Solutions Manual

Principles of Electric Machines and Power Electronics

Second Edition

P. C. Sen

Solutions Manual to Accompany

PRINCIPLES OF ELECTRIC MACHINES AND POWER ELECTRONICS

SECOND EDITION

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PREFACE

This text book can be used for either a single semester length course or for a two-semester course sequence. The instructor should feel free to choose the topics for the course. The selection would be guided by the objectives of the course and the curriculum. The flexible organization of the material allows sections and entire chapters to be skipped without loosing continuity. The instructor may also choose the sequence of coverage - for example, some may prefer to discuss synchronous machines before induction machines.

It is also possible to offer a short course in power electronics based on Chapter 10 on Power Semiconductor Converters and sections on motor speed control in Chapters 4, 5, 6 and 7.

The Second Edition incorporates nearly twice as many problems as the First Edition. When assigning problems, the instructor may select some from this text book, while others can be freshly formulated or selected from another book. Students may be critical if all problems are assigned from the same text book.

P.C. Sen

September, 1996

CONTENTS

CHAPTER 1

$\boxed{1.1}$ (a) $H = \dfrac{Ni}{l} = \dfrac{250 \times 100}{0.5} = 50{,}000 \text{ At/m}.$

$$B = \mu_0 H = 4\pi 10^{-7} \times 50{,}000 = 0.062832 \text{ T}$$

(b) $L = \dfrac{N\phi}{i} = \dfrac{NB\pi r^2}{i} = \dfrac{N\pi r^2}{i}\mu_0 \dfrac{Ni}{l} = \dfrac{N^2\mu_0 \pi r^2}{l}$

$$= \frac{250^2 \times 4\pi 10^{-7} \times 3.1416 \times (2.5 \times 10^{-2})^2}{0.5} \text{ H}$$

$$= 308.43 \ \mu\text{H}$$

$\boxed{1.2}$ (a) $\mathcal{R}_{thick} = \dfrac{70 \times 10^{-2}}{2000 . 4\pi . 10^{-7} \times 15 \times 10 \times 10^{-4}}$

$$= 18568.03$$

$$\mathcal{R}_{thin} = \frac{80 \times 10^{-2}}{2000 . 4\pi . 10^{-7} \times 10 \times 10 \times 10^{-4}}$$

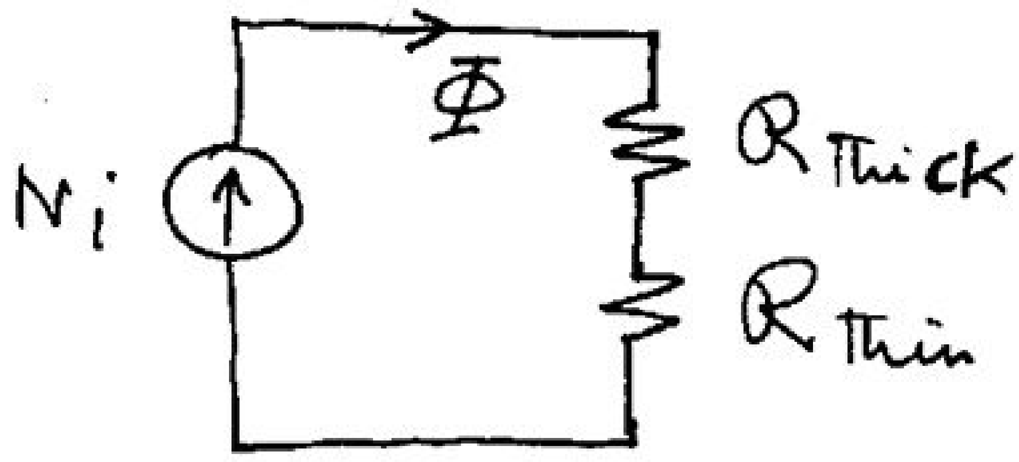
$$= 31830.91$$

$$\mathcal{R}_{thick} + \mathcal{R}_{thin} = 50398.94$$

$$\Phi = \frac{500 \times 1}{50398.94} = 0.009921 \text{ Wb}.$$

(b) $B_{thick} = \dfrac{0.009921}{150 \times 10^{-4}} = 0.6614 \text{ T}.$

$$B_{thin} = \frac{0.009921}{100 \times 10^{-4}} = 0.9921 \text{ T}$$



1.3 $B_{thick} = \frac{0.012}{15 \times 10 \times 10^{-4}} = 0.8\,T$

$B_{thin} = \frac{0.012}{10 \times 10 \times 10^{-4}} = 1.2\,T$

$H_{Thick} = \frac{0.8}{2000 \times 4\pi \times 10^{-7}} = 318.31\ At/m$

$H_{thin} = \frac{1.2}{2000 \times 4\pi \times 10^{-7}} = 477.46\ At/m$

$F = 318.31 \times 2 \times 35 \times 10^{-2} + 477.46 \times 2 \times 40 \times 10^{-2}$

$= 604.79\ At.$

$i = \frac{604.79}{500} = 1.2096\ A.$

1.4 (a) The two mmfs aid each other.

$F = 600 \times 0.28 + 300 \times 0.56 = 336\ At.$

$H = \frac{336}{2\pi \times 20 \times 10^{-2}} = 267.38\ At/m.$

$B = 1.14\,T \rightarrow$ from Fig. 1.7

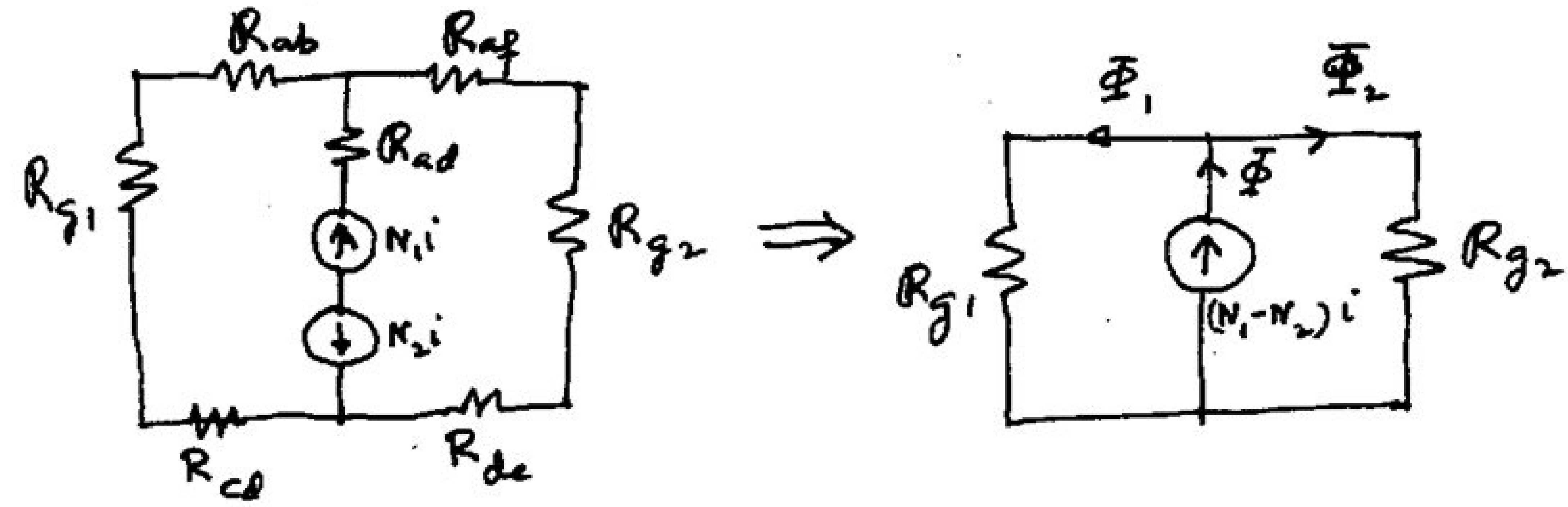
(b) $\Phi = 1.14 \times 2 \times 2 \times 10^{-4} = 0.000456\ Wb.$

(c) $\mu_r = \frac{B}{\mu_0 H}$

$= \frac{1.14}{4\pi \times 10^{-7} \times 267.38}$

$= 3393$

$\boxed{1.5}$ MMFs of the two coils oppose each other.



$$A_{g1} = A_{g2} = 2.5 \times 2.5 \times 10^{-4} = 6.25 \times 10^{-4}\ m^2$$

$$\mathcal{R}_{g1} = \frac{0.05 \times 10^{-2}}{4\pi\, 10^{-7} \times 6.25 \times 10^{-4}} = 0.637 \times 10^{6}\ At/Wb.$$

$$\mathcal{R}_{g2} = \frac{0.1 \times 10^{-2}}{4\pi\, 10^{-7} \times 6.25 \times 10^{-4}} = 1.274 \times 10^{6}\ At/Wb$$

$$\Phi_1 = \frac{(700-200)\,0.5}{0.637 \times 10^{6}} = 0.392 \times 10^{-3}\ Wb.$$

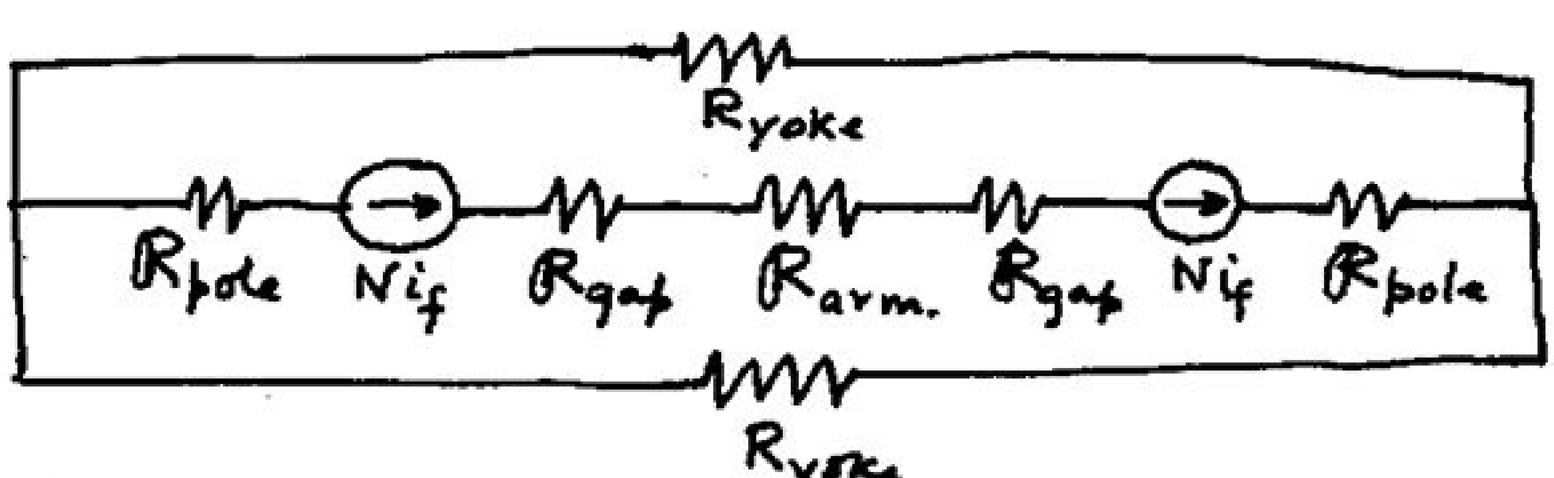
$$\Phi_2 = \frac{500 \times 0.5}{1.274 \times 10^{6}} = 0.196 \times 10^{-3}\ Wb$$

$$\Phi = \Phi_1 + \Phi_2 = 0.588 \times 10^{-3}\ Wb$$

$$B_{g1} = \frac{0.392 \times 10^{-3}}{6.25 \times 10^{-4}} = 0.627\ Wb/m^2$$

$$B_{g2} = \frac{0.196 \times 10^{-3}}{6.25 \times 10^{-4}} = 0.3135\ Wb/m^2$$

$\boxed{1.6}$ (a)



(b) Armature : For Si-steel, $H = 275\ A{\cdot}t/m$ for $B = 1.1T$

$$F_{arm.} = 275 \times 0.2 = 55\ A{\cdot}t$$

Airgap : $H = \frac{1.1}{4\pi 10^{-7}} = 8.75 \times 10^5$ A.t/m.

$$F_{gap} = 2 \times 0.1 \times 10^{-2} \times 8.75 \times 10^5 = 1750 \text{ A.t}$$

Pole : For cast steel, $H = 800$ A.t/m for $B = 1.1$ T

$$F_{pole} = 2 \times 10 \times 10^{-2} \times 800 = 160 \text{ A.t}$$

Yoke : Cross-section of yoke is half of that of armature. Hence flux density is same in yoke as in the armature. MMF for upper yoke is same as the MMF for lower yoke.

$$F_{yoke} = 800 \times \frac{160}{2} \times 10^{-2} = 640 \text{ A.t}$$

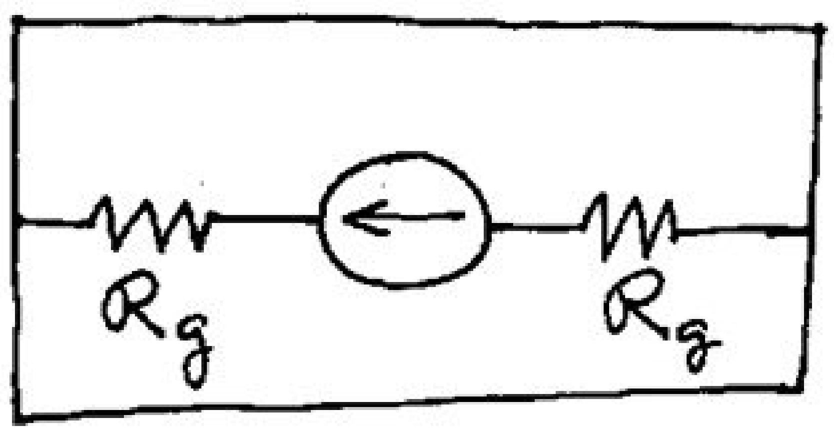
Total mmf required is

$$F = 55 + 1750 + 160 + 640 = 2605 \text{ At}$$

Ampere-turn per pole $= 1302.5$.

(c) Armature flux, $\Phi = BA = 1.1 \times 400 \times 10^{-4} = 0.044$ Wb

$\boxed{1.7}$ (a)

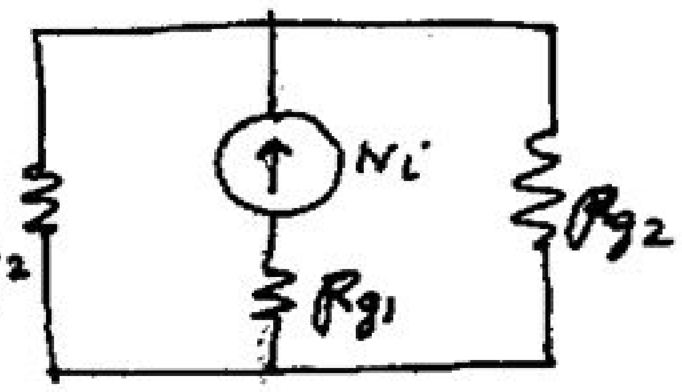


(b) $$R_g = \frac{l_g}{\mu_0 A_g} = \frac{0.25 \times 10^{-3}}{4\pi 10^{-7} \times 500 \times 10^{-4}} = 39.7886 \times 10^3 \text{ At/Wb.}$$

$$\Phi = \frac{Ni}{2 R_g} = \frac{500 \times 5}{2 \times 39.7886 \times 10^3} = 0.0314 \text{ Wb.}$$

$$B_g = \frac{0.0314}{500 \times 10^{-4}} = 0.628 \text{ T.}$$

$\boxed{1.8}$ $B = 1.4\,T$ throughout. $H_c = 0$.



$$Ni = H_{g1} g_1 + H_{g2} g_2$$

$$H_{g1} = H_{g2} = \frac{B}{\mu_0}$$

$$g_1 = g_2 = g$$

$$Ni = 2\frac{B}{\mu_0} g$$

$$g = \frac{\mu_0 Ni}{2B} = \frac{4\pi 10^{-7} \times 500 \times 20}{2 \times 1.4} \rightarrow 4.5\ \text{mm}$$

$\boxed{1.9}$ (a) Mean length of core, $l_c = 2\pi \frac{(10+6)}{2} \times 10^{-2}\,m = 0.503\,m$

For cast steel $H_c = 1000\ At/m$ at $B = 1.2T$.

$$i = \frac{H_c l_c}{N} = \frac{1000 \times 0.503}{200} = 2.51\ A.$$

(b) $$A_c = \pi (2 \times 10^{-2})^2 = 1.26 \times 10^{-3}\ m^2$$

$$\Phi_c = A_c B_c = 1.26 \times 10^{-3} \times 1.2 = 1.51 \times 10^{-3}\ Wb$$

(c) $$Ni = H_c l_c + H_g l_g = H_c l_c + \frac{B_g}{\mu_0} l_g$$

$$i = \frac{1000 \times 0.503}{200} + \frac{1.2 \times 2 \times 10^{-3}}{4\pi 10^{-3} \times 200} = 12.06\ A$$

$\boxed{1.10}$ (a) $B = \mu_r \mu_0 H = \mu_r \mu_0 \frac{Ni}{2\pi r}$

$$B_{max} = 2000 \times 4\pi \times 10^{-7} \times \frac{200 \times 2.0}{2\pi . 6 \times 10^{-2}} = 2.666T \Big|_{\text{at inside.}}$$

$$B_{min} = 2000 \times 4\pi \times 10^{-7} \frac{200 \times 2.0}{2\pi . 10 . 10^{-2}} = 1.6\ T \Big|_{\text{at outside}}$$

(b) $$\Phi = \int B\,dA = \int \mu H\,(10-6)\,10^{-2}\,dr$$

$$= \int \mu \frac{Ni}{2\pi r} \times 0.04\,dr = \frac{\mu Ni \times 0.04}{2\pi} \ln\frac{r_2}{r_1}$$

$$= \frac{2000 \times 4\pi \times 10^{-7} \times 200 \times 2.0 \times 0.04}{2\pi} \ln\frac{10}{6} = 3.269 \times 10^{-3}\ Wb.$$

(c) $$B\Big|_{\text{centre}} = 2000 \times 4\pi \times 10^{-7} \times \frac{200 \times 2.0}{2\pi \times 8 \times 10^{-2}} = 2.0\,T.$$

$$B\Big|_{\text{avg}} = \bar{B} = \frac{\Phi}{A} = \frac{3.269 \times 10^{-3}}{4 \times 4 \times 10^{-4}} = 2.0432\,T.$$

1.11 (a)

(b) Mean length $l_c = 4 \times 30 = 120$ cm.

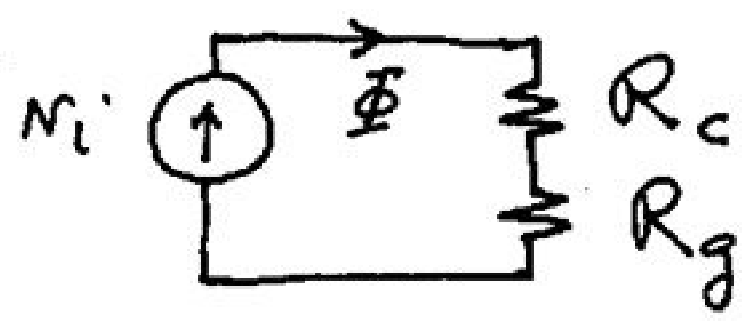
$$R_c = \frac{l_c}{\mu_c A_c} = \frac{120 \times 10^{-2}}{2000 \times 4\pi \times 10^{-7} \times 10 \times 5 \times 10^{-4}} = 95.49 \times 10^3 \text{ At/Wb.}$$

$$\Phi = \frac{400 \times 1.5}{95.49 \times 10^3} = 6.2832 \times 10^{-3} \text{ Wb.}$$

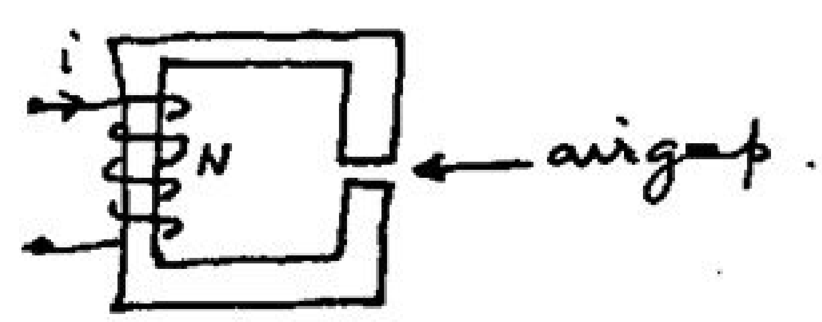
$$B = \frac{6.2832 \times 10^{-3}}{10 \times 5 \times 10^{-4}} = 1.257 \text{ T.}$$

(c) $$L = \frac{N^2}{R_c} = \frac{400^2}{95.49 \times 10^3} = 1.6756 \text{ H.}$$

1.12 (a)



$\Rightarrow$



(b) $$R_c = \frac{(120-1) \times 10^{-2}}{2000 \times 4\pi \times 10^{-7} \times 50 \times 10^{-4}} = 94.697 \times 10^3 \text{ At/Wb.}$$

$$R_g = \frac{1 \times 10^{-2}}{4\pi \times 10^{-7} \times 50 \times 10^{-4} \times 1.1} = 1446.86 \times 10^3 \text{ At/Wb.}$$

$$\Phi = \frac{400 \times 1.5}{(94.697 + 1446.86) \times 10^3} = 0.3892 \times 10^{-3} \text{ Wb.}$$

$$B = \frac{0.3892 \times 10^{-3}}{50 \times 10^{-4}} = 0.078 \text{ T}$$ $\rightarrow$ significantly reduced due to airgap.

(c) $$L = \frac{400^2}{R_c + R_g} = \frac{400^2}{1541.56 \times 10^3} = 0.1038 \text{ H.}$$

1.13

$$R_g = \frac{l_g}{\mu_0 A_g}, \quad R_c = 0.$$

$$L = \frac{N^2}{R_g} = \frac{N^2 \mu_0 A_g}{l_g}$$

$$l_g = \frac{N^2 \mu_0 A_g}{L} = \frac{100^2 \times 4\pi 10^{-7} \times 5 \times 10^{-4}}{10 \times 10^{-3}}$$

$$= 0.1257 \times 10^{-3} \text{ m}$$

$$= 0.1257 \text{ mm.}$$

$\boxed{1.14}$ (a) $F = H_c l_c + H_g l_g$, $B_c = B_g = 0.5\ T$

$l_g = 0.15 \times 10^{-2}\ m$, $l_c = (4 \times 8 - 0.15)\,10^{-2} = 0.3185\ m$

For cast steel, $H_c = 350\ A.t/m$ for $B_c = 0.5\,T$.

$$F = 350 \times 0.3185 + \frac{0.5}{4\pi 10^{-7}} \times 0.15 \times 10^{-2} = 708.3\ A.t$$

If mmfs of coils aid each other,

$$i = \frac{708.3}{350 + 150} = 1.01\,A$$

If mmfs of coils oppose each other,

$$i = \frac{708.3}{350 - 150} = 3.54\ A$$

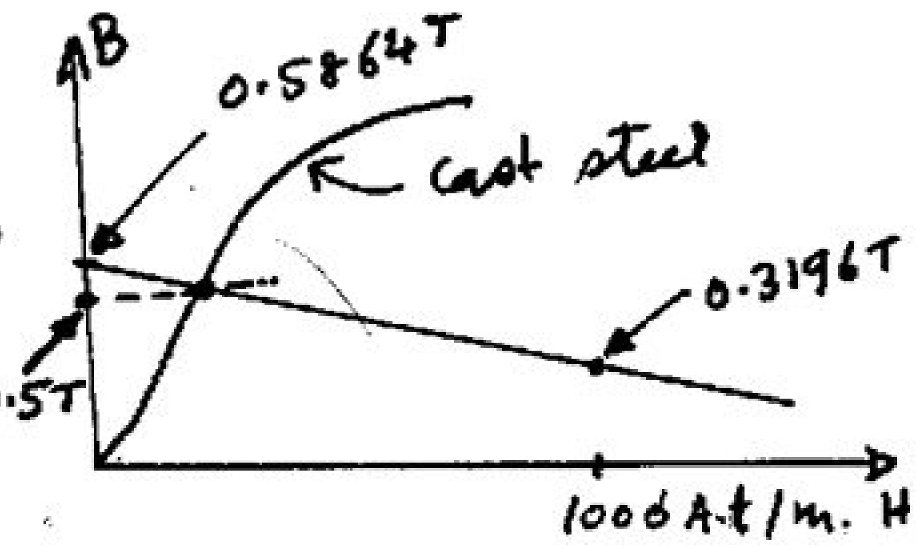
(b) $$L_A = \frac{\lambda_A\,(= N_A B A)}{i_A \text{ to produce } \lambda_A} = \frac{350 \times 0.5 \times 4 \times 10^{-4}}{708.3/350} = 34.6\,mH$$

$$L_B = \frac{\lambda_B\,(= N_B B A)}{i_B} = \frac{150 \times 0.5 \times 4 \times 10^{-4}}{708.3/150} = 6.35\ mH$$

Note that $\frac{L_A}{L_B} = \frac{34.6}{6.35} = \left(\frac{N_A}{N_B}\right)^2$

(c) Load line method.

Intersection on B-axis



$$c = \frac{N i \mu_0}{l_g} = \frac{350 \times 2 \times 4\pi 10^{-7}}{0.15 \times 10^{-2}}$$

$$= 0.5864\,T.$$

Slope, $m = -\mu_0 \frac{l_c}{l_g} = -4\pi 10^{-7} \frac{0.3185}{0.15 \times 10^{-2}}$

$$= -0.2668 \times 10^{-3}$$

Load line equation,

$$B = 0.5864 - 0.2668 \times 10^{-3} \times H$$

For $H = 1000\ A.t/m \longrightarrow B = 0.5864 - 0.2668 = 0.3196$

From intersection of B−H curve & load line.

$$B \simeq 0.5\,T$$

$\boxed{1.15}$ (a) $B_v = 0.6\,T$, $B_h = \frac{1.5}{1.0} \times 0.6 = 0.9\,T$.

$l_v = 7 \times 2 = 14$ cm, $l_h = 7.5 \times 2 = 15$ cm.

$H_v = 400 \times \frac{0.6}{0.8} = 300$ A·t/m.

$H_h = 400 + \frac{1000-400}{2} = 700$ A·t/m.

$F = H_h l_h + H_v l_v = 700 \times 0.15 + 300 \times 0.14 = 147$ A·t

$F = I_1 N_1 + I_2 N_2 = 2 \times 200 + I_2 \times 100 = 147$ A·t

$I_2 = -2.53$ A

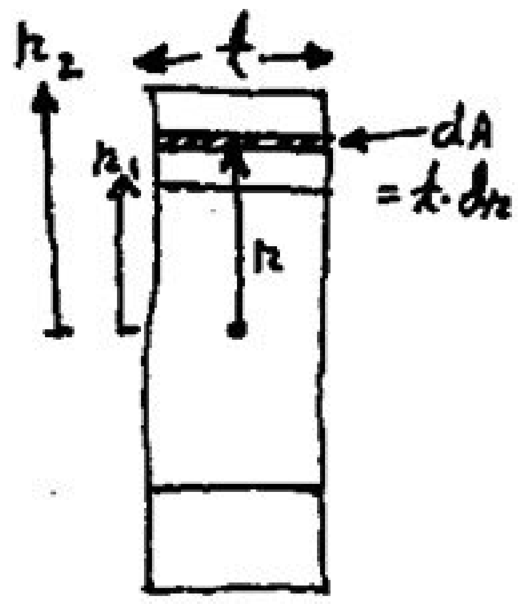
(b) $F = 0.5 \times 200 + 1.96 \times 100 = 296$ A·t. Obviously, flux density will be higher now.

Solve by Trial & Error:

B_v	B_h	H_v	H_h	$F = H_h l_h + H_v l_v$
0.8	1.2	400	1600	$= 240 + 56 = 296$ A·t.

$\Phi = B_v A_v = 0.8 \times (1.5 \times 10^{-2})^2 \rightarrow 0.18$ mWb.

$\boxed{1.16}$ (a) $H_r = \frac{Ni}{2\pi r}$, $B_r = \mu \frac{Ni}{2\pi r}$



$$\lambda = N\Phi = N\int B_r\, dA = N\int B_r\, t\, dr$$

$$= \frac{\mu N^2 i}{2\pi} t \int_{r_1}^{r_2} \frac{dr}{r} = \frac{\mu N^2 i t}{2\pi} \ln\left(\frac{r_2}{r_1}\right)$$

$$L = \frac{\lambda}{i} = \frac{\mu N^2 t}{2\pi} \ln\left(\frac{r_2}{r_1}\right)$$

$$\mu = \frac{1.0}{100} = 10^{-2}$$

$$L = \frac{10^{-2} \times 100^2 \times 3 \times 10^{-2}}{2 \times 3.1416} \ln\left(\frac{6}{4}\right) \rightarrow 193.6 \text{ mH}.$$

(b) Inner periphery of the core will saturate first. Hence $H = 100$ At/m at $r = 4$ cm

$$i = \frac{Hl}{N} = \frac{100 \times 2\pi\, 4 \times 10^{-2}}{100} = 0.2513\,A$$

(c) This will occur if $H > 100$ At/m around the outer periphery.

$$i_{min} = \frac{100 \times 2\pi\, 6 \times 10^{-2}}{100} = 0.377\,A$$

1·17 From equation 1·40

$$E = 4.44\, N f A B_{max}$$

$$B_{max} = \frac{E}{4.44\, N f A}$$

If this is substituted in equations 1·33 and 1·34,

$$P_h = K_h f \frac{E^2}{(4.44\, N f A)^2} = K_1 \frac{E^2}{f}$$

$$P_e = K_e f^2 \frac{E^2}{(4.44\, N f A)^2} = K_2 E^2$$

Hysteresis loss: $\frac{P_{h(60)}}{P_{h(50)}} = \left(\frac{E_{60}}{E_{50}}\right)^2 \times \left(\frac{f_{50}}{f_{60}}\right) = \left(\frac{110}{100}\right)^2 \left(\frac{50}{60}\right) = 1.008$

Eddy current loss

$$\frac{P_{e(60)}}{P_{e(50)}} = \left(\frac{E_{60}}{E_{50}}\right)^2 = \left(\frac{110}{100}\right)^2 = 1.21$$

1·18 Area of B-H loop $= 20 \times 2.4 = 48$

Volume of core $= 15 \times 10^{-2} \times 10 \times 10^{-4} = 15 \times 10^{-5}\ m^3$

$$P_h = 15 \times 10^{-5} \times 48 \times 400 = 2.88\ W$$

1·19 $E_{rms} = 4.44\, N f A B_{max}$

$= 4.44 \times 500 \times 60 \times 5 \times 10^{-4} \times 1.2$

$= 79.92\ V.$

1·20 (a) $\Phi = BA \times 0.95 = 1.2 \times 25 \times 10^{-4} \times 0.95 \sin 377t$

$= 28.5 \times 10^{-4} \sin 377t$

$e_1 = N_1 \frac{d\Phi}{dt} = 200 \times 28.5 \times 10^{-4} \times 377 \cos 377t$

$= 214.89 \cos 377t.$

$E_1 = \frac{214.89}{\sqrt{2}} = 151.973\ V.$

(b) $B_{max} = 1.2\,T$, $H_{max} = \frac{1.2}{10000 \times 4\pi \times 10^{-7}} = 95.49\ At/m$

$i_{max} = \frac{95.49 \times 90 \times 10^{-2}}{200} = 0.4297\ A$

$i_1 = 0.4297 \sin 377t.$

(c) $E_2 = \frac{N_2}{N_1} E_1 = \frac{400}{200} \times 151.973 = 303.946\ V.$

→ From equation 1.40 (same flux links both windings)

1.17 From equation 1.40

$$E = 4.44\, N f A B_{max}$$

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Eddy current loss

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$$= 4.44 \times 500 \times 60 \times 5 \times 10^{-4} \times 1.2$$

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1.20 (a) $\Phi = BA \times 0.95 = 1.2 \times 25 \times 10^{-4} \times 0.95 \sin 377t$

$$= 28.5 \times 10^{-4} \sin 377t$$

$$e_1 = N_1 \frac{d\Phi}{dt} = 200 \times 28.5 \times 10^{-4} \times 377 \cos 377t$$

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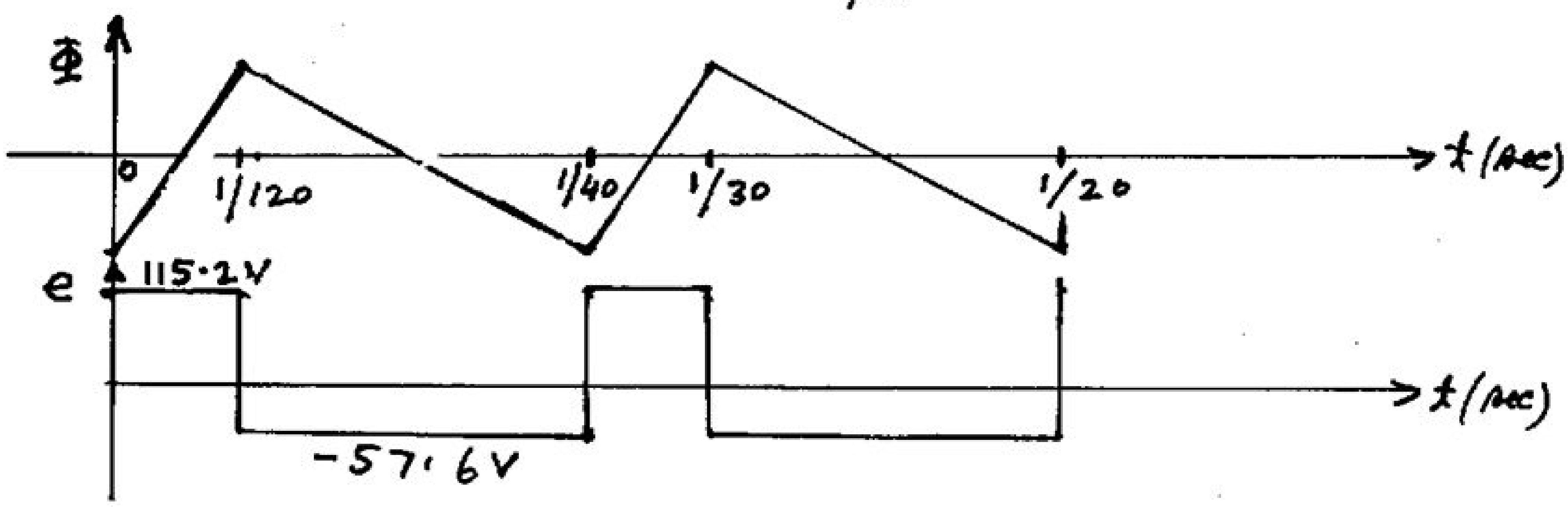
(c) $E_2 = \frac{N_2}{N_1} E_1 = \frac{400}{200} \times 151.973 = 303.946\ V.$

→ From equation 1.40 (same flux links both windings)

$\boxed{1.21}$ $0 < t < \frac{1}{120}$ sec $\rightarrow$ $e = N\frac{d\Phi}{dt} = 400 \times \frac{2.4\times10^{-3}}{1/120} = 115.2\,V$

$\frac{1}{120} < t < \frac{1}{40}$ sec $\rightarrow$ $e = 400 \times \frac{2.4\times10^{-3}}{1/60} = 57.6\,V$



$\boxed{1.22}$ From equation 1.42, total flux swing during each half cycle is

$$\Delta\Phi = \frac{1}{500}\left[45\times\frac{1}{360} + 90\times\frac{1}{360} + 45\times\frac{1}{360}\right]$$

$$= 1.0\times10^{-3}\ \text{Wb}.$$

During the positive half cycle of the input voltage, the flux swings from -0.5×10^{-3} Wb to 0.5×10^{-3} Wb.

$0 < t < \frac{1}{360}$

$$\Phi(t) = -0.5\times10^{-3} + \frac{45}{500}t$$

$$\Phi\Big|_{\frac{1}{360}} = -0.5\times10^{-3} + \frac{45}{500}\times\frac{1}{360} = -0.25\times10^{-3}\ \text{Wb}.$$

$\frac{1}{360} < t < \frac{1}{180}$

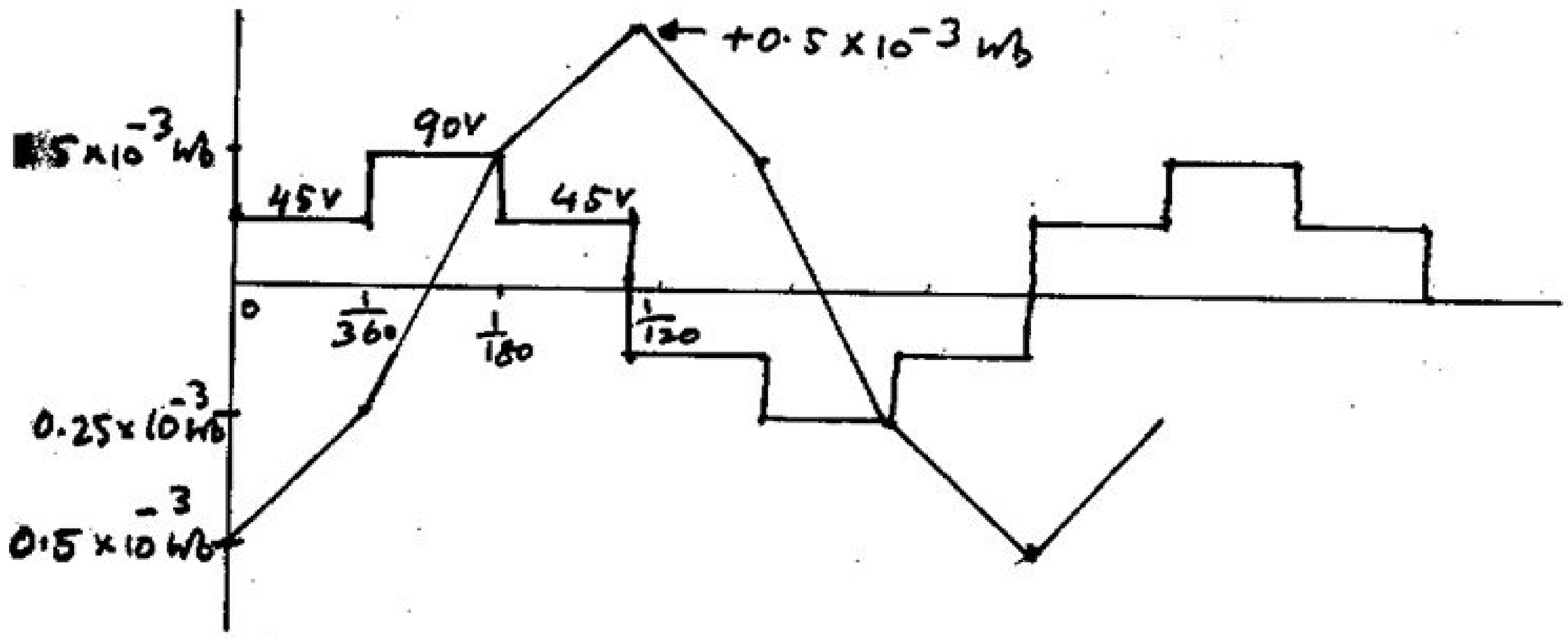
$$\Phi(t) = -0.25\times10^{-3} + \frac{90}{500}\left(t - \frac{1}{360}\right)$$

$$\Phi\Big|_{\frac{1}{180}} = -0.25\times10^{-3} + \frac{90}{900}\left(\frac{1}{180} - \frac{1}{360}\right) = 0.25\times10^{-3}\ \text{Wb}$$

$\frac{1}{180} < t < \frac{1}{120}$

$$\Phi(t) = 0.25\times10^{-3} + \frac{45}{500}\left(t - \frac{1}{180}\right)$$

$$\Phi\Big|_{\frac{1}{120}} = 0.25\times10^{-3} + \frac{45}{500}\left(\frac{1}{120} - \frac{1}{180}\right) = 0.5\times10^{-3}\ \text{Wb}$$



$\boxed{1.23}$ When core is unsaturated, core reluctance is zero and coil inductance is infinite. When core is saturated, core reluctance is infinite and coil inductance is zero.

$$\Phi_{sat} = B_{sat} \times A = 1.5 \times 2 \times 10^{-4} = 3 \times 10^{-4} \text{ Wb}$$

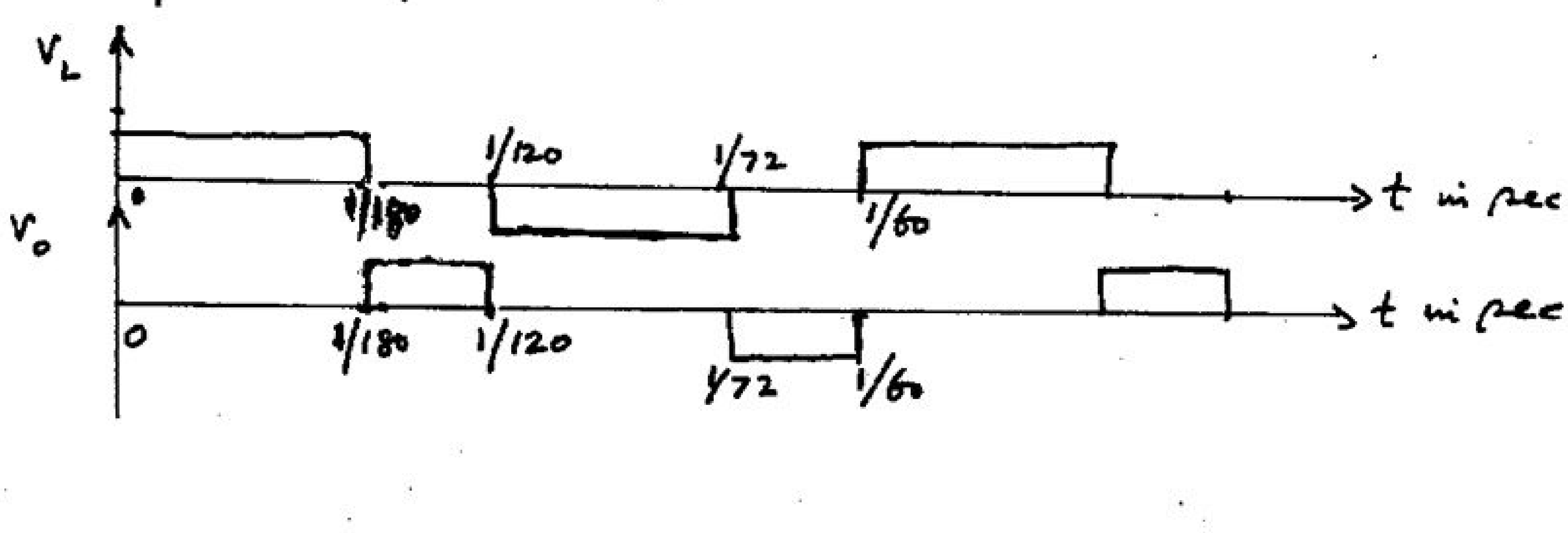
Flux swing, $\Delta\Phi = 2\Phi_{sat} = 6 \times 10^{-4}$ Wb.

Time t_s to swing from negative to positive saturation →

$$E t_s = N \Delta\Phi$$

$$t_s = \frac{1000 \times 6 \times 10^{-4}}{108} = \frac{1}{180} \text{ sec.}$$

All the input voltage will be across the coil (infinite inductance) during 0 to t_s and all the input voltage will be across the resistance during t_s to $\frac{1}{120}$ sec (zero inductance of coil during this period).



1.24 Operating point before the Keeper was inserted was:

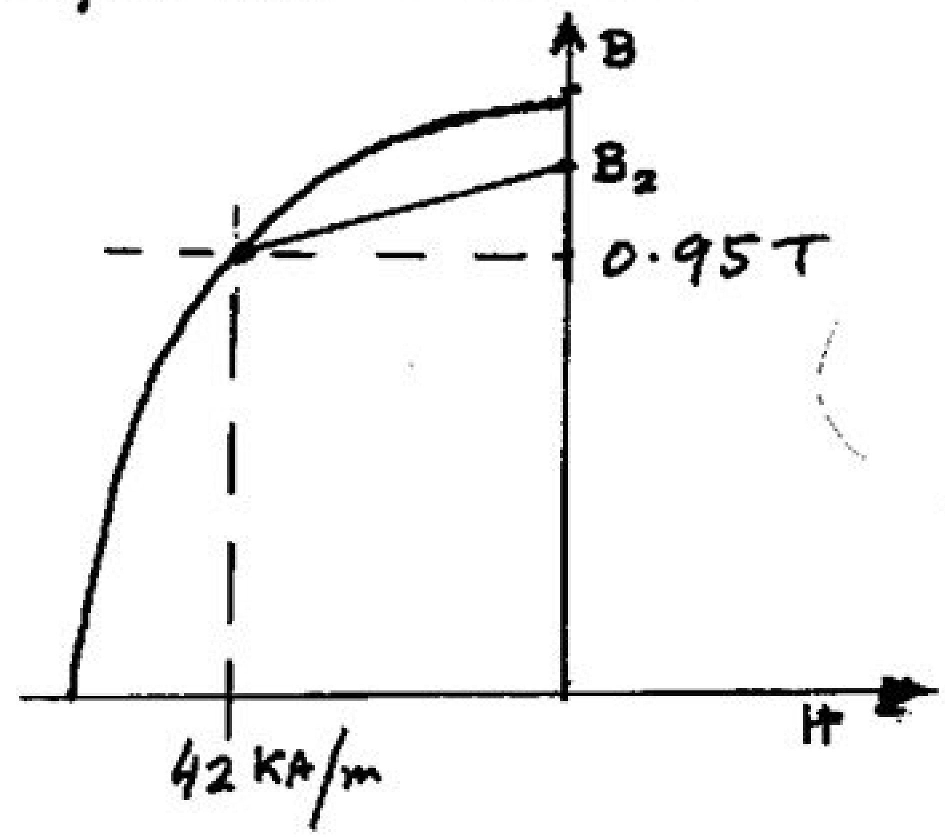
$B_m = 0.95\ T$, $H_m = 42\ kA/m$

Now, $\mu_{rec} = 4 \times 4\pi \times 10^{-7}$

The new operating point is

$$B_2 = 0.95 + 16\pi 10^{-7} \times 42 \times 10^3$$

$$= 1.16\ T.$$



1.25 (a) From Fig. 1.24, product of $H_m B_m$ is maximum at approximately

$$B_m = 0.45\ T,\ H_m = -350\ kA/m,\ H_m B_m\Big|_{max} = 157.5 \times 10^3\ J/m^3$$

(b) $$l_m = \frac{0.4 \times 10^{-2} \times 0.8}{350 \times 10^3 \times 4\pi \times 10^{-7}} = 0.007276\ m = 0.7276\ cm$$

$$A_m = \frac{0.8 \times 2.5 \times 10^{-4}}{0.45}\ m^2 = 4.4444\ cm^2$$

(c) $$V\Big|_{sam.} = l_m A_m = 0.7276 \times 4.4444 = 3.2338\ cm^3$$

$$V\Big|_{Aln} = 6.06 \times 2.105 = 12.7563\ cm^3.$$

$$\frac{V|_{Sam}}{V|_{Aln}} = \frac{3.2338}{12.7563} = 0.2535$$

1.26 (a) From Fig 1.24, $H_m B_m$ product is maximum at approximately, $B_m = 0.6T$, $H_m = 480\ kA/m$, $H_m B_m\Big|_{max} = 288 \times 10^3\ J/m^3$

(b) $$l_m = \frac{0.4 \times 10^{-2} \times 0.8}{480 \times 10^3 \times 4\pi \times 10^{-7}} = 0.005305\ m = 0.5305\ cm$$

$$A_m = \frac{0.8 \times 2.5 \times 10^{-4}}{0.6}\ m^2 = 3.3333\ cm^2$$

(c) $$V\Big|_{Neod..} = 0.5305 \times 3.3333 = 1.7683\ cm^3$$

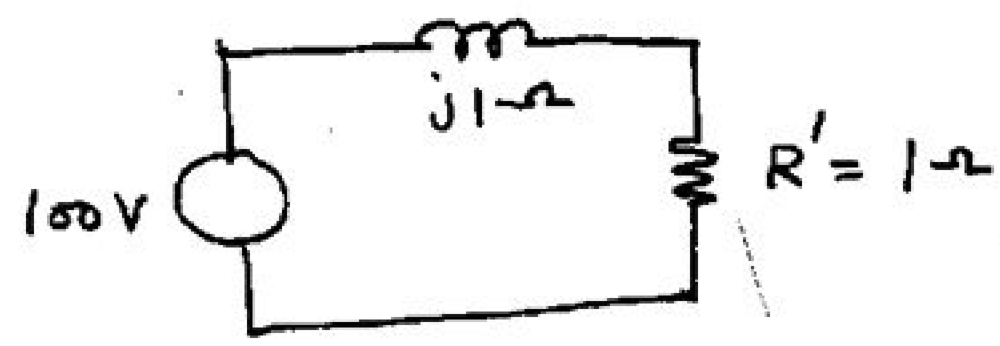
$$\frac{V|_{Neod}}{V|_{Aln}} = \frac{1.7683}{12.7563} = 0.1386$$

CHAPTER 2

2.1. Referred resistance

$R' = a^2 R_L = 1$

$a = \frac{1}{\sqrt{R_L}}$



Load voltage.

$V_L = \frac{1}{a} V' = \frac{1}{a} \frac{|1|}{|1+j1|} \times 100 = \frac{70.71}{a}$

Load power

$P_L = \frac{V_L^2}{R_L}$

R_L	0.5 Ω	1.0 Ω
a	1.41	1
V_L	50 V	70.7 V
P_L	5000 W	5000 W

$\boxed{2.2}$ (a) $V_2 = \frac{N_2}{N_1} V_1 = \frac{500}{1000} \times 220 = 110\,V$.

(b) $I_2 = \frac{5000}{110} = 45.4545\,A$

$Z_2 = \frac{110}{45.4545} = 2.42\,\Omega$

(c) $Z_2' = \left(\frac{1000}{500}\right)^2 \times 2.42 = 9.68\,\Omega$

$\boxed{2.3}$ (a) 10 KVA.

(b) $I_2 = \frac{10\,000}{110} = 90.91\,A$

$Z_2 = \frac{110}{90.91} = 1.21 \angle -\cos^{-1} 0.8$

$= 1.21 \angle -36.87^\circ\ \Omega$.

$\boxed{2.4}$ (a) $V_{H(\text{rated})} = 1000\text{V}$, $I_{H(\text{rated})} = \dfrac{100\times10^3}{1000} = 100\text{A}$.

$V_{L(\text{rated})} = 100\text{V}$, $I_{L(\text{rated})} = \dfrac{100\times10^3}{100} = 1000\text{A}$.

(b) From open circuit test,

$$R_{cL} = \frac{100^2}{400} = 25\,\Omega.$$

$$I_{cL} = \frac{100}{25} = 4\text{A}.$$

$$I_{mL} = \sqrt{6^2-4^2} = 4.47\text{A}$$

$$X_{mL} = \frac{100}{4.47} = 22.37\,\Omega$$

Turns ratio $a = \dfrac{1000}{100} = 10$

Refer to high voltage side,

$R_{cH} = 25\times10^2 = 2500\,\Omega$, $X_{mH} = 2237\,\Omega$

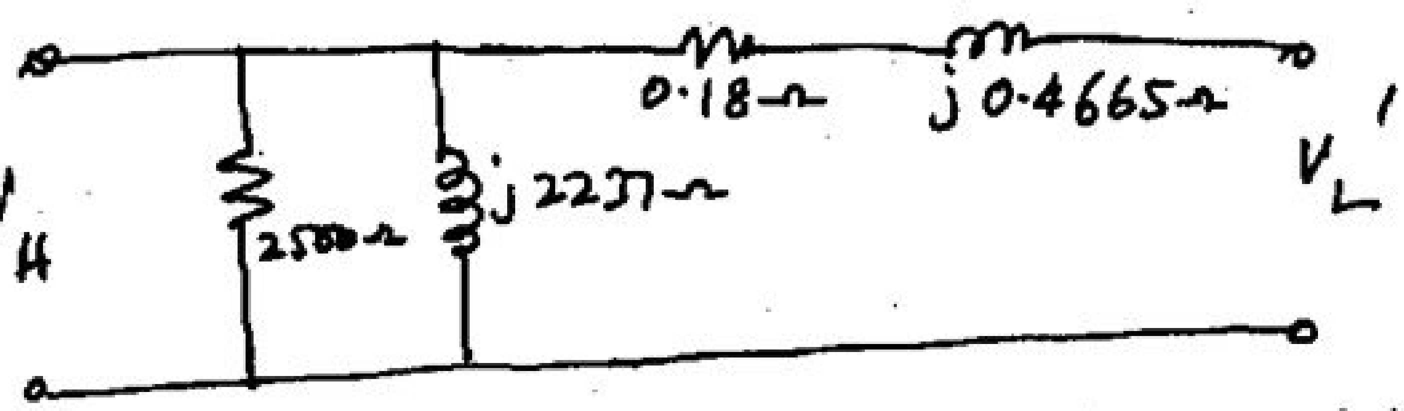
From short circuit test,

$$R_{eq\,H} = \frac{1800}{100^2} = 0.18\,\Omega.$$

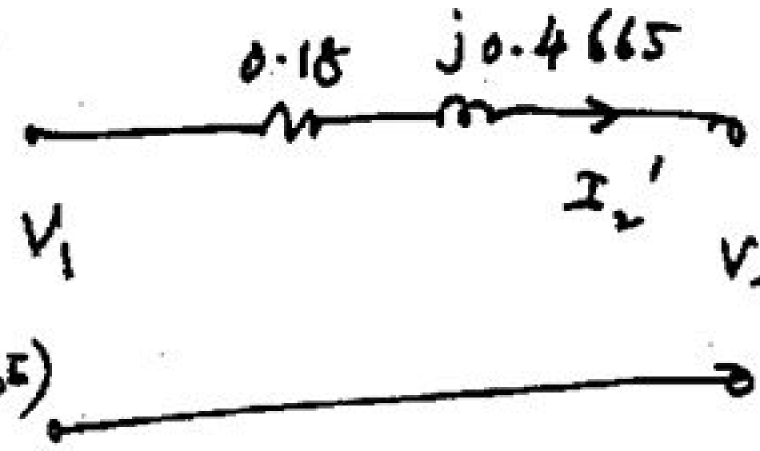
$$Z_{eq\,H} = \frac{50}{100} = 0.5\,\Omega$$

$$X_{eq\,H} = \sqrt{0.5^2 - 0.18^2} = 0.4665\,\Omega$$

Equivalent circuit referred to H.V. side



(C) $V_1 = V_2' + I_2' Z_{eq\,H}$

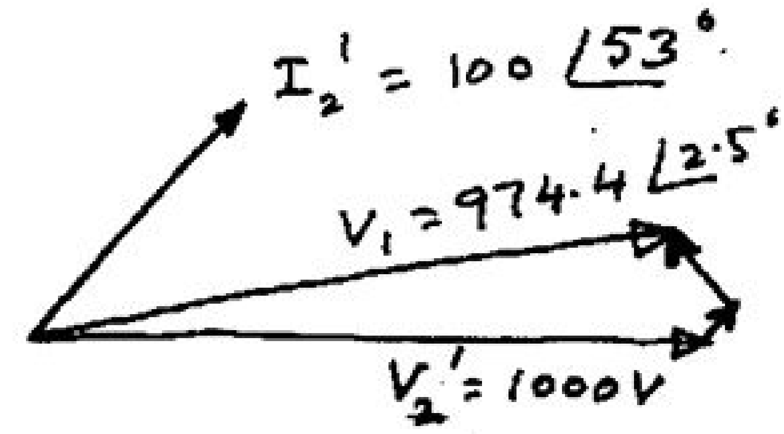


$= 1000\angle 0^\circ + 100\angle 53^\circ (0.18 + j0.4665)$

$= 974.4\angle 2.5^\circ$

$$V.R = \frac{974.4 - 1000}{1000}\times100\,\% = -2.56\,\%$$

(d)



2.5 (a) From the open circuit test.

$R_{CL} = \frac{220^2}{650} = 74.46\,\Omega$, $I_{CL} = \frac{220}{74.46} = 2.95A$

$I_{mL} = \sqrt{9.5^2 - 2.95^2} = 9.03A$, $X_{mL} = \frac{220}{9.03} = 24.36\,\Omega$

$a = \frac{220}{440} = 0.5$

From the short circuit test.

$R_{eqH} = \frac{950}{55^2} = 0.314\,\Omega$, $Z_{eqH} = \frac{37.5}{55} = 0.68\,\Omega$

$X_{eqH} = \sqrt{0.68^2 - 0.314^2} = 0.6032\,\Omega$

Base quantities.

$V_{b(H)} = 440V$, $I_{b(H)} = \frac{25000}{440} = 56.82A$

$Z_b(H) = \frac{440}{56.82} = 7.744\,\Omega$.

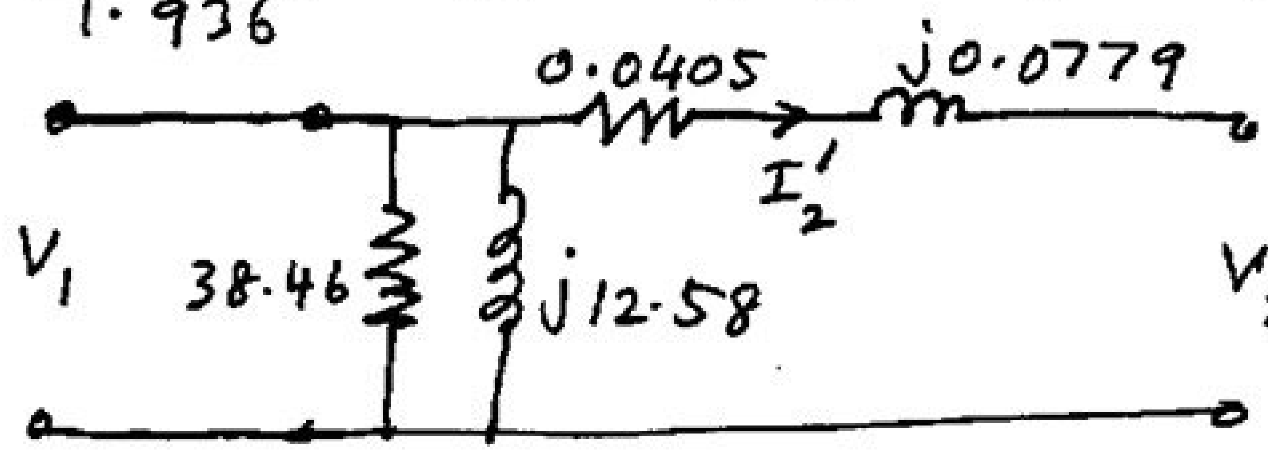
$V_b(L) = 220V$, $I_b(L) = \frac{25000}{220} = 113.64A$

$Z_b(L) = \frac{220}{113.64} = 1.936\,\Omega$.

$R_{eq} = \frac{0.314}{7.744} = 0.0405$ pu, $X_{eq} = \frac{0.6032}{7.744} = 0.0779$ pu

$Z_{eq} = \frac{0.68}{7.744} = 0.0878$ pu,

$R_{CL} = \frac{74.46}{1.936} = 38.46$ pu, $X_{mL} = \frac{24.36}{1.936} = 12.58$ pu.

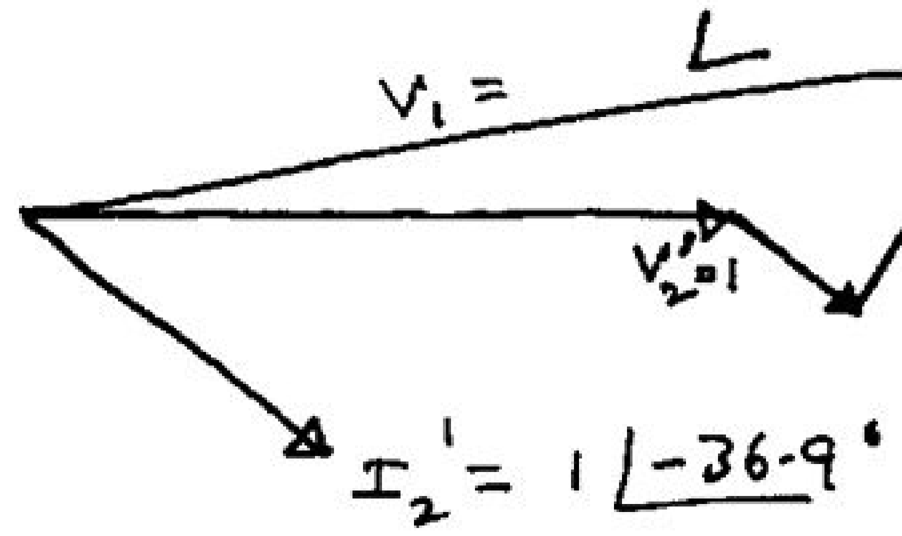


(b) $V_1 = V_2' + I_2' Z_{eq} = 1\angle 0 + 1\angle -36.9 \, (0.0405 + j0.0779)$

$= 1.08 \angle 2^\circ$

$$VR = \frac{1.08 - 1}{1} \times 100\% = 8\%$$

(c)



$\boxed{2.6}$ No load test from 120V side.

$a = \dfrac{2400}{120} = 20$, $V_{oc} = 120V$.

$$I_{cL} = \frac{120}{64000/20^2} = 0.75A.$$

$$I_{mL} = \frac{120}{9600/20^2} = 5.0A$$

$$I_{oc} = \sqrt{5^2 + 0.75^2} = 5.056 A$$

$$P_{oc} = 120 \times 0.75 = 90W.$$

Short circuit test from 2400V side.

$$I_{sc} = \frac{10\,000}{2400} = 4.17A.$$

$$V_{sc} = 4.17 \times \sqrt{5^2 + 25^2} = 106.3V.$$

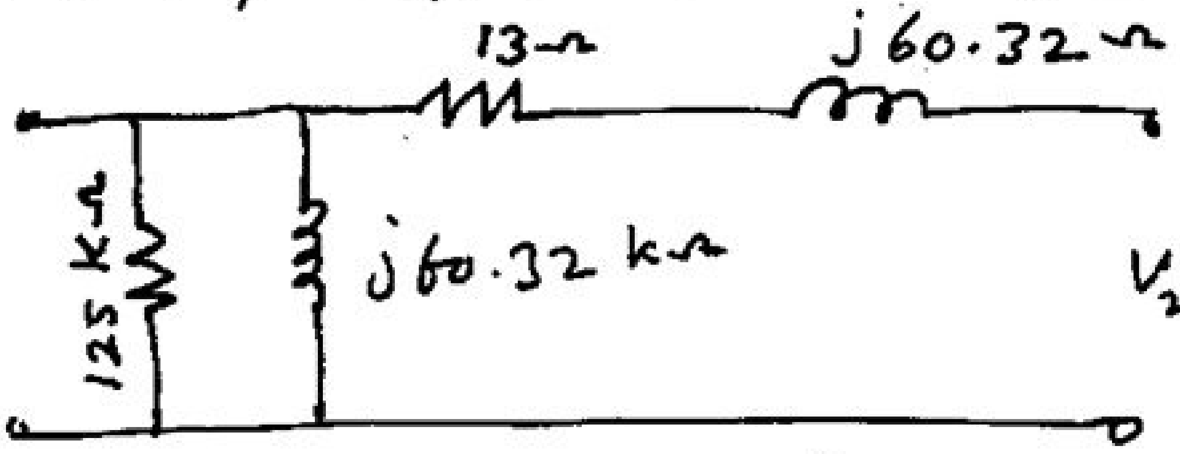
$$P_{sc} = 4.17^2 \times 5 = 86.94W$$

$\boxed{2.7}$ (a) $n = \dfrac{11000}{2200} = 5$

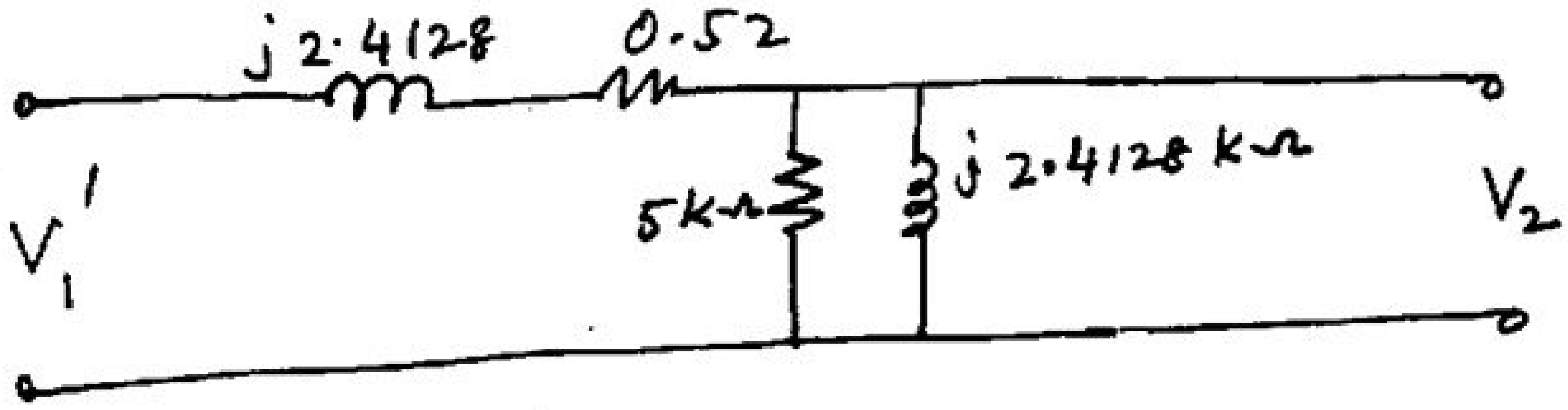
$X_{L_1} = 377 \times 0.08 = 30.16\,\Omega$, $X_{L_2}' = 5^2 \times 377 \times 0.0032 = 30.16\,\Omega$

$X_{m_1} = 377 \times 160 = 60.32\,k\Omega$, $R_{c_1} = 125\,k\Omega$.

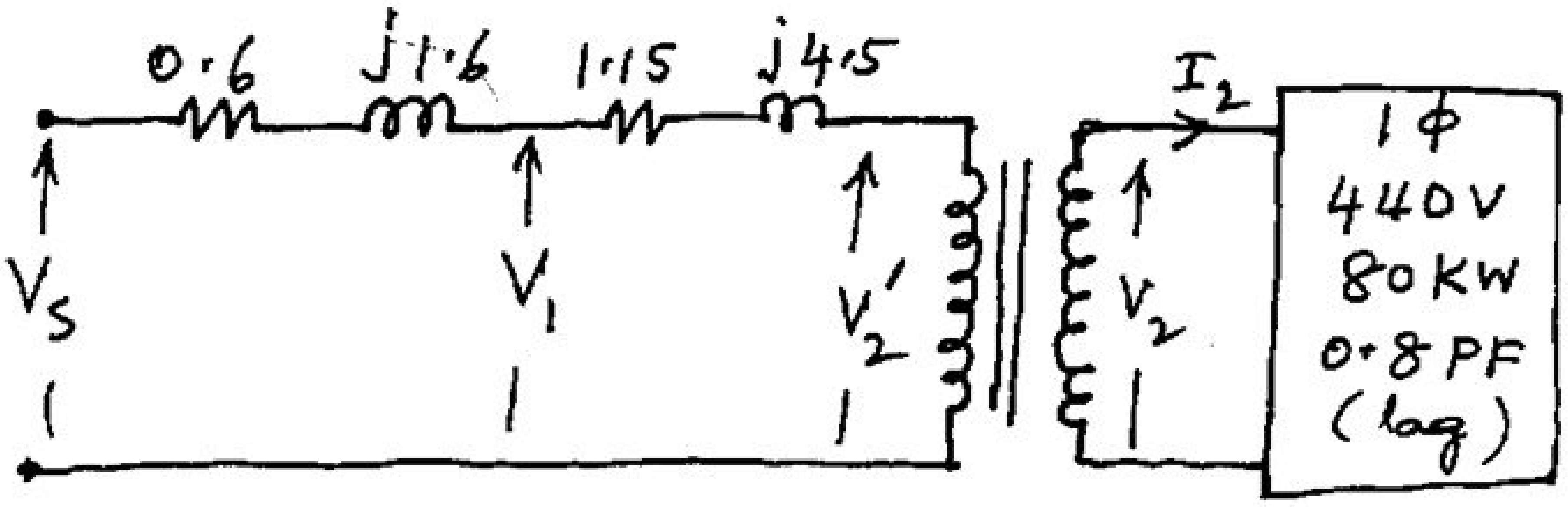
$R_1 = 6.0\,\Omega$, $R_2' = 5^2 \times 0.28 = 7.0\,\Omega$



(b)



$\boxed{2.8}$ (a)



(b) $I_2 = \dfrac{80\,000}{440 \times 0.8} = 227.27\,A.$

$\phi = \cos^{-1} 0.8 = 36.9^\circ$, $a = \dfrac{2200}{440} = 5$

$I_2' = 227.27/5 = 45.45\,A$

$V_1 = 2200\angle 0^\circ + 45.45\angle{-36.9}\,(1.15 + j\,4.5)$

$= 2368.3\angle 3.2^\circ$ V.

(c) $V_S = 2200\angle 0^\circ + 45.45\angle{-36.9}\,(1.75 + j\,6.1)$

$= 2436.3\angle 4.1^\circ$ V.

2.9 (a) $I_{1(\text{rated})} = \dfrac{3000}{240} = 12.5\,A \rightarrow$ full-load current.

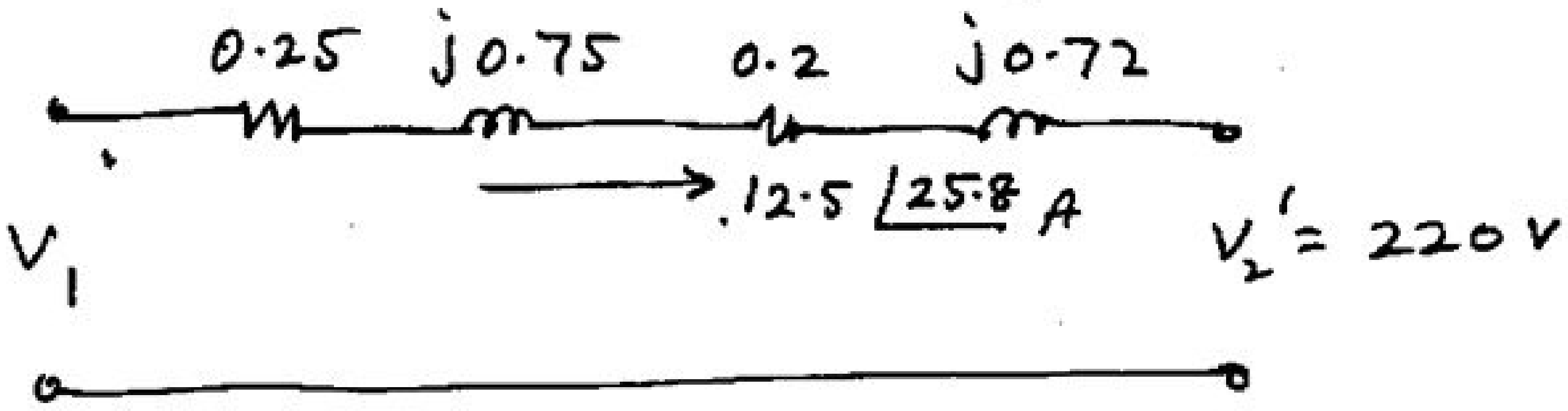
$a = \dfrac{240}{120} = 2$

$I_{2(\text{rated})} = 12.5 \times 2 = 25\,A \rightarrow$ full-load current.

$R'_{LV} = 0.05 \times 2^2 = 0.2\,\Omega$

$X'_{LV} = 0.18 \times 2^2 = 0.72\,\Omega$

Equivalent Circuit referred to HV side



$$V_1 = 220\angle 0^\circ + 12.5\angle 25.8\,(0.45 + j1.47)$$

$$= 217.9\angle 5^\circ \text{ V.}$$

$$VR = \frac{217.9 - 220}{220} \times 100\,\% = -0.95\,\%$$

(b) $I_H = \left| \dfrac{217.9}{0.45 + j1.47} \right| = \dfrac{217.9}{1.5373} = 141.74\,A$

$I_L = 2I_H = 283.48\,A.$

2.10 (a)

(i) $I_{HV}\big|_{NL} = \dfrac{11000}{57600} + \dfrac{11000}{j16.34} = 0.7\angle -74.2^\circ\,A$

$I_{HV(\text{rated})} = \dfrac{300 \times 10^3}{11 \times 10^3} = 27.27\,A.$

$I_{HV}\big|_{NL}$ in % $= \dfrac{0.7}{27.27} \times 100 = 2.57\,\%$

(ii) $P_{NL} = P_{core} = \dfrac{11000^2}{57600} = 2100\,W$

(iii) $PF\big|_{NL} = \cos 74.2^\circ = 0.27$ lagging

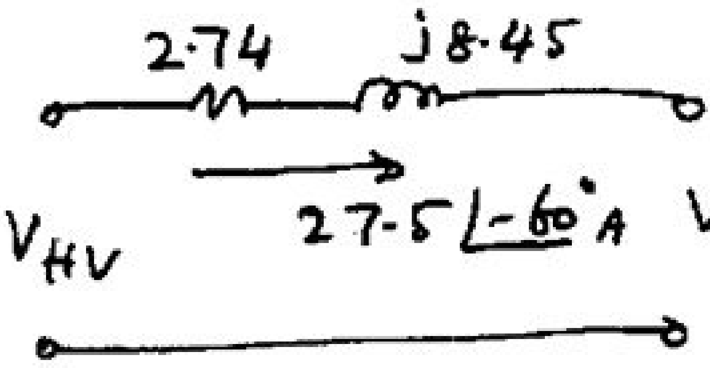
(iv) $P_{cu,FL} = 27.27^2 \times 2.784 = 2070\,W.$

(b) $I_{LV} = \dfrac{2200}{16\angle 60^\circ} = 137.5\angle -60^\circ$ A.

Referred to HV side, $\left(a = \dfrac{11000}{2.2} = 5 \right)$

$I_{LV}' = \dfrac{137.5}{5} = 27.50\angle -60^\circ$ A.

$V_{HV} = 11000\angle 0^\circ + 27.5\angle -60^\circ \times (2.74 + j\,8.45)$



$= 11{,}239\angle 0.3^\circ$ V.

$VR = \dfrac{11239 - 11000}{11000} \times 100\% = 2.17\%$

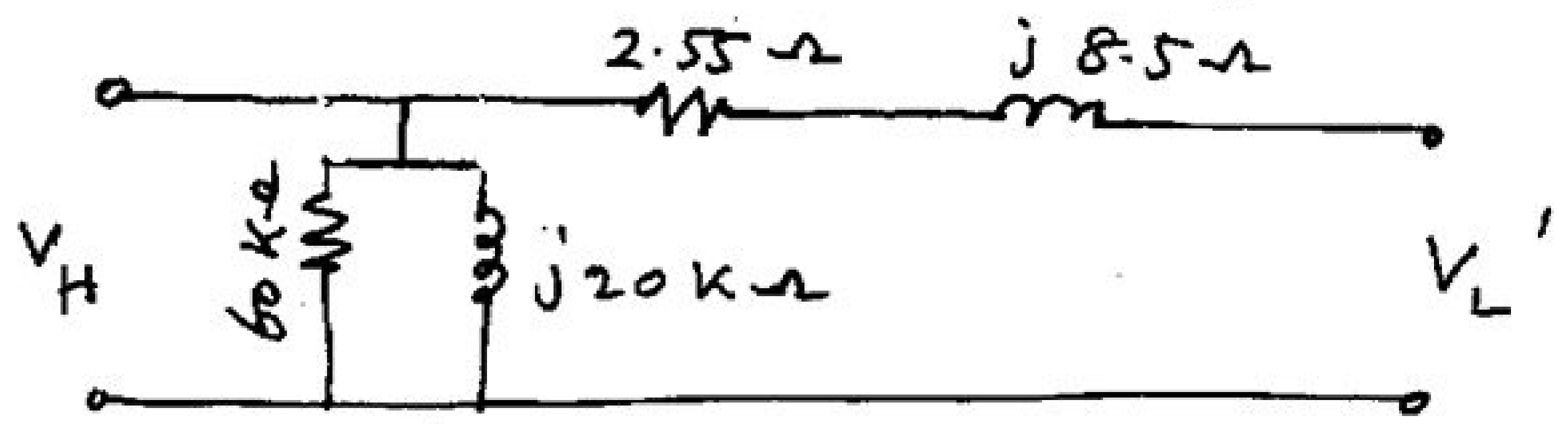
$\boxed{2.11}$ (a) $n = \dfrac{11}{2.2} = 5$

$R_{HV} = 1.3\,\Omega$, $R_{LV}' = 0.05 \times 5^2 = 1.25\,\Omega$

$X_{HV} = 4.5\,\Omega$, $X_{LV}' = 0.16 \times 5^2 = 4.0\,\Omega$

$R_{eq(HV)} = 1.3 + 1.25 = 2.55\,\Omega$, $X_{eq(HV)} = 4.5 + 4 = 8.5\,\Omega$

$R_{c(HV)} = 2.4 \times 5^2 = 60\,k\Omega$, $X_{m(HV)} = 0.8 \times 5^2 = 20\,k\Omega$



(b) $I_{o(HV)} = \dfrac{11000}{60\,000} + \dfrac{11000}{j\,20\,000} = 0.5797$ A.

$I_{HV(rated)} = \dfrac{250 \times 10^3}{11 \times 10^3} = 22.73$ A

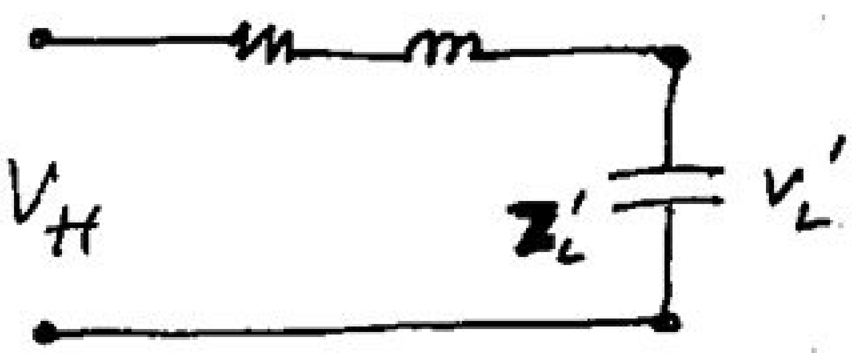
$I_{o(HV)} = \dfrac{0.5797}{22.73}$ pu $= 0.0255$ pu.

(c) $Z_{eq(HV)} = 2.55 + j\,8.5 = 8.8743\,\Omega$

$V_{HV} = 22.73 \times 8.8743 = 201.7$ V.

$$P_{core} = \frac{201.7^2}{60\times10^3} = 0.678\,W, \quad P_{cu} = 22.73^2 \times 2.55 = 1314\,W.$$

(d) $Z_L' = 15 \times 5^2 \angle{-90} = -j375\,\Omega$



$$V_L' = \frac{11000}{2.55 + j8.5 - j375} \times (-j375)$$

$|V_L'| = 11255$ V.

$$VR = \frac{11000 - 11255}{11255} \times 100\% = -2.27\%.$$

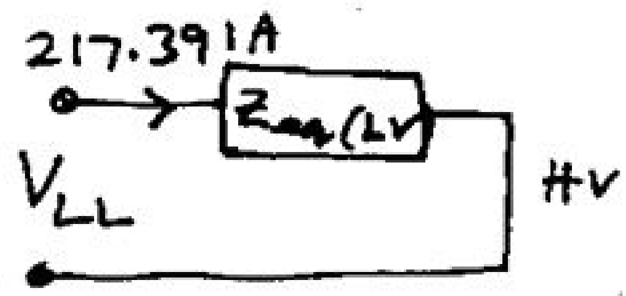
$\boxed{2.12}$ (a) $I_{HV} = \dfrac{100000}{2300} = 43.478\,A$, $I_{LV} = \dfrac{2300}{460} = a = \dfrac{2300}{460} = 5$

$$I_{LV} = \frac{100000}{460} = 217.391\,A$$

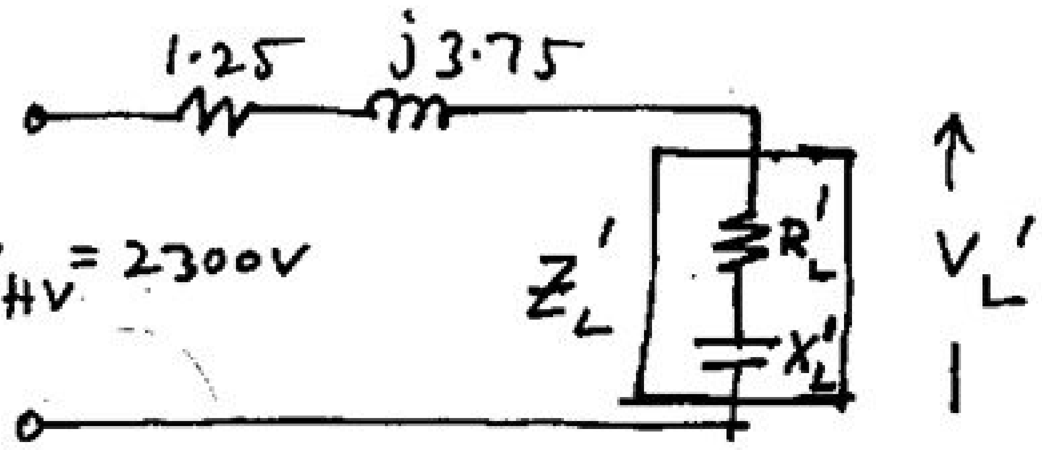
$$Z_{eq(LV)} = \frac{1.25 + j3.75}{5^2} = 0.05 + j0.15\,\Omega = 0.158\angle{71.6^\circ}$$

$V_{LL} = 217.391 \times 0.158 = 34.348\,V$

$P_{cu} = (217.391)^2 \times 0.05 = 2362.94\,W$



(b) $X_L' = -j3.75$ for PF = 1



$$1.25 + R_L' = \frac{2300}{43.478} = 52.9\,\Omega$$

$R_L' = 52.9 - 1.25 = 51.65\,\Omega$

$Z_L' = 51.65 - j3.75 = 51.79\angle{-4.15^\circ}\,\Omega$

$$Z_L = \frac{51.79}{5^2} = 2.07\,\Omega.$$

$V_L' = 43.478 \times 51.79 = 2229.99\,V$

$$V_L = \frac{2229.99}{5} = 445.998\,V$$

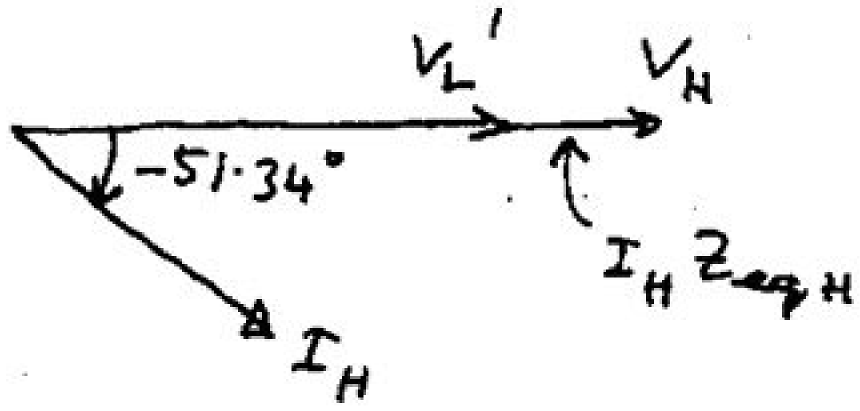
$$VR = \frac{460 - 445.998}{445.998} \times 100\% = 3.14\%$$

$\boxed{2.13}$ $\theta_{eq} = \tan^{-1}\frac{5}{4} = 51.34°$

For worst case $VR \rightarrow \theta_2 = -51.34°$

and V_L' and V_H are in phase.

$I_H = \frac{25000}{2300} = 10.87A$



$Z_{eqH} = \sqrt{4^2+5^2} = 6.4\,\Omega$

$I_H Z_{eqH} = 10.87 \times 6.4 = 69.6\,V$

$VR = \frac{69.6}{2300} \times 100\% = 3.03\%$

$\boxed{2.14}$ (a) $P_{out} = 25 \times 0.85 = 21.25\,kW$

$P_{cu} = I_H^2 R_{eqH} = 10.87^2 \times 4 = 472.63\,W$

$P_{core} = \frac{230^2}{450} = 117.56\,W$

$Eff = \frac{21,250}{21,250 + 472.63 + 117.56} \times 100\% = 97.3\%$

(b) $X = \sqrt{\frac{117.56}{472.63}} = 0.499$

$P_{cu} = P_{core} = 117.56\,W$

$P_{out} = 25 \times 0.499 = 12.475\,kW$

$Eff = \frac{12475}{12475 + 117.56 + 117.56} \times 100\% = 98.15\%$

$\boxed{2.15}$ (a) $P_{out} = 10 \times 0.8 = 8\,kW$

$P_{core} = 100\,W$, $P_{cu,FL} = 60 \times 2^2 = 240\,W$

$Eff = \frac{8000}{8000 + 100 + 240} \times 100\% = 95.92\%$

(b) $X = \sqrt{\frac{100}{240}} = 0.6455$

$$Eff_{max} = \frac{10 \times 10^3 \times 0.6455 \times 0.9}{(10^4 \times 0.6455 \times 0.9) + 100 + 100} \times 100\%$$

$= 96.67\%$

(c) $E_{24\,hrs} = 0 + 10 \times 0.7 \times 0.8 \times 10 + 10 \times 0.9 \times 0.9 \times 8$

$= 120.8$ KWh.

$E_{core} = 100 \times 24 \times 10^{-3} = 2.4$ KWh

$E_{Cu} = (240 \times 0.7^2 \times 10 + 240 \times 0.9^2 \times 8) \times 10^{-3}$

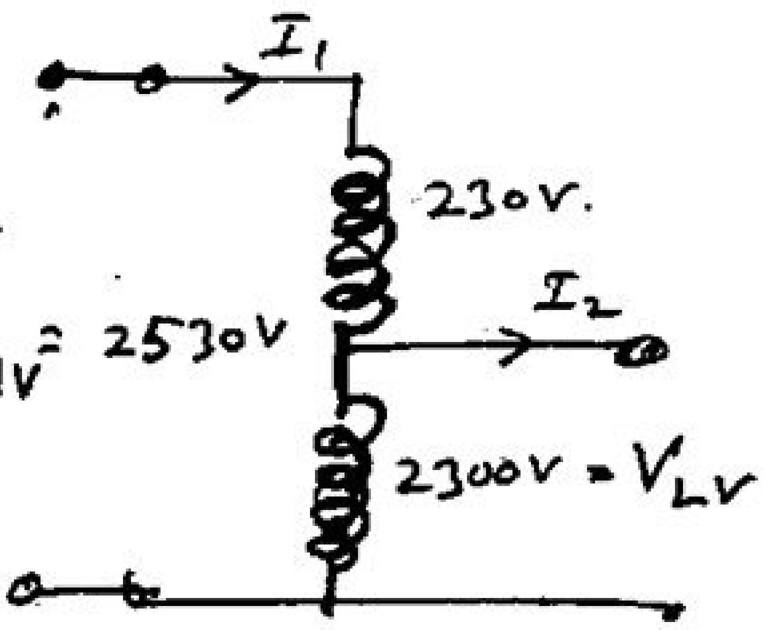
$= 2.7312$ kWh.

$$Eff_{all\ day} = \frac{120.8}{120.8 + 2.4 + 2.7312} \times 100\% = 95.93\%$$

$\boxed{2.16}$ (a) For maximum KVA rating, the high voltage side should carry the high current.

