

ELECTRIC MACHINES

Steady State, Transients,
and Design with MATLAB[®]



ION BOLDEA
LUCIAN TUTELEA



CRC Press
Taylor & Francis Group

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Contents

Preface	xvii

Part I Steady State

1 Introduction	3
1.1 Electric Energy and Electric Machines	3
1.2 Basic Types of Transformers and Electric Machines	6
1.3 Losses and Efficiency	13
1.4 Physical Limitations and Ratings	17
1.5 Nameplate Ratings	19
1.6 Methods of Analysis	21
1.7 State of the Art and Perspective	22
1.8 Summary	25
1.9 Proposed Problems	26
References	27
2 Electric Transformers	31
2.1 AC Coil with Magnetic Core and Transformer Principles	32
2.2 Magnetic Materials in EMs and Their Losses	38
2.2.1 Magnetization Curve and Hysteresis Cycle	38
2.2.2 Permanent Magnets	41
2.2.3 Losses in Soft Magnetic Materials	42
2.3 Electric Conductors and Their Skin Effects	45
2.4 Components of Single- and 3-Phase Transformers	51
2.4.1 Cores	52
2.4.2 Windings	54
2.5 Flux Linkages and Inductances of Single-Phase Transformers	59
2.5.1 Leakage Inductances of Cylindrical Windings	61
2.5.2 Leakage Inductances of Alternate Windings	62
2.6 Circuit Equations of Single-Phase Transformers with Core Losses	64
2.7 Steady State and Equivalent Circuit	65
2.8 No-Load Steady State ($I_2 = 0$)/Lab 2.1	67
2.8.1 Magnetic Saturation under No Load	70

2.9	Steady-State Short-Circuit Mode/Lab 2.2	71
2.10	Single-Phase Transformers: Steady-State Operation on Load/Lab 2.3	74
2.11	Three-Phase Transformers: Phase Connections	78
2.12	Particulars of 3-Phase Transformers on No Load	82
2.12.1	No-Load Current Asymmetry	82
2.12.2	Y Primary Connection for the 3-Limb Core	83
2.13	General Equations of 3-Phase Transformers	84
2.13.1	Inductance Measurement/Lab 2.4	86
2.14	Unbalanced Load Steady State in 3-Phase Transformers/Lab 2.5	87
2.15	Paralleling 3-Phase Transformers	91
2.16	Transients in Transformers	94
2.16.1	Electromagnetic (R,L) Transients	94
2.16.2	Inrush Current Transients/Lab 2.6	95
2.16.3	Sudden Short Circuit from No Load ($V'_2 = 0$)/Lab 2.7	96
2.16.4	Forces at Peak Short-Circuit Current	97
2.16.5	Electrostatic (C,R) Ultrafast Transients	99
2.16.6	Protection Measures of Anti-Overvoltage Electrostatic Transients	102
2.17	Instrument Transformers	102
2.18	Autotransformers	103
2.19	Transformers and Inductances for Power Electronics	106
2.20	Preliminary Transformer Design (Sizing) by Example	108
2.20.1	Specifications	108
2.20.2	Deliverables	109
2.20.3	Magnetic Circuit Sizing	109
2.20.4	Windings Sizing	110
2.20.5	Losses and Efficiency	112
2.20.6	No-Load Current	112
2.20.7	Active Material Weight	113
2.20.8	Equivalent Circuit	113
2.21	Summary	114
2.22	Proposed Problems	115
	References	118
3	Energy Conversion and Types of Electric Machines	121
3.1	Energy Conversion in Electric Machines	121
3.2	Electromagnetic Torque	122
3.2.1	Cogging Torque (PM Torque at Zero Current)	123
3.3	Passive Rotor Electric Machines	124
3.4	Active Rotor Electric Machines	127
3.4.1	DC Rotor and AC Stator Currents	128
3.4.2	AC Currents in the Rotor and the Stator	128
3.4.3	DC (PM) Stator and AC Rotor	129

3.5	Fix Magnetic Field (Brush–Commutator) Electric Machines	131
3.6	Traveling Field Electric Machines	132
3.7	Types of Linear Electric Machines	134
3.8	Summary	139
3.9	Proposed Problem	140
	References	141
4	Brush–Commutator Machines: Steady State	143
4.1	Introduction	143
4.1.1	Stator and Rotor Construction Elements	143
4.2	Brush–Commutator Armature Windings	146
4.2.1	Simple Lap Windings by Example: $N_s = 16, 2p_1 = 4$	148
4.2.2	Simple Wave Windings by Example: $N_s = 9, 2p_1 = 2$	150
4.3	Brush–Commutator	152
4.4	Airgap Flux Density of Stator Excitation MMF	154
4.5	No-Load Magnetization Curve by Example	155
4.6	PM Airgap Flux Density and Armature Reaction by Example	160
4.7	Commutation Process	164
4.7.1	AC Excitation Brush-Commutation Winding	166
4.8	EMF	168
4.9	Equivalent Circuit and Excitation Connection	170
4.10	DC Brush Motor/Generator with Separate (or PM) Excitation/Lab 4.1	171
4.11	DC Brush PM Motor Steady-State and Speed Control Methods/Lab 4.2	173
4.11.1	Speed Control Methods	175
4.12	DC Brush Series Motor/Lab 4.3	181
4.12.1	Starting and Speed Control	183
4.13	AC Brush Series Universal Motor	184
4.14	Testing Brush–Commutator Machines/Lab 4.4	187
4.14.1	DC Brush PM Motor Losses, Efficiency, and Cogging Torque	188
4.15	Preliminary Design of a DC Brush PM Automotive Motor by Example	191
4.15.1	PM Stator Geometry	192
4.15.2	Rotor Slot and Winding Design	193
4.16	Summary	195
4.17	Proposed Problems	197
	References	201

5	Induction Machines: Steady State	203
5.1	Introduction: Applications and Topologies	203
5.2	Construction Elements	204
5.3	AC Distributed Windings	206
5.3.1	Traveling MMF of AC Distributed Windings	206
5.3.2	Primitive Single-Layer Distributed Windings ($q \geq 1$, Integer)	209
5.3.3	Primitive Two-Layer 3-Phase Distributed Windings ($q = \text{Integer}$)	211
5.3.4	MMF Space Harmonics for Integer q (Slots/Pole/Phase)	212
5.3.5	Practical One-Layer AC 3-Phase Distributed Windings	216
5.3.6	Pole Count Changing AC 3-Phase Distributed Windings	220
5.3.7	Two-Phase AC Windings	221
5.3.8	Cage Rotor Windings	222
5.4	Induction Machine Inductances	225
5.4.1	Main Inductance	225
5.4.2	Leakage Inductance	227
5.5	Rotor Cage Reduction to the Stator	229
5.6	Wound Rotor Reduction to the Stator	230
5.7	Three-Phase Induction Machine Circuit Equations	230
5.8	Symmetric Steady State of 3-Phase IMs	234
5.9	Ideal No-Load Operation/Lab 5.1	236
5.10	Zero Speed Operation ($S = 1$)/Lab 5.2	238
5.11	No-Load Motor Operation (Free Shaft)/Lab 5.3	241
5.12	Motor Operation on Load ($1 > S > 0$)/Lab 5.4	243
5.13	Generating at Power Grid ($n > f_1/p_1, S < 0$)/Lab 5.5	244
5.14	Autonomous Generator Mode ($S < 0$)/Lab 5.6	245
5.15	Electromagnetic Torque and Motor Characteristics	246
5.16	Deep-Bar and Dual-Cage Rotors	252
5.17	Parasitic (Space Harmonics) Torques	253
5.18	Starting Methods	256
5.18.1	Direct Starting (Cage Rotor)	256
5.18.2	Reduced Stator Voltages	257
5.18.3	Additional Rotor Resistance Starting	258
5.19	Speed Control Methods	259
5.19.1	Wound Rotor IM Speed Control	261
5.20	Unbalanced Supply Voltages	264
5.21	One Stator Phase Open by Example	265
5.22	One Rotor Phase Open	269
5.23	Capacitor Split-Phase Induction Motors	270
5.24	Linear Induction Motors	275
5.24.1	End and Edge Effects in LIMs	277

5.25	Regenerative and Virtual Load Testing of IMs/Lab 5.7	280
5.26	Preliminary Electromagnetic IM Design by Example	282
5.26.1	Magnetic Circuit	283
5.26.2	Electric Circuit	287
5.26.3	Parameters	287
5.26.4	Starting Current and Torque	290
5.26.5	Breakdown Slip and Torque	291
5.26.6	Magnetization Reactance, X_m , and Core Losses, p_{iron}	291
5.26.7	No-Load and Rated Currents, I_0 and I_n	293
5.26.8	Efficiency and Power Factor	294
5.26.9	Final Remarks	294
5.27	Summary	295
5.28	Proposed Problems	298
	References	300
6	Synchronous Machines: Steady State	303
6.1	Introduction: Applications and Topologies	303
6.2	Stator (Armature) Windings for SMs	306
6.2.1	Nonoverlapping (Concentrated) Coil SM Armature Windings	307
6.3	SM Rotors: Airgap Flux Density Distribution and EMF	314
6.3.1	PM Rotor Airgap Flux Density	317
6.4	Two-Reaction Principle via Generator Mode	318
6.5	Armature Reaction and Magnetization Reactances, X_{dm} and X_{qm}	320
6.6	Symmetric Steady-State Equations and Phasor Diagram	323
6.7	Autonomous Synchronous Generators	325
6.7.1	No-Load Saturation Curve/Lab 6.1	325
6.7.2	Short-Circuit Curve: (I_{sc} (I_F))/Lab 6.2	326
6.7.3	Load Curve: V_s (I_s)/Lab 6.3	326
6.8	Synchronous Generators at Power Grid/Lab 6.4	331
6.8.1	Active Power/Angle Curves: P_e (δ_v)	332
6.8.2	V-Shaped Curves	333
6.8.3	Reactive Power Capability Curves	334
6.9	Basic Static- and Dynamic-Stability Concepts	335
6.10	Unbalanced Load Steady State of SGs/Lab 6.5	339
6.10.1	Measuring X_d , X_q , Z_- , and X_0 /Lab	340
6.11	Large Synchronous Motors	344
6.11.1	Power Balance	345
6.12	PM Synchronous Motors: Steady State	346
6.13	Load Torque Pulsations Handling by Synchronous Motors/Generators	349

6.14 Asynchronous Starting of SMs and Their Self-Synchronization to Power Grid	352
6.15 Single-Phase and Split-Phase Capacitor PM Synchronous Motors	353
6.15.1 Steady State of Single-Phase Cageless-Rotor PMSMs	354
6.16 Preliminary Design Methodology of a 3-Phase PMSM by Example	357
6.17 Summary	363
6.18 Proposed Problems	366
References	370

Part II Transients

7 Advanced Models for Electric Machines	375
7.1 Introduction	375
7.2 Orthogonal (dq) Physical Model	376
7.3 Pulsational and Motion-Induced Voltages in dq Models	378
7.4 dq Model of DC Brush PM Motor ($\omega_b = 0$)	379
7.5 Basic dq Model of Synchronous Machines ($\omega_b = \omega_r$)	380
7.6 Basic dq Model of Induction Machines ($\omega_b = 0, \omega_r, \omega_1$)	382
7.7 Magnetic Saturation in dq Models	383
7.8 Frequency (Skin) Effect Consideration in dq Models	386
7.9 Equivalence between dq Models and AC Machines	387
7.10 Space Phasor (Complex Variable) Model	389
7.11 High-Frequency Models for Electric Machines	392
7.12 Summary	393
7.13 Proposed Problems	396
References	398
8 Transients of Brush-Commutator DC Machines	401
8.1 Introduction	401
8.2 Orthogonal (dq) Model of DC Brush Machines with Separate Excitation	401
8.3 Electromagnetic (Fast) Transients	404
8.4 Electromechanical Transients	406
8.4.1 Constant Excitation (PM) Flux, Ψ_{dt}	406
8.4.2 Variable Flux Transients	410
8.4.3 DC Brush Series Motor Transients	411
8.5 Basic Closed-Loop Control of DC Brush PM Motor	413
8.6 DC-DC Converter-Fed DC Brush PM Motor	413
8.7 Parameters from Test Data/Lab 8.1	415
8.8 Summary	417

8.9	Proposed Problems	418
	References	419
9	Synchronous Machine Transients	421
9.1	Introduction	421
9.2	Phase Inductances of SMs	422
9.3	Phase Coordinate Model	423
9.4	dq0 Model—Relationships of 3-Phase SM Parameters	425
9.5	Structural Diagram of the SM dq0 Model	427
9.6	pu dq0 Model of SMs	430
9.7	Balanced Steady State via the dq0 Model	432
9.8	Laplace Parameters for Electromagnetic Transients	436
9.9	Electromagnetic Transients at Constant Speed	437
9.10	Sudden 3-Phase Short Circuit from a Generator at No Load/Lab 9.1	439
9.11	Asynchronous Running of SMs at a Given Speed	442
9.12	Reduced-Order dq0 Models for Electromechanical Transients	446
9.12.1	Neglecting Fast Stator Electrical Transients	446
9.12.2	Neglecting Stator and Rotor Cage Transients	447
9.12.3	Simplified (Third-Order) dq Model Adaptation for SM Voltage Control	447
9.13	Small-Deviation Electromechanical Transients (in PU)	449
9.14	Large-Deviation Electromechanical Transients	453
9.14.1	Asynchronous Starting and Self-Synchronization of DC-Excited SMs/Lab 9.2	453
9.14.2	Asynchronous Self-Starting of PMSMs to Power Grid	455
9.14.3	Line-to-Line and Line-to-Neutral Faults	455
9.15	Transients for Controlled Flux and Sinusoidal Current SMs	456
9.15.1	Constant d -Axis (ψ_d) Flux Transients in Cageless SMs	457
9.15.2	Vector Control of PMSMs at Constant ψ_{d0} ($i_{d0} = \text{const}$)	460
9.15.3	Constant Stator Flux Transients in Cageless SMs at $\cos \psi_1 = 1$	461
9.15.4	Vector Control of SMs with Constant Flux (ψ_s) and $\cos \varphi_s = 1$	464
9.16	Transients for Controlled Flux and Rectangular Current SMs	465
9.16.1	Model of Brushless DC Motor Transients	465
9.16.2	DC-Excited Cage Rotor SM Model for Rectangular Current Control	468

9.17 Switched Reluctance Machine Modeling for Transients	469
9.18 Split-Phase Cage Rotor SMs	475
9.19 Standstill Testing for SM Parameters/Lab 9.3	477
9.19.1 Saturated Steady-State Parameters, L_{dm} and L_{qm} , from Current Decay Tests at Standstill	478
9.19.2 Single Frequency Test for Subtransient Inductances, L_d'' and L_q''	481
9.19.3 Standstill Frequency Response Tests	481
9.20 Linear Synchronous Motor Transients	483
9.21 Summary	486
9.22 Proposed Problems	489
References	492
10 Transients of Induction Machines	495
10.1 Three-Phase Variable Model	495
10.2 dq (Space Phasor) Model of IMs	497
10.3 Three-Phase IM–dq Model Relationships	499
10.4 Magnetic Saturation and Skin Effects in the dq Model	500
10.5 Space Phasor Model Steady State: Cage and Wound Rotor IMs	501
10.6 Electromagnetic Transients	507
10.7 Three-Phase Sudden Short Circuit/Lab 10.1	509
10.7.1 Transient Current at Zero Speed	512
10.8 Small-Deviation Electromechanical Transients	512
10.9 Large-Deviation Electromechanical Transients/Lab 10.2	514
10.10 Reduced-Order dq Model in Multimachine Transients	516
10.10.1 Other Severe Transients	518
10.11 m/N_r Actual Winding Modeling of IMs with Cage Faults	518
10.12 Transients for Controlled Magnetic Flux and Variable Frequency	522
10.12.1 Complex Eigenvalues of IM Space Phasor Model	522
10.13 Cage Rotor Constant Stator Flux Transients and Vector Control Basics	524
10.13.1 Cage-Rotor Constant Rotor Stator Flux Transients and Vector Control Basics	529
10.13.2 Constant Rotor Flux Transients and Vector Control Principles of Doubly Fed IMs	532
10.14 Doubly Fed IM as a Brushless Exciter for SMs	533
10.15 Parameter Estimation in Standstill Tests/Lab 10.3	537
10.15.1 Standstill Flux Decay for Magnetization Curve Identification: $\Psi_m^*(I_m)$	538

10.15.2 Identification of Resistances and Leakage Inductances for Standstill Flux Decay Tests	540
10.15.3 Standstill Frequency Response Tests	541
10.16 Split-Phase Capacitor IM Transients/Lab 10.4	542
10.16.1 Phase Variable Model	543
10.16.2 dq Model	544
10.17 Linear Induction Motor Transients	545
10.18 Summary	549
10.19 Proposed Problems	553
References	557

Part III FEM Analysis and Optimal Design

11 Essentials of Finite Element Method in Electromagnetics	561
11.1 Vectorial Fields	561
11.1.1 Coordinate Systems	561
11.1.2 Operations with Vectors	563
11.1.3 Line and Surface (Flux) Integrals of a Vectorial Field	564
11.1.4 Differential Operations	565
11.1.5 Integral Identities	567
11.1.6 Differential Identities	568
11.2 Electromagnetic Fields	569
11.2.1 Electrostatic Fields	569
11.2.2 Fields of Current Densities	570
11.2.3 Magnetic Fields	571
11.2.4 Electromagnetic Fields: Maxwell Equations	572
11.3 Visualization of Fields	573
11.4 Boundary Conditions	576
11.4.1 Dirichlet's Boundary Conditions	576
11.4.2 Neumann's Boundary Conditions	576
11.4.3 Mixed Robin's Boundary Conditions	577
11.4.4 Periodic Boundary Conditions	577
11.4.5 Open Boundaries	577
11.4.5.1 Problem Truncation	577
11.4.5.2 Asymptotical Boundary Conditions	577
11.4.5.3 Kelvin Transform	579
11.5 Finite Element Method	581
11.5.1 Residuum (Galerkin's) Method	582
11.5.2 Variational (Rayleigh–Ritz) Method	583
11.5.3 Stages in Finite Element Method Application	583
11.5.3.1 Domain Discretization	583
11.5.3.2 Choosing Interpolation Functions	584

11.5.3.3	Formulation of Algebraic System Equations	584
11.5.3.4	Solving Algebraic Equations	584
11.6	2D FEM	584
11.7	Analysis with FEM	586
11.7.1	Electromagnetic Forces	589
11.7.1.1	Integration of Lorenz Force	589
11.7.1.2	Maxwell Tensor Method	590
11.7.1.3	Virtual Work Method	590
11.7.2	Loss Computation	591
11.7.2.1	Iron Losses	591
References		592
12	FEM in Electric Machines: Electromagnetic Analysis	595
12.1	Single-Phase Linear PM Motors	595
12.1.1	Preprocessor Stage	597
12.1.2	Postprocessor Stage	602
12.1.3	Summary	609
12.2	Rotary PMSMs (6/4)	609
12.2.1	BLDC: Preprocessor Stage	610
12.2.2	BLDC Motor Analysis: Postprocessor Stage	616
12.2.3	Summary	632
12.3	The 3-Phase Induction Machines	632
12.3.1	Induction Machines: Ideal No Load	636
12.3.2	Rotor Bar Skin Effect	642
12.3.3	Summary	649
References		650
13	Optimal Design of Electric Machines: The Basics	651
13.1	Electric Machine Design Problem	651
13.2	Optimization Methods	653
13.3	Optimum Current Control	659
13.4	Modified Hooke–Jeeves Optimization Algorithm	664
13.5	Electric Machine Design Using Genetic Algorithms	670
References		674
14	Optimization Design of Surface PMSMs	675
14.1	Design Theme	675
14.2	Electric and Magnetic Loadings	675
14.3	Choosing a Few Dimensioning Factors	677
14.4	A Few Technological Constraints	678
14.5	Choosing Magnetic Materials	678
14.6	Dimensioning Methodology	681
14.6.1	Rotor Sizing	685
14.6.2	PM Flux Computation	686

14.6.3	Weights of Active Materials	692
14.6.4	Losses	693
14.6.5	Thermal Verification	694
14.6.6	Machine Characteristics	694
14.7	Optimal Design with Genetic Algorithms	694
14.7.1	Objective (Fitting) Function	696
14.7.2	PMSM Optimization Design Using Genetic Algorithms: A Case Study	697
14.8	Optimal Design of PMSMs Using Hooke–Jeeves Method . . .	709
14.9	Conclusion	710
	References	715
15	Optimization Design of Induction Machines	717
15.1	Realistic Analytical Model for Induction Machine Design . . .	717
15.1.1	Design Theme	717
15.1.2	Design Variables	718
15.1.3	Induction Machine Dimensioning	719
15.1.3.1	Rotor Design	721
15.1.3.2	Stator Slot Dimensions	724
15.1.3.3	Winding End-Connection Length	724
15.1.4	Induction Machine Parameters	725
15.2	Induction Motor Optimal Design Using Genetic Algorithms	729
15.3	Induction Motor Optimal Design Using Hooke–Jeeves Algorithm	739
15.4	Machine Performance	742
15.5	Conclusion	750
	References	751
Index		753

Preface

From Energy to Electric Machines

Electric machines are devices that convert mechanical energy to electric energy in electric generators, and electric energy to mechanical energy in electric motors. These machines are reversible in that they can switch easily from the generating to the motoring operation modes.

Energy is essential to modern life for heating/cooling, for producing various industrial and home appliances, and for transporting goods and people. Electric energy is most adequate for temperature (heat) and motion control as it is clean, easy to transport across long distances, and easy to process through power electronics.

Electric generators (with the exception of fuel cells, photovoltaic panels, and batteries) are used as sources of electric energy in almost all electric power plants. They are driven by prime movers, such as hydraulic, gas, and steam turbines; diesel engines; and wind or wave turbines.

On the other hand, electric motors produce motion control that is necessary in all industries to increase productivity, save energy, and reduce pollution.

Electric motor drives, digitally controlled by power electronics via digital signal processors (DSPs) are used in a wide variety of applications from overhead cranes to people and goods movers, from internal combustion engines to hybrid electric cars, and from home appliances to info gadgets. The power of the motors may vary from hundreds of megawatts per unit to a few fractions of a watt per unit. Homes, cars, ships, aircraft, robots, desktop and notebook computers, and mobile phones, all have one or more electric motor drives in them.

Subject

This book aims to cover standard and new electric machines in a comprehensive manner in terms of

1. Topologies
2. Steady-state modeling and performance
3. Preliminary design and testing
4. Modeling for transients
5. Control essentials

6. Finite element analysis
7. Optimal design methodologies

How to Use This Book

This book would cover three semesters using the items listed above under topologies, steady-state modeling and performance, and preliminary design and testing (Chapters 1 through 6); modeling for transients and control essentials (Chapters 6 through 10); and finite element analysis and optimal design methodologies (Chapters 11 through 15): one semester for an undergraduate course and two semesters for a graduate course.

The three parts are independent, but an effort as been made to unify the symbols throughout the book. This book includes several worked out numerical examples and proposed problems with hints to find a solution.

The CD Version/MATLAB® and Simulink® Programs

This book contains a total of 17 MATLAB and Simulink programs dedicated to the three parts in this book. They are on the CD that comes along with this book.

Addressee

This book is dedicated to all electrical and mechanical engineering students and research & development engineers in industry who are interested in the exploitation, design, testing, and manufacturing of electric machines for generating electricity, or in constant or variable speed motors for motion control.

Content of the Chapters**Part I Steady State**

Chapter 1 is an introduction to the subject and deals with the consumption of electric energy and its applications through electric machines; basic types of transformers and electric machines with illustrations; descriptions of principles, power ranges, and typical applications; and state-of-the-art methods of analyses of electric machines.

Chapter 2 provides comprehensive coverage of single-phase and 3-phase power transformers. It also provides several examples, from topologies to steady-state modeling, performance, and testing, to transients and preliminary electromagnetic design methodologies.

Chapter 3 investigates the energy conversion process in the main types of rotary and linear motion electric machines.

Chapter 4 covers dc and ac brush machines in terms of topology, steady-state modeling, and characteristics in generating/motoring/braking operation modes.

Chapters 5 and 6 provide thorough coverage of topologies, steady-state modeling, and performance in various operation (motor/generator) modes, and the preliminary design of 3 (and 1)-phase induction and synchronous (with PM and dc-excited rotor) machines.

Part II Transients

Chapters 7 through 10 deal with electric machine transients as follows:

Chapter 7: Advanced (dq, space phasor) electric machine models for transients.

Chapter 8: DC brush machine transient modeling, transfer functions, and control essentials.

Chapter 9: Synchronous machine transient modeling, transfer functions, and control essentials.

Chapter 10: Induction machine transient modeling, transfer functions, and main control techniques.

Part III FEM Analysis and Optimal Design

Chapters 11 through 15 cover finite element analysis and the optimal design of electrical machines as follows:

Chapter 11: Finite element method essentials and a linear machine case study.

Chapter 12: Finite element method in the analysis of PM synchronous and induction machines with case studies.

Chapter 13: The basics of optimal design methodologies for electric machines.

Chapters 14 and 15: Optimal design of PM synchronous and induction machines with Hooke-Jeeves and genetic algorithms, respectively, along with case studies.

MATLAB and Simulink Programs on the accompanying CD

Chapter 2—Transformer:

- Example 1—Magnetic circuit
- Example 2—Transformer steady state
- Example 3—Transformer unbalanced load current
- Example 4—Transformer unbalanced load impedance

Chapter 4—DC brush motor:

- Example 5—Separately excited dc motor
- Example 6—Series excitation dc motor

Chapter 5—Induction machines—steady state:

- Example 7—Induction motor characteristics

Chapter 6—Synchronous machines—steady state:

- Example 8—Synchronous motor characteristics

Chapters 7 through 10—Transients:

- Example 9—No-load transformer grid connection
- Example 10—Loaded transformer grid connection
- Example 11—DC motor transients
- Example 12—Induction motor transients
- Example 13—Synchronous motor transients

Chapter 14—Optimal design of induction machines:

- Example 14—Induction machine optimal design by the Hooke–Jeeves method
- Example 15—Induction machine optimal design by genetic algorithms

Chapter 15—Optimal design of PM synchronous machines:

- Example 16—PM synchronous machine optimal design by the Hooke–Jeeves method
- Example 17—PM synchronous machine optimal design by genetic algorithms

The computer simulation programs in MATLAB and Simulink should be instrumental for seminars and homework assignments in facilitating a quantitative assessment of various parameters and performance indices of electric machines. Various chapters contain sections with the label “Lab,” which means that they may constitute the subject of laboratory work.

That this book provides “comprehensive” coverage of electric machines is more than evident by the detailed coverage of computer simulation programs. Many parts of this book have been used in the classroom for quite a few years, in progressively modified forms.

We have made every effort to introduce ready (safe)-to-use (in industry) expressions of parameters, modeling, and characteristics, to make this book directly applicable for research & development, in the industry of electric machines for modern (distributed) power systems and industrial motion control via power electronics.

Ion Boldea
Lucian Tutelea
Timisoara, Romania

MATLAB® is a registered trademark of The MathWorks, Inc. For product information, please contact:

The MathWorks, Inc.

3 Apple Hill Drive

Natick, MA 01760-2098 USA

Tel: 508 647 7000

Fax: 508-647-7001

E-mail: info@mathworks.com

Web: www.mathworks.com

Part I

Steady State

Introduction

1.1 Electric Energy and Electric Machines

Electric energy represents a key element in modern society. Fossil fuels such as coal, natural gas, or nuclear fuel are burned in a combustor to produce heat, which is then transformed into mechanical energy in a turbine (prime mover). Alternatively, wind and hydroturbines transform wind and hydro energies into mechanical energy. An electric generator is driven directly or through transmission by the turbine to produce electric energy.

Electric energy is measured in joule or kWh:

$$1 \text{ kWh} = 3.6 \times 10^6 \text{ J} \quad (1.1)$$

The current global energy consumption is about 16×10^{12} kWh per year with a projected increase of 2%–3% per year.

Electric power is measured either in W and MW, or in GW (1 GW = 10^{12} W).

The total installed power in power plants all over the world today is around 3700 GW (800 GW in the United States), out of which 50 GW is installed in wind turbine generators. Installed power tends to increase faster (above 4% per year) than the consumption of electric energy (in kWh) because of the limited availability of various fuel power plants and the daily (or monthly) peak power requirements (Figure 1.1).

With the exception of solar and fuel cells, which contribute negligibly, practically all electric energy is “produced” or rather converted from mechanical energy through electric generators: constant-speed-regulated ac (synchronous) generators are mostly used, but in recent times variable speed-regulated ac (synchronous and induction) or dc output (switched reluctance) generators are being used for small hydro and wind energies.

Electric energy is converted into controlled mechanical work again in electric motors (about 60%), or into controlled heat (for lighting, cooling, heating, etc.). Electric machines are reversible and can function either as generators or as motors (Figure 1.2). They either convert mechanical energy into electric energy (generator mode) or the process is reversed (motor mode).

In both modes, energy conversion ratios and electric machine costs are paramount, because, ultimately, a more costly electric machine means more

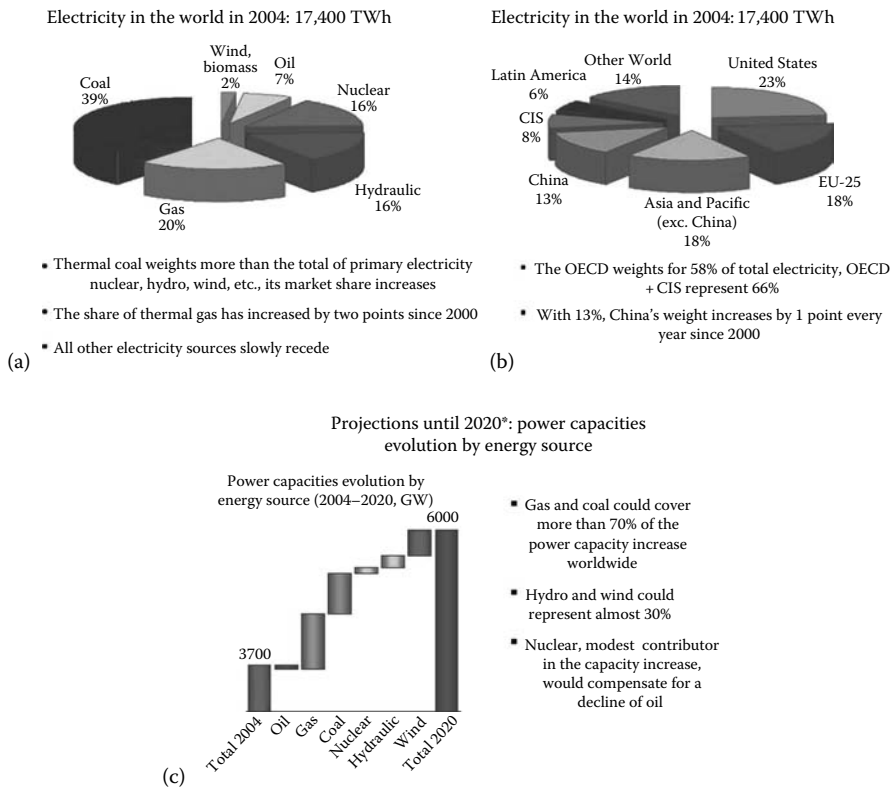


FIGURE 1.1
Electric energy in the world. *Note: Forecasts extracted from the EnerFuture Forecasts Service.

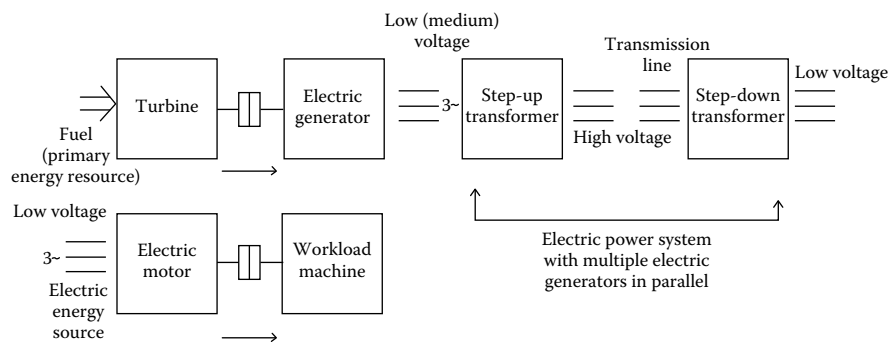


FIGURE 1.2
Generator/motor operation mode.

active materials, which in turn means more energy to produce them, leading to more thermal and chemical pollution.

Variable speed control of electric motors through power electronics is currently the key solution to increased productivity, consuming less energy in both residential and industrial applications: from info gadgets and home appliances to the new electric or hybrid automobiles, public transportation, pumps, compressors, and industrial drives (Figure 1.3). Refs. [1–34] deal with the core topics of this book in more detail.

Electric machines do convert mechanical energy into electric energy, but not directly. They need to store energy in a magnetic form.

Electric machines are systems of coupled electric and magnetic circuits that convert energy, based on the electromagnetic induction (Faraday's) law for bodies in relative motion.

They have, in general, a fixed part called the stator and a movable part, called the rotor with an airgap layer of 0.2 mm (for smallest, sub-Watt units) to about 20 mm (for largest power turbine generators of 1700 MW/unit).

As electric generators produce electric power at low and medium voltages (below 28 kV in general), and as electric energy transmission at hundreds of kilometers requires high voltage (to reduce copper losses and weight) and costs of transmission lines, while consumers require low voltage electric energy (for lower costs and human safety), voltage step-up and step-down are required. Voltage step-up and step-down are also required to match local source of electric energy to motor voltage requirements, such as

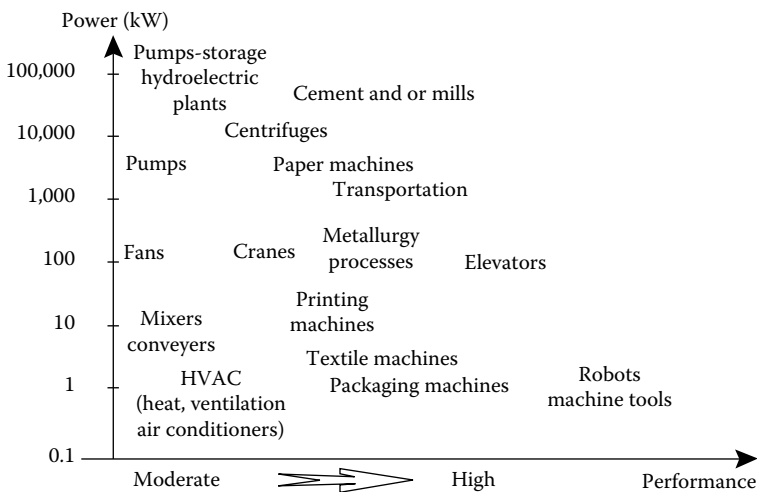


FIGURE 1.3

Electric motor drive applications.

those of electric automobiles or other passive loads with (or without) power electronics, for example, furnaces in metallurgy, etc. Also, electric separation for equipment safety is required in many applications.

The electric transformer, which is a system of coupled electric and magnetic circuits, performs voltage (or current) step-up or step-down based on Faraday's law, but for relatively fixed bodies (with no mechanical to electric energy conversion).

Electric power transformers, based on Faraday's law, are primarily used with electric machines, and, therefore, are dealt within detail. Power electronics transformers are dealt with in a separate paragraph, as they deserve special attention.

1.2 Basic Types of Transformers and Electric Machines

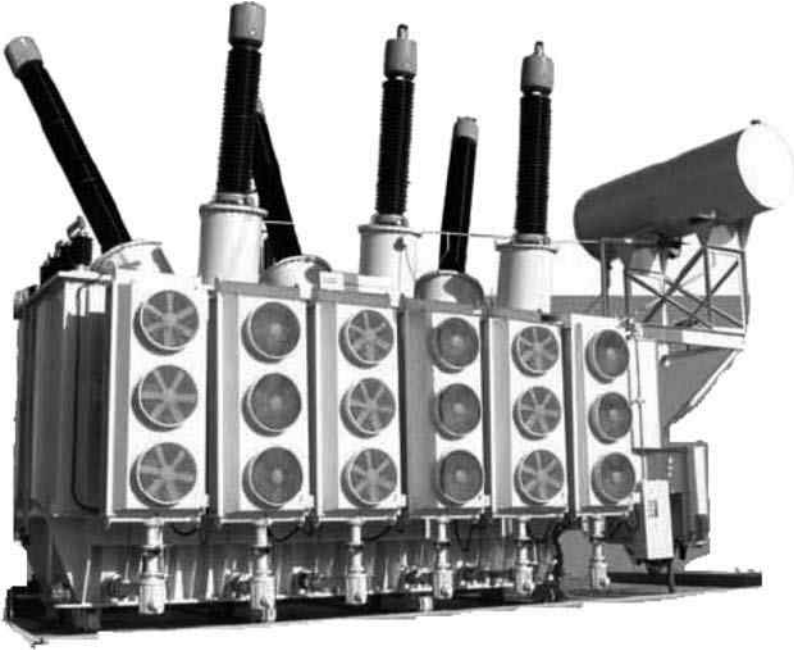
This book introduces the basic types of transformers and electric machines; representative illustrations of the main types of transformers (by applicability) and electric machines (by principle) are also provided.

It is needless to say that transformers and electric machines represent a worldwide business regulated technically by national and international standards. The IEC (International Electrotechnical Commission) and the IEEE (Institute of Electrical and Electronics Engineers) issue some of the most approved international standards in the field.

The main types of electric transformers are

- *Power transformers* (Figure 1.4)—3-phase and 1-phase—at 50 (60) Hz for power transportation and distribution to consumers (residential or industrial) at the required voltage level [1]
- *Power transformers for special applications*: autotransformers, phase-shifting transformers, HVDC (high voltage direct current) transmission line transformers, industrial power electronics motor drive transformers, traction transformers, reactors, earthing transformers, welding transformers, locomotive transformers, furnace transformers, etc. [1]
- Voltage and current measurement transformers (to measure high ac voltages and currents with 100 V, 5 A instruments)
- Power electronics power and control transformers and reactors (at high switching frequency—from kHz to MHz) [2]

Electric machines may be classified based on principle into two main categories:

**FIGURE 1.4**

1100 MVA, 345/19 kV, 3-phase generator step-up transformer.

- Fixed magnetic fields of the stator and the rotor in the airgap (brush-commutator electric machines)
- Moving magnetic fields in the airgap

For all electric machines, the objective is to produce an electromagnetic torque (or electric power) that does not show ripple during steady-state operation. For pure traveling and fixed magnetic field machines, this is feasible. But when the moving magnetic field speed is not constant in time, this is not possible (such as in single-phase alternating current machines).

- *Fixed magnetic field machines* (Figure 1.5) are all provided on the rotor shaft with a cylindrical (or disk-shaped) mechanical commutator realized with copper sectors that are insulated from each other and that connect all rotor coils (placed in a slotted laminated cylindrical or disk-shaped core) in series and on which act mechanically pressured electric brushes that collect (for generator mode) or “introduce” direct current (dc) in the rotor to (from) a dc power source. The stator is made of a thin laminated soft-iron core with salient poles ($2p_1$ poles)—semiperiods—that hold dc-fed coils (or permanent magnets [PMs]) that produce a fixed heteropolar magnetic flux density distribution in the airgap, aligned with stator poles. On the

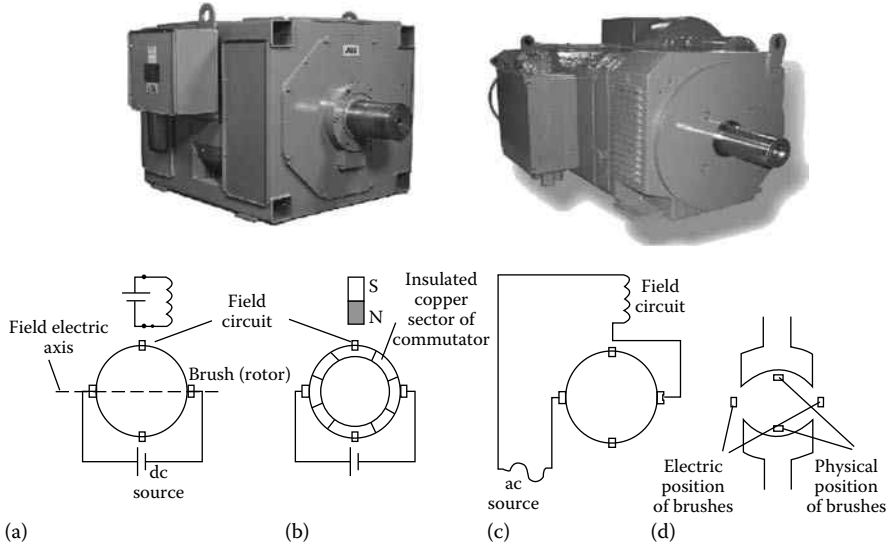


FIGURE 1.5
Brush-commutator cylindrical electric machine: (a) dc-excited (two poles: $2p_1 = 2$), (b) PM (dc)-excited (two poles: $2p_1 = 2$), (c) ac-series-excited (universal motor) (two poles: $2p_1 = 2$), and (d) stator and rotor field axes.

other hand, the mechanical brush-commutator changes the brush dc currents into ac currents in the rotor ($f = n \cdot p_1$, where f is the frequency in Hz and n is the speed in rps), whose magnetic flux density in the airgap is fixed, physically 90 shifted to brushes (if the coils are symmetric).

To produce maximum torque (based on the $\vec{f} = \vec{j} \times \vec{B}$ principle), the two field axes (maximums) have to be shifted by 90 electric degrees ($\alpha_e = p_1 \cdot \alpha_m$, where α_m is the mechanical angle and α_e is the electric angle), that is, for symmetric rotor coils, the brush physical axis coincides with the stator pole axis (Figure 1.5d). The “electric” position of brushes is, in fact, along the rotor current field axis.

Brush-commutator machines are connected to dc sources, but can also be connected to ac (1-phase) sources, with the field excitation circuit connected in series to brushes (universal motor). Also, dc series excitation brush-commutator machines are still used by urban public transportation and in electric or diesel locomotives (where as multiple dc-excited generators still exist in some parts of the world, generating powers up to 6–8 MW/unit).

The speed and power limitations are rather severe as dictated by the safe (spark-free) mechanical commutation of rotor currents from dc to ac in the mechanical commutator. The most popular contemporary brush-commutator machines are small-power machines with PM excitation to drive

printers, small ventilators on info gadgets, and auxiliaries (wind wipers, fuel pumps, door openers) on electric vehicles or in small robots. Though the tendency is to replace them with brushless (moving field) electric motors, they may last long, up to 1 kW and 30,000 rpm in motors used universally in construction tools (vibrators, etc.) and for various home appliances (dryers, vacuum cleaners, some washing machines, kitchen mixers, etc.).

- *Moving (mobile) field electric machines* may be classified into two main categories based on the way the rotor currents are produced:

a. Induction machines

b. Synchronous machines

Both have a uniformly slotted, laminated, soft-iron cylindrical core in the stator which holds $2p_1$ -pole (semiperiods) 3-phase winding coils, which, when an alternative (sinusoidal) voltage is fed at frequency f_1 , produce in the airgap a magnetic flux density wave field at constant speed (called synchronous speed) n_1 :

$$n_1 = f_1/p_1 \quad (1.2)$$

The rotor of the induction machine (IM) holds in the uniformly slotted laminated rotor core aluminum (copper) bars short-circuited by end rings (cage rotor or wound rotor) or 3-phase windings (as in the stator), connected to insulated copper rings and then to stator brushes which are not commutators, (but only a mechanical power switch) as they do not change the frequency of current in the rotor (Figure 1.6).

The wound rotor may be connected to a variable impedance or through a frequency changer to the same ac power source as the stator.

The traveling field (at n_1 , rps) of stator currents produces in the rotor, which rotates at speed n (rps), electromagnetic forces (emfs) at frequency f_2 :

$$f_2 = f_1 - n \cdot p_1 = S \cdot f_1; \quad S = 1 - \frac{n \cdot p_1}{f_1} \quad (1.3)$$

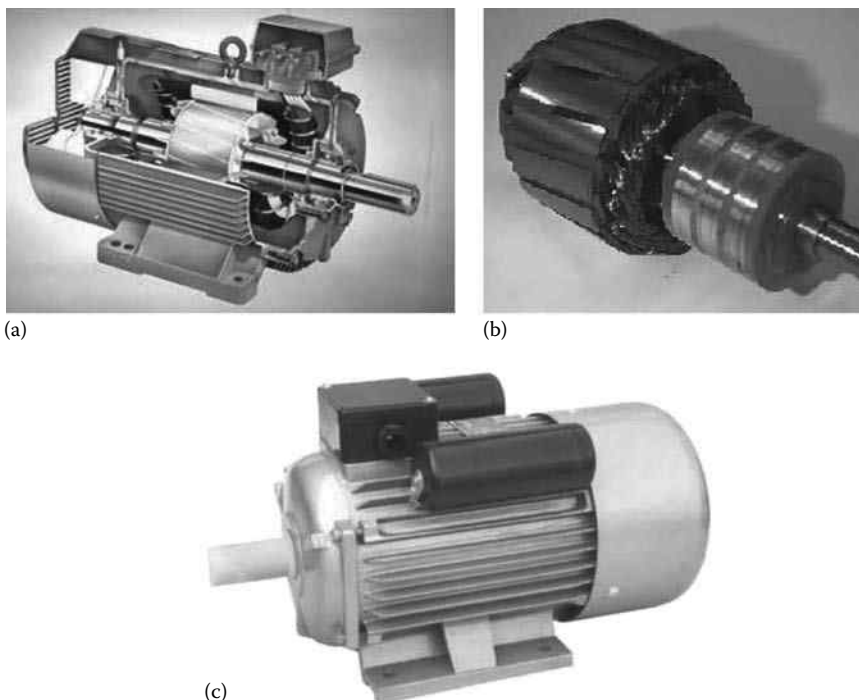
where

S is called slip

p_1 is called pole pairs or magnetic field periods per (one) mechanical revolution

Now, the corresponding rotor currents will have the same frequency f_2 but their traveling field speed with respect to the stator will be the same, $n_1 = f_1/p_1$, as that of the stator currents, under steady state.

So the magnetic fields produced by the stator and the rotor in the airgap will be at standstill with each other at any speed. Consequently, the machine develops a constant steady-state torque at any speed if the rotor currents are nonzero. The rotor current is zero for the cage rotor at the rotor speed, $n_0 = n_1 = f_1/p_1$, called the standard ideal no-load (or synchronous) speed.

**FIGURE 1.6**

Induction machines (IM): (a) 3-phase with cage rotor, (b) 3-phase with wound rotor, and (c) 1-phase supply capacitor IM.

So, to develop torque, $n \neq n_1$ ($S \neq 0$), and thus the IM is also called the asynchronous machine. For $n < n_1$, the machine acts as a motor and, for $n > n_1$, as a generator. The wound rotor IM may reach a zero rotor current at any speed, provided the rotor power electronics supply can produce (or absorb) electric power at a frequency f_2 , according to Equation 1.3, called also called the “theorem of frequencies.”

The wound rotor IM doubly fed (in the stator at constant voltage and frequency, and in the rotor at variable frequency (f_2) and voltage (V_2)) may work as a motor and a generator both for $n < f_1/p_1$ (subsynchronous) and for $n > f_1/p_1$ (supersynchronous). The latter is currently the workhorse of the variable speed wind generator industry and in pump storage hydroelectric power plants (which generate electricity at peak consumption hours and pump water back into the upper reservoir during off-peak hours) generating up to 400 MW/unit.

At low powers (up to 2–3 kW) in hand tools, small-power compressors, and washing machines, the 1-phase power grid (50 (60) Hz)–supplied IM, with a main and an auxiliary winding in the stator and cage rotor, is still widely used, owing to its ruggedness and overall low cost (of motor and capitalized losses), and constant speed.

When variable speed is needed, the 3-phase cage rotor IM is used, with power electronics variable voltage and frequency supply to the stator.

The cage rotor IM is, thus, not only the workhorse but has also recently become (at variable V_1 and f_1) the racehorse of industry.

A synchronous machine (Figure 1.7) has about the same stator morphology but the rotor completely resembles the stator of the brush-commutator machine. That is, the rotor has a laminated core with the dc-fed electric coils placed around salient poles or in slots or has permanent magnets to produce a heteropolar magnetic flux density in the airgap with the same number of poles, $2p_1$, as the ac stator winding currents.

The rotor frequency is $f_2 = 0$, and, thus, mandatorily, the rotor speed n is

$$n = n_1 = f_1/p_1 \quad (1.4)$$

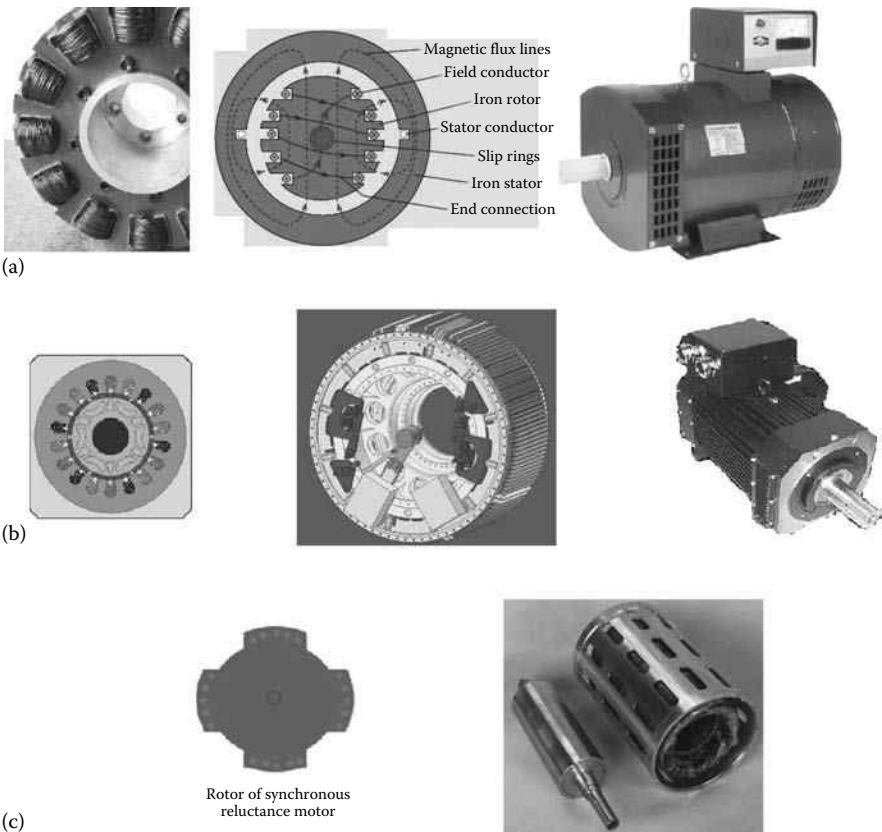


FIGURE 1.7

Synchronous machines: (a) Synchronous generator with dc rotor excitation, (b) PMSM, and (c) RSM.

The slip is $S = 0$ during steady state and this is why the machine is called synchronous (SM). The trouble is that the rotor field is fixed to the rotor and thus the machine can work only at synchronism, $n = n_1 = f_1/p_1$. To change the speed, the stator frequency has to be changed accordingly, from zero, for synchronous starting.

So the SM fed from the standard 50 (60) or 400 Hz sources (the latter on aircraft) cannot be started as such. It may start as an induction motor first, up to a speed $n < n_1$, with a cage placed on rotor poles and the excitation winding connected to a 10/1 resistance. Then, the self-synchronization starts by switching the field winding to the dc supply. DC excitation may be produced by slip rings and brushes but the synchronous motor may be also brushless.

For the cage rotor PMSM, the self-starting at the power grid as an induction motor and self-synchronization take place in one step.

It is also possible to use a passive magnetically anisotropic rotor (Figure 1.7c) and produce torque based on the energy conversion principle:

$$T_e = - \frac{\partial W_m}{\partial \theta_r} \quad (1.5)$$

When W_m —magnetic energy stored in the machine—varies with the rotor position due to rotor anisotropy, electromagnetic (reluctance) torque occurs.

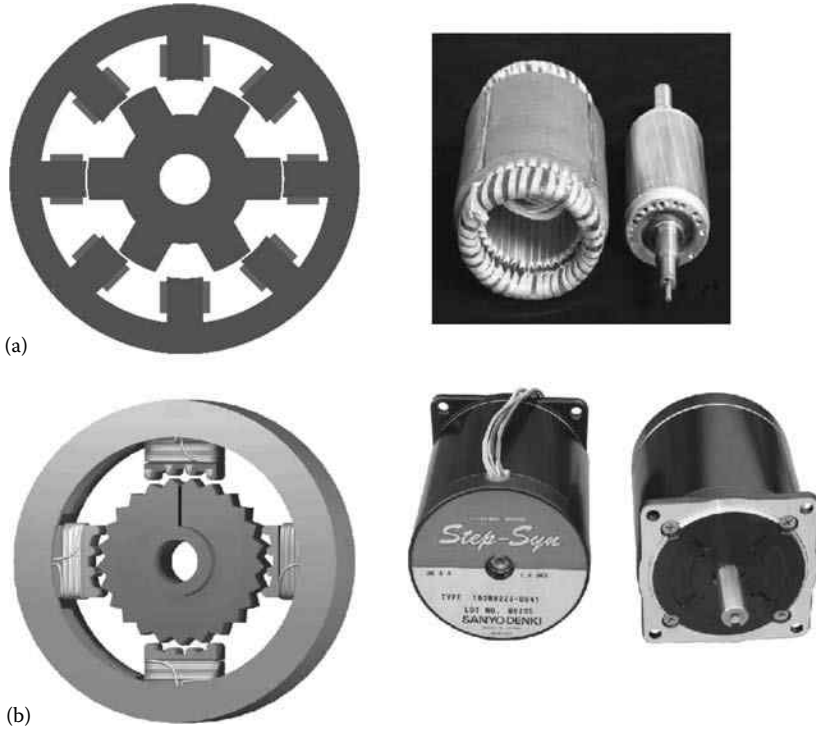
This is the so-called reluctance synchronous machine (RSM), which, due to large magnetic saliency in the rotor, is now considered competitive, based on good performance at lower cost, for various power grid and variable speed applications at small powers (below 100 kW) and down to 200 W or less.

With the same PM or anisotropic rotor, with or without a cage, the SM may have only a main and an auxiliary capacitor winding in the stator, when only a 1-phase ac supply is available. More appliances and automotive light-starting torque applications are the target of such motors.

Now, if IMs and SMs qualify for constant speed (traveling) field machines, which may be supplied either from standard ac power grid or through variable voltage and frequency power electronics, there are double saliency machines with passive anisotropic rotors without a cage, which have jumping stator field and are totally dependent on power electronics. They need stator-position-triggered current pulsed control. They are called switched reluctance machines (Figure 1.8a) or stepper motors when the current pulse sequence is independent of the rotor position, but frequency ramping is limited so that the motor does not lose steps (Figure 1.8b).

In thermally or chemically aggressive environments, the SRM and stepper motors have found good markets.

This section has attempted a qualitative classification (characterization) of electric machines, with their applications, described later in this book.

**FIGURE 1.8**

(a) Switched reluctance motors and (b) stepper motors.

For more info on this subject see [3] and visit www.abb.com/motors, www.siemens.com, www.ge.com. Please also note that for every rotary-motion electric machine there is a linear-motion counterpart. They will be treated in the following chapters. For more on linear electric machines, see [4]. The torque production of various electric machines will be revisited in a more rigorous manner in Chapter 3.

1.3 Losses and Efficiency

Energy losses in electric machines produce heat, harm the environment, and cost money, as more input energy is required. Losses in transformers are electric in nature and occur as copper (winding), P_{copper} , and core (magnetic), P_{core} , losses; electric machines add mechanical losses, p_{mec} .

The power efficiency of an electric machine, η_e , is defined as the output/input power:

$$\eta_e = \frac{\text{output power}}{\text{input power}} = \frac{\text{output power}}{\text{output power} + \sum \text{losses}} = \frac{P_2}{P_2 + \sum p} \quad (1.6)$$

Also $\eta_e = \frac{P_2}{P_1}$.

$$\sum p = p_{\text{copper}} + p_{\text{core}} + p_{\text{mec}} \quad (1.7)$$

For an electric machine with frequent acceleration–deceleration sequences, energy efficiency per duty cycle, EE, is more relevant

$$EE = \frac{\text{energy output}}{\text{energy output} + \text{energy losses}} = \frac{W_2}{W_2 + \sum W} \quad (1.8)$$

with

$$W_2 = \int P_2 dt; \quad \sum W = \sum \int p dt \quad (1.9)$$

P_2 in efficiency formula (1.6) is active power, or time average in ac, per cycle period T in ac machines. For 3-phase machines as generators, P_{2e} is the electric power:

$$P_{2 \text{ electric}} = \frac{1}{T} \sum_n \int_0^T v_i(t) \cdot i_i(t) dt \quad (1.10)$$

For sinusoidal stator voltages and currents,

$$v_i(t) = V_1 \sqrt{2} \cos \left(\omega_1 t - (i-1) \frac{2\pi}{3} \right); \quad i = 1, 2, 3 \quad (1.11)$$

$$i_i(t) = I_1 \sqrt{2} \cos \left(\omega_1 t - (i-1) \frac{2\pi}{3} - \varphi_1 \right); \quad i = 1, 2, 3 \quad (1.12)$$

$$P_{2e} = 3V_1 I_1 \cos \varphi_1 \quad (1.13)$$

where

V_1 and I_1 are the rms values of sinusoidal voltages and currents, respectively

φ_1 is the time lag angle between phase voltage and current phasors

For dc current (dc brush–commutator) machines as generators, the output power P_{2e} is

$$P_{2e} = V_{\text{dc}} I_{\text{dc}} \quad (1.14)$$

For an electric motor operation, P_{2e} is the mechanical (shaft) power P_{2m} :

$$P_{2m} = T_{\text{shaft}} 2\pi n \quad (1.15)$$

where

T_{shaft} is the shaft torque [Nm]

n is the speed (rps)

The electromagnetic torque, T_e , is

$$T_e = T_{\text{shaft}} \pm \frac{p_{\text{mec}}}{2\pi n} \quad (1.16)$$

The “+” sign is valid for motor operations and the “−” sign for generator operations.

For ac motors and for motors supplied from power electronics, more energy conversion performance indexes may be added.

The first one is EEF , the ratio between the active output power, P_2 , and the apparent RMS input power, S_{RMS} , or peak apparent power, S_{peak} :

$$EEF_{\text{RMS}} = \frac{P_2}{S_{\text{RMS}}}; \quad EEF_{\text{peak}} = \frac{P_2}{S_{\text{peak}}} \quad (1.17)$$

$$S_{\text{RMS}} = 3V_{\text{RMS}}I_{\text{RMS}} \quad (1.18)$$

$$S_{\text{peak}} = 3V_{\text{peak}}I_{\text{peak}} \quad (1.19)$$

For a sinusoidal operation,

$$S_{\text{RMS}} = 3V_1I_1; \quad EEF_{\text{RMS}} = \eta \cos \varphi_1 \quad (1.20)$$

Operation at the unity power factor ($\varphi_1 = 0$, Figure 1.9a) for sinusoidal voltages and currents is ideal in ac machines because the absorbed stator current is minimum, for a given active power, and thus the copper losses in the ac supply grid are minimum. For SMs with a dc rotor excitation, this is feasible for all load levels if the dc field current can be controlled. Not so is the case in power grid connected IMs, which always show a lagging power factor angle (Figure 1.9b).

The peak apparent power, S_{peak} , is very useful in the rating of power switches or power electronics that connect the electric machines to the power grid.

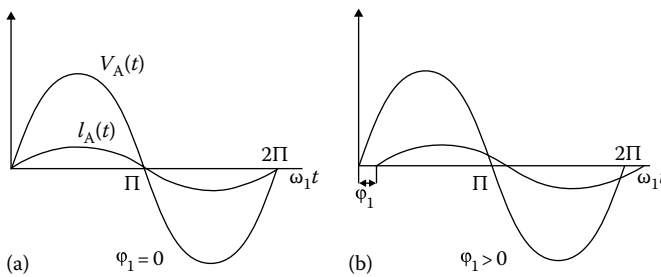


FIGURE 1.9

(a) Unity power factor (synchronous motors) and (b) lagging power factor in ac machines (induction motors).

Example 1.1 A directly driven permanent magnet synchronous wind generator (PMSG) of $S_n = 4.53$ MVA is connected to a $V_{nl} = 3.53$ kV (line voltage), 50 Hz power grid, and works at a rated efficiency $\eta = 0.96$ and power factor $\cos \varphi = 0.5$.

Calculate

- Rated stator phase current (star connection): I_n
- Number of pole pairs p_1 , if the rated speed $n_n = 15$ rpm
- Total losses and shaft (input) power
- Rated shaft torque

Solution:

- From Equation 1.20

$$S_n = \sqrt{3} V_{nl} I_n = 4.5 \times 10^6 \text{ VA}$$

$$I_n = \frac{4.5 \times 10^6}{\sqrt{3} \times 3500} = 743.18 \text{ A}$$

- Based on Equation 1.4

$$p_1 = \frac{f_1}{n_1} = \frac{50}{15/60} = 200 \text{ polepairs!}$$

So the machine shows 400 poles.

- The total losses, $\sum p$, from Equation 1.17

$$\begin{aligned} \sum p &= P_{2e} \cdot \left(\frac{1}{\eta_n} - 1 \right) = S_n \cos \varphi_n \left(\frac{1}{\eta_n} - 1 \right) \\ &= 4.5 \times 10^6 \times 0.5 \left(\frac{1}{0.96} - 1 \right) = 0.09375 \times 10^6 \text{ W} \end{aligned}$$

The shaft power, P_{1m} , is

$$P_{1m} = \frac{\sum p}{1 - \eta_n} = \frac{0.09375 \times 10^6}{1 - 0.96} = 2.343 \times 10^6 \text{ W} = 2.343 \text{ MW}$$

- The shaft torque, T_{shaft} , from Equation 1.16 is

$$T_{\text{shaft}} = \frac{P_{1m}}{2\pi n} = \frac{2.343 \times 10^6}{2\pi} \times \frac{15}{60} = 1.4928 \times 10^6 \text{ Nm!}$$

1.4 Physical Limitations and Ratings

Physical limitations on electric machines are of electric, mechanical, and thermal origins. The maximum and time-average temperature of electric conductors is limited by the class of electric insulation. PMs irreversibly lose their hard magnetic properties above a certain temperature.

The insulation materials have been classified (standardized) into four classes whose maximum safe temperature limits given by the IEC 317 Standard are

Class A: 105C (less and less used today)

Class B: 130C

Class F: 155C

Class H: 180C

With special insulation materials, even higher safe temperatures are feasible. Electric conductor insulation materials are dealt within IEC standards 317-20, 51, 13, 26 for the insulation classes mentioned above. Among the mechanical limitations, we mention here the maximum rotor shear stress, f_t (N/cm²), that guarantees the rotor mechanical integrity, dynamic airgap error, and maximum safe speed (in rps).

The electromagnetic rotor shear stress, f_t , may be calculated as

$$f_t = K_e \cdot A_1 \cdot B_{g1} \cdot \cos \gamma_1 \quad (1.21)$$

$$T_e = f_t \cdot \pi D_r L \cdot \frac{D_r}{2} \quad (1.22)$$

where

L is the axial lamination stack length (m)

D_r is the rotor diameter (m)

A_1 is the fundamental of stator slot ampere-turns/m of rotor periphery (peak value) (from 2×10^3 to 2×10^5 A_{turns}/m)

B_{g1} is the peak value of fundamental resultant flux density in the airgap (from 0.2 T to maximum 1.1 T)

B_{g1} is limited by the magnetic saturation in the machine magnetic cores, which corresponds to a $B_{\text{coremax}} \approx (1.2 - 2.3)$ T as the airgap magnetic flux passes from the stator to the rotor core. Values above 2.0 T correspond to special soft magnetic materials such as Hyperco.50

K_e is a form (single-phase) factor for sinusoidal A and B_g (1-phase machine)

$K_e = 1$ (it is around 1/2 for a 1-phase machine)

The phase angle between A_1 and B_{g1} is γ and its optimum value is $\gamma_1 = 0$, which, in ac machines, occurs only in synchronous machines.

For practical machines the rotor shear stress—or tangential specific force—is $f_t = 0.1 - 10 \text{ N/cm}^2$, due to the magnetic saturation, B_{coremax} , and winding temperature limitation, $A_{1 \text{ max}}$; mechanically, the rotor materials can handle more, with the exception of heavily loaded electric machines with extremely high torque densities (Nm/m^3 or Nm/kg of rotor or of the entire machine). From Equation 1.22

$$T_e = 2f_t (\text{rotor volume}) \quad (1.23)$$

Note that f_t value increases with the rotor diameter.

The stator outer to rotor diameter is, in general,

$$\frac{D_{\text{out}}}{D_r} \approx 2 - K_p (p_1 - 1) \quad (1.24)$$

For $p_1 = 1-4$, $K_p \approx 0.1-0.2$.

So, in fact, the torque per motor volume is approximately

$$\frac{T_e}{\text{motor volume}} \approx \frac{2f_t}{(2 - K_p (p_1 - 1))^2} \quad (1.25)$$

With an average specific weight of $\gamma_{\text{an}} = 8 \times 10 \text{ kg/m}^3$, the torque/weight of motor (Nm/kg) can be calculated and ranges from 0.2 Nm/kg , in small- or high-speed motors, to 3 Nm/kg in typical 1000–3600 rpm kW motors and to considerably more in large-torque (diameter) generators/motors.

The temperature limitations are also related to the current density in the windings and to the cooling system, torque, and speed of the electric machine. In transformer design, current densities vary from 2.5 to 4 A/mm^2 , while in forced air-cooled electric motors, it is 5–8 A/mm^2 . For forced water (or air)-cooled electric machines, average values of $j_{\text{cor}} = 8-16 \text{ A/mm}^2$ are typical.

It is evident that high torque/volume is in contradiction to low losses (and high-efficiency) attempts. So lower active material weight (and cost) is in contradiction to low losses (and temperatures). This is why a global cost function is to be defined and then minimized through design optimization.

GlobalcostinU.S.dollars(euro) = materials and fabrication manpower costs

+ capitalized loss energy costs $\left(\sum p_a t_{\text{an}} \times \text{energy costs} \right)$

+ capitalized maintenance and repair costs

For a very few small-power applications, the motor initial cost is maximum in the global cost. It depends also on the average number of hours/day for the average life of the electric machine (above 10–15 years, in general).

Alternatively, all components in global cost may be converted into joules and, even better, in CO_2 pollution weight as machine materials fabrication, losses, maintenance, and repair means energy (and, consequently, pollution).

Such a joule global cost analysis has demonstrated recently that a 100 L refrigerator is superior to a desktop computer. In complex (say vehicular) applications, not the motor but the entire vehicle (system) global cost (in U.S. dollars or euro or joule or CO₂ weight) should be subject to optimization.

Example 1.2 A small 3-phase PM synchronous motor, used for active power steering in a modern car, has a rotor diameter, $D_r = 0.030$ m, and a stack length (stator and rotor average), $L = 0.06$ m. The sintered NeFeB PMs produce a peak (sinusoidal) airgap flux density, $B_g = 0.7$ T, and, for rated stator current, the specific tangential force is $f_t = 1.2$ N/cm².

Calculate

- Rated electromagnetic torque, T_e
- With zero mechanical losses, the rated (mechanical) power P_{2m} at 3000 rpm
- If the rated efficiency is $\eta_n = 0.9$ and power factor $\cos(\varphi_n) = 0.8$, $V_{nl} = 18\sqrt{3}$ V (star connection) —42 V dc bus—determine the rated phase current
- Rated stator Ampere-turns/m (A_1)

Solution:

- From Equation 1.23 the electromagnetic torque T_e is

$$T_e = f_t \pi D_r L \frac{D_r}{2} = 1.2 \cdot 10^4 \cdot \pi \cdot 0.003 \cdot 0.06 \cdot \frac{0.03}{2} = 1.01736 \text{ Nm}$$

- The electromagnetic power, $P_{2e} = P_{2m}$, (zero mechanical losses) is

$$P_{2m} = T_e 2\pi n = 1.01736 \cdot 2\pi \cdot \frac{3000}{60} = 319.45 \text{ W}$$

- The input electric power from Equation 1.7:

$$P_{1e} = \frac{P_{2m}}{\eta_n} = \sqrt{3} V_{nl} I_n \cos \varphi_n; \quad I_n = \frac{319.45}{0.9\sqrt{3} \times 18 \times 0.8} \approx 14.245 \text{ A}$$

- From Equation 1.22

$$A_1 = \frac{f_t}{K_e B_{gPM}} = \frac{1.2 \times 10^2}{1 \times 0.7} = 1.714 \times 10^4 \text{ A/m}$$

1.5 Nameplate Ratings

Sample nameplates for a transformer and an induction machine are given in Figure 1.10.

The manufacturers provide basic information for a transformer (Figure 1.10a) such as



FIGURE 1.10
Sample nameplates: (a) A transformer and (b) an induction machine.

- Voltage rating, V (kV)
- Apparent power, VA (kVA or MVA)
- Current rating, A
- Temperature rise, °C
- Short-circuited voltage rating, %
- No-load current rating, %
- Connection diagrams (such as $Y_{dx}; x = 1, 3, 5, 7, 9, 11$ or $Y_{yx}; x = 0, 2, 4, 6, 8, 10, 12$)
- Serial number
- Weight, kg
- Insulation class
- Cooling information

- High and low voltage markings (Hi, Xi, ANSI standards, UVW and, respectively, uvw in IE-IEC standards; ABC, abc in some national standards)

For electric machines (Figure 1.10b), the nameplates contain data such as

- Power: (rated, active, mechanical) power for motors and apparent power (in kVA or MVA) for power grid-connected electric machines; for variable speed power electronics fed electric machine the base power, P_b , at base speed, n_b , concept is used and corresponds to the full converter voltage, assigned duty cycle load torque at rated motor winding temperature.
- Information on motor environment and heat transfer marked as OPEN (drip-proof, splash-proof, dust-proof, water-cooled, encapsulated).
- Speed (rated and synchronous) in revolutions per minute for grid-connected (constant speed) motors and base speed, n_b , and maximum speed, n_{max} , for variable speed machines.
- Line-to-line stator (and rotor, if applied) voltage.
- Rated (line stator and rotor (if applied)) currents.
- Efficiency at full load (and at 25% load).
- Volt amperes in ac machines.
- Maximum allowable temperature rise in the hot spot.
- Extreme ambient temperature and altitude.
- Service factor indicates how much overrated power the machine can continuously sustain without overheating: this is 1.15 for many motors.
- Supply frequency in Hz (constant or variable, in stator or rotor).
- Torque is given only for variable speed motors at base and maximum speed.
- Rotor inertia (in kg m^2).

1.6 Methods of Analysis

Again, electric transformer and electric machines represent systems of coupled electric and magnetic circuits at standstill, and, respectively, in relative motion with each other.

So the magnetic field spatial distribution and time variation in the magnetic circuits and the spatial distribution of windings and their current time variations in the electric circuits have to be solved first.

Accounting for magnetic saturation in magnetic circuits and for skin (field) effect in electric circuits is also necessary as they notably affect the performance. The magnetic field distribution may be approached by analytical and numerical (finite element) methods.

Then, from calculated magnetic energy or flux in various parts of the machine, the machine's various self- (L_i) and mutual inductances are calculated (W_m is the magnetic energy (Joule) and I_i , current (A))

$$L_i = \frac{2W_{mi}}{I_i^2} = \frac{\Psi_i}{I_i}; \quad R_i = \rho \frac{l_{\text{con}}}{A} K_{\text{skin}} \quad (1.26)$$

Resistances of machine phases can be calculated based on winding geometry, accounting for skin effect by a frequency-dependent coefficient $K_{\text{skin}} \geq 1.0$. (ρ is the electric resistivity ($\Omega \text{ m}$), l_{con} is the conductor length (m), and A is the conductor cross section (m^2)).

Moreover, the electric transformers or electric machines are reduced to electric circuits that are only coupled electrically and that contain resistances, inductances, and “electromagnetic forces” (emfs) produced by relative motion between magnetic field axes and windings physical axes.

Thus, circuit models of transformers and electric machines are used in a few, now widely recognized, forms [5–34]:

- Phase coordinate models (with phase phasor models for sinusoidal steady state in ac machines)
- Orthogonal (dq) axes models [5]
- Space vector (complex variable) models [12–14]
- Spiral vector models [28]

In this book, we will use the above models as follows:

Part 1—steady state: analytical field model (circuit model) and phasor form for ac machines

Part 2—transients: orthogonal and space vector models

Part 3—finite element (FE) analysis and analytical and FE field/circuit models for optimal design

1.7 State of the Art and Perspective

Invented in the nineteenth century (dc brush machine by Faraday in 1831, induction machine by Ferrari in 1886 and Tesla in 1887, transformer

development by Dolivo Dobrovolski in 1890), electric machines had become a mature field by 1930 when strong electric power systems became available all over the world.

Brush-commutator machines have been used as variable speed motors before 1965 as they required only variable dc voltage from brush-commutator generators (Ward-Leonard machine group).

The development of power electronics after 1965 (based on thyristors, then GTOs or bipolar transistors) led to variable speed drives with ac motors of medium-large power.

But the development of IGBT and MOSFETs after 1980 produced a revolution in sub-kW to MW/unit variable speed drives with ac motors in the foreground.

This era coincided with the industrial development of permanent magnet brush-commutator and synchronous motors and the latter has gained widespread acceptance with torques from milli-Nm (for mobile phones) to more than mega-Nm (for directly driven wind generators).

Advanced-vector and direct torque and flux control field oriented (FOC, DTFC) of ac motor drives with IGBTs (or IGCTs for very large powers) pulse-width-modulated (PWM) static power convertors (power electronics) have led to fast (millisecond range) and robust torque control response.

A very good proportion of all electric motors is used in variable speed drives for various applications (Figure 1.3). The electric car is one such example (Figure 1.11).

Variable speed generators [34] with power electronics, cage rotor and wound rotor induction—dc-excited or PM synchronous—up to 5 MW/unit are already in use (50,000 MW total installed wind power) with a steady (above 10%) annual growth, to produce more “green” energy.

Electric machines are tightly standardized—see IEEE and IEC standards—from specifications to installment, maintenance, and repair.

The numerous electric machine manufacturers have their proprietary design methodologies. For finite element analysis and design backup with power electronics control, there are distinct, small engineering companies that produce and upgrade their dedicated software annually (such as “Vector Fields,” “Ansoft,” “CEDRAT,” “FEMM,” and “SPEED Consortium” for design code, etc.)

The following subjects are considered as “hot” in the field of electric machines:

- Less time-consuming finite element-coupled circuit models for analysis, to be directly used for modeling in optimal design methodologies.
- Lower-cost high-performance PMs with remnant flux density well above $B_r = 1.3$ T.
- Distributed (weaker) power systems will lead to lower average power/unit generators that will have, in part, to operate at variable

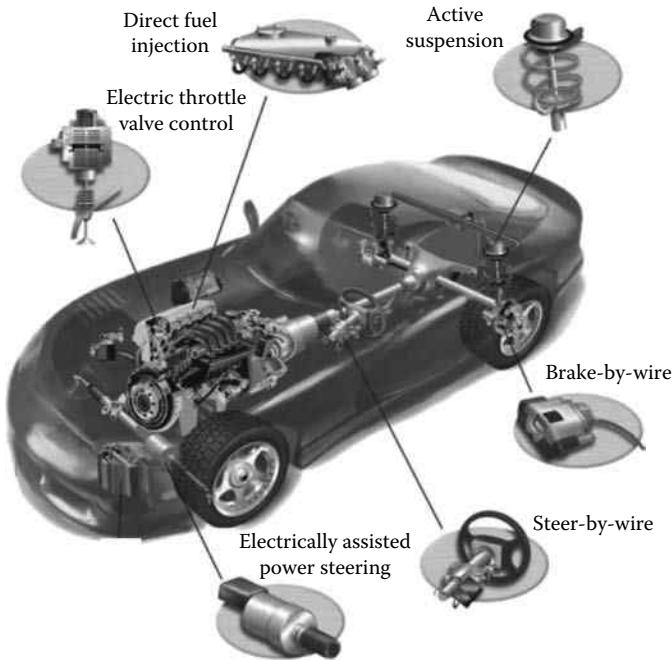


FIGURE 1.11
Automotive electric motor applications.

speed to be stable and flexible, with operation for lower losses. Design, fabrication, and application of variable speed generator systems, especially for wind and small hydro applications, seems to be a promising field.

- Higher-efficiency, lower-weight (and cost) PM synchronous and switched reluctance motors for home appliance microrobots, and, even more, for the more electric automobiles, aircraft, and vessels.
- Magnetic composite soft materials for high-frequency (speed) electric motors with relative permeability above 500–800 and lower specific losses and costs.
- Small displacement linear oscillatory PM brushless motor/generator systems for compressors and hybrid electric vehicle (HEV) actuators.
- Better power grid and high-frequency electric transformers in various applications, from distributed power systems to industrial- and automotive-power-electronics-controlled systems for better power quality.

1.8 Summary

- Electric energy is a key element of the civilization level.
- Electric energy is produced in electric power plants where a prime mover (turbine) rotates an electric generator.
- The prime mover is driven by the heat of a combustor that burns fossil or nuclear fuels, or by the kinetic energy of wind or water.
- Electric energy produces heat, while burning fuels and thus pollute the environment chemically.
- Electric energy is thus limited, for sustainable development.
- As 60% of electric energy is converted to mechanical work, it is essential to use it wisely in electric motors.
- Electric motors/generators are used in all industries from home appliances, info gadgets, and robotics to transportation, pumps, ventilators, compressors, and industrial processes.
- Electric machines convert mechanical energy to electric energy or vice versa, with magnetic energy storage.
- Electric transformers step up (at generators end) or step down (at consumers end) the voltage (and current level) in alternating current (ac) power systems. They do not contain parts in motion, but as electric machines, they work on the principle of Faraday's law of electromagnetic induction and are associated, in most cases, with electric machines, in applications. This is why they are dealt with here.
- Electric transformers are used in electric power systems, various industries, power electronics, and for ac voltage and current measurements.
- Electric machines and transformers are limited by the maximum allowable temperature of insulation materials, magnetic loading (due to magnetic saturation), and the skin effect and temperature of electric conductors, but more importantly by the mechanical tangential and radial stresses on the rotor and stator materials. Thus, limited N/cm^2 , Nm/m^3 , Nm/kg , and kVA/kg are typical to electric transformers and electric machines.
- The initial cost of electric machines tends to be in conflict with the capitalized losses costs and maintenance/repair costs over the operation life of the machine. Thus, global costs optimization is required.

- Global costs may be expressed not only in U.S. dollars (euro, etc.) but also in joule, or even better, in kilograms of CO₂ sprayed in the environment during the fabrication of machine materials, the fabrication of machines, human lifestyle, CO₂ by-production, capitalized losses translated back into energy capacity and thus CO₂ exhaust, and, finally, maintenance and repair costs translated into kilograms of CO₂, as per the entire life of the respective electric machine.
- The minimum total energy in joule/person/year and the total CO₂ in weight/person/year criteria for a target living standard seems a wiser way to approach technologies of the future, including electric machines.

1.9 Proposed Problems

1.1 A large lossless (3-phase) 1 MW, 6 kV line voltage synchronous motor (star connection) operates at 3600 rpm at 60 Hz. Calculate

- a. Input active power, P_{1e}
- b. Number of pole pairs, p_1
- c. Rated torque
- d. Stator phase current at $\cos \varphi_1 = 1$ and $\cos \varphi_1 = 0.9$

Hints: Equations 1.13 through 1.15.

1.2 An old electric motor of 50 kW and rated efficiency $\eta_n = 0.91$ is replaced by a new one with 0.94 rated efficiency. For 2500 h per year of operation at full power, calculate

- a. Input energy/year in the two cases
- b. Total energy losses/year in the two cases
- c. Energy cost savings in U.S. dollars if 1 kWh costs \$0.1

Hints: Example 1.1, loss costs = $\sum p(W)$ hours 10^{-3} energy cost (USD/kWh).

1.3 The new motor in Example 1.2 costs \$2000 and should be in operation for 15 years. If the energy costs increase by 1% per year from \$0.1/kWh in the first year and the maintenance and repair costs over the 15 years of life is about \$1000 for the old motor and \$500 for the new one, for

2500 h/year at full power, calculate the overall cost savings with the new motor over the 15 years of the motor's life.

Hints: Add the cost of losses for 15 years (considering the energy cost increase per year) to the maintenance cost for the new and old motor and find the difference; then add the initial cost (for the new motor only).

- 1.4 An electric 3-phase hydro generator of $S_n = 215$ MVA at 15 kV line voltage (star connection) operates at the power grid at $f_1 = 50$ Hz, at $n_n = 75$ rpm. The copper losses in the rotor field winding is $P_{exc} = 0.01 S_n$, the stator copper losses $P_{cos} = 0.0033 S_n$ at $\cos \varphi_1 = 1$ and $p_{core} = p_{mec} = 0.015 S_n$. Calculate

- Total rated losses at unity power factor ($\cos \varphi_1 = 1$)
- Rated efficiency
- Rated shaft torque
- Electromagnetic torque
- Rated phase current
- Number of pole pairs

Hints: Add all losses, use Equation 1.6 for efficiency, Equation 1.15 for shaft torque, Equation 1.16 for electromagnetic torque, Equation 1.13 for rated current, and Equation 1.1 for the number of pole pairs.

- 1.5 A 3-phase PM synchronous electric motor is designed with $A_1 = 2 \times 10^4$ A_{turns}/m and an airgap PM flux density fundamental $B_{gPM1} = 0.8$ T, for a specific tangential force $f_t = 2$ N/cm². For a 100 Nm torque and a rotor diameter D_r to stack length L ratio $D_r/L_1 = 1$, the following is required:

- Rotor diameter D_r and stack length
- The ratio torque/rotor volume
- The ratio power/rotor volume at $n = 6000$ rpm

Hints: Example 1.2.

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2

Electric Transformers

Electric transformers step up or step down the input ac voltage, transmitting the input power minus losses, $\sum p$. These have thus a primary and a secondary. A typical 1-phase transformer has a laminated silicon-iron core and a primary and a secondary coil (winding) (Figure 2.1).

- To understand the voltage step-up or step-down ability of the transformer, we consider here the secondary switch open (zero secondary current: $I_2 = 0$). In other words, we have an ac (primary) coil with a laminated soft iron (lossless) core for the time being.
- After elucidating the principles, we treat the main characteristics (and loss mechanisms) of magnetic core and electric conductors in ac magnetic fields (or currents).
- Then we discuss the construction of single-phase transformers and their main leakage inductance expressions.
- The circuit model equations of the single-phase transformer are derived, and then used to describe no-load, short-circuit, and on-load operation modes.
- Three-phase transformer topologies, connections, and general equations are introduced later.
- Unbalanced load operation of 3-phase transformers and the conditions to connect them in parallel are treated in some detail.
- Electric transformer transients are treated in general, with applications for inrush current, sudden short circuit, electrodynamic forces, and transformer behavior at ultra fast voltage pulses (of commutation and atmosphere nature).
- Special transformers, such as autotransformers, 3-winding transformers, and transformers for power electronics, are also introduced.
- An example of the preliminary electromagnetic design methodology is given at the end of this chapter.

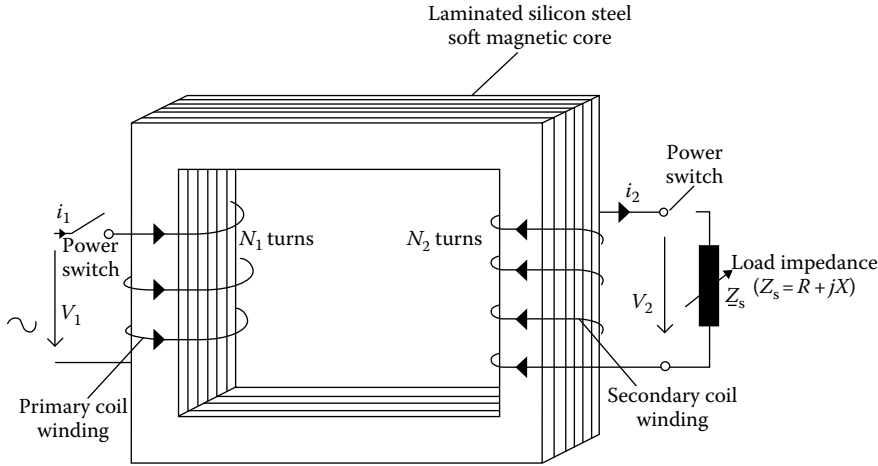


FIGURE 2.1
Single-phase transformer.

2.1 AC Coil with Magnetic Core and Transformer Principles

The ac coil with ideal (lossless) magnetic core in Figure 2.2, which corresponds to a transformer on no load, contains an airgap, which brings more generality to the subject, as even practical transformers have small (0.1–0.2 mm) airgaps between laminations, while all electric machines have airgaps from 0.2 to 20 mm, in general.

As the frequency is considered to be less than 30 kHz (in most cases much lower: 50(60) Hz in today’s electric power systems), the magnetic circuit (Ampere’s) law, for the average magnetic field path ABCD in Figure 2.2, is

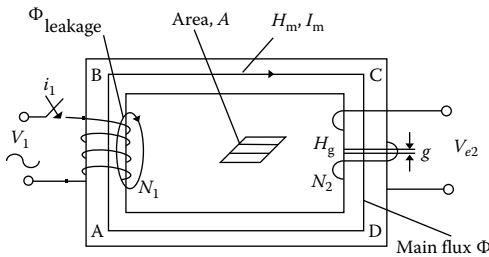


FIGURE 2.2
The magnetic core ac coil (transformer on no load).

written as

$$\oint \vec{H} \cdot d\vec{l} = \sum N_i \cdot I_i \quad (2.1)$$

or approximately

$$H_m \cdot l_m + H_g \cdot g = N_1 \cdot i_{10} \quad (2.2)$$

where

H_m and H_g are the magnetic fields (in A/m) in the magnetic core and airgap, respectively

l_m and g are their path lengths in the core and the airgap, respectively

The magnetic core, B_m (H_m), relationship is given here as

$$B_m = \mu_m(H_m) \cdot H_m \quad (2.3)$$

where

B_m is the flux density (in T)

μ_m is the magnetic permeability (in H/m)

For soft magnetic materials, typically found in transformers and electric machines, μ_m is large, that is, $\mu_m > 1000\mu_0$, where $\mu_0 = 1.256 \cdot 10^{-6}$ H/m is the air permeability. The magnetic permeability of soft magnetic materials, μ_m , decreases with H_m and B_m , a phenomenon called magnetic saturation. The flux density is thus limited to $B_{\max} = 1.6\text{--}1.8$ T in 50(60) Hz standard transformers, to limit the maximum value of H_m , and thus (from Equation 2.2) the no-load current, i_{10} .

Now, if the cross section core area is A , the flux (Gauss) law at the airgap is

$$\Phi_A = \int_A B_m \, dA \quad (2.4)$$

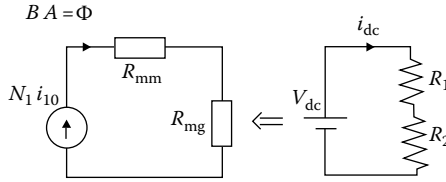
or approximately

$$\Phi_A = B_m \cdot A \approx B_g \cdot A \Rightarrow B_m \approx B_g \quad (2.5)$$

Combining Equations 2.2, 2.3, and 2.5 yields

$$N_1 \cdot i_{10} = \Phi[R_{mm} + R_{mg}]; \quad R_{mm} = \frac{l_m}{\mu_m \cdot A}, \quad R_{mg} = \frac{g}{\mu_0 \cdot A} \quad (2.6)$$

where R_{mm} and R_{mg} are the so-called magnetic reluctances of the magnetic core and the airgap zone, respectively. They resemble the electric resistances. Φ is the current and $N_1 i_{10}$ is the voltage in dc circuits under steady state (Figure 2.3).


FIGURE 2.3

The equivalent magnetic circuit of a magnetic core coil with airgap.

For ac current in the coil, $R_{mm}(\mu_m)$ depends on the momentary value of the current; but, in general, for practical designs, a single value of flux density in the magnetic core is used (in general, at $B_{\max} \times 0.867$).

The ac current produces an ac magnetic field (H_m and B_m are ac), say, sinusoidal as the supply voltage V_1 is sinusoidal:

$$V_1(t) = V\sqrt{2} \cdot \cos(\omega_1 t + \gamma_0) \quad (2.7)$$

The ac magnetic field crosses the area of each coil turn, shown in Figure 2.2, once and, as it varies with frequency f_1 , it induces an electromagnetic force (emf)-induced voltage, V_e , according to Faraday's law:

$$\oint \vec{E} \cdot d\vec{l} = -N_1 \cdot \frac{d\Phi}{dt} \quad (2.8)$$

or

$$V_{e1} = -N_1 \cdot \frac{d\Phi}{dt} \quad (2.9)$$

Making use of Equation 2.6 in Equation 2.9 yields the emf V_e expression:

$$V_{e1} = \frac{-N_1^2 \cdot \frac{di_{10}}{dt}}{R_{mm} + R_{mg}} = -L_{1m} \cdot \frac{di_{10}}{dt} \quad (2.10)$$

L_{1m} is the so-called main inductance:

$$L_{1m} = \frac{N_1^2}{R_{mm} + R_{mg}} \quad (2.11)$$

V_{e1} is called self-induced voltage, but a similar induced voltage, produced by the same flux, Φ , occurs in the secondary coil (Figure 2.2):

$$V_{e2} = -N_2 \cdot \frac{d\Phi}{dt} = -\frac{N_2}{N_1} \cdot L_{1m} \cdot \frac{di_{10}}{dt} \quad (2.12)$$

The ratio of the two emfs is

$$\frac{V_{e2}}{V_{e1}} = \frac{N_2 \cdot \Phi}{N_1 \cdot \Phi} = \frac{N_2}{N_1} \quad (2.13)$$

because the same ac magnetic flux is involved.

Now coming back to Faraday's law (Equation 2.8), for the coil complete circuit as a sink, we find

$$i_{10} \cdot R_1 - V_1 = V_{e1} - L_{1l} \cdot \frac{di_1}{dt} \quad (2.14)$$

The additional term in Equation 2.14 is related to the leakage (partly in air) flux, Φ_l in Figure 2.2, which does not embrace the secondary coil. In general, $L_{1l} < L_m/500$ but is, still, very important.

With an ac voltage, V_1 (Equation 2.7), the steady-state solution of Equation 2.14 is obtainable in complex numbers:

$$\underline{V}_1 = V\sqrt{2} \cdot e^{j(\omega_1 t + \gamma_0)}; \quad \underline{I}_{10} = \frac{\underline{V}_1}{\underline{Z}_{10}}; \quad \underline{Z}_{10} = R_1 + j\omega_1 (L_{1l} + L_{1m}) \quad (2.15)$$

or

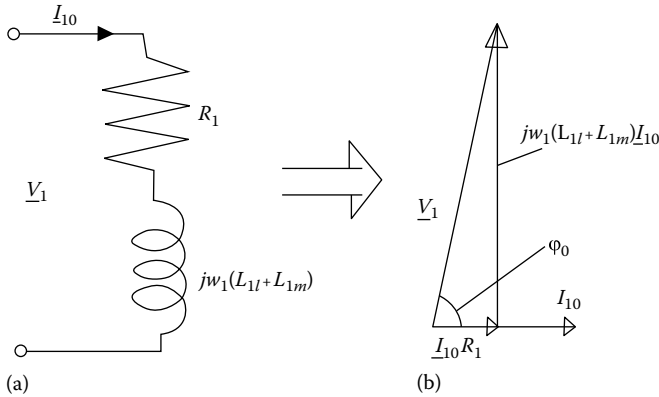
$$\underline{I}_{10} = I_{10}\sqrt{2} \cdot \cos(\omega_1 t + \gamma_0 - \varphi_0); \quad I_{10} = \frac{V}{|\underline{Z}_{10}|} \quad (2.16)$$

where φ_0 is the voltage/current (power factor) angle and is in general large ($\cos \varphi_0 < 0.05$), as the coil resistance should be small in comparison with the large reactance due to the large magnetic permeability of the soft magnetic core.

A phasor diagram corresponding to the ac (sinusoidal) coil with magnetic core under steady state is shown in Figure 2.4.

Remarks:

- The ac coil with soft lossless magnetic core is a high inductance R, L circuit and may be treated as such.
- The presence of the airgap, g , decreases the inductance, L , and thus increases the current absorbed for a given voltage, frequency, and coil geometric data.
- For an ideal magnetic core (no losses and univoque $B_m(H_m)$ relationship), the ac magnetic field, H_m , is in phase with the coil current, I_1 , and both vary sinusoidally in time.
- The equivalent magnetic circuit of the coil with ideal magnetic core, though valid also in ac, resembles the purely resistive electric circuit. With magnetic flux, Φ , replacing the current, the coil magnetomotive force, $N_1 \cdot i_{10}$, replaces the voltage and the place of electric resistance (Figure 2.4) is taken by the magnetic reluctance.


FIGURE 2.4

(a) Equivalent circuit and (b) the phasor diagram of the ac coil with soft ideal (lossless) magnetic core.

- For sinusoidal voltage, and ideal magnetic core, the ac coil draws sinusoidal current under steady state and behaves as an $R_1 + j \cdot X_0$ impedance, which may be illustrated in complex variables (phasors) with the corresponding phasor diagrams (Figure 2.46).
- As the primary ac coil “sends” the same flux, Φ , through the secondary coil, the latter induces an emf in the secondary V_{e2} . Approximately,

$$V_{e2} = V_{e1} \cdot \frac{N_1}{N_2} \approx V_1 \cdot \frac{N_2}{N_1} \quad (2.17)$$

because the voltage drop on the coil resistance and leakage reactance ($\omega_1 \cdot L_{1l}$) are less than 0.1% of V_1 .

- With $N_2 > N_1$ the voltage is stepped up and with $N_2 < N_1$ the voltage is stepped down.
- Now, neglecting the transformer losses and leakage fluxes, for the time being, the input and output power are equal to each other:

$$V_1 \cdot I_1 \approx V_2 \cdot I_2 \quad (2.18)$$

So, stepping up the voltage means stepping down the current and vice versa.

This illustrates the transformer basic principles.

Example 2.1 An ideal transformer, with a magnetic core uniform cross section, $A = 1.5 \cdot 10^{-2}$ m, mean flux path length $l_m = 1$ m, and $N_1 = 500$ turns, has a manufacturing airgap, $g = 2 \cdot 10^{-4}$ m, and operates at no load and a

core flux density, $B_m = 1.5$ T (peak value), when supplied from an ac voltage source at 60 Hz. Calculate the following:

- The primary voltage, V_1 , equal to emf V_{e1} (RMS)
- For an average magnetic core permeability, $\mu_m = 2000 \cdot \mu_0$, calculate the primary (magnetization) current, I_{10} (RMS)
- Neglecting the leakage inductance, calculate the primary coil inductance, $L_0 = L_{1m}$
- With a design current density, $j_{cor} = 3$ A/mm², for a rated current, $I_n = 100 \cdot I_{10}$, and a mean turn length of $l_{ct} = 0.6$ m, determine the primary coil resistance, R_1 , and then the circuit power factor on no load
- The number of turns required in the secondary to halve the primary voltage

Solution:

- From Equation 2.9, $\underline{V}_{e1} = -N_1 \cdot \frac{d\Phi}{dt} = -N_1 \cdot A \cdot \frac{dB_m}{dt} = -N_1 \cdot A \cdot \omega_1 \cdot \underline{B}_m$ (because $d/dt \rightarrow j\omega_1$ in complex numbers (phasors) for sinusoidal variables)

$$(V_{e1})_{\text{peak}} = 500 \cdot 10^{-2} \cdot 2\pi \cdot 60 \cdot 1.5 = 2826 \text{ V}$$

or

$$(V_{e1})_{\text{RMS}} = \frac{(V_{e1})_{\text{peak}}}{\sqrt{2}} = 2004 \text{ V}$$

- From Equation 2.6, the primary (magnetization) current on no load, i_{10} (RMS), is

$$\begin{aligned} (i_{10})_{\text{RMS}} &= \frac{\Phi_{\text{max}}}{N_1 \sqrt{2}} (R_{\text{mm}} + R_{\text{mg}}) = \frac{B_{\text{max}}}{N_1 \sqrt{2}} \left(\frac{l_m}{\mu_m} + \frac{g}{\mu_0} \right) \frac{1}{A} \\ &= \frac{1.6}{500 \sqrt{2}} \left(\frac{1}{2000} + 2 \cdot 10^{-4} \right) \cdot \frac{1}{1.5 \cdot 10^4} \cdot \frac{1}{1.56 \cdot 10^{-6}} = 84.32 \text{ A} \end{aligned}$$

- The main primary coil inductance, L_{1m} , is Equation 2.11

$$L_{1m} = \frac{N_1 \cdot \frac{\Phi_{\text{max}}}{\sqrt{2}}}{\omega_1 \cdot (i_{10})_{\text{RMS}}} = \frac{N_1^2 \cdot \frac{B_{\text{max}} \cdot A}{\sqrt{2}}}{\omega_1 \cdot (i_{10})_{\text{RMS}}} = \frac{500^2 \cdot \frac{1.5}{\sqrt{2}} \cdot 10^{-2}}{2\pi 60 \cdot 84.32} = 0.0837 \text{ H}$$

- The primary winding (coil) resistance is known as

$$R_1 = \rho_{Co} \cdot \frac{l_{ct} \cdot N_1}{A_{Co}} = \rho_{Co} \cdot \frac{l_{ct} \cdot N_1}{I_n / j_{cor}} = \frac{2.1 \cdot 10^{-8} \cdot 0.6 \cdot 500}{100 \cdot 84.32 / 3 \cdot 10^6} = 0.224 \cdot 10^{-2} \Omega$$

The power factor, $\cos \varphi_1$, is (from Figure 2.4b)

$$\cos \varphi_{10} = \frac{R_1 \cdot (i_{10})_{\text{RMS}}}{(V_1)_{\text{RMS}}} = \frac{0.224 \cdot 10^{-2} \cdot 84.32}{2004} = 9.43 \cdot 10^{-5}$$

The ideal magnetic core coil—or transformer on no load—is essentially a large reactance with a very small coil resistance. Here, the core loss is neglected, which explains why $\cos \varphi_{10}$ is much smaller than 0.05.

e. The number of turns to halve the voltage in the secondary is, from (2.13),

$$N_2 = N_1 \cdot \frac{V_{e2}}{V_{e1}} = 500 \cdot \frac{1}{2} = 250 \text{ turns}$$

Note: When an ac coil with magnetic core and airgap (eventually multiple airgaps) is used as a reactor in power systems or in power electronics (for voltage boosting), the total airgap is notably larger to allow large currents without running in too heavy magnetic saturation, which, as we will show in the next paragraph, means very large core losses, that is, overheating.

2.2 Magnetic Materials in EMs and Their Losses

Magnetic core materials in electric machines are defined by a few characteristics out of which the flux density (B_m , in T) and the magnetic field (H_m , in A/m) are paramount (Equation 2.3).

Magnetic permeability, μ_m ($\mu_m = B_m/H_m$), is defined as a scalar in homogeneous materials and as a tensor in nonhomogeneous materials.

2.2.1 Magnetization Curve and Hysteresis Cycle

A magnetic material is characterized by its relative magnetic permeability, μ_{mrel} :

$$\mu_{\text{mrel}} = \frac{\mu_m}{\mu_0} \quad (2.19)$$

Some magnetic materials have $\mu_{\text{mrel}} > 1$ (they are ferromagnetic or soft) while nonmagnetic materials have $\mu_{\text{mrel}} < 1$ (they are paramagnetic with $\mu_{\text{mrel}} \approx 1$ or superconducting with $\mu_{\text{mrel}} \approx 0$). Soft magnetic materials that make the magnetic cores of transformers and electric machines include alloys of iron, nickel, cobalt, and one rare earth element or they are soft steels with silicon, with $\mu_{\text{mrel}} > 2000$ at $B_m = 1.0$ T.

For frequencies above 500 Hz, compressed, injected, soft powder materials that contain iron particles suspended in an epoxy or a plastic matrix are used.

Soft magnetic materials are characterized by

- B_m (H_m) and H_m (B_m) curves
- Saturation flux density, B_{sat}
- Temperature variation of permeability
- Hysteresis cycle
- Electric conductivity
- Curie temperature
- Loss coefficients

As, from power transformers to power electronics coils and transformers, frequency ranges from 50 (60) Hz to 1 MHz, choosing adequate soft magnetic materials is a very challenging task [1]. We will detail more here on soft magnetic materials for power transformers and electric machines, where fundamental frequency is lower than 4 kHz (in 240,000 rpm small electric motors).

