

John Walker
Joseph Awange

Surveying for Civil and Mine Engineers

Acquire the Skills in Weeks

Second Edition

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 Springer

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*This book is dedicated to all civil and mine engineering students
(past and present) of Curtin University, Australia.*

Foreword



I am pleased to present a foreword to this book, aimed at explaining surveying methods and practices to Civil and Mining Engineers. The authors have made a sharp distinction between engineering surveying, a profession in its own right, and the surveying knowledge requirements expected of Civil and Mining Engineers. The book is the result of close cooperation between the Departments of Civil Engineering, Mining Engineering, and Spatial Sciences at Curtin University and the University of Western Australia; the course has been presented to undergraduate and postgraduate students over a ten year period.

Through theory and practice, using lecture material, workshops and practical exercises, the half semester course provides the Civil and Mining Engineer with an understanding of surveying, enabling an informed discussion with surveyors. Additionally the course promotes and examines practical skills that will enable a Civil or Mining Engineer to perform simple, inexpensive, survey tasks that should be part of an engineer's skill set. The survey ability is also expected by many Civil and Mining Engineering associations, and the course can be proffered by tertiary institutes as an example to professional accreditation bodies.

As Mine Managers, graduate Mining Engineers have a particular responsibility for all surveying on a mine site. This book aims at giving them the ability to examine survey planning and to understand the presented survey result, again through general theoretical knowledge and targeted practical workshops and exercises.

Theory moves from basic plane surveying, through to geodetic surveying principals, and includes error analysis, coordinate transformation and examples of least squares adjustments. With the increasing sophistication and precision of survey equipment and methods, the Engineer has to have a corresponding appreciation of methodologies and results. The ability to move between local coordinate and national grid systems is fundamental to large scale engineering projects.

Five practical exercises focus on basic manipulative skill sets and take the student through differential levelling to plane 3D surveying, including coordinate set-out. Corresponding reduction and data interpretation allows the production of earthworks plans and the determination of quantities. DEM production; survey control design and coordinate adjustment illustrate tasks on a small engineering site. A practical examination incorporates circular curve calculation, set-out and quality assurance. The capabilities and limitations of hand-held GPS receivers in Civil and Mining Engineering are also contrasted with a generous chapter on modern GNSS techniques.

Some of the chapters include information that is missing from standard survey texts; strike and dip analysis from a survey point of view. Calculating inclined angles in mining. Methods of volume determination in road embankments and open pit mines. Circular and transition curves, and superelevation related to a local (Australian) context. Vertical curves related to road design standards. Many of the solved problems are calculated in great detail, allowing the student to use a “follow me” approach to reaching a result.

All calculations can be carried out on simple, non-programmable calculators. Examples of the use of conformal coordinate transformation techniques are expanded from the calculator to spreadsheets (Microsoft Excel) and numerical computing environments (Mathworks MATLAB). Matrix solutions of over-determined resection and trilateration problems are illustrated by example.

The “hands on” approach to the course is designed to provide Civil and Mining Engineers with a knowledge and understanding of surveying techniques that can be applied in discussions with professional surveyors. And allow them to carry out standard survey tasks with confidence.

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Joseph wishes to express his sincere thanks to Professors Bernhard Heck and Hansjoerg Kutterer of Karlsruhe Institute of Technology (KIT, Germany) for hosting him during the period of his Alexander von Humboldt Fellowship (June–September 2019) and Prof. Herick Othieno of Maseno University (Kenya) for hosting him during December 2019–January 2020. Parts of this book were written during these periods. He is also grateful to Prof. Ian Fitzsimons (Head of School, Earth and Planetary Sciences-EPS, Curtin University, Australia) for the support and motivation that enabled the preparation of this edition. He also wishes to acknowledge the support of Alexander von Humboldt that facilitated his stay at KIT. To all, he says, “ahsante sana” (Swahili for thank you very much). Special thanks go to his family, namely Mrs Naomi Awange, Lucy Awange and Ruth Awange who had to contend with his long periods of absence from home.

John wishes to thank Joseph with whom he has taught engineering surveying to civil and mine engineering students for over a decade for the opportunity to be introduced to his publisher, Springer Nature, which has brought this book to fruition. We perceived a “gap” in available literature that would allow a half semester unit to provide a permanent record for the engineer’s bookshelf. The result is a compilation of lecture notes, workshop presentations and field practical exercises in to an expanded presentation of surveying techniques at the undergraduate level.

John also wishes to thank the Discipline of Spatial Sciences within the school of EPS and the Discipline of Civil Engineering within the School of Civil and Mechanical Engineering at Curtin University for the opportunity to engage with undergraduates at Curtin over the last 20 years.

Preface

What's it all about?

The structure of this book is to provide a sequence of theory, workshops and field practical sessions that mimic a simple survey project, designed for civil and mine engineers. The format of the book is based on a number of years of experience gained in presenting the course at undergraduate and post graduate levels. The course is designed to guide engineers through surveying tasks that the engineering industry feels is necessary for students to have in order to demonstrate competency in surveying techniques such as; data gathering and reduction, and report presentation. The course is not designed to make engineers become surveyors, rather, it is designed to allow an appreciation of the civil and mine engineering surveyor's job. There are many excellent textbooks available on the subject of civil engineering surveying, but they address the surveyor, not the engineer. Hopefully this book will distil many parts of the standard textbook. A lot of the material presented is scattered through very disparate sources and has been gathered into this book to show what techniques lie behind a surveyor's repertoire of observational and computational skills, and provide an understanding of the decisions made in terms of the presentation of results. The course has been designed to run over about 6 weeks of a semester, providing a half unit load which complements a computer aided design (CAD) based design project. The following is an example of a generic course structure.

Preface to 2nd edition

What's changed?

Thanks to feedback from users of the first edition. New chapters have been added to address issues raised. Chapter orders have been re-arranged.

Chapter 3 has been expanded with mass haul diagrams (MHD) supplementing the embankment volume calculations.

The Bowditch adjustment of an open linked traverse has been added to Chapter 5.

Chapter 7 discusses underground mine survey. It has been added to address a shortfall in the first edition. Attention has been paid to modern free station techniques of traversing, as well as an explanation of control transfer by traditional methods. Wall stations and associated free station positioning is introduced. The use of single wire, Weisback triangle and Weiss quadrilateral horizontal control, with associated errors, has been explored. Vertical control transfer has also been addressed.

Chapter 9, Earthworks on a sloping site, is a re-visit of Chapter 3. It examines embankment calculations in more general terms; it pays more attention to slope intercepts in cross section area calculations. Strike and dip of a surface is examined mathematically from a survey point of view to supplement the geologists view of dipping planes. The author's interest has led to an examination of circular fill pads on a sloping site and segued into the calculation of ring tanks.

Circular curves, Chapter 10, have been supplemented by the inclusion of transition, or spiral, curves in Chapter 11 and the imposition of superelevation to create a composite curve in Chapter 12.

GNSS in Chapter 14 now includes discussion of precise measuring techniques for engineering.

A major expansion of Chapter 15, setting out engineering structures, now includes more detailed discussion of horizontal and vertical control; the maintenance of verticality and the establishment of grade and batter control in earthworks.

In all the book now contains links to over 180 equations, 450 illustrations and 64 tables.

Curtin University offers courses in Civil and Mining Engineering at undergraduate and postgraduate levels. Offerings may be found at:

<https://study.curtin.edu.au/offering/course-ug-bachelor-of-engineering-civil-and-construction-engineering-honours-bachelor-of-science-mining--bb-ccemnv2/>

The Civil Engineer. A role in ambitious construction projects.

Civil engineers design and construct infrastructure such as:

- bridges
- roads
- harbours
- highways
- dams
- buildings.

As a built environment becomes increasingly complicated, ambitious construction projects are completed by teams of people with different skills, working together. The civil engineer plays an important role in this process.

The Mining Engineer. A stepping stone to Mine Management

Mining engineers plan and direct the engineering aspects of extracting mineral resources from the earth.

Mining engineers plan and manage operations to exploit minerals from underground or open-pit mines, safely and efficiently. They design and direct mining operations and infrastructure including:

- drilling
- blasting
- loading and hauling
- tunnel creation and maintenance
- access road planning and maintenance
- water and power supplies.

Mining engineers may supervise other engineers, surveyors, geologists, scientists and technicians working on a mine site and may find employment in metropolitan or regional locations or in different countries.

The tasks with which a Mining Engineer may be associated include:

- conduct investigations of mineral deposits and undertake evaluations in collaboration with geologists, other earth scientists and economists to determine whether the mineral deposits can be mined profitably
- determine the most suitable method of mining the minerals taking into account factors such as the depth and characteristics of the deposit and its surroundings
- prepare the layout of the mine development and the procedure by which the minerals are to be mined
- prepare plans for mines, including tunnels and shafts for underground operations, and pits and haulage roads for open-cut operations, using Computer-Aided Design (CAD) packages
- plan and coordinate the employment of mining staff and equipment with regard to efficiency, safety and environmental conditions
- talk to geologists and other engineers about the design, selection and provision of machines, facilities and systems for mining, as well as infrastructure such as access roads, water and power supplies
- coordinate with the operations supervisor to make sure there is proper implementation of the plans

- operate computers to assist with calculations, prepare estimates on the cost of the operation and control expenditure when mines begin production
- oversee the construction of the mine and the installation of the plant machinery and equipment
- make sure that mining regulations are observed, including the proper use and care of explosives, and the correct ventilation to allow the removal of dust and gases
- conduct research aimed at improving efficiency and safety in mines
- establish first aid and emergency services facilities at the mines.

In broader terms a qualification in Engineering is a prerequisite to an advancement in Engineering Management. For Mining Engineers in Western Australia, it is a statutory requirement proscribed by Act and regulation (the Mining Act). For Civil Engineers advancement in Management depends on demonstrated competencies in a number of fields; investigation, design, analysis, costing, quantity surveying, tender documentation and presentation, project coordination and management, project delivery, safety and human resource management.

Essentially, Engineering is about management of the project and the resulting built environment, about the integration of diverse groups of skilled professionals to achieve a common goal.

Then, “Why Surveying?”

You will have a team of surveyors intimately involved in controlling the placement of your design. They ensure not only adherence to the design dimensions, but also, through Licences Surveyors, ensure the legal location of the project in terms of cadastral boundaries. Encroachment on another property is an expensive business.

So, “Why Me? Won’t the Surveyor do it?”

Yes, BUT, sometimes you may need to do a simple task on a small job; check an invert level before a pour, understand the origin of survey data, calculate rough quantities, understand quantities before signing off on invoices, evaluate proposed survey control for a project, check structural setout. All the little things that your industry expects you to be able to do in the field.

As projects become larger and more complex; as surveying becomes more automated both in the field and in the design process; as machine control relies on both terrestrial data and GNSS (GPS) data, the need for basic surveying techniques seems a distant memory. It’s not, you still have to understand the basics, and understand positioning from a whole new spectrum of positional precision and accuracy.

Hopefully you will find this course provides that understanding.

Curtin University (Australia)

Curtin University (Australia), Karlsruhe Institute of Technology (Germany), Kyoto University (Japan) and Federal University of Pernambuco (Brazil)

J.Walker

J.L. Awange

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Joseph wishes to express his sincere thanks to Prof. B. Heck (Karlsruhe Institute of Technology (KIT), Germany) for hosting him during the period of his Alexander von Humboldt Fellowship (June-September 2015), Prof. Y. Fukuda (Kyoto University, Japan) for hosting him during the period of his Japan Society of Promotion of Science (JSPS) Fellowship (October-November 2015), and Prof R. Goncalves of Federal University of Pernambuco (Brazil) for hosting him during his Science Without Boarder (December 2015 – March 2016). Parts of this book were written during these periods. He is also grateful to Prof. B. Veenendaal (Head of Department, Spatial Sciences, Curtin University, Australia) for the support and motivation that enabled the preparation of this edition. He also wishes to acknowledge the support of Alexander von Humboldt that facilitated his stay at KIT, JSPS that supported his stay at Kyoto University, and Capes for supporting his stay in Brazil. To all, he says, “ahsante sana” (Swahili for thank you very much). Special thanks go to his family, namely Mrs Naomi Awange, Lucy Awange and Ruth Awange who had to contend with his long periods of absence from home.

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Dedication

This book is dedicated to all civil and mine engineering students (past and present) of Curtin University, Australia.

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1 Chapter 1 Fundamental Surveying

1.1 Introductory Remarks

“Indeed, the most important part of engineering work—and also of other scientific work—is the determination of the method of attacking the problem, whatever it may be, whether an experimental investigation, or a theoretical calculation. ... It is by the choice of a suitable method of attack, that intricate problems are reduced to simple phenomena, and then easily solved.”
Charles Proteus Steinmetz.

This chapter introduces the basics of surveying that will be covered in the book. The aim of the chapter is such that once a civil or mine engineering student has covered it and undertaken the necessary workshop, they should:

- ✓ Be able to define surveying.
- ✓ Know various types of surveying.
- ✓ Be able to classify errors in surveying.
- ✓ Know the basic rules of recording field notes.
- ✓ Recognize instrumentation relevant for undertaking engineering surveys.
- ✓ Know the fundamental trigonometrical formulae used in surveying.

More details can be found in Uren and Price (2010, Chapter 1) and Awange and Kiema (2013, Chapter 3).

1.2 Definitions

1.2.1 Surveying

Surveying is traditionally defined as the determination of the location of points on or near the Earth’s surface. A more modern definition is *“the collection, processing, and management of spatial information”*. (Uren and Price 2010). Surveying is important for numerous applications that includes, e.g. land ownership, engineering, mining, marine navigation, mapping and many more.

1.2.2 Engineering and Mine Surveying

Engineering surveying is that branch of surveying that deals mainly with construction, deformation monitoring, industrial and built environment (see Figure 1-1) whereas Mine surveying is undertaken to support mining activities through provision of control points for mining locations. These controls are used for infrastructure construction and also for coordination of points within the mining areas. Besides engineering and mine surveying, other types of surveying include:

- Cadastral surveying, which deals with property boundary determination. Whether for property ownership or development of land, knowledge of who owns what property will always be required.
- Topographic surveying, which deals with the generation of maps at various scales. These maps support variety of uses, ranging from reconnaissance to assisting flood management in civil engineering, for exploration and rescue purposes in mining, mapping spatially changing features, e.g., changes in wetland perimeter in environmental monitoring, to soil type maps for assisting land management decisions. Chapter 4 explores the role of topographical maps in detail.
- Photogrammetric surveying, which uses photogrammetric technology for mapping purposes.
- Control surveying, which is performed to provide horizontal and vertical controls. These in turn provide a framework upon which subsequent locations are based. This type of

surveying is more accurately undertaken than the other types of survey and often involves use of more precise equipment.

- Hydrographic surveying, which is undertaken for marine purposes and could also find use in measuring changes in sea level. This is a vital indicator for monitoring the impact of climate change.
- Satellite surveying (see e.g., Seeber 2003). Although this can be undertaken locally to support development activities, its functions are globally oriented and will be discussed in detail in the Chapter 10.
- Inertial surveying systems, which consist of three accelerometers that are orthogonally mounted in known directions, relative to inertial space, on a stable platform used to measure changes in the three-dimensional position, as well as, the length and direction of the gravity vector (Cross 1985). An in-depth exposition of this system is presented, e.g., by Cross 1985 who list the advantages of the system as being faster, independent of refraction of the measuring signals, and independent of external organization unlike the global navigation satellite system (GNSS) discussed in Chapter 10 (see e.g., Awange 2012). Like GNSS, the systems are all-weather and all-day but are expensive to purchase and can only be used in interpolative mode (Cross 1985). Its usage includes (Cross 1985): provision of photogrammetric control, densification of national control networks, route surveys e.g., for pipelines, powerlines and roads, cadastral surveys, fixing of navigation aids and geophysical surveys. For geophysical surveys, an immediate application that would support mining activities is the measurement of gravity. In some remote-sensing activities, it can give information about the position and attitude of the sensors and it can be combined with a satellite navigation system to give real-time positions offshore (Cross 1985).

1.3 Plane and Geodetic Surveying

The definition of surveying given in Section 1.1 is achieved through distances and angular measurements, which are converted into coordinates to indicate the horizontal position, while heights are measured relative to a given reference, i.e., the vertical dimension.

Surveying plays a pivotal role in determining land ownership, engineering, mapping, marine navigation, and environmental monitoring through height determination of anthropogenic land subsidence among other uses. For example, surveying plays a crucial role in monitoring deformation of structures such as dams, bridges, buildings and many others. The main strength of surveying is that it operates locally, at levels by which most development activities take place.

Surveying can take on the form of plane surveying, where a flat horizontal surface is used to define the local surface of the Earth and the vertical is taken to be perpendicular to this surface (Figure 1-1). Plane surveying, therefore, adopts a horizontal plane as a computational reference and the vertical direction is defined by the local gravity vector, which is considered constant. All measured angles are plane angles, and the method is applicable for areas of limited size.

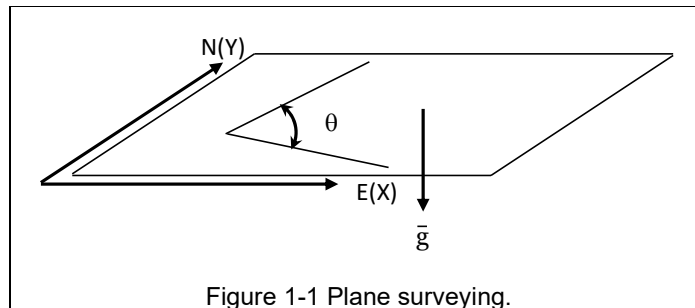


Figure 1-1 Plane surveying.

In contrast to plane surveying, *geodetic surveying* uses a curved surface of the Earth as the computational reference, e.g., an ellipsoid of revolution (Figure 1-2). The ellipsoid of revolution is considered to approximate the figure of the Earth and forms the surface upon which GNSS positioning (Chapter 14) is undertaken. Both could be used for civil and mine engineering at

local scale (plane surveying) and global or regional scale (geodetic surveying) as will be discussed in subsequent chapters.

Geodetic surveying, Figure 1-2, is based on the ellipsoid of revolution with a and b being the semi-major and semi-minor axes of the ellipsoid respectively. The position of a point on the reference ellipsoid will be defined by the latitude ϕ , longitude λ , and height h .

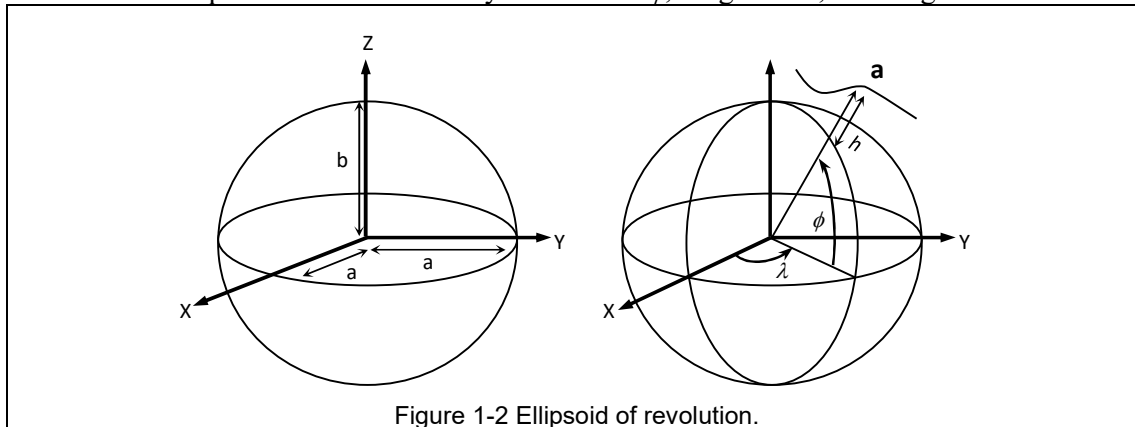


Figure 1-2 Ellipsoid of revolution.

The distinction between plane and geodetic surveying, therefore, is that plane surveying assumes a flat horizontal surface of the Earth where the vertical is considered to be perpendicular to this surface. Geodetic surveying on the other hand accounts for the true form of the Earth as illustrated in Figure 1-2.

1.4 Measuring Techniques

1.4.1 Plane Surveying Measurements and Instruments

Plane surveying measures linear and angular quantities (Figure 1-5). Linear measurements take the form of horizontal distances, e.g., measured directly by a level or indirectly using a Total Station. In measuring horizontal or slope distances with a Total Station (Figure 1-3), use is made of electromagnetic distance measurement (EDM, Uren and Price 2010). Indirect measurement of height differences (vertical distance) using a Total Station instrument makes use of the slope distance and elevation or vertical angle. Height differences are directly measurable using a level (Figure 1-4).



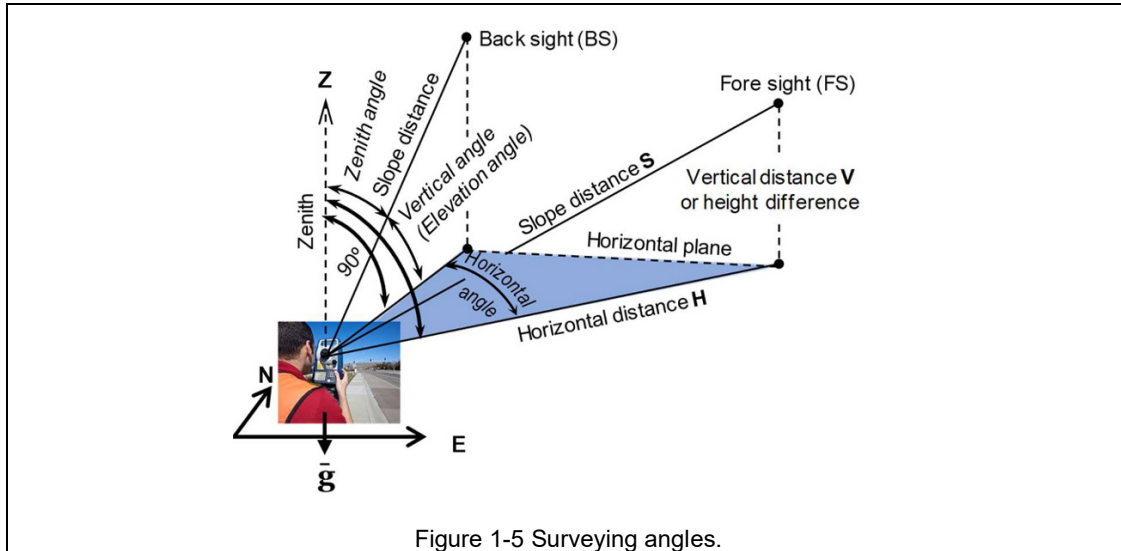
Figure 1-3 Total Station and prism

Figure 1-4 Level and staff.

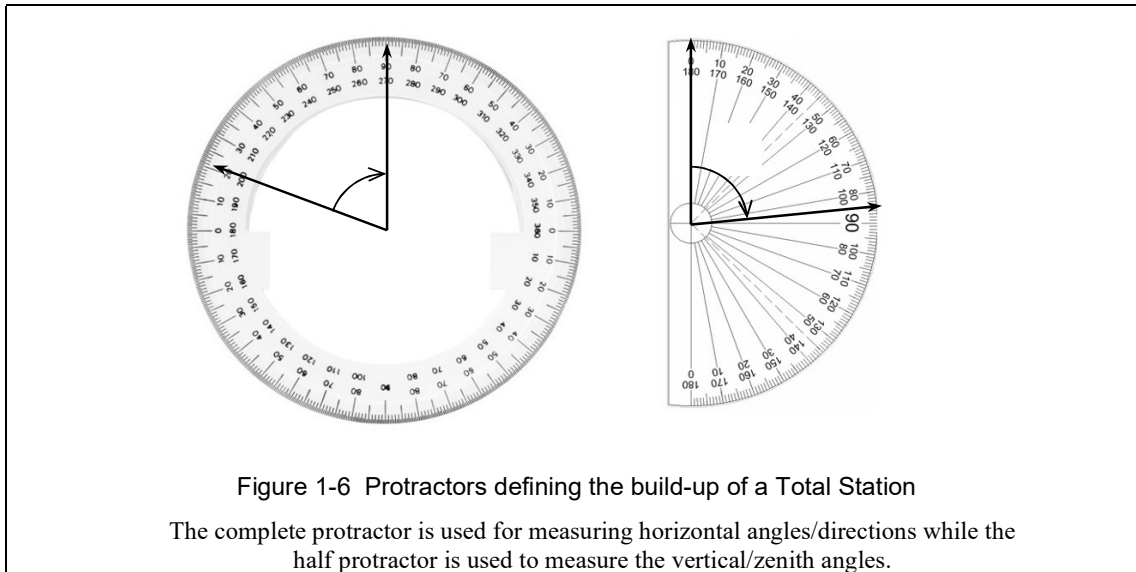
Angular measurements are of three types; vertical or elevation angles, zenith angles, and horizontal angles (Figure 1-5). Vertical or elevation angle to a point is measured with reference to a horizontal plane while the zenith angle is measured with respect to the zenith or vertical direction, where the vertical angle α is given as:

$$\alpha = 90^\circ - z,$$

and z is the zenith angle. The horizontal angle is obtained in a horizontal plane by taking the



difference between two directions. A Total Station is used to measure the angles discussed above. One can view a Total Station as made of two protractors. A 360° protractor marked in degrees placed in the horizontal plane and used to measure the horizontal angles, and a half circle protractor in the vertical plane used to measure the vertical/zenith angles (Figure 1-6).



1.4.2 Geodetic Measuring Techniques

For geodetic surveying, measurement methods that cover wider spatial extent such as regions, continental or the entire globe are essential. Such methods are useful for large scale engineering projects such as inter-city highway construction. They include but are not limited to global positioning methods, e.g., by Global Navigation Satellite Systems (GNSS) discussed in Chapter 14 (see also Awange 2012), Satellite altimetric methods, Satellite Laser Ranging (SLR) and Lunar Laser Ranging, Interferometric Synthetic Aperture Radar (InSAR), and Very Long Baseline Interferometry (VLBI).

1.4.3 Basic Measuring Principles and Error Management

In surveying, as well as geodesy, there are basic measuring principles that must be adhered to in order to achieve the desired outcome that satisfy both the client and the operator (surveyor). These include completing the measuring task within the shortest possible time and at the least possible cost. This may be beneficial to engineering projects whose monetary budget for excavation is limited. Further, the task must be completed according to instructions and using instruments of appropriate precision. The records of the field notes are essential and form part of legal evidence in a court of law in case of disputes. The following should be taken into consideration.

- ✓ Field notes are permanent records of work done in the field and must be thorough, neat, accurate and guarded carefully.
- ✓ Mistakes in field books are never erased but crossed out with one horizontal line through the middle.
- ✓ Specific field note formats exist for different types of surveys. This is particularly important for cadastral surveys, where notes may be used as evidence in court cases.

Finally, for mitigation and management purposes, in order to achieve accurate results, types of errors in surveying measurements and their sources that should be taken care of include:

- ❑ Natural
 - Due to the medium in which observations are made.
 - Factors: Wind, temperature, humidity, etc.
- ❑ Instrumental
 - Due to imperfections in instrument construction or adjustment.
 - May be reduced or eliminated by calibration and/or observation procedure.
- ❑ Personal
 - Limitations in operator ability.
 - Can be improved with practice.
 - Examples: Ability to read vernier scale, ability to accurately point cross-hairs, etc.
- ❑ Mistakes, also called gross errors or blunders.
 - Usually, but not always, large magnitude.
 - Examples: Reading a tape incorrectly, transposing numbers, i.e., A distance of 15.369m is read while 15.396m is recorded in the field book.
- ❑ Systematic (deterministic) errors
 - Errors that follow some physical or geometric law.
 - Their effects can be mathematically modelled and, thus, corrected.
 - Examples: Refraction of the line of sight, thermal expansion of a steel band.
- ❑ Random errors (what is left over)
 - Errors that can't be modelled and corrected: random variations.
 - Governed by probabilistic or stochastic models.

1.5 Measurement Types

1.5.1 Linear Measurements

Linear measurements deal with distances (slope, horizontal or vertical), which are normally required for plotting the positions of details when mapping and also for scales of the maps (see Chapter 4). Horizontal and vertical distances are useful for mapping, provision of controls, and monitoring spatial and vertical changes of features. Slopes and vertical distances are essential for setting out construction structures, where vertical distances are useful in height transfer from floor to floor in multistorey building or in mining, i.e., transferring distances from the surface to underground.

Distance measurements can be undertaken using, e.g., tapes (for short distances) or electromagnetic distance measuring (EDM). Errors associated with tape measurements include instrumental errors (e.g., incorrect length where the tape is either too short or too long), natural errors (e.g., the expansion or contraction of the steel tape caused by temperature changes) or personal errors such as wrong reading of the tape or poor alignment while measuring the distances. EDM is nowadays the most common tool used for measuring distances. It can be classified according to either radiation source (optical or microwave), measurement principle (phase or pulse), or whether a reflector is required or not (i.e., reflectorless). The operating principle involves the signal being emitted from the Total Station to some reflector, which reflects the signals back to the emitter. The distance is then obtained from the basic equation:

$$\text{speed} = \frac{\text{Distance}}{\text{time}},$$

since the speed of light c is known, and the time the signal takes to travel from the emitter and back is measured, say Δt , the distance measured by the EDM instrument then becomes:

$$d = \frac{c\Delta t}{2}.$$

The division by 2 is because the signal travels twice the distance (i.e., from the emitter to the reflector and back). The phase method, where the signal travels in a sinusoidal form is the most common method of distance measurement found in surveying instruments. However, the pulse method using pulsed laser is becoming more common particularly for reflectorless instruments. Errors associated with EDM are elaborately discussed, e.g., in Uren and Price (2010).

1.5.2 Traversing

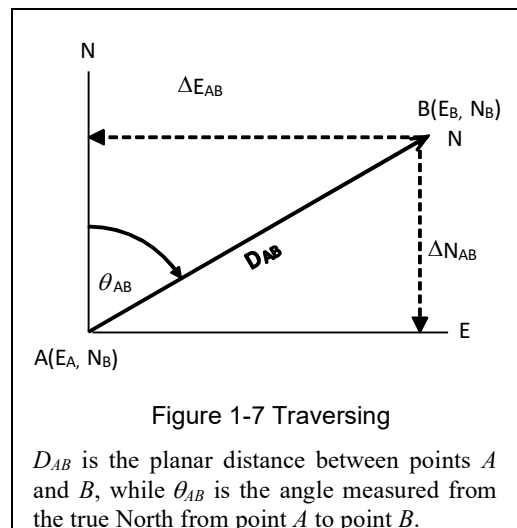
Traversing, which is covered in detail in Chapter 5, is a survey technique used to determine the planar positions (Easting and Northing: E_B and N_B , Figure 1-7) of control points or setting out points using measured angle θ_{AB} and distance D_{AB} (Fig. 1.6). The position of point B obtained relative to that of A is given as:

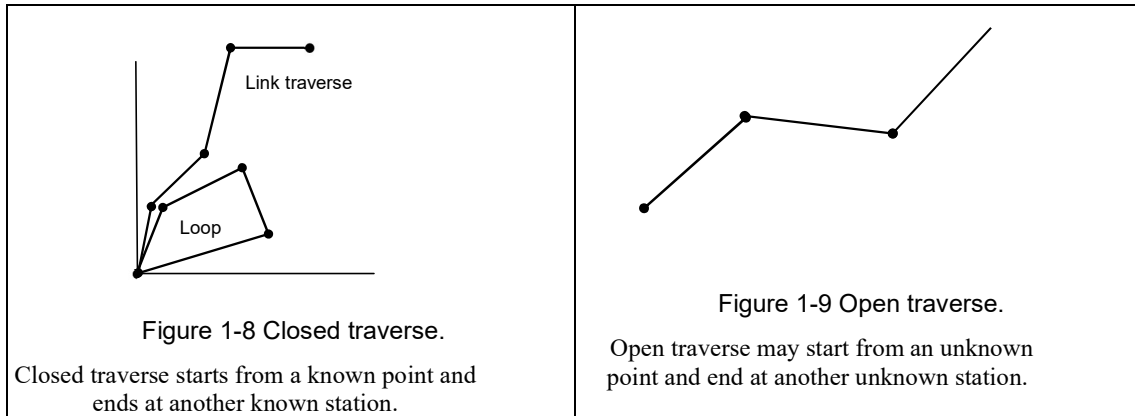
	$\Delta E_{AB} = E_B - E_A = D_{AB} \sin \theta_{AB}$	Eqn 1-1
	$\Delta N_{AB} = N_B - N_A = D_{AB} \cos \theta_{AB}$	Eqn 1-2

where E_A and N_A (E_A is the known planar position of point A).

Applications of traversing include the establishing of control points useful for construction purposes or for delineating feature boundaries, horizontal control for generation of topographic maps (see e.g., Chap. 4) and also for detail maps for engineering works, establishment of planar positions of points during construction (set-out), for area and volume computations, and ground control needed for photogrammetric mapping discussed in Section 1.2.2.

A traverse can take the form of either open or closed. A closed route can start from a known point and end at another known point (e.g., from A to B in Figure 1-8). This type of traverse is also known as link traverse. If the traverse starts from a known point and closes at the same point, then it is known as a loop traverse. An open traverse starts from a known point but does not end at a known point in Figure 1-9.





1.6 Concluding Remarks

Most of the details covered in this introductory Chapter will be expounded in subsequent chapters. Of importance is that the students should know the purpose of surveying for civil and mine engineering, types of measurements, and errors that underpin these measurements.

1.7 Reference for Chapter 1

1. Awange JL (2012) Environmental monitoring using GNSS, global navigation satellite system, Springer, Berlin.
2. Awange JL and Kiema JBK (2013) Environmental Geoinformatics, Springer, Berlin.
3. Cross PA (1985) Inertial surveying: principles, methods and accuracy. International Journal of Remote Sensing 6(10): 1585-1598.
4. Seeber, G (2003) Satellite geodesy. Berlin New York, Walter de Gruyter. ISBN 978-3-11-017549-3.
5. Uren J, Price WF (2010) Surveying for Engineers. Palgrave Macmillan Ltd. pp. 816.



2 Chapter 2 Levelling

2.1 Introductory Remarks

This Chapter introduces you to the theory and practical skills of levelling, a process of determining elevations (heights) or differences in elevations. It can be performed using either differential levelling using a level (discussed in this Chapter) or trigonometric levelling using a Total Station (discussed in Chapter 6). Levelling can be used in all aspects of surveying. Particularly for engineering and mining, levelling finds use in:

- ✓ Establishing vertical controls.
- ✓ To establish heights of points during constructions.
- ✓ Route surveys.
- ✓ For contouring purposes.
- ✓ For road cross-sections or volumes of earthwork in civil engineering works.
- ✓ For provision of levels of inclined surface during construction.
- ✓ For designing decline box cut in mining amongst other tasks, e.g., rescuing.

In Chapter 3 where we discuss the representation of relief and vertical sections, you will employ the skills of levelling that you will have learnt in this Chapter. Working through the materials of this Chapter and the workshop materials in Appendix A1-1, you should:

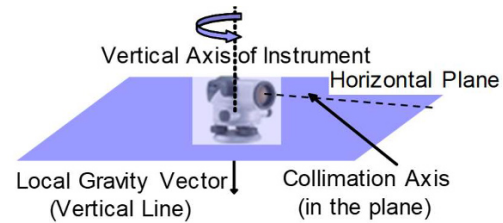
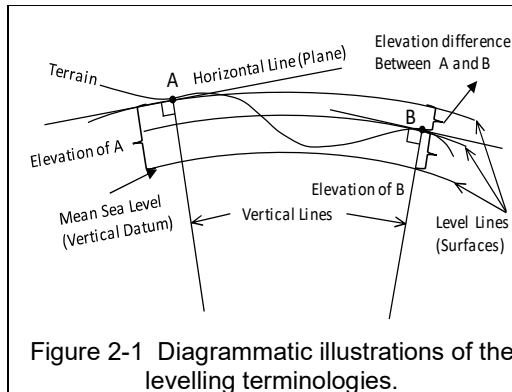
1. Be able to differentiate between heights, datum and bench marks (BM).
2. Know and understand the use of the levelling equipment.
3. Understand the field procedures for levelling.
4. Be able to calculate reduced levels (RL).
5. Know sources of errors in levelling and how to manage them.
6. Know the various levelling methods.

Essential references include, e.g., Uren and Price (2010), Schofield and Breach (2007), and Irvine and Maclellan (2006).

2.2 Definitions of Levelling Terminologies

- Level surface (see Figure 2-1).
 - A (curved) surface orthogonal to the plumb line everywhere.
 - More correctly an equipotential surface for which gravitational potential is constant.
 - A still body of water unaffected by tides is a good analogy.
 - They are not equidistant apart but converge and diverge due to changes in density.
- Vertical line (see Figure 2-1 and Figure 2-2).
 - The direction of gravity.
 - Therefore, the direction indicated by a plumb line.
 - In general, it deviates from a line emanating from the geometric centre of the Earth.
 - In reality it is curved, but this can be neglected in small plane surveys.
- Horizontal plane (see Figure 2-2).
 - A plane tangent to a level surface (orthogonal to the plumb line).
 - The collimation axis (line of sight) of a levelling instrument that is in correct adjustment. Once levelled, it defines a horizontal plane as the instrument is rotated.
- Vertical datum.
 - Any level surface to which heights are referenced.
 - The vertical datum in Australia is the Australian Height Datum (AHD).
- Mean Sea Level (MSL).
 - Mean height of ocean level taken with data from coastal tide gauges over a 19-year period.
 - 30 tide gauges used in Australia.
 - 97,230 km of two-way levelling used with the tide gauge data to define the AHD.

- Tidal datum.
 - Average of all high waters observed over a 19-year period.
 - Mean high water (MHW).
- Elevation.
 - The vertical distance (height) above the datum.
- Bench mark (BM).
 - A permanent monument or feature for which elevation is known.
 - Vertical control.
 - A set of benchmarks used to “control the heights” of a project.



2.3 Example: The Australian Height Datum (AHD)

Heights in Australia are referenced to the Australian Height Datum (AHD) defined as mean sea level at 30 tide gauges around Australia, observed between 1966 and 1968 (Figure 2-3).

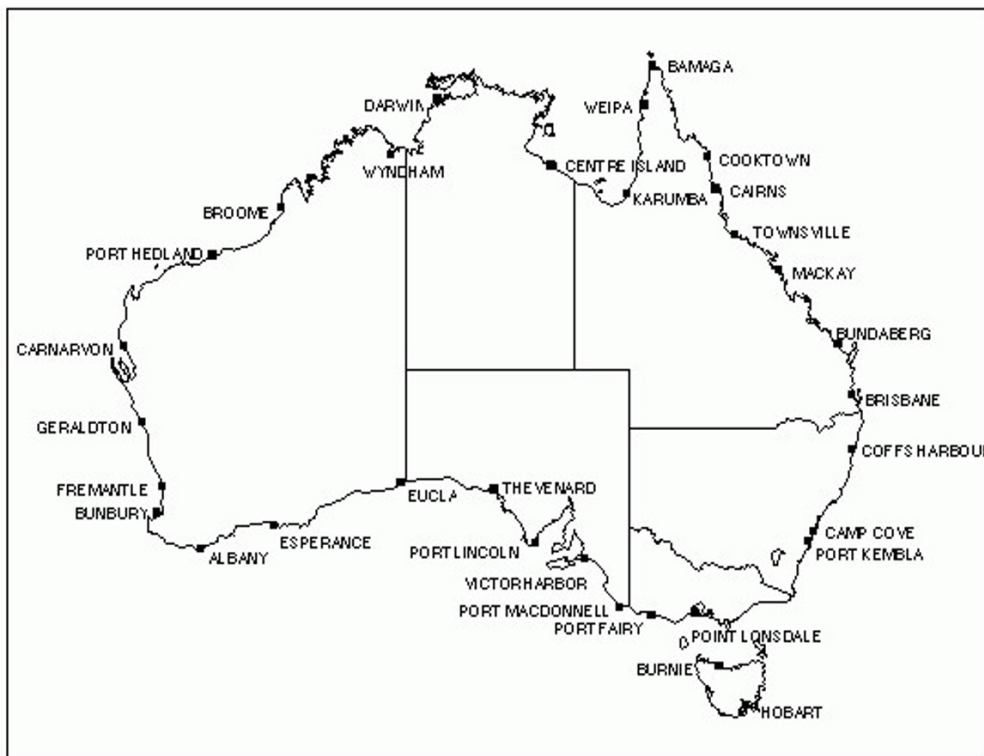
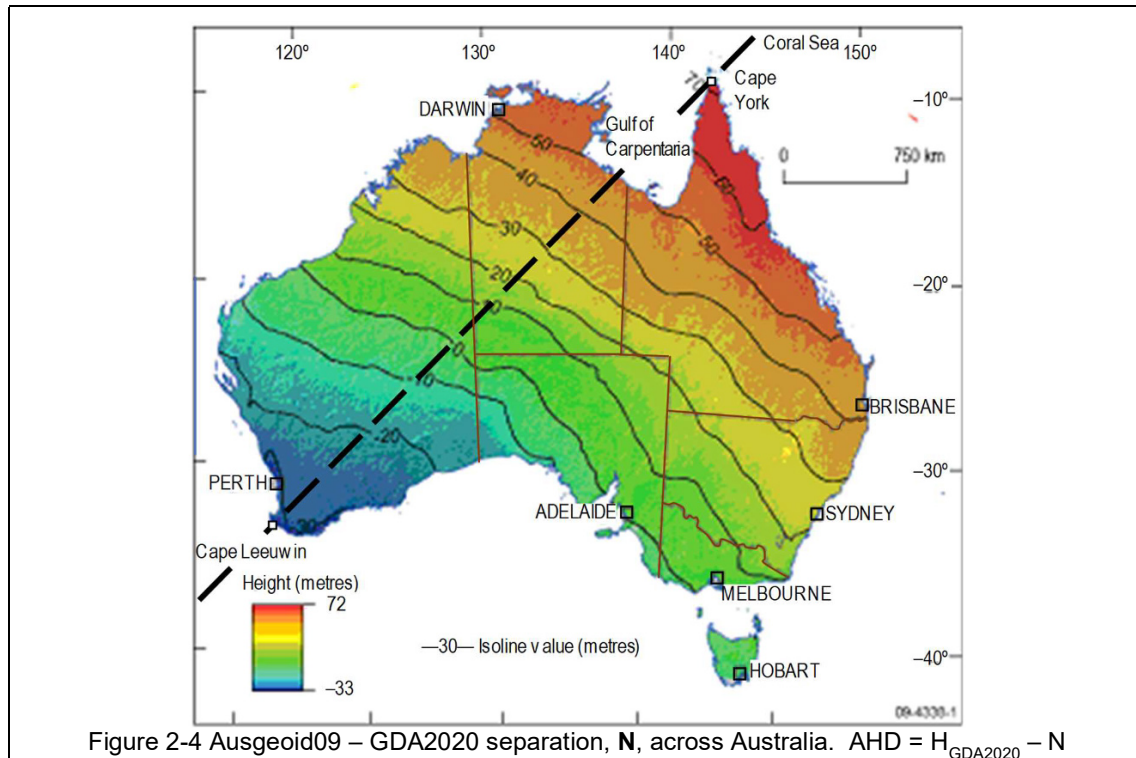


Figure 2-3 Map of tide gauges in Australia. Image source: Geoscience Australia

<http://www.ga.gov.au/geodesy/datums/ahd.jsp>

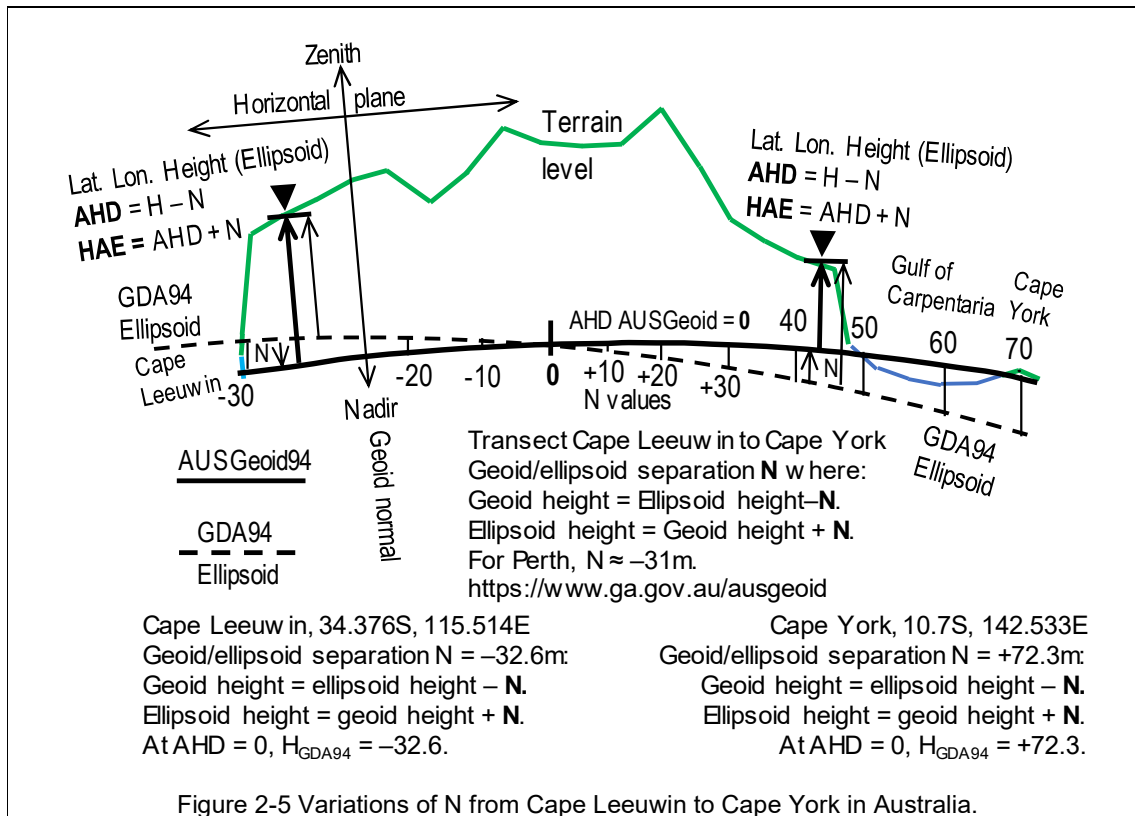
Height control is established by **Bench Marks**, vertical control points tied to the AHD by differential levelling. The AHD, almost coincident with the geoid (an equipotential surface approximating the mean sea level), varies from the ellipsoid, the idealised mathematical shape of the Earth by a geoid-ellipsoid separation, N . The AHD N value is modelled to an accuracy of about 0.03m over Australia (Figure 2-4). The difference between the geoid and the AHD is due to the difference in water density between Northern and Southern Australia and accounts for a tilt of $\pm 0.5\text{m}$.

(see, e.g., <http://www.ga.gov.au/scientific-topics/positioning-navigation/geodesy/geodetic-datums/geoid>).



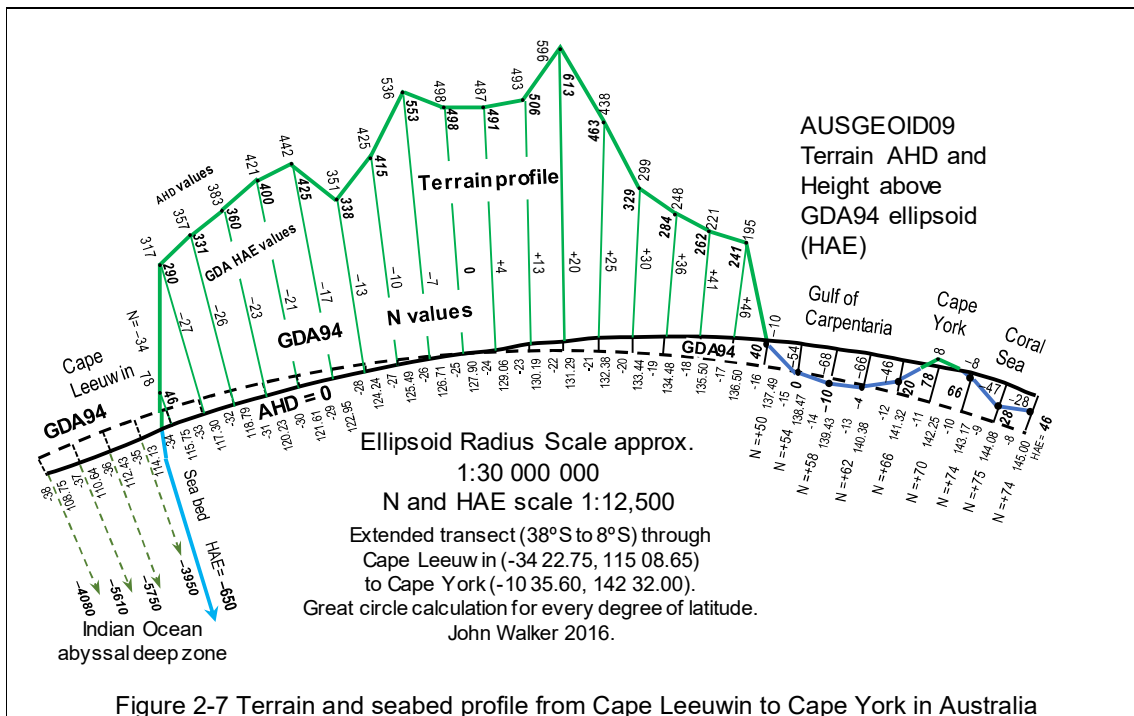
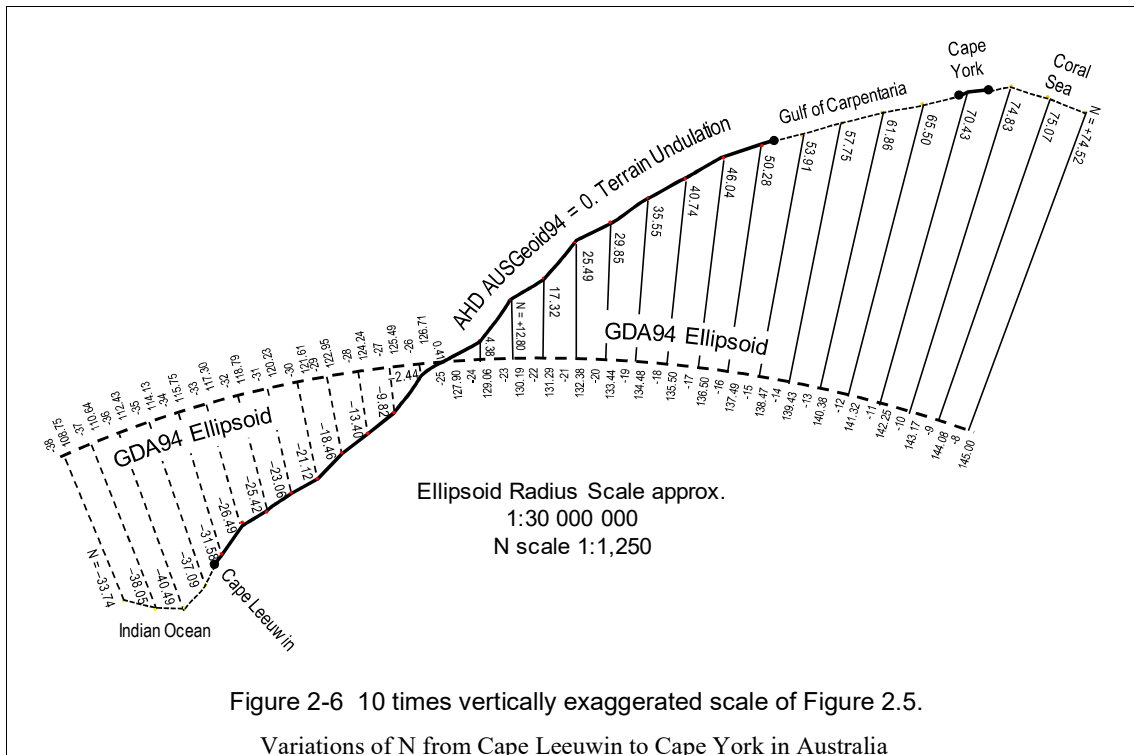
The important point to remember is that GPS/GNSS heights (see Chapter 14) refer to the **ellipsoid** and differential levelling heights refer to the **geoid**. Thus, any GPS/GNSS height must be corrected to the AHD by the separation value, N , before using GNSS derived vertical control. The current model of the AHD/geoid separation is Ausgeoid09, which is an upgrade from the original Ausgeoid98 (Featherstone et al. 2001). Illustrated in Figure 2-4 are the 10m isolines of AHD and Geodetic Datum Australia (GDA2020) separation over the AHD model area (<http://www.ga.gov.au/ausgeoid/>). The variation of the N values across Australia from Cape Leeuwin to Cape York are shown in Figure 2-5 to Figure 2-7. The model definitions are the AHD, Ausgeoid09/GDA2020 separation scale is 1:1,250, and the terrain profile, i.e., the AHD/GDA2020 separation is scale 1:12,500.

Note the “gentle” slope between the -30m and -20m isolines (10m change) over some 750Km in south-western Australia compared with the $+10\text{m}$ to $+20\text{m}$ isolines (10m change) over about 210Km or less in central Australia. Down to a 10m change over 150Km near the Tanami Desert, NT.



Height transfer between the GDA2020 Ellipsoid and the Australian Height Datum is performed as follows:

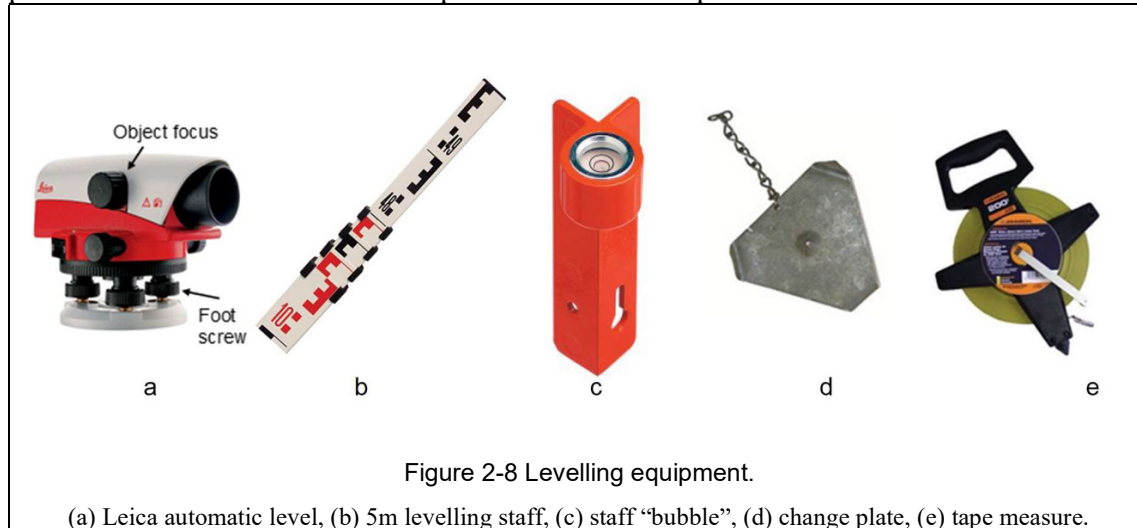
1. AHD (roughly height above mean sea level on the local geoid) is transferred from Benchmarks (BM) using differential levelling or trigonometric height differencing.
2. GNSS (GPS) height surveying is referenced the GDA2020 ellipsoid. This is very similar to the WGS84 ellipsoid used by GNSS satellites (see Chapter 10 for more discussion).
3. GDA2020 ellipsoid heights are transferred to the AHD using the AUSgeoid09 grid ellipsoid interpolation of the AHD – ellipsoid separation value, N .
4. N is realised to about 0.03m (1 sigma) over Australia, compatible with $k = 12\sqrt{d}$ (3rd order) levelling.
5. GNSS RTK surveys over a local area can have the observed ellipsoidal heights transferred to AHD using a single N value for the area.



2.4 Instrumentation: Automatic Level and Staff

The instruments one needs to carry out levelling procedure discussed in Section 2.5 are a level and staff (Figure 1-4). In addition, one may need to have a measuring tape and a change plate (a metallic/plastic plate used on soft ground), see Figure 2-8. The staff “bubble” ensures the staff is held vertically. Before being used, the instrument must be setup and levelled as discussed below.

The following discusses the automatic level with circular levelling “bubble”. Older “tilting plate” levels use a tube level that is parallel to the telescope.



2.4.1 Setting the instrument for area levelling

Leica manual, page 11. “Setting up the tripod”.

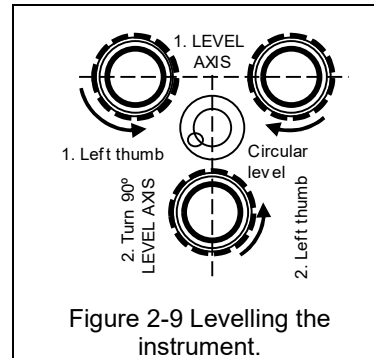
For differential levelling with an automatic level (Section 2.5.2), it is only necessary to roughly centre the circular level bubble on the instrument plate. The automatic compensator (pendulum) will provide the final levelling.

2.4.2 Levelling the Instrument

Leica manual, page 12. “Levelling up”.

Mount the level on the tripod head using the centre bolt (e.g., Figure 1-4)

1. Centre the three foot-screws on the base plate (there is a mid-point mark on screw housing), see Figure 2-9.
2. Centre the circular level using the foot screws,
 - turn the level until its axis is parallel to two of the foot screws
 - simultaneously turn the two parallel foot screws in opposite direction
 - move the bubble along its axis towards the centre following your LEFT thumb,
 - using ONLY the remaining foot screw
 - move the bubble towards the centre following your LEFT thumb
 - you may need to repeat the procedure to centre the bubble correctly.



2.4.3 Focusing the Instrument

Leica manual, page 13. “Focusing telescope”.

Correct focus of the eyepiece reticule and the object focus is critical to accurate observations. The following points apply to all optical observations.

The eyepiece focus is set once for EACH individual observer.

But the object focus (see Figure 2-8a. Figure 2-10), using the focussing knob on the side of the telescope, must be done for EACH observation.

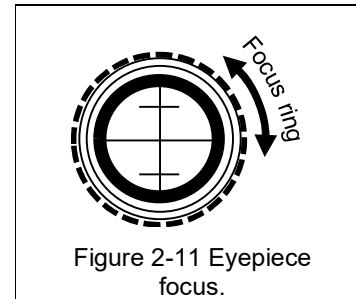


2.4.3.1 Eyepiece focus

1. Place a light-coloured card in front of the OBJECT lens
2. Looking through the eyepiece, turn the EYEPIECE (diopetre) focus ring until the reticule engravings are sharp and dark (Figure 2-11).

2.4.3.2 Object focus

1. Point to the level staff, the point to be focused.
2. Looking through the eyepiece, turn the OBJECT focusing knob (Figure 2-12) until the image is in sharp focus on the reticule (Figure 2-13).
3. Observing the object, move your head up and down slightly to confirm there is no parallax in the focus between the line of sight of the reticule and the object.



2.4.3.3 Backsights and Foresights

“Sight” is the word meaning either an observation or a reading.

A **Backsight (BS)** is the **first** sight (reading) taken after setting the instrument in position for a reading. It is the beginning sight for the instrument set-up.

A **Foresight (FS)** is the **last** sight (reading) taken before moving the instrument to a new position. It is the ending sight for the instrument set-up prior to moving to the next set-up.

An **Intermediate sight (IS)** is any sight taken between the BS and FS.

A **Change point (CP)** is the common position between two instrument setups. A CP has a **FS** from the previous instrument position and a **BS** from the next instrument position.



2.4.4 Reading the Staff Interval

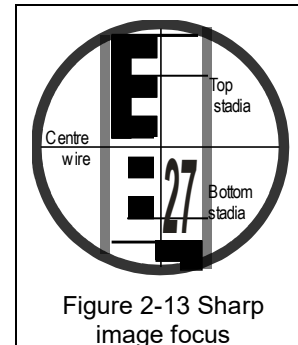
Leica manual, page 15. “Height reading”.

The face of the staff is marked with an E pattern of 1cm graduations (see Figure 2-14). The observer is expected to interpolate the millimetre readings. Because of the eye’s ability to discriminate proportions, it is relatively easy to interpolate to the millimetre.

The levelling staff should be extended only to the amount needed for the exercise. Ensure the correct section segments are extended. Each segment is about 1m long.

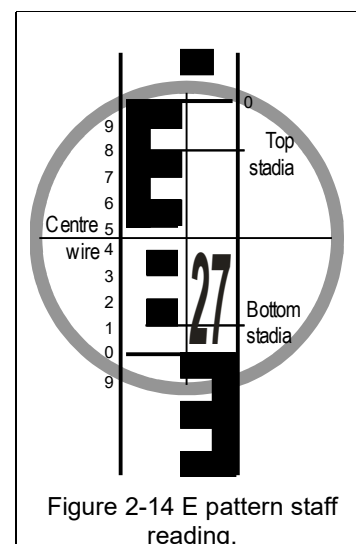
Object focus:

1. Focus on the staff with the OBJECT focus knob; the depth of field is shallow and requires critical focusing
2. Check for parallax in reticule.
3. Gently tap the side of the level to ensure freedom of the automatic pendulum compensator.



2.4.4.1 Staff reading

1. Using the **centre** cross hair, read the staff, estimating millimetres.
The illustrated staff on reads 2.745, the decimal point is inferred in the 27
2. Record the reading in **metres**.
3. The two stadia wire, used for measuring distance, can be used as a check of the centre wire reading,



Top wire (TW) = 2.780

Mean = 2.745 = centre wire

Bottom wire (BW) = 2.710

Distance = (TW – BW) × 100 (stadia constant) = 0.07 × 100 = 7m

Recording top and bottom wires can be a useful method of blunder detection.

The observer and the booker must work together to ensure that the readings have been recorded correctly. The booker should be able to reduce the readings, calculating RL from either rise/fall (Section 2.5.3) or height of collimation method (Section 2.5.7), and check for consistency. Furthermore, the booker should ensure an adequate description of the observed point. The final reduced level (RL) and all checks can be concluded quickly, before leaving the job. There is nothing worse than having to return to the job to fix a blunder.

Figure 2-15 shows an upright and an inverted staff combination. The inverted staff is read in a negative (down) sense. $\Delta H = BS - FS = 1.3 - (-1.285) = 2.585$ rise from base to underside.

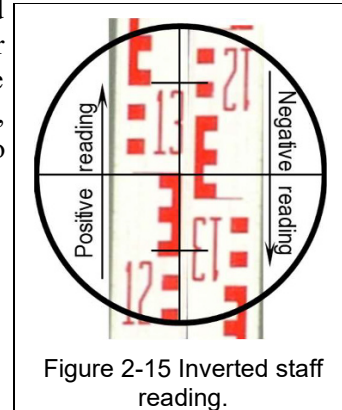


Figure 2-15 Inverted staff reading.

2.4.4.2 Staff handling

1. Make sure the required staff sections are fully extended and locked.
2. When transferring heights, ensure that the base of the staff is on a stable point; a peg, a recoverable point or a firmly embedded change plate (Figure 2-8(d)).
3. Minimum reading on the staff occurs when it is held vertical. This is achieved by using the “staff bubble”, a circular level held on the side of the staff. Slightly rocking the staff backwards and forwards to find the minimum reading is another method that could be used in the absence of a bubble.

2.4.5 Collimation Checks (Two peg test)

Leica manual, page 21. “Checking and adjusting line-of-sight”.

To check that the instrument’s line of sight is horizontal it is necessary to perform a line-of-sight test. This is commonly called the “two peg test”. It should be performed on any instrument that is to be used in production prior to any project. The object of the test is to ensure that the line of collimation of the level is within manufacturer’s recommendations. Outside this limit the instrument should be re-tested and if still deficient, adjusted or sent for rectification.

The principle of the test is to find the difference in height between two points:

1. Eliminating collimation error by observing, Δh , from the mid-point (Figure 2-16)
 - line of sight (LoS) errors are cancelled (deflections, δ , cancelled).

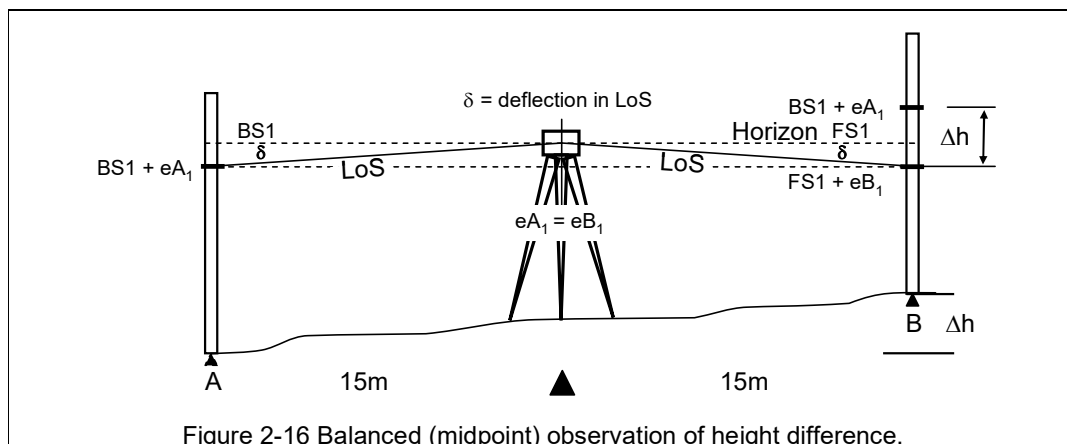
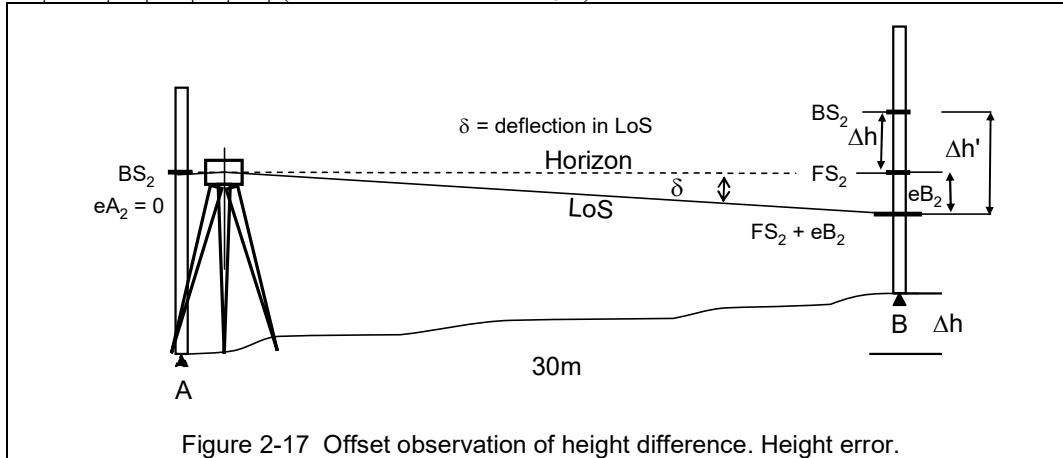


Figure 2-16 Balanced (midpoint) observation of height difference.

2. From a point close to A, 1 – 2m (Figure 2-17), determine $\Delta h'$ by observing over the full (30m) line-of-sight. The error in the line-of-sight is the discrepancy between the two observed height differences;
- $|\text{error}| = |\Delta h'| - |\Delta h|$ (unbalanced deflection, δ).



3. The recommended maximum error for the Leica Level NA720/724 is 3mm (0.003m).
4. Example of record sheet using the rise and fall booking method:
5. Table 2-1 shows an error of 4 mm, outside manufacturer's specifications. The test would

Table 2-1 Booking and analysis of a collimation test.

BS	IS	FS	Rise	Fall	Remarks
1.742					<i>A₁. Mid point. Equal error</i>
		1.622	0.120		<i>B₁. Mid point. Equal error</i>
1.580					<i>A₂. At staff. Zero error</i>
		1.456	0.124		<i>B₂. 30m to B. All error</i>
$\Delta h = 0.120$		$\Delta h' + eB_2 = 0.124$		$ \text{Error} = 0.124 - 0.120 = 0.004$	

be repeated to confirm the result before any decision on adjustment is made. Or extra care could be used to balance foresight/back-sight observations.

6. If this measurement was confirming that the reticule needed adjusting, then the reading to B₂ would have to be adjusted so that the staff reading was:
- $1.580 - 0.120 = \mathbf{1.460}$.

2.5 Measuring and Reduction Techniques

2.5.1 Basic Rules of Levelling

In using an automatic level for vertical control and height differences, you need to remember the following *six* rules of levelling:

1. The instrument must be properly levelled, pendulum free; check circular bubble, pendulum “tap”.
2. Backsight distance should be approximately equal to the foresight distance. Stadia wires can be used to approximate the distances (see staff reading above).
3. Instrument, staff and change plate must be stable, and the staff kept on the change plate.
4. Staff must be vertical. Check that the staff bubble is centred.
5. Remove parallax, **read the centre wire**; sharp, coincident focus of reticule and object. Reticule (cross wire) focus set with eyepiece ring; object focus to eliminate parallax.
6. Close the traverse.

For all exercises in Appendix (A2-1), we assume that the Earth is FLAT. All calculations are based on plane trigonometry, and the horizontal plane is perpendicular to the local **geoid** normal. Distance measurements are generally less than 100 m and level observations are rarely over 50 m, generally 30 m is sufficient. These limitations then allow us to ignore the effects of Earth’s curvature and scale factor. Heights will be referenced to the Australian Height Datum (AHD). The AHD was defined as mean sea level at 30 tide gauges around Australia, observed between 1966 and 1968 (Figure 2-3).

2.5.2 Levelling Specifications in Australia

The Intergovernmental Committee on Surveying and Mapping (ICSM) through its Special Publication 1 (SP1), Version 2.1 (2014) provides guidelines for Differential Levelling to achieve various levels of misclose. https://www.icsm.gov.au/sites/default/files/2018-02/Guideline-for-Control-Surveys-by-Differential-Levelling_v2.1.pdf

Allowable misclose: When conducting differential levelling or Total Station differential levelling, errors propagate in proportion to the square root of the travelled distance. A misclose assessment should be undertaken to verify that forward and backward runs of a levelling traverse, including any individual bays, are within the maximum allowable misclose. The allowable misclose is calculated using the formula:

$r_{mm} = n\sqrt{k} \text{ (} k \text{ in km)}$	Eqn 2-1

Three standards, using empirical values, are recommended:

$$n = 2\text{mm}\sqrt{k}, n = 6\text{mm}\sqrt{k} \text{ and } n = 12\text{mm}\sqrt{k}.$$

Distance in km shows that a 250m (0.25km) level run between known points should result in a misclose of $12\sqrt{(0.25)} = 6\text{mm}$.

Equipment: For the recommended maximum allowable misclose of $12\sqrt{k}$ (forward and back). A 2 mm/km optical level; collimation check at start of project; wood or fibreglass staff, calibrated within 5 years; staff bubble attached and accurate to 10' verticality, telescopic tripod; standard change plate; thermometer accurate to 1°C.

Observation techniques: Two-way levelling; foresight approx. equal to backsight; staff readings to 1 mm; temperature recorded at start and at 1-hour intervals, or at pronounced changes to conditions; maximum sight length 80 m; minimum ground clearance 0.3 m. One of the automatic levels used for the levelling exercises is the Leica NA720/NA724, a general-purpose surveying and construction level. Accuracy is quoted as 2.0 mm/km for the NA724, meeting SP1 specifications.

Refer to the “Leica NA720/724/728/730 Instruction Manual, English, Version 1.0” by Leica Geosystems. This manual is contained in most instrument cases.

2.5.3 Differential Levelling: The “Rise and Fall” Method

Levelling procedure:

- ✓ A horizontal line of sight is established using some form of levelling mechanism:
 - Spirit level tube
 - Swinging pendulum
- ✓ A graduated staff is read through the telescope of the level
- ✓ The elevation of points can be established by first reading the staff on a Bench Mark (BM)
- ✓ The staff is then moved to the desired point, the level is turned, and the staff is read again
- ✓ The reading at the benchmark (BM) is called the backsight (BS), see BS1 in .
- ✓ The reading taken after turning the instrument and moving the staff is the foresight (FS), see FS1 in Figure 2-18.
- ✓ To continue levelling, the staff is kept on the point at B and the instrument moved to the midpoint between B and the next point, C, i.e., STN 2.
- ✓ B is called the change point (CP) or turning point (TP)
- ✓ The staff at B is carefully turned toward the instrument and a BS reading taken
- ✓ Then the staff is moved to C and a FS reading is made
- ✓ The procedure is repeated as many times as needed
- ✓ The levelling should always end on a BM (i.e., D) as a check!

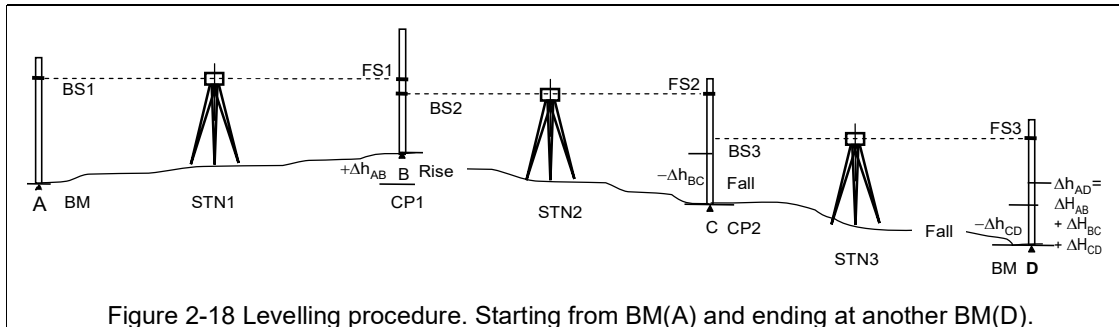


Figure 2-18 Levelling procedure. Starting from BM(A) and ending at another BM(D).

2.5.4 Booking and Field Reduction Procedures for Levelling

Two note reduction methods for calculating elevations from the BS and FS observations exist. Each of the methods uses only two equations for the computations. These methods are

- Rise and Fall method (i.e., a fall is simply a negative Rise);

$\text{Rise (or Fall)} = BS - FS. \text{ Elev} = \text{Previous elev} + \text{Rise (or Fall)}$	Eqn 2-2

- Height of Collimation method;

$HI = \text{Elev} + BS. \text{ Elev} = HI - BS$	Eqn 2-3

In what follows, a detailed examination of the methods is presented.

2.5.5 Rise and Fall Booking and Reduction Procedures

The “rise and fall” method of booking allows three sets of checks to be applied to the booked values. It only checks the calculations, not the booked observed values. Transfer a control point level (bench mark) at A to a new control points at B and C, checked by closing to control point at D. Referring to Table 2-2, the field book recording and checking by “rise and fall” observations, using Eqn 2-2 are:

Control point A has a known RL of 10.125

Control point D has a known RL of 8.930

difference $\Delta h_{AC} = 8.930 - 10.125 = -1.195$

Table 2-2 Rise and fall, checking calculations.

BS	IS	FS	Rise	Fall	RL	Remarks
1.742					10.125	A. Control point.
1.580		1.522	0.220		10.345	B. Change point 1
0.625		1.892		0.312	10.033	C. Change point 2
		1.732		1.107	8.926	D. Observed R.L. point
Sum		Sum	Sum	Sum		
3.947		5.146	0.220	1.419		
$\Sigma BS - \Sigma FS$			$\Sigma Rise - \Sigma Fall$		Last RL - First RL	
	-1.199		Diff =	-1.199	-1.199	$\delta h'_{AD}$ Last - First
Three checks agree. The math checks agree with the booked observations.						

However, Table 2-3 shows there is a discrepancy (misclose) between the booked observations and reductions and the known RLs of A and D of 0.004. This may need to be investigated, or accepted (with justification in relation to job specifications).

Table 2-3 Establishment of misclose.

					10.125	A. BM (fixed)
					8.930	D. BM (fixed)
					-1.195	δh_{AD} Last - First
			Misclose		+0.004	$\delta h_{AD} - \delta h'_{AD}$

2.5.5.1 Three wire booking: using the stadia wires

Leica manual, page 16. “Line levelling”.

At each level reading, the three wires, the horizontal centre wire and the two stadia wires, may be booked to provide, a) a check against blunder readings and b) a calculation of distance to the staff. (Stadia constant 100).

Table 2-4 Three wire readings.

BS	IS	FS	Rise	Fall	RL	Remarks
1.542						Top wire
1.742					10.125	A. Control point.
1.943						Bottom wire
1.653		1.627				Top wire
1.590		1.522	0.220		11.345	B. Change point 1
1.407		1.417				Bottom wire

The above snippet of field book, Table 2-4, shows:

BS to A. The middle wire (MW) agrees with the mean of the top wire (TW) and the bottom wire (BW). Distance = $(1.943 - 1.542) \times 100 = 40.1$ m

FS to CP1. MW agreement. $(TW+BW)/2 = MW$. Distance = $(1.627 - 1.417) \times 100 = 21$ m

After **changing** observation station, reading: **BS to CP1**. MW disagrees with $(TW+BW)/2 = 1.580$. Is it a booking blunder?

- **Check.** Distance 24.6 m. Further checking: $(TW - MW) \times 200 = 12.6$ m,
 $(MW - BW) \times 200 = 36.6$.

- **Solution.** Re-observe BS to check before the next FS reading.

Do not take the “mean” value between TW and BW to get a “refined” MW reading. Use it as a **check**.

Could you “assume” a misreading of the mid-wire, correct the reading to 1.580 and then use this as the “correct” BS reading to the next fore station?

2.5.6 Area Levelling: The “Rise and Fall” Method

The “rise and fall” method of booking will be used instead of the illustrated “height of collimation” method in § 2.5.7. Again, we are only checking the calculations, not the booked observed values. In the two levelling exercises in Appendix A2-1, there will normally be a mix of “line” and “area” levelling observations. All observations, other than the back sight (BS) and the fore sight (FS) are considered intermediate sights (IS) and are booked in the IS column of the “rise and fall” booking sheet.

Exercise 1 is the transfer of reduced levels (RL) to temporary control points (temporary benchmarks, TBM) from existing BMs. You may have to use intermediate sights (IS) to establish these TBM and will have to use them to answer some of the exercise questions.

Exercise 2, the area levelling and mapping exercise, will use mainly intermediate sights (IS) to pick up cross section levels, but will also require a number of change points (CP) to carry observations through the area.

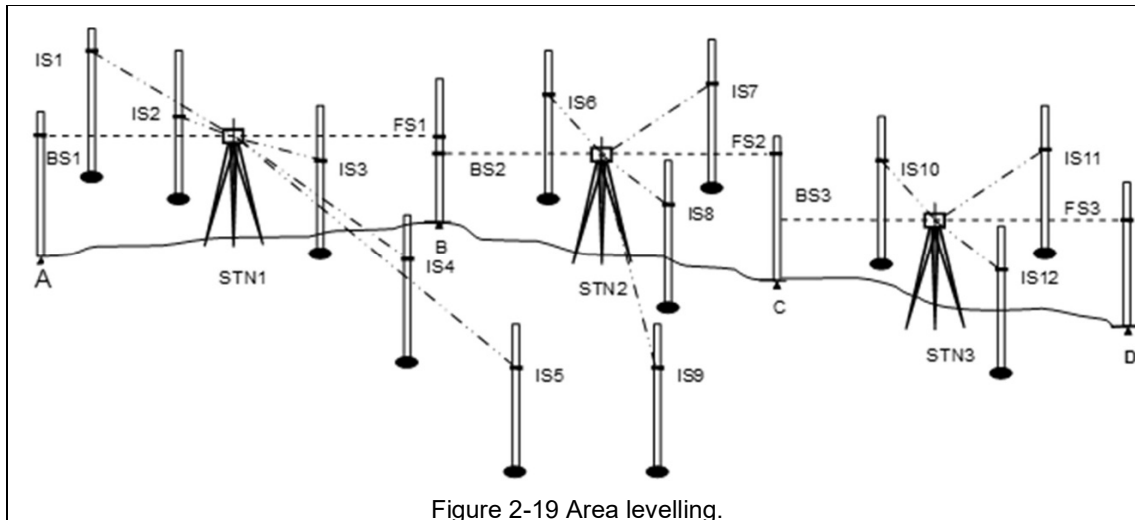


Figure 2-19 Area levelling.

Table 2-5 is an example of booking intermediate sights (IS) between back sights and fore sights, and checking the reductions for observations taken in area, Figure 2-19.

Note that while the checks agree with each other, no maths problems, there is still a discrepancy between the calculated RL of BM C and its true value. This is likely a random error mis-closure.

The check sums only validate the mathematics of the reduced observed values. It does not mean that the individual observed values are correct. A blunder reading itself can't be isolated between check points.

Table 2-5 Rise and fall method. Check calculations.

BS	IS	FS	Rise	Fall	RL	Remarks
1.742					10.125	A. Control point.
	1.628		0.114		10.293	IS1
	1.205		0.423		10.662	IS2
	1.585			0.380	10.282	IS3
	1.625			0.040	10.242	IS4
	1.550		0.075		10.317	IS5
1.580		1.522	0.028		10.345	B. Change point 1
	1.635			0.055	10.290	IS6
	1.640			0.005	10.285	IS7
	1.825			0.185	10.100	IS8
	1.745		0.080		10.180	IS9
0.625		1.892		0.147	10.033	C. Change point 2.
	1.254			0.629	9.404	IS10
	1.365			0.111	9.293	IS11
	1.158		0.207		9.500	IS12
		1.732		0.574	8.926	D. Observed R.L.
Sum		Sum	Sum	Sum	8.930	D. BM RL 8.930
3.947		5.146	0.927	2.126		closed to 0.004m
$\Sigma BS - \Sigma FS$			$\Sigma Rise - \Sigma Fall$		Last RL - First RL	
	-1.199		Diff =	-1.199	-1.199	$\delta h'_{AD}$ Last - First
Three checks agree. The maths agrees with the booked observations.						

2.5.7 Area Levelling: The “Height of Collimation” Method

This method, see Table 2-6, using the area in Figure 2-19, is easy to use both for level pick-up and for level set-out. A staff back sight (BS) to a known level (point or BM) is taken and added to the known RL. This becomes the “height of collimation” of the level. It is the RL of the observed horizontal plane, the instrument horizon, i.e., from Eqn 2-3:

$HI \text{ or } HC = \text{Known RL} + BS. \text{ RL any point} = HI - IS \text{ or } FS$	Eqn 2-4

The RL of any point = HC – IS/FS, the staff reading. This method allows change points (CP) to be introduced as temporary bench marks (TBM). The next instrument set-up has a new HC based on the RL of the CP and the new FS.

From CP2 (Table 2-7), you wish to establish a design level of RL10.5 over an area. The instrument has been moved and the RL of BM B is used as a control. Additional levels, for other design RLs, are also illustrated. Store the HC in your calculator and use that value, minus the required level, to find the staff reading.

The method is used for setting-out of vertical control and is the basis of setting-out using rotating laser levels and detectors. Once the HC has been established from a known RL, it is merely a case of raising or lowering the staff until the required level coincides with the base of the staff. The established level is marked by be a peg driven to level, or a line on a stake.

There is no easy way to check the RL values of intermediate levels compared with the “rise and fall” method.

Table 2-6 Height of collimation booking and reduction.					
BS	IS	FS	HC	RL	Remarks
					.
1.742			11.867	10.125	RL A. BM . HC for STN1.
	1.628			10.293	IS1
	1.205			10.662	IS2
	1.585			10.282	IS3
	1.625			10.242	IS4
	1.550			10.317	IS5
1.580		1.522	11.925	10.345	B. CP 1. HC. For STN2. RLB + FS
	1.635			10.290	IS6
	1.640			10.285	IS7
	1.825			10.100	IS8
	1.745			10.180	IS9
0.625		1.892	10.658	10.033	C. CP 2. HC. For STN3. RLC + FS
	1.254			9.404	IS10
	1.365			9.923	IS11
	1.158			9.950	IS12
		1.732		8.926	D. BM D. RL 8.930
Sum		Sum			closed to 0.004m
3.947		5.146			
$\Sigma BS - \Sigma FS = -1.199$				-1.199	$\delta h'_{AD}$ Last - First

2.5.7.1 Using HC to set out design levels.

Table 2-7 Calculate IS to set out of design levels.					
BS	IS	FS	HC	RL	Remarks
1.370				10.345	RL B.
			11.715		HC. For CP2. RLB + FS
	1.215			10.500	Finish level required.
Set IS	1.215				$11.715 - 10.5 = 1.215$
	1.315			10.400	Calculated IS. Design RL
	1.415			10.300	Design RL
	1.515			10.200	Design RL
	1.615			10.100	Design RL

Distribution of misclose? Not in this course.

If the traverse misclose is within allowable limits, then there is an argument for the distribution of a correction through the points. It is carried out in proportion to the number bays; above we have -0.004 over 3 bays (A – B, B – C, C – D). The correction is $+0.004/3$ per bay cumulative as, say, $+0.001$ A to B, $+0.002$ B to C, $+0.004$ C to D.

Uren and Price (2010), section 2.4, p49 gives details. However, it is hard to differentiate between random errors and systematic errors (collimation caused by unequal BS/FS). Thus, distribution of misclose is **not** used in this course. Miscloses must always be judged on tolerances and the aim of the task, and an engineering decision made.

2.6 Examples of Levelling for Height Control

Two exercises were designed for the Curtin University Civil and Mine engineering courses. The exercises illustrate the use of appropriate practical exercises in available area:

Establishment and transfer of levels using **differential** levelling is used to find the level of new and established control points. The method transfers the **orthometric** height throughout a level network. Control levelling must always close back to a known point or to, at least, a previously established intermediate point in the network. Students use this differential method in Field Practical 1 to set level control for their area levelling exercise in Field Practical 2 (see Appendix A2-1).

2.6.1 Civil Engineering for a Road Excavation Exercise

The exercise (Figure 2-20) area is about 100m × 50m on a sloping site, falling to the south east over a height difference of some 3.5m. The area features pine trees, paths and service boxes. These features lend themselves to various forms of mapping which are explored through the course.

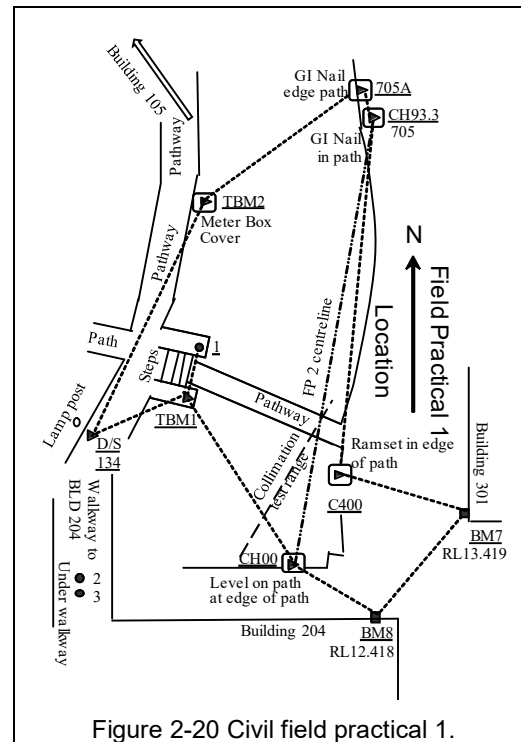


Figure 2-20 Civil field practical 1.

- a. Exercise 1 involves a level collimation test, then establishment of vertical control to be used in exercise 2. It is an opportunity for students to become familiar with instrument setup, focussing and staff reading and recording.

The level collimation exercise provides a practical example of balanced back sight/fore sight observation carried out by each student as an independently (moving and re-setting the level). Booking and field reduction, and evaluation of results, is practiced. The exercise includes an introduction to site reconnaissance, location of control points and establishment of task objectives.

- b. Exercise 1 then examines level runs of two type for control establishments:
 - an open traverse from an established benchmark, via an inversion observation to check portal clearance.
It introduces change points for the rise and fall method of booking.
 - a closed loop traverse, finishing at a different benchmark, traverses through some of the open loop marks. The exercise requires several change points to be used, giving team members the opportunity to carry out observation, booking and staff handling tasks. The exercise timing allows field checking of results before task completion.

- c. Exercise 2 (Figure 2-21) incorporates field set-out of a perpendicular offset grid along an established centre line. The task continues from control established in exercise 1 and introduces area levelling observation, booking and reduction. Multiple change points allow rotation of observation tasks.
- the chainage and offset method of field mapping is used to locate trees, services and feature boundaries. From this field data, a scaled plan of the exercise area is produced showing feature and the boundaries of a design excavation along the centre line.
 - field data is used to calculate the volume of a cutting of designated slope and batter intercepts.

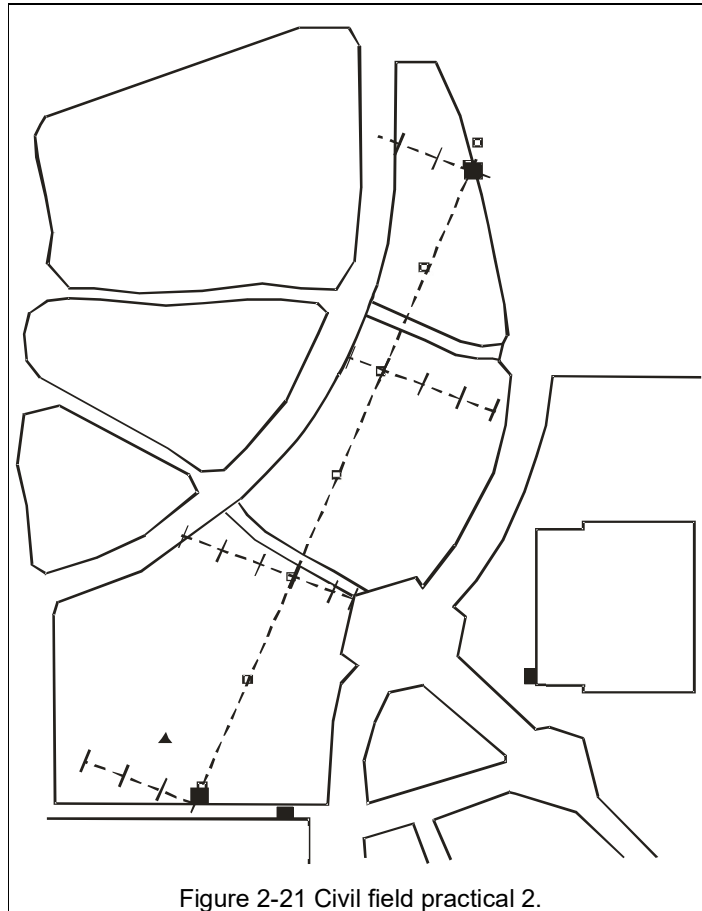


Figure 2-21 Civil field practical 2.

2.6.2 Mining Engineering Surveying for a Site Development

The exercise area is about $120 \text{ m} \times 90 \text{ m}$ on a sloping site, falling to the south east over a height difference of some 6 m. The area features pine trees, paths and service boxes. These features lend themselves to various forms of mapping which are explored through the course.

- a. Exercise 1 (Figure 2-22) involves a level collimation test, then establishment of vertical control to be used in exercise 2. It is an opportunity for students to become familiar with instrument setup, focussing and staff reading and recording. The level collimation exercise provides a practical example of balanced back sight/fore sight observation carried out by each student independently (moving and re-setting the level). Booking and field reduction, and evaluation of results, is practiced. The exercise includes an introduction to site reconnaissance, location of control points and establishment of task objectives.
- b. Exercise 1 then examines level runs of two type for control establishments:
 - an open traverse from an established benchmark, via an inversion observation to check portal clearance. It introduces change points for the rise and fall method of booking.
 - a closed loop traverse, finishing at a different benchmark, traverses through some of the open loop marks. The exercise requires several change points to be used, giving team members the opportunity to carry out observation, booking and staff handling tasks. The exercise timing allows field checking of results before task completion.

- c. Exercise 2 (Figure 2-23) incorporates field set-out of a rectangular grid along over a designated area. The task continues from control established in exercise 1 and introduces area levelling observation, booking and reduction. Multiple change points allow rotation of observation tasks.

- grid layout of a typical blasting grid. Area levelling, using the Height of Collimation method illustrates field practice.
- grid method (estimating position in grid cell) of field mapping is used to locate trees, services and feature boundaries. From this field data, a scaled plan of the exercise area is produced showing feature and the boundaries of a design excavation area.
- field data is used to calculate the volume of an excavation down to a design level.

2.6.3 Grid Layout for levelling

The survey area should be gridded in a rectangular pattern aligned N-S and E-W, or as dictated by the main axis. This will allow easy integration to the local coordinate system for the GIS/CAD systems. A plan of the work area allows **determination** of the grid area, and its alignment. The base control points should be referenced to known control to allow incorporation of the grid coordinates into the local survey grid system. The alignment axis can be indicated by survey stakes, ranging poles or pathways. Ranging poles and cloth tape will be used to place chaining arrows at the grid intersections.

The height of collimation method of recording will allow quick reduction of the DEM levels.

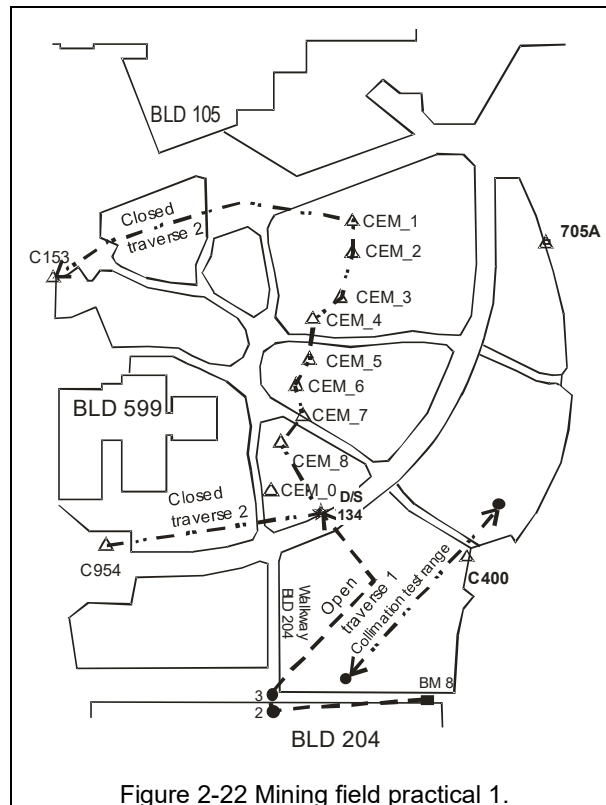


Figure 2-22 Mining field practical 1.

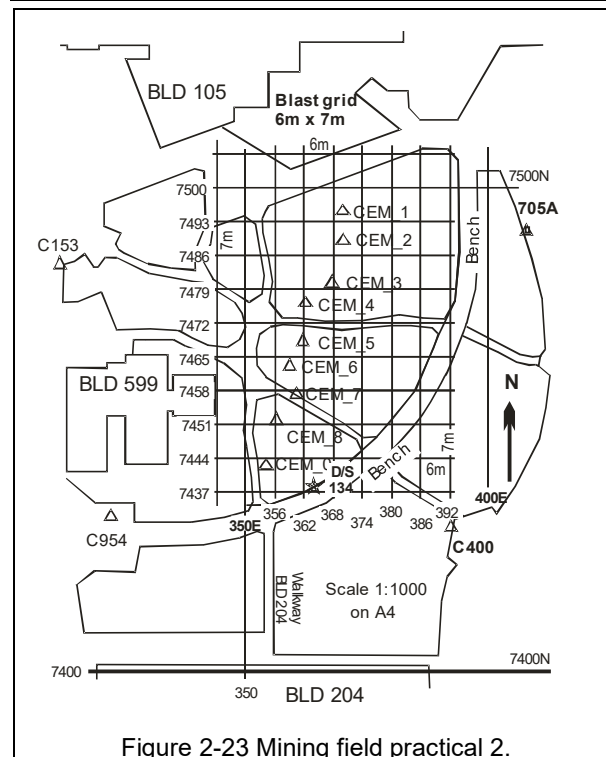


Figure 2-23 Mining field practical 2.

2.6.4 Orthogonal Grid Layout Methods

The requirement for a grid is that it should be orthogonal, i.e., the grid lines should be perpendicular to each other. How is this achieved?

3 – 4 – 5 triangle: The traditional technique is to use the dimensions of a plane right angle triangle as control. This method has been used since antiquity and continues today in the building industry. The best traditional triangle is the “3 – 4 – 5” triangle. 3 and 4 are the perpendicular axes and 5 is the length of the hypotenuse.

Establish the **grid origin** and lay out the extension of the main axis, marking it with stakes or ranging poles. Lay the tape on the ground from the origin. Place a survey pin at, say, 6.0m. From the origin lay a second tape at about right angles to the main axis. Measure down 8.0m. Check the diagonal from the 6m pin and adjust the 8m alignment until the diagonal distance reads 10.0m. Extend the alignment with a range pole and you have your main grid axes. It works with a 30m tape.

Optical square: It is a small hand instrument (Figure 2-24, Figure 2-25) used for laying off a right angle by means of two pentaprisms separated vertically by a small gap. The pentaprisms deflect the line of sight through the prism faces by 90° to the left and right.

If two range poles are coincident in the prisms, then a point also coincident through the gap is perpendicular to the line of the range poles (Figure 2-26). Hanging a plum bob from the prism allows a reasonable intersection.

Grid orientation: Grid orientation may be calculated by measuring in from known points to a convenient integer coordinate, or by examination of a site plan. Often plans will be plotted over existing features that can be used as a base line.

2.6.5 Height from Vertical Inclination

Part of the practical tasks involves finding the height of trees using a tape and **clinometer**. The enclosed case clinometer comprises a pendulous calibrated wheel supported on dual sapphire bearings in a fluid filled capsule. The face of the wheel is graduated 0° to ±90° and is read against an index on the capsule. The periphery of the wheel, viewed through the magnifying eyepiece, shows elevation and depression from the horizon. It is graduated in degrees (LHS) and percentage of horizontal distance (RHS) to the target. Typical performance is quoted as: accuracy ¼°, graduation 0.5°.

The illustration shows the simultaneous view using both eyes, reading to the top of a tree trunk. The clinometer wheel is viewed with one eye, and the target with the other. (Tricky ‘til you get the hang of it.) The elevation reading is 7° (about 12%).

The height of the tree, above the observer’s eye, is the horizontal distance to target × tan(angle).

Assume H distance = 50m:

$$Ht = 50 \tan(7^\circ) = 6.1\text{m} \quad (50 \times 12\% = 6).$$

Take a second reading to the base of the tree, say -4°. $Ht = -3.5$.

Tree height = $6.1 - -3.5 = 9.6\text{m}$.



Figure 2-24 Optical square.

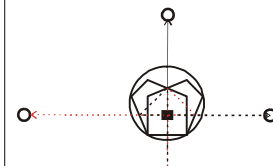


Figure 2-25 Optical path through square.

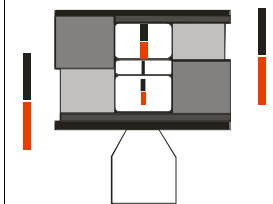


Figure 2-26 Aligning to perpendicular.

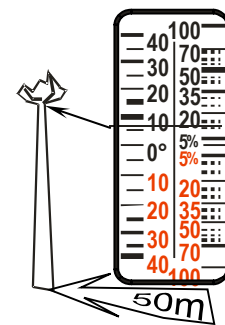
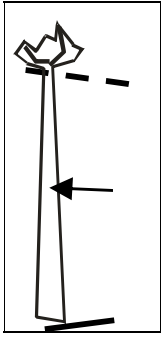


Figure 2-27 Tree elevation.



Always measure from the longest possible distance away. The inclination angles are lower; the pointing errors associated target definition are minimised. Read to both base and top, you are thus independent of eye height, especially on sloping ground.

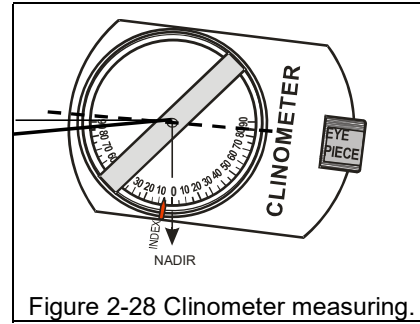


Figure 2-28 Clinometer measuring.

2.7 Errors in Levelling and Management Strategies

The best analytical discussion of errors in levelling is contained in Ghilani (chapter 9, pp151, *Error propagation in elevation determination*). As with any set of measurement observations, differential levelling is subject to three types of errors (i) systematic errors, (ii) random errors and (iii) blunders or mistakes and errors in datum total transfer.

2.7.1 Systematic Errors in Level Observations

1. Collimation errors, minimised by one of the rules of levelling:

Back sight and foresight approximately equal:

- an unbalance observation will introduce either a positive or negative systematic error depending on:

- a) collimation error,

The Leica manufacturer's maximum error for example is 0.003 (3 mm) in 30 m = 0.0001 radians = 21" arc.

- b) difference between length of total back sights and total fore sights, error = distance × collimation error (radians).

- maximum errors; 0.001m at 10m, 0.005 m at 50m.

In an extended line levelling exercise, the difference in the sum of the distances covered as back sights and foresight may contribute to a significant systematic error.

2. Earth curvature and refraction, h_{CR} , minimised by one of the rules of levelling:

Back sight and foresight approximately equal:

- the staff reading error, in metres, combining curvature and refraction, is:

$$- h_{CR} = CR \left(\frac{D}{1000} \right)^2, \text{ where } CR = 0.0675 \text{ for } D \text{ in metres.}$$

Eqn 2-5

An unbalanced set of observations will introduce a positive systematic error, e_{CR} , that must be **subtracted** from the difference in elevation

$$- e_{CR} = \frac{CR}{1000^2} (\Sigma D_{BS}^2 - \Sigma D_{FS}^2)$$

Eqn 2-6

CR is quoted as 0.0675 by Ghilani (2010). It is close to an empirical value of $k=1/14$ (0.07) developed by Bomford's investigation of curvature and refraction of radio and light waves in EDM (e.g., Bomford, §3.5, (1.34), p52).

The effect is minimal, for example (Ghilani 2010, p147). Take transfer of levels on a sloping site, where BS/FS can vary in the ratio of 5:1. A 1:6 slope (9.5°), with a 5 m staff, has a BS = 8m and FS = 21m. Over 150m traverse, say 5 × 30m bays.

- $\Sigma_{BS} = 105$, $\Sigma_{FS} = 40$. $dS^2 = (105^2 - 40^2) = 9,425$; $e_{CR} = 0.0675/10^6 \times 9425m$
- $e_{CR} = 0.00064m$, under 1mm.

3. Staff calibration and mechanical construction errors:

It is generally assumed that staff markings have been manufactured accurately. They can be checked against a certified measuring tape (\$100) or by a certified testing authority.

Telescopic segmented staffs can suffer from worn locking mechanisms, especially over the base and 1st segment which are the most commonly extended sections. Use a tape cross the segment join to find any wear, making sure the staff is vertical to load the lock. (Figure 2-29).

4. If using two staffs, make sure they have similar base segment lengths, a worn staff foot could introduce a systematic error between staves, a staff datum error. Complete each pair of level bays by leapfrogging staves. Ensure an even number of bays.

5. Temperature expansion: staffs should be manufactured at a standard temperature, about 25°C,
 - a fibreglass staff temperature coefficient is about 10 ppm/°C
 - an aluminium staff temperature coefficient is about 25 ppm/°C

At 35°C, $\delta T = 10$, over a height difference of 50 m there would be an error of:

- 0.013 m using an aluminium staff
 $((50(dH) \times 25/10^6) \times 10 (\delta T) = 0.0125)$ and
- 0.005 m using a fibreglass staff.



Figure 2-29 Worn staff.

2.7.2 Random Errors in Level Observations

1. Observer reading errors:

single reading error:

$$\pm 1.2 \text{ mm}/30 \text{ m} = 0.00004 \text{ radians} = 8'' \text{ arc}$$

2. Instrument level error: usually expressed as the:

- standard deviation in mm per km of double run levelling:
 - $\pm 2 \text{ mm}/\text{km} = 2.0^{-6} \text{ radians} = 0.4'' \text{ arc}$

this value agrees with the declared compensator settling accuracy.

3. Staff verticality, staff bubble error, δ
 may be of the order of 5' arc, (Ghilani 2010, p149)

$$e_{LS} = \frac{h}{2} \sin^2 \delta, \text{ at } h = 5\text{m} \quad e_{LS} = 0.005\text{mm}$$

- (1/200mm).

when $\delta = 1.5^\circ$, at $h = 5\text{m}$ then $e_{LS} \approx \pm 0.002\text{m}$

- (2mm).

The analysis (Figure 2-30) shows that a staff bubble should be used for all observations. But at normal observation levels, where $h = 2\text{m}$, a non-verticality of 2.5° is needed to produce a 2mm error.

4. Lack of a staff bubble can be overcome by rocking the vertical staff slowly backwards and forwards and reading the minimum height value on the mid wire. (Also, check verticality against vertical cross hair).

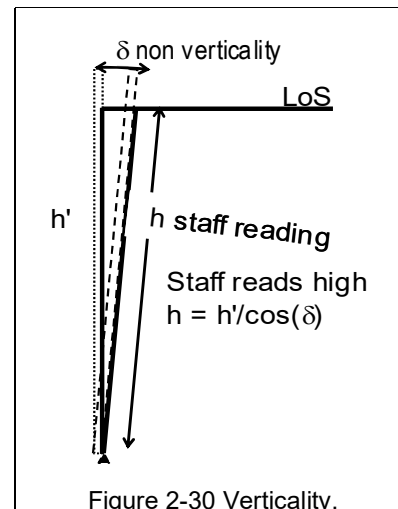
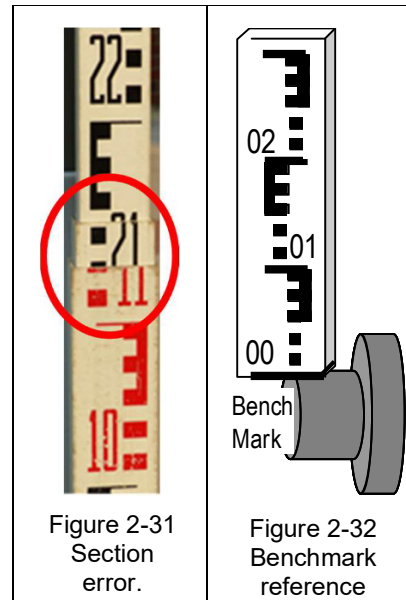


Figure 2-30 Verticality.

2.7.3 Observation Blunders and Mistakes

Likely sources of error in observations will come from:

1. Gross reading error of centre wire
 - read and record top and bottom stadia wires,
 - check mean.
2. Reading top or bottom stadia wire instead of centre wire.
3. Parallax in reading staff,
 - check reticule and object focus.
4. Unstable change points
5. Staff section not extended, or not locked (Figure 2-31).
6. Incorrect recording of Benchmark data
7. Not using BM correctly
 - ensure correct reference surface (Figure 2-32).
8. Transcription errors in field book
 - poor character formation, printing, transposition.



2.8 Concluding Remarks

This chapter has introduced you to levelling, i.e., the surveying procedure used to determine elevation differences. Arguably, most tasks that you will undertake in civil engineering and mining operations will require some sort of height (elevation) information. In the chapters ahead, we will see how this information becomes vital for the design/construction projects.

2.9 References to Chapter 2

1. Uren and Price (2010) Surveying for engineers. Fifth edition, Palgrave Macmillan, Chaps 2.
2. Ghilani, C (2010) Adjustment Computations Spatial Data Analysis. 5th ed., John Wiley, New Jersey 672 p.
3. Bomford, G (1980) Geodesy. Fourth edition, Clarendon Press, Oxford.
4. Featherstone, W., Kirby, J., Kearsley, A. et al. Journal of Geodesy (2001) 75: 313. doi:10.1007/s001900100177



3 Chapter 3 Relief and Vertical Sections

3.1 Introductory Remarks

In this chapter, we introduce you to the concept of relief and vertical sections and their applicability to civil and mine engineering. This chapter begins by providing definitions to terminology that will be used here and also in the workshop and field practical. We hope that by engaging the materials in this Chapter, the workshop and the field practical, you should:

- ✓ Be able to define relief and know the elements in representation of relief.
- ✓ Know the importance of relief in civil and mine engineering.
- ✓ Understand the nature of vertical sections (longitudinal and cross sections).
- ✓ Understands the characteristics of contours.
- ✓ Understand the surveying procedures for tracing contours in the field.
- ✓ Be able to interpolate contour lines from survey data.
- ✓ Be able to distinguish between DTM and DEM and know their uses.
- ✓ For the civil engineers, be in a position to design a foot/cyclic path from levelling data, i.e., RLs and FLs (e.g., field practical 2).
- ✓ For the mine engineers, be able to undertake the establishment of a gridded area to provide levels for the design of blasting operations in an open cast mine site.

Essential references include Schofield and Breach (2007) and Irvine and Maclellan (2006).

3.2 Definitions

1. **Relief:** A relief is a general term applied to the shape of ground in a vertical plane. The representation of a relief on a map is the showing of heights and shape of the ground above a datum, which is normally the mean sea level (MSL). Relief can be represented by heights, shapes, spot heights and contours. Representation of heights is largely a factual matter in which variation will arise from the type, density and accuracy of the information provided while the representation of shape is largely artistic and the methods vary on different maps. Maps designed for architectures and engineers for design purposes usually represent relief by either spot heights or contours or both.
 - In engineering, representation of relief is necessary in order to allow designs of constructions to be drawn in a manner that “harmonizes” with site.
 - When it is not possible for the engineer to visit the site, he/she may rely on a graphical representation of relief in form of a map.
 - All topographical maps have some representation of relief.
 - The extent of relief shown on a map will depend on the scale and purpose of the map.
2. **Scale:** A scale is the ratio of a distance on the ground to the same distance on a map, e.g., 1:20,000. 1 unit on the map represent 20,000 units on the ground.
 - 1 unit on the map is equivalent to 20,000 units on the ground.
 - 1 m on the map is equivalent to 20,000 m on the ground.
 - 1 mm on the map is equivalent to 200 m on the ground.
3. **Spot heights:** These are less accurate heights and normally without any definite mark on the ground.
 - They are selected to indicate the height of the ground at random points such as tops of hills and slopes, bottom of valleys, ridge points etc., to supplement information by contours.
 - They are shown by a dot with the height value.
 - The accuracy will vary but should be as accurate as the contours. Spot heights are written with decimal point on the point on the ground, and the height value is written on the slope so that it may be read from the bottom right corner of the map.

4. **Contours:** A contour is a line on the map joining points of equal heights and is the standard method of showing relief on topographical maps. Contouring combines an accurate indication of height with good indication of shapes, especially when used in conjunction with spot heights. Heights for establishing contours can be obtained from angular measurements (known as trigonometric heighting, see Chap 6), levelling discussed in Chap 2, and from gridding as we shall see in Sect. 3.4.12.

Uses of contours:

- Used for constructing longitudinal and cross-section for initial investigations (see below for definitions of longitudinal and cross sections).
- Used for generating DEM and DTM (will be discussed in a later section).
- Used to construct route lines of constant gradient.
- For computing volumes (see Sect. 3.4).
- Delineate the limits of constructed dams, roads, railways, tunnels, etc.
- Delineate and measure drainage areas.

Properties of a contour:

- They are normally continuous lines (brown). Every fourth or fifth contour depending on the vertical interval is called index and is normally made thicker for easier reading.
 - Horizontal separation indicates the steepness of the ground.
 - Highly irregular contours define rugged terrain.
 - Concentric close contours represent hills or hollows.
 - Contour lines crossing a stream form V's pointing upstream.
 - The edge of a body of water forms a contour line.
 - Vertical distance between contours is called interval. The choice of this vertical interval depends on (i) the type of scale of the map and type of country mapped, e.g., on a 1:50,000 map with average relief, a contour interval may be 10 or 20 m while at 1:12,000 scale it is probably 0.5 m, (ii) type of project involved, e.g., contouring an airstrip requires an extreme small interval, and type of terrain; flat or undulating. The interval can be used to calculate gradients through "Gradient = Vertical interval/horizontal equivalent". For the purpose of cost, the smaller the interval the more the cost.
5. **Longitudinal and cross sections** (Irvine and MacLennan 2006, Chapter 6):
- Longitudinal section – Vertical section along the centre line of the complete length of the work.
 - Cross sections – Vertical sections drawn at right angles to the centre line of the works. Information given by the sections provides data for:
 - (a) Determining suitable gradients for construction work.
 - (b) Calculating volumes of earthworks.
 - (c) Supplying details of depth of cutting or heights of filling required.
 Example: Levelling is undertaken at the centre line and cross sections. See Table 3-1.
6. **Formation and Inverted Levels** (Irvine and MacLennan 2006, Chapter 6):
- *Formation level* of any construction works is the level to which the earth is excavated or deposited to accommodate works (e.g., the straight line rising from CH00 with RL of 12m to CH93.3 with RL of 14.25m in Figure 3-1). Formation width is the width of the design feature, e.g., a road.
 - *Inverted level* is the inside of the bottom of a pipe but for practical purposes maybe considered to be the bottom of excavation.

Table 3-1 Reduced levels for a road design.

BATTER INTERCEPTS FROM CROSS SECTION LEVELLING

RL	RMK	Above DATUM RL 12.00	$m = 4$ $b = 5$	$h_s =$ $(W_L - b/2)/m$	W_L	Slope k @ 5m or offset interval	CL Grade Final Level	$h =$ CUT/Fill	Slope k @ 5m or offset interval	W_R	$h_R =$ $(W_R - b/2)/m$
13.191	00 15L	1.191	15L			15.9		1.19			
12.876	00 10L	0.876	10L			8.4		0.88			
12.281	00 5L	0.281	5L	0.18	3.2	17.8		0.28			
12.00	CH 00 CL	0.000	0				0.00	0.00			
12.00	00 5R	0.000	5R					0.00	∞ or level	2.5	0
12.80	CH 15 CL	0.800	15				0.36	0.44			
14.585	30 15L	2.585	15L			11.1		1.86			
14.132	30 10L	2.132	10L			13.0		1.41			
13.747	30 5L	1.747	5L	1.20	7.3	14.4		1.02			
13.40	CH 30 CL	1.400	30				0.72	0.68			
12.96	30 5R	0.960	5R					0.24	-11.4	3.9	0.35
12.725	30 7R	0.725	7R					0.00	-8.5		
13.90	CH 45 CL	1.900	45				1.09	0.81			
14.614	60 6L	2.614	6L			33.3		1.17			
14.584	60 5L	2.584	5L	1.20	7.3	17.6		1.14			
14.30	CH 60 CL	2.300	60				1.45	0.85			
14.129	60 5R	2.129	5R					0.68	-29.2	5.2	0.68
13.884	60 10R	1.884	10R					0.50	-20.4		
13.711	60 15R	1.711	15R					0.32	-28.9		
14.50	CH 75 CL	2.500	75				1.81	0.69			
15.177	90 9.5L	3.117	9.5L			14.1		1.05			
14.858	90 5L	2.858	5L	0.73	5.4	9.0		0.73			
14.30	CH 90 CL	2.300	90				2.17	0.13			
14.153	90 1.8R	2.153	1.8R					-0.02	-12.2	2.5	0
14.25	CH 93.3	2.250	93.3				2.25	0.00			
Grade/Slope = 1:41.5		Grade slope = run/rise = 93.3/2.25 = 41.5									

The plotted longitudinal profile of reduced data in Table 3-1 is shown in Figure 3-1.

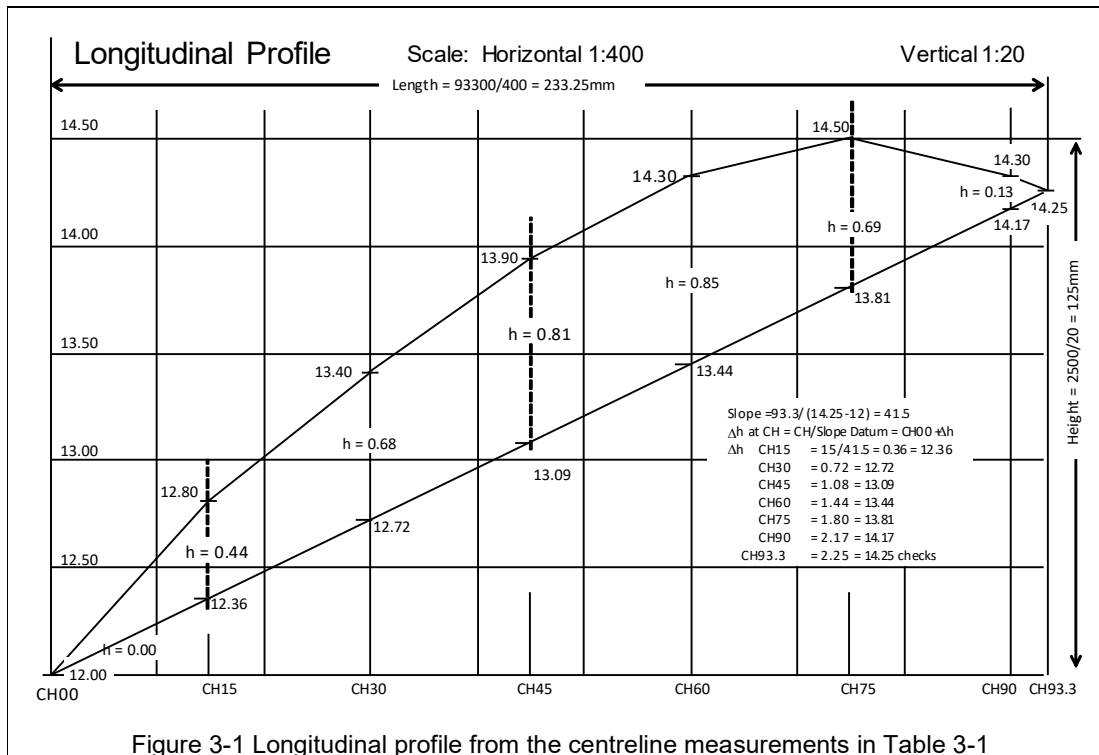


Figure 3-1 Longitudinal profile from the centreline measurements in Table 3-1

7. Embankment and Cutting

- Embankment – The formation level of the proposed works is higher than the actual ground level such that the side slope (batters) slope downwards from the formation level to the ground (see e.g., Figure 3-2).
- Cutting is the opposite, i.e., when the formation level is lower than the actual ground (see e.g., Figure 3-3).

8. Design gradient, batter gradient, and surface gradient

Design gradient is the slope of the design feature, e.g., a road (rising or falling as discussed in Chapter 9). Batter gradients are the side slopes in Table 3-1 Reduced levels for a road design. above, while surface slope (k) is the slope of the actual ground surface (see §3.4).

9. Chainage – This is the linear measurement from the start of the structure, e.g., road along the centreline at given intervals. They are normally specified at the design stage and useful in specifying points at which reduced levels should be taken for the generation of longitudinal and cross-sectional profiles.

10. Digital Elevation and Digital Terrain Models

- DEM (Digital Elevation Model) is the representation of terrain surface numerically by very dense network of points of known X, Y, Z coordinates. It includes natural terrain features, vegetation, roads, buildings etc. Triangulation Irregular Network (TIN) are favoured for DEM modelling. Delaunay triangulations are used to create the best conditioned triangular network. DEM input required to generate surfaces for engineering design can be ordered (grids), semi-ordered (profiles or contours) or random pointing.
- DTM (Digital Terrain Model) is a filtered version of DEM and contains the “bare Earth”.

Uses of DEM and DTM

- DEM can be used for:
 - ✓ Landscaping
 - ✓ City modelling
 - ✓ Visualization applications.
- DTM can be used for
 - ✓ Flood modelling (from contour lines)
 - ✓ Drainage modelling (from contours and cross sections)
 - ✓ Land use studies
 - ✓ Slope models or volume calculations (from cross sections).

3.3 Application: Road Design

Design of new roads will generally fall into either (i) roads following existing routes, e.g., those involved with either upgrading of an existing road or a short deviation or (ii) roads following new routes, e.g., those roads linking two points for the first time. For road design along existing routes, a *cross section* survey of the existing road is generally carried out as a first step in the design. This survey includes, e.g., road cross sections every 25m along the formation where the measured locations will include:

- ✓ Centre line, edge of bitumen, edge of formation, bottom of drain, top of drain, any other changes of grade, fence, etc.
- ✓ Road furniture (to all fixtures in the road and road reserve).
- ✓ Improvements such as buildings and bridges.

In the example of Australia, a road design for an existing route will generally be undertaken in 3 stages as follows:

1. **Stage 1** (Horizontal Control, see Chapter 5) - Control points are placed approximately every 250 m in the existing bitumen road and connected to Australian Map Grid (AMG) via adjacent Standard Surveying Marks (SSM's). The requirements are generally traversing with an accuracy of 1:10,000 to 1:20,000 or better (Chapter 5).
2. **Stage 2** (Vertical Control, see Chapter 1) - Control points are connected to the Australian Height Datum (AHD) via adjacent Bench Marks. The requirements are generally a level traverse with an accuracy of $12\text{mm}/\sqrt{k}$, where k is the traverse distance in kilometres.
3. **Stage 3** (Detail Survey) - A cross sectional survey is carried out every 20 m along the alignment of the road. From the control points established in stages 1 and 2 above, an EDM radiation survey of not more than 125 m is carried out. Distance and angular measurements are undertaken together with the heights of the instrument and target. From these measurements, a topographical map showing features and contours/cross sections can be generated. These types of surveys could be carried out using GPS surveying (Chapter 14). For Australia, the main roads department (MRD; www.mainroads.wa.gov.au) has formulated a set of procedures covering a cross section survey of this nature.

3.4 Calculation of Road Cross Sections in Cut and Fill Operations

The calculation of road cross section area is presented as a series of materials developed for the Curtin University Civil Engineering Surveying workshops (see Appendix A1-2) to try to distil and simplify material presented in standard textbooks. A simplified profile of an embankment, in either cut or fill, is used as an illustration. See Figure 3-2.

The basic components of embankment design are:

1. Pavement, design width. This is given the label ***b***. It is symmetrical about the centre line.
2. Design finish level of the formation centre line, the datum.
3. Surface level at formation centre line.
4. The difference between centreline design finish level and surface level, either the depth of cut or fill, ***h***.
5. The formation **batter** slope, the constructed gradient of the cut or fill operation, labelled ***m***. The slope is expressed as a **rise** and **run** ratio of **1:*m***, e.g. 1:3 means that for every vertical change in height there will be a 3-fold increase in horizontal deviation. Normally the batter slope, ***m***, is designed to mimic the natural repose of the formation construction material. In road construction using natural materials the slope may be different for cut operations as opposed to fill.
6. The **cross fall** of the natural surface from the design centre line is designated ***k***. It goes outwards beyond the batter slope intercepts. For calculations, ***k*** has a sign such that **+*k*** represents a **rise** from the centreline outwards while **-*k*** represents a **fall** from the centreline outwards. If there is **no** cross fall, then the slope is **flat**, and ***k* = 0:1** or ***k* → ∞** (***k* = 10,000** will suffice in general calculations).
7. Formation width, the distance from the centreline at which the batter slope intercepts the surface is designated ***w*** and expressed as a distance left, ***W_L***, and right, ***W_R***, of the centreline. The formation width depends on ***h***, ***b/2***, ***m*** and ***k***.

The calculations are expressed as distances either side, **left** or **right** of the centre line. The designation of left and right is taken in the sense of observing in the direction of the centreline progress, i.e., of increasing chainage. Figure 3-2 shows the definitions given above for a section in fill. Note that the cross-section distances are measured horizontally away from the centreline. The vertical distances are labelled, in this case, from some arbitrary vertical datum starting at 10.0. The finish level is at 10.3 and the centre line surface level is about 10.08. Thus ***h* = 0.22**.

Cross section diagrams are often shown with a **vertical exaggeration** which helps to illustrate the formation section. In Figure 3-2 a vertical exaggeration of 10:1 is evident. Vertical exaggeration at a nominated scale also allows one to plot the road section, showing natural surface and the pavement width at design level. By plotting the formation batters (at the exaggerated scale) it is possible to scale off the formation width intercept with the natural surface.

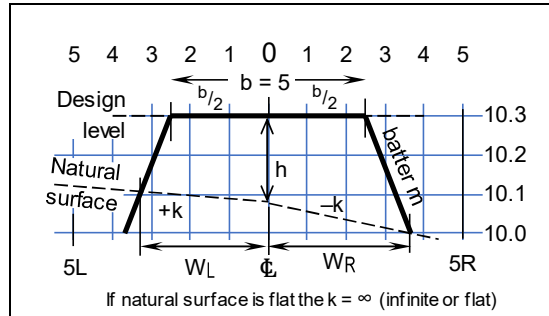


Figure 3-2 Definitions of a sectional fill.

3.4.1 Calculation of Cut Formation Width W_L , and, W_R , Flat Ground

The development of the calculation of formation width will start with a cut embankment referenced to a flat natural surface (Figure 3-3).

The design components are:

pavement width, $b = 5$

batter slope, $m = 3$,

depth (cut/ fill), $h = 0.25$,

cross slope is **flat**,

$k = \infty = 10,000$. It is ignored here.

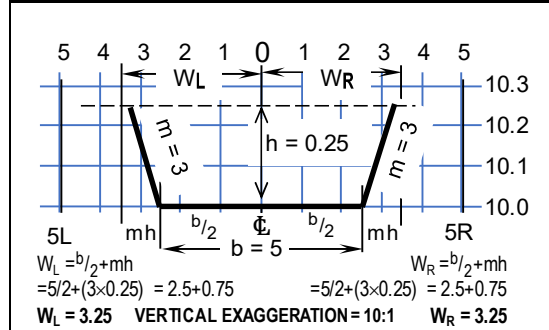


Figure 3-3 Formation width for a flat surface.

Flat formation surface:	$W_L / W_R = \frac{b}{2} + mh$	Eqn 3-1

3.4.2 Calculation of Formation Width, W_L , and, W_R , Incorporating Cross Fall. Sloping Ground

Figure 3-4 introduces a constant surface slope, k , that accounts for determining formation width W_L and W_R . As seen from the exaggerated diagram, compared with a level surface, the batter width increases as we “chase” an upward (k positive) slope to intercept the ground. Likewise, the batter width will decrease against falling (k negative) slope.

3.4.2.1 Formation width with constant cross fall

In the diagram, Figure 3-4:	$A_1B = \frac{W_L}{k}$	Eqn 3-2

The difference in height between the ground at the centreline and the formation batter intercept is due to the slope of $1:k$ over the distance w_1 .

Similarly:	$C_1B = \frac{W_R}{k}$	Eqn 3-3

The extended batter, of slope m , over a horizontal base $b/2$ intercepts the centreline at G with a height:

	$GE = \frac{b}{2m}$	Eqn 3-4

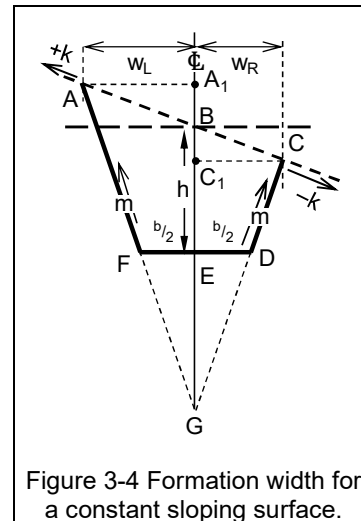


Figure 3-4 Formation width for a constant sloping surface.

Triangles AA₁G and FEG are similar. Using similar triangles:

$\frac{AA_1}{FE} = \frac{GA_1}{GE}$ and substituting:	$\frac{W_L}{\frac{b}{2}} = \left\{ \frac{\frac{b}{2m} + h + \frac{W_L}{k}}{\frac{b}{2m}} \right\} \Leftrightarrow w_1 = m \left[\frac{b}{2m} + h + \frac{W_L}{k} \right]$ $W_L \left(1 - \frac{m}{k} \right) = \frac{b}{2} + mh$ $W_L = \left(\frac{b}{2} + mh \right) \left(\frac{k}{k-m} \right)$	Eqn 3-5

As the denominator is less than the numerator, then $\left(\frac{k}{k-m} \right) > 1$. The batter “chases” the slope.

Note that the slope of **k** has been taken as positive. If the slope is falling from the centre line (and is kept positive) then, by inspection, $w < \left(\frac{b}{2} + mh \right)$.

Thus, **w**₂ must be expressed in terms of triangles CC₁G and DEG.

$\frac{CC_1}{DE} = \frac{GC_1}{GE}$ and substituting:	$\frac{W_R}{\frac{b}{2}} = \left\{ \frac{\frac{b}{2m} + h - \frac{W_R}{k}}{\frac{b}{2m}} \right\} \Leftrightarrow w_2 = m \left[\frac{b}{2m} + h - \frac{W_R}{k} \right]$ $W_R \left(1 + \frac{m}{k} \right) = \frac{b}{2} + mh$ $W_R = \left(\frac{b}{2} + mh \right) \left(\frac{k}{k+m} \right)$	Eqn 3-6

As the denominator is greater than the numerator, then $\left(\frac{k}{k+m} \right) < 1$. The batter “retreats”.

This then means applying one of the two rules, determined by inspection of **k**, for a rising or a falling slope from the centre line.

3.4.2.2 Unified formula using sign of **k**

However, if we apply the **sign** of the slope to **k** then we can express the left and right formation widths with one formula (keeping **m** positive):

From Eqn 3-5	$W_{L/R} = \left(\frac{b}{2} + mh \right) \left(\frac{k}{k-m} \right) \text{ and, if } k \text{ is negative then}$ $W_{L/R} = \left(\frac{b}{2} + mh \right) \left(\frac{-k}{-k-m} \right) \Leftrightarrow \left(\frac{b}{2} + mh \right) \left(\frac{k}{k+m} \right)$ $W_{L/R} = \left(\frac{b}{2} + mh \right) \left(\frac{k}{k-m} \right)$	Eqn 3-7

This returns to the original statement for W_R , as stated above, where the slope is falling from the centreline. It is best to have only **one** formula to work with and use the **sign** of k to control the denominator. $W_{L/R} = \left(\frac{b}{2} + mh \right) \left(\frac{k}{k-m} \right)$

3.4.3 Calculation of Formation Width, W_L , and, W_R , Constant Cross Fall Ground

The development of the calculation of formation width now moves on to calculating formation width against a constantly sloping natural surface (Figure 3-5).

In Figure 3-5 the cross slope has been calculated from a level profile measured in the field:

10.5 RL at 5m Left
10.25 RL centreline, Φ
10.0 RL at 5m Right

The slope k is calculated using run/rise **from** the centreline:

$$k_L = 5 / (10.5 - 10.25) \Phi - 5L \\ = 5 / 0.25 = 20,$$

- a **rise** away from the centreline:

$$k_R = 5 / (10.0 - 10.25) \Phi - 5R \\ = 5 / -0.25 = -20,$$

- a **fall** away from the centreline.

The components of the design are:

pavement width, $b = 5$

batter slope, $m = 3$

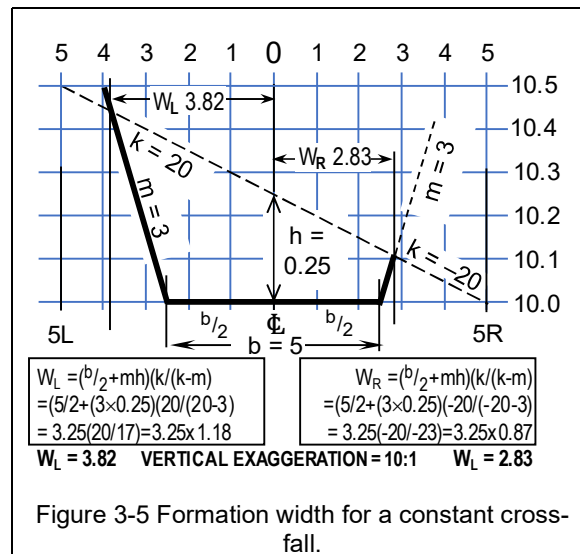
depth (cut or fill), $h = 0.25$

cross slope is **1:20**, $k_L = +20$, $k_R = -20$.

Applying Eqn 3-7:

$$W_{L/R} = \left(\frac{b}{2} + mh \right) \left(\frac{k}{k-m} \right) \text{ results in } W_L = 3.82 \text{ and } W_R = 2.83.$$

The calculation of W_L and W_R are also shown in the Figure 3-6 and Figure 3-7, demonstrating the operation of the **sign** of k in the results.



3.4.4 Calculation of Formation Width, W_L , and W_R , Varying Cross Fall Ground

The development of the calculation of formation width then moves on to calculating formation width against a varying sloping natural surface (Figure 3-6).

In Figure 3-6 the cross slope has been calculated from a level profile measured in the field:

10.35 RL at 5m Left
10.25 RL centreline
10.05 RL at 5m Right

Slope k is calculated using run/rise **from** the centreline:

$$k_L = 5 / (10.35 - 10.25) \\ = 5 / 0.1 = 50,$$

- a **rise** away from the centreline:

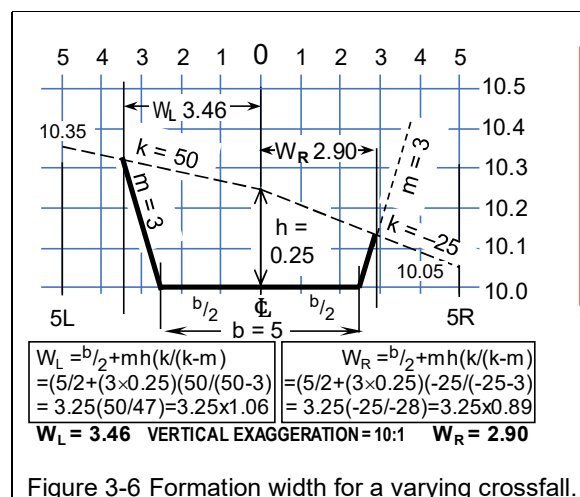
$$k_R = 5 / (10.05 - 10.25) \\ = 5 / -0.2 = -25,$$

- a **fall** away from centreline.

The components of the design are:

pavement width, $b = 5$

batter slope, $m = 3$



depth (cut or fill), $h = 0.25$

cross slope varies, $k_L = +50$, $k_R = -25$.

Applying Eqn 3-7 results in $W_L = 3.46$ and $W_R = 2.90$.

Another case to consider is where the depth, h , is such that the batter intercept falls outside the first cross section.

3.4.5 Batter Width Falls Outside the First Cross Section

Calculations with varying different cross falls. A big grade change at 5L.

Using $k=50$ in Figure 3-7, the batter intercept, $W_L = 5.76\text{m}$, is outside the 5L section. The intercept is between 5L and 10L. As the field notes reveal, RLs at:

10L = 11.3.

5L = 10.8

CL = 10.7

Datum = 10.0

The resultant slopes, from CL to 5L (k_{L1}) and 5L to 10L (k_{L2}) are:

$$k_{L1}(\text{CL} - 5\text{L}) = 5/(10.8 - 10.7) = +50$$

$$k_{L2}(5\text{L} - 10\text{L}) = 5/(11.3 - 10.8) = +10.$$

What is the height of the batter slope above the datum at profile offset 5L?

Find the height at which a batter of $m = 4$ intercepts the 5L cross section. Here $b = 5$, so $b/2 = 2.5$.

From the edge of the formation the batter slope rises from 2.5L at slope of

$m = 4$ until it intercepts 5L, at distance of 2.5m. See Figure 3-8.

As $w = mh$ then

$$h = w/m$$

The height of batter intercept at 5L:

$$h_{wL=5} = (5 - 2.5)/4 = 0.625 \text{ at } 5\text{L}.$$

Turning to the design:

h_{5L} is the surface height at 5L above the design datum.

From field book and design level

$$h_{5L} = (10.8 - 10.0) = 0.8$$

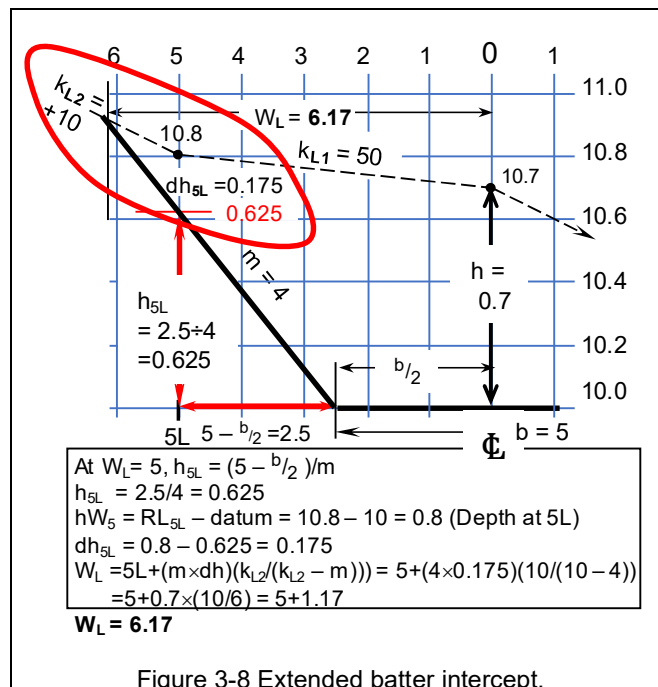
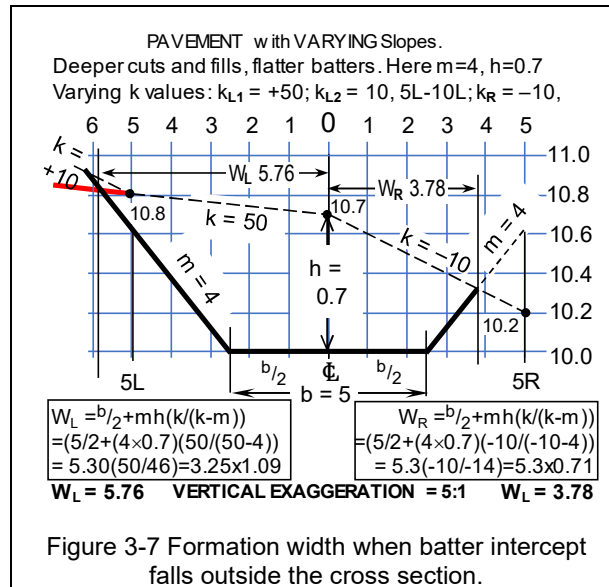
The depth of the batter intercept at 5L, called dh_{5L} , between the 5L surface and the 5L batter intercept is:

$$dh_{5L} = 0.8 - 0.625 = 0.175$$

below the surface at 5L.

What is the formation width increment, dw_L , from 5L, where the batter is still below the slope, to the rising

$k_{L2} = +10$ slope surface intercept? Reiterating: $dh_{5L} = 0.175$, $k_{L2} = +10$, $m = 4$.



dw_L is then calculated using the formula:

$$\begin{aligned} dw_L &= (m \times dh_{5L})(k_L / (k_L - m)) = (4 \times 0.175)(10 / (10 - 4)) \\ dw_L &= 1.17. \end{aligned}$$

Total $W_L = W_{L=5} + dw_L = 5 + 1.17 = 6.17$ as shown in Figure 3-7 and Figure 3-8.

3.4.6 Derivation of AREA Formula using Formation Widths

Having calculated the formation widths, W_L and W_R , it is a simple matter to calculate the AREA of the section.

A **CONSTANT** slope is assumed from the batter intercept to the centreline. Individual slopes, k , can vary as they only affect the formation widths W_L and W_R .

In practical terms **assuming** a constant slope makes little difference to the calculated area compared with methods that more closely follow the surface profile, which rely on using a matrix cross multiplication method. In Figure 3-9;

$\text{Area } ABCDFA = \text{Area } ABG + \text{Area } CBG - \text{Area } DFG$

$$\begin{aligned} A &= \frac{1}{2} W_1 (h + EG) + \frac{1}{2} W_2 (h + EG) - \frac{1}{2} FD \cdot EG \\ EG &= \frac{b}{2m}, FD = b \end{aligned}$$

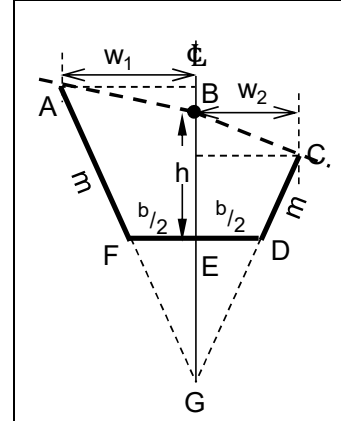


Figure 3-9 Area computations.

$$\begin{aligned} A &= \left\{ \frac{1}{2} W_1 \left(\frac{b}{2m} + h \right) + \frac{1}{2} W_2 \left(\frac{b}{2m} + h \right) \right\} - \frac{1}{2} b \left(\frac{b}{2m} \right). \text{ Factor out } \frac{1}{2} \\ A &= \frac{1}{2} \left(\left(\frac{b}{2m} + h \right) (W_1 + W_2) - \left(\frac{b^2}{2m} \right) \right). \text{ Factor out } \frac{1}{m} \\ \text{Area} &= \frac{1}{2m} \left(\left(\frac{b}{2} + mh \right) (W_1 + W_2) - \left(\frac{b^2}{2} \right) \right) \end{aligned}$$

Eqn 3-8

Note the process in Eqn 3-8:

You have to calculate $\left(\frac{b}{2} + mh \right) (w_1 + w_2)$, subtract $\frac{b^2}{2}$ and then divide the result by $\frac{1}{2m}$.

Using the values in Figure 3.6:

$$\text{Area} = \frac{1}{2 \times 3} \left\{ \left(\frac{5}{2} + 3 \times 0.25 \right) (3.46 + 2.9) - \frac{5^2}{2} \right\} = \frac{1}{6} \{ (3.25)(6.36) - 12.5 \} = 1.36$$

3.4.7 Area by Coordinates

Referring to Figure 3-10 we can see that the section provides an XY coordinate set if we can determine the h_L and h_R above the datum. This drawing replicates Figure 3-6.

Having determined W_L and W_R , we can then calculate the height of the batter intercept with the ground slope above the datum level.

Basically $w = mh$
($b/2$ merely allows for the formation width;
 k introduces the effect of slope.)

Now $w = b/2 + mh$,
thus $w - b/2 = mh$, and
 $h = (w - b/2)/m$.

By following around Figure 3-10 the XY coordinates of the cross section can be tabulated:

Remember, centreline is at $W = 0$.

Coordinates are **negative** to the left of centreline and positive above the zero-height datum.

The area of a closed, non-intersecting polygon can be found from:

$Area = \frac{(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_4 - x_4y_3) \dots (x_ny_{n+1} - x_{n+1}y_n)}{2}$	Eqn 3-9

{w, h} represents the {X, Y} coordinates above the datum in Figure 3-10.

The coordinates of an n vertex polygon are best written as a $2 \times (n+1)$ matrix where $(n+1)=1$, the start vertex coordinate. The cross multiplication of the matrix represents **twice** the area of the polygon.

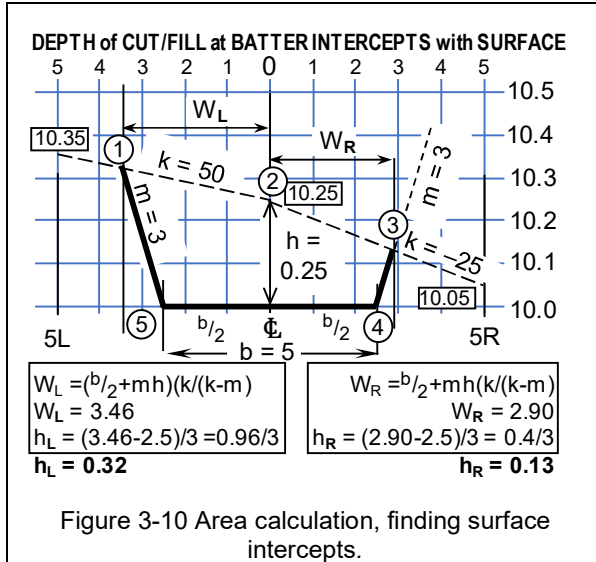
And remember to **close** back to the start point.

n	Point	w	h	above datum
1	1	-3.46	0.32	w_L, h_L
2	2	0.0	0.25	CL, h
3	3	2.9	0.13	w_R, h_R
4	4	2.5	0.0	$b/2_R$, datum
5	5	-2.5	0.0	$b/2_L$, datum
6	1	-3.46	0.32	w_L, h_L (return to 1)

Matrix cross multiplication method

All the zero elements drop out.

$\begin{array}{cc} -3.46 & 0.32 \\ 0.0 & 0.25 \\ 2.9 & 0.13 \\ 2.5 & 0.0 \\ -2.5 & 0.0 \\ -3.46 & 0.32 \end{array}$	$\Sigma +ve$	$\Sigma -ve$	$2 \text{ Area} =$ $(-3.46 \cdot 0.25 + -2.5 \cdot 0.32)$ $-(2.9 \cdot 0.25 + 2.5 \cdot 0.13).$ $\Sigma -ve = -1.66 - 1.05 = -2.71$ taking absolute value: Area = $\frac{1}{2} 2.71 m^2 = 1.36$



3.5 Calculation of Embankment Volume from Road Cross Sections in Cut and Fill Operations

After calculating the various cross section areas there are a number of ways to find the volume of the earthwork.

a) **Calculation of volume from mean areas.**

$$\text{Volume} = V = \left(\frac{A_1 + A_2 + A_3 \dots A_{n-1} + A_n}{n} \right) \times L. \text{ The method is not recommended.}$$

b) **Calculation of volume from end areas. This is the recommended method for the practical exercise in Appendix A2.**

$Volume = \Sigma V = D_{12} \left(\frac{A_1 + A_2}{2} \right) + D_{23} \left(\frac{A_2 + A_3}{2} \right) + D_{34} \left(\frac{A_3 + A_4}{2} \right) + \dots$	Eqn 3-10

And, where D is the common distance **between** sections:

$Volume = \Sigma V = \frac{D}{2} (A_1 + A_n + 2A_2 + 2A_3 + \dots + 2A_{n-1})$	Eqn 3-11

c) **Calculation of volume from Simpsons Rule. You need an ODD number of sections.**

$Volume = \Sigma V = \frac{D}{3} (A_1 + A_n + 4A_2 + 2A_3 + 4A_4 + \dots + 4A_{n-1})$	Eqn 3-12

It is a variation of the prismoidal formula.

n must be an **odd** number of sections.

D is the common distance **between** sections.

Main Roads WA require volumes by the prismoidal formula to be used but accept end area volumes for Mass Haul Diagrams. https://www.mainroads.wa.gov.au/Documents/67-08-90_Earthworks%20Volume%20Calculations%20Standard.RCN-D12%5E23434852.DOCX

3.6 Plan Scale, Horizontal and Vertical

For plan drawings to be constructed on paper the **scale** is defined as the representation of the relationship between a unit of measure on the paper and the unit of measure on the ground.

It is a **representative** fraction between the paper units and the ground units.

a scale of 1:1000 means that 1mm on paper = 1000mm (1m) on the ground.

For example, the mining practical exercise area at Curtin University (Australia) is about 120m × 110m wide. The gridded area is about 80m × 60m wide.

The Civil Engineering grid exercise area is 100m × 40m.

An A4 paper sheet is 297 mm × 210 mm from which about 10 mm margin must be excluded. Thus, the drawing area is about 275 mm × 190 mm. You must also make allowance for the title blocks, revision panels, scale bars etc.

In reality the useful plotting area is about 250mm × 190mm

At a scale of 1:400 this represents an area of 100m × 76m;

at a scale of 1:500 this represents an area of 125m × 95m

To be presented on an A4 sheet of paper at a scale of 1:500 the plan size will be:

1 m = 1000 mm/500 = 2.0 mm,

a tree of diameter 600mm = 1.2mm (can you draw a 0.6mm radius?)

0.1 m = 100 mm/500 = 0.2 mm. This is below the limit of hand plotting.

The engineering plan can thus be presented at 1:400, but the mining site plan would have to be trimmed in width (the E/W extent) to fit at 1:500. An A3 sheet, inserted in landscape orientation and Z folded to A4, provides scope for larger areas.

A triangular scale rule, Staedtler MARS 561 98-4 or similar, with scales of 1:100, :200, :250, :300, :400 and 1:500 is recommended for hand plotting.

3.7 Plan Sketching and Drawing

The ability to present hand drawn plans is of great importance in the presentation of initial observations. A neat, accurate field sketch, produced without resort to CAD programs, enhances the understanding of field data by its immediacy.

The tools of trade are: 1) a 360° protractor; 2) a well graduated ruler; 3) a good compass for arcs and circles; 4) 30° and 45° triangles; 5) pencils and eraser. Good blank paper is essential, and accurately printed graph paper is handy. (Photocopies of graph paper result in grid shrinkage; thus, the scale plots incorrectly.)

The field sketch drawings in a field book don't have to be any particular scale, but the ratio between measured distances should be maintained so that the drawing is conformal. If you make a scale of 80mm = 105m then 75m is about 60mm (57). This example (105/0.08) has a scale of 1:1312 which can be stored in a calculator memory for use during the sketch.

3.7.1 Plotting radiations

Choose a scale that allows your plan to fit on an A4 plot area of about 250mm × 190mm. The exercises generally specify a suitable scale. Any control points should be plotted, this can provide orientation for the radiation point.

Use the protractor aligned with the control, or a grid, to provide a North. Align the directions accurately. You can then read angles which will help with the reduction of observations, especially of directions either side of 360°. For a radiation (side shot) drawing, the protractor can be fixed over the observation point, directions radiated and then, the protractor removed, the distances to points scaled in with the ruler.

3.7.2 Vertical Exaggeration

You can only use vertical exaggeration in profile (elevation) drawings. Any **plan** drawings must use the same scale in both the X and Y axes.

It can be seen from the previous discussion on embankments that, compared with the width of an embankment, the depth can be small.

At a scale as large as 1:200 a full batter width of 6.8 m = 6800/200 = 34 mm.

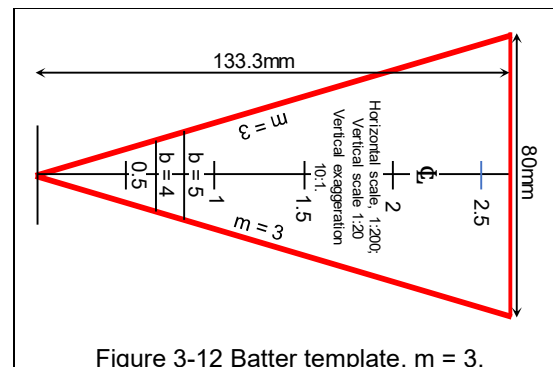
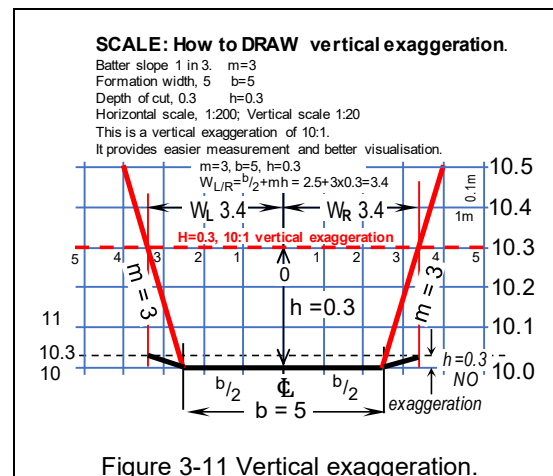
However, the corresponding depth of 0.3 m at the same scale = 300/200 = 1.5 mm.

This is extremely difficult to plot, and almost impossible to measure to extract cross section data.

By having a vertical exaggeration of, say, 10:1 then the vertical scale = 1:20. Thus the depth of 0.3 m = 300/20 = 15 mm. This is a much easier profile to draw. Figure 3-11.

A handy technique is to cut a thin acetate sheet (from a clear binder cover) at the appropriate vertically exaggerated slope. Something like Figure 3-12. It is marked at $h=0.5$ increments and $b=4$, $b=5$ widths.

An $m=4$ template is sized 100mm wide × 125mm high.



3.8 Calculation of Embankment Volume from Batter Slopes: Mining

3.8.1 Batters, Ramps and Benches

As an introduction to mine and civil earthworks development you should have studied engineering mechanics, geology, and mechanics of solids. Mining geomechanics and mine geotechnical engineering come later in your course. This introduction to the surveying side of batter slopes will give some mathematical tools for batter design. Correctly calculated and constructed batter slopes are an integral part of slope stability. As an engineer and a manager, you will be responsible for the design and implementation of mine walls in both cut and fill operations. As such, the calculation of slopes, areas and volumes follows the formulas used in road design.

In open pit design (Figure 3-13), the main question to be answered is the origin of design boundary.

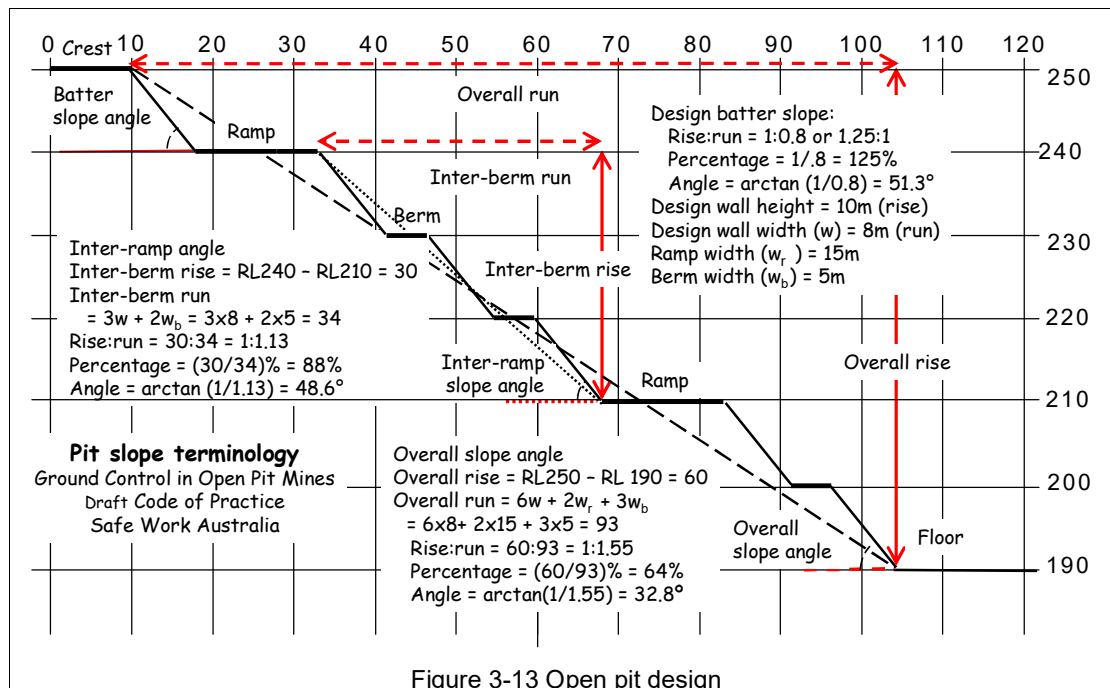


Figure 3-13 Open pit design

Does the design start at the ground level crest boundary?

If so, the distance to the toe of the batter will depend on the depth to a bench, and the design batter slope either in slope, 1:m, percentage or angle values.

In Figure 3-13, the batter slope angle, the resulting inter-ramp slope angle, and the final overall slope angle are quite different.

Berm width is concerned with pit wall safety and is designed to contain hazardous rock falls, prevent material spillage on to lower levels, provide access to the slope wall and access to surveying and stability monitoring points. Once bench blasting has commenced, the berm may be incorporated every 10 to 20 metres, depending on material stability. Berm heights of 30m may be acceptable in massive stable formations.

Ramp width is determined by traffic requirements. Haul trucks, excavators and large machinery must have access to the mine works. The ramp width would also incorporate some of the berm functions. The ramp inclination and routing is a specialist design area.

The width of the road will depend on the width of the largest vehicle to use the road. Is it single or dual lane, straight or curved? A curved, dual lane road for a mid-sized off highway truck would incorporate an allowance for vehicle overhang and a width expansion factor to the

truck width. Standard practice is to then add an allowance for windrows on either side to allow road maintenance as well.

So, a mid-sized off-highway truck, 100t capacity is likely to be 6.9m wide. A 20% width factor and dual road spacing will require nearly 30m road width, plus 10m of windrow. The largest, +300t payload, trucks are about 10m wide and would require a total width of over 50m.

Maximum grade in-pit is about 10%, has a slope of 1:10 which is about 5.7° .

3.8.2 Calculate Cross Section Areas for Volume: Mining

The Field Practical survey area is of restricted size. However, it allows the collection of data, which can demonstrate the principles of computations used in engineering and mining.

In Field Practical 2, ground levels are determined by using the grid intersections representing a drill and blast field. Typically blasting is carried out on a 6m by 7m grid (personal communications with AusIMM). The data collected can form cross sections, the datum of which is specified by a bench level.

The **crest boundary** set for the excavation is the 350E grid line and the 7500N grid line (Figure 3-14).

From field booking, cross sections for the nominated sections can be generated.

Figure 3-14 shows alternate cross sections and the inferred toe of the batter for a slope of 1:1.25 (39°) from the crest to the bench level of 15.5m.

The task is to calculate the volume bounded by the surface, the batter slope and the bench.

Remember, the blast grid is on a $6\text{m} \times 7\text{m}$ pattern.

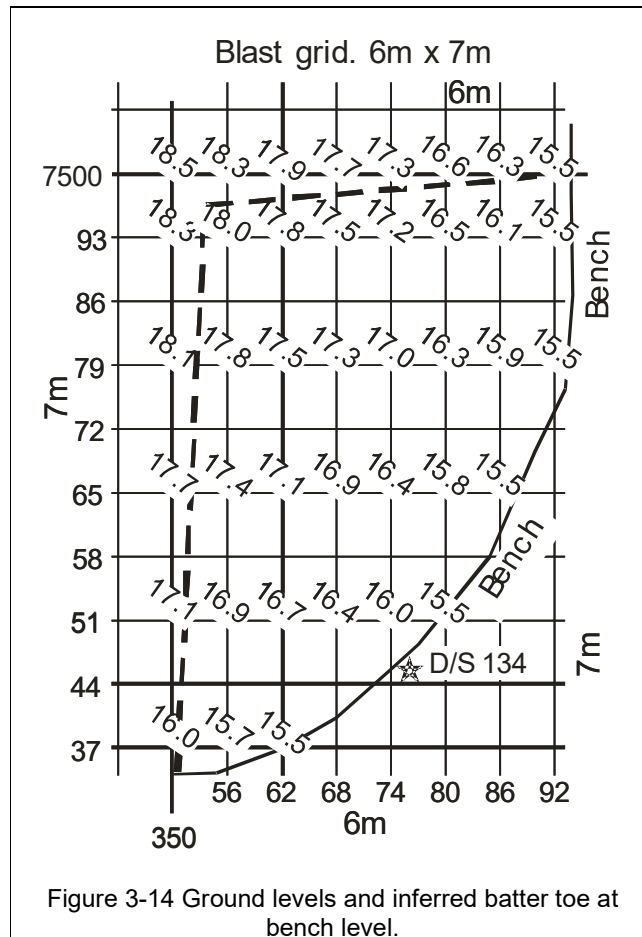


Figure 3-14 Ground levels and inferred batter toe at bench level.

3.8.2.1 Generate Cross Sections

The crest boundary set at 350E.

The RL is 18.3 m

The height to bench is 2.8m, the toe of the batter is located at 353.5E. Ground RLs provide the Y ordinate and the grid E value provides the $\times$ ordinate.

Note the vertical exaggeration and off-set datum for height. A horizontal scale of 1:250 will make the $\times$ axis (45 m) a maximum width of 180 mm at 7493N.

A vertical scale of 1:50 and a height of 3m will require a vertical axis of 60 mm.

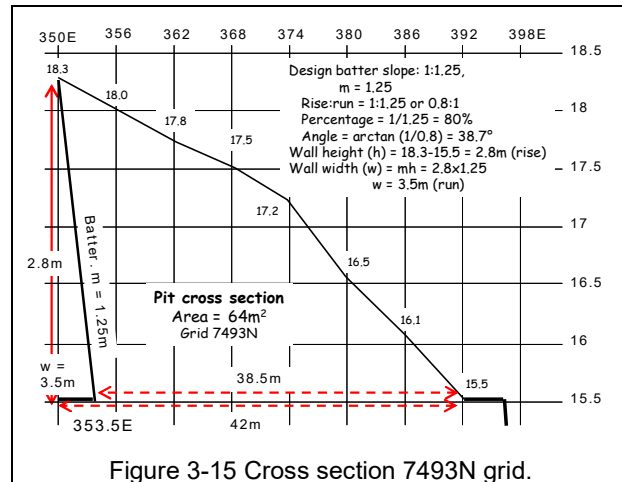


Figure 3-15 Cross section 7493N grid.

3.8.2.2 Area by matrix cross multiplication

Using the matrix cross multiplication method in §3.4.7, the area of the section 7493N is calculated as 64 m². Instead of using “raw” coordinates for the matrix, it may be easier to use translated offsets, i.e.,

350, 18.3 $\leftrightarrow$ 0, 2.8

356, 18.0 $\leftrightarrow$ 6, 2.5.

362, 17.8 $\leftrightarrow$ 12, 2.3

etc. (see dE and h in Table 3-2), just a datum shift.

Table 3-2 shows the process of producing the same result for both actual and shifted coordinates.

Again, note the vertical exaggeration and offset datum for height. A horizontal scale of 1:250 will make your $\times$ axis (35 m) a maximum width of 140 mm at 7451N.

A vertical scale of 1:50 and a height of 2 m will require a vertical axis of 40 mm.

Similarly, at section 7451N, the observed profile provides a section area of 27 m².

Taking the 7451N section (Figure 3-16) and the crest boundary set at 350E the RL is 17.1 m.

The height to bench is 1.6 m, the toe of the batter is located at 352E. Ground RLs provide the Y ordinate and the grid E value provides the $\times$ ordinate.

Taking the E/W section lines, the following cross section areas have been calculated: using the mean end area, the volume between each pair of grid lines is calculated and summed in Table 3-3. Remember, the grid spacing is 7m proceeding south from 7500N, the table has been abbreviated by using every second grid line below 7493N. Cross grid levels are taken every 6m from 350E eastwards.

Refer to §3.5 and Eqn 3-10 and Eqn 3-11 for methods of volume calculation.

E	RL	+ve	-ve	dE	h	+ve	-ve
350	18.3		6515	0	2.8		17
356	18.0	6300	6516	6	2.5	0	30
362	17.8	6337	6550	12	2.3	14	41
368	17.5	6335	6545	18	2.0	24	48
374	17.2	6330	6536	24	1.7	30	51
380	16.5	6171	6369	30	1.0	24	36
386	16.1	6118	6311	36	0.6	18	25
392	15.5	5983	5479	42	0.0	0	0
353.5	15.5	6076	5425	3.5	0.0	0	0
350	18.3	6469		0	2.8	10	
Sums		56119	56246			120	248
Half difference = 64						Half difference = 64	

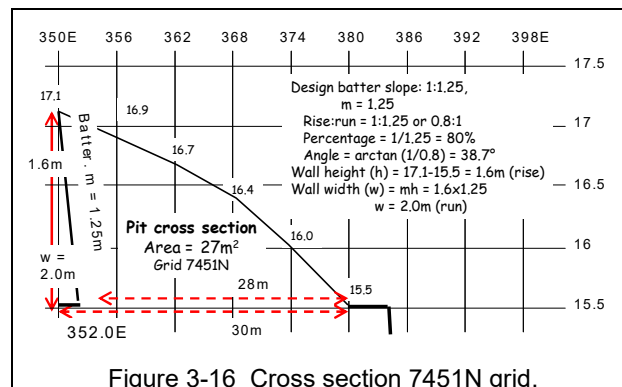


Figure 3-16 Cross section 7451N grid.

3.8.2.3 Batter slope at 7500N

The volume calculated so far allows for the batter on the 350E crest line. There is also the batter slope projecting into the excavation from the 7500N section (Figure 3-17). We now make allowances for the volume of material left by the batter associated with the excavation from the bench to the crest of the 7500N grid line.

What about the batter to the 7500N crest line?

How can we provide for the volume of material left behind the batter to the bench?

One way would be to calculate the area of each triangle at each cross section between 350E and 398E, where the bench is exposed.

Your field book calculations will provide the difference in height, h , at each grid E. Since we know the batter slope, m , and, using the stockpile example, the triangle area is $\frac{1}{2}h^2m$. Calculated values for the grid areas and the volume are shown on Figure 3-18.

The volume of the batter is about 99 m^3 , shown in Figure 3-18. Subtracting this value from Table 3-3 results in an excavation volume:

$$- 2661 - 99 = 2562 \text{ m}^3.$$

Note that this value is the in-situ volume. The material to be excavated will be subject to a bulking, or “swell”, factor for transport. The batter in our example is predicated on a stable rock material. Common values of bulking factor for “rock” are between 40% and 80%. Thus the 2600 m^3 could bulk out to some $4,000 \text{ m}^3$, depending on your blast effort.

Figure 3-17 is a diagram of the batter associated with the 7500N crest.