$I_{2300v(wdg)} = \frac{25000}{2300} = 10.87\,A$

$I_{230v(wdg)} = \frac{25000}{230} = 108.7\,A$.



(b) $V_{HV} = 2300 + 230 = 2530\,v$.

$V_{LV} = 2300\,v$.

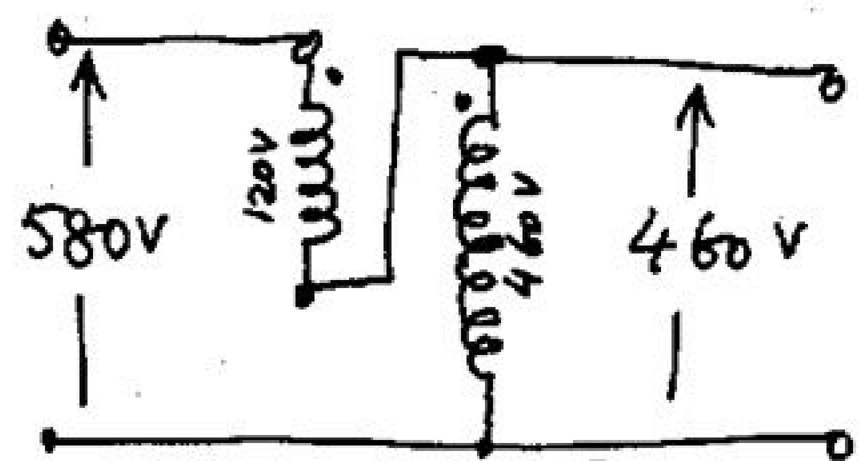
(c) $KVA\big|_{HV} = 2530 \times 108.7 = 275.01$ KVA

$KVA\big|_{LV} = 2300\,I_2 = 2300 \times n\,I_1$

$= 2300 \times \frac{2530}{2300} \times 108.7$

$= 275.01$ KVA.

2.17 (a)



(b) $I_{120V} = \frac{10\times10^3}{120} = 83.33\,A$

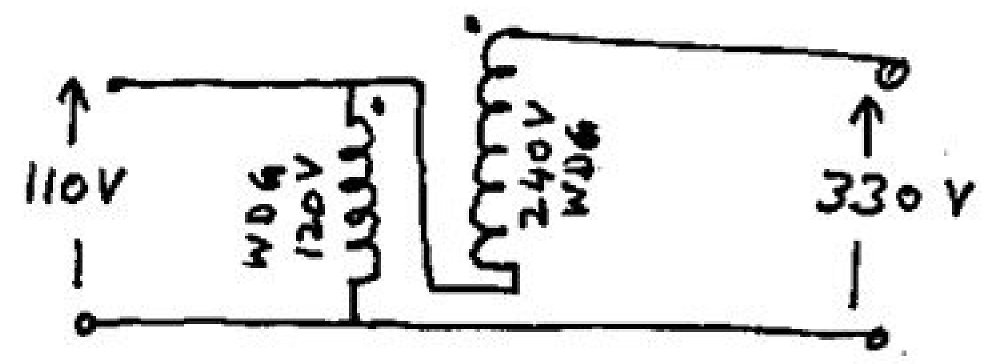
$KVA|_{max} = 580 \times 83.33 = 48.33\ KVA$

(c) Transformer losses when rated current flows.

$P_{loss} = 9\times10^3(1-0.96) = 360\,W$

$Eff = \frac{48330 \times 0.9}{48330\times0.9 + 360} \times 100\% = 99.2\%$

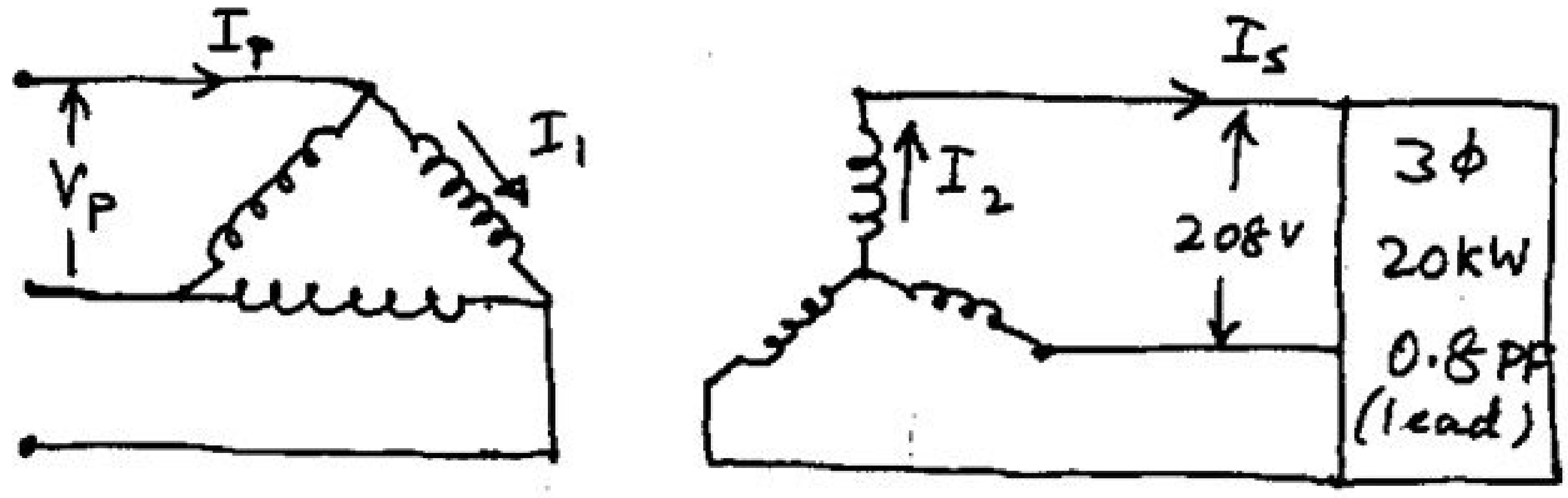
2.18 (a)



(b) $I_{HV} = \frac{3000}{240} = 12.5\,A$

$Rating = (120+240)\times 12.5 = 4500\ VA = 4.5\ KVA$

2.19 (a)

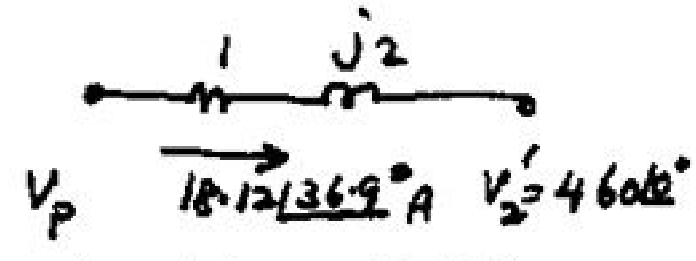


(b) $Load\ KVA = \frac{20}{0.8} = 25$

$I_2 = I_S = \frac{25\times10^3}{\sqrt{3}\times208} = 69.4\,A$

$a = \frac{460}{120} = 3.83$

$I_1 = \frac{69.4}{3.83} = 18.12\,A$



(c) $Z_{eq_1} = 1.0 + j2.0$, $\phi = \cos^{-1} 0.8 = 36.9°$

$$V_p = 460\angle 0° + 18.12\angle 36.9 \, (1 + j2.0)$$
$$= 454.5\angle 5°$$

(d) $$VR = \frac{454.5 - 460}{460} \times 100\% = -1.2\ \%$$

Note: This problem can also be solved by deriving the single phase equivalent circuit i.e. transforming Δ- connection into its equivalent Y- connection.

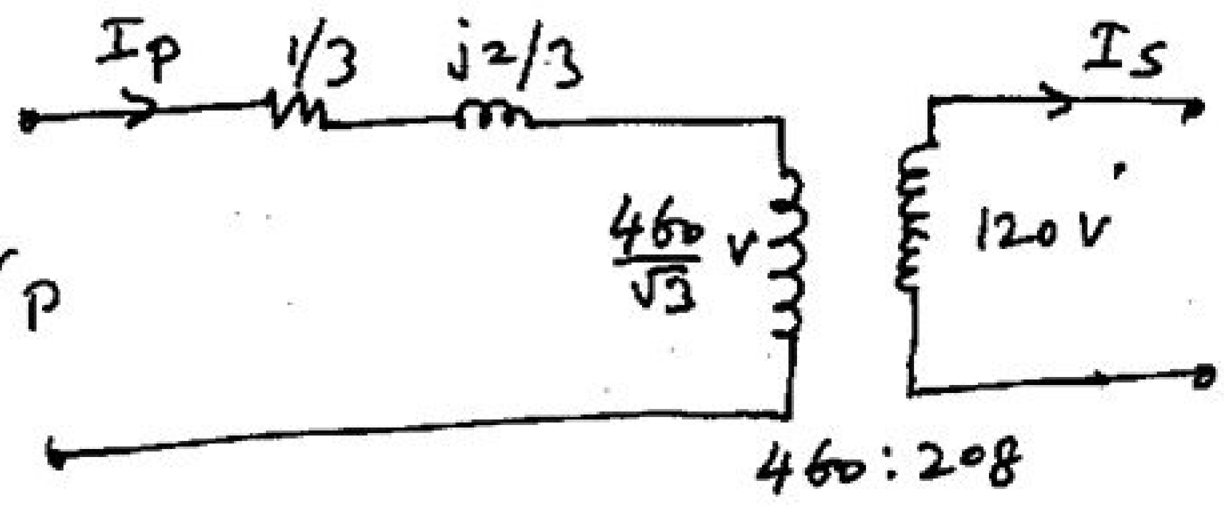
$$Z_Y = \frac{Z_\Delta}{3} = \frac{1}{3} + j\frac{2}{3}$$

Y- phase voltage of primary $= \frac{460}{\sqrt{3}}$

Y - phase voltage of secondary $= 120V$

Turns ratio $a' = \frac{460/\sqrt{3}}{120} = \frac{460}{208} = \frac{V_{LL}|_{pri}}{V_{LL}|_{sec.}}$

Equivalent circuit is:



$$I_s = 69.4\ A \ , \ I_p = 69.4 \times \frac{208}{460} = 31.38 A$$

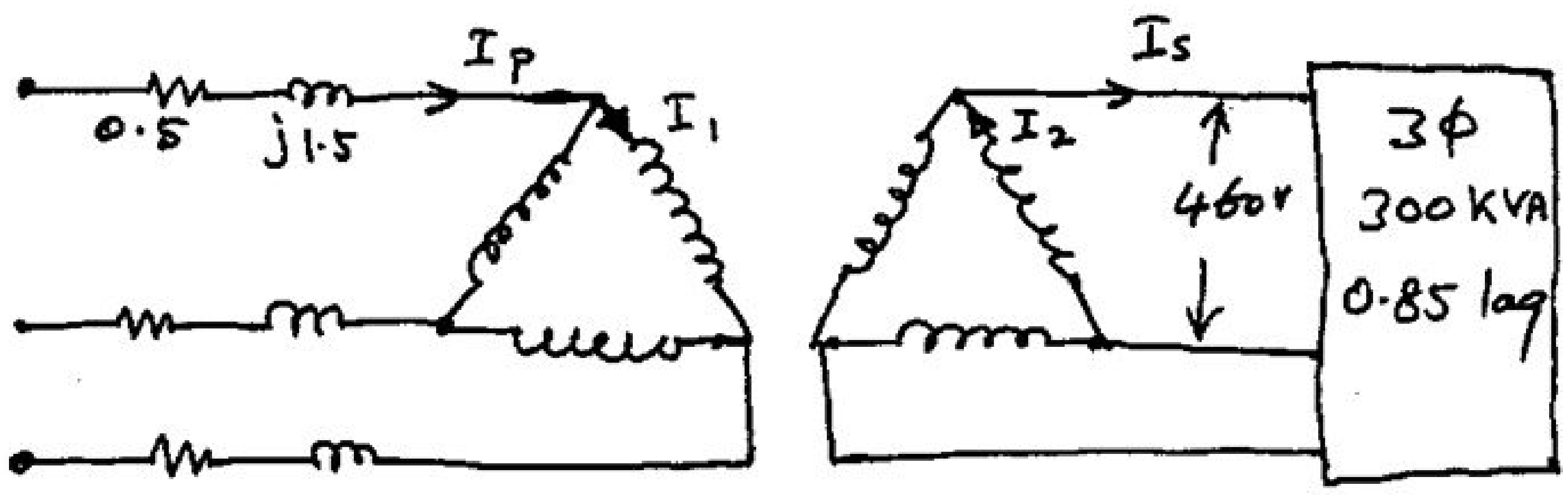
$$V_p = \frac{460}{\sqrt{3}}\angle 0° + 31.38\angle 36.9° \left(\frac{1}{3} + j\frac{2}{3}\right)$$

$$\sqrt{3}V_p = 460 + 18.12\angle 36.9 \, (1 + j2)$$

$= 454.5 \rightarrow$ Primary line-to-line voltage.

$$VR = \frac{454.5 - 460}{460} \times 100\% = -1.2\ \%$$

$\boxed{2.20}$ (a)



(b) Equivalent primary Y-voltage $= \frac{2300}{\sqrt{3}} = 1328\,V$.

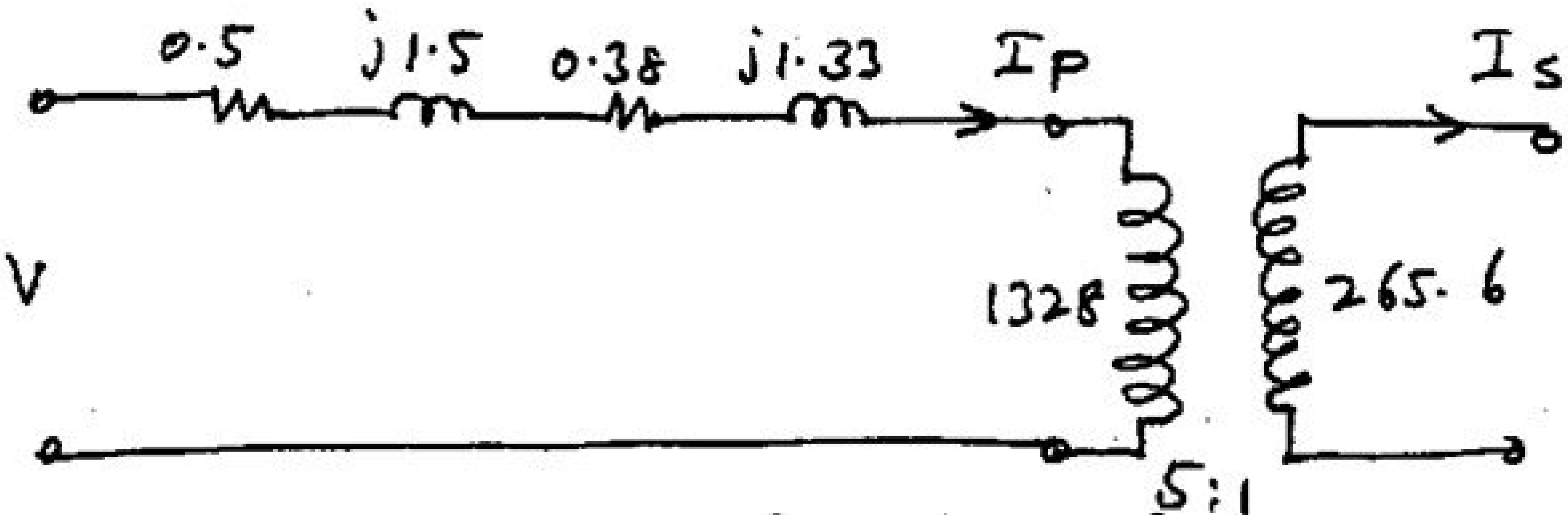
Equivalent secondary Y-voltage $= \frac{460}{\sqrt{3}} = 265.6$

Equivalent turns ratio, $a' = \frac{2300}{460} = 5 = a$

$Z_{eq1(\Delta)} = (0.045 + j0.16)\,5^2 = 1.13 + j4.0\,\Omega$

$Z_{eq1(Y)} = \frac{1.13 + j4.0}{3} = 0.38 + j1.33\,\Omega$

Equivalent 1ϕ circuit is



(c) $I_s = \frac{300 \times 10^3}{\sqrt{3} \times 460} = 376.54\,A$

$\phi = \cos^{-1} 0.85 = 31.8^\circ$

$I_p = \frac{376.54}{5} = 75.3 \angle -31.8^\circ\ A$

$V = 1328\angle 0^\circ + 75.3\angle -31.8\ (0.88 + j\,2.83)$

$= \mathbf{1497.4 \angle 5.6^\circ\ V}$

Sending end voltage (Line to Line) $= \sqrt{3} \times 1497.4 = \mathbf{2593.5\,V}$

(d) $I_1 = 75.3/\sqrt{3} = 43.48\ A$

$I_2 = 376.54/\sqrt{3} = 217.4\,A$

2.21

(a) Maximum secondary line current, $I_L = \frac{250 \times 10^3}{460} = 543.48\text{A}$

(b) $\phi = \cos^{-1} 0.8 = 36.87°$

$P_1 = 250 \cos(30 - 36.87) = 248.205 \text{ KW}$

$P_2 = 250 \cos(30 + 36.87) = 98.205 \text{ KW}$

$P = 248.205 + 98.205 = 346.41 \text{ KW}$

(c) $I_P = 543.48 \times \frac{460}{230} = 1086.95 \text{ A}.$

(d) $P = 750 \times 0.8 = 600 \text{ KW}$

$\% \text{ increase} = \frac{600 - 346.41}{346.41} \times 100 = 73.2\%$

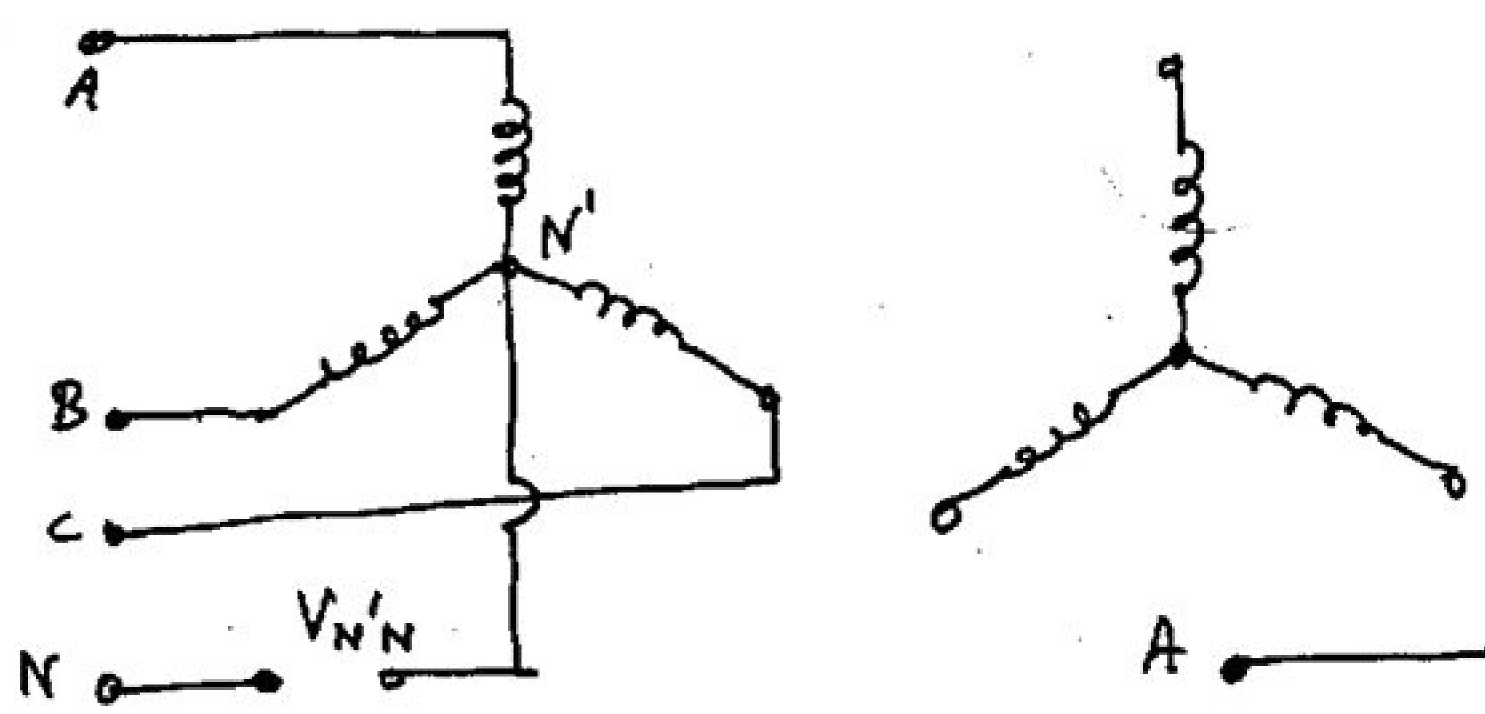
2.22 (a) The voltage is a 3rd harmonic voltage.

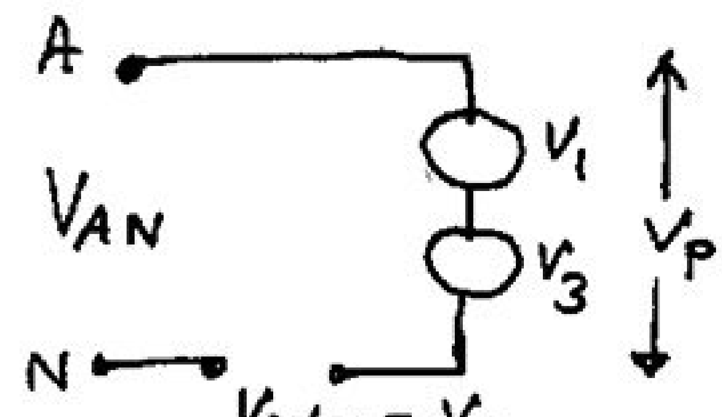
(b) Voltages induced in primary and secondary windings have the same wave-form.

Ratio of phase-voltages $\rightarrow$ 10

Ratio of line-voltages $\rightarrow$ 10

(c)





$V_{N'N} = V_3 = 1200 \text{ V}$

$V_{AN} = \frac{4000}{\sqrt{3}} = 2310 \text{V}.$

$V_P = \sqrt{V_{AN}^2 + V_{N'N}^2} = V_{AN}\sqrt{1 + \left(\frac{V_{N'N}}{V_{AN}}\right)^2} = V_{AN}\sqrt{1 + \left(\frac{1200}{2310}\right)^2}$

$= 1.13 V_{AN} = 1.13 \times 2310 = 2610 \text{V}.$

$$\frac{V_{L-L}}{V_{L-N}} = \frac{\sqrt{3}\,V_{AN}}{1.13\,V_{AN}} = 1.52$$

The ratio is the same on both sides of the transformer

2.23

(a)

	High voltage	Low Voltage
Base Voltage V_b	2100 V	210 V
Base Volt-ampere S_b	200 KVA	200 KVA
Base current $I_b = S_b / V_b$	95.24 A	952.4 A
Base impedance $Z_b = V_b / I_b$	22.05	0.2205 Ω

(b) $Z_{eq} = (0.25 + j\,1.5)/22.05 = 0.01134 + j\,0.068$

(c) $I_m = \left| (0.025 - j0.075) \frac{210}{952.4} \right| = 0.01743$ pu.

(d) $P_{core} = 0.025 \times 210^2 = 1102.5$ W

$$= \frac{1102.5}{200{,}000} = 0.00551 \text{ pu.}$$

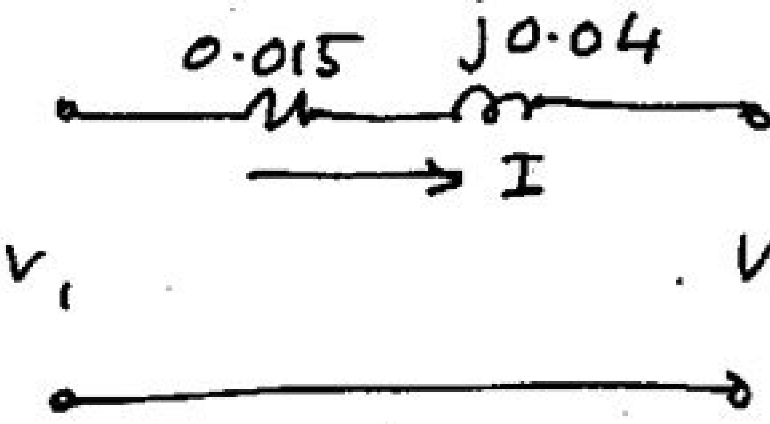
$P_{cu} = 0.01134$ pu.

$P_{Loss} = P_{core} + P_{cu} = 0.01685$ pu.

$\boxed{2.24}$

(a) $$\text{Eff} = \frac{1\times 0.85}{1\times 0.85 + 0.015 + 0.01}\times 100\% = 97.14\%$$

(b) $I = 1\angle{-31.8^\circ}$



$$V_1 = 1\angle 0^\circ + 1\angle{-31.8}\,(0.015 + j\,0.04)$$

$$= 1.034\angle 1.24 \text{ pu.}$$

$$VR = \frac{1.034 - 1.0}{1.0}\times 100\% = 3.4\%$$

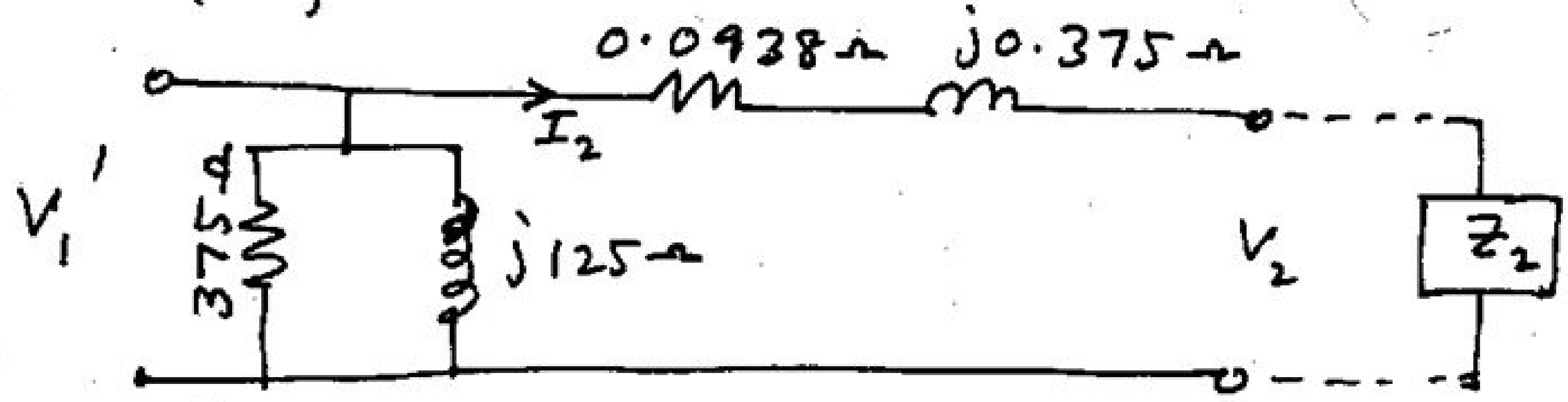
$\boxed{2.25}$ (a) For low voltage side

$$I_b = \frac{10000}{250} = 40A,\quad Z_b = \frac{250}{40} = 6.25\,\Omega$$

$$Z_{eq(LV)} = (0.015 + j0.06)\times 6.25 = 0.0938 + j0.375\,\Omega$$

$$R_{c(LV)} = 6.25\times 60 = 375\,\Omega$$

$$X_{m(LV)} = 6.25\times 20 = 125\,\Omega$$



(b) $V_1' = 250$ V

$$|I_2| = \left|\frac{250}{0.0938 + j0.375 - j5}\right| = 54.04 \text{ A.}$$

$$V_2 = 54.04\times 5 = 270.2 \text{ V.}$$

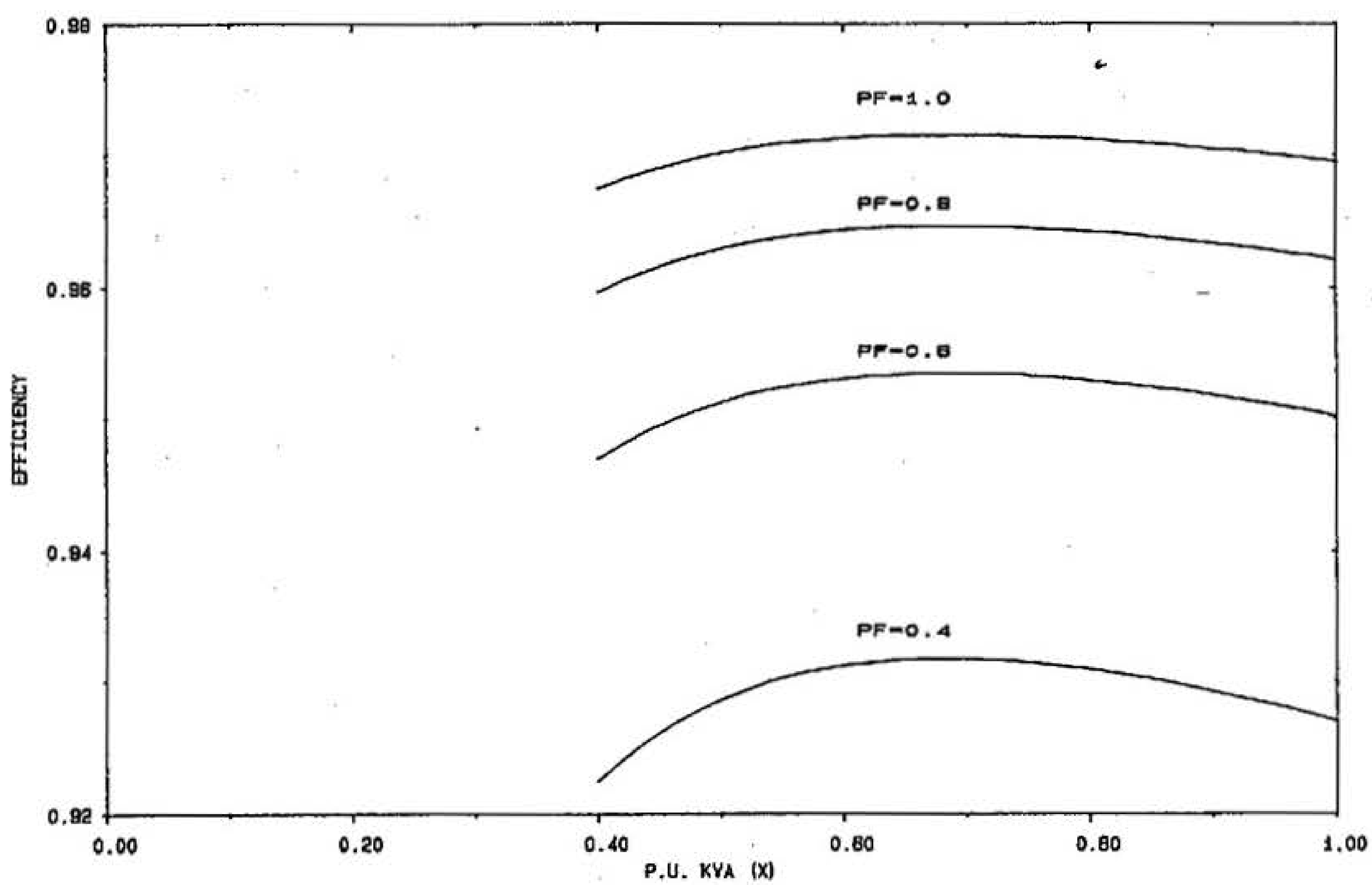
$$VR = \frac{250 - 270.2}{270.2}\times 100\% = -7.48\%$$

2.26

(a)

P.F	X	Efficiency	P.F	X	Efficiency
1.0000	0.0000	0.0000	0.6000	0.0000	0.0000
	0.1000	0.9073		0.1000	0.8545
	0.2000	0.9485		0.2000	0.9170
	0.3000	0.9617		0.3000	0.9378
	0.4000	0.9675		0.4000	0.9470
	0.5000	0.9702		0.5000	0.9512
	0.6000	0.9713		0.6000	0.9530
	0.7000	0.9715		0.7000	0.9534
	0.8000	0.9712		0.8000	0.9528
	0.9000	0.9704		0.9000	0.9517
	1.0000	0.9695		1.0000	0.9501
0.80000	0.0000	0.0000	0.4000	0.0000	0.0000
	0.1000	0.8868		0.1000	0.7966
	0.2000	0.9364		0.2000	0.8805
	0.3000	0.9526		0.3000	0.9095
	0.4000	0.9597		0.4000	0.9225
	0.5000	0.9630		0.5000	0.9286
	0.6000	0.9644		0.6000	0.9312
	0.7000	0.9646		0.7000	0.9317
	0.8000	0.9642		0.8000	0.9309
	0.9000	0.9633		0.9000	0.9292
	1.0000	0.9621		1.0000	0.9270

(b)



CHAPTER 3

$\boxed{3.1}$ $W_f = \int i\,d\lambda = \int (\lambda^{3/2} + 2.5\,\lambda(x-1)^2)\,d\lambda = \frac{2}{5}\lambda^{5/2} + 2.5\frac{\lambda^2}{2}(x-1)^2$

$$f_m = -\frac{\partial W_f}{\partial x}(\lambda, x)\Big|_{\lambda} = -\frac{2.5}{2}\lambda^2 \times 2(x-1) = -2.5\,\lambda^2(x-1)$$

$$f_m\Big|_{x=0.6m} = -2.5\,\lambda^2(0.6-1) = \lambda^2$$

$\boxed{3.2}$ (a) $i = \left(\frac{\lambda g}{1.2}\right)^2$

$$W_f = \int_0^{\lambda} i\,d\lambda = \int_0^{\lambda}\left(\frac{\lambda g}{1.2}\right)^2 d\lambda = \frac{g^2}{1.2^2} \times \frac{\lambda^3}{3}$$

$$f_m = -\frac{\partial W_f}{\partial g}(\lambda, g)\Big|_{\lambda = \text{constant}} = -\frac{\lambda^3 2g}{1.2^2 \times 3}$$

For $i = 2A$ and $g = 10$ cm.

$$\lambda = \frac{1.2 \times 2^{\frac{1}{2}}}{10 \times 10^{-2}} = 16.97 \text{ Wb-turn}$$

$$f_m = -\frac{16.97^3 \times 2 \times 0.1}{1.2^2 \times 3} = -226.25\text{ N.}$$

(b) $W_f' = \int_0^i \lambda\,di = \int_0^i \left(\frac{1.2\,i^{\frac{1}{2}}}{g}\right)di = \frac{1.2}{g} \times \frac{2}{3} \times i^{3/2}$

$$f_m = \frac{\partial W_f'}{\partial g}(i, g)\Big|_{i = \text{constant}} = -\frac{1.2 \times 2}{3}\,i^{3/2}\frac{1}{g^2}\Big|_{i = \text{constant}}$$

$$= -\frac{1.2 \times 2}{3} \times 2^{3/2} \times \frac{1}{0.1^2}\text{ N}$$

$$= -226.25\text{ N.}$$

3.3 (a) For cast steel (Fig 1.7), $H_c = 350$ A.t/m at $B = 0.5T$

(i) $H_g = \frac{0.5}{4\pi 10^{-7}}$

$Ni = H_c l_c + H_g l_g$

$500\, i = 350 \times 60 \times 10^{-2} + \frac{0.5}{4\pi 10^{-7}} \times 1 \times 10^{-3}$

$i = 1.22\,A.$

$V = 4 \times 1.22 = 4.88\,V.$

(ii) $W_f = \frac{B^2}{2\mu_0} V_g + V_c(\int H\,dB)$

$= \frac{0.5^2}{2 \times 4\pi 10^{-7}} \times 5 \times 5 \times 10^{-4} \times 1 \times 10^{-3} + 5 \times 5 \times 10^{-4} \times 60 \times 10^{-4} \times \left(\frac{0.5 \times 350}{2}\right)$

$= 0.2487 + 0.1313$

$= 0.38\,J$

(iii) $f_m = \frac{B^2}{2\mu_0} \times \text{Area} = \frac{0.5^2}{2 \times 4\pi 10^{-7}} \times 25 \times 10^{-4} = 248.7\,N.$

(iv) $L = \frac{\lambda}{i} = \frac{NAB}{i} = \frac{500 \times 25 \times 10^{-4} \times 0.5}{1.22} = 0.5123\,H$

(b) (i) $Ni = H_c l_c.$

$H_c = \frac{500 \times 1.22}{60 \times 10^{-2}} = 1017$ A.t/m.

From Fig (1.7) $\longrightarrow B_c = 1.2\,T.$

$f_m = \frac{1.2^2}{2 \times 4\pi 10^{-7}} \times 25 \times 10^{-4} = 1432.4\,N.$

$W_f = V_c \int H\,dB = 25 \times 60 \times 10^{-6} \times \frac{1.2 \times 1000}{2} = 0.9\,J$

(ii) Arms moves slowly. Therefore i remains essentially constant.

$$dW_e = i\,d\lambda = i\,NA\,dB$$

$$= 1.22 \times 500 \times 25 \times 10^{-4}\,(1.2 - 0.5)$$

$$= 1.0675\ \text{J}$$

Energy flow is from source to actuator.

Increase in stored energy

$$dW_f = 0.9 - 0.38 = 0.52\ \text{J}.$$

Mechanical energy produced is

$$dW_m = 1.0675 - 0.52 = 0.5475\ \text{J}$$

$\boxed{3.4}$

(a) Flux density is same at all three airgaps

$$Ni = H_g\ 2g = \frac{B_g}{\mu_0} \times 2g$$

$$g = \frac{Ni\,\mu_0}{2B_g} = \frac{600 \times 15 \times 4\pi\,10^{-7}}{2 \times 1.4} = 4.04\ \text{mm}$$

(b) $$f_m = \frac{B^2}{2\mu_0} \times \text{Area} = \frac{1.4^2}{2 \times 4\pi\,10^{-7}} \times (8 \times 80 \times 10^{-4} + 16 \times 80 \times 10^{-4} + 8 \times 80 \times 10^{-4})$$

$$= \frac{1.4^2}{8\pi\,10^{-7}}\,(0.256) = 199.64\ \text{KN}$$

(c) $$f_m \propto B^2 \propto \frac{1}{g^2}$$

$$\frac{199.64 \times 10^3}{1000 \times 9.81} = \left(\frac{g_{max}}{4.04}\right)^2 \longrightarrow g_{max} = 18.225\ \text{mm}$$

$\boxed{3.5}$ (a) $g = 10\ mm = 10 \times 10^{-3}\ m$

(i) $Ni = H_g \cdot 2g = \frac{B_g}{\mu_0} \times 2g$

$$i = \frac{1.25 \times 2 \times 10 \times 10^{-3}}{2500 \times 4\pi\ 10^{-7}} = 7.9577\ A$$

(ii) $W_f = W_{fg} = \frac{B_g^2}{2\mu_0} \times V_g = \frac{1.25^2}{2 \times 4\pi \times 10^{-7}} \times 80 \times 40 \times 10^{-6} \times 10 \times 10^{-3}$

$= 19.8\ J$

(iii) $f_m = \frac{B_g^2}{2\mu_0} \times Area = \frac{1.25^2}{2 \times 4\pi \times 10^{-7}} \times 80 \times 40 \times 10^{-6} = 1980\ N.$

(iv) $Mass = \frac{1980}{9.81} = 201.64\ kg.$

(b) $g = 5\ mm = 5 \times 10^{-3}\ m.$

$$f_m = \frac{B_g^2}{2\mu_0} \times Area \propto B_g^2 \propto \left(\frac{Ni}{2g}\right)^2 \propto \left(\frac{i}{g}\right)^2$$

Force remains same for $g = 5\ mm$ and $g = 10\ mm$.

$$\left(\frac{i_2}{g_2}\right)^2 = \left(\frac{i_1}{g_1}\right)^2 \rightarrow \left(\frac{i_2}{5}\right)^2 = \left(\frac{7.9577}{10}\right)^2$$

$i_2 = 3.97789\ A.$

$\boxed{3.6}$ (a) $N = 2500$, $B_g = 1.5\ T$, $i = \frac{15}{10} = 1.5\ A$

$Ni = H_g g = \frac{B_g}{\mu_0} \times g$

$2500 \times 1.5 = \frac{1.5}{4\pi 10^{-7}} \times g \rightarrow g = 3.1416\ mm$

$\rightarrow g = 3.1416\ mm$

$W_f = \frac{B_g^2}{2\mu_0} \times Volume = \frac{1.5^2}{2 \times 4\pi \times 10^{-7}} \times (0.0016)(3.1416 \times 10^{-3})$

$= 4.5\ J$

(b) $$f_m = \frac{B_g^2}{2\mu_0} \times A_g = \frac{(\mu_0 H_g)^2}{2\mu_0} \times A_g = \frac{\mu_0 A_g}{2} \times H_g^2$$

$$= \frac{\mu_0 A_g}{2}\left(\frac{Ni}{g}\right)^2 \propto \frac{1}{g^2}$$

$$= \frac{4\pi \cdot 10^{-7} \times 0.0016 \times (2500 \times 1.5)^2}{2g^2}$$

$$= \frac{0.0141372}{g^2}$$

(c) $g = 3.1416$ mm

$$f_m = \frac{0.141372}{(3.1416 \times 10^{-3})^2} = 1432.38\ \text{N}$$

or $$f_m = \frac{B_g^2}{2\mu_0} \times A_g = \frac{1.5^2}{2 \times 4\pi\, 10^{-7}} \times 0.0016 = 1432.38\ \text{N}.$$

(d)(i) $$f_m\Big|_{5\,mm} = \frac{0.0141372}{(5 \times 10^{-3})^2} = 565.488\ \text{N}.$$

or $$B_g = \mu_0 H_g = \mu_0 \frac{Ni}{g} = 4\pi 10^{-7} \times \frac{2500 \times 1.5}{5 \times 10^{-3}} = 0.9425\ \text{T}$$

$$f_m = \frac{B_g^2}{2\mu_0} \times A_g = \frac{0.9425^2}{2 \times 4\pi\, 10^{-7}} \times 0.0016 = 565.488\ \text{N}.$$

(ii) λ (hence B_g) stays constant during the motion of the plunger. Therefore the force (f_m) stays constant.

$$W_{mech} = f_m \times g = 565.488 \times 5 \times 10^{-3} = 2.8276\ \text{J}.$$

or $$W_f\Big|_{5\,mm} = \frac{B_g^2}{2\mu_0} \times A \times g = \frac{0.9425^2}{2 \times 4\pi\, 10^{-7}} \times 0.0016 \times 5 \times 10^{-3}$$

$$= 2.8276\ \text{J}.$$

$$W_f\Big|_{10\,mm} = \frac{B_g^2}{2\mu_0} \times \text{Volume} = \frac{B_g^2}{2\mu_0} \times 0 = 0.$$

$$dW_m = W_f\Big|_{final} - W_f\Big|_{initial} = 2.8276\ \text{J}.$$

Note: $dW_e = ei\,dt = i\,d\lambda = i \times 0 = 0$.

$\boxed{3.7}$ (a)(i) $f_m = 550 = \frac{B^2}{2\mu_0} \times$ pole area.

$$B = \sqrt{\frac{550 \times 2 \times 4\pi 10^{-7}}{2 \times 5 \times 5 \times 10^{-4}}} = 0.5258\,T$$

$$Ni = H_g l_g = \frac{B}{\mu_0} 2g$$

$$i = \frac{0.5258 \times 2 \times 1 \times 10^{-3}}{4\pi\, 10^{-7} \times 400} = 2.09\,A$$

$$V = 5 \times 2.09 = 10.45\,V \text{ (dc)}$$

(ii) $W_f = \frac{B^2}{2\mu_0} \times$ volume of airgap

$$= \frac{0.5258^2}{2 \times 4\pi 10^{-7}} \times 2 \times 5 \times 5 \times 10^{-4} \times 10^{-3}$$

$$= 0.55\,J.$$

or $W_f = \frac{1}{2}\lambda i$ (linear system)

$$= \frac{1}{2} NABi = \frac{1}{2} \times 400 \times 5 \times 5 \times 10^{-4} \times .5258 \times 2.09$$

$$= 0.55\,J$$

(b) $L = \frac{N^2}{R_g} = \frac{N^2}{2g/\mu_0 A_g} = \frac{N^2 \mu_0 A_g}{2g}$

$$= \frac{400^2 \times 4\pi 10^{-7} \times 5 \times 5 \times 10^{-4}}{2 \times 10^{-3}} = 0.2513\,H$$

$$Z = \sqrt{5^2 + (2\pi 60 \times 0.2513)^2} = 94.87\,\Omega$$

$f_m \propto B_{rms}^2$, $B_{rms} \propto I_{rms}$.

Hence, same rms current is required to produce the same average torque.

$$I_{rms} = 2.09\,A.$$

$$V_{rms} = 2.09 \times 94.87 = 198.3\,V.$$

3.8

(a) $L = 4.5 + 18\theta\ \mu H$, $K_s = 0.65 \times 10^{-3}$ N.m/rad.

$$T = \frac{1}{2} i^2 \frac{dL}{d\theta} = \frac{1}{2} i^2 \times 18 \times 10^{-6} = 9 \times 10^{-6} i^2$$

$$T\Big|_{avg} = 9 \times 10^{-6} \left| i^2 \right|_{avg} = 9 \times 10^{-6} I_{rms}^2 \text{ N.m.}$$

In the equilibrium position,

$$0.65 \times 10^{-3}\theta = 9 \times 10^{-6} I_{rms}^2$$

$$\theta = 13.85 \times 10^{-3} I_{rms}^2$$

Deflection (θ) is proportional to square of the rms current. Scale on the ammeter will be nonlinear.

(b) $\theta = 13.85 \times 10^{-3} \times 10^2 = 1.385$ radians.

$$= \frac{180}{\pi} \times 1.385 = 79.35^\circ.$$

(c) $L = 4.5 + 18\,(1.385)\,\mu H = 29.43\,\mu H$

$$Z = 0.015 + j\,377 \times 29.43 \times 10^{-6} = 0.015 + j0.011$$

$$|Z| = 0.0186\,\Omega.$$

$$|V| = 0.0186 \times 10 = 0.186\text{ V}.$$

3.9

(a) $T = \frac{1}{2} i^2 \frac{dL}{d\theta} = \frac{1}{2} i^2 \left(+0.6 \sin 2\theta + 0.8 \sin 4\theta \right)$

Let $i = \sqrt{2}\,10 \sin \omega t$, $\theta = \omega_m t - \delta$.

$$T = 100 \sin^2 \omega t \left[0.6 \sin(2\omega_m t - 2\delta) + 0.8 \sin(4\omega_m t - 4\delta) \right]$$

$$= 30 \sin(2\omega_m t - 2\delta) - 30 \cos 2\omega t \sin(2\omega_m t - 2\delta)$$
$$+ 40 \sin(4\omega_m t - 4\delta) - 40 \cos 2\omega t \sin(4\omega_m t - 4\delta)$$

$$T = 30 \sin(2\omega_m t - 2\delta) + 40 \sin(4\omega_m t - 4\delta)$$
$$- 15\left[\sin(2\omega_m t - 2\delta + 2\omega t) + \sin(2\omega_m t - 2\delta - 2\omega t)\right]$$
$$- 20\left[\sin(4\omega_m t - 4\delta + 2\omega t) + \sin(4\omega_m t - 4\delta - 2\omega t)\right]$$

For $\omega_m \neq 0$, average torques are produced when

$$\omega_m = \pm\omega = \pm 377 \text{ rad/sec.}$$

or $\omega_m = \pm \frac{\omega}{2} = \pm 188.5$ rad/sec.

(b) For $\omega_m = \pm 377$ rad/sec.

$$T_{avg} = 15 \sin 2\delta$$
$$T_{max} = 15 \text{ N.m.}$$
$$P_{max} = 15 \times 377 = 5655 \text{ W}$$

For $\omega_m = \pm 188.5$ rad/sec.

$$T_{avg} = 20 \sin 4\delta$$
$$T_{max} = 20 \text{ N.m.}$$
$$P_{max} = 20 \times 188.5 = 3770 \text{ W}$$

Note: At lower speed more torque but less power is available.

(C) For $\omega_m = 0$

$$T_{avg} = 30 \sin 2\delta + 40 \sin 4\delta$$

For maximum torque, $\frac{dT_{avg}}{d\delta} = 0 \rightarrow$

$$0 = 60 \cos 2\delta + 160 \cos 4\delta \rightarrow \delta = 25.86^\circ.$$

$$T_{max} = 30 \sin 51.72 + 40 \sin 103.44 = 62.45 \text{ N.m.}$$

$\boxed{3.10}$ (a) $\phi = \int \frac{v}{N} dt = \frac{1}{N} \int \sqrt{2} \times 120 \sin \omega t = \frac{\sqrt{2} \times 120}{200\, \omega} \cos \omega t$

$= \frac{\sqrt{2} \times 120}{200 \times 377} \cos \omega t = 2.25 \times 10^{-3} \cos \omega t = \phi_m \cos \omega t.$

(b) $Ni = \phi \mathcal{R} \rightarrow i = \frac{\phi \mathcal{R}}{N} \rightarrow i^2 = \frac{\phi^2 \mathcal{R}^2}{N^2}$

$L = \frac{N^2}{\mathcal{R}} \rightarrow \frac{dL}{d\theta} = N^2 \frac{d}{d\theta} \left(\frac{1}{\mathcal{R}}\right) = \frac{N^2}{\mathcal{R}^2} \cdot \frac{d\mathcal{R}}{d\theta}.$

$T = \frac{1}{2} i^2 \frac{dL}{d\theta} = -\frac{1}{2} \times \frac{\phi^2 \mathcal{R}^2}{N^2} \times \frac{N^2}{\mathcal{R}^2} \cdot \frac{d\mathcal{R}}{d\theta} = -\frac{1}{2} \phi^2 \frac{d\mathcal{R}}{d\theta}$

(c) $\mathcal{R} = 2 \times 10^5 - 10^5 \cos 4\theta = \mathcal{R}_0 - \mathcal{R}_a \cos 4\theta$

$T = -\frac{1}{2} \phi^2 \frac{d\mathcal{R}}{d\theta} = \frac{1}{2} \phi_m^2 \cos^2 \omega t \cdot 4 \mathcal{R}_a \sin 4(\omega_m t + \delta)$

$= \phi_m^2 \mathcal{R}_a \left[\sin 4(\omega_m t + \delta) + \cos 2\omega t \sin 4(\omega_m t + \delta)\right]$

$= \phi_m^2 \mathcal{R}_a \left[\sin 4(\omega_m t + \delta) + \frac{1}{2} \sin \{2(2\omega_m + \omega)t + 4\delta\}\right.$
$\left. + \frac{1}{2} \sin \{2(2\omega_m - \omega)t + 4\delta\}\right]$

For average torque,

$\omega_m = 0$ and $\omega_m = \pm \frac{\omega}{2}.$

(d)(i) $\omega_m = 0. \rightarrow T_{avg} = \phi_m^2 \mathcal{R}_a \sin 4\delta.$

$T_{max} = \phi_m^2 \mathcal{R}_a = (2.25 \times 10^{-3})^2 \times 10^5 = 0.506$ N.m.

$P_{max} = T \times \omega_m = 0.$

(ii) $\omega_m = \pm \frac{\omega}{2} = \pm \frac{2\pi 60}{2} = \pm 188.5$ rad/sec.

$T_{avg} = \phi_m^2 \mathcal{R}_a \times \frac{1}{2} \sin 4\delta = 0.253 \sin 4\delta.$

$T_{max} = 0.253$ N.m.

$P_{max} = 0.253 \times 188.5 = 47.69$ W.

$\boxed{3.11}$ (a) $e_2 = \frac{d\lambda_2}{dt} = \frac{d}{dt}(L_{sr}\, i_s)$

Let, $\theta = \omega_m t + \delta = \left(\frac{3600}{60} 2\pi\right) t + \delta = 120\pi t + \delta$

$i = \sqrt{2} \times 5 \sin \omega t = 7.07 \sin 2 \times 60 \pi t$

$= 7.07 \sin 120 \pi t.$

$e_2 = \frac{d}{dt}\left[0.08 \cos(120\pi t + \delta)\, 7.07 \sin 120\pi t\right]$

$= 213.2 \cos(240\pi t + \delta) = \sqrt{2} \times 150.8 \cos(240\pi t + \delta)$

$E_2\big|_{rms} = 150.8\ V$ and $f_2 = 120\ Hz.$

(b) $T = i_s i_r \frac{dL_{sr}}{d\theta} = i^2 \frac{dL_{sr}}{d\theta}$ & $i = \sqrt{2} \times 5 \sin \omega t.$

$T = 50 \sin^2 \omega t \left[-0.08 \sin(\omega_m t + \delta)\right]$

$= -4\left(\frac{1 - \cos 2\omega t}{2}\right) \sin(\omega_m t + \delta)$

$= -2 \sin(\omega_m t + \delta) + \sin(\omega_m t + 2\omega t + \delta)$

$+ \sin(\omega_m t - 2\omega t + \delta)$

For average torque,

1. $\omega_m = 0$, $T_{avg} = -2 \sin \delta \rightarrow T_{max} = 2\ N.m.$

2. $\omega_m = \pm 2\omega = \pm 240\pi\ rad/sec.$,

$T_{avg} = 1 \sin \delta \longrightarrow T_{max} = 1.0\ N.m.$

CHAPTER 4

4.1 (a) 120 V machine → lap winding
240 V machine → wave winding

(b) 120 V machine
parallel paths, $\overset{\parallel\ \text{paths}}{a} = \overset{\text{poles}}{p} = 4$
coils in series in each path $= \frac{120}{4} = 30$
Total coils $= 30 \times 4 = 120$

240 V machine
$a = 2$, coils in each path $= \frac{240}{4} = 60$
Total coils $= 60 \times 2 = 120$

(c) 120 V machine
$I_a = 4 \times I_c = 4 \times 5 = 20\ A$
$KW = 120 \times 20 \times 10^{-3} = 2.4$

240 V machine
$I_a = 2 \times I_c = 2 \times 5 = 10\ A$
$KW = 240 \times 10 \times 10^{-3} = 2.4$

4.2 (a) $N = 300$, $p = 4$, $a = 2$, $\Phi = 0.025$ Wb.
$$K_a = \frac{Np}{\pi a} = \frac{300 \times 4}{\pi \times 2} \doteq 190.99$$
$$E_a = 190.99 \times 0.025 \times \frac{1000}{60} \times 2\pi = 500\text{V}.$$
(b) $I_a = 2 \times 25 = 50\ A.$
$P = 500 \times 50 = 25\ KW$

4.3 (a) $\omega_m = \frac{1200}{60} \times 2\pi = 125.66$ rad./sec.
$K_a\phi = 114/125.66 = 0.907$ V/rad./sec.

(b) $E_a = 114\ V$
$I_a = 114/(0.2+2) = 51.82\ A$

(c) $T = K_a\phi I_a = 0.907 \times 51.82 = 47$ N·m
$P = I_L^2 R_L = 51.82^2 \times 2 = 5370.6$ W

4.4 (a) $k_a\phi = 0.907$ V/rad./sec. ← depends on If and not on speed

(b) $E_a = 114 \times \frac{800}{1200} = 76\,V$

$I_a = 76/2.2 = 34.55\,A$

(c) $T = 0.907 \times 34.55 = 31.33$ N·m

$P = 34.55^2 \times 2 = 2387.41\,W$

4.5 (A) $I_a\big|_{rated} = \frac{6000}{120} = 50\,A$

$E_a\big|_{1500\,rpm} = 120 + 50 \times 0.2 = 130\,V.$

$E_a\big|_{1200\,rpm} = 130 \times \frac{1200}{1500} = 104\,V.$

$I_f \simeq 0.64\,A$ → from magnetization curve.

(b.) $R_f = \frac{120}{0.64} = 187.5\,\Omega.$

$R_{fc} = 187.5 - 100 = 87.5\,\Omega.$

4.6 (a) $R_f\,(min) = 100\,\Omega$

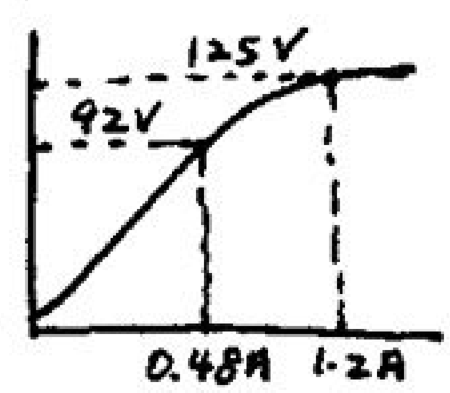
$I_f\,(max) = 120/100 = 1.2\,A$

$E_a\,(max) = 125\,V$ (from magnetization curve)

$R_f\,(max) = 250\,\Omega$

$I_f\,(min) = 120/250 = 0.48\,A$

$E_a\,(min) = 92\,V$ (from curve)



(b) $V_t\big|_{NL} = E_a = 120\,V$

$I_f = 1.0\,A$ (from curve)

$R_f = 120/1.0 = 120\,\Omega$

$R_{fc} = 120 - 100 = 20\,\Omega$

No AR → $V_t = 120 - 50 \times 0.2 = 110\,V$

$I_f\,(AR) = 0.1\,A$

$I_f\,(eff) = 1 - 0.1 = 0.9\,A$

$E_a = 117.5\,V$

$V_t = 117.5 - 10 = 107.5\,V$

$\boxed{4.7}$ (a) $I_{f(max.)} = \frac{120}{100} = 1.2\,A$, $E_{a\,(max)} = 125 \times \frac{1500}{1200} = 156.25\,V$

$I_{f\,(min)} = \frac{120}{250} = 0.4AA$, $E_{a(min)} = 92 \times \frac{1500}{1200} = 115\,V.$

(b) $V_t\Big|_{\substack{NL \\ 1500\,rpm}} = E_a\Big|_{1500} = 120\,V.$

$E_a\Big|_{1200} = 120 \times \frac{1200}{1500} = 96\,V.$

$I_f \simeq 0.534\,A \rightarrow$ from magnetization curve

$R_f = \frac{120}{0.534} = 224.72\,\Omega$, $R_{fc} = 224.72 - 100 = 124.72\,\Omega$

No AR $\rightarrow$ $V_t = 120 - 50 \times 0.2 = 110\,V.$

$I_{f\,(AR)} = 0.1\,A \longrightarrow I_{f\,(eff)} = 0.534 - 0.1 = 0.434\,A$

$E_a\Big|_{1200} = 83.8\,V$, $E_a\Big|_{1500} = 83.8 \times \frac{1500}{1200} = 104.75\,V.$

$V_t = 104.75 - 50 \times 0.2 = 94.75\,V.$

$\boxed{4.8}$ (a) $I_a\Big|_{rated} = \frac{6000}{120} = 50\,A$

At $I_f = 1.0\,A$ and 1200 rpm $\rightarrow E_a = 120\,V.$

$V_t = 120 - 50 \times 0.2 = 110\,V.$

$P_o = 110 \times 50 = 5500\,W$, $P_{in} = 120 \times 50 + 400 = 6400\,W$

$Eff. = \frac{5500}{6400} \times 100\% = 85.94\%$

(b) At $I_f = 1.0\,A$ and 1500 rpm $\rightarrow E_a = 120 \times \frac{1500}{1200} = 150\,V$

$V_t = 150 - 50 \times 0.2 = 140\,V.$

$P_o = 140 \times 50 = 7000\,W$

$P_{in} = 150 \times 50 + 400 \times \frac{1500}{1200} = 8000\,W$

$Eff. = \frac{7000}{8000} \times 100\% = 87.5\%$

$\boxed{4.9}$ (a) self - excited

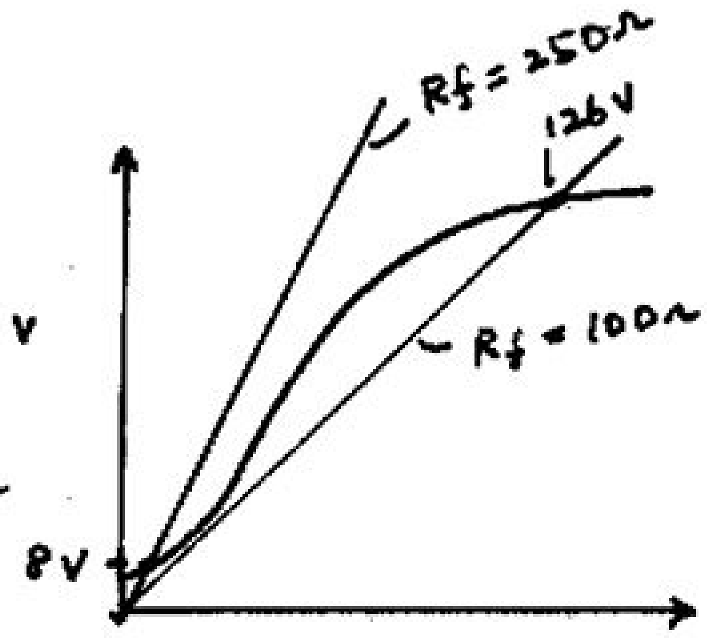
$R_f = R_{fw} = 100\,\Omega$ Line

$V_t|_{NL} = E_a(max) = 126\ V$

$R_f = R_{fw} + R_{fc(max)} = 100 + 150$

$= 250\,\Omega$ line

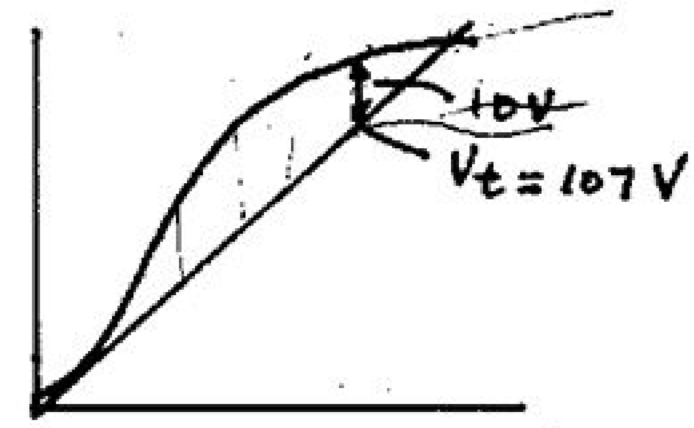
$V_t|_{NL} = E_a(min) = 8\ V$



(b) $V_t|_{NL} = E_a = 120\,V$

$R_f = 120/1 = 120\,\Omega$

$R_{fc} = 120 - 100 = 20\,\Omega$

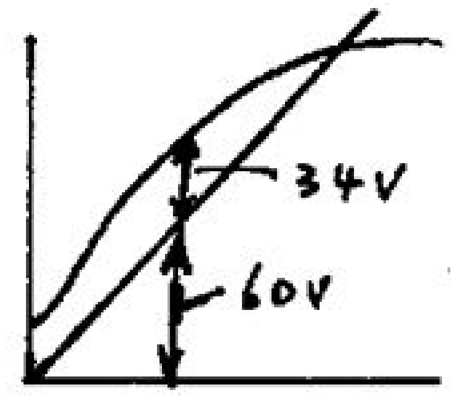


(i) 50A flows

$I_aR_a = 50 \times 0.2 = 10\,V$

$V_t = 107\ V$

$I_aR_a|_{max}$ = max vertical distance between $120\,\Omega$ line + E_a Curve = 34



$I_a = \frac{34}{0.2} = 170\,A$

$V_t = 60\,V$

(ii) $I_a = 50\,A \longrightarrow I_{f(AR)} = 0.1\,A$

From triangle abc

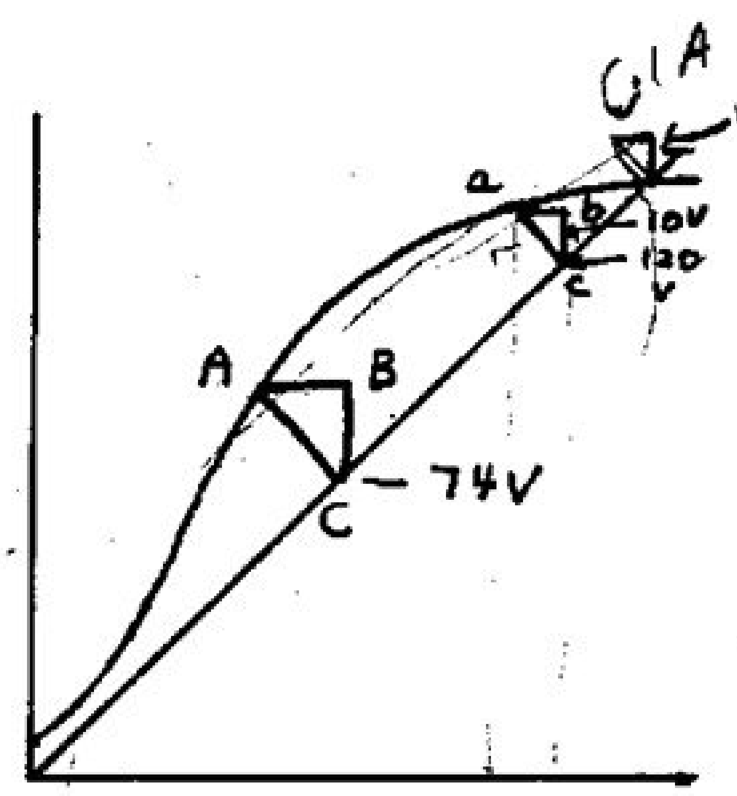
$V_t = 102$

From triangle ABC

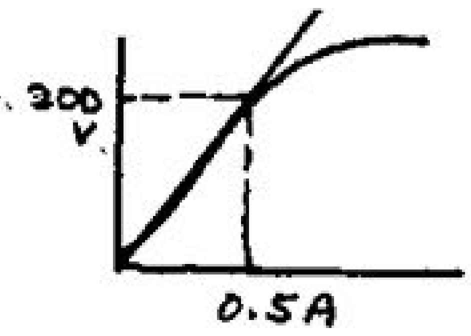
$V_t = 74\,V$

$BC = 16\ V$

$I_a = \frac{16}{0.2} = 80\,A$

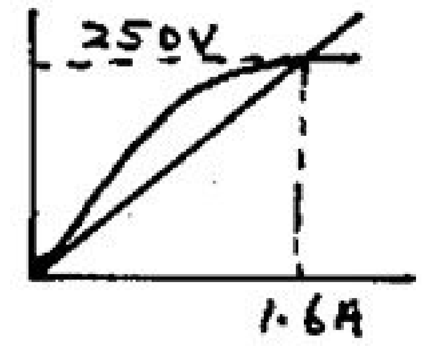


$\boxed{4.10}$ (a) E_a (residual) = 10 V



(b) R_f (crit.) = $200/0.5 = 400\Omega$

(c) $R_f = \dfrac{250}{1.6} = 156.25\,\Omega$



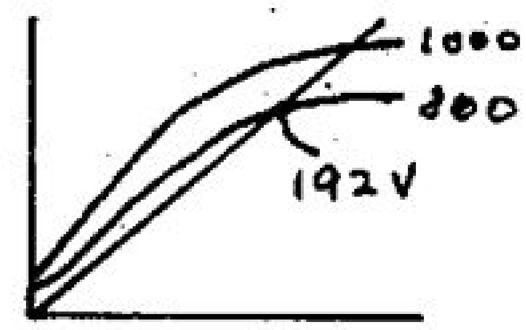
$R_{fc} = 156.25 - 133 = 23.25\,\Omega$

(d) $E_a \propto n$

Draw magnetization curve for 800 rpm

Field Resistance line for $133\,\Omega$ intersects this mag. curve at

$E_a = 192\text{V}$

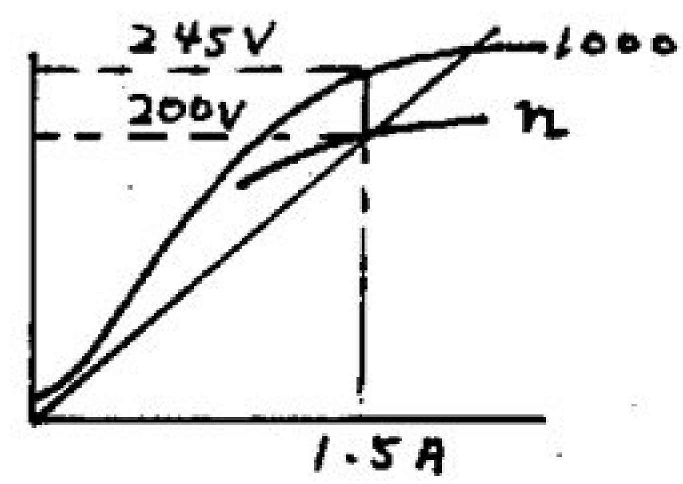


(e) Magnetization curve at n rpm will intersect the field resistance line for $133\,\Omega$ at $E_a = 200\text{V}$. Therefore, $I_f = 1.5\text{A}$.

At 1000 rpm (for $I_f = 1.5\text{A}$), $E_a = 245\text{ V}$

$$\frac{n}{1000} = \frac{200}{245}$$

$$n = \frac{200}{245} \times 1000 = 816.3 \text{ rpm}$$



$\boxed{4.11}$ (a) Rated load : $V_t = 250$ V

$I_a = I_a(\text{rated}) = \dfrac{10000}{250} = 40\text{A}$

$E_a = V_t + I_a R_a = 250 + 40 \times 0.25 = 258\text{V}$

(b) Developed power $= E_a I_a = 258 \times 40 = 10{,}320\text{ W}$

$$\omega_m = \frac{1000}{60} \times 2\pi = 104.72 \text{ rad/sec.}$$

$$T = \frac{10320}{104.72} = 98.55 \text{ N-m}$$

(c) From magnetization curve

for $E_a = 258\text{V}$, $I_f = 1.86$ A

(d) $P_{out} = V_t I_t = 250(40 - 1.86) = 9535\text{ W}$

$P_{in} = E_a I_a + P_{rotational} = 10{,}320 + 500 = 10820\text{W}$

$$\eta = \frac{9535}{10820} \times 100 = 88.12\,\%$$

4.12 (a) $I_a|_{FL} = 24000/240 = 100\,A$

$E_a = V_t + I_aR_a = 225 + 100 \times 0.12 = 237\,V$

$\omega_m = 1000/60 \times 2\pi = 104.67$ rad./sec.

$$T = \frac{E_aI_a}{\omega_m} = \frac{237 \times 100}{104.67} = 226.43\ N\cdot m$$

(b) $E_a|_{NL} = 240\ V$

$E_a|_{FL} = 237\ V$

$\Delta E_a|_{AR} = 240 - 237 = 3\,V$

(c) MMF required at full-load $= 600 \times 2.2 = 1320$ A-t

MMF provided by shunt-field winding $= 600 \times 1.8$
$= 1080$ A-t

MMF provided by series field winding

$N_{sr}I_{sr} = N_{sr}I_a = 1320 - 1080 = 240$ A-t

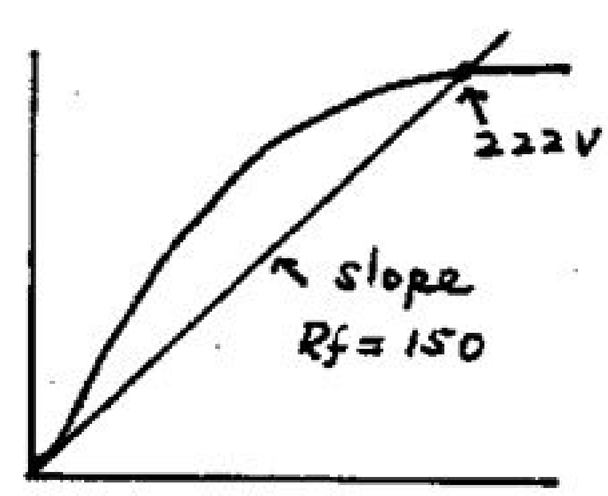
$$N_{sr} = \frac{240}{100} = 2.4 \text{ turns/pole}$$

4.13 1(a) $E_a|_{max}$ will occur at $R_{fc} = 0$

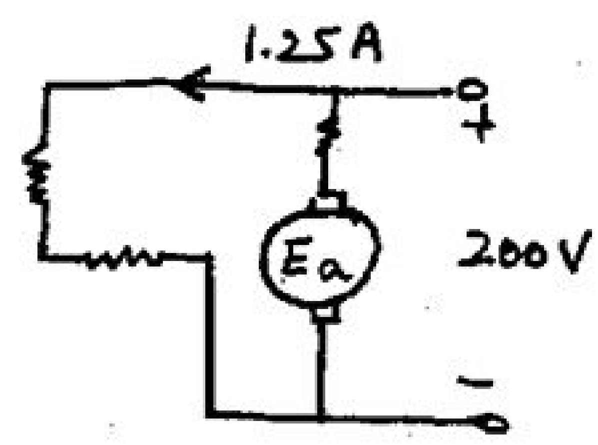
Draw field resistance line for

$R_f = R_{fw} = 150\,\Omega$

$E_a(max) = 222\,V$



(b) $I_a(\text{rated}) = \frac{20,000}{200} = 100\,A$



$V_t(\text{rated}) = 200\ V$

$R_f = \frac{200}{1.25} = 160\,\Omega$

$R_{fc} = 160 - 150 = 10\,\Omega$

(c) $E_a = V_t + I_aR_a = 200 + 100 \times 0.1 = 210\,V$

$P_{dc} = E_aI_a = 210 \times 100 = 21000\ W$

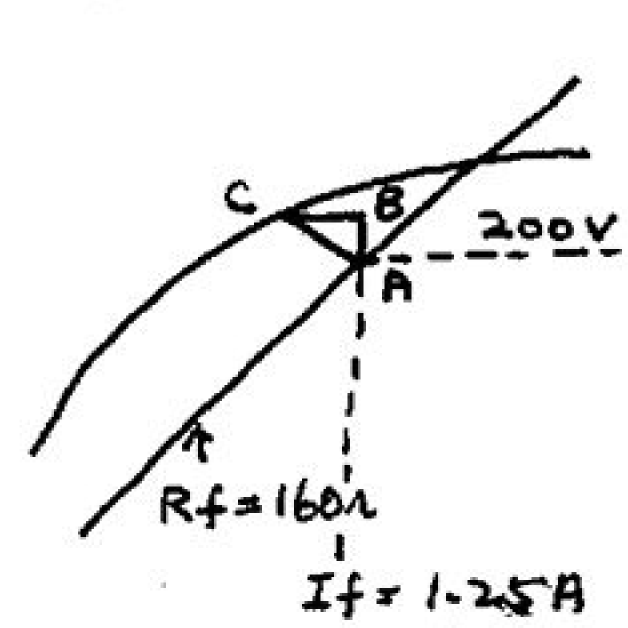
$\omega_m = \frac{1800}{60} \times 2\pi = 188.5$ rad./sec.

$$T = \frac{E_aI_a}{\omega_m} = \frac{21000}{188.5} = 111.41\ N\cdot m$$

4.13 Continued

(d) Draw field resistance line at $R_f = 160\Omega$

At $V_t = 200V$, $I_f = 1.25A$ draw triangle ABC with a vertical line $AB = 10V$ and horizontal line BC where C is on magnetization curve

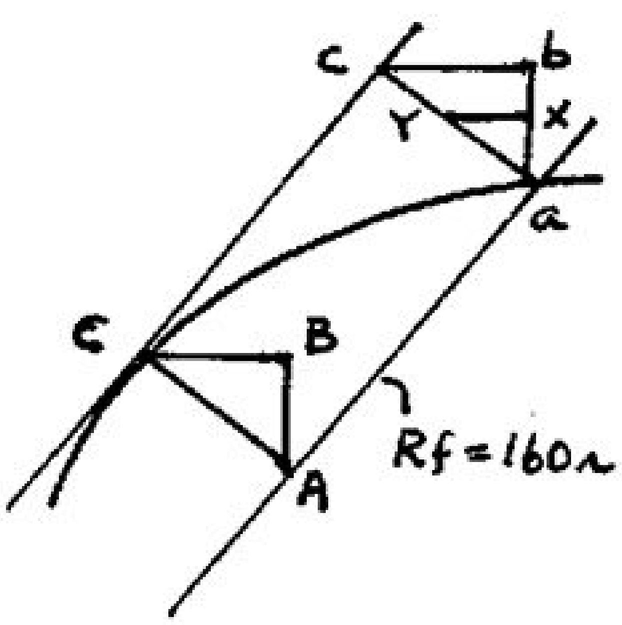


$BC = 0.09A = I_{f(AR)}$

(e) $AB = 25V$

$I_a = \frac{25}{0.1} = 250A$

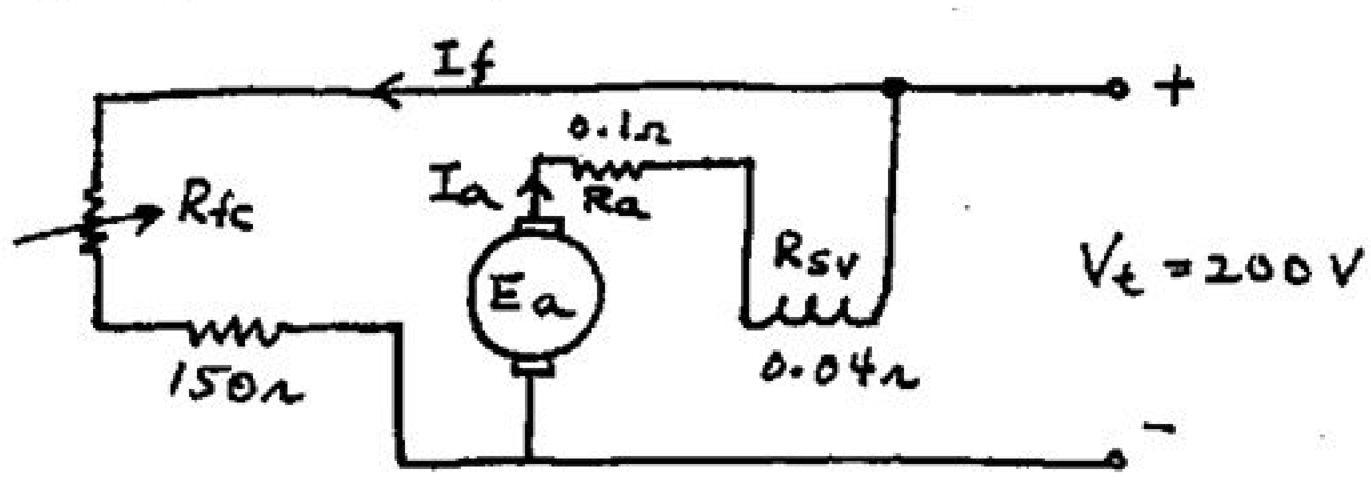
For full-load, construct triangle aXY such that $aX = 10V$, $XY = 0.09A$. Draw Cc parallel to Aa such that Cc is tangent to magnetisation curve. Draw Δ abc. This is the largest Δ that will fit inside the magnetisation curve and field resistance line at C. Δ ABC is same as Δ abc



$AB = 25V = I_{a(max)} R_a$

$I_{a(max)} = \frac{25}{0.1} = 250A$

4.13 (2 a)



(b) $E_a = V_t + I_a(R_a + R_{sv})$

$= 200 + 100(0.1 + 0.04)$

$= 214V$

From the magnetization curve for $E_a = 214V$

$I_{f(eff)} = 1.25A = I_f + \frac{N_{sr} I_a}{N_f} - I_{f(AR)}$

4.13 continued

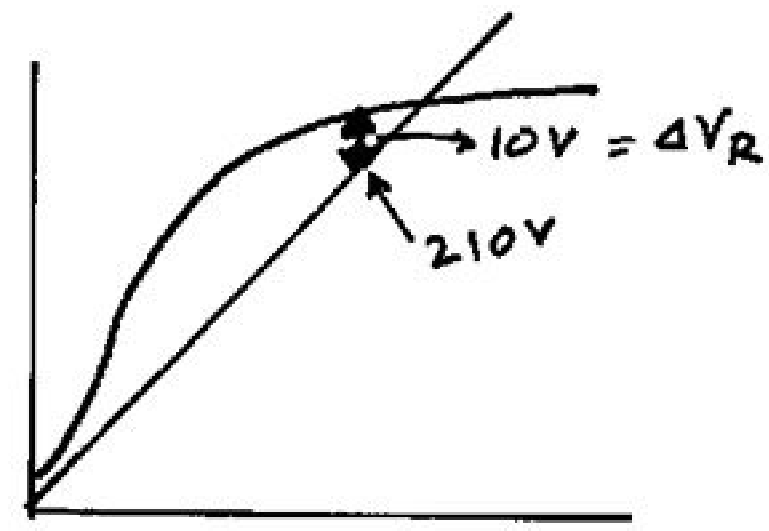
Now $I_{f(AR)} = 0.09\ A$ at full-load value of I_a (from part 1d)

$V_t|_{NL} \simeq E_a = 200\ V \rightarrow$ from magnetization curve $I_f = 1\ A$

Hence $1.25 = 1 + \dfrac{N_{sr} \times 100}{1200} - 0.09$

$\Rightarrow N_{sr} = 4.08$ turns / pole

$\boxed{4.14}$ (a) $I_a\Big|_{rated} = \dfrac{20{,}000}{200} = 100\ A.$

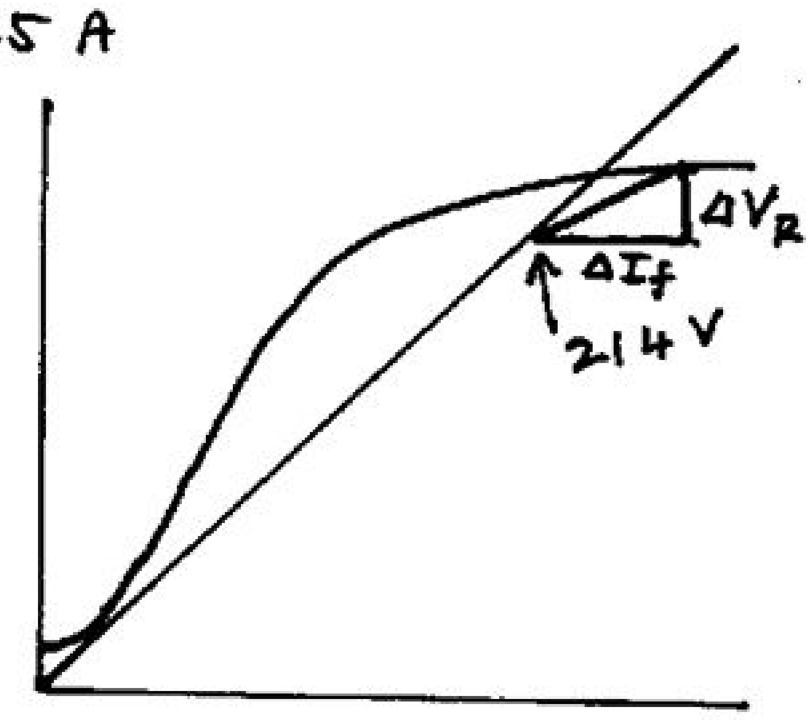


$\Delta V_R = I_a R_a = 100 \times 0.1 = 10 V.$

$V_t = 210\ V.$

(b) $\Delta I_f = + \dfrac{N_{sr} I_a}{N_f} = \dfrac{3 \times 100}{1200} = +0.25\ A$

$\Delta V_R = 100\,(0.1 + 0.03) = 13\ V.$



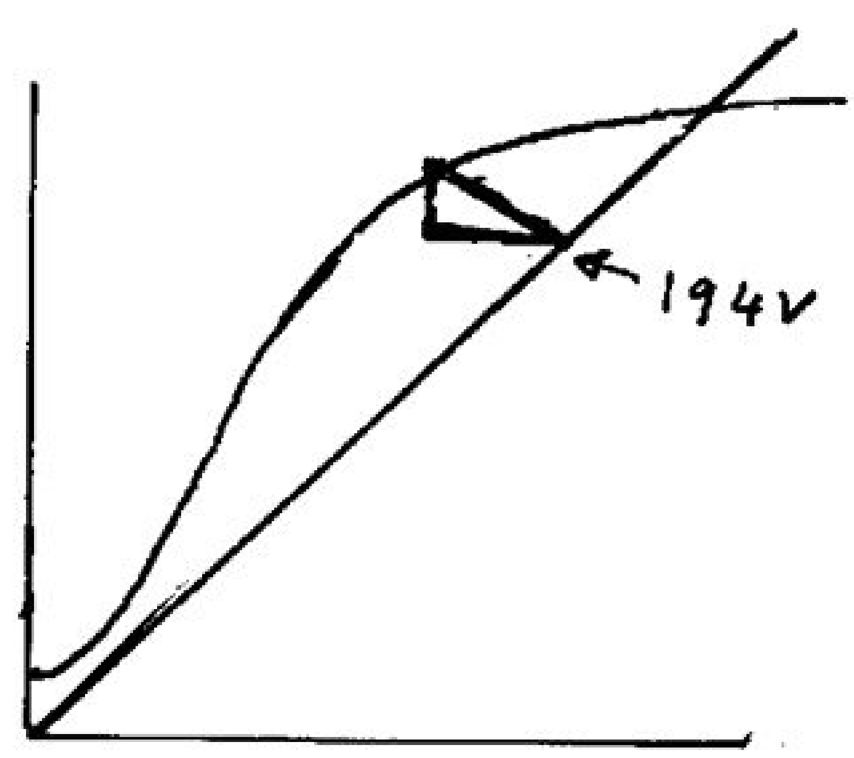
The right-angle triangle with sides $\Delta V_R = 13\ V$ and $\Delta I_f \rightarrow 0.25\ A$ with points on the resistance line and magnetization curve gives

$V_t \simeq 214\ V.$

(c) $\Delta I_f = -\ 0.25 A$

$\Delta V_R = 13\ V.$

$V_t \simeq 194\ V.$



$\boxed{4.15}$ (a) $E_a = 106\text{V} \rightarrow V_t = 106 - 20 \times 0.3 = 100\text{V}$

(b) $E_a = 184\text{V} \rightarrow V_t = 184 - 40 \times 0.3 = 172\text{V}$

(c) $E_a = 208 \rightarrow V_t = 208 - 60 \times 0.3 = 190\text{V}$

$\boxed{4.16}$ (a) $T = \dfrac{100 \times 746}{2000 \times 2\pi/60} = 356.2\ \text{N.m}$

(b) $P_{in} = \dfrac{100 \times 746}{0.9} = 82{,}888.9\text{W}$

$I_a = \dfrac{82888.9}{440} = 188.4\text{A}$.

$\boxed{4.17}$ $200 = K_a\Phi \times \dfrac{1800}{60} \times 2\pi \rightarrow K_a\Phi = 1.061$

$I_a = \dfrac{T}{K_a\Phi} = \dfrac{100}{1.061} = 94.25\text{A}$

(a) $E_a = 220 - 94.25 \times 0.1 = 210.575\text{V}$

$n = \dfrac{210.575}{1.061 \times 2\pi} \times 60\ \text{rpm} = 1895.2\ \text{rpm}.$

(b) $E_a = 220 + 94.25 \times 0.1 = 229.425\ \text{V}$

$n = \dfrac{229.425}{1.061 \times 2\pi} \times 60\ \text{rpm} = 2064.9\ \text{rpm}.$

$\boxed{4.18}$ $R_{fc} = 0\Omega \rightarrow I_f = \dfrac{200}{150} = 1.3333\text{A}$

From the magnetization curve $\rightarrow E_a = 218\text{V}$.