Silicon steel laminations and soft magnetic powders are used for the purpose.

A graphical representation of the magnetization curve, B_m (H_m), and of the hysteresis cycle of a standard silicon (3.5) steel M19 are shown in Figure 2.5a and b.

The magnetization process implies the magnetization dipoles in the material being gradually oriented by an external magnetic field (mmf). For a monotonous rising and falling of mmf level, the hysteresis cycle is obtained. So, the magnetization curve represents either an average curve through the middle of the hysteresis cycle or the tips of subsequent hysteresis cycles of lower and lower magnitudes.

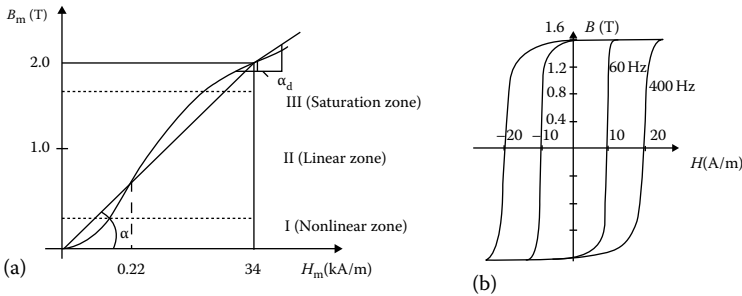


FIGURE 2.5

(a) Magnetization curve and (b) hysteresis cycle of deltamax tape-wound core 0.5 mm strip.

The presence of the hysteresis cycle is related to the energy consumed in the material to orient the magnetic microdipoles. Hence, hysteresis losses occur; they are reasonably small in soft magnetic materials but increase with frequency as the number of cycles per second increases and because the hysteresis cycle itself becomes larger as frequency increases (Figure 2.5b).

The presence of magnetic saturation and, implicitly, the $B_m(H_m)$ nonlinearity with the hysteresis cycle suggest three different magnetic permeabilities (Figure 2.6).

- The normal permeability, μ_n , is

$$\mu_n = \frac{B_m}{H_m} = \tan \alpha_n; \quad \mu_{nrel} = \frac{\mu_n}{\mu_0} \quad (2.20)$$

- The differential permeability, μ_d , is

$$\mu_d = \frac{dB_m}{dH_m} = \tan \alpha_d; \quad \mu_{drel} = \frac{\mu_d}{\mu_0} \quad (2.21)$$

- The incremental permeability, μ_i , is

$$\mu_i = \frac{\Delta B_i}{\Delta H_i} = \tan \alpha_i; \quad \mu_{irel} = \frac{\mu_i}{\mu_0} \quad (2.22)$$

Close to origin and as the material saturates (Figure 2.5a), the nonlinearity of the magnetization curve makes the three permeabilities different from each other. Only for the linear zone II in Figure 2.5a, they are identical (Figure 2.7).

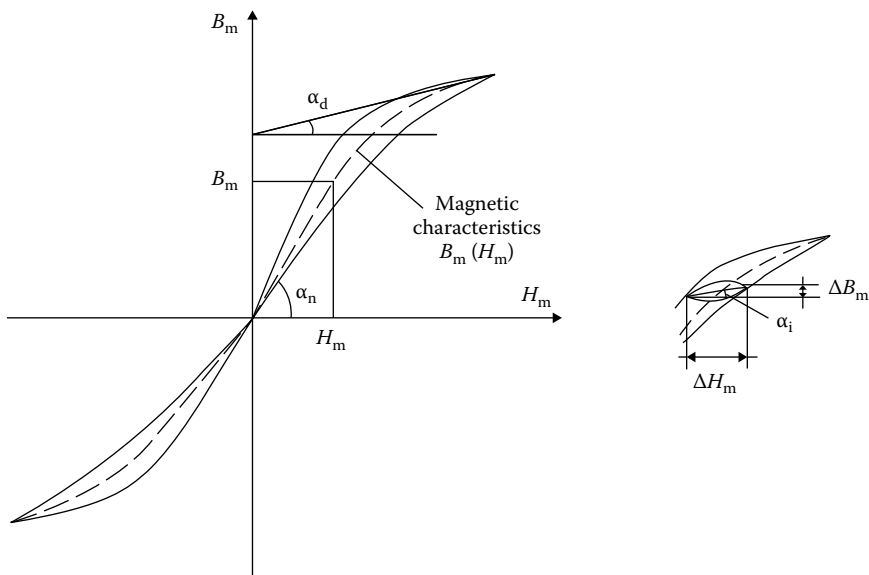
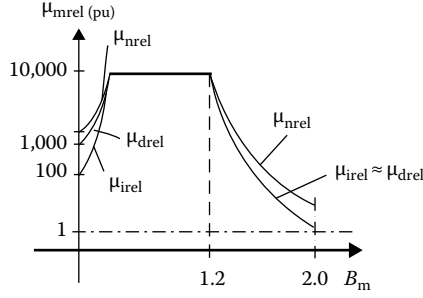


FIGURE 2.6

The three different permeabilities in soft magnetic materials.

**FIGURE 2.7**

The three magnetic permeabilities of soft silicon steel core sheets.

A few remarks are in order:

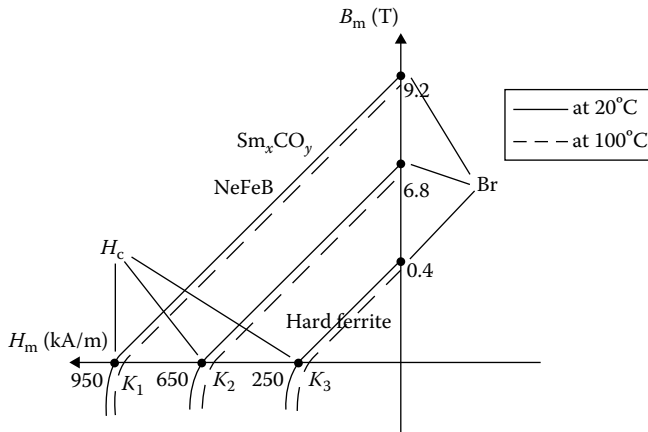
1. All magnetic permeabilities decrease notably for silicon above 1.2 T or so, but the differential permeability, which shows up in large ac transients, and the incremental permeability, which appears in small deviation ac transients over dc magnetization, are about equal to each other and notably smaller than the normal permeability, corresponding to dc magnetization.
2. As, in electric machines (at least), mixed dc–ac magnetization is common, all three permeabilities are to be used adequately if correct current response is expected.
3. Also, the magnetization curve varies with frequency and, for frequencies notably above 50–60 Hz, pertinent measurements are needed. In essence, the permeability decreases with frequency and in contrast to hysteresis cycle area. Smaller peak flux densities are adopted for frequencies above 200 Hz.

2.2.2 Permanent Magnets

PMs are solid, more magnetic materials with an extremely wide hysteresis cycle and a recoil permeability, $\mu_{\text{rec}} \approx (1.05\text{--}1.3) \cdot \mu_0$ (Figure 2.8).

Only the second quadrant of the PM hysteresis cycle is given, with the “knee” (demagnetization) point (K_1, K_2, K_4) in the third quadrant. The remnant flux density, B_r ($H_m = 0$), and the coercive field, H_c ($B_m = 0$), add to characterize the PM fully.

Also, the B_m/H_m curve moves downward for sintered and bonded NeFeB and Sm_xCo_y materials and upward (they improve!) for hard ferrites, when the PM temperature rises. Sm_xCo_y may work up to 300 °C while NeFeB only to 120 °C, before very important damage of demagnetization will occur. Eddy currents from external fields produce losses in PMs. PMs are used to

**FIGURE 2.8**

Permanent magnet characteristics.

produce dc-type magnetic fields to replace dc (excitation coils) in brush-commutator and in synchronous machines.

They are magnetized in a special magnetic enclosure capable of producing flux densities in the magnet of $3B_r$, with magnetic fields of $3H_c$ in a few millisecond-long pulses. The low recoil permeability, $\mu_{\text{rec}}/\mu_0 = (1.05 - 1.3)$, allows the PMs to hold a lot of stored magnetic energy for years, unless temperature or too large overcurrent mmfs demagnetize them. For more on PMs in electric machines, see [3].

2.2.3 Losses in Soft Magnetic Materials

Traditionally, soft magnetic material losses have been divided into hysteresis losses, P_h (in W/kg or W/m³), and eddy current losses, P_{eddy} , (in W/kg or W/m³). Hysteresis losses per hysteresis cycle are proportional to hysteresis area and the frequency of the magnetic field, f , (current) in sinusoidal operation mode:

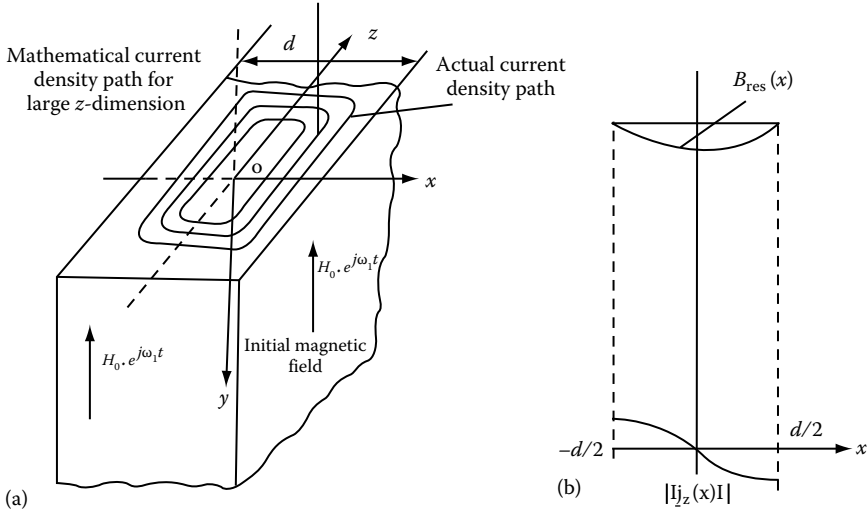
$$P_h \approx K_h \cdot f \cdot B_m^2 \text{ [W/kg]} \quad (2.23)$$

where

B_m is the maximum ac flux density

K_h accounts for hysteresis-involved loop contour (shape) and for frequency (more on hysteresis cycle approximations in [4])

Hysteresis losses depend also on the magnetic field character such as ac as in transformers and in fix field machines and moving field as in induction

**FIGURE 2.9**

(a) Eddy current path in a soft iron sheet and (b) resultant flux density, B_{res} , and eddy current density versus sheet depth.

and synchronous machines. They are 10%–30% larger in traveling fields than in ac fields for $B_m < 1.6$ T. However, in traveling fields, core losses have a maximum of around 1.6 T and then decrease to smaller values by 2.0 T in silicon steel sheets.

Eddy current losses in soft magnetic materials are produced by ac fields parallel to soft iron sheets that cannot penetrate the sheets completely because the eddy currents are induced by this field in a plane perpendicular to the field direction (Figure 2.9a). The induced current density paths are closed (because $\text{div } \vec{J} = 0$), and its amplitude decreases along the sheet depth.

Making use of Ampere's and Faraday's laws, the induced magnetic field, H_y , equations are

$$\text{rot } \vec{H} = \vec{J}; \quad \text{rot } (\sigma_{\text{iron}} \vec{J}) = -\frac{dB_{res}}{dt}$$

or, with single dimensional current density, J_z ,

$$\frac{\partial H_y}{\partial x} = J_z; \quad H_{0y} = H_0 \cdot e^{j\omega_1 t} \quad (2.24)$$

$$\frac{1}{\sigma_{\text{iron}}} \cdot \frac{\partial J_z}{\partial x} = j\omega_1 \cdot \mu_m \cdot (H_{0y} + H_y) = j\omega_1 B_{res}$$

where

B_{res} is the resultant flux density

H_{0y} is the initial external magnetic field

Eliminating H_y from Equation 2.24 we obtain

$$\frac{\partial^2 J_z}{\partial x^2} - j\omega_1 \cdot \mu_m \cdot \sigma_{\text{iron}} \cdot J_z = j\omega_1 \cdot \sigma_{\text{iron}} \cdot B_0 \quad (2.25)$$

The boundary condition (Figure 2.9b) is

$$(\partial J_z)_{x=\pm d/2} = 0 \quad (2.26)$$

This way, the eddy current density within the sheet is obtained; then, power dissipated per unit weight, with γ_{iron} as iron mass density (in kg/m^3), is

$$\begin{aligned} P_{\text{eddy}} &= \frac{2 \cdot \gamma_{\text{iron}}}{d \cdot \sigma_{\text{iron}}} \cdot \frac{1}{2} \cdot \int_{d/2}^0 (J_z(x))^2 dx \\ &= \frac{\gamma_{\text{iron}} \cdot d \cdot \omega_1}{\delta \cdot \mu_m} B_0^2 \cdot \left(\frac{\sinh \frac{d}{\delta} - \sin \frac{d}{\delta}}{\cosh \frac{d}{\delta} - \cos \frac{d}{\delta}} \right); \quad \frac{\text{W}}{\text{kg}} \end{aligned} \quad (2.27)$$

With δ , the so-called depth of field penetration in the sheet is

$$\delta = \sqrt{\frac{2}{\omega_1 \cdot \mu_m \cdot \sigma_{\text{iron}}}} \quad (2.28)$$

The depth of field penetration means, in fact, an $e(2.781)$ times reduction of current density from the sheet surface.

For well designed, low losses, sheets, $\delta < d/2$. With $\sigma_{\text{iron}} = 10^6 (\Omega \cdot \text{m})^{-1}$, $\mu_m = 2000 \cdot \mu_0$, $f_1 = 60 \text{ Hz}$, and $\delta = 2.055 \cdot 10^{-3} \text{ m}$. So 0.35 to 0.5 mm thick sheets provide for small skin (Field) effect. For $\delta < d/2$,

$$P_{\text{eddy}} \approx K_w \cdot \omega_1^2 \cdot B_m^2; \quad \frac{\text{W}}{\text{kg}}; \quad k_w = \frac{\sigma_{\text{iron}} \cdot d_{\text{iron}}^2}{24} \quad (2.29)$$

Equation 2.29 is valid for ac fields. For traveling fields of same amplitude, the eddy current losses are about two times larger. As traveling and ac fields coexist in most electric machines (not in transformers), it is recommended to run tests on the magnetic core in conditions very similar to those in the respective machine.

A more complete formula of total soft material losses is given as [5]

$$\begin{aligned} P_{\text{iron}} &\approx K_h \cdot f \cdot B_m^2 \cdot K(B_m) + \frac{\sigma_{\text{iron}}}{12} \cdot \frac{d^2 \cdot f}{\gamma_{\text{iron}}} \cdot \int_{1/f} \left(\frac{dB}{dt} \right)^2 \cdot dt \\ &\quad + K_{\text{ex}} \cdot f \int_{1/f} \left(\frac{dB}{dt} \right)^{1.5} \cdot dt; \quad \frac{\text{W}}{\text{kg}} \end{aligned} \quad (2.30)$$

where

$$K(B_m) = 1 + \frac{0.65}{B_m} \cdot \sum_1^n \Delta B_i$$

B_m is the maximum flux density

f is the frequency

K_{ex} is the excess loss coefficient

ΔB_i being flux density variation during integration time step

The formula is valid for $\delta > d/2$ conditions and includes an excess loss term while it accepts even nonsinusoidal field variation. It has, however, to be ranked against experiments that by regression methods may lead to best K_h , K_{ex} , K choices for the assigned frequency range.

Original data for typical M19, 0.5 mm thick sheet magnetization curve and losses (for ac fields) are given in Tables 2.1 and 2.2.

Note: As properties of magnetic materials improve by the year, the reader is urged to visit the sites of leading magnetic-materials producers such as Hitachi, Vacuum Schmelze, mag-inc.com, Magnet sales manufacturing Inc., Hognas AB, etc.

2.3 Electric Conductors and Their Skin Effects

Electric currents in transformers and electric machines flow in electric conductors characterized by high electrical conductivity and by the magnetic permeability of the air.

Pure electrolytic copper (seldom aluminum) is used to make electric conductors. The electric conductivity of copper conductor, ρ_{Co} , is

$$\rho_{Co} \approx 1.8 \cdot 10^{-8} \cdot \left(1 + \frac{1}{273} \cdot (T - 20^\circ) \right), \quad [\Omega \cdot m] \quad (2.31)$$

For dc currents, the current spreads uniformly over the electric conductor cross section. Round wire conductors are produced with up to 3 mm uninsulated diameter. Above 6 mm² cross section, rectangular cross section conductors are used. Round copper (magnetic) bare diameters are standardized from 0.3 up to 3 mm. Values from 0.3 to 1.5 mm are as follows: 0.3, 0.32, 0.33, 0.35, 0.38, 0.40, 0.42, 0.45, 0.48, 0.5, 0.53, 0.55, 0.58, 0.6, 0.63, 0.65, 0.67, 0.7, 0.71, 0.75, 0.8, 0.85, 0.9, 0.95, 1.0, 1.05, 1.1, 1.12, 1.15, 1.18, 1.2, 1.25, 1.3, 1.32, 1.35, 1.40, 1.45, 1.5.

For dc current conductors, the dc resistance only produces the known copper losses, $(P_{Co})_{dc}$:

$$(P_{Co})_{dc} = R_{dc} \cdot I_{dc}^2 \quad (2.32)$$

There is dc current in the stator of brush-commutator motor and in the synchronous machine rotor. The vicinity of the magnetic circuit (slot walls)

TABLE 2.1

B–H Curve for Silicon (3.5%) Steel (0.5 mm thick) at 50 Hz

B (T)	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
H (A/m)	22.8	35	45	49	57	65	70	76	83	90
B (T)	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
H (A/m)	98	106	115	124	135	148	162	177	198	220
B (T)	1.05	1.1	1.15	1.2	1.25	1.3	1.35	1.4	1.45	1.5
H (A/m)	237	273	310	356	417	482	585	760	1050	1340
B (T)	1.55	1.6	1.65	1.7	1.75	1.8	1.85	1.9	1.95	2.0
H (A/m)	1760	2460	3460	4800	6160	8270	11170	15220	22000	34000

TABLE 2.2
Typical Core Loss |W /lb) of As-Sheared 29 Gage M19 Fully Processed CRNO at Various Frequencies

Induction (KG)	50 Hz	60 Hz	100 Hz	150 Hz	200 Hz	300 Hz	400 Hz	600 Hz	1000 Hz	1500 Hz	2000 Hz
1.0	0.008	0.009	0.017	0.029	0.042	0.074	0.112	0.205	0.465	0.900	1.451
2.0	0.031	0.039	0.072	0.119	0.173	0.300	0.451	0.812	1.786	3.370	5.318
4.0	0.109	0.134	0.252	0.424	0.621	1.085	1.635	2.960	6.340	11.834	18.523
7.0	0.273	0.340	0.647	1.106	1.640	2.920	4.450	8.180	17.753	33.720	53.971
10.0	0.404	0.617	1.182	2.040	3.060	5.530	8.590	16.180	36.303	71.529	116.702
12.0	0.687	0.858	1.648	2.860	4.290	7.830	12.203	23.500	54.258	108.995	179.321
13.0	0.812	1.014	1.942	3.360	5.060	9.230	14.409	27.810	65.100	131.918	
14.0	0.969	1.209	2.310	4.000	6.000	10.920	17.000				
15.0	1.161	1.447	2.770	4.760	7.150	13.000	20.144				
15.5	1.256	1.559	2.990	5.150	7.710	13.942	21.619				
16.0	1.342	1.667	3.179	5.466	8.189						
16.5	1.420	1.763	3.375	5.788	8.674						
17.0	1.492	1.852	3.540	6.089	9.129						

does not alter the uniform distribution of the dc (excitation) current in the conductors in slots. In contrast, in transformers and all electric machines, either on rotor or on stator, ac currents flow into copper-conductor coils connected in windings. Even if the conductor is surrounded by air, as the frequency increases and once the conductor radius approaches the field penetration depth, formula (2.28), for copper, gives

$$\delta_{Co} = \sqrt{\frac{2}{\mu_0 \cdot \omega_1 \cdot \sigma_{copper}}} \tag{2.33}$$

Skin effect occurs in the sense that the current density decreases with the radius inside the conductor.

For $\sigma_{copper} = 5.55 \cdot 10^7 \text{ S}$, $f_1 = 60 \text{ Hz}$, the penetration depth, $\delta = 6.16 \cdot 10^{-3} \text{ m}$. So, if the conductor diameter, $d_{con} \ll \delta$, and, for maximum 3 mm at 60 Hz, all standardized round magnetic wires qualify for low skin effect. The conductors in air such as in the end connections of coils in electric machines are typical for the case in point. In the vicinity of laminated magnetic cores, in the transformer's windows or in machine slots, Figure 2.10a and b, the situation differs.

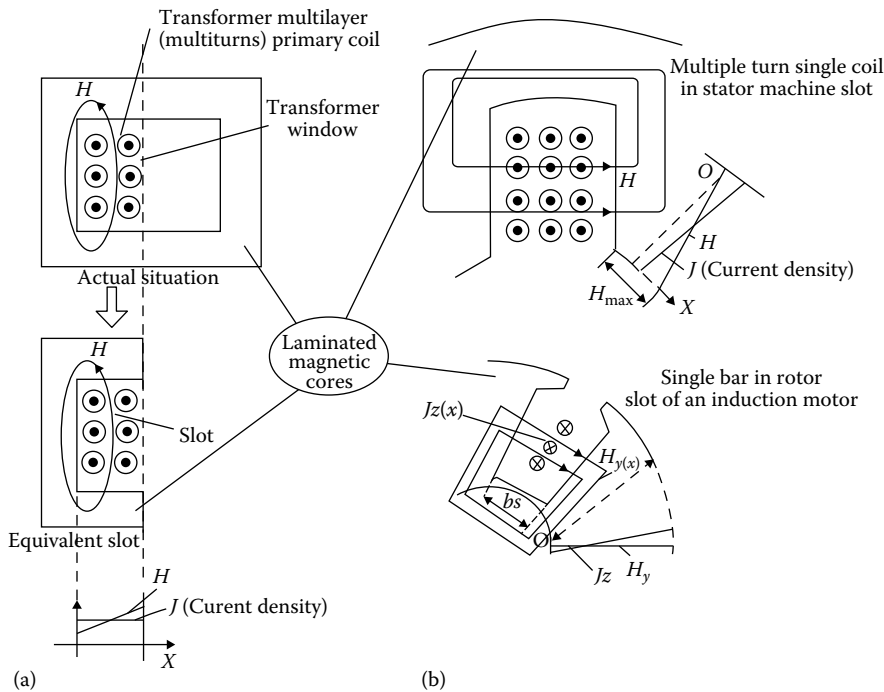


FIGURE 2.10
Placement of ac coils in the vicinity of laminated iron core walls (a) in transformers and (b) in electric machines.

It seems obvious that for both situations in Figure 2.10, the conductor(s) traveled by ac current of frequency, f , are placed in some kind of open slot (with three soft iron core walls). In reality, for electric machinery, the slots may be semi-closed, or even closed, but these cases will be treated in the respective chapters in the book. For the single conductor (bar) in the rotor slot, applying the same Ampere's and Faraday's laws at point A (Figure 2.10b), we get

$$\frac{\partial H_y}{\partial x} = J_z; \quad \frac{1}{\sigma_{Al}} \cdot \frac{\partial J_z}{\partial x} = \mu_0 \cdot \frac{\partial H_y}{\partial t} \quad (2.34)$$

with the boundary condition:

$$(H_y)_{x=0} = 0 \quad \text{and} \quad b_s \cdot \int_0^h J_z(x) dx = I\sqrt{2} \quad (2.35)$$

The ac current in the conductor bar in Figure 2.10b may be expressed in complex number terms:

$$\underline{I} = I\sqrt{2} \cdot e^{j\omega_1 t} \quad (2.36)$$

As the bar current is sinusoidal, the magnetic field $\underline{H}_y$ varies the same way. So,

$$\underline{H}_y = H_y(x) \cdot e^{j\omega_1 t} \quad (2.37)$$

Consequently, from Equation 2.34, after eliminating J_z , we obtain an equation similar to Equation 2.26:

$$\frac{d^2 \underline{H}_y(x)}{dx^2} = \underline{\gamma}^2 \cdot \underline{H}_y(x); \quad \underline{\gamma} = \pm \beta \cdot (1+j); \quad \beta = \sqrt{\frac{\omega \cdot \mu_0 \cdot \sigma_{Al}}{2}} \quad (2.38)$$

Again, the depth of its own field penetration in the conductor bar, δ_{Al} , is

$$\delta_{Al} = \frac{1}{\beta} \cdot \sqrt{\frac{2}{\omega \cdot \mu_0 \cdot \sigma_{Al}}} \quad (2.39)$$

The solution of Equation 2.38 is

$$\underline{H}_y(x) = \underline{A}_1 \cdot \sinh \beta(1+j) + \underline{A}_2 \cdot \cosh \beta \cdot (1+j) \quad (2.40)$$

Finally, with the boundary conditions (Equation 2.35), we get

$$\begin{aligned} \underline{H}_y(x) &= \frac{I\sqrt{2}}{b_s} \cdot \frac{\sinh \beta \cdot (1+j) \cdot x}{\sinh \beta \cdot (1+j) \cdot h}; \\ \underline{J}_z(x) &= \frac{I\sqrt{2}}{b_s} \cdot \beta \cdot (1+j) \cdot \frac{\cosh \beta \cdot (1+j) \cdot x}{\cosh \beta \cdot (1+j) \cdot h} \end{aligned} \quad (2.41)$$

The gradual amplitude reduction of current density, J_z , and its magnetic field, H_y , with slot depth is visible also in Figure 2.10b.

When frequency goes down, the field penetration depth δ_{Al} increases, and the current density becomes more uniform over the entire cross section of the conductor.

To calculate the active and reactive powers that penetrate the slot, the Poynting vector $\underline{S}$ definition is used:

$$\underline{S} = \frac{1}{2} \cdot \int_{A_{\text{upper}}} (\underline{E}_z \cdot \underline{H}_y) dA = P + j \cdot Q = P_{dc} \cdot \varphi(\xi) + j \cdot Q_{dc} \cdot \Psi(\xi) \quad (2.42)$$

with

$$\sigma_{Al} \cdot \underline{E}_z = J_z \quad (2.43)$$

From Equations 2.40 through 2.42 we obtain

$$\xi = \beta \cdot h; \quad \varphi(\xi) = \xi \frac{\sinh(2\xi) + \sin(2\xi)}{\cosh(2\xi) - \cos(2\xi)}; \quad \Psi(\xi) = \frac{3}{2\xi} \cdot \frac{\sinh(2\xi) - \sin(2\xi)}{\cosh(2\xi) - \cos(2\xi)} \quad (2.44)$$

$$P_{dc} = \frac{L_{\text{stack}} \cdot I^2}{\sigma_{Al} \cdot h \cdot b_s} = R_{dc} \cdot I^2; \quad Q_{dc} = \frac{\omega \cdot \mu_0 \cdot L_{\text{stack}} \cdot h_s}{3b_s} \cdot I^2 = \frac{1}{2} \cdot X_{slc} \cdot I^2 \quad (2.45)$$

Equating Equation 2.41 with Equation 2.44, we may identify the ac resistance, R , and slot leakage reactance, X_{sl} :

$$R_{\text{mono}} = R_{dc} \cdot \varphi(\xi); \quad X_{sl\text{mono}} = X_{slc} \cdot \Psi(\xi) \quad (2.46)$$

A graphical representation of the ac resistance and slot leakage reactance skin effect coefficients $\varphi(\xi)$ and $\Psi(\xi)$ dependence on ξ , the ratio between conductor height, h_s , and its own ac field penetration depth, δ , is shown in Figure 2.11.

While the single conductor case is typical to cage rotors of induction motors, in the stator of electric machines and in transformers (Figure 2.10a), there are multiple electric conductors in series in coils, arranged into, say, m layers.

For the situation in Figure 2.12, the ac resistance factor, K_{Rp} , for the conductors in layer p , on same rationale, is

$$K_{Rp} = \varphi(\xi) + \frac{I_u \cdot (I_u + I_p \cdot \cos(\gamma_{up}))}{I_p^2} \cdot \Psi'(\xi) \quad (2.47)$$

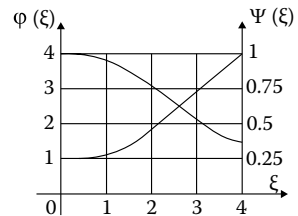
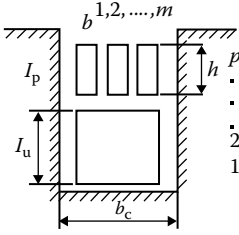


FIGURE 2.11

Skin effect ac resistance and reactance coefficients, $\varphi(\xi)$ and $\Psi(\xi)$, of a single conductor in slot.

**FIGURE 2.12**

Multiple-layer conductors in slots.

with

$$\Psi'(\xi) = \frac{\sinh \xi + \sin(\xi)}{\xi \cdot (\cosh(\xi) + \cos(\xi))} \quad (2.48)$$

And γ_{up} is the time lag angle between the currents in the p layer and in the layers beneath it.

If all the conductors are connected in series in the slot, ($I_u = (p-1) \cdot I_p$),

$$K_{Rp} = \varphi(\xi) + p \cdot (p-1) \cdot \Psi'(\xi) \quad (2.49)$$

The average value of K_{Rp} for all m layers, K_{Rm} , is

$$K_{Rm} = \varphi(\xi) + \left(\frac{m^2 - 1}{3} \right) \cdot \Psi'(\xi); \quad \xi = \frac{h}{\delta_{Al}} \quad (2.50)$$

Note: In most electric machine stators, there are two multiple-turn typical coils in a slot, which do not necessarily belong to the same phase; this influences the ac resistance coefficient, K_{Rm} .

As for a given slot in Figure 2.12 (total slot height: $h_{\text{slot}} = m \cdot h$), the number, m , of conductor layers increases, the allowable conductor height, h , decreases; consequently, there is an optimal conductor height, when K_{Rm} is minimum, called critical conductor height, h_{critical} .

For large transformers and electric machines at power grid, (50(60) Hz), or for small power transformers at higher frequencies or small power high speed (frequency) motors, a single turn (bar) is made of quite a few conductors in parallel. To attenuate the proximity effect (mutually induced eddy currents), the elementary conductors are all placed in turn in all positions within the slot. This way, the Roebel bar was born (Figure 2.13). For Roebel bars, $K_{Rm} \approx \varphi(\xi)$, which is a great advantage as by design K_{Rm} is kept as $1 < K_R \leq 1.1$, to limit ac skin effect losses.

2.4 Components of Single- and 3-Phase Transformers

Again, the transformer is a static apparatus for electric ac energy (power) transfer, with step-up or step-down voltage, by magnetically and (eventually) electrically coupled electric circuits, at a given frequency.

The single-phase power transformer (at 50(60) Hz frequency) contains quite a few components such as [6]

- The closed laminated soft iron core
- The low- and one (or more) high-voltage windings

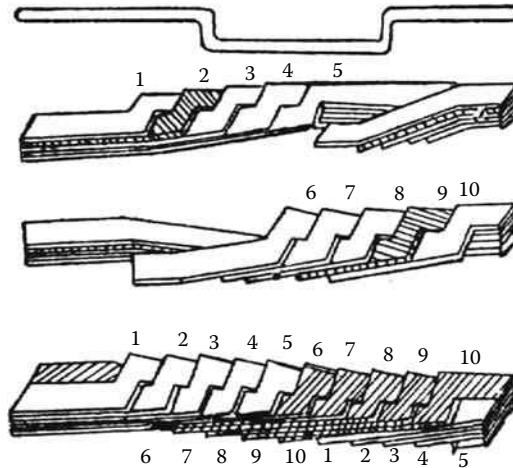


FIGURE 2.13
Transposed elementary conductors (Roebel bar).

- Oil tank (if any) with conservator
- Terminals, low- and high-voltage oil to air bushings
- Cable connections
- Coolers
- Radiators
- Fans
- Forced oil (forced air) heat exchanges
- Oil pumps
- Off-circuit and on-load tap changes
- Accessories

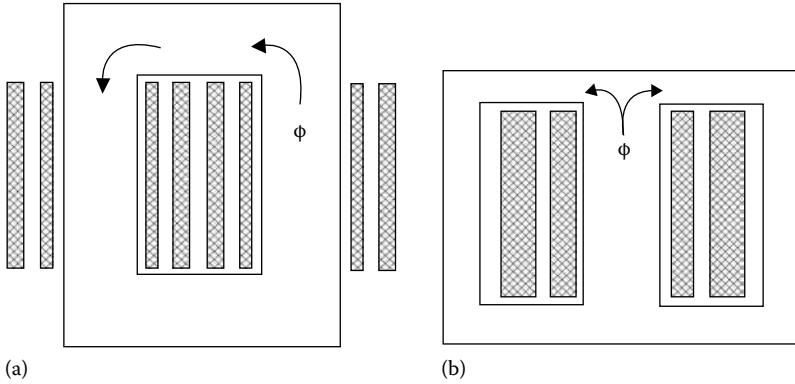
We will treat mainly the magnetic core and the windings in some detail.

2.4.1 Cores

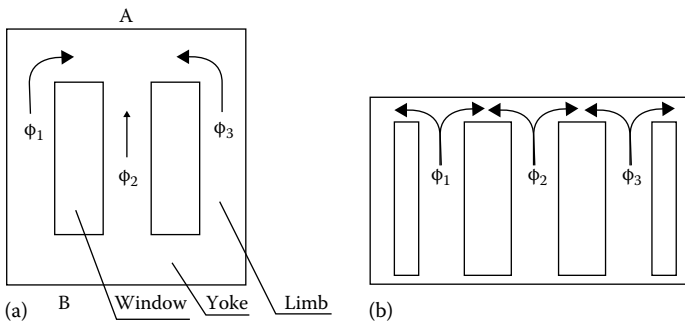
There are two practical laminated core configurations for single-phase power transformers: core type and shell type (Figure 2.14a and b).

It is evident that the shell configuration allows for a lower height and wider topology, which may be advantageous for transportation permits.

The 3-phase power transformers are built with three limbs (Figure 2.15a) or with five limbs (Figure 2.15b). The limbs are surrounded by the

**FIGURE 2.14**

Single-phase power transformer magnetic cores (a) core type and (b) shell type.

**FIGURE 2.15**

Three-phase power transformer cores (a) with three limbs and (b) with five limbs.

low- (high-) voltage windings of one phase. In the five-limb configuration, the height of the yoke is halved and thus, again, the transformer is less tall, but wider. In normal, symmetric operations, the phase voltages and currents are symmetric (same amplitude, 120° phase shift between phases). So, the magnetic flux in all limbs is sinusoidal, and their summation in points A and B (Figure 2.15a) is zero, according to Gauss's flux law.

The outer limbs and the yokes of the five-limb 3-phase transformer "see" half of the central limb's flux. The same is true for the single-phase shell core (Figure 2.14a).

The thin (0.5 mm thick for 50(60) Hz) oriented-grain silicon steel sheets that make the magnetic cores are stacked to achieve low-core losses and low-magnetization current.

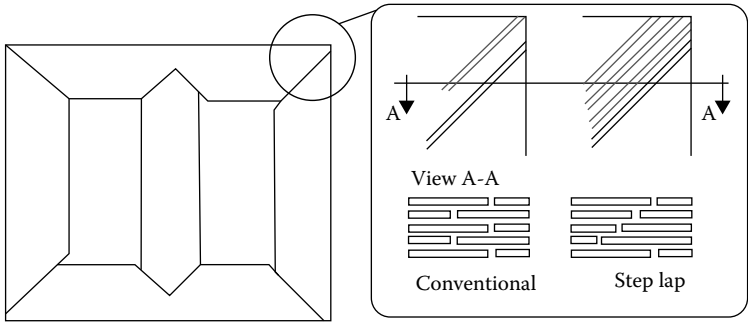


FIGURE 2.16
Stacking pattern with conventional or step-up joints.

The connection between limbs (columns) and yokes is arranged in general by 45° joints, to achieve a large cross-over section and low-flux density in the non-preferred magnetization direction.

They are laid in packets of two or four where each packet (layer) has the joint displaced relative to the adjacent one (Figure 2.16).

The core is earthed, in general, in one point. The magnetic core made of thin insulated lamination layers is glued together for small power and wrapped with steel straps around the limbs or epoxy-cured stacked for large powers. Holes through the lamination core are to be avoided to reduce additional losses. Clamps with curved tie bolts keep the yoke laminations tight.

The cross section of limbs is quadratric or polygonal but the yoke’s interior sides are straight to allow the winding location in the transformer window. While the stacked cores are typical for large power, single-phase distribution transformer use wound (less expensive) cores (Figure 2.17).

2.4.2 Windings

To increase the space factor, practical transformers use rectangular cross section (or the least flattened round) below 6 mm² area conductors.

As the current rating increases two or more times, elementary conductors (strands) are connected in parallel to form a turn. Each strand (Figure 2.18a) is insulated by paper lapping or by an enamel. If two or more strands are insulated in a common paper covering, they are considered a cable—the smallest visible conductor. A few cables in parallel carry the phase currents. As explained in Section 2.3, the continuously transposed cable

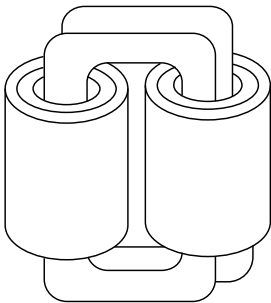
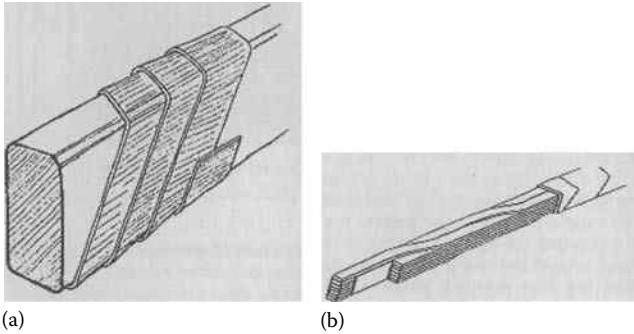


FIGURE 2.17
Single-phase distribution transformer with wound core.

**FIGURE 2.18**

(a) Conductor strand with insulation paper lapping and (b) continuously transposed cable.

(Roebel bar) is used to reduce proximity effect in large current rating transformers (Figure 2.18b).

By transposing at a 10 cm pitch the insulated strands (up to a hundred!), all of them experience the same emf (produced by their currents) and thus circulating currents (proximity effects) are avoided.

Windings are divided into four main types:

- Layer (cylindrical) windings
- Helical windings
- Disc windings
- Foil windings

The level of current and the number of turns per phase determine the winding type.

Layer windings (Figure 2.19a) have turns that are arranged axially, close to each other in single or multiple layers and are used mainly for small and medium power/unit, or, in large transformers, as regulating windings (Figure 2.19a).

The helical winding (Figure 2.19b) is similar to layer windings but with spacers between each turn or thread and is suitable for large currents that share several parallel strands.

All cables (made of one or more strands in a common paper cover) in a disc belong to the same turn and are in parallel.

To avoid circulating currents between strands, each of them changes position along the winding transposition zone, to experience to same total ac magnetic field. The helical winding has a high space factor and is mechanically robust and easy to manufacture from a continuously transposed cable. Disc windings (Figure 2.19c) are used for a large number of turns (and lower

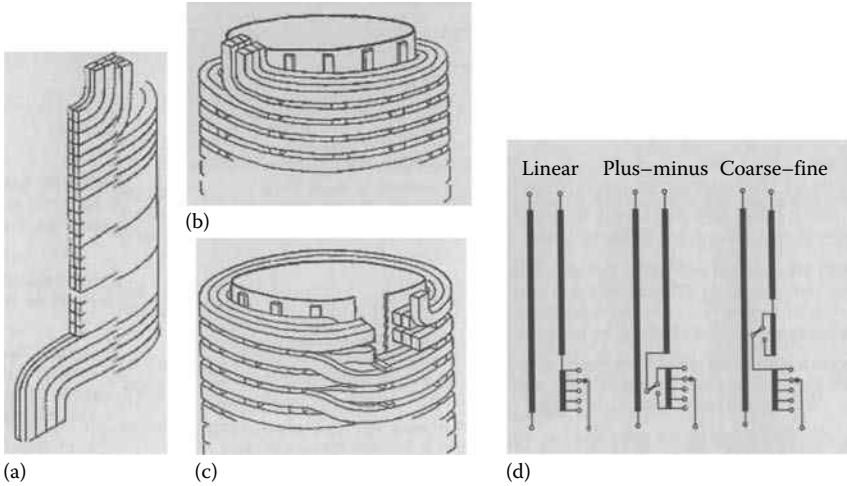


FIGURE 2.19 Transformer windings: (a) Regulating winding in layer type design, (b) double-threaded helical winding, (c) conventional and interleaved disc winding, and (d) three basic ways to arrange voltage regulation.

currents). It is built with a number of discs connected in series. The turns in a disc are wound radially like a spiral.

Helical windings have one turn per disc while disc windings have two or more turns per disc. The capacitance between segments of conventional disc windings are lower than that between them and earth and thus the distribution of fast front (atmospherically or commutation) voltage pulse will be nonuniform. To counteract this demerit, interleaved disc windings are used (Figure 2.19c).

Foil windings are made of thin aluminum or copper sheets from 0.1 to 1.2 mm in thickness, with the main merit of small electrodynamic axial forces during transformer sudden short circuit due to the beneficial influence of eddy currents induced in the neighboring foils by the current in one foil, at the cost of additional losses. Low voltage, distribution transformers have foil windings, due to ease in manufacturing and large space factor and in large transformers that experience frequent strong overcurrents.

Tapping for turn-ratio limited range regulation of the voltage is feasible for not-so-large currents (Figure 2.19d). In this case, a small part of the winding will be left without current, and the uncompensated large axial force during large overcurrents has to be considered in the mechanical design. For large regulating range (as in the locomotive transformers, for example), the layer and helical regulating turns are arranged in a separate winding shell whose height is almost equal to the main winding's axial height, to avoid uncompensated large axial forces. They are located by the winding

neutral star point where the electric potential between phases is small. There are no-load and on-load tap changers. The latter evolved dramatically, from mechanical to electronic (thyristor) commutation under load.

Windings may also be classified as

- Concentric and biconcentric cylindrical (Figure 2.20a)
- Alternate windings (Figure 2.20b)

Alternate windings are used in large transformers to reduce leakage reactance as the rather opposing currents in subsequent primary/secondary coils reduce the leakage (air field) magnetic field and energy. This way, the voltage regulation (voltage reduction with load) is decreased, as needed in large voltage ac transmission power lines.

The windings and the cores produce heat by winding and core losses and oil, with high heat capacity (1.8 kW s/kg K), is needed to transport this heat to the heat exchangers. For a 20 inlet–outlet oil temperature differential and 180 kW of losses (30–50 MVA transformer) an $180/(1.8 \times 20) = 5 \text{ kg/s}$ oil flow rate is required. Ten times more is needed for a 400 MVA transformer.

The oil circulation through heat exchangers is done naturally through thermo siphon effect or it is pumped in a controllable way. Unfortunately, the loss of auxiliary power source that supplies the electric motor pumps leads to immediate transformer tripping. The transformer tank and its oil conservator are shown in Figure 2.21a and b.

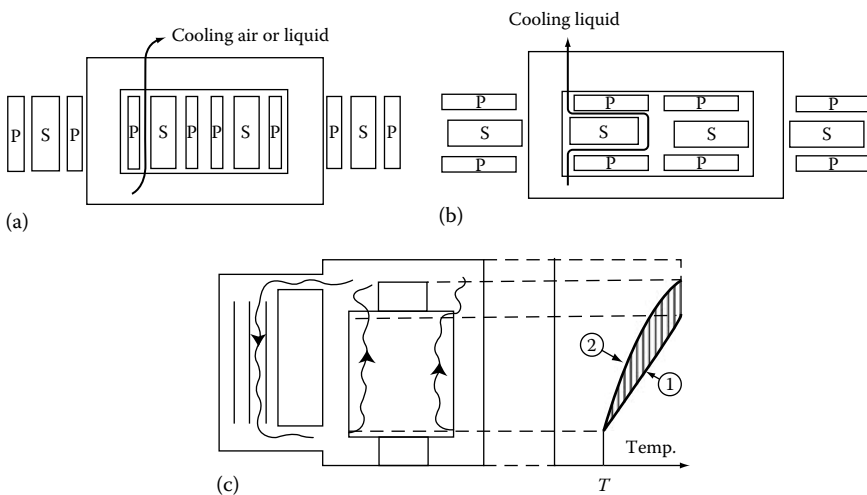
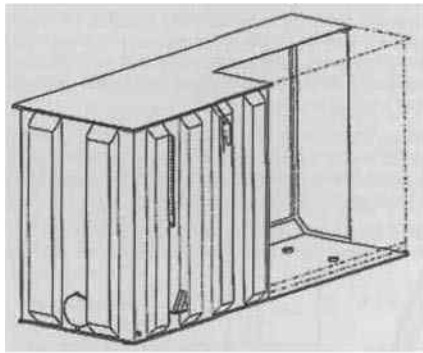
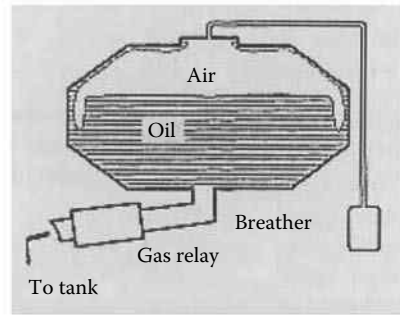


FIGURE 2.20

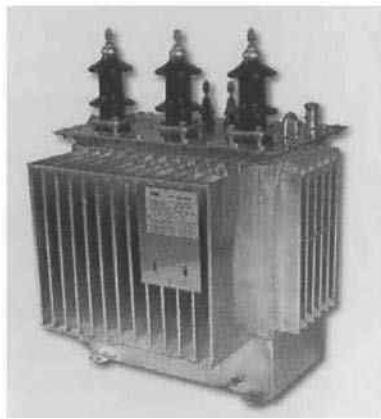
(a) Cylindrical windings, (b) alternated windings, and (c) cooling oil circulation.



(a)



(b)



(c)

FIGURE 2.21

(a) Transformer tank, (b) oil conservator, and (c) corrugated tank.

The tank is designed to cope with temperature and oil expansion during operation, and, in most cases, a separate expansion vessel, called conservator, is added (Figure 2.21b).

For small and medium powers, a corrugated tank is used. Figure 2.21c shows the cover, corrugated walls (which act as enhanced-area heat transmitters), and bottom box. They are hermetically sealed, in general. The lack of oil contact with air humidity is a definite advantage of corrugated tanks. Oil to air bushings are clearly visible in Figure 2.21c and their construction and geometry depend on the current and voltage levels.

High-current terminals with low voltage and high current (30 kA) for furnaces and electrolysis are in the form of flat bar palms or cylindrical studs mounted in panels of plastic laminate.

2.5 Flux Linkages and Inductances of Single-Phase Transformers

As already inferred in Section 2.1, the transformer operates based on Faraday's, Ampere's, and Gauss's laws applied to coupled magnetic/electric circuits at standstill.

According to Figure 2.22, under load, there is a current, I_2 , in the transformer, that is its secondary. The primary and secondary mmfs together produce the main magnetic flux (coupling) through the core, $\Phi_m = B_m \cdot A$, where A is the average laminated core cross-section area and B_m is the average flux density.

So the emfs induced in the transformer primary and secondary, V_{e1} and V_{e2} , are proportional to the number of turns, N_1 and N_2 :

$$V_{e1} = -N_1 \cdot \frac{d\Phi_m}{dt}; \quad V_{e2} = -N_2 \cdot \frac{d\Phi_m}{dt} \quad (2.51)$$

We may thus define main self L_{11m} , L_{22m} and mutual L_{12m} inductances for transformers in relation to main flux linkages, Ψ_{11m} , Ψ_{12m} , Ψ_{22m} :

$$\begin{aligned} \Psi_{11m} &= L_{11m} \cdot i_1 = (B_m \cdot A)_{i_2=0} \cdot N_1; & L_{11m} &= \frac{N_1^2}{R_m} \\ \Psi_{12m} &= L_{12m} \cdot i_1 = (B_m \cdot A)_{i_2=0} \cdot N_2; & L_{12m} &= \frac{N_1 \cdot N_2}{R_m} \\ \Psi_{22m} &= L_{22m} \cdot i_2 = (B_m \cdot A)_{i_1=0} \cdot N_2; & L_{22m} &= \frac{N_2^2}{R_m} \end{aligned} \quad (2.52)$$

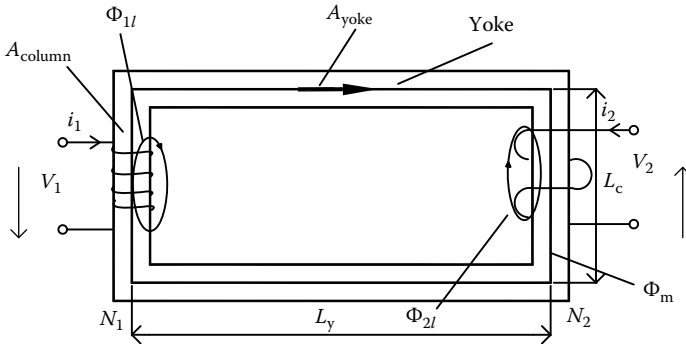


FIGURE 2.22

Single-phase transformer main, Φ_m , and leakage fluxes, Φ_{1l} and Φ_{2l} .

where R_m is the resultant magnetic reluctance for the flux lines that flow in the laminated core and embrace both windings:

$$R_m \approx \frac{2 \cdot L_{\text{column}}}{\mu_{rc} \cdot \mu_0 \cdot A_{\text{column}}} + \frac{2 \cdot L_y}{\mu_{ry} \cdot \mu_0 \cdot A_{\text{yoke}}} + \frac{g_{cy}}{\mu_0 \cdot A_{\text{yoke}}} \quad (2.53)$$

where

μ_{rc} and μ_{ry} are relative (P.U.) values of magnetic permeability in the limb (column) and yoke, respectively

g_{cy} is the equivalent airgap due to yoke/column stacking of laminations,

A_{column} and A_{yoke} are the cross-section cores of coil limbs and yokes

It is evident that μ_{rc} and μ_{ry} depend on the average flux density, B_{mc} and B_{uj} , in the limb and yoke. According to Ampere's law (Figure 2.21),

$$N_1 \cdot i_1 + N_2 \cdot i_2 = R_m \cdot \Phi_m = N_1 \cdot i_{01} \quad (2.54)$$

The current, i_{01} , is called the magnetizing current and is, in general, less than 2% of the rated primary current, I_{1n} ($I_{01}/I_{1n} < 0.02$). The rated current, I_{1n} , is defined as the current in the primary under full load that, by design, allows for safe operation under a given overtemperature of a transformer over its entire life (above 10–15 years in general), for the average duty cycle (load power versus time).

Now if, at first approximation, i_{01} is neglected in Equation 2.53,

$$N_1 \cdot i_1 + N_2 \cdot i_2 = 0; \quad i_2 = -\frac{N_1}{N_2} \cdot i_1 \quad (2.55)$$

Consequently, and evidently, from Equations 2.51 and 2.55,

$$V_{e1} \cdot i_1 = -V_{e2} \cdot i_2 \quad (2.56)$$

Now, as V_{e1} and V_{e2} are in phase (see Equation 2.50), it follows from Equation 2.56 that the currents, i_1 and i_2 , are shifted in time at ideally 180° (in reality a little less or more), and that their instantaneous mmfs are almost equal in amplitude.

The sign $\ominus$ comes from the fact that we adopted the sink/source association of power signs for primary/secondary (Figure 2.22).

It is also evident from Figure 2.22 that a part of the magnetic flux lines of both primary and secondary mmfs flow partly through air without embracing the other winding. These are called flux leakage lines and they produce corresponding fluxes, Φ_{1l} and Φ_{2l} , and magnetic energy (in the air within each winding and the space between the two), W_{m1l} and W_{m2l} , for which we may define leakage inductances, L_{1l} and L_{2l} :

$$L_{1l} = \frac{N_1 \cdot \Phi_{1l}}{i_1} = \frac{\Psi_{1l}}{i_1} = \frac{2W_{m1l}}{i_1^2}; \quad L_{2l} = \frac{N_2 \cdot \Phi_{2l}}{i_2} = \frac{\Psi_{2l}}{i_2} = \frac{2W_{m2l}}{i_2^2} \quad (2.57)$$

These leakage inductances are calculated in Sections 2.5.1 and 2.5.2 for cylindrical (layer) and alternate windings, respectively.

2.5.1 Leakage Inductances of Cylindrical Windings

Because the magnetic core is rather flat and the coils are circular, the leakage flux lines show true three-dimensional paths (Figure 2.23a).

To a first approximation, inside and outside the transformer window, the magnetic field path may be considered linear (vertical), as shown in Figure 2.23b for concentric windings and in Figure 2.23c for biconcentric windings.

With this gross simplification (to be corrected finally by Rogowski's coefficient for the leakage inductance), the magnetic field, H_x , varies only with x (horizontal variable):

$$H_x \cdot L_c = N_2 \cdot i_2 \cdot \frac{x}{a_2} \quad (2.58)$$

where a_1 and a_2 are the radial thickness of the two windings, respectively. In between the windings (δ) space,

$$H_{xm} = \frac{N_2 \cdot i_2}{L_c} \approx -\frac{N_1 \cdot i_1}{L_c}, \quad a_2 < x < a_2 + \delta \quad (2.59)$$

The magnetic field paths of each winding fill their own volume plus half of the interval between them, and thus the magnetic energy of each of them may be calculated separately:

$$\begin{aligned} L_{2l} &= K_{\text{Rog}2} \cdot \frac{2W_{m2l}}{i_2^2} = \frac{2K_{\text{Rog}2}}{i_2^2} \cdot \frac{1}{2} \cdot \mu_0 \cdot \int_{a_2}^{a_2+\delta} H_x^2 \cdot \pi \cdot (D+2x) \cdot L_c \cdot dx \\ &= \frac{\mu_0 \cdot N_2^2}{L_c} \cdot \pi \cdot D_{2av} \cdot a_{r2} \cdot K_{\text{Rog}2}; \\ a_{r2} &= \frac{a_2}{3} + \frac{\delta}{2} \end{aligned} \quad (2.60)$$

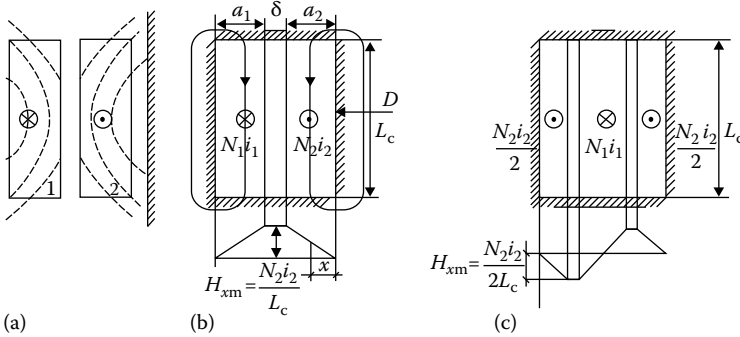


FIGURE 2.23

Leakage field of cylindrical windings: (a) Actual flux path, (b) computational flux path (concentric winding), and (c) the case of biconcentric winding.

In a similar way, for the primary,

$$L_{1l} = \mu_0 \cdot \frac{N_1^2}{L_c} \cdot \pi \cdot D_{1av} \cdot a_{r1} \cdot K_{Rog1}; \quad a_{r1} = \frac{a_1}{3} + \frac{\delta}{2} \quad (2.61)$$

The average diameters of turns, D_{2av} and D_{1av} , are

$$D_{2av} \approx D + 3 \frac{a_2}{2} \quad (2.62)$$

$$D_{1av} \approx D + 2 \cdot (a_2 + d) + 3 \cdot \frac{a_1}{2} \quad (2.63)$$

where

$D_{2av} < D_{1av}$ corresponds to the lower voltage winding, which is placed closer to the core

D is the outside diameter of the core insulation cylinder

K_{Rog1} and K_{Rog2} are the Rogowski's coefficients, which are larger than 1

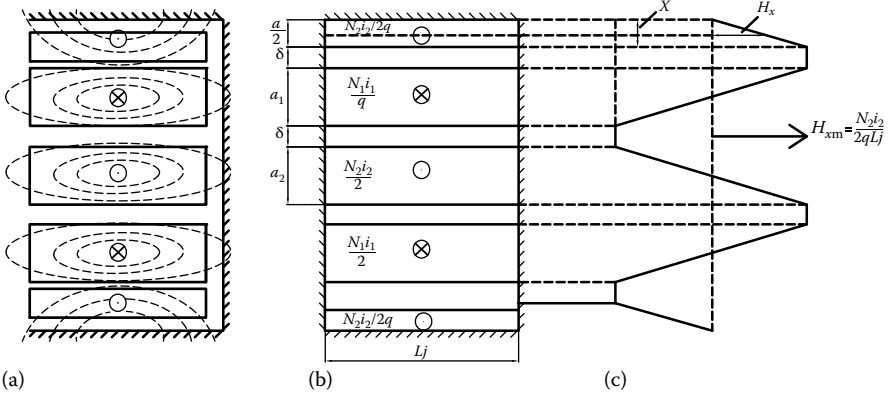
(Today, finite element 3D analysis allows us to calculate with more precision the leakage inductances at the cost of considerable computation time).

A few remarks are in order:

- Full use of almost 180° phase shift of i_1 and i_2 was made to calculate L_{1l} and L_{2l} .
- The leakage inductance is inversely proportional to the column (limbs) height, L_c , and is proportional to turn average diameter and radial thickness of the windings, a_1 and a_2 , and the insulation layer, δ , between them.
- To reduce the leakage inductances, the transformer core should be slim and tall.
- To increase leakage inductances (and reduce short-circuit current), the transformer with cylindrical windings should be less tall and wide.
- The biconcentric winding—where the two halves of the low voltage winding alternate radially around the high voltage winding—reduces two times the maximum leakage field between the windings and thus reduces more than two times the leakage inductance.
- Reducing the leakage magnetic energy by large L_c , the electrodynamic forces between windings will also be reduced.

2.5.2 Leakage Inductances of Alternate Windings

The alternate winding with its actual and computational leakage field paths and leakage field variation along the winding are all shown in Figure 2.24,

**FIGURE 2.24**

Alternate windings leakage field: (a) actual field paths, (b) computational field paths, and (c) field vertical distribution.

for the division of both windings in q separately insulated coils in series ($2q$ half coils).

The leakage magnetic field, H_x , alternates along vertical dimension, and is

$$H_x = \frac{N_2 \cdot i_2}{2 \cdot q \cdot L_y} \cdot \frac{2x}{a_2}; \quad 0 \leq x \leq \frac{a_2}{2} \quad (2.64)$$

$$H_x = \frac{N_2 \cdot i_2}{2 \cdot q \cdot L_y}; \quad \frac{a_2}{2} < x < \frac{a_2}{2} + \delta \quad (2.65)$$

Proceeding as for the cylindrical windings, we get the two leakage inductances:

$$L_{2l} = \frac{\mu_0 \cdot N_2^2}{2 \cdot q \cdot L_y} \cdot \pi \cdot D_{av} \cdot a_{r1} \cdot K_{Rog}; \quad a_{r2} = \frac{a_2}{6} + \frac{\delta}{2} \quad (2.66)$$

$$L_{1l} = \frac{\mu_0 \cdot N_1^2}{2 \cdot q \cdot L_y} \cdot \pi \cdot D_{av} \cdot a_{r2} \cdot K_{Rog}; \quad a_{r1} = \frac{a_1}{6} + \frac{\delta}{2} \quad (2.67)$$

The average turn diameter, D_{av} , is now the same, and so is the Rogowski's coefficient, K_{Rog} .

Note: It is already evident that the leakage inductances are much smaller for the alternate windings, more than $2q$ times smaller, for $L_y = L_c$. For power transmission transformer and other applications when low voltage regulation (voltage reduction with load) is necessary, alternate windings are advantageous, at the costs of higher short-circuit current (in relative values: pu). The inductance expressions in relation to transformer geometry are to be essential even in this preliminary electromagnetic design, described at the end of this chapter.

While the above equations are valid for both transients and steady state, we now continue with steady state.

2.6 Circuit Equations of Single-Phase Transformers with Core Losses

The circuit equations of a single-phase transformer stem from the circuit form of Faraday's law applied to primary and secondary circuits of a transformer (Figure 2.23) whose main self and mutual main inductances, L_{11m} , L_{22m} , and L_{12m} , and leakage inductances, L_{1l} and L_{2l} , have been defined and calculated in the previous section.

$$i_1 \cdot R_1 - V_1 = -\frac{d\Psi_1}{dt}; \quad \text{sink} \quad (2.68)$$

$$i_2 \cdot R_2 + V_2 = -\frac{d\Psi_2}{dt}; \quad \text{source} \quad (2.69)$$

The total flux linkages, in the absence of the core loss effect on them, are

$$\Psi_1 = \Psi_{1m0} + L_{1l} \cdot i_1; \quad \Psi_2 = \Psi_{2m0} + L_{2l} \cdot i_2 \quad (2.70)$$

where Ψ_{1m0} and Ψ_{2m0} are the main flux linkages.

$$\Psi_{1m0} = L_{11m} \cdot i_1 + L_{12m} \cdot i_2; \quad \Psi_{2m0} = L_{22m} \cdot i_2 + L_{12m} \cdot i_1 \quad (2.71)$$

Now, the iron losses may be considered as produced in a purely resistive (R_{iron}) short-circuited special winding that embraces the yoke. Then, we may write

$$-\frac{d\Psi_{1m0}}{dt} = R_{\text{iron}} \cdot i_{\text{iron}} \quad (2.72)$$

Equation 2.72 show that the fictitious core loss winding current produces the core loss in the resistance, R_{iron} , reduced to the primary (same number of turns: N_1). We may now admit that the core loss eddy current, i_{iron} , produces a reaction field through the main inductance, L_{11m} . Consequently, the resultant main flux, Ψ_{1m} , is

$$\Psi_{1m} = \Psi_{1m0} + L_{11m} \cdot i_{\text{iron}} \quad (2.73)$$

Eliminating i_{iron} from Equations 2.72 and 2.73 yields

$$\Psi_{1m} = \Psi_{1m0} - \frac{L_{11m}}{R_{\text{iron}}} \cdot \frac{d\Psi_{1m0}}{dt} \quad (2.74)$$

The Ψ_{1m} and Ψ_{2m} will replace Ψ_{1m0} and Ψ_{2m0} in Equation 2.70 to yield from Equations 2.68 and 2.69:

$$i_1 \cdot R_1 + L_{1l} \frac{di_1}{dt} - V_1 = V_{e1} = -\frac{d\Psi_{1m}}{dt} \quad (2.75)$$

$$i_2 \cdot R_2 + L_{2l} \frac{di_2}{dt} + V_2 = V_{e2} = -\frac{d\Psi_{2m}}{dt} = \frac{N_2}{N_1} \cdot V_{e1} \quad (2.76)$$

and

$$V_{e1} = -\frac{d\Psi_{1m}}{dt} = -L_{11m} \cdot \frac{di_{01}}{dt} + \frac{L_{11m}^2}{R_{\text{iron}}} \cdot \frac{d^2 i_{01}}{dt^2}; \quad (2.77)$$

$$i_{01} = i_1 + i'_2; \quad i'_2 = \frac{N_2}{N_1} \cdot i_2 \quad (2.78)$$

Multiplying Equation 2.76 by N_1/N_2 we obtain

$$i_2^2 \cdot R'_2 + L'_{2l} \cdot \frac{di'_2}{dt} + V'_2 = V_{e1} \quad (2.79)$$

$$R'_2 = R_2 \cdot \frac{N_1^2}{N_2^2}; \quad L'_{2l} = L_{2l} \cdot \frac{N_1^2}{N_2^2}; \quad V'_2 = V_2 \cdot \frac{N_1}{N_2} \quad (2.80)$$

Now, as both the primary and the secondary windings show the same emf, V_{e1} , it means that the new Equations 2.75 and 2.79 refer to the transformer with the same number of turns (N_1) or with the secondary reduced to the primary. It is evident that the actual and reduced secondaries are equivalent in terms of losses.

2.7 Steady State and Equivalent Circuit

Under steady state, the load is unchanged and the input voltage, $V_1(t)$, is sinusoidal in time. If we neglect magnetic saturation (and hysteresis cycle) nonlinearities, the output voltage, V_2 , and the input and output currents, i_1 and i_2 , are also sinusoidal in time:

$$V_1(t) = V_1 \sqrt{2} \cdot \cos(\omega_1 t + \gamma_0) \quad (2.81)$$

Consequently, complex variables may be used: $V_1(t) \rightarrow \underline{V}_1$, with $d/dt = j\omega_1$. So, in complex variables (phasors), Equations 2.74, 2.75, and 2.79 become

$$\begin{aligned} I_1 \underline{Z}_1 - \underline{V}_1 &= \underline{V}_{e1}; \quad \underline{Z}_1 = R_1 + jX_{1l}; \quad X_{1l} = \omega_1 L_{1l}; \quad I'_2 \underline{Z}'_2 + \underline{V}'_2 = \underline{V}_{e1} \\ \underline{Z}'_2 &= R'_2 + jX'_{2l}; \quad X'_{2l} = \omega_1 L'_{2l}; \quad \underline{V}_{e1} = -\underline{Z}_{1m} I_{01} \\ \underline{Z}_{1m} &= R_{1m} + jX_{1m}; \quad X_{1m} = \omega_1 L_{1m} \end{aligned} \quad (2.82)$$

with $R_{1m} = \omega_1^2 \frac{L_{11m}^2}{R_{iron}}$.

The load equation is

$$\underline{V}'_2 = \underline{Z}'_s \underline{I}'_2; \quad \underline{Z}'_s = \underline{Z}_s \frac{N_1^2}{N_2^2} \quad (2.83)$$

With $\underline{Z}_s$ being the load impedance characterized by amplitude, Z_s , and phase angle, φ_2 .

As in the first two equations of (Equation 2.82) $\underline{V}_{e1}$ shows twice it means that the primary and reduced secondary magnetically electric circuits may be electrically connected, to form the equivalent circuit of the transformer under steady state with R_{1m} , a series resistance which relates to core losses (Figure 2.25).

R_{1m} is the equivalent core loss series resistance (Figure 2.25b) that is convenient to use with the equivalent circuit, as it is calculated from measured (or precalculated in the design stage) core losses:

$$p_{iron} = I_{01}^2 R_{1m} = \frac{(X_{1m} I_{01})^2}{R_{iron}} \quad (2.84)$$

A few remarks are in order:

- In regular transformers, L_{1l} and L'_{2l} are not far away from each other in value ($X_{1l} \approx X'_{2l}$) and are at least 100 times smaller than main inductance, L_{11m} ; so $X_{1l}(X'_{2l}) \ll X_{1m}$.
- The core loss series resistance, $R_{1m} \ll R_{iron}$ but much smaller than X_{1m} in value and a few tens of times larger than the primary winding resistance, R_1 .
- Now R_1 and R'_2 are not far away from each other in numbers ($R_1 \approx R'_2$) as the secondary is reduced to primary and the design current

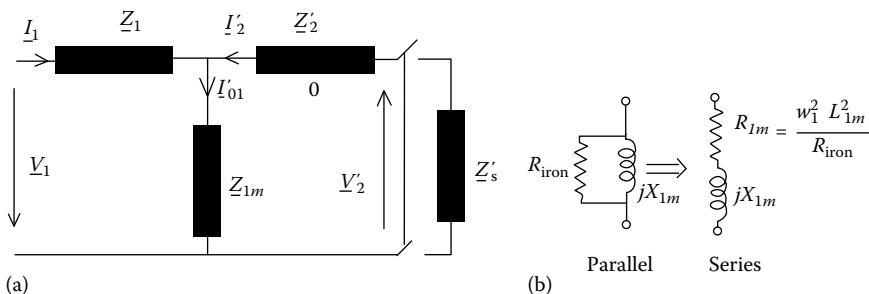


FIGURE 2.25

Transformer steady-state (a) equivalent circuit and (b) core loss parallel resistance, R_{iron} , and series resistance, R_{1m} .

density is not much different for the two; so the primary and secondary winding losses are not far away from each other.

For steady-state thorough analysis, we first discuss no-load and short-circuit operation modes and tests.

2.8 No-Load Steady State ($I_2 = 0$)/Lab 2.1

Under no load, the secondary current is zero or the secondary terminals are disconnected from any load. In fact, the transformer degenerates into the case of an ac coil with magnetic (laminated) core.

From Equation 2.82, with $I'_2 = 0$ we obtain

$$\underline{I}_{10}\underline{Z}_1 - \underline{V}_1 = \underline{V}_{e10}; \quad \underline{V}_{e10} = -\underline{Z}_{1m}\underline{I}_{10}; \quad \underline{V}'_{20} = \underline{V}_{e1} \quad (2.85)$$

Finally,

$$\underline{I}_{10} = \frac{\underline{V}_1}{\underline{Z}_0}; \quad \underline{Z}_0 = \underline{Z}_1 + \underline{Z}_{1m} = R_0 + j\omega_1 L_0 \quad (2.86)$$

with

$$R_0 = R_1 + R_{1m} \quad (R_1 \ll R_{1m}) \text{ and } L_0 = L_{1l} + L_{1m} \quad (2.87)$$

The equivalent circuit under no load (from Equation 2.25) is shown in Figure 2.26a. R_0 and L_0 (X_0) are the no-load resistance and inductance (reactance), and for standard power transformers (at 50(60) Hz) $X_0/R_0 > 20$, so the power factor at no load is very small.

The no-load current at rated voltage, V_{1m} , is almost equal to magnetizing current, I_{01} , under rated load in most power transformers because the voltage drop on $\underline{Z}_1$ is small (less than 1%–2%).

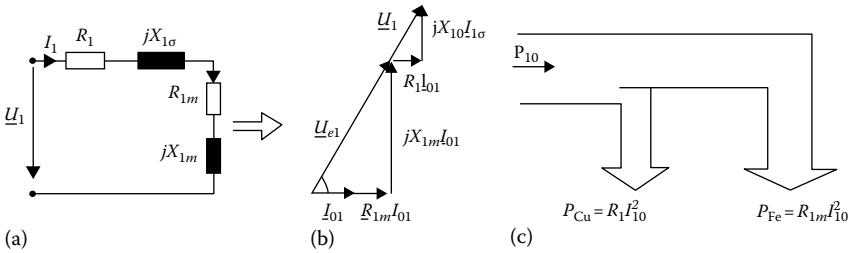


FIGURE 2.26

No-load case: (a) Equivalent circuit, (b) phasor diagram, and (c) power breakdown.

Consequently, the magnetic flux density in the core is about the same under no load and load. And so are the core losses.

So, it is sufficient to measure the core losses under no load and then use them as constant under various loads, for same (rated) frequency and voltage, V_1 , level.

Now, the iron losses are much smaller than copper losses because $I_{10} < 0.02I_{1n}$ and $R_{1m} > R_1$. So the measured no-load power, P_0 , is

$$P_0 = R_1 I_{10}^2 + R_{1m} I_{10}^2 \approx R_{1m} I_{10}^2 = p_{\text{iron}} \quad (2.88)$$

The no-load test at full voltage serves to measure, in fact, core losses ($P_0 = P_{\text{iron}}$) valid also for load conditions.

The no-load characteristics to be drawn, after measurements, refer to P_0 and I_{10} measured directly for growing values of input voltage, V_1 , from $0.1V_{1n}$ to $1.05V_{1n}$ in 0.02–0.05 pu steps by using a variable output voltage additional transformer source (Figure 2.27).

From the measured I_{10} , P_0 , and voltage, V_1 , we may calculate

$$R_1 + R_{1m} = f_1(V_1) \approx R_{1m} \quad \text{and} \quad X_{1m} + X_{1l} = f_2(V_1) \approx X_{1m} \quad (2.89)$$

$$R_1 + R_{1m} \approx R_{1m} = \frac{P_0}{I_{10}^2}; \quad X_{1m} + X_{1l} \approx X_{1m} = \sqrt{\frac{P_0^2}{I_{10}^2} - (R_{1m} + R_{1m})^2} \quad (2.90)$$

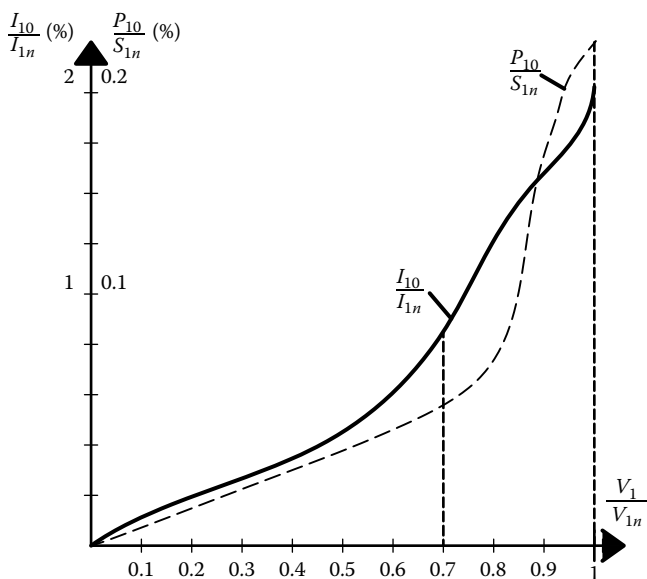


FIGURE 2.27

No-load current, I_{10} , and core losses vs. input voltage.

The function, $X_{1m}(V_1)$, may be considered the no-load magnetization curve of the transformer and may be used to verify the design calculations of main inductance variation with load (magnetization current)

$$L_{11m}(I_{10}) = \frac{X_{1m}(I_{10})}{\omega_1} = \frac{N_1^2}{R_m(I_{10})} \quad (2.91)$$

where, again, R_m is the magnetic core reluctance, which is dependent on the level of flux density, B_m , and on the stacking airgap, g_e , during core assembly from laminations.

Example 2.2 No-Load Transformer

A 1-phase transformer is rated $S_n = 10$ MVA with $V_{1n}/V_{2n} = 35/6$ kV, and has the design primary resistance, $R_1 = 0.612 \Omega$ and leakage primary reactance, $X_{1l} = 25 \Omega$; under no load and at rated voltage, the transformer absorbs the power, $P_e = 25$ kW, for the no-load current, $I_{10} = 0.02I_{1n}$ (I_{1n} : rated current). The magnetization (core loss) series and parallel resistances, R_{1m} and R_{iron} , magnetization reactance, X_{1m} , and power factor, $\cos \varphi_0$, are required (Figure 2.28).

Solution:

The definition of rated apparent power is

$$S_n = V_{1n}I_{1n} \approx V_{2n}I_{2n}$$

Consequently, the rated current, I_{1n} , is found first:

$$I_{1n} = \frac{S_n}{V_{1n}} = \frac{10 \cdot 10^6 \text{ VA}}{35 \cdot 10^3 \text{ V}} = 285.7 \text{ A}$$

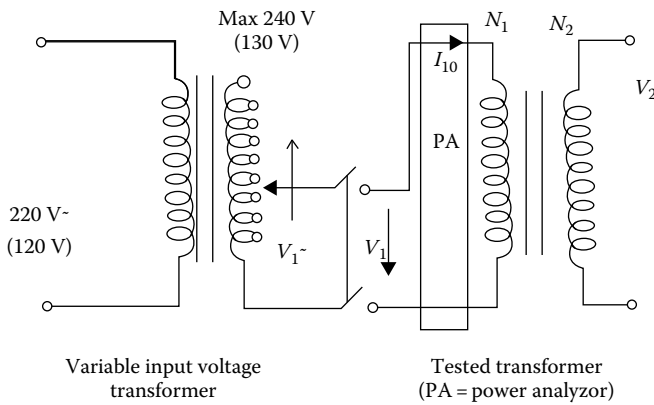


FIGURE 2.28

Lab arrangement for no-load transformer testing.

So, the rated no-load current, $I_{10} = 0.02I_{1n} = 0.02 \cdot 285.7 = 5.714$ A.
From Equation 2.89,

$$R_{1m} = \frac{P_0}{I_{10n}^2} - R_1 = \frac{22 \cdot 10^3}{5.714^2} - 0.612 = 673.8 - 0.612 = 673.2 \Omega$$

So,

$$R_{1m} >> R_1$$

Also

$$\begin{aligned} X_{1m} &= \sqrt{\frac{V_{1n}^2}{I_{10n}^2} - (R_1 + R_{1m})^2} - X_{1l} = \sqrt{\frac{(35 \cdot 10^3)^2}{5.714^2} - 673.8^2} - 2.5 \\ &= 6088 - 2.5 = 6085 \Omega \end{aligned}$$

Notice that $X_{1m} \approx 10R_{1m}$ in our case.

So, the power factor, $\cos \varphi_0$, at no load is simply

$$\cos \varphi_0 = \frac{P_0}{V_{1n}I_{10}} = \frac{22,000}{35,000 \cdot 5.714} = 0.11$$

As expected, the power factor at no load is low because the core losses are moderate and the machine core is properly saturated. Also, $\frac{X_{1l}}{X_{1m}} = \frac{L_{1l}}{L_{11m}} = \frac{2.5}{6063} = \frac{1}{2420}! < \frac{1}{1000}$.

2.8.1 Magnetic Saturation under No Load

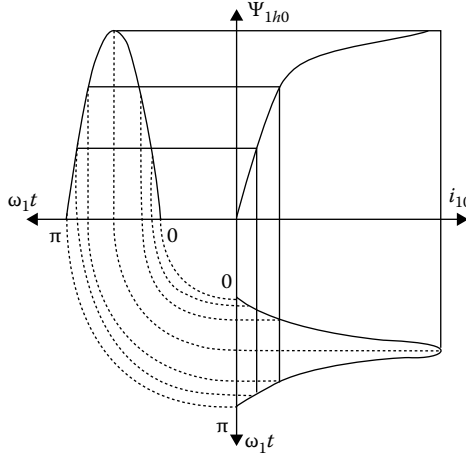
The presence of magnetic saturation, let alone the hysteresis cycle (delay), leads not only to a reduction in the magnetization reactance (inductance), X_{1m} (L_{11m}), and an increase in the no-load current at rated voltage, but the nonlinear dependence of main flux Ψ_{1m} of i_{10} (Figure 2.26): $L_{11m} = \frac{\Psi_{1m}}{\Psi_{10}}$, results in a nonsinusoidal current waveform for sinusoidal voltage input:

$$V_1(t) = V_1\sqrt{2} \cdot \cos(\omega_1 t) = \frac{d\Psi_{10}}{dt} \approx \frac{d\Psi_{1m}}{dt} \quad (2.92)$$

So

$$\Psi_{1m}(t) \approx \frac{V\sqrt{2}}{\omega_1} \sin \omega_1 t \quad (2.93)$$

But when we look for each instantaneous value of flux, $\Psi_{1m}(t)$, in the nonlinear magnetization curve, $\Psi_{1m}(I_{10})$, for the corresponding instantaneous current value, a nonsinusoidal current waveform is obtained (Figure 2.29).

**FIGURE 2.29**

No-load current actual waveform with magnetic saturation considered.

It is evident that 3rd, 5th, 7th (odd numbers in general) harmonics are present and, using a digital oscilloscope or a wide frequency band current sensor with galvanic separation and a computer interface, the $I_{10}(t)$ wave can be captured and analyzed as part of the “no-load transformer test.” Now, if the current instrument, in the previous current and power measurements, measures the RMS value of current, then, still, the R_{1m} and X_{1m} calculated values are acceptable even for industrial tests, which are standardized (see IEEE, NEMA, and IEC standards on transformer testing).

The influence of hysteresis cycle produces also flux/current delays and additional harmonics in the current wave form. But, once the transformer is loaded, as $I_{10n} < 0.02I_{1n}$, the influence of the no-load (magnetization) current becomes too small to really count.

2.9 Steady-State Short-Circuit Mode/Lab 2.2

The steady-state short circuit refers to the already short-circuited secondary situation $V'_2 = 0$ of transformer under sinusoidal input voltage. From Equation 2.92 we obtain

$$I_{1sc}Z_1 - \underline{V}_1 = \underline{V}_{e1}; \quad \underline{V}_{e1} = -\underline{Z}_{1m}I_{01sc} = I'_{2sc}Z'_2; \quad I_{01sc} = I_{1sc} + I'_{2sc} \quad (2.94)$$

Now, if we consider $I_{01sc} \approx 0$, because I_{1sc} and $I'_{2sc} \gg I_{10}$, from Equation 2.94 we obtain

$$I_{1sc} = -I'_{2sc}; \quad I_{1sc} = \frac{V_1}{Z_{sc}}; \quad Z_{sc} = Z_1 + Z'_2 \quad (2.95)$$

So the transformer (with reduced secondary) degenerates into its short-circuit impedance, Z_{sc} (Figure 2.30).

As the short-circuit impedance is small, the steady state short-circuit current is 10–30 times larger than the rated current.

So we should not supply the short-circuited transformer from the rated voltage, but at a much lower voltage, such that not to surpass rated current, I_{1n} .

In fact, the rated short-circuit voltage, V_{1scn} , is defined as the low voltage for which, under short circuit, the transformer absorbs rated current, $I_{1n} \approx I'_{2n}$.

So (from Equation 2.94),

$$V_{1scn} = |Z_{sc}| I_{1n} \approx (0.03 - 0.12) V_{1n} \quad (2.96)$$

Under short-circuit tests we again use the arrangement in Figure 2.28, below 12% rated voltage and up to rated primary current I_{1n} !

Again, we measure power, P_{sc} , current, I_{1sc} , and voltage, V_{1sc} , to calculate the short-circuit impedance (from Equation 2.95), $|Z_{sc}|$, resistance, R_{sc} , and reactance, X_{sc} :

$$R_{sc} = \frac{P_{sc}}{I_{1sc}^2}; \quad X_{sc} = \sqrt{\frac{V_{1sc}^2}{I_{1sc}^2} - R_{sc}^2} \quad (2.97)$$

And power factor, $\cos \varphi_{sc}$, is

$$0.3 > \cos \varphi_{sc} = \frac{R_{sc}}{|Z_{sc}|} > \cos \varphi_0 \quad (2.98)$$

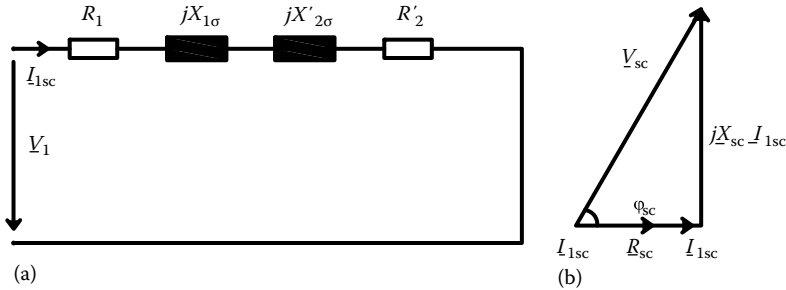


FIGURE 2.30

Steady state of short-circuited transformer. (a) Equivalent circuit and (b) phasor diagram.

The power factor at short circuit is essential when paralleling transformers to avoid circulating currents between them.

Example 2.3 Transformer Under Short Circuit

For the transformer in Example 2.2, the rated short-circuit voltage, $V_{1nsc} = 0.04 V_{1n}$, while the active power measured at rated current, I_{1n} is $P_{scn} = 100$ kW. Calculate Z_{sc} , R_{sc} , X_{sc} , $\cos \varphi_{sc}$, R'_2 , X'_2 .

Solution:

From Equation 2.95, at rated current, the short-circuit impedance, $|Z_{sc}|$ is

$$|Z_{sc}| = \frac{V_{1scn}}{I_{1n}} = \frac{0.04 \cdot 35,000 \text{ V}}{285.7 \text{ A}} = \frac{1400}{285.7} = 4.9 \Omega$$

The short-circuit resistance, R_{sc} (Equation 2.96) is

$$R_{sc} = \frac{P_{scn}}{I_{1n}^2} = \frac{100 \cdot 10^3}{285.7^2} = 1.225 \Omega$$

As $R_1 = 0.612 \Omega$ (from Example 2.2), $R'_2 = R_{sc} - R_1 = 1.225 - 0.612 = 0.613 \Omega \approx R_1$, as expected.

Now, the power factor at short circuit is (Equation 2.97)

$$\cos \varphi_{sc} = \frac{R_{sc}}{Z_{sc}} = \frac{1.225}{4.9} = 0.25$$

The short-circuit reactance, X_{sc} , is

$$X_{sc} = Z_{sc} \sin \varphi_{sc} = 4.9 \cdot 0.968 = 4.744 \Omega$$

With $X_{1l} = 2.5 \Omega$, $X'_{2l} = X_{sc} - X_{1l} = 4.744 - 2.5 = 2.244 \Omega$, not far away from X_{1l} .

In general, in measurements, we may not separate R_1 from R'_2 and X_{1l} from X'_{2l} ; so $R_1 \approx R'_2$ and $X_{1l} \approx X'_{2l}$ in measured R_{sc} and X_{sc} . Now, the tests may be repeated at voltages up to V_{1nsc} such that the current is 0.25, 0.5, 0.75, $1.0 \cdot I_{1n}$, and for each current value, R_{sc} and X_{sc} are measured. Finally, the average values of R_{sc} and X_{sc} are taken as the final results. The ac winding losses at rated current (and short-circuit rated voltage tests) are the same as those at rated load. So, at given load,

$$p_{\text{copper}} = P_{scn} \frac{I_1^2}{I_{1n}^2} \quad (2.99)$$

The short-circuit test, up to rated current is useful to determine R_{sc} and copper winding ac losses P_{copper} for given load (current).

The no-load and short-circuit tests serve thus to separate iron and copper losses, to be used in assessing efficiency under load.

2.10 Single-Phase Transformers: Steady-State Operation on Load/Lab 2.3

On (under) load the transformer, supplied at given (rated) voltage, delivers power to an ac impedance load in the secondary. The energy transfer is done electromagnetically with iron and copper losses.

Active and reactive power break down under load is shown in Figure 2.31.

The operation under load is characterized by efficiency η versus load factor, $K_s = I_1/I_{1n}$ and load voltage regulation (drop) for given power factor, $\cos \varphi_2$ (1, 0.8, 0.6).

The efficiency η is defined as

$$\eta = \frac{\text{output active power}}{\text{input active power}} = \frac{P_2}{P_2 + p_{\text{copper}} + p_{\text{iron}}} \quad (2.100)$$

Based on no-load and short-circuit mode considerations, Equation 2.99 becomes

$$\eta = \frac{V'_2 I'_2 \cos \varphi_2}{V'_2 I'_2 \cos \varphi_2 + p_{0n} + p_{\text{sc}} \frac{I_1^2}{I_{1n}^2}} \quad (2.101)$$

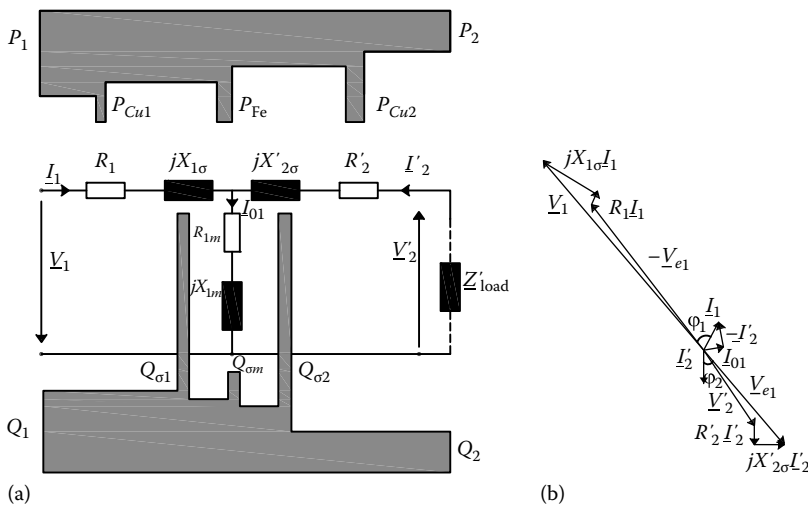


FIGURE 2.31

(a) Active and reactive power balance and (b) phasor diagram under load.

Now, if $I'_2 \approx I_1$ (which means $I_1/I_{1n} > 0.2$) and, with load factor, $K_s = I_1/I_{1n}$, Equation 2.101 becomes

$$\eta = \frac{S_{2n} K_s \cos \varphi_2}{S_{2n} K_s \cos \varphi_2 + p_{0n} + p_{scn} K_s^2} \quad (2.102)$$

The maximum efficiency is obtained for $\partial\eta/\partial K_s = 0$, for K_{sk} :

$$K_{sk} = \sqrt{\frac{p_{0n}}{p_{scn}}} \quad (2.103)$$

In fact, for the critical (maximum efficiency) load factor, copper and core losses are equal to each other.

Note: A transformer with full load operation most of the time has to be designed with K_{sk} close to unity while one with discontinuous and partial load operation (distribution transformer in schools, public institutions, or companies with large human resources, etc.) should be designed with K_{sk} around 0.5. Typical curves of efficiency versus load factor, $K_s = I_1/I_{1n} \approx P_2/P_{2n}$, are shown in Figure 2.32.

For $I_1/I_{1n} > 0.2$, the magnetization current, I_{01} , may be neglected and thus $I_2 = -I_1$ and thus the equation (Equation 2.82) becomes

$$-V'_2 = V_1 - I_1 Z_{sc} \quad (2.104)$$

Equation 2.104 leads to the simple equivalent scheme in Figure 2.33a and its corresponding phasor diagram (Figure 2.33b).

The secondary voltage variation, ΔV , under load is

$$\Delta U = V_1 - V'_2 \approx \overline{AB} \quad (2.105)$$

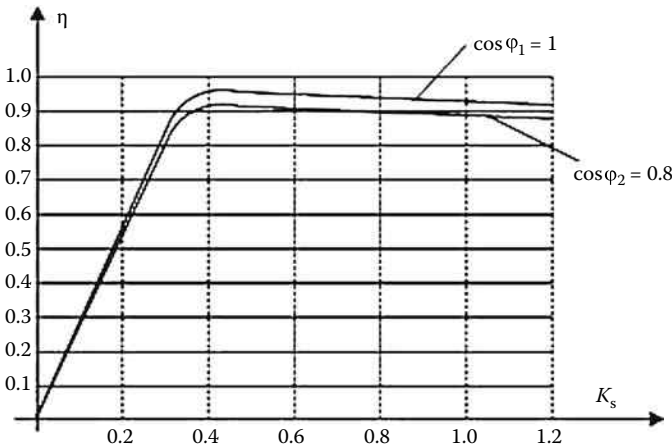
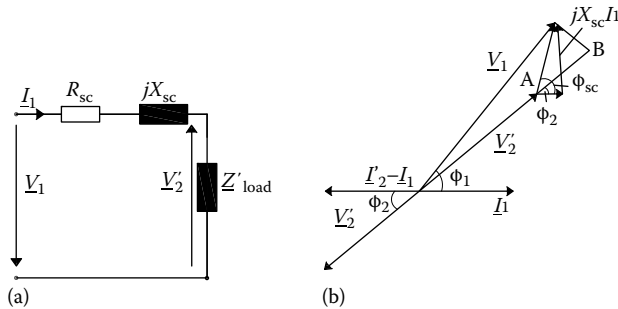


FIGURE 2.32

Efficiency versus load factor.

**FIGURE 2.33**

(a) Simplified equivalent circuit and (b) phasor diagram with neglected magnetization current ($I_{01} = 0$).

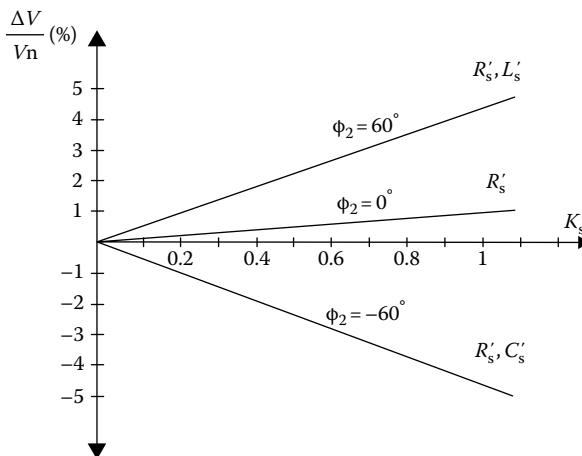
From Figure 2.33b, $\overline{AB}$ is

$$\Delta U \approx I_1 R_{sc} \cos \varphi_2 + I_1 X_{sc} \sin \varphi_2 = K_s V_{1scn} \cos(\varphi_{sc} - \varphi_2) \quad (2.106)$$

As, in general, $\varphi_{sc} = (80 - 85)^\circ$, the voltage regulation (drop) may be positive if $\varphi_2 \geq 0$ (resistive-inductive load) or negative for $\varphi_2 < \varphi_{sc} - 90^\circ$ (Figure 2.34), in general, for resistive-capacitive load such as overhead long power transmission lines without load.

Example 2.4 Transformer on Load

For the transformer in Examples 2.3 and 2.4, with $p_{on}=22$ kW, $S_n=10$ MVA, $V_{1nsc}=1400$ V, $\cos \varphi_{sc}=0.25$ (with $R_{sc} \approx 1.20 \Omega$, $X_{sc} = 4.7\Omega$), and $V_{1n}/V_{2n}=35/6$ kV, calculate the rated efficiency, voltage regulation and secondary voltage at pure resistive, inductive, and capacitive rated load.

**FIGURE 2.34**

Voltage regulation versus load factor, $K_s = I_1/I_{1n}$.

Solution:

Pure resistive, inductive, and capacitive loads means $\varphi_2 = 0, +90^\circ, -90^\circ$

The efficiency for rated load, ($K_s = K_{sn}=1$), and resistive load, ($\varphi_2 = 0$), is, from Equation 2.102,

$$\eta = \frac{10 \cdot 10^6 \cdot 10 \cdot 10}{10 \cdot 10^6 \cdot 10 \cdot 10 + 22 \cdot 10^3 + 100 \cdot 10^3} = 0.9879$$

For $\varphi_2 = 90^\circ$ (pure inductive and capacitive load), the active power output, P_2 , is zero and thus the efficiency is zero.

The voltage rated current regulation, ΔV_n , (from Equation 2.105) is

$$\begin{aligned} \Delta V &= (V_1 - V'_2) = V_{1nsc} \cdot \cos(\varphi_{sc} - \varphi_2) \\ &= 1400 \cdot 0.25 = +350 \text{ V}; \quad \varphi_2 = 0 \\ &= 1400 \cdot \cos(85^\circ - 90^\circ) = +1394 \text{ V}; \quad \varphi_2 = 90^\circ \\ &= 1400 \cdot \cos(85^\circ - (-90^\circ)) = -1394 \text{ V}; \quad \varphi_2 = -90^\circ \end{aligned}$$

The secondary voltage is

$$V_{2'} = (V_1 - \Delta V_n) \cdot \frac{V_{2n}}{V_{1n}} = 5940 \text{ V}, 5761 \text{ V}, 6239 \text{ V}$$

The no-load secondary voltage is 6000 V.

Example 2.5 Dual Output Voltage 1-Phase Residential Transformers

In the United States and some other countries, dual 120-V and 240-V output voltages are used for residential power supplies (Figure 2.35).

Let us consider the case of a 15-kVA, 2400/240/120-V, 60-Hz transformer. The load 1 absorbs 1.5 kW at 120 V and 0.867 PF leading, while load 3 absorbs 4 kW at 0.867 PF lagging. What is the maximum admissible load for the load 2 at unity PF and 120 V and the currents, I_1 , I_2 , I_3 (Figure 2.35)?

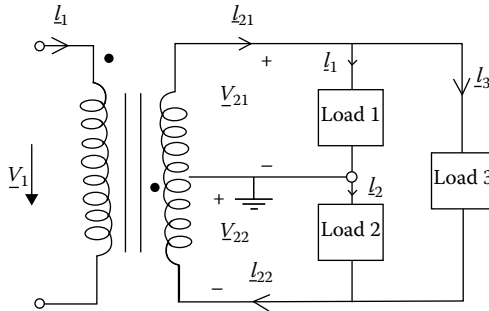


FIGURE 2.35

For dual voltage single-phase residential transformer.

Solution:

The current in load 2 is approximately the RMS value of I_1 (for load 1), that is,

$$I_1 \approx \frac{P_1}{V_1 \cdot \cos \varphi} = \frac{1500}{120 \cdot 0.867} = 14.41 \text{ A}$$

$$\underline{I}_1 = I_1 (\cos \varphi - j \sin \varphi) = 14.41 \cdot (0.867 - j0.5)$$

Similarly,

$$I_3 \approx \frac{P_3}{V_3 \cdot \cos \varphi} = \frac{4000}{240 \cdot 0.867} = 19.22 \text{ A}$$

$$\underline{I}_3 = I_3 (\cos \varphi + j \sin \varphi) = 19.22 \cdot (0.867 + j0.5)$$

We still cannot calculate I_2 .

For the 120-V output circuit load 2, the power factor is unity, so only active power $P_{2\max}$ is delivered.

Now, the apparent total power should be equal to $S_n = 10 \text{ kVA}$:

$$S_n = \sqrt{(P_1 + P_{2\max} + P_3)^2 + (Q_1 + Q_3)^2} = 10.000 \text{ VA}$$

where

$$Q_1 = P_1 \cdot \tan \varphi_1 = 1500\sqrt{3} \text{ VA}; P_1 = 1500 \text{ W}$$

$$Q_3 = P_3 \cdot \tan \varphi_3 = 4000\sqrt{3} \text{ VA}; P_3 = 4000 \text{ W}$$

Finally,

$$P_{2\max} = 10^3 \cdot \sqrt{15^2 - (1.5 + 4)^2} \cdot 3 - 5.5 \cdot 10^3 = 6.0866 \text{ kW}$$

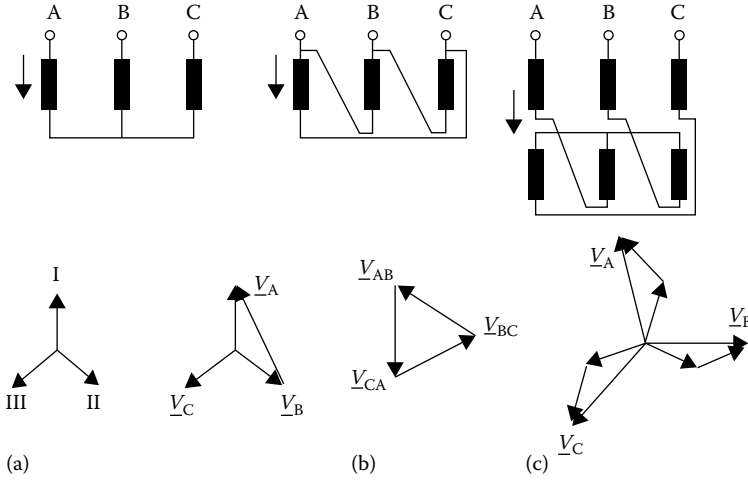
So, the current in load 2, $I_{2\max}$ (purely active), is

$$I_2 = I_{2\max} = \frac{P_{2\max}}{120 \text{ V} \cdot \cos \varphi_2} = \frac{6086.6}{120 \cdot 1} = 50.722 \text{ A}$$

We end the 1-phase transformer steady-state analysis to proceed with the 3-phase transformer steady state, by starting with the 3-phase connections and their order number.

2.11 Three-Phase Transformers: Phase Connections

The phase connections pertaining to either primary or secondary (or tertiary) 3-phase windings may be

**FIGURE 2.36**

Three phase connections: (a) Star (λ), (b) delta (Δ), and (c) zig-zag (Z).

- Star (λ) connections (Figure 2.36a)
- Delta (Δ) connections (Figure 2.36b)
- Zig-zag (Z) connections (Figure 2.36c)

For the stator connection, the beginnings or the ends of the three phases are connected to a neutral point (o) which may be available or not.

Capital letters (ABC) are used here to denote high voltages side and lower case letters (abc) for lower voltage side phase beginnings and XYZ and xyz for phase ends. (Different national standards use other denotations).