Grid	Area	Grid Spacing	Vol
7500	70		
		7	470
7493	64		
		14	840
7479	56		
		14	672
7465	40		
		14	469
7451	27		
		14	210
7437	3		
	Sum volumes		2661

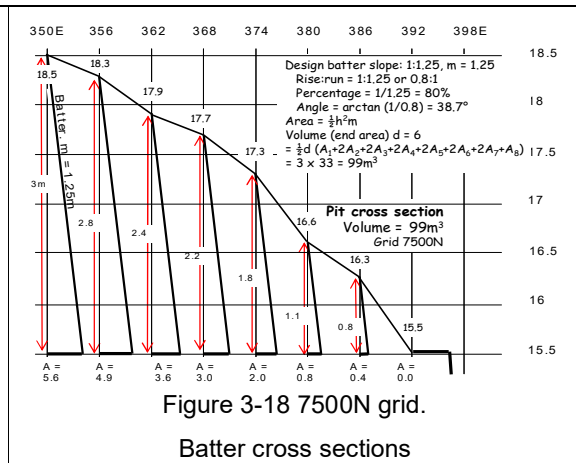
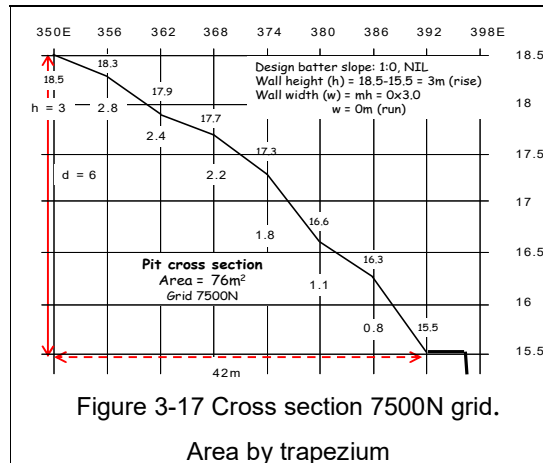


Figure 3-18 shows the batters from the floor of the excavation to the crest of the 7500N grid line. The area of each triangle is shown on the X axis.

The preceding calculations show that excavation volume calculations are not that hard. The example is not designed to replace the methods presented in other formal mining engineering units.

3.8.3 Cross Section Area by Trapezium

An alternate approach to calculating area is to obtain the area of each **trapezium** formed between the adjacent grid lines.

At the 7493N cross section, Figure 3-15, it can be seen that each pair of E grids form a trapezium.

$$h_{350} = 18.3 - 15.5 = 2.8, h_{356} = 18.0 - 15.5 = 2.5; h_{362} = 2.3, h_{368} = 2.0, h_{374} = 1.7, h_{380} = 1.0, h_{386} = 0.6, h_{392} = 0.$$

Area of trapezium, $Area = \frac{1}{2}d(h_n + h_{n+1})$, where **d** is the column spacing (6m on the grid).

$$A = 3(2.8+2.5) + 3(2.5+2.3) + 3(2.3+2.0) + 3(2.0+1.7) + 3(1.7+1.0) + 3(1.0+0.6) + 3(0.6+0) = 69m^2.$$

Incorporating batters requires the subtraction of $\frac{1}{2}h^2m$ from each cross section.

$$A = \frac{1}{2}h^2m = \frac{1}{2} \times 2.8^2 \times 1.25 = 4.84 \times 1.25$$

$$Area\ batter = 4.9$$

$$Section\ area = 69 - 4.9 = 64m^2, \text{ agreeing with the area calculated in Figure 3-15.}$$

Again, the volume is calculated by the end area method in §3.5 for all the cross sections. And the volume of the batter to the crest of 7500N must be excluded as shown in §3.8.2.3.

3.8.4 Block Volume

Another approach to calculating volume is to calculate the volume of each **block** formed between the adjacent grid plans. The approach is to find the volume of a block by calculating the mean height (depth) of each grid area, multiplied by the area itself.

A block volume calculated by grid depths.

Taking blocks 1 – 4, 8, 9, 15, 16 in Figure 3-19.

Taking a section of the area for volume calculations:

Volume block = average depth $\times$ area

$$V = \Sigma (\text{vertices})/4 \times A \text{ or } \Sigma (\text{vertices}) \times A/4$$

$$V_1 = (3 + 2.8 + 2.5 + 2.8) \times (6 \times 7)/4 = 11.1 \times 10.5 = 116.6, \text{ similarly}$$

$$V_2 = (2.8 + 2.4 + 2.3 + 2.5) \times 10.5 = 105.0,$$

$$V_3 = (2.4 + 2.2 + 2.0 + 2.3) \times 10.5 = 93.4$$

$$V_4 = (2.2 + 1.8 + 1.7 + 2.0) \times 10.5 = 80.8$$

$$V_8 = (2.8 + 2.5 + 2.4 + 2.7) \times 10.5 = 109.0$$

$$V_9 = (2.5 + 2.3 + 2.2 + 2.4) \times 10.5 = 98.7$$

$$V_{15} = (2.7 + 2.4 + 2.3 + 2.6) \times 10.5 = 105.0$$

$$V_{16} = (2.4 + 2.2 + 2.0 + 2.3) \times 10.5 = 93.4$$

$$\text{Total volume} = 802.2m^3.$$

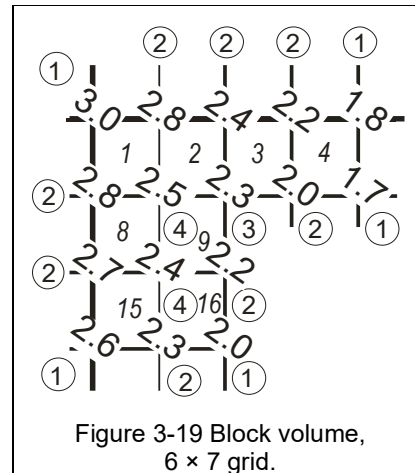


Figure 3-19 Block volume, 6 x 7 grid.

3.8.4.1 Block volume method

Several the depths have been used repeatedly for each block. As with the end area method of calculating volumes, so a method can be articulated for the calculation of block volume.

Volume = (single grid area)/4 $\times$ { $(\Sigma \text{ depths used once}) + 2(\Sigma \text{ depths used twice}) + 3(\Sigma \text{ depths used thrice}) + 4(\Sigma \text{ depths used four times})$ } + ΣV . (Uren J and B Price)

Figure 3-20 annotates the number of times each depth is used.

$$\begin{aligned}
 V &= 42/4 \times \{(3.0+1.8+1.7+2.0+2.6) \\
 &+ 2(2.8+2.4+2.2+2.0+2.2+2.3+2.7+2.8) \\
 &+ 3(2.3) + 4(2.5+2.4)\} \\
 &= 10.5 \times (11.1+2 \times 19.4+3 \times 2.3+4 \times 4.9) \\
 &= 10.5 \times (11.1+38.8+6.9+19.6) \\
 &= 10.5 \times 76.4 = 802.2 \text{ m}^3.
 \end{aligned}$$

The formula includes a value $+\Sigma V$, all the extra little bits of volume outside a whole block.

Do they matter?

By inspection block 52, near D/S 134 (Figure 3-20), seems to have the highest $h = 0.5$ in one corner. $V_{52} = (0.5 + 0 + 0 + 0) \times 10.5 = 5 \text{ m}^3$.

V_{55} would be considered a whole block as it's mostly inside the boundary.

$V_{56} =$ Maybe 1.5 m^3 . They would all seem insignificant and total less than 10 m^3 . Does it matter?

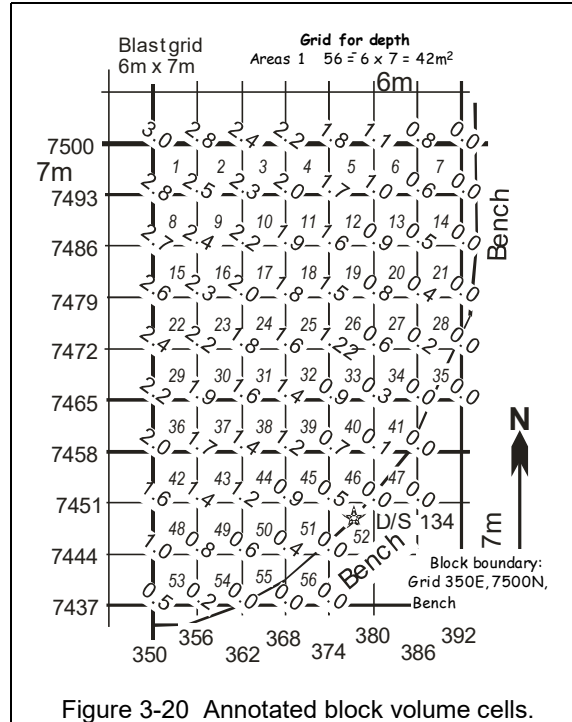


Figure 3-20 Annotated block volume cells.

3.8.4.2 Block volume value

Full block volume, over 56 blocks (Figure 3-20), is calculated to be 2853 m^3 using the methodology illustrated in §3.8.4.

3.8.4.3 Pyramid volume for batter calculations

What about the volume of the batter slope of 39° , $m = 1.25$ (Figure 3-13)

Area of triangle, $a = \frac{1}{2} h^2 m$ (or $\frac{1}{2} h b$)

Volume of a pyramid, $V_p = \frac{1}{3} a h$

So, the batter volume for each batter from the top left, 350E, 7500N, along each wall, could be treated as a **pyramid**. By inspection, at the top corner of block 1 (Figure 3-21), $h = 3$, so the toe, b (width), is, say 4, from crest. Area = $\frac{1}{2} (3 \times 4)$. $a = 6$. This is the base area of the pyramid.

Assuming a straight gradient from crest to toe along grid line

Length of 350E grid = $7500 - 7437 = h = 63$. $V_{350E} = \frac{1}{3} (6 \times 63) = 126 \text{ m}^3$

Length of 7500N grid = $350 - 392 = h = 42$. $V_{7500N} = \frac{1}{3} (6 \times 42) = 84 \text{ m}^3$.

Excavation volume, allowing for gradient batters, = $2853 - 126 - 84 = 2643 \text{ m}^3$

3.8.4.3.1 Compare methods

How does this gradient line approximation compare with batter volumes calculated from the individual cross sections?

Compare the straight-line gradient volume with the volumes related to cross section spot heights.

Figure 3-21 shows the batter volume along the 350E grid from 7500N down to 7437N. This profile volume calculates as 196 m^3 compared with the straight gradi-

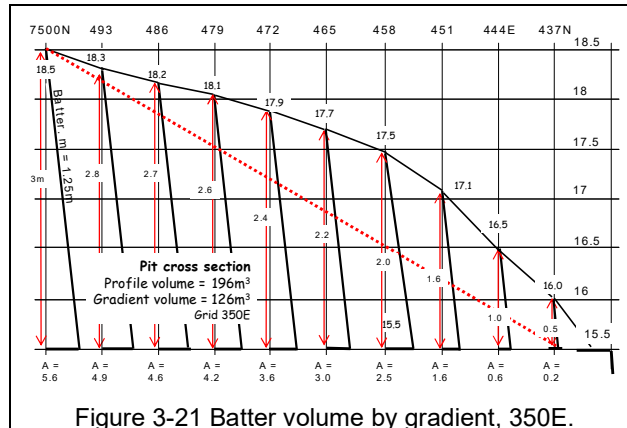


Figure 3-21 Batter volume by gradient, 350E.

ent value of 126 m^3 , i.e., 70 m^3 more because of the difference between the profile and the straight gradient.

Similarly, Figure 3-22 shows the difference of only 15 m^3 along the 7500N grid between 350E and 398E because the profile and gradient are more similar.

The difference in batter volume between the two methods is 85 m^3 . The excavation volume using end areas, accounting for batter ground profile is 2562 m^3 . See §3.8.2.3 and Table 3-3.

The volume calculated by block volume and gradient batters is 2643 m^3 (§3.8.4.2). Subtracting a further 85 m^3 results in an excavation volume of 2558 m^3 , which agrees well with the 2562 m^3 previously calculated in §3.8.2.3

All this data is, of course, reduced in GIS and packages such as Surpac™ etc.

The purpose of the last few pages has been to present a few methods that can be used for hand calculation of excavation volumes. No heavy maths is needed, just the tedious application of a few basic formulae.

They could also form the basis of small computer programs in MATLAB™ or spreadsheet manipulation in Microsoft Excel™.

3.8.5 Volume by Contour

Another approach to obtaining volume is to calculate the area of each **contour** formed between the boundary and the contour line. The **volume** is then calculated by the end area method, the distance being the contour interval.

Previously Figure 3-14 showed grid heights. The grid can now be contoured, a method that can be hand drawn by inspection and estimates of contour line position.

Alternately, contours are calculated and plotted using GIS and mapping programs. They are also a popular source of algorithm development by computer programmers.

Figure 3-23 shows contours generated by calculating contour values at appropriate linear grid intercepts.

Until computerisation of mapping, areas were determined using a mechanical integrator called a **planimeter**.

An approach available to the programmer is to use area by matrix cross multiplication, the coordinate pairs being the E and N grid intercepts used to plot the contours. The area between the boundary and the 18m contour is about 120 m^2 .

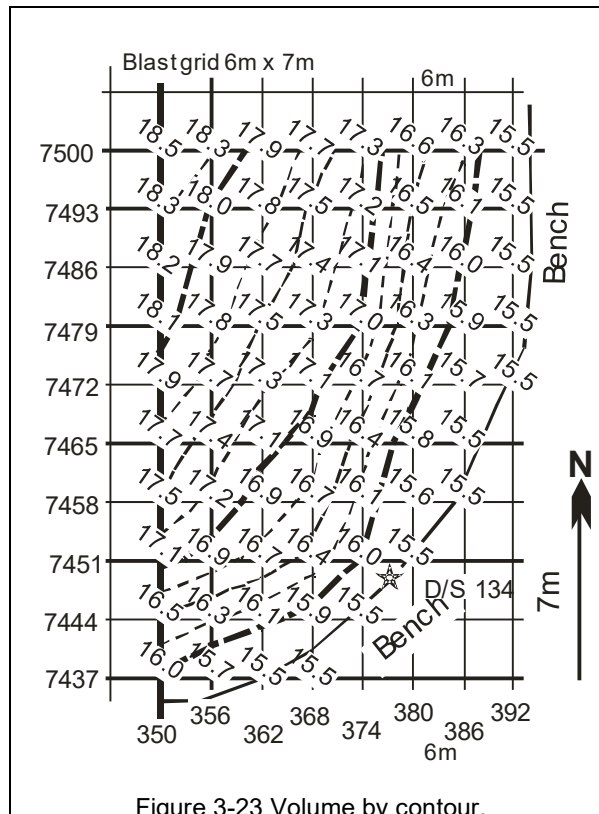
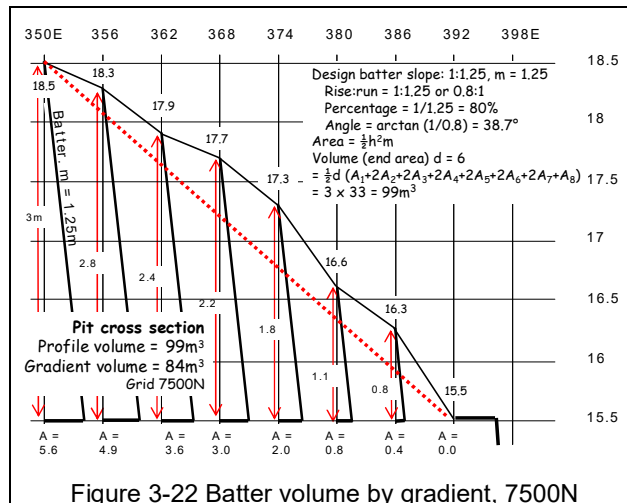


Figure 3-26 (page 67) is a larger image of Figure 3-23 and can be used as an exercise in contour mapping and data extraction as detailed below.

Although hard to pick out, the coordinates for the 18m contour boundary are:

350, 7476; 352, 7479; 356, 7493; 361, 7500; 350, 7500; 350, 7476.

Notice that the figure closes back to the start point at 350, 7476.

Continuing a calculation of contour areas, the following area values were realised:

RL 18.5: $A_1=0$, RL 18.0: $A_2=117$, RL 17.5: $A_3=490$, RL 17.0: $A_4=928$,
RL 16.5: $A_5=1319$, RL 16.0: $A_6=1758$, RL 15.5: $A_7=2295$.

Volume by end area = $d/2(A_1 + A_7 + 2(A_2+A_3+ A_4+ A_5+A_6))$. See Eqn 3-11.

where d is the contour interval of 0.5m

$$V = \frac{1}{2} (0.5) \times (0 + 2295 + 2(117 + 490 + 928 + 1319 + 1758)) = 0.25 \times (2295 + 2 \times 4609) \\ = 0.25 \times (2295 + 9218)$$

$$V = 0.25 \times 11513 = 2878\text{m}^3.$$

Note relatively good agreement in the volume using the three methods:

batter volumes have been excluded from the calculations.

- End area = 2857, recalculated for no batter.
- Block = 2853, from §3.8.4.2.
- Contour = 2878.

3.8.6 Missing Grid Data

Occasionally grid points may be missed, either being unobservable or inadvertently. What does one do?

If the data is missing inside a block of levels it may be best to **interpolate** the missing value. This will involve a judgement of the variation in slope around the data point. Generally, a **linear** interpolation is the best estimate.

Linear interpolation can be used to **calculate** the level values of contours on the blast grid.

Say we are missing a level at 356E, 7493N (Figure 3-24).

interpolating on 7493 between 350 and 362 (18.3 – 17.9)

- gives 18.05

interpolating on 356 between 7500 and 7483 (18.3 – 17.8)

- gives 18.1

interpolating diagonally (18.5 – 17.7) gives 18.1

interpolating diagonally (18.2 – 17.9) gives 18.05.

A value of 18.0 or 18.1 would seem reasonable and wouldn't make any significant difference to calculations.

Having said that, **bicubic** interpolation is a powerful tool, but is not considered here. To interpolate a point in a square, we need an array of 16 points, and derive slopes, to get a "better" answer. This is probably true for relatively homogeneous data fields, but our simple need doesn't justify it. Surpac and other mining and GIS packages uses this stuff, and many more specialised routines such as Lerchs-Grossman algorithm as well, in the generation of ore body estimates.

Similarly, missing data adjacent, but outside, the data field may be determined by **linear extrapolation** of the preceding values.

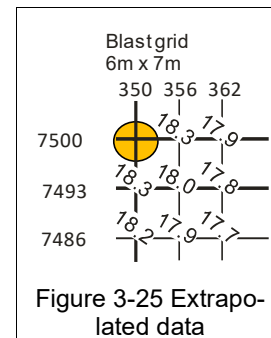
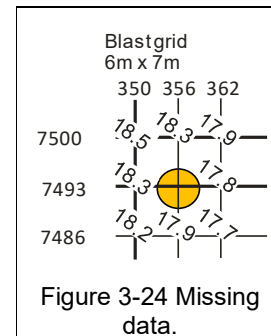
A level is missing at 350E, 7500N (Figure 3-25).

extrapolating on 7500 from 362 to 350 (17.9 – 18.3) = 18.7

extrapolating on 350 from 7486 to 7493 (18.2 – 18.3) = 18.4

diagonally (17.7 – 18.0) = 18.3

Meaning the 3 values results in a value of about 18.5



The methods will help **generate** a complete set of grid values from a **few** missed points. Try calculating one of your **missing** grid lines from your practical session. Check your results with data gathered by the other members of your class.

Different? But what difference would it make in the overall scheme of things.

Don't extend your extrapolation beyond your data boundary. Certainly, not more than one grid interval.

The calculated values should not be too far from real (missing) values and should not cause any serious errors in area or volume. Your calculations need to be made from defensible estimates backed up by observed data.

In general, when drawing profiles or plans from measured data, you can only join the points by a **line**. Don't be tempted to "pretty" the drawing up by using curves or splines to make it look "nice". There is generally less truth in a pretty curve than a straight line as there are no extra artefacts to distort the data.

The area (Figure 3-26) is reproduced here below to allow you to plot a contour over the area. A contour interval of 0.5 m should suffice.

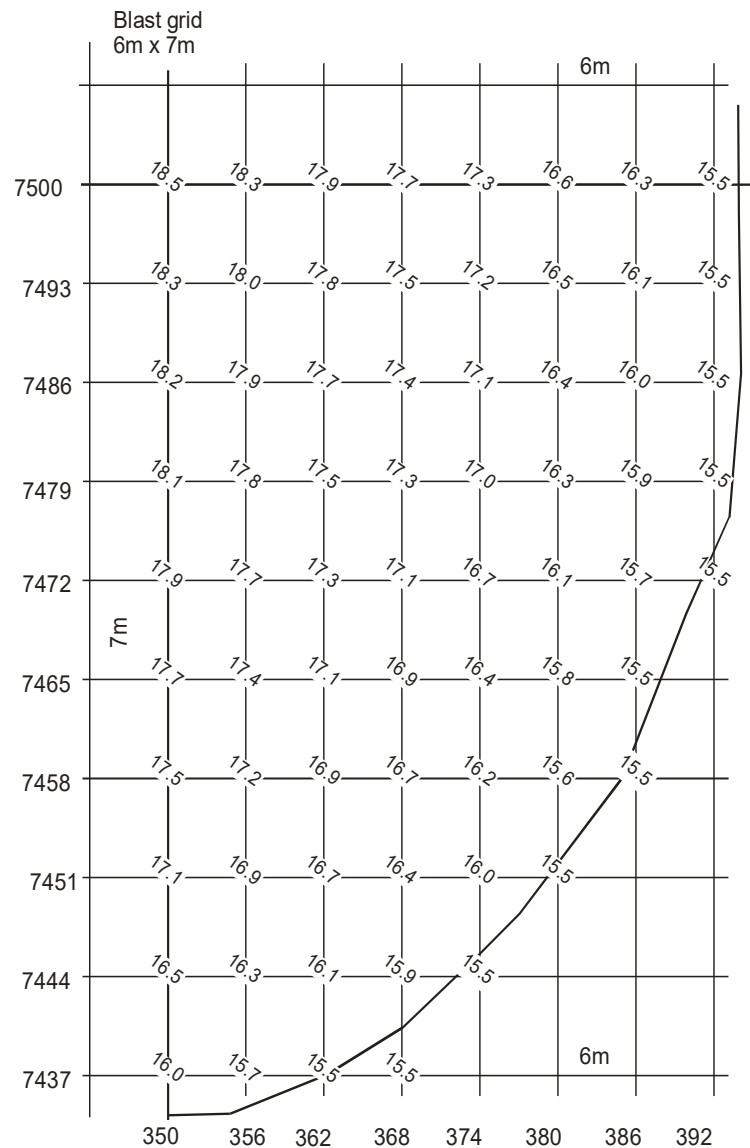


Figure 3-26 Contour exercise.

3.8.7 Stockpile Volumes

Angle of repose for free stockpiles.

Material for transport is stored in a **stockpile**. Any free standing, free flowing material will exhibit a tendency to form a heap where the slope of the free-standing material is often called the **angle of repose**.

Most free-standing mined material seems to stabilise at about:

25°, granite and blast furnace slag

35°; iron ore, coal, sand and gravel.

40°, manganese ore

These angles represent the following slopes (1:m) and percentage slope.

40° = 1:1.2, 84%

35° = 1:1.4, 70%

25° = 1:2.1, 47%

If the height, **h**, and the length of the crest, **ℓ**, of an unrestrained stockpile is known then an approximation of the volume can be arrived at. The stockpile material has a known angle of repose, which can be expressed as the slope, **m**, where:

$$\text{Slope } m = \cot(\text{angle of repose})$$

$$\text{Radius } r = mh$$

$$\text{Triangular area } A = rh = mh^2$$

$$\text{Conic volume } V = \pi r^2 \frac{h}{3} = \frac{\pi}{3} m^2 h^3$$

The volume is cross sectional area × crest length
+ volume of the cone (half cone at each end.)

Figure 3-27 shows an iron ore stockpile,
repose angle 35° slope, $m = 1.4$

Crest length, ℓ , = 50m; height, h = 25m,

Triangular end area = $mh^2 = 1.4 \times 25^2 = 875\text{m}^2$,

Length = 50,

Body volume = $\ell \times A = 50 \times 875 = 43,750 \text{ m}^3$.

Conic volume, $r = mh = 35$.

$$V = 3.14 \times 35^2 \times 25/3 = 32,070\text{m}^3.$$

Total volume = $75,820 \text{ m}^3$.

The stockpile is 120m long, 70m wide and 25m high.

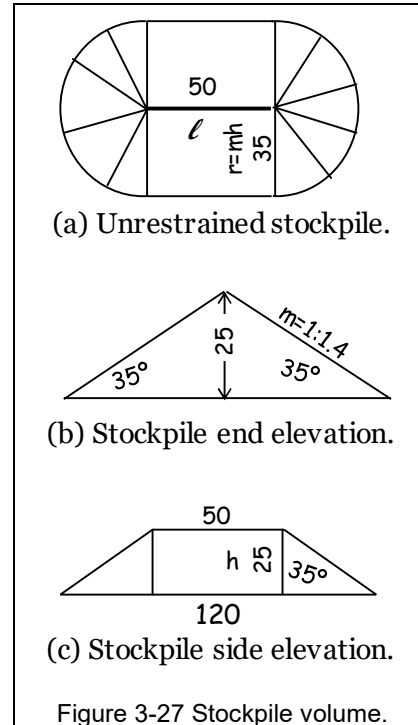


Figure 3-27 Stockpile volume.

3.9 Mass Haul Diagram

The tools are now in place to plan an optimum material movement strategy. Earthworks costs involve the removal of surplus (cut) material and the placement and compaction of fill material. Included in the job must be a knowledge of extra waste material for disposal, and the requirement for extra fill (borrow) material.

Mass haul diagrams (MHD) are a graphical representation of cumulative cut and fill **volumes** over the length of the works area.

Initially a route survey will provide a ground profile of the centreline, and a planned grade, or construction, line will be determined. Often the start and end levels of the grade line will be determined by existing levels. For example, a new road is needed between two existing bridges. The bridge levels determine the grade line, not necessarily a linear change. It may involve a vertical curve as discussed in Chapter 13. Vertical Curves

3.9.1 Definitions

- 1) *Haul*. The volume of material $\times$ distance moved. Expressed in “station meters”.
- 2) *Station metre*. 1m^3 moved over a station distance of 100m.
- 3) *Bank volume*. The amount of material required for cut or fill, not allowing for swell or compaction. Also called *in-situ* volume.
- 4) *Swell*. An expansion factor applied to bank cut material, depending on material type: soil, 10% - rock, 40%.
- 5) *Compaction, shrink*. A contraction factor applied to bank fill material, depending on material type: soil, 10%; clay, 20%. The factor depends on the geotechnical requirements of fill type, density, moisture content etc.
- 6) *Waste*. Material excavated from cut, but not used in embankment fill.
- 7) *Borrow*. Material needed for the formation embankment that cannot be obtained from the site excavation. Carried from a “borrow” pit.
- 8) *Free haul distance/volume*. Haul distance or volume to be carried, generally a contracted distance based on close movement of material.
- 9) *Overhaul distance/volume*. Distance/material volume outside the free haul specification.
- 10) *Economic haul distance*. Distance beyond which it is more economical to waste or borrow material rather than fill or excavate in situ material.

3.9.2 Basic Mass Haul Concepts

A mass haul diagram (MHD) is a graph of accumulated material **volumes** versus longitudinal distance (chainage).

Figure 3-28 a) shows an idealized longitudinal section imposed on a design grade line. The vertical axis represents height above design. Intermediate chainages have not been shown. Given symmetrical formation design, the vertical axis could represent end area at each chainage.

Figure 3-28 b) represents the idealized accumulated volume over the grade line, starting at zero, and accumulating the volume between each chainage station. Cut is a positive volume and, in this diagram, becomes a maximum at the point where the long section and the grade line intersect. From then onwards the formation has to be *filled*, depleting the surplus cut until it is exhausted at the end of the grade. The average haul distance, EF, is the perpendicular bisector of the ordinate CD between the

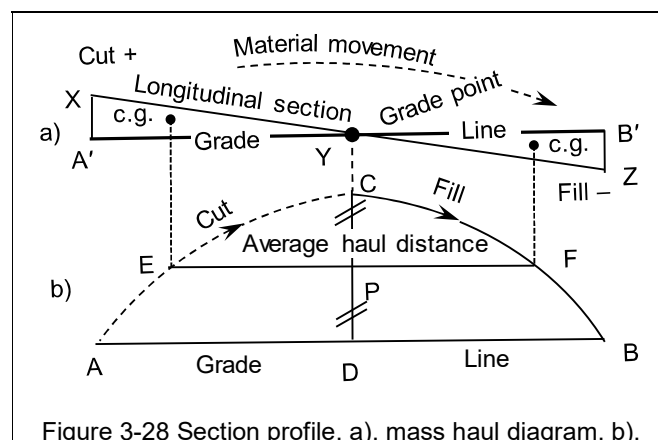


Figure 3-28 Section profile, a), mass haul diagram, b).

zero-line AB and the maximum fill at C. It represents the distance between the mass centres (c.g.) of the cut and fill sections.

3.9.3 Generation of a Mass Haul Diagram

The first task in generating a mass haul diagram (MHD) is to draw a profile of the longitudinal surface and the design grade line. Referring back to Chapter 3 Relief and Vertical Sections, page 46 refresh profiling from Table 3-1 and Figure 3-1.

Note that only observed/calculated survey values have been plotted. The plots are joined by straight lines. There is nothing in the survey data that allows generation of curves between the observations. Diagrams in many texts show curves, and then interpolate an ordinate value from the curve intercept. Looks nice, but not defensible.

3.9.3.1 Route survey and grade design

A gravity-assisted bulk rail hopper loading railroad is planned over an undulating route 1200m long. It is to be constructed as a road formation with a steady rising slope of 1.25% (1:80) between Stn 0 and Stn 12. The preliminary planning assumes there is no cross fall over the centreline and levels are taken every 100m. Formation width is 8m and the cut and fill batter slopes are equal at 1:3 (A rise/fall of 1m vertical in 3m horizontal).

Figure 3-29 illustrates the profile and grade determined from the field book. With no cross fall, simplified equations can be applied to find formation width at the crest or toe if cut/fill sections. Vertical exaggeration is about 10:1.

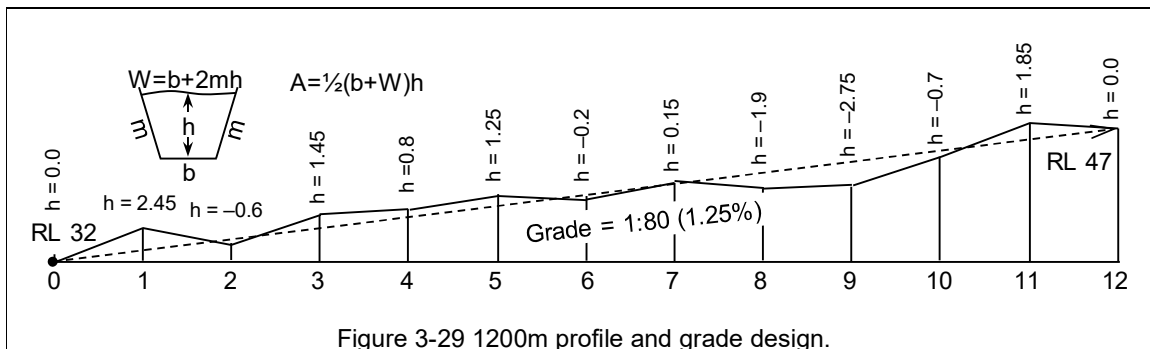


Figure 3-29 1200m profile and grade design.

Visual inspection of the profile shows some alternating sections of cut (+h) and fill (-h) and, with symmetrical formation shape there is more fill available than cut material required.

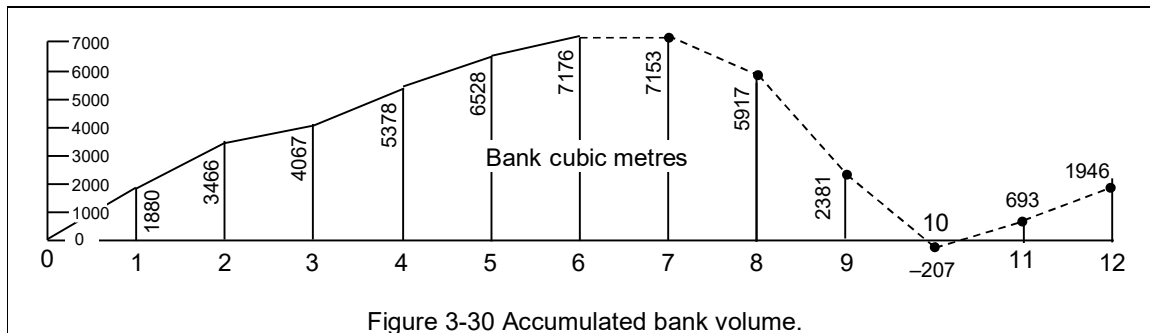
With a formation width of $b = 8\text{m}$ and batter slopes, $m = 3$, in cut and fill then Table 3-4 shows the amount of cut (+) and fill (-) at each 100m section. The volume of cut and fill is calculated by the end area method between chainages. The accumulated volume at each station shows the total amount of cut or fill available at that station. The volume here is expressed as *bank cubic metres*. That is the volume of material with no accounting for cut material swelling as excavated or shrinking as compacted.

Table 3-4 Bank volume cut/fill calculations. Formation width $b = 8$, batter $m = 3$.

STN	0	1	2	3	4	5	6	7	8	9	10	11	12
GL	32	35.7	33.9	37.2	37.8	39.5	39.3	40.9	40.1	40.5	43.8	47.6	47
Grade	32	33.25	34.5	35.75	37	38.25	39.5	40.75	42	43.25	44.5	45.75	47
h	0	2.45	-0.6	1.45	0.8	1.25	-0.2	0.15	-1.9	-2.75	-0.7	1.85	0
W	8	22.7	11.6	16.7	12.8	15.5	9.2	8.9	19.4	24.5	12.2	19.1	8
A Cut	0	37.6		17.9	8.3	14.7		1.3				25.1	0
A Fill			-5.9				-1.7		-26.0	-44.7	-7.1		
V Cut	0	1880	1586	601	1331	1150	648					900	1253
V Fill								-23	-1238	-3536	-2588		
Σ Vol	0	1880	3466	4067	5378	6528	7176	7153	5917	2381	-207	693	1946

Table 3-4 is graphed in Figure 3-30 and shows that there is a surplus of nearly 2000m³ of cut over fill. However this is unswollen cut and uncompacted fill, the bank surplus (waste).

Note also that the plan of the works area (formation width × length), very hard to draw, has a surface area of some 18,000m². The surface material will have to be removed, stockpiled and returned as part of the environmental remediation. However the formation surface itself accounts for 9600m², meaning there will be plenty of remediation material available.



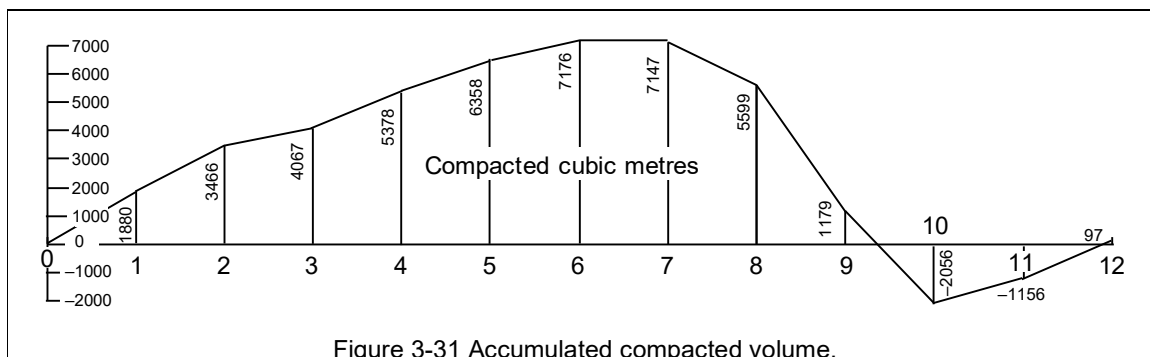
3.9.3.2 Bulk and shrink in cut and fill

As bank material is excavated (cut) it will swell by a factor which depends on the material. Similarly, in fill, material must be compacted and thus bank material must have a shrinkage factor incorporated. In a normal soil/gravel regime there is a swell factor of some 10% and a shrinkage factor in fill of some 15-20%.

For this exercise the two will be combined to have a compaction factor of, say 25%. That means that 1000m³ of bank material will compact to 800m³. Excavation swell is incorporated in this factor. The 25% compaction factor (1.25) means dividing the fill volume by 0.8. Table 3-5 shows the effect of the compaction factor on the fill volume, resulting in a surplus of about 100m³.

STN	0	1	2	3	4	5	6	7	8	9	10	11	12
GL	32	35.7	33.9	37.2	37.8	39.5	39.3	40.9	40.1	40.5	43.8	47.6	47
Grade	32	33.25	34.5	35.75	37	38.25	39.5	40.75	42	43.25	44.5	45.75	47
h	0	2.45	-0.6	1.45	0.8	1.25	-0.2	0.15	-1.9	-2.75	-0.7	1.85	0
V Cut	0	1880	1586	601	1331	1150	648					900	1253
V Fill								-23	-1238	-3536	-2588		
Fill/0.8								-29	-1548	-4420	-3235		
Σ Vol	0	1880	3466	4067	5378	6358	7176	7147	5599	1179	-2056	-1156	97

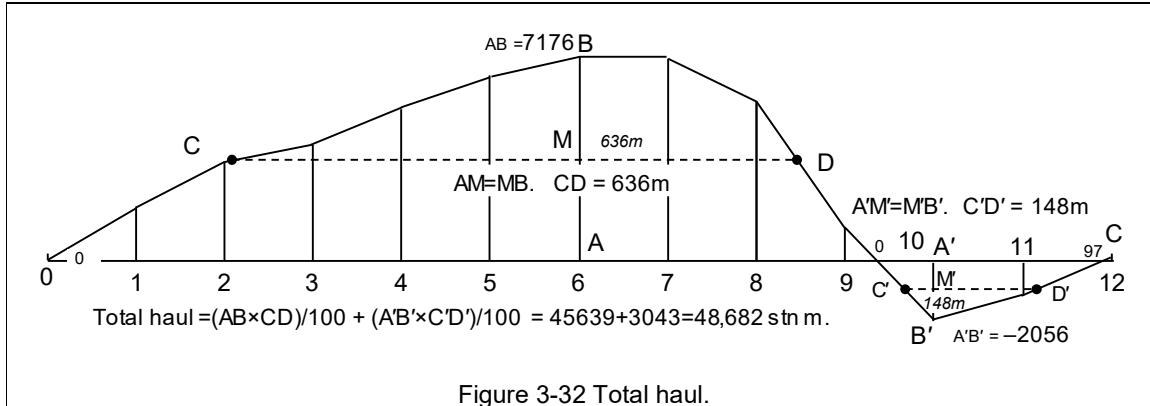
Table 3-5 is graphed in Figure 3-31 and shows that there is a surplus of nearly 100m³ of cut over compacted fill. This meets the definition of waste.



3.9.3.3 Calculating haul in station metres

The maximum mass ordinates in both cut (+) and fill (–) allows calculation of the average haul distance for the cut and fill. M and M' are the mid ordinates respectively and the lengths CD and C'D' represent the average haul distance. Distance is scaled from the mass haul diagram.

The total haul is the amount of material that must be moved over the project length. The principle of haul derived from Figure 3-28, is applied to Figure 3-32. The maximum cut ordinate AB, 7176m^3 , is at Stn 6. M is the midpoint of the ordinate. CD is the perpendicular to AB at M and spans the mass haul curve from just after Stn 2 to between Stn 8 and Stn 9. The length of CD, 636m, has been scaled from Figure 3-32. Similarly, the fill ordinate A'B', -2056m^3 has a mid-ordinate span, C'D', of 148m. Total haul volume is about 49,000 stn m.



3.9.3.4 Balancing lines

Because the MHD in the previous section has balanced at almost zero volume there is only one balancing line; the project can be completed with available material. Most of the fill will be supplied by cut from Stn 0 up to about Stn 9+40. The remainder of the fill will come from the Stn 12 cut back towards Stn 9+40. Re-examining Figure 3-29 shows where the profile cut will come from.

There seems to be very little waste for disposal. The project of course will now be subject to much more rigorous geological and geotechnical review, including the choice of machinery and construction methods.

What happens if there is a surplus of material at one end? There is the ability to use the MHD to determine which of a number of scenarios may be the best. Generally the lesser total haul will be cheaper.

3.9.3.4.1 Worked example 3.1

A set of volumes over a 1200m section have been calculated. Examine the two available MHD balance lines to determine the lower haul volume.

Table 3-6 shows the station and accumulated volumes and Figure 3-34 is the resultant mass haul diagram with the initial balance line joining the 1200m section.

Table 3-6 Fill/cut/fill volumes.													
STN	0	1	2	3	4	5	6	7	8	9	10	11	12
V Cut	0					200	1400	2200	1500	600			
V Fill		-1100	-2300	-1500	-400						-500	-700	-1800
Σ Vol	0	-1100	-3400	-4900	-5300	-5100	-3700	-1500	0	900	400	-300	-2100

In the absence of original survey data, surface profile, cross section profiles and areas a method of visualizing the actual profile could employ plotting the section volumes as illustrated in Figure 3-33.

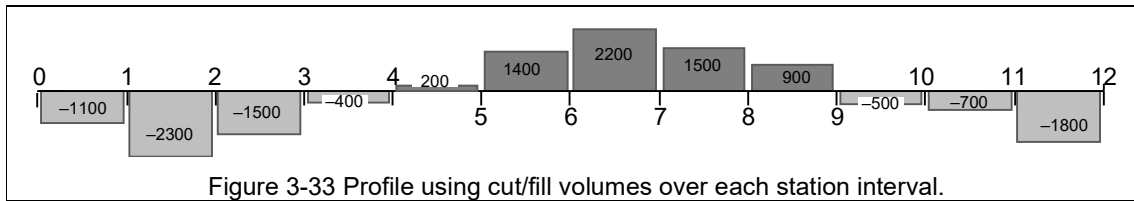


Figure 3-33 Profile using cut/fill volumes over each station interval.

The MHD in Figure 3-34 shows two sets of balance lines: one starting at Stn 0 with accumulated volume 0, the other starting at Stn 12 at a fill volume -2100 . Overall there is a requirement to “borrow” 2100m^3 for the job. Balancing from which end, Stn 0 or Stn 12, will be answered by which solution supplies the lower haul value.

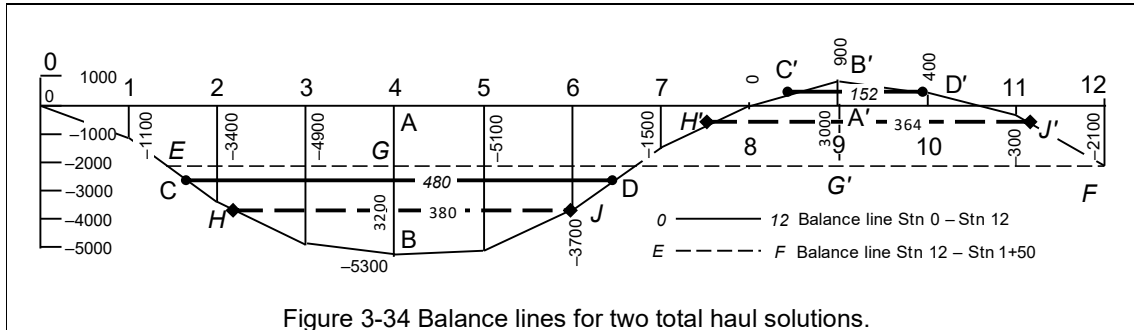


Figure 3-34 Balance lines for two total haul solutions.

- Balance line 0-12:
Balancing from the Stn 0 end forward allows placement of borrow from Stn 12.
Cut balance line CD = 480m, mass ordinate AB = 5300m^3
Cut balance line C'D' = 152m, mass ordinate A'B' = 900m^3
Haul = $(480 \times 5300)/100 + (152 \times 900)/100 = 25,440 + 1,368 = 26,808$ stn m.
 - Balance line E-F (from Stn 1+50 – Stn 12):
Balancing from the Stn 12 end back allows placement of borrow from Stn 0.
Cut balance line HJ = 380m, mass ordinate GB = $5300 - 2100 = 3200\text{m}^3$
Cut balance line H'J' = 364m, mass ordinate G'B' = $2100 + 900 = 3000\text{m}^3$
Haul = $(380 \times 3200)/100 + (364 \times 3000)/100 = 12,160 + 10,920 = 23,080$ stn m.
- Thus, placing borrow from the Stn 0 end requires the lesser haul of on site material. The required borrow of 2100m^3 is a separate exercise.

3.9.3.5 Auxiliary balance lines

The principals applied to the previous example can be incorporated in more complex diagrams. Figure 3-36 shows a 1200m section with two cut sections. The minimum volume ordinate at Stn 7 is 3000m^3 . An auxiliary balance line is drawn parallel to the base line through the Stn 7 ordinate at D.

Figure 3-35 is a section volume profile that generates the MHD in Figure 3-36.

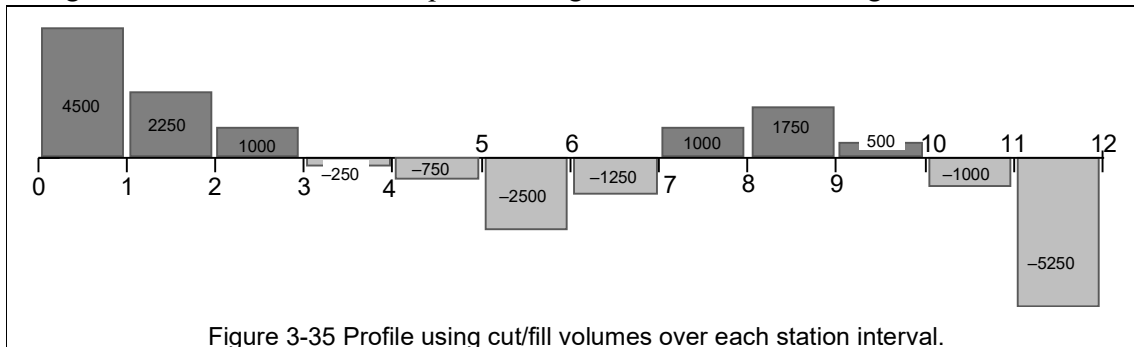


Figure 3-35 Profile using cut/fill volumes over each station interval.

The MHD shows that there are three haul areas to be considered:

- ACEBA. A-C balances E-B with an ordinate of 3000m^3 at D-Stn 7. The scaled mid-ordinate haul distance is 1136m.
- CFDGC. C-F balances F-D with an ordinate value of $(7750-3000) = 4750\text{m}^3$ at F-G. The scaled mid-ordinate haul distance is 420m.
- DJEKD. D-J balances J-E with an ordinate value of $(6250-3000) = 3250\text{m}^3$ at J-K. The scaled mid-ordinate haul distance is 268m.

The three hauls are:

$$\text{ACEBA} = \text{A-C} = \text{E-B. } (3000 \times 1136) / 100 = 34080 \text{ stn m.}$$

$$\text{CFDGC} = \text{C-F} = \text{F-D. } (4750 \times 420) / 100 = 19950 \text{ stn m.}$$

$$\text{DJEKD} = \text{D-J} = \text{J-E. } (3250 \times 268) / 100 = 8710 \text{ stn m.}$$

Total haul = 62,740 stn m.

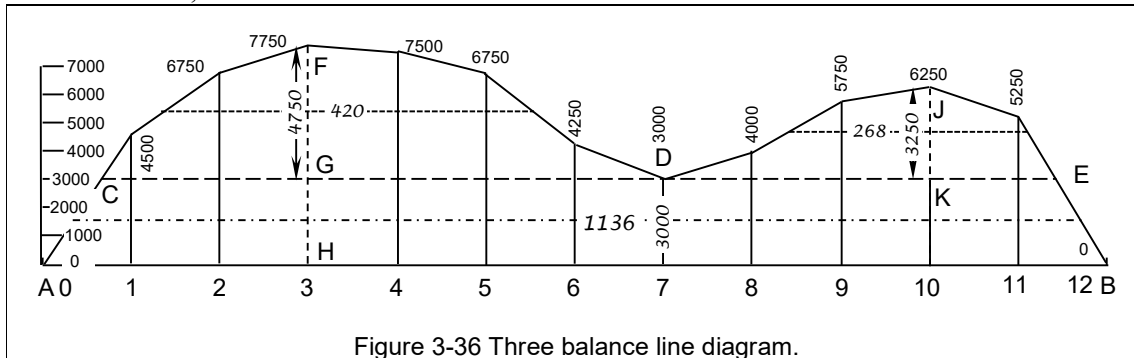


Figure 3-36 Three balance line diagram.

Figure 3-37 is a section volume profile that generates the MHD in Figure 3-38.

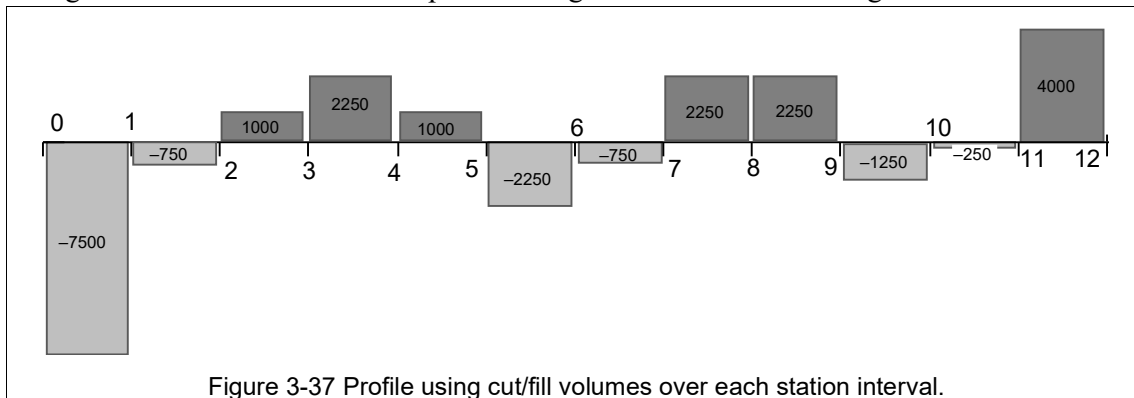


Figure 3-37 Profile using cut/fill volumes over each station interval.

The MHD with five haul areas, all fill areas, is illustrated in Figure 3-38. The five hauls are:

- ABEDCA. C-F = B-E. $(2500 \times 1150) / 100 = 28750 \text{ stn m.}$
- CDHGFA. C-F = D-H. $(1500 \times 825) / 100 = 12375 \text{ stn m.}$
- FLGMF. F-M = G-M. $(4250 \times 270) / 100 = 11745 \text{ stn m.}$
- GNHOG. G-O = H-O. $(3000 \times 225) / 100 = 6750 \text{ stn m.}$
- DPEQD. D-Q = E-Q. $(1500 \times 155) / 100 = 2325 \text{ stn m.}$

Total haul = 61,675 stn m.

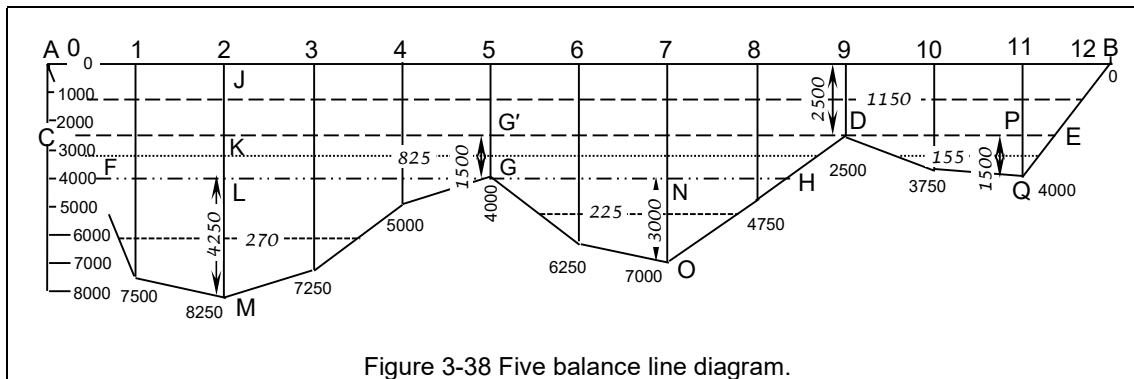


Figure 3-38 Five balance line diagram.

3.9.4 The Economies of Haul

The economies of earthworks rests on understanding not only the quantities of material to be moved but also the geotechnical issues of soil and rock structure of the cut material. The cut material may not be suitable for fill. Rock will not compact to fill. It cannot be economically crushed if suitable borrow material is available. Indeed, this section has only addressed the quantification of haul operations. The exciting part of dirt-working civil engineering is the planning and execution of earthworks projects.

Cut and fill operations require a combination of earth moving equipment. The linear nature of the illustrated project requires equipment to break up the consolidated cut material so that it can be loaded and moved to the fill area for compaction to the desired formation profile.

Figure 3-29 shows the maximum cut to be about 2.5m. The MHD, Figure 3-32, shows a long cut required from Stn 0 and a short cut from Stn 12 to provide the fill. Which machinery?

As illustrated in Figure 3-39, general loaded haul distances are summarised for various types of earth moving machinery. For example, a bulldozer (track-type tractor with ripper blades and push blade) is economical out to about 100m. This machine can rip and push cut in-situ ground material.

A wheel loader generally cannot rip in-situ ground. It can carry loose material between 50m and 180m. Over lesser distances it is too slow to load.

A wheel-tractor scraper works well out to about 3000m. These machines can scoop, carry and spread fill material at relatively high speed. Some machines have twin engines for better self-loading capability in suitable soil. Single tractor engine machines require push assistance in the load phase. Elevator equipped scrapers can self-load finely prepared material, and provide good depth control while spreading. Spreading depth is controlled by bowl lift, contributing to fill of a suitable depth for compaction.

The bulldozer can then perform two tasks:

- ripping and pushing (hauling) fill over short distances;
- ripping soil for scrapers, then pushing them for the cut part of the haul.

3.9.4.1 Scraper capacity

Scrapers come in a number of general carrying capacity. Here the Caterpillar™ range is used as an example. The smaller CAT 621K has a bowl capacity of 13 – 18m³ and the larger CAT 631K carries 18 – 26m³. The largest, the CAT 657G has a capacity of 24 – 34m³. The machines have a loaded top speed of 55km/h. The scrapers have single or twin engines for extra tractive

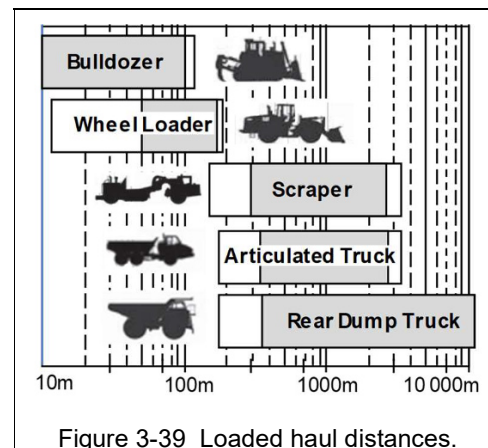


Figure 3-39 Loaded haul distances.

power. Scrapers can also operate in tandem push/pull configuration to load 2 machines sequentially, effectively having 4 engines in the cut operation.

3.9.4.2 Track dozer capacity

The bulldozer must be matched to the scraper model to ensure efficient performance of the scraper. One or two dozers may be used to push a scraper in cut.

Scraper	Bowl capacity	Dozer	Blade capacity (U type)
621/627	15m ³ /20m ³	D8/D9	12m ³ /16m ³
631/637	24m ³	D9/D10	16m ³ /22m ³
657	34m ³	D10/D11	22m ³ /34m ³

3.9.4.3 Earthmoving performance reference

The Caterpillar Performance Handbook, January 2017, SEBD0351-47 is a 2440-page document covering all their product lines. The handbook also includes all the performance specifications and calculation methods and examples for all forms of earthworks.

3.10 Concluding Remarks

This chapter has presented you with a direct application of elevations (heights) that were treated in Chapter 2. Using heights, we have seen how profiles of linear features such as roads, sewer and storm water lines could be obtained. This becomes important for civil engineers who must employ the knowledge of vertical sections (longitudinal and cross sections) to compute earthworks (cuts and fills) during the design of their features. For those in mining, computing volumes of stockpiles in addition to designing Ramps and Berms are daily routines that will call upon the knowledge of computing volumes learnt in this Chapter. For example, in designing a Box Cut at a given dipping gradient, one requires to know how to obtain heights and areas both which lead to the blast volumes to be removed. This chapter has presented several methods that grid the given area upon which the heights are obtained with respect to a given bench. This is surely a chapter that you would like to revisit more often.

3.11 Reference to Chapter 3

1. Irvine and Maclellan (2006) Surveying for Construction. Fifth edition, McGraw, Chaps. 5 and 6.
2. Schofield and Breach (2007) Engineering Surveying. Sixth edition, Elsevier, Chap. 3.
3. Banister and Raymond (1972) Surveying. Third Edition, Pitman Paperbacks, Chap. 5.
4. https://www.albancat.com/content/uploads/2017/08/SEBD0351_ED47.pdf



4 Chapter 4 Total Station: Measurements and Computations

4.1 Introductory Remarks

This Chapter introduces you to the Total Station instrument used for measuring angles and distances that are needed to generate planar coordinates from the traverse method discussed in the next chapter. Once you have completed this chapter, workshop materials in Appendix A1-3 and the field practical in Appendix A2-3, you should:

- ✓ Be familiar with the Total Station instrument and its parts.
- ✓ Be able to set and operate the instrument.
- ✓ Be able to record (book) angular and distance measurements in a manner that can be understood by other professionals.
- ✓ Understand the errors associated with Total Station operations.

LASER SAFETY: This part is based on:

- (i) Australian Standard 2211-2004 Laser Safety.
- (ii) AS2397-1993 – Guide to the Safe Use of Lasers in the Construction Industry.

Total Stations use a Class 1 or Class 3 laser for distance measurements. The major hazard when using a laser is to the eye, i.e., the retina – associated with lasers in the visible and near infrared spectrum. The lens of the eye will focus the Laser beam onto the retina of the eye causing a microscopic burn or blister to be formed. This can lead to temporary or permanent blindness.

Laser safety rules include the following operator responsibility:

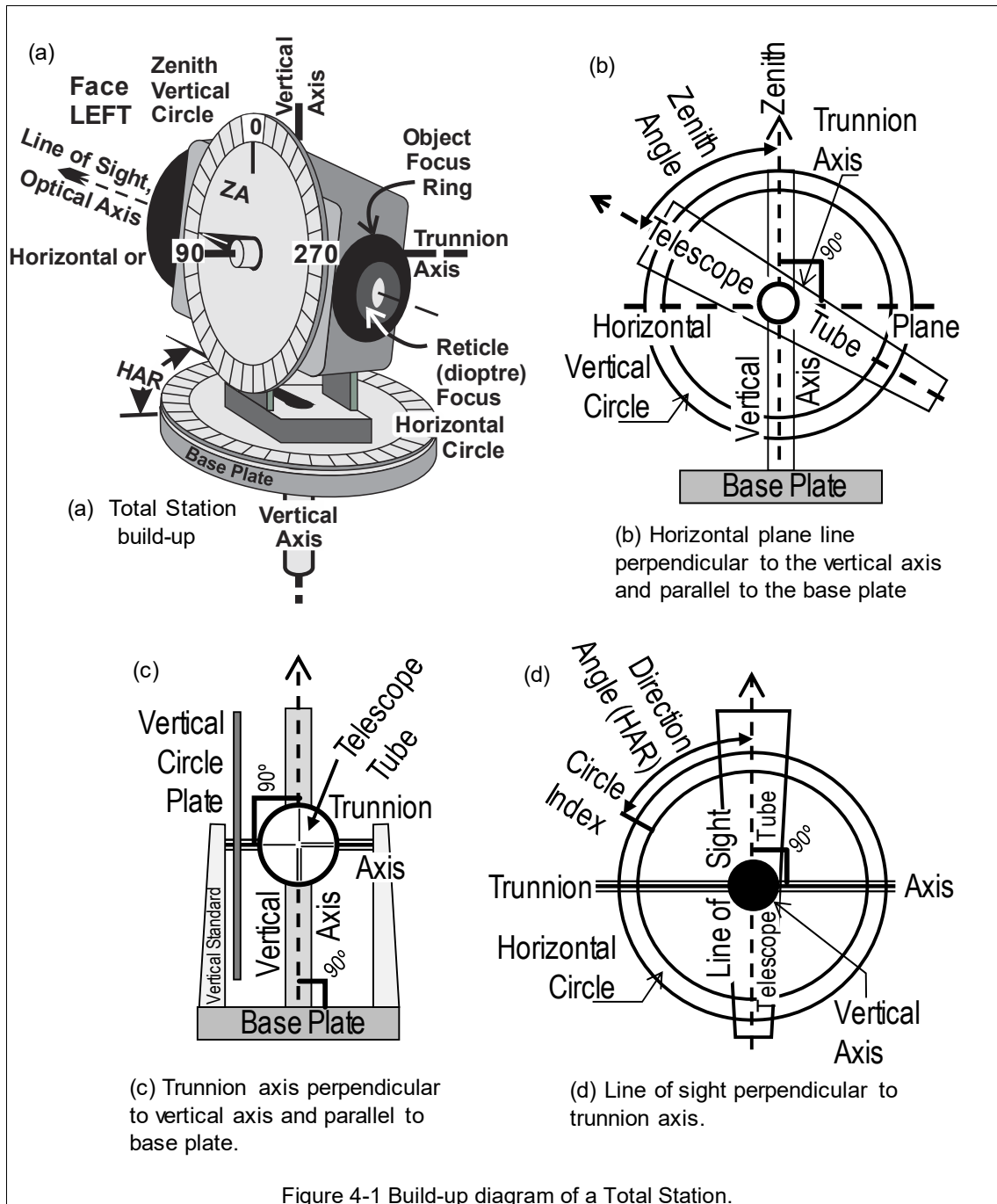
- ✓ Do not point the laser beam at another person. If the laser beam strikes an eye it could cause serious injury.
- ✓ Do not look directly into a laser beam – it could cause serious eye injury.
- ✓ Do not look at a laser beam through a telescope, binoculars or other optical instruments.
- ✓ Sight targets so that the reflected laser beam does not deviate from the working observation path.

4.2 Instrumentation and Operation

4.2.1 The Total Station

- ✓ Telescope (see Figure 4.1a containing line of sight): 30x or 40x magnification; short so it can be plunged.
- ✓ Graduated glass encoder circles are detected for horizontal and vertical angular displacement.
- ✓ Circle clamps and tangent slow-motion screws allow fine rotation to the target once a direction has been clamped.
- ✓ The vertical circle “normal” position is on the left (Face Left, FL) as indicated in Figure 4-1a. Some instruments are marked as Face I.
- ✓ When the telescope is plunged, the vertical circle is on the right (Face Right, FR or Face II).
- ✓ An optical plummet in the tribrach or theodolite itself allows precise centring over a point.
- ✓ The circular bubble in the tribrach allows coarse levelling with the tripod legs.
- ✓ The levelling head comprises 3 foot screws used to precisely level the instrument.
- ✓ In electronic instruments, the level bubbles may be internal and only visible on the LCD display.
- ✓ An EDM may be mounted atop the telescope (for older instruments) or be inside the instrument and measure on the collimation axis (in modern instruments).
- ✓ Electronic data recording devices may be externally connected or inside the instrument.
- ✓ Important relations between axes (see Figure 4-1b, c, and d):
 - The horizontal or trunnion axis must be orthogonal to the vertical axis,

- The collimation axis must be orthogonal to the Trunnion axis,
- The standing or vertical axis must be orthogonal to the plane containing the horizontal circle,
- The collimation axis must intersect the trunnion axis.



4.2.2 Setting over a Point Using the Optical or Laser Plummet

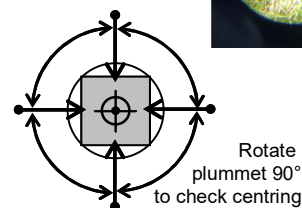
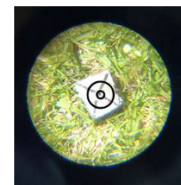
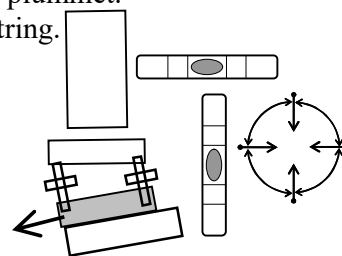
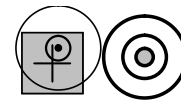
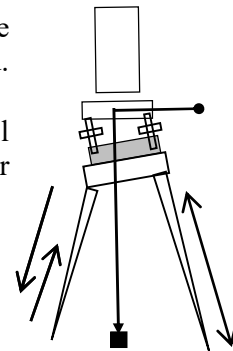
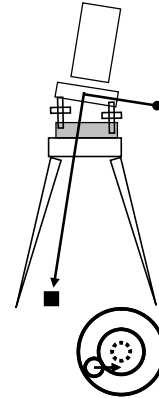
Centring accurately over a control point helps eliminate large random errors in your observations. This applies to Total Stations, levels, prism sets, laser scanners and GNSS receivers. The method outlined below does not require the use of a plumb bob, however one must be able to place the head of the tripod fairly over the point to start with.

1. Extend the tripod legs to a mid-chest height and clamp the legs.
2. Stand over the point and project one leg about a step in front of yourself, keeping the tripod head roughly horizontal, make a short step backwards and project the legs equidistantly so that the tripod is set as an equilateral triangle roughly over the point with the head level. Set the tripod feet firmly in the ground surface.
3. Ensure that the tripod clamps are secure. Mount the instrument on the tripod, in the centre of the tripod head, and check that the plate clamp bolt is secure. Ensure the foot-screws are in the centre of their run.
4. Look through the optical plummet or turn on the laser plummet. You should be able to see the control point in the plummet eyepiece or illuminated by a red laser dot. Ensure that the reticule is in sharp focus and focused on the ground or point.
5. Using the foot-screws, drive the reticule or laser until it is over the point. The tribrach and the plate will not be parallel to the tripod head. The circular level may be well off centre.
6. By adjusting the length of each tripod leg, centre the circular level using the legs only. The reticule should stay fairly well centred over the point.
7. Once the plate circular level is reasonably centred, use the foot screws to centre the plate level bubble. The Trimble M3 uses an electronic level display to provide levelling guidance. Refer to §2.4.2 on levelling with the foot-screws. Check plate level through 90°.
8. Check the optical plummet. The reticule or pointer will have moved off the point. Loosen the plate bolt slightly and move the instrument and tribrach across the tripod head to re-centre the plummet. Do not “twist” the tribrach around its axis during re-centring.
9. Check and re-centre the plate level using the footscrews, check centring through the optical plummet, check focus and parallax, check plate level through 90°.

This is an iterative process.

10. Check centring over the point by observing the optical plummet through 4 quadrants. Any collimation error will show as the reticule path describes a small circle around the point. Slight collimation can be eliminated by centring the point inside the described reticule circle.

The commonality of Sokkia TS and prism sets allows the semi-forced centring technique to be used in traversing control points. The system allows swapping the instrument and prism sets between tribrachs without having to re-centre over the point or re-measure heights. Any re-levelling is insignificant.



4.2.3 Preparing Trimble M3 DR 5" for Survey Observations

The Trimble M3 DR 5" Total Station, or similar, is used in some tertiary institution's Surveying courses and is typical of most surveying modern instruments. The setup, control and output of survey information is displayed on a 90 mm QVGA colour touch screen. The operating system is Windows® Embedded CE 6.0. The system displays and programs are run by software such as Trimble Field Access, which allows common survey tasks to be carried out. The software stores data, provides special survey job solutions; it includes the COGO program to solve most complex trigonometric problems.

The operation of the M3, help files etc., are accessed via:

<http://www.trimble.com/Survey/trimblem3.aspx?tab=Overview>

and are recommended for your introduction to the instrument.

Trimble M3 DR 1D Users Manual can be access via:

trl.trimble.com/docushare/dsweb/Get/Document-267977/TrimbleM3_DR_1D.pdf.

Trimble M3 data sheet:

trl.trimble.com/docushare/dsweb/Get/Document-262358/022543-155J_TrimbleM3_DS_0414_LR.pdf

Trimble Digital Fieldbook V7.01:

trl.trimble.com/docushare/dsweb/Get/Document-517810/TDFv701_Help_English.pdf



4.2.4 Preparing Sokkia SET530RK3 for Survey Observations

The Sokkia SET530RK3 Total Station used in teaching civil and mine engineering surveying units at Curtin University (Australia) is typical of most surveying instruments. It is controlled via 2 keypads and screens. The measured data can be read off either screen. The instrument has several built-in functions for:

- setting instrument parameters,
- solving repetitive computational tasks,
- recording observational data.

For the field exercises in civil and mine engineering units, we recommend that **all observational data** be recorded in a field survey book to provide students with booking practise. The complexity and expertise needed for data recording, download and manipulations precludes the use of many of the instrument's features.

Do not record the H distance from the SHV screen. The instrument MAY have a scale factor (SF) embedded in a JOB. The SF affects the horizontal distance (H) only.

4.2.4.1 Preparing the instrument for observations

After levelling and positioning the instrument over its designated point, insert a charged battery and turn the instrument on. Allow time for the display to boot. It should normally boot to page 1 of the measurement screen.

The screen shows the instrument is in **measure mode** (Figure 4-2) **(Meas)**

the prism constant **PC -30**

is set for the normal target

atmospheric correction is zero **ppm 0**

last slope distance: **S 55.450m**

current vertical angle: **ZA 95° 34' 32"**

current direction: **HAR 195° 37' 45"**

the current display page is **page 1** **P1**

the highlighted menu labels are controlled by the function key (**[F1]** – **[F4]**)

[F1] initiates a **distance measurement** to the target prism

the distance stays on the screen until **F1** is used for the next distance measure.

The **ZA** and **HAR** readings vary as the telescope is moved.

[F2] toggles between the vector screen

slope distance **S**, VA and HAR and the

rectangular screen, **SHV** (Figure 4-3) **SHV**

slope **S**, horizontal **H** and vertical **V** distance

These rectangular coordinates refer to the measurement at the last **[F1]** press.

The **FUNC** button toggles through the three (3) pages of the **Meas** screen. (Figure 4-4) **FUNC**

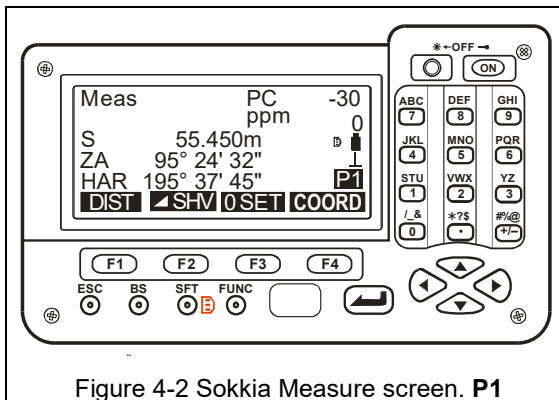


Figure 4-2 Sokkia Measure screen. P1

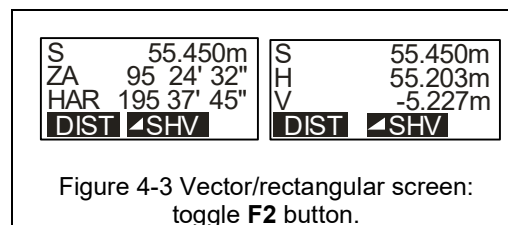


Figure 4-3 Vector/rectangular screen: toggle F2 button.

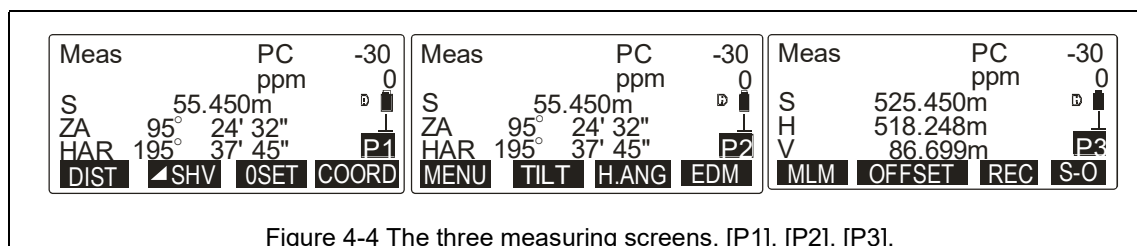


Figure 4-4 The three measuring screens. [P1], [P2], [P3].

The **MENU** screen (Figure 4-5) is accessed through the **[F1]** function key of the **Meas** screen **P2** (Figure 4-4). It allows memory and instrument configuration to be set.

Return to the **Meas** screen by using **[F1]** function key.

Function key **[F2] TILT** activates a graphics screen to allow accurate instrument levelling, or to allow a levelling check.

Return to the **P2 Meas** screen using the **ESC** button.



If **Out of range** is displayed on the **Meas** screen, then the instrument is out of level and the tilt compensator cannot provide corrections. Check and re-level the instrument using the plate bubble or the **TILT** screen (Figure 4-6).

4.2.4.2 Setting the Horizontal Angle Reading

Function key **[F3] H.ANG** on **P2** allows Horizontal Angle Reading (HAR) input when aligning with a target of known bearing. It requires bearing to be input in the **D.MMSS** format, followed by the **ENTER** key.



195° 37' 45" is input from the keypad as:

195.3745

Note that on the **P1 Meas** screen (Figure 4-4) the function key **[F3] 0SET** allows a direction of **0° 00' 00"** to be set. After the first **[F3]** key press the **0SET** flashes. Press the **[F3]** key a second time to confirm the setting. The display is a bit slow in updating.

Function key **[F4] EDM** displays two pages of information (Figure 4-8):

The first page allows setting of the “distance measuring mode”, the method by which the slope distance is determined.

Fine “r” is the default setting, a repeat measure to a prism, displays to 0.001m,

precision $\pm(3\text{mm} + \text{dist.} \times 2\text{ppm})$

Fine “AVG” for more precise distance, averages over 3 – 9 readings, displays to 0.0001m (0.1mm),

precision $\pm(2\text{mm} + \text{dist.} \times 2\text{ppm})$

Fine “s”, Rapid “s” are single measures. They are slightly faster measurements, but of less accuracy,

displays to 0.001m (1mm).

The second page, Figure 4-9, accessed by the down button  allows input of current atmospheric conditions to compensate for deviation from the instrument’s default atmospheric condition of zero ppm correction, **ppm 0**.

[F1] 0ppm allows the reset of default condition.

Changes to these screens must be confirmed using the enter before exiting the **EDM** screen using the **ESC** button. You are returned to the **P2 MEAS** screen.

EDM atmospheric correction is discussed in §4.3.1.

Standard atmosphere for the Sokkia SET530 is:

Pressure 1013hPa,
temperature 15°C,
relative humidity (RH) 0%.

Due to the small contribution that humidity makes to most measurements, it can be ignored. The SET530 is capable of measuring to 4000m on a single prism, but even at Newman, with an approaching cyclone and RH = 75% the error only equates to 1ppm (0.004m) by ignoring RH correction.

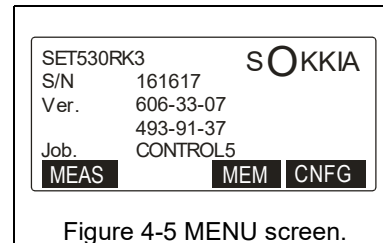


Figure 4-5 MENU screen.

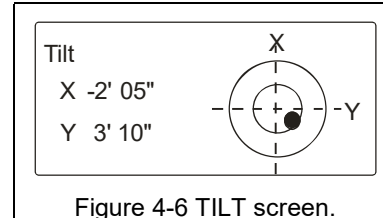


Figure 4-6 TILT screen.

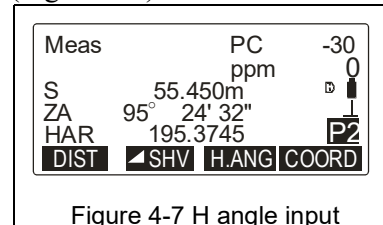


Figure 4-7 H angle input

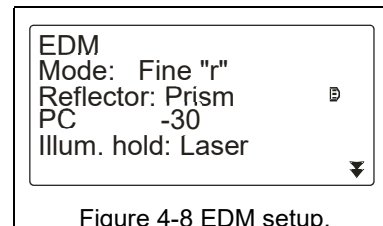


Figure 4-8 EDM setup.

EDM also allows input for non-standard prism constants. These constants arise when target prisms such as 360° prisms and 360° mini prisms are used. The normal Sokkia APS12-MAR target set using an AP01AR 62mm diameter prism has a prism constant of -30 mm. This is shown on the screen as **PC - 30**. The distance measured through this prism is 30 mm too long because of the length of mechanical construction of the target, the reflected beam in the prism and the change of light velocity through the prism glass. This extra length is subtracted by the on-board controller. Most circular 62mm and “peanut” prisms have a PC - 30. The PC is marked on the prism, must be set on the instrument and recorded in the field book.

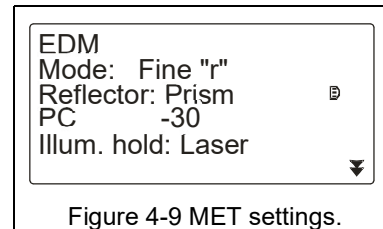


Figure 4-9 MET settings.

Prominent exceptions are the Leica Geosystems GPR121/111, GPH1P Circular prisms with a PC of -34.4. (4.4mm different from most others). If you work with Leica Total Stations use only their prism sets and be very mindful of the use of Additive Prism Constants (APC) for their various prism types. Table 4-1, page 88 summarises various APCs.

The **SFT** button toggles through the prism types in the EDM menu in the **Meas** screen, circular prism, **PC -30** (or as defined in setup) **SFT**
flat reflector sheet, **PC 0** and 
reflectorless, **PC 0**.

The 360° Mini-prism, used in Field Practical 3 (see Appendix A2-3), has a **PC +3**. The circular prism, **PC -30**, can be used and a correction of 0.027 (27 mm) is applied to calculations of the recorded field slope distance data. This method may eliminate the need to set-up a new PC value in the Total Station.

ESC button escapes the current operation,

BS button activates Back Sight menu screen.

use **H.ANG** function on the **P2 Meas** screen
to input orientation for this exercise.

FUNC button toggles the **Meas** screen.

The square red plastic window is used by a remote controller handset.

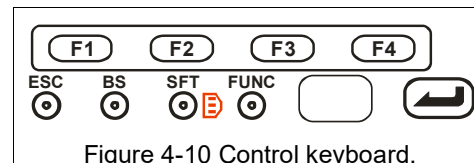


Figure 4-10 Control keyboard.

4.2.4.3 Setting initial direction using a tubular (trough) compass

The Sokkia Total Stations are equipped with a tubular compass that is attached to a slot in the instrument carrying handle. The CP7/CP8 compass allows setting the instrument to the local magnetic north direction. The compass comprises a tube containing a compass needle, needle pivot and a clamping screw. A properly balanced free needle lies horizontally aligned in the magnetic field.

A pointer on the south end of the needle is viewed through a window and aligned between two index lines, the lubber lines. Attached to the handle of the set-up TS, the instrument is turned towards North until the south end of the needle swings freely and stabilises between the lubber lines. Clamped, and using the slow-motion screws, the TS is aligned to 360° MAGNETIC. (Aligned with the magnetic meridian.)

Adding the magnetic declination of the place to 360° results in the TRUE meridian direction of the pointing. Appendix A2-3 and Field Practical 3 demonstrate the use of the compass for azimuth setting.

True direction (T) = Magnetic (M) + deviation (D).

Setting magnetic North, M = 360°, where, in Perth deviation is about - 1° 35', T = 358° 25'.



Figure 4-11 Tubular compass.

Canberra's deviation is $+12^\circ 20'$. $T = 12^\circ 20'$. Clamped on magnetic north, the TRUE direction is entered using the **[F3] H.ANG** function and thus sets the alignment of the TS closely with the geographic meridian. (True North).

The compass needle must be balanced against the magnetic inclination of the area. Perth's inclination is -66° . A compass balanced in Japan (inclination $+49^\circ$) must be re-balanced.

4.3 Measurements

4.3.1 Distance Measurements

Distance is a linear measurement of length and can either be vertical, horizontal or slope (slant). It can be measured either directly (e.g., using a measuring tape, chaining, and pacing) or indirectly using e.g., an electronic distance measurement (EDM) device fitted in the Total Station. Indirect distance measuring can take on the form of geometrical (e.g., optical) or electronic (e.g., wave based). Distances find use as follows:

- Horizontal and Vertical Distances
 - ✓ Mapping
 - ✓ Control surveys
 - ✓ Engineering design works
- Slopes and Vertical Distances
 - ✓ Setting out construction sites
 - ✓ Vertical distances are useful in height transfer from floor to floor in multistorey building and in mining (surface to underground)

EDM is the most common method of distance measurement now and exists in Total Stations, hand-held distance meters (Disto) and laser scanners. They can be classified according to:

- ✓ Radiation source: optical (visible and NIR) or microwave
- ✓ Measurement principle: phase difference or pulse
- ✓ Whether a reflector is required or not (reflector-less)

4.3.1.1 Phase difference method

(Figure 4-12) is the most common (now) method of distance measurement found in surveying instruments. In Figure 4-12, the

- emitted signal $y = \arcsin(\omega_m t)$
- received signal $y = \arcsin(\omega_m t - \phi)$

Note that ω_m denotes the angular frequency of the modulating wavelength. The EDM instrument measures the phase difference between the emitted and received signals, $\Delta\phi$. The known modulating wavelength, λ_m , can be used to convert the phase difference into the elapsed time between emission and reception. Only fractions of one complete cycle (wavelength) can be measured, thus, there remains an unknown integer number of cycles (the ambiguity), n , between the instrument and reflector. The two-way flight time (time between emission and reception), Δt , is given by:

$$\Delta t = \left(\frac{\Delta\phi}{2\pi} + n \right) \frac{\lambda_m}{v}$$

- The distance is derived by multiplying the time difference by the velocity. However, since Δt represents the round-trip (i.e. two-

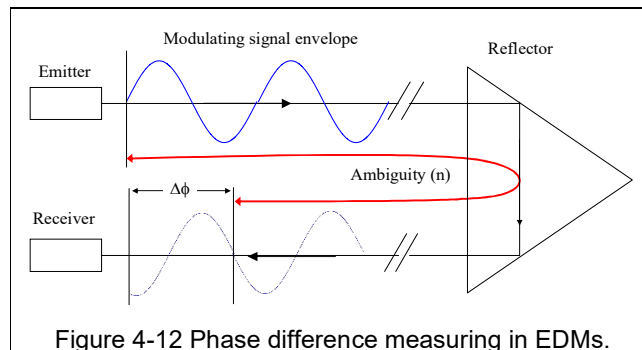


Figure 4-12 Phase difference measuring in EDMs.

way flight) time, it must be divided by 2 to obtain the distance between instrument and reflector,

$$d = \left(\frac{c\Delta t}{2} \right)$$

4.3.1.2 Pulse method

(Figure 4-13) is becoming more common in EDMs, particularly reflectorless instruments. Instead of continuous amplitude modulation, the carrier signal (visible or NIR light) is modulated into discrete pulses. The distance is derived from the two-way flight time of the pulse.

$$2d = v(t_r - t_e),$$

$$d = \frac{v(t_r - t_e)}{2} = d = \frac{c\Delta t}{2}$$

Since $c \approx 300 \text{ mm/ns}$, 10 mm range resolution requires 67 pulse time interval measurement resolution.

4.3.1.3 Reflector-less EDM

Many new Total Stations offer two means of EDM

- ✓ Phase-difference based, using a prism
- ✓ Reflector-less, using a pulsed laser

Laser scanners (terrestrial and airborne) are reflector-less. Laser scanner systems featuring the phase-difference method and the pulse method exist. One terrestrial laser scanner features both.

4.3.2 Prisms

Prisms used for EDM are glass corner cubes that reflect the incident electromagnetic (EM) radiation back to the instrument.

- ✓ This is because the apex angle of the cube is 90° .
- ✓ Multiple prism clusters are used for long distances.
- ✓ Omni-directional prisms are useful for topographic surveys.

4.4 Prism Constant, Why All the Fuss?

4.4.1 How a Prism is Measured, and its PC is Determined?

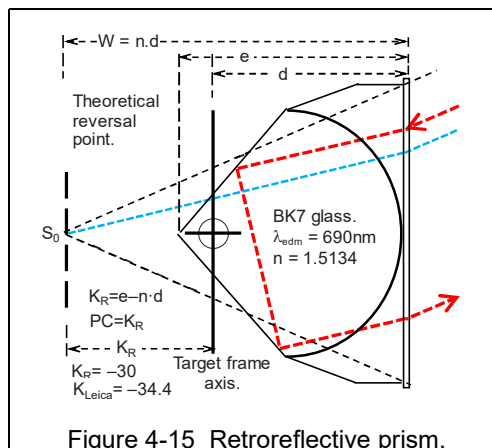


Figure 4-15 Retroreflective prism.

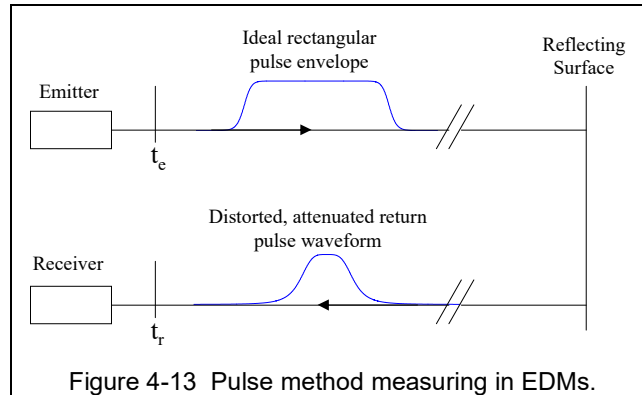


Figure 4-13 Pulse method measuring in EDMs.

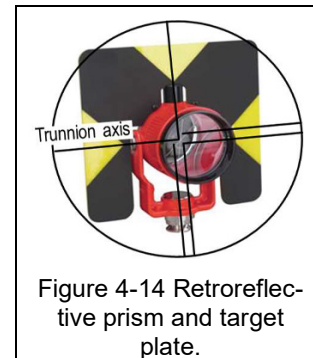


Figure 4-14 Retroreflective prism and target plate.

Distance is measured using a modulated light beam reflected to the instrument by a **corner prism**. The corner prism is used to ensure that the reflected beam is parallel to the incoming measuring beam. It is reflected off the internal faces of the cube.

The velocity of the signal through the prism is retarded by a factor known as the **refractive index** of the glass. It is dependent on the composition of the glass and on the wavelength, λ , of the signal.

BK7 is a borosilicate fused glass with excellent characteristics for prisms. It has high transmissivity, typically over 95% from far infrared (2,000nm)

through the visible spectrum (red, 700nm – violet, 380nm) to the near ultraviolet (350nm).

Refractive index, n_d , ranges from 1.513 at 700nm to 1.536 at 365nm. The n_d curve is flatter at the red end of the visible spectrum, allowing the use of red lasers for EDM. The Abbe number, $V_d = 65$, is a measure of low dispersion.

The optical path length of the light within the prism in each case in Figure 4-16 is $(2a + 2b)$, which is equivalent in both cases to twice the base-apex distance, d , since $b=b'$.

Since the index of refraction (n) of the glass is higher than that of air (≈ 1.52), the measured distance is longer. Why?

The base-apex distance d (Figure 4-16) can be determined from the vacuum-equivalent flight time of the signal in the glass, Δt^* , as:

$$d = \frac{c\Delta t^*}{2n_g}$$

which can be rearranged as:

$$\Delta t^* = \frac{2n_g d}{c}$$

For the same flight time in air, which is assumed by the instrument, the biased base-apex distance, d^* , is given by:

$$d^* = \frac{c\Delta t^*}{2n_a} = \frac{c}{2n_a} \cdot \frac{2n_g d}{c} = \frac{n_g}{n_a} d$$

Assuming the index of refraction of air, n_a , is 1, then the zero of the prism, i.e., the plane to which the distance measurement refers (the air-equivalent path) is given by $(n_g \cdot d)$. Furthermore, the apex of the prism may be offset from the vertical axis of the target or pole upon which it is mounted. These offsets represent systematic errors that must be modelled and applied as corrections to the measured distances. A measured distance is corrected as follows

- ✓ Subtract the air-equivalent path distance ($n_g \cdot d$) to reduce the distance to the face of the prism (referred to as the Additive Constant, AC). Figure 4-17.
- ✓ Add the offset from the prism face to the vertical axis, b .

Thus, absolute prism constant is given by (Figure 4-17) as:

	$PC = -n_g d + b$	Eqn 4-1

In practice, the combined effect (i.e. prism constant) is determined (through calibration) and input to the Total Station instrument rather than determining the individual components. Prism-EDM combinations are calibrated together since the estimated prism constant will also contain a component due to the offset of the EDM zero reference from the Total Station vertical axis.

4.4.1.1 Examples: Typical 62mm diameter prism

Mechanically BK7 is a good glass. It also has a very low level of bubbles and inclusions, is acid resistant and has low water absorption. Its relatively low melting point and annealing temperatures make it relatively inexpensive to produce.

But one must pay for the precision of the machining of the 90° prism corners.

The ray of light is reflected off the theoretical reversal point, a path about 1.5 times the depth of the prism. The target frame vertical axis depends on the design **prism constant**, K_r .

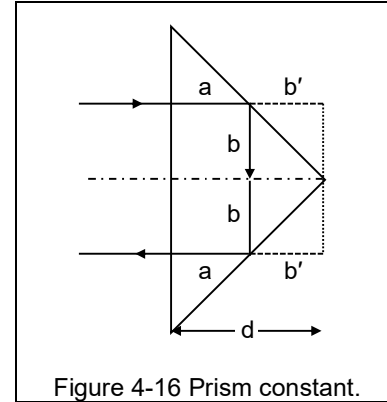


Figure 4-16 Prism constant.

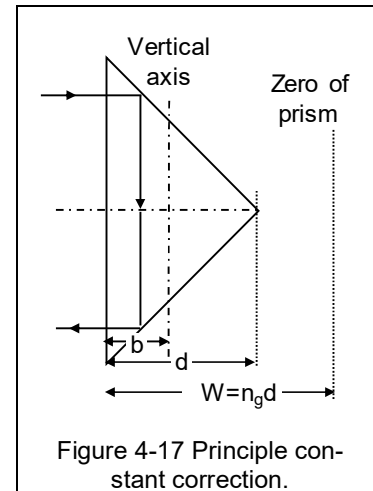


Figure 4-17 Principle constant correction.

A typical 62mm prism has a depth of 47.5mm. The Sokkia EDM has a wavelength of 690nm. With $n_g = 1.5134$, $W = 71.9\text{mm}$ and prism constant, $K_r = -30$, the plumb axis of the prism is $71.9 - 30 = 41.9\text{mm}$ behind the face of the prism.

Most prism set manufacturers provide a $PC = -30$.

Be aware that the Leica Geosystems standard round 62mm prisms, the GPR series, have a $PC -34.4\text{mm}$. This value shows on the Leica EDM setting screen as CIRC 0. It is handled internally in the EDM software to display the correct distance.

There are a number of other prism sets available and the EDM **PC** setting in the Total Station has to be set for the prism. The PC , or K_r , depends on the mechanical construction of the prism.

Peanut prisms of about 25mm diameter use manufacturer specific carriers to provide a $PC -20$ or a $PC 0$, allowing measurements to a corner or restricted face.

360° prisms have PC s depending on the physical size of the 6-prism cluster. Two different 360° prisms used at Curtin are:

Bear 360 mini-prism, $PC = +3\text{mm}$, used in field exercises, and

Bear 360 large prism, $PC = +13\text{mm}$

Reflectorless observations, or observations to target sheets, use a $PC = 0$.

For all instruments other than Leica Geosystem products, use the PC specified for the particular prism and set this in the EDM PC .

4.4.2 The Leica Geosystems Total Station Prism Constant Conundrum

Leica instruments have a strong presence in the mining industry, and in general surveying. If you are using Leica equipment, use Leica targets and enter the correct constant in the EDM.

If you are using a non-Leica prism then DEFINE or set USER according to the following:

Leica USER $PC = \text{Prism offset } (K_r \text{ or } PC, \text{ generally negative}) + 34.4$

$PC -30 \quad \text{USER } PC = -30 + 34.4 = +4.4$

$PC -40 \quad \text{USER } PC = -40 + 34.4 = -5.6$

Source: Leica FAQ – Prism Offsets. May 2002.

Finding the PC for a Leica defined prism: $PC = \text{Leica constant } -34.4$.

Table 4-1 presents:

- the Leica prism, the constant defined in the EDM software and the corresponding PC for other EDMs because the prism is defined, there is no need to provide a USER PC to the instrument
- other prisms. Set the Leica EDM to 0 (circular, GPR series). Then set Leica USER PC as tabled.

Note that reflective tape, $PC = 0$, can be set in the Leica EDM with constant $+34.4$ or with constant 0.0 and the Leica USER PC as $+34.4$.

Table 4-1 Prism offsets.			
Leica Prism	Leica constant	Other EDM PC	Leica USER PC
GPR121 (62mm)	0.0	-34.4	
GMP101 (mini prism)	+17.5	-16.9	
Reflective tape	+34.4	0.0	
GRZ4/GRZ122 (360° prism)	+23.1	-11.3	
GRZ101 (360° mini prism)	+30.0	-4.4	
Other prisms	Set Leica constant	Other EDM PC	Set Leica USER PC
62mm circular (PC -30)	0.0	-30	+4.4
62mm circular (PC -40)	0.0	-40	-5.6
Bear 360° mini prism	0.0	+3	+37.4
Bear 360° large prism	0.0	+13	+47.4
Reflective tape	0.0	0	+34.4

Be sure you KNOW how to set the correct PC in YOUR EDM. Check the prism manufacturer's specifications. It should be labelled on the prism or specified in a catalogue.

As a precaution, always book the prism used, its PC and the PC you set in the EDM menu. And check it during your observations.

4.4.3 Prism Constant Calibration:

- ✓ A simple method for checking or determining the prism constant involves measurement of three distances between points set out on a long line. Set three tripods on a level straight line, attach a second tribrach base. Test only one prism. Use forced centring methods.
- ✓ For each distance the following equation can be written $AC = AB + BC$.

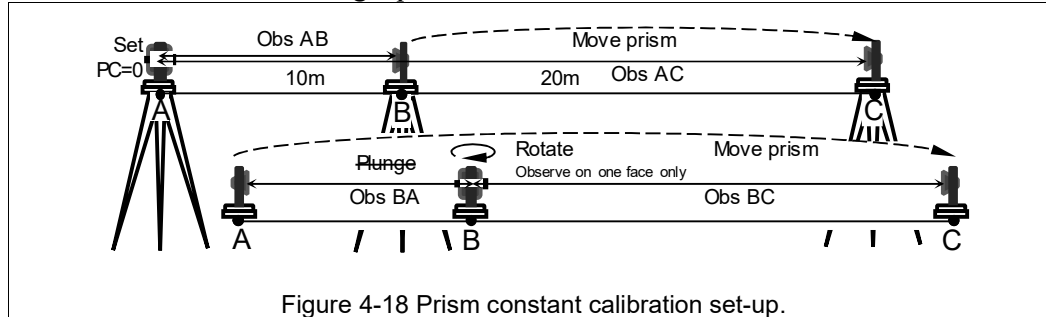


Figure 4-18 Prism constant calibration set-up.

From the condition that $AC = AB + BC$, the prism constant, PC, is determined as:

$AB = AB^{obs} + PC, BC = BC^{obs} + PC, AC = AC^{obs} + PC$ $AB + BC = AC$ $AB^{obs} + PC + BC^{obs} + PC = AC^{obs} + PC$ $PC = AC^{obs} - AB^{obs} - BC^{obs}$	Eqn 4-2

4.4.3.1 Scale error

Perhaps the second-most important instrumental systematic error (particularly in phase-difference instruments) is the scale error. The time-of-flight equation for the phase-difference method can be recast in terms of modulating frequency, f_m ,

$\Delta t = \left(\frac{\Delta \phi}{2\pi} + n \right) \frac{\lambda_m}{v} = \left(\frac{\Delta \phi}{2\pi} + n \right) \frac{1}{f_m}$	Eqn 4-3

- ✓ Scale (i.e. distance-dependent) errors can exist due to errors in the modulating frequency caused by the oscillator – usually expressed as parts per million (ppm = 10^{-6}).
- ✓ In calibrating the scale error, care must be taken to properly model and remove atmospheric effects (particularly temperature), which have a similar systematic effect on distances. Usually corrected under the total station EDM correction menu.

4.4.3.2 Atmospheric effects.

Accurate measurements are essential to accurate atmospheric EDM corrections. Precision of measurement is specified to correct distance measurements to manufacturer's tolerances.

Handheld specialist weather stations provide accurate values; time stamped local weather station recordings provide less accurate data. Forecast data is better than no data.

Weather reports and forecasts provide pressure at Mean Sea Level (MSL or 0 AHD).

Pressure decreases at 1hPa every 9m above MSL pressure.

Temperature, t , pressure, p , and humidity (partial pressure of water vapour), e , affect the index of refraction of air, which is calculated by

$(n_a - 1) \times 10^6 = \frac{273.15 n_g p}{(273.15 + t) \times 1013.25} - \frac{11.27 e}{273.15 + t}$ $n_g = (n_g - 1) \times 10^6 = 287.6155 + \frac{4.88660}{\lambda^2} + \frac{0.06800}{\lambda^4}$	Eqn 4-4

where N_g is the group refractive index and λ is the carrier wavelength (nm).

At the observation site the effects are:

- Temperature (largest): 1 ppm per °C (1 ppm is 1 mm at 1km).
- Absolute pressure: 0.3 ppm per hPa. (Decreases at 1hPa every 9m above MSL pressure).
- Partial pressure of water vapour: 0.04 ppm per hPa.
- Atmospheric variations in Temperature (t), Pressure (p) and Humidity (e)
- result in variations in refraction.

4.4.3.2.1 Sokkia SET 530RK3 atmospheric correction factor example

Manufacturers provide instrument atmospheric correction factors which depend on the EDM wavelength. Correction equations, similar to Eqn 4-5, are provided for each EDM machine.

The distance correction is $D = d + dC$, where C in ppm. For the Sokkia SET530RK3:

$C = 282.59 - \frac{0.2942 p}{1 + 0.003661 t} + \frac{0.0416 e}{1 + 0.003661 t} \text{ where}$ $e = h \frac{E}{100} \text{ and } E = 6.11 \times 10^{\left(\frac{7.5t}{t+273.3}\right)}, \text{ and :}$	Eqn 4-5

t – Air Temperature in degrees Celsius (C°).

p – Absolute air pressure in Hectopascals (hPa) (actual pressure at observation site).

e – Water vapour pressure (hPa).

h – Relative humidity, RH, (%).

E – Saturated water vapour pressure (hPa).

Consider: $t = 35^\circ\text{C}$, $d = 1234.660$, and pressure, $p = 975\text{hPa}$. Correction due to humidity is small and can be ignored. $T =$

$C = 282.59 - \frac{0.2942 \times 975}{1 + 0.003661 \times 35} + \text{ignore humidity}$ $C = 282.59 - 254.265 = 28.3 \text{ ppm}$ $D = 1234.660 + (1234.66 \times 28.3 \times 10^{-6}) = 1234.660 + 0.034$ $D = 1234.694$	

Next, let us consider $t = 15^\circ\text{C}$, $d = 1234.660$, and pressure, $p = 1020\text{hPa}$. Correction due to humidity can be ignored.

$C = 282.59 - \frac{0.2942 \times 1020}{1 + 0.003661 \times 15} + \text{ignore humidity}$ $C = 282.59 - 254.463 = -1.9 \text{ ppm}$ $D = 1234.660 + (1234.66 \times -1.9 \times 10^{-6}) = 1234.660 - 0.002$ $D = 1234.658$	

4.4.3.3 Periodic (Cyclic) errors

- ✓ These exist due to electrical or optical crosstalk (interference) within the EDM instrument.
- ✓ They are on the order of a few mm and behave as sinusoids with wavelength equal to the unit length, which for most EDMs is 10 m.
- ✓ They can have both sine and cosine components.

4.4.4 Summary of Major Error Sources

- Systematic
 - ✓ Instrumental
 - Prism constant
 - Scale
 - Cyclic
 - ✓ Natural: index of refraction
- Random
 - ✓ EDM precision (measure of random error dispersion) is typically specified in the format of $\pm(a \text{ mm} + b \text{ ppm})$
 - ✓ There is a constant part and a distance-dependent part
 - ✓ Example: $\pm(2 \text{ mm} + 2 \text{ ppm})$.

4.5 Angular Measurements

Surveyors in Australia, as in many Commonwealth countries, use the sexagesimals system for measuring and reporting angles.

- ✓ There are 360° in one circle
- ✓ Each degree is divided into 60 minutes (arcminutes), $1^\circ = 60'$
- ✓ Each minute is divided into 60 seconds (arcseconds), $1' = 60''$

We are interested in how these angles are measured:

- ✓ The Total Station is levelled so that horizontal angles or directions are measured in a horizontal plane (see Figure 4-1 (a)).
- ✓ Zenith angles (see Figure 4-1 (b)) are referenced to the zenith, which for a levelled instrument coincides with the local gravity vector.
- ✓ Elevation/depression angles are referenced to the horizontal plane.
- ✓ If the instrument is not properly levelled or not in correct adjustment, systematic errors will exist in the angles.
- ✓ These errors will propagate into subsequently computed co-ordinates.

To measure an angle, let us consider Figure 4-19. We are interested in measuring the angle between Stn 1 and Stn 3 with the instrument set at Stn 2. Once the instrument has been set and levelled at Stn 2 as discussed in §4.1, the instrument's telescope is pointed to Stn 1 while on Face Left (FL).

Ensure that the crosshairs of the instrument accurately bisect the centre of the reflector at Stn 1 then lock the instrument. The slow-motion screw is used to perfect the bisection. With the instrument locked, the reading on the instrument is now set at $0^{\circ} 00' 00''$ (or any arbitrary reading). This is the back sight (BS) reading.

Taking into consideration that angular measurements in surveying are performed clockwise, unlock the instrument and slowly turn it towards Stn 3. Bisect the reflector's centre using the crosshairs and then lock the instrument and sight the target with the slow-motion screws. The horizontal reading of the instrument (HAR for Sokkia) will now indicate $60^{\circ} 20' 40''$, i.e., the fore sight (FS) reading. The equivalent vertical angle can also be read on the instrument given as ZA (for Sokkia).

The horizontal angle Stn 1-Stn 2 – Stn 3 is given as the difference between FS and BS, i.e., angle (Stn 1-Stn 2-Stn 3) = FS-BS = $60^{\circ} 20' 40'' - 0^{\circ} 00' 00'' = 60^{\circ} 20' 40''$.

In §4.6.3, Table 4-2, measurement of angles in both Face Left (FL) and Face Right (FR) and the booking of the measured angles are illustrated. Students are encouraged to study this example. Note that in this Example, the booking format is just one type of the several in use.

Previously, using theodolites, the angle Stn 1-Stn 2- Stn 3 would be measured several times with different starting circle readings to Stn 1 and the mean taken. For example, if, instead of the reading Stn 2 - Stn 1 being $0^{\circ} 00' 00''$, the moveable horizontal circle is now be rotated to be set to $60^{\circ} 20' 40''$ and, if the reading to Stn 3 now read as $120^{\circ} 40' 30''$, the measured horizontal angle Stn 1 - Stn 2 - Stn 3 is FS-BS = $120^{\circ} 40' 30'' - 60^{\circ} 20' 40'' = 60^{\circ} 19' 50''$.

The measured horizontal angle Stn 1-Stn 2 - Stn 3 now becomes the mean of the two angles $(60^{\circ} 20' 40'' + 60^{\circ} 19' 50'')/2 = 60^{\circ} 20' 15''$. The advantage of repeated measurement lies in the increased redundancies and improved precision for less precisely graduated instruments.

For multiple angle measurements with a Total Station a new horizontal direction (HAR) must be entered. This provides only a mathematical rotation of the direction. It does not rotate the (fixed) horizontal encoder circle.

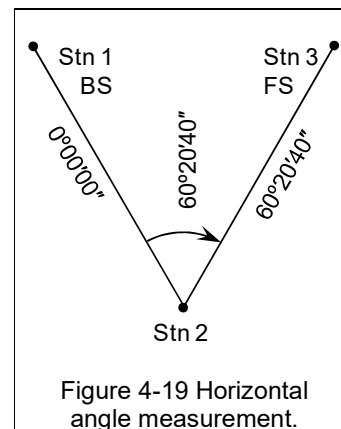
4.6 Combined Total Station Measurements

Measurement with the Total Station provides the vector components; direction, vertical angle, and distance. Proper recording of this data is vital to the successful conduct of observations at a point. Apart from the measurements, other components of the session need to be measured and recorded.

4.6.1 Station Records

All information pertinent to the occupied station needs to be recorded in a Field Book.

1. Task, personnel, date.
2. Name, description, location, positional data (coordinates, RL), access and station diagram (for later station recovery).
3. Instrument data; make, model, serial number
4. Set-up data; height of instrument (HI) above station, prism constant, atmospheric corrections applied.
5. Target data; prism make, model, S/N. Height of target (HT) above ground point.
6. Control points observed; name, description, location, positional data (coordinates, RL), access and station diagram (for later point recovery).
7. Orientation method; points used, method used (random, magnetic meridian, local grid). Points used to establish orientation, back-sight (BS) description.



4.6.2 Recording Observations

Observations are recorded in a field book for later reduction of the data. The data must be recorded accurately by the booker, ensuring consistency in multiple measurements. To stable targets distances will not vary by more than a few millimetres, and over reciprocal lines the agreement with the forward observations should be very similar.

It is basically impossible to re-set over a point once the instrument has been moved. The entire task must be repeated. Be sure you have all the data before you move to the next point.

4.6.3 Example Recording of Observations from an Initial Station (Table 4-2)

Stn 1001, 25mm peg at G/L			Task: <i>Determine position by distances</i>				21/11/2016	
TS Sokkia	SET530RK3	S/N 08/7		Prisms: APS12. PC -30				
GROUP: <i>Fri 3-6</i>		Observer:	<i>B Lee</i>		Booker:		<i>A. Smith</i>	
BRG	360° <i>Mag meridian</i> 358°21'10"			PC -30	PPM = 10		HI	1.550
AT/TO			HAR	ZA	S	H	V	Remarks
AT		1001						<i>HI = 1.585</i>
<i>B/S: SE 105</i>		<i>L</i>	19 25 10		<i>B/S to corner BLD 105</i>			
		<i>Mean</i>	19 25 23					
<i>SE 105</i>		<i>R</i>	199 25 36					
<i>Mean incl angle: SE 105 - 705A 42 52 57</i>								
705A		<i>L</i>	62 18 35	90 36 50	65.428			<i>HT = 1.587</i>
		<i>Mean</i>	62 18 20					
705A		<i>R</i>	242 18 45	269 23 20	65.429			
<i>Incl. Angle 705A - C400</i>				76 22 20				
C400		<i>L</i>	138 40 35	92 11 20	59.583			<i>HT = 1.535</i>
		<i>Mean</i>	138 40 40					
C400		<i>R</i>	318 40 45	267 48 30	59.584			

4.7 Computations

4.7.1 Manipulating VECTORS

Coordinate system calculations.

There are two basic survey calculations which confront us when using co-ordinates.

- (1) The Join Calculation, § 4.7.1.1

Given the co-ordinates of two points A and B, determine the **bearing** and **distance** between them.

- Convert RECTANGULAR to POLAR vector.

- (2) The Polar Calculation, § 4.7.1.2

Given the co-ordinates of a point A (E_A , N_A) together with the bearing and distance from point A to point B, determine the co-ordinates of point B (E_B , N_B).

- Convert POLAR vector to RECTANGULAR coordinates.

In Figure 4-20, the coordinates of points A and B are represented by (E_A , N_A) and (E_B , N_B), respectively. The bearing of AB is shown as θ_{AB} (i.e. an angle clockwise from the north in the first quadrant). Distance $AB = D_{AB}$.

Notice that the direction of the vector is indicated by the vector subscripts. To calculate the intended vector, the convention is:

The bearing θ_{AB} is the direction **from A to B**.

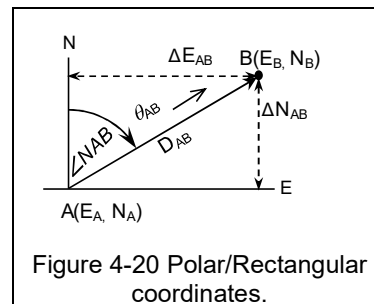
The distance $AB = D_{AB}$ is the distance **from A to B**.

Self-evident maybe, but in vector terms, the distance $BA = D_{BA}$ is negative. ($D_{BA} = -D_{AB}$).

In expressing the coordinate **differences**, i.e. ΔE_{AB} and ΔN_{AB} , the difference convention is:

the point you are going **to** minus the point you are coming **from**.

$$\Delta E_{AB} = E_B - E_A \text{ and } \Delta N_{AB} = N_B - N_A$$



4.7.1.1 The Join calculation

(The polar vector *joining* the two rectangular coordinates)
[RECTANGULAR to POLAR].

Given: The co-ordinates of A (E_A , N_A) and the co-ordinates of B (E_B , N_B). (See Figure 4-21).

Find: The bearing and distance A to B,
i.e. θ_{AB} , D_{AB} .

Solution: Find the **angle** formed with reference to the **North/South** axis of the reference frame.

$$\Delta E_{AB} = E_B - E_A$$

$$\Delta N_{AB} = N_B - N_A$$

$\tan \angle A = \frac{\Delta E_{AB}}{\Delta N_{AB}},$ $\theta_{AB} = \arctan\left(\frac{\Delta E_{AB}}{\Delta N_{AB}}\right) \text{ in the correct quadrant.}$	Eqn 4-6

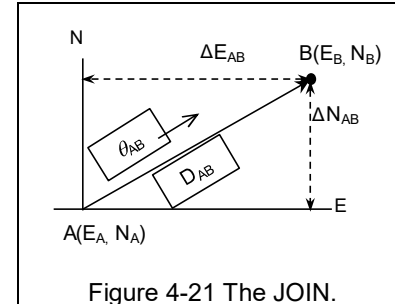


Figure 4-21 The JOIN.

Hypotenuse from Pythagoras.	$D_{AB} = \frac{\Delta E_{AB}}{\sin(\theta_{AB})} = \frac{\Delta N_{AB}}{\cos(\theta_{AB})} = \sqrt{\Delta E_{AB}^2 + \Delta N_{AB}^2}$	Eqn 4-7

4.7.1.1.1 From the arctangent angle, find the bearing of AB in all 4 quadrants?

Evaluate the **angle** generated. Examine the **sign** of the result.

Tan is positive in the 1st (I) and 3rd (IV) quadrants.

Thus, our method uses the **sign** of the NORTH component (ΔN).

$\Delta N > 0$ (**positive**): $\theta_{AB} = 360 + \angle A$.

If $\theta > 360$: $\theta = 360 \text{ MOD } \theta$ Quadrant I, IV

$\Delta N < 0$ (**negative**): $\theta_{AB} = 180 + \angle A$.

($90 < \theta < 270$) Quadrant II, III

The $360 \text{ MOD } \theta$, or $360 \text{ MODULO } \theta$, operator reduces the value of θ to a range $0 \rightarrow 360$. ($0 > \theta > 360$)

$\Delta N = 0$ (**zero**): $\arctan\left(\frac{\Delta E}{0}\right)$ cannot be evaluated; calculator

gives "Divide by 0" error message.

$\tan(90^\circ)$ is undefined as $\lim_{\theta \rightarrow 90^\circ} [\tan \theta] \rightarrow \infty$ (tends to infinity)

thus for $\Delta N = 0$, $\theta = 90^\circ$ or 270° .

(For convenience we say $\tan(90^\circ) = \infty$).

$\Delta E = 0$:

$\text{Atan}\left(\frac{0}{\Delta N}\right)$ is evaluated correctly as 0° ; and if $\Delta N < 0$ (negative) then $\theta = 180^\circ$

Test statement **if** $\Delta N = 0$ **AND** **if** $\Delta E > 0$ **then** $\theta = 90^\circ$
 else if $\Delta E < 0$ **then** $\theta = 270^\circ$

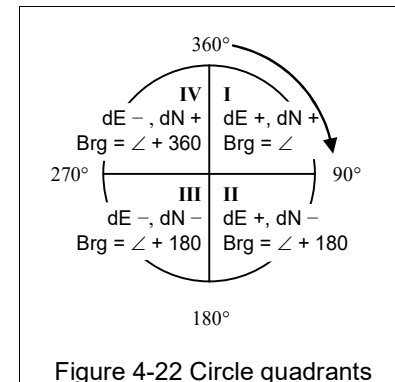


Figure 4-22 Circle quadrants

**4.7.1.1.2 Illustrating the bearing of AB in all 4 quadrants. Eqn 4-6,
Figure 4-23 - Figure 4-26**

Quadrant 1: ($0 \rightarrow 90^\circ$). Tan $\angle A$ is **POSITIVE**

$$\tan \angle A = \frac{\Delta E_{AB}}{\Delta N_{AB}}, \Delta N > 0 (+ve)$$

$$\theta_{AB} = 360 + \arctan\left(\frac{\Delta E}{\Delta N}\right)$$

$$\theta_{AB} = 360 + (+\angle), \theta_{AB} = 360 \text{ MOD } \theta$$

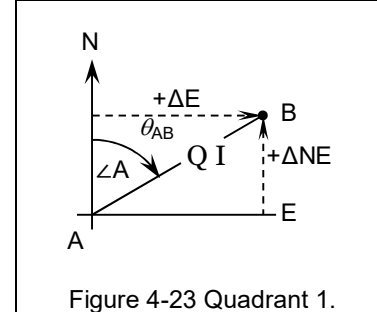


Figure 4-23 Quadrant 1.

Quadrant 2: ($90^\circ \rightarrow 180^\circ$). Tan $\angle A$ is **NEGATIVE**

$$\tan \angle A = \frac{\Delta E_{AB}}{\Delta N_{AB}}, \Delta N < 0 (-ve)$$

$$\theta_{AB} = 180 + \text{Atan}\left(\frac{\Delta E}{-\Delta N}\right)$$

$$\theta_{AB} = 180 + (-\angle)$$

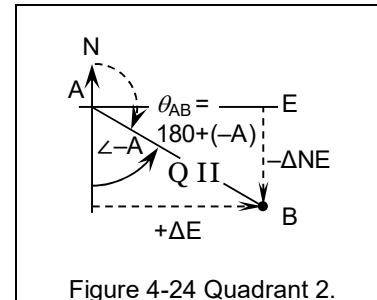


Figure 4-24 Quadrant 2.

Quadrant 3: ($180^\circ \rightarrow 270^\circ$). Tan $\angle A$ is **POSITIVE**

$$\tan \angle A = \frac{\Delta E_{AB}}{\Delta N_{AB}}, \Delta N < 0 (-ve)$$

$$\theta_{AB} = 180 + \text{Atan}\left(\frac{-\Delta E}{-\Delta N}\right)$$

$$\theta_{AB} = 180 + (+\angle)$$

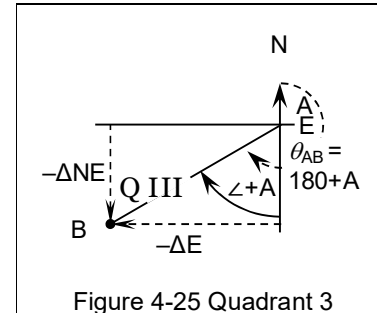


Figure 4-25 Quadrant 3

Quadrant 4: ($270^\circ \rightarrow 360^\circ$). Tan $\angle A$ is **NEGATIVE**

$$\tan \angle A = \frac{\Delta E_{AB}}{\Delta N_{AB}}, \Delta N > 0 (+ve)$$

$$\theta_{AB} = 360 + \text{Atan}\left(\frac{-\Delta E}{\Delta N}\right)$$

$$\theta_{AB} = 360 + (-\angle)$$

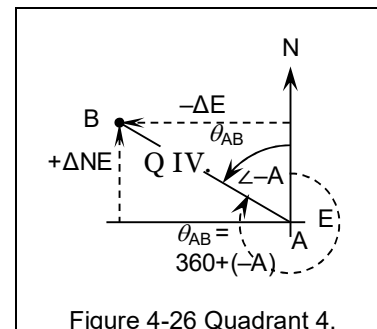


Figure 4-26 Quadrant 4.

4.7.1.1.3 Join calculation example

Given: The co-ordinates A (E_A , N_A) and B (E_B , N_B)

Find: The bearing and distance A to B

Format: Refer Eqn 4-6, Eqn 4-7

$$\text{To: } E_B \quad N_B \quad \tan \angle NAB = \frac{\Delta E_{AB}}{\Delta N_{AB}}, \theta_{AB} = \arctan \left(\frac{\Delta E_{AB}}{\Delta N_{AB}} \right) \text{ in quadrant}$$

$$\text{— From: } \frac{E_A \quad N_A}{\Delta E_{AB} \quad \Delta N_{AB}} \quad D_{AB} = \sqrt{\Delta E_{AB}^2 + \Delta N_{AB}^2}$$

Difference $\Delta E_{AB} \quad \Delta N_{AB}$

$$\begin{aligned} \text{CHECK: } E_B &= E_A + D_{AB} \cdot \sin(\theta_{AB}) \\ N_B &= N_A + D_{AB} \cdot \cos(\theta_{AB}) \end{aligned}$$

Method: **to find θ_{AB} , D_{AB} .**

1) Find the coordinate difference

$$\Delta E_{AB} = E_B - E_A \text{ and}$$

$$\Delta N_{AB} = N_B - N_A$$

2) calculate the Tangent of the **angle**:

$$\tan \angle A = \frac{\Delta E_{AB}}{\Delta N_{AB}}$$

using **sign** of ΔE_{AB} and ΔN_{AB}

3) find the Arctangent of the **angle**:

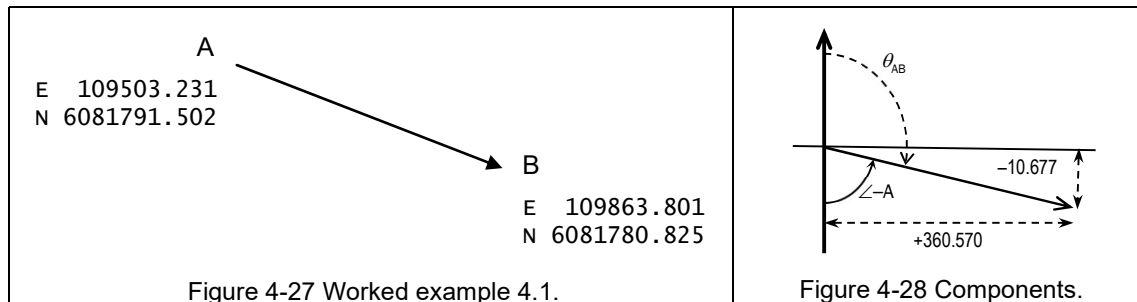
$$\angle A = \arctan \left(\frac{\Delta E_{AB}}{\Delta N_{AB}} \right)$$

calculate θ_{AB} **in correct quadrant**

4) and distance using Pythagoras' theorem $D_{AB} = \sqrt{\Delta E_{AB}^2 + \Delta N_{AB}^2}$.

4.7.1.1.4 Worked example 4.1

Compute the bearing and distance from A to B (see Figure 4-27)



	Easting	Northing
B	109863.801	6081780.825
– A	109503.231	6081791.502
	+360.570	–10.677

Figure 4-27

$\Delta N < 0$ Figure 4-28

$$\angle A = \arctan(360.571/-10.677) = \arctan(-33.7707) = -88^\circ 18' 14'' \text{ in quadrant II (Figure 4-24)}$$

$$\theta_{AB} = 180 + (-88^\circ 18' 14''),$$

$$\theta_{AB} = \mathbf{91^\circ 41' 46''}$$

$$D_{AB} = \mathbf{360.728}$$

CHECK.

$$\begin{aligned} E_B &= E_A + D_{AB} \cdot \sin(\theta_{AB}) \\ &= 109503.231 + 360.728 \cdot \sin(91^\circ 41' 46'') \\ &= 109503.231 + 360.570 \end{aligned}$$

$$E_B = 109863.801 \quad \checkmark$$

$$\begin{aligned} N_B &= N_A + D_{AB} \cdot \cos(\theta_{AB}) \\ &= 6081791.502 + 360.728 \cdot \cos(91^\circ 41' 46'') \\ &= 6081791.502 + (-10.677) \end{aligned}$$

$$N_B = 6081780.825 \quad \checkmark$$

4.7.1.2 The Polar calculation

(The coordinates created by a *polar* vector from a point)

[POLAR to RECTANGULAR]

Polar Calculation.

Given: A (E_A, N_A)

- together with the polar vector
- the bearing/distance A to B (Figure 4-29).

Find: The coordinates of B (E_B, N_B)

Now:	$\Delta E_{AB} = D_{AB} \sin \theta_{AB}$	Eqn 4-8
	$\Delta N_{AB} = D_{AB} \cos \theta_{AB}$	Eqn 4-9

$$E_B = E_A + \Delta E_{AB}$$

$$N_B = N_A + \Delta N_{AB}$$

Format:

	Brg/Dist	E	N
A	θ_{AB}	E_A	N_A
	D_{AB}	$\pm \Delta E_{AB}$	$\pm \Delta N_{AB}$
B	SUM	E_B	N_B

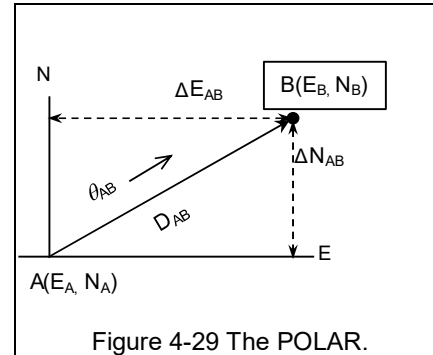


Figure 4-29 The POLAR.

4.7.1.2.1 Worked example 4.2.

Calculate the coordinates of B. Figure 4-30 .

Given: $E_A = 548979.504$, $N_A = 6444421.068$

$$\theta_{AB} = 221^\circ 03' 00''$$

$$D_{AB} = 522.211$$

$$\sin(221^\circ 03' 00'') = -0.675617$$

$$\cos(221^\circ 03' 00'') = -0.754137$$

$$\Delta E_{AB} = 522.211 \times -0.675617 = -342.945 \text{ (Eqn 4-8)}$$

$$\Delta N_{AB} = 522.211 \times -0.754137 = -393.818 \text{ (Eqn 4-9)}$$

Find: Co-ordinates of B (E_B, N_B)

	Brg/Dist	E	N
A		548979.504	6444421.068
	$221^\circ 03' 00''$, 522.211	$\Delta E -342.945$	$\Delta N -393.818$
B		548636.559	6444027.250

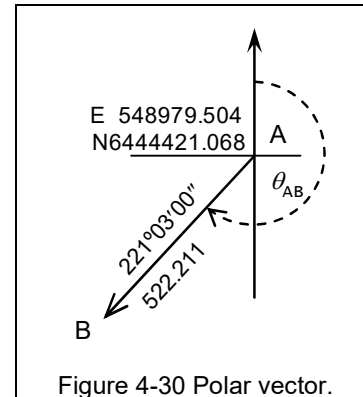


Figure 4-30 Polar vector.

4.7.2 Using the POLAR and RECTANGULAR Function on a Calculator

Calculators approved for engineering courses have built in functions to solve the join and polar problems, which are problems of conversion of a vector between a rectangular coordinate system to a polar coordinate system.

The methods are illustrated in the Appendix A3: **Survey Calculations on the HP 10s+.**

The HP10s+ is well suited to the surveying course because it stores the results of the **POL**(and **REC**(functions in memory for later recording or use in continued calculations.

4.7.3 Reduction of Electronically Measured Distance to the Spheroid

We have examined various aspects of errors that can occur in EDM derived distances, and it is best to examine the steps necessary to reduce a raw distance measured by EDM to the eventual mapping plane. In Australia, the eventual mapping projection, the MGA2020, is based on the GDA2020 using the GRS80 datum. From these final coordinates we can transform to other required projections.

Chapter 12 on coordinate transformations illustrates the transformation process between various coordinate sets. However, to produce coordinates, the observed distances must be reduced to the mapping plane, eventually to the MGA2020. The process is referred to as the “Reduction of distance to the spheroid/ellipsoid”.

Under normal observation conditions, measuring over distances up to a few hundred metres, there are four corrections to be made to a measured slope distance before it is incorporated in the map projection calculations.

4.7.3.1 Reduction to the plane

Mapping on the local plane, regardless of height above the geoid or the ellipsoid.

Reduction of slope distance to horizontal distance, as shown previously:

	$Horizontal\ distance, H = Slope\ distance, S \times \sin(Zenith\ angle, ZA)$	Eqn 4-10

- the level terrain distance from the observation point.

This is where we stop the distance reductions in producing our plan on the local plane.

The continuation of reduction of distances and directions for mapping to a projection is the purview of the surveyor.

The local plane may be expanded to a local projection over an extended area, the main object being that the scale factor over the projection is very close to 1, and the grid convergence spread evenly across a central meridian.

Mapping on the projection, defined by projection parameters, requires:

1. Reduction of level terrain distance to geoidal (sea level) distance.
 - application of a height correction.
2. Reduction of geoidal distance to ellipsoidal distance.
 - application of geoid/ellipsoid height correction.
3. Application of a point or line **scale factor** applicable to the distance from the central meridian.

4.7.3.2 Reduction to the ellipsoid

Reduction of the wave path distance, d_1 , (observed EDM distance) to the ellipsoidal chord distance, d_3 , using Clarke’s (1966) formula:

	$d_3 = \left[\left(d_2^2 - (h_A - h_B)^2 \right) / (1 + h_A R_\alpha)(1 + h_B R_\alpha) \right]^{\frac{1}{2}}$	Eqn 4-11

The reduced wavepath distance, d_1 , and the reduced slope distance, d_2 , are inseparable out to 15Km. Note that h_A and h_B are ellipsoidal heights; R_α is the radius of curvature the line azimuth. Figure 4-31 illustrates the relationship between various distances.

Height values on the AHD, H_A and H_B must incorporate the ellipsoid/geoid separation N . Over a small area common values for N (from Geoscience Australia) and R_α (6,366,000 at 32°S) would suffice for the reduction.

The ellipsoidal distance, s , equals the ellipsoidal chord distance, d_3 , for EDM to 4,000m.

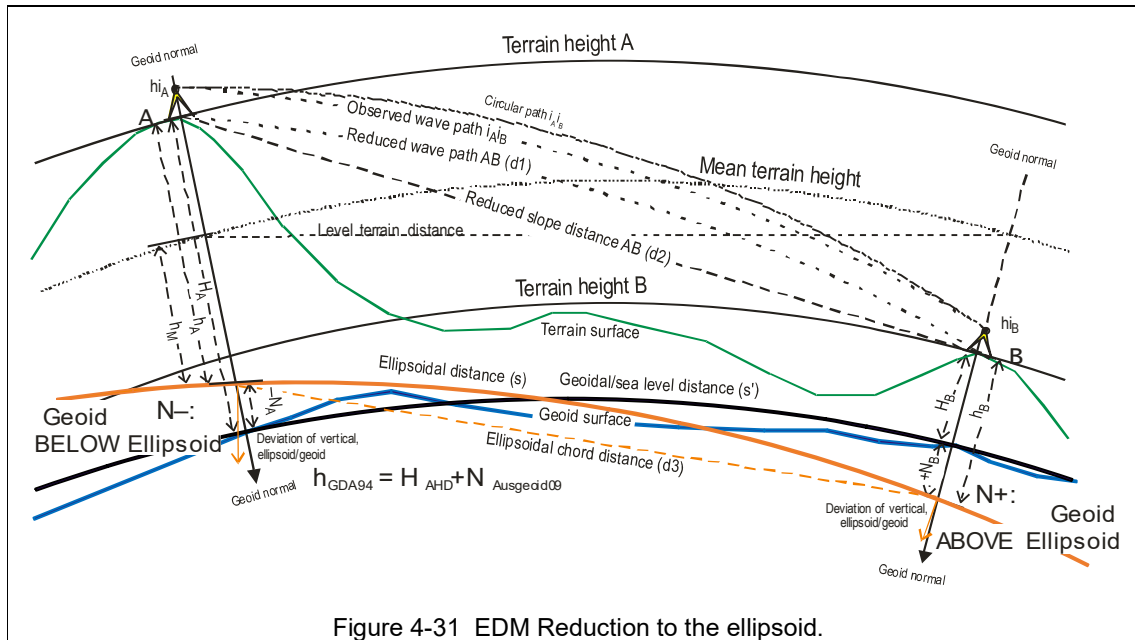


Figure 4-31 EDM Reduction to the ellipsoid.

4.8 Observation Blunders and Mistakes

The most likely sources of errors in observations will come from:

1. Incorrectly plumbed over point;
 - plate levels, unchecked centring collimation, **damaged plummet assembly**.
2. Incorrect prism constant setting in EDM.
3. Incorrect height of instrument and target (HI, HT).
4. Incorrect EDM meteorological settings.
5. Incorrect observation parameters set for horizontal distance (H);
 - scale factor, C&R correction, sea level correction
6. Parallax in pointing to target,
 - check reticule and object focus.
7. Gross data recording errors from data screen
8. Incorrectly identified data point.
9. Incorrect recording of control point data
10. Transcription errors in field book
 - poor character formation, printing, transposition

4.9 Concluding Remarks

This Chapter introduced you to the Total Station instrument used by surveyors to measure distances and angles. The chapter elaborated on distance and angular measurement procedures and their associated errors. Finally, the concept of computing rectangular coordinates (Easting and Northings) given polar coordinates (distances and bearings) and vice versa was introduced. Note that while computing the bearings, it is important to choose the correct quadrant out of the four quadrants. Students should make sure they go over this chapter thoroughly as it forms the foundation needed to understand the remaining chapters of the book.

4.10 Reference to Chapter 4

1. Clark, D. (1966). Plane and Geodetic Surveying. London, England, Constable and Company, Ltd.

2. ISCM. GDA2020 - Technical Manual, V2.4. <http://www.icsm.gov.au/datum/gda2020-and-GDA2020-technical-manuals>
3. Sokkia Co., Ltd. Sokkia Series 30RK Reflectorless Total Station Users Manual. Kana-gawa, Japan. 6th ed.

5 Chapter 5 Traversing

5.1 Introductory Remarks

This Chapter introduces you to the theory and practical skills of traversing, a process of determining horizontal controls. For civil and mine engineering, traverse finds use in:

- ✓ Establishing horizontal controls i.e., known positions (Easting and Northings).
- ✓ Horizontal controls for topographic and detail (i.e. pre-engineering) surveys (i.e., the part that will be undertaken in the practical discussed in Appendix A2-3).
- ✓ To establish planar coordinates of points during constructions (set-out).
- ✓ For topographical purposes.
- ✓ For area and volume computations
- ✓ Ground control for photogrammetric mapping.

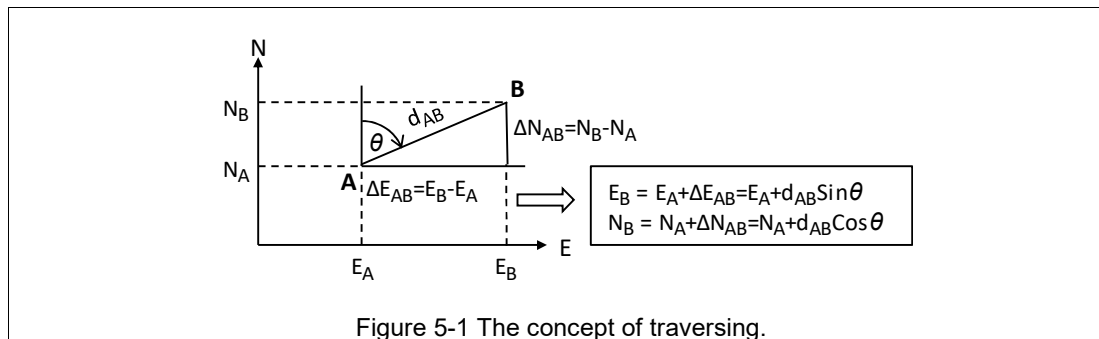
Working through the materials of this Chapter and the workshop materials in Appendix A2-3 and A2-4, you should:

- Know what control survey entails and their roles in engineering and mine surveying.
- Explain what traverse is and know the field methods and computational methods related to it.
- Know the types of traverses and perform all the calculations that are required to obtain traverse coordinates from measured angles and distances.
- Understand the usefulness of traversing in Civil Engineering.

Essential references include Uren and Price (2010, Chapter 6), Schofield and Breach (2007, Chapter 6) and Irvine and Maclellan (2006, Chapter 8).

5.2 Definition and Applications

Traversing is a surveying technique that employs a Total Station (Chapter 4) to determine the planar positions (Easting and Northing: E_B and N_B in Figure 5-1 below) of control points or setting out points using measured angles and distances (d_{AB} and θ below). For vertical controls, you have already learnt the levelling technique in Chapter 2.

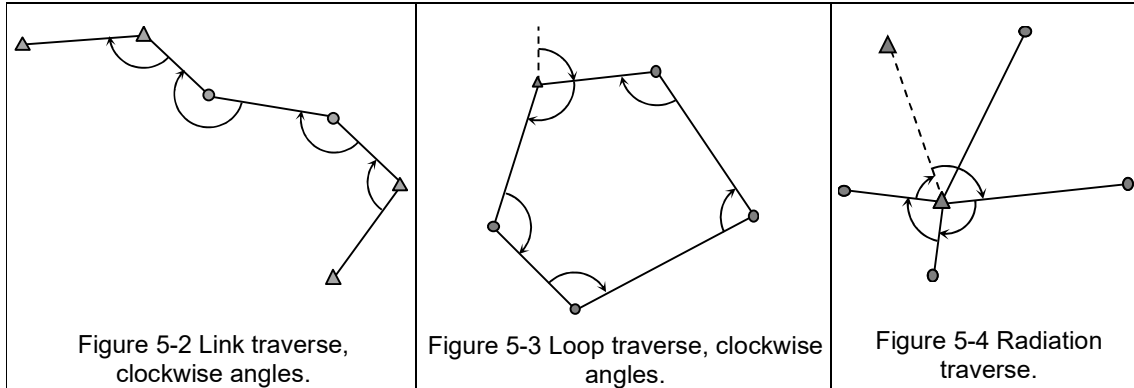


Control points.

- ✓ For ALL civil and mine engineering works, control survey is carried out to fix positions of reference points used in mapping and setting out.
- ✓ For most construction sites, controls are made up of vertical and horizontal positions.
- ✓ For large engineering projects, the Earth's curvature should be considered. This is done using global navigation satellite systems GNSS (and will be covered in Chapter 14).

Traverses can either be open or closed.

- Figure 5-2).
- If it starts and ends at the same point, i.e., Figure 5-3, it is called a loop traverse.
- Another type of traverse includes radiation, or side shot, traverse where all points are uniquely measured and no redundancy exists (Figure 5-4).



5.3 Traverse Procedure

Traverse procedure entails angular and distance measurements as illustrated in Figure 5-5. The Total Station (Fig 4.1a) is placed at point TPA and pointed at a target stationed at TP1. The known direction from TPA-TP1 is set in the instrument as described in §4.1. Then measure the angles TP1-TPA-B on face left (FL) and right (FR) (see example 5.1 in §5.4). Measure the distance TPA-B several times. The target at B then moves to C, that of TP1 to TPA and the instrument to B. Set the bearing B-TPA (obtain by adding or subtracting 180° to the forward bearing TPA-B). Now, measure the angle TPA-B-C on both faces (left and right) as before, and the distance B-C several times. The target at C then moves to D, that at TPA to B, and the instrument to C. Measure the angle B-C-D and the distance C-D as done in the previous step. Next, move the instruments and the targets to measure the angle C-D-E and the distance D-E. Finally, with the Total Station at E, the final angle D-E-F (known target) is measured. The final angles should be the measured direction E-F known target, which can be compared to the known direction to get the angular misclose.

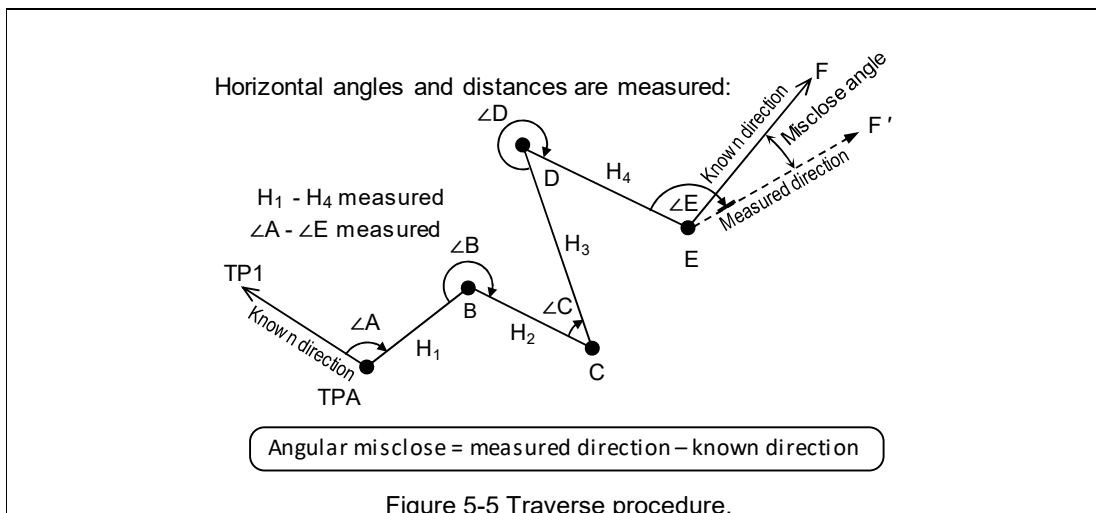


Figure 5-5 Traverse procedure.

In the Figure 5-5, the points occupied by the Total Station are known as traverse stations. They can either be temporary or permanent points. They can either comprise:

- Wooden stakes driven flush with the ground and with a nail driven on top
 - ✓ Large nail on soil
 - ✓ Concrete nail
 - ✓ Cut-cross scribed in concrete
- If they are to be used later, stations should be referenced or witnessed with appropriate horizontal steel band or tape measurements and the necessary field notes made.
 - ✓ Geodetic control points may be used in the traverse. In Australia, they are known as Standard Survey Marks (SSMs). At Curtin University, the points are known as Curtin Survey Marks (CSMs).

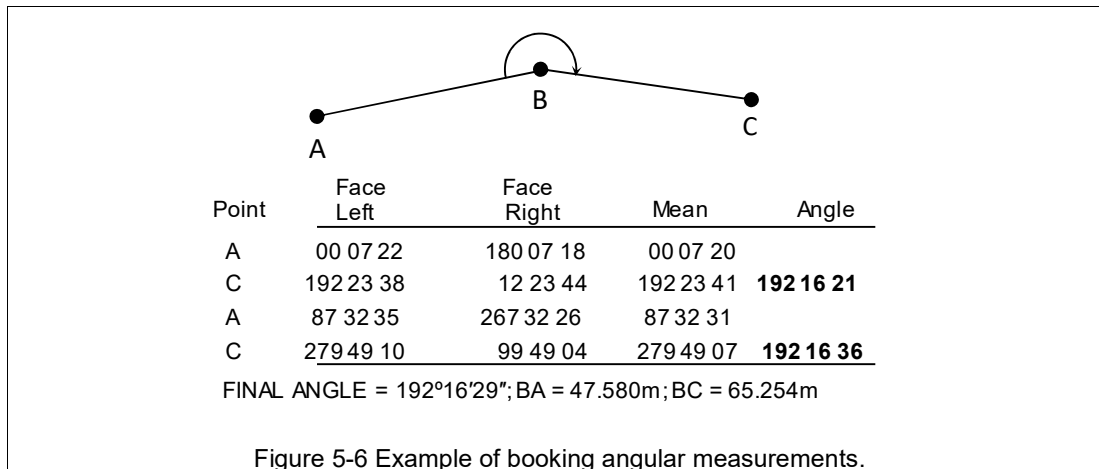
5.4 Field Notes Reduction

Field reduction for traversing involves correcting the measured angles and distances for effects of random errors.

- Angles
 - ✓ Reduce FL and FR readings to their mean value
 - ✓ This is usually already done in the field book
 - ✓ Must be careful to use the mean value (rather than the FL value) for the traverse computations—this emphasises the need for neat, accurate field notes.
- Distances
 - ✓ If repeat measurements are made, their mean values should first be calculated
 - ✓ All measured distances must be corrected.
 - ✓ When steel band are used, they must be corrected for slope, length, temperature, tension and sag.
 - ✓ When EDM are used, the slope distances must be corrected for the prism constant, scale errors and cyclic errors and reduced to horizontal distances (see §4.3).

Example 5.1 Angular measurements and reduction. The Total Station will be set and levelled at point B. The instrument is then pointed at a target placed at point A and the first reading entered in the instrument as discussed in §4.2. This reading is booked under face left (FL) as $0^\circ 07' 22''$. Rotate the instrument on face left (FL) still to point at a target at point C. Lock the instrument and aim the cross hairs to bisect the centre of the target at point C. The reading on the instrument is $192^\circ 23' 38''$ and is booked under face left (FL). Now, the reading on face left is complete. Unlock the telescope and turn it to face you. With the telescope facing you, rotate the instrument so that the telescope is once again facing point C. The instrument is now on Face Right (FR) and the reading displayed differs with that of FL by 180° . This reading, $12^\circ 23' 44''$ is now booked under the column of Face Right but under the row of C. Finally, rotate the instrument back to the starting point A and take the last reading $180^\circ 07' 18''$ which is recorded under the column of Face Right under the row of A. This completes one set of reading measured on both faces (FL and FR).

Taking a closer look at the readings of A, Figure 5-6, you will notice that the difference occurs only in the seconds' part. The mean of these seconds are taken and the face left reading with the mean of the seconds, i.e., $0^\circ 07' 20''$ is booked under the mean face left value. This is repeated for face right and indicated as $192^\circ 23' 41''$. Now the measured angle ABC is obtained by taking the difference between the two means to give $192^\circ 16' 21''$. Another set of reading is taken as shown in Fig. 5.6 and finally the mean of the sets is taken. Several readings of the distances are also taken, and their means taken. The angles and distances are then subsequently used in the Bowditch adjustment as discussed in §5.5.4.



5.4.1 Angular Misclose

Traverse adjustment is done in 2 steps:

- ✓ Balance the angles
- ✓ Traverse adjustment

If no gross errors exist in the angles and all systematic effects have been eliminated by

- ✓ Instrument calibration,
- ✓ Reducing FL/FR observations to their mean,
- ✓ Careful levelling,
- ✓ Careful instrument and target centring,

then only random errors will exist. The random errors cause the sum of interior angles to deviate from the theoretical value. The sum of interior angles of a traverse should equal

$\Sigma_{internal\ angles} = (n - 2) \times 180$	Eqn 5-1

where n is the number of angles (or sides).

For an instrument with stated accuracy of δ in seconds (") (e.g. 1" for a SET1, 5" for a SET530), the allowable angular misclose, ϵ (epsilon), is:

$\epsilon = \delta \sqrt{n}$	Eqn 5-2

If the angular misclose is found to be acceptable, then a correction can be applied to each angle so that the sum of the adjusted angles is correct. The textbooks suggest several methods for determining the corrections

- ✓ Arbitrary: Not recommended.
- ✓ Larger corrections for angles observed in poor conditions: subjective.
- ✓ Average (mean) value: logical and recommended.

It should be noted that corrections should not be applied in proportion to the size of the angle, since errors due to pointing, reading and instrument and target centring are independent of the size of the angle. If the total angular misclose is $\pm\epsilon$, then the correction applied to each angle is given by

$c = \mp \frac{\epsilon}{n}$	Eqn 5-3

Note the negative/positive sign! It is the opposite from the plus/minus sign of the misclose, ϵ , indicating that the correction is the opposite sign from the misclose.

Usually the corrections are rounded to the nearest second. Due to the accumulation of rounding errors, one or two corrections may have to be (arbitrarily) modified by $\pm 1''$ so that the sum

of adjusted angles is correct. The sum of adjusted angles should be computed afterward as a check.

5.4.2 Bearing and Coordinates Computations

The bearing of each line around the traverse is calculated in the clockwise direction. Two methods can be used, i.e.,

- ✓ Computing the back bearing and subtracting the interior angle, e.g.,
 $(180^\circ + 60^\circ 38' 32'') - 81^\circ 53' 34'' = 158^\circ 44' 58''$.
- ✓ Deflection of the interior angle (see example below).

Example 5.2 (Deflection of the interior angle method): This is done in three steps.

Step 1: Initial bearing and clockwise interior angles are given. (Figure 5-7a)

$$AB = 60^\circ 38' 32''$$

$$\angle A = 49^\circ 03' 23''$$

$$\angle B = 81^\circ 53' 34''$$

$$\angle C = 49^\circ 03' 03''$$

Step 2: Deflection angles are calculated from clockwise interior angles (Figure 5-7b)
 (Defl. angle = $180^\circ - \text{int. angle}$).

$$B' = 180^\circ - 81^\circ 53' 34'' = 98^\circ 06' 26''$$

$$C' = 180^\circ - 49^\circ 03' 03'' = 130^\circ 56' 57''$$

$$A' = 180^\circ - 49^\circ 03' 23'' = 130^\circ 56' 37''$$

Step 3: Calculated bearings (Figure 5-7c)
 (Previous bearing + deflection angle,
 e.g., $BC = AB + \text{defl}_B$)

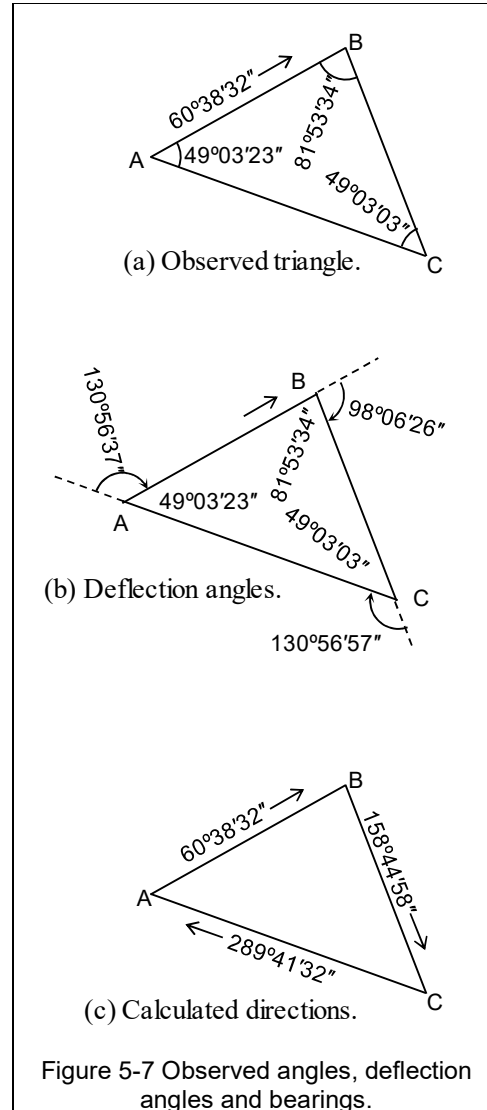
$$\begin{aligned} BC &= 60^\circ 38' 32'' + 98^\circ 06' 26'' \\ &= 158^\circ 44' 58'' \end{aligned}$$

$$\begin{aligned} CA &= 158^\circ 44' 58'' + 130^\circ 56' 57'' \\ &= 289^\circ 41' 55'' \end{aligned}$$

$$\begin{aligned} AB &= 289^\circ 41' 55'' + 130^\circ 56' 37'' \\ &= 420^\circ 38' 32'' \end{aligned}$$

$$AB - 360^\circ = 60^\circ 38' 32''. \text{ Check.}$$

(Always add the last angle to check against the initial bearing.)



5.4.3 Traverse Misclose: Bowditch Adjustment

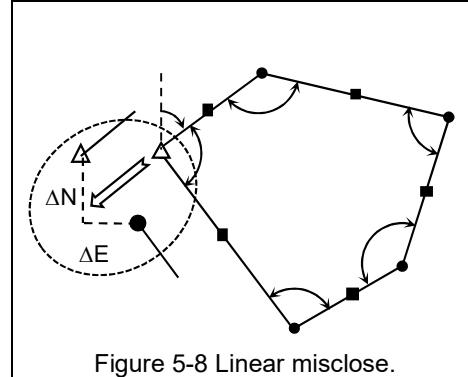
To compute the Eastings and Northings (E_B and N_B) of station B given those of A, measured distance d_{AB} and angles θ (see § 5.5, Figure 5-11 and Table 5-1). This calculation is performed for each traverse line. Like the loop levelling where $\Delta H=0$, in traversing, the sum of ΔN and

$\sum_{i=1}^n \Delta E_i = 0$	Eqn 5-4
$\sum_{i=1}^n \Delta N_i = 0$	Eqn 5-5

ΔE of all n traverse sides should equal zero, i.e.,

Due to random errors, the sum will deviate from zero, leading to a linear misclose (Figure 5-8), i.e.,

$\sqrt{\left(\sum_{i=1}^n \Delta E_i\right)^2 + \left(\sum_{i=1}^n \Delta N_i\right)^2} \neq 0$	Eqn 5-6



The fractional linear misclosure, the ratio of the sum of side lengths to misclosure length, gives a measure of traverse precision.

$1 : \frac{\sum_{i=1}^n d_i}{\sqrt{\left(\sum_{i=1}^n \Delta E_i\right)^2 + \left(\sum_{i=1}^n \Delta N_i\right)^2}}$	Eqn 5-7

Each line of the traverse is adjusted with a correction **opposite** in sign to the misclosure.

- The question then arises on how the overall traverse misclosure should be distributed to each line. The options include equal adjustment, transit method, least-squares method or Bowditch method.
 - ✓ In the Bowditch method, the corrections to each line are proportional to the side length.
 - ✓ Recommend method although modern software incorporates least squares solutions.
- Bowditch method:

$c_{E_i} = \mp d_i \frac{\sum_{i=1}^n \Delta E_i}{\sum_{i=1}^n d_i}, \quad c_{N_i} = \mp d_i \frac{\sum_{i=1}^n \Delta N_i}{\sum_{i=1}^n d_i}$	Eqn 5-8

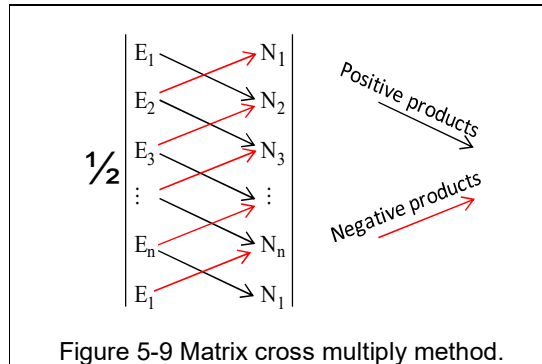
- Note that the correction has the **opposite sign** from the misclosure (workshop in Appendix A1-4 will treat this in detail).
- After adjusting each side, the sum of ΔN and ΔE should be calculated as a check and should both equal zero.

5.4.4 Area Computations

The area of the closed figure should be calculated and reported in both m² and ha. The area is computed using coordinates' methods (see § 3.4.7). The general formula is given as:

$Area = \left \frac{1}{2} \left(\sum_{i=1}^n (E_i N_{i+1} - E_{i+1} N_i) \right) \right $	Eqn 5-9

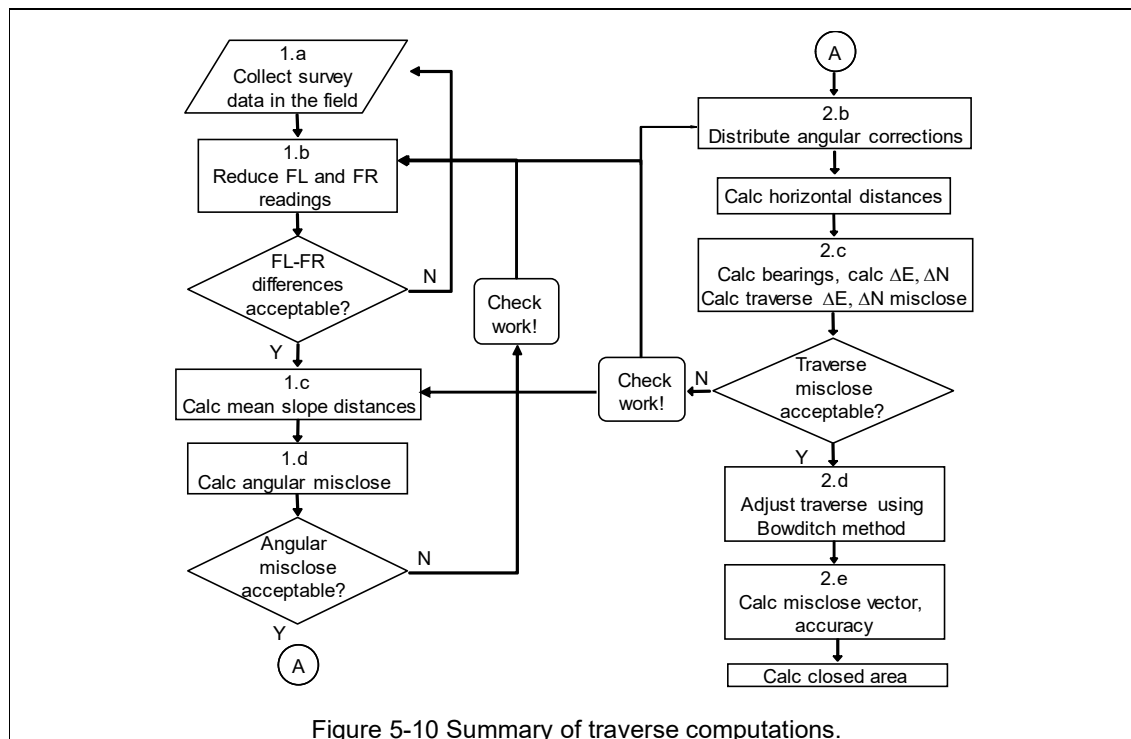
The figure must close back to the start point, i.e., E₁ and N₁ must appear twice. The formula is computed as shown in Figure 5-9:



Remember to halve your answer.

The result may be positive or negative, it depends on the coordinate order. Area is always positive.

Some texts use a N, then E, column order to provide a positive answer in a clockwise traverse. However, the presented column format maintains the normal E, N coordinate order used in traversing, but gives a negative answer in a clockwise traverse. It saves transcription errors.



5.4.5 Summary of the Traverse Computation.

The summary in Figure 5-10 should be processed in two phases:

1. Field data collection and checking. As the observations are recorded the data should be checked for validity. Catch the blunder in the field. This is the bookers main task.
 - a. Have all points been observed in sequence?
 - b. FL/FR differences close, both horizontal and zenith angles?
 - c. Do slope distances agree?

- d. Calculate angular misclose, simple task for closed loop ($\Sigma_{\text{interior angles}} = (n-2) \times 180^\circ$).
Pre-calculate closure angle for linked traverse. Sum observed angles and compare.
2. Office calculations involve:
 - a. Calculation of horizontal (and vertical) distances, sum of lengths to be adjusted.
 - b. Distribution of angular misclose as equal angle corrections, agree with final bearing.
 - c. Calculate traverse, ΔE , ΔN . Sum and compare coordinates misclose.
 - d. Proportional distribution of ΔE , ΔN misclose over each leg. $\Sigma \text{ corrections} = -\Sigma \text{ misclose}$. Adjusted final coordinates = target fixed coordinates.
 - e. Calculate misclose vector, calculate proportional accuracy of closure.

5.5 Worked Example 5.1: Set-out and Adjustment of Control Points for Site Control

A control point, **CEM_0**, was established in Field Practical A2-3-1. The point was determined by observing to 705A and C400. Extracts from the field book refer to the back sight to 705A.

Following the observation of a digital elevation model (DEM) for the design area, there is a need for the placement of site control points to be used by other contractors. For this practical exercise you will establish two extra control points, **CEM_A** and **CEM_B**, Figure 5-11, using your initial control point, **CEM_0**, established in Field Practical A2 - 3 - 1. Prism sets with target plates will be set up on tripods over the control points **CEM_A** and **CEM_B**.

The triangle of points so established will be fully observed by reading directions and distances on both faces (FL and FR) of the Total Station at each control point. The observations from each control point will be made using the semi-forced centring technique whereby instruments, and targets are swapped between tribrachs without disturbing the tripod set-up over the control point. Orientation of the control network will be established from the back sight (BS) observations to 705A.

At the initial control point, **CEM_0**, include face left (FL)/face right (FR) observations to your BS orientation point in your round of observations to the two other introduced control points. This will allow the mean angle between your BS and first control point to be calculated, and the **bearing** to the first control point to be established.

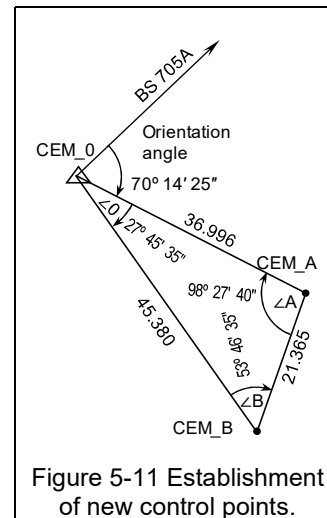


Figure 5-11 Establishment of new control points.

5.5.1 Field Book Observations and Reduction

Table 5-1 Calculation of mean angles and distances from field book, part a.							
AT/TO		HAR	ZA	S	H	V	Remarks
AT	CEM_0	HAR			RL	16.035	HI = 1.585
705A	L	47 51 50					B Sight
	Mean	47 51 53					
705A	R	227 51 55					B Sight
	Incl. Angle	70 14 27					Orientation angle to 705A
CEM_A	L	118 06 15	95 06 28	37.144	36.996	-3.307	HT = 1.563
	Mean	118 06 20			RL A = RL CEM_0 + HI + V - HT		= 12.75
CEM_A	R	298 06 25	264 53 32	37.144	36.997		
	Incl. Angle	27 45 35					Angle at _00
CEM_B	L	145 51 50	95 16 04	45.571	45.379	-4.184	HT = 1.576
	Mean	145 51 55			RL B = RL CEM_0 + HI + V - HT		= 11.86
CEM_B	R	325 52 00	264 43 34	45.572	45.380		
AT	CEM_A	Change station to CEM_A					HT = 1.563
CEM_B	L	0 01 25	92 21 02	21.383	21.365	-0.877	HT = 1.576
	Mean	00 01 30					
CEM_B	R	180 01 35	267 38 58	21.382	21.364		

<i>Incl. Angle</i>		98 27 40					<i>Angle at _A</i>
CEM_0	L	98 29 04	84 53 31	37.143	36.995	3.307	HI = 1.585
<i>Mean</i>		98 29 10					
CEM_0	R	278 29 16	275 06 29	37.144	36.996		
AT	CEM_B	<i>Change station to CEM_B</i>					HT = 1.576
CEM_0	L	0 01 25	84 43 57	45.573	45.381	4.184	HI = 1.585
<i>Mean</i>		00 01 30					
CEM_0	R	180 01 35	275 06 04	45.572	45.380		
<i>Incl. Angle</i>		53 46 35					<i>Angle at _B</i>
CEM_A	L	43 48 01	87 38 58	21.384	21.366	0.877	HT = 1.563
<i>Mean</i>		43 48 05					
CEM_A	R	233 48 09	272 21 02	21.383	21.365		
<i>Σ angles =</i>		179 59 50					

5.5.2 Network Adjustment Method

- a. at each control point, calculate directions and distances (Example 5.1 in § 5.4) as follows;
 1. obtain the mean direction to each the control points from the mean of the FL/FR observations,
 2. obtain the mean **angle** between the control points from the difference of the mean directions,
 3. obtain the mean vertical angle (as a zenith distance) from the FL/FR observations,
 4. obtain the mean slope distance of each observation,
 5. the resulting **horizontal** distance and **vertical** height difference to the other two control point targets.
 6. obtain the RLs of the two control points from the initial control point.
- b. for the network, calculating internal angles;
 1. the **sum** of the mean internal angles,
 2. the angular **misclose** for the network, compared with the sum of the internal angles of the plane figure,
 3. the **adjusted** internal angles after the even **distribution** of angular misclose as discussed in § 5.4.1.
- c. for the network, calculating adjusted bearings;
 1. the **bearing** of the first control line by the addition of the mean angle to the BS bearing,
 2. the **bearing** of the other control lines by the addition of the adjusted internal angles (see § 5.4.3),
 3. perform a **check** of the initial control line bearing using the last internal angle.
- d. for the network, calculate and adjust the coordinate misclose;
 1. the ΔE and ΔN coordinate differences from the adjusted bearings and mean **horizontal** distances,
 2. the **sum** of the ΔE s and ΔN s to calculate the **misclose** in ΔE and ΔN ,
 3. the misclose **corrections** in ΔE and ΔN , distributed by the **Bowditch** adjustment, (i.e., § 5.4.3),
 4. the corrected ΔE s and ΔN s for the network,
 5. the adjusted coordinates of each control point from your initial control point coordinates,
 6. the misclose ratio calculated from the misclose vector and the sum of the network distances.

The angular misclose is distributed **evenly** between each of the internal angles. (§ 5.5.3).

The **coordinate** misclose is distributed **in proportion** to the individual network **distances** (see § 5.5.4).

5.5.3 Angular Misclose Correction

Table 5-2 Correction of angular misclose, corrected bearings. Part b and c.					
AT CEM_0	HAR				
Brg to BS 705A	47 51 53	Calculated BRG from CEM_0 to 705A			
∠BS to A	70 14 27	Mean angle at CEM_0 between BS and CEM_A			
BRG 00 - A	118 06 20			118 06 20	Brg_0 - _A
BRG A - 00				298 06 20	Back BRG
Mean observed angle		correction	Corr angle	Corr BRG	
∠A	98 27 40	+4	98 27 44		
BRG A - B				199 38 36	BRG_A - _B
BRG B - A				19 38 36	Back BRG
∠B	53 46 35	+3	53 46 38		
BRG B - 00				325 51 58	BRG_B - _0
BRG 00 - B				145 51 58	Back BRG
∠00	27 45 35	+3	27 45 38		
BRG 00 - A				118 06 20	Check BRG_0 - _A
SUM Obs angles	179 59 50	Too small, 10" has to be added to make 180°			
Correct Sum	180 00 00				
Misclose	-00 00 10				
Correction	+10"	Σ = +10"			

Note that the angular misclose (Table 5-2) is distributed nearly evenly by integer values. Because of the way the figure has been traversed in a clockwise order (0 – A – B – 0), the adjusted angles are subtracted from individual back bearings. Traversing in the anticlockwise direction (0 – B – A – 0) would allow the adjusted angles to be added.

5.5.4 Adjusted Coordinates using the Bowditch Adjustment

The proportional coordinate adjustments in Easting and Northing are called **corrections** and

have the opposite SIGN from the miscloses. Calculate a correction, $v_{(E/N)} = \left(\frac{-misclose_{(E/N)}}{\Sigma D} \right)$,

as a constant for ΔE and ΔN (Eqn 5-8) which can be stored in memory. It can be retrieved easily

and multiplied by each traverse distance to calculate the adjustment $v_n = \left(\frac{-misclose}{\Sigma D} \right) d_n$. It

helps avoid keystroke errors associated with the $v_n = -misclose \times \left(\frac{d_n}{\Sigma D} \right)$ method.

Watch the SIGN of the corrections and make sure they have been applied correctly to the coordinate differences, including their sign.

Table 5-3 details the results of:

1. Calculating unadjusted coordinate differences from **adjusted** bearing and observed mean horizontal distance. Sum of distances.
2. Summing unadjusted coordinate differences to find linear miscloses. Calculate misclose vector and ratio.
3. Distributing misclose as a **correction** to each set of unadjusted coordinates in proportion to individual traverse distances. Sum corrections equal negative misclose.
4. Summing unadjusted coordinates and corrections; ensure adjusted coordinates close.

Table 5-3 Bowditch adjustment to coordinate differences. Adjusted coordinates. Part d.

		Adjusted coordinates			E	N	Re-
CEM_0					354.303	7442.780	
	<i>Adj. BRG</i>	<i>H. DIST</i>	<i>dE</i>	<i>dN</i>			
00 - A	118 06 20	36.996	32.663	-17.429			
		<i>Correction</i>	0.004	-0.004			
CEM0_A	<i>Corrected dE & dN</i>		32.638	-17.443	386.491	7425.347	
A - B	199 38 36	21.365	-7.182	-20.122			
		<i>Correction</i>	0.003	-0.002			
CEM0_B	<i>Corrected dE & dN</i>		-7.180	-20.124	379.761	7405.223	
B - 00	325 51 58	45.380	-25.464	37.562			
		<i>Correction</i>	0.006	-0.006			
CEM_0	<i>Corrected dE & dN</i>		-25.458	37.557	354.303	7442.780	<i>check</i>
<i>Sum network distance</i>		103.741					
	<i>Misclose</i>	ΣdE & ΣdN	-0.013	0.012	<i>Sum of coordinate differences</i>		
	<i>Corrections, v (nu)</i>	ΣvE & ΣvN	0.013	-0.012	<i>Sum of coordinate corrections</i>		
	<i>Misclose vector dist</i>	0.017	<i>Direction</i>	313.3°			
	<i>Misclose ration</i>	1:5947					
	<i>Correction $\Sigma vE / \Sigma Dist$</i>	vE/m	$1.223E^{-04}$				
	<i>Correction $\Sigma vN / \Sigma Dist$</i>	vN/m	$-1.154E^{-04}$				

Note that the coordinate misclose is distributed nearly proportionally to the traverse leg distances by integer values. The misclose in dE and dN is the difference between the finish and start coordinates. Without adjustment, the final coordinate values for CEM-0 would have been 354.290 and 7442.792. This shows that 0.013 (13 mm) would have to be **added** to the E coordinate and 0.012 (12 mm) **subtracted** from the N coordinate to close the figure.