$\dfrac{n}{1800} = \dfrac{200}{218} \longrightarrow n = 1651.4\ \text{rpm}.$

$R_{fc} = 200\Omega \rightarrow I_f = \dfrac{200}{150+200} = 0.5714\text{A}$

From the magnetization curve $\rightarrow E_a = 150\text{V}$.

$\dfrac{n}{1800} = \dfrac{200}{150} \rightarrow n \doteq 2400\ \text{rpm}.$

Note: If the field current decreases, the speed increases.

$\boxed{4.19}$ (a) $I_a\Big|_{FL} = \dfrac{20 \times 1000}{200} = 100\,A.$

$E_a = 200 - 100 \times 0.1 = 190\,V.$

$n = \dfrac{190}{218} \times 1800 = 1568.8$ rpm.

(b) $218 = K_a \Phi\, 1800$

$190 = K_a (0.9\,\Phi)\, n$

$\rightarrow n = \dfrac{190}{218} \times \dfrac{1800}{0.9} = 1743.1$ rpm.

Note: Speed increases due to armature reaction effect.

$\boxed{4.20}$ (a) $R_{fc} = 50\,\Omega \rightarrow I_f = \dfrac{200}{150+50} = 1\,A$.

$E_a\Big|_{1800} = 200\,V \rightarrow$ from the magnetization curve.

No-load, $V_t = E_a = 200\,V \longrightarrow n = 1800$ rpm.

(b) $I_a\Big|_{FL} = 100\,A.$

$I_{f(eff.)} = I_f + \dfrac{N_{sr}}{N_f} I_a = 1 + \dfrac{5}{1200} \times 100 = 1.4167\,A$

From the magnetization curve, the corresponding E_a is:

$E_a\Big|_{1800} = 220\,V.$

The actual E_a is

$E_a = 200 - 100\,(0.1 + 0.05) = 185\,V.$

Thus, $n = \dfrac{185}{220} \times 1800 = 1513.6$ rpm.

$\boxed{4.21}$ (a) Same as problem 4.20. → $n = 1800$ rpm

(b) $I_{f(eff)} = 1 - \frac{5 \times 100}{1200} = 0.5833A$.

From the magnetization curve, the corresponding E_a is → $E_a = 153V$.

The actual E_a is 185V (problem 4.20)

Thus, $n = \frac{185}{153} \times 1800 = 2176.5$ rpm.

$\boxed{4.22}$ (a) $I_a = 50A$

The equivalent I_f is → $I_f = \frac{5}{1200} \times 50 = 0.2083A$

The corresponding E_a at 1800 rpm → $E_a\big|_{1800} = 54V$.

The actual E_a is → $E_a = 200 - 50(0.1 + 0.05) = 192.5V$

$$n = \frac{192.5}{54} \times 1800 = 6416.7 \text{ rpm.}$$

$$54 = K_a\Phi \times \frac{1800}{60} \times 2\pi \rightarrow K_a\Phi = 0.2865$$

$$T = 0.2865 \times 50 = 14.3239 \text{ N.m}$$

(b) $I_a = 100A \rightarrow I_f = \frac{5}{1200} \times 100 = 0.4167A$.

→ $E_a\big|_{1800} = 112V$ from the magnetization curve.

$$E_a\big|_{actual} = 200 - 100(0.1 + 0.05) = 185V.$$

$$n = \frac{185}{112} \times 1800 = 2973.2 \text{ rpm.}$$

$$K_a\Phi = \frac{112 \times 60}{1800 \times 2\pi} = 0.5942.$$

$$T = 0.5942 \times 100 = 59.42 \text{ N.m}$$

Note: In series motor → Low torque, high speed and high torque, low speed.

$\boxed{4.23}$ (a) $V_t < E_a \longrightarrow I_a$ into the machine

$\longrightarrow$ motor

(b) $R_a = \dfrac{240 - 230}{40} = 0.25\,\Omega$

(c) $I_a^2 R_a = 40^2 \times 0.25 = 400\,W$

$E_a I_a = 230 \times 40 = 9200\,W$

(d) $\omega_m = \dfrac{1200}{60} \times 2\pi = 125.7$ rad./sec.

$T = \dfrac{9200}{125.7} = 73.19$ N.m

(e)(i) Load thrown-off $E_a = V_t = 240\,V$

$240 = K\phi_{NL} n_{NL}$

Before load is thrown off,

$230 = K\phi_{FL} 1200$

$\phi_{NL} = \phi_{FL}$

so $\dfrac{n}{1200} = \dfrac{240}{230}$

$n = \dfrac{240}{230} \times 1200 = 1252.2$ rpm

(ii) $240 = K\phi_{NL} n_{NL}$

$230 = K\phi_{FL}\, 1200 = K(0.9)\phi_{NL}\, 1200$

$n = \dfrac{240}{230} \times 1200 \times 0.9 = 1127$ rpm

$\boxed{4.24}$ $P_{Load} = P_{dev.} = E_a I_a = (200 - I_a \times 0.1) I_a = 15360\,W$

$\longrightarrow I_a = 80A$ or $1920A \longrightarrow I_a = 80A$.

$E_a = 200 - 80 \times 0.1 = 192V$.

(a) $R_{fc} = 0 \longrightarrow I_f = \dfrac{200}{150} = 1.3333A$.

From the magnetization curve $\longrightarrow E_a\big|_{1800} = 218V$.

$K_a\Phi = \dfrac{218}{1800 \times 2\pi/60} = 1.1565 \longrightarrow n = \dfrac{192}{1.1565 \times 2\pi} \times 60 = 1585.4$ rpm.

4.24 Continued :

$R_{fc} = 200\Omega \rightarrow I_f = \frac{200}{350} = 0.5714A.$

$\rightarrow E_a\Big|_{1800} = 150V \rightarrow K_a\Phi = \frac{150}{1800 \times 2\pi/60} = 0.7958$

$n = \frac{192}{0.7958 \times 2\pi} \times 60 = 2303.9$ rpm.

Speed range $1585.4 < n < 2303.9$ rpm.

(b) $P_L = 15360W$, $P_{rot} = 300W$

$R_{fc} = 0 \rightarrow I_t = I_a + I_f = 80 + \frac{200}{150} = 81.3333A$

$P_{in} = 200 \times 81.3333 = 16266.6W.$

$\text{Eff.} = \frac{15360}{16266.6 + 300} \times 100\% = 92.72\%.$

$R_{fc} = 200\Omega \rightarrow I_t = 80 + \frac{200}{350} = 80.5714A$

$P_{in} = 200 \times 80.5714 = 16114.28W.$

$\text{Eff.} = \frac{15360}{16114.28 + 300} \times 100\% = 93.58\%.$

$\boxed{4.25}$ $10 = K_a\Phi\, 10 \rightarrow K_a\Phi = 1.0 \rightarrow I_a = \frac{T}{K_a\Phi} = \frac{25}{1} = 25A$

$200 = K_a\Phi\,\omega_m + I_a R_a.$

$\omega_m = \frac{200 - 25 \times 0.2}{1.0} = 195$ rad/sec $= 1862.1$ rpm.

$\boxed{4.26}$ (a) $T = 300 = K_a\Phi I_a$ --(1) and $E_a = 600 - 0.5I_a = K_a\Phi\,\omega_m$ --(2)

$\omega_m = \frac{1500}{60} \times 2\pi = 157.08$ rad/sec --(3).

From (1), (2) & (3) $\rightarrow \frac{300}{600 - 0.5I_a} = \frac{I_a}{157.08} \rightarrow I_a = 84.49A$ or $1115.51A$

$I_a = 84.49A \rightarrow K_a\Phi = \frac{300}{84.49} = 3.5507.$

(b) $K_a\Phi = 0.9 \times 3.5507 = 3.1956.$

$I_a = \frac{300}{3.1956} = 93.8791A \rightarrow E_a = 600 - 0.5 \times 93.8791 = 553.06V.$

$n = \frac{553.06}{3.1956 \times 2\pi} \times 60$ rpm $= 1652.7$ rpm.

$\boxed{4.27}$ (a) $E_a = 230 - (0.2)(200) = 190\ V$

(b) $E_a I_a = 190 \times 200 = 38,000\ W$

$P_{load} = 38,000 - 500 = 37,500$

$$T_L = \frac{37500}{2\pi\ 1200/60} = 298.4\ Nm$$

(c) $I_f = \frac{230}{115} = 2\ A$

$I_t = I_a + I_f = 200 + 2 = 202 A$

$P_{in} = V_t I_t = 230 \times 202 = 46,460\ W$

$$\eta = \frac{37500}{46460} \times 100\% = 80.71\%$$

$\boxed{4.28}$ (1a) $I_a|_{FL} = \frac{23000}{230} = 100 A$

$E_a = 230 - 100 \times 0.1 = 220 V$

(1b) $E_{a(NL)} = V_t = 230 = K\phi_{NL}\, 1500$

$E_{a\,(FL)} = 220 = K\phi_{FL}\, 1480$

$$\phi_{FL} = \phi_{NL} \frac{1500}{1480} \times \frac{220}{230} = 0.97\ \phi_{NL}$$

% reduction = 3%

(2a) $E_a = 230 + 100 \times 0.1 = 240\ V$

(b) Since I_f and I_a are same for both motoring and generating conditions, machine flux remains same.

$$\frac{E_a(\text{gen})|_{FL}}{E_a(\text{motor})|_{FL}} = \frac{240}{220} = \frac{K\phi_{FL}\, n}{K\phi_{FL}\, 1480}$$

$$n = \frac{240}{220} \times 1480 = 1614.6\ \text{rpm}$$

(c) $E_a(\text{gen})|_{NL} = K\phi_{NL}\, 1614.6 = V_t$

$E_a(\text{motor})|_{NL} = K\phi_{NL}\, 1500 = 230$

$$V_t = \frac{1614.6}{1500} \times 230 = 247.6\ V$$

4.29 (a) $I_a = \dfrac{10\,000}{250} = 40\,A \rightarrow E_a = 250 - 0.25 \times 40 = 240\,V$

$\omega_m = \dfrac{1200}{60} \times 2\pi = 125.7$ rad/sec.

$P_m = E_a I_a = 240 \times 40 = 9600\,W.$

$T = \dfrac{9600}{125.7} = 76.37$ N.m.

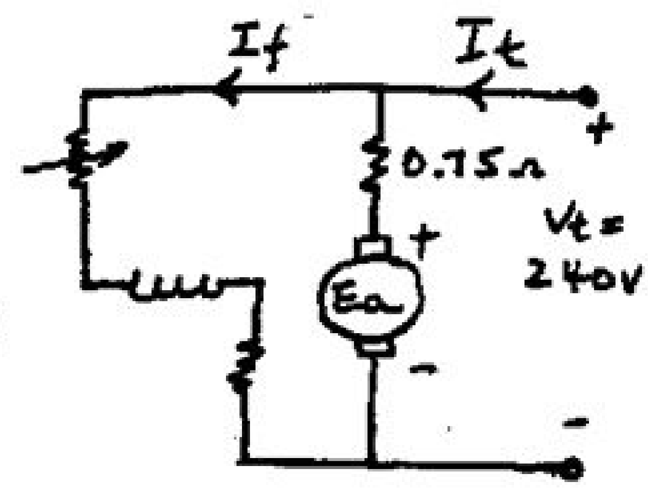
(b) $E_a = 250 - 0.25 \times 4 = 249\,V.$

(i) $P_{rot} = E_a I_a = 249 \times 4 = 996\,W.$

(ii) $\dfrac{E_2}{E_1} = \dfrac{\Phi_2 n_2}{\Phi_1 n_1} \rightarrow \dfrac{249}{240} = \dfrac{\Phi_1 n_2}{\Phi_1 1200} \rightarrow n_2 = 1245$ rpm

(iii) $\dfrac{249}{240} = \dfrac{1.1 \Phi_1 n_2}{\Phi_1 1200} \rightarrow n_2 = 1131.82$ rpm.

4.30 (a) Use subscript 1 for $I_f = 1A$
and subscript 2 for $I_f = 0.7A$



$I_{t_1} = 7A$, $I_{f_1} = 1A$, $I_{a_1} = 7 - 1 = 6A$

& $n = 1200$ rpm

$E_{a_1} = 240 - 6 \times 0.75 = 235.5\,V$

Now $E_a = K_a \phi \omega_m = K_f I_f \omega_m$ for magnetic linearity

$= k I_f n$ where n is speed in rpm

$E_{a_1} = 235.5 = k\,1200$

$k = \dfrac{235.5}{1200}$

With $I_{f_2} = 0.7A$

$E_{a_2} = k I_{f_2} n_2 = \dfrac{235.5}{1200} \times 0.7\, n_2$

Also $E_{a_2} = 240 - I_{a_2} \times 0.75$

So $240 - I_{a_2} \times 0.75 = \dfrac{235.5}{1200} \times 0.7\, n_2$ —— (1)

4.30 continued:

Now $T = K_a \phi I_a = K_f I_f I_a \rightarrow$ magnetic linearity

$$T_1 = K_f \, 1 \cdot 6$$

$$T_2 = K_f \, 0.7 \, I_{a_2}$$

$$\frac{T_1}{T_2} = \frac{6}{0.7 I_{a_2}} = \frac{n_1}{n_2} \rightarrow \text{from load characteristic } (T \propto n)$$

$$\frac{6}{0.7 I_{a_2}} = \frac{1200}{n_2} \quad --- \; 2$$

From 1 + 2, $n_2 = 1681.4$ rpm

$I_{a_2} = 12.01$ A

(b) $I_{a_2} = 12.01$ A

$I_t = 12.01 + 0.7 = 12.71$ A

$E_{a2} = 240 - 0.75 \times 12.01 = 230.99$ V

$P_{dev} = E_{a2} I_{a2} = 230.99 \times 12.01 = 2774.22 \text{ W} = P_{out}$.

$P_{input} = V_t I_t = 240 \times 12.71 = 3050.4$ W

$Eff = \frac{2774.22}{3050.4} \times 100\% = 91 \%$

$\boxed{4.31}$ $T = \frac{E_{a1} \times I_{a1}}{\omega_{m1}} = \frac{235.5 \times 6}{1200 \times 2\pi / 60} = 11.244$ N.m.

(a) $E_{a2} = 240 - I_{a2} \times 0.75 = \frac{235.5}{1200} \times 0.7 \, n_2 \quad --- \; 1$

$T_1 = K_f \times 1 \times 6 = T_2 = K_f \times 0.7 \times I_{a2} = 11.244 \quad --- \; 2$

From 2 $\rightarrow$ $K_f = \frac{11.244}{6} = 1.874$

$I_{a2} = \frac{11.244}{1.874 \times 0.7} = 8.5714$

From 1 $\rightarrow$ $240 - 8.5714 \times 0.75 = \frac{235.5}{1200} \times 0.7 \, n_2$

$n_2 = 1700.25$ rpm.

4.31 Continued:

(b) $I_{a2} = 8.5714A$, $I_t = 8.5714 + 0.7 = 9.27A$

$E_{a2} = 240 - 8.5714 \times 0.75 = 235.57V$

$P_{dev} = 233.57 \times 8.5714 = 2002W = P_{out}$.

$P_{in} = 240 \times 9.27 = 2224.8W$

$Eff. = \frac{2002}{2224.8} \times 100\% = 90\%$

$\boxed{4.32}$ (a) $R_a = 5/40 = 0.125\Omega$, $I_a = \frac{125}{0.125} = 1000A$.

$I_{a(rated)} = 5000/125 = 40A$.

(b) $I_a = 40 \times 1.5 = \frac{125}{0.125 + R_{ex}} \rightarrow R_{ex} = 1.9583\Omega$.

(c) $E_a = 125 - 40 \times 0.125 = 120V$.

$125 = K_a \Phi n$.

$120 = K_a (0.9\Phi) 1800$

$n = \frac{125}{120} \times 0.9 \times 1800 = 1687.5$ rpm.

$\boxed{4.33}$ (a) Before the field circuit broke

$E_{a1} = V_t - I_a R_a = 120 - 10 \times 0.1 = 119 = K\Phi N$

Immediately after the field circuit broke

$E_{a2} = K(0.05\Phi)N = 0.05 \times 119 = 5.95V$.

And $E_{a2} = 120 - I_{a2} R_2 \rightarrow I_{a2} = \frac{120 - 5.95}{0.1} = 1140.5A$.

(b) In the steady state after the field circuit broke, the armature current will be such that the developed torque is equal to load torque.

Before the field circuit broke, $T_1 = K\Phi I_{a1} = K\Phi 10 = T_{Load}$.

After the break (steady state) $\rightarrow K\Phi 10 = K(0.05\Phi) I_a$

$\rightarrow I_a = \frac{10}{0.05} = 200A \rightarrow$ steady state current.

$E_a = 120 - 200 \times 0.1 = 100V$.

Hence, $\frac{100}{119} = \frac{K(0.05\Phi)N}{K\Phi 1800} \longrightarrow N = 30252.1$ rpm.

4.34 (a) $R_a + R_{sr} = 5/5 = 1\Omega$

$5 = k_{sr} 5^2 \rightarrow k_{sr} = 5/25 = 0.2$

$T = 0.2 \times 10^2 = 50\ N{\cdot}m$

(b) $E_a = k_{sr} I_a \omega_m$

$= 0.2 \times 10 \times 300/60 \times 2\pi = 62.83\,V$

$120 = 62.83 + 10(1 + R_{ext})$

$$R_{ext} = \frac{120 - 62.83}{10} - 1 = 4.72\,\Omega$$

4.35 (a) $E_a = 230 - 40(0.25 + 0.1) = 216\ V$

$P = E_a I_a = 216 \times 40 = 8640\ W$

$$\omega_m = \frac{1200}{60} \times 2\pi = 125.66\ \text{rad./sec.}$$

$$T = \frac{8640}{125.66} = 68.76\ N{\cdot}m$$

(b) $E_a = k_{sr} I_a \omega_m = K I_a n$

$E_a = 230 - 20(0.25 + 0.1) = 223\,V$

$$\frac{216}{223} = \frac{40 \times 1200}{20 \times n}$$

$n = 2478$ rpm $\quad \omega_m = 259.5$ rad./sec.

$P = 223 \times 20 = 4460\ W$

$T = 4460/259.5 = 17.19\ N{\cdot}m$

Note: (a) 1200 rpm, 68.76 N.m → low speed high torque

(b) 2324.66 rpm, 17.19 N.m → high speed low torque

4.36 Total resistance required at start

$$R_{T1} = R_a + R_{ae1} = \frac{200}{400} = 0.5\,\Omega.$$

$R_{ae1} = 0.5 - 0.15 = 0.35\,\Omega.$

With I_a reduced to 200A,

$E_a = 200 - 0.5 \times 200 = 100\,V$

4.36 continued :

Total resistance required after switching out the first resistor R_1,

$$R_{T2} = \frac{200-100}{400} = 0.25\,\Omega$$

$$R_{ae2} = 0.25 - 0.15 = 0.1\,\Omega.$$

With I_a reduced to 200A,

$$E_a = 200 - 0.25 \times 200 = 150\,V.$$

Total resistance required after switching out the second resistance R_2.

$$R_{T3} = \frac{200-150}{400} = 0.125\,\Omega$$

This is less than R_a and therefore I_a will not increase to 400 A. Thus two resistances are required in the starter box.

$$R_1 = 0.35 - 0.1 = 0.25\,\Omega$$

$$R_2 = R_{e2} = 0.1\,\Omega.$$

$\boxed{4.37}$ (a) Full load $I_a = 5000/250 = 20\,A$

Lowest speed 200 rpm, $I_{fm} = 0.8\,A$, $I_a = 20\,A$

From curve at 1200 rpm

$I_{fm} = 0.8$, $E_{am} = 250\,V$

$$E_{am}\Big|_{300\,rpm} = 250 \times \frac{200}{1200} = 41.67\,V$$

$$E_{ag} = 41.67 + 20\,(2 \times 0.5) = 61.67\,V$$

From curve $\longrightarrow$ $I_{fg} = 0.103\,A$

Highest speed : 1200 rpm, $I_{fm} = 0.8\,A$, $I_a = 20\,A$

$$E_{am}\Big|_{1200} = 250\,V$$

$$E_{ag} = 250 + 20\,(2 \times 0.5) = 270\,V$$

From curve $\longrightarrow$ $I_{fg} = 1.2\,A$

Hence required range is $0.103\,A < I_{fg} < 1.2\,A$

4.37 continued:

(b) $I_{fg} = 1.0\,A$, $I_{fm} = 0.2\,A$

From curve, $E_{ag} = 262\,V$ @ $I_{fg} = 1.0\,A$

$E_{am} = 262 - 20\,(2 \times 0.5) = 242\,V$

From curve, E_{am} at 1200 rpm and $I_{fm} = 0.2\,A$ is 120 V

Hence $n = 1200 \times \frac{242}{120} = 2420$ rpm

$\boxed{4.38}$ (a) $V_t = \frac{\sqrt{2} \times 265}{\pi}(1 + \cos 30°) = 222.57\,V$

$E_a = 222.57 - 40 \times 0.25 = 212.57\,V$

$N = 212.57/0.18 = 1180.93$ rpm

(b) $T = 68.75$ N·m

(c) $P = 222.57 \times 40 = 8902.8\,W$

$\boxed{4.39}$ (a) $V_t = \frac{3\sqrt{6} \times 277}{2\pi}(1 + \cos\alpha)$

$= 324\,(1 + \cos\alpha)$

$\underline{\alpha = 0°}$

$V_t = 324\,(1 + \cos 0) = 648\,V$

$E_a = 648 - 16.5 \times 0.0874 = 646.6\,V$

$N_o = \frac{646.6}{0.33} = 1959$ rpm

$\underline{\alpha = 30°}$

$V_t = 324\,(1 + \cos 30) = 604.6\,V$

$E_a = 604.6 - 16.5 \times 0.0874 = 603.16\,V$

$N_o = \frac{603.16}{0.33} = 1827.8$ rpm

(b) $E_a = 0.33 \times 1800 = 594\,V$

$V_t = 608.4\,V$

$608 = 324\,(1 + \cos\alpha)$

$\rightarrow \alpha = 28.8°$

(c) 2.18 %

$\boxed{4.40}$ (a) $T_{LOAD} = T_{motor} = K I_a^2 = 200$ N.m

If torque is constant, I_a and flux remain const.

$V_t = \alpha V = 0.5 \times 400 = 200V$

$E_a = 200 - 40 \times 0.75 = 170V$

$E_a = K_a \phi \omega_m$, $T = K_a \phi I_a$

$K_a \phi = T/I_a = 200/4 = 5$

$\omega_m = 170/5 = 34.0$ rad./sec. $\rightarrow$ 324.8 rpm

$P_{out} = HP = \frac{E_a I_a}{746} = \frac{170 \times 40}{746} = 9.12$

$P_{in} = 170 \times 40 + 40^2 \times 0.75 = 6800 + 1200 = 8000W$

$Eff = \frac{6800}{8000} \times 100\% = 85\%$

(b) $\omega_m = 34$ rad./sec.

$E_a = 170V$

$I_a(R_{ext} + R_a + R_{sr}) = V_t - E_a$

$R_{ext} + 0.75 = \frac{400 - 170}{40} = 5.75$

$R_{ext} = 5\Omega$

$HP = 9.12$

$Eff = \frac{6800}{6800 + 40^2 \times 5.75} = \frac{6800}{6800 + 9200} = 42.5\%$

CHAPTER 5

$\boxed{5.1}$ (a) $\frac{120f}{P} \approx 1746$

$$P = \frac{120 \times 60}{1746} \rightarrow 4$$

(b) $$n_s = \frac{120 \times 60}{4} = 1800 \text{ rpm}$$

$$s = \frac{1800 - 1746}{1800} = 0.03$$

(c) $$f_2 = 0.03 \times 60 = 1.8 \text{ Hz}$$

(d) (i) 1800 rpm
(ii) 0 rpm

$\boxed{5.2}$ (a) $0.03 = \frac{1200 - n}{1200}$, $n_s = \frac{120 \times 60}{6} = 1200$ rpm.

$n = 1164$ rpm $\rightarrow$ direction same as the rotating field.

(b) $f_2 = 0.03 \times 60 = 1.8$ Hz

(c) 1200 rpm

(d) 1200 rpm

(e) (i) $n_{sl} = 0.03 \times 1200 = 36$ rpm $\rightarrow$ direction same as the rotor motion.

(ii) 1200 rpm

(iii) 0 rpm

$\boxed{5.3}$ (a) $-0.03 = \frac{1200 - n}{1200}$

$n = 1236$ rpm $\rightarrow$ direction same as the rotating field.

(b) $f_2 = 0.03 \times 60 = 1.8$ Hz.

(c) 1200 rpm.

(d) 1200 rpm.

(e) (i) $n_{sl} = -0.03 \times 1200 = -36$ rpm $\rightarrow$ direction opposite to rotor motion.

(ii) 1200 rpm.

(iii) 0 rpm

$\boxed{5.4}$ (a) $n_s = \frac{120 \times 60}{6} = 1200$ rpm.

$s = 1.5 = \frac{1200 - n}{1200}$

$n = -600$ rpm $\rightarrow$ direction opposite to the rotating field.

(b) $f_2 = 1.5 \times 60 = 90$ Hz.

(c) 1200 rpm

(d) 1200 rpm.

(e) (i) $n_{sl} = 1.5 \times 1200 = 1800$ rpm

$\rightarrow$ direction opposite to rotor motion (or same as the rotating field).

(ii) 1200 rpm

(iii) 0 rpm.

$\boxed{5.5}$ (a)(i) $n_s = \frac{120 \times 60}{6} = 1200$ rpm

$$s = \frac{1200 - 1140}{1200} = 0.05$$

(ii) $E_{2s} = s E_2 = s \frac{E_1}{a}$

$$a = \frac{1}{0.5} = 2.0$$

$$E_{2s} = 0.05 \times \frac{1}{2.0} \times \frac{208}{\sqrt{3}} = 3 \text{ V}$$

$$f_2 = 0.05 \times 60 = 3 \text{ Hz}$$

(iii) - slip rpm $n_2 = s n_s = 0.05 \times 1200 = 60$ rpm

- with respect to stator $\rightarrow$ 1200 rpm.

(b) Inverted induction motor

(i) opposite

(ii) $E_{1s} = s E_1 = s\, a E_2 = 0$

$$s = \frac{1200 - 1164}{1200} = 0.03$$

$$E_{1s} = 0.03 \times 2 \times \frac{208}{\sqrt{3}} = 7.2 \text{ V}$$

$$f_1 = 0.03 \times 60 = 1.8 \text{ Hz}$$

5.6 (a) <u>No load test</u>

$$Z_{NL} = \frac{460/\sqrt{3}}{40} = 6.64\ \Omega$$

$$R_{NL} = \frac{P_{NL}}{3I_1^2} = \frac{4200}{3\times 40^2} = 0.875\ \Omega$$

$$X_{NL} = (6.64^2 - 0.875^2)^{1/2} = 6.58\ \Omega$$

$$x_1 + x_m = 6.58\ \Omega$$

<u>Blocked-rotor test</u>

$$R_{BL} = \frac{8000}{3\times 140^2} = 0.136\ \Omega$$

$$R_1 = \frac{0.152}{2} \quad \text{(from resistance between two stator terminals)}$$

$$= 0.076\ \Omega$$

$$R_2' = 0.136 - 0.076 = 0.06\ \Omega$$

$$Z_{BL} = \frac{100/\sqrt{3}}{140} = 0.412\ \Omega$$

$$X_{BL} = (0.412^2 - 0.136^2)^{1/2} = 0.389\ \Omega$$

$$x_1 + x_2' = 0.389\ \Omega$$

$$x_1 = x_2' = 0.1945\ \Omega$$

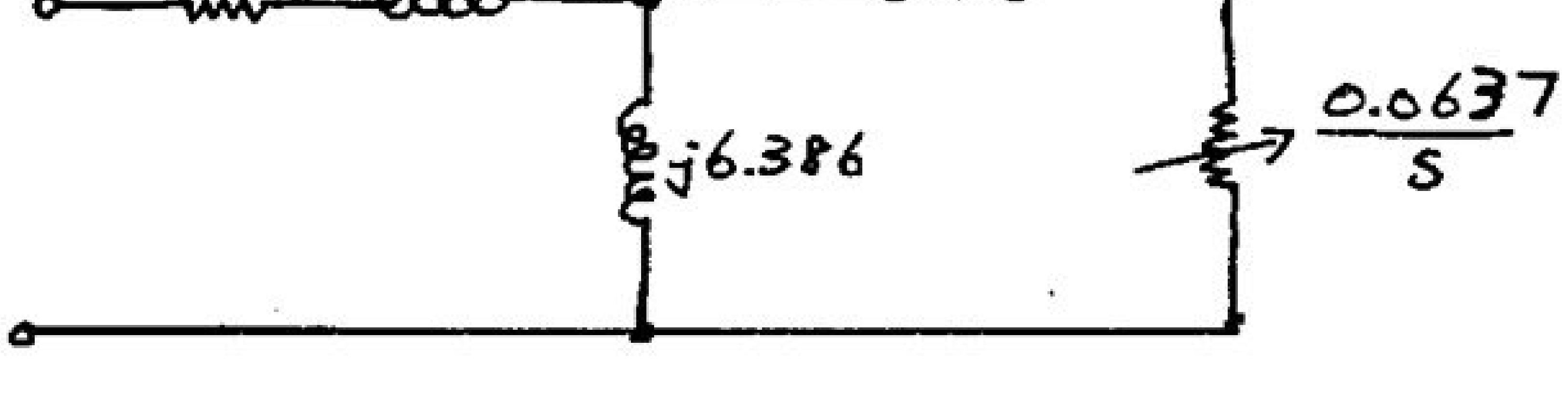
$$X_m = 6.58 - 0.1945 = 6.3855\ \Omega$$

Precise value for R_2'

$$R = R_{BL} - R_1 = 0.136 - 0.076 = 0.06\ \Omega$$

$$R_2' = \left(\frac{0.1945 + 6.3855}{6.3855}\right)^2 \times 0.06 = 0.0637\ \Omega$$

Equivalent circuit is



$\boxed{5.6}$ (b) $n_s = \frac{120f}{P} = \frac{120\times 60}{8} = 900\,rpm$

$S = \frac{900-873}{900} = 0.03$

$\frac{R_2'}{S} = \frac{0.0637}{0.03} = 2.123\Omega$

Input Impedance Z_1

$Z_1 = 0.076 + j0.195 + \frac{(j6.386)(2.123+j0.195)}{2.123+j(6.386+0.195)} = 2.121\angle 27.16^\circ$

Input Current $I_1 = \frac{460/\sqrt{3}}{2.121\angle 27.16^\circ} = 125.22\angle -27.16^\circ$

Input Power $P_{in} = 3\times\frac{460}{\sqrt{3}}\times 125.22\cos 27.16^\circ = 88.767\,KW$

Stator cu-loss, $P_{st} = 3\times 125.22^2\times 0.076 = 3.575\,KW$

Airgap Power $P_{ag} = 88.767 - 3.575 = 85.192\,KW$

Rotor-cu loss $= sP_{ag} = 0.03\times 85.192 = 2.556\,KW$

Mechanical Power developed

$P_{mech} = (1-S)P_{ag} = 0.97\times 85.192 = 82.636\,KW$

$P_{out} = P_{mech} - P_{rotational\ loss}$

From no-load test

$P_{rot} = 4200 - 3\times 0.076\times 40^2 = 3835.2\,W$

$P_{out} = 82.636\times 10^3 - 3835.2 = 78.80\,KW$

$Eff = \frac{P_{out}}{P_{in}}\times 100\% = \frac{78.80}{88.767}\times 100 = 88.77\%$

$\boxed{5.7}$ (a) $P_{rot} = 500 - 3\times 6.5^2 \times \frac{0.54}{2} = 465.78\ W$

(b) From no-load test $R_1 = \frac{0.54}{2} = 0.27\Omega$

$V_1 = \frac{208}{\sqrt{3}} = 120.1\ V$

$Z_{NL} = \frac{120.1}{6.5} = 18.48\ \Omega$

$R_{NL} = \frac{500}{3\times 6.5^2} = 3.94\ \Omega$

$X_{NL} = \sqrt{18.48^2 - 3.94^2} = 18.05\ \Omega$

$X_1 + X_m = X_{NL} = 18.05\ \Omega$

From blocked - rotor test

$R_{BL} = \frac{1250}{3\times 25^2} = 0.6667\ \Omega$

$R_2' = 0.6667 - 0.27 = 0.3967\ \Omega$

$Z_{BL} = \frac{44/\sqrt{3}}{25} = 1.0162\ \Omega$

$X_{BL} = \sqrt{1.0162^2 - 0.6667^2} = 0.7669\ \Omega$

$X_1 = X_2' = 0.3834\ \Omega$

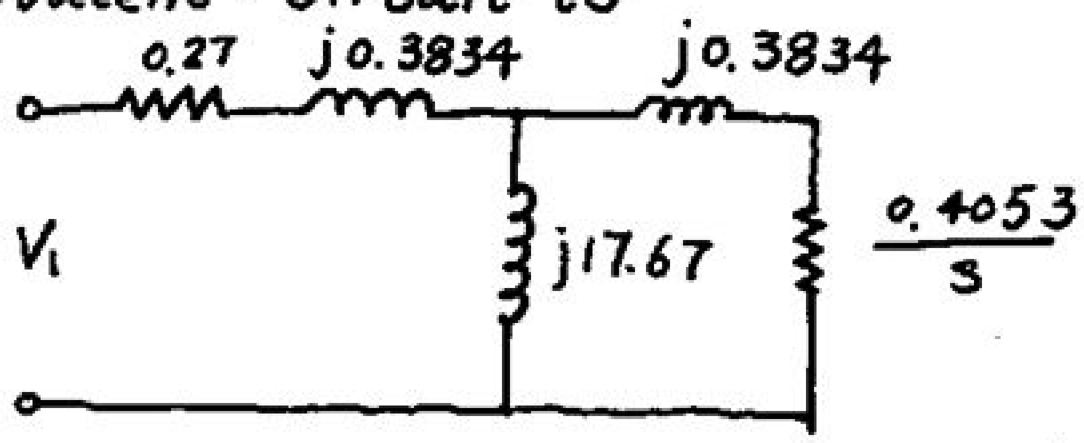
$X_m = 18.05 - 0.3834 = 17.6666\ \Omega$

Precise value for R_2'

$R = R_{BL} - R_1 = 0.6667 - 0.27 = 0.3967\ \Omega$

$R_2' = \frac{0.3834 + 17.6667}{17.6667} \times 0.3967 = 0.4053\ \Omega$

Equivalent Circuit is



(c) Neglecting X_m, $s_b \approx \frac{R_2'}{\sqrt{R_1^2 + (X_1 + X_2')^2}} = \frac{0.4053}{\sqrt{0.27^2 + 0.7668^2}}$

$= 0.4985$

Maximum torque occurs at 49.85% of synchrouns speed.

It is class-D type motor

(d) $\frac{R_2'}{s} = \frac{0.4053}{0.1} = 4.053\ \Omega$

$Z_1 = 0.27 + j0.3834 + \frac{j17.67(4.053 + j0.3834)}{4.053 + j18.05} = 4.273\angle 21.83^\circ$

5.7 Continued:

$$I_1 = \frac{208/\sqrt{3}}{4.273\angle 21.83^\circ} = 28.104\angle -21.83^\circ$$

$$P_s = 3\times 120.1\times 28.104\times\cos(-21.83^\circ) = 9396.809\,W$$

$$P_{ag} = 9396.809 - 3\times 28.104^2\times 0.27 = 8757.045\,W$$

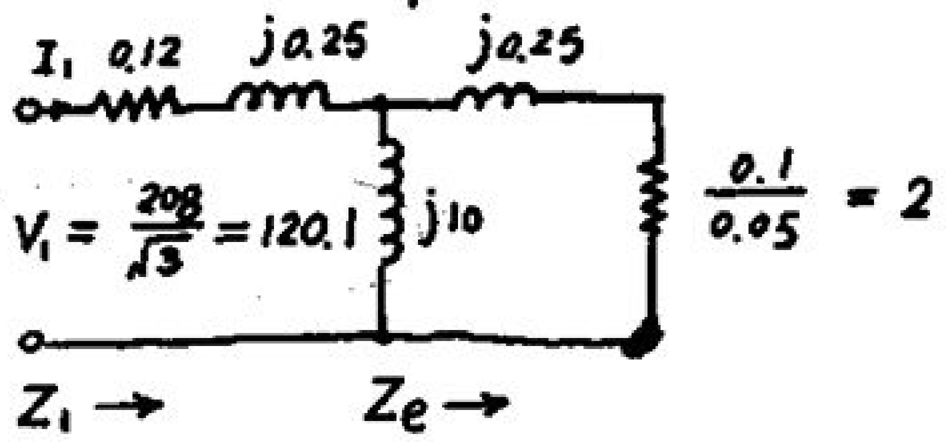
$$P_{mech} = (1-0.1)\times 8757.045 = 7881.341\,W$$

$$P_{out} = P_{shaft} = P_{mech} - P_{rot} = 7881.341 - 465.78 = 7415.561\,W$$

$$HP = \frac{7415.561}{746} = 9.94\ (Hp)$$

$\boxed{5.8}$ (a) $n_s = \frac{120\times 60}{4} = 1800$ rpm $\quad \omega_s = \frac{1800}{60}\times 2\pi = 188.5$ rad/sec

(b)



$$Z_1 = 0.12 + j0.25 + R_e + jX_e = 0.12 + j0.25 + \frac{j10\times(2+j0.25)}{2+j10.25}$$

$$= 0.12 + j0.25 + 1.9538 + j0.8517 = 2.1314\angle 23.5533^\circ\,\Omega$$

$$R_e = 1.9538,\ X_e = 0.8517$$

$$I_1 = \frac{120.1}{2.1314\angle 23.5533^\circ} = 56.3479\angle -23.5533^\circ$$

(c) $P_1 = 3\times(56.3479)^2\times 0.12 = 1143.0309\ W$

(d) $P_s = 3\times 120.1\times 56.3479\times\cos(-23.5533) = 18610.9794\,W$

$P_{ag} = P_s - P_1 = 17467.9485\,W$

(e) $P_2 = sP_{ag} = 0.05\times 17467.9485 = 873.3974\,W$

(f) $P_{mech} = (1-s)P_{ag} = 16594.5511\,W$

$P_{shaft} = P_{mech} - P_{rotational} = 16194.5511\,W$

(g) $T = \frac{P_{ag}}{188.5} = \frac{17467.9485}{188.5} = 92.6682\ N.m$

$T_{shaft} = \frac{P_{shaft}}{188.5} = \frac{16194.5511}{188.5} = 85.9127\ N.m$

(h) $Eff = \frac{P_{shaft}}{P_s} = 0.8702\times 100\% = 87.02\%$

$\boxed{5.9}$ (a) To operate the motor at the same flux level, in order to avoid saturation, the supply voltage should be proportional to frequency

$$V = \frac{50}{60}\times 460 = 383.33\,V\ (\text{Line})$$

5.9 Continued

(b) (i) $n_s = \frac{120 \times 50}{4} = 1500 \text{ rpm} \qquad n = 1500(1-0.03) = 1455 \text{ rpm}$

$f_2 = sf_1 = 0.03 \times 50 = 1.5 \text{ Hz}$

(ii) In the equivalent circuit, resistances will remain same but the reactance will be proportional to frequency

$R_1 = 0.25\,\Omega$, $R_2' = 0.2\,\Omega$, $X_1 = X_2' = 0.5 \times \frac{50}{60} = 0.42\,\Omega$

$X_m = 30 \times \frac{50}{60} = 25\,\Omega$

$\frac{R_2'}{s} = \frac{0.2}{0.03} = 6.67\,\Omega$

$Z_1 = 0.2 + j0.42 + \frac{(j25) \times (6.67 + j0.42)}{6.67 + j25.42} = 6.687\angle 21.181°$

$V_1 = \frac{383.33}{\sqrt{3}} = 221.32 \qquad I_1 = \frac{221.32}{6.687\angle 21.181°} = 33.097\angle -21.181°$

$PF = \cos(-21.181°) = 0.9324$

$I_2 = \frac{j25}{6.67 + j25.42} \times 33.097\angle -21.181° = 31.475\angle -6.479°$

$\omega_s = \frac{1500}{60} \times 2\pi = 157.1 \text{ radian/sec.}$

$P_{ag} = 3 \times 31.475^2 \times 6.67 = 19823.427 \text{ W}$

$T = \frac{19823.427}{157.1} = 126.184 \text{ N.m.}$

$P_{mech} = (1 - 0.03) \times 19823.427 = 19228.724 \text{ W}$

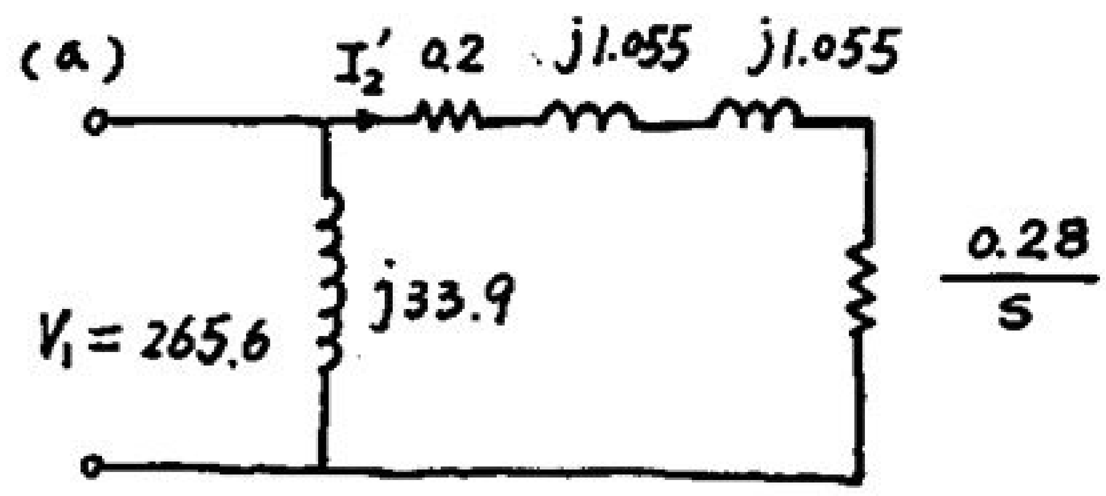
$P_{rot} = 1700 \times \frac{1455}{1740} = 1421.552 \text{ W}$

$P_{out} = P_{mech} - P_{rot} = 17807.172 \text{ W}$

$P_{in} = 3 \times 221.32 \times 33.097 \times 0.9324 = 20489.568 \text{ W}$

$Eff = \frac{17807.172}{20489.568} = 86.9\%$

$\boxed{5.10}$ (a)



$n_s = \frac{120 \times 60}{6} = 1200 \text{ rpm} \qquad \omega_{syn} = \frac{1200}{60} \times 2\pi = 125.66 \text{ rad/sec}$

$T_{st} = \frac{3}{125.66} \times \frac{265.6^2}{(0.48)^2 + (2.11)^2} \times 0.28 = 100.7 \text{ N.m.}$

5.10 continued:

(b) $\frac{0.28}{S_b} = \sqrt{0.2^2 + (2.11)^2} = 2.12$

$S_b = 0.132$

$T_b = \frac{3 \times 265.6^2}{125.66 \times (2.1212^2 + 2.11^2)} \times 2.1212 = 399.1 \text{ N.m.}$

(c) $T = \frac{1}{\omega_{syn}} \times \frac{V_1^2}{(R_1 + \frac{R_2'}{s})^2 + (X_1 + X_2')^2} \times \frac{R_2'}{s}$

Near synchronous speed i.e. s very small,

$R_1 + \frac{R_2'}{s} >> X_1 + X_2'$ and $\frac{R_2'}{s} >> R_1$

Hence $T = \frac{3}{\omega_{syn}} \times \frac{V_1^2}{R_2'} \times S = \frac{3}{125.66} \times \frac{265.6^2}{0.28} \times S = 6014.8S$

$6014.8S = 1.8\,\omega_m = 1.8\,(1-S)\,\omega_{syn} = 1.8 \times (1-S) \times 125.66$

$S = \frac{226.188}{6240.988} = 0.0362 \quad n = (1 - 0.0362) \times 1200 = 1156 \text{ rpm}$

$\boxed{5.11}$ $V_1 = \frac{208}{\sqrt{3}} = 120.1\text{V} \quad X_1 = X_2' = 377 \times 0.25 \times 10^{-3} = 0.09425\,\Omega$

$X_m = 377 \times 15 \times 10^{-3} = 5.655\,\Omega$

$n_s = \frac{120 \times 60}{6} = 1200 \text{ rpm} \quad \omega_s = \frac{1200}{60} \times 2\pi = 125.664 \text{ rad/sec}$

From equation 5.55 with V_{th} replaced by V_1,

$T_m \simeq \frac{3}{125.664} \times \frac{120.1^2}{0.11} \times S = 3130S$

$T_L = 12.7 \times 10^{-3} \times (125.664\,(1-S))^2 = 200.55\,(1-S)^2$

$T_m = T_L$

$3130S = 200.55\,(1-S)^2 \quad S = 17.553,\ 0.05697$

$n = 1200\,(1 - 0.05697) = 1131.6 \text{ rpm} \rightarrow 118.5 \text{ rad/sec}$

$T = 3130 \times 0.05697 = 178.32 \text{ N.m.} \quad P = 178.32 \times \frac{1131.6}{60} \times 2\pi = 21.131 \text{ KW}$

$\boxed{5.14}$ (a) $P = 4$

(b) $I_2' = \frac{265.59}{0.07 + 0.05 + j\,(1.2 + 1.1)} = \frac{265.59}{0.12 + j2.3} = 115.3235\text{A}$

$T_{st} = \frac{3}{188.496} \times (115.3235)^2 \times 0.2 = 42.33 \text{ N.m.}$

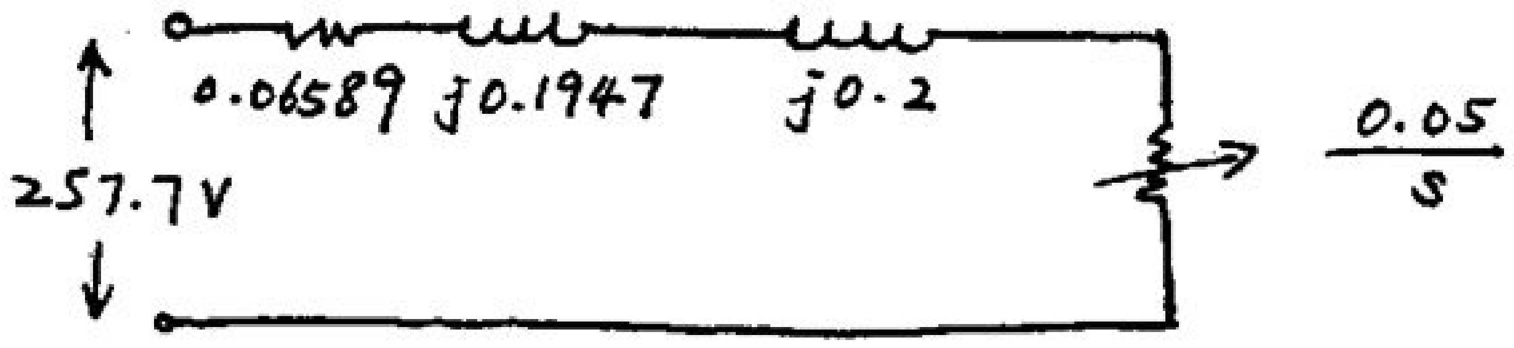
(c) $S_{T_{max}} = 1 = \frac{0.2 + R'_{ext}}{\sqrt{(0.25)^2 + (2.3)^2}} = \frac{0.2 + R'_{ext}}{2.3135}$

$\rightarrow R'_{ext} = 2.1135 = R_{ext}.$

$\boxed{5.12}$ (a) Thevenin Eq. cct.

$$V_{th} = \frac{x_m}{x_1 + x_m} V_1 = \frac{6.5}{0.2 + 6.5} \times 265.6 = 257.7$$

$$R_{th} + jX_{th} = \frac{(j6.5)(j0.2 + 0.07)}{0.07 + j0.2 + j6.5} = 0.06589 + j0.1947$$



(b) $$T_{st} = \frac{3 \times 257.7^2 \times 0.05}{94.25\left[(0.06589 + 0.05)^2 + (0.19467 + 0.2)^2\right]}$$

$$= 624.7 \text{ N.m}$$

$$T_{max} = \frac{3 \times 257.7^2}{2 \times 94.25\left[0.06589 + \sqrt{0.06589^2 + (0.1947 + 0.2)^2}\right]}$$

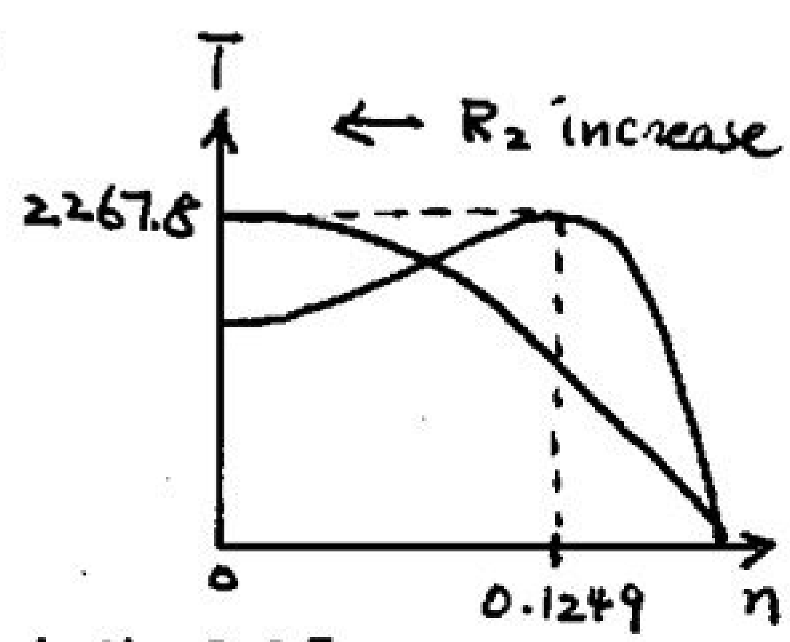
$$= 2267.8 \text{ N.m}$$

$$S_{T\,max} = \frac{0.05}{\sqrt{0.06589^2 + (0.1947 + 0.2)^2}} = 0.1249$$

speed in rpm for which max torque occurs

$= (1 - S_{T\,max})\, n_s = (1 - 0.1249)900 = 787.5$ rpm

(c) $$S_{T\,max} = \frac{r_2'}{\sqrt{r_1^2 + (x_1 + x_2)^2}} \propto r_2'$$



$$S_{T\,max} = k\, r_2'$$

$$\frac{S_{T\,max}}{r_2'} = \frac{S|start}{r_2' \text{ at start}}$$

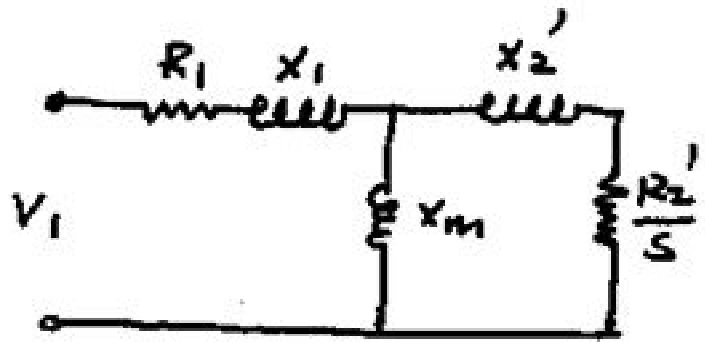
$$\therefore\ r_2'|_{start} = \frac{S|start}{S_{T\,max}} \times r_2' = \frac{1 \times 0.05}{0.1249} = 0.4\,\Omega$$

$$\therefore\ r_2|_{ext} = (0.4 - 0.05)/1.2^2 = 0.243\,\Omega$$

$\boxed{5.13}$ (a) $n = 1760 \simeq \dfrac{120 \times 60}{P}$

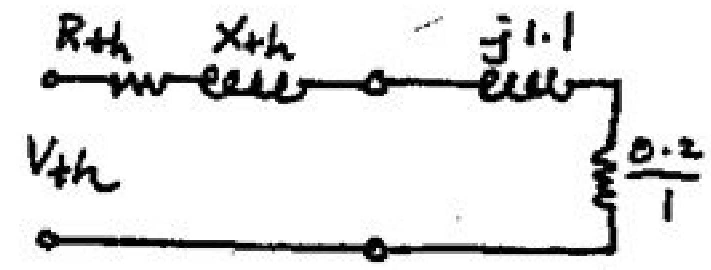
$P = 4.09 \rightarrow 4$

(b) $V_1 = \dfrac{460}{\sqrt{3}} = 265.59$



$V_{th} = \dfrac{35}{1.2 + 35} V_1$

$= 0.967 \times 265.59 = 256.79 \text{V} \Rightarrow$



$R_{th} = 0.967^2 \times 0.25 = 0.234 \, \Omega$

$X_{th} = 1.2 \Omega = X_1$

$\omega_{syn} = \dfrac{1800}{60} \times 2\pi = 188.496$ rad./sec.

$T_{st} = \dfrac{3}{188.496} \quad \dfrac{256.79^2}{(0.234 + 0.2)^2 + (1.2 + 1.1)^2} \times 0.2$

$= 38.31$ N.m

(c) $S_{T_{max}} = 1 = \dfrac{R_2' + R_{ext}'}{\sqrt{0.234^2 + (1.2 + 1.1)^2}} = \dfrac{R_2' + R_{ext}'}{2.31}$

$R_{ext}' = 2.31 - 0.2 = 2.11 \, \Omega = R_{ext}.$

$\boxed{5.14}$ See on page 68.

$\boxed{5.15}$

$S = \dfrac{1800 - 1746}{1800} = 0.03$

$P_{shaft} = 100 \times 746 = 74600 \text{ W}$

$P_{mech} = 74600 + 3500 = 78100 \text{ W}$

$P_{ag} = \dfrac{78100}{1 - 0.03} = 80515.5$

$P_{in} = 80515.5 + 3000 = 83515.5 \text{ W}$

$\eta = \dfrac{74600}{83515.5} \times 100 = 89.3 \%$

$\boxed{5.16}$ (a) $P_{ag} = 40 - 0.5 = 39.5 \text{ kW}$

$P_{cu2} = (0.025)(39.5) = 0.9875 \text{ kW}$

(b) $P_{mech} = (1 - 0.025)(39.5) = 38.513 \text{ kW}$

5.16(b) continued

$$P_{shaft} = 38.513 - 2.5 = 36.01\ kW$$

$$= \frac{36.01}{0.746}\ HP = 48.27\ HP$$

(c) $$\eta = \frac{36.01}{40} \times 100 = 90.03\ \%$$

(d) $$n_s = \frac{(120)(60)}{6} = 1200\ ,\quad n = 1200 \times 0.975 = 1170\ rpm$$

$$\omega = 2\pi\ \frac{1170}{60} = 122.5\ rad./sec.$$

$$T = \frac{36010}{122.5} = 293.96\ N.m$$

$\boxed{5.17}$ (a) $$n_{s\,(SM)} = \frac{120 \times 60}{4} = 1800\ rpm$$

$$n_s\ (IM) = \frac{120 \times 60}{6} = 1200\ rpm$$

Speed $n = 1800\ rpm = n_{s\,(SM)}$

$$S = \frac{1200 - 1800}{1200} = -0.5$$

$$f_2 = 0.5 \times 60 = 30\ Hz$$

$$P_{IM} \equiv P_g = P_2/S = \frac{1}{-0.5} = -2\ p.u.$$

$$P_{SM} = -P_{out} = -P_{mech} = -(1-S)P_g$$

$$= -(1+0.5)(-2) = 3\ p.u.$$

$$\begin{array}{c} \qquad\qquad\qquad\quad \downarrow P_{IM} = -2\,p.u. \\ P_{SM} = 3\,p.u. \longrightarrow \Sigma \longrightarrow P_{load} = P_2 = 1\,p.u. \end{array}$$

(b) $n = 1800 \qquad S = \dfrac{1200-(-1800)}{1200} = 2.5$

$$f_2 = 2.5 \times 60 = 150\ Hz$$

$$P_{IM} = \frac{1}{2.5} = 0.4\ p.u.$$

$$\begin{array}{c} \qquad\qquad\quad \downarrow P_{IM} = 0.4\,pu \\ P_{SM} = 0.6 \longrightarrow \Sigma \longrightarrow P_{load} = 1\,pu \end{array}$$

$$P_{SM} = -(1-2.5)0.4 = 0.6\ p.u.$$

$\boxed{5.18}$ (a) $n_s = \dfrac{120 \times 60}{4} = 1800$ rpm

$s = \dfrac{1800 - 1710}{1800} = 0.05$

Induction machine is motoring

(b) $R_2'/s = 0.35/0.05 = 7\Omega$

$Z_1 = (0.25 + j0.55) + j\,\dfrac{38\,(7 + j1.1)}{7 + j(38 + 1.1)} = 7.207\,\underline{/22.5^\circ}$

$I_1 = \dfrac{208/\sqrt{3}}{7.207\,\underline{/22.5^\circ}} = 16.66\,\underline{/-22.5}$ A

(c) $P_m = \sqrt{3} \times 208 \times 16.66 \cos(22.5) = 5544.36$ W (into machine)

$Q_m = \sqrt{3} \times 208 \times 16.66 \sin(22.5) = 2298.8$ VAR into machine

(d) $P_{ag} = 5544.36 - 3 \times 16.66^2 \times 0.25 = 5336.2$ W

$P_{cu2} = s P_{ag} = 0.05 \times 5336.2 = 266.81$ W

(e) $P_{mech} = (1 - s) P_{ag} = (1 - 0.05) 5336.2 = 5069.39$ W

$P_{shaft} = 5069.39 - 225 = 4844.39$ W

$E_a I_a = P_{shaft} - P_{rot\,dc\,m/c} = 4844.39 - 225$
$= 4619.39$ W

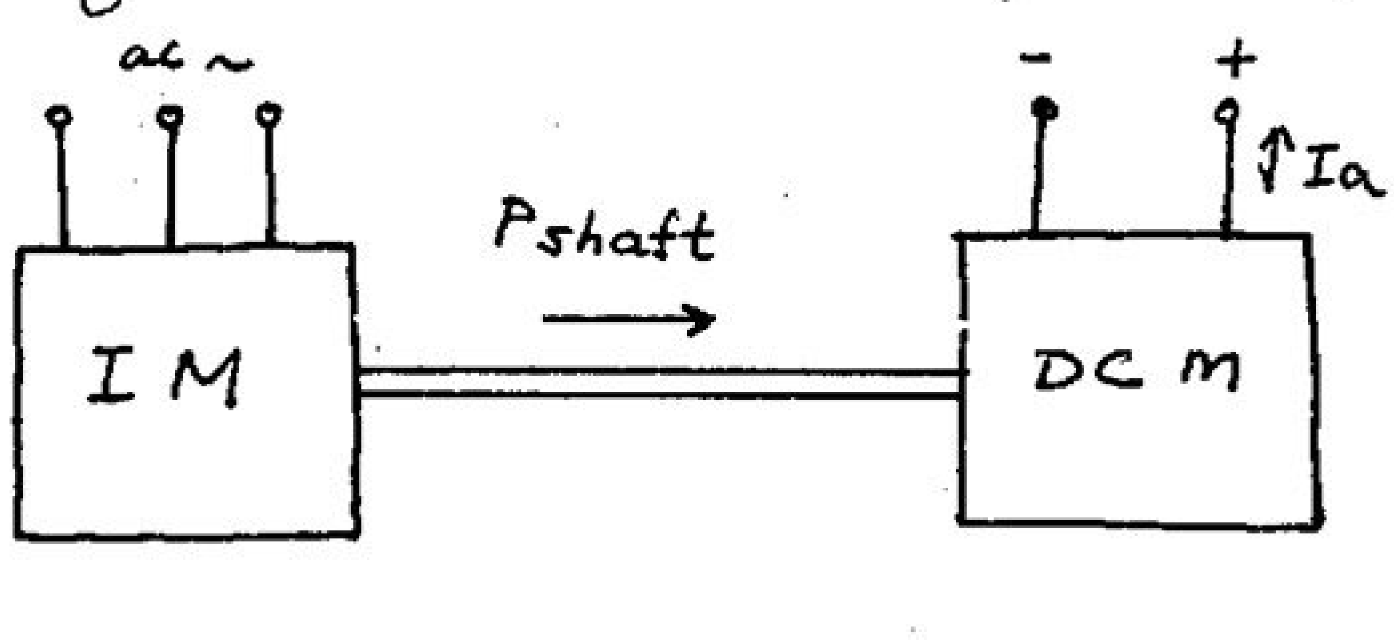
For the dc m/c (generator)

$E_a = V_t + I_a R_a$

$\dfrac{4619.39}{I_a} = 220 + I_a \times 0.4$ (quadratic equation)

$I_a = 20.25$ A or -570.25 A

Realistic value of $I_a = 20.25$A it flows out of the dc m/c (ie generator)



$\boxed{5.19}$ (a) $S = \dfrac{1800 - 1890}{1800} = -0.05$

Induction machine operates in the generating mode

(b) $\dfrac{R_2'}{S} = \dfrac{0.35}{-0.05} = -7.0\,\Omega$

$$Z_1 = (0.25 + j0.55) + \frac{j38\,(-7 + j1.1)}{-7 + j(38 + 1.1)} = 6.75\,\angle 155.8$$

$$I_1 = \frac{208/\sqrt{3}}{6.75\,\angle 155.8} = 17.8\,\angle -155.8$$

(c) $P_m = \sqrt{3} \times 208 \times 17.8 \cos(155.8) = -5849.2\,W$ (delivered to supply)

$Q_m = \sqrt{3} \times 208 \times 17.8 \sin(155.8) = 2628.7\,VAR$ (draws from supply)

(d) $P_{ag} = 5849.2 + 3 \times 17.8^2 \times 0.25 = 6086.83\,W$

$P_{cu2} = sP_{ag} = 0.05 \times 6086.83 = 304.34\,W$

(e) $P_{mech} = (1 - s) P_{ag} = 1.05 \times 6086.83 = 6391.2\,W$

$P_{shaft} = 6391.2 + 225 = 6616.2\,W$

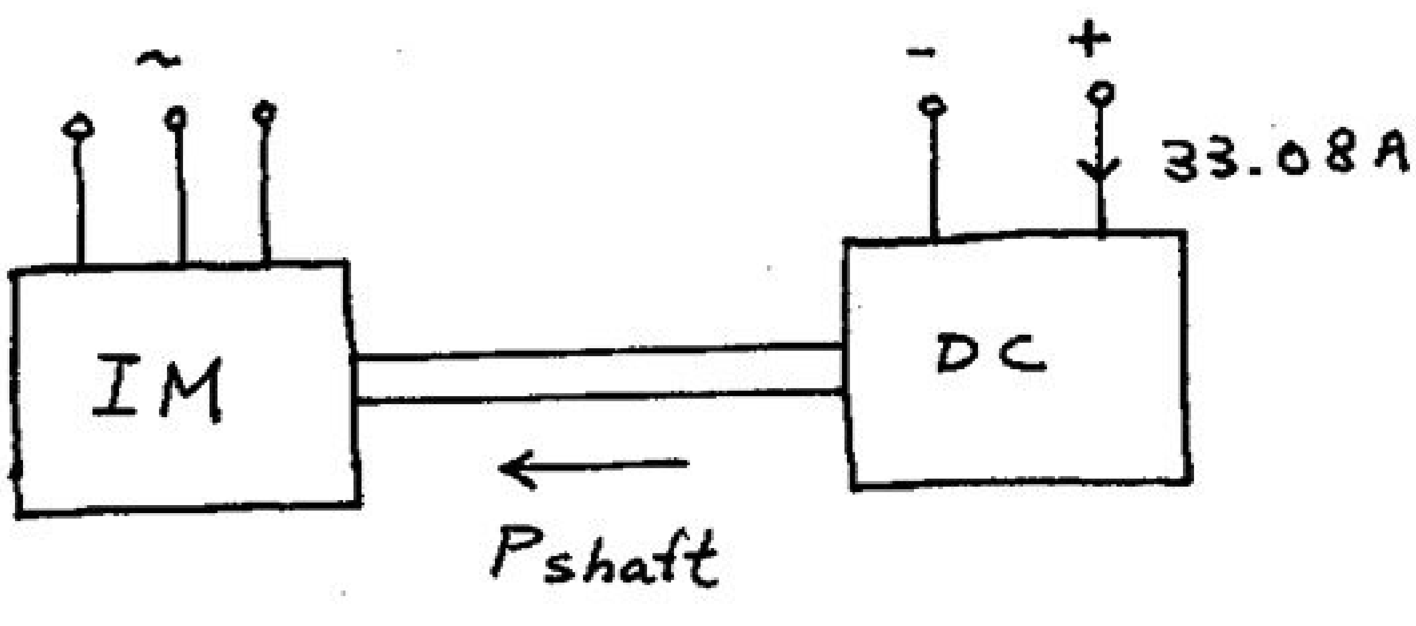
$E_a I_a = 6616.2 + 225 = 6841.2\,W$

$V_t = E_a + I_a R_a$

$$220 = \frac{6841.2}{I_a} + I_a(0.4) \qquad \text{(quadratic equation)}$$

$I_a = 33.08,\ 516.9$

$I_a = 33.08$ flows into the dc machine



5.20 (a) Because of phase sequence reversal, the rotor is now rotating opposite to the rotating field.

Mode of operation is plugging

(b) $$S = \frac{1800 - (-1710)}{1800} = 1.95$$

$$\frac{R_2'}{S} = \frac{0.35}{1.95} = 0.1795\ \Omega$$

$$Z_1 = (0.25 + j\,0.55) + \frac{j\,38\,(0.1795 + j\,1.1)}{0.1795 + j\,(38 + 1.1)}$$

$$= 1.67\ \angle 75.5^\circ\ \Omega$$

$$I_1 = \frac{208/\sqrt{3}}{1.67\ \angle 75.5} = 71.91\ \angle -75.5^\circ\ A$$

(c) $P_m = \sqrt{3} \times 208 \times 71.91 \times \cos 75.5 = 6486.4\,W$ (into machine)

$Q_m = \sqrt{3} \times 208 \times 71.91 \times \sin 75.5 = 25080.8$ VAR (into m/c)

(d) $P_{ag} = 6486.4 - 3 \times 71.91^2 \times 0.25 = 2608.1\ W$

$P_{cu2} = 1.95 \times 2608.1 = 5085.8\ W$

(e) $P_{mech} = (1 - 1.95)\,2608.1 = -2477.7\ W$ (into IM)

$P_{shaft} = 2477.7 + 225 = 2702.7$

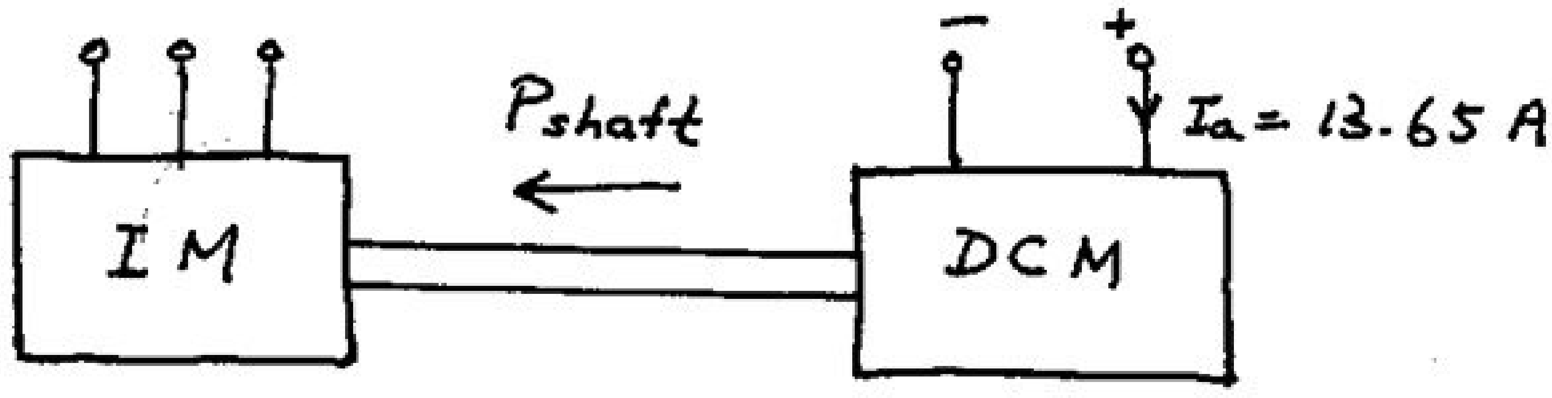
$E_a I_a = 2702.7 + 225 = 2927.7\ W$

$V_t = E_a + I_a R_a$

$$220 = \frac{2927.7}{I_a} + I_a \times 0.4 \rightarrow \text{(quadratic)}$$

$I_a = 13.65\,A, \quad 536.35\,A$

$I_a = 13.65\ A$ into the dc machine

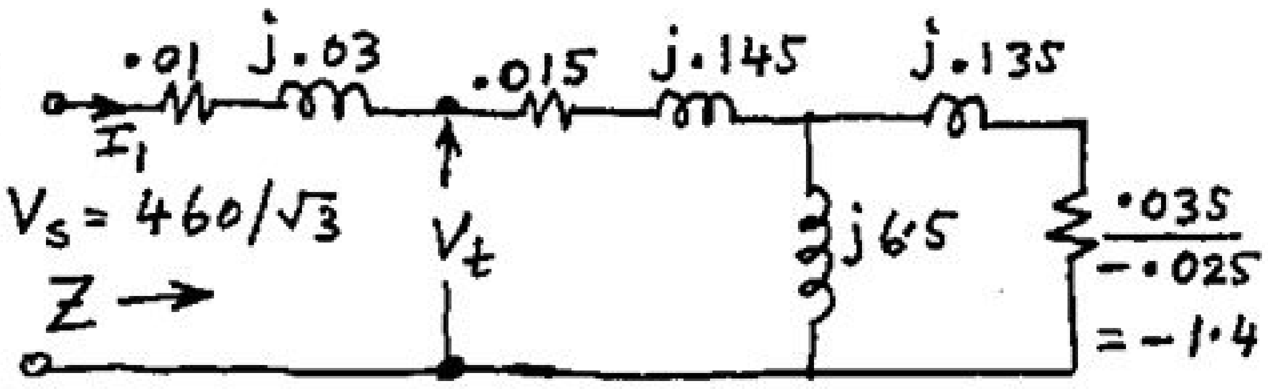


$\boxed{5.21}$

(a) $n_s = \frac{120 \times 60}{8} = 900\,rpm$

$n = (1 + 0.025)\,900 = 922.5\,rpm.$

(b)



$$Z = 0.01 + j0.03 + 0.015 + j0.145 + \frac{(j6.5)(-1.4 + j0.135)}{j6.5 + (-1.4 + j0.135)}$$

$$= 0.025 + j0.175 + (-1.2863 + j0.4037) = -1.2613 + j0.5787 = 1.39\angle 155.35^\circ$$

If we take V_s as a reference then $V_s = \frac{460}{\sqrt{3}} = 265.5811\angle 0^\circ\,V$

$$I_1 = \frac{V_s}{Z} = \frac{265.5811\angle 0}{1.39\angle 155.35^\circ} = 191.066\angle -155.35^\circ$$

$$V_t = V_s - I_1 Z_f = 265.5811 - (191.066\angle -155.35) \times (0.01 + j0.03)$$

$$= 264.931 + j6.007 = 264.999\angle 1.299^\circ$$

(c) Power from Source

$$P_s = 3 \times V_s \times I_1 \cos\theta_1 = 3 \times 265.5811 \times 191.066 \times (-0.9089) = -138.5913\,KW$$

The negative sign means the power is delivered to the source.

$$PF = \cos\theta_1 = \cos(-155.35^\circ) = -0.9089$$

(d) From Fig 5.23 (b)

$$P_{ag} = P_s + 3 I_1^2 (0.01 + 0.015) = 138.591 \times 10^3 + 3 \times (191.066)^2 \times 0.025$$

$$= 141.3383\,KW$$

$$P_{mech} = (1 - S) P_{ag} = (1 - (-0.025)) \times 141.3383 \times 10^3 = 144.8718\,KW$$

$$P_{rot} = 3\,KW$$

$$P_{shaft} = P_{mech} + P_{rot} = 144.8718 + 3 = 147.8718\,KW$$

$$P_{in} = P_{shaft} = 147.8718\,KW$$

$$Eff = \frac{P_s}{P_{in}} = \frac{138.5913}{147.8718} = 0.9372$$

$\boxed{5.22}$ (a) $n_s = -1800\,rpm \quad n = 1740\,rpm \quad S = \frac{-1800 - 1740}{-1800} = 1.9667$

(b) $f_2 = 1.9667 \times 60 = 118\,Hz$

(c) $\frac{R_2'}{S} = \frac{0.2}{1.9667} = 0.1017\,\Omega$

$$T = \frac{3}{188.5} \times \frac{(261.3)^2}{(0.24 + 0.1017)^2 + (0.49 + 0.5)^2} \times 0.1017 = 100.75\,N.m$$

Opposite to rotor motion

5.23

$$n_s = \frac{120 \times 60}{8} = 900 \text{ rpm}$$

Full-load speed, $n_{FL} = 900(1-0.03) = 873$ rpm

$$T_{600rpm} = \left(\frac{600}{873}\right)^2 T_{FL} = T'$$

$$S_{600rpm} = \frac{900-600}{900} = \frac{1}{3}$$

with $R_2 = 0.02\,\Omega$, slip for torque T' is

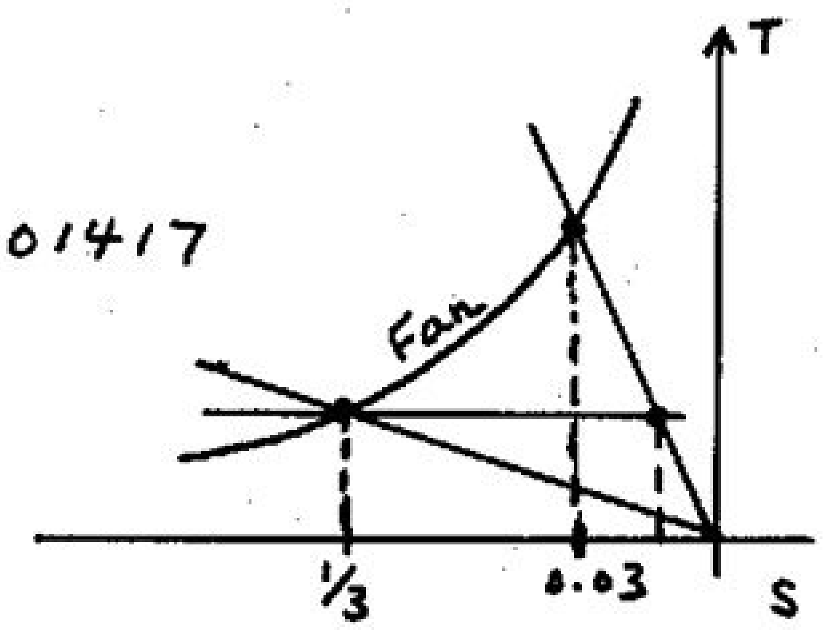
$$s\Big|_{0.02\Omega} = 0.03 \times \frac{T'}{T_{FL}} = 0.03 \times \left(\frac{600}{873}\right)^2 = 0.01417$$

To develop the same torque at $s = \frac{1}{3}$

$$\frac{R_2 + R_{ext}}{1/3} = \frac{R_2}{0.01417}$$

$$3(0.02 + R_{ext}) = 0.02 / 0.01417$$

$$\Rightarrow R_{ext} = 0.45\,\Omega$$



5.24 (a)

$$\frac{T_m}{T_{st}} = \frac{S_{Tm}^2 + 1}{2\,S_{Tm}} = \frac{2.5}{1.75}$$

$$1.75\,S_{Tm}^2 - 5\,S_{Tm} + 1.75 = 0$$

$$S_{Tm} = \frac{-(-5) \pm \sqrt{5^2 - 4(1.75)(1.75)}}{2(1.75)} = 2.45, 0.408$$

Take $S_{Tm} = 0.408$

(b)

$$\frac{T_m}{T_{FL}} = \frac{S_{Tm}^2 + S_{FL}^2}{2\,S_{Tm}\,S_{FL}} = \frac{0.408^2 + S_{FL}^2}{2 \times 0.408\,S_{FL}} = 2.5$$

$$S_{FL} = 1.955,\ 0.085$$

Take $S_{FL} = 0.085$

5.24 (c) $T \propto \frac{I_2^2 R_2}{S}$

$$\frac{T_{st}}{T_{FL}} = \frac{I_{2ST}^2}{I_{2FL}^2} \frac{S_{FL}}{S_{ST}} = \left(\frac{I_{2ST}}{I_{2FL}}\right)^2 \frac{0.085}{1} = 1.75$$

$$\frac{I_{2ST}}{I_{2FL}} = \sqrt{\frac{1.75}{0.085}} = 4.53$$

$$I_{2ST} = 4.53 \, pu$$

(d) $\frac{T_m}{T_{FL}} = \left(\frac{I_{2max}}{I_{2FL}}\right)^2 \frac{S_{FL}}{S_{Tmax}} = \left(\frac{I_{2max}}{I_{2FL}}\right)^2 \frac{0.085}{0.408} = 2.5$

$$\frac{I_{2max}}{I_{2FL}} = \sqrt{\frac{2.5 \times 0.408}{0.085}} = 3.46$$

$$I_{2max} = 3.46 \, pu$$

$\boxed{5.25}$ (a) $\frac{T_{max}}{T_{FL}} = 2.5 = \frac{S_{T(max)}^2 + S_{FL}^2}{2 S_{Tmax} S_{FL}} = \frac{S_{Tmax}^2 + 0.04^2}{2 \times 0.04 \times S_{Tmax}}$

solving $S_{Tmax} = \frac{0.2 \pm \sqrt{0.2^2 - 4(0.0016)}}{2}$

$$= 0.19165, \; 0.0083$$

Take $S_{Tmax} = 0.19165 \rightarrow n = (1 - 0.19165)1800 = 1455$ rpm

(b) $\frac{T_{max}}{T_{st}} = \frac{S_{Tmax}^2 + 1}{2 S_{Tmax}} = \frac{0.19165^2 + 1}{2 \times 0.19165} = 2.7$

$\therefore T_{st} = 2.5/2.7 = 0.9259$

(c) For same max. torque

$$\frac{R_2}{S_{Tmax}} = \frac{R_2 + R_{ext}}{S_{ST}} = \frac{0.5 + R_{ext}}{1}$$

$\Rightarrow R_{ext} = 2.109$

(d) For same full-load torque

$\frac{R_2}{S_{FL}} = \frac{R_2 + R_{ext}}{S'_{FL}} \quad \frac{0.5}{0.04} = \frac{0.5 + 2.109}{S'_{FL}} \Rightarrow S'_{FL} = 0.20872$

$n = (1 - 0.20872) 1800 = 1424.3$ rpm

$\boxed{5.26}$ (a) slip $s = \dfrac{1800-1710}{1800} = 0.05$

$$\frac{|I_2'|_{start}}{|I_2''|_{start}} = \frac{|E/(4+j1.5)|}{|E/(0.5+j4.5)|} = \frac{4.528}{4.272} = 1.06$$

$$\frac{|I_2'|_{FL}}{|I_2''|_{FL}} = \frac{\left|E/\left(\frac{4}{0.05}+j1.5\right)\right|}{\left|E/\left(\frac{0.5}{0.05}+j4.5\right)\right|} = \frac{\sqrt{10^2+4.5^2}}{\sqrt{80^2+1.5^2}} = 0.137$$

(b) $T_{st} = T_{st}\,(outer) + T_{st}\,(inner)$

$$= \frac{3E^2}{\omega_s\,|4+j1.5|^2}\times 4 + \frac{3}{\omega_s}\,\frac{E^2}{|0.5+j4.5|^2}\times 0.5$$

$$= k\left[\frac{4}{4^2+1.5^2} + \frac{0.5}{0.5^2+4.5^2}\right]$$

where $k = 3\dfrac{E^2}{\omega_s}$

$$= k\left[0.21918 + 0.02439\right] = 0.24357\,k$$

$T_{FL} = T_{FL}\,(outer) + T_{FL}\,(inner)$

$$= k\left[\frac{4/0.05}{(4/0.05)^2+1.5^2} + \frac{0.5/0.05}{(0.5/0.05)^2+4.5^2}\right]$$

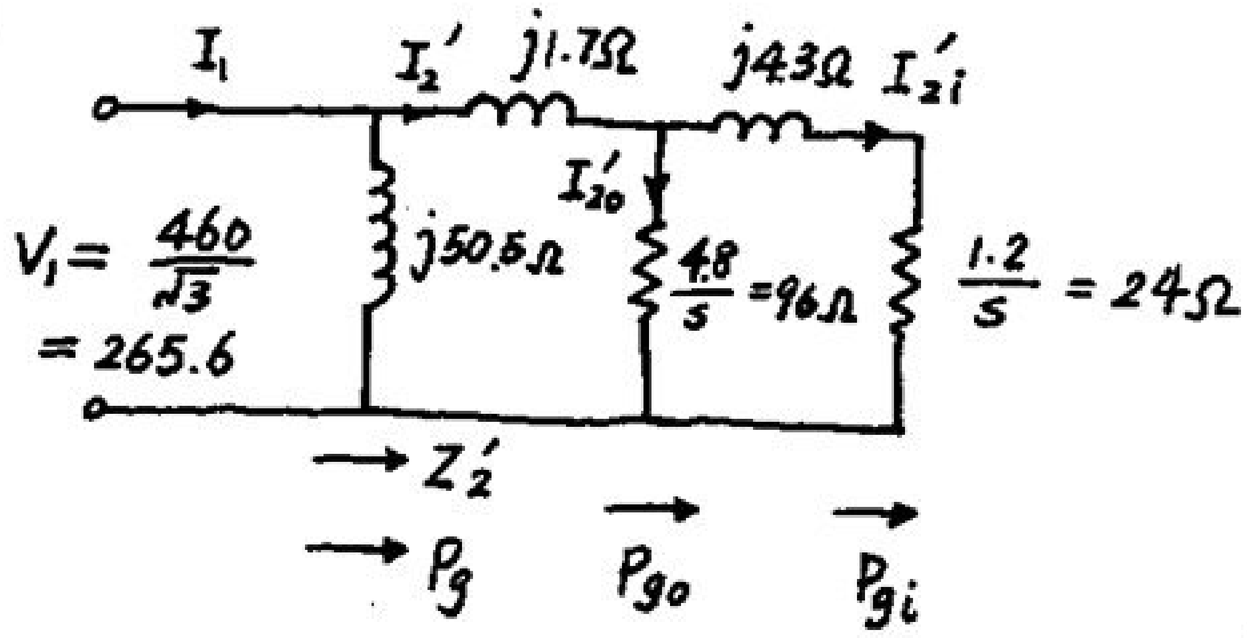
$$= k\left[0.012496 + 0.08316\right] = 0.09566\,k$$

$$\frac{T_{st}}{T_{FL}} = \frac{0.24357k}{0.09566k} = 2.5463 \text{ or } 255\%$$

(c) $$\frac{T_{st}\,(outer)}{T_{st}\,(inner)} = \frac{0.21918}{0.02439} = 8.986$$

$$\frac{T_{FL}\,(outer)}{T_{FL}\,(inner)} = \frac{0.012496}{0.08316} = 0.15$$

$\boxed{5.27}$



$S = 0.05$, $\frac{4.8}{0.05} = 96\Omega$, $\frac{1.2}{0.05} = 24\Omega$

$$Z_2' = j1.7 + \frac{96(24 + j4.3)}{120 + j4.3} = 19.805\angle 12.98^\circ$$

$$I_2' = \frac{265.6}{19.805\angle 12.98^\circ} = 13.411\angle -12.98^\circ$$

$$|I_{20}'| = \left|\frac{24 + j4.3}{20 + j4.3}\right| |I_2'| = 2.723 \text{ A} \quad |I_{2i}'| = \left|\frac{96}{120 + j4.3}\right| |I_2'| = 10.722\text{A}$$

$$P_{go} = 3|I_{20}'|^2 x$$

$$P_{gi} = 3 \times (10.722)^2 \times 24 = 8277.058 \text{ W}$$

$$\omega_s = \frac{1800}{60} \times 2\pi = 188.5$$

$$T_o = \frac{P}{188.5} = 11.33 \text{ N.m} \qquad T_i = \frac{8277.058}{188.5} = 43.91 \text{ N.m}$$

$$T = T_o + T_i = 55.24 \text{ N.m}$$

or $P_g = 3 \times 265.6 \times |I_2'| \cos\theta_2' = 3 \times 265.6 \times 13.41 \times 0.9744 = 10412.824\text{W}$

$$T = \frac{P_g}{\omega_s} = \frac{10412.824}{188.5} = 55.24 \text{ N.m.}$$

$\boxed{5.28}$ $I_Y = 245.9 = \frac{460/\sqrt{3}}{Z_{st}} = \frac{V_L}{\sqrt{3}\,Z_{st}}$

$$I_\Delta = \sqrt{3} \times \frac{V_L}{Z_{st}} = \sqrt{3} \times \sqrt{3}\, I_Y = 3I_Y = 3 \times 245.9 = 737.7\text{A}$$

$\boxed{5.29}$ (a) $I_{rated} = \frac{200 \times 746}{\sqrt{3} \times 460 \times 0.85 \times 0.9} = 244.8 \text{ A}$

(b)

$I \propto V_{motor}$

$$\frac{I_{\text{Full Voltage}}}{I_{\text{Reduced voltage}}} = \frac{460}{V} = \frac{6}{2} = 3$$

$$V = \frac{460}{3} = 153.3 \text{ V}$$

(c) $T \propto V^2$

$$\frac{T_{153.3V}}{T_{460V}} = \left(\frac{153.3}{460}\right)^2 = 0.1111$$

Starting torque is substantially reduced at a reduced starting voltage.