For the star connection (Figure 2.36a), the line voltage, $\underline{V}_{AB} = \underline{V}_A - \underline{V}_B$; similarly, $\underline{V}_{BC} = \underline{V}_B - \underline{V}_C$ and $\underline{V}_{CA} = \underline{V}_C - \underline{V}_A$. For symmetric line voltages (equal amplitudes and 120° phase shift), the line voltage $V_{l\gamma}$ is

$$V_{l\gamma} = V_{ph\gamma} \cdot \sqrt{3}; \quad I_{l\gamma} = I_{ph\gamma} \quad (2.107)$$

For the delta (Δ) connection (Figure 2.36b), the line currents are: $\underline{I}_{AB} = \underline{I}_A - \underline{I}_B$; $\underline{I}_{BC} = \underline{I}_B - \underline{I}_C$; $\underline{I}_{CA} = \underline{I}_C - \underline{I}_A$; the line and phase voltages are equal to each other. For symmetrical phase currents (same amplitude and 120° phase shift), the line current, $I_{l\Delta}$, is

$$I_{l\Delta} = I_{ph\Delta} \cdot \sqrt{3}; \quad V_{l\Delta} = V_{ph\Delta} \quad (2.108)$$

Now, the apparent 3-phase power, S , has the same formula for star and delta connections (for symmetric voltages and currents):

$$S_{3ph} = 3 \cdot V_{ph} \cdot I_{ph} = \sqrt{3} \cdot V_l \cdot I_l \quad (2.109)$$

RMS values are considered (Equations 2.106 through 2.108) for all sinusoidal variables. It should be noticed that the Y connection implies zero current summation but the Δ connection allows for a circulation (homopolar, or 3 ν harmonic component) current between phases, without affecting the external power source.

This occurs when the load is unbalanced in the secondary of the transformer.

The Z connection (Figure 2.36c) used for the transformer secondary has each phase made of two half-phases. Half-phases of separate transformer limbs are connected in opposite series so that each phase voltage is made of the vectorial difference between the voltages of the two component half-phases and thus $V_a = V_{sb} \cdot \sqrt{3}$ (instead of $2V_{sb}$ in the case of Y or Δ connection).

To obtain the same no-load secondary voltage, the Z connections require $2/\sqrt{3}$ times more turns (and more copper losses) than the star or delta connections. But for a Z connection, with an available neutral point, single-phase load is possible without asymmetrization of the unloaded phase voltages, because the neutral circulating current produces emfs in phase on all half-phases and they cancel each other on each phase.

Connection schemes combine Y(Y_0), Δ in the primary or secondary or Z (in secondary) to obtain combinations such as $Y\Delta$, Yy_0 , Δy_0 , Δy , Y_z . A scheme (combination) of connections is characterized by the phase-shift angle, β , between the primary and the secondary homologous line voltages ($\underline{V}_{AB}, \underline{V}_{ab}$), ($\underline{V}_{BC}, \underline{V}_{bc}$), with the secondary voltage lagging the primary voltage.

The connection scheme order $n = \beta^\circ/30^\circ$ and is an integer.

It may be demonstrated that when the primary and secondary connections are the same (Yy or $\Delta\Delta$), n is an even number (0, 2, 4, 6, 8, 10), and, when they are different (Δy), n is an odd number (1, 3, 5, 7, 9, 11). Let us consider the connection scheme, Δy , in Figure 2.37 and try to find its order, n .

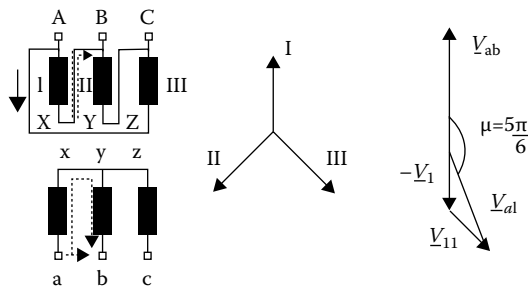


FIGURE 2.37
Connection scheme, Δy_5 .

First we choose a positive direction for $\int \underline{dl} \cdot \underline{E}$ along the transformers columns I, II, and III. We also suppose that the primary line voltages are symmetric. The emfs produced by the primary and secondary windings placed around the same transformer limb are in phase if they are coiled and travel in the same direction, because they are flown by the same flux.

The primary line voltage, say $\underline{V}_{AB}$, for the Δ connection is obtained from the emf of phase A placed on a single limb, while $\underline{V}_{ab}$ is obtained by subtracting emfs from windings on two limbs (Figure 2.37).

The lagging angle, β , between $\underline{V}_{ab}$ and $\underline{V}_{AB}$ is 150° , so $n = 150/30 = 5$. It is evident why n is odd: the two line voltages are composed differently. To connect transformers in parallel and avoid circulating current, the connection scheme order has to be the same.

Example 2.6 Connection Scheme with Given Order, n (Y Δ 7)

Solution:

Building a certain connection scheme is the typical problem at the design stage. First we draw the phase windings, connected only in the primary but not the secondary, with marked beginnings and ends (Figure 2.38).

By setting the position of the three limbs (emfs) we may first obtain $\underline{V}_{AB}$ from 2-phase voltages, $\underline{V}_A$ and $\underline{V}_B$, and then draw $\underline{V}_{ab}$ phasor lagging $\underline{V}_{AB}$ by $7 \times 30^\circ = 210^\circ$.

It is now evident that $\underline{V}_{ab}$, obtained from a single-limb winding (Δ connection), corresponds to limb I traveled in the negative (vertical) direction. This is how we can place the letters a and b and then c is completed univoquely. We should mention that n should be valid for all three line voltage pairs. We should then verify it, for one more pair ($\underline{V}_{BC}$, $\underline{V}_{bc}$).

It may be demonstrated, however, that n is met for all three line voltage pairs, after being set for one pair, if the order of phases is A.B.C in the primary and a.b.c or b.c.a or c.a.b in the secondary.

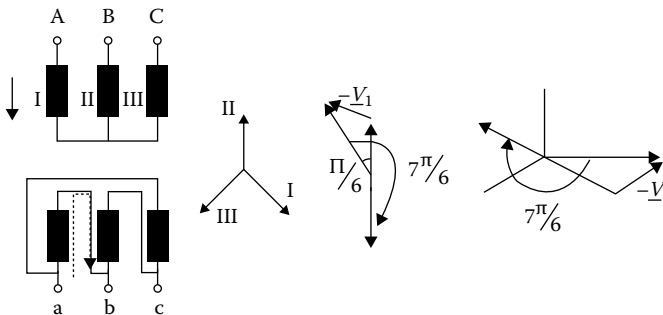


FIGURE 2.38
Connection scheme, Y Δ 7.

2.12 Particulars of 3-Phase Transformers on No Load

The no-load steady-state operation of 3-phase transformers is heavily influenced by the core configuration (3 limb, 5 limb, or 3×1 phase) and by the connection scheme.

We treat here only the asymmetry of the no-load phase currents produced by the 3-limb core and the no-load current waveform in the presence of magnetic saturation in Yy0 connection scheme.

2.12.1 No-Load Current Asymmetry

The no-load current asymmetry occurs mainly by 3-limb/3-phase transformers because the magnetic reluctances corresponding to the magnetic fluxes in phases A and C (Figure 2.39a) are larger than that in phase B. Not so is the case in 5-limb cores or 3×1 -phase transformer groups.

If we neglect the voltage drops along the primary impedance, Z_1 , for sinusoidal voltages, the emfs and thus the magnetic fluxes in the three limbs, Φ_A , Φ_B and Φ_C , are sinusoidal in time. So, $\Phi_A + \Phi_B + \Phi_C = 0$ and they are symmetric (equal in amplitude, with 120° phase shift) for symmetric voltages, V_A , V_B , V_C . Let us consider the sum of the currents to be zero, that is, a star connection ($I_{0A} + I_{0B} + I_{0C} = 0$).

Applying Kirchoff's second theorem to the magnetic circuits in Figure 2.39a we obtain

$$N_1 \cdot (I_{0A} - I_{0B}) = (R_{mc} + R_{mj}) \cdot \Phi_A - R_{mc} \cdot \Phi_B \quad (2.110)$$

$$N_1 \cdot (I_{0A} - I_{0C}) = (R_{mc} + R_{mj}) \cdot (\Phi_A - \Phi_C) \quad (2.111)$$

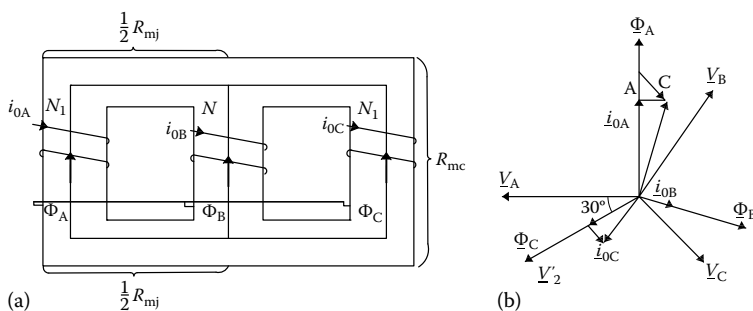


FIGURE 2.39

Three-limb transformer on no load: (a) The core and (b) phasors at no load.

With the summation of core fluxes and currents being zero, we may find the currents:

$$\begin{aligned}
 I_{0A} &= \left(\frac{R_{mc} + R_{my}}{N_1} \right) \cdot \underline{\Phi}_A + \frac{1}{3} \cdot \frac{R_{my}}{N_1} \cdot \underline{\Phi}_B \\
 I_{0B} &= \left(R_{mc} + \frac{R_{my}}{3} \right) \cdot \frac{\underline{\Phi}_B}{N_1} \\
 I_{0C} &= \left(\frac{R_{mc} + R_{mj}}{N_1} \right) \cdot \underline{\Phi}_C + \frac{1}{3} \cdot \frac{R_{my}}{N_1} \cdot \underline{\Phi}_B
 \end{aligned} \tag{2.112}$$

So, only the current in phase B, placed in the middle column, is in phase with the corresponding flux, Φ_B , and it is the smallest of all the three currents ($I_{0A} = I_{0C} > I_{0B}$). The phasor diagram of Equation 2.112—Figure 2.38b—with voltage phasors ($\underline{V}_A, \underline{V}_B, \underline{V}_C$) at 90° ahead of Φ_A, Φ_B, Φ_C illustrates the fact that only I_{0B} is purely reactive (magnetization) while I_{0A} and I_{0C} have an active component—one positive and the other negative. It simply means a circulation of active power between phases A and C.

A few consequences of this situation are as follows:

- For 3-limb core 3-phase transformers, the active power, P_0 , related to core losses, is calculated by algebraically adding up the active power, on all phases (it is negative on one lateral limb).
- The standardized no-load current, $I_{10n} = (I_{0A} + I_{0B} + I_{0C})/3$.
- So far, magnetic saturation was neglected. It will come up next.

2.12.2 Y Primary Connection for the 3-Limb Core

The Y connection is generally used in the primary, for protection through neutral-point voltage monitoring. Because the summation of currents is zero, in the presence of magnetic saturation (which is notable in practical transformers), the third time harmonic of no-load current can not occur. But it shows up in the core flux, which departs from sinusoidal waveform characteristic to the saturated single-phase transformer on no load and thus produces a third order ($3f_1$ -frequency) flux harmonic in the core flux (Figure 2.40a). The third harmonic means a $3 \cdot 120^\circ = 360^\circ$ phase shift between phases.

The third time harmonic of magnetic flux may not close path through the 3-limb core because it has to observe Gauss's law (zero flux on a closed area around the central limb). Consequently, the magnetic flux at $3f_1$ frequency (150, 180 Hz for power grid supply) closes through the oil tank walls, inducing notable eddy current losses. To reduce these additional rather severe no-load losses, a tertiary winding with small rating, with series phase connection is added (Figure 2.40b).

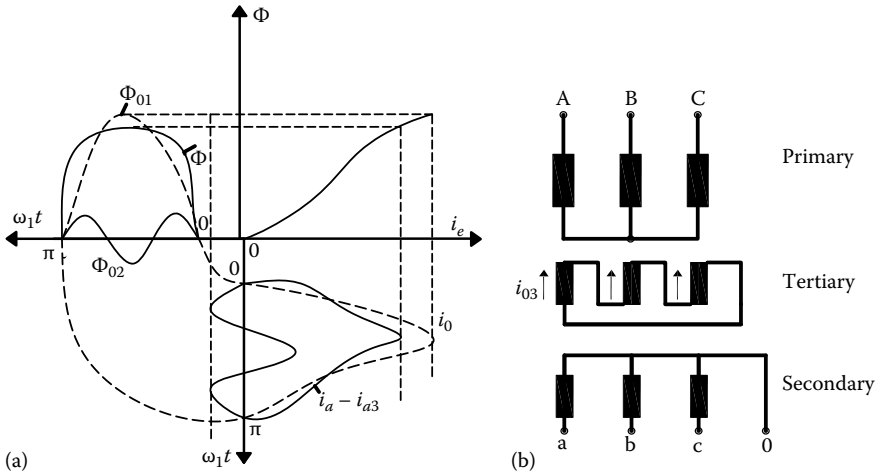


FIGURE 2.40
(a) Third flux time harmonics for the no load of a 3-limb 3-phase transformer with Yy0 connection and (b) tertiary winding.

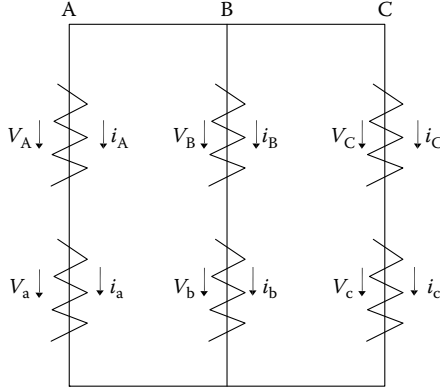
The current induced in the short-circuited winding severely reduces the resultant mmf at $3f_1$ frequency, and thus the eddy current losses in the oil tank are practically zero (for dry transformers there is no oil tank). The electromagnetic interference of the $3f_1$ flux lines in the air is reduced. It is important to note that, for the 3×1 -phase transformer group, the third time harmonic of flux closes paths in the core and thus no additional eddy current losses occur in the oil tank. The situation is almost similar in the 5-limb 3-phase transformers.

The tertiary series-connected winding will also prove useful for unbalanced load, to destroy rated frequency zero sequence (homopolar) secondary current field for Y primary connections (such as in Yy0 connection scheme).

2.13 General Equations of 3-Phase Transformers

For the 3×1 -phase transformer group, we only have to put together the equations of each of the three 1-phase transformers as there is no magnetic coupling between them (they have separate cores).

But as most transformers have 3-limb cores, with a few with 5-limb cores, there is scope to analyze the coupling between phases, at least to measure correctly the 3-phase transformer parameters and study the transients for the general (any) case.

**FIGURE 2.41**

Three-phase 3-limb core transformer phase voltages and currents.

Figure 2.41 shows the 3-limb core 3-phase transformer with its phase voltages and currents.

Again, the primary is sink and the secondary is source. Consequently, in phase coordinates,

$$\frac{d\Psi_{A,B,C}}{dt} = V_{A,B,C} - R_1 \cdot i_{A,B,C}; \quad \frac{d\Psi_{a,b,c}}{dt} = -V_{a,b,c} - R_2 \cdot i_{a,b,c} \quad (2.113)$$

There is magnetic coupling from each of the six windings. Let us consider that interphase coupling occurs only through main (core) flux. So,

$$\begin{aligned} \Psi_A &= L_{1l} \cdot i_A + L_{AAm} \cdot i_A + L_{ABm} \cdot i_B \\ &\quad + L_{ACm} \cdot i_C + L_{Aam} \cdot i_a + L_{Abm} \cdot i_b + L_{Acm} \cdot i_c \\ \Psi_a &= L_{aAm} \cdot i_A + L_{aBm} \cdot i_B \\ &\quad + L_{aCm} \cdot i_C + L_{2l} \cdot i_a + L_{aam} \cdot i_a + L_{abm} \cdot i_b + L_{acm} \cdot i_c \end{aligned} \quad (2.114)$$

Similar expressions for phases, B, b, C, and c can be obtained. And thus,

$$|\Psi_{A,B,C}| = |L_{ABCabc}| \cdot |i_{ABCabc}|^T \quad (2.115)$$

L_{ABCabc} is a 6×6 matrix with some of the terms equal to each other such as $L_{ABm} = L_{BCm}$, $L_{abm} = L_{bcm}$, $L_{AAm} = L_{CCm}$, $L_{aam} = L_{ccm}$, etc.

Adding the consumer equations and considering the primary voltages as inputs and flux (Ψ_{ABCabc}) as the variable vector and (i_{ABCabc}) as the dummy variable vector (to be eliminated through Equation 2.114), the transformer equations can be solved by numerical methods for any steady state or transient (with symmetric or asymmetric input voltages and balanced or unbalanced load). However, the transformer parameters have to be known, either from design or through measurements.

2.13.1 Inductance Measurement/Lab 2.4

Let us first consider that $i_A + i_B + i_C = 0$ and $i_a + i_b + i_c = 0$ and also, approximately, that $L_{ABm} = L_{ACm}$, $L_{abm} = L_{acm}$, and $L_{ABm} = L_{abm}$. In this case, from Equation 2.113, we get

$$\begin{aligned}\Psi_A &= L_{1l} \cdot i_A + (L_{AAm} - L_{ABm}) \cdot i_A + (L_{aam} - L_{abm}) \cdot i_a \\ \Psi_a &= L_{2l} \cdot i_a + (L_{aam} - L_{abm}) \cdot i_a + (L_{Aam} - L_{Abm}) \cdot i_A\end{aligned}\quad (2.116)$$

Let us call

$$L_{1mc} = L_{AAm} - L_{ABm}; \quad L_{2mc} = L_{aam} - L_{Abm}; \quad L_{12mc} = L_{Aam} - L_{Abm} \quad (2.117)$$

The cyclic inductances, L_{1mc} , L_{2mc} , and L_{12mc} , are valid when all currents exist and current summation is zero. In this particular case, the influence of other phases in 1-phase flux is “hidden” in the cyclic inductances. To measure all inductances in Equation 2.116 we have to supply only one phase, with the other phases open and measure the current on the supplied phase, i_{A0} (RMS), and the voltages on all phases, V_{A0} , V_{B0} , V_{C0} , V_{a0} , V_{b0} , V_{c0} (RMS):

$$\begin{aligned}L_{1l} + L_{AAm} &\approx \frac{V_{A0}}{\omega_1 \cdot I_{A0}}; \quad L_{ABm} = \frac{-V_{B0}}{\omega_1 \cdot I_{A0}}; \quad L_{ACm} = \frac{-V_{C0}}{\omega_1 \cdot I_{A0}} \\ L_{Aam} &= \frac{V_{a0}}{\omega_1 \cdot I_{A0}}; \quad L_{Abm} = \frac{-V_{b0}}{\omega_1 \cdot I_{A0}}; \quad L_{Acm} = \frac{-V_{c0}}{\omega_1 \cdot I_{A0}}\end{aligned}\quad (2.118)$$

To find the cyclic inductances, L_{1mc} , L_{2mc} , and L_{12mc} , directly, we supply first all primary phases, with secondary phases open, and then do the same test with all secondary phases active and the primary phases open:

$$L_{1l} + L_{1mc} = \frac{V_{A03}}{\omega_1 \cdot I_{A03}}; \quad L_{12mc} = \frac{V_{a03}}{\omega_1 \cdot I_{A03}}; \quad L_{2l} + L_{2mc} = \frac{V_{a03}}{\omega_1 \cdot I_{a03}} \quad (2.119)$$

For balanced input voltages and balanced load, in steady state and transients, the single-phase transformer equations (with cyclic inductances, however) may be applied safely.

However, under unbalanced input voltages or loads in the secondary, the general equations of 3-phase transformer, with measured inductances are to be used and solved numerically. Magnetic saturation has to be considered, especially if the input voltage and (or) the frequency vary as in power electronics associated applications.

For steady-state unbalanced load, the method of symmetrical components proved very practical and intuitive.

2.14 Unbalanced Load Steady State in 3-Phase Transformers/Lab 2.5

In 3-phase power distribution systems, often, single-phase consumers are served. For electric ac locomotives, induction furnaces, and institutional and residential consumers, if care is not exercised at 3-phase feeding transformer connections scheme, corroborated with magnetic core configuration, the phase voltages of secondary and primary may become unbalanced. First, the neutral potential departs from earth (ground) potential; second, the less-loaded secondary phase voltage may be large and thus endanger the remaining voltage-sensitive consumers on the respective phases. The method of symmetrical components proves very instrumental to treat such situations.

In essence, the voltages and currents in the primary and secondary are decomposed into direct, inverse, and zero sequences (+, −, 0) (Figure 2.42):

$$\begin{pmatrix} I_{a+} \\ I_{a-} \\ I_{a0} \end{pmatrix} = \frac{1}{3} \cdot \begin{pmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix}; \quad \begin{aligned} I_{b+} &= I_{a+} \cdot a^2; & I_{c+} &= I_{a+} \cdot a \\ I_{b-} &= I_{a-} \cdot a; & I_{c-} &= I_{a-} \cdot a^2 \\ I_{b0} &= I_{c0} = I_{a0}; & a &= e^{j \frac{2\pi}{3}} \end{aligned} \quad (2.120)$$

The inverse transformation is

$$\begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{pmatrix} \cdot \begin{pmatrix} I_{a+} \\ I_{a-} \\ I_{a0} \end{pmatrix} \quad (2.121)$$

If the magnetization current +/− components are neglected:

$$\begin{aligned} I_{A+,B+,C+} &= -\frac{N_2}{N_1} \cdot i_{a+,b+,c+} \\ I_{A-,B-,C-} &= -\frac{N_2}{N_1} \cdot i_{a-,b-,c-} \end{aligned} \quad (2.122)$$

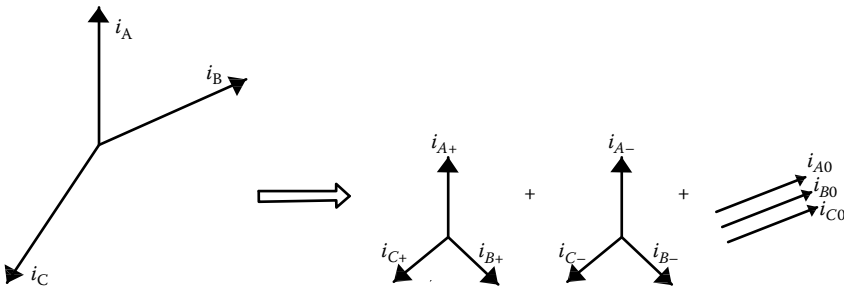


FIGURE 2.42

Symmetrical components superposition.

The direct and inverse components of secondary currents—due to unbalanced load—are reflected in the primary as in Equation 2.122 because the transformer is insensitive to the sequence (order) of phases; it behaves basically through the short-circuit impedance, $Z_{sc} = Z_+ = Z_-$, as known from previous paragraphs. The secondary zero sequence current component, $I_{0a} = I_{0b} = I_{0c}$ —due to single-phase unbalanced loads—typical for Y_0, Δ, Z_0 secondary connection cannot occur in the Y -connected primary unless a tertiary series connected winding is added. If the zero sequence current can occur in the primary, the transformer behavior towards it is the same as for $+/-$ sequences. So, for example, a single-phase load (in the secondary) means no unbalance in the phase voltages in the primary and secondary, and no neutral point potential deviation.

On the contrary, if the zero sequence current cannot occur in the primary, then, from its point of view, the transformer is open in the primary and the entire secondary zero sequence mmf, $N_2 \cdot i_{a0} = N_2 \cdot i_{b0} = N_2 \cdot i_{c0}$, produces zero sequence fluxes in the cores. The level of this flux, Φ_{a0} , depends on the core configuration of the 3-phase transformer (Figure 2.43a through c).

So, for the 3-limb core (Figure 2.43a) the uncompensated zero sequence secondary produces close flux paths through the oil tank walls. The transformer has a reasonably low zero sequence impedance, Z_0 , due to the predominant air zone of flux lines.

On the contrary, for 3×1 -phase transformer groups, the uncompensated secondary zero sequence mmfs close flux paths entirely through iron and the transformer impedance is the no-load impedance, $Z_0 = Z_{no\ load} = R_1 + R_{1m} + j(X_{1l} + X_{1m})$, treated in a previous section.

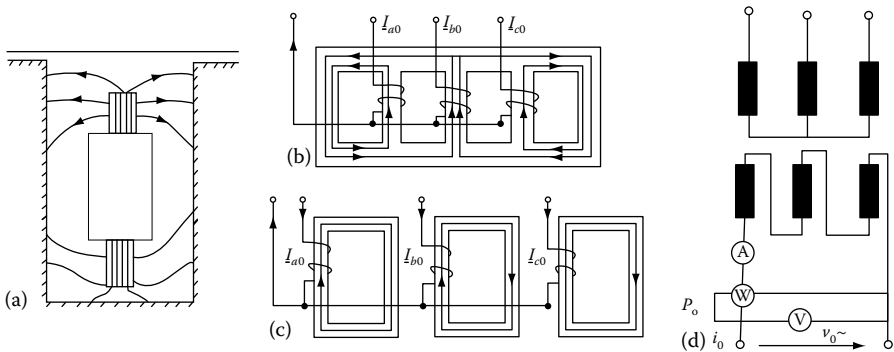


FIGURE 2.43
Zero sequence secondary mmf uncompensated flux lines. (a) 3-limb transformer, (b) 5-limb transformer, (c) 3×1 -phase transformer group, and (d) zero sequence impedance measurement.

The secondary neutral point potential moves by a value, V_{a0} (same for all phases):

$$V_{a0} = -I_{a0} \cdot Z_0 \quad (2.123)$$

The larger the zero sequence impedance for a given load unbalance degree (I_{a0}), the larger unbalancing phase voltage (by $V_{a0} = \underline{V}_{b0} = \underline{V}_{c0}$).

So, if the load in the secondary has a single-phase notable component (y0—connection scheme) and the 3×1 -phase transformer group is used, a tertiary series winding is required if a Y connection is used in the primary. Not so is the case for Δ connection in the primary which allows for zero sequence currents, and thus $Z_0 \rightarrow Z_{sc}$. The 3-limb transformer, instead, may operate with Yy0 connection scheme and limited zero sequence secondary current because $Z_{sc} < Z_0 \ll Z_{no \text{ load}}$. For symmetrical input voltages, and neglecting the voltage on the short-circuit impedance, the voltages will equal the emfs. Consequently, for the Yy0 connection scheme (and no tertiary series winding), $V_{A-} = 0$ and thus the A, a phases voltages will be

$$\begin{aligned} \underline{V}_A &= \underline{V}_{A+} + \underline{V}_{A0} = -(\underline{V}_{ea+} - \underline{Z}_0 \cdot I_{a0}) \cdot \frac{N_1}{N_2} \\ \underline{V}_a &= \underline{V}_{a+} + \underline{V}_{a0} = \underline{V}_{ea+} - \underline{Z}_0 \cdot I_{a0} \end{aligned} \quad (2.124)$$

We should mention that the zero sequence impedance, Z_0 (Equation 2.123), should be measured (calculated) in the secondary (Figure 2.47d) with secondary phases supplied in series from a single-phase ac source, and the primary open. Now, the primary currents “reflect” only the + and – sequence secondary current (Yy0 without tertiary series winding):

$$\begin{aligned} I_A &= I_{A+} + I_{A-} \approx -\frac{N_2}{N_1} \cdot (I_{a+} + I_{a-}) \\ I_B &= a^2 \cdot I_{A+} + a \cdot I_{A-} \approx -\frac{N_2}{N_1} \cdot (a^2 \cdot I_{a+} + a \cdot I_{a-}) \\ I_C &= a \cdot I_{A+} + a^2 \cdot I_{A-} \approx -\frac{N_2}{N_1} \cdot (a \cdot I_{a+} + a^2 \cdot I_{a-}) \end{aligned} \quad (2.125)$$

With Z_0 secondary connection, even for single-phase load, when large zero sequence currents occur, their mmfs produced by the half-phases cancel effects on each core limb and thus the zero sequence flux, $\phi_{a0} = \phi_{b0} = \phi_{c0} = 0$ ($\underline{Z}_0 = 0$).

The Yz0 connection scheme may be single-phase fully loaded without neutral point potential displacement from ground.

Example 2.7 Yy0 Connection with Pure Single-Phase Load

A 3-phase transformer with Yy0 connection scheme and line voltages, $V_{1nl}/V_{2nl} = 6000/380$ V, is purely resistive, loaded only on phase a (in the secondary) at $I_a = 120$ A ($I_b = I_c = 0$). The homopolar impedance is approximated by a reactance, X_0 . Calculate the current distribution between the two

phases and the neutral potential in the secondary, (V_{a0}) and primary (V_A) and the phase voltages in the secondary, V_b, V_c , for $X_0 = 1 \Omega$.

Solution:

Making use of Equation 2.123, with $\underline{Z}_0 = jX_0$, that $V_{ea+} = V_{2ln}/\sqrt{3} = 380/\sqrt{3} = 220 \text{ V}$

The phase a current, I_a , is in phase with V_a .

From Equation 2.120 with

$$\underline{I}_b = \underline{I}_c = 0; \underline{I}_{a+} = \underline{I}_{a-} = \underline{I}_{a0} = \frac{\underline{I}_a}{3}; \underline{I}_{a+} = \underline{I}_{a-} = \underline{I}_{a0} = \frac{120}{3} = 40 \text{ A}$$

Now $N_2/N_1 = V_{2phn}/V_{1phn} = \frac{380}{6000} = 0.0633$, and, thus, from Equation 2.125,

$$\underline{I}_A = \underline{I}_{A+} + \underline{I}_{A-} = 2 \cdot \underline{I}_{A+} = -2 \frac{N_2}{N_1} \cdot \underline{I}_{a+} = -2 \cdot 0.0633 \cdot 40 = -5.064 \text{ A}$$

$$\underline{I}_B = a^2 \cdot \underline{I}_{A+} + a \cdot \underline{I}_{A-} = (a + a^2) \cdot \underline{I}_{A+} - \underline{I}_{A+} = \frac{-\underline{I}_A}{2} = +2.532 \text{ A}$$

$$\underline{I}_C = a \cdot \underline{I}_{A+} + a^2 \cdot \underline{I}_{A-} = (a + a^2) \underline{I}_{A+} = +2.532 \text{ A}$$

So, the current of phase A in the primary is divided into two equal currents in phases B and C, opposite in phase, while in the secondary there is current only in phase a. From the rectangular triangle in Figure 2.44, we get

$$V_a = V_{\text{eat}}^2 - X_0^2 \cdot I_{a0}^2 = 220^2 - (1 \cdot 40)^2 = 216 \text{ V}$$

Solving the triangles in Figure 2.44, we get $V_b = 258 \text{ V}$, $V_c = 191 \text{ V}$.

So the secondary phase voltages are unbalanced but not very severely, a sign that the transformer has a 3-limb core.

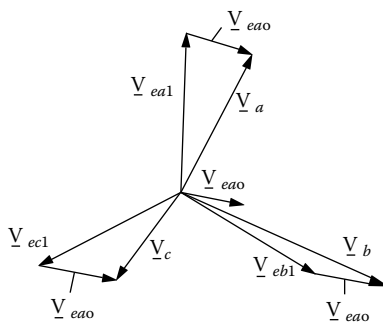


FIGURE 2.44

Secondary voltages phasor diagram for a single-phase resistive load and Yy0 connection scheme.

The secondary null point displacement from ground, $V_{a0} = i_{a0} \cdot X_0 = 1 \cdot 40 = 40$ V. For the neutral point of primary, $V_{A0} = N_1/N_2 \cdot V_{a0} = \frac{6000}{380} \cdot 40 = 631.58$ V.

The other way around, any measured neutral point potential is an indication of unbalanced load or of nonsymmetric input voltages. But this latter case is left out due to lack of space.

2.15 Paralleling 3-Phase Transformers

The increase in input electric power of a company is handled through an additional transformer, to be connected in parallel with existing one(s).

The homologous terminals are to be connected together, first in the primary and then, after checking that safe paralleling is feasible, the homologous terminals of the secondary are connected together.

In essence, optimal parallel operation takes place when

- There is no circulation current between transformers at no load
- Each transformer is loaded in the same proportion to the rated condition (same load factor, $K_{S1} = K_{S2}$)
- The currents in all transformers in parallel are in phase

It is intuitive to consider the following paralleling conditions:

- The rated line voltages of all transformers are the same both in the primary and secondary
- The transformer ratio, N_1/N_2 , is the same
- The connection scheme has the same order, n , so that the homologous secondary line voltages are in phase ($n_1 = n_2$)

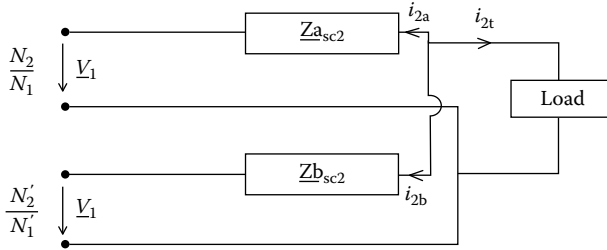
The above conditions guarantee zero circulating current at no load.

To see that the loading factor, K_s , is the same for all transformers, we have to take up the simplified equivalent circuit that represents the transformer as a short-circuit impedance, $\underline{Z}_{1sc}$, but as seen from the secondary (Figure 2.45),

$$\underline{Z}_{sc2} = \frac{N_2^2}{N_1^2} \cdot \underline{Z}_{1sc}.$$

We may now calculate simply the current in the secondary of the first transformer, I_{2a} , as a function of load current, I_{2t} :

$$I_{2a} = \frac{-V_1 \cdot \left(\frac{N_2}{N_1} - \frac{N'_2}{N'_1} \right)}{\underline{Z}_{asc2} + \underline{Z}_{bsc2}} + \frac{I_{2t} \cdot \underline{Z}_{bsc2}}{\underline{Z}_{asc2} + \underline{Z}_{bsc2}} \quad (2.126)$$


FIGURE 2.45

Transformers in parallel, seen from the secondary.

Also, for the second transformer,

$$I_{2b} = \frac{+V_1 \cdot \left(\frac{N_2}{N_1} - \frac{N'_2}{N'_1} \right)}{Z_{asc2} + Z_{bsc2}} + \frac{I_{2t} \cdot Z_{asc2}}{Z_{asc2} + Z_{bsc2}} \quad (2.127)$$

While the second components in Equations 2.126 and 2.127 represent the contribution to the load current, the first terms (equal in amplitude and opposite in phase) represent the circulation current apparent from no load (when $I_{2t} = 0$). The circulating current is due to different transformer ratios. The standard transformer ratios may be different with at most 1% error to avoid notable circulating currents (the denominators in Equation 2.126 contain the short-circuit impedances, which are small).

Now, to provide the same loading factor,

$$\frac{I_{2a}}{I_{2an}} = \frac{I_{2b}}{I_{2bn}} \quad (2.128)$$

By making use of Equations 2.126 and 2.127, with $N_1/N_2 = N'_1/N'_2$, Equation 2.128 becomes

$$Z_{bsc2} \cdot I_{2bn} = Z_{asc2} \cdot I_{2an} \quad (2.129)$$

But this means, in fact, that

The short-circuit power factor is the same for both transformers:

$$\cos \varphi_{sca} = \cos \varphi_{scb}$$

The secondary (and primary) rated short-circuit voltages are the same.

$$V_{nsca2} = V_{nsb2}, (V_{1nsca} = V_{1nsb})$$

If paralleling is required on short notice and the rated short-circuited voltages and their power factors have errors above +10%, then, if feasible, additional series impedances are added to the transformer with lower rated short-circuit voltage to come close to fulfilling (2.129).

Example 2.8 Heating Test through Identical Transformers in Parallel on No Load/Lab 2.6

Let us consider two identical power distribution (say residential) transformers with provision for 5% of rated voltage variation. Calculate in relative values (per unit or pu) of the circulating current between the two transformers in parallel, without load, one connected on the +5% tap and the other on the -5% tap (Figure 2.46), if the rated short-circuit voltage is 5%.

Solution:

From the connection in Figure 2.46 it is clear that the transformer rated turn ratio, K_n , is modified to $1.05 \cdot K_n$ for the first transformer and $0.95 \cdot K_n$ for the second transformer ($K_n = N_{1n}/N_{2n}$).

Now applying Equations 2.126 and 2.127, we are left only with the circulating current, I_{2a} ($I_{2t} = 0$):

$$I_{2a} = -I_{2b} = \frac{-\left(\frac{1}{1.05 \cdot K_n} - \frac{1}{0.95 \cdot K_n}\right) \cdot V_1}{\underline{Z}_{asc2} + \underline{Z}_{bsc2}}$$

But, seen from the secondary, and with number of turns slightly changed:

$$\underline{Z}_{asc2} = Z_{1sc} \cdot \frac{1}{(1.05 \cdot K_n)^2}$$

$$\underline{Z}_{bsc2} = Z_{1sc} \cdot \frac{1}{(0.95 \cdot K_n)^2}$$

So, with $I_{2n}/K_n \approx I_{1n}$ and $Z_{1sc} \cdot I_{1n} = V_{1nsc}$,

$$I_{2a} \approx \frac{\frac{0.1}{K_n} \cdot I_{2n}}{\frac{Z_{1sc}}{K_n^2} \cdot I_{2n} \cdot \left(\frac{1}{1.05^2} + \frac{1}{0.95^2}\right)} \approx \frac{0.1 \cdot I_{2n}}{2 \cdot \frac{V_{1nsc}}{V_1}} = \frac{0.1}{2 \cdot 0.05} \cdot I_{2n} = I_{2n}$$

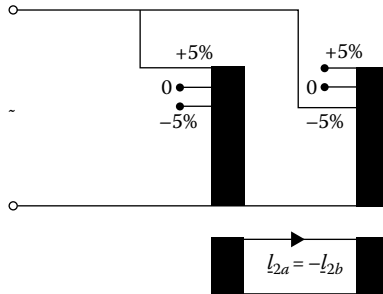


FIGURE 2.46

Tapped identical transformers in parallel for heating test.

So, for the case in point, the provoked circulating current is the rated current and thus rated heating of both transformers is obtained. Only the losses of two identical transformers are absorbed from the power grid.

The magnetic saturation conditions are not identical and the rated losses may be considered the average losses in our case. If v_{1nsc} (in percent) $\neq 5\%$, then the provoked circulating current is different from the rated value, but by a known ratio.

2.16 Transients in Transformers

Input voltage or load variations are accompanied by intervals of time when the amplitudes of the voltages and currents vary. The time intervals when the voltages or currents are not only sinusoidal may be assimilated with transients.

The inrush current of a transformer which is on no load and is directly connected to the power grid, a sudden short circuit at the secondary terminals, and atmospheric or power electronics steep front voltage pulses, all produce transients.

For slow transients (up to a few kilohertz), the model developed so far of the transformer may be extrapolated but above that (from tens of kilohertz to microsecond voltage pulses of modern power electronics), the capacitors between the turns and the oil tank (earth) play a key role. We will use the term electromagnetic transients for slow transients (wherein only resistances and inductances describe the transformer equations and equivalent circuits) and electrostatic transients for fast transients (wherein transformer series and parallel parasitic capacitors play a key role).

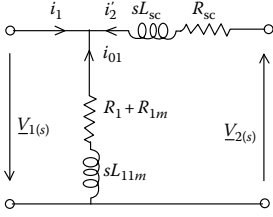
2.16.1 Electromagnetic (R,L) Transients

We return to Equations 2.75 and 2.79—valid for electromagnetic transients—to obtain, for single-phase transformer:

$$\begin{aligned} i_1 \cdot R_1 - V_1 &= -(L_{1l} + L_{11m}) \cdot \frac{di_1}{dt} - L_{11m} \cdot \frac{di'_2}{dt} \\ i'_2 \cdot R'_2 + V'_2 &= -L_{11m} \cdot \frac{di_1}{dt} - (L'_{2l} + L_{11m}) \cdot \frac{di'_2}{dt} \end{aligned} \quad (2.130)$$

Replacing d/dt by s (Laplace operator), we obtain in matrix format:

$$\begin{vmatrix} R_1 + s \cdot \underbrace{(L_{1l} + L_{11m})}_{L_1} & s \cdot L_{11m} \\ s \cdot L_{11m} & R'_2 + s \cdot \underbrace{(L'_{2l} + L_{11m})}_{L'_2} \end{vmatrix} \cdot \begin{vmatrix} i_1 \\ i'_2 \end{vmatrix} = \begin{vmatrix} V_1 \\ -V'_2 \end{vmatrix} \quad (2.131)$$

**FIGURE 2.47**

Transformer structural diagram for electro-magnetic transients.

This leads to two eigenvalues of s from the characteristic equation:

$$s^2 \cdot (L_1 \cdot L'_2 - L_{11n}^2) + s \cdot (L_1 \cdot R'_2 + L'_2 \cdot R_1) + R_1 \cdot R'_2 = 0 \quad (2.132)$$

With $R_1 = R'_2 = R_{sc}/2$, $L_{1l} = L'_{2l}$ ($L_1 = L'_2$) and $L_{1l} \cdot L_{2l}$ neglected, we get, from Equation 2.132:

$$s^2 \cdot L_{sc} \cdot L_{11m} + s \cdot L_1 \cdot R_{sc} + \frac{R_{sc}^2}{4} = 0 \quad (2.133)$$

$$s_{1,2} = \frac{(-L_1 \pm \sqrt{L_1^2 - L_{sc} \cdot L_{11m}}) \cdot R_{sc}}{2 \cdot L_{sc} \cdot L_{11m}} \approx \begin{cases} -\frac{R_{sc}}{L_{sc}} \\ -\frac{R_1}{L_{11m}} \end{cases} \quad (2.134)$$

So, the electromagnetic transients of transformers are always stable and are characterized by a fast time constant, $T_{sc} = L_{sc}/R_{sc}$, of tens of milliseconds corresponding to the load variation process and a large time constant, $T_m = L_{11m}/R_1$, corresponding to magnetization (voltage variation) as pictured in Figure 2.47.

We are now investigating two particular electromagnetic transients with important practical consequences.

2.16.2 Inrush Current Transients/Lab 2.6

A single-phase transformer with open secondary is connected to the power grid:

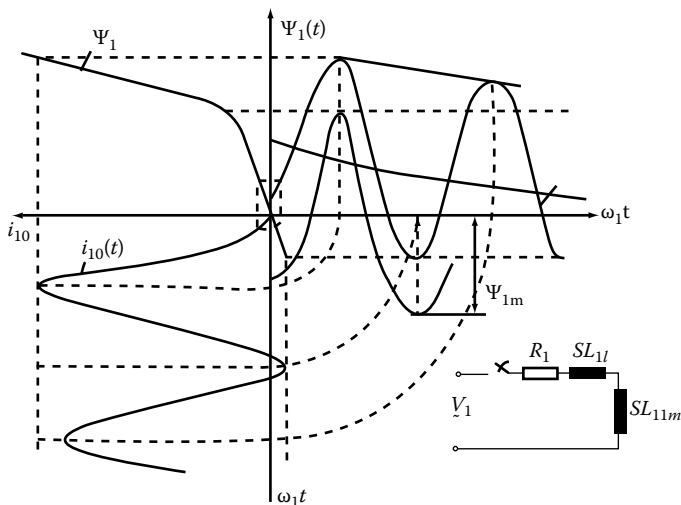
$$V_{1t} = V_1 \sqrt{2} \cos(\omega_1 t + \gamma_0) = R_1 \cdot \frac{\Psi_{10}}{L_1(i_{10})} + \frac{d\Psi_{10}}{dt}; \quad \Psi_{10} = L_1(i_{10}) \cdot i_{10} \quad (2.135)$$

At time zero, $\Psi_{10(t=0)} = \Psi_{1rem}$, where Ψ_{1rem} is the remnant flux in the transformer core, corresponding to the material hysteresis loop. As seen from Equation 2.135, only the parallel branch in Figure 2.47 is active ($i'_2 = 0$). With $\Psi_{10}(i_{10})$, a nonlinear function, due to magnetic saturation, Equation 2.135 may be solved either numerically or graphically to get realistic results.

By considering $L_1(i_{10})$ constant, only in the first right-side term of Equation 2.135, we may solve Equation 2.134 analytically to get

$$\Psi_{10}(t) = \Psi_{1m} \cdot [\sin(\omega_1 t + \gamma_0 - \varphi_0) - \sin(\gamma_0 - \varphi_0) \cdot e^{\frac{-t}{T_0}}] + \Psi_{1rem}$$

$$\varphi_0 = \tan^{-1} \left(\omega_1 \cdot \frac{L_{10}}{R_1} \right) \quad (2.136)$$

**FIGURE 2.48**

Inrush current waveform—a graphical solution.

With $\Psi_m = V_1 \sqrt{2} / \omega_1$ and $T_0 = L_{10} / R_1 \approx T_m$, as the rather slow time constant in the previous paragraph. Now, adding the nonlinear $\Psi_{10}(i_{10})$ function and neglecting hysteresis, we may graphically find the inrush current waveform (Figure 2.48).

As Equation 2.136 points out, there is no transient (exponential) current component if $\gamma_0 = \phi_0$, but for $\gamma_0 - \phi_0 = \pi/2$ the maximum transient current is expected as the peak flux amplitude almost doubles ($\approx 2\Psi_{1m}$) with respect to rated value (already placed in the magnetic saturation zone).

So the peak current may be up to 5–6 times the rated value, though the steady-state no-load current is less than $0.02 \cdot I_{1 \text{ rated}}$. To avoid overcurrent protection tripping, a short duty resistor is connected in series, to limit the current below the tripping value. After the transients (a few seconds) are gone, the resistance is short-circuited by a switch.

2.16.3 Sudden Short Circuit from No Load ($V'_2 = 0$)/Lab 2.7

In this case, the transformer is represented only by R_{sc} and sL_{sc} (series impedance) in Figure 2.47.

The current initial value corresponds to no load, i_{10} , and may be neglected this time ($i_{10} \approx 0$). So the transformer equation becomes

$$V_1 \sqrt{2} \cdot \cos(\omega_1 t + \gamma_0) = R_{sc} \cdot i_1 + L_{sc} \frac{di}{dt}; \quad (i_1)_{t=0} = 0 \quad (2.137)$$

The solution is straightforward as magnetic saturation plays no role:

$$i_{1sc}(t) = \frac{V_1\sqrt{2}}{Z_{sc}} \cdot \left[\cos(\omega_1 t + \gamma_0 - \varphi_{sc}) - \cos(\gamma_0 - \varphi_{sc}) \cdot e^{\frac{-t}{T_{sc}}} \right] \quad (2.138)$$

where φ_{sc} is the short-circuit power factor angle

$$\cos \varphi_{sc} = \frac{R_{sc}}{Z_{sc}}; \quad Z_{sc} = \sqrt{R_{sc}^2 + \omega_1^2 \cdot L_{sc}^2}; \quad T_{sc} = \frac{L_{sc}}{R_{sc}} \quad (2.139)$$

Again, there is no transient for $\gamma_0 - \varphi_{sc} = \pi/2$.

This time, the transient phenomenon is fast and thus short lived (1–7 periods). The peak value of the i_{1sc} may be found for $\partial i_{isc}/\partial t = 0$ and is expressed as

$$i_{sk} = \frac{V_{1n}\sqrt{2}}{Z_{sc}} \cdot K_{sc}; \quad \frac{i_{sk}}{I_{1n}} = \frac{V_{1n}\sqrt{2} \cdot K_{sc}}{Z_{sc} \cdot I_{1n}} = \frac{K_{sc} \cdot \sqrt{2}}{\frac{V_{1nsc}}{V_{1n}}} \quad (2.140)$$

The short-circuit coefficient, $K_{sc} = 1.2$ – 1.8 , is larger for larger transformers, in general. So, with $V_{1nsc}/V_{1n} = 0.04$ – 0.12 and $K_{sc} = 1.5$, the $i_{sk}/I_{1n} = \frac{2}{0.04-0.12} = 50$ – 17 . The short-lived peak short-circuit current may damage the windings through mechanical deformation by huge electrodynamic forces.

Because the steady-state short circuit current is $\frac{I_{1scn}}{I_{1n}} = \frac{V_{1n}}{Z_{sc}I_{1n}} = \frac{V_{1n}}{V_{1nsc}} = 25$ – 8.5 , even the steady-state short-circuit at rated voltage may produce rapid winding over heating, and if by any reason the temperature protection does not trip the transformer in due time, the transformer is designed to withstand the short circuit until the winding temperature reaches 250°C . The corresponding time varies from 5 to 8 s in small transformers and from 20 to 30 s and more in large transformers.

2.16.4 Forces at Peak Short-Circuit Current

Due to the very large peak short-circuit current, i_{sk} , the neighboring primary and secondary phases exert on each other electrodynamic forces (as between parallel conductors).

The leakage field of primary in the zone of secondary (calculated with zero secondary current) interacts with the secondary current to produce these forces whose general iBl formula is

$$d\vec{F} = i \left(d\vec{l} \times \vec{B} \right) \quad (2.141)$$

The leakage flux density has two components: one which is axial (vertical in Figure 2.23), B_a , and one which is radial, B_r , which shows up only close

to the yokes. So the force increment has two components: one radial $d\bar{F}_r$ and one axial $d\bar{F}_a$:

$$d\bar{F}_r = i \left(d\bar{l} \times \bar{B}_a \right); \quad d\bar{F}_a = i \left(d\bar{l} \times \bar{B}_r \right) \quad (2.142)$$

As B_r changes sign over the axial coordinate, F_a acts with two opposite forces, each acting on half the windings.

The radial forces squeeze the low voltage winding to the core and elongate the high voltage winding.

Approximate design expressions of radial and axial forces may be obtained from leakage magnetic energy variation along the desired direction:

$$F_r = -\frac{\partial W_{ml1}}{\partial a_{1r}}; \quad F_a = \frac{1}{2} \frac{\partial W_{ml1}}{\partial L_c}; \quad W_{ml1} = L_{11} \cdot \frac{i_{sk}^2}{2} \quad (2.143)$$

According to leakage inductance expressions derived in previous sections

$$\frac{\partial L_{11}}{\partial a_{1r}} = \frac{-L_{11}}{a_{1r}}; \quad \frac{\partial L_{11}}{\partial L_c} = \frac{-L_{11}}{L_c} \quad (2.144)$$

Consequently,

$$F_r = \frac{L_{11} \cdot i_{sk}^2}{2 \cdot a_{1r}}; \quad F_a = \frac{L_{11} \cdot i_{sk}^2}{4 \cdot L_c} \quad (2.145)$$

So

$$\frac{F_r}{F_a} = \frac{2 \cdot L_c}{a_{1r}} \geq 1, \quad \text{in general} \quad (2.146)$$

By design, the transformer windings have to withstand the peak short-circuit current forces without mechanical deformation.

Example 2.9 Peak Electrodynamic Forces

Peak electrodynamic forces of 1-MVA, 60-Hz, single-phase core transformer have the limb (window) height, $L_c = 1.5$ m, the cylindric (multilayer) windings radial thickness, $a_1 = a_2 = 0.1$ m, and the insulation layer in between, $\delta = 0.01$ m. The rated short-circuit (peak transient overcurrent) factor, $K_{sc} = 1.6$. Calculate the radial and axial forces exerted on the two windings, F_r and F_a , for the peak sudden short-circuit current.

Solution:

The radial variable, $a = \frac{\delta}{2} + \frac{a_1}{3} = 0.01 + \frac{0.1}{3} = 0.0433$ m

According to Equation 2.145, both F_r and F_a may be calculated with peak short-circuit current, i_{sk} , from Equation 2.140 $i_{sk} = \frac{K_{sc}\sqrt{2}}{\sqrt{V_{1nsc}}} \cdot I_{1n}$. To solve the problem with the given data, we need to assume that $L_{1l} = L_{2l} = L_{sc}/2$:

Generally,

$$F_r = \frac{1}{2} \frac{\omega_1 L_{sc}}{2\omega_1} \cdot \left(\frac{K_{sc} \sqrt{2}}{\frac{V_{1nsc}}{V_{1n}}} \right)^2 \cdot I_{1n}^2 \cdot \frac{1}{a_{1r}} \approx \frac{1}{4} \frac{V_{1n} I_{1n} K_{sc}^2}{\omega_1 \frac{V_{1nsc}}{V_{1n}}} \frac{1}{L_{a1}} = \frac{S_n \cdot K_{sc}^2}{4 \cdot \omega_1 \cdot a_{1r} \cdot \frac{V_{1nsc}}{V_{1n}}} \\ = \frac{10^6 \cdot 1.6^2}{4 \cdot \pi \cdot 60 \cdot 0.0433 \cdot 0.04} = 3.62 \cdot 10^6 \text{ N} = 3620 \text{ kN!} \quad (2.147)$$

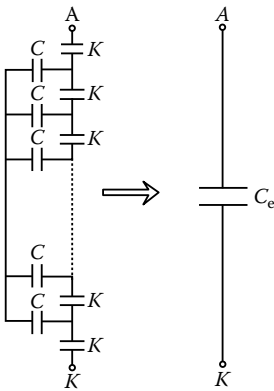
The axial electrodynamic force, F_a , is much smaller:

$$F_a = F_r \cdot \frac{a_{1r}}{2L_c} = F_r \cdot \frac{0.0433}{2 \cdot 1.5} = F_r \cdot 0.01443 = 3620 \text{ kN} \cdot 0.01443 \approx 52.25 \text{ kN}$$

Note: As in power distribution transformers winding, tapping of at least $\pm 5\%$ are provided ($\pm 10\%$ for generator transformers), notably larger axial forces are expected if the primary and secondary windings are left with unequal vertical lengths.

2.16.5 Electrostatic (C,R) Ultrafast Transients

Steep front, overvoltage pulses with microsecond-long fronts may occur due to atmospheric discharges (thunders) and commutation by electromagnetic or power electronic fast power switches. For such ultrafast voltage pulses, the power transformer may not be represented any longer mainly by its electromagnetic model developed so far.



With microsecond front voltage pulses, equivalent frequencies of $10\text{--}10^3$ kHz may be considered, and thus at least in the first few microseconds after the transients' initiation, the resistances and inductances of the transformer may be neglected. It follows that for super high frequencies the transformer model is represented by its inter-turn, interwinding and turn-to-earth capacitors. In a real transformer, there are very many parallel (C) and series (R) parasitic capacitors, interconnected to form a very complicated net in a multilayer or alternate winding 3-phase transformer.

Only for a single-layer winding a simple representation such as in Figure 2.49 is feasible.

FIGURE 2.49
Super high-frequency (electrostatic) equivalent circuit for a single-layer winding transformer.

Let us now lump all capacitors into only the equivalent capacitor, C_e , that represents the transformer for microsecond front voltage atmospheric wave from a thunder, propagating along an overhead transmission line toward a transformer (Figure 2.50).

Let us consider V_d , the direct voltage wave, V_r , the reflected one, due to transformer equivalent capacitor, $C_e = (10^{-7} - 10^{-10}) F$, I_d the direct wave current, I_r , the reflected wave current, I_t , the transformer current, and Z , the transmission line impedance (about 500Ω). From Figure 2.50 it follows that:

$$\begin{aligned} V_d &= ZI_d; \quad V_r = ZI_r; \quad i_t = i_d - i_r; \\ V_t &= V_d + V_r = \frac{1}{C_e} \cdot \int i_t dt \end{aligned} \quad (2.148)$$

Eliminating i_d , i_r , V_d , and V_r from Equation 2.148 and solving the resulting first-order differential equation in V_t for zero initial transformer voltage ($(V_t)_{t=0} = 0$), we obtain

$$\begin{aligned} V_t &= 2 \cdot V_d \cdot \left(1 - e^{\frac{-t}{C_e Z}}\right) \\ i_t &= \frac{2V_d}{Z} \cdot e^{\frac{-t}{C_e Z}} \end{aligned} \quad (2.149)$$

The time constant of the process, $T_e = (50-0.05) \mu s$, in general.

So the transformer representative capacitor, C_e , charges at double voltage pulse level, $2V_d$, within microseconds, when the resistances and inductances in the transformer do not participate yet.

Now the problem is how this double voltage pulse is distributed along the winding from terminal to the neutral point (star connection). This process also takes place very quickly and thus we still neglect the resistances and inductances in the transformer but now we need to apply the distributed capacitor model (Figure 2.51).

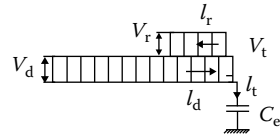


FIGURE 2.50
Microsecond front voltage propagation to a transformer.

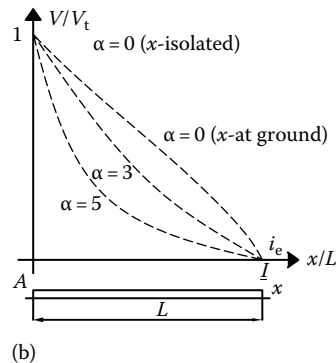
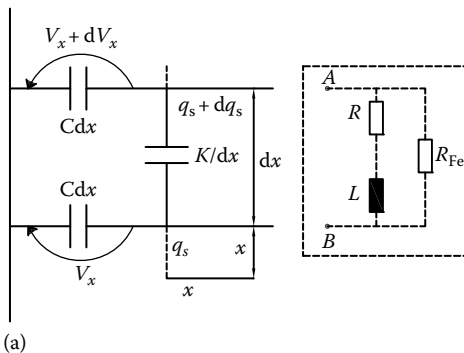


FIGURE 2.51

(a) Distributed capacitor model of the transformer and (b) “initial” voltage distribution along the winding.

According to Figure 2.51, we may write

$$\begin{aligned} V(x) + dV - V(x) &= q(x) \cdot \frac{dx}{K} \\ q(x) + \Delta q - q(x) &= V(x) C dx \end{aligned} \quad (2.150)$$

or

$$\frac{dV(x)}{dx} = \frac{q}{K}; \quad \frac{dq(x)}{dx} = C \cdot V(x) \quad (2.151)$$

Eliminating q (the electric charge local value) from Equation 2.150 yields:

$$\frac{d^2V(x)}{dx^2} - \frac{C}{K}V(x) = 0 \quad (2.152)$$

This is a wave equation with x , the distance from the neutral point to phase terminal (entry):

$$V(x) = A \sinh(\alpha \cdot x) + B \cosh(\alpha \cdot x); \quad \alpha = \sqrt{\frac{C}{K}} \quad (2.153)$$

With the boundary conditions:

$$V = 0 \quad \text{for} \quad x = 0 \text{—for earthed neutral point}$$

$$\frac{dV}{dx} = 0 \quad \text{for} \quad x = 0 \text{—for isolated neutral point}$$

$$V = V_{\max} = 2V_d \quad \text{for} \quad x = l \text{—for the phase entry} \quad (2.154)$$

So

$$\begin{aligned} V(x) &= V_{\max} \cdot \frac{\sinh(\alpha x)}{\sinh(\alpha L)}; & \text{earthed neutral point} \\ V(x) &= V_{\max} \cdot \frac{\cosh(\alpha x)}{\cosh(\alpha L)}; & \text{isolated neutral point} \end{aligned} \quad (2.155)$$

For the single-phase transformer, the neutral point is replaced by the phase winding end, which may be, again, earthed or isolated.

It is now evident (Figure 2.51b) that if $\alpha = \sqrt{\frac{C}{K}} > 5$, the initial voltage distribution for both neutral situations is highly nonlinear along the winding length, and thus the first 10% of winding may experience more than 50% of the initial voltage, which, if large in itself, will stress heavily the first 10% of the winding. The phenomenon is also common for power electronics-supplied transformers and electric machines; when repeated (by pulse width modulation), such voltage pulses may lead not only to voltage doubling but tripling, with the same nonuniform voltage distribution along the winding length. Special measures are needed to deal with the situation.

Now, after this “initial” distribution installs itself—in a few microseconds or so—the resistances and inductances, including R_{iron} (core loss resistance) show up in the equivalent circuit (Figure 2.51a), and thus resonance phenomena may occur which may lead to even 20%–40% more overvoltage. Antiresonant measures have to be taken to avoid this phenomenon.

2.16.6 Protection Measures of Anti-Overvoltage Electrostatic Transients

There are external (to transformer) measures to reduce the impact of $V_{\text{max}} = 2V_d$ over-voltage level and thus implicitly reduce electrostatic stress in the windings of the transformer. Spark gaps are typical of such measures on the current bushings for medium- or low-voltage transformers. In essence, the spark gaps suffer ionization at high-voltage levels, and thus the electric charges flow to the earth through the spark gap, and not through the transformer. Better external protection is obtained through close-to-terminal surge arresters with short ground connectors.

Internal measures, to reduce initial overvoltage stress on the first 10% of transformer winding, have evolved from thinner and taller cables in the first 10% turns (to reduce C and increase K), a metallic protection ring at the phase entry and close to the oil tank, and a cylindrical vertical metallic screen at phase entry and interleaved first 10% turns, to random wound coils in electric machines and ladders of ZnO (zinc oxide) varistor elements around the sensitive (say voltage regulating) parts of the windings.

2.17 Instrument Transformers

To reduce the sensed ac voltages and currents to lower levels, acceptable for available instruments (transducers, or sensors), potential (voltage) transformers and, respectively, current transformers are used.

They also provide conduction isolation from high-voltage lines or medium-voltage power sources for transformers and electric machines. Voltmeters range to 600 V (or 1 kV) while ammeters are built up to 5 A.

If a 34-kV line voltage is to be measured, a 350:1 potential transformer works under no load (open secondary), so,

$$V_1 = V_{20} \frac{N_1}{N_2} \quad (2.156)$$

However, due to voltage drop along R_1 and X_{1l} , V_{20} has a small amplitude error and a small phase error with respect to the measured voltage, $V_1 \cdot N_2/N_1$. They should be minimized by design, but a correction factor

may be given in precision-sensitive applications. The primary is designed for no-load current and the secondary for no load. So the voltage transformer is small in volume.

The current transformer has, in general, a round-shaped low loss magnetic core with a multiturn secondary winding connected over a small resistance (shunt) whose voltage is proportional to the secondary current. Now the primary is the high current carrying cable which may be twisted to go through the inside of the round core 2, 3, N_1 times ($N_1 < 5$ in general). The current transformer operates basically under short-circuit conditions in the secondary but current fed (and limited) in the primary. Still, if the core has high permeability, the magnetization current, i_{10} , is negligible.

$$N_1 \cdot i_1 + N_2 \cdot i_2 = N_1 \cdot i_{10} \approx 0; \quad i_1 = \frac{-N_2 \cdot i_2}{N_1}; \quad N_2 > N_1 \quad (2.157)$$

The current, $i_2 \leq 5$ A, is, in fact, measured. For $I_1=10$ kA and $I_2 = 5$ A, we would need $N_2/N_1 = 1000/5 = 200/1$. Again, the current transformer has to show small winding losses, besides small magnetization current, to secure small phase and amplitude errors between i_1 and i_2 .

The current transformer is designed to a small voltage rating and is small in volume.

Now being heavily underrated in current, the voltage transformer is to be protected against short circuit, while the current transformer, being underrated in voltage, should be protected against open secondary, when, in addition, the large uncompensated mmf in the primary would produce excessive core losses and heavily saturate the core.

2.18 Autotransformers

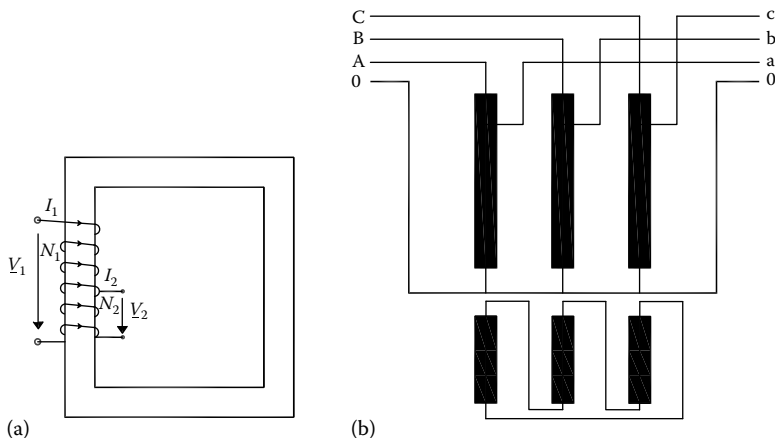
For step up and step down of voltages in ratios of up to 2:1 (or 1:2), as needed in long transmission lines, to compensate for voltage drop due to long lines reactance, a transformer with a single but tapped (Figure 2.52) winding (called autotransformer) is used, to cut transformer costs and losses. The autotransformer is used also as lower-cost variable voltage supply.

Neglecting the magnetization current, we may write

$$(N_1 - N_2 \cdot I_1) - N_2 \cdot I_s = 0; \quad K_T = \frac{N_1}{N_2} \quad (2.158)$$

where I_s is the secondary current and I_1 the primary current. The load current part of the winding, I_2 , is

$$I_2 = I_1 + I_s \quad (2.159)$$

**FIGURE 2.52**

The step-down autotransformer: (a) Single phase and (b) three phase (Y0y0) with tertiary winding.

From Equation 2.158,

$$I_s = I_1 \cdot (K_T - 1); \quad I_2 = I_s \cdot \frac{K_T}{K_T - 1} \quad (2.160)$$

If all losses are neglected, the input and output power are identical:

$$V_1 \cdot I_1 = V_2 \cdot I_2 \quad (2.161)$$

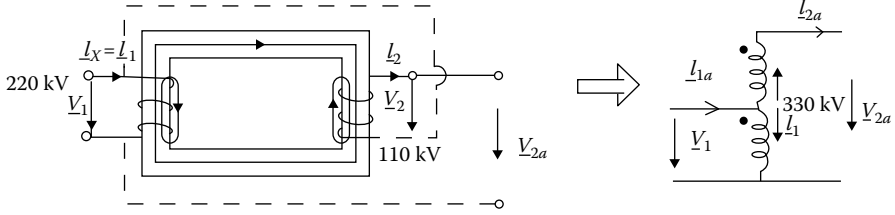
Consequently, the electromagnetic power, S_e , transmitted through electromagnetic induction (through the core) is

$$S_{en} = V_2 \cdot I_s = V_2 \cdot I_2 \cdot \left(\frac{K_T}{K_T - 1} \right)^{-1} = S_n \cdot \left(\frac{K_T}{K_T - 1} \right)^{-1} \quad (2.162)$$

For $K_T = 2$, $S_e = S_n/2$ and $I_s = I_1$, so only half of the power is transmitted through the core and thus the transformer core design rating for design is 50% of that of a corresponding 2-winding transformer. The rest of the power is transmitted directly—by wire—from the primary to the load.

It follows that the autotransformer is notably less costly; as expected, the core losses, and especially the copper losses, are smaller, and so is the short-circuit rated voltage, V_{1scn} . So the voltage regulation is smaller than that in a 2-winding transformer of the same rating. But the short-circuit current will be larger and thus a stronger over current protection system is required.

Though we treated only the step-down autotransformer, the step-up transformer may be treated in a similar way and has similar performance.

**FIGURE 2.53**

From transformer to step-up autotransformer.

Example 2.10 From Transformer to Autotransformer

Figure 2.53 shows how to connect a transformer into a step-up autotransformer. Let us consider the transformer ratings: 220/110 kV, 50 MVA. Find the autotransformer rated output voltage and power.

Solution:

The rated input current, I_x , is

$$I_1 = I_x = \frac{S_n}{V_1} = \frac{50 \cdot 10^6}{220 \cdot 10^3} = 227 \text{ A}$$

$$I_{2a} = I_2 = \frac{S_n}{V_2} = \frac{50 \cdot 10^6}{110 \cdot 10^3} = 454 \text{ A}$$

By scalar addition, the input current, I_{1a} , and the output voltage, V_{2a} , of the autotransformer connections are

$$I_{1a} = I_1 + I_2 = 227 + 454 = 681 \text{ A}$$

$$V_{2a} = V_1 + V_2 = (220 + 110) \cdot 10^3 = 330 \text{ kV}$$

So the total delivered power of the autotransformer, S_{an} , is

$$S_{an} = V_1 \cdot I_{1a} = V_{2a} \cdot I_{2a} = 220 \cdot 10^3 \cdot 681 = 150 \text{ MVA}$$

So, as expected in this case, three times more power is available in the step-up autotransformer connection. However, we should notice that, because the input current is three times larger, the feeding cable has to be designed accordingly. Also, the output voltage is three times larger, so the insulation in the secondary winding has to be enforced. But, as the rated losses are the same and the power is tripled, the efficiency of the autotransformer is notably increased in comparison with the original transformer.

2.19 Transformers and Inductances for Power Electronics

Power electronics manages to change voltage/current waveforms (amplitude and frequency) or, in other words, electric power parameters, through fast static-power semiconductor-controlled switches.

The modern static power-controlled switches perform on/off commutation within microseconds.

In power electronics, electric power fast processing, by semiconductor-controlled power switches (or rectifiers: SCRs), electric energy storage elements such as inductors and capacitors are used. For galvanic separation of ac voltage, large step-up or step-down ratios and high-frequency electric transformers are used for powers in the 1–100 kHz frequency range and power in the kilowatt to hundreds of kilowatt range. In more distributed electric power systems, power electronics is used to eliminate current or voltage harmonics and to compensate reactive power or voltage drop long power transmission lines. Moreover, high voltage dc power line ties are also used to make the standard ac power line more flexible.

In all these applications, transformers are used together with IGCT power electronics in the tens and hundreds of MVA for switching frequency up to 1 kHz.

Figure 2.54 presents the storage inductance (or inductor), L , used in a dc–dc converter with dc voltage boost.

The inductor, L , is connected to the input dc power source for the interval, T_{on} , through the IGBT (insulated gate bipolar transistor) power switch and then, when the inductor charging circuit is opened (during time T_{off}), the output voltage, V_{out} , is

$$V_{\text{out}} = V_{\text{in}} - L \frac{di}{dt} \quad (2.163)$$

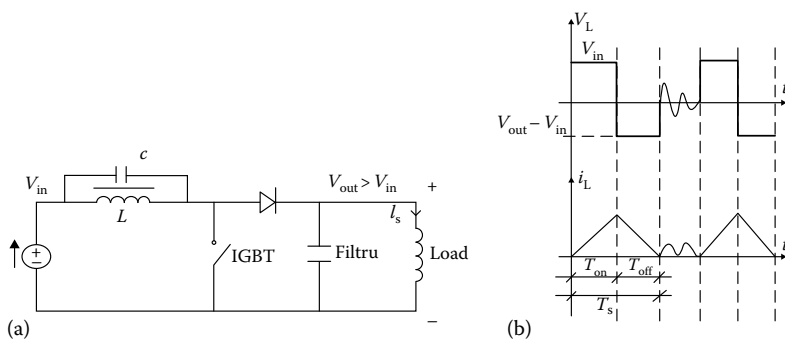


FIGURE 2.54

Boost dc–dc converter: (a) The equivalent circuit and (b) storage inductance voltage and current.

If the switching period, $T_s > T_{on} + T_{off}$, a zero inductor current interval occurs.

The pulsations in the inductor current during the ideal zero current time interval (Figure 2.53b) are due to the reactor parasitic capacitor, C , that acts at high commutation frequencies of the IGBT: $(1/T_s) = (10\text{--}20)$ kHz.

Such a reactor, Figure 2.55, is typical for a boost dc–dc converter for 60 kW power (220–500 Vdc), switched at 10 kHz, in a full hybrid electric vehicle (HEV).

As seen in Figure 2.55, the core has to have multiple airgaps to allow for large currents without heavy magnetic saturation, and should be made of very thin (less than 0.1 mm thick) silicon steel laminations or from a soft composite material, to reduce the core losses at the rather large switching frequency of 10 kHz.

A low volume (and low weight) electric welding apparatus contains a rectifier and a 20–100 kHz step-down transformer and a fast-diode rectifier.

A contact-less battery charging system for an HEV (Figure 2.56) or a mobile phone battery (in situ) charging system imply also the use transformers with small or large airgap at 20–50 kHz.

At the other end of the scale, to compensate the voltage drop along long ac transmission lines, a two stage ac–ac power electronics converter and a series-connected transformer is used. In this case, the transformer fundamental frequency is 50(60) Hz but, due to the converter fast switching, there are harmonics, etc.

All the above examples of transformers with power electronics suggest that special materials and configurations and models are needed which are tied to the application power and frequency range [1].

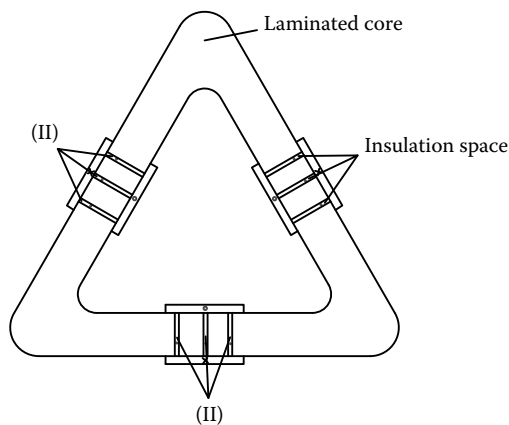
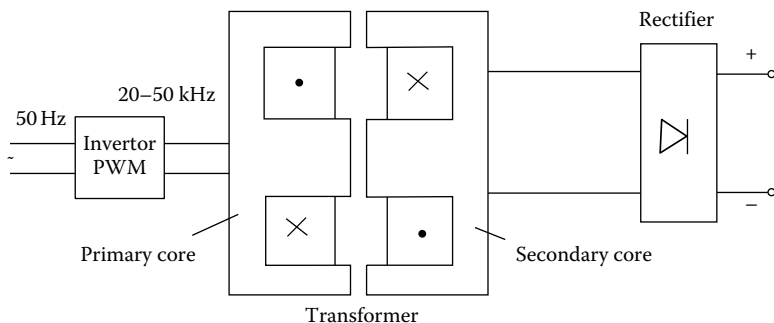


FIGURE 2.55

Storage reactor with multiple gaps in a dc–dc boost converter.

**FIGURE 2.56**

Battery charger made of high-frequency inverter + transformer + high-frequency diode rectifier.

2.20 Preliminary Transformer Design (Sizing) by Example

By design, we mean here dimensioning the transformer for given specifications.

So, by design we mean synthesis, while calculating performance for a given geometry is called analysis.

The design (synthesis) uses the analysis iteratively. Analysis implies a mathematical model.

The mathematical models may be of field distribution (finite element) type or of circuit type. As FE models are computation-time prohibitive, they are, in most cases, used for performance validation, after design optimization based on analytical models. Here, we will approach the preliminary (general) transformer design by way of a case study and making full use of transformer parameters, rather realistic expressions of the circuit model, already derived in this chapter.

2.20.1 Specifications

- Transformer rated kVA: $S_n = 100 \text{ kVA}$
- Number of phases: 3
- Connections scheme: Yz0
- Magnetization current, $i_{01} < 0.015I_n$
- Frequency, $f_1 = 50 \text{ Hz}$
- Line voltages, V_{1l}/V_{2l} : 6000/380 V
- Rated short-circuit voltage, $V_{1scn} < 0.045 \cdot V_1$

- Rated current density, $j_{co} = (3\text{--}3.5) \text{ A/mm}^2$
- Cylindrical (layer) windings 3-limb iron core

2.20.2 Deliverables

- Core geometry
- Winding design
- Resistance, reactances, and the equivalent circuit
- Losses and efficiency
- Rated no-load current and rated short-circuit voltage

2.20.3 Magnetic Circuit Sizing

The 3-limb magnetic core with its main geometrical variables is shown in Figure 2.57.

The magnetization curve ($B_m(H_m)$) of the laminated silicon steel sheet is given in Table 2.3.

The core losses at $B_m = 1.5 \text{ T}$, at 50 Hz, $p_{\text{iron}} = 1.12 \text{ W/kg}$ and depend on $(B_m/1.5)^2$.

We start the magnetic circuit design by setting the limb and yoke flux density values, $B_c = B_y = 1.4 \text{ T}$. We may find, from the magnetization curve, through interpolation, the magnetic field in the column, $H_c = H_y$.

Based on the magnetic circuit law:

$$H_c \cdot \left(L_c + L_w + \pi \frac{D_c}{4} \right) = N_1 \cdot I_{01} \quad (2.164)$$

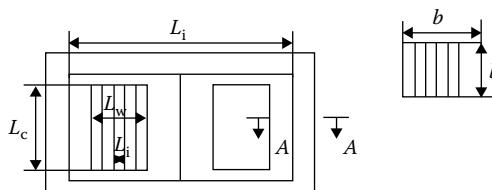


FIGURE 2.57

The 3-limb magnetic core.

TABLE 2.3

Silicon Steel Sheet Magnetization Curve

B (T)	0.1	0.15	0.2	0.3	0.4	0.5	0.6	0.7	0.9	1.4	1.5	1.6
H (A/m)	35	45	49	65	76	90	106	124	177	760	1340	2460

With L_w being the window length and D_c being the core column external diameter.

The phase emf, V_{e1} , is almost equal to the phase voltage, $V_{1l}/\sqrt{3}$ (star connection in the primary):

$$V_{e1} = \pi\sqrt{2}f_1 B_c \times A_c \times N_1 = K_e \cdot \frac{V_{1n}}{\sqrt{3}}; \quad K_e = 0.985-0.95 \quad (2.165)$$

2.20.4 Windings Sizing

The window room of the core has to be enough to place the primary and secondary windings of the two phases plus an insulation space, L_i , $10 \text{ mm} < L_i < 60 \text{ mm}$ (for $V_{1nl} = 6 \text{ kV}$).

We may assume $N_1 I_1 = N_2 I_2$, and, thus,

$$(L_c - b) \cdot (L_w - L_i) = \frac{2 \cdot 2N_1 I_{1n}}{j_{Co} \cdot K_{fill}} \quad (2.166)$$

$K_{fill} \approx 0.5$ is the winding filling factor.

For star connection of phases, we adopt an initial value for the limb core area, $A_c = 10^{-2} \text{ m}^2$.

From Equation 2.165 the number of turns, N_1 , of primary is

$$N_1 = \frac{6000}{\sqrt{3} \cdot \pi\sqrt{2} \cdot 50 \cdot 1.4 \cdot 10^{-2}} \approx 1085 \text{ turns}$$

On the other hand, standard industrial experience with tens of kVA transformer designs yields the primary emf per turn [7]:

$$E_{turn} = K_E \cdot \sqrt{S_n \text{ (in kVA)}} \quad (2.167)$$

where

$K_E = 0.6-0.7$ for 3-phase industrial transformers

$K_E = 0.45$ for 3-phase distribution (residential) transformers

$K_E = 0.75-0.85$ for 1-phase transformers

For our case

$$E_{turn} = 0.45\sqrt{100} = 4.5 \text{ V/turn}$$

According to this rationale N_1 would be

$$N_1 = \left(\frac{V_{1n}}{\sqrt{3}} \right) \cdot \frac{K_E}{E_{turn}} = \frac{6000 \cdot 0.97}{\sqrt{3} \cdot 4.5} = 778 \text{ turns} \quad (2.168)$$

We stick with $N_1 = 1085$ turns (from Equation 2.165).

The rated current, I_{1n} , is

$$I_{1n} = \frac{S_n}{\sqrt{2} \cdot V_{1nl}} = \frac{100 \cdot 10^3}{\sqrt{3} \cdot 6000} = 9.6334 \text{ A} \quad (2.169)$$

The number of turns, N_2 , in the secondary is

$$N_2 = N_1 \cdot \frac{V_{2n}}{V_{1n}} \cdot \frac{2}{\sqrt{3}} = 1085 \cdot \left(\frac{\frac{380}{\sqrt{3}}}{\frac{6000}{\sqrt{3}}} \right) \cdot \frac{2}{\sqrt{3}} = 80 \text{ turns} \quad (2.170)$$

The $2/\sqrt{3}$ factor is due to the z_0 connection in the secondary.

The winding radial width in the window, a_1 and a_2 , are

$$a_1 = \frac{N_1 \cdot I_{1n}}{j_{Co} \cdot K_{fill} \cdot L_c}; \quad a_2 = \frac{N_2 \cdot I_{2n}}{j_{Co} \cdot K_{fill} \cdot L_c} = \frac{2}{\sqrt{3}} \cdot a_1 \quad (2.171)$$

The average diameters of the turns are

$$D_{av2} = D_{core} + a_2; \quad D_{av1} = D_{core} + 2 \cdot a_2 + \delta + a_1 \quad (2.172)$$

The leakage reactance expressions (derived earlier in this chapter) are

$$\begin{aligned} X_{1l} &= \omega_1 \mu_0 \frac{N_1^2 \cdot \pi}{L_c} \cdot D_{av1} \cdot \left(\frac{a_1}{3} + \frac{\delta}{2} \right) \\ X'_{2l} &= \omega_1 \mu_0 \frac{N_2^2 \cdot \pi}{L_c} \cdot D_{av2} \cdot \left(\frac{a_2}{3} + \frac{\delta}{2} \right) \end{aligned} \quad (2.173)$$

The primary and secondary resistances are straightforward:

$$\begin{aligned} R_1 &= \rho_{Co} \cdot \frac{\pi \cdot D_{av1} \cdot N_1}{\frac{I_{1n}}{j_{Co}}} \\ R'_2 &= \rho_{Co} \cdot \frac{\pi \cdot D_{av2} \cdot N_1}{\frac{I_{1n}}{j_{Co}}} \cdot \frac{2}{\sqrt{3}} \end{aligned} \quad (2.174)$$

The distance between windings, δ , is assigned to: $\delta = 0.012$ m. We may eliminate a_1 and a_2 from Equations 2.171 through 2.174 to remain with only one unknown, the column (limb) height, L_c , in the short-circuit (rated) voltage expression:

$$V_{1scn} = I_{1n} \sqrt{(R_1 + R'_2)^2 + (X_{1l} + X'_{2l})^2} \quad (2.175)$$

For $\delta = 0.012$, it follows that for $L_c = 0.5$ m, $D_{av1} = 0.2028$ m, $D_{av2} = 0.14768$ m, $X'_{2l} = 6.5 \Omega$, $X_{1l} = 7.245 \Omega$, $R'_2 = 4.666 \Omega$, $R_1 = 4.8185 \Omega$.

The short-circuit rated voltage, V_{1scn} , is then

$$V_{1scn} = 160.87 \text{ V} \quad (2.176)$$

Let us now check the short-circuit rated voltage in percents:

$$\frac{V_{1scnl}}{V_{1n}} = 4.638\% \quad (2.177)$$

This value is close to the desired 4.5% and thus the value of L_c of 0.5 m holds.

2.20.5 Losses and Efficiency

The rated copper losses P_{copper} is

$$P_{\text{copper}} = 3 (R_1 + R'_2) I_{1n}^2 = 2640 \text{ W} \quad (2.178)$$

To calculate the core losses, we first need to finish up the magnetic core geometry, with window length, L_w , computation:

$$L_w = 2 (2a_1 + 2a_2 + \delta) + D \quad (2.179)$$

where D the radial distance between neighboring phases ($D = 0.1$ m is ok for 6 kV).

So, from Equation 2.179, L_w is

$$L_w = 0.27 \text{ m} \quad (2.180)$$

The core weight, G_{iron} , for same limb and yoke cross-section area, A_c , is

$$G_{\text{iron}} = A_c [3 (L_c + D_c) + 4L_w] \gamma_{\text{iron}} = 225.49 \text{ kg} \quad (2.181)$$

The specific core loss at 1.4 T is

$$(P_{\text{iron}})_{1.4 \text{ T}} = (P_{\text{iron}})_{1.5 \text{ T}} \times \left(\frac{B_c}{1.5} \right)^{1.7} = 1.0 \text{ W/kg} \quad (2.182)$$

So the iron losses, P_{iron} , is

$$p_{\text{iron}} = G_{\text{iron}} (P_{\text{iron}})_{1.45} = 225.44 \cdot 1.0 = 225.44 \text{ W} \quad (2.183)$$

The rated efficiency, η_n , is thus

$$\eta_n = \frac{S_n}{S_n + p_{\text{iron}} + p_{\text{copper}}} = \frac{100,000}{100,000 + 2,640 + 225.44} = 0.972 \quad (2.184)$$

2.20.6 No-Load Current

The rated no-load current (I_{10n}) equal to the magnetization current, I_{01} :

$$\begin{aligned} I_{10} \approx I_{01} &= (H_c)_{1.4 \text{ T}} \cdot \frac{L_c + L_w + \frac{\pi}{4} \cdot D_c}{N_1} = 200.0 \cdot \frac{0.5 + 0.27 + \frac{\pi}{4} \cdot 0.1288}{1085} \\ &= 0.16 \text{ A} \frac{I_{d0}}{I_{1n}} = \frac{0.16}{9.6344} = 1.66\% \end{aligned} \quad (2.185)$$

2.20.7 Active Material Weight

The copper weight, G_{copper} , is

$$\begin{aligned} G_{\text{copper}} &= 3 \cdot \left(\pi \cdot D_{\text{av1}} + \pi \cdot D_{\text{av2}} \cdot \frac{4}{3} \right) \cdot \frac{I_n}{\rho_{\text{Co}}} \cdot N_1 \cdot \gamma_{\text{Copper}} \\ &= 3 \cdot \left(\pi \cdot 0.2028 + \pi \cdot 0.14768 \cdot \frac{4}{3} \right) \cdot \frac{9.6344}{3.2 \cdot 10^6} \cdot 1085 \cdot 8900 \\ &= 109.32 \text{ kg} \end{aligned} \quad (2.186)$$

The total active material weight, G_a , is

$$G_a = G_{\text{iron}} + G_{\text{copper}} = 225.44 + 109.32 = 334.72 \text{ kg} \quad (2.187)$$

The kVA/kg in the transformer is

$$\frac{S_n}{G_a} = \frac{100 \text{ kVA}}{334.72 \text{ kg}} \approx 0.29 \frac{\text{kVA}}{\text{kg}} \quad (2.188)$$

2.20.8 Equivalent Circuit

From the equivalent circuit, only the magnetization reactance, X_m , and core loss resistance, R_{1m} , are to be calculated as

$$X_m = \frac{V_{1n}}{\sqrt{3} \cdot I_{10}} - X_{1l} = \frac{6000}{\sqrt{3} \cdot 0.16} - 7.245 = 21,676 - 7.245 \approx 21,670 \Omega \quad (2.189)$$

$$R_{1m} = \frac{p_{\text{iron}}}{3 \cdot I_{10}^2} = \frac{225.44}{3 \cdot 0.16^2} = 2935.4 \Omega \quad (2.190)$$

The equivalent circuit in numbers is pictured in Figure 2.58.

Note: As the results of the preliminary design are reasonable, they could be a good start for the thermal and mechanical design, and for a design optimization code as done in Part 3 of this book.

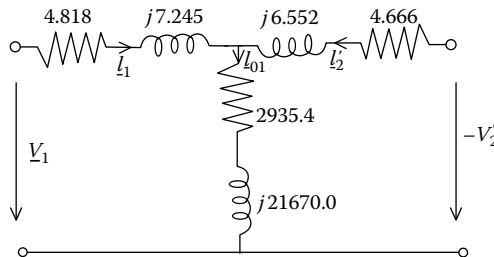


FIGURE 2.58

The equivalent circuit.

2.21 Summary

- Electric transformers are a set of magnetically coupled electric circuits capable to step up or step down the voltage in ac power transmission. They are based on Faraday's law for bodies at standstill. Transformers serve also for galvanic separation.
- The transformer ratio, $K_T = N_1/N_2 = (V_{1n}/V_{2n})_{ph}$, reflects the voltage step up ($K_T < 1$) or step down ($K_T > 1$).
- Transformers may be classified as single-phase or 3 (or multiple)-phase configurations. Alternatively, we distinguish power transformers (for power systems, industry and power distribution), measurement transformers, autotransformers, and transformers for power electronics.
- Magnetic circuits of transformers are made of thin sheets of silicon steel at industrial frequency (50(60) Hz) or from soft ferrites or permalloy, etc., at high frequencies as in association with power electronics.
- Magnetic circuits are characterized by magnetic saturation and (hysteresis and eddy current) losses. The thin sheets provide for low eddy current losses and allow thus for high efficiency.
- Electric circuits are flown by ac currents close to the magnetic core and exhibit skin effects which increase the resistance and decrease the leakage inductance. In transformers, skin effect is reduced through strand transposition (up to Roebel bars or Litz wire).
- To study transformer steady state and transients, leakage and main inductances and resistances are defined and calculated to form an equivalent circuit with the secondary reduced to the primary.
- While the large main (magnetization) reactances lead to very low no-load current (up to 2% of rated current), the leakage reactances and resistances determine the short-circuit current, which is large.
- The power, p_0 , at no load and rated voltage almost equals the core losses under load. The short-circuit winding losses at rated current, I_{1n} (and at short-circuit rated voltage $V_{1scn} = Z_{sc} \cdot I_{1n} \approx (0.04-0.12) \cdot V_{1n}$), equal those at rated load. So, from these two tests not only the transformer parameters for the equivalent circuit but also the losses under specified load factor, $K_s = I_1/I_{1n}$, can be calculated.

- Under load, there is a secondary voltage variation, ΔV_2 , of secondary voltage, V_2 , from its no-load value, V_{20} , which is called voltage regulation and is proportional to the load factor, K_s , short-circuit rated voltage and to $\cos(\varphi_{sc} - \varphi_2)$; φ_{sc} , φ_2 - short-circuit and load power factor angles:

$$\Delta V_2 = \Delta V_1 \cdot \frac{N_2}{N_1} = \frac{N_2}{N_1} \cdot (K_s \cdot V_{1nsc} \cdot \cos(\varphi_{sc} - \varphi_2)) = V_{20} - V_2$$

- ΔV_2 should be small for power transmission and distribution to secure pretty constant output voltage with load but it may be intentionally large if the short-circuit current has to be limited.
- Transformers are often connected in parallel; to share the load fairly they have to have the same primary and secondary rated voltages and connection scheme and order, n , and the same V_{1nsc} and $\cos \varphi_{sc}$.
- Three-phase transformers sometimes supply unbalanced loads and then the connection scheme and the magnetic core type have to be corroborated wisely to attenuate the zero sequence uncompensated load current emf that unbalances the phase voltages and moves the neutral potential from ground level. Similar aspects occur in 3-phase transformers connected to weak power grids with unbalanced voltages.
- Transformers undergo electromagnetic and ultra-high frequency (electrostatic) transients. Adequate means of transformer protection are needed to avoid transformer thermal or mechanical damage due to severe transients.
- See more on transformers in dedicated books and standards.

2.22 Proposed Problems

- 2.1** The laminated silicon core of an ac coil is made of 0.5 mm thick sheets with an electric conductivity, $\sigma = 1 \cdot 10^6 \text{ } (\Omega \cdot \text{m})^{-1}$, and a relative permeability, μ_{rel} :

$$\mu_{rel} = 3000; \quad \text{for } B < 0.8 \text{ T}$$

$$\mu_{rel} = 3000 - (B - 0.8)^2 \cdot 10^3; \quad \text{for } 0.8 \leq B < 2 \text{ T}$$

Calculate the eddy current losses per unit volume, in such a core at 60 Hz and at 600 Hz.

Hint: Use Equations 2.27 through 2.29.

- 2.2** In the open slot of an electric machine there is a single rectangular copper bar with $h = 0.020$ m (height) and $b = 0.005$ m (width). Copper conductivity, $\sigma_{\text{Co}} = 5 \cdot 10^7$ ($\Omega \cdot \text{m}$)⁻¹.

- Calculate the skin effect resistance and reactance coefficients, K_R and K_x , of the conductor for 60 Hz and for 1 Hz.
- Replace the single conductor by two conductors with $h' = h/2$ in height, connected in series and calculate again the skin effect coefficients in the same conductor as for (a).

Hint: Use Equations 2.44 through 2.50 with $m = 2$.

- 2.3** The cylindrical (multilayer) winding of a transformer is characterized by $N_1 = 100$ turns, radial thickness, $a_{1r} = 0.01$ or 0.03 m, and the distance between windings, $\delta_{is} = 0.005$ m, core diameter, $D = 0.1$ m.

- Calculate the winding leakage inductance for $a_{1r} = 0.01$ and 0.03 m for a column height $L_c = 0.08$ m.
- For the same copper volume and $L_c = 0.15$ m, determine the winding radial thickness, a_{1r} , and, again, the leakage inductance; compare and discuss the results for cases (a) and (b).

Hint: Use Equations 2.61 through 2.63.

- 2.4** A single-phase 1-MVA $V_{1nl}/V_{2nl} = 110/20$ kV transformer is characterized by no-load current, $I_{10} = 0.01 \cdot I_{1n}$, no-load power factor, $\cos \varphi_0 = 0.05$, short-circuit rated voltage, $V_{1scn} = 0.04 \cdot V_{1n}$ and $\cos \varphi_{sc} = 0.15$.

Calculate

- The rated and no-load currents, I_{1n} , I_{10n}
- The short-circuit resistance, R_{sc} , and reactance, X_{sc}
- Rated copper losses, p_{co}
- Iron losses, p_{iron}
- Rated efficiency at $\cos \varphi_2 = 1$ and $\cos \varphi_2 = 0.8$ lagging
- Load factor, $K_{scn} = I_1/I_{1n}$, for maximum efficiency
- Voltage drop, ΔV , in percent of rated voltage, at rated load and $\cos \varphi_2 = 1$ and $\cos \varphi_2 = 0.867$, lagging and leading
- The short-circuit rated voltage, V_{1scn} , in Volt

Hint: See Examples 2.2 through 2.4.

- 2.5** A 3-limb core 3-phase 60 Hz, 220 V/RMS per phase transformer, with star primary connection, operates on no load. The magnetic reluctances of limbs and yokes are $R_{mc} = R_{my} = 1$ [H]⁻¹.

- a. Neglecting all voltage drops in the transformer, express the limb flux phasors $\underline{\Phi}_A, \underline{\Phi}_B, \underline{\Phi}_C$ with their calculated amplitude
- b. Calculate in complex number terms the three-phase currents on no load
- c. Determine the active power for each phase, noting that the voltage/flux phase angle is 90°

Hint: See Figure 2.39 and Equations 2.112.

- 2.6 A 3-phase 20/0.38 kV, 500-kVA, Yy0 transformer is loaded at rated secondary current, I_{2n} , only on phase a in the secondary at $\cos \varphi_2 = 0.707$ lagging. Determine
- a. The rated phase currents in the primary
 - b. The current in the phase a of secondary
 - c. The currents in phases A, B, C of primary for the single-phase load
 - d. No-load reactance/phase, X_m , if the no-load current is $I_{10} = 0.01 \cdot I_{1n}$
 - e. The zero sequence emf per phase in the secondary and primary if $X_0 = 0.1 \cdot X_m$ (or the neutral potential to ground)
 - f. The secondary phase voltages, V_a, V_b, V_c
 - g. The primary phase voltages, V_A, V_B, V_C

Hint: Check Example 2.7.

- 2.7 Two 3-phase Yy6 transformers of $S_{na} = 500$ kVA and $S_{nb} = 300$ kVA, $V_{1n1}/V_{2n1} = 6000/380$ V, $V_{nsca} = 1.1 \cdot V_{nscb} = 0.04 \cdot V_{1n}$, $\cos \varphi_{sca} = \cos \varphi_{scb} = 0.3$, with same number of turns and transformer ratio, operate in parallel.
- a. With the first transformer tapped +5% and second at -5% in the primary, on no load, in parallel, calculate the circulating current between the two transformers in the primary and secondary.
 - b. With both transformers tapped at 0%, and the first transformer loaded at rated current and the second one in parallel, and both supplying a resistive load, calculate the secondary current in the second transformer and in the load. Discuss the results.

Hint: See Equations 2.120 through 2.127 and Example 2.6.

- 2.8 A 1-phase 60-Hz transformer under no load is connected suddenly to the power grid. The primary resistance, $R_1 = 0.1 \Omega$, and the primary voltage is 220 V (RMS). The following are required:
- a. The approximate total flux-linkage amplitude during steady-state no load, Ψ_{1m0}

- b. The magnetization curve is given by $I_0 = a \cdot \Psi_{1m0} + b \cdot \Psi_{1m0}^2$
 With $\Psi_{1m0} = 0.95$ Wb for $i_0 = 0.1$ A and $\Psi_{1m0} = 1.8$ Wb for $i_0 = 50$ A. Determine the inductance function, $L_{10}(i_0) = \Psi_{1m0}/i_0$
- c. For zero remnant flux and $\gamma_0 - \varphi_0 = 90^\circ$ represented in a graph, the inrush current versus time, for a constant time constant, $T_0 = L_{10} \cdot (0.2 \text{ A}) / R_1$, and constant $\varphi_0 = (\tan^{-1}(\omega_1 \cdot T_0))$

Hint: Check Figure 2.48 and Equation 2.136.

- 2.9 The inter-turn and turn/earth capacitances, K and C per unit winding length (height in meters), of a 3-phase transformer are $C = 10 \mu\text{F} = 25 K$.

- a. Calculate the initial distribution of an atmospheric microsecond front voltage of 1 MV along the winding height for an isolated and earthed neutral
- b. After using metal screens connected at primary phase terminal $C = K$; calculate again the initial voltage distribution along the winding height for the two cases and discuss the results

Hint: Check Equations 2.150 and 2.155 and Figure 2.51.

- 2.10 A 10-kVA, 220/110 V single-phase autotransformer is considered. Determine

- a. The electromagnetic rated power, S_{em}
- b. The rated primary, secondary and load currents
- c. Calculate the ratio of copper losses of the autotransformer to transformer with the same voltages and design current density (it means both resistances are proportional to turns squared only)

Hint: Check Equations 2.158 through 2.162 and Example 2.8.

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3

Energy Conversion and Types of Electric Machines

3.1 Energy Conversion in Electric Machines

Electric machines are sets of magnetically and electrically coupled electric circuits with one movable element (rotor), which convert electric energy into mechanical energy (motor mode) or vice versa (generator mode). They are based on the law of energy conversion and on Faraday's law for bodies in relative motion. In the following text, we will discuss the energy conversion principle of electric machines work, and introduce the basic types of electric machines, by making use of the frequency theorem [1].

The energy conversion in electric machines involves energy in four forms:

$$\begin{array}{ccccccc}
 \text{Electric} & & & & \text{Stored} & & \text{Energy} \\
 \text{energy} & & & & \text{magnetic} & & \text{loss} \\
 \text{from the} & = & \text{Mechanical} & + & \text{energy} & + & \text{in the electric} \\
 \text{power source} & & \text{energy} & & & & \text{machine} \\
 & & \text{Motor} & & & & \\
 \xrightarrow{\quad} & & \xrightarrow{\quad} & & & & \\
 \xleftarrow{\text{Generator}} & & \xleftarrow{\text{Generator}} & & & &
 \end{array} \quad (3.1)$$

There are three main reasons for the loss of energy:

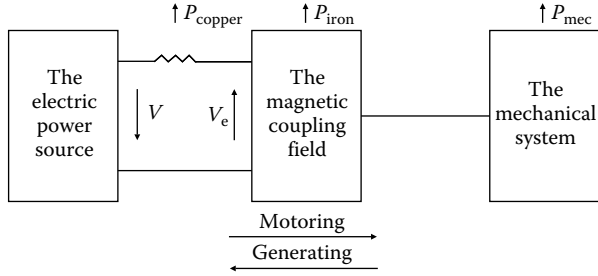
- Magnetic core hysteresis and eddy current losses (as in transformers): p_{iron}
- Winding losses (as in transformers): p_{copper}
- Mechanical losses (windage, bearing, ventilator losses): p_{mec}

Figure 3.1 portrays Equation 3.1 with specified losses.

According to Figure 3.1, the net electric energy converted into magnetic energy, dW_e , is

$$dW_e = (V - Ri) i dt \quad (3.2)$$

To transform the electric energy into magnetic energy (in the electric machine), the coupling magnetic field (of the net of the magnetic/electric

**FIGURE 3.1**

Energy conversion in electric machines.

circuits that make the electric machine) has to produce a reaction in the electric circuit, which manifests itself by the electromotive force (emf), V_e :

$$-V_e = V - Ri; \quad dW_e = (-V_e)i \, dt \quad (3.3)$$

If the electric energy is transmitted to the coupling magnetic field through a few electric circuits, Equation 3.3 will contain their summation.