5.5.5 Calculation of AREA by Coordinates Method

With the adjustment completed, the AREA of the figure is calculated by coordinate method (matrix cross multiplication, see § 3.4.7, Eqn 3-9). Being triangular, a few methods present themselves. These do not scale well to increased vertices (polygons) and in general the calculation of area by coordinates is used. Its downfall is that, apart from an independent re-calculation, there is no method of checking the results. Being a triangle, there are **four** sets of coordinates in a closed figure, returning to the start point.

Reiterating Eqn 5-9:

$$Area = \left| \frac{1}{2} \left(\sum_{i=1}^n (E_i N_{i+1} - E_{i+1} N_i) \right) \right|$$

It is realized by forming a matrix of coordinates and cross multiplying.

$$Area = \frac{1}{2}(E_1 N_2 - E_2 N_1 + E_2 N_3 - E_3 N_2 + E_3 N_4 - E_4 N_3),$$

Point/n	E	N
0 1	354.303	7442.780
A 2	386.491	7425.347
B 3	379.761	7405.223
0 4	354.303	7442.780

E

354.303

386.491

379.761

354.303

N

7442.780

7425.347

7405.223

7442.780

$(E_1 N_2 + E_2 N_3 + E_3 N_4) = 8319352.34$

$(E_2 N_1 + E_3 N_2 + E_4 N_3) = 8320117.41$

$8319352.34 - 8320117.41 = -765.08$

$|Area| = \frac{1}{2}(-765.08) = \mathbf{382.54}$

Figure 5-12 Area calculation by matrix.

AREA = 382.54

There is no independent way to check the results. Just a re-calculation.

5.5.6 Calculation of Area by Double Longitude Coordinates

The method of area by double longitude coordinates utilizes the general formula:

$$2Area = \sum_{p=1}^n (E_{p+1} - E_p)(N_p + N_{p+1}) \text{ where } (E_{p+1} - E_p) = \delta E^s, (N_p + N_{p+1}) = \Sigma N^s \quad \text{Eqn 5-10}$$

The check of the calculations is:

$$2Area = \sum_{p=1}^n (N_{p+1} - N_p)(E_p + E_{p+1}) \text{ where } (N_{p+1} - N_p) = \delta N^s, (E_p + E_{p+1}) = \Sigma E^s \quad \text{Eqn 5-11}$$

The areas will be the same, but of opposite signs

Once the Bowditch adjustment has been completed the adjusted coordinate differences in Easting and Northing are available for the double longitude calculation. The **sums** of the adjacent adjusted Eastings and Northings coordinates are tabulated in Table 5-4.

Table 5-4 Area by double longitude coordinates.						
Column		1	2	3	4	
Point	p	$E/\delta E^s$	$N/\delta N^s$	ΣE^s	ΣN^s	
0	1	354.303	7442.780			
Differences		32.188	-17.443	740.794	14868.127	-12914.26
A	2	386.491	7425.347			
		-6.730	-20.124	766.252	14830.570	-15420.06
B	3	379.761	7405.223			
		-25.458	37.557	734.064	14848.003	27569.24
0	4	354.303	7442.780			
					2 Area =	-765.08
					Area =	382.54

Area calculation and check:

$$2Area = \sum_{p=1}^n (N_{p+1} - N_p)(E_p + E_{p+1}) \Rightarrow \sum_{p=1}^n (\delta N^s)(\Sigma E^s) \quad \text{Eqn 5-12}$$

2A = sum inner products, columns 2 and 3

$$2A = (-17.443 \times 740.794) + (-20.124 \times 766.252) + (37.557 \times 734.064) = -765.075$$

$$2Area = \sum_{p=1}^n (E_{p+1} - E_p)(N_p + N_{p+1}) \Rightarrow \sum_{p=1}^n (\delta E^s)(\Sigma N^s) \quad \text{Eqn 5-13}$$

Check. 2A = sum outer products, columns 1 and 4

Check calculation:

$$2A = (32.188 \times 14868.127) + (-6.730 \times 14830.570) + (-25.458 \times 14848.003) = 765.075$$

$$|Area| = \frac{1}{2}(765.075) = 382.538$$

The check calculations **must** produce the same result with the **opposite** sign.

This IS an independent way to check the results.

5.5.7 Bowditch Adjustment of an Open Linked Traverse

Figure 5-2, an open linked traverse, page 101, joins two known networks of control. It is a method used to provide extra control points, perhaps needed on a building site. Figure 5-5 also shows the linked traverse. It shows the angular misclose between the observed bearing C-D', using observed angles, and the final observed bearing to the known, calculated (true) bearing of the final control line C-D.

Reduced fieldbook observations, Figure 5-13 show: mean observed angles A, 1, 2, 3, 4, C; mean horizontal observed distances H_{AB} , H_1 to H_5 , H_{CD} ; observed BRG CD'.

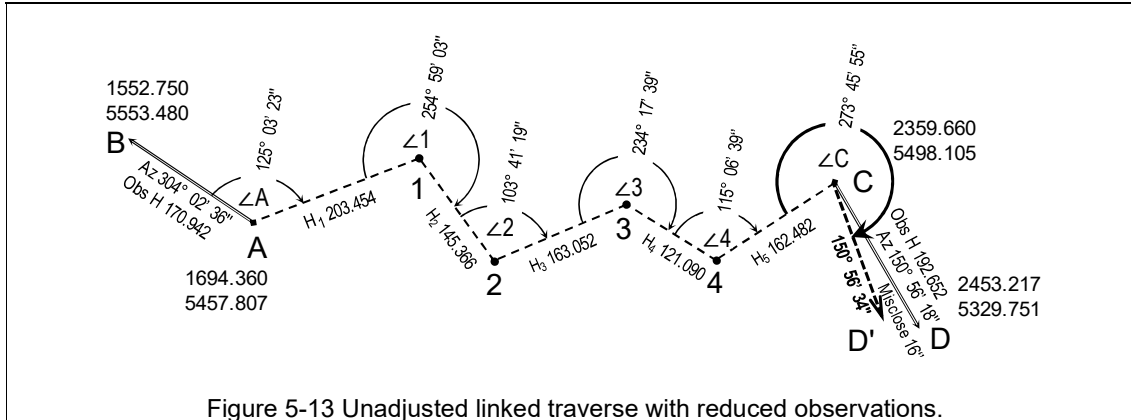


Figure 5-13 Unadjusted linked traverse with reduced observations.

Brg AB and Brg CD are calculated from known coordinates of A, B, C and D. The task is to determine the adjusted coordinates of the 4 new stations between the known points A and C. The Bowditch adjustment is justified for a number of reasons:

1. Points A, B, C and D are accepted as true, plane coordinates,
2. Observations were taken with a single instrument under suitable condition. It means that all observations have similar characteristics of angular and distance errors. They are of equal "weight". The adjustment is of random errors.
3. The observations have been reduced to horizontal angles and distances.

5.5.7.1 Methods to determine angular misclose and corrections distribution

Figure 5-10 shows the process of adjusting the observations using the Bowditch method. Using the join method, the bearings and horizontal distances of the two fixed lines, A-B and C-D have been calculated as:

$$\text{BRG AB} = 304^\circ 20' 36'' \quad H_{AB} = 170.900$$

$$\text{BRG CD} = 150^\circ 56' 18'' \quad H_{CD} = 192.801$$

1. Find angular difference: $\text{BRG}_{CD} - \text{BRG}_{AB}$
 - a. These are the forward bearings from C-D and A-B
 $\text{BRG}_{CD} = 150^\circ 56' 18''$: Since $\text{BRG}_{CD} < \text{BRG}_{AB}$,
add 360° to avoid negative angles.
 $\text{BRG}_{CD} = 510^\circ 56' 18''$
 $\text{BRG}_{AB} = -304^\circ 20' 36''$
Difference = $206^\circ 53' 42''$
 - b. Sum observed angles $(A+1+2+3+4+C) = 1106^\circ 53' 58''$
Reduce sum by $(n-1) \times 180^\circ$.
There are $n = 6$ angles: $1106^\circ 53' 58'' - 900^\circ = 206^\circ 53' 58''$.
 - c. Calculate and distribute misclose as a correction.
 $\angle \text{DCD}'$ is misclose between:

observed angle BCD'	$206^\circ 53' 58''$	from reduced sum of observed angles
known angle BCD	$-206^\circ 53' 42''$	from difference of calculated bearings

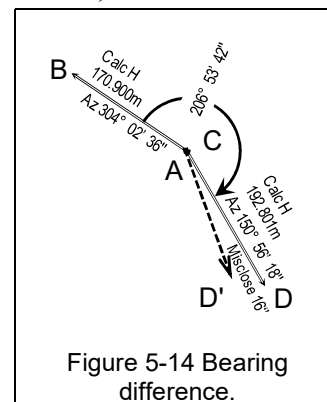


Figure 5-14 Bearing difference.

- Misclose angle $+16''$
 Total correction required $-16''$
2. Find the final observed bearing, compare with calculated bearing to determine misclose.
 - b. Add the initial $BRG_{AB} + \Sigma(\text{angles})$: $304^\circ 02' 36'' + 1106^\circ 53' 58'' = 1410^\circ 56' 34''$
 - c. Reduce sum by $(n+1) \times 180^\circ$.
 There are $n = 6$ angles.
 Observed BRG_{CD} : $1410^\circ 56' 34'' - 1260^\circ = 150^\circ 56' 34''$.
 - d. Calculate and distribute misclose as a correction.
 $\angle DCD'$ is misclose between:

observed $BRG_{CD'}$	$150^\circ 56' 34''$
and known BRG_{CD}	$-150^\circ 56' 18''$
Misclose angle	$+16''$
Total correction required	$-16''$
 3. Each measured angles is reduced by about $16/6 = 2.7''$. ($4 \times 3''$ plus $2 \times 2'' = 16''$.)
 See Figure 5-15 Misclose diagram. Figure 5-14 for the above calculations.

5.5.7.2 Adjust observed angles, calculate adjusted bearings

Table 5-5 is the calculation of the adjusted angles and bearings between A and C.

The angular misclose over the observed angles is distributed equally over the six observed angles. Do not use fractions. Use integer corrections and adjust to meet misclose.

Did the misclose meet project angular misclose tolerances?

Table 5-5 Misclose and adjusted BRGs.						
Calc. BRG AB	304 02 36	OBS angle	Corr'n "	ADJ angle	Line	ADJ BRG
Observed angles	$\angle A$	125 03 23	-3	125 03 20	A - 1	69 05 56
	$\angle 1$	254 59 03	-3	254 59 00	1 - 2	114 04 56
	$\angle 2$	103 41 19	-2	103 41 17	2 - 3	67 46 13
	$\angle 3$	234 17 39	-2	234 17 37	3 - 4	122 03 50
	$\angle 4$	115 06 39	-3	115 06 36	4 - C	57 10 26
	$\angle C$	273 45 55	-3	273 45 52	C - D'	150 56 18
	$\Sigma \text{OBS} \angle$	1106 53 58	$\Sigma \text{ADJ} \angle$	1106 53 42	Agrees with BRG C-D	
$BRG_{CD'} = (\Sigma \text{OBS} \angle + BRG_{AB}) \text{ MOD } 360$		150 56 34				
Calc. BRG CD	150 56 18		$\Sigma \text{Corr'n}$	$(-3 \times 4 + -2 \times 2) = -16$		
	Misclose	+16	-16	Allowed misclose $= 15'' \sqrt{n}$		$15 \sqrt{6} = 37'' \checkmark$

5.5.7.3 Check for scale factor in horizontal distances

The opening and closing observations allow for checking the presence of horizontal scale factor in the coordinate frame.

If your reference coordinates are modelled on, say UTM (e.g. MGA2020) then there will be a point or line scale factor in the horizontal distances. Your observed horizontal distances must also incorporate the scale factor. The scale factor may be introduced via:

1. Adjusting the scale factor in the JOB parameters in the total station program. The displayed H distance incorporates the scale factor,
2. Applying the scale factor to the horizontal distance calculated from ZA and slope distance, this is the H distance with a $SF = 1$

From Figure 5-14 and Figure 5-13

$$H_{AB(\text{calc})} = 170.900, H_{AB(\text{obs})} = 170.942. S.F. = 170.900/170.942 = 0.999754$$

$$H_{CD(\text{calc})} = 192.801, H_{CD(\text{obs})} = 192.850. S.F. = 192.801/192.850 = 0.999746$$

Accepted mean S.F for the area = 0.99975 to reduce observed H distances to projection.

The observed H distances in Figure 5-13 have been reduced to projection distances using S.F. 0.99975 as shown in Table 5-6.

5.5.7.4 Determine coordinate misclose and distribute corrections proportionally

The adjusted bearings and observed horizontal distances, reduced by projection scale factor, are used to calculate the coordinate differences, ΔE and ΔN for each line to be adjusted. Line AB and CD are **not** part of the misclose corrections.

Once the miscloses have been calculated, a vector can be found over the hypotenuse. Divided into the total observed distance, the figure produced represents the “accuracy” of the survey. Quoted as 1:n, the result can give an indication of the precision of the survey before any more calculations are attempted.

A survey accuracy of, say, 1:10,000 would be met in this example as it approaches 1:15,000. If the accuracy specification is not met, then it is time to check the survey observations and reductions.

Table 5-6 Proportional linear misclose distribution. Adjusted coordinate differences.								
Line	Adj BRG	Reduced H dist	ΔE	ΔN	vE	vN	Adj. ΔE	Adj. ΔN
A - 1	69 05 56	203.403	190.019	72.565	+0.009	-0.008	190.028	72.557
1 - 2	114 04 56	145.330	85.254	-117.697	+0.007	-0.006	85.261	-117.703
2 - 3	67 46 13	163.011	150.895	61.671	+0.008	-0.007	150.903	61.664
3 - 4	122 03 50	121.060	102.593	-64.266	+0.005	-0.004	102.598	-64.270
4 - C	57 10 26	162.441	136.502	88.058	+0.007	-0.007	136.510	88.051
Σ obs H dist.		795.245			Adj. $\Sigma DE, \Sigma DN$		655.300	40.298
A - C'	Obs. $\Sigma DE, \Sigma DN$		655.263	40.330				
A - C	Calc. DE, DN		655.300	40.298	True $\Delta E, \Delta N$ between A & C			
	Misclose vE, vN		-0.037	0.032	+0.037	-0.032	Check Σ corr's	
Corr'n= -vE/m, -vN/m			+4.63 ⁻⁰⁵	-4.08 ⁻⁰⁵	Correction $\Delta E, \Delta N$ per metre H distance			
Misclose vector		311.4°	0.049	1:16,200	Accuracy		= Σ obs dist/misclose vector	

5.5.7.5 Calculate final adjusted coordinates

The adjusted coordinates are presented in table form using the adjusted coordinate differences between the stations. Check that the coordinates of the end point agree with the original data.

Table 5-7 is a continuation of Table 5-6, providing final adjusted coordinates of the new control points 1 – 4. Figure 5-15 is a diagram of the misclose between C and C'.

Table 5-7 Adjusted coordinates.				
Point	Adj ΔE	Adj ΔN	E	N
Station A			1694.360	5457.807
1	190.028	72.557	1884.388	5536.364
2	85.261	-117.703	1969.649	5412.661
3	150.903	61.664	2120.522	5474.325
4	102.598	-64.271	2223.150	5410.054
Station C	136.510	88.051	2359.660	5498.105

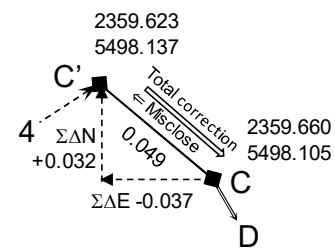


Figure 5-15 Misclose diagram.

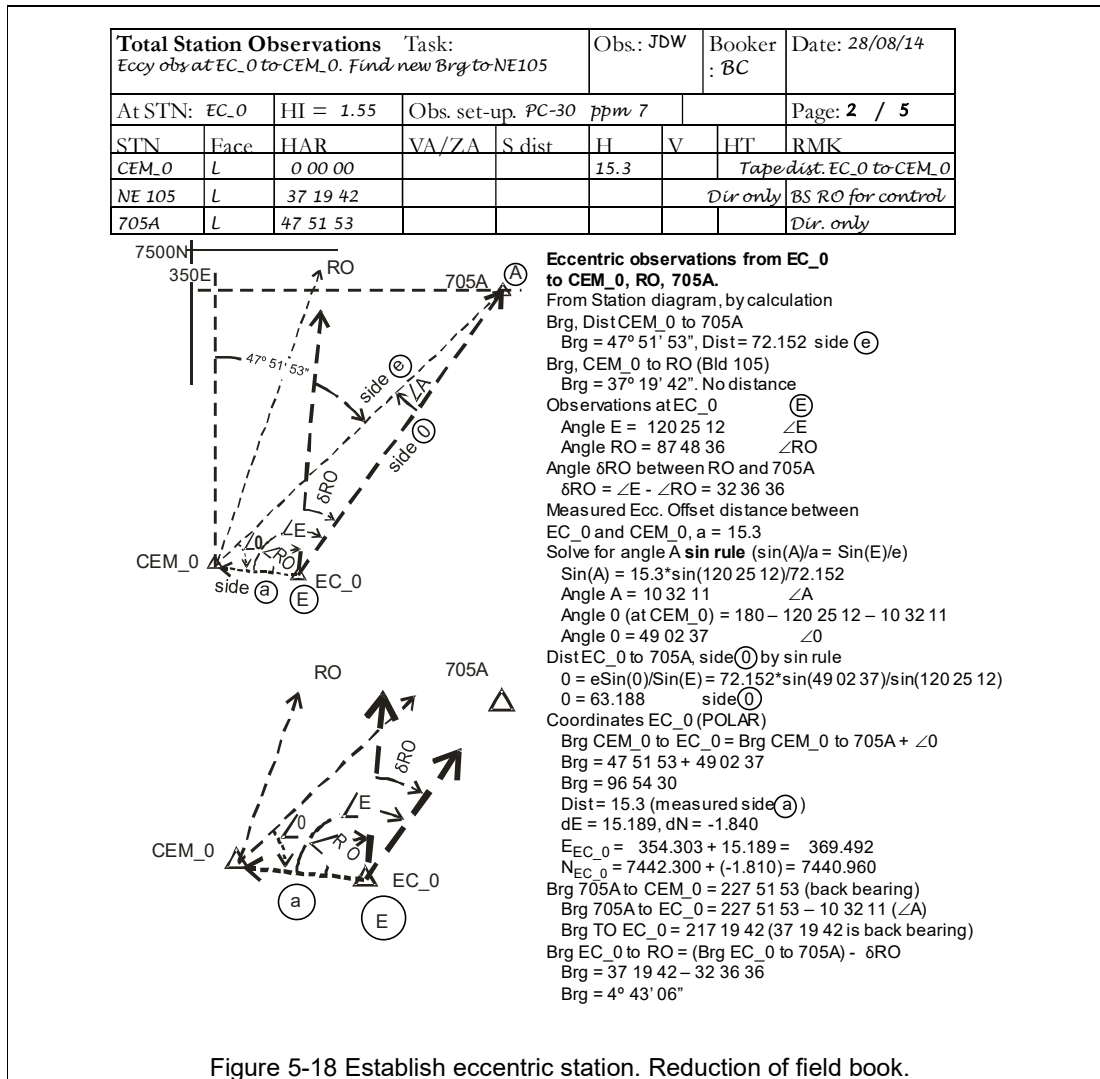


Figure 5-18 Establish eccentric station. Reduction of field book.

In this example the offset distance, 15.3m, is far too large. But it allows a scaled graphic to be presented.

Instead let side $a = 2.5$ m and leave the angles as above. By Sin rule for angles and sides:

$$\text{Angle } A = 1^{\circ} 42' 55'' \quad (2.5 \times \sin(120^{\circ} 25' 12'') / 72.152 = 0.02988 = 1.7122^{\circ})$$

$$\text{Angle } 0 = 57^{\circ} 51' 53'' \quad (180 - 120^{\circ} 25' 12'' - 1^{\circ} 42' 55'')$$

$$\text{Side } 0 = 70.852\text{m}$$

Coordinates EC_0

$$\text{Brg CEM}_0 \text{ to EC}_0 = \text{Brg 705A} + \angle 0 = 47^{\circ} 51' 53'' + 57^{\circ} 51' 53''$$

$$\text{Brg} = 105^{\circ} 43' 46''$$

Dist, a , = 2.5. Then, by POLAR calculation:

$$dE = 2.406, dN = -0.678$$

$$E_{EC_0} = 354.303 + 2.406 = 56.709$$

$$N_{EC_0} = 7442.800 + (-0.678) = 7442.122$$

$$\text{Brg 705A to EC}_0 = \text{Back bearing} - \angle A = 227^{\circ} 51' 53'' - 1^{\circ} 42' 55''$$

$$\text{Brg} = 226^{\circ} 08' 58''. \text{ Reverse bearing EC}_0 \text{ to 705A} = 46^{\circ} 08' 58''$$

$$\text{Brg EC}_0 \text{ to RO} = 46^{\circ} 08' 58'' - 32^{\circ} 36' 36'' \text{ (Brg EC}_0 \text{ to 705A} - \delta RO)$$

$$\text{Brg} = 13^{\circ} 32' 22''. \text{ This can be compared with the original Brg of } 19^{\circ} 27' 28''.$$

You now have an orientation from the eccentric station to the RO in the original coordinate system.

5.6 Sources of Errors in Traversing

- Mistakes and systematic errors in taping.
- Inaccurate centring of the Total Station or prism affecting the distances.
- Total Station not level or not in adjustment
- Incorrect use of the Total Station
- For manual recording, mistakes in reading and booking.

5.7 Concluding Remarks

This chapter has presented you with the method employed by surveyors to determine horizontal positions of points (Easting and Northings). In your professional work, you will more often be dealing with positions of points. Understanding how they are obtained is therefore the first step to properly constrain your structures in their absolute and relative positions. For mining students, knowing the correct positions of points can be crucial for several tasks that include, e.g., rescue missions. In summary, therefore, positions are derived from measured angles and distances which must be corrected and adjusted due to the presence of random errors. It is the finally adjusted coordinates (Easting and Northings) that are used for setting out of structures.

5.8 Reference for Chapter 5

1. Irvine and Maclellan (2006) Surveying for Construction. Fifth edition, McGraw, Chaps. 5 and 6
2. Schofield and Breach (2007) Engineering Surveying. Sixth edition, Elsevier, Chap. 3
3. Uren J, Price WF (2010) Surveying for Engineers. Palgrave Macmillan Ltd. pp. 816.



6 Chapter 6 Total Station Differential Levelling

6.1 Introductory Remarks

Total Station differential levelling (also known as trigonometric heighting) is a variant of conventional differential levelling. Differences in height can be determined by making a series of zenith angle and slope distance observations to a prism mounted on a fixed height pole. This technique is conducted in the same fashion as conventional two-way differential levelling.

The Intergovernmental Committee on Surveying and Mapping (ICSM) through its Special Publication 1, Version 2.0 (2013) provides guidelines for Total Station Differential Levelling to achieve various levels of misclosure.

6.1.1 Allowable misclose:

When conducting differential levelling or Total Station differential levelling, errors propagate in proportion to the square root of the travelled distance. A misclose assessment should be undertaken to verify that forward and backward runs of a levelling traverse, including any individual bays, are within the maximum allowable misclose.

The allowable misclose is calculated using the formula $r_{mm} = n\sqrt{k}$ (k in km). Three standards, using empirical values, are recommended: $2\text{mm}\sqrt{k}$, $6\text{mm}\sqrt{k}$ and $12\text{mm}\sqrt{k}$

For the recommended maximum allowable misclosure of $12\sqrt{k}$ (forward and back) the following requirements must be met.

6.1.2 Equipment:

An EDM Total Station accuracy requirement; distance $\pm 3\text{mm} + 2\text{ppm}$, zenith angle $5''$, daily calibration of index errors of vertical circle and level compensator, compensator accuracy $2.5''$, atmospheric measurement accurate to $t = 1^\circ\text{C}$, $P = 1\text{Hpa}$, $RH = 2\%$ (N/A to $12\sqrt{k}$ levelling), telescopic tripod, fixed height reflector with bi-pod support; staff bubble attached and accurate to $10'$ verticality, fixed height ensured; permanently mounted, balanced and tilting prism, general prism; standard change plate.

6.1.3 Observation Techniques:

Two-way levelling; avoid fixed rod index error, observer same rod on first BS/last FS; total distance foresight approx. equal to backsight; F/L, F/R rounds of observations, 3; height difference readings to 1mm; atmospheric conditions N/A; maximum sight length 120m; minimum ground clearance 0.5m.

Check the specifications of your Total Station to ensure it meets these requirements.

6.2 Errors Propagation in Trigonometric Levelling

Compared with differential levelling, where we are concerned generally only with a horizontal plane through the level, trigonometric levelling involves extra sources of systematic and random errors.

The difference in height between instrument and target, reading the elevation angle, α , and the slope distance, S , is:

	$\Delta h = S \sin \alpha $	Eqn 6-1

However, the Total Station reading for vertical angle is generally expressed as the **zenith angle, Z**, where the zenith direction is 0° , and the horizon is 90° . The formula then becomes:

	$\Delta h = S \cos Z$	Eqn 6-2

Using the cosine function gives the resultant Δh the correct elevation or depression sign.

The complete Δh between two ground points must incorporate the height of both the instrument (hi) and the target (ht), and the correction for curvature and refraction (h_{CR}). This was discussed in differential levelling, given:

$h_{CR} = CR \left(\frac{D}{1000} \right)^2, \text{ where } CR = 0.0675 \text{ for } D \text{ in metres. } D = S \sin Z \text{ metres.}$	Eqn 6-3

Thus, in Figure 6-1:

$\Delta h = hi + S \cos Z + h_{CR} = CR \left(\frac{S \sin Z}{1000} \right)^2 - ht$	Eqn 6-4

Trigonometric levelling is achieved by measuring the elevation angle between the Total Station and the target and measuring the slope distance to the target.

The sources of error include:

- digital reading accuracy,
- (DIN 18723, ISO 17123)
- Vertical automatic compensator error,
- distance reading accuracy, the slope distance, S
- set-up accuracy over the measuring point and target point (centring accuracy),
- accuracy of height of instrument (hi) and target (ht).

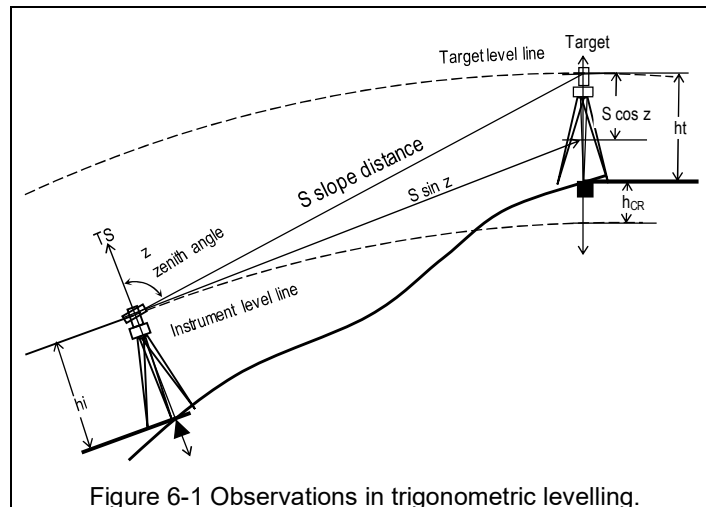


Figure 6-1 Observations in trigonometric levelling.

Continuing the analytical discussion of errors in levelling,

- Ghilani 2010,
- *Error propagation in trigonometric levelling*, chapter 9.4, pp159 et seq.

As with any set of measurement observations, trigonometric levelling is subject to three types of errors:

Systematic errors,
Random errors, and
Blunders or mistakes.

6.2.1 Systematic Errors in Level Observations

1. Vertical circle collimation errors,
 - minimised by taking the mean of Face Left (FL) and Face Right (FR) zenith angle readings to target,
2. Earth curvature and refraction, h_{CR} , corrected by Eqn 6-3:

The effect is minimal over typical lines of maximum distance $D = S \sin Z = 120$, for example: (Ghilani 2010, p153)

$$h_{CR} = 0.0009\text{m} (>1\text{mm})$$
3. Deviation from instrument manufacturer's standard atmosphere for distance measurement.

The slope distance is used to calculate the difference in height, Δh

 - the velocity of light (the distance measuring method) varies with air density.
 - causes errors in distance reading.

Deviations from the standard atmosphere by local meteorological conditions introduces a correction, expressed in parts per million (ppm) of slope distance. At higher elevations, local pressure is significantly lower than sea level pressure.

Within acceptable limits atmospheric pressure decreases at about 1hPa/9m of altitude. (Aviation uses 1hPa/30ft.)

Correction (ppm) from manufacturer's standard atmosphere. Sokkia SET530 series.	deviations are of the order of: ¹
Instruments calibration standard atmosphere:	
temperature 15°C, (Leica 12°C)	1ppm/°C
local pressure 1013.25hPa, (Leica 1012hPa)	1ppm/3.6hPa, pressure drops approx. =1hPa/9m altitude. 1ppm/32m altitude AHD
RH 0%.	0.1ppm/10% change in RH, insignificant.

At Newman, altitude 550m, MSL pressure 1010, temp 35°C the station pressure is about $1010 - (550/9) = 949\text{hPa}$

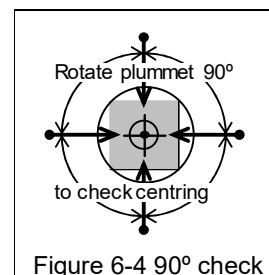
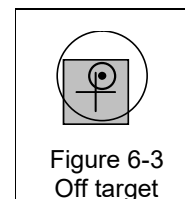
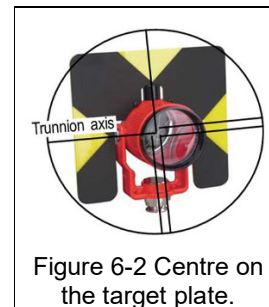
ppm correction = +20 for temperature (35 – 25) and
= +18 for pressure (1013-949)/3.6
= +38ppm

which means that, unless the meteorological conditions are dialled into the instrument, there is a slope distance error of 0.004m (4mm) over 100m distance.

4. Incorrect scale factor (S.F.) dialled or programmed into instrument computer JOB.
 - only affects displayed horizontal distance,
 - record slope distance and zenith angle
 - check and note scale factor setting or change to S.F = 1 in field book.

6.2.2 Random Errors in Observations

1. Digital reading accuracy, the ability to define the accuracy of pointing to the target.
 - specified by the manufacturer, the Sokkia SET530RK3 has a pointing accuracy of $\pm 5''$
 - in accordance with DIN 18723, ISO 17123
2. Sighting and pointing errors to the target.
Errors will tend to be random, and are affected by:
 - parallax error in focussing the eyepiece and the object pointing to the centre of the prism, instead of intersecting the target plate. (Intersect the plate, not the prism. Figure 6-2).
3. EDM errors,
 - coincidence of measuring point with vertical axis,
 - errors in distance calculation (oscillator errors)
 - errors associated with target; prism, flat plate, reflectorless
 - varies; from $\pm 2\text{mm} \pm 2\text{ppm}$ to a prism, to $\pm 10\text{mm} \pm 2\text{ppm}$ for reflectorless readings.
4. Centring errors, ability to centre over a point. (Figure 6-3).
This error may be introduced by:
 - imprecise centring of reticule point. In order of 2 – 5mm.
 - non-perpendicular optical plummet,
 - check misalignment of plummet by 90° observations through the plummet to the ground point (Figure 6-4).



¹ Rüeger, J.M., Introduction to Electronic Distance Measurement, School of Surveying, UNSW, 1984

Measurement of height of instrument trunnion axis and target axis, Figure 6-5, probably of the order of $\pm 5\text{mm}$.

$$\text{Recalling that } \Delta h = h_i + S \cos Z + CR \left(\frac{S \sin Z}{1000} \right)^2 - h_t$$

where $CR = 0.0675$

the standard error of the Δh observation, combining partial derivatives (Ghilani 2010, p160), is:

$\sigma_{\Delta h} = \sqrt{\sigma_{h_i}^2 + \sigma_{h_t}^2 + \left[\left(\cos Z + \frac{CRS \sin^2 Z}{500,000} \right) \sigma_s \right]^2 + \left[\left(\cos Z + \frac{CRS \sin Z \cos Z}{500,000} - S \sin Z \right) \frac{\sigma_z}{206265} \right]^2}$	Eqn 6-5



Figure 6-5 Trunnion axis, height of instrument HI

For example, where $h_t = h_i$, $\sigma_{h_t/h_i} = \pm 0.005$ and $z = 88^\circ \pm 5''$,
 $S = 150 \pm 0.01\text{m}$
 $\Delta h = 5.235\text{m}$
 $\sigma_{\Delta h} = \pm 0.008$.

However, analysis of the observations shows that the contribution of the error in measuring instrument and target height, $\sigma_{h_t/h_i} = \pm 0.005\text{m}$, contributes $\pm 0.0071\text{m}$ while the TS contributes $\pm 0.0036\text{m}$ to the error budget. (They combine to produce $\sigma_{\Delta h} = \pm 0.008$.)

6.3 Calculation of Three Dimensional Coordinates from Observations

6.3.1 Capture of points for a digital elevation model (DEM). “Resection by distances.”

There are two calculation tasks to be carried out from your observations:

1. Calculation of observation point coordinates including:
 ENH coordinates,
 orientation of backsights,
 check calculations.
2. Calculation of data point coordinates taken from the observation point:
 ENH coordinates.

6.3.2 Calculation of Observed Distances

Two methods are available, both of which rely on the reduction of the observed **slope distance**, **S**, into its **horizontal distance**, **H**, and **vertical distance**, **V**, components.

H and V are the rectangular coordinates of the polar vector r, θ

r = slope distance, **S**

θ = zenith angle, **Z**.

Horizontal distance	$H = S \sin Z $	Eqn 6-6

Correct sign of elevation difference, target/instrument.	$V = S \cos Z$	Eqn 6-7

Remember: The zenith angle, Z, is from the ZENITH (Figure 4-1(b)) and is at 90° when horizontal. DO NOT try to calculate angles of elevation or depression. Too risky.

6.3.3 Direct Calculation of Horizontal X, Y Coordinates by Intersection of Horizontal Distances to Known Points

The method relies on measuring horizontal distances from the unknown point to two known control points. It is analogous to finding the point of intersection of the radii from the two known points. The horizontal distance between the two controls is calculated by the JOIN method, it also provides the bearing from one point to another.

Two other distances are required: the **perpendicular distance, h** , from the known base line to the apex of the triangle (the “height” of the triangle); the **projected distance, p** , along the base line to the perpendicular intersection point.

H_1 is calculated, H_2 and H_3 are measured from Point 1. Note that the distances are **opposite** their respective control points.

To conform with survey convention, we re-label the coordinate frame as E (east) and N (north). Direction from 0° is a clockwise rotation through to 360° . Again, it is a polar to rectangular conversion with:

r indicating **horizontal distance, H**
 θ indicating direction and

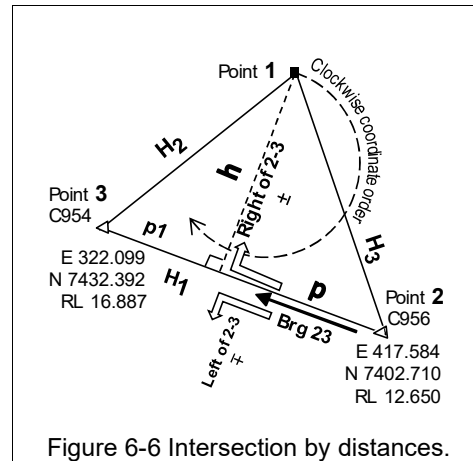


Figure 6-6 Intersection by distances.

	$E = H \sin \theta$	Eqn 6-8

	$N = H \cos \theta$	Eqn 6-9

With intersection point 1 label the two known points in **clockwise** order as 2 and 3. (See Figure 6-6).

Solve the coordinates of point 1 directly by the equations:

Pt 1 to left/right line 2-3; use $\pm$	$E_1 = E_2 + p \sin \theta_{23} \pm h \cos \theta_{23}$	Eqn 6-10

Pt 1 to left/right line 2-3; use $\mp$	$N_1 = N_2 + p \cos \theta_{23} \mp h \sin \theta_{23}$	Eqn 6-11

Note the equations require finding the distance along the known line, 2-3, to the perpendicular projection to the unknown point, 1. The distance p is the projection to the perpendicular. h is the height of the perpendicular. p_1 is the remainder of distance 2-3. See Figure 6-6.

H_1 is calculated distance 2-3; H_2 and H_3 are the reduced horizontal distances 1-3 and 2-3.

p	$p = \frac{H_1^2 + h_3^2 - H_2^2}{2H_1}$	Eqn 6-12

h	$h = \sqrt{H_3^2 - p^2}$	Eqn 6-13

6.3.3.1 An alternate method, avoiding bearings calculations.

It may be more convenient to use a coordinate difference form rather than the bearing from 2 to 3. The modern scientific calculator makes this step redundant. However the problem can be solved with a four function plus $\sqrt{\quad}$ calculator. It may be more convenient on your HP10s calculator. It also guards against input errors, and problems determining the bearing.

1. Calculate the POLAR between 2 and 3. (Bearing 2-3 and distance).

$E_3 - E_2 =$ opposite side;

$N_3 - N_2 =$ adjacent side;

H_1 is the hypotenuse found by Pythagoras. It is the side **opposite** point 1.

	$H_1 = \sqrt{(E_3 - E_2)^2 + (N_3 - N_2)^2}$	Eqn 6-14

The **Sin θ_{23}** and **Cos θ_{23}** can found directly from coordinate differences:

Calculating $\sin(\theta_{2-3})$ directly from opposite side $E_3 - E_2$	$a = \frac{E_3 - E_2}{H_1}$	Eqn 6-15

Calculating $\cos(\theta_{2-3})$ directly from opposite side $N_3 - N_2$	$b = \frac{N_3 - N_2}{H_1}$	Eqn 6-16

2. Calculate the horizontal distances from measured slope distance: Eqn 6-6 and Eqn 6-7.

Opposite point 2	$H_2 = S_{13} \sin Z_{13}$	
Opposite point 3	$H_3 = S_{12} \sin Z_{12}$	

3. We know, from the POLAR, that $E_1 = E_2 + dE_{21}$ and $N_1 = N_2 + dN_{21}$.

Working with Eqn 6-10 and expanding:

E_1	$E_1 = E_2 + p \frac{E_3 - E_2}{H_1} \pm h \frac{N_3 - N_2}{H_1}$ $\Rightarrow E_1 = E_2 + ap \pm bh$	Eqn 6-17

Similarly, working with Eqn 6-11 and expanding:

N_1	$N_1 = N_2 + p \frac{N_3 - N_2}{H_1} \mp h \frac{E_3 - E_2}{H_1}$ $\Rightarrow N_1 = N_2 + bp \pm ah$	Eqn 6-18

4. Then, by specifying that the coordinate points are in clockwise order,

1 is to the **right** of **line 2 – 3** Figure 6-6 the intersection by distance solution becomes:

$E_1 = E_2 + ap + bh$	1 is to the right of line 2 – 3
$N_1 = N_2 + bp - ah$	1 is to the right of line 2 – 3

5. Calculate bearings from point 1 to points 2 and 3 to provide orientation and checks:

Bearing 1-2	$\theta_{12} = \arctan \left(\frac{\Delta E_{12}}{\Delta N_{12}} \right) = \arctan \left(\frac{E_2 - E_1}{N_2 - N_1} \right) \text{ in correct quadrant}$	Eqn 6-19

Bearing 1-3	$\theta_{13} = \arctan \left(\frac{\Delta E_{13}}{\Delta N_{13}} \right) = \arctan \left(\frac{E_3 - E_1}{N_3 - N_1} \right) \text{ in correct quadrant}$	Eqn 6-20

6.3.3.2 Worked example 6.1

Sample calculation sheet from field book data:

Table 6-1 Observations for intersection distances.							
At STN 1001						HI	1.535
To STN	Face	HAR	VA (Z)	S dist	H=S sinZ	V=S cosZ	
C956	L	120 20 30	92 30 30	70.184			HT 1.598
Mean Dirn		120 20 35	92 30 25	70.183	70.116	-3.070	
C956	R	300 20 40	267 29 40	70.182			
Mean angle		137 50 45					
C954	L	258 11 20	87 55 10	36.272			HT 1.750
Mean Dirn		258 11 30	87 55 20	36.272	36.248	1.315	
C954	R	78 11 40	272 04 30	36.272			

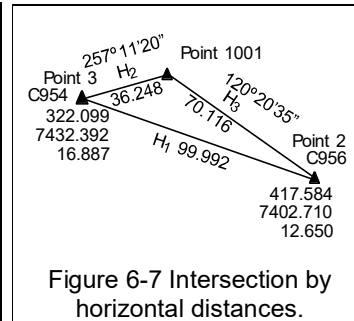


Figure 6-7 Intersection by horizontal distances.

- Enumerate data set and calculate parameters (from Figure 6-7 and Table 6-1):

From supplied data:

$$\begin{aligned}
 \Delta E_{23} &= E_3 - E_2 = -95.485 & \Delta N_{23} &= N_3 - N_2 = 29.682 \\
 H_1 &= H_{23} = \sqrt{(\Delta E_{23})^2 + (\Delta N_{23})^2} = 99.992 & & \text{from calculated join 2 - 3, Pythagoras} \\
 H_2 &= H_{13} = 36.248 & & \text{from reduction of slope distance} \\
 H_3 &= H_{12} = 70.116 & & \text{from reduction of slope distance} \\
 a &= \Delta E_{23}/H_1 = -0.95493 & b &= \Delta N_{23}/H_1 = 0.29684 \\
 p &= (H_3^2 + H_1^2 - H_2^2)/(2H_1) = 68.0091 & h &= \sqrt{(H_3^2 - p^2)} = 17.0591
 \end{aligned}$$

- Calculate coordinates of STN 1:

$$\begin{aligned}
 E_1 &= E_2 + ap + bh = 357.704 & \text{note + sign} \\
 N_1 &= N_2 + bp - ah = 7439.188 & \text{note - sign}
 \end{aligned}$$

- Calculate RL STN 1001 and check:

(Back from RLs 2 & 3) Figure 6-8.

$$\begin{aligned}
 RL_1 &= RL_2 - V_{21} + HT_2 - HI \\
 &= 12.650 - (-3.070) + 1.598 - 1.535 \\
 &= 15.783 \\
 RL_1 &= RL_3 - V_{31} + HT_3 - HI \\
 &= 16.887 - (+1.315) + 1.750 - 1.535 \\
 &= 15.787 \checkmark \\
 \text{Mean } RL_1 &= 15.785
 \end{aligned}$$

- Calculate BRG₁₂ and BRG₁₃

$$\begin{aligned}
 \theta_{12} &= \arctan \left(\frac{E_2 - E_1}{N_2 - N_1} \right) \text{ in correct quadrant} \\
 &= \arctan (-1.64152) \\
 &= -58.65059 + 180 \text{ (quadrant 2)} = 121.34941 \\
 &= 121^\circ 20' 58''
 \end{aligned}$$

$$\begin{aligned}
 \theta_{13} &= \arctan \left(\frac{E_3 - E_1}{N_3 - N_1} \right) \text{ in correct quadrant} \\
 &= \arctan (5.238931) \\
 &= 79.193453 + 180 \text{ (quadrant 3)} = 259.19345 \\
 &= 259^\circ 11' 36''
 \end{aligned}$$

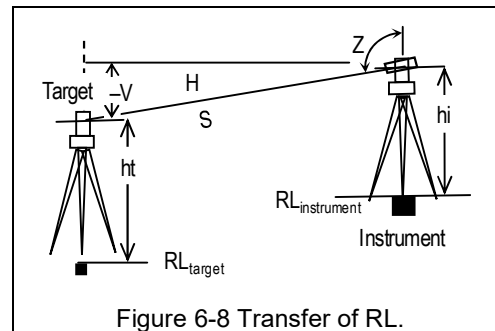


Figure 6-8 Transfer of RL.

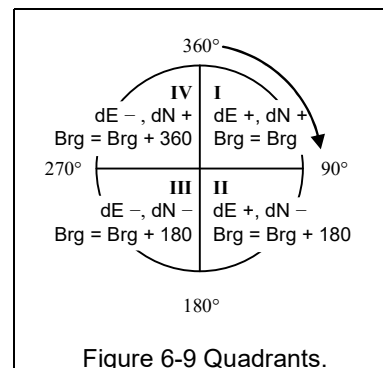


Figure 6-9 Quadrants.

5. Calculate angle between BRG₁₂ and BRG₁₃ and check with mean observed angle.
 $= 259^\circ 11' 36'' - 121^\circ 20' 58''$
 Calculated angle $= 137^\circ 50' 39''$
 From field sheet:
 Mean angle $= 137^\circ 50' 45'' \checkmark$ (6" difference over 70m = 0.002m (2mm))
6. Summarize data:
- | | | |
|------------|------------|------------|
| STN 1001 | C956 | C954 |
| E 357.704 | E 417.584 | N 322.099 |
| N 7439.188 | N 7402.710 | N 7432.392 |
| RL 15.785 | RL 12.650 | RL 16.887 |
- HAR₁₂ 120° 20' 35" Mean direction (FL/FR) to C956
 BRG₁₂ 121° 20' 58" BRG to C956.
 Rotation angle from mean direction +1° 00' 23"
 BRG₁₃ 259° 11' 36" BRG to C954.

6.3.4 Orientation of a Local Coordinate System to a Grid System

- Initial orientation of directions was by alignment to true north using a tube (trough) compass to define magnetic north. The Total Station horizontal circle was clamped, and the true direction of magnetic north was entered into the Total Station after allowance was made for magnetic variation (deviation).
- Once the coordinates of the new control point are determined, the calculated GRID BEARING is compared to an observed control point bearing and the rotation angle determined. +1° 00' 23"
- The initial BS bearing can be corrected to give a true bearing for future observations.

6.3.5 Observation, Booking and Calculation of Terrain Points for DTM.

- Aim at your back sight on FL.
 In this case to STN 2, C956 has a BRG of 121° 20' 58" (§ 6.3.4(3))
 Set the calculated bearing on the Total Station using **H.ANG**
 Set the prism on a prism pole and adjust HT to the same height as the HI.
 Observe and book on FL to significant points, as illustrated in a booking sheet
 Calculate the horizontal distance, H and the vertical distance, V, to each point.
 see observations in Table 6-2.

Table 6-2 DEM observations.

At STN 1001		Calculate horizontal, H, and vertical, V, distances				RL 15.785	HI	1.535
To STN	Face	HAR	VA (Z)	S dist	H = S sinZ	V = S cosZ		
2, C956	L	121 20 58	92 33 36	70.186	70.116	-3.135	HT	1.598
NE Corner 204		143 35 09	93 50 00	50.327	50.215	-3.364	HT	HI
N face 204		154 30 50	94 19 33	44.876	44.748	-3.385		
CH 00		155 31 20	95 40 55	39.747	39.552	-3.395		
705, CH93.3		44 52 37	91 14 30	71.903	71.886	-1.558		
Path P1		34 59 41	90 22 45	88.404	88.402	-0.585		
P2		52 55 07	90 33 14	61.550	61.547	-0.595		
P3		76 07 40	91 07 11	31.470	31.464	-0.615		
S Valve 1								
Tree T1 dia 0.4								

2. Reduction of observations to coordinates

E station point = E STN 1 + dE (= H Sin(HAR))

N station point = N STN 1 + dN (= H Cos(HAR))

RL station point = RL STN 1 + V + HI – HT note that if HT = HI then

RL of point = RL STN 1 + V, see Table 6-3.

Table 6-3 Reduction of observations.

A+ STN 1001	Radiate from 1001		357.704	7439.188	15.785		Calc
To STN	dE	dN	E	N	V	RL	
2, C956	59.880	-36.478	417.584	7402.711	-3.135	12.587	Check
Cnr 204	29.808	-40.410	387.513	7398.778	-3.364	12.412	
N Face 204	19.225	-40.393	376.959	7398.795	-3.385	12.400	
CH00	16.388	-35.997	374.092	7403.191	-3.935	11.850	
705, CH93.3	50.722	50..594	408.426	7490.129	-1.558	14.230	
Path P1	50.699	72.420	408.426	7511.608	-0.585	15.200	
P2	49.101	37.110	406.805	7476.298	-0.595	15.190	
P3	30.546	7.544	388.250	7446.732	-0.615	15.170	
S Valve 1							
Tree T1 dia 0.4							

6.3.6 Feature Codes for Point and Line Pickup

In general, a **feature** is either a **point** or be part of a **line** or **polygon** (a closing line). Prefixing a feature with a symbol can be used to tell processing software how to display and process the collected data.

A **point** indicator is generally a plus sign, +.

A **line** indicator could be a slash mark, /.

Polygon indicators would depend on the specific software.

During the survey data collection process, it is necessary to provide a description of each feature. This description is generally a short letter code or a numeric code which is associated with the feature. Codes are generally grouped around specific activities from which a particular code may be chosen.

Many integrated survey packages have suggested codes. Survey companies may have specific codes to be used with data pickup. These codes are often held in either the Total Station itself, or in an external data recorder attached to the Total Station.

Codes should be simple and easy to remember. Often, they are an acronym of the full name. Whether a feature is a point, or part of a line.

Data collection for this field practical will be recorded on a feature pickup sheet attached to Field Practical 3. Use the To STN column to record sequential observation numbers. The Remarks column can be used to record the feature code.

Some suggested codes are listed in Table 6-4 :

Table 6-4 Sample feature codes.

Nr	CODE	Description	Nr	CODE	Description	Nr	CODE	Description
1	PM	Perm survey mark	160	DH	Drill hole	411	EP	Edge path
2	BM	Bench mark	170	WROW	Windrow	512	CABL	Cable pit
4	STN	Instrument station				609	RET	Retaining wall
5	SM	Survey mark	201	TREE	Tree >2m	713	ELP	Electric pole with light
101	CONT	Contour	204	SHRB	Shrub <5m	716	EPIT	Electricity pit
102	TOP	Top cut/fill	401	CB	Centre bitumen road	741	SPIT	Sewerage pit
103	TOE	Toe cut/fill	403	EB	Edge bitumen road	751	SV	Stop valve
104	ES	Existing surface	407	INV	Invert kerb/channel	762	UPIT	Unclassified pit
150	GRID	Grid intersection	410	PATH	Pedestrian path	851	TXT1	Text 1

6.3.7 DTM File Output for CAD Processing

As part of reporting results of Field Practical 3 appendix A2-3-1 details the observation and computational tasks of the exercise. Part of that process is to produce a file suitable for input to a CAD package.

This requires the coordinates be assembled in a text file. The format will be ENZP which can be made up in an Excel spreadsheet and saved as a TXT file. Completing part of the table, Table 6-3, we produce an ENZP file as shown.

This is a file of tab (→) separated fields. The exact format will be defined by your CAD software. The P (description) field may be of limited length.

Table 6-5 ENZP tab delimited (*.txt) file for CAD

E	N	Z	P
357.704→	7439.188→	15.785→	STN1001
376.959→	7398.795→	→	RO NE B105
417.584→	7402.711→	12.587→	2, C956
387.513→	7398.778→	12.412→	NE BLD 204
376.959→	7398.795→	12.400→	N face 204
374.092→	7403.191→	11.850→	CH 00
408.426→	7490.129→	14.230→	705, CH93.3
408.426→	7511.608→	15.200→	Path P1
406.805→	7476.298→	15.190→	P2
388.250→	7446.732→	15.170→	P3

6.4 Concluding remarks

This Chapter presented an alternative method to height determination that was covered in Chapter 1 (i.e., spirit levelling). In trigonometric heighting, abbreviated trig. heighting, rather than using a level as we saw in Chapter 2, a total station is used instead. Whereas in Chapter 2 the spirit levelling measured height differences, trig. heighting presented in this chapter measures angles and slope distances, which are then subjected to the traditional “sine” “tangent” and “cosine” formulae to derive the heights, hence its name trigonometric heighting. The advantages of trig. heighting are rapidness in generating heights and the low cost involved. For various engineering tasks that require rapid generation relief, e.g., during reconnaissance for planning purposes, the method is handy. Moreover, with the current use of robotic total station that once set automatically tracks the target, spot heights and random heights of points of interest can be obtained. The disadvantage with the method is the low accuracy compared to the spirit levelling, which makes it unsuitable for use in obtaining vertical controls.

6.5 Reference to Chapter 6

1. Ghilani, C., Adjustment Computations Spatial Data Analysis, Wiley, 2010, 5th ed
2. Rüeger, J.M., Introduction to Electronic Distance Measurement, School of Surveying, UNSW, 1984



7 Chapter 7 Underground Mining Survey: Transferring Traversing and Levelling Measurements

7.1 Introductory Remarks

Surface traversing has been introduced and practiced during the course. In underground workings traversing techniques remain the same as above ground. The conditions are physically more difficult; it's dark, often cramped for space, there are shorter observation lines, the need for more control points, the types of control point installation, continuous mine working activity. It's tricky.

Whilst surveying of the mine should be left to the surveyors, this section is intended to provide the mine engineer an insight in to the various techniques of:

- a. Transferring surface coordinates to underground workings
- b. Transferring benchmarks and RL underground
- c. Establishing recoverable control points in the workings
- d. Checking traverse directions
- e. Understanding limitations of transfer techniques.

The mine manager, within his many duties, is also responsible for all aspects of mine surveying. As a mine engineer you should be conversant with the mine manager's responsibilities as you progress your career. Familiarity with Mining Act, Regulations and Codes of Practice is essential.

7.1.1 Western Australian Mines Act and Regulations

The information in this section is a generalised precis of parts of the Mines Act and Regulations. In no way does it claim to be authoritative. You must act within current legislation, guided by your manager. Neither does it pertain to any other State authority.

In Western Australia mine surveying is governed by the requirements in the *Mines Safety and Inspection Act 1994* and the *Mines Safety and Inspection Regulations 1995* relating to surveys at mining operations. Both are enforceable documents and make punitive provision for breaches of the Act and the Regulations. The *Act* sets out broad duties, and is supported by a further tier of statute, commonly referred to as regulations, together with lower tiers of non-statutory codes of practice and guidelines.

The *Regulations* support the Act, they provide more specific requirements for a range of activities.

The *Mine survey – code of practice* provides guidance on meeting the requirements in the Act and Regulations relating to surveys at mining operations. The Code applies to all mine surveyors employed on mine sites in Western Australia, particularly at underground and surface mining operations. The provisions of this *code of practice* apply to all mining operations as defined in the Act [s. 4(1)].

This *code of practice* was developed by the Mines Survey Board. It is issued by Resources Safety under the Mines Safety and Inspection Act 1994, with endorsement by the Mining Industry Advisory Committee (MIAC) and approval from the Minister for Mines and Petroleum in the State of Western Australia.

The recommended reference for this publication is: Department of Mines and Petroleum, 2011, *Mines survey — code of practice* (2nd edition): Resources Safety, Department of Mines and Petroleum, Western Australia, 20 pp. ISBN 978 1 921163 70 8 © Department of Mines and Petroleum 2011.

http://www.dmp.wa.gov.au/Documents/Safety/MSH_COP_MineSurvey.pdf

The *Mines survey – code of practice* is the mine surveyor's professional handbook. Mine engineers should be familiar with its contents. Always check for the latest edition. With the

introduction of the new Geodetic Datum (GDA2020) and Map Grid (MGA2020) on 1 January 2020, certain provisions of the above documents will change in line with that date.

7.2 Mines Survey Compilation and Tolerances

The Western Australian code of practice sets out recommendations and minimum standards of accuracies for mine surveys in line with the 2nd edition.

With the introduction of the new Geodetic Datum of Australia 2020 (GDA2020) and corresponding Map Grid of Australia 2020 (MGA2020) on January 1, 2020 the various Acts and Regulations and the Mine Survey - Code of Practice are being amended.

7.2.1 Survey Compilation (COP §1.3, Compilation. §1.4 Conversion to MGA2020)

The survey should be conducted on the Map Grid of Australia 2020 (MGA2020) based on the Geodetic Datum of Australia 2020 (GDA2020). Any mine survey conducted on a local grid must be converted to MGA2020 in mine documents and plans. The transformation must be documented and certified by the mine surveyor.

Levels are to be based on the Australian Height Datum (AHD71) or a local height datum. The conversion between local and AHD71 must be documented. There are many instances where a local height datum is used.

The relationship between local grids and heights and MGA2020/AHD71 must be clearly defined.

7.2.2 Baselines

The minimum surface baseline length is 250m. (COP §2.3). It must be substantially marked by a monument (survey mark) at each end. Its relationship between MGA2020 and any local grid in terms of coordinates and orientation must be clearly stated.

An underground baseline should be established in the underground workings of each level off a shaft or at least every 750 m of traverse in a decline access. The baseline should be suitably positioned, as long as practicable, but at least 100 m in length. (COP §2.4).

7.2.3 Accuracy

Minimum accuracy of a traverse is 1:5000 or within 0.5m absolute accuracy to the mine datum. (COP §2.1), (COP §2.5).

The standard for level misclose is 12mm/km of closed loop survey. RLs of forward stations placed by total station differential levelling should be checked by re-survey or differential levelling. One off measurements of instrument and station heights are unacceptable. A second set of independent observations must be used. (COP §2.7).

Note that 12mm/km is a “tighter” requirement than the traditional Class 3 “ $12\sqrt{K}$ ” specification. E.g. over a 200m double run the mine accuracy is 2.4mm, $12\sqrt{K} = 5.4\text{mm}$.

Instrumentation used for control surveys and traversing should meet to the following precision specifications:

- a. angular measurements (horizontal and vertical) — maximum standard error of $\pm 5''$;
- b. distance measurements — maximum standard error of $\pm (3 \text{ mm} + 5 \text{ ppm})$;
- c. GPS positioning — maximum standard error of horizontal positioning of $\pm (10 \text{ mm} + 10 \text{ ppm})$; and
- d. levelling — maximum standard error of $\pm 4 \text{ mm}$ per kilometre of double run levelling.

7.2.3.1 Instruments at Curtin

These instrument requirements are generally met by mid-range instruments.

The total stations, Sokkia SET530RK3, used by Spatial Sciences at Curtin Bentley campus are 5" instruments, or the better Sokkia SET1 and FX101, which are 1" instruments. Suitable Leica Geosystems instruments are used at Bentley and WASM Kalgoorlie.

The distance accuracy requirement can be met in fine mode to both prisms ($\pm (2 \text{ mm} + 2 \text{ ppm})$) and reflective sheet (range 100m – 500m), reflectorless white (200m) or gray (100m) to $\pm (3 \text{ mm} + 2 \text{ ppm})$. Both instruments have the same performance.

The Trimble R10 GNSS receivers meet the accuracy requirements using static GNSS surveying and real time kinematic (RTK) surveying. Note that code differential GNSS positioning, including DGPS (RTCM104) and SBAS (WAAS, EGNOS etc) differential, do not meet the requirements.

The Leica NA724 automatic levels have stated accuracy of $\pm 2\text{mm}$ per km of double run levelling.

Sokkia GYRO X II gyro stations mounted on total stations have 15" autonomous azimuth accuracy.

7.2.4 Connecting Surface and Underground Surveys

The definition of various access methods to mineworks underground are detailed in the Code of Practice Appendix 2, Glossary.

We will examine the connection of surface and underground surveys by:

- a. traversing through an adit or a decline or incline ramp;
- b. plumbing or plummetering one or more vertical shafts; or
- c. using a gyro theodolite to establish azimuth.

7.2.5 Levelling

Examine transfers of level from the surface by:

- a. differential levelling through ramps and adits;
- b. vertical plumbing; or
- c. EDM using prisms and retroreflective targets.

7.3 Establishment of Control Points for Traversing

Survey control stations are marks established either at (or just below) ground level or on permanent pillars. Their positions are established by the extension of an existing control network through conventional traversing or differential GNSS. The stations are in protected places and do not generally interfere with engineering activities. They are safely and easily accessible, cover wide areas and can provide an overall network of redundantly adjusted coordinates.

7.3.1 Backs

In underground mining traversing the traditional control stations are known as “backs”. Backs are ring bolts anchored in the ceiling of the tunnel. Floor based control stations are untenable. They are intrusive to the mine operations, difficult to find and maintain and are probably unstable due to mine movement exacerbated by continual heavy traffic. Backs were suitable when ceilings were 2 – 3 metres high; They could be easily found and were reasonably accessible both to plumb-bobs and levelling staves.

Differential levels are read from an inverted staff fitted with a hook in its base suspended from the back’s ring bolt. The hook must be installed in the base of the level with an allowance for the offset from the zero of the staff foot.

Using backs, the survey instrument, a theodolite, was set up under a suspended plumb-bob and measurements, including distance taping, made between the plumb-bob strings. Targets could be similarly installed for more precise work. This may include semi-forced centring. A number of major problems arise:

1. Installation of backs for control in the roof is limited by height. High marks require scaffolding for access.
2. Generally undertaken near the centreline of the drive, thus limiting mining work during survey.
3. Plumbing inaccuracies due to plumb line accuracy and ventilation air currents.

7.3.2 Wall Brackets

Later techniques involved the installation of wall brackets as permanent control point installations. They consisted of special tribrach mounting brackets grouted to the side of the tunnel. Notwithstanding access to the brackets, problems persist:

1. Difficult and expensive to install as a permanent solution.
2. Ensuring protection against damage to the projecting brackets.
3. Difficulty of sight line distances close to the wall.
4. Horizontal refraction of the line of sight due to horizontal temperature gradients across the drive.

7.3.3 Wall Stations

Wall stations in underground mining are small diameter tubes (sleeves) drilled and grouted into mine walls. They are in fixed positions, so have known coordinates. They are surveyed by the “free station” method of positioning which uses resection from an arbitrarily positioned total station to prisms mounted on a stem inserted in the wall station tube.

Survey control is projected forward from the free station to desirably placed new wall station tubes.

7.4 Wall Stations in Mining Traversing

The latest method, introduced to underground mining in Western Australia, is the use of wall station targets positioned by resection from an auxiliary station. Modern total stations, using specialised software, establish a position by measuring to fixed targets. As the station’s coordinates are known, further targets can be installed by radiation from the total station.

The total station can then be moved forward; its position re-established from new targets and further targets can be installed. This method of establishing control using targets relies on a specific set of target prisms attached to a standardised stem. A small diameter hole is drilled into the tunnel wall. A 10mm diameter aluminium sleeve is installed in the hole, flush with the wall. The prism and its stem are temporarily installed in the sleeve while the survey measurements are taken.

At the end of the survey the prism and stem are retrieved, leaving only the sleeve in place. With suitable marking (for finding it) the sleeve now acts as a semi-forced centring mount for the prism and stem. And since the prisms are standardised the coordinates of the prism are held between epochs.

The siting of the total station is arbitrary, ensuring that set-up can be made in a safe place. It is up to the surveyor to optimise the traverse configuration when siting the total station.

7.4.1 Position by Resection. Also Known as a Free Station Position

An example of a resection to two known points is illustrated in §16.5.1. The example considers only 2D coordinates in X and Y (Easting and Northing). The result was obtained by least squares.

The determination of horizontal coordinates and height (ENH) was shown in §6.3 by means of the average of two height differences from the control stations. The observations were of zenith angle and slope distances.

Although shown as two operations, the normal resection observations to the two control stations essentially consisted of one observed angle, two zenith angles and two slope distances. These observations can be solved and analysed by least squares as a unique solution. Observation to a third (and subsequent) points provides an overdetermined solution. This results in a set of coordinates for the observation station (the total station) combined with residuals expressing the accuracy of the result.

7.4.1.1 Resection to ground control points

The calculation of a least squares position by resection measurement is a standard program on modern total stations. The program requires:

1. Coordinates of the known target points; E_i , N_i , RL_i . Height of targets, instrument; HT_i , HI .
2. Observed horizontal direction to each point, H_i .
3. Observed vertical angle, the zenith angle, ZA_i .
4. Observed slope distance, S_i .

Individual manufacturers have slightly different observation methodologies. Basically, the known stations are held as a data file in the instrument. Once started, the program guides the operator through the observation steps to the job completion where the observing station coordinates; E_0 , N_0 and RL_0 , are displayed with corresponding standard deviations σE , σN . Other manufacturers may show σRL and σAz .

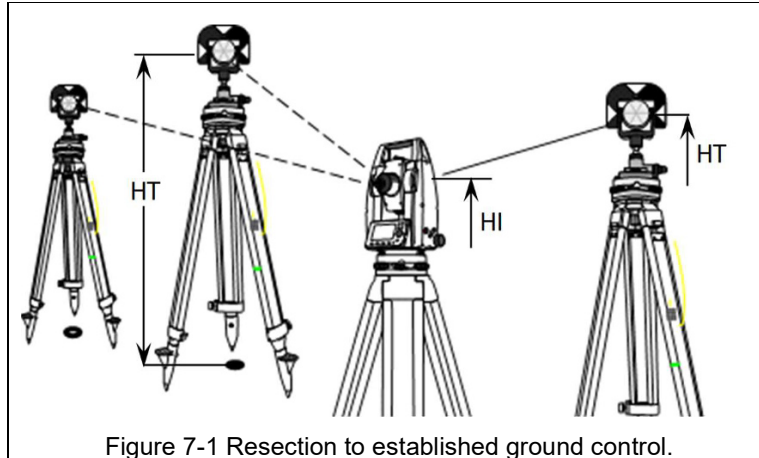


Figure 7-1 Resection to established ground control.

The point of the observations is to find the coordinates of the total station from the coordinates of the targets. Traditionally the target RL values come from the RL of the control point and the target height. The free station control point, under the total station, has its RL fixed by subtracting the instrument height from the total station E , N , Z coordinates.

7.4.1.2 Resection to wall stations

Normal resection ground control points are difficult to install and maintain in mining. Inevitably control moved forward rapidly as drives are developed.

The erection of traditional bracketed wall stations is expensive and time consuming. Installations use a mounting plate to which the tribrach base is fixed by a 5/8" bolt, allowing forced centring of the targets or total station. Some brackets are mounted on a tube that is grouted into a drill hole in the drive wall. These are about 30mm diameter and require a drilling rig. Whilst the setup was suitable for carrying control forward, it was cumbersome to establish new stations quickly. Prism targets could be mounted on the brackets and the total station used in free station configuration.

What was needed was a way of establishing a simple, robust and inexpensive method of mounting targets in the drive wall with a repeatable method of establishing the prism coordinates. The total station is located by resection (free station), new targets are installed and coordinated from the free station.

In the early 2000s surveyors came up with the idea of using a small diameter (10mm) sleeve glued or puttied into the drive wall. A suitable prism was mounted on a stainless-steel stem about 200mm long and 12mm diameter. The bottom 60mm of the stem was reduce to 7.5mm to allow it to slip into the wall tube. Figure 7-2.

A 10mm aluminium sleeve has an internal diameter of about 8mm. For temporary or one-off setups, the stem may be inserted in an 8mm hole drilled directly into the rock face.

The sleeve hole is small enough to allow the use of a portable battery-operated drill driving a hole into the wall using a 12mm diameter drill bit. A temporary 8mm hole is even easier.

The requirement of the prism was that it can be observed from any direction, in any orientation, while maintaining its orientation centre. The centre of the prism is the control point. The wall station coordinates are re-established in subsequent surveys because of the standard dimensions of the stem and prism are inserted in the wall station sleeve. Local industry surveyors have settled on the Leica Geosystems GPR1 prism and GPH1 prism holder attached to a special stainless-steel stem, CR Kennedy (Leica distributors) Product Code 602009.

The Leica GPR1 prism has a stated centring accuracy of 1mm. The “prism constant” of -34.4mm means that the prism holder can be aligned in any direction and still maintain a constant coordinate, the reflective centre of the prism.

The surveyors also use the Leica Geosystems total stations; the Leica Viva TS15 is fairly modern. However, most of the older instruments, the TCRA1200 and TCRA1100 series, are more than adequate. Just a bit old. The system has been chosen to work in Western Australian mines because Leica Geosystems equipment is popular in the underground industry.

The combination of these high order prisms and total stations means that the resultant traversing and set-out is of a high accuracy. Figure 7-3 shows the orientation of wall stations in a drive. However, there are certain errors in azimuth that must be checked by traversing and incorporation of gyro-theodolite azimuth observation to “tighten” the coordinate system to required levels.

Traverse network design, observation and adjustment is the responsibility of the mine surveyor. The surveyor reports to the mine manager, who is ultimately responsible for the survey presentation.

Ignoring environmental issues in the mine such as subsidence and geological movement in the drives we now have a collection of control points that are “floating”, Figure 7-4, somewhere away from the wall station sleeve. In that the surveyor uses only one style of prism and stem, the “floating” point can be re-established by re-installing the prism.

The problem faced by a mine manager is to preserve the coordinates of the wall sleeve, rather than the floating point. Whether this is a problem depends on an interpretation of the Mining Regulations. All other control points; monuments, wall brackets and backs have coordinates associated with a physical object. Why not sleeved wall stations.

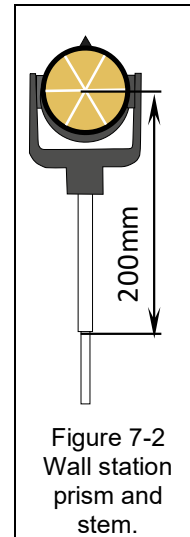


Figure 7-2
Wall station
prism and
stem.

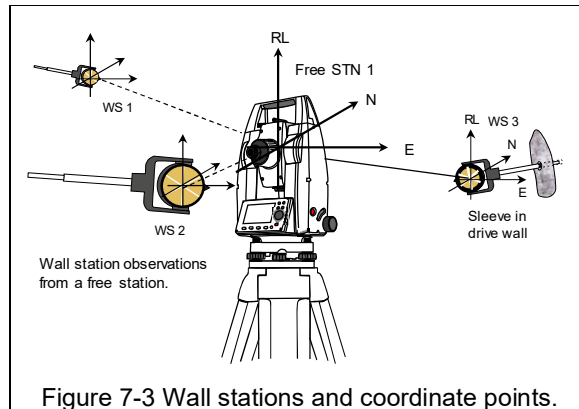


Figure 7-3 Wall stations and coordinate points.

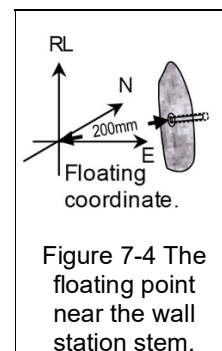


Figure 7-4 The
floating point
near the wall
station stem.

7.4.2 Projecting from a Floating Point to the Sleeve

The answer to the problem of sleeve coordinates is to project the prism coordinates along the 200mm length of the stem to the mouth of the sleeve.

How, when the vector from the prism to the sleeve is not known? All we know is its length. We don't know its directions in terms of an azimuth or inclination. The hole should be drilled off horizontal so that the sleeve can be cleaned and drained when re-used.

7.4.2.1 Two-point offset station observations

To find the coordinates of an inaccessible point a technique known as the two-station offset measurement or the "hidden point" can be used. Some total stations have this software routine built in, the Sokkia "Offset Station" routine. The method used the Sokkia RT500-K two-point target. The Sokkia prism pole appears to be about 2100mm long. The prisms are located at 1200mm and 1800mm from the foot.

Two targets are fixed to a prism pole, forming a straight line. The foot of the pole, the offset point indicator, is set at a measured distance from the second prism. In Figure 7-5 P1 and P2 are separated by 5 units. P2 is 1 unit from the foot.

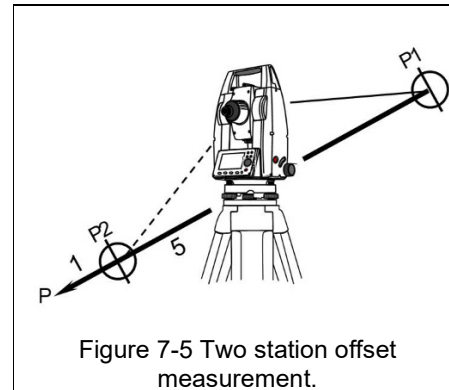


Figure 7-5 Two station offset measurement.

From the known total station coordinates (E, N, Z)

and alignment (azimuth) the routine requires prism P1 and P2 to be observed. The distance from P2 to the foot is entered and the program calculates the coordinates (E, N, Z) of the offset point.

Basically, the polar vector between P1 and P2 is projected by the distance P2 - P.

A total station was set up on a control point with coordinates E 105.0, N 110.0 and RL 10.0. Instrument height, HI 1.7.

A backsight in the coordinate system was set in the machine for azimuth orientation. Instrument coordinates are thus: E 105.0, N 110.0, Z 11.7. (Figure 7-6).

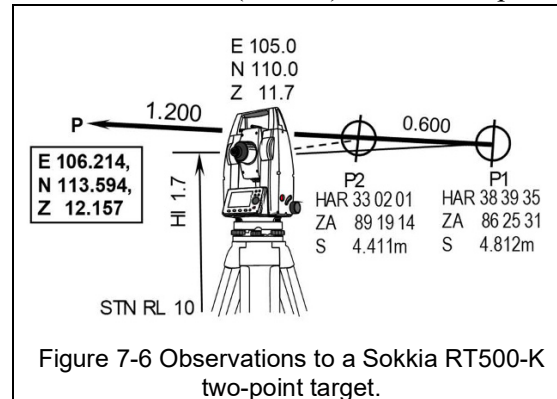


Figure 7-6 Observations to a Sokkia RT500-K two-point target.

With the foot of the two-point target on the unknown point readings were taken to:

P1: HAR 38° 39' 35", ZA 86° 25' 31", S 4.812

P2: HAR 33° 02' 01", ZA 89° 19' 14", S 4.411

P2 to P: 1.200.

The resultant coordinates are: E 106.214, N 113.594, Z 12.157.

This is the RL of P because the foot of the prism pole was on the point. HT = 0.

Solution of the foot point coordinates involves:

- Calculate coordinates of P1 and P2 from the two sets of HAR, ZA and S distance. (Join method).
- Calculate the Azimuth and ZA between P1 and P2.
- Calculate dE, dN and dH from P2 for the offset slope distance P2 to P, S = 1.2m.
 $dE = S \sin(AZ) \sin(ZA)$; $dN = S \cos(AZ) \sin(ZA)$; $dH = S \cos(ZA)$.
- Calculate coordinates of P: $E_{P2} + dE$, $N_{P2} + dN$, $Z_{P2} + dH$.

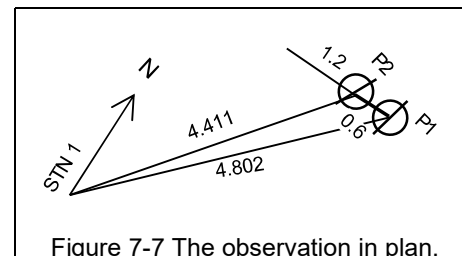


Figure 7-7 The observation in plan.

A number of points have to be made about the technique.

1. It doesn't fit with the 200mm offset of the wall station target.
2. Finding a suitable two-target prism set; the Sokkia RT500-K seems difficult to source and expensive.
3. The main requirement is that the prism centre is ON the centreline of the prism pole. The prisms are reflective sheet, so $\pm 5\text{mm}$ (stated accuracy of total station).
4. The pole must be held steady while the two prisms are read. The AP71 tripod pole stand or similar is suitable.
5. A precision total station is needed for meaningful observations. At 5m there is an angle of only $5\frac{1}{2}^\circ$ to measure the 600mm between the two prisms. Further away, the cut angle decreases.

However, the technique points the way to a solution of the wall station offset from the inserted stem tube.

Point 3, above, may not matter that much as we will develop a technique that relies on coordinate differences. Measurements to similar targets should produce adequate accuracy for the differences. The final resultant 3D vector will be accurate enough. This argument is yet to be tested in the field.

7.4.2.2 Designing a two-prism target pole for wall station sleeves

The wall station target, Figure 7-2, the Leica GPR1, is offset 200mm from the wall tube by the special stem. A second GPR1 target would need to be set co-axially with the first target at a set distance. A suitable bracket and carrier must be designing and manufacturing for attachment to the first prism and stem.

The stem and tube should supply enough rigidity to maintain the co-axial alignment of the prisms, but it would be heavy. Perhaps a lighter structure would help, using different reflectors.

Two light-weight Leica prisms come to the fore. The GMP111 mini-prism combined with the GMP112 hidden point pole. The 30cm GLS115 poles can be combined with the GPM111/112 to provide prism separations of 400, 700 or 1000mm. The two prisms maintain EDM accuracy of the total station at 1 – 2mm (depending on TS model).

Another contender is a pair of GPR105 double sided reflectors. However, they use a reflective sheet which degrades EDM accuracy to 5mm.

A special stem, designed to screw into the base of GPM111 reflector, has been designed to fit in to the wall tube and provide the same 200mm offset used by the GPR1 prism and stem.

The set-up is illustrated in Figure 7-10. The assembly is fairly compact. Whether the GPM111 would be a suitable alternate to the GPR1 for initial wall station survey is open to discussion. For the purpose of finding the sleeve coordinates it is probably adequate.

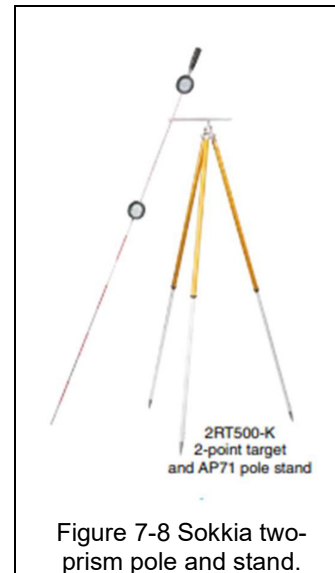


Figure 7-8 Sokkia two-prism pole and stand.

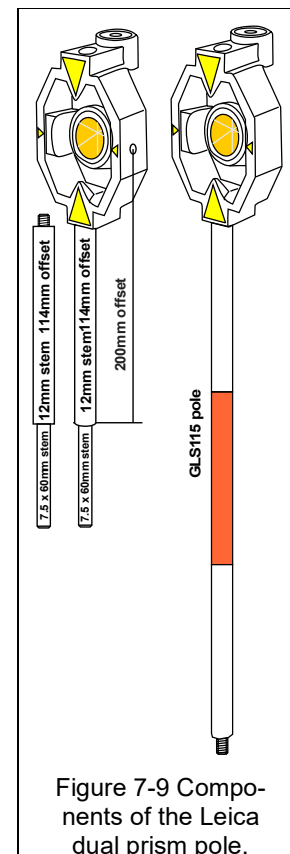


Figure 7-9 Components of the Leica dual prism pole.

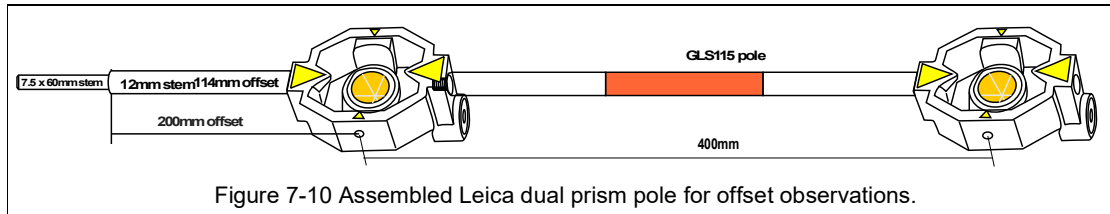


Figure 7-10 Assembled Leica dual prism pole for offset observations.

The remaining problem to be solved is the one regarding simplification of the observations. Used in conjunction with establishing a wall station the extra observation to the second prism adds little time to the task.

However, the dual pole is envisaged being used on established wall station sleeves. The task is now to find a quicker method of projecting the wall station prism coordinates, the floating point, to the sleeve.

7.4.2.3 Direction cosines to the rescue

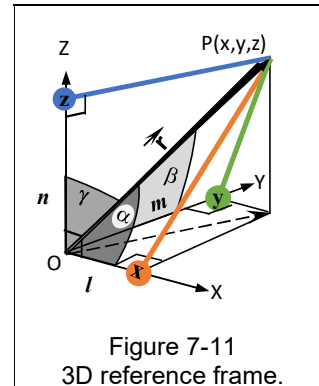
The method of transferring coordinates to the sleeve that are outlined in §7.4.2.1 is cumbersome and relies on combining a 2D position (azimuth and horizontal distance) and a height difference from ZA and slope distance.

It is worth examining a 3D projection using direction cosines. Once established, the direction cosine values can be added to the sleeve coordinates to allow projection of the sleeve to an offset prism. In that the GPR1 and stem combination should allow the original point to be returned, use of different stems and prisms could be accommodated. Another advantage is that any tectonic movement in the sleeve could be detected in later surveys.

7.4.2.3.1 Direction cosines? Didn't tell me about them at school!

The prism and stem produce a 3D vector in the X, Y and Z axes aligned with the mine survey. In Figure 7-11 the vector r (O to P) represent the wall station stem from the sleeve to the prism. Call the coordinates of O (0, 0, 0) and P(x, y, z), we can calculate the cosines of the inclined angles:

1. $r = \sqrt{dx^2 + dy^2 + dz^2}$ In this case $r=0.2m$
 2. $\cos \alpha = \cos \angle XOP = dx/r = l$
 3. $\cos \beta = \cos \angle YOP = dy/r = m$
 4. $\cos \gamma = \cos \angle ZOP = dz/r = n$
- Eqn 7-1

Figure 7-11
3D reference frame.

An example choosing some more sensible numbers.

Figure 7-12, Figure 7-13 and Figure 7-14, show plans of the 3 planes in the XYZ frame.

P(1.5, 4, 2.6) has a radius of 5 from origin O.

Direction cosines:

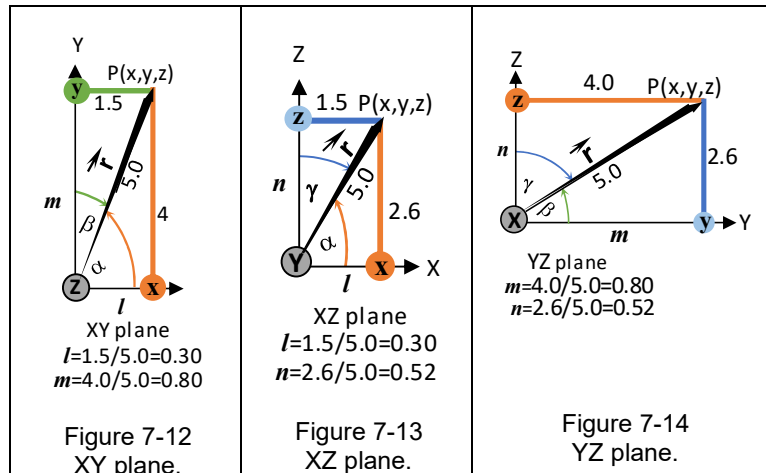
$$r = \sqrt{(dx^2 + dy^2 + dz^2)}.$$

$$r = \sqrt{(1.5^2 + 4^2 + 2.6^2)} = 5.$$

$$l = 1.5/5 = 0.3$$

$$m = 4.0/5 = 0.8$$

$$n = 2.6/5 = 0.52$$

Figure 7-12
XY plane.Figure 7-13
XZ plane.Figure 7-14
YZ plane.

For any value of r we can find dx , dy and dz from either the origin or point P.

$$dx = lr$$

$$dy = mr$$

$$dz = nr$$

This still implies that we must know the origin, which we are trying to find. However, all is not lost. If we use the dual prism pole, we establish the vector r from the coordinates of P1 and P2. Let's work on distance $r = 1$ between P1 and P2.

Using Eqn 7-1 it then means that that dX , dY and dZ are the direction cosines:

1. $r = 1m$
2. $\cos \alpha = \cos \angle XOP = (P_2x - P_1x) = l$
3. $\cos \beta = \cos \angle YOP = (P_2y - P_1y) = m$
4. $\cos \gamma = \cos \angle ZOP = (P_2z - P_1z) = n$

Eqn 7-2

The offset from P2 to the sleeve is 0.2m. Thus the coordinates of P, the sleeve, are $P(P_2x+0.2l, P_2y+0.2m, P_2z+0.2n)$

l , m and n are the cosines of the angles α , β and γ , called the direction cosines. There is no need to calculate the angle by the arccosine (arccos). In terms of azimuth, Az , and zenith angle, ZA , the values of the direction cosines are:

$$l = \sin(Az) \sin(ZA)$$

$$m = \cos(Az) \sin(ZA)$$

$$n = \cos(ZA)$$

Eqn 7-3

Note that the vector is from P1 to P2 towards P. The vector will have to be reversed if the coordinates of P are to be projected outwards from the stem sleeve.

7.4.2.4 Simulation of solution

Returning to §7.4.2.1 and the configuration in Figure 7-6; a GPR1 wall station, P2, has coordinates E108.000, N113.75, Z12.00

From an arbitrary free station, (STN1), determined by resection, the TS coordinates has coordinates E105.0, N110.0, Z11.7. The resection program provides the azimuth alignment. Using the dual prism assembly, similar to Figure 7-10, with a displacement of 1.000 between prisms ($3 \times$ GLS115 poles), Figure 7-15. Az is the HAR generated by the free station software in the TS for STN1.

The measurement to P2 was:

Az $38^\circ 39' 35''$, ZA $86^\circ 25' 31''$, $S4.812m$

Side-shot to P1:

Az $46^\circ 06' 18''$, ZA $87^\circ 45' 55''$, $S5.541m$

Coordinates of P1 by side-shot from STN1: E108.992, N113.836, Z11.916.

The coordinate differences between P1 and P2 (towards the sleeve ($P_2 - P_1$)) are:

dE , -0.992 , dN -0.086 , dZ $+0.084$.

The resulting distance between P1 and P2 is

$$r = \sqrt{(dE^2 + dN^2 + dZ^2)}.$$

$$r = \sqrt{(-0.992^2 + -0.086^2 + 0.084^2)} = 0.999 = 1.000m, \text{ the distance between P1 and P2.}$$

In this case the direction cosines between P1 and P2 (towards the sleeve) are:

$$l = -0.992, m = -0.086, n = +0.084.$$

Using the direction cosines prism, P1, offset of 0.2m, the coordinates of the sleeve, P, are:

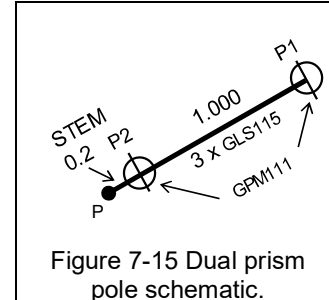


Figure 7-15 Dual prism pole schematic.

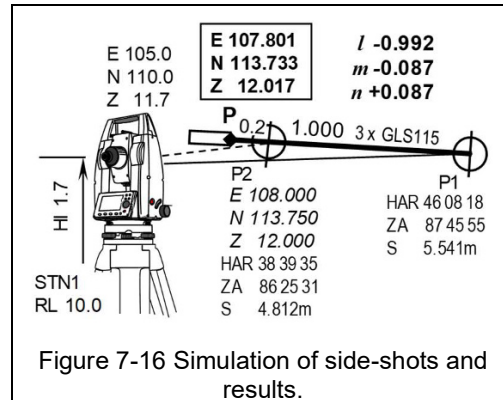


Figure 7-16 Simulation of side-shots and results.

$$P_E = P_1E + 0.2l, P_N = P_1N + 0.2m, P_Z = P_1Z + 0.2n.$$

$$P(108+0.2 \times -0.992, 113.75+0.2 \times -0.086, 12+0.2 \times +0.084)$$

$$P(107.801, 113.733, 12.017).$$

If we record the direction cosines in **reverse** (away from the sleeve), along with the sleeve coordinates, we can now re-establish P2, or any other coordinates associated with that sleeve, no matter what the stem offset to the prism.

The sleeve coordinates may then be recorded as:

$$P(E107.801, N113.733, RL12.017; l\ 0.992, m\ 0.086, n\ -0.084).$$

7.4.2.5 Repeating the observations with 1 × GLS115 pole

The side-shot observations become, Figure 7-17:

Coordinates of P1:

$$E108.397, N113.785, Z11.965.$$

The coordinate differences between P1 and P2:

$$dE, -0.397, dN -0.035, dZ +0.035.$$

The resulting distance between P1 and P2 is

$$r = \sqrt{(dE^2 + dN^2 + dZ^2)}.$$

$$r = \sqrt{((-0.397)^2 + (-0.035)^2 + 0.035^2)} = 0.400.$$

This is the distance between P1 and P2 using 1 × GLS115 pole.

In this case, dividing dE, dN and dZ by 0.4, the direction cosines between P1 and P2 are:

$$l = -0.992, m = -0.086, n = +0.084.$$

Using direction cosines and prism, P2, offset 0.2, the coordinates of the sleeve, P, are:

$$P_E = P_1E + 0.2l, P_N = P_1N + 0.2m, P_Z = P_1Z + 0.2n.$$

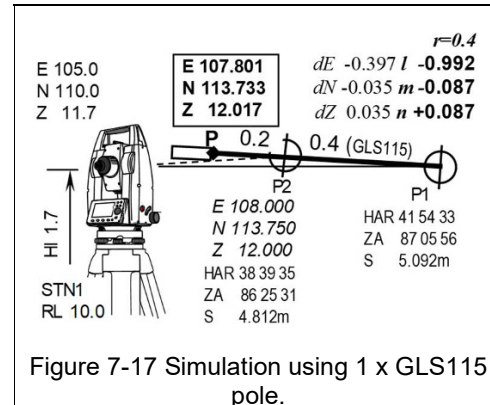
$$P(108+0.2 \times -0.992, 113.75+0.2 \times -0.086, 12+0.2 \times +0.084)$$

$$P(107.801, 113.733, 12.017).$$

If we ignore the l, m and n components we can just use the dE, dN, dZ values, halve them and add to the P2 coordinate ($0.2/0.4 = 0.5$).

The sleeve coordinates should still be recorded, with the **reversed** direction cosines, as:

$$P(E107.801, N113.733, RL12.017; l\ 0.992, m\ 0.086, n\ -0.084).$$



7.4.3 General Procedure in the Field

1. Set up TS approx. perpendicular to sleeve alignment some 10 to 20 m away.
2. Insert the stem into the sleeve and align the prisms to the TS. Check prism constant. (We are looking for a relative position between the stations.)
3. Set instrument as a free station to established control. HAR set as azimuth. Use P2, the wall station, in your resection.
4. Measure to P1, the outer prism. Record E, N, Z of target.
5. Remeasure to P2 and confirm wall station coordinates. (Blunder detection).
6. Calculate dE, dN, dZ between P1 and P2 ($P_2 - P_1$). Reduce these components to the unit radius, $r = 1$. (Divide each by the distance between P1 and P2). These are the l, m, n direction cosines from P1 to P2.
7. Multiply the 3 cosines by 0.2 (stem offset from sleeve to wall station prism).
8. Add $0.2l, 0.2m, 0.2n$ to wall station prism coordinates (E, N, RL).
9. Voilà. E, N, RL of sleeve.
10. Record the coordinates of the sleeve, and the direction cosines, l, m, n , **away** from the face.

7.4.3.1 Comments on wall station traversing

Several tests of wall station traversing, either by wall mounted brackets or wall station sleeves, has revealed some horizontal refraction problems due to large horizontal temperature gradients across the drive.

Maintenance of azimuth control in long drives; the Superconducting Super Collider (SSC) near Dallas, TX., a ring of radius 13.9km (87km) ¹ or the EUROTUNNEL (49.9km) showed significant azimuth deviations ². Mine wall station traversing has shown some deviations ³, the correction of which were implemented by gyro-theodolite observations.

The need for good traverse planning by the mine surveyor is important. The free station nature of the measurements may allow the more optimum siting of the observation point. But the proximity to the wall may be the temperature gradient influencer. A zig-zag pattern of wall stations, as suggested by Shepherd L, and Jaroz ⁴, still recommends gyro-azimuth corrections. Whether the accuracy requirements of the Code of Practice is met is the main question.

7.5 Azimuth Transfer from the Surface Coordinate Frame

Establishment of underground control from surface connections by way of adits and drives does not allow for transfer of control points from the surface to unconnected underground workings. The transfer of control is through a vertical shaft by way of a connected vertical projection. This is accomplished by either shaft plumbing with a free hanging wire or by zenith optical observations from the underground station below the shaft.

Sighting to the wire is taken at a close distance, of the order of 2 – 3metres.

The free hanging wire is suspended down the shaft and straightened with a heavy plumb bob or weight. The wire is restrained from swinging by immersing the weight in a fluid, generally water although high viscosity oil and other liquids have been used. Water is cheaper. The aim of the exercise is to reduce swing displacement of the wire caused by ventilation air currents.

Zenith transfers rely on an optical plummet or a theodolite, fitted with a right angle eyepiece, pointed to the zenith. The transferred point is marked on a horizontal target plate adjacent the upper level control point. It is measured and assigned X, Y coordinates which are then referred to the lower level.

The method relies on good visibility and is limited to about 250m. The question of safety for the operator working under an open shaft is important.

7.5.1 Physics of the Suspension Wire

The wire used for suspension of the weight must be such that it can safely stand the strain.

The wires are retained on a windlass mounted on a frame for installation adjacent the shaft. The windlass drum reel holds a substantial amount of piano wire for shaft deployment.

Determine the length of the plumbing wire, L

1. The weight suspended from should be about $L/3$ kg.
2. The weight should be limited to about half the breaking strain of the wire.
3. Single strand 304 stainless steel spring wire has a tensile strength of about 1900MPa (1mm-1.5mm diameter). Music (piano) wire has a higher tensile strength: (2200Mpa for 1.3mm). $\text{MPa} = \text{Pa}/\text{mm}^2 = 9.8\text{N}/\text{mm}^2$.
4. In general, thin wires have a tensile strength of $200\text{kg}/\text{mm}^2$. (2000MPa)
5. Minimum cross-sectional area of the wire = maximum load (plumb weight)/tensile strength.
6. For accurate sighting the diameter of the wire should be about 40" wide in the telescope of the TS. 0.6mm from 3m distance. Aiming at the centre of the wire is difficult.
7. Putting some figures to the plumbing problem: $L = 60\text{m}$ ⁵
 Plumb mass = $60/3 = 20\text{kg}$ (1). Breaking strain = 40kg (2)
 Cross-section area = $40/200 = 0.2\text{mm}^2$ (4), (5)

Radius, $r = 0.25\text{mm}$, diameter, $d = 0.5\text{mm} = 25\text{SWG}$

Width of wire at $3\text{m} = 1.7^{-4}$ radians $= 35''$.

When stored on a reel, high tensile wire attains a coil “set” so that it can create twists and kinks if not handled correctly. Kinks lead to wire fracture, a real safety issue in deployment.

The wire should be deployed down the shaft using a light weight, a 16oz plumb bob. The full weight is attached once the wire has reached the floor on the drive, and all is in readiness for the survey. It’s a matter of safety, and making sure the wire has not snagged any projections in the shaft.

7.5.1.1 Telescope reticule markings.

Survey pointings to wires are made from close range. Figure 7-18 shows the reticule view of a 0.5mm wire at distances of 3.5m, 5m and 10m distances. The gap between the reticule pairs is about $48''$. At 3.5m the gap to the reticule wires either side of the plumb wire is $9''$. At 10m the gap is about $20''$. Both these gaps provide a pointing accuracy of under $1''$.⁶

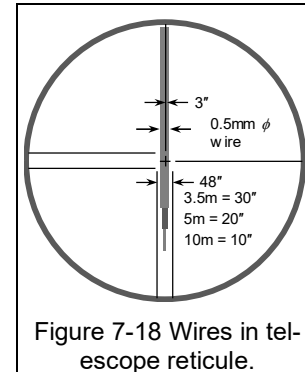


Figure 7-18 Wires in telescope reticule.

7.5.2 Student Practical Sessions in Mine Surveying

Spatial Sciences at Curtin University has manufactured small “windlasses” for use in mine surveying techniques. They are attached to railings and fences over voids allowing about a 7m wire drop. The wire is “10LB” trace nylon coated multi-strand fishing trace with a 2.5kg weight. Two frames were made for the Weisbach triangle and one for a single wire deployment.

The set-up is quick and easy.

7.6 Azimuth Transfer Methods

Azimuth transfer between the surface and the underground works via shafts employ two methods: the single wire and the two wire. The two-wire transfer uses special observation techniques that will be examined in detail so that the engineer has an appreciation of the work involved and the results expected. Azimuth is also established and confirmed using gyro stations.

7.6.1 Single Wire

A single wire coordinate transfer through a shaft implies that a traverse can be performed between two sets of coordinates, which are related to the surface traverse. No underground azimuth is available until after the traverse is run. The join between the two sets of coordinates is compared with the surface azimuth and a rotation is applied to the underground traverse.

Generally, a reasonable initial direction can be found, even if it’s a suspect magnetic direction.

S_A and S_B are the mine surface coordinates. U_A is connected by a traverse to U_B via U_1 , U_2 and U_3 . The direction of the polar between U_A and U_B is calculated and rotated to coincide with the direction $S_A - S_B$. Figure 7-19 Shows the adjusted underground traverse.

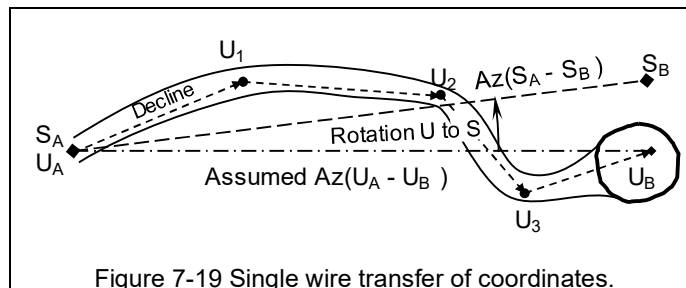


Figure 7-19 Single wire transfer of coordinates.

Table 7-1 is an example of an adjusted single wire traverse.

Table 7-1 Single wire traverse adjusted coordinates.								
At S_A		Az	Dist			E100.000	N500.000	S_A
To S_B		83°15'30"	132.195			231.281.	515.519	S_B
Line	BRG	Az	Dist					
U_A-U_B	94°13'22"		132.192					
Rotation	-10°57'52"	Az	Dist	dE	dN	E	N	Station
UA – U1	75°30'25"	64°32'33"	39.25	35.439	16.871	135.439	516.871	U1
U1-U2	91°45'35"	80°47'43"	45.28	44.697	7.243	180.136	524.114	U2
U2-U3	145°15'50"	134°17'58"	29.85	21.364	-20.847	201.500	503.267	U3
U3-UB	78°36'05"	67°38'13"	32.20	29.778	12.251	231.278	515.519	UB

7.6.2 The Weisbach Triangle

In mine development azimuth of a traverse is carried forward from two wires suspended in a single shaft. Initially co-planing was used whereby the theodolite line of sight was made coincident with the two suspended wires. This involved moving the theodolite into position, and then recalculating its coordinates, and thus forward azimuth.

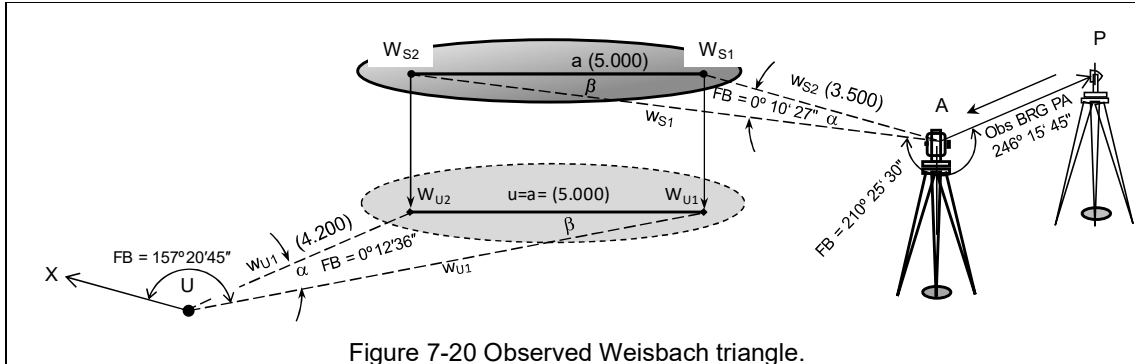


Figure 7-20 Observed Weisbach triangle.

By establishing a station almost in line with the two wires a very acute angle between the station and the two wires allows solution of the azimuth between the two wires to be found by the sine rule for triangle. In about 1853 the “Weisbach Triangle” was introduced. It has been named after its originator Professor Dr. h.c. J L Weisbach (1806-1871)⁷ after he introduced theodolites into mine surveying instead of the compass and protractor.

7.6.2.1 Measuring the Weisbach triangle

The simplified calculations on the triangle, using the Sine rule, relies on having a very acute (small) angle between the observing point and the two suspended wires.

Distance **between** the wires need not be measured better than 0.01m if the angle is kept below 30' (Ogundange, 2016). The distance **to** the close wire should be measured accurately, with the distance to the far wire as a check of coordinate calculation. At small angles the distance to the far wire $\approx$ sum 2 measured distances (a and w_{S2}).

If the coordinates of the far wire are used then the distance between the wires need to be accurate as well; this is needed to project the close wire's coordinates to the far wire by the azimuth of the wire direction.

7.6.2.1.1 Worked example 7.1

Figure 7-20 is an observed Weisbach triangle. The main task is to transfer the azimuth PA to the underground line, UX.

By the Sine rule: $\frac{\sin a}{a} = \frac{\sin b}{b}$ and for small angles, $\sin a = \text{radians} = \text{arc seconds}(')/206265$.

Since 206265 is a constant, $\sin a$ and $\sin b$ can be expressed in seconds (''):

Calculate angle β $\frac{\alpha''}{a} = \frac{\beta''}{w_{S2}}$, thus $\beta'' = \frac{\alpha'' w_{S2}}{a}$. w_{S2} = distance A- W_{S1}	Eqn 7-4

1. Surface observations

(Figure 7-21):

Az AP = $66^{\circ}15'45''$ by traverse

$\angle PAW_1 = 210^{\circ}25'30''$ from FB

dist. a ($W_{S1} - W_{S2}$) = 5.00 measured

$\angle \alpha = 0^{\circ}10'27'' = 627''$ from FB

dist. w_{S2} (A- W_{S1}) = 3.500 measured

By Eqn 7-4: $\angle \beta = 627 \times 3.5/5 = 439 = 0^{\circ}07'19''$

Taking Az AW_2 from FB from Figure 7-21:

Az ($W_2 - W_1$) = Az ($AW_2 - 180 - \beta$) = ($276^{\circ}41'15'' - 180$) - $0^{\circ}07'19'' = 96^{\circ}33'56''$

Az ($W_1 - W_2$) = $276^{\circ}33'56''$.

$\angle W_{S1} = 180 - \alpha - \beta = 180 - 0^{\circ}10'27'' - 0^{\circ}07'19'' = 179^{\circ}42'14''$

Distance w_1 (A - W_{S2}) = $a \times \sin(W_{S1}) / \sin \alpha = 5.0 \times \sin(179^{\circ}42'14'') / (627/206265) = 8.501\text{m}$.

Check by sin rule: $a/\sin A = b/\sin B = c/\sin C$.

2. Underground obs. (Figure 7-22):

$\angle XUW_1 = 157^{\circ}20'45''$ from FB

Dist. $u = (W_1 - W_2) = 5.000$ measured

$\angle \alpha = 0^{\circ}12'36'' = 756''$ from FB

Dist. w_{U2} (A- W_{U1}) = 4.200 measured

By Eqn 7-4: $\angle \beta = 756 \times 4.2/5 = 635 = 0^{\circ}10'35''$

Taking Az AW_2 from FB from surface calculations:

Az ($W_{U1} - U$) = Az ($W_2W_1 + 180 - \beta$) = ($96^{\circ}33'56'' + 180$) - $0^{\circ}10'35'' = 276^{\circ}23'21''$

Az ($U - X$) = Az ($(W_{U1} - U) - 180 - \angle U$) = ($276^{\circ}23'21'' - 180$) - $157^{\circ}20'45'' = 299^{\circ}02'36''$

$\angle W_{U2} = 180 - \alpha - \beta = 180 - 0^{\circ}12'36'' - 0^{\circ}10'35'' = 179^{\circ}36'49''$

Distance w_{U2} (U - W_{U1}) = $u \times \sin(W_{U2}) / \sin \alpha = 5.0 \times \sin(179^{\circ}36'49'') / (756/206265) = 9.200\text{m}$.

Check by sin rule: $a/\sin A = b/\sin B = c/\sin C$.

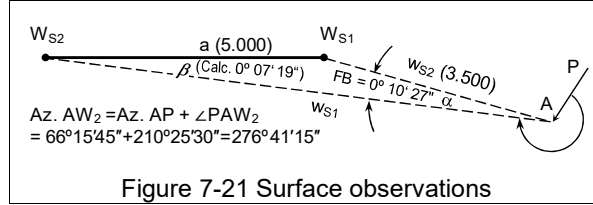


Figure 7-21 Surface observations

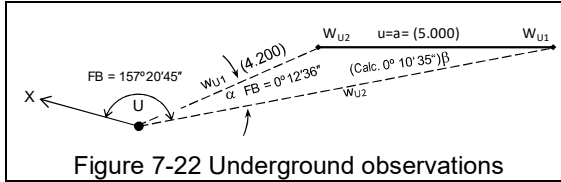


Figure 7-22 Underground observations

7.6.2.2 Error analysis of the calculated angle angle β

Error analysis can be performed on the triangle, thus providing an estimate of the standard deviation of the calculated angle β . A derivation, developed by the author, is tedious and you are recommended to read Ogundare, 2016, p456. His simplified formula is recommended.

Taking α and β as the acute angles, knowing σ_α and with corresponding sides a and b .

Ogundare (12.10)	$\sigma_\beta^2 = \frac{b^2}{a^2} \sigma_\alpha^2$	Eqn 7-5

Using the surface observations of the Worked Example 7.1, Figure 7-21 and Eqn 7-5 :

$a = W_1 - W_2 = 5$, $b = A - W_2 = 8.5$, $\sigma_\alpha = \pm 5''$

$\sigma_\beta^2 = 72$, $\sigma_\beta = \pm 8.5''$

The analysis shows that better results for $\sigma\beta$ rely on a better value for $\sigma\alpha$. This implies multiple rounds for angle readings and for the observation station being close to the near wire.

7.6.3 The Weiss Quadrilateral

The Weiss quadrilateral is an extension of the Weisbach triangle at the surface which is transferred to a braced quadrilateral at the underground level. Its task is to transfer azimuth via an auxiliary base when, due to mine layout constraints, a suitable Weisbach triangle cannot be observed. It takes advantage of development in EDM, especially highly accurate total stations that now offer accuracy at reasonable prices.

The braced quadrilateral, W_{U1} , U_R , U_Q and W_{U2} in Figure 7-23 shows the figure being observed for angles and distances from stations Q and R. Station U connects the quadrilateral to the continuing underground traverse, with azimuth determined from U_Q and U_R .

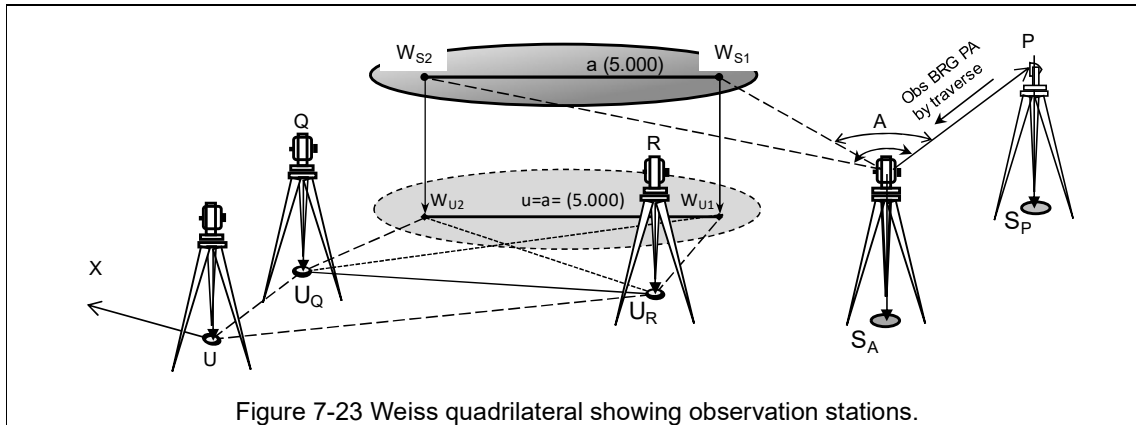


Figure 7-23 Weiss quadrilateral showing observation stations.

The underground quadrilateral should be relatively symmetrical, the baseline Q-R being fairly parallel to W1-W2. Figure 7-24 shows the baseline Q-R with an east/west orientation and the angles observed to wires W1 and W2 from underground stations Q and R.

Ogundare (2016) recommends multiple rounds of angle observations to W1 and W2 from Q and R. Forced centring between U, Q and R for distance recording. He also suggests the use of two total stations and autocollimation to each other to form a more accurate backsight over the baseline for the observation of the angles.

The distance W1-W2 is known from the surface measurements and will form a check for the quadrilateral calculations. The azimuth and the coordinates are also known.

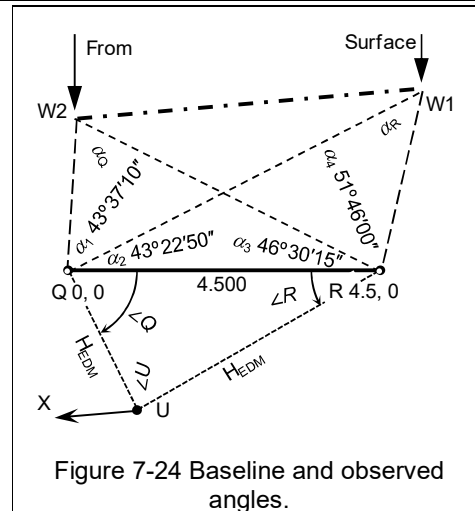


Figure 7-24 Baseline and observed angles.

7.6.3.1 Calculating the underground coordinates

The solution suggested is one whereby the local coordinates of W1 and W2 are calculated from the baseline coordinates and the observed angles. Stations W1 and W2, along with Q and R, can then be rotated and translated to the surface frame.

The “Intersection by angles” solution uses a 4.5m baseline oriented E/W. $QR = 90^\circ$, 4.5m.

The solution is clockwise circular from the intercept, 1, (W2) through 2 (R) to 3 (Q). Figure 7-27 illustrates the circular order of Eqn 7-6 and Eqn 7-7. Note the difference in coordinate order between the solutions of E_1 (N_3-N_2) and N_1 (E_2-E_3).⁸

The E/W orientation provides computationally simple results. N_2 and $N_3 = 0$. E_1 (Eqn 7-6) eliminates 3 elements (they are zero). Similarly N_1 (Eqn 7-7), because of zeros, eliminates many elements. Similarly, a N/S orientation with E_2 and $E_3 = 0$ is as computationally simple.

If coordinates are assigned to Q and R that result in a skewed alignment (nor E/W or N/S) then Eqn 7-6 and Eqn 7-7 still provide coordinates of W1 and W2 directly, without the need to calculate the bearing of Q-R.

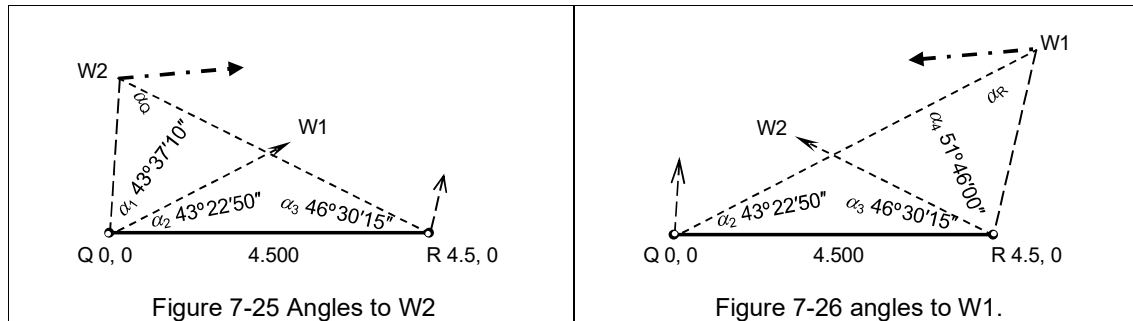
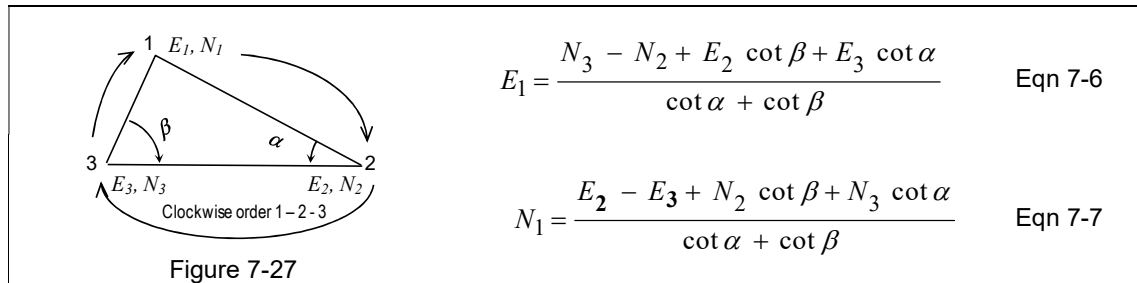


Figure 7-25 show baseline QR and observed angles $\beta = \alpha_1 + \alpha_2$ at Q and $\alpha = \alpha_3$ at R.
Figure 7-26 show baseline QR and observed angles $\beta = \alpha_2$ at Q and $\alpha = \alpha_3 + \alpha_4$ at R.



Using Eqn 7-6 and Eqn 7-7 in conjunction with Figure 7-27 the coordinates of W1 and W2 are as follows:

In Figure 7-25:

R (E_2, N_2) = 4.500, 0.000

Q (E_3, N_3) = 0.000, 0.000

$\alpha = \alpha_3 =$

$\beta = \alpha_1 + \alpha_2 = 87^\circ 00' 00''$

W2: $E_1 = 0.236$ $N_1 = 4.494$

Bearing W1 – W2 = $265^\circ 00' 20''$

Distance W1 – W2 = 5.000m

In Figure 7-26

R (E_2, N_2) = 4.500, 0.000

Q (E_3, N_3) = 0.000, 0.000

$46^\circ 30' 15''$ $\alpha = \alpha_3 + \alpha_4 = 98^\circ 16' 15''$

$\beta = \alpha_2 = 43^\circ 22' 50''$

W1: $E_1 = 5.217$ $N_1 = 4.930$

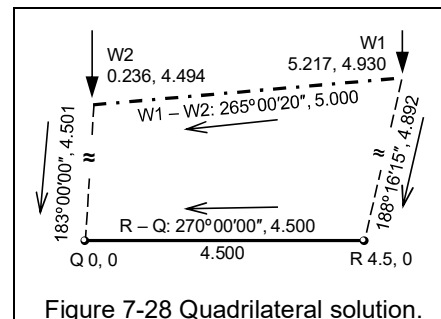
The sides of the quadrilateral, W2-Q and W1-R, are solved by the polar function from the above coordinates (Figure 7-28):

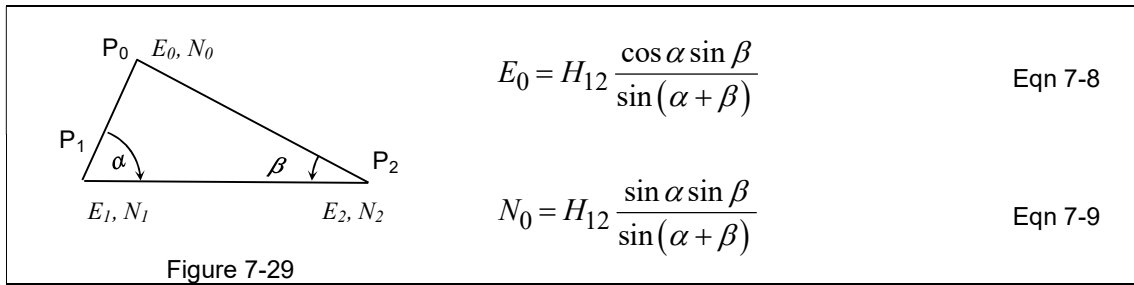
W2-Q Brg $183^\circ 00' 00''$, Dist 4.501m

W1-R Brg $188^\circ 16' 15''$, Dist 4.892m

Distances were confirmed by the application of the sin rule for sides.

Awange and Paláncz (2016 Sect. 17.2) present the conventional solution to the planar intersection in Figure 7-29 using Eqn 7-8 and Eqn 7-9 to calculate the intersection at P_0 .



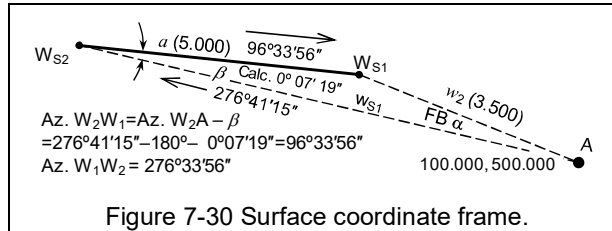


Allowing P_0 to represent W_1 (from Figure 7-26), with $\alpha = 43^\circ 22' 50''$, $\beta = 98^\circ 16' 15''$, $H_{12} = 4.5$, results in $P_0\{5.217, 4.930\}$. Similarly P_0 as W_2 (Figure 7-25) gives $P_0\{0.236, 4.494\}$.

7.6.3.2 Connecting to the surface

Worked example 7.1, § 7.6.2.1.1, page 144 provides a surface bearing of $276^\circ 33' 56''$ from W_1 to W_2 . Reiterating, in Figure 7-30: the underground bearing, W_1 - W_2 is $265^\circ 00' 20''$.

The quadrilateral must be rotated by $11^\circ 33' 36''$ to align it with the surface azimuth.



Furthermore, the underground coordinate system must be translated to the surface mine grid. Assuming the mine coordinates of to be: $A(100.000, 500.000)$.

7.6.3.2.1 Calculate coordinates of W_1 and W_2

Az. A - $W_1 = 276^\circ 41' 15'' + 0^\circ 10' 27'' = 276^\circ 51' 42''$ Distance 3.500m (Figure 7-21).

$$E_{W1} = E_A + dE = 100.000 + (-3.475) = 96.525$$

$$N_{W1} = N_A + dN = 500.000 + 0.418 = 500.418$$

Az. W_1 - $W_2 = 276^\circ 33' 56''$ Distance 5.000m

$$E_{W2} = E_{W1} + dE = 96.525 + (-4.968) = 91.557$$

$$N_{W2} = N_{W1} + dN = 500.418 + 0.572 = 500.990$$

7.6.3.2.2 Rotate underground figure around W_1

The origin of the rotation is W_1 . The traverse W_1 - R - Q - W_2 - W_1 is rotated by $11^\circ 33' 36''$, starting at W_1 . Table 7-2 is the closed traverse. It confirms the coordinates of W_2 and closes back to the start at W_1 .

Table 7-2 Quadrilateral traverse coordinates.								
At W_1	Rotation	$11^\circ 33' 36''$				E96.525	N500.418	W_1
Line	BRG	Az	Dist	dE	dN	E	N	Station
$W_1 - R$	$188^\circ 16' 15''$	$199^\circ 49' 51''$	4.982	-1.690	-4.686	94.835	495.732	R
$R - Q$	$270^\circ 00' 00''$	$281^\circ 33' 36''$	4.500	-4.409	0.902	90.426	496.634	Q
$Q - W_2$	$3^\circ 00' 00''$	$14^\circ 33' 36''$	4.501	1.131	4.356	91.557	500.990	W_2
W_2 - W_1	$85^\circ 00' 20''$	$96^\circ 33' 56''$	5.000	4.967	0.572	96.525	500.418	W_1

7.6.3.3 Continuing the underground traverse

Thus far, coordinates for Q and R have been transformed into the surface grid. The task now is to connect the quadrilateral to the rest of the underground network. Figure 7-31 shows the continuation from U to X (and onwards).

The Weiss quadrilateral was chosen because station U could not be configured for the Weisbach triangle alignment. The stations Q, R and U should be set up simultaneously so that forced centring observations may be made.

Apart from providing coordinates, the object of the exercise is to transfer azimuth from the surface to the underground workings.

The survey field book observations have been reduced as shown in Figure 7-31. Because all the distances are very short, special care needs to be taken in pointing the telescope for direction reading. While making readings to the wires and across the baseline, the angles to U should have been incorporated in the round of directions. Multiple distance observations to U would also allow an analysis of the distance standard deviation. Stations R, Q and U should use forced centring to eliminate the need to mark R and Q. Station U needs to be established as a permanent mark to construct the line U-X.

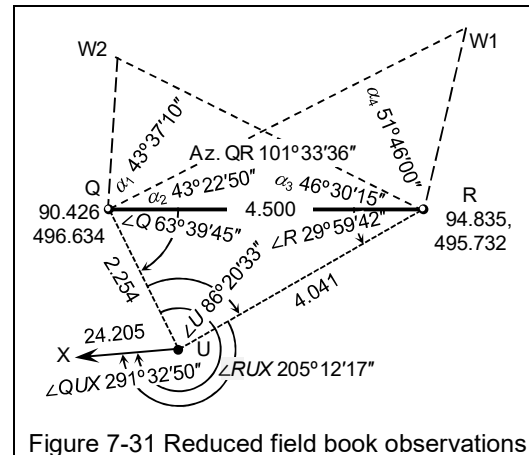


Figure 7-31 Reduced field book observations

7.6.3.3.1 Reducing the observations

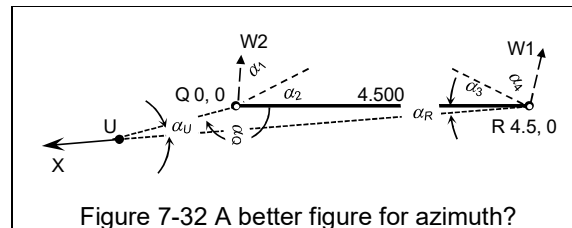
Find azimuths to U, the coordinates of U. Then calculate azimuth and coordinates of X. From Figure 7-31 and Table 7-2.

1. Az QR 101°33'36"
 + ∠Q +63°39'45"
 Az QU 165°13'21" Dist QR 2.254
 ∠QUX 291°32'50"
 Az UX 276°46'11"
 Coords Q 90.426 496.634
 dE, dN (QU) 0.572 -2.179
 Coords U **91.001 494.454**
2. Az UX 276°46'11" Dist 24.205
 Coords U 91.001 494.454
 dE, dN (UX) -24.036 2.853
 Coords X **66.965 497.307**

Az RQ	281°33'36"	
- ∠R	-29°59'42"	
Az RU	251°33'54"	Dist RU 4.041
∠RUX	205°12'17"	
Az UX	276°46'11"	Checks ✓
Coords R	94.835 495.732	
dE, dN (RU)	-3.834 -1.278	
Coords U	91.001 494.454	Checks ✓

7.6.3.4 Accuracy of azimuth transfer

Analysis of observations by Belton⁹ (personal communications, 2019) indicate that the situation of station U may provide better azimuth accuracy if modelled on Figure 7-32 than Figure 7-31.



7.6.3.4.1 Comparison of error ellipse between Figure 7-31 and Figure 7-32

Using the least squares program Move3¹⁰ to generate error ellipses for the two scenarios, the results showed a slight advantage for azimuth using the Weisbach triangle underground.

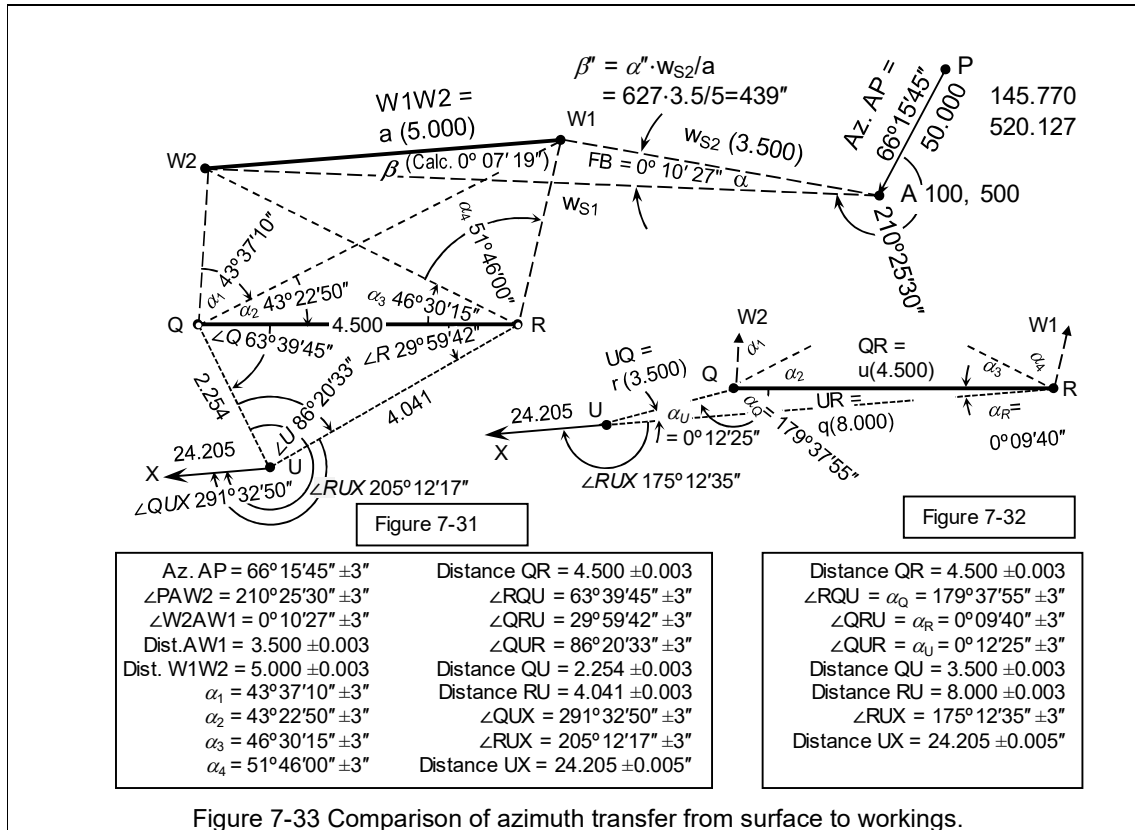


Figure 7-33 Comparison of azimuth transfer from surface to workings.

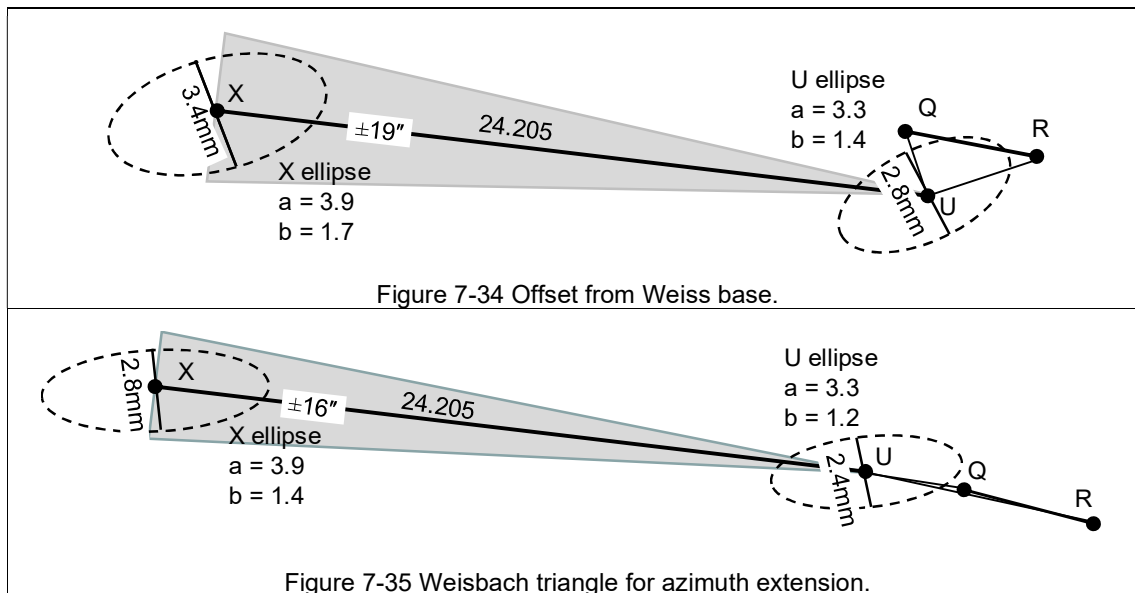


Figure 7-35 Weisbach triangle for azimuth extension.

The simplistic results show that, allowing for very small errors in the transfer wires from the surface, there is already some $\pm 20''$ uncertainty over the projected base UX.

7.6.3.5 Air currents, spiral set and scale reading errors

Practical uncertainty in the azimuth transfer, taking into account wire spacing, spiral radius of the wire reel, air currents and plumbbob swing readings, can produce quite large errors. Ogundare (2016, p489, Ex 12.7), using work by Chrzanowski and Robinson, 1981, estimates the standard deviation of an azimuth transfer 300m deep as being $\pm 28''$.

a. Air current influence:

- h air chamber height at observation station, m ,
- H length of plumb line, m ,
- d wire diameter (m),
- v air velocity, $m \cdot sec^{-1}$,
- p weight on plumbline, kg .

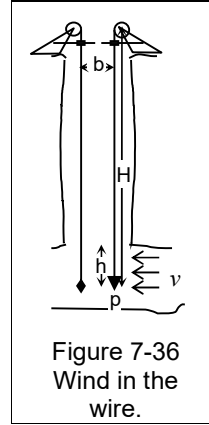


Figure 7-36
Wind in the
wire.

Air current influence, e_{air}	$e = \frac{30h(H)(d)v^2}{p} mm. \quad e_{air\ current} = \frac{1}{2}e$	Eqn 7-10

b. Spiral shape of plumbline due to storage reel:

- d wire diameter, mm ,
- E modulus of elasticity of the wire; Youngs modulus ($kg \cdot mm^{-2} = N^3 mm^2 \cdot g^{-1}$).
- R spiral radius, mm , (wire reel radius)

Spiral shape influence, e_{spiral}	$r_{spiral} = \frac{\pi d^4 E}{64 R p} mm. \quad e_{spiral} = 2r_{spiral}$	Eqn 7-11

c. Scale reading. With reasonable small amplitude parallel plumbbob swings (10cm, $\pm 10^\circ$) read 10 times estimated to 0.2mm then $e_{scale} = 0.2mm$. Distance between wires, b .

Standard deviation, σ_{Az}	$\sigma_{Az} = \pm \frac{206265}{b} \sqrt{2(e_{air}^2 + e_{spiral}^2 + e_{scale}^2)}$	Eqn 7-12

7.6.4 Azimuth by Gyroscope

Directional reliability of survey base lines in underground workings is a constant issue for the surveyor. The preceding sections have shown techniques traditionally used to bring surface coordinates to underground. The implication is that the coordinates will provide azimuth (direction) within the mine grid system. However, with short observation lines, the reliability of direction transfer is problematic.

The only instrument able to provide an independent direction underground is the gyroscopically aligned theodolite or total station. There are very few manufacturers of these instruments. The original gyro attachment, The Leica GAK-1 was fitted to modified Wild T2 and T16 theodolites. They have been discontinued and are rarely seen. A lack of spare suspension wires was one reason. The GAK-1 was introduced about 1963 as the first truly portable gyro attachment. The battery and inverter power supply were the BIG items.

The MOM (Magyar Optikai Művek (Hungarian Optical Works)) produced the G1-B1, Gi-B2 and Gi-B3 gyro attachment, located below the theodolite between the legs of the tripod.

Similar in construction was the British Aircraft Corporation's Precision Indicator of the Meridian (PIM).

One other gyro system currently available is the DMT GYROMAT 3000/5000. Their accuracy for azimuth is quoted at 3.5" to 2.5" arc. (www.gyromat.de)

Sokkia, with its series of GYRO X models, market a gyro station fitted to total station with modified software to assist in azimuth determination. (Figure 7-38).

The essential mechanical features of these gyroscopes is shown in Figure 7-39.

Basically, a gyroscope points to one point in space. Figure 7-37. Suspended vertically, as a pendulum, and constrained in the drift and topple axes, the spin axis will align itself with the meridian through the earth's spin axis. The true geographic north.

The gyroscope motor is suspended by a suspension tape housed in a cylindrical body. The pendulum precesses (oscillates) across the local meridian. By observing and timing the oscillations the alignment to true north can be found.

An optical tube, aligned with the spin axis, carries an index which is projected on to a scale. The scale is read multiple times during the observation session and uses the turning points of the oscillations to determine the direction of north.

The gyro rotor is electrically driven by a 115VAC, 3 phase, 400Hz motor at about 20,000 rpm. (The gyro motor has its origins in aircraft gyroscopic instruments.)

The suspension wire is made of a high modulus of elasticity Co, Cr, Ni steel alloy, designed to maintain spin axis alignment. The GAK-1 tape was 0.4mm thick; it is non-magnetic and corrosion resistant. The same wire is used in watch springs.

Both the GAK-1 and the GYRO X provide azimuth accuracy to about 15-20" arc. It takes about 15 minutes to determine one observation. It is a specialised task best left to the mine surveyor.

From the mine's engineers' point of view, the determination and maintenance of azimuth accuracy is an integral part of the mine plan. When gyro observations have to be taken there may be a need for some interruptions to nearby work. Careful attention to normal traversing, and an analysis of direction errors, will determine the frequency of gyro observations.

Schofield and Breach (Sixth edition, chapter 13) provides a full discussion of underground surveying methods, including a detailed section relating to gyro azimuth.

Ogundare (2016) devotes a section on setting up and using the Sokkia GYRO X system, probably the only suspension gyro attachment readily available in Australia.

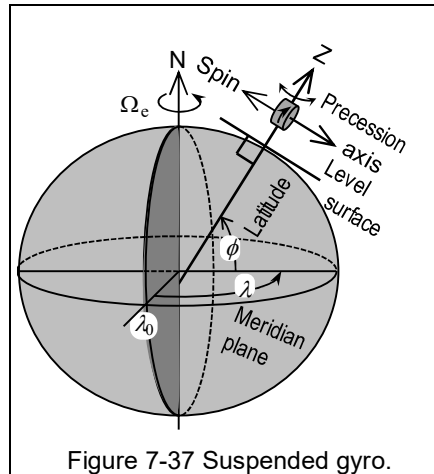


Figure 7-37 Suspended gyro.



Figure 7-38 Sokkia GYRO XII Automated Gyro Station.

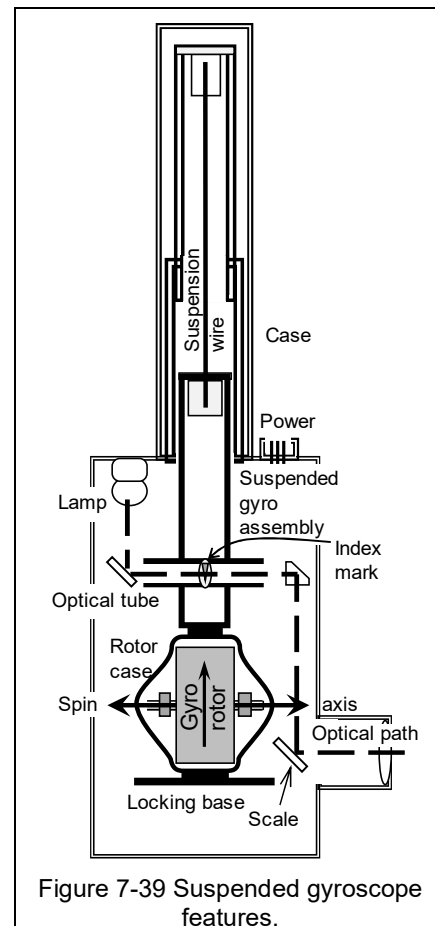


Figure 7-39 Suspended gyroscope features.

A number of “underslung” gyro units are being produced in China and India. As with the DMT GYROMATs, MOM and PIM units the main issue would seem to be with the centring accuracies over the survey mark. On short survey lines centring accuracy has a strong influence on azimuth standard deviations of direction.¹¹

Underslung units were available to military users, including a Kern DKM1 MIL theodolite mounted on a Lear Siegler azimuth gyro unit, manufactured in 1984.

The suspended gyroscope, mounted on top of the total station, is fixed over the survey point using the optical or laser plummet through the tribrach. Centring accuracies of about 1mm or better are achievable.

7.7 Vertical Control Transfer by Shaft Plumbing

A level control network is essential in mining operations. The common method is differential levelling through adits and declines. It is subject to the standards of levelling experienced in surface levelling.

However, level can be transferred by using tapes or wires suspended in vertical shafts. Transfer by vertical EDM is also possible. See §7.5.1 regarding piano (music) wire.

7.7.1 Taping

The use of steel tapes (to about 100m) and steel bands or wires provide direct reading of difference in height between levels using either a level or a total station (set at $ZA = 90^\circ$). The height of collimation of the surface BM and the tape reading are recorded. The underground readings are also recorded.

$$RL_{U/G} = RL_{BM} + FS - (T_S - T_U) - FS_{U/G}.$$

The method will be the same, no matter what depth is plumbed. The questions that arise depend on the magnitude of the systematic errors introduced by the vertical wire.

Normal supported horizontal measurements using a calibrated tape have corrections applied for temperature and tension deviations from calibrated values.

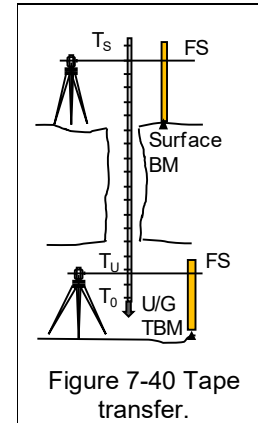


Figure 7-40 Tape transfer.

7.7.1.1 Temperature correction

Steel bands and tapes are generally calibrated or manufactured to a standard $T_s = 20^\circ\text{C}$ and a tension (pulling force) $P_s = 50\text{N}$ (5.1kg). Fibreglass tapes are standardised at $P_s = 20\text{N}$ (2.04kg).

There are various accuracy standards in play: EC Class 1 is printed to about 1:10,000, Class 2 is about 1:5,000.¹² Japanese Industry Standards (JIS) Class 1¹³ should be equivalent.

The coefficient of linear (thermal) expansion for steel tapes, CLE_{steel} is about $11.6 \times 10^{-6} \text{ m/m/}^\circ\text{C}$.

Temperature correction.	$C_{temp} = CLE_{Steel} \cdot L_{measured} (T_m - T_s)$	Eqn 7-13

If the temperature is above standard, then the tape will read short. The correction must be applied. $L_{true} = L_{measured} + C_{temp}$

Thus, for accurate work, the standard temperature of the tape should be known. The mean temperature between the bottom and top of the shaft is probably the best indicator of environmental temperature T_m .

e.g. A shaft is plumbed by tape with $L_{measured} = 205.520\text{m}$. With $T_s = 20^\circ\text{C}$ and $T_m = 40^\circ\text{C}$, what is the temperature correction to be applied to the tape, thus the true length of the tape?

From Eqn 7-13: $C_{temp} = 11.6 \times 10^{-6} \times 205.52 \times (40 - 20) = 0.048\text{m}.$

$$L_{true} = 205.520 + 0.048 = 205.568\text{m}.$$

How accurately do we need to know the temperature?

Eqn 7-13 can be rewritten: $L_t = L_m (1 + \alpha (T_m - T_s))$ where $\alpha = CLE_{steel}$.

Find the partial derivative of L with respect to Δt . $L_t = L_m (1 + \alpha (\Delta t)) \Rightarrow \delta L = L_m \cdot \alpha \cdot \delta (\Delta t)$ $\delta (\Delta t) = \frac{\delta L}{L_m \alpha}$	Eqn 7-14

To keep within Class 1 accuracy, $(\delta L/L)$ of 1:10,000, then, from Eqn 7-14:

$$\delta (\Delta t) = \frac{1}{10^4 \cdot 11.6^{-6}} = \pm 9^\circ\text{C}. \text{ Temperature read to } \pm 3^\circ\text{C} \text{ will provide 1:30,000 accuracy.}$$

7.7.1.2 Tension correction, horizontal

Tapes used for horizontal measurement are normally held under tension for making accurate measurements. The tension is normally supplied by using a spring balance to supply a force to the tape. New tapes should be tensioned to 50N (5kg). The standard correction for tension in horizontal tapes is shown in Eqn 7-15.

Tension correction.	$C_{tension} = \frac{(P_m - P_s)L}{AE}$	Eqn 7-15

Where:

$C_{tension}$ is the elongation in tape length due to pull; or the correction to be applied due to applying a tension which differs from standard tension; meters;

P_m tension applied to the tape during measurement; N, newtons;

P_s standard tension, when the tape is the correct length, often 50 N; newtons (5Kg);

L measured or erroneous length of the line; metres

A cross-sectional area of the tape; m^2 ;

E modulus of elasticity of the tape material; Youngs modulus (Nm^{-2}).

$E_{steel} = 193\text{GPa}$ derived from 45 samples to average an Imperial value of 28Mpsi. ¹⁴
 (1 GPa = 10^9 N m^{-2} = 10^6 N cm^{-2} = 10^3 N mm^{-2} .)

The value for A can given by Eqn 7-16: (helpful for determining A of nylon coated tapes where $SG_{nylon} = 1.12$ compared with $SG_{steel} = 7.8$).

Cross sectional area tape	$A_{mm^2} = \frac{W}{L \cdot U_w} \times 10^6$	Eqn 7-16

Where:

W total weight of the tape; kilograms;

U_w unit weight of the tape; kilograms per cubic metre, kg m^{-3} .

For steel tapes, the value for $U_w = 7866. \text{kg} \cdot \text{m}^{-3}$.

Steel tapes are about 9 – 13mm wide and 0.2mm thick. Many are coated with nylon to protect their markings, making them up to 0.5mm thick. For calculations, A is between 1.8mm^2 and 2.6mm^2 . A 13mm wide 100m tape has a weight of about 2kg.

Old survey bands, usually 100m long, are $\frac{1}{8}"$ or $\frac{1}{12}"$ (3.2mm or 2.1mm) wide and 0.5mm thick. The corresponding weights are 1.25kg and 0.85kg (mass 12.3N and 8.2N).

Piano (music) wire has slightly higher yield tensile strength and modulus of elasticity ($E = 210\text{GPa}$) and a unit weight of $7800 \text{ kg} \cdot \text{m}^{-3}$ (ASTM A228 Steel).

7.7.1.3 Tension correction in vertical shafts

Used in vertical shaft plumbing, a tape will hang vertical under its own mass. (Figure 7-41). (Shepherd, 1968).

Consider the tape AB, length l , fixed at A and freely suspended. It will lengthen by a value, say s , due to the tension, P , caused by the gravitational acceleration, g , pulling on the mass, ml , of the tape (Figure 7-42).

The tension at B is zero, and at A is the maximum value $mg l$. The stress varies along the length AB.

A mass, M , may be attached to the tape. The tension at any point, P,

$$P_p = M g + m g (l - x) \quad \text{Eqn 7-17}$$

The strain (elongation over length due to tension) can be characterised at any point P (length x):

$$\frac{\delta s}{\delta x} = \frac{P_p}{AE}, \quad \delta s = \frac{P_p \delta x}{AE}$$

$$P_p = \frac{AE \delta s}{\delta x}$$

Eqn 7-18

Equating Eqn 7-17 and Eqn 7-18 and integrating the result:

Equating:

$$\text{Integrating:} \quad \frac{AE \delta s}{\delta x} = Mg + mgl - mgx$$

$$AEs = Mg x + mglx - \frac{1}{2}(mgx^2) + d$$

$$\text{When } x=0 \text{ and } s=0 \text{ then } d=0 \quad s = \frac{gx}{AE} \left[M + \frac{1}{2}m(2l - x) \right]$$

$$\text{When } x = l: \quad s = \frac{gl}{AE} \left[M + \frac{1}{2}ml \right] \quad \text{Eqn 7-19}$$

$$s = \frac{mgl^2}{2AE} \quad \text{Eqn 7-20}$$

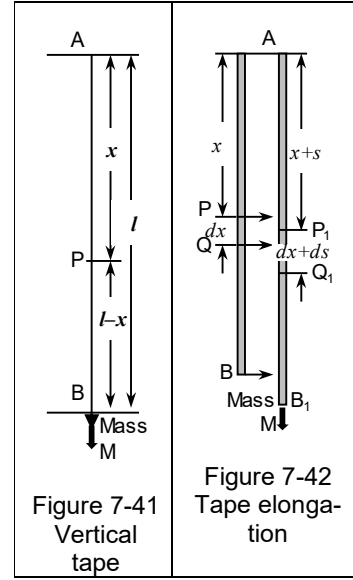
$$\text{When } M = 0 \text{ (no extra mass)} \quad \text{Eqn 7-21}$$

If a tape has been standardised at a certain tension, P_s then, compared with the unweighted vertical tape, a standardisation factor must be introduced:

Assuming $x = l$ then

$$s_l = \frac{-xP_s}{AE}$$

$$\text{The general equation, including tension correction, will be:} \quad s = \frac{gx}{AE} \left(M + \frac{1}{2}m(2l - x) - \frac{P_s}{g} \right) \quad \text{Eqn 7-22}$$



7.7.1.3.1 Worked example 7.2

Several surveyor's 100m steel bands are joined for a shaft plumbing exercise. The $\frac{1}{8}$ " (3.2mm×0.5mm) bands have been standardised at 68°F at a standard tension of 15lbs force. With no systematic errors joining the bands, the depth is read as 502.5m. The average temperature of the shaft is 30°C. What is the corrected length of the unweighted band?

1. $T_s = 68^\circ\text{F} = 20^\circ\text{C}$, $T_m = 30^\circ\text{C}$.
2. From Eqn 7-13: $C_{temp} = CLE_{steel} \times L_{measured} \times (T_m - T_s)$
 $= 11.6 \times 10^{-6} \times 502.5 \times (30 - 20)$
 $C_{temp} = 0.058\text{m}$.
3. $P_s = 15\text{ lbf} = 6.8\text{ kg f} = 67\text{ N}$. (convert pounds force to Newtons).
4. $g = 9.81$, $x = l$, $A = 3.2 \times 0.5 = 1.6\text{ mm}^2$, $E = 193 \times 10^3\text{ N mm}^{-2}$ (193GPa)
 $m = A \times U_w = 1.6 \times 10^{-6}\text{ m}^2 \times 7866\text{ kg m}^{-3} = 0.013\text{ kg m}^{-1}$
5. From Eqn 7-22:

$$s = \frac{9.81 \times 502.5}{1.6 \times 193 \times 10^3} \left(0 + \frac{0.013}{2} \times (1005 - 502.5) - \frac{67}{9.81} \right)$$

 $s = -0.057\text{m}$.
6. $L = l + s + C_{temp} = 502.25 + (-0.057) + 0.058$
 $L = 502.251\text{m}$.

7.7.1.3.2 Worked example 7.3

A 1000m deep shaft is to be plumbed for coordinate transfer and also to attempt to transfer height using piano wire on a windlass. Using §7.5.1 determine the wire to be used.

1. Plumb weight $= L/3 = 1000/3 = 330\text{ kg}$
2. Minimum wire area $= 2 \times 330/210 = 3.14\text{ mm}^2$.
3. Wire diameter $= 2\text{ mm}$ (14SWG or 80mil).
4. $g = 9.81$, $A = 3.14\text{ mm}^2$, $E = 210 \times 10^3\text{ N mm}^{-2}$ (210GPa)
 $m = A \times U_w = 3.14 \times 10^{-6}\text{ m}^2 \times 7800\text{ kg m}^{-3} = 0.0245\text{ kg m}^{-1}$
5. Weight of suspended wire $= 24.5\text{ kg}$ (tension 25kg or 240N).

Assuming the exercise is possible, how to measure and correct the wire for the 330kg tension mass? From a suitable surface bench mark a height of collimation mark is made at the top of the wire (a paint mark). Similarly the HC of the unknown lower BM is marked. The exercise task is to determine the stretch in the vertical wire due to the suspended weight.

The practical task is measuring the length of the wire while it is being extracted from the shaft. Perhaps a 100m horizontal surface to support the wire to allow it to be measured in bays. The tape is marked as it is measured over the bay. (EDM or calibrated tape.) Assume the measured length, $l = 1000\text{ m}$.

Piano wire is springy and holds a set from being coiled, so it will need some form of measured tensioning when layed horizontal for measurement. Perhaps 25kg (240N), the weight of the wire. Or 5kg (50N), the value used to calibrate steel tapes (T_m).

Having measured the length of the wire, what is the effect of temperature and of the plumbing weight on the vertical wire?

- a. Temperature variation with depth. Geothermal gradients over Australia vary from $10^\circ - 35^\circ\text{C/km}$ ¹⁵. Starting at a sub-collar temperature of, say $T_s = 20^\circ\text{C}$, the temperature gradient is relatively linear below about 100m. Assume the gradient is 20°C/km , thus $T_{1000} = 40^\circ\text{C}$.

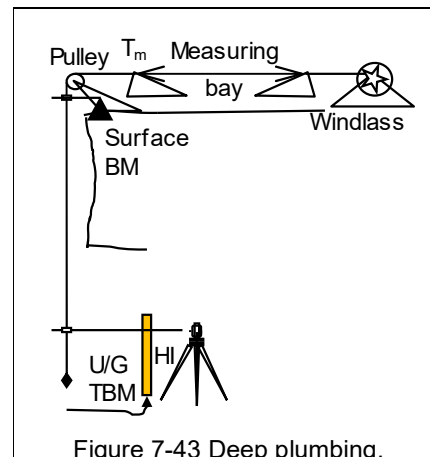


Figure 7-43 Deep plumbing.

From Eqn 7-13:

$$6. \quad C_{temp} = CLE_{steel} \cdot L_{measured}(T_m - T_s) = 11.6^{-6} \times 1000 \times (40 - 20) \\ C_{temp} = 0.23m.$$

- b. The previous Worked Example 7.2 considered a tape measured verically *in situ*. Here, in reality, how much did the wire stretch in the shaft, compared with its measured length during retrieval? Referring to §7.7.1.2 and Eqn 7-15:

$$7. \quad M_{plumb} = 330kg, m_{tape} = 24.5kg, P_s = 50N,$$

$$d_{wire} = 2mm, A = 3.14mm^2, E = 210 \times 10^3$$

$$8. \quad g = 9.81, x = l = 1000m, d = 2mm, A = 3.14mm^2, E = 210 \times 10^3 N \text{ mm}^{-2} (193GPa)$$

$$m = A \times U_w = 3.14E^{-6} m^2 \times 7800kg \text{ m}^{-3} = 0.0245kg \text{ m}^{-1}$$

9. From Eqn 7-22:

$$s = \frac{9.81 \times 1000}{3.14 \times 210 \times 10^3} \left(330 + \frac{0.0245}{2} \times (2000 - 1000) - \frac{50}{9.81} \right) = 3.576m$$

$$10. \quad L = l + s + C_{temp} = 1000 + 3.576 + 0.230$$

$$L = 1003.806m.$$

7.7.2 EDM in Vertical Shafts.

Height transfer by EDM is covered in Ogundare (2016). The EDM method is arranged using Talbot Hobson “spheres”. These are accurately made open spherical adapter rings that can hold mirrors or prisms in the centre of a mounting cage. The technique sets out to use a zenith plummet to ensure verticality between a surface plate target to the underground instrument. At a vertical accuracy of 1:100,000 the angular accuracy is about 2", but the technique is limited to shallow shafts due to optical limitations.

Is there another way to transfer levels using EDM?

Referring back to 7.4.1.2, page 134, the use of a precision prism may be considered.

Figure 7-44 proposes using a diagonal eyepiece (Leica GFZ3 etc.) on a total station to allow near zenith observations of slope distance to a surface prism positioned over the shaft.

Using automatic target recognition (ATR) to the prism would overcome the need for accurate visual sighting to the prism (no diagonal eyepiece). In fact, using the laser guide light to point to the top of the shaft the sight line would be sufficiently accurate for ATR target acquisition due to EDM beam divergence of 1°20' (1.5gon)¹⁶.

By way of examining some of the numbers (a thought game) suppose a 5m diameter shaft is 500m deep. To avoid grazing the wall suppose the prism, jutting over the edge, and the TS have a horizontal displacement of 4m.

The zenith angle is $\arctan(4/500) = 0^\circ 27' 30''$.

If the slope distance, $S1 = 500m$ then:

$V1 = 500 \times \cos(0^\circ 27' 30'') = 499.994$ to the trunnion axis of the TS. If the reading was in error by 30" then $V1$ is still $\pm 0.5mm$.

Expected EDM accuracy ($\pm(2mm + 2ppm)$) over 500m = $\pm 3mm$.

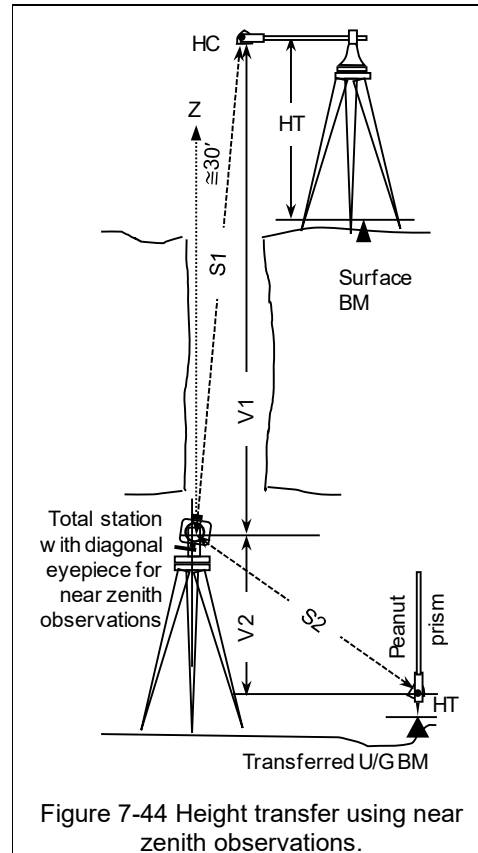


Figure 7-44 Height transfer using near zenith observations.

The transfer of the height of instrument (HI) from the trunnion axis to the underground BM is achieved by observing a peanut prism on the BM where $HI = V2 + HT$. Thus, the RL of the transferred underground benchmark:

$$RL_{U/G} = RL_{surface} + HT_{surface} - V1 - V2 - HT_{prism}.$$

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7.8.1 Essential references.

Essential references include Uren and Price (2010, Chapter 6), Schofield and Breach (2007, Chapter 6) and Irvine and Maclellan (2006, Chapter 8).

7.9 End notes

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8 Chapter 8 Strike And Dip to an Embedded Plane

8.1 Introductory Remarks

Mining engineers are involved in the extraction of minerals from ore bodies that often incorporate deposition seams.

Civil engineers may also need an understanding of slopes associated with earth-works on sloping terrain. Typical use could be on bulk earth structures such as water reservoir dam walls, tank bunds and the like.

The ability to calculate depths and directions of drill holes, the slopes of drives and the calculation of inclined angles is not well covered in most surveying texts. This chapter attempts to show some of the simple calculation techniques available for use in the field, or as an adjunct to normal mine planning software. Field measurement of strike and dip of an exposed seam can be made using Total Stations, automatic levels or geologists' compasses, e.g. the Brunton type. There are few references to strike and dip. Schofield and Breach (2007). Shepherd (1968, 1981) covers the subject in detail.

8.2 Strike and Dip

Geologists describe the orientation of a seam of mineralisation in terms of the direction of the horizontal plane intersecting the seam, the **strike**, and the maximum inclination of the plane, the **dip**. Exploratory drilling will intersect different geological structures, some of which may warrant development into a mine. Having located the seam, we now need to describe its location and orientation so that a mine may be planned to expose the seam.

8.3 Strike and Dip to a Plane

The relationship between the ground surface and the inferred seam, defined by three drill holes, DH0, DH1, and DH2, is illustrated in Figure 8-1.

Plane P_0, P_1, P_2, P_3 is the **horizontal** plane surface through S_0 , (below DH1) which is the “highest” intersect point.

The **dipping** plane, the intersected seam, is defined below the surface of points DH0, DH1 and DH2 at S_0, S_1 , and S_2 .

The line S_{strike} to S_3 defines the direction, as yet unknown, of the **strike line**.

The lines $S_0 - S_1$, and $S_0 - S_2$ are the **apparent** strike lines from S_0 . Apparent dip angle, α .

The **true, or maximum, dip**, θ , from S_0 (DIP_{true}) is perpendicular to the strike line. So, if you find the direction of maximum dip; then you have defined the strike direction.

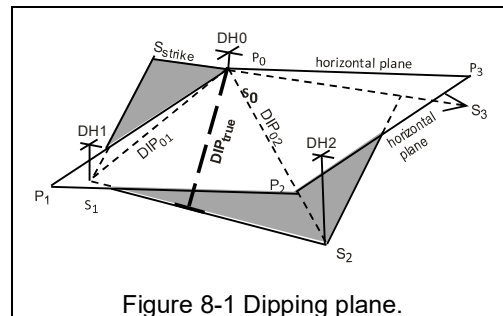


Figure 8-1 Dipping plane.

8.4 Deriving Strike and Dip of a Plane

The dipping plane is represented by the plane $ap-bdra$ containing S_0, S_1 , and S_2 in . $rcqbdr$ is the plane containing S_1 and S_2 at a depth ac below S_0 . Angles at c and q are right angles.

There are 4 angles to consider:

1. The **true dip**, θ_{max} (theta), is the slope ad where $ac = \text{rise}$ and $cd = \text{run}$. Slopes (dips) are expressed as a value of $1:k$, $k = \text{rise/run}$.
2. The **apparent dip** from a (S_0) to b at S_2 (α_2).

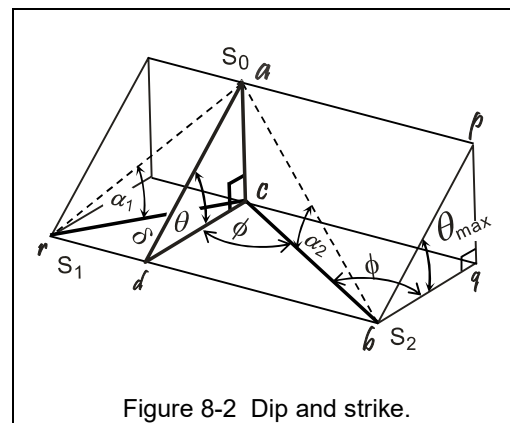


Figure 8-2 Dip and strike.

3. The apparent dip from a to r at S_1 (α_1) (“apparent” means as observed or measured).
4. The included angle, ϕ (phi), is between the apparent dip to b , S_2 , and the true dip to d .

ϕ is the angle dcb . The line $bp = ad \cos \phi = \frac{cd}{cb}$

From we will re-label $\theta_{\max}$ as θ for the development of the true dip formula.

1. $\tan \theta = \frac{ac}{cd} = \frac{pq}{qb}$, maximum dip, and, using α ($1/k$) to denote apparent dip (k = slope):

2. $\tan \alpha_2 = \frac{ac}{cb} = \frac{pq}{cb}$, because $ac = pq$. [$\tan \alpha_2 = 1/k_2$]

$\frac{pq}{cb}$ is the cross product of the apparent and true dips at b (S_2).

$$\frac{pq}{cb} = \frac{pq}{bq} \times \frac{bq}{cb} = \tan \theta \cos \phi$$

Eqn 8-1

3. $\tan \alpha_1 = \frac{ac}{cr} = \frac{pq}{cr}$, [$\tan \alpha_1 = 1/k_1$]

and again, is the cross product of the apparent and true dips at r (S_1).

$$\frac{pq}{cr} = \frac{pq}{bq} \times \frac{bq}{cr} = \tan \theta \cos \delta$$

Eqn 8-2

There are now three dips defined: true dip, $\tan \theta$; apparent dip, $\tan \alpha_1$; and apparent dip, $\tan \alpha_2$. They are related by the common side, ac .

The two apparent dips are defined by their respective **cross** products.

$\tan \alpha_2 = \frac{pq}{cb}$, $\tan \theta = \frac{pq}{qb}$, $\cos \phi = \frac{bq}{cb}$. Thus, by cross product:

$$\tan \alpha_2 = \tan \theta \cos \delta$$

Eqn 8-3

The included angle between the two apparent dips can be designated Δ (capital delta), the angle rcb . This is the measured angle at DH0 between DH1 and DH2.

ϕ is the included angle between the true dip and apparent dip to S_2 , (see Figure 8-3).

Label the angle between the true dip and the apparent dip to S_1 , as δ (delta).

So $\delta = \Delta - \phi$, thus $\phi = \Delta - \delta$.

$$\tan \alpha_2 = \tan \theta \cos \phi \Rightarrow \tan \theta \cos (\Delta - \delta)$$

$$\text{Expanding: } \cos (\Delta \pm \delta) \Leftrightarrow (\cos \Delta \cos \delta \mp \sin \Delta \sin \delta)$$

$$\tan \alpha_2 = \tan \theta (\cos \Delta \cos \delta + \sin \Delta \sin \delta)$$

Eqn 8-4

$$\tan \theta = \frac{\tan \alpha_2}{(\cos \Delta \cos \delta + \sin \Delta \sin \delta)} \text{ and, } \alpha_2 = \frac{1}{k_2}$$

$$\tan \theta = \frac{1}{k_2 (\cos \Delta \cos \delta + \sin \Delta \sin \delta)}$$

Eqn 8-5

Apparent dip $\tan \alpha_1$ is defined by its cross product such that:

$$\tan \theta_1 = \frac{pq}{qr}, \tan \theta = \frac{pq}{bq}, \cos \delta = \frac{bq}{cr}. \text{ So :}$$

$$\tan \theta_1 = \tan \theta \cos \delta$$

Eqn 8-6

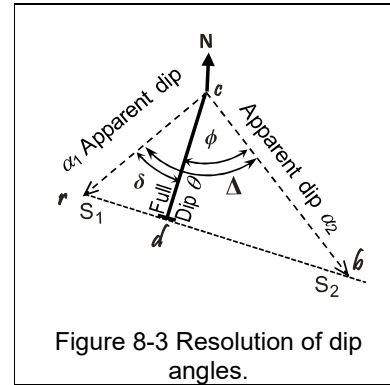


Figure 8-3 Resolution of dip angles.

$$\tan \theta = \frac{\tan \alpha_1}{\cos \delta} \text{ and similarly, } \tan \alpha_1 = \frac{1}{k_1}$$

$$\tan \theta = \frac{1}{k_1 \cos \delta}$$

Eqn 8-7

As Δ is known, there are two expressions with **tan** θ common, linked by **sin** δ and **cos** δ , which will allow the calculation of the angle δ .

Remember, the preceding equation to calculate the deflection angle, δ , which is measured from the true dip, $\tan \theta$, to the apparent dip at S_1 , $\tan \alpha_1$.

Thus, the true dip, (following Eqn 8-6), $\tan \theta = \frac{\tan \alpha_1}{\cos \delta}$.

Dip angle: $\theta = \arctan \left(\frac{\tan \alpha_1}{\cos \delta} \right)$

Eqn 8-8

Knowing the orientation (azimuth) of either of the apparent dips and the included angle between the two apparent dip lines, then the azimuth of the true dip can be calculated.

The **strike line** has **no** dip and is perpendicular to the true dip. If the plane is dipping down, then the strike azimuth is the dip azimuth $- 90^\circ$.

Solve for the angle δ since it is common in both Eqn 8-7 and Eqn 8-5.

$$\tan \theta = \frac{1}{k_1 \cos \delta} \text{ and } \tan \theta = \frac{1}{k_2 (\cos \Delta \cos \delta + \sin \Delta \sin \delta)}, \text{ inverting and equating:}$$

$$k_1 \cos \delta = k_2 (\cos \Delta \cos \delta + \sin \Delta \sin \delta). \quad k_1 \cos \delta = k_2 \cos \Delta \cos \delta + k_2 \sin \Delta \sin \delta$$

$$k_1 \cos \delta - k_2 \cos \Delta \cos \delta = k_2 \sin \Delta \sin \delta. \text{ And since values } \Delta, k_1 \text{ and } k_2 \text{ are known:}$$

$$\frac{k_2 \sin \Delta \sin \delta}{k_1 \cos \delta - k_2 \cos \Delta \cos \delta} \Rightarrow \frac{\sin \delta}{\cos \delta} = \tan \delta$$

Eqn 8-9

8.4.1 Strike and Dip of a Plane, Worked Example 8.1 by Calculation

A geologist returns from a field trip with the data on three drill holes: Figure 8-4.

DH0, RL100, intercepted a seam, S_0 , at a depth of 10m

DH1, RL95, 72m from DH0, azimuth 063° intercepted the seam, S_1 , at a depth of 22m

DH2, RL90, 111m from DH0, azimuth 130° intercepted the seam, S_2 , at a depth of 32m

8.4.1.1 Calculate the strike and dip of the seam

Figure 8-5 shows the two planes.

Invert level (IL) of seam intersects and depth (dIL) below S_0 datum.