5.30 (a) For same flux, V/f is constant

$$\frac{V}{50} = \frac{460}{60}$$

$$V = \frac{460}{60} \times 50 = 383.3 \quad \text{(line-to-line)}$$

(b) R_1, L_1 negligible

$$V_1 \simeq E_1$$

From equation 5.81, if V/f is constant and $(E \approx V)$, torque depends on f_2 or slip rpm

$$n_{slip} = \frac{120 f_2}{P}$$

slip rpm with 60 Hz operation at full-load torque

$$= 1800 - 1755 = 45 \text{ rpm}$$

For full-load torque with 50 Hz supply speed is

$$n = n_{s\,(50\,Hz)} - 45 = \frac{120 \times 50}{4} - 45$$

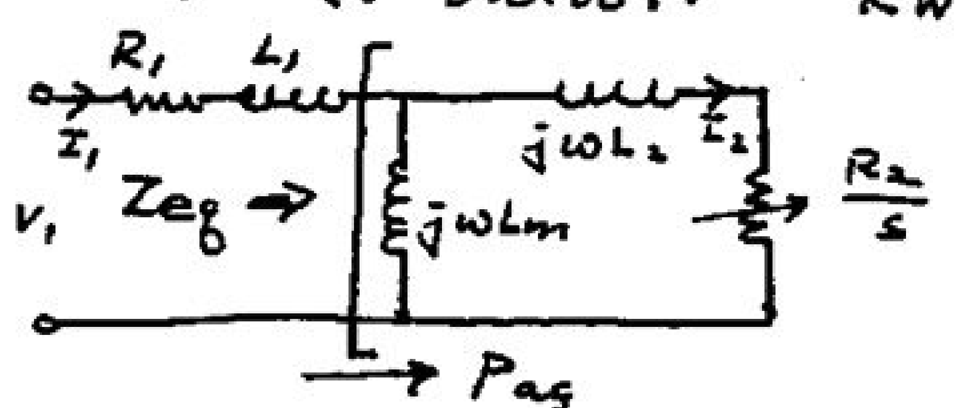
$$= 1455 \text{ rpm}$$

5.31 (a) $T_{rated} = \dfrac{\text{rated power}}{\text{rated speed}} = \dfrac{50 \times 745.7}{1180 \times 2\pi/60} = 301.73$ N·m

$\text{Slip} = \dfrac{1200 - 1180}{1200} = 0.01667 \qquad P_{ag} = \dfrac{50 \times 745.7}{(1 - 0.01667)} = 37.9.17$ kW

$Z_{eq} = R_{eq} + j X_{eq}$

$P_{ag} = 3 I_1^2 R_{eq} \Rightarrow I_1 = \sqrt{\dfrac{P_{ag}}{3 R_{eq}}}$



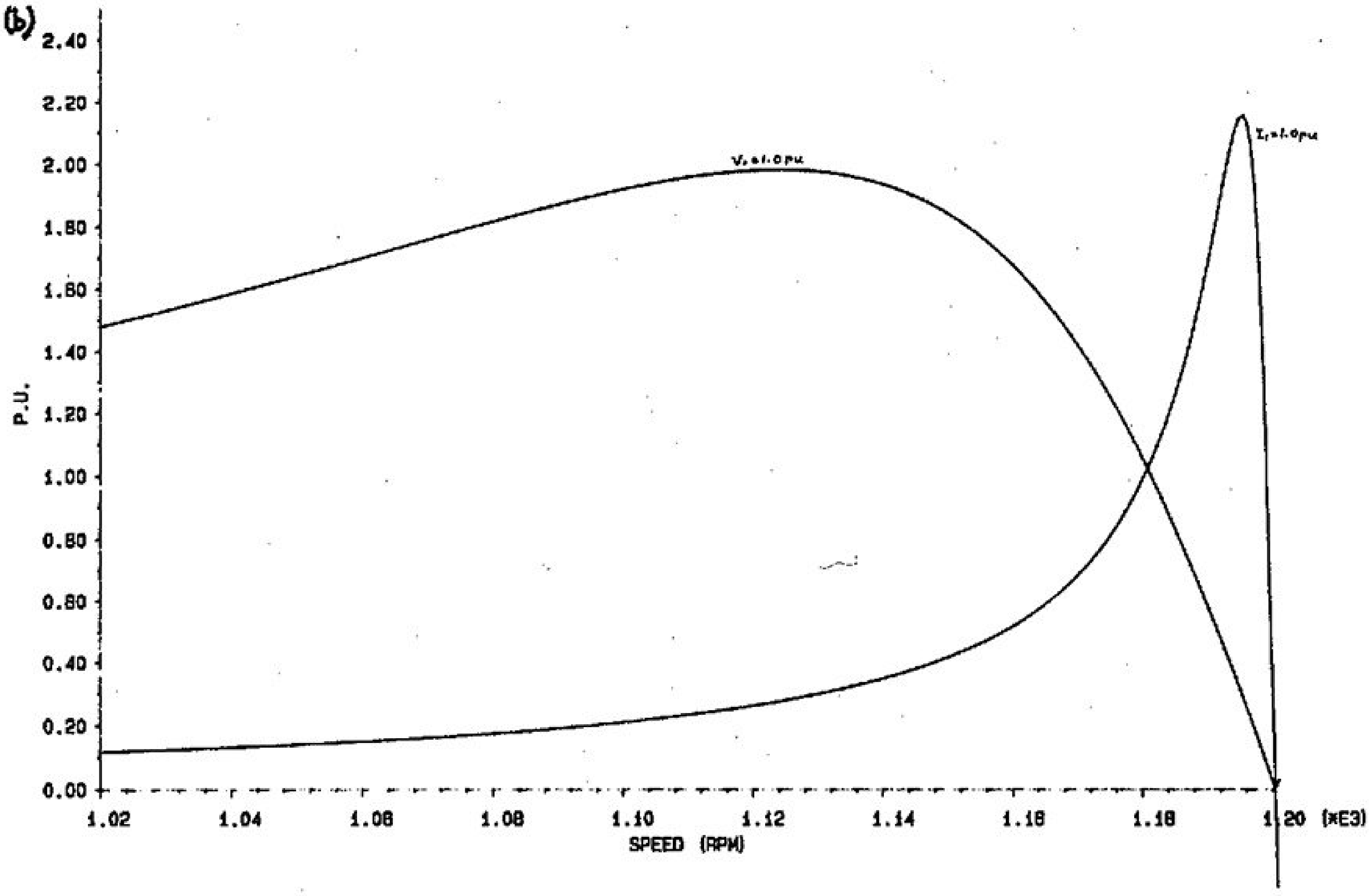
$$Z_{eq} = \frac{j\omega L_m (R_2/s + j\omega L_2)}{R_2/s + j\omega (L_m + L_2)}$$

$$= j\,\frac{71.644 - 6.36738}{4.2419 + j\,17.267} = 3.828 + j\,1.3085$$

$$\therefore \; I_1 = \sqrt{\frac{37917}{3\,(3.828)}} = 57.46 \text{ A}$$

5·31 continued:

(b)



$\boxed{5.32}$ (a) From example 5.4,

$T_{FL} = 163.11 \text{ N.m} = 1 \text{ pu}.$

(i) $T_{max} = 431.68 \text{ N.m}$ (from ex. 5.4)

$$= \frac{431.68}{163.11} = 2.65 \text{ pu}.$$

(ii) $E = 265\angle 0^\circ - (0.25 + j0.5)\, 42.75 \angle -19.7$

$= 248.27 \angle -3.8^\circ \text{ V}$

$$\frac{E}{f} = \frac{248.27}{60} = 4.138$$

From equation 5.86

$$T_{max} = 3 \times \frac{4}{4\pi} (4.138)^2 \frac{1}{4\pi \times 1.33 \times 10^{-3}}$$

$= 978.17 \text{ N.m}$

$$= \frac{978.17}{163.11} = 6 \text{ p.u.}$$

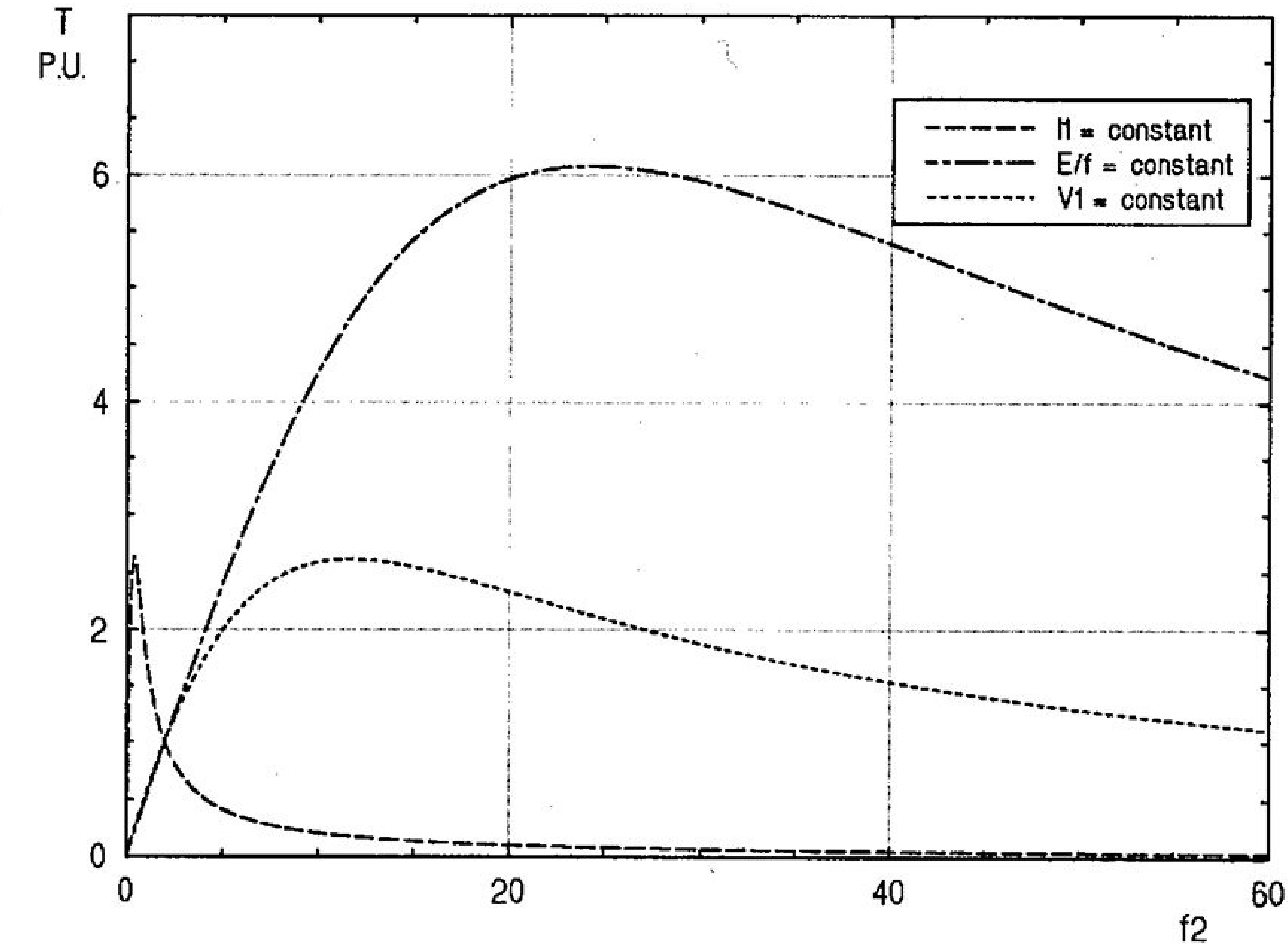
5.32 continued :

a(iii) From equation 5.90

$$T_{max} = 3 \times \frac{4\,(79.58 \times 10^{-3})^2}{4\,(79.58 + 1.33)\,10^{-3}} \times 42.76^2$$

$$= 429.35 \text{ N.m} = 2.63 \text{ pu.}$$

(b)



$\boxed{5.33}$ (a) Fundamental torque, T_1

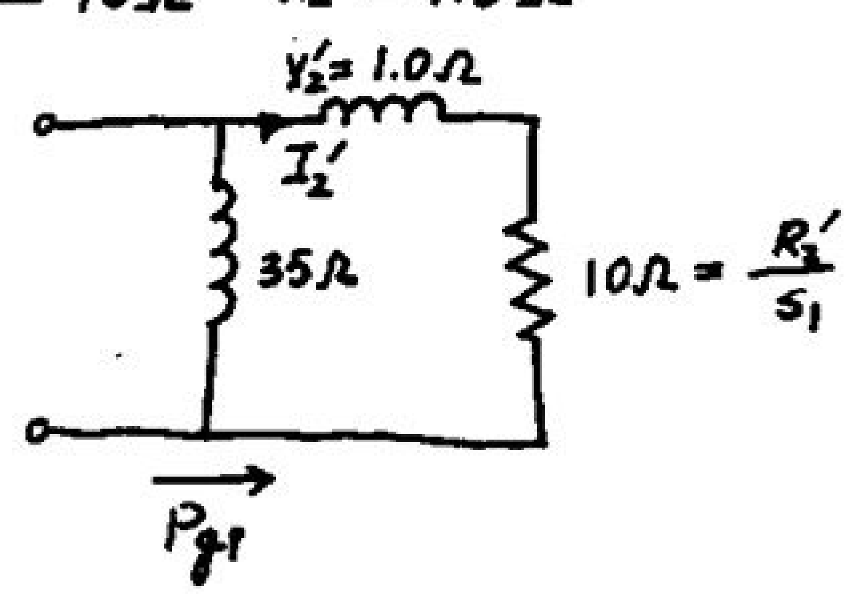
$n = 1710$ rpm , $n_s = \frac{120 \times 60}{4} = 1800$ rpm

$s_1 = \frac{1800 - 1710}{1800} = 0.05$ $\quad \frac{R_2'}{s} = \frac{0.5}{0.05} = 10\Omega$ $\quad X_2' = 1.0\Omega$

$I_2' = \left| \frac{100}{10 + j1.0} \right| = \frac{100}{10.05} = 9.95$ A

$P_{g_1} = 3 \times 9.95^2 \times 10 = 2970.22$ W

$T_1 = \frac{2970.22}{\frac{1800}{60} \times 2\pi} = 15.757$ N.m



5.33 Continued:

(b) Fifth harmonic torque, T_5

$$n_{s(5)} = -\frac{120\times(5\times60)}{4} = -9000 \text{ rpm}$$

$$S_5 = \frac{-9000-1710}{-9000} = 1.19 \qquad \frac{R_2'}{S_5} = \frac{0.5}{1.19} = 0.42\,\Omega$$

$$5X_2' = 5\times1 = 5\,\Omega \qquad I_2' = \frac{15}{0.42+j5} = 2.99\text{A}$$

$$P_{g5} = 3\times2.99^2\times0.42 = 11.2606 \text{ W}$$

$$T_5 = \frac{11.2606}{\frac{-9000}{60}\times2\pi} = -0.01195 \text{ N.m}$$

(c) Seventh harmonic torque, T_7

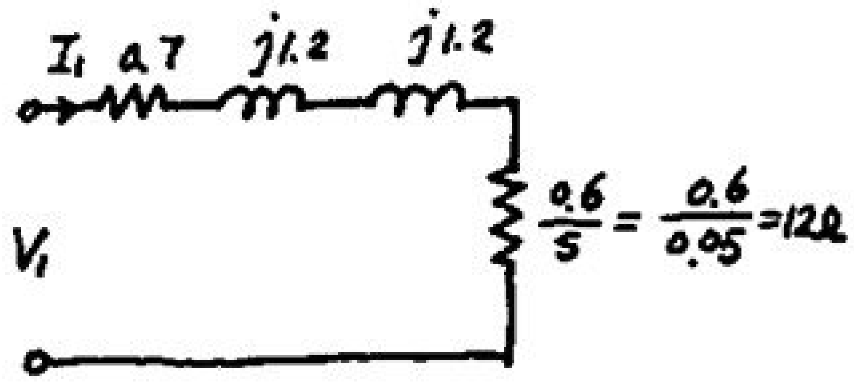
$$n_{s(7)} = \frac{12\times(7\times60)}{4} = 12{,}600 \text{ rpm} \qquad S_7 = \frac{12600-1710}{12600} = 0.8643$$

$$\frac{R_2'}{S_7} = \frac{0.5}{0.8643} = 0.5785\,\Omega \qquad 7X_2' = 7\times1 = 7\,\Omega$$

$$I_2' = \frac{10}{0.5785+j7} = 1.4237\text{A} \qquad P_{g7} = 3\times1.4237^2\times0.5785 = 3.518 \text{ W}$$

$$T_7 = \frac{3.5178}{\frac{12600}{60}\times2\pi} = 0.002666 \text{ N.m}$$

$\boxed{5.34}$ (a) $h=1$ $\quad S = \frac{1800-1710}{1800} = 0.05$

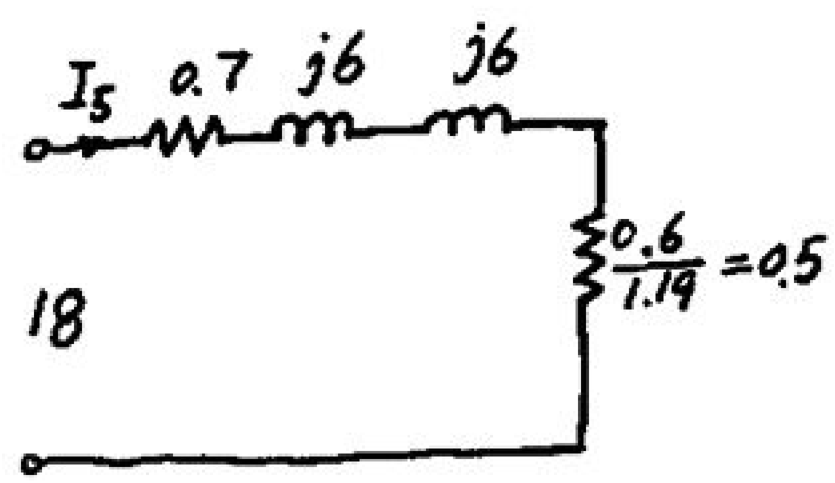


$$I_1 = \frac{100}{\sqrt{12.7^2+2.4^2}} = \frac{100}{12.9248} = 7.737\text{A}$$

$$P_{g1} = 3\times7.737^2\times12 = 2155 \text{ W}$$

$$T_1 = \frac{2155}{\frac{1800}{60}\times2\pi} = 11.433 \text{ N.m}$$

(b) $h=5$ $\quad S = \frac{-9000-1710}{-9000} = 1.19$



$$I_5 = \frac{18}{\sqrt{1.2^2+12^2}} = \frac{18}{12.06} = 1.49\text{A}$$

$$P_{g5} = 3\times1.49^2\times0.5 = 3.33 \text{ W}$$

$$T_5 = \frac{3.33}{\frac{-9000}{60}\times2\pi} = -0.0035 \text{ N.m}$$

5.35 (a) - Limited number of slots.

- Non-sinusoidal distribution of winding conductors in slots.
- Alters T-n characteristics at low speed causing crawing effect.

(b) $n_1 = \frac{120 \times 60}{8} = 900$ rpm.

$n_5 = -\frac{120 \times 60}{5 \times 8} = -180$ rpm $\rightarrow$ opposite to n_1.

5.36 (a) $V_s = 2 \times 50 \times 10^{-2} \times 50 = 50$ m/sec. $= \frac{50 \times 60 \times 60}{1000}$ km/hr $= 180$ km/hr

$V = (1 - 0.25) \times 180 = 135$ km/hr

(b) right to left

5.37 (a) $V_s = 2 \times 30 \times 10^{-2} \times 50$ m/sec. $= 30$ m/sec

$= 30 \times 60 \times 60 \times 10^{-3}$ km/hr $= 108$ km/hr.

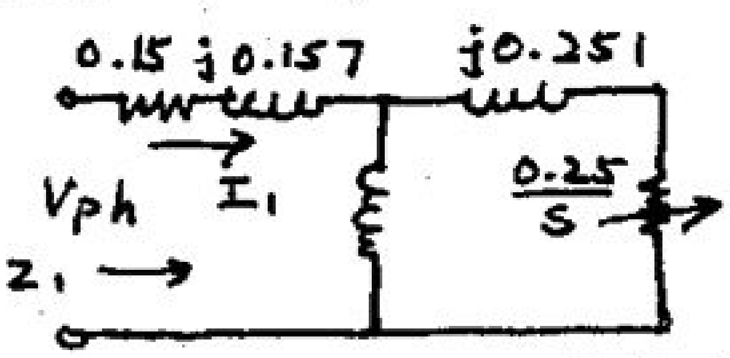
$s = \frac{108 - 75}{108} = 0.3056$

(b) $X_1 = 2\pi 50 \times 0.5 \times 10^{-3} = 0.157\,\Omega$ $\quad X_2' = 2\pi 50 \times 0.8 \times 10^{-3} = 0.251\,\Omega$

$X_m = 2\pi 50 \times 5.0 \times 10^{-3} = 1.57\,\Omega$ $\quad V_{ph} = 300/\sqrt{3} = 173.2$ V

$Z_1 = 0.15 + j\,0.157 + \frac{j1.57(0.818 + j\,0.251)}{0.818 + j\,(1.57 + 0.251)}$

$= 0.89 \angle 42.4^\circ$



$I_1 = \frac{173.2 \angle 0^\circ}{0.89 \angle 42.4} = 194.6 \angle -42.4^\circ$ A

$P_{in} = 3VI_1 \cos\theta = 3 \times 173.2 \times 194.6 \cos 42.4 = 74.67$ kW

$Pf = \cos 42.4 = 0.73845$ lag

$P_g = P_{in} - 3I_1^2 R_1 = 74.67 \times 10^3 - 3(194.6)^2 0.15 = 57.63$ kW

$P_{mech} = (1 - s) P_g = (1 - 0.3056)\,57.63 = 40$ kW

$P_{rotor\ cu\ loss} = s P_g = 0.3056\,(57.63) = 17.63$ kW

Thrust $F = P_g / V_s = \frac{57.63}{30} = 1.921$ kN

Check $F \times V = P_{mech}$

$1.921 \times 10^3 \times 75 \times 10^3 / (60 \times 60) = 40$ kW $= P_{mech}$

5.38 (a) $f_1 = f_n - f_2 \qquad f_n = f_1 + f_2$

speed greater than synchronous speed → generating mode

(b) $f_n = 60 + 5 = 65\ Hz$

(c) $slip = -\frac{5}{60} = -\frac{1}{12}$

(d) $V_s = 2 \times 30 \times 10^{-12} \times 60 = 36\ m/sec.$

$= \frac{36 \times 60 \times 60}{1000}\ Km/hr = 129.6\ Km/hr$

$V = (1-s)V_s = (1-(-\frac{1}{12}))\times 129.6 = 140.4\ km/hr \rightarrow 39 m/hr$

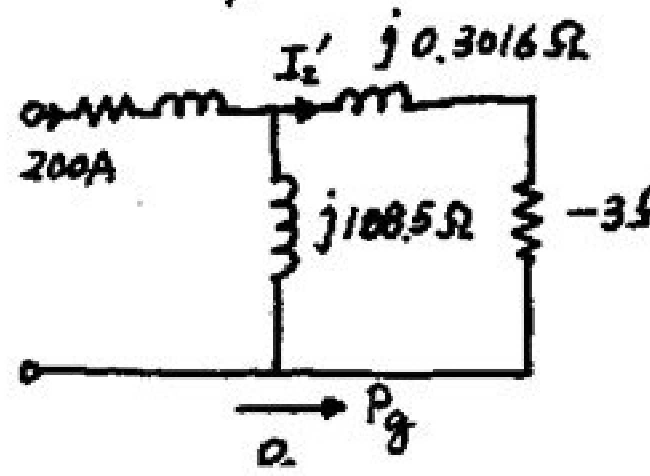
(e) $X_2' = 2\pi \times 60 \times 0.8 \times 10^{-3} = 0.3016\ \Omega$

$X_m = 2\pi \times 60 \times 5 \times 10^{-3} = 1.885\ \Omega \qquad \frac{R_2'}{s} = \frac{0.25}{-1/12} = -0.25 \times 12 = -3\Omega$

$|I_2'| = \frac{|1.885|}{|-3 + j2.1866|} \times 200 = 101.26A$

$P_g = 3 \times 101.26^2 \times (-3) = -92{,}286.7W$

Power flows from rotor to stator



(f) Synchronous velocity $v_s = 2 \times 30 \times 10^{-2} \times 60 = 36\ m/sec$

Thrust $= \frac{92{,}286.7}{36} = 2563.52\ N$

5.39 (a) $f_1 = f_2 + f_n$, $n < n_s$ → motoring

$V_s = 2 \times 30 \times 10^{-2} \times 60 = 36 m/sec. = \frac{36 \times 60 \times 60}{1000} km/hr = 129.6\ km/hr$

$s = \frac{5}{60} = \frac{1}{12} \qquad v = (1 - \frac{1}{12}) \times 129.6 = 118.8\ km/m$

$X_2' = 2\pi \times 60 \times 0.8 \times 10^{-3} = 0.3016\Omega$

$X_m = 2\pi \times 60 \times 5 \times 10^{-3} = 1.885\Omega \qquad \frac{R_2'}{s} = \frac{0.25}{1/12} = 0.25 \times 12 = 3\Omega$

$I_2' = \left|\frac{1.885}{3 + j2.1866}\right| \times 200 = \frac{1.885}{3.723} \times 200 = 101.26\ A$

$P_{g1} = 3 \times 101.26^2 \times 3 = 92{,}286.7W$

$F = \frac{P_{g1}}{v_s} = \frac{92{,}286.7}{36} = 2563.52\ N$

5.40 $f_1 = f_n + f_2 = 0 + f_2 = f_2 = 5\ Hz \qquad X_2' = 2\pi 5 \times 0.8 \times 10^{-3} = 0.0251\ \Omega$

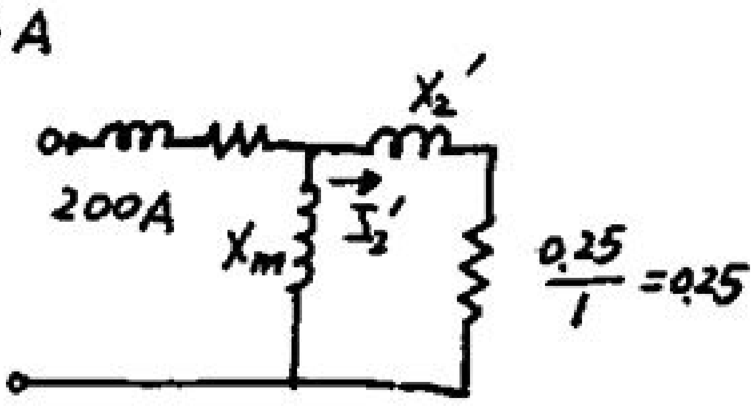
$X_m = 2\pi 5 \times 50 \times 10^{-3} = 0.1571\ \Omega \qquad s = 1$

$I_2' = \frac{0.1571}{0.25 + j0.1822} \times 200 = \frac{3.142}{0.3093} = 101.584\ A$

$P_g = 3 \times 101.584^2 \times 0.25 = 7739.5\ W$

$v_s = 2 \times 30 \times 10^{-2} \times 5 = 300 \times 10^{-2} = 3.0 m/sec.$

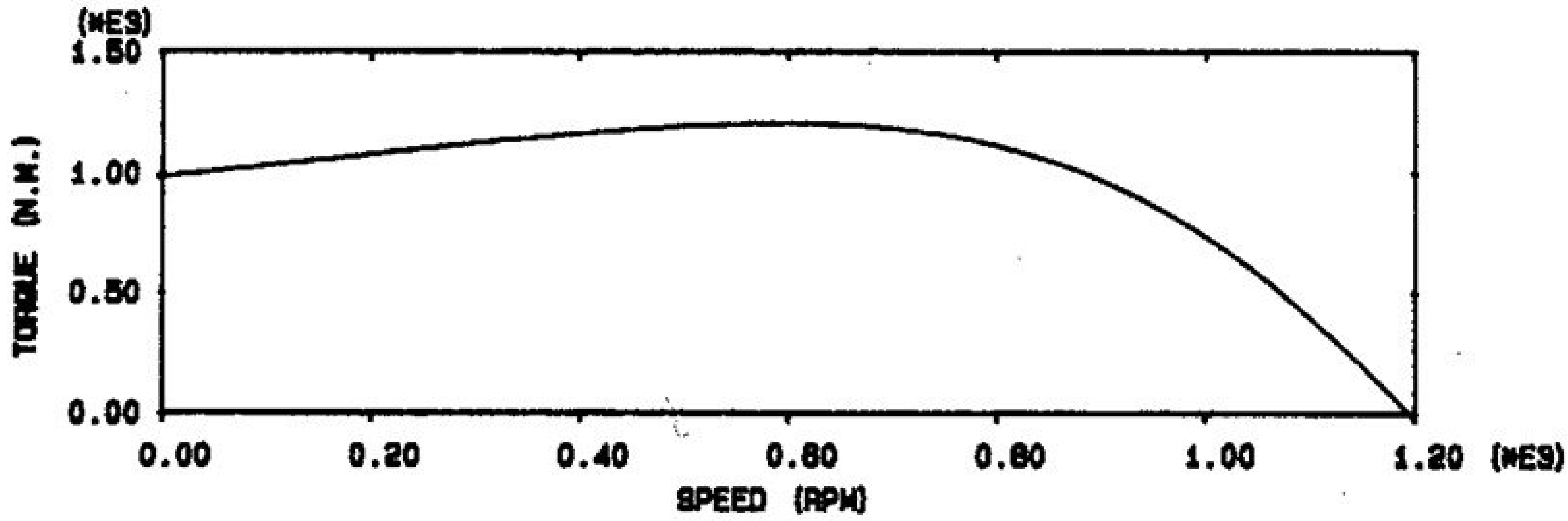
$F_{start} = \frac{7739.5}{3} = 2579.8\ N$

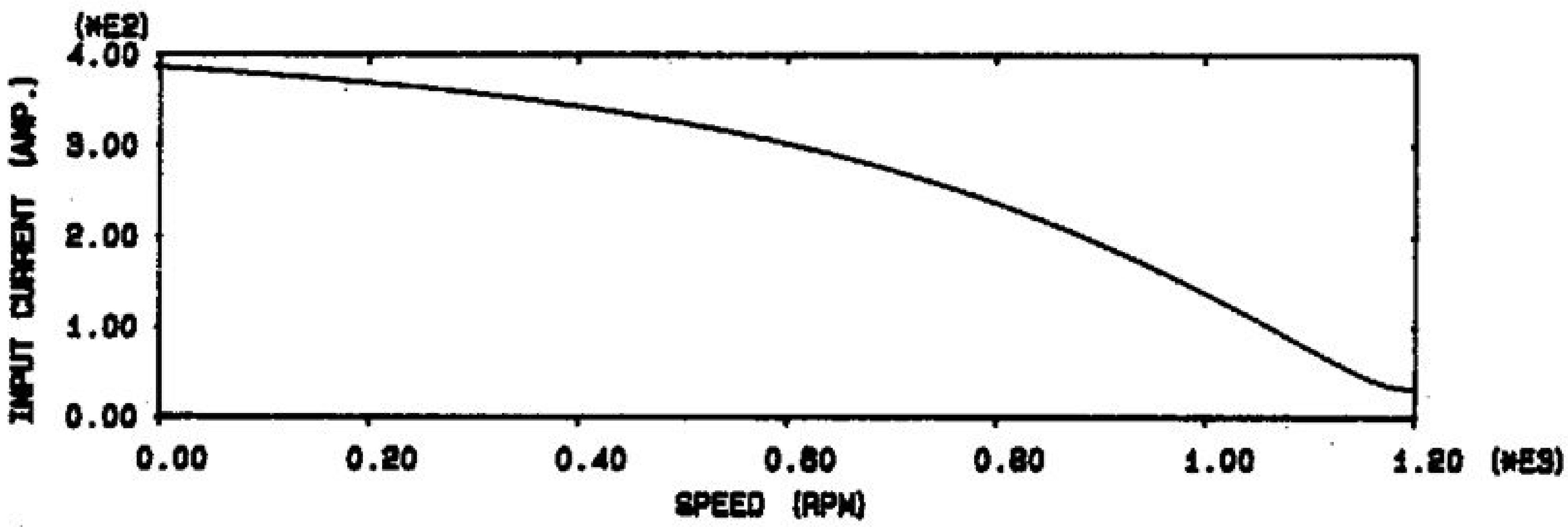


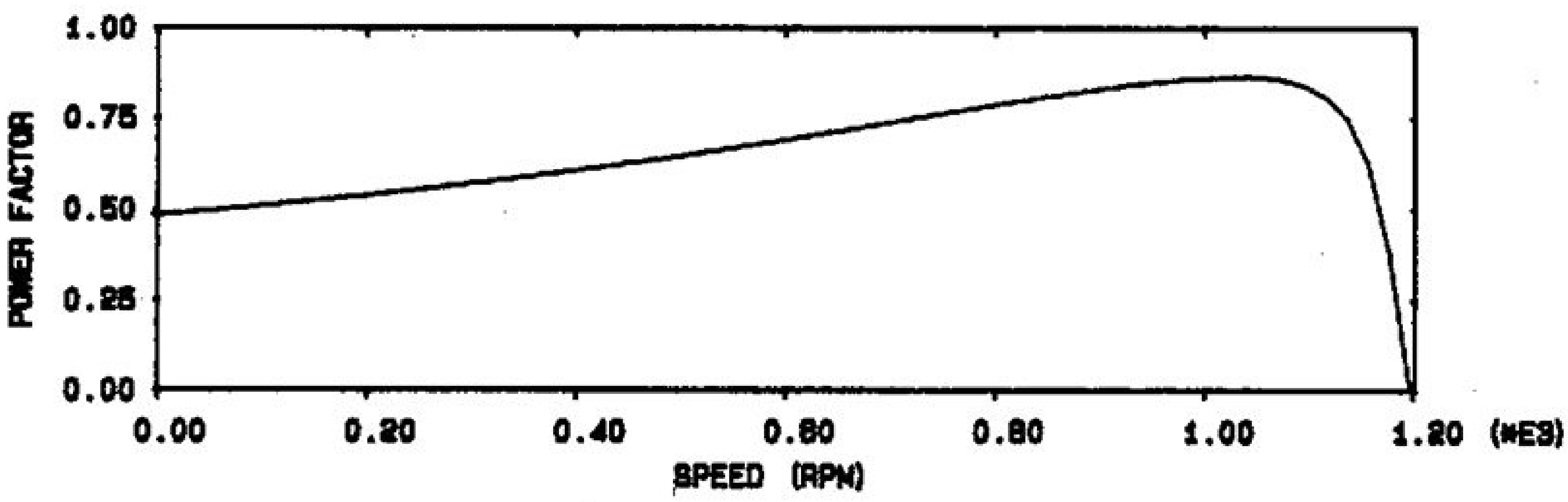
5.41

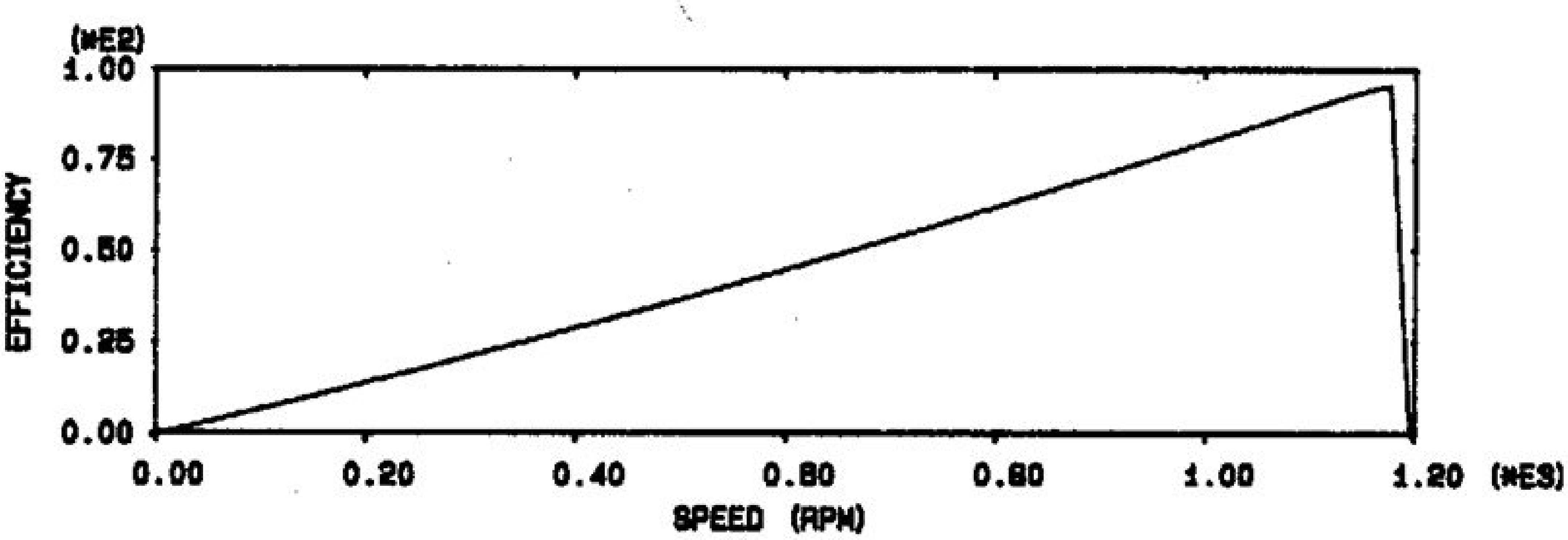
a) SPEED (RPM)	TORQUE (N.M.)	CURRENT (A)	PF	EFFICIENCY
0	991.70	389.07	0.489	0.00
40	1010.10	386.08	0.499	2.75
80	1028.69	382.86	0.510	5.53
120	1047.42	379.39	0.521	8.35
160	1066.18	375.64	0.532	11.20
200	1084.87	371.59	0.544	14.09
240	1103.33	367.20	0.556	17.02
280	1121.39	362.43	0.569	19.98
300	1130.20	359.89	0.576	21.47
320	1138.83	357.25	0.583	22.97
340	1147.24	354.49	0.590	24.49
360	1155.39	351.61	0.597	26.01
380	1163.25	348.60	0.605	27.54
400	1170.75	345.45	0.612	29.08
420	1177.86	342.17	0.620	30.63
440	1184.52	338.74	0.628	32.19
460	1190.67	335.15	0.636	33.77
480	1196.23	331.39	0.644	35.35
500	1201.14	327.46	0.652	36.94
520	1205.32	323.35	0.660	38.54
540	1208.69	319.04	0.669	40.15
560	1211.13	314.53	0.678	41.77
580	1212.57	309.81	0.687	43.41
600	1212.88	304.87	0.696	45.05
620	1211.95	299.68	0.705	46.70
640	1209.65	294.25	0.714	48.37
660	1205.85	288.56	0.724	50.04
680	1200.40	282.60	0.733	51.73
700	1193.15	276.36	0.743	53.43
720	1183.94	269.81	0.753	55.14
740	1172.59	262.96	0.762	56.86
760	1158.94	255.79	0.772	58.59
780	1142.79	248.28	0.782	60.33
800	1123.97	240.43	0.791	62.09
820	1102.27	232.22	0.801	63.85
840	1077.52	223.64	0.810	65.63
860	1049.53	214.69	0.819	67.42
880	1018.10	205.36	0.828	69.22
900	983.08	195.64	0.836	71.03
920	944.31	185.53	0.844	72.86
940	901.65	175.04	0.851	74.69
960	854.98	164.16	0.858	76.54
980	804.23	152.91	0.864	78.40
1000	749.35	141.30	0.868	80.26
1020	690.35	129.36	0.870	82.14
1040	627.26	117.13	0.871	84.03
1060	560.19	104.66	0.867	85.92
1080	489.29	92.02	0.859	87.82
1100	414.78	79.35	0.842	89.71
1120	336.95	66.84	0.810	91.59
1140	256.13	54.86	0.748	93.42
1160	172.72	44.13	0.627	95.12
1180	87.18	36.08	0.389	96.27
1200	5.26	32.99	7.453	3.37
c) 1025	674.95	126.33	0.871	82.62

(b) 5.41 continued:









6.1 Induction motor (IM)

$P_{IM} = \frac{1000 \times 746}{0.85} = 877.6471\ KW \quad S_{IM} = \frac{P_{IM}}{0.7} = 1253.7815\ KVA$

$Q_{IM} = S_{IM} \sin(\cos^{-1} 0.7) = 895.3791\ KVAR$

Lighting and heating load (LH)

$P_{LH} = 100\ KW \quad Q_{LH} = 0$

Synchronous motor (SM)

$P_{SM} = 300 \times 746 = 223.8\ KW$

Factory Power

$P_F = 877.6471 + 100 + 223.8 = 1201.45\ KW$

$PF_F = 0.95 \quad S_F = \frac{1201.45}{0.95} = 1264.68\ KVA$

$Q_F = 1264.68 \sin(\cos^{-1} 0.95) = 394.9\ KVAR$

Sychronous motor has to provide

$Q_{SM} = 895.3791 - 394.9 = 500.48\ KVAR$

$S_{SM} = \sqrt{223.8^2 + 500.48^2} = 548.24\ KVA$

$PF_{SM} = \cos\left(\tan^{-1} \frac{500.48}{223.8}\right) = 0.4082$ (leading)

6.2 (a) 180Hz supply

$P_{60N} = 6, \quad P_{180N} = 18, \quad n = \frac{120 \times 60}{6} = 1200\ rpm$

(b) 400Hz supply

$P_{60N} = 6, \quad P_{400N} = 40, \quad n = \frac{120 \times 60}{6} = 1200\ rpm$

6.3 (a) See graph Fig P6.3

(b) $V_b = \frac{15 \times 10^3}{\sqrt{3}} = 8660.5\ V$

$I_b = \frac{195 \times 10^6}{\sqrt{3} \times 15 \times 10^3} = 7505.8\ A$

$Z_b = \frac{8660.5}{7505.8} = 1.1538\ \Omega$

$X_s\big|_{unsat} = \frac{18.75 \times 10^3}{\sqrt{3} \times 7000} = 1.5465\ \Omega$

$= \frac{1.5465}{1.1538} = 1.3404\ p.u.$

$X_s\big|_{sat} = \frac{15 \times 10^3}{\sqrt{3} \times 700} = 1.2372\ \Omega \rightarrow 1.0723\ p.u$

(a)

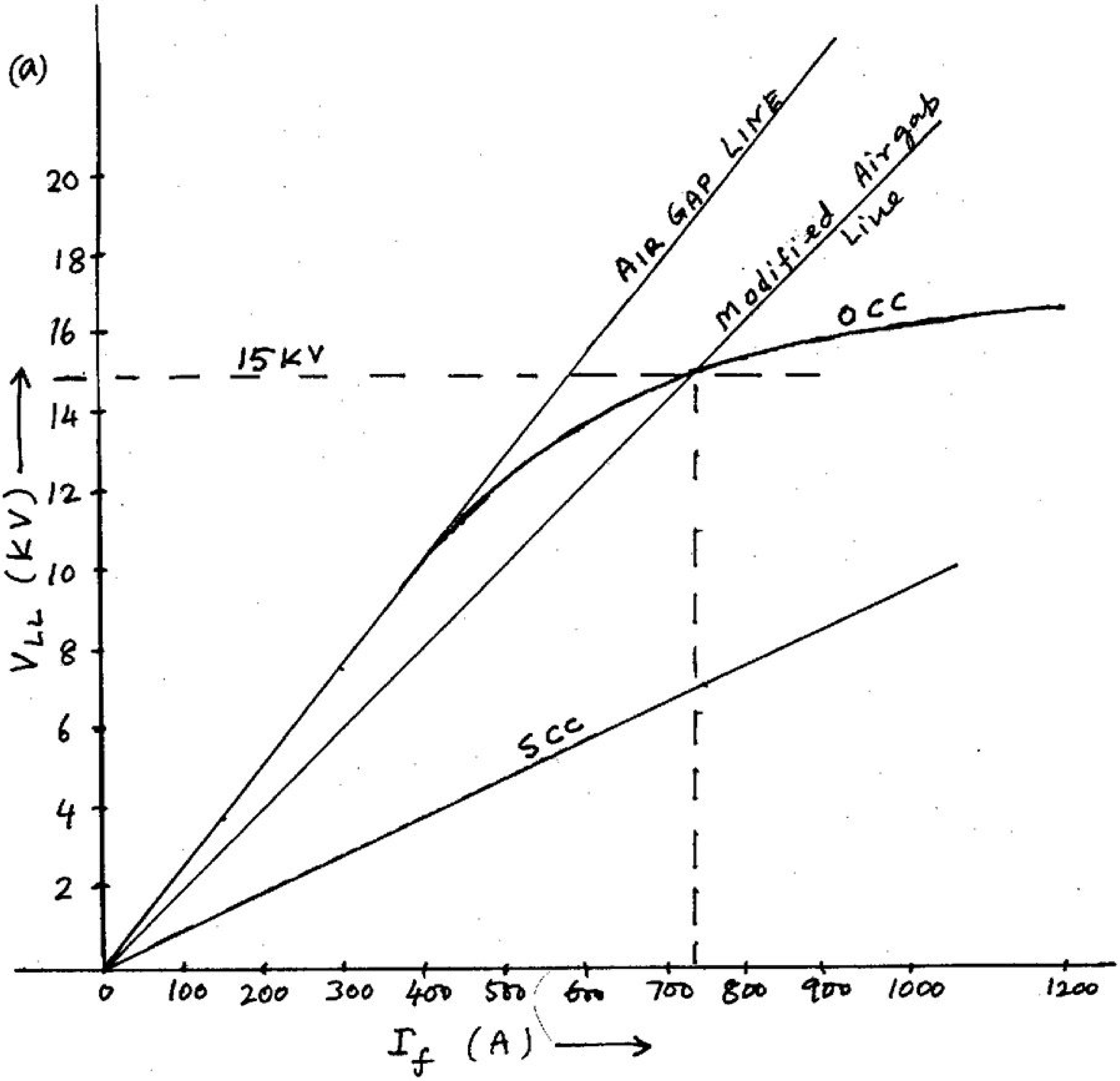


Fig. P6.3

(c) $I_a = \frac{100}{195} \angle \cos^{-1} 0.8 = 0.5128 \angle 36.87^\circ$ p.u.

$E_f = 1\angle 0^\circ + 0.5128 \angle 36.87 \times 1.0723 \angle 90$

$= 0.8016 \angle 33.3^\circ$ p.u.

$I_f = 0.8016 \times 750 = 601.2$ A

(d) Load removed and $I_f = 601.2$ A

From occ $\rightarrow$ $V_t = 13.6$ kV (LL)

$VR = \frac{13.6 - 15}{15} \times 100\% = -9.33\%$

$\boxed{6.4}$ (a) $3600 = \dfrac{120 \times 60}{P} \rightarrow P = 2$

(b) $V_b = \dfrac{25}{\sqrt{3}}\,KV = 14{,}434.2\ V \qquad I_b = \dfrac{750 \times 10^6}{\sqrt{3} \times 25 \times 10^3} = 17320\,A$

$Z_b = \dfrac{14434.2}{17320} = 0.8334\,\Omega$

$X_s|_{unsat} = \dfrac{30KV/\sqrt{3}}{10KA} = 1.732\,\Omega = \dfrac{1.732}{0.8334}\ P.U. = 2.0782\ P.U.$

$X_s|_{sat} = \dfrac{25KV/\sqrt{3}}{10KA} = 1.4434\,\Omega = \dfrac{1.4434}{0.8334}\ P.U. = 1.732\ P.U.$

(c) $I_a = \dfrac{E_f}{X_s} = \dfrac{k_1 f}{K_2 f} = \dfrac{K_3 n}{K_4 n} = constant = \dfrac{30KV/\sqrt{3}}{X_s|_{unsat}} = \dfrac{30KV/\sqrt{3}}{1.732} = 10{,}000\,A$

(d) $V_t = 1\,PU,\ I_a = 1\,PU\ \angle -\cos^{-1} 0.9 = 1\angle -25.84^\circ\ P.U.$

$E_f = 1\angle 0^\circ + 1\angle -25.84^\circ \times 1.732\angle 90^\circ = 1 + 1.732\angle 64.16^\circ = 1 + 0.7549 + j1.5588$

$= 2.3473\,\angle 41.6133^\circ$

$\boxed{6.5}$ (a) $\theta = \cos^{-1} 0.85 = 31.8^\circ$

$P = 0.5\ p.u. = V_t I_a \cos\theta$

$0.5 = 1 \times I_a \times 0.85$

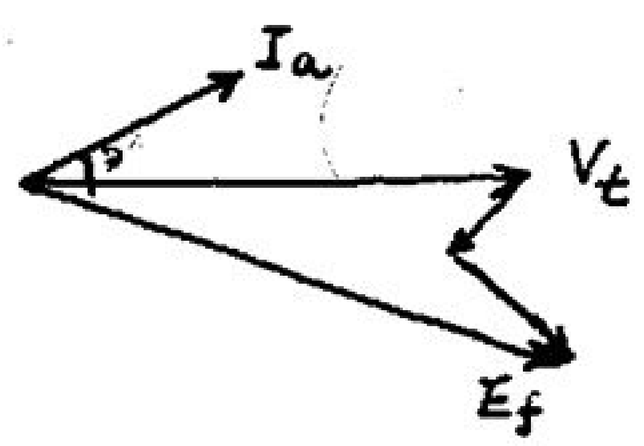
$I_a = 0.588\ \angle +31.8^\circ\ p.u.$

$E_f = 1\angle 0^\circ - 0.588\angle 31.8^\circ \cdot 0.84\angle 89.8^\circ$

$= 1.3272\ \angle -18.5^\circ$

$I_f = 1.3272 \times 200 = 265.44\ A$

(b)



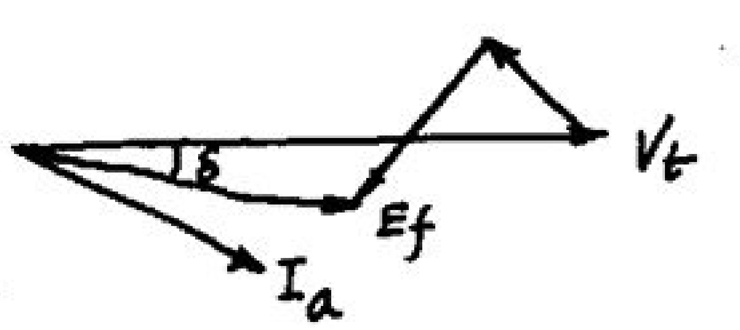
(c) From OCC (Fig. E6.2) at $I_f = 265\,A$

$V_t = E_f = 15.45\ KV$

$\boxed{6.6}$ (a) $I_a = 0.588\angle 0^\circ\ P.U. \quad E_f = 1\angle 0^\circ - 0.588\angle -31.8^\circ \times 0.84\angle 89.8^\circ = 0.8488\angle -29.57^\circ$

$I_f = 0.8488 \times 200 = 169.76\,A$

(b)

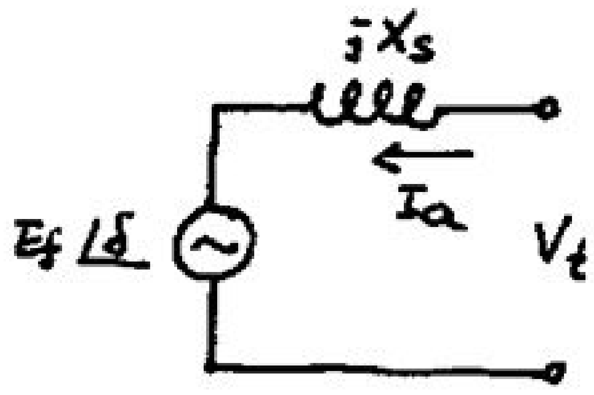


(c) From OCC (Fig E6.2) at $I_f = 169.76\,A \quad V_t = E_f = 12.85\,KV$

$\boxed{6.7}$ $E_f|_{max} = 2.5 \text{ pu}$ $\quad I_a = \dfrac{V_t\angle 0^\circ - E_f\angle 0^\circ}{jX_s} = \dfrac{1-2.5}{1.2\angle 90^\circ} = 1.25\angle 90^\circ$

$Q|_{max} = V_t I_a = 1 \times 1.25 = 1.25 \text{ p.u.}$

$\boxed{6.8}$ (a) $\quad V_t = \dfrac{2300}{\sqrt{3}} = 1328 \text{ V}$



$$165.8\times10^3 = 3\,\frac{V_t\,E_f}{X_s}$$

$$= \frac{3\times1328\times E_f}{11}\,\sin 15^\circ$$

$$E_f = 1769\,\angle{-15^\circ} \text{ V/ph}$$

(b) $$I_a = \frac{V_t\angle 0 - E_f\angle{-\delta}}{X_s\angle 90} = \frac{1328\angle 0^\circ - 1769\angle{-15^\circ}}{11\angle 90^\circ}$$

$$= 54.14\,\angle 39.8 \text{ A}$$

(c) $\quad PF = \cos 39.8 = 0.77$ leading

(d) $\quad P_{in} = 0$ and so $\delta = 0$

(i) $$I_a = \frac{1328\angle 0^\circ - 1769\angle 0^\circ}{11\angle 90^\circ}$$

$$= -40.1\,\angle{-90^\circ} = 40.1\,\angle 90^\circ \text{ A}$$

$PF = \cos 90^\circ = 0$ leading

(ii)

(iii) $$I_a = \frac{V_t\angle 0^\circ - E_f\angle 0^\circ}{X_s\angle 90^\circ}$$ is minimum ie. zero

if $V_t = E_f$ Hence E_f has to be reduced from 1769 V to 1328 V by reducing I_f

$$I_f|_{new} = \frac{1328}{1769}\times I_f\,(old) = 0.75\,I_f\,(old)$$

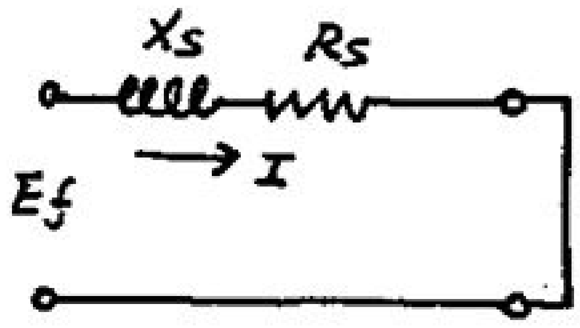
$\boxed{6.9}$ (a) (i) Rated Voltage $V_t = \frac{11}{\sqrt{3}} = 6.35\,kV$

$E_f = V_t = 6.35\,kV$

(ii) $Z_s = 1.5 + j15 = \sqrt{2.25 + 225}\,\angle \tan^{-1} 10^\circ$

$= 15.06\,\angle 84.3^\circ\ \Omega$

$I_a = \frac{6350}{15.06} = 421.65\,A$



$I_a|_{rated} = \frac{2000}{\sqrt{3} \times 11} = 105\,A$

(b) (i) $E_f = 6350 + 105\,\angle -37 \cdot 15.06\,\angle 84.3^\circ$

$= 7510\,\angle 9^\circ\ V$

(ii) $E_f \propto I_f$

% increase in $I_f = \frac{7510 - 6350}{6350} \times 100 = 18.27\%$

(iii) $P_{max} = \frac{3 \times 7510 \times 6350}{15} = 3 \times 3.18 \times 10^6$ watts

$= 9540\,kW$

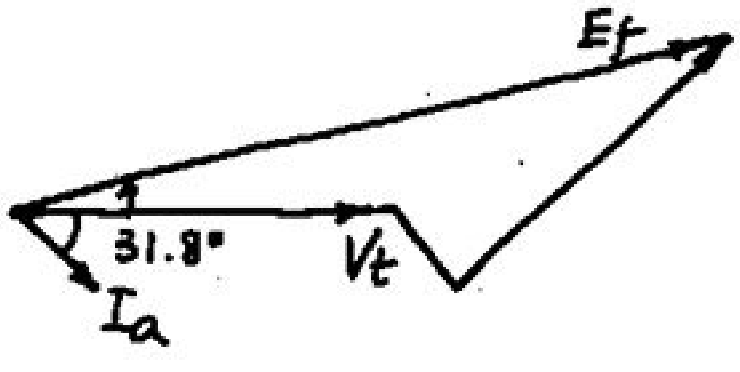
$\boxed{6.10}$ (a) $n_s = \frac{120 \times 60}{2} = 3600\,rpm$

(b) $V_{base} = \frac{12 \times 10^3}{\sqrt{3}} = 6928.4\,V/phase$ $\quad I_{base} = \frac{120 \times 10^6}{\sqrt{3} \times 12 \times 10^3} = 5773.7\,A$

$Z_{base} = \frac{6928.4}{5773.7} = 1.2 \quad R_a = 0.015 \times 1.2 = 0.018\,\Omega \quad X_s = 0.85 \times 1.2 = 1.02\,\Omega$

(c) $E_f = 1\angle 0^\circ + 1\angle -31.8^\circ \times 0.018 + 1\angle -31.8^\circ \times 1.02\angle 90^\circ = 1.774\,p.u.$

$= 1.774 \times 6928.4 = 12290.9\,V$



(d)

(i) $P_{Ra} = 3I^2 R_a = 3 \times 5773.7^2 \times 0.018 = 1.8\,MW$

(ii) P_{out} = Power to Bus $= 120 \times 0.85 = 102\,MW$

Turbine power $P_{in} = \frac{102}{0.92} = 110.8696\,MW$

Total loss $= 110.8696 - 102 = 8.896\,MW$

Rotational loss $= 8.8696 - 1.8 = 7.0696\,MW$

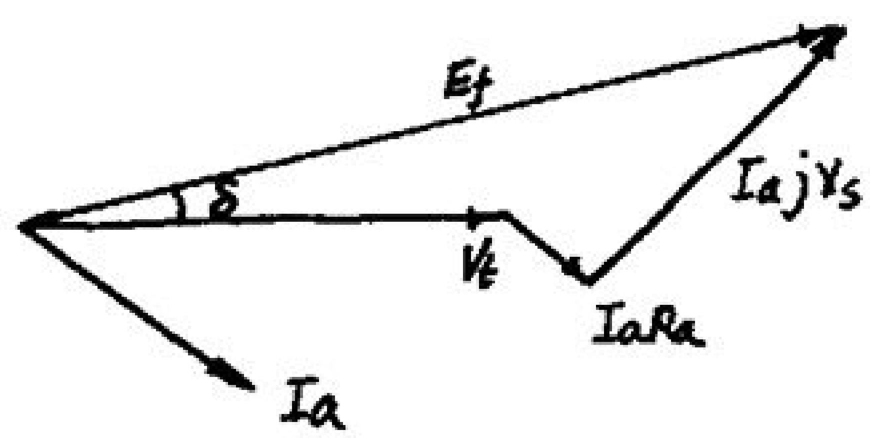
(iii) Torque $= \frac{110.8696 \times 10^6}{\frac{3600}{60} \times 2\pi} = 294.09\,KN.m$

6.11 (a) $V_t|_{rated} = \frac{14\times10^3}{\sqrt{3}} = 8083\,V/phase \quad I_a|_{rated} = \frac{10\times10^6}{\sqrt{3}\times14\times10^3} = 412.4A$

$\theta = \cos^{-1} 0.85 = 31.8^\circ$

$E_f = 8083\angle0^\circ + (412.4\angle-31.8^\circ)\times2 + (412.4\angle-31.8^\circ)\times20\angle90^\circ$

$= 8083 + 700 - j434.6 + 4346.3 + j7009.9 = 13129.3 + j6575.3 = 14{,}683.8\angle26^\circ V$



(b) $P = 10\times0.85 = 8.5MW \quad n_s = \frac{120\times60}{2} = 3600\,rpm$

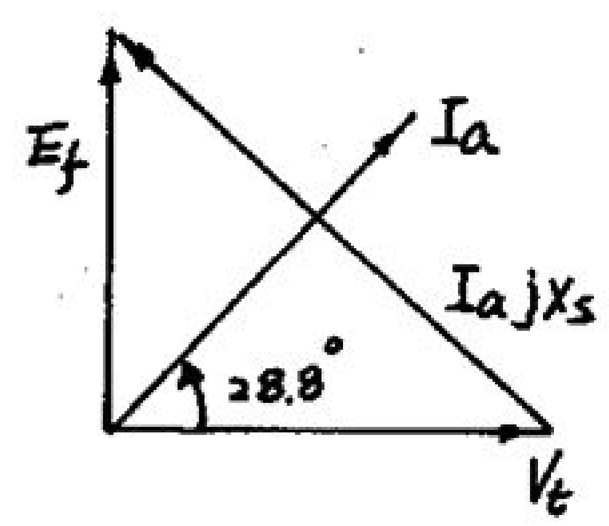
$\omega_s = \frac{3600}{60}\times2\pi = 377.0\,rad/sec. \quad T = \frac{8.5\times10^6}{377} = 22.55\,KN \quad \delta = 26.6^\circ$

(c) $P_{max} = \frac{3\times8083\times14683}{20} = 817.8MW$

(d) $I_a = \frac{14683\angle90^\circ - 8083\angle0^\circ}{20\angle90^\circ} = \frac{j14683 - 8083}{j20} = \frac{16760.83\angle118.8^\circ}{20\angle90^\circ}$

$= 838.04\angle28.8^\circ A$

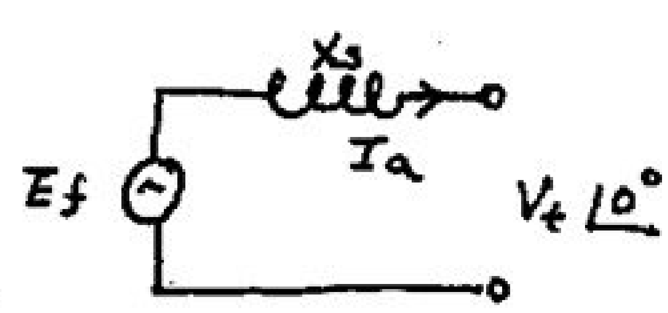
$PF = \cos 28.8^\circ = 0.876$ (leading)



6.12 (a) $10{,}000 = \sqrt{3}\times208\times I_a\times0.8$

$I_a = \frac{10000}{\sqrt{3}\times208\times0.8} = 34.7A$

$\theta = 36.87^\circ$



$E_f = V_t + I_a jX_s$

$= 120\angle0^\circ + 34.7\angle-36.87^\circ \cdot 1.5\angle90^\circ$

$= 156.86\angle+15.4^\circ \quad V$

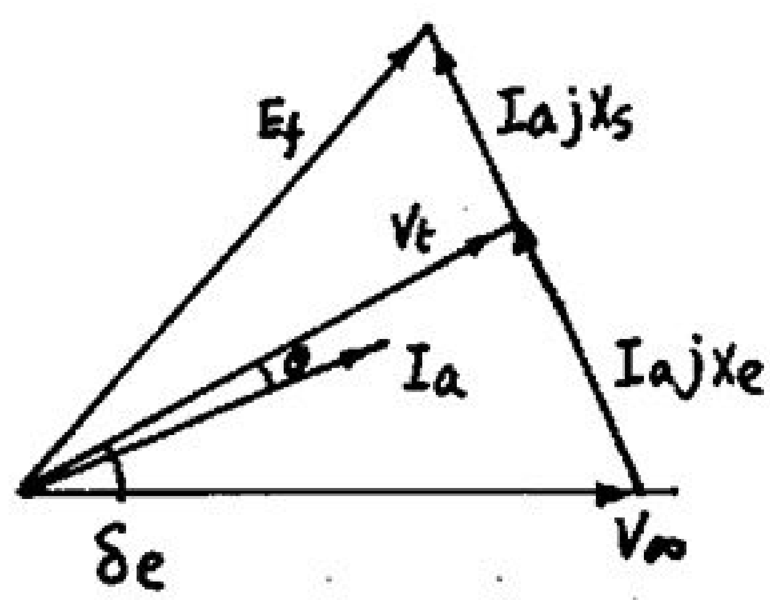
(b) $I_a = \dfrac{10{,}000}{\sqrt{3}\times 208} = 27.76\ A$

$E_f = 120\angle 0^\circ + 27.76\angle 0^\circ \cdot 1.5\angle 90^\circ$

$= 127\angle 19.1^\circ$

$\dfrac{I_f(b)}{I_f(a)} = \dfrac{127}{156.86}\times 100 = 80.96\ \%$

$\boxed{6.13}$ (a)



$V_t = V_\infty + I_a jX_e \longrightarrow$ Take V_∞ as ref. $\rightarrow V_t$ leads V_∞

$E_f = V_t + I_a jX_s = V_\infty + I_a j(X_s + X_e) \longrightarrow E_f$ leads V_t

(b) $P = 0.8\ p.u. = \dfrac{V_\infty V_t}{X_e}\sin\delta_e = \dfrac{1\times 1}{0.25}\sin\delta_e \quad \delta_e = 11.537^\circ$

Alli $\quad PF = \cos\theta = \cos\frac{\delta}{2} = 0.9949$ (lag)

$I_a = \dfrac{P}{V_t\cos\theta} = \dfrac{0.8}{1\times 0.9949} = 0.8041\ P.u.$

(c) $E_f = V_\infty + I_a j(X_s + X_e) = 1 + 0.8041\angle 5.7685^\circ \times 1.75\angle 90^\circ = 1.6424\angle 58.5^\circ$

$\boxed{6.14}$ (a) $P_{max} = \dfrac{V_t V_\infty}{X_e}\sin 90^\circ = \dfrac{1\times 1}{0.25} = 4\ P.U = 4\times 500 = 2000\ MW$

(b) $P_{max} = \dfrac{E_f V_\infty}{X_s + X_e}\sin 90^\circ = \dfrac{1\times 1}{1.5+0.25} = 0.5714\ p.u. = 0.5714\times 500 = 285.714\ MW$

$\boxed{6.15}$ (a) $1800 = \dfrac{1200\times 60}{P} \longrightarrow P = 4$

(b) $V_b = \dfrac{6600}{\sqrt{3}} = 3810.6\ V/ph \quad I_b = 1500A \quad Z_b = \dfrac{3810.6}{1500} = 2.54\Omega$

$X_s = 0.95\times 2.54 = 2.4134\Omega \quad R_a = 0.012\times 2.54 = 0.0305\ \Omega$

(c) (i) $T = \dfrac{2000\times 746}{1800\times 2\pi/60} = 79{,}152.87\ N.m$

(ii) $P_{in} = \sqrt{3}\times 6600\times 1350\times 1 = 15.432\ MW$

$P_{out} = 20000\times 746 = 14.92\ MW \quad Eff = \dfrac{14.92}{15.32} = 96.68\%$

(iii) $P_{loss} = 15.432 - 14.92 = 512\,KW$ $\quad P_{rot} = 512\times10^3 - 3\times1350^2\times0.0305 = 345.24\,KW$

(iv) $P_f = 120\times5.5 = 660\,W$

(v) $E_f = 1\angle0° - (0.015)\times1.0\angle0° - (j0.95)\times1\angle0° = 1.368$ pu.

$= 1.368 \times 3810.6 = 5212.9\ V/ph$

$\boxed{6.16}$ (a) $V_{base} = \dfrac{2300}{\sqrt{3}} = 1327.95\ V/ph$ $\quad I_{base} = \dfrac{1\times10^6}{\sqrt{3}\times2300} = 251.03A$

$Z_{base} = \dfrac{1327.95}{251.03} = 5.29\,\Omega$ $\quad X_s = \dfrac{5.03}{5.29} = 0.95$ p.u.

(b) Power from supply

$P_{in} = \dfrac{500\times746\times10^{-6}}{0.95} = 0.3926\ MW = 0.3926$ p.u.

$I_a = \dfrac{0.3926}{1\times0.85} = 0.4619$ p.u. $\quad \theta = \cos^{-1}0.85 = 31.8°$

$E_f = 1 - 0.462\angle-31.8°\times0.95\angle90° = 1 - 0.439\angle58.2° = 1 - 0.2313 - j0.3731$

$= 0.7687 - j0.3731 = 0.8545\angle-25.9°$ p.u.

(c) New excitation $E_f' = 0.6E_f$

Same power transfer $\quad E_f\sin\delta = E_f'\sin\delta'$

$\sin\delta' = \dfrac{E_f'}{E_f}\sin\delta = 0.6\sin25.9°$ $\quad \delta' = 15.2°$

$I_a = \dfrac{E_f' - V_t}{jX_s} = \dfrac{0.6\times0.8548\angle15.2° - 1\angle0°}{0.95\angle90°} = \dfrac{0.5129 - 1\angle0°}{0.95\angle90°} = 0.551\angle75.1°$ p.u

P.F. $= \cos75.1° = 0.257$ $\quad P_{max} = \dfrac{1\times0.5129}{0.95} = 0.5399$ p.u. $> P_{in}\ (= 0.3926)$

Synchronism will not be lost due to reduction of I_f

$\boxed{6.17}$ (a) E_f lags V_t, hence the machine is operating as a motor

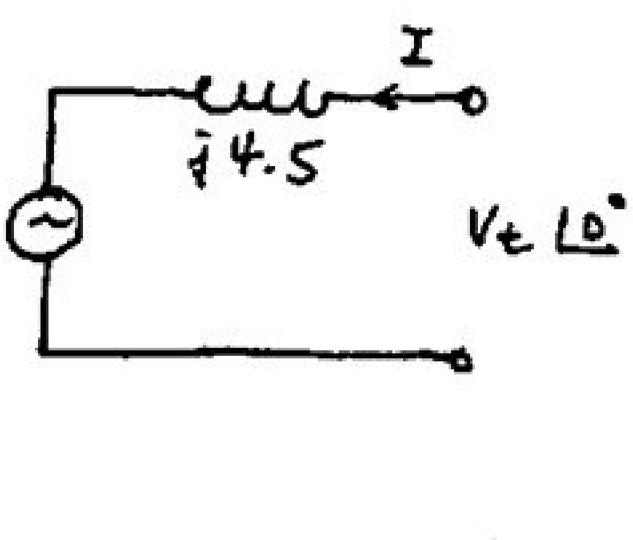
(b) $V_t = 1\angle0°$, $E_f = \dfrac{3450}{2300}\angle-20° = 1.5\angle-20°$ $\quad I_a = \dfrac{1\angle0° - 1.5\angle-20°}{0.9\angle90°} = \dfrac{0.6567\angle128.6°}{0.9\angle90°} = 0.73\angle38.6°$

PF $= \cos38.6° = 0.7815$ (leading)

$P = 1\times0.73\times0.7815 = 0.5705$ p.u. $= 5.705$ MW

$\boxed{6.18}$ (a) $\quad I = 250\angle-36.87°$ given

$P_{out} = P_{in} \rightarrow$ no losses



$P_{in} = 3VI\cos\theta = 3\,\dfrac{2300}{\sqrt{3}}\times250\times0.8$

$= 796.8$ kw

(b) $\bar{E}_f = V_t\angle 0^\circ - jI\,4.5 = \frac{2300}{\sqrt{3}} - (250)(4.5)\angle -36.87+90$

$= 1112\angle -54^\circ$

$$P = \frac{3|E_f||V_t|}{X_S}\sin\delta \qquad \delta = -90^\circ\Big|_{max\ power}$$

$$P_{max} = \frac{3(1112)(1328)}{4.5} = 984.5\ kW$$

$$T = \frac{P_{max}}{\omega_m} = \frac{984.5}{2\pi 60/12/2} = 15.67\ kN$$

$$\bar{I} = \frac{\vec{V}_t - E_f\angle\delta}{jX_s} = \frac{1328 - 1112\angle -90}{4.5\angle 90} = 384.9\angle -50^\circ A$$

$$Pf = \cos(-50^\circ) = 0.642\ lagging$$

$\boxed{6.19}$ (a) $V_t\Big|_{rated} = \frac{2300}{\sqrt{3}} = 1327.9\ V/ph = 1\ p.u.$

$P_{out} = 1000hp = 0.746\ MW = 0.746\ p.u.$

$P_{in} = P_{out} = 0.746\ p.u. \longrightarrow$ no losses

$0.746 = VI\cos\theta = 1 \times I_a \times 0.85$

$$I_a = \frac{0.746}{0.85} = 0.8776\ p.u.$$

$\cos\theta = 0.85 \rightarrow \theta = 31.8^\circ$

$$E_f = V_t - I_a X_s$$
$$= 1 - 0.8776\angle 31.8^\circ \cdot 0.8\angle 90^\circ$$
$$= 1.4968\angle -23.8^\circ$$

(b)

$$P_{max} = \frac{1.4968 \times 1}{0.8} = 1.871\ p.u. = T_{max}$$

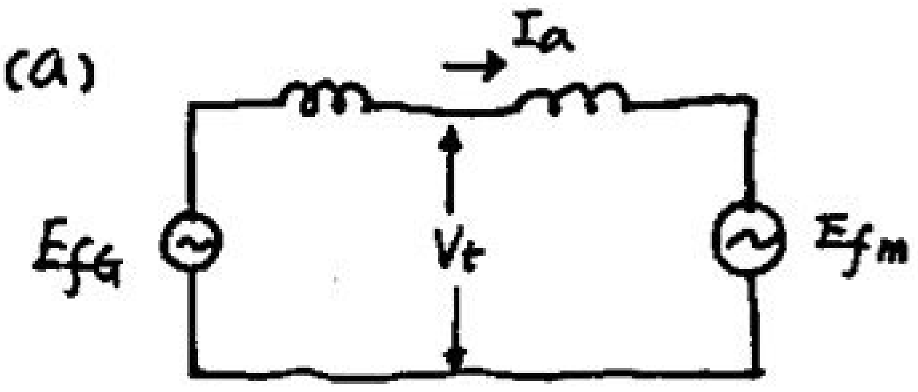
(c)

$$0.746 = \frac{1 \times E_f}{0.8}\sin 90^\circ \rightarrow E_f = 0.5968\ p.u.$$

$\frac{I_f|_{new}}{I_f|_{part(a)}}$ Reduction in $I_f \rightarrow \frac{0.5968}{1.4968} = 0.3987$

$\simeq 40\%$

6.20 (a)



$$V_b = \frac{2300}{\sqrt{3}} = 1328 \text{ V}/\phi$$

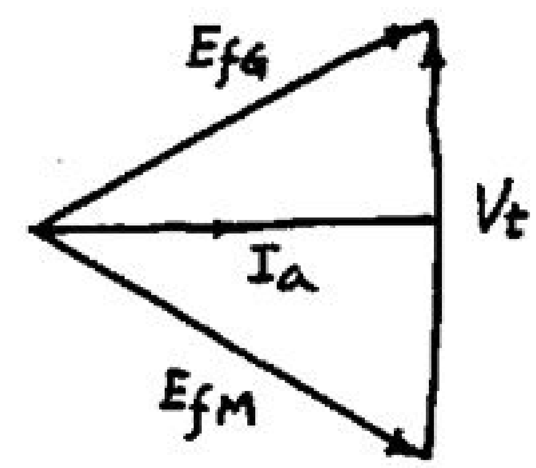
Generator: $I_b = \frac{10^6}{\sqrt{3} \times 2300} = 251\text{A}$ $\quad Z_b = \frac{1328}{251} = 5.291$ $\quad X_S = 0.9 \times 5.29 = 4.762\,\Omega$

Motor: $I_b = \frac{500 \times 10^3}{\sqrt{3} \times 2300} = 125.5\text{A}$ $\quad Z_b = \frac{1328}{125.5} = 10.582$ $\quad X_S = 0.8 \times 10.582 = 8.465\,\Omega$

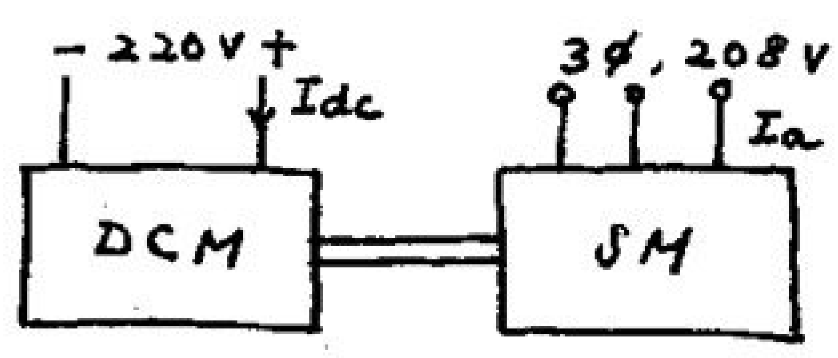
(b) $I_a = \frac{500 \times 746}{\sqrt{3} \times 2300 \times 1} = 93.6\text{A}$

(c) $E_{fG} = 1328\angle 0^\circ + 93.6\angle 0^\circ \times 4.762\angle 90^\circ = 1400.8\angle 18.55^\circ$

$E_{fM} = 1328\angle 0^\circ - 93.6\angle 0^\circ \times 8.465\angle 90^\circ = 1546.4\angle -30.8^\circ$



6.21 (a)

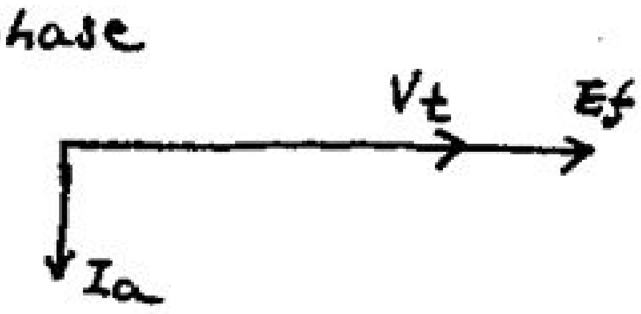


$$V_t\big|_{rated} = \frac{208}{\sqrt{3}} = 120\text{V}$$

$$E_f = 1.25 \times 120 = 150\text{V}$$

$I_{dc} = 0 \quad$ ie $\quad V_t = E_a$

$P = 0 \rightarrow \delta = 0 \quad E_f + V_t$ in phase



$$I_a = \frac{E_f\angle 0^\circ - V_t\angle 0^\circ}{X_s\angle 90^\circ}$$

$$= \frac{150 - 120}{3\angle 90}$$

$$= 10\angle -90^\circ \text{ A}$$

$PF = \cos(-90) = 0$ lagging

6.21 (b) DC machine is motor, Synchronous machine is generator

DC machine $\rightarrow E_a < V_t \rightarrow$ excitation of dc is to be reduced

$$I_{dc} = \frac{8000}{220} = 36.36\ A$$

Syn. machine

$$8000 = \frac{3 \times 120 \times 150}{3} \sin\delta$$

$$\delta = 26.4^\circ$$

$$I_a = \frac{150\angle 26.4 - 120\angle 0}{3\angle 90} = 22.74\angle -12.14^\circ$$

$$PF = \cos(-12.14) = 0.9776 \text{ lagging}$$

(c) DC machine is generator

Syn. machine is motor

$E_a > V_t \rightarrow$ excitation of dc machine is to be reduced

$I_{dc} = 36.36\ A$ into the bus

Syn. machine

$$8000 = \frac{3 \times 120 \times 150}{3} \sin\delta$$

$$|\delta| = 26.4^\circ$$

$$I_a = \frac{120\angle 0^\circ - 150\angle -26.4}{3\angle 90^\circ} = 22.74\angle 12.14$$

$$PF = \cos 12.14 = 0.9776 \text{ leading}$$

$\boxed{6.22}$ (a) Rated KVA $= \sqrt{3} \times 4.6 \times 62.75 = 500$ KVA

(b) Capability Curve

Radius for armature heating is S rating

$S = |V_t||I_a| = 1 \times 1 = 1$ p.u.

$i_f = 7.5\ A$ produces $E_f = 1.0$ p.u.

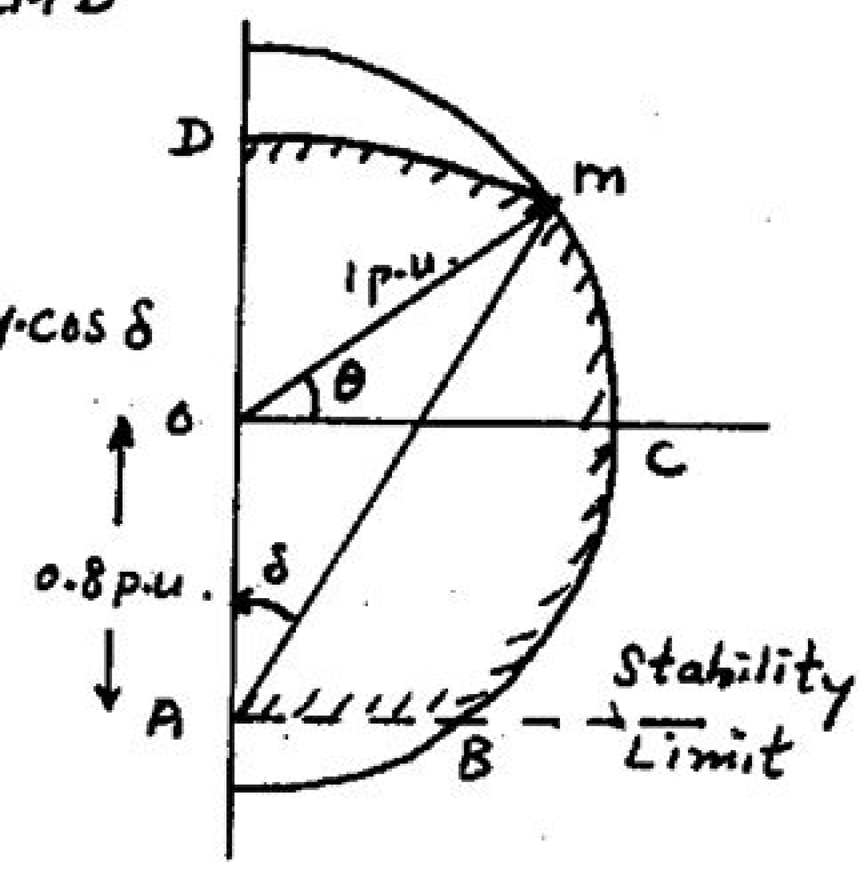
6.22(b) continued

$\therefore$ rated i_f of 15A will produce $E_f = 2$ p.u.
(at constant V_t)

$$\therefore \quad \frac{|V_t||E_f|}{X_s} = \frac{1 \times 2}{1.25} = 1.6 \text{ p.u}$$ – gives the radius for field heating

$$\frac{|V_t|^2}{X_s} = \frac{1}{1.25} = 0.8 \text{ p.u.}$$

The capability curve is ABCMD



(c) from OMA

$$OM^2 = OA^2 + AM^2 - 2 \cdot OA \cdot AM \cdot \cos\delta$$

$$\cos\delta = \frac{0.8^2 + 1.6^2 - 1^2}{2 \times 0.8 \times 1.6}$$

$$= 0.859$$

$$\delta = 31^\circ$$

$$1 \times \cos\theta = 1.6 \sin\delta$$

$$= 0.818$$

$$\theta = 35^\circ$$

$\boxed{6.23}$ (a)

$$V_t = 1 \angle 0^\circ \text{ p.u.}$$

$$I_a = 0.8 \angle -25.84^\circ \text{ p.u.}$$

$$\tan\delta = \frac{I_a X_q \cos\phi}{V_t + I_a X_q \sin\phi}$$

$$= \frac{0.8 \times 0.7 \times 0.9}{1 + 0.8 \times 0.7 \times 0.4936}$$

$$= 0.405$$

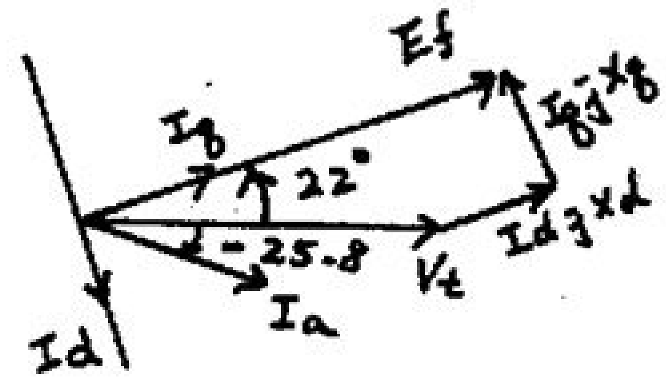
$$\delta = 22.052^\circ$$

$$\psi = 25.84 + 22.052 = 47.89$$

$$I_d = 0.8 \cdot \sin 47.89 = 0.5935$$

6.23 (a) Continued

$I_q = 0.8 \cos 47.89 = 0.5364$



$E_f = V_t \cos\delta + I_d X_d$

$= 1 \cdot \cos 22.05 + 0.5935 \times 1$

$= 1.5204$ p.u.

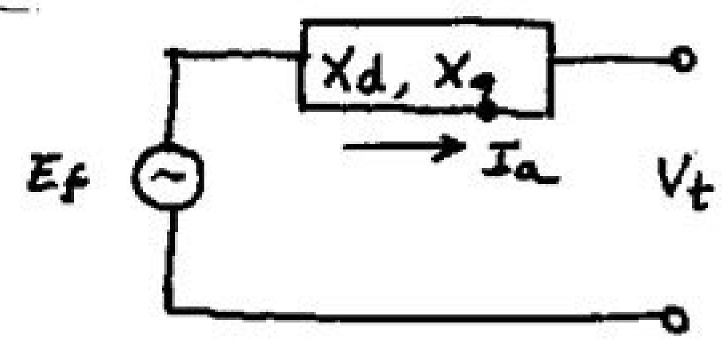
(b) $E_f = 0 \qquad 2\delta = 90°$

$P_{max} = \frac{1^2(1-0.7)}{2 \times 1 \times 0.7} = \frac{0.3}{1.4} = 0.2143$ p.u.

$= 21.43$ MW

$0 = V_t + I_d jX_d + I_q jX_q$

$V_t = -I_d jX_d - I_q jX_d$



For phasor diagram V_t is reference, q-axis is at +45°. I_q is along q-axis. I_d is along positive d-axis to satisfy the voltage equation.

$I_d X_d = I_q X_q = V_t \cos 45° = 1/\sqrt{2}$

$I_d = \frac{1}{1 \times \sqrt{2}} = 0.707$ p.u.

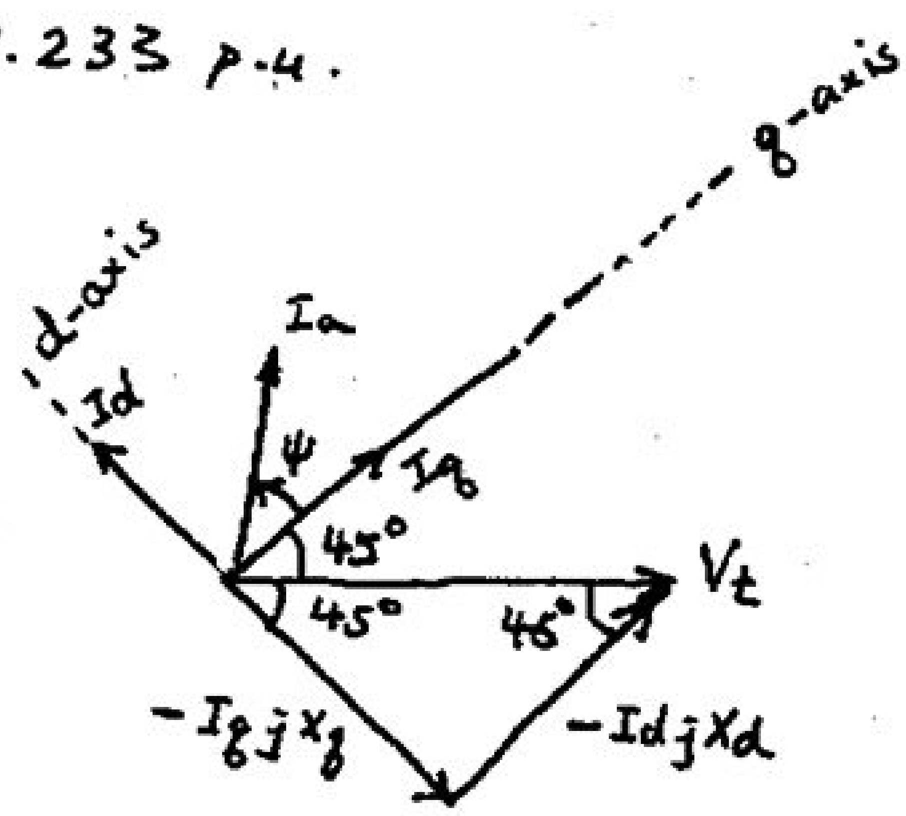
$I_q = \frac{1}{0.7 \times \sqrt{2}} = 1.01$ p.u.

$I_a = \sqrt{1.01^2 + 0.707^2} = 1.233$ p.u.

$\psi = \tan^{-1}\frac{0.707}{1.01} = 35°$

$\phi = 45° + 35° = 80°$

PF $= \cos 80 = 0.1736$



$\boxed{6.24}$ (a) $P = 1 = \frac{V_t E_f}{0.95} \sin 25° + \frac{V_t^2}{2}\left(\frac{1}{0.45} - \frac{1}{0.95}\right) \sin 50°$

$Q = 0 = \frac{V_t E_f}{0.95} \cos 25° - V_t^2 \left(\frac{\sin^2 25°}{0.45} + \frac{\cos^2 25°}{0.95}\right)$

Solving for E_f & V_t

$V_t = 0.9823$ pu. $E_f = 1.299$ pu.

(Or From phasor diagram, $V_t \sin\delta = I_q X_q$ $V_t = \frac{0.9226 \times 0.45}{\sin 25°} = 0.9824$ P.U.