3.2 Electromagnetic Torque

According to Faraday's law, the emf, V_e , is

$$V_e = \frac{-d_s \Psi}{dt} = -\frac{\partial \Psi}{\partial t} - \frac{\partial \Psi}{\partial \theta_r} \frac{d\theta_r}{dt} \quad (3.4)$$

where 's' refers to the total (substantial) flux-linkage time derivative. V_e contains the pulsational emf and the motion emf (θ_r is the rotor position). It turns out that only the motion emf participates directly in the electro-magneto-mechanical energy conversion and torque production in electric machines. From Equations 3.3 and 3.4,

$$dW_e = i \, d_s \Psi \quad (3.5)$$

Thus, when the flux linkage is constant ($d_s \Psi = 0$), there is no electric energy transfer between the electric machine and the power source. On the other hand, the mechanical energy increment, dW_{mec} , is defined by the electromagnetic torque, T_e , and the rotor angle increment, $d\theta_r$:

$$dW_{mec} = T_e d\theta_r \quad (3.6)$$

Denoting the stored magnetic energy increment by dW_{mag} and combining the above equations, we obtain

$$dW_e = i d\Psi_s = dW_{\text{mag}} + T_e d\theta_r \quad (3.7)$$

Now, if $\Psi = \text{const}$, as stated above, the energy transfer from the electric power source is zero, and thus all of the stored magnetic energy comes from the mechanical energy conversion:

$$T_e = \left(-\frac{\partial W_{\text{mag}}}{\partial \theta_r} \right)_{\Psi = \text{const}} \quad (3.8)$$

In practice, such a situation occurs when the mechanical energy of the rotor is converted into magnetic stored energy and finally into iron and copper losses in the electric machine in the generator mode, supplying a braking resistor, for instance, on board an urban people mover. The electromagnetic torque is negative and brakes the electric generator almost to zero speed.

3.2.1 Cogging Torque (PM Torque at Zero Current)

The torque developed in a PM machine for zero current (zero electric energy transfer from the power source) is due to the variation of PM-produced magnetic energy in the airgap with the rotor position, due to the stator and the core rotor slot openings (or saliencies). It is an almost zero-loss bidirectional conversion of the magnetic energy of PMs to mechanical energy and back. The average torque being zero, no net mechanical energy is produced but torque ripple occurs. This torque will be discussed in Section 3.3 through a finite element analysis. But in most cases, $dW_e \neq 0 (d\Psi \neq 0)$, so there is an electric energy transfer from (to) the electric power source, so we have to refer to Equation 3.7 in a modified form:

$$dW_{\text{mag}} = i d\Psi - T_e d\theta_r; \quad I = \frac{\partial W_{\text{mag}}}{\partial \Psi} \quad (3.9)$$

Now, Ψ and θ_r are the independent variables. However, the relationship between flux linkages and currents in an electric machine (via inductances) is rather straightforward. Further on, we introduce a new energy function, W'_{mag} , called coenergy:

$$W'_{\text{mag}} = -W_{\text{mag}} + i\Psi \quad (3.10)$$

or

$$dW_{\text{mag}} = i d\Psi + \Psi di - dW_{\text{mag}} \quad (3.11)$$

By substituting Equation 3.11 in Equation 3.9, to eliminate W_{mag} , we obtain

$$T_e = \left(+\frac{\partial W'_{\text{mag}}}{\partial \theta_r} \right)_{I = \text{const}}; \quad \Psi = +\frac{\partial W'_{\text{mag}}}{\partial i} \quad (3.12)$$

In general, $\Psi(i)$ functions are nonlinear due to the magnetic saturation of magnetic cores in the electric machines as in Figure 3.2, from Equations 3.9 and 3.11:

$$W_{\text{mag}} = \int_0^{\Psi_m} i d\Psi; \quad W'_{\text{mag}} = \int_0^{i_m} \Psi di \quad (3.13)$$

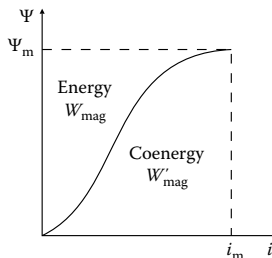


FIGURE 3.2

Magnetic energy and coenergy with a single excitation (current) source.

Equation 3.12 implies nonzero energy transfer from (to) the electric power source ($dW_e = i d\Psi \neq 0$).

The electromagnetic torque is nonzero according to Equations 3.9 and 3.12 only when the magnetic energy (or coenergy) of magnetic fields in the electric machine varies with respect to the rotor (mover) position, θ_r . The application of this general principle has led to numerous practical configurations of electric machines with rotary or linear motion. To classify electric machines, we will subsequently use two principles: with passive or active rotors and with fixed or traveling magnetic fields produced by the fixed part (stator) and/or the rotor in the small air space between them called the airgap.

3.3 Passive Rotor Electric Machines

The passive rotor is made of a soft magnetic material and it does not have any windings or permanent magnets. In order to produce torque (magnetic coenergy variation with the rotor position), it has to have a magnetic saliency, that is, at least one self- or mutual-inductance should vary with the rotor position (Figure 3.3):

$$L(\theta_r) = L_0 + L_m \cos 2\theta_r \quad (3.14)$$

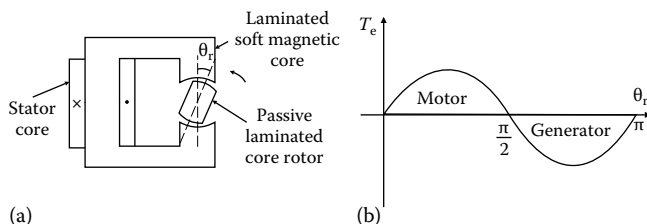
In this primitive configuration, both the stator and the rotor have magnetic saliency. The coenergy is calculated for a single inductance:

$$W'_{\text{mag}} = \int_0^i \Psi di = \int_0^i i L(\theta_r) di = \frac{L(\theta_r) i^2}{2} \quad (3.15)$$

From Equation 3.11, the electromagnetic torque, T_e , is

$$T_{e1} = \frac{\partial W'_{\text{mag}}}{\partial \theta_r} = \frac{i^2}{2} \frac{\partial L}{\partial \theta_r} = -i^2 L_m \sin 2\theta_r \quad (3.16)$$

It is evident that the torque varies with the rotor position and is maximum at $\pi/4$. Left free, the rotor goes back to the $\theta_r = 0$ position. The rotor tends

**FIGURE 3.3**

The single-phase reluctance—passive rotor—electric machine: (a) The configuration and (b) the electromagnetic torque vs. the rotor position, θ_r .

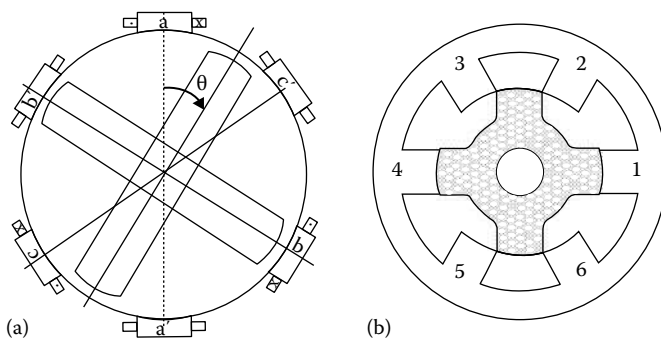
to align the stator field axis and this is the principle of reluctance electric machines. In the single-phase configuration of Figure 3.3, the reluctance machine cannot rotate continuously but may be used for a limited angle motion (ideally 0° – 90°). The average torque per revolution is zero.

However, if we place three stators *a*, *b*, and *c* (Figure 3.4a) along the rotor periphery, spatially shifted by 120° and flowed by symmetrical ac currents i_a , i_b , and i_c :

$$i_{a,b,c} = I\sqrt{2} \sin\left(\omega_1 t - (i-1)\frac{2\pi}{3}\right) \quad (3.17)$$

then the average torque per revolution will not be zero and the machine may rotate continuously, as desired:

$$L_{a,b,c} = L_0 + L_m \cos\left(2\theta_r + (i-1)\frac{2\pi}{3}\right) \quad (3.18)$$

**FIGURE 3.4**

(a) Elementary 3-phase reluctance (passive rotor) machine and (b) switched reluctance machine ($(2p_1)_{\text{rotor}} = 4$).

The rotor still has two salient poles ($2p_1 = 2$). In reality, multiple pole pair ($p_1 > 1$) configurations could be built. The machine coenergy formula now shows only three terms as no magnetic coupling between the three stator phases is considered:

$$W'_{\text{mag}} = \sum_{a,b,c} L_{a,b,c}(\theta_r) \frac{i_{a,b,c}^2}{2} \quad (3.19)$$

with the final electromagnetic torque, T_e :

$$T_{e3} = \left(\frac{\partial W'_{\text{mag}}}{\partial \theta_r} \right)_{i=\text{const}} = \frac{3I^2 L_m}{2} \cos(2\theta_r - 2\omega_1 t) \quad (3.20)$$

For constant angular rotor speed, ω_r :

$$\theta_r = \int \omega_r dt = \omega_r t + \theta_{r0} \quad (3.21)$$

So, finally, Equation 3.20 becomes

$$T_e = 3I^2 \frac{L_m}{2} \cos 2[\theta_0 + (\omega_r - \omega_1)t] \quad (3.22)$$

Only for $\omega_r = \omega_1$, the average torque is nonzero and there are no torque pulsations:

$$T_{e3}(t) = 3I^2 \frac{L_m}{2} \cos 2\theta_0 \quad (3.23)$$

Thus, the stator current angular frequency, ω_1 , and the rotor angular speed should be equal to each other to obtain, in a 2-pole, 3-phase ac machine, a constant instantaneous (ideal) torque. This is the principle of 3-phase (multi-phase) ac machines with passive rotors (reluctance machines).

Again, multiple pole pair configurations are feasible, but then $\omega_r = p_1 \Omega = 2\pi n p_1$ (n in rps).

Note: In practical reluctance synchronous machines ($\omega_1 = \omega_r$), distributed windings are used in the stator- and mutual (interphase)-inductances are nonzero and bring in more torque.

It is feasible to supply the three phases of the salient pole stator with sequences of the same polarity current pulses such that only one phase produces the torque at a time, as triggered by the adequate rotor position (when its torque is positive, Figure 3.3).

In this case, the stator is also magnetically salient (6, 12 poles) and uses concentrated coils (multiples of two per phase) and the rotor may have 4, 8 poles for 3-phase machines. These are called switched reluctance machines (Figure 3.4b) and may be built with one, two, three, four, and more phases, working in sequence with controlled current pulses to produce low pulsation torques with the rotor position. Stepper reluctance motors work on the same

principle, but their voltage (current) pulses are open-loop (feed-forward) referenced.

The majority of electric machines have an active rotor (with dc or ac coils or with PMs) but can still retain the magnetic rotor saliency and thus develop a reluctance torque component.

3.4 Active Rotor Electric Machines

A primitive active rotor single-phase electric machine is shown in Figure 3.5. The flux linkages of the stator/rotor circuits are

$$\psi_1 = L_{11}i_1 + L_{12}i_2; \quad \psi_2 = L_{12}i_1 + L_{22}i_2 \quad (3.24)$$

A coupling between the stator and the rotor windings is evident. The magnetic conergy is thus

$$W'_{\text{mag}} = \frac{1}{2}L_{11}i_1^2 + L_{12}i_1i_2 + \frac{1}{2}L_{22}i_2^2 \quad (3.25)$$

with the torque

$$T_e = \frac{\partial W'_{\text{mag}}}{\partial \theta_r} = \frac{1}{2}i_1^2 \frac{\partial L_{11}}{\partial \theta_r} + \frac{1}{2}i_1^2 \frac{\partial L_{22}}{\partial \theta_r} + i_1i_2 \frac{\partial L_{12}}{\partial \theta_r} \quad (3.26)$$

The first two components of the torque are reluctance torques, as explained earlier. The third term is new and is called an interaction torque. We may draw the conclusion that at least the mutual-inductance, if not the stator self-inductances, has to vary with the rotor position to secure a nonzero torque. This case could be extended to 3-phase machines also. Now we have to specify what types of stator/rotor currents—ac or dc—are used.

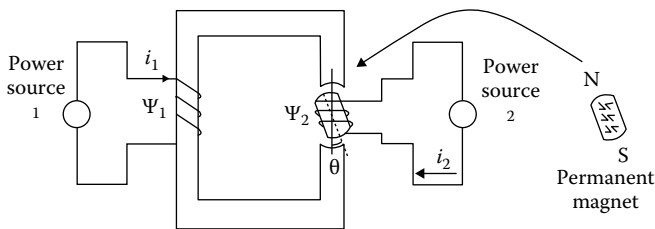


FIGURE 3.5

Primitive active rotor single-phase electric machine.

3.4.1 DC Rotor and AC Stator Currents

Let us consider that self-inductances above are constant (L_{11} and L_{22}) and $L_{12} = L_m \cos \theta_r$ (2 poles) while $i_1 = I\sqrt{2} \sin \omega_1 t$ and $i_2 = I_2 = \text{const}$. From Equation 3.26, we get

$$T_{e1} = II_2 \sqrt{2} L_m \frac{1}{2} [\cos(\omega_1 t + \theta_r) - \cos(\omega_1 t - \theta_r)] \quad (3.27)$$

A nonzero torque is obtained only if either $\omega_1 = \omega_r$ or $\omega_1 = -\omega_r$ ($\theta_r = \omega_r t$). However, this means that one term in Equation 3.27 is constant while the other one varies with $2\omega_1$. This is typical to the single-phase synchronous ($\omega_1 = \omega_r$) motor with active ($I_2 = \text{const}$ or PM) rotor. To eliminate the pulsating torque in Equation 3.27 with $\theta_r - \omega_1 t = \theta_0 = \text{const}$ ($\omega_1 = \omega_r$), we may imagine three stators axially shifted by 120° with each other and fed through 3-phase ac currents ($i_{a,b,c} = I\sqrt{2} \sin(\omega_1 t - (i-1)(2\pi/3))$). If we neglect the coupling between phases,

$$T_{e3} = I_2 \left(i_a \frac{\partial L_{ma}}{\partial \theta_r} + i_b \frac{\partial L_{mb}}{\partial \theta_r} + i_c \frac{\partial L_{mc}}{\partial \theta_r} \right) \quad (3.28)$$

with

$$L_{ma,mb,mc} = L_m \cos \left(\theta_r - (i-1) \frac{2\pi}{3} \right); \quad i = 1, 2, 3 \quad (3.29)$$

the torque becomes

$$T_{e3} = I_2 I \sqrt{2} L_m \frac{3}{2} \sin((\omega_1 - \omega_r)t - \theta_0) \quad (3.30)$$

Again, only for $\omega_1 = \omega_r$, the instantaneous torque is constant; this is the case for primitive synchronous 3-phase machines with dc (or PM) rotor excitation.

Note: We mentioned the PM here directly, but the PM may be modeled by a constant (dc) mmf (current) ideal (superconducting) coil, where mmf refers to ampereturns or magnetomotive force.

So speed (ω_r) control may be operated only through frequency (ω_1) control, performed, in general, through power electronics.

3.4.2 AC Currents in the Rotor and the Stator

Let us consider the same single-phase configuration, but the rotor current, i_2 , is

$$i_2 = I_2 \sqrt{2} \sin \omega_2 t \quad (3.31)$$

The torque from Equation 3.25 becomes

$$T_{e1} = -II_2 \sin \omega_1 t (2L_m \sin \omega_2 t) \sin \theta_r$$

For constant speed ω_r , $\theta_r = \omega_r t$, it could be demonstrated that the torque may have a nonzero average component when

$$\omega_1 \mp \omega_2 = \omega_r \quad (3.32)$$

Even in this case, the torque has three pulsating additional components.

To eliminate the pulsating components of the torque, we will place three windings on the stator and on the rotor, supplied by ac symmetric currents of frequency ω_1 and ω_2 , respectively.

This is how we obtain a total torque:

$$T_{e3} = 3L_m I_1 I_2 \sin((\omega_1 - \omega_2 - \omega_r)t + \gamma) \quad (3.33)$$

Again, with $\omega_1 = \omega_2 + \omega_r$ ($\omega_2 \geq 0$), the torque is

$$T_{e3} = 3L_m I_1 I_2 \sin(\gamma) \quad (3.34)$$

where γ is the phase angle between the stator and the rotor currents if represented at the same frequency (ω_1). Now this is, in fact, the so-called doubly fed induction machine where the rotor frequency currents of frequency $\omega_2 = \omega_1 - \omega_r$ should be provided from outside by a PWM inverter.

It is also possible to place short-circuited windings (bars in rotor slots with end rings) and thus “induce” emfs in the rotor at exactly $\omega_2 = \omega_1 - \omega_r$ frequency by motion.

This is the cage-rotor induction machine, the workhorse of the industry.

3.4.3 DC (PM) Stator and AC Rotor

When the stator is dc-fed and has $2p_1$ poles surrounded by dc coils (or PMs) and the rotor is ac-fed through n ac coils uniformly placed along the rotor periphery (with $2\pi/2p_1n$ angle span), with currents showing a $2\pi/n$ time lag (Figure 3.6),

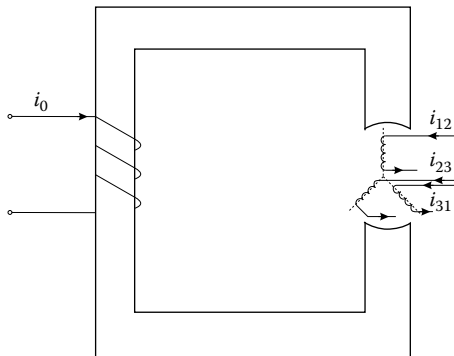
$$I_{2i} = I_2 \sqrt{2} \sin \left(\omega_2 t - (i-1) \frac{2\pi}{n} \right) \quad (3.35)$$

The mutual-inductances between the stator winding and the rotor coils are

$$L_{12i} = L_m \cos \left(\omega_r t - (i-1) \frac{2\pi}{n} \right); \quad i = 1, n \quad (3.36)$$

So the torque T_{en} is

$$T_{en} = I_2 I_0 \sqrt{2} L_m \sum_{i=1}^n \cos \left(\omega_2 t - (i-1) \frac{2\pi}{n} \right) \cos \left(\theta_r - (i-1) \frac{2\pi}{n} \right) \quad (3.37)$$

**FIGURE 3.6**

DC (PM) stator and ac multiphase rotor machine.

Finally,

$$T_{en} = I_2 I_0 \sqrt{2} L_m \frac{n}{2} \sin(\omega_2 t - \theta_r) \quad (3.38)$$

The electromagnetic average torque is nonzero only if $\omega_2 = \omega_r$ (because $\theta_r = \omega_r t + \theta_0$):

$$T_{en} = -I_2 I_0 \sqrt{2} L_m \frac{n}{2} \sin \theta_0 \quad (3.39)$$

So the rotor currents frequency ω_2 is equal to the rotor speed ω_r . This condition is met by the brush-commutator machines that transforms dc brush currents, through a mechanical commutator, into ac currents of $\omega_2 = \omega_r$.

The ac currents in the rotor coils of the practical brush-commutator machines are trapezoidal rather than sinusoidal, but the principle above still holds.

The frequency theorem-based classification $\omega_1 = \omega_2 + \omega_r$ has led to the identification of ac stator synchronous machines (SMs) with dc (PM) rotor excitation, or with a passive (magnetically salient) rotor, to the switched reluctance machines (similar to SMs, but with passive rotors and sequential position-triggered current control pulses) and to induction machines with ac stators and ac rotors in both configurations: doubly fed or with cage-rotor.

Finally, the brush-commutator machines have been also identified as dc (PM) stator and ac multiphase current rotor machines. No machine can apparently escape the frequency theorem principle, so we have them all.

However, we may identify them also by the type of the magnetic field they show in the airgap: traveling (moving) or fixed.

3.5 Fix Magnetic Field (Brush-Commutator) Electric Machines

As the angle between the two field axes is 90° , the interaction between them to produce a torque is optimum. Also, the variation of the rotor current (to vary the torque) does not lead to any emf in the stator dc (field) winding.

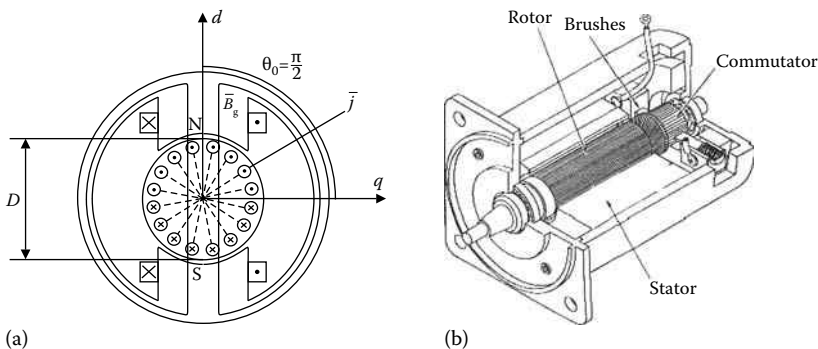


FIGURE 3.7
Two-pole ($2p_1 = 2$) electric machine (a) with a fixed magnetic field and (b) without rotor slots.

So the torque (rotor) and the field (stator) current controls are decoupled by machine topology. Permanent magnets may replace the dc excitation stator winding, but the principle of fixed magnetic fields interaction still holds.

We may now calculate the torque by the (BIL) tangential force formula:

$$T_e = \frac{D}{2} F_t = \frac{D}{2} B_{\text{gav}} (A \times \pi D) L \quad (3.40)$$

where B_{gav} is the average stator-produced airgap flux density, A is the average rotor current loading in A turns/m, D is the rotor diameter, and L is the laminated core stack length (about the same in the stator and the rotor).

If A would be independent of D and L , with given B_{gav} , the torque would be proportional to the rotor volume. So it is the torque that decides the size of the machine.

3.6 Traveling Field Electric Machines

Let us reclaim the synchronous machine example, with dc (or PM) rotor excitation (Figure 3.8).

First of all, the dc or PM rotor heteropolar magnetic field has its axis in the rotor pole axis d .

This excitation field becomes a traveling field only when the rotor moves, say at speed ω_r .

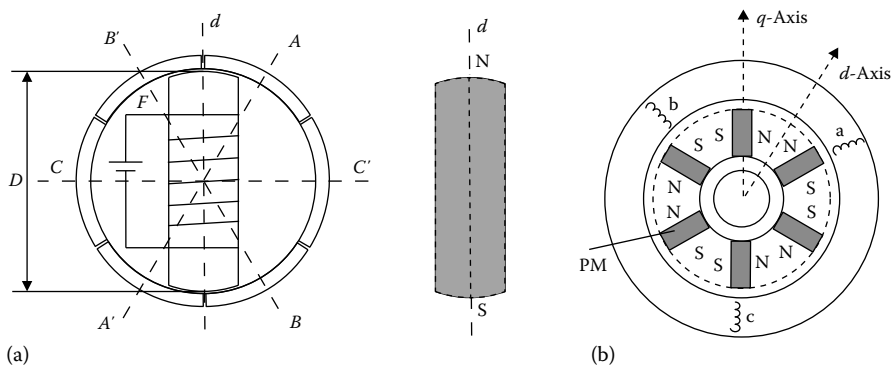


FIGURE 3.8

Traveling field electric synchronous machines (a) with dc rotor excitation and $2p_1 = 2$ and (b) with PM rotor excitation and $2p_1 = 6$.

But with respect to the rotor, it is (if curvature is neglected)

$$B_F^r(x_r) = B_{Fm} \sin \frac{\pi}{\tau} x_r; \quad \tau = \frac{\pi D}{2p_1} \quad (3.41)$$

With respect to the stator

$$x_r = x_s - vt = x_s - \omega_r \frac{\tau}{\pi} t \quad (3.42)$$

at a constant speed ω_r , where x_s is the stator coordinate. So, with respect to the stator (x_s),

$$B_F^s(x_s, t) = B_{Fm} \sin \left(\frac{\pi}{\tau} x_s - \omega_r t \right) \quad (3.43)$$

To the stator, this is truly a traveling field at rotor speed ω_r .

Let us suppose that the stator currents produce a linear current density $A_s(x_s, t)$. Consequently, the torque (as in Equation 3.40) is

$$T_e(t) = \frac{D}{2} L \int_0^{2p_1\tau} B_F^s(x_s, t) A_s(x_s, t) dx \quad (3.44)$$

To obtain a constant instantaneous (ideal) torque (to get rid of time dependence), it is evident that $A_s(x_s, t)$ should be of the form

$$A_s(x_r, t) = A_{ms} \sin \left(\frac{\pi}{\tau} x_s - \omega_r t - \theta_0 \right) \quad (3.45)$$

In such conditions,

$$T_e(t) = \frac{\pi D^2 L}{4} B_{Fm} A_{ms} \cos \theta_0 \quad (3.46)$$

So both the rotor airgap field and the stator current loading (also the stator magnetomotive force) travel at rotor speed under steady state to produce constant instantaneous torque.

As the stator mmf $F_s(x, t) = \int A_s(x_s, t) dx$, F_s and A_s are 90° phase-shifted. So the maximum torque is obtained at $\theta_0 = 0$ (Equation 3.46), but this means that the excitation rotor field axis and the stator mmf (and field) axis are both running at rotor speed and are 90° phase-shifted; as for the brush-commutator machine, however, both the fields are at a standstill (are fixed).

A similar rationale may be applied to the doubly fed and cage-rotor induction machines whose stator and rotor currents produce magnetic fields traveling at speed ω_1 with respect to the stator, but the frequency of the rotor currents $\omega_2 = \omega_1 - \omega_r \neq 0$. This is why induction machines are also called asynchronous machines.

3.7 Types of Linear Electric Machines

For all types of rotary motion electric machines, that have cylindrical or disk-shaped rotors, there is at least one linear version of it, which is obtained by cutting it longitudinally and spreading it into a plane (flat type) or even rerolling it along an axial axis (tubular types for limited excursion motion)—Figures 3.9 and 3.10.

Three-phase linear induction and synchronous motors have found application in urban people movers on wheels for propulsion and in very rapid interurban transportation (at 400–500 km/h) on magnetic suspension (MAGLEVs) [3].

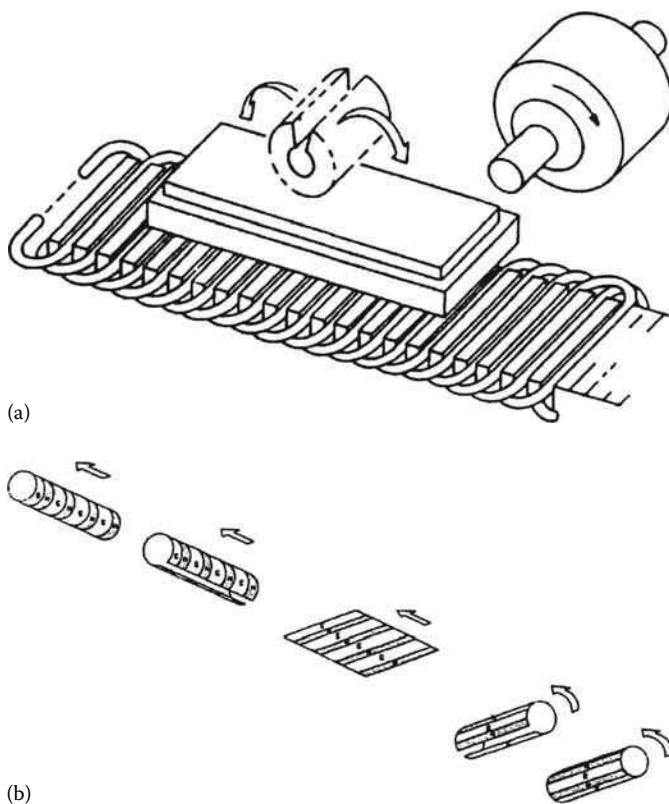
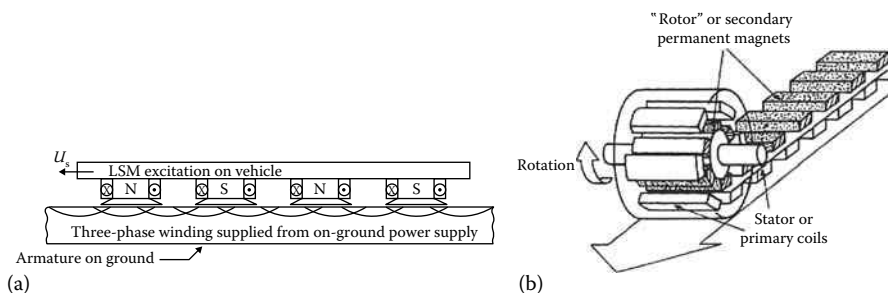


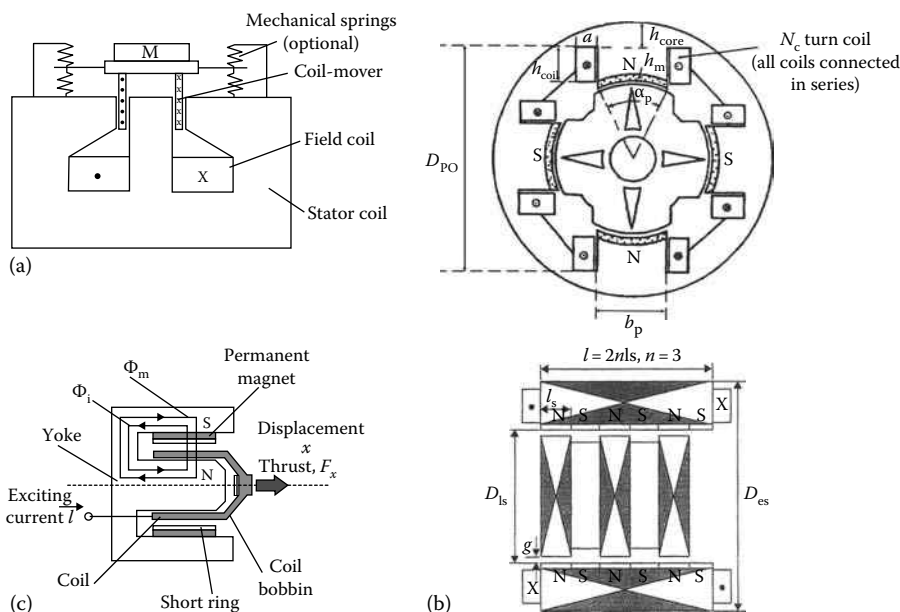
FIGURE 3.9

Obtaining a 3-phase linear induction machine (with traveling field) from a rotary machine: (a) Flat and (b) tubular.

**FIGURE 3.10**

Three-phase linear flat synchronous machines (with traveling field): (a) With dc heteropolar excitation and (b) with PM excitation.

Single-phase synchronous (PM-mover or stator PM and iron-mover) tubular configurations for linear oscillatory motion are used to drive small-power compressors (e.g., refrigerators) or linear generators (e.g., Stirling engine prime mover)—Figure 3.11a and b. They may be assimilated with PM plunger solenoids connected to the grid or power electronics-controlled.

**FIGURE 3.11**

Linear oscillatory motor/generator: (a) With PM-mover, (b) with stator-PM and iron (reluctance)-mover, and (c) with stator-PM and coil-mover (the microphone/loudspeaker).

The loudspeaker/microphone is a typical case of a linear oscillatory stator-PM, coil-mover single-phase synchronous machine (Figure 3.11c). It may be used as an electrodynamic vibrator with frequencies up to around 500 Hz.

Example 3.1 The Loudspeaker/Microphone as a Linear PM Motor

Let us consider the tubular configuration in Figure 3.12 that contains

- A tubular inner and an outer soft magnetic composite shell of soft ferrite or Somaloy 550 etc.
- A tubular PM, radially magnetized, placed also on the stator

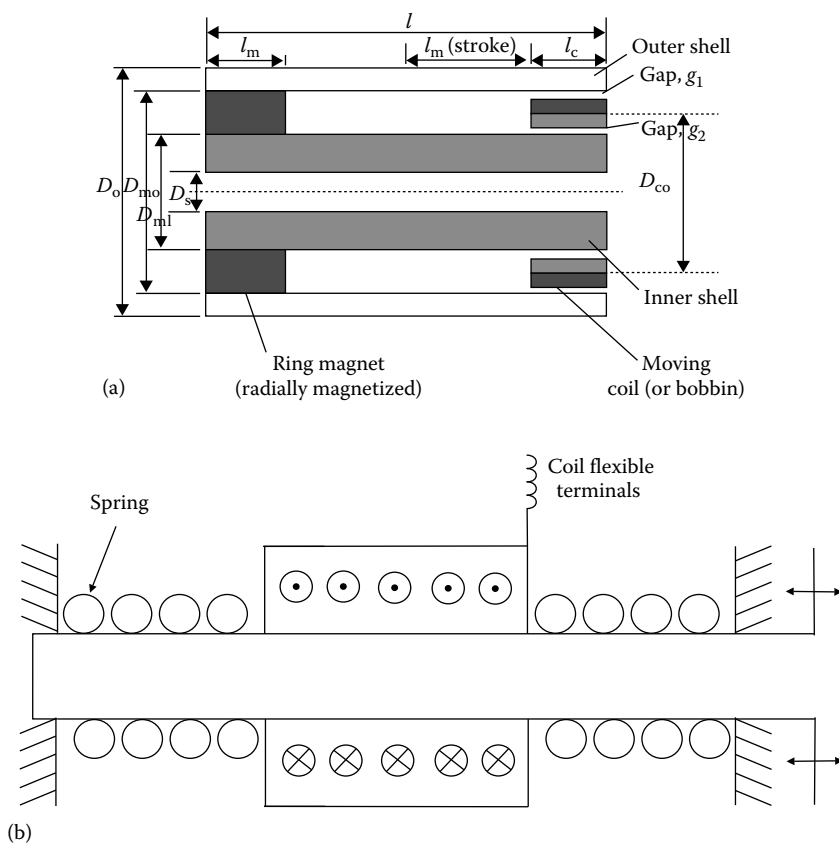


FIGURE 3.12

The loudspeaker/microphone as (a) a linear PM machine and (b) a mover in the middle with springs.

- A mover that contains only a nonmagnetic shell that holds the tubular copper multiturn coil, which constitutes the mover, connected by flexible terminals to an ac (controlled or uncontrolled) power source

The PM flux paths close axially through the two shells and radially through the PM and the coil-mover to produce a unipolar magnetic field B_{PM} in the coil. Then, when the coil carries a current i , a BIL force is developed. The force changes the sign when the current changes the polarity, and thus an oscillatory motion is produced.

To increase the efficiency, the energy necessary to accelerate and decelerate the mover at stroke ends is stored in the mechanical springs, which, in the compressor drives, may be designed to be mechanically resonant at the imposed electric frequency of the currents in the coil ($f_m = f_e$). Very good efficiencies could be obtained down to 20 W power or less in this situation.

The emf in the coil comes from the $B_{PM}Ul$ formula (l —mean coil turn length, U —linear speed):

$$V_e = B_{PM}\pi D_{arc}N_c \frac{dx}{dt} \quad (3.47)$$

F_e springs from the *BIL* formula:

$$F_e = B_{PM}\pi D_{arc}i_cN_c \quad (3.48)$$

where N_c is the turns in series per coil.

As the oscillatory motion is quasi-sinusoidal,

$$x = x_1 \cos \omega_r t \quad (3.49)$$

It follows from Equation 3.47 that the emf is sinusoidal:

$$V_e(t) = -B_{PM}\pi D_{mar}N_c\omega_r x_1 \sin \omega_r t \quad (3.50)$$

As the coil inductance does not vary with the mover position $L_c = \text{const}$, the motion and the voltage equations are

$$M_{mover} \frac{dU}{dt} = F_e - F_{load} - K_{spring}(X - l_{stoke}/2) \quad (3.51)$$

$$\frac{dx}{dt} = U$$

$$i_c R_c + L_c \frac{di_c}{dt} = V_c(t) - V_e(t) \quad (3.52)$$

Under mechanical resonance conditions,

$$\omega_r = \sqrt{K_{spring}/M_{mover}} = \omega_1; \quad F_{load} \approx K_{load}U(t) \quad (3.53)$$

If $V_e = V_{em} \cos \omega_r t$, then the current under harmonic (steady-state sinusoidal) motion will have the same frequency, ω_r , and we have a synchronous single-phase PM machine.

This time, the emf is sinusoidal due to the harmonic motion imposed by the mechanical springs.

As the mechanical springs move back and forth, they store the mover kinetic energy at stroke ends, and thus the electromagnetic force is responsible mainly for startup and then to cover the load force (for the compressor, this load may be considered proportional to the speed ω_r : $F_{load} \approx K_{load}U(t)$).

So, for steady-state harmonic motion,

$$\underline{V}_1 = V_0 \sqrt{2} e^{j(\omega_r t + \gamma)} \quad (3.54)$$

$$\underline{U}_1 = j\omega_r \underline{X}_1$$

and with $\frac{d}{dt} = j\omega_r$ in Equations 3.51 and 3.52, we obtain in complex members,

$$j\omega_r \underline{I}_1 = \frac{\underline{V}_1 - R_c \underline{I}_1 - j\omega_r K_{PM}}{L_c}; \quad K_{PM} = B_{PM} \pi D_{arc} N_c \quad (3.55)$$

$$-\omega_r^2 \underline{X}_1 = \frac{K_{PM} \underline{I}_1 - K_{spring} \underline{X}_1 - j\omega_r C_{load} \underline{X}_1}{M_{mover}} \quad (3.56)$$

with

$$K_{spring} = M_{mover} \omega_r^2 \quad (3.57)$$

and mechanical resonance conditions,

$$\underline{I}_1 = \underline{X}_1 j\omega_r \frac{C_{load}}{K_{PM}} = \underline{U}_1 \frac{C_{load}}{K_{PM}} \quad (3.58)$$

From Equation 3.55, $\underline{I}_1$ may be calculated and from Equation 3.58, $\underline{X}_1$ may be calculated as sinusoidal current and position phasors (amplitudes and phases included), respectively.

For sinusoidal resonant motion, with the load force proportional to the speed, Equation 3.58 shows the current $\underline{I}_1$ in phase with linear speed $\underline{U}_1$, that is, with the emf V_{e1} , and thus the maximum force per current is obtained (best efficiency).

Example 3.2 Linear Compressor Coil-Mover PM Linear Motor

In a numerical example with $P_n = 125$ W, 120 V, 60 Hz compressor load with $C_{load} = 86.8$ N s/m, $l_{stroke} = 0.01$ m, $R_c = 6.3$ Ω , $L_c = 91.6$ mN, $K_{PM} = 78.6$ Wb/m, and $M_{mover} = 0.57$ kg, at resonance conditions ($K_{spring} = M_{rotor} \omega_r^2 = 0.57(2\pi 60)^2 = 76772$ N/m), we get the motion amplitude X_1 from Equations 3.55 and 3.58:

$$X_1 = 4.8 \cdot 10^{-3} \text{ m} \quad \text{for } I_n = 1.46 \text{ A (RMS)}$$

The copper losses $p_{\text{copper}} = R_c I_n^2 = 6.3 \times 1.46^2 = 13.4 \text{ W}$; with 4 W more for iron and mechanical spring loss, the efficiency is

$$\eta_n = \frac{P_n - \sum p}{P_r} = \frac{120 - 13.4 - 5}{120} = 0.877$$

The power factor

$$\cos \varphi_n = \frac{P_n}{V_n I_n \eta_n} = \frac{120}{120 \times 1.46 \times 0.877} = 0.817$$

This performance is quite satisfactory as the maximum speed $|U| = \omega_r X_1 = 2 \times \pi \times 60 \times 4.8 \times 10^{-3} = 1.76 \text{ m/s}$.

- The above performance has been secured by the spring's job of converting the mover kinetic energy to the spring's potential energy to relieve the electromagnetic force and source from handling the mover acceleration and deceleration through the oscillatory motion.
- If the load increases, the motion amplitude (X_1) decreases, as expected, but the efficiency remains good.
- Designed with an electric frequency equal to mechanical eigen (resonance) frequency, the electric frequency, ω_1 , should be kept constant or varied slightly through power electronics to track the small mechanical resonance frequency variations due to temperature and wearing (K_{spring} changes) and thus maintain a high efficiency.

More information on linear electric machines can be found in [2].

3.8 Summary

- This book deals only with electromagnetic machines that use magnetic energy storage.
- There are also electrostatic machines with electrostatic energy conversion, but they are used in sub mm (10^{-3} m) diameter micromachines. We will not discuss them here (see *IEEE Transactions on Micro-electromechanical Systems for Knowledge Acquisition*).
- There are also piezoelectric (traveling) field machines for a large torque (Nm or more) at very small speeds—rotary and linear (see [4,5]). However, they are not discussed in this book.
- In this chapter, the electromagnetic torque of electric machines is derived from the stored magnetic energy (coenergy), based on the generalized force concept.

- The main types of electric machines are derived, with respect to passive and active (magnetically or electrically) rotors, based on the frequency theorem $\omega_1 = \omega_2 + \omega_r$ (with ω_1 —stator electric frequency, ω_2 —rotor electric frequency, and ω_r —rotor mechanical in electric terms ($\omega_r = \Omega_1/p_1$ —pole pairs or electric periods per revolution)); 3-phase and single-phase machines are introduced.
- For $\omega_2 = 0$ (dc rotor excitation), synchronous machines are obtained while brush-commutator machines correspond to $\omega_1 = 0$ (dc stator excitation). Finally, for induction machines, $\omega_2 \neq 0$, it is positive for motoring and negative for generating.
- The same practical machines are classified into fixed (brush-commutator) and traveling (ac synchronous and induction) magnetic field machines.
- The linear electric machines are counterparts of the rotary machines in all configurations (principles).
- The case of the loudspeaker as a linear oscillatory-motion PM machine is discussed in detail for steady state, in a small compressor application, to envisage the general energy conversion details of all other electric machines.
- As this is an introductory chapter, only one proposed problem is included.

3.9 Proposed Problem

- 3.1 A U-shaped plunger solenoid (Figure 3.13) with a soft composite material core is needed to activate an internal combustion engine (ICE)

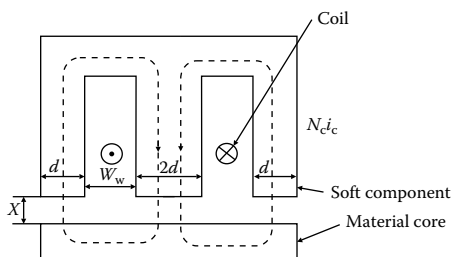


FIGURE 3.13
Tubular plunger solenoid.

valve. The travel starts from 0.5×10^{-3} m airgap and ends at 8.5×10^{-3} m. For the geometrical data in Figure 3.13 and an ideal magnetic core (infinite permeability):

a. Derive the expression of thrust (from the energy formula) as a function of airgap, for a given coil current i_c , number of turn/coil N_c , and active area at the airgap A :

- $W_w = 20 \times 10^{-3}$ m
- $d = 20 \times 10^{-3}$ m
- Stack depth $L = 0.05$ m

b. Calculate the coil ampere turns $N_c i_c$ for the same thrust $F_x = 500$ N for $x_{\min} = 0.5 \times 10^{-3}$ m and $x_{\max} = 8.5 \times 10^{-3}$ m.

Hint: First calculate the coil inductance, noticing that there are two airgap magnet circuit branches in parallel: $L_c = (\mu_0 N_c^2 / 2x) dL/2$ and $F_x = (i_c^2 / 2)(\partial L_c / \partial x)$.

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Brush–Commutator Machines: Steady State

4.1 Introduction

Brush–commutator electric machines are commercially also called dc machines, but today any machine can be supplied from a dc source, provided a power electronic converter is available.

Also, besides dc brush–commutator machines, there is the ac brush–commutator series (universal) motor still in use in many home appliances (e.g., vacuum cleaners, home robots, and hair dryers), for construction (vibration) tools up to 1 kW at 30,000 rpm. They operate with fixed magnetic fields (dc stator and ac rotor currents, Chapter 3).

Though considered a “doomed species,” due to brush–commutator scintillation and wearing limitations, faced with faster power electronic (static or brushless) commutation in traveling field machines, the brush–commutator PM small motors and a few megawatts, low speed ones (less than 150 rpm) for special applications (automotive and metallurgy drives, respectively) will die hard, for single quadrant variable speed drives, due to overall lower cost.

We will give preference to PM small dc motors and to ac brush–commutator (universal) motors because of their potential for the future.

The dc motors still in use for rail, urban, or marine transportation or metallurgy will be discussed only briefly because most probably these motors will not be in use in the next decade or so. A dc brush–commutator PM motor with its main parts is shown in Figure 4.1.

4.1.1 Stator and Rotor Construction Elements

PM dc brush machines (which contain a stator and a rotor as main parts) may be built with [1–4]

- Radial airgap (or cylindrical rotor/stator) (Figure 4.1)
- Axial airgap (or disk-shaped rotor/stator)
- Slotted thin-sheet silicon steel rotor core (Figure 4.2)
- Slotless rotor core
- Surface PM $2p_1$ poles in the stator (Figure 4.2a)



FIGURE 4.1
Typical dc brush–commutator PM motor with its mains parts.

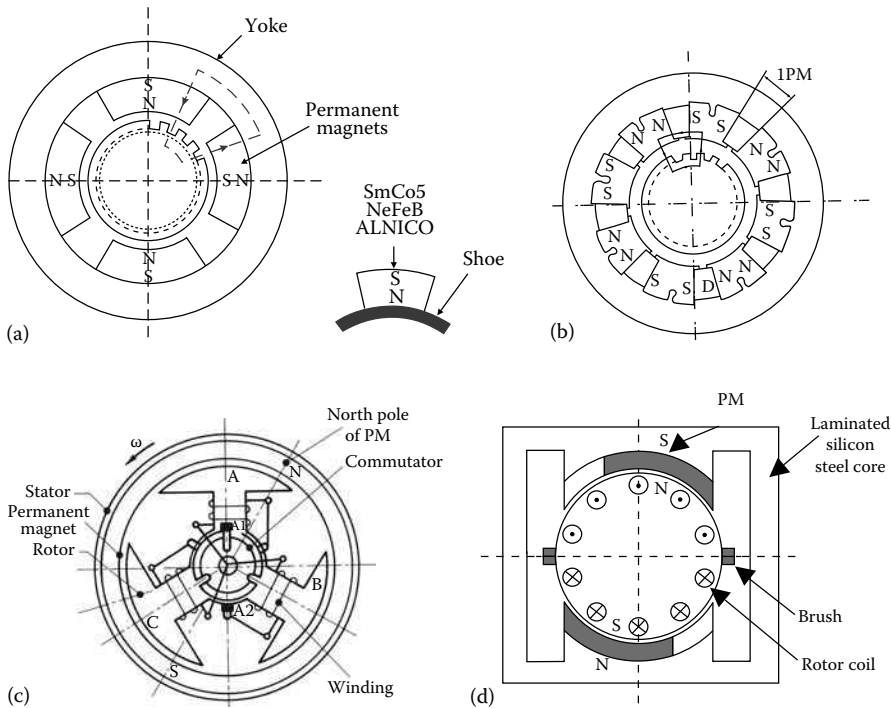


FIGURE 4.2
PM dc brush machines: (a) with surface PMs, (b) with interior PMs, (c) with external rotor (for ventilators), and (d) hybrid PM–iron (reluctance) stator poles.

- Interior PM $2p_1$ poles in the stator (Figure 4.2b)
- Interior rotor (Figure 4.2a,b)
- External rotor (Figure 4.2c)

The machine has a stator and a mover (rotor) with an air layer in between.

The axial-airgap configuration with a disk-shaped rotor is most often (with rotor windings in the airgap) used (with $2p_1 \geq 4$) to reduce axial length, volume, and rotor electric time constant, and thus obtain an ultrafast rotor-current (torque) control with power electronics at moderate costs.

The stator contains either a laminated (or solid iron) back iron and radially magnetized surface strong PM stators (Figure 4.2a) or laminated thin-sheet silicon steel poles with radially deep PMs (with tangential magnetization provided by “easy-to-demagnetize” ALNICO PMs [$B_r = 0.8T$, $H_c = 80 \text{ kA/m}$] or Ferrite PMs [$B_r = 0.4T$, $H_c = 350 \text{ kA/m}$]) (Figure 4.2b).

In slotless rotors or in rotor silicon steel sheet cores, there are uniform slots to hold identical coils that span π/p_1 radians and are all connected in series through the brush-commutator insulated copper sectors. They are called the armature winding. In slotless rotors with radial airgap, there is a laminated silicon iron back core in the rotor to complete the PM magnetic flux path.

For automobile engine-starter motors, hybrid surface PM/re reluctance stator poles with cylindrical slotted rotors are used to secure a high starting torque and a large torque up to 300–400 rpm where the engine ignites at low ambient temperatures (Figure 4.2d) (see [4] for more details).

The dc (or ac) electromagnetic excitation stator with laminated silicon steel core (made of 0.5 mm thick sheets) shows salient poles and concentrated coils that produce the excitation field (Figure 4.3).

AC-excited stators, where excitation coils are connected in series at commutator brushes in the universal motor, lack the interpoles in general and have, again, salient poles with concentrated coils placed in a laminated silicon steel (or soft composite material) stator core (Figure 4.4).

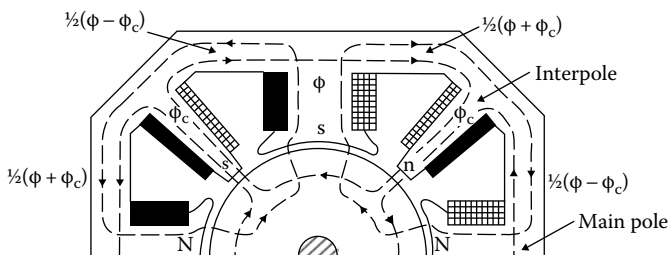


FIGURE 4.3

Cross section of a dc brush-commutator machine with interpoles to ease the commutation process.

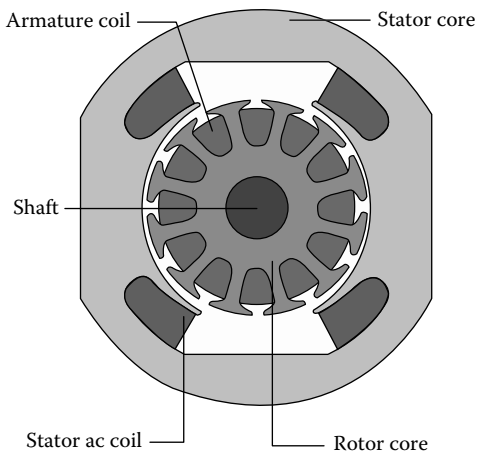


FIGURE 4.4
Cross section of a universal (ac brush-commutator single-phase) motor.

4.2 Brush-Commutator Armature Windings

These two types of armature windings that are placed in the uniform slots of the rotor silicon steel sheet core are made of lap coils or wave coils (Figure 4.5).

The span of the coils $y_c \approx \tau$ (τ = pole pitch; $\tau = \pi D / 2p_1$), to embrace all the stator pole flux and thus to produce maximum electromagnetic force (emf).

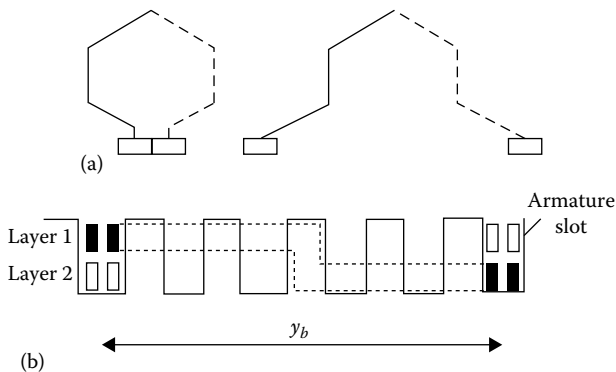
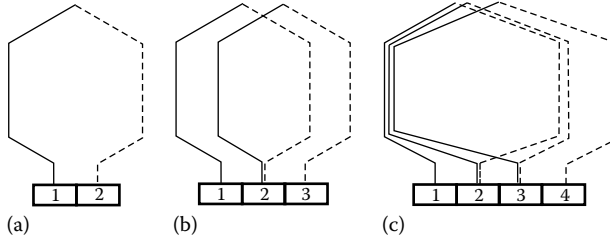


FIGURE 4.5
(a) Lap and wave coils and (b) their placement in two layers in slots.

**FIGURE 4.6**

Lap coils: (a) simple, (b) double, and (c) triple in steps.

The step of the coils at the commutator y_c is

$$y_c = m; \quad m = 1, 2 \text{—lap winding} \quad (4.1)$$

$$y_c = \frac{k - m}{p_1}; \quad m = 1, 2 \text{—wave winding} \quad (4.2)$$

K is the number of commutator copper sectors along the periphery. For $m = 1$, we obtain simple windings; for $m = 2$, we obtain what are called double windings.

The coils may have 2, 4, or even 6 ends, i.e., they may be simple, double, or triple coils (Figure 4.6).

For multiple end coils, it is possible that part of the turns be placed in adjacent slots to ease the commutation process.

The emfs induced in the coils, sides placed in a certain rotor slot are ac and trapezoidal in time, but we consider here only the fundamental.

In this latter case, each slot emf is characterized by an electric phase angle α_{ec} :

$$\alpha_{ec} = \frac{2\pi}{N_s} p_1 \quad (4.3)$$

where

p_1 is the pole pairs in the stator

N_s is the number of slots on the rotor

If the winding is fully symmetric, after every two poles (a period), the emfs are in phase. In general, however, t emfs are in phase:

$$t = \text{LCD}(N_s, p_1) \leq p_1 \quad (4.4)$$

so the number of distinct phase emfs in slots is N_s/t and they form a regular polygon with N_s/t sides with a phase shift angle α_{et} :

$$\alpha_{et} = \frac{2\pi}{N_s} t \quad (4.5)$$

If all the slot emfs are placed as phasors one after the other, we end up with t polygons.

4.2.1 Simple Lap Windings by Example: $N_s = 16$, $2p_1 = 4$

Let us proceed directly with an example.

Consider a simple lap winding for $N_s = 16$, $2p_1 = 4$. According to Equations 4.3 through 4.5,

$$\alpha_{ec} = \frac{2\pi}{N_s} p_1 = \frac{2\pi}{16} 2 = \frac{\pi}{4}$$

$$\alpha_{et} = \frac{2\pi}{16} t = \frac{2\pi}{16} 2 = \frac{\pi}{4} = \alpha_{ec}$$

Consequently, the order of the slots in the emf polygon is the same with the physical order of slots along the rotor periphery, and the polygon has $N_s/t = 16/2 = 8$ sides. So, there are two polygons that overlap completely because $t = p_1$ (Figure 4.7).

A side of the polygon contains the forward and backward sides of a coil situated under the neighboring excitation (PM) poles.

The brush-commutator, made up of insulated copper sectors has $K = uN_s = 1 \cdot N_s = 16$ sectors connects in series all coils, with a span y , equal to the pole pitch $\tau = N_s/2p_1 = 16/2 \cdot 2 = 4$ slot pitches (Figure 4.7).

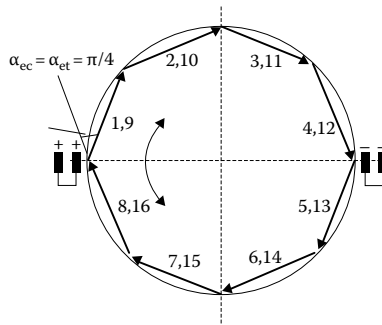
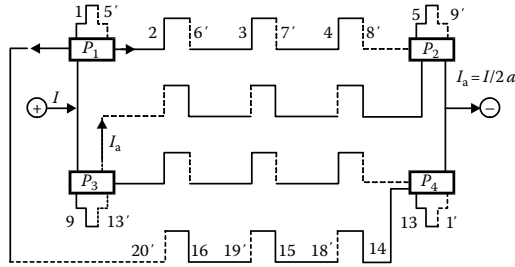


FIGURE 4.7

Elementary emf polygons: $N_s = 16$, $2p_1 = 4$, $m = 1$ (simple lap winding), and $u = 1$ (2-end (simple) coils).

**FIGURE 4.9**

The current paths composition for simple lap winding: $N_s = 16$, $m = 1$, $2p_1 = 4$, and $2a = 4$.

We should also note that in any time instant on each current path, one coil is short-circuited (it commutates or it changes current polarity while the rotor moves with one commutator sector; or the coil switches from one (+) coil path to the next (-)).

So out of $4(N_s/2a)$ coils per current path, only $3 = ((N_s/2a) - 1)$ produce emfs in series (Figure 4.9). The commutating coils are 1–5', 5–9', 9–13', and 13–1'. The nonuniformity of the airgap between various stator poles, due to manufacturing imperfections, may produce emfs/current paths which differ from each other. As all the current paths are in parallel at the brushes, circulating currents may occur which then circulate through the brush–commutator contact. To divert these currents, equipotential connections at the collector side are made between all (or most) polygon corners (Figure 4.7) as seen in Figure 4.8.

Note: For the double lap windings ($m = 2$), in essence, two simple lap windings are built: one for the even number of slots and one for the odd number of slots and then they are connected in parallel by doubling the brushes' span. So we end up with $2a = 2p_1m$ current paths but still with $2p_1$ brushes, albeit of double span.

4.2.2 Simple Wave Windings by Example: $N_s = 9$, $2p_1 = 2$

For large voltage and small (medium) current such as in universal motors for some applications, supplied at 220 V (110 V), 50(60) Hz, wave windings are more appropriate.

Although $2p_1 = 2$, 4poles, $2p_1 = 2$ is more common.

Let us consider here the case of $2p_1 = 2$.

The coil step at the commutator y_c is (Equation 4.1):

$$y_c = \frac{k - m}{p_1} = \frac{9 - 1}{1} = 8; \quad k = N_s = 9, \quad m = 1$$

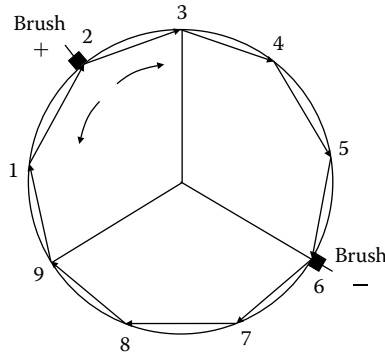


FIGURE 4.10

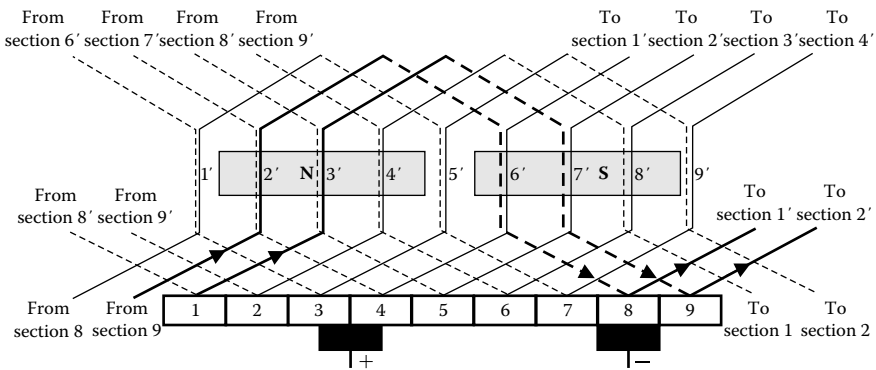


FIGURE 4.11

Now the coil span $y = \text{integer}(N_s/2p_1) = \text{integer}(9/2) = 4$ slot pitches.

This time, $t = \text{LCD}(9, 1) = 1$; all slot emfs have distinct phase angles, so, still, the order of polygon sides corresponds to the physical order of the slots (Figure 4.10).

For $2p_1 = 4$, this is not the case.

The number of current path $2a = 2$ (it is $2a = 2$ even for $2p_1 = 4$ poles; for double wave winding ($m = 2$), however, $2a = 2m$).

The winding layout is shown in Figure 4.11.

We should notice that now the coil 6-9' is short-circuited at brush (+) and coil 1-5' at brush (-).

So one current path contains three active slot (coil) emfs in series and the other contains four. This is very important in a realistic design when calculating the average emf per path V_{ea} .

So there is always some circulation current between current paths through the brush–commutator which has to be taken care of for the design. Also, the emf/path (no-load voltage) has notable time pulsations (N_s of them per revolution).

To improve commutation, it is possible to increase the number of commutator sectors by adopting double coils ($u = 2$, $K = N_s u = 9 \times 2 = 18$).

Note: The $N_s = 3$, $2p_1 = 2$ combination used for the micromotor in Figure 4.9c is an extreme case when one coil commutes (is short-circuited) all the time.

As seen already in Figure 4.9c, the short-circuited (commutating) coil does not have all the sides exactly in the neutral axis.

The brushes are moved away from their ideal position to improve commutation in small machines with a preferred direction of motion.

For more on windings of heavy duty (in transportation or metallurgy) brush–commutator motors, which have combined simple lap/double wave windings, see [3].

Airgap (slotless) windings are built in quite a few configurations which show notable peculiarities [1–3,5].

4.3 Brush–Commutator

Brush–commutator of small machines consists of K hard-drawn or silver copper wedge-shaped segments (sectors) insulated from one to another and connected to the armature coil ends (Figure 4.12a and b). The commutator segments are insulated also from the shaft by a die-cast resin holder, which is fixed on the motor shaft. The silver–copper segments can survive the flood soldering of armature-coil ends to the tongs at 300°C .

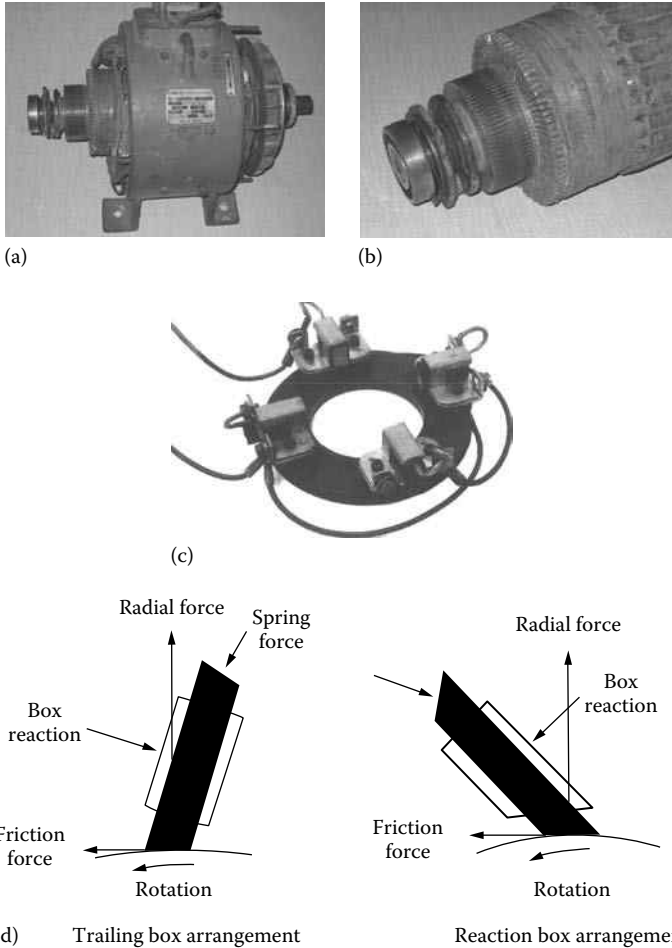
Spacers between copper segments are made from shellac-bound mica splitting (90° mica) or from epoxi-resin-bound fine mica (samcanite). They should wear as slowly as copper segments and be mechanically hard and elastic.

The brush gear consists of $2a$ brush holders, fitted to a yoke of insulating material which houses brushes of suitable electric conductivity and hardness (Figure 4.12c).

Good sliding friction quality and adequate (large) electric conductivity characterize good brushes. Brushes are made of

Natural graphite (good for large-voltage, small motors)

Hard carbon (low cost, used for fractional power and low-speed machines)

**FIGURE 4.12**

(a) Brush-commutator motor, (b) rotor, (c) the brush gear, and (d) brush positioning.

Electrographite (good conductivity, good for industrial and traction motors)

Metal graphite (high conductivity, good for low-voltage motors such as automobile actuators, etc.)

The brushes are pressed to the commutator by mechanical springs and are placed radially for bidirectional motion (inclined by 30° – 40° for narrow brushes) or in trailing box or reaction (inclined by 10° – 15°) box, for a preferred direction of motion (Figure 4.12d).

4.4 Airgap Flux Density of Stator Excitation MMF

When the armature current is zero (no load) and the machine is excited, through the stator heteropolar excitation mmf ($N_F i_F$ /per pole), the flux lines flow through the stator pole radially, then through the rotor teeth, then below the slots through the back iron, and then back to the airgap, stator poles, and stator back iron (Figure 4.13a).

The excitation airgap flux density $B_{gF}(x)$ distribution (Figure 4.13b) reveals ripples due to rotor slot openings. In fact, slotting leads to a reduction in the average value of the airgap flux density which is calculated by considering an increase in the equivalent airgap g_e (from g) by $K_C > 1$. K_C is the so-called Carter coefficient (derived by conformal transformation to determine magnetic field distribution in the airgap with open rotor slots):

$$K_C \approx \frac{\tau_s}{\tau_s - \gamma b_s}; \quad \gamma \approx \frac{(b_s/g)^2}{5 + b_s/g}; \quad g_e = K_C g \quad (4.8)$$

where

τ_s is the rotor slot pitch (in meters)

b_s is the rotor slot opening span (in meters)

g is the airgap

g_e is the equivalent airgap

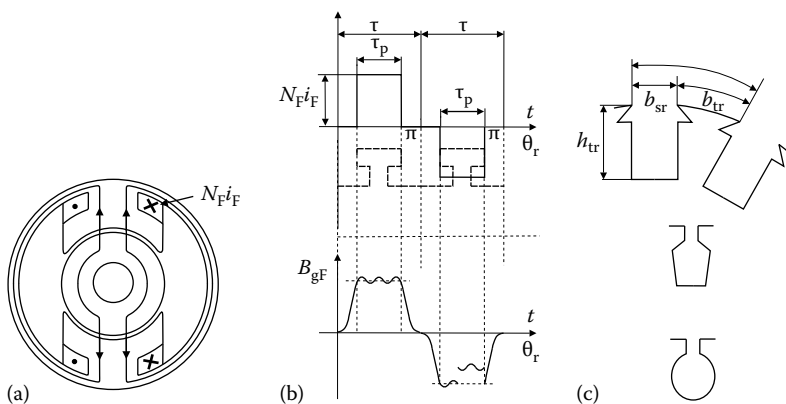


FIGURE 4.13

(a) Flux line of stator excitation field ($2p_1 = 2$ poles), (b) excitation mmf and flux, and (c) rotor slots.

The open-slot configuration, used for preformed armature coils insertion in slots, may increase the equivalent airgap by as much as 20%–30%, so it has to be considered in any practical design.

4.5 No-Load Magnetization Curve by Example

The relationship between the excitation (or PM) magnetic pole-flux, ϕ_{pole} , and the pole mmf $N_{\text{F}}i_{\text{F}}$, at zero armature current, is called the no-load magnetization curve and is a crucial design target.

The magnetic flux line path in Figure 4.14a is decomposed in a few components that can be represented by unique flux density/magnetic field

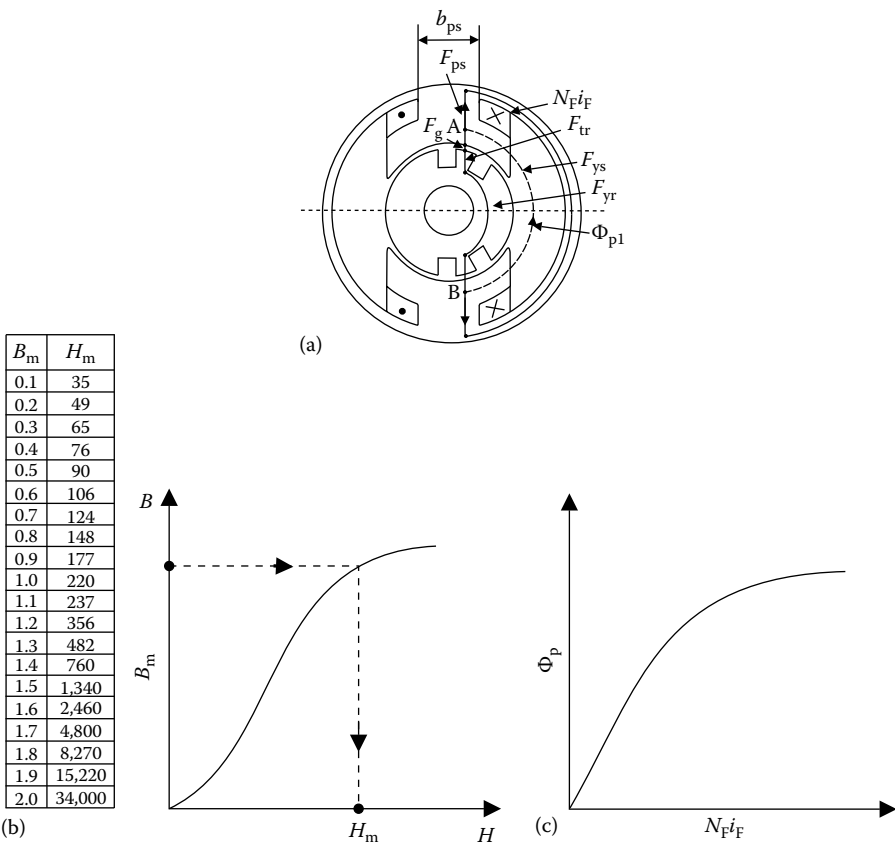


FIGURE 4.14

(a) Excitation flux line and its mmf contributions, (b) magnetic core $B(H)$ curve and table, and (c) no-load magnetization curve $\phi_p(N_{\text{F}}i_{\text{F}})$.

values: the two airgaps, the yoke and the poles in the stator and the two teeth and the yoke in the rotor, characterized by their corresponding mmf contributions: $2F_g$, F_{ys} , F_{ps} , F_{tr} , and F_{yr} (Figure 4.14).

Ampere's law along the flux line yields

$$2N_F i_F = 2F_g + 2F_{tr} + F_{yr} + 2F_{ps} + F_{ys} \quad (4.9)$$

Let us start with a given airgap flux density B_{gF} . Then the pole flux ϕ_p is

$$\phi_p \approx \tau_p L_e B_{gF}; \quad \frac{\tau_p}{\tau} = 0.65\text{--}0.75 \quad (4.10)$$

where

τ_p is the stator pole shoe

τ is the pole pitch ($\tau = \pi D / 2p_1$)

L_e is the equivalent stack length

$L_e = K_{\text{fill}} L$, where $K_{\text{fill}} > 0.9$ is the lamination filling factor and L is the measured stack length

Let us consider an example with $B_{gF} = 0.5$ T, stack length $L_e = 0.05$ m, rotor diameter $D = 0.06$ m, $2p_1 = 2$, airgap $g = 1.5 \times 10^{-3}$ m, $K_c = 1.2$, and $\tau_p/\tau = 0.7$; $(\phi_{pg})_{0.5\text{ T}}$ is

$$(\phi_{pg})_{0.5\text{ T}} = 0.5 \times \frac{\pi 0.06}{2} \times 0.7 \times 0.05 = 1.648 \times 10^{-3} \text{ Wb}$$

What interests us from now on is only the airgap flux density $B_{gF} = 0.5$ T.

The airgap mmf: $F_g = g K_c H_g = g K_c B_{gF} / \mu_0 = 1.5 \times 10^{-3} \times 1.2 \times 0.5 / 1.256 \times 10^{-6} = 716$ A turns. The rotor teeth flux density B_{tr} is calculated by equalizing the flux along a rotor tooth pitch to the one through the rotor tooth (b_{tr} -width):

$$B_{gF} \tau_s = B_{tr} b_{tr} \quad (4.11)$$

As in rectangular (open) slot rotors, the tooth width, b_{tr} , varies radially with the magnetic flux density; magnetic field, B_{tr} ; H_{tr} determined at the airgap, H_{t0} ; slot middle, H_{tm} ; and slot bottom, H_{t1} , are considered and averaged. Thus, we obtain H_{tav} as

$$H_{\text{tav}} = \frac{1}{6} (H_{t0} + 4H_{tm} + H_{t1}) \quad (4.12)$$

Thus, the rotor tooth mmf F_{tr} is

$$F_{tr} = h_{sr} H_{\text{tav}} \quad (4.13)$$

For our case, $b_{tr}/\tau_s = 0.4$ (0.40–0.55 in general); also the rotor diameter D per rotor slot height h_{tr} is large.

So B_{tr} is constant along the rotor tooth height (not so, in general).

So from Equation 4.12, $B_{tr} = B_g \tau_s / \tau_{tr} = 0.5 / 0.4 = 1.25 \text{ T}$.

From the table in Figure 4.14, $H_{tr}(B_{tr} = 1.25 \text{ T})$ is $H_{tr} = 417 \text{ A/m}$.

For a rotor slot height $h_{tr} = 0.01 \text{ m}$, F_{tr} is

$$F_{tr} = 417 \times 0.012 = 5.3 \text{ A turns}$$

For the rotor yoke, we first need an average thickness, h_{yr} :

$$h_{yr} = \frac{(D_{\text{rotor}} - D_{\text{shaft}})}{2} - h_{tr} = \frac{(0.06 - 0.01)}{2} - 0.012 = 0.025 - 0.012 = 0.013 \text{ m} \quad (4.14)$$

So the rotor yoke average flux density B_{yr} is

$$B_{yr} = \frac{(B_g \tau \times (\tau/2))}{h_{yr}} = \frac{(0.5 \times 0.0471)}{0.013} = 1.81 \text{ T} \quad (4.15)$$

$$\tau \approx \pi \frac{D_r}{2p_1} = \pi \frac{0.06}{2} = 0.0942 \text{ m}$$

The value of B_{yr} is close to the limit in practice because the armature current mmf will add additional yoke flux.

From the table in Figure 4.14, $H_{yr} = 8300 \text{ A/m}$.

The average magnetic field path in the rotor back iron l_{yr} is

$$l_{yr} = \frac{\pi(D - 2h_{sr} - h_{yr})}{2} = \frac{\pi(0.06 - 2 \times 0.012 - 0.013)}{2} = 0.0361 \text{ m} \quad (4.16)$$

So the rotor yoke mmf F_{yr} is

$$F_{yr} = l_{yr} \times N_{yr} = 0.0361 \times 8310 = 300 \text{ A turns} \quad (4.17)$$

As seen in Figure 4.14a, there is some leakage flux ϕ_{pe} , which closes the paths directly between the stator poles (or PMs), which is proportional to the total mmf F_{pp} between the stator poles through the rotor (points AB):

$$F_{pp} = 2F_g + 2F_{tr} + F_{yr} = 2 \times 716 + 2 \times 5.3 + 300 = 1742 \text{ A turns} \quad (4.18)$$

As the first approximation, we may consider

$$\phi_{pe} = \phi_{pg} \frac{F_{pp} - 2F_g}{2F_g} = 1.648 \times 10^{-3} \times \frac{1742 - 1432}{1432} = 0.3574 \times 10^{-3} \text{ Wb} \quad (4.19)$$

So the total stator pole flux, ϕ_{PF} , is

$$\phi_{PF} = \phi_{pg} + \phi_{pe} = (1.648 + 0.3574) \times 10^{-3} = 2.01 \times 10^{-3} \text{ Wb} \quad (4.20)$$

Now the flux densities in the stator pole shoe and pole body are

$$B_{\text{pshoe}} = B_{\text{gF}} \times \frac{\phi_{\text{pF}}}{\phi_{\text{pe}}} = 0.5 \times \frac{2.01 \times 10^{-3}}{1.648 \times 10^{-3}} = 0.61 \text{ T} \quad (4.21)$$

$$B_{\text{pbody}} = B_{\text{pshoe}} \times \frac{\tau_p}{b_{\text{pstator}}} = \frac{0.61}{0.5} = 1.22 \text{ T} \quad (4.22)$$

(with $\tau_p = 0.7\tau$ in our case)

The stator pole body width $b_{\text{pstator}}/\tau_p \approx 0.4\text{--}0.55$ to leave enough room for the excitation coils, while avoiding too heavy magnetic saturation (we take here $b_{\text{pstator}}/\tau_p = 0.5$).

Now it is reasonable to assume that the excitation coil radial height h_{cF} is equal to or smaller than the rotor slots.

From the table in Figure 4.14, at $B_{\text{pbody}} = 1.22 \text{ T}$, we find $H_{\text{pb}} = 360 \text{ A/m}$ and with pole body height $h_{\text{cF}} = h_{\text{tr}} = 0.012 \text{ m}$ (and neglecting the mmf in the pole shoe because B_{pshoe} is small (0.61 T)), the pole body mmf F_{ps} is

$$F_{\text{ps}} = h_{\text{tr}} \times H_{\text{pbody}} = 0.012 \times 360 = 5 \text{ A turns} \quad (4.23)$$

Finally, the flux density in the stator back iron B_{ys} is

$$B_{\text{ys}} = B_{\text{gF}} \times \frac{(\tau_p/2)}{h_{\text{ys}}} \quad (4.24)$$

The stator core is designed for $B_{\text{ys}} \leq 1.4 \text{ T}$ to leave room for armature flux contribution before a too heavy magnetic saturation is reached.

With $\tau_p = 0.7\tau = 0.7 \times 0.0942 = 0.066 \text{ m}$, $B_{\text{gF}} = 0.5 \text{ T}$, and $B_{\text{ys}} = 1.3 \text{ T}$, we obtain the stator yoke, h_{ys} :

$$h_{\text{ys}} = \frac{B_{\text{gF}}}{B_{\text{ys}}} \frac{\tau_p}{2} = \frac{0.5}{1.3} \times 0.066 = 0.02536 \text{ m} \quad (4.25)$$

Now for $B_{\text{ys}} = 1.3 \text{ T}$ from the table in Figure 4.14b, $H_{\text{ys}} = 482 \text{ A/m}$.

The length of the flux path in the back iron, l_{ys} , is approximately

$$l_{\text{ys}} = \frac{\pi(D_{\text{rotor}} + 2g + 2h_{\text{pshoe}} + 2h_{\text{pbody}} + h_{\text{ys}})}{2p_1} \quad (4.26)$$

$$h_{\text{pshoe}} \approx \frac{(\tau_p - b_{\text{pbody}})}{2\sqrt{3}} = \frac{(0.066 - 0.06612)}{2\sqrt{3}} = 0.0095 \text{ m} \quad (4.27)$$

The $\sqrt{3}$ factor stands for a 30° angle of the pole shoe geometry.

$$l_{\text{ys}} = \frac{\pi(0.06 + 2 \times 0.0015 + 2 \times 0.00095 + 2 \times 0.12 + 0.02536)}{2 \times 1} = 0.206 \text{ m}$$

The external diameter of the machine D_{out} is

$$D_{\text{out}} = D_{\text{rotor}} + 2g + 2h_{\text{pshoe}} + 2h_{\text{pbody}} + 2h_{\text{ys}} = 0.15672 \text{ m} \quad (4.28)$$

The ratio $D_{\text{rotor}}/D_{\text{out}} = 0.06/0.15672 = 0.3828$. This is too low a value for a close to optimal design. For two poles, this design ideally recommends $(D_{\text{rotor}}/D_{\text{out}})_{2p_1=2} \approx 0.45\text{--}0.55$.

The mmf in the stator yoke F_{ys} is

$$F_{\text{ys}} = h_{\text{ys}} \times H_{\text{ys}} = 0.206 \times 482 = 99.292 \text{ A turns} \quad (4.29)$$

Now the total field (excitation) A turns per pole $N_{\text{F}}i_{\text{F}}$ is Equation 4.10:

$$\begin{aligned} N_{\text{F}}i_{\text{F}} &= F_{\text{g}} + F_{\text{tr}} + F_{\text{pr}} + (F_{\text{yr}} + F_{\text{ys}})/2 = 716 + 5.3 + 5 \\ &+ (300 + 99.292)/2 = 926.3 \text{ A turns/pole} \end{aligned} \quad (4.30)$$

The total contribution of iron may be assessed through a saturation coefficient $K_{\text{s}} > 0$:

$$1 + K_{\text{s}} = \frac{N_{\text{F}}i_{\text{F}}}{F_{\text{g}}} = \frac{926.3}{716} = 1.294 \quad (4.31)$$

Values of $K_{\text{s}} = 0.2\text{--}0.4$, or even more, are considered feasible.

The computation sequence above may be easily mechanized by building a computer code where the airgap flux density is set at 10(20) values to finally yield the entire $N_{\text{F}}i_{\text{F}}(\phi_{\text{pg}})$ curve, that is the no-load magnetization curve, which is used in design and performance assessment.

Note that in this paragraph, a preliminary machine sizing was, in fact, done.

The rotor diameter D_{rotor} and stack length L have been given, but they may be assigned starting values based on tangential force f_{tn} ; $L/D_{\text{r}} = 0.5\text{--}2.5$.

Let us consider here the inverse process, that is, to calculate a feasible rated torque for the case study here. For a rated tangential force $f_{\text{t}} = 0.7 \text{ N/cm}^2$, the rated torque would be

$$T_{\text{e}} = f_{\text{t}} \pi D_{\text{r}} L_{\text{e}} \frac{D_{\text{r}}}{2} = 7 \times 10^3 \pi 0.06^2 \frac{0.05}{2} = 1.978 \approx 2 \text{ N m} \quad (4.32)$$

We may even calculate the slot mmf required to produce this torque (from the BIL formula [Chapter 1]):

$$T_{\text{e}} \approx B_{\text{gF}} \frac{\tau_{\text{p}}}{\tau} L \frac{D_{\text{r}}}{2} \pi D_{\text{r}} A \quad (4.33)$$

$$A = \frac{2 \times 2.2}{0.5 \pi 0.06^2 \times 0.7 \times 0.05} = 22242 \text{ A turns/m} \quad (4.34)$$

This is a rather large value for $D_r = 0.06$ m, but let us see what it requires in terms of current density if the slot depth $h_{tr} = 0.012$ m and the tooth/slot (b_{tr}/b_{sr}) ratio is unity:

$$A = \frac{2N_c i_c N_s}{\pi D_r}; \quad 2N_c i_c = K_{fill} \frac{\pi D_r}{N_s} \frac{1}{2} h_{tr} j_{corotor} \quad (4.35)$$

where K_{fill} is the rotor slot copper fill factor $K_{fill} = 0.4\text{--}0.6$; $K_{fill} > 0.45$ only for preformed coils. The slot width $b_{tr} \approx (\pi D_r / N_s) 1/2$ (half of rotor slot pitch).

So from Equation 4.23, we can determine the rated current density $j_{corotor}$, then the A turns per coil $N_c i_c$ (there are two coils in every rotor slot because the armature winding has two layers), and then the number of slots, which has to be an even number for simple lap windings (low voltage) and an odd number for simple wave windings (large voltage):

$$j_{corotor} = \frac{2A}{K_{fill} h_{tr}} = \frac{2 \times 22242}{0.45 \times 0.012} = 8.2377 \times 10^6 \text{ A/m}^2 = 8.2377 \text{ A/mm}^2 \quad (4.36)$$

Forced air cooling is required to secure thermally safe operation at this current density.

The number of slots N_s is our choice, and we should consider a commutation which favors large N_s , but then the commutator geometry limitations have to be considered.

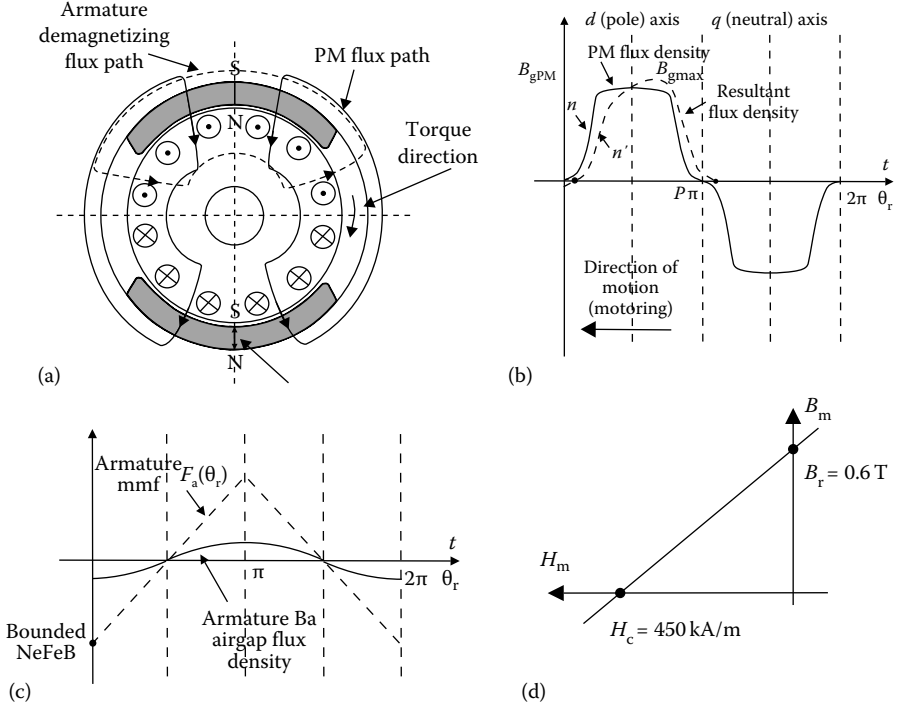
For our case ($D_{rotor} = 0.06$ m), we may safely choose $N_s = 12, 16$, or 18 .

4.6 PM Airgap Flux Density and Armature Reaction by Example

Both surface PMs and interior PMs (Figure 4.2) have been used, but today strong surface PMs of NeFeB or of hard Ferrites (for micromotors) are preferred. The main reason is improved brush commutation, because the total magnetic airgap includes the PM radial (axial, in the case of axial airgap configurations) height h_{PM} (Figure 4.15a).

The PM may exhibit radial or parallel magnetization and both have merits and demerits. Rounded PM corners—to be studied by FEM—are provided to reduce PM flux density gradients at their ends and thus reduce the PM (cogging) torque at zero current due to slot openings. The PM span τ_{PM} per pole pitch τ may now be larger: $\tau_{PM}/\tau = 0.66\text{--}0.85$.

Larger values of this ratio lead to more torque per given current (more of the periphery is active) but the leakage flux between the poles now takes

**FIGURE 4.15**

(a) PM airgap flux density, (b) its distribution, (c) armature mmf and flux density, and (d) PM demagnetization curve.

from the useful PM flux density, B_{gPM} :

$$B_{gPM} = \frac{B_m}{1 + K_{IPM}} \quad (4.37)$$

where B_m is the flux density in the PM.

The leakage factor for surface PMs is in general $K_{IPM} \approx 0.15-0.3$ and may be determined with good precision by numerical methods such as FEM.

A few remarks are in order:

- The current polarity in all coils beneath a PM pole is the same due to the brush-commutator operation if the brushes are in the ideal position (neutral axis of PM flux).
- The mmf of rotor currents rises stepwise in each slot (approximately linearly) from the d (pole(PM)) axis position (Figure 4.14c). And so does its flux density:

$$F_a(\theta_r) = F_{am} \left(\frac{2\theta_r}{\pi} - 1 \right); \quad 0 \leq \theta_r \leq \pi, \text{ in electric radians} \quad (4.38)$$

The maximum armature mmf F_{am} corresponds here to half the pole pitch:

$$F_{\text{am}} = \frac{2N_c i_c N_s}{4\pi} \quad (4.39)$$

For contemporary magnets, the recoil permeability $(\mu_{\text{PM}})_{\text{pu}} = 1.05\text{--}1.2$.

Now the armature flux density B_a in the airgap is

$$B_a(\theta_r) \approx \frac{4F_a(\theta_r) \cdot \mu_0}{gK_c + h_{\text{PM}}(\mu_{\text{PM}})_{\text{pu}}} \quad (4.40)$$

$(\mu_{\text{PM}})_{\text{pu}}$ is the relative value of PM permeability.

Now, for the surface PM stator along the entire periphery, the $B_a(\theta_r)$ is linear (Equation 4.40 and Figure 4.15) since the total magnetic airgap, $gK_c + h_{\text{PM}}(\mu_{\text{PM}})_{\text{pu}}$, is all along the same (uniform).

In contrast, for excited stators or interior PM stators, the magnetic airgap becomes much larger between the poles, and thus in Equation 4.39, F_{am} is reduced by τ_p/τ and the airgap is increased for $0 < \theta_r < 1 - (\pi/2)(\tau_p/\tau)$ and $\pi > |\theta_r| > \pi(\tau_p/\tau)$ (between poles).

- The armature (rotor mmf) flux density in the airgap brings up additional flux per trailing half a pole and a reduction on the entry half a pole for motoring (Figure 4.15a and b).

In the surface PM stator case, it is unlikely that magnetic saturation in the rotor teeth of the trailing half pole will occur. This is in contrast to the excited stator, where this highest rotor tooth flux density, $B_{\text{trmax}} = B_{\text{gmax}}(\tau_s/b_{\text{tr}})$, becomes a major design variable (limitation).

- As expected, the large magnetic airgap of the surface PM stator will lead to a notably smaller rotor commutating coil (leakage) inductance, L_a , which in turn will ease the commutation process.
- For the PM stator, the excitation flux density is assigned a value—for given PMs and machine geometry and thus—so the no-load magnetic curve loses its meaning. But sizing the PM to produce the calculated PM airgap flux density B_{gPM} is necessary.

Let us consider the same rotor example as in the previous Section 4.5, now with $B_{\text{gPM}} = 0.5\text{ T}$.

As a rule, for surface PM design, $B_{\text{gPM}} \approx (0.5\text{--}0.8)B_r$, where B_r is the remnant flux density of PMs (lower values are adequate for slotless (in air) armature windings).

Let us consider bonded (lower cost) NeFeB magnets with $B_r = 0.8 \text{ T}$ and $H_c = 650 \text{ kA/m}$:

$$(\mu_{rPM})_{pu} = \frac{B_r}{(\mu_0 H_c)} = 1.05$$

The PM pole may be replaced by a coil in air with an mmf $\theta_{PM} = H_c h_{PM}$, whose radial thickness is equal to the magnet thickness.

The pole mmf is replaced by the PM mmf $h_{PM} \times H_m$, where H_m is the actual magnetic field in the PM corresponding to B_m , but with positive sign.

Now as the flux density in the PM, B_m (with leakage flux coefficient $K_{IPM} = 0.3$) from Equation 4.36,

$$B_m = B_{gPM}(1 + K_{IPM}) = 0.5(1 + 0.3) = 0.65 \text{ T}$$

$$|H'_m| \approx \frac{B_m}{\mu_0} = \frac{0.65}{1.25 \times 10^{-6}} = 0.5175 \times 10^6 \text{ A/m} \quad (4.41)$$

So the mmf in the PMs is

$$F_{PM} = H'_m \times h_{PM} = 0.5175 \times 10^6 \times h_{PM} \quad (4.42)$$

Now for the case in point, the balance of mmf is

$$2H_c \times h_{PM} = 2\theta_{PM} = 2F_g + 2F_{tr} + F_{yr} + F_{ys} + 2F_{PM} \quad (4.43)$$

For $B_{gPM} = 0.5 \text{ T}$ as in Section 4.5., $F_g = 716 \text{ A turns}$, $F_{tr} = 5.3 \text{ A turns}$, and $F_{yr} = 300 \text{ A turns}$.

The above interpretation of $B_m(H'_m)$ linear curve through origin (Equation 4.41) for PMs is valid if the PM is first replaced by a coil in air with an mmf of $H_c h_{PM}$. But F_{yr} (mmf in the stator yoke) is not known because the PM height is not known. Even in Section 4.5., it was moderate ($F_{ys} = 99 \text{ A turns}$). As the PM height h_{PM} is to be smaller, F_{ys} will be even smaller (smaller flux line length in the stator yoke).

So the only unknown in Equation 4.43 is the magnet height h_{PM} , but the equation has to be solved iteratively.

For a conservative solution, let us keep $F_{ys} = 99 \text{ A turns}$ and, consequently, from Equation 4.43

$$2h_{PM}(0.650 \times 10^6 - 0.5175 \times 10^6) = 2 \times 716 + 2 \times 5.8 + 300 + 99 = 920.8 \quad (4.44)$$

So the magnet thickness is

$$h_{PM} = 3.4747 \times 10^{-3} \text{ m}$$

Note that the mechanical airgap $g = 1.5 \text{ mm}$.

As the total stator yoke thickness $h_{ys} = 0.025$ m (because the total flux per pole holds), the external diameter of the stator is

$$\begin{aligned} D_{out} &= D_{rotor} + 2g + 2h_{PM} + 2h_{ys} \\ &= 0.06 + 2 \times 0.0015 + 2 \times 0.00347 + 2 \times 0.025 \approx 0.12 \text{ m} \end{aligned} \quad (4.45)$$

(instead of 0.156 m as it was with dc excitation)

So the surface PM stator leads to a reduced stator outer diameter for the same rotor, same airgap flux density, same torque, and same rotor with same copper losses.

So, the PM dc brush motor is not only smaller but also with larger efficiency as the excitation losses are considered zero.

Note: To be fair, we should add the PM magnetization energy losses instead of excitation losses. However, the PM magnetization energy losses are so small when spread over the operational life of the PM machine that they can be neglected.

4.7 Commutation Process

Commutation can be defined as a group of phenomena related to the rotor coils current polarity reversal, when each one of them passes through the zero excitation (PM) flux density (neutral) axis.

In fact, this polarity reversal of current reduces to the coil switching from the (+) current path to the subsequent (−) current path.

During this interval, the respective rotor coil is short-circuited by the brush (brushes), and this phenomenon is known as brush commutation.

This is very similar to the soft (slow) power electronic commutation which means also applying a zero voltage to the respective coil.

The commutation is good if there is no visible scintillation and the commutator surface remains clean and undeteriorated for continuous duty at maximum rotor current and speed for which the machine was designed.

The behavior of the copper brush contact phenomenon with speed, current, etc. is a science in itself, and so it warrants separate treatment.

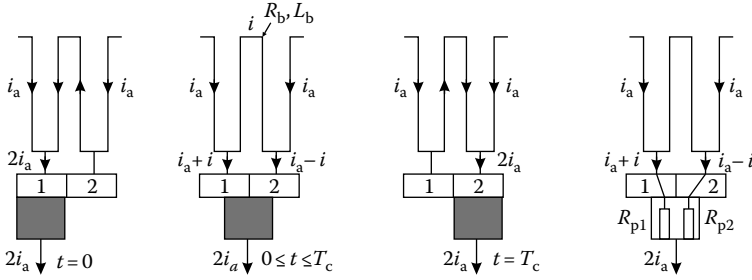
This is beyond our scope here.

Also, the brush span here is considered equal to the copper segment span at the commutator, and the intersegment insulation thickness is neglected.

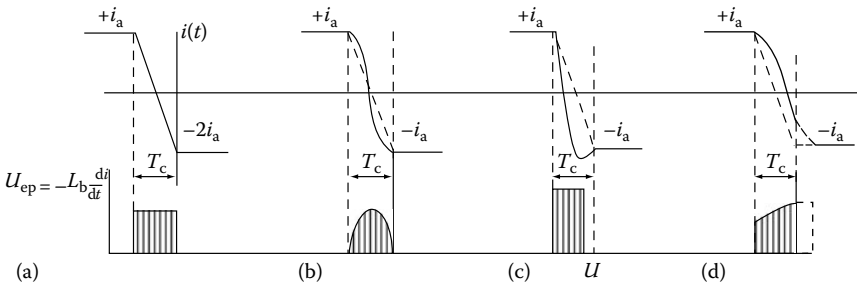
The unfolding of commutation process in time is illustrated in Figure 4.16.

Let us denote the rotor coil resistance and inductance as R_c and L_c , respectively, and the brush–segment contact resistance corresponding to copper segment 1 and 2 as R_{b1} and R_{b2} , respectively:

$$R_{b1} = R_b \frac{A_b}{A_{b1}} = R_b \frac{1}{1 - t/T_c}; \quad R_{b2} = R_b \frac{A_b}{A_{b2}} = R_b \times T_c/t \quad (4.46)$$

**FIGURE 4.16**

Rotor coil current reversal during commutation.

**FIGURE 4.17**

Commutation and self-emf ($-L_c(di/dt)$) during commutation: (a) linear, (b) resistive, (c) early, and (d) late.

A_b —brush (segment) total area.

Now the commutating coil circuit (voltage) equation is (Figure 4.16)

$$R_c i + R_{b1}(i_a + i) - R_{b2}(i_a - i) = -L_c \frac{di}{dt} + V_{er} + V_{ec} = \sum V_{ei} \quad (4.47)$$

$-L_c(di/dt) > 0$ is the self-induced coil voltage

$V_{er} > 0$; $V_{er} = B_{amax}\pi D_r n L N_c$ is the motion-induced voltage by the remaining (non-commutating) armature coils

$V_{ec} < 0$ is the motion-induced voltage by interpoles (Figure 4.3) on stator with their coils in series at brushes (PM motors lack interpoles). V_{ec} has opposite sign in comparison with V_{er} and $-L_c(di/dt)$ to effect total emf cancellation and thus provide for safe, in time, resistive commutation (Figure 4.17b).

In fact, the interpole mmf/pole $F_{interpole} = N_{interpole} i_{brush} > \text{armature mmf per half a pole}$ (Equation 4.39) to secure $|V_{ec}| > V_{er}$ for brushes placed in the neutral electric axis.

Note that the interpole current is the current at the brushes: $i_{\text{brush}} = 2aI_c$. For an intuitive interpretation of current commutation, let us consider a few particular simplified cases:

Linear commutation: $\sum V_{ei} = 0$ and $R_c = 0$.

In this case, the current $i(t)$ solution from Equation 4.46 with Equation 4.47 is

$$i(t) = i_c(1 - 2t/T_c) \quad (4.48)$$

The current variation is linear (Figure 4.17a) and the self-induced emf, $e = -L_c(di/dt) = L_c 2i_c nK$, is constant, where n is the speed in rps and K is the number of commutator copper segments (sectors). The zeroing of the resultant emf presupposes, in fact, the existence of interpoles.

Resistive commutation: $\sum V_{ei} = 0$ but $R_c \neq 0$.

Again, from Equation 4.46 with Equation 4.47, it follows that (Figure 4.17b)

$$i(t) = \frac{i_c(1 - 2t/T_c)}{1 + \frac{R_c}{R_b} \frac{t}{T_c} (1 - \frac{t}{T_c})} \quad (4.49)$$

For resistive commutation, total zero emf is needed, so interpoles are necessary (in excited stators).

But, again, the commutation current variation from $+i_c$ to $-i_c$ is completed exactly within commutation time $T_c = 1/(nK)$; for $n = 60$ rps and $K = 24$, $T_s \approx 0.7 \times 10^{-3}$ s.

Early commutation means that the interpoles are too strong, and thus total emf becomes negative at some point (Figure 4.17c); an increase of interpoles' airgap solves the problem.

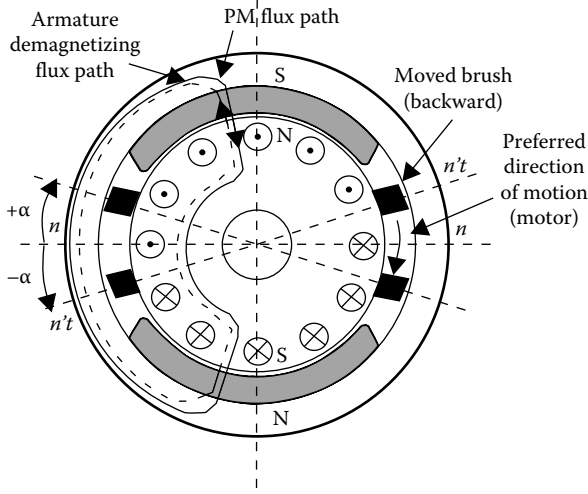
Late commutation means a leftover emf between adjacent segments (Figure 4.17d), and thus scintillation occurs. The solution is to enforce the interpoles or reduce the airgap of interpoles.

For the PM motors, which do not have interpoles, and a single direction of motion, it is feasible to move the brushes in countermotion direction-wise by a small angle ($-\alpha$) to produce a negative PM flux density in the commutating coil (see n, n' , Figures 4.15b and 4.18).

This produces a negative V_{er} to cancel $-L_c(di/dt)$ and produces again zero total emf in the commutating coil short-circuited by brushes. The α/π part of rotor (armature) mmf now "demagnetizes" the magnets (Figure 4.18). This is the "price" for better commutation.

4.7.1 AC Excitation Brush-Commutation Winding

As already mentioned, the universal motor is ac series-excited and supplied. In contrast to dc brush commutation, where the excitation (or PM) flux in the

**FIGURE 4.18**

Brushes shifted countermotion-wise by α from neutral axis for better commutation.

commutating coil is constant, and thus no emf from excitation is induced, in the latter, the ac excitation induces a stator frequency (speed independent) additional emf V_e in the commutation coil with its pole flux, which is always there. This emf, V_e , which occurs even at zero speed, has to be limited to less than 3–4 V to eventually secure safe commutation. The smaller the number of turns per coil ($N_c = 1, 2$), the better.

Reducing the ac frequency is another solution.