IL $S_0 = 100 - 10 = 90$ (the highest)

IL $S_1 = 95 - 22 = 73 \quad \text{dIL}_{S_0 - S_1} = 90 - 73 = 17$

IL $S_2 = 90 - 32 = 58 \quad \text{dIL}_{S_0 - S_2} = 90 - 58 = 32$

Horizontal distances and apparent dips ($\tan \alpha = \text{rise/run}$). α is the apparent dip angle.

$\text{ds}_1 = 72\text{m}$, $\text{dip}_{S_1} (\tan \alpha_1) = 17/72 = 0.236 = 1:4.24 (k_1)$

$\text{ds}_2 = 111\text{m}$, $\text{dip}_{S_2} (\tan \alpha_2) = 32/111 = 0.288 = 1:3.47 (k_2)$

Included angle between apparent dip lines

$\Delta = \text{Az}_{S_2} - \text{Az}_{S_1} = 130 - 63 = 67^\circ$

See the dip plan in Figure 8-6.

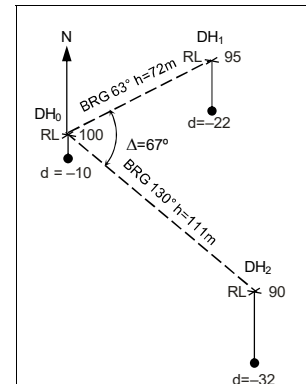


Figure 8-4 Field sketch.

8.4.1.2 Calculate the strike and dip directions

Evaluate Eqn 8-4: $\tan \alpha_1 = (\cos \Delta \cos \delta + \sin \Delta \sin \delta)$

$$\frac{1}{4.24} = \tan \theta (\cos 67 \cos \delta + \sin 67 \sin \delta)$$

$$\tan \theta = \frac{1}{4.24 (\cos 67 \cos \delta + \sin 67 \sin \delta)}$$

Evaluate Eqn 8-3: $\tan \alpha_2 = \tan \theta \cos \delta$

$$\frac{1}{3.47} = \tan \theta \cos \delta$$

$$\tan \theta = \frac{1}{3.47 \cos \delta}$$

We now have two equal expressions for $\tan \theta$. Invert each one and express the equality:

$$4.24 (\cos 67 \cos \delta + \sin 67 \sin \delta) = 3.47 \cos \delta$$

$$4.24 (0.39 \cos \delta + 0.92 \sin \delta) = 3.47 \cos \delta$$

$$\Rightarrow 1.65 \cos \delta + 3.90 \sin \delta = 3.47 \cos \delta$$

Gather terms :

$$3.90 \sin \delta = 3.47 \cos \delta - 1.65 \cos \delta$$

$$\Rightarrow 3.90 \sin \delta = 1.82 \cos \delta$$

Now, as $\tan \delta$ can be expressed in terms of $\sin \delta$ and $\cos \delta$, solve for δ ,

$$\tan \delta = \frac{\sin \delta}{\cos \delta} = \frac{1.82}{3.90} = 0.467. \text{ Thus } \delta = \arctan(0.467) = 25.03, \delta = 25^\circ 01'$$

Azimuth to full dip = $Az_{S2} - \delta = 130 - 25 = 105^\circ$

Strike azimuth = dip direction - $90 = 105 - 90 = 15^\circ$

8.4.1.3 Calculate the full dip angle

Now evaluate full dip $\tan \theta$ using Eqn 8-8, Dip S_2 , where $\tan \alpha_2 = 1:3.47$

From Eqn 8-3 :

$$\tan \alpha_2 = \tan \theta \cos \delta. \delta = 25^\circ. \text{ Full dip, } \tan \theta = 1 : x$$

$$\frac{1}{3.47} = \frac{1}{x} \cos 25 \Rightarrow x = 3.47 \times \cos 25 = 3.14$$

$$\text{Rate of full dip, } \theta = 1 \text{ in } 3.14 = \arctan\left(\frac{1}{3.14}\right) = 17.6^\circ.$$

Having solved for the orientation of the strike and true dip you are now able to calculate depths to seam from any point in the exploration area.

Volume of overburden and all the other tasks associated with mine development can now be calculated.

8.4.2 Strike and Dip of a Plane, the Graphical Solution

The previous complex calculations can be simplified by using a graphical solution derived from the observed data. It also allows visualisation of the observations and can be done in the field. Referring to the plan in Figure 8-6, it can be seen that the true dip, θ , is less than any apparent dip, α_1 . The slope is at its steepest point at $\theta_{\max}$.

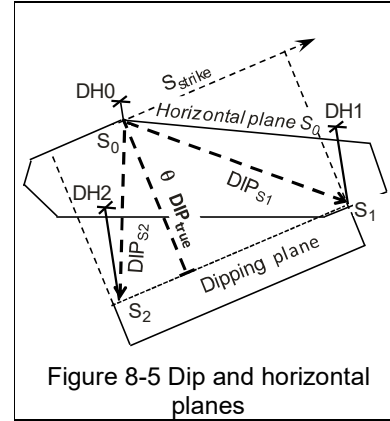


Figure 8-5 Dip and horizontal planes

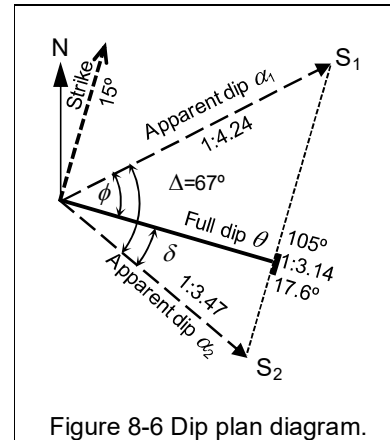


Figure 8-6 Dip plan diagram.

A geologist returns from a field trip with the following information on three different drill holes: Figure 8-7 is the field sketch.

DH0, RL100, intercepted a seam, S_0 at a depth of 7m.

$$ILS_0 = 100 - 7 = 93$$

DH1, RL95, 70m from DH0, azimuth 070° intercepted the seam, S_1 , at a depth of 25m.

$$ILS_1 = 95 - 25 = 70, \text{ rise to } RLS_0 = 93 - 70 = 23, \text{ run} = 70$$

DH2, RL90, 100m from DH0, azimuth 130° intercepted the seam, S_2 , at a depth of 38m.

$$ILS_2 = 90 - 38 = 52, \text{ rise to } RLS_0 = 93 - 52 = 41, \text{ run} = 100$$

From this information, where $\tan = \text{rise/run}$:

$$\text{dip}_1 (\tan a_1) = 23/70 = 0.33 = 1:3.03.$$

$$\text{Azimuth, } Az_1 = 070$$

$$\text{dip}_2 (\tan a_2) = 41/100 = 0.41 = 1:2.44.$$

$$\text{Azimuth, } Az_2 = 130.$$

Draw a plan closely to scale. You are drawing dip angles, not horizontal distances. Designed to fit in an A5 field book. See Figure 8-8. Using a scale of 20mm per dip unit.

- 1) From S_0 ,
 - a) draw a line on a bearing of 70° towards S_1 for a distance 3.03 units (say $3.03 \times 20 = 60.6\text{mm}$)
 - b) draw a line on a bearing of 130° towards S_2 for a distance of 2.44 units ($2.44 \times 20 = 48.8\text{mm}$).
- 2) Join S_1 and S_2 . From this base line, draw a perpendicular through S_0 . Measure the distance from the base to S_0 (46mm) and convert to dip units $46/20 = 2.30$.
 the **true** dip, θ , is $1:2.30 = \arctan(1/2.30) = 23.5^\circ$
 measure the bearing of the true dip from $S_0 = 111^\circ$
 calculate bearing of strike = $111 - 90 = 21^\circ$

With a carefully drawn diagram it is then possible to find the apparent dip in any direction by scaling from S_0 to the strike line between S_1 and S_2 . Figure 8-9 illustrates the next two problems.

1. What is the apparent dip on a bearing of 90° from S_0 ?
 Draw a line at a bearing of 90° to intercept the strike line between S_1 and S_2 .
 It measures 50mm and at a scale of 1:20.
 The dip = 1:2.5 (22°).
2. What is the azimuth of the 1:4 dip line?
 The scale factor requires an 80mm line be drawn to intercept the strike line (extended past S_1).
 Measure its azimuth as 57° .

The angle between apparent and full dip, recall Figure 8-9, $\delta = 111 - 57 = 54^\circ$. Because of symmetry to the dipping plane it means that there is another azimuth to the 1:4 apparent dip.

$$\text{It is true dip} + \delta, = 111 + 54 = 165^\circ$$

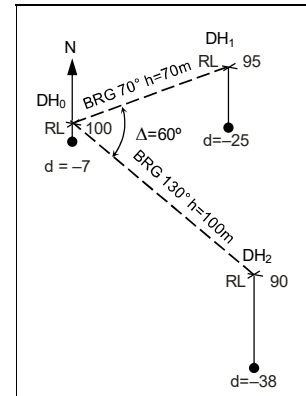


Figure 8-7 Field sketch.

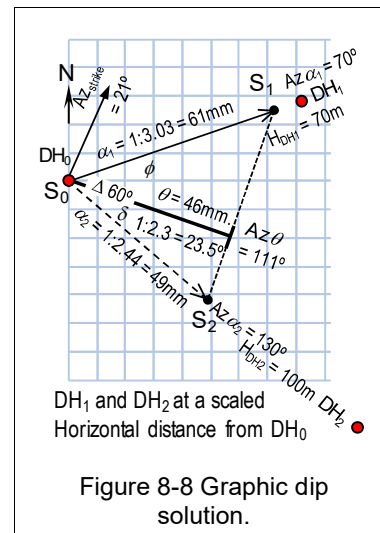


Figure 8-8 Graphic dip solution.

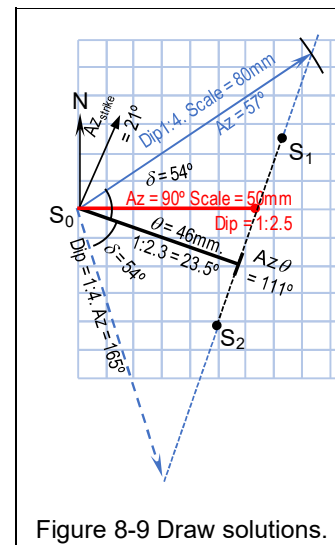


Figure 8-9 Draw solutions.

8.4.3 Outlier Problems

Blythe application of the above formula will sometimes result in “funny” answers. Two problems arise:

1. The angle Δ , the angle between the directions to the two drill holes, is greater than 90° . In this case $\cos\Delta$ in Eqn 8-4 will produce a negative value ($\cos>90$ is negative). The resulting value of δ will be wrong because, in this case, $\Delta = 90 - \delta$.
2. $\tan\delta$ is negative, thus δ is negative. This means that the maximum dip, θ , direction is “behind” the dip to the second drill hole. What we must do is accept the negative answer and move forward with the calculation of the direction of the true dip. From Eqn 8-4 dip direction, cd , = (direction cr (to S_1) $-\delta$). This hold true in the graphical solutions illustrated in Section 0 (Figure 8-11).

Az to $S_2 = 80$, $\delta = -25$. Az dip = $80 - (-25) = 105^\circ$

8.4.3.1 The case of included angle $\Delta > 90^\circ$

By using a graphic solution check, the mathematical solution to a calculation arising from Field Practical 4 can be shown to be incorrect as illustrated in Figure 8-10.

From DH3, the two directions to DH2 and DH1 are 78.6° and 178.2° . Thus $\Delta = 99.6^\circ$.

Referring to §8.4, and re-assigning the k subscripts:
 $k_1 = \alpha_1 = 5.1$, $k_2 = \alpha_2 = 9.2$. (Eqn 8-9)

$$\begin{aligned}\tan \delta &= \frac{k_1 \sin \Delta \sin \delta}{k_2 \cos \delta - k_1 \cos \Delta \cos \delta} \\ &= \frac{5.1 \sin(99.6) \sin \delta}{9.2 \cos \delta - 5.1 \cos(99.6) \cos \delta} \\ &= \frac{5.03 \sin \delta}{10.05 \cos \delta} \\ \tan \delta &= 0.50. \delta = 26.58^\circ \\ \tan \theta &= 1 / (k_2 \cos \delta) = 1 / (9.2 \cos 26.58^\circ) \\ &= 1 / (9.2 \times 0.89) = 1 / 8.23 = 6.93^\circ\end{aligned}$$

Dip $\theta = 8.23$. (True dip)

Check by diagram:

By measurement we see that the angle $\Delta \approx 99^\circ$. OK.

We measure δ , between DH1 and the full dip, as 63° .

By scaling we measure dip, θ , as 4.1 .

Calculation by Eqn 8-9 gives results of $\delta = 26.6^\circ$ and $\theta = 8.2$.

What has gone wrong?

The problem is that, as Δ is greater than 90° , it is in quadrant 2 where $\cos\Delta$ is negative. So $\delta = 90 - \delta = 90 - 26.6 = 63.4^\circ$. (This agrees with our diagram.)

Recalculating with $\delta = 63.4^\circ$

True dip $\theta = 9.2 \times \cos(63.4) = 4.1 = 13.8^\circ$. (Agrees!)

Dip azimuth = $AZ_{DH1} - \delta = 178.2 - 63.4 = 114.8^\circ$.

Strike azimuth = $114.8 - 90 = 24.8^\circ = 25^\circ$.

Check: $\phi = \Delta - \phi$. We see that $\phi = 99.6 - 63.4 = 36.2^\circ$.

$\theta = k_1 \cos \phi = 5.1 \times \cos(36.2) = 4.1$. (Agrees!)

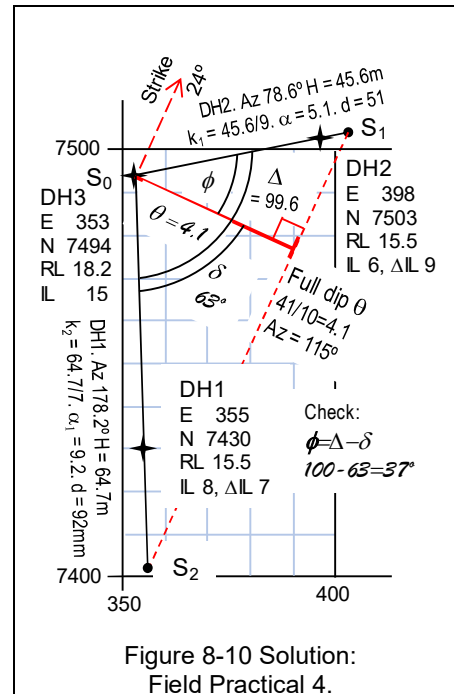


Figure 8-10 Solution:
Field Practical 4.

Grid Brg to $DH_1 = Az_{DH_1} - Az_{Strike} = 63 - 15 = 48^\circ$.

Sight to DH_1 , note the direction, subtract 48° , turn to new direction. Then Zero Set the TS [0 SET]. E.g., HAR to $DH_1 = 105^\circ$, turn to HAR $(105-48) = 57^\circ$. Zero Set to strike azimuth.

Thus, any bearing and horizontal distance (H) can be converted to local dE and local dN by the standard rectangular formulae, $dE = H \sin(\text{brg})$, $dN = H \cos(\text{brg})$.

The depth (h) to the seam at any point (reference RL S_0) can be calculated:

$$dH = dE/\theta$$

8.5.1 Worked Example 8.2

Regard point DH_P in Figure 8-12.

At DH_0 a sight was taken to P,
Az (HAR) = 73, ZA = $99^\circ 20'$,

slope distance = 74m.

Grid Brg from strike line = 73° ,

Dist 73m. dE = 70, dN = 21.

$$h_P = dE_{S_0}/\text{true dip}$$

$$h_P = 70/3.14 = 22.3\text{m below } S_0$$

$$IL\ S_P = RL\ S_0 - h_P = 90 - 22.3 = 67.7$$

The profile between DH_0 and DH_P is shown in Figure 8-13.

IL $S_P = 68\text{m}$, the RL of the seam at P.

From DH_1 , RL = 100, the difference in height to P, $dH = 74 \times \cos(99.33^\circ) = -12\text{m}$

$$RL\ DH_P = RL\ DH_0 - dH = 100 - 12 = 88\text{m}$$

$$\text{Depth to seam at } DH_P = 88 - 68 = 20\text{m}.$$

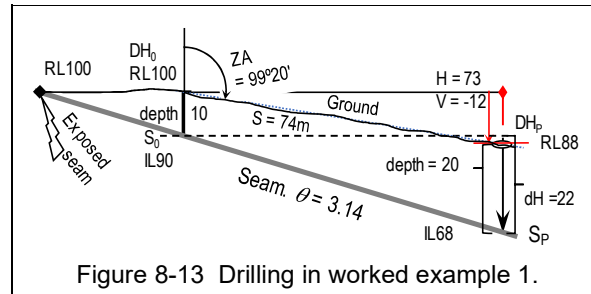


Figure 8-13 Drilling in worked example 1.

8.6 Seam Thickness

Having calculated the true dip, θ , the **thickness** of the seam can be calculated.

From the worked example in §8.4.1, the seam was intercepted 10m vertically below DH_0 . Continued vertical drilling resulted in an **apparent** seam thickness(d_a) of 16m. What is the **true** thickness (d_t) of the seam?

In Figure 8-14,

Depth to seam, $h = 10\text{m}$

True dip, $\theta = 1:3.16 = 17.6^\circ$

true thickness, $d_t = \text{apparent } (d_a)\cos\theta$

$$\text{seam } (d_t) = 16 \cdot \cos(17.6) = \mathbf{15.2m}.$$

8.6.1 Worked Example 8.3

Consider an inclined drill hole, DH_1 , which is inclined away from the strike line. Drilling starts at +7 from the strike line through DH_1 at an inclination of $+10^\circ$ from the vertical away “down slope” (Inclination 27.6° perpendicular to seam).

At DH_1 :

Inclination angle to seam normal, $I = 27.6^\circ$

Advance from strike, $s = 7$

$$\text{Vertical depth to seam} = h + s/\theta = 10 + (7/3.16) = 12.2$$

$$\text{Incline depth to seam from } DH_1: ab = 12.2 \cdot \cos(17.6) = 11.6$$

$$\text{Seam thickness, } bc = d_t = 15.2$$

$$\text{Distance to top seam } (S_{1T}) = ab/\cos I = 11.6/\cos(27.6) = 13.1$$

$$\text{Distance to base seam } (S_{1B}) = (ab+bc)/\cos I = (11.6 + 15.2)/\cos(27.6) = 30.2$$

$$\text{Apparent core interval } S_{1B} - S_{1T} = 30.2 - 13.1 = 17.1.$$

$$\text{Or, core length} = d_t/\cos I = 15.2/\cos(27.6) = 17.1\text{m}.$$

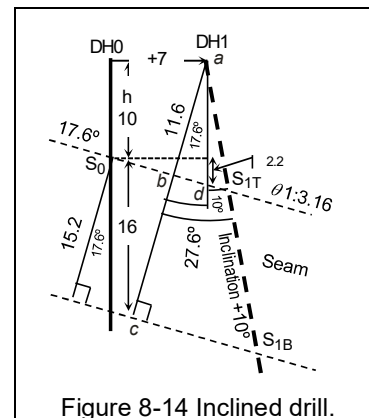


Figure 8-14 Inclined drill.

8.7 Direction of Any Slope Over the Dipping Surface

Investigate the **direction** of desired apparent dip. **Draw** a design apparent dip of 1:6.

The direction of **any** apparent dip can be realised graphically by scaling the dip until it meets the strike line perpendicular to the true dip. There will be two intercepts as illustrated.

Use a compass to draw the dip radius of 6 units, α_6 , to the same scale used for the true dip, θ , to intercept the extended strike line through S_1 , and S_2 . Using Worked Example 1 in §8.4.1

Figure 8-15 shows an apparent dip of 1:6 ($\alpha = 6, 9.5^\circ$). A protractor shows an azimuth of about 47° or 163° .

Calculate the dip deflection angle, δ , between true dip, θ , and apparent dip, α_6 , as a check. We have the expression (Eqn 8-3) from §8.4, page 160:

$$\tan \alpha_6 = \tan \theta \cos \delta \text{ so that } \cos \delta = \frac{\tan \alpha_6}{\tan \theta}$$

$\tan \alpha_6$ is the apparent dip, 1:6. $1/6 = 0.6667$

$$\tan \theta \text{ is the true dip, } 1:3.14. \quad 1/3.14 = 0.3185 \quad \cos \delta = \frac{0.6667}{0.3185} = 0.5233 \quad \delta = 58.4^\circ$$

Azimuth of apparent dip 1:6 = true dip $\pm \delta$

$$= 105 + 58 = 163^\circ \text{ or}$$

$$= 105 - 58 = 47^\circ.$$

This agrees with the graphical solution and it is up to the engineer to pick the required direction.

Any point on a line of bearing of 47° (or 163°) will have an apparent dip of 1:6 on this seam plane.

At a distance (H) of 100m, the depth (h) to the plane, below DH_0 at S_0 , will be:

$$h = H_{\text{dist.}} \tan \alpha_6.$$

$$H = 100, h = 100/6 = 16.7\text{m. And}$$

from the Figure 8-15, RL $S_0 = 90\text{m}$.

$$\text{IL } S_6 = 90 - 17 = 73\text{m.}$$

We still require the RL at the surface at the chosen point, DH_6 , to establish the drill depth to the seam. This would be achieved using the Zenith Angle (ZA) and slope distance when setting the position of DH_6 from DH_0 as shown in Worked Example 8.1. (§8.4.1, p161).

Note that we have used azimuth to define the direction to S_6 . The grid direction to S_6 is azimuth – strike = $47 - 15 = 32^\circ$ or $163 - 15 = 148^\circ$.

As a check of depth to S_6 : BRG 32° , $H = 100$. dE = 53, $h = 53/3.14 = 16.8$

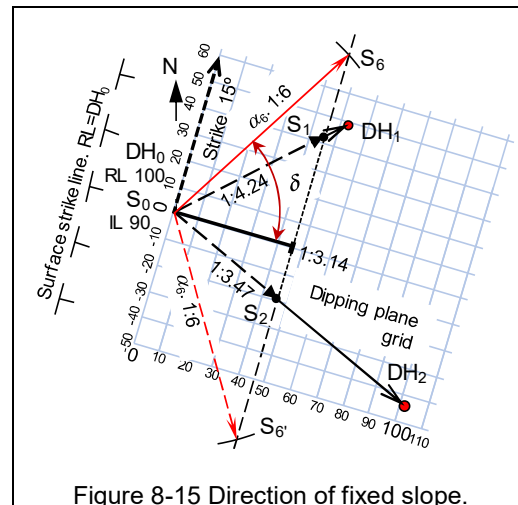


Figure 8-15 Direction of fixed slope.

8.8 Horizontal Angles Projected on to an Inclined Plane

The deflection angles that have been developed for dipping plane have been **horizontal** angles. The question arises as to the value of the included angle on the inclined plane. Theodolites measure horizontal angles and inclinations (vertical angles). Only a **sextant**, sighting to two targets on an inclined plane, can read the inclined angle. The astronomic “method of lunars” solved longitude (time) problems by using a sextant to measure the inclined angle between a bright star or planet and the moon.

Inclined angles are needed for the manufacture of pipe-work or ducting elbows along any inclined surface. The angular difference between horizontal and inclined may not be great, but it may be significant on steep inclinations.

In Figure 8-16:

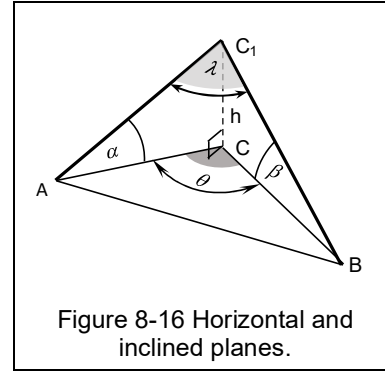
AC_1 and BC_1 are inclined to horizontal plane ACB .

inclinations at A and B are α and β respectively.

horizontal angle $ACB = \theta$

incline angle $AC_1B = \lambda$

height difference $CC_1 = h$



$$AC = h \cot \alpha, \quad AC_1 = h \csc \alpha. \quad \csc(\operatorname{cosec}) = \frac{1}{\sin}, \quad \cot = \frac{1}{\tan}$$

$$BC = h \cot \beta, \quad BC_1 = h \csc \beta$$

In triangle ACB , by cos rule for sides, in Figure 8-16,

$$AB^2 = AC^2 + BC^2 - 2AC \cdot BC \cos \theta$$

$$AB^2 = h^2 \cot^2 \alpha + h^2 \cot^2 \beta - 2h^2 \cot \alpha \cot \beta \cos \theta$$

Similarly, in triangle AC_1B , by cos rule for sides:

$$AB^2 = h^2 \csc^2 \alpha + h^2 \csc^2 \beta - 2h^2 \csc \alpha \csc \beta \cos \lambda$$

By similarity, gathering terms and cancelling h^2 :

$$h^2 \cot^2 \alpha + h^2 \cot^2 \beta - 2h^2 \cot \alpha \cot \beta \cos \theta = h^2 \csc^2 \alpha + h^2 \csc^2 \beta - 2h^2 \csc \alpha \csc \beta \cos \lambda$$

$$2h^2 \csc \alpha \csc \beta \cos \lambda = h^2 \csc^2 \alpha + h^2 \csc^2 \beta - h^2 \cot^2 \alpha + h^2 \cot^2 \beta - 2h^2 \cot \alpha \cot \beta \cos \theta$$

$$2 \csc \alpha \csc \beta \cos \lambda = \csc^2 \alpha + \csc^2 \beta - \cot^2 \alpha + \cot^2 \beta - 2 \cot \alpha \cot \beta \cos \theta$$

Thus, expressing the similarity in terms of the inclined angle λ :

$$\cos \lambda = \frac{(\csc 2\alpha - \cot 2\alpha) + (\csc 2\beta - \cot 2\beta) + 2 \cot \alpha \cot \beta \cos \theta}{2 \csc \alpha \csc \beta}$$

Because, from general trigonometric proof:

$$1 + \cot^2 A = \csc^2 A, \text{ then } \csc^2 A - \cot^2 A = 1, \text{ and:}$$

$$\csc^2 \alpha - \cot^2 \alpha = 1 = \csc^2 \beta - \cot^2 \beta$$

Thrashing valiantly through the enlightening jungle of trigonometric obfuscation to the sun-lit up-lands,

$$\cos \lambda = \frac{1 + 1 + 2 \cot \alpha \cot \beta \cos \theta}{2 \csc \alpha \csc \beta} = \frac{2(1 + \cot \alpha \cot \beta \cos \theta)}{2 \csc \alpha \csc \beta} = \frac{\frac{1}{1} + \frac{1}{\tan \alpha} \cdot \frac{1}{\tan \beta} \cdot \frac{\cos \theta}{1}}{\frac{1}{\sin \alpha} \cdot \frac{1}{\sin \beta}}$$

Extract the **inclined** angle, λ , in terms of inclination and **horizontal** angle, θ .

$$\text{Inclined angle } \lambda: \quad \cos \lambda = \sin \alpha \sin \beta + \cos \alpha \cos \beta \cos \theta$$

Eqn 8-10

Extract the **horizontal** angle, θ , in terms of inclination and **inclined** angle, λ .

$$\text{Horizontal angle } \theta: \quad \cos \theta = \frac{\cos \lambda - \sin \alpha \sin \beta}{\cos \alpha \cos \beta}$$

Eqn 8-11

The formula is sensitive to the sign of the inclination of the two lines at the intersection (the bend). A continuing falling decline will have one positive inclination to, and one negative inclination away from, the intersection.

8.8.1 Worked Example 8.4

A pipe is to be installed along two intersecting declines. The coordinates of the pipe supports provide the following information, see Figure 8-17:

The horizontal angle at the bend is 63°

one decline, A-C₁, rises at a gradient of 1:3.

the second decline, B-C₁, rises on a gradient of 1:4

Both declines are rising **to** the intersection, so each gradient is positive to the high point.

Determine the inclined angle at the intersection, C₁:

Data:

$$C = \theta = 63^\circ$$

$$A = \alpha = \arctan(1/3) = 18.43^\circ$$

$$B = \beta = \arctan(1/4) = 14.03^\circ$$

Using Figure 8-10:

$$\cos \lambda = \sin \alpha \sin \beta + \cos \alpha \cos \beta \cos \theta$$

Solution:

$$\begin{aligned} \cos \lambda &= \sin(18.43)\sin(14.03) \\ &\quad + \cos(18.43)\cos(14.03)\cos(63) \\ &= 0.316 \cdot 0.242 + 0.949 \cdot 0.97 \cdot 0.454 \\ &= 0.077 + 0.417 \end{aligned}$$

$$\cos \lambda = 0.494$$

$$\lambda = 60.36^\circ$$

Inclined angle, C₁ = $60^\circ 21'$

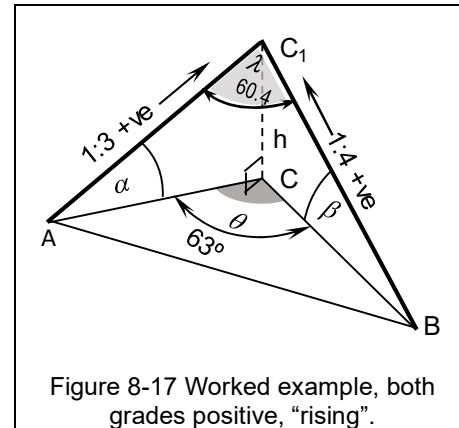


Figure 8-17 Worked example, both grades positive, "rising".

8.8.2 Worked Example 8.5

A pipe is to be installed along two intersecting declines. The coordinates of the pipe supports provides the following information:

The horizontal angle at the bend is 63°

one decline, A - C₁, rises at a gradient of 1:3.

the second decline, C₁-B, continues rising on a gradient of 1:4

One decline rises **to** the intersection, C₁,

the other continues rising **from** C₁

so one gradient is positive A-C₁

the other is negative B-C₁. It falls **to** the intersection)

Determine the inclined angle at intersection, C₁:

See Figure 8-18.

Data:

$$C = \theta = 63^\circ$$

$$A = \alpha = \arctan(1/3) = 18.43^\circ$$

$$B = \beta = \arctan(1/-4) = -14.03^\circ$$

Using Figure 8-10:

$$\cos \lambda = \sin \alpha \sin \beta + \cos \alpha \cos \beta \cos \theta$$

Solution:

$$\begin{aligned} \cos \lambda &= \sin(18.43)\sin(-14.03) \\ &\quad + \cos(18.43)\cos(-14.03)\cos(63) \\ &= 0.316 \cdot -0.242 + 0.949 \cdot 0.97 \cdot 0.454 \\ &= -0.077 + 0.417 \end{aligned}$$

$$\cos \lambda = 0.340$$

$$\lambda = 70.14^\circ$$

Inclined angle, C₁ = $70^\circ 08'$

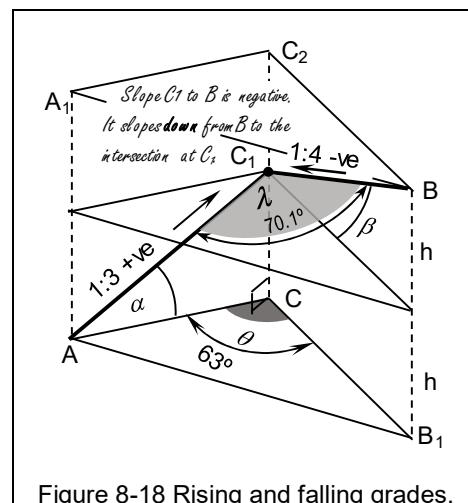


Figure 8-18 Rising and falling grades.

Note that only the **sine** changes sign as the **sign** of the inclination changes. If both inclinations were negative, it wouldn't matter as the product of the two negative sines is a positive, as in Worked example 1.

8.9 Concluding Remarks

This chapter is a demonstration of elements used to calculate on the dipping plane. We hope we have shown that the process can, in simple cases, be solved in the field, either by simple mathematics or by graphical means.

Courses in Structural Geology introduce the subject of a dipping plane. They have not been investigated because the references to this chapter cover the subject more fully from the surveying and computational aspect.

8.10 Reference for Chapter 7

- 1 Shepherd, F.A, Surveying Problems and Solutions, Edward Arnold, 1968
- 2 Shepherd, F.A, Engineering Surveying, Problems and Solutions, Edward Arnold, 2nd ed, 1983



9 Chapter 9 Earthworks on a sloping site

9.1 Revisit Earthworks

Chapter 3 Relief and Vertical Sections,
§3.4 Calculation of Road Cross Sections in Cut and Fill Operations.

Chapter 8 Strike and Dip to an Embedded Plane,
§8.3 Strike and Dip to a Plane.

These two sections explored the following:

- a. earthworks formation calculations on a sloping site and,
- b. strike and dip on a sloping site.

As far as embankments (roads) are concerned we may consider any formation alignment to be in the direction of a local strike line, which means that the batters of the formation follow the true dip of the site (90° to the strike), the ground level.

9.1.1 Introduction

Earthworks involve two components; the embankment itself (the formation), and the natural surface on which the embankment is formed. §3.4 considered embankments in either full cut (the survey exercises) or full fill. Obviously, the formation can be in cut and fill as the alignment sections proceed through the natural terrain. We will try to develop some form of grand unified theory of earthworks which will handle cut/fill embankments. Further we will calculate approximate intersection points between embankment and natural surface to enable flow of water during rainfall and avoid the roads flooding. From this information we will then develop the calculation of cut/fill areas at cross sections.

A word on the **symbols** of the four design components being studied.

1. The formation width, **b**, refers to the “flat” portion of the embankment design. On a road it may include the running pavement and associated kerbs, pathways and shoulders out to the start of the batter slopes that form the cutting or the embankment. Normal *cross fall* is about -2% from the centreline. The formation is assumed flat in this discussion, as the emphasis is on methodology, not final design.
2. A formation centreline may be in *cut* or *fill*. It is defined as the sign of the difference in height, **h**, between the surface level and the design level at the centreline of the formation. If *design level* – *surface level* is positive, **+h**, then it is a *fill* formation. Otherwise, if *design level* – *surface level* is negative, **–h**, then it is a *cut* formation.
3. For the formation batter slope, we will use the symbol **m**.
4. For the natural slope, previously identified as *cross slope* we have used the symbol **k**; it is the *true* (maximum) dip, θ , of the slope perpendicular to the cross-section alignment.

And the **sign** of the two slopes? How can we define the sign of the slope?

Response, “look at the slope AWAY from the centreline.”

This works in relation to the RL of the points, $RL_{to} - RL_{from}$ the traverse levels. As with levelling; a fall is negative, a rise is positive.

the definition, and thus the sign, of h . The revised formula changes the sign of the operator in the variable ($b/2 + mh$) to a negative to become:

$$W = \left(\frac{b}{2} - mh \right) \left(\frac{k}{k-m} \right) \quad \text{Eqn 9-2}$$

In essence, m and h will always be of an opposite sign. If you are *cutting* down to the formation level, then h will be negative, but m will be positive as you have to cut up to the surface from the formation. Contrarywise, if you are filling up to a formation level from the surface, h will be positive. But the slope of the batter, m , will be negative as you make the batter downwards to the surface.

The product, $-mh$ will always be positive. It extends formation width past its edge at $b/2$.

9.1.3.1 Formation width

Eqn 9-2 enables calculation of the formation width between the design pavement and the ground profile.

The various scenarios of formation and slope parameters will be investigated in subsequent sections.

9.1.3.2 Crest and toe heights

From Eqn 9-2 we can calculate the height of the crest or depth to the toe of the embankment. We will call this value fill, f , representing cut/fill height *from* the reference surface (the formation or the ground) *to* the crest or toe of the embankment.

W is a function of $b/2$, h , m and k .

f is a function of:

- W , $b/2$ and m , components of the design surface, or
- h and k , components of the natural surface.
- f relates to both the design and the natural surface.

- By ignoring ground slope;
calculate toe/crest height with respect to the formation level using W_L , W_R and m .

$$W = \left(\frac{b}{2} - mh \right) \Rightarrow h_W = - \left(\frac{W - \frac{b}{2}}{m} \right)$$

However, since we want to know the value of f from the formation to the toe or crest, the sign of h_W is changed to reflect the reference surface as the *from* surface. Maintain the sign of m .

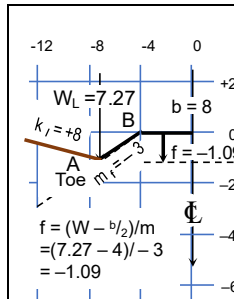


Figure 9-2 Fall to toe, W_L , from formation. (Eqn 9-3)

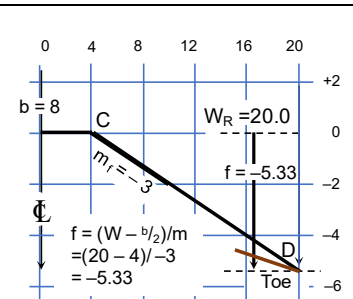


Figure 9-3 Fall to toe, W_R , from formation. (Eqn 9-3)

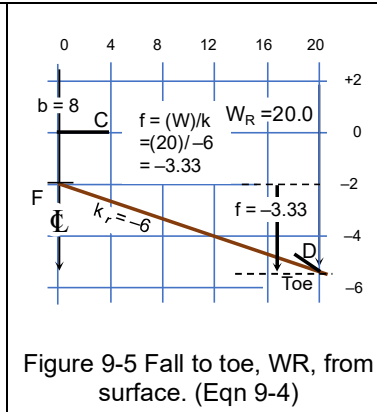
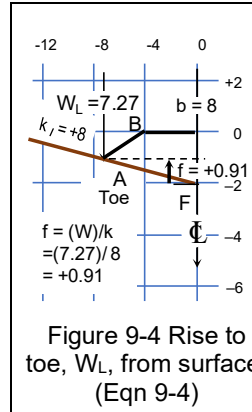
$$\text{From formation to toe/crest} \quad f_W = \left(\frac{W - \frac{b}{2}}{m} \right) \quad \text{Eqn 9-3}$$

2. By ignoring batter slope m ; calculate crest/toe height with respect to the ground centreline using k . Here $b/2=0$.

$$W = \left(\frac{b}{2} \frac{k}{(k-m)} - mh \frac{k}{(k-m)} \right)$$

$$W = \left(0 - \frac{mhk}{(k-m)} \right) \Rightarrow \left(0 - \frac{hk}{k} \right)$$

$$W = k(-h) \Rightarrow h_w = -\frac{W}{k}$$



From surface profile, to toe/crest $f_w = \left(\frac{W}{k} \right)$

Eqn 9-4

9.1.4 Cut and Fill on a Flat Surface

Examine earthworks calculations under *fill* or *cut* scenarios. Units are metres.

9.1.4.1 Fill on flat. See Figure 9-6

$W_{L/R}$ = half embankment width, left or right

$b = 8$ (formation width)

$h = +2$ (fill at centreline, (design – surface))

$m = -3$ formation batter slope

k = no slope, approaches ∞ (flat).

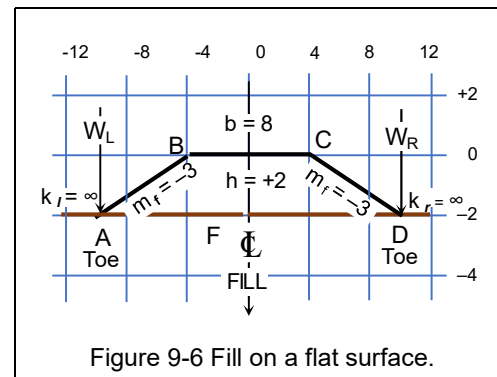
$$k/(k-m) = 1$$

$$W = ((8/2 - (-3 \times +2)) \times 1) = 10$$

Embankment width is thus 20m, and the area (trapezium) is 28m² of *fill* above the surface.

$$A = (((W_L + W_R) + b)/2) \times h.$$

Because h is positive (+2) we can say that the area is *fill*. (Design level – surface level)



9.1.4.2 Cut in flat. See Figure 9-7

W = half embankment width

$b = 8$ (formation width)

$h = -2$ (cut at centreline, (design – surface))

$m = +3$ formation batter slope

k = no slope, approaches ∞ (flat).

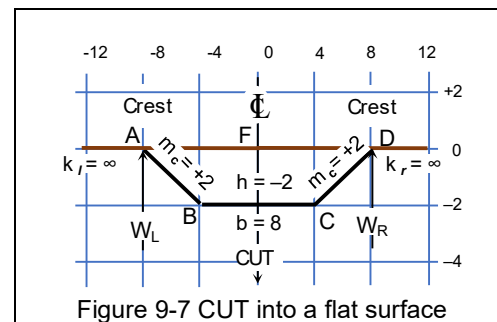
$$k/(k-m) = 1$$

$$W_L = ((8/2 - (+3 \times -2)) \times 1) = 10 = W_R.$$

Embankment width is thus 20m, and the area (trapezium) is 28m² of *cut* below the surface.

$$A = (((W_L + W_R) + b)/2) \times h.$$

Because h is negative (-2) we can say that the area is *cut*. (Design level – surface level).



9.1.5 The Effect of a Sloping Surface

What happens when a batter slope must intercept a surface slope? Two values of surface slope, k_l on the left and k_r on the right are defined, as in §9.1.1. As illustrated in Figure 9-8, the left and right embankment width now differ according to the surface slope and the depth of fill (or cut, as illustrated in Figure 9-9).

9.1.5.1 Full fill calculations. Figure 9-8

We will also calculate the depth from the surface centreline level to each embankment toe.

Using Eqn 9-2, with the signed value of mh . Values are confirmed by inspection.

W_L = half embankment width, left

$b = 8$ (formation width)

$h = +2$ (fill at centreline, (design – surface))

$m = -3$ formation batter slope

$k = +8$ (sloping up from the centreline)

$$k/(k-m) = 8/(8-(-3)) = 8/11 = 0.73$$

$$W_L = ((8/2 - (-3 \times +2)) \times (8/11)) = 7.27$$

Embankment width, W_E , is $7.27 + 20 = 27.27\text{m}$, but the area is not a simple trapezium.

Because h is positive (+2) we can say that the area is full fill. (Design level – surface level).

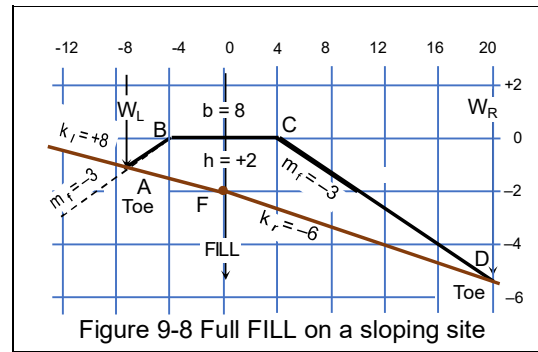


Figure 9-8 Full FILL on a sloping site

9.1.5.2 Full cut calculations

As illustrated in Figure 9-9, the left and right embankment width now differ according to the surface slope and the depth of cut, (or fill, as previously illustrated in Figure 9-8).

We will also calculate the depth from the surface centreline level to each embankment crest.

Using Eqn 9-2 with the signed value of mh , the following formation widths are calculated. They are slightly different because the two ground slopes now provide a cut surface, rather than the fill surface previously illustrated.

W_L = half embankment width, left

$b = 8$ (formation width)

$h = -2$ (cut at centreline, (design – surface))

$m_c = +3$ formation batter slope

$k = +8$ (sloping up from the centreline)

$$k/(k-m) = 8/(8-(+3)) = 8/5 = 1.6$$

$$W_L = ((8/2 - (+3 \times -2)) \times (8/5)) = 16.0$$

By inspection, this is correct.

Embankment width, W_E , is $16 + 6.7 = 22.7\text{m}$, but the area is not a simple trapezium.

Because h is negative (-2), from the diagram, the area is full cut. (i.e., Design level – surface level).

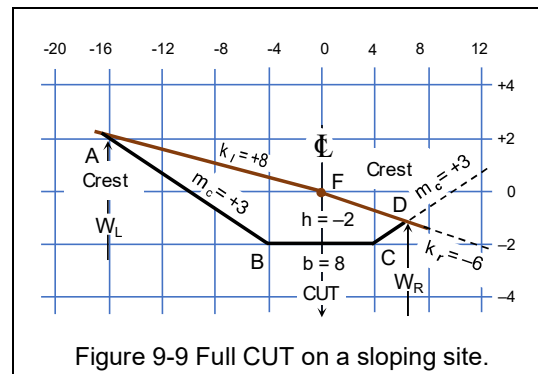


Figure 9-9 Full CUT on a sloping site.

W_R = half embankment width, right

$b = 8$ (formation width)

$h = -2$ (cut at centreline, (design – surface))

$m_c = +3$ formation batter slope

$k = -6$ (sloping down from the centreline)

$$k/(k-m) = -6/(-6-(+3)) = -6/-9 = 0.67$$

$$W_R = ((8/2 - (+3 \times -2)) \times (-6/-9)) = 6.67$$

By inspection, this is correct

9.1.6 Heights to Toes or Crests

The height of toes and crests of embankments with reference to either the ground level centreline, or the formation design level, are of necessity used in one of the methods of calculating area. Combined with the known levels of the ground survey or the design plan, one can then find the approximate RL of the points. This is of assistance in initial set-out for the earthworks.

9.1.6.1 Calculating using surveyed centreline ground level (GL) and ground slopes (k)

$$f_w = \frac{W}{k} \quad \text{Recalling Eqn 9-4.}$$

Calculating the height of the cut or fill to the toe of the embankment from the centreline surface level (GL), using the ground slope (k) values. This method refers itself to the established route survey of the centreline; the **height** in this case is a value above or below the centreline point itself at the calculated width from the centreline, W_L or W_R . The height can be referred to as f_L or f_R . RL of the batter intersection with the ground is the centreline RL + height. The height value may be either positive, above the GL, or negative, below the GL.

Referring to the Fill calculations in §9.1.5.1 and to Figure 9-4, Figure 9-5 and Figure 9-8.

Using Eqn 9-4:

$W_L = 7.27$, half embankment width, left $k = +8$ (sloping <i>up</i> from the centreline) $f_{7.3L} = W_L/k = 7.27/+8 = +0.91$ above GL $RL_{7.3L} = -2 + 0.91 = -1.09$	$W_R = 20$, half embankment width, right $k = -6$ (sloping <i>down</i> from the centreline) $f_{20R} = W_R/k = 20/-6 = -3.33$ below GL $RL_{7.3L} = -2 - 3.33 = -5.33$

9.1.6.2 Calculating using the formation design level (FL) and batter slope (m)

$$f_w = \frac{W - \frac{b}{2}}{m} \quad \text{Recalling Eqn 9-3.}$$

Alternately, the height of the cut or fill to the toe of the embankment can be calculated from the centreline formation level and the formation batter slope. Calculations start at the edge of the formation; at $b/2$ from the centreline. RL of the batter intersection with the ground is the formation level (FL) + height. The height value may be either positive, above the formation, or negative, below the formation.

Figure 9-3 and Figure 9-8.

Using Eqn 9-3:

$W_L = 7.27$, half embankment width, left $= 20$, half embankment width, right $b = 8$ (formation width) $m_f = -3$ formation batter slope $f_{7.3L} = (W_L - b/2)/m = (7.27 - 4)/-3 = 3.27/-3$ $= -1.09$ below formation level (FL) $RL_{7.3L} = -2 + 0.91 = -1.09$	W_R $b = 8$ (formation width) $m_f = -3$ formation batter slope $f_{20R} = (W_R - b/2)/m = (20 - 4)/-3 = 16/-3$ $= -5.33$ below formation level (FL) $RL_{20R} = -2 - 3.33 = -5.33$

Referring to §9.1.5.1 and Figure 9-2,

Figure 9-10 shows the results of calculations in §9.1.5.1.

The sum of the two values equals the centreline height difference between ground and design levels ($h = +2$), as:

$$h_{L(\text{ground} - \text{formation})} \text{ using } k = 0.91 - (-1.09) = +2 \text{ and}$$

$$h_{R(\text{ground} - \text{formation})} \text{ using } m = -3.3 - (-5.3) = +2$$

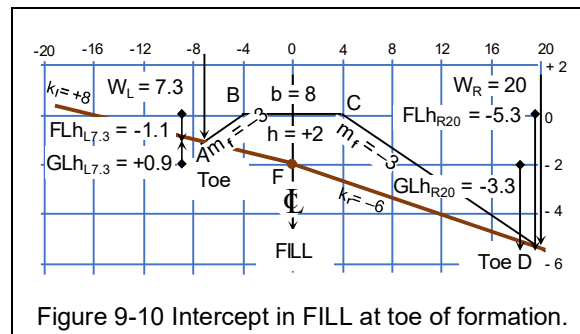


Figure 9-10 Intercept in FILL at toe of formation.

Using the same technique on the cut operation in Figure 9-11 and §9.1.5.2 results in the following, as shown:

Using Eqn 9-4:

Calculating using centreline design level:

$$h_{16L} = (16 - 4)/+3 \quad h_{6.7R} = (6.7 - 4)/+3$$

$$= 4 \quad = 0.9$$

Using Eqn 9-3:

Calculating using centreline formation level:

$$h_{16L} = (16)/+8 \quad h_{6.7R} = (6.7)/-6$$

$$= 2 \quad = -1.1$$

In both cases $h = -2$ as:

$h_L(\text{ground} - \text{formation})$ using $k = 2 - 4 = -2$ and

$h_R(\text{ground} - \text{formation})$ using $m = -1.1 - 0.9 = -2$

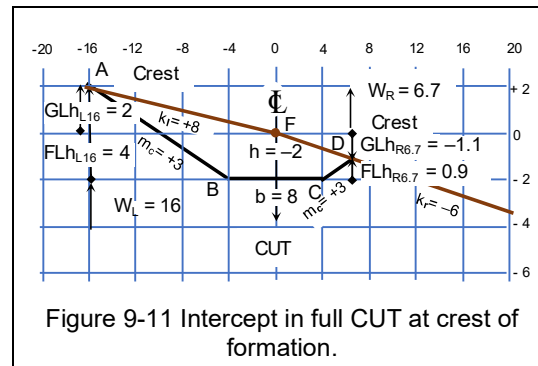


Figure 9-11 Intercept in full CUT at crest of formation.

9.1.7 Cross Section Area Calculations

§3.4.6 and §3.4.7 illustrated methods of finding the cross-section area using either the derived area formula (Eqn 3-8): or the matrix cross multiplication method (Eqn 3-9).

$$A = \frac{1}{2|m|} \left[\left(\frac{b}{2} + |mh| \right) (W_L + W_R) - \frac{b^2}{2} \right] \quad \text{Eqn 9-5}$$

$$A = \left| \frac{(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + \dots + (x_ny_{n+1} - x_{n+1}y_n)}{2} \right| \quad \text{Eqn 9-6}$$

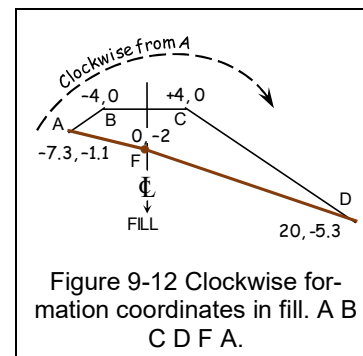


Figure 9-12 Clockwise formation coordinates in fill. A B C D F A.

Conforming to the revised Eqn 9-2 in §9.1.3 does not require a re-write of the area formula with the change of sign. The formula only works in full cut, or full fill, which means that, within our definitions, the product of mh is negative. However, since area is an absolute value, we can treat m and h as absolutes. Here both m and h are treated as absolutes.

9.1.7.1 Derived area from formula or from coordinates

Taking the W_L and W_R values from §9.1.4.1, as well as the design parameters, the area, by Eqn 9-4 is:

$$\text{Area} = \frac{1}{2|3|} \left[\left(\frac{8}{2} + |3 \times 2| \right) (7.3 + 20) - \frac{8^2}{2} \right] = \frac{1}{6} \left[(10)(27.3) - \frac{64}{2} \right] = \frac{241}{6}$$

$$= 40.2 \text{ m}^2$$

Figure 9-10 is interpreted, and Figure 9-12 provides a table of closed coordinates, following the embankment. Note that the use of height values, rather than RL, can simplify the matrix.

Further, W_L is negative and W_R is positive from the centreline, as illustrated in Figure 9-12.

Follow the formation in a clockwise direction: A B C D F A.

Calculation of area by matrix cross multiplication method: (revisit §3.4.7)

Point	Offset	height	+ve sum	-ve sum
A	-7.3	-1.1		4.4
B	-4	0	0	0
C	4	0	0	0
D	20	-5.3	-21.2	0
F	0	-2	-40	14.6
A	-7.3	-1.1	0	
			-61.2	-19.0
Sum = 2Area =			-80.2	Area = -40.1

Area = 40.1 (area is absolute.)

9.1.7.2 Notes on area calculations

Note that the two area values, by formula or matrix cross multiplication in § 9.1.7.1 are nearly identical, indicating either is acceptable. The difference is in the rounding used to one decimal place (1dp). Exact values give identical answers for each method.

Of interest is that the area of the triangle AFDA (Figure 9-11), the ground profile, is 3m^2 . This means that the centreline ground coordinate must be used in the coordinate area calculations to account for the two differing surface slopes. The derived area method, §9.1.7.1, using W_L/W_R , makes allowances for the ground slopes during the calculations.

Are we in cut or fill? Our definition of h , depth with respect to the height difference between design and surface, would suggest that if h is positive, then we have a fill area. If h is negative, then we have a cut area (as discussed in §9.1.1).

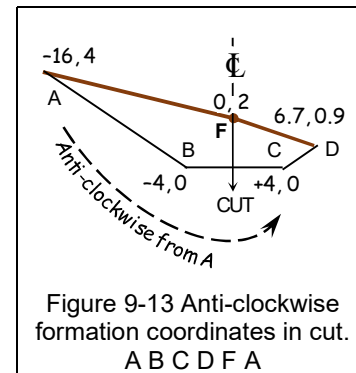
Run the coordinate calculation for the cut in Figure 9-9 and Figure 9-11 in *clockwise* sequence and examine Figure 9-13.

The area for A F D C B A is -32.4m^2 (negative).

However, if we follow the *formation* (ABCD) and then return via the *surface* (DFA), it is anti-clockwise and gives a positive result:

A B C D F A has an area 32.4m^2 . It is positive.

So, the coordinate order can matter here. What matters is still the question - are we in cut (h negative) or fill (h positive)? Of course, if we draw the profile, we then have a visual answer. And we may have to define a rule: *follow the formation* from the left-hand intercept. If the result is negative, we are in **fill**, if positive, then we are in **cut**.



9.1.7.3 Following the formation surface, what happens?

Previously we looked at the direction order of points, either clockwise or anti-clockwise. But is it better to follow the *formation* surface? In Figure 9-12 we see ABCD as the formation surface and it runs clockwise. But in Figure 9-13 we see that the formation surface ABCD is anti-clockwise. Thus, the two areas were of opposite sign.

9.1.8 Combined Cut and Fill in a Cross Section

§ 9.1.4 and § 9.1.5 were devoted to full cut or fill. By that, we mean that there was no intersection across the design formation level by the natural surface. Examining a more normal situation, where cut and fill occur in the cross section, we need to see how the size of each can be calculated.

The depth of cut or fill is defined by the symbol h . Remember that the sign of h represents the depth *from* the surface to the design. $h = +ve$ is fill to the design, $h = -ve$ is cut from the surface. Surface cross slope is k . And formation width is b . What are the values of h where the slope, k , causes the intercept between k and the design level to be less than $b/2$?

By introducing a new definition, say formation intercept, I , between the centreline and $b/2$, we can test and calculate for areas in combined cut and fill.

Additionally, how are we going to calculate each area? Can we define a value of the resultant area to use in *mass haul* diagrams?

Figure 9-14 shows the surface profile in two situations, where the minimum depth of cut or fill results in an intersection at the left or right formation width. Either $W_L = b/2$ or $W_R = b/2$.

9.1.8.1 Minimum h for full cut. Figure 9-14

For a full *cut* minimum depth, the value occurs at the edge of the design formation ($b/2 = 4$) where the surface cross section slope, k , is negative. This is the slope from the surface centreline ($F_{C(cut)}$) through the shoulder at C to C'. The result for h is negative because k is negative.

$$h_{min} = b/2/k = 4/-6 = -0.67. \text{ This is a cut.}$$

From here, using Eqn 9-2:

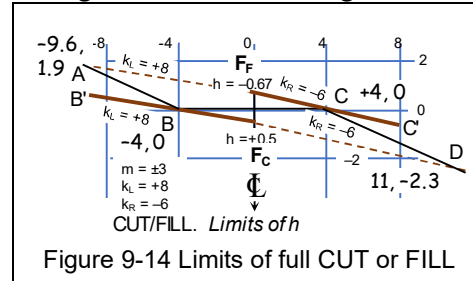
$$W_L = 9.6, \quad W_R = 4.0 \text{ (formation shoulder).}$$

The formula works.

From §9.1.6.2 the values of heights above design level are (Eqn 9-3) in Figure 9-15:

$$f_L = (W_L - b/2)/m = (9.6 - 4)/3 = 1.9,$$

$$f_R = (W_R - b/2)/m = (4 - 4)/-3 = 0.$$



9.1.8.2 Minimum h for full fill. Figure 9-14

For a full *fill* minimum height, h , the value occurs at the edge of the design formation ($b/2 = 4$) where the surface cross section slope, k , is positive. This is the slope from the surface centreline ($F_{F(fill)}$) through the shoulder at B to B'. Note that the result for h is positive because k is positive.

$$h_{min} = b/2/k = 4/8 = 0.5. \text{ This is fill. From here, using Eqn 9-2:}$$

$$W_L = 4.0 \text{ (the formation shoulder), } W_R = 11.0. \text{ The formula works.}$$

Again, from §9.1.6.2, batter heights by batter slope, the values of heights above design level are (Eqn 9-3) in Figure 9-16:

$$f_L = (W_L - b/2)/m = (4 - 4)/3 = 0,$$

$$f_R = (W_R - b/2)/m = (11 - 4)/-3 = -2.3$$

9.1.8.3 Cut/Fill area calculations

1) Area by Eqn 9-6.

The natural surface A F_F C will need all *cut* to realise the design formation C B A.

The area of cut is bounded by A F_F C B A = 8.3m².

The natural surface B F_C D will need all *fill* to reach the design formation B C D.

The area of fill is bounded by B C D F_C B = 8.4m².

2) Area by coordinates. Following the design formation:

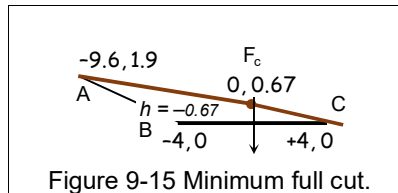


Figure 9-15 Minimum full cut.

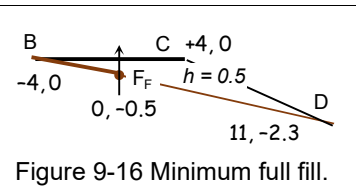


Figure 9-16 Minimum full fill.

CUT Figure 9-15.

Point	Offset	height
W _L A	-9.6	1.9
B	-4	0
W _R C	4	0
F _C	0	0.67
W _L A	-9.6	1.9

Area = 8.3

FILL Figure 9-16

Point	Offset	height
W _L B	-4	0
C	4	0
W _R D	11	-2.3
F _F	0	-0.5
W _L B	-4	0

Area = 8.4

Note that the areas are nearly the same because of the near symmetry of the two slopes, $k = 8$ and $k = -6$.

9.1.8.4 Cut and fill on grade, $h = 0$. Figure 9-17

Returning to our diagrams, what happens if the formation centreline is *on grade*? Figure 9-17 shows the situation where $h = 0$.

The cut and fill batters intercept the surface outside the formation width. Applying the standard formation width and height formulas (Eqn 9-2 results in the intercepts, as shown.)

Calculating the cut/fill area now becomes problematic.

By derived Eqn 9-4: $f_W = \frac{W}{k}$.

Recalling Eqn 9-5, where $W_L = 6.4$, $W_R = 8$ and $h = 0$, the area is 4.27m^2 . This is patently wrong.

Using coordinates method, following the formation, there are 3 answers:

$$\begin{array}{llll} \text{CUT:} & A B F A & = & 1.6 \\ \text{FILL:} & F C D F & = & -2.67 \\ \text{Area:} & A B C D F A & = & -1.07 \end{array}$$

The difference between the cut (1.6) and the fill (-2.67) is -1.07 . This agrees with the clockwise path around the formation, following the formation outline (ABCD) and the surface (DFA). Because of the *sign* of the result we can say that there is a deficiency of 1.07 cut in the cross-section area, comparing the amount of material from the cut and the required fill for the embankment.

9.1.8.5 A small cut, between $h = 0$ and $h = -0.67$

Now examine the situation where there is a small amount of *cut* as shown in Figure 9-18, where $h = -0.5$.

In this situation, using Eqn 9-3 to calculate W_L and W_R still works. F_L and f_R from :

$W_L = 8.8$, and $f_L = 1.6$. This is correct because:

$$(b/2 - mh) = 5.5, (k/(k-m)) = 8/(8-3) = 8/5 = 1.6.$$

And: $W_R = 5.0$, and $f_R = -0.3$. This is correct because:

$$(b/2 - mh) = 2.5, (k/(k-m)) = -6/(-6 - (-3)) = -6/-3 = 2.$$

But, in the diagram, the surface intercepts the formation at:

$$I_R = 3, 0 \text{ as } I_R = kh = -6 \times -0.5 = 3.$$

Using coordinates, following the formation, there are 3 area answers:

$$\begin{array}{llll} \text{CUT:} & A B I_R F A & = & 6.15 \\ \text{FILL:} & I_R C D I_R & = & -0.17 \\ \text{Area:} & A B C D F A & = & 5.98 \text{ Cut difference.} \end{array}$$

Point	Offset	height	Point	Offset	height
W_L A	-8.8	1.6	I_R	-3	0
B	-4	0	C	4	0
I_R	3	0	W_R D	5	-0.33
-h F_F	0	0.5	I_R	3	0
W_L A	-8.8	1.6			
Area = 6.15 cut			Area = -0.17 fill		

Note that the intersection point between cut and fill, I_R , is not included in the matrix Area figure, A B C D F_F A. That calculation produces only the difference between cut and fill.

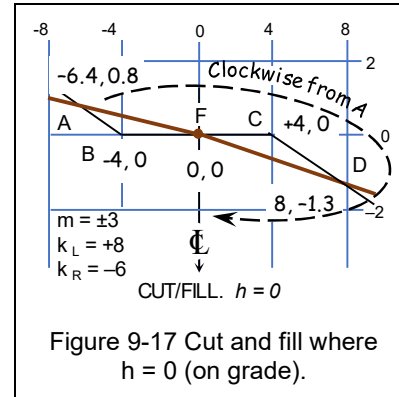


Figure 9-17 Cut and fill where $h = 0$ (on grade).

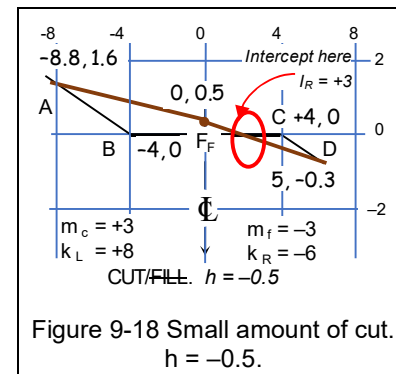


Figure 9-18 Small amount of cut. $h = -0.5$.

9.1.8.6 A small fill, between $h = 0$ and $h = +0.5$

Examine the situation where there is a small amount of *fill* as shown in Figure 9-19 where $h = +0.3$

In this situation, Eqn 9-2 for W_L and W_R still works:

$W_L = 5.0$, and $h_L = 0.3$ This *is* correct.

$(b/2 - mh) = 3.1$, $(k/(k-m)) = 8/(8-3) = 8/5$.

At $W_L = 5$, where does the $k = +8$ surface intercept the formation shoulder?

It does that at $h = 5/8 = 0.62$ above the centreline ground level. With a *fill* of $h = +0.3$ then the formation is *on grade* at

$h_L = 0.32$ above the left formation shoulder.

The formation intercepts the grade surface at

$I_L = +8 \times 0.3 = 2.4$, less than the required $W_L = 4$

And:

$W_R = 9.8$, and $h_R = -1.9$

$(b/2 - mh) = 4.9$, $(k/(k-m)) = -6/(-6 - (-3)) = -6/-3 = 2$.

Using coordinates, following the formation, there are 3 answers:

CUT: A B I_L A = 0.26

FILL: I_L C D F I_L = -5.70

Area: A B C D F A = -5.44. Fill difference.

Point	Offset	height	Point	Offset	height
W_L A	-5	0.32	I_L	-2.4	0
B	-4	0	C	4	0
I_L	-2.4	0	W_R D	9.8	-1.93
W_L A	-5	0.32	-h F _C	0	-0.3
			I_L	-2.4	0
Area = 0.26 cut			Area = -5.70 fill		

Again, note that the intersection point between cut and fill, I_L , is not included in the Area figure, A B C D F_C A. The calculation produces only the difference between cut and fill.

9.1.9 Summary of Cross Section Area Calculations on Simple Formation Design

Eqn 3-8, page 55 in §3.4.6 (see §9.1.7.1) can be safely applied to all area calculations involving either full cut or full fill, where the half formation widths, W_L and W_R are greater than the half formation width, $b/2$.

However, between minimum values of $h = b/2 \pm k$ Eqn 3-8 fails and we must resort to the matrix method of area determination.

By following the formation profile from left to right and returning to the start point via the surface profile, the result is the difference between the cut and any fill in the cross-section. We have seen how the *sign* of the result can determine cut or fill area.

To calculate cut **and** fill at a cross-section we must calculate the coordinate of the intersection of the natural surface with the formation level. Again, by following the formation and return paths through the intersection point, it means that a cut area and a fill area will be determined. The resultant sum, either positive or negative, will be the same as the area determined by the previously mentioned formation path.

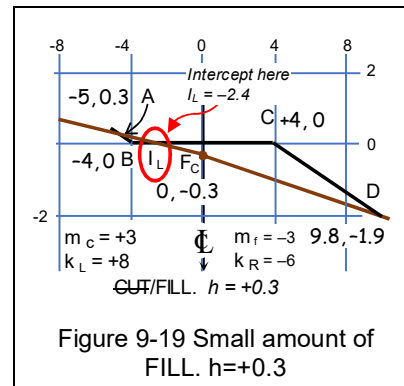


Figure 9-19 Small amount of FILL. $h = +0.3$

9.1.10 Intersection of Two Lines

The preceding sections have been predicated on the intersection of two straight lines originating from the centreline of the formation. For our purposes this is generally sufficient to calculate intercepts and section areas prior to volume calculations. §3.4.5 examined the case of a batter intercept occurring outside an initial offset of 5m. The batter then continued to meet a ground slope different from the initial ground slope. This realised a different batter intercept that could be argued is insignificant. However, it was part of the survey exercise, and an illustration of techniques to handle the problem.

9.1.10.1 Traditional line-line intersection method using points and slopes

Figure 9-20 Line-line intersection. shows the right-hand portion of a ground survey, G1, G2, G3, and the design embankment line, P1, P2. The example provides points common to two intersecting of lines. The surface on which an embankment will be formed is a surveyed ground surface slope (line 2). The designed formation surface comprises a formation pavement and embankment batters (line 1). We have successfully applied Eqn 9-2 to determine W_R where $h = +1$, $k = -6$. $W_R = 14$. The two methods to determine the two relative h coordinate (-2.33 from G1 and -3.33 from P1) at W_R . $h = 0.67$ ($2/3$) reference the x axis, level 0.

The formulas for calculating w and h are an extraction of the general solution of two intersecting straight lines, $y = ax + b$ and $y = cx + d$. But be very careful about applying straight linear solutions to our problems. In these above linear equations, the slopes, a and c , are defined as rise/run.

We have effectively turned the equation axes by 90° , so that batter slope, m , and ground slope, k , are defined as offset(run)/rise (x/y or w/h). This means that $m = 1/a$ and $k = 1/c$ in the linear equations.

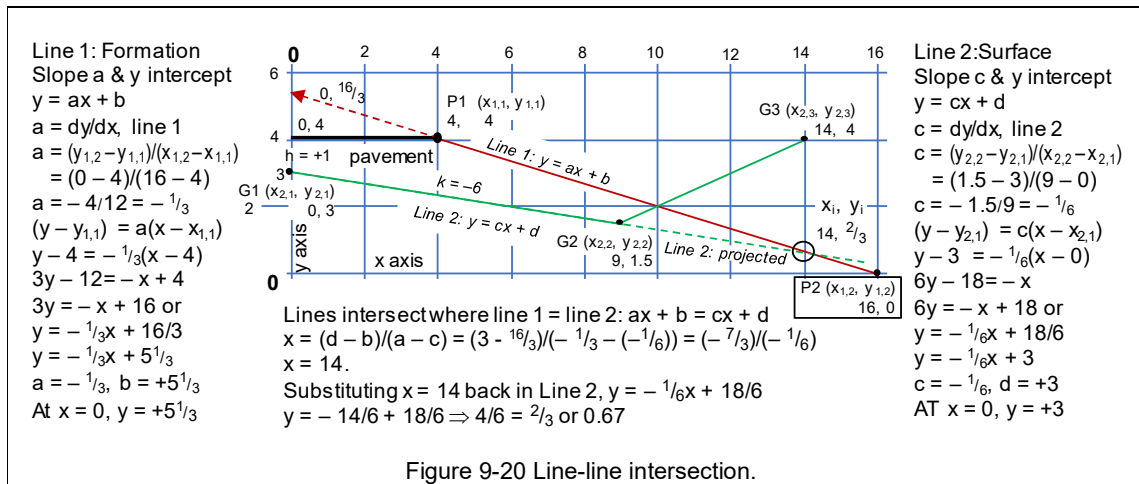


Figure 9-20 Line-line intersection.

9.1.10.2 Illustration of solving the intersection of the two lines

In Figure 9-20 Line-line intersection. above:

The intersection has a common y intercept. Thus, from $y = ax + b$ and $y = cx + d$:

$$ax + b = cx + d$$

a and c are the slopes of the Line 1 and Line 2. b and d are the y intercepts at $x = 0$.

Check the y value by substituting $x = 14$ back in Line 1, $y = -1/3x + 16/3$

$$y = -14/3 + 16/3 \Rightarrow 2/3 = 2/3 \text{ or } 0.67$$

The intersection of the two lines is $(14, 0.67)$. This value also agrees with the visual interpretation of the figure.

Note that the y result is the actual value reference the coordinate frame ($y = 0$ on the x axis, the y axis is the centreline).

9.1.10.3 Comparing the line-line intersection with Eqn 9-2

We have the following, where $m = 1/a$ and $k = 1/c$:

$b/2 = 4$ (half pavement width, $h = +1$ (fill), $m = -3$ (batter slope), $k = -6$ (ground slope).

$$W_R = (b/2 - mh)(k/(k - m)) \Rightarrow (4 - 3 \times 1)(-6/(-6 - -3)) = 7 \times 2 = 14.$$

Height from ground centreline:

$$h_R = W_R/k = 14/-6 = -2.33.$$

However, the RL of the centreline, reference the ground slope $k = -6$, at $x_{2,1}$, $y_{2,1}$ is 3.

Thus, in the coordinate frame, RL at the toe, $RL_{WR} = 3 - 2.33 = 0.67$.

9.1.10.4 Line-line intersection method using 2 sets of points on each line

In Figure 9-21 the example provides points common to two intersecting of lines. The surface on which the embankment will be formed is a surveyed ground surface slope (line 2 and line 3). The designed formation surface comprises a formation pavement and embankment batters (line 1). We cannot apply Eqn 9-2 to determine the W coordinates, W_R , for the embankment width, because the illustrated intercept is between line 1 and line 3. Previously, in Figure 9-20 we saw that the intersection of line 1 and line 2 was at $W_R = 14$.

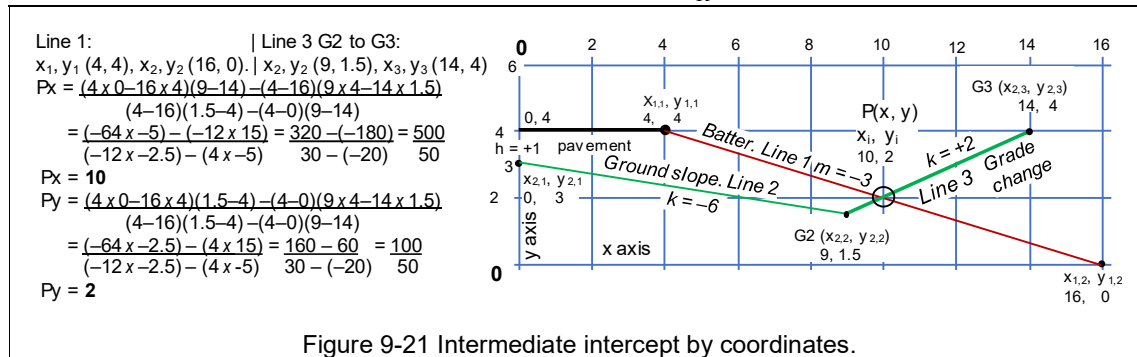


Figure 9-21 Intermediate intercept by coordinates.

Previously calculated, $W_R = 14$, using line 2 assuming a constant grade of $k = -6$. It was surveyed out to only 9m before a change in grade was noted. As line 3 shows, the intercept is somewhere across line 3 between a new $P_{2,2}$ (9, 1.5) and $P_{2,3}$ (14, 4). The slope of line 3, $k = +2$.

The method detailed in §3.4.5 involves calculating h at $W_R = 9$, at the change of grade and then applying the change of grade, k , for the next section, to the equation. Figure 9-22 shows the intersection.

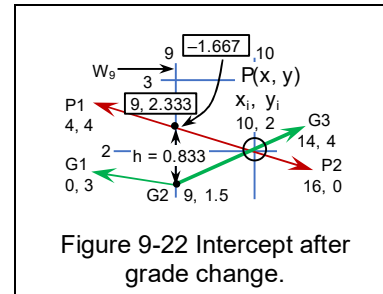


Figure 9-22 Intercept after grade change.

In keeping with Eqn 9-2:

$$W_1 = W_0 + \left(-mh \left(\frac{k}{(k - m)} \right) \right)$$

$$W_0 = G_2x - \frac{b}{2} = 9 - 4 = 5. \text{ Offset distance } G_2x - P_1x.$$

$$k_3 = \frac{dx}{dy} = \frac{G_4x - G_3x}{RL_{G_4} - RL_{G_3}} = \frac{14 - 9}{4 - 1.5} = \frac{5}{2.5} = +2$$

$$\text{At } W_R = 9, h_{R9} = \frac{W_R - b/2}{m} = \frac{9 - 4}{-3} = -1.667. \text{ RL}_{R9} = RL_{P1} - h_{R9} = (4 - 1.667) = 2.333$$

$$RL_{G2} = 1.5, h_{R9} = RL_{R9} - RL_{G2} = 2.333 - 1.5 = 0.833, m = -3$$

$$W_I = \left(W_R - \left(-mh_{R9} \frac{k_3}{(k_3 - m)} \right) \right) = \left(9 - (-3 \times 0.833) \times \left(\frac{2}{2 - (-3)} \right) \right) = 9 - \left(-2.5 \times \frac{2}{5} \right) = 10$$

And continuing, to find the RL at the $W_1 = 10$ by Eqn 9-3.

$$h_{W10} = \frac{W_{10} - \frac{b}{2}}{m} = \frac{10 - 4}{-3} = \frac{6}{-3} = -2$$

$$RL_{W10} = RL_{W1} + h_{W10} = 4 + (-2) = 2.$$

Figure 9-22 illustrates the method.

9.1.10.5 Line intercepts by matrix determinant

The method of line-line intercepts involves using a matrix determinant method to solve the position of the intercept $P(x_i, y_i)$ between line 1 and line 3. See:

https://en.wikipedia.org/wiki/Line-line_intersection

$$P_x = \frac{\begin{bmatrix} \begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \end{pmatrix} \begin{pmatrix} x_1 & 1 \\ x_2 & 1 \end{pmatrix} \\ \begin{pmatrix} x_3 & y_3 \\ x_4 & y_4 \end{pmatrix} \begin{pmatrix} x_3 & 1 \\ x_4 & 1 \end{pmatrix} \end{bmatrix}}{\begin{bmatrix} \begin{pmatrix} x_1 & 1 \\ x_2 & 1 \end{pmatrix} \begin{pmatrix} y_1 & 1 \\ y_2 & 1 \end{pmatrix} \\ \begin{pmatrix} x_3 & 1 \\ x_4 & 1 \end{pmatrix} \begin{pmatrix} y_3 & 1 \\ y_4 & 1 \end{pmatrix} \end{bmatrix}}, P_y = \frac{\begin{bmatrix} \begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \end{pmatrix} \begin{pmatrix} y_1 & 1 \\ y_2 & 1 \end{pmatrix} \\ \begin{pmatrix} x_3 & y_3 \\ x_4 & y_4 \end{pmatrix} \begin{pmatrix} y_3 & 1 \\ y_4 & 1 \end{pmatrix} \end{bmatrix}}{\begin{bmatrix} \begin{pmatrix} x_1 & 1 \\ x_2 & 1 \end{pmatrix} \begin{pmatrix} y_1 & 1 \\ y_2 & 1 \end{pmatrix} \\ \begin{pmatrix} x_3 & 1 \\ x_4 & 1 \end{pmatrix} \begin{pmatrix} y_3 & 1 \\ y_4 & 1 \end{pmatrix} \end{bmatrix}} \quad \text{Eqn 9-7}$$

Solution of the determinants produces coordinates for the intercept at $P(x, y)$.

$$P_x = \frac{(x_1 y_2 - y_1 x_2)(y_3 - y_4) - (y_1 - y_2)(x_3 y_4 - x_4 y_3)}{(x_1 - x_2)(y_3 - y_4) - (y_1 - x_2)(x_3 - x_4)} \quad \text{Eqn 9-8}$$

$$P_y = \frac{(x_1 y_2 - y_1 x_2)(x_3 - x_4) - (x_1 - x_2)(x_3 y_4 - x_4 y_3)}{(x_1 - x_2)(y_3 - y_4) - (y_1 - x_2)(x_3 - x_4)} \quad \text{Eqn 9-9}$$

Note that the denominator is the same in both Eqn 9-8 and Eqn 9-9 And only part of each numerator changes. As illustrated in Figure 9-21 the solution is $P_{xy}(10, 2)$.

Using the original line coordinates; Line 1 from (4, 4) to (16, 0) and line 2 from (0, 3) to (12, 1) results in the intersection at (14, 0.67) as shown in Figure 9-20

9.1.11 Design Profile Imposed on the Ground Surface

Why use coordinates?

Surface coordinates, in the form of distance along the centreline, y , offset from the centreline, x , and height, z or RL, are established by initial survey of the ground. The survey may be quite detailed. Depending on cross section topography, the cross sections may be every 5 metres from the centreline, as in the civil engineering exercise, or every 10 metres, or to some defined boundary. The profile interval, along the formation route could vary between 25 metres and 100 metres. Again, this depends on terrain. The survey should always be carried out across a significant change in topological profile, crossing a valley floor or a ridge line, even if it is not on a chainage point.

The surveyed long section profile will then be used to plan the (road) formation design and it is this design that must be examined in terms of providing coordinates of the design profile. There is a formation width, b . There are cut and fill batter slopes, m_c and m_f and there may be other constraints such as cut-off drains, defined below the top of the formation.

The derived intersection coordinates allow use of the matrix method of area calculation, as used in §3.4.7 (p 56), and subsequently.

Figure 9-23 is a sample of a formation design.

The essential parameters are:

Formation width, CD. $b = 8$. RL = 0.

Fill batter slopes, CF and GH. $m_{\text{fill}} = -3$.

Cut batter slopes, BC and EG. $m_{\text{cut}} = +2$.

Drainage line location re CL, B = -8, E = +8.

The end coordinates of the lines are:

C = -4, 0; D = +4, 0. Pavement.

B = -8, -1.33; E = +8, -1.33. Drain lines.

H = -16, -4; J = +16, -4. Toe coordinates for fill.

A = -20, +4.67; G = +20, +4.67. Crest coordinates for cut.

The coordinates define the points on a line that may be extended to meet an intersecting surface profile and, using matrix determinants, can be used to calculate intersections.

Introducing a surveyed ground cross section, how do the intersections look?

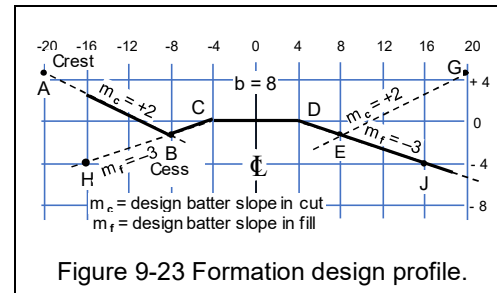


Figure 9-23 Formation design profile.

9.1.11.1 Intersections at $h = 0$

The ground cross section has left, centre and right relative height coordinates. The design profile is imposed on such that the formation centreline is on the ground at F. Thus $h = 0$.

Figure 9-24 shows the section:

L = -20, +5 ($k_L = +4$)

F = 0, 0, CL ($h = 0$)

R = 20, -2 ($k_R = -10$).

An Excel solution of the matrix determinant, Eqn 9-8 and Eqn 9-9, provides:

Left intercept of line F-L with B-A:

$I_{Lx} = -21.33$, $I_{Ly} = 5.33$, the crest intercept.

Right intercept of line F-R with E-G:

$I_{Rx} = 8.89$, $I_{Ry} = -0.89$. This is a crest intercept outside the drain line at E = +8, -1.33.

F-R also intercepts the profile D-J at $P_x = 5.7$, $P_y = -0.57$. But note that P_x is inside the drainage line at E = +8, and thus cannot be used because of the design limitations.

The figure shows that, on the left, the excavation is all cut (I_L B C F I_L). Cut area = 32.6m^2 .

Cut (1.1m^2) and fill (0.8m^2) on the right of the Δ almost cancel out. Balance of 32.9m^2 cut.

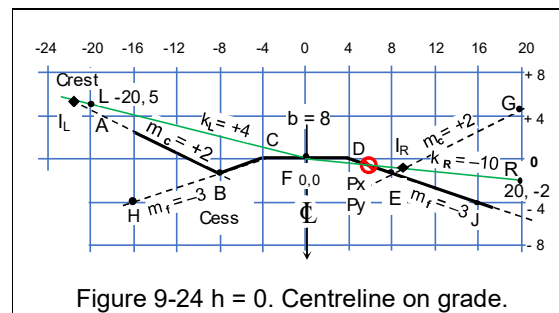


Figure 9-24 $h = 0$. Centreline on grade.

9.1.11.2 Intersection at $h = -2$

The design profile has a cut of $h = -2$; below the ground.

Figure 9-25 shows the section:

L = -20, +7 ($k_L = +4$)

F = 0, +2, CL ($h = +2$)

R = 20, 0 ($k_R = -10$).

Using new values for L, F and R, Eqn 9-8 and Eqn 9-9, provide the following results.

Left intercept, line F-L with B-A:

$I_{Lx} = -29.33$, $I_{Ly} = 9.33$,

the crest intercept.

Right intercept of line F-R with D-J:

$I_{Rx} = 12.22$, $I_{Ry} = 0.29$.

This is a crest intercept outside the drain line at E = +8, -1.33.

From the Figure 9-25, the excavation is all fill. (I_L B C D E I_R F I_L).

The area of fill = 104m^2 .

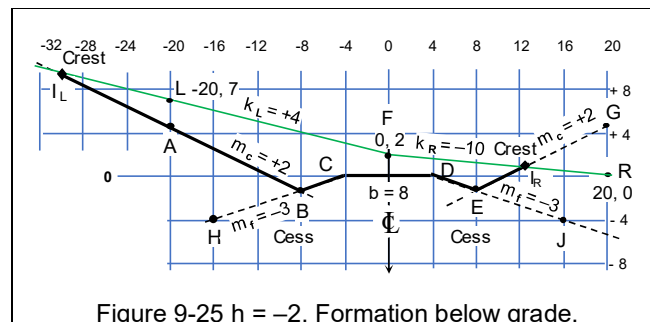


Figure 9-25 $h = -2$. Formation below grade.

9.1.11.3 Intersection at $h = +2$

The design profile has a fill of $h = +2$; above the ground.

Figure 9-26 shows the section:

$$L = -20, +3 \quad (k_L = +4)$$

$$F = 0, -2, \quad CL \quad (h = +2)$$

$$R = 20, -4 \quad (k_R = -10).$$

Using new values for L, F and R, Eqn 9-8 and Eqn 9-9 provide the following results.

Left intercept of line F-L with B-A:

$$I_{Lx} = -13.33, I_{Ly} = 1.33, \text{ the crest intercept.}$$

Right intercept of line F-R with D-J:

$$I_{Rx} = 14.29, I_{Ry} = -3.43.$$

This is a crest intercept outside the drain line at $E = +8, -1.33$.

F-R also intercepts the profile C-B at:

$$P_x = -5.7, P_y = -0.57.$$

But note that the indicated $P_x = -5.7$ is inside the drain line at $B = -8$, and thus cannot be defined as the toe of the fill because of the design limitations.

From the figure, the excavation is more fill than cut ($I_L B P_x I_L$). The area of cut = 5m^2 . The area of fill = 28m^2 , thus requiring a fill balance of 23m^2 .

Figure 9-27 shows the intercepts above the formation in the vicinity of F, the ground profile centre point: Line F-L does intercept the profile C-B and line F-R does intercept D-E, but at positions above the formation on the *other* side of the centreline. The y scale is exaggerated 4 times to make the intersection more visible.

In this case fill batter DE intercepts ground FR to the *left* of centreline. Similarly, fill batter CB intercepts ground FL on the *right* of centreline. This is an artefact of the maths.

$$P_{Lx} = +1.1, \quad P_{Ly} = 1.7$$

$$P_{Rx} = -2.9, \quad P_{Ry} = 2.3.$$

Returning to Figure 9-26 with calculated coordinates as summarised in Figure 9-28.

A	-13.3	1.3	Crest
B	-8	-1.3	Defined drain line (4m from B)
G	-5.7	-0.6	Intersection cut with ground profile
C	-4	0	Formation left shoulder
D	4	-3.4	Toe
F	0	-2	Surface centreline

The task is now to design a matrix array to calculate either the cut area and the fill area, or the difference between cut and fill.

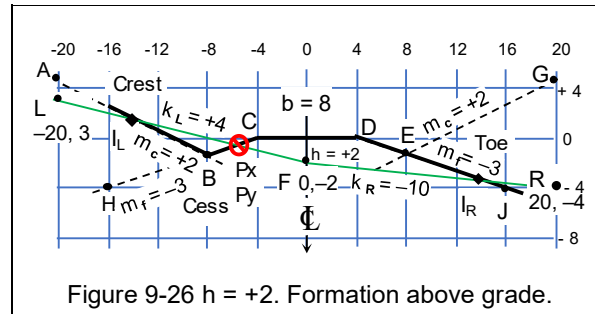


Figure 9-26 $h = +2$. Formation above grade.

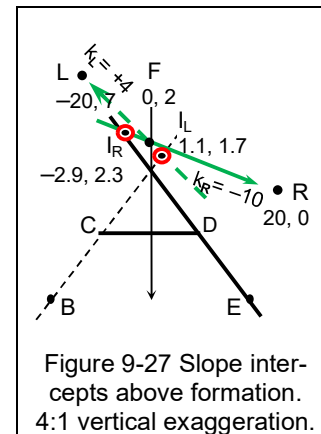


Figure 9-27 Slope intercepts above formation. 4:1 vertical exaggeration.

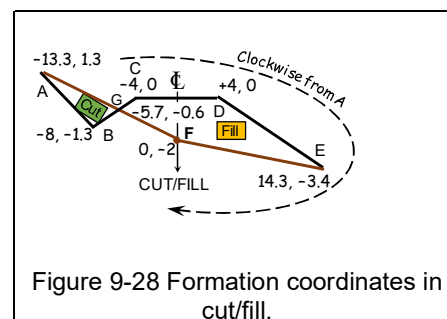


Figure 9-28 Formation coordinates in cut/fill.

How did we get these figures?

The formation level is $RL = 0$, the ground $CL_{RL} = -2$.

Left side W values are negative.

$$b = 8, m_{fill} = -3, m_{cut} = +2, k_L = +4, k_R = -10, W_{drain} = 8, h = +2(\text{fill}).$$

$$C = -b/2 \text{ at formation } RL = 0. \quad (-4, 0)$$

$$D = +b/2 \text{ at formation } RL = 0. \quad (+4, 0)$$

$$E = W_R = 14.3 \text{ by formula, } h_R = (W_R - b/2)/m_{fill} = -3.4. \quad (+14.3, -3.4)$$

$$F = CL \text{ ground (below formation), } h = 2. \quad (0, -2)$$

$$G = W_L = 5.7 \text{ by formula, } h_L = (W_L - b/2)/m_{fill} = -0.6. \quad (-5.7, -0.6)$$

$$B = W_{drain} = 8 \text{ (defined), } h_{drain} = (W_{drain} - b/2)/m_{fill} = -1.3. \quad (-8, -1.3)$$

$$A = W_{drain} + (m_{cut} \times h_{drain}) \times (k_L / (k_L - m_{fill})) = 13.3, h_A = W_A / k_L - h = 1.3. \quad (-13.3, 1.3)$$

The matrix method works by advancing around the formation (here clockwise from left to right). In, following the formation, the figure is:

A B C D E F A, and the signed area is -22.9m^2 , which is the difference between (G not used):

$$\text{Cut } A B G A = +5.1 \text{ and}$$

$$\text{Fill } G C D E F G = -28.$$

So, the area, -22.9 , of the formation is the deficit of fill required compared with the amount of cut available to be used in fill. This is useful for the start calculations of a mass haul diagram.

9.2 Circular Earthworks on a Sloping Site

9.2.1 Reprise of Strike and Dip

Chapter 8 Strike and Dip to an Embedded Plane,
§8.3 Strike and Dip to a Plane

What are the calculations in relation to an isolated structure, say a tank pad on a slope? Or the construction of a spoil heap, defining the toe of the embankment inside a boundary?

Recapping the sloping site; the slope of the surface is expressed as a ratio of the vertical distance compared with the horizontal distance.

Definitions:

- θ = full dip ($\theta_{\max}$) positive down, direction 0° .
- α = apparent dip at angle δ from full dip,
- δ = angular deviation from direction of full dip,
- Δ = direction of apparent dip, α (Δ same as δ).
- b = formation width (road) or pad diameter,
- R_f = radius formation, $R_f = \frac{1}{2}$ formation width b .
- R_Δ = radius in direction of apparent dip.

9.2.2 The Sign of Things

Figure 9-29 shows that the slope is 1:4 at full dip θ . We have previously used the symbol, k , to denote slope. Here $k = 4$. Continuing with the concept of the slope being perpendicular to the alignment of the formation we can then consider the slope as a dipping plane (Chapter 8) and can say that the true dip is, $\theta = 4$, being considered positive downwards as illustrated and accepted in structural geology.

Remember that the *angle* of the dip (slope) = $\arctan(1/k)$ so that if $k = 4$, $\text{dip} = 14^\circ$.

Dip is often expressed as a depression angle (downwards); for example, in angles of repose, or from field readings using an inclinometer in a geologist's compass. For construction we want the site to have a fairly flat slope. Building on any slope greater than 1:4 approaches our batter slope limit of 1:3 or 1:2.

And returning to the *apparent* dip on a sloping plane, Figure 9-30 illustrates how the slope *decreases* from the true dip as the direction of the dipping plane retreats to the strike direction. The *length* of the apparent dip vector represents the apparent dip value; it can be calculated as:

$$\alpha = \frac{\theta}{\cos \delta} \text{ or can be scaled with reasonable accuracy} \quad \text{Eqn 9-10}$$

Now impose an *embankment* profile over the sloping plane. As an illustration, Figure 9-31 sets the batter slope as 1:3, where, from §8.4, we say $m = 3$. The batter and the slope meet at 4m from the beginning of the slope. In terms of our formation we can see that the half formation width, $R_f = \frac{b}{2} = 1$ from the centre line. The formation depth, h , is zero ($h = 0$).

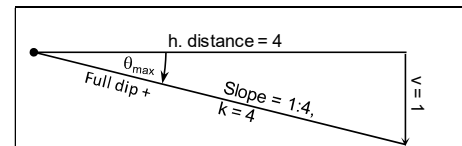


Figure 9-29 Full dip on the plane.

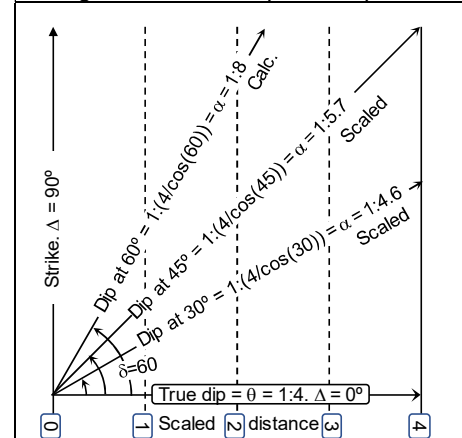


Figure 9-30 Apparent dip over the plane.

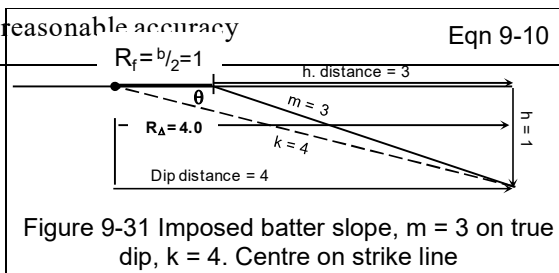


Figure 9-31 Imposed batter slope, $m = 3$ on true dip, $k = 4$. Centre on strike line

Note that we are also defining the embankment slope direction to be the same as the dip direction (typical linear embankment profile). It is perpendicular to the local strike line.

Taking the standard formation width formula and applying a new convention for the radius of the pad, $R_f = b/2 = 1$. Replace width, W , used in road formations, with R_Δ to denote radius in the dip direction from the centre line (pad centre). Here, at $\theta_{\max} = k = 4$ at $\Delta = 0^\circ$, the full dip line. And, by the same geological definition, the batter slope $m = 3$ will be positive downwards. Amending Eqn 9-1 in line with our new definitions:

$$R_\Delta = (R_f + mh) \left(\frac{k}{k-m} \right) \quad \text{Eqn 9-11}$$

Using Eqn 9-11 we find that:

$$R_\Delta = (R_f + mh) \left(\frac{k}{k-m} \right) = (1 + 3 \times 0) \left(\frac{4}{4-3} \right) = 4. \text{ Radius at full dip as with Figure 9-31.}$$

What is the height, h_Δ , between the toe and the formation level? Again, amending Eqn 9-3 with our new definitions:

$$h_\Delta = \frac{(R_\Delta - R_f)}{m} \quad \text{Eqn 9-12}$$

In Figure 9-31, $h_\Delta = \frac{(R_\Delta - R_f)}{m}$, $R_f = \frac{b}{2}$. Then $h_\Delta = \frac{4-1}{3} = 1$

Now take an apparent dip direction at $\delta = 60^\circ$ (Figure 9-30) from the true dip. (Eqn 9-10).

Apparent dip: $\alpha = \frac{4}{\cos 60} = 8$

Thus $k_{60} = 8$ at 60° from the dip line.

Figure 9-32 shows how the embankment slope, $m = 3$, intercepts the apparent slope, $k = 8$,

at: $R_{60} = (1 + 3 \times 0) \left(\frac{8}{8-3} \right) = 1.60$ from the

centre point of the circular pad where $R_f = 1$.

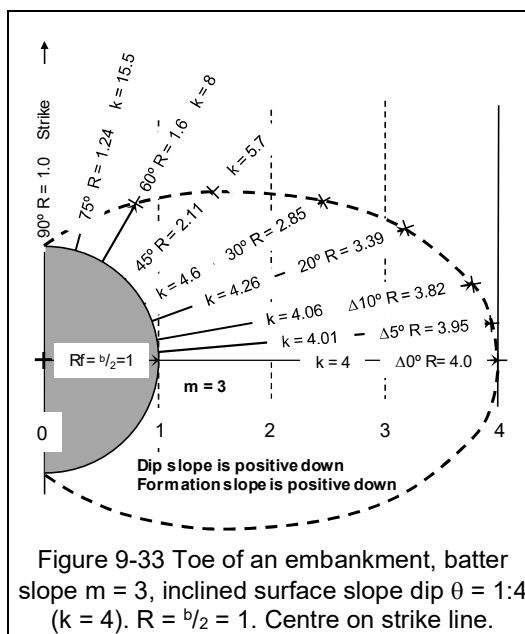
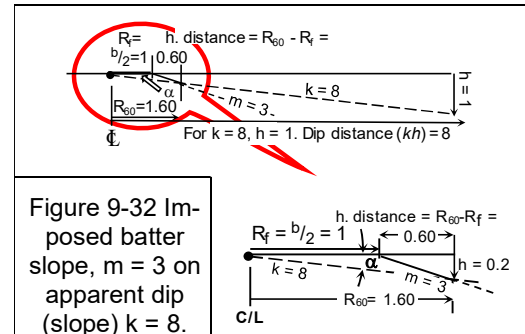
In Figure 9-32 height, using Eqn 9-12, $h_{60} = (1.6 - 1) / 3 = 0.2$

Since we are working from the pad built on the strike line, we see that $h_{\text{pad}} = 0$ at the centre.

See how it works. Devise a table of deflections from true dip to the toe of the embankment for any design pad radius (R_f), embankment slope (m) and true dip, θ . (maximum k).

Only one half of the toe intercepts need calculating. The other half of the fill is a mirror of the quadrant. The other half of the works would be in cut, assuming a uniform slope.

Using a unit radius $R_f = (b/2) = 1$ and true dip, $\theta = 4$, we can find the radius, R_Δ , from any apparent dip angle, α . Since we are working with varying slopes (apparent dips) we can call them



slopes of k . The embankment slope, m , will be a constant value. Denote the radius of the formation width, R_Δ , in the dip direction, Δ .

Consolidate Eqn 9-10 and Eqn 9-11. Find values for k and R at $\Delta = 30^\circ$ from the dip line.

$$\alpha = k_\Delta = \frac{\theta}{\cos \delta} \text{ apparent dip in direction } \Delta.$$

$$\text{For } \Delta = 30^\circ, \theta = 4 : k_\Delta = \frac{4}{\cos 30} = 4.62$$

$$R_\Delta = (R_f + mh) \left(\frac{k}{k - m} \right) \text{ from pad centre.}$$

$$R_\Delta = (1 + 3 \times 0) \left(\frac{4.62}{4.62 - 3} \right) = 2.85$$

Figure 9-33 shows, for various Δ , the toe of the embankment in the first quadrant, and reflected in the second quadrant.

Table 9-1 is a set of values used to plot the first quadrant of Figure 9-33. The opposite side of the dip line reflects the first quadrant.

Table 9-1 Radius to toe of embankment

True dip $\theta = 4$	$m = 3$	Apparent dip α $k_0 = 4$ at $\Delta = 0$	$R_f = b/2 = 1$
$\Delta = 0$	$m = 3$	$k_0 = 4.0$	$R_0 = 4.0$
$\Delta = 5$	$m = 3$	$k_5 = 4.01$	$R_5 = 3.95$
$\Delta = 10$	$m = 3$	$k_{10} = 4.06$	$R_{10} = 3.82$
$\Delta = 20$	$m = 3$	$k_{20} = 4.26$	$R_{30} = 3.39$
$\Delta = 30$	$m = 3$	$k_{30} = 4.62$	$R_{30} = 2.85$
$\Delta = 45$	$m = 3$	$k_{45} = 5.66$	$R_{45} = 2.11$
$\Delta = 60$	$m = 3$	$k_{60} = 8$	$R_{60} = 1.60$
$\Delta = 90$	$m = 3$	$k_{90} = \infty$	$R_{90} = 1.0$

9.3 Volume in circular embankments

The easiest way to calculate the volume of the imposed embankment is to use the end area method, as seen in our road volumes.

9.3.1 Segment Areas. Area between Dip Plane and Plane of the Embankment

However, this time we calculate the area of the segments through the embankment. We are working from the strike line, where $h = 0$, so half the pad will be fill. Area of the semi-circular pad: $A = \frac{1}{2} \pi R^2$.

But what's this segment bit?

The distances to the toe of the embankment are measured from the centre of the pad. Depending on the apparent dip angle, δ , we form a **sector** of angle 2δ and radius R_Δ . A **segment** is the area between the arc and the chord of the sector. The chord is the strike-line on the plane of the slope at a given contour level. See Figure 9-34.

The area of the segment is (area sector – area triangle). Developing the equation from trigonometry:

Area sector	$\frac{1}{2} 2\delta^R R_\Delta^2$, δ^R is in radians	Eqn 9-13
Area triangle	$\frac{1}{2} R_\Delta^2 \sin 2\delta$, δ in degrees	Eqn 9-14
Area segment	$\frac{1}{2} R_\Delta^2 (2\delta^R - \sin 2\delta)$	Eqn 9-15

$$\text{For } \delta = \Delta = 30 = 0.524^R, k_{30} = 4.62, R_{30} = 2.85$$

$$\text{Segment area: } R_{30} = \frac{1}{2} (2.85^2 (2 \times 0.524 - \sin (2 \times 30))) = 0.736$$

Figure 9-35 is an illustration of the segments generated from Table 9-1. At this stage the difference in height between each segment has not been calculated.

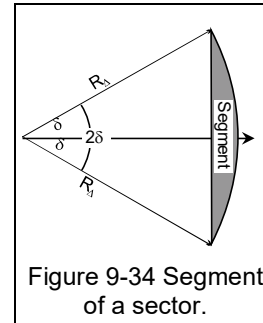


Figure 9-34 Segment of a sector.

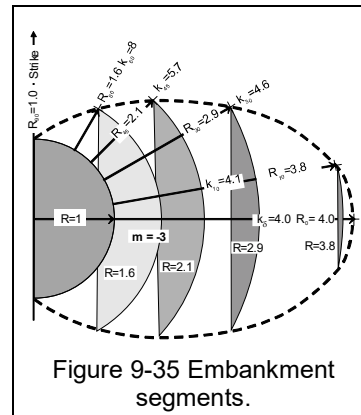


Figure 9-35 Embankment segments.

9.3.2 End Area Volume, finding dh

Having found the sector areas at each apparent dip, the next task is to find the vertical distance between each sector. Referring to Figure 9-36 we can see that the vertical height to the toe is the distance from the perimeter of the pad to the toe divided by the batter slope, Eqn 9-12, using values from Table 9-1.

$$h_{\Delta} = \frac{(R_{\Delta} - R_f)}{m}$$

e.g. $\Delta = 60$, $R_{\Delta} = 1.60$, $m = 3$, $R_f = 1$

$$h_{60} = \left(\frac{1.60 - 1}{3} \right) = 0.20$$

$$h_{30} = \left(\frac{2.85 - 1}{3} \right) = 0.62$$

$$dh_{(h_{60}-h_{30})} = 0.62 - 0.20 = 0.42$$

$$Volume_{(R_{60}-R_{30})} = \frac{1}{2} (1.57 + 0.74) \times 0.42 = 0.485$$

R_{60} , R_{30} from Table 9-2. Note that the height intervals between the segments are not regular. The heights depend on the apparent dips, α , or k_{Δ} .

Remember the end area formula, Eqn 3-10. See §3.5, page 56.

$$Volume = \Sigma V = D_{12} \left(\frac{A_1 + A_2}{2} \right) + D_{23} \left(\frac{A_2 + A_3}{2} \right) + D_{34} \left(\frac{A_3 + A_4}{2} \right) + \dots$$

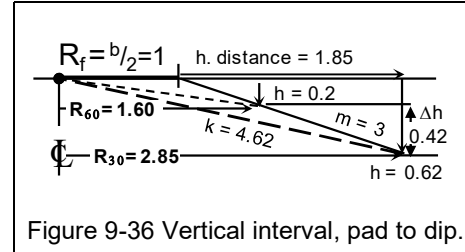


Figure 9-36 Vertical interval, pad to dip.

9.3.2.1 Generate the table of values. Strike on pad centre, $h = 0$.

The volume, 0.934, calculated on a unit radius = 1, is a good approximation of the value achieved when δ is iterated every 10 degrees. Using a 1° iteration makes little difference.

Volume from Table 2-2 = 0.934

Volume for D every 10° = 0.9442

Volume for D every 5° = 0.9446

Volume for D every 1° = 0.9447

Table 9-2 Sector areas, heights, volumes at unit radius.

Δ	R_{Δ}	Area	h_{Δ}	dh	End area volume
90	1.0	1.57	0	---	
60	1.60	1.57	0.20	0.2	0.314
45	2.13	1.29	0.376	0.176	0.252
30	2.85	0.74	0.618	0.242	0.246
20	3.39	0.32	0.796	0.178	0.094
10	3.83	0.05	0.942	0.146	0.027
5	3.95	0.01	0.985	0.043	0.001
0	4	0	1	0.015	0.000
				Total	0.934

Using $R_f = 10\text{m}$ for the pad radius (a 20m diameter pad) there is a difference of less than 11m^3 between Table 9-2 calculation intervals (935m^3) and dips calculated at 1° intervals (945m^3).

9.3.3 Strike through Back of Pad. $h = R/k$

Consider the back of the pad on the strike line. Previous discussion from the pad centre, on the strike line, means that $h = 0$.

From Figure 9-36 we also see that $h = \frac{R_f}{k}$ and the dip direction continues around to the back of the pad.

Apparent dip $k_{\Delta} = \frac{\theta}{\cos \Delta}$. k_{Δ} varies from $k_0 = 4$ (true dip), to

$k_{90} = \infty$ through to $k_{180} = -4$ (true rise). See Figure 9-37.

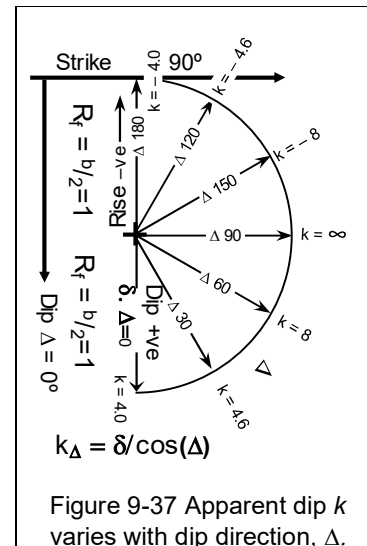


Figure 9-37 Apparent dip k varies with dip direction, Δ .

The sign of the k_{Δ} values is consistent because $\cos\Delta$ is negative in the second quadrant ($\Delta = 90^{\circ} \rightarrow 180^{\circ}$).

Table 9-3 shows how the application of the standard formation width formula produces the distance from the centre of the pad to the toe of the embankment. Figure 9-38 is the resultant plan of the toe of the embankment.

True dip $\theta = 4$ $m = 3$. $h_{CL} = 0.25$	Apparent dip	$R_f = 1$. $R_{\Delta} =$ $(R_f + mh)(k/(k-m))$
Δ $\cos(\Delta)$		
$\Delta = 180$: $\cos = -1.00$	$K_{180} = -4.0$	$R_{180} = 1.00$
$\Delta = 150$: $\cos = -0.87$	$K_{150} = -4.62$	$R_{150} = 1.01$
$\Delta = 135$: $\cos = -0.71$	$K_{135} = -5.66$	$R_{135} = 1.14$
$\Delta = 120$: $\cos = -0.50$	$K_{120} = -8.0$	$R_{120} = 1.27$
$\Delta = 90$: $\cos = 0.00$	$K_{90} = \infty$	$R_{90} = 1.75$
$\Delta = 60$: $\cos = 0.50$	$K_{60} = 8.0$	$R_{60} = 2.80$
$\Delta = 45$: $\cos = 0.71$	$K_{45} = 5.66$	$R_{45} = 3.73$
$\Delta = 30$: $\cos = 0.87$	$K_{30} = 4.62$	$R_{30} = 4.99$
$\Delta = 0$: $\cos = 1.00$	$K_0 = 4.0$	$R_0 = 7.00$

Radius = 1. $h_{CL} = R_f/k = 1/4 = 0.25$. (2m diameter pad).

At $\Delta = 0$, $R_f = 1$, $m = 3$, $k = 4$,

$$R_{\Delta} = ((1+3 \times 0.25) \times (4/(4 - (+3)))) = 7$$

At $\Delta = 180$, $R_f = 1$, $m = -3$, $k = -4$,

$$R_{\Delta} = ((1-3 \times 0.25) \times (-4/(-4 - (-3)))) = 1$$

9.3.3.1 Volume from segment areas. $\Delta > 90^{\circ}$

The method used in § 9.3.1 is expanded to include the segment where apparent dip angle, Δ , is greater than 90° . Note that at $\Delta = 180$ the area = 3.14, the area of a circle (the pad). The maximum segment area occurs at about $\Delta = 70$.

The two shaded segments in Figure 9-39 show a segment area at $\Delta = 120^{\circ}$ (4.09) compared with the area at $\Delta = 60^{\circ}$ (4.81).

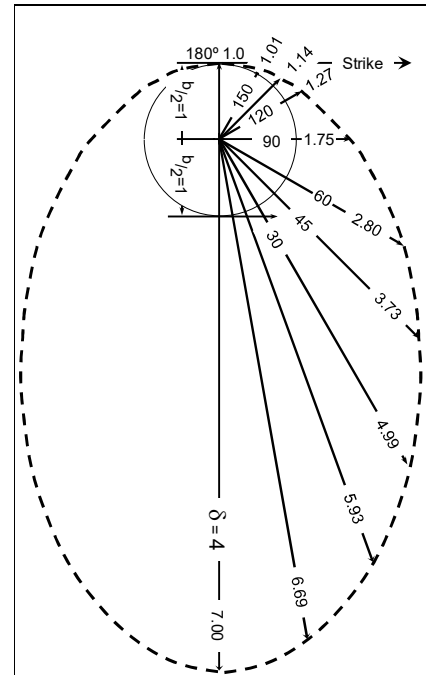


Figure 9-38 Toe of an embankment batter slope $m = 3$, inclined surface slope of dip $\theta = 1:4$ ($k = 4$). $R = b/2 = 1$. Back of pad on strike line.

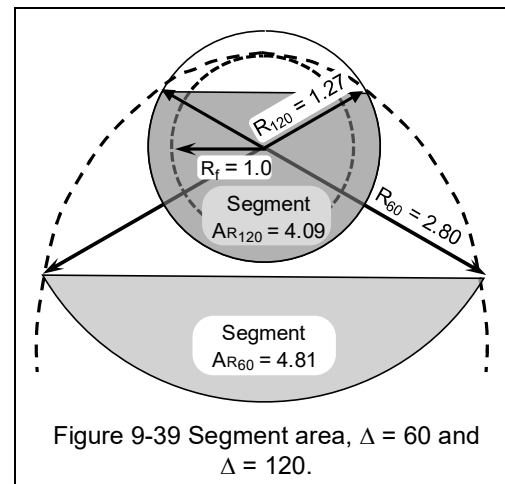


Figure 9-39 Segment area, $\Delta = 60$ and $\Delta = 120$.

9.3.3.2 Fixed dh intervals for contour lines

The initial toe calculations have been carried out at various dip deflection values, δ , away from the true dip. How do we find an apparent dip direction to correspond with an h value?

The unit radius example has a $h = 1$ for full dip. Find all the parameters for $h = 0.5$, halfway down the batter.

Let's turn the formulae around to find, initially, R_Δ (radius to toe) and from R_Δ find δ . R_f is radius of the pad ($b/2$). See Figure 9-40, refer to Eqn 9-12.

- a) What is the toe radius for $h = 0.5$?

$$h_\Delta = \frac{(R_\Delta - R_f)}{m}. R_f = \frac{b}{2} = \text{pad radius}.$$

$$R_\Delta - R_f = h_\Delta m. \text{ Thus}$$

$$R_\Delta = h_\Delta m + R_f. \text{ Dip distance for design } h.$$

$$\text{For } h_\Delta = 0.5, m = 3, R_f = 1$$

$$R_\Delta = (0.5 \times 3) + 1 = 2.5$$

- b) What is the k_Δ for $R_\Delta = 2.5$? Using Eqn 9-11:

$$R_\Delta = R_f \left(\frac{k_\Delta}{(k_\Delta - m)} \right) \text{ from centre of pad}$$

$$R_\Delta (k_\Delta - m) = R_f k_\Delta \Rightarrow R_\Delta k_\Delta - R_f k_\Delta = R_\Delta m.$$

$$\therefore k_\Delta = \frac{R_\Delta m}{R_\Delta - R_f}.$$

$$\text{For } R_\Delta = 2.5, R_f = 1, m = 3$$

$$k_\Delta = 2.5 \times 3 / (2.5 - 1) = 7.5 / 1.5 = 5.0$$

- c) What is the dip angle, δ , for the apparent dip $k_\Delta = 5$. Using Eqn 9-10 for dip angle δ :

$$k_\Delta = \frac{\theta}{\cos \delta} \text{ apparent dip at angle } \delta$$

$$\cos \delta = \frac{\theta}{k_\Delta}. \text{ For } k_\Delta = 5.0, \theta = k_0 = 4$$

$$\cos \delta = \frac{4}{5} = 0.8. \theta = 36.9^\circ (36^\circ 52')$$

Thus, any contour line can be imposed over the embankment by choosing the appropriate value of h below the pad level, where $h = 0$ at the surface. Actual contour values will have to be determined by offset from the surveyed RL of the pad.

Figure 9-41 shows the three contours just calculated.

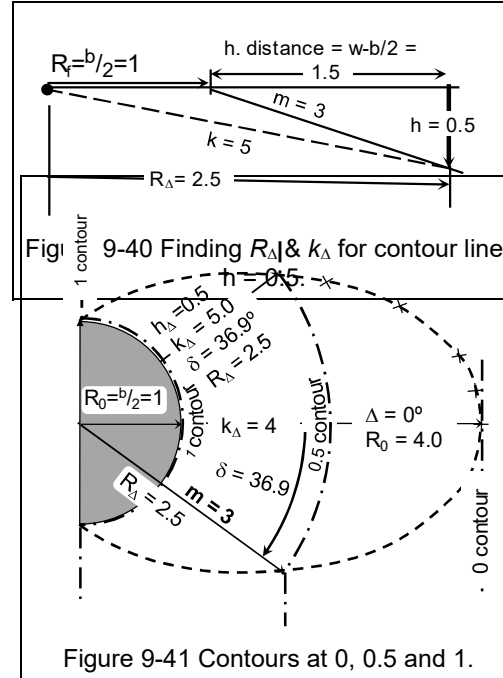
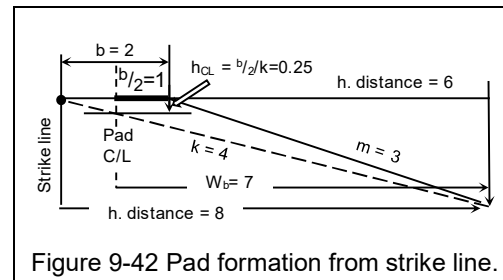


Figure 9-41 Contours at 0, 0.5 and 1.



9.3.4 Pad in Full Fill from Strike Line

We have previously examined the pad being in half fill, half cut. The centre line of the pad is on the strike line. That means, from a theoretical position, the cut and the fill would be equal.

What happens if the "back" of the pad is on the strike line? There will be all fill from the strike line to the toe. Just like a normal road formation, apply

the same formula as for our normal road formations. The height, h_{CL} , at the centre line depends on the formation width, (the radius of the pad, R_f ($b/2$)) and the true dip (θ). See Figure 9-42.

$$h_{CL} = \frac{R_f}{\theta} = \frac{1}{4} = 0.25$$

The apparent dip angle, δ , and the radius of the pad start at the **centre** of the pad at height, h_{CL} , below the formation level where $h = 0$, our reference height on the strike line. shows the variation limits on k , remembering $k_{max} = \pm\theta$, the true dip.

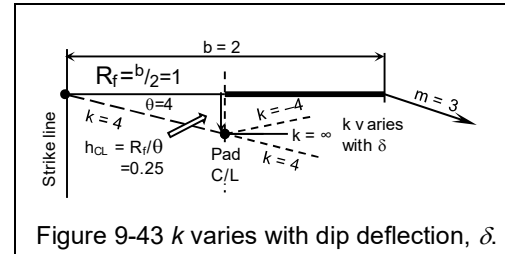


Figure 9-43 k varies with dip deflection, δ .

9.3.4.1 Comparative table of embankment volumes

The table produces a volume of 6.18. This value is a little high compared with a more involved set of apparent dips.

Volume from Table 9-4 = 6.18

Volume for D every 10° = 6.112

Volume for D every 5° = 6.115

Volume for D every 1° = 6.11

9.3.5 Full Fill versus Cut and Fill

Examine the difference between the two methods.

Using the example parameters of $R_f = 1$, dip $\theta = 4$, batter $m = 3$ it shows that:

1. A *cut/fill* operation centred on the strike line has a fill volume of about 1.
Neglecting bulking factors, there is a corresponding cut volume of 1.
There is a fill height of 1 needing a compaction layer every 0.2m,
Thus, 5 compaction passes.
Material handled volume is 2.
The work dimensions are about 3m×4m fill and 3m×4m cut. Work area = 24.
2. The *fill only* operation has a volume of 6.2 and covers dimensions of 5m×8m.
Work area = 40.
There is a fill height of 2 needing a compaction layer every 0.2m,
Thus, 10 compaction passes.

There is 3 times as much material to be moved in the full fill operation, and it must be “borrowed” from another site. The work area is 67% bigger than the cut/fill operation.

The cut/fill does not require haulage of material, just on-site transfer from cut to fill.

Fill only must factor in access to suitable “borrow” material, borrow pit equipment, haulage machinery, haulage distance and borrow site reclamation.

In practical terms a cut/fill pad design is more economical than a full fill design. However, for the design of waste dumps, you now have methods of calculating simple designs.

9.3.6 Earthworks Construction, Calculations on Compaction

Site stability of earthworks construction in fill requires mechanical compaction of the fill material in relatively thin horizontal layers called *lifts*. Lifts vary between 100mm – 500mm and depend on the nature of the fill material and the purpose of the structure. Typically, a 200mm

Table 9-4 Sector areas, heights, volumes at unit radius.

Δ	R_Δ	Area	h_Δ	dh	Volume by end area
180	1.0	AR_Δ	0	First differences	
		3.14			
150	1.01	3.43	0.02	0.02	0.067
135	1.14	3.74	0.048	0.028	0.099
120	1.27	4.09	0.091	0.043	0.169
90	1.75	4.81	0.250	0.159	0.708
70	2.35	4.99	0.451	0.201	0.986
60	2.80	4.81	0.600	0.149	0.729
45	3.73	3.96	0.909	0.309	1.355
30	4.99	2.56	1.331	0.422	1.314
0	7.0	0.0	2.0	0.669	0.755
				Total	6.18

lift allows mixing and spreading, watering (optimum water content) and compaction (optimum density) in roadworks and earth wall construction.

How many deflections angles, δ , are there when compaction is required every 200mm, $dh = 0.2$? The real question is: how many lifts (compaction passes) will be needed for the height of the embankment? Secondly, what is the compaction area for each level, and the total area?

These answers have a direct bearing on resource allocation to the task. The following example shows the calculation method for a pad of unit radius ($R_f = 1$). Measurements can be scaled.

Extrapolating to a 10m radius pad: height/distance $\times 10$, area $\times 100$ and volume $\times 1000$.

9.3.6.1 Work comparison on a 20m diameter pad

An exercise calculation for a 20m circular pad to form the base for a 15m circular tank will allow a comparison of effort between the cut/fill and full fill methods. The design allows a 2.5m clearance around the tank for access, or for a containment bund. A 15m tank that is 3m high has a capacity of 530,000litres. Filled with water, its base pressure is 29kPa. Will the fill material handle the stress?

9.3.6.2 Solution of compaction works using unit radius, $R_f = 1$

For a given height, h , down from the reference surface (pad where $h = 0$) the height of the pad centre line, h_{CL} , depends on R_f (strike to pad centre distance) and true dip θ . Replace apparent dip, α , in the direction Δ with the symbol k_Δ .

§ 9.3.3.1 and Figure 9-35 demonstrate the methods involved. We chose the dip angle, Δ . Now we have to calculate the dip angle for varying heights.

Use a sequence of methods to find:

1. Equation to find the dip distance, R_Δ , from the pad centre for the compaction height, h :
Refer to Eqn 9-3 page 173.

$$h_\Delta = (R_\Delta - R_f) / m. R_f = \frac{b}{2} = \text{pad radius.}$$

$$R_\Delta - R_f = h_\Delta m. \text{ Thus}$$

$$R_\Delta = h_\Delta m + R_f. \text{ Dip distance for design } h.$$

Example: For $h_\Delta = 1.0$, $m = 3$, $R_f = 1$

$$R_\Delta = (1 \times 3) + 1 = 4.0$$

2. Find the apparent dip, k_Δ , for the dip distance, R_Δ .
See Figure 9-44 and recall Eqn 9-4, page 174.

$$f_w = \left(\frac{W}{k} \right) \text{ from surface centre-line to toe.}$$

$$k_\Delta = \left(\frac{R_\Delta}{(h_\Delta - h_{CL})} \right) \text{ apparent dip from pad centre}$$

Example: For $R_\Delta = 4$, $h_\Delta = 1.0$, $h_{CL} = 0.25$

$$k_\Delta = 4 / (1.0 - 0.25) = 4 / 0.75 = 5.33$$

3. Find the dip angle, δ , for the apparent dip, $k_\Delta = 5.33$. Using Eqn 9-10.

$$k_\Delta = \frac{\theta}{\cos \delta} \text{ apparent dip at angle } \delta. \cos \delta = \frac{\theta}{k_\Delta}.$$

For $k_\Delta = 5.33$, $\theta = k_0 = 4$

$$\cos \delta = 4 / 5.33 = 0.75. \delta = 0.723^R,$$

$$\delta^\circ = 41.4^\circ.$$

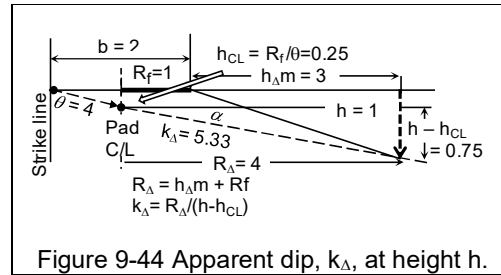


Figure 9-44 Apparent dip, k_Δ , at height h .

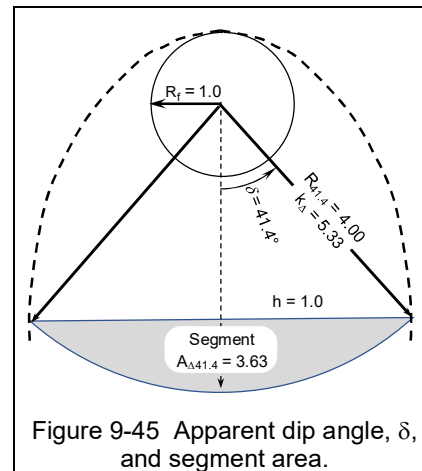


Figure 9-45 Apparent dip angle, δ , and segment area.

4. Find segment area, see and Eqn 9-15.

$$\text{Area segment} = \frac{1}{2} R_{\Delta}^2 (2\delta^R - \sin(2\delta)).$$

$$\text{For } \delta=41.4 = 0.723^R, R_{\Delta} = 4$$

$$A_{R_{41.4}} = \frac{1}{2} (4^2 (1.445 - \sin(82.8))) = 8(0.453) = 3.63$$

5. Table of area and volume for each lift.

Table 9-5 presents the results of the area and volume calculations for a unit radius pad and a 0.2m compaction lift (10 layers). The volume calculates to 6.1m^3 . The area to be compacted is 32m^2 .

9.3.6.3 Solution of compaction works using radius, $R_f = 10$

A more “real world” comparison of a 10m radius pad (Table 9-6) of the two methods is shown in Figure 9-46 and Figure 9-47 with the following results for the example:

20m diameter pad,
 $R_f = 10\text{m}$.
 Compaction lift = 0.2m

Table 9-5 Lift (h down from pad), dip deflection, lift area, lift volumes.

h	Δ	Area	dVol
0.0	180.0	3.14	
0.2	97.2	4.66	0.78
0.4	74.2	5.00	0.97
0.6	60.0	4.82	0.98
0.8	49.7	4.32	0.91
1.0	41.4	3.63	0.79
1.2	34.3	2.82	0.64
1.4	27.8	1.96	0.48
1.6	21.4	1.14	0.31
1.8	14.4	0.42	0.16
2.0	0.0	0.00	0.04
Total		31.90	6.07

Table 9-6 Comparison of fill versus cut/fill for a circular pad on a sloping site.

	Full fill (Figure 9-46)	Cut/fill (Figure 9-47)
h_{CL}	2.5	0
Plan area	3,321	998 cut, 998 fill
No lifts of 0.2m	20	10
Compact area	30,733	4,645
Compact volume	6115	929
Cut volume	0	929

9.3.6.4 Illustration of full fill versus cut/fill of a 20m circular pad on a sloping site.

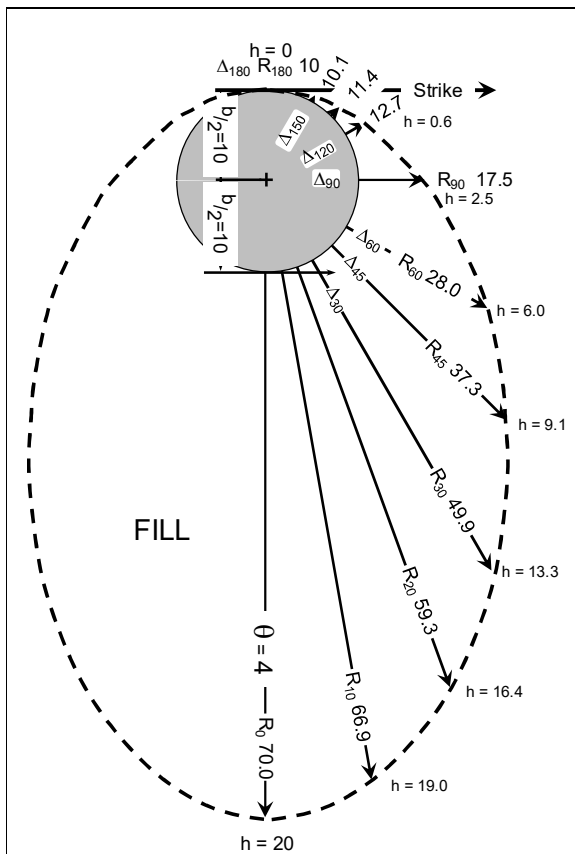


Figure 9-46 Full fill for 10m radius pad.

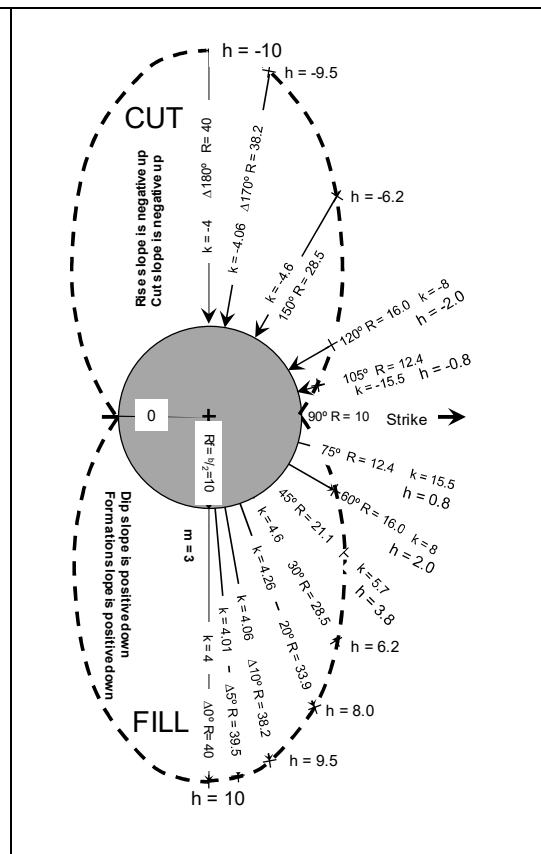


Figure 9-47 Cut/fill for 10m radius pad

9.4

9.4 Earthworks for Water Storage

Water storage, either for agriculture or mining, is mainly accomplished by the construction of compacted earth wall dams. Agricultural dams provide stock water, reservoir capacity for irrigation, flood mitigation and flow diversion. These types of dams take advantage of topography and streamlines, where overland flow from rain events is captured and retained for long term use.

In mining the purpose of artificial reservoirs includes water supply for habitation, mine processes and effluent treatment. Some structures can be massive, e.g. the Alcoa Residue Storage Areas (RSA), the “mud lakes” at their alumina refineries at Wagerup, Pinjarra and Kwinana in Western Australia.

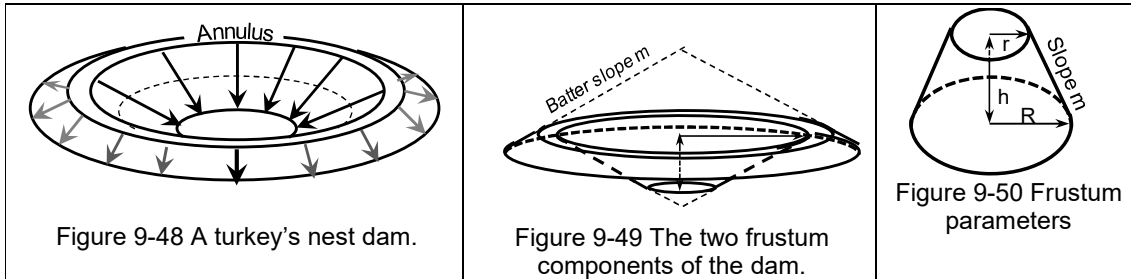
The main feature of these mining reservoirs is that they are constructed on relatively flat land. They cover large areas and the walls are constructed from compacted material excavated from within the reservoir area. Notwithstanding the environmental, geotechnical and economic issues the construction is based on cut and fill operations. The inside of the reservoir is the borrow pit; the retaining wall is a compacted formation akin to a road formation.

A popular form of earthen reservoir in both mining and agriculture is the “Turkey’s nest” form. It is a circular, or ring, dam. They are generally shallow, have low walls and are economical to build. The trick is to achieve a balance of cut/fill such that a desired capacity can be achieved without excessive extra material borrowing, or excess disposal.

9.4.1 The Turkey’s Nest Concept

We will examine the mathematics behind the structure and apply some design parameters.

Basically, a circular turkey’s nest dam (Figure 9-48) is a composed of two *frustums* (Figure 9-49), one inserted in the other. The rings of the dam can be considered as an *annulus*.



A frustum is a cone that has been truncated by a plane parallel to the cone base. Let R and r be the major and minor radii, h is the vertical distance between them (Figure 9-50).

9.4.2 The equations for a frustum and an annulus:

Volume frustum:	$V_{(f)} = \frac{\pi}{3} h (R^2 + Rr + r^2)$	Eqn 9-16
Surface area frustum:	$A_{(fs)} = \pi (R + r) \sqrt{(R - r)^2 + h^2}$	Eqn 9-17
Area annulus:	$A_{(ann)} = \pi (R - r)^2$	Eqn 9-18
Volume annulus:	$V_{(ann)} = 2\pi R_{centroid} \times \text{cross section area}$	Eqn 9-19

Interestingly, as $\pi/3 = 1.05$ then the volume of a frustum is $h(R^2 + Rr + r^2) + 5\%$.

9.4.3 Turkey's Nest Parameters

B = base diameter,

D = excavation depth,

F = excavation diameter,

Cf = bund crest width,

C = crest diameter

h = bund height,

W = crest width,

m = batter slope,

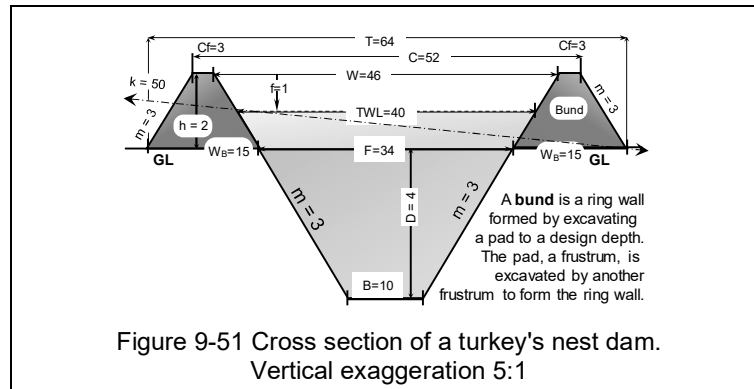
k = surface slope,

W = bund width,

f = freeboard.

T = bund toe diameter,

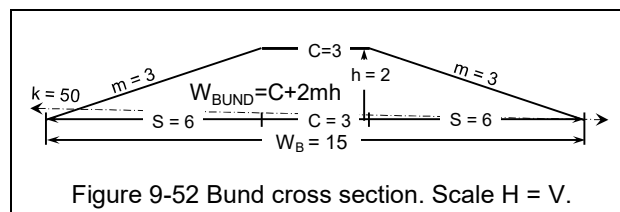
TWL = top water level diameter.



9.4.4 Design Premises

The values used in this section are taken from a Government of South Australia publication (2011) addressing water storage on farms in the Eyre Peninsular. The area is of relatively low rainfall and is one pertinent to this discussion. Only gross figures are taken to provide a sensible set of example calculations.

The basic design requirement for a dam is its capacity. Agricultural storage dams need to factor stock drinking requirements, evaporation, rainfall, storage period, catchment capacity, rainfall, catchment run-off.



A dam designed for a 250mm annual rainfall area, with 3 years storage capacity, allowing for 50% seepage and evaporation losses will need a capacity of some 1750m³ to maintain 500 dry sheep equivalent animals (wethers and dry ewes). (Government of South Australia, 2011).

Mining and industry storage are mainly concerned with process capacity.

Referring to the two components of the dam are:

1. The excavation below the ground.

This should meet the capacity requirements of the dam

2. The embankment bund above the ground.

It should be designed to use the excavated material. This style of ring dam uses a minimum height embankment with a crest roadway. The volume inside the embankment is considered as extra storage.

Assume a natural surface slope of 1:50 as illustrated. $k = \theta = 50$, is used with normal embankment calculation procedures. For normal planning it is problematic as to whether to include ground slope in initial calculations. However, we will develop the slope argument later.

9.4.5 Basic Equations

All volumes calculated are Bank Cubic Metres (BCM) and do not take excavation swell or fill compaction factors into account.

We start by calculating the cut and fill profiles as formations on a flat surface.

The values of these parameters are aligned to agricultural requirements and are meant to be applicable to inexpensive construction methods using basic earthmoving equipment.

The 6 parameters that we have are:

1. B, base diameter. From design volume requirements.
2. D, depth of excavation. Cut depth.

3. h , bund height. Fill. To balance cut volume.
4. C , crest formation width. Access to bund wall.
5. m , batter slope. Value depends on material.
 $m = 3$ is a compromise that limits erosion (scouring and rilling) from rainfall, allows stock access to water's edge, makes easy excavation with limited equipment, provides a stable embankment with reduced seepage when full.
6. f , freeboard between top of embankment and design top water level (TWL), the full charge level. Freeboard allows for the introduction of catchment inlets and spillways, and the protection of the top of the embankment from wind and wave action.

The volume of the frustum $V_{(f)} = \frac{\pi}{3}h(R^2 + Rr + r^2)$ was introduced in §9.4.2, Eqn 9-16.

The calculations start from the desired volume of the dam. This is best taken to be the excavation volume. The capacity above ground level (bund height minus freeboard) can be considered a bonus volume.

9.4.5.1 Excavation volume, below level ground

The design in creates an excavation volume of 1670m^3 . This is 95% of required volume. In the example $B = 10$, $D = 4$ floor to ground level and batter slope, $m = 3$. F is the diameter at ground level.

Where $B = 10$, $h = D = 4$, $m = 3$

$$F = B + 2mh,$$

$$F = 10 + (2 \times 4 \times 3) = 34\text{m}.$$

$$\text{Where } r = \frac{B}{2} = 5, R = \frac{F}{2} = 17, h = D = 4$$

$$V = \frac{\pi}{3}h(R^2 + Rr + r^2) = 1671\text{m}^3.$$

9.4.5.2 Embankment volume, above level ground

For the embankment, find the diameters associated with the bund:

T , at the toe of the bund at ground level;

W , the excavation width at the top of the inside of the bund;

C , the crest width at the top of the bund;

F , the excavation width at ground level.

Where $B = 10$, $D = 4$, $m = 3$, $h = 2$, $C_f = 3$.

$$W = B + 2m(D + h) = 10 + 2 \times 3 \times (4 + 2) = 46. \quad \text{Formation inside width at top of bund.}$$

$$C = W + 2C_f = 46 + 2 \times 3 = 52. \quad \text{Formation width outside crest.}$$

$$T = C + 2mh = 52 + 2 \times 3 \times 2 = 64. \quad \text{Formation width at toe of bund.}$$

The volume of fill for the bund is the volume of bund frustum minus the excavation volume between the crest and ground level.

T , C , W and F are the diameters, h is bund height.

$$\text{Where } R = \frac{T}{2} = 32, r = \frac{C}{2} = 26, h = 2 \quad (\text{Figure 9-53})$$

$$V_{BUND} = 5303\text{m}^3$$

$$\text{Where } R = \frac{W}{2} = 23, r = \frac{F}{2} = 17, h = 2$$

$$V_{CUT} = 2532\text{m}^3 \quad (\text{Figure 9-54})$$

$$V_{FILL} = V_{BUND} - V_{CUT} = 2770\text{m}^3. \quad \text{Volume to create } 2\text{m high bund, } 3\text{m crest, } m = 3.$$

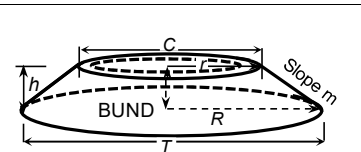


Figure 9-53 Bund volume.

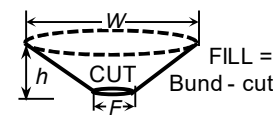


Figure 9-54 Cut volume.

The bund volume can also be calculated from annulus or by centroid. See § 9.4.5.5.1.

9.4.5.3 Optimising bund fill volume

On flat ground we see that, for a 2m bund height, we are deficient some 1100m³ of material. Lowering the height of the bund to 1.5m and recalculating results in a $V_{\text{FILL}} = 1630\text{m}^3$, which is balanced by the excavation of 1670m³. See Figure 9-55.

For $h = 1.5$

$$W = B + 2m(D + h) = 10 + 2 \times 3 \times (4 + 1.5) = 43. \text{ Formation width at top of bund.}$$

$$C = W + 2Cf = 43 + 2 \times 3 = 49. \text{ Formation width outside crest}$$

$$T = C + 2mh = 49 + 2 \times 3 \times 1.5 = 58. \text{ Formation width at toe of bund.}$$

Recalculating:

$$\text{Where } R = \frac{T}{2} = 9, r = \frac{C}{2} = 24.5, h = 1.5$$

$$V_{\text{BUND}} = 3380\text{m}^3$$

$$\text{Where } R = \frac{W}{2} = 21.5, r = \frac{F}{2} = 17, h = 1.5$$

$$V_{\text{CUT}} = 1754\text{m}^3$$

$$V_{\text{FILL}} = 1626\text{m}^3. \text{ Volume to create a 1.5m high bund, 3m crest and slope } m=3.$$

9.4.5.3.1 Storage capacity at the lower bund height

Allowing for a bund height of $h=1.5$ and freeboard of, say, 0.5m means an increase in holding capacity of:

$$TWL = B + 2m(D + h) = 10 + 2 \times 3 \times (4 + 1) = 40. \text{ Formation width at top water level.}$$

$$\text{Where } R = 20, r = 17, h = 5$$

$$V = \frac{\pi}{3} h (R^2 + Rr + r^2) = 1078\text{m}^3.$$

With freeboard of 0.5m the holding capacity of the design to $1671 + 1078 = 2750\text{m}^3$.

Allowing for freeboard of 1.0m reduces the capacity ($h=0.5$) to $1671 + 472 = 2144\text{m}^3$, which is still in excess of desired capacity.

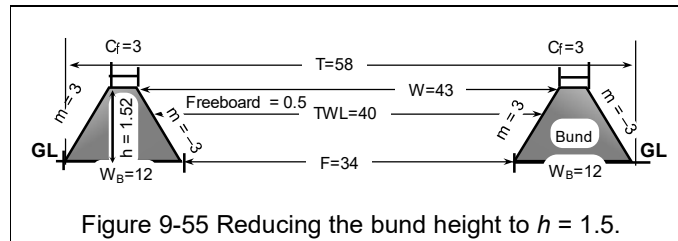


Figure 9-55 Reducing the bund height to $h = 1.5$.

9.4.5.4 Applying ground slope to the bund

Is it possible to apply the methods of § 9.3 to the problem of a bund on a sloping site?

Here is what's happening with the ground slope, k .

The true dip is, $\theta = 50$ towards the toe. As we rise with fill from the GL the radius decreases on the outside of the bund and increases from the inside toe. The apparent dip, α , decreases away from the true dip line.

Let the toe of the downstream bund be defined as GL with RL = 0.

Retain the bund height as $h = 2\text{m}$ and see if the slope can provide both extra excavation material and use less fill material on the bund.

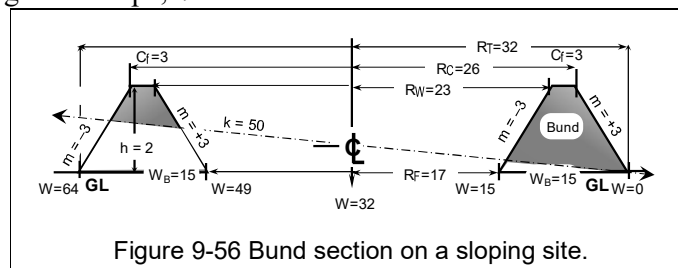


Figure 9-56 Bund section on a sloping site.

9.4.5.4.1 Slope intersection with bund walls

By using the true dip, $\theta = 50$ and renaming it k , we can find where the ground slope intersects the bund walls of batter slope, m . By applying the normal formation (road) width formula we can arrive at a solution for W where the ground slope intersects the bund wall.

Then W is the distance from the toe of the down-slope bund ($W = 0$) to the toe of each up-slope bund. W at the bund toes is akin to the centreline offsets used in roadworks. W_n is the batter intercept at h_n . Review § 3.4 and § 3.4.5 to refresh the following formula. Also § 9.2.2 and Eqn 9-2.

Conventionally $k = +50$ as it rises from $W = 0$, the downstream toe.

Similarly m will be either positive or negative, $m = \pm 3$ depending on its batter slope direction to the top of the bund from the toe of the batter. The “inside” batter has slope $m = +3$ (up), the “outside” batter, $m = -3$ (down). The SIGN of m now follows our convention from road formations: down from the centreline is negative, up is positive See Figure 9-57.

Using Eqn 9-11 and Eqn 9-4:

$$W = W + \left(mh \left(\frac{k}{k-m} \right) \right) \text{ and } h = \frac{W}{k}$$

Where $W = 64$, $m = -3$, $k = 50$

$$h_{64} = \frac{64}{50} = 1.28 \quad (\text{Eqn 9-4})$$

$$W = 64 + \left(-36 \times 1.28 \left(\frac{50}{50 - (-3)} \right) \right) = 60.38 \quad (\text{Eqn 9-11})$$

Where $W = 60.38$, $k = 50$

$$h_{60.38} = \frac{60.38}{50} = 1.21$$

Note that $k = 50$, $m = \pm 3$, so that $k/(k \pm m)$ is close to 1 (0.94 or 1.06), thus the effort to find the various heights from GL may not be worthwhile. The calculations for W_n follow the formula. shows the cross section of the dam where the ground slope intersects the bund batters.

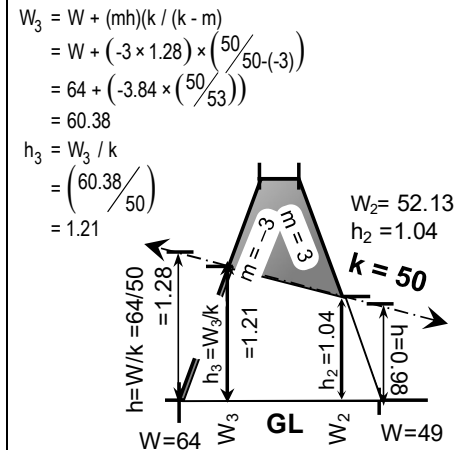


Figure 9-57 W_3 and h_3 derived from $W=64$, the batter toe.

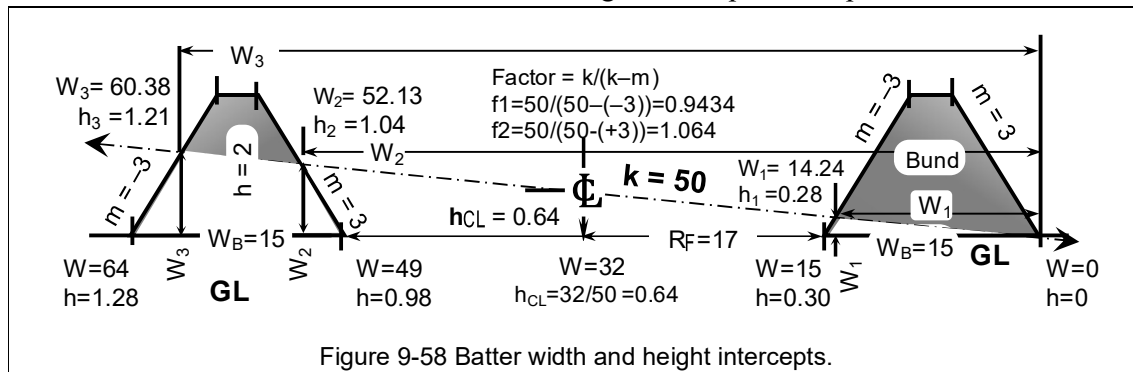


Figure 9-58 Batter width and height intercepts.

Table 9-7 shows the batter intercepts and heights above the level base.

9.4.5.5 Bund volume by annulus, slicing into lifts

Strictly, an annulus is the shape formed by two concentric circle in the form of a washer. Annular volume is then the volume of a tube. Assuming a lift of 0.2m for compaction, we will create a number of full and partial annular volumes to calculate bund volume. The method is similar to that used in § 9.3

Table 9-7 shows that above $h = 1.2$ the ring is fully formed to $h = 2.0$. Below $h = 1.2$ the ring is partial, and we must devise a method of calculation across the sloping site.

9.4.5.5.1 Volume of an annulus. End area

There are two methods of calculating volume of an annulus, one of which is better suited to our problem. We also need to know the area of each compaction lift as part of our construction estimate.

Find the area of each annulus, top and bottom, and multiply the mean area by the lift. Find the volume of the bund.

From Figure 9-59:

At GL: $h = 0$, $R_T = R_0 = 32$, $R_F = r_0 = 17$.

At $h = 2$; $R_W = R_2 = 23$, $R_C = r_2 = 26$.

$$\text{Area annulus} = \pi(R^2 - r^2) \quad (\text{Eqn 9-18})$$

$$A_0 = \pi(32^2 - 17^2) = 735\pi = 2309.1\text{m}^2 \quad \text{GL}$$

$$A_2 = \pi(26^2 - 23^2) = 147\pi = 461.8\text{m}^2. \quad h = 2\text{m}$$

$$V = \frac{1}{2}(A_0 + A_2)h = 1385.4 \times 2 = 2771\text{m}^3$$

The two plane rings are shown in Figure 9-60.

Compare this method, § 9.4.5.5.1, with § 9.4.5.5.2 below which will be used for the build-up of compaction layers of 0.2m. It gives the area of each layer.

9.4.5.5.2 Volume by rotation of cross section area at the centroid radius

The bund cross section is a symmetric trapezium. The centroid is near the centre of the cross-section figure, so for any design there is a common centroid radius. It is on the centreline of the crest because each batter is equal.

We are not interested in the y value because we are rotating about the y axis. (Shepherd 1983) and, as an example, Figure 9-61 shows a the centroid of a non-symmetric trapezium:

$$y = \frac{2a+b}{3(a+b)}h = 1.46 \quad \text{after Shepherd (1983, p345)}$$

$$x = \frac{b}{2} + \frac{(2a+b)(2t+a-b)}{6(a+b)} = 4.23$$

Table 9-7 Comparing values for W.

W	$h = W/k$		$W = mh \times (k/k \pm m)$	$h = w/m$
15	0.30	W_1	14.24	0.28
32	0.64	W_{CL}	32	0.64
49	0.92	W_2	52.13	1.04
64	1.28	W_3	60.38	1.21

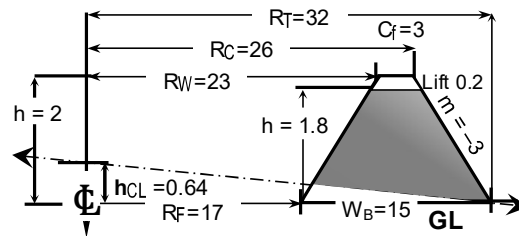


Figure 9-59 Lift section in a bund.

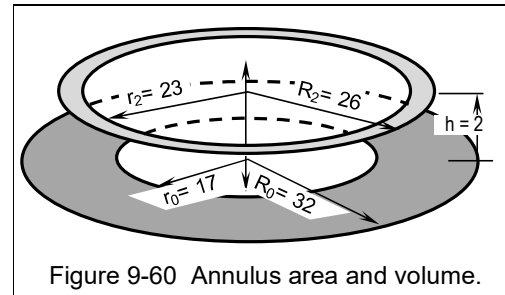


Figure 9-60 Annulus area and volume.

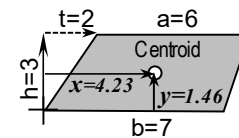


Figure 9-61 Trapezium centroid.

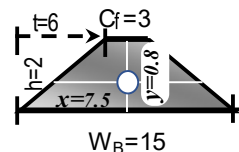


Figure 9-62 Symmetric trapezium

For example, with the bund profile being a symmetric trapezium, Figure 9-62 shows the symmetric centroid location.

Where $a = 3, b = 15, h = 2, t = 6$

$$y = \frac{2a+b}{3(a+b)} h = 0.78. \text{ Ignored.}$$

$$x = \frac{b}{2} + \frac{(2a+b)(2t+a-b)}{6(a+b)} = 7.5$$

$x = 7.5$ stands to reason because of the symmetric nature of the bund ($2t+a-b=0$).

With $R_F = 17$, the radius of centroid revolution, R_C , is: $R_C = 17 + 7.5 = 24.5$.

Circumference $= 2\pi R_C = 153.94\text{m}$.

Annulus area (trapezium),

$$A = \frac{1}{2}(a+b) \times h = ((15+3)/2) \times 2 = 18\text{m}^2.$$

$$\text{Volume} = 2\pi R_C \times A = 153.94 \times 18 = 2771\text{m}^3.$$

The circumference will stay the same for any height as the radius is constant as shown in Figure 9-63.

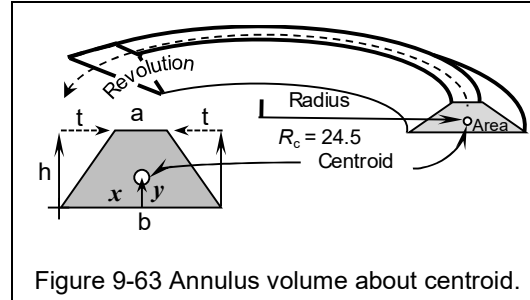


Figure 9-63 Annulus volume about centroid.

Table 9-7 and show that the annular area is over a complete circle above $h = 1.2$. Below this value the portion of area decreases to the ground level. At $h = 0.6$ less than half the wall will be fill.

Assuming a lift of 0.2m for compaction, the last lift for the bund will be from $h = 1.8$.

Find the bund radii for a given $h = 1.8$ above GL.

Where $R_T = 32, R_F = 17, \text{lift} = 0.2$,

$$h = 1.8, m = 3$$

$$R_{1.8} = R_T - mh = 32 - 5.4 = 26.6\text{m}$$

$$r_{1.8} = R_F + mh = 17 + 5.4 = 22.4\text{m}$$

$$C = \text{circumference} = 2\pi R_c = 153.94\text{m}$$

Where $R = 26.6, r = 22.4, h = l = 0.2$

$$b = 26.6 - 22.4 = 4.2\text{m}$$

$$a = b - 2mh = 4.2 - 2 \times 3 \times 0.2 = 4.2 - 1.2 = 3\text{m}$$

$$A = (b+a) / 2 \times l = (4.2+3) / 2 \times 0.2 = 0.72\text{m}^2$$

$$V = A \times C = 0.72 \times 153.94 = 110.84\text{m}^3.$$

9.4.5.6 Compaction layers, area and fill volume

Figure 9-58 shows that the upstream side of the bund intercepts the ground at about 1.2m . At the centre of the crest ($R=56.5$) the intercept, $h=1.1$. Thus, the full diameter of the bund will be built above this level, $h = 1.2$.

Below that level the bund runs into the surface slope and does not occupy a full circle. We revert to the method used in §9.3. This will involve calculating the apparent dip, α , for the lift height and then determining the dip deflection angle δ . We will use the same method of calculating the area of each segment for each lift. The lift will be in the form of a “truncated” annulus.

1. Find the radius at $h = 1$.

Where $R_T = 32, R_F = 17, h = 1.0, m = 3$

$$R_{h=1} = R_T - mh = 32 - 3 = 29\text{m}$$

$$r_{h=1} = R_F + mh = 17 + 3 = 20\text{m}$$

2. Find the apparent dips at $h = 1$. Figure 9-64. This is above the h_{CL} , so the slope is rising. There are two dips, one to the inside of the bund, the other to the outside. From Eqn 9-12:

$$k_{\Delta} = \left(\frac{R_{\Delta}}{h_{CL} - h_{\Delta}} \right) \text{ apparent dip at centre point}$$

$$\text{Where } R_T = 29, h_{\Delta} = 1.0, h_{CL} = 0.64$$

$$k_{R_T} = \frac{29}{0.64 - 1} = \frac{29}{-0.36}$$

$$k_{R=29} = -80.56 \text{ (} k \text{ rising)}$$

$$\text{Where } r_T = 20, h_{\Delta} = 1.0, h_{CL} = 0.64$$

$$k_{r_T} = \frac{20}{0.64 - 1} = \frac{20}{-0.36}$$

$$k_{r=20} = -55.56 \text{ (} k \text{ rising)}$$

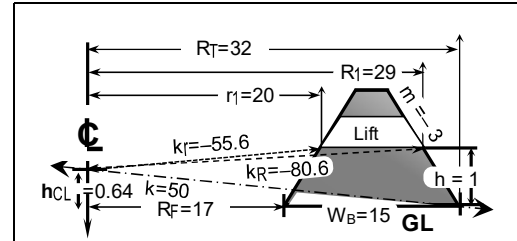


Figure 9-64 Example of rising apparent dips.

3. Now we must see how the game plays out with two slopes as we find the apparent dip deflection angles, δ . Derived from Eqn 9-10:

$$k_{\Delta} = \frac{\theta}{\cos \delta}, k_{\Delta} \text{ apparent dip } \alpha$$

$$\cos \delta = \frac{\theta}{k_{\Delta}}, \text{ apparent dip } (k_{\Delta}) \text{ deflection angle } \delta$$

$$\text{Where } k_{R=29} = -80.56, \theta = k_0 = 50$$

$$\cos \delta = \frac{50}{-80.56} = -0.6207$$

$$\delta_{R=29} = 2.24 \text{ radians}, \delta^{\circ} = 128.37^{\circ} (128^{\circ}22')$$

$$\text{Where } k_{r=20} = -55.56, \theta = k_0 = 50$$

$$\cos \delta = \frac{50}{-55.56} = -0.900$$

$$\delta_{r=20} = 2.69 \text{ radians}, \delta^{\circ} = 154.16^{\circ} (154^{\circ}10')$$

There are two different deflection angles arising from two different apparent dip angles. The dip angles differ because they use different radii to the same RL or lift level.

We now generate two segments, based on the two radii, and find the truncated annulus area by subtraction. Refer to § 9.3.1 for the proof.

4. Segment areas. Area of a segment from Eqn 9-15. Figure 9-65.

$$\delta_{R=29} = 2.24 \text{ radians}, \delta^{\circ} = 128.37^{\circ} (128^{\circ}22')$$

$$A_{R=29} = \frac{1}{2} 29^2 (2 \times 2.24 - \sin(2 \times 128.37)) = 2293.5 m^2$$

$$\delta_{R=20} = 2.69 \text{ radians}, \delta^{\circ} = 154.16^{\circ} (154^{\circ}10')$$

$$A_{R=20} = \frac{1}{2} 20^2 (2 \times 2.69 - \sin(2 \times 154.10)) = 1233.1 m^2$$

$$\text{Truncated annulus area} = 1060.4 m^2$$

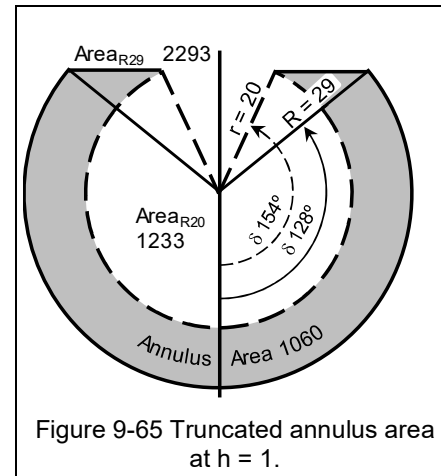


Figure 9-65 Truncated annulus area at $h = 1$.

9.4.5.6.1 Area of a partial annulus

The area of partial annulars only works when the lift is below certain limits of h . The value of h_{Δ} is used to calculate an apparent dip, α , through the equations:

$$k_{\Delta} = \left(\frac{R_{\Delta}}{(h_{CL} - h_{\Delta})} \right) \alpha \text{ from } h_{CL}$$

$$k_{\Delta} = \left(\frac{\theta}{\cos \delta} \right), k_{\Delta} : \text{apparent dip } \alpha$$

$$\cos \delta = \left(\frac{\theta}{k_{\Delta}} \right), \text{ dip deflection angle } \delta$$

at apparent dip k_{Δ}

The limiting factor on k_{Δ} , the apparent dip, is the fact that it cannot be greater than the true dip, θ . See Figure 9-66.

If $k_{\Delta} > \theta$ then $\cos \delta > 1$, which is an error. Table 9-8 shows the limits for this exercise.

1. h must be above 0.3m to calculate the downslope bund.
2. h between 0.3m and almost 1.0m can calculate the truncated annulus areas (max $h=1.04$)
3. h between 1.0m and 1.27m will allow only the area of the external segment to be calculated.
4. Above $h=1.21$ the areas are a full annulus.

In practical terms compaction and earthworks will start at GL. To get an idea of the theoretical numbers we can construct a table of areas and volumes working around the h limits. Note the incorrect k value (-53) for $r=30.2$ at $h=1.21$. I used the deflection angle of 170.3° which fits with $R = 28.4^{\circ}$. The theoreticals are meaningless near the full circle.

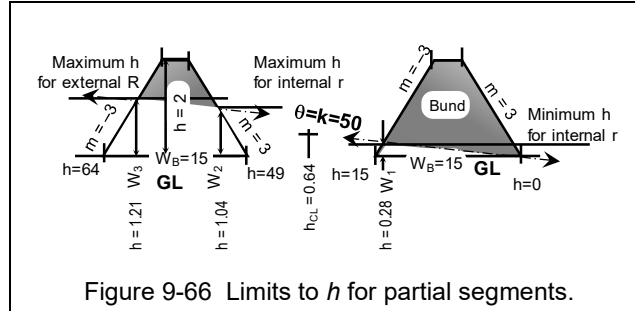


Figure 9-66 Limits to h for partial segments.

Table 9-8 Compaction lifts

h	dh	R	k	δ°	A	r	k	δ	A	Area	V
0		32	50	0	3217	0		0	0	0	0
0.3	0.3	31.1	92	56.8	517	17.9	52.6	18.2	7	510	77
0.5	0.2	30.5	218	76.7	1038	18.5	132	67.8	285	753	126
0.7	0.2	29.9	-498	95.8	1583	19.1	-318	99.0	687	896	165
1.0	0.3	29	-81	128.4	2293	20	-56	154.1	1233	1060	293
1.2	0.2	28.4	-51	170.3	2531	20.6	-37	170.3	1331	1200	226
Annulus		R	k	δ°	A	r	k	δ	A	Area	
1.4	0.2	27.8		180		21.2		180		1016	222
1.6	0.2	27.2		180		21.8		180		831	185
1.8	0.2	26.6		180		22.4		180		646	148
2.0	0.2	26		180		23.0		180		462	111
									Totals	7374	1553

9.4.6 Lining the Dam (frustum) Wall

The estimate of required storage accounts for seepage through the wall. Proper sealing of the interior will reduce or eliminate this problem. Environmental policies may require sealing the dam to prevent contamination.

The basic design parameter was the provision of about 1750m^3 storage (§ 9.4.5.1). The excavated (storage) volume for the given design parameters was 1670m^3 (§ 9.4.5.3) and this

would be used to construct the bund wall to a height of 1.5m. The preceding calculations, Table 9-8, on a sloping site, show the excavation would provide enough material for a 2m bund.

Allowing for a 1m freeboard, the 2m bund ($h=1$) could support an extra 1000m^3 water.

Where $R_F = 17$, $h = 1$, $m = 3$

$$r_{h=1} = R_F + mh = 20$$

$$V_{h=1} = \frac{\pi}{3} h (R^2 + Rr + r^2) = 1077\text{m}^3.$$

This then takes the capacity to $1671 + 1077 = 2750\text{m}^3$, which greatly exceeds our 3 year, 250mm rainfall, design storage requirement (1750m^3) in § 9.4.4.

Note that part of the excavation preparation is over 3200m^2 (at $h = 0$, $R = 32$) of stripping overburden to bare soil. This would need to be moved out to a radius of 35m - 40m, to be spread back over the exterior wall as part of the site remediation program.

9.4.6.1 The equation for the frustum surface area

The wall of a cylinder has an area of $A = 2\pi rh$. A frustum has a surface area of $2\pi d(R+r)/2$ where d is the length of the side, not the height, h of the cylinder. From Eqn 9-17 the interior lining area (Figure 9-67) is:

$$A_{fs} = \pi(R+r)\sqrt{(R-r)^2 + h^2}$$

For $R = 23$, $r = 5$, $h = 6$ (depth)

$$A_{WALL} = \pi(28)\sqrt{(18)^2 + 6^2} = 1670\text{m}^2$$

Line the floor, $r = 5$

$$A_{FLOOR} = \pi r^2 = 78\text{m}^2$$

$$\text{Lining area} = 1750\text{m}^2$$

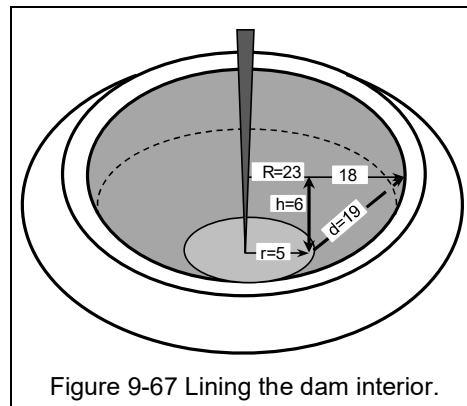


Figure 9-67 Lining the dam interior.

9.4.6.2 Clay and Bentonite Lining

Say 1750m^2 surface area. With a clay lining of 300mm (0.3m) the compacted volume is about 525m^3 . This would mean access to 700-800 m^3 of a suitable clay borrow pit.

Bentonite can be mixed with 150-200mm good topsoil at about 7Kg/m^2 or applied as a pure blanket at 10Kg/m^2 and covered with 100mm good topsoil. If livestock access the site, the cover should be 450mm. Some 12-18 tonnes of bentonite are needed. Bulk bentonite costs \$160-\$200 tonne.

At least we are now able to make some judgements on completing the dam job.

As a "roughie", how much overburden do we need to rehabilitate the external wall? Say the average height of the crest above ground is 1.32m at the centre of the dam. Then the radius $R = 26 + (3 \times 1.32) = 30\text{m}$. See Figure 9-68.

$R = 26$, $r = 30$, $h = 1.32$ and from Eqn 9-17:

$$A_{EXTERIOR} = 750\text{m}^2.$$

Overburden stripping before excavation covered an area of 3200m^2 , so there is adequate material for rehabilitation.

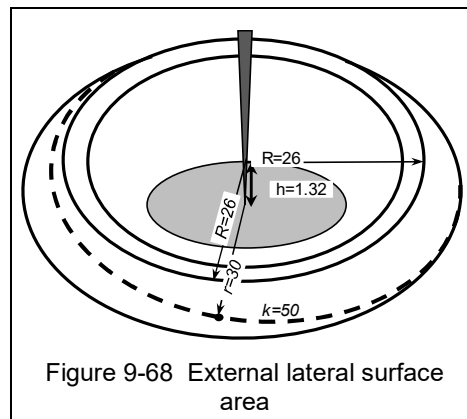


Figure 9-68 External lateral surface area

9.4.6.3 Lining with geofabric material

Complete sealing of the dam, e.g. to stop seepage, is generally accomplished using an impervious sheet of geotextile material as a liner. Information available from Australian geomembrane suppliers, the Department of Civil & Engineering Environment at the University of Illinois at Urbana-Champaign and the Fabricated Geomembrane Institute (FGI) indicate that quite detailed base preparation is needed for successful liner installation.

Installation involves good subgrade slope preparation, with suitably graded and compacted base material below 10mm (3/8") diameter about 150mm deep.