$E_f = V_t \cos\delta + I_d X_d = 0.9824 \cos 25° + 0.4302 \times 0.95 = 1.299$ P.U.)

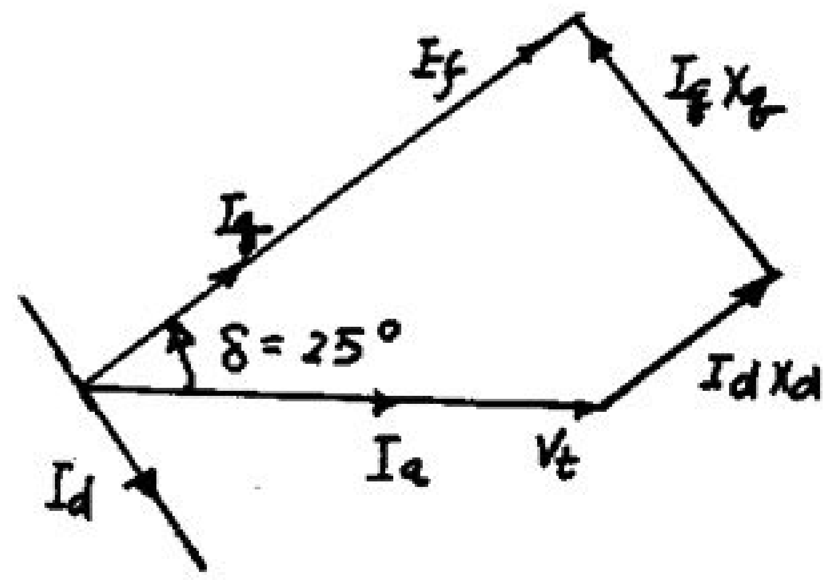
(b) $P = V_t I_a \cos\theta$

$1 = 0.9823 \times I_a \times 1.0$

$I_a = 1.018$ P.U

$I_d = 1.018 \sin 25° = 0.4302$ pu

$I_q = 1.018 \cos 25 = 0.9226$ pu.



$\boxed{6.25}$ $P = \frac{|V_t||E_f|}{X_d} \sin\delta + \frac{V_t^2 (X_d - X_q)}{2 X_d X_q} \sin 2\delta$

$= \frac{1^2}{1.5} \sin\delta + \frac{1^2 (1.5 - 1.0)}{2 \times 1.5 \times 1.0} \sin 2\delta$

$= 0.667 \sin\delta + 0.1667 \sin 2\delta$

For maximum power transfer

$\frac{dP}{d\delta} = 0 = 0.667 \cos\delta + 0.3334 \cos 2\delta$

$\rightarrow \cos^2\delta + \cos\delta - 0.5 = 0.$

$\cos\delta = \frac{-1 \pm \sqrt{1 - 4 \times 1 \times 0.5}}{2} = 0.366, -1.366.$

$\cos\delta = 0.366 \rightarrow \delta = 68.53°$

$\therefore P_{max}\big|_{\delta_{max}} = 0.667 \sin 68.53 + 0.1667 \sin 2 \times 68.53 = 0.734$ pu

$= 0.734 \times 40$ MW $= 29.37$ MW.

$V_t \sin\delta = I_q X_q \rightarrow I_q = \frac{1 \times \sin 68.53}{1} = 0.931$ pu.

$V_t \cos\delta = |E_f| - |I_d X_d| \rightarrow I_d = \frac{1 - \cos 68.53}{1.5} = 0.423$ pu.

$I_a = \sqrt{0.423^2 + 0.931^2} = 1.023$ pu

$\phi = \delta - \tan^{-1}\frac{I_d}{I_q} = 68.53 - \tan^{-1}\frac{0.423}{0.931}$

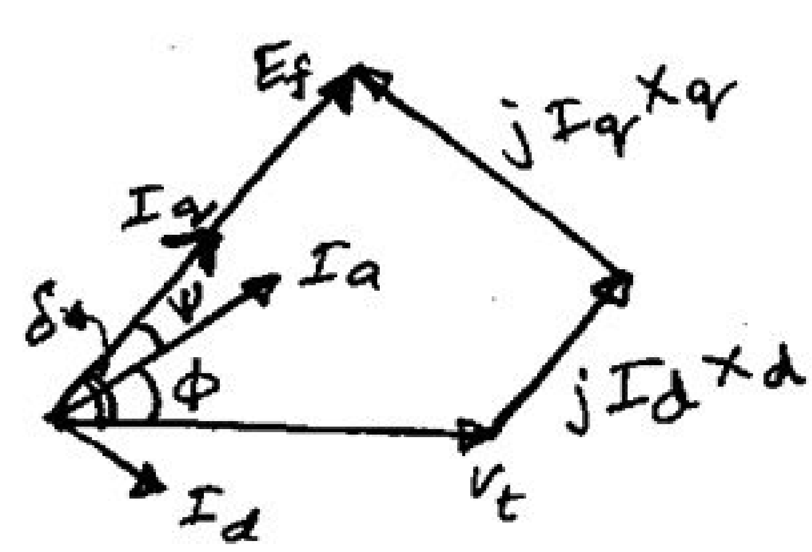
$= 44.1°$

PF $= \cos 44.1° = 0.718.$

check: $P = V_t I_a \cos\phi$

$= 1 \times 1.023 \times 0.718$

$= 0.7345$ pu



$\boxed{6.26}$ (a) $P = 100 \times 0.8 = 80\,MW = \frac{80}{200} = 0.4\,pu$

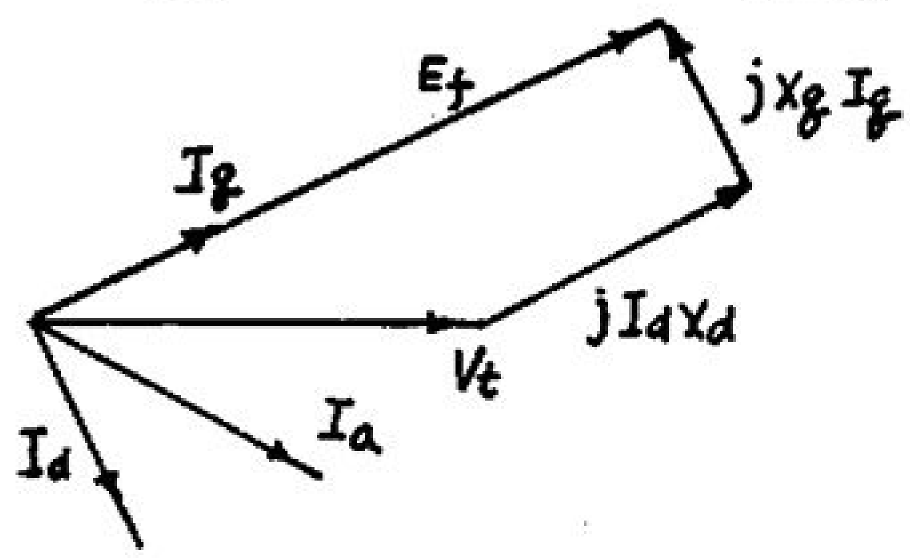
$$I_a = \frac{P}{V.PF} = \frac{0.4}{1.0 \times 0.8} = 0.5\,pu = 0.5\angle{-36.87^\circ}\,pu$$

From Fig 6.27C, $\psi = \phi + \delta$

$$Tan\,\delta = \frac{0.5 \times 0.85 \times 0.8}{1 + 0.5 \times 0.85 \times \sin 36.87} = 0.2709 \qquad \delta = 15.16^\circ$$

$\psi = 15.16 + 36.87 = 52.03$ $\quad I_d = 0.5 \times \sin 52.03 = 0.3942$ $\quad I_q = 0.5 \times \cos 52.03 = 0.3076$

$E_f = V_t \cos\delta + I_d X_d = 1 \times \cos 15.16 + 0.3942 \times 1.45 = 1.5368\,pu$



(b) $P = \frac{1 \times E_f}{1.45}\sin\delta + \frac{1^2(1.45 - 0.85)}{2 \times 1.45 \times 0.85}\sin 2\delta = 0.6897 E_f \sin\delta + 0.2434 \sin 2\delta$

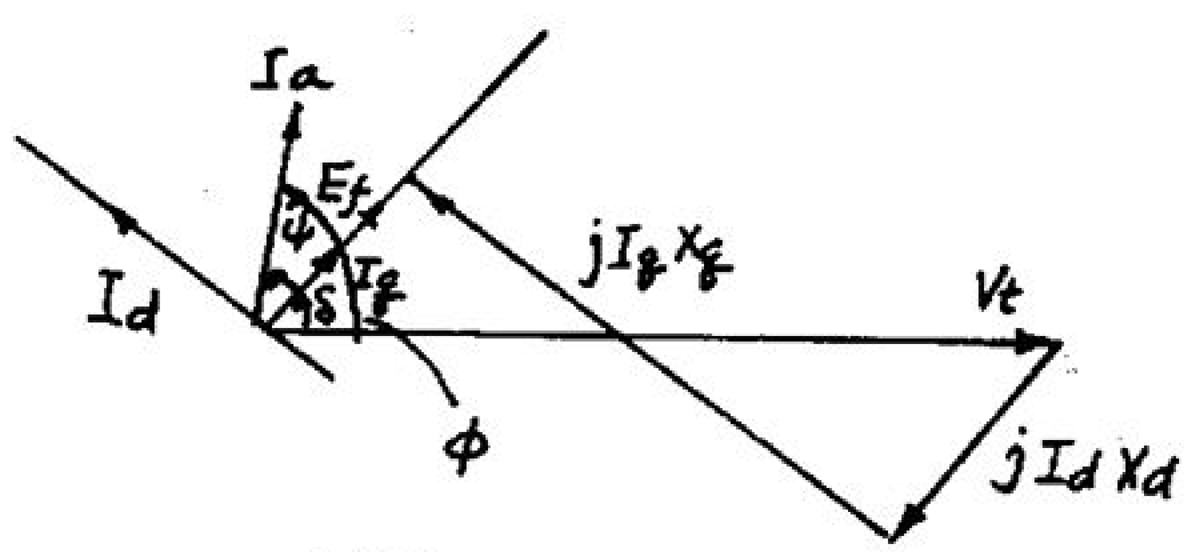
At static stability limit

$\frac{dp}{d\delta} = 0 \rightarrow 0.6897 E_f \cos\delta + 0.4868 \cos 2\delta \rightarrow 0.6897 E_f = -0.4868 \frac{\cos 2\delta}{\cos\delta}$

From ① & ②

$P = P_{max} = 0.4 = -0.4868 \frac{\cos 2\delta}{\cos\delta}\sin\delta + 0.2434 \sin 2\delta$

$0.4 = -0.4868 \cos 2\delta \tan\delta + 0.2434 \sin 2\delta$ By trial & error $\delta = 53^\circ$



$$E_f = -\frac{0.4868}{0.6897} \times \frac{\cos 2 \times 53^\circ}{\cos 53^\circ} = -\frac{0.2756}{0.6018} = 0.3232\,pu$$

$I_q X_q = V_t \sin\delta = 1 \times \sin 53^\circ = 0.7986\,pu$

$I_q = \frac{0.7986}{0.85} = 0.9396\,pu$

$I_d X_d = V_t \cos\delta - E_f = 1 \times \cos 53^\circ - 0.3232 = 0.2786$

$I_d = \frac{0.2786}{1.45} = 0.1921\,pu$ $\quad I_a = \sqrt{0.1921^2 + 0.9396^2} = 0.959\,pu$

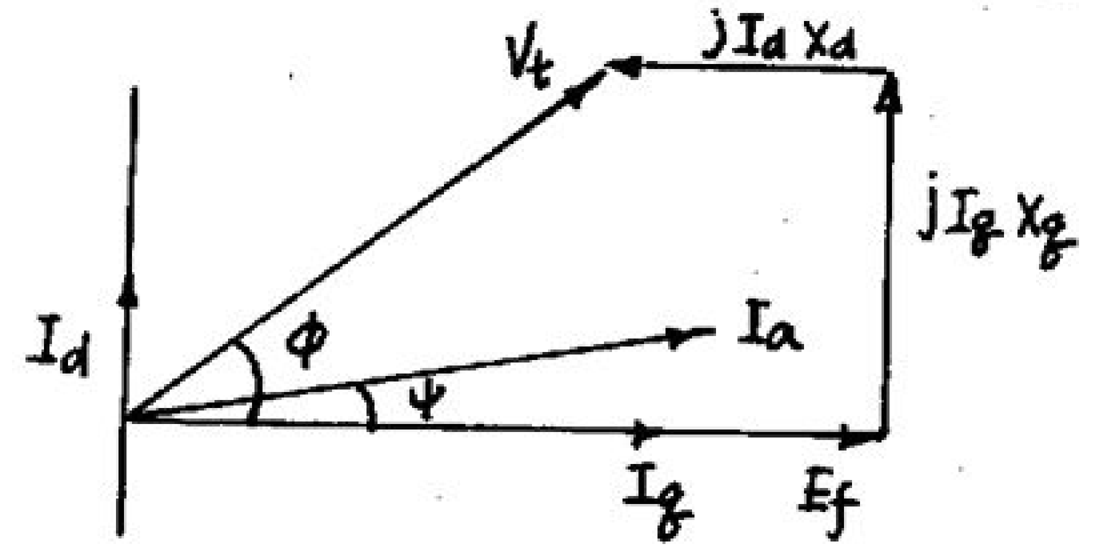
$\psi = \tan^{-1}\frac{0.1921}{0.9396} = 11.555^\circ$ $\quad \phi = 53^\circ + 11.555^\circ \rightarrow P.F. = \cos 64.555^\circ = 0.4296$ (leading)

$\boxed{6.28}$ (a) $V_t = 1\angle 0°\ p.u.$ $I_a = \frac{P}{V \times PF} = \frac{0.72}{1 \times 0.9} = 0.8\angle -25.84°\ pu$

From phasor diagram in Fig. 6.27d $\psi = \phi - \delta$

$\tan\delta = \frac{0.8 \times 0.7 \times 0.9}{1 - 0.8 \times 0.7 \times 0.4359} = 0.6668 \qquad \delta = 33.69°$

Phasor I_a is between phasors V_t & E_f. The phasor diagram is drawn



$\psi = 33.69 - 25.84 = 7.85°$ $I_q = 0.8\cos 7.85° = 0.7925A$ $I_d = 0.8\sin 7.85° = 0.1093A$

$E_f = V_t\cos\delta + I_d X_d$ (from phasor diagram)

$= 1.0 \times \cos 33.69° + 0.1093 \times 1.0 = 0.9414\ p.u.$

CHECK $P = \frac{1 \times 0.9414}{1.0}\sin 33.69° + \frac{1^2(1-0.7)}{2 \times 1 \times 0.7}\sin 2 \times 33.69°$

$= 0.5222 + 0.1978 = 0.72\ P.U. \longrightarrow 72\ MW$

(b) $E_f = 0$, $2\delta = 90°$ $P_{max} = \frac{1^2(1-0.7)}{2 \times 1 \times 0.7}\sin 90° = 0.2143\ pu \rightarrow 21.43\,MW$

$V_t = E_f + jI_d X_d + jI_q X_q = jI_d X_d + jI_q X_q$

$|I_d X_d| = |I_q X_q| = V_t \cos 45° = \frac{1}{\sqrt{2}}$

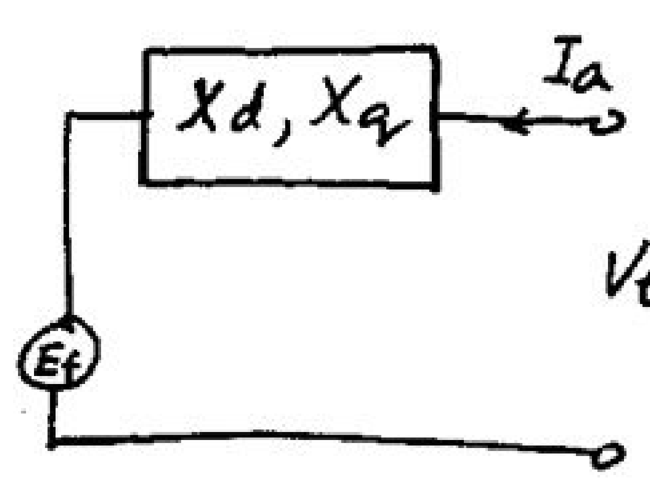
$I_d = \frac{1}{1 \times \sqrt{2}} = 0.7072$ $I_q = \frac{1}{0.7 \times \sqrt{2}} = 1.0103$

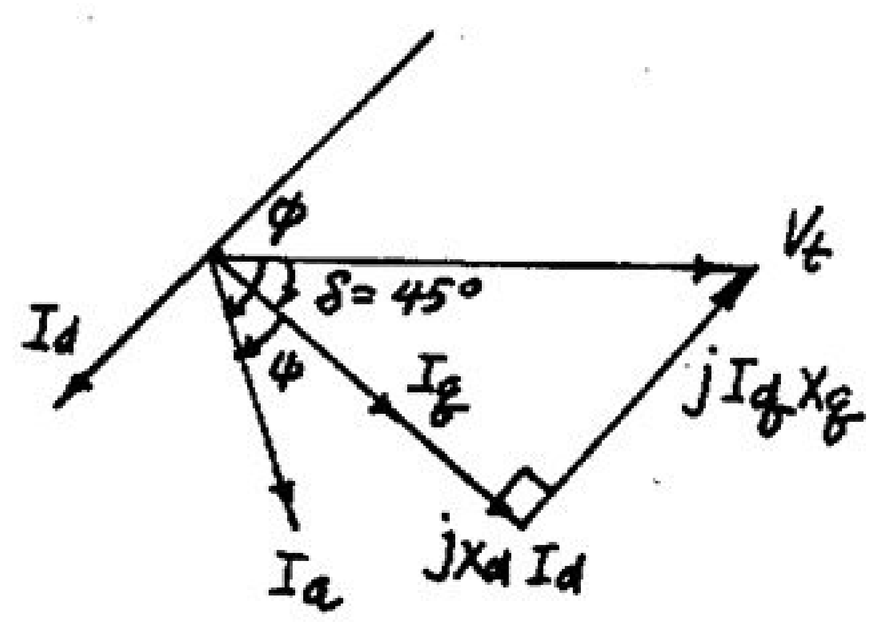
$I_a = \sqrt{0.7072^2 + 1.0103^2} = 1.2331\ P.U.$

$\psi = \tan^{-1}\frac{0.7072}{1.0103} = 34.984°$

$\phi = 45° + 34.984° = 79.984°$

P.F. $= \cos 79.984° = 0.1739$



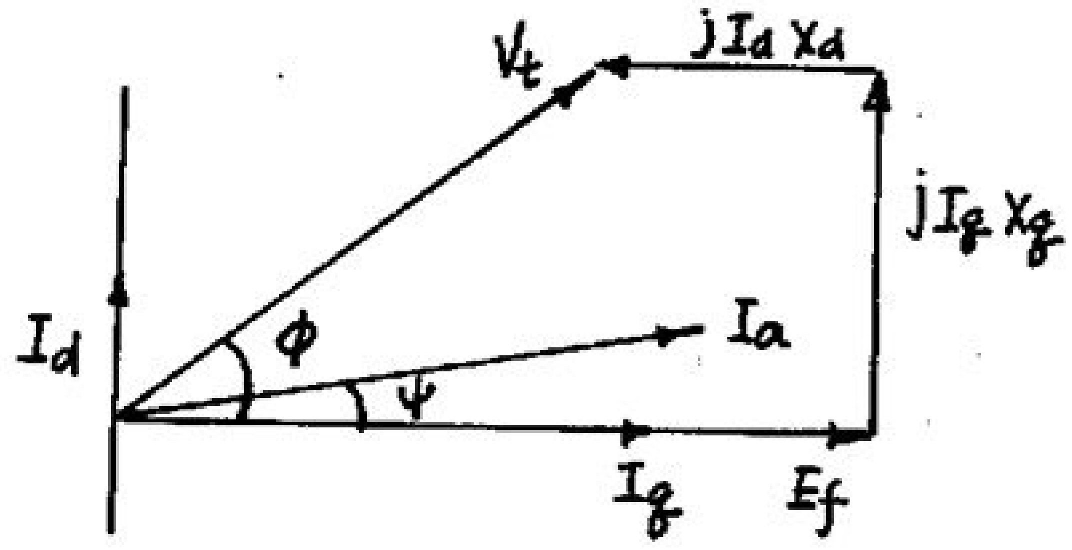


6.28 (a) $V_t = 1\angle 0°$ p.u. $I_a = \frac{P}{V \times PF} = \frac{0.72}{1 \times 0.9} = 0.8\angle -25.84°$ pu

From phasor diagram in Fig. 6.27d $\psi = \phi - \delta$

$\tan\delta = \frac{0.8 \times 0.7 \times 0.9}{1 - 0.8 \times 0.7 \times 0.4359} = 0.6668 \qquad \delta = 33.69°$

Phasor I_a is between phasors V_t & E_f. The phasordiagram is drawn



$\psi = 33.69 - 25.84 = 7.85° \quad I_q = 0.8\cos 7.85° = 0.7925A \quad I_d = 0.8\sin 7.85° = 0.1093A$

$E_f = V_t\cos\delta + I_d X_d$ (from phasor diagram)

$= 1.0 \times \cos 33.69° + 0.1093 \times 1.0 = 0.9414$ p.u.

CHECK $P = \frac{1 \times 0.9414}{1.0}\sin 33.69° + \frac{1^2(1-0.7)}{2 \times 1 \times 0.7}\sin 2 \times 33.69°$

$= 0.5222 + 0.1978 = 0.72$ P.U. $\longrightarrow$ 72 MW

(b) $E_f = 0$, $2\delta = 90°$ $P_{max} = \frac{1^2(1-0.7)}{2 \times 1 \times 0.7}\sin 90° = 0.2143$ pu $\rightarrow$ 21.43 MW

$V_t = E_f + jI_d X_d + jI_q X_q = jI_d X_d + jI_q X_q$

$|I_d X_d| = |I_q X_q| = V_t\cos 45° = \frac{1}{\sqrt{2}}$

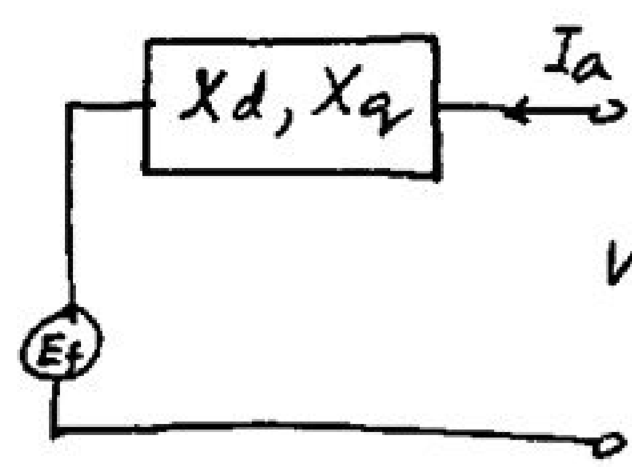
$I_d = \frac{1}{1 \times \sqrt{2}} = 0.7072 \quad I_q = \frac{1}{0.7 \times \sqrt{2}} = 1.0103$

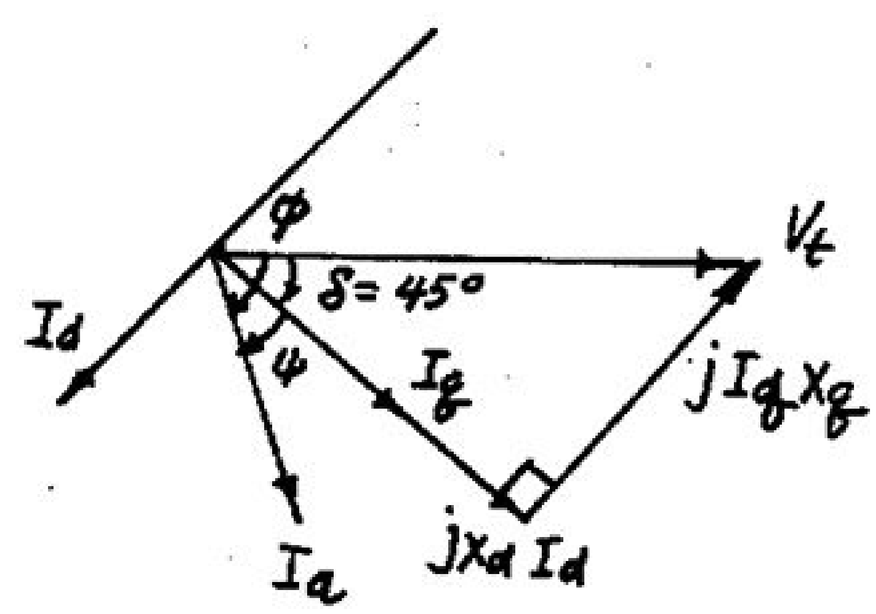
$I_a = \sqrt{0.7072^2 + 1.0103^2} = 1.2331$ P.U.

$\psi = \tan^{-1}\frac{0.7072}{1.0103} = 34.984°$

$\phi = 45° + 34.984° = 79.984°$

P.F. $= \cos 79.984° = 0.1739$





$\boxed{6.29}$ $T = \frac{P}{\omega_{syn}} = \frac{P}{1.0} = P$ in pu

$$T = \frac{|V_t||E_f|}{X_d}\sin\delta + \frac{V_t^2(X_d - X_q)}{2X_dX_q}\sin 2\delta$$

For maximum torque

$$\frac{dT}{d\delta} = \frac{|V_t||E_f|}{X_d}\cos\delta + \frac{V_t^2(X_d - X_q)}{2X_dX_q} \times 2 \times \cos 2\delta = 0$$

$$E_f\cos\delta + \frac{V_t(X_d - X_q)}{X_q}\cos 2\delta = 0$$

$$\cos\delta = -\frac{V_t(X_d - X_q)}{E_f X_q}\cos 2\delta = -\frac{1\times(0.9-0.65)}{1\times 0.65}\times\cos 2\delta = -0.3846\cos 2\delta$$

$$= -0.3846\cos(2\cos^2\delta - 1)$$

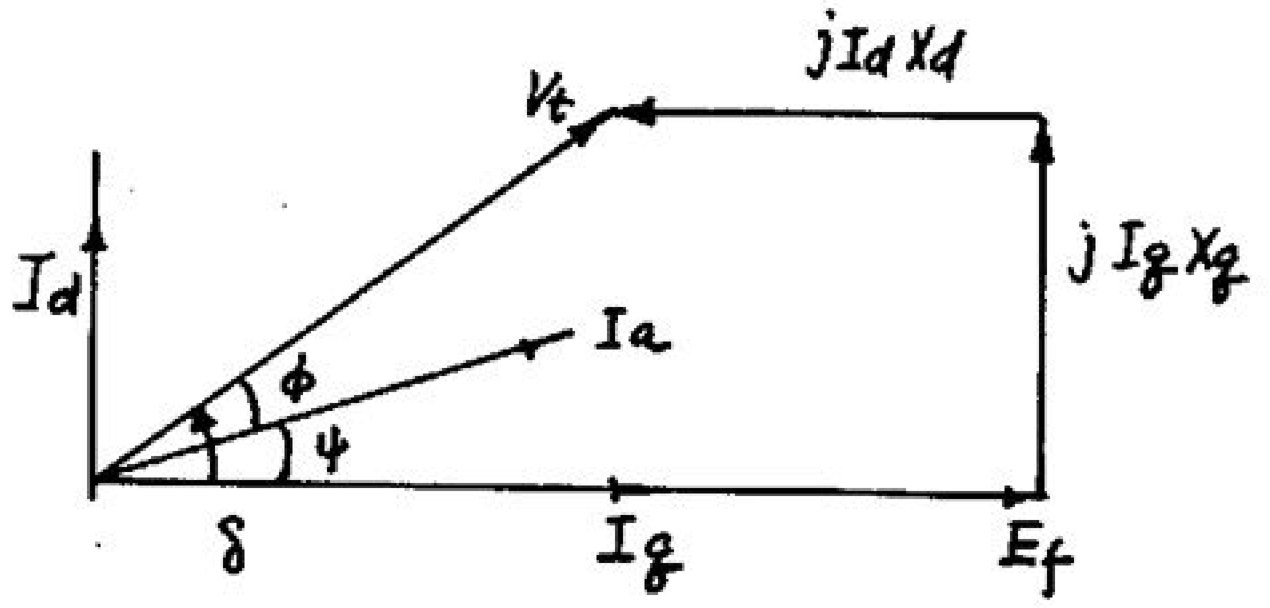
$$0.7692\cos^2\delta + \cos\delta - 0.3846 = 0$$

$$\cos\delta = \frac{-1 \pm \sqrt{1^2 - 4(0.7692)(-0.3846)}}{2\times 0.7692} = 0.3105$$

$$\cos\delta = 0.3105 \rightarrow \delta = 71.9^\circ$$

$$T_{max} = \frac{1\times 1}{0.9}\sin 71.9^\circ + \frac{1^2(0.9-0.65)}{2\times 0.9\times 0.65}\sin 2\times 71.9^\circ = 1.18 \text{ P.U.}$$

$$V_t = E_f + jI_dX_d + jI_qX_q$$



$$I_qX_q = V_t\sin\delta \qquad I_q = \frac{1\times\sin 71.9^\circ}{0.65} = 1.462 \text{ P.U.}$$

$$I_dX_d = E_f - V_t\cos\delta = 1 - 1\times\cos 71.9^\circ \qquad I_d = \frac{1 - 1\times\cos 71.9^\circ}{0.9} = 0.766$$

$$I_a = \sqrt{0.766^2 + 1.462^2} = 1.65 \qquad \psi = \tan^{-1}\frac{0.766}{1.462} = 27.65^\circ$$

$$\phi = \delta - \psi = 71.9 - 27.65 = 44.25^\circ \qquad \text{P.F.} = \cos 44.25^\circ = 0.716.$$

$\boxed{6.30}$ (a) $1 = \frac{|V_t||E_f|}{X_s} = \frac{1\times E_f}{0.9}$

$$E_f = 0.9 \text{ p.u.}$$

(b) $P = \frac{1\times E_f}{0.9}\sin\delta + \frac{1^2(0.9-0.6)}{2\times 0.9\times 0.6}\sin 2\delta$

$$= 1.11\,E_f\sin\delta + 0.2778\sin 2\delta$$

$$\frac{dP}{d\delta} = 1.11\, E_f \cos\delta + 0.5556 \cos 2\delta = 0$$

Given $P_{max} = P_{rated} = 1.0\ p.u.$

$$\therefore 1 = -\frac{0.5556 \cos 2\delta \sin\delta}{\cos\delta} + 0.2778 \sin 2\delta$$

$$\Rightarrow 1 = -0.5556 \cos 2\delta \tan\delta + 0.2778 \sin 2\delta$$

$\delta = 65.5^\circ \leftarrow$ trial + error

$$\therefore |E_f| = -\frac{0.5556}{1.11}\,\frac{\cos(2\times 65.5^\circ)}{\cos(65.5^\circ)} \simeq 0.79\ p.u.$$

$$\therefore E_f = 0.79 \angle 65.5^\circ$$

$\boxed{6.31}$ (a) $f = \frac{np}{120} = \frac{300\times 4}{120} = 10\,Hz = \frac{1800\times 4}{120} = 60\,Hz$

(b) $X_S = 2\pi\times 60\times 3.85\times 10^{-3} = 1.4514\,\Omega$ $\quad I_a = \frac{125\times 746}{\sqrt{3}\times 480\times 0.85} = 131.96\,A$

$\phi = \cos^{-1} 0.85 = 31.79^\circ$

$E_f = \frac{480}{\sqrt{3}} \angle 0^\circ - 131.96 \angle 31.79^\circ \times 1.4514 \angle 90^\circ = 277.14 \angle 0^\circ - 191 \angle 121.79^\circ$

$= 277.14 - 100.62 - j\,162.3471 = 176.52 - j\,162.3471$

$= 239.8247 \angle -42.6^\circ$

(c) (i) $P_{max}\big|_{1800\,rpm} = \frac{3\times 277.14\times 239.8247}{1.4514} = 137.381\,KW$

(ii) $P_{max}\big|_{300\,rpm} = \frac{3\times\left(277.14\times\frac{300}{1800}\right)\times\left(239.8247\times\frac{300}{1800}\right)}{1.4514\times\frac{300}{1800}}$

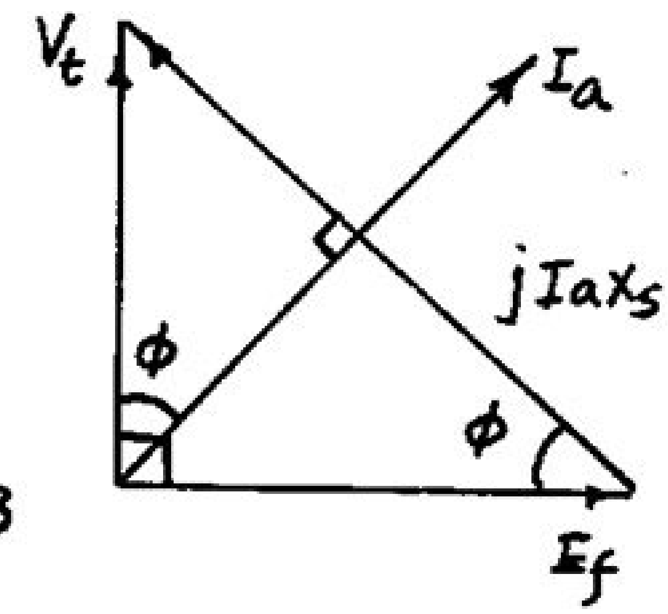
$= P_{max}\big|_{1800} \times \frac{300}{1800} = 22.897\,KW$

$\boxed{6.32}$ (a) $n_b = \frac{120f}{P} = \frac{120\times 60}{4} = 1800\ rpm$ $\quad \frac{V_t}{f} = \frac{208/\sqrt{3}}{60} = \frac{120}{60} = 2\,V/Hz$

(b) $T_{max} = \frac{3K_1 P}{8\pi^2 L_S}\left(\frac{V_t}{f}\right) = \frac{3\times 1.9\times 4\times 2}{8\pi^2\times 20\times 10^{-3}} = 28.88\ N.m.$

(c) $\delta = 90°$ and V_t leads E_f, $X_s = 377 \times 20 \times 10^{-3} = 7.54\Omega$

From the phasor diagram,



$$I_a = \frac{\sqrt{120^2 + 114^2}}{7.54} = 21.9519A$$

$$P.F. = \cos\phi = \frac{E_f}{I_a X_S} = \frac{114}{21.9519 \times 7.54}$$

$$= 0.6887 \text{ leading}$$

$$\text{Power} = \frac{3E_f V_t}{X_s} \sin 90° = \frac{3 \times 114 \times 120}{7.54} = 5443$$

or $\text{Power} = T\omega = 28.88 \times \frac{1800}{60} \times 2\pi = 5443W$

$\boxed{6.33}$ (a) $V_s = \dfrac{500 \times 1000}{60 \times 60} = 138.9 \ m/sec.$

$$f = \frac{V_s}{2\tau_p} = \frac{138.9}{2 \times 50 \times 10^{-2}} = 138.9 \ Hz$$

(b) $T = \text{Thrust} = \dfrac{\text{Power}}{V_s} = \dfrac{5.00 \times 10^6}{138.9} = 35997 \ N = 36 \ KN$

$\boxed{6.34}$

The rotor has four poles. Therefore, the inductance variation for a phase will repeat every 90° (= 360° ÷ 4). Since $\theta_s = \theta_r = 40°$, the inductance will increase for 40° duration and decrease for 40° duration.

There are six stator poles and three stator phases. Each phase is thus excited at an interval of 60°. The inductance profile and current & torque variations will be same for each phase, except that these are shifted by 60° with respect to each other.

The inductance profile of phase-a winding is shown in Fig. P6.34A.

(a) The waveforms of current and torque produced by different phases and the total torque $T = T_a + T_b + T_c$ are shown in Fig. P6.34 a. The resultant torque has considerable ripple. (40° width)

(b) Waveforms for 30° width are shown in Fig P6.34b. Torque ripple is eliminated for this current width. However, average value of the resultant torque is reduced compared to 40° width of current.

(C) Waveforms for 20° width are shown in Fig P6.34C. For this current width, there are zero torque intervals. Average value of the resultant torque is further reduced.

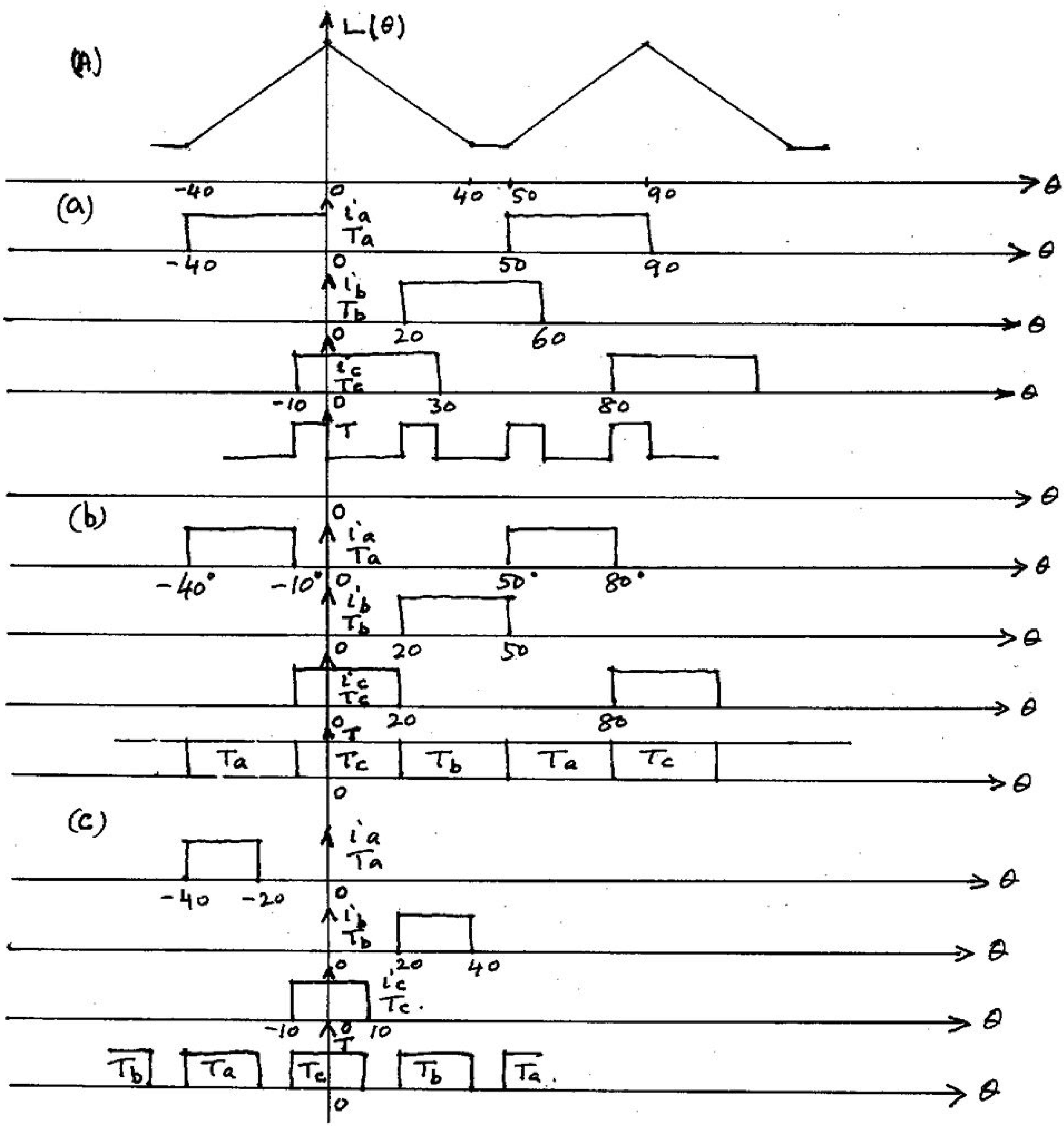


Fig P6.34

3Φ SRM, 6/4 Poles, $\theta_s = \theta_r = 40°$.

(a) 40° current width

(b) 30° current width

(c) 20° current width

(A) Inductance profile of phase-a

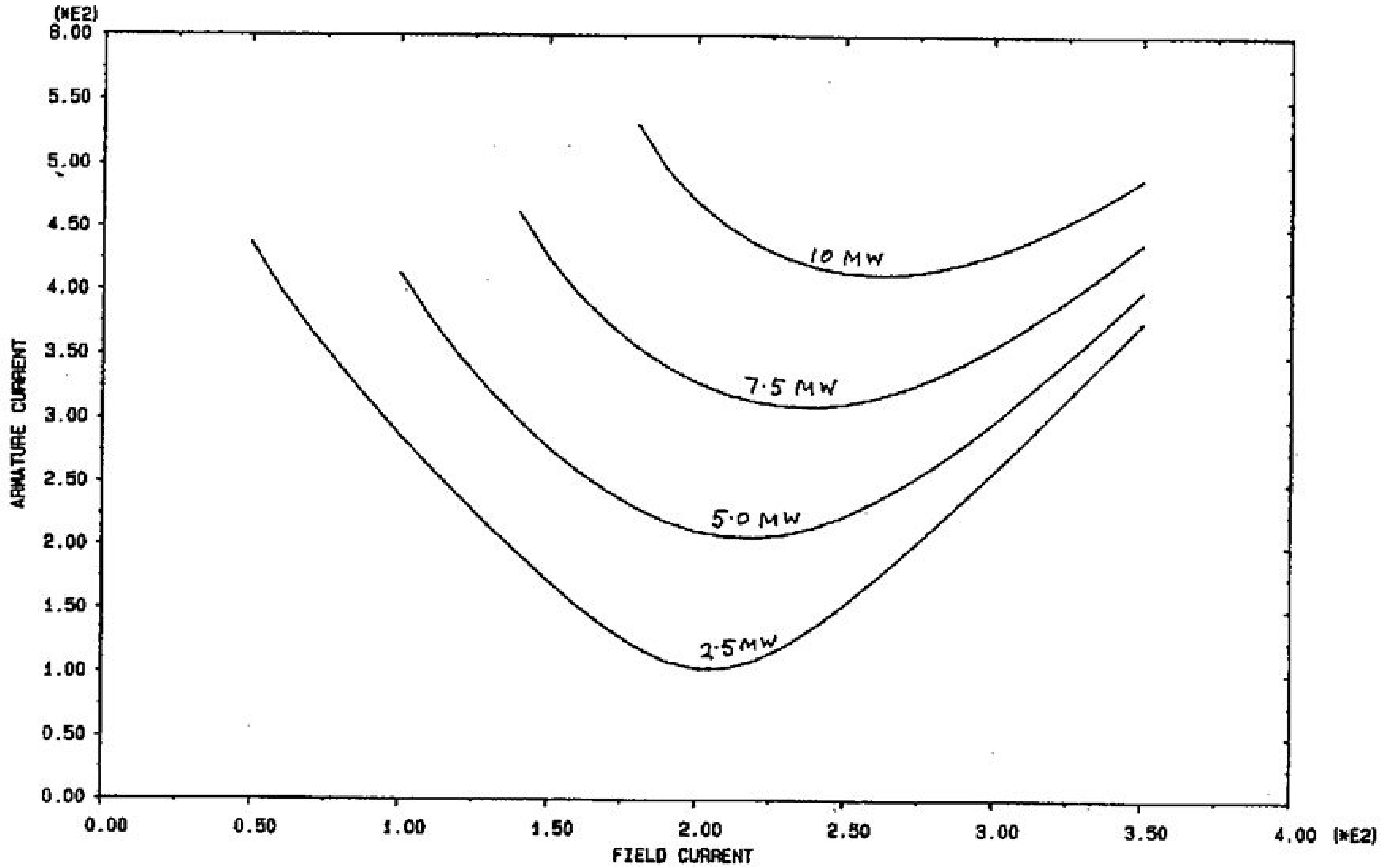
6.35

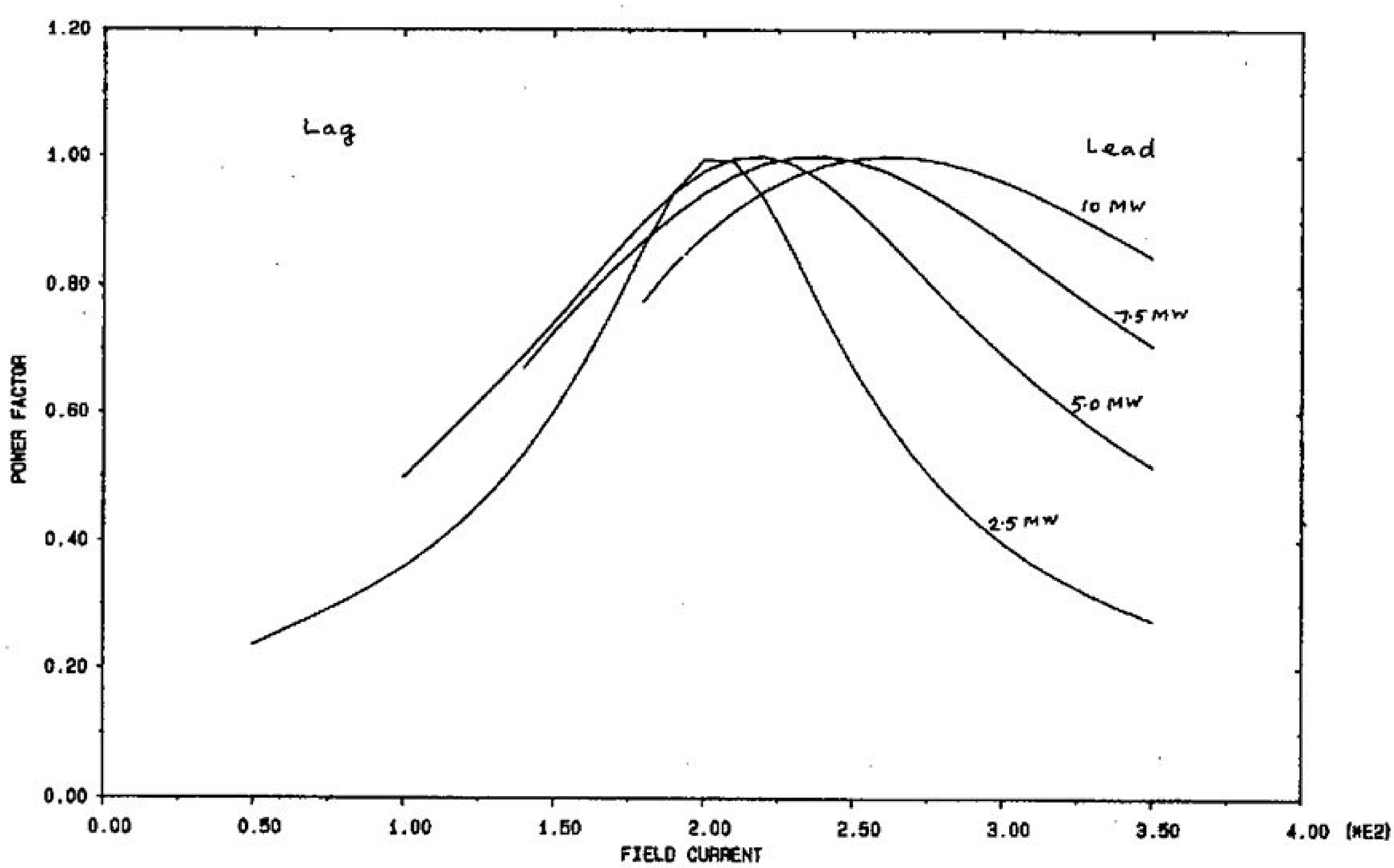
(a)

P(MW)	I_f(A)	1.(A)	PF
2.5	50.0	436.1	0.236 LAG
	60.0	398.7	0.258 LAG
	70.0	367.6	0.280 LAG
	80.0	339.2	0.303 LAG
	90.0	312.5	0.329 LAG
	100.0	286.8	0.359 LAG
	110.0	262.0	0.393 LAG
	120.0	238.1	0.433 LAG
	130.0	214.9	0.479 LAG
	140.0	192.7	0.535 LAG
	150.0	171.6	0.600 LAG
	160.0	152.0	0.678 LAG
	170.0	134.5	0.766 LAG
	180.0	119.9	0.859 LAG
	190.0	109.2	0.943 LAG
	200.0	103.6	0.994 LAG
	210.0	104.0	0.990 LEAD
	220.0	110.2	0.935 LEAD
	230.0	121.3	0.849 LEAD
	240.0	136.1	0.757 LEAD
	250.0	153.5	0.671 LEAD
	260.0	172.7	0.596 LEAD
	270.0	193.1	0.533 LEAD
	280.0	214.5	0.480 LEAD
	290.0	236.5	0.435 LEAD
	300.0	259.0	0.398 LEAD
	310.0	281.9	0.365 LEAD
	320.0	305.0	0.338 LEAD
	330.0	328.4	0.313 LEAD
	340.0	351.9	0.292 LEAD
	350.0	375.6	0.274 LEAD
5.0	100.0	412.8	0.499 LAG
	110.0	377.7	0.545 LAG
	120.0	348.0	0.592 LAG
	130.0	321.9	0.640 LAG
	140.0	298.5	0.690 LAG
	150.0	277.5	0.742 LAG
	160.0	258.9	0.796 LAG
	170.0	242.7	0.849 LAG
	180.0	229.2	0.899 LAG
	190.0	218.6	0.943 LAG
	200.0	211.1	0.976 LAG
	210.0	207.0	0.995 LAG
	220.0	206.3	0.999 LEAD
	230.0	209.0	0.986 LEAD
	240.0	214.9	0.959 LEAD
	250.0	223.6	0.921 LEAD
	260.0	234.9	0.877 LEAD
	270.0	248.3	0.830 LEAD

P(MW)	I_f(A)	1.(A)	PF
	280.0	263.5	0.782 LEAD
	290.0	280.2	0.735 LEAD
	300.0	298.1	0.691 LEAD
	310.0	317.0	0.650 LEAD
	320.0	336.8	0.612 LEAD
	330.0	357.1	0.577 LEAD
	340.0	378.1	0.545 LEAD
	350.0	399.5	0.516 LEAD
7.5	140.0	460.9	0.671 LAG
	150.0	425.0	0.727 LAG
	160.0	397.1	0.778 LAG
	170.0	374.4	0.826 LAG
	180.0	355.7	0.869 LAG
	190.0	340.3	0.908 LAG
	200.0	328.2	0.942 LAG
	210.0	319.1	0.969 LAG
	220.0	313.0	0.987 LAG
	230.0	309.8	0.998 LAG
	240.0	309.4	0.999 LEAD
	250.0	311.6	0.992 LEAD
	260.0	316.4	0.977 LEAD
	270.0	323.4	0.956 LEAD
	280.0	332.5	0.930 LEAD
	290.0	343.5	0.900 LEAD
	300.0	356.1	0.868 LEAD
	310.0	370.2	0.835 LEAD
	320.0	385.6	0.802 LEAD
	330.0	402.0	0.769 LEAD
	340.0	419.4	0.737 LEAD
	350.0	437.6	0.706 LEAD
10.0	180.0	530.6	0.777 LAG
	190.0	495.2	0.832 LAG
	200.0	470.0	0.877 LAG
	210.0	450.9	0.914 LAG
	220.0	436.4	0.944 LAG
	230.0	425.8	0.968 LAG
	240.0	418.4	0.985 LAG
	250.0	414.0	0.996 LAG
	260.0	412.4	0.999 LAG
	270.0	413.2	0.997 LEAD
	280.0	416.4	0.990 LEAD
	290.0	421.7	0.977 LEAD
	300.0	429.0	0.961 LEAD
	310.0	438.0	0.941 LEAD
	320.0	448.6	0.919 LEAD
	330.0	460.6	0.895 LEAD
	340.0	473.9	0.870 LEAD
	350.0	488.4	0.844 LEAD

(b)


(*E2)
6.00
5.50
5.00
4.50
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.50
0.00
ARMATURE CURRENT
10 MW
7·5 MW
5·0 MW
2·5 MW
0.00
0.50
1.00
1.50
2.00
2.50
3.00
3.50
4.00 (*E2)
FIELD CURRENT


1.20
1.00
0.80
0.60
0.40
0.20
0.00
POWER FACTOR
Lag
Lead
10 MW
7·5 MW
5·0 MW
2·5 MW
0.00
0.50
1.00
1.50
2.00
2.50
3.00
3.50
4.00 (*E2)
FIELD CURRENT

CHAPTER 7

7.1 a) Block Rotor Test ($V = 52V$, $I = 8.0A$, $P = 255W$)

$P = 8^2 (R_1 + R_2') = 64 (2 + R_2') = 255$

$\therefore \; R_2' = 1.98 \,\Omega$

$|Z| = \frac{52}{8} = 6.5 \,\Omega = \sqrt{(2 + 1.98)^2 + (x_1 + x_2')^2}$

$\Rightarrow \; x_1 + x_2' = 5.139 \,\Omega \qquad x_1 = x_2' = 2.57 \,\Omega$

No Load Test

$P = I^2 R_{NL}$

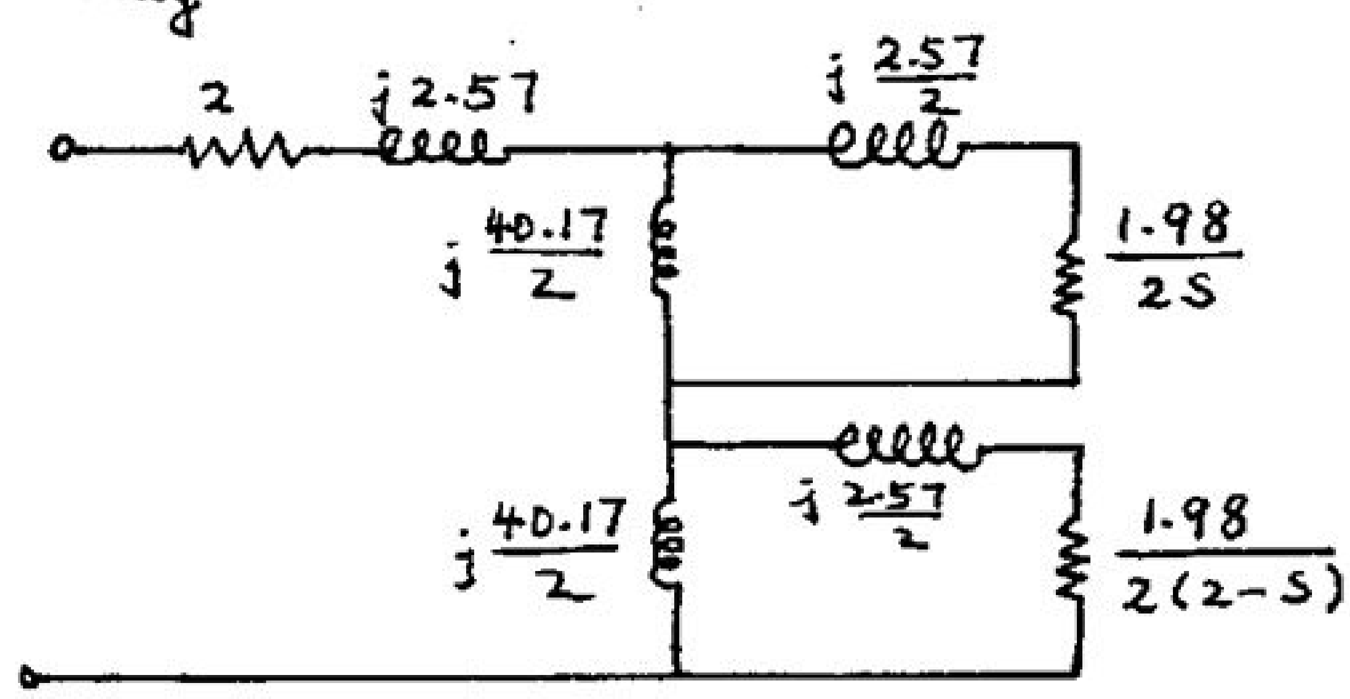
$100 = 4.5^2 R_{NL} \; \Rightarrow \; R_{NL} = 4.938 \,\Omega$

$Z_{NL} = \frac{110}{4.5} = 24.\dot{4}$

$24.44^2 = R_{NL}^2 + \left(\frac{X_{mag}}{2} + x_1 + \frac{x_2'}{2} \right)^2$

$= 4.938^2 + \left(\frac{X_{mag}}{2} + 2.57 + 1.285 \right)^2$

$\Rightarrow \; X_{mag} = 40.17 \,\Omega$



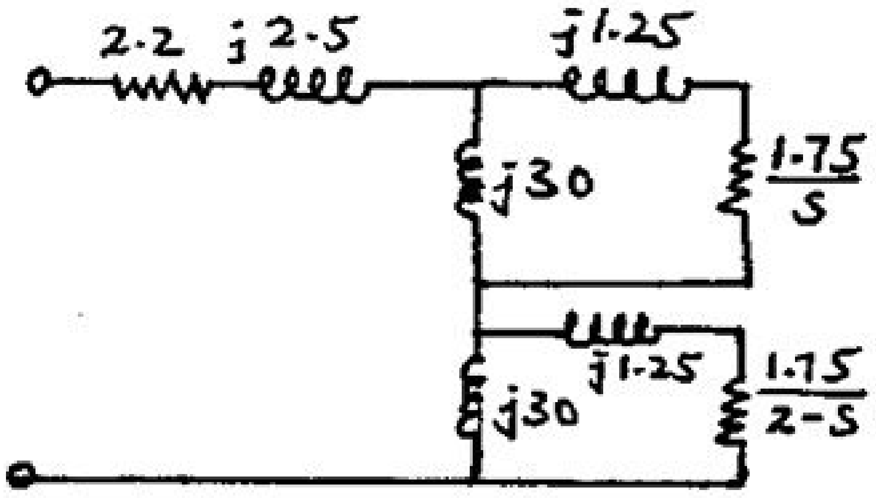
(b) $P_{rot} = 100 - 4.5^2 \left(2 + \frac{1.98}{2}\right)$
$= 39.45 \, W$

7.2 Speed

$S = 0.04$

$\therefore \; n = (1 - 0.04)\, 1800 = 1728 \text{ rpm}$

7.2 continued



Input Current

$$Z_f = R_f + jX_f$$

$$= \frac{j30(1.75/0.04 + j1.25)}{1.75/0.04 + j(30+1.25)}$$

$$= 13.62 + j\,20.27 \ \Omega$$

$$Z_b = R_b + jX_b$$

$$= \frac{j30\,(1.75/(2-0.04) + j1.25)}{1.75/(2-0.04) + j(30+1.25)}$$

$$= 0.8222 + j1.2335 \ \Omega$$

$$Z_{in} = 2.2 + j2.5 + R_f + jX_f + R_b + jX_b$$

$$= (2.2 + 13.62 + 0.8222) + j(2.5 + 20.27 + 1.2335)$$

$$= 29.21 \angle 55.26 \ \Omega$$

$$I_{in} = \frac{115}{29.21\angle 55.26} = 3.937 \angle -55.26^\circ \ A$$

Power factor

$$PF = \cos 55.26 = 0.5736 \text{ lagging}$$

Input Power

$$P_{in} = VI\cos\theta$$

$$= 115 \times 3.937 \times 0.5736$$

$$= 260\,W$$

Developed Torque

$$\omega_{syn} = \frac{1800}{60} \times 2\pi = 188.5 \text{ rad./sec.}$$

$$T = \frac{I^2}{\omega_{syn}}(R_f - R_b)$$

7.2 continued

$$= \frac{3.937^2}{188.5}(13.62 - 0.8222)$$

$$= 1.0523 \text{ N-m}$$

<u>Output Power</u>

$$P_{mech} = T\, \omega_{syn}(1 - 0.04)$$

$$= 1.0523(188.5)(1 - 0.04)$$

$$= 190.42 \text{ W}$$

$$P_{out} = P_{mech} - P_{rot}$$

$$= 190.42 - (20 + 15)$$

$$= 155.42 \text{ W}$$

<u>Efficiency</u>

$$Eff = \frac{P_{out}}{P_{in}} = \frac{155.42}{260} = 0.5977 \simeq 60\%$$

<u>Rotor cu-loss</u>

$$P_{gf} = I^2 R_f = 3.937^2(13.62) = 211.1 \text{ W}$$

$$P_{gb} = I^2 R_b = 3.937^2(0.8222) = 12.74 \text{ W}$$

$$P_2 = s\, P_{gf} + (2 - s)\, P_{gb}$$

$$= 0.04 \times 211.1 + (2 - 0.04) \times 12.74$$

$$= 33.42 \text{ W}$$

$\boxed{7.3}$ At rated speed

$$Z_f = 13 + j\,16.79$$

$$Z_b = 0.61 + j\,1.55$$

$$\frac{\phi_f}{\phi_b} = \frac{E_{mf}}{E_{mb}} = \frac{I_m Z_f}{I_m Z_b} = \frac{Z_f}{Z_b} = \frac{\sqrt{13^2 + 16.79^2}}{\sqrt{0.61^2 + 1.55^2}}$$

$$= \frac{21.235}{1.6657} = 12.7481$$

$\boxed{7.4}$

$$\phi_m = \tan^{-1}\frac{4.8}{2.8} = 59.74^\circ$$

$$\phi_a \text{ (required)} = 59.74^\circ - 90 = -30.26^\circ$$

$$-30.26 = \tan^{-1}\left(\frac{6 - X_c}{8}\right)$$

$$\Rightarrow X_c = 10.67\,\Omega$$

$$C = \frac{10^6}{377 \times 10.67}\,\mu F = 248.6\,\mu F$$

$$\frac{N_a}{N_m} = \frac{I_m}{I_a} = \frac{V/Z_m}{V/Z_{a(total)}} = \frac{Z_{a\,(total)}}{Z_m}$$

$$= \frac{|8 + j6 - j\,10.67|}{|2.8 + j\,4.8|} = \frac{9.263}{5.557}$$

$$= 1.67$$

$\boxed{75}$ (a) Running condition → only main winding in circuit

standstill test → $V = 41.0V$, $I = 5.8A$, $W = 115W$

$P = 5.8^2(R_1 + R_2') = 5.8^2(1.85 + R_2') = 115W \rightarrow R_2' = 1.57\Omega$

$Z_n = \frac{41}{5.8} = 7.07 = \sqrt{(1.85+1.57)^2 + (X_1 + X_2')^2}$

$\rightarrow X_1 + X_2' = 6.19\Omega \rightarrow X_1 = X_2 = 3.1\Omega$

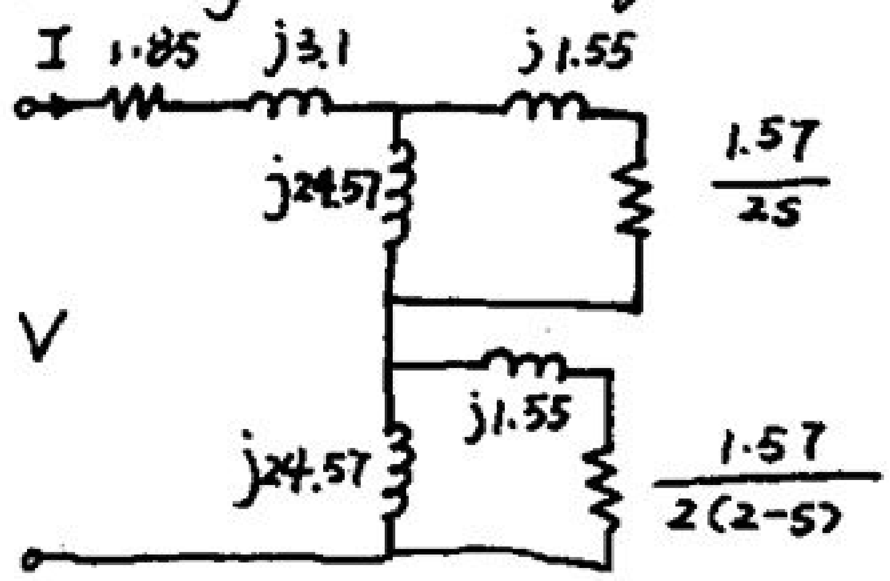
No load test → $V = 120V$, $I = 4.0A$, $W = 110W$

$110 = 4^2 R_{NL} \rightarrow R_{NL} = 6.875\Omega$

$Z_{NL} = \frac{120}{4} = 30\Omega$

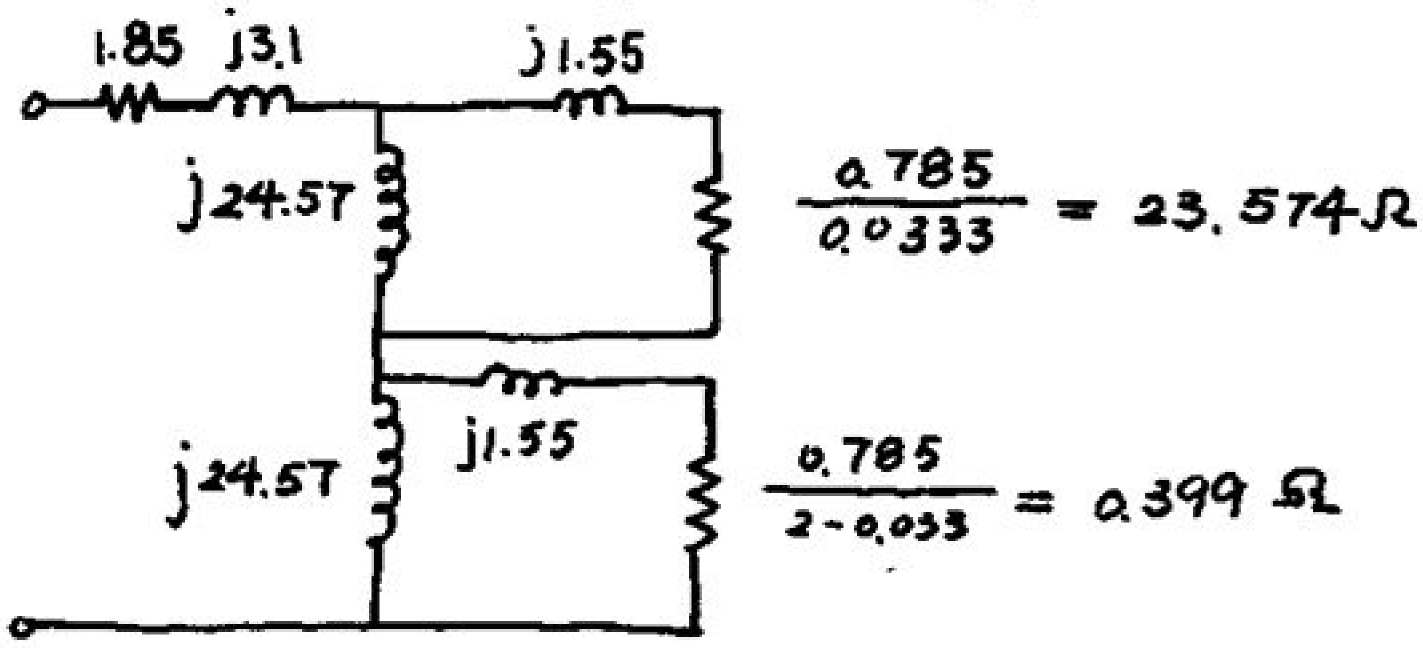
$30^2 = R_{NL}^2 + \left(\frac{X_{mag}}{2} + 3.1 + \frac{3.1}{2}\right)^2 \rightarrow X_{mag} = 49.13\Omega$

Running condition equivalent circuit



(b) $P_{rot} = 110 - 4^2(1.85 + \frac{1.57}{2}) = 67.84W$

(c)



$S = \frac{1800-1740}{1800} = 0.0333$

$Z_f = R_f + jX_f = \frac{j24.57\,(23.574 + j1.55)}{23.574 + j(24.57+1.55)} = \frac{29.57\angle 90^\circ \times 23.625\angle 3.76^\circ}{35.185\angle 47.93^\circ}$

$= 16.4975\angle 45.83^\circ = 11.5 + j11.83$

$Z_b = \frac{24.57\angle 90^\circ\,(0.399 + j1.55)}{0.399 + j26.12} = \frac{24.57\angle 90^\circ \times 1.6\angle 75.56^\circ}{26.123\angle 89.125^\circ}$

$= 1.505\angle 76.435^\circ = 0.353 + j1.463$

$Z_{in} = (R_1 + R_f + R_b) + j(X_1 + X_f + X_b) = 1.85 + 11.5 + 0.353 + j(3.1 + 11.83 + 1.463)$

$= 13.703 + j16.393 = 21.366\angle 50.11^\circ$

$$I_{in} = \frac{120}{21.366} \angle -50.11^\circ = 5.62 \angle -50.11^\circ$$

$$P_{in} = VI\cos\theta = 120 \times 5.62 \cos 50.11^\circ = 432.5\,W \qquad W_{syn} = \frac{1800}{60} \times 2\pi = 188.5\ rad/sec.$$

$$T = \frac{I^2}{\omega_{syn}} (R_f - R_b) = \frac{5.62^2}{188.5} (11.5 - 0.353) = 1.868\ N.m$$

$$P_{mech} = 1.868 \times 188.5 (1 - 0.0333) = 340.4\ W$$

$$P_{out} = 340.4 - P_{rot} = 340.4 - 67.84 = 272.56\,W$$

$$\eta = \frac{272.56}{432.5} \times 100\% = 63\%$$

$\boxed{7.6}$ (a) From tests

$$Z_m = \frac{41}{5.8} = 7.07\,\Omega, \quad R_m = \frac{115}{5.8^2} = 3.42\,\Omega, \quad X_m = \sqrt{7.07^2 - 3.42^2} = 6.19\,\Omega$$

$$\theta_m = \tan^{-1} \frac{X_m}{Z_m} = 61.1^\circ$$

$$Z_a = \frac{38}{7.0} = 5.43\,\Omega, \quad R_a = \frac{200}{7^2} = 4.08\,\Omega$$

$$X_a = \sqrt{5.43^2 - 4.08^2} = 3.58 \qquad \theta_a = \tan^{-1} \frac{X_a}{R_a} = 41.27^\circ$$

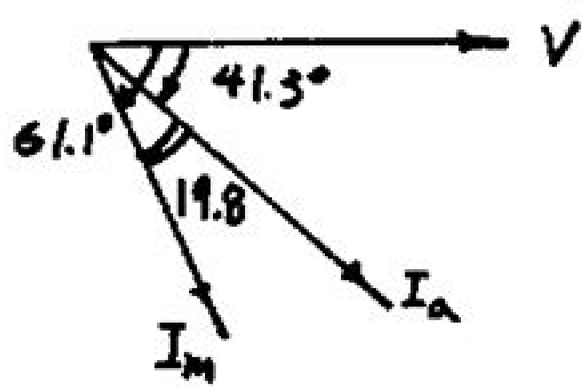
$$Z_m = 3.42 + j6.19 = 7.07 \angle 61.1^\circ \qquad Z_a = 4.08 + j3.58 = 5.43 \angle 41.3^\circ$$

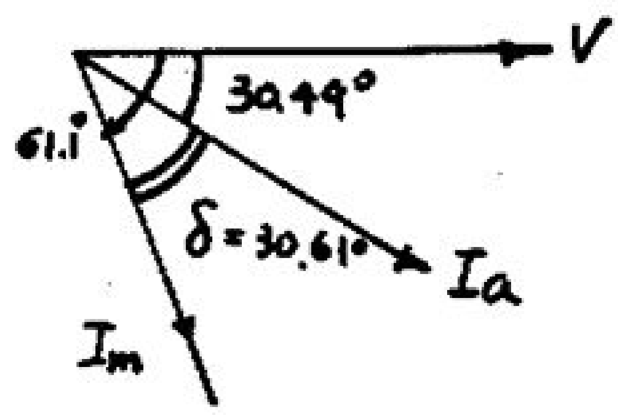
(b) (i) Without R_s

$$I_m = \frac{120}{7.07 \angle 61.1^\circ} = 16.97 \angle -61.1^\circ A \qquad I_a = \frac{120}{5.43 \angle 41.3^\circ} = 22.14 \angle -41.3^\circ A$$

With R_s

$$I_a = \frac{120}{4.08 + 2.00 + j3.58} = \frac{120}{7.06} \angle -30.49^\circ = 17 \angle -30.49^\circ \quad I_m = 16.97 \angle -61.1^\circ A$$





(ii) Starting current I_{st}

Without R_s

$$I_{st} = I_m + I_a = 16.97 \angle -61.1^\circ + 22.14 \angle -41.3^\circ = 38.53 \angle -49.8^\circ$$

With R_s

$$I_{st} = 16.97 \angle -61.1^\circ + 17 \angle -30.49^\circ = 32.77 \angle -45.77^\circ$$

Starting torque T_{st}

$T_{st} \propto I_m I_a \sin\alpha$ where $\alpha = \angle \theta_a - \theta_m = K I_a \sin\alpha \rightarrow I_m$ remaining same

$$T_{st}\big|_{No\,R_s} = K\,22.14 \sin 19.8^\circ = K7.5 \qquad T_{st}\big|_{with\,R_s} = K17 \sin 30.61^\circ = K8.66$$

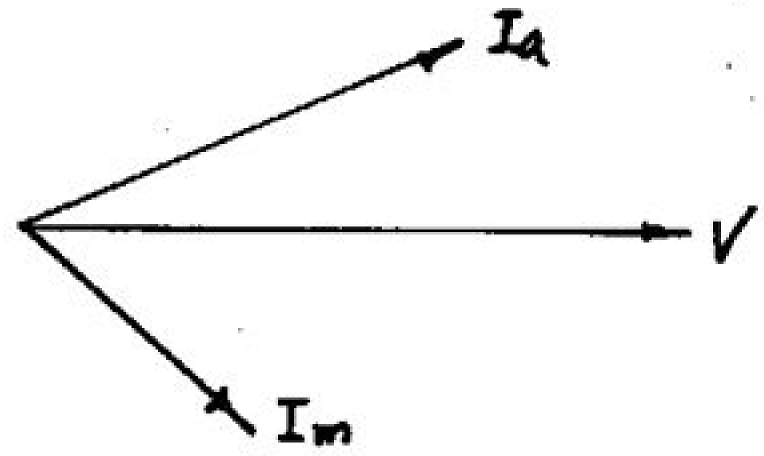
$$\frac{T_{st}|_{\text{with } R_s}}{T_{st}|_{\text{No } R_s}} = \frac{8.66}{7.5} = 1.15 \qquad \frac{I_{st}|_{\text{with } R_s}}{I_{st}|_{\text{No } R_s}} = \frac{32.77}{38.53} = 0.85$$

$\boxed{7.7}$ $X_c = \frac{1}{\omega C} = \frac{10^6}{377 \times 500} = 5.31\,\Omega$

With $C_s = 500\,\mu F$

$$I_a = \frac{120}{4.08 + j\,3.58 - j5.31} = 27.09\angle 22.98^\circ A$$

With C_s



With C_s → $I_{st} = 16.97\angle -61.1^\circ + 27.09\angle 22.98^\circ = 33.42\angle -7.34^\circ A$

$T_{st} = K\,27.09 \sin 84.08^\circ = 26.95\,K$

$$\frac{I_{st}|_{\text{with } C_s}}{I_{st}|_{\text{No } C_s}} = \frac{33.43}{38.53} = 0.87 \qquad \frac{T_{st}|_{\text{with } C_s}}{T_{st}|_{\text{No } C_s}} = \frac{26.95}{7.5} = 3.59$$

$\boxed{7.8}$ (a) $\phi_m = \tan^{-1}\frac{4.8}{5.5} = 41.11^\circ \quad \phi_a(\text{required}) = 41.11^\circ - 90^\circ = -48.89^\circ$

$$-48.89 = \tan^{-1}\frac{(5 - X_c)}{8.5}$$

$$-1.15 = \frac{5 - X_c}{8.5}$$

$X_c = 5 + 8.5 \times 1.15 = 14.78 \qquad C = \frac{10^6}{377 \times 14.78} = 179.47\,\mu F$

$$I_m = \frac{120}{5.5 + j4.8} = \frac{120}{7.3\angle 41.11^\circ}$$

(b) Without C_s $\quad I_a = \frac{120}{8.5 + j5} = \frac{120}{9.86\angle 30.47^\circ} = 12.17\angle -30.47^\circ$

With C_s $\quad I_a = \frac{120}{8.5 + j5 - j14.28} = \frac{120}{8.5 - j9.78} = \frac{120}{12.96\angle -49.01^\circ} = 9.26\angle 49.01^\circ$

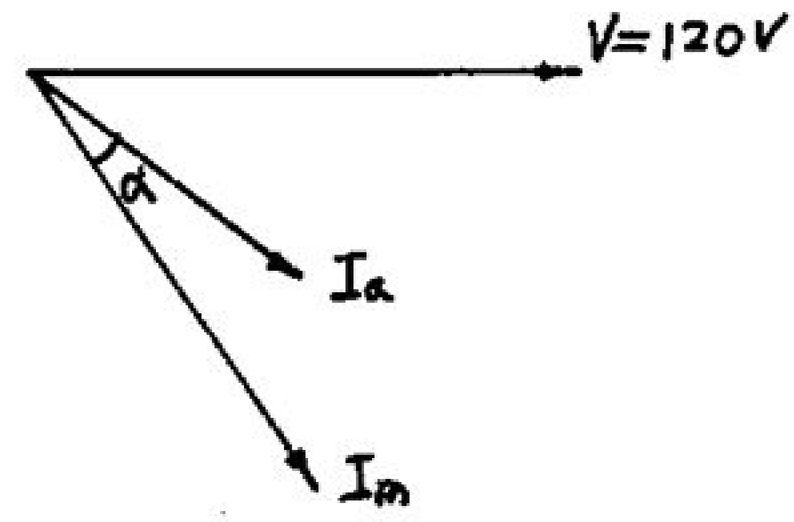
$T_{st} \propto I_m I_a \sin\alpha = K I_a$ if I_m is constant

$T_{st}|_{\text{without } C_s} = K\,12.17 \sin(-30.47^\circ - (-41.11^\circ)) = K\,2.25$

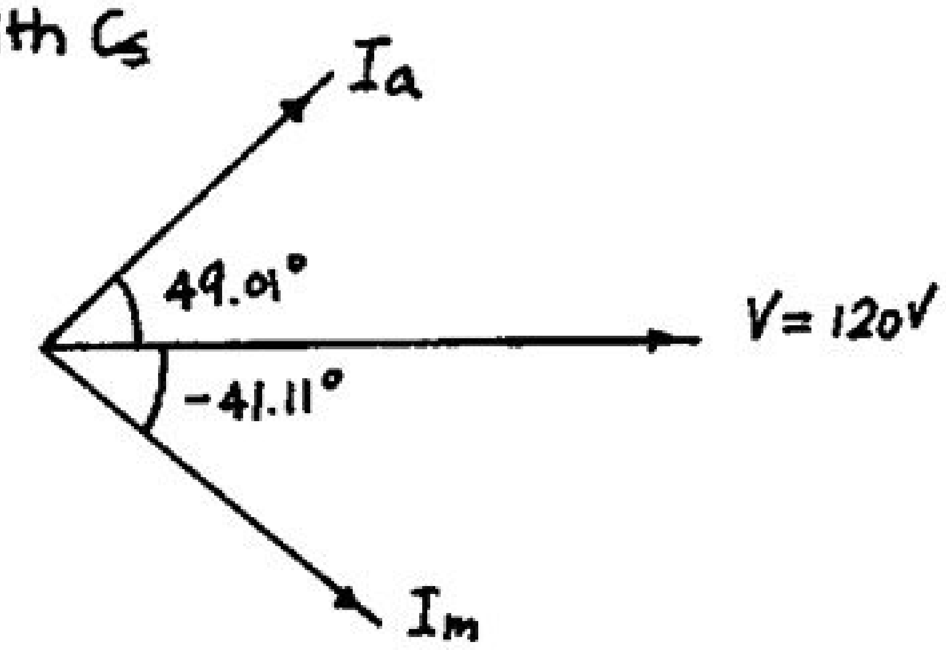
$T_{st}|_{\text{with } C_s} = K\,9.26 \sin(49.01^\circ - (-41.11^\circ)) = K\,9.26 \sin 90.12^\circ = K\,9.26$

$$\frac{T_{st}|_{\text{with } C_s}}{T_{st}|_{\text{without } C_s}} = \frac{9.26}{2.25} = 4.12$$

(c) Without C_s

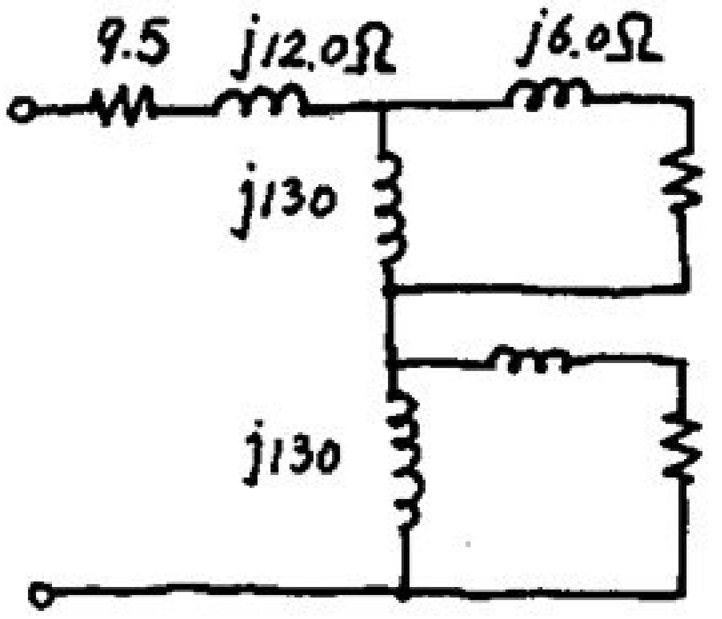


With C_s



$\boxed{7.9}$ (a) $S = \frac{1800-1710}{1800} = 0.05$

(b)



$\frac{0.5\times 10.8}{0.05} = 108\,\Omega$

$\frac{0.5\times 10.8}{2-0.05} = 2.77\,\Omega$

$$Z_f = R_f + jX_f = \frac{j130\,(108+j6.0)}{108+j136} = \frac{130\angle 90^\circ \times 108.167\angle 93.18^\circ}{173.666\angle 51.546^\circ}$$

$$= 80.97\angle 41.634^\circ = 60.517 + j53.794$$

$$Z_b = R_b + jX_b = \frac{j130\,(2.77+j6.0)}{2.77+j136} = \frac{j130 \times 6.609\angle 65.219^\circ}{136.028\angle 88.833^\circ}$$

$$= 6.316\angle 66.386^\circ = 2.53 + j5.787$$

$$Z_{in} = 9.5 + j12.0 + 60.517 + j53.794 + 2.53 + j5.787 = 101.916\angle 44.61^\circ$$

$$I_{in} = \frac{230}{101.916\angle 44.61^\circ} = 2.257\angle -44.61^\circ$$

(c) Input $P.F. = \cos 44.61^\circ = 0.712$ Lagging

(d) $P_{in} = 230 \times 2.257 \times 0.712 = 369.606\,W$

(e) $T = \frac{I^2}{\omega_{syn}}(R_f - R_b) = \frac{2.257^2}{188.5}(60.517 - 2.53) = 1.567\,N.m.$

(f) $P_{mech} = T \cdot \omega_m = 1.567 \times 188.5 \times (1-0.05) = 280.61\,W$

$P_{shaft} = P_{mech} - P_{core} - P_{F\&W} = 280.61 - 35 - 10 = 235.61\,W$

$T_{shaft} = \frac{P_{shaft}}{\omega_m} = \frac{235.61}{\frac{1710}{60}\times 2\pi} = \frac{235.61}{179} = 1.316\ N.m.$

(g) $Eff = \frac{P_{shaft}}{P_{in}} = \frac{235.61}{369.606} = 63.75\%$

(h) Rotor frequency

$\longrightarrow sf_1 = 0.05 \times 60 = 3$ Hz

$\longrightarrow (2-s)f_1 = (2-0.05) \times 60 = 117$ Hz

$P_{gf} = I^2 R_f = 2.257^2 \times 60.517 = 308.28$ W

$P_{gb} = I^2 R_b = 2.257^2 \times 2.53 = 12.89$ W

$P_2 = 0.05 \times 308.28 + (2-0.05) \times 12.89 = 40.55$ W

(i) $\frac{\Phi_f}{\Phi_b} = \frac{E_{mf}}{E_{mb}} = \frac{IZ_f}{IZ_b} = \frac{Z_f}{Z_b} = \frac{\sqrt{60.517^2 + 53.794^2}}{\sqrt{2.53^2 + 5.787^2}} = \frac{80.97}{6.312} = 12.81$

$\boxed{7.10}$ (a) $|Z_m| = \frac{32}{4} = 8\,\Omega$, $R_m = \frac{80}{4^2} = 5\,\Omega$, $X_m = \sqrt{8^2 - 5^2} = 6.245$

$Z_m = 5 + j6.245 = 8\angle 51.32^\circ\ \Omega$

$|Z_a| = \frac{40}{4} = 10\,\Omega$, $R_a = \frac{128}{4^2} = 8\,\Omega$, $X_a = \sqrt{10^2 - 8^2} = 6\,\Omega$

$Z_a = 8 + j6 = 10\angle 36.87^\circ\ \Omega$

(b) $R_a(\text{total}) = \frac{X_a}{X_m}(R_m + |Z_m|)$

$= \frac{6}{6.245}(5 + 8) = 12.49\,\Omega$

$R_a(\text{ext}) = 12.49 - 8 = 4.49\,\Omega$

(c) $I_m = \frac{120}{8\angle 51.32} = 15\angle -51.32^\circ$ A

(i) Without R_a (ext)

$I_a = \frac{120}{10\angle 36.87} = 12\angle -36.87$ A

$I = 15\angle -51.32 + 12\angle -36.87 = 26.78\angle -44.9^\circ$

7.10 (c) continued

$$T_s = K I_a \sin\alpha = K I_a \sin(\theta_a - \theta_m)$$
$$= K 12 \sin(-36.87 + 51.32) = K 12 \sin 14.45$$
$$= K\, 2.99$$

(ii) With R_a (ext)

$$I_a = \frac{120}{12.49 + j6} = 8.66 \angle -25.66 \text{ A}$$

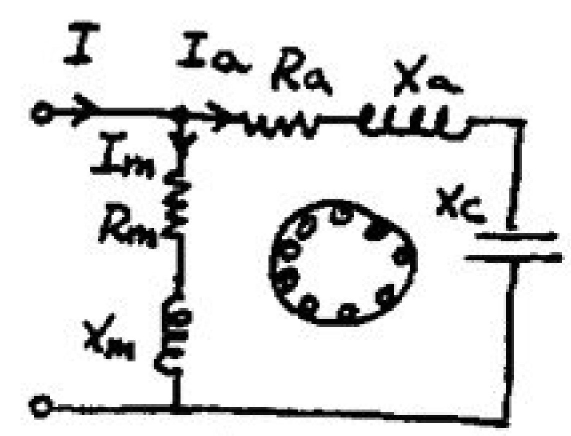
$$I = 15 \angle -51.32 + 8.66 \angle -25.66 = 23.11 \angle -42^\circ$$

$$T_s = K\, 8.66 \sin(-25.66 + 51.32) = K\, 8.66 \sin 25.66$$
$$= K\, 3.75$$

$$\frac{T_s \mid \text{with } R_a(\text{ext})}{T_s \mid \text{without } R_a(\text{ext})} = \frac{3.75}{2.99} = 1.2542$$

$$\frac{I \mid \text{with } R_a(\text{ext})}{I \mid \text{without } R_a(\text{ext})} = \frac{23.11}{26.78} = 0.86296$$