Note: The commutating inductance is placed with its sides in excitation (PM) neutral axis, and thus its inductance L_c refers to the d axis magnetic reluctance main field, L_{cm} , and the leakage inductance, L_{cl} :

$$L_c = L_{cl} + L_{cm} \quad (4.50)$$

The coil leakage inductance L_{cl} is having the same formula as in ac rotary machines and includes the slot leakage component L_{cls} and the coil end-connection component L_{cle} , with L_{cls} from Chapter 2 (Equation 4.44):

$$L_{cls} = 2\mu_0 N_c^2 \left(\frac{h_{rs}}{3b_{rs}} + \frac{L_{end}}{L_e} \ln \frac{L_{end}}{r_{end}} \right)$$

L_{cm} is

$$L_{cm} \approx \frac{N_c^2}{R_{gm}} = \frac{\mu_0 N_c^2 \tau_p L_{stack}}{g_m}; \quad g_m = gK_c + h_{PM} \quad (4.51)$$

where

h_{rs} is the slot height

b_{rs} is the rotor slot average width (valid for open slots)

L_{end} is the coil end-connection length at one machine end

r_{end} is the radius of coil end-connection bundle

L_e is the magnetic machine stack length

R_{gm} is the airgap magnetic reluctance

It is thus evident that the surface PM stator, with a much larger magnetic airgap (due to $h_{PM} > g$), has a smaller commutating coil inductance.

4.8 EMF

As already discussed, the emf at the brushes corresponds to one of the $2a$ current paths in parallel and thus collects coil emfs (V_{ec}) from all coils, on a current path, in series.

But, again, the regular flux density in the airgap is nonuniform due to the armature reaction (see Section 4.7) as shown in a different form in Figure 4.19.

The motion emf in a coil, produced by excitation, from BIL formula is

$$V_{ea}(x) = 2N_c(B_a(x) + B_{gF}(x)) \times L_{stack} \times \pi \times D_r \times n \quad (4.52)$$

where

$B_a(x)$ is the local armature flux density in the airgap

$B_{gF,PM}$ is the local excitation (PM) flux density in the airgap

Factor 2 comes from the fact that there are two active sides for each coil, and they are considered here exactly one pole-pitch aside (diametrical coils).

As evident from Figure 4.19, the local resultant flux density varies while the coil moves under the pole, so its emf, which is the voltage drop between neighboring commutator segments, is nonuniform.

This nonuniformity may be large in non-PM motors, and thus it may endanger the life of insulation layer between the commutator segments, which has to stay below 25 V.

Compensating windings, placed in stator pole slots and connected, again in series with the armature (at brushes), destroy the armature reaction under stator poles for all rotor currents and thus eliminate the problem (Figure 4.19b).

In addition, compensation windings, used only in very heavy duty transportation motors, reduce the magnetic saturation produced by the armature-reaction field.

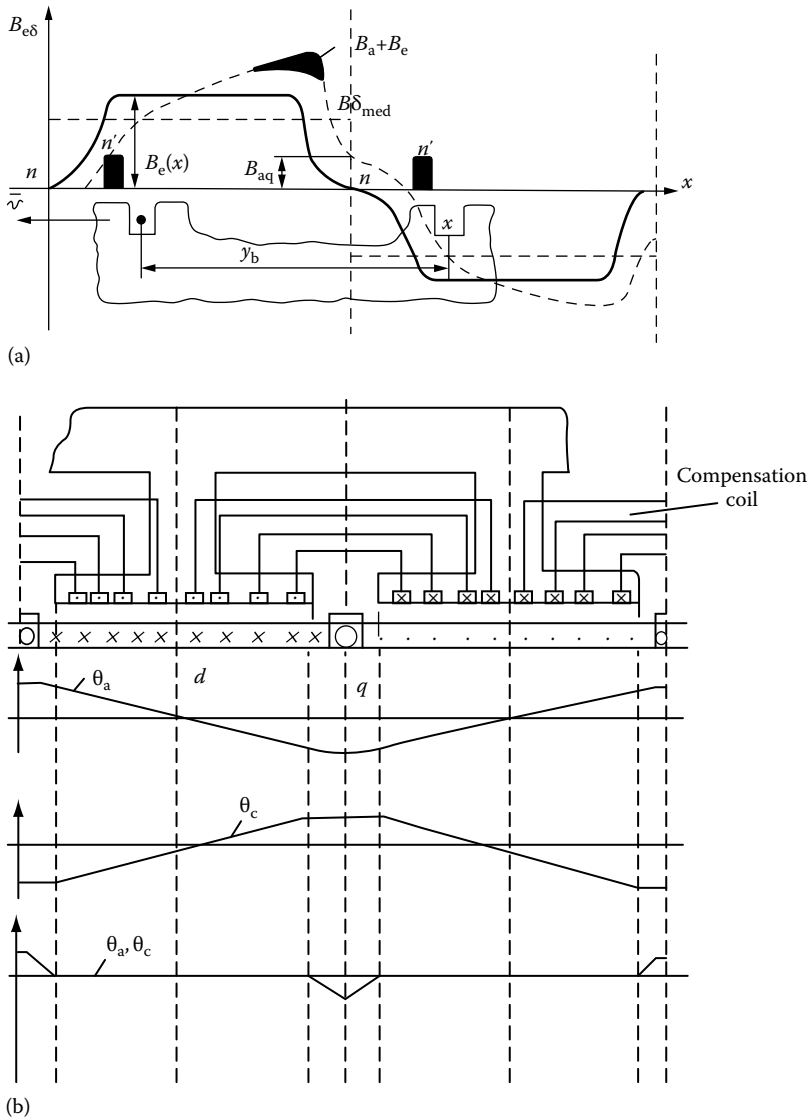


FIGURE 4.19
(a) Resultant nonuniform flux density in the airgap and (b) compensation winding.

Now we may define an average total airgap flux density B_{gan} :

$$B_{\text{gan}} = \phi_p / (\tau L_e) \quad (4.53)$$

where L_e is the magnetic stack length ($L_e = L_{\text{stack}} K_{\text{filliron}}$).

There are ideally $N_s/2a$ (in reality $\approx N_s/2a - 1$) active coils in series per current path, and the brush emf V_{ea} is

$$V_{\text{ea}} = K_e n \phi_p \quad (4.54)$$

$$K_e = \frac{N p_1}{a}; \quad N = N_s 2N_c \quad (4.55)$$

where N is the total slot conductors per rotor. So, the no-load (average) voltage V_{ea} (at brushes) is proportional to speed n (rps) and total flux/pole through a proportionality coefficient which depends on the number of rotor slots N_s , number of pole pairs p_1 , and the inverse of current path pairs a .

It is now evident why lap windings (with larger $2a = 2p_1 m$) are preferred for low-voltage, high-current applications.

4.9 Equivalent Circuit and Excitation Connection

As we have settled that stator excitation (or PM) and armature mmfs and airgap magnetic fields have orthogonal electric axes (d and q ; $\alpha_{\text{edq}} = 90^\circ = p_1 \alpha_{\text{gdq}}$), the equivalent circuit is straightforward, especially for separate or PM excitation (Figure 4.20).

C is the compensation winding and K is the interpole winding.

The commutation (K) and compensation (C) stator windings, both in the armature windings axis, are lumped into the armature winding. To eliminate the excitation power source, shunt or series excitation connection is used

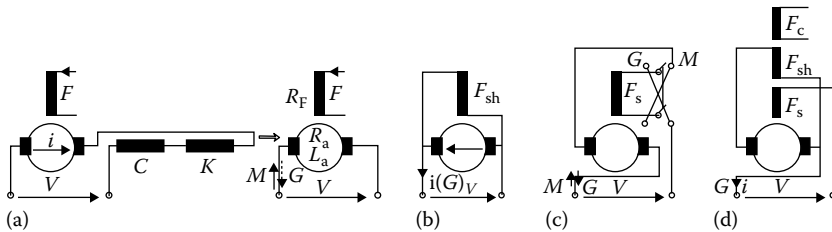


FIGURE 4.20

Equivalent circuits and excitation connection options: (a) separate excitation (or PM), (b) shunt excitation, (c) series excitation, and (d) mixed excitation.

(Figure 4.20b and c). Mixed excitation is used today only for a few remaining dc generators in vessels or in Diesel–electric locomotives (Figure 4.20d).

In the following sections, we will consider only the separate (and PM) excitation in some detail for both motor and generator modes and, in short, dc series and ac series brush motors.

4.10 DC Brush Motor/Generator with Separate (or PM) Excitation/Lab 4.1

Let us consider a separate excitation dc brush machine (Figure 4.21) [4].

The machine is fed with a dc source in the stator and then in the rotor, separately, in this sequence. This is the motor mode. The general equation is

$$V_a = R_a i_a + L_a \frac{di_a}{dt} + V_{ea} + \Delta V_{\text{brush}} \quad (4.56)$$

For steady state (Equation 4.54),

$$V_{ea} = K_e n \phi_p; \quad i_a = \text{const} \quad (4.57)$$

The brush voltage drop ΔV_{brush} (around 1 V) is not negligible for low-voltage (automotive) motors.

Multiplying by the armature current, i_a , we obtain the power balance:

$$V_a i_a = R_a i_a^2 + V_{ea} i_a + \Delta V_{\text{brush}} i_a \quad (4.58)$$

The electromagnetic power $V_{ea} i_a$ is equal to the product of electromagnetic torque T_e and mechanical speed ($2\pi n$):

$$V_{ea} i_a = T_e 2\pi n \quad (4.59)$$

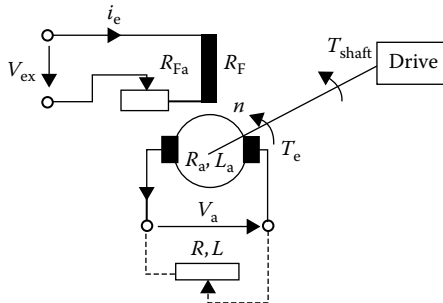


FIGURE 4.21

DC brush machine with separate excitation.

Making use of Equation 4.57, for V_{ea} ,

$$T_e = K_e \phi_p i_a / 2\pi \quad (4.60)$$

The electromagnetic torque is proportional to pole flux ϕ_p and brush (input (armature)) current i_a . And, with brushes in neutral electric axis, any change in the armature current i_a will be effect-less on the excitation circuit current (due to 90° electric phase shift between armature and excitation circuits).

So the excitation circuit equation is

$$V_F = R_F i_F + L_{Ft} \frac{di_F}{dt} \quad (4.61)$$

$L_{Ft}(i_F)$ is the transient inductance (Chapter 2) in the presence of magnetic saturation:

$$L_{Ft}(i_F) = L_F(i_F) + \frac{\partial L_F}{\partial i_F} i_F \quad (4.62)$$

For the PM excitation, ϕ_p is constant in general if the magnetic saturation in the machine magnetic cores is negligible. Otherwise, a small pole flux decrease with armature current occurs. This may lead to motor self-overspeeding with load if the supply voltage stays constant. However, this does not happen if closed-loop torque (speed) control is performed through power electronics, but it is still important for refined machine design (sizing).

Now, if we add the mechanical losses, p_{mec} , and the rotor iron losses (the stator excitation (or PM) field produces, in any point of the rotor core, a traveling field at frequency $\omega_r = 2\pi n p_1$, and thus both hysteresis and eddy current core losses occur), we get the complete power balance (Figure 4.22). Generating mode (Figure 4.22a) is used in contemporary motor drives for regenerative braking with bidirectional power electronics supply.

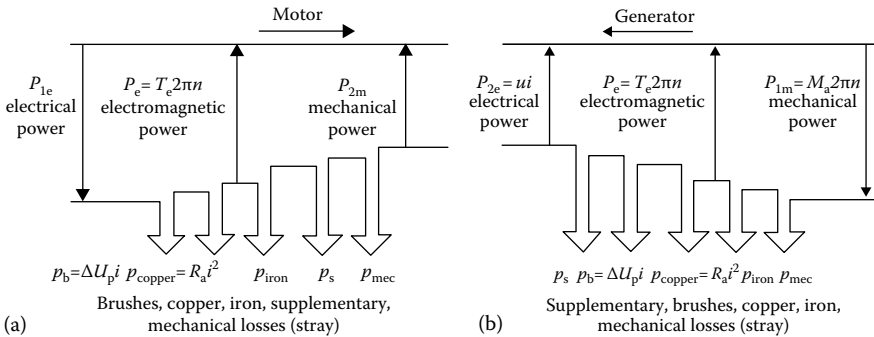


FIGURE 4.22

DC brush machine power balance: (a) for motoring and (b) for generating.

It is generator braking because the torque is negative ($i_a < 0$) and the input power is negative ($i_a < 0$) (Equation 4.59).

The excitation losses are traditionally left out just because they are obtained from a different source, but the total efficiency for generating is

$$\eta_{tg} = \frac{P_{2\text{electrical}}}{P_{1\text{mechanical}}} = \frac{V_a i_a}{V_a i_a + p_{\text{mec}} + p_{\text{iron}} + p_{\text{copper}} + p_{\text{brush}} + p_{\text{excitation}} + p_s} \quad (4.63)$$

For the motor mode (Figure 4.22b),

$$\begin{aligned} \eta_{tm} &= \frac{P_{2\text{mechanical}}}{P_{1\text{electrical}} + p_{\text{excitation}}} \\ &= \frac{T_{\text{shaft}} 2\pi n}{T_{\text{shaft}} 2\pi n + p_{\text{mec}} + p_{\text{iron}} + p_{\text{copper}} + p_{\text{brush}} + p_{\text{excitation}} + p_s} \\ &= \frac{T_{\text{shaft}} 2\pi n}{V_{1a} i_a + p_{\text{excitation}}} \end{aligned} \quad (4.64)$$

Torque T_{shaft} is

$$T_{\text{shaft}} = T_e \mp \frac{p_{\text{mec}} + p_{\text{iron}}}{2\pi n} \quad (4.65)$$

where

p_s is the stray losses

p_{mec} is the mechanical losses in W

and n is the speed in rps

Sign (−) is for motor and sign (+) is for generator.

For the generator (Equation 4.63),

$$P_{1\text{mechanical}} = T_{\text{shaftgen}} 2\pi n \quad (4.66)$$

4.11 DC Brush PM Motor Steady-State and Speed Control Methods/Lab 4.2

Dc brush PM (or any dc(ac)) motor is characterized at steady state by

Speed/current: $n = f_i(i_a)$; i_F and $V_a = \text{const}$;

Speed/torque curve: $n = f_{T_e}(T_e)$; i_F and $V_a = \text{const}$;

Efficiency versus torque curve: $\eta_m = f_n(T_e)$; f or $V_a = \text{const}$.

The current/speed curve is obtained directly from the armature circuit Equation 4.56 at steady state ($di_a/dt = 0$):

$$i_a = \frac{V_a - K_e \Phi_p n - \Delta V_{\text{brush}}}{R_a + R_{\text{add}}} \quad (4.67)$$

Similarly, the speed/torque curve is obtained from Equation 4.66 and torque formula 4.60:

$$n = n_{0i} - \frac{2\pi(R_a + R_{add})T_e}{(K_e\Phi_p)^2} - \frac{\Delta V_{brush}}{K_e\Phi_p} \quad (4.68)$$

$$n_{0i} = \frac{V_a}{K_e\Phi_p} \quad (4.69)$$

The speed n_{0i} for zero armature current ($i_a = 0$) is called the ideal no-load speed (in rps).

The ideal no-load speed is proportional to the dc armature supply voltage V_a . When V_a is changed, n_{0i} is changed, and, consequently, speed n is also changed almost in proportion.

It is also possible to reduce (vary) the current by adding a resistance, R_{add} , in series with R_a (Equations 4.67 and 4.68).

All these are shown in Figure 4.23.

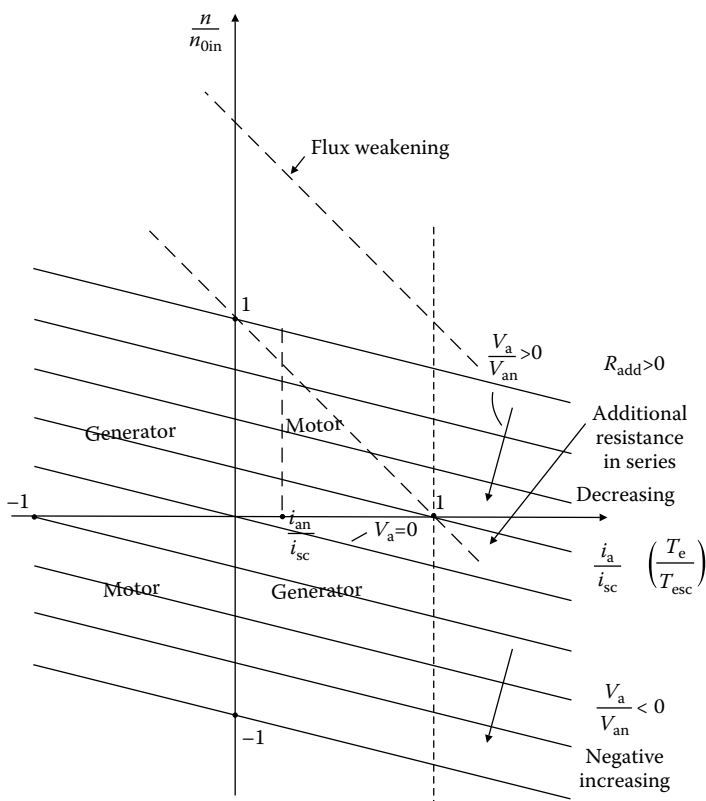


FIGURE 4.23

Per unit (pu) speed, n/n_{0in} , versus current, i_a/i_{an} , or torque, T_e/T_{en} .

The short-circuit torque, T_{esc} , corresponds to the short-circuit ($V_a = 0$) current, i_{sc} :

$$T_{\text{esc}} = K_e \phi_p i_{\text{sc}} \quad (4.70)$$

$$i_{\text{sc}} = \frac{V_a}{R_a} \quad (4.71)$$

obtained at rated voltage V_a . The ideal rated no-load speed is

$$n_{0\text{in}} = \frac{V_{\text{an}}}{K_e \phi_p} \quad (4.72)$$

$$\frac{i_a}{i_{\text{an}}} = 1 - \frac{n}{n_{0\text{in}}}; \quad \frac{i_{\text{sc}}}{i_{\text{an}}} > \frac{20}{1} \quad (4.73)$$

$$\frac{n}{n_{0\text{in}}} = 1 - \frac{T_e}{T_{\text{esc}}} \quad (4.74)$$

The four quadrant operation is shown in Figure 4.23. The fact that the speed/current and speed/torque in (pu) are identical (Equations 4.73 and 4.74) indicates that torque control is the same as current control.

When the current is positive, it is motor (or generator) in forward (backward) motion direction (quadrants 1 and 4) mode, depending on the armature voltage polarity (+) or (−).

For motor/generator operation in reversal motion, the voltage is negative (quadrant 3) or positive (quadrant 2).

From this, we may derive motor starting and speed control methods for dc brush PM motors.

4.11.1 Speed Control Methods

Speed may be controlled for a given torque (current) (Equation 4.67)—positive or negative—for motoring and regenerating by controlling the armature voltage V_a (positive or negative): a four-quadrant ac–dc or dc–dc static converter (chopper) is required (Figure 4.23).

This easy voltage-control method of speed has secured the dc brush PM motor a future in niche applications. This method maintains low losses when controlled speed decreases, so it is efficient in energy conversion.

Speed for a given torque (current) may also be reduced by an additional resistance R_{add} in series to armature, but this will lead to an increase of losses in R_{add} , so it is energy intensive and it should be used only in short-duty small power motors to reduce the initial motor costs.

Note: For separate excitation, there is one more speed control method, above the rated speed, i.e., by weakening the pole flux ϕ_p (by reducing the field current i_F), and thus increasing the no-load ideal speed (Figure 4.23).

This method would be good for maintaining constant electromagnetic power, $P_e = T_e \times 2\pi n$, when speed increases above n_{0in} for rated voltage; 2 to 1 to 3 to 1 constant power speed range (above the rated speed) is practical in urban transportation drives with dc–dc converter supply.

All of the above are explained in numerical Example 4.1.

Example 4.1 DC Brush PM Motor/Generator/Countercurrent Braking

Let us consider a small automotive dc brush PM motor with rated voltage $V_{dc} = 42\text{ V}$, rated power $P_n = 55\text{ W}$, and rated efficiency of $\eta_n = 0.9$ at $n_n = 30\text{ rps}$ (1800 rpm). The brush voltage drop at rated current is $\Delta V_{brush} = 1\text{ V}$ and the core losses p_{iron} and mechanical losses (at rated speed) p_{mecn} are $p_{iron} = p_{mecn} = 0.01P_n$.

Calculate the following:

- a. The rated armature current, I_{an}
- b. The copper rated losses, p_{con}
- c. The armature resistance, R_a
- d. The rated emf V_{en} and the rated electromagnetic torque, T_{en}
- e. The rated shaft torque, T_{shaft}
- f. Ideal no-load speed, n_{0in}
- g. Calculate the input power and efficiency at half rated $n_n/2$ and rated current (torque) and the input (reduced) voltage V'_a ; notice that core losses and mechanical losses are proportional to the required speed n
- h. Calculate the voltage required at $n_n/2$ and the rated torque in the regenerative braking
- i. For the same $n_n/2$ and $T_{en}(i_{an})$, calculate the required additional resistance R_{add} and efficiency
- j. At zero speed, determine the voltage V_a for rated but braking torque T_{en} (negative), called the countercurrent braking
- k. For n_n and generator mode on resistive load at armature terminals and for rated voltage V_{an} , calculate the load R_{load} . This may be a designated dc generator mode or it may be a braking regime called dynamic braking because the machine takes the energy from the load machine (kinetic energy) which decelerates gradually
- l. Draw the characteristics for investigated motor (generator braking) modes

Solution:

- a. The rated i_{an} springs from the input electric power $P_{1e} = V_{an}i_{an}$.

$$P_{1e} = \frac{P_n}{\eta_n} = \frac{55 \text{ W}}{0.9} = 61.11 \text{ W}$$

$$I_{an} = \frac{P_{e1}}{V_{an}} = \frac{61.11}{42} = 1.455 \text{ A}$$

- b. The rated copper loss is the only unknown component of all losses $\sum_p$:

$$\sum_p = P_{e1} - P_n = 61.11 - 55 = 6.11 \text{ W}$$

$$P_{\text{coppern}} = \sum_p - p_{\text{iron}} - p_{\text{mec}} - p_{\text{brushes}} = 6.11 - 0.01 \times 2 \times 55$$

$$- 1 \times 1.455 = 3.555 \text{ W}$$

- c. The armature resistance R_a is

$$R_a = \frac{P_{\text{coppern}}}{I_{an}^2} = \frac{3.555}{1.455^2} = 1.68 \Omega$$

- d. For the rated emf V_{ean} , we make use of Equation 4.56 with $di_a/dt = 0$:

$$V_{ean} = V_{an} - R_a I_{an} - \Delta V_{\text{brush}} = 42 - 1.68 \times 1.455 - 1 = 38.556 \text{ V}$$

The torque T_{en} is (Equation 4.59)

$$T_{en} = \frac{V_{ean} I_{an}}{2\pi n_n} = \frac{38.556 \times 1.455}{2\pi \times 30} = 0.2977 \text{ N m}$$

- e. The shaft rated torque T_{shaftn} (Equation 4.66) is

$$T_{\text{shaftn}} = T_{en} - \frac{(p_{\text{mecn}} + p_{\text{iron}})}{2\pi n_n} = 0.2977 - \frac{0.02 \times 55}{2\pi \times 30}$$

$$= 0.2918 \text{ N m} = \frac{P_n}{2\pi \times n_n}$$

- f. The ideal no-load speed $n_{0n} = V_{an}/(K_e \Phi_p)$ (4.72), but with $V_{ean} = K_e \Phi_p n_n$ from Equation 4.54, it follows that

$$n_{0n} = \frac{V_{an}}{V_{ean}} n_n = \frac{42}{38.556} \cdot 30 = 32.679 \text{ rps} \approx 1960 \text{ rpm}$$

- g. At half rated speed $n_n/2 = 15 \text{ rps (900 rpm)}$ and rated motor torque $T_{en} = 0.2977 \text{ N m}$, the required voltage, from Equation 4.57, is

$$V_a = R_a I_{an} + V_{\text{ean}} \frac{n_n/2}{n_n} + \Delta V_{\text{brush}} = 1.68 \times 1.455 + 38.556/2 + 1 = 22.722 \text{ V}$$

As both core losses and mechanical losses are reduced $((n_n/2)/n_n)^2 = 4$ times, we may calculate the total losses for this case $\sum p$:

$$\begin{aligned} \left(\sum p \right)_{n_n/2} &= p_{\text{coppern}} + (p_{\text{iron}} + p_{\text{mecn}}) \left(\frac{n_n/2}{n_n} \right)^2 + \Delta V_{\text{brush}} \cdot I_{an} \\ &= 3.555 + 2 \times 0.01 \times 55 \times 1/4 + 1.0 \times 1.455 = 5.286 \text{ W} \end{aligned}$$

Now the input electric power $P_{1e} = V_a I_{an} = 22.722 \times 1.455 = 33.06 \text{ W}$. So the efficiency $\eta_{\text{motor}} = (P_{1e} - \sum p)/P_{1e} = (33.06 - 5.286)/33.06 = 0.84$.

This is still a very good value.

- h. For the regenerative braking, the torque $T_e = -T_{\text{en}} = -0.2977 \text{ N m}$ and the current $I'_a = -I_{an} = -1.455 \text{ A}$; $\Delta V_{\text{brush}} = 1 \text{ V}$. Now from Equation 4.56

$$\begin{aligned} V'_a &= R_a I'_a + V_{\text{ean}} \frac{n_n/2}{n_n} - \Delta V_{\text{brush}} \\ &= 1.68 \times (-1.455) + \frac{38.556}{2} - 1 = 15.8336 \text{ V} \end{aligned}$$

The losses are the same as for motoring at $n_n/2$ and $+T_{\text{en}}$ torque, so the efficiency is

$$\eta'_{\text{gen}} = \frac{V'_a |I'_a|}{V'_a |I'_a| + \sum p} = \frac{15.8336 \times 1.455}{15.8336 \times 1.455 + 5.286} = 0.813$$

The shaft torque T_{shaft} is larger than the electromagnetic torque:

$$(T_{\text{shaft}})_{\text{regen}} = T_e - \frac{p_{\text{mec}} + p_{\text{iron}}}{2\pi n/2} = -0.2977 - \frac{0.02 \times 55}{2\pi 30/2} \frac{1}{4} = -0.3006 \text{ N m}$$

- i. At $n_n/2$ and rated torque T_{en} and current I_{an} , with additional series resistance R_{add} at rated voltage V_{an} , Equation 4.56 becomes

$$V_{an} = (R_a + R_{\text{add}}) I_{an} + \frac{1}{2} V_{\text{ean}}^* + \Delta V_{\text{brush}}$$

so

$$R_{\text{add}} = (42 - \frac{1}{2} \times 38.556 - 1)/1.455 - R_a = 13.249 \Omega$$

Now total losses are

$$\begin{aligned} \left(\sum p \right) R_{\text{add}}^* &= \left(\sum p \right) R_a, I_{an}, n_n/2 + R_{\text{add}} I_{an}^2 = 5.286 \\ &+ 13.249 \times 1.455^2 = 33.33 \text{ W} \end{aligned}$$

So, the efficiency is

$$(\eta_n)_{n_n/2, R_{\text{add}}, I_{\text{an}}} = \frac{V_{\text{an}} I_{\text{an}} - (\sum p)_{R_{\text{add}}}}{V_{\text{an}} I_{\text{an}}} = \frac{61.11 - 33.43}{61.11} = 0.4529$$

This is not an acceptable value for sustained operation.

- j. At zero speed, the voltage Equation 4.56 degenerates in

$$V_a'' = R_a I_a + \Delta V_{\text{brush}}$$

For rated braking, torque $T_e = -T_{\text{en}} = -0.2977 \text{ Nm}$, the current $I_a' = -I_{\text{an}} = -1.455 \text{ A}$, and thus the voltage $(V_a'')_{n_n=0, T_e} = 1.68 \times (-1.455) + 1 = -1.444 \text{ V}$. The four-quadrant converter has to be able to deliver negative current at negative voltage (third quadrant).

This is called the countercurrent braking (which is traditionally performed with $-V_{\text{an}}$ and a large additional R_{add} to limit the current, at the price of very large losses at low (or zero) speed).

The method corresponds to motoring in the reverse direction at zero speed and may be used as a controllable loading machine at zero speed for the convenient testing of other variable speed drives near zero speed.

- k. The generator mode on a resistive load R_{load} at n_n makes use of Equation 4.57:

$$\begin{aligned} R_{\text{load}} I_{\text{an}} &= (V_{\text{an}})_{\text{gen}} = R_a (-I_{\text{an}}) + V_{\text{ean}} - \Delta V_{\text{brush}} \\ &= 1.68 \times (-1.455) + 38.556 - 1 = 35.116 \text{ V} \end{aligned}$$

So, the load resistance $R_{\text{load}} = (V_{\text{an}})_{\text{gen}} / I_{\text{an}} = 35.116 / 1.455 = 24.13 \Omega$.

The output electric power in the load resistance $P_{2e} = (V_{\text{an}})_{\text{gen}} I_{\text{an}} = 35.116 \times 1.455 = 51.09 \text{ W}$.

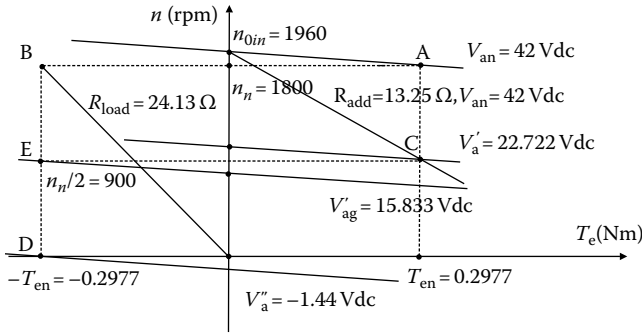
The fact that at rated current there is a voltage drop from no-load voltage V_{ean} to full generator load, V_{an} , ΔV :

$$\Delta V = \frac{V_{\text{ean}} - (V_{\text{an}})_{\text{gen}}}{V_{\text{ean}}} = \frac{38.556 - 35.116}{38.556} = 0.0892$$

shows that the generator design has to be dealt with separately if given rated load voltage or rated voltage regulation is to be met.

- l. The mechanical speed/torque curves for the instances explained above are shown in Figure 4.24.

A—motor/rated; *B*—generator/rated (on R_{load}); *C*—motor/rated torque at 50% rated speed, either produced voltage V_a or with additional resistance R_{add} ; *D*—counter-rated current braking at zero speed with reduced negative

**FIGURE 4.24**

Motor/generator/braking speed/torque curves of dc brush PM machine.

voltage V'_a ; and E —regenerative braking at rated torque, 50% rated speed and reduced voltage V'_a .

Note: The separately excited dc brush machine has flux weakening (reducing ϕ_p) as an additional method for speed control above the rated speed and constant electromagnetic power; up to 3:1 max speed/rated speed ratios are feasible. In such cases, the dc brush PM motor is not suitable, and if the dc-excited brush motor is ruled out, ac machines with power electronic control are most suitable for such cases.

Example 4.2 Derivation of Rotor Parameters R_a and L_a

The rotor resistance and leakage inductance are essential for machine design. R_a contains the brush-commutator component. So the rotor coils, resistance R_{ar} , considering the ideal number of coils in series per current path $N_s/2a$, is

$$R_{ar} \approx \left(\rho_{c0} \frac{l_{\text{coil}} N_c}{A_{\text{copper}}} \frac{N_s}{2a} \right) \frac{1}{2a}; \quad l_{\text{coil}} = 2(L_{\text{stack}} + L_{\text{end}}) \quad (4.75)$$

ρ_{c0} —copper electric resistivity in Ohm meter, L_{end} —coil end-connection length at one machine end, N_s —number of rotor slots, and A —copper wire (cable) cross section:

$$A_{\text{copper}} = \frac{I_{an}}{2aj_{\text{cor}}}; \quad i_c = \frac{I_a}{2a} \quad (4.76)$$

where

j_{cor} is the design current density in the rotor

I_{an} is the rated current at the terminals

The rotor inductance, L_a , has to consider the magnetic energy of the armature flux produced by the rotor current in the airgap and in the iron of the machine. The axis of this flux is along the neutral axis of the stator poles.

An exact value may be obtained by FEM, but at least for surface PM rotor with the total magnetic airgap $g_m = K_{cg} + h_{PM}(\mu_{rPM})_{pu}$ ($(\mu_{rPM})_{pu} = -1.05-1.2$), its main component L_{am} comes from the armature magnetic energy in a cylindrical large airgap g_m as produced by a triangular mmf variation (Figure 4.15c and Equations 4.39 and 4.40):

$$W_{ma} = \frac{L_a i_a^2}{2} = \frac{1}{2} H_0 L_e 4p_1 \int_0^{\frac{\pi}{2}} g_m \left(\frac{2N_c i_c N_s}{4p_1 g_m} \frac{2\theta}{\pi} \right)^2 \frac{D_r}{2} d\theta_r \quad (4.77)$$

The integral is taken only for a half a pole during which the armature mmf varies from zero to its maximum and then the result is multiplied by $4p_1$ (the number of half poles per periphery). Finally:

$$L_{am} \approx \frac{\mu_0 N_c^2 N_s^2}{24a^2} \frac{L_e \tau}{g_m} \quad (4.78)$$

The total rotor (armature) inductance L_a also contains the leakage inductance, corresponding to the slot leakage field and the end-connection leakage field L_{al} .

L_{al} is related to one coil leakage inductance L_{cl} Equation 4.51, with $N_s/2a$ coils in series and with $2a$ paths in parallel:

$$L_{al} = \left(L_{cl} \frac{N_s}{2a} \right) \frac{1}{2a} \quad (4.79)$$

Finally,

$$L_a = L_{am} + L_{al} \quad (4.80)$$

For surface PM stators, L_{am} and L_{al} may have comparable values.

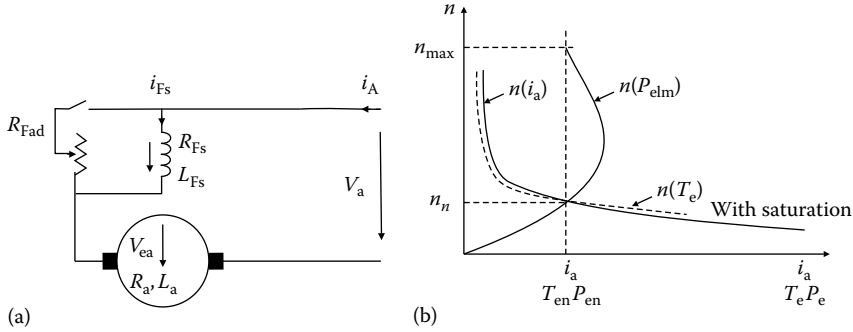
Note: For airgap (slotless) windings, the total magnetic gap g_m extends over the windings depth h_{tr} , while the slot leakage component in L_{cl} (first term in Equation 4.79) is taken out.

For excited brush machines with interpoles and compensation poles, the expression of L_a takes a more important (more involved) mathematical form.

4.12 DC Brush Series Motor/Lab 4.3

With the excitation circuit in series with the armature (at brushes), the dc brush series motor is convenient for wide constant power speed range, so it is needed in traction applications or ICE starters (Figure 4.25).

This machine is used mainly as a motor, and it may be used for self-excitation regenerative braking only after switching the terminals of the field winding first (to reverse the excitation field) (Figure 4.20).

**FIGURE 4.25**

DC brush series machine: (a) equivalent circuit and (b) natural characteristic curves for rated voltage and no R_{Fad} .

Alternatively, the excitation may be separated first and then supplied separately (at low voltage) for regenerative braking, as done routinely in standard Diesel–electric or electric locomotives. (Recent and new traction drives use induction machines with full power electronic control.)

We will deal here only with the motor mode for which the voltage equation is

$$V_a = (R_a + R_{Fs}^e) i_a + (L_a + L_{Fs}) \frac{di_a}{dt} + V_{ea} \quad (4.81)$$

$$V_{ea} = K_e \phi_p(i_{Fs}) n; \quad R_{Fs}^e = \frac{R_{Fs} R_{Fad}}{R_{Fs} + R_{Fad}}; \quad i_{Fs} = i_a \frac{R_{Fad}}{R_{Fad} + R_{Fs}} \quad (4.82)$$

For steady state, $di_a/dt = 0$.

Now the pole flux is produced by the field current i_{Fs} , which is proportional (Equation 4.82) to the armature current (and equal to it when no flux weakening is performed by adding R_{Fad} in parallel with the field winding R_{Fs}).

The torque T_e is

$$T_e = \frac{V_{ea} i_a}{2\pi n} = \frac{K_e \phi_p(i_{Fs}) i_a}{2\pi} \quad (4.83)$$

The dc brush series motor may be described by the curves:

- $n(i_a)$
- $n(T_e); n(P_{em})$
- $\eta(P_{em})$

$$\eta = \frac{P_{em} - p_{mec} - p_{iron}}{P_{em} + p_{copper} + p_{brush}} \quad (4.84)$$

Magnetic saturation occurs above a certain i_a value and, from then on, the pole flux ϕ_p stays constant, and thus the characteristics at high current degenerate into those of dc brush separate excitation motors. Let us neglect the saturation and consider a linear relationship between i_{Fs} and ϕ_p :

$$\phi_p \approx K_\phi i_{Fs} \quad (4.85)$$

From Equations 4.81 through 4.84, we derive

$$i_a = \frac{V_a}{R_a + \frac{R_{Fs} R_{Fad}}{R_{Fs} + R_{Fad}} + K_e K_\phi \frac{R_{Fad}}{R_{Fad} + R_{Fs}} n} \quad (4.86)$$

$$T_e = K_e K_\phi \frac{R_{Fad}}{R_{Fad} + R_{Fs}} i_a^2; \quad P_{elm} = T_e 2\pi n \quad (4.87)$$

Graphical representation of $n(i_a)$, $n(T_e)$, and $n(P_{em})$ is shown in Figure 4.25b.

It is evident from Equations 4.86 and 4.87 that the electromagnetic (developed) power is maximum at certain speed for constant voltage V_a (Figure 4.25b).

Also, the ideal no-load speed is $n_{0i} \rightarrow \infty$ because at zero current, the flux tends to zero (in fact to a permanent small value) ϕ_{prem} . So, a dc brush series motor should not be left without a load, a condition always fulfilled in traction drives.

$n(i_a)$ and $n(T_e)$ are mild, allowing wide speed variation with appropriate control to provide rigorously constant electromagnetic power over an $n_{max}/n_n = 3:1$ speed range.

4.12.1 Starting and Speed Control

Two effective methods of starting (limiting the current) and speed control are (for given torque) from Equations 4.86 and 4.87:

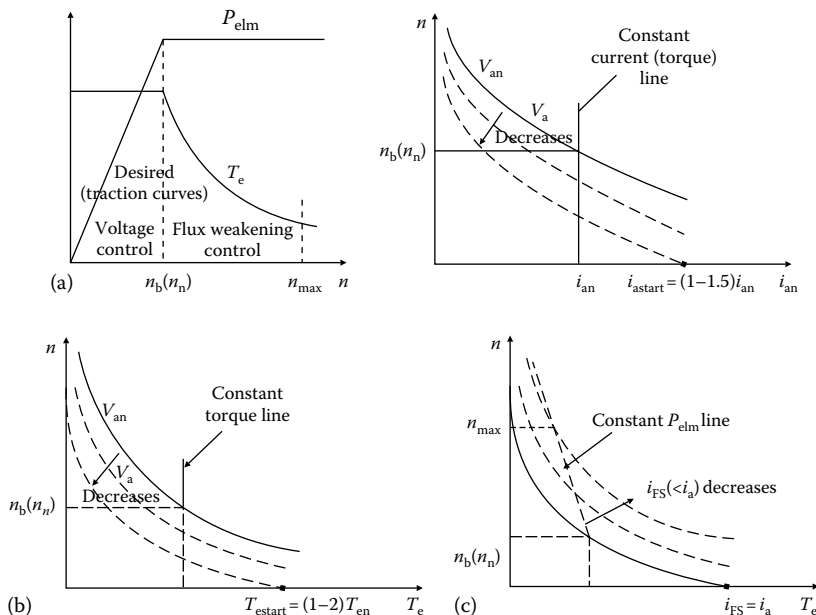
- Voltage control (by voltage reduction below V_{an}): $n < n_n$

The torque at a given speed is proportional to the voltage squared (with neglected saturation), and thus $n(i_a)$ and $n(T_e)$ curves fall below those at V_{an} , if $V_a < V_{an}$, to produce the desired current (torque) at any speed below the rated speed n_n (Figure 4.26a). For example, constant torque up to the base (rated) speed may be provided this way (Figure 4.26b).

- Field weakening speed control: $n_n < n < n_{max}$

By modifying stepwise the resistance R_{Fad} in parallel with the field winding R_{Fs} , the field current i_{Fs} becomes smaller than the armature current i_a , and thus flux weakening occurs.

Above the base speed, we may modify R_{Fad} (Figure 4.25a) to maintain $P_{elm} = P_{en}$, if so desired, up to the maximum speed n_{max} (Figure 4.26c).

**FIGURE 4.26**

Speed control of dc brush series motor curves: (a) Traction motor torque (power) speed envelopes, (b) voltage reduction control under base speed, n_b , for constant torque limit ($n(i_a)$ and $n(T_e)$ curves), and (c) flux weakening above base speed, n_b , for constant P_{elm} limit.

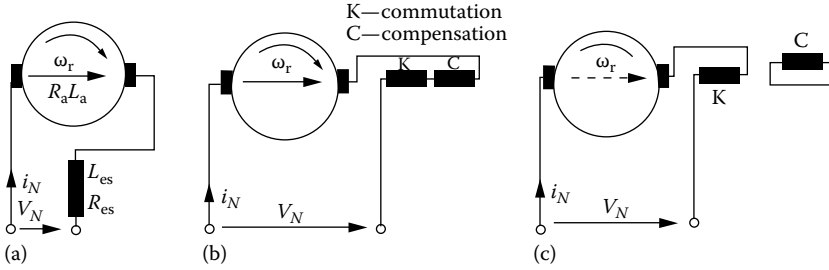
4.13 AC Brush Series Universal Motor

As already discussed earlier, the universal motor is ac series-excited at constant (grid) frequency, but at a variable voltage amplitude, for speed control. For home appliances or construction tools (below 1 kW), a triac ac voltage changer (soft starter) produces the required voltage. For 2(3)-fixed-speed simpler appliances, winding tapping or additional series resistance is used.

Right after 1900, universal motors in the tenths of kilo watts, with variable output voltage tapping of winding under load, in configurations provided with interpoles and series compensation or short circuit (Figure 4.27), have been used in Switzerland, for example, in the mountain railroad locomotives at 16.33 Hz; some of them are apparently still in use today!

The machine voltage equation should take into consideration the fact that the ac excitation induces a pulsation V_{ep} and a motion emf V_{er} into the rotor circuit:

$$(R_a + R_{Fs})i - V = V_{ep} - V_{er} \quad (4.88)$$

**FIGURE 4.27**

AC brush series motor: (a) low power, (b) medium power with series compensation C, and (c) medium power with short-circuit compensation C; K—interpole (commutation) winding.

The pulsation (transformer-like) self-induced voltage (emf) V_{ep} is

$$V_{ep} = -(L_a + L_{Fs}) \frac{di}{dt} \quad (4.89)$$

The motion emf V_{er} is, as in dc, with neglected magnetic saturation:

$$V_{er} = K_e n \phi_p \approx K_e K_\phi n i \quad (4.90)$$

The electromagnetic torque T_e , as for dc, is

$$T_e(t) = \frac{V_{er} i}{2\pi n} \approx \frac{K_e K_\phi i^2}{2\pi} \quad (4.91)$$

The torque expression is similar to the formula for the dc brush series motor, but the current is ac and, during steady state, at stator (supply) frequency ω_1 .

For steady state (constant speed and constant torque load),

$$V(t) = V_1 \sqrt{2} \cos \omega_1 t; \quad i(t) = I_1 \sqrt{2} \cos(\omega_1 t - \varphi_1) \quad (4.92)$$

Current i in the stator and at the rotor brushes is indeed at frequency ω_1 , but in the rotor, one more frequency, $\omega_r = 2\pi n p_1$, occurs as in the case of dc brush machines.

For rated speed, $\omega_r = (3-6)\omega_1$, in order to secure a large motion emf, V_{er} , which, being in phase with the current, leads to a very good power factor.

Now with Equation 4.92 in Equation 4.91, the instantaneous steady-state torque, $T_e(t)$, is

$$T_e(t) = \frac{K_e K_\phi I_1^2}{2\pi} (1 - \cos 2(\omega_1 t - \varphi_1)) \quad (4.93)$$

As expected, from a single-phase winding ac machine (of any kind), the torque pulsates at $2\omega_1$ and care must be exercised in damping the frame vibration because of this.

As with ac sinusoidal terminal voltage and current circuits (leaving out the true $\omega_1 + \omega_r$ frequency in the rotor), phasors may be used:

$$v \rightarrow \underline{V} = V_1 \sqrt{2} e^{j\omega_1 t}; \quad i \rightarrow \underline{I} = I_1 \sqrt{2} e^{j(\omega_1 t - \varphi_1)} \quad (4.94)$$

With Equation 4.94 in Equations 4.88 through 4.90, we obtain

$$\underline{V} = (R_a + R_{Fs})\underline{I} + j\omega_1(L_a + L_{Fs})\underline{I} + K_e K_\phi n \underline{I} \quad (4.95)$$

With $R_a + F_{Fs} = R_{ae}$, with $\omega_1(L_a + L_F) = X_{ae}$, and by introducing a series resistance R_{iron} to account for iron losses (both in the stator and the rotor), Equation 4.95 becomes

$$\underline{V} = \underline{I}(R_{ae} + jX_{ae} + R_{iron} + K_e K_\phi n) \quad (4.96)$$

The average torque T_{eav} (from Equation 4.93) is

$$T_{eav} = \frac{K_e K_\phi I^2}{2\pi}; \quad \cos \varphi = \frac{(R_{ae} + R_{iron} + K_e K_\phi n)I}{V} \quad (4.97)$$

With current from Equation 4.96 in Equation 4.97, the torque speed curve is

$$T_{ean} = \frac{K_e K_\phi}{2\pi} \frac{V^2}{(R_{ae} + R_{iron} + K_e K_\phi n)^2 + X_{ae}^2} \quad (4.98)$$

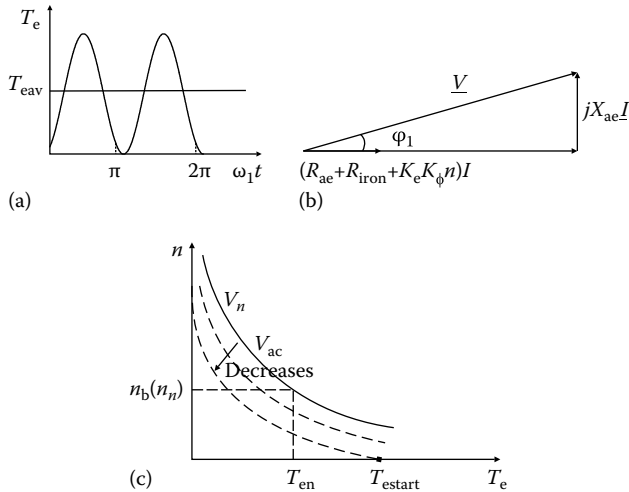
The $n(T_{ean})$ curve (Figure 4.28c) resembles the dc brush series motor and speed control is handled by amplitude voltage control easily through a triac soft starter (with an adequate power filter to attenuate harmonics current in the rather weak (residential) power sources (transformers)) [7].

As already explained in the paragraph on commutation, ac current commutation is more difficult because of the additional ac excitation emf in the commuting coil present at all speeds. This ac emf has to be reduced below 3.5 V at start and to 1.5–2.5 V at full speed running.

Frequency reduction is another way to improve commutation, which is the main design limiting factor.

Due to overall reduced cost, the universal motor is still heavily used in home appliances, from hair dryers to vacuum cleaners and some washing machines, and for handheld tools with dual or variable speed.

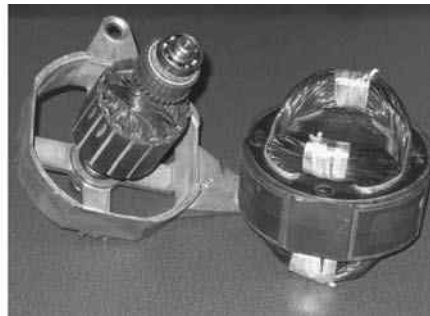
One contemporary automotive small dc brush PM motor and one handheld tool universal motor are shown in Figure 4.29.

**FIGURE 4.28**

AC brush series (universal) motor characteristics: (a) instantaneous torque pulsations at $2\omega_1$ frequency, (b) the phasor diagram at rated speed: $\cos \phi_1 > 0.9$ for $(p_1 n_n / f_1) \approx (3 \div 6)$, and (c) speed control by ac voltage reduction: $n(T_e)$ curves.



(a)



(b)

FIGURE 4.29

(a) Automotive small dc brush PM motor and (b) handheld universal contemporary motor.

4.14 Testing Brush-Commutator Machines/Lab 4.4

Testing is done for acceptability and performance, and there are international and national standards for testing, such as IEEE or IEC Standards.

Here, we concentrate on modern methods to assess the performance and for heating (rated duty loading).

It all starts with the efficiency definition, say, for motor mode:

$$\eta_{\text{motor}} = \frac{P_{2\text{mechanical}}}{P_{1\text{electrical}}} = \frac{P_{1\text{electrical}} - \sum p}{P_{1\text{electrical}}} \quad (4.99)$$

Let us first consider the dc brush PM motor.

4.14.1 DC Brush PM Motor Losses, Efficiency, and Cogging Torque

First we should note that the PM field is always there and so are the iron losses due to it, situated mainly in the rotor at frequency $f_r = p_1 n$, variable with speed.

Let us suppose that we are able to attach to the shaft a driving machine (an identical twin motor, for example) with a torque-meter or at least an encoder with speed estimation down to very low speeds (1–2 rpm) (Figure 4.30a and b).

With a complete test rig having twin motors, a torque-meter, and an encoder, we can run M_2 as the motor and M_1 as the generator, and thus first at rated voltage V_{an} and rated current I_{a2n} , machine 2 is loaded.

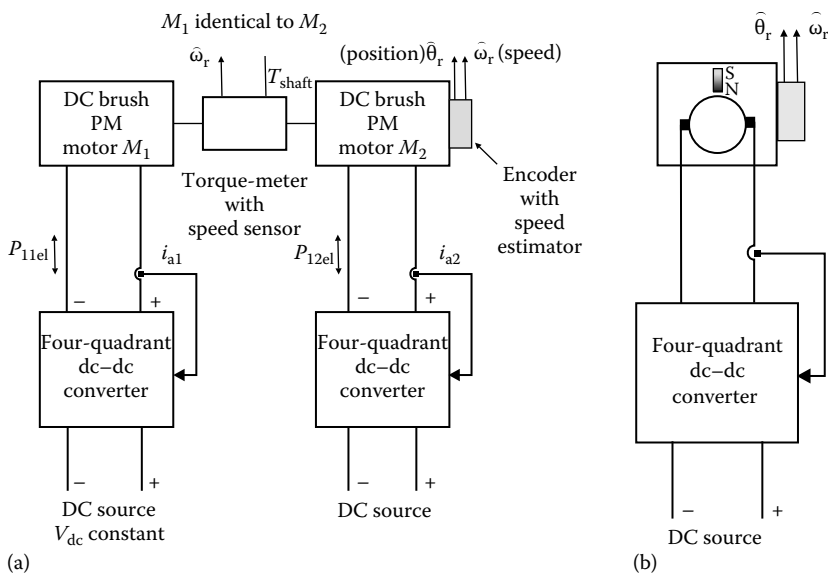


FIGURE 4.30

(a) Testing rig for small dc brush PM motors: complete, with twin motors M_1 and M_2 and (b) minimal.

We can measure rather precisely the electric input power in both machines P_{11el} and P_{12el} ; their difference is twice the total losses of each of the twin machines:

$$2 \left(\sum p \right) = P_{12el} - |P_{11el}| \quad (4.100)$$

$P_{12el} = V_{an}i_{a1n}$, $P_{11el} = V_{an}i_{a2}$, and $i_{a2} = i_{a1n}$.

While driving machine 1, at zero current ($i_{a2} = 0$), by machine one in the speed closed-loop control mode at very low speed (1–20 rpm), we may record the rotor position θ_r simultaneously with the torque from the torque-meter.

This is the cogging torque of the machine 2: $T_{cogg}(\theta_r)$.

The number of periods of cogging torque is the lowest common multiplier (LCM) of the stator PM poles $2p_1$ and the rotor slot number N_s . The larger the LCM, the smaller the cogging torque.

The shaping of the PM pole ends might also reduce the peak cogging torque value considerably.

The cogging torque produces speed pulsations, frame noise, etc., and it has to be reduced in most, if not all, of the drives used today.

PM pole flux ϕ_{PM} may also be determined under no load by measuring the no-load voltage V_{ea2} and speed of M_2 (on no load) when driven by M_1 :

$$K_e \phi_{PM} = \frac{V_{ea2}(V)}{n(rps)} \quad (4.101)$$

The same schematic in Figure 4.30a may be used to program the loading (through the current i_{a1} in M_1) according to real duty cycles and thus perform the heating or endurance tests. Only the losses in the two machines are absorbed from the power grid.

The minimal test rig (Figure 4.30b) requires a priori specific tests, according to the voltage equation of M_2 :

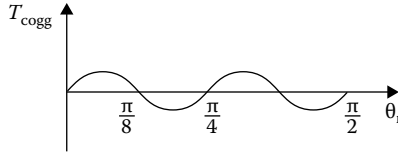
$$V_{a2} = R_a I_{a2} + V_{ea2} \quad (4.102)$$

R_a is measured in advance, with brushes on commutator (it contains part of brush voltage drop contribution), at rated current, quickly (by stalling the motor with a wrench), to avoid overheating influence on R_a .

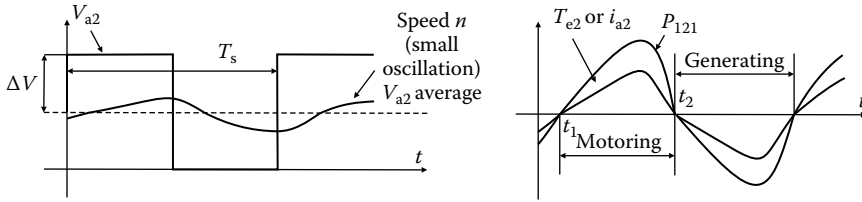
Then the motor running on-load, with V_{a2} and n measured (Equation 4.102), delivers V_{ea2} . Thus,

$$K_e \phi_p = \frac{V_{ea2}}{n}$$

Now we may pulsate the input voltage up and down with a certain amplitude and frequency until the motor cannot follow, and thus speed pulsations are small.


FIGURE 4.31

Typical cogging torque, $T_{\text{cogg}}(i_a = 0)$, versus position for $N_s = 8$ rotor slots and $2p_1 = 2$; $\text{LCM}(8, 2) = 8$.


FIGURE 4.32

Artificial loading of dc brush PM motor.

The machine switches from the motor mode to the generator mode, with the total average power from the converter equal to the total losses in the machine (Figure 4.32).

This is known as artificial loading.

$$\sum p = \frac{1}{T_s} \int_0^{T_s} P_{12e} dt \quad (4.103)$$

$$i_{a2}(\text{RMS}) = \sqrt{\frac{1}{T_s} \int_0^{T_s} i_{a2}^2 dt} \quad (4.104)$$

$$(P_{\text{an}12e})_{\text{motoring}} = \frac{1}{T_s} \int_{T_1}^{T_2} P_{e12e}(t) dt \quad (4.105)$$

Now the efficiency is (with Equations 4.103 through 4.105)

$$\eta_{\text{motor}} = \frac{(P_{\text{an}12e})_{\text{motor}} - \sum p}{(P_{\text{an}12e})_{\text{motor}}} \quad (4.106)$$

If the endurance (heating) test at the rated duty cycle is desired, then the voltage amplitude ΔV pulsation amplitude is the output of a closed-loop current regulator which monitors the $(i_{a2})_{\text{RMS}}$ current. Only the voltage V_{a2} , current

i_{a2} , and speed n are measured ($P_{12e} = V_{a2} \cdot i_{a2}$) and processed using Equations 4.103 through 4.105.

The $K_e \phi_p$ coefficient in the emf V_{ea2} may be also measured by free deceleration of M_2 with V_{ea2} and speed n measured ($K_e \phi_p = V_{ea2}/n$).

The cogging torque may not be measured with this minimal (but contemporary) test rig.

Testing of universal motor is somewhat similar but artificial loading is not feasible. For full loading, the complete test rig (Figure 4.30a), but with a dc brush PM loading machine (M_1) and a universal motor (M_2), supplied through an ac triac (soft starter), is necessary.

4.15 Preliminary Design of a DC Brush PM Automotive Motor by Example

As for any electric machine design, we start with specifications such as

- Rated voltage: $V_{dc} = 36 \text{ V}$
- Rated continuous torque: $T_{en} = 0.5 \text{ Nm}$, constant to maximum speed, n_{max}
- Maximum speed: $n_{max} = 3600 \text{ rpm} = 60 \text{ rps}$
- Peak torque at maximum speed (T_{emax}) $_{n_{max}} = 0.6 \text{ Nm}$
- Short duty cycle: 10% self-ventilation, $j_{comax} \approx 4.5 \text{ A/mm}^2$
- Surface PM stator

Solution:

The design items are

- Rotor diameter, D_r , and stack length, L_{stack}
- PM stator design
- Rotor slot and armature winding design
- R_a , L_a , $K_e \phi_p$, rated losses, and efficiency

Given the small torque, a 2-pole motor is chosen for start.

The tangential specific force is chosen at $f_{tmax} = 0.3 \times 10^4 \text{ N/m}^2$ (for low current density), and thus (Equation 4.13),

$$T_{emax} = f_{tmax} \pi D_r L_{stack} \frac{L_{stack}}{2} = f_{tmax} \frac{\pi D_r^3}{2} \frac{L_{stack}}{D_r} \quad (4.107)$$

With $\lambda_s = L_{\text{stack}}/D_r = 1.2$, we obtain the rotor diameter:

$$D_r = \sqrt[3]{\frac{2T_{\text{emax}}}{f_{\text{tmax}}\pi\lambda_s}} = \sqrt[3]{\frac{2 \times 0.6}{0.3 \times 10^4 \times \pi \times 1.2}} = 0.0474 \text{ m}$$

The stack length is

$$L_{\text{stack}} = D_r\lambda_s = 1.2 \times 0.0474 = 0.05685 \text{ m} \approx 0.06 \text{ m} \quad (4.108)$$

4.15.1 PM Stator Geometry

First, we have to consider the PM's material and its remnant flux density, B_r , and coercive field, H_c . Let us consider lower cost, bonded NeFeB, with $B_r = 0.6 \text{ T}$ and $H_c = 450 \text{ kA/m}$. For surface PMs, let us consider an airgap flux density $B_{\text{gPM}} = 0.45 \text{ T}$, with a fringing (leakage) $K_{\text{IPM}} = 0.15$. From Equation 4.25, the PM flux density, B_m , is

$$B_m = B_{\text{gPM}}(1 + K_{\text{IPM}}) = 0.45(1 + 0.15) = 0.5175 \text{ T}$$

For an airgap $g = 1 \text{ mm}$, let us find the PM thickness, h_{PM} , if the iron saturation factor $1 + K_s = 1.15$ (Equation 4.31):

$$h_{\text{PM}}(H_c - H_m) \approx F_g(1 + K_s); \quad F_g = K_{\text{cg}} \frac{B_{\text{gPM}}}{\mu_0}; \quad H_m \approx \frac{B_m}{\mu_0 \times 1.05} \quad (4.109)$$

The saturation factor K_s value is low due to the large total (magnetic) airgap $g_m \approx K_{\text{cg}} + h_{\text{PM}}$.

Also, due to large g_m , with semiclosed rotor slots, $K_c \approx 1.03$ – 1.06 ; let us consider $K_c = 1.05$.

From Equation 4.109,

$$h_{\text{PM}} \approx \frac{F_g K_s}{B_r - B_m} = \frac{1.05 \times 10^{-3} 0.45}{0.6 - 0.5175} = 5.727 \times 10^{-3} \text{ m}$$

The PM radial height has been chosen large in order to reduce the rotor coil inductance L_c and thus ease the process of commutation at 3000 rpm. The

pole span of the PM is $\tau_p = 0.7\tau$, with $\tau = \frac{\pi D_r}{2p_1} = \frac{\pi 0.0474}{2} = 0.0744 \text{ m}$.

The stator yoke flux density B_{ys} is

$$B_{\text{ys}} = \frac{(\tau/2)B_m}{h_{\text{ys}}}; \quad B_{\text{ys}} = 1.4 \text{ T} \quad (4.110)$$

So, $h_{\text{ys}} = ((0.0744 \cdot 0.7)/2)(0.5175/1.4) = 0.0096 \text{ m}$.

So, the stator outer diameter, D_{out} , is

$$\begin{aligned} D_{\text{out}} &= D_r + 2g + 2h_{\text{PM}} + 2h_{\text{ys}} \\ &= 0.0474 + 2 \times 0.001 + 2 \times 0.005727 + 2 \times 0.00096 = 0.08 \text{ m} \end{aligned}$$

The ratio $D_r/D_{out} = 0.0474/0.08 = 0.5925$, which is in the practical interval for 2-pole motors.

Realistically, now the torque is (Equation 4.61)

$$T_{\text{emax}} = \frac{p_1}{2\pi} \frac{N}{a} \phi_p i_a \frac{\tau_p}{\tau}; \quad i_a = 2ai_c; \quad N = N_c N_s \quad (4.111)$$

We first have to settle the rotor yoke depth, h_{yr} , after considering a shaft diameter of 10 mm for 0.6 N m:

$$h_{yr} \approx \frac{\tau_p B_{gPM}}{2B_{yr}} = \frac{0.7 \times 0.0747}{2} \frac{0.45}{1.5} = 0.0078 \text{ m}$$

4.15.2 Rotor Slot and Winding Design

The slot height left, h_{tr} (Figure 4.33), is

$$h_{tr} = \frac{D_r - D_{\text{shaft}} - 2h_{ys}}{2} = \frac{0.0474 - 0.01 - 2 \times 0.0078}{2} \approx 0.011 \text{ m}$$

From Equation 4.111, the A turns per rotor periphery Ni_a for a simple lap winding ($2a = 2p_1 = 2$) is

$$\begin{aligned} Ni_{\text{amax}} &= \frac{T_{\text{emax}} \times 2\pi \times a \times 0.85}{p_1 \times B_{gPM} \times l_{\text{stack}} \times \left(\frac{\tau_p}{\tau}\right)} = \frac{0.6 \times 2\pi \times 1 \times 0.85}{1 \times 1.05688 \times 0.06 \times 0.7} \\ &= 4037 \text{ A turns/periphery} \end{aligned}$$

Now the number of turns should be an even number for a simple lap winding. A slot pitch τ_s easy to get through stamping is $\tau_s = (0.009-0.012) \text{ m}$.

With 16 rotor slots, $\tau_s = \pi D_r / N_s = \pi \times 0.0474 / 16 = 0.0093 \text{ m}$. With the rotor teeth area equal to the slot area and a slot fill factor, K_{fill} , the current density is

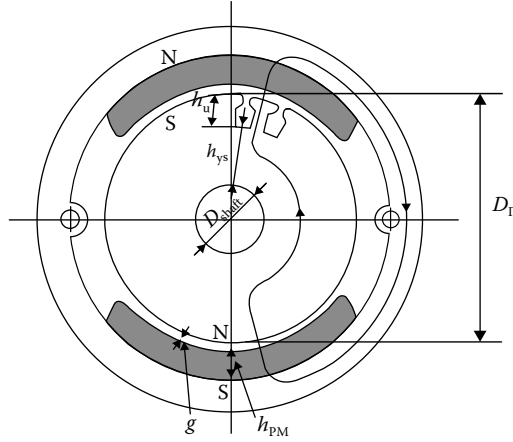
$$\begin{aligned} j_{\text{cormax}} &= \frac{Ni_{\text{amax}}}{K_{\text{fill}} \times \frac{1}{2} \times \frac{\pi}{4} (D_r^2 - (D_r - 2h_{tr})^2)} \\ &= \frac{4037 \times 10^6}{0.45 \times \frac{1}{2} \times \frac{\pi}{4} (47.4^2 - (47.4 - 2 \times 11)^2)} = 14.27 \times 10^6 \text{ A/m}^2 \end{aligned}$$

For 10% duty cycle,

$$(j_{\text{cor}})_e = j_{\text{cormax}} \sqrt{0.1} = 14.27 \times 10^6 \sqrt{0.1} \text{ A/m}^2 = 3.532 < 4.0 \text{ A/mm}^2$$

The slot active area, A_{slota} , is

$$\begin{aligned} A_{\text{slota}} &= \frac{\pi(D_r^2 - (D_r - 2h_{tr})^2)}{2 \times 4 \times N_s} = \frac{\pi(47.4^2 - (47.4 - 21.1)^2) \times 10^{-6}}{2 \times 4 \times 16} \\ &= 39.29 \times 10^{-6} \text{ m}^2 \end{aligned}$$

**FIGURE 4.33**

DC brush PM motor cross section.

With the active slot height $h_{tra} = 8 \text{ mm} < h_{tr} = 11 \text{ mm}$, the average width $b_{tsa} = A_{\text{slot}}/h_{tr} = 39.29/8 \approx 5 \text{ mm}$, with an average tooth width $b_{tra} \approx \tau_s - b_{ta} \approx 9.3 - 5 = 4.3 \text{ mm}$, which will secure a tooth flux density below 1.2 T on no load ($B_{gPM} \approx 0.45 \text{ T}$).

It is feasible to further decrease the rotor tooth width in order to increase the slot active area and thus further reduce the peak current density and copper losses. The number of conductors, N , per rotor periphery may be calculated from the emf expression:

$$V_{\text{ean}} = \frac{p_1}{a} N \phi_p n_{\text{max}} = (0.9-0.92) V_{\text{dcn}} = (32.4-33.12) \text{ V}$$

$$\phi_p = \tau_p B_{gPM} L_{\text{stack}} = 0.7 \times 0.0744 \times 0.45 \times 0.05689 = 1.333 \times 10^{-3} \text{ Wb}$$

$$N = \frac{32.4}{1 \times 1.333 \times 10^{-3} \times 60} \approx 405 \text{ turns}$$

There are 16 slots and 2 coils per slot, so the number of turns per coil is

$$N_c = \frac{N}{2N_s} = \frac{405}{2 \times 16} \approx 12.66 = 12 \text{ turns/coil}$$

$$\text{So, } N = N_c 2N_s = 12 \times 2 \times 16 = 384 \text{ and } i_{\text{amax}} = \frac{N i_{\text{amax}}}{N} = \frac{4037}{384} = 10.51 \text{ A.}$$

$$\text{The coil current } i_{\text{cmax}} = \frac{i_{\text{amax}}}{2a} = \frac{10.51}{2 \cdot 1} = 5.255 \text{ A.}$$

So, the coil wire diameter, d_{c0} , is

$$d_{c0} = \sqrt{\frac{i_{\text{cmax}}}{j_{\text{cormax}}} \frac{4}{\pi}} = \sqrt{\frac{4.2 \times 4}{14.27 \times 10^6 \pi}} = 0.685 \times 10^{-3} \text{ m}$$

Note: We now proceed with the motor parameters: R_c , L_c , R_a , and L_a . But we will first stress on copper losses:

$$\begin{aligned}
 p_{\text{copper}} &= R_a I_a^2 = \frac{1}{2a} \rho_{c0} \frac{l_{\text{coil}}}{\frac{\pi d_{c0}^2}{4}} N_c \frac{N_s}{2a} I_a^2 \\
 &= \frac{1}{2} 2.1 \times 10^{-8} \frac{0.337 \times 4}{\pi \times (0.685)^2 \times 10^{-6}} \times 12 \times \frac{16}{2} \times 5.255^2 = 75.57 \text{ W} \\
 l_{\text{coil}} &= 2(l_{\text{stack}} + 1.5\tau) = 2(0.05688 + 1.5 \times 0.0744) = 0.337 \text{ m}
 \end{aligned}$$

The electromagnetic power, P_{elmax} , is

$$P_{\text{elmax}} = T_{\text{emax}} \times 2 \times \pi \times n_{\text{max}} = 0.6 \times 2 \times \pi \times 60 = 226.08 \text{ W}$$

As we can see, the efficiency of this preliminary design is

$$\eta < \frac{226.08}{226.08 + 75.57} = 0.749$$

Note: We also notice that the stack length is still too small, so the active coil length (0.06 m) is much smaller than the end-connection length (0.11 m).

A longer stack length, with same or slightly lower rotor diameter D_r , should bring smaller mmf Ni_{amax} and, thus, notably smaller copper losses.

Also, an axial airgap disk-shaped rotor solution might be considered for $2p_1 = 4$ poles.

The above preliminary design, to be finished by calculating R_c , L_c , R_a , and L_a , serves at least as a good start for a detailed optimal design code that accounts for all losses, commutator design, costs, thermal and mechanical limitations, etc.

For more on design, see [8–10].

4.16 Summary

- Electric machines convert electric energy to mechanical energy or vice versa via stored magnetic energy (or coenergy) in a set of magnetically coupled electric circuits with a fixed part (the stator), a movable part (the rotor), and an airgap in between.
- The dc brush machines are supplied from a dc voltage source at the two terminals of the rotor, but they contain, on the rotor, a mechanical commutator made of electrically insulated copper

segments that perform series connection of all identical coils placed in uniform slots of silicon sheet soft iron core of the rotor. Brushes, fixed to the stator, press the commutator segments such that the current in the rotor coils changes polarity when the coils move from one pole to the next.

- A pole pitch, τ , is the circumferential extension of the positive magnetic flux produced by the $2p_1$ concentrated coils placed on salient poles or by $2p_1$ PM poles on the stator.
- The dc excitation on the stator produces a trapezoidal heteropolar flux density distribution with $2p_1$ poles in the airgap (p_1 electrical periods); this is how the electrical angle, $\alpha_e = p_1 \alpha_m$ (α_m —the mechanical angle), is defined.
- The stator dc excitation (or PM) magnetic field axes are fixed, with their maxima along the middle of the $2p_1$ stator pole (d axis). The rotor current magnetomotive force has its $2p_1$ maxima along the neutral axis (q axis, 90° electrical degrees away from the d axis), irrespective of speed, due to the brush–commutator principle, which acts as the rectifier (for generator mode) or as the inverter (for the motor mode) for the $f_r = p_1 n$ frequency coil emfs and currents, I_c . The brush–commutator machine operates with fixed orthogonal magnetic fields [6].
- The lap and wave armature (rotor) windings have a $2a$ parallel current path via the brushes ($2a = 2p_1 m$ for the lap windings and $2a = 2m$ for the wave windings; $m = 1, 2$ is the winding order), and the $m = 2$ brush span covers two copper segments of the commutator. Lap windings are preferred for larger current and low voltage motors, and wave windings for lower current and large voltage motors.
- Because of motion, the dc heteropolar field of the stator produces hysteresis and eddy currents in the laminated core of the rotor at a frequency $f_r = p_1 n$.
- The voltage drop along the brush copper segment contact varies from 0.5 V (in metal brushes) to 1.5–2 V for carbon brushes.
- The dc excitation of the stator may be supplied separately, shunted, or in series to the (+)(–) brushes of the commutator.
- The brushes commute the coils (one by one) from one current path (+) to the next one (–), when the coils move from under one stator pole to the next.

The process of current reversal through a short circuit should be terminated in time $T_c = (1/kn)$ (k is the number of commutator segments, n is speed in rps), which corresponds to the angle rotation by one commutator segment.

- Proper commutation is limited by coil inductance, speed, and current level. Commutator wear and scintillation are the major demerits of dc brush machines.
- The motor characteristics of a dc brush PM motor, for example in automotive applications at low power, relate, speed, n , to, armature (rotor) brush current, i , electromagnetic torque, T_e , and electromagnetic power, and are rather rigid (linear).

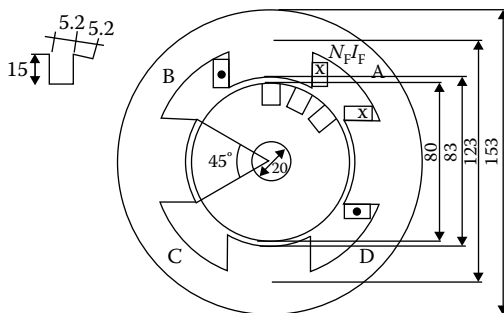
So speed drops only a little with torque.

- Dc brush PM motors exhibit an LCM ($N_s, 2p_1$) period pulsating torque at zero rotor current, which is called *cogging torque*. Cogging torque is a result of PM magnetic energy conversion to mechanical energy and back, for a zero average value per revolution. Cogging torque may be reduced by increasing the number of rotor slots, decreasing slot opening width and shaping the PM ends adequately. Cogging torque reduction means less noise, less vibration.
- Speed effective control (and starting) is performed through armature voltage control, or via series additional resistance R_{ad} at brushes; in this latter case the initial cost of R_{ad} is small but the Joule losses are large.
- Dc and ac brush series motors eliminate the need for excitation power source and are characterized by mild $n(I_a)$, $n(T_e)$ curves and an easy to obtain (through voltage control) 3:1 n_{max}/n_{base} speed ratio at constant electromagnetic power, so typical for traction motor or home appliances or handheld tools.
- The ac brush series (universal) motor is ac-fed (at f_1), and thus exhibits $2f_1$ steady-state full torque pulsations.
- Testing of brush-commutator machines is done according to the evolutionary standards: here, only two (one complete and one minimal) test rigs, containing four-quadrant dc-dc converters, are introduced and shown capable to load the machine and measure the losses and parameters (R_a , L_a , and V_{ea}).

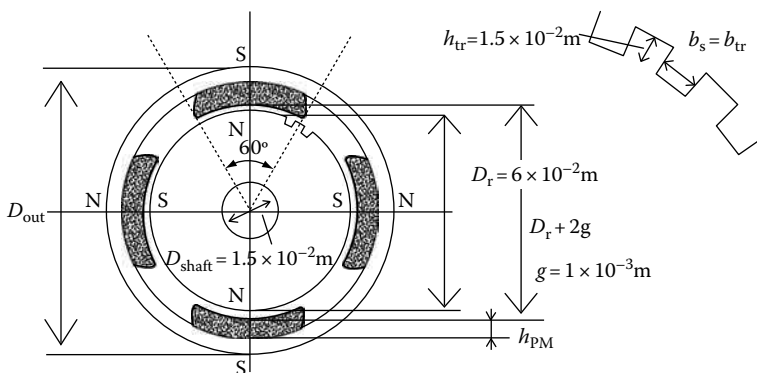
4.17 Proposed Problems

- 4.1 Build a simple lap winding for $2p_1 = 4$ poles and $N_s = 24$ slots, including the computation of t , α_{ec} , and α_{et} , drawing the emf polygon, armature winding overlay with copper segments and brushes, the coils in series in all four current paths, and the commutating four coils at the four brushes.

Hint: See Section 4.2: lap windings.

**FIGURE 4.34**

DC-excited brush-commutator motor geometry.

**FIGURE 4.35**PM stator brush-commutator motor geometry with $N_s = 16$ slots and $2p_1 = 4$ poles.

- 4.2 Build a simple wave winding for $2p_1 = 4$ poles and $N_s = 17$ slots, revealing all information as in Example 4.1.

Hint: See Section 4.2: wave windings.

- 4.3 For the dc-excited brush-commutator motor in Figure 4.34, calculate the pole excitation A turns $N_F I_F$ for an airgap flux density $B_g = 0.70$ T; note that the pole shoe was eliminated. All dimensions are given in millimeters.

Hint: See Section 4.5.

- 4.4 For the PM stator brush-commutator geometry in Figure 4.35, calculate the PM radial thickness h_{PM} and the outer stator diameter D_{out} for airgap flux density under PM, $B_{gPM} = 0.8$ T, by using NeFeB PMs

with $B_r = 1.2T$ and $H_c = 960 \text{ kA/m}$. The rotor has $N_s = 16$ slots and $2p_1 = 4$.

Hint: See Section 4.7.

- 4.5** In Example 4.4, consider a current density $J_{cr} = 5 \text{ A/mm}^2$ and a slot filling factor $k_{fill} = 0.45$. Calculate the slot A turns $2N_c I_c$ ($N_c I_c$ —A turns/coil) and the maximum of the armature-reaction flux density. Draw the PM armature and resultant flux density distribution along the rotor's complete periphery.

Hints: See Equations 4.38 through 4.40 and Figure 4.15.

- 4.6** A 4-pole surface PM stator dc brush-commutator motor has the following data:

Turns/rotor coil: $N_c = 15$

Airgap: $g = 1 \times 10^{-3} \text{ m}$

Rotor diameter: $D_r = 0.06 \text{ m}$

Number of slots: $N_s = 16$

Number of poles: $2p_1 = 4$

PM radial thickness: $h_{PM} = 4 \times 10^{-3} \text{ m}$

Stack length: $L_e = 0.06 \text{ m}$

Coil end-connection length: $L_{end} = 0.06 \text{ m}$

Radius of coil bundle: $r_{end} = 0.01 \text{ m}$

Rotor open slot height: $h_{rs} = 1.5 \times 10^{-2} \text{ m}$

Rotor slot average width: $b_{rs} = 6 \times 10^{-3} \text{ m}$

Simple lap winding

Calculate the coil inductance, L_c , and the armature-reaction inductance, L_a .

Hints: See Equations 4.50, 4.51, 4.78, and 4.80.

- 4.7** A small dc brush PM small motor is fed at 12 V dc and is rated at $P_n = 25 \text{ W}$; $\eta_n = 0.8$, $n = 30 \text{ rps}$, and $\Delta V_{brush} = 0.5 \text{ V}$, with $P_{iron} = P_{mec}/2 = 0.025P_n$.

Calculate

- The rated input current I_{an} and copper losses p_{con}
- The armature resistance R_a and the rated emf
- The ideal no-load speed at rated voltage
- Rated electromagnetic torque T_{en} and shaft torque T_{shaftn}

Hint: See Example 4.1.

- 4.8** A dc brush series urban traction (streetcar) motor has the data: $V_{an} = 500$ V dc, $P_a = 50$ kW, $n_n = 1800$ rpm, and $\eta_n = 0.93$; the total copper losses (R_a and R_{Fs} ; $R_a = R_{Fs}$) is $p_{cop} = 0.04 P_n$, $p_{iron} = p_{mec}$, and $\Delta V_{brush} = 2$ V. The magnetic saturation is neglected.

Calculate

- Rated current and armature and series-excitation copper losses, R_a and R_{Fs} .
- Iron, brush, and mechanical losses (p_{iron} , p_{mec} , and p_{brush}).
- The emf, V_{ean} , the rated electromagnetic torque, T_{en} , and electromagnetic power, P_{elm} .
- For rated torque, calculate the required voltage V_a values at 900 rpm and at standstill.
- For rated voltage, at $3n_n = 5400$ rpm and $T_{en}/3$, calculate the required resistance R_{fields} in parallel with the field circuit. Determine the electromagnetic power again.

Hints: See Section 4.13 and Equations 4.81 through 4.87.

- 4.9** A home appliance (washing machine) 2-pole universal motor has the rated power $P_n = 350$ W at $n_n = 9000$ rpm, at 50 Hz and $V_n = 220$ V (RMS). The core loss $p_{iron} = p_{copper}/2$, $p_{mec} = 0.02 P_n$, the rated efficiency $\eta_n = 0.9$, and rated power factor $\cos \varphi_n = 0.95$ lagging.

Calculate

- Rated current I_n
- Total winding resistance R_{ae} and core loss resistance R_{iron}
- The motion-induced emf V_{er}
- The total machine inductance L_{ae}
- The electromagnetic torque
- The shaft torque
- Starting current and torque at rated voltage

Hints: See Section 4.14 and Equations 4.96 through 4.101.

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Induction Machines: Steady State

5.1 Introduction: Applications and Topologies

Induction (asynchronous) machines are provided with electric windings that are placed in uniform slots located along the periphery of the stator and the rotor (with slot openings toward an airgap) in silicon–steel sheet (laminated), soft magnetic cores. Ac currents of different frequencies, f_1 and f_2 , flow through the electrical windings in the stator and the rotor ($|f_2| < |f_1|$) [1–3].

Induction machines (IMs), as classified in Chapter 3, are ac current starters, ac current rotors, and traveling field machines.

The stator full power winding is also called the primary winding and is a 1-, 2-, or 3-phase distributed winding. Most IMs, above 100 W, are 3-phase machines. The rotor winding is called the secondary winding.

The rotor is built in two configurations:

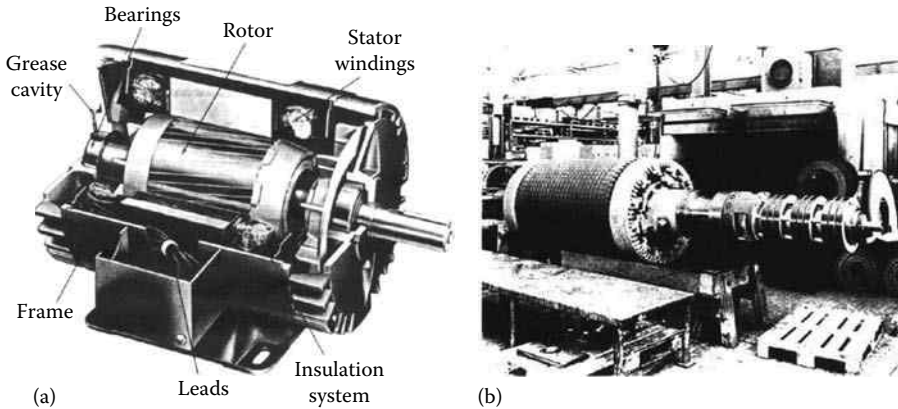
- Cage rotor: with uninsulated aluminum (brass or copper) bars, short-circuited by end rings
- Wound rotor: with a 3-phase distributed winding connected to slip rings and, via brushes, to an impedance or to a second power source of frequency f_2 such that,

$$f_2 = f_1 - np_1 \quad (5.1)$$

according to the frequency theorem (Chapter 3), rippleless (ideal) torque at steady state is obtained.

Being reversible, the IM operates as a motor or as a generator. Though it is predominantly used as a motor up to 30 MW for cage rotors, it has been used at 400 MW as a motor and a generator for wound rotors used in pump storage hydropower plants [1]. The IM is the “workhorse,” but recently it has become the “race horse” of the industry in power electronics variable speed (via variable frequency f_1 or f_2) applications.

The IM may also be built as a flat or as a tubular linear induction motor for applications in urban transportation and industrial transportation on wheels or for magnetic levitation (for clean rooms).

**FIGURE 5.1**

The induction machine: (a) 3-phase with cage rotor and (b) 3-phase with wound rotor.

5.2 Construction Elements

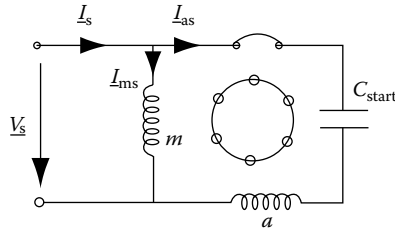
As any electric machine, the IM has a fixed part, the stator (primary) and a moving part, the rotor (secondary) (Figure 5.1).

The main part of the stator is the stator core (stack) made of a soft, *nonoriented grain* silicon-steel sheet (laminated core). The stator core (stack) is provided toward its interior (airgap) side with uniform slots that will host the ac-distributed 1-, 2-, or 3-phase winding supplied at a frequency f_1 .

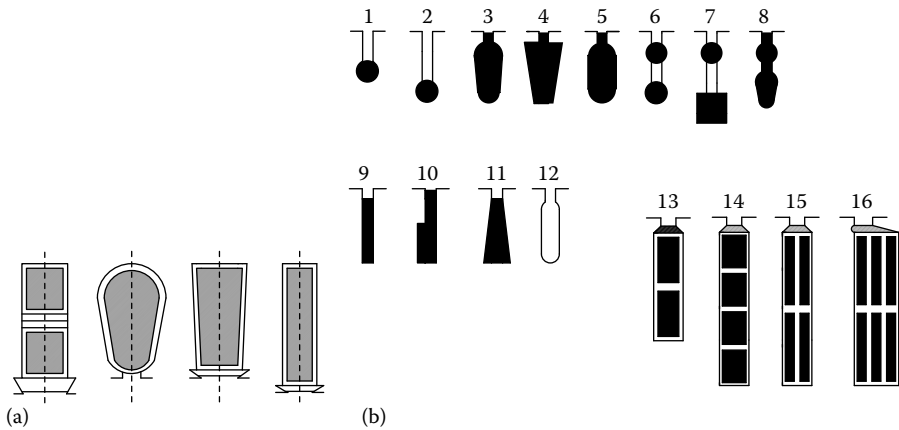
The rotor also has two main parts: the laminated core with uniform slots which host either uninsulated conductors (bars) short-circuited at stack ends by conductor end rings (the cage rotor)—Figure 5.1a—or a 3-phase ac winding connected to three copper slip rings (the wound rotor)—Figure 5.1b—and via brushes, to either an impedance or a separate power source at frequency f_2 (according to Equation 5.1). Besides 3-phase, single-phase supply, small-power induction machines are built (Figure 5.2).

The airgap—the radial distance between the stator and the rotor—has values ranging from 0.3 mm (at 100 W) to 2.0 (3.0) mm at large powers (thousands of kW).

In general, the stator and rotor slots are semiclosed (Figure 5.3) but for large machines, on one side (either rotor or stator), they are open, to allow preformed coils. For low-power machines the teeth are rectangular, while for high-power machines, the slots are rectangular, to allow preformed coils with a large copper fill factor.


FIGURE 5.2

Single-phase supply capacitor IM.


FIGURE 5.3

Typical slots for induction machines: (a) stator and (b) rotor: 1–5 (single cage), 6–8 (double cage), 9–12 (deep bars), 13–16 (wound rotor).

The cage rotors are more robust than the wound rotors. The wound rotor IMs are used only in limited variable speed applications. For small machines, die-cast aluminum cage rotors are typical, but copper round-bar cage rotors may be preferred for low-power applications when high efficiency is top priority (small pumps, refrigerators). For large cage rotor IMs, copper or brass bars are common. The frame is made of steel but mostly, even up to 100 kW, of aluminum (Figure 5.1a). Ball bearings are used for small- and medium-power IMs, and slip bearings for large-power IMs.

To reduce torque pulsations, noise, and additional rotor core and cage losses, magnetic field harmonics are reduced by reducing slot openings/airgap ratio to less than (4–5)/1, by slot skewing, winding coil chording, or increasing the number of slots.

Only a few combinations of N_s and N_r numbers of slots avoid the so-called synchronous torque pulsations and should be observed in any practical design. The stator (or wound rotor) ac distributed windings constitute

the main part of an IM (or a synchronous machine), and thus will be treated in some detail, for practical use, in what follows.

5.3 AC Distributed Windings

We use the generic term of ac distributed windings to refer to the ac full-power windings that produce a traveling airgap field in induction and synchronous machines. They contain interleaved phase coils rather than tooth-wound coils that are used for some permanent magnet synchronous machines, for salient stator, salient rotor steppers and switching reluctance machines, which are discussed in Chapter 6.

A sinusoidal flux density distribution in the airgap along the rotor periphery is needed. By interleaved phase coils, we mean multiple slot pitch spans for coils in the winding. The 3-phase case is the most used for high power while 2-phase windings (one is main, the other auxiliary (or starting)) are used (below a few kW) are suitable for single-phase residential (or rural) ac power grids at 50 (60) Hz.

Designing ac windings means assigning coils to various slots and to machine phases, and then connecting them in Y (or Δ) for 3-phase machines.

We will first treat the ideal, traveling magnetization force (mmf) by superposition of phase components, and then the mmf of primitive single-layer and two-layer chorded-coil ac windings (with integer count of slots per pole per phase, $q = 1\text{--}15$).

Further on, the mmf space harmonic content of the integer q windings is calculated with distribution, chording, and skewing winding factor expressions.

Rules for building practical one- and two-layer 3-phase ac distributed windings are presented and applied for integer and fractional $q \geq 1$.

Two-phase (main and auxiliary (or starting)) distributed windings that are similar to a sinusoidal space distribution for small, single-phase ac power grid-supplied ac motors are treated in some detail.

5.3.1 Traveling MMF of AC Distributed Windings

As already discussed in Chapter 3, traveling field ac machines require traveling magnetomotive forces:

$$F_{sf}(x, t) = F_{sfm} \cos\left(\frac{\pi}{\tau}x - \omega_1 t - \theta_0\right) \quad (5.2)$$

where

x is the coordinate along the stator core periphery after unfolding it in a plane

τ is the spatial half-period (pole pitch) of an mmf fundamental (ideal) wave

ω_1 is the angular frequency of stator currents

θ_0 is the angular stator position with respect to the phase A axis at $t = 0$

We may decompose it into terms:

$$F_{sf}(x, t) = F_{sfm} \left[\cos \left(\frac{\pi}{\tau} x - \theta_0 \right) \cos(\omega_1 t) + \sin \left(\frac{\pi}{\tau} x - \theta_0 \right) \sin(\omega_1 t) \right] \quad (5.3)$$

This mere trigonometric exercise indicates that we are now having two mmfs at a standstill, with a spatial sinusoidal distribution and sinusoidal currents. Both the space and the time angles between the two mmf components are 90° . So 2-phase windings with a sinusoidal spatial distribution and 90° space phase lag, through which ac currents flow with 90° time phase lag, can produce a traveling mmf.

Now, according to Ampere's law, the airgap flux density, $B_{gs}(x, t)$, is simply:

$$B_{gs}(x, t) = \frac{\mu_0 F_{sf}(x, t)}{g_m(x) (K_s + 1)} \quad (5.4)$$

where g_m is the magnetic (total) airgap which may vary (as in the stator) with the rotor position—for salient pole rotors—and K_s is the magnetic saturation factor $0 < K_s < 0.8$, in general, and accounts for iron mmf requirements as discussed already in Section 4.6.

g_m is a constant (nonsalient pole, dc-excited or surface-PM pole rotors) if the mmf is a pure traveling wave, and so is the airgap flux density wave (Equation 5.4). Thus, the condition for a rippleless ideal torque may be obtained through two orthogonal phases (Equation 5.3). Their arrangement is typical for single-phase ac power grid-connected small ac (synchronous and induction) motors.

We may decompose Equation 5.2 also into m terms:

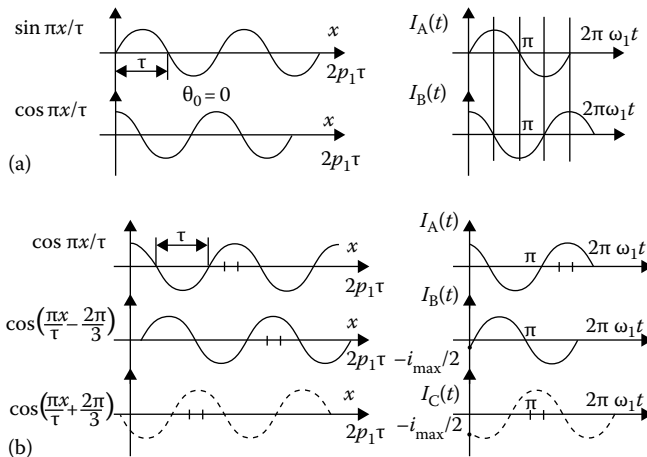
$$\begin{aligned} F_{sf}(x, t) = \frac{2}{m} F_{sfm} \left[\cos \left(\frac{\pi}{\tau} x - \theta_0 \right) \cos(\omega_1 t) + \cos \left(\frac{\pi}{\tau} x - \theta_0 - \frac{2\pi}{m} \right) \right. \\ \times \cos \left(\omega_1 t - \frac{2\pi}{m} \right) + \cos \left(\frac{\pi}{\tau} x - \theta_0 - (m-1) \frac{2\pi}{m} \right) \\ \left. \times \cos \left(\omega_1 t - (m-1) \frac{2\pi}{m} \right) \right] \quad (5.5) \end{aligned}$$

Consequently, phase windings m with a spatial sinusoidal distribution and sinusoidal currents dephased spatially, and in time by $2\pi/m$, also produce a traveling mmf.

For the 2-phase case, $m = 4$ in Equation 5.5. The most common case is $m = 3$ phases. The situation for 2- and 3-phases is illustrated in Figure 5.4.

The pole pitch, τ , calculated at a stator interior diameter, D_{is} , is

$$\tau = \frac{\pi D_{is}}{2p_1} \quad (5.6)$$

**FIGURE 5.4**

Ideal multiphase mmfs: (a) 2-phase machine and (b) 3-phase machine.

The number of pole pairs, p_1 (or mmf electrical periods per one mechanical revolution), leads to the same definition of the electrical angle, α_e , as for brush-commutator machines:

$$\alpha = p_1 \alpha_g; \quad \alpha_g, \text{ geometrical angle} \quad (5.7)$$

A pure sinusoidal distribution of mmf is feasible only with slotless windings of unequal numbers of turns (cosinusoidal variation of turns/coil on the periphery for each phase).

As this is not practical even for slotless ac windings (used for some small-power PMSMs), the ac windings are placed in a limited number of slots:

$$N_s = 2p_1 q m \quad (5.8)$$

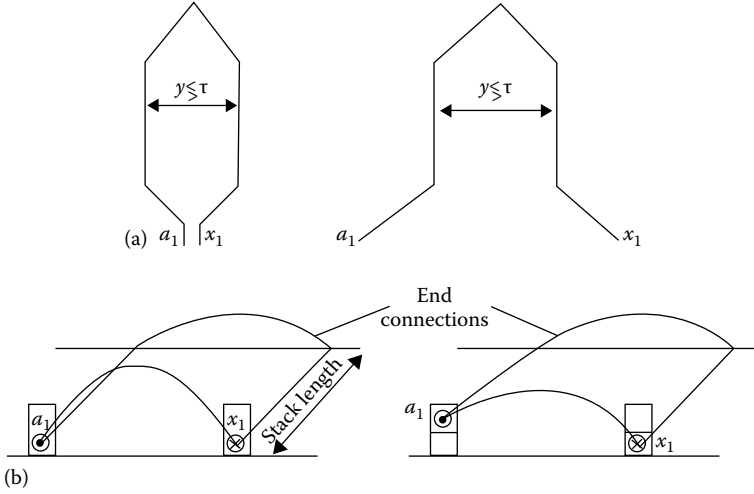
The total number of slots, N_s , has to be divisible by m (number of phases), to retain some symmetry of the phase windings, with an integer q , characteristic of interleaved phase coils in distributed windings:

$$q = \frac{N_s}{2p_1 m} = \text{Integer} \quad (5.9)$$

N_s has to be divisible by $2p_1 m$; $q = 1, 2, \dots, 12$ or even more in turbo-generators or large two-pole induction machines. It is also feasible to have $q = a + b/c$, $a \geq 1$, for fractional q distributed windings.

The ac distributed windings are made of lap or wave coils as for brush-commutator windings (Chapter 4), Figure 5.5, placed in one or two layers in uniform slots along the stator periphery.

Single-layer windings make use of diametrical coils (full pitch span: $y = \tau$), while two-layer windings often make use of chorded coils

**FIGURE 5.5**

Lap and wave single-turn (bar) coils placed in (a) one layer in slots and (b) two layers in slots.

($2\tau/3 \leq y \leq \tau$) to reduce the coil end-connection length (and copper losses) and to reduce some mmf space harmonics, as shown later. Unfortunately, the mmf fundamental amplitude is also reduced by chorded coils.

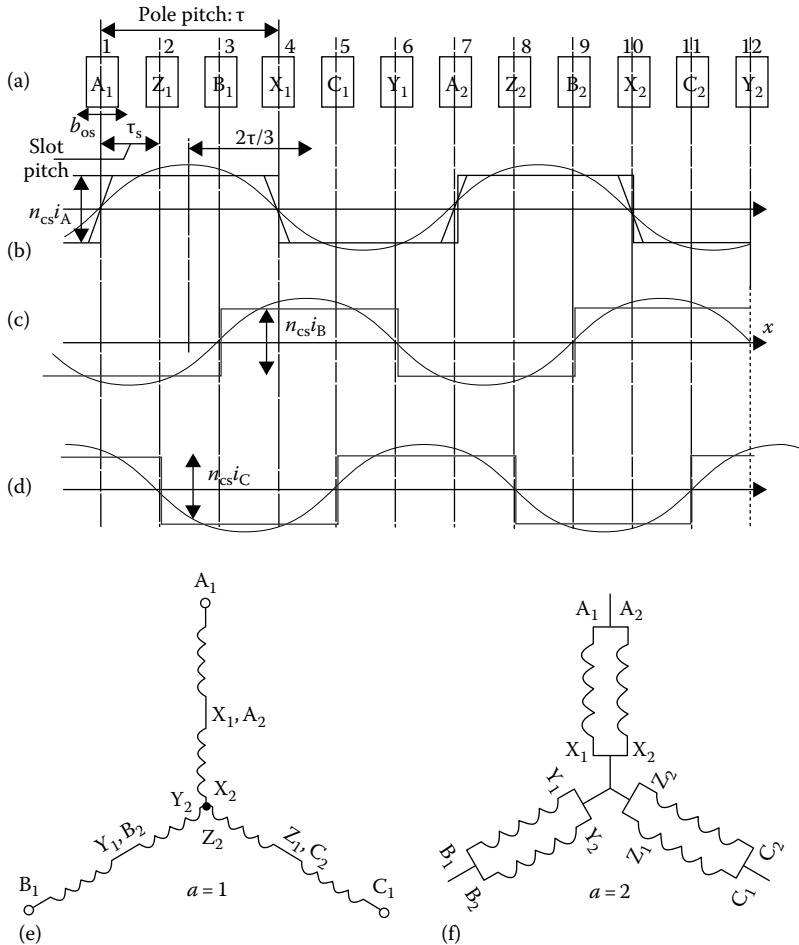
5.3.2 Primitive Single-Layer Distributed Windings ($q \geq 1$, Integer)

Let us consider a primitive $2p_1 = 4$ poles 3-phase distributed winding with $q = 1$ slot/pole/phase: $N_s = 2p_1mq = 2 \times 2 \times 3 \times 1 = 12$ slots in all. There are four poles, so each pole has $N_s/2p_1 = 12/(2 \times 2) = 3$ slot pitches τ_s , or three slots, one per phase ($q = 1$).

For single-layer windings, one coil fully occupies two slots, so there are, in all, six coils; two coils per phase; and four slots per phase, one pole pitch, τ , or three slot pitches, τ_s , apart. For phase A we have slots 1, 4, 7, and 10. Phases B and C are placed in slots by moving $2/3$ and $4/3$ of a pole pitch from phase A to the right (Figure 5.6a and b). If the slot opening is considered zero, the mmf “jumps” by $n_c I_{A,B,C}$ along the middle of each slot. For $q = 1$, a rectangular heteropolar mmf distribution for each phase is obtained (Figure 5.6b through d).

The coils of a phase may all be connected in series (Figure 5.6e) to form a single current path, $a = 1$, or some of them (in our case all of them) in parallel to form a current paths: $1 < a \leq p_1$ (Figure 5.6f).

The rectangular phase mmf distribution may be used as it is for rectangular current control in PMSMs with $q = 1$, or it may be decomposed into


FIGURE 5.6

Single-layer primitive distributed 3-phase winding ($2p_1 = 4$, $m = 3$, $q = 1$ slot/pole/phase ($N_s = 12$ slots in all)): (a) phase coil allocation to slots, (b–d) phase mmfs rectangular distribution, (e) series star connection ($a = 1$ current path), and (f) parallel star connection ($a = 2$ current paths).

harmonics for each phase:

$$F_{A\nu}(x, t) = \frac{2}{\pi} \frac{n_c I \sqrt{2}}{\nu} \cos \omega_1 t \cos \frac{\nu \pi}{\tau} x \quad (5.10)$$

For the fundamental ($\nu = 1$), we obtain the maximum mmf amplitude as expected ($q = 1$).

For $q \geq 1$, a multistep rectangular mmf distribution is expected, with lower harmonics content. Two-layer windings with chorded coils further reduce the phase mmf space harmonics content for $q > 1$.

5.3.3 Primitive Two-Layer 3-Phase Distributed Windings ($q = \text{Integer}$)

Let us consider again $2p_1 = 4$ poles, $m = 3$ phases, but now q changes from 1 to 2 slots/pole/phase: $N_s = 2p_1mq = 2 \times 2 \times 3 \times 2 = 24$ slots. The pole pitch span in slot pitches, τ , is then $\tau = N_s/2p_1 = 24/(2 \times 2) = 6$ slot pitches, τ_s .