The liner must be anchored to the top of the bund by trenching, this involves about 2m extra radius to the liner size. For the above model the anchor requires another 170m² of liner. This takes the total area to about 1900m² to seal the dam. Supply cost of 1mm (40mil) flexible liner is approximately \$10/m².

9.4.7 Setting a Dam Out. The Survey Aspects

All earthworks jobs, as with any survey work, rely on a stable, recoverable set of controls. The dam will be built on a site that does not need any tie to a national coordinate system. **The only requirement would be for it to be located within a cadastral boundary.**

The starting point is the centre of the dam. From there we can radiate out to the wall toe radius. Radial extension from the centre can then provide recoverable control points that allow quick reestablishment of inner and outer radii and levels as the job progresses. A radiation every 30° at a radius of 40m have a chord of 21.7m. A second set of radial points at, say, 50m will provide recoverable radial lines back into the dam during the job. Construction points in the job site will occasionally need to be re-established.

Most, if not all, survey can be carried out with an automatic level and 50m tape. And a whole heap of pegs, stakes and flagging tape. A total station and prism pole may be handy for referencing the centre to more distant objects, such as fence-lines, boundary corners etc.

9.4.7.1 Initial measurements

After setting the level up at the centre, the first task is to find the true dip direction.

1. Point the level to what looks like upstream. Set the compass plate on the level to 0°.
Have your assistant traverse around you at, say, 30m and record the level reading and direction either side of your zero. The lowest staff reading will be the dip direction. Mark it. Check downstream for the highest reading and mark it. The two marks should align through the level. This should be your true dip direction.
Tip. Read the 3 wires. (Top – bottom) × 100 = distance. Mid = RL. BS – FS = fall.
Dip, θ , = fall/(Dist_{BS}+Dist_{FS})
2. Reset your level plate direction to zero to the upstream peg. Place a peg (for RL) and a stake at, say 40m and another stake at 50m. This is your alignment to the centre of the dam. Read the RL of the peg.
3. Work around the centre at 30° intervals. Stake and peg at 40m, stake at 50m. Read RL.
4. The design called for a downstream toe at 32m. Set a peg at 32m downstream. Read RL.
This peg will be the first to go, but it defines your dam RL.

In one setup you have established the dip direction (180°), the RL of the downstream toe, the recoverable centre of the dam and RLs of the reference points at 30° around the dam.

You have only used a level, staff and 50m tape. Easy.

It may be a good idea to establish some control points a bit further away. They can be coordinated using a handheld GPS. Take both the UTM and lat/long coordinates. Also use the GPS on the dam centre. Bearing and stadia distance as well as RL can be obtained to sufficient accuracy.

9.4.7.2 Toe distances and levels on the sloping site

What are the RLs of points around the outside of the dam wall, the toe, when it is located on a sloping site?

We have decided the downstream to, the low point, has an RL of zero. The RL of the 30° steps around the embankment toe can be staked and ground levels taken.

The result of our exercise example shows the toe radii and theoretical RLs in Table 9-9, and are illustrated in Figure 9-70.

1. Find apparent dip, $k\Delta$, at dip deflection angles, δ .

True dip, θ , $k_0 = 50$, dip $\delta = 60^\circ$, $h = 2$

$$k_\Delta = \frac{\theta}{\cos \delta} = \frac{50}{\cos(60)} = 100$$

2. Find radius, R_Δ , to toe at apparent dip, $k_\Delta = 100$.

Where $R_{CREST} = 26$, $R_{TOE} = 32$, $m = 3$

$$h_{CL} = h - \frac{R_{TOE}}{\theta} = 1.36$$

$$R_\Delta = (R_{CREST} + mh_{CL}) \left(\frac{k}{k-m} \right)$$

$$R_{\Delta=60} = (26 + 3 \times 1.36) \left(\frac{100}{97} \right) = 31.0m$$

3. Find RL_{TOE} from R_Δ and m . Ground $RL=0$.

$$RL_{TOE} = h - \left(\frac{(R_\Delta - R_{CREST})}{m} \right)$$

$$RL_{TOE} = 2 - \left(\frac{(31 - 26)}{3} \right) = 0.33m$$

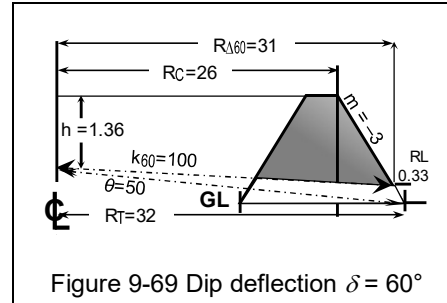


Figure 9-69 Dip deflection $\delta = 60^\circ$

Table 9-9 Initial dam set-out.

Initial setout.					
$R_{toe} = 32$, $R_{crest} = 26$, $h_{crest} = 2$, True dip, $k = 50$, $m = 3$, Dip dir'n = 180° , $h_{CL} = 1.36$					
δ	Δ°	$k\Delta$	R_{TOE}	RL_{TOE}	
0	180	50	32.0	0.00	
30	150	57.7	31.7	0.09	
60	120	100	31.0	0.33	
90	90	10000	30.1	0.64	
120	60	-100	29.2	0.93	
150	30	-57.7	28.6	1.14	
180	0	-50	28.4	1.21	

9.4.8 Dam Control and Survey Set-out

Figure 9-70 shows a combination of survey control using offset pegs and stakes and the exterior toe line as proposed in §9.4.7.

The main purpose of the offsets is to allow quick re-establishment of any construction control points as the job progresses.

9.4.8.1 Running the job

The best helpers that an engineer has are a good dirt-working foreman and experienced machine operators. They can work with minimum of set-out once the job boundaries have been defined. Apart from proper compaction of the lifts (to maintain optimum soil density) the other task is nicely graded banks at the correct inclination.

A slope of 1V:3H ($m = 3$) is generally accepted for agricultural dam walls.

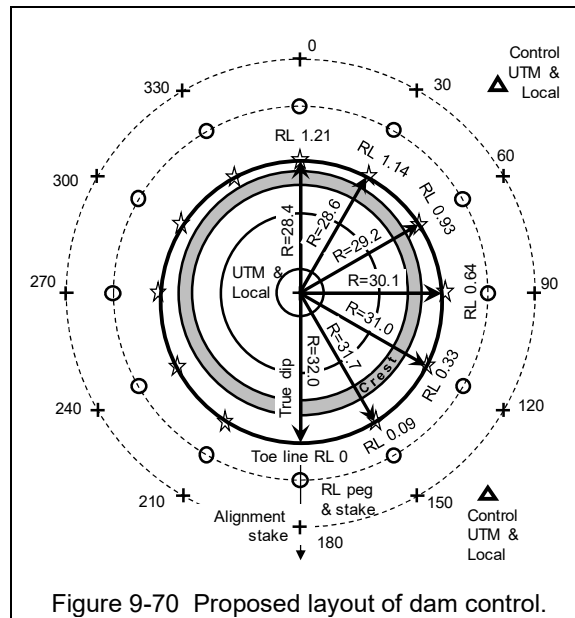


Figure 9-70 Proposed layout of dam control.

9.4.8.2 Check measurements of slope utilise a method of in-situ measurement

Set a level up on the crest, find the RL of a point on the edge of the wall in line with your set-out stakes and measure the distance from the control peg to the test point. Knowing the batter design, the RL of the point can be compared with the design. Corrections can then be issued.

An automatic level, staff and tape are all that's needed. The use of a TS is OK if you ensure the vertical circle is locked at a ZA of 90°. A staff is essential. A tape is helpful, and quicker than EDM for set-out.

In Figure 9-71 a BS is taken to a control RL peg. Use the HC method of reduction.

BS	IS	FS	RL	
			0.50	Peg
2.20			2.70	HC
		1.50	1.20	P1

$Dist\ P1 = 40 - 12.5 = 27.5$

$Off\ set\ from\ crest\ (R_c = 26) = 27.5 - 26 = 1.5\ Fall = 1.5/3 = 0.5$

$Design\ RL = 2 - 0.5 = 1.5.$

$P1 = 0.3\ low.$

P1 is 12.5m from the 40m peg. It is thus 27.5m from the CL. The crest radius is 26m.

Thus, P1 is 1.5m from the crest. Its RL should be $RL_{crest} - 1.5/m = 2 - 1.5/3 = 1.5$.

The $RL_{P1} = 1.2$. It is 0.3m low.

Put a stake at the point, lift the staff till it reads $2.7 - 1.5 = 1.2m$ (or measure up 0.3). Mark the level, put some flagging on the mark and make a note, "Fill 0.3" on the stake. Don't be too precious about stakes. They get bowled over all the time.

Batter boards and slope rails may be used to control the slope. They are adequately explained in your Civil Engineering Surveyor's textbooks. (Uren and Price 2010, p484) (Schofield and Breach (2007, p503).

Never upset the machine operators. They are your friends, and the only ones who can make your job look good.

Here endeth the lesson.

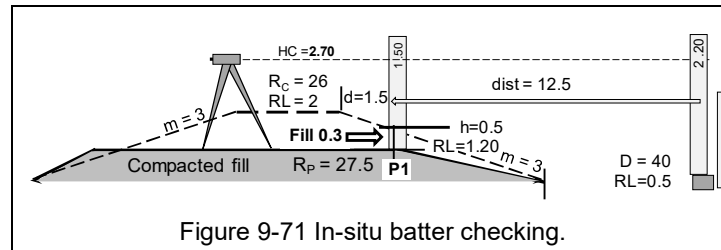


Figure 9-71 In-situ batter checking.

9.4.9 References

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10 Chapter 10 Circular Curves

10.1 Introductory Remarks

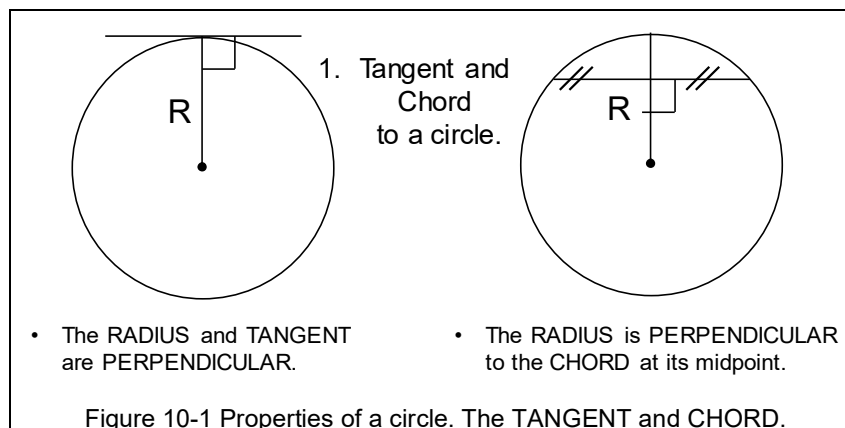
At the end of this Chapter and having gone through the workshop and field practical materials, you should:

- Appreciate the geometric properties of a circular curve.
- Differentiate between different types of horizontal curves.
- Design curves of constant radii to join straight sections of roads and railways.
- Understand the base mapping necessary to assist an engineer in selecting a road or rail alignment.
- Setting out circular curves using either chord length and deflection angle; offset from the long chord; or coordinates methods.
- Understand how simple curves can be combined as compound and reverse curves.
- Horizontal Curves are made up of circular and transitional curves. They are used to:
 - ✓ ensure that vehicles travel safely from one section of the road or track to another.
 - ✓ Achieve horizontal alignment, i.e., the combination of horizontal curves and straight used in design.
- Circular curves have uniform radius.
- Transition curves have varying radius and are used:
 - ✓ To introduce radial force gradually and uniformly in order to minimize passenger discomfort
 - ✓ To gradually introduce super elevation, which involves raising one of the road channels relative to the other and enabling the effect of the radial force to be reduced.
 - ✓ In conjunction with circular curves to form composite curves or in pairs to form wholly transitional curves.

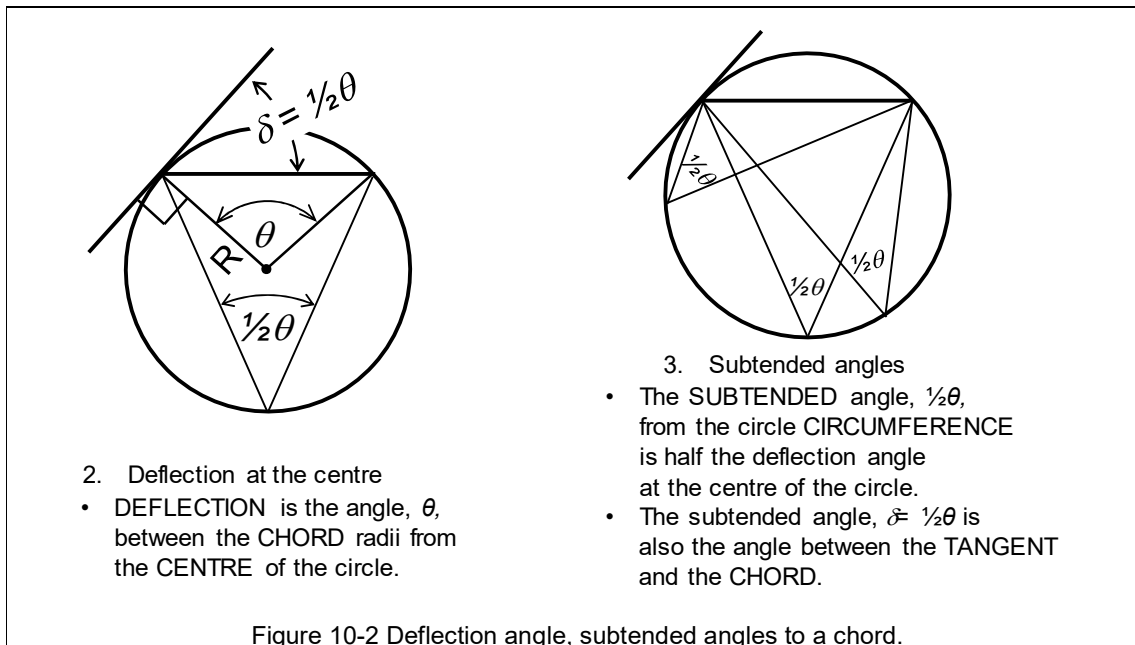
Circular curves are used in

- Used in Road and Rail alignment.
- A straight road would be ideal but terrain constraints, e.g., hills, buildings, rivers, etc. do not permit.
- Horizontal alignment thus joins the respective straight lines (called tangents). Alignments are chosen on digital maps which show contours, natural surface features and soil types.
- In very hilly country, alignments may be completely made up of curves (circular and transition).

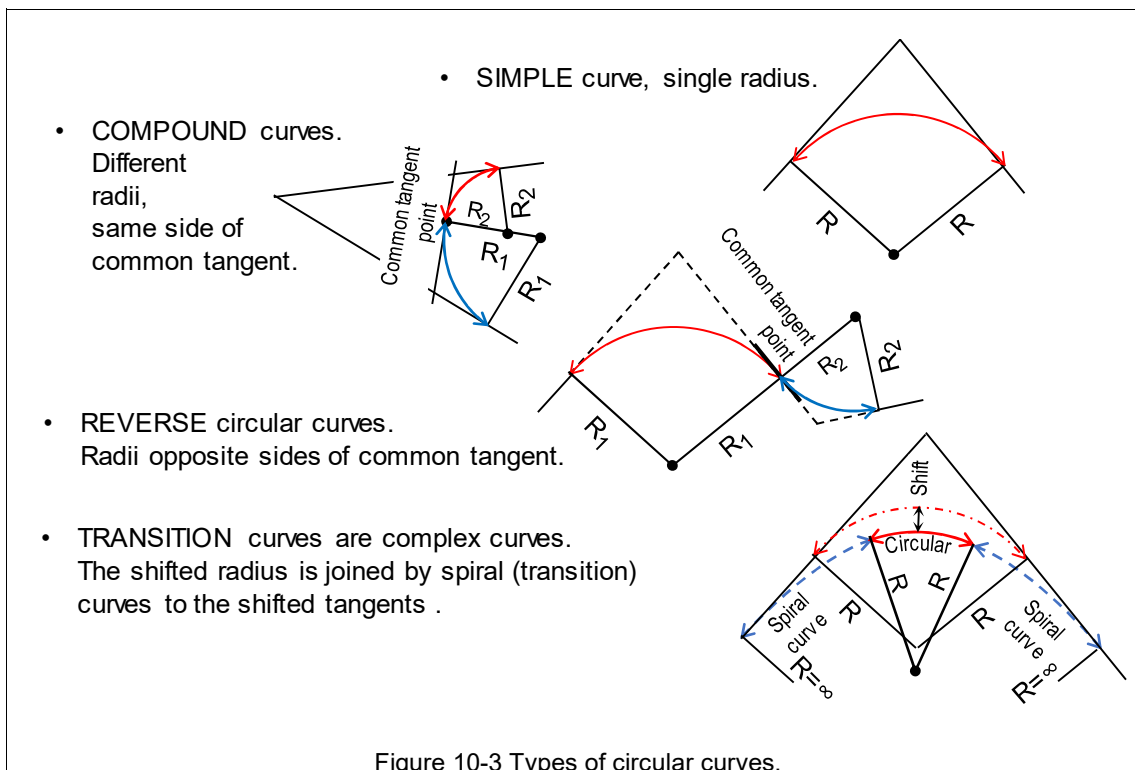
The **tangent** and **chord** properties of a circle are shown in Figure 10-1.



The **deflection** and **subtended angles** properties of a circle are shown in Figure 10-2.



Types of circular curves are shown Figure 10-3.



10.2 Generating a Circular Curve - A Process of Visualisation

The circular curve is the simple figure that joins two straight lines together. It is a thing of beauty once it has been laid out in the field. But it is also a bit of a mystery in its generation, until the basics are mastered. The development of the circular curve is best illustrated by a build-up of the elements of the curve.

10.2.1 Basic Formulae. Arc length

All CIRCULAR calculations are carried out in RADIANs. $360^\circ = 2\pi$ (6.2832) radians.

$Arc, S = R\theta^{radians}$	Eqn 10-1

Figure 10-4 shows a simple curve of given RADIUS, R , joins two intersecting straight lines.

The straight lines differ in direction by a DEFLECTION angle, Δ (delta) degrees, at the INTERSECTION POINT (IP).

The curve has a CENTRE at the intersection of two RADII that are PERPENDICULAR to the two straight lines and are TANGENT POINTS to the curve. The radii are PERPENDICULAR to the straight lines AT the TANGENT POINTS.

The DEFLECTION angle, θ (theta), at the centre of the arc is equal to the deflection angle, Δ , at the intersection point (IP).

The LENGTH, S , of the ARC between the two tangent points, TPs, is given by the expression: Eqn 10-1
 $S = R\theta^{radians}$

The ingoing curve starts at the tangent point, the POINT OF CURVATURE, PC, and its outgoing ends at the other tangent point, the POINT OF TANGENCY, PT.

Example:

Radius, $R = 200$

Deflection angle at IP, $\Delta = 80^\circ$

then $\theta = 80 \cdot \pi/180 = 1.3963$ radians

Arc, $S = 279.253$

10.2.2 The Tangent Distance

The line O-IP, joining the IP and the centre, O, bisects the deflection angle, Δ .

The triangles O-PC-IP and O-PT-IP are equal.

The length of the straight line from the PC to the IP and IP to PT is the TANGENT DISTANCE, TD.

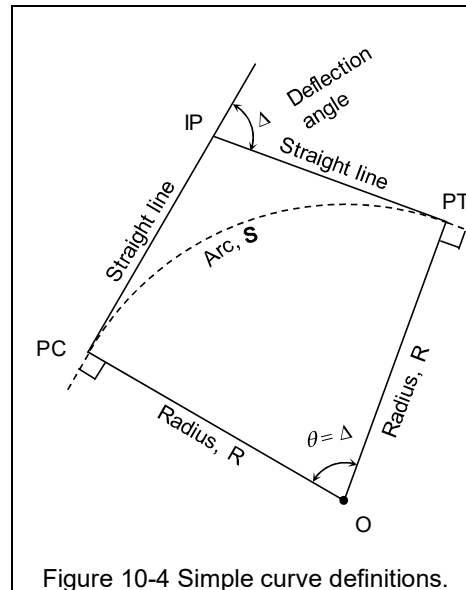


Figure 10-4 Simple curve definitions.

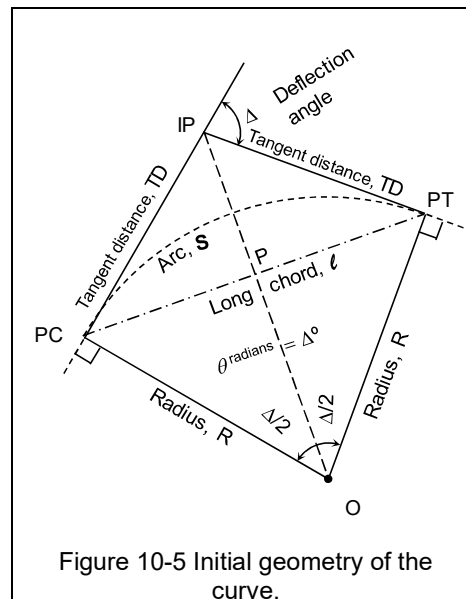


Figure 10-5 Initial geometry of the curve.

From plane trigonometry, the TD is the opposite side, and the radius, R , is the adjacent side of the triangle PC-O-IP from the half deflection angle $\Delta/2$.

In Figure 10-5 and Figure 10-6, in triangle O-PC-IP,

$\tan\left(\frac{\Delta}{2}\right) = \frac{opp}{adj} = \frac{TD}{R}$ $TD = R \tan\left(\frac{\Delta}{2}\right)$	Eqn 10-2

From Eqn 10-2:

$$TD = R \tan(\Delta/2) = 200 \tan(40^\circ) = 167.820$$

The two tangent distances are equal:

$$PC \text{ to } IP = IP \text{ to } PT.$$

10.2.3 The Long Chord

The LONG CHORD, Figure 10-5, from PC to PT is made up of two half chords, PC – P, and P – PT.

As shown in Figure 10-7, in the triangle O-PC-P

$\sin\left(\frac{\Delta}{2}\right) = \frac{opp}{hypot} = \frac{\ell/2}{R}$ $\text{Long chord, } \ell = 2R \sin\left(\frac{\Delta}{2}\right)$	Eqn 10-3

The long chord, length ℓ , joining PC to PT: Eqn 10-3

$$\ell = 2R \sin(\Delta/2) = 400 \sin(40^\circ) = 257.115$$

The ANGLE between the TANGENT STRAIGHT and the CHORD at the tangent point is the CHORD DEFLECTION ANGLE, labelled δ (delta), and is half the TANGENT deflection angle. $\delta^\circ = \Delta^\circ/2$.

Notice in Figure 10-8, the difference between the arc length, S , and the chord, ℓ .

$$\text{Arc length, } S = 279.253$$

$$\text{Long chord } \ell = 257.115$$

10.2.4 Other Chords

The other chords we are concerned with join any two points on the arc of circle radius R . Consider first, the chord starting at the PC.

Thus, in , any point, p , on the arc at an arc distance, s , from the tangent point, is tangential to the arc at p towards the centre, O .

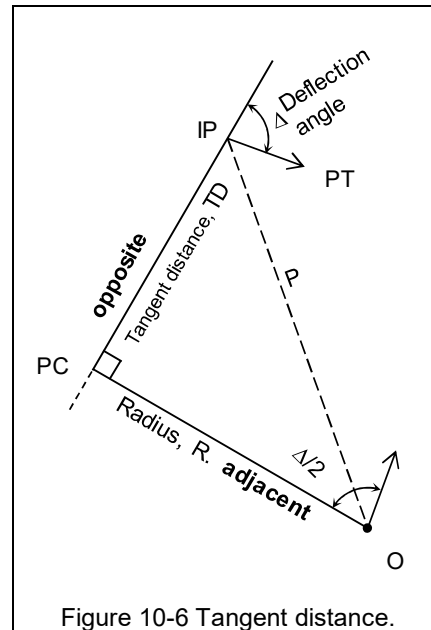


Figure 10-6 Tangent distance.

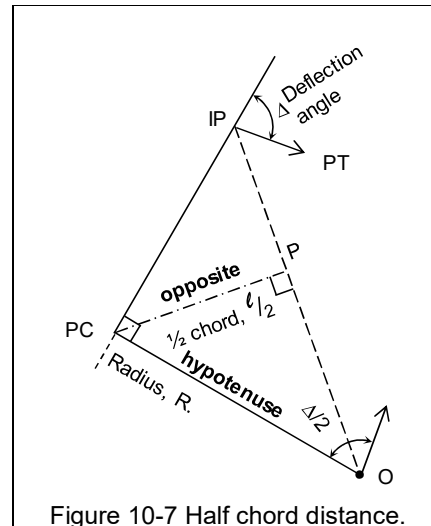


Figure 10-7 Half chord distance.

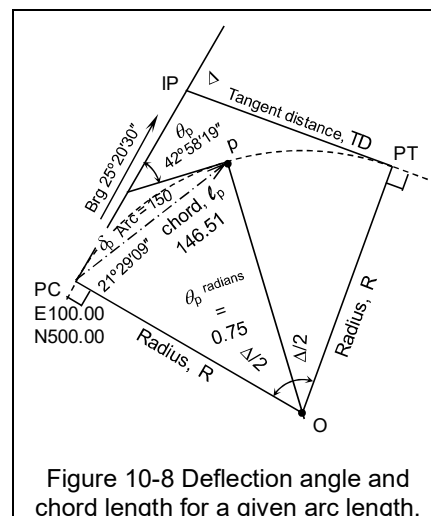


Figure 10-8 Deflection angle and chord length for a given arc length.

10.2.5 The Chord Deflection Angle

It follows that, from Eqn 10-1, then the TANGENT deflection:

Tangent deflection angle:	$\theta_{p(\text{radians})} = \frac{S_p}{R}$	Eqn 10-4

and the angle, in degrees, is:

Radians to degrees:	$\Delta_p^\circ = \theta_p \frac{180}{\pi}$	Eqn 10-5

The CHORD deflection from the tangent point to **p** is *half* the tangent deflection angle:

Chord deflection angle:	$\delta_p^\circ = \frac{\Delta_p}{2}$	Eqn 10-6

and the chord DISTANCE is (from Eqn 10-3):

$$\ell_p = 2R \sin(\delta_p^\circ)$$

From these equations we can now start to find some solutions to common circular curve problems.

10.2.5.1 Worked example 10.1

Suppose point *p* is 150m around the curve from the PC, Figure 10-8.

Find coordinates of *P*.

arc length, s = 150

s_p = 150, *R* = 200:

remember, as *S_p* = *Rθ_p*, then *θ_p* = *s_p*/*R*

$$\theta_p = 150/200 = 0.75 \text{ radians}$$

$$\Delta_p = 42.972^\circ = 42^\circ 58' 19''$$

$$\delta_p = 21.486^\circ = 21^\circ 29' 09'' \text{ (half } \Delta)$$

$$\begin{aligned} \ell_p &= 2R \sin(\delta_p) = 400 \sin(21^\circ 29' 09'') \\ &= 146.509 \end{aligned}$$

The BEARING of the CHORD at the tangent point is the BEARING from the PC to the IP PLUS the TANGENT deflection angle, *δ_p*.

$$\text{BRG}_{\text{PC} - \text{IP}} = 35^\circ 20' 30''$$

$$\delta_p = 21^\circ 29' 09''$$

$$\text{BRG}_{\text{PC} - p} = 56^\circ 49' 39''$$

$$\ell_p = 146.509$$

The vector between the PC and *p* can be converted to rectangular coordinates:

$$\delta E_p = \ell_p \sin(\text{BRG}),$$

$$\delta N_p = \ell_p \cos(\text{BRG})$$

If the coordinates of the PC are known, then: *E_p* = *E_{PC}* + *δE_p* and *N_p* = *N_{PC}* + *δN_p*.

Note that the points along the curve RADIATE from the PC.

All the calculated *δE_s* and *δN_s* are added to the PC coordinates.

Find coordinates of *p* from PC E100.00, N500.00

$$\text{BRG}_p = 56^\circ 49' 39'', \ell_p = 146.509 \text{ converts to } \delta E_p = 122.632, \delta N_p = 80.16$$

$$E_p = 222.632$$

$$N_p = 580.164$$

10.2.6 Chainage

In a lot of survey work the word chainage is used to express a linear distance from some start point, as opposed to a distance between two points (see). Usually, they are the actual distance from the start point of the road or railway to points along the development. In the initial design of the curve several CHAINAGE values will be determined. These chainages generally apply from the start of the project along the straight lines defining path of the project.

In Figure 10-10, the chainage increases from Z along the straight to point TP1 where the circular curve begins. The chainage can either:

- (1) Increase along the straight to the intersection point (IP) This is used only for initial road set out before curves are designed

$$CH_{IP} = CH_{TP1} + TD.$$

- (2) Increase along the curve to TP2 - Used for road construction.

$$CH_{TP2} = CH_{TP1} + S.$$

In curve design this means that chainages will be determined for each of the IPs. When the curve radii are determined then the THROUGH CHAINAGE of the straights and curves will be calculated.

Chainage intervals at set distances, depending on design, will be used (e.g. every 50m). Figure 10-11 shows CH_{PC} , tangent distance, and arc length:

Chainage PC = chainage IP minus the tangent distance.

$$\begin{aligned} CH_{PC} &= CH_{IP} - TD. \\ &= 1496.27 - 167.820 \\ &= 1328.450 \end{aligned}$$

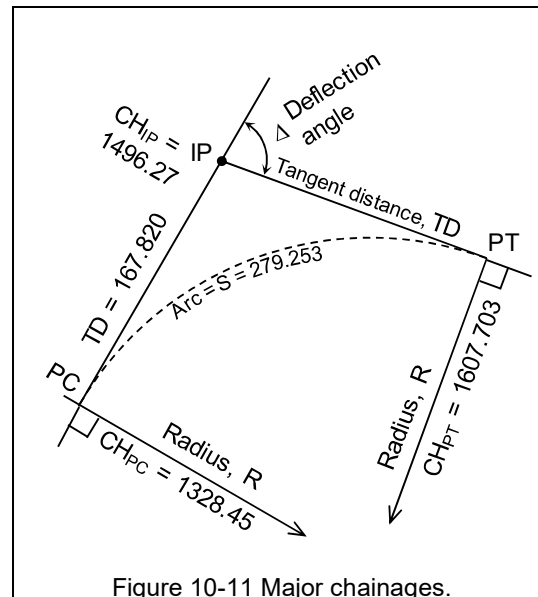
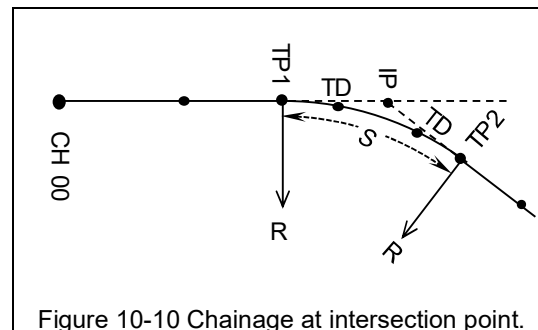
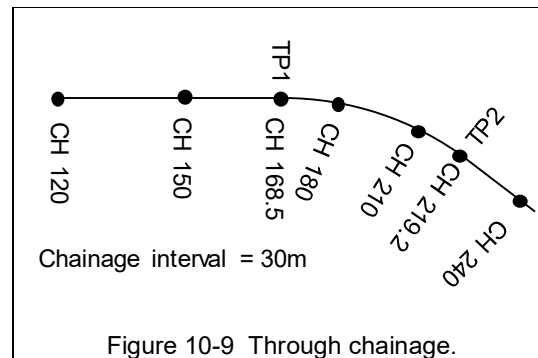
The through chainage of the next point on the curve will be a multiple of the chainage interval, say 50m, = CH1350.

The chainage PT = chainage PC plus the arc length, S.

$$\begin{aligned} CH_{PT} &= CH_{PC} + S \\ CH_{PT} &= 1328.450 + 279.253 \\ &= 1607.703. \end{aligned}$$

Chainage PT does NOT equal $CH_{IP} + TD$.

Other than chainages defining changes in direction, i.e. at the PC and PT, all the other calculations on the curve are generally carried out at defined chainage intervals.



Thus, in Figure 10-12, with a chainage interval of 50m, the points on the curve will be marked by even 50m chainages around the arc from the PC to the PT. The through arc distances, from the PC, can be tabulated for calculation of chord deflection and distance:

$$\text{CH}_{\text{PC}} = 1328.450,$$

the next points, until $\text{CH}_{\text{PT}} = 1607.703$, are:

$$\text{CH}_{1350} = 1350.000 \quad \text{s} = 21.550$$

$$\text{CH}_{1400} = 1400.000 \quad \text{s} = 71.550$$

$$\text{CH}_{1450} = 1450.000 \quad \text{s} = 121.550$$

$$\text{CH}_{1500} = 1500.000 \quad s = 171.550$$

$$\text{CH}_{1550} = 1550.000 \quad \text{s} = 221.550$$

$$\text{CH}_{1600} = 1600.000 \quad \text{s} = 271.550$$

$$\text{CH}_{\text{PT}} = 1607.703, \quad s = 279.253.$$

This is the arc length from PC to PT.

It is a check of your maths.

A table (Table 10-1) of deflection angles, bearings and chord lengths, coordinate differences, δE_p , δN_p , and coordinates, E_p , N_p can now be compiled using the derived formulas:

Initial bearing, PC-IP = $35^{\circ} 20' 30''$.

radius $R = 200\text{m}$,

deflection angle, $\Delta = 80^\circ$,

chainage interval 50m.

Check: Deflection angle from the PC - PT, $\delta = 40^\circ = \frac{1}{2}\Delta$.

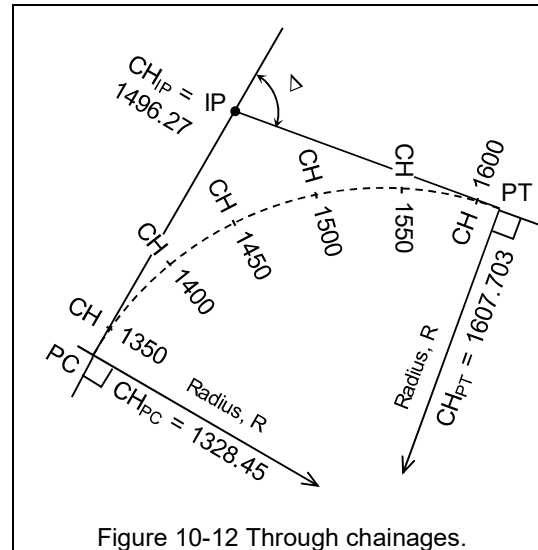


Figure 10-12 Through chainages.

Table 10-1 Solution of curve setout.

Chainage	s	$\theta_p = s_p/R$	$\delta_p = \Delta/2$	Brg ℓ_p	$\ell_p = 2R \cdot \sin(\delta_p)$	δE_p	δN_p	E_p	N_p
CH _{PC}	0.000	0.0	0 00 00	35 20 30	0.00	0.00	0.00	100.00	500.00
CH ₁₃₅₀	21.550	0.108	3 05 12	38 25 42	21.54	13.39	16.87	113.39	516.87
CH ₁₄₀₀	71.550	0.378	10 14 56	45 35 26	71.17	50.84	49.80	150.84	549.80
CH ₁₄₅₀	121.550	0.608	17 24 39	52 45 09	119.69	95.28	72.44	195.28	572.44
CH ₁₅₀₀	171.550	0.858	24 34 22	59 54 52	166.34	143.93	83.38	243.93	583.38
CH ₁₅₅₀	221.550	1.108	31 39 21	66 59 51	209.93	193.24	82.03	293.24	582.03
CH ₁₆₀₀	271.550	1.358	38 53 48	74 14 18	251.17	241.73	68.23	341.73	568.23
CH _{PT}	279.253	1.396	40 00 00	75 20 30	257.12	248.75	65.06	348.75	565.06

$$\text{Distance PC} - \text{PT} = \ell_p = 2R\sin(\frac{1}{2}\Delta) = 257.12.$$

10.2.7 In, Through and Out Arcs. Setout and Checking Tools

Calculating CHORDS; for the first arc (in arc), the chaining interval (through arc), and the last arc (out arc), allow checking of the curve set-out. On long curves the through chainage deflection angle and chord is required to advance the Total Station set-up stations. Refer Table 10-1:

$$\begin{aligned} \text{In arc} &= 1350 - 1328.45 = 21.55 \\ \delta &= \frac{1}{2}(21.55/200)^R = 0.054^R = 3^\circ 08' 12''. \end{aligned}$$

$$\text{in chord} = 2R\sin(\delta), \quad = 21.54$$

Through arc = Chainage interval = 50
 $\delta = \frac{1}{2}(50/200) = 0.125^R = 7^\circ 09' 43''$
 through chord = 49.87

$$\begin{aligned} \text{Out arc} &= 1607.703 - 1600 = 7.703 \\ \delta &= \frac{1}{2}(7.703/200) = 0.019^{\text{R}} = 1^{\circ} 06' 12'' \\ \text{out chord} &= 7.702 \end{aligned}$$

10.2.8 Calculating a Circular Curve: Definitions and Equations

The simple curve.

- 1) ASB is a simple curve of radius R connecting two straights EA and BF (See Figure 10-13)
- 2) A and B are the tangent points and the straights are produced to meet in D (the intersection point or point of intersection, PI or IP).
- 3) AD, BD are tangents to the curve and AD = BD.
 Δ is the deflection angle.
- 4) The deflection angle, Δ , equals the angle at the centre, $\angle ACB$
- 5) Triangles ADC and BDC are congruent and $\angle ACD = \angle BCD = \frac{\Delta}{2}$
- 6) Point S, on arc ASB, has a deflection angle $\angle DAS = \delta$, which is half of angle $\angle ACS = 2\delta$.

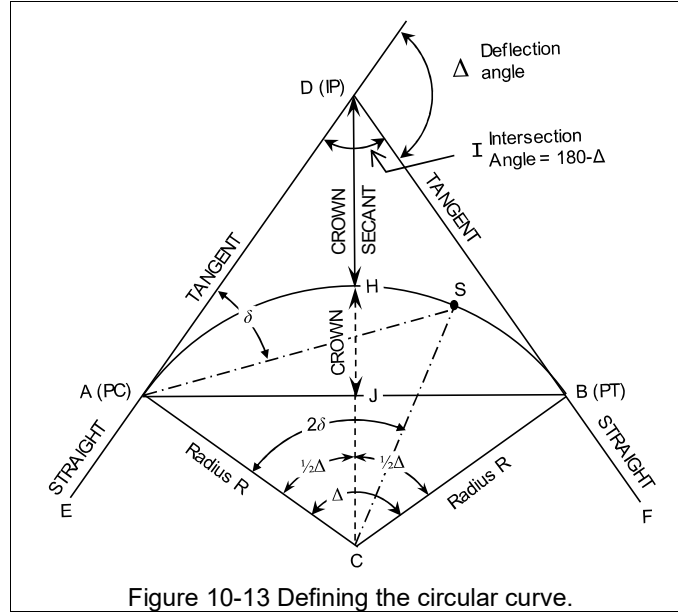


Figure 10-13 Defining the circular curve.

By definition:

The **DIRECTION** of the **RADIUS** is **PERPENDICULAR** to the **TANGENT**.
Brg Radius = Brg Tangent $\pm 90^\circ$

Tangent Distance AD	$TD = R \tan \frac{\Delta}{2}$	Eqn 10-7
Radius (R)	$R = TD \cot \frac{\Delta}{2}, \left(\cot = \frac{1}{\tan} \right)$	Eqn 10-8
Arc length AB (S)	$S = R\Delta^{\text{radians}}$	Eqn 10-9
Secant Distance DC	$DC = R \sec \frac{\Delta}{2}, \left(\sec = \frac{1}{\cos} \right)$	Eqn 10-10
Crown Secant DH	$DH = R \sec \frac{\Delta}{2} - R = \left(R \left(\sec \frac{\Delta}{2} - 1 \right) \right)$	Eqn 10-11
Crown Distance JH	$JH = R - R \cos \frac{\Delta}{2} = \left(R \left(\cos \frac{\Delta}{2} - 1 \right) \right) = R \text{ versine } \frac{\Delta}{2}$	Eqn 10-12
Long Chord AB	$AB = 2R \sin \frac{\Delta}{2}$	Eqn 10-13
Chord AS	$AS = 2R \sin \delta, \text{ where } \delta = \frac{\Delta}{2}$	Eqn 10-14

10.3 Laying Out of Circular Curves

- Lay out principal points - TP₁, IP and TP₂.
- Lay out midpoint of curve, the crown (H), if required. Measuring from the IP to the crown, the crown secant distance, is a good check.
- Peg out centre line of a circular curve by:
 - ✓ Deflection angles and chords – will be covered in this Chapter, workshop and practicals.
 - ✓ Tangent offsets (Uren & Price, p602)
 - ✓ Chord offsets (Uren & Price, p608)
 - ✓ Radiations using coordinates (Uren & Price, p610)

10.3.1 Setting Steps using Deflection Angle and Chord Method (Figure 10-14):

- 1 Set up Total Station at TP₁
- 2 Point to intersection Point, - IP, and set horizontal direction to zero [0 SET].
- 3 Turn deflection angle of 1st point, measure and mark chord length, $\ell_1 = 2R\sin\delta_1$.
- 4 Turn deflection angle of 2nd point, measure and mark chord length, $\ell_2 = 2R\sin\delta_2$.
- 5 Continue until you reach TP₂.

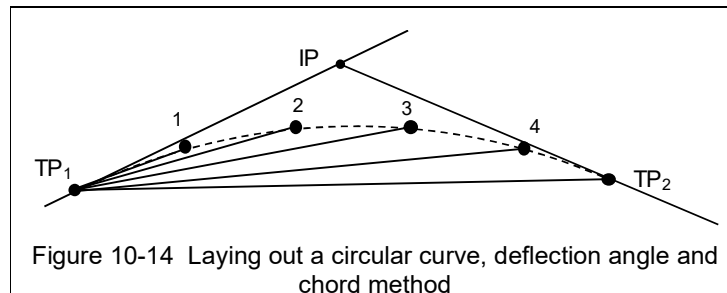


Figure 10-14 Laying out a circular curve, deflection angle and chord method

10.3.1.1 Checks – Deflection/Chord method

- ✓ Carry out visual check - does the curve look right.
- ✓ Measure distance to midpoint of curve, the crown, and TP₂.
- ✓ Measuring deflection angle to midpoint of curve, the crown, and TP₂.
- ✓ Measuring distances between individual pegged points. The in, through, and out chords.
- ✓ Check radii to the pegged points, if the centre point is accessible.
- ✓ Repeat pegging from TP₂ - pegs should be at the same points to those from TP₁.

10.3.2 Setting Steps using Coordinates Method (Figure 10-15):

- 1 Calculations - Determine the coordinates of all the points to be pegged – TP₁, TP₂, IP, crown and all chainage points.
- 2 Set Total Station on any known coordinated control point, CP₁, and back sight to the second coordinated control point, CP₂.
- 3 Set BS bearing, [H.ANG], in the Total Station.
- 4 Calculate the bearing and distance to each point on the curve to be pegged from CP₁.
- 5 Turn off bearing and measure distance to each point on the curve to be pegged.

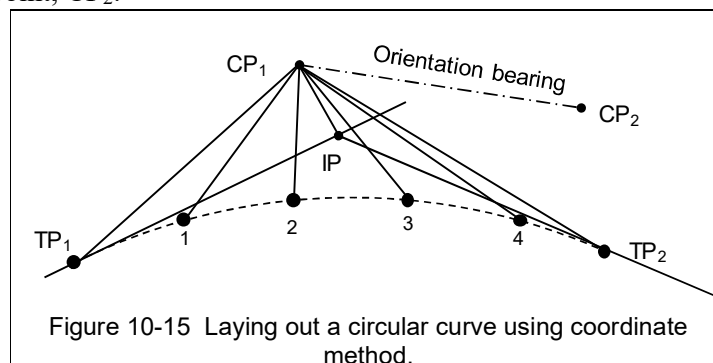


Figure 10-15 Laying out a circular curve using coordinate method.

10.3.2.1 Checks – Coordinates method

- ✓ Measure bearing and distance to third control point - check on control points.
- ✓ Carry out visual check - does the curve look right.
- ✓ Measuring distances between individual pegged points. The in, through, and out chords.
- ✓ Check radii to the pegged points, if the centre point is accessible.
- ✓ Repeat pegging from a second control point - pegs should agree.

10.4 Setting Out Road Sumps and Drains

Road drainage storm water pits, often called sumps, and pipework are integrated into the design of road profiles to provide drainage. Pits are located by reference to centreline chainage with a specified offset from the centreline to the centre of the sump. Standard drawings of concrete pits cover both rectangular and circular plans. Circular stormwater pits for typical roads are about 1200mm diameter and 1200mm high. Location of the pit in relation to the road centreline will be specified in detailed road drawings.

There are two methods of set-out that can be considered.

10.4.1 Offset by Calculation of Coordinates

In fully developed road designs all positional elements of the design will eventually be listed in a coordinate data base. The listing is generally produced by CAD packages in formats suitable for the job. Referring to § 6.3.7, page 129, for an example of a simplified point file (ENZP).

Files for curves are differently structured, carrying data necessary to set the curve out. Typically it contains: Chainage, Easting, Northing, Radius, Tangent length 1, Tangent Length 2, Deflection Angle Bearing, Description

Table 10-2 shows a fully developed transition curve which will be examined in the next Chapter 11. Transition curves and superelevation.

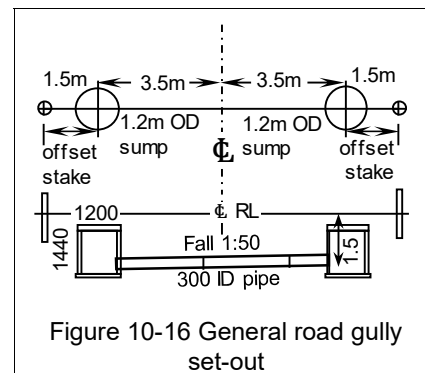
Table 10-2 Transition curve file. Refer § 11.4, p248

CHAINAGE	Easting,	Northing,	Radius,	TD 1,	TD 2,	Deflection,	Bearing,	Description
1900.000	396097.568	6457231.611					154.57'21"	Straight 1
1920.000	396106.034	6457213.491					154.51'21"	Straight 1
1940.000	396114.500	6457195.371					154.57'21"	Straight 1
1960.000	396122.967	6457177.252					154.57'21"	Straight 1
1966.208	396125.595	6457171.628					154.57'21"	Straight (SSD)
1980.000	396131.433	6457159.132					154.57'21"	Straight 1
2000.000	396139.899	6457141.013					154.57'21"	Straight 1
2020.000	396148.366	6457122.893					154.51'21"	Straight 1
2036.208	396155.227	6457108.209					154.57'21"	Straight
2036.208	396155.227	6457108.209					154.51'21"	Spiral 1 (TS)
2040.000	396156.833	6457104.774					154.55'11"	Spiral 1
2060.000	396165.467	6457086.734					153.36'16"	Spiral 1
2066.208	396168.264	6457081.192					152.48'26"	Spiral 1 (SC)
2066.208	396168.264	6457081.192	400.000	97.668	97.668	19'03'05'	152.48'26"	Arc
2080.000	396174.777	6457069.035	400.000	97.668	97.668	19.03'05'	150'49'54"	Arc
2100.000	396184.957	6457051.822	400.000	97.668	97.668	19.01'05"	141.58'01"	Arc
2120.000	396195.985	6457035.140	400.000	97.668	97.668	19.03'05"	145.06'08"	Arc
2140.000	396207.832	6457019.029	400.000	97.668	97.668	19.01'05"	142.14'14"	Arc
2160.000	396220.470	6457003.531	400.000	97.668	97.668	19'03'05'	139.22'22"	Arc
2180.000	396233.867	6456988.683	400.000	97.668	97.668	19.03'05'	136.30'28"	Arc
2199.212	396247.419	6456975.069	400.000	97.668	97.668	19.03'05'	133.45'21"	Arc
2199.212	396247.419	6456975.069					133.45'21"	Spiral 2 (CS)
2200.000	396247.989	6456974.524					133.38'40"	Spiral 2
2220.000	396262.718	6456960.996					131.48'35"	Spiral 2
2229.212	396269.599	6456954.810					131'36'26"	Spiral 2 (ST)
2229.212	396269.599	6456954.870					131.36'26"	Straight 2
2229.212	396269.599	6456954.870					131.36'26"	Straight 2
2240.000	396271.665	6456941.701					131.36'26"	Straight 2
2260.000	396292.619	6456934.421					131.36'26"	Straight 2
2280.000	396307.573	6456921.146					131.36'26"	Straight 2
2281.544	396308.728	6456920.121					131.36'26"	Straight 2
2290.000	396315.051	6456914.506					131.36'26"	Straight 2
2299.212	396321.939	6456908.389					131.36'26"	Straight (ESD)

Other parameters of the curve are:

1. Radius 400m
2. Design speed 70Kmh⁻¹
3. Spiral length 30m
4. Tangent runout 70m
5. IP 396196.571 6457019.724
6. Projection (MGA2020 Zone 50 f=0.9996)

From this data any structures, pits and culverts, can be incorporated in the road design. Note that this is a plan of the curve. No elevation data has been supplied for the vertical curve design. Road drainage will have to be installed at the curve low point, and at other chainages to collect and divert surface run-off.



10.4.1.1 Worked example 10.1

Three gullies and drainpipes are to be installed in the roadway at the following chainages:

1. 1970 on the straight
2. 2130 on the curve
3. 2265 on the straight.

Offset stakes are to be set on the centreline chainage and 5.00m either side of, and perpendicular to, the centreline. On the curve the alignment is towards the centre (perpendicular to the tangent at the chainage)

1. CH 1970 is halfway between CH1960 and CH 1980:

1960.000 396122.967 6457177.252

154.57'21" Straight

1980.000 396131.433 6457159.132

154.57'21" Straight

CH 1970 CL 396127.200 6457168.192 (midway)

CH 1970 5R: $E = 396127.200 + 5 \times \sin(152^\circ 48' 26'' + 90^\circ) = 396122.753$ (Brg away from Φ)
 $N = 6457168.192 + 5 \times \cos(152^\circ 48' 26'' + 90^\circ) = 6457165.071$

CH 1970 5L: $E = 396127.200 + 5 \times \sin(152^\circ 48' 26'' - 90^\circ) = 396131.647$ (Brg towards Φ)
 $N = 6457168.192 + 5 \times \cos(152^\circ 48' 26'' - 90^\circ) = 6457170.477$

2. CH 2130 is on the circular arc halfway between CH2120 and CH 2140

2120.000 396195.985 6457035.140 400.000 19.03'05" 145.06'08" Arc

2140.000 396207.832 6457019.029 400.000 19.01'05" 142.14'14" Arc

There are a couple of ways to approach the problem:

- i. By deflection from CH 2120. Eqn 10-4 and Eqn 10-6.
 BRG CH 2120 145°06'08" arc = 10m, radius = 400m,
 Defl'n angle at centre, $\Delta = 10/400 = 0.025^R = 1^\circ 25' 57''$
 Half deflection angle, $\delta = \Delta/2 = 0^\circ 42' 58''$.
 Chord = 10.000m (same as arc distance, $R = 400$)
 Brg to CH 2130 = $145^\circ 06' 08'' - 0^\circ 42' 58'' = 144^\circ 23' 10''$

2130 CL: $E = 396195.985 + 10 \times \sin(144^\circ 23' 10'') = 396201.808$

$N = 6457035.140 + 10 \times \cos(144^\circ 23' 10'') = 6457027.010$

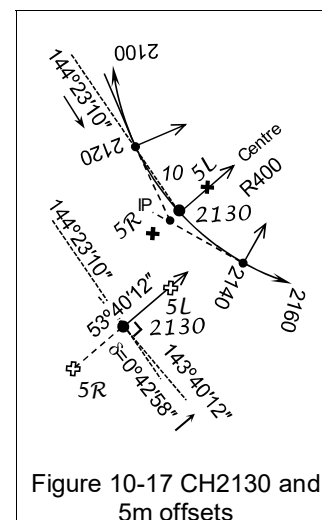
δ is used again to find the Bearing AT CH2130. The Brg across the road = $144^\circ 23' 10'' - \delta - 90 = 53^\circ 40' 12''$. See Figure 10-17. By using $D = -5$ the dE and dN are established away from the centre.

2130 5R: $E = 396201.808 - 5 \times \sin(53^\circ 40' 12'') = 396197.780$

$N = 6457027.010 - 5 \times \cos(53^\circ 40' 12'') = 6457024.048$

2130 5L: $E = 396201.808 + 5 \times \sin(53^\circ 40' 12'') = 396205.836$

$N = 6457027.010 + 5 \times \cos(53^\circ 40' 12'') = 6457029.972$



- ii. Calculate the curve centre coordinates from the perpendicular to the SC and CS (arc tangent points). The curve turns left.

Brg at SC: $152^{\circ}48'26''$, Brg at CS (end of arc): $133^{\circ}45'21''$. $R = 400\text{m}$

By bearing from calculated centre coordinates using radii:

$5L = 395\text{m}$, $CL = 400\text{m}$, $5R = 405\text{m}$ to offsets and centreline.

The centre is perpendicular to any bearing on the arc. § 10.2.1

2120.000 396195.985 6457035.140 400.000 19.03'05" 145.06'08" Arc

CENTRE: $E = 396195.985 + 400 \times \sin(145^{\circ}06'08'' - 90) = 396524.055$

$N = 6457035.140 + 400 \times \cos(145^{\circ}06'08'' - 90) = 6457263.986$

Check from CH2140:

2140.000 396207.832 6457019.029 400.000 19.03'05" 142.14'14" Arc

CENTRE: $E = 396207.832 - 400 \times \sin(142^{\circ}14'14'' - 90) = 396524.053$

$N = 6457019.029 - 400 \times \cos(142^{\circ}14'14'' - 90) = 6457263.986$

Working from each end of the arc (SC and CS) the coordinates are confirmed as:

CENTRE 396524.054 6457263.986 400.000 19.03'05" 142.14'14" Arc

BRG from centre to CH 2130 is BRG 2120 – deflection, Δ , to CH 2130 + 90°

Brg CH 2130 = $145^{\circ}06'08'' - 1^{\circ}25'57'' + 90 = 233^{\circ}40'11''$

2130 CL: $E = 396524.054 + 400 \times \sin(233^{\circ}40'11'') = 396201.808$

$N = 6457263.986 + 400 \times \cos(233^{\circ}40'11'') = 6457027.010$

Similarly, for 2130 5R, $R = 405$: $E = 396197.780$, $N = 6457024.048$ and

2130 5L, $R = 395$: $E = 396205.836$, $N = 6457029.972$

3. CH 2265 is 5m from CH 2260 towards CH2280: point moves 5/20 down from CH2260

2260.000 396292.619 6456934.421 131.36'26" Straight

2280.000 396307.573 6456921.146 131.36'26" Straight

$dE = 14.954$, $dE' = 0.25dE = 3.738$ CH 2265E = $E + dE'$

$dN = -13.275$, $dN' = 0.25dN = -3.319$ CH 2265N = $N + dN'$

CH 2265CL = 396296.357 6456931.102

As for CH 1970, calculate the L and R offsets for CH 2265 $\perp$. Bearings $131^{\circ}36'26'' \pm 90^{\circ}$.

Similarly, for 2265 5R, $D = 5$: $E = 396293.037$, $N = 6456927.363$ and

2265 5L, $D = 5$: $E = 396299.677$, $N = 6456934.841$

These methods rely on a fair amount of pre-computation, especially amending positions to an existing plan. And a return to the field, with the associated finding control, setting up etc.

10.4.2 In Field Offset by Taping

Establishment of extra points in a road construction imply that most of the road has been completed, but that chainage points are still extant. The use of tapes and perpendicular offsets offers a quick method of establishing set-out points.

Two methods of perpendicular setout are examined which are suitable for the job. § 2.6.4, p42, discusses offset methods used in Field Practical 2. Review Figure 2-26 and Figure 2-25.

1. CH 1970 is halfway between CH1960 and CH 1980:

Use sighting stakes or sighting rods at the chainages. Lay the tape between the two established chainages. Keep it straight and mark the center distance. (Takes care of slope distances).

CH 1970 5L and 5R offsets placed perpendicular to the straight.

- Pythagoras to the rescue. Swing the diagonal to intercept the offset. Check the measurement from the other chainage.

- Set the $\perp$ mark and stand over it, or use a plumb bob. Sight to stakes on the centreline and use an optical square to establish the offsets. Always check a diagonal distance.

Figure 10-18 shows the method.

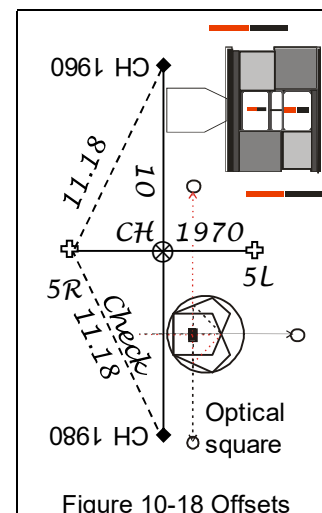


Figure 10-18 Offsets

2. CH 2130 is on the circular arc halfway between CH2120 and CH 2140.

The centre chainage is placed by bisecting the chord between CH 2120 and CH 2140. An offset from the chord to the arc is used to establish the 5L and 5R marks as well as the Φ .

The offset distance is the crown distance, see § 10.2.8 and Eqn 10-12, are shown in Figure 10-19. The arc is radius 400m with the associated coordinates of the end chainages (Table 10-2):

2120.000	396195.985	6457035.140	400.000	145.06'08"
2140.000	396207.832	6457019.029	400.000	142.14'14"

Arc length, $s = 20$.

$\Delta = 20/400 = 2^\circ 51' 53''$, $\delta = \Delta/2 = 1^\circ 25' 57''$.

Crown = $400 - 400\cos(1^\circ 25' 57'') = 0.125\text{m}$, half shoe length.

- i. Lay the tape straight between the two centreline pegs, pick half way and mark it. Then tape 125mm away from the centre. Now 5L is $5 - 0.125 = 4.875\text{m}$ and 5R is 5.125m from the chord mark. The respective diagonals are 11.125m and 11.238m . Figure 10-20.

The offset is small because the radius is very large. But the chord and crown method is quite useful for setting out curves using a tape and offset only. Then the “halving and quartering” technique can be used. If, for example, the curve is to be pegged every 5m then the half chord crown is placed at 0.125m . The chord between that crown and the start is halved (at 5m). The crown is then $\frac{1}{4} \times 0.125 = 0.031$. From that crown to the start the half chord is 2.5m and the crown is $\frac{1}{4} \times 0.031 = 0.008\text{m}$.

The method (Figure 10-21) can be quite useful for setting out short radius arcs, say kerbs for intersections where a number of pegs is needed to guide the kerbing machine.

- ii. Lay the tape straight between the two centreline pegs, pick half way and mark it. As in the preceding example tape 125mm away from the centre. Now 5L is $5 - 0.125 = 4.875\text{m}$ and 5R is 5.125m from the chord mark. Rather than tape diagonals, use an optical square to set the perpendicular to the chord. Figure 10-22

Its probably a good idea to put a line stake in a bit further away than the desired left (5L) offset. The line from there over the mid chord can provide a better alignment for the Φ and 5L/5R marks.

Remember that the measurements are being taken from the chord line. And use an alternate method to check the work.

3. CH 2265 is 5m from CH 2260 towards CH2280. As with placing CH 1970, lay the tape on the line and measure down the 5m. Then proceed with the offsets either by taping the diagonals, or by using the optical square.

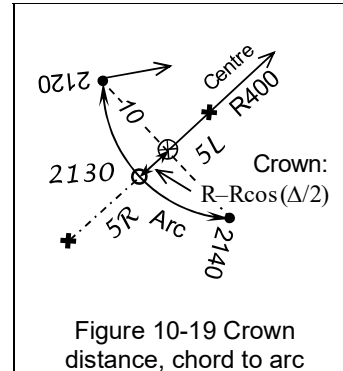


Figure 10-19 Crown distance, chord to arc

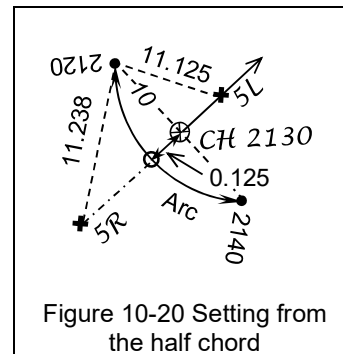


Figure 10-20 Setting from the half chord

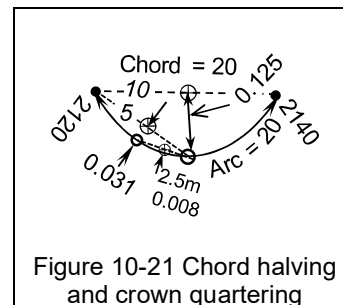


Figure 10-21 Chord halving and crown quartering

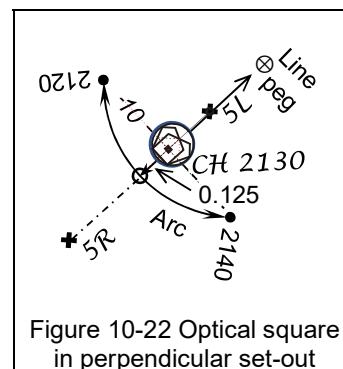


Figure 10-22 Optical square in perpendicular set-out

10.4.3 Opinion on Field Methods for Set-outs

A large number of set-outs are initiated in-field after the works plans have been implemented. The engineer and surveyor must produce results without access to many of the office systems, so they need to develop methodologies to produce results on site. The requirements of the precision necessary for the work will dictate how the task is carried out. Checking the work is always the most important of the job.

10.5 Calculations for a Field Practical Exercise

Field practical 5 involves setting out the centre line of a designated circular curve, for which your group will be given full instructions.

Prior to Field Practical 5 each group member will be expected to calculate a full set of circular curve data for presentation to examiners prior to the commencement of the Field Practical.

Figure 10-23 shows a curve designed around a set of parameters, defined by the:

centre line; Curve radius, deflection angle at PI.

point of curvature;

PC coordinates, PC chainage.

point of intersection;

deflection angle, PI coordinates, orientation, PC to PI, PI chainage.

formation data;

pavement, shoulder, and formation width.

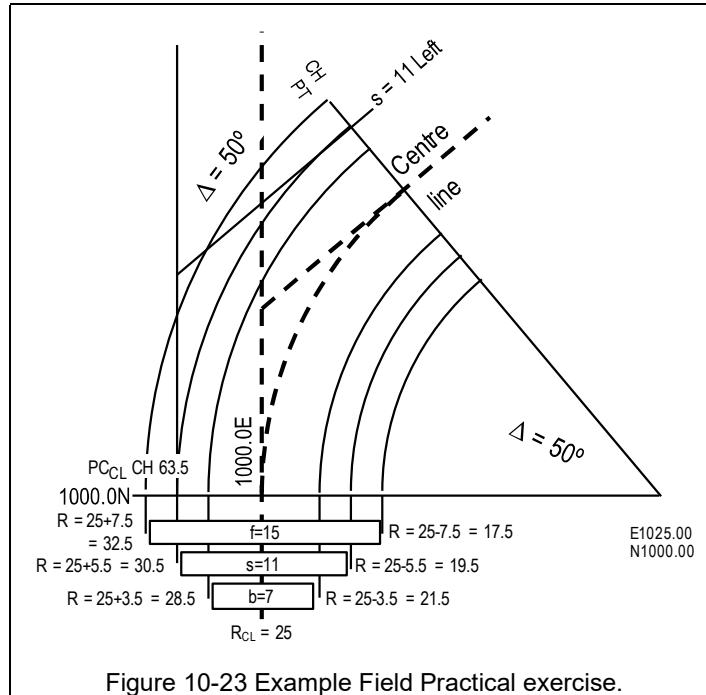


Figure 10-23 Example Field Practical exercise.

10.5.1 Calculation of a Design Curve

The following specifications are an **example** of the curve data that you will use to calculate design curve, refer to Figure 10-23.

Design parameters for curve turning to the RIGHT

CURVE	M1
Initial direction of tangent	Brg IP = 360
Chainage PC (TP1)	CH = 63.5
Coordinates of PC (TP1)	E = 1000.000
Coordinates of PC (TP1)	N = 1000.000
Radius design Centre line	R = 25.0
Design deflection angle Δ	$\Delta = 70^\circ$
Through chaining interval	s = 10
Centreline offset	o = 0
Pavement width	b = 7
Shoulder width	s = 11
Formation width	f = 15

Always draw a diagram and label all the given data.

With curve diagrams, it is often necessary to exaggerate the deflection angle which allows labelling when the deflection angle, Δ , is small and the radius, R, is large.

10.5.2 Curve Radius from Design Profile

A simple road profile is illustrated in Figure 10-24, showing that the design is symmetrical around the centre line. The analogy of symmetry is carried over from the Section 3.4.

You are assigned curve M1 left shoulder kerb, $s = 11$, you will have to derive YOUR parameters:

deflection angle, $\Delta = 50^\circ$

radius = CL radius $\pm$ (width/2)
 $= 25 + 5.5$

(curve turns right, centre is to right radius increases)

$R = 30.5$

$CH_{PC} = 63.5$

(the centre is perpendicular to the tangent point, all chainages are the same)

$PC\ E = CL\ E \pm (\text{width}/2)$
 $= 1000.00 - 5.5$

(curve turns right, centre is to right, E decreases)

$PC\ E = 994.5$

$PC\ N = CL\ N = 1000.00$

(Brg IP = 360° , so, no N offset due to bearing or formation (width/2 distance).

Figure 10-25 shows the different chainages at the PT and the Easting coordinates of the PC.

The coordinates of the formation IP are then:

$IP\ E = PC\ E = 994.5 = 994.50E$

(Brg = 360° , straight N of PC E)

$TD = R \cdot \tan(\Delta/2) = 30.5 \tan(25) = 14.22$

$IP\ N = PC\ N + TD$
 $= 1000.00 + 14.22 = 1014.22N$

Arc length, $S = R \cdot \Delta_{\text{radians}}$
 $= 30.5 \cdot (50 \cdot \pi / 180)$
 $= 30.5 \times 0.8727 = 26.616,$

and thus,

$CH_{PT} \text{ (right formation)} = CH_{PC} + S$
 $= 63.5 + 26.62 = 90.116$

(End of curve.)

In many cases, you will be given the coordinates and chainage of the IP, the deflection angle, Δ , and the radius, R , on the design centreline.

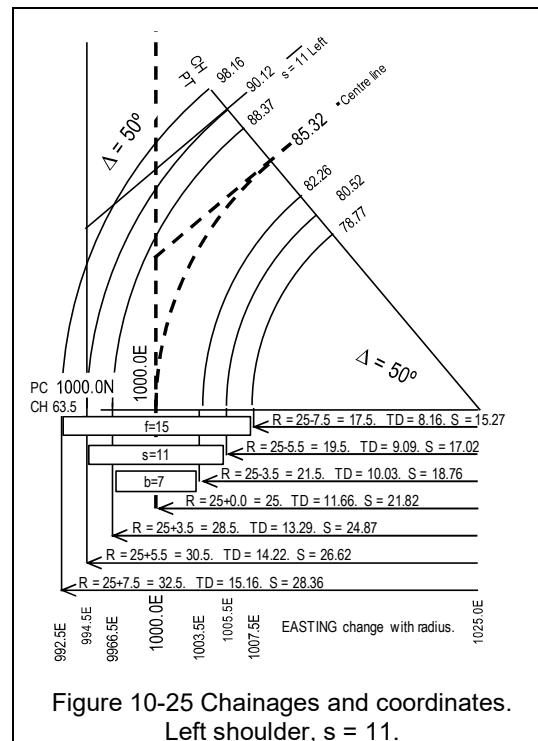
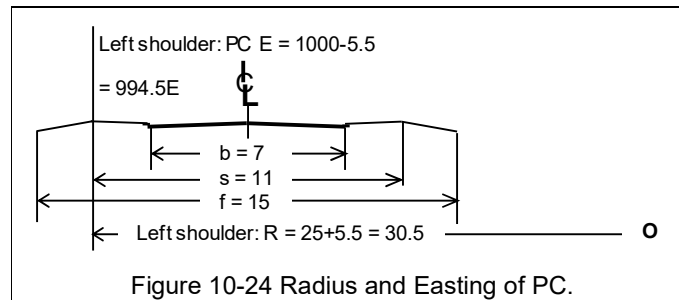
Calculate backwards to find the coordinates of the point of curvature, PC, and chainage of the PC.

- Calculate the tangent distance, TD and subtract it from the CH_{IP} :
 $CH_{PC} = CH_{IP} - TD.$
- Reverse the bearing to the IP,
 - use the TD to calculate the rectangular coordinate differences, dE and dN.

These coordinate differences, added to the IP coordinates, result in the coordinates of the PC.

10.5.3 Initial Curve Calculations

From your allocated curve, there are some parameters that you will have to deduce and calculate, as follows:



10.5.3.1 Curve M1, centre line calculations

Based on **CENTRE LINE** calculations, centre line offset = 0

Initial direction of tangent	360°	
Chainage & coords of PC (TP1) =	Ch 63.5	E1000.000 N1000.000
Radius	R = 25	
Design deflection angle Δ	$\Delta = \mathbf{50}$	
Through chainage interval	s = 5	
Arc length, $S = R \cdot \Delta^{\text{radians}}$	= 21.817	
Design TD = $R \tan(\Delta/2)$	= 14.222	
Long chord = $2R \sin(\Delta/2)$	= 21.131	
Secant distance = $R/\cos(\Delta/2)$	= 27.584	
Crown secant = $R/\cos(\Delta/2) - R$	= 27.584 - 25	= 2.584
Crown distance = $R - R \cos(\Delta/2)$	= 25 - 22.658	= 2.342
CH PT = CH PC + S	= 63.5 + 21.817	= 85.317
CH Crown = CH PC + S/2	= 63.5 + 21.817/2	= 74.408
CH IP = CH PC + TD	= 63.5 + 14.222	= 77.722
E coordinates of IP, E IP + dE	= 1000.000 + 0.000	= E1000.000.
N coordinates of IP, N IP + dN	= 1000.000 + 14.222	= N1014.222.

With the parameters calculated create a table of values. Note that the chainage of the PC is 63.5; the through chainage is 5m and the calculated table must reflect this specification:

Remember that the centre deflection angle, Δ = arc length(s)/radius(R), (Eqn 10-4) is evaluated in **radians** and has to be converted to an angle in **degrees**. The **chord deflection angle** (δ) is **half** the centre deflection angle (Eqn 10-6). Table 10-3 shows the centre deflection angle in radians, chord deflection in degrees.

The bearing of the IP is 360°, so the chord deflection angle becomes the bearing to each chainage point. In the field, it may be necessary to add the chord deflection angles to a nominated bearing before calculating the chainage point coordinates. Bearing (θ) = Initial Brg + δ .

The chord length $\ell = 2R \cdot \sin(\delta)$. $dE = \ell \sin(\theta)$, $dN = \ell \cos(\theta)$.

Note: Chainage coordinates **radiate** from the PC coordinates.

(Ch E = PC E + dE, Ch N = PC N + dN).

10.5.3.2 In, through and out arcs for Curve M1

Referring to Section 10.2.7, and Table 10-3, the following are the three arcs for radius, R=25m:

In arc	=	1.5	$2 \delta^\circ = 0.0600$	$\delta^\circ = 1^\circ 43' 07''$	$\ell \text{ chord} = 1.500$
Ch interval	=	5	$2 \delta^\circ = 0.2000$	$\delta^\circ = 5^\circ 43' 47''$	$\ell \text{ chord} = 4.992$
Out arc	=	0.317	$2 \delta^\circ = 0.0128$	$\delta^\circ = 0^\circ 22' 00''$	$\ell \text{ chord} = 0.317$

Table 10-3 Set-out for curve M1, centre line.										
Point	Ch	s	$2 \delta^R$	δ°	$\delta^\circ(\text{DMS})$	$\ell \text{ chord}$	dE	dN	E	N
CH _{PC}	63.50	0.00	0.000	0.000		0.00	0.0	0.0	1000.00	1000.00
CH	65.00	1.50	0.0600	1.719	$1^\circ 43' 07''$	1.50	0.04	1.50	1000.04	1001.50
CH	70.00	6.50	0.2600	7.448	$7^\circ 26' 54''$	6.48	0.84	6.43	1000.84	1006.43
CH	75.00	11.50	0.4600	13.178	$13^\circ 10' 40''$	11.40	2.60	11.10	1002.60	1011.10
CH	80.00	16.50	0.6600	18.908	$18^\circ 54' 27''$	16.20	5.25	15.33	1005.25	1015.33
CH	85.00	21.50	0.8600	24.637	$24^\circ 38' 13''$	20.84	8.69	18.95	1008.69	1018.95
CH _{PT}	85.32	21.82	0.8728	25.000	$25^\circ 00' 00''$	21.13	8.93	19.15	1008.93	1019.15

10.5.4 Roads in Mining Operations

Properly designed and well-maintained roads are a vital part of safe and efficient mining operations. The machinery is BIG, with some tipper trucks now spreading up to 10 metres width.

Two-way traffic around a bend calls for pavement widths of 40 metres, formation widths of 60 metres, and inside radius of about 100 metres. Gradients must be constant; this may put an extra layer of complexity on curve design.

10.5.5 Haul Road Example

The example of a mining haul road curve (Section 10.5.6) shows larger radius curves, longer through chainages and increased formation and pavement widths, Figure 10-26.

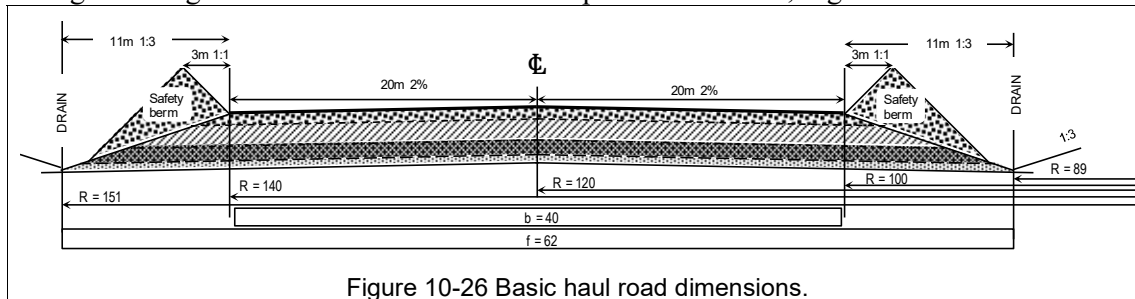


Figure 10-26 Basic haul road dimensions.

From your allocated curve, these are some of the parameters you will have to deduce and calculate as follows:

10.5.6 Curve T5, centre line calculations

Based on **CENTRE LINE** calculations, centre line offset = 0

Initial direction of tangent	360°	
Chainage & coords of PC (TP1) =	Ch 211.00	E994.000 N3111.000
Radius	R = 105	
Design deflection angle Δ	$\Delta = 81$	
Through chainage interval	s = 25	
Arc length, $S = R \cdot \Delta^{\text{radians}}$	= 148.440	
Design TD = $R \tan(\Delta/2)$	= 89.678	
Long chord = $2R \sin(\Delta/2)$	= 136.384	
Secant distance = $R / \cos(\Delta/2)$	= 138.084	
Crown secant = $R / \cos(\Delta/2) - R$	= 138.084 - 105	= 30.084
Crown distance = $R - R \cos(\Delta/2)$	= 105 - 79.843	= 25.157
CH PT = CH PC + S	= 211 + 148.440	= 359.440
CH Crown = CH PC + S/2	= 211 + 148.44/2	= 285.220
CH IP = CH PC + TD	= 211 + 89.678	= 300.678
E coordinates of IP, E IP + dE	= 994.000 + 0.000	= E994.000
N coordinates of IP, N IP + dN	= 3111.000 + 89.678	= N3200.678

With the parameters calculated it is time to create a table of values. Note that the chainage of the PC is 211, that the through chainage is 25m and the table must reflect this specification: Remember that the centre deflection angle (2δ) = arc length(s)/radius(R) is evaluated in **radians** and has to be converted to an angle in **degrees**. The **chord deflection angle** (δ) is **half** the centre deflection angle. Table 10-4 shows the centre deflection angle in radians, chord deflection in degrees.

The bearing of the IP is 360°, so the chord deflection angle becomes the bearing to each chainage point. In the field, it may be necessary to add the chord deflection angles to a nominated bearing before calculating the coordinates of the chainage points.

$$\text{Brg}(\theta) = \text{Initial Brg} + \delta.$$

10.5.6.1 Set-out table for Curve T5

The chord length $\ell = 2R \cdot \sin(\delta)$. $dE = \ell \sin(\theta)$, $dN = \ell \cos(\theta)$.

Note: Chainage coordinates **radiate** from the PC coordinates.

(Ch E = PC E + dE, Ch N = PC N + dN).

Table 10-4 Set-out for curve T5, centre line.										
Point	Ch	s	$2\delta^R$	δ°	$\delta^\circ(\text{DMS})$	ℓ chord	dE	dN	E	N
CH _{PC}	211.00	0.00	0.000	0.000		0.00	0.0	0.0	994.00	3111.00
CH	225.00	14.00	0.1333	3.820	3° 49' 10"	13.99	0.93	13.96	994.93	3124.56
CH	250.00	39.00	0.3714	10.641	10° 38' 26"	38.78	7.16	38.11	1001.16	3149.11
CH	275.00	64.00	0.6095	14.462	17° 27' 41"	63.01	18.91	60.11	1012.91	3171.11
CH	300.00	89.00	0.8476	24.282	24° 16' 56"	86.36	35.51	78.92	1029.51	3189.72
CH	325.00	114.00	1.0857	31.103	31° 06' 12"	108.48	56.04	92.89	1050.04	3203.89
CH	350.00	139.00	1.3238	37.924	37° 55' 27"	129.07	79.33	101.81	1073.33	3212.81
CH _{PT}	359.44	148.44	1.4137	41.500	41° 30' 00"	136.38	88.57	103.71	1082.57	3214.71

10.5.6.2 In, through and out Arcs for Curve T5

Referring to Section 10.2.7, and Table 10-4, the following are the three arcs for radius, R=105m:

In arc	= 14	$2\delta^R = 0.1333$	$\delta^\circ = 3^\circ 49' 10''$	ℓ chord = 13.990
Ch interval	= 25	$2\delta^R = 0.2381$	$\delta^\circ = 6^\circ 49' 15''$	ℓ chord = 24.941
Out arc	= 9.44	$2\delta^R = 0.0899$	$\delta^\circ = 2^\circ 34' 32''$	ℓ chord = 9.437

10.5.6.3 Resulting tangent and arc distances for the T5 formation

Note that the radii involved in the four other calculations, with, for example, formation width **f=62** and pavement width **b = 40** are:

Left formation:	105 + 31	= 136	TD = 116.15	Arc, S = 192.26
Left pavement:	105 + 20	= 125	TD = 106.76	Arc, S = 176.11
Centreline:	105 + 0	= 105	TD = 89.68	Arc, S = 148.44
Right formation:	105 - 20	= 85	TD = 72.60	Arc, S = 120.16
Right pavement:	105 - 31	= 74	TD = 63.20	Arc, S = 104.62

10.6 Concluding Remarks

It is essential for the students to know at what point the horizontal alignment (circular curve) starts, its length and where it ends. Basic circular geometrical properties (tangency, chord, radius and subtended angles) are the employed in the design stage. Chainages (road length intervals from some starting point) to the IP (intersection point of the straights), the end of the circular curve and the chainage intervals along the circular curve will always be specified for the given horizontal alignment. With these parameters, the chainage of the start of the curve, plus the length of the curve, can be computed. This is followed by the ingoing, through and outgoing arc lengths, plus their deflection angles and chords required for the actual setting out. A table of chords and deflection angles is then tabulated. It should be pointed out that other methods (i.e., coordinates approach) could be used in setting out of the horizontal alignments. However, these coordinates themselves are generated from initial bearing, deflection angles and chord distances. This chapter has synthesised these materials in a manner that we hope will be easier for the students to follow and understand.

10.7 References to Chapter 10

1. Uren and Price (2006, 2010) Surveying for engineers. Fourth edition, Palgrave Macmillan, Chaps 12.
2. Schofield and Breach (2007) Engineering Surveying. Sixth edition, Elsevier, Chap. 10.

3. Irvine and Maclellan (2006) *Surveying for Construction*. Fifth edition, McGraw, Chap. 11.

11 Chapter 11. Transition curves and superelevation

11.1 Road Design incorporating Transition Curves and Superelevation

11.1.1 Introduction

The circular curve is a fundamental element in road design. Its radius is a function of many factors including: operating (design) speed, curve super elevation, radial forces and tyre friction and occupant riding comfort.

These design criteria are published by: Ausroads Ltd, Sydney, 2016. Edition 3.2

AGRD03-16: Guide to Road Design Part 3: Geometric Design. ISBN 978-1-925451-24

The Austroads *Guide to Road Design* seeks to capture the contemporary road design practice of member organisations (refer to the Austroads *Guide to Road Design Part 1: Introduction to Road Design* (Austroads 2015a)). In doing so, it provides valuable guidance to designers on the production of safe, economical and efficient road designs.

The purpose of this guide is to provide the information necessary to enable designers to develop safe and coordinated road alignments that cater for the traffic demand at the chosen speed. This guide also presents information leading to the choice of appropriate cross-section standards, which will enable designers to balance the needs of all road users and the environment in which the road is constructed. For guidance on the design of unsealed roads, designers should refer to the ARRB *Unsealed Roads Manual: Guidelines to Good Practice* (Giummarra 2009).

Austroads is the association of Australian and New Zealand road transport and traffic authorities. Austroads members are the six Australian state and two territory road transport and traffic authorities, the Australian Government Department of Infrastructure and Regional, the Australian Local Government Association, and New Zealand Transport Agency.

11.1.2 The Fundamentals of Uniform Circular Motion

Uniform circular motion is the path taken by a body moving at constant linear speed around a centre point of constant radius. To maintain the circular path the body is subject to an **acceleration** towards the centre of the circle.

A particle moves, Figure 11-1, from point A, at time t_i with velocity $\mathbf{v}_i$ to point B, at t_f , velocity $\mathbf{v}_f$.

$\mathbf{v}_i$ and $\mathbf{v}_f$ are the two velocity vectors. If the magnitudes are the same ($v_i = v_f = v$), then the only change is **direction**. and Figure 11-2 shows the vector subtraction.

$$\text{The average acceleration, } \bar{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{dv}{dt} \quad \text{Eqn 11-1}$$

Consider Figure 11-1 with sides dr and r similar to Figure 11-2 so that:

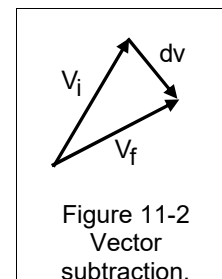
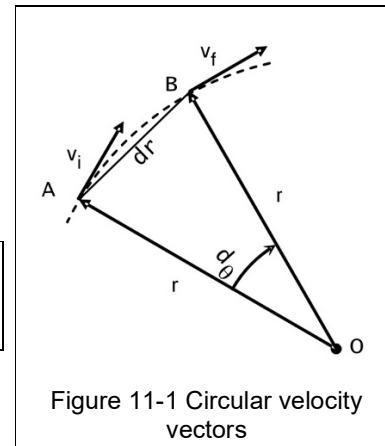
$$\frac{dr}{r} = \frac{dv}{v} \quad \text{And, as } \bar{a} = \frac{dv}{dt} \text{ by substitution we have:}$$

$$\bar{a} = \frac{v \cdot dv}{t \cdot dt} \quad (\text{Figure 11-2})$$

In the limit, as $\lim_{t \rightarrow 0} \left(\frac{dr}{dt} \right) = v$ and $d\mathbf{v}$ points to the centre of the circle.

$$\text{The magnitude of the acceleration is: } a_r = \frac{v^2}{r} \quad \text{Eqn 11-2}$$

It is called centripetal (centre-seeking) acceleration.



11.1.3 Forces Involved in Uniform Circular Motion

From Newton's Second Law of Motion we know that, to change the path of a body of mass, m , in uniform linear motion, we must apply an acceleration, a , to the body. The amount of **force**, F , needed to apply that acceleration is: $F = ma$

Newton's second law of motion:	$F = ma$	Eqn 11-3

A body in uniform **circular** motion is subject to a centripetal acceleration, a_r towards the centre of the circle. If the body has mass, m , then the force needed to maintain that path is:

From Eqn 11-3 and Eqn 11-2:	$F = m \frac{v^2}{r}$	Eqn 11-4

The **force**, F , needed to maintain this circular path can be called:

- a) tension, if the body is attached to a string;
- b) gravity, if it is the moon circling the earth or
- c) friction, if it is a vehicle driving around a circular curve.

11.1.4 The Forces on a Body in Uniform Circular Motion

Consider a body of mass, m , stationary on the surface of the earth. The force applied by the surface of the earth to maintain the body in its static position is $F = ma$. We are holding the body against the acceleration of gravity, $g \approx 9.8 \text{ m} \cdot \text{sec}^{-2}$, towards the centre of the earth, opposing the force applied **by** the body **to** the surface.

We can call this force the **weight**, $W = mg$, in Newtons ($\text{kg} \cdot \text{m} \cdot \text{sec}^{-2}$).

Please note that we normally call the **mass** of a body its **weight**.

Now let us call the body a vehicle and the surface of the earth a road. The forces on a vehicle (Figure 11-3) following a circular (flat) curve are:

- friction, F , keeping the vehicle on its circular path;
- weight, W , of the vehicle on the road,
- reaction, n , of road to weight of the vehicle

Assume a vehicle of

mass, $m = 1500 \text{ Kg}$

speed, $v = 50 \text{ Km} \cdot \text{h}^{-1} = 50000/3600 \text{ ms}^{-1} = 50/3.6 = 13.9 \text{ ms}^{-1}$

radius, $r = 100 \text{ m}$

What is the frictional force needed to keep the vehicle on the road?

From Eqn 11-4:

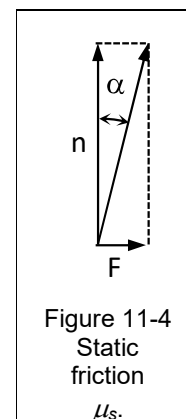
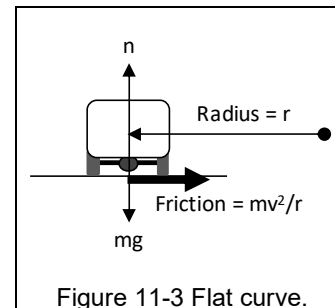
$$F = m \frac{v^2}{r} = 1500 \frac{13.9^2}{100} = 2893 \text{ Newtons } (\text{kg} \cdot \text{m} \cdot \text{s}^{-2})$$

See Figure 11-3.

Now examine the resultant vector diagram of forces, Figure 11-4, $n = W = mg$:

From Eqn 11-4:	$\tan(\alpha) = \frac{F}{n} = \frac{\frac{mv^2}{r}}{mg} = \frac{v^2}{rg} = \mu_s$	Eqn 11-5

In this case $\tan(\alpha)$ represents the **coefficient of static friction**, μ_s



In the previous example, and using Eqn 11-5 and where $n = mg = 14700N$

$$\tan(\alpha) = \frac{2893}{147700} = 0.197 \text{ or } = \frac{13.9^2}{100 \times 9.8} = 0.197$$

$$\mu_s = \tan(\alpha) = 0.197. (\alpha = 11^\circ)$$

The **maximum** coefficient of static friction between tyre and pavement depends very much on both surfaces but can be estimated at $\mu_{s(\max)} = 0.7$ for an asphalt pavement. Specialist tyres can have $\mu_s > 1.0$. In the above example the vehicle will maintain its path because $\mu_s < \mu_{s(\max)}$. Conversely, if we decide on a design coefficient of static friction, we are then able to specify a **maximum** speed for a given radius.

$$\text{Now, } \mu_s = \frac{v^2}{rg}$$

$$v = (\mu_s \cdot rg)^{\frac{1}{2}}$$

$$v_{\max} = (\mu_{s(\max)} \cdot rg)^{\frac{1}{2}} m \cdot s^{-1} = 11.27 (\mu_{s(\max)} \cdot rg)^{\frac{1}{2}} Kmh^{-1}$$

For: $\mu_s = 0.7$ and $r = 100$, **Maximum** speed = $94 Kmh^{-1}$

We are also able to specify **minimum** radius for a give speed for the design coefficient of static friction.

$$\text{Now, } \mu_s = \frac{v^2}{rg}$$

$$r_{\min} = \frac{v^2}{\mu_{s(\max)} \cdot g} m \cdot s^{-1} = \frac{v^2}{127 \cdot \mu_{s(\max)}} Kmh^{-1}$$

For: $\mu_s = 0.7$ and $v = 50$, **Minimum** radius = $28m$

It should be noted that operating a vehicle at these limits will result in very uncomfortable driving conditions, with no margin for safety. Beyond these figures the vehicle would skid tangentially from the curve.

11.1.5 Forces on a Canted Surface

If, however, the vehicle is placed on a cross fall (Figure 11-5), or slope, that equals $\tan(\alpha)$ as it traverses the curve then the frictional force, F , will be balanced by the resultant force through the vehicle in a direction perpendicular to the slope. The vehicle occupants will then be subject only to a force acting vertically through the seat. There will be no apparent side force.

This phenomenon is clearly demonstrated by a bicycles and motorbikes “leaning” in to a turn, where the frictional force is reduced substantially.

The cross fall slope needed to reduce F to 0 is:

From Eqn 11-5 and Figure 11-4: Converting to Kph	$\tan(\alpha) = \frac{v^2}{rg} m \cdot s^{-1}$ $= \frac{v^2}{(3.6^2 \times 9.8)r} = \frac{v^2}{127 \cdot r} Km \cdot h^{-1}$

Adverse cross fall (Figure 11-6) on a circular curve refers to the normal cross fall, used on a straight road, being incorporated into a curve. Adverse Crossfall has been described as: A slope on a curved pavement which generates forces detracting from the ability of a vehicle to maintain a circular path. Adverse cross fall,

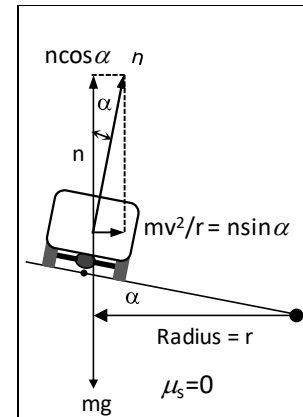


Figure 11-5 Vehicle on cross-fall.

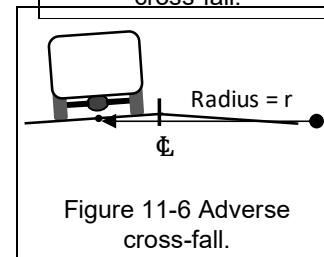


Figure 11-6 Adverse cross-fall.

especially at speed, results in a very uncomfortable experience. It also adversely affects the stability of high load vehicles.

11.1.6 Coefficients of Friction

Pavement design confines itself to the coefficient of **static** friction. This is the coefficient of friction that will prevent an object moving on a surface, up to the point where the applied force, F , overcomes the weight of the object, $n = mg$.

$$F_{\max} = \mu_s \cdot n$$

This situation occurs when the point of contact of the tyre on the pavement is not moving. Once the body starts moving the coefficient of friction is called the coefficient of **kinetic** (moving) friction, μ_k . It is less than μ_s .

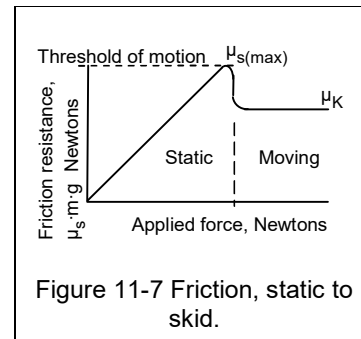
When a tyre starts to skid on a pavement (the point of contact is moving) it is under the influence of kinetic friction.

Figure 11-7 shows that an object will not move until the applied force matches the frictional resistance of the body at the **maximum** coefficient of static friction, $\mu_{s(\max)}$.

Once the body starts moving, a lesser applied force is needed because the coefficient of kinetic friction, $\mu_k < \mu_{s(\max)}$.

Note the characteristically sharp break at $\mu_{s(\max)}$.

The third type of friction coefficient is **rolling** friction, μ_R . This is the friction needed to maintain the static contact between tyre and pavement as the tyre rolls along. Values of μ_R can vary between 0.02 and 0.06.



11.1.7 Practical Design Considerations for Circular Road Curves

The derivation of the formulae involved in uniform circular motion is developed and expanded into a practical set of road design standards.

The recommendations are contained in “Rural Road Design, A Guide to the Geometric Design of Rural Roads”, Austroads, Sydney 2003. ISBN 0 85588 655 2.

This document has been superseded by Austroads, Guide to Rural Road Design Part 3: Geometric Design, AGRD03-16, 2017. No comparisons with the previous version have been made. Recommendations may have changed in this 360 page document. However, the previous version provides enough data for illustrative purposes.

Note that there are slightly different terminologies used in “Rural Road Design”.

Coefficient of static friction, μ_s , is called variously:

side friction factor, f , (eqn 9.1, p35) and

longitudinal friction factor, F , (8.3.1, p26).

Cross fall ($\tan(\alpha)$) is denoted as e (m.m^{-1}).

The sign indicates slope from the centreline of the pavement.

Minimum radius of a turn takes into account the cross fall, or elevation, e .

This equation can then address the problem of adverse cross fall.

$$R_{\min} = \frac{v^2}{127(e_{\max} + \mu_{s(\max)})}$$

Eqn 11-6

11.1.8 Recommended Values for Coefficient of Static Friction

Whilst a value of $\mu_{S(\max)} = 0.7$ may be achieved, much lower values are used in the design of curves. This is to take into account such factors as:

- adequate safety margin against skidding,
- wearing surface polishing,
- wearing surface texture,
- surface contamination from oil spill, loose aggregate etc,
- wet weather,
- driver behaviour.

11.1.8.1 Side friction values

Recommended static coefficient values.

Table 11-1 (Rural Road Design, Table 9.1, p37)		
Operating Speed (Kmh ⁻¹)	$\mu_{S(\text{design})}$	$\mu_{S(\max)}$
50	0.30	0.35
60	0.24	0.33
70	0.19	0.31
80	0.16	0.26
90	0.13	0.20
100	0.12	0.16
110	0.12	0.12
120	0.11	0.11
130	0.11	0.11

11.1.9 Recommended Minimum Radii Values of Horizontal Curves

Table 11-2 is based on differing values of superelevation, e , which are based on terrain. Care must be exercised in choosing higher values of superelevation ($e = 0.1$, or 10%), recognising:

- a higher level of driver discomfort,
- tendency of slow vehicles to track towards the centre,
- stability of high load commercial vehicles,
- difference between inner and outer formation levels,
- length available to develop superelevation.

11.1.9.1 Minimum radii values of horizontal curves

Table 11-2 (Rural Road Design, Table 9.2, p38)								
Operating Speed (Kmh ⁻¹)	Minimum Radius, m, (rounded up)							
	Flat terrain, $e = 0.03$		Undulating terrain, $e = 0.06$		Rolling terrain, $e = 0.07$		Mountainous terrain, $e = 0.10$	
	r(des)	r(abs)	r(des)	r(abs)	r(des)	r(abs)	r(des)	r(abs)
50	60	52	56	40	53	47	49	44
60	105	79	95	73	91	71	83	66
70	175	113	154	104	148	102	133	94
80	265	173	229	157	219	153	194	140
90	315	219	335	245	319	236	277	213
100	525	415	437	358	414	342	--	--
110	635	635	529	529	501	501	--	--
120	810	810	667	667	--	--	--	--
130	950	950	782	782	--	--	--	--

11.1.10 Superelevation. Road Profile on a Composite Curve

A **composite** curve is a **transitional** curve incorporating **superelevation** in the road design. The purpose of introducing superelevation into the transverse road profile (the road cross-section) is to provide for a comfortable, stable and safe vehicular passage around a curve.

In determining the generation of superelevation, we will consider only the carriageway of the entire road profile. The carriageway consists of the traffic lane(s) and the road shoulders and is carried on the road pavement. Figure 11-8 illustrates a typical two lane two-way rural road. Urban roads will share the same characteristics in relation to the carriageway.

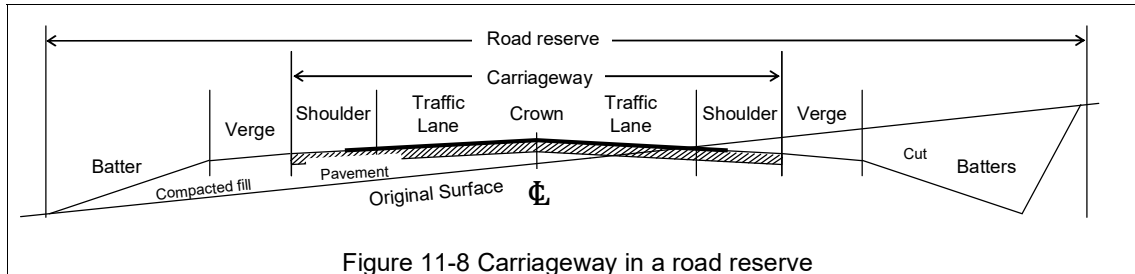


Figure 11-8 Carriageway in a road reserve

The crown of the road, the centreline of the carriageway, is the point from which the slope of the road cross section is developed.

The slope, or cross fall, or camber, is expressed either as a percentage or as the equivalent number. i.e. $3\% = 0.03$. It is also the tangent of the slope, so $\arctan(0.03) = 1.7^\circ$. The purpose of cross fall is to provide water drainage from the pavement surface.

By convention the **sign** of the slope of indicates the relationship between the centreline and endpoint of the slope. A straight section of road has a, say, -3% slope. This shows that the edge of the carriageway is below the centreline. (Figure 11-9)

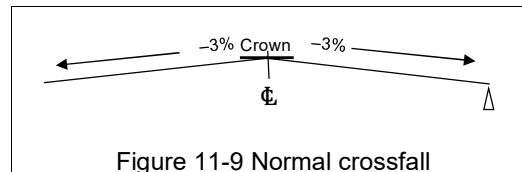


Figure 11-9 Normal crossfall

The minimum drainage cross fall depends on the pavement surface material.

Elevated crossfall (Figure 11-10) can thus be considered as a **positive** slope, rising above the road centreline, at the normal road cross fall.

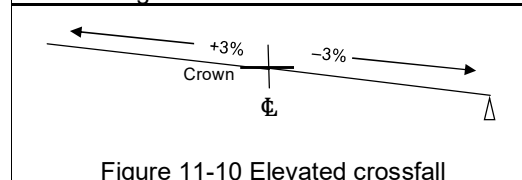


Figure 11-10 Elevated crossfall

Superelevation (Figure 11-11) can be increased over the normal slope. It is needed to overcome the side forces acting on a vehicle following the circular curve portion of composite curve.

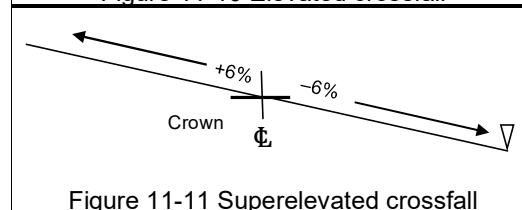


Figure 11-11 Superelevated crossfall

11.1.10.1 Developing superelevation between the straight and circular curve

Figure 11-12 shows that **prior** to the Tangent to Spiral (TS) point, and **after** the Spiral to Tangent (ST) point, there is a portion of the straight called the **Tangent Runout, T_{ro}** . This is the length over which the normal cross fall is elevated to, or reduced from, the level cross fall section at the beginning and end of the spiral.

The formula for length of superelevation development, L_e , provides the answer at a chosen (maximum recommended) rate of rotation, ω . However, the main requirement of superelevation development is to provide a **constant rate** of rotation ($< \omega_{max}$) around the spiral. This rate should then be extended back along the Tangent Runout, T_{ro} .

11.1.10.2 Length of development of superelevation based on rate of rotation

The criterion used in developing a road profile from normal cross fall on a straight to a fully superelevated cross fall on a circular curve is the **rate of rotation** of the cross fall.

The maximum rate of rotation, ω , recommended is:

operating speed $< 80 \text{Kmh}^{-1}$ $\omega = 3.5\% \text{sec}^{-1}$
 $= 0.035 \text{radians} \cdot \text{sec}^{-1}$ and

operating speed $> 80 \text{Kmh}^{-1}$ $\omega = 2.5\% \text{sec}^{-1}$
 $= 0.025 \text{radians} \cdot \text{sec}^{-1}$.

It follows that the length required to develop the superelevation depends on:

operating speed,

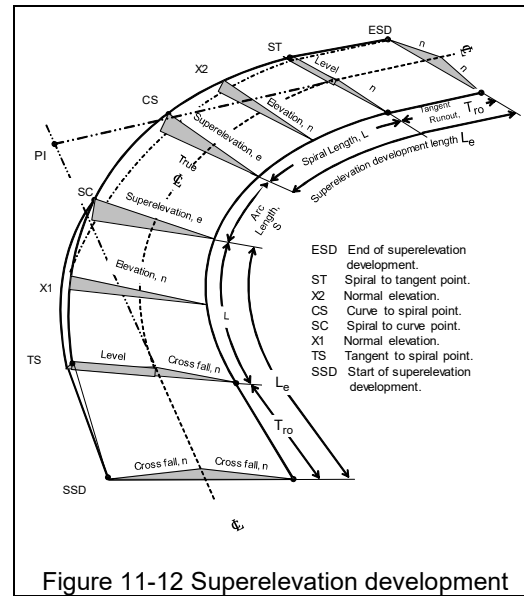
change of relative grade from the straight to the curve.

v = operating speed ($\text{m} \cdot \text{sec}^{-1}$)

e_1 = normal cross fall

e_2 = superelevated cross fall

ω = rate of rotation



Length of superelevation development:
$$L_e = \frac{v(e_2 - e_1)}{\omega}$$
 Eqn 11-7

Ensure that e_1 , e_2 and ω operate in either radians or percentage.

Converting $v \text{ m} \cdot \text{sec}^{-1}$ to Kmh^{-1} :
$$L_e = \frac{v(e_2 - e_1)}{3.6\omega}$$
 Eqn 11-8

For $\omega = 0.035 \text{radians} \cdot \text{sec}^{-1}$, $v < 80 \text{Kmh}^{-1}$: $L_{e(\min)} = v(e_2 - e_1)/0.126$

For $\omega = 0.025 \text{radians} \cdot \text{sec}^{-1}$, $v > 80 \text{Kmh}^{-1}$: $L_{e(\min)} = v(e_2 - e_1)/0.09$

Table 11-3 (Rural Road Design, Table 9.3, p43)

Operating Speed (Kmh^{-1})	Rate of rotation $\text{R} \cdot \text{sec}^{-1}$	Length of super elevation development from normal cross fall to required superelevation			
		-0.03 to $+0.03$	-0.03 to $+0.05$	-0.03 to $+0.07$	-0.03 to $+0.10$
50	0.035	24	32	40	52
60	0.035	29	38	48	62
70	0.035	33	44	56	72
80	0.025	53	71	89	116
90	0.025	60	80	100	130
100	0.025	67	89	111	--
110	0.025	73	98	122	--
120	0.025	80	107	--	--
130	0.025	87	116	--	--

11.1.10.3 Length of superelevation development based on rate of rotation

11.1.10.4 **Caution.** Use of Table 11-1 to Table 11-3

The tables have been incorporated into this course manual to provide a means of checking your calculations of the derived formulae. They are **not** to be used for other than course work within units provided by the Discipline of Spatial Sciences.

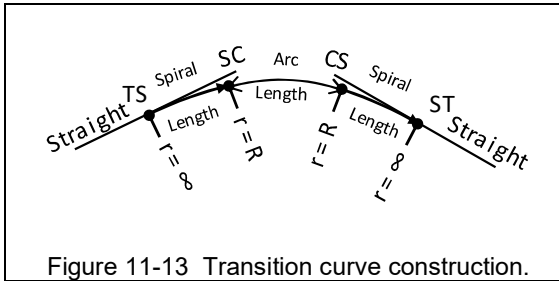


Figure 11-13 Transition curve construction.

11.2 The Transition or Spiral Curve

Points around the transition curve (Figure 11-13):

Straight to TS

TS (Tangent to Spiral) Start spiral

TS – SC. Spiral curve. ∞ to R

SC (Spiral to Curve) Start circular

SC – CS. Circular curve, R constant

CS (Curve to Spiral) End circular

CS – ST. Spiral curve. R to ∞

ST (Spiral to Tangent) End spiral

TS to straight

11.2.1 Introduction to Transition Curves

Much of the derivation of the transition curve presented here is based on the permitted use of the late J.A.L. Cavill's publication, "Survey Engineering, A Guide to First Principles."

The transition curve is a curve that joins a straight path to a circular curve in a gradual manner. The transition curve is part of a spiral. It is designed to provide uniform change of radial acceleration from zero, on the straight, to the design radial acceleration of the circular curve.

Because the change from a straight section of road directly to a curve may be too abrupt, a spiral curve is introduced between the straight and curve and again between the curve and straight. The spiral gradually introduces super-elevation from zero at the TS to its maximum at SC. The spiral from the CS to the ST reverses this procedure.

The distance between the TS and SC, and CS and TS, is called the **length, L** , of the transition curve.

The curve used to achieve this transition is the **Clothoid**, or Euler's Spiral, curve. This figure has the property that its curvature varies linearly with its arc length.

Clothoid **Spiral Angle**

$$\Phi = \frac{L}{2R} \text{ (radians)}$$

Eqn 11-9

11.2.2 Introducing the Transition Spiral

The transition curve is inserted into a space between the tangents of the circular curve and its arc by **shifting** the circular curve “inwards” away from the tangents. **The radius, R , of the curve does not change.** The centre of the curve moves away from the PI. Figure 11-14.

In making such a shift the deflection angle, Δ , of the circular curve decreases by an amount 2Φ , dependent on the length, L , of the transition spiral.

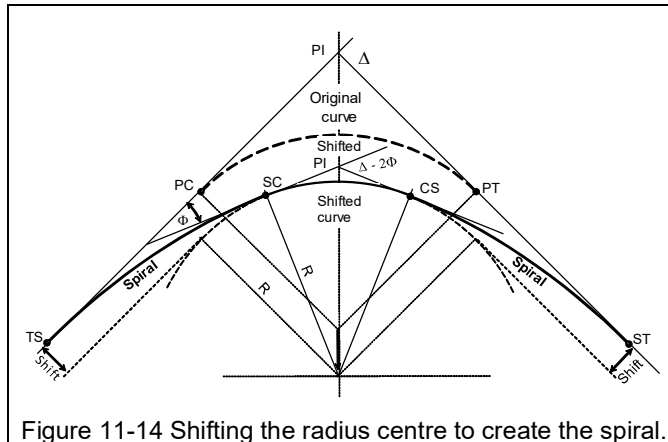


Figure 11-14 Shifting the radius centre to create the spiral.

$$\text{Shifted circular deflection angle } S_{\Delta} = R(\Delta - 2\Phi)$$

Eqn 11-10

The **length** of the shifted curve is then $L = S_{\Delta}R$.

11.2.3 Equations for the Clothoid

The Clothoid, or Euler's spiral, is also called the Cornu curve.

The parametric form of the curve's equations is:

$$x = \int_0^t \sin u^2 du$$

$$y = \int_0^t \cos u^2 du$$

These forms are derived from the Fresnel Integrals.

The clothoid spiral property:

Curvature varies linearly with arc length.

Thus, we will develop formulae that use the arc length (ℓ) along the transition curve between the start of the transition, TS and the start of the circular curve, SC. (Figure 11-14).

At SC the arc length, ℓ , equals the **length** of the transition curve, L .

The equation for the Clothoid is:

$d\phi/d\ell = s$. The curvature (s) is linearly related to arc length ℓ .

ℓ any length around the spiral from TS

p any point on spiral at distance ℓ from TS

r radius of curvature at p

ϕ (phi), tangential angle (radians) between tangents through TS and p

L total **length** of spiral from TP to SC

R radius at L , radius of circular curve

Φ (phi), the **spiral angle** (radians) between tangents through TS and SC.

NOTE: both ϕ and Φ are measured in **RADIANS**.

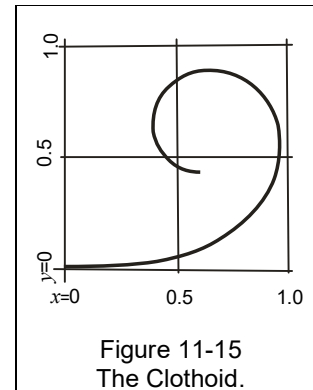


Figure 11-15
The Clothoid.

11.2.4 Developing the Spiral Angle

The curvature increases with length (Figure 11-16) so that:

$r\ell = RL = \text{a constant}$, thus:

$$\frac{1}{r} = \frac{\ell}{LR}$$

At p , $d\phi = \frac{d\ell}{r}$

so that $\frac{d\phi}{d\ell} = \frac{\ell}{LR}$

and integrating:

$$\phi = \frac{\ell^2}{2LR} + C$$

When $\ell = 0$ then:

$$\phi = 0 \text{ and } C = 0$$

And, when $\ell = L$:

$$\Phi = L/2R, \text{ (Eqn 11-9) the Spiral Angle in radians.}$$

It follows that, for any point p on the spiral:

$$\phi = \frac{\ell^2}{2LR} \text{ and, substituting } R = \frac{L}{2\Phi} = \frac{\ell^2}{2L(L/2\Phi)} = \frac{\ell^2}{2L^2/2\Phi}$$

$$\phi = (\ell/L)^2 \Phi$$

Eqn 11-11

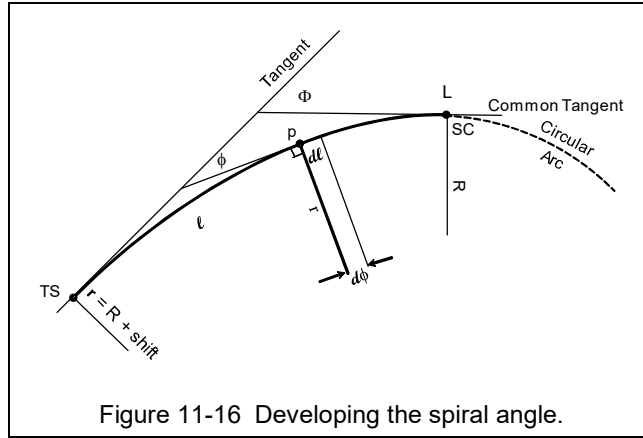


Figure 11-16 Developing the spiral angle.

11.2.5 Cartesian Coordinates of a Point on the Spiral

Cartesian coordinates of points along the spiral (Figure 11-17), referenced to the tangent, can be developed from a power series.

In let $p(x, y)$

be any point on the spiral.

It is offset from the TS by:

x along the tangent

y perpendicular to the tangent.

Given: from § 11.2.4 and Eqn 11-11

$$\Phi = L/2R, \phi = \ell^2/2LR, \phi = (\ell/L)^2 \Phi.$$

Near p , using $\cos$ and $\sin$ power series:

$$dx = \cos \phi d\ell = \left(1 - \frac{\phi^2}{2!} + \frac{\phi^4}{4!} - \frac{\phi^6}{6!} + \frac{\phi^8}{8!} - \dots\right) d\ell$$

$$dy = \sin \phi d\ell = \left(1 - \frac{\phi^3}{3!} + \frac{\phi^5}{5!} - \frac{\phi^7}{7!} + \dots\right) d\ell$$

By substituting $\phi = (\ell/L)^2 \Phi$ in the above equations, integrating and then restoring ϕ the following expressions are derived:

$$x = \ell \left(1 - \frac{\phi^2}{10} + \frac{\phi^4}{216} - \frac{\phi^6}{9360} + \frac{\phi^8}{685540} - \dots\right)$$

$$y = \ell \left(\frac{\phi}{3} - \frac{\phi^3}{42} + \frac{\phi^5}{1320} - \frac{\phi^7}{75600} + \frac{\phi^9}{6894720} - \dots\right)$$

To find x and y at SC it is necessary to substitute Φ and L for ϕ and ℓ

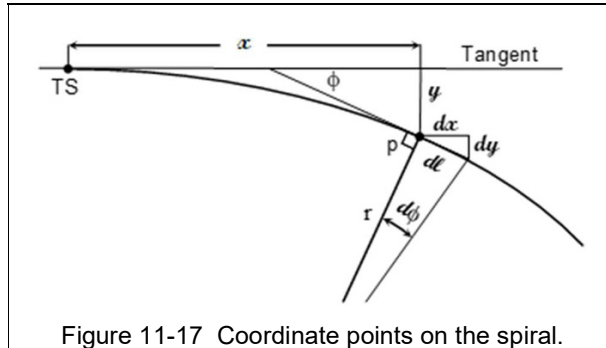


Figure 11-17 Coordinate points on the spiral.

$$x = L \left(1 - \frac{\Phi^2}{10} + \frac{\Phi^4}{216} - \dots \right) \quad \text{Eqn 11-12}$$

$$y = L \left(\frac{\Phi}{3} + \frac{\Phi^3}{42} - \dots \right) \quad \text{Eqn 11-13}$$

When the spiral length, L , and the radius, R , of the arc are known the calculations of ϕ for various values of ℓ can be carried out.

11.2.5.1 Worked example 11.1:

Given $L = 60$, $R = 150$ and $\ell = 25$: (25m from TS)

$$\phi = \frac{\ell^2}{2LR} = \frac{25^2}{(2 \times 60 \times 150)} = 0.03472 \text{ radians}$$

The corresponding x and y are found:

$$x = 25 \left(1 - \frac{0.03472^2}{10} + \frac{0.03472^4}{216} - \dots \right) = 25(1 - 0.000121 + 6.73^{-9}) = 25 \times 0.9998794$$

$$x = 24.997$$

$$y = 25 \left(\frac{0.03472}{3} - \frac{0.03472^3}{42} + \dots \right) = 25(0.0115741 - 9.97^{-7}) = 25 \times 0.0115740$$

$$y = 0.289$$

Generally, sufficient terms should be used in the determination of x and y so that the change in value is less than 1mm between successive terms.

$$y = \ell \left(\sum_{n=1}^{\infty} (-1)^{n-1} \frac{\phi^{2n-1}}{(4n-1)(2n-1)!} \right) \quad \text{Eqn 11-14}$$

The general formula for n terms is thus:

$$x = \ell \left(\sum_{n=0}^{\infty} (-1)^n \frac{\phi^{2n}}{(4n+1)(2n)!} \right) \quad \text{Eqn 11-15}$$

11.3 Design and Set-out of Full Transition Curve. Worked Example 11.2

The introduction of the transition spiral into a previously designed circular curve makes use of the original design parameters.

Figure 11-18 is an existing circular curve that has to incorporate a transition curve. The chainage of the IP is known and the bearings of the tangents of the two straights are known.

The design speed for the curve is known, thus the radius of the circular curve is determined from road design recommendations.

Chainage IP	= 2150.63m
Deflection angle, D	= 81°
Design speed	= 50Kmh ⁻¹
Radius R	= 150m.

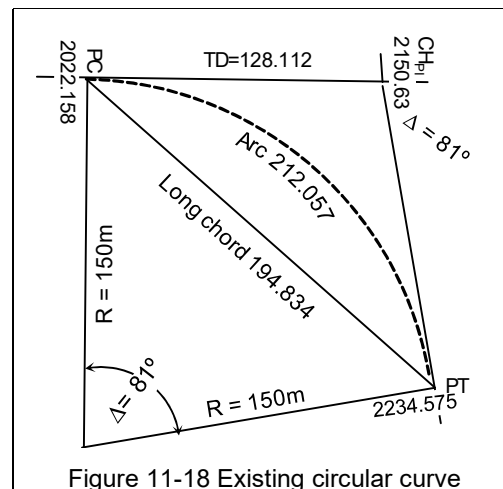


Figure 11-18 Existing circular curve

11.3.1 Length of Transition Spiral

The length of the transition spiral is calculated from:

$$L = \frac{V^3}{cR} \quad \text{Eqn 11-16}$$

where:

V in $m.s^{-1}$,

R in m and

c in $m.s^{-3}$.

c is an empirical figure for radial acceleration around the curve which does not result in discomfort for the human body.

Transformed into Km/h the empirical formula becomes:

$$L = \frac{V^3}{3.6^3 cR} = \frac{V^3}{46.66cR} = \frac{0.0214V^3}{cR} \approx \frac{V^3}{47cR} \quad \text{Eqn 11-17}$$

c is generally 0.3 to a maximum of 0.6 except in mountainous areas where $c_{\max}$ is 0.9 for $V_{\max}$ 14m.sec⁻¹. (50Km/hr).

11.3.1.1 Calculate transition spiral length

At a design speed of 50Km/h,

$c = 0.3$,

a circular curve is designed with radius $R = 150m$

The length of the transition spiral is thus:

$$L = \frac{V^3}{46.66cR} = \frac{50^3}{46.66 \cdot 0.3 \cdot 150} = \frac{125000}{2099.5} = 59.54m.$$

The designer would then have the choice of using:

- the calculated value
- changing the value to, say, 60m for the design length
- using a shorter value of 50m, with a consequent value of $c = 0.36$.

Choose:	$L = 60m$,
Design Radius:	$R = 150m$

11.3.1.2 The spiral angle

The spiral angle Φ is calculated from Eqn 11-11:

Continuing the calculations for a spiral length:

$L = 60$

$R = 150$

$$\Phi = \frac{L}{2R} = \frac{60}{2 \times 150} = 0.2^R = 11^\circ 27' 33''$$

11.3.1.3 Cartesian coordinates of spiral to curve point, SC, on the spiral

Cartesian coordinates of Spiral to Curve point, SC, along the spiral, referenced to the tangent, have been developed from a power series.

Referring to Figure 11-19,

To find X and Y at SC it is necessary to use Φ and L:

$L = 60$

$R = 150$

$\Phi = 0.2^R$

$\Phi = 11^\circ 27' 33''$

Φ radians in Eqn 11-12 and Eqn 11-13:

$$\begin{aligned}
 X &= L \left(1 - \frac{\Phi^2}{10} + \frac{\Phi^4}{216} - \dots \right) \\
 &= 60 \left(1 - \frac{0.2^2}{10} + \frac{0.2^4}{216} - \dots \right) \\
 &= 60(1 - 0.004 + 7.41 \times 10^{-7} - \dots) \\
 &= 60 \times (0.996) \text{ Note: 2nd term unnecessary.} \quad \boxed{X = 59.760} \\
 Y &= L \left(\frac{\Phi}{3} - \frac{\Phi^3}{42} + \dots \right) = 60 \left(\frac{0.2}{3} - \frac{0.2^3}{42} + \dots \right) = 60(0.067 - 0.0002 + \dots) \\
 &= 3.988586 \text{ Note the second term is necessary.} \quad \boxed{Y = 3.989}
 \end{aligned}$$

11.3.1.4 The shift distance

Figure 11-19 shows that:

Shift $S = Y - R(1 - \cos \Phi)$	Eqn 11-18

Where, from § 11.3.1.4:

$$Y = 3.989.$$

The mid ordinate, or crown distance (§ 10.2.8 formula (vi))

$$\begin{aligned}
 R(1 - \cos \Phi) &= 150 \times (1 - 0.980067) \\
 &= 150 \times 0.019933 = 2.990
 \end{aligned}$$

$$\text{Shift } S = 3.988586 - 2.990013 = 0.998573 = 1.00.$$

The value for the shift can be found directly from the approximation:

Shift $S = \frac{L^2}{24R}$	Eqn 11-19

Using the approximation (Eqn 11-19) and values from § 11.3.1.1:

$$\text{Shift} = \frac{L^2}{24R} = \frac{60^2}{24 \times 150} = \frac{3600}{3600}$$

Shift = 1.000

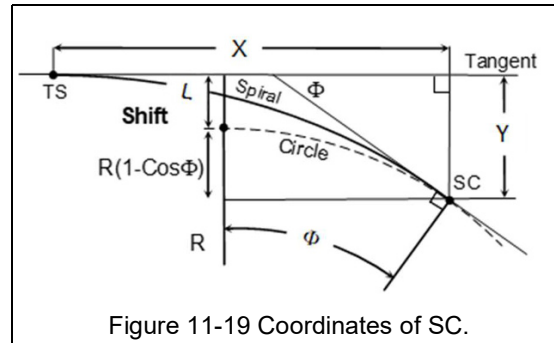
11.3.1.5 Derivation of the shift distance approximation

$$\text{Shift } S = Y - R(1 - \cos \Phi)$$

By expanding $\cos \Phi$,

substituting $L/2R$ for Φ , or $L/2\Phi$ for R
where appropriate and

subtracting $R(1 - \cos \Phi)$ from the series
for Y it can be shown that:



$$\begin{aligned}
 \text{Shift} &= \frac{L^2}{24R} \cdot \left(1 - \frac{\Phi^2}{28} + \frac{\Phi^4}{1320} - \dots \right) = 1 \left(1 - 0.001429 + 1.21 \times 10^{-6} \dots \right) = 1 \times 0.99857 \\
 &= 0.99986 \text{ or } 1.000
 \end{aligned}$$

The approximation $L^2/24R$ (Eqn 11-19) finds the Shift sufficiently well from the Transition Length, L , and the Circular Radius, R , without using further terms.

11.3.1.6 Tangent distance; TS to PI

Recall the formula for Tangent Distance for a **circular curve** (Eqn 9-2):

$$TD = R \tan\left(\frac{\Delta}{2}\right) \text{ where } \Delta \text{ is the deflection angle between the two straights.}$$

By introducing a transition from the straight to the circular curve, the Tangent Distance from the Transition Start (TS) to the Point of Intersection (PI) of the two curves will be significantly longer than the circular curve Tangent Distance from Tangent Point (TP) to the PI.

We will continue with the problem already started:

Assume: $\Delta = 81^\circ$.

$L = 60\text{m}$, given

$R = 150\text{m}$, given

$S = 1.000\text{m}$ (§ 11.3.1.4)

$X = 59.760\text{m}$ (§ 11.3.1.3)

$Y = 3.989\text{m}$ (§ 11.3.1.3)

$\Phi = 0.2^R$ (§ 11.3.1.2)

$\Phi = 11^\circ 27' 33''$

From Figure 11-20 Tangent distance to PI. (not drawn to scale). It can be seen that:

Tangent Distance;

$$TD = (R + S) \tan\left(\frac{\Delta}{2}\right) + (X - R \sin \Phi)$$

$$(R + S) \tan\left(\frac{\Delta}{2}\right) = (150 + 1) \tan\left(\frac{81^\circ}{2}\right)$$

$$= 151 \times (0.8451) = 128.966\text{m}$$

$$(X - R \sin \Phi)$$

$$= 59.760 - 150 \sin(11^\circ 27' 33'')$$

$$= 59.760 - 150 \times (0.1987)$$

$$= 59.760 - 29.800 = 29.960$$

$$TD = 128.966 + 29.960$$

$$TD = 158.926\text{m}$$

Note: a **check** of the Transition Tangent Distance can be made using approximations:

$$TD = TD \text{ (circular curve)} + \text{shift} + \text{half Transition Length } (L/2)$$

$$= 150 \times \tan(40.5^\circ) + 1.0 + 60/2 = 128.11 + 1 + 30$$

$$= 159.11$$

11.3.1.7 Circular distance: SC to CS

From Figure 11-20 (not drawn to scale) it can be seen that:

Circular Distance;

$$Arc = R(\Delta - 2\Phi)^{\text{radians}}$$

$$= 150(81 - 2 \times (11^\circ 27' 33''))^R = 150(81 - 22^\circ 55' 06'')^R$$

$$= 150(58^\circ 04' 54'')^R = 150 \times 1.01372$$

$$= 152.058$$

$$Arc = 152.058$$

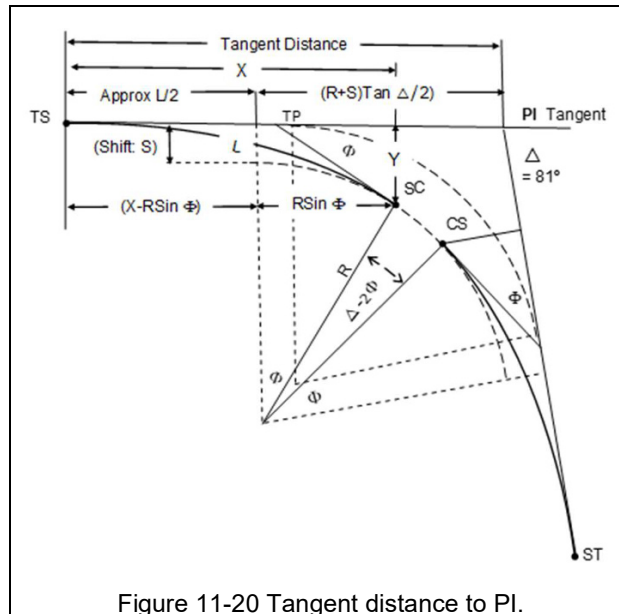


Figure 11-20 Tangent distance to PI.

11.3.1.8 Tangential components of transition curve: TS to SC

Setting out the basic points of the transition curve requires:

Deflection of chord from TS to SC: the Tangential angle δ

Distance from TS to SC: the Long Chord

Distance from TS to the intercept of the common tangent at SC with the straight from TS to PI: the Long Tangent

Distance from the straight/common tangent intercept to SC: the Short Tangent.

From Figure 11-21:

Tangential angle $\delta = \arctan ((Y/X))$

Long Chord $= \sqrt{(X^2 + Y^2)}$

Long Tangent $= X - Y \cot \Phi$

Short Tangent $= Y \cdot \operatorname{cosec} \Phi$

Again, taking figures already calculated (Figure 11-22):

$X = 59.760\text{m}$ (§11.3.1.3)

$Y = 3.989\text{m}$

$\Phi = 0.2^{\text{R}}$ (§11.3.1.2)

$\Phi = 11^{\circ}27'33''$

Tangential angle $\delta = \arctan \left(\frac{Y}{X} \right)$

$$= \arctan \left(\frac{3.989}{59.760} \right)$$

$$= \arctan (0.06675) = 3.819^{\circ}$$

$$\delta = 3^{\circ}49'08''$$

Long chord $= \sqrt{X^2 + Y^2}$

$$= \sqrt{59.7^2 + 3.989^2} = \sqrt{3587.17}$$

$$\text{Long Chord} = 59.896$$

Long tangent $= X - Y \cot \Phi$

$$= 59.76 - \frac{3.98}{\tan(11^{\circ}27'33'')}$$

$$= 59.76 - \frac{3.98}{0.20271}$$

$$= 59 - 19.678$$

$$\text{Long Tangent} = 40.082$$

Short tangent $= Y \csc \Phi \left(\csc = \frac{1}{\sin} \right)$

$$= \frac{3.989}{\sin(11^{\circ}27'33'')} = \frac{3.989}{0.19867}$$

$$\text{Short Tangent} = 20.079$$

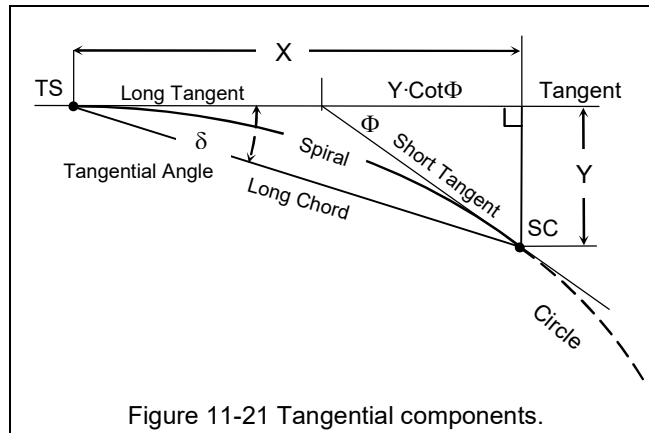


Figure 11-21 Tangential components.

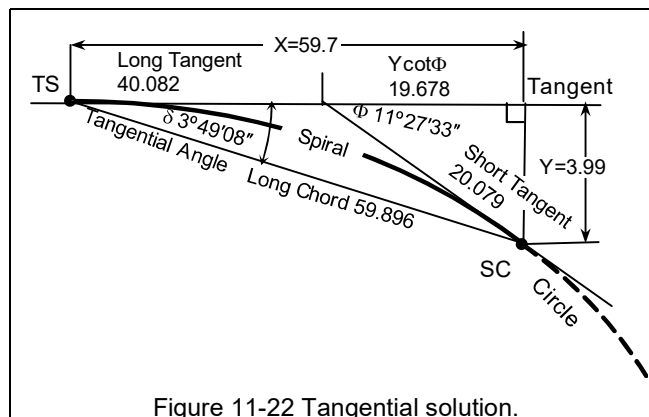


Figure 11-22 Tangential solution.

11.3.1.9 Chainages from TS to ST

The chainage around the curve (sometimes called the “through chainage”) generally starts with the definition of the chainage of the PI. From this point the chainage of the TS is determined by subtracting the Tangent Distance.

Note that the two spirals are of equal length. The arc is not to scale.

The through chainages of the other points are then found by adding the lengths of the respective curves.

Continuing the example, assign a value for the through chainage of the PI. (Figure 11-23)

Chainage PI = 2150.63.

PI	=	2150.63	2150.63	PI
– TD	=	<u>–158.926</u>		
TS	=	1991.704	1991.704	TS
+ L	=	<u>+60.000</u>		
SC	=	2051.704	2051.704	SC
+ Arc	=	<u>+152.058</u>		
CS	=	2203.762	2203.762	CS
+ L	=	<u>+60.000</u>		
ST	=	2263.762	2263.762	ST

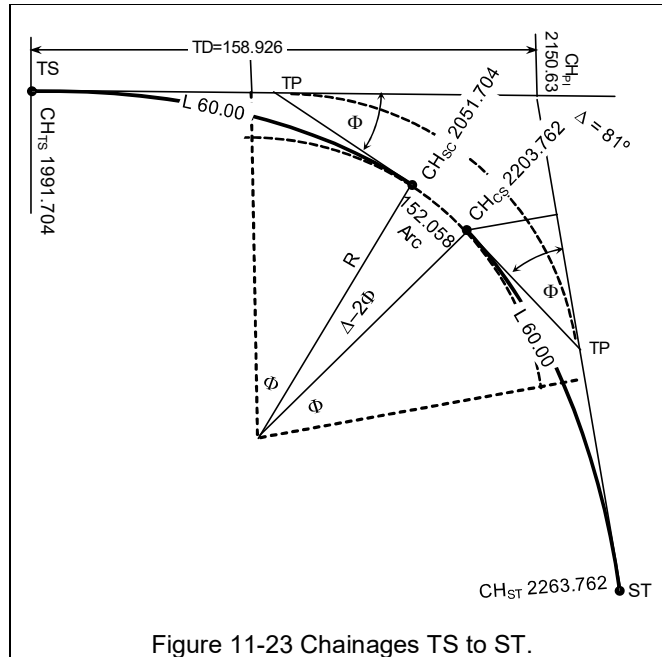


Figure 11-23 Chainages TS to ST.

11.3.2 Set out Calculations for the Spiral and the Arc

Practical set out methods for pegging the two transition spirals and the circular arc are covered in your practical session.

Modern mapping systems, such as Civilcad, produce coordinates in accordance with the road design. Combined with control points, the centre line coordinates can be set out using by radiation using a Total Station.

However it is possible to provide some initial set-out data which can be used in conjunction with setting out the TS, SC, CS and ST points.

In general, by setting up on the PI the TS can be set by measuring the Tangent Distance (TD) back along the straight. The ST is then set by measuring the TD forward, through the deflection angle, down the connecting straight.

11.3.2.1 Deflection calculations for the spiral and the arc

The Length of the spiral, $L = 60\text{m}$. Since the circular Arc Length (PC to TP) $\approx 152\text{m}$ (§11.3.1.7) it would make sense to use arc lengths of $\ell = 150\text{m}$. The extra 2m to finish the circular curve are immaterial.

11.3.2.1.1 Spiral arc

For the spirals, prepare a table of deflection angles and cumulative chords using the power series in § 11.2.5.

$$\phi_n = \frac{\ell_n^2}{2LR} \text{ where } \ell_n \text{ is arc length chainage, } n, \text{ and } \phi \text{ in radians.}$$

$$L = 60m, R = 150m \text{ from design}$$

$$\text{Cumulative deflection } \delta_n = \arctan \left(\frac{y_n}{x_n} \right)$$

See § 11.3.1.1

$$\text{Cumulative chord } d_n = \sqrt{y_n^2 + x_n^2}$$

A table of y_n and x_n allows the use of the rectangular to polar function to find deflection and cumulative chord distance (d).

Modifying Eqn 11-11 and Eqn 11-12 for the power series to generate x and y is illustrated in § 11.2.5.1.

$$x = \ell \left(1 - \frac{\phi^2}{10} + \frac{\phi^4}{216} - \dots \right) \quad y = \ell \left(\frac{\phi}{3} + \frac{\phi^3}{42} - \dots \right)$$

Table 11-4 Spiral arc.

ℓ	$\phi = \ell^2/2LR$ radians	x	y	δ ° ' "	Cumulative chord d
0	0.0	0	0	0 00 00	0.000
15	0.0125	15.000	0.062	0 14 19	15.000
30	0.05	29.992	0.500	0 57 18	29.997
45	0.1125	44.943	1.686	2 08 54	44.975
60	0.2	59.760	3.989	3 49 06	59.893

11.3.2.1.2 Circular arc

For the arc, prepare a table of deflection angles and cumulative chords (ℓ) and deflection angles (δ) using the formulae in §10.2.4 and § 10.2.5 (PC to TP).

Note: A difference in terminology between the symbols used for a clothoid and for a circle.

For the clothoid I have used spiral length (ℓ) and cumulative chords (d).

For the circle I have used arc distance (S) and cumulative chords (ℓ). $\delta = \Delta/2$, both in radians.

Table 11-5 Circular arc.

Arc Dist. S	Deflection (δ) ° ' "	Cumulative Chord ℓ	Run = $\times$ (dN) $\ell \cos(\delta)$	Offset = Y (dE) $\ell \sin(\delta)$
0	0 00 00	0.000	0.00	0.00
15	2 51 53	14.994	14.975	0.749
30	5 43 46	29.950	29.800	2.990
45	8 35 40	44.831	44.328	6.700
60	11 27 33	59.601	58.413	11.841
75	14 19 26	74.221	71.914	18.363
90	17 11 19	88.656	84.696	26.200
105	20 03 13	102.869	96.633	35.274
120	22 55 06	116.826	107.603	45.494
135	25 46 59	130.490	117.499	56.759
150	28 38 52	143.828	126.211	68.955
152.058	29 02 27	145.630	127.321	70.694

11.3.2.2 Setout Calculations for the shifted PI and shifted crown

The PI for the shifted arc, in the direction of the curve centre point, can also be set out. Referring to where S = shift = 1.000 (§11.3.1.4):

$$\begin{aligned}\text{Shifted secant} &= S(\sec(\Delta/2)) = 1.000/\cos(81^\circ/2) \\ &= 1/(0.760406)\end{aligned}$$

$$\boxed{\text{Shifted Secant} = 1.315}$$

From Figure 17.15: (Shift exaggerated)

The Crown Secant for the original curve

$$\begin{aligned}&= R \times (\sec(\Delta/2) - 1) = 150 \times ((1/\cos(40.5^\circ) - 1)) \\ &= 150 \times 0.315087\end{aligned}$$

$$\boxed{\text{Crown secant (orig)} = 47.263}$$

The Crown Secant for the shifted curve

$$\begin{aligned}&= R \times (\sec((\Delta - 2\Phi)/2) - 1) \\ &= 150 \times ((1/\cos(29^\circ 02' 27'') - 1)) \\ &= 150 \times 0.143806\end{aligned}$$

$$\boxed{\text{Crown secant (shifted)} = 21.571}$$

PI to PI_{shifted}

$$\begin{aligned}&= \text{Crown Secant} \\ &+ \text{Shifted Secant} \\ &- \text{Shifted Crown Secant} \\ &= 47.263 + 1.315 - 21.571\end{aligned}$$

$$\boxed{\text{PI to PI}_{\text{shifted}} = 27.007}$$

PI to Crown_{shifted} This is the centre of the alignment of the circular curve, E.

$$\begin{aligned}&= \text{Crown Secant} + \text{Shifted Secant. Also} = (R + S) \times \sec(\Delta/2) - R \\ &= 47.263 + 1.315\end{aligned}$$

$$\boxed{\text{PI to Crown}_{\text{shifted}} = 48.578}$$

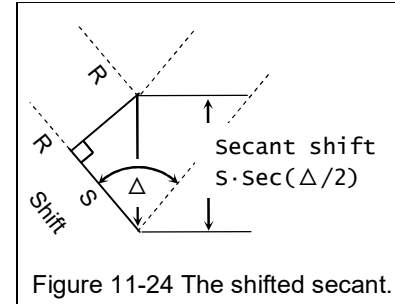


Figure 11-24 The shifted secant.

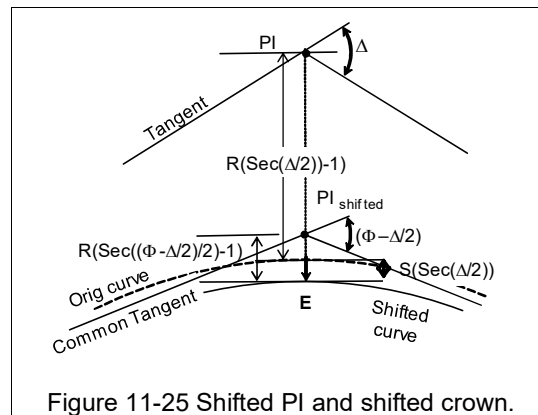


Figure 11-25 Shifted PI and shifted crown.

11.3.3 Diagram of a Circular Arc with Two Spirals

This general diagram, Figure 11-26, is drawn with the shift, S , greatly exaggerated. Refer to Figure 11-25 for the relationship between the PI and shifted PI and the Secant shift.

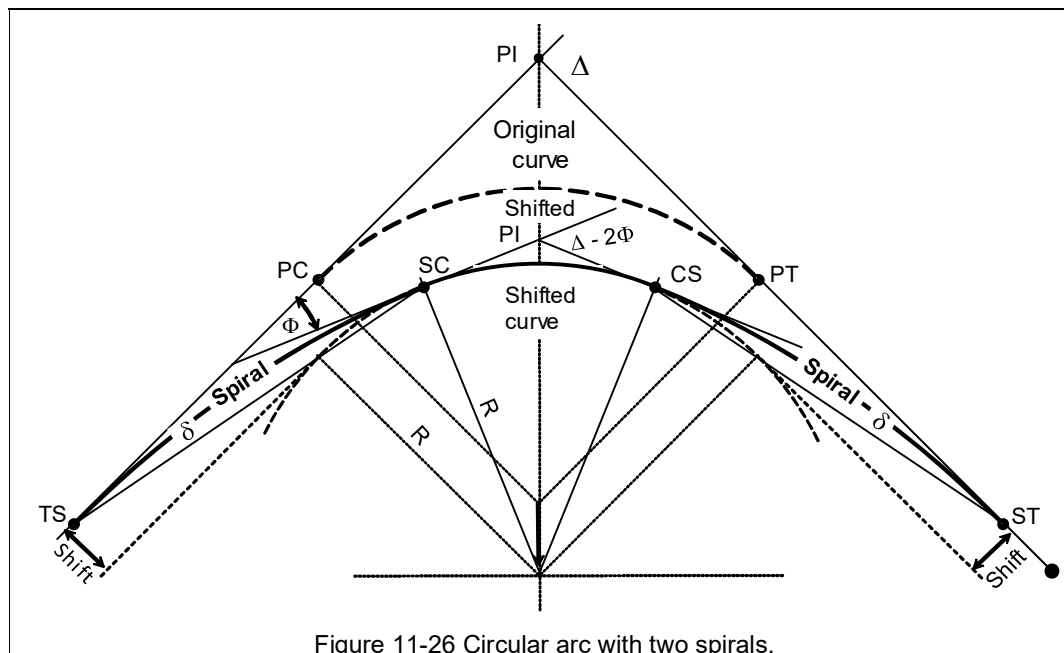


Figure 11-26 Circular arc with two spirals.

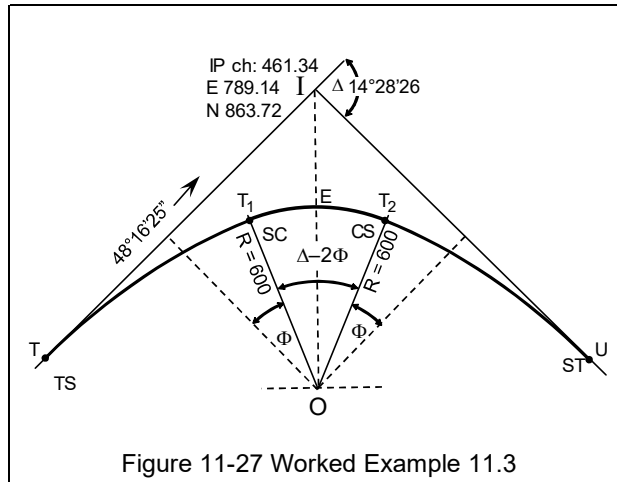
11.4 Worked Example 11.3. Circular Curve with Transition Spirals

This worked example illustrates a problem set for the Curtin University Engineering Control Surveying 281 unit. The nomenclature will follow the format use in the unit text, “Surveying for Engineers”, Uren, J and W.F. Price, The McMillan Press Ltd, Third Edition. Chapter 11, Transition Curves.

Two straights, intersecting at I, with a deflection angle of $14^{\circ}26'28''$, are to be joined by a composite curve consisting of a central circular arc and two transition curves of equal length. The diagram, Figure 11-27, is drawn with the deflection angle, Δ , greatly exaggerated.

The exercise will then continue with the application of super elevation in to, around and out of the compound curve.

Chainage at IP	= 461.34m
E_I	789.14
N_I	863.72
Bearing T to I, θ_{TI}	$48^{\circ}16'25''$
Deflection, Δ	= $14^{\circ}28'26''$
Design speed	= 85Km/h
Circular radius	= 600m
Radial acceleration (c)	= 0.3msec^{-3}
Crossfall	= $(-0.03) - 3\%$
Superelevation	= $(+0.06) + 6\%$



The curve is to be pegged at every even 20m chainage. Superelevation around a curve is examined in Chapter 12. Calculations of Superelevation on a Composite Curve Page 260.

11.5 Step by Step Solution:

11.5.1 Transition Curve Length

- 1) Find length of transition curves T – T₁, T₂ – U.

Eqn 11-17, § 11.3.1.

$$L = \frac{V^3}{3.6^3 c R} = \frac{85^3}{46.66 \times 0.3 \times 600} = \frac{614125}{8398.08} = 73.127$$

$$L = 75\text{m} \quad (5\text{m rounding AGRD03-16 recommendation, §7.5.4})$$

11.5.2 Spiral angle

- 2) Find the Spiral angle, $\Phi = \frac{L}{2R}$

Eqn 11-9, § 11.2.2

$$= 75 / (2 \cdot 600) = 0.0625^R = 3^{\circ}34'52''$$

$$\Phi = 3^{\circ}34'52''$$

11.5.3 Length of Circular Curve

- 3) Find length of the circular curve T₁ – T₂.

- a) Shifted curve deflection angle = $\Delta - 2\Phi$

§ 11.2.2

$$\Delta_s = 14^{\circ}28'26'' - 2 \times (3^{\circ}34'52'') = 7^{\circ}18'43'' = 0.127617^R$$

$$\Delta_s = 7^{\circ}18'43''$$

- b) Arc length, S, = $R \Delta_s^R$

$$= 600 \cdot (0.127617) = 76.570\text{m}$$

$$\text{Arc, S} = 76.57\text{m}$$

11.5.4 Tangent Distance

- 4) Find tangent distance
- $T - I$
- .

§ 11.3.1.6

$$TD = (R+S)\tan(\Delta/2) + (X-R\sin\Phi)$$

Three **components** are needed:

- a) Spiral angle,
- $\Phi = 3^\circ 34' 52''$
- (
- 0.0625^R
-)

§ 11.5.2

- b) X and Y Cartesian coordinate of
- T_1
- ,

§ 11.3.1.3

Using radians for Φ

$$X = L \left(1 - \frac{\Phi^2}{10} + \frac{\Phi^4}{216} - \dots \right)$$

$$= 75 \times (1 - (0.0625)^2/10 + (0.0625)^4/216 - \dots) = 75 \times (1 - 0.000391 + 7.06^{-8} - \dots)$$

$$= 75 \times 0.99961 = 74.971$$

Note: 2nd term unnecessary.

$$\mathbf{X = 74.971}$$

$$Y = L \left(\frac{\Phi}{3} - \frac{\Phi^3}{42} + \dots \right) \text{ Y coordinate for Long Chord}$$

$$= 75 \times (0.0625/3 - (0.0625)^3/42 + \dots) = 75 \times (0.020833 - 0.000006 + \dots)$$

$$= 75 \times 0.020833 = 1.561$$

Note: 2nd term unnecessary.

$$\mathbf{Y = 1.561}$$

- c) Shift,
- $S = L^2/24R$

§ 11.3.1.5

$$= 75^2/(24 \times 600) = 0.391\text{m}$$

$$\mathbf{\text{Shift, } S = 0.391}$$

Combine the 3 components, and $\Delta = 14^\circ 26' 28''$, to find Tangent Distance:

§ 11.3.1.6

$$TD = (R+S)\tan(\Delta/2) + (X-R\sin\Phi)$$

$$(R+S)\tan(\Delta/2) = (600 + 0.391)\tan(14^\circ 28' 26''/2) = 600.391 \times (0.126985)$$

$$= 76.240\text{m}$$

$$(X-R\sin\Phi) = 74.971 - 600 \times \sin(3^\circ 34' 52'') = 74.971 - 37.476$$

$$= 37.495\text{m}$$

$$TD = 76.240 + 37.495 = 113.735$$

$$\mathbf{TD = 113.735}$$

11.5.5 Find Through Chainages: Figure 11-28

- 1) Start of tangent spiral,
- T
- , from IP.

$$T = IP - TD$$

$$461.34 - 113.735$$

$$\mathbf{T = 347.605}$$

- 2) End of first spiral,
- T_1
- , from
- T

$$T_1 = T + L$$

$$347.605 + 75$$

$$\mathbf{T_1 = 422.605}$$

- 3) Find the chainage of shifted

crown secant at E.

$$E = T_1 + S/2$$

$$422.605 + (76.57/2)$$

$$\mathbf{E = 460.390}$$

- 4) Find through chainage of
- T_2
- around the circular curve

$$T_2 = T_1 + S$$

$$422.605 + 76.570$$

$$\mathbf{T_2 = 499.175}$$

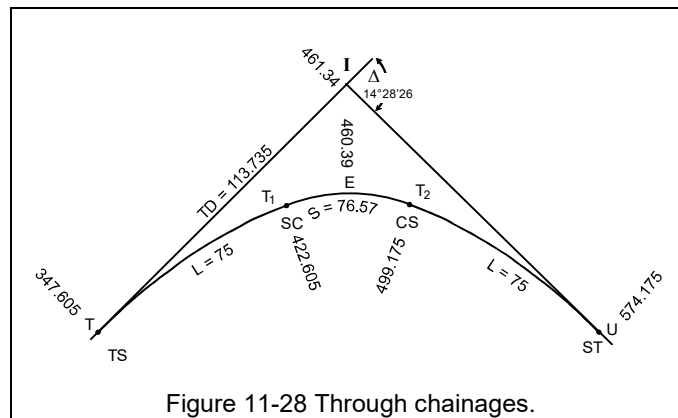


Figure 11-28 Through chainages.

- 5) Find through chainage of U. along the second symmetrical spiral, length L

$$U = T_2 + L$$

$$499.175 + 75$$

$$U = 574.175$$

11.5.6 Determine 20m Chainage Points on First Spiral:

Prepare a table of chords and deflection angles for set out of the spiral from T to T₁.

Use the power series in § 11.2.5

$\phi_{CH} = \ell_{CH}^2/2LR$ and ϕ is in radians

$$x = \ell \left(1 - \frac{\phi^2}{10} + \frac{\phi^4}{216} - \dots \right)$$

$$y = \ell \left(\frac{\phi}{3} - \frac{\phi^3}{42} + \dots \right)$$

$$\delta_{CH} = \text{Atan}(y_{CH}/x_{CH}).$$

$$d_{CH} = \sqrt{y^2 + x^2}$$

A table of x_{CH} and y_{CH} (Table 11-6) will allow the use of the rectangular to polar function to find deflection angle or tangential angle (δ) and cumulative chord distance (d).

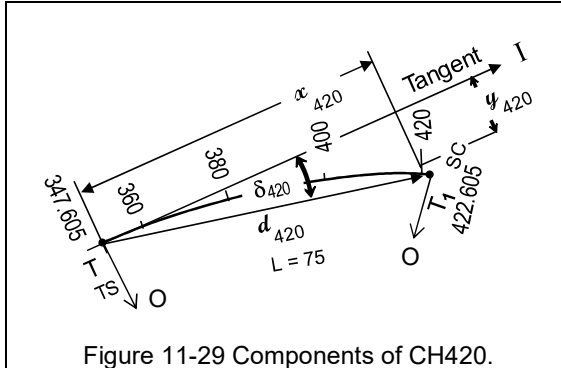


Figure 11-29 Components of CH420.

11.5.6.1 Chainage points on spiral curve, T to T₁

Table 11-6 Deflection angle and chord. T to T₁

Stn	Chain	Arc length	Dist. ℓ	$\phi = \ell^2/2LR$ radians	x run	y rise	δ ° ' "	Cumulative Chord d
T	347.605		0.00	0.0	0.00	0.00	0 00 00	0.00
360	360	12.396	12.395	0.001707	12.395	0.007	0 01 57	12.395
380	380	20	32.395	0.011661	32.395	0.126	0 13 22	32.395
400	400	20	52.395	0.030503	52.391	0.533	0 34 57	52.393
420	420	20	72.395	0.058234	72.371	1.405	1 06 44	72.385
T ₁	422.605	2.604	75	0.0625	74.971	1.562	1 11 37	74.987

11.5.7 Determine 20m Chainage Points on Circular Curve, T₁ – T₂

Prepare a table of chords and deflection angles for setout of the circular curve from T₁ in direction of the common tangent. § 11.3.2.1.2

Review formula:

s_{CH} is the arc distance from SC.

$$\Delta_{CH} = \frac{s_{CH}}{R}. \text{ Thus } \delta_{CH} = \frac{\Delta_{CH}}{2}$$

$$\ell_{CH} = 2R \sin(\delta_{CH})$$

$$x_{CH} = \ell \sin \delta$$

$$y_{CH} = \ell \cos \delta$$

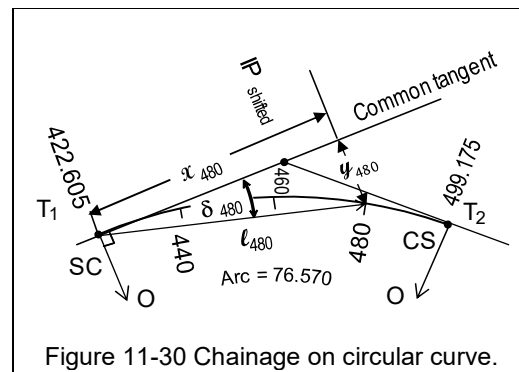


Figure 11-30 Chainage on circular curve.

Table 11-7 Points on circular curve.

Stn	Chain	Arc Dist. s	Deflection ($\delta=\Delta/2$)	Cumulative Chord ℓ	Run = $\times \ell \cos(\delta)$	Offset = $y \ell \sin(\delta)$
T ₁	422.605	0.00	0° 00' 00"	0.000	0.00	0.00
440	440	17.396	0 52 22	17.395	17.393	0.252
460	460	37.396	1 47 10	37.340	37.372	1.165
480	480	57.396	2 44 26	57.374	57.308	2.743
T ₂	499.175	76.570	3 39 21	76.518	76.362	4.879

x and y are shown in Table 11-7, but do not figure in coordinate creation for set-out. The x , y components are aligned with the common tangent.

They would have to be rotated to the bearing of the common tangent to produce coordinates.

11.5.7.1 Chainage points on circular curve T₁ to T₂

Note that the deflection angle to T₂, $\delta_{T2} = \frac{1}{2}(\Delta - 2\Phi)$, i.e. half the shifted deflection angle.

Thus, the calculation check is correct. $(\Delta - 2\Phi) = 7^\circ 18' 43''$ § 11.5.3

11.5.8 Determine 20m Chainage Points on Second Spiral, T₂ to U:

Prepare a table of chords and deflection angles for set-out of the spiral from U in direction T₂.

Use the power series in § 11.2.5

$\phi_{CH} = \ell_{CH}^2/2LR$ and ϕ is in radians

$$x = \ell \left(1 - \frac{\phi^2}{10} + \frac{\phi^4}{216} - \dots \right)$$

$$y = \ell \left(\frac{\phi}{3} - \frac{\phi^3}{42} + \dots \right)$$

$$\delta_{CH} = \arctan(y_{CH}/x_{CH}).$$

$$d_{CH} = \sqrt{(y^2 + x^2)}$$

It is necessary to set the second spiral out from the end of the spiral, U, back towards the circular curve, T₂. You cannot calculate **from** T₂ to U.

The sign of y , in this case, is negative as it is in the 4th quadrant with respect to the direction from U to IP. The tangential angles are thus negative and have to be **subtracted** from the bearing U to I.

11.5.8.1 Chainage points on spiral curve T₂ to U

Table 11-8 Deflection angle and chord. U to T₂.

Stn	Chain	Arc length	Dist. ℓ	$\phi = \ell^2/2LR$ radians	x run	-y rise	δ ° ' "	Cumulative Chord ℓ
U	574.175		0.00	0.0	0.00	0.00	0 00 00	0.00
560	560	14.174	14.174	0.001928	13.174	-0.008	-0 02 13	13.174
540	540	20	34.174	0.012228	33.174	-0.135	-0 14 01	33.174
520	520	20	54.174	0.032609	53.169	-0.557	-0 36 00	53.172
500	500	20	74.174	0.06113				
T ₂	499.175	20.826	75	0.0625	74.971	-1.562	-1 11 38	74.987

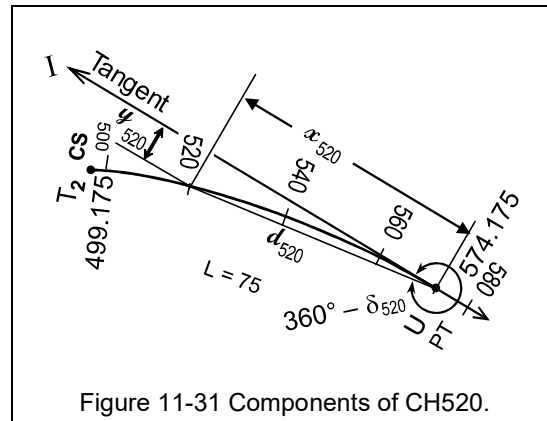


Figure 11-31 Components of CH520.

Calculate the bearing of the shifted crown secant from the IP

$$\text{Intercept angle, } \angle I = 180 - \Delta = 180 - 14^\circ 28' 26'' = 165^\circ 31' 34''$$

$$\text{Brg IT} \quad 228^\circ 16' 25''$$

$$- \angle I/2 \quad 82^\circ 45' 47''$$

$$\text{Brg I - E} \quad \underline{145^\circ 30' 38''}$$

$$(R+S)\sec(\Delta/2) - R = 600.391/\cos((14^\circ 28' 26'')/2) - 600$$

$$\text{Distance IP to E} \quad \underline{5.212}$$

11.5.9.2 Calculate a traverse sheet: I, T, T1, T2, U, I

The traverse closes within 0.002m.

Table 11-9 Traverse sheet closure.						
Stn	Brg	Dist	dE	dN	E	N
I					789.140	863.720
T	228 16 25	113.736	-84.885	-75.700	704.255	788.020
T1	49 28 02	74.987	56.993	48.733	761.248	836.753
T2	55 30 38	76.518	63.068	43.329	824.316	880.082
U	61 33 15	74.987	65.934	35.718	890.250	915.800
I	242 44 51	113.736	-101.111	-52.081	789.139	863.719

If distance to 3d.p. is used the close is exact.

11.5.10 Calculation of Chainage Station Coordinates:

Now that the traverse has been proved, the intermediate chainages can be introduced to a table to produce coordinated for each point.

My technique is to use a join from each of the transition points.

11.5.10.1 For the first spiral, T to T1

Table 11-10 Coordinates T to T1. Radiations from T.

- Use the coordinates of T calculated in the traverse.
- From the TS at T, to each chainage, add the deflections to the initial bearing, T to IP.
- Using the tangential distance, calculate the coordinates of each chainage, adding the dEs and dNs to the initial coordinates of T. Coordinates are radiated from T.
- Check calculated coordinates of T1 agree with the traverse values.

Deflection angle and distance come from Table 11-6 (deflection and chord length).

Table 11-10 Coordinates T to T1.							
Stn	Defl'n	Brg	Dist	dE	dN	E	N
T	Brg to I	48 16 25	0.00			704.255	788.020
360	0 01 56	48 18 22	12.395	9.256	8.245	713.511	796.265
380	0 13 22	48 29 47	32.395	24.261	21.467	728.517	809.488
400	0 34 58	48 51 22	52.393	39.455	34.472	743.711	822.493
420	1 06 45	49 23 09	72.385	54.948	47.120	759.203	835.140
T1	1 11 37	49 28 02	74.987	56.993	48.733	761.248	836.753

11.5.10.2 For the circular arc, T₁ to T₂

Table 11-11 Coordinates T₁ to T₂. Radiations from T₁.

- Use the coordinates of T₁ calculated in the traverse.
- From the SC at T₁, to each chainage, add the deflections to the initial bearing of the common tangent.
- Using the chord distance, calculate the coordinates of each chainage, adding the dEs and dNs to the initial coordinates of T₁.
- Check calculated coordinates of T₂ agree with the traverse values.

Deflection angle and distance come from Table 11-7 (deflection and chord length).

- The shifted crown secant can be set out from the IP

Table 11-11 Coordinates T₁ to T₂.

Stn	Defl'n	Brg	Dist	dE	dN	E	N
T ₁	Brg CT	51 51 17	0.00			761.248	836.753
440	0 52 22	52 43 39	17.395	13.834	10.545	775.083	847.298
460	1 47 10	53 38 27	37.389	30.110	22.166	791.358	858.920
480	2 44 26	54 35 43	57.374	46.764	33.239	808.012	869.993
T ₂	3 39 21	55 30 38	76.518	63.069	43.329	824.317	880.082

11.5.10.3 For the second spiral, T₂ to U

Table 11-12 Coordinates T₂ to U. Rearranged in chainage order after calculation.

Note that you work backwards from U to T₂.

- Use the coordinates of U calculated in the traverse. Radiations from U.
- From the ST at U, to each chainage, **add** the (negative) deflections to the initial bearing, U to IP.
- Using the tangential distance, calculate the coordinates of each chainage, adding the dEs and dNs to the initial coordinates of U.
- Check calculated coordinates of T₂ agree with the traverse values.

Deflection angle and distance come from Table 11-8 (deflection and chord length).

Table 11-12 Coordinates T₂ to U. Rearranged in chainage order after calculation.

Stn	Defl'n	Brg	Dist	dE	dN	E	N
T ₂	-1 11 37	241 33 14	74.987	65.933	35.719	824.317	880.082
500	-1 10 03	241 36 40	74.162	65.225	35.296	825.026	880.505
520	-0 37 22	242 07 29	54.172	47.886	25.328	842.364	890.473
540	-0 14 52	242 29 59	34.175	30.313	15.780	859.937	900.021
560	-0 02 33	242 42 18	14.175	12.596	6.500	877.654	909.301
U	Brg to I	242 44 51	0.000	0.000	0.000	890.250	915.801

11.5.11 Set-out of Chainage Coordinates from Other Control Points

Figure 11-33 represents the composite curve. Its associated control points for set-out are:

G E727.61 N893.83

H E940.57 N886.28

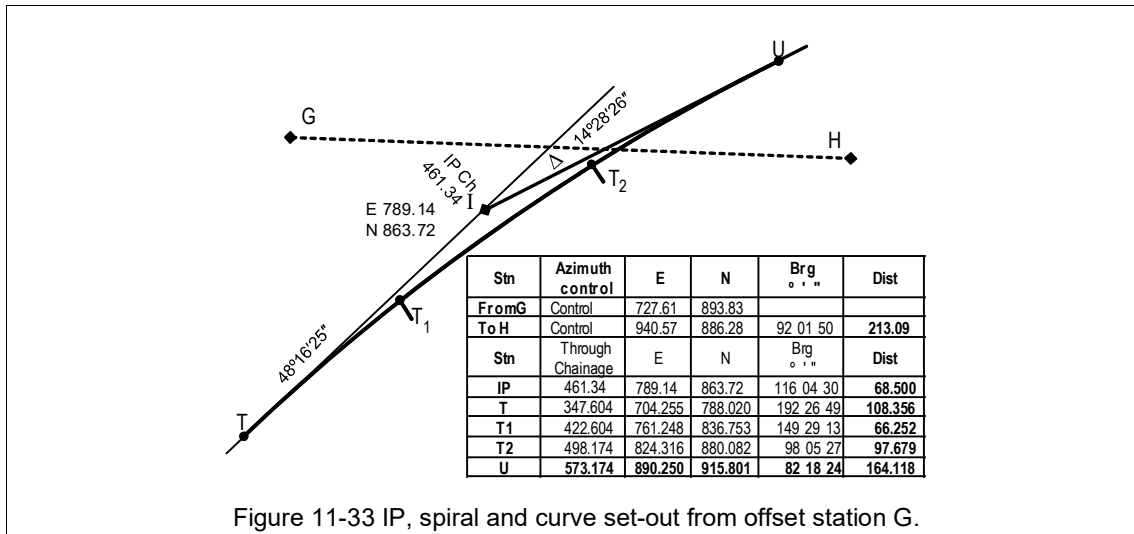


Figure 11-33 IP, spiral and curve set-out from offset station G.

The centre line is to be set-out from Control Point G, orientation by the Bearing G-H.

Brg GH = 92°01'50"

Dist GH = 213.09

Table 11-13 shows the set-out bearings and distances from G.

Table 11-13 Set out of chainage stations from control point G.

Stn	Azimuth control	E	N	Brg o ' "	Dist
From G	Control	727.61	893.83		
To H	Control	940.57	886.28	92 01 50	213.090
Stn	Through chainage	E	N	Brg o ' "	Dist
IP	461.34	789.140	863.720	116 04 30	68.500
T (TS)	347.605	704.256	788.021	192 26 49	108.356
360	360	713.511	796.265	188 13 21	98.578
380	380	728.517	809.488	179 23 02	84.347
400	400	743.711	822.493	167 16 53	73.132
420	420	759.203	835.140	151 42 21	66.653
T1 (SC)	422.605	761.248	836.753	149 29 13	66.252
440	440	775.083	847.298	134 25 36	66.475
460	460	791.358	858.920	118 42 22	72.681
480	480	808.012	869.993	106 30 49	83.861
T2 (CS)	499.175	824.317	880.082	98 05 27	97.679
500	500	825.026	880.505	97 47 21	98.323
520	520	842.364	890.473	91 40 33	114.803
540	540	859.937	900.021	87 19 17	132.472
560	560	877.654	909.301	84 06 47	150.839
U (TS)	574.175	890.250	915.801	82 18 24	164.118

11.6 Equations for the Clothoid Spiral Curve

11.6.1 General Formula

The following formulae relate to a compound curve generated by the Clothoid Spiral curves between two straights of deflection angle, Δ , joined by a circular curve of radius, R .

$$\text{Length of transition spiral } L = \frac{V^3}{3.6^3 c R} \quad (0.3 > c > 0.9, R \text{ in metres, } V \text{ in Km.h}^{-1}) \quad \S 11.3.1$$

$$\text{Spiral Angle} \quad \Phi = L/2R \text{ (radians).} \quad \S 11.2.4$$

$$\text{Projected tangent distance} \quad X = L \left(1 - \frac{\Phi^2}{10} + \frac{\Phi^4}{216} - \dots \right) \quad \S 11.2.5$$

$$\text{Projected tangent offset} \quad Y = L \left(\frac{\Phi}{3} - \frac{\Phi^3}{42} + \dots \right) \quad \S 11.2.5$$

$$\text{Shift Distance} \quad S = \frac{L^2}{24R} \cdot \left(1 - \frac{\Phi^2}{28} + \frac{\Phi^4}{1320} - \dots \right) = L^2/24R \text{ (approx)} \quad \S 11.3.1.4$$

$$\text{Shift Distance} \quad S = Y - R(1 - \cos\Phi) \quad \S 11.3.1.5$$

$$\text{Tangent Distance} \quad TD = (R+S)\tan(\Delta/2) + (X-R\sin\Phi) \quad \S 11.3.1.6$$

$$\text{Shifted Arc length} \quad s = R(\Delta - 2\Phi)^R \quad \S 11.2.2$$

$$\text{Shifted Crown Secant} \quad = (R+S)\sec(\Delta/2) - R \quad \S 11.3.2.2$$

$$\text{Tangential angle} \quad \delta = \arctan((Y/X))$$

$$\text{Long Chord} \quad = (X^2 + Y^2)^{1/2}$$

$$\text{Long Tangent} \quad = X - Y \times \cot\Phi$$

$$\text{Short Tangent} \quad = Y \times \operatorname{cosec}\Phi$$

11.6.2 Spiral Deflections and Chords

Spiral deflection angles and chord distances to a point, p, at length, ℓ , on the spiral:

$$\text{Tangential angle} = \phi^R \quad = \ell^2/2LR \text{ or } \phi = (\ell/L)^2 \Phi \text{ for } \ell_1, \ell_2, \dots, \ell_n$$

$$\text{Where} \quad x = \ell \cdot \left(1 - \frac{\phi^2}{10} + \frac{\phi^4}{216} - \dots \right) \text{ for } \ell_1, \ell_2, \dots, \ell_n$$

$$y = \ell \cdot \left(\frac{\phi}{3} - \frac{\phi^3}{42} + \dots \right) \text{ for } \ell_1, \ell_2, \dots, \ell_n$$

$$\text{Deflection angle} \quad \delta^\circ = \arctan(y/x) \text{ for } \ell_1, \ell_2, \dots, \ell_n$$

$$\text{Chord distance} \quad d = (y^2 + x^2)^{1/2}$$

11.6.3 Circular Deflections and Chords

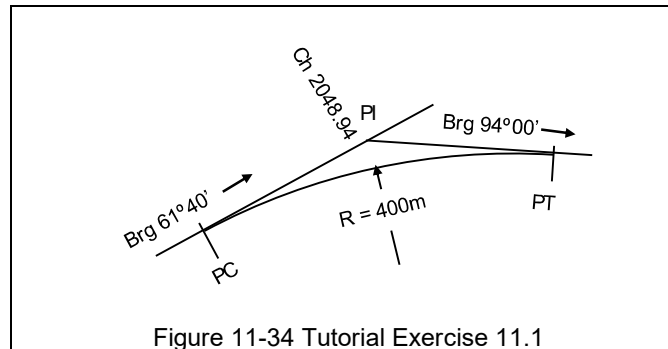
Circular deflection angles and chord distances to a point, p, at arc length, s, on the shifted arc, centre, O:

$$\text{Deflection angle} \quad \delta^r = s/2R \text{ (radians) for } s_1, s_2, \dots, s_n \quad \S 11.3.2.1.2$$

$$\text{Chord length} \quad \ell = 2 \times R \sin(\delta) \text{ for } s_1, s_2, \dots, s_n \quad \S 10.2.4$$

11.7 Tutorial Exercises 11.1

A simple curve in Figure 11-34 is to be improved by the introduction of Transition Spirals at each end.