$\boxed{7.11}$ (a)



$$Z_m = 5 + j6.25 = 8 \angle 51.32^\circ\ \Omega$$
$$Z_a = 8 + j6 = 10 \angle 36.87^\circ\ \Omega$$
$$X_c = X_a + \frac{R_a R_m}{|Z_m| + X_m}$$
$$= 6 + \frac{8 \times 5}{8 + 6.25} = 8.81 = \frac{1}{\omega C}$$
$$C = \frac{10^6}{377 \times 8.81}\ \mu F = 301.1\ \mu F$$

(b) $$I_m = \frac{120}{8 \angle 51.32} = 15 \angle \overset{\theta_m}{-51.32} \text{ A}$$

(i) Without C

7.11 (b) (i) Continued

$$I_a = \frac{120}{10 \angle 36.87} = 12 \angle \overset{\theta_a}{-36.87} \text{ A}$$

$$I_{st} = I_m + I_a = 15 \angle -51.32 + 12 \angle -36.87 = 26.8 \angle -45$$

$$T_{st} = k' I_m I_a \sin(\theta_a - \theta_m)$$

$$= k\, I_a \sin(\theta_a - \theta_m) \rightarrow I_m \text{ is constant}$$

$$= k\, 12 \sin(-36.87 - (-51.32))$$

$$= k\, 2.99$$

(ii) With C

$$I_a = \frac{120}{8 + j6 - j8.81} = 14.15 \angle 19.35 \text{ A}$$

$$I_{st} = 15 \angle -51.32 + 14.15 \angle 19.35 = 23.8 \angle -17^\circ \text{ A}$$

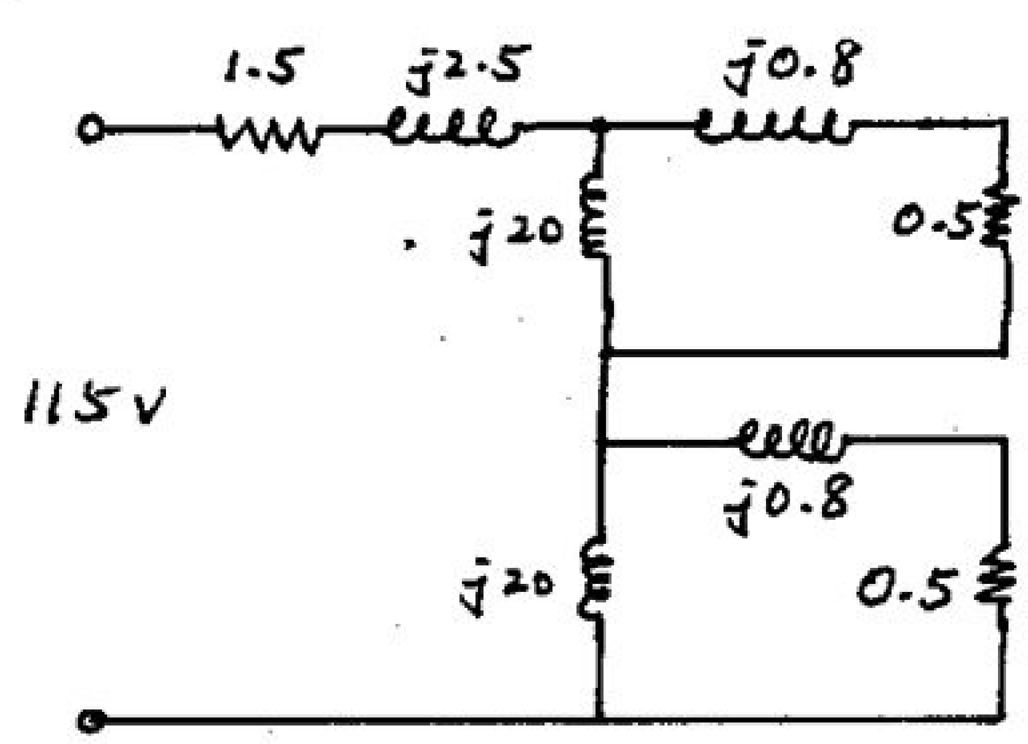
$$T_{st} = k\, 14.15 \sin(19.35 - (-51.32))$$

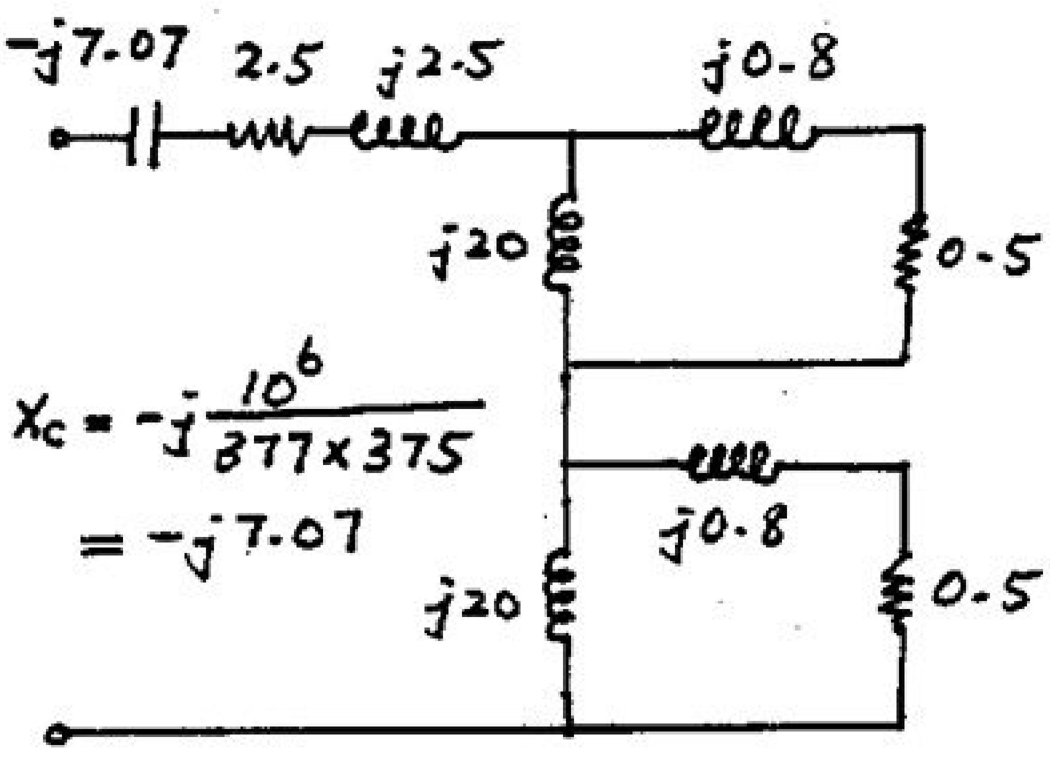
$$= k\, 13.35$$

$$\frac{T_{st}|_c}{T_{st}|} = \frac{13.35}{2.99} = 4.46$$

$$\frac{I_{st}|_c}{I_{st}|} = \frac{23.8}{26.8} = 0.89$$

$\boxed{7.12}$ (a)





$$X_c = -j\frac{10^6}{377 \times 375} = -j7.07$$

7.12 (a) Continued

at starting

$$Z_m = 1.5 + j\,2.5 + 2\left[\, j20 \,//\, j0.8 + 0.5 \,\right] = 2.424 + j4.061$$

$$= 4.729 \;\underline{/59.17^\circ}\;\Omega$$

$$R_f = R_a = (2.424 - 1.5)/2 = 0.462\,\Omega$$

$$Z_a = 2.5 + j\,(2.5 - 7.074) + 2\,(j20 // j0.8 + 0.5)$$

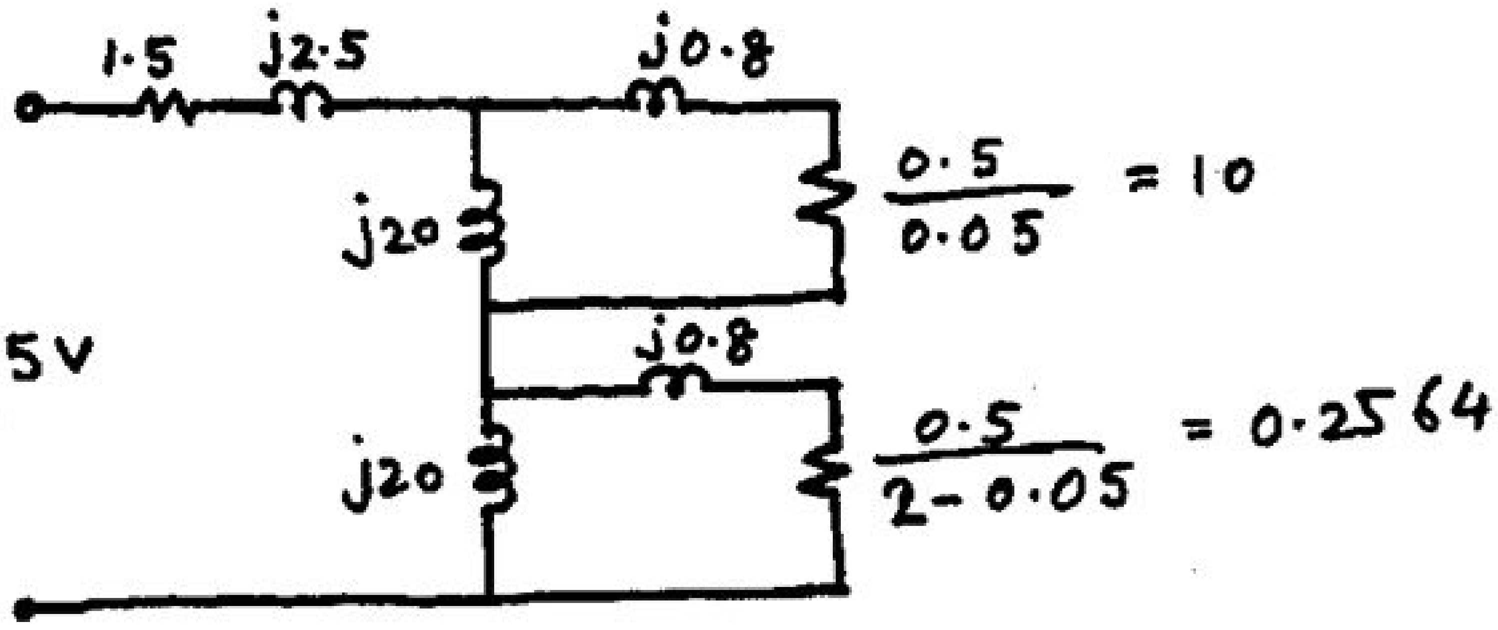
$$= 3.424 - j\,3.013 = 4.561 \;\underline{/-41.35^\circ}\;\Omega$$

$$T_{st} = \frac{2n\,|I_a|\,|I_m|\,(R_f + R_b)\,\sin(\theta_a - \theta_m)}{\omega_s}$$

$$= \frac{2 \times 1 \times (115)^2 \times 2 \times 0.462\,\sin(59.17 + 41.35)}{4.729 \times 4.561 \times 1800 \times 2\pi/60}$$

$$= 5.91\ N\cdot m$$

(b)



Full load slip $S = \dfrac{1800 - 1710}{1800} = 0.05$

$$R_f + jX_f = j20 // j0.8 + \frac{0.5}{0.05} = 7.51 + j4.38$$

$$= 8.69 \;\underline{/30.25}$$

$$R_b + jX_b = j20 // j0.8 + \frac{0.5}{1.95} = 0.237 + j\,0.772$$

$$= 0.808 \;\underline{/72.93}$$

$$Z_{main} = 1.5 + j2.5 + 7.51 + j4.38 + 0.237 + j0.772$$

$$= 9.247 + j\,7.652 = 12.003 \;\underline{/39.61}$$

$$I_m = \frac{115}{12.003} = 9.58\ A$$

7.12(b) continued

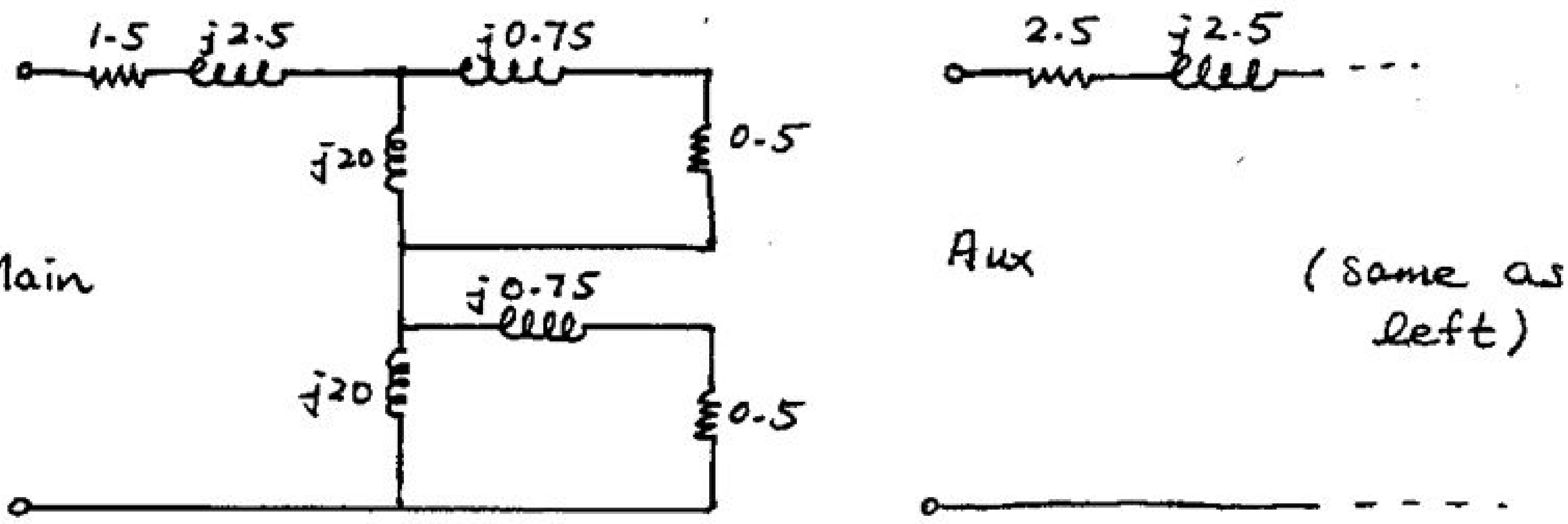
$$T_{int} = \frac{I_m^2}{\omega_s}(R_f - R_b)$$

$$= \frac{9.58^2\,(7.51 - 0.237)}{1800 \times 2\pi / 60}$$

$$= 3.54 \text{ N.m}$$

(c) Ratio of $\dfrac{T_{st}}{T_{int}} = \dfrac{5.91}{3.54} = 1.67$

$\boxed{7.13}$ (a)



At starting :

$$R_f + jX_f = R_b + jX_b = j20 \,//\, (0.5 + j0.75) = 0.464 + j0.734$$

$$Z_m = 1.5 + j2.5 + 2(0.464 + j0.734) = 2.43 + j3.97$$

$$= 4.65\,\underline{/58.34} = R_m + jX_m$$

$$Z_a = 2.5 + j2.5 + 2(0.464 + j0.734) = 3.43 + j3.97$$

$$= 5.24\,\underline{/49.2} = R_a + jX_a$$

(b) $I_m = \dfrac{120}{4.65}\,\underline{/-58.34^\circ} = 25.81\,\underline{/-58.34}$

$$I_a = \frac{120}{5.24}\,\underline{/-49.2} = 22.9\,\underline{/-49.2}$$

$$I_{st} = I_m + I_a \rightarrow |I_{st}| = \boxed{48.55\text{ A}}$$

7.13 (b) continued

$$T_{st} = \frac{2 \frac{N_a}{N_m} |I_a||I_m| (R_f + R_b) \sin(\theta_a - \theta_m)}{\omega_s}$$

$$= \frac{2 \times 1 \times 22.9 \times 25.81 \times 2 \times 0.464 \times \sin 9.16}{1800 \times 2\pi/60}$$

$$= 0.926 \text{ N-m}$$

(c) $\left.\frac{T_{st}}{I_{st}}\right|_{max}$ when $X_c = X_a + \left[\frac{-X_m R_a + |Z_m|\sqrt{R_a(R_a+R_m)}}{R_m}\right]$

$R_a + jX_a = 3.43 + j3.97$, $R_m + jX_m = 2.43 + j3.97$

$|Z_m| = 4.65$

$\Rightarrow X_c = 6.95\,\Omega = \frac{1}{\omega C} \Rightarrow \boxed{C = 382\ \mu F}$

$Z_a = 3.43 + j3.97 - j6.95 = 3.43 - j2.98 = 4.544 \angle 40.98$

$I_a = \frac{120}{4.544} = 26.41 \angle 40.98$

$I_{st} = I_m + I_a = 25.81 \angle -58.34 + 26.41 \angle 40.98$

$|I_{st}| = \boxed{33.81\ A}$

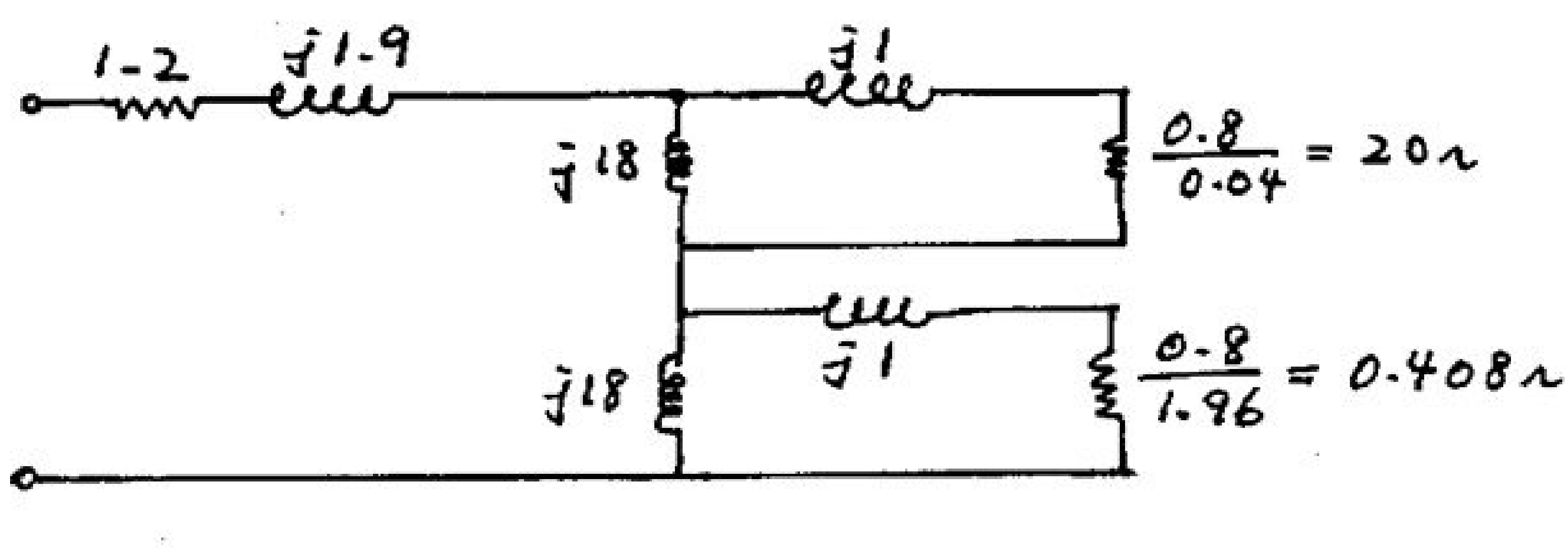
$$T_{st} = \frac{2 \times 1 \times 26.41 \times 25.81 \times 2 \times 0.464 \sin 99.32}{1800 \times 2\pi/60}$$

$$= \boxed{6.62\ \text{N-m}}$$

(d) without C $\longrightarrow \frac{T_{st}}{I_{st}} = \frac{0.926}{48.55} = 0.01907$ N-m/A

with C $\longrightarrow \frac{T_{st}}{I_{st}} = \frac{6.62}{33.81} = 0.1958$ N-m/A

$\boxed{7.14}$ (a)



7-14(a) Continued

$$R_f + jX_f = \frac{1}{\frac{1}{j18} + \frac{1}{20+j1}} = 8.515 + j9.911$$

$$R_b + jX_b = \frac{1}{\frac{1}{j18} + \frac{1}{0.408+j1}} = 0.366 + j0.955$$

$$Z_{in} = 10.08 + j12.77 = 16.2 \angle 51.48 \ \Omega$$

$$\cos\theta = 0.623$$

$$I_m = \frac{120}{16.2} = 7.41 \text{ A}$$

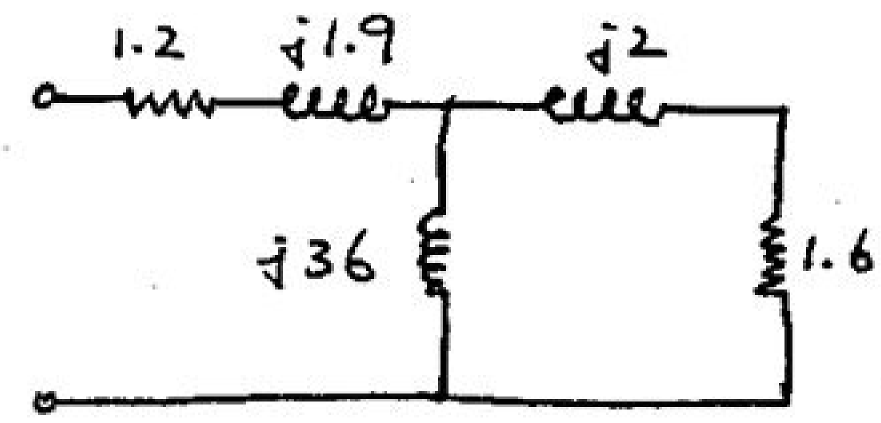
$$P_{out} = I_m^2 (R_f - R_b)\frac{\omega}{\omega_s} - P_{rot} = 429.9 - 40 = 389.9 \text{ W}$$

$$P_{in} = 120 \times 7.41 \times \cos\theta = 554.2 \text{ W}$$

$$\eta = \frac{389.9}{554.2} \times \frac{100}{1} = 70.35\%$$

(b) At Standstill

$$Z_{in} = 1.2 + j1.9 + \frac{1}{\frac{1}{j36} + \frac{1}{1.6+j2}}$$



$$= 2.633 + j3.855$$

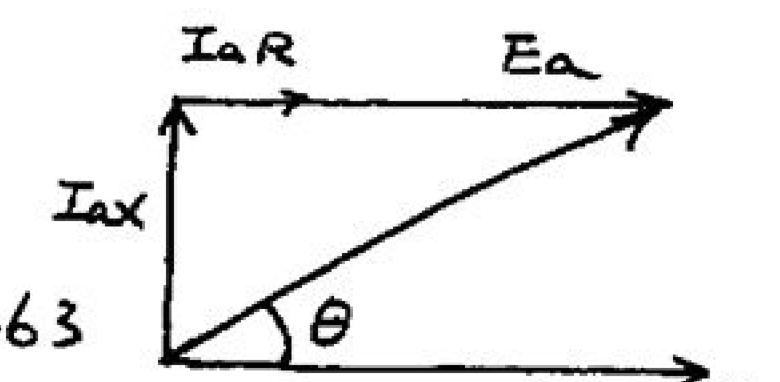
$\equiv r + jx$ for series ac motor

$$I_a = 7.41 \text{ A}$$

$$E_a = \sqrt{V_t^2 - (I_a x)^2} - I_a R$$

$$= \sqrt{120^2 - (7.41 \times 3.855)^2} - 7.41 \times 2.63$$

$$= 97.21$$



Output power $= E_a I_a - P_{rot} = 7.41 \times 97.21 - 40$

$$= 680.3 \text{ W}$$

7.14(b) continued

$$\cos\theta = \frac{97.21 + 7.41 \times 2.633}{120} = 0.973$$

$$P_{in} = 120 \times 7.41 \times 0.973 = 865.3\ W$$

$$\eta = \frac{680.3}{865.3} \times \frac{100}{1} = 78.6\ \%$$

$\boxed{7.15}$ (a) DC Supply

(i) $E_a = V_t - I_a R = 400 - 20 \times 1.6 = 368\ V$

$P_{mech} = E_a I_a = 368 \times 20 = 7360 W$

(ii) $P_{in} = V_t I_a = 400 \times 20 = 8000 W$

$$\eta = \frac{P_{mech}}{P_{in}} = \frac{7360}{8000} = 92\%$$

(b) AC supply

(i) $V_t^2 = (E_a + I_a R)^2 + (I_a X)^2$

$400^2 = (E_a + 20 \times 1.6)^2 + (20 \times 10)^2$

$E_a = 314.41 V$

$$n = \frac{314.41}{368} \times 2000 = 1708.75\ rpm.$$

(ii) $PF = \cos\theta = \frac{E_a + I_a R}{V_t} = \frac{314.41 + 32}{400} = 0.866$

(iii) $P_{mech} = 314.41 \times 20 = 6288.2 W$

(iv) $P_{in} = V_t I_a \cos\theta = 400 \times 20 \times 0.866 = 6928 W$

$$\eta = \frac{6288.2}{6928} \times 100\% = 90.77\%$$

(V) $T_{st} = K_d \Phi_d I_a = K_{sr} I_a^2$

$E_a = K_{sr} I_a \omega_m \longrightarrow K_{sr} = \frac{368}{20 \times \frac{2000}{60} \times 2\pi} = 0.0879$

$$I_a|_{st} = \frac{400}{(1.6^2 + 10^2)^{\frac{1}{2}}} = 39.49\ A$$

$T_{st} = 0.0879 \times 39.49^2 = 137 N.m$

$\boxed{7.16}$ (a) <u>Uncompensated Motor</u>

(i) $V^2 = (E_a + I_aR)^2 + (I_aX)^2$

$120^2 = (E_a + 1.6\times5)^2 + (1.6\times25)^2$

$E_a = 105.14\ V$

$PF = \dfrac{E_a + I_aR}{V} = \dfrac{105.14 + 1.6\times5}{120} = 0.943$

(ii) $P_{mech} = 105.14 \times 1.6 = 168.224\ W$

(iii) $P_{in} = 120 \times 1.6 \times 0.943 = 181.06$

$P_{out} = P_{mech} - P_{rot} = 168.224 - 30 = 138.224$

$\eta = \dfrac{P_{out}}{P_{in}} = \dfrac{138.224}{181.06} = 0.7634$ (or 76.34%)

(b) <u>Compensated Motor</u>

(i) $E_a = \sqrt{V^2 - (I_aX)^2} - I_aR = \sqrt{(120)^2 - (1.6\times3)^2} - 1.6\times5.5$
$= 111.104$

$E_a = k_a \Phi_d \omega_m$, since I_a is the same so is Φ_d

$\therefore\ n = 1800 \dfrac{E_a\ (\text{compensated})}{E_a\ (\text{uncompensated})} = 1902\ rpm$

(ii) $PF = \dfrac{111.104 + 1.6\times5.5}{120} = 0.9992$

(iii) $P_{mech} = 111.104 \times 1.6 = 177.77$, $P_{in} = 120 \times 1.6 \times 0.9992$
$= 191.85$

7.16 (b) (iii) continued

$$P_{out} = 177.77 - 30 = 147.77, \quad \eta = \frac{147.77}{191.85} = 77.02\%$$

(c) $T = K_a \phi_d I_a = K_{sr} I_a^2$ (for linear magnetic property)

$$E_a = K_{sr} I_a \omega_m$$

$$K_{sr} = \frac{105.14 \times 60}{1.6 \times 1800 \times 2\pi} = 0.3486$$

$$I_a|_{st} = \frac{120}{\sqrt{5^2 + 25^2}} = 4.71A, \quad T_{st} = 0.3486 \times 4.71 = 7.734 \text{ N.m}$$

7.17

Speed	Zin	Iin	PF	Pin	T	Pmech	Pout	Pg	P2	Eff
1600.	18.16	6.33	0.806	586.70	2.281	382.25	347.25	498.46	116.21	59.19
1605.	18.44	6.24	0.803	575.68	2.248	377.92	342.92	490.11	112.18	59.57
1610.	18.73	6.14	0.799	564.51	2.215	373.38	338.38	481.57	108.20	59.94
1615.	19.03	6.04	0.796	553.20	2.179	368.61	333.61	472.87	104.26	60.31
1620.	19.34	5.95	0.792	541.74	2.143	363.62	328.62	463.98	100.36	60.66
1625.	19.67	5.85	0.788	530.14	2.106	358.39	323.39	454.90	96.51	61.00
1630.	20.00	5.75	0.784	518.38	2.068	352.93	317.93	445.65	92.72	61.33
1635.	20.35	5.65	0.779	506.47	2.028	347.22	312.22	436.20	88.98	61.65
1640.	20.71	5.55	0.774	494.42	1.987	341.27	306.27	426.57	85.30	61.95
1645.	21.08	5.46	0.769	482.21	1.945	335.06	300.06	416.75	81.68	62.23
1650.	21.47	5.36	0.763	469.86	1.902	328.60	293.60	406.73	78.13	62.49
1655.	21.87	5.26	0.756	457.35	1.857	321.88	286.88	396.51	74.64	62.73
1660.	22.28	5.16	0.749	444.70	1.811	314.88	279.88	386.10	71.22	62.94
1665.	22.71	5.06	0.742	431.90	1.764	307.62	272.62	375.49	67.87	63.12
1670.	23.15	4.97	0.733	418.95	1.716	300.07	265.07	364.67	64.60	63.27
1675.	23.61	4.87	0.725	405.85	1.666	292.24	257.24	353.66	61.41	63.38
1680.	24.08	4.78	0.715	392.60	1.615	284.13	249.13	342.44	58.31	63.46
1685.	24.57	4.68	0.704	379.21	1.563	275.72	240.72	331.01	55.29	63.48
1690.	25.07	4.59	0.693	365.67	1.509	267.01	232.01	319.37	52.36	63.45
1695.	25.58	4.50	0.681	351.98	1.453	258.00	223.00	307.53	49.53	63.36
1700.	26.11	4.40	0.668	338.16	1.397	248.68	213.68	295.47	46.79	63.19
1705.	26.64	4.32	0.653	324.19	1.339	239.05	204.05	283.20	44.15	62.94
1710.	27.19	4.23	0.638	310.07	1.279	229.10	194.10	270.72	41.62	62.60
1715.	27.74	4.14	0.621	295.82	1.218	218.83	183.83	258.03	39.20	62.14
1720.	28.30	4.06	0.602	281.44	1.156	208.22	173.22	245.12	36.89	61.55
1725.	28.86	3.98	0.583	266.91	1.092	197.29	162.29	231.99	34.70	60.80
1730.	29.42	3.91	0.561	252.25	1.027	186.02	151.02	218.65	32.63	59.87
1735.	29.98	3.84	0.538	237.47	0.960	174.41	139.41	205.09	30.68	58.71
1740.	30.52	3.77	0.514	222.55	0.892	162.45	127.45	191.32	28.87	57.27
1745.	31.05	3.70	0.487	207.51	0.822	150.15	115.15	177.33	27.18	55.49
1750.	31.56	3.64	0.459	192.34	0.750	137.49	102.49	163.12	25.63	53.28
1755.	32.03	3.59	0.429	177.05	0.677	124.47	89.47	148.70	24.23	50.53
1760.	32.47	3.54	0.397	161.65	0.603	111.09	76.09	134.06	22.96	47.07
1765.	32.87	3.50	0.363	146.13	0.527	97.35	62.35	119.20	21.85	42.67
1770.	33.21	3.46	0.328	130.51	0.449	83.24	48.24	104.13	20.89	36.96
1775.	33.50	3.43	0.291	114.78	0.370	68.76	33.76	88.85	20.09	29.41
1780.	33.72	3.41	0.252	98.94	0.289	53.91	18.91	73.36	19.45	19.11
1785.	33.87	3.40	0.213	83.01	0.207	38.68	3.68	57.65	18.98	4.43
1790.	33.95	3.39	0.172	66.99	0.123	23.07	-11.93	41.74	18.67	-17.81
1795.	33.94	3.39	0.131	50.87	0.038	7.08	-27.92	25.62	18.54	-54.89

CHAPTER 8

$\boxed{8.1}$ (a) $$\frac{\omega_m(s)}{\omega_m^*(s)} = \frac{\frac{k_p s + k_i}{s} \frac{2.733}{1+0.0157s}}{1 + \frac{k_p s + k_i}{s} \frac{2.733}{1+0.0157s}}$$

$$= \frac{(k_p s + k_i)(2.733)}{s(1+0.0157s) + (k_p s + k_i)(2.733)}$$

(b) $$\frac{\omega_m(s)}{\omega_m^*(s)} = \frac{k_i(0.0157s+1)(2.733)}{s(1+0.0157s) + k_i(0.0157s+1)2.733}$$

$$= \frac{2.733 k_i}{s + 2.733 k_i}$$

$$= \frac{1/\tau}{s + 1/\tau}$$

$$\tau = \frac{1}{2.733 k_i} = 1$$

$$k_i = \frac{1}{2.733} = 0.3659$$

$$k_p = 0.0157 \times 0.3659 = 0.005745$$

$\boxed{8.2}$ (a) $$\frac{\theta(s)}{\theta_{ref}(s)} = \frac{\frac{10}{s(s+5)}}{1 + \frac{10}{s(s+5)}} = \frac{10}{s^2 + 5s + 10}$$

(b) $$\theta_{ref}(s) = \frac{\pi}{s}$$

$$\theta(s) = \frac{\pi\, 10}{s(s^2 + 5s + 10)}$$

$$= \frac{\pi\, \omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

$$\omega_n = \sqrt{10} = 3.162 \text{ rad./sec}$$

$$\zeta = \frac{5}{2\sqrt{10}} = 0.791$$

$$\theta(t) = \pi\left[1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_n\sqrt{1-\zeta^2}\, t + \cos^{-1}\zeta)\right)$$

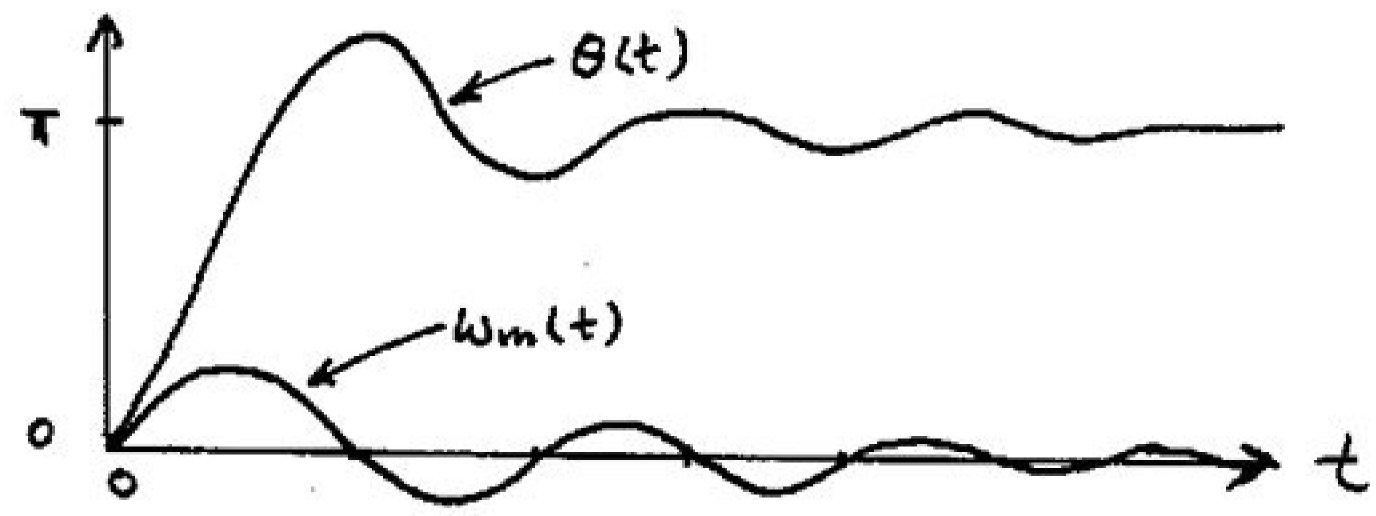
8.2(b) continued

$$= \pi \left[1 - 1.6345 e^{-2.5t} \sin(1.9345t + 37.7°) \right]$$

$$\omega_m(t) = \frac{d\theta(t)}{dt} \quad \text{or} \quad \omega_m(s) = s\theta(s) = \frac{\pi\, 10}{s^2 + 5s + 10}$$

$$\omega_m(t) = \frac{\pi \omega_n^2}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \sin \omega_n \sqrt{1-\xi^2}\, t$$

$$= 51.35 e^{-2.5t} \sin 1.9345t$$



8.3 (a)

$$E_{1n} = 50 \cos(\alpha + 120) = 50 \cos(30 + 120) = -43.3 \text{ V}$$
$$E_{2n} = 50 \cos\alpha = 50 \cos 30° = 43.3 \text{ V}$$
$$E_{3n} = 50 \cos(\alpha - 120) = 50 \cos(30 - 120) = 0 \text{ V}$$

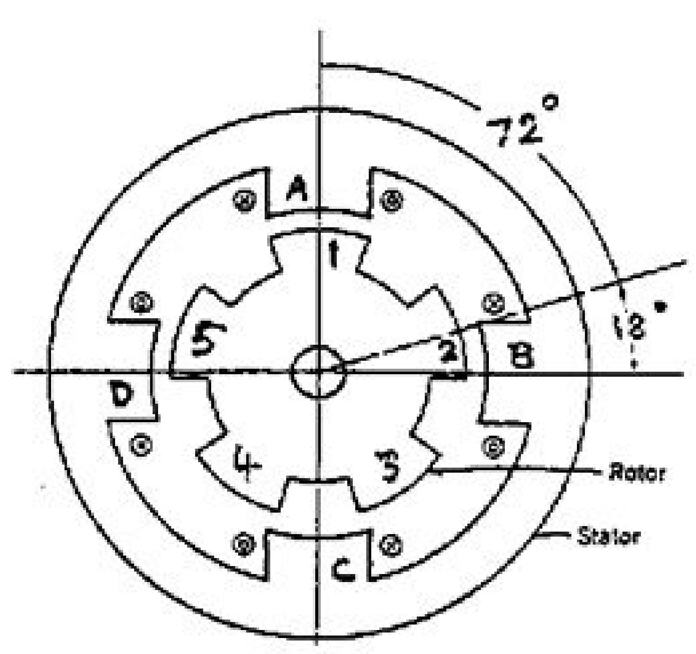
(b)

$$E_{12} = \sqrt{3} \times 50 \times \sin(30 - 120) = -86.6 \text{ V}$$
$$E_{23} = \sqrt{3} \times 50 \times \sin(30 + 120) = 43.3 \text{ V}$$
$$E_{31} = \sqrt{3} \times 50 \times \sin 30° = 43.3 \text{ V}$$

8.4 $E = 10 \sin(20 - 0) = 10 \times 0.342 = 3.42 \text{ V}$

8.5 Sequence is A, B, C, D, A

8.6 4 phases on stator
5 poles on rotor
Sequence A, B, C, D, A



$\boxed{8.7}$ (a) $\Delta\theta = 15°$

(b) $n = \frac{360°}{15} = 24$

(c) $rps = \frac{120}{24} = 5$

$rpm = 5 \times 60 = 300\ rpm$

$\boxed{8.8}$ (a) $\Delta\theta = 45°$

(b) $n = \frac{360}{45} = 8$

(c) $rps = \frac{120}{8} = 15$

$rpm = 15 \times 60 = 900$

$\boxed{8.9}$ $\tau_p = \frac{360}{8} = 45°$

$\Delta\theta = \frac{45}{3} = 15°$

$\boxed{8.10}$ (a) $n = \frac{360}{2} = 180$

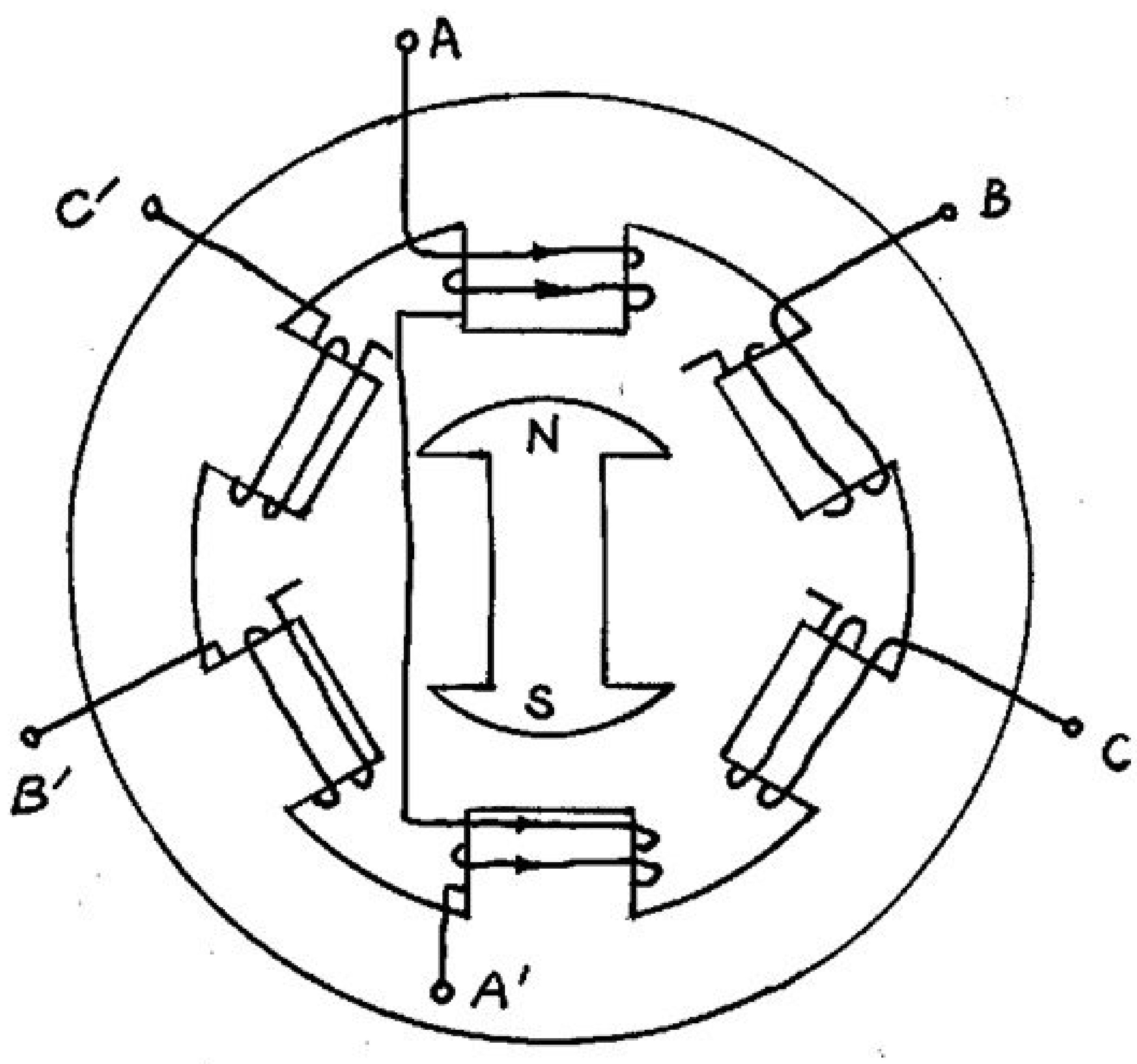
(b) $2 = \frac{360}{3x} \longrightarrow x = 60$

$\boxed{8.11}$ sequence is A, A+B, B, B-A, -A, -A-B, -B, -B+A, A

$\boxed{8.12}$ (a) 3

(b) $\Delta\theta = \frac{360}{6} = 60°$ A, B, C and A

(c)



8.13

(a)

Signal	Phases	Angle
1000	a	0°
1100	ab	45°
0100	b	90°
0110	bc	135°
0010	c	180°
0011	cd	225°
0001	d	270°
1001	da	315°

(b) $\frac{720}{60}\left(\frac{\text{rev}}{\text{sec}}\right) \times 8 \left(\frac{\text{nibbles}}{\text{sec}}\right) = 96$ nibbles/sec.

8.14 (a)

Signal	Phase	Angle
1000	A	0°
0100	B	90°
0010	C	180°
0001	D	270°
1000	A	360°

(b) 4 phase/revolution

$rps = \frac{100}{4} = 25$

$rpm = 25 \times 60 = 1500$ rpm

8.15

(a) $T_{on} = \frac{50}{10+2} = 4.167$ msec.

$\tau_{off} = \frac{50}{10+2+5} = 2.94$ msec.

(b) $V_s = (10+2)5 = 60\ V$

(c) Transistor

$V_{CE\,(max)} = 60 + 5 \times 5 = 85\,V$

$I_c = 5A$

Diode

$V = 60\,V$

$I = 5\,A$

(d) $t_{on} = 3(4.167 + 2.94) = 21.321$ msec.

Max. stepping rate $= \frac{1}{21.321 \times 10^{-3}} = 46.9$ steps/sec

8.16 When transistor is on

$$I = 5 = \frac{V_s}{2 + R_{ext}}$$

When transistor is off

$$i = 5e^{-t/\tau} - \frac{V_s}{2 + R_{ext}}(1 - e^{-t/\tau})$$

$$= 5e^{-t/\tau} - 5(1 - e^{-t/\tau})$$

$$0 = 5e^{-0.001/\tau} - 5(1 - e^{-0.001/\tau})$$

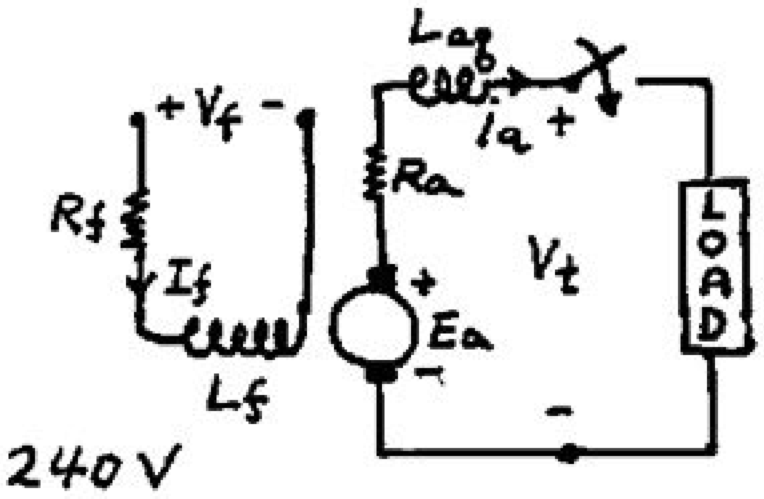
$$e^{-0.001/\tau} = 0.5$$

$$\tau = \frac{-0.001}{\ln(0.5)} = 1.4427 \text{ msec.}$$

$$\tau = \frac{L_w}{R_w + R_{ext}} = \frac{50}{2 + R_{ext}} = 1.4427$$

$$\Rightarrow R_{ext} = 32.66\,\Omega$$

$$V_s = 5(2 + 32.66) = 173.3\ V$$



9.1 (a) $K_g\Big|_{1200rpm} = 100 \times \frac{1200}{1000} = 120\ V/A$

@ 1200 rpm $I_f = 2A$

$\therefore E_a\Big|_{1200} = K_g\Big|_{1200} \times I_f = 120 \times 2 = 240V$

$$E_a(t) = \underbrace{(R_a + R_L)}_{R_T} i_a(t) + \underbrace{(L_{ag} + L_L)}_{L_T} \frac{d}{dt} i_a(t)$$

Take Laplace Transform

$$I_a(s) = \frac{E_a|_{1200}}{R_T} \times \frac{1}{S(1 + \frac{L_T}{R_T}S)}$$

$$i_a(t) = \mathcal{L}^{-1}\{I_a(s)\} = \frac{240}{2}(1 - e^{-t/\tau_{at}})$$

where $\tau_{at} = L_T/R_T = \frac{(10+10)10^{-3}}{0.2 + 1.8} = 0.01$ sec

$$V_t(t) = R_L i_a + L_L \frac{di_a}{dt}$$

$$= 1.8 \times 120(1 - e^{-100t}) + 10 \times 10^{-3}(120 \times 100 e^{-100t})$$

$$= 216 - 96 e^{-100t}\ V$$

(b) $V_t(\infty) = 216\ V$

(c) $T = K_f I_f i_a$

$E_a = K_f I_f \omega_m = K_g I_f$

$K_f \omega_m = K_g$

$\therefore K_f = \frac{K_g}{\omega_m} = \frac{100}{1000 \times 2\pi/60} = \frac{3}{\pi}$

$\therefore T = \frac{3}{\pi}(2) i_a$

$= 229.2(1 - e^{-100t})\ N \cdot m$

or $T = \frac{E_a i_a(t)}{\omega_m} = \frac{240}{1200 \times 2\pi/60} \times 120(1 - e^{-100t})$

$= 229.2(1 - e^{-100t})\ N \cdot m$

9.2 (a) $V_t = Ri + K_m\omega_m$

$V_t(s) = RI(s) + K_m\omega_m(s)$ .... (1)

$T = K_m i = J\frac{d\omega_m}{dt} + B_m\omega_m + B_L\omega_m \; (= T_L)$

$K_m i = J\frac{d\omega_m}{dt} + B\omega_m$

$K_m I(s) = JS\omega_m(s) + B\omega_m(s)$ .... (2)

From ① & ②

$V_t(s) = K_m\omega_m(s) + \frac{R(JS+B)}{K_m}\omega_m(s)$

$$\frac{\omega_m(s)}{V_t(s)} = \frac{K_m}{K_m^2 + RB + RBS\tau_m} \qquad \left(\tau_m = \frac{J}{B} = \frac{2.5}{0.25} = 10\,sec\right)$$

$$= \frac{K_m}{(K_m^2+RB)(1+\frac{RB}{K_m^2+RB}S\tau_m)} = \frac{2}{2^2+0.4\times0.25}\times\frac{1}{(1+\frac{0.4\times0.25S}{2^2+0.4\times0.25})\times(10)}$$

$$= 0.488\times\frac{1}{1+0.244S}$$

$$\omega_m(s) = \frac{200}{s}\times0.488\times\frac{1}{1+0.244S} = \frac{97.6}{s}\times\frac{1}{1+0.244S}$$

$$\omega_m(t) = 97.6\,(1-e^{-t/0.244})$$

(b) $\omega_m(\infty) = 97.6$ rad/sec

(c) $0.95\times97.6 = 97.6(1-e^{-t/0.244}) \rightarrow e^{-t/0.244} = 0.05 \rightarrow t = 0.732$ sec.

(d) $i_a(t) = \frac{V_t - K_m\omega_m}{R_a}$

$$= \frac{200 - 2(97.6 - 97.6e^{-t/0.244})}{0.4} = 500 - 488 + 488e^{-t/0.244}$$

$$= 12 + 488e^{-t/0.244}$$

$i_a|_{ss} = 12A$

9.3 (a) Because rotational losses are neglected, in steady state motor does not produce any torque. Therefore, before the voltage was changed, $I_a = 0$

$E_a = V_t$

$K_m \omega_m = 100\,V$

$$K_m = \frac{100}{\frac{1500}{60} \times 2\pi} = 0.637 \ V/rad/sec.$$

After the voltage was changed,

$$V_t = e_a + R_a i_a = K_m \omega_m + R_a i_a$$

Also $T = K_m i_a = J \frac{d\omega_m}{dt}$

$$\text{or } V_t = K_m \omega_m + R_a \frac{J}{K_m} \frac{d\omega_m}{dt}$$
$$= 0.637\,\omega_m + 0.5 \times 1.1/0.637 \frac{d\omega_m}{dt}$$

$$V_t = 0.64\,\omega_m + 0.86 \frac{d\omega_m}{dt}$$

$$V_t(s) = 0.64\,\omega_m(s) + 0.86\,(s\,\omega_m(s) - \omega_{mo})$$

where $\omega_{mo} = \frac{1500}{60} \times 2\pi = 157.1$ rad./sec.

$$\frac{120}{s} = 0.64\,\omega_m(s) + 0.86\,s\,\omega_m(s) - 0.86 \times 157.1$$

$$\Rightarrow \omega_m(s) = \frac{120 + 135.11 s}{s(0.64 + 0.86 s)}$$

$$= \frac{120 + 135.11 s}{s\, 0.64\left(1 + \frac{0.86}{0.64} s\right)} = \frac{139.1 + 157.1 s}{s(s + 0.744)}$$

$$= \frac{A}{s} + \frac{B}{s + 0.744}$$

$$A = 187 \ , \quad B = -29.9$$

$$\therefore \omega_m(t) = \mathcal{L}^{-1}\{\omega_m(s)\} = 187 - 29.9\, e^{-0.744 t} \ \text{rad./sec.}$$

(b) $$\omega_m\big|_{t=1} = 187 - 29.9\, e^{-0.744}$$
$$= 172.95 \ \text{rad./sec.}$$

(c) $$\omega_m(\infty) = 187 \ \text{rad./sec.}$$

$\boxed{9.4}$ (a) steady state

$$T = B\omega_m = K_m I_a$$

$$I_a = \frac{B\omega_m}{K_m}$$

$$V_t - I_a R_a = E_a = K_m\omega_m$$

$$200 = I_a R_a + K_m\omega_m = \frac{B\omega_m}{K_m}R_a + K_m\omega_m = \omega_m\left(K_m + \frac{BR_a}{K_m}\right)$$

$$= \omega_m\left(2 + \frac{0.2\times0.5}{2}\right) = \omega_m(2+0.05)$$

$$\omega_m = \frac{200}{2.05} = 97.56 \text{ rad/sec.} \qquad n = \frac{97.56}{2\pi}\times60 = 931.64 \text{ rpm}$$

(b)(i) After the voltage changes

$$V_t = e_a + i_a R_a = K_m\omega_m + i_a R_a$$

Also, $T = K_m i_a = J\frac{d\omega_m}{dt} + B\omega_m$

$$V_t = K_m\omega_m + \frac{R_a}{K_m}\left(J\frac{d\omega_m}{dt} + B\omega_m\right) = \omega_m\left(K_m + \frac{R_a B}{K_m}\right) + \frac{R_a J}{K_m}\frac{d\omega_m}{dt}$$

$$= \omega_m\left(2 + \frac{0.5\times0.2}{2}\right) + \frac{0.5\times2}{2}\frac{d\omega_m}{dt} = \omega_m\times2.05 + 0.5\frac{d\omega_m}{dt}$$

$$V_t(s) = 2.05\,\omega_m(s) + 0.5\left[s\omega_m(s) - \omega_{m0}\right]$$

$$\frac{100}{s} = 2.05\,\omega_m(s) + 0.5\left[s\omega_m(s) - 97.56\right]$$

$$\omega_m(s) = \frac{\frac{100}{s} + 48.78}{2.05 + 0.5s} = \frac{100 + 48.78s}{s(2.05+0.5s)} = \frac{100+48.78s}{0.5s\left(s+\frac{2.05}{0.5}\right)} = \frac{200 + 0.97565s}{s(s+4.1)}$$

$$= \frac{A}{s} + \frac{B}{s+4.1}$$

$$A = \left.\frac{200 + 97.56s}{s+4.1}\right|_{s=0} = \frac{200}{4.1} = 48.78$$

$$B = \left.\frac{200+97.56s}{s}\right|_{s=-4.1} = \frac{200 - 97.56\times4.1}{-4.1} = 48.78$$

$$\omega_m = 48.78 + 48.78\,e^{-4.1t}$$

(ii) $\omega_m(\infty) = 48.78$ rad/sec $\rightarrow \frac{48.78}{2\pi}\times60 = 465.82$ rpm

$\boxed{9.5}$ (a) $K_m = K_f i_f = 1 \times 2 = 2$

No-load $\rightarrow$ $V_t = E_a = 220\ V = K_m \omega_{mo}$

$$\omega_{mo} = \frac{220}{2} = 110 \text{ rad./sec.}$$

(b) After voltage reversal $\rightarrow$ $V_t = -220\,V$

$$V_t = e_a + R_a i_a = K_m \omega_m + R_a i_a$$

$$T = K_m i_a = J \frac{d\omega_m}{dt}$$

$$V_t = K_m \omega_m + R_a \frac{J}{K_m} \frac{d\omega_m}{dt}$$

$$= 2\omega_m + \frac{0.4 \times 4.5}{2} \frac{d\omega_m}{dt}$$

$$= 2\omega_m + 0.9 \frac{d\omega_m}{dt}$$

$$V_t(s) = 2\,\omega_m(s) + 0.9\,(s\,\omega_m(s) - \omega_{mo})$$

$$-\frac{220}{s} = 2\,\omega_m(s) + 0.9\,s\,\omega_m(s) - 0.9 \times 110$$

$$\omega_m(s) = \frac{-244.44 + 110s}{s\,(s + 2.222)}$$

$$= \frac{A}{s} + \frac{B}{s + 2.222}$$

where $A = -110$ $\quad B = 220$

$$\omega_m(t) = \mathcal{L}^{-1}\{\omega_m(s)\} = -110 + 220\,e^{-2.222t} \text{ rad./sec.}$$

(c) $0 = -110 + 220\,e^{-2.222t}$

$t = 0.315$ sec.

$\boxed{9.6}$ (a) $K_m = K_f i_f = 1 \times 2 = 2 V/rad/sec$

$$\omega_{mo} = \frac{450}{60} \times 2\pi = 47.124 \text{ rad/sec.}$$

$$E_a = K_m \omega_m = 2 \times 47.124 = 94.248 V \qquad I_a = \frac{100 - 94.248}{0.5} = 11.504 A$$

(b) $T = K_m I_a = 2 \times 11.504 = 23 N.m \qquad T_B = 0.1 \times 47.124 = 4.7124 N.m.$

$$T_L = 23 - 4.7124 = 18.2876 N.m$$

(c) $T = K_m i_a = 0 = J \frac{d\omega_m}{dt} + B_m \omega_m + T_L$

$$J(s\omega_m(s) - \omega_{mo}) + B\omega_m(s) + \frac{T_L}{s} = 0$$

$$Js\omega_m(s) - J\omega_{mo} + B\omega_m(s) + \frac{18.2876}{s} = 0$$

$$\omega_m(s) = \frac{J\omega_{mo} - \frac{18.2876}{s}}{B + Js} = \frac{94.248s - 18.2876}{2(s+0.05)s}$$

$$= \frac{A_1}{s} + \frac{A_2}{s+0.05}$$

$$A_1 = s\omega_m(s)\Big|_{s=0} = \frac{94.248s - 18.2876}{2(s+0.05)}\Big|_{s=0} = -182.876$$

$$A_2 = (s+0.5)\omega_m(s)\Big|_{s=-0.05} = \frac{94.248s - 18.2876}{2s}\Big|_{s=-0.05} = 230$$

$$\omega_m(s) = -\frac{182.876}{s} + \frac{230}{s+0.05}$$

$$\omega_m(t) = -182.876 + 230e^{-0.05t}$$

$$\omega_m(\infty) = -182.876 \text{ rad/sec}$$

CHECK: $\omega_m = -182.876 + 230 = 47.124 = \omega_{mo}$

$T_B = 0.1 \times 182.876 = 18.2876 = T_L$

$\boxed{9.7}$ T_L removed

$$T = K_m i_a = J\frac{d\omega_m}{dt} + B_m \omega_m$$

$$V_t = R_a i_a + K_m\omega_m = K_m\omega_m + R_a\left(\frac{J}{K_m}\frac{d\omega_m}{dt} + \frac{B_m}{K_m}\omega_m\right)$$

$$= 2\omega_m + 0.5\times\left(\frac{2}{2}\frac{d\omega_m}{dt} + \frac{0.1}{2}\omega_m\right) = 2\omega_m + 0.5\frac{d\omega_m}{dt} + 0.025\omega_m$$

$$100 = 2.025\omega_m + 0.5\frac{d\omega_m}{dt}$$

$$\frac{100}{s} = 2.025\omega_m(s) + 0.5\left[s\omega_m(s) - 47.124\right]$$

$$= 2.025\omega_m(s) + 0.5s\,\omega_m(s) - 23.562$$

$$\omega_m(s) = \frac{\frac{100}{s} + 23.562}{2.025 + 0.5s}$$

$$\omega_m(s) = \frac{100 + 23.562s}{0.5s(s+4.05)} = \frac{200 + 47.124s}{s(s+4.05)} = \frac{A_1}{s} + \frac{A_2}{s+4.05}$$

$$A_1 = s\omega_m(s)\big|_{s=0} = \frac{200}{4.05} = 49.38$$

$$A_2 = (s+1.0125)\omega_m(s)\big|_{s=-4.05} = \frac{200 - 47.124\times 4.05}{-4.05} = \frac{9.15}{-4.05} = -2.26$$

$$\omega_m(s) = \frac{49.38}{s} - \frac{2.26}{s+4.05}$$

$$\omega_m(t) = 49.38 - 2.26\,e^{-4.05t}$$

$$\omega_m(\infty) = 49.38$$

$$\omega_{mo} = 49.38 - 2.26 = 47.12$$

$$i(t) = \frac{V_t - K_m\omega_m}{R_a} = \frac{100 - 2\omega_m}{0.5} = 200 - 4\omega_m$$

$$= 200 - 4(49.38 - 2.26e^{-4.05t}) = 2.48 + 9.04e^{-4.05t}$$

$i\big|_0 = 2.48 + 9.04 = 11.52\text{ A}$ $\quad$ $i\big|_\infty = 2.48\text{ A} \rightarrow$ No load.

$\boxed{9.8}$ (a) $K_f I_f = 1\times 2 = 2$

$$100 = 2\omega_o + 0.5\times 2.469 = E_a + I_a R_a$$

$$\omega_o = \frac{100 - 1.24}{2} = 49.38 \text{ radian/sec} = \frac{49.38}{2\pi}\times 60 = 471.54 \text{ rpm}$$

$$T = K_f I_f I_a = 2\times 2.469 = 4.938 \text{ N.m}$$

(b) $100 = K_m\omega_m + R_a i_a$

$$K_m i_a = J\frac{d\omega_m}{dt} + B\omega_m + T_L$$

$$100 = K_m\omega_m + R_a\left[\frac{J}{K_m}\frac{d\omega_m}{dt} + B\frac{\omega_m}{K_m} + \frac{T_L}{K_m}\right]$$

$$= 2\omega_m + 0.5\left[\frac{2}{2}\frac{d\omega_m}{dt} + \frac{0.1}{2}\omega_m + \frac{10}{2}\right] = 2\omega_m + 0.5\frac{d\omega_m}{dt} + 0.025\omega_m + 2.5$$

$$97.5 = 0.5\frac{d\omega_m}{dt} + 2.025\omega_m$$

In s domain

$$\frac{97.5}{s} = 0.5\left[s\omega_m(s) - \omega_o\right] + 2.025\omega_m(s) = 0.5\left[s\omega_m(s) - 49.38\right] + 2.025\omega_m(s)$$

$$\omega_m(s) = \frac{\frac{97.5}{s} + 24.69}{0.5s + 2.025} = \frac{195 + 49.38s}{s(s+4.05)} = \frac{A_1}{s} + \frac{A_2}{s+4.05} = \frac{48.15}{s} + \frac{1.2319}{s+4.05}$$

$$\omega_m(t) = 48.15 + 1.2319\,e^{-4.05t}$$

$$\omega_m|_{ss} = \omega_m(\infty) = 48.15 \text{ rad/sec} \rightarrow 459.8 \text{ rpm}$$

$$I_a|_{ss} = \frac{V_t - K_m\omega_m}{R_a} = \frac{100 - 2\times 48.15}{0.5} = 7.4A$$

$$T|_{ss} = K_m I_a = 2\times 7.4 = 14.8 \text{ N.m}$$

$\boxed{9.9}$ (a) $\omega_{mo} = \frac{471.569}{60}\times 2\pi = 49.3827$ rad/sec

$$I_a = \frac{100 - 2\times 49.3827}{0.5} = 2.48A$$

$$K_m = K_f I_f = 2$$

$$T = K_m I_a = 2\times 2.48 = 4.96 \text{ N.m}$$

(b) I_f reduced to 1A Find $\omega_m(t)$

$$K_m = K_f i_f = 1\times 1 = 1$$

$$100 = K_m\omega_m + R_a i_a$$

$$K_m i_a = J\frac{d\omega_m}{dt} + B\omega_m$$

$$100 = K_m\omega_m + R_a\left(\frac{J}{K_m}\frac{d\omega_m}{dt} + \frac{B}{K_m}\omega_m\right) = K_m\omega_m + \frac{R_aJ}{K_m}\frac{d\omega_m}{dt} + \frac{R_aB}{K_m}\omega_m$$

$$= 1\times\omega_m + \frac{0.5\times 2}{1}\frac{d\omega_m}{dt} + \frac{0.5\times 0.1}{1}\omega_m = \omega_m + \frac{d\omega_m}{dt} + 0.05\omega_m$$

$$\frac{100}{s} = \omega_m(s) + s\,\omega_m(s) - \omega_{mo} + 0.05\omega_m(s)$$

$$\frac{100}{s} + 49.38 = \omega_m(s)(1.05 + s)$$

$$\omega_m(s) = \frac{100 + 49.38s}{s(s+1.05)} = \frac{A_1}{s} + \frac{A_2}{s+1.05}$$

$$A_1 = \frac{100}{1.05} = 95.24$$

$$A_2 = \frac{100 + 49.38(-1.05)}{-(1.05)} = \frac{100 - 51.85}{-1.05} = -45.86$$

$$\omega_m(t) = 95.24 - 45.86\,e^{-1.05t}$$

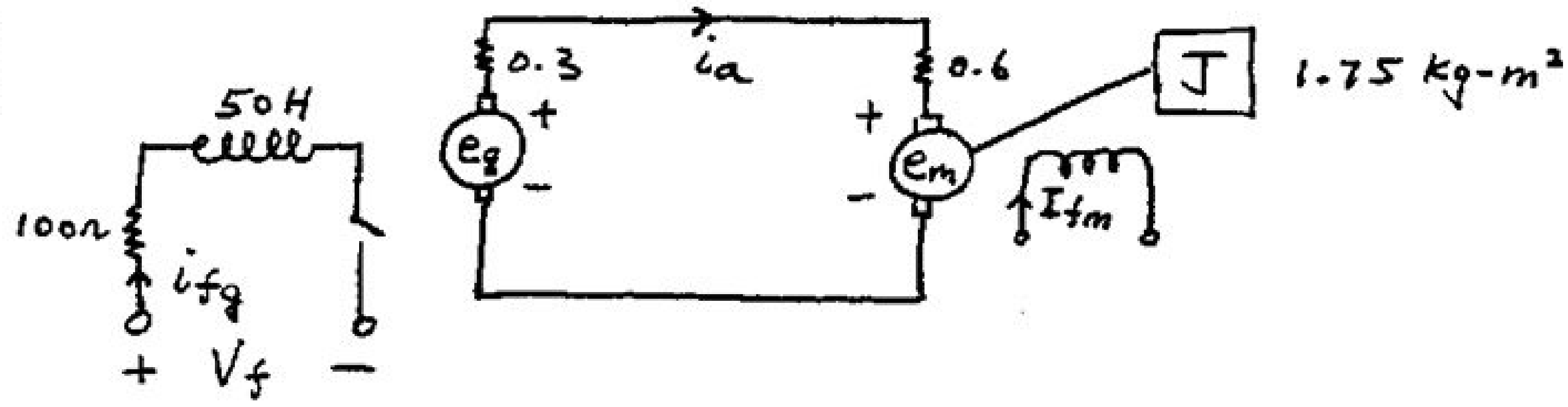
$$\omega_m|_0 = 95.24 - 45.86 = 49.38 \text{ rad/sec}$$

CHECK : $\omega_m|_\infty = 95.24$ rad/sec $= 909.4729$ rpm

$$I_a|_\infty = \frac{100 - 1\times 95.24}{0.5} = 9.52A$$

$$T = K_m I_a = 1\times 9.52 \text{ N.m}$$

9.10



$$V_f = R_f\, i_{fg} + L_f \frac{di_{fg}}{dt}$$

$$V_f(s) = R_f\, I_{fg}(s) + L_f\, s\, I_{fg}(s)$$

$$\frac{50}{s} = 100\, I_{fg}(s) + 50 s\, I_{fg}(s)$$

$$I_{fg}(s) = \frac{50}{s(100+50s)}$$

$$e_g = k_g\, i_{fg}$$

$$E_g(s) = K_g\, I_{fg}(s) = \frac{5000}{s(100+50s)}$$

$$i_a = \frac{e_g - e_m}{0.3+0.6} = \frac{e_g - k_m\omega_m}{0.9}$$

$$k_m i_a = J\frac{d\omega_m}{dt}$$

$$k_m \frac{e_g - k_m\omega_m}{0.9} = J\frac{d\omega_m}{dt}$$

$$\frac{1.1 e_g}{0.9} - \frac{(1.1)^2}{0.9}\omega_m = 1.75\frac{d\omega_m}{dt}$$

$$1.2222\, e_g - 1.3444\, \omega_m = 1.75\frac{d\omega}{dt}$$

$$1.2222\, E_g(s) - 1.3444\, \omega_m(s) = 1.75\, s\, \omega_m(s)$$

$$\omega_m(s) = \frac{1.2222\, E_g(s)}{1.3444 + 1.75s}$$

$$= \frac{1.2222 \times 5000}{s(100+50s)(1.3444+1.75s)}$$

$$= \frac{69.84}{s(2+s)(0.7682+s)}$$

$$= \frac{A}{s} + \frac{B}{2+s} + \frac{C}{0.7682+s}$$

$$= \frac{45.457}{s} + \frac{28.349}{2+s} - \frac{73.8057}{0.7682+s}$$

$$\omega_m(t) = 45.457 + 28.349e^{-2t} - 73.806\, e^{-0.7682t}$$

$\boxed{9.11}$ (a) $I_{dco}\big|_{max} = \frac{\sqrt{2}\,(1)}{0.2} = 7.07$ p.u.

(b) $$i_{sc}(t) = \sqrt{2}\left[\frac{1}{0.8} + \left(\frac{1}{0.35} - \frac{1}{0.8}\right)e^{-t/2.5} + \left(\frac{1}{0.2} - \frac{1}{0.35}\right)e^{-t/0.07}\right] \times \sin\omega t + 7.07\,e^{-t/0.25}$$

$$= \sqrt{2}\,(1.25 + 1.607e^{-0.4t} + 2.143\,e^{-14.3t})\sin\omega t + 7.07e^{-4t} \text{ p.u.}$$

(c) $$i_{sc}\big|_{t=0.2} = \sqrt{2}\,(1.25 + 1.607e^{-0.08} + 2.143\,e^{-2.86})\sin\omega t + 7.07e^{-0.8}$$

$$= \sqrt{2}\,(2.862)\sin\omega t + 3.182$$

$$I_{sc}\big|_{t=0.2} = \sqrt{2.862^2 + 3.182^2} = 4.28 \text{ p.u.}$$

$\boxed{9.12}$ (a) From pre-fault condition

$$E_i'' = V_t + I_a j X_d'' = 1 + 1\angle 0^\circ \cdot 0.3\angle 90^\circ = 1.044 \text{ p.u.}$$

$$I_{ac}\Big|_{max} = \frac{E_i''}{X_d''} = \frac{1.044}{0.3} = 3.48 \text{ p.u.}$$

(b) $$I_{dc}\Big|_{max} = \sqrt{2} \times 3.48 = 4.921 \text{ p.u.}$$

(c) $$I_{sc}\Big|_{max} = \sqrt{3.48^2 + 4.921^2} = 6.027 \text{ p.u.}$$

(d) From pre-fault condition

$$E_i = V_t + I_a j X_d = 1 + 1\angle 0^\circ\, 1\angle 90^\circ = 1.414 \text{ p.u.}$$

$$E_i' = 1 + 1\angle 0 \cdot 0.4\angle 90^\circ = 1.077 \text{ p.u.}$$

$$i_{sc}(t) = \sqrt{2}\left[\frac{1.414}{1} + \left(\frac{1.077}{0.4} - \frac{1.414}{1}\right)e^{-t/1.5} + \left(\frac{1.044}{0.3} - \frac{1.077}{0.4}\right)e^{-t/0.03}\right]\sin\omega t + 4.921 e^{-t/0.2}$$

$$= \sqrt{2}\left[1.414 + 1.2785 e^{-0.6667t} + 0.7875 e^{-33.3t}\right] \cdot \sin\omega t + 4.921 e^{-5t}$$

$$i_{sc}\Big|_{t=0.5} = \sqrt{2}\left[1.414 + 1.2785 \times 0.716 + 0.7875 \times 0\right] \times \sin\omega t + 4.921 \times 0.0821$$

$$= \sqrt{2}\,(2.3294) + 0.404$$

$$I_{sc}\Big|_{t=0.5} = \sqrt{2.3294^2 + 0.404^2} = 2.364 \text{ p.u.}$$

$\boxed{9.13}$ (a) Prefault conditions ; $V_t = 1.0$ p.u. $I_a = 1\angle 0^\circ$ p.u.

$$E_i = V_t + I_a j X_d = 1 + (1)(1.2)\angle 90^\circ = 1.562\angle 50.2^\circ$$

$$E_i' = 1.0 + (1)(0.4)\angle 90^\circ = 1.077\angle 21.8^\circ$$

$$E_i'' = 1.0 + (1)(0.25)\angle 90^\circ = 1.0308\angle 14^\circ$$

Rms s/c current

$$I_{sc}(t) = \frac{|E_i|}{X_d} + \left(\frac{|E_i'|}{X_d'} - \frac{|E_i|}{X_d}\right)e^{-t/\tau_d'} + \left(\frac{|E_i''|}{X_d''} - \frac{|E_i'|}{X_d'}\right)e^{-t/\tau_d''}$$

$$= \frac{1.562}{1.2} + \left(\frac{1.077}{0.4} - \frac{1.562}{1.2}\right)e^{-t/1.2} + \left(\frac{1.0308}{0.25} - \frac{1.077}{0.4}\right)e^{-t/0.025}$$

$$= 1.302 + 1.391e^{-t/1.2} + 1.4307\,e^{-t/0.025}\ \text{p.u.}$$

(b) $I_{sc}(0) = 1.302 + 1.391 + 1.4307 = 4.1237$ p.u.

$$I_{sc}(0.5) = 1.302 + 1.391(0.6592) + 1.4307(2.06\times10^{-9})$$

$$= 2.219\ \text{p.u.}$$

$$I_{sc}(5) \cong 1.302 + 1.391(0.0155)$$

$$= 1.3236\ \text{p.u.}$$

$\boxed{9.14}$ From prefault condition

$$E_i = V_t + I_a jX_d = 1 + 1\angle{-36.87^\circ}\times1\angle90^\circ = 1 + 1\angle53.13^\circ = 1+0.6+j0.8 = 1.79\angle26.6^\circ\text{pu}$$

$$E_i' = 1 + 1\angle{-36.87^\circ}\times0.5\angle90^\circ = 1 + 0.5\angle53.13^\circ = 1+0.3+j0.4 = 1.36\angle17.1^\circ\text{p.u.}$$

$$E_i'' = 1 + 1\angle{-36.87^\circ}\times0.25\angle90^\circ = 1+0.25\angle53.13^\circ = 1+0.15+j0.2 = 1.17\angle9.9^\circ\text{p.u.}$$

Worst fault condition

$$I_{dc}\big|_{max} = \sqrt{2}\cdot\frac{1.17}{0.25} = \sqrt{2}\times4.68 = 6.618\ \text{p.u.}$$

$$i_{sc}(t) = \sqrt{2}\left[\frac{1.79}{1} + \left(\frac{1.36}{0.5} - \frac{1.79}{1}\right)e^{-t/2} + \left(\frac{1.17}{0.25} - \frac{1.36}{0.5}\right)e^{-t/0.2}\right]\sin\omega t + 6.618\,e^{-t/0.5}$$

$$= \sqrt{2}\left[1.79 + (2.72-1.79)e^{-0.5t} + (4.68-2.72)e^{-5t}\right]\sin\omega t + 6.618\,e^{-2t}$$

$$= \sqrt{2}\left[1.79 + 0.93e^{-0.5t} + 1.98e^{-5t}\right]\sin\omega t + 6.618\,e^{-2t}$$

$$i_{sc}\big|_{t=0.5} = \sqrt{2}\left[1.79 + 0.93\times0.7788 + 1.98\times0.0821\right]\sin\omega t + 6.618\times0.3679$$

$$= \sqrt{2}\,(2.677)\sin\omega t + 2.435$$

$$I_{sc}\big|_{0.5} = \sqrt{2.677^2 + 2.435^2} = 3.6188\ \text{pu}$$

$$i_{sc}\big|_{0.5} = \sqrt{2}\,(2.677)\sin 2\pi\times60\times0.5 + 2.435 = 2.435\ \text{p.u.}$$

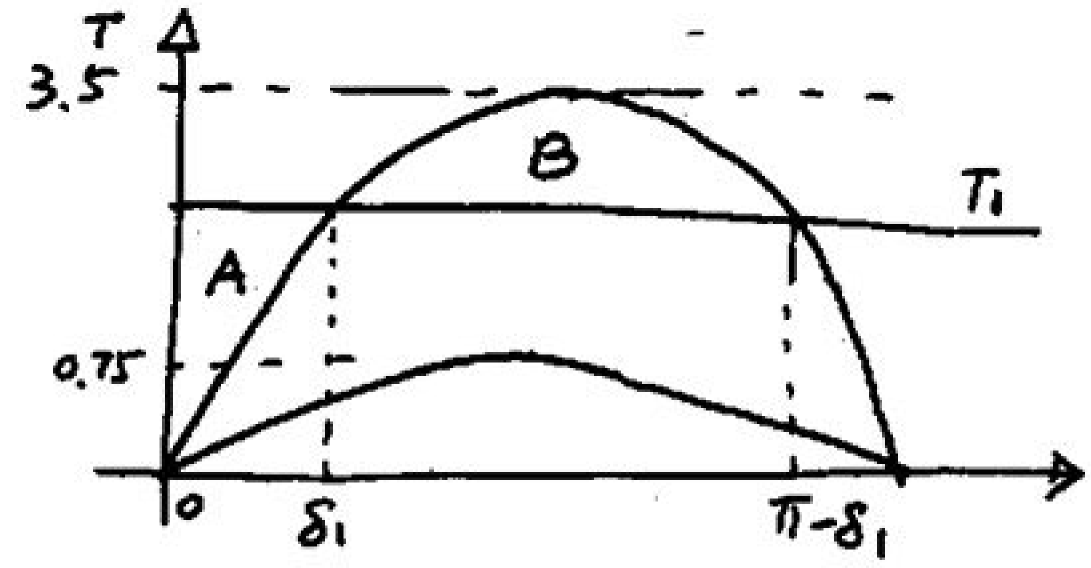
$\boxed{9.15}$ (a) $E_f = V_t - I_a jX_d = 1\angle 0^\circ - 0.5\angle -90^\circ \times 0.8\angle 90^\circ = 1 - 0.4 = 0.6$

$$T = \frac{V_t E_f}{\omega_s X_d}\sin\delta = \frac{1\times 0.6}{1\times 0.8}\sin\delta = 0.75\sin\delta$$

$T_{max} = 0.75\,p.u.$

(b) $E_i' = V_t - I_a jX_d' = 1 - 0.5\angle -90^\circ \times 0.25\angle 90^\circ = 1 - 0.125 = 0.875$

$$T = \frac{1\times 0.875}{1\times 0.25} = \frac{V_t E_i'}{\omega_s \cdot X_d'}\sin\delta = 3.5\sin\delta$$



$$A = T_1\delta_1 - \int_0^{\delta_1} 3.5\sin\delta\, d\delta$$
$$= T_1\delta_1 - 3.5(1-\cos\delta_1)$$

$$B = \int_{\delta_1}^{\pi-\delta_1} 3.5\sin\delta\, d\delta - T_1(\pi - 2\delta_1)$$
$$= 3.5(2\cos\delta_1) - T_1(\pi - 2\delta_1)$$

Area A = Area B

$T_1(\pi - \delta_1) - 3.5 - 3.5\cos\delta_1 = 0$

Also, $T_1 = 3.5\sin\delta_1$ Hence $3.5\sin\delta_1(\pi-\delta_1) - 3.5 - 3.5\cos\delta_1 = 0$

$(\pi - \delta_1)\sin\delta_1 - \cos\delta_1 = 1$

Solving for δ_1 (Trial & error)

$\delta_1 = 46.5^\circ$ or 0.8116 radian

Therefore, $T_1 = 3.5\sin 46.5^\circ = 3.5\times 0.7254 = 2.5388\ p.u.$

The torque T_1 can not be sustained for long, because steady state stability limit is 0.75 p.u.

$\boxed{9.16}$ Initially $T_1 = 1$ pu.

At unity p.f, $E_i' = V_t - I_a j X_d' = 1 - 1\angle 0^\circ \times 0.4\angle 90^\circ = 1.08\angle -21.8^\circ$ pu

$$T = \frac{|V_t||E_i'|}{W_{syn.} X_d'} \sin\delta = \frac{1 \times 1.08}{1 \times 0.4} \sin\delta = 2.7 \sin\delta.$$

$$\sin\delta_0 = \frac{1}{2.7} \rightarrow \delta_0 = 21.74^\circ = 0.3794 \text{ radian.}$$

$$\text{Area "A"} = T_2(\delta_1 - 0.3794) - \int_{0.3794}^{\delta_1} 2.7 \sin\delta \, d\delta$$

21.74°

$$= T_2(\delta_1 - 0.3794) - 2.7(\cos(0.3794) - \cos\delta_1)$$

$$= T_2(\delta_1 - 0.3794) - 2.7(0.9289 - \cos\delta_1)$$

$$\text{Area "B"} = 2.7(2\cos\delta_1) - T_2(\pi - 2\delta_1)$$

Area A = Area B.

$$T_2(2.7622 - \delta_1) - 2.7\cos\delta_1 - 2.7 \times 0.9289 = 0 \quad \text{--- (1)}$$

$$\text{But } T_2 = 2.7 \sin\delta_1 \quad \text{--- (2)}$$

Substituting (2) in (1),

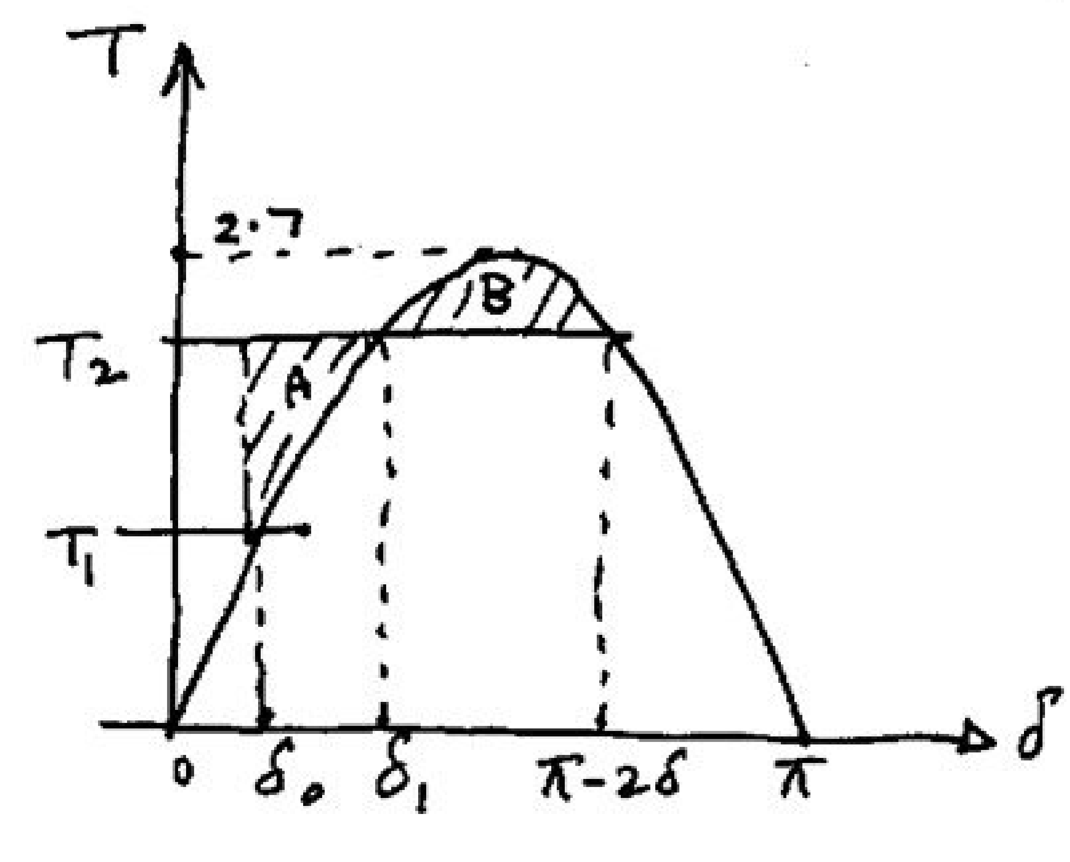
$$(2.7622 - \delta_1)\sin\delta_1 - \cos\delta_1 - 0.9289 = 0.$$

$$\delta_1 = 0.977 \text{ radian}$$

$$\rightarrow 56^\circ.$$

So, $T_2 = 2.7 \sin 56 = 2.2384$ pu.

An additional load torque of 1.2384 pu can be added.



<u>Steady State</u>

$$E_f = V_t - I_a j X_d$$

$$= 1 - 1\angle 0^\circ \times 1.2\angle 90$$

$$= 1.562 \text{ pu}$$

$$T_{max} = \frac{|V_t||E_f|}{W_{syn} X_d} = \frac{1 \times 1.562}{1 \times 1.2} = 1.3017 \text{ pu}.$$

Thus, the new torque (2.2384 pu) can not be sustained for long.