The chorded-coil span is taken as $y/\tau = 5/6$ ($y/\tau = 4/6 = 2/3$, the lowest for $q = \text{integer}$ and distributed windings).

Disregarding the second layer, let us allocate phase coils to slots in the first layer, observing that there are $q = 2$ neighboring slots for each phase under each pole; in all $N_s/m = 8$ coils per phase (two layers). The $2/3$ pole phase lag of the phase mmf is provided as for the single-layer winding case (Figure 5.7a). Once the allocation of phase coils to slots in the first layer is done, the distribution for the second layer is moved to the left by $(1 - y/\tau)mq$ slots (Figure 5.7a). Then, for the moment when the phase A current is maximum, its mmf distribution is rectangular but with two steps/polarity (Figure 5.7b). If we add the contribution of all 3-phase mmfs for

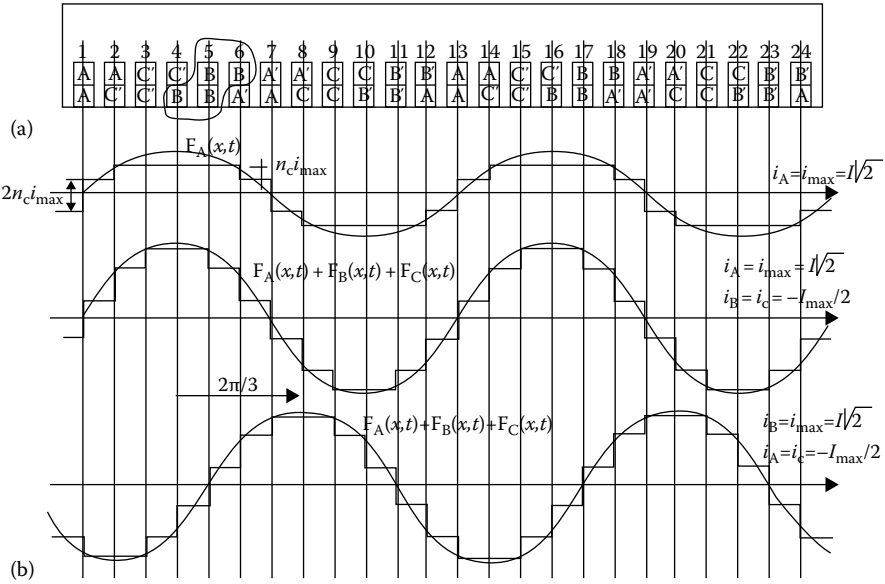


FIGURE 5.7

Two-layer winding ($N_s = 24$, $2p_1 = 4$ poles, $y/\tau = 5/6$): (a) allocation of phase coils to slots and (b) mmf distribution.

$I_A = I_{\max} = -2I_B = -2I_C$ (when I_A is maximum and the currents are symmetric: $I_A + I_B + I_C = 0$), we obtain the 3-phase mmf distribution with 3 steps/polarity (Figure 5.7b). For the later moment, when the current in phase B is maximum ($I_B = I_{\max} = -2I_A = -2I_C$), the 3-phase mmf distribution peaks have moved to the right by $(2/3)\tau$ or $2\pi/3$ electrical radians. So the mmf is traveling, as expected.

5.3.4 MMF Space Harmonics for Integer q (Slots/Pole/Phase)

The geometrical stepwise representation of phase mmfs in Figure 5.7b suggests that generalizing Equation 5.10 is straightforward for $q > 1$ and $y/\tau < 1$ and, for only the fundamental ($v = 1$):

$$F_{A1}(x, t) = \frac{2}{\pi} n_c q I \sqrt{2} K_{q1} K_{y1} \cos\left(\frac{\pi}{\tau} x\right) \cos(\omega_1 t) \quad (5.11)$$

with

$$K_{q1} = \frac{\sin \frac{\pi}{6}}{q \sin \frac{\pi}{6q}} \leq 1; \quad K_{y1} = \sin y \frac{\pi y}{2\tau} \leq 1 \quad (5.12)$$

K_{q1} is known as the winding distribution factor, and K_{y1} is the chording factor (for $q = 1$ and $y = \tau$; $K_{q1} = K_{y1} = 1$). Keeping the windings symmetric implies $(y/\tau) > (2/3)$, because all phases have the same number of slots per pole under each pole ($q = \text{integer}$).

With all coils in series, we have W_1 turns per phase:

$$W_1 = 2p_1 q n_c \quad (5.13)$$

where n_c is the turns per coil (for single-layer windings $W_1 = p_1 q n_c$ because there is 1 coil/slot). Thus F_{A1} in Equation 5.11 becomes

$$F_{A1}(x, t) = \frac{2}{\pi p_1} W_1 I \sqrt{2} K_{q1} K_{y1} \cos\left(\frac{\pi}{\tau} x\right) \cos \omega_1 t \quad (5.14)$$

Adding the 3-phase contributions, with $2\pi/3$ time and space phase shift, yields

$$F_1(x, t) = F_{1m} \cos\left(\frac{\pi}{\tau} x - \omega_1 t\right) \quad (5.15)$$

with

$$F_{1m}(x, t) = \frac{3W_1 I \sqrt{2} K_{q1} K_{y1}}{\pi p_1} (\text{A turns/pole}) \quad (5.16)$$

The derivative of the pole mmf, $F_1(x, t)$, with respect to x is called the linear current density, $A(x, t)$:

$$A_1(x, t) = \frac{\partial F_1(x, t)}{\partial x} = -A_{1m} \sin\left(\frac{\pi}{\tau}x - \omega_1 t\right) \quad (5.17)$$

$$A_{1m} = \frac{\pi}{\tau} F_{1m} \quad (5.18)$$

A_{1m} is the peak value of the linear current density or current loading, a key design factor; $A_{1m} = (1,000\text{--}50,000)$ A/m, from small to very large machines and increases with the rotor diameter (and torque).

The space harmonics content of 3-phase winding mmf is obtained from Equation 5.10 as

$$F_v(x, t) = \frac{3W_1 I \sqrt{2} K_{qv} K_{yv}}{\pi p_1 v} \left[K_{BI} \cos\left(v \frac{\pi}{\tau} x - \omega_1 t - (v-1) \frac{2\pi}{3}\right) - K_{BII} \cos\left(v \frac{\pi}{\tau} x + \omega_1 t - (v+1) \frac{2\pi}{3}\right) \right] \quad (5.19)$$

with

$$K_{qv} = \frac{\sin \frac{v\pi}{6}}{q \sin \frac{v\pi}{6q}}; \quad K_{yv} = \sin \frac{v\pi y}{2\tau}; \quad K_{BI} = \frac{\sin(v-1)\pi}{3 \sin(v-1) \frac{\pi}{3}}; \\ K_{BII} = \frac{\sin(v+1)\pi}{3 \sin(v+1) \frac{\pi}{3}} \quad (5.20)$$

Due to the full phase symmetry with q = integer only, odd order space harmonics survive in the mmf. For a star connection, at least, $3k$ harmonics may not occur as their current summation is zero. So we are left with $v = 3k \pm 1 = 5, 7, 11, 13, 17, 19, \dots$, all prime numbers!

We should notice in Equation 5.20 that for $v_d = 3k + 1$ (7,13,19), $K_{BI} = 1$ and $K_{BII} = 0$. But the remaining first term in Equation 5.19 represents direct (forward) traveling waves:

$$\frac{v\pi}{\tau} x - \omega_1 t = \text{const.}; \quad \frac{dx}{dt} = \frac{\omega_1 \tau}{\pi v} = \frac{2\tau f_1}{v}; \quad \omega_1 = 2\pi f_1 \quad (5.21)$$

On the contrary, for $v_i = 3k - 1$ (5,11,17,...), $K_{BI} = 0$ and $K_{BII} = 1$ and we are left only with the second term in Equation 5.19, which refers to inverse (backward) traveling waves:

$$\frac{dx}{dt} = -\frac{\omega_1 \tau}{\pi v} = -\frac{2\tau f_1}{v} \quad (5.22)$$

We should notice that the traveling speed of space harmonics, v , is v times lower than that of the fundamental.

The mmf space harmonics, as derived above, refer to $\pi/3$ phase belt sequences, which are predominant in the industry today. In some applications, $2\pi/3$ phase belt sequences are used (such as in 2/1 ratio, pole count changing induction machine winding).

Example 5.1 MMF Harmonics for Integer q

Let us consider a stator with an interior diameter, $D_{is} = 0.12$ m; the number of stator slots, $N_s = 24$; $2p_1 = 4$; $y/\tau = 5/6$; two-layer winding, one current path, ($a = 1$); the slot area, $A_{slot} = 120 \text{ mm}^2$; total copper filling factor, $k_{fill} = 0.45$; the rated current density, $j_{con} = 5 \text{ A/mm}^2$; and the number of turns per coil, $n_c = 20$. Calculate:

- The rated RMS current, I_n , and wire gauge
- The pole pitch, τ , and the slot pitch, τ_s
- K_{q1} , K_{y1} , and $K_{w1} = K_{q1}K_{y1}$
- The number of turns/phase, W_1 , mmf, and current loading fundamental amplitudes, F_{1m} , A_{1m}
- K_{q7} , K_{y7} , F_{7m} ($v = +7$)

Solution:

- The slot copper area, A_{cos} , is covered by an mmf of $2n_c I$ (2 coils/slot):

$$A_{cos} = A_{slot} k_{fill} = \frac{2n_c I_n}{j_{con}}$$

So the rated current, I_n , is

$$I_n = \frac{A_{slot} k_{fill}}{2n_c} j_{con} = \frac{120 \times 10^{-6} \times 0.45 \times 5 \times 10^6}{2 \times 20} = 6.75 \text{ A}$$

The copper wire bare diameter, d_{Co} , is

$$d_{Co} = \sqrt{\frac{I_n}{j_{con}} \frac{4}{\pi}} = \sqrt{\frac{6.75 \times 4}{5 \times 10^6 \times \pi}} = 1.3 \times 10^{-3} \text{ m}$$

- The pole pitch, τ , is Equation 5.6

$$\tau = \frac{\pi D_{is}}{2p_1} = \frac{\pi \times 0.12}{2 \times 2} = 0.0942 \text{ m}$$

with the slot pitch, τ_s

$$\tau_s = \frac{\tau}{3q} = \frac{0.0942}{3 \times 2} = 0.0157 \text{ m}$$

c. From Equation 5.12,

$$K_{q1} = \frac{\sin \frac{\pi}{6}}{q \sin \frac{\pi}{6q}} = \frac{0.5}{2 \times \sin \frac{\pi}{12}} = 0.9659$$

$$K_{y1} = \sin \frac{y\pi}{2\tau} = \sin \frac{5\pi}{12} = 0.9659$$

$$K_{w1} = K_{q1}K_{y1} = 0.9659 \times 0.9659 = 0.9329$$

d. The number of turns per phase, W_1 (with $a = 1$ current paths), from Equation 5.13 is

$$W_1 = 2p_1 q n_c = 2 \times 2 \times 2 \times 20 = 160 \text{ turns/phase}$$

According to the F_{1m} and A_{1m} expressions (Equations 5.16 through 5.18)

$$\begin{aligned} F_{1m} &= \frac{3W_1 I \sqrt{2} K_{q1} K_{y1}}{\pi p_1} = \frac{3 \times 160 \times 6.75 \sqrt{2} \times 0.9659 \times 0.9659}{\pi \times 2} \\ &= 678 \text{ A turns/pole} \end{aligned}$$

$$A_{1m} = F_{1m} \frac{\pi}{\tau} = 678 \times \frac{\pi}{0.0942} = 22,622.8 \text{ A turns/m}$$

e. From Equation 5.20,

$$K_{q7} = \frac{\sin \frac{7\pi}{6}}{2 \sin \frac{7\pi}{6 \times 2}} = -0.2588$$

$$K_{y7} = \sin \frac{7\pi}{2} \frac{5}{6} = 0.2588$$

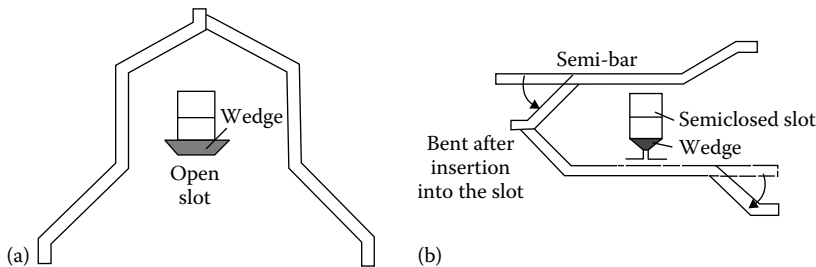
$$\begin{aligned} F_{7m} &= \frac{3W_1 I \sqrt{2} K_{q7} K_{y7}}{\pi p_1 \times 7} = \frac{3 \times 160 \times 6.75 \sqrt{2} \times 0.2588^2}{\pi \times 2 \times 7} \\ &= 6.96 \text{ A turns/pole} \end{aligned}$$

As seen above, $F_{7m}/F_{1m} = 6.96/678 \approx 0.01$! The chording and distribution factors have contributed massively to this practically good result.

It may be shown that for a 120° phase belt and $q = 2$

$$K_{q1} = \left(\frac{\sin \frac{\nu\pi}{3}}{q \sin \frac{\nu\pi}{3q}} \right)_{\nu=1} = 0.867$$

This is almost 10% smaller than K_{q1} above ($K_{q1} = 0.9569$) for 60° belt windings. This partly explains why 60° belt windings are preferred in the industry.

**FIGURE 5.8**

Bar (single turn) coils: (a) continuous bar and (b) semi-bar.

5.3.5 Practical One-Layer AC 3-Phase Distributed Windings

As already mentioned, windings are made by lap and wave coils. Wave coils are used for single-turn (bar) coils of large ac (synchronous or induction) machines (Figure 5.8).

Multi-turn coils are made of lap coils. But the phase-belt lap coils under a single pole may be made of unequal concentric coils (Figure 5.9a) or of identical (chain) coils (Figure 5.9b) for single-layer windings. For double-layer winding flexible coils, made of round wire, the coils are identical and mechanically flexible (for small machines). They may be preformed for rectangular cross-section conductors and twisted to fit with one side in one layer and the other side in another layer.

Rules to build single-layer winding by example are as follows (for $N_s = 24$, $2p_1 = 4$, $m = 3$):

- Calculate the slot-turn, self-induced emf angle shift between neighboring slots:

$$\alpha_{es} = \frac{2\pi}{N_s} p_1 = \frac{2\pi}{24} 2 = \frac{\pi}{6} \quad (5.23)$$

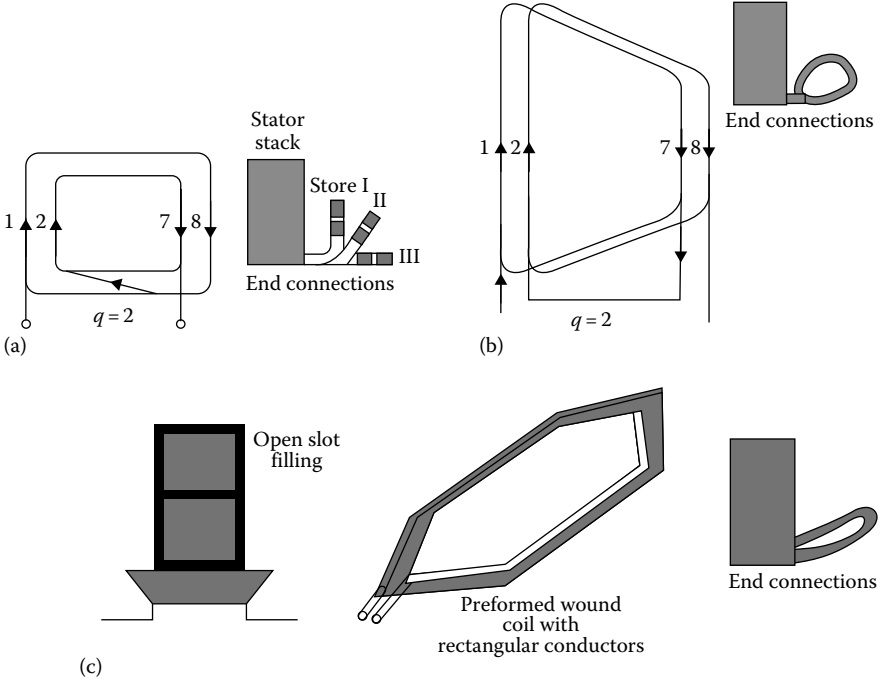
- Calculate the number of emfs in phase, t :

$$t = \text{LCD}(N_s, p_1) = \text{LCD}(24, 2) = 2 = p_1 \quad (5.24)$$

- Build a star of N_s/t arrows with an angle among them:

$$\alpha_{et} = \frac{2\pi}{N_s} t = \frac{2\pi}{24} 2 = \frac{\pi}{6} = \alpha_{es} \quad (5.25)$$

so the order of arrows is the natural order of slots.

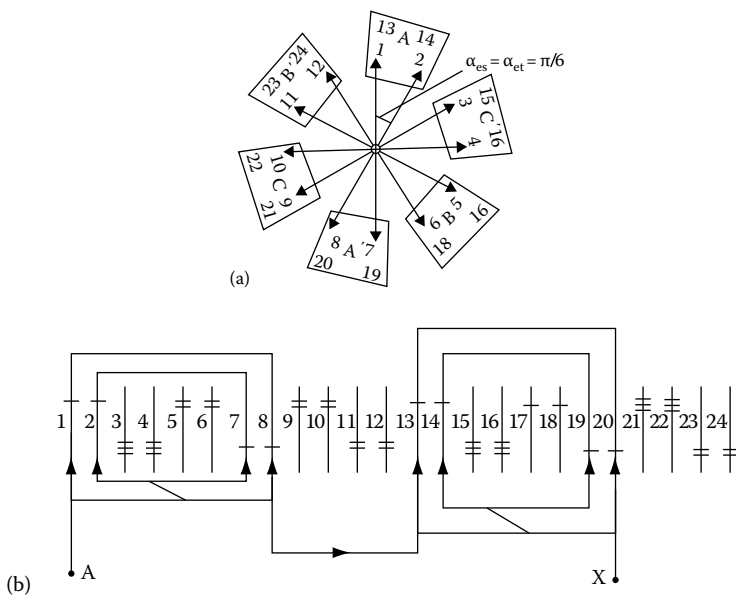
**FIGURE 5.9**

AC winding practical coils: (a) concentric coil phase belt/single layer, (b) identical (chain) coil phase belt/single layer, and (c) preformed coil/two layer.

- Select $N_s/2m = 24/(2 \times 3) = 4$ arrows to represent the entry coil sides of phase A, and take another opposite $N_s/2m$ to represent the exit coil sides for the same phase.
- Move 120° and arrange for phase B slot allocation and then again for phase C (Figure 5.10).

For the two-layer windings, the basic rules by example are

- Consider the case of $N_s = 27, 2p_1 = 4$.
- Build the slot emf phasors star as for the single-layer case.
- Choose $N_s/m = 27/3 = 9$ arrows for each phase, after dividing them into two, almost (or completely), opposite in phase groups to represent phase A.
- For the case in point, where $t = \text{LCD}(N_s, p_1) = \text{LCD}(27, 2) = 1$, $\alpha_{ec} = 2\alpha_{et}$ and thus the order of arrows is not the same

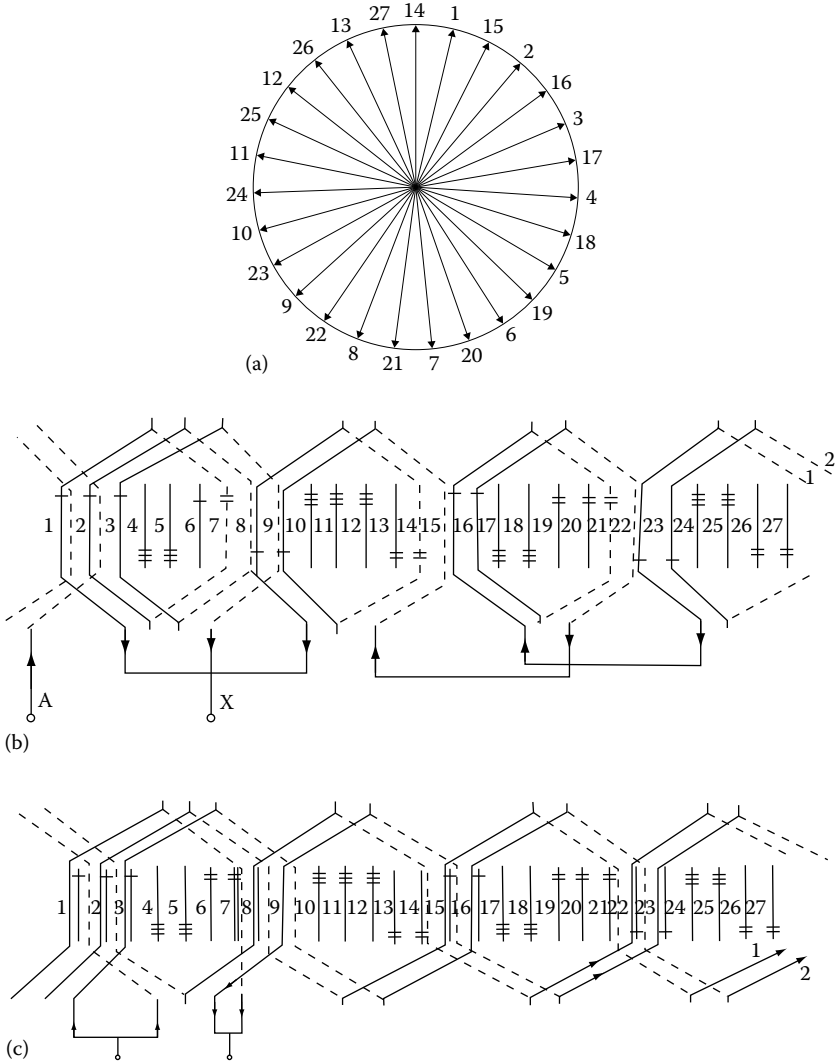
**FIGURE 5.10**

Single-layer 24-slot, 4-pole, 3-phase winding: (a) emf phasors (star of arrows) and (b) winding layout with only phase A coils shown.

as the natural order (Figure 5.11); this is so because $t=1$ and $q = N_s/2p_1m = 27/(2 \times 2 \times 3) = 2 + 1/4$, that is fractional q .

- Then the next $N_s/m = 9$ arrow group is allocated to phase C and then the last one to phase B (Figure 5.11a).
- The slot allocation to phases in the slot emf phasor star (Figure 5.11a) refers to the first layer. The coil span $y = \text{integer}(N_s/2p_1) = \text{integer}(27/4) = 6$ slot pitches. This is implicitly a chorded-coil winding.
- For bar-coils, either lap (Figure 5.11b) or wave coils (Figure 5.11c) are used. The latter results in shorter additional connection cables between coils in a phase.
- The winding factor, $K_{w1} = K_{q1}$, K_{y1} , may be calculated as such for $q = \text{integer}$. For $q = a + b/c$, $a \geq 1$, which is fractional; the distribution factor, K_{q1} , is calculated as for integer $q = ac + b = 2 \times 4 + 1 = 9$ (in our case), as seen in Figure 5.11a.

$$K_{q1} = \frac{\sin \frac{\pi}{6}}{(ac + b) \sin \frac{\pi}{6(ac+b)}} \quad (5.26)$$

**FIGURE 5.11**

Distributed fractional 3-phase, two-layer windings: $N_s = 27$ slots, $2p_1 = 4$ poles: (a) slot-emf phasor star, (b) winding layout with lap bar coils, and (c) with wave bar coils

- Fractional $q > 1$ is used to reduce the order (and amplitude) of the first slot harmonic of the airgap flux density:

$$(\nu_s)_{\min} = (2qkm \pm 1)_{k=1} = 2(ac + b)m \pm 1 \quad (5.27)$$

Note. Pole count ($2p_1$) changing windings, used for induction machines (IMs) to produce two synchronous speeds ($\omega_1 = f_1/p_1$), will be discussed in the next section.

5.3.6 Pole Count Changing AC 3-Phase Distributed Windings

In some applications, two-speed operation (in the 2:1 ratio for example) is required but the total motor cost is paramount (hairdryers or hand-drill-tools).

Let us consider again the $N_s = 24$ slot, $2p_1 = 4$ poles, two-layer winding. Changing the number of poles changes the ideal no load (synchronous or field) speed, $n_1 = f_1/p_1$. To reduce the number of poles from $2p_1 = 4$ to $2p_1 = 2$, each phase winding is divided into two parts ($A_1 - X_1$, $A_2 - X_2$ for phase A), each part referring to the two neighboring poles of the four-pole winding (Figure 5.12). By reversing the connection of 1-phase half winding

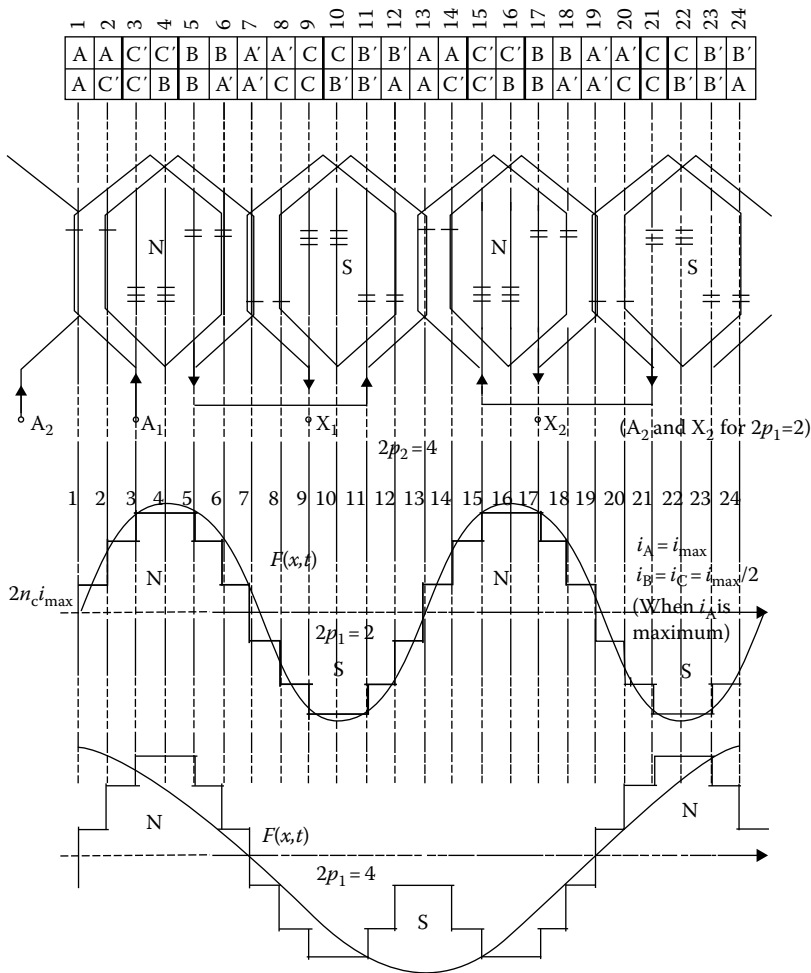


FIGURE 5.12
Pole changing winding: $N_s = 24$, $2p_1 = 4$, $2p_1 = 2$.

the number of poles is reduced from $2p_1 = 4$ to 2 (or vice versa). We now have four terminals for each phase in the terminal box. The pole switching (changing) may be done via an electromechanical power switch or, in variable speed drives, by two twin PWM inverters serving each winding half.

When operated at a constant voltage and frequency power grid, the connection between the two-half windings for $2p_1$ and $2p'_1$ leaves two main alternatives: the same torque or the same power for both speeds. For the case of constant power, the series triangle connection for $2p_1 = 4$ (small speed) is transformed into a parallel star connection for $2p'_1 = 2$ (high speed).

Winding utilization is worse at high speeds ($2p'_1 = 2$ in our case) as the neighboring low span coil groups form a large span pole. More on pole changing windings can be found in Ref. [4, Chapter 4].

5.3.7 Two-Phase AC Windings

When only a single-phase supply is available (residential applications), 2-phase windings are needed: main winding (m) and auxiliary winding (aux). The latter has either a resistor or a capacitor in series. As already shown in Section 5.3.1, the two windings are displaced by 90° (electrical), and they are symmetrical for a certain speed by choosing a pertinent capacitor, C .

Symmetrization for the start (C_{start}) and separately for the rated speed ($C_{\text{run}} \ll C_{\text{start}}$) is typical for heavy-start, high-efficiency and power factor applications. For light and infrequent starting, a single capacitor, C , may be used, and, for bidirectional motion, the capacitor may be switched from one winding to another (Figure 5.13).

For some single-capacitor 2-phase windings, the auxiliary winding is disconnected after starting, so it is designed to occupy only 33.3% of the periphery with the main winding occupying the other 66.6%. For reversible motion, they have to be identical (Figure 5.14).

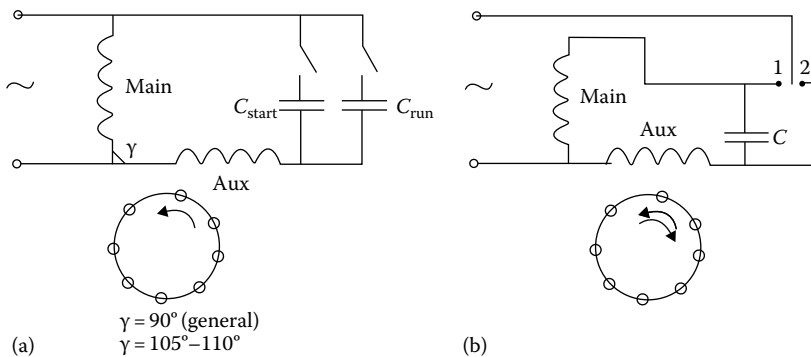


FIGURE 5.13

Two-phase induction motor: (a) unidirectional motion and (b) bidirectional motion: 1—forward; 2—backward.

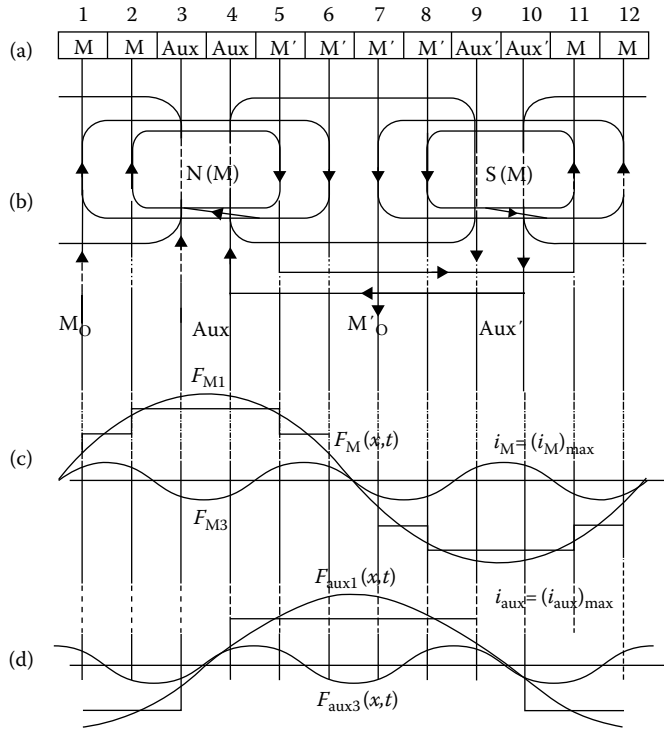


FIGURE 5.14 Capacitor start 2-phase winding: $N_s = 12$ slots, $2p_1 = 2$ poles: (a) slot/phase allocation, (b) coil connections, (c) main phase mmf, and (d) auxiliary phase mmf.

To reduce torque pulsations due to mmf space harmonics, the two windings are built with different numbers of turns/coils so as to produce a quasi-sinusoidal mmf spatial distribution.

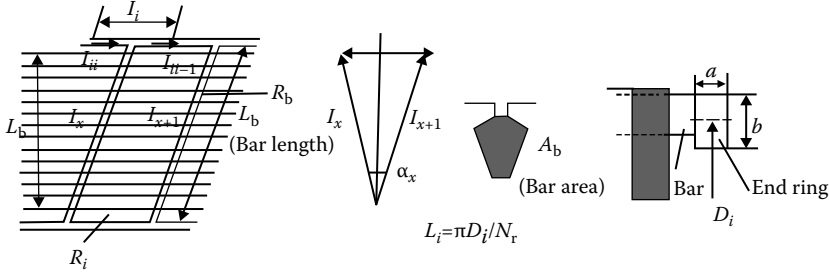
The pole count changing windings may also be built for 2-phase windings [4, Chapter 4].

Three-phase and 2-phase distributed windings are characteristic not only of IMs but also of synchronous machine stators.

Note. Typical windings for linear induction motors will be discussed in Section 5.24.

5.3.8 Cage Rotor Windings

Faultless cage rotor windings are symmetric windings, and thus the time phase shift between currents in neighboring end ring segments, I_i and I_{i+1} , is $\alpha_{er} = 2\pi p_1 / N_r$ (Figure 5.15).

**FIGURE 5.15**

Rotor cage geometry.

Consequently, the end ring and cage-bar currents, I_r and I_b , are related by

$$I_r = \frac{I_b}{2 \sin \frac{\alpha_{er}}{2}} \quad (5.28)$$

Let us denote by R_b and R_r the bar and ring segment resistances:

$$R_b = \rho_b \frac{l_b}{A_b}; \quad R_r = \rho_r \frac{l_r}{A_r}; \quad A_r = a \times b; \quad l_r = \frac{\pi D_{ring}}{N_r} \quad (5.29)$$

We may lump the bar and ring resistances into an equivalent bar resistance, R_{be} :

$$R_{be} I_s^2 = R_b I_b^2 + R_r I_r^2 \quad \text{or} \quad R_{be} = R_b + \frac{R_r}{2 \sin^2 \frac{\pi p_1}{N_r}} \quad (5.30)$$

When the number of rotor slots per pole pair (N_r/p_1) becomes small or fractional, the above expressions are less trustworthy.

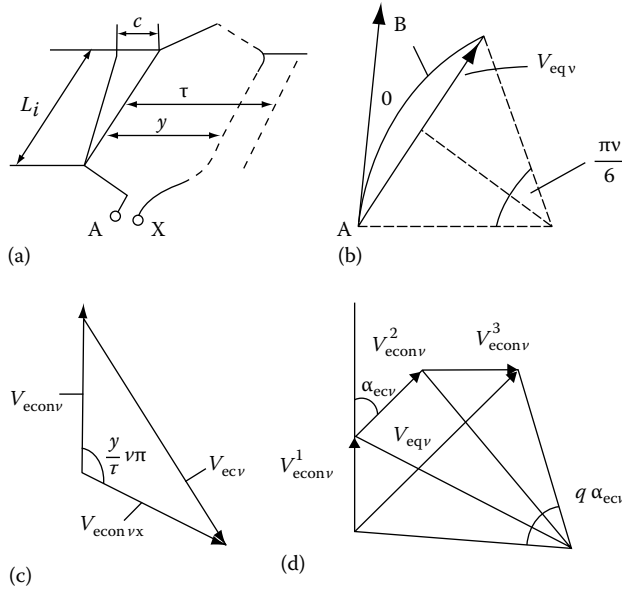
Let us consider that the rotor dc or PM excitation is zero (as in ims) and the airgap is uniform (salient poles) and that the machine stator is 3-phase ac current fed to produce a sinusoidal mmf, and thus a sinusoidal airgap flux density is marked by mmf and slot harmonics (Figure 5.16).

For low-power machines, the stator slots may be skewed (Figure 5.17a) by a length c along the rotor axis. The flux density along the rotor axis is phase shifted due to skewing (Figure 5.17b) with an emf reduction/turn/slot by the so-called the skewing factor, K_{cv} :

$$K_{cv} = \frac{\overline{AB}}{AOB} = \frac{\sin \frac{c\sqrt{\pi}}{2\tau}}{\frac{c\sqrt{\pi}}{2\tau}} \quad (5.31)$$

FIGURE 5.16

Airgap flux density of a stator ac winding with uniform airgap and a passive iron rotor.


FIGURE 5.17

Winding factor components: (a and b) skewed slots, (c) with chorded coils, and (d) distribution factor K_{qv} derivation.

Thus, the emf is self-induced by the airgap flux density harmonic, B_{gv} , in a conductor in slot, V_{econv} :

$$V_{econv} = B_{gv} \frac{UL_e}{\sqrt{2}} K_{cv} = \frac{1}{2\sqrt{2}} \omega_1 K_{cv} \Phi_v; \quad U = 2\tau_v f_1 \quad (5.32)$$

where Φ_v is the pole flux for the harmonic, v .

A coil with span y (Figure 5.17) leads to another coil emf reduction by the already defined chording factor, K_{yv} :

$$V_{ecoilv} = 2K_{yv} V_{econv} n_c; \quad K_{yv} = \sin \frac{vy}{\tau} \frac{\pi}{2} \quad (5.33)$$

There are n_c turns/coil, q coils/phase belt/pole (Figure 5.17d):

$$V_{eqv} = qK_{qv} V_{ecoilv}; \quad K_{qv} = \frac{\sin \frac{v\pi}{6}}{q \sin \frac{v\pi}{6q}} \quad (5.34)$$

We have just reobtained the distribution factor, K_{qv} , of the mmf, as expected.

With all $2p_1$ phase belts in series, the phase emf RMS value V_{ev} is

$$V_{ev} = \pi\sqrt{2}f_1 W_1 K_{wv} \Phi_v; \quad K_{wv} = K_{qv} K_{yv} K_{cv}; \quad W_1 = 2pq n_c \quad (5.35)$$

$K_{w\nu}$ is the total winding factor for the space harmonic, ν . For $\nu = 1$, the fundamental component is obtained. It is very similar to the emf in a transformer winding with the total winding correction factor, $K_{w\nu}$, and with the polar flux, Φ_ν .

5.4 Induction Machine Inductances

In order to investigate the IMs using the equivalent electric circuit theory, we need to define the phase self and mutual inductances and phase resistances for the stator and rotor, as previously done for the transformer. The self-inductance of a winding considered here is made of two main components:

- Main inductance (L_{ssm} for the stator)
- Leakage inductance (L_{sl} for the stator)

L_{ssm} refers to the main magnetic field that crosses the airgap and surrounds both stator and rotor windings, while L_{sl} refers to the leakage flux that surrounds only one winding as in the transformer.

5.4.1 Main Inductance

For a single-phase ac distributed winding, L_{ssm} may be calculated either from its flux linkage or from its magnetic energy. Let us use the flux linkage route as in Equation 5.4:

$$B_{g1} = \frac{\mu_0 F_{m1\text{phase}}}{g_e}; \quad g_e = K_c g (1 + K_s) \quad (5.36)$$

F_{m1} is the amplitude of a single-phase mmf per pole (Equation 5.10).

$$F_{m1\text{phase}} = \frac{2W_1 I \sqrt{2} K_{q1} K_{y1}}{p_1 \pi} \quad (5.37)$$

$K_c = (1.1\text{--}1.3)$ —Carter coefficient to account for the slot opening effect on airgap

$K_s = (0.2\text{--}0.5)$ (larger for $2p_1 = 2$ poles)—magnetic saturation factor (as in Chapter 4). But the fundamental flux per pole, Φ_1 , is

$$\Phi_1 = \frac{2}{\pi} B_{g1} L_e \tau; \quad \Psi_{ssm1} = \Phi_1 W_1 K_{w1} \quad (5.38)$$

and thus, from Equations 5.36 through 5.38, L_{ssm} is

$$L_{ssm} = \frac{\Psi_{ssm}}{I \sqrt{2}} = \frac{4\mu_0}{\pi^2} (W_1 K_{w1})^2 \frac{\tau L_e}{p_1 g_e} \quad (5.39)$$

For space harmonics, Equation 5.39 becomes simply

$$L_{ssm\nu} = \frac{\Psi_{ssm\nu}}{I\sqrt{2}} = \frac{4\mu_0}{\pi^2} \left(\frac{W_1 K_{w1\nu}}{\nu} \right)^2 \frac{\tau L_e}{p_1 g_e} \quad (5.40)$$

For a 3-phase current supply, the so-called cyclic main (magnetization) inductance is $L_{1m\nu}$

$$L_{1m\nu} = \frac{V_{e\nu}}{\omega_1 I} = \frac{6\nu_0}{\pi} \left(\frac{W_1 K_{w\nu}}{\nu} \right)^2 \frac{\tau L_e}{p_1 g_e} \quad (5.41)$$

with $V_{e\nu}$ from Equation 5.35.

The harmonics ($\nu > 1$) in $L_{1m\nu}$ are also part of the leakage field as they, in fact, do not couple tightly with the rotor.

For the 3-phase connection, the airgap magnetic flux density is

$$(B_{g1})_{3\text{-phase}} = \frac{\mu_0 F_{1m}}{K_{cg} (1 + K_s)}; \quad F_{1m} = \frac{3\sqrt{2} w_1 I_0 K_{w1}}{p_1 \pi} \quad (5.42)$$

The main inductance field corresponds to the magnetization current, I_0 , which is about the same as the no-load current, I_{10} (as for the transformer), that is, with zero rotor currents. Also, we may “build” (calculate) the no-load magnetization curve as for the brush-commutator machine discussed in Section 4.3. We do not repeat the process here but introduce it as a proposed problem. Industrial design experience has synthesized the no load magnetization curve as $B_{g1} (I_0/I_n)$ with $2p_1$ poles as parameter (Figure 5.18) or as l_{1m} (p.u.) versus I_0/I_n ; (l_{1m} (p.u.) = $\omega_1 L_{1m} I_n / V_n$).

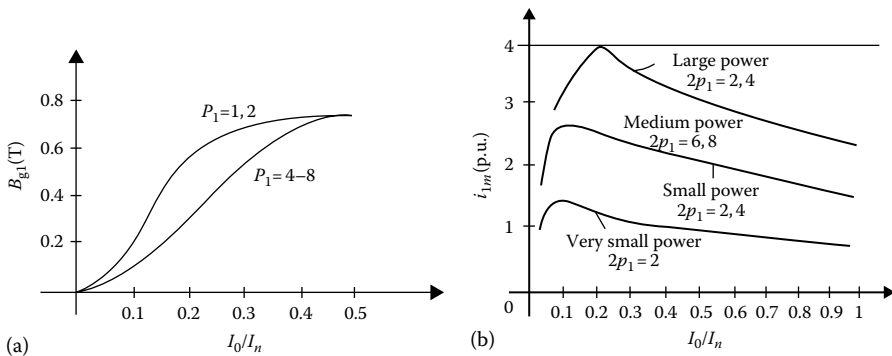


FIGURE 5.18

Magnetization characteristics: (a) typical IM magnetization curves and (b) magnetization inductance.

5.4.2 Leakage Inductance

The leakage inductance is related to a single phase as no leakage flux coupling between phases is considered. This is not to be done for dual windings in the same slots. Leakage flux lines are illustrated in Figure 5.19, and they lead to the following components of phase leakage inductance:

- Slot leakage inductances: $L_{\text{s slot}}, L_{\text{r slot}}$ which refer to the flux lines that cross the slot
- Zig-zag leakage inductances: $L_{\text{zls}}, L_{\text{zlr}}$
- End-connection leakage inductance: $L_{\text{els}}, L_{\text{elr}}$
- Skewing leakage inductance: L_{skewr}
- Differential leakage inductance: $L_{\text{dls}}, L_{\text{dlr}}$

The last subscript “s” stands for the stator and “r” for the rotor. The differential leakage stator inductance, L_{dls} , refers to the space harmonics, ν , of the main inductance, $L_{1\text{m}}$ (Equation 5.41):

$$L_{\text{dls}} = \sum_{\nu > 1} L_{1\text{m}\nu} = 2\mu_0 \frac{W_1^2 L_c}{p_1 q} \lambda_{\text{dls}}; \quad \lambda_{\text{dls}} = \frac{3q}{\pi^2 g K_c} \sum_{\nu > 1} \frac{K_{w\nu}^2}{\nu^2} \quad (5.43)$$

for $\nu = km \pm 1$. For single-phase machines, L_{dls} tends to be larger.

Note. As the rotor cage-induced currents may reduce L_{dls} in Equation 5.43, a more detailed assessment of L_{dls} is required for a rigorous design (see Ref. [4, Chapter 6]).

The slot leakage inductance may be calculated, in the absence of a skin effect (Chapter 2), by considering a linear magnetic field distribution from the slot bottom to the slot top for rectangular slots (Figure 5.19).

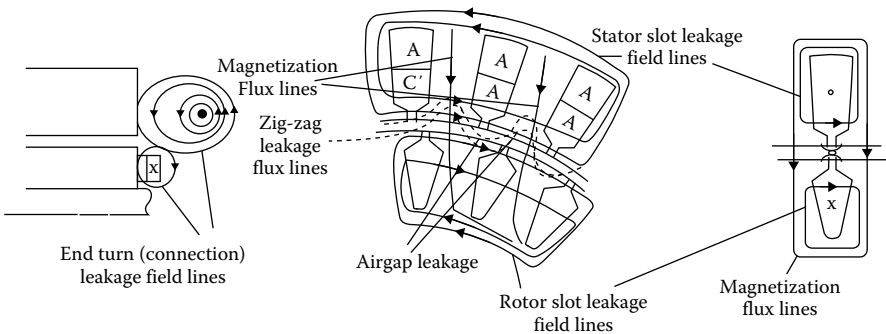


FIGURE 5.19

Classification of leakage flux lines.

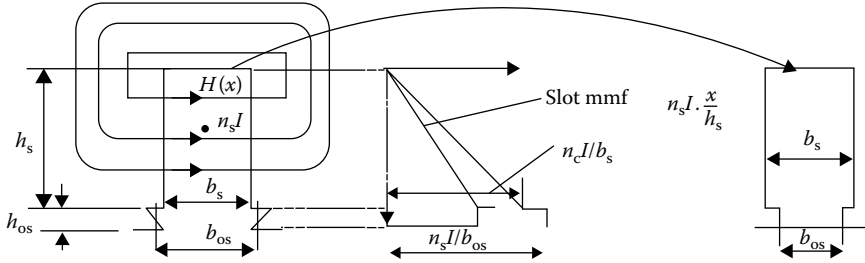


FIGURE 5.20
Rectangular slot leakage field.

Ampere's law, along the contours, in Figure 5.20 leads to

$$\begin{aligned} H(x) b_s &= \frac{n_s I x}{h_s} \quad 0 \leq x \leq h_s; \quad n_s \text{—conductors/slot} \\ H(x) b_s &= n_s I \quad h_s \leq x \leq h_s + h_{os} \end{aligned} \quad (5.44)$$

The magnetic energy in the slot volume, W_{ms} , leads to the leakage inductance for slot $L_{s\text{slot}}$:

$$L_{s\text{slot}} = \frac{2}{I^2} W_{ms} = \frac{2}{I^2} \frac{1}{2} L_e b_s \mu_0 \int_0^{h_s+h_{os}} H^2(x) dx = \mu_0 n_s^2 L_e \lambda_s; \quad \lambda_s = \frac{h_s}{3b_s} + \frac{h_{os}}{b_{os}} \quad (5.45)$$

λ_s is called the slot geometrical permeance coefficient:

$$\lambda_s = 0.5 - 2.5; \quad \text{for } \frac{h_s}{b_s} < 6; \quad h_{os} = (1 - 3) 10^{-3} \text{m} \quad (5.46)$$

which depends on the slot aspect ratio, h_s/b_s ; deeper slots lead to larger λ_s .

Now a phase occupies $N_s/m = 2p_1q$ slots, and thus the phase leakage inductance, $L_{s\text{ls}}$, is

$$L_{s\text{ls}} = L_{s\text{slots}} 2p_1q = 2\mu_0 \frac{W_1^2 L_e}{p_1q} \lambda_s \quad (5.47)$$

For trapezoidal or round slots, separate expressions are available (see Ref. [4, Chapter 6]).

The zig-zag leakage inductance, $L_{z\text{ls},r}$, is [4]

$$L_{z\text{ls},r} = 2\mu_0 \frac{W_s^2 L_e}{p_1q} \lambda_{z\text{s},r}; \quad \lambda_{z\text{s},r} = \frac{5g \frac{K_c}{b_{0\text{s},r}}}{5 + 4g \frac{K_c}{b_{0\text{s},r}}} \frac{3\beta_y + 1}{4} < 1; \quad \beta_y = \frac{y}{\tau} \quad (5.48)$$

$\beta_y = 1$ for cage rotors or full-pitch stator coils, and $b_{0s,r}$ is the stator (rotor) slot opening at the airgap.

The end-connection leakage inductance refers to a 3D magnetic field at machine axial ends.

$$L_{els,r} = 2\mu_0 \frac{W_1^2 L_e}{p_1 q} \lambda_{es,r} \quad (5.49)$$

while the geometrical permeance coefficient, $\lambda_{es,r}$, [Ref. 4, Chapter 6] is

$$\lambda_{es} \approx 0.67 \frac{q}{L_e} (l_{es} - 0.64\tau); \quad \text{single-layer windings} \quad (5.50)$$

where l_{es} is the coil end-connection length.

$$\lambda_{es} \approx 0.34 \frac{q}{L_e} (l_{es} - 0.64\tau); \quad \text{double-layer windings} \quad (5.51)$$

For cage rotors with end rings attached to the rotor stack (core) [4]:

$$\lambda_{ering} \approx \frac{3D_{ring}}{2L_e \sin^2\left(\frac{\pi p_1}{N_r}\right)} \log 4.7 \frac{D_{ring}}{a + 2b} < (1.5 - 2) \quad (5.52)$$

Short stacks (L_e/τ —small) lead to relatively large λ_{es} , and thus larger leakage inductance, besides larger copper losses. This is why the stack length (L_e) per pole pitch (τ) ratio should be larger than unity if possible.

The skewing leakage inductance, L_{skew} , is due to the fact that the stator–rotor magnetic coupling is reduced by the inclination of rotor slots [4]:

$$L_{skew} = (1 - K_{skew}^2) L_{lm}; \quad K_{skew} = K_{cl} = \frac{\sin \alpha_{skew}}{\alpha_{skew}}; \quad \alpha_{skew} = \frac{c}{\tau} \frac{\pi}{2} \quad (5.53)$$

Skewing increases the leakage inductance, and this will reduce the peak (breakdown) torque as shown later in this chapter.

The total leakage inductance, $L_{sls,r}$, is

$$L_{s,rl} = L_{s,rlslot} + L_{zls,r} + L_{dls,r} + L_{els,r} + L_{skews,r} = 2\mu_0 \frac{W_s^2 L_e}{p_1 q} \sum \lambda_{is,r} \quad (5.54)$$

The rotor leakage inductance (L_{rl}) is already reduced to the stator but let us derive this reduction, as it is necessary for the equivalent circuit of the IM, be it with the cage or the wound rotor.

5.5 Rotor Cage Reduction to the Stator

The rotor cage may be considered as a multiphase winding with $m_r = N_r$ (rotor slots) phases, and $W_2 = 1/2$ turns per phase and whose winding factor K_{w2} (for axial slots) is unity.

The reduction to the stator means mmf equivalence with a 3-phase equivalent winding, with W_1 turns per phase and I'_r current:

$$(F_{1m})_r = \frac{3W_1K_{w1}I'_r\sqrt{2}}{\pi p_1} = N_r \frac{1}{2} \frac{I_b\sqrt{2}}{\pi p_1} K_{\text{skewr}} \quad (5.55)$$

So,

$$I'_r = K_i I_b; K_i = \frac{N_r K_{\text{skewr}}}{6W_1K_{w1}} \quad (5.56)$$

For loss equivalence,

$$R_{be} I_b^2 N_r = 3R'_r I_r^2 \quad (5.57)$$

$$R'_r = R_{be} \frac{N_r}{3K_i^2}$$

In a similar way, for the cage leakage inductance

$$L'_{rl} = L_{bel} \frac{N_r}{3K_i^2} \quad (5.58)$$

5.6 Wound Rotor Reduction to the Stator

As for the transformer, the wound rotor 3-phase winding may be reduced to the stator by conserving the fundamental mmf, winding losses and magnetic leakage energy, and power.

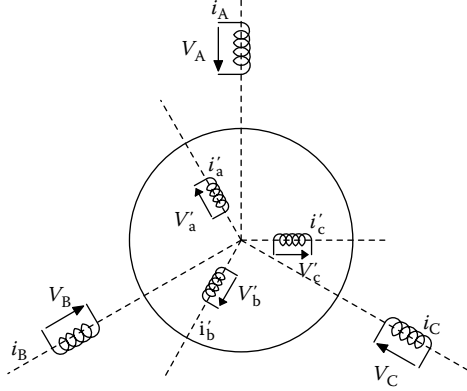
$$\frac{I'_r}{I_r} = K_i = \frac{V_r}{V'_r}; \quad K_i = \frac{W_2 K_{w1r}}{W_1 K_{w1s}}; \quad \frac{R'_r}{R_r} = \frac{L'_{rl}}{L_{rl}} = \frac{1}{K_i^2} \quad (5.59)$$

$$\frac{3W_1 K_{w1s} I'_r \sqrt{2}}{\pi p_1} = \frac{3W_2 K_{w1r} I_r \sqrt{2}}{\pi p_1} \quad (5.60)$$

By this reduction, implicitly, the mutual inductance between the stator and rotor equivalent three phases (L_{srsm}) is equal to the main self-inductance, L_{ssm} .

5.7 Three-Phase Induction Machine Circuit Equations

The 3-phase induction machine may be represented by three stator and three rotor equivalent windings with magnetic coupling between them (Figure 5.21).

**FIGURE 5.21**

The 3-phase induction machine with reduced rotor windings.

Let us neglect for the time being magnetic saturation, iron losses, and mmf space (or emf time) harmonics. For phase coordinates: stator coordinates for the stator, rotor coordinates for the rotor, there are no motion-induced voltages:

$$\begin{aligned}
 I_{A,B,C}R_s - V_{A,B,C} &= -\frac{d\Psi_{A,B,C}}{dt} \\
 I'_{a,b,c}R'_r - V'_{a,b,c} &= -\frac{d\Psi'_{a,b,c}}{dt}
 \end{aligned} \tag{5.61}$$

Because the mmf distribution is sinusoidal along the stator bore, the mutual inductance between stator and rotor phases varies sinusoidally with the angle between the respective phases (rotor position electrical angle, Θ_{er}). The self-inductance and stator–stator and rotor–rotor mutual inductances do not depend on the rotor position. Consequently, Ψ_A and Ψ'_a flux linkages are

$$\begin{aligned}
 \Psi_A &= L_{sl}I_A + L_{ssm} \left(I_A + I_B \cos \frac{2\pi}{3} + I_C \cos \frac{-2\pi}{3} \right) \\
 &\quad + L_{ssm} \left(I'_a \cos \Theta_{er} + I'_b \cos \left(\Theta_{er} + \frac{2\pi}{3} \right) + I'_c \cos \left(\Theta_{er} - \frac{2\pi}{3} \right) \right)
 \end{aligned} \tag{5.62}$$

$$\begin{aligned}
 \Psi'_a &= L'_{ler}I'_a + L_{ssm} \left(I_A + I_B \cos \frac{2\pi}{3} + I_C \cos \frac{-2\pi}{3} \right) \\
 &\quad + L_{ssm} \left(I_A \cos (-\Theta_{er}) + I_B \cos \left(-\Theta_{er} + \frac{2\pi}{3} \right) + I_C \cos \left(-\Theta_{er} - \frac{2\pi}{3} \right) \right)
 \end{aligned} \tag{5.63}$$

Let us consider

$$\begin{aligned} I_A + I_B + I_C &= 0 \\ I'_a + I'_b + I'_c &= 0 \end{aligned} \quad (5.64)$$

Then, with Equation 5.64, Ψ_A, Ψ'_a become

$$\Psi_A = (L_{sl} + L_{1m}) I_A + L_{1m} \left(I'_a \cos \Theta_{er} - \frac{1}{\sqrt{3}} (I'_b - I'_c) \sin \Theta_{er} \right) \quad (5.65)$$

$$\Psi_A = L'_{sl} I'_a + L_{1m} \left(I_A \cos (-\Theta_{er}) - \frac{1}{\sqrt{3}} (I_B - I_C) \sin (-\Theta_{er}) \right) \quad (5.66)$$

$L_{1m} = 3L_{ssm}/2$ —the cyclic main inductance

To eliminate I'_b, I'_c, I_B , and I_C from the above equations, and for phase segregation we need to observe that for symmetric direct sequence currents (see also [6])

$$jI_{A,a'+} = -\frac{1}{\sqrt{3}} (I_{B,b'+} - I_{C,c'+}) \quad (5.67)$$

while for inverse sequence currents

$$jI_{A,a'-} = \frac{1}{\sqrt{3}} (I_{B,b'-} - I_{C,c'-}) \quad (5.68)$$

So, from Equations 5.65 through 5.68 for direct and inverse sequences, the flux current relationships become

$$\begin{aligned} \Psi_{A+} &= L_{sl} I_{A+} + L_{1m} (I_{A+} + I'_{a+} e^{j\Theta_{er}}) \\ \Psi'_{a+} &= L'_{sl} I'_{a+} + L_{1m} (I'_{a+} + I_{A+} e^{-j\Theta_{er}}) \end{aligned} \quad (5.69)$$

$$\begin{aligned} \Psi_{A-} &= L_{sl} I_{A-} + L_{1m} (I_{A-} + I'_{a-} e^{-j\Theta_{er}}) \\ \Psi'_{a-} &= L'_{sl} I'_{a-} + L_{1m} (I'_{a-} + I_{A-} e^{j\Theta_{er}}) \end{aligned} \quad (5.70)$$

Let us consider symmetric (positive sequence) currents in the three phases; drop the “+” subscript and denote

$$I_a^s = I'_a e^{j\Theta_{er}} : \Psi_a^{1s} = \Psi'_a e^{j\Theta_{er}} \quad (5.71)$$

Now from Equations 5.69 and 5.70, we get

$$\begin{aligned}\Psi_A &= L_{sl}I_A + L_{1m}(I_A + I_a^{'s}) \\ \Psi_a' &= L_{er}'I_a + L_{1m}(I_a^{'s} + I_A)\end{aligned}\quad (5.72)$$

The total time derivative of Ψ_a' in Equation 5.70 becomes

$$\frac{d\Psi_a'}{dt} = \frac{d}{dt}(\Psi_a^{'s}e^{-j\Theta_{er}}) = \frac{d\Psi_a^{'s}}{dt}e^{-j\Theta_{er}} - j\frac{d\Theta_{er}}{dt}\Psi_a^{'s}e^{-j\Theta_{er}}; \quad \frac{d\Theta_{er}}{dt} = \omega_r = 2\pi p_1 n \quad (5.73)$$

Finally, the rotor equation in stator coordinates ($I_a^{'s}$, $\Psi_a^{'s}$) becomes

$$I_a^{'s}R_r' - V_a^{'s} = -\frac{d\Psi_a^{'s}}{dt} + j\omega_r\Psi_a^{'s}; \quad V_a^{'s} = V_a'e^{j\Theta_{er}} \quad (5.74)$$

Let us add the stator equation:

$$I_A R_s - V_A = -\frac{d\Psi_A}{dt} \quad (5.75)$$

A few remarks are in order:

- Equations 5.74 and 5.75 are valid for symmetric (direct) sequence transients and steady state.
- For the negative sequence symmetric currents, ω_r , becomes $(-\omega_r)$.
- In the rotor equation, too, the variables are all reduced to stator coordinates, including the rotor voltage, $V_a^{'s}$.
- For steady state, complex number variables may be used because the voltages and currents in Equations 5.74 and 5.75 are all sinusoidal at frequency ω_1 :

$$V_{A,B,C} = V_s\sqrt{2} \cos\left(\omega_1 t - (i-1)\frac{2\pi}{3}\right); \quad i = 1, 2, 3 \quad (5.76)$$

so $d/dt \rightarrow j\omega_1$.

- Multiplying rotor equation by $3(I_a^{'s})^*$, we notice that only the speed-containing term refers to electromagnetic power:

$$P_{em} = T_e \frac{\omega_r}{p_1} = -\text{Real}(3j\omega_1\Psi_a^{'s}(I_a^{'s})^*) \quad (5.77)$$

The electromagnetic torque of the stator or rotor is then

$$T_e = -3p_1 \text{Imag}(\Psi_a^{'s}(I_a^{'s})^*) = 3p_1 \text{Imag}(\Psi_A I_A^*) \quad (5.78)$$

Equation 5.78 reflects Newton's principle of action and reaction. From now on, we may abandon the A, a phase subscript, as the 3-phase equations are obtained from three single-phase segregated equations (as for transformers) with the influence of other phases lumped in (via L_{1m}).

5.8 Symmetric Steady State of 3-Phase IMs

By symmetric steady state, we mean symmetric sinusoidal voltages in the stator at frequency, ω_1 , eventually also in the rotor, at frequency, ω_2 , and constant speed $\omega_r = \omega_1 - \omega_2$. In complex variables,

$$V_{A,B,C} = V_s \sqrt{2} \cos \left(\omega_1 t - (i-1) \frac{2\pi}{3} \right) \quad (5.79)$$

becomes

$$\underline{V}_s = V_s \sqrt{2} e^{j\omega_1 t}; \quad \underline{V}_r^{1s} = V_r^l \sqrt{2} e^{j(\omega_1 t - \gamma)} \quad (5.80)$$

Also in Equations 5.74 and 5.75, $d/dt \rightarrow j\omega_1$ and thus

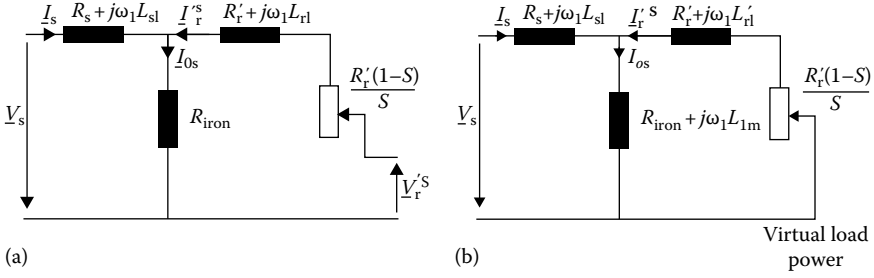
$$\begin{aligned} I_s \underline{Z}_{sl} - \underline{V}_s &= \underline{V}_{es}^0; \quad S = \frac{(\omega_1 - \omega_r)}{\omega_1}; \quad \underline{Z}_{sl} = R_s + j\omega_1 L_{sl} \\ I_r^{1s} \underline{Z}_{rl}' - \frac{V_r^s}{S} &= \underline{V}_{es}^0; \quad \underline{Z}_{rl}' = \frac{R_r'}{S} + j\omega_1 L_{rl} \\ \underline{V}_{es}^0 &= -j\omega_1 L_{1m} I_{01}; \quad \underline{I}_{0s} = \underline{I}_s + \underline{I}_r^s \end{aligned} \quad (5.81)$$

If we add the core losses, as for the transformer, via a resistor in series with $j\omega_1 L_{1m}$, the emf V_{er}^0 becomes V_{er} :

$$\underline{V}_{er} = -\underline{Z}_{1m} I_{01}; \quad \underline{Z}_{1m} = R_{iron} + j\omega_1 L_{1m} \quad (5.82)$$

S is called the slip of the IM. It is the p.u. difference of ideal no-load electrical speed, ω_1 , and the electrical rotor speed, ω_r , a kind of p.u. speed regulation.

Equations 5.81 resemble those of a transformer (with rotor sink association of signs) but the slip, S , is the new variable directly related to the machine speed. The apparent load, if the rotor is short-circuited ($V_r^s = 0$),

**FIGURE 5.22**

The IM equivalent circuit for symmetric steady state: (a) with wound rotor (b) with cage rotor.

is $R'_r(1-S)/S$ and is speed dependent. The equivalent circuit for Equations 5.81 is thus straightforward (Figure 5.22). Note that in the equivalent circuit, all variables are at ω_1 . There is no motion and the mechanical power of the real machine is equal to the active power in $R'_r(1-S)/S$.

The electromagnetic torque, from Equation 5.77, is

$$T_e = -3p_1 \text{Imag}(\underline{\Psi}_r^s (I_r^s)^*) = 3p_1 L_{lm} \text{Imag}(\underline{I}_s I_r^s) \quad (5.83)$$

Only for the cage rotor (or short-circuited rotor), the electromechanical power is

$$P_{elm} = T_e \frac{\omega_r}{p_1} = T_e \frac{\omega_1}{p_1} (1-S) = 3R'_r (I_r^s)^2 \frac{1-S}{S} \quad (5.84)$$

$$T_e = \frac{3p_1 R'_r (I_r^s)^2}{\omega_1 S} = \frac{p_1}{\omega_1} P_{elm}; \quad P_{elm} = \frac{3R'_r (I_r^s)^2}{S} = \frac{p_{corotor}}{S} \quad (5.85)$$

P_{em} is the electromagnetic power, or the power in the total secondary equivalent resistance. P_{em} is the total active power that passes from the stator to the rotor or vice versa. P_{em} may be either positive (for the motor mode) or negative (for the generator mode) depending on $S \gtrless 0$.

The motor produces torque ($T_e > 0$) in the direction of motion while the generator brakes the rotor ($T_e < 0$ for $\omega_r > 0$). The difference between the generator and the braking modes is that only for the generator $P_{em} < 0$ for $\omega_r > 0$, and thus a part of the kinetic energy from the rotor is returned back to the electric power source. The basic operation modes for positive ideal no-load speed, ω_1 , for the cage rotor are shown in Table 5.1.

Let us discuss in detail a few particular symmetric steady-state operation modes.

TABLE 5.1Operation Modes ($f_1/p_1 > 0$) for Cage Rotor IMs

S	$-\infty \leftarrow$	0	++++	1	$\rightarrow +\infty$
n	$+\infty \leftarrow$	f_1/p_1	++++	0	$\rightarrow -\infty$
T_e	—	0	++++	++++	++++
P_{em}	—	0	++++	++++	++++
Operation mode	Generator		Motor		Braking

5.9 Ideal No-Load Operation/Lab 5.1

The ideal no load corresponds to the zero rotor current ($I_r^s = 0$); from Equation 5.81:

$$\underline{V}_r^s = S_0 \underline{V}_{es} \approx -S_0 \underline{V}_{s0} \quad (5.86)$$

or

$$S_0 = \frac{\omega_1 - \omega_{r0}}{\omega_1} = -\frac{\underline{V}_r^s}{\underline{V}_{s0}} \quad (5.87)$$

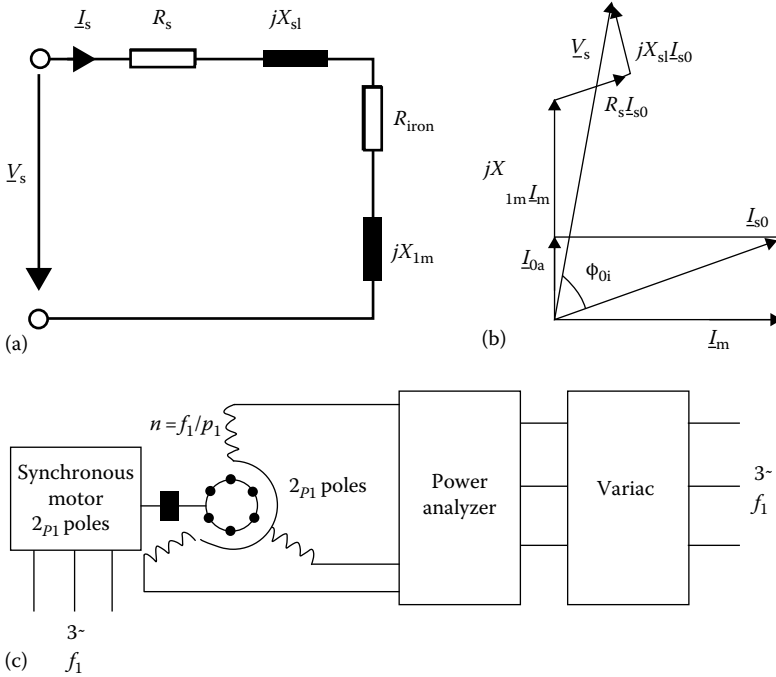
For $\underline{V}_{r0}^s$ (rotor voltage in stator coordinates, at stator frequency) about in phase with the stator voltage $\underline{V}_{s0}$, the machine slip at ideal no load is $S_0 < 0$, so $\omega_{r0}/\omega_1 > 1$, which is a supersynchronous operation. For the rotor voltage in stator coordinates, $\underline{V}_{r0}^s$ in phase opposition with the stator voltage $S_0 > 0$ and thus $\omega_{r0}/\omega_1 < 1$, which is a undersynchronous operation. So the wound rotor (doubly fed) induction machine can operate as motor or generator for $\omega_r > \omega_1$, provided the rotor-side PWM converter supplies rotor voltages with an adequate phase angle, at frequency $f_2 = Sf_1$.

For a short-circuited rotor (cage rotor), $S_0 = 0$ ($\underline{V}_{r0}^s = 0$) and the ideal no load speed is $n_0 = \omega_1/2\pi p_1 = f_1/p_1$. With the rotor circuit open (electrically: $I_r^s = 0$), the equivalent circuit of Figure 5.22 gains a simplified form (Figure 5.23a) with the phasor diagram of Figure 5.23b. The similarities with a transformer at no-load are clearly visible.

Active power absorbed at an ideal load represents the stator copper losses, p_{Co} , plus iron losses p_{iron} :

$$P_0 = 3R_s I_{s0}^2 + 3R_{iron} I_{s0}^2 = p_{Co} + p_{iron} \quad (5.88)$$

When recording measurements under an ideal no-load operation, the IM is driven by a synchronous motor (with a cage in the rotor) with the same number of poles, $2p_1$, to develop exactly the ideal no-load speed, $n_0 = f_1/p_1$. Alternatively, if a variable speed drive is available, the driving motor speed is increased until the stator current, I_{s0} , of the tested IM is minimum (which corresponds to the ideal no load speed). With R_s measured previously (say

**FIGURE 5.23**

Ideal no-load operation (cage rotor, $V_r^{1s} = 0$): (a) equivalent circuit, (b) phasor diagram, and (c) test rig.

in dc for small machines) and P_0 and I_{s0} , V_{s0} measured via a power analyzer, the iron losses, p_{iron} , may be calculated from Equation 5.88.

Further on,

$$X_{sl} + X_{lm} = \frac{V_{s0} \sin \varphi_0}{I_{s0}}; \quad \sin \varphi_0 = \sqrt{1 - \left(\frac{P_0}{3V_{s0}I_{s0}} \right)^2} \quad (5.89)$$

As for the transformer, the iron losses under load will be only slightly smaller than those under ideal no load for, given stator voltage V_{s0} and frequency f_1 . Additional, stray core losses occur on load.

Example 5.2 Ideal No Load of IM

A $2p_1 = 4$ pole cage rotor IM is driven at synchronism at $n_0 = 1800$ rpm, for $f_1 = 60$ Hz, at a rated voltage $V_s = 120$ V (phase RMS) and draws a current $I_{s0} = 4$ A. The power analyzer shows an input power $P_0 = 40$ W; the stator parameters are $R_s = 0.12 \Omega$ and $X_{sl} = 1 \Omega$. Calculate the iron losses p_{iron} , R_{iron} , X_{lm} , and $\cos \varphi_0$.

Solution:

From Equation 5.88, the core losses, p_{iron} , are

$$p_{\text{iron}} = P_0 - 3R_s I_{s0}^2 = 40 - 3 \times 0.12 \times 4^2 = 34.24 \text{ W}$$

Also from Equation 5.88, the core series resistance, R_{iron} , is

$$R_{\text{iron}} = \frac{p_{\text{iron}}}{3I_{s0}^2} = \frac{34.24}{3 \times 4^2} = 0.7133 \Omega \gg R_s$$

The power factor, $\cos \varphi_0$, is straightforward from Equation 5.89

$$\cos \varphi_0 = \frac{P_0}{3V_{s0}I_{s0}} = \frac{40}{3 \times 120 \times 4} = 0.0277; \quad \sin \varphi_0 \approx 1$$

Again, from Equation 5.104

$$X_{1m} = \frac{V_{s0} \sin \varphi_0}{I_{s0}} - X_{sl} = \frac{120 \times 1}{4} - 1 = 29 \Omega.$$

5.10 Zero Speed Operation ($S = 1$)/Lab 5.2

This time, if we neglect the magnetization current, the equivalent circuit remains with the so-called shortcircuit impedance (Figure 5.24a through c):

$$\underline{Z}_{sc} = R_{sc} + jX_{sc}; \quad R_{sc} = R_s + R_{rstart}^s; X_{sc} = X_{sl} + X_{rlstart}^s \quad (5.90)$$

Both rotor resistance and leakage inductance, R_{rstart}^s and $X_{rlstart}^s$, are in a power grid-connected IM, influenced by skin effect coefficients, $K_R(S\omega_1)$, $K_X(S\omega_1)$, derived in Chapter 2. They are thus different from R_r^s and X_{rl}^s values for rated (load) conditions at rotor (slip) frequency:

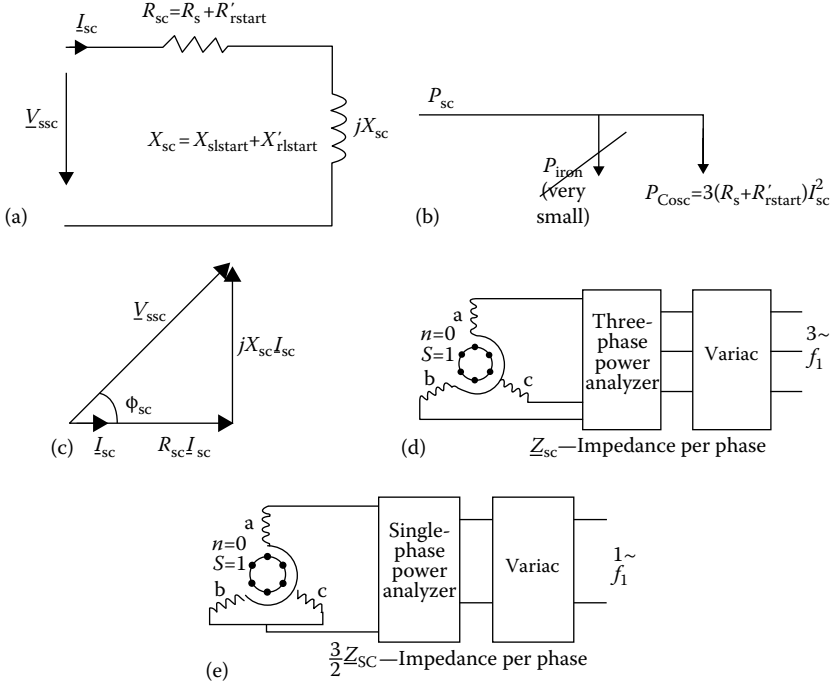
$$f_{2n} = S_n f_{1n} = (0.005 \div 0.05) f_{1n} \quad (5.91)$$

As expected, for the rated voltage, V_{sn} , at zero speed, the starting current, I_{start} , is large:

$$I_{start} = \frac{V_{sn}}{|Z_{sc}(S = 1)|} \quad (5.92)$$

In general, for well-designed cage rotor IMs to be connected directly at a power grid, $I_{start}/I_{rated} = (4.5\text{--}7.5)$.

Testing, however, at zero speed (Figure 5.24d) implies a lower voltage, obtained through a Variac, such that the current does not reach values above a rated current.


FIGURE 5.24

Zero speed operation ($s = 1$): (a) simplified equivalent circuit, (b) power flow, (c) phasor diagram, (d) 3-phase zero-speed testing, and (e) single-phase zero-speed testing.

And we measure V_{sc} , I_{sc} , and P_{sc} to obtain

$$P_{sc} \approx 3R_{scstart}I_{sc}^2; \quad \cos \phi_{sc} = \frac{P_{sc}}{3V_{sc}I_{sc}} \quad (5.93)$$

$$X_{sc} = \frac{V_{sc} \sin \phi_{sc}}{I_{sc}}; \quad R'_{rstart} = R_{scstart} - R_s \quad (5.94)$$

The starting torque is

$$T_{estart} \approx \frac{3R'_{rstart}I_{sc}^2}{\omega_1} p_1 \quad (5.95)$$

In general, for cage rotor IMs,

$$\frac{T_{estart}}{T_{erated}} = (0.7-2.5) \quad (5.96)$$

Because the voltage $V_{sc} \ll V_{sn}$, the core losses are neglected at zero-speed testing around the rated current. We may not separate X_{sl} from $X'_{rlstart}$ in

X_{sc} , so, in general, in the industry we adopt the equality condition: $X_{sl} = X_{rlstart}^{/s} = X_{scstart}/2$.

Now if we represent graphically the measured short-circuit current, I_{sc} , versus voltage, we get the results as shown in Figure 5.25.

While the zero-speed test at the rated frequency and rated current is good to estimate the starting current and torque for the rated voltage by proportionality, the skin effect makes the use of R_{sc} in calculating copper losses under load much less reliable.

Tests at zero speed with an PWM inverter at low frequency $f'_1 = S_n f_1 = f_{2n}$ should be proper for the purpose. To avoid shaft stalling (due to starting torque), a single phase frequency test at zero speed may be performed (Figure 5.24e).

Note. For closed slots on the rotor (to reduce noise and rotor surface additional core losses at the cost of lower breakdown torque and rated power factor), the I_{sc} (V_{sc}) straight line from tests at zero speed (Figure 5.25a) intersects the abscissa at E_s (6–12 V in 220 V per phase, 50(60) Hz IMs). This additional emf, E_s , is due to the closed rotor slot bridge's early magnetic saturation. E_s is 90% above the rotor current and is equal to

$$(E_s)_{\text{closeslots}} \approx \frac{4}{\pi} \pi \sqrt{2} (f W_1) \Phi_{\text{bridge}} K_{w1}; \quad \Phi_{\text{bridge}} = B_{\text{sbridge}} h_{\text{or}} L_e \quad (5.97)$$

$B_{\text{sbridge}} = (2 - 2.2)$ T is the magnetic saturation flux density of the iron core material, $h_{\text{or}} = (0.3 - 1)$ mm is the rotor bridge width, and L_e is the stack length.

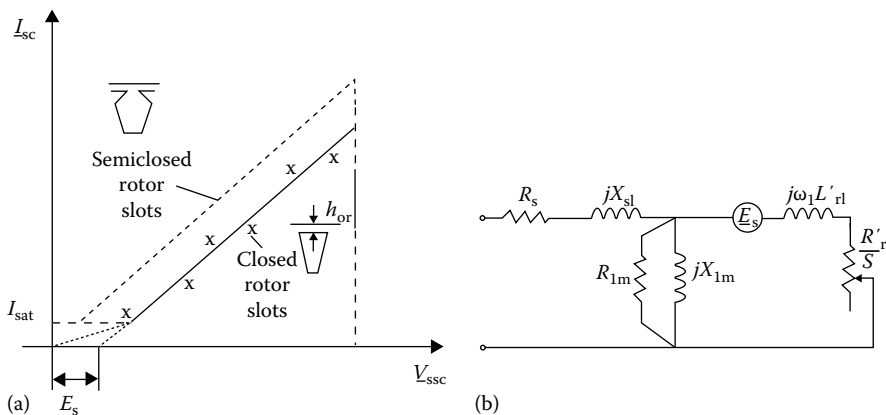


FIGURE 5.25

The case of closed rotor slots: (a) V_{sc} vs. I_{sc} and (b) the equivalent circuit with E_s added for closed rotor slots.

Example 5.3 Zero-Speed Operation of IM

The same IM (with semiclosed slots) as that referred to in Example 5.2 is tested at zero speed and rated current (star connection) $I_n = 12$ A, for phase voltage $V_{sc} = 20$ V (rated voltage is 120 V) and power $P_{sc} = 155$ W. The design values of $R_s = 0.12 \Omega$, $R_r^s = 0.12 \Omega$, and $X_{sl} = X'_{rl} = 1 \Omega$ hold. Calculate $R_{scstart}$, $X_{scstart}$, R_{rstart}^s , $X_{rlstart}^s$ and determine the skin effect factors, and K_R and K_X , and the starting torque at the rated voltage.

Solution:

From Equation 5.93:

$$R_{scstart} = \frac{P_{sc}}{3I_n^2} = \frac{155}{3 \times 12^2} = 0.358 \Omega$$

So

$$R_{rstart}^s = R_{scstart} - R_s = 0.358 - 0.12 = 0.238 \Omega$$

So

$$K_R = \frac{R_{rstart}^s}{R_r^s} = \frac{0.238}{0.12} = 2.38 > 1!$$

Also from Equations 5.93 and 5.94

$$\cos \varphi_{sc} = \frac{P_{sc}}{3V_{sc}I_n} = \frac{155}{3 \times 20 \times 12} = 0.2153; \quad \sin \varphi_{sc} = 0.976$$

$$X_{scstart} = \frac{V_{sc} \sin \varphi_{sc}}{I_n} = \frac{20}{12} \times 0.976 = 1.627 \Omega$$

Now

$$X_{rlstart}^s = X_{scstart} - X_{sl} = 1.627 - 1 = 0.627 \Omega$$

The skin effect factor on reactance K_X is

$$K_X = \frac{X_{rlstart}^s}{X'_{lr}} = \frac{0.627}{1} = 0.627 < 1$$

The starting torque, T_{estart} , Equation 5.95 is for rated voltage (120 V):

$$T_{estart} = \frac{3R_{rstart}^s I_{start}^2}{\omega_1} p_1 = \frac{3 \times 0.238 \times \left(12 \times \frac{120}{20}\right)^2 \times 2}{2\pi \times 60} = 19.646 \text{ Nm}$$

5.11 No-Load Motor Operation (Free Shaft)/Lab 5.3

When no load is applied to the shaft, the IM works in a no-load motor mode. The equivalent circuit (Figure 5.25) may not consider the rotor current zero in

principle, because then there would be no torque to cover mechanical losses. However, I'_{r0} is so small that for power balance calculations in a first instance, it may be neglected.

The test arrangements in Figure 5.26 allow again to measure P_{0n} , I_0 , and V_0 ; the test should be driven to the decreasing voltage until the stator current starts rising (Figure 5.26b):

$$P_{0n} = p_{\text{iron}} + p_{\text{mec}} = f \left(\left(\frac{V_s}{V_{\text{sn}}} \right)^2 \right) \quad (5.98)$$

The intersection of the rather straight line represented by Equation 5.98 with a vertical axis leads to p_{mec} (Figure 5.26b). Then p_{iron} at the rated voltage is segregated simply.

Now that we have p_{mec} , which has been considered constant, while core losses depend on voltage squared ($V \approx \omega_1 W_1 K_{w1} \Phi$), we may approximately write (from Figure 5.26a)

$$\frac{R'_r}{S_{0n}} I'_{r0} \approx V_{\text{sn}}; \quad p_{\text{mec}} = \frac{3R'_r I_{r0}^2}{S_{0n}} (1 - S_{0n}) \approx 3V_{\text{sn}} I'_{r0} \quad (5.99)$$

In Equation 5.99, the rotor circuit at no load with a very small slip S_{0n} ($S_{0n} < 10^{-3}$) is considered purely active. From Equation 5.99, we may first calculate I'_{r0} , and then, with R'_r known, S_{0n} !

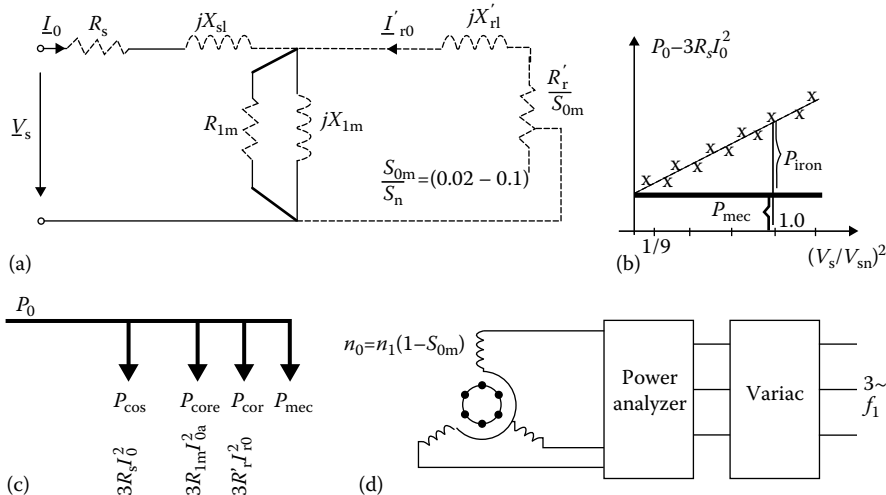


FIGURE 5.26

No-load motor operation: (a) equivalent circuit, (b) no load loss segregation, (c) power balance, and (d) test arrangement.

Example 5.4

The same motor as that referred to in Examples 5.2 and 5.3 is tested as motor on no load at $V_{sn} = 120 \text{ V}$, 60 Hz , with $I_0 = 4.2 \text{ A}$; after tests, $P_{0n} = 95 \text{ W}$, $p_{mec} = 50 \text{ W}$, and $R_s = 1.2R'_r = 0.12 \Omega$. Calculate the iron losses and the slip on no load, S_0 .

Solution:

From Equation 5.98 at the rated voltage,

$$\begin{aligned} p_{\text{iron}} &= P_{0n} - 3R_s I_0^2 - p_{\text{mec}} = 95 - 3 \times 0.12 \times 4.1^2 - 50 \\ &= 39 \text{ W} \approx 40 \text{ W (at ideal noload)} \end{aligned}$$

Now from Equation 5.99

$$I'_{r0} \approx \frac{p_{\text{mec}}}{3V_{sn}} = \frac{50}{3 \times 120} = 0.1388 \text{ A}$$

and

$$S_{0n} = \frac{R'_r I'_{r0}}{V_{sn}} = \frac{0.1 \times 0.1388}{120} = 1.15 \times 10^{-4}!$$

Now it is clear why, in reality, S_{0n} may be neglected.

Note. A digital scope recording of no-load IM stator current, with star and delta phase connections, would show different shapes with visible time harmonics, mainly due to magnetic saturation of the machine iron cores.

5.12 Motor Operation on Load ($1 > S > 0$)/Lab 5.4

On load operation takes place when the IM drives a mechanical load (pump, compressor, drive-train, machine tool, etc.). For motoring, as seen in Table 5.1, $0 < n < n_0 = f_1/p_1$ or $1 > S > 0$. The complete equivalent circuit should be used for the load operation, while the power balance is reiterated in Figure 5.27.

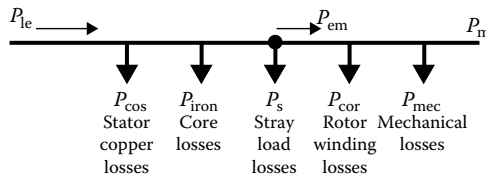


FIGURE 5.27

Power balance for motoring.

The efficiency, η , is

$$\eta = \frac{\text{Shaft power}}{\text{Input electric power}} = \frac{P_m}{P_{1e}} = \frac{P_m}{P_m + p_{\cos} + p_{\text{iron}} + p_s + p_{\text{Cor}} + p_{\text{mec}}} \quad (5.100)$$

The rated speed, n_n , is

$$n_n = \frac{f_1}{p_1} (1 - S_n) \quad (5.101)$$

The rated slip, $S_n = (0.06\text{--}0.006)$, with the larger values for low-power (below 200 W) IMs.

The stray load losses refer to additional losses on the stator and rotor surface, due to slotting and magnetic saturation and in the rotor cage, due to stator and rotor mmfs space harmonics (see Ref. [4, Chapter 11]).

The second definition of slip S (the first one is contained in Equation 5.101) from Equation 5.85 is

$$S = \frac{p_{\text{Cor}}}{P_{\text{em}} - p_s} \approx \frac{p_{\text{Cor}}}{P_{\text{em}}} \quad (5.102)$$

So, the larger the slip (the lower the speed n), the larger the rotor winding losses, p_{Cor} , for a given electromagnetic power (or torque).

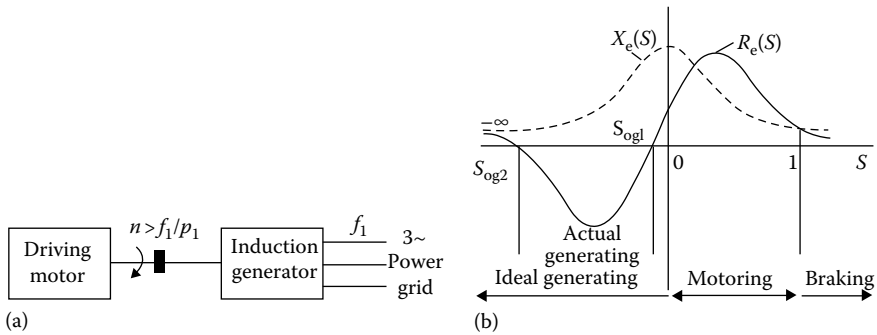
5.13 Generating at Power Grid ($n > f_1/p_1$, $S < 0$)/Lab 5.5

As already shown in Equation 5.85 and Table 5.1, when the IM is driven above the no load ideal (synchronous or zero rotor current) speed, $S < 0$ and the electromagnetic torque becomes negative (Figure 5.28).

The driving motor may be a diesel engine, a hydraulic, or a wind turbine; in the lab, it should be a variable speed drive.

To calculate performance based on the equivalent circuit, we may calculate the IM equivalent resistance and reactance, R_e and X_e , as the function of slip S (Figure 5.28b).

It is evident from Figure 5.27b that while R_e changes sign to allow for generating active power for $S < 0$, X_e remains positive, and thus the IM always draws reactive power from the power grid. The latter is the main drawback of an IM as a generator. The situation today is mended by introducing a two-stage PWM converter interface whose dc link capacitor produces the necessary reactive power to IM and also can deliver a substantial amount to the power grid in a controlled manner.

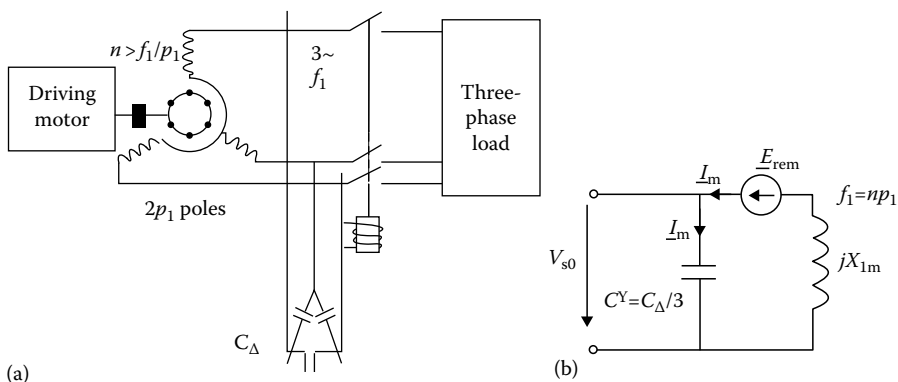

FIGURE 5.28

Induction generator: (a) at power grid and (b) equivalent IM, $R_e(s)$, $X_e(s)$.

5.14 Autonomous Generator Mode ($S < 0$)/Lab 5.6

As already demonstrated, the IM operates as a generator for $S < 0$ or at the supersynchronous speed ($n > f_1/p_1$). The frequency is fixed for a power grid connection and the reactive power is drawn from the grid for machine magnetization.

For the autonomous generator mode, still $S < 0$ but the output frequency, f_1 , is in relation to the capacitor (or synchronous condenser (Figure 5.29a)), which provides the reactive power, and to machine parameters. The capacitors are delta connected to reduce their capacitance. Let us explore the ideal no load operation with capacitor excitation (Figure 5.29b).


FIGURE 5.29

Autonomous induction generator with cage rotor: (a) with capacitor self-excitation and (b) equivalent circuit with capacitor excitation under ideal no load.

The dc remanent magnetization in the rotor produces on the stator (by motion) an emf, $\underline{E}_{\text{rem}}$, at frequency $f_1 = p_1 n_0$ (n_0 —rotor speed). This way ac currents flow into the machine magnetization reactance and capacitors. They add to the initial $\underline{E}_{\text{rem}}$ and thus voltage buildup takes place, until the voltage settles at a certain value, V_{s0} .

Changing the speed will change both the frequency, $f'_1 = n'_0 p_1$, and the settling voltage, V'_{s0} . Neglecting the stator resistance, R_s , and the stator leakage reactance, X_{sl} , for zero rotor currents (ideal no-load operation), leads to the equivalent circuit in Figure 5.29b.

The machine equation becomes

$$V_{s0} = |jX_{1m}I_m + E_{\text{rem}}| = -\frac{j}{\omega_1 C_Y} I_m = V_{s0}(I_m) \quad (5.103)$$

Due to magnetization inductance saturation, X_{1m} depends on I_m , and thus $V_{s0}(I_m)$ is a nonlinear function that starts at E_{rem} (Figure 5.30a). Graphically, the intersection of $V_{s0}(I_m)$ with the capacitor straight line leads to self-excitation voltage V_{s0} (point *A*) for a given speed, n_0 . If the speed is reduced to n'_0 , the operation point *A* moves to *A'* at smaller voltage and smaller frequency $f'_1 < f_1$.

E_{rem} is mandatory for the self-excitation to initiate, but magnetic saturation is also necessary to provide a safe intersection of no-load magnetization curve, $V_{s0}(I_m)$, with the capacitor line (Figure 5.30a).

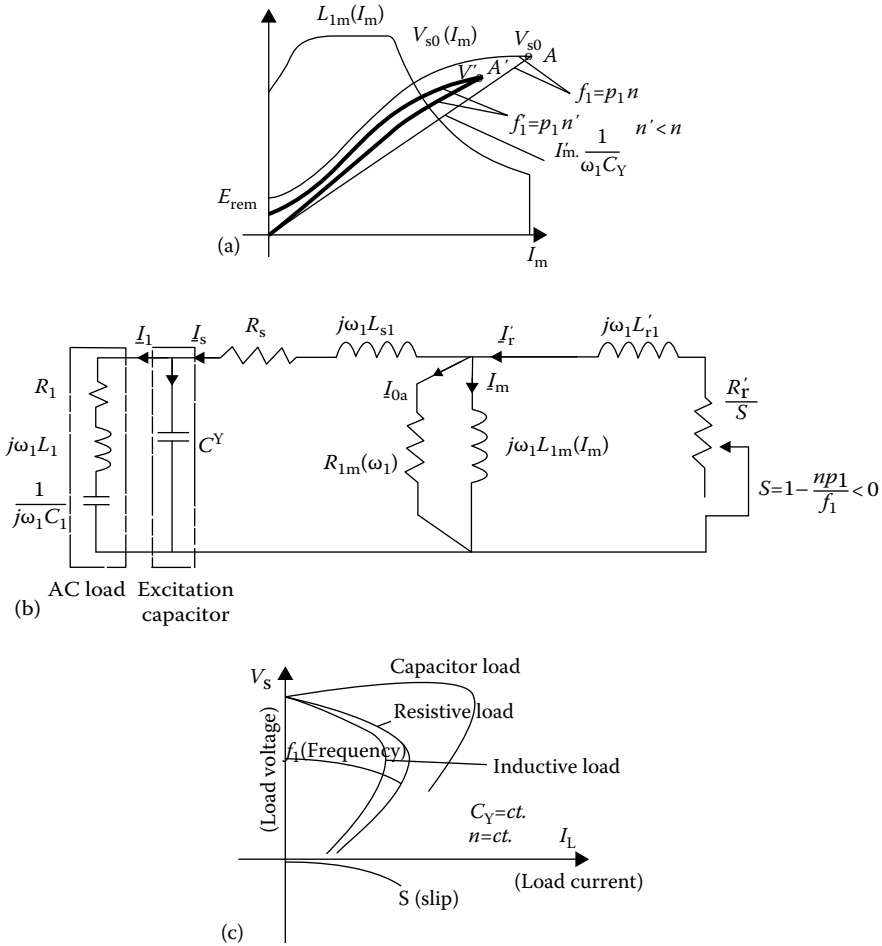
To calculate the performance on load, the complete equivalent circuit (Figure 5.30b), with $L_{1m}(I_m)$ given—magnetic saturation considered—has to be solved, iteratively in general, to obtain the external characteristic $V_s(I_L)$ for resistive–inductive, R_L, L_L , or resistive–capacitive, R_L, C_L , loads (Figure 5.30c), for, say, a constant speed (see Ref. [5]).

It should be noticed that, as expected, slip *S* increases with load, but also frequency f_1 decreases with load, at a constant speed. Frequency-sensitive loads should be avoided in such a simple scheme. A rather notable voltage drop (regulation) is already visible with a pure resistive load.

A fully variable single capacitor connected to the IM terminals through a PWM voltage-source inverter may provide a constant voltage and, to some extent, constant frequency, for constant speed driving by a prime mover.

5.15 Electromagnetic Torque and Motor Characteristics

By electromagnetic torque, T_e , we understand here the interaction torque between the stator current produced fundamental airgap flux density and the fundamental (sinusoidal) rotor currents.


FIGURE 5.30

Autonomous induction generator: (a) self-excitation on no load, (b) complete equivalent circuit with load, and (c) $V_L(I_L)$ curves, slip $s(I_L)$, and frequency $f_1(I_L)$ for constant speed operation.

Its basic expression has already been derived in Equation 5.85, for the singly fed (cage or shortcircuited rotor) IM.

The wound rotor IM may be considered as singly fed only if a passive impedance R'_L, L'_L, C'_L is lumped into the rotor circuit. Let us consider only an additional rotor resistance R'_L (even a diode rectifier with a single resistance load qualifies). Then Equation 5.85 becomes

$$T_e = \frac{3R'_{re}I_r^2}{S}; \quad R'_{re} = R'_r + R'_L \quad (5.104)$$

From the equivalent circuit (Figure 5.21), the rotor current, I'_r is

$$I'_r = \frac{-I_s \underline{Z}_{1m}}{\frac{R'_{re}}{s} + jX'_{rl} \underline{Z}_{1m}} \quad (5.105)$$

and with $(\underline{Z}_{1m} + jX_{sl}/\underline{Z}_{1m}) \approx (X_{1m} + X_{sl})/X_{1m} = C_1 \approx (1.02-1.08)$, the stator current is

$$I_s \approx \frac{V_s}{R_s + \frac{C_1 R'_{re}}{s} + j(X_{sl} + C_1 X'_{rl})} \quad (5.106)$$

Substituting I_s into Equation 5.105 yields the torque, T_e :

$$T_e = 3V_s^2 \frac{p_1}{\omega_1} \frac{\frac{R'_{re}}{s}}{\left(R_s + C_1 \frac{R'_{re}}{s}\right)^2 + (X_{sl} + C_1 X'_{rl})^2} \quad (5.107)$$

The maximum torque is obtained for $\partial T_e / \partial s = 0$ at

$$s_k = \frac{\pm C_1 R'_{re}}{\sqrt{R_s^2 + (X_{sl} + C_1 X'_{rl})^2}} \approx \frac{\pm R'_{re}}{\omega_1 L_{sc}} \quad (5.108)$$

$$T_{ek} = \frac{3p_1}{\omega_1} \frac{V_s^2}{2C_1 \left[R_s \pm \sqrt{R_s^2 + (X_{sl} + C_1 X'_{rl})^2} \right]} \approx 3p_1 \left(\frac{V_s}{\omega_1} \right)^2 \frac{1}{2L_{sc}} \quad (5.109)$$

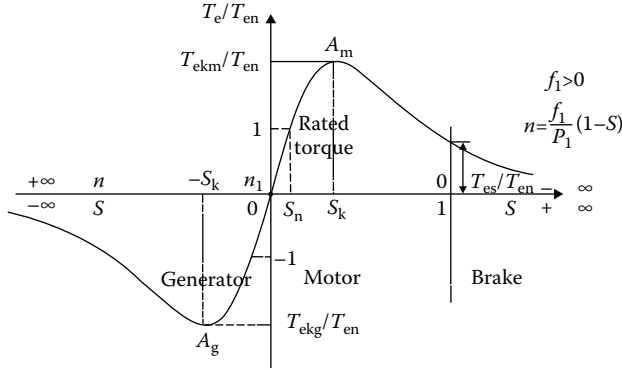
A few remarks are in order:

- The peak (breakdown) torque is independent of the rotor total resistance, R'_{re} , and in general is inversely proportional to the short-circuit inductance, L_{sc} .
- The sign $\pm$ in Equations 5.108 and 5.109 refers to motor and generator modes, respectively, as seen in Figure 5.31.
- The neglect of R_s in Equations 5.107 through 5.109 is valid only for $f_1 > 5$ Hz, even for large IMs.
- While the torque expression (Equation 5.109) with C_1 coefficient approximation produces small errors, the I_s expression (Equation 5.106) will exaggerate the power factor computation results and thus $\underline{I}_s$ is

$$\underline{I}_s \approx I'_r + \frac{V_s}{R_s + jX_{sl} + \underline{Z}_{1m}} = I_{sa} - jI_{sr} \quad (5.110)$$

With I'_r from Equation 5.110, a better precision in the power factor, $\cos \varphi_1$, calculation is obtained

$$\cos \varphi_1 \approx \frac{I_{sa}}{\sqrt{I_{sa}^2 + I_{sr}^2}} = \frac{|R_e|}{Z_e} \quad (5.111)$$

**FIGURE 5.31**

Torque vs. slip S for constant voltage amplitude V_s and frequency f_1 .