Given:

- Operating speed = 85 Km h^{-1}
- Radius of circular curve = 400m
- Rate of radial acceleration (c) = 0.3
- Coordinates of PI: E 1219.12, N 1118.15
- Chainage at PI = 2048.94
- Bearing of initial tangent = $61^\circ 40'$
- Bearing of final tangent = $94^\circ 00'$

Calculate:

- a) Transition length, do not round to integer or suitable length
- b) Chainage of TS, SC, CS and ST spiral control points
- c) Show closure of tangents and spiral control points

The centre line is to be pegged at even 20m intervals.

Calculate:

- d) Data to set out centreline pegs of spirals from TS and ST
- e) Data to set out centreline pegs of circular curve from SC
- f) Coordinates of all centreline pegs.

Carriageway width is 10m.

The road formation has a normal cross fall is -0.03 (-3%). Superelevation around the curve is to be a maximum of +0.07 (+7%).

Calculate:

- g) Tangent runout
- h) Chainage at SSD and ESD
- i) Rate of rotation of superelevation development at operating speed
- j) Cross fall and RL of each even 20m chainage from SSD to ESD, include spiral control points.
- k) Chainages where inside cross fall deviates from normal cross fall.

11.7.1 Answers to Tutorial Exercises 11.1:

Calculate:

- a) Tangent Length, $TL = 171.13$
 b) Chainage $TS = 2048.94 - 171.13 = 1877.81$
 $SC = 1877.81 + 109.69 = 1987.50$
 $CS = 1987.50 + 116.04 = 2103.54$
 $ST = 2103.54 + 109.69 = 2213.23$
 c) Show closure of tangents and spiral control points

Stn	Brg	Dist	dE	dN	E	N
PI					1219.12	1118.15
TS	241 40 00	171.13	-150.629	-81.218	1068.491	1036.932
SC	64 17 06	109.60	98.746	47.555	1167.237	1084.487
CS	77 50.00	115.63	113.033	24.370	1280.270	1108.857
ST	91 22 54	109.60	109.568	-2.643	1389.838	1106.214
PI	274 00 00	171.13	-170.713	11.937	1219.125	1118.151

The centre line is to be pegged at even 20m intervals.

- d) 1) Data to set out centreline pegs of spirals from TS and SC.

Stn	Defl'n	Brg	Dist	dE	dN	E	N
TS	Brg to I	61 40 00	0.00			1068.491	1036.932
1880	0 00 04	61 40 04	2.19	1.928	1.039	1070.419	1037.971
1900	0 06 26	61 46 26	22.19	19.551	10.495	1088.042	1047.427
1920	0 23 15	62 03 15	42.19	37.270	19.772	1105.761	1056.704
1940	0 50 31	62 30 31	62.19	55.167	28.708	1123.658	1065.640
1960	1 28 13	63 08 13	82.17	73.303	37.129	1141.794	1074.061
1980	2 16 21	63 56 21	102.13	91.746	44.868	1160.237	1081.800
SC	2 37 06	64 17 06	109.60	98.746	47.555	1167.237	1084.487

- 2) Data to set out centreline pegs of spirals
- from ST back to CS.**

Stn	Defl'n	Brg	Dist	dE	dN	E	N
CS	Brg to I						
1880	357 22 54	271 22 54	109.60	-109.568	2.643	1280.27	1108.857
1900	358 06 31	272 06 31	93.19	-93.127	3.429	1296.711	1109.643
1920	358 49 59	272 49 59	73.21	-73.121	3.618	1316.717	1109.832
1940	359 23 00	273 23 00	53.22	-53.127	3.141	1336.711	1109.355
1960	359 45 35	273 45 35	33.23	-33.158	2.179	1356.680	1108.393
1980	359 57 43	273 57 43	13.23	-13.198	0.914	1376.640	1107.128
ST	360 00 00	274 00 00	0.00	0.000	0.000	1389.838	1106.214

- e) Data to set out centreline pegs of circular curve from SC to CS

Brg Common Tangent at SC = Brg TS to IP + Spiral angle

$$= 61\ 40\ 00 + 7\ 51\ 22 = 69\ 31\ 22$$

Stn	Defl'n	Brg	Dist	dE	dN	E	N
SC	Brg CT	69 31 22	0.00			1167.237	1084.487
2000	0 53 43	70 25 05	12.50	11.777	4.189	1179.014	1088.676
2020	2 19 38	71 51 00	32.50	30.874	10.121	1198.111	1094.608
2040	3 45 10	73 16 32	52.50	50.243	15.090	1217.480	1099.577
2060	5 11 07	74 42 29	72.50	69.839	19.086	1237.076	1103.573
2080	6 37 30	76 08 52	92.50	89.616	22.099	1256.853	1106.586
2100	8 03 27	77 34 48	112.50	109.506	24.116	1276.743	1108.603
CS		77 50 00	116.04	113.033	24.370	1280.270	1108.857

Calculate:

- g) Tangent runout = 47.01
 h) Chainage SSD = 1877.81 – 47.01 = 1830.80
 ESD = 2213.23 + 47.01 = 2260.24
 i) $\omega_{85} = 0.014 \text{ radians.sec}^{-1}$ ($\omega_{85} < \omega_{80(\text{max})} = 0.025$)
 j) Cross fall and RL of each even 20m chainage from SSD to ESD,
 include spiral control points.

Stn	Through Chainage	Incremental distance, d	e _{Left} outside CL	e _{Right} inside CL	RL _{Left}	RL _{Right}
SSD	1830.80	0	-0.030	-0.030	-0.15	-0.15
1840	1840.00	9.2	-0.024	-0.030	-0.12	-0.15
1860	1860.00	29.2	-0.011	-0.030	-0.06	-0.15
TS	1877.81	47.01	0.000	-0.030	0.00	-0.15
1880	1880.00	49.2	0.001	-0.030	0.01	-0.15
1900	1900.00	69.2	0.014	-0.030	0.07	-0.15
1920	1920.00	89.2	0.027	-0.030	0.13	-0.15
1940	1940.00	109.2	0.040	-0.040	0.20	-0.20
1960	1960.00	129.2	0.052	-0.052	0.26	-0.26
1980	1980.00	149.2	0.065	-0.065	0.33	-0.33
SC	1987.50	156.7	0.070	-0.070	0.35	-0.35
2000	2000.00	12.5	0.070	-0.070	0.35	-0.35
2020	2020.00	32.5	0.070	-0.070	0.35	-0.35
2040	2040.00	52.5	0.070	-0.070	0.35	-0.35
2060	2060.00	72.5	0.070	-0.070	0.35	-0.35
2080	2080.00	92.5	0.070	-0.070	0.35	-0.35
2100	2100.00	112.5	0.070	-0.070	0.35	-0.35
CS	2103.54	116.04	0.070	-0.070	0.35	-0.35
2120	2120.00	16.46	0.059	-0.059	0.30	-0.30
2140	2140.00	36.46	0.047	-0.047	0.23	-0.23
2160	2160.00	56.46	0.034	-0.034	0.17	-0.17
2180	2180.00	76.46	0.021	-0.030	0.11	-0.15
2200	2200.00	96.46	0.008	-0.030	0.04	-0.15
ST	2213.23	109.69	0.000	-0.030	0.00	-0.15
2220	2220.00	116.46	-0.004	-0.030	-0.02	-0.15
2240	2240.00	136.46	-0.017	-0.030	-0.09	-0.15
2260	2260.00	156.46	-0.030	-0.030	-0.15	-0.15
ESD	2260.24	156.7	-0.030	-0.030	-0.15	-0.15

- k) Chainages where inside cross fall sags below normal cross fall.
 = TS + T_{ro} = 1877.81 + 47.01 = 1924.82
 = ST – T_{ro} = 2213.23 – 47.01 = 2166.22

Other value s as checks:

spiral angle, $\Phi = 7^\circ 51' 21.5''$

tangent length, L = 109.69

shifted deflection angle = $\Delta - 2\Phi = 16^\circ 37' 17''$

circular arc, s = 116.04

11.8 References to Chapter 11

Ausroads Ltd, Sydney, 2016. Edition 3.2 AGRD03-16: Guide to Road Design Part 3: Geometric Design. ISBN 978-1-925451-24

12 Chapter 12. Calculations of Superelevation on a Composite Curve

12.1 Superelevation. Road Profile on a Composite Curve

Referring to § 11.1.10 Superelevation. Road Profile on a Composite Curve, page 235 the introduction is reiterated. The recommended design values and equations are used in this chapter.

The purpose of introducing superelevation into the transverse road profile (the road cross-section) is to provide for a comfortable, stable and safe vehicular passage around a curve.

In determining the generation of superelevation we will consider only the carriageway of the entire road profile. The carriageway consists of the traffic lane(s) and the road shoulders, and is carried on the road pavement. Figure 12-1 illustrates a typical two lane two way rural road. Urban roads will share the same characteristics in relation to the carriageway.

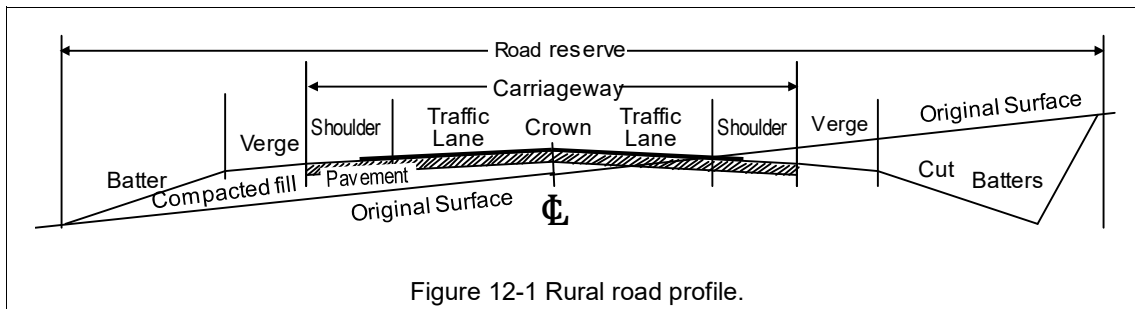


Figure 12-1 Rural road profile.

The crown of the road, the centreline of the carriageway, is the point from which the slope of the road cross section is developed.

The slope, cross fall, or camber, is expressed either as a percentage or the equivalent number. i.e. $3\% = 0.03$. It is also the tangent of the slope, so $\text{atan}(0.03) = 1.7^\circ$. The purpose of cross fall is to provide water drainage from the pavement surface. The minimum drainage cross fall depends on the pavement surface material.

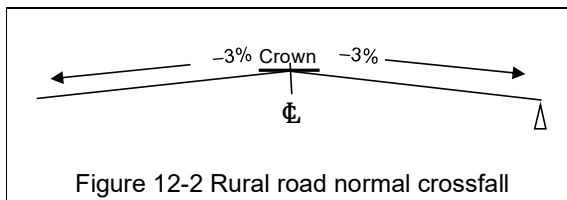


Figure 12-2 Rural road normal crossfall

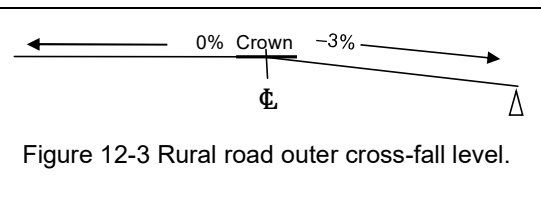


Figure 12-3 Rural road outer cross-fall level.

By convention the **sign** of the slope of indicates the relationship between the centreline and endpoint of the slope. A straight section of road has a, say, -3% slope. This shows that the edge of the carriageway is below the centreline. (Figure 12-2).

The elevation of the outer cross rises to level about the crown is shown in Figure 12-3.

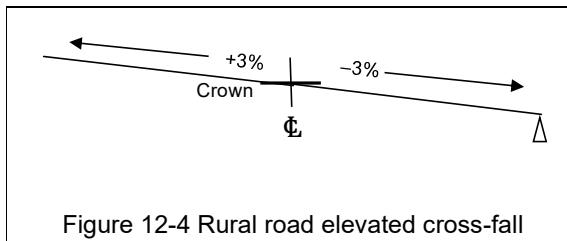


Figure 12-4 Rural road elevated cross-fall

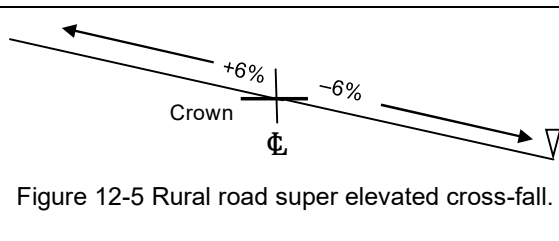


Figure 12-5 Rural road super elevated cross-fall.

The outer cross fall continues to rise about the crown. Elevation (Figure 12-4) can thus be considered as a **positive** slope, rising above the road centreline, at the normal road cross fall.

Superelevation is increased to a maximum over the normal slope. The elevation pivots about the crown level, the centre of the road stays at the crown level and the inner shoulder falls as shown in Figure 12-5. The superelevation is needed to overcome the side forces acting on a vehicle following the circular curve portion of composite curve.

12.1.1 Developing Superelevation between the Straight and Circular Curve

§ 11.1.10.2 showed the method of calculating the length of superelevation development using Eqn 11-7.

$$L_e = \frac{v(e_2 - e_1)}{\omega}$$

Table 11-3 (p236) provided the recommended values of L_e derived from the “Rural Road Design” book.

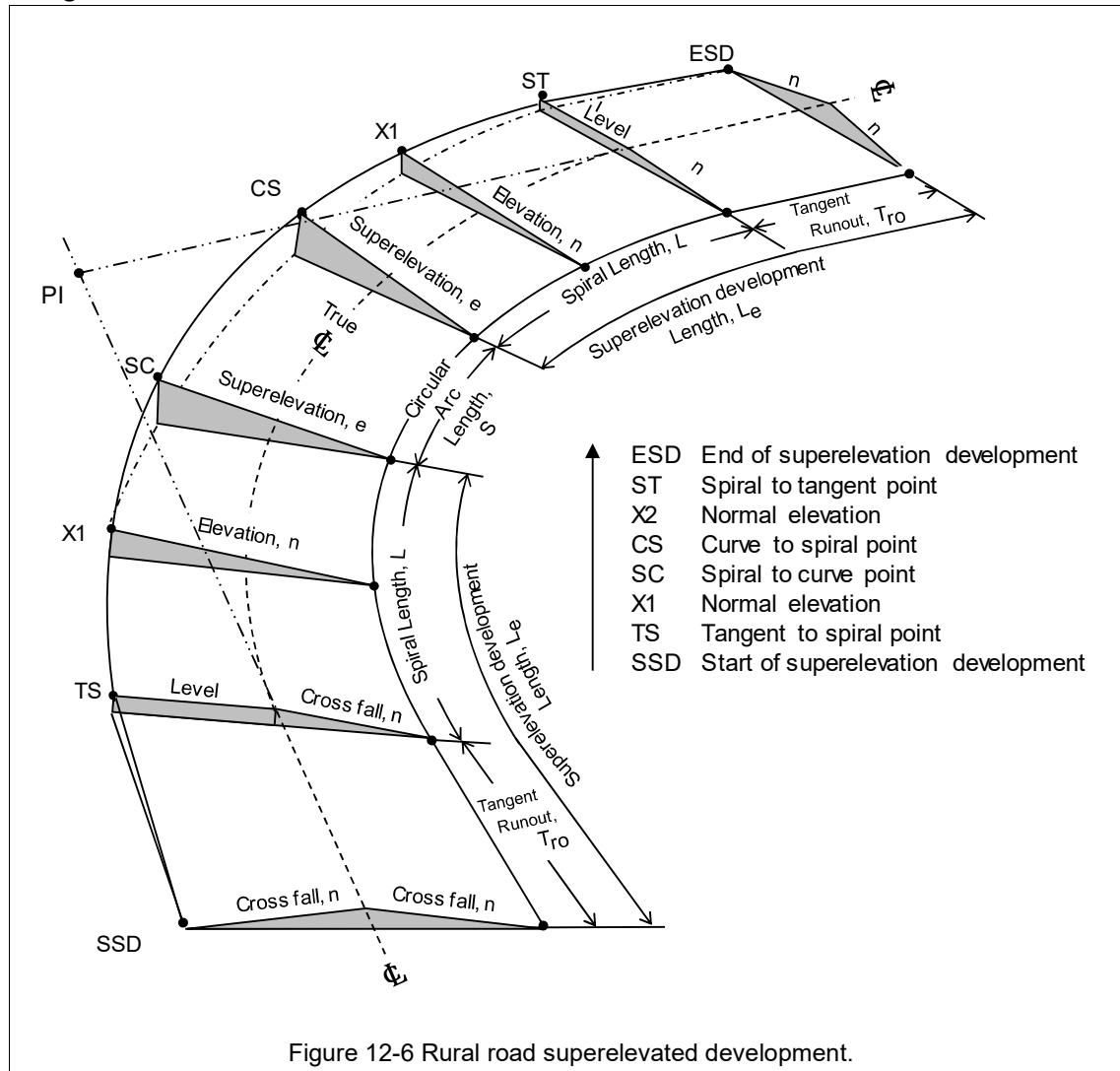


Figure 12-6 shows that **prior** to the Tangent to Spiral (TS) point, and **after** the Spiral to Tangent (ST) point, there is a portion of the straight called the **Tangent Runout, T_{ro}** . This is the length over which the normal cross fall is elevated to, or reduced from, the level cross fall section at the beginning and end of the spiral. (Figure 12-3)

The formula for length of superelevation development, L_e , provides the answer at a chosen (maximum recommended) rate of rotation, ω . The main requirement of superelevation development is to provide a **constant rate** of rotation ($< \omega_{max}$) around the spiral. This rate should then be extended back along the Tangent Runout, T_{ro} .

12.1.1.1 The length of superelevation development:

Superelevation development starts at the SSD, where a road profile is in normal crossfall configuration. The outer crossfall rises on the straight around to the start of the tangent spiral, TS and continues to rise until full superelevation is realised at the start of the circular curve, SC.

The rate of rotation from e_1 (-0.03 or -3%) to e_2 ($+0.06$ or 6%) is constant at rate ω . The length depends on the speed, v .

$$L_e = \frac{v(e_2 - e_1)}{\omega}. \quad (L = vt \text{ where } t = \frac{(e_2 - e_1)}{\omega}).$$

It can be seen that, for a given length, L , the rate of rotation, ω , can be found:

$$\omega = \frac{v(e_2 - e_1)}{L}$$

In the case of L being the length of the transition spiral:

$e_1 = 0$ at TS

$e_2 =$ superelevation cross fall at SC

$v =$ design speed, msec^{-1} .

An expression for the Tangent Runout, T_{ro} , can be developed from:

the normal cross fall, e_1 , at the SSD

the superelevation cross fall, e_2 , at SC and

the length of the tangent spiral, L .

$$T_{ro} = \frac{L(-e_1)}{e_2} \quad (\text{note that } e_1 \text{ is a negative value at SSD})$$

Thus the length of superelevation development, L_e , can be found. And it provides a constant rate of rotation from SSD to SC.

$$L_e = L + \frac{L(-e_1)}{e_2}$$

12.2 Worked Example 12.1 Superelevation on a Transition Curve

The pavement profile of the road designed in Worked Example 11.2 (§ 11.4, p248) has the following cross falls:

$e_1 = -0.03$, (-3%) normal cross fall at SSD

$e_2 = +0.06$, ($+6\%$) superelevation cross fall at SC

Using the data from Worked Example 11.2, determine the chainage of SSD and ESD.

Show that the rate of rotation of the development of the superelevation, ω , does not exceed the design maximum, $0.025 \text{ radians} \cdot \text{sec}^{-1}$.

Prepare a table of cross fall values and RLs either side of the centre line at the even 20m chainages. Carriageway width, W , for set out = 10m.

1) Previous design data. (p248)

Length of spiral $L = 73.13\text{m}$. TS to SC

Chainage at T $= 348.54$. TS

Chainage at U $= 573.20$. ST

Design speed $v = 85 \text{Kmh}^{-1}$

2) Calculate tangent runout, T_{ro}

$$T_{ro} = \frac{L(-e_1)}{e_2} = 73.13 \frac{-(-0.03)}{0.06}$$

$$T_{ro} = 36.57$$

3) Calculate chainages

$$\begin{aligned} \text{SSD} &= \text{TS} - T_{\text{ro}} = 348.54 - 36.57 \\ \text{SSD} &= \mathbf{311.97} \\ \text{ESD} &= \text{ST} + T_{\text{ro}} = 573.20 + 36.57 \\ \text{ESD} &= \mathbf{609.77} \end{aligned}$$

4) Calculate rate of rotation of superelevation development.

$$\omega = \frac{v(e_2 - e_1)}{L}$$

For L use either (i) the change on the T_{ro} , (ii) the spiral length, L , or (iii) the length of superelevation development, $L_e = L + T_{\text{ro}}$.

Choosing method (iii).

$$\begin{aligned} L_e &= 73.13 + 36.57 = 109.70 \\ e_1 &= -0.03 \\ e_2 &= +0.06 \\ v &= 85 \text{Kmh}^{-1} = 85/3.6 = 23.6 \text{msec}^{-1} \\ \omega &= v \frac{e_2 - e_1}{L_e} = 23.6 \frac{0.06 - (-0.03)}{109.7} \end{aligned}$$

$$\omega = \mathbf{0.019 \text{radians} \cdot \text{sec}^{-1}}$$

$$\omega < 0.025 \text{radians} \cdot \text{sec}^{-1}. \text{ Design criteria met.}$$

5) Calculate the linear rate of slope change per metre of curve length.

$$\delta e = \frac{(e_2 - e_1)}{L_e} = \frac{0.09}{109.7} = 0.00082 \text{radial rate/m to } e_2.$$

Super elevation starts after the normal cross fall of the outer carriageway exceeds $-e_1$, i.e. when the outer cross fall $= +0.03$.

This distance is either:

- $(0.03 - (-0.03)) / \delta e$ (rotate from -0.03 to $+0.03 = 0.06$)
 $= 0.06 / 0.00082 = 73.13 \text{m from SSD, or}$
- $(0.03 - 0) / \delta e$ (rotate from -0.03 to level $= 0.03$)
 $= 0.03 / 0.00082 = 36.57 \text{m from T (TS point).}$

Chainage elevation rising above normal cross fall, X1

$$\begin{aligned} &= \text{SSD} + 73.13 = 385.1 \text{ or} \\ &= \text{T} + 36.57 = 385.1. \end{aligned}$$

Similarly, the superelevation falls to normal crossfall, X2 after:

$$\begin{aligned} &= \text{ESD} - 73.13 = 536.64 \text{ or} \\ &= \text{U} - 36.357 = 536.63. \end{aligned}$$

6) Calculate cross fall and RL at each chainage. Add X1 and X2 to Table 12-1:

- Tabulate slope at each chainage.
 The change in slope for the superelevation is linear:
 $\delta e = 0.00082 \text{radians/m}$
- Tabulate RLs at each chainage. Table 12-1.
 Centreline is elevation $= 0.00$
 Carriageway, $W = 10$
 $e_{\text{CH}} = e_1 + \delta e \times S$ (through distance) to $e_{\text{max}} = 0.06$. ($e_1 = -0.03$)
 $\text{RL} = e \times W/2$
 $e_{\text{max}} = +0.06$ and -0.06 from T1 to T2, superelevated section.

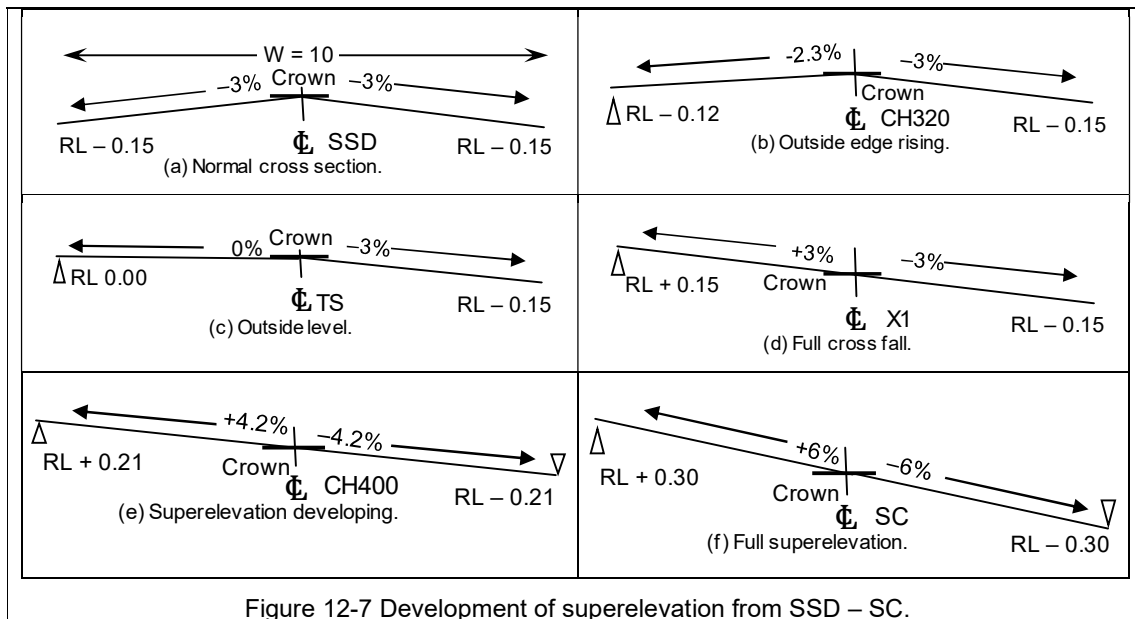
Table 12-1 Chainages and RLs reference ϕ level.

Stn	Through Chainage	Incremental distance, d	Through distance, S.	e_{Left} outside CL	e_{Right} inside CL	RL _{Left}	RL _{Right}
SSD	311.97	0.00	0.00	-0.03	-0.03	-0.15	-0.15
320	320.00	8.03	8.03	-0.023	-0.03	-0.12	-0.15
340	340.00	28.03	28.03	-0.007	-0.03	-0.04	-0.15
T	348.54	36.57	36.57	0.00	-0.03	0.00	-0.15
360	360.00	11.46	48.03	+0.009	-0.03	+0.05	-0.15
380	380.00	31.46	68.03	+0.025	-0.03	+0.13	-0.15
X1	385.10	36.57	71.13	+0.03	-0.03	+0.15	-0.15
400	400.00	51.46	88.03	+0.042	-0.042	+0.21	-0.21
420	420.00	71.46	100.03	+0.058	-0.058	+0.29	-0.29
T1	421.67	73.13	109.7	+0.06	-0.06	+0.30	-0.30
440	440.00	18.33	128.03	+0.06	-0.06	+0.30	-0.30
460	460.00	38.33	148.03	+0.06	-0.06	+0.30	-0.30
480	480.00	58.33	168.03	+0.06	-0.06	+0.30	-0.30
500	500.00	78.33	188.03	+0.06	-0.06	+0.30	-0.30
T2	500.11	78.44	188.14	+0.06	-0.06	+0.30	-0.30
520	520.00	19.89	208.03	+0.044	-0.044	+0.22	-0.22
X2	536.63	36.52	224.66	+0.03	-0.03	+0.15	-0.15
540	540.00	39.89	228.03	+0.027	-0.03	+0.14	-0.15
560	560.00	59.89	248.03	+0.011	-0.03	+0.05	-0.15
U	573.20	73.13	261.23	0.00	-0.03	+0.00	-0.15
580	580.00	6.80	268.03	-0.006	-0.03	-0.03	-0.15
600	600.00	26.8	288.03	-0.022	-0.03	-0.11	-0.15
ESD	609.77	36.57	297.80	-0.03	-0.03	-0.15	-0.15

The following diagrams (Figure 12-7) show the pavement cross section at selected chainages. They illustrate the development of the superelevation from the start of superelevation development, SSD, through to the fully developed superelevation at the start of the circular curve, SC.

Note that the decrease in elevation on the inside of the turn does not occur until the outside elevation, $e_{Left} > +0.03$. In this case, where normal cross fall, $n = \frac{1}{2}e$, superelevation, it occurs near chainage 385m, which is half way around the spiral from TS. ($TS + L/2 = 348.54 + 73.13/2 = 385.10$).

At the end of the circular curve the process is reversed back to the ESD. Refer to Figure 12-6.



12.3 Comments on superelevation calculation.

The purpose of both the transition curve and superelevation calculations is not to design the roads with suitable parameters, but to show how those parameters are incorporated into a set of field layout coordinates.

The design of a road is properly the task of a road traffic engineer.

12.4 References to Chapter 12

Ausroads Ltd, Sydney, 2016. Edition 3.2 AGRD03-16: Guide to Road Design Part 3: Geometric Design. ISBN 978-1-925451-24

13 Chapter 13. Vertical Curves

13.1 Introductory Remarks

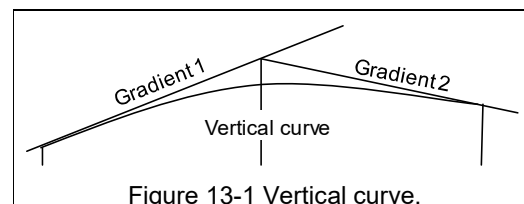
At the end of this Chapter and Exercise 6 (Appendix A2-5), you should:

- ✓ Understand and appreciate the geometric properties of a vertical curve.
- ✓ Understand what gradients are and the limitations that are imposed on their values.
- ✓ Appreciate the role of vertical curves in improving safety and comfort of passengers travelling from one intersecting gradient to another.
- ✓ Be able to design a vertical curve.

Exercise 6 involves calculating the volume occupied by a series of vertical curves through modelled terrain.

13.1.1 Definition

The vertical curve, Figure 13-1, is a curve joining two different grades (or gradients) in a vertical profile. The purpose of the curve is to smooth the passage from one gradient to another.



13.1.2 Uses

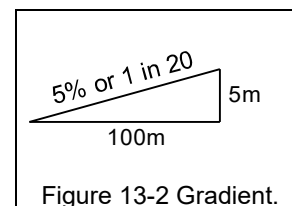
- Introduced between two intersecting gradients to smooth the passage from one gradient to another in the vertical plane.
- A vehicle travelling along a vertical curve experiences a radial force that tries to force the vehicle from the centre of curvature of the vertical curve. This forces the vehicle to leave the road in case of a crest design, while the vehicles could come into contact with a road surface for a sag design. Both result in discomfort and danger to passengers and are minimized through restricting the gradients and choosing a suitable type and length of curve.
- Adequate visibility: Vertical curves enables vehicles travelling at a given design speed to stop or overtake safely.
- Considerations include safety, riding comfort and sight distance.

13.1.3 Components of a Vertical Curve

The **gradient** or grade of a slope is expressed as a **percentage**. A slope is expressed as a ratio of **vertical** to **horizontal**, generally 1: n.

The gradient of a slope of 1 vertical in 20 horizontal (1:20)
 $= 1/20 = 0.05 = 5\%$, i.e.,

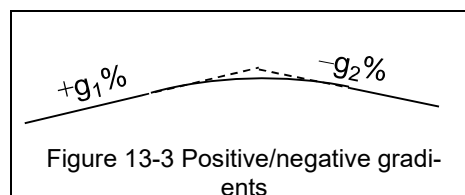
Gradient = 5m vertical for every 100m horizontal (see Figure 13-2).



The slope **angle** = arctangent (slope). $1:20 = 0.05 = \arctan(0.05) = 2.862^\circ = 2^\circ 52'$.

A rising gradient in the direction of the running chainage is positive gradient $+g_1\%$ while a falling gradient in the direction of the running chainage is negative gradient $-g_2\%$. See Figure 13-3.

In road design a vertical curve is generally a **parabolic** curve in the longitudinal profile of the carriageway to provide a change of grade at a specified vertical acceleration. In developing examples of the vertical curve the design criteria used is that which is published in Austroads, 2003.



13.2 Elements of the Vertical Curve

Two gradients intersect at a point V, the Point of Intersection (PI). The gradients are **tangential** to a parabolic section running from the initial Point of Curvature (PC) to the final Point of Tangency (PT).

Notations: (Figure 13-4)

G_1 Initial gradient, expressed as a percentage

G_2 succeeding gradient, expressed as a percentage

V or PI Vertex or Point of Intersection

PC Point of Curvature, the start of the curve

PT Point of Tangency, the end of the curve

L Length of the curve. It is measured **horizontally** in units of 100m.

A vertical curve with a horizontal distance of 350m has $L=3.5$. The arc length of the parabola can be found by summing the chord lengths between chainage stations.

A Gradient difference.

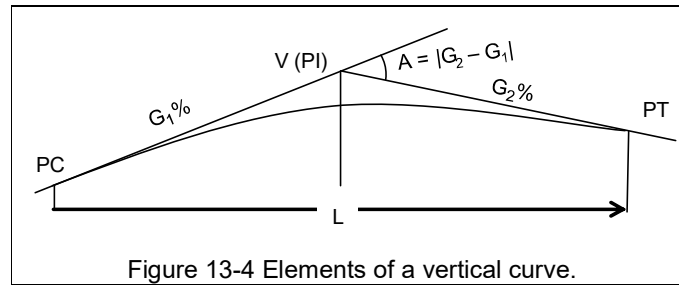


Figure 13-4 Elements of a vertical curve.

13.2.1 Two Properties of a Parabola:

Figure 13-5 shows the **first** of two properties of a parabola. The horizontal projections of the tangents at PC and PT are equal. They are equal to half the horizontal projection of the curve, i.e., $\frac{1}{2}L$.

The **second** property is that the rate of change of grade of the curve is constant. The gradient at any point along the parabola can be expressed using the horizontal length of the curve.

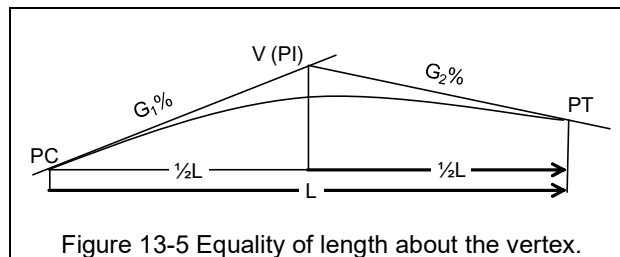


Figure 13-5 Equality of length about the vertex.

$$\text{Rate of change of grade } r\% = \frac{G_2 - G_1}{L}$$

Eqn 13-1

where $r\%$ can be read as **r “percent per 100”**

Figure 13-6 shows a curve between two grades, $G_1 = +5\%$, $G_2 = -4\%$, joining a parabolic curve of length = 300m. $L = 3$. Find r .

From Equation 9.1:

$$r\% = \frac{(-4) - (+5)}{3} = -3\%$$

To find the gradient of the curve at $PC + 100$ where: (Figure 13-7)
final gradient = Initial gradient + gradient change.

At $PC + 100$: $L = 1$, $r = -3$

$$\begin{aligned} G_{100} &= 5 + (-3 \times 1) \\ &= 2\% \end{aligned}$$

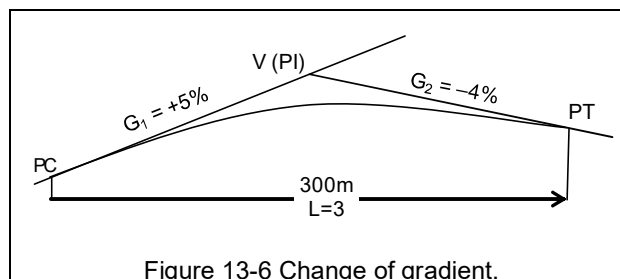


Figure 13-6 Change of gradient.

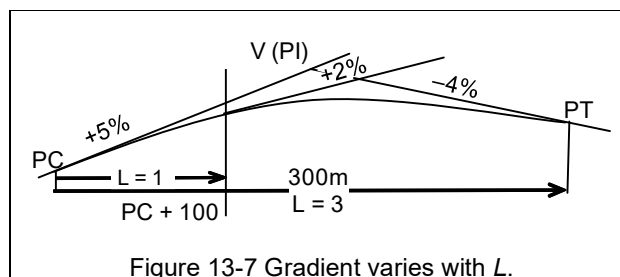


Figure 13-7 Gradient varies with L .

13.2.2 Sign of r

If $r > 0$ (positive) = sag curve

If $r < 0$ (negative) = crest curve

If the signs of G_1 and G_2 are **opposite**, then there will be a high or low point. (Figure 13-8).

If G_1 and G_2 are numerically equal, but opposite sign, then the high or low point will be at the midpoint. Otherwise it must be calculated.

Crest curves, Figure 13-9, occur when the succeeding gradient is less than the initial gradient.

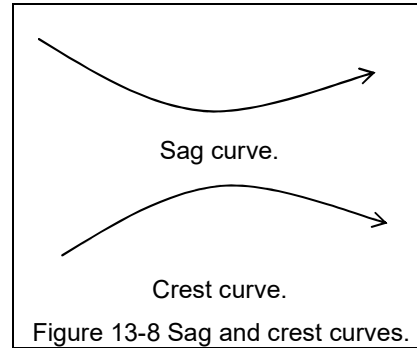


Figure 13-8 Sag and crest curves.

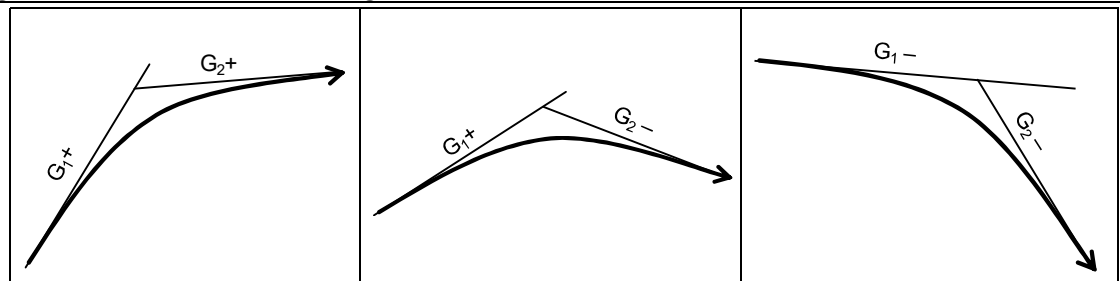


Figure 13-9 Configuration of crest curve.

Sag curves, Figure 13-10, occur when the succeeding gradient is greater than the initial gradient.

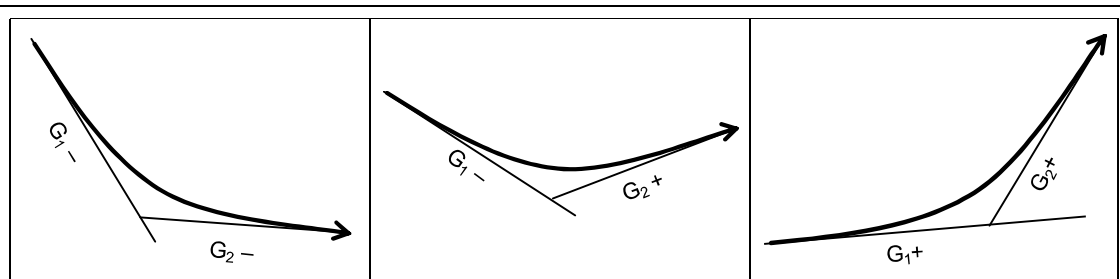


Figure 13-10 Configuration of sag curve.

13.3 Calculus of the Parabola

The parabola is a conic curve of a second-degree equation. If there is no xy term in the expression, then the principal axis is either horizontal, $y = \pm kx^2$ or vertical, $x = \pm ky^2$. The shape, or rate of change of gradient, is given by the constant, k .

The general form of the parabola, with its principal axis parallel to the vertical axis is

$$y = ax^2 + bx + c$$

Eqn 13-2

This equation will be developed to enable calculation of the vertical coordinates, the RLs, of points on the curve.

Reference Figure 13-11, y will be the RL of any point on the curve at a horizontal distance, x , from the PC.

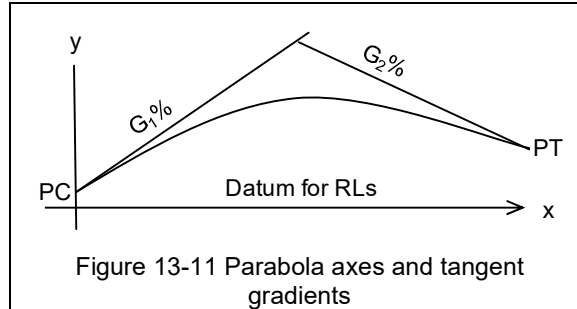


Figure 13-11 Parabola axes and tangent gradients

13.3.1 Developing the General Equation

Convert the general equation: $y = ax^2 + bx + c$ into an expression covered by the elements of the vertical curve.

1. Find the value of y at $x = 0$

$$y = c = RL \text{ of } PC$$

Eqn 13-3

2. The first derivative of Eqn 13-2 provides the gradient:

$$\frac{dy}{dx} = 2ax + b, \text{ and again}$$

$$\text{When } x = 0, \frac{dy}{dx} = b \text{ i.e.:}$$

$$b = \text{gradient of curve when } x = 0$$

$$b = G_I$$

3. The second derivative of Eqn 13-2 of a curve provides the rate of change of grade:

$$\frac{d^2y}{dx^2} = 2a.$$

We have defined r as the rate of change of gradient. Thus:

$$r = 2a \text{ or } a = \frac{1}{2}r$$

Eqn 13-4

4. Substitute derived constants into Eqn 13-2 gives the expression:

$$y = \frac{1}{2}rx^2 + G_Ix + RL_{PC}$$

Eqn 13-5

13.3.2 Deriving Data from the General Equation

The general parabolic Eqn 13-2, for the vertical curve is expressed in terms related to the design of the curve (i.e., Eqn 13-5)

$$y = \frac{1}{2}rx^2 + G_Ix + RL_{PC} \quad \text{where}$$

r = rate of change of gradient (as percent per 100)

G_I = initial gradient of slope (as a percentage)

RL_{PC} = RL of Point of Curvature, at the start of vertical curve.

13.3.3 High or Low Point of the Vertical Curve

The high/low point of the curve is the point at which the gradient of the curve is zero.

Find the value of x where slope = 0

Take first derivative, using Eqn 13-5, $\frac{dy}{dx} = rx + G_1$

Gradient is zero when $\frac{dy}{dx} = 0$, i.e.,

$$rx + G_1 = 0$$

$$x = \frac{-G_1}{r}$$

Eqn 13-6

where x = distance from PC in units of 100.

$$\text{Chainage high/low point} = \text{Chainage PC} + 100 \times \frac{-G_1}{r}$$

Eqn 13-7

Note: If the high/low point is on the parabola between the PC and PT then

$\frac{-G_1}{r}$ will be positive because G_1 and r will be opposite signs.

Taking the vertical curve in Figure 13-6 as an example, remember G_1 is negative:

$$r = -3$$

$$G_1 = 5$$

$$x = \frac{-5}{-3} = 1.67, \text{ the chainage of the high point is thus:}$$

$$\text{CH}_{\text{PC}} + 100x = \text{CH}_{\text{PC}} + 167.$$

13.3.4 Level of High or Low Point of the Vertical Curve

The height (Δh) of the high/low point of the curve is the product of the distance at which the gradient of the curve is zero, $\times$ and gradient of the line AP.

Line AP is parallel to the tangent through the mean point of the **parabola** AP. The gradient of the curve at the mean point will be the mean of the gradients at the end points, A and, in this case, P, where the gradient is zero.

Examine the gradients:

$$\text{Gradient at A} = G_1$$

$$\text{Gradient at P} = 0$$

$$\text{Gradient AP} = \frac{1}{2}(G_1 + 0) = \frac{1}{2}G_1$$

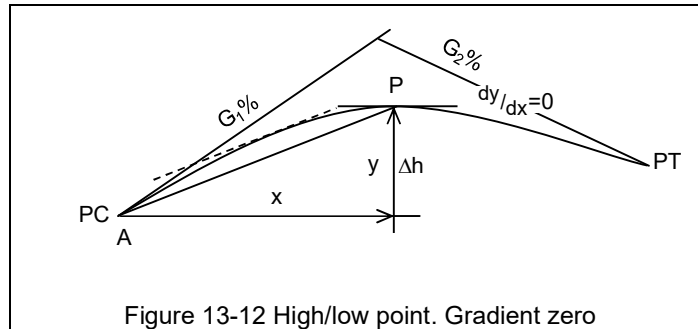


Figure 13-12 High/low point. Gradient zero

$$\Delta h = \frac{1}{2} G_1 x$$

Eqn 13-8

$$RL_P = RL_{\text{PC}} + \frac{1}{2} G_1 x$$

Eqn 13-9

The RL of the low point will use the same formula because G_1 is negative.

It can also be seen that, substituting x from Eqn 13-6:

$$\Delta h = \frac{-\frac{1}{2}G_1^2}{r}$$

Eqn 13-10

13.3.4.1 Worked Example 13.1

Two intersecting profile gradients consist of a +5% gradient meeting a -3% gradient.

They are to be joined by a parabolic vertical curve 200m long. $L = 2$.

Chainage of the PC = 1400m,

RL PC = 299m.

Calculate the chainage and RL of the highest point.

1. Calculate $r\%$ using Eqn 13-1:

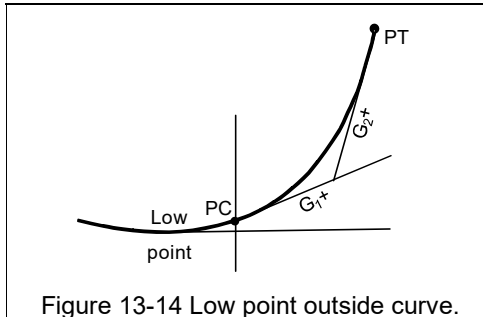
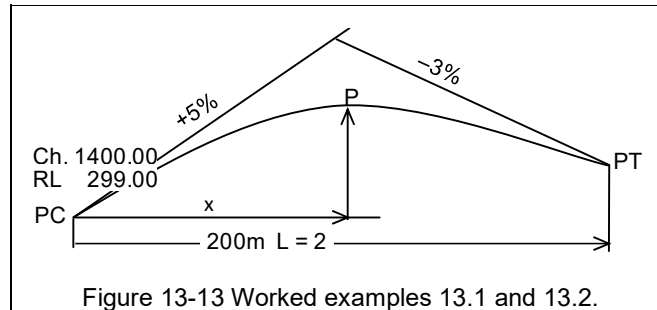
$$r = \frac{G_2 - G_1}{L} = \frac{-3 - (+5)}{2} = -4$$

2. Chainage to high point, P: (Eqn 13-6): $x = \frac{-G_1}{r} = \frac{-5}{-4} = 1.25, = 125\text{m}.$

Chainage P = $CH_{PC} + x = 1400 + 125 = 1525\text{m}.$

3. Δh of high point, P: (Eqn 13-8): $\Delta h = \frac{1}{2}G_1 x = \frac{1}{2} \times 5 \times 1.25 = 3.125\text{m}$

RL of high point, P = $RL_P + \Delta h = 299 + 3.125 = 302.125\text{m}.$



Note that the high/low point of the parabola $\left(\frac{dy}{dx} = 0\right)$ may not occur on the section of parabola under investigation (see Figure 13-14).

13.3.5 The Midpoint of the Vertical Curve

The “external” distance, e , is the vertical distance from the vertex, or point of intersection, of the two gradients to the midpoint of the curve. In Figure 13-15, the gradient G_1 has been produced to point C vertically above the PT.

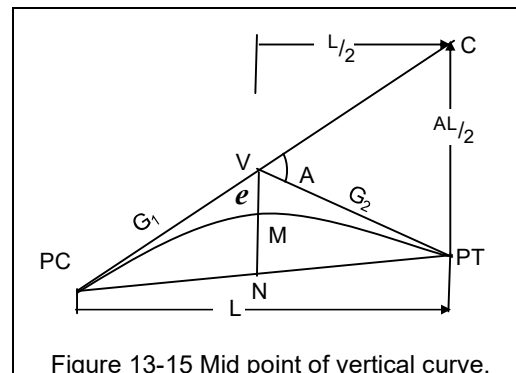
Horizontal distance PC to PT = L (units of 100).

Angle A is the algebraic change of grade.

$$A = |G_2 - G_1|$$

N is the midpoint of the chord PC to PT, and is vertically below V.

Note that, if $|G_1| = |G_2|$ then M is the high point, P, as in Figure 13-15.



1. The Δh of the PT with reference to PC = $\Delta h(\text{PC to V}) - \Delta h(\text{V to PT})$
remember that G_2 is negative
 $\Delta h_{\text{PC-PT}} = \frac{1}{2}(G_1 L) + \frac{1}{2}(G_2 L) = \frac{1}{2}(G_1 + G_2)L$.
2. It follows that Δh between C and PT
 $= \Delta h(\text{PC to C}) - \Delta h(\text{PC to PT})$
 $= G_1 L - \frac{1}{2}(G_1 + G_2)L = \frac{1}{2}(G_1 - G_2)L$, but $G_1 - G_2 = |G_2 - G_1| = A$
 $= \frac{1}{2}AL$ or $\frac{AL}{2}$.
3. Similarly: $VN = \Delta h(\text{PC to V}) - \Delta h(\text{PC to N})$
 $= \frac{1}{2}G_1 L - \frac{1}{2}(\frac{1}{2}(G_1 + G_2)L)$
 $= \frac{1}{4}(G_1 - G_2)L$
 $= \frac{1}{4}AL$ or $\frac{AL}{4}$.

One of the properties of the parabola is:

Offsets from the tangent vary with the square of the distance from the PC.

In general terms: $e = \left(\frac{n}{m}\right)^2 \cdot \frac{AL}{2}$ and $\frac{AL}{2}$ is a constant for the curve.

The offset to V from the tangent at M (this is e), which is $\frac{1}{2}L$ from the PC

$$h_{\text{VM}} = e = \left(\frac{1}{2}\right)^2 L \times \frac{A}{2} = \frac{1}{4}L \times \frac{A}{2} = \frac{AL}{8}$$

This shows that $e = \frac{1}{2}$ the difference between the RL of V and the RL of N.

Using the squared offset method, a number of other “ e ” values can be found such that, if L is the length of the curve:

$$\text{for } \frac{1}{4}L: \quad e = \frac{1}{16} \cdot \frac{AL}{2}$$

$$\text{for } \frac{3}{4}L: \quad e = \frac{9}{16} \cdot \frac{AL}{2}$$

13.3.5.1 Worked Example 13.2

Two intersecting profile gradients consist of a +5% gradient meeting a –3% gradient. They are to be joined by a parabolic vertical curve 200m long. (Figure 13-16).

Find the offset, e , from the initial gradient G_1 at the midpoint and also the $\frac{1}{4}$ and $\frac{3}{4}$ point of length of the curve.

From § 13.3.4.1 Worked Example 13.1:

$$L = 200$$

$$G_1 = +5\%$$

$$G_2 = -3\%$$

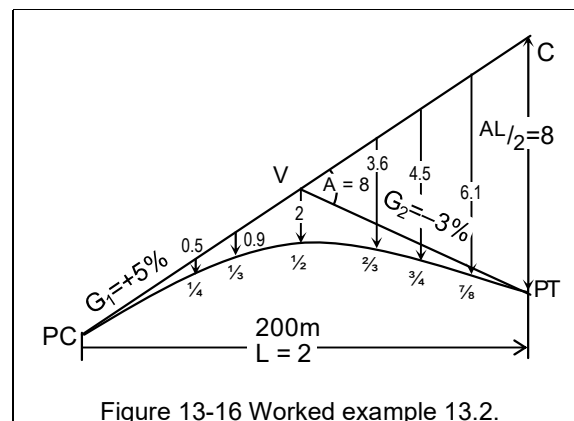
$$A = |G_2 - G_1| = 8$$

$$\frac{AL}{2} = 8$$

$$e_{\text{midpoint}} = \frac{1}{4} \times \frac{AL}{2} = \frac{AL}{8} = \frac{2 \cdot 8}{8} = 2$$

$$e_{\frac{1}{4}} = \frac{1}{16} \times \frac{AL}{2} = \frac{1}{16} \times 8 = 0.5$$

$$e_{\frac{3}{4}} = \frac{9}{16} \times \frac{AL}{2} = \frac{9}{16} \times 8 = 4.5$$



Similarly, the offset from the initial gradient can be found at any portion of the length of the parabola once $\frac{AL}{2}$ has been evaluated.

e.g. $e \frac{1}{3} = \frac{1}{9} \times 8 = 0.89$

$$e \frac{2}{3} = \frac{4}{9} \times 8 = 3.56.$$

The technique has limited application compared with modern calculation methods, but the mid-point value of $e = \frac{AL}{8}$ is a handy check of the calculation.

13.3.6 Pass a Curve Through a Fixed Point

It is often necessary to have a curve pass through a fixed point when two gradients are being joined. A practical example is the requirement to provide a certain amount of “clearance” below an overpass, or a bridge.

The task may be to find the **length** of the parabola joining two designated grades.

Or it may be necessary to find the **gradient** of the succeeding gradient if the length of the parabola is fixed.

The governing variable in these cases is the value r , the rate of change of gradient ($r\%$), used in the parabolic formula:

$$y = \frac{1}{2}rx^2 + G_1x + RL_{PC}, \text{ (Eqn 13-5), where } r = \frac{G_2 - G_1}{L} \text{ and } x \text{ is units of 100.}$$

13.3.6.1 Worked Example 13.3

A road is to be redesigned using a parabolic vertical curve to provide clearance beneath a bridge (see Figure 13-17).

The PC has chainage 1140.00m, with an RL of 125.604m.

The initial gradient, $G_1 = -4\%$.

A bridge at chainage 1300.00 has an RL = 128.5m at its lowest point.

You must provide **4.5m** clearance from the underside of the bridge. The parabola will finish with a succeeding gradient, $G_2 = +3.6\%$.

What is the length of the curve?

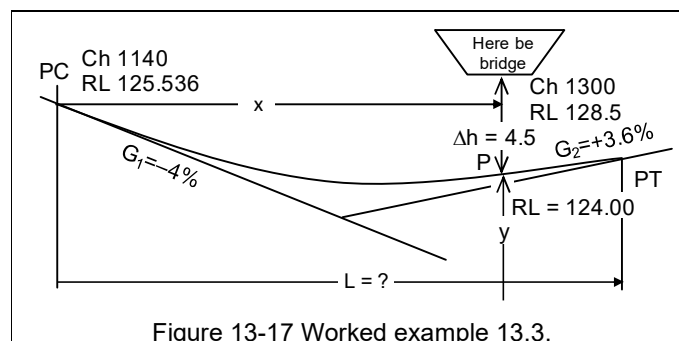


Figure 13-17 Worked example 13.3.

Formula of vertical curve: (Eqn 13-5), $y = \frac{1}{2}rx^2 + G_1x + RL_{PC}$,

Data: $x = (1300 - 1140)/100 = 1.6$ (units of 100 to allow for grade %)

$y = 128.5 - 4.5 = 124.0$ (RL of clearance)

$RL_{PC} = 125.536$

$G_1 = -4\%$

$G_2 = +3.6\%$

Using the formula of vertical curve above, find $r\%$: to allow calculation of L

$$124.0 = \frac{1}{2}r \times (1.6)^2 + (-4 \times 1.6) + 125.536$$

$$r = \frac{2(124 + 6.4 - 125.536)}{1.6^2} = 3.8$$

But: $r = \frac{G_2 - G_1}{L}$

thus $L = \frac{G_2 - G_1}{r} = \frac{3.6 - (-4)}{3.8} = 2.0$ units of 100.

Length of curve = $L \times 100 = 200\text{m}$

13.4 Calculate the RLs of Points Along the Vertical Curve

Once a vertical curve has been designed it is necessary to generate a tabulated list of RLs at intermediate stations. As with the horizontal curve, they will most likely be at regular whole number chainages. Remember that for the vertical curve, we measure the chainages horizontally from the PC.

A constant pegging interval allows a check of the calculations to be performed using the second difference between RLs.

The first difference (Δ') is the difference between successive RLs. The second difference (Δ'') is the difference between successive first differences, the Δ' 's. As the parabola is a second-degree equation the Δ'' must be constant for a constant interval.

The first and second differences can be tabulated in the pegging data.

Chord length can be tabulated to find approximate arc length.

13.4.1 Worked Example 13.4. Single Vertical Curve

Figure 13-17 shows the design of a vertical curve to be pegged at 20m intervals from the PC to the PT over a length of 200m.

Using the vertical curve in Worked example 13.3, and Figure 13-18 for visualisation:

Tabulate the RLs, show the second difference similarities.

Find the chainage and RL for the low point. Use chords to find arc length.

Ch PC = 1140.00m,

RL PC = 125.536m.

Ch PT = 1340.00m.

$G_1 = -4\%$, $G_2 = +3.6\%$.

$L = 200$,

$r = (G_2 - G_1)/L$

$= (3.6 - (-4))/2$

$r = +3.8$ (sag curve).

Create the formula for the curve

in terms of x from:

$x = 0, 0.2, 0.4, \dots, 1.8, 2$:

$RL_{Ch} = \frac{1}{2}rx^2 + G_1x + RL_{PC}$

$= 1.9x^2 - 4x + 125.536$.

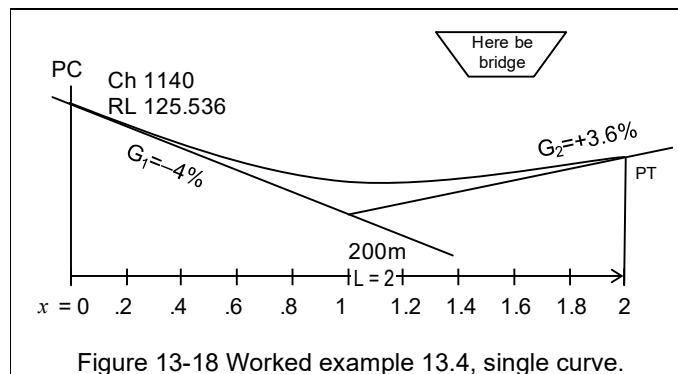


Table 13-1 shows the results of the calculations for RL at each chainage as well as the first and second RL differences between the chainages. The arc length is the summation of the chords between each chainage pair.

Table 13-1 Worked example 13.4. Table of chainages, RL, first & second differences, chords. Checks.					
Chainage	x	RL	Δ'	Δ''	Chord
1140	0.0	125.536			
			-0.724		20.013
1160	0.2	124.812		0.152	
			-0.572		20.008
1180	0.4	124.240		0.152	
			-0.420		20.004
1200	0.6	123.820		0.152	
			-0.268		20.002
1220	0.8	123.552		0.152	
			-0.116		20.000
1240	1.0	123.436		0.152	
			0.036		20.000
1260	1.2	123.472		0.152	
			+0.188		20.001
1280	1.4	123.660		0.152	
			+0.340		20.003
1300	1.6	124.000		0.152	
			+0.492		20.006
1320	1.8	124.492		0.152	
			+0.644		20.010
1340	2.0	125.136			
Arc Length =					200.047

First difference,

$$\Delta' = RL_{n+1} - RL_n$$

Second difference,

$$\Delta'' = r(dx)^2$$

where:

$$dx = \text{peg interval}/100$$

$$= 20/100$$

$$= 0.2$$

$$\text{i.e., } \Delta'' = r(dx)^2$$

$$= 3.8 \times (0.2)^2$$

$$= \mathbf{0.152}$$

$$\text{Check } \Delta'' = \Delta'_{n+1} - \Delta'_n$$

e.g.,

$$\Delta'_{1160} - \Delta'_{1140} = -0.572 - (-0.724)$$

$$= 0.152$$

The **chord** of each interval (dx) is calculated using the first difference Δ' .

$$\text{Chord}_{(n, n+1)}$$

$$= \sqrt{(100dx)^2 + (\Delta')^2_{(n, n+1)}}$$

$$\text{e.g., Chord}_{(1140, 1160)}$$

$$= \sqrt{(100 \cdot 0.2)^2 + (-0.724)^2}$$

$$= 20.013.$$

$$\text{Arc length} = \Sigma(\text{Chords})$$

Find the low point:

$$\text{Distance to low point (Eqn 13-6): } x = \frac{-G_1}{r} = \frac{-(-4)}{3.8} = 1.053 \quad (\text{distance} = 100x = 105.3)$$

Chainage low point = 1140 + 105.3 = **1245.30**, checks by inspection against pegging sheet.

RL of low point at distance x (Eqn 13-9):

$$RL = \frac{1}{2}G_1x + RL_{PC} = -2 \times 1.053 + 125.536 = -2.105 + 125.536$$

RL low point = **123.431**, checks against pegging sheet between Ch1240 & 1260.

13.4.2 Worked Example 13.5. Multiple Vertical Curves

Figure 13-19 shows the design of two vertical curves, joined by straight sections, to be pegged at 15m intervals over a length of 105m.

The RL at CH 00 is 10.000

Vertical Curve design:

1. The path rises at a slope of +2% between CH 00 and CH 15.
2. The path increases, through a **sag** curve, to a slope of +5% over a length of 15m.
3. The path continues a straight slope (5%) to CH75.
4. The path flattens, via a **crest** curve, to a slope, 0%, between CH 75 and CH105.

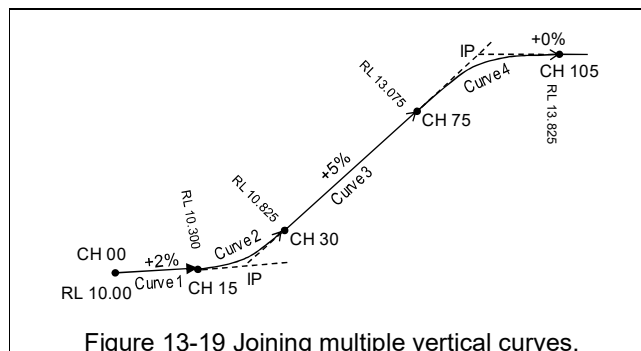


Figure 13-19 Joining multiple vertical curves.

The calculation is broken into four separate parts, the main task being to calculate the RLs at each change of grade. The formula for calculating the RLs over multiple curves replaces the term for the level at the “point of curvature”, RL_{PC} , with the expression, “beginning of vertical curve”, RL_{BVC} .

The RL at the **end** of one vertical curve is the RL at the **beginning** of the next vertical curve.

Only one formula is used: Eqn 13-5. $RL_{CH} = \frac{1}{2}rx^2 + G_1x + RL_{BVC}$.

By definition, the rate of change on a straight slope is, $r = 0$, because $G_1 = G_2$.

1. Straight gradient: Ch00 to CH15, $L = 15m = 0.15$

CH 00, $RL_{BVC} = 10.000$, $G_1 = +2\%$, $G_2 = +2\%$

CH 15, $x = 0.15$ (15m)

$$r = (G_2 - G_1)/L = (2 - 2)/0.15 = 0$$

$RL_{CH15} = \frac{1}{2}rx^2 + G_1x + RL_{CH00} = 0 + 2 \times 0.15 + 10 = 10.300$. This is RL_{BVC} for curve 2.

2. Sag curve, CH15 to CH30, $L = 15m = 0.15$

CH 15, $RL_{BVC} = 10.300$, $G_1 = +2\%$, $G_2 = +5\%$

CH 30, $x = 0.15$ (CH30 - CH15 = 15m)

$$RL_{CH30} = \frac{1}{2}rx^2 + G_1x + RL_{CH15}$$

$$= \frac{1}{2}(20 \times 0.15^2) + 2 \times 0.15 + 10.300 = 0.225 + 0.3 + 10.3 = 10.825. RL_{BVC} \text{ curve 3.}$$

3. Straight gradient, CH30 to CH75, $L = 45m = 0.45$

CH 30, $RL_{BVC} = 10.825$, $G_1 = +5\%$, $G_2 = +5\%$,

$$r = (G_2 - G_1)/L = (5 - 5)/0.45 = 0$$

CH 45, $x = 0.15$ (CH45 - CH30 = 15m)

$$RL_{CH45} = \frac{1}{2}rx^2 + G_1x + RL_{CH30} = 0 + 5 \times 0.15 + 10.825 = 0.75 + 10.825 = 11.575$$

CH 60, $x = 0.3$ (CH60 - CH30 = 30m)

$$RL_{CH60} = \frac{1}{2}rx^2 + G_1x + RL_{CH30} = 0 + 5 \times 0.30 + 10.825 = 1.50 + 10.825 = 12.325$$

CH 75, $x = 0.45$ (CH75 - CH30 = 45m)

$$RL_{CH75} = \frac{1}{2}rx^2 + G_1x + RL_{CH30} = 0 + 5 \times 0.45 + 10.825 = 2.25 + 10.825 = 13.075$$

RL_{CH75} is the RL_{BVC} for curve 4.

4. Crest curve, CH75 to CH105, $L = 30m = 0.30$

CH 75, $RL_{BVC} = 13.075$, $G_1 = +5\%$, $G_2 = +0\%$,

$$r = (G_2 - G_1)/L = (0 - 5)/0.3 = -16.67$$

CH 90, $x = 0.15$ (CH90 - CH75 = 15m)

$$RL_{CH90} = \frac{1}{2}rx^2 + G_1x + RL_{CH75}$$

$$= \frac{1}{2}(-16.67 \times 0.15^2) + 5 \times 0.15 + 13.075 = -0.188 + 0.75 + 13.075 = 13.638$$

CH 105, $x = 0.30$ (CH105 - CH75 = 30m)

$$RL_{CH105} = \frac{1}{2}rx^2 + G_1x + RL_{CH75}$$

$$= \frac{1}{2}(-16.67 \times 0.30^2) + 5 \times 0.30 + 13.075 = -0.750 + 1.50 + 13.075 = 13.825.$$

Table 13-2 shows the results of solving the above problem (§ 13.4.2), including first and second differences, and chord lengths.

Table 13-2 Vertical curve profile with multiple slopes.					
Chainage	x	RL	Δ'	Δ''	Chord
00	0.0	10.000			
			+0.300		15.003
15	0.15	10.300		0.225	
			+0.525		15.009
30	0.15	10.825		0.225	
			+0.750		15.019
45	0.15	11.575		0.000	
			+0.750		15.019
60	0.3	12.325		0.000	
			+0.750		15.019
75	0.45	13.075		-0.187	
			+0.563		15.010
90	0.15	13.638		-0.376	
			+0.187		15.001
105	0.3	13.825			
Arc Length =					105.080

Note the two sets of formula.

One for the curve:

$$y = \frac{1}{2}rx^2 + G_1x + RL_{PC}$$

and one for the straight:

$$y = G_1x + RL_{PC}$$

The curve formula can be used for the straight because, as $G_2 = G_1$.

$$\text{and } r = (G_2 - G_1)/L,$$

then $r = 0$ and thus

$$\frac{1}{2}rx^2 = 0.$$

This may be of help in generalizing a spreadsheet application.

13.5 Vertical Curves in Rural Road Design

To the surveyor, the common use of the vertical curve is in rural road design and construction.

Austrroads, 2003, Chapter 10, has a comprehensive treatment of vertical alignment to which you are referred.

The vertical profile of a road is a series of grades joined by parabolic curves.

Grades should be as flat as possible taking into consideration:

- terrain
- longitudinal drainage
- economy.

Steeper grades produce greater variations in vehicle speed between vehicles with varying power to weight ratios.

These variations lead to higher relative speeds between vehicles, increasing risk of rear end accident rates. They also lead to traffic queuing and overtaking requirements, which increase traffic safety problems, especially at higher traffic densities.

There is an additional freight cost due to increased power requirements and lower speeds.

13.5.1 Effects of Grade on Vehicle Type

Table 13-3 Effects of grades on traffic speeds. (Austrroads, 2003, Table 10.1, p54).					
Grade %	Reduction in Vehicle Speed compared to Flat Grade				Road Type Suitability
	Up hill		Down hill		
	Light vehicle	Heavy vehicle	Light vehicle	Heavy vehicle	
0 - 3	Minimal	Minimal	Minimal	Minimal	For use on all roads
3 - 6	Minimal	Some reduction on high speed roads	Minimal	Minimal	Use on low-moderate speed roads. (Including high traffic volume roads).
6 - 9	Largely unaffected	Significantly slower	Minimal	Minimal for straight alignment. Substantial for winding alignment	For use on roads in mountainous terrain. Usually need to provide auxiliary lanes in high traffic volume.
9 - 12	Slower	Much slower	Slower	Significantly slower for straight alignment. Much slower for winding alignment	Need to provide auxiliary lanes for moderate-high traffic volumes. Consider run-away vehicle facilities for high commercial traffic.
12 - 15	10-15Km/h slower	15% max negotiable	10-15Km/h slower	Extremely slow	Satisfactory on low volume roads. Little/no commercial traffic.
15 - 33	Very slow	Not negotiable	Very slow	Not negotiable	Only in extreme cases & over short lengths. No commercial traffic.

13.5.2 Grades in Rural Road Design

There are two limiting grades in road design.

The first is the minimum grade. It can be zero except in cut where it should be a minimum of 0.5% to provide longitudinal road drainage. An absolute minimum of 0.33% (1 in 300) may be used as long as the table drains maintain a minimum of 0.5% to ensure drainage from the cutting.

Maximum grades affect vehicle performance, and therefore safety. The length of the maximum grade also affects the performance, especially heavy vehicles. A short length of steep grade may be tolerated before performance degradation becomes a problem.

Terrain also affects maximum grades, with the ability to use higher grades over short distances. This may have an effect on construction costs.

Table 13-4 General maximum grades (%) (Austroads, 2003, Table 10.2, p55).

Operating Speed (Kmh ⁻¹)	Terrain		
	Flat	Rolling	Mountainous
60	6 - 8	7 - 9	9 - 10
80	4 - 6	5 - 7	7 - 9
100	3 - 5	4 - 6	6 - 8
120	3 - 5	4 - 6	--
130	3 - 5	4 - 6	--

13.5.3 Length of Vertical Curves in Rural Road Design

The length of a vertical curve should be as great as possible, within economic constraints.

The minimum length of vertical curves is governed by:

1. Sight distance:
a requirement in all situations for driver safety.
2. Appearance:
generally required in situations of low embankment and flat terrain.
3. Riding comfort:
A general requirement to limit increased vertical acceleration through a sag curve to 0.05g. Through approaches to floodways, low standard roads and intersections the increase in vertical acceleration may rise to 0.1g.

13.5.4 “K” Value in Vertical Curve Design

The minimum vertical curvature allowed in a vertical road profile and can be expressed as a single value, the **K** value, to allow computerised road design.

$K = L/A$ where:

K = length of vertical curve for 1% change of grade (metres)

L = length of vertical curve (metres)

A = algebraic change of grade (%).

13.5.4.1 Worked Example 13.6

Worked example 12.4 (§13.4.1) used a sag curve of length $L = 200\text{m}$. From the example:

$$A = 9.6\%$$

$$K = L/A = 200/9.6 = 21$$

From Austroads, 2003, Table 10.5, p59, this K value represents an operating speed of 110Km/h for a 0.05g sag curve, but (Table 10.6, p59) only 70Km/h for “Headlight sight distance”.

13.5.5 Sight Distance in Vertical Curve Design

Austroads, 2003, Chapter 8, has a comprehensive treatment of sight distances to which you are referred.

Sight distance is the distance necessary for a driver to see, react to and avoid an object on a road. A number of parameters are incorporated in various scenarios to calculate sight distance.

The 3 main parameters are:

Object height: intersection line, 0.0m; object, 0.2m; taillights, 0.6m

Driver eye height: car, 1.05m; truck, 2.4m

Reaction time: normal 2.5 seconds; alert, 2.0 seconds.

Various types of sight distances can be determined:

1. Stopping sight distance (**SSD**). The distance necessary for an alert driver, travelling on a wet pavement at design speed, to perceive, react and brake to a stop before reaching a hazard on the road ahead.
2. Overtaking sight distance (**OSD**). The distance necessary for a driver to overtake safely a slower vehicle without interfering with the speed of an oncoming vehicle.
3. Manoeuvring sight distance. The distance needed for an alert driver to perceive, react, brake and manoeuvre to avoid an object. It is generally about 6% less than the SSD.
4. Headlight sight distance. The distance that can safely be assumed for the visibility of an object on a bituminous road. This distance is between 120m and 150m. This distance corresponds to a SSD speed range between 80Km/h and 90Km/hr, or a manoeuvring sight distance speed of 100Km/h.

Active visibility objects such as retro-reflective targets (reflectors), car headlights, tail lights and large or light coloured objects may allow SSD at design speed at night.

Increase in light output has little effect on visibility. A fivefold output increase will only allow a 15Km/h increase in speed. A tenfold increase in light output will allow a reduction of only 50% in object size.

Beam intensity is limited by the requirements of driving visibility and glare to oncoming vehicles.

5. Horizontal curve perception distance. This is the distance needed for a driver to perceive and react to the presence of a horizontal curve that may require speed adjustment.

A driver needs to see enough the curve to judge its curvature. The minimums are:

5 degrees of curve, (radius $\times 5^\circ$ in radians = $R \times 0.09$, 9% radius.)

about 80m of curve length

the whole curve.

Horizontal curves “hidden” over a crest curve can cause particular perception problems.

These various sight distances produce a recommended range of “K” values that are needed to be incorporated into the design of the vertical profile curve. The designer’s task is to ensure that the design exceeds the minimum values for the design operating speed in various profile configurations.

The surveyor’s task is to apply the design to the parabolic curve formula to calculate the RLs of the vertical stations.

13.5.6 The Arc Length of the Vertical Curve

The task may be to find the **length** of the parabola joining two designated grades.

This is mostly done by summing the chord lengths between each pegging station, if indeed that level of accuracy is required. In general, the **length** of the vertical curve, L, will suffice.

The derivation of the arc length of any parabola is not examined as the effort needed to perform the calculation provides no advantage over the summation of chord lengths.

Worked example 9.3 produced an answer of 200.0480m at 20m intervals (10 steps). Iterating at a $dx = 1\text{m}$ (200 steps) produced an answer of 200.0485m

13.6 Concluding Remarks

For Engineers who will be working on road design and construction, besides the horizontal road alignment that was presented in Chapter 9, the vertical road alignment will also be part of your task. This is necessitated by the need for driver/passenger comfort as the vehicle passes a crest or sag, safe stoppage distance in case of hazards or objects, safe overtaking distance or headlight

sight distance. Vertical road alignments occur where gradient changes and is modelled using a parabolic curve (of quadratic type). Essential is for the students to first determine the constant terms of the parabolic equation then apply it to obtain the reduced levels (RLs) at given chainages along the road. This Chapter presented you with the concepts and worked examples on how this is achieved.

13.7 References for Chapter 13

1. Uren and Price (2010) Surveying for engineers. Fourth edition, Palgrave Macmillan, Chap. 14.
2. Austroads, Rural Road Design - A Guide to the Geometric Design of Rural Roads, Sydney 2003. ISBN 0 85588 655 2. *Supersede by:*
3. Austroads, Guide to Rural Road Design Part 3: Geometric Design, AGRD03-16, 2016.



14 Chapter 14 Global Navigation Satellite System

14.1 Introductory Remarks

In your career in mining and civil engineering, global navigation satellite system (GNSS) mode of positioning will be a routine task. This is because this satellite system is fast revolutionizing these industries in a manner that its inventors never fathomed. Numerous books that cover the GNSS topics exist (see, e.g., Awange 2012 and the references therein). Their applications to Engineering works have also been treated in books e.g., Uren and Price (2010, Chapter 7) and Irvine and MacLennan (2006, Chapter 9). In this Chapter, we provide the fundamental of GNSS and refer the interested students to these books. In particular, the book by El-Rabbany (2006) is suitable for beginners as it is void of many equations found in other GNSS books while Awange (2012, 2017) offers examples of its application to environment, an area that could interest both mining and civil engineers dealing with environmental impacts of their operations. In §14.1, we show how the GNSS is revolutionizing the mining and civil engineering operations. §14.3 then presents the students with the basic operations of GNSS with GPS as an example. The errors that underpin the GNSS operations are discussed in §14.4 before concluding the chapter in §14.5.

Throughout history, position (location) determination has been one of the fundamental tasks undertaken by humans on a daily basis. Each day, one deals with positioning, be it going to work, the market, sports, church, mosque, temple, school or college, one must start from a known location and move towards another known destination. Often the start and end locations are known since the surrounding physical features form a reference upon which we navigate ourselves. In the absence of these reference features, for instance in the desert or sea, one then requires some tool that can provide knowledge of position. To mountaineers, pilots, sailors, etc., the knowledge of position is of great importance. The traditional way of locating one's position has been the use of maps or compasses to determine directions. In modern times, however, the entry into the game by Global Navigation Satellite Systems (GNSS; Hofmann-Wellenhof 2008) comprising the US's Global Positioning System (GPS), Russia's Global Navigation Satellite System (GLONASS), the European's Galileo, and the proposed Chinese's Compass has revolutionized the art of positioning, see, e.g., Awange (2012) and Awange and Kiema (2013). The use of GNSS satellites can be best illustrated by a case where someone is lost in the middle of the desert or ocean and is seeking to know his or her exact location (see, e.g., Figure 14-1).

In such a case, one requires a GNSS receiver to be able to locate one's own position. Assuming one has access to a hand-held GNSS receiver (e.g., Figure 14-1), a mobile phone or a watch fitted with a GPS receiver, one needs only to press a button and the position will be displayed in terms of geographical longitude, λ and latitude, ϕ . One then needs to locate these values on a given map or press a button to send his/her position as a short message service (sms) on a mobile phone as is the case in search and rescue missions. For mining and engineering tasks, use of GNSS is already revolutionising the way operations are carried out (see, e.g., § 14.2 and also Uren and Price 2010, Chapter 7).

The increase in civilian use has led to the desire of autonomy by different nations who have in turn embarked on designing and developing their own systems. In this regard, the European nations are developing the Galileo system, the Russians are modernizing their GLONASS system while the Chinese are launching a new Compass system. All of these systems form GNSS with desirable positional capability suitable for mining operations and civil engineering tasks. These GNSS desirable capabilities are:

- ✓ **Global:** This enables their use anywhere on earth.
- ✓ **All weather:** This feature makes GNSS useful during cloudy and rainy periods.

- ✓ **Able to provide 24-hour coverage:** This enables both day and night observation and can thus enable, e.g., rescue mission in mine operations to be undertaken at any time.
- ✓ **Cheaper:** Compared to other terrestrial surveying observation techniques such as levelling and traversing discussed in Chapters 1 and 5 respectively, GNSS are economical due to the fact that as an operation, few operators are needed to operate the receivers and process data. Less time is therefore required to undertake a GNSS survey to obtain a solution since it combines both procedures (levelling and traversing) of into a single operation to obtained three-dimensional position.
- ✓ **Able to use a global common reference frame** (e.g., WGS 84 Coordinate System discussed in Awange 2012).

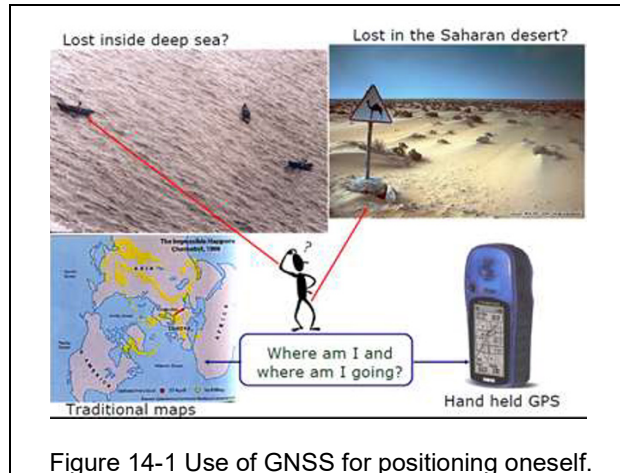


Figure 14-1 Use of GNSS for positioning oneself.

14.2 GNSS, A Revolution to Mine and Civil Engineering Industries

14.2.1 Measuring principle and GNSS Family

Position is calculated by accurately measuring the distance of a receiver from the satellite by determining the delay in the radio signal transmitted by the satellite. This delay is measured by matching and comparing the received signals by an equivalent receiver generated signal. More precise measurements use phase instead of timing measurements. The signals, however, are subject to various sources of errors, which are discussed in § 14.4. For detailed discussion on the measuring principle, we refer to Awange (2012, Section 3.3).

The Global Positioning System or GPS is the oldest and most widely used GNSS system, and as such will be extensively discussed in this chapter. The development of GPS satellites dates back to 1960's (Hofmann-Wellenhof 2001, Leick 2003). By 1973, the US military had embarked on a program that would culminate into the NAVigation System with Timing And Ranging (NAVSTAR) GPS, which became fully operational in 1995. The overall aim was to develop a tool that could be used to locate points on the Earth without using terrestrial targets, some of which could have been based in domains hostile to the US. GPS satellites were therefore primarily designed for the use of the US military operating anywhere in the world, with the aim of providing passive real-time 3D positioning, navigation and velocity data. The civilian applications and time transfer though the predominant use of GPS is, in fact, a secondary role. Not to be left behind, the Russians developed GLONASS that was first launched in 1982 has also been in operation after attaining full operational capability in 1995. Full constellation should consist of 21 satellites in 3 orbits plus 3 spares orbiting at 25,000 km above the Earth surface. Other members of the GNSS family comprise the People's Republic of China's Beidou (also known as Compass) and the European's Galileo under development.

14.2.2 Applications to Mine and Civil Engineering Operations

With the GNSS receivers undergoing significant improvement to enhance their reliability and the quality of signals tracked and the plethora of GNSS satellites discussed in section 14.2.1 above, the world will soon be proliferated with various kinds of receivers that will be able to

track several or all GNSS satellites. Indeed, that they will revolutionize mine and civil engineering operations is indisputable. Civil engineering and mining works will benefit enormously from these enhanced and improved GNSS satellites. First, the possibilities of combining some of the main GNSS satellite systems could impact positively on engineering and mining accuracies by meeting demands of various users, e.g., for open pit mining where the haul tracks and machines employ GPS navigation system. Furthermore, safety and rescue missions in mining will benefit from improved positioning accuracy. Evidently, mining companies are increasingly demanding their employees to acquaint themselves with the use of hand-held GNSS receiver (e.g., Figure 14-1) for rescue purposes besides its use in prospecting and exploration. Such tasks stand to benefit from the expanded and improved GNSS system. For civil engineering works, the capability to achieve cm-mm level positioning accuracies in real time will significantly reduce operational costs since the levelling tasks (Chapter 1) and traversing task (Chapter 5) are now capable of being undertaken using GNSS receiver. This will also benefit post construction monitoring of the structures for deformation. While standalone (single) receivers such as that in Figure 14-1 have been known to have 3-15 m accuracies, the expanded and improved GNSS system is already pushing this accuracy to sub-meter level. This is expected to benefit reconnaissance works where stations will be identified to within a radius of less than 3m.

14.3 GPS Design and Operation

In general, GPS is comprised of *space*, *control* and *user segments*, which are described in the following subsections.

14.3.1 Space Segment

This segment was designed to be made up of 24 satellites plus 4 spares orbiting in a near circular orbit at a height of about 20,200 km above the Earth's surface. In June 2011, the orbits were adjusted to supply a 27-slot constellation² ("Expandable 24") for improved baseline coverage. As of September 2016, there were 31 operational GPS satellites in space³. Each satellite takes about 11 hours 58 minutes 2 seconds to orbit around the Earth (i.e., two sidereal orbits the Earth per day, Agnew and Larson 2007). The expandable 24 constellation consists of 6 orbital planes inclined at 55° from the equator, each orbit plane containing 4 + 1 satellites (Fig. 10.2). With this setup, and an elevation of above 15° , about 4 to 8 satellites can be observed anywhere on the Earth at any time (Hofman-Wellenhof et al. 2001, 2008, Leick 2003). This is important to obtain 3D positioning in real-time. The satellites themselves are made up of solar panels, internal components (atomic clock and radio transmitters) and external components such as antenna. The orientation of the satellite

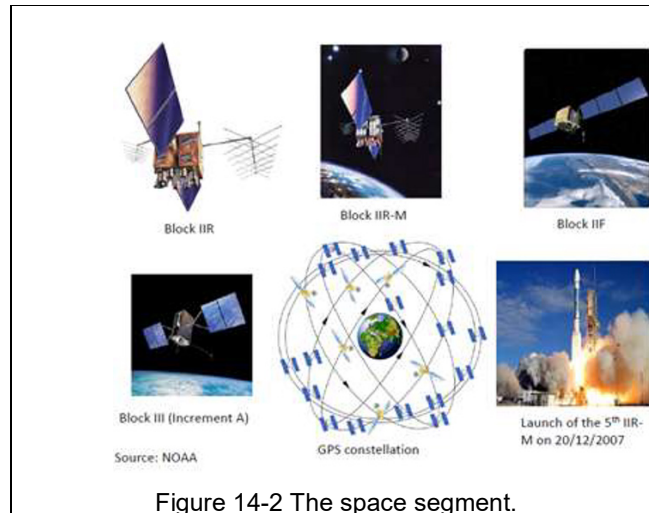


Figure 14-2 The space segment.

² <http://www.gps.gov/systems/gps/space/>

³ <http://adn.agi.com/SatelliteOutageCalendar/SOFCalendar.aspx>

<http://www.gps.gov/systems/gps/space/>

in space is such that the solar panels face the sun to receive energy to power the satellite while the antennas face the Earth to transmit and receive radio signals.

14.3.2 Control Segment

The GPS control segment consists of *master*, *monitor* and *ground* stations. The master control station is located at Colorado Springs (Schriever Air Force Base, Colorado) with a backup station at Vandenberg, California. The ground stations are made up of four antennae located at *Cape Canaveral, Florida*, *Ascension Island* in the Atlantic Ocean, *Diego Garcia* in the Indian Ocean, and *Kwajalein Island* in the Pacific Ocean. By September 2016, nine monitoring stations of the NGA (National Geospatial-Intelligence Agency), six USAF monitoring stations and seven (AFSCN) remote tracking stations are part of the network, enabling every satellite to be seen by at least two monitoring stations and thus improve the accuracy of the computed satellite orbital parameters known as *ephemeris* (see Fig. 10.3)⁴.

These stations monitor the orbital parameters and send the information to the master control station at Colorado Springs. The information obtained from these monitoring stations tracking the satellites are in-turn used to control the satellites and predict their orbits. This is done through the processing and analysis of the information by the master station, which computes the satellite ephemeris and clock parameters and transmits them to the satellites via the monitoring stations. The satellite ephemeris consists of satellite positions and velocities predicted at given times.

There are eleven command and control antennas distributed across the world that augment the control system by monitoring and tracking the satellites in space and transmitting correction information to individual satellites through ground antennas. These stations form the part of the International GNSS Service (IGS) network of over 400 monitoring sites covering all GNSS transmissions. The ground control network is therefore responsible for tracking and maintaining the satellite constellation by monitoring satellite's health and signal integrity and maintaining the satellite orbital configuration.

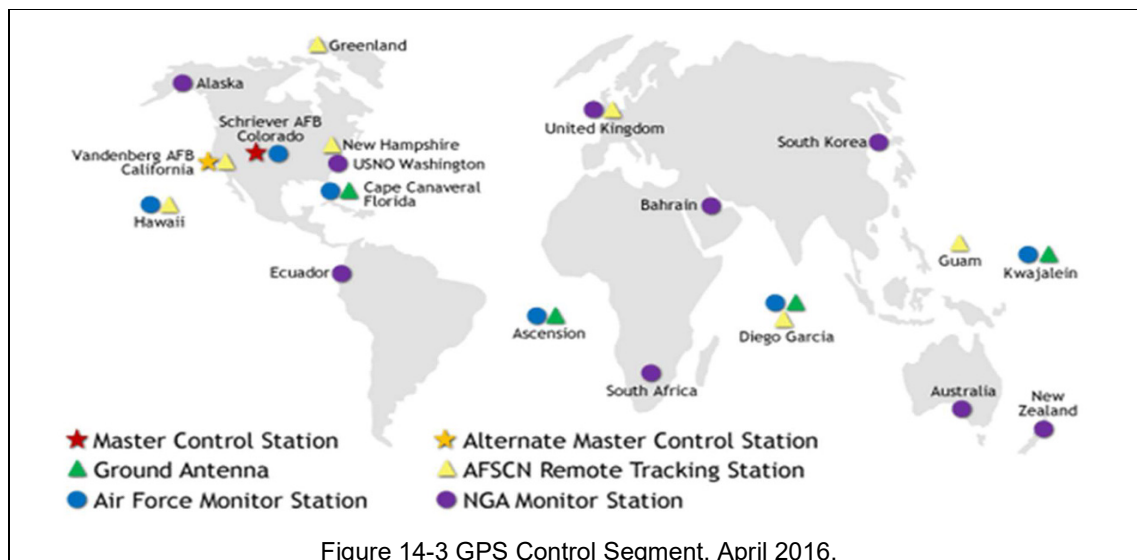


Figure 14-3 GPS Control Segment, April 2016.

14.3.3 User Segment

The user segment consists of receivers (most of which consist of 12 channels), which are either hand-held (also available in wrist watches, mobile phones, etc.) or mountable receivers e.g., in

⁴ www.gps.gov/multimedia/images/GPS-control-segment-map.pdf

vehicles, or permanently positioned. The availability of 12 channels enable receivers to track and process data from 12 satellites in parallel, thus improving on positioning accuracy. These receivers are employed by a wide range of users to meet their daily needs. So wide and varied are the uses of GPS that Awange and Grafarend (2005) termed it the Global Problem Solver (GPS). For military purposes, it is useful in guiding fighter planes, bombers and missiles, as well as naval and ground units.

Civilian use covers a wide range of applications, such as mining, where it is used to guide heavy machinery, or locating positions to agriculture in what has become known as “*precision farming*”. Using GPS and GIS, farmers can integrate location, amount of fertilizer and yield expected and analyse the three for optimum output. Modern car tracking companies use GPS to locate stolen vehicles or trucks that have veered away from predestined routes, while in the aviation sector, GPS can be used in both aircrafts and airports to guide landings and take offs. GPS is also widely used in sports such as mountaineering. The list of uses is therefore only limited to our imaginations (see the applications to civil and mine engineering operations in Section 10.2.2).

14.4 Errors in GPS Measurements

Just like any other measurement, the accuracy derived from GPS measurements are subject to errors that degrade the quality of the derived parameters, including those of interest to civil and mine operations. This section considers some of the most significant errors that undermine GPS observations and discusses how these errors could be minimized and/or avoided.

14.4.1 Ephemeris Errors

As the satellites move along their orbits, they are influenced by external forces such as solar and lunar (moon) gravitational attraction, as well as periodic solar flares (Irvine 2006, p.180).

For shorter baselines (i.e., distances less than 30 km between two receivers on Earth), orbital errors tend to cancel through differencing techniques (see Awange 2012). Over long baselines however, e.g., over 1000 km, orbital errors no longer cancel owing to different receivers sensing different components of the error due to significant changes in the vector directions.

Using the data from the monitoring stations, the master control station predicts satellite positions (broadcast ephemeris), which are transmitted to the user as part of the navigation message together with the data signals during positioning. The accuracy of the broadcast ephemeris has improved tremendously, i.e., from 20-80 m in 1987 to 2 m currently, see e.g., El-Rabbany (2006, p.16). El-Rabbany (2006, p.16) attributes this improvement to superior operational software and improved orbital modelling. *Broadcast ephemeris* are useful for real-time positioning (i.e., if the GPS receiver is expected to deliver results while collecting measurements in the field). *Precise ephemeris* on the other hand are useful for post processing tasks which are required later. In such cases, the post-processed positions of the satellites by other global tracking stations are available 17 hours to 2 weeks later with accuracies of meters.

For civil and mine operation tasks, this error should be eliminated when determining the controls (vertical and horizontal discussed in Chapters 2 and 5 respectively). For other tasks, such as machine guidance in mining and general road survey in civil engineering, the error might be insignificant and use of correct positioning procedures that minimize it should suffice.

14.4.2 Clock Errors

Satellite clocks are precise atomic clocks (i.e., rubidium and caesium time standards). In contrast, the receivers cannot include atomic clocks since the cost would be too high for users to afford and for safety concerns. Clocks within the receivers are thus less precise and as such subject to errors. This is not to say that the satellite clocks are error free, but that the magnitude of the receiver errors are much higher.

The satellite and receiver clocks also must be synchronized to measure the time taken by the signal to travel from the satellites to the receiver. Since the synchronization is normally not perfect, errors are likely to occur. This error is also known as time offset, i.e., the difference between the time as recorded by the satellite and that recorded by the receiver (see, e.g., US Army Corps of Engineers 2007).

Satellite clock errors are small in magnitude and easily corrected because the ground control stations closely monitor the time drift and can determine second order polynomials which accurately model the time drift (US Army Corps of Engineers 2007). These second order polynomials are included in the broadcast message. The receiver clock error is determined as a nuisance unknown along with the required coordinate parameters in the observation equation and this explains the need of the fourth satellite.

14.4.3 Atmospheric Errors

The atmosphere is the medium above the Earth by which the GPS signal passes before it reaches the receiver. *Charged particles* in the ionosphere (50-1000 km) and *water vapour* in the troposphere (1-8 km) affect the speed of the GPS signals, leading to an optical path length between the satellite and the receiver and a delay in the corresponding time the GPS signal takes to reach the receiver, see e.g., Belvis (1992). One of the key tasks of geodetic GPS processing software therefore is to “correct” the ranges between the satellite and the receiver to remove the effects of the Earth's atmosphere, thereby reducing all optical path lengths to straight-line path lengths (Belvis et al., 1992).

The *ionosphere* is made of *negatively charged electrons*, *positively charged atoms* and *molecules* called ions. The charged particles are a result of free electrons that occur high in the atmosphere and are caused by solar activity and geomagnetic storms. The number of free electrons in the column of a unit area along which the signal travels between the sending satellite and the receiver make up what is known in GPS literatures as the Total Electron Content (TEC). Free electrons in the ionosphere delay the GPS code measurements, thus making them too long on the one-hand while advancing the GPS phase measurements, making them too short on the other hand, thus resulting in incorrect ranges (i.e., error in the measured ranges) (Hofman-Wellenhof et al., (2001, pp. 99-108; Leick 2003 p. 191).

The size of the delay or advance (which can amount to tens of meters) depends on the TEC and carrier frequency, i.e., the ionosphere is a dispersive medium (Leick 2003 p. 191). The error effect of the ionospheric refraction on the GPS range value depend on sunspot activity, time of the day, satellite geometry, geographical location and the season. Ionospheric delay can vary from 40-60 m during the day to 6-12 m at night (US Army Corps of Engineers 2007). GPS operations conducted during periods of high sunspot activity or with satellites near the horizon produce range results with the highest errors, whereas GPS observations conducted during low sunspot activity, during the night, or with satellites near the zenith produce range results with the smallest ionospheric errors. Ionospheric effects are prominent over longer baselines (> 30-50 km) although high ionospheric activities can affect shorter distances. Ionospheric errors can be significantly reduced through:

- ✓ Use of dual frequency. Since signal speed through the ionosphere is dependent on frequency (dispersive medium), ionospheric effects which cause a delay of about 2-80 m can be modelled using frequency combination. They are removed mostly by comparing the signal delays of the L1 and L2 frequencies and exploiting the known dispersion relations for the atmosphere (Brunner and Gu 1991).

The disadvantage with using dual frequencies is the increased noise and as such, this approach is useful mainly for longer baselines. For very short baselines (< 5 km), where the atmosphere is assumed uniform, the ionosphere error is minimized once differencing techniques are used (see, e.g., Awange 2012).

- ✓ *Modelling.* GPS navigation messages contain ionospheric correction parameters that are used in correction models during data processing. Several ionospheric correction models, such as the Klobuchar model, are available in commercial software and can be used to reduce ionospheric error. However, this only gives an approximate value that usually does not remove more than 50% of the error. Moreover, modelling is generally inefficient in handling short-term variations in the ionospheric error.

The second medium, the *troposphere*, also known as the neutral atmosphere, consists of 75% of the total molecular mass of the atmosphere, as well as all the water vapour and aerosols. The troposphere is a non-dispersive medium, i.e., the refraction is independent of the frequency of the signal passing through it. Tropospheric errors vary significantly with *latitude* and *height* and are dependent on *climatic zone*. The neutral atmosphere is therefore a mixture of *dry gases* and *water vapour*. Water vapour is unique in this mixture because it is the only constituent that possesses a dipole moment contribution to its refractivity, thus leading to separate treatment between the dipole and non-dipole contribution to refractivity by the water vapour and other constituents in the atmosphere (Belvis et al., 1992).

The tropospheric errors thus comprise two parts: the *hydrostatic part*, commonly referred to in various GPS text as “dry part”, and the dipole component known as the “*wet part*”. According to Belvis et al., (1992) both hydrostatic and wet delays are smallest for paths oriented along the zenith direction and increase approximately inversely with the sine of the elevation angle, i.e., either delay will tend to increase by about a factor of 4 from zenith to an elevation of about 15°.

The *hydrostatic part* contributes 90% of the tropospheric error. It is easily modelled out to a few millimetres or better given surface pressure. The remaining 10% occurs from the wet part, resulting from the *water vapour*, which depends on the refractivity of the air through which the signal is travelling. The refractivity of air depends on (1) the density of *air molecules* (dry component) and (2) the *density of the water vapour* (wet component). Above 50 km altitude, the density of molecules is very low and hence its effect is small. Although the wet delay is always much smaller than the hydrostatic delay, it is usually by far more variable and more difficult to remove (Belvis et al., 1992).

The tropospheric delay therefore depends on *temperature*, *pressure* and *humidity* and affects signals from satellites at lower elevations more than those at higher elevation. For example, El-Rabbany and Belvis (1992) indicates that tropospheric delay results in pseudorange errors of about 2.3 m for satellites at the zenith (i.e., satellites directly overhead), 9.3 m at 15° elevation and 20–28 m for 5° elevation. Therefore, the lower the elevation angle of the incoming GPS signal, the greater the tropospheric effect because the signal travels a longer path through the troposphere.

Tropospheric delay can be problematic, especially when stations are widely distributed at different altitudes. For example, cold (dense) air can accumulate in mountain basins on clear calm nights whilst mountain tops may be considerably warmer. Tropospheric delay can also exhibit short-term variations, e.g., due to the passing of *weather fronts*. The *hydrostatic part* can be modelled by employing surface meteorological data or by acquiring them from external sources such as the European Centre for Medium Weather Forecast (ECMWF) and the National Center for Environmental Prediction (NCEP).

Whereas the hydrostatic delay can be modelled from the surface meteorological data, the wet component cannot be accurately determined in the same manner, but is instead measured from water vapour radiometers (WVR) (Elgered et al., 1991; Resch 1994; Ware et al., 1986) or by directly estimating the time varying *zenith wet delay* (ZWD) as unknowns from the GPS observations (Herring et al., 1990; Tralli et al., 1988). These estimation techniques usually assume azimuthal symmetry of the atmosphere, and they exploit the form of the elevation dependence of the delay (i.e., the mapping function) and the fact that the delay changes little over

short periods of time (Belvis et al., 1992). These analyses typically constrain the variations in the *zenith wet delay* to between 1 and 20 mm per hour, depending on location and time of year, leading to the recovery of ZWD from GPS data with an accuracy between 5 and 20 mm (Belvis et al., 1992).

In most processing software, ZWD is estimated using Integrated Precipitate Water Vapour (IPWV) models. Many models, e.g., Saastamoinen, Hopfield, and Magnet, have been proposed to model tropospheric errors, see e.g., (Hofmann-Wellenhof et al. 2001). Some of these models depend on real meteorological data input. However, the best observational principle is to keep the baselines as short as possible.

14.4.4 Multipath

Consider now a satellite signal that is meant to travel straight to the receiver being reflected by a surface, as shown in Figure 10.4. The measured pseudorange reaching the receiver ends up being longer than the actual pseudorange had the signal travelled directly. In urban areas, the presence of buildings contributes greatly to the multipath effect. Multipath errors can be avoided by placing the receiver in a place without reflective or refractive surfaces. The best practice is to place the receiver at least 3 m from reflecting walls and in addition use GPS antennas with ground planes which discard indirect reflected signals (which are of a lower power). A choke ring antenna also provides a means of reducing multipath while other receivers have in-built filtering mechanisms. Good mission planning also helps to reduce the effect of multipath.

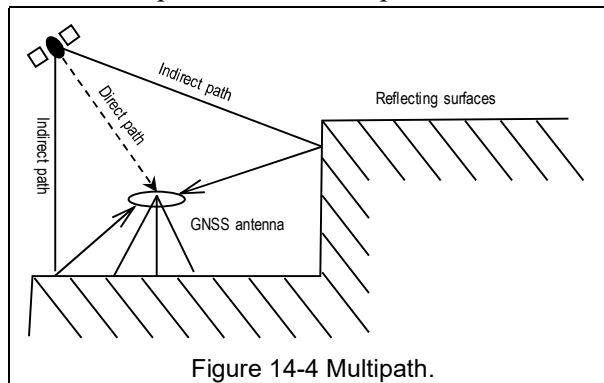


Figure 14-4 Multipath.

14.4.5 Satellite Constellation “Geometry”

Dilution of precision (DOP) depends on the distribution of the satellites in space (see Figure 10.5). With clear visibility and a large number of satellites, the value of DOP is low, indicating a good geometry. With obstructions and fewer satellites, however, the DOP values becomes high, indicating poor geometry which may negatively affect positioning accuracy.

Also, used to measure the geometric strength is the Position Dilution of Precision (PDOP) which can be used essentially as an expression of the quality of the satellite’s geometry. Usually, a PDOP value of less than 6 but greater than 1 is desirable. For a detailed discussion of this topic, we refer the reader to Awange (2012, 2018).

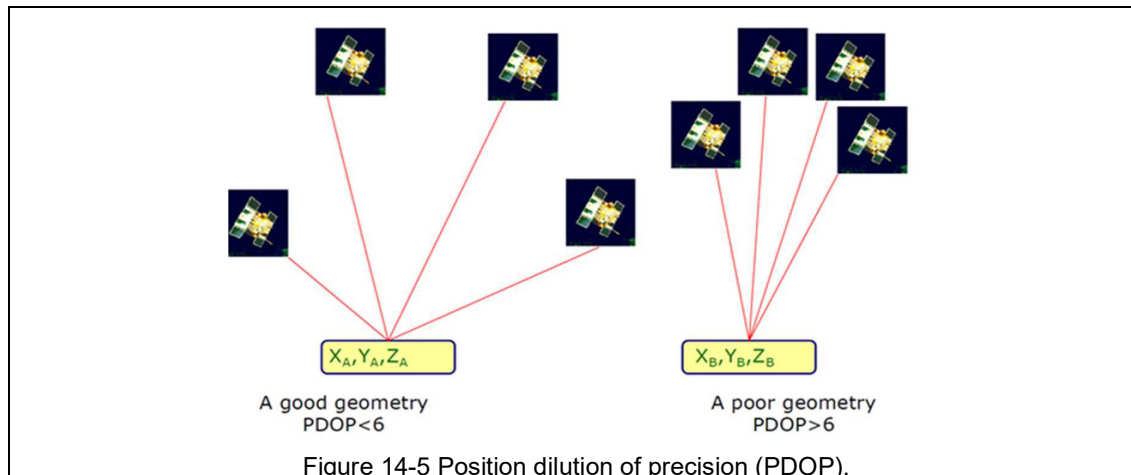


Figure 14-5 Position dilution of precision (PDOP).

14.4.6 Other Sources of Errors

Other sources of errors which may degrade accuracy include hardware errors due to variations in the antenna phase centres with satellite altitude. This error is greater at elevations below 15° but less between 15° and 60° . In addition, there is receiver noise (e.g., signal processing, clock synchronization, receiver resolution, signal noise, etc.) and cycle slips which results when the receivers lose lock on a satellite due to, for example, signal blockage by buildings. Radio interference, severe ionospheric disturbance, and high receiver dynamics can also cause signal loss El-Rabbany (2006, p.25).

14.5 GNSS measuring techniques useful to Engineering

In what follows, three commonly used GNSS positioning techniques that would be useful to engineering, i.e., RK (Relative Kinematic), RTK (Real Time Kinematic), and PPP (Precise Point Positioning) are discussed following Goncalves and Awange (2017), with examples on how they can be employed to map the Spring tide line in Fig. 13-6 that represents some shoreline indicators in a coastal profile. The mean high water (MHW) and mean low water (MLW) both take into account a tidal datum defined by the intersection of the coastal profile with a special vertical elevation defined by the tidal constituents of a particular beach. The same case applies to the spring tide line, which by definition is dependent on a specific epoch related to the tide. Figure 14-6 also represents the vegetation line, dune, berm, beach face, and wet beach. For this figure, for example, it is desirable to map the spring tide line and, based on this limit, to build a buffer zone (see a GNSS receiver mapping of this boundary). Notice that a spring tide line position is not fixed during time and is also dependent on sediment transport and geomorphological characteristics of a beach that can be affected by climate change and anthropogenic factors. This fact highlights the importance of monitoring its position.

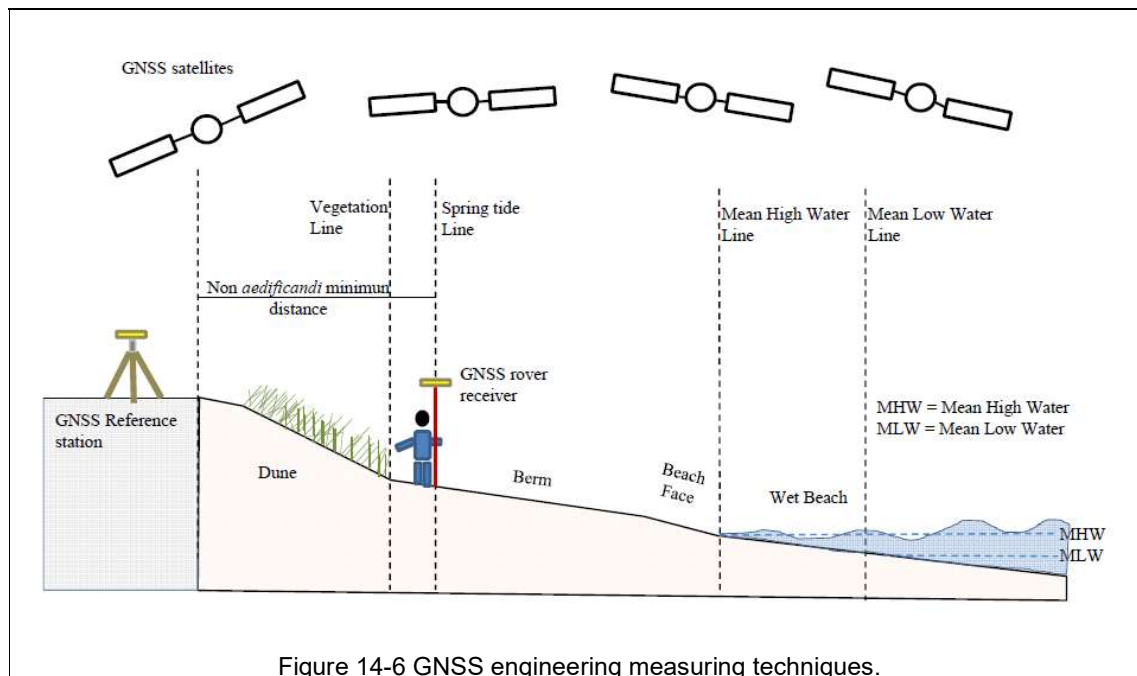


Figure 14-6: GNSS reference static stations and a person with a rover receiver tracking four satellites in a beach profile with some shoreline indicators (e.g., MHW, MLW, vegetation line, and spring tide line).

14.5.1 Relative Kinematic (RK)

The basic concept of RK positioning is to determine the coordinates of unknown points (e.g., the spring tide line's position as shown in Fig. 13-6) with respect to a known point (the station whose position is precisely known, e.g., a base or reference station) [Fig. 13-7(a)]. In this technique, the reference receiver at a known Station A remains fixed, and the second receiver (known as the roving receiver) moves collecting data that are later processed using a given vendor software, e.g., Trimble Business Center, to determine the positions. The roving receiver can be mounted on a vehicle (Fig. 13-8) or simply held by the observer. Although the reference station collects data during the entire period of observation, the roving receiver can collect data at a point between 5 and 10 min before moving to another point or collect kinematically. Both receivers (reference and rover) observe the same GNSS satellites simultaneously to determine a baseline, i.e., the distance between the known Point A and the roving receiver at a Point B.

The main purpose of having two receivers, one fixed and the other roving, is to enable elimination or minimization of common errors, such as satellite clock error, ionospheric delay, tropospheric delay, and ephemeris errors, among others. The advantage of this method is that it provides more accurate results compared with the RTK and PPP methods discussed next, i.e., centimeter-level accuracy. This is because it permits post-observation processing, which is a period that precise known satellite positions could be used and more errors eliminated by deselecting poor satellites. Moreover, common errors in both reference and roving stations cancel out by using baseline processing techniques during data processing. The disadvantage of the method is its reliance on two receivers, increasing the hardware and manpower costs for the authority undertaking the monitoring campaign. Furthermore, the obtained roving positions will be relative to that of the reference station due to the nature of the method, in which the baseline vector is computed and its elements added to the position of the reference station to obtain the position of the rover. Any errors in the reference station or wrong input of the reference station will propagate to the roving station, hence, impacting the shoreline determination. An example of the application of the RK method to shoreline mapping, particularly when the reference station is near the shoreline, is presented in Goncalves et al. (2012a). In Fig. 1, for example, the spring tide line position could be mapped by having a reference station fixed on the ground to the left, and the roving station mounted on a mobile device, as shown in Figure 14-7. Details of the RK method can be found in Leick (2004), Hofmann-Wellenhof et al. (2008), and Awange (2012,2018).

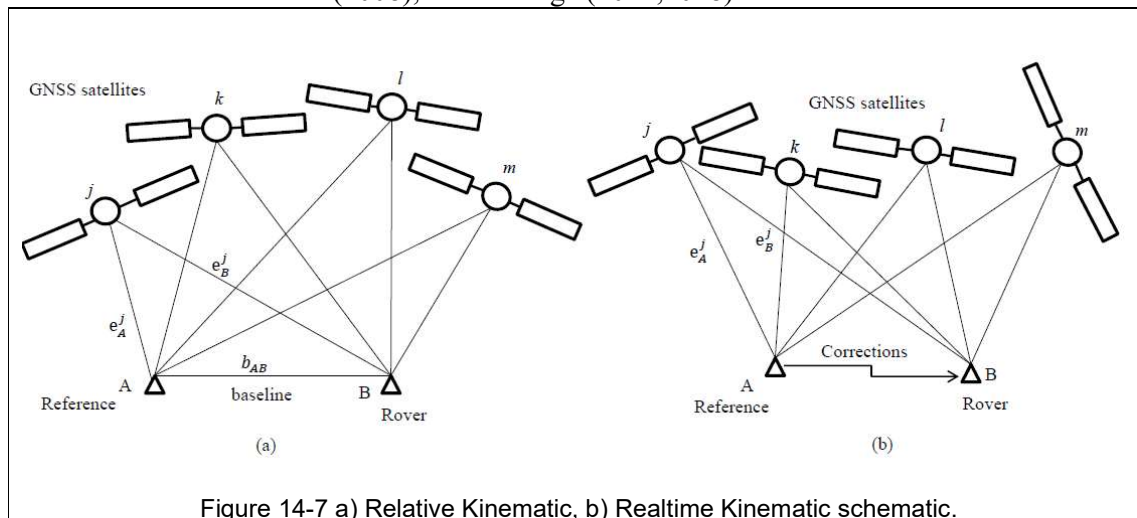


Figure 14-7: (a) RK (Note: A = known reference (fixed) station; B = unknown roving station; b_{AB} = baseline; and e_A and e_B = geometric distances between the satellites and the receivers);

b) RTK method, in which the carrier phase and other information (e.g., base station coordinates and its antenna heights) are sent from Reference Receiver A attached to a radio transmitter via communication link to Roving Receiver B attached to a radio receiver; Roving Receiver B combines and processes the received data to get the coordinates of the rover [(a) and (b) adapted from Hofmann-Wellenhof et al. 2008].

14.5.2 Real-time Kinematic (RTK)

Figure 14-7(b) shows the basic configuration for the RTK method, which is considered a real-time positioning using two or more receivers. The basic configuration for this technique consists of a receiver fixed at a Point A with known coordinates and a rover at a Point B [similar to the configuration of the RK method in Figure 14-7(a)]. The difference between RTK and RK methods, however, is that RTK is capable of delivering real-time positions in the field as opposed to RK, thus alleviating the need for post-processing the data. Similar to the RK technique discussed previously, RTK also requires more than two receivers, with one placed at a reference (base station) station and both measuring carrier phases.

For RTK, the receiver at the control station attached to a radio transmitter samples data every second and transmits these raw data together with its position (along with the antenna height) via the communication link (e.g., satellites, mobile phones, radio) to the roving receiver attached to a radio receiver (Awange 2012,2018). Using its radio receiver, the rover receives the transmitted data from the base receiver and uses its built-in software to combine and process the GNSS measurements obtained at both the base and roving receiver to obtain its position in real time (El-Rabbany 2006, p. 76). Some GNSS manufacturers provide a handheld controller that can be used to operate both the roving and the base receivers. Normally, the user carries the roving receiver attached to the radio link in a backpack. This method requires fixing the integer ambiguity at the start of the survey (initialization) before undertaking it.

One of the limitations of this method is that any loss of lock by the receivers to the satellite due to obstruction or loss of communication between the receivers could cause loss of accuracy in positioning. In addition, similar to the RK method, RTK requires the use of at least two receivers, increasing the costs. Reliance on radio link also means that the method can only be used to monitor shorter shoreline lengths in which radio communication is permissible. Furthermore, compared with RK, the quality of the obtained positions at the rover are degraded to latency (i.e., the control station data reaches the rover after some delay). Its use in monitoring the spring tide line in Figure 14-7, for example, would be the same as the RK method in terms of configuration, but the roving receiver in Figure 14-7(b) will provide the positions in real time (no need for post-processing). This method could be useful for shoreline monitoring in cases in which instant decisions are to be made in real time. Its accuracy is to the centimeter level, which is way beyond the required accuracy for shoreline mapping that in most cases ranges from decimeters to meters. The RTK method has been used in coastal applications to determine modifications in dunes by obtaining positions used to generate digital terrain models (Portz et al. 2015) and also when collecting bathymetric data of ocean bottom topography (Dugan et al. 2001).

Figure 14-8: Example of a shoreline survey using a vehicle with a GNSS receiver; the line represents the shoreline indicator mapped. Image after Goncalves and Awange (2017).

14.5.3 Precise Point Positioning (PPP)

Unlike the RTK and RK methods, PPP only needs a single GNSS receiver during the shoreline monitoring to achieve centimetre-level accuracy (static); thus, it becomes attractive for shoreline mapping in terms of reduced costs of the equipment and personnel. This accuracy is achieved by addressing the factors that bedevil the RTK and RK methods, i.e., orbital errors, clock errors, and atmospheric errors (ionosphere and troposphere) (Leandro et al. 2011; Juan et al. 2012). During postprocessing, the PPP method exploits the precise GNSS orbital and clock parameters provided, e.g., by the International GNSS Service, Jet Propulsion Laboratory, and other commercial sources (Awange 2012,2018).

From the PPP method, it is possible to obtain the position of a station or trajectory to centimeter-level accuracy in two ways: estimation close to real time (latency in seconds) mode or post-processing. Using the precise GNSS orbit (accurate ephemerides data), clock information products, and the dual-frequency code pseudorange and/or carrier phase observations, the PPP method processes the ionospheric-free combinations of undifferenced code and phase observations to estimate the positions, integer ambiguity N , and the tropospheric effect (Hofmann-Wellenhof et al. 2008, pp. 166–169).

Its advantages compared with RK and RTK are the following (Gao 2006; Awange 2012, 2018):

1. It uses only a single receiver to achieve centimetre-level accuracy, hence, removing the requirement of a reference station and simultaneous observations to the same satellites from both the rover and the reference station. Furthermore, the operating range limit is not an issue.
2. Its use of a global reference frame gives it a global outlook enabling it to provide much greater consistency.
3. Its use of a single receiver reduces equipment cost and also makes the approach less labour intensive.
4. Along with the provision of location-based data, PPP needs to estimate clock and tropospheric effect parameters, providing a new way for precise time transfer and water-vapor estimating using a single GNSS receiver.

Its disadvantage, however, is the convergence time, which is rather long. The PPP sometimes requires around 30 min to obtain centimetre-level accuracy or to succeed in the first ambiguity fixing (Li and Zhang 2014).

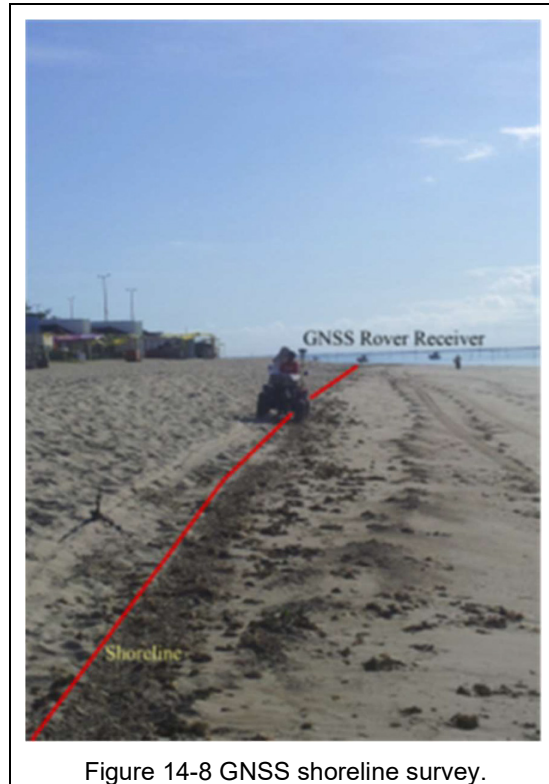


Figure 14-8 GNSS shoreline survey.

14.6 Concluding Remarks

In summary, this chapter has presented the basics of GPS satellites by looking at areas of applications in civil and mine operations, and the associated errors. In general, the measuring principle and the errors discussed in this chapter are also valid for all the other GNSS systems (GLONASS, Galileo and Compass). They only differ in design, signal structure and coordinate systems.

With the anticipated GNSS systems that will comprise various Global Positioning Satellites, civil and mine operations and management tasks requiring use of satellites will benefit from increased number of satellites. Increased number of satellites will have additional advantages compared to GPS system currently in use. Some of the advantages include additional frequencies, which will enable modelling of ionospheric and atmospheric errors to better resolutions; additional signals that will benefit wider range of tasks; and a wider range of satellites from which the users will be able to choose from. GNSS will offer much improved accuracy, integrity and efficiency performances for all kinds of user communities over the world. Detailed use of GNSS to environmental monitoring and management is presented in Awange (2012,2018).

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15 Chapter 15 Setting Out Of Engineering Structures

15.1 Introductory Remarks

In the preceding chapters, methods and techniques for establishing horizontal controls (i.e., traversing; Chapter 5) and vertical control (i.e., levelling; Chapters 1 and 6) were presented. This Chapter now employs the skills learnt in the previous Chapters to set out the structure physically on the ground. This include, e.g., setting out of buildings for civil engineers or haul roads for open pit mining. Of importance is to know the task required of you, the accuracy for setting it out and checking for errors, the booking of field notes and reading of plans, and last but not least, the communication structure and team working while setting out the structures. This chapter therefore prepares you with the skills necessary to practically set out engineering structures. At the end of the Chapter you should:

- ✓ Understand the basic aims of setting out civil works.
- ✓ Appreciate the key important considerations.
- ✓ Understand the stages in setting out horizontal and vertical positions.
- ✓ Distinguish between various setting out methods.

Definition: Setting out can be defined as the establishment of *marks* and *lines* to define the positions and heights of key points to enable construction to be carried out. The procedure adopted should ensure that a specific design feature, i.e., building, a road, etc., is correctly positioned both in *absolute* and *relative* terms at the construction stage. The two main aims of a setting out survey are to:

- a) Place structures in their correct *relative* and *absolute* position, i.e., the structure must be correct size, in correct plan position and at correct level.
- b) Be carried out rapidly in an efficient manner to minimize construction cost and delay **but be fully checked**.

In Figure 15-1 for example, the tunnel is to be set out in the correct location (horizontally) relative to the trees and correct height to enable the train to pass through.

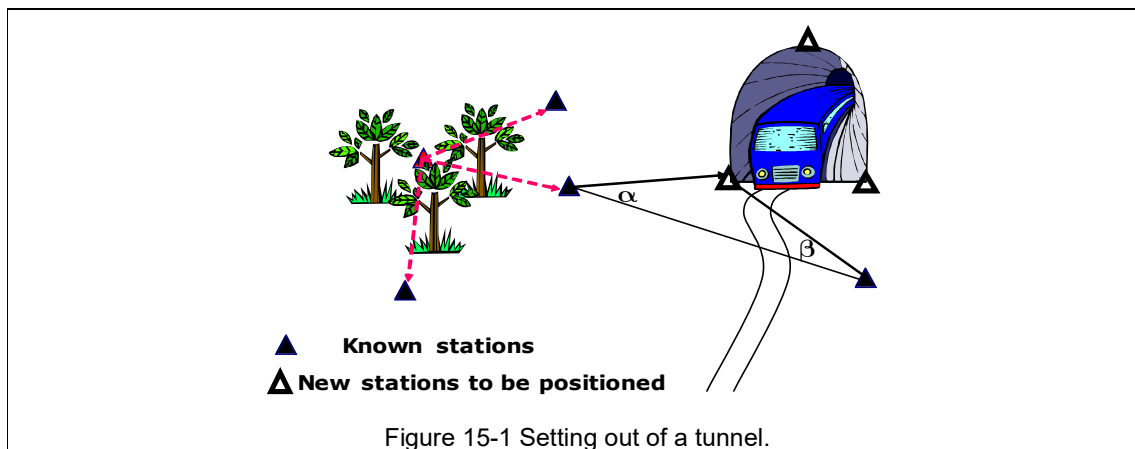


Figure 15-1 Setting out of a tunnel.

Important Considerations:

- ✓ Recording and filing information – do it once, do it right.
- ✓ Care of instrument – protect, store, clean, check and maintain the instrument. During an engineering project, it is important that the surveyor checks and maintains his/her equipment on a regular basis. The checks and adjustments can be summarized as follows:
 - EDM calibration to determine:
 - a) Additive Constant.
 - b) Scale Factor.
 - c) Cyclic Error

- Theodolite calibration checks for:
 - a) Horizontal collimation.
 - b) Vertical collimation.
 - c) Optical plummet.
 - Level adjustment for collimation.
 - ✓ Maintaining accuracy – right gear for the job.
 - ✓ Regular site inspection – what is missing? Check control points and setout pegs for signs of movement on a regular basis.
 - ✓ Error detection – independent checks, use other stations. Every survey mark placed must be checked for gross errors.
 - ✓ Communication on site – know what has been done and when. The engineer or foreman must be informed what marks have been placed and their relationship to the structure.
 - Diagrams.
 - Plans or sketches
- Communication also ensures good teamwork spirit besides ironing out misunderstanding that may occur during setting out e.g., using the correct plan etc.

15.2 Control Surveying

Control Surveys are performed to provide a network of accurately coordinated points so that the surveyor can carry out:

- a) Site surveys
- b) Setouts of engineering structures
- c) Monitoring of existing structures for deformation.

Due to the different surveying techniques used, control surveying is generally divided into:

- Horizontal Control network with accuracies greater than 1:10000, however should get better than 1:15000 with the right observing techniques. Horizontal controls are established using the traverse method discussed in Chapter 5.
- Vertical control network with accuracies to third order levelling ($12\sqrt{K}m$ in mm) or better. Chapter 2 provides the method and procedures needed to establish vertical controls.

The exception to this is a GPS survey which provides both horizontal and vertical coordinates – a true 3-dimensional survey. Control surveys invariably include redundant observations which requires the observations to be adjusted using the following methods (See Chapter 5):

1. Bowditch method - The value of the adjustments found by this method are directly proportional to the length of the individual traverse lines.
2. Transit method - Using this method, adjustments are proportional to the values of ΔE and ΔN for the various lines.
3. Equal adjustment - Used for an EDM traverse, where an equal distribution of the misclose is acceptable by adjusting each measured distance by the same order of magnitude.
4. Least squares adjustment - This is the preferred and generally accepted form of adjusting control networks. Its main advantages over the other methods are that:
 - ✓ Takes into account the different quality of angle and distances.
 - ✓ Least squares adjustment will produce a unique solution.
 - ✓ Makes the sum of the squares of the corrections (residuals) a minimum.

The main disadvantage with the Least Squares Adjustments is that it cannot be carried out by manual calculation methods. You require:

- Access to a computer.
- Least Squares Program.

15.3 Undertaking an Engineering Construction Project

15.3.1 Personnel involved

In the course of any engineering construction project the involvement of people can be broadly divided into three categories:

1. **Client or Developer** – Provides the finance and impetus for the development.
2. **Project Engineer** – Provides the professional expertise and is responsible for:
 - a) The plans and specifications for the project.
 - b) The letting of contracts for the various stages of construction.
 - c) The general overall management of construction.

The Project Engineer will use the services of a Surveyor to:

- ✓ Carry out any site surveying required for the project design phase.
- ✓ Provide the initial project control.
- ✓ Carry out check surveys of structures or survey setouts during the course of construction.

3. **Contractor** – carries out the actual construction of the project. For very large projects it is possible to have a number of different contractors for different stages of the development. The contractor uses the services of a surveyor to:

- ✓ Provide additional survey control as required.
- ✓ Carry out the necessary setout surveys for construction.

15.3.2 Recording and Filing of Information

Field notes are an extremely important facet of an engineering survey. They:

- a) Provide documentary evidence of the work carried out and the dimensions used.
- b) If any problems arise during construction, field notes can be used (in a court of law) to prove or verify the correct location of survey marks.
- c) Allow work to be checked at a later date in the office.
- d) Allows other surveyors to carry out work on site using previously set out marks.
- e) Allows the draftsmen to draw up plans showing the location of survey marks and their relationship to the structures being built.

Field notes must be written showing all survey works carried out on a project. They should include:

- a) Date, plan references and plan amendment date used for set-out.
- b) Clear diagram of the survey carried out and all field measurements observed.
- c) Location of all survey marks placed during the course of a survey and checks carried out.
- d) Reference to any electronic data used or stored. The reference should include name of the files used and where they are stored or backed up.
- e) All field notes should be cross referenced in the job file/summary sheet for the project.

15.3.3 Plans

During the course of a project the surveyor will produce or use a number of plans and diagrams from surveys carried out in the field. These can be summarized as follows:

- **Initial survey site plan.** This type of plan will generally show:
 - ✓ All existing structures on the site and directly adjacent to the site.
 - ✓ Levels and site contours.
 - ✓ Location of all services and invert levels to sewerage and drainage lines.
 - ✓ Any other information requested by the project manager.
- **Survey control plans** showing the location off:
 - ✓ Horizontal control and coordinate list
 - ✓ Vertical control and RL's.

- **Engineering design plans.** The project plans are drawn up once the project design has been finalized and incorporates the information provided from the initial site survey. The design plans are used by both the contractor and the surveyor to set out the construction for the engineering project.

NOTE: - Project design is subject to change during the construction phase. It is always the surveyor's responsibility to ensure that he/she is using the most up to date plan available.

- 1 A construction plan will list the dates that any amendments have been made to the design.
 - 2 Before any setout work commences, the amendment dates on the surveyor's plans should always be checked with the site foreman's or site engineer's plans to ensure that he has the most up to date information available.
- **Set out Plans.** As soon as a practicably possible the survey must provide a sketch or plan to the site foreman or site engineer showing:
 1. All **survey marks** placed during the course of a construction survey.
 2. The relationship of the set-out marks to the actual structure.
 3. A record of work carried out during the engineering construction.

It must be remembered that one of the most common errors on a construction site is associated with misunderstanding the location of survey setout marks in relation to the structure.

15.3.4 Setting out Methods and Procedures

15.3.4.1 Accuracy of the setting out.

The accuracy of the survey work carried out for a construction project will depend on the project specifications. These specifications will vary from project to project and is dependent on the precision required for construction. The accuracy of a *control survey* for a construction site is:

- a) To a greater accuracy than that of the construction specifications.
- b) Dependent on the equipment and observation procedures used.

15.3.4.2 Procedure and Methods.

During the life of a project the surveyor will generally set out the **initial horizontal control** network and then add additional control points as required. Initial controls are generally carried out by a control traverse that is either link or closed (see Chapter 5). In what follows, we present the methods for setting out (i) the horizontal controls, and (ii) the engineering structure.

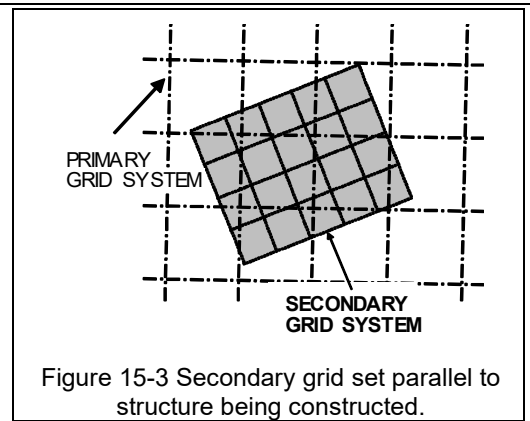
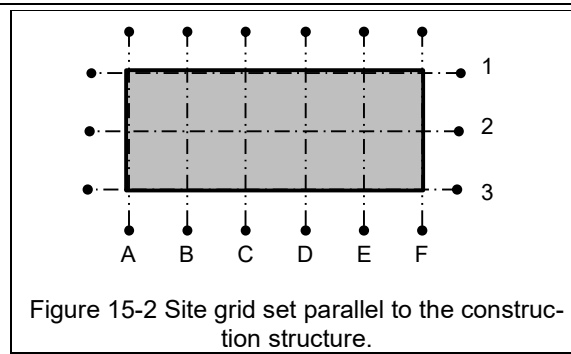
15.3.4.3 Stages in Setting out: Horizontal control.

In general, a preliminary survey will first be carried out. In this regard,

- Check the design – what can be set out and from where?
- Perform a reconnaissance – on site and adjacent to site; are they protected or not protected?
- Survey the stations – are they temporary or permanent?
- Check the Bench Marks – Are there TBM, State BMs, or site BM?
- Collect and study the plans – site surveys, both cadastral and topography:

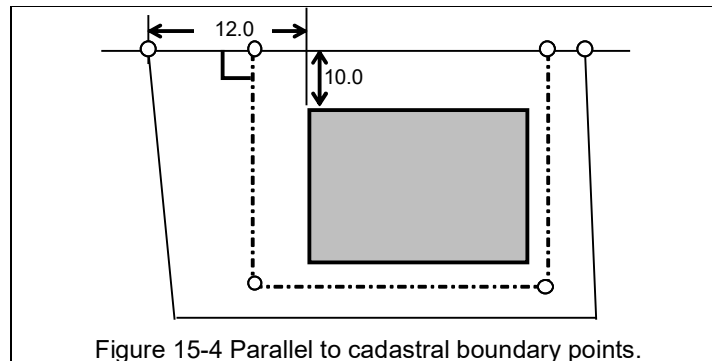
15.3.4.4 Coordinate Systems.

For large engineering projects a grid system is generally used. This grid may be based on (i) national coordinate system, e.g., Map Grid of Australia (MGA) coordinates for Australia, which are generally used for projects covering very large areas, (ii) arbitrary grid – usually designed to be parallel to the structure and referred to as a '**SITE GRID**' (Figure 15-2), and (iii) secondary grid – grid established inside a structure. These types of grids are generally set out parallel to the structure itself to facilitate construction (Figure 15-3).

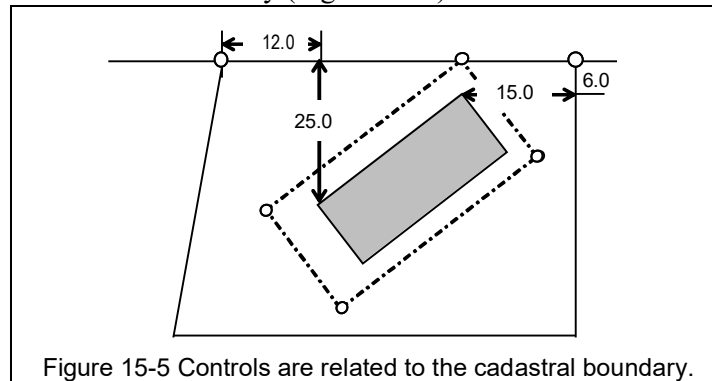


In the case of small buildings or structures, the control is either set out:

- Parallel to a one cadastral boundary - In this case the boundary marks have been used for the baseline for the horizontal control (Figure 15-4).



- Related to the cadastral boundary (Figure 15-5).



Horizontal controls MUST be brought near the feature to be set. In Figure 15-1, you will note that the horizontal controls have been brought near the tunnel to be set. In general, the principle of working from whole to part is followed. Normally, the outer (closed traverse) is set by bearing and distances measurements. They are then used to set up site stations that are subsequently used to set up differing grids (Figure 15-6):

- Survey grid: Usually related to mapping in the area.
- Site grid: Usually related to survey grid at several points on site, but distance set to true for site (Figure 15-2).
- Structure grid: can vary between parts of large construction, i.e., in either scale or direction (mathematical transformation).
- Secondary grid: established within structure (Figure 15-3).

Reference marks with known heights are required and accurate methods must be adopted.

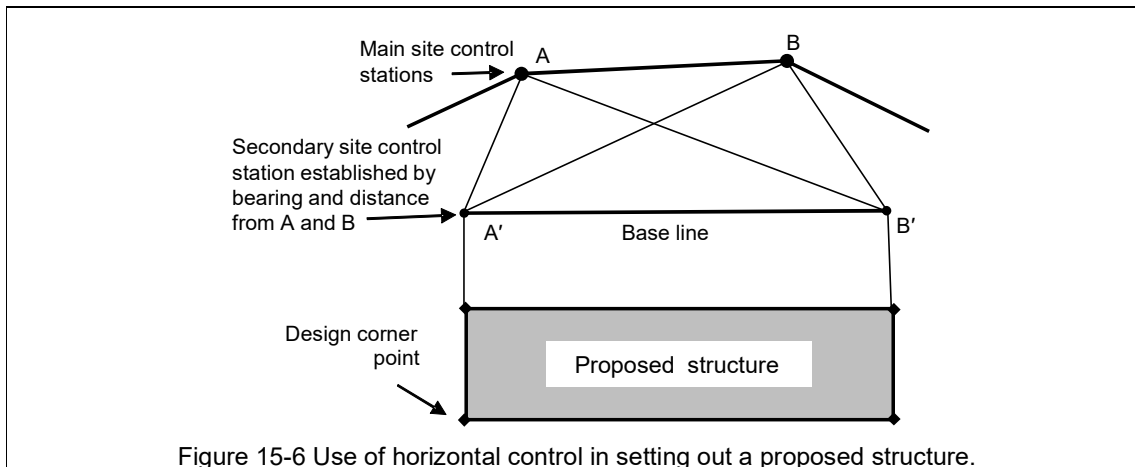


Figure 15-6 Use of horizontal control in setting out a proposed structure.

15.3.5 Methods for establishing horizontal control.

Some of the methods include (Awange and Grafarend 2005; Uren and Price 2010, p 272 onwards).

- Traverse method discussed in Chapter 5. This includes linked, closed and braced quadrilateral or triangular network – these types of network are used in situations where high accuracy control is necessary – 2nd order or better (Figure 15-7).

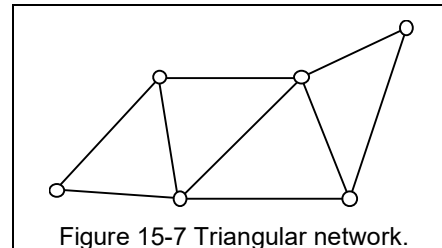


Figure 15-7 Triangular network.

- Intersection is the measurement of distances or angles from two known points to a third point. (Figure 15-8).

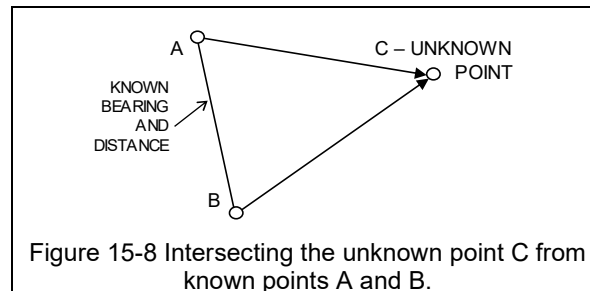
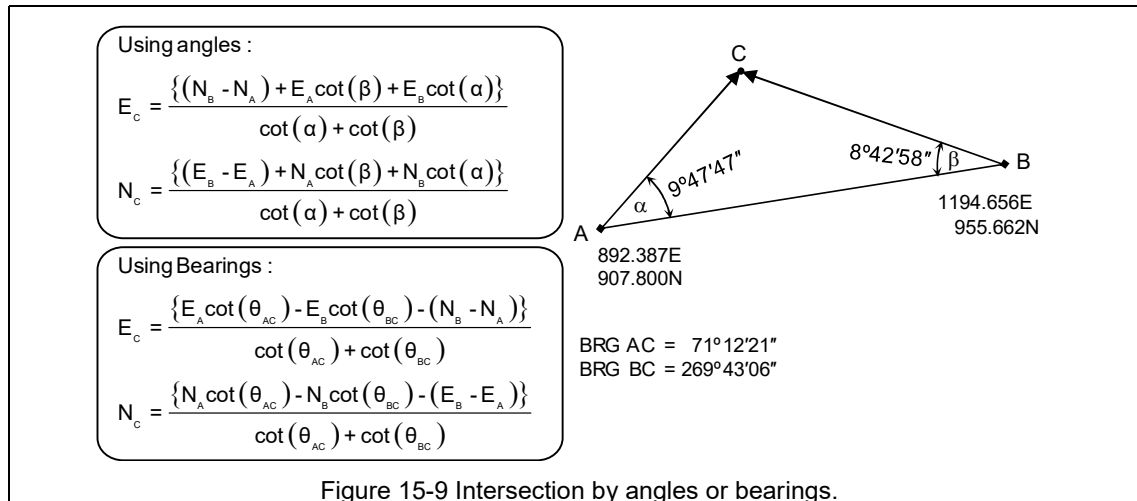


Figure 15-8 Intersecting the unknown point C from known points A and B.

15.3.5.1 Intersection Worked Example 15.1:

Given the coordinates of control points at an open pit mine site as A (892.387E(m), 907.800N(m)) and B (1194.656E(m), 955.662(m)), with only angles measured from points A and B to the unknown point C as $\alpha=9^{\circ}47'47''$ and $\beta=8^{\circ}42'58''$, locate station C without occupying it. Two formulæ can be used based on either angles or bearings as follows (Figure 15-9):

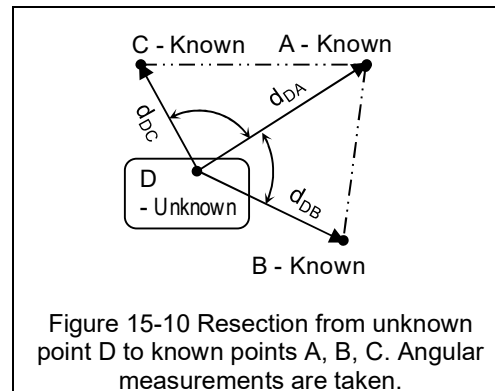


Using either measurement leads to the following position of C:

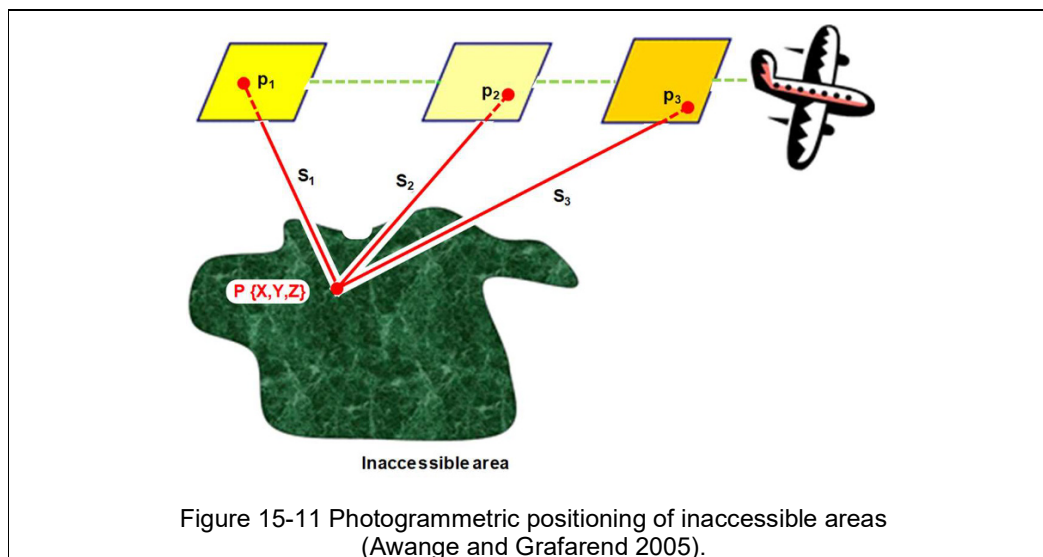
Angle method: **E_C=1038.434m, N_C=905.764m** and

Bearing method: **E_C=1038.436m, N_C=905.764m.**

- Resection is the measurement of angles or distances from an unknown point (D) to three or more known points (A, B, C) in Figure 15-10. Resection by distances is undertaken in Field Practical 3 and is discussed in § Calculation of Three Dimensional Coordinates from Observations 6.3, p 123. It is also used in Mine Surveying, § 7.3.



- For inaccessible areas, Photogrammetry method (Figure 15-11) is used (see Awange and Grafarend 2005, p. pp.168-171 for details).



15.3.5.2 Methods for setting out engineering structures

Set out points for structures can be either carried out by

- ✓ **Baseline method** - This method involves the setting up of a baseline parallel to the structure. This is especially effective where a horizontal grid system has been established as part of the control (see Figure 15-4). The advantages with this system are (i) dimensions are read directly from the plan, no calculations, (ii) only simple calculations required, and (iii) only use 90° (i.e., perpendicular offsets from the baseline).
- ✓ **Radiation from a point** (Figure 15-12) - The main advantage of this method is that it is very quick in the field.

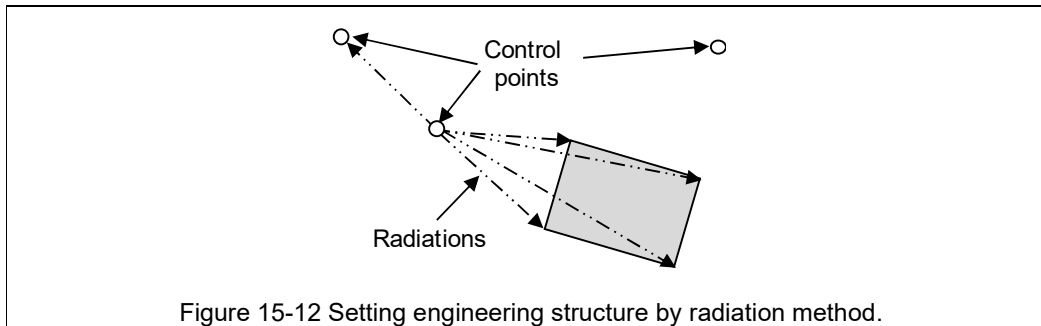


Figure 15-12 Setting engineering structure by radiation method.

- **Radiation from a resected point** (see Figure 15-13) - This method is a variation of the radiation point method. Instead of setting up on a known control point. This method involves:

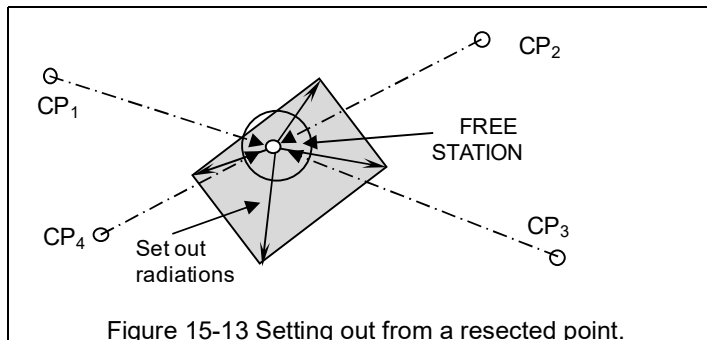


Figure 15-13 Setting out from a resected point.

- ✓ The instrument is set up in any convenient location on the site.
- ✓ A resection is carried out to the surrounding control points.
- ✓ The coordinates of the instrument (Free Station) are calculated using either Total Station software or computer (calculator) program.

- ✓ The coordinates for the Construction points to be set out are calculated and used to calculate a bearing and distance from the Instrument station.
- ✓ The last step should involve a check on your work. It should involve:
 - (i) Distance check between each set out point
 - (ii) New resection and radiation to set out points.
- ✓ Main disadvantages using the Resection method are:
 - ❖ May involve coordinate calculations for set out points.
 - ❖ Set out must be checked from a different location.
 - ❖ Resection must be to three or more control points, where both distances and angles are read (need redundancy to ensure gross errors are detected).
 - ❖ Need resection software on Total Station or in calculator.

15.3.6 Stages in Setting out: Structural Horizontal Control.

Having decided on a method for establishing horizontal control points on site it is necessary to establish the structure control points. These points will include the slab boundaries, column centrelines, interior structural boundaries (lift wells, poured concrete columns etc). These points will have to be transferred to each floor of a building as it rises. The structure control must be checked across diagonals to check the orthogonality of the plan.

Discounting the use of highly specialised automatic coordinate tracking systems employed by surveyors the standard methods used rely on the corners being referenced by on-line offset points as implied in Figure 15-2. The advantage of the offset points is that they are accessible to the various site contractors. The offset controls have to be removed enough from the structure that they don't interfere with the site work, and aren't in a position to be disturbed. The sight boards may be used to provide protection to the control points. Figure 15-14 illustrates setout control that may be suitable for a single floor structure. The offset control points should be set in concrete, be accurately marked and annotated. A 3-6mm drill hole, with a grouted nail or crosshead screw on line at distance, can be used.

The use of sight boards is to provide a sight line above the floor for contractors to use in daily work. They should also be set as RL references, set at equal levels at some convenient height above the RL. Say "Floor + 500mm" should be written on the boards.

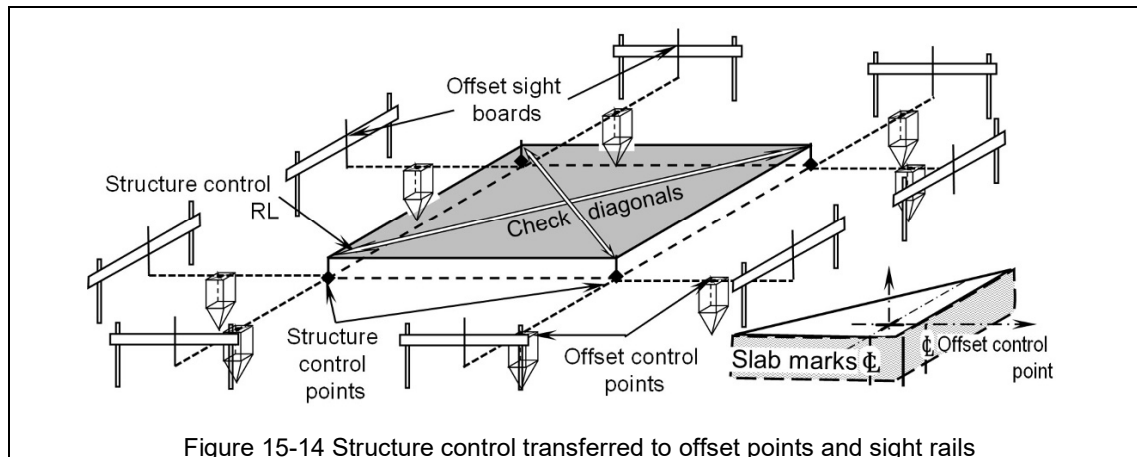


Figure 15-14 Structure control transferred to offset points and sight rails

15.3.6.1 Projecting offset horizontal control points

With the structure control points established there are a few points to be made about the projection of these points to the offset control points and the sight boards. A line is projected from a control point by using the line of sight through a total station or a theodolite. The traditional method is that, having set up over the control, the back sight is made (to the other control point). The line is sighted past the control to the extended offset point and carefully measured to the planned distance. The sight board mark is also set on line. The telescope is then plunged (transited on its horizontal axis) to point to the "behind" point and sight board. The lines should be checked on face left and face right to guard against collimation errors in the telescope.

If control points have been set by coordinates using total station vectors (bearing and distance method) it is as well to set up on the end line points to ensure that the intermediate points ARE on line, not wobbling about a mean line. Check measurements with a EC class 1 tape. (Refer to the discussion in § 7.7.1).

Remember also that EDM is subject to centring errors over the point as well as errors in the total station ($\pm 2\text{mm}$) and 2-5mm in the prism. Plus any error in non-verticality of the prism pole. An error of 3-5mm is quite easily achieved.

15.3.6.2 External multilevel control transfer

The limitation on transferring control with total stations is the angle of elevation that can be used on the telescope. Total stations are limited to just over 30° elevation (60° ZA) before it becomes impossible to sight through the tube. The offset control point must be between 15 and 20m from the structure to allow a visible elevation of 10 – 12m.

The only answer to this is to use a right-angle eyepiece attached to the telescope eyepiece. This works well with elevated sights, but depressed sights of much more than 35° are limited by the line of sight intercepting the TS body.

The use of forced centring methods with prisms, tribrachs and tripods was covered in Chapter 4 and § 4.3 and the Field Practical 3. The process illustrated in Figure 15-15 relies on ensuring the required line is maintained through the building. This may mean re-observing from station 4 back to station 1 to “fine tune” the line through 2 and 3. The coordinates of 2 and 3 will be by slope distance and zenith angle recordings, as with normal traversing. With the line being established from 1 to 4 (0 SET), the coordinates of 2 and 3 are the ΔN coordinates of the line. From 2 and 3 other coordinates can be measured by tape on line. FL/FR mean position may be needed. As elevation increases the trunnion axis tilt error increases direction error.

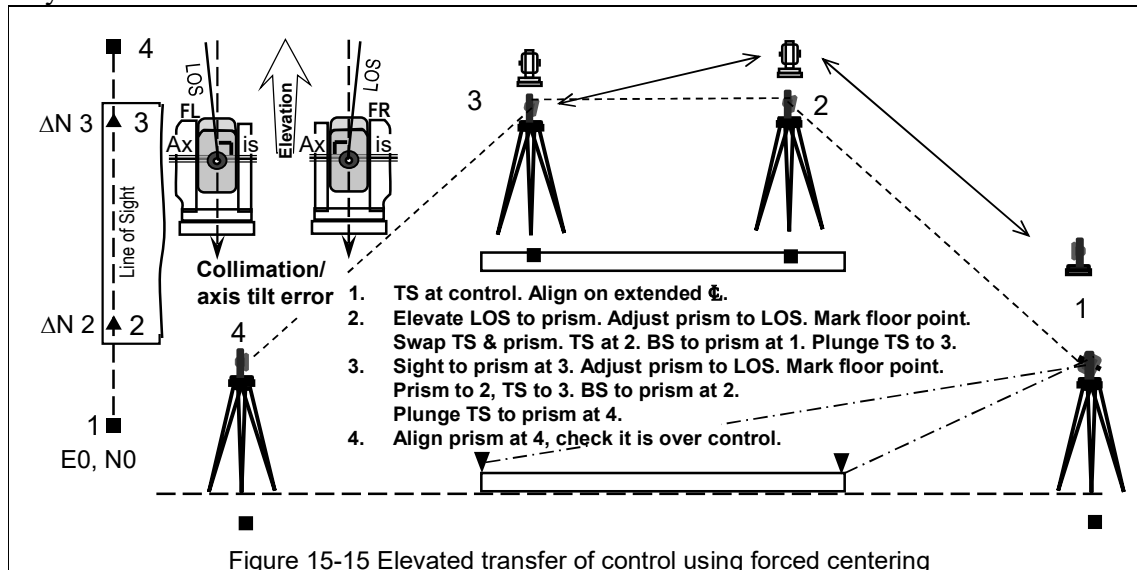


Figure 15-15 Elevated transfer of control using forced centering

15.3.6.3 In-building multilevel control transfer

Multilevel control transfer inside the building is accomplished through voids (holes) left in the corners of the building. The base control point is projected vertically to transparent plates placed over corresponding voids in the higher floor slabs.

The usual instrument used for this vertical transfer is the zenith plummet. Through a pendulous optical system, similar to the automatic level, a line of sight is projected vertically upwards. It is much the same as the centering plummet on a total station.

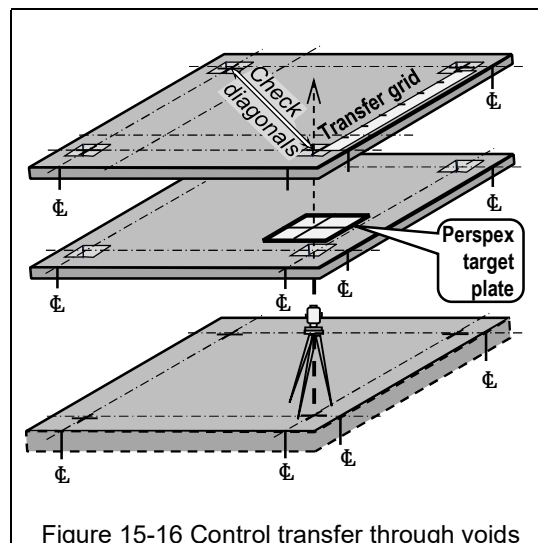


Figure 15-16 Control transfer through voids

15.3.6.3.1 Optical plummets

Optical plummets can be of a nadir/zenith arrangement where the vertical is projected either down (plumb over the ground mark) or upwards through the tribrach centre via a pentaprism. The instrument is levelled using the tribrach. Simple optical plummets, as illustrated in Figure 15-17, are levelled by plate bubbles. They have an accuracy of about 1:10,000 (10mm/100m).

Pendulum compensated plummets, Figure 15-18, can meet 1:200,000 accuracy (0.5mm/100m).

15.3.6.3.2 Laser plummets

Laser plummets for building sites can project a beam over 100m. They have typical accuracies of 1:40,000 (2.5mm/100m).

Both types of plummets have a centering accuracy of about 1- 1.5mm/1.5m over a control point.

The zenith point is transferred to a perspex target plate set over the slab void. It should be referenced to the slab as it will have to be moved before the next floor zenith is projected. The void itself must be protected for safety reasons. No gapping hole.

The structure grid should be established from the plates and checked by diagonals in a manner similar to the base slab.

Depending on the construction of the plummet, and the arrangement of the zenith cross hairs, it should always be checked for verticality by observing in the four quadrants, as discussed in Chapter 4, § 4.2.2.

15.3.6.3.3 Total station plumbing

A total station or a theodolite can be used instead of a zenith plummet. A right angle eyepiece is attached to the eyepiece lens and the telescope is pointed and clamped to the zenith, $ZA = 0^\circ$ (90° elevation). After setting over the control point and carefully levelling the tribrach plate bubbles align to the grid axis direction and set direction to 0° (0 SET).

Mark the perspex target plate using the *horizontal* cross hair. Turn the the instrument to direction 180° and again mark the horizontal cross hair. Then mark the horizontal cross hair at 90° and 270° . As the zenith is clamped at 0° the method eliminates the non-verticality of the automatic vertical circle setting. Figure 15-19 shows a marked perspex target. This type marking may also apply to zenith plumbing instruments if they project lines instead of center dots and circles. Dots can also be plotted if the instrument is taken through 90° rotations.

15.3.6.4 Free station control transfer

Increased height of buildings means that other survey techniques need to be introduced. The free station, by resection from established control, enables a total station to be located at a convenient location. Once the total station coordinates have been established, other points are set out by radiation, Figure 15-13. Also refer to Chapter 16, § 16.6 page 345.

Free station establishment relies on having a number of suitably established control points adjacent the site. This allows a selection of the most suitable points, from a geometric perspective, available for an overdetermined least squares solution of the station coordinates. The calculations are carried out by in-built total station programs. Different control points will be needed as the building progresses. The established control stations themselves must be adjusted by least squares methods and be subject to periodic checking to ensure their stability.

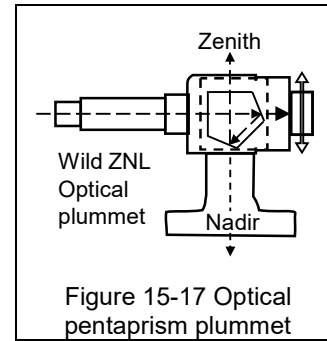


Figure 15-17 Optical pentaprism plummet

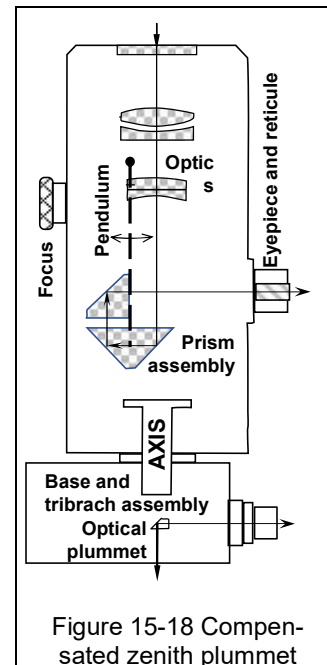


Figure 15-18 Compensated zenith plummet

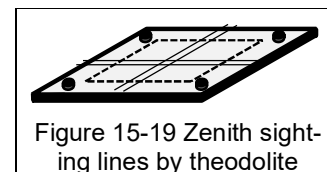


Figure 15-19 Zenith sighting lines by theodolite

High specification equipment is needed for this type of surveying. The TS can meet 0.5" accuracy with EDM to $\pm 0.5\text{mm}$. High precision prisms need to be permanently installed for control. Depending on the set-out, special targets may be needed.

15.3.7 Stages in Setting out: Vertical Control.

Transfer heights from state BMs to site. These helps with determining:

- ✓ Drainage, roads, and service levels.
- ✓ Setting heights on slab of buildings (Figure 15-20). The staff is placed on the datum point and the height read as 3.173m. Next the height is moved to the slab and the reading 1.74 m read to set the slab's reduced level at 16.5m
- ✓ Control from floor to floor level. This includes vertical plumbing, concrete thickness, layout of pillars and elevators etc.

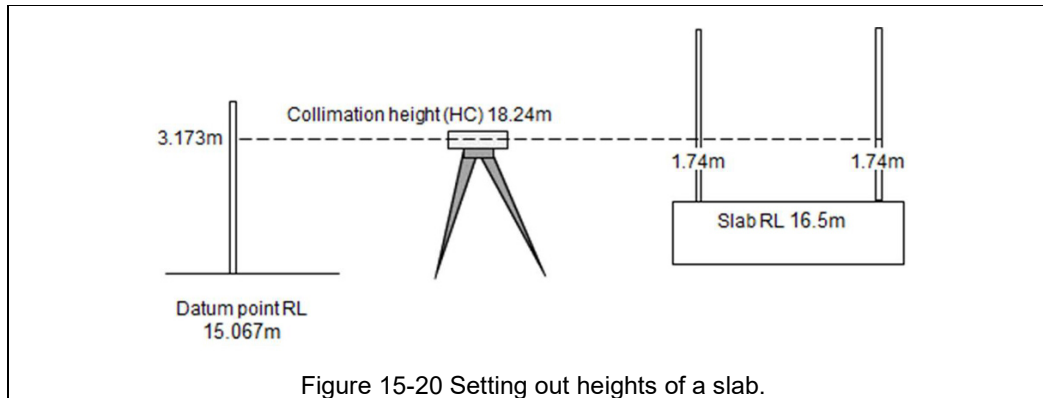


Figure 15-20 Setting out heights of a slab.

For vertical control, it is important to:

- ✓ Ensure that there are an adequate number of points available on site.
- ✓ Ensure benchmarks are solid and well protected.

The accuracies associated with levelling on site can generally be given as:

- For soft surfaces ± 0.010 m.
- For hard surfaces ± 0.005 m.
- For levels on pegs or structures usually given by the Project specifications.

NOTE: When providing levels on site it is always important to ensure that you use at least two BM's in your levelling traverse. This ensures that:

- ✓ BM has not been disturbed.
- ✓ RL transcription errors are found

Vertical control on a construction site can be provided by:

- 1) Spirit levelling (Chapter 1). This technique is generally used where high accuracy's over relatively small areas are required. The generally accepted accuracy for vertical control on most project sites is third order levelling - $12\sqrt{\text{km}}$ given in mm.
- 2) Trigonometric heighting (see Chapter 6 Total Station Differential Levelling for details). A theodolite can be used to measure the difference in elevation between two points (Figure 15-21). This method is called trigonometric heighting and for short distances of less than 200 to 300 m, it will give a reasonable accurate answer. It generally tends to be used in situations where:
 - ❖ High accuracy vertical control is not necessary – 4th order or less.
 - ❖ It is difficult or impossible to carry out a spirit levelling survey due to physical obstructions – large water bodies, hills valleys etc.
 - ❖ It is necessary to bring in vertical control over long distances.
 - ❖ This may very well be the case in remote areas of the state.
 - ❖ The existing vertical control in the area is by trigonometric heighting.

15.3.7.1 Corrections to trigonometric heighting

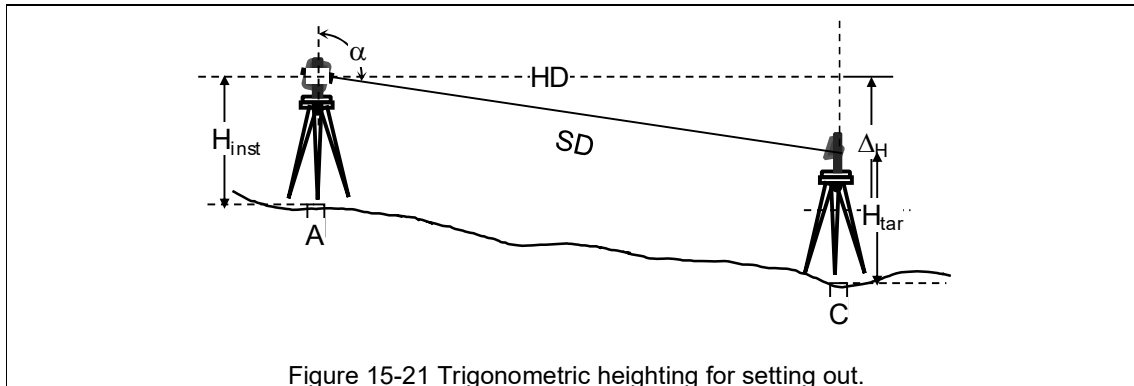


Figure 15-21 Trigonometric heighting for setting out.

There are several corrections associated with trigonometric heighting that must be applied over long distances. Major ones are:

- 1) Curvature of the Earth over long distances must be corrected: Point A and B, in Figure 15-22, are the same relative level. Radius of the earth, taken as 6375Km, compared with distance, is such that deflection, q , is a very small angle. So radians can be used instead of angle functions.

With units in Km the approximate formulae for curvature (c), within 0.01m:.

$c_{\text{metres}} = 0.0784 \text{Km}^2 \text{ to } 20\text{Km}$	Eqn 15-1
$c_{\text{metres}} = \frac{\text{Km}^2}{2R} \times 1000 \text{ to } 90\text{Km}$	Eqn 15-2

- 2) Vertical refraction (Figure 15-23): An error occurs due to the bending of a light ray as it passes through changing air densities – refraction. The angular correction is referred to as the coefficient of refraction (k) and varies significantly over a 24-hour period.