10.1 (a) From equation 10.1

$$V_{dc} = \frac{\sqrt{2}V_p}{\pi}$$

From Fig 10.17

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{\pi}(\sqrt{2}V_p \sin\theta)^2 d\theta} = \frac{V_p}{\sqrt{2}}$$

$$\frac{V_{rms}}{V_{dc}} = \frac{V_p}{\sqrt{2}} \times \frac{\pi}{\sqrt{2}V_p} = \frac{\pi}{2} = 1.5706$$

$$RF = \left\{\left(\frac{V_{rms}}{V_{dc}}\right)^2 - 1\right\}^{\frac{1}{2}} = (1.5706^2 - 1)^{\frac{1}{2}} = 1.2114$$

(b) From equation 10.2

$$V_{dc} = \frac{V_p}{\sqrt{2}\pi}(1 + \cos 90^\circ) = \frac{V_p}{\sqrt{2}\pi}$$

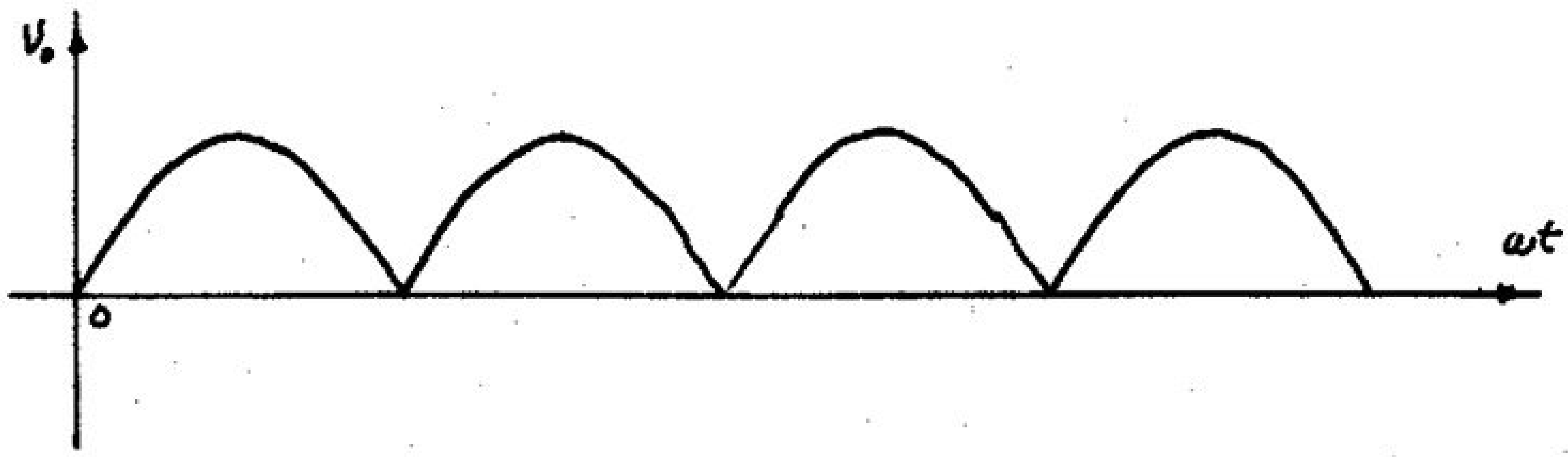
$$V_{rms} = \left\{\frac{1}{2\pi}\int_{\frac{\pi}{2}}^{\pi}(\sqrt{2}V_p \sin\theta)^2 d\theta\right\}^{\frac{1}{2}} = \frac{V_p}{2}$$

$$\frac{V_{rms}}{V_{dc}} = \frac{V_p}{2} \times \frac{\sqrt{2}\pi}{V_p} = \frac{\pi}{\sqrt{2}} = 2.2218$$

$$RF = (2.2218^2 - 1)^{\frac{1}{2}} = 1.984$$

RF is a measure of ripple content

10.2



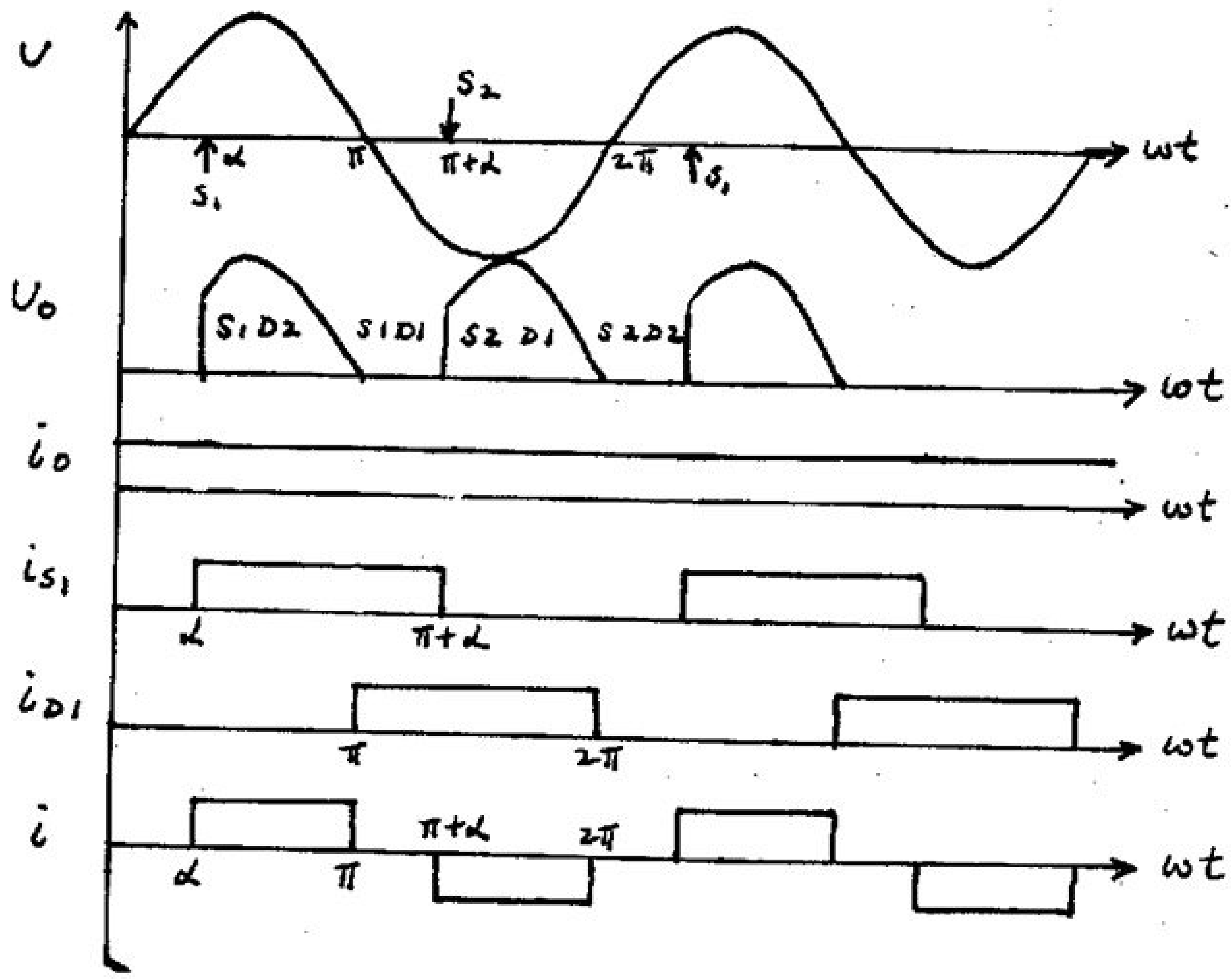
$$V_{dc} = \frac{1}{\pi}\int_0^{\pi}\sqrt{2}V_p \sin\theta\, d\theta = \frac{2\sqrt{2}}{\pi}V_p$$

$$V_{rms} = V_p$$

$$\frac{V_{rms}}{V_{dc}} = \frac{\pi}{2\sqrt{2}} = 1.111$$

$$RF = (1.111^2 - 1)^{\frac{1}{2}} = 0.4838$$

10.3 (a)



(b) $V_0 = \dfrac{\sqrt{2} \times 120}{\pi} \times (1 + \cos 60°) = 81.02\,V$

$P_{motor} = 81.02 \times 15 = 1215.3\,W$

(c) $V = 120\,V, \quad \alpha = 60°$

$I = 15\sqrt{\dfrac{120}{180}} = 12.25\,A$

$VA = 120 \times 12.25 = 1469.7\,W$

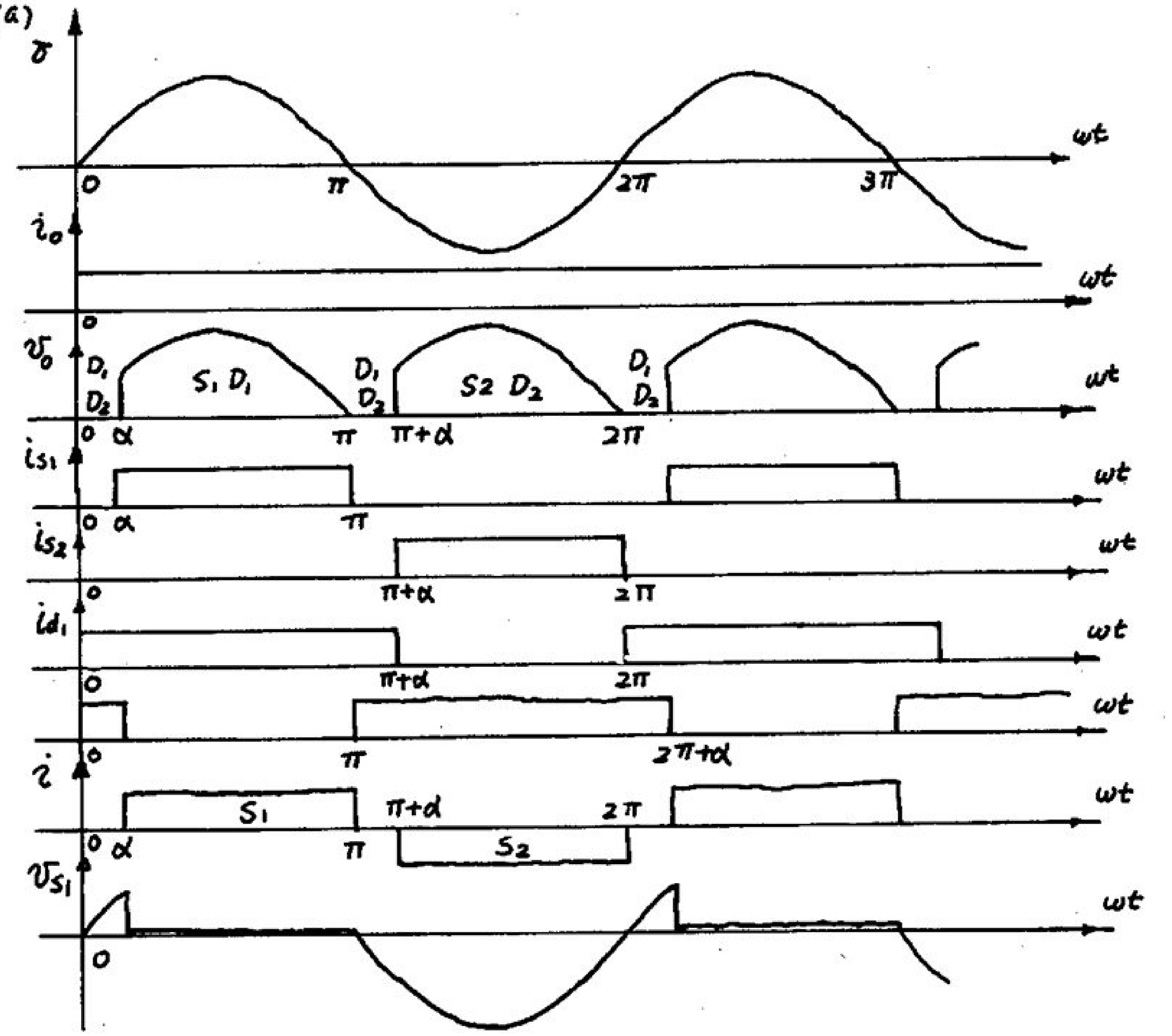
$P_s = P_{motor} = 1215.3\,W$

$PF = \dfrac{1215.3}{1469.7} = 0.827$

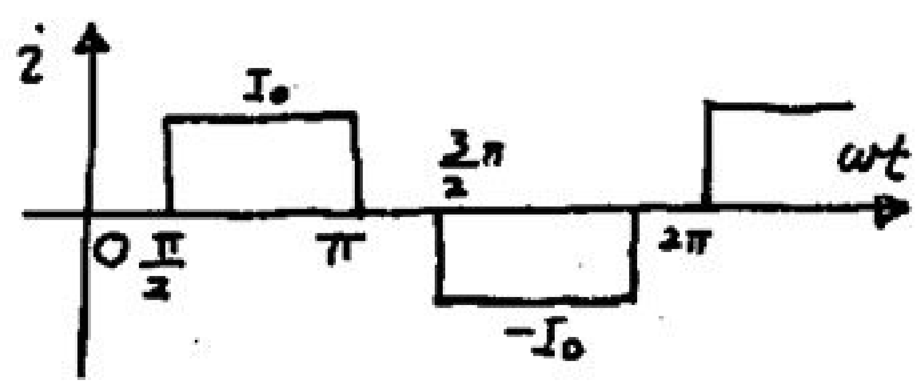
(d) $I_{SCR} = \dfrac{15}{\sqrt{2}} = 10.61\,A$

$I_{diode} = 10.61\,A$

10.4 (a)



(b) $V_0 = \frac{\sqrt{2} \times 208}{\pi}(1 + \cos 90°) = 93.62\,V$



$I_0 = \frac{93.62}{10} = 9.362\,A$

$I = \sqrt{\frac{1}{\pi}\int_{\pi/2}^{\pi} I_0^2\, d\theta} = \frac{I_0}{\sqrt{2}} = \frac{9.362}{\sqrt{2}} = 6.62\,A$

$P_0 = I_{0R}^2 R = I_0^2 R = 9.362^2 \times 10 = 876.47\,W$

or $P_0 = V_0 I_0 = 93.62 \times 9.362 = 876.47\,W \longrightarrow I_0$ is ripple-free because $L \to \infty$

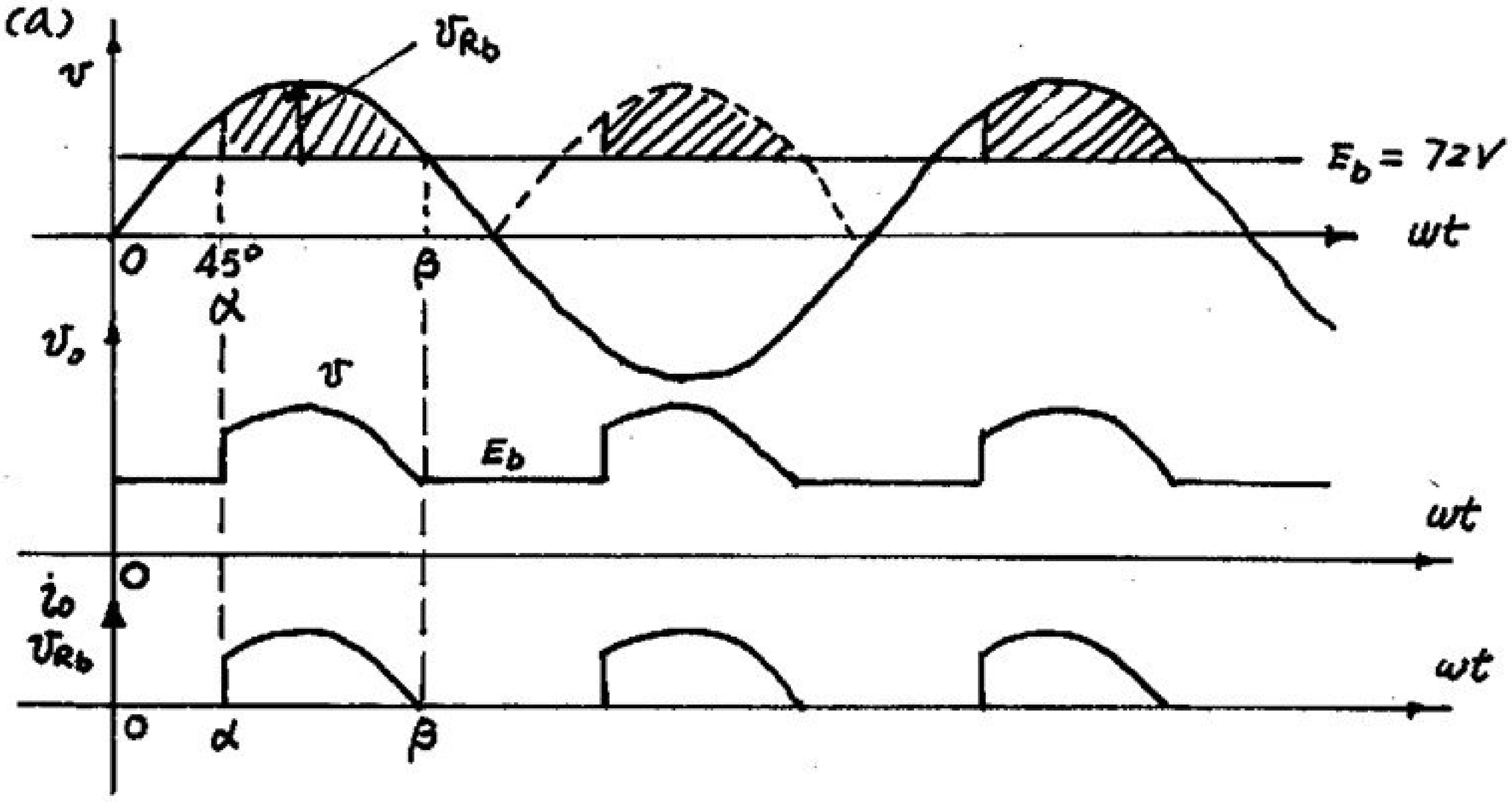
Supply $VA = 208 \times 6.62 = 1376.96$

supply $PF = \frac{876.47}{1376.96} = 0.637$

(c) $0 < \alpha < 180°$

(d) No $\rightarrow v_0$ can not be negative, because of D_1 & D_2

10.5 (a)



(b) i_o discontinuous

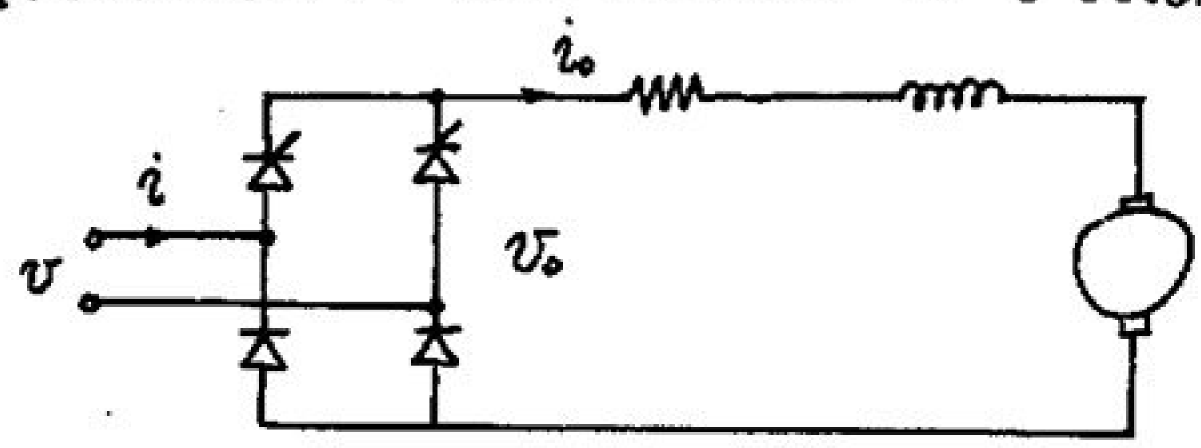
$\alpha = 45°$, $E_b = 7.2 \times 10 = 72$ volt

$\sqrt{2}\, 120 \sin\beta = 72 \longrightarrow \beta = 25°$

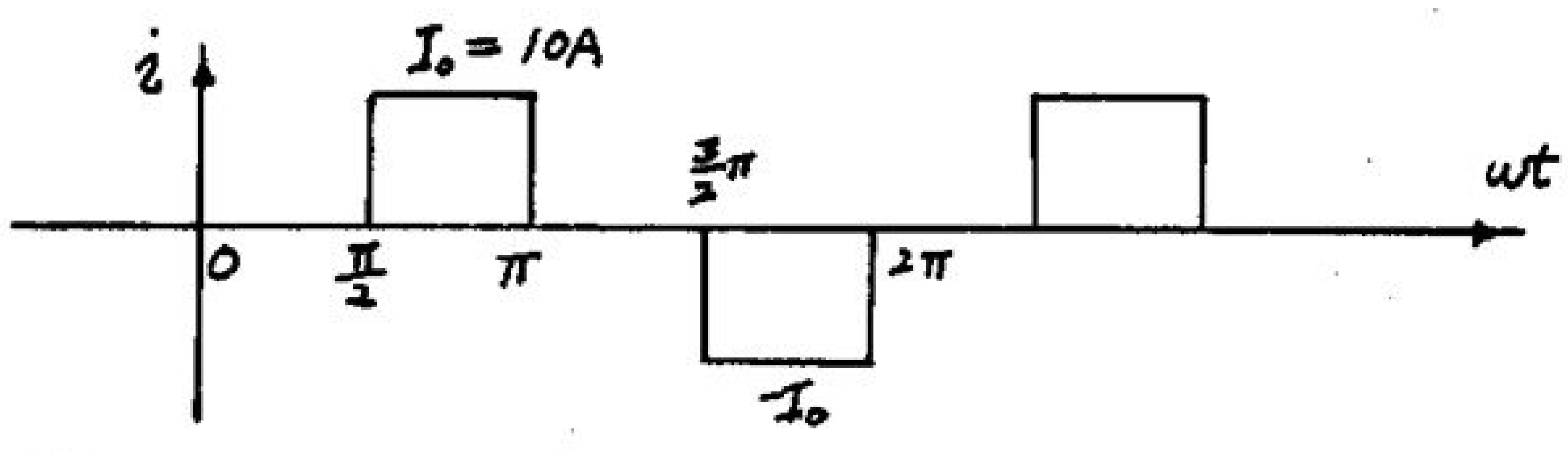
$$I_o = \frac{1}{\pi}\int_{\alpha = 45°}^{\beta = 180° - 25°} \frac{\sqrt{2} \times 120 \sin\theta - 72}{0.6}\, d\theta = 72.9A$$

(c) $P = E_b I_o = 72 \times 75.9 = 5465\ W$

10.6 (a) This is a 1ϕ semi-converter $\rightarrow$ i becomes zero at π, 2π



(b)



$i = \Sigma(a_n \sin n\theta + b_n \cos n\theta) = \Sigma\, C_n \sin(n\theta + \phi_n)$

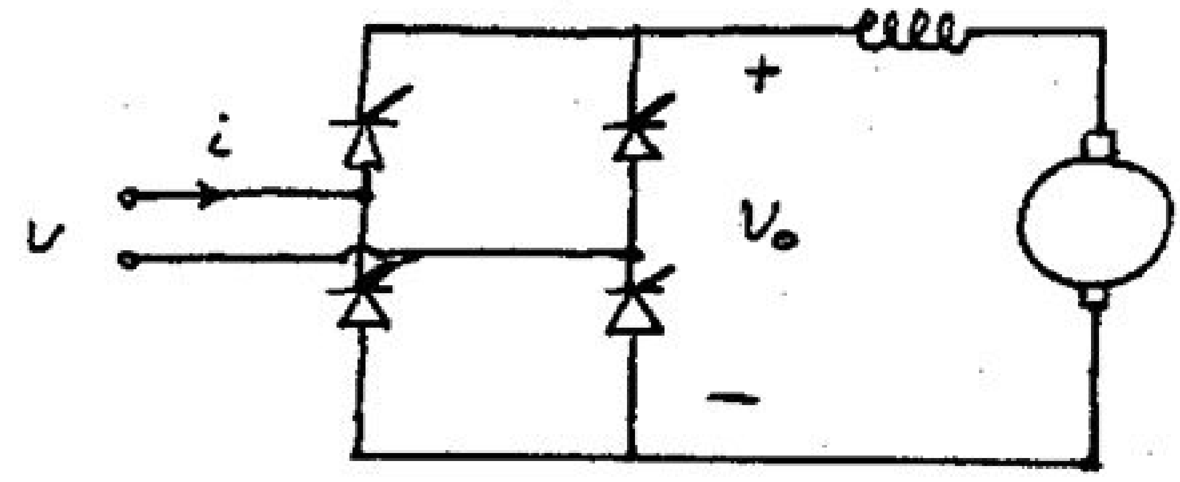
$a_1 = \frac{2}{\pi}\int_{\frac{\pi}{2}}^{\pi} I_o \sin\theta\, d\theta = \frac{2I_o}{\pi}\Big[-\cos\theta\Big]_{\pi/2}^{\pi} = \frac{2I_o}{\pi}(-\cos\pi + \cos\frac{\pi}{2}) = \frac{2I_o}{\pi}$

$b_1 = \frac{2}{\pi}\int_{\pi/2}^{\pi} I_o \cos\theta\, d\theta = \frac{2I_o}{\pi}\big[\sin\theta\big]_{\pi/2}^{\pi} = \frac{2I_o}{\pi}(\sin\pi - \sin\frac{\pi}{2}) = -\frac{2I_o}{\pi}$

$c_1 = \left[\left(\frac{2I_o}{\pi}\right)^2 + \left(-\frac{2I_o}{\pi}\right)^2\right]^{\frac{1}{2}} = \frac{2\sqrt{2}}{\pi} I_o \qquad \phi_1 = \tan^{-1}\frac{b_1}{a_1} = \tan^{-1}(-1) = -45°$

RMS Fundamental current $I_1 = \frac{c_1}{\sqrt{2}} = \frac{2I_o}{\pi} = \frac{2 \times 10}{\pi} = 6.37A \quad \phi_1 = -45°$

10.7 (a)



(b)(i) $V_o = 0.055 \times 1000 = 55\text{ V}$

$$I_o = \frac{5 \times 746}{110} = 33.9\text{ A}$$

$$V_o = \frac{2\sqrt{2}\,V}{\pi}\cos\alpha = 108.02\cos\alpha = 55\text{ V}$$

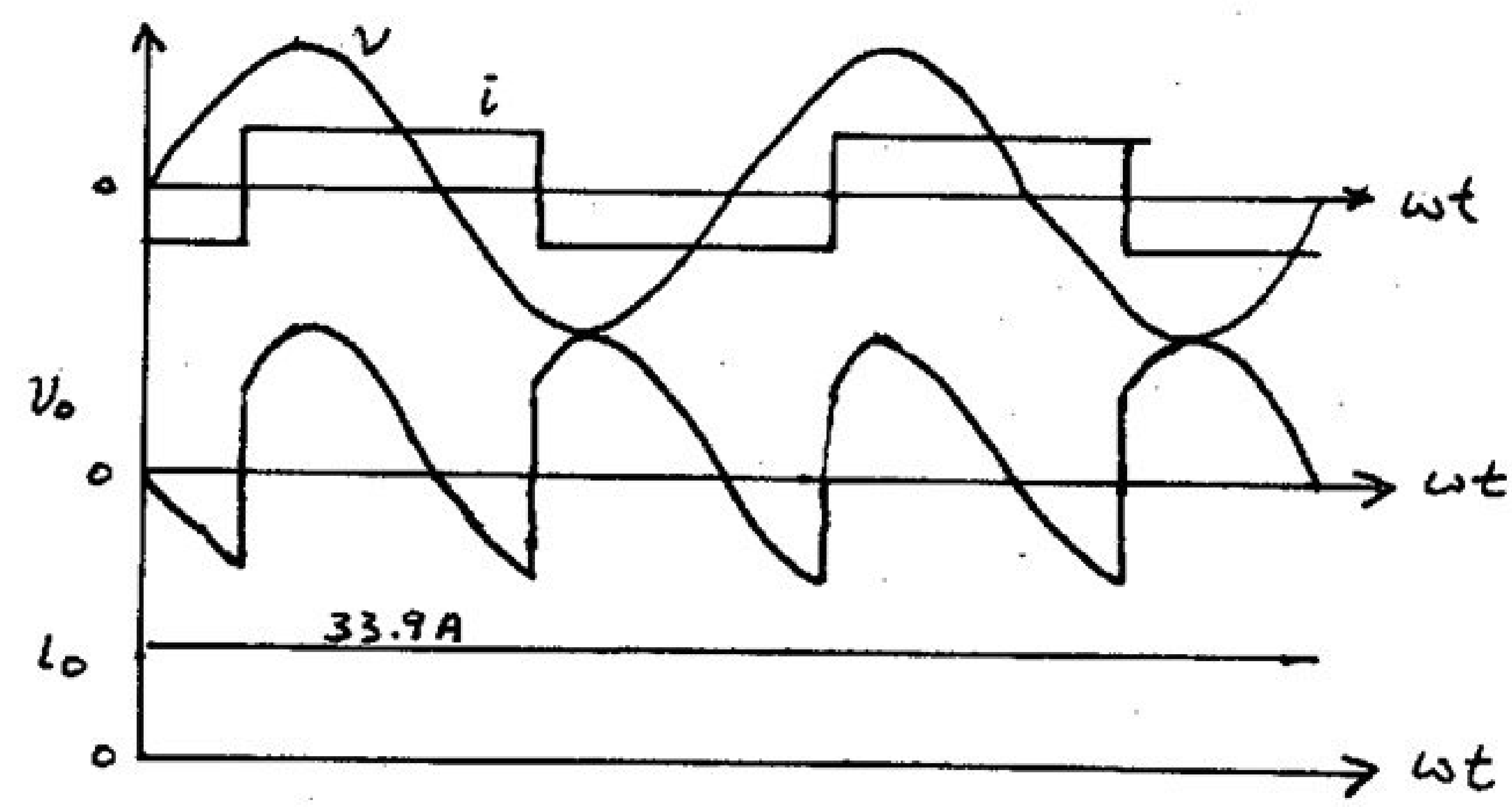
$\alpha = 60^\circ \qquad \cos\alpha = 0.5$

(ii) $I = I_o = 33.9\text{A}$

$$I_{Th} = \frac{33.9}{\sqrt{2}} = 23.97\text{ A}$$

(iii) $$PF = \frac{55 \times 33.9}{120 \times 33.9} \simeq 0.5$$

(iv)



$\boxed{10.8}$ (a) $V_o = \frac{2\sqrt{2}\times 120}{\pi}\cos 60^\circ = 54V \quad E_a = 0.055\times 200 = 11V$

$I_o = \frac{V_o - E_a}{R} = \frac{54-(-11)}{1} = 65A$

(b) DC machine is delivering power $P_a = 11\times 65 = 715W$

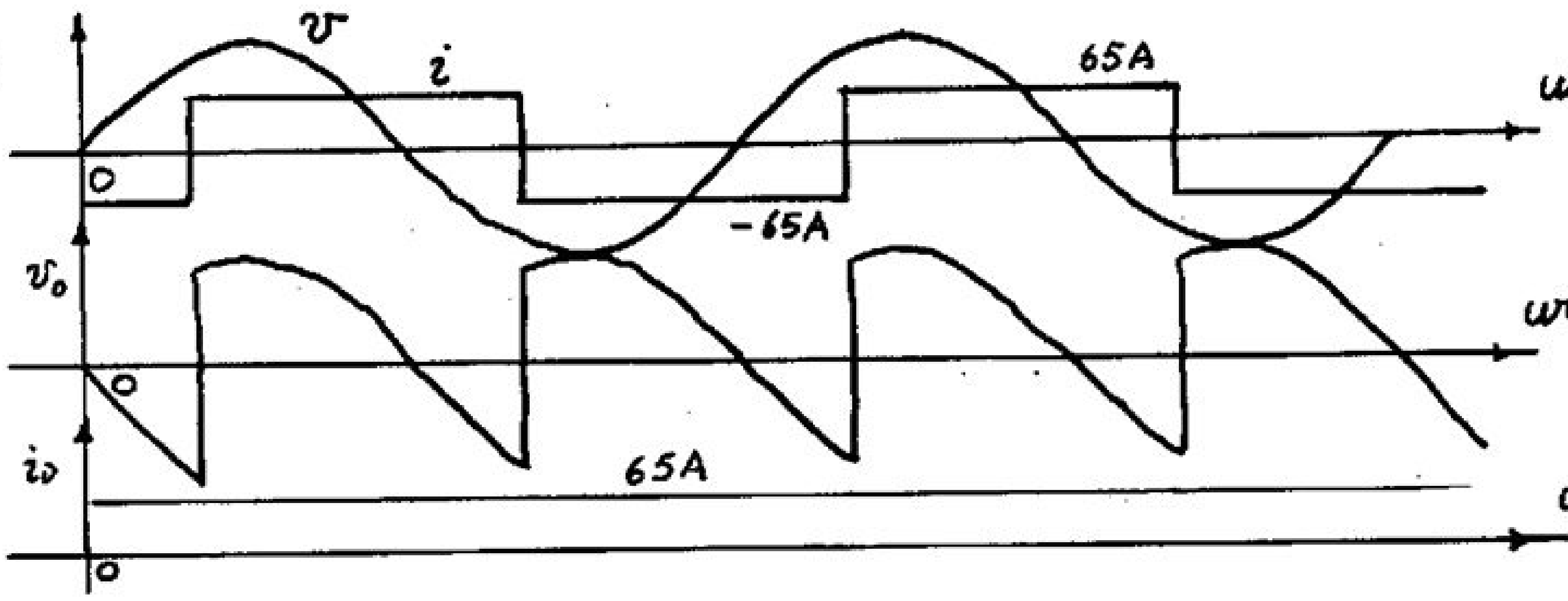
(c) AC supply is delivering power $P_{ac} = P_o = V_o I_o = 54\times 65 = 3510W$

(d) All powers are going to the resistance R, $P_R = 65^2\times 1 = 4225W (= P_a + P_{ac})$

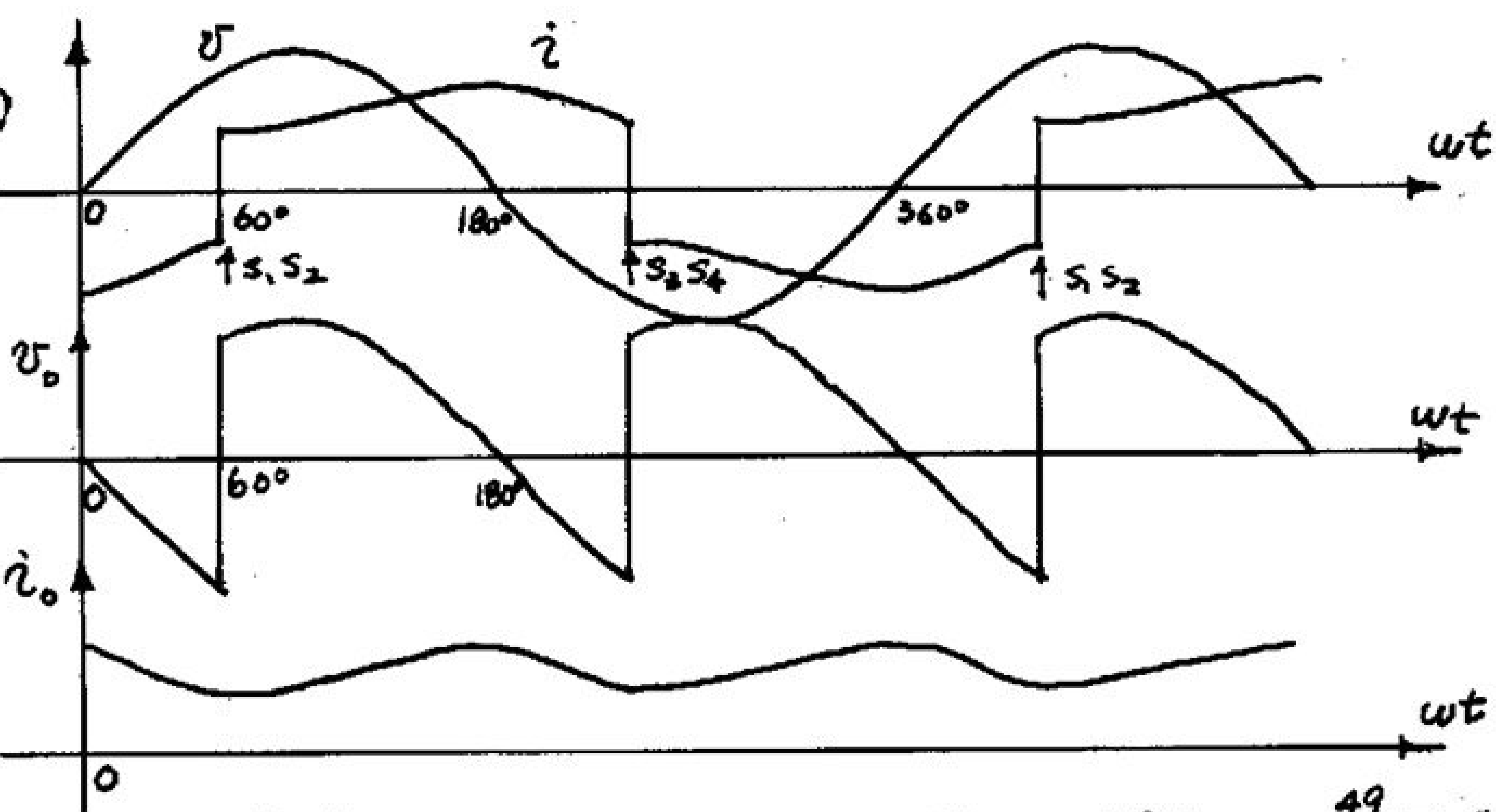
(e) Supply volt-ampere $S = 120\times 65 = 7800VA$

$P.F. = \frac{P_{ac}}{S} = \frac{3510}{7800} = 0.45$

(f)



$\boxed{10.9}$ (i)



(ii) $V_o = \frac{2\times\sqrt{2}\times 120}{\pi}\cos 60^\circ = 54V \quad E_a = 54 - 0.25\times 20 = 49V \quad n = \frac{49}{0.1} = 490\ rpm$

(iii) $I_o = 20A \quad I_{o(rms)} = \sqrt{20^2 + (0.2\times 20)^2} = 20.4A$

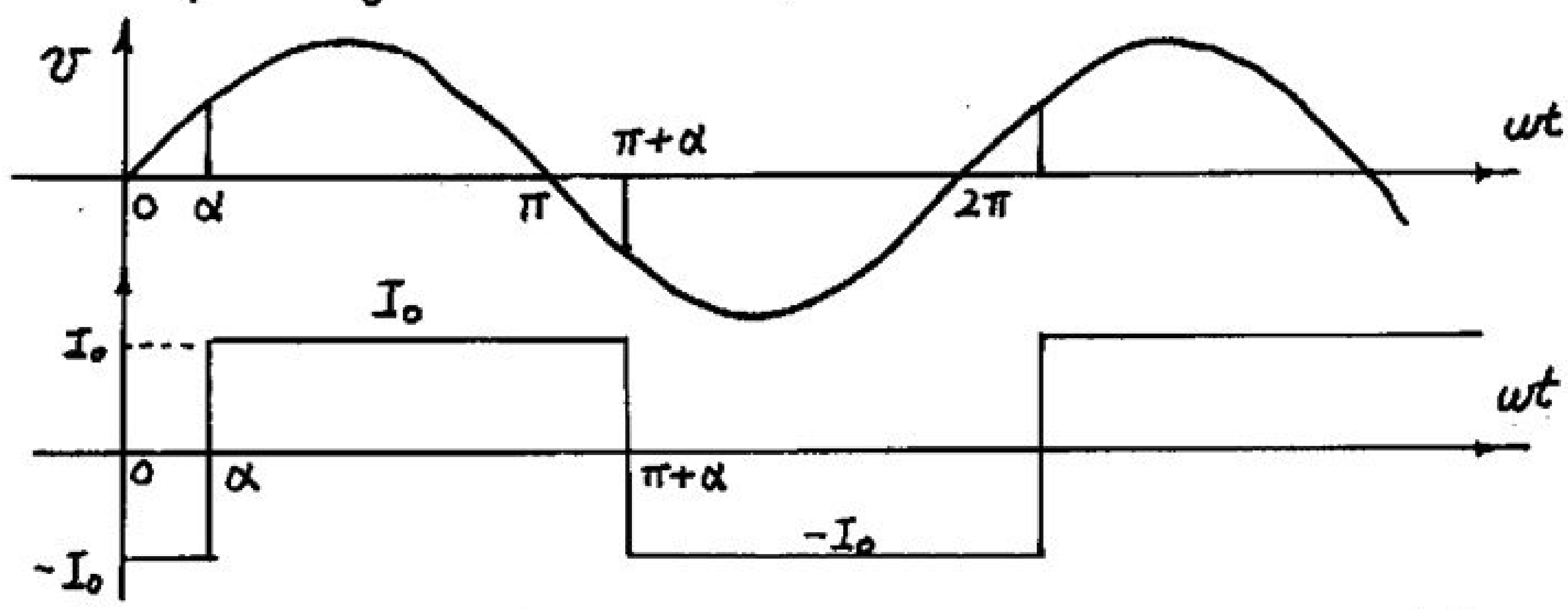
ripple content

$P_R = 20.4^2\times 0.25 = 105W \quad P_a = E_a I_o = 49\times 20 = 980W$

(iv) $P_s = P_o = P_R + P_a = 105 + 980 = 1085W$

$PF = \frac{1085}{120\times 20.4} = \frac{1085}{2448} = 0.443$

$\boxed{10.10}$ (a) Refer to Fig 10.21, the waveforms are drawn as follows.



(b) supply current i may be represented by the Fourier as follows

$$i = \sum_{n=1}^{\infty} (A_n \sin n\theta + B_n \cos n\theta) = \Sigma\, I_n \sin(n\theta + \phi_n)$$

where $I_n = (A_n^2 + B_n^2)^{\frac{1}{2}}$ $\quad \phi_n = \tan^{-1}\frac{B_n}{A_n}$

For fundamental current

$$A_1 = \frac{2}{\pi}\int_{\alpha}^{\pi+\alpha} I_0 \sin\theta d\theta = \frac{2I_0}{\pi}\left[-\cos\theta\right]_{\alpha}^{\pi+\alpha} = \frac{4I_0}{\pi}\cos\alpha$$

$$B_1 = \frac{2}{\pi}\int_{\alpha}^{\pi+\alpha} I_0 \cos\theta d\theta = \frac{2I_0}{\pi}\left[\sin\theta\right]_{\alpha}^{\pi+\alpha} = -\frac{4I_0}{\pi}\sin\alpha$$

$$i_1 = A_1 \sin\theta + B_1 \cos\theta$$

$$= I_1 \sin(\theta + \phi_1) \text{ where } I_1 = \sqrt{a_1^2 + b_1^2} = \frac{4I_0}{\pi} \quad \phi_1 = \tan^{-1}\frac{b_1}{a_1} = -\alpha$$

$$i_1(\omega t) = \frac{4I_0}{\pi}\sin(\omega t - \alpha)$$

Hence the phase angle ϕ_1 of the fundamental component of supply current is α, i.e. the same as the firing angle.

(c) Average output voltage $V_0 = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} \sqrt{2}\, V \sin\theta\, d\theta = \frac{2\sqrt{2}}{\pi} V\cos\alpha$

Average output current $= I_0 =$ output rms current

Power delivered by ac source at output $= V_0 I_0$

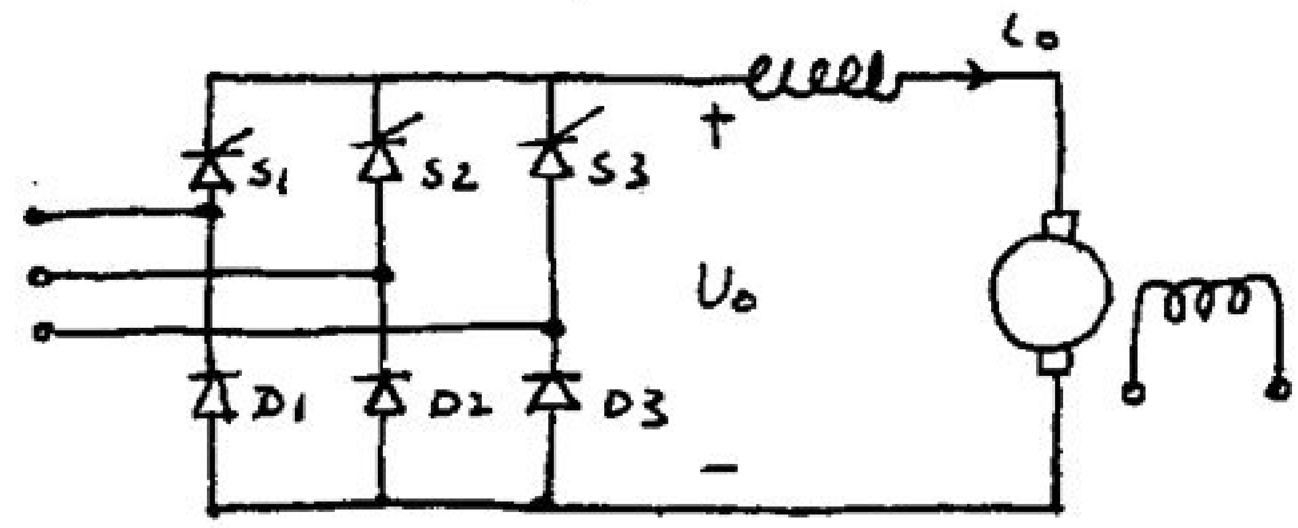
rms input current $= I_0$

Input voltamperes $= V I_0$

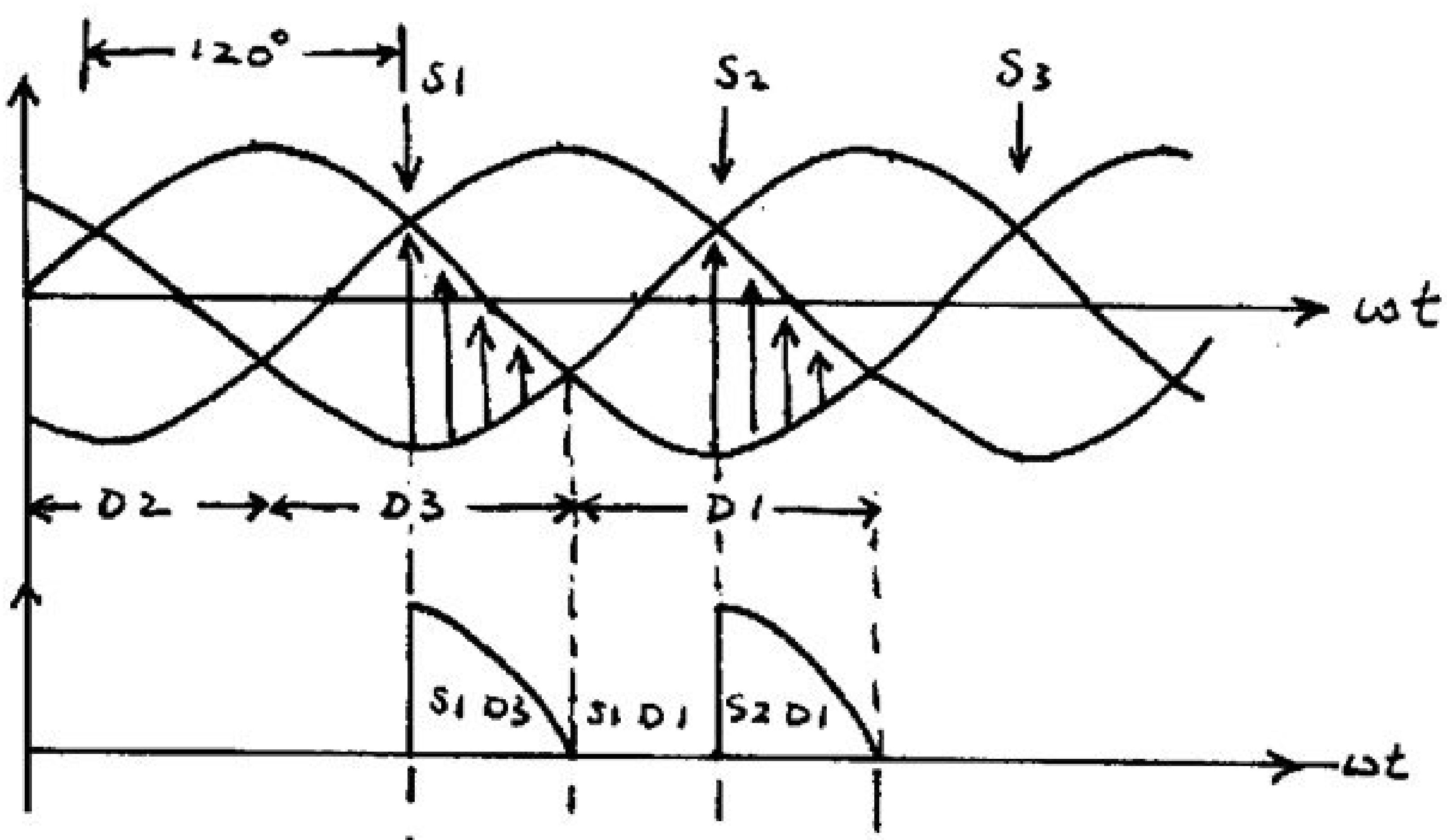
$$\text{Supply powerfactor} = \frac{\text{input power}}{\text{input voltamps}} = \frac{V_0 I_0}{V I_0} = \frac{V_0}{V}$$

$$= \frac{2\sqrt{2}}{\pi}\cos\alpha \quad \propto \cos\alpha$$

$\boxed{10.11}$ (a)



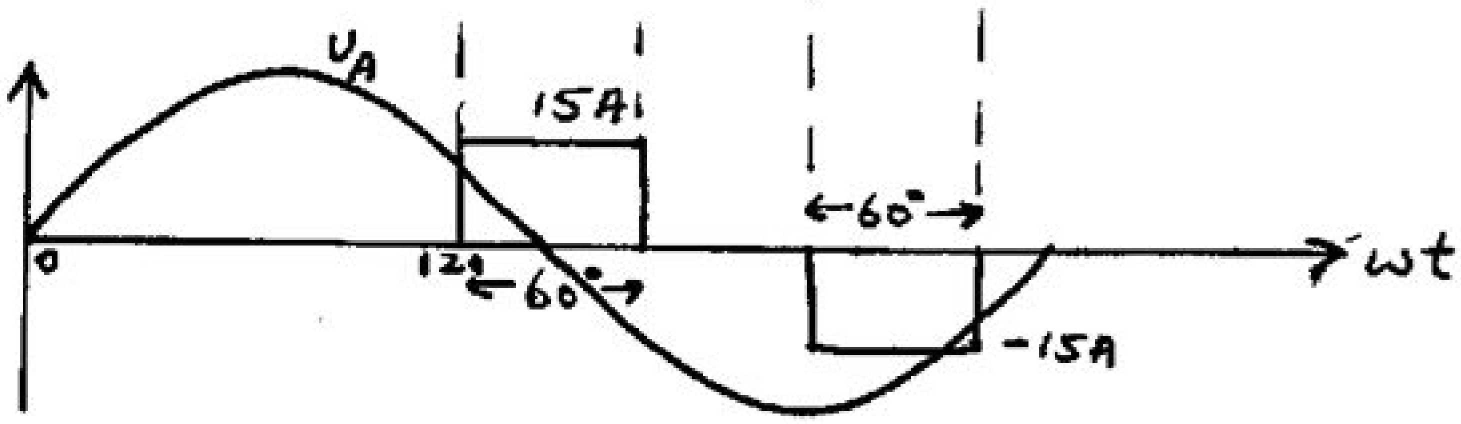
(b)



(c) $V_o = \dfrac{3\sqrt{2} \times 208}{2\pi} (1 + \cos 120°)$

$= 70.21\ V$

(d)



(e) Width $= 60°$

$I_A = 15\sqrt{\dfrac{60}{180}} = 8.66\,A$

$P_o = 70.21 \times 15 = 1053.15\ W$

Input $VA = \sqrt{3} \times 208 \times 8.66 = 3119.8$

$PF = \dfrac{1053.15}{3119.8} = 0.34$

$\boxed{10.12}$ (a) From equation 10.10

$$V_o = \frac{3\sqrt{2} \times 110}{\pi} \cos 50^\circ = 95.4729\ V$$

$$E_a = 0.1 \times 900 = 90\ V$$

$$I_o = \frac{95.4729 - 90}{0.2} = \frac{5.4729}{0.2} = 27.3643\ A$$

(b) Each thyristor conducts load current for 120° in each cycle

$$I_{Th} = \frac{27.3643}{\sqrt{3}} = 15.8\ A$$

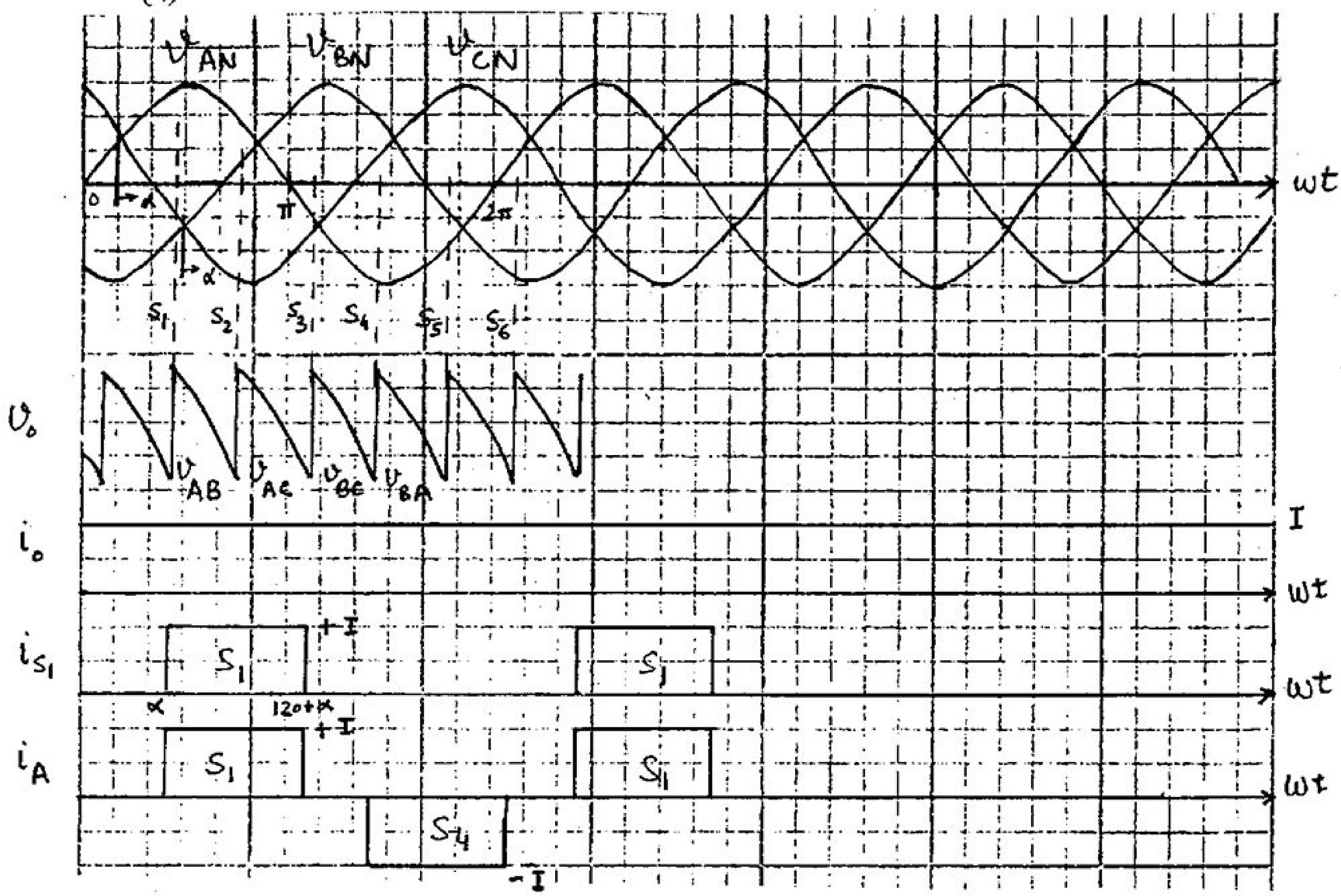
$$I_{Line} = \sqrt{\frac{2}{3}} \times 27.3643 = 22.3429\ A$$

(c) Supply Volt-ampere $= \sqrt{3} \times 110 \times 22.3429 = 4256.77\ VA$

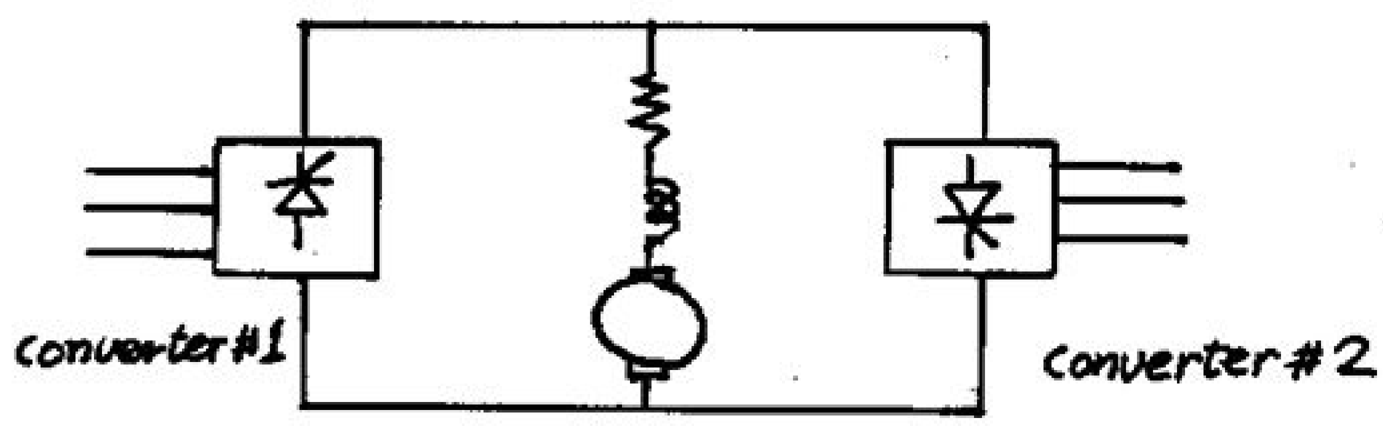
$$P_{out} = V_o I_o = 95.4729 \times 27.3643 = 2612.55\ W$$

$$PF = \frac{2612.55}{4256.77} = 0.6137$$

(d)

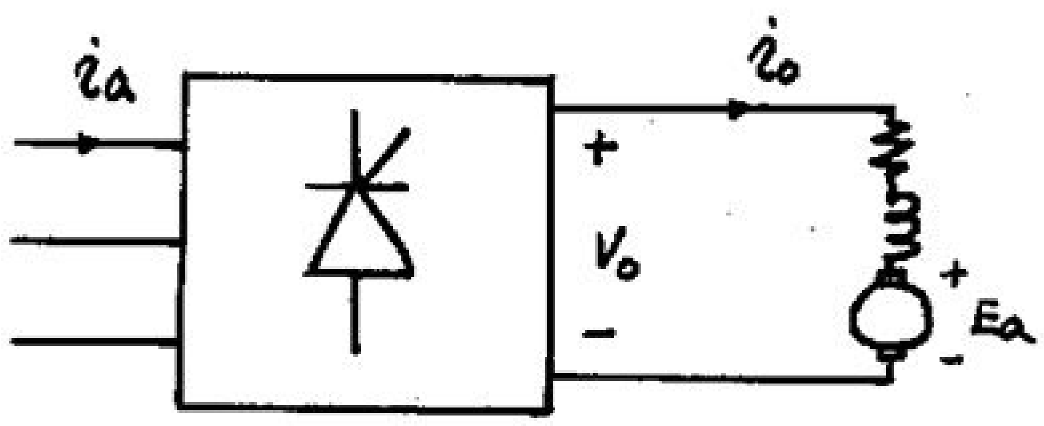


1013 (a)



Converter #1 can be used for motoring and converter #2 for regenerative braking

(b) (i)



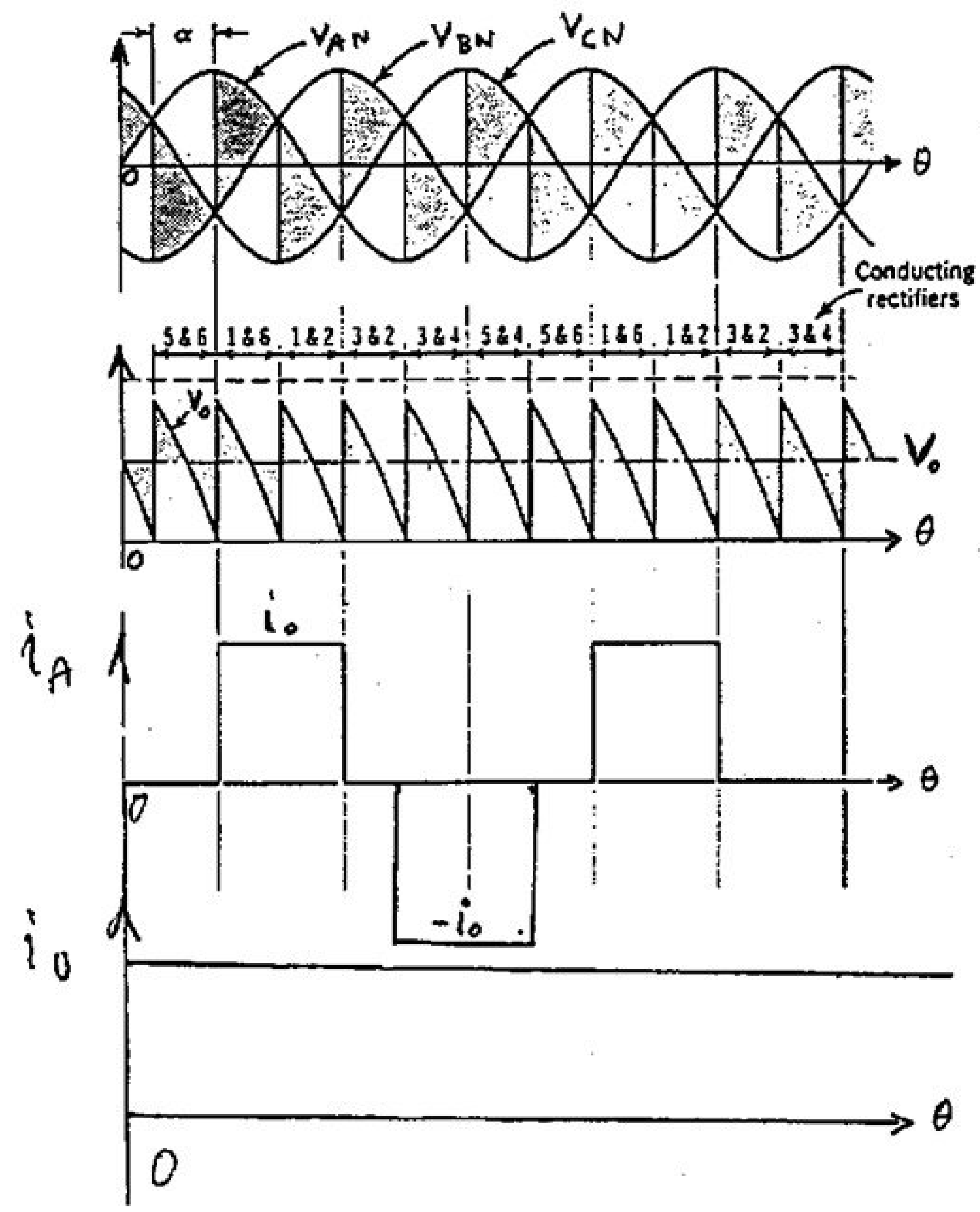
(ii) $V_o = \frac{3\sqrt{6}\times 277}{\pi}\cos 60^\circ = 323.9631\,V$

$E_a = 323.9631 - 130\times 0.1 = 310.9631\,V$

$n = \frac{310.9631}{0.3} = 1036.5438$ rpm

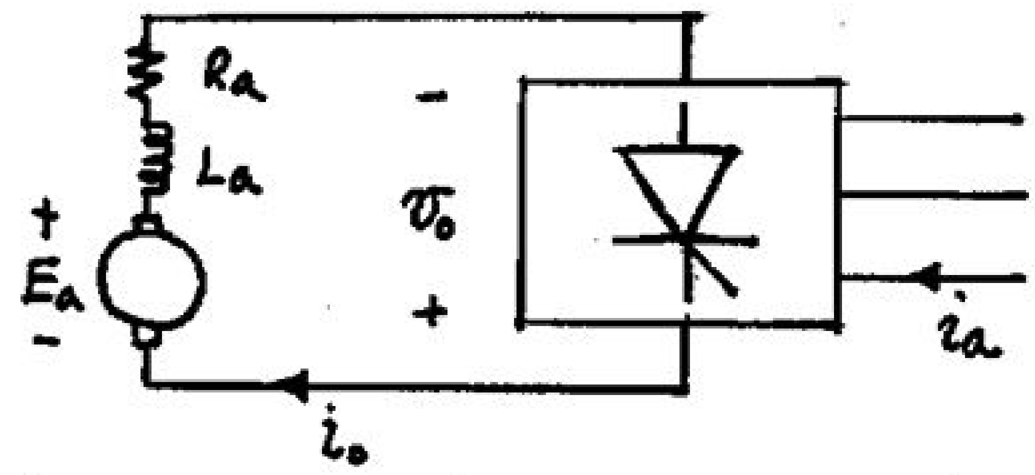
(iii) $P_s = P_o = V_o I_o = 323.9631\times 130 = 42.1152$ KW

(iv)



10·13

(c) (i)



(ii) $V_o = \frac{3\sqrt{6}\times 277}{\pi}\cos 120^\circ = -323.9631\,V$

$V_o = -E_a + I_a R_a$

$E_a = -V_o + I_a R_a = 323.9631 + 130\times 0.1 = 336.9631\,V$

$n = \frac{336.9631}{0.3} = 1123.21$ rpm

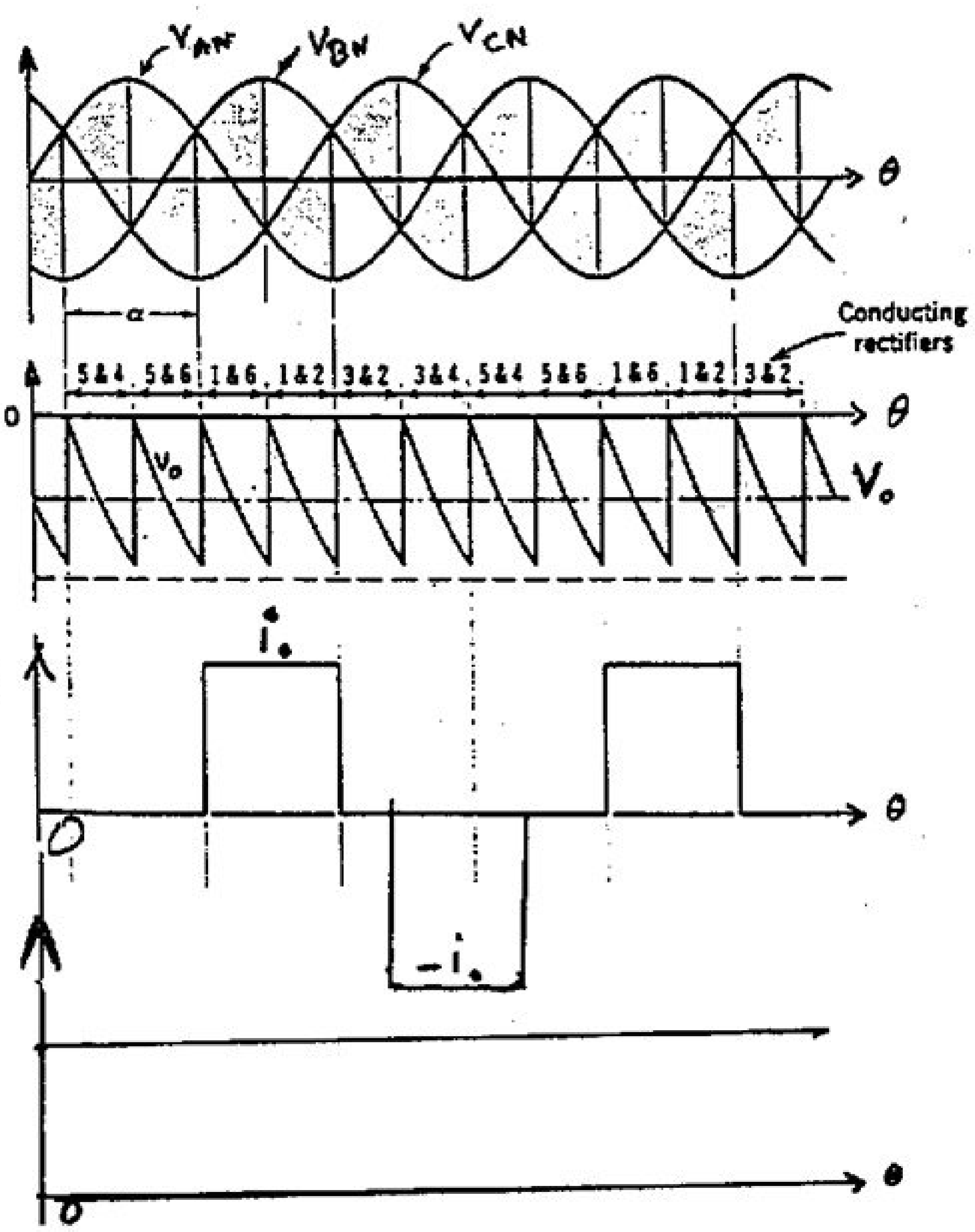
(iii) $P_a = 336.9631\times 130 = 43{,}805.2\,W$ (delivered by E_a)

$P_R = 130^2\times 0.1 = 1690\,W$ (absorbed by R_a)

$P_{ac} = P_a - P_R = 42115.2\,W \rightarrow$ fed back to supply

or $V_o I_o = P_{ac} = 323.9631\times 130 = 42115.2\,W$

(iv)



$\boxed{10.14}$

$$P_{av} = \frac{1}{\pi}\int_{\alpha}^{\pi} \frac{V_R^2}{R}\, d\theta$$

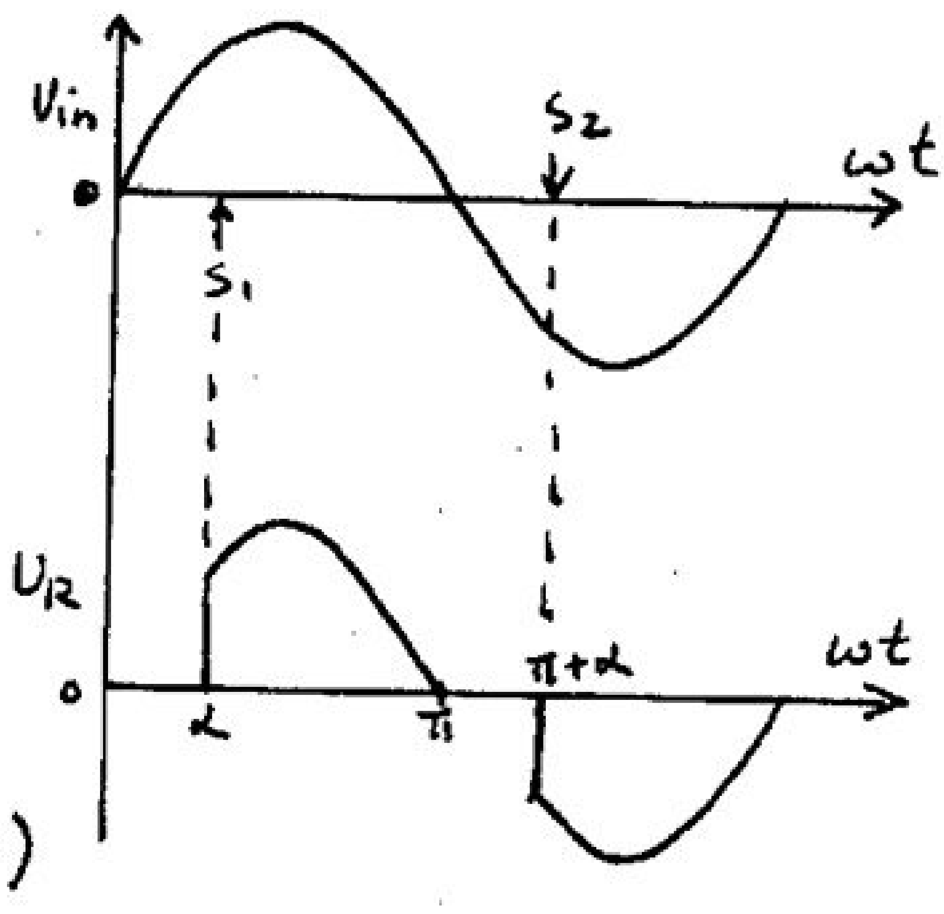
$$= \frac{1}{\pi R}\int_{\alpha}^{\pi} (\sqrt{2}\, V_{RMS} \sin\theta)^2 d\theta$$

$$= \frac{2 V_{RMS}^2}{\pi R}\int_{\alpha}^{\pi} \frac{1-\cos 2\theta}{2}\, d\theta$$

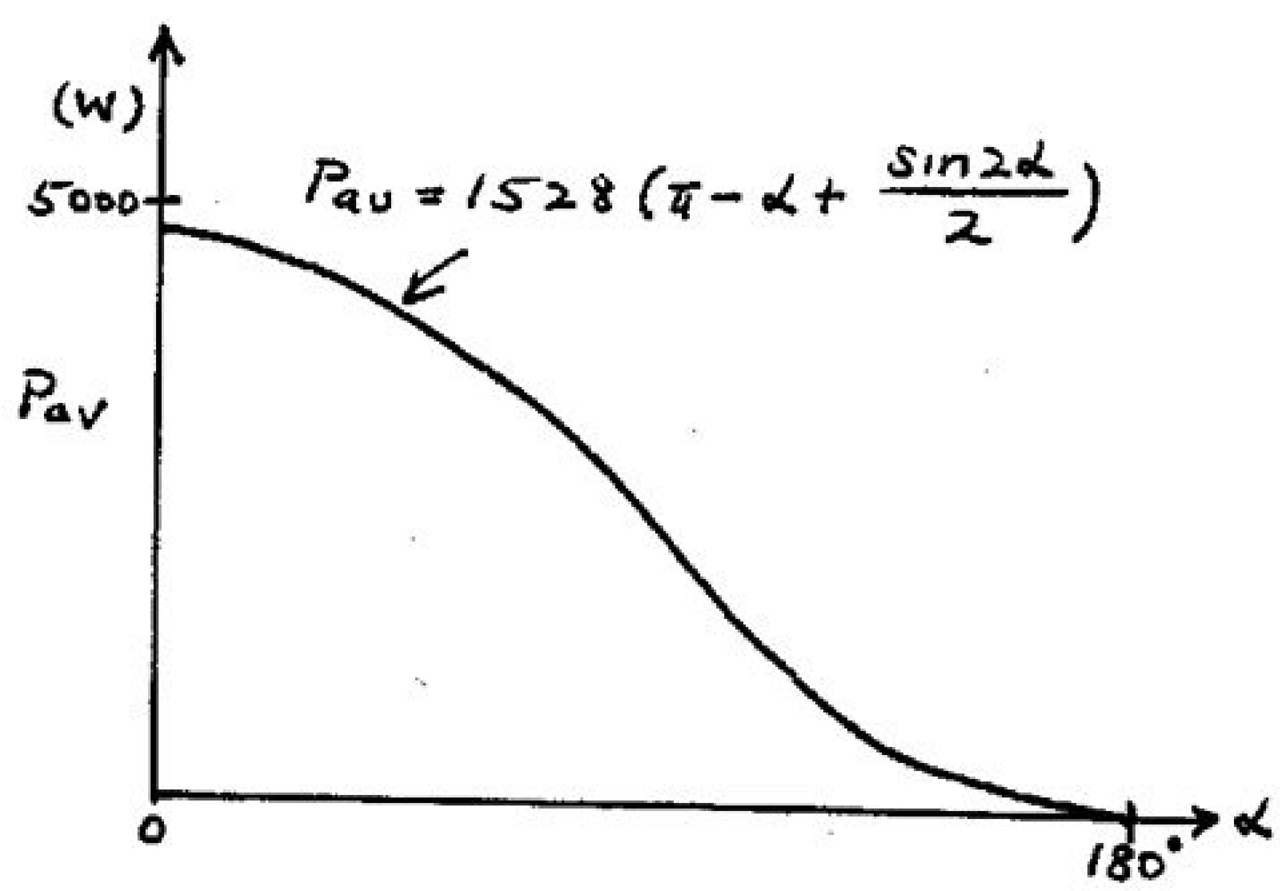
$$= \frac{V_{RMS}^2}{\pi R}\left[\theta - \frac{\sin 2\theta}{2}\right]_{\alpha}^{\pi}$$

$$= \frac{120^2}{\pi 3}\left[\pi - \alpha + \frac{\sin 2\alpha}{2}\right]$$

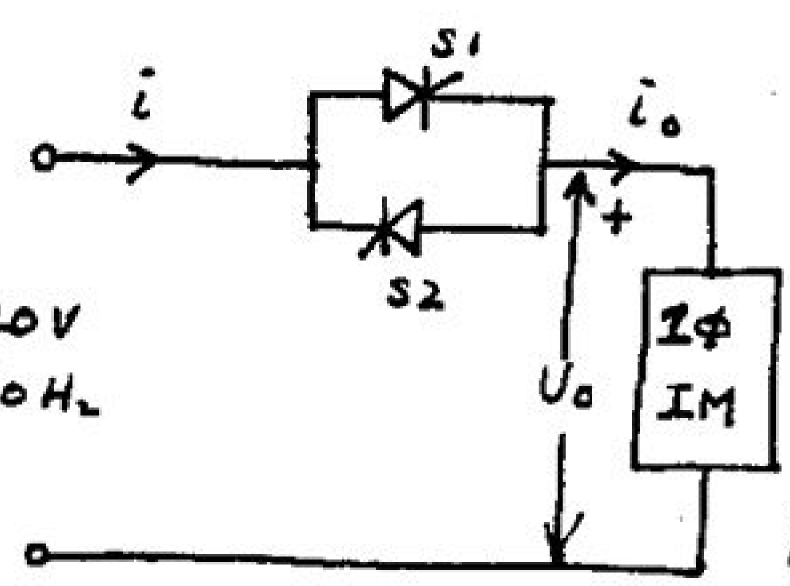
$$= 1528\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)$$

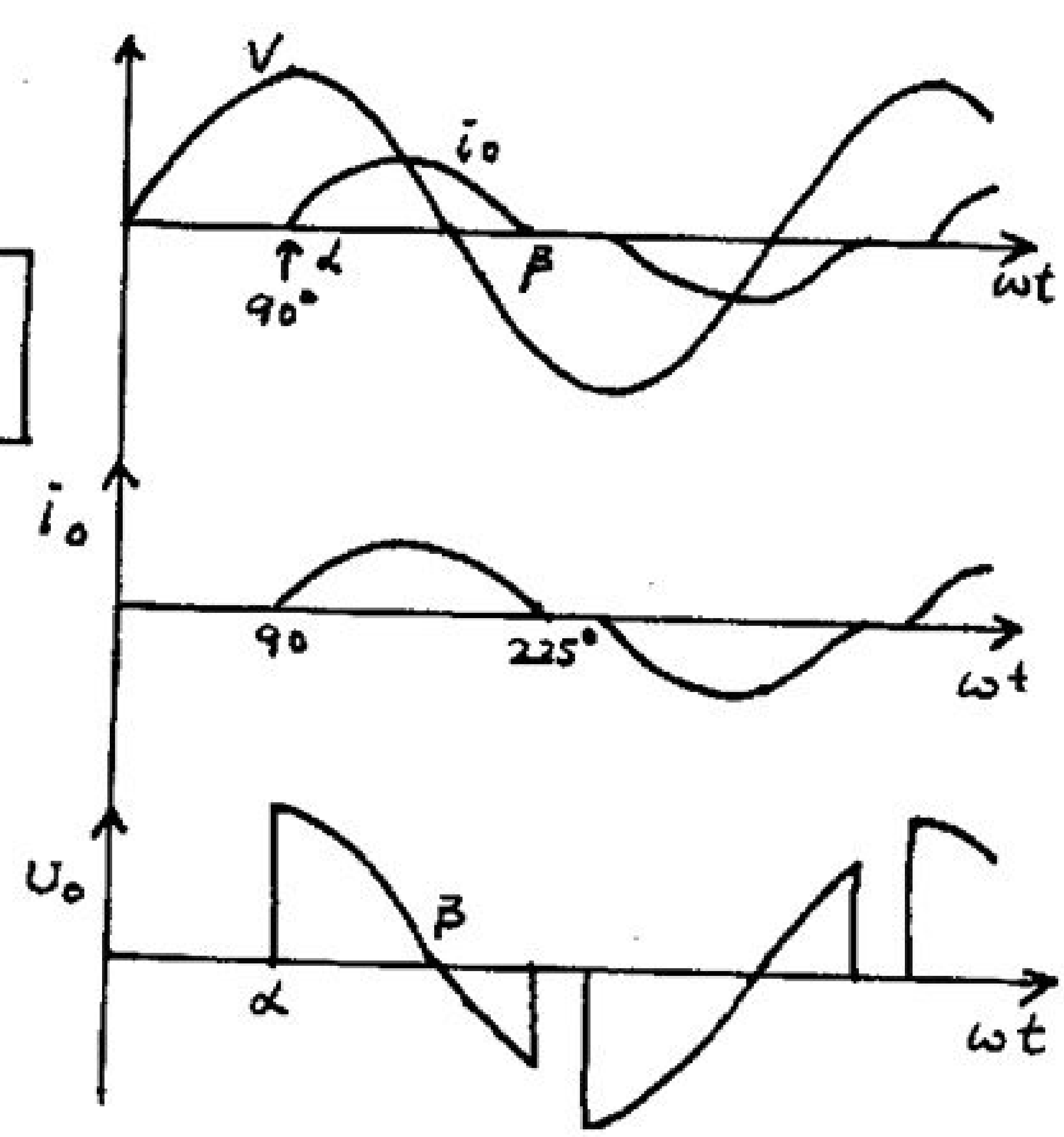


α	P_{av}
0°	4800 W
30°	4662
60°	3854
90°	2400
120°	938
150°	138
180°	0



$\boxed{10.15}$ (a)(i)





10-15

a(ii) $V_{o\,(rms)} = \sqrt{\frac{1}{\pi}\int_\alpha^\beta (\sqrt{2}V\sin\theta)^2 d\theta}$

$$= \left[\frac{2V^2}{\pi}\int_\alpha^\beta \frac{1-\cos 2\theta}{2}\,d\theta\right]^{1/2}$$

$$= \frac{V}{\sqrt{\pi}}\left[\int_\alpha^\beta (1-\cos 2\theta)\,d\theta\right]^{1/2}$$

$$= \frac{V}{\sqrt{\pi}}\left[\theta - \frac{\sin 2\theta}{2}\right]_\alpha^{\beta\ 1/2}$$

$$= \frac{V}{\sqrt{\pi}}\left[(\beta-\alpha) - \frac{\sin 2\beta - \sin 2\alpha}{2}\right]^{1/2}$$

$\alpha = 90°$

$\beta = 90 + 135 = 225°$

$$V_{o\,(rms)} = \frac{120}{\sqrt{\pi}}\left[\frac{135(\pi)}{180} - \frac{\sin 450 - \sin 180}{2}\right]^{1/2}$$

$$= 92.2\text{ V}$$

(b)(i) $\phi = \cos^{-1} 0.7 = 45.6°$

$\alpha_{max} = 45.6°$

(ii) $\alpha = 15° < \phi$

current is sinusoidal and full voltage is across the load

$V_{o\,(rms)} = 120\text{ V}$

$$\text{Input VA} = \frac{1 \times 746}{0.7 \times 0.75} = 1420.95$$

$$I_o = I = \frac{1420.95}{120} = 11.84\text{ A}$$

$$I_{SCR\,(rms)} = \frac{11.84}{\sqrt{2}} = 8.37\text{ A}$$

10.16 (a) AC converter

Input ac → fixed voltage & fixed frequency

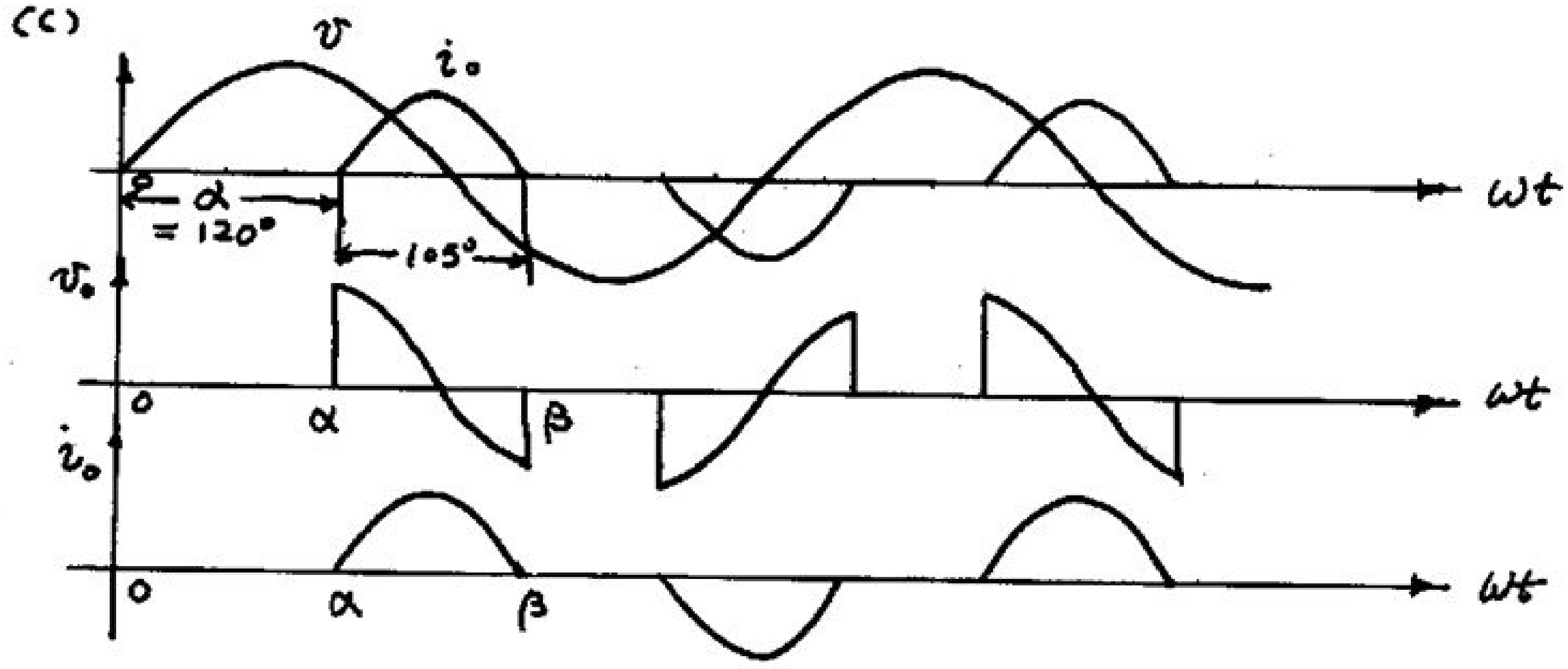
Output ac → variable voltage but fundamental frequency same as input frequency

(b)

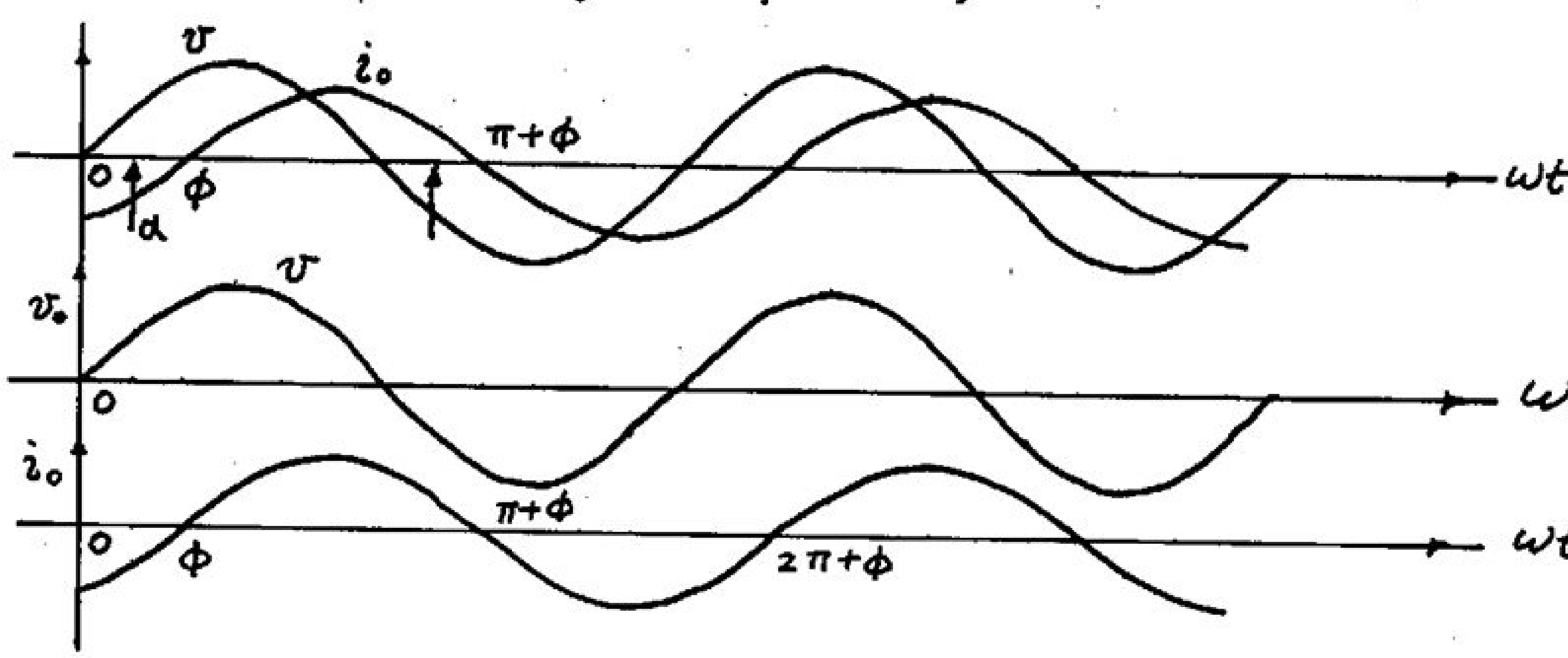
First half-cycle → D_1, T, D_4

2nd half-cycle → D_2, T, D_3

(c)



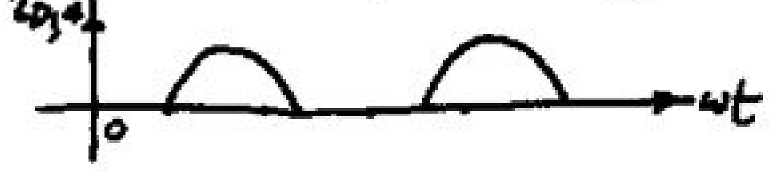
(d) $\alpha = 30° < \phi\ (= 60°)$ Thyristor is fired at 30°, but will turn on at 60°. current sinusoidal



(e) $V_o(rms) = 120V$ $\quad I_o(rms) = \frac{120}{12} = 10A$

$I_{Th} = 10A$ → thyristor carries both halfcycle of output current

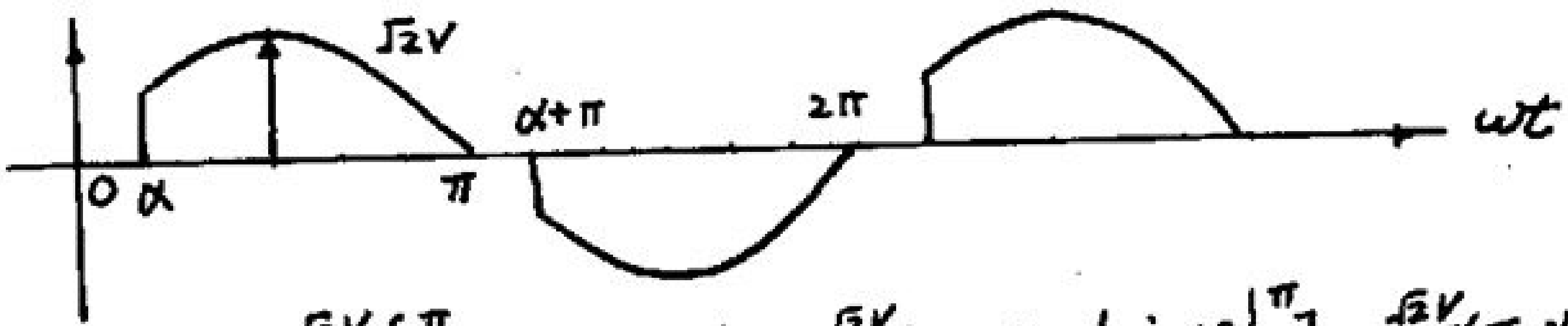
$I_{diode} = \frac{10}{\sqrt{2}} = 7.07A$



(f) - SCR conducts during both half-cycles, hence better utilization of SCR.