Typical $I_s(S)$, $T_e/T_{en}(S)$, and $\cos \varphi_1(S)$ and, finally, efficiency $\eta(S)$ or versus speed are obtained from the equivalent circuit and constitute the steady-state curves of the IM (Figure 5.32).

For highly variable loads (from 25% to 125% rated load), the design of the IM should provide a large plateau of high efficiency to save energy.

Example 5.5 IM Torque and Performance

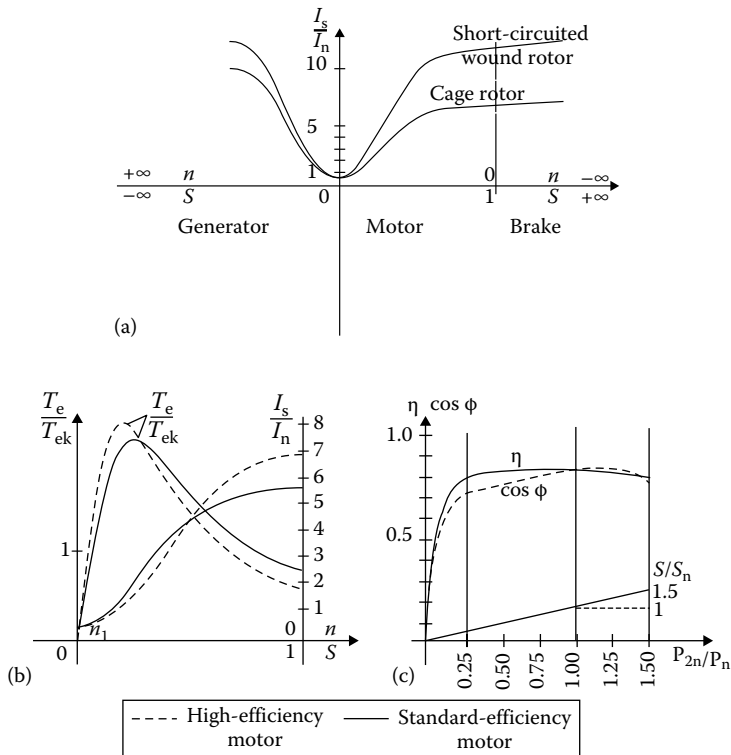
An IM with deep bars is characterized by a rated $P_n = 25 \text{ kW}$, at $V_{\text{sline}} = 380 \text{ V}$ (star connection), $f_1 = 50 \text{ Hz}$, efficiency $\eta_n = 0.92$, $\cos \varphi_1 = 0.9$, $p_{\text{mec}} = 0.005 P_n$, $p_s = 0.005 P_n$, $p_{\text{iron}} = 0.015 P_n$, $p_{\text{cosn}} = 0.03 P_n$, $2p_1 = 4$ poles, starting current (at rated voltage) $I_{\text{sc}} = 5.2 I_n$ at $\cos \varphi_{\text{sc}} = 0.4$, and the no load current $I_{0n} = 0.3 I_n$. Calculate:

- Rotor cage rated losses, p_{corn} , electromagnetic power, P_{em} , slip, S_n , speed, n_n , rated current, I_n , rotor resistance, R'_r , for rated load
- Stator resistance, R_s , and rotor resistance at start, R'_{rstart}
- Rated and starting electromagnetic torque, T_{en} and T_{estart}
- Breakdown torque, T_{ek}

Solution:

- The rotor cage losses, p_{corn} , is the only unknown component of losses so

$$\begin{aligned}
 p_{\text{corn}} &= \frac{P_n}{\eta_n} - (p_{\text{cosn}} + p_{\text{iron}} + p_s + p_{\text{mec}}) - P_n \\
 &= 25,000 \left[\left(\frac{1}{0.92} - 1 \right) - (0.03 + 0.015 + 0.005 + 0.015) \right] = 594 \text{ W}
 \end{aligned}$$


FIGURE 5.32

IM steady-state characteristics: (a) stator current vs. slip (speed), (b) electromagnetic torque vs. slip (speed), and (c) efficiency and power factor vs. slip (speed).

The rated current, I_n , comes directly from the input power, P_n/η_n :

$$I_n = \frac{P_n}{\eta_n \sqrt{3} V_{\text{line}} \cos \phi_n} = \frac{25,000}{0.92 \times \sqrt{3} \times 380 \times 0.9} = 45.928 \text{ A}$$

The rated slip, S_n , from Equation 5.102 is

$$S_n = \frac{p_{\text{corn}}}{\frac{P_n}{\eta_n} - p_{\text{cosn}} - p_{\text{iron}} - p_s} = \frac{549}{25,000 \left[\frac{1}{0.92} - 0.05 \right]} = 0.02106$$

The rated speed, n_n , is then

$$n_n = \frac{f_1}{p_1} (1 - S_n) = \frac{50}{2} (1 - 0.02106) = 24.4735 \text{ rps} = 1468.4 \text{ rpm}$$

The electromagnetic power, P_{em} , from Equation 5.85 is

$$P_{em} = \frac{p_{corn}}{S_n} = \frac{549}{0.02106} = 26,083 \text{ W}$$

To find the rotor resistance, we first use the rated rotor current, I'_{rn} . By considering the magnetization current as purely reactive and the rotor current (at small slip) as purely active, the latter is simply

$$I'_{rn} = \sqrt{I_n^2 - I_{0n}^2} = 45.928\sqrt{1 - 0.3^2} = 43.81 \text{ A}$$

So the rotor resistance reduced to the stator, R'_r , is simply

$$R'_r = \frac{p_{corn}}{3I_{rn}^2} = \frac{0.03 \times 25,000}{3 \times 43.81^2} = 0.1185 \Omega$$

b. The stator resistance, R_s , is straightforward:

$$R_s = \frac{p_{cosn}}{3I_n^2} = \frac{0.03 \times 25,000}{3 \times 45.928^2} = 0.1185 \Omega$$

The rotor resistance, R'_{rstart} , at start is simply

$$R'_{rstart} = \frac{V_{sline} \cos \varphi_{sc}}{\sqrt{3} I_{sc}} - R_s = \frac{380 \times 0.4}{\sqrt{3} \times 5.2 \times 45.928} - 0.1185 = 0.25 \Omega$$

So the skin-effect resistance factor, K_R , is

$$K_R = \frac{R'_{rstart}}{R'_r} = \frac{0.25}{0.1185} = 2.109$$

This result suggests a strong skin effect (deep-bars rotor cage).

c. The rated electromagnetic torque

$$T_{em} = \frac{P_{em} p_1}{\omega_1} = \frac{26,083 \times 2}{2 \times \pi \times 50} = 166.133 \text{ Nm}$$

The starting torque, T_{estart} , is

$$T_{estart} \approx \frac{3R'_{rstart} I_{sc}^2 p_1}{\omega_1} = \frac{3 \times 0.25 \times (5.2 \times 45.928)^2 \times 2}{2 \times \pi \times 50} = 272.47 \text{ Nm}$$

d. For the breakdown torque, we need the short-circuit reactance, X_{sc}

$$X_{scstart} = \frac{V_{sline} \sin \varphi_{sc}}{\sqrt{3} I_{sc}} = \frac{380 \times \sqrt{1 - 0.4^2}}{\sqrt{3} \times 5.2 \times 45.928} = 0.843 \Omega$$

This is an underestimated value for rated torque calculation but not so bad for the breakdown torque, when some notable skin effect still manifests.

Apparently, from Equation 5.109, the breakdown torque T_{ek} is

$$T_{ek} \approx 3 \left(\frac{V_{sline}}{\sqrt{3}} \right)^2 \frac{p_1}{\omega_1} \frac{1}{2X_{sc}} = 3 \left(\frac{380}{\sqrt{3}} \right)^2 \frac{2}{2 \times \pi \times 50} \frac{1}{2 \times 0.843} = 548.54 \text{ Nm}$$

Now, the $T_{ek}/T_{em} = 548.54/166.133 = 3.3$, a value which is larger than usual, a signal that the problem data are not entirely coherent with an industrial motor with a deep-bar cage rotor.

5.16 Deep-Bar and Dual-Cage Rotors

For heavy and frequent starting torque application IMs, connected directly to the local power grid, the skin effect is used to reduce the starting current down to around $5 \times I_n$ and increase the starting torque above $2 \times T_{en}$; deep-bar and dual-cage rotors are used for this (Figure 5.33a and b).

In both cases, the equivalent rotor resistance, R'_r , and leakage reactance, X'_{rl} , depend on the slip (on rotor frequency, in fact):

$$\begin{aligned} R'_r(\omega_2) &= K_R(\omega_2) R'_{r0}; & S &= \frac{\omega_2}{\omega_1} \\ X'_{rl}(\omega_2) &= K_X(\omega_2) X'_{rl0} \end{aligned} \quad (5.112)$$

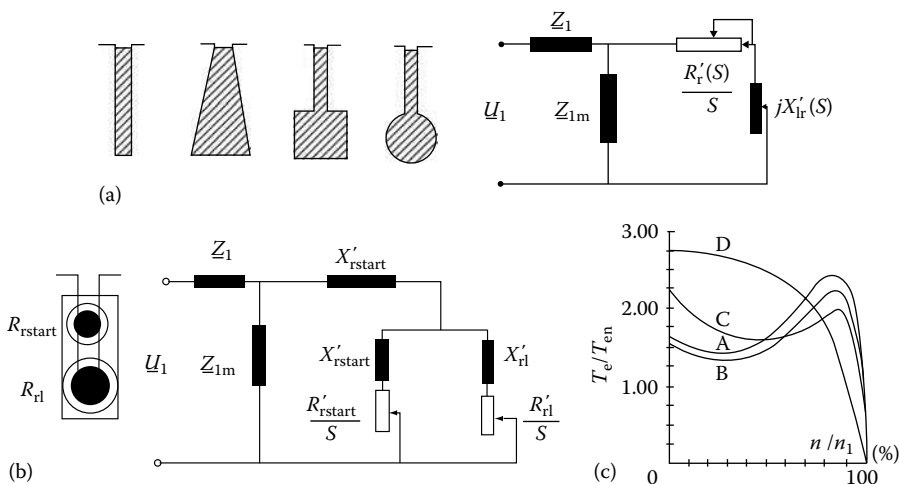


FIGURE 5.33

Deep rotor bar and dual-cage IMs (a) deep-bar, (b) dual-cage rotor, and (c) torque-speed curve types (NEMA standards).

As derived in Chapter 2,

$$K_R = \xi \frac{\sinh 2\xi + \sin 2\xi}{\cosh 2\xi - \cos 2\xi}; \quad K_X = \frac{3}{2\xi} \frac{\sinh 2\xi - \sin 2\xi}{\cosh 2\xi - \cos 2\xi} \quad (5.113)$$

$$\xi = h_s \sqrt{\frac{\omega_2 \mu_0 \sigma_{AC}}{2}} \quad (5.114)$$

To calculate performance, $I_s(S)$, $T_e(S)$, $\eta(S)$, and $\cos \varphi(S)$, we can still make use of the equivalent circuit (Figure 5.33a) but with variable parameters.

For the dual-cage rotor, the upper-starting cage may be made of brass (of higher resistivity), while the inner (running) cage is made of aluminum. In such a case, separate end rings may be necessary due to different thermal expansion rates.

At start and low speeds, the starting cage works, as the field penetration depth is small (ω_2 is large), while at high speeds (low slips), the running cage is prevalent.

The constant parameter dual-cage concept may also be used to simulate the deep-bar effect.

The NEMA standards suggest four main designs, and while A and B refer to low skin effect, C and D refer to the deep-bar and dual-cage rotors used for heavy and frequent starts.

5.17 Parasitic (Space Harmonics) Torques

As already mentioned in Section 5.3 on ac windings, their mmf and airgap magnetic field (flux density) show space harmonics in the order of $\nu = km \pm 1$, $m = 3$ and $k \geq 1$ with the pole pitch $\tau_\nu = \tau/\nu$, and their synchronous speed with respect to stator $n_{1\nu} = n_1/\nu$. Among these harmonics, the first-slot harmonics $\nu_c = 2qm \pm 1$ ($k = 2q$) are very important as their distribution factor is $K_{q\nu_c} = K_{q1}$. The open slots tend to magnify these mmf harmonic effects. The currents induced in the rotor by these harmonic fields produce their own fields, full of space harmonics, of ν' .

If for the wound rotor, with three phases in the rotor and stator, both stator and rotor windings mmf have the same orders, for the cage rotor the relationship between a stator harmonic, ν , and a rotor harmonic, ν' , is

$$p_1\nu - p_1\nu' = K_2 N_r \quad (5.115)$$

where N_r is the number of rotor slots.

The slip for the ν mmf space harmonic in the stator S_ν is

$$S_\nu = \frac{n_{1\nu} - n}{n_{1\nu}} = \frac{\frac{n_1}{\nu} - n_1}{\frac{n_1}{\nu}} = 1 - \nu(1 - S) \quad (5.116)$$

Now, the ν' rotor mmf harmonic speed with respect to the rotor is

$$n_{2\nu,\nu'} = \frac{f_{2\nu}}{\nu' p_1} = \frac{f_{2\nu} S_\nu}{\nu' p_1} = \frac{n_1}{\nu'} (1 - \nu(1 - S)) \quad (5.117)$$

But, with respect to the stator, the ν' rotor (produced by ν stator) harmonic moves at speed $n_{1\nu,\nu'}$:

$$n_{1\nu,\nu'} = n_{2\nu,\nu'} + n = \frac{n_1}{\nu'} (1 + (\nu' - \nu)(1 - S)) \quad (5.118)$$

or with Equation 5.115:

$$n_{1\nu,\nu'} = \frac{n_1}{\nu'} \left[1 + \frac{K_2 N_r}{p_1} (1 - S) \right] \quad (5.119)$$

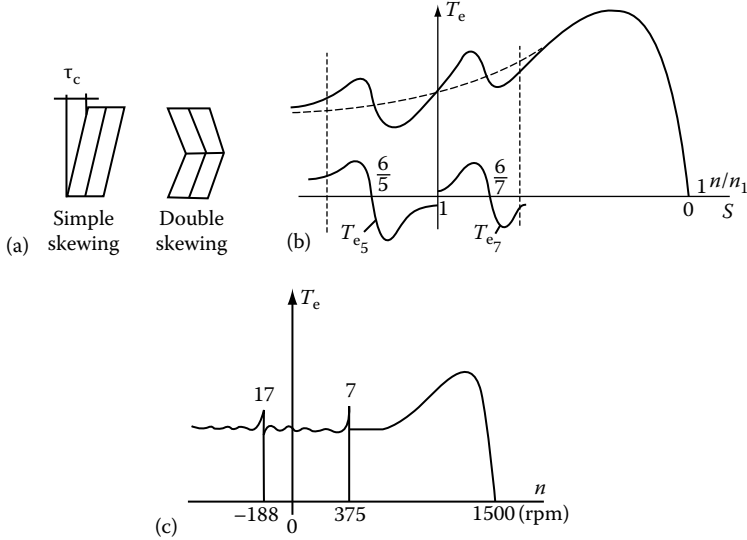
So every stator mmf space harmonic, ν , produces an infinity of harmonics ν' in the cage rotor mmf whose traveling speed with respect to the stator is $n_{1\nu,\nu'}$. These harmonics act upon an equivalent circuit of an IM with parameters corresponding to the same frequency, but in most cases, the magnetization branch may be neglected.

The main effects of mmf space harmonics are parasitic torques and uncompensated radial forces that produce noise and vibration.

- The parasitic torques manifest themselves between stator and rotor mmf harmonics of the same order (according to the frequency theorem (Chapter 3)).
- *Asynchronous parasitic torques* are produced by the stator harmonic, ν , in the stator and one in the rotor produced by the former ($\nu = \nu'$).

With their synchronism $S_\nu = 0$ for $\nu = 2km - 1$ (5, 11, 17), they are inverse harmonics (see Section 5.3 on ac windings), and for $\nu = 2km + 1$ (7, 13, 19,...), they are direct:

$$\begin{aligned} S_5 = 0 &= 1 - (-5)(1 - S_{05}); & S_{05} &= \frac{6}{5} \\ S_7 = 0 &= 1 - (+7)(1 - S_{07}); & S_{07} &= \frac{6}{7} \end{aligned} \quad (5.120)$$

**FIGURE 5.34**

Asynchronous parasitic torques: (a) slot skewing and (b), and (c) synchronous parasitic torques for $N_s = 36$, $2p_1 = 4$, $N_r = 16$ (not practical).

On the torque/speed (slip) curve, the fifth and seventh asynchronous torques tend to be the most visible (Figure 5.34b). Coil chording is used to attenuate the fifth harmonic.

$$K_{y5} = \sin \frac{\pi}{2} \frac{5y}{\tau} = 0; \quad \frac{y}{\tau} = \frac{2K_1}{5} \approx \frac{5}{6} \quad (5.121)$$

To attenuate the first-slot harmonic $\nu_c = 2qm \pm 1 = (N_s/p_1) \pm 1$, slot skewing is used

$$K_{cv} = \frac{\sin \frac{\pi}{2} \frac{c}{\tau} \nu}{\frac{\pi}{2} \frac{c}{\tau} \nu} = 0; \quad \frac{c}{\tau} = \frac{2K_2}{\frac{N_{c1}}{p_1} \pm 1} \quad (5.122)$$

Skewing by one to two slot pitches is typical.

- *Synchronous parasitic torques* occur when the two (stator and rotor) harmonics, ν_1 and ν' , have different origins (Equation 5.116).

$$n_{1\nu_1} = \frac{n_1}{\nu_1} = n_{1\nu, \nu'} = \frac{n_1}{\nu'} \left[1 + \frac{K_2 N_r}{p_1} (1 - S) \right] \quad (5.123)$$

ν' and ν_1 should be equal to each other but one may be positive and the other positive or negative:

$$\nu_1 = \nu'; \quad S = 1 \quad (5.124)$$

$$\nu_1 = -\nu'; \quad S = 1 + \frac{2p_1}{K_2 N_r} \quad (5.125)$$

So synchronous parasitic torques occur at zero speed ($S = 1$) or at close to zero speeds (Equation 5.125). They appear to us as oscillatory at respective slips (Equations 5.124 and 5.125) on the $T_e(S)$ curve (Figure 5.34c) and, if at standstill ($S = 1$), they may lead to unsafe starting.

By choosing right combinations between the stator and the rotor slot numbers, N_s , N_r , and pole pairs p_1 , the main synchronous parasitic torques may be suppressed. In general,

$$\begin{aligned} N_s &\neq N_r; \quad 2N_s \neq N_r, N_r \pm 2p_1 \\ N_r &\neq \frac{N_s}{2} \pm p_1 \end{aligned} \quad (5.126)$$

For more on parasitic torques in IMs, see Ref. [4, Chapter 10] and Ref. [7].

5.18 Starting Methods

The starting methods refer to the machine connected to the power grid without a frequency changer (PWM converter). It is, in fact, a transient process, both in terms of electrical variables (flux linkages, current) and mechanical variables (torque, speed).

These transients will be investigated in Chapter 10 of the book. Here, the main starting methods and their current and torque versus speed characteristics are given.

For the cage rotor IM, these are

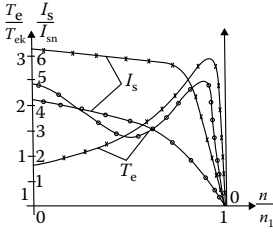
- Direct starting to 3-phase ac power grid
- Reduced stator voltage (soft starter by star/delta connection switch or autotransformer)

For the wound rotor IM

- Controlled rotor additional resistance

5.18.1 Direct Starting (Cage Rotor)

For simplification, let us consider the IM with a large inertia at shaft. Thus, the starting will be slow, and the machine will experience only mechanical

**FIGURE 5.35**

Torque and current vs. speed n/n_1 . x—single-cage rotor; 0—dual-cage rotor.

transients. Typical steady-state characteristics versus speed are shown in Figure 5.35.

What interests us most, for frequent starts (say, for compressor loads), is the energy for a start from zero speed to almost ideal no load speed $n_1 = f_1/p_1$, under no load at shaft. However, the investigation may be solved numerically under load if $T_{\text{load}}(\omega_r)$ is known:

$$\frac{J}{p_1} \frac{d\omega_r}{dt} = T_e(\omega_r) - T_{\text{load}}; \quad \omega_r = \omega_1(1 - S) \quad (5.127)$$

We may define a so-called electromechanical time constant:

$$\tau_{\text{em}} = \frac{2\omega_1}{T_{\text{ek}}} \frac{J}{p_1} \quad (5.128)$$

τ_{em} is a few tens of milliseconds for small machines and seconds for large machines.

Now, for no load $T_{\text{load}} = 0$, and thus Equation 5.127 may be integrated from zero to ideal starting time, t_p , that corresponds roughly to synchronous speed n_1 , to obtain rotor winding losses:

$$W_{\text{cor}} = \int_0^{t_p} P_{\text{em}} S dt = \int_0^{t_p} \frac{J}{p_1} \frac{d\omega_r}{dt} \frac{\omega_1}{p_1} S dt = -\frac{J}{p_1} \int_0^1 \omega_1^2 S dS = \frac{J}{2} \left(\frac{\omega_1}{p_1} \right)^2 \quad (5.129)$$

So the rotor energy winding losses for direct starting are equal to the kinetic energy of the rotor. Now, the stator copper losses W_{cos} is

$$W_{\text{cos}} \approx W_{\text{cor}} \frac{R_s}{R'_r} \quad (5.130)$$

The total winding energy losses for no load starting is

$$W_{\text{co}} = W_{\text{cos}} + W_{\text{cor}} = \frac{J}{2} \left(\frac{\omega_1}{p_1} \right)^2 \left(1 + \frac{R_s}{R'_r} \right) \quad (5.131)$$

Larger rotor resistance (deep-bar-cage) rotors lead to lower energy losses for frequent starts, as expected.

5.18.2 Reduced Stator Voltages

The starting current (Equation 5.92) (at zero speed) is $(5-8)I_n$, where I_n is the rated current. In many situations, the weak power grid or the application requires reduced starting current for light (low load at low speed) starts.

For such cases, there are three main devices: the soft (thyristor) starter (Figure 5.36a), the star–delta switch (Figure 5.36b), and the autotransformer (Figure 5.36c). Soft starters may reduce the starting current to $(2.5\text{--}3)I_n$ and, by special control, provide additional torque below 33% of rated speed during starting; they are commercial to about 1.5 MVA/unit.

The star–delta switch relies on phase voltage increase by $\sqrt{3}$ times after a settled time after motor start initialization. Unfortunately, the machine is designed for delta connection, so star connection means voltage and current reduction by $\sqrt{3}$ and thus a torque reduction by three times occurs, because the stator current is proportional to the phase voltage and the torque with the stator voltage squared:

$$V_Y^{\text{ph}} = \frac{V_{\Delta}^{\text{ph}}}{\sqrt{3}}; \quad I_Y^{\text{ph}} = \frac{I_{\Delta}^{\text{ph}}}{\sqrt{3}}; \quad T_{eY} = \frac{T_{e\Delta}}{3} \quad (5.132)$$

This explains why only light starts are feasible. A similar situation is obtained with the autotransformer that produces, with K_2 closed and K_1 open, low voltage and then, with K_2 open and K_1 closed, full voltage.

5.18.3 Additional Rotor Resistance Starting

Wound rotor IMs are applied at large power loads (above 100 kW) when limited (10%–20%) speed control range (interval) is required.

To cut costs, a diode-rectifier variable resistance is used (Figure 5.37). The static switch K_1 (or the dc–dc converter) controls the on–off process in the resistance current, eventually with a stator current close-loop regulator for limiting the stator current at the desired value.

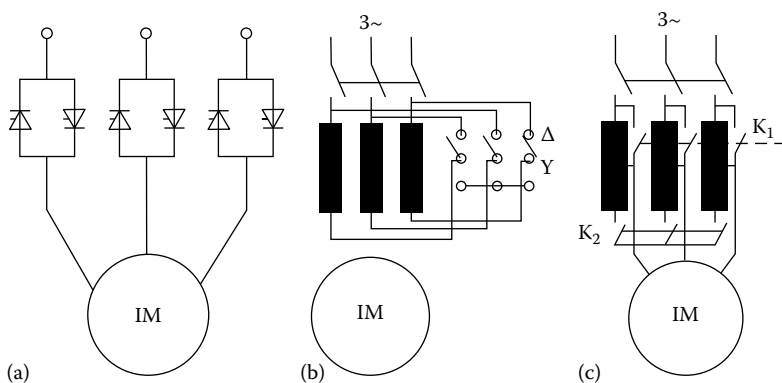
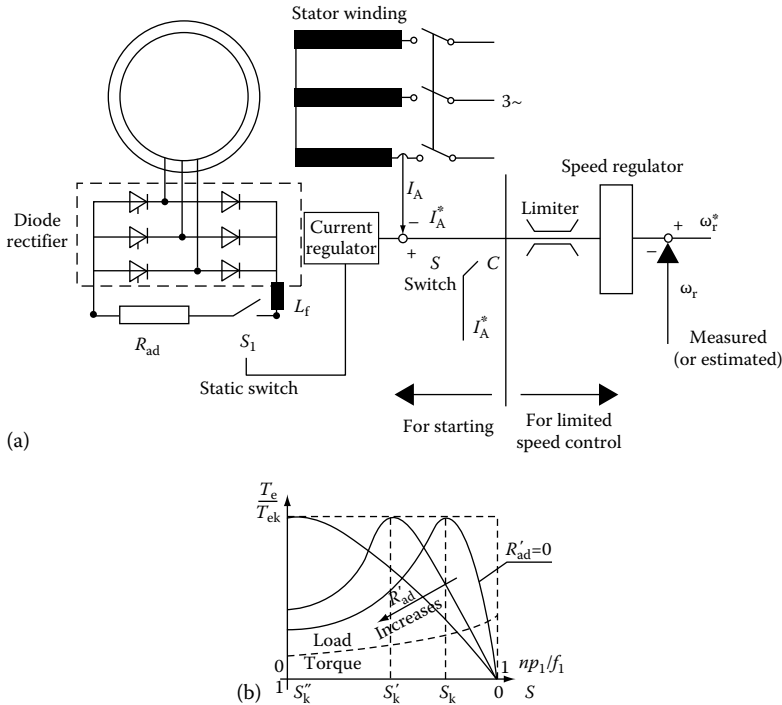


FIGURE 5.36

Reduced voltage starting: (a) with soft starter, (b) with star–delta switch, and (c) with autotransformer.


FIGURE 5.37

Additional rotor resistance starting of wound rotor IMs: (a) wound rotor IM starting with additional controlled resistance and (b) torque–speed characteristics (for limited speed control).

An additional speed regulator may be used to produce the reference current and thus limited speed range close-loop control with the same device is obtained (Figure 5.37).

As the breakdown torque—independent of rotor total resistance (Figure 5.37)—moves toward zero speed, the method is characterized by large torque at start, and thus heavy starts are approachable provided energy consumption is not a problem in the application, due to the small number of starts (only a few per hour).

5.19 Speed Control Methods

For now, we are concerned with cage rotor IMs for which the speed n is

$$n = \frac{f_1}{p_1} (1 - S) \quad (5.133)$$

The speed control methods are then related to the terms in Equation 5.133:

- Changing the slip S for a given torque: by reducing voltage for limited slip (speed) control range (10%–20%) (see Section 5.18) (Figure 5.38a)
- Pole count (pair) p_1 changing: by using two different stator windings or a pole changing winding (with $p_1/p_1' = 1/2$ in general) for constant power or torque applications (Figure 5.38b)
- Stator frequency, f_1 , and voltage, V_s , control (Figure 5.38c): to control the flux linkage level within a reasonable (or desired) magnetic saturation of motor cores

The characteristics in Figure 5.38 are drawn by analysis of torque expression in Section 5.15. While reduced voltage speed control is applicable only

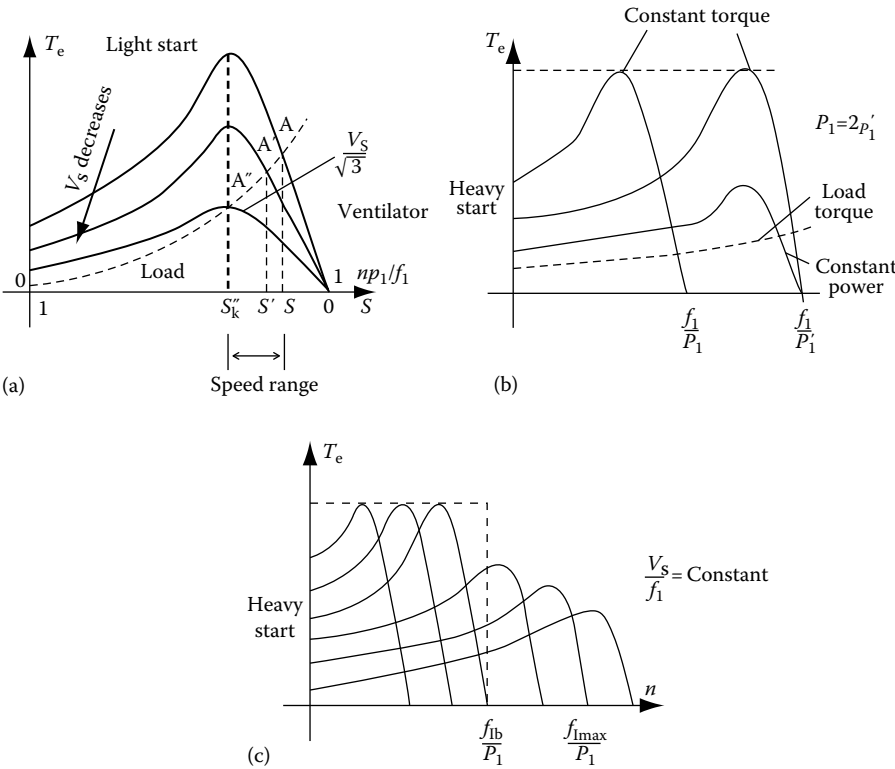


FIGURE 5.38 Speed control methods: (a) with soft starter, (b) with changing pole, and (c) variable frequency and voltage.

to light starts and provides for limited speed control, pole changing (2:1) and frequency (and voltage) control provide for heavy starts, wide speed control range (up to 1000/1), and good energy conversion. Frequency and voltage coordinated control with tight flux control—called vector control or direct torque and flux control—to produce fast torque response, with PWM converter supplies for IMs, are now a worldwide industry and will be treated in Chapter 10 as a mere brief introduction in *Electric Drives*, a separate topic in itself.

5.19.1 Wound Rotor IM Speed Control

Wound rotor IM speed control may be accomplished either through a diode rectifier and dc–dc converter and a fixed resistor, for starting and limited speed control range (10%–15%), to limit the total energy loss, or variable frequency ω_2 and voltage V_r' in the rotor via a bidirectional two-stage PWM converter (Figure 5.39). The rotor flux linkage in the machine is controlled together with frequency $\omega_2 = \omega_1 + \omega_r$ ($\omega_1 = \text{constant}$, $\omega_r = \text{variable}$). As already shown, this is a limited speed range control method ($\pm 30\%$, $|S_{\max}| < 0.3$), for which the rotor-connected converter rating is about $|S_{\max}| P_n$ (P_n —stator rated power). At maximum (oversynchronous) speed, the total machine power is $P_n + |S_{\max}| P_n$ for rated torque. This method is used for variable speed wind generators up to 5 MW, or by hydrogenerator systems (including pump storage up to 400 MW) to cut the equipment costs but retain good performance.

More on the doubly fed induction generator (DFIG) can be found in Ref. [5, Chapter 3].

Example 5.6 V/f Speed Control

An induction motor with cage rotor has the following design data: $P_n = 5.5 \text{ kW}$, $V_{nl} = 440 \text{ V}$ (stator), $f_{1b} = 60 \text{ Hz}$, $\eta_n = 0.92$, $\cos \varphi_n = 0.9$, $2p_1 = 4$,

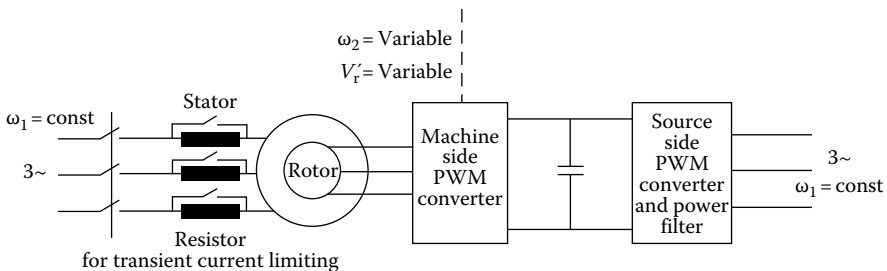


FIGURE 5.39

Variable speed wound rotor IM with variable frequency and voltage in the rotor for under and supersynchronous motor–generator operation modes.

$I_{\text{start}}/I_n = 6/1$, $I_{0n}/I_n = 0.33$, $p_{\text{iron}} = p_{\text{mec}} = p_s = 0.015P_n$, $R_s = 1.2R'_r$, and $X_{\text{sl}} = X'_{\text{rl}}$, and skin effect is negligible.

Calculate rated stator current, rated rotor current, R_s , R'_r , X_{1m} , X_{sc} and critical slip S_k , and breakdown torque at base frequency f_{1b} and full voltage and at $f_{1\text{max}} = 2f_{1b} = 120$ Hz. To preserve breakdown torque at start and at all frequencies, at $f_{\text{min}} = S_k f_{1b}$, determine the required f_{min} and stator voltage and V/f dependence $V_s = V_0 + kf$.

Solution:

The problem in its first part is similar to the previous numerical example (Example 5.5). Efficiency is

$$\eta_n = \frac{P_n}{\sqrt{3}V_{\text{nl}}I_n \cos \varphi_n} = \frac{5500}{\sqrt{3} \times 440 \times I_n \times 0.9} = 0.92$$

The rated current, I_n , is

$$I_n = 8.726 \text{ A}$$

The rotor-rated current, I'_{rn} , is

$$I'_{\text{rn}} = \sqrt{I_n^2 - I_{0n}^2} = 8.726\sqrt{1 - 0.33^2} = 8.226 \text{ A}$$

The copper stator and rotor losses, $p_{\text{Cos}} + p_{\text{Cor}}$, are

$$\begin{aligned} p_{\text{Cos}} + p_{\text{Cor}} &= \frac{P_n}{\eta_n} - P_n - p_{\text{mec}} - p_{\text{iron}} - p_{\text{ps}} = 5500 \left(\frac{1}{0.92} - 1 - 3 \times 0.015 \right) \\ &= 230.76 \text{ W} \end{aligned}$$

But also

$$p_{\text{Cos}} + p_{\text{Cor}} = 3I_n^2 \left[1.2R'_r + R'_r \left(\frac{I'_{\text{rn}}}{I_n} \right)^2 \right] = 3 \times R'_r \times 159$$

So

$$R'_r = 0.4836 \Omega; \quad R_s = 1.2 \times 0.4836 = 0.588 \Omega$$

The short-circuit reactance is calculated from the starting impedance:

$$\begin{aligned} X_{\text{sc}} &= \sqrt{\left(\frac{V_{\text{nl}}}{6\sqrt{3}I_n} \right)^2 - (R_s + R'_r)^2} = \sqrt{\left(\frac{440}{\sqrt{3} \times 6 \times 8.726} \right)^2 - (0.4836 + 0.588)^2} \\ &= 4.72 \Omega \end{aligned}$$

The critical slip (Equation 5.108) is

$$(S_k)_{60 \text{ Hz}} \approx \frac{R'_r}{\sqrt{R_s^2 + X_{\text{sc}}^2}} = \frac{0.4836}{\sqrt{0.588^2 + 4.72^2}} = 0.1067$$

and the breakdown torque is

$$(T_{ek})_{60\text{ Hz}} \approx \frac{3}{2} p_1 \frac{(V_{nl}/\sqrt{3})^2}{2\pi f_{1b}} \frac{1}{R_s + \sqrt{R_s^2 + X_{sc}^2}} = 100.345 \text{ Nm}$$

For $f_{1\text{ max}} = 120 \text{ Hz}$,

$$(S_k)_{120\text{ Hz}} = \frac{0.4836}{\sqrt{0.588^2 + (4.72 \times 2)^2}} = 0.0511!$$

Though the critical slip decreases twice the corresponding frequency, $\omega_2 = S f_1$ remains the same. The breakdown torque is now

$$(T_{ek})_{120\text{ Hz}} = \frac{3}{2} \times 2 \frac{(440/\sqrt{3})^2}{2\pi 120} \frac{1}{0.588 + \sqrt{0.588^2 + (4.72 \times 2)^2}} = 25.55 \text{ Nm}$$

The critical rotor frequency

$$f_{2k} = f_{1b} (S_k)_{f_{1b}} = 60 \times 0.1067 = 6.402 \text{ Hz}$$

Now, the machine has to develop a breakdown torque of 100.345 Nm at zero speed ($S = 1$) and $f'_1 = f_2 = 6.402 \text{ Hz}$, and we need to calculate the required stator voltage to do so

$$(T_{ek}) = 100.345 = \frac{3}{2} p_1 \frac{(V'_{ph})^2}{2\pi f'_1} \frac{1}{R_s + \sqrt{R_s^2 + (X_{sc} S_k)^2}}$$

$$V'_{ph} = 42.448 \text{ V}$$

To find an appropriate formula for $V_s(f_1) = V_0 + kf$, we notice that at 60 Hz $V_s = V_{nl}/\sqrt{3} = 440/\sqrt{3} = 254.33 \text{ V}$. So

$$42.448 = V_0 + k \times 6.402$$

$$254.33 = V_0 + k \times 60$$

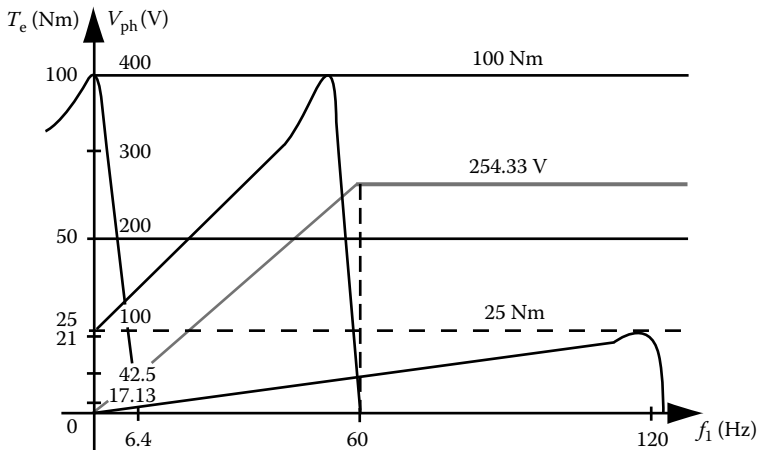
Consequently

$$k = \frac{254.33 - 42.448}{60 - 6.402} = 3.9545$$

and

$$V_0 = 17.13 \text{ V}$$

The torque–speed curves for 120, 60, and 6.402 Hz at 254.33, 254.33, and 42.448 V (phase voltage RMS), respectively, are shown in Figure 5.40.

**FIGURE 5.40**

$V_s = V_0 + kf_1$ and torque–speed curves for V/f control for constant break-down torque from 60 Hz to zero speed (6.402 Hz).

5.20 Unbalanced Supply Voltages

In real, local power grids, the 3-phase line voltages are not purely balanced (equal amplitude, 120° phase shifts). For a 3-phase power supply, we may decompose them in forward (+) and backward (–) components

$$\begin{aligned}\underline{V}_{a+} &= \frac{1}{3} (\underline{V}_a + a\underline{V}_b + a^2\underline{V}_c); \quad a = e^{j\frac{2\pi}{3}} \\ \underline{V}_{a-} &= \frac{1}{3} (\underline{V}_a + a^2\underline{V}_b + a\underline{V}_c) \\ \underline{V}_{b+} &= a^2\underline{V}_{a+}; \quad \underline{V}_{c+} = a\underline{V}_{a+} \\ \underline{V}_{b-} &= a\underline{V}_{a-}; \quad \underline{V}_{c-} = a^2\underline{V}_{a-}\end{aligned}\quad (5.134)$$

Now, the slip for the direct ($\oplus$) component is $S_+ = S$, but for the reverse ($\ominus$) component, S_- is

$$S_- = \frac{-\frac{f_1}{p_1} - n}{-\frac{f_1}{p_1}} = 2 - S \quad (5.135)$$

So, if we neglect the skin effect (the rotor-slip frequency is large for the $\ominus$ component: $f_{2-} = S_-f_1$) and the magnetic saturation (which may be considered constant or only dependent on the $\oplus$ component), we do have two different fictitious machines with their $\underline{V}_+$ and $\underline{V}_-$ voltages and S and $2-S$ slip values.

The torque is composed of two components, and making use of torque/electromagnetic power (copper loss, slip) definition, it becomes

$$T_e = T_{e+} + T_{e-} = \frac{3R'_r (I'_{r+})^2}{S} \frac{p_1}{\omega_1} + \frac{3R'_r (I'_{r-})^2}{2-S} \frac{p_1}{-\omega_1} \quad (5.136)$$

The phase voltage unbalance index (in percent) can be defined as

$$V_{\text{unbalance}} = \frac{\Delta V_{\text{max}}}{V_{\text{av}}}; \quad \Delta V_{\text{max}} = V_{\text{max}} - V_{\text{min}}; \quad V_{\text{av}} = \frac{V_a + V_b + V_c}{3} \quad (5.137)$$

where V_{max} and V_{min} are the maximum and minimum of the three voltages, respectively, and

$$V_{\text{unbalance}} (\%) = \frac{V_{a-}}{V_{a+}} 100 \quad (5.138)$$

The efficiency is

$$\eta = \frac{T_e \omega_1 (1 - S)}{3p_1 \text{Re} [V_{a+} I_{a+}^* + V_{a-} I_{a-}^*]} \quad (5.139)$$

A small voltage unbalance leads to a notable phase current unbalance. The $\ominus$ component torque is in general negative and small but it contributes to more losses and lower efficiency, for a given torque.

NEMA standards recommend an IM derating with a voltage unbalance as in Equation 5.137 to 97.5% at $\Delta V_{\text{unbalance}} = 2\%$ and to 75% at $\Delta V_{\text{unbalance}} = 5\%$ voltage!

5.21 One Stator Phase Open by Example

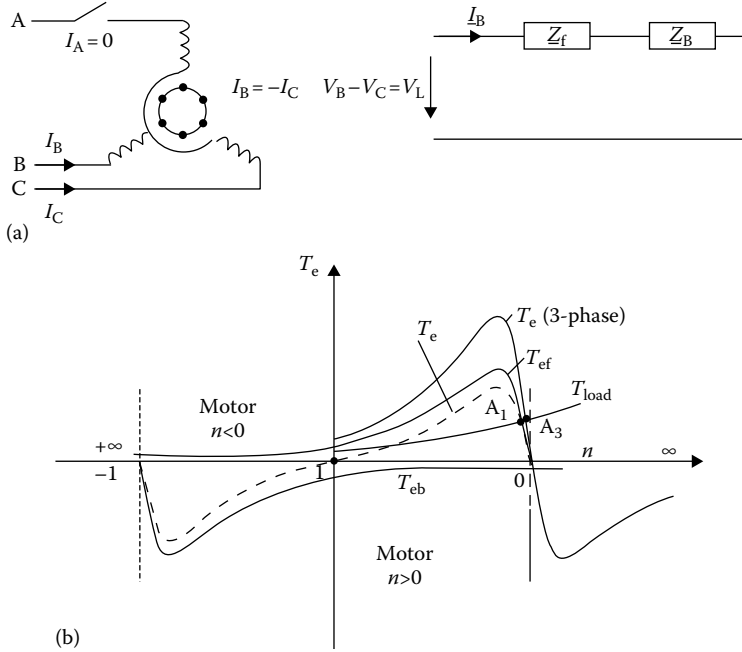
Let us consider the case when phase A is open (Figure 5.41): when $I_A = 0$; $I_B + I_C = 0$ (star connection).

For $V_{\text{nl}} = 220\text{V}$, $f_1 = 60\text{Hz}$, $2p_1 = 4$, $R_s = R'_r = 1\ \Omega$, $X_{sl} = X'_{rl} = 2.5\ \Omega$, $X_{1m} = 75\ \Omega$, $R_{1m} = \infty$ (no core losses), and $S = 0.03$. Calculate stator current and torque, and power factor before and after opening phase A.

Solution:

First, the $\oplus$ and $\ominus$ stator currents are calculated

$$\begin{aligned} I_{A+} &= \frac{1}{3} (I_A + aI_B + a^2I_C) = I_B \frac{a - a^2}{3} = j \frac{I_B}{\sqrt{3}} \\ I_{A-} &= \frac{1}{3} (I_A + a^2I_B + aI_C) = -I_{A+} \end{aligned} \quad (5.140)$$


FIGURE 5.41

One stator phase is open (a) equivalent circuit and (b) torque–speed curve.

So, in fact, the two equivalent impedances, $\underline{Z}_+$ (S) and $\underline{Z}_-$ ($2 - S$), are related to their phase voltage components (Figure 5.42a)

$$\underline{V}_{A+} = \underline{Z}_+ \underline{I}_{A+}; \quad \underline{V}_{A-} = \underline{Z}_- \underline{I}_{A-} \quad (5.141)$$

with $\underline{V}_{B+} = a^2 \underline{V}_{A+}$ and $\underline{V}_{B-} = a \underline{V}_{A-}$.

The line voltage, $\underline{V}_B - \underline{V}_C$, which is known, has the expression

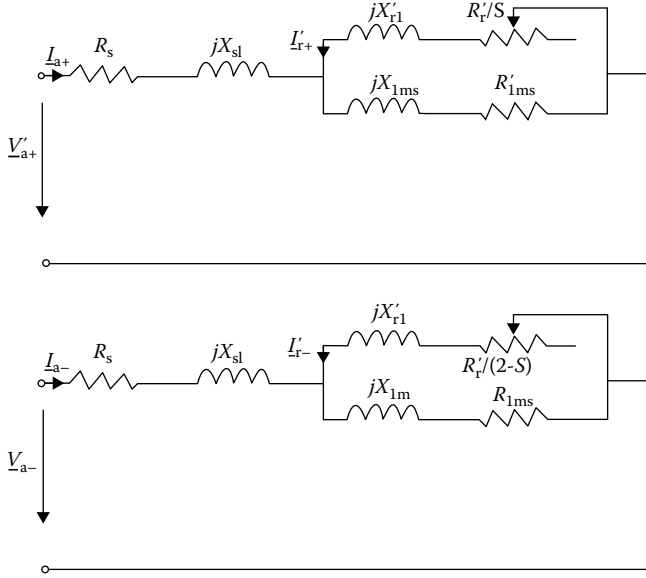
$$\begin{aligned} \underline{V}_B - \underline{V}_C &= \underline{V}_{B+} + \underline{V}_{B-} - \underline{V}_{C+} - \underline{V}_{C-} = (a^2 - a) \underline{I}_{A+} (\underline{Z}_+ + \underline{Z}_-) \\ \underline{V}_B - \underline{V}_C &= \underline{I}_B (\underline{Z}_+ + \underline{Z}_-) \end{aligned} \quad (5.142)$$

Equation 5.136 is reflected in Figure 5.41a.

For zero speed ($S = 1$): $(\underline{Z}_+)_{S=1} = (\underline{Z}_-)_{S=1} = \underline{Z}_{sc}$ and thus

$$\underline{I}_{sc1} = (\underline{I}_B)_{S=1} = \frac{\underline{V}_{nl}}{2\underline{Z}_{sc}} = \frac{\sqrt{3}}{2} \underline{I}_{sc3} \quad (5.143)$$

Consequently, at standstill ($S = 1$), because $\underline{Z}_+ = \underline{Z}_-$, the direct and inverse torque components are equal, so the resultant torque is zero, and the phase B (C) current is a bit smaller than for the 3-phase operation.

**FIGURE 5.42**

Unbalanced voltage IM $\oplus$ and $\ominus$ component equivalent circuits of IM.

From Figure 5.42 and Example 5.5, before opening phase A

$$\underline{V}_A = \underline{Z}_+(S) \underline{I}_A; \quad c_1 \approx \frac{X_{sl} + X_{1m}}{X_{1m}} = \frac{75 + 2.5}{75} = 1.033$$

$$I_s = I_A = I_B = I_C = \frac{V_{nl}/\sqrt{3}}{\sqrt{\left(R_s + c_1 \frac{R_r'}{S}\right)^2 + \left(X_{sl} + c_1 X_{lr}'\right)^2}} = 3.55 \text{ A}$$

The no load current is

$$I_{0n} \approx \frac{V_{nl}/\sqrt{3}}{\left| R_s + j(X_{1m} + X_{sl}) \right|} = \frac{220}{\sqrt{3}\sqrt{1^2 + 77.5^2}} = 1.64 \text{ A}$$

So the rotor current, I'_{rn} , is

$$I'_{rn} \approx \sqrt{I_s^2 - I_{0n}^2} = \sqrt{3.55^2 - 1.64^2} = 3.148 \text{ A}$$

$$\cos \varphi_n \approx \frac{I'_{rn}}{I_n} = \frac{3.148}{3.55} = 0.886$$

The torque for symmetric steady state at $S_n = 0.03$ is

$$T_{e3} = \frac{3R'_r I_m^2}{S_n} \frac{p_1}{\omega_1} = \frac{3 \times 1 \times 3.148^2}{0.03} \frac{2}{2\pi \times 60} = 5.261 \text{ Nm}$$

For the same slip $S = 0.03$ but with phase A open, based on Equation 5.142, we can calculate I_B and then I_{A+} and I_{A-} :

$$\begin{aligned} I_B &= \frac{V_{BC}}{\underline{Z}_+ + \underline{Z}_-} = \frac{V_{BC}}{\left[2(R_s + jX_{sl}) + \frac{R'_r}{2-S} + jX'_{rl} + j\frac{X_{lm}R'_r/S}{\frac{R'_r}{S} + j(X_{lm} + X_{rl})} \right]} \\ &= \frac{220}{\left[2(1 + j2.5) + \frac{1}{2-0.03} + j2.5 + \frac{j75/0.03}{1/0.03 + j(75+2.5)} \right]} = 4.705 - j2.752 \end{aligned}$$

So

$$\cos \varphi_1 = \frac{4.706}{\sqrt{4.706^2 + 2.752^2}} = \frac{4.706}{5.45} = 0.8633$$

The current $I_B = 5.45$ A is notably larger than for 3-phase symmetric (balanced) voltages ($I_n = 3.55$ A). Now, for the calculation of torque components, we need I'_{r+} and I'_{r-} (Figure 5.42)

$$\begin{aligned} I'_{r+} &= I_{A+} \left| \frac{jX_{lm}}{j(X_{lm} + X'_{rl}) + R'_r/S} \right| = \frac{5.45}{\sqrt{3}} \left| \frac{j75}{j(75 + 2.5) + 1/0.03} \right| = 2.8 \text{ A} \\ I'_{r-} &= I_{A-} = \frac{I_B}{\sqrt{3}} = \frac{5.45}{\sqrt{3}} = 3.15 \text{ A} \end{aligned}$$

The torque (Equation 5.136) is

$$\begin{aligned} T_e &= T_{e+} + T_{e-} = \frac{3p_1}{\omega_1} R'_r \left[\frac{(I'_{r+})^2}{S} - \frac{(I'_{r-})^2}{2-S} \right] = \frac{3 \times 2 \times 1}{2\pi \times 60} \left(\frac{2.8^2}{0.03} - \frac{3.15^2}{2-0.03} \right) \\ &= 4.16 - 0.08 = 4.08 \text{ Nm} \end{aligned}$$

So, the torque with one phase open is reduced for $S = 0.03$ from 5.261 to 4.08 Nm. The power factor is a bit reduced from 0.88 to 0.8633. The copper losses for 3-phase and 2-phase operations are

$$P_{Co3} = 3(R_s I_n^2 + R'_r I_m^2) = 3(1 \times 3.55^2 + 1 \times 3.148^2) = 67.53 \text{ W}$$

$$\begin{aligned} P_{Co1} &= V_{BC} I_B \cos \varphi_1 - T_e \frac{\omega_1}{p_1} (1 - S) \\ &= 220 \times 5.45 \times 0.8633 - 4.08 \times \frac{2\pi \times 60}{2} (1 - 0.03) \\ &= 1035.1 - 745.61 = 289.5 \text{ W} \end{aligned}$$

The copper losses are much higher for one phase open, and thus the motor gets overheated if it is not disconnected in due time.

This example illustrated the case of a single-phase winding induction motor which may not start as such but it may operate acceptably in terms of power factor and torque but at higher copper losses.

5.22 One Rotor Phase Open

It is not seldom that some rotor bars or end rings break, and thus the rotor winding becomes asymmetric. An exact treatment of one or more broken bars needs bar-by-bar circuit simulation circuits (see Ref. [4, Chapter 13]), but for a 3-phase wound rotor, the case of one phase open is easy to handle (Figure 5.43a and b).

In effect now, the rotor current, I'_{br} , is decomposed into $\oplus$ and $\ominus$ components:

$$\begin{aligned} I'_{ar+} &= -I'_{ar-} = -\frac{j}{\sqrt{3}} I'_{br} \\ \underline{V}'_{ar+} &= \underline{V}'_{ar-} = \frac{1}{3} (\underline{V}'_{ar} - \underline{V}'_{br}) \end{aligned} \quad (5.144)$$

because $\underline{I}'_{br} = -\underline{I}'_{cr}$; $\underline{V}'_{br} = \underline{V}'_{cr}$ (Figure 5.43a).

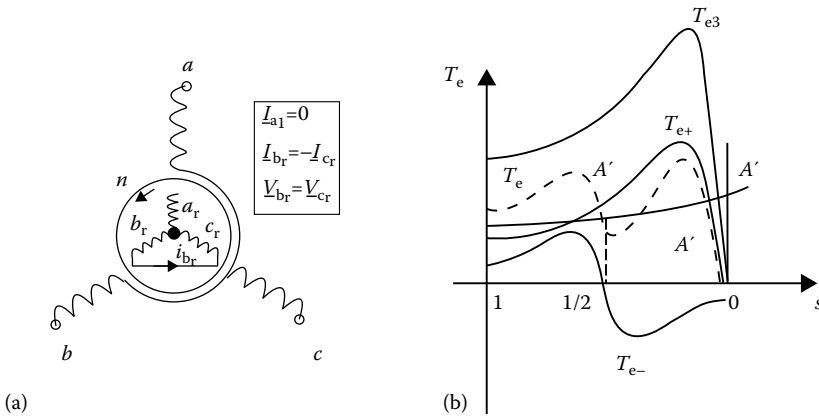


FIGURE 5.43

One rotor phase open: (a) one rotor phase open and (b) torque/speed components and operation points A' , A'' , and A''' .

For this situation, $\oplus$ and $\ominus$ equations “spring” from the rotor whose $\ominus$ mmf component leads to an n' speed with respect to the stator, which is considered short-circuited for the $\ominus$ component (infinite power source):

$$\begin{aligned} I'_{r+} R'_r - \underline{V}_{r+} &= -jS\omega_1 \underline{\Psi}'_{r+}; & \underline{\Psi}'_{r+} &= L'_r I'_{r+} + L_{1m} I_{s+} \\ I_{s+} R_s - \underline{V}_s &= -j\omega_1 \underline{\Psi}_{s+}; & \underline{\Psi}_{s+} &= L_s I_{s+} + L_{1m} I'_{r+} \end{aligned} \quad (5.145)$$

$$n' = n - S \frac{f_1}{p_1} = \frac{f_1}{p_1} (1 - 2S) = \frac{f'_1}{p_1}; \quad f'_1 = f_1 (1 - 2S) \quad (5.146)$$

and

$$\begin{aligned} I'_{r-} R'_r - \underline{V}_{r-} &= -jS\omega_1 \underline{\Psi}'_{r-}; & \underline{\Psi}'_{r-} &= L'_r I'_{r-} + L_{1m} I_{s-} \\ I_{s-} R_s &= -j(1 - 2S) \omega_1 \underline{\Psi}_{s-}; & \underline{\Psi}_{s-} &= L_s I_{s-} + L_{1m} I'_{r-} \end{aligned} \quad (5.147)$$

The unknowns of Equations 5.145 through 5.147 are $I'_{r+} = -I'_{r-}$, $V'_{r+} = V'_{r-}$, I'_{s+} , I'_{s-} for given slip and motor parameters. The relationship between rotor and stator $\ominus$ components (from Equation 5.147) is

$$I_{s-} = -\frac{j\omega_1 (1 - 2S) L_{1m} I'_{r-}}{R_s + j\omega_1 (1 - 2S) L_s} \quad (5.148)$$

$$L_s = L_{1m} + L_{sl}; \quad L'_r = L'_{rl} + L_{1m}.$$

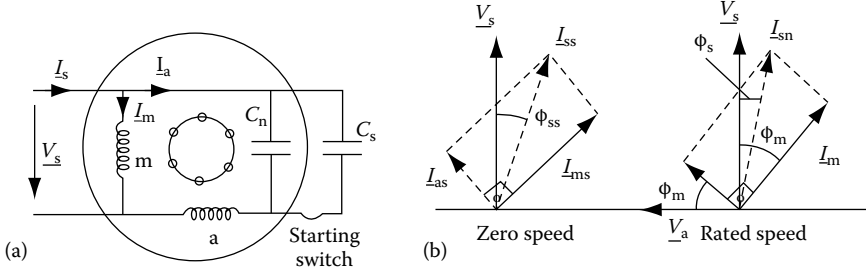
The torque is composed of two components:

$$T_e = 3p_1 L_{1m} \text{Imag}[I_{s+} I'^*_{r+} - I_{s-} I'^*_{r-}] \quad (5.149)$$

It is clear from Equation 5.148 that the stator $\ominus$ current $I_{s-} = 0$ (its synchronism) for $S = 1/2$. The $\ominus$ torque component is positive for $S > 1/2$ and negative for $S < 1/2$, and it is zero at synchronism ($S = 0$). The $\ominus$ torque component is also called single-phase monoaxial (Georges') torque (Figure 5.43b). If the load torque at low speeds is large, the machine may accelerate only to point A''' (around 50% rated speed) due to one rotor phase open—instead of accelerating to A' . Limit A corresponds to the symmetric rotor operation.

5.23 Capacitor Split-Phase Induction Motors

Induction motors connected directly to a single-phase ac power grid are called split-phase motors. Such a motor has a main winding always on, during starting and operation, and an auxiliary winding (displaced by 90° (electrical) in general) for starting and also for running.

**FIGURE 5.44**

The dual capacitor IM: (a) equivalent scheme and (b) phasor diagram for zero and rated speeds.

The auxiliary phase has a series-connected resistance or a capacitor (C_{start}) or two (a running capacitor $C_n < C_{start}$), to produce at a desired slip ($S = 1$ and $S = S_n$) an almost traveling resultant mmf (Figure 5.44a and b). The two windings have different numbers of turns, $W_m \neq W_a$ ($a = W_a/W_m$), for unidirectional motion and $W_m = W_a$ for bidirectional motion, when the capacitor is switched from one phase to the other.

It may be demonstrated that if the amplitudes of the main and the auxiliary windings mmfs, F_{1m} and F_{1a} , are not equal to each other or their time phase angle of currents I_m and I_a is not 90° , the total mmf may be decomposed into a positive (+) mmf, F_{1+} , and an inverse (−) mmf, F_{1-} .

For steady state, we can use the symmetrical components theory (Figure 5.45a):

$$\begin{aligned} E_m &= E_{m+} + E_{m-}; & E_a &= E_{a+} + E_{a-} \\ \underline{A}_{m+} &= \frac{1}{2} (\underline{A}_m - j\underline{A}_a); & \underline{A}_{m-} &= \underline{A}_{m+}^* \end{aligned} \quad (5.150)$$

The machine acts as two separate machines for the two components:

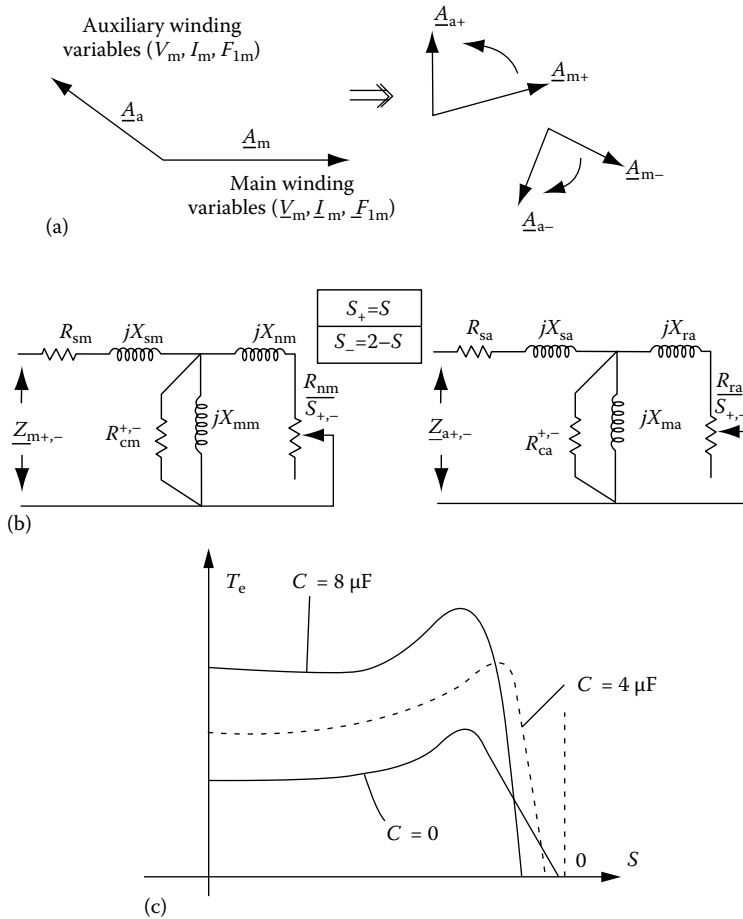
$$\begin{aligned} \underline{V}_{m+} &= \underline{Z}_{m+} \underline{I}_{m+}; & \underline{V}_{m-} &= \underline{Z}_{m-} \underline{I}_{m-} \\ \underline{V}_{a+} &= \underline{Z}_{a+} \underline{I}_{a+}; & \underline{V}_{a-} &= \underline{Z}_{a-} \underline{I}_{a-} \\ \underline{V}_m &= \underline{V}_{m+} + \underline{V}_{m-}; & \underline{V}_a &= \underline{V}_{a+} + \underline{V}_{a-} \end{aligned} \quad (5.151)$$

The $\oplus$ and $\ominus$ impedances of the machine correspond to S and $2 - S$ slip values (as shown in Figure 5.42a). $\underline{Z}_{m\pm}$ and $\underline{Z}_{a\pm}$ have the rotor reduced to the main and auxiliary windings, respectively (Figure 5.41b).

The relationship between $\underline{V}_m$ and $\underline{V}_a$ voltages and the source voltage $\underline{V}_s$ is

$$\underline{V}_{sn} = \underline{V}_s; \underline{V}_a = \underline{V}_s - (\underline{I}_{a+} + \underline{I}_{a-}) \underline{Z}_a \quad (5.152)$$

where $\underline{Z}_a$ is the auxiliary resistance or capacitance in series with the auxiliary winding.


FIGURE 5.45

The symmetrical components model: (a) the $+/-$ model components, (b) equivalent impedances $\underline{Z}_{m+}$ and $\underline{Z}_{a+}$, and (c) typical T_e (ω_r) curve.

Solving Equations 5.151 and 5.152 with $\underline{Z}_{m+}$ and $\underline{Z}_{a+}$ from Figure 5.45b, the machine currents I_{a+} , I_{m+} , and I_{rm+} can be found (see Figure 5.45a).

So the torque T_e is

$$T_e = T_{e+} + T_{e-} = \frac{2p_1}{\omega_1} R_{rm} \left[\frac{I_{rm+}^2}{S} - \frac{I_{rm-}^2}{2-S} \right] \quad (5.153)$$

When $Z_a = \infty$ (the auxiliary phase is open), $I_{rm+} = I_{rm-}$ and thus at zero speed ($S = 1$), the torque is zero (as demonstrated in Section 5.21).

A typical T_e (ω_r) characteristic is shown in Figure 5.45c for a split-phase capacitor induction motor. The positive sequence torque, T_{e+} , has the

synchronism at $(+\omega_1/p_1)$ while the negative sequence has it at $(-\omega_1/p_1)$. The negative sequence torque, T_{e-} , is not large, but due to its large slip $(2 - S)$, it produces large rotor winding losses.

Full symmetrization of the two windings is thus required for reducing winding losses:

$$I_{rm-} = 0 \quad (5.154)$$

This leads to the required turns ratio a and the required capacitor C for a given slip. Now, a , once chosen, say for symmetrization at $S = 1$, may not be changed for symmetrization at a rated slip S_n , but a smaller capacitor, C_n , should be chosen.

For the case of the main and the auxiliary windings using the same quantity of copper ($R_{sm} = R_{sa}/a^2$, $X_{sm} = X_{sa}/a^2$), the symmetrization conditions are (Ref. [4, Chapter 24])

$$\begin{aligned} a &= X_{m+}/R_{m+} = \tan \varphi_+ \\ X_C &= 1/\omega_1 C = Z_+ a \sqrt{a^2 + 1} \end{aligned} \quad (5.155)$$

The $T_e(\omega_r)$ for two different capacitors (and for open auxiliary phase: $C = 0$) are shown in Figure 5.45 only to illustrate that a very large capacitor is not adequate for running conditions. Designing split-phase permanent capacitor motors is thus a hard-to-bargain compromise between good starting and running performance, even with two capacitors $C_{start} \gg C_{run}$.

As split-phase capacitor motors with efficiency above 85% at 100 W are used predominantly in home appliances such as refrigerator compressors, a numerical example is solved here to grasp a feeling of magnitudes.

Example 5.7 Capacitor-Run Split-Phase IM

Let us consider a capacitor run ($C = 4 \mu\text{F}$), split-phase IM fed at 230 V and 50 Hz with $n_n = 940 \text{ rpm}$ ($2p_1 = 6$). The main and auxiliary winding parameters are $a = 1.73$, $R_{sm} = 34 \Omega$, $X_{sm} = 35.9 \Omega$, $R_{sa} = R_{sm}/a^2$, $X_{sa} = X_{sm}/a^2$, $X_{rm} = 29.32 \Omega$, $R_{rm} = 23.25 \Omega$, and $X_{mm} = 249 \Omega$.

Calculate the supply current, torque, power factor, input power, and efficiency at $S = 0.06$ (core and mechanical losses are neglected).

Solution:

From Equations 5.151 and 5.152, we may derive [4, pp. 84]

$$\begin{aligned} I_+ &= \frac{V_s}{2} \frac{(1 - j/a) (\underline{Z}_- + 2\underline{Z}_a^m)}{\underline{Z}_+ \underline{Z}_- + \underline{Z}_a^m (\underline{Z}_+ + \underline{Z}_-)} \\ I_- &= \frac{V_s}{2} \frac{(1 + j/a) (\underline{Z}_+ + 2\underline{Z}_a^m)}{\underline{Z}_+ \underline{Z}_- + \underline{Z}_a^m (\underline{Z}_+ + \underline{Z}_-)} \end{aligned} \quad (5.156)$$

$$T_e = \frac{2p_1}{\omega_1} \left[I_{m+}^2 \operatorname{Re}(\underline{Z}_+) - I_{m-}^2 \operatorname{Re}(\underline{Z}_-) - R_{sm} (I_{m+}^2 - I_{m-}^2) \right] \quad (5.157)$$

$$\underline{Z}_a^m = -\frac{j}{2\omega C a^2} = -\frac{j}{2\pi 50 \times 4 \times 10^{-6} \times 1.73^2} = -j133.4 \Omega$$

$$(\underline{Z}_+)_{S=0.06} = R_{sm} + jX_{sm} + \frac{jX_{mm} (R_{rm}/S + jX_{rm})}{R_{rm}/S + j(X_{mm} + X_{rm})} = 139.4 + j197.75$$

$$(\underline{Z}_-)_{S=0.06} = R_{sm} + jX_{sm} + \frac{jX_{mm} (R_{rm}/(2-S) + jX_{rm})}{R_{rm}/(2-S) + j(X_{mm} + X_{rm})} = 46.6 + j50.25$$

So from Equations 5.156 and 5.157

$$I_{m+} = 0.525 - j0.794$$

$$I_{m-} = 0.1016 + j0.0134$$

$$T_{e+} = \frac{2 \times 3}{2\pi 50} 0.9518^2 (139.4 - 34) = 1.8245 \text{ Nm}$$

$$T_{e-} = -\frac{2 \times 3}{2\pi 50} 0.1025^2 (46.6 - 34) = -2.53 \times 10^{-3} \text{ Nm}$$

$$T_e = T_{e+} + T_{e-} = 1.822 \text{ Nm}$$

The main and the auxiliary winding currents, I_m and I_a , are (Equation 5.151)

$$I_m = I_{m+} + I_{m-} = 0.525 - j0.794 + 0.1016 + j0.0134 \approx 0.62 - j0.78$$

$$I_a = j \frac{I_{m+} + I_{m-}}{a} = 0.466 + j0.274$$

The total stator current, I_s , is

$$I_s = I_a + I_m \approx 1.092 - j0.506$$

So the motor power factor, $\cos \varphi_s$, is

$$\cos \varphi_s = \frac{\operatorname{Re}(I_s)}{I_s} = \frac{1.092}{1.204} \approx 0.90!!$$

The input active power, P_{1e} , is

$$P_{1e} = V_s \operatorname{Re}(I_s) = 230 \times 1.092 = 251.16 \text{ W}$$

Now the mechanical power, P_{out} , is

$$P_{\text{out}} = T_e \frac{\omega_1}{p_1} (1 - S) = 1.822 \times \frac{2\pi 50 (1 - 0.06)}{3} = 179.26 \text{ W}$$

So the efficiency, $\eta_{S=0.06}$, is

$$\eta_{S=0.06} = \frac{P_{\text{out}}}{P_{1e}} = \frac{179.26}{251.16} = 0.7137$$

Notes:

- The reverse $\ominus$ torque component is small.
- The power factor is good (due to the capacitor's presence).
- The efficiency is not very high, partly due to $2p_1 = 6$ ($2p_1 = 2$ would lead to better efficiency, in general).
- The phase shift between I_m and I_a is about $30.45 - (-51.52) \approx 82^\circ$, not far away from the ideal 90° .
- The ratio of m and a mmf amplitudes is $I_a W_a / I_m W_m = a I_n / I_m = 1.73 \times 0.54 / 0.996 = 0.9308$. The ratio of mmf amplitudes is not far away from unity. So the situation is not far away from the symmetrization conditions for $S = 0.06$.
- The capacitor split-phase stator may also be built with a cage PM (or reluctance) rotor as a synchronous motor.

5.24 Linear Induction Motors

By the imaginary process of unrolling a cage-rotor conventional 3-phase induction motor and spreading the cage (now turned ladder), a short-primary long-secondary single-sided linear induction motor is obtained (Figure 5.46).

In applications such as urban and interurban people movers or in industrial short-distance transport, the secondary ladder secondary on ground may be replaced, for cost reasons, by an aluminum sheet over one to four pieces of solid back iron (Figure 5.46).

With the solid back iron on ground, there will be skin effect and eddy currents in it from the primary traveling field. So the solid iron contributes to thrust but also leads to an increase in the magnetization current.

The mechanical airgap $g = 1\text{--}15$ mm, 1 mm for short distance (say, a clean room) transport and 8–15 mm for urban and interurban people movers.

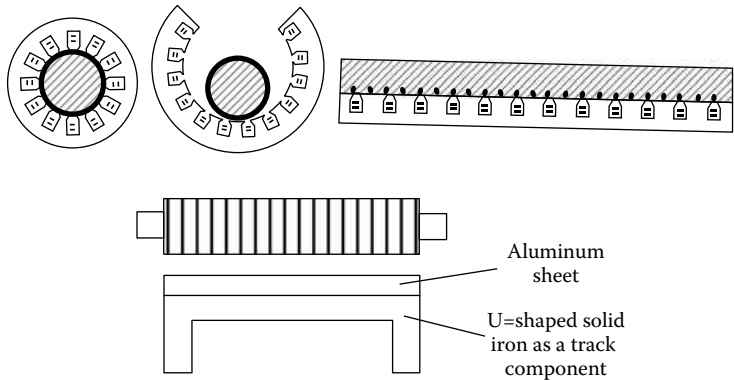


FIGURE 5.46
Obtaining a linear induction motor with flat geometry.

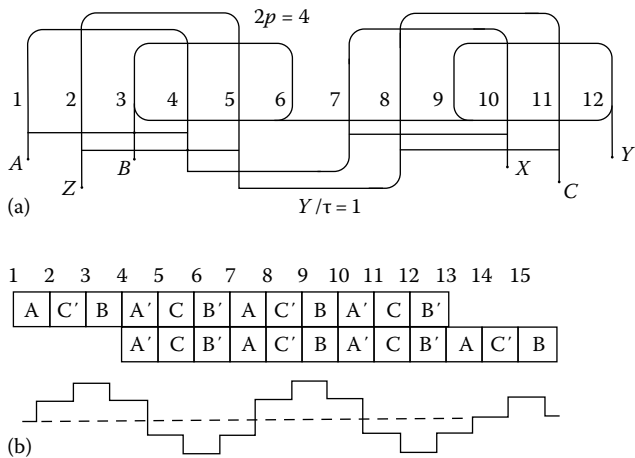


FIGURE 5.47
Linear induction motor windings: (a) single-layer, full-pitch windings ($N_1 = 12$ slots, $2p_1 = 4$, $q = 1$) and (b) double-layer, full-pitch windings ($N_1 = 15$ slots, $2p_1 + 1 = 5$, $q = 1$).

The 3-phase windings stem from those of rotary machines by cutting and unrolling to obtain

- Single layer $2p_1 = 2, 4, 6, \dots$ (even number) windings (Figure 5.47a)
- Double-layer full-pitch (or chorded-coil) windings with $2p_1 + 1$ poles, with half-wound end poles, to destroy the additional (Gauss law caused) back iron ac (fix) flux density to almost zero in the $2p_1 - 1$ central poles.

5.24.1 End and Edge Effects in LIMs

When the number of poles is small, say $2p_1 = 2, 4$, the single-layer winding (Figure 5.47a) appears more adequate as it allows full utilization of all primary cores, but phase B is in a different position with respect to phases A and C relative to the limited core length along the direction of motion.

A so-called **static end effect** occurs in such cases, which is characterized by unbalanced phase currents. A 2-phase winding would avoid this inconvenience.

For $2p_1 > 4$, in general, $2p_1 + 1$ double-layer windings with half-filled end poles (Figure 5.47b) are to be used as the static end effect is thus diminished.

Because the aluminum sheet is a continuum, the second current density has a longitudinal component J_x (Figure 5.48) besides the useful, transverse, component J_y . On an overall basis, this effect produces an increase, K_T , in the equivalent resistance of the secondary, when reduced to the primary. This effect accounts also for the current density lines returning outside the active zone (as in end rings) and is called the transverse edge effect.

Approximately

$$K_T = \frac{1}{1 - \lambda \frac{\tanh \frac{\pi}{\tau} a_e}{\frac{\pi}{\tau} a_e}} > 1; \quad a_e = a + g_e$$

$$\lambda = \frac{1}{1 + \tanh \frac{\pi}{\tau} a_e \tanh \frac{\pi}{\tau} (c - a_e)} \quad (5.158)$$

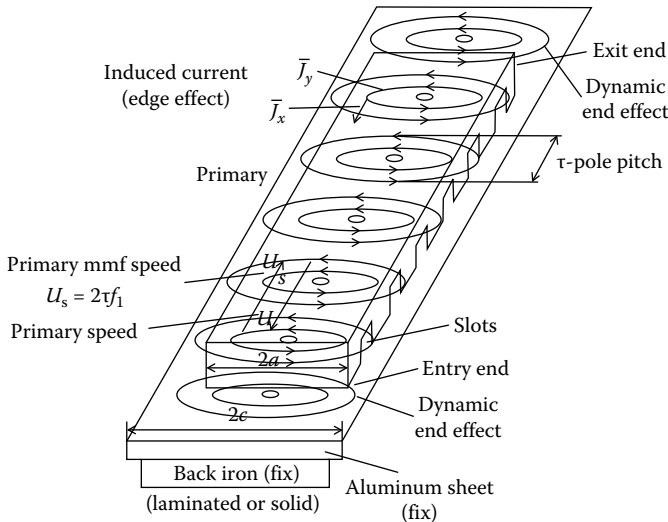


FIGURE 5.48

LIM with secondary current density paths reflecting both edge effect and longitudinal (dynamic) end effect.

In general, the aluminum overhang is $c - a_e \leq \tau/\pi$; τ is the pole pitch of the primary winding.

The coefficient $K_T > 1$ may be lumped into the aluminum conductivity $\sigma_{Ale} = d_{Ale}\sigma_{Al}/K_T$ and then we may suppose that the secondary current density has only the transverse component J_y , which is thrust producing.

Solving the airgap flux density distribution, B_g , with pure traveling mmf and only axial x variation, and accounting for Gauss flux law [10], the Poisson equation yields

$$\frac{\partial B_g}{\partial x^2} - \frac{\mu_0 \sigma_{Ale} U}{g_e} \frac{\partial B_g}{\partial x} - \frac{\mu_0 \sigma_{Ale}}{g_e} \frac{\partial B_g}{\partial t} = \frac{\mu_0}{g_e} \frac{\partial A_s}{\partial x} \quad (5.159)$$

$$A(x, t) = A_m e^{j(\omega_1 t - \frac{\pi}{\tau} x)} \quad \text{stator current sheet} \quad (5.160)$$

with

$$A_m = \frac{3W_1 k_{W1} I \sqrt{2}}{p_1 \tau}; \quad U \quad \text{speed (m/s)} \quad (5.161)$$

In the absence of a dynamic end effect, the airgap flux density is also a traveling wave:

$$B_{gc}(x, t) = B_g e^{j(\omega_1 t - \frac{\pi}{\tau} x)} \quad (5.162)$$

In this case, Equation 5.159 yields

$$B_{gc} = \frac{\mu_0 F_{1m}}{g_e (1 + jSG_e)}; \quad F_{1m} = A_{1m} \frac{\tau}{\pi} \quad (5.163)$$

where S is the slip, as for rotary IMs.

$$S = \frac{U_s - U}{U_s}; \quad U_s = \tau \frac{\omega}{\pi} = 2\tau f_1 \quad (5.164)$$

and

$$G_e = \frac{2f_1 \mu_0 \sigma_{Ale} \tau^2}{\pi g_e} = \frac{X_{1m}}{R_r^1}; \quad g_e = g + d_{Ale} \quad (5.165)$$

G_e is called the equivalent goodness factor.

X_{1m} and R_r^1 are magnetization and secondary resistances reduced to the primary (in the absence of the end effect).

d_{Ale} is the equivalent aluminum plate thickness, which may also include the back iron contribution with its conductivity and field penetration depth.

The larger the goodness factor, G_e , the better the conventional performance. The concept may also be extended to cage rotor induction machines, not which has been done so far! However, the larger the goodness factor and

the lower the number of machine poles $2p_1 + 1$, the larger the damaging consequence of dynamic end effects in producing additional secondary losses, lower thrust, and lower power factor.

Considering only the active (primary) length, a simplified solution for the airgap flux density, B_g (Equation 5.159) is

$$\begin{aligned} \underline{B}_g(x, t) = & \underbrace{\underline{A}e^{\gamma_1 x}}_{\text{Entry end effect wave}} + \underbrace{\underline{B}e^{\gamma_2(x-L_p)}}_{\text{Exit end effect wave}} \\ & + \underbrace{\underline{B}_{gc}e^{-j\frac{\tau}{\pi}x}}_{\text{Conventional traveling field}} \end{aligned} \quad (5.166)$$

where L_p is the machine primary length.

With

$$\begin{aligned} \gamma_{1,2} = \pm \frac{a_1}{2} \left(\sqrt{\frac{b_1 + 1}{2}} \pm 1 + j\sqrt{\frac{b_1 - 1}{2}} \right) &= \gamma_{1,2r} \pm j\gamma_i \\ a_1 = \frac{\pi}{\tau} G_e (1 - s); \quad b_1 = \sqrt{1 + \frac{16}{G_e^2 (1 - s)^4}} \end{aligned} \quad (5.167)$$

Coefficients $\underline{A}$ and $\underline{B}$ are obtained from boundary conditions at primary entry and exit ends. The thrust is simply $\bar{j} \times \bar{B}l$ along the primary length:

$$F_x \approx -a_e d_{Al} \text{Re} \left[\int_0^{L_p} \underline{A}^*(x) \underline{B}_g(x) dx \right] = F_{xc} + F_{xend} \quad (5.168)$$

The end effect thrust, F_{xend} , is additional to the conventional one, F_{xc} , and both are given by

$$F_{xend} = \frac{a_e \mu_0 \tau}{g_e} A_m^2 \text{Re} \left[-\frac{j(\gamma_1 \tau) (e^{\gamma_2 \tau - j\pi} - 1)}{(\gamma_2 \tau - \gamma_1 \tau) \left(\frac{\gamma_2 \tau}{\pi} - j \right)} \right] \quad (5.169)$$

$$F_{xc} = 2a_e p_1 \frac{\tau^2 A_m^2}{g_e \pi} \frac{\mu_0 S G_e}{1 + S^2 G_e^2} \quad (5.170)$$

We may extract in the same way an end effect reactive power, Q_{end} , and the secondary aluminum losses, P_{Alend} . To simplify the optimization design, in the absence of reliable end effect compensation schemes despite more than 40 years of efforts [10], an optimum goodness factor G_e was proposed [10] such that the dynamic end effect thrust is zero at zero slip ($S = 0$) (Figure 5.49). G_e depends only on the number of pole pairs, p_1 .

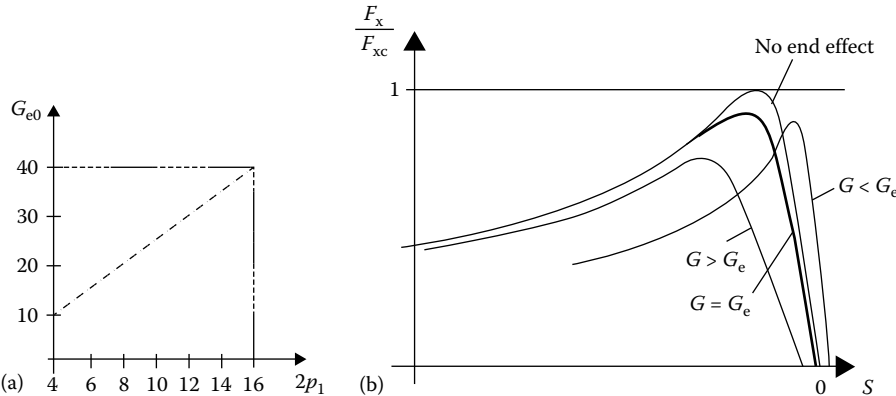


FIGURE 5.49 Dynamic end effect in LIMs: (a) the optimum goodness factor vs. pole number $2p_1$ and (b) thrust/slip with end effect.

Example: For $2p_1 = 12$, at $S = 0.07$ and $U = 110$ m/s, efficiency $\eta_2 = 0.89$ and $\cos \varphi_2 = 0.82$ (these are secondary efficiency and power factor: primary losses and primary leakage reactive power were not considered). This is still very good performance at high speeds.

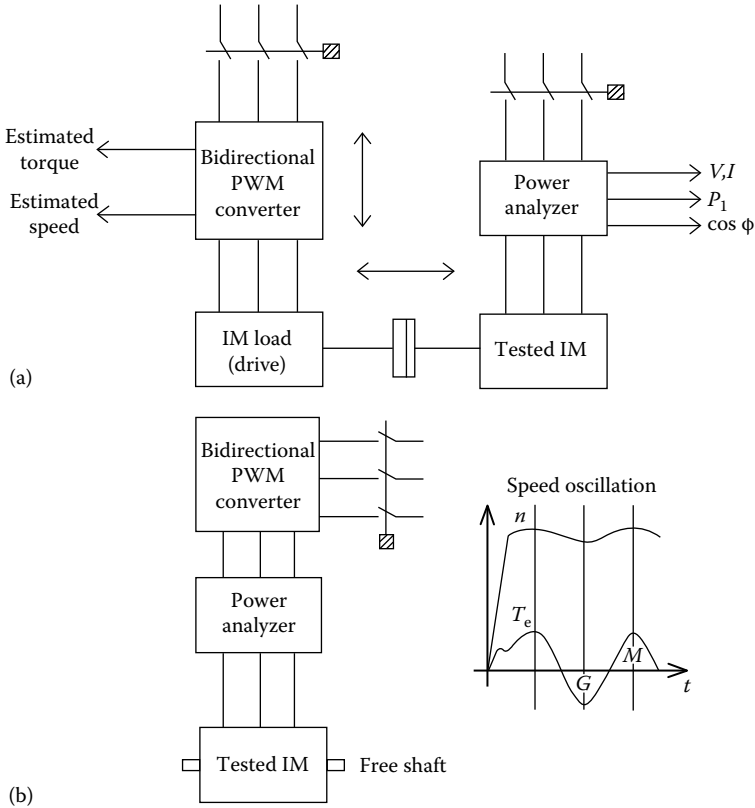
To simplify the design of LIM with end effect (which is notable even for urban people movers with LIM ($U_{\max} < 30$ m/s)), dynamic end effect correction terms depending on SG_e and p_1 may be introduced in the rotary IM equivalent circuit to streamline the LIM design [8].

We conclude the presentation of LIM theory, urging the interested reader to follow the abundant pertinent literature [8–14].

5.25 Regenerative and Virtual Load Testing of IMs/Lab 5.7

The availability of PWM bidirectional power flow two or single stage converters (frequency changers) allows for regenerative and virtual load testing for performance and for temperature (endurance) tests (Figure 5.50a).

The load (drive) IM may act as a motor or a generator with the tested machine as either a generator ($S < 0$) or a motor ($S > 0$). With the load (drive) system yielding electromagnetic torque and speed estimations and with the power analyzer delivering the input power, $\cos \varphi$, current of the tested IM, the latter may be tested as a motor and a generator up to 150% rated torque (thrust), if needed by the application.

**FIGURE 5.50**

IM testing: (a) IM regenerative braking testing and (b) virtual load testing.

For virtual loading (say for vertical shaft IMs where coupling a driver is not easy), the tested IM itself (with a free shaft) is “driven” by the bidirectional PWM converter, and the speed reference is oscillated with an amplitude and frequency that produces the desired RMS stator current. If we recover the motor input active power (positive or negative), its average after a few cycles represents the machine energy loss, W_{loss} . But if we integrate and average only positive power, we get the input energy for motoring, W_{motor} .

So, the efficiency η_m is

$$\eta_m = \frac{W_{\text{motor}} - W_{\text{loss}}}{W_{\text{motor}}} \quad (5.171)$$

This test is called artificial loading by mechanically forcing the IM to switch from motor to generator by notable speed oscillation. It is also possible for PWM to have two frequencies f_1 and $f'_1 = (0.8-0.9)f_1$, and thus the machine will have speed n about constant $f'_1/p_1 < n < f_1/p_1$, but the

machine will be electrically forced to switch from a motor to a generator. Thus, the frequency mixing method was introduced in 1929, with a transformer and an ac generator as power sources at f_1 and f'_1 , but nowadays, this test can be done elegantly with power electronics.

For more on IM testing, see Ref. [4, Chapter 22], ANSI-IEEE 112, IEC Publication 34, Part 2, IEC Publication 37, ANSI-NEMA Publication MG1, standards IEEE Standard 114 and 839/1986 for single-phase IMs.

5.26 Preliminary Electromagnetic IM Design by Example

It is needed to design (size) a cage rotor conduction motor at $P_n = 5 \text{ kW}$, $V_{nl} = 380 \text{ V}$ (star connection), and $f_1 = 50 \text{ Hz}$ with $2p_1 = 4$ poles and an efficiency around (above) 85%. The breakdown torque is $T_{ek}/T_{en} \geq 2.25$ and the starting torque is $T_{es}/T_{en} \geq 1.2$. The starting current is $I_{\text{stator}}/I_n < 6.5$ and $\cos \varphi_n \geq 0.85$.

Solution:

The above data are called “main specifications.” A realistic analytical model of the IM is needed. Such a model will then require a set of variables to size the machine to suit the specifications.

Let us mention here a possible set of variables:

- Stator outer diameter: D_{out}
- Stator inner diameter: D_{is}
- Airgap: g
- Shaft diameter: D_{shaft}
- Stator core axial length: L_e
- Stator slots height: h_{ss}
- Stator slot width/slot pitch: $b_{\text{ss}}/\tau_{\text{ss}}$
- Rotor slot height: h_{sr}
- Rotor slot width/slot pitch: $b_{\text{sr}}/\tau_{\text{sr}}$
- Number of stator slots: N_s
- Number of rotor slots: N_r
- Number of turns per phase W_s (and the number of current paths a)

Earlier in this chapter, we have introduced (derived) analytical expressions of machine parameters, R_s , R_r , L_{sl} , L_{rl} , L_{lm} , and R_{iron} , that depend on the above variables and have entered the expressions for the above design specifications.

As the relationships are nonlinear we cannot derive directly a single set of variables above—which define the machine geometry for the potential manufacturer—from the specifications and from analytical expressions.

But if we add some additional data from past design experience, we can produce a preliminary initial set of variables that can lead to a rather complete machine sizing. This initial design serves as a good basis for design optimization.

The main preliminary design issues are

- Magnetic circuit
- Electric circuit
- Parameters
- Starting torque and current
- Magnetizing reactance X_m
- No load current
- Rated current
- Efficiency and power factor

5.26.1 Magnetic Circuit

As the basic design constant, we choose the tangential specific force, f_t , at rated torque. This specific force, f_t , increases with rotor diameter and has values in the interval:

$$f_t = 0.3 - 3 \text{ N/cm}^2 \quad \text{for } D_{is} = (0.05 - 0.5) \text{ m} \quad (5.172)$$

Also, the ratio between the stator core length and the stator bore diameter, L_e/D_{is} , is

$$\frac{2p_1 L_e}{\pi D_{is}} = \frac{L_e}{\tau} = 0.5 - 3 \quad (5.173)$$

A longer core means a relatively shorter end-connection length and thus lower stator winding losses.

The fundamental amplitude of airgap flux density, about the same at no load and at load, is

$$B_{g1} = (0.4 - 0.8) \text{ T} \quad (5.174)$$

Smaller values are characteristic of small power (below 0.5 kW) and large frequency (speed) IMs.

From industrial experience and design optimization results,

$$\begin{aligned} k_D &= \frac{D_{is}}{D_{out}} \approx 0.5-0.6; 2p_1 = 2 \\ k_D &= \frac{D_{is}}{D_{out}} \approx 0.6-0.67; 2p_1 = 4 \\ k_D &= \frac{D_{is}}{D_{out}} \approx 0.67-0.72; 2p_1 = 6 \\ k_D &= \frac{D_{is}}{D_{out}} \approx 0.7-0.75; 2p_1 \geq 6 \end{aligned} \quad (5.175)$$

The airgap, g , is mechanically limited at the lower end, and at the upper end by the necessity to limit additional core and cage losses due to space harmonics caused by the windings in slots and by slot opening: $g = (0.2-2.5)$ mm, with the larger values suited for the MW power range.

The design (rated) current densities depend on the duty cycle, type of cooling, machine size, and the desired efficiency.

For the case in discussion, $f_{bt} = 1.7 \text{ N/cm}^2$, $L_e/\tau = 0.820$, $B_{g1} = 0.75 \text{ T}$, $D_{is}/D_{out} = 0.623$, and $g = 0.4 \text{ mm}$.

Though the rated slip is not yet known, we may choose an initial value $S_n = 0.025$, as the speed regulation is small with IMs.

So the rated torque, T_{en} , is

$$T_{en} \approx \frac{P_n}{2\pi \frac{f_1}{p_1} (1 - S_n)} = \frac{5000}{2\pi \frac{50}{2} \times (1 - 0.025)} = 32.66 \text{ Nm} \quad (5.176)$$

But

$$\begin{aligned} T_{en} &\approx f_{tn} \frac{D_{is}}{2} \left(\frac{L_e}{\tau} \frac{2\pi}{2p_1} \right) \pi D_{is}^2 = 1.7 \times 10^4 \times D_{is}^3 \times \frac{\pi}{2} \times 0.828 \times \frac{3.14}{2 \times 2} \\ D_{is} &= 0.12345 \text{ m} \end{aligned} \quad (5.177)$$

So the stator stack length, L_e , is

$$L_e = \frac{L_e}{\tau} \frac{\pi D_{is}}{2p_1} = 0.828 \times \pi \times \frac{0.1345}{2 \times 2} = 0.08 \text{ m} \quad (5.178)$$

The rotor diameter, D_{out} , is thus

$$(D_{out})_{2p_1=4} = \frac{D_{is}}{k_D} = \frac{0.12345}{0.623} = 0.198 \text{ m} \quad (5.179)$$

The shaft diameter is chosen in relation to the breakdown torque, $D_{shaft} = 30 \text{ mm}$.

We consider that magnetic saturation does not flatten the sinusoidal air-gap flux density, and the slot opening will be accounted for only by the Carter coefficient, K_c .

Thus, it is rather handy to calculate the stator back iron (yoke) height, h_{ys} ($B_{ys} = 1.5 \text{ T}$)

$$h_{ys} = \frac{\frac{\Phi_p}{2}}{B_{ys}L_e} = \frac{\frac{B_{g1}\tau}{\pi}}{B_{ys}} = \frac{\frac{0.75 \times \pi \times 0.12345}{\pi \times 2 \times 2}}{1.5} = 15.428 \times 10^{-3} \text{ m} \quad (5.180)$$

For the rotor yoke, h_{yr} ($B_{yr} = 1.6 \text{ T}$) is

$$h_{yr} = \frac{\frac{\Phi_p}{2}}{B_{yr}L_e} = \frac{\frac{B_{g1}\tau}{\pi}}{B_{yr}} = \frac{\frac{0.75 \times 0.0969}{\pi}}{1.6} = 14.5 \times 10^{-3} \text{ m} \quad (5.181)$$

$$\tau = \frac{\pi D_{is}}{2p_1} = \pi \times \frac{0.12345}{2 \cdot 2} = 0.0969 \text{ m} \quad (5.182)$$

As the outer and shaft diameters, D_{out} and D_{shaft} , are now known, the total radial height of stator and rotor slots, h_{ss} and h_{sr} , may be calculated as

$$h_{ss} = \frac{D_{out} - D_{is}}{2} - h_{ys} = \frac{(0.198 - 0.1234)}{2} - 0.0154 \approx 21.6 \text{ mm} \quad (5.183)$$

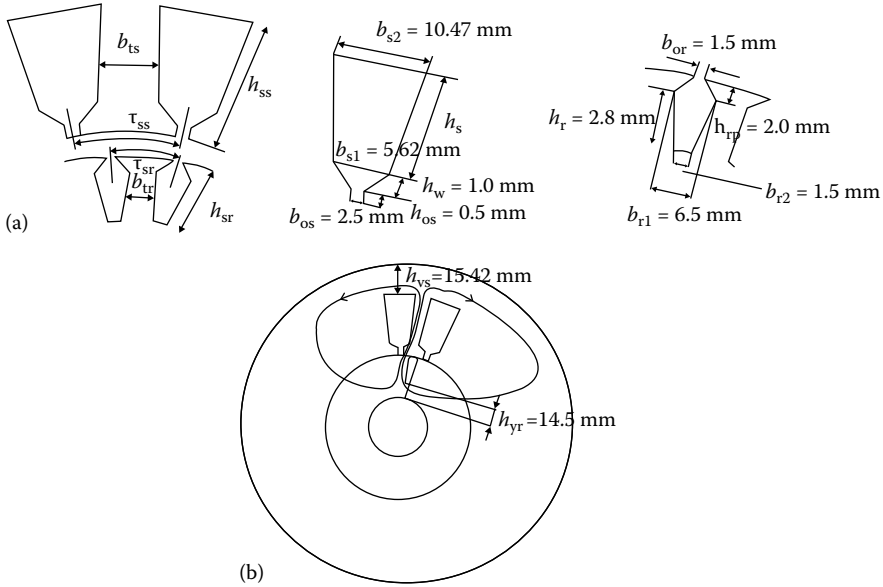
$$h_{sr} = \frac{D_{out} - D_{shaft}}{2} - h_{yr} - g = \frac{(0.12345 - 0.03)}{2} - 0.145 - 0.4 \approx 31.5 \text{ mm} \quad (5.184)$$

It is now time to choose the number of stator slots per pole and phase $q = 3$ (the pole pitch is $\tau = 0.09687 \text{ m}$) and thus with $2p_1 = 4$, $N_s = 2p_1 q m_1 = 2 \times 2 \times 3 \times 3 = 36$ slots. The rotor slot number $N_r = 30$ (there are tables of suitable N_s , N_r , $2p_1$, combinations to minimize parasitic torque effects [4, Chapter 15]). The tooth flux density in the stator and rotor are again chosen, at $B_{ts,r} = 1.5 \text{ T}$ and thus the tooth/slot pitch ratio is

$$\frac{b_{ts}}{\tau_{ss}} = \frac{b_{tr}}{\tau_{sr}} = \frac{B_{g1}}{B_{ts,r}} = \frac{0.75}{1.5} = 0.5 \quad (5.185)$$

Now, we can completely size the stator if in Figure 5.51 we adopt $h_{0s} = 0.5 \text{ mm}$, $b_{0s} = 2.5 \text{ mm}$, $h_w = 1 \text{ mm}$, $h_{rp} = 2 \text{ mm}$, and $b_{0r} = 1.5 \text{ mm}$. The active slot top and bottom widths are

$$\begin{aligned} b_{s1} &= \frac{\pi (D_{is} + 2(h_{0s} + h_w))}{N_s} - b_{ts} \\ &= \frac{\pi \times (123.4 + 2 \times (0.5 + 1))}{36} - 0.5387 \times \frac{96.87}{9} = 5.62 \text{ mm} \\ b_{s2} &= \frac{\pi (D_{is} + 2h_{ss} + h_{ys})}{N_s} - b_{ts} \\ &= \frac{\pi \times (123.4 + 2 \times 21.6)}{36} - 5.4 = 10.47 \text{ mm} \end{aligned}$$

**FIGURE 5.51**

The magnetic circuit: (a) typical stator and rotor slots and calculated geometry (b) cross-section.

$$b_{r1} = \frac{\pi(D_{is} - 2g - 2h_{rp}))}{N_r} - b_{tr} = \frac{\pi \times (123.4 - 0.8 - 4)}{30} - 6.48 = 6.5 \text{ mm}$$

$$b_{r2} = \frac{\pi(D_{ir} - 2g - 2h_{sr}))}{N_r} - b_{tr} = \frac{\pi \times (123.4 - 0.8 - 60)}{30} - 6.48 = 1.5 \text{ mm !!}$$
(5.186)

The slot active areas (filled with coils) in the stator and rotor are

$$A_{ssa} = (h_{ss} - h_{os} - h_w) \frac{(b_{s1} + b_{s2})}{2} = \frac{(21.5 - 1.5) \times (5.62 + 10.47)}{2}$$

$$= 160.90 \text{ mm}^2$$
(5.187)

$$A_{sra} = h_{rp} \frac{(b_{or} + b_{r1})}{2} + (h_{sr} - h_{rp}) \frac{(b_{r1} + b_{r2})}{2} = 2 \times \frac{(1.5 + 6.5)}{2} + 28 \times \frac{8}{2}$$

$$= 120 \text{ mm}^2$$
(5.188)

The ratio of stator and rotor slot areas is

$$\frac{A_{ss}}{A_{sr}} = \frac{N_s A_{ssa}}{(N_r A_{sra})} = \frac{36 \times 160.90}{(30 \times 120)} = 1.609 > 1$$
(5.189)

As expected, $A_{ss}/A_{sr} > 1$ as less slot space is available in the interior rotor, to avoid very heavy magnetic saturation: even values of 2/1 or a bit more are feasible.