Atmospheric refraction, k , is a variable coefficient due to atmospheric variations. Correction, r : (using Eqn 15-2):

Experimentally, by Saastamoinen, J, and others, the value of between 1/7 and 1/8 ($k = 0.13$) has been determined. At dawn and dusk it may be $k = 0$. With ground level, grazing rays, k may vary between ± 3 .

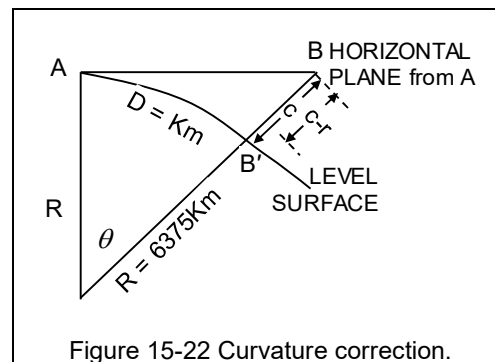


Figure 15-22 Curvature correction.

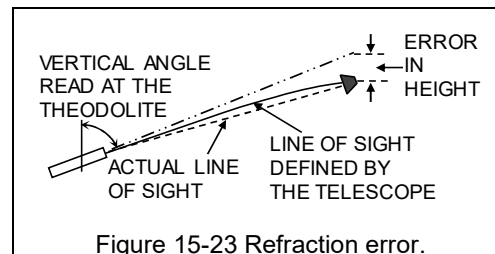


Figure 15-23 Refraction error.

$k = \frac{R_{\text{earth}}}{R_{\text{refracted line of sight}}}$	Eqn 15-3
$r_{\text{metres}} = k \left(\frac{\text{Km}^2}{2R} \right) \times 1000$	

15.3.7.4.1 Worked example 15.2

Using Figure 15-25 to illustrate the corrections:

Assume $ZA = 88^\circ$, $D = 6000\text{m}$. $k = 0.13$. $HA = 150\text{mAHD}$ and $hi = ht$

$H = 5996.345$ ($Km = 6$), $V = 209.397$ and from:

Eqn 15-1 curvature $c = 2.822\text{m}$.

Eqn 15-3, refraction $r = 0.367\text{m}$

$(c-r) = 2.457$. Compare with Eqn 15-4 where $(c-r) = 2.457$

$HB = 150 + 209.397 + 2.457 = 361.854\text{m AHD}$.

These corrections need to apply over distances greater than 250m, where $C\&R = 0.003\text{m}$, as shown in Table 15-1 (H m is horizontal distance in metres), for determining height difference where no other means of height measurement exists.

Table 15-1 C&R

H m	C&R
200	0.003
400	0.011
600	0.025
1000	0.068
1500	0.154
2000	0.273
3000	0.614
4000	1.092
5000	1.706

With the advent of GNSS, discussed in Chapter 14 Global Navigation Satellite System, the transfer of height is now based on ellipsoidal height corrected to geoidal height § 2.2. Expecting heights to $\pm 0.03\text{m}$ over Australia, the consideration of EDM should be examined for lines longer than 700m. However you must check whether some GNSS techniques are allowed on construction sites. Main Roads WA have limits on RTK in road construction.

15.3.8 Maintaining verticality in structures

Verticality is a local issue governed by the local gravity vector. The plumb bob is the true indicator of that vector through the nadir. The use of plumb lines in mining has been examined in Chapter 7 Underground Mining Survey: Transferring Traversing and Levelling Measurements, specifically § 7.5, page 141.

Verticality of structures is a different issue as the transfer of the plumb line is upwards as a building is constructed.

The main reason for vertical alignment is one of structural integrity and design conformity. However an oft overlooked issue is the one complying with a cadastral (property) boundary. A building leaning over a boundary line, even by a few centimetres, is a very costly mistake. Buy the 2cm strip of land, or move the building. It's a seller's market.

15.3.8.1 Plumb bob and spirit level

Historically the plum line has been the reference from which vertical angles have been measured. The "accuracy" of the inclined angle has depended on the quality of the manufacture of angles, the perpendicular 90° to provide an horizon (horizontal plane) and the subsequent subdivision. Modern manufacturing techniques provide better repeatability and accuracy.

The plumb bob, the mass at the end of the line, maintains the gravity vector. Typical surveying plumb bobs are carrot shaped and weigh between 200g and 500g. They are made of brass, with a replaceable steel tip. The tip is hard, and fairly sharp. Watch your toes.

The plumb bob line provides a vertical sight plane allowing alignment of tripods above or below control marks. Use a second string at 90° from the first for the second intersecting plane.

The easiest way to check the verticality of a wall is to suspend a plum line from the top of the wall at an offset distance. Then compare the offset at the base. If they are the same then that plane of the wall is vertical.

The spirit level uses fluid filled vials mounted on a frame to show the horizontal plane. The "bubble" in the vial finds its way to the top of the curved vial tube. The radius of the curve decides the sensitivity of the level to angular displacement from the horizontal. Levels in

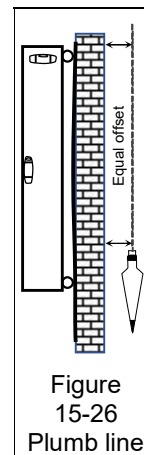


Figure 15-26
Plumb line

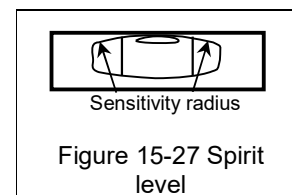


Figure 15-27 Spirit
level

survey instruments typically have a sensitivity of 30" per 2mm of displacement. (0.15mm/m, $r = 13,750\text{mm}$)

The modern spirit level was invented in 1661 by French scientist Melchisédech Thévenot. Although used in France from that time it did not come in to widespread use until the early 18th century. https://en.wikipedia.org/wiki/Melchisédech_Thévenot.

Spirit levels, or builder's levels, are up to about 2.4m in length. They incorporate 2 or 3 vial sets: one for horizontal plane, one for a vertical plane (to mimic the plumb line) and sometimes one set at 45°. The user must be aware of methods used to test the vial alignment.

There are a range of sensitivities, measured as the inclination needed to move the bubble 2mm; builders levels at about 10mm/m (10/1000 = 35'), 1mm/m (3') and 0.5mm/m (1.5'). Workshop levels have sensitivities from 1 to 0.05mm/m (10") and machinist/engineers levels are available to 0.01mm/m (0.2"). <https://www.leveldevelopments.com/sensitivity-explained/>

These figures imply that you have to know the verticality specifications for a wall before deciding on the accuracy of the level. A normal builders level used on a 5m tiltup wall could be out of vertical by 50mm.

The other consideration with levels is the "flatness" of the surface being measured. If the surface is irregular, then the measuring face of the level may rock on the undulations. It may be necessary to insert similar diameter short rods between the level and the wall. Figure 15-26.

15.3.8.2 Theodolites and total stations for vertical alignment

The total station is the common instrument used for checking verticality. The main requirement is that the trunnion axis of the telescope is truly perpendicular to the vertical axis of the instrument so that the reticule in the telescope sweeps out a plane in line with the local gravity vertical. Careful levelling of the instrument will not guarantee that the vertical axis is perpendicular to the gravity normal (horizon). The trunnion axis must be parallel to the horizon, and the vertical axis must be perpendicular to the trunnion axis. This is a problem of manufacturing tolerances, and the condition and maintenance of the instrument.

Automatic compensators (tilt sensors) in the instrument provide mathematical corrections to measured horizontal and vertical directions due to slight mislevelment of the instrument. They do not provide a physical correction to misalignment. For accurate vertical alignment the sighting should be done using both faces of the telescope.

Vertical alignment is achieved by setting the line of sight either on, to check column centerline, or offset by a measured distance from the edge of the column. Accurate marking of the column centreline to well defined contrasting targets is necessary. The column targets should be set prior to delivery to site. After one column has been set the theodolite has to be moved perpendicular to the base line to observe and set the other column vertical.

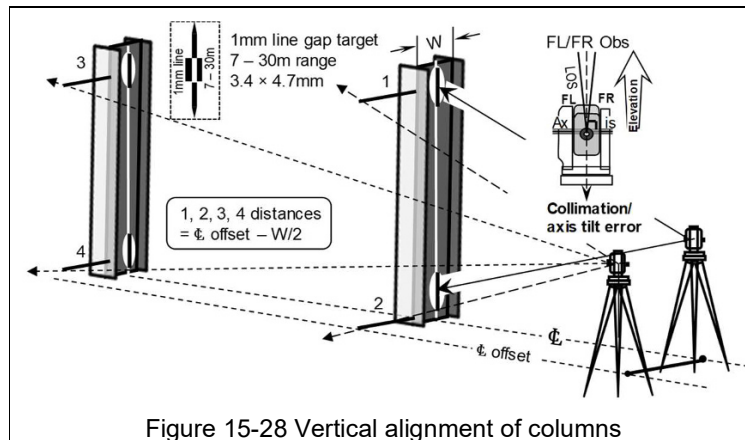


Figure 15-28 Vertical alignment of columns

The Figure 15-28 insert shows the optimum size of a target with a 1mm wide centreline mark. The target is shown in a 10mm x 20mm frame. It is only 3.5mm wide and 4.7mm high. It will work between 7m and 30m from the telescope. Closer than 7m, it's too wide a gap for the vertical wire to be aimed accurately. Over 30m, the gap gets too small. The vertical wire covers the 1mm line at about 65m.

Table 15-2 shows overall target dimensions for some line gap with minimum, optimum and maximum sighting distances. These dimensions should provide less than 0.5" sighting accuracy. The figures refer to about a 24 power telescope with a 3" cross hair and are the result of extensive observations by trained observers. (Walker 2020, appendix C.)

Line gap	W (mm)	H (mm)	Min (m)	Optimum	Max (m)
0.5	1.7	2.3	3	8	15
1	3.4	4.7	7	15	30
1.5	5.1	7.3	10	25	45
2	6.7	9.3	13	32	60
3	10.0	14.0	20	48	90

15.3.8.2.1 Errors due to mis-levelment of the trunnion axis

If the trunnion axis is not level, then the azimuth of the line of sight will suffer as telescope elevation increases. This is the reason that both faces of the telescope are used. Without a diagonal eyepiece elevation is limited to about 30°. The azimuth error, dAz , is a function of elevation and axis tilt.

Because the tilt angle, ν , is small; $\sin \nu$, $\tan \nu$ and ν radians are considered equal. If ν is in seconds then the azimuth error is in seconds.

Azimuth error	$dAz = \nu \tan h$	Eqn 15-5

Consider Figure 15-29 where the theodolite plate is mis-levelled by 1 division, $n = 30''$. A tall column, located 50m away, has $\odot$ targets at the base and an elevation of 30°. (The height to the target is 29m). The theodolite is aimed at the base target, then elevated to the upper target. A -1 division error is noted on the plate bubble. What is the tilt offset error at the upper target?

By Eqn 15-5: $dAz = -30'' \times \tan 30^\circ = -17'' = -17/206265 = -8.2E^{-5}$ radians.

$$\begin{aligned} \text{Tilt offset error} &= \theta r (dAz \times H \text{ dist}) = -8.2E^{-5} \times 50\text{m} \\ &= -0.004\text{m (4mm)}. \end{aligned}$$

The LOS will be 4mm left of the true vertical.

When the second face is observed the error will be +0.004m, thus cancelling the mis-levelment out. However you must NOT try to re-level the plate bubble between observations.

If a much shorter base line, say 15m, is used to the 29m target then a steep sight, using a diagonal eyepiece, of 62.6° is observed. In this case the dAz error is 58" ($2.8E^{-4}$ radians) and the offset error will be about the same 4mm because of the shorter base line ($2.8E^{-4} \times 15 = 0.004$).

15.3.8.3 Laser levels in vertical and horizontal alignment

Laser plummets were discussed in § 15.3.6.3.2 when used for vertical transfer of horizontal control.

Laser levels, on the other hand, act to project a plane of light using a laser beam. The configuration allows for either horizontal or vertical alignment of the beam which can be seen as a red or a green line on a surface. Green lines are more easily perceived by the eye.

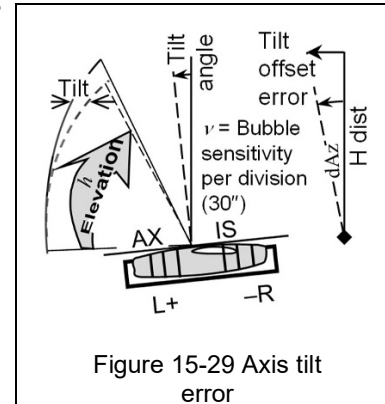


Figure 15-29 Axis tilt error

15.3.8.3.1 Line laser

Line lasers transmit a continuous line, where a laser beam is spread to a line by a cylindrical lens or a Powell lens. The spread, or divergence, of the beam is up to 120° . Through prisms both vertical and horizontal lines can be produced. The plumb line for the planes is established by a pendulum system. The units will self-level from about $\pm 3-5^\circ$ of horizontal. Many line lasers have plumbing lasers for zenith and nadir spots. They are typically used for set-out of fittings in rooms. They are limited to about 10 – 20m range. Typical level accuracy is quoted at 1mm/10m (1:10,000). Beam spot size is about 2mm/5m, divergence about $0.5 - 2\text{mRad}$ ($1.7' - 7'$). At its intended range beam divergence is not a problem.

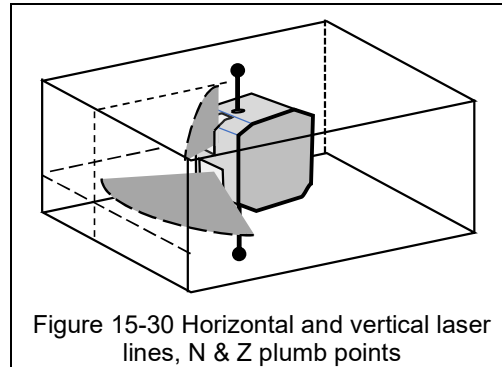


Figure 15-30 Horizontal and vertical laser lines, N & Z plumb points

15.3.8.3.2 Rotating beam laser

Rotating beam lasers, Figure 15-31, project a beam of light through 360° . Normally the horizontal plane is scanned. Using eye protective laser glasses, the spot can be seen. Due to vision persistence the spot can be seen on a surface as a discontinuous line, the length and brightness depending on the rotor speed. Self-levelling is via servo motors, again about $\pm 3-5^\circ$ range.

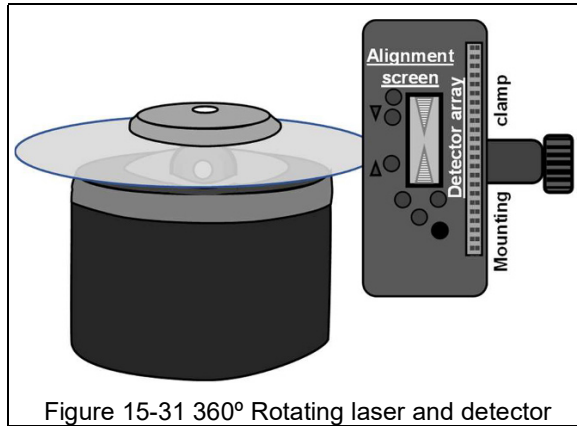


Figure 15-31 360° Rotating laser and detector

The scanning dot is captured by a photodiode detector which indicates a vertical offset from the beam plane. A liquid crystal display (LCD) provides direction to the collimation level.

Level accuracy can be up to 1:30,000, detector range to 120m ($\pm 4\text{mm}$) with a detector accuracy of 2mm (influenced by laser dot size). Some higher powered levels can be detected out to 600m.

However the attraction of long range levelling must be tempered by the intrusion of earth curvature and atmospheric refraction ($c-r$) corrections § 15.3.7.1 and Eqn 15-4. Because the beam is close to the ground temperature profiles make the refraction constant, k , unpredictable. Using $k = 0.13$; at 120m $c-r$ is 0.001m (1mm), 200m = 0.003, 300m = 0.006, 500m = 0.017, 600m = 0.025 At 600m $c-r$ can vary from -0.056m at $k = +3$; 0.000, $k = 0$; +0.0112, $k = -3$..

These values would not upset machine control systems for most earthworks applications. However it is a point that has to be made for work over large areas.

15.3.8.3.3 Grade laser

Rotating grade lasers have a mechanism that allows the X or Y axis to be tilted up to about $\pm 25\%$ grade (1:4 or 14°) This feature is useful in laser guided machine control.

Additionally, the lasers can be mounted on tilting plates mounted on a tripod to provide greater tilt ranges for slope grading. Alignment to the grade line, normal to the slope alignment, will be a big part of the set-up process. The alignment aspect of laser set-up and use is very lightly treated in advertising material.

15.3.9 A long straight line

Levelling discussed in this section is concerned with work over a restricted area where the local gravity normal provides the horizontal line of sight. But the earth curves, and we have discussed earth's curvature and refraction in § 15.3.7.1. Re-iterating Eqn 15-1, with Figure 15-22, corrections for curvature out to 20Km are;

$$c_{\text{metres}} = \frac{\text{Km}^2}{2R} = (0.0785 \text{Km}^2) \text{ to } 20\text{Km} ;$$

Examples of a long straight line are the four orthogonal vacuum measuring tubes of the two LIGO gravity wave detectors, one (LLO) at Livingston, Louisiana and the other (LHO) at Hanford, Washinton. Each vacuum beam tube is 4Km long. With mean WGS84 radius of 6369Km the difference in height due to earth is 1.256m at 4Km.

The beam tubes are 1250mm diameter fabricated in 20m sections of welded spiral wound stainless steel. Positioning was to $\pm 75\text{mm}$. <https://dcc.ligo.org/public/0073/P990007/001/P990007-00.pdf>)

Support were established about every 15m (300 per tube). After 300m the curvature was 0.007m (7mm). The simplified formula has been slightly modified to take account of actual mean earth radius.

Refraction is not an issue as the laser is fired in a vacuum of 10^{-9}Torr . <https://www.ligo.caltech.edu/page/vacuum> .

15.3.10 Grade control and set out

Vertical control is set by establishing benchmarks as in § 15.3.6. Grade control is needed to maintain design slopes for earthworks batters and drainage slopes.

The use of GNSS and laser guided machine control, especially on large earthworks sites, roads, dams, mines etc has relieved the labour intensive setout of grade control setout. However ther are still requirements for slope guidance on many jobs. The main methods of visual signalling for the machine operators or drainage layers are:

- "Boning rods" or traveller rods for trenching,
- Cut/fill indicator stakes for formation profile,
- Sight rails for batter cut/fill control,
- Optical pentaprism for slope profile,
- GNSS or laser guided grade control.

15.3.10.1 Trench and pipelaying control

The efficiency of flow through piping systems relies on accurate grade control of both the trench base material and the fall of the pipes themselves. Large diameter concrete pipes are generally less than 2.4m in length and are difficult to align.

The control of the grade line relies on the accurate placement of the design invert level RLs established on line and grade by traditional differential measurements. Definition of the "invert level" depends on the stage of construction. Initial subgrade? Final formation grade? Inside diameter of pipe?

Over relatively short distances of trench the easiest control method is to sight between horizontal rails at the top set at the start and end of the trench section. The rails are established at the same vertical offset from the grade level. A T shaped rod, a vertical rod with a horizontal cross piece, is constructed such that its length (collimation height) is the same as the sight rail

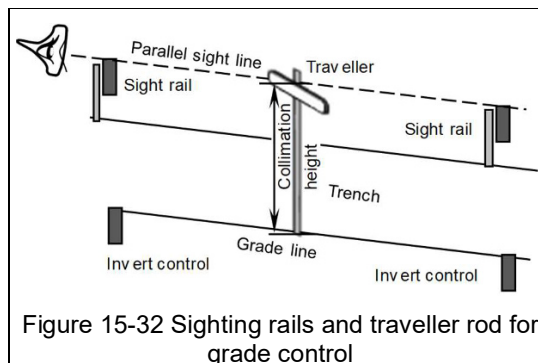


Figure 15-32 Sighting rails and traveller rod for grade control

offset. Sighting over the rails and moving the travelling T rod until it coincides with the sight line establishes any intermediate grade point.

The use of an inclined laser beam station replaces the traveller with a beam detector set to the collimation height

Pipe lasers are used inside pipes from about 150mm ID. Grades are set from the user interface, the levelling mechanism setting the input grade. A green or red target (matching the beam) is used to pack the pipe to the desired grade. Typically they work up to about 150m with an accuracy of 1:20,000 (5mm/100m).

15.3.10.1.1 Drainage grade control. Worked Example 15.1

Grade control is to be established for a drain under a road. Specifications are shown in Figure 15-33. Basically, the bedding level at the centre of the pipe is 1.3m below the centreline RL. Level stakes marked with RL, offset instructions (DOWN 0.5)

$$\text{RL}_{\text{BED}} = 14.935 - 1.3 = 13.635$$

Slope = 1:40 from R to L

$$\text{RL}_{9.5\text{R}} = 13.635 + 9.5/40 = 13.873$$

$$\text{RL}_{9.5\text{L}} = 13.635 - 9.5/40 = 13.398$$

Checks: Left is lower RL. Fall agrees with marked values.

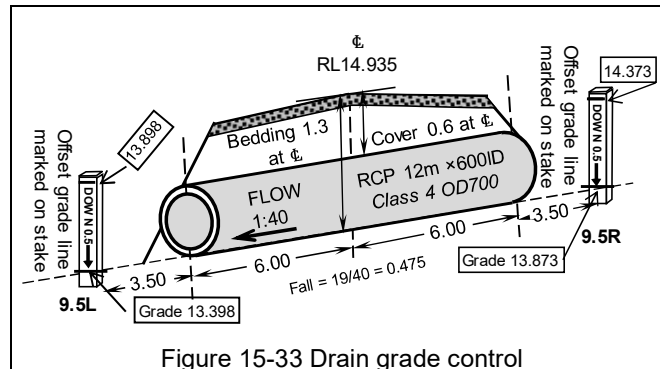


Figure 15-33 Drain grade control

15.3.10.2 Cut/fill stakes for level

As with any formation construction, control needs to be established outside the formation area. They should be at a constant offset either side of the centreline. From this control the formation design points, the centreline, the shoulders lines, toes in shallow cut and fill can be quickly re-established.

The design points are generally marked with 25mm square painted wooden stakes (a disposable item). At each station (chainage) a level is used. Establish HC reference the Stn RL, the RL of the formation and compare it with the design RL. If the formation is low then mark the design RL on the stake (use flagging tape too) and write the fill amount on the stake.

If the formation is high, determine how high. Then add an amount to it to get a “nice” cut figure for the machine operator to estimate.

In Figure 15-34 at Stn 100 the formation is 0.2m high. Add, say, 0.4 to the existing formation level. Mark the 0.6m offset on the stake (with flagging) and write “Cut 0.6m”.

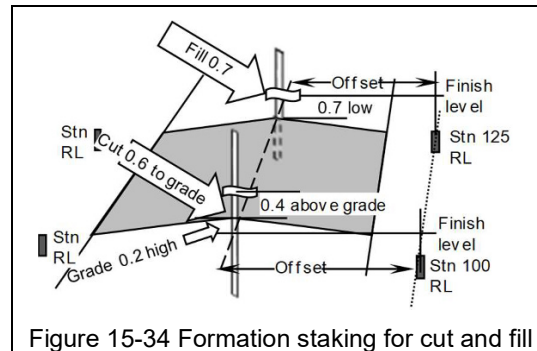


Figure 15-34 Formation staking for cut and fill

Setting the job out, and continually monitoring the progress (replacing stakes), requires a thorough understanding of the control offsets, their RLs, the design finish RLs and subgrade thicknesses. The method shown in Figure 15-20 is used. § 9.4.8 Dam Control and Survey Set-out, page 209 and Figure 9-71 also illustrate the setout method.

The equipment needed is an automatic level, staff, cloth tape, offset tape, top painted stakes (1200 x 25 x 25), sledgehammer, flagging tape and Texta pens. And a very familiar calculator. You have to be able to work closely with a survey assistant (or a junior-to-you engineer) so that you can do the thinking and they can do the work. Normally a site survey technician will do the work. But you may have to do it too.

15.3.10.3 Staking for toe/crest profile

The toe of a fill profile, and the crest of a cut profile, often indicate the boundary of the embankment cross section. The typical road cross section in Figure 15-35 shows a typical road section imposed on an existing surface. The toe of the fill is shown as IA0L (left) INTERFACE earthworks. Similarly, the crest of the cut is the IA0R (right) INTERFACE earthworks. These two points roughly correspond to the calculated W_L and W_R that were calculated in Field Practical 2 in Chapter 3 Relief and Vertical Sections, page 46, and in §3.4.4. The calculation of the drain line (IF0R) is considered in § 9.1.10, page 182.

The cross section surface profile (RLs at 10L, CL and 10R) is an approximation of the real ground section and is used for initial project planning for quantities, based on W_L and W_R . Once the profile design had been finalised the marking of the earthworks interfaces at each chainage can be undertaken. Note that the survey surface differ, although the 10L and 10R levels may be sufficiently accurate for planning.

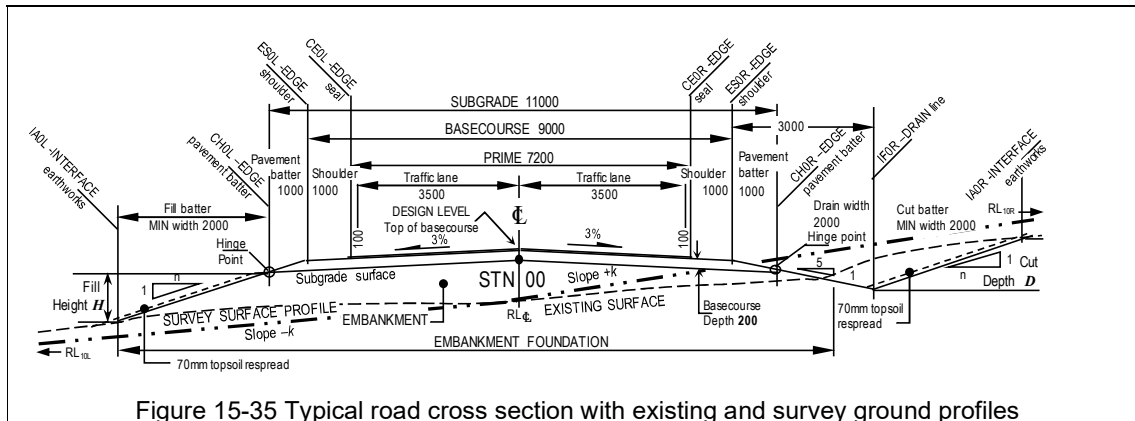


Figure 15-35 Typical road cross section with existing and survey ground profiles

It can be seen that the IA0L point is at the intersection of the ground and the defined batter slope from the CH0L hinge point, 5.5m (5500) from the centreline. Here the fill height H is the level below the subgrade hinge point. The fill batter width is thus dependent on the depth, H , and the design batter slope, $m = n$ (1:n). IA0L is thus $5.5 + mH$ from the centre line. Nice in theory, but the actual point is found iteratively by a bit of trial and error. Note that the value of n will vary according to H or D and the minimum batter width, in this case 2000mm, up to a nominated value, probably 1:5.

15.3.10.3.1 Setting a toe in fill

The survey profile resulted in an RL=10.0 at offset 10L. $W_L = 9.5$ was calculated from the centreline. Figure 15-36 shows the existing surface and the left toe area. An automatic level has been set reference a benchmark and has HC= 12.30.

The left pavement batter level, CH0L, set at 5.5m from the Φ is staked as § 15.3.10.2. Fill 0.5m required.

Batter slope, $m = 5$ from specifications.

5. Set the stake. Read the existing surface; staff = 1.8, RL = $12.3 - 1.8 = 10.5$. Design level = 11.0. Mark the stake with "Fill 0.5".

6. Read the existing surface at $W_L = 9.5$ m. Staff = 2.0. RL = $12.3 - 2.0 = 10.3$. DH from CH0L design = $11 - 10.3 = 0.7$. But $h = 0.8$ at W_L , below the existing surface. Ignore this W_L .

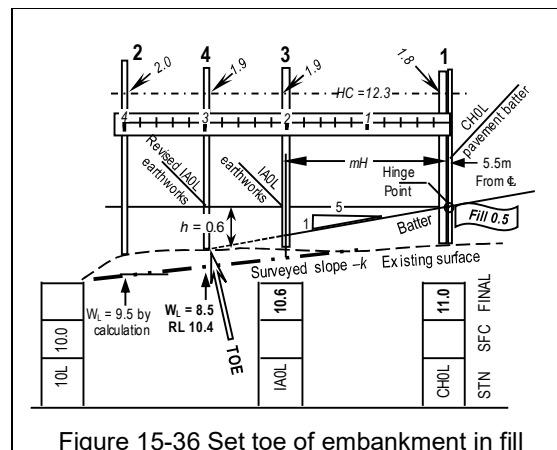


Figure 15-36 Set toe of embankment in fill

7. Move is to the IA0L distance, 2.000 from CH0L (specified) at 7.5m from $\mathcal{L}$. Staff = 1.9. RL = 10.4, too low by 0.2 from IA0L level of 10.6.
8. Move out $5 \times 0.2 = 1.0\text{m}$ (mh) and check level at 8.5 from $\mathcal{L}$. Staff = 1.9, no change. RL = 10.4, $h = 0.6$. $mh = 5 \times 0.6 = 3\text{m}$ and is the intersect between the batter slope and the existing surface. Stake this as the new toe $W_L = 8.5\text{m}$ from $\mathcal{L}$ at RL = 10.4.

15.3.10.3.2 Setting the crest in cut

The right pavement batter level, CH0R, set at 5.5m from the $\mathcal{L}$ is staked as § 15.3.10.2.

Batter slope, $m = 5$ from CH0R – IF0R (drain); $m = 3$ to cut surface, shown in Figure 15-37.

1. Set the stake. Read the existing surface; staff = 1.5, RL = 12.3 – 1.5 = 10.8. Design level = 11.0. Mark the stake with “Fill 0.2”.
2. Set stake and read surface at IF0R, drain line: Staff = 1.1, RL = 11.2, Design level = 10.6. Cut depth = 0.6. Mark the stake at 0.3 above ground with “Cut 0.9”.
3. Surface at $W_R = 12.2\text{m}$. Staff = 0.3. RL = 12.0. DH from IF0R design = 12.0 – 10.6 = 1.4. Find m from $w = W_R - \text{IF0R} = 12.2 - 7.5 = 4.7$.

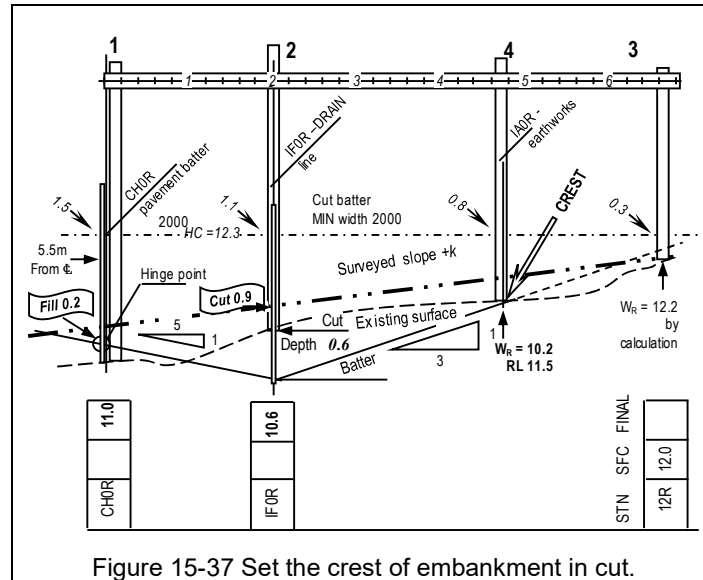


Figure 15-37 Set the crest of embankment in cut.

- Slope $m = w/h = 4.7/1.4 = 3.4$ from drain to W_R , slope is flatter than $m = 3$ cut batter.
4. Move in about 2m to $W_R = 10.2$ (2.7m from drain). Staff = 0.8, RL = 11.5. $h = 0.9$ above drain. $w = 0.9 \times 3 = 2.7$ from drain. $W_R = \text{IF0R} + w = 7.5 + 2.7 = 10.2$. On batter. This is the intersection of the cut batter from the drain to the surface. Stake this crest at $W_R = 10.2$ from $\mathcal{L}$ at RL = 11.5.

15.3.10.3.3 Comments on toe/crest pegging

The process is hard to describe. But in the field the process of iterating to the intersect point becomes one of visualising the cross section and making informed “guesses” as to the intersect. You have to be very familiar with your calculator because all the work is done “on the fly” with only the final positions booked for records. For this type of formation pegging results to 0.1m should suffice.

15.3.10.4 Sight rails for batters in cut and fill

It is essential to maintain specified slopes in cut and fill:

1. Observe environmental and property boundaries,
2. Preserve control over quantities of cut and fill volumes,
3. Meet contractual obligations.

The previous section detailed the placement of toes and crests according to plan specifications. But how is the slope of the batter maintained? Especially in fill, the batter can be controlled by pegging the crest of the next lift after compaction of a lift. Considerable survey resources are expended to ensure the machine operators are maintaining the slopes.

The use of sighting rails provides a visual indication of the slope so that the operator can trim to it. The rails can also be used in association with “boning rods” (§ 15.3.10.1) to allow supervisors to check progress without survey pegging. The sight rail also provides machine operators with correct slope indication.

Sight rails must be visible to the operators, so 100-150mm thin boards are needed, and be painted white. Set two stakes back from the toe marker and carry the toe RL on to the stakes. The stake line must be perpendicular to the centreline. (On a cross section.)

Decide on the height of the traveller, say $H_T = 1.2\text{m}$, and measure the offset to the first stake. Figure 15-38 shows a sight board set out with stakes at 0.6m and 2.4m from the toe peg. With batter slope $m = 3$, the figure shows the height offsets from the toe RL. Use round figures for offset that work with the slope. For $m = 3$ use 0.6, 1.2, 1.8 etc. For $m = 4$ use 0.8, 1.2, 1.6 etc.

Sight rails for cut are set similarly, but are often used on much steeper slopes to assist with wall drilling and excavation.

The set out is the same, but best done as the crest is pegged. The rail should be very firmly mounted and braced. Again, much of the field maths are carried out “on the fly” with a hand calculator. The results of job must be recorded.

15.3.10.5 The total station in profile and slope set out

One tends to think and set out in terms of cross section strings; each string being treated in isolation from established offsets. And we also tend to think in terms of automatic levels, staff and tapes. However, there is no need to discount the use of a total station being used in a locally oriented grid system.

Referring to Chapter 8 Strike And Dip to an Embedded Plane, page 159 et seq, a batter can be considered as a dip plane and techniques discussed can be adapted to formation setout.

The total station can be set up over a station (chainage) point of known RL. It is sighted along the chainage line (to the back sight) and then set in coordinate mode. Through a number of screens the E, N and Z(RL) values, along with the the instrument (HI) and target (HI) heights, are entered in the coordinate screen.

Curtin Sokkia total stations are set to a E, N, Z coordinate display system to conform to Australian norms. (N, E, Z is

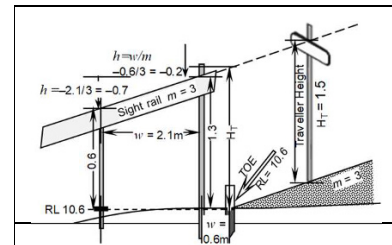


Figure 15-38 Sight rails in fill

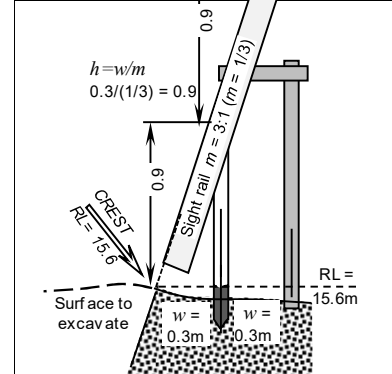


Figure 15-39 Sight rail, steep cut

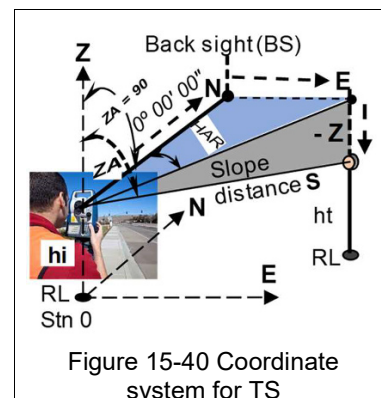


Figure 15-40 Coordinate system for TS

default). As observations are made the display will show the E, N, Z(RL) coordinates of foot of the prism. The E coordinates will be the offset from the centreline and the N coordinates will be the distance (chainage) from the TS. The Z coordinate is the RL at the foot of the staff.

Assume a TS is set up over a chainage point Stn 0 (Figure 15-40), RL = 12.05m. HT = HI. The TS back sight is the forward Stn 0+25m. Sight to that line and set the TS coordinate screen as shown in Figure 15-41. Follow from the **Meas** screen.

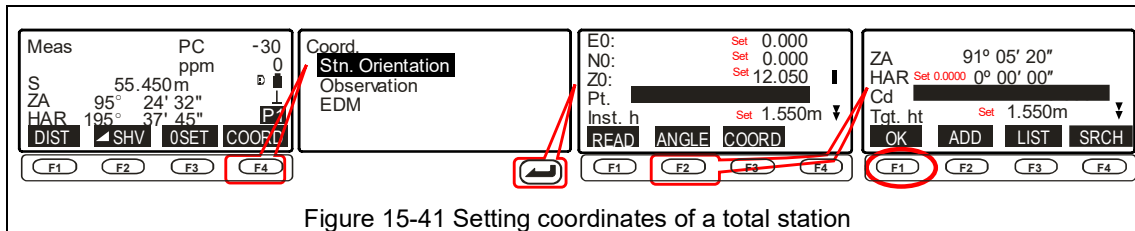


Figure 15-41 Setting coordinates of a total station

Press the **OK [F1]** button to finish station set-up. The **Coord** screen reappears. Press the down arrow and select **Observation**. The coordinates of the prism are updated as it is sighted. When the point has been noted the next point is calculated using the **OBS [F1]** key.

Send the survey assistant to measure the profile (Figure 15-37). The measurements to CF0R (pavement hinge) and IF0R (drain line) are illustrated in Figure 15-42. Because you have design data, the cut/fill values can be marked on the station stake.

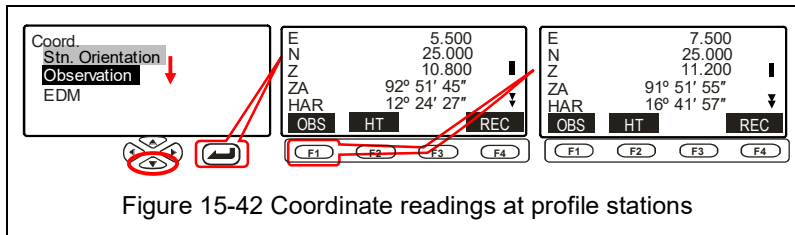


Figure 15-42 Coordinate readings at profile stations

The total station can also be used as a level by locking the vertical circle to $ZA = 90^\circ$ using the vertical circle clamp. Set alignment using the **OSET** button. The height of collimation is through the instrument. The staff provides the level for HC and provides H distance using the EDM in reflectorless mode. (Laser-safe eye protection as PPE for the survey assistant).

From HAR the distance and offset can be calculated using the **Rec()** function on a calculator.

Theodolites used this method by reading the staff with the stadia lines; the method calculated horizontal distance by tacheometry.

If a defined level is to be set out then flagging tape can be tied to the staff to show the level reference. It saves reading a staff value each time a level is set.

Modern surveying techniques use automatic tracking to follow the rover prism. The rover incorporates coordinate displays to mimic the TS display. The surveyor can measure and place points as a solo operator. The staking can then be marked with cut/fill instructions.

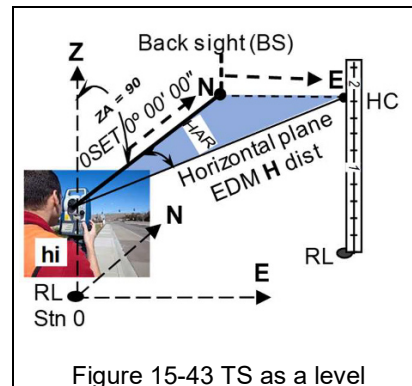


Figure 15-43 TS as a level

15.3.10.6 The pentaprism in large fill operations

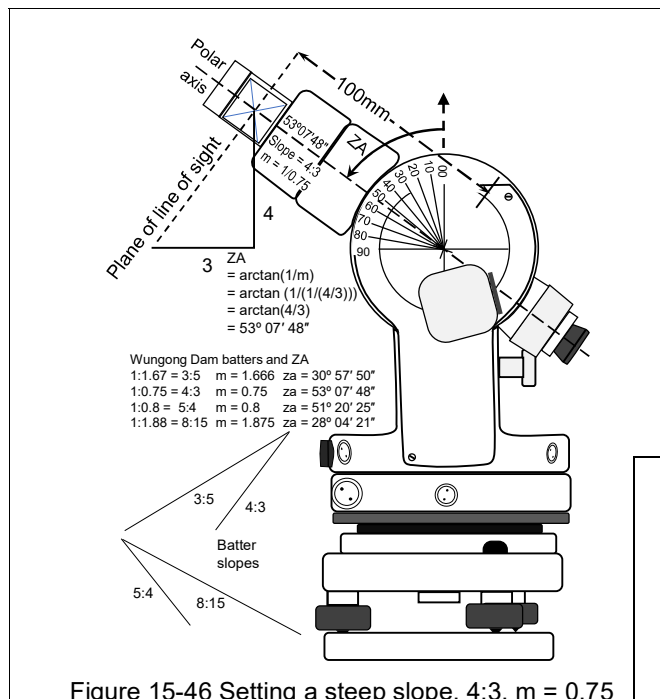
Figure 15-44 shows the Wungong Brook earth cored dam completed in 1979. The dam is located near the City of Armadale in Western Australia. The crest is some 425m long and rises about 66m above the streamline.

The zone 1 material is compacted clay, zone 2 is sand and gravel filter material and zone 3 is graded and compacted rock. Faced with rip rap.

The left abutment has a slope of about 1:3, the right abutment rises at 1:4.5. The steep slopes presented challenges in surveying batter lines on the rock abutments. The delineation of the various zone boundary lines prior to the laying of compacted material were set on swept bare rock. Four different batter slopes were involved in the design (Figure 15-47).

The boundary lines were painted on the exposed rock and were marked using a Wild GPP5 pentaprism attached to a Wild T1A theodolite. Balanced by a counter weight..

A pentaprism, Figure 15-45 and Figure 15-46, is a rotatable optical attachment fitted to the front of the telescope. The prism turns the line of sight through 90°, projecting an inclined plane perpendicular to the telescope LOS.



show about the extent of zenith angles for a straight telescope. An elevation greater than about 35° (ZA < 55°) makes the telescope too difficult to observe. Above this angle there is a need for a diagonal or a right-angle elbow attachments for the eyepiece of the telescope.

15.3.10.6.1 The pentaprism on site, marking the batter line

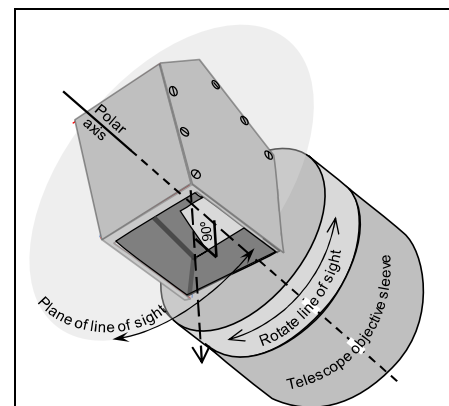


Figure 15-45 Pentaprism attachment

The clay core (zone 1) and the two filter batters (zone 2) in Figure 15-47

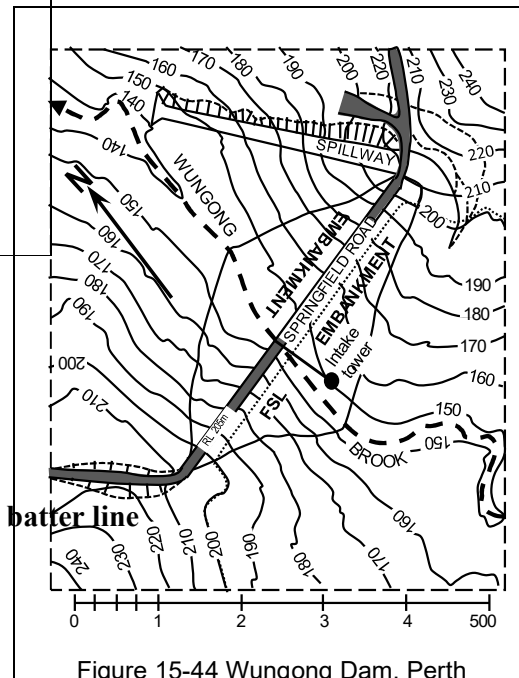


Figure 15-44 Wungong Dam, Perth

- 5) Set and clamp the zenith angle (ZA).
- 6) Have the survey assistant stand the staff at the batter toe point.
- 7) Rotate the prism upward until you see the staff in the crosshair centre. Read the staff. It is behind you.
- 8) Tie flagging around the staff at the reading. Assistant is now ready to mark the batter toe.
- 9) Not a calculator in sight.

To set a batter toe, the assistant is directed across the slope until the flagging is in the cross hairs. The foot of the staff marks the toe.

The pentaprism is a rare beast, infrequently used but very handy when employed. Some have been adapted for attachment to total stations.

15.4 Concluding Remarks

Whereas in your professional career as civil and mine engineers you might not be the ones carrying out the survey for setting out the structures, a task normally undertaken by surveyors within the company, knowledge and understanding of what they do is of great importance. This is because the overall responsibility of overseeing the construction could be squarely on you and hence the need to understand the types of controls used, data collected together with their accuracies and how they are used to realize the setting out of the structure. Be an active civil and mine engineer who can regularly visit the site for inspection and remember that a successful setting out is a function of “Good Surveying Practices”. These can be summarised as follows:

- ✓ Keep clear and accurate records (Field notes + electronic information).
- ✓ Adopt logical filing system for all information associated with the job.
- ✓ Look after all survey instruments, calibrated and serviced regularly.
- ✓ Be familiar with the construction site.
- ✓ Check all plans and drawings for dates and latest design amendments.
- ✓ Place all control points in safe locations and protect from accidental damage.
- ✓ Inspect the site regularly. Make sure pegs placed and control points have not been knocked or moved.
- ✓ Plan your setting out schedule to correspond to the construction program.
- ✓ Work to survey specifications for the project.
- ✓ Maintain the required accuracy at all times.
- ✓ Check all field work by an independent method.
- ✓ Communicate results to contractor – notes and plans.

15.5 References to Chapter 15

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16 Chapter 16 Coordinate Transformation And Least Squares Solutions

16.1 Introduction

Coordinate transformation plays a significant role in civil engineering and mining operations, for example, where coordinates obtained in the GNSS system discussed in Chapter 10 needs to be transformed into local systems, e.g., local mine grid system. This Chapter offers fundamentals of transforming the coordinates from one system to another and provides the basics of the least squares solutions, the method that is employed for such transformation.

The matter of coordinate transformation from one system to another depends on a number of factors; coordinate system and datum, orientation, area (size), purpose and accuracy requirements. The transformation parameters and methods also depend on the “accuracy” of the initial observations.

Coordinate transformations rely on having a number of points known in both coordinate systems. And they rely on the observational accuracy with which each system has been observed and calculated.

It is important to remember that, for the purposes of this discussion, the transformation is from one set of plane coordinates into another. The transformation of plane coordinates to geographic coordinates, latitude and longitude, depends on the datum, the chosen “figure of the earth” and the projection, the mathematical methods and formulae, of the transformation (see e.g., Awange and Grafarend 2005, Awange and Palacz 2016 for more details).

16.2 Definitions

Datum is used to describe, for a particular region or the whole earth, a simple set of parameters that will best locate a point on the surface of the earth. In using a datum, again, the purpose for which it will be utilised must be examined. The position’s place on the earth is also determined by temporal parameters describing the earth’s rotation in space. Location can thus be described, in order of complexity, as:

1. Plane, the “flat earth”, measured in linear units, of infinite radius,
2. Spherical, the “round earth”, of a single radius, and
3. Elliptical, the ellipsoidal or “spheroidal”, earth, of two radii; the major and the minor semi-axes.

Projection is the mathematical set of parameters that are used to basically transform between a point’s position on the earth to a plane. It is the projection that allows us to represent the earth on maps of varying scales and uses. Again, in order of complexity, we have commonly used:

1. Tangent plane, mapping from a single point outward to all other points
 - Plane, stereographic, orthographic,
2. Cylindrical, wrapping the earth in a cylinder, either around the equator or a meridian (over the poles),
 - Mercator, **Transverse Mercator** (Gauss-Kruger, UTM, MGA2020 etc)
3. Conical, placing a cone above a pole so that the cone is tangential the earth on a parallel of latitude,
 - Lamberts Conic Conformal (single parallel, or World Aeronautic Chart 1:1,000,000 with two parallels).

The **spherical** earth. Until the introduction of **geodesy**, the study of the **shape** of the earth, in the 19th century the enlightened opinion of map makers (cartographers) was that the earth was “round”. It was a ball with a spin axis through the poles, the North and South poles, and an equator perpendicular to the polar axis at the maximum radius. The tricky bit being to determine the radius. Position on earth was determined by increasingly accurate observations of the stars, planets, the Moon and the Sun. Figure 16-1 illustrate an elliptical earth. For the spherical earth

point M moves to O so that $OE = ON = OP = \text{radius}$.

The expressions used to describe a position on the earth are (see Figure 16-2):

Latitude, the angular distance, $A'MP$, from the parallel of the equator, EQ, to the position, P.

A **parallel** of latitude is the small circle perpendicular to the polar axis, NOS;

Longitude, the angular distance, AOB, between a **reference meridian**, NGBS, and the meridian through the point, NPAS (Figure 16-2).

A **meridian** is the great circle perpendicular to the equator through the poles (a great circle).

Height, the **elevation** above (or below) the radius of the earth. Since most of the earth is covered in water the natural height datum is “sea level”, the declared radius of the earth,

- **Sea level** is the long term average of observations of the ocean’s level against the land (recall Chapter 1).

Measurement of position:

Latitude, “the latitude of the place is the elevation of the pole”. The (N or S) pole has a declination of 90° from the equator. By measuring elevation to a celestial body of known declination the latitude can be calculated. A fairly simple exercise.

Longitude is the angular distance between a reference meridian and the point. An exercise in time (tricky that.). Reference meridians include Ferro (from the 2nd century AD the western edge of the Old World), Paris (20°E of Ferro, $2^\circ 20'$ $20''\text{E}$ Greenwich), Greenwich (Airey transit telescope, 1844) and now the IERS Reference Meridian (IRM) some 102m east of Greenwich.

Height. Sea level, and its corresponding equipotential (equal gravity) surface under the land, is expressed by the Australian Height Datum (AHD09), Earth Gravitational Model (EGM1996). Recall §2.2 and §2.3, page24, discussing the AHD.

Quick and dirty on the spherical earth. Many navigation problems, on land, sea and in the air, can be solved to a satisfactory level of accuracy using simple spherical trigonometry and either the mean radius of the earth at the line midpoint, or the “1 minute of arc of latitude = 1 nautical mile” definition. The nautical mile has been variously defined as: 1) 6080 feet (1853.2m), based on the Clarke (1866) spheroid and 2) 1852m (First International Extraordinary Hydrographic Conference, Monaco (1929). This implies a radius of about 3,437.7nm or 6,366,707m.

A spherical triangle is formed by the intersections of three great circles on the sphere. Solution of the spherical triangle involves the arc length of each side and the angle between them. The sum of the 3 angles is greater than 180° by the “spherical excess”, quite a small number in navigation, and ignored except to calculate area of the triangle.

The easiest treatment of spherical trigonometry is found in Mackie, 1978.

The results of calculations, compared with the “true” distance over an ellipsoidal earth, are of sufficient accuracy to allow flight and sail planning, especially considering the vagaries of wind and wave.

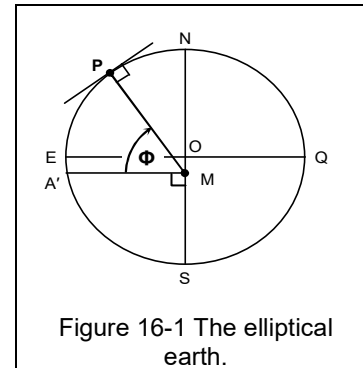


Figure 16-1 The elliptical earth.

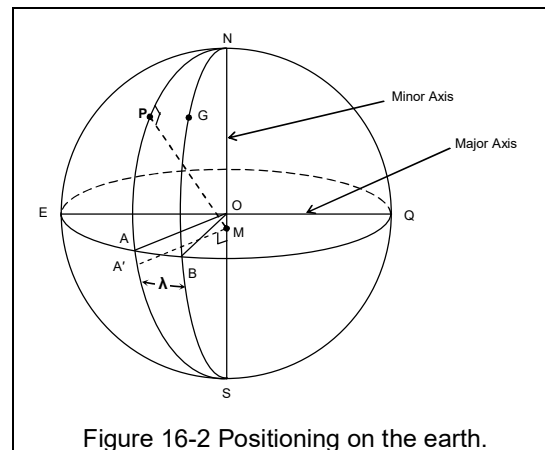


Figure 16-2 Positioning on the earth.

Mean and tricky on the ellipsoidal earth. Geodetic computations on the ellipsoid are outside the scope of this document. The conversion between positions on the ellipsoid, and the direct problem of generating a position from an azimuth and ellipsoidal distance are adequately handled by Vincenty's Inverse formula and Vincenty's Direct formula. These methods superseded the Robbin's and Robbin's Reverse methods employed previously.

The reader is referred to the Geoscience Australia website (<http://www.ga.gov.au/scientific-topics/positioning-navigation/geodesy/geodetic-techniques/calculation-methods>).

16.3 Transformation Methods

The methods explored here are of three types:

1. Block or Datum shifts; 2 parameters, x shift, y shift
2. Conformal transformations; 4 parameters, θ rotation, s scale, x shift, y shift
3. Affine transformations; 6 parameters, θ rotation, α skew, s_x scale, s_y scale, x shift, y shift

16.3.1 Block Shift

Transforming between some local coordinate system and a national system from low accuracy transformation parameters.

By this we may mean transforming hand held GPS coordinates into a local grid system to incorporate UTM gathered data in a local map. Or transforming local coordinates to UTM to allow recovery of points by GPS.

A handheld GPS can typically gather at the 3-5m accuracy level. Using its data, even on the most accurate set of coordinates, the resultant transformation cannot be expected to be any better than the basic GPS accuracy.

Datum shifting is a powerful tool in computations and, for teaching purposes, is employed on the realization of the UWA grid, which is based on the Perth Coastal Grid. The same technique is applied to Map Grid of Australia (MGA2020) coordinates for computational purposes.

Taking, for example, GNSS coordinates of points known in an example UWA grid and also in the MGA2020. It is a simple matter to find the shift in Easting and Northing between them. The UWA grid is translated from the Perth Coastal Grid (PCG2020). Figure 16-3.

	JO1	UWA grid	MGA2020	Shift
E:	1069.120		388270.020	dE = 387200.9
N:	5290.789		6461092.588	dN = 6455801.8
	JO2	UWA grid	MGA2020	Shift
E:	1188.113		388388.505	dE = 387200.4
N:	5333.960		6461137.047	dN = 6455803.1

The difference in shift values between the two coordinate differences is due, mainly, to the rotation of about 38 minutes of arc between the grid bearings.

The mean shift could be represented, at the 1m level, as 387,200E and 6,455,802N. This is well within the accuracy of the handheld GPS. At a scale of 1:1,000 these values can be plotted to the mm level.

By way of example, a GPS UTM reading of E388285, N6461225 could be transformed to UWA grid as:

$$388285 - 387200 = E1085,$$

$$6461225 - 6455802 = N5423, \text{ on the north-western edge of the oval.}$$

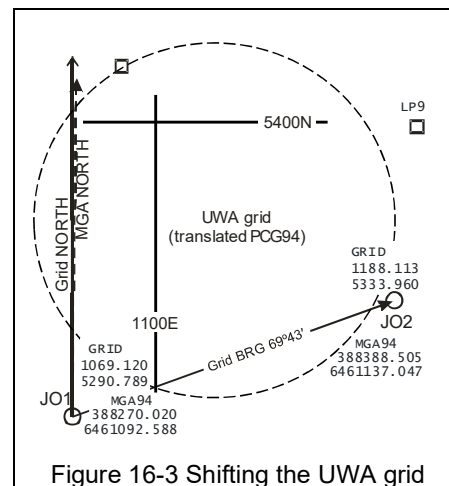


Figure 16-3 Shifting the UWA grid

Conversely, a scaled point, say LP9, at UWA E1196, N5397 becomes, in UTM coordinates:

$$1196 + 387200 = E388396$$

$$5397 + 6455802 = N6461199$$

16.3.2 Conformal Transformation

A conformal transformation between two systems maintains angles between the systems, and so the shape of a small area.

Transforming between two coordinate systems uses four (4) parameters; datum shift in E and N, scale change equal in E and N and angle rotation between the systems.

There are a number of different coordinate systems that can be used for an exploration or mining project. Generally an existing mine site will be based on a Local Mine Grid (Flat Earth system), but should be connected into the Geodetic Datum of Australia (GDA). It is quite probable that an initial exploration program will be based on MGA2020 coordinates. In both situations it then becomes necessary to be able to transform a set of coordinates from one system to another.

Generally a Local Mine Grid is based on a selected datum point located in a safe area on site and is given an arbitrary set of coordinates. In addition, the coordinate system will be orientated by selecting a second datum point in a particular direction. Common Local Mine Grid coordinate grid orientations can be based on; the direction of the ore body strike, Magnetic North, True North (Figure 16-4) or any other direction that seemed appropriate at the time.

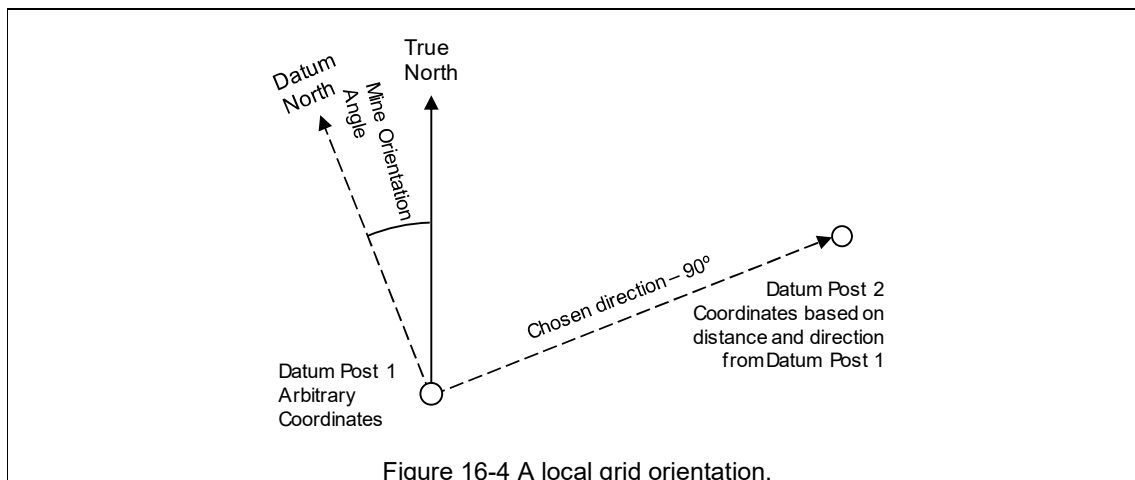


Figure 16-4 A local grid orientation.

The common Coordinate systems used today by the mining industry today tend to be either:

1. The Map Grid of Australia 2020 (MGA2020), based on the 2020 Geodetic Datum of Australia from 1 January 2020 onwards, the new standard or,
2. The Map Grid of Australia2000 (MGA200), based on the 2000 Geodetic Datum of Australia from January 2000 until January 2020, and now superseded, or,
3. Local Mine Grid, a plane grid system based on arbitrary coordinates and orientation.

However, if you are in Australia and are using data or information that precedes January 2000 then there are a number of other coordinate systems that could of been used apart from a Local Mine Grid. These systems are:

1. The Australian Map Grid 84 (AMG84) Coordinates, based on the 1984 Australian Geodetic Datum. This coordinate system was the standard System from 1984 to until 1st January 2000 in Western Australia, South Australia and Queensland (Kirby 2010). This system was replaced by the MGA2000 system after this date. Note that coordinates in this system will vary from an MGA2000 coordinate of the same point by approximately 200m in a South Westerly direction.

2. Australian Map Grid 66 (AMG66) Coordinates. This system of coordinates replaced the individual State based systems in 1966 and was used up until the introduction of the AMG84 system in 1984, except NSW, Victoria and Tasmania. These states continued to use the AGD66 system up until 1st January 2000.
3. Coordinate systems predating 1966 were generally State or regionally based systems.

Given the number of coordinate systems in existence it is often a common requirement to be able to transform a set of points coordinates from one system to another. Invariable for mining or exploration areas set up after 1st January 2000 this will involve the transformation between MGA2020 and the Local mine Grid system.

For small areas it is possible to use a 2 Dimensional Transformation based on 2 points having coordinates in either system. This allows the following parameters to be determined that can then be applied to convert coordinates from one system to the other.

- A) A rotation shift (θ) between the two systems.
- B) A datum shift, E_o , N_o , from one system to the other.
- C) A scale change (S) between the two systems.

This process gives rise to four transformation parameters (Uren et al, 2006) written as:

Conformal transformation parameters	$a = S \cos(\theta)$	Eqn 16-1
	$b = S \sin(\theta)$	
	$E_o =$ origin coordinate shift in E	
	$N_o =$ origin coordinate shift in N .	

These parameters can be determined by writing the relationship for two common coordinated points in terms of four equations:

Parameter relationships	$e_1 = E_1 a - N_1 b + E_o$	Eqn 16-2
	$n_1 = N_1 a + E_1 b + N_o$	
	$e_2 = E_2 a - N_2 b + E_o$	
	$n_2 = N_2 a + E_2 b + N_o$	

These four equations can be used to solve for the four transformation parameters **a**, **b**, **E_o** and **N_o**. Once these four parameters are known they can then be used to convert any point in the secondary coordinate system to the primary coordinate system. These parameters can also be expressed as **a**, **b**, **c**, and **d**.

Any transformation parameters calculated will always be based on data from actual survey work so that any errors within either system will manifest themselves in the calculated parameters. This means that the transformation parameters are always going to be a “best fit” only. Any small errors in the transformation parameters resulting from the two survey systems will increase as you progress out from the datum points used.

Generally, it is good practice to calculate the transformation parameters based on points outside of the subject area. Always work from without to within. A minimum of 2 points is required to calculate the transformation parameters but it is always good practice to have 1 or 2 other dual coordinated points located outside the area to check the transformation parameter accuracies.

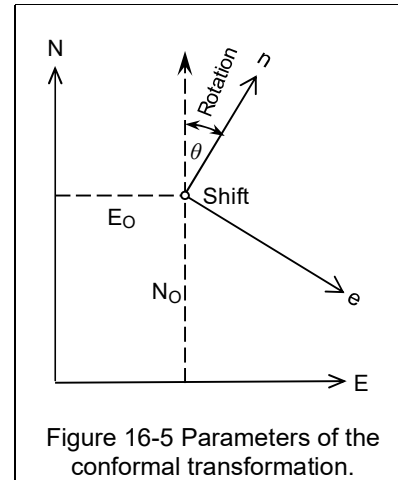
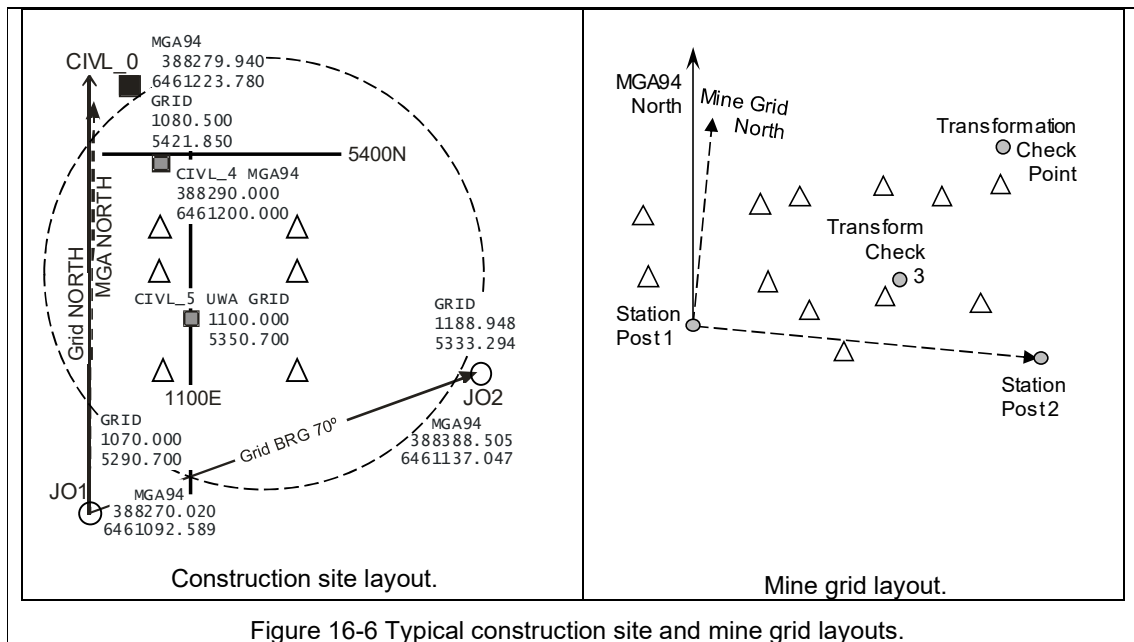


Figure 16-5 Parameters of the conformal transformation.



A two dimensional (2D) transformation described above can only be used over small areas of approximately 5 kilometres or less when converted from MGA2020 coordinates to Local Mine Grid Coordinates or vice versa. Over large areas this type of transformation will not accurately model the scale variations of the MGA projection onto a “Flat Earth” system such as a Local Mine Grid system. In addition, these errors will increase in size as you move out from the central meridian of your MGA zone towards the zone boundary. Another source of error will be the height above the ellipsoid and variation of heights across the transformation area.

In Table 16-1 a transformation area having a uniform height over the area less than 370m above the ellipsoid and within 40km of the central meridian will have an error of 0ppm due to the transformation parameters used. In the case where the area in question ranges in height between 145m and 595m the errors associated with the transformation will be as much as 25ppm at some 50Km from the central meridian.

Table 16-1 Approximate maximum relative errors in parts per million (ppm) of a 2D Transformation based on the distance from the central meridian and height (ellipsoid) range over the area.

Table Courtesy of Dr Michael Kuhn, Curtin University.

	0 ppm	5 ppm	10 ppm	15 ppm	20 ppm	25 ppm
0 km	170	125 – 215	80 – 260	35 – 305	10 – 350	55 – 395
10 km	180	135 – 225	90 – 270	45 – 315	0 – 360	45 – 405
20 km	200	155 – 245	110 – 290	65 – 335	20 – 380	25 – 425
30 km	245	200 - 290	155 – 335	110 – 380	65 - 425	20 – 470
40 km	300	255 - 345	210 - 390	165 – 435	120 - 480	75 – 525
50 km	370	325 - 415	280 - 460	235 – 505	190 - 550	145 – 595

The 2D transformation is only suitable over small areas reasonably close to the central meridian. If areas are larger or more accuracy is required then other types of coordinate transformations need to be used. The most rigorous and accurate transformation is the grid to file transformation that can deliver accuracies of between 0.02 to 0.2m. Information regarding this type of transformation, software and data grid files can be obtained from the Geoscience Australian website. (www.ga.gov.au)

- 4: Subtract (16.7) from (16.5) and solve for **a**.

$$(16.5) \quad -118.948 = -118.485a + 44.459b$$

$$(16.7) \quad \text{Subtract } 16.224796 = 16.681553a + 44.458b \\ -135.193 = -135.166553a$$

$$a = \frac{-135.193}{-135.166553} = 1.00019416$$

- 5: Solve for **b** by substituting the value of **a** (1.00019416) in (16.5), (16.6) or (16.7).

$$\text{Now, (16.6) is } -43.294 = -44.458a + (-118.485)b$$

$$\text{Substitute } a \text{ value: } -43.294 = -44.459 \times 1.00019416 + (-118.485)b$$

$$-43.294 = -44.4666318 + (-118.485)b$$

$$1.17263 = -118.485b$$

$$b = \frac{1.17263}{-118.485} = -0.0098969$$

- 6: Solve for the shifts **E_o**, (c), and **N_o**, (d), using the calculated values of **a** and **b**. You can use either equations (1) and (2) for point one or equations (3) and (4) for point 2. A good check on the problem is to solve for both sets of equations and **check** to see that the **E_o** and **N_o** are the same for both points.

$$e_1 = E_1a - N_1b + E_o, \text{ re-arrange formula}$$

$$E_o = e_1 - E_1a + N_1b$$

$$E_o = 1070.000 - 388270.020 \times 1.00019416 + 6461092.589 \times -0.0098969$$

$$= 1070.000 - 388345.405 + (-63944.655)$$

$$E_o = -451,220.060 \text{ (note this is a minus shift)}$$

$$n_1 = N_1a + E_1b + N_o, \text{ re-arrange formula}$$

$$N_o = n_1 - N_1a - E_1b$$

$$N_o = 5290.000 - 6461092.589 \times 1.00019416 + 388270.020 \times -0.0098969$$

$$= 5290.000 - 6462347.046 - (-3842.662)$$

$$N_o = -6,453,214.385$$

- 7: **Check.** The final step of the process should be a check of your transformation parameters using a third common point (CIVL_0).

$$a = 1.00019416 \quad b = -0.0098969 \quad E_o = -451,220.060 \quad N_o = -6,453,214.385$$

$$e_3 = E_3a - N_3b + E_o$$

$$= 388279.940 \times 1.00019416 - 6461223.780 \times -0.0098969 + -451220.060$$

$$= 388355.327 - (-63945.953) + -451220.060$$

$$e_3 = 1081.220 \quad \text{error } -0.020$$

$$n_3 = N_3a + E_3b + N_o$$

$$= 6461223.78 \times 1.00019416 + 388279.94 \times -0.0098969 + (-6453214.385)$$

$$= 6462478.263 + (-3842.760) + (-6453214.385)$$

$$n_3 = 5421.118 \quad \text{error } -0.018$$

Note: The check grid coordinates will always be a little different from the given surveyed coordinates. This is because of the small survey errors that exist in both coordinate networks.

The **rotation** of the construction grid from MGA2020 can be determined from:

Rotation angle $\theta = \arctan\left(\frac{a}{b}\right).$	Eqn 16-3

$$\theta = \arctan\left(\frac{b}{a}\right) = \arctan\left(\frac{-0.0098969}{1.00019416}\right) = \arctan(-0.009894958) = -0.566921^\circ$$

$$\theta = -0^\circ 34' 01''$$

The **scale factor** between MGA2020 and the construction grid can be determined from:

Scale factor	$S = \left(\frac{a}{\cos b} \right)$	Eqn 16-4

$$S = \left(\frac{a}{\cos \theta} \right) = \left(\frac{1.00019416}{\cos(-0^\circ 34' 01'') } \right) = \frac{1.00019416}{0.999951}, \text{ or}$$

$$S = \sqrt{a^2 + b^2} = \sqrt{1.00019416^2 + (-0.0098969)^2}$$

$$\mathbf{S = 1.0002431}$$

Thus, if any point is derived in MGA 94 it can be transformed into local coordinates.

Point CIVL_4 has been assigned MGA2020 coordinates $E_4 = 388,290.000$, $N_4 = 6,461,200.000$.

What are its local coordinates (e_4 , n_4)?

$$\begin{aligned} e_4 &= E_4a - N_4b + E_0 \\ &= 388290.000 \times 1.00019416 - 6461200.000 \times -0.0098969 + -451220.060 \\ &= 388365.389 - (-63945.718) + -451220.060 \end{aligned}$$

$$\mathbf{e_4 = 1091.047}$$

$$\begin{aligned} n_4 &= N_4a + E_4b + N_0 \\ &= 6461200.000 \times 1.00019416 + 388290.000 \times -0.0098969 + (-6453214.385) \\ &= 6462454.478 + (-3842.859) + (-6453214.385) \end{aligned}$$

$$\mathbf{n_4 = 5397.234}$$

16.3.2.1.2 Transform Construction grid coordinates to MGA2020

Calculate the transformation parameters needed to transform other construction grid coordinates to MGA2020.

1. To determine the transformation parameters for the opposite direction, construction grid to MGA2020, the transformation equations (Eqn 16-2) need to be rewritten and solved as in the above example.

$$E_1 = e_1a - n_1b + e_0$$

$$N_1 = n_1a + n_1b + n_0$$

$$E_2 = e_2a - e_2b + e_0$$

$$N_2 = n_2a + n_2b + n_0$$

2. The parameters of the transformation, calculated as previously demonstrated, are:

$$\mathbf{a = 0.9997080.994363} \quad \mathbf{b = 0.0098921} \quad \mathbf{e_0 = 387,252.662} \quad \mathbf{n_0 = 6,455,793.549}$$

$$\mathbf{\theta = + 0^\circ 34' 01''} \quad (\text{same rotation as previous, opposite sign})$$

$$\mathbf{S = 0.9997569} \quad (\text{reciprocal of previous scale})$$

3. **Check.** Compare construction grid point 3 (CIVL_0, $e_3 = 1081.2$, $n_3 = 5421.1$) with corresponding MGA2020 coordinates:

$$\begin{aligned} E_3 &= e_3a - n_3b + e_0 \\ &= 1081.5 \times 0.999708 - 5421.1 \times 0.00989207 + 387252.662 \\ &= 1080.884 - 53.626 + 387252.662 \end{aligned}$$

$$\mathbf{E_3 = 388,279.920} \quad \mathbf{error 0.020}$$

$$\begin{aligned} N_3 &= n_3a + e_3b + n_0 \\ &= 5421.1 \times 0.999708 + 1081.5 \times 0.00989207 + 6455793.549 \\ &= 5419.517 + 10.695 + 6455793.549 \end{aligned}$$

$$\mathbf{N_3 = 6,461,223.762} \quad \mathbf{error 0.018}$$

Note the small survey errors that exist in both coordinate networks.

Using the derived parameters, any local point can be transformed to MGA2020 coordinates.

16.3.2.1.3 Transform another point

Point CIVL_5 has been assigned local coordinates $e_5 = 1100.000$, $n_5 = 5350.000$.

What are its MGA2020 coordinates?

$$E_5 = e_{5a} - n_5b + e_0$$

$$= 1100.0 \times 0.999708 - 5350.0 \times 0.00989207 + 387252.662$$

$$= 1099.679 - 52.923 + 387252.662$$

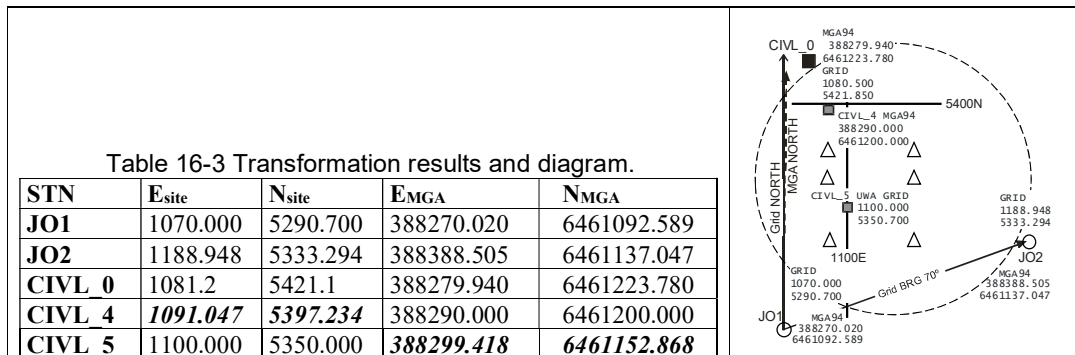
$$E_5 = \mathbf{388,299.418}$$

$$N_5 = n_{5a} + e_5b + n_0$$

$$= 5350.0 \times 0.999708 + 1100.0 \times 0.00989207 + 6455793.549$$

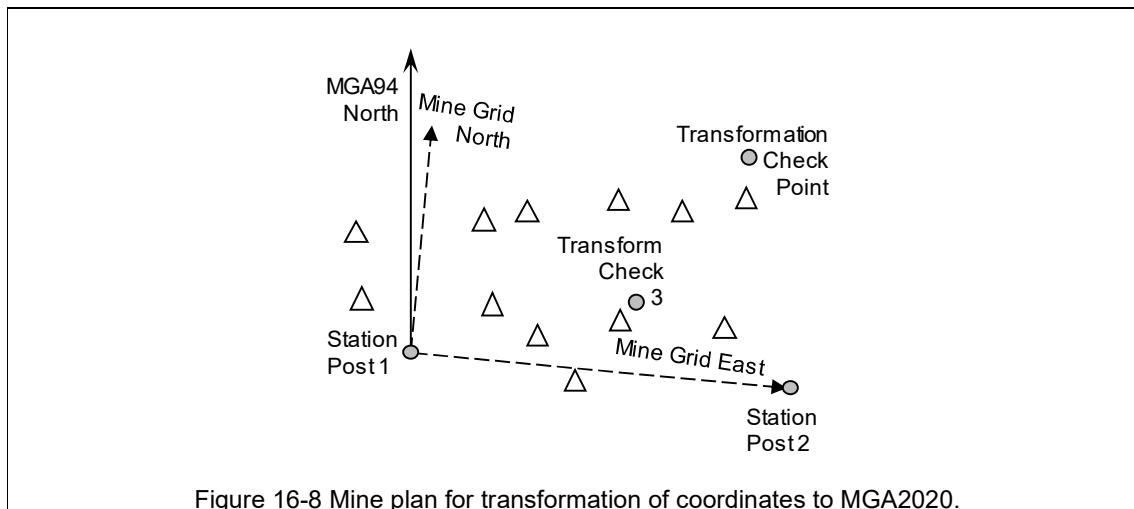
$$= 5348.437 + 10.881 + 6455793.549$$

$$N_5 = \mathbf{6,461,152.868}$$



16.3.3 Matrix Method of Coordinate Transformations - the Unique Solution

Transform mine grid coordinates to MGA2020. The following worked example is the determination of the transformation parameters required to change GNSS MGA2020 coordinates into a local mine grid coordinate system. As shown in Figure 16-8 of the mine plan, station 1 and station 2 are the original mine datum stations. The surveyor of the day set out his baseline approximately east and defined the bearing of the line as 90° . Station 3 is another point on site having both MGA2020 coordinates and Mine Grid coordinates. An extra transformation check point has been provided.



Whilst the preceding Worked Example 16.1 (§16.3.2.1) was a direct solution of a unique conformal transformation, a matrix method is introduced to solve the unique solution. It will form the basis of a Least Squares solution of an over-determined transformation, from the simple conformal to the Affine and beyond.

General observations in surveying can be reduced to the matrix form:

$$\mathbf{A} \cdot \mathbf{x} = \mathbf{b} + \mathbf{v}$$

$\mathbf{x}$ a vector of terms to be computed, including coordinates

$\mathbf{b}$ a vector relating to the observations

$\mathbf{A}$ a **matrix** of coefficients

$\mathbf{v}$ a vector of residuals, to be a minimum.

Taking the original conformal transformation, and recognizing that there ARE the minimum four (4) transformation parameters,

Eqn 16-2 (from §16.3.2.1 modified for W.E. 16.1) and the $\mathbf{A}$ matrix of coefficients is:

(point) a b E_o N_o

$\begin{aligned} e_1 &= E_1 a - N_1 b + E_o + 0N_o \\ n_1 &= N_1 a + E_1 b + 0E_o + N_o \\ e_2 &= E_2 a - N_2 b + E_o + 0N_o \\ n_2 &= N_2 a + E_2 b + 0E_o + N_o \end{aligned}$	$Matrix \mathbf{A} = \begin{matrix} 1 & \begin{bmatrix} E_1 & -N_1 & 1 & 0 \\ N_1 & E_1 & 0 & 1 \end{bmatrix} \\ 2 & \begin{bmatrix} E_2 & -N_2 & 1 & 0 \\ N_2 & E_2 & 0 & 1 \end{bmatrix} \end{matrix}$	Eqn 16-5

The solution is unique, using two points, so there will be no residuals.

And thus no $\mathbf{v}$ matrix of residuals.

16.3.3.1 Worked example 16.2. Matrix based conformal transformation

Mine plan data relating to Figure 16-8 is shown in Table 16-4, Calculate the transformation parameters needed to transform other mine grid coordinates to MGA2020.

Table 16-4 Mine grid data for transformation.				
Station	E_{mine}	N_{mine}	E_{MGA}	N_{MGA}
Post 1	50000.000	125000.000	377990.614	6458835.443
Post 2	51235.300	125000.000	379218.354	6458701.729
3	50684.385	125224.717	378695.153	6458984.707
Check point			379042.990	6459442.390

1. Identify the $\mathbf{x}$ matrix (Eqn 16-1) of the parameters of the transformed points X:

$4 \times 1 \text{ matrix}$	$\mathbf{x} = \begin{bmatrix} a \\ b \\ E_o \\ N_o \end{bmatrix}$	Eqn 16-6

2. Form the $\mathbf{A}$ (4×4) matrix (Eqn 16-5) of the observation coefficients for E and N.

The solution comprises the control points 1 and 2, considered **fixed**.

a b E_o N_o

$$\mathbf{A} = \begin{matrix} E_1 \\ N_1 \\ E_2 \\ N_2 \end{matrix} \begin{bmatrix} 377990.614 & -6458835.443 & 1 & 0 \\ 6458835.443 & 377990.614 & 0 & 1 \\ 379218.354 & -6458701.729 & 1 & 0 \\ 6458701.729 & 379218.354 & 0 & 1 \end{bmatrix}$$

3. Form the $\mathbf{b}$ matrix (4×1); the matrix of the observed coordinates in the **Mine** system.

$$\mathbf{b} = \begin{matrix} e_1 \\ n_1 \\ e_2 \\ n_2 \end{matrix} \begin{bmatrix} 50000.000 \\ 125000.000 \\ 51235.300 \\ 125000.000 \end{bmatrix} \quad n_1 \text{ and } n_2 \text{ are on an East/West alignment}$$

4. Because there are no residuals (unique solution) we have the matrix:

$$\mathbf{A} \cdot \mathbf{x} = \mathbf{b} \quad \text{then:}$$

$$\mathbf{x} = \mathbf{A}^{-1} \cdot \mathbf{b} \quad (\mathbf{A} \text{ inverse} \times \mathbf{b})$$

The $\mathbf{A}$ matrix is a 4×4 and is a major exercise to invert by hand. Using the **Matlab** = **inv**($\mathbf{A}$), or Microsoft **Excel** the inversion routine {=MINVERSE(A11:A44)} produces $\mathbf{A}$ inverse (displayed to 6DP).

$$\mathbf{A}^{-1} = \begin{bmatrix} -0.000805 & 8.766838^{-5} & 0.000805 & -8.766838^{-5} \\ -8.766838^{-5} & -0.000805 & 8.766838^{-5} & 0.000805 \\ 260.969573 & -5232.220538 & 261.969573 & 5232.220538 \\ 5232.220538 & -260.969573 & -5232.220538 & 261.969573 \end{bmatrix}$$

5. Now find the $\times (4 \times 1)$ matrix: $\mathbf{x} = \mathbf{A}^{-1} \cdot \mathbf{b} \quad (4 \times 4) \times (4 \times 1) = (4 \times 1)$

$$\{\mathbf{x}_{11}:\mathbf{x}_{41}\} = \{\text{MMULT}(\mathbf{A}^{-1}_{11}:\mathbf{A}^{-1}_{44}, \mathbf{b}_{11}:\mathbf{b}_{41})\}$$

$$\mathbf{x} = \begin{bmatrix} a \\ b \\ E_o \\ N_o \end{bmatrix} = \begin{bmatrix} 0.994363 \\ 0.108297 \\ 373611.013 \\ 6338362.031 \end{bmatrix}$$

6. And the rotation (Eqn 16-3):

$$\theta = \arctan\left(\frac{b}{a}\right) = \arctan\left(\frac{0.108297}{0.994363}\right) = \arctan(0.1089109) = 6.215638^\circ$$

$$\theta = 6^\circ 12' 56''$$

7. Find the scale factor from Eqn 16-4:

$$S = \sqrt{a^2 + b^2} = \sqrt{0.994363^2 + 0.108297^2}$$

$$S = 1.000243$$

8. Calculating the coordinates of point 3, using coefficients of observations (MGA2020 coordinates of Station 3):

$$\begin{aligned} e_3 &= E_3 a - N_3 b + E_o + 0N_o \\ n_3 &= N_3 a + E_3 b + 0E_o + N_o \end{aligned} \quad \mathbf{A}_3 = \begin{bmatrix} 378695.153 & -6458984.707 & 1 & 0 \\ 6458984.707 & 378695.153 & 0 & 1 \end{bmatrix}$$

$$\text{Result Matrix} = \mathbf{A}_3 \cdot \mathbf{x} \quad \{\mathbf{a}_{11}:\mathbf{a}_{21}\} = \{\text{MMULT}(\mathbf{A}_{311}:\mathbf{A}_{324}, \mathbf{x}_{11}:\mathbf{x}_{41})\}$$

$$\mathbf{e}_3 = 50,684.403 \quad (\text{error } 0.018)$$

$$\mathbf{n}_3 = 125,224.722 \quad (\text{error } 0.005)$$

Reversing the transformation. Mine grid to MGA2020

To determine the transformation parameters for the opposite direction, Local Mine Grid to MGA2020, the transformation equation need to be rewritten and solved as in the preceding example.

Eqn 16-5 in the mine system (§ 16.3.2.1, W.E. 16.1) and the $\mathbf{A}$ matrix of coefficients is:

$$\begin{aligned} E_1 &= e_1 a - n_1 b + e_o + 0n_o \\ N_1 &= n_1 a + e_1 b + 0e_o + n_o \\ E_2 &= e_2 a - n_2 b + e_o + 0n_o \\ N_2 &= n_2 a + e_2 b + 0e_o + n_o \end{aligned} \quad \text{(point)} \quad \begin{matrix} a & b & e_o & n_o \end{matrix} \quad \text{Matrix } \mathbf{A} = \begin{bmatrix} e_1 & -n_1 & 1 & 0 \\ n_1 & e_1 & 0 & 1 \\ e_2 & -n_2 & 1 & 0 \\ n_2 & e_2 & 0 & 1 \end{bmatrix}$$

1. Form the $\mathbf{A}$ (4×4) matrix of the observation coefficients for e and n in the mine system.

The solution comprises the control points 1 and 2, considered **fixed**.

$$\mathbf{A} = \begin{matrix} & a & b & e_o & n_o \\ \begin{matrix} e_1 \\ n_1 \\ e_2 \\ n_2 \end{matrix} & \begin{bmatrix} 50000.000 & -125000.000 & 1 & 0 \\ 125000.000 & 50000.000 & 0 & 1 \\ 51235.300 & -125000.000 & 1 & 0 \\ 125000.000 & 51235.300 & 0 & 1 \end{bmatrix} \end{matrix}$$

2. Form the **b** matrix (4×1); the matrix of the observed coordinates in the **MGA** system.

$$\mathbf{b} = \begin{bmatrix} E_1 \\ N_1 \\ E_2 \\ N_2 \end{bmatrix} = \begin{bmatrix} 377990.614 \\ 6458835.443 \\ 379218.354 \\ 6458701.729 \end{bmatrix}$$

3. Because there are no residuals (unique solution) we have the matrix:

$$\mathbf{A} \cdot \mathbf{x} = \mathbf{b}, \text{ then:}$$

$$\mathbf{x} = \mathbf{A}^{-1} \cdot \mathbf{b}$$

Find the $\mathbf{A}^{-1}$ matrix (shown to 5dp):

$$\mathbf{A}^{-1} = \begin{bmatrix} -0.00081 & 0 & 0.00081 & 0 \\ 0 & -0.00081 & 0 & 0.00081 \\ 41.47600 & -101.19000 & -40.47600 & 101.19000 \\ 101.19000 & 41.47600 & -101.19000 & -40.47600 \end{bmatrix}$$

4. Now find the $\times (4 \times 1)$ matrix: $\mathbf{x} = \mathbf{A}^{-1} \cdot \mathbf{b} \quad (4 \times 4) \times (4 \times 1) = (4 \times 1)$

$$\{x_{11}:x_{41}\} = \{\text{MMULT}(\mathbf{A}^{-1}_{11}:\mathbf{A}^{-1}_{44}, \mathbf{b}_{11}:\mathbf{b}_{41})\}$$

$$\mathbf{x} = \begin{bmatrix} a \\ b \\ e_o \\ n_o \end{bmatrix} = \begin{bmatrix} 0.993880 \\ -0.108244 \\ 314766.042 \\ -6340012.647 \end{bmatrix}$$

The parameters of the transformation are:

$$\begin{aligned} \mathbf{a} &= 0.993880 \quad \mathbf{b} = -0.108244 & \mathbf{e_o} &= 314,766.094 & \mathbf{n_o} &= -6,340,012.647 \\ \theta &= -6^\circ 12' 56'' & & & & \text{(same rotation as previous, opposite sign)} \\ \mathbf{S} &= 0.999757 & & & & \text{(reciprocal of previous scale)} \end{aligned}$$

5. Calculating the coordinates of point **3**, using coefficients of observations (MINE coordinates of Station 3):

$$\begin{aligned} E_3 &= e_3 a - n_3 b + e_o + 0 n_o \\ N_3 &= n_3 a + e_3 b + 0 e_o + n_o \end{aligned} \quad \mathbf{A}_3 = \begin{bmatrix} 50684.385 & -125224.717 & 1 & 0 \\ 125224.717 & 50684.385 & 0 & 1 \end{bmatrix}$$

$$\mathbf{Result Matrix} = \mathbf{A}_3 \cdot \mathbf{x} \quad \{a_{11}:a_{21}\} = \{\text{MMULT}(\mathbf{A}_3_{11}:\mathbf{A}_3_{24}, \mathbf{x}_{11}:\mathbf{x}_{41})\}$$

$$\mathbf{E}_3 = 378,695.135 \quad (\text{error } 0.018)$$

$$\mathbf{N}_3 = 6,458,984.704 \quad (\text{error } 0.003)$$

Note the small survey errors that exist in both coordinate networks.

16.3.4 Least Squares Solution to Coordinate Transformation

Taking the original conformal transformation, and introducing the third point, Station 3, the transformation becomes over-determined because there are more coefficients, six (6), than the minimum four (4) transformation parameters required. We saw a small transformation error between our forward and backward solutions in Worked Example 12.2. Now we will distribute that error to provide a better estimate of the transformation parameters (a , b , E_o , N_o) to allow a “better” solution of unknown transformation points.

Eqn 16-5 (modified from W.E. 16.1) and the overdetermined **A** matrix of coefficients is:

	a	b	E_o	N_o	
$ \begin{aligned} e_1 &= E_1 a - N_1 b + E_o + 0 N_o \\ n_1 &= N_1 a + E_1 b + 0 E_o + N_o \\ e_2 &= E_2 a - N_2 b + E_o + 0 N_o \\ n_2 &= N_2 a + E_2 b + 0 E_o + N_o \\ e_3 &= E_3 a - N_3 b + E_o + 0 N_o \\ n_3 &= N_3 a + E_3 b + 0 E_o + N_o \end{aligned} $	$ \text{Matrix } \mathbf{A} = \begin{bmatrix} E_1 & -N_1 & 1 & 0 \\ N_1 & E_1 & 0 & 1 \\ E_2 & -N_2 & 1 & 0 \\ N_2 & E_2 & 0 & 1 \\ E_3 & -N_3 & 1 & 0 \\ N_3 & E_3 & 0 & 1 \end{bmatrix} $	Eqn 16-7			

This solution is non-unique, using three points, so there will be residual errors. There will be a **v** matrix of residuals.

16.3.5 Worked Example 16.3. Over-determined Least Squares Conformal Transformation

The use of Microsoft Excel™ to solve these problems

- Form the **x** matrix (Eqn 16-6) of the parameters of the transformed points X:

$$\mathbf{x} = \begin{bmatrix} a \\ b \\ E_o \\ N_o \end{bmatrix} \quad \text{is a } 4 \times 1 \text{ matrix}$$

- Form the **A** (6×4) matrix (Eqn 16-7) of the observation coefficients for E and N. The solution comprises the control points 1, 2 and 3, considered **fixed**.

	a	b	E_o	N_o
E_1	377990.614	-6458835.443	1	0
N_1	6458835.443	377990.614	0	1
E_2	379218.354	-6458701.729	1	0
N_2	6458701.729	379218.354	0	1
E_3	378695.153	-6458984.707	1	0
N_3	6458984.707	378695.153	0	1

- Form the **b** matrix (6×1); the matrix of the observed coordinates in the **Mine** system.

$$\mathbf{b} = \begin{bmatrix} e_1 \\ n_1 \\ e_2 \\ n_2 \\ e_3 \\ n_3 \end{bmatrix} = \begin{bmatrix} 50000.000 \\ 125000.000 \\ 51235.300 \\ 125000.000 \\ 50684.385 \\ 125224.717 \end{bmatrix}$$

- Because there are now residuals (over-determined solution) we must generate the **v** matrix (6×1) of residuals, **v**, (Greek letter, nu).

Residuals matrix	$\mathbf{v} = \begin{matrix} e_1 \\ n_1 \\ e_2 \\ n_2 \\ e_3 \\ n_3 \end{matrix}$	$\begin{bmatrix} v_{e_1} \\ v_{n_1} \\ v_{e_2} \\ v_{n_2} \\ v_{e_3} \\ v_{n_3} \end{bmatrix}$	Eqn 16-8

5. Solve the matrix $\mathbf{A} \cdot \mathbf{x} = \mathbf{b} + \mathbf{v}$.

The matrix must first be **normalised** to the form by using the **transpose** of $\mathbf{A}$, $\mathbf{A}^T$:

$\mathbf{A}^T \cdot \mathbf{A} \cdot \mathbf{x} = \mathbf{A}^T \cdot \mathbf{b}$, then, minimising $\mathbf{A}^T \mathbf{A}$ by least squares, leads to:

$$\hat{\mathbf{x}} = (\mathbf{A}^T \mathbf{A})^{-1} \cdot \mathbf{A}^T \mathbf{b} \quad (\text{Invert}(\mathbf{A}^T \mathbf{A}) \text{ times } (\mathbf{A}^T \mathbf{b}))$$

The generation of the $\mathbf{A}^T \cdot \mathbf{A}$ matrix $(4 \times 6) \times (6 \times 4)$ is a 4×4 and is a major exercise to calculate, then invert, by hand. Using Microsoft **Excel**, the matrix transpose, multiplication and inversion routines produce (as in algebraic software such as MATLAB or Mathematica):

$$(\mathbf{A}^T \mathbf{A}) = \begin{bmatrix} 1.2558^{14} & 0 & 1135904.121 & 19376521.880 \\ 0 & 1.2558^{14} & 19376521.880 & 1135904.121 \\ 1135904.121 & -19376521.880 & 3 & 0 \\ 19376521.880 & 1135904.121 & 0 & 3 \end{bmatrix}$$

6. Find the $\mathbf{N}^{-1}$ (inverse) matrix: ($\mathbf{N}$ inverse being the inverse of $\mathbf{A}^T \cdot \mathbf{A}$).

$$\{\mathbf{N}^{-1}_{11} : \mathbf{N}^{-1}_{41}\} = \{\text{MINVERSE}(\text{MMULT}(\text{TRANSPOSE}(\mathbf{A}_{11} : \mathbf{A}_{64}), \mathbf{A}_{11} : \mathbf{A}_{64}))\}$$

This can generate the $\mathbf{N}^{-1}$ matrix in one operation from the original $\mathbf{A}$ matrix.

$$(\mathbf{A}^T \mathbf{A})^{-1} = \mathbf{N}^{-1} = \begin{bmatrix} 1.2512^{-6} & 1.1016^{-15} & -0.47375 & -8.08131 \\ -2.5362^{-15} & 1.2512^{-6} & 8.08131 & -0.47375 \\ -0.47375 & 8.08131 & 52375271.9 & 0.07614 \\ -8.08131 & -0.47375 & -0.07614 & 52375271.4 \end{bmatrix}$$

7. Now find the $\mathbf{A}^T \cdot \mathbf{b}$ $(4 \times 6) \times (6 \times 1) = (4 \times 1)$ matrix:

$$\{\mathbf{A}^T \mathbf{b}_{11} : \mathbf{A}^T \mathbf{b}_{41}\} = \{\text{MMULT}(\mathbf{A}^T_{11} : \mathbf{A}^T_{46}), \mathbf{b}_{11} : \mathbf{b}_{41}\}$$

$$\mathbf{A}^T \mathbf{b} = \begin{bmatrix} 2.48104^{12} \\ -8.39152^{11} \\ 151919.685 \\ 375224.717 \end{bmatrix}$$

8. Finally the $\mathbf{x}$ (4×1) matrix: $\mathbf{x} = \mathbf{N}^{-1} \cdot \mathbf{A}^T \cdot \mathbf{b}$ $(4 \times 4) \times (4 \times 1) = (4 \times 1)$

$$\{\mathbf{x}_{11} : \mathbf{x}_{41}\} = \{\text{MMULT}(\mathbf{N}^{-1}_{11} : \mathbf{N}^{-1}_{44}), \mathbf{A}^T \mathbf{b}_{11} : \mathbf{A}^T \mathbf{b}_{41}\}$$

$$\mathbf{x} = \begin{matrix} a \\ b \\ E_o \\ N_o \end{matrix} \begin{bmatrix} 0.994361 \\ 0.108300 \\ 373630.072 \\ -6338348.973 \end{bmatrix}$$

9. And the rotation (Eqn 16-3):

$$\theta = \arctan\left(\frac{b}{a}\right) = \arctan\left(\frac{0.108300}{0.994361}\right) = \arctan(0.1089142) = 6.215821^\circ$$

$$\theta = 6^\circ 12' 57''$$

10. Find the scale factor from Eqn 16-4:

$$S = \sqrt{a^2 + b^2} = \sqrt{0.994361^2 + 0.108300^2}$$

$$S = 1.000241$$

11. Calculating the adjusted coordinates of the three points, using coefficients of observations. (MGA2020 coordinates of the stations) provides the adjusted mine coordinates and their residuals:

Result Matrix: $\mathbf{b}_{\text{calc}} = \mathbf{A} \cdot \mathbf{x}$ $\mathbf{V} = \mathbf{b}_{\text{obs}} - \mathbf{b}_{\text{calc}}$ (residuals)

$$\mathbf{b}_{\text{calc}} = \begin{matrix} e_1 \\ n_1 \\ e_2 \\ n_2 \\ e_3 \\ n_3 \end{matrix} \begin{bmatrix} 49999.996 \\ 124999.997 \\ 51235.293 \\ 125000.001 \\ 50684.396 \\ 125224.721 \end{bmatrix} \quad \mathbf{V} = \mathbf{b}_{\text{obs}} - \mathbf{b}_{\text{calc}} \begin{bmatrix} 0.004 \\ 0.003 \\ 0.007 \\ -0.001 \\ -0.011 \\ -0.004 \end{bmatrix}$$

The residuals do not show any major discrepancies in the transformation.

12. Calculate the adjusted mine grid coordinates of the MGA2020 check point, which should also have previously observed mine grid coordinates:

The $\mathbf{A}_{\text{ck}}$ (2×4) matrix of the observation coefficients for E and N is:

$$\mathbf{A} = \begin{matrix} E_{ck} \\ N_{ck} \end{matrix} \begin{matrix} a & b & E_o & N_o \end{matrix} \begin{bmatrix} 377990.614 & -6458835.443 & 1 & 0 \\ 6458835.443 & 377990.614 & 0 & 1 \end{bmatrix}$$

Result matrix: $\mathbf{b}_{\text{calc}} = \mathbf{A} \cdot \mathbf{x}$

$$\mathbf{b}_{\text{calc}} = \begin{matrix} e_{ck} \\ n_{ck} \end{matrix} \begin{bmatrix} 50980.705 \\ 125717.493 \end{bmatrix}$$

16.3.5.1 The use of MATLAB to solve the problems

Your course will introduce you to the MATLAB environment as a programming and problem-solving tool. There are many excellent books on the subject; there is plenty of expertise around the Spatial Science departments.

Compare the spreadsheet solution with the MATLAB solution in parallel.

Using MATLAB as a matrix calculator (Worked Example 12.3):

We will use the comma (,) as the variable separator, although a space or a tab will do the same.

Entry of a multi-dimensional matrix requires the use of the Enter key at the end of each line; using the symbol  to denote the  key.

Refer to the matrix formation on the spreadsheet page.

1. Form the $\mathbf{A}$ (6×4) matrix of the observation coefficients for E and N.

The solution comprises the control points 1, 2 and 3, considered **fixed**.

Enter the a, b, E_o , N_o coefficients for each row:

```
EDU>> A = [ 377990.614, -6458835.443, 1, 0
6458835.443, 377990.614, 0, 1
379218.354, -6458701.729, 1, 0
6458701.729, 379218.354, 0, 1
378695.153, -6458984.707, 1, 0
6458984.707, 378695.153, 0, 1];
```

Note a) the matrix is initiated with an opening square bracket [

b) the matrix is finalized with an closing square bracket]

c) the semi-colon, ;, after the closing bracket suppresses the display of the $\mathbf{A}$ matrix.

2. Form the $\mathbf{b}$ matrix (6×1); the matrix of observed coordinates in the Mine system. ($\mathbf{b}_{\text{obs}}$)

```
EDU>> b = [ 50000
125000
51235.3
125000
50684.385
125224.717];
```

3. Solve the matrix $\mathbf{A} \cdot \mathbf{x} = \mathbf{b} + \mathbf{v}$.

The matrix must first be **normalised** to the form:

$A^T A \cdot x = A^T b + v$, then

$$x = (A^T A)^{-1} \cdot A^T b + v$$

The x and the v matrix can be handled as normal calculations.

```
EDU>>x = (inv(A'*A))*(A'*b) ←
```

Note a) the **inv(A'*A)** function finds the inverse of the bracketed expression,

b) A' (A prime) is the **transpose** of the A matrix,

c) no semi-colon, the x vector (matrix) is displayed to 4 dp with scaling,

d) extra (unnecessary) brackets have been included for clarity.

```
x =
1.0e+006 *      The answers to 4 or 3dp derived from format long
0.0000          a      0.9944
0.0000          b      0.1083
0.3736          Eo 373630.072
-6.3383          No -6338348.973
```

4. Find the rotation in radians: $\theta = \arctan\left(\frac{b}{a}\right)$

```
EDU>>Rotation = atan(x(2)/x(1)) ← define Rotation: x(2) = b, x(1) = a
Rotation =
```

0.1085 radians, to be converted to degrees.

```
EDU>>R_degrees = Rotation * 180/pi ←
R_degrees =
```

6.2158 | $\theta = 6^\circ 12' 57''$

5. And the scale factor, $SF = \sqrt{a^2 + b^2}$, define SF using the **norm(vector)** function.

```
EDU>>SF = norm(x(1:2)) ← x(1:2) represents the first two elements, a and b, of x vector.
```

1.0002 | SF = 1.000241

6. Calculate the adjusted coordinates of the three points, b_{calc}

Result Matrix: $b_{calc} = A \cdot x$

```
EDU>>bcalc = A*x ← define bcalc, adjusted values of transformed coordinates.
bcalc =
```

```
1.0e+005 *      These are the values to 3dp.
0.5000          e1 49999.996
1.2500          n1 124999.997
0.5124          e2 51235.293
1.2500          n2 125000.000
0.5068          e3 50684.396
1.2522          n3 125224.720
```

7. Calculate the residuals.

Residuals matrix: $v = b - b_{calc}$ (**residuals**)

```
EDU>>v = b-bcalc ← define v, residuals between observed and calculated coordinates.
v =
```

```
0.0045
0.0034
0.0068
-0.0003
-0.0113
-0.0031
```

8. Check that the sum of the residuals vector is a minimum.

```
EDU>>sumv = sum(v) ←
sumv =
```

0.0000 | Sum residuals.

16.4 Coordinate Translations – a VERY HELPFUL Technique

You will have noted that working in MGA coordinates, and even in the Local Mine Grid, that you have some serious numbers to manipulate. Used directly in the program, these numbers are well outside the capability of even a 12-digit calculator. In fact, Microsoft Excel introduces errors, even with its 16-digit computations.

What to do? Translate the axes of one or both coordinate systems. That's what!

Why use numbers like E378,000, N6,500,000 (MGA) when the work area is only covering an area of dE < 1,300 and dN < 300? Move (translate) the ORIGIN up close to the work site.

1. This is the original data (Table 16-5):

Table 16-5 Original mine data.				
Station	E _{mine}	N _{mine}	E _{MGA}	N _{MGA}
Post 1	50000.000	125000.000	377990.614	6458835.443
Post 2	51235.300	125000.000	379218.354	6458701.729
Station 3	50684.385	125224.717	378695.153	6458984.707

By examination we can choose some minimum numbers for both systems that will leave the system in the first quadrant:

False origin mine False origin MGA

e_{mine} 50000 E_{MGA} 377,900

n_{mine} 125000 N_{MGA} 6,458,700

2. Subtracting BOTH sets of false origins, the TRANSLATED coordinates are (Table 16-6):

Table 16-6 Translated mine data.				
Station	E _{mine}	N _{mine}	E _{MGA}	N _{MGA}
Post 1	0.000	0.000	390.614	135.443
Post 2	1235.300	0.000	1318.354	11.729
Station 3	684.385	224.717	795.153	284.707

Interesting things now happen:

The $\mathbf{x}_{\text{orig}}$ matrix was:

$$\mathbf{x}_{\text{orig}} = \begin{bmatrix} a \\ b \\ E_o \\ N_o \end{bmatrix} = \begin{bmatrix} 0.994361 \\ 0.108300 \\ 373630.072 \\ -6338348.973 \end{bmatrix}$$

becomes translated $\mathbf{x}_{\text{shift}}$:

$$\mathbf{x}_{\text{shift}} = \begin{bmatrix} a \\ b \\ E_o \\ N_o \end{bmatrix} = \begin{bmatrix} 0.994361 \\ 0.108300 \\ -75.439 \\ -144.496 \end{bmatrix}$$

There is almost no change for scale factor, s , or the rotation, θ .

However, the shifts, $\mathbf{E}_o$ and $\mathbf{N}_o$, are much nicer to deal with.

But it is the residuals of the transformations that provide food for thought:

The original coordinates,		residual	shifted coordinates		residuals are:
e_1	$\begin{bmatrix} 49999.996 \\ 124999.997 \end{bmatrix}$	$\begin{bmatrix} 0.004 \\ 0.003 \end{bmatrix}$	e_1	$\begin{bmatrix} -0.004 \\ -0.003 \end{bmatrix}$	$\begin{bmatrix} 0.004 \\ 0.003 \end{bmatrix}$
n_1			n_1		
e_2	$\begin{bmatrix} 51235.293 \\ 125000.001 \end{bmatrix}$	$\begin{bmatrix} 0.007 \\ -0.001 \end{bmatrix}$	e_2	$\begin{bmatrix} 1235.293 \\ 0.003 \end{bmatrix}$	$\begin{bmatrix} 0.007 \\ -0.000 \end{bmatrix}$
n_2			n_2		
e_3	$\begin{bmatrix} 50684.396 \\ 125224.721 \end{bmatrix}$	$\begin{bmatrix} -0.011 \\ -0.004 \end{bmatrix}$	e_3	$\begin{bmatrix} 684.396 \\ 224.720 \end{bmatrix}$	$\begin{bmatrix} -0.011 \\ -0.003 \end{bmatrix}$
n_3			n_3		

$$\mathbf{b}_{\text{calc}} = \mathbf{V} = \mathbf{b}_{\text{obs}} - \mathbf{b}_{\text{calc}}$$

This shows that some errors have crept into the original transformation because of computational limitations of the Excel program, did not arise in MATLAB

Using a 12 digit calculator on the original problem would have provided meaningless answers, even those programmed to do least squares adjustments.

3. Finally ADD the false origins to the transformed points. This returns you to the original reference frame.

16.5 Resection to a Point by EDM – a Least Squares Solution

The aim of surveying is to find 2D or 3D coordinates of a position.

A Total Station measures distances and directions directly to targets set over known points.

Modern Total Station positioning methods use the intersection of measurements between known points from the unknown observation point occupied by the TS.

A similar methodology (different instrument) uses resection distances (pseudo ranges) between a GNSS station and the orbiting satellites.

All surveying equations that involve observations can be reduced to the form:

$$\mathbf{Ax} = \mathbf{b} + \mathbf{v}.$$

The final coordinates are the initial (estimated) coordinates plus the iterated corrections, the $\mathbf{x}$ vector, such that the sum of the residuals ($\mathbf{v}$ vector) is a minimum.

A a matrix of coefficients (a design matrix), calculating coordinate differences,

x a vector of the terms to be computed, generally including coordinate differences,

b a vector of the differences between observations and calculated results,

v a vector of residuals, to be a minimum (or an error vector).

Resection is also a function found in all TS software and this section is intended as an introduction to the method of resection - including a least squares solution - to calculate the most likely unknown station.

Recalling Section 6.3.3, intersection by distances, direct solution, it can be seen that a Total Station will give horizontal distances to two known points. The unknown point can be calculated directly.

However, the TS also shows directions. The **directions** to the known points thus form the **angle** between the observation point and the two known points.

Whilst we may consider this angle to be **redundant** information (it isn't used in the solution by distances); it can still be used as a check of our calculations. By observing the directions from the solved unknown point to the two known points we can get the included angle at the unknown point. There will be a difference between the observed and calculated angles.

But, more than a check, it can now be included in the solution of the unknown coordinates so that the coordinate values we calculate are as close to the true value as we can make, and prove, them from ALL in information available.

The method we use is the method of **Least Squares** where the sum of the squares of the differences between the calculated and the true values, the **residuals**, is a minimum. From this information we can further derive information about the "accuracy" of our observations, the standard deviations.

Most least squares solutions are beyond the scope of hand calculation. Surveyors will be introduced to least squares software in later years of their course. The software will do the work for them. However this section, with a simple problem, will be solved manually using matrix manipulation to show the process.

The ability to understand and manipulate matrices is, along with partial differentiation and linearization of functions, vital to the understanding of least squares solution. **Matlab** will be the software of instruction for matrix work. Microsoft **Excel** can also manipulate matrices, as long as you know the rules, and aren't too ambitious.

All the calculations have been made in Microsoft Excel to full precision and then displayed as shown. The numbers have not been truncated.

The basic structure of the solutions presented comes from Schofield and Breach, 2007, Ghilani, 5th ed and WAIT, 1982.

The design matrix of coefficients uses equations that are generally non-linear. They have to be linearised by the Taylor expansion, using partial derivatives. Some linearisations are shown, but other linearisations are beyond the scope of these examples.

16.5.1 Resection by Observation from an Unknown Point to Two Known Points

The following least squares solution follows Schofield and Breach (2007) chapter 7, Rigorous methods of control, section 7.7, Design matrix and observations vector (page 235).

In this example we are considering only the errors at position X. We are assuming that control stations A and B are fixed (basically error free). $\delta E_A, \delta N_A, \delta E_B, \delta N_B = 0$

In

Figure 16-9 the adjusted coordinates of X are to be calculated from the following observations:

Control A E1000.000, N5000.000, fixed

points B E1160.000, N5190.000, fixed

At station X the following **observations** were made:

XA = 295.055 ± 0.01 m (H_3 distance)

XB = 304.864 ± 0.01 m (H_2 distance)

$\angle AXB = 48^\circ 53' 15'' \pm 10''$

Initial coordinates for X have been calculated from the method described in Section 6.3.3, intersection by distances.

Figure 16-9 has been annotated to help you confirm the coordinates of X (point 1),

Initial values for calculation:

X₀ E1281, N4910.

The most likely values of the x coordinates are:

$E_X = E_{X0} + \delta E_X$ the initial values + small error

$N_X = N_{X0} + \delta N_X$

All surveying equations that involve observations can be reduced to the form:

$$\mathbf{Ax} = \mathbf{b} + \mathbf{v}$$

A a **matrix** of coefficients (or a design matrix)

x a vector of the terms to be computed, including coordinates

b a vector of the differences between observed and calculated values

v a vector of residuals, to be a minimum (or an error vector).

Since we are combining an angle observation and two distance observations the **b** vector will include δ (angle), δs_{XA} and δs_{XB} .

$$\mathbf{b} = \begin{bmatrix} \delta \angle \text{ seconds of arc} \\ \delta H_{XA} \text{ metres} \\ \delta H_{XB} \text{ metres} \end{bmatrix}$$

The aim of the adjustment is to reduce the difference between the observed and the calculated angles to a minimum. Because we are taking careful observations, the difference will most likely be seconds of arc.

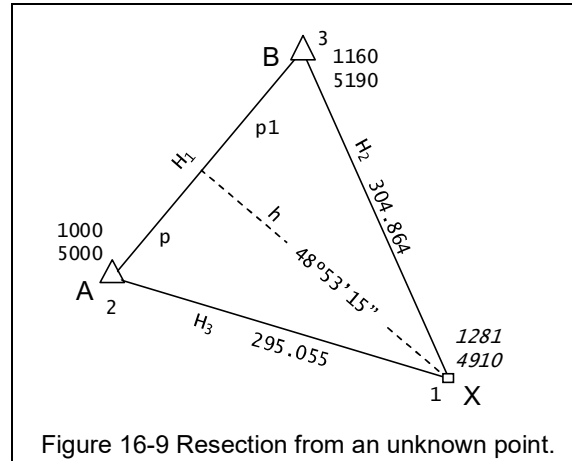


Figure 16-9 Resection from an unknown point.

16.5.1.1 Angle equation

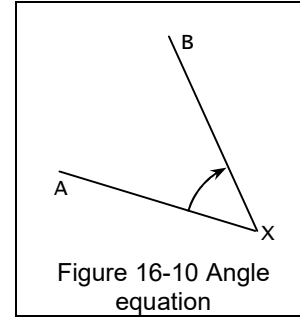
The design matrix will consider the effect of the angle AXB and the length of the observed lines on δE and δN between the observed and calculated coordinates.

Angle AXB, from observations, and from coordinates, needs to be minimised such that:

$$\text{Observed } \angle AXB - (\theta_{XB} - \theta_{XA}) = 0$$

$$\angle AXB - \arctan \left[\frac{E_B - E_X}{N_B - N_X} \right] + \arctan \left[\frac{E_A - E_X}{N_A - N_X} \right] = 0.$$

This is non-linear.



16.5.1.2 Distance equation

The design matrix and the effect of the direction and length of the lines XA and XB on δE and δN between the observed and calculated coordinates.

The two distances, XA and XB, need to be minimised such that:

$$\text{Observed distance} - \text{calculated distance} = 0.$$

$$l_{ij} - \left[(E_j - E_i)^2 + (N_j - N_i)^2 \right]^{1/2} = 0 \text{ where } l_{ij} \text{ is a generic term for distances } H_{XA} \text{ or } H_{XB}$$

The form is non-linear.

1. Form the **A** (3×2) matrix of the observation coefficients for δE and δN .

In general the solution should include the uncertainties of the control points A and B. However, they are considered **fixed** and so we only consider the uncertainties in the measurements from **X**, **vis**;

angle **AXB**

distance **XA**, s_{XA}

distance **XB**, s_{XB}

Fixed points A and B have not appeared in the **x** matrix, and thus do not appear in the **A** matrix.

From the **calculated** bearings and distances from X to A and B, the angle AXB can be found as the difference between the two calculated bearings.

The coefficients of the design matrix:

$$\mathbf{A} = \begin{bmatrix} \frac{\partial \angle AXB}{\partial E_X} & \frac{\partial \angle AXB}{\partial N_X} \\ \frac{\partial s_{XA}}{\partial E_X} & \frac{\partial s_{XA}}{\partial N_X} \\ \frac{\partial s_{XB}}{\partial E_X} & \frac{\partial s_{XB}}{\partial N_X} \end{bmatrix} = \begin{bmatrix} -407.5 & 397.5 \\ 0.9523 & -0.3050 \\ 0.3967 & -0.9180 \end{bmatrix}$$

Evaluating A_{11} as an example

note that $\sin 1'' = 0.000004848137 = 1/206265$

$$\begin{aligned} \frac{\cos \theta_{XA}}{s_{XA} \sin 1''} - \frac{\cos \theta_{XB}}{s_{XB} \sin 1''} &= \frac{\cos(287^\circ 45' 34'')}{295.061/206265} - \frac{\cos(336^\circ 37' 44'')}{305.026/206265} = \frac{0.3050}{0.00143} - \frac{0.9179}{0.00148} \\ &= 213.228 - 620.739 \\ &= -407.511. \end{aligned}$$

2. Form the **x** matrix of the errors in Easting and Northing of the point X:

$$\mathbf{x} = \begin{bmatrix} \delta E_X \\ \delta N_X \end{bmatrix} \text{ is a } 2 \times 1 \text{ matrix.}$$

3. The bearings and distances from the initial coordinates of X to A and B are **calculated**.

$$\theta_{XA} = 287^\circ 45' 34'' \quad s_{XA} = 295.061$$

$$\theta_{XB} = 336^\circ 37' 44'' \quad s_{XA} = 305.026$$

$$\angle AXB = 48^\circ 52' 09'' \text{ (calculated angle, to be used later)}$$

4. The **W** (weight) matrix is a square 3×3 matrix with the weights of the observations on the diagonal. Since they are assumed uncorrelated, the rest of the matrix is filled with zeros. Form the **W** matrix from the standard deviations of the observations. The weight is $1/\sigma^2$:

$$\sigma_\theta = \pm 10'' \quad (\sigma_\theta)^2 = 100 \quad W_\theta = 0.01$$

$$\sigma_s = \pm 0.01 \text{ m} \quad (\sigma_s)^2 = 0.0001 \quad W_s = 10,000$$

$$\mathbf{W} = \begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 10000 & 0 \\ 0 & 0 & 10000 \end{bmatrix} \begin{array}{l} \text{refers to } \angle AXB \\ \text{refers to } s_{XA} \\ \text{refers to } s_{XB} \end{array}$$

5. Form the **b** matrix (3×1); the matrix of the difference between observed and calculated observations. The sense is **obs – calc** in the same order as the **v** and **W** matrices. Units are converted to seconds and metres.

$$\text{observed } \angle AXB \quad 48^\circ 53' 15''$$

$$\text{calculated } \angle AXB \quad 48^\circ 52' 09''$$

$$\text{obs} - \text{calc} = \quad \quad \quad 66''$$

$$\mathbf{b} = \begin{bmatrix} 66'' \\ -0.006 \\ -0.162 \end{bmatrix} \begin{array}{l} \text{refers to difference } \angle AXB \\ \text{refers to difference } s_{XA} \\ \text{refers to difference } s_{XB} \end{array}$$

6. The matrix of unknown, the **x** matrix (3×1), results from manipulation of the **A**, **W** and **b** matrices. This discussion is NOT a lesson in matrix manipulation.

$$\hat{\mathbf{x}} = (\mathbf{A}^T \mathbf{W} \mathbf{A})^{-1} \cdot (\mathbf{A}^T \mathbf{W} \mathbf{b}) \text{ (see Awange and Grafarend 2005).}$$

- 6.1 Start by calculating the product $\mathbf{A}^T \mathbf{W}$ as it is used in both matrices;
 $(2 \times 3) \cdot (3 \times 3) = (2 \times 3)$

$$\mathbf{A}^T = \begin{bmatrix} -407.5 & 0.9523 & 0.3967 \\ 397.5 & -0.3050 & -0.9180 \end{bmatrix} \quad \mathbf{W} = \begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 10000 & 0 \\ 0 & 0 & 10000 \end{bmatrix}$$

$$\mathbf{A}^T \mathbf{W} = \begin{bmatrix} -4.075 & 9523.5 & 3966.9 \\ 3.975 & -3050.2 & -9179.5 \end{bmatrix}$$

Now calculate the product $(\mathbf{A}^T \mathbf{W}) \mathbf{A}$;

$$(2 \times 3) \times (3 \times 2) = (2 \times 2)$$

$$\mathbf{A}^T \mathbf{W} = \begin{bmatrix} -4.075 & 9523.5 & 3966.9 \\ 3.975 & -3050.2 & -9179.5 \end{bmatrix} \quad \mathbf{A} = \begin{bmatrix} -407.5 & 397.5 \\ 0.9523 & -0.3050 \\ 0.3967 & -0.9180 \end{bmatrix}$$

$$\mathbf{A}^T \mathbf{W} \mathbf{A} = \begin{bmatrix} 12303.9 & -8166.1 \\ -8166.1 & 10936.8 \end{bmatrix} \text{ and call it } \mathbf{M}.$$

- 6.2 Using matrix software, find the inverse $(\mathbf{A}^T \mathbf{W} \mathbf{A})^{-1}$, a (2×2) matrix. Call it $\mathbf{M}^{-1}$:

$$(\mathbf{A}^T \mathbf{W} \mathbf{A})^{-1} = \begin{bmatrix} 9.916 \times 10^{-5} & 4.467 \times 10^{-5} \\ 4.467 \times 10^{-5} & 1.116 \times 10^{-4} \end{bmatrix} = \mathbf{M}^{-1}$$

- 6.3 To **check** the inverse, we know that $\mathbf{M}^{-1} \times \mathbf{M} = \mathbf{I}$ (the identity matrix).

$$\mathbf{M}^{-1} = \begin{bmatrix} 1.611 \times 10^{-4} & 1.203 \times 10^{-4} \\ 1.203 \times 10^{-4} & 1.813 \times 10^{-4} \end{bmatrix} \quad \mathbf{M} = \begin{bmatrix} 12303.9 & -8166.1 \\ -8166.1 & 10936.8 \end{bmatrix} \quad \mathbf{M}^{-1} \mathbf{M} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \mathbf{I}$$

- 6.4 Now, for the second time, use $(\mathbf{A}^T \mathbf{W})$ to calculate the product $(\mathbf{A}^T \mathbf{W}) \mathbf{b}$;

$$(2 \times 3) \times (3 \times 1) = (2 \times 1)$$

$$\mathbf{A}^T \mathbf{W} = \begin{bmatrix} -4.075 & 9523.5 & 3966.9 \\ 3.975 & -3050.2 & -9197.5 \end{bmatrix} \mathbf{b} = \begin{bmatrix} 66 \\ -0.006 \\ -0.162 \end{bmatrix}$$

$$\mathbf{A}^T \mathbf{W} \mathbf{b} = \begin{bmatrix} -969.7 \\ 1769.9 \end{bmatrix}$$

7. The $\mathbf{x}$ matrix is the product of $(\mathbf{A}^T \mathbf{W} \mathbf{A})^{-1}$ and $(\mathbf{A}^T \mathbf{W} \mathbf{b})$;

$$(2 \times 2) \times (2 \times 1) = (2 \times 1)$$

$$(\mathbf{A}^T \mathbf{W} \mathbf{A})^{-1} = \begin{bmatrix} 1.611^{-4} & 1.203^{-4} \\ 1.203^{-4} & 1.813^{-4} \end{bmatrix} \quad \mathbf{A}^T \mathbf{W} \mathbf{b} = \begin{bmatrix} -969.7 \\ 1769.9 \end{bmatrix}$$

$$\mathbf{x} = \begin{bmatrix} 0.057 \\ 0.204 \end{bmatrix} = \begin{bmatrix} \delta E_x \\ \delta N_x \end{bmatrix}$$

8. **Adjusted values.** We started by saying that the most likely values of the X coordinates are:

$$E_x = e_x + \delta E_x$$

$$N_x = n_x + \delta N_x$$

8.1 From the initial estimated coordinates, add the errors to find the adjusted coordinates:

$$E_x = 1281 + 0.057 = \mathbf{1281.057}$$

$$N_x = 4910 + 0.204 = \mathbf{4910.204}$$

9. The $\mathbf{v}$ matrix of **residuals** is defined by: $\mathbf{v} = \mathbf{Ax} - \mathbf{b}$;

$$(3 \times 2) \times (2 \times 1) = (3 \times 1) - (3 \times 1)$$

$$\mathbf{A} = \begin{bmatrix} -407.5 & 397.5 \\ 0.9523 & -0.3050 \\ 0.3967 & -0.9180 \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} 0.057 \\ 0.204 \end{bmatrix} \quad \mathbf{Ax} = \begin{bmatrix} 58 \\ -0.008 \\ -0.165 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 66 \\ -0.006 \\ -0.162 \end{bmatrix}$$

$$\mathbf{v} = \mathbf{Ax} - \mathbf{b} = \begin{bmatrix} -8 \\ -0.0022 \\ -0.0026 \end{bmatrix}$$

Now we must look at the quality of the adjustment by examining the **standard** error of a unit weight. This allows a check against blunders; it is also an assessment of whether the weights assigned to adjustment are appropriate.

The **variance** factor σ_0^2 is the sum of the weighted squares of the residuals ($\sum (w_i V_i^2)$) divided by the degrees of freedom ($m - n$)

σ_0^2 can also be defined: $\hat{\sigma}_0^2 = \mathbf{v}^T \mathbf{W} \mathbf{v} / (m - n)$,

m = number observations,

n = number of parameters.

Start by forming the $\mathbf{v}^T \mathbf{W}$ matrix:

$$(3 \times 3) \times (1 \times 3) = (3 \times 1)$$

$$\mathbf{v}^T = [-8 \quad -0.0022 \quad -0.0026] \quad \mathbf{W} = \begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 10000 & 0 \\ 0 & 0 & 10000 \end{bmatrix}$$

$$\mathbf{v}^T \mathbf{W} = [-0.08 \quad -21.76 \quad -25.57]$$

Then form the $\sigma_0^2 = \mathbf{v}^T \mathbf{W} \mathbf{v}$ matrix and divide by the degrees of freedom:

$$(3 \times 1) \cdot (1 \times 3) = (1 \times 1)$$

$$\sigma_0^2 = \mathbf{v}^T \mathbf{W} \mathbf{v} = [-0.08 \quad -21.76 \quad -25.57] \mathbf{v} = \begin{bmatrix} -8 \\ -0.0022 \\ -0.0026 \end{bmatrix}$$

$$\sigma_0^2 = 0.7528 / (m - n)$$

Degrees of freedom

$$m = \text{number observations} = 3,$$

n = number of parameters = 2

$m - n = 1$ degree of freedom

Thus $\sigma_0^2 = 0.7528$ and at the 5% level of the is acceptable on the χ^2 test.

$\sigma_0 = 0.868$, an acceptable figure for the **standard error**

10. The variance-**covariance** matrix is an indication as to whether the various off diagonal elements of the adjustment show any correlation between each other. By examining the upper diagonal of the inverse of the **normal** equation matrix, $(A^T W A)^{-1}$, we can find the **variances** of the parameters **on** the diagonal, and the **covariances** off the diagonal.

$$(A^T W A)^{-1} = \begin{bmatrix} 9.916^{-5} & 4.467^{-5} \\ 4.467^{-5} & 1.116^{-4} \end{bmatrix} = \mathbf{M}^{-1} \quad (\text{See paragraph 6.2})$$

The correlation coefficient between any two parameters can be found from:

$$r_{ij} = \sigma_{ij} / (\sigma_{ii} \sigma_{jj})$$

$$\mathbf{r}_{ij} = \begin{bmatrix} \sigma_1 & \sigma_{12} \\ & \sigma_2 \end{bmatrix} = \begin{bmatrix} 0.0127 & 0.704 \\ & 0.0135 \end{bmatrix}$$

11. Once the initial coordinates have been adjusted, we have a better estimate of the coordinates. By **iterating**, feeding the adjusted coordinates back in at the start of the exercise, a new set of adjusted coordinates can be found. The iterations continue until the adjustment **converges** so that the “improvement” in coordinates stabilises.

By using $E_X = 1281.057$, $N_X = 4910.204$ as the new estimate for the adjustment produces the following “improved” coordinates:

$E_X = 1281.057$, $N_X = 4910.204$, which shows no change.

This result is to be expected given accurate measurements and a good initial estimate. The other factor is that there is only one degree of freedom, so there is hardly any over-determination.

16.6 Least Squares Solution of an Over-determined Trilateration Problem

Plane coordinates from distance observations is also known as “Intersection by distances.” Ghilani, 5th ed, Section 14.2, p242, explains the adjustment of an intersection’s coordinates by least squares solution.

Observation of two distances from the unknown control point to **two** known points produces a unique solution to the point’s plane position (see e.g., Awange and Grafarend 2005, Awange and Palancz 2016 for exact solutions).

It only requires two observations, the distances, to produce the two unknowns, the coordinates in E and N.

If more distance observations are made, say to a third known point, then there are three (3) observations for the two unknowns. The system is **over-determined** (i.e., more observations than unknowns).

The third observation (distance), in conjunction with one of the other distances, could be used to check the first answer. In all, we could arrive at three sets of coordinates for the same point using the three combinatorial pairs of observed distances.

16.6.1 Overdetermined Coordinate Example

By way of example, observations to 3 control points (Figure 16-11) produced the following results: - this could be the “easy” way, given the simplicity of the calculations

Refer to Section 6.3.3:

Point	Fixed coordinates		Measured horizontal distance, l , from U.
	E	N	l
A	1188.113	5333.960	152.420m
B	1069.120	5290.789	138.580m
C	1046.174	5348.466	84.110m

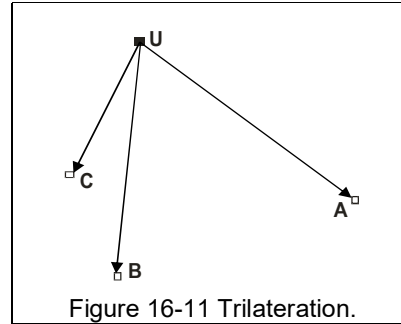


Figure 16-11 Trilateration.

The three sets of coordinates calculated for the unknown point U, using the distance pairs UA, UB; UA, UC and UB, UC are:

U_{AB}		U_{AC}		U_{BC}	
U_{E1}	U_{N1}	U_{E2}	U_{N2}	U_{E3}	U_{N3}
1069.248	5429.369	1069.235	5429.353	1069.178	5429.369

The average of these observations from U, which will be used as the initial coordinates, is:

U_E	U_N	S_E	S_N
1069.220	5429.364	0.037	0.009

Is there a better way of doing this, allowing us to find the “most likely” values of the unknown point, U?

Again, we tackle the problem with a least squares solution by using an observation equation of the measurements (observations) to produce an answer that minimises the sum of the squares of residuals.

It is the normal matrix solution that we are after:

$A \cdot x = b + v$, it looks similar and is solved using the same matrix manipulation techniques used previously, but with non-linear functions.

16.6.1.1 Linearize non-linear functions

Since we are using **distances** (trilateration) to find the coordinates of U the errors in the distances must be minimised.

For any line, IJ , there is the observed distance, l_{ij} , plus its associated residual v_{ij} , (see Figure 16-12).

By plane trigonometry it stands that:

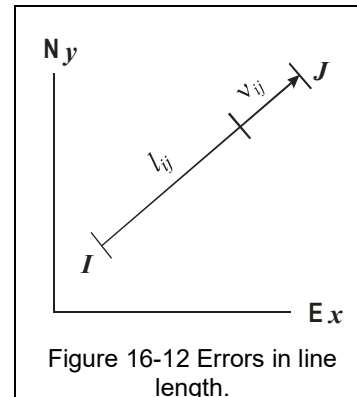


Figure 16-12 Errors in line length.

$$l_{ij} + v_{ij} = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2} \quad \text{Eqn 16-9}$$

Eqn 16-9 is a non-linear function with unknown parameters:

$$x_i, x_j, y_i, y_j$$

and the function can be expressed as:

$$F(x_i, y_i, x_j, y_j) = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2} = \left[(x_j - x_i)^2 + (y_j - y_i)^2 \right]^{\frac{1}{2}} \quad \text{Eqn 16-10}$$

Eqn 16-10 can be linearized and solved by a first-order Taylor series expansion by finding the partial derivatives of the elements of the function:

in essence, the final function = initial function + corrections

$$F() = F()_0 + d(F)$$

$$F(x_i, y_i, x_j, y_j) = F(x_{i0}, y_{i0}, x_{j0}, y_{j0})$$

$$= \left(\frac{\partial F}{\partial x_i} \right)_0 dx_i + \left(\frac{\partial F}{\partial y_i} \right)_0 dy_i + \left(\frac{\partial F}{\partial x_j} \right)_0 dx_j + \left(\frac{\partial F}{\partial y_j} \right)_0 dy_j \quad \text{Eqn 16-11}$$

Thus, for the start coordinates, i : $x_i = x_{i0} + dx_i$ $y_i = y_{i0} + dy_i$
 and, for the end coordinates, j : $x_j = x_{j0} + dx_j$ $y_j = y_{j0} + dy_j$
 Evaluating the partial derivatives is illustrated using $\partial F/\partial x_i$, using Eqn 16-10,

$$F(x_i, y_i, x_j, y_j) = \left[(x_j - x_i)^2 + (y_j - y_i)^2 \right]^{\frac{1}{2}}$$

The partial derivative with respect to x_i is:

$$\left(\frac{\partial F}{\partial x_i} \right) = \frac{1}{2} \left[\left[(x_j - x_i)^2 + (y_j - y_i)^2 \right]^{-\frac{1}{2}} \left[2(x_j - x_i)(-1) \right] \right] \text{note that } (y_j - y_i)^2 \text{ is a constant and cancels.}$$

$$\frac{\partial F}{\partial x_i} = \frac{-2(x_j - x_i)}{2 \left[(x_j - x_i)^2 + (y_j - y_i)^2 \right]^{\frac{1}{2}}} = \frac{-(x_j - x_i)}{\sqrt{(x_j - x_i)^2 + (y_j - y_i)^2}} = \frac{(x_i - x_j)}{IJ} \quad (16.1)$$

And, similarly,
$$\frac{\partial F}{\partial y_i} = \frac{(y_i - y_j)}{IJ} \quad (16.2)$$

But
$$\left(\frac{\partial F}{\partial x_j} \right) = \frac{1}{2} \left[(x_j - x_i)^2 + (y_j - y_i)^2 \right]^{-\frac{1}{2}} \left[2(x_j - x_i)(+1) \right], \text{ rearrange to:}$$

$$= \frac{(x_j - x_i)}{\sqrt{(x_j - x_i)^2 + (y_j - y_i)^2}} \text{ Note the change of subscript order from (15.1)}$$

With respect to point x_j ,
$$\frac{\partial F}{\partial x_j} = \frac{(x_j - x_i)}{IJ} \quad (16.3)$$

and y_j ,
$$\frac{\partial F}{\partial y_j} = \frac{(y_j - y_i)}{IJ} \quad (16.4)$$

$$\left(\frac{(x_i - x_j)}{IJ} \right)_0 dx_i + \left(\frac{(y_i - y_j)}{IJ} \right)_0 dy_i + \left(\frac{(x_j - x_i)}{IJ} \right)_0 dx_j + \left(\frac{(y_j - y_i)}{IJ} \right)_0 dy_j = k_l + v_l \quad (16.5)$$

Now the initial parameters are calculated using the approximate (initial) values, $k_l = l_{ij} - IJ_0$
 and $IJ_0 = F(x_{i0}, y_{i0}, x_{j0}, y_{j0}) = \sqrt{(x_{j0} - x_{i0})^2 + (y_{j0} - y_{i0})^2} \quad (16.6)$

16.6.1.2 First Iteration, i_0 ,

Start with the estimates of $\mathbf{Ax}=\mathbf{b}+\mathbf{v}$

In the first iteration, we have used two measured distances to define three approximate coordinates of U and taken the average position. We can say that (from figure 12.11) $k_l = l_{ij} - l_{j0} = 0$ with, in this case, k_l = the difference between the measured and calculated distance to the third (overdetermined) point. Normally we would calculate a unique position of U, say from A and B. Then CU is the overdetermining distance. Distances AU and BU were used to compute the unique coordinates of U, so their residual distance errors are 0.000. The residual distance for CU, $l_{CU} - CU_0$ (measured distance – calculated distance) is: $84.110 - 84.129 = -0.019$.

However, since point U is averages from 3 pairs of distances from A, B and C we will start by calculating distances from AU, BU and CU. The residuals are shown in the $\mathbf{b}_0$ matrix below.

The initial coordinates of U_0 , from the averaged solution using three distances, U-A, U-B and U-C, are $U_{e0} = 1069.220$, $U_{n0} = 5429.364$. See §0.

And because we have turned a non-linear function into a linear function by the Taylor expansion we can use the normal linear matrix methods for the solution of the least squares adjustment.

$$\mathbf{A}_0 = \begin{bmatrix} \frac{x_U - x_A}{AU} & \frac{y_U - y_A}{AU} \\ \frac{x_U - x_B}{BU} & \frac{y_U - y_B}{BU} \\ \frac{x_U - x_C}{CU} & \frac{y_U - y_C}{CU} \end{bmatrix} = \begin{bmatrix} \frac{1069.220 - 1188.113}{152.42} & \frac{5429.364 - 5333.960}{152.42} \\ \frac{1069.220 - 1069.120}{138.580} & \frac{5429.364 - 5290.789}{138.580} \\ \frac{1069.220 - 1046.174}{84.110} & \frac{5429.364 - 5348.466}{84.110} \end{bmatrix} = \begin{bmatrix} -0.78004 & 0.62593 \\ 0.00072 & 0.99996 \\ 0.27400 & 0.96181 \end{bmatrix}$$

The three calculated distances to the average position of U_0 are:

$$AU_0 = 152.438$$

$$BU_0 = 138.575$$

$$CU_0 = 84.117$$

The $\mathbf{b}$ matrix elements are the residual distances from U to A, B and C. Since U is an estimated position the distances AU, BU and CU will all have residual distance for errors. For example, distance CU, $l_{CU} - CU_0$ (measured – calculated distance) is: $84.110 - 84.117 = -0.007$.

$$\mathbf{b}_0 = \begin{bmatrix} l_{AU} - AU_0 \\ l_{BU} - BU_0 \\ l_{CU} - CU_0 \end{bmatrix} = \begin{bmatrix} 154.420 - 154.438 \\ 138.580 - 138.575 \\ 84.110 - 84.117 \end{bmatrix} = \begin{bmatrix} -0.018 \\ 0.005 \\ -0.007 \end{bmatrix}$$

Normalize the matrix $\mathbf{Ax} = \mathbf{b}$ by using the transpose of $\mathbf{A}$. Thus $\mathbf{A}^T \mathbf{A} = \mathbf{A}^T \mathbf{b}$.

Solve for $\mathbf{x}$, finding the inverse of $\mathbf{A}^T \mathbf{A}$, so that $\mathbf{x}_0 = ((\mathbf{A}^T \mathbf{A})^{-1}) \mathbf{A}^T \mathbf{b}$.

$$\mathbf{A}^T \mathbf{A} = \begin{bmatrix} 0.684 & -0.224 \\ -0.224 & 2.317 \end{bmatrix} (\mathbf{A}^T \mathbf{A})^{-1} = \begin{bmatrix} 1.511 & 0.146 \\ 0.146 & 0.446 \end{bmatrix} \mathbf{A}^T \mathbf{b} = \begin{bmatrix} 0.0126 \\ -0.0129 \end{bmatrix} \mathbf{x}_0 = \begin{bmatrix} 0.0171 \\ -0.0039 \end{bmatrix}$$

At the end of the first iteration the coordinates of U become, as $U_1 = (E_0 + x_{E0}, N_0 + x_{N0})$:

$$E_1 = 1069.220 + (+0.0171) = 1069.2371$$

$$N_1 = 5429.364 + (-0.0039) = 5429.3601$$

16.6.1.3 Second Iteration, i_1 :

Run the second iteration, where now, using the U_1 iterated coordinates as the initial values:

$$\mathbf{A}_1 = \begin{bmatrix} \frac{1069.237 - 1188.113}{152.42} & \frac{5429.360 - 5333.960}{152.42} \\ \frac{1069.237 - 1069.120}{138.580} & \frac{5429.360 - 5290.789}{138.580} \\ \frac{1069.237 - 1046.174}{84.110} & \frac{5429.360 - 5348.466}{84.110} \end{bmatrix} = \begin{bmatrix} -0.779923 & 0.625903 \\ 0.000845 & 0.999936 \\ 0.274201 & 0.961765 \end{bmatrix}$$

Using the new coordinates for U , the distances AU_1 , BU_1 and CU_1 are calculated, and shown to 4DP, to create the $\mathbf{b}$ matrix:

$$AU_1 = 152.4226$$

$$BU_1 = 138.5711$$

$$CU_1 = 84.1175$$

$$\mathbf{b}_1 = \begin{bmatrix} l_{AU} - AU_1 \\ l_{BU} - BU_1 \\ l_{CU} - CU_1 \end{bmatrix} = \begin{bmatrix} 154.420 & -154.423 \\ 138.580 & -138.571 \\ 84.110 & -84.118 \end{bmatrix} = \begin{bmatrix} -0.003 \\ 0.009 \\ -0.008 \end{bmatrix}$$

Normalize the matrix $\mathbf{Ax} = \mathbf{b}$ by using the transpose of $\mathbf{A}$. Thus $\mathbf{A}^T\mathbf{A} = \mathbf{A}^T\mathbf{b}$.

Solve for $\mathbf{x}$, finding the inverse of $\mathbf{A}^T\mathbf{A}$, so that $\mathbf{x}_1 = ((\mathbf{A}^T\mathbf{A})^{-1})\mathbf{A}^T\mathbf{b}$.

$$\mathbf{A}^T\mathbf{A} \begin{bmatrix} 0.68347 & -0.22359 \\ -0.22359 & 2.31662 \end{bmatrix} ((\mathbf{A}^T\mathbf{A})^{-1}) \begin{bmatrix} 1.511 & 0.146 \\ 0.146 & 0.446 \end{bmatrix} \mathbf{A}^T\mathbf{b} \begin{bmatrix} 4.28^{-7} \\ -3.74^{-6} \end{bmatrix} \mathbf{x}_1 = \begin{bmatrix} 1.0^{-7} \\ -1.6^{-6} \end{bmatrix}$$

The dE (x_{E1}) and dN (x_{N1}) corrections are now effectively zero.

At the end of the second iteration the coordinates of U become, as $U_2 = (E_1 + x_{E1}, N_1 + x_{N1})$:

$$1069.2371 + (-0.000) = 1069.2371$$

$$5429.3601 + (-0.000) = 5429.3601$$

16.6.1.4 Third Iteration, i_2 :

The third iteration where now, using the U_2 iterated coordinates as the initial values: The $\mathbf{A}$ matrix changes slightly.

$$\mathbf{A}_1 = \begin{bmatrix} \frac{1069.237 - 1188.113}{152.42} & \frac{5429.360 - 5333.960}{152.42} \\ \frac{1069.237 - 1069.120}{138.580} & \frac{5429.360 - 5290.789}{138.580} \\ \frac{1069.237 - 1046.174}{84.110} & \frac{5429.360 - 5348.466}{84.110} \end{bmatrix} = \begin{bmatrix} -0.779929 & 0.625885 \\ 0.000839 & 0.999960 \\ 0.274191 & 0.961733 \end{bmatrix}$$

Using the new coordinates for U , the distances AU_2 , BU_2 and CU_2 are calculated, and shown to 4DP, to create the $\mathbf{b}$ matrix:

$$AU_1 = 152.4216$$

$$BU_1 = 138.5744$$

$$CU_1 = 84.1175$$

$$\mathbf{b}_1 = \begin{bmatrix} l_{AU} - AU_1 \\ l_{BU} - BU_1 \\ l_{CU} - CU_1 \end{bmatrix} = \begin{bmatrix} 154.420 & -154.422 \\ 138.580 & -138.574 \\ 84.110 & -84.115 \end{bmatrix} = \begin{bmatrix} -0.002 \\ 0.006 \\ -0.005 \end{bmatrix}$$

Normalize the matrix $\mathbf{Ax} = \mathbf{b}$ by using the transpose of $\mathbf{A}$. Thus $\mathbf{A}^T\mathbf{A} = \mathbf{A}^T\mathbf{b}$.

Solve for $\mathbf{x}$, finding the inverse of $\mathbf{A}^T\mathbf{A}$, so that $\mathbf{x}_1 = ((\mathbf{A}^T\mathbf{A})^{-1})\mathbf{A}^T\mathbf{b}$.

$$\mathbf{A}^T \mathbf{A} \begin{bmatrix} 0.68347 & -0.22361 \\ -0.22361 & 2.31658 \end{bmatrix} ((\mathbf{A}^T \mathbf{A})^{-1}) \begin{bmatrix} 1.511 & 0.146 \\ 0.146 & 0.446 \end{bmatrix} \mathbf{A}^T \mathbf{b} \begin{bmatrix} 3.46^{-6} \\ -3.12^{-6} \end{bmatrix} \mathbf{x}_1 = \begin{bmatrix} -5.7^{-6} \\ -1.9^{-6} \end{bmatrix}$$

The dE (x_{E1}) and dN (x_{N1}) corrections are still now effectively zero.

At the end of the second iteration the coordinates of U become, as $U_3 = (E_2 + x_{E2}, N_2 + x_{N2})$:

$$1069.2371 + (-0.000) = 1069.2371$$

$$5429.3601 + (-0.000) = 5429.3601$$

Final least squares derived coordinates. The best estimate for U is thus:

E_U 1069.237

N_U 5429.360.

We started by saying we would have residuals, $\mathbf{v}$, from the estimates, where $\mathbf{Ax} = \mathbf{b} + \mathbf{v}$.

Thus, at the end of the iterations, $\mathbf{v} = \mathbf{A}_2 \mathbf{x}_2 - \mathbf{b}_2$.

$$\mathbf{A}_2 \mathbf{x}_2 = \begin{bmatrix} -3.24^{-6} \\ -1.90^{-6} \\ -3.38^{-6} \end{bmatrix} \cdot \mathbf{b}_2 = \begin{bmatrix} -0.0016 \\ 0.0056 \\ -0.0047 \end{bmatrix} \cdot \mathbf{v} = \mathbf{Ax} - \mathbf{b} = \begin{bmatrix} 0.0016 \\ -0.0056 \\ 0.0047 \end{bmatrix}$$

The variance, and hence the standard deviation, is found from the sum the residuals squared, Σv^2 , divided by the number of redundant observations ($m - n = I$).

$$\mathbf{v}^2 = \begin{bmatrix} 2.7^{-6} \\ 3.1^{-5} \\ 2.2^{-5} \end{bmatrix} \cdot \Sigma v^2 = 5.57^{-5}$$

$$\Sigma v^2 = \sigma_0^2 = 0.000056, \text{ thus } \sigma_0 = \pm 0.0075.$$

16.7 Affine Coordinate Transformation with Weights

The conformal transformation merely translates one set of coordinates to another by a scaling, shift and rotation. The basic SHAPE of the figure remains, the transformation is CONFORMAL.

The next transformation is designed to work where there are different scales in X and Y. It is popularly used in photogrammetry the interior orientation to account for differential shrinkage in photo film images, or in non-conformal maps.

16.7.1 Developing the Affine Transformation

An affine transformation is used in photogrammetry to compensate for uneven film shrinkage derived from the camera fiducial mark system and arbitrarily measured fiducial marks on the film (Figure 16-13). It makes allowances for two different scale factors; one in the X, the other in the Y direction.

The camera has four (or 8) fiducial marks that provides an overdetermined solution as only 3 fiducials are needed to generate the 6 parameters of the affine transformation; 2 scaling and 1 skew angle, 1 rotation, and 2 translation. The transformation allows other points (pass points) measured on the film to be coordinated in the camera fiducial system.

The 6-parameter affine transformation equation is:

Affine transformation parameters	$E = ax + by + c$ $N = dx + ey + f$	Eqn 16-12

In matrix form:

$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} a & b \\ d & e \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ f \end{bmatrix}$	Eqn 16-13

The coordinate reference frame x, y of points on the image is **non-orthogonal** (not at right angles), Figure 16-14.

Start by assuming the points are measured **along** the x and y (non-orthogonal) axes.

1. Make the figure orthogonal by *skewing* the Oy axis around to Oy' to be perpendicular to the Ox axis. Call the skew angle α , positive anticlockwise. The frame is now $x'Oy'$. Project the coordinates of B in the orthogonal frame, including s_x and s_y as scale factors for the coordinates. See Figure 16-15.

$$x'_B = s_x x + s_y (y \sin \alpha)$$

$$y'_B = s_x x + s_y (y \cos \alpha) \text{ but, starting at O, } x = 0.$$

$$y'_B = s_x 0 + s_y (y \cos \alpha)$$

Matrix form:	$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & s_y \sin \alpha \\ 0 & s_y \cos \alpha \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$	Eqn 16-14

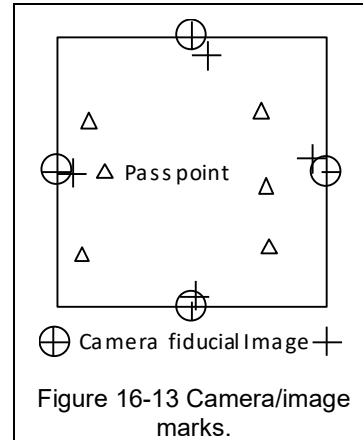


Figure 16-13 Camera/image marks.

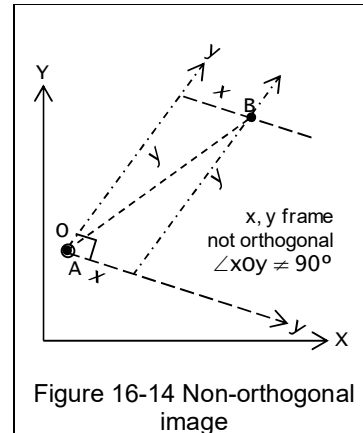


Figure 16-14 Non-orthogonal image

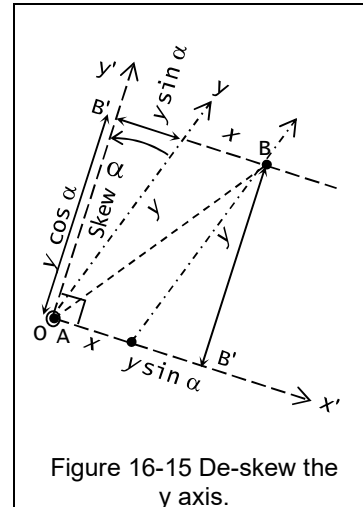


Figure 16-15 De-skew the y axis.

2. Now rotate the figure in the $x'Oy'$ frame into the $x''Oy''$ system. This is parallel the X, Y system but shifted (translated) from its origin. Figure 16-16

$$\begin{aligned}x''_B &= x' \cos \theta + y' \sin \theta \\y''_B &= -x' \sin \theta + y' \cos \theta\end{aligned}$$

Matrix form: $\begin{bmatrix} x'' \\ y'' \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix}$	Eqn 16-15

This is the normal rotation process used in conformal transformations. We have turned the skewed projection into a conformal projection via two matrix operation, scaling and de-skewing.

3. Next shift the figure in the $x''Oy''$ frame to the XY system. The two systems are parallel. Call the shifts X_o and Y_o .

$$\begin{aligned}X_B &= x''_B + X_o \\Y_B &= y''_B + Y_o\end{aligned}$$

Matrix form: $\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} x'' \\ y'' \end{bmatrix} + \begin{bmatrix} X_o \\ Y_o \end{bmatrix}$	Eqn 16-16

4. Combining Eqn 16-13, Eqn 16-14 and Eqn 16-15, and renaming X_o and Y_o as c and f : (partitioned for clarity), results in the matrix:

$$\begin{aligned}\begin{bmatrix} X \\ Y \end{bmatrix} &= \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} s_x & s_y \sin \alpha \\ 0 & s_y \cos \alpha \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ f \end{bmatrix} \\ &= \begin{bmatrix} s_x \cos \theta & s_y \cos \theta \sin \alpha + s_y \sin \theta \cos \alpha \\ -s_x \sin \theta & -s_y \sin \theta \sin \alpha + s_y \cos \theta \cos \alpha \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ f \end{bmatrix}\end{aligned}$$

Eqn 16-17

5. Substituting the elements of Eqn 16-17 into Eqn 16-12 gives the coefficients:

$$a = s_x \cos \theta \quad (16.1)$$

$$\begin{aligned}b &= s_y (\cos \theta \sin \alpha + \sin \theta \cos \alpha) \\ &= s_y \sin(\theta + \alpha)\end{aligned} \quad (16.2)$$

$$d = -s_x \sin \theta \quad (16.3)$$

$$\begin{aligned}e &= s_y (\cos \theta \sin \alpha - \sin \theta \cos \alpha) \\ &= s_y \cos(\theta + \alpha)\end{aligned} \quad (16.4)$$

From these elements we deduce: 2 scales, 1 rotation angle and 1 skew angle.

$$s_x = \sqrt{a^2 + d^2}, \text{ scale in } x,$$

$$s_y = \sqrt{b^2 + e^2}, \text{ scale in } y,$$

$$\tan \theta = \frac{-d}{a}, \text{ or } \cos \theta = \frac{a}{s_x}, \text{ rotation angle, } \theta$$

$$\tan(\theta + \alpha) = \frac{b}{e}, \text{ or } \cos(\theta + \alpha) = \frac{e}{s_y}, \text{ sum of rotation and skew, derive skew angle, } \alpha.$$

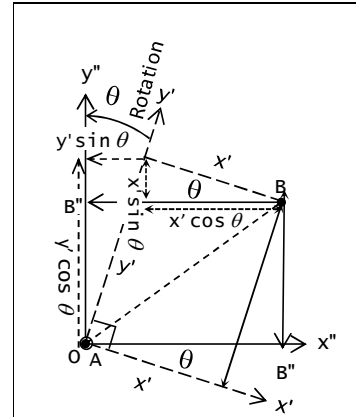


Figure 16-16 Rotate axes

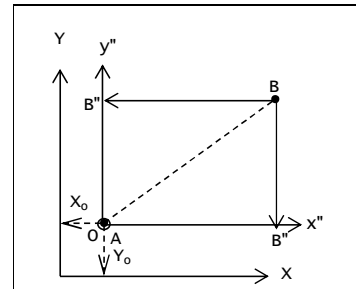


Figure 16-17 Shift to origin.

16.7.2 Dealing with the Skew Angle

The derivation started with the x and y coordinates being measured along their non-orthogonal axes. Coordinate measuring systems are **orthogonal** in x and y so the skew angle $\alpha = 0$.

Eqn 16-17 reduces to (because $\sin\alpha = 0$, $\cos\alpha = 1$):

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} s_x \cos \theta & s_y \sin \theta \\ -s_x \sin \theta & s_y \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ f \end{bmatrix} \quad \text{Eqn 16-18}$$

6. Substituting the elements of Eqn 16-18 into Eqn 16-12 gives the coefficients:

$$a = s_x \cos \theta \quad (16.5)$$

$$b = s_y \sin \theta \quad (16.6)$$

$$d = -s_x \sin \theta \quad (16.7)$$

$$e = s_y \cos \theta \quad (16.8)$$

7. From these elements we deduce: 2 scales, 1 rotation angle, skew angle = 0.

$$s_x = \sqrt{a^2 + d^2} ,$$

$$s_y = \sqrt{b^2 + e^2} ,$$

$$\tan \theta = \frac{b}{a} .$$

The shift elements X_o and Y_o are represented by c and f .

The mathematical model is of the form:

$$ax + by + c = X$$

$$dx + ey + f = Y$$

in the matrix form:

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} a & b \\ d & e \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ f \end{bmatrix}$$

and, because it's overdetermined:

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} X \\ Y \end{bmatrix} + \begin{bmatrix} V_x \\ V_y \end{bmatrix}$$

It can be expressed in the matrix form $\mathbf{Ax} = \mathbf{b} + \mathbf{v}$.

In the **affine** transformation there are three (3) common points required for a unique solution.

This provides the 6 parameters, a, b, c, d, e and f.

The observation equations for one point are:

$$ax + by + c = X + V_x$$

$$dx + ey + f = Y + V_y$$

Expanding the equations to include all parameters in each line, they become:

$$ax + by + 1c + 0d + 0e + 0f = X + V_x$$

$$0a + 0b + 0c + dx + ey + 1f = Y + V_y$$

Eqn 16-19

The standard deviations of the observations, if known, can be included as a vector of precisions (**p**), generating a **weight** matrix of the observations.

Noting that naming conventions can change from author to author, in general, observations in surveying can be reduced to the matrix form: $\mathbf{AWx} = \mathbf{Wb} + \mathbf{v}$

A a **matrix** of coefficients;

p a **vector** of observation precisions, the standard deviation;

W a diagonal weight matrix of the inverse of the observation variances ($W = 1/p^2$).

x a vector of terms to be computed, including coordinates;

b a vector relating to the observations;

v a vector of residuals, to be a minimum.

Taking the original affine transformation, and recognizing there ARE a minimum six (6) transformation parameters, the equations are: the **A** matrix of coefficients and the **p** vector.

$$\text{Observations} = \begin{bmatrix} a_{x1} + b_{y1} + c = X_1 + v_{x1} \\ d_{x1} + e_{y1} + f = Y_1 + v_{y1} \\ a_{x2} + b_{y2} + c = X_2 + v_{x2} \\ d_{x2} + e_{y2} + f = Y_2 + v_{y2} \\ a_{x3} + b_{y3} + c = X_3 + v_{x3} \\ d_{x3} + e_{y3} + f = Y_3 + v_{y3} \end{bmatrix} \quad \mathbf{A} = \begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_2 & y_2 & 1 \\ x_3 & y_3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_3 & y_3 & 1 \end{bmatrix} \quad \mathbf{p} = \begin{bmatrix} p_{x1} \\ p_{y1} \\ p_{x2} \\ p_{y2} \\ p_{x3} \\ p_{y3} \end{bmatrix} \quad \text{Eqn 16-20}$$

The form of the least squares solution is as shown. Because this solution is unique, using three points shown here, there will be no residuals.

And thus the **v** vector = 0.

$$\mathbf{A} = \begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_2 & y_2 & 1 \\ x_3 & y_3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_3 & y_3 & 1 \end{bmatrix} \quad \mathbf{W} = \begin{bmatrix} W_{x1} & & & & & \\ & W_{y1} & & & & \\ & & W_{x2} & & & \\ & & & W_{y2} & & \\ & & & & W_{x3} & \\ & & & & & W_{y3} \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} X_1 \\ Y_1 \\ X_2 \\ Y_2 \\ X_3 \\ Y_3 \end{bmatrix} \quad \text{Eqn 16-21}$$

16.7.3 Worked example 16.4. Over-determined Least Squares Affine Transformation

The following example will use 4 common points, thus over-determining the transformation. It will result in a least squares solution with the introduction of the **v** vector, a set of residuals. The standard deviation of each observation has been estimated and this will allow the introduction of **weighted** observations in the solution.

The calibrated coordinates of the four camera fiducial marks, X and Y, are supplied by the manufacturer of a 230mm (9inch) format aerial camera. The coordinates are in millimetres (mm) from the camera principle point (PP).

The film (image) fiducial marks and two pass points have been read in a coordinating digitiser (read in inches) with estimates for the observations of the fiducials. Calculate the adjusted coordinates of the fiducials and the two pass points. The film coordinate system has been rotated 90° to the camera system. (Ghilani, section 18.5).

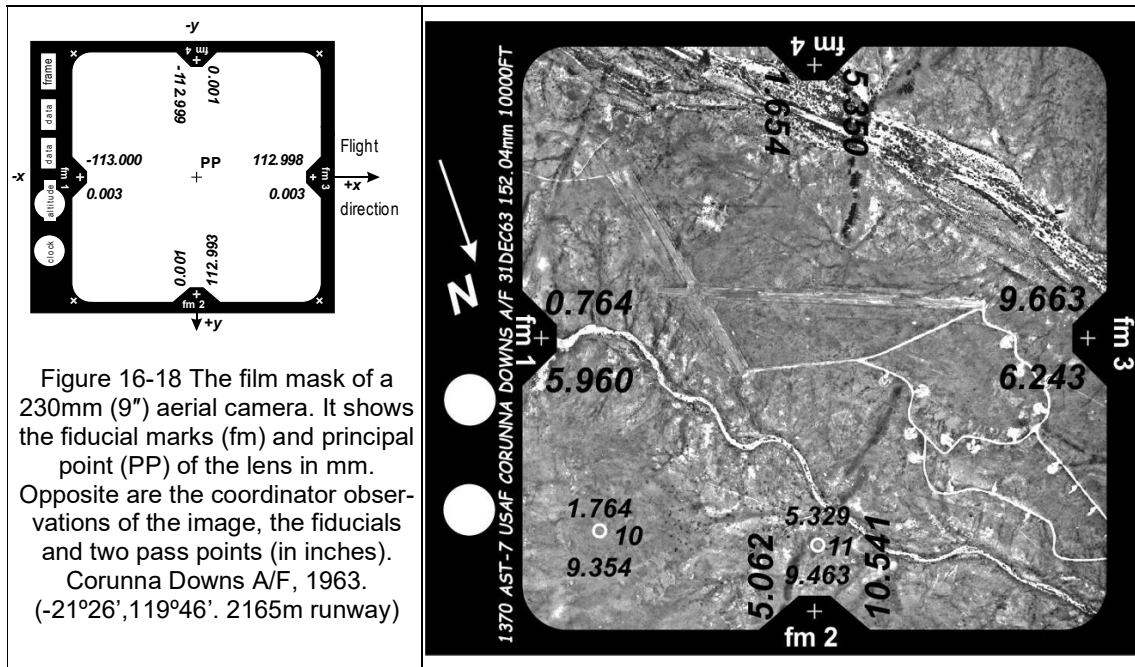
Standard deviations of the fiducial observations, σ_x , σ_y , generate a weight matrix, **W**. This results in a weighted matrix, **Wax** = **Wb** + **v**

The solution of the adjustment is shown on the right-hand side. Note that the two pass points, 10 and 11, are now in the camera coordinate reference frame.

Point	X	Y	x	y	σ_x	σ_y	X	Y	$\pm\sigma_x$	$\pm\sigma_y$
	orig	orig	obs	obs	obs	obs	adj	adj	adj	adj
1	-113.000	0.003	0.764	5.960	0.104	0.112	-112.899	0.052	0.132	0.141
2	0.001	112.993	5.062	10.541	0.096	0.120	-0.085	112.936	0.125	0.147
3	112.998	0.003	9.663	6.243	0.112	0.088	113.115	0.033	0.139	0.118
4	0.001	-112.999	5.350	1.654	0.096	0.104	-0.085	-113.042	0.125	0.134
10			1.746	9.354			-85.193	85.470	0.134	0.154
11			5.329	9.463			5.803	85.337	0.107	0.123

Parameters from the resultant **x** matrix.

$$\mathbf{x} = \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} = \begin{bmatrix} 25.372 \\ 0.822 \\ -137.183 \\ -0.810 \\ 25.402 \\ 150.724 \end{bmatrix} \quad \begin{aligned} \text{Scale in } x &= s_x = \sqrt{a^2 + d^2} = 25.384 \\ \text{Scale in } y &= s_y = \sqrt{b^2 + e^2} = 25.415 \\ \text{Rotation} &= \tan \theta = \frac{b}{a} = 0.0324, \theta = 1^\circ 51' \\ \text{Shift in } X_o &= d = -137.183 \\ \text{Shift in } Y_o &= f = 150.724 \end{aligned} \quad \begin{aligned} &\text{The scales in } x \text{ and } y \text{ are about 25.4.} \\ &\text{This is the conversion factor of inch} \\ &\text{measurements on the photo into the} \\ &\text{metric (mm) dimensions of the} \\ &\text{camera fiducial frame.} \\ &\text{Difference in scale could be due to} \\ &\text{temporal uneven image shrinkage.} \end{aligned}$$



The Affine transformation will be carried out using Matlab. Excel is a valuable tool but at this level it starts to get unwieldy.

1. Form the $\mathbf{x}$ matrix of the parameters of the transformed points X :

$$\mathbf{x} = \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix}$$

2. Form the $\mathbf{A}$ (8×6) matrix of the observation coefficients for x and y . These are the measurements taken by the digitizer. The standard deviation, σ_{x_i} , σ_{y_i} , of each observation is included here as the $\mathbf{p}$ vector, which will allow generation of the **weight** matrix. The solution comprises the x y control points 1, 2, 3 and 4, which are considered **fixed**.

$$\mathbf{A} = \begin{matrix} x_1 & \begin{bmatrix} 0.764 & 5.960 & 1 & 0 & 0 & 0 \end{bmatrix} \\ y_1 & \begin{bmatrix} 0 & 0 & 0 & 0.764 & 5.960 & 1 \end{bmatrix} \\ x_2 & \begin{bmatrix} 5.062 & 10.451 & 1 & 0 & 0 & 0 \end{bmatrix} \\ y_2 & \begin{bmatrix} 0 & 0 & 0 & 5.062 & 10.451 & 1 \end{bmatrix} \\ x_3 & \begin{bmatrix} 9.663 & 6.243 & 1 & 0 & 0 & 0 \end{bmatrix} \\ y_3 & \begin{bmatrix} 0 & 0 & 0 & 9.663 & 6.243 & 1 \end{bmatrix} \\ x_4 & \begin{bmatrix} 3.530 & 1.654 & 1 & 0 & 0 & 0 \end{bmatrix} \\ y_4 & \begin{bmatrix} 0 & 0 & 0 & 3.530 & 1.654 & 1 \end{bmatrix} \end{matrix} \quad \mathbf{p} = \begin{bmatrix} 0.104 \\ 0.112 \\ 0.096 \\ 0.120 \\ 0.112 \\ 0.088 \\ 0.096 \\ 0.104 \end{bmatrix}$$

3. Using the p vector, generate the diagonal W matrix where $W_{ii} = \frac{1}{p^2}$

$$W = \begin{bmatrix} 92.456 & & & & & & & \\ & 79.719 & & & & & & \\ & & 108.507 & & & & & \\ & & & 69.444 & & & & \\ & & & & 71.719 & & & \\ & & & & & 129.132 & & \\ & & & & & & 108.507 & \\ & & & & & & & 92.456 \end{bmatrix}$$

4. Form the $\mathbf{b}$ matrix (8×1); the matrix of the fixed coordinates in the **Camera** system (the fiducial marks).