- Higher on-state conduction drop. During current conduction voltage drops occur in one thyristor and two diodes – all three in series.

10.17 (a)



$$a_1 = \frac{2}{\pi}\int_\alpha^\pi \sqrt{2}\sin\theta\sin\theta\, d\theta = \frac{\sqrt{2}V}{\pi}\int_\alpha^\pi (1-\cos 2\theta)\, d\theta = \frac{\sqrt{2}V}{\pi}\left[(\pi-\alpha) - \frac{1}{2}\sin 2\theta\Big|_\alpha^\pi\right] = \frac{\sqrt{2}V}{\pi}(\pi-\alpha+\frac{1}{2}\sin 2\alpha)$$

$$b_1 = \frac{2}{\pi}\int_\alpha^\pi \sqrt{2}\sin\theta\cos\theta\, d\theta = \frac{\sqrt{2}V}{\pi}\int_\alpha^\pi \sin 2\theta\, d\theta = \frac{\sqrt{2}V}{\pi}\left(-\frac{\cos 2\theta}{2}\right)\Big|_\alpha^\pi = \frac{\sqrt{2}}{2\pi}V(\cos 2\alpha - 1)$$

$$C_1 = \sqrt{a_1^2+b_1^2} = \frac{\sqrt{2}V}{\pi}\sqrt{(\pi-\alpha+\frac{1}{2}\sin 2\alpha)^2 + \frac{1}{4}(\cos 2\alpha - 1)^2}$$

(b) There is no even order hamonics for $n = 3, 5, 7, 9, \cdots$

$$a_n = \frac{2}{\pi}\int_\alpha^\pi \sqrt{2}V\sin n\theta\sin\theta\, d\theta = \frac{\sqrt{2}V}{\pi}\int_\alpha^\pi [\cos(n-1)\theta - \cos(n+1)\theta]\, d\theta$$
$$= \frac{\sqrt{2}V}{\pi}\left[\frac{\sin(n-1)\theta}{n-1}\Big|_\alpha^\pi - \frac{\sin(n+1)\theta}{n+1}\Big|_\alpha^\pi\right] = \frac{\sqrt{2}V}{\pi}\left[\frac{\sin(n+1)\alpha}{n+1} - \frac{\sin(n-1)\alpha}{n-1}\right]$$

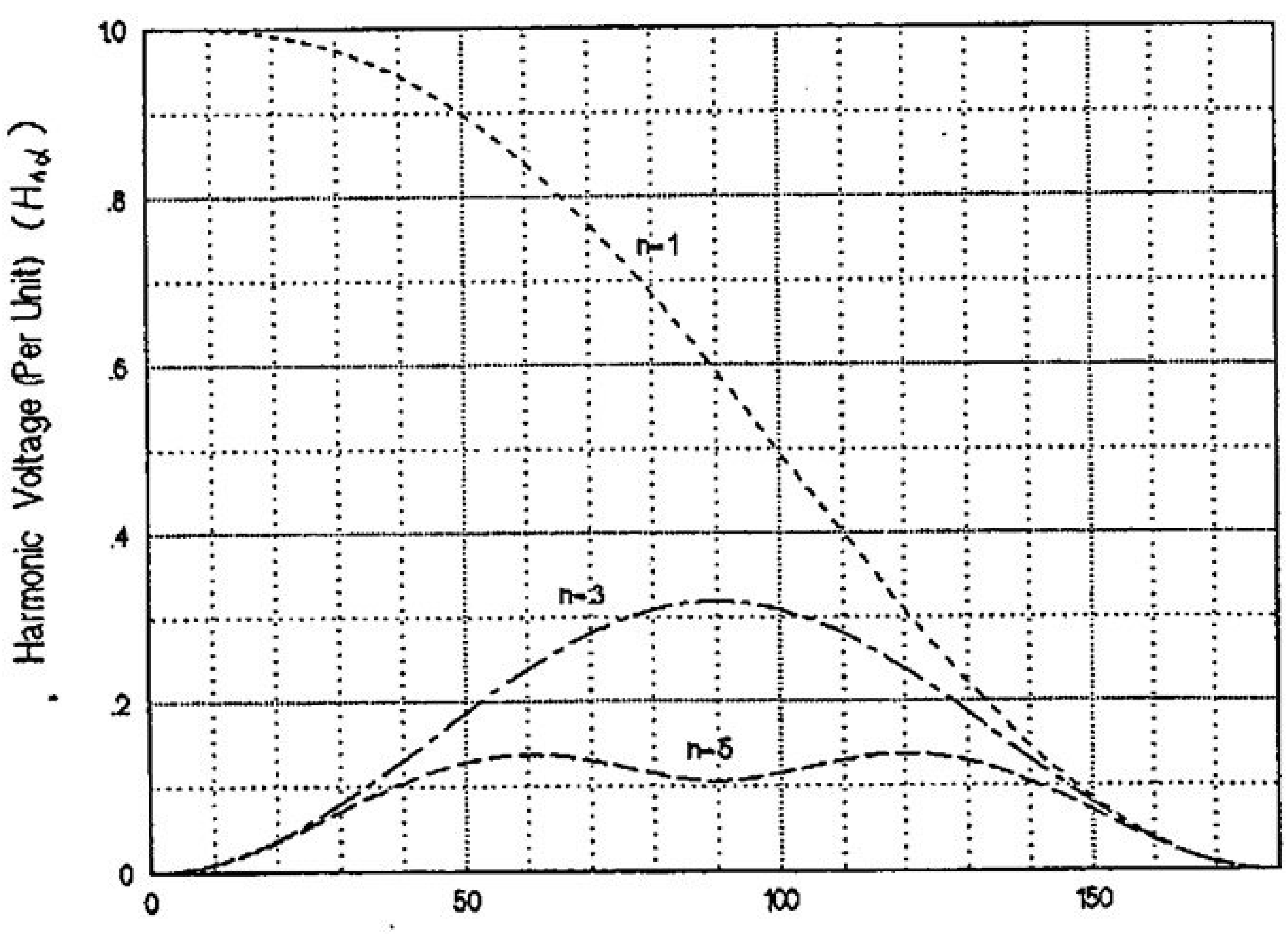
$$b_n = \frac{2}{\pi}\int_\alpha^\pi \sqrt{2}V\sin n\theta\cos\theta\, d\theta = \frac{\sqrt{2}V}{\pi}\int_\alpha^\pi [\sin(n+1)\theta - \sin(n-1)\theta]\, d\theta = \frac{\sqrt{2}V}{\pi}\left[\frac{\cos(n+1)\alpha - 1}{n+1} + \frac{1-\cos(n-1)\alpha}{n-1}\right] \quad \text{(note } n = \text{odd)}$$

$$C_n = \sqrt{a_n^2 + b_n^2}$$

(c) $H_{1\alpha} = \dfrac{\sqrt{a_1^2+b_1^2}}{\sqrt{2}V}$ $\quad H_{3\alpha} = \dfrac{\sqrt{a_3^2+b_3^2}}{\sqrt{2}V}$ $\quad H_{5\alpha} = \dfrac{\sqrt{a_5^2+b_5^2}}{\sqrt{2}V}$

where $a_3 = \dfrac{\sqrt{2}V}{\pi}\left[\dfrac{\sin 4\alpha}{4} - \dfrac{\sin 2\alpha}{2}\right]$, $\quad a_5 = \dfrac{\sqrt{2}V}{\pi}\left[\dfrac{\sin 6\alpha}{6} - \dfrac{\sin 4\alpha}{4}\right]$

$b_3 = \dfrac{\sqrt{2}V}{\pi}\left[\dfrac{\cos 4\alpha - 1}{4} + \dfrac{1-\cos 2\alpha}{2}\right]$, $\quad b_5 = \dfrac{\sqrt{2}V}{\pi}\left[\dfrac{\cos 6\alpha - 1}{6} + \dfrac{1-\cos 4\alpha}{4}\right]$



(d) Maximum 3rd harmonic voltage occurs at $\alpha = 90°$

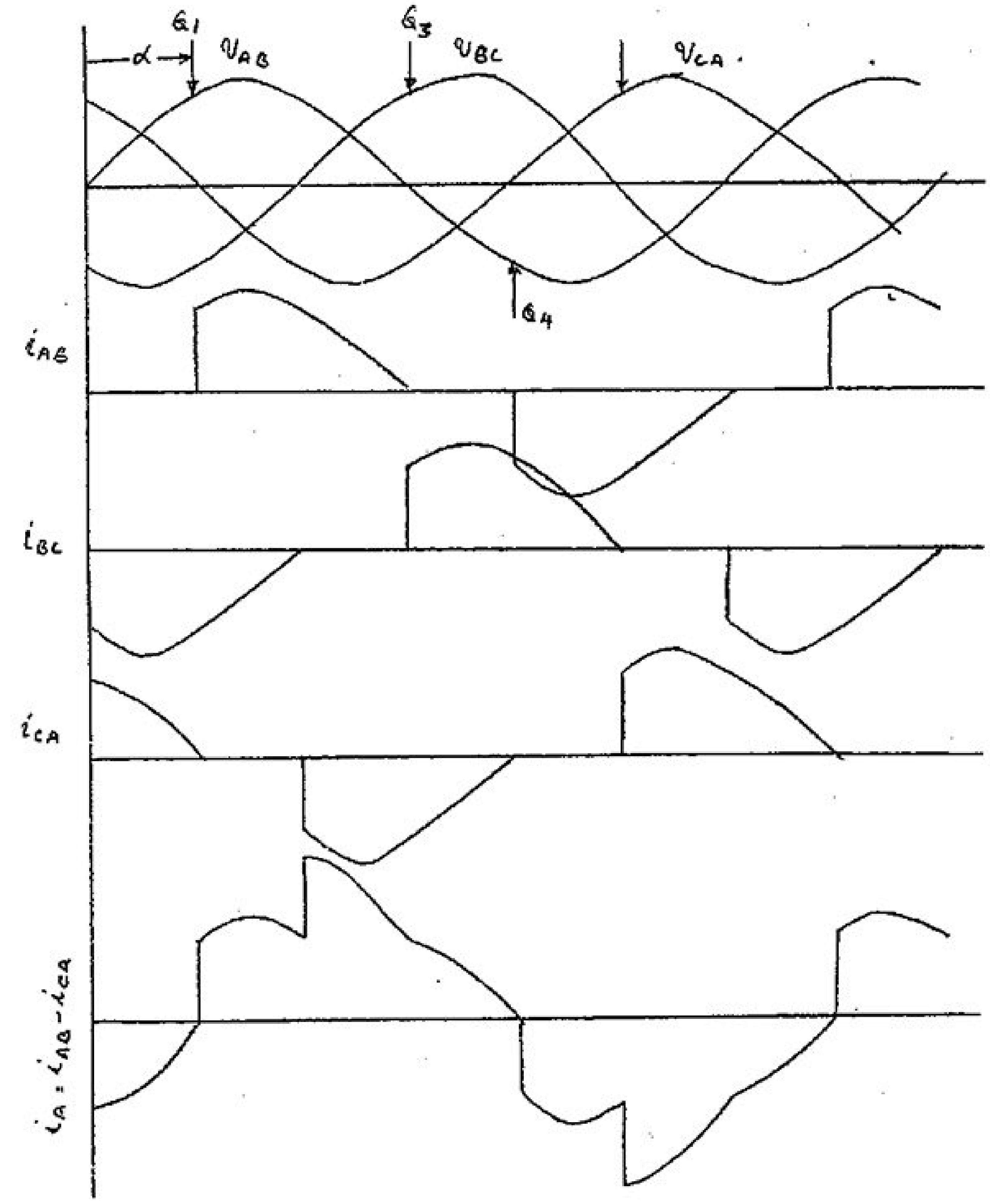
$H_{3\alpha} = 0.32 = \dfrac{V_3}{V}$ for $\alpha = 90°$ $\; V_3 = 0.32 \times 120 = 38.4V$ $\; I_3 = \dfrac{38.4}{10} = 3.84A$

$H_{1\alpha} = 0.6 = \dfrac{V_1}{V}$ $\; V_1 = 0.6 \times 120 = 72V$ $\; I_1 = \dfrac{72}{10} = 7.2A$

$H_{5\alpha} = 0.11 = \dfrac{V_5}{V}$ $\; V_5 = 0.11 \times 120 = 13.2$ $\; I_5 = \dfrac{13.2}{10} = 1.32A$

$\boxed{10.18}$ (a) $0^\circ < \alpha < 180^\circ \longrightarrow \phi = 0^\circ \longrightarrow \alpha_{min} = 0^\circ$

(b)



(c) Ratings for maximum output

R load $\longrightarrow \phi = 0^\circ$

Hence $\alpha_{min} = 0^\circ \longrightarrow$ current is sinusoidal

$I_{ab} = \frac{200}{10} = 20\,A \longrightarrow$ rms phase current

$I_R = \frac{20}{\sqrt{2}} = 14.14\,A \longrightarrow$ rms SCR current

$V_{SCR}\big|_{peak} = \sqrt{2} \times 200 = 282.8\,V \rightarrow$ max. voltage across SCR

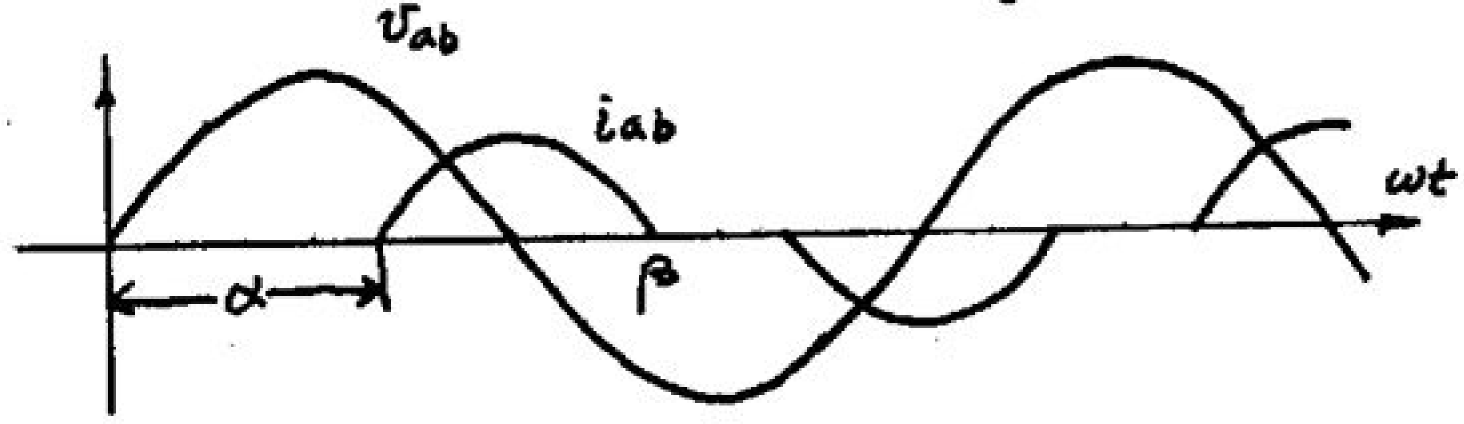
10.19 (a) Because of Δ - connection, each phase can be independantly considered

$$L\frac{di_{ab}}{dt} = v_{ab}$$

$$i_{ab} = \int_\alpha^\theta v_{ab}\,d\theta + i(\alpha)$$

$$= \frac{1}{\omega L}\int_\alpha^\theta \sqrt{2}V\sin\theta\,d\theta + 0$$

$$= \frac{\sqrt{2}V}{\omega L}(\cos\alpha - \cos\omega t)$$



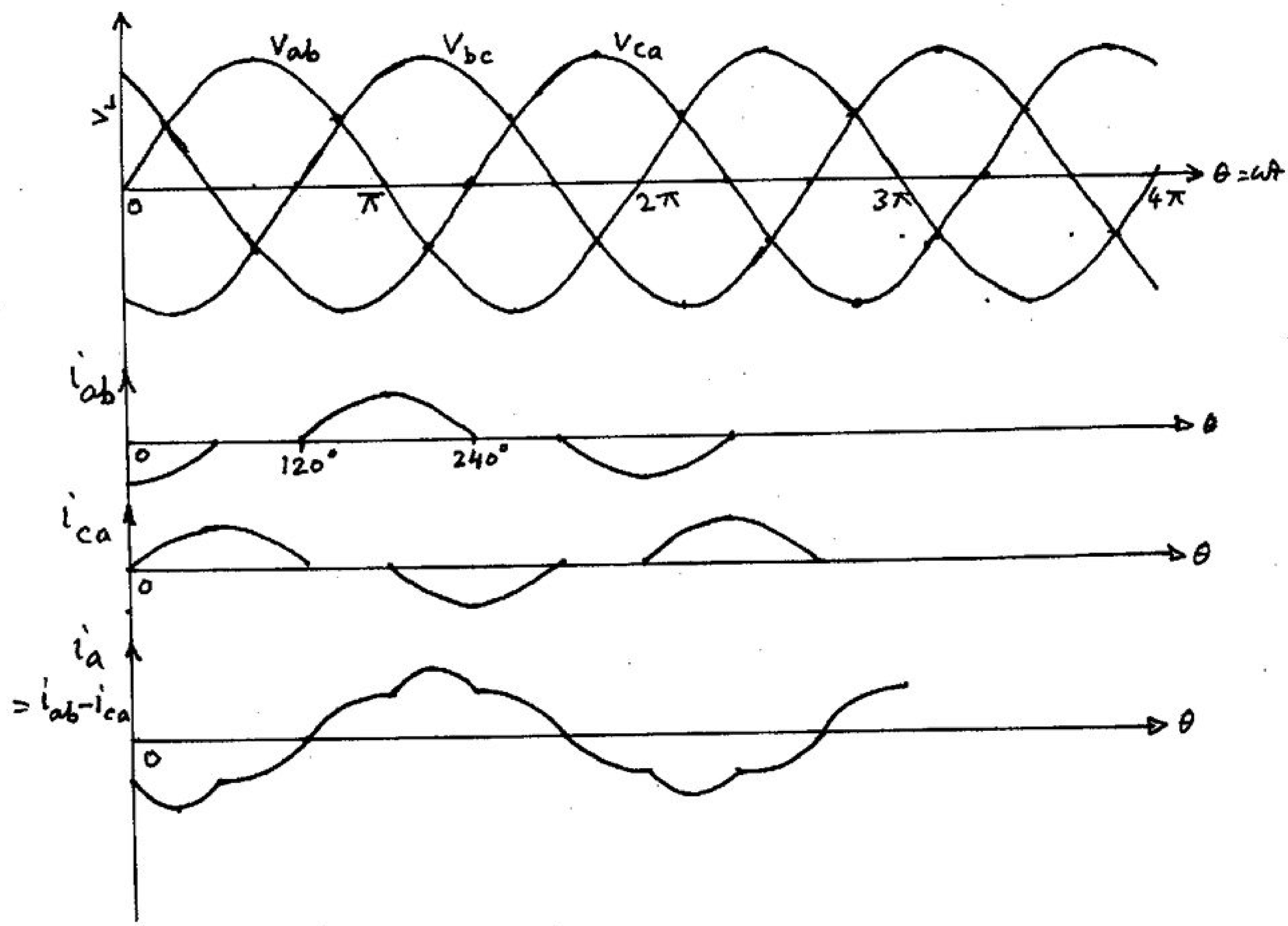
(b) For $L \rightarrow \ \phi = \frac{\pi}{2}$

For current control $\frac{\pi}{2} < \alpha < \pi$

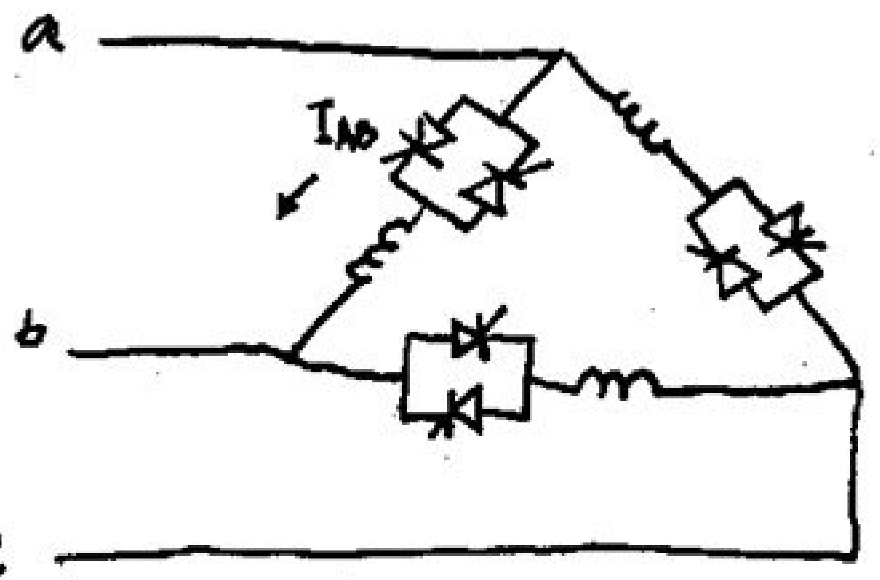
(d) $\alpha = 60° < \phi \longrightarrow$ hence current is sinusoidal

(i) $I_L = \frac{208}{\omega L} = \frac{208}{2\pi\, 60 \times 10 \times 10^{-3}} = 55.17\,A$

(ii) $I_{SCR} = \frac{55.17}{\sqrt{2}} = 39\,A$, $V_{SCR}\Big|_{peak} = \sqrt{2}\times 208 = 294\,V$



$\boxed{10.20}$ (a)



(b) $P_{out} = 5hp = 5 \times 746 = 3730\,W$

$P_{out} = VA_{in}\,\eta\,P.F. \longrightarrow$ Input $VA = \frac{5hp}{0.85 \times 0.9} = 6.536\,Hp = 4875.86\,W$

(c) $P.F. = 0.85 \rightarrow \cos\varphi = 0.85 \rightarrow \phi = 31.79° \longrightarrow$ full load range $0 \le \alpha \le 31.79°$

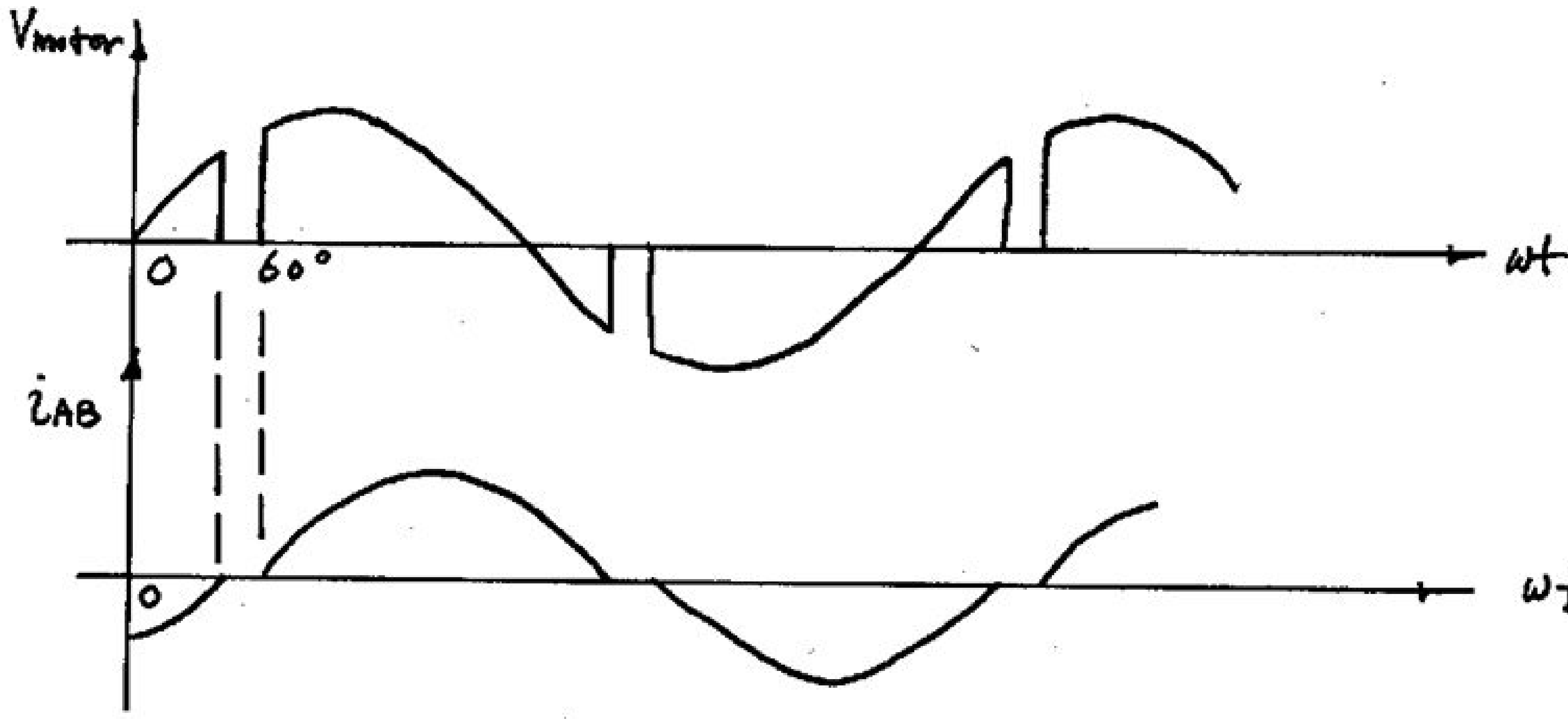
(d) voltage rating: $V_{SCRpeak} = 208\sqrt{2} = 294.16\,V$

Current rating:

$VA = \sqrt{3}\,I_L V_L$, $I_L = \frac{4879.86}{\sqrt{3} \cdot 208} = 13.534A$, $I_{phase} = \frac{I_L}{\sqrt{3}} = 7.81386A$

$I_{SCRrms} = \frac{I_{phase}}{\sqrt{2}} = 5.525A$

(e) $PF = 0.8 < 0.85 \longrightarrow$ current discontinuous



$\boxed{10.21}$ (a)

$$V_o = E_a + I_a R_a$$

$$= 0.05 \times 1200 + 125 \times 0.15$$

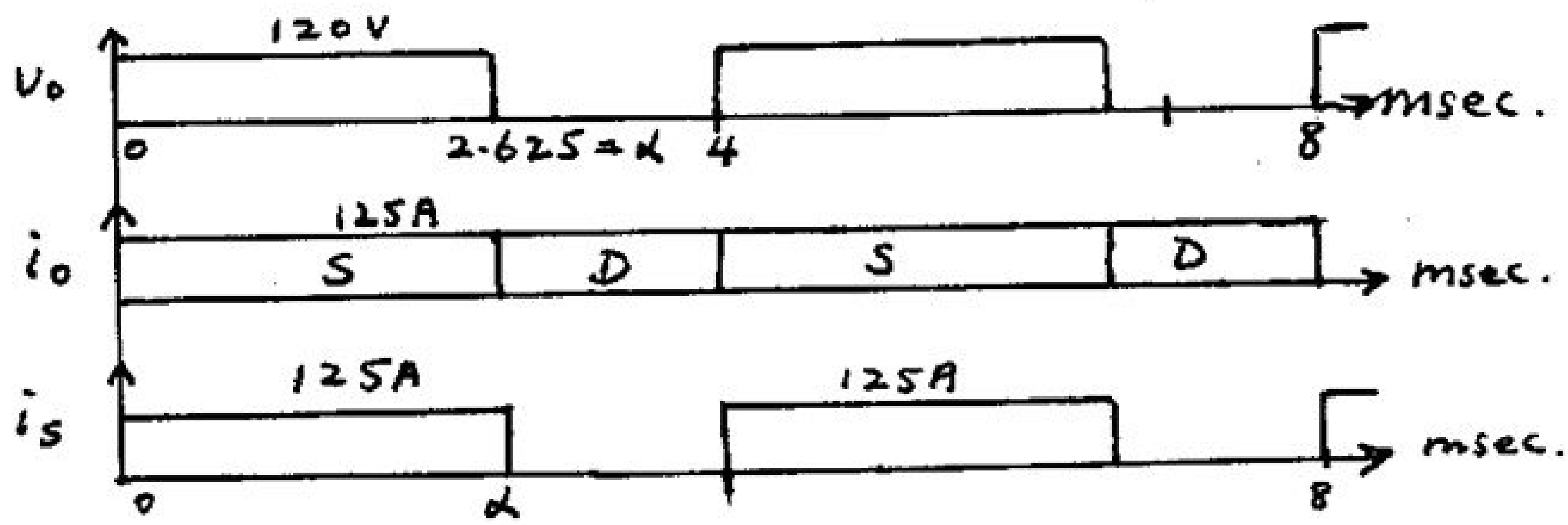
$$= 78.75\,V$$

$$T = \frac{10^3}{250}\,msec = 4\,msec$$

$$\alpha = \frac{78.75}{120} = 0.6563$$

$$t_{on} = \alpha T = 0.6563 \times 4 = 2.625\,msec.$$

(b)



(c) $$E_a I_o = 60 \times 125 = 7500\,W$$

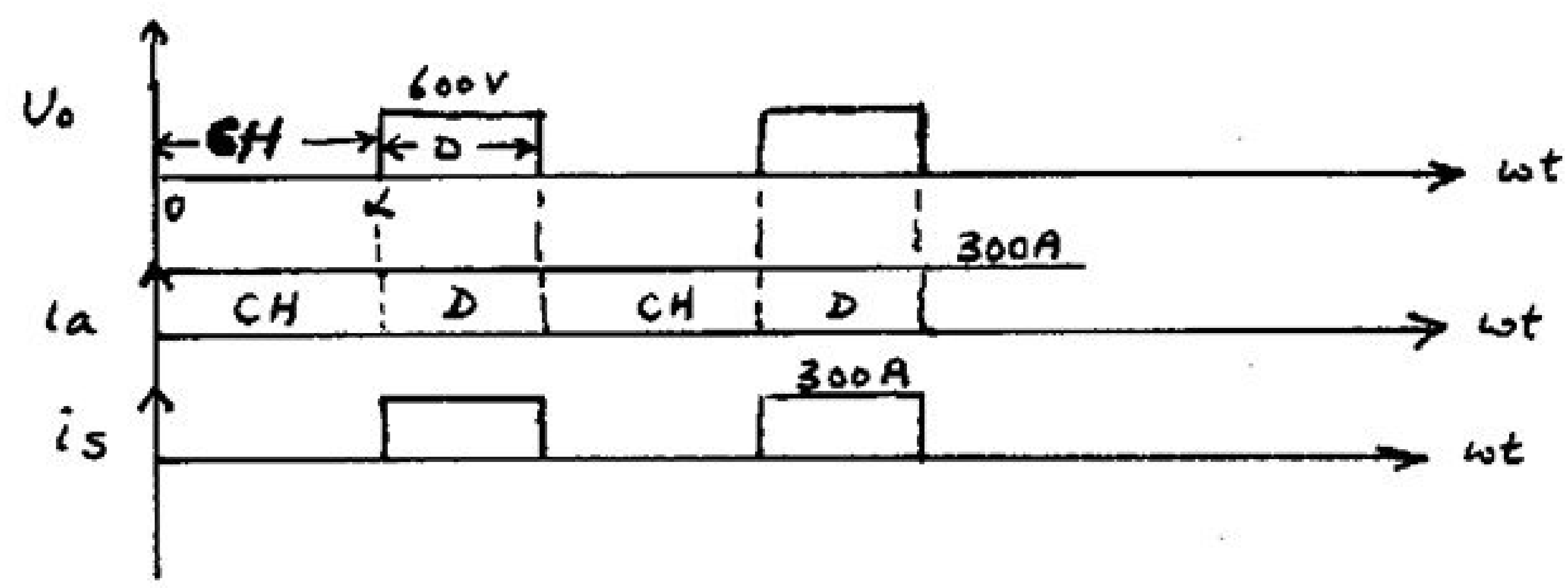
$$T = \frac{7500}{1200/60 \times 2\pi} = 59.683\ N\cdot m$$

$$P_o = V_o I_o = 78.75 \times 125 = 9844\,W$$

$$I_s = 125 \times 0.6563 = 82.03\ A$$

$$P_s = 120 \times 82.03 = 9844\ W$$

$\boxed{10.22}$ (a)



(b) $$V_o = (1-\alpha)\,600 = E_a = 0.3 \times 800 = 240\ V$$

$$\alpha = 1 - \frac{240}{600} = 0.6$$

(c) $$I_s = (1-\alpha)\,300 = (1-0.6)\,300 = 120\ A$$

$$P_s = 600 \times 120 = 72\ KW$$

Or $P_s = P_a = E_a I_a = 240 \times 300 = 72\ kW$

$\boxed{10.23}$ (a) during $t_{on} \rightarrow v_L = v = L\frac{I_2 - I_1}{t_{on}} = L\frac{\Delta I}{t_{on}}$

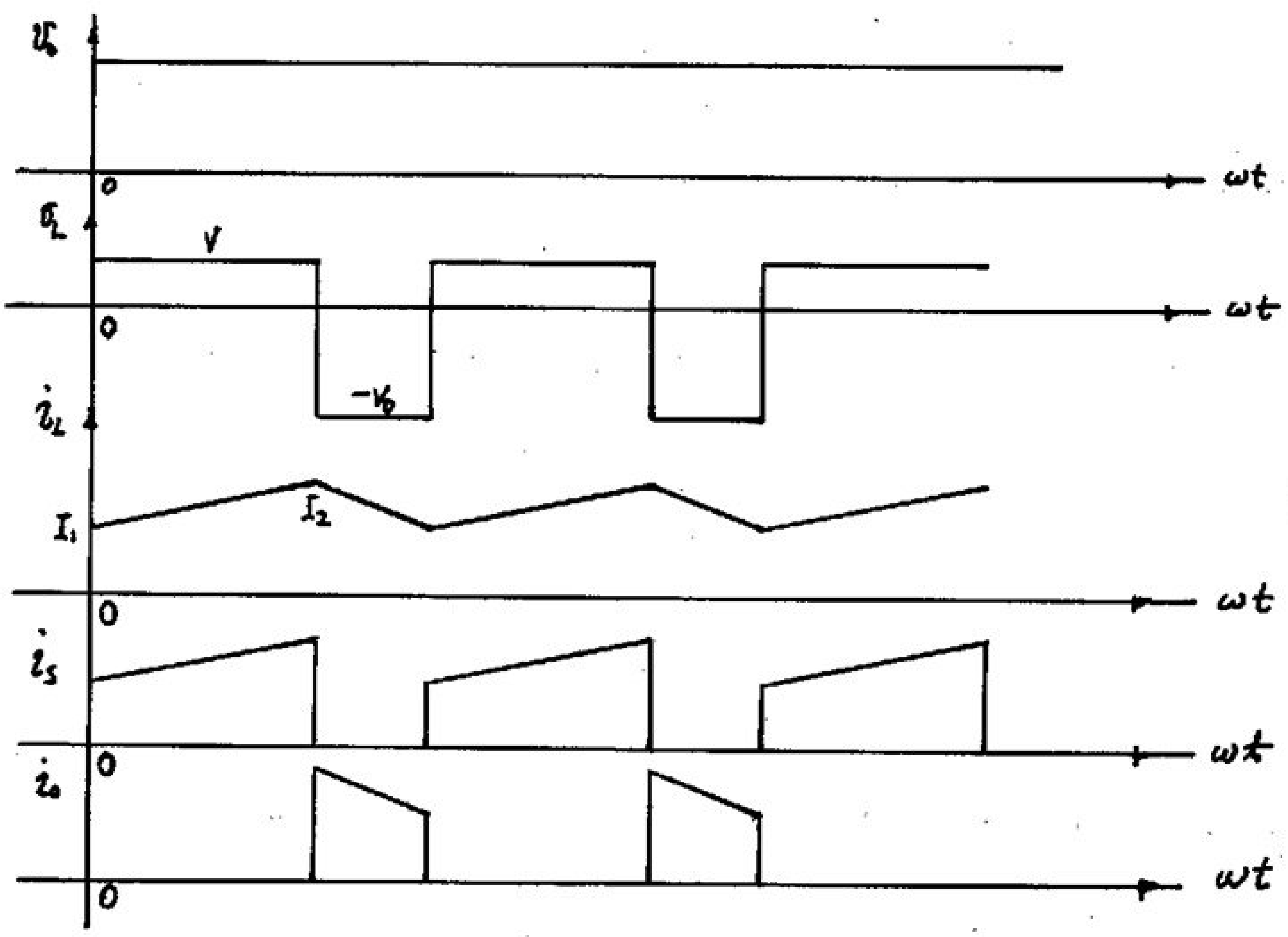
during $t_{off} \rightarrow v_L = -v_o = L\frac{I_1 - I_2}{t_{off}} = -L\frac{\Delta I}{t_{off}}$

$$\frac{V t_{on}}{L} = \frac{V_o t_{off}}{L}$$

$$V_o = \frac{t_{on}}{t_{off}} V = \frac{t_{on}}{T - t_{on}} V = \frac{\alpha}{1-\alpha} V$$

α	V_o
0	0
0.5	V
1.0	∞

(b)



(c) • This is a step-down, step-up chopper

• Polarity of V_o is the same as that of V

• Higher conduction losses → two devices conduct at a time

$\boxed{10.24}$ (a) $I = 0.5A \quad R_t = 100 \times R_b = 100 \times 0.1 = 10\Omega, \quad V_R = 0.5 \times 10 = 5V$

$$V_{o1} = 100 \times V_{b1} + V_R = 100 \times 2.5 + 5 = 255V$$

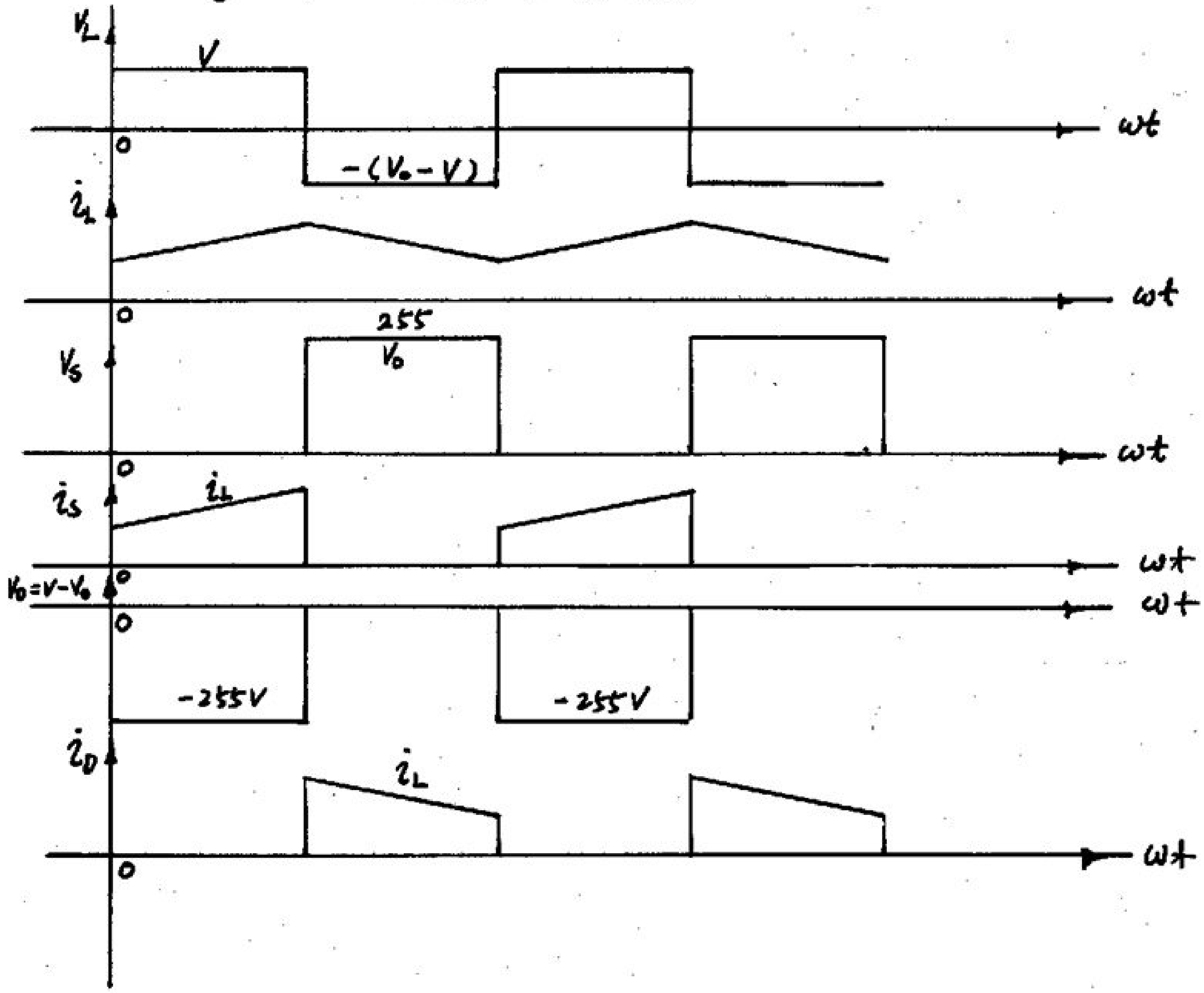
$$V_{o2} = 100 \times V_{b2} + V_R = 100 \times 3.2 + 5 = 325V$$

$$V_o = \frac{1}{1-\alpha} V \longrightarrow \alpha = \frac{V_o - V}{V_o}$$

$$\alpha_1 = \frac{255 - 150}{255} = 0.4118 \qquad \alpha_2 = \frac{325 - 150}{325} = 0.5385$$

α changes from 0.4118 to 0.5385

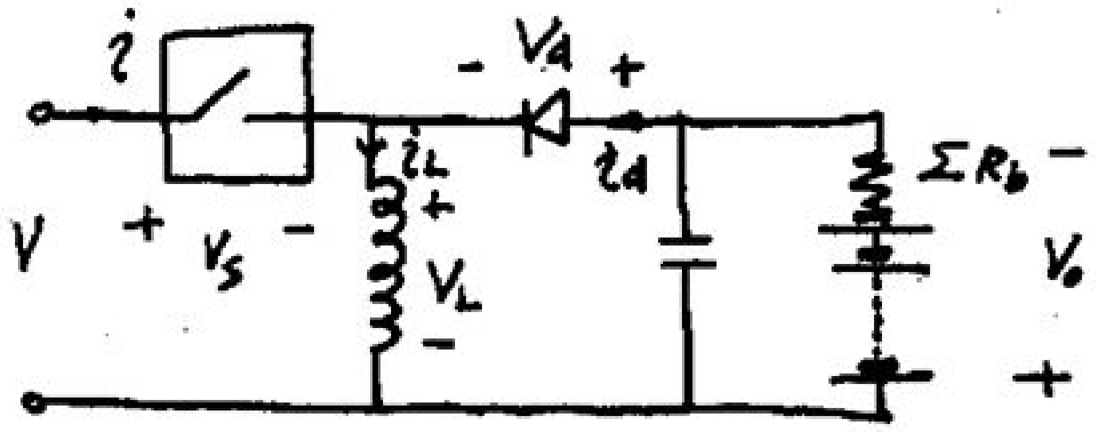
(b)



$\boxed{10.25}$ (a) $V_{o1} = 100 V_{b1} + IR = 100 \times 1.2 + 100 \times 0.1 \times 0.5 = 125V$

$V_{o2} = 100 \times 3.2 + 5 = 325\ V$

$V = 150V$ Need a Buck-Boost Converter

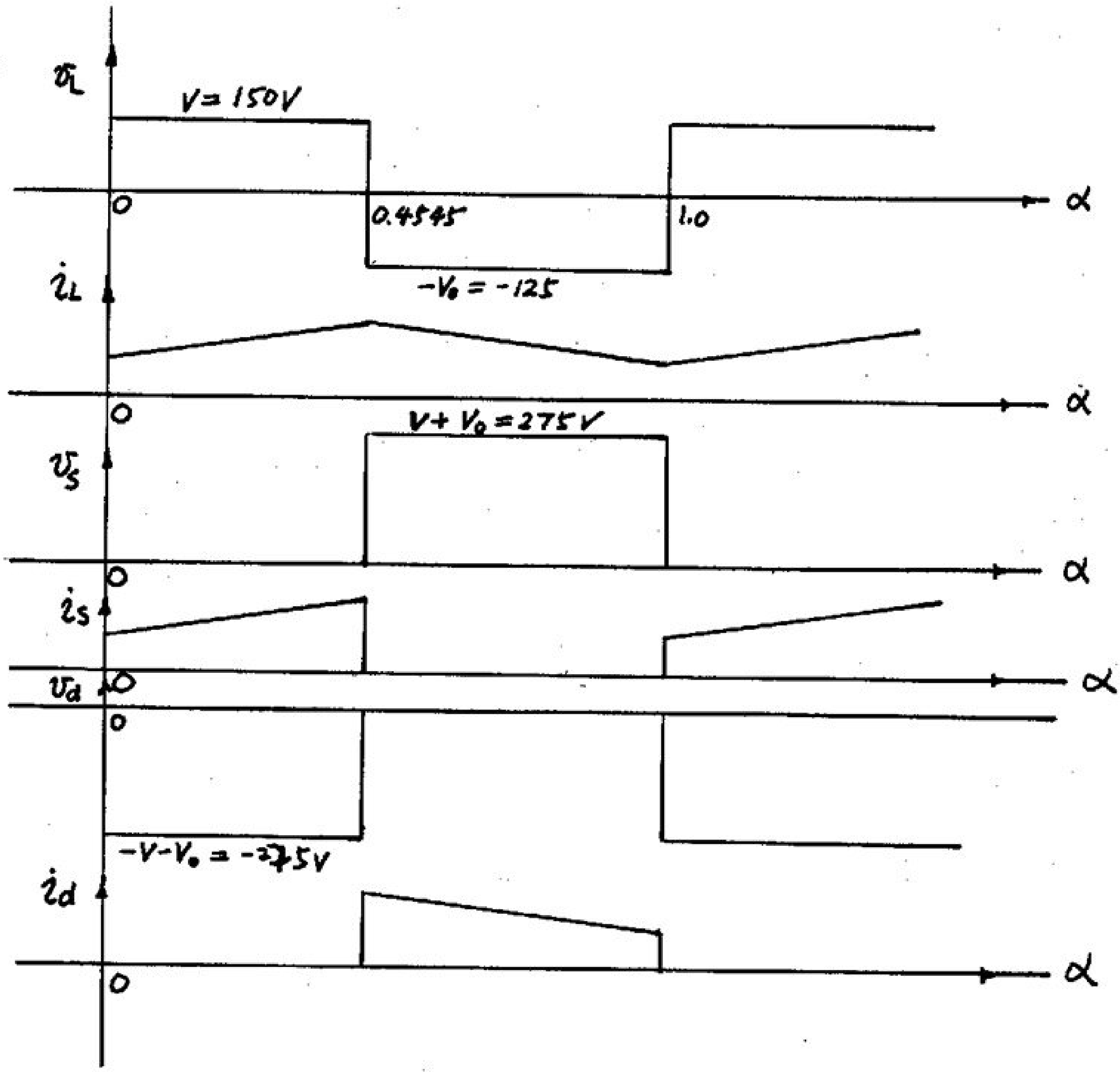


(b) $V_o = \dfrac{\alpha}{1-\alpha} V$

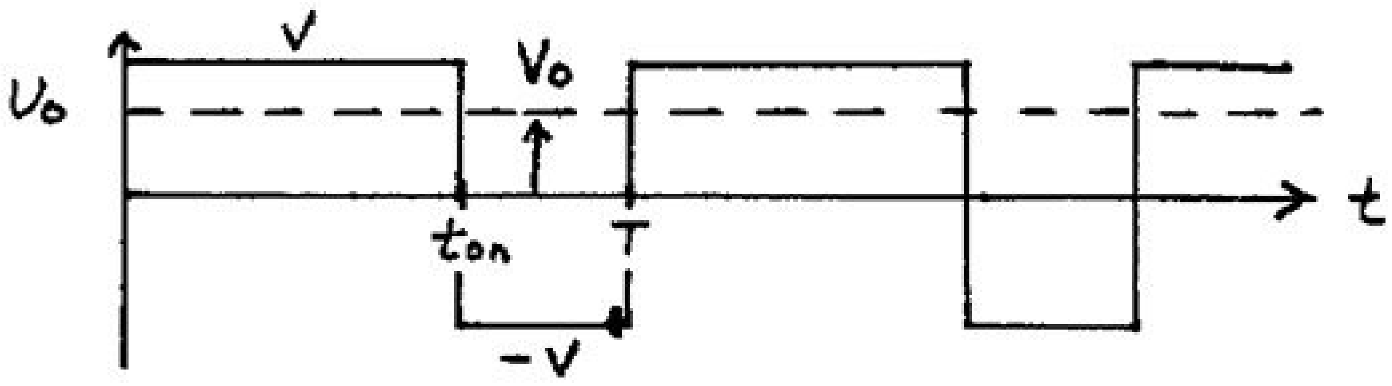
$\alpha = \dfrac{V_o}{V + V_o}$

$\alpha_1 = \dfrac{125}{150 + 125} = 0.4545 \qquad \alpha_2 = \dfrac{325}{152 + 325} = 0.6842$

(c)



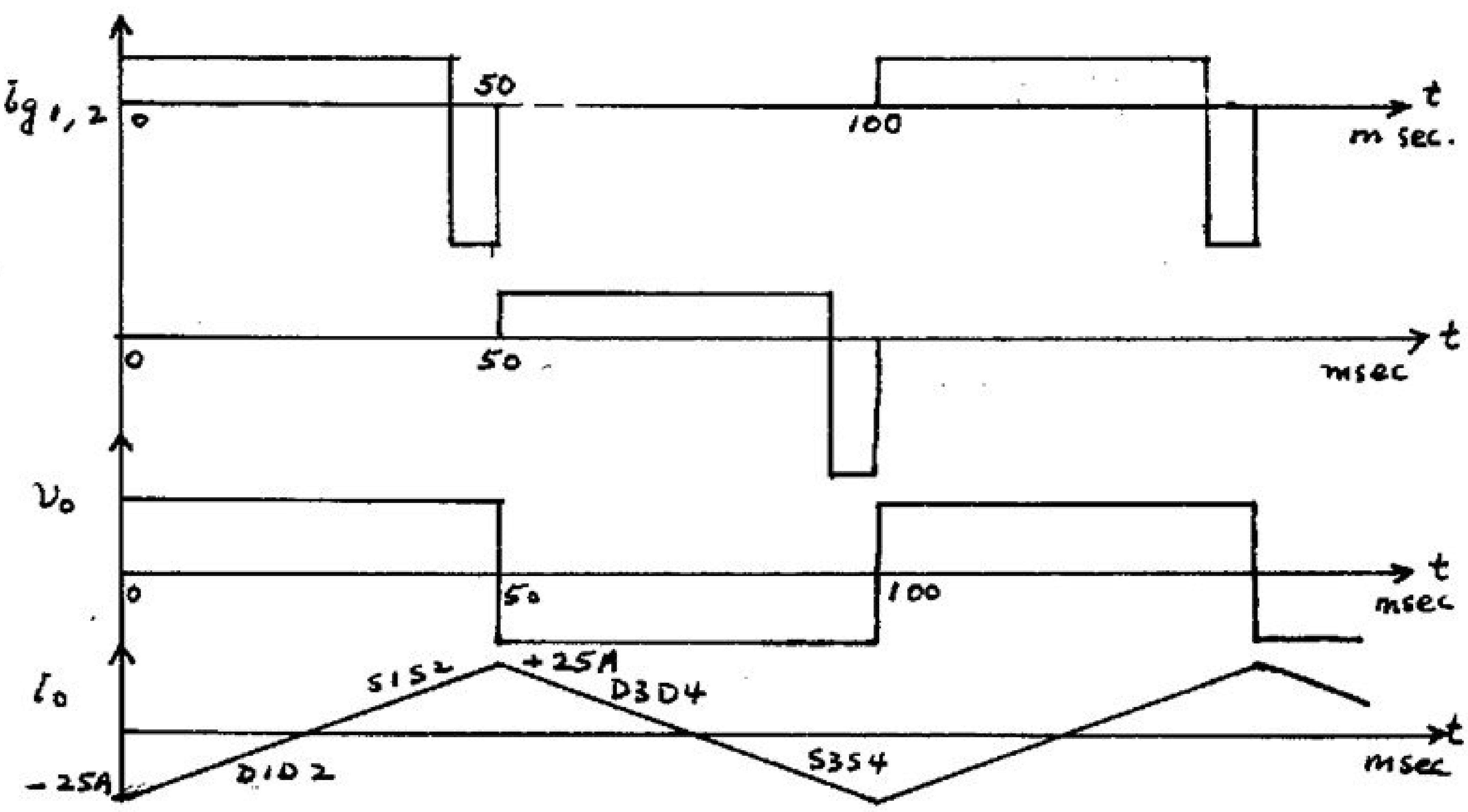
$\boxed{10.26}$ (a)



(b) $$V_o = \frac{1}{T}\int_0^T v_o \, dt = \frac{1}{T}\left[\int_0^{t_{on}} V\, dt + \int_{t_{on}}^{T} -V\, dt\right]$$

$$= V(2\alpha - 1)$$

$\boxed{10.27}$

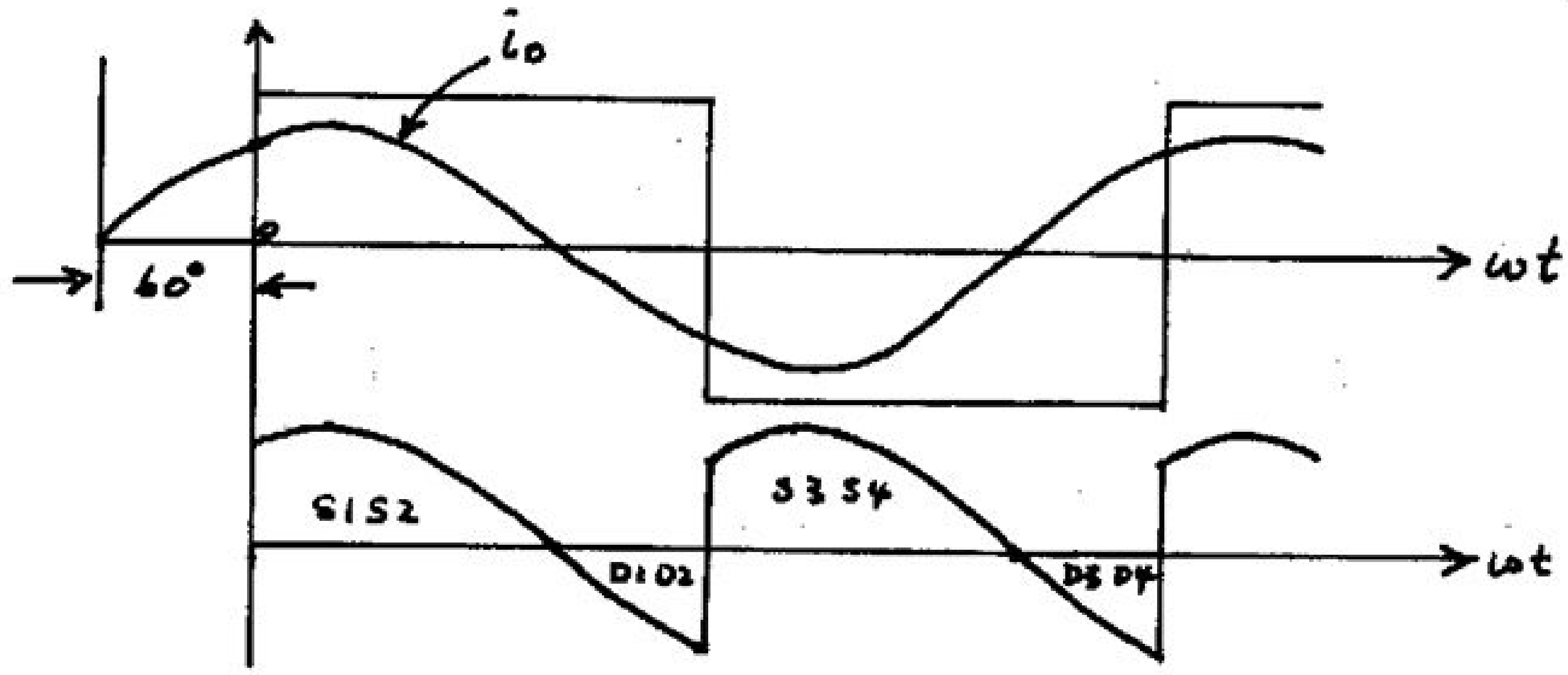


$$L\frac{di}{dt} = V, \longrightarrow \Delta i = \frac{V \cdot \Delta t}{L}$$

$$2\, I_{o(max)} = \frac{V \times \frac{T}{2}}{L} = \frac{100 \times 50 \times 10^{-3}}{100 \times 10^{-3}} = 50\,A$$

$$I_{o(max)} = 25\,A$$

10.28 (a)



(b) $$|I_s|_{avg} = I_s = \frac{1}{\pi}\int_0^{\pi} 400 \sin(\omega t + 60°)\, d\omega t$$

$$= \frac{400}{\pi} = 127.3 \text{ A}$$

$$P_s = 300 \times 127.3 = 38.2 \text{ Kw}$$

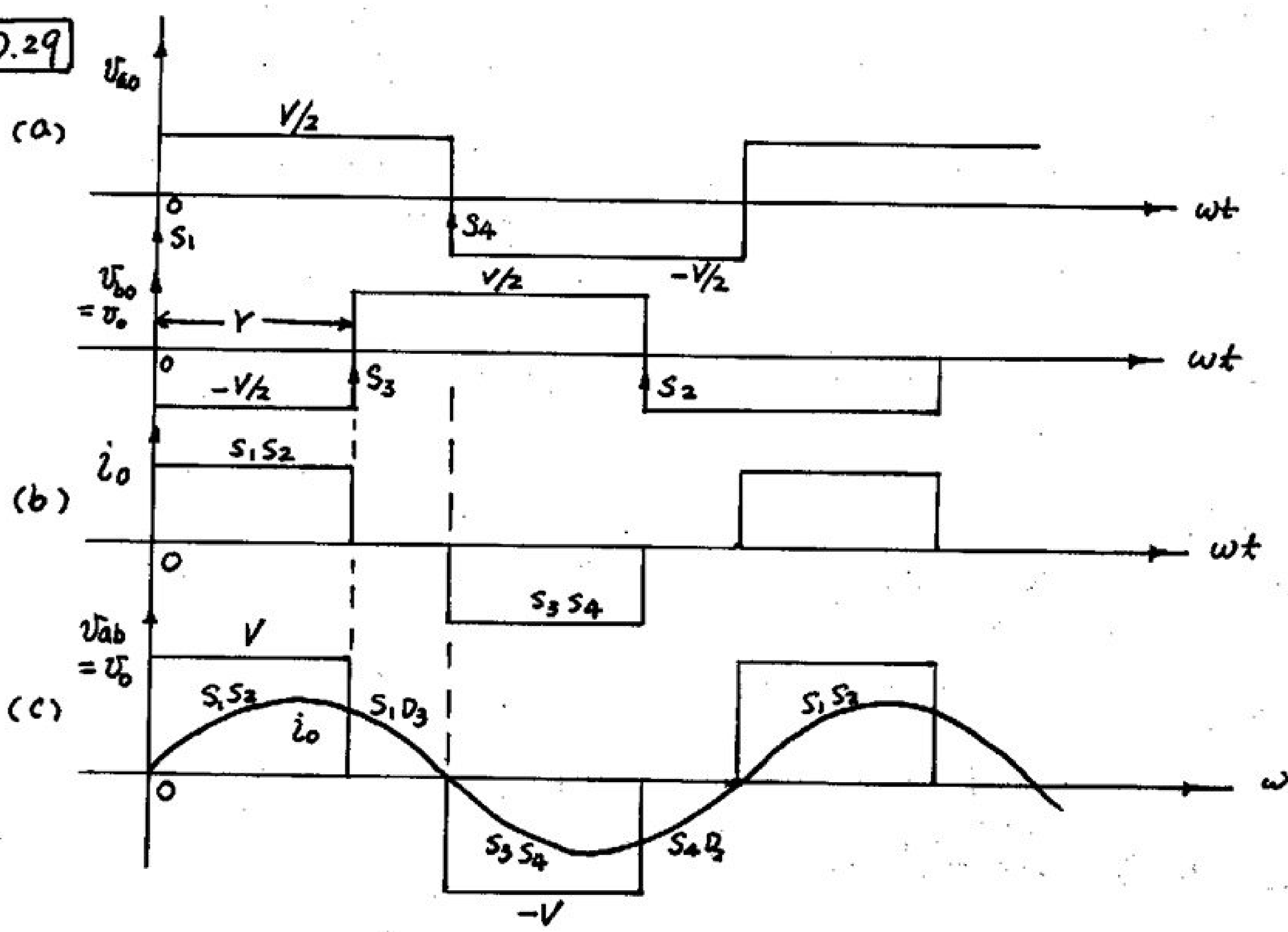
(c) $$V_{o1} = \frac{4V}{\pi\sqrt{2}} = \frac{4 \times 300}{\pi\sqrt{2}} = 270.14$$

$$P_{out} = V_{o1} I_o \cos\theta$$

$$= 270.14 \times \frac{400}{\sqrt{2}} \cos 60°$$

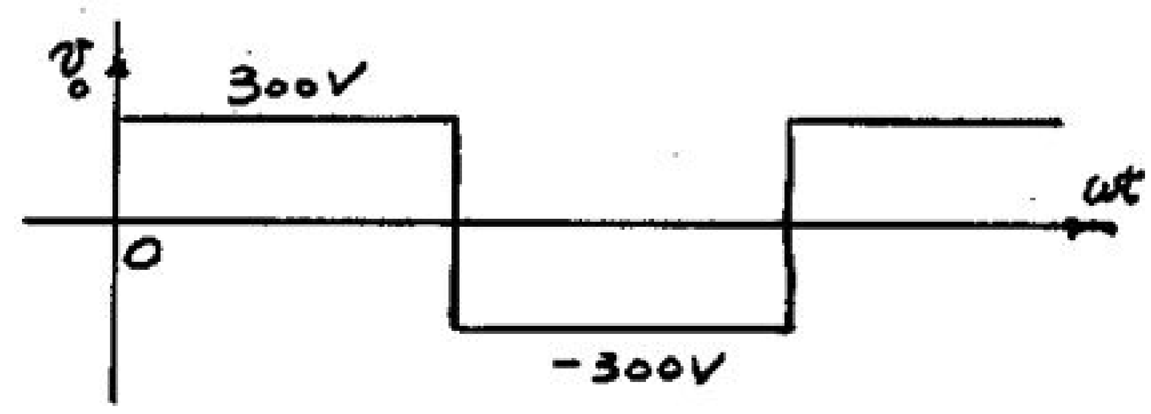
$$= 38.2 \text{ kW}$$

10.29



10.30 (a)

$$v_{o1} = \frac{4\,300}{\pi}\sin\omega t$$



$$V_{o1} = \frac{4\times 300}{\pi\sqrt{2}} = 270V$$

(b) $Z_1 = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2} = \sqrt{0.5^2 + (10-10.5)^2} = 0.707\Omega$

$\phi_1 = \tan^{-1}\frac{-0.5}{0.5} = -45^\circ$

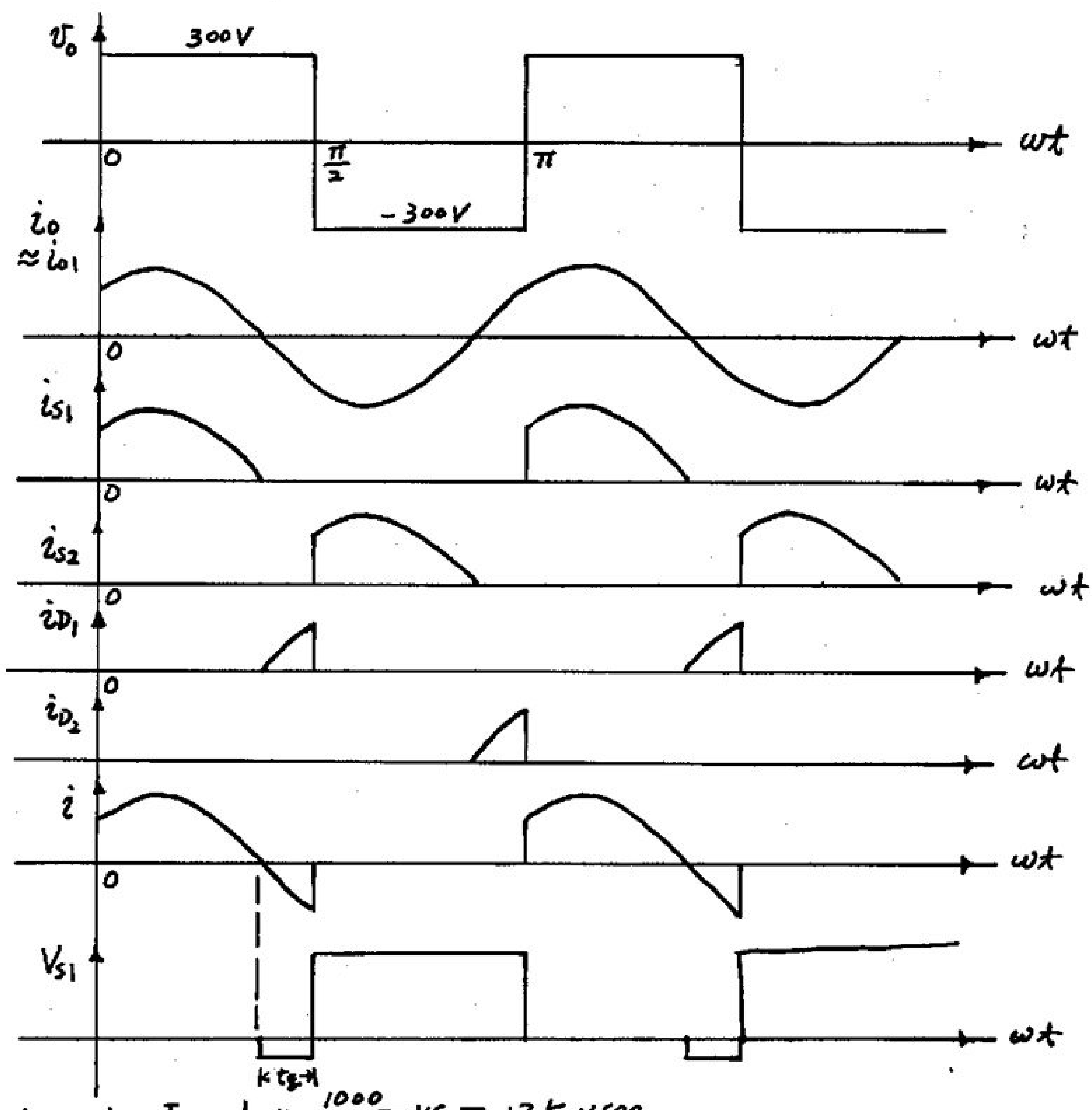
$i_{o1} = \frac{\sqrt{2}V_{o1}}{|Z_1|}\sin(\omega t + \phi_1) \longrightarrow I_{o1} = \frac{270}{0.707} = 382A$

$Z_3 = \sqrt{0.5^2 + (30 - \frac{10.5}{3})^2} = \sqrt{0.5^2 + (30-3.5)^2} = \sqrt{0.5^2 + 26.5^2} \approx 26.5\Omega$

$V_{o3} = \frac{4\times 300}{3\pi\sqrt{2}} = 90V \qquad I_{o3} = \frac{90}{26.5} = 3.4A \ll I_{o1}$

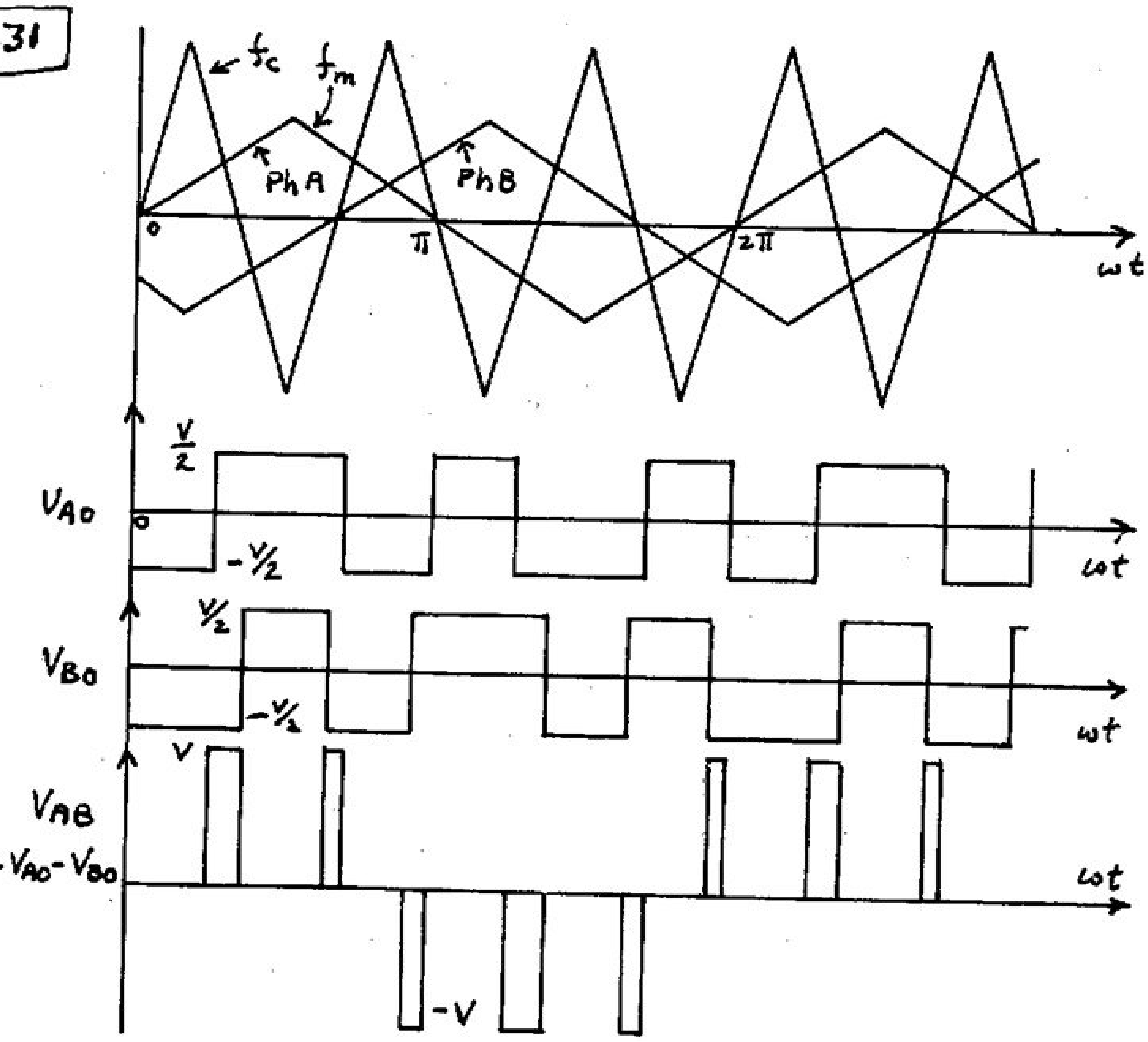
It is justified to assume $i_o \simeq i_{o1}$

(c)

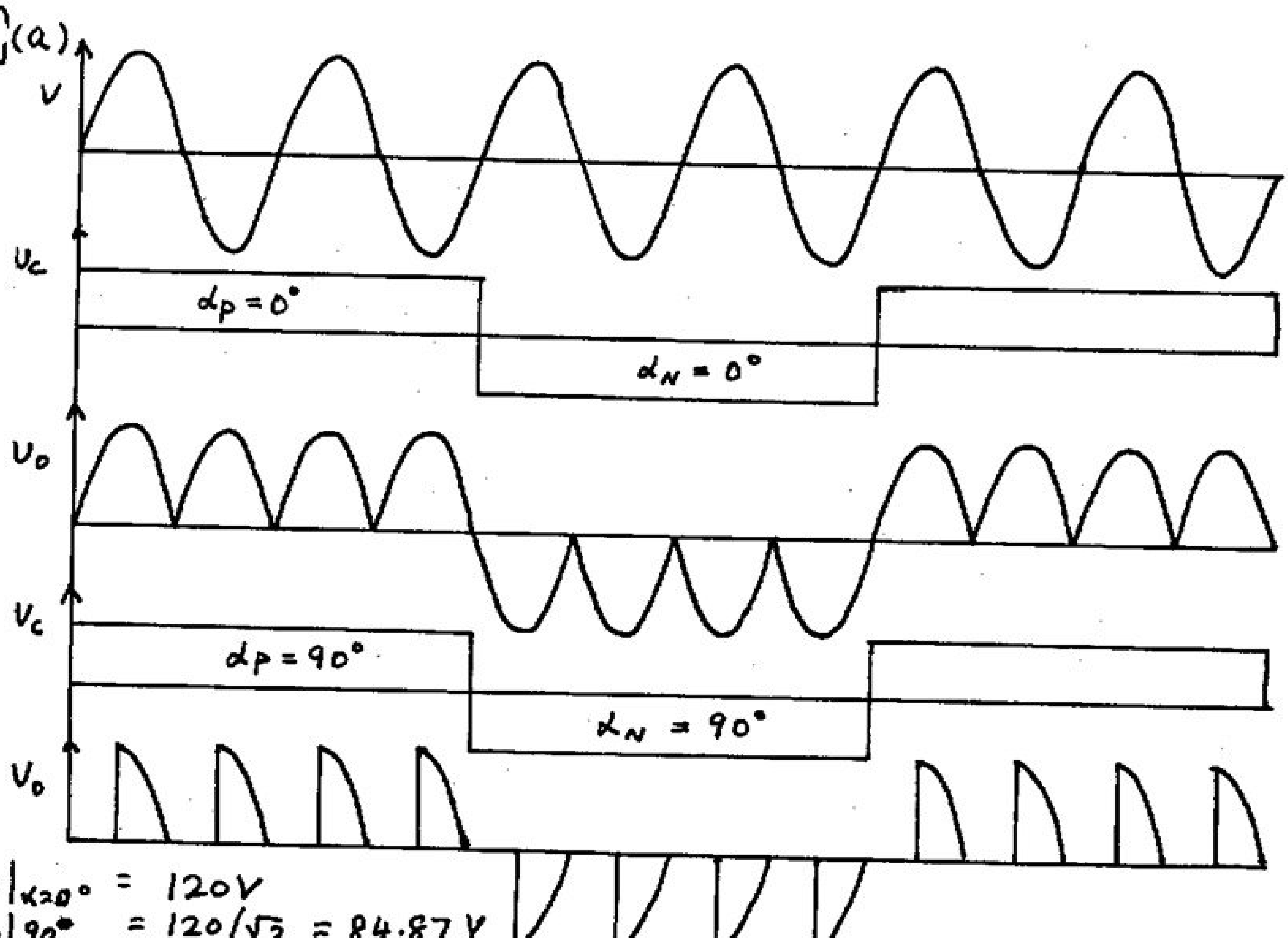


(d) $t_q = \frac{1}{4}\times\frac{T}{2} = \frac{1}{4}\times\frac{1000}{2}\mu s = 125\,\mu sec.$

10.31



10.32 (a)



(b) $V_o|_{\alpha=0°} = 120V$

$V_o|_{90°} = 120/\sqrt{2} = 84.87\ V$

APPENDIX A

$\boxed{A.1}$ $n = \frac{36}{4\times 3} = 3$ slots/pole/phase

$\alpha = \frac{180}{3\times 3} = 20$, $\gamma = 180 - 140 = 40$

$\therefore$ distribution factors are

$$k_{d1} = \frac{\sin(3\times 20/2)}{3\sin(20/2)} = 0.9598$$

$$k_{d3} = \frac{\sin(3\times 3\times 20/2)}{3\sin(3\times 20/2)} = 0.6667$$

$$k_{d5} = \frac{\sin(3\times 5\times 20/2)}{3\sin(5\times 20/2)} = 0.2176$$

The pitch factors are :

$$k_{p1} = \cos(40/2) = 0.9397$$

$$k_{p3} = \cos(3\times 40/2) = 0.5$$

$$k_{p5} = \cos(5\times 40/2) = 0.1736$$

The winding factors are :

$$k_{w1} = k_{d1}k_{p1} = 0.9598 \times 0.9397 = 0.9019$$

$$k_{w3} = k_{d3}k_{p3} = 0.6667\times 0.5 = 0.3333$$

$$k_{w5} = 0.2176\times 0.1736 = 0.0378$$

From eq. A.27, the rms fundamental voltage is

$$E_1 \propto B_{1(max)}k_{w1} = k\times 1\times 0.9019 = 0.9019\,k$$

The 3rd + 5th harmonic voltages are

$$E_3 = k\,B_{3max}k_{w3} = k\times 0.4\times 0.3333 = 0.13332\,k$$

$$E_5 = k\,B_{5max}k_{w5} = k\times 0.2\times 0.0378 = 0.0076\,k$$

Phase voltage is

$$E_{Ln} = (E_1^2 + E_3^2 + E_5^2 + \cdots)^{1/2}$$

$$= k\,(0.9019^2 + 0.13332^2 + 0.0076^2 + \cdots)^{1/2}$$

$$\simeq 0.9118\,k$$

A.1 continued

$$\frac{E_{ln}}{E_1} = \frac{0.9118}{0.9019} = 1.01$$

line-to-line voltage is

$$E_{ll} = \sqrt{3}\,(E_1^2 + E_5^2 + \cdots)^{1/2}$$

$$= \sqrt{3}\,K\,(0.9019^2 + 0.0076^2 + \cdots)^{1/2}$$

$$= \sqrt{3}\,K\,(0.9019)$$

$$\frac{E_{ll}}{E_1} = \frac{\sqrt{3} \times 0.9019}{0.9019} = \sqrt{3}$$

$$\frac{E_{ll}}{E_{ln}} = \frac{\sqrt{3} \times 0.9019}{0.9118} = \sqrt{3}\,(0.98914)$$

Note that the line-to-line voltage is slightly lower than $\sqrt{3}$ times the phase voltage due to the absence of 3rd harmonic component in the line-to-line voltage.

$\boxed{\text{A.2}}$ (a) $n = \frac{108}{6 \times 3} = 6$ slots/pole/phase

Each slot has two coil-sides (double layer)

Total coils = 108

Total turns = $108 \times 30 = 3240$

Turns per phase $N_{ph} = 3240/3 = 1080$

$$\alpha = \frac{180}{(3)(6)} = 10^\circ$$

$$\gamma = 180 - 150 = 30^\circ$$

$$\phi_1 = 0.01 \text{ Wb}$$

$$\phi_3 = 0.01 \times 0.3/3 = 0.001 \text{ Wb}$$

$$\phi_5 = 0.01 \times 0.2/5 = 0.0004 \text{ Wb}$$

$k_{d1} = 0.9561$, $k_{p1} = 0.966$, $k_{w1} = 0.9234$

$k_{d3} = 0.644$, $k_{p3} = 0.707$, $k_{w3} = 0.4553$

A.2 Continued

$Kd_5 = 0.197$, $Kp_5 = 0.259$, $Kw_5 = 0.051$

6 - pole 1200 rpm $\rightarrow$ $f_1 = 60$ Hz

$E_1 = 4.44 N_{ph} f_1 \Phi_1 K_{w1} = 4.44 \times 60 \times 0.01 \times 1080 \times 0.9234$

$= 2656.7$ V/phase

$E_3 = 4.44 \times 3 \times 60 \times 0.001 \times 1080 \times 0.4553 = 393$ V/ph

$E_5 = 4.44 \times 5 \times 60 \times 0.0004 \times 1080 \times 0.051 = 29.3$ V/ph

(b) $E_{L-n} = \sqrt{2656.7^2 + 393^2 + 29.3^2} = 2685.8$ V

(c) $E_{L-L} = \sqrt{3}\sqrt{2656.7^2 + 29.3^2} = 4602$ V

$\boxed{A.3}$ (a) Total conductors $= 96 \times 20 = 1920$

Total turns $= 1920/2 = 960$

Turns per phase $N_{ph} = \frac{960}{3} = 320$

(b) To make 3rd harmonic voltage zero,

$K_{p3} = \cos 3\gamma/2 = 0 = \cos 90°$

$\gamma = \frac{90 \times 2}{3} = 60°$

Coil - span $= 180 - 60 = 120°$

(c) $f = \frac{n \times P}{120} = \frac{750 \times 8}{120} = 50$ Hz

(d) $\alpha = \frac{180}{96/8} = 15°$

$n = \frac{96}{8 \times 3} = 4$ slots/pole/phase

$\gamma = 60°$

$Kd_1 = \frac{\sin(4 \times 15/2)}{4 \sin 15/2} = 0.958$

$Kd_3 = 0.653$

A.3 (d) continued

$$K_{d5} = 0.205$$

$$K_{p1} = \cos 60/2 = 0.866$$

$$K_{p3} = \cos(3 \times 60/2) = 0$$

$$K_{p5} = \cos(5 \times 60/2) = 0.866$$

$$K_{w1} = 0.8296$$

$$K_{w3} = 0$$

$$K_{w5} = 0.5655$$

$$E_1 = 4.44 \times 50 \times 0.12 \times 320 \times 0.8296 = 7072 \text{ V/phase}$$

$$E_3 = 0$$

$$E_5 = 4.44 \times 5 \times 50 \times \frac{0.12 \times 0.15}{5} \times 320 \times 0.5655$$

$$= 723 \text{ V/phase}$$

$$E_{LN} = \sqrt{7072^2 + 723^2} = 7108.9 \text{ V}$$

$$E_{LL} = \sqrt{3}\sqrt{7072^2 + 723^2} = 12.313 \text{ kV}$$

$\boxed{A.4}$ (a)

$$12 \text{ kV} = (E_1^2 + E_3^2 + E_5^2)^{1/2}$$

$$= (E_1^2 + (0.25E_1)^2 + (0.05E_1)^2)^{1/2}$$

$$= 1.032 E_1$$

$$\therefore E_1 = 11.63 \text{ kV}$$

$$E_3 = 0.25 \times 11.63 = 2.907 \text{ kV}$$

$$E_5 = 0.05 \times 11.63 = 0.5814 \text{ kV}$$

(b)

$$E_{LL} = \sqrt{3}\,(E_1^2 + (0.05E_1)^2)^{1/2}$$

$$= \sqrt{3}\,(11.63^2 + 0.5814^2)^{1/2} = 20.169 \text{ kV}$$

(c)

$$n = \frac{96}{(8)(3)} = 4 \text{ slots/pole/phase}, \quad \alpha = \frac{180}{12} = 15^\circ$$

$$\gamma = 180 - 135 = 45^\circ$$

A.4 (c) continued

$K_{d_1} = 0.9577 \qquad K_{p_1} = 0.924 \qquad K_{w_1} = 0.885$

$K_{d_3} = 0.653 \qquad K_{p_3} = 0.383 \qquad K_{w_3} = 0.25$

$K_{d_5} = 0.205 \qquad K_{p_5} = -0.383 \qquad K_{w_5} = -0.079$

$$E_h = 4.44 h f N \phi_h K_{wh}$$

$$\phi_h \propto B_{h max}/h \qquad \therefore \quad E_h \propto B_{h max} K_{wh}$$

$$B_{h(max)} = K \frac{E_h}{K_{wh}}$$

$$B_{1 max} = K\left(\frac{11.63}{0.885}\right) = 13.14K$$

$$B_{3 max} = K\left(\frac{2.907}{0.25}\right) = 11.63K$$

$$B_{5 max} = K\left(\frac{0.5814}{-0.0791}\right) = 7.3595K$$

$$B_1 : B_3 : B_5 = 1 : 0.885 : 0.56$$