Because there are now residuals (over-determined solution) we generate the $\mathbf{v}$ vector (8×1) of residuals, $\mathbf{v}$ (nu).

$$\mathbf{b} = \begin{bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \\ x_3 \\ y_3 \\ x_4 \\ y_4 \end{bmatrix} = \begin{bmatrix} -113.000 \\ 0.003 \\ 0.001 \\ 112.993 \\ 112.998 \\ 0.003 \\ 0.001 \\ -112.999 \end{bmatrix}$$

$$\mathbf{v} = \begin{bmatrix} v_{X1} \\ v_{Y1} \\ v_{X2} \\ v_{Y2} \\ v_{X3} \\ v_{Y3} \\ v_{X4} \\ v_{Y4} \end{bmatrix}$$

16.7.4 Affine Transformation using MATLAB.

The manipulation of the previously defined matrices becomes unwieldy in Excel and the exercise continues using MATLAB.

MATLAB (Matrix Laboratory), by The Math Works, Inc is installed on the laboratory computers and is the tool of choice for solving matrix problems. You will use it extensively during your degree course.

- a. Pre-dimension the $\mathbf{x}$ matrix of the transformation parameters a to f ; it is a column vector of dimension 6×1 . This is not strictly necessary as the $\mathbf{x}$ vector will be generated as the result of matrix manipulation. The input is shown as a transposed row vector to make it a column vector, using the transpose operator, the single quote, `'`. Notice also that the output has been suppressed with the semicolon operator, `;`

```
EDU> x = [zeros(1,6)]'; % Transpose x[1x6] row vector to x[6x1] column vector.
```

- b. Form the $\mathbf{A}$ matrix of observations for the six (6) unknown $\mathbf{x}$ elements.

The elements are separated by a comma for clarity. A space will do. `↵` enter key at end of line.

```
EDU> A = [0.764, 5.96, 1, 0, 0, 0 ↵
0, 0, 0, 0.764, 5.96, 1 ↵
5.062, 10.541, 1, 0, 0, 0 ↵
0, 0, 0, 5.062, 10.541, 1 ↵
9.663, 6.243, 1, 0, 0, 0 ↵
0, 0, 0, 9.663, 6.243, 1 ↵
5.35, 1.654, 1, 0, 0, 0 ↵
0, 0, 0, 5.35, 1.654, 1]; ↵
```

- c. Form the **b** matrix of the fiducials. The input is shown as a transposed row vector to make it a column vector, using the transpose operator, the single quote, `'`.

```
EDU> b = [-113, 0.003, 0.001, 112.996, 112.998, 0.003, .001, -112.999]';
```

- d. Form the **p** matrix of the observation errors. It is a transposed row vector.

```
EDU> p = [0.104, 0.112, 0.096, 0.12, 0.112, 0.088, 0.096, 0.104]';
```

- e. Form the **W** (weight) matrix of observations errors.

This is a diagonal matrix where the weight of an observation, W_{ii} , is generated from $W = 1/\sigma_i^2$.

The function `diag(p)` turns the **p** vector column into a square matrix with **p** on the diagonal.

`diag(p)^2` squares the error on the diagonal and

`inv(diag(p)^2)` produces the required weight matrix **W** directly.

```
EDU> W = inv(diag(p)^2) | Output format edited for this page.
```

```
W =
 92.4556      0      0      0      0      0      0      0
      0  79.7194      0      0      0      0      0      0
      0      0 108.5069      0      0      0      0      0
      0      0      0 69.4444      0      0      0      0
      0      0      0      0 79.7194      0      0      0
      0      0      0      0      0 129.1322      0      0
      0      0      0      0      0      0 108.5069      0
      0      0      0      0      0      0      0 92.4556
```

- f. Weighted observations in surveying can be reduced to the matrix form:

$$\mathbf{W}\mathbf{A}\mathbf{x} = \mathbf{W}\mathbf{b} + \mathbf{v}$$

W a matrix of **uncorrelated** observation precisions,

A a matrix of coefficients,

x a vector of terms to be computed,

b a vector relating to the observations and

v a vector of residuals, to be a minimum.

- g. To solve the parameters vector, **x**, the matrix form is first normalized:

$$\mathbf{A}^T\mathbf{W}\mathbf{A}\mathbf{x} = \mathbf{A}^T\mathbf{W}\mathbf{b} + \mathbf{v}$$

Calculate the six unknown transformation parameters, x_1 to x_6 being elements *a* to *f*.

The left-division operator, `\`, can solve $\mathbf{A}\mathbf{x} = \mathbf{b}$ for a unique solution ($\mathbf{x} = \mathbf{A} \backslash \mathbf{b}$). It is slightly more accurate, using the LU factorisation approach, and less computationally intensive, than $\mathbf{x} = \text{inv}(\mathbf{A}) * \mathbf{b}$ which is the standard way of solving for **x**.

Consider now our normalised observation equations, using the `\` operator.

For this exercise we will form matrices by solving the normalized **A** and **b** matrices, and assign labels ("names") to the result.

```
EDU> N=A' *W*A; | N = A^TWA | Output suppressed
```

```
EDU> Cb=A' *W*b; | Cb = A^TWb | Output suppressed
```

- h. We now have the solution in the form $\mathbf{N}\mathbf{x} = \mathbf{C}\mathbf{b} + \mathbf{v}$ which leads to the solution of **x** by the left division operator

```
EDU>> x=N\Cb | Equivalent to EDU> x=(A' *W*A) \ (A' *W*b) .
```

```
x =
 25.3715      a
  0.8222      b
-137.1827      c
 -0.8100      d
 25.4020      e
-150.7236      f
```


- i. Now it is time to calculate:
- $\mathbf{b}_{adj} = \mathbf{A}\mathbf{x}$; the adjusted coordinates of the fiducials;
 - $\mathbf{v} = \mathbf{b}_{adj} - \mathbf{b}$; residuals;
 - $\Sigma \mathbf{v}$; the sum of the residuals.

After completing these calculations an analysis of the results will be made.

```
EDU> b_adj=A*x                                     % Adjusted fiducial coordinates

b_adj =
-112.8986      X1
    0.0524      Y1
   -0.0853      X2
   112.9363      Y2
   113.1152      X3
    0.0334      Y3
   -0.0852      X4
  -113.0415      Y4

EDU> v=b_adj-b                                     % Residual = adjusted - true

v =
    0.1014      VX1
    0.0494      VY1
   -0.0863      VX2
   -0.0567      VY2
    0.1172      VX3
    0.0304      VY3
   -0.0862      VX4
   -0.0425      VY4

EDU> sum_v=sum(v)                                   % Sum residuals

sum_v =
    0.0267
```

- j. Analyze the standard deviation of the transformation parameters, the $\mathbf{x}$ vector. Standard error Wolf p212 (12.15).
- Find **reference standard deviation of observations**:
 - $\mathbf{SE} = \mathbf{v}^T \mathbf{W} \mathbf{v}$ the standard error of the observations
 - degrees of freedom = (no. obs- no. parameters) of obs. matrix
 - Reference standard variance of obs, $\sigma_o^2 = \mathbf{SE}/\text{dof}$
 - Reference standard deviation, $\sigma_o = \sqrt{\sigma_o^2}$,

```
EDU> SE=v'*W*v                                     % Standard error Wolf p212 (12.15)

SE =
    4.3656

EDU> [m,n]=size(A);                               % Size of A matrix; m rows, n columns

EDU> dof=m-n                                       % Degrees of freedom

dof =
    2

EDU> Ref_SV=(SE/dof)                               % Reference standard variance of observations

Ref_SV =       $\sigma_o^2$ , Used in the formation of the standard deviation of
    2.1828      the calculated coordinates.

EDU> Ref_SD=sqrt(SV)                               % Reference standard deviation of observations

Ref_SD =
    1.4774       $\sigma_o$ 
```

Broadly speaking the reference standard deviation, $\sigma_o = 1.48$, shows that there is no evidence of gross error in the observations. Schofield & Breach, p251 (7.9.2)

- 2) Find **variance-covariance matrix** of the observations ($\mathbf{VCV}_{\text{obs}}$) from the square root of the elements of the **inverted** weighted set of OBSERVATIONS.
 $\mathbf{VCV}_{\text{obs}} = \sqrt{(\text{abs}(\mathbf{A}^T \mathbf{W} \mathbf{A})^{-1})}$. Remember $\mathbf{N} = \mathbf{A}^T \mathbf{W} \mathbf{A}$ Wolf p224, (13.21)

EDU» format short

EDU» Sqrt_VCV_obs_parameters=sqrt(abs(inv(N))) % VCV matrix

```
Sqrt_VCV_obs_parameters =
    0.0171    0.0015    0.0387         0         0         0
    0.0015    0.0153    0.0378         0         0         0
    0.0387    0.0378    0.1375         0         0         0
         0         0         0    0.0158    0.0026    0.0376
         0         0         0    0.0026    0.0177    0.0424
         0         0         0    0.0376    0.0424    0.1464
```

- 3) Find the estimated standard deviation of the elements of the observed parameters through the expression:

$$S_i = S_o \sqrt{q_{xx}} \quad \text{Wolf p224, (13.24)}$$

EDU» SD_obs_parameters=diag(Sqrt_VCV_obs_parameters*Ref_SD)

SD_obs_parameters =

```
    0.0253    ±σa
    0.0226    ±σb
    0.2031    ±σc
    0.0234    ±σd
    0.0262    ±σe
    0.2163    ±σf
```

- k. Analyze the standard deviation of the adjusted coordinates of the fiducial points, the **b_adj** vector.

- 1) The **reference standard deviation of observations** has been determined

- a) Display it again, σ_o ,

EDU» Ref_SD % Call Reference standard deviation of observations

Ref_SD =

```
    1.4774    σo
```

- 2) Find variance-covariance matrix of the weighted set of **calculations**

$$\sigma^2_{\text{calc}} = \mathbf{A} \mathbf{N}^{-1} \mathbf{A}^T$$

EDU» VCV_b_adj=A*inv(N)*A'; % Variance-covariance matrix of the 8 adjusted fiducials

EDU» Sqrt_VCV_b_adj=sqrt(abs(VCV_b_adj)) % Sqrt of VCV of adjusted fiducials

Sqrt_VCV_b_adj =

```
    0.0895         0    0.0489         0    0.0570         0    0.0489         0
         0    0.0953         0    0.0631         0    0.0462         0    0.0546
    0.0489         0    0.0847         0    0.0526         0    0.0451         0
         0    0.0631         0    0.0992         0    0.0495         0    0.0585
    0.0570         0    0.0526         0    0.0938         0    0.0525         0
         0    0.0462         0    0.0495         0    0.0802         0    0.0428
    0.0489         0    0.0451         0    0.0525         0    0.0848         0
         0    0.0546         0    0.0585         0    0.0428         0    0.0908
```

- 3) Find the estimated standard deviation of the elements of the adjusted coordinates through the expression

$$S_i = S_o \sqrt{q_{xx}} \quad \text{Wolf p2 24, (13.24)}$$

EDU» SD_b_adj=diag(Sqrt_VCV_b_adj*Ref_SD) % Standard deviation of adjusted fiducials

```
SD_b_adj =
    0.1322      ±σx1
    0.1408      ±σy1
    0.1252      ±σx2
    0.1465      ±σy2
    0.1385      ±σx3
    0.1185      ±σy3
    0.1252      ±σx4
    0.1342      ±σy4
```

- l. To calculate the adjusted coordinates of the pass points 10 & 11. We form the observation equations and use the transformation parameters, **x**.

- 1) Form the observation equations for points 10 & 11.

```
EDU» App=[1.746, 9.354, 1, 0, 0, 0      % Observation equation of points 10 & 11
0, 0, 0, 1.746, 9.354, 1
5.329, 9.463, 1, 0, 0, 0
0, 0, 0, 5.329, 9.463, 1];
```

- 2) Calculate the pass point coordinates in the fiducial system.

```
EDU» b_adj_pp=App*x      % Adjusted pass point coordinates of 10 & 11
b_adj_pp =
   -85.1932      X10
    85.4703      Y10
     5.8025      X11
    85.3371      Y11
```

- m. Analyze the standard deviation of the adjusted coordinates of the pass points, the **b_adj_pp** vector.

- 1) The **reference standard deviation of observations** has been determined

- a) Display it again, σ_o ,

```
EDU» Ref_SD      % Call Reference standard deviation of observations
Ref_SD =
    1.4774      σo
```

- 2) Find variance-covariance matrix of the weighted set of **calculations**

$$\sigma^2_{calc} = \mathbf{AN}^{-1}\mathbf{A}^T$$

```
EDU» VCV_b_adj=A*inv(N)*A'; % Variance-covariance matrix of the 8 adjusted fiducials
```

```
EDU» Sqrt_VCV_b_adj_pp=sqrt(abs(VCV_b_adj_pp)) % Sqrt of VCV of adjusted pass points
```

```
Sqrt_VCV_b_adj_pp =
    0.0180      0      0.0106      0
      0      0.0238      0      0.0159
    0.0106      0      0.0114      0
      0      0.0159      0      0.0151
```

- 3) Find the estimated standard deviation of the elements of the adjusted coordinates through the expression:

$$\mathbf{S}_i = \mathbf{S}_o \sqrt{q_{xx}} \quad \text{Wolf p224, (13.24)}$$

```
EDU» SD_b_adj_pp=diag(Sqrt_VCV_b_adj_pp*Ref_SD) % Standard deviation of pass points
```

```
SD_b_adj_pp =      Standard deviation of pass points
    0.1341      ±σx10
    0.1543      ±σy10
    0.1069      ±σx11
    0.1228      ±σy11
```

n. Present the results in a table.

The solution for the adjustment is shown on the right-hand side. Note that the two pass points are now in the camera coordinate frame.

Point	X orig	Y orig	x obs	y obs	σ_x obs	σ_y obs	X adj	Y adj	$\pm\sigma_x$ adj	$\pm\sigma_y$ adj
1	-113.000	0.003	0.764	5.960	0.104	0.112	-112.899	0.052	0.132	0.141
2	0.001	112.993	5.062	10.541	0.096	0.120	-0.085	112.936	0.125	0.147
3	112.998	0.003	9.663	6.243	0.112	0.088	113.115	0.033	0.139	0.118
4	0.001	-112.999	5.350	1.654	0.096	0.104	-0.085	-113.042	0.125	0.134
10			1.746	9.354			-85.193	85.470	0.134	0.154
11			5.329	9.463			5.803	85.337	0.107	0.123

16.7.5 MATLAB Script used for this Affine Adjustment

```
% Affine transformation of 4 fiducial points and two pass points on a photograph
%
% Form of Least Squares solution
%      Ax = b + v
% For weighted matrix:
%      WAx = Wb + v
% Solution of over-determined matrix requires normalising:
%      A'WAx = A'Wb + v
%
% Initialise and form observation equations
x=zeros(1,6)'; % Initialise transformation parameters vector
A=[.764 5.96 1 0 0 0 % Observation equations of fiducials
0 0 0 .764 5.96 1
5.062 10.541 1 0 0 0
0 0 0 5.062 10.541 1
9.663 6.243 1 0 0 0
0 0 0 9.663 6.243 1
5.35 1.654 1 0 0 0
0 0 0 5.35 1.654 1];
b=[-113 .003 .001 112.993 112.998 .003 .001 -112.999]'; % Fiducial coordinates
p=[.104 .112 .096 .12 .112 .088 .096 .104]'; % Sdev of fiducial observations
%
% Form weight matrix
W=inv(diag(p)^2); % Weight matrix W = 1/p^2 on diagonal
N=A'*W*A; % Normalized weighted observations
Cb=A'*W*b; % Normalized weighted fiducials
%
% Solve for transformation parameters
x=N\Cb % Transformation vector using left-division operator
%
% Calculate transformed fiducial coordinates
b_adj=A*x % Adjusted fiducial coordinates
%
% Calculate residuals: adjusted - true
v=b_adj-b % Residual = adjusted - true
sum_v=sum(v) % Sum residuals
%
% Check transformation, is the Reference Standard Deviation acceptable?
% Reference Standard deviation
SE=v'*W*v % Standard error
[m,n]=size(A); % Dimension of A matrix; m rows, n columns
dof=m-n % Degrees of freedom
Ref_SV=SE/dof % Reference Standard variance
Ref_SD=sqrt(Ref_SV) % Reference Standard deviation
%
% Analysis of transformation parameters
VCV_obs_parameters=inv(N) % Variance-covariance matrix
of 6 observed parameters. (N = A'*W*A)
Sqrt_VCV_obs_parameters=sqrt(abs(VCV_obs_parameters)) % VCV matrix of obs;
avoid complex number results
SD_obs_parameters=diag(Sqrt_VCV_obs_parameters*Ref_SD) % Sdev of the observed
transformation parameters
%
% Analysis of adjusted fiducials (8 points)
VCV_b_adj=A*inv(N)*A'; % VCV matrix of the 8 adjusted fiducials
Sqrt_VCV_b_adj=sqrt(abs(VCV_b_adj)) % VCV adjusted fiducials
SD_b_adj=diag(Sqrt_VCV_b_adj*Ref_SD) % Sdev of adjusted fiducials
```

```

%
% Form observation equation for the pass points 10 & 11
App=[1.746 9.354 1 0 0 0          % Observation equation of points 10 & 11
0 0 0 1.746 9.354 1
5.329 9.463 1 0 0 0
0 0 0 5.329 9.463 1];
%
% and transform them into the fiducial system
b_pp_adj=App*x          % Adjusted coordinates of 10 & 11
%
% Analysis of adjusted passpoints
VCV_obs_pp=App*inv(N)*App'; % VCV matrix of the adjusted passpoints 10 & 11
Variance_obs_pp=VCV_obs_pp*Ref_SV          %Variance of 10 & 11
SD_obs_pp=diag(sqrt(abs(Variance_obs_pp))) % Sdev adjusted passpoints 10 & 11
% END

```

16.8 Concluding Remarks

This Chapter presented you with skills and techniques that you can employ to transform coordinates from one system to another through the application of the least squares solution. Other efficient techniques for obtaining the transformation parameters include, e.g., the Procrustes method that solves overdetermined systems in closed form (exact solution, see e.g., Awange and Grafarend 2005; Awange and Palancz 2016). In general, with increased use of GNSS for modern civil engineering and mine operations necessitates that the students be equipped with the knowledge and skills to transform such coordinates into local systems.

An introduction to least squares solutions should help in understanding that no answer is “correct”, but that there are methods used by surveyors to produce the “best” solution.

16.9 References to Chapter 16

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17 APPENDICES

A1 Workshop Materials

Scheduled workshops are designed to introduce the aims, methods, techniques and calculations applicable to each field exercise. Part of the workshop will involve a demonstration of the exercise equipment; its care and handling, set-up, reading and recording, and data recording format. Through attendance at the lecture and workshop, and reading the field practical instructions, you should be able to conduct the field exercises with minimum assistance. Practical time is limited, a thorough understanding of the exercise will allow maximum benefit for the group.

A1-1 Workshop 1 - Levelling

Aim: To introduce differential spirit levelling as a survey technique. During this workshop, students will get the opportunity to engage with the levelling equipment, understand the tasks below and be able to perform the field exercise also described below. The workshop runs for 2 hours. During the first 1½ hours, the students are introduced to the instruments, levelling procedure and reduction of the measurements using rise and fall method. In the remaining 30 minutes, students go to the field where a demonstration of setting out a level and using it is carried out. The students get hands on experience on how to handle the instrument. Once they have attended the lecture and this workshop, the students should be ready to carry out the field practical 1 in Section A2-2.

Class: Differential levelling, booking readings, inverted staff is negative, rise & fall calculation method. Calculation of RLs. Mention height of collimation method, Establishment of temporary bench marks (TBM) for exercise. Change points, loop closure, checks, acceptable misclose, Collimation error theory, test for collimation error, acceptable error (manufacturer). Best practices; 6 rules of levelling, including balanced sights (especially BS/FS).

Field: Automatic level, unpack, set-up, repacking instrument, tripod stability, Instrument levelling using T method across foot-screws, Staff, safety, extension, check joints, verticality, read staff (stadia wires), solid CPs. Booking in the field, equipment record, units of measure (metres), neatness. Concept of Back sight (BS), intermediate sight (IS) and fore sight (FS). Collimation test (two peg test), acceptable errors. Transfer of level via change point (CP). Traverse closure to known bench marks, acceptable errors, open/closed traverse. Field checks of readings. $\Sigma BS - \Sigma FS$, $\Sigma Rise - \Sigma Fall$, $EndRL - StartRL$. Inspect survey area, position of established bench marks (BM), vertical control points.

Method: Set up 6 levels & 4 staffs (one inverted). Each group member read staff 1 (BS) and 2 (FS), establish as CP. Change to another level and read staff 2 (BS), inverted staff (IS) and 3 (FS). Calculate difference in height between all points, check results.

e.g., Figure 17-1. Rise/fall=BS-FS.

$$BS = 1.30, FS = -1.285.$$

$$Rise = 1.3 - (-1.285) = 2.585$$

$$BS = -1.285, FS = 1.30.$$

$$Fall = -1.285 - 1.30 = -2.585$$

Any horizontal line across staves will produce the same result. The structure does not move.

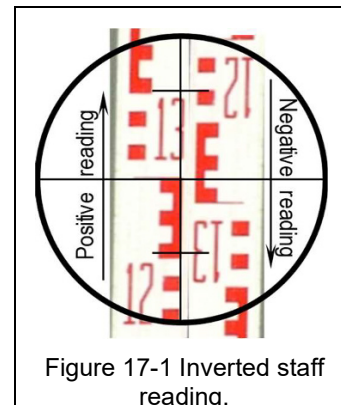


Figure 17-1 Inverted staff reading.

The workshop proceeds in steps as follows (Chapter 2 Levelling) :

Step 1: Introduce the students to the levelling equipment below.

Step 2: Introduce students to various parts of the Level and the cross-hairs.

Step 3: Show the students how to read a staff, Figure 17-2.

Step 4: Demonstrate on the board how a two-peg test is undertaken, Figure 17-3.

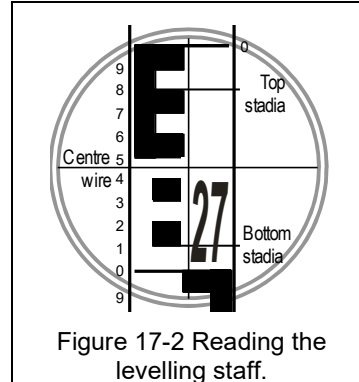


Figure 17-2 Reading the levelling staff.

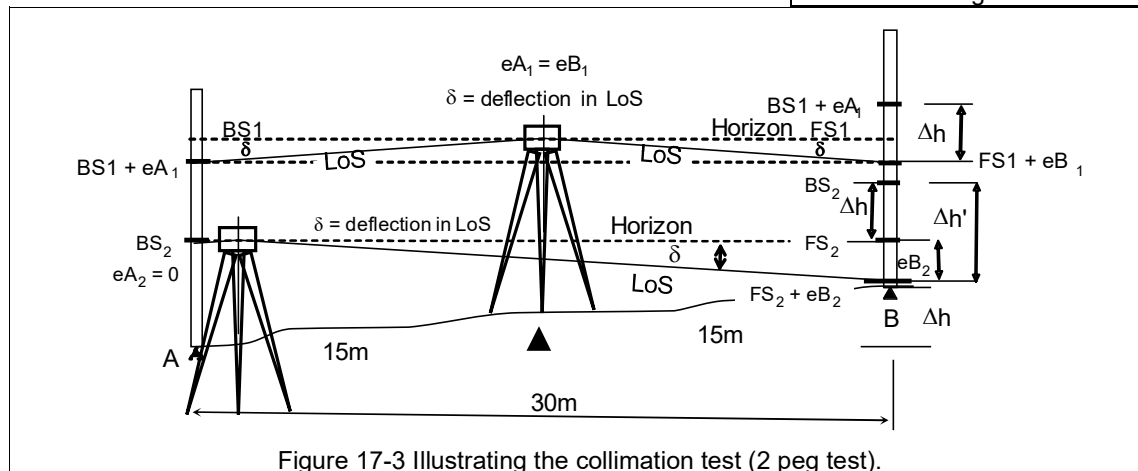


Figure 17-3 Illustrating the collimation test (2 peg test).

Step 5: Demonstrate the levelling procedure (see §2.5) using a level and staff.

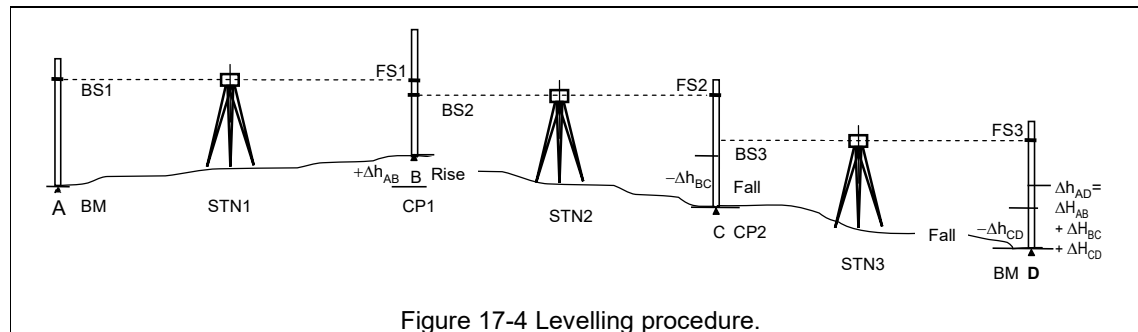


Figure 17-4 Levelling procedure.

Step 6: Perform a general field reduction example based on rise and fall on the board for the students. After that, let each student perform the following reductions bases on rise and fall method. Include the inverted staff (see

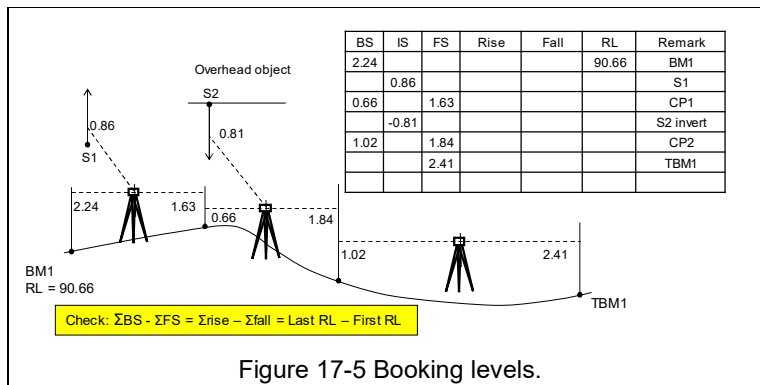


Figure 17-5 Booking levels.

Figure 17-1). The lecturer can then go around and check the performance of each student and assist where necessary. After all the students have completed, the lecture auto-populates the values on the board based on the student's answers. In case of students who are still not to task with rise and fall, the lecturer/tutor should identify them for follow on so as to give them private tutoring. Following this step, the students should proceed to the 30 minutes field demonstration where a tutor has assembled the equipment ready for the demonstration. Students are encouraged to do further readings from books such as Uren and Price (2010).

A1-2 Workshop 2 – Earthworks: Relief Representation and Vertical Sections

This workshop aims at enhancing the students understanding of the materials discussed in Chapter 3 Relief and Vertical Sections (page 46). The students should therefore consult Chapter 3. The materials can be presented in the manner discussed below.

Aim: Use area levelling to generate a digital terrain model (DTM). Understand specifications of design surface parameters. Extraction and calculation of combined DTM and design surface data. Horizontal and vertical scale. Vertical scale exaggeration. Feature measurement, extraction and plotting. Reporting.

Class: Point positioning methods: random, grid, cross section. Equipment, granularity. Recording observations on field sheets. Feature position measurement techniques; chainage/offset, radiation. Plotting DTM, extraction of vertical height differences. Calculation of survey longitudinal profile and of design surface profile, (longitudinal elevation). Calculation of surface cross sections profile, slope extraction. Extraction of height differences between survey and design profile, discuss design cross section parameters; formation width, batter slope. Calculation batter intercepts, cross section area, from height differences. Calculate longitudinal volume from cross section area, volume by end area versus Simpson's rule. Plot longitudinal terrain and design surface profile to scale. Plot cross sections to scale, Plot plan of area and features to scale. (Trees, pathways, walls, appurtenances.)

Field: Inspect area, define limits. Original BM not accessible. Confirm previously established TBMs. Inspect established longitudinal centre line (CL) control points (CH00, CH93.3) Demonstrate establishment of cross section profile control. Booking in the field, equipment record, units of measure (metres), neatness. Sketch of area. Recording of cross section and profile levels as intermediate sights (IS). Use of CL control points as change points (CP). Field reduction of RLs. Closure of level loop to established TBM, misclose checks. Feature positioning by chainage and offset, recording in FB. Vertical measurement of trees by clinometer and tape. Vertical measurement of trees by clinometer and tape (see §2.6.5).

Method: Provide 30 m tapes, clinometers, flagging pins for demonstration.

Discuss team management, division of tasks. Concurrent tasks

A1-3 Workshop 3 – Total Station and Angular Measurements

This workshop aims at enhancing the students understanding of the materials discussed in Chapter 4 Total Station: Measurements and Computations. The students should therefore consult Chapter 4. The materials can be presented in the manner discussed below.

- Aim:** Introduce the students to a Total Station, the instrument used to measure angles and distances. This workshop will also aim at facilitating the understanding of bearings and how they are computed. Units of distances and angles will also be given.
- Class:** Use of Total Stations to measure angles and distances. Recording observations on field sheets. Computing bearings. Field demonstration.
- Field:** Setting out and levelling a Total Station. Measure angles and distances. Establish control and orientation to be used in practical A2-3-1.

The workshop is to be presented in 4 steps as follows:

- Step 1:** Demonstrate to the students in class the Total Station measuring procedure of face left (FL) and face right (FR).
- Step 2:** Discuss with the students the properties of angles, horizontal directions and bearings.
- Step 3:** Introduce the two bearing computation methods (i) deflection angle method and (ii) the back-bearing method. Use can be made of a hypothetical triangle before the students are given the examples in the slides below to compute individually. After all the students have completed, the lecture auto-populates the values on the board based on the students' answers. In case of students who are still not to task with rise and fall, the lecturer/tutor should identify them for follow on so as to give them extra tutoring.
- Step 4:** Once the students have grasped the bearing computation skills, in the remaining half an hour, they should proceed for a field demonstration by the tutor on how to use set up, level and use the Total Station to measure angles and distances.

Workshop slides:

Units of measurement.

Metre, imperial foot, US survey foot, chains & links.

Conversions: 1 foot = 0.3048m, 1m = 3.28084ft = 39.37007874 inches

1chain = 66 feet, 1chain = 100links. 1link = 0.201168m.

1 US survey foot = 1200/3937m (defined, used in some States Plane Coordinate Systems).

Angular measurement.

Circular; (radians), plane; (degrees/grads/mils).

Units of angular measurement.

Degrees, sexagesimal system: 1 circle = 360°, 1° = 60', 1' = 60".

Other units of angular measurement.

Grad/gon, decimal system: 1 circle = 400g, (= 360°),

mils, military system: 1 circle = 6400 mils, 1mil = 3' 22.5", approx. 1/1000 radian.

Units of area measurement.

Metric (metres², hectares), imperial (Acres, ac.; roods, r; perch, p.)

Conversions: 1 are = 100m², 1 Ha = 100are = 10,000m²

10000m² × 0.3048² sq feet = 107,639.104ft²

1 acre = 10sq chains = 66 × 660 ft = 43,560sq ft. = 43560 × 0.3048² = 4046.856m²

1Ha = 107639.104/43560 = 2.471acres (2ac. 1r. 35p.)

Angle calculations:

Forward and back bearings; interior/exterior angles, deflection angles, addition, subtraction, clockwise positive/ anticlockwise negative.

Bearings and angles, close angles to check bearing.

Coordinate calculations:

Polar to rectangular: Brg (θ), distance (S) to dE, dN,
 $dE = S \sin(\theta)$, $Dn = S \cos(\theta)$.

Rectangular to polar: dE, dN to Brg (θ), distance (S)

$S = \sqrt{(dE^2 + dN^2)}$, angle = $\text{Atan}(dE/dN)$, Brg (θ) = angle + quadrant.
HP10S+, POL(, REC(, functions. (Appendix A5-3)

A1-4 Workshop 4 – Traversing

This workshop aims at enhancing the students understanding of the materials discussed in Chapter 5 Traversing. The students should therefore consult Chapter 5. The materials can be presented in the manner discussed below.

Aim: Use a Total Station to establish horizontal controls. In this regard, the aim is to develop good observational and computational skills to measure distances and angles in order to establish the easting and northing coordinates of points that are to be used for other engineering tasks.

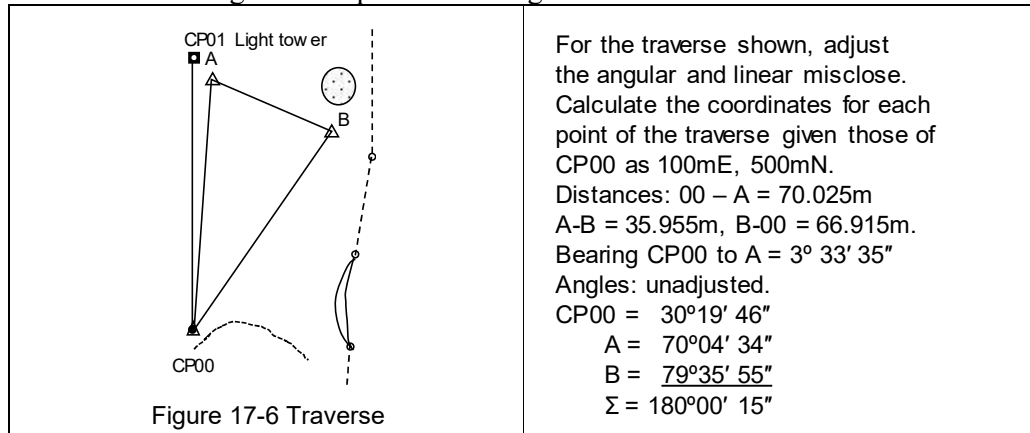
Class: Point positioning methods: Use of Total Stations to measure angle and distances. Recording observations on field sheets. Balance the angles. Compute the bearings. Perform the Bowditch adjustment to correct for the traverse misclose. Compute the coordinates of given points (stations). Compute the area using coordinates method.

Field: Occupy your station established in practical A2-4. Orient to a given reference. Undertake feature pick up using the Total Station.

The workshop is to be presented in 2 steps as follows:

Step 1: Demonstrate to the students in class the traverse procedure.

Step 2: Let the students independently compute the Bowditch adjustment, Figure 17-7, of the traverse network in Figure 17-6, and the subsequent area using the coordinates method. Note, this workshop builds onto workshop 3; hence the students should be able to compute the bearings. After all the students have completed, the lecture auto-populates the values on the board based on the students answers. In case of students who are still not to task with the Bowditch adjustment, the lecturer/tutor should identify them for follow on so as to give them private tutoring.



Line	Forward Bearing	Horizontal Distance	Calculated		Adjustments		Coordinates		
	Adjusted		ΔE	ΔN	δE	δN	E	N	
CP00							100.000	500.000	CP00
00-A	3 33 36	70.025							A
A-B	113 29 06	35.595							B
B-00	213 53 16	66.915							CP00
	Sums								

Figure 17-7 Bowditch adjustment exercise.

Discuss team management, division of tasks. Concurrent tasks

A1-5 Workshop 5 – Horizontal Curves

Chapter 10 Circular Curves covers the theoretical and practical aspects of horizontal circular curves that form the last practical exercise of the semester.

Calculating a circular curve. Definitions and formulae. Refer to Section 10.2

Recall, using: Figure 17-8, Figure 17-9 and Figure 17-10.

Definitions:

- (i) AB is a simple curve of radius R connecting two straights AE and BF
- (ii) A and B are the tangent points and the straights are produced to meet in D (the intersection point, IP, or point of intersection, PI).
- (iii) AD, BD are tangents to the simple curve and $AD = BD$.
- (iv) Δ is the deflection angle, both at the centre and the intersection point.
- (v) The deflection angle (Δ) equals the angle at the centre, $\angle ACB$
- (vi) Triangles ADC and BDC are congruent and $\angle ACD = \angle BCD = \Delta/2$

DIRECTION of RADIUS is PERPENDICULAR to TANGENT.

Brg Radius = Brg Tangent $\pm 90^\circ$

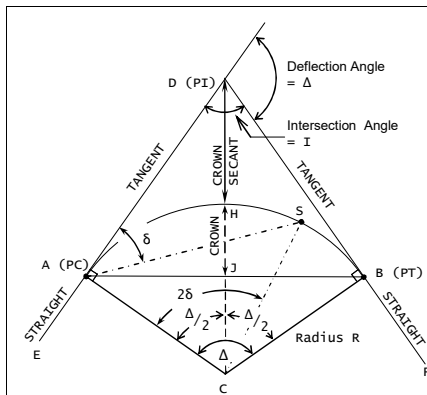


Figure 17-8 Geometry of the curve.

Curve formulae.

- (i) Tangent Distance AD. $TD = R \cdot \tan \Delta/2$
- (ii) Radius (R). $R = \text{Tangent Distance (AD)} \cdot \cot \Delta/2$
($\cot = 1/\tan$)
- (iii) Arc deflection angle Δ , $\Delta = s/R$ (s = any arc length)
- (iv) Arc length (AB). $S = R \cdot \Delta$ (Δ measured in radians)
- (v) Secant Distance (DC) = $R \cdot \sec \Delta/2$ ($\sec = 1/\cos$)
- (vi) Crown Secant (DH) = $R \cdot \sec \Delta/2 - R = R \cdot (\sec \Delta/2 - 1)$
- (vii) Crown Distance (JH) = $R - R \cdot \cos \Delta/2 = R \cdot (1 - \cos \Delta/2)$
 $= (R - \text{Vers } \Delta/2)$

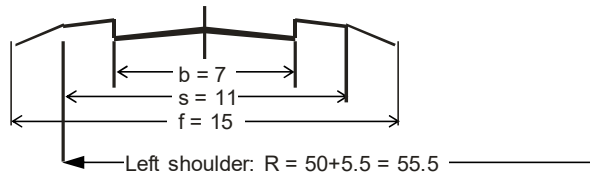


Figure 17-9 Formation definitions. Pavement, b; shoulder, s; formation, f.

	CURVE	EX2	LEFT SHOULDER
Initial direction of tangent	Brg IP =	360	Design TD = 38.86
Chainage PC (TP1)	CH =	511.00	CH IP = 549.86
Coordinates of PC (TP1)	E =	1050.000	Long chord = 63.67
Coordinates of PC (TP1)	N =	5511.000	Arc length, S = 67.81
Radius design Centre line	R =	50	Secant distance = 67.75
Design deflection angle Δ	Δ =	70	Crown secant = 12.25
Through chaining interval	s =	10	Crown distance = 10.04
Centreline offset	o =	0	CH PT (TP2) = 578.81
Pavement width	b =	7	IP E = 1044.50
Shoulder width	s =	11	IP N = 5549.862
Formation width	f =	15	

Figure 17-10 Example curve for solution, curve EX2.

A2 Field Practical Materials

A2-1 Instructions for Field Practical Sessions

All field practical sessions are conducted with a supplied set of “task specifications” which will include a version control. These specifications can change annually and supersede examples contained in this document. Using the correct control version of a document is basic to quality control of a task. The documentation will generally be supplied electronically on your LMS.

A2-1-1 Introductory Remarks

Field practical sessions are designed with staged specific tasks to be undertaken within the overall objectives of the unit.

Objectives The field practical exercises are designed to enable an engineer to undertake surveying tasks applicable to the exploration, measurement, design and layout of points over a project field area. The resultant information will enable the engineer to integrate field data into terrain models developed in Computer Aided Design (CAD) packages.

Learning Outcomes By the end of the semester the successful engineer will be apply basic surveying techniques to:

- position points on a plane surface;
- use modern surveying equipment such as:
- automatic level, Total Station and GPS;
- carry out team based survey tasks;
- record field data in a specific format;
- perform field reductions and checks of recorded data
- produce field maps and reduce data for CAD systems;
- communicate results to client through written reports.

Workshop sessions are conducted to demonstrate the methodologies, calculations and field equipment to be used in each specific task. It is thus essential to attend the workshops so that the field practical can be undertaken with a full understanding of the task.

A2-1-2 Conduct of Field Practical Sessions.

Within a field practical class, students will form groups of 3 (maximum 4) members and will remain in that group for the duration of the unit.

Each group member is expected to contribute fully to each exercise by participating in the observation, data recording (booking), equipment manipulation (survey assistant), checking and reduction of data. Each group member will be expected to submit an entire report (individual submission) or contribute equally to preparation of a group report.

For each field exercise the group should nominate a leader. The leader has the responsibility of collating and distributing recorded field data in a timely manner so that each member can carry out the required checking, reduction and calculation of field data. For a group submission the leader will be responsible for the submission of the report to the unit supervisor by the due date.

For individual submissions each team member is to ensure they have all the recorded field necessary for preparation of their report, submitted individually by the due date.

Location of Field Exercises and Survey Control.

Within unit outcomes CVEN2000 integrates field surveying and CAD competence. To this end Field Practicals 1 and 2, Area Levelling, and Field Practicals 3 and 4, 3D Surveying and Control, of the survey component will be undertaken in the treed area of the campus between buildings 204 (Engineering) and 105 (Library).

The practical field examination will be conducted on the Edinburg Oval (South).

Survey control is provided by existing bench marks and control points adjacent to the exercise area. Data will be provided for each exercise.

Establishment of control points is a vital task in every job. Additional temporary field control values will be established by each group to either installed points, known to the supervisors, or nominated points as required for the task.

Control and Topographic Survey.

Task 1. Determinations of Vertical Control/Vertical Sections (Field Practical 1 and 2):

Field Practical 1 and 2 will use an automatic level and staff to provide vertical control and ground profile levels. Other topographic data will be measured by use of tapes and clinometer.

Task 2. Determination of Horizontal Control/Digital Terrain Model (Field Practical 3 and 4):

Field Practical 3 and 4 will use a Total Station, prism sets and tapes for the measurement of position and level of control points and topographic features.

Field Examination.

Establishment of Horizontal Curves (Field Practical 5: - Field examination):

The field examination will incorporate survey control and curve chainage points set-out using GPS, curve chainage points set-out using a Total Station. Set-out will be checked using measuring tapes

Optional Assessment (Individual Practical 6: - Assignment):

An optional assessment may be available to students who wish to replace a poorly assessed field practical with a better mark.

The optional assessment will comprise calculation of a determined survey task similar to other field practicals without the need for field work.

A2-1-3 Field Checks

The following checks are to be performed and recorded in the field directly in the field book.
Automatic level and staff

- Two peg (collimation) test,
- Staff systematic error from wear.

Total Station

- Check plate level (bubble),
- Check optical plummet.

Observations for survey control

- Differential levelling check sums and traverse misclose,
- Mean horizontal directions and vertical angles on Total Station (FL/FR),
- Mean slope distances,
- Prism constant.

Observations for topographic pick-up

- Backsight direction applied to Total Station on FL,
- Close of level traverse to established control.

A2-1-4 Computations and Plan Drawings

All computations can be carried out on your HP10s+ calculator. Competency in its use in surveying is essential. The HP10s+ is the only calculator allowed in the final examination.

Appendix A5 has been written for the HP10s+. It is an expansion of the User Guide to incorporate routines appropriate to the computational requirements of the course.

Drawings to nominated scale must be submitted. Neat, inked drawings with title block and scale are expected. Drawing scale is such that they fit on an A4 sheet.

CAD drawings, on A4, may be submitted. They must be to scale and correctly annotated. Calculation by the CAD program must be confirmed by hand calculations.

Computations and drawings will be checked by the supervisors, who will have “model” answers. Submissions will be checked against your field book data.

A2-1-5 Equipment

Exercise field equipment will be issued from the Survey Store (206.150) at the start of the practical period. You should check that you have received all the equipment you signed for. Equipment will be checked on return to the store at the end of the exercise.

Any damage to the equipment, or deficiencies, must be reported to the field supervisor and/or the survey technical officer.

A2-1-6 Hygiene, Safety and Dress

Observe personal hygiene, do not litter in the field.

Personal safety.

At all times you are expected to work in a safe manner, and to respect the safety of others.

Hazards.

Potential equipment hazards, and safe handling procedures, will be part of a briefing.

Be safety aware.

Footwear.

Enclosed footwear is mandatory. Safety footwear is encouraged as it will be needed when you work on an industrial site.

Clothing.

Hats, long sleeved shirts and long trousers are encouraged for sun protections. Sunscreen lotion is available at the Survey Store.

Traffic areas.

The exercise areas are common areas with other members of the University. Exercise care when measuring across pathways and pedestrian access areas.

Be aware of, and give way to, vehicle movements on roadways adjacent the exercise area. The field task on Edinburgh Oval (South) is kept within the confines of the oval.

Apart from crossing Dumas Road, do not survey on the road reserve outside the wooden posts.

A2-1-7 Workshops

Scheduled workshops are designed to introduce the aims, methods, techniques and calculations applicable to each field exercise. Part of the workshop will involve a demonstration of the exercise equipment; its care and handling, set-up, reading and recording, and data recording format.

Through attendance at the lecture and workshop, and reading the field practical instructions, you should be able to conduct the field exercises with minimum assistance. Practical time is limited, a thorough understanding of the exercise will allow maximum benefit for the group.

A2-1-8 Field Practical Instructions

The instructions should be treated as a work order from your senior engineer. The results from your exercises must be treated as the basis for the preparation of a tender document, the awarding of which may impact on your continued employment.

A2-1-9 Field Books and Data Recording

The **original** field book and field sheet observations are the provable records of group work. Each group member must sign the record book for each exercise, (no sign = no mark), task, references and equipment used must be recorded, all observations must be recorded **without** erasure, field drawings neat and legible, errors are ruled out and corrections recorded.

Use a fair copy of the field book with your work. The **original**, as submitted with your reports, is the one evaluated for group marks.

Additional field sheets will be loaded to Blackboard with the exercise field task description and are to be incorporated with the group field book

A2-2 Establishment of Vertical Control and Vertical Sections for a Construction Site

A2-2-1 Field Practical 1: Establishment of Vertical Control

Motivation: In your profession as a civil engineer, dealing with heights of points on topographic surfaces will be a daily routine. Whether you are constructing roads, buildings or in mining, you will need some height information. The term given to these heights with respect to some known Benchmark is Reduced Levels (RL).

Aims (1) Establishment of vertical control for construction site.

(2) Design a road/drainage to serve a constructed Computer Building.

Task: Establish vertical control for construction site.

Duration: one practical session of two (2) hours.

In this exercise, **Civil Engineers** are required to establish the RLs of CH00, TMB1, TBM2, deck spike D/S134, 705A, CH93.3 (705) and C400 marked on the attached diagram (Figure 17-11). This type of survey is necessary when, for design work, levels of points are required such that floor levels can be made compatible with new works.

Mining Engineers will establish vertical control for a blast pattern shown in Figure 17-12

Outcome: Following the lecture, workshop and this practical, you are expected to be able to:

1. Set-up the levelling instrument in a suitable location; stable and convenient height, reticule focussed (eyepiece focus).
2. Observe and record staff readings. Use correct object focussing and removal of parallax against reticule. Make careful interpolation of millimetres between staff graduations.
3. Carry out a levelling exercise.
4. Perform the reductions necessary to obtain RLs from levelling measurements.
5. Perform all the necessary checks pertaining to levelling.
6. Know the possible sources of errors in levelling and how they could be minimized or avoided.

Group: 3-4 students per group.

Equipment:

- 1 automatic level and tripod;
- 1 staff, staff bubble and change plate.
- 2 pegs, hammer.

Field Work:

1. Each group member will be expected to use the level to observe portions of the exercise, and have those observations recorded in the group field book.
2. Start by undertaking a **two-peg** (collimation check) test. This will help you in checking the vertical collimation of the level. Use the provided levelling staves that have been set up for the session.
3. Each group member will make, record and reduce independent observations.
4. Use the rise and fall method of booking.
5. Read the staff carefully. Pay particular attention to focussing and ensure you read the middle wire.
6. Initial open traverse to D/S134. Start the levelling by observing **BM8** (RL 12.418m) as the backsight.
Proceed with levelling through the points in the following order: BM8, CH00, 2 and 3, then a change point, CP1, to deck spike D/S134.
7. During the initial open traverse check path clearance under access walkway to Building 204. Before leaving your first set-up, take two intermediate shots; one underneath the suspended walkway to building 204, i.e., inverted reading; the other immediately below.

This will allow calculation of clearance. Your tutor will show you these points, which you should name 2 and 3.

8. Closed traverse. Restart the levelling by observing **BM8** (RL 12.418m) Proceed with levelling through the points in the following order: BM8, CH00, TBM1, 1, D/S134, TBM2, 705A, CH93.3 (705), C400 and **BM7** (i.e., close the traverse at the second bench mark, BM7 of RL 13.419m). RLs of CH00 and CH93.3 (705) are required for Field Practical 2, Task 1. RLs of deck spike D/S134, 705A and C400 are required for Field Practical 3, Task 2. Mining engineers will perform a limited closed traverse from C954, D/S134, nominated CEM peg, to C153 to establish a blast grid for Field Practical 2.
9. Enter the observations obtained on the attached booking sheet,
calculate the reduced levels RL using the rises and falls method.
Apply all checks.

Misclosure should be less than ± 0.015 m.

Remember the **six** rules of levelling:

1. Instrument properly levelled, pendulum free; check circular bubble, pendulum “tap”.
2. Backsight distance approximately equal to foresight distance;
3. Instrument, staff and change plate stable. Staff kept on change plate.
4. Staff vertical; check staff bubble is centred.
5. Remove parallax, **read centre wire**; sharp coincident focus of reticule and object.
6. Close the traverse.

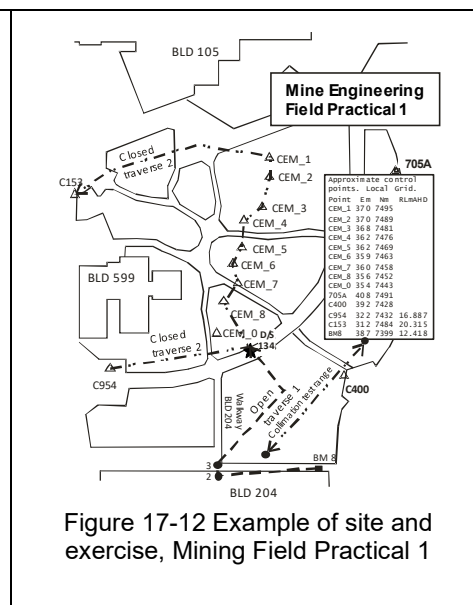
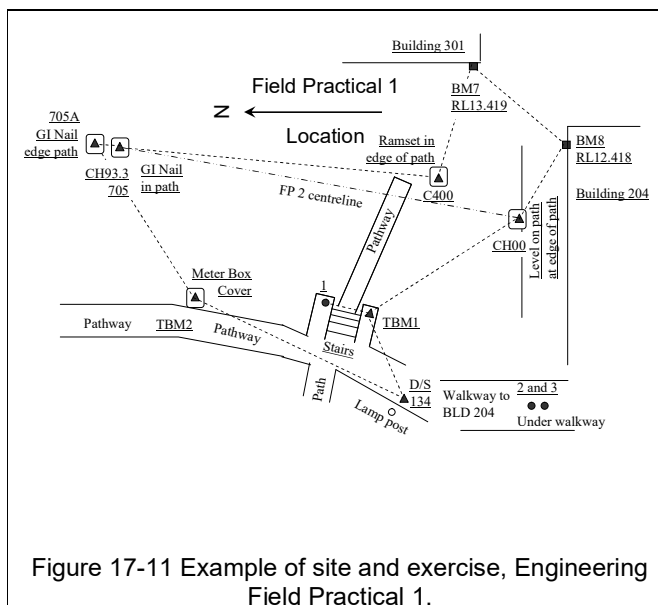
Requirements: On completion of Field Practical 2, area levelling, each person is to submit an individual report including these Field Practical 1 observations:

1. A copy of your group’s field notes
2. Individually **calculated** Height of Collimation test (from the two-peg test),
3. Individually **calculated** RLs of **ALL** points, including checks.
4. Statement of **conformity to required misclosure.**

Note: Field notes will be assessed on the neatness of booking and correctness of recording.

Whereas you may share field notes, the REDUCTIONS and REPORT MUST BE DONE INDIVIDUALLY BY EACH GROUP MEMBER. ANY SORT OF JOINT WORK WILL BE CONSIDERED AS **PLAGIARISM**.

Submission Date: Combined with Practical exercise 2, area levelling, individual submission is required by the commencement of practical 3.



A2-2-2 Field Practical 2: Vertical Sections

Task: Measure area grids to serve a constructed Computer Building or mine site.

Duration: one practical session of two (2) hours.

Undertake levelling of cross sections to support a road/drainage or other design requirements using gridded points to allow preparation of nominated longitudinal and cross sections profiles. Pick up detail of trees and pathways to allow nomination of arboreal and pedestrian disturbance. Calculate volumes, draw plans.

Outcome: Following the lecture, workshop and this practical, you are expected to be able to:

1. Set out a nominated survey grid for section observations.
2. Carry out differential levelling survey over a gridded area.
3. Check levels against control and complete survey to nominated accuracy.
4. Measure the location and height of trees, obstructions and adjacent structures (sumps and paths) within the survey area.
5. Calculate all levels and provide checks.
6. Calculate excavation batters, cross section area and excavation volume to design surface.
7. Draw plans and sections to illustrate report.
8. Present a report on Task 1.

Group: 3 - 4 students per group.

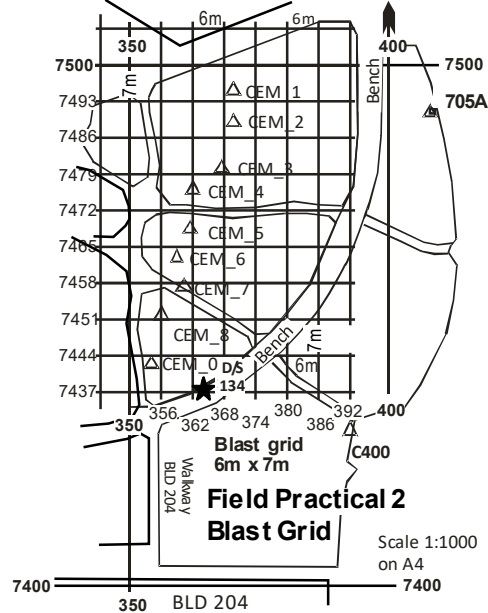
Equipment:

- 1 automatic level and tripod,
- 1 metric staff and staff level and 1 change plate,
- 1 30m tape,
- 1 set chaining arrows,
- 1 optical square, 1 Clinometer.

Comments:

1. The vertical control for the exercise will be that which you established during Laboratory 1, at CH00, TBM1, 1, TBM2, and CH93.3 (705), or the various CEM_8 to CEM_1 points.
2. For the **Civil Engineers** this exercise you will be using a 90m section of centre line which will be pegged or otherwise marked at 15m intervals (see Figure 17-13). Temporary bench marks have been established during Task 1 at the 00 chainage (CH00) on the centreline and at the nominal centre line chainage of 93.3m (CH93.3); this point is located outside the work area on the edge of the brick pathway.
3. Cross section points will be marked at 5m intervals either side of the centre line at each long section 30metre interval. i.e. CH00, CH30, CH60 and CH90.

4. **Mining Engineers** will produce blast grid RLs for investigation in a GIS package (Figure



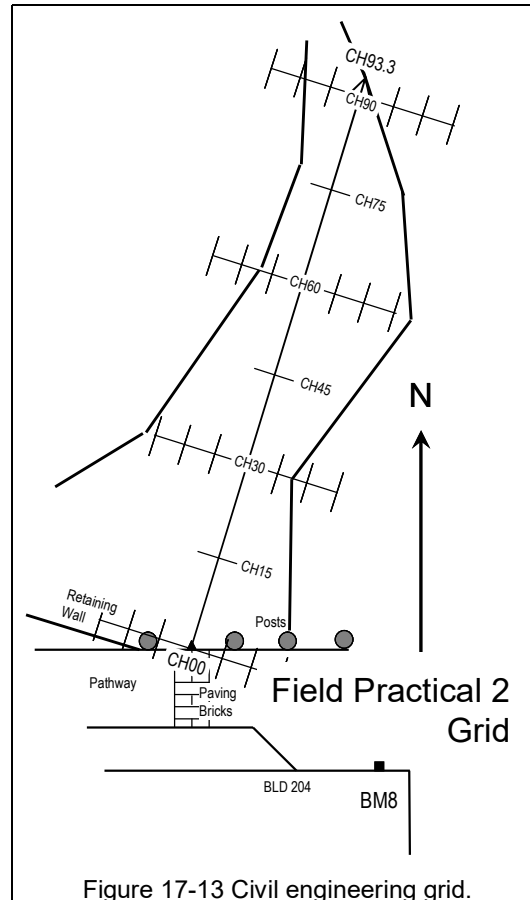
- 17-14).
5. Blast grid intersections will be based on nominated origin coordinates, set out as specified, nominally a $6\text{m} \times 7\text{m}$ grid.
 6. Most required observations will be intermediate sights.
 7. See lecture notes on the presentation of the profile and cross-sectional data.
- Refer pages 38 – 44 of Irvine & Maclellan, or similar in Uren & Price
 Irvine, W and Maclellan, F Surveying for Construction, 5th Edition, McGraw-Hill.
 Uren, J and Price, W F (2006) Surveying for Engineers, 4th Edition, Macmillan.

Field Work:

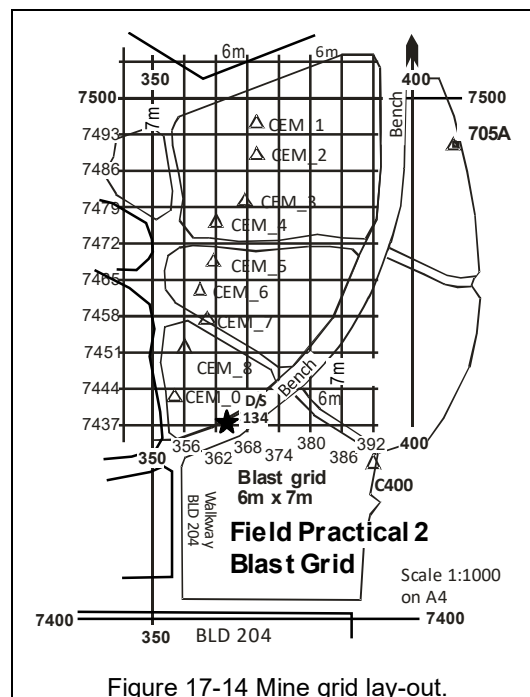
Print your own copies of levelling and pick-up sheets. These must be submitted with the original field book.

Civil:

1. Centre line pegs will be provided at 15m intervals, or the mine origin will be marked.
2. Share with other groups in marking 5, 10 and 15 metre offsets either side of the centre line to the limit of works on the 00m, 30m, 60m and 90m chainages.
3. Use any of your established benchmarks as a check vertical control. Close the circuit CH00 – CH93.3 to at least one other bench mark. Level along the centre line at 15m intervals. The level at CH00 is on the existing path. Simultaneously level the 5, 10 and 15 metre offsets either side of the centre line on 00m, 30m, 60m and 90m chainage.
4. Civil groups should measure one representative pine tree using the tape for circumference and the use the staff, tape and **clinometer** to determine economic height.

**Mining:**

1. Share with other groups in marking the 7m offset north grid South from the 7500N origin at 6m intervals from 350E to the limit of works (bench level).
2. Use an assigned CEM bench mark and close to D/S134 as a check. As directed, level across the site's North grid line at 6m intervals.
3. Pick up all trees, pathways and major appurtenances within the work area by longitudinal chainage and tape offset. Pick up sufficient detail of pathways etc. to provide for a site plan of the area for presentation to the client.



Submission of report.

The report is to be in a format suitable for submission to the supervising engineer.

Group submission of field observations, Field Practicals 1 and 2. (5 marks)

Individual submission addressing the following points: (20 marks)

1. Reduce levels by the Height of Collimation method, applying all checks, including confirmation of group work in Task 1.
2. **Civil:** Plot the longitudinal section using a horizontal scale of 1:400 and a vertical scale of 1:20, including the surface profile and the design pavement.
3. The drainage trench has a **pavement** width of two (2) metres and batters of 1 in 3 which will be suited to unsupported excavation. Assuming a design surface rising grade of **1:40** from a level **-0.50m** (0.50m below) at CH00 towards CH93.3, calculate the grade finish level and the batter intercepts with the natural surface.
 Plot cross-sections using a horizontal scale of 1:200 and vertical scale of 1:20. Show this design on each of the four cross-section sketches.
 Calculate the area of each cross section. Calculate the volume of excavation using the end area method.
 Draw a plan of the area showing the centreline, offset points and levels, trees, pathways and batter intercepts with the natural surface. Plan scale 1:400.
 Determine the trees to be removed for the earthworks. Calculate the volume of tree for sale. Determine total volume of saleable timber.
 A good copy of the original field notes. This to include the collimation checks and level control traverse from Task 1.
 Reduced levels, all plots and a discussion on the reasons for any difference in the centre line levels.
4. **Mining:** Prepare a table of 3D coordinates, E, N and RL, for input to a GIS.
 Plot cross-sections using a horizontal scale of 1:500 and vertical scale of 1:20. Draw batters of nominated slope. See Section 3.8.
 Calculate the area of each cross section. Calculate the volume of excavation using the end area method.
 Draw a plan of the area showing the blast grid and levels, trees, pathways Plan scale 1:500.
 A good copy of the original field notes. This to include the collimation checks and level control traverse from Task 1.
 Reduced levels, all plots and a discussion on the reasons for any difference in the centre line levels.

DETAIL PICKUP SHEET

Date:.....
Group:.....
Job:.....

Left side features	Chainage	Right side features

Example of a Marking Guide

Name _____ ID _____ Submission _____ date _____

Marking Sheet: Class _____ Group _____ Tutor _____

Task 1. Profile Levelling & Detail Pickup. Individual Mark (/25)

Original Field Notes: Date, Group members, Equipment, S/N Task methods (R/F and checks). Neatness of booking Correctness of recording	5 (group)	FB for practical 1 and 2, level records, pick-up sheet Comment
Submission: 1 - Reduction of level sheet to produce control points and profiles. 3mks 2 – Longitudinal plot to correct scale 2mks 3 – Plot cross sections, to correct scale 2mks 4 – Cross section area, volume. 3 mks 6 - Plan of area; features, trees, path, batter line, to correct scale 3mks 7 - Tree numbers, selected tree calculations, harvest volume 2mks	15 (individual)	Including results from Field Practical 1. Comment
Presentation: Language, 1mk Complete discussion of the work, misclose, specifications achieved etc 3mks, Visual quality 1mk.	5 (individual)	Report in suitable format to project engineer. Comment

Submission of Practical 1 report. Combined Field Practicals 1 and 2.

The report is to be in a format suitable for submission to the chief engineer including:

Group submission of field observations (Field Book), Field Practicals 1 and 2.

Individual submission addressing the following points:

The submission of a report on the field practical session should present the exercise as a report to your field supervisor. You can thus assume that the supervisor is competent in field surveying. Thus, the report does not need lengthy explanations of methods, procedures and calculation used to determine results.

However, the supervisor will be keenly interested as to whether your task has been accomplished within the required accuracy. Calculations will be checked.

You should start with:

a signed cover sheet describing the task, the dates and the team members:

a short “executive summary” of the results:

- summary of task;
- description of site;
- calculated volumes;
- calculated tree removal;
- accuracy achieved for any level traverse between control points, instrument compliance with task specifications, -were specifications met, and if not, how were they minimised, effect of inaccuracies.
- any site-specific structures likely to hinder or impact on the project.

The report itself should table the following material. You should use and submit the Summary of calculated values to check that you have made the required calculations.

1. Reduced levels calculated by the Rise and Fall method, showing all checks, including confirmation of group work in Field Practical 1.
2. Calculated centreline grade finish levels at each 15m chainage
3. Calculated batter intercepts at each 30m cross section with the natural surface.
4. Calculated area of each cross section.
5. Calculated volume of excavation using the end area method.
6. Plot and label the longitudinal section of the centreline and grade line using a horizontal scale of 1:400 and vertical scale of 1:20.
7. Plot and label the cross-sections using a horizontal scale of 1:200 and vertical scale of 1:20.

Show this design on each of the four cross-sections.

8. Draw and label a 1:400 scale plan of the area showing the centreline, cross section points and levels, trees, pathways and batter intercepts with the natural surface. Show any impediments in the works area that may impact on the task.
9. Determine and indicate the trees to be removed for the earthworks. Calculate the volume of a representative tree for sale. Determine total volume of saleable timber.
10. A good copy of the **original** field book (from group submission). This to include the level collimation (two peg test) checks and level control traverse from Field Practical 1.
11. Reduced levels, all plots, and a discussion on the reasons for any difference in the centre line levels closure.
12. A brief discussion of any problems encountered and methods used to rectify or minimise them.

A short conclusion should précis your exercise results.

Calculation sheets and methods should be included in an appendix.

Task 1. Summary of calculated values, Field Practical 1. Individual Report									
Name:		ID:			Sign:				
Observation date:				Observation Time:					
Class:		Group:		Submission date:					
From FIELD BOOK and calculation, provide the following information. You used the control points BM8 and BM7.									
BM8 RL				BM7 RL				BM7 observed RL	
Collimation test, Error				Level traverse BM8 to BM7. Misclose: Accept/Reject?					
RL CH00:			RL TBM1:			Portal clearance: Pass/Fail?			
RL: Point 1:			RL D/S134:			RL TBM2:			
RL 705 (Ch93.3)			RL 705A:			RL C400:			
Task 1. Summary of calculated values, Field Practical 2. Individual Report									
Observation date:					Observation Time:				
Class:		Group:		Submission date:					
Chain RL	00	15	30	45	60	75	90	93.3	
Surface									
Grade									
C/L h									
W _{left}									
W _{right}									
Area									
Misclose Ch93.3 (Prac1 – Prac2)					Excavation Volume: To grade level.				
Single tree volume:			No trees for removal:				Total volume:		
Submission Check List									
Group original FB Cross sections, profiles?/.			Copies all records/Plan?				Report Due date?		

A2-3 Establishment of Horizontal Control for a Construction Site

A2-3-1 Field Practical 3: Establishment of Horizontal Control

Motivation: In your profession as a civil or mining engineer, dealing with the three-dimensional (3D) coordinates of points on topographic surfaces will be a daily routine. Whether you are designing or constructing roads, buildings or in mining, you will need positional information. The term given to these positions with respect to some known plane surface is Coordinates expressed in terms Easting (E (X)), Northing (N (Y)) and height (RL).

Aims: (1) **Establishment of horizontal control for a construction site.**

(2) **Generation of a Digital Terrain Model**

(3) **Establishment of construction site boundary control.**

Task: **Establishment of horizontal control for construction site.**

Observation and recording of DTM objects.

Duration: **one practical session of two (2) hours.**

In this exercise you will be expected to use a **Total Station** to establish the position a control point by observation to existing control points. This type of survey is necessary when, for design work, control points for building corners points must be established for new works.

Furthermore, you will be expected to locate structures, line features and trees in 3D to enable a digital elevation model (DEM) of the existing site to be constructed in a CAD package.

Outcome: Following the lecture, workshop and this practical, you are expected to be able to:

1. Set-up the Total Station over a given control point.
2. Establish preliminary orientation of the Total Station to true North.
3. Observe and record readings to prisms set up over existing control points.
4. Observe and record readings to a prominent feature to establish site orientation.
5. Carry out a 3D topographic survey of the features of the survey site.
6. Calculate the control point 3D coordinates from the targeted control points.
7. Calculate the orientation correction to be applied to the topographic observations.
8. Calculate the 3D coordinates of the topographic DEM.
9. Perform checks pertaining to angular measurement to control points, the difference between observed and calculated angles.
10. Know the possible sources of errors in Total Station observations and how they could be minimized or avoided.

Group: 3 - 4 students per group.

Equipment:

- 1 Total Station and tripod
- 1 Prism set, prism pole and prism bubble
- 1 Offset tape
- 1 30m tape
- Control points will be occupied by extra 360° prism sets.

Comments:

1. The three-dimension (3D) control points for this exercise will be marked by prism sets established over stations 400 and 705A.
2. **Civil.** Each group will set-up a Total Station over a designated control point in the exercise area. Initial NORTH orientation will be provided by the use of an attached magnetic trough compass oriented to magnetic north. The S end of compass needle bisects lubber lines. A horizontal deviation allows orientation to true north. <http://www.ga.gov.au/oracle/geomag/agrfform.jsp>. From AGRF10, declination, $D = -1.647$ (1 April 2016), $-1^{\circ} 38' 50''$ at $-32^{\circ} 00' 21''$, $115^{\circ} 53' 40''$. True direction = Magnetic direction + D. Magnetic direction = True direction - D, $360^{\circ}M = 358^{\circ} 21' 10''$ True.

Mining. Each group will set-up a Total Station over a designated control point in the exercise area. Orientation will be provided by calculating an approximate grid bearing to 705A from the group's assigned control point. The coordinates are approximate, and those of 705A and C400 will be refined after the completion of the exercise when revised coordinates will be issued.

3. Each group will observe horizontal directions, zenith distance and slope distances to the two control points. Observations to be taken on Face left (FL) and Face right (FR) and slope distance will be recorded at each pointing.
4. After observation of the control points each group will observe: a) a prominent backsight point for Practical 4 orientation (direction only, FL/FR), b) the North-East corner, and a second point, of Building 204 will be observed to provide location and orientation of the building.
5. After observation of the control points each group will pick up topographic details, pathways, walls, light posts, ground level structures and trees within the survey area. Horizontal directions, zenith distance and slope distances on FL only. Sufficient points will be observed to provide a detailed site and contour plan.

Field Work:

1. Set-up Total Station over the designated control point. Draw station diagrams to allow re-location of the point for Field Practical 4. The group may be given a different station for FP4, using another team's work. Make sure your diagram is complete.
2. **Civil.** Install trough compass and align Total Station to Magnetic North. Set H.ANG on TS screen p2. Key 358.2110 then  (ENTER) key. Confirm 358° 21' 10" alignment on screen. Observe and book a prominent recoverable back sight point. Remove trough compass.
3. **Mining.** Enter your calculated grid bearing from your assigned control point to 705A.
4. Confirm the prism constant (PC) is set to +3 to match the 360° mini-prism targets.
5. From your observation point, measure and record FL and FR directions and slope distances to two selected control points. Bisect the 360° mini-prism.
6. Measure and record height of instrument (HI) and targets (HT). Sketch and measure location of control points.
7. Observe a prominent point (edge BLD105) by direction only, FL/FR, for use as backsight orientation for FP4.
8. After completing control station observations, check that TS is on FL, place the prism on the prism pole and adjust the prism HT to match the TS HI. Change the prism constant (PC) to -30.
9. Observe the NE corner of building 204 to provide a datum for the position of the proposed building, pick another point on the north face of 204 to provide orientation.
10. Commence the topographic survey; reading and recording FL direction, VA and slope distance to each chosen topographic point. Use a feature codes for points. Referring to the sketch plan, pick up sufficient feature points to complete a topographic plan of pathways, trees, light poles and service points within the works area. Obscured points may have to be recorded to an offset point to provide location. Pick up two points used in Field Practical 1 to provide integration of the area levelling into your DEM. (Include deck spike D/S134). Pick up corners of building site for calculation of RLs to be incorporated in design. Measure distances from corner points on brick paving.

Field Data Reduction:

1. Reduce field observation to control points. Use mean FL/FR zenith angle and mean slope distances to calculate horizontal and vertical distances to the control points. Use mean FL/FR directions to determine directions to control points.

2. Calculate horizontal coordinates of your control point using the calculated horizontal distances by the method of “intersection by distances”.
3. Calculate the RL of your control point using the vertical distance from the instrument to the one prism. Allow for HI and HT. Check your control RL by calculating the to the second prism set. Check RL of the established control points against RLs determined in Field Practical 1.
4. Check the mean angle from the instrument to the two control points against the angle calculated by the Cosine rule from three (3) distances.
5. Calculate the bearings to your two control points. Determine the rotation angle to be applied to one of your mean directions to align your DEM observations.
6. Apply the rotation angle to your DEM observations. Calculate the horizontal and vertical distances from the VA and the slope distance. Distances to trees should be adjusted by an estimate of diameter. Calculate the rectangular (dE, dN) coordinates to each DEM point from your station. Calculate E and N coordinates of each DEM point for your site plan. Calculate RL for each DEM point from the vertical distance (dH) from your station.
7. Provide a DTM file in PENZ (ascii.txt) format for processing in CAD.
8. Calculate the orientation of your Backsight (BLD105). This value will be used in FP4.
9. Calculate the corner and face coordinates (E, N) of building 204 to use as a datum for your Civil Computer Lab location. Offset distances will be advised.

10. You may have to swap data with another group in FP4. Ensure it is complete.

Location of works

Control point coordinates:
PCG2020, AHD

C954

PCG2020 Modified for CVEN2000

E: 57322.099m **322.099**

N: 257432.392m **7432.392**

AHD: 16.887m

~~C956~~

~~PCG94 Modified for CVEN2000~~

~~E: 57417.584m **417.584**~~

~~N: 257402.710m **7402.710**~~

~~AHD: 12.650m~~

705A

PCG2020 Modified for CVEN2000

E: 57407.808m **407.810**

N: 257491.205 **7491.198**

AHD: As determined during FP1

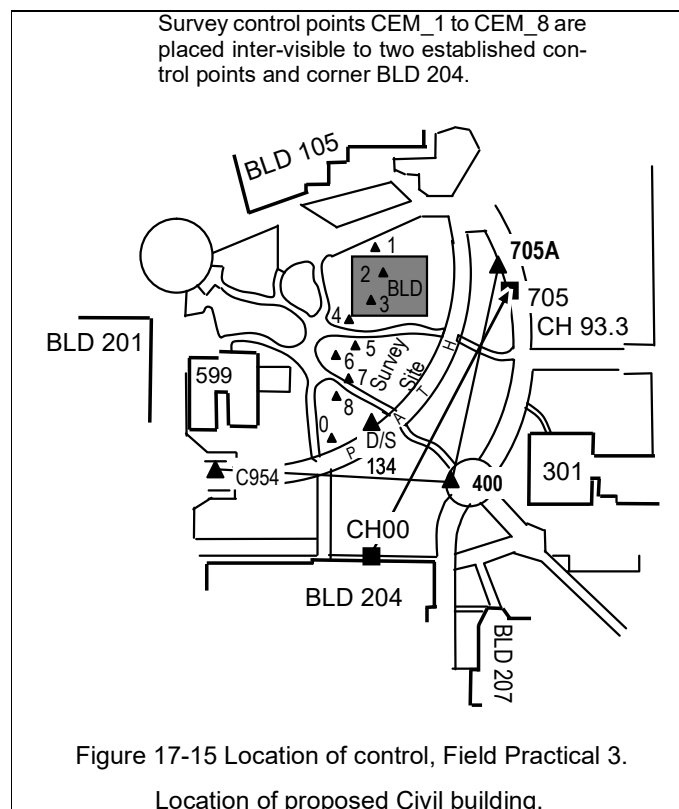
C400

PCG2020 Modified for CVEN2000

E: 57392.107m **392.107**

N: 257428.494m **7428.494**

AHD: As determined during FP1



A2-3-2 Field Practical 4: Establishment and Adjustment of Construction Site Boundary Control

Task: Establishment and adjustment of construction site boundary control.

Duration: one practical session of two (2) hours.

In this exercise you will be expected to use a **Total Station** to fully observe a horizontal triangular control network and adjust the coordinates of the network using the Bowditch adjustment. This type of survey is necessary when, for design work, control points for building corners points must be established for new works.

Your group may be allocated a different control station from the one observed in FP3. You may have to swap your field results with another group.

Outcome: Following the lecture, workshop and this practical, you are expected to be able to:

1. Set-up the Total Station over an established control point.
2. Establish preliminary orientation of the Total Station using established back sights (BS).
3. Position two (2) extra control points adjacent the proposed computer building.
4. Sequentially observe and record readings to prisms set up over the 3 control points.
5. Calculate and adjust the 3 internal observed angles and calculate the adjusted bearings.
6. Calculate the point 3D coordinates from the established control point by traverse.
7. Calculate the misclosure of the traverse.
8. Adjust the traverse's coordinates using the Bowditch adjustment.
9. Know the possible sources of errors in Total Station observations and how they could be minimized or avoided.

Group: 3 - 4 students per group.

Equipment:

- 1 Total Station and tripod
- 2 Prism sets and tripods, PC -30.
- 1 Offset tape
- 1 30m tape
- 3 pegs and hammer, flagging.

Comments:

1. The backsight from the previously established control point (Task 1) will be used to provide orientation for the exercise.
2. Two other inter-visible control points will be established near the proposed computer building to provide set-out control.
3. By traverse each group will observe horizontal directions, zenith distance and slope distances at the three control points. Observations to be taken on Face left (FL) and Face right (FR) and slope distance will be recorded at each pointing.
4. A full calculation of the coordinates of the traverse will be made using the Bowditch adjustment.
5. A report will be made of the results.

Field Work:

1. Set-up Total Station over the previously established control point. Record instrument height, HI. Ensure the PC -30 is set in the Total Station.
2. On FL, observe the Backsight established in Field Practical 3 and enter the calculated bearing to the Total Station. This is the orientation of the traverse network.
3. Establish the two other building control points, A and B, set-up prism sets over the points. Record the target heights, HT A and HT B.
4. After setting the back bearing on Face Left (FL), record the horizontal distance and direction to both points A and B of your triangle. Repeat these measurements on Face Right (FR). Use an appropriate booking table (Field notes). Directions are read in sequence FL A – FL B – FR B – FR A. Slope distance is read at each pointing.

5. Swap the prism on A to the initial CP and move your Total Station to point A on the triangle. **DO NOT** remove the instrument tribrachs from the tripod, use the tribrach lock to release the instruments. Ensure that they lock securely in the new position. Read the direction and distance of A-CP, read the direction and distance of line A-B. Now swap the Total Station and the prism between A and B, read a direction and distance B-A and then read the direction and distance of line B-CP.
6. Use the mean FL/FR BRG initial CP to A as the fixed orientation of your control adjustment.
 - (i) Calculate your angular misclose. Adjust angles, calculate adjusted bearings of sides.
 - (ii) Calculate the linear misclose from adjusted (see appropriate lecture).
 - (iii) Find the coordinates for the two unknown corners of the triangle A and B using Bowditch adjustment.
 - (iv) Plot the control points as well as the detail pickup features. Scale 1:500
7. Calculate the plane area of the triangle using the coordinate method.

Submission of report.

The report is to be in a format suitable for submission to the chief engineer.

Group submission of field observations, Field Practicals 3 and 4. (5 marks)

Group submission addressing the following points: (20 marks)

1. Calculation of established control point in Practical 3.
2. Back bearings to control points. Rotation angle for orientation of DEM pickup.
3. 3D coordinates of all DEM points. Coordinates of NE corner building 204.
4. Report on Bowditch adjustment of control points A and B. Angular adjustment of traverse angles. Adopted bearings of traverse. Linear misclose. Final 3D coordinates. Area of triangle.
5. A good copy of the original field notes. This is to include the station diagrams, checks and control traverse from Task 1. (FP1)
6. **Original field book of FP3 and FP4.** This is the document used to check your work.

Example of Marking Guide

Tutor _____ **Class** _____ **Group** _____ **ID** _____

Submission date: _____ **ID** _____

ID _____

Task 2. Marking Sheet: **ID** _____

Survey Control, DEM and Setout for a Construction Site. _____ **Group Mark (/25)**

Field Notes: Date, Group members, Equipment, Make, model, S/N Task methods (L/R face, S dist, L face for pickup). Neatness of booking Correctness of recording	5 (group)	Good copy of Field Book, Total Station records, feature pick-up sheet Comments:
Submission: 1 – Calculation of control points established in Practical 3. 3 mks 2 – Back bearings to control points. Rotation angle for orientation of DEM pickup. 1 mk 3 – 3D coordinates of DEM points. Coords of NE corner BLD 204 3 mk 4 – Report on Bowditch adjustment of control points A and B. 3 mks 6 – Adjusted coordinates by Bowditch. 3 mks 7 – Misclose ratio. 1 mk 8 – Area calculation. 1 mk 9 – Plot of control points, DEM points to suitable scale. Point list for CAD export. 2 mks 10 – Brg/dist/RL building and site corners 3 mks	20 (group)	Comments:

Submission of report. Combined Field Practicals 3 and 4.

The report is to be in a format suitable for submission to the chief engineer including:

Group submission of field observations, Field Practicals 3 and 4.

Group submission addressing the following points:

The submission of a report on the field practical session should present the exercise as a report to your field supervisor. You can thus assume that the supervisor is competent in field surveying. Thus, the report does not need lengthy explanations of methods, procedures and calculation used to determine results.

However, the supervisor will be keenly interested as to whether your task has been accomplished within the required accuracy of the task.

You should start with:

a signed cover sheet describing the task, the dates and the team members:

a short “executive summary” of the results:

- summary of task;
- description of site;
- calculated initial control point and orientation;
- calculated coordinates of CAD design datum;
- accuracy achieved for traverse between building set-out control points, instrument compliance with task specifications, were specifications met, and if not, how were they minimised, effect of inaccuracies.
- any site-specific structures likely to hinder or impact on the project.

The report itself should table:

1. Three dimensional (3D) coordinates of initial control point calculated by the “Intersection by Distances” method from two established control points, showing all checks, including confirmation of group work in Field Practical 3.
2. Calculated orientation to one of the two control points.
3. Calculated 2D coordinates of the north-east corner of building 204.
4. Calculated 3D coordinates of all points used to create the site digital elevation model (DEM). The format to be suitable for input to the AutoCAD Revit BIM program.
5. Adjusted coordinates of the building control points established in Field Practical 4. Include statement of traverse closure adjustment, coordinate misclosure and traverse accuracy using the Bowditch adjustment procedure. Calculated area of enclosed triangle using the coordinate cross multiplication method.
6. Draw and label a plan of the area showing the all control points used or established, DEM points and feature code. Show any impediments in the works area that may impact on the building location.
7. Show a tie to the centreline of the path design (Field Practical 2) including the design finish level at CH90.
8. The original field book and a good copy of the original field notes.
9. A brief discussion of any problems encountered and methods used to rectify or minimise them.

A short conclusion should précis your exercise results.

Calculation sheets and methods should be included in an appendix.

Task 2. Summary of calculated values, Field Practical 3				
Observation date:		Observation Time:		
Class:		Group:		
Student ID		Signature		
Student ID		Signature		
Student ID		Signature		
Student ID		Signature		
From FIELD BOOK and calculation, provide the following information You will have occupied one of control points CEM 1 to CEM 8. You used CPs C400 and 705A.				
Name CP 1: _____ HI _____		Name CPs used: 2: _____, 3: _____		
Coordinates 2:	E:	N:	RL:	HT:
Coordinates 3:	E:	N:	RL:	HT:
Mean Horizontal distance H3: 1 - 2:				
Mean Horizontal distance H2: 1 - 3:				
Mean direction 1 to 2				
Mean direction 1 to 3				
Mean angle at 1				
Mean direction 1 to Back Sight				
Mean direction 1 to CNR 204			H Dist	
Resulting calculated coordinates of Control Point 1, orientation correction and BS BRG				
Calculated coords 1: _____	E:	N:	RL:	
Calculated Orientation, 1 to 2				
Orientation rotation angle				
Adopted BRG 1 to Back Sight				
Adopted BRG 1 to CNR 204		Coords CNR 204	E	N
SE corner Design building	E	N		
RLs of site corners:	NW	NE	SE	SW
RL D/S 134 (FP1): _____	RL D/S134 (FP3): _____		Misclose: _____	

Task 2. Summary of calculated values, Field Practical 4 and Bowditch adjustment			
Observation date:		Observation Time:	
Class:		Group:	
Student ID		Signature	
Student ID		Signature	
Student ID		Signature	
Student ID		Signature	
Initial Mean Bearing CP00 - A			
Unadjusted angle CP00			
Unadjusted angle A			
Unadjusted angle B			
Angular Misclose		Correction per angle	
Adjusted angle CP00			
Adjusted angle A			
Adjusted angle B			
Adjusted BRG CP00 - A			
Adjusted BRG A - B			
Adjusted BRG B - CP00			
Misclose d_E		Misclose d_N	
Linear Misclose		Misclose ration 1:	
Adjusted E_{CP00}		Adjusted N_{CP00}	
Adjusted E_A		Adjusted N_A	
Adjusted E_B		Adjusted N_B	
Adjusted E_{CP00}		Adjusted N_{CP00}	
Area CP00-A-B-CP00; m^2			
BRG/H Dist from CP_A to SE corner building		BRG	H Dist
BRG/H Dist from CP_B to SE corner building		BRG	H Dist

A2-4 Establishment of Horizontal Curves Field Examination

A2-4-1 Calculation and Set-out of a Simple Curve—Assessed Field Practical 5

Group work component: To be assessed in the field

Individual component:

Calculations and drawing of a nominated curve to be submitted at the field practical session.

Objective:

Individually: to calculate a nominated curve, posted to Blackboard, and draw all elements at a scale of 1:1000. (Fits on A4)

Group component, assessed in field examination:

As a group, work together to set out a nominated simple circular curve, check set-out, report misclosures, compare TS coordinates with handheld GPS. Check GPS accuracy against a known control point. Evaluate misclose.

Objective:

As a group: Set out a simple circular curve by deflection and chord measurement method. The curve has been pre-calculated prior to the practical session and is to be set out by each group in the field. The group will then check the curve for set-out precision.

The practical will be conducted on the Edinburgh Oval South opposite the John Curtin Centre.

Group: 4 students per group.

Equipment:

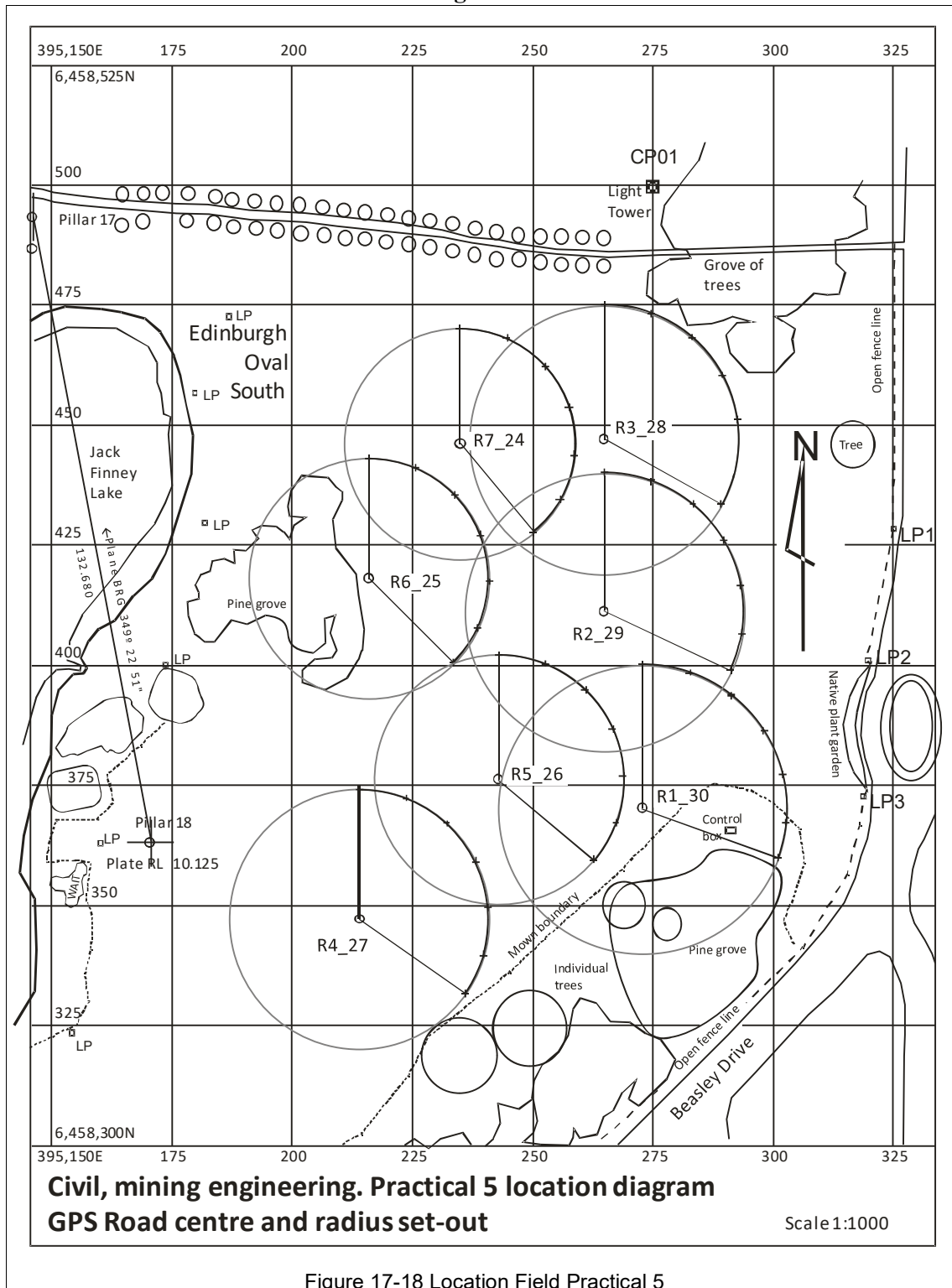
- 1 Sokkia SET530 Total Station and tripod, peanut prism.
- 50m tape, 8m offset tape, 1 set chaining arrows, 2 pegs, 1 hammer, flagging
- Hand-held Garmin 76/72H GPS.

Comments:

1. For the individual portion, each group will be given data describing a simple circular curve. The curve describes a circular path having a centreline, left and right pavement offsets and offsets to back of kerb. Each member of the group will calculate a designated offset. The curves will be about 25m radius, deflection $< 90^\circ$ and have 5 or 6 chainage points.
2. All elements of the group field curve have been pre-calculated ready for field set-out.
3. In the field your tutor will provide one of the pre-calculated curve designs and you will lay that design out by deflection angle and chord measurement.
4. On completion of the set-out you will be required to evaluate the set-out using a hand-held GPS.
5. Suggested reading:
Uren, J and Price, W F (2006/2010), Chapter 12, Circular curves, setting out by traditional methods.
Irvine and MacLennan, 5th Edition, definitions and formula used in workshop:

Field Work: (See Figure 17-16).

1. A position for each group's tangent point (TP1) using the handheld GPS will be indicated on a diagram of the area (Field sheet A2-4-2, Figure 17-18). Peg TP1 at that position using the MGA coordinates from the GPS. (Field sheet A2-4-4).
2. Peg the centre point, O, using the GPS. DO NOT USE THE TAPE OR PRISM.
3. Set the Total Station over the TP1, sight on centre, O, on Face RIGHT and set 0° using the F3 button [0.SET]. Plunge telescope on to face LEFT so that the TS is pointing to 360° .
4. Turn the TS to 90° and peg the intersect point (IP) at the **calculated** tangent distance using the provided prism. DO NOT USE THE TAPE.
5. You will then peg TP2 at the appropriate bearing using the 50m tape, noting it's through chainage. DO NOT USE THE PRISM.

A2-4-2 Location of Field Work: Edinburgh Oval South

A2-4-3 Curve Set-out Data

LIST OF BEARINGS and CHORD DISTANCES – FOR EACH GROUP

Road centre lines bearings and chord distances on Edinburgh Oval South: **$\Delta = 110^\circ$ Road 1 (Group 1) R = 30m**

Point	Bearing	Chord
Centre	180° 00' 00"	30.000
PC 100	90° 00' 00"	0.000
CH 110	99° 32' 57"	9.954
CH 120	109° 05' 54"	19.632
CH 130	118° 38' 52"	28.766
CH 140	128° 11' 49"	37.102
CH 150	137° 44' 47"	44.411
PT 157.60	145° 00' 00"	49.149

 $\Delta = 114^\circ$ Road 2 (Group 2) R = 29m

Point	Bearing	Chord
Centre	180° 00' 00"	29.000
PC 100	90° 00' 00"	0.000
CH 110	99° 52' 42"	9.951
CH 120	109° 45' 25"	19.606
CH 130	119° 38' 08"	28.680
CH 140	129° 30' 51"	36.904
CH 150	139° 23' 34"	44.033
PT 157.70	147° 00' 00"	48.643

 $\Delta = 119^\circ$ Road 3 (Group 3) R = 28m

Point	Bearing	Chord
Centre	180° 00' 00"	28.000
PC 100	90° 00' 00"	0.000
CH 110	100° 13' 53"	9.947
CH 120	110° 27' 46"	19.578
CH 130	120° 41' 39"	28.586
CH 140	130° 55' 32"	36.684
CH 150	141° 09' 25"	43.617
PT 158.15	149° 30' 00"	48.251

 $\Delta = 125^\circ$ Road 4 (Group 4) R = 27m

Point	Bearing	Chord
Centre	180° 00' 00"	27.000
PC 100	90° 00' 00"	0.000
CH 110	100° 36' 37"	9.943
CH 120	111° 13' 14"	19.546
CH 130	121° 49' 51"	28.480
CH 140	132° 26' 28"	36.441
CH 150	143° 03' 05"	43.156
PT 158.90	152° 30' 00"	47.899

 $\Delta = 130^\circ$ Road 5 (Group 5) R = 26m

Point	Bearing	Chord
Centre	180° 00' 00"	26.000
PC 100	90° 00' 00"	0.000
CH 110	101° 01' 06"	9.938
CH 120	112° 02' 12"	19.511
CH 130	123° 03' 18"	28.363
CH 140	134° 04' 25"	36.170
CH 150	145° 05' 31"	42.644
PT 158.99	155° 00' 00"	47.128

 $\Delta = 135^\circ$ Road 2 (Group 2) R = 25m

Point	Bearing	Chord
Centre	180° 00' 00"	25.000
PC 100	90° 00' 00"	0.000
CH 110	101° 27' 32"	9.933
CH 120	112° 55' 05"	19.471
CH 130	124° 22' 38"	28.232
CH 140	135° 50' 11"	35.868
CH 150	147° 17' 44"	42.074
PT 158.90	157° 30' 00"	46.194

 $\Delta = 140^\circ$ Road 7 (Group 7) R = 24m

Point	Bearing	Chord
Centre	180° 00' 00"	24.000
PC 100	90° 00' 00"	0.000
CH 110	101° 56' 11"	9.928
CH 120	113° 52' 23"	19.425
CH 130	125° 48' 35"	28.085
CH 140	137° 44' 47"	35.528
CH 150	149° 40' 59"	41.436
PT 158.64	160° 00' 00"	45.105

Set-out instructions

1. Set the PC and centre point by **GPS**.
2. Calculate TD and set IP with **prism**
3. Set the chainage points out using the Total Station for bearing and the 50m tape for chord distance.
4. Calculate in chord, through chord and out chord distances for checks between chainages.

A2-4-4 Curve Coordinate Data for Each Group

Coordinates are Map Grid of Australia (MGA2020), Zone 50.

Road centre lines on Edinburgh Oval South. Set only PC and Centre with GPS. **$\Delta = 110^\circ$ Road 1 (Group 1) R = 30m**

Point	Easting	Northing
Centre	395,273.0	6,458,370.0
PC 100	395,273.0	6,458,400.0
CH 110	395,282.8	6,458,398.3
CH 120	395,291.6	6,458,393.6
CH 130	395,298.2	6,458,386.2
CH 140	395,302.2	6,458,377.1
CH 150	395,302.9	6,458,367.1
PT 157.60	395,301.2	6,458,359.7

 $\Delta = 114^\circ$ Road 2 (Group 2) R = 29m

Point	Easting	Northing
Centre	395,265.0	6,458,411.0
PC 100	395,265.0	6,458,440.0
CH 110	395,274.8	6,458,438.3
CH 120	395,283.4	6,458,433.4
CH 130	395,289.9	6,458,425.8
CH 140	395,293.5	6,458,416.5
CH 150	395,293.7	6,458,406.6
PT 157.70	395,291.5	6,458,399.2

 $\Delta = 119^\circ$ Road 3 (Group 3) R = 28m

Point	Easting	Northing
Centre	395,265.0	6,458,447.0
PC 100	395,265.0	6,458,475.0
CH 110	395,274.8	6,458,473.2
CH 120	395,283.3	6,458,468.2
CH 130	395,289.6	6,458,460.4
CH 140	395,292.7	6,458,451.0
CH 150	395,292.4	6,458,441.0
PT 158.15	395,289.5	6,458,433.4

 $\Delta = 125^\circ$ Road 4 (Group 4) R = 27m

Point	Easting	Northing
Centre	395,214.0	6,458,347.0
PC 100	395,214.0	6,458,374.0
CH 110	395,223.8	6,458,372.2
CH 120	395,232.2	6,458,366.9
CH 130	395,238.2	6,458,359.0
CH 140	395,240.9	6,458,349.4
CH 150	395,239.9	6,458,339.4
PT 158.90	395,236.1	6,458,331.5

 $\Delta = 130^\circ$ Road 5 (Group 5) R = 26m

Point	Easting	Northing
Centre	395,243.0	6,458,376.0
PC 100	395,243.0	6,458,402.0
CH 110	395,242.8	6,458,400.1
CH 120	395,261.1	6,458,394.7
CH 130	395,266.8	6,458,386.5
CH 140	395,269.0	6,458,376.8
CH 150	395,267.4	6,458,367.0
PT 158.99	395,262.9	6,458,359.3

 $\Delta = 135^\circ$ Road 2 (Group 2) R = 25m

Point	Easting	Northing
Centre	395,216.0	6,458,418.0
PC 100	395,216.0	6,458,443.0
CH 110	395,225.7	6,458,441.0
CH 120	395,233.9	6,458,435.4
CH 130	395,239.3	6,458,427.1
CH 140	395,241.0	6,458,417.3
CH 150	395,238.7	6,458,407.6
PT 158.90	395,233.7	6,458,400.3

 $\Delta = 140^\circ$ Road 7 (Group 7) R = 24m

Point	Easting	Northing
Centre	395,235.0	6,458,446.0
PC 100	395,235.0	6,458,470.0
CH 110	395,244.7	6,458,467.9
CH 120	395,252.8	6,458,462.1
CH 130	395,257.8	6,458,453.6
CH 140	395,258.9	6,458,443.7
CH 150	395,255.9	6,458,434.2
PT 158.64	395,250.4	6,458,427.6

Set-out instructions

1. Except for the PC and CENTRE point, do not use the GPS to set out these other chainage points.
2. Check and record these chainages after being set out by TS and tape using the hand held GPS.
3. The GPS displays to metres only.
4. Calculate BRG/DIST misclose.

A2-4-5 Curve Check Observations**Formulae to calculate the mis-closures on the curve**

$$BRG = \text{Atan}\left(\frac{dE}{dN}\right) \text{ in correct quadrant}$$

$$\text{Dist} = \sqrt{dE^2 + dN^2}$$

Class _____ Group _____ Road number _____ TD _____
 Out arc _____ Out chord _____.

Poi nt	TS E	TS N	GPS E	GPS N	Mis- close dE	Mis- close dN	BRG	Dist
PC								
110								
120								
130								
140								
150								
PT								

Check taping of radii and chords between chainages of the TS set locations on the curve:**Check of Pillar 18 (record last 3 digits of E and N)**

Road number _____ Radius: _____ Pillar 18 observations

Point	Radius to	Error	Chord	_____	E 395 170.5	N 6 458 362.9	Ht 10.125
Centre	to:	to:	Dist	Error	1		
PC			0.0	0.0	2		
110					3		
120					4		
130					5		
140					6		
150					Average		
PT							
					Misclose		
					dE	dN	
					Brg	Dist.	

A2-4-6 Example Group Assessment Form**Example of Marking Guide**

Assessment Marking Sheet: Group **Class** _____ **Date:** _____

Group + Individual / Total

Student _____	ID _____	Mark _____	_____ + _____ / _____
Student _____	ID _____	Mark _____	_____ + _____ / _____
Student _____	ID _____	Mark _____	_____ + _____ / _____
Student _____	ID _____	Mark _____	_____ + _____ / _____

Laboratory 5. Setting of a horizontal curve.**Assessment form.**

Set-up of instrument Correct centring and orientation Levelling circular bubble, Levelling plate bubble	3.0 (group)	
Set-out of TP1, Centre peg using GPS. IP and TP2 using TS and prism/tape 3mks Set-out of pegs (arrows) on radius at chord distances 5mks Checking distance between chainages and radius from centre peg. 2mks Checking set-up from GPS at each chainage and at Pillar 18. Calc δE , δN , BRG, dist 2mks	12.0 (group)	Obs Brg to IP: _____ TD: _____. Obs TD _____ Long chord _____. Obs LC _____ Brg to PT: _____ Obs Brg to PT: _____
Individual's calculations parameters set-out details chainage point coordinates 8mks Sketch of the curve (1:200) 2mks	10.0 (individual)	

A2-4-7 Definitions and Formulae.

The simple curve. See Section 10.2.8

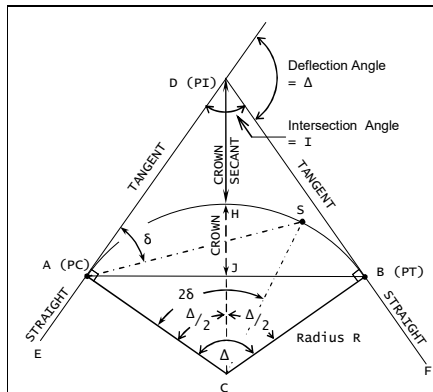


Figure 17-19 Geometry of the curve.

Formulae.

- (i) Tangent Distance AD. $TD = R \cdot \tan \frac{\Delta}{2}$
- (ii) Radius (R). $R = \text{Tangent Distance (AD)} \cdot \cot \frac{\Delta}{2}$
($\cot = 1/\tan$)
- (iii) Arc deflection angle Δ , $\Delta = \frac{s}{R}$ (s = any arc length)
- (iv) Arc length (AB). $S = R \cdot \Delta$ (Δ measured in radians)
- (v) Secant Distance (DC) = $R \cdot \sec \frac{\Delta}{2}$ ($\sec = 1/\cos$)
- (vi) Crown Secant (DH) = $R \cdot \sec \frac{\Delta}{2} - R = R \cdot (\sec \frac{\Delta}{2} - 1)$
- (vii) Crown Distance (JH) = $R - R \cdot \cos \frac{\Delta}{2} = R \cdot (1 - \cos \frac{\Delta}{2})$
= $(R - \text{Vers } \frac{\Delta}{2})$

Curve formulae.

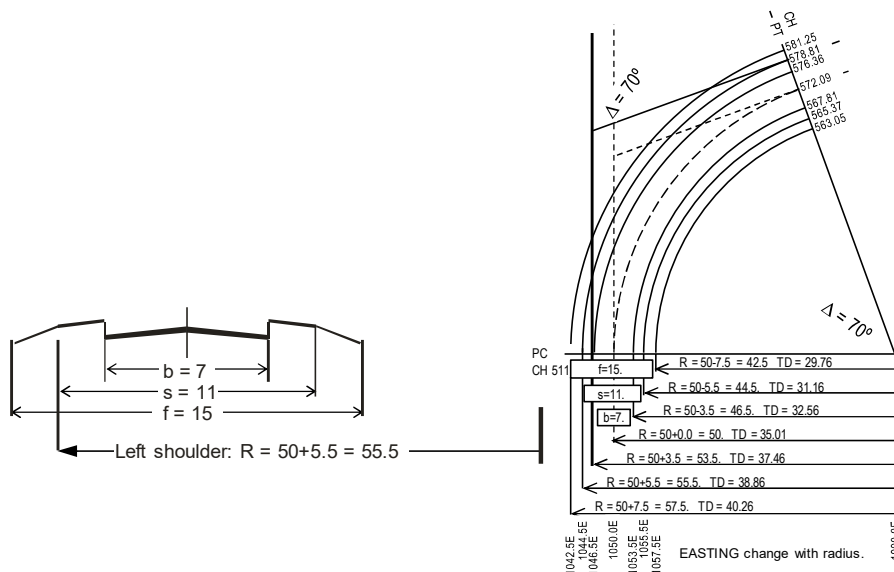


Figure 17-20 Formation definitions. Pavement, b; shoulder, s; formation, f.

	CURVE	EX2	LEFT SHOULDER
Initial direction of tangent	Brg IP =	360	Design TD =
Chainage PC (TP1)	CH =	511.00	CH IP =
Coordinates of PC (TP1)	E =	1050.000	Long chord =
Coordinates of PC (TP1)	N =	5511.000	Arc length, S =
Radius design Centre line	R =	50	Secant distance =
Design deflection angle Δ	Δ =	70	Crown secant =
Through chaining interval	s =	10	Crown distance
Centreline offset	o =	0	CH PT (TP2) =
Pavement width	b =	7	IP E =
Shoulder width	s =	11	IP N =
Formation width	f =	15	

Figure 17-21 Example curve for solution, curve EX2.

Example design parameters for individual curve calculations.

Designed to illustrate a single carriageway road.

Design parameters for curve turning to the RIGHT							
MONDAY CLASS 1 LAB	CURVE	M1	M2	M3	M4	M5	M6
Initial direction of tangent	Brg IP =	360	360	360	360	360	360
Chainage PC (TP1)	CH =	63.50	106.50	107.50	106.50	107.50	105.50
Coordinates of PC (TP1)	E =	1011.000	1012.000	1013.000	1014.000	1015.000	1016.000
Coordinates of PC (TP1)	N =	1018.000	3118.500	3119.500	3120.500	3121.500	3116.750
Radius design Centre line	R =	25	21.5	22.5	23.5	24.5	25.5
Design deflection angle Δ	Δ =	50	85	82	74	72	68
Through chaining interval	s =	5	5	5	5	5	5
Centreline offset	o =	0	0	0	0	0	0
Pavement width	b =	5	5	5	5	5	5
Formation width	f =	8	8	8	8	8	8
Design parameters for curve turning to the RIGHT							
TUESDAY CLASS 4 LAB	CURVE	T1	T2	T3	T4	T5	T6
Initial direction of tangent	Brg IP =	360	360	360	360	360	360
Chainage PC (TP1)	CH =	510.50	511.00	512.00	505.00	506.00	507.00
Coordinates of PC (TP1)	E =	999.000	998.000	997.000	996.000	995.000	1000.000
Coordinates of PC (TP1)	N =	3110.000	3111.000	3112.000	3105.000	3106.000	3107.000
Radius design Centre line	R =	24	25	26	21.5	22.5	23.5
Design deflection angle Δ	Δ =	77	73	70	82	81	79
Through chaining interval	s =	5	5	5	5	5	5
Centreline offset	o =	0	0	0	0	0	0
Pavement width	b =	5	5	5	5	5	5
Formation width	f =	8	8	8	8	8	8
Design parameters for curve turning to the RIGHT							
THURSDAY CLASS 2 LAB	CURVE	W1	W2	W3	W4	W5	W6
Initial direction of tangent	Brg IP =	360	360	360	360	360	360
Chainage PC (TP1)	CH =	408.00	409.00	410.00	411.00	412.00	413.00
Coordinates of PC (TP1)	E =	999.000	998.000	997.000	996.000	995.000	994.000
Coordinates of PC (TP1)	N =	3108.000	3109.000	3110.000	3111.000	3112.000	3113.000
Radius design Centre line	R =	24.5	25.5	26.5	22	23	24
Design deflection angle Δ	Δ =	73	69	66	80	80	76
Through chaining interval	s =	5	5	5	5	5	5
Centreline offset	o =	0	0	0	0	0	0
Pavement width	b =	5	5	5	5	5	5
Formation width	f =	8	8	8	8	8	8
Design parameters for curve turning to the RIGHT							
FRIDAY CLASS 3 LAB	CURVE	F1	F2	F3	F4	F5	F6
Initial direction of tangent	Brg IP =	360	360	360	360	360	360
Chainage PC (TP1)	CH =	207.00	208.00	207.50	211.00	211.00	212.00
Coordinates of PC (TP1)	E =	998.000	997.000	996.000	995.000	994.000	993.000
Coordinates of PC (TP1)	N =	3107.000	3108.000	3109.000	3110.000	3111.000	3112.000
Radius design Centre line	R =	25	24	23	22	21	20
Design deflection angle Δ	Δ =	68	69	72	77	81	83
Through chaining interval	s =	5	5	5	5	5	5
Centreline offset	o =	0	0	0	0	0	0
Pavement width	b =	5	5	5	5	5	5
Formation width	f =	8	8	8	8	8	8

A2-5 Computation of Design Parameters of Vertical Curves

A2-5-1 Practical 6

Task: Solve an assigned survey task

An optional practical assessment, in the form of a desktop exercise, may be made available to students who may wish to replace an existing filed practical mark. The task is designed to represent the amount of work involved in the calculation and reporting of the other assessed reports.

Objective: (REFER TO Appendix A1-2, Chapter 3, Figure 3-1, Table 3-1)

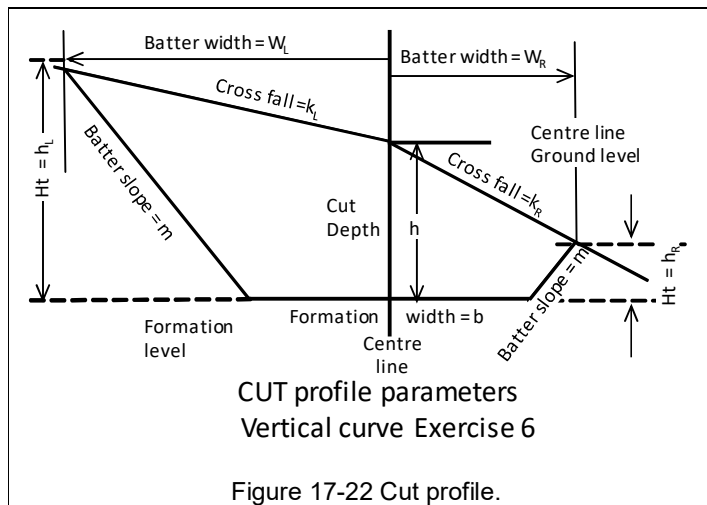
To calculate the formation level of a series of vertical curve between CH 00 and CH 95 of the previously conducted area levelling exercise.

Individual: Individual design and calculation effort.

Equipment: Nil

A2-5-2 Task:

1. You are given the parameters of a vertical curve designed to link the path at CH 00 through to the path at CH 95 through the pine grove to the North of BLD 204.
2. You are provided with the Ground Level (GL) and cross fall on the centre line at CH 00, CH 15, CH 30, CH 45, CH 60, CH 75 CH 90 and CH 95.
3. Your design is for a path excavated to formation level defined by the vertical curve. The side batters have a slope of 1:3 to the surface.
4. Calculate all the parameters of the path at each chainage point using the parameters
formation width; **b = 3.0m**
batter slope, **m = 3**,
the ground slope, k_{Left} and k_{Right}



- a) the RL of the path
- b) depth of cut, **h**, at the centre line between GL and path RL
- c) the batter width, W_{Left} and W_{Right} using the batter slope, **m**, at the intersection with the ground slope, **k**,
- d) RL of batter slope **at LEFT and RIGHT** intersection with slope **k**
- e) cross section area by formula or coordinate cross multiplication
- f) volume by end area method, or your choice
- g) Draw and label longitudinal profile of centre line GL and path formation level
scale horizontal 1:400, Vertical 1:20.

Vertical Curve design parameters of design surface:

1. The path falls at a slope of -1% between CH 00 and CH 15.
2. The path increases, through a **sag** curve to a slope of $+4\%$ between (CH 15 – CH 30).
3. The path maintains a straight slope between CH30 and CH80.
4. The path again falls, via a **crest** curve, to a slope, -1% , between CH 80 and CH95.

Ground RL and cross slopes

CH	GL	K left	K right
0	11.85	15	Flat (infinite)
15	13.083	14	-15
30	13.444	12	-10
45	14.152	10	-13
60	14.301	15	-25
75	14.535	12	-18
90	14.306	9	-12
95	14.215	Flat (infinite)	Flat (infinite)

Submission:**To your tutor**

1. Calculated path RL at designated chainages. (2mks)
2. Calculated h at each chainage. (4mks)
3. Calculated batter intercepts, distance from CL and intercept RL. (6mks)
4. Calculated end areas at each chainage (6mks)
5. Calculated volume of excavation (3mks)
6. Draw and label longitudinal profile of centre line GL and path formation level scale horizontal 1:400, Vertical 1:20 (4mks)

Assessment 4: Summary of calculated values, Vertical Curve							
Submission date:							
Class:			Group:				
Student ID:			Signature				
Mark Total 25			Mark:				
Comment:							
CH	Finish RL	h	W_L	Ground RL_L	W_R	Ground RL_R	End area
00							
15							
30							
45							
60							
75							
90							
95							
Volume; m^3							
Attach drawing longitudinal profile of centre line GL and path formation level. Scale horizontal 1:400, Vertical 1:20							
Attach calculation sheets for evaluation/confirmation of your answers.							

Example of Marking Guide**Assignment Marking Sheet: Individual.** Group _____ Class _____

Student _____ ID _____ Mark _____

Practical 6. Vertical Curve calculations.

Calculated path RL at designated chainages (2mks)		
Calculated h at each chainage. (4mks)		
Calculated batter intercepts, distance from CL and surface intercept RL. (6mks)		
Calculated end areas at each chainage (6mks)		
Calculated volume of excavation (3mks)		
Draw and label longitudinal profile of centre line GL and path formation level. Scale horizontal 1:400, Vertical 1:20 (4mks)		

A2-6 Examples of Interim Reports

Practical A2-1-1.

Determinations of Vertical Control/Vertical Sections (Field Practical 1 and 2):

The task of Field Practical 1, carried out by the survey team on 5 March 2015, was to establish vertical control over the survey area including:

1. instrument test
2. open traverse to bench mark, D/S134
3. closed traverse through designated control points CEM_8 to CEM_1,
4. check of portal clearance.

The task of Field Practical 2, carried out by the survey team on 12 March 2015, was to observe ground levels over the designated survey field using spacing for a blasting grid of $6\text{m} \times 7\text{m}$. The observation tasks included:

1. establish a $6\text{m} \times 7\text{m}$ grid from provided survey lines,
2. using control RLs established in task 1, observe designated grid lines
3. determine position of trees and paths based on grid layout,
4. check RL of D/S134

The team members for task 1 and task 2 were:

S. Wright, 12345k	I.M. Bold, 23457f
B. Careful, 24578h	U. Slessor, 05146d

Executive Summary:

Prior to the survey being undertaken the equipment was inspected and tested:

Level:	Leica NA724, Asset no 16 (S/N on record)
	Collimation test over 40m line: error 0.004m (4mm). Outside manufacturer's specification.
	Balanced observations taken to ameliorate apparent error.
Staff:	Crain 5 section 5m length fibreglass. Good condition, section extension tested for wear.
	No error measured over section joint.

Task 1 (Practicals 1 and 2) was carried out by the team to determine the surface levels of the designated survey area so that the levels could be incorporated into a Digital Elevation Model (DEM) for further processing in a GIS program. Initial calculated values are designed to provide a check of subsequent GIS results.

The site itself is approximately $63\text{m} \times 42\text{m}$ (2650m^2) between the Civil Engineering building 204 and the Library, building 105 on Curtin Campus. The site slopes down in a generally easterly direction from an RL of 18.5m to a girding bench at 15.5m.

Vertically excavated in-situ volume of material to a bench level of 15.5m is approximately 2900m^3 .

Vertical drilling to bench level involves some 85m over 55 drill holes; using a 250mm (10") rotary drill produces 4m^3 of spall for explosives stemming.

All measurements were recorded in the field book, calculated by either the rise and fall method (practical 1) and the height of collimation method for practical 2. The reductions were checked for agreement between the readings, reductions and resulting levels.

The closed traverse to determine control benchmark RLs and D/S134 was run between station C954 and C153. The overall vertical misclose was 0.013m (13mm) which was within specifications.

Comparing the level at D/S134 against the open traverse showed a misclose of 0.010m (10mm).

The portal clearance was determined to be 3.210m, agreeing with the advised clearance.

Practical 2 was initiated by the set-out of the $6\text{m} \times 7\text{m}$ blast grid being flagged by sprinkler flags. From control point CEM_8, RL determined in practical 1, the nominated lines of the blast field were observed and the observations were closed back to CEM_8. D/S134 was levelled and found to agree within 0.015m (15mm) of practical 1 observations. Reduction of grid levels was by the height of collimation method. The closure, through two change points, back to CEM_8 was confirmed by the rise and fall method, with a misclose of 0.010m (10mm).

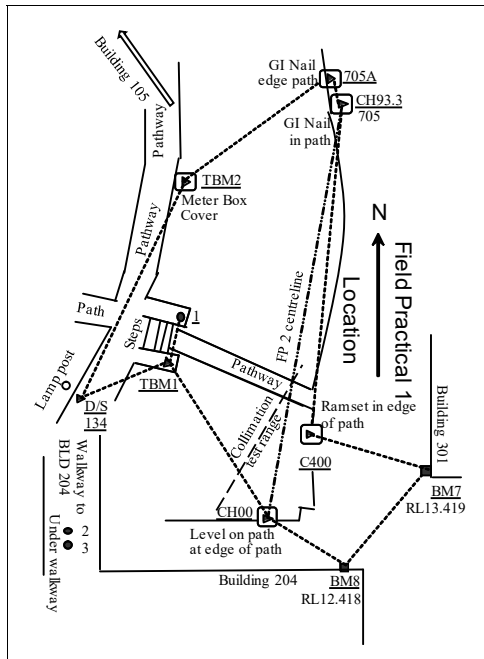
The approximate positions of trees and pathways within the survey area were recorded in relation to the blast grid coordinates. A representative tree was measured to ascertain total tree volume over the site.

A précis of results, with examples of reduction methods, is appended, together with a copy of the reduced field book. Plans and cross sections are attached.

Signed. S.Wright 17/08/2015

A3 Civil Engineers Practicals

A3-1 Civil Engineers Practical 1



Location, instructions.

Appendix

A2-2

Description of task. FP 1

A2-2-1

Field work.

- Collimation test by each team member
- Open traverse from BM8. Sequence:
 - BM8, CH00, clearance at 2 & 3, D/S134.
- Closed traverse from BM8. Sequence: BM8, CH00, TBM1, 1, D/S134, TBM 2, 705, 705A, C400, BM7, BM8.

Measurement and reduction:

§ 2.5

- Instrument set-up

§ 2.4.1

- Collimation test

§ 2.4.5

- Differential Levelling rise & fall;

§ 2.5.3

- Area levelling

§ 2.5.6

Calculation of observations

§ 2.5.4

Recording of Practical Tasks:

ALL observations are to be entered in supplied Field Book.

Record readings as provided on **Rise and Fall** booking sheet in the FB. Records must be NEAT.

- Record readings in metres, staff read (estimated) to millimetre level (3DP).
- Staff sections extended correctly (check that there is not a closed section, 1m error)

Observe the rules of levelling

§ 2.5.1

Sequence of Practical tasks:

1. Collimation check, individual observations (move instrument & re-level)
Team members each observe from mid-point, then from **close** (2 – 3m) to one staff.
2. Set level; observe BM8, BM8, points 2 & 3, and change point for open traverse to D/S134
BS BM8 (plug), 3 intermediate sights (IS) to portal, FS to firm CP1 (BS = FS distance).
3. Move level to allow sight to D/S134 and CP1
BS to CP1, FS to D/S134. End of open traverse.
4. Return level to allow back sight to BM8
BS to BM8, IS to CH00 FS to a change point (CP1)
5. Move level to allow sight to CP1, TBM1, D/S 134 and TBM2. Points inter-visible?
BS to D/S134, IS to TBM1, D/S 134, TBM1 and FS to TBM2 as CP2. (BS = FS distance).
6. Move level to allow sight to CP2, (TBM2) and forward to about 705/705A
BS to CP2, IS to 705, FS to 705A as CP3.
7. Move level to allow sight to CP3 (705A) and forward to about C400
BS to CP3, FS to C400 as CP4.
8. Move level to allow sight to CP4, with sight line to close on BM7
BS to CP4, IS to BM8 (check), FS to BM7. Complete closed traverse.
9. Calculate rise/fall and RLs to find misclose. Acceptable at 0.015m?
10. Calculate RLs of each point.
Compare levels D/S134 between open and closed traverse. Check misclose to BM8.

Field reduction of Level observations:

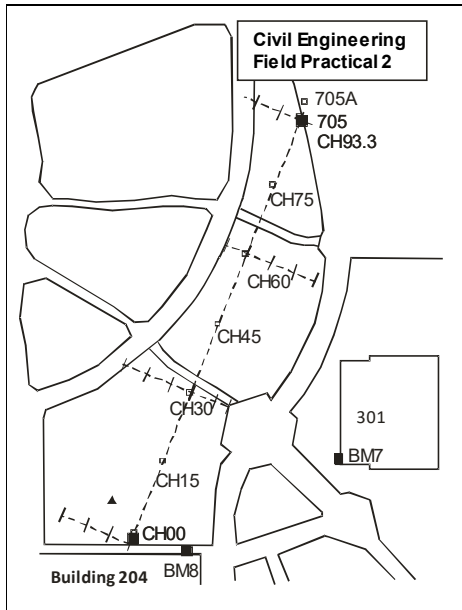
ALL observations should be reduced in the field. The booker has plenty of time between level readings to calculate the rise/fall between points.

Each team member checks their own collimation observations. Acceptable at 0.003m?

Draw and label a sketch of the field set-out. Indicate level position and CPs used.

Ensure FB is completed correctly. It's worth 25% of evaluation and is the only acceptable record of observation. Any loose sheets **MUST** be submitted with the FB. No erasures of data. Strike the error out and write the new entry adjacent the error. Any "fair copy" must have original data in support.

A3-2 Civil Engineers Practical 2



Location, instructions.

Appendix	A2-2
Description of task. FP 2	A2-2-2
Field work.	
Establishment of perpendicular offset grid	§2.6.4
Area levelling of grid from CH00	
Recording and reduction by Height of Collimation	§ 2.5.7
Mapping of trees, paths in exercise area,	
Measure tree with tape and clinometer.	§ 2.6.5
Field data reduction:	§ 2.5.4
Recording of Practical Tasks:	
ALL observations are to be entered in supplied Field Book.	
Record readings as provided on Rise and Fall booking sheet in the FB. Records must be NEAT.	
- Record readings in metres, staff read (estimated) to millimetre level (3DP).	
- Staff sections extended correctly (check that there is not a closed section, 1m error)	§ 2.7.3
Observe the rules of levelling	§ 2.5.1
Because of the compressed dimensions of the site, groups	

may start anywhere along the centreline. Start at the nearest established TBM (Field Practical 1) and sequentially survey the centreline and offsets. Include FP1 TBMs during the traverse

Sequence of Practical tasks:

Starting at CH00 as TBM, or any other TBM.

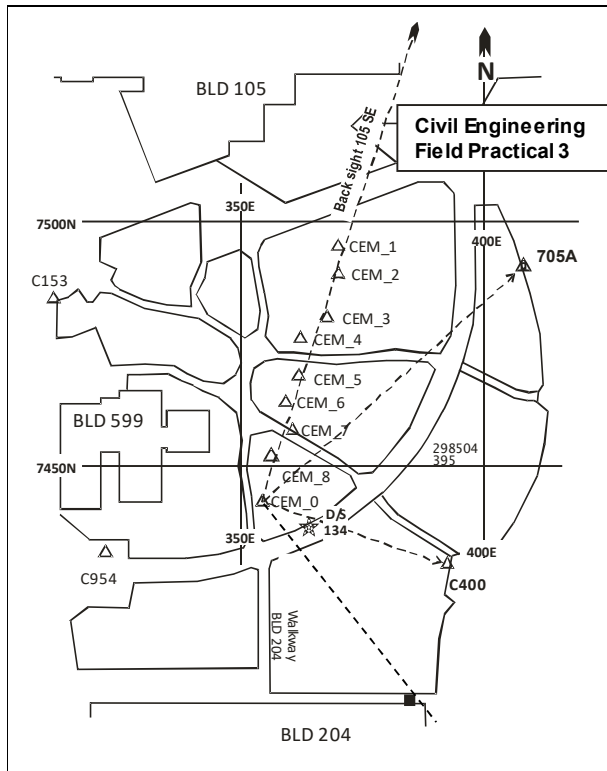
1. Sequentially level centreline and offsets. TBM 2 could allow a start at CH 45 or CH 60.
This method requires accurate booking of chainage and offsets. CL pegs may be used as CPs.
2. Complete the traverse of the centreline and close to a TBM.
3. Set level to allow BS on your control point (CH00) with an extensive view of site;
observe CH00 as BS, IS to grid intersections and a FS to firm change point (CP1)
4. Move level to allow sight to CP1 and remainder of survey field
BS to CP1, IS to grid intersections and a FS to firm change point CP2.
5. Observe the assigned grid intersections, using change points as necessary, include levelling to D/S134 as a check.
6. Calculate RLs by HC method.
7. Use Rise and Fall method to check between CH00, the CPs and D/S134.
Establish misclosure with values from FP1.
8. Record position of trees by using chainage and offset method from the centreline, CH00 to CH93.3.

Measure one tree for timber volume using clinometer and tape.

Field recording of Level observations. The booker should "keep up" with observations, ensuring correct identification of grid lines. Levels need to be correctly placed

Draw and label a sketch of the field set-out. Indicate level position and CPs used. Complete FB.

A3-3 Civil Engineers Practical 3



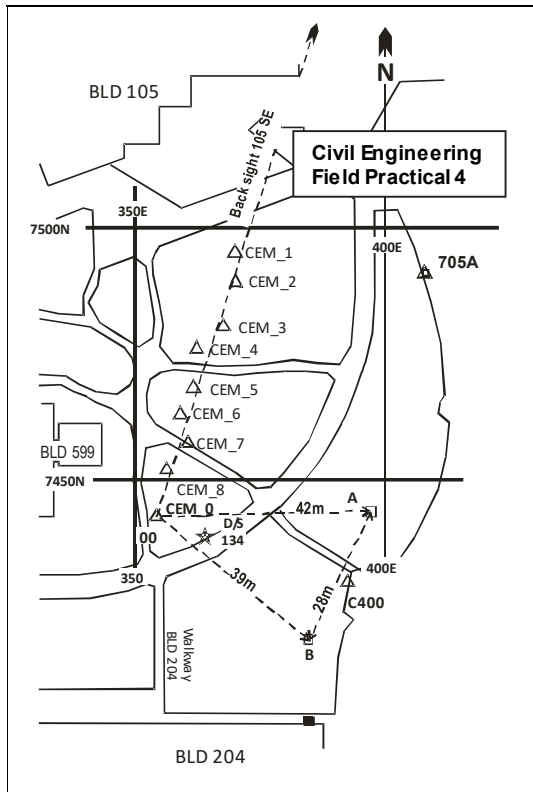
Location, instructions

Appendix	A2-3
Description of task. FP 3	A2-3-1
Field work.	
Set-up Total Station	§ 4.2.4
Azimuth for orientation	§ 4.2.4.3
Instrument height	
Target height, prism constant	
Observation, booking,	§ 4.3.1
Backsight (RO), NE204, D/S134	
Pick up trees, paths in designated area surrounding CEM_1 to CEM_4	
Recording of TS observations	
Booking	§ 4.6.2
Field data reduction:	§ 4.7
Recording of Practical Tasks:	
ALL observations are to be entered in supplied Field Book.	
Record readings on provided Traverse booking sheet in the FB.	
Records must be NEAT.	
- Record angles to second, distances in metres to millimetre level (3DP).	
- Ht instrument, target in metres to mm.	
Control point data:	
360° mini-prisms set over points.	
705A: E407.808, N7491.205, RL from FP1.	
C400: E392.107, N7428.494, RL from FP1.	

Sequence of Practical tasks:

1. Set and level TS over designated control point (CEM_1 – CEM_8)
2. Draw station diagrams. Measure and record HI of TS, HT of prisms.
3. Set Prism Constant (PC) as PC **+3** (360° mini-prism).
4. Set and align TS to magnetic meridian with tube compass.
5. Set HAR to read TRUE AZIMUTH using H.ANG function on TS § 4.2.4.2
6. Establish **Back Sight** feature.
Observe HAR only for reference alignment to BS on FL.
7. Read and record in sequence: HAR, ZA, Slope distance
FL: 705A
FL: C400
FR: C400
FR: 705A
FR: BS. HAR only.
8. Change TS PC to PC **-30**
9. Set prism on prism pole and set to height of instrument (HI)
10. Observe and book on **Face Left only**: ZA, HAR, Slope Distance for:
NE corner of Building 204, for establishment of proposed building set-out
D/S134 (comparison of level from FP1 and 2)
11. Using the supplied reflector held at approximately height of instrument (HI), continue the topographic survey of features; path edges, trees, sumps etc. within area containing CEM_1 to CEM_4. (topography plan and levels for building design).

A3-4 Civil Engineers Practical 4



Location, instructions.

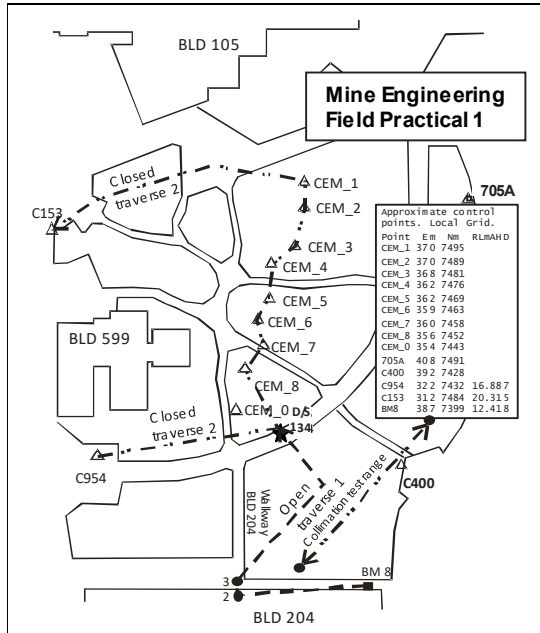
Appendix	A2-3
Description of task. FP 4	A2-3-2
Field work.	
Set-up Total Station	§ 4.2.4
Azimuth for orientation calculated from FP3	
Instrument height (HI)	
Set up prisms over control points, layout	A2-3-2
Target height (HT), prism constant	
Observation, booking,	§ 4.3.1
Backsight (RO), azimuth from FP3	
Recording of TS observations	
ALL observations are to be entered in Field Book.	
Record as provided on FB booking sheet in FB:	
height of instrument at 00 (HI),	
height of targets A and B (HT).	§ 4.2.2
Booking	§ 4.6.2
Semi-forced centring around network	A2-3-2
Field data reduction:	§ 4.7
Recording of Practical Tasks:	
Prism constant correction PC -30mm	
TRUE bearing of BACKSIGHT	
Sequence of observations	
Calculation of observations	§ 5.5
Reduction of observations	§ 5.5.1
Adjustment of observations	§ 5.5.2
Bowditch method	§ 5.5.4
Calculation of area	§ 5.5.5

Sequence of Practical Tasks:

1. Set and level TS over designated control point (CVEN_1 – CVEN_8) (**STN 00**)
2. On lower grassed area set two control points, A and B, pegged as designated in the figure. Triangle sides about 45m, 25-30m and 45m. Ensure inter-visibility between points. Use flagging tape to aid identification of **your** targets.
3. Set-up and level prism set over each point (**A and B**).
4. Draw and label a sketch of the field set-out.
5. Set Prism Constant (PC) as **PC -30**.
6. Re-establish Field Practical 3 **Back Sight** feature pointing.
7. Set HAR to read **TRUE BRG** to backsight using **H.ANG** function on TS (enter as **D.MMSS**) (§ 4.2.4.2). Calculated after FP 3.
8. From **STN 00**, read and record: Slope distance, ZA, HAR in sequence; FL: BS – FL: A – FL: B. Plunge TS; FR: B – FR: A – FR: BS.
9. Swap TS with prism at **A. Remove instrument and prism from tribrach using lock.**
10. At **STN A**. Sight to left of **STN B** and use **0 SET** on TS screen P1. Read and record: Slope distance, ZA, HAR in sequence; FL: B – FL: 00. Plunge TS; FR: 00 – FR: B.
11. Swap TS with prism at **B. Remove instrument and prism from tribrach using lock.**
12. At **STN B**. Sight to left of **STN 00** and use **0 SET** on TS screen P1. Read and record: Slope distance, ZA, HAR in sequence; FL: 00 – FL: A. Plunge TS; FR: A – FR: 00.
13. If requiring any further Topographic features for FP3. Set prism pole and prism to HI. Re-establish **STN 00**, set BS true BRG and observe **Face Left only**.

A4 Mining Engineers Practicals

A4-1 Mining Engineers Practical 1



Location, instructions.

Reference:

Appendix A2-2

Description of task. FP 1

A2-2-1

Field work.

Collimation test by each team member

Open traverse from BM8. Observations sequence:

BM8, portal clearance at 2 & 3, D/S134

Closed traverse from C954.

Observations sequence: C954, D/S134, CEM_8

– CEM_1, C153

Measurement and reduction: § 2.5

Instrument set-up § 2.4.1

Collimation test § 2.4.5

Differential Levelling rise & fall; § 2.5.3

Area levelling § 2.5.6

Calculation of observations § 2.5.4

ALL observations are to be entered in supplied

Field Book. Record readings as provided on **Rise and Fall** booking sheet in the FB.

Records must be NEAT.

Record readings in metres, staff read (estimated) to millimetre level (3DP).

Staff sections extended correctly (check that there is not a closed section, 1m error)

Comply with rules of levelling § 2.5.1

Sequence of Practical Tasks:

1. Collimation check, individual observations (move instrument & re-level)
Each team member observe from mid-point, then from **close** (2 – 3m) to one staff.
2. Set level; observe BM8, portal points 2 & 3, and change point for open traverse to D/S134
BS BM8 (plug), two intermediate sights (IS) to portal, FS to firm CP1 (BS = FS distance).
3. Move level to allow sight to D/S134 and CP1
BS to CP1, FS to D/S134. (BS = FS distance). End of open traverse.
4. Move level to allow sight to control point C954 and D/S134
BS to C954, FS to D/S134. (BS = FS distance). D/S134 is CP1
5. Move level to allow sight to CP1, D/S134 and forward to about CEM_4. Points inter-visible.
BS to D/S134, IS to CEM_8 and other CEM points, FS to CEM_4 as CP2.
6. Move level to allow sight to CP2, D/S134 and forward to about CEM_4
BS to D/S134, IS to CEM_8 and other CEM points, FS to CEM_4 as CP3.
7. Move level to allow sight to CP3 (CEM_4) and forward to about CEM_1
BS to CP3, IS to remaining CEM points, FS to CEM_1 as CP4.
8. Move level to allow sight to CP3, with sight line to close on C153
BS to CP4, FS to C153. (BS = FS distance) Complete closed traverse.
9. Calculate rise/fall and RLs to find misclose. Acceptable at 0.015m?
10. Calculate RLs of each point.

Compare levels D/S134 between open and closed traverse

Field reduction of Level observations:

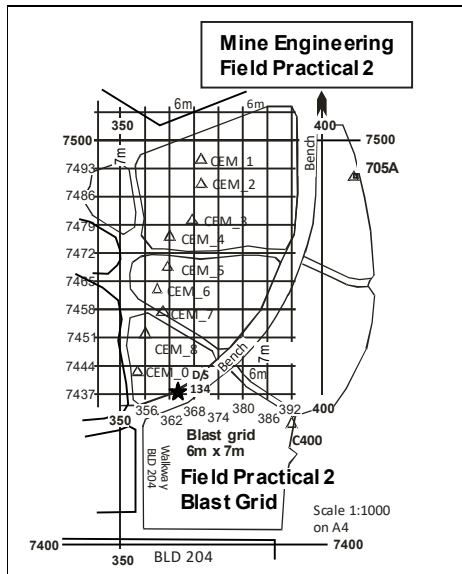
ALL observations should be reduced in the field. The booker has plenty of time between level readings to calculate the rise/fall between points.

Each team member check their own collimation observations. Acceptable at 0.003m?

Draw and label a sketch of the field set-out. Indicate level position and CPs used.

Ensure FB is completed correctly. It's worth 25% of evaluation and is the only acceptable record of observation. Any loose sheets **MUST** be submitted with the FB. No erasures of data. Strike the error out and write the new entry adjacent the error. Any "fair copy" must have original data in support.

A4-2 Mining Engineers Practical 2



Location, instructions.

Reference:

Appendix A2-2

Description of task. FP 2

§ A2-2-2

Field work.

Establishment of blast grid

§ 2.6.3

Area levelling of grid from assigned control BM

Recording and reduction by Height of Collimation (HC)

Locate trees within grid squares.

Measurement and reduction (HC)

Area levelling, HC method

§ 2.5.7

Calculation of observations

ALL observations are to be entered in supplied Field

Book. Record readings as provided on **Height of Collimation (HC)** booking sheet in the FB.

Records must be NEAT.

Record readings in metres, staff read (estimated) to millimetre level (3DP).

Staff sections extended correctly (check that there is not a closed section, 1m error)

Comply with rules of levelling

§ 2.5.1

Sequence of Practical Tasks:

Because of the amount of data to be gathered, the grid lines will be split between the anticipated two survey classes, in line with the plotting requirements of the assignment. Your starting bench mark is an assigned BM from CEM_1 to CEM_8. Level determined in FP1.

Starting at 350E grid line.

- 1st practical class: Level cross section at 7500N grid line, then every 14m south. i.e., at 7486N, 7472N, 7458N and 7444N.
- 2nd practical class: to interleave with the 1st practical class. Level cross section at 7500N and 7493N grid lines and then every 14m south. i.e., at: 7479N, 7465N and 7451N.
3. The check vertical datum by closing the level of D/S134.

Sequence of Practical Tasks:

ALL observations are to be entered in supplied Field Book.

Record readings as provided on **Rise and Fall** booking sheet in the FB. Records must be NEAT.

Re-label the **Fall** column heading as **HC**. The **Rise** column will not be used.

$HC = RL_{\text{start BM}} + BS$

$RL_{\text{point}} = HC - IS/FS$. FS is read at CP before moving level.

Record readings in metres, staff read (estimated) to millimetre level (3DP).

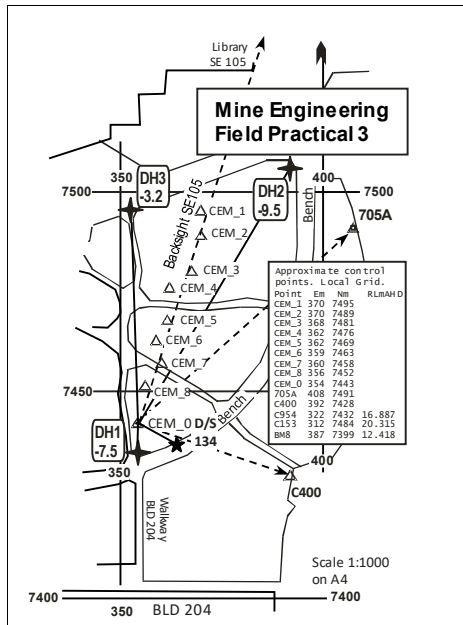
Staff sections extended correctly (check that there is not a closed section, 1m error)

1. Set level to allow BS on your control point (CEM_1 – CEM_8 as assigned) with an extensive view of site;
observe CEM_ BM as BS, IS to grid intersections and a FS to firm change point (CP1)
2. Move level to allow sight to CP1 and remainder of survey field
BS to CP1, IS to grid intersections and a FS to firm change point CP2.
3. Observer the assigned grid intersections, using change points as necessary, until levelling is closed at D/S134
4. Calculate RLs by HC method.
5. Use Rise and Fall method to check between CEM_ BM, the CPs and D/S134.
Establish misclosure with values from FP1.
6. Record position of trees by estimation from grid intersections.
Measure one tree for timber volume using clinometer and tape.

Field recording of Level observations. The booker should “keep up” with observations, ensuring correct identification of grid lines. Levels need to be correctly placed

Draw and label a sketch of the field set-out. Indicate level position and CPs used. Complete FB.

A4-3 Mining Engineers Practical 3



Location, instructions

Appendix

A2-3

Description of task. FP 3

A2-3-1

Field work.

Set-up Total Station

§ 4.2.4

Grid bearing for orientation

§ 4.7.1

Calculate BRG to 705A

from supplied coordinates

§ 4.7.1.1

Instrument height (HI)

Target height (HT), prism constant

Observation, booking,

§ 4.3.1

Backsight (RO), DH1, DH2, DH3, D/S134

Pick up trees, paths in site area surrounding

CEM_1 to CEM_4 to bench wall.

Recording of TS observations

Booking

§ 4.6.2

Field data reduction:

§ 4.7

Intersection by distance: calculations

§ 6.3.3

DTM observations

§ 6.3.5

Control point data: grid as supplied

360° mini-prisms set over points.

705A: E408, N7491, RL from FP1.

C400: E392, N7428, RL from FP1.

Recording of Practical Tasks:

ALL observations are to be entered in supplied Field Book.

Record readings on provided **Traverse** booking sheet in the FB. Records must be NEAT.

- Record angles to second, distances in metres to millimetre level (3DP).
- Ht instrument, target in metres to mm.

Sequence of Practical Tasks

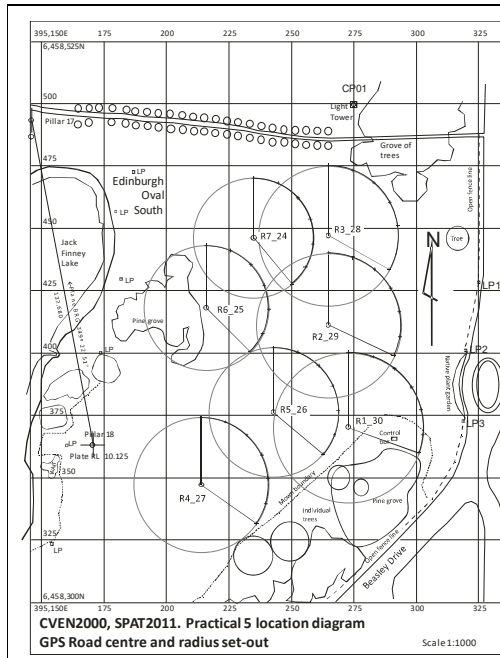
ALL observations to be entered in Field Book.

Record as provided on FB booking sheet

Height of instrument, height of targets at 705A, C400

1. Set and level TS over designated control point (CEM_1 – CEM_8)
2. Draw station diagrams.
3. Set Prism Constant (PC) as PC +3.
4. Set and align TS to 705A
5. Set HAR to read TRUE AZIMUTH using H.ANG function on TS § 4.2.4.2
6. Establish **Back Sight** feature. Observe HAR only for reference alignment to BS on FL.
7. Read and record in sequence: Slope distance, ZA, HAR
FL: 705A
FL: C400
FR: C400
FR: 705A
FR: BS. HAR only.
8. Change TS PC to PC -30
9. Set prism on prism pole and set to height of instrument (HI)
10. Observe and book on **Face Left only**: ZA, HAR, Slope Distance for:
D/S134 (comparison of level from Prac 1 and 2)
DH1, DH2 and DH3 for calculation of seam level.
11. Using the supplied reflector held at approximately height of instrument (HI), continue the topographic survey of features; path edges, trees, bench wall. (topography plan and levels for excavation).

A4-5 Civil and Mining Engineers. Practical 5



Field examination

Appendix

A2-4

Description of task. FP 5

A2-4-1

Set-out and check assigned circular curve.
use allocated sprinkler pins for marking curve.

Field work.

Locate PC and centre point (O) by hand-held **GPS**
from given coordinates of assigned curve.

MGA coordinates

use 25mm pegs.

Set-up TS and conduct examination tasks as specified for assigned curve.

A2-4-1

Mini prism constant correction **PC-20mm**

Orientation as specified to radius centre.

Calculate tangent distance

set out and mark **IP** by TS and **prism**

Set out and mark curve by TS bearing and **taped**
distance

Inform examiner on completion of set-out

Examiner will evaluate curve using TS.

Check chainage points,

check radius; check in, through and out chords
observations with GPS, check mis-closures.

Check hand-held GPS at known control point.

Set and level Total Station (TS) over designated PC

Group: _____ **Curve no.** _____. **MGA E:** _____, **MGA N** _____

1. Set-up

- Set BRG 360° to centre on face left (**FR**). Use 0_SET on FR and plunge TS to point 360 on FL.
- Set out and mark tangent distance to **IP** using mini-prism.
- Set out chainages using given BRG and distance, by 60m cloth tape. Mark with sprinkler flags.

2. Concurrent with set-out, other team members

Calculate tangent distance to **IP**

BRG _____

TD _____

chainage, coords IP

CH_{IP} _____ . **E** _____, **N** _____

Arcs and chords:

In arc _____, **in chord** _____

through arc _____, **through chord** _____

out arc _____, **out chord** _____

3. On completion of chainage set-out; **before dismantling TS setup.**

4. Advise examiner that set-out complete so that task can be evaluated.

- Team to measure and record
radius to each chainage
chord distance **between** each chainage point.
- calculate errors in both radii and individual chord distances.

5. Begin GPS pickup of chainage points and Pillar 18.

- Record last 3 digits of MGA coordinates of each point, include TP1 and centre point.
- Record last 3 digits of MGA coords and height at Pillar 18. Take 6 observations, average.
Compare with known coordinates.
- Calculate misclose distances between set-out and observed coordinates.
- Present completed calculations to examiner for evaluation and marking.

The exercise relies on teamwork within the group; some working with the TS on set-out, others performing pre-calculations, check measurements, observations with GPS, calculations of misclose distances. Rotate tasks.

A5 Survey Calculations on the HP 10s+, HP300s+

A5-1 Introduction

The HP 10s+ Scientific Calculator is the calculator specified for use in engineering examinations. Consult your user manual, not particularly informative, on the basic functions of the calculator.

A new model, the HP300s+, has been introduced. It features a 3-line display. The screen does not provide enough contrast for easy viewing. The use of mid blue printing on a shiny black case makes for difficult viewing. The comma key, $\square$, is now $\text{SHIFT } \square$. Access to the Pol and Rec functions is now via $\text{SHIFT } \square$ and $\text{SHIFT } \square$. The y , x and r , θ components are stored in the X and Y memory registers.

HP10s+. Set-up for surveying.

Getting up the calculator ready for survey computations involves the MODE key for setting base parameters.

First press MODE 1 COMP for computations,

Next press MODE 1 Deg for degrees mode,

Third press MODE 1 Fix, press 1 for

Fix 0*9?

choose 6 to maintain precision if you are not using memory locations for Ans storage.

A5-1-1 Time/angle Calculations:

You can use the $\square$ key to manipulate angles. The display uses the degree $^\circ$ symbol as the unit (minutes, seconds) separator. ($125^\circ 28' 37''$).

$125^\circ 28' 37''$ is input as 125 $\square$ 28 $\square$ 37 $\square$. The top line, the display **input** line, will show $125^\circ 28' 37''$. Pressing $\square$ puts $125^\circ 28' 37''$ in the second line as the **output**, or result line. Importantly, it is the line that fills the Ans memory, after the $\square$ key is used. Using the $\square$ key will toggle the Ans line between $125^\circ 28' 37''$ and 125.476944 , but the Ans memory only holds $125^\circ 28' 37''$. This means you can't convert DMS to decimal degrees to use for following calculations (converting to radians, say).

A5-1-2 DMS to Decimal Degrees: on Ans line.

The $\square$ key toggles between Degrees, Minutes,

Seconds (DMS) and decimal degrees

display on the Ans line: $125^\circ 28' 37'' \square 125.476944$.

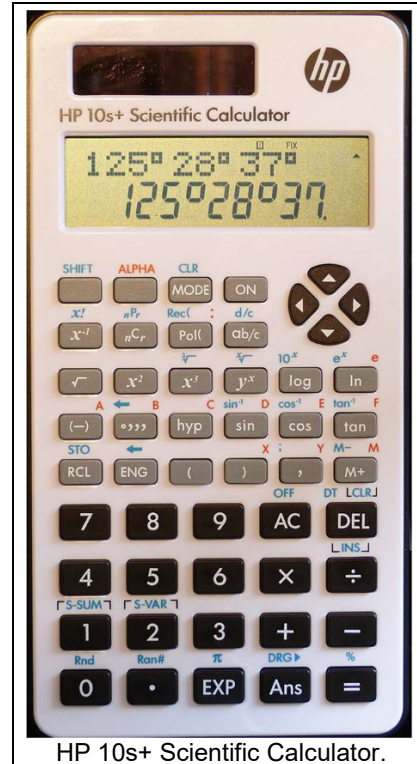
The degrees display expressed as degrees, minutes, seconds ($125^\circ 28' 37''$) returns an Ans to any problem in the same format: degrees, minutes, seconds.

If you input **decimal** degrees, you can convert them to DMS after pressing $\square$ to put them in the Ans line, then use $\square$ to toggle to DMS.

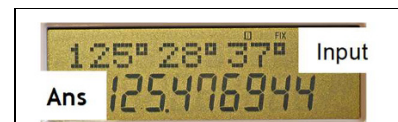
$125.4769 \square 125.4769 \square 125^\circ 28' 36.8''$,

but the Ans memory still holds the decimal input, 125.4769.

If you want to convert DMS to decimal and use the decimal value in chain calculations you must do it on the input line: $125+28/60+37/3600 \square 125.476944$, which will be in the Ans buffer.



HP 10s+ Scientific Calculator.



A5-1-3 Degrees to Radians:

A nasty, tricky conversion with this calculator, and there are several ways to handle it:

1. The old faithful: $\text{Deg} \times \pi \div 180$ [=] unfortunately it provides a funny looking answer if you use the $^{\circ}'''''$ input:
 $125^{\circ}28'37'' \times \pi \div 180$ [=] $2^{\circ}11'23.95$ This is, in fact, the correct answer to the problem of dividing $125^{\circ}28'37''$ by 57.29578 ($180/\pi$). It is expressed in degrees, minutes, seconds! However, it is incorrect in this case because radians are decimals. You have to use the $^{\circ}'''$ key to get 2.189986 , the correct answer in radians.
2. The new faithful, using a memory [STO] (SHIFT RCL) in a memory location A - D, to store the number of degrees in a radian. 57.29578 ($\pi/180$).
 $180 \div \pi$ [=] 57.29578 [STO] (SHIFT RCL) A
 $125^{\circ}28'37'' \times \text{[A]}$ [=] $2^{\circ}11'23.95$ then use the $^{\circ}'''$ key to get 2.189986 .
3. Frogging around using the DRG key (SHIFT Ans), probably forget it.
4. Work in decimal degrees and use either 1 or 2 above. But you must convert DMS to decimal degrees to decimals first;
 $125 + 28 \div 60 + 37 \div 3600$ [=] 125.476944 , then $\text{[Ans]} \div \text{[A]}$ [=] 2.189986 ,
5. Consult h20331.www2.hp.com/Hpsub/downloads/10s_03_operating_modes.pdf.

A5-1-4 Why Radians to Degrees?

A little bit easier, and where is it needed? With circular curves, that's where.

Curve or arc length is an expression of the distance around the arc of radius generated by an angle between the end radii of the arc.

Arc distance, $S = R \theta$ (radians) where θ is the arc deflection angle (at the centre).

If we are given radius, R, and arc length, S, then:

arc deflections, θ , = S/R expressed in **radians**.

In this book, concerning circular curves, we tend to use the symbol Δ (delta) because it denotes the **deflection** angle between two tangents joined by a circular curve. It is the same as the **arc** deflection angle at the centre between the two radii joining the PC and the PT.

Radians to Degrees:

The radian answer will come from a computations like, for example:

$$328.498 \div 150 \text{ [=]} 2.189986.$$

The usual conversion with this calculator :

1. The old faithful: $\text{Radians} \times 180 \div \pi$ [=]
 $\text{[Ans]} \times 180 \div \pi$ [=] 125.460556 is the answer in decimal degrees.
 It can be converted to DMS using the $^{\circ}'''''$ $125^{\circ}28'37''$
2. The new faithful, using a memory [STO] (SHIFT RCL) in a memory location A - D, to store the number of degrees in a radian. 57.29578 ($\pi/180$).
 $180 \div \pi$ [=] 57.29578 [STO] A. then, from the previous answer (scroll to it):
 $\text{[Ans]} \times \text{[A]}$ [=] 125.460556 , then use the $^{\circ}'''''$ key to get $125^{\circ}28'37''$. But the value in Ans will still be in decimal degrees.

A5-2 Vector Conversion and Axes Rotation

Normal polar vectors are represented in a mathematically oriented (anticlockwise) frame. Zero-degree axis is along the + x axis, pointing horizontally to the right. The vector rotates anticlockwise to 90° through the + y axis, 180° through - x, 270° through - y and back to 360° at +x.

Surveying polar vectors are represented in an azimuthally (clockwise) oriented frame. Zero degrees points vertically up to North (N, +y). It then rotates **clockwise** to 90° on the East axis

(E, +x), 180° on the South axis (-N, -y), 270° on the West axis (-E, -x) and returns to 360° on the N axis.

To use the SHIFT **Pol()** and **Rec()** functions we have to swap our rectangular coordinate orientation to make the calculator think it's working in math orientation. The polar coordinates don't change, it's only the rectangular coordinates that need understanding.

A5-2-1 Them (the US) and Us (Australia)

A word on the difference between rectangular/polar coordinate order using the

SHIFT **Pol()** and **Rec()** functions in azimuth vector mode.

The HP10S+ (and all its ilk) conform to the American order of:

N (Latitude distance, y), E (Departure distance, x) for rectangular coordinates and:

r (Distance), θ (whole circle bearing) for polar coordinates.

A5-2-2 The Importance of the SIGN of the Rectangular Coordinates

It is imperative that you maintain the SIGN of the N and E in the rectangular coordinates. The sign is necessary to produce the required ATN2() angle, and vital to produce the ATAN() angle. Especially for the ATAN, because the answer; positive in two, negative in two quadrants, is the angle referenced the North/South axis. Combined with the SIGN of the latitude difference (ΔN or Δy), the BEARING of the vector can be ascertained. Refer to § 4.3.1 Manipulating Vectors for revision.

ATAN() function marked as $\tan^{-1}$ (SHIFT **tan^F**):

In using the ATAN() function examine the SIGN of the δN .

If δN is POSITIVE the ANGLE is in quadrants I and IV. If necessary, ADD 360°.

If δN is NEGATIVE, the angle is in quadrants II or III, so ADD 180°.

ATAN2() function.

The returned angle (stored in memory F (RCL **tan^F**)) is either positive, (0 clockwise to 180°) or negative (0 anticlockwise to -180°)

if positive it is the BEARING or,

if negative, the BEARING is $360^\circ + \text{RCL } \tan^F$.

A5-3 Converting between Polar and Rectangular Coordinates using Function Keys

A5-3-1 Trigonometric Conversion to Rectangular Coordinates

The examples in this manual, converting bearing/distance to dE, dN via the standard method:

$$dE = \text{distance} \times \sin(\text{brg})$$

$$dN = \text{distance} \times \cos(\text{brg})$$

is tedious and error prone from having to key in the distance and bearing twice, and record each answer.

A5-3-2 Trigonometric Conversion using the **Rec()** Function Key

To convert from the polar BRG/DIST coordinates to dE, dN you should make use of the **Rec()** function; (SHIFT **Pol()**). However, there are a couple of gotchas with these two functions:

1. the input is in the sequence **r, θ** , i.e. distance, bearing. Note the comma separator.
2. the answers, dE and dN, are held in two memory locations, E and F.

cos^E holds the dN and **tan^F** holds dE. The display show the dN value (memory E)

dE is accessed by recalling memory F (RCL **tan^F**)

dN is accessed by recalling memory E (RCL **cos^E**)

Example: Convert the following BRG/DIST to dE, dN:

BRG 125° 15' 20", distance 102.536m.

Conventionally:

$102.536 \times \sin 125^\circ 15' 20'' = 83.729420$, dE, then re-keying for dN

$102.536 \times \cos 125^\circ 15' 20'' = -59.186278$, dN.

Note the requirement to repeatedly key the same data.

Using **Rec()**: input the distance, the comma, then the BRG in either DMS, using the **°** key for each value of D, M and S; or use decimal degrees.

There is no need to close the parenthesis **)** key before enter, **=**:

$\text{Rec}(102.536, 125^\circ 15' 20'' = -59.186278$, dN, then, to retrieve dE

$\text{RCL } \tan^F = 83.729420$, dE.

dN and dE will be retained in **cos^E** and **tan^F** until overwritten.

A5-4 Converting between Rectangular and Polar Coordinates Using Function Keys

A5-4-1 Trigonometric Conversion to Polar Coordinates

The examples in this manual, converting dE, dN to bearing/distance via the standard,

$\theta = \text{Atan}(\tan^{-1})(dE/dN)$, converted to a whole circle bearing in the correct quadrant.

$r = \text{Sqrt}(\sqrt{(dE^2 + dN^2)})$, Pythagoras, is also tedious and error prone from having to key in the dE and dN twice, and record each answer.

Conventionally we calculate and record the dE and dN between two points, then go ahead through a number of steps to get the result.

Let's try an example and attack it three ways: twice conventionally, then using the **Pol()** function key.

$E_A = 135.872$ $N_A = 7450.124$

$E_B = 125.126$ $N_B = 7540.067$

$dE_{AB} = -10.746$ $dN_{AB} = 89.943$.

Finding the tangent,

using $-10.746 \div 89.943 = -0.119476$,

find **ATAN()**, $\tan^{-1}\text{Ans} = -6.813156$,

it is in quadrant 4. (dE -ve, dN+ve), so add 360:

$360 + \text{Ans} = 353.186844$ **°** $353^\circ 11' 12.6$

Then, using Pythagoras for distance, using the **x²** key and the **√** keys

$(-10.746)^2 + 89.943^2 = 8205.219765$. (Note the use of parentheses for the negative dE_{AB}).

$\sqrt{\text{Ans}} = 90.5827$ (Note: $-10.746^2 + 89.943^2 = 7974.2667$; and $\sqrt{\text{Ans}} = 89.2987$).

BIMDAS at work.

Shortcutting it a bit, and knowing the angle is in quadrant 4 (+360)

$360 + \tan^{-1}(-10.746 \div 89.943) = 353.186844$ **°** $353^\circ 11' 12.6$

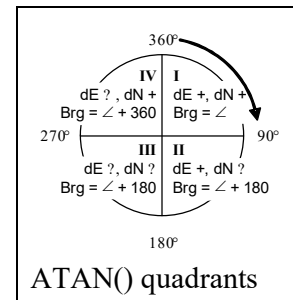
Straight in to Pythagoras: $\sqrt{(-10.746)^2 + 89.943^2} = 90.5827$

or, shorter (?) by entering the E and N values directly

$360 + \tan^{-1}((125.126 - 135.872) \div (7540.067 - 7450.124)) =$

353.186844 **°** $353^\circ 11' 12.6$

$\sqrt{((125.126 - 135.872)^2 + (7540.067 - 7450.124)^2)} = 90.5827$



A5-4-2 Trigonometric Conversion using the $\boxed{\text{Pol}}$ Function Key

Using the $\boxed{\text{Pol}}$ function key: – converting rectangular(dN, dE) to polar (r, θ)

Note: we have to input dN, then dE,

$\boxed{\text{Pol}}(89.943 \boxed{,} -10.746) \boxed{=}$ 90.5827, distance, r,
then, to retrieve bearing, θ ,

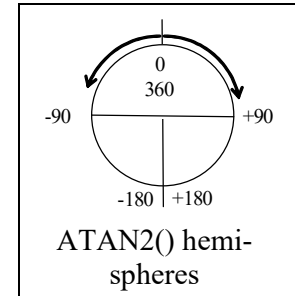
RCL $\boxed{\tan^F}$ F= -5.813155, bearing, θ , is negative,
this means θ is in the western hemisphere, add 360;
360+ RCL $\boxed{\tan^F}$ $\boxed{=}$ 353.186844 $\boxed{0^\circ}$ 353° 11' 12.6

The $\boxed{\text{Pol}}$ function uses the ATN2(dN,dE) function, providing a bearing between

0 and +180 clockwise for quadrants 1 and 2 and
0 and -180 anti-clockwise for quadrants 4 and 3.

Thus a negative result requires the addition of 360°.

r and θ will be retained in $\boxed{\cos^E}$ and $\boxed{\tan^F}$ until overwritten.



A5-4-3 The ATAN2() Function in Spreadsheets.

ATAN2(dN,dE) is a handy function in Excel, note the order: dN, dE the best way to get decimal degrees in the correct bearing is to use

=MOD(DEGREES(ATAN2(dN, dE)),360).

A5-5 Using the $\boxed{\text{M+}}$ Key for Accumulate Operations

A few sets of calculations lend themselves to the accumulation of results.

- Rise & Fall levelling,
- Areas by coordinate cross multiplication.

Using the example on Table A3-5.1,

$\text{RL}_{\text{BM}} = 10.125$

Load the memory with RL_{BM} :

10.125 $\boxed{\text{STO}}$ $\boxed{\text{M+}}$

this puts 10.125 in M+

Get the first rise/fall:

1.742 - 1.628 $\boxed{=}$ 0.114 (note rise)

$\boxed{\text{M+}}$ puts result in M+ (don't use the = key).

Use RCL $\boxed{\text{M+}}$ M= 0.239 to see RL

Next rise/fall:

1.628 - 1.205 $\boxed{=}$ 0.423 (note rise)

$\boxed{\text{M+}}$ puts result in M+ (don't use the = key).

Use RCL $\boxed{\text{M+}}$ M= 0.662 to see RL.

Next rise/fall:

1.205 - 1.1585 $\boxed{=}$ -0.380

(note fall) $\boxed{\text{M+}}$ puts result in M+ (don't use the = key).

Use RCL $\boxed{\text{M+}}$ M= 0.282 to see RL..

The $\boxed{\text{M+}}$ key is a tricky beast to get accustomed to, but may be useful for chain calculations.

Table A3-5.1

BS	IS	FS	Rise	Fall	RL
1.742				BM	10.125
	1.628		0.114		10.239
	1.205		0.423		10.662
	1.585			0.380	10.282
	1.625			0.040	10.242
	1.550		0.075		10.317
1.580		1.522	0.028		10.345
	1.635			0.055	10.290
	1.640			0.005	10.285
	1.825			0.185	10.100
	1.745		0.080		10.180
		1.892		0.147	10.033
Sum		Sum	Sum	Sum	
3.322		3.414	0.720	0.812	
$\Sigma \text{ BS} - \Sigma \text{ FS}$			$\Sigma \text{ Rises} - \Sigma \text{ Falls}$		
	-0.092		Diff =	-0.092	-0.092

A5-6 Statistics, Mean and Standard Deviations

During the field practical sessions there are occasions when the mean of a set of observations is required. A good check is to evaluate the standard deviation (SD) of the set to form an opinion of the acceptability of the observation, a form of blunder detection.

Field practical 4, establishment of control, produces four horizontal, and four vertical, the distances for each leg of the control traverse. A check of the mean and the SD may be warranted to guard against mistakes.

The easiest process for using the statistics functions would require that you calculate the horizontal and vertical distances of each observation using the Rec(function to convert zenith angle (ZA) and slope distance. If both FL and FR observations are taken, then the absolute value need to be assigned to the horizontal distance, otherwise the FR distance will be negative.

($\sin(ZA = 90) = +1$, $\sin(ZA = 270) = -1$).

Using $\text{Rec}(125.257, 89^\circ 30' 45'' = 1.066$, $\text{RCL } \tan^{-1} F = 125.252466$ and making a table for the four observations, the SD mode can be then be initiated.

Set SD mode: **MODE** 2 **=**.

Clear registers (Clr):

CLR (**SHIFT** **MODE** 1 **=**).

Stat clear... **=** 0.0000.

The **M+** key acts as the **DT** key.

n = input count.

125.2525 **M+** n=1.00)

125.2544 **M+** n=2.00)

125.2514 **M+** n=3.00)

125.2503 **M+** n=4.00)

Find mean: S-VAR (**SHIFT** 2) 1 **=** $\bar{x}$ 125.2521

Find standard deviation of mean: S-VAR (**SHIFT** 2) 2 **=** σ_n 0.0015

Obs	ZA	S dist	H dist	V dist
A – B FL	89° 30' 45"	125.257	125.2525	+1.066
A – B FR	270° 29' 30"	125.259	125.2544	+1.075
B – A FL	90° 29' 30"	125.256	125.2514	-1.075
B – A FR	269° 30' 15"	125.255	125.2503	-1.084

This method will help catch number transposition blunders, although the booker must always be alert for anomalies in the data recording.

A5-7 Concluding remarks

It is hoped that this expansion on the capabilities of the HP 10S+ calculator, mandated for some courses, will be of use in allowing you to explore its vector capabilities.

A5-8 References Appendix A5

1. Hewlett-Packard, HP 10s+ Scientific Calculator User Guide, Hewlett-Packard Development Company, 2012.

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