5.26.2 Electric Circuit

It is known from industrial practice that the emf in the stator, V_{en} , is about 0.93–0.98 of the supply rated voltage, V_n :

$$\frac{V_{en}}{V_n} \approx (0.93 - 0.98) \quad (5.190)$$

The smaller values are valid for sub-kW IM and for larger number of poles $2p_1$ (when the stator leakage reactance in p.u. is increasing). But V_{en} is, from Equation 5.34,

$$V_{en} = \pi \sqrt{2} f_1 W_s k_{w1s} \frac{2}{\pi} B_{g1} \tau L_e \quad (5.191)$$

With Equation 5.9, the stator winding factor, k_{w1s} , is

$$k_{w1s} = \frac{\sin \frac{\pi}{6}}{q \sin \frac{\pi}{6}} \sin \frac{y}{\tau} \frac{\pi}{2} \quad (5.192)$$

With $q = 3$ and y/τ (coil span/pole pitch) = 8/9, $k_{w1} = 0.925$. The only unknown in Equation 5.191 is the number of turns per phase, W_s :

$$W_s = \frac{\frac{380}{\sqrt{3}} \times 0.97}{\pi \sqrt{2} \times 50 \times 0.925 \times \frac{2}{\pi} \times 0.75 \times 0.0968 \times 0.0825} = 252 \text{ turns/phase}$$

Now, the number of turns/coil, W_c for a two layer winding is

$$W_c = \text{Integer} \left(\frac{W_s}{2p_1 q} \right) = \text{integer} \left(\frac{252}{2 \times 2 \times 3} \right) = 21 \text{ turns/coil}$$

So the number of turns per phase is $W_s = 252$ turns, indeed. At this point in design, we may not calculate the rated current, I_n , unless we assume certain values for rated efficiency and power factor (in traditional design methods η_n and $\cos \varphi_n$ are assumed initial values, based on experience). Here, we first proceed with machine parameters and then calculate I_n , η_n , and $\cos \varphi_n$.

5.26.3 Parameters

Knowing the number of turns per phase and the stator slot area, we may calculate the stator resistance:

$$R_s = \rho_{Co} l_{cs} \frac{W_s}{A_{Co}} = \frac{2.1 \times 10^{-8} \times 0.4353 \times 252}{1.408 \times 10^{-6}} = 1.63682 \Omega$$

$$A_{Co} = \frac{A_{ss} k_{fill}}{2W_c} = \frac{160.9 \times 0.45}{2 \times 21} = 1.408 \text{ mm}^2 \quad (5.193)$$

The coil turn length, l_{cs} , is

$$l_{cs} \approx 2L_e + 2l_{ec} = 2L_e + \pi y = 2 \times (0.0825 + 0.13518) = 0.4353 \text{ m} \quad (5.194)$$

The rotor aluminum bar resistance, R_b , is

$$R_b = \rho_{Al} \frac{L_e + 0.01}{A_{sra}} = \frac{3.5 \times 10^{-8} \times 0.0925}{120 \times 10^{-6}} = 0.27 \times 10^{-4} \Omega \quad (5.195)$$

The end ring-to-bar current ratio (Equation 5.24) is

$$\frac{I_r}{I_b} = \frac{1}{2 \sin \frac{\alpha_{esr}}{2}} = \frac{1}{2 \sin \frac{2 \times \pi \times 2}{2 \times 30}} = 2.38 \quad (5.196)$$

So the end ring area, A_{ring} , is

$$A_{ring} \approx A_{sra} \frac{I_r}{I_b} = 120 \times 2.38 = 286.62 \text{ mm}^2 \quad (5.197)$$

The rotor bar/ring relationships are illustrated in Figure 5.52.

The end ring cross-sectional dimensions, a and b , are $a = 10 \text{ mm}$ and $b = 28.6 \text{ mm}$. The length of the end ring segment, corresponding to a bar, is

$$L_{ring} \approx \frac{\pi (D_{is} - b - 2g)}{N_r} = \frac{\pi \times (123.4 - 28.6 - 0.8)}{30} \approx 9.92 \text{ mm} \quad (5.198)$$

So the end ring segment resistance, R_r , is

$$R_r = \rho_{Al} \frac{L_{ring}}{A_{ring}} = \frac{3.5 \times 10^{-8} \times 0.00992}{286 \times 10^{-6}} = 0.1211 \times 10^{-5} \Omega \quad (5.199)$$

The equivalent bar resistance, R_{be} (Equation 5.26), is

$$R_{be} = R_b + \frac{R_b}{2 \sin^2 \alpha_{esr}} = 0.27 \times 10^{-4} + \frac{0.1211 \times 10^{-5}}{2 \sin^2 \frac{\pi}{15}} = 0.4083 \times 10^{-4} \Omega \quad (5.200)$$

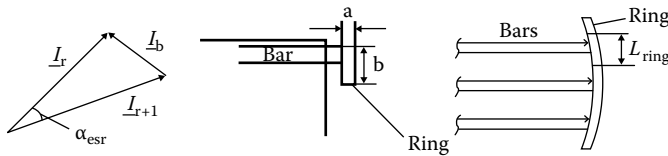


FIGURE 5.52

The rotor end ring sizing.

The reduction factor to the stator is (Equation 5.51)

$$R'_r = R_{be} \frac{N_r}{3k_i^2}; \quad k_i = \frac{N_r k_{skew}}{6W_s k_{w1s}} \quad (5.201)$$

With 1 rotor slot pitch ($c = \tau_{ss}$) skewing (Equation 5.48),

$$k_{skew} = \sin \frac{\frac{\pi}{2} \frac{c}{\tau}}{\frac{\pi}{2} \frac{c}{\tau}} \quad (5.202)$$

So

$$k_i = \frac{30 \cdot 0.95}{6 \cdot 252 \cdot 0.925} = 0.0203 \quad (5.203)$$

and

$$R'_r = 0.4083 \times 10^{-4} \frac{30}{3 \times 0.0203^2} = 0.99 \, \Omega < R_s = 1.636 \, \Omega$$

Leakage reactances

According to Equation 5.49, the stator leakage reactance (see Section 5.6.1) is

$$X_{sl} = 2\mu_0\omega_1 \frac{W_1^2 L_e}{p_1 q} \sum \lambda \quad (5.204)$$

$$\sum \lambda = \lambda_{sls} + \lambda_{slc} + \lambda_{sld} + \lambda_{slz}$$

We will consider here the differential leakage lumped into the zig-zag leakage λ_z . The slot permeance coefficient is

$$\lambda_{sls} = \frac{hs}{3(b_{s1} + b_{s2})} + \frac{h_{0s}}{b_{0s}} + \frac{2h_w}{b_{0s} + b_{s2}} = 1.347 \quad (5.205)$$

The zig-zag permeance coefficient is

$$\lambda_{slz} = \frac{5g/b_{0s}}{5 + 4g/b_{0s}} = 0.1418 \quad (5.206)$$

The end-connection permeance coefficient is

$$\lambda_{sec} = 0.34 \frac{g}{L_e} (l_{ac} - 0.64y) = 0.99 \quad (5.207)$$

So the total stator leakage reactance X_{sl} is (Equation 5.204)

$$X_{sl} = 2 \times 1.256 \times 10^{-6} \times 2\pi \times 50 \times 252^2 \frac{0.0825}{2 \times 3} \times (1.347 + 0.1418 + 0.99) \approx 0.752 \, \Omega \quad (5.208)$$

The rotor cage leakage reactance is calculated in the same way (see again Section 5.6.1)

$$\lambda_{rls} = \frac{2h_{rp}}{3(b_{r1} + b_{r2})} + \frac{2h_r}{3(b_{r1} + b_{r2})} = 2.506 \quad (5.209)$$

For the end ring,

$$\lambda_{ring} = \frac{L_{ring}}{L_e} \ln \left(\frac{L_{ring} N_r}{2\sqrt{\frac{ab}{\pi}}} \right) = 0.331 \quad (5.210)$$

So

$$\begin{aligned} X_{rl} &= X_{be} \frac{N_r}{3k_t^2} \\ X_{be} &= \mu_0 \omega_1 L_e (\lambda_{rls} + \lambda_{ring}) = 0.922 \times 10^{-4} \Omega \\ X_{rl} &\approx 2.21 \Omega \end{aligned} \quad (5.211)$$

Note. The skewing leakage inductance was neglected. Also, the main reactance, X_{1m} , should be reduced by k_{skew} .

5.26.4 Starting Current and Torque

Due to the skin effects, the rotor parameters, R'_r and X'_{rl} , have to be corrected when calculating starting current and torque.

Let us consider the rotor slot as rectangular.

The skin effect parameter, ξ (Equation 5.104), is

$$(\xi)_{start} = h_{rs} \sqrt{\frac{\omega_1 \mu_0 \sigma_{se}}{2}} = 2.25 \quad (5.212)$$

Approximately from Equation 5.209

$$\begin{aligned} \phi(\xi) &= (\xi)_{start} = 2.25 \\ \psi(\xi) &\approx \frac{3}{2\xi} = 0.66 \end{aligned} \quad (5.213)$$

Because the actual rotor slot is thinner toward the bottom, we adopt $\phi(\xi) = 1.5$, $\psi(\xi) = 0.8$ (smaller skin effect).

The skin effect in the end ring is, in general, smaller because it is placed mostly in air, but we will still consider it to be the same as is in the slot zone.

So

$$\begin{aligned} R'_{rstart} &= \phi(\xi) R'_r = 1.5 \times 0.99 = 1.485 \Omega \\ X'_{rlstart} &= \psi(\xi) X'_{rl} = 0.8 \times 2.215 = 1.772 \Omega \end{aligned} \quad (5.214)$$

So the starting current, I_{start} , is simply (Equation 5.56)

$$I_{\text{start}} = \frac{V_{\text{rl}}/\sqrt{3}}{\sqrt{(R_s + R'_{\text{rstart}})^2 + (X_{\text{sl}} + X'_{\text{rlstart}})^2}} = 47.12 \text{ A} \quad (5.215)$$

The starting torque is obtained applying Equation 5.95 for $S = 1$:

$$T_{\text{estart}} \approx \frac{3p_1}{\omega_1} I_{\text{start}}^2 R'_{\text{rstart}} \approx 63 \text{ Nm} \quad (5.216)$$

5.26.5 Breakdown Slip and Torque

S_k , T_{ek} (Equations 5.108 and 5.109):

$$S_k = \frac{R'_r}{\sqrt{R_s^2 + (X_{\text{sl}} + X_{\text{rl}})^2}} = \frac{0.99}{\sqrt{1.636^2 + (1.70 + 2.215)^2}} = 0.2338 \quad (5.217)$$

$$T_{\text{ek}} \approx \frac{3p_1}{2} \frac{\left(\frac{V_{\text{nl}}}{\sqrt{3}}\right)^2}{\omega_1} \cdot \frac{1}{X_{\text{sl}} + X'_{\text{rl}}} = \frac{3 \times 2}{2} \times \frac{220^2}{2\pi \times 50} \times \frac{1}{(1.70 + 2.215)} = 118 \text{ Nm} \quad (5.218)$$

Note. The rated torque, still to be calculated, will be about 32 Nm, so the starting and breakdown torques are large (at the price of lower efficiency, perhaps).

5.26.6 Magnetization Reactance, X_m , and Core Losses, p_{iron}

The magnetization reactance may be calculated after we calculate the iron mmf contribution coefficient, K_s , to magnetic saturation (Section 5.4.6):

$$K_s = \frac{2H_{\text{ts}}h_s + H_{\text{ys}}l_{\text{ys}} + H_{\text{yr}}l_{\text{yr}} + 2H_{\text{tr}}l_r}{2 \frac{B_{\text{gl}}}{\mu_0} g K_c} \quad (5.219)$$

The Carter coefficient, K_c (which accounts for both stator and rotor slotting), is

$$K_c = K_{\text{cs}} K_{\text{cr}}$$

$$K_c = \frac{\tau_{\text{ss}}}{\tau_{\text{ss}} - \gamma_{\text{s}} g} \frac{\tau_{\text{sr}}}{\tau_{\text{sr}} - \gamma_{\text{r}} g}; \quad \gamma_{\text{s,r}} = \frac{(b_{0\text{s,r}}/g)^2}{5 + b_{0\text{s,r}}/g} \quad (5.220)$$

$$\tau_{\text{ss}} = \frac{\pi D_{\text{is}}}{N_s} = \frac{\pi \cdot 0.1234}{36} = 0.0107 \text{ m}; \quad b_{0\text{s}} = 2.5 \text{ mm} \quad (5.221)$$

$$\tau_{\text{sr}} = \frac{\pi \cdot (D_{\text{is}} - 2g)}{N_s} = \frac{\pi (0.1234 - 2 \times 0.0004)}{30} = 0.01283 \text{ m}; \quad b_{0\text{r}} = 1.5 \text{ mm} \quad (5.222)$$

Finally, $K_c = 1.497$

The stator wire diameter is

$$d_{co} = \sqrt{\frac{4}{\pi} \cdot A_{co}} = \sqrt{\frac{4}{\pi} \cdot 1.408} = 1.34 \text{ mm} \quad (5.223)$$

As the stator slot opening is $b_0 = 2.5 \text{ mm}$, there is enough room to insert the turns one by one in the slot.

Now with $B_{ts} = B_{tr} = 1.5 \text{ T} = B_{ys} = B_{yr}$ from the $B(H)$ curve of the silicon steel (Chapter 4), $H_{ts} = H_{tr} = 500 \text{ A/m}$.

The average field path lengths in the stator and rotor yokes based on conservative industrial design experience is

$$l_{ys} = \frac{\pi \cdot (D_{out} - h_{ys})}{2p} = \frac{\pi \cdot (0.198 - 0.0154)}{4} = 0.143 \text{ m} \quad (5.224)$$

$$l_{yr} = \frac{\pi \cdot (D_{shaft} - h_{yr})}{2p_1} = \frac{\pi \cdot (0.03 + 0.0154)}{4} = 0.03564 \text{ m} \quad (5.225)$$

So finally K_s (Equation 5.219) is

$$K_s = \frac{2 \times 500 \times 0.0215 + 500 \times 0.9433 + 500 \times 0.03564 + 2 \times 500 \times 0.028}{2 \times \frac{0.75}{1.256 \times 10^{-6}} \times 0.4 \times 10^{-3} \times 1.497} = 0.194$$

The magnetic saturation coefficient, K_s , is rather small (values from 0.3 through 0.5 are common for $2p_1 = 4$) but the Carter coefficient is rather large, so an increase in airgap or a reduction of b_{0s} , say, to 2 mm is necessary to keep not only the magnetization current (and power factor) but the additional core (stray) losses within practical limits.

The magnetization reactance (Equation 5.41) is

$$X_{1m} = \frac{6\mu_0\omega_1}{\pi^2} (W_s k_{w1s})^2 \frac{\tau L_e}{p_1 g K_c (1 + K_s)} = 65.77 \Omega \quad (5.226)$$

The specific core losses at 1.5 T, 50 Hz are considered here:

$$(p_{iron})_{1.5 \text{ T}, 50 \text{ Hz}} = 4.2 \text{ W/kg} \quad (5.227)$$

As 1.5 T was considered in both parts of stator iron core (yoke and teeth), we only need to calculate the total iron weight. Stator yoke weight, G_{ys} , is

$$G_{ys} \approx \pi (D_{int} - h_{ys}) \times h_{ys} \times L_e \gamma_{iron} = 5.536 \text{ kg} \quad (5.228)$$

The stator teeth weight is

$$G_{ts} = h_{ss} \times L_e \times b_{ts} \times N_s \times \gamma_{iron} = 2.606 \text{ kg} \quad (5.229)$$

The rotor iron losses are negligible because the rotor (slip) frequency $f_2 = S_n f_1 < (1.5-2)$ Hz in our case. So the total core loss, p_{iron} , is

$$p_{\text{iron}} = (p_{\text{iron}})_{1.5 \text{ T}, 50 \text{ Hz}} \times G_{\text{ts}} + (p_{\text{iron}})_{1.5 \text{ T}, 50 \text{ Hz}} G_{\text{ys}} = 34.1964 \text{ W} \quad (5.230)$$

To account for additional core losses, we may double the value of fundamental core losses above:

$$p_{\text{iron}t} = 2p_{\text{iron}} = 68.4 \text{ W}$$

5.26.7 No-Load and Rated Currents, I_0 and I_n

Based on

$$\frac{V_{\text{nl}}}{\sqrt{3}} = I_0 \sqrt{(R_s + R_{\text{iron}})^2 + (X_{\text{sl}} + X_{1\text{m}})^2} \quad (5.231)$$

and

$$3R_{\text{iron}}I_0^2 = p_{\text{iron}t} \quad (5.232)$$

we may iteratively calculate both the unknown I_0 and R_{iron} . But, neglecting R_{iron} in Equation 5.231 does not produce unacceptable errors:

$$I_{0i} \approx \frac{(V_{\text{nl}}/\sqrt{3})}{\sqrt{R_s^2 + (X_{\text{sl}} + X_{1\text{m}})^2}} = 3.26 \text{ A} \quad (5.233)$$

The iron loss resistance, R_{iron} , is

$$R_{\text{iron}} = \frac{p_{\text{iron}t}}{3I_0^2} = 2.158 \Omega \quad (5.234)$$

I_{0i} is the ideal no-load current which is close to the motor no-load current I_0 because the iron losses plus mechanical losses do not add much to I_{0i} and the flux density in the airgap is about the same for ideal and motor no-load modes.

The rated current I_n contains the rated slip which is still unknown:

$$I_n = \frac{(V_{\text{nl}}/\sqrt{3})}{\sqrt{(R_s + C_1 \frac{R'_r}{S_n})^2 + (X_{\text{sl}} + C_1 X'_{\text{rl}})^2}}; \quad C = 1 + \frac{1.7}{65.7} = 1.0258 \quad (5.235)$$

$$I'_{\text{rn}} = \sqrt{I_n^2 - I_{0i}^2} \quad (5.236)$$

To avoid tedious iteration calculations, we vary S_n from 0.01 to 0.06 and calculate from Equation 5.235 I_n , and I'_{rn} from Equation 5.236, and then

$$T_{\text{en}} = \frac{3R'_r(I'_{\text{rn}})^2}{S_n} \frac{p_1}{\omega_1} \quad (5.237)$$

For nominal power and, say (with $p_{\text{mecn}} = 0.01P_n$),

$$p_{\text{mecn}} + P_n = T_{\text{en}} \frac{2\pi f_1}{1 - S_n} = 5050 \text{ W} \quad (5.238)$$

We vary slip until Equation 5.238 is satisfied (with I_n from Equations 5.235 through 5.237). Let us consider $S_n = 0.05$ and obtain $I_n = 9.868 \text{ A}$, $I'_{\text{tn}} = 9.314 \text{ A}$, and $T_{\text{em}} = 32.82 \text{ Nm}$. The developed shaft power is

$$P_{\text{shaft}} = T_{\text{e}} \frac{2\pi f_1}{1 - S_n} - p_{\text{mec}} = 4845 \text{ W} < 5000 \text{ W} \quad (5.239)$$

We are close but not quite there. A slightly larger slip would do it, perhaps $S_n = 0.053$.

5.26.8 Efficiency and Power Factor

The efficiency, η_n , is simply

$$\eta_n \approx \frac{P_n}{T_{\text{e}} \frac{2\pi f_1}{p_1} + p_{\text{iron}} + 3R_s I_n^2} = 0.85 \quad (5.240)$$

We should note that we did not enter exactly the rated power value but the efficiency is about 85%.

The power factor, $\cos \varphi_n$, is

$$\cos \varphi_n = \frac{P_n}{\eta_n 3(V_{nl}/\sqrt{3})I_n} = 0.875 \quad (5.241)$$

5.26.9 Final Remarks

- The stator resistance should be reduced by 20%–30% by deepening the stator slots with mild additional magnetic saturation of the stator yoke. The efficiency may be increased by a few percent.
- The rather large $T_{\text{start}}/T_{\text{en}} = 63/32.82$ and $T_{\text{ek}}/T_{\text{en}} = 118/32.82$ are responsible to a great extent for the lower efficiency. In essence, the machine is a bit too small (in volume) for the tasks. For design optimization, the design should go back and change the specific force (rotor shear stress) f_{tn} (in N/cm^2) and the stack length (pole pitch) and redo the whole design. As is shown in Part III of this book, design optimization with various methods may improve performance notably, observing the important constraints (more on IM design in Refs. [15–18]).

5.27 Summary

- Induction machines are ac stator and ac rotor traveling magnetic field machines with $2p_1$ poles.
- They have both the stator and the rotor cores made of nonoriented grain silicon steel sheets, with stamped uniform (for three phases) slots, around the periphery toward the airgap.
- The IM airgap should be small enough to reduce the magnetization current, I_{0s} , but large enough to reduce supplementary (additional) load losses, p_s , due to mmf and slot opening magnetic field space harmonics. In general, $g = 0.2\text{--}2.5\text{ mm}$ for powers from 100 W to 30 MW, but $g = (1\text{--}15)\text{ mm}$ for linear induction machines.
- The stator slots are filled, in general, with 3-phase single- or double-layer ac windings with traveling mmf and thus the traveling airgap field. The IM windings have the number of slots/pole/phase $q \geq 2$, integer or even fractional.
- The rotor slots are filled either with aluminum uninsulated die-cast bars and end rings (the cage rotor) or with a 3-phase ac winding with the same number of poles, $2p_1$, as in the stator. The cage rotor adapts to any number of stator poles and thus pole changing stator windings may be used with cage rotor, to change the ideal no-load or synchronous speed $n_1 = f_1/p_1$: the speed for zero rotor current and torque.
- The ac winding mmf shows space harmonics ν ($\nu > 1$) above the fundamental due to winding placement in slots. The mmf harmonics, $\nu = -(5, 7, 11, 17, \dots)$, are inverse rotating at n_1/ν ($\nu < 0$) and they are forward rotating for $\nu = +(7, 13, 19, \dots)$. These harmonics produce their own rotor cage currents and asynchronous parasitic torques that may hamper starting under heavy loads. Chording the coils in the winding to $y/\tau \approx 0.8$ leads to harmonic reduction to acceptable levels in industry. The first slot harmonics $\nu_c = 2kqm \pm 1$ also produce parasitic torques, asynchronous and synchronous ones, that are reduced by slot skewing and, respectively, by choosing proper stator and rotor count combinations ($N_s \neq N_r$).
- Rotor 3-phase or cage windings may be mathematically reduced to the stator as done for transformers. This way the mutual stator/rotor phase inductance amplitudes are equal to the stator phase self main flux inductance.
- The main and leakage magnetic fields in the IM are “translated” into main L_{1m} and leakage inductances L_{sl} , L_{rl} ; in general,

$I_{1m}(\text{p.u.}) = L_{1m}\omega_n I_n / V_n = (1.2 - 4)$, $I_{sl}(\text{p.u.}) = I_{rl}(\text{p.u.}) = 0.03-0.1$, which smaller for linear induction motors with large airgap

- Once the mutual self and leakage inductances and resistances of stator/rotor are defined (calculated), the machine may be considered as an “ensemble” of coupled electric circuits with only stator/rotor mutual inductances to be dependent on rotor electrical position $\Theta_{er} = p_1 \Theta_r$; Θ_r —mechanical rotor position angle.
- After changing the rotor variables to stator coordinates (by counter-rotation along Θ_{er}), the equations of an IM become independent of the rotor position for steady state and phasors may be applied. The obtained equations, allowing for phase apparent segregation (star connection) resemble those of the transformer, but to the cage rotor an additional resistance $R'_r(1 - S)/S$ is added, which corresponds to the mechanical power developed by the actual IM.
- S is the slip; $S = (n_1 - n)/n_1$; for $S = 0$ (and cage rotor) the synchronous operation (zero torque) is obtained. For motoring $1 > S > 0$ ($n = 0$ to n_1), and for generating $S < 0$ ($n > n_1$ for $n_1 > 0$).
- IM ideal no-load motor and stalled rotor operation modes are used to segregate the losses and assess equivalent circuit parameters.
- Self-excited induction generator operation with capacitors is dependent on magnetic saturation, speed, and machine parameters. Voltage regulation is substantial, so a controlled capacitor would be needed for voltage (or frequency)-sensitive loads even at a constant speed.
- The electromagnetic torque has a breakdown value for motoring ($S = S_k$) and for generating ($S = -S_k$), which is independent of rotor resistance R'_r , but S_k is proportional to R'_r .
- Speed control (variation) for a given torque may be done by pole-changing windings or frequency f_1 control through full power PWM converters. Only for starting, the reduced voltage method is feasible. Rotor resistance increases, feasible with wound rotors, produces large torque at all speeds but for very large losses; so it is used for limited speed range control. Supplying the wound rotor from a variable frequency f_2 PWM converter ($f_2 = f_1 - np_1$) allows for limited speed control for lower than rated rotor PWM converter rating (and costs).
- Deep bars or a dual cage in the rotor allows for heavy starts at reduced starting currents (down to $5I_n$); I_n is the stator rated current.
- Unbalanced supply voltages lead to negative sequence current components I_{s-} , besides the direct sequence I_{s+} , and they contribute to

larger losses and lower torque and power factor in the machine. Machine derating is recommended in the NEMA standards in relation to voltage unbalance index.

- The one stator phase open—which is equivalent to a single-phase supply—leads to zero starting torque and to larger loss and speed for given torque and nonzero speed.
- Asymmetric rotor circuits (broken bars or one rotor phase open) lead to additional reverse sequence stator currents at frequency $f'_1 = f_1(1 - 2S)$ which, in small machines (with large R_s (in p.u.)), produce Georges' effect (torque) that is zero at $S = 1/2$ and thus the motor, during starting on load, may be "arrested" around 50% of rated speed. This phenomenon is typical to asynchronous starting of synchronous motor with the dc excitation circuit short-circuited (see Chapter 6). Broken bars have to be diagnosed early and due measures of rotor replacement be taken as soon as feasible.
- For low-power and residential applications (refrigerator compressors, heater's circulating pumps), capacitor split-phase IMs are used. They have a main and a starting (or permanent) orthogonally placed auxiliary winding. The mmf of such a 2-phase winding has a positive $\oplus$ and a negative $\ominus$ sequence. Destroying the negative sequence stator current component (symmetric operation) is met at a single slip S for

$$a = \frac{X_{m+}}{R_{m+}} = \tan \varphi_+; \quad a = \frac{W_a}{W_m}; \quad X_C = \frac{1}{\omega_1 C} = Z_+ a \sqrt{a^2 + 1}$$

where

W_a is the auxiliary winding turns

W_m is the main winding turns

C is the capacitor in series with the auxiliary winding

X_{m+} , R_{m+} are the total $\oplus$ sequence impedance components as seen in the main winding

- To design a good capacitor IM, a hard-to-get compromise between starting and running performance and motor total costs has to be worked out.
- Linear induction motors may be obtained by cutting and unrolling a rotary IM.
- LIMs have a longitudinally (along motion direction) open magnetic circuit. Consequently, the traveling mmf of the primary at linear speed $U_s = 2\pi f_1$ induces at primary entry and exit, at small slip values $S = 1 - U/U_s$, for a small number of poles $2p_1$ ($2p_1 + 1$), end effect secondary currents which reduce the thrust and efficiency

and decrease the power factor. These dynamic end effects may be reduced to reasonable proportions if the LIM is designed at the optimum goodness factor, $G_e(2p_1) = X_m/R'_r$ (Figure 5.49), where X_m is the magnetizing reactance and R'_r is the secondary resistance.

- The thrust of LIM suits well for urban and suburban transportation or in industrial short travel transport with wheels or with magnetic, controlled suspension (MAGLEV).
- Induction motor full-load testing shows low-energy consumption by using bidirectional PWM converter motor loads/drivers for regenerating braking or for virtual load (of the free-shaft IM). For a full account of IM industrial testing, see IEEE-112B Standard, Ref. [4, Chapter 22] and Ref. [15, Chapter 7].
- PWM converters provide variable frequency and voltage that recently transformed the IM from the workhorse into the racehorse of industry.

5.28 Proposed Problems

- 5.1 Build a two-layer $2p_1 = 2$, $q = 15$ turbogenerator 3-phase ac winding with chorded coils ($y/\tau = 36/45$), and calculate its distribution and chording factors $k_{q\nu}$ and $k_{y\nu}$ for $\nu = (-5), (+7), (-11), (+13), (-17), (+19), 6q \pm 1$.

Hint: See Section 5.3.1, Equation 5.20.

- 5.2 Build a 2-phase single-layer winding for a capacitor-split IM for $2p_1 = 2$, $N_s = 24$ with 16 slots for the main winding and 8 slots for the auxiliary winding.

Hint: See Figure 5.14 for clues.

- 5.3 For the winding in Example 5.1, calculate the ratio between the differential leakage, L_{sld} , and the main (fundamental) inductance, L_{1m} , and discuss the result.

Hint: See Equations 5.41 and 5.43.

- 5.4 A 3-phase cage rotor induction motor of $P_n = 1.5$ kW, $V_{nl} = 220$ V (star connection), $f_1 = 60$ Hz, $2p_1 = 4$ has a rated efficiency $\eta_n = 0.85$ and a power factor $\cos\varphi_n = 0.85$. The iron, stray, and mechanical losses are $p_{iron} = p_s = p_{mec} = 1.0\%$ of P_n and $p_{cosn} = 1.2p_{corn}$. Calculate:

- The rated phase current I_n .
- The no-load current $I_0 = I_n \sin \varphi_n$, and the rated rotor current $I'_{rn} = \sqrt{I_n^2 - I_0^2}$.

- c. The total losses, $\sum p_n$, and the stator and rotor winding losses, $p_{\cos n}$, $p_{\cos n}$, R_s , R_r , and R_{sc} .
- d. The electromagnetic power $P_{elm} = P_1 - p_{\cos n} - p_{iron} - p_s$ and the rated slip S_n .
- e. The rated electromagnetic torque, T_{en} , and the shaft torque, T_{shaft} .
- f. If the peak torque is $T_{ek} = 2.2T_{en}$, calculate approximately the short-circuit reactance, X'_{sc} , of the machine, the critical slip, S_k ($C_1 = 1.05$).
- g. If there is no skin effect, calculate the starting current, I_{start} , and torque, T_{estart} .
- h. Calculate the stator current, I_s , the delivered electric power (as generator), P_1 , and the absorbed reactive power, Q_s , for $S = -0.03$ and determine the rotor speed (in rps) required for this, at a power grid.

Hints: Check Example 5.5 and Equations 5.108 and 5.109.

- 5.5 For a cage rotor 3-phase induction machine with $N_s = 36$ slots, $N_r = 16$ slots and $2p_1 = 4$, $f_1 = 60$ Hz. Calculate the speeds (in rps) where teeth harmonics ($2q_{1,2}m \pm 1$) and mmf space harmonics ($km \pm 1$) produce synchronous parasitic torques.

Hints: Check Equations 5.115 through 5.119.

- 5.6 A 3-phase IM with $V_{nl} = 220$ V (star connection), $f_1 = 60$ Hz, $2p_1 = 4$, $R_s = R'_r = 1\Omega$, $X_{sl} = X'_{rl} = 2.5\Omega$, $X_m = 75\Omega$, $R_{iron} = \infty$ (no iron losses) remains in two-phases (B and C) because phase A gets disconnected while working at slip $S_n = 0.04$.

Calculate:

- a. The stator current, I_s , the magnetization current, I_0 , the rotor current, I'_r at $S = 0.03$ (from the equivalent circuit) with 3-phase balanced steady state. Also electromagnetic torque is needed, and so is the direct impedance, Z_+ .
- b. With phase A open, calculate for the same slip $S = 0.03$, the inverse impedance, Z_- , the stator current, and electromagnetic torque.
- c. The starting ($S = 1$) current for three phases closed and with the case of phase A open. Will the motor start with phase A open?

Hints: See Section 5.21.

- 5.7 Resolve Example 5.6, but for $S = 0$ ($n = n_1 = f_1/p_1$). Discuss the results.
- 5.8 A flat 3-phase linear induction motor with an aluminum sheet on an ideal laminated core has the following data:

- Stack length: $2a = 0.20$ m
- Mechanical airgap: $g = 0.01$ m

- Aluminum plate thickness: $d_{AL} = 6 \text{ mm}$
- Pole pitch: $\tau = 0.25 \text{ m}$
- Number of poles: $2p_1 = 8$ (single-layer winding)
- The aluminum plate width: $2e = 0.36 \text{ m}$
- Rated frequency: $f_n = 50 \text{ Hz}$
- $\rho_{Al} = 3.5 \times 10^{-8} \Omega \text{ m}$

Calculate:

- a. The ideal synchronous speed $U_s = 2\tau f_n$.
- b. The goodness factor G_e , after calculating the edge factor k_T .
- c. The primary mmf amplitude, F_{1m} , for which the conventional primary airgap flux density, B_{gc} , is 0.6 T at $S = 0.1$.
- d. The current loading amplitude A_m , the end effect coefficients γ_1 and γ_2 .
- e. The end effect thrust, F_{xend} , the conventional thrust, F_{xc} and the total thrust for $S = 0.1, 0.05, 0.01$ for A_m of point d above. Discuss the results.

Hints: Check Section 5.24, Equations 5.158 through 5.170.

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Synchronous Machines: Steady State

6.1 Introduction: Applications and Topologies

As shown in Chapter 3, synchronous machines (SMs) are characterized by ac multiphase winding currents in the stator, and dc (or PM) excitation or magnetically salient (reluctance) passive rotors. These are basically traveling field machines in which both stator and rotor fields are rotating “synchronously” at the rotor electric speed, ω_r :

$$\omega_1 = \omega_r = 2\pi p_1 n_1; \quad n_1 = \frac{f_1}{p_1} \quad (6.1)$$

where

ω_1 is the stator frequency

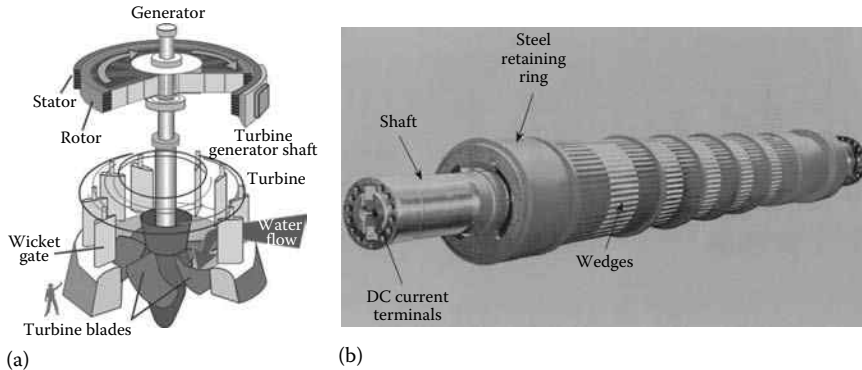
p_1 denotes pole pairs

There are also unipolar rotor-position-triggered or ramped-frequency stator-current pulse multiphase machines with reluctance (and eventually with PMs also) rotors that have a stepping stator field whose average speed is equal to the rotor speed. These are called stepper machines when the current pulses are initiated at a ramping reference frequency independent of the rotor position, or are called switched reluctance machines when the current pulses in each phase (one or two at a time) are rotor-position-triggered.

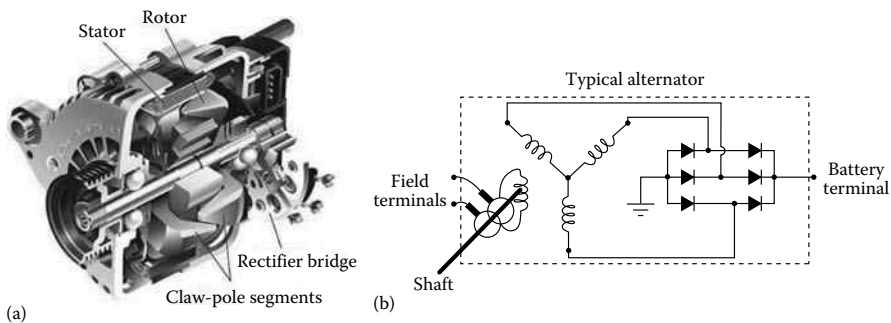
SMs require stator frequency control for controlling speed, which is carried out by frequency changers (pulse width-modulated [PWM] inverter/rectifier).

SMs have widespread applications. Some examples are as follows:

Power systems electric power plants use synchronous generators (SGs) with salient rotor poles (hydrogenerators, $2p_1 > 4$, Figure 6.1a) or nonsalient rotor poles (turbogenerators, $2p_1 = 2, 4$, Figure 6.1b) with dc rotor excitation, to power up to 770 MVA (hydrogenerators) and 1700 MVA (turbogenerators) [1]. Power electronics is used only to supply, through brushes and copper slip rings, energy to the excitation heteropolar excitation on the dc rotor.

**FIGURE 6.1**

Large SGs for power systems. (a) Rotor cross section of a hydrogenerator ($S_n \leq 770$ [MW], $f = 50(60)$ [Hz], $V_{nl} \leq 24$ [kV]). (b) Rotor cross section of a turbogenerator ($S_n \leq 1500$ [MW], $f = 50(60)$ [Hz], $V_{nl} \leq 28$ [kV]).

**FIGURE 6.2**

Automotive alternator with diode rectifier dc output with battery backup ($S_n = 0.5 - 2.5$ [kW], $V_{dc} = 14, 28, 42$ [V]).

Automobile alternators with single ring-shaped coil heteropolar dc excitation claw-pole rotor and with ac stator and diode rectifier dc output to the on-board battery (Figure 6.2).

Permanent magnet synchronous generators (PMSGs), which have been introduced recently for full bidirectional converter control to the power grid for wind energy conversion (up to 3 MVA at 16 rpm, transmission-less driving, Figure 6.3).

Automotive permanent magnet synchronous small power actuators and starters/alternators in hybrid electric vehicles (HEV) (Figure 6.4) [2–11].

Hard disks or other information gadget drives (Figure 6.5).

Single-phase micro-PMSMs (permanent magnet synchronous motors) with parking PM and PWM inverter control for self-starting micropower motors

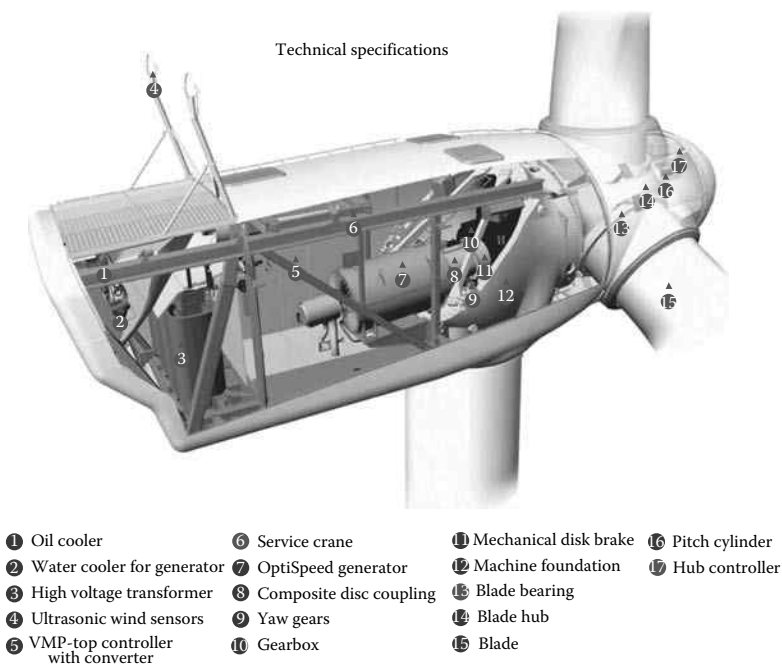


FIGURE 6.3
PMSG for renewable energy (wind generator)—3 MW at 16 rpm.



FIGURE 6.4
Automotive electric power steering assist PMSM; and $V_{dc} = 12\text{ V}$, $T_{en} = \text{up to } 1\text{ Nm}$, $n = 2000\text{ rpm}$, $n_{max} = 4000\text{ rpm}$, and $T_{en,max} = 0.3\text{ Nm}$ with PWM inverter control for variable speeds.

(for disk drives, mobile-phone ringers, or automotive climate-control air blowers.) (Figures 6.5 and 6.6).

Special configuration PM-assisted reluctance rotor SMs (PM-RSMs), which have been introduced recently as automotive starters for achieving large torque density (Figure 6.7a) or as transverse flux (TF) and flux reversal (FR) PMSMs for large-torque low-speed motors/generators with lower copper losses (Figure 6.7a and b) [12].

Linear SMs, as counterparts of rotary ones, which have been proposed for people movers (up to 400–550 km/h), for industrial (limited) excursion applications, and for linear oscillatory motions (Figure 6.8a through f) as in refrigerator compressors or mobile-phone ringers.

The applications and configurations presented so far illustrate the unusually large spectrum of power, speed, and topological diversity of synchronous machines.

The stator magnetic core that houses the ac 1- or 3-phase windings is made of silicon sheets of a thickness of 0.5 mm or less (up to 150 Hz fundamental frequency), and 0.1 mm (from 500 Hz to 3 kHz fundamental frequency), which are provided with uniform slots which are open (Figure 6.9a), semiopen (Figure 6.9b), or semiclosed (Figure 6.9c).

In the semiopen slot, each of the two preformed multistrand cable (bar) of the coil is inserted one by one in the slot.

The rotor core of the SMs is in general laminated for interior PM and reluctance rotor configurations, and so are the pole shoes of the salient pole rotor SGs or those of solid soft iron in large turbogenerators ($2p_1 = 2, 4$) and in the yoke of surface PM rotors.

To visualize the construction elements, we show in Figure 6.10 a dissection into a PMSM that identifies the additional elements, such as the shaft and the frame.



FIGURE 6.5

Axial airgap PM synchronous (dc brushless) hard disk motor and drive (embedded power electronics control).

6.2 Stator (Armature) Windings for SMs

There are two main types of armature (full power) windings:

- Distributed ac windings—(described in Chapter 5), used for all power plant generators, autonomous medium-power generators, large SMs, and many PMSMs with sinusoidal current control.
- Nonoverlapping (tooth-wound or circular) coil windings—used in PMSMs (from hard disk to industrial servodrives and large-torque,

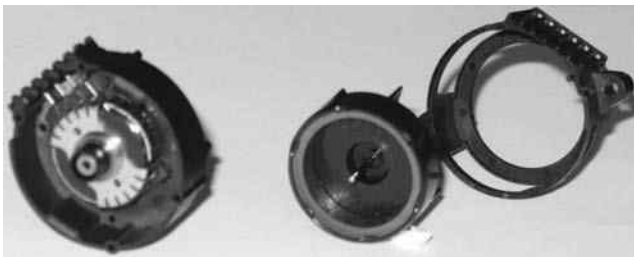
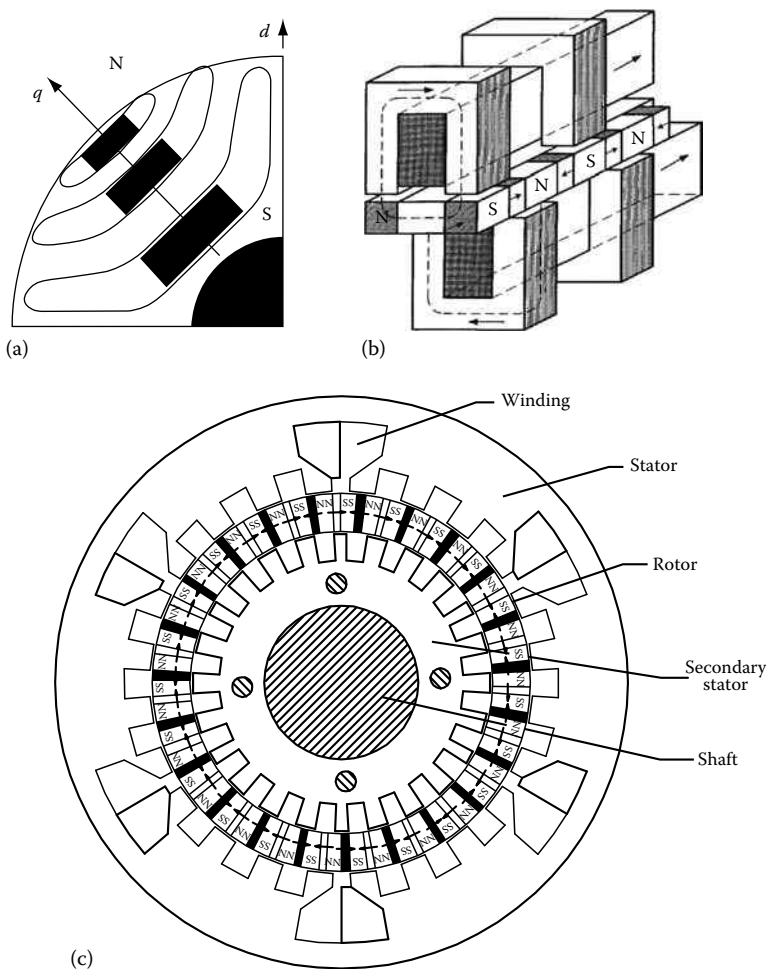


FIGURE 6.6

Automobile climate hot-air-blow micro-PMSM : single phase, with parking PM and PWM inverter control.

**FIGURE 6.7**

Special configuration PMSMs: (a) PM-RSM, (b) TF, and (c) flux reversal machine (FRM).

low-speed applications) and, for saliency, in dual stepper and switched reluctance machines, with unipolar current pulses; circular coils embrace all poles on the periphery in the TF-PMSM to produce “torque magnification.”

6.2.1 Nonoverlapping (Concentrated) Coil SM Armature Windings

In PMSMs, to reduce the end-coil length (and losses), and to reduce the torque at zero current, tooth-wound coil windings with $N_s \neq 2p_1$ but

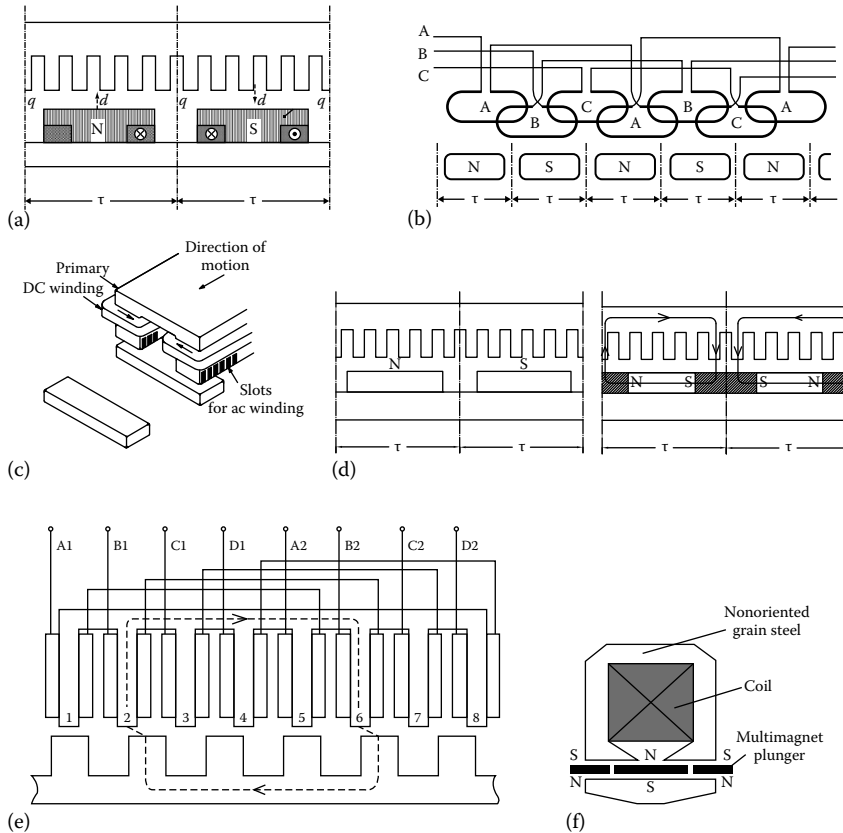


FIGURE 6.8

(a) Linear SMs with dc excitation on board and 3-phase ac-controlled winding on ground (Transrapid Maglev, in Germany). (b) Linear SMs with superconducting dc excitation on board and 3-phase ac-controlled winding on ground (LSM-Japanese Maglev, in Japan). (c) Linear homopolar SM with dc and ac 3-phase-controlled windings on board (Magnibus Maglev, in Romania). (d) Linear flat PMSM for industrial use. (e) Linear flat switched reluctance motor (SRM). (f) Linear tubular oscillatory, PM single-phase SM. (From Boldea, I. and Nasar, S.A., *Linear Electric Actuators and Generators*, Cambridge University Press, Cambridge, U.K., 1997; Boldea, I. and Nasar, S.A., *Linear Motion Electromagnetic Devices*, CRC Press, Taylor & Francis Group, New York, 2001.)

$N_r = 2p_1 + 2k$ (Figure 6.11a and b) are used. For all these PMSMs, $q < 0.5$. For the case in Figure 6.11b and c, $N_s = 6$ slots and $2p_1 = 4$ poles ($k = 1$). There is one coil per phase for the single-layer winding and two coils per phase for the two-layer winding.

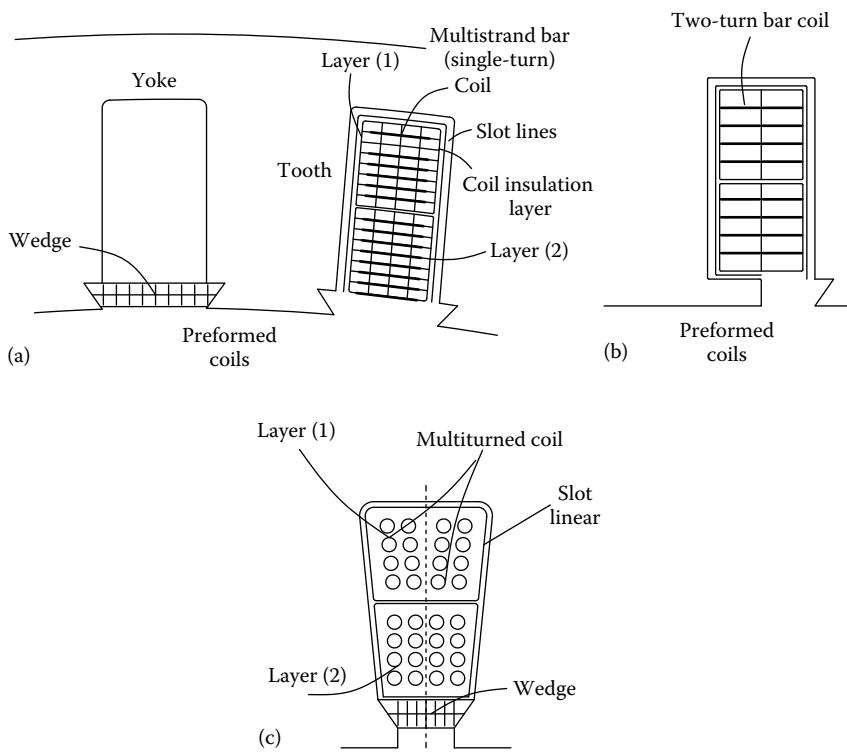


FIGURE 6.9 Typical slot shapes for armature (stator) windings: (a) open slots (for large-power SGs), (b) semiopen slots (for two-turn bar coils), and (c) semiclosed slots for low-torque SMs.

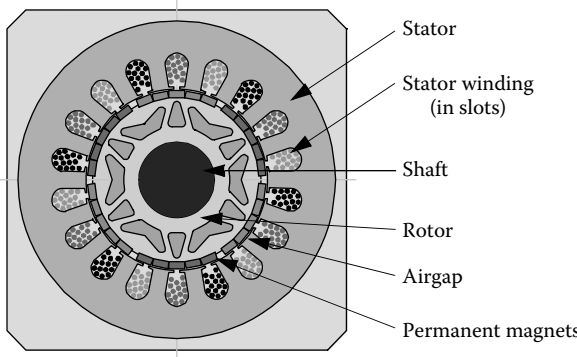


FIGURE 6.10 Open cross section in a PMSM.

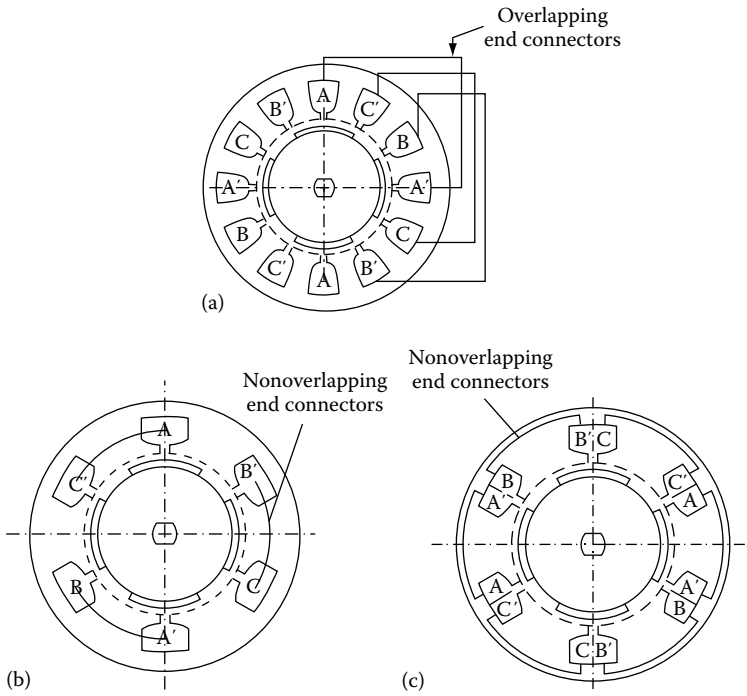


FIGURE 6.11 Four-pole PMSM windings: (a) distributed with $q = 1$ ($N_s = 12$), $2p_1 = 4$; (b) tooth-wound, single-layer ($N_s = 6$), $2p_1 = 4$; and (c) tooth-wound dual-layer ($N_s = 6$), $2p_1 = 4$.

There are many other combinations of N_s and $2p_1$ such as $3/2, 3/4, 6/4, 6/8, 9/8, 9/10, 9/12, 12/10, 12/14, 24/16, 24/22, \dots, 36/42$, and so on.

It has been demonstrated that the number of periods of the torque at zero current (cogging torque) is a multiple of the LCM of N_s and $2p_1$; the larger the LCM the smaller the cogging torque.

These machines operate as $2p_1$ pole machines. So the stator mmfs with $N_s \neq 2p_1$ have rather large winding factors for $2p_1$ periods (Tables 6.1 and 6.2), but they also have sub- and super-harmonics, which may be included in the differential leakage inductance (explained in Chapter 5). It is feasible to produce with such windings a sinusoidal emf (by motion) in the stator phases.

Basically, similar tooth coil windings are used for salient poles passive rotors ($N_s, 2p_1$; $N_s = 2p_1 + 2k$; $k = \pm 1, \pm 2$) of SRMs (Figure 6.12).

Stepper motors use large numbers of N_s and $2p_1$, and, thus, are good for refined step-by-step motion or slewing; based on reluctance torque (see Chapter 3), they use single-polarity current pulses in open-circuit sequences

TABLE 6.1
Winding Factors of Concentrated Windings

		2p-Poles									
N_s	2	4	6	8	10	12	14	16	18	20	24
slots	*	*	*	*	*	*	*	*	*	*	*
3	*	*	*	*	*	*	*	*	*	*	*
6	*	0.86	*	*	*	*	*	*	*	*	*
9	*	0.866	*	0.866	0.5	0.5	0.5	0.5	0.5	0.5	0.5
12	*	0.736	0.617	0.866	0.96	0.96	0.96	0.96	0.96	0.96	0.96
15	*	*	*	0.866	0.866	0.866	0.866	0.866	0.866	0.866	0.866
18	*	*	0.247	0.481	0.383	0.621	0.866	0.866	0.866	0.866	0.866
21	*	*	*	*	0.473	0.543	0.676	0.647	0.866	0.866	0.866
24	*	*	*	*	0.248	0.468	0.397	0.565	0.622	0.521	0.866

Source: Boldea, I., *Variable Speed Generators*, Chapter 10, CRC Press, Taylor & Francis Group, New York, 2005.

* One layer.

** Two layers.

TABLE 6.2
LCM of N_s and $2p_1$

N_s	$2p_1$							
	2	4	6	8	10	12	14	16
3	6	12	*	*	*	*	*	*
6	*	12	*	24	30	*	*	*
9	*	36	18	72	90	36	126	144
12	*	*	*	24	60	*	84	48
15	*	*	30	120	30	60	210	240
18	*	*	*	72	90	36	126	144
21	*	*	*	168	210	84	42	336
24	*	*	*	*	120	*	168	48

* One layer.

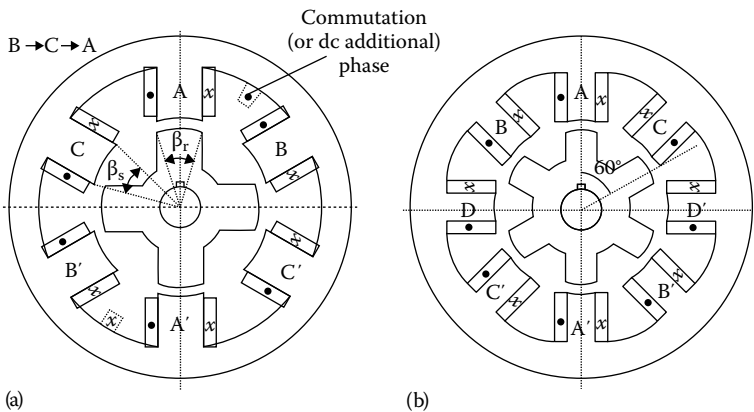
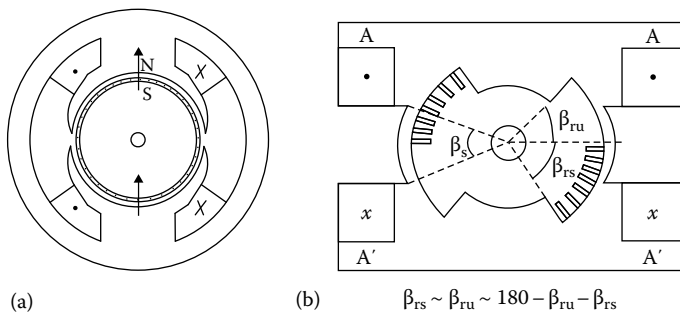


FIGURE 6.12
SRMs: (a) $N_s = 6$, $2p_1 = 4$; 3 phases; and (b) $N_s = 8$, $2p_1 = 6$; 4 phases.

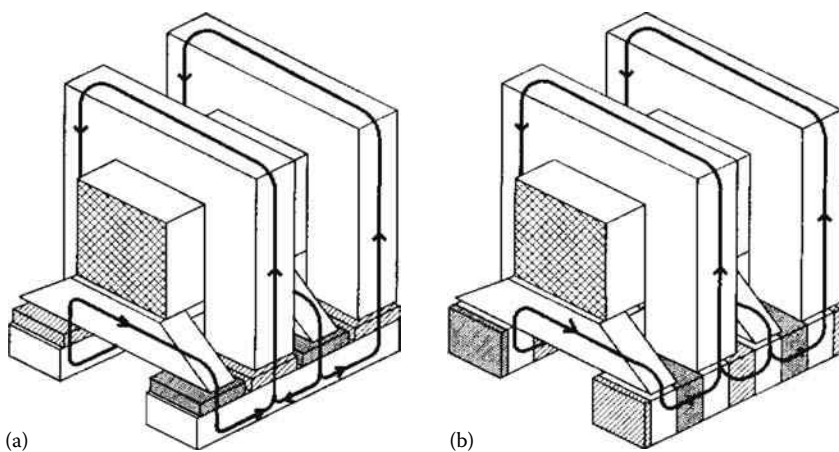
of phases. In contrast, SRMs have their phase current pulses triggered by the rotor position to preserve synchronism, with rotor motion, phase by phase.

For single-phase configurations, the numbers of stator slots, N_s , and rotor poles are the same (both for PMSMs and SRMs) Figure 6.13. Both need a self-starting positioning of the rotor by a parking PM or a stepped airgap. The cogging torque is large with $N_s = 2p_1$, but the rotor poles may be used to reduce total double-frequency torque pulsations, typical to ac single-phase machines.

The same numbers of stator salient poles, N_s , and rotor poles, $2p_1$, ($N_s = 2p_1$) are typical for the so-called TF-PMSM made of 2(3) or more single-phase units placed axially on the shaft with a $2\pi/3$ phase shift for the 3-phase case (Figure 6.14). TF-PMSMs enjoy the merit of acquiring the highest torque per watt of copper losses for given PM weight. The PM flux fringing (leakage) is one of their their main demerits, besides difficulty in manufacturing.

**FIGURE 6.13**

Two-pole, two-slot single-phase tooth-wound machines: (a) PMSM and (b) SRM.

**FIGURE 6.14**

TF-PMSMs: (a) with surface PM rotors and (b) with interior PM rotors (flux concentration).

All these tooth-wound machines, or even TF-PMs [12] or reluctance SMs do not have a cage on the rotor, so they cannot be connected directly to the ac power grid. They are fully dependent on full power electronics variable-frequency control. Even if they had a cage rotor, because $N_S \neq 2p_1$ ($N_S = 2p_1 + 2k; k = \pm 1, \pm 2$), the stator mmf has a very rich space harmonics content that would lead to large torque pulsations and additional (eddy current) losses in the PMs themselves. It goes without saying that axial airgap disk-shaped rotor configurations are also feasible.

We now return to the SM's distributed armature ac windings—with dc or PM rotor excitation.

6.3 SM Rotors: Airgap Flux Density Distribution and EMF

For radial airgap (cylindrical rotor/stator) SMS, there are six basic types of active rotors (Figure 6.15a through d) and variable reluctance (passive) rotors (Figure 6.15e and f).

We will now refer only to active rotors and armature mmfs.

We already showed in Chapter 5 that a 3-phase armature winding produces a traveling mmf/pole distribution:

$$F_1(x_1, t) = F_{1m} \cos \left(\omega_1 t - \frac{\pi}{\tau} x_s - \delta_i \right); \quad F_{1m} = \frac{3(W_s k_{\omega_{1s}}) I \sqrt{2}}{\pi p_1} \quad (6.2)$$

This wave travels at peripheral speed:

$$U_s = \frac{\tau}{\pi} \omega_1 = 2\tau f_1 \quad (6.3)$$

which corresponds to the already defined synchronous speed, $n_1 = f_1/p_1$. Now, as demonstrated in Chapter 3, to have a rippleless torque, the rotor-produced mmf should travel at the same speed along the stator. If the rotor has a dc heteropolar winding (Figure 6.15a) or PMs (Figure 6.15b), their mmf is fixed to the rotor. So, for $\omega_1 = \omega_r = 2\pi p_1 n_1$, the same number of poles has to be produced by the rotor's dc excitation or PMs.

With rotor poles, the rotor mmf distribution is single stepped (Figure 6.16a) for salient poles and multiple stepped (Figure 6.16b) for nonsalient poles. So the airgap maximum flux density, $B_{g_{Fm}}$, occurs above the rotor pole shoe:

$$B_{g_{Fm}} = \frac{\mu_0 W_F I_F}{K_{c8}(1 + K_{s0})}; \quad \text{for } |x| < \tau_p/2; \quad \frac{\tau_p}{\tau} = 0.65 - 0.75 \quad (6.4)$$

and is zero otherwise for salient poles (Figure 6.16a), and

$$B_{g_{Fm}} = \frac{\mu_0 (n_{cp}/2) W_{cF} I_F}{K_{c8}(1 + K_{s0})}; \quad \text{for } |x| < \tau_p/2; \quad \frac{\tau_p}{\tau} = 0.30 - 0.4 \quad (6.5)$$

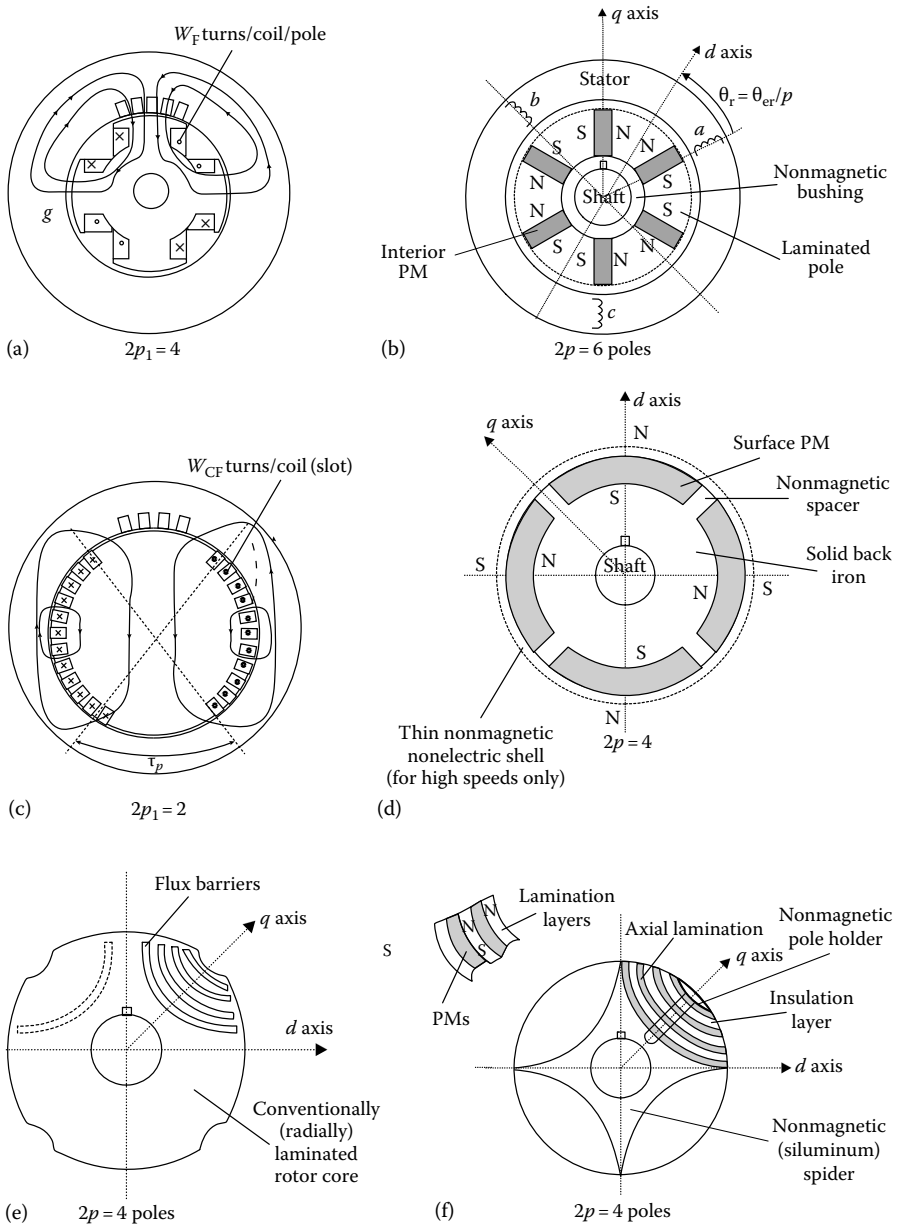
and is stepwise decreasing otherwise (Figure 6.16b).

We may decompose this distribution into a fundamental and harmonics:

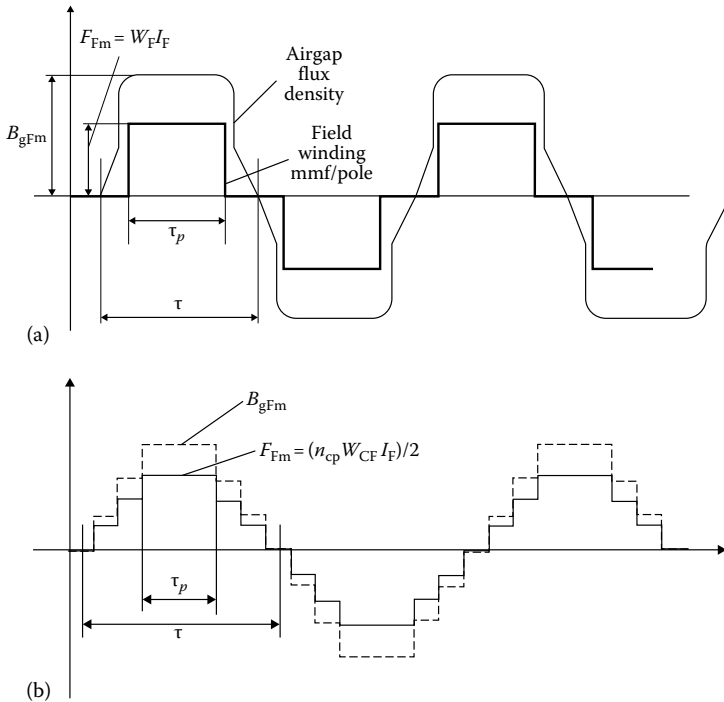
$$B_{g_{Fv}}(x_r) = K_{Fv} B_{g_{Fm}} \cos v \frac{\pi}{\tau} x_r; \quad v = 1, 3, 5 \quad (6.6)$$

with

$$K_{Fv} = \frac{4}{\pi} \sin v \frac{\tau_p}{\tau} \frac{\pi}{2} \quad (6.7)$$

**FIGURE 6.15**

(a,b) Salient and (c,d) nonsalient poles. (a,c) DC-excited and (b,d) PM rotors. (e) Flux barrier and (f) axially laminated rotors.

**FIGURE 6.16**

Excitation mmf and airgap flux density for (a) salient poles and (b) nonsalient poles.

for salient rotor poles and

$$K_{F_v} = \frac{8}{v^2 \pi^2} \frac{\cos v \frac{\tau_p}{\tau} \frac{\pi}{2}}{\left(1 - v \frac{\tau_p}{\tau}\right)}, \quad (6.8)$$

for nonsalient rotor poles.

It is evident that the harmonics content is lower for nonsalient poles than for salient poles.

To reduce the harmonics for the salient pole excitation, the airgap above rotor poles may be gradually increased from the pole center to the pole shoe ends:

$$g(x_r) = \frac{g}{\cos \frac{\pi}{\tau} x_r} \quad (6.9)$$

This measure will lead to a less harmonics content in the emf induced by motion by the dc excitation flux density rotor in the stator (armature) ac windings.

Note: The PM airgap flux density is very close to that of salient poles, but, for surface PM poles, the airgap g_m includes the PM radial thickness and

is constant along the rotor periphery; it is not so for interior PM poles. Trapezoidal, rather than sinusoidal, PM airgap flux density distribution might be intended in variable frequency-fed PMSMs (the so-called brushless dc PM machines) with $q = 1$ or $q < 1$ tooth-wound windings, for rectangular bipolar current control.

We will continue by retaining the fundamental of the dc excitor or the PM airgap flux density in rotor coordinates, x_r :

$$B_{g_{F_1}}(x_r) = K_{F_1} B_{g_{F_m}} \cos \frac{\pi}{\tau} x_r \quad (6.10)$$

If the speed ω_r is constant, the relationship between the rotor x_r and the stator coordinate x_s is

$$\frac{\pi}{\tau} x_r = \frac{\pi}{\tau} x_s - \omega_r t - \theta_0 \quad (6.11)$$

Substituting Equation 6.10 in Equation 6.11 we obtain ($\theta_0 = 0$):

$$B_{g_{F_1}}(x_s) = B_{g_{F_{m1}}} \cos \left(\frac{\pi}{\tau} x_s - \omega_r t \right) \quad (6.12)$$

So, seen from the point of view of the stator, the dc rotor-excited or PM-produced flux density looks like a traveling wave at rotor speed.

The emf induced by this field in a stator phase winding is

$$E_{A1}(t) = -\frac{d}{dt} W_s k_{w_1} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} l_{\text{stack}} \cdot B_{g_{F_1}}(x_s, t) dx \quad (6.13)$$

With Equation 6.13 we obtain

$$E_{A1}(t) = E_1 \sqrt{2} \cos \omega_r t \quad (6.14)$$

$$E_1 = \pi \sqrt{2} \left(\frac{\omega_r}{2\pi} \right) B_{g_{F_{m1}}} \cdot l_{\text{stack}} W_s k_{w_1} \cdot \frac{2\tau}{\pi} \quad (6.15)$$

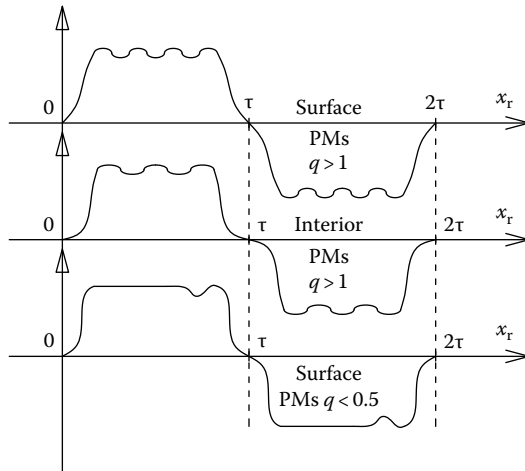
For three symmetric phases:

$$E_{A,B,C,1}(t) = E_1 \sqrt{2} \cos \left(\omega_r t - (i-1) \frac{2\pi}{3} \right); \quad i = 1, 2, 3 \quad (6.16)$$

6.3.1 PM Rotor Airgap Flux Density

The extreme PM rotor configurations are shown in Figure 6.15a and b. A few analytical methods to calculate the PM airgap flux density, including the stator slot openings, have been introduced [12], but ultimately the 2D or 3D finite element method (FEM) has to be used for new configurations, at least for the validation of results.

Results such as in Figure 6.17 are obtained for surface PM (nonsalient) poles, interior PM (salient) poles, and surface PM poles, but for tooth-wound coil windings, $q < 0.5$.

**FIGURE 6.17**

PM rotor-produced airgap flux density distribution.

6.4 Two-Reaction Principle via Generator Mode

Let us start with a dc rotor-excited 3-phase SG with no load that is driven at speed ω_r (Figure 6.18a). When a balanced ac load is connected to the stator, the emfs that occur in the stator have the angular frequency, ω_r . So balanced 3-phase stator currents at this frequency are expected.

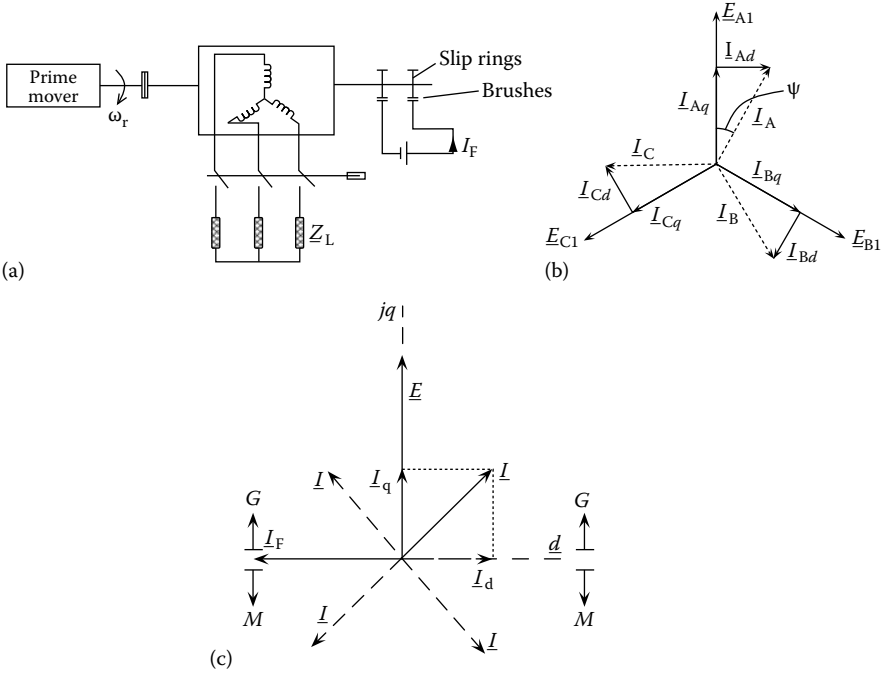
The phase shift angle between the 3-phase emfs and currents, Ψ , depends on the nature of the load (power factor) and on machine parameters (Figure 6.18b).

We may now decompose each phase current into two components, one in phase with the emfs and one at 90° to it: I_{Ad} , I_{Bd} , I_{Cd} , I_{Aq} , I_{Bq} , and I_{Cq} . With sinusoidal emf and current-time variations, phasors may be used for steady-state SGs. The d -axis phase components of phase currents I_{Ad} , I_{Bd} , and I_{Cd} produce a traveling mmf, $F_{ad}(x_s, t)$, aligned to the rotor poles (or to the dc excitation flux density) but opposite in sign if defined as

$$F_{ad}(x_s, t) = -F_{adm} \cos\left(\omega_1 t - \frac{\pi}{\tau} x\right) \quad (6.17)$$

Similarly, for q axis, phase components whose mmf is aligned with the interpole rotor axis is obtained as

$$F_{aq}(x_s, t) = F_{aqm} \cos\left(\omega_1 t - \frac{\pi}{\tau} x - \frac{\pi}{2}\right) \quad (6.18)$$

**FIGURE 6.18**

SG principle: (a) SG at load, (b) emfs and current phasors, and (c) Generator (G)–Motor (M) operation mode divide.

Comparing Equation 6.18 with Equation 6.2:

$$F_{adm} = \frac{3\sqrt{2}I_d W_s k_{w1s}}{\pi p_1}; \quad I_d = I \cos \delta_i = |I_{A,B,C,d}|; \quad \psi = \frac{\pi}{2} - \delta_i \quad (6.19)$$

$$F_{aqm} = \frac{3\sqrt{2}I_q W_s k_{w1s}}{\pi p_1}; \quad I_q = I \sin \delta_i = |I_{A,B,C,q}| \quad (6.20)$$

The emfs in Equations 6.14 through 6.16 may be expressed as phasors:

$$\underline{E}_{A,B,C} = -j\omega_r M_{F_a} \cdot \underline{I}_{F_{A,B,C}} \quad (6.21)$$

Equation 6.21 shows that the emfs are produced by the dc rotor excitation through motion. In other words, they are produced by a fictitious 3-phase ac winding flowed by symmetric fictitious currents $\underline{I}_{FA}$, $\underline{I}_{FB}$, and $\underline{I}_{FC}$ of frequency ω_r .

Comparing Equations 6.4 and 6.5 and having I_F as the RMS values of $I_{FA,B,C}$, with Equation 6.21, the mutual inductance, M_{Fa} , is

$$M_{Fa} = \mu_0 \frac{\sqrt{2}}{\pi} \frac{W_s W_F k_{w1s} \cdot \tau l_{stack}}{g k_c (1 + k_s)} \cdot k_{F1} \quad (6.22)$$

$$W_F = \frac{n_{cp}}{2} W_{CF} ; \quad \text{for nonsalient rotor poles} \quad (6.23)$$

The phasor diagram in Figure 6.18c concerns 1-phase phasors, the fictitious field current $\underline{I}_F$, which is ahead of emf $\underline{E}$ (Equation 6.21) by 90° , along axis d . Now, in the case of nonsalient rotors the active powers P_{elm} and reactive powers Q_{elm} are produced by the interaction between $\underline{E}_{A,B,C}$ and $\underline{I}_{A,B,C}$:

$$\underline{S}_n = P_{elm} + jQ_{elm} = 3 \operatorname{Re} (\underline{E} \cdot \underline{I}^*) + 3 \operatorname{Im} (\underline{E} \cdot \underline{I}^*) \quad (6.24)$$

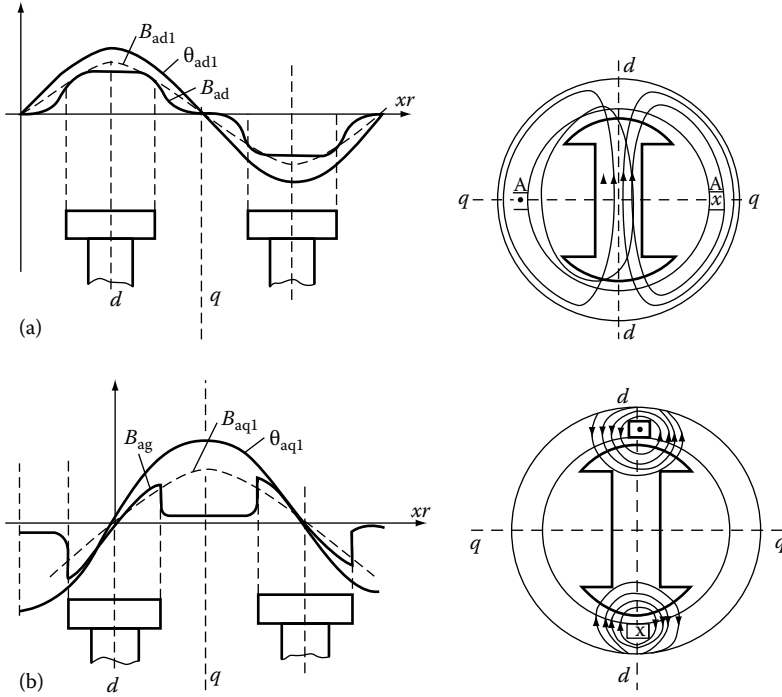
Motoring and generating modes are defined solely with respect to the active power (here, positive for the generator and negative for the motor mode). The reactive power, Q_{elm} , may either be positive or negative depending on the excitation (field) current, I_F , level. For an underexcited machine $Q_{elm} < 0$ and for an overexcited machine $Q_{elm} > 0$. So the SG has an extraordinary property of switching from the leading to the lagging power factor just by changing the field current, I_F . This is the key property for achieving voltage control in electric power systems under load source variations.

6.5 Armature Reaction and Magnetization Reactances, X_{dm} and X_{qm}

In the previous section, we have introduced the d and q rotor axes-aligned stator winding mmfs for steady state: $F_{ad}(x_s, t)$ and $F_{aq}(x_s, t)$. Consequently, their airgap flux density distributions, B_{ad} and B_{aq} , can be calculated simply if the equivalent airgap variation along the rotor periphery is defined mathematically. Figure 6.19a and b illustrates these configurations for axes d and q both, for the salient pole rotor configuration (the nonsalient pole configuration is a particular case of the latter and the PM rotors fall into the same category).

Extracting the fundamentals B_{ad1} and B_{aq1} from the nonsinusoidal airgap flux densities B_{ad} and B_{aq} leads to

$$B_{ad1} = \frac{2}{2\pi} \int_0^\pi B_{ad}(x_r) \sin \frac{\pi}{\tau} x_r dx_r \quad (6.25)$$

**FIGURE 6.19**

d and q axes armature-reaction flux densities and their flux lines.

$$B_{ad}(x_r) = 0; \quad 0 \leq x_r \leq \left(\frac{\tau - \tau_p}{2}\right) \text{ and } \left(\frac{\tau + \tau_p}{2}\right) \leq x_r \leq \tau$$

$$= \frac{\mu_0 F_{adm} \sin \frac{\pi}{\tau} x_r}{k_c g (1 + k_{sd})}; \quad \left(\frac{\tau - \tau_p}{2}\right) < x_r < \left(\frac{\tau + \tau_p}{2}\right) \quad (6.26)$$

$$B_{aq1} = \frac{2}{2\tau} \int_0^{\tau} B_{aq}(x_r) \sin \frac{\pi}{\tau} x_r dx_r \quad (6.27)$$

with

$$B_{aq}(x_r) = \begin{cases} \frac{\mu_0 F_{aqm} \sin \frac{\pi}{\tau} x_r}{k_c g (1 + k_{sq})}; & 0 \leq x_r < \frac{\tau_p}{2} \text{ and } \left(\tau - \frac{\tau_p}{2}\right) < x_r < \tau \\ \frac{\mu_0 F_{aqm} \sin \frac{\pi}{\tau} x_r}{k_c (6g)}; & \frac{\tau_p}{2} < x_r < \left(\tau - \frac{\tau_p}{2}\right) \end{cases} \quad (6.28)$$

The airgap between rotor poles for the q axis has been taken as $6g$ (other approximations are welcome).

Finally:

$$B_{ad1} = \frac{\mu_0 F_{adm} \cdot k_{d1}}{k_{cg} (1 + k_{sd})}; \quad k_{d1} = \frac{\tau_p}{\tau} + \frac{1}{\pi} \sin \frac{\tau_p}{\tau} \pi < 1 \quad (6.29)$$

$$B_{aq1} = \frac{\mu_0 F_{aqm} \cdot k_{q1}}{k_{cg} (1 + k_{sq})}; \quad k_{q1} = \frac{\tau_p}{\tau} - \frac{1}{\pi} \sin \frac{\tau_p}{\tau} \pi + \frac{2}{3\pi} \cos \frac{\tau_p}{\tau} \frac{\pi}{2} < 1 \quad (6.30)$$

For obtaining a uniform airgap, the condition is $k_{d1} = k_{q1} = 1$, as is for induction machines. In general, $k_{d1} = 0.8 - 0.92$ and $k_{q1} = 0.4 - 0.6$ for regular salient pole rotor dc-excited SMs.

Note: For interior PM rotors similar expressions may be derived, but with dedicated equivalent airgap variations and with $k_{d1} < k_{q1}$ if the PMs are placed along axis d .

The magnetization inductances along the two axes correspond to B_{ad1} and B_{aq1} and may be derived directly in relation to the already known magnetization inductance, L_{1m} , of IMs (with a uniform airgap), because only the coefficients k_{d1} and k_{q1} are new:

$$L_{1m} = \frac{6\mu_0 (W_s \cdot k_{w1s})^2 \tau L_e}{\pi^2 k_{cg} (1 + k_s)}; \quad B_{g1} = \frac{\mu_0 F_{1m}}{k_{cg} (1 + k_s)} \quad (6.31)$$

$$L_{dm} = L_{1m} \cdot k_{d1} = L_{1m} \cdot \frac{B_{ad1}}{B_{g1}}; \quad L_{qm} = L_{1m} \cdot k_{q1} = L_{1m} \cdot \frac{B_{aq1}}{B_{g1}} \quad (6.32)$$

Magnetic saturation is denoted by the coefficient k_s (k_{sd} and k_{sq} on axes d and q), but more involved treatments are used in industry. So now we state that the symmetric stator current components, I_d and I_q , self-induce ac emfs in the stator phase:

$$\underline{E}_{ad} = -j\omega_r L_{dm} I_d; \quad \underline{E}_{aq} = -j\omega_r L_{qm} I_q \quad (6.33)$$

In other words, the above terms are valid strictly for balanced stator voltages and currents under steady state.

As for the IMs, stator cyclic inductances (with all phases flowed by balanced currents), called synchronous inductances L_d and L_q , are

$$L_d = L_{dm} + L_{sl}; \quad L_q = L_{qm} + L_{sl} \quad (6.34)$$

$$\underline{E}_d = -j\omega_r L_d I_d; \quad \underline{E}_q = -j\omega_r L_q I_q \quad (6.35)$$

L_{sl} is the stator phase leakage inductance (same as for IMs).

6.6 Symmetric Steady-State Equations and Phasor Diagram

Based on the expression of phase emf $\underline{E}_{A,B,C}$ (Equation 6.21), produced by three fictitious ac stator currents, $\underline{I}_{F_{A,B,C}}$, and also on Equations 6.33 through 6.35, where the armature reaction is sensed in the stator phases as two self-induced emfs, $\underline{E}_{ad}$ and $\underline{E}_{aq}$, the phasor phase equation of SM—as a generator—under symmetric steady state—is

$$\begin{aligned} \underline{I}_s R_s + \underline{V}_s &= \underline{E} + \underline{E}_d + \underline{E}_q; \quad \underline{E} = -j\omega_r M_{Fa} \underline{I}_F \\ \underline{E}_d &= -jX_d \underline{I}_d; \quad \underline{E}_q = -jX_q \underline{I}_q; \quad X_d = \omega_r L_d; \quad X_q = \omega_r L_q \\ \underline{I}_s &= \underline{I}_d + j\underline{I}_q; \quad \underline{I}_d = \frac{\underline{I}_F}{I_F} \underline{I}_d; \quad \underline{I}_q = -j\frac{\underline{I}_F}{I_F} \underline{I}_q \end{aligned} \quad (6.36)$$

The last equation in 6.36 clarifies the fact that $\underline{I}_F$ and $\underline{I}_d$ phasors are along the same direction ($I_d \geq 0$) and that $\underline{I}_q$ is lagging for the generator mode and leading for the motor mode by 90° . There are three such equations (one for each phase), with all phasors shifted by $\pm 120^\circ$ (electrical). We may combine all emfs into one term:

$$\underline{I}_s R_s + \underline{V}_s = -j\omega_r \underline{\Psi}_{s0} = \underline{E}_{res}$$

where

$$\underline{\Psi}_{s0} = M_{Fa} \underline{I}_F + L_d \underline{I}_d + L_q \underline{I}_q \quad (6.37)$$

which is the resultant phase flux-linkage phasor.

Multiplying Equation 6.37 by $\underline{I}_s^*$ we may directly obtain the output active and reactive powers of the generator:

$$3\underline{V}_s \underline{I}_s^* = P_s + jQ_s \quad (6.38)$$

$$P_s = 3X_{Fa} \cdot I_E I_q - 3I_s^2 R_s + 3(X_d - X_q) I_d I_q \quad (6.39)$$

$$Q_s = -3X_{Fa} \cdot I_F I_d - 3(X_d I_d^2 + X_q I_q^2) \quad (6.40)$$

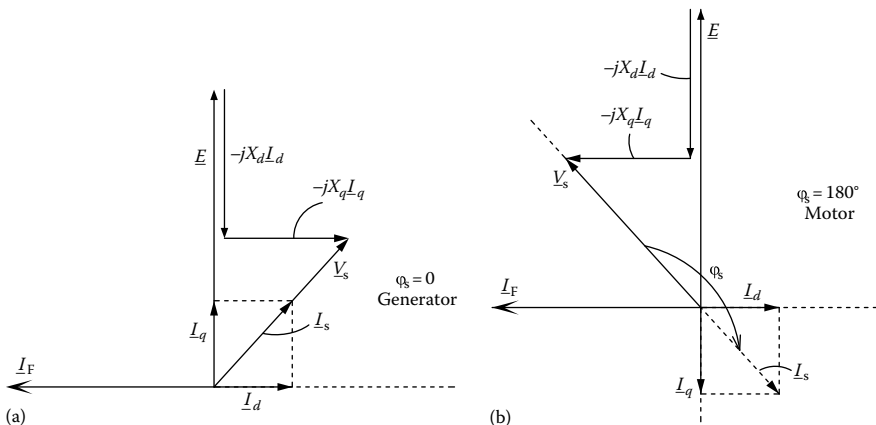
As we neglected the core losses so far, the first and last terms added in Equation 6.39 represent the electromagnetic (active) power:

$$P_{elm} = T_e \cdot \frac{\omega_r}{p_1} = 3\omega_r M_{Fa} I_F I_q + 3\omega_r (L_d - L_q) I_d I_q \quad (6.41)$$

So the electromagnetic torque, T_e , is

$$T_e = 3p_1 [M_{Fa} I_F + (L_d - L_q) I_d] I_q; \quad M_{Fa} I_F = \Psi_{PM_d} \quad (6.42)$$

With PMs placed along axis d (instead of electric dc excitation), the PM flux linkage (as seen from the stator phases), Ψ_{PM_d} , comes along, when $L_d < L_q$ as well.

**FIGURE 6.20**

Phasor diagrams of SMs at unity power factor for (a) generator and (b) motor.

The electromagnetic torque has two components:

- The interaction torque (between the excitation [or PM] field and the stator mmf)
- The reluctance torque (for $L_d \neq L_q$) due to stator-produced magnetic energy variation with the rotor position via rotor magnetic saliency

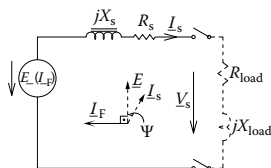
The leading power factor in Equation 6.40 means the positive reactive power Q_s and may be obtained only for negative, demagnetizing, I_d . Changing the sign of torque T_e implies changing the sign of I_q in Equation 6.42, which in fact means a shift of 180° in its phase, as the stator phase current component along the direction of dc-excited or PM-produced emf. For large SMs, we may neglect the stator resistance when analyzing the phasor diagram. For the motoring and generating modes, the relations in Equation 6.36 lead to simplified phasor diagrams as in Figure 6.20, for unity power factor.

Unity power factor may be maintained when load (I_q) varies only if the field current can be varied. For the nonsalient pole rotor SMs, $X_d = X_q = X_s$, and, thus, the voltage equation in Equation 6.36 becomes

$$I_s R_s + \underline{V}_s = \underline{E} - jX_s I_s \quad (6.43)$$

which leads to the equivalent circuit in Figure 6.21.

For nonsalient pole rotor ($X_d \neq X_q$) SMs, the equivalent circuit is more complicated to include the reluctance torque (power). For PMSMs, simply $I_E = \text{const.}$

**FIGURE 6.21**

Equivalent circuit of SM with nonsalient pole rotor.

6.7 Autonomous Synchronous Generators

ASGs have found many applications in automobiles, trucks, buses, diesel-electric locomotives, avionics (at 400 Hz), vessel cogeneration, wind generators, standby sources for telecom, hospitals, and remote areas, etc. The power per unit varies from a few mega volt amperes to 1 kW or less. A few characteristics of ASGs that define their operation are

- No-load saturation curve ($E(I_F)$, $I_s = 0$, $n_1 = \text{const.}$)
- Short-circuit curve ($I_{sc}(I_F)$, $V_s = 0$, $n_1 = \text{const.}$)
- External (load) curve ($V_s(I_s)$ for $n_1 = \text{const.}$, $\cos \varphi_s = \text{const.}$)

6.7.1 No-Load Saturation Curve/Lab 6.1

The no-load saturation (magnetization) curve may be obtained through computation by the same analytical method as is used for the brush-commutator machine, point by point, for given pole flux Φ_p . The FEM computation of this curve is also common. This curve may also be obtained through a dedicated test (which is standardized).

$$E_1(I_F) = \frac{\omega_r}{\sqrt{2}} \frac{2}{\pi} \tau B_{g_{\text{Fm}}}(I_F) \cdot k_{1F} \times l_{\text{stack}} \cdot W_s k_{w_{1s}} (V_{\text{RMS}}) \quad (6.44)$$

The experimental arrangement (Figure 6.22a) consists a prime mover (a variable speed drive with refined speed control) that drives the ASG at $n_1 = \frac{f_{1n}}{p_1}$. A variable dc-dc source can provide a variable field I_F within very large limits (1–100). The measured variables are the stator emf E_1 , the frequency f_1 , and the field current I_F . For a monotonous variation of I_F from zero up and

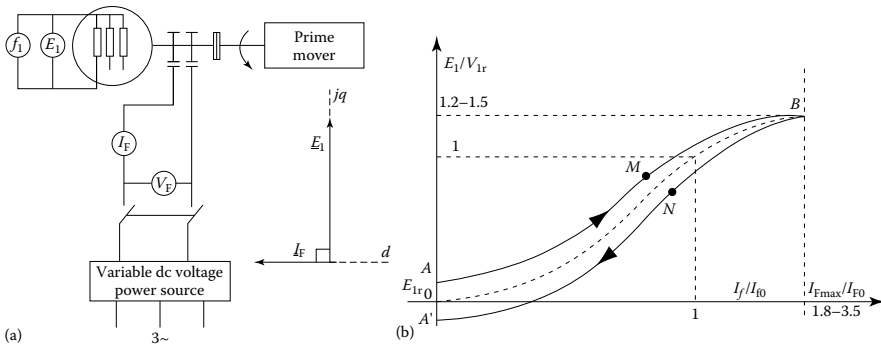


FIGURE 6.22

No-load-curve test: (a) test rig and (b) the characteristic curves.

down, even the cycle of hysteresis of $E_1(I_F)$ is obtained (Figure 6.22b). The test should be done for up to $E_{1\max}/U_n = (1.2-1.5)$, depending on the low-power factor of load envisaged for rated voltage and current. The average (dotted) curve in Figure 6.22b constitutes the no-load saturation curve. For maximum emf, $E_{1\max}$, it is essential for the maximum field current to be limited as the cooling system of the machine should handle the situation adequately.

6.7.2 Short-Circuit Curve: ($I_{sc}(I_F)$)/Lab 6.2

The short-circuit curve is obtained experimentally with the SG already short-circuited at stator terminals and driven at rated speed, $n_1 = f_{1n}/p_1$, by a refined speed-controlled prime mover (Figure 6.23a). The curve is linear (Figure 6.23b) due to a low amount of magnetic saturation in the SG on account of the “purely” demagnetized (I_d only) content of the stator current, I_{sc} , if R_s is neglected (for no torque: $I_q = 0$) (Figure 6.23c and d). From Equation 6.43 with $I_{sc} = I_d$, $V_s = 0$:

$$\underline{E}_1(I_F) = jX_{dmsat}I_{3sc} \quad (6.45)$$

The resultant emf produced by the main (airgap) field in the machine is given by

$$\underline{E}_{1res}(I_F) = \underline{E}_1(I_F) - jX_{dmunsat}I_{3sc} = -jX_{sl}I_{sc3} \quad (6.46)$$

E_{1res} refers to a low value of E_1 in the no-load curve (Figure 6.23e), which justifies nonsaturation claims for the magnetic circuit. An equivalent I_{F0} field current may be defined for it:

$$I_{F0} = I_F - I_{3sc} \cdot \frac{X_{dm}}{X_{Fa}} \quad (6.47)$$

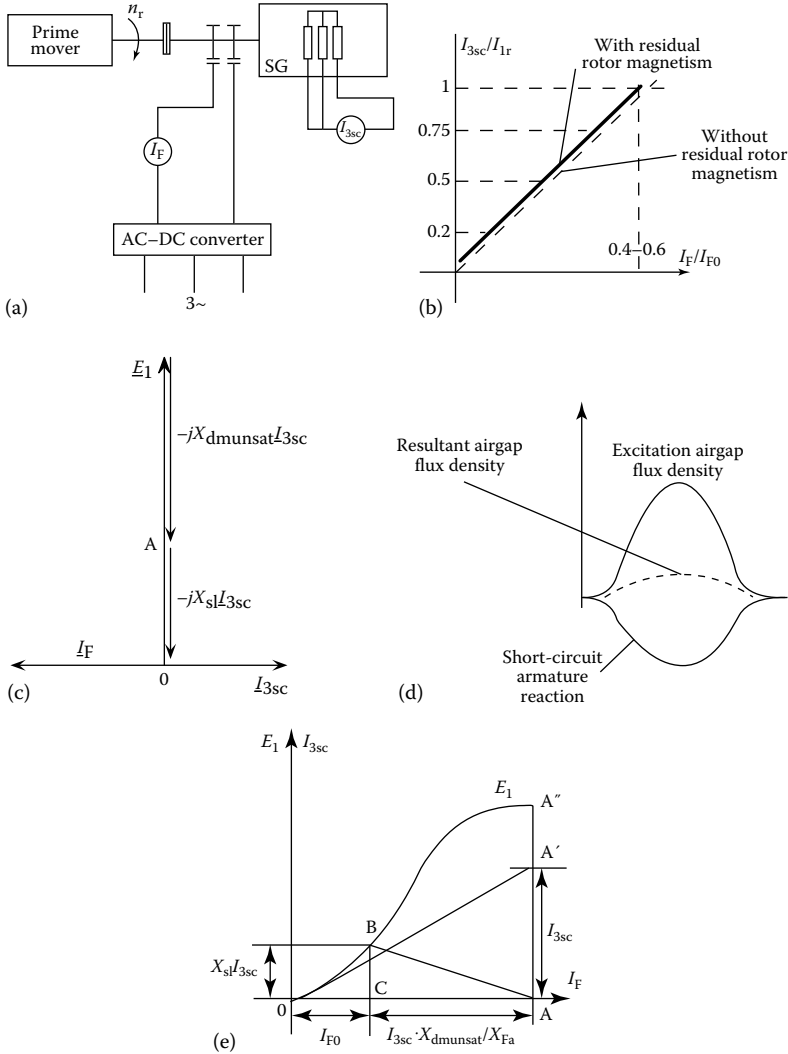
This is how the short-circuit triangle ABC (Figure 6.23e) was born.

While X_d in Equation 6.46 corresponds to I_{F0} , the saturated value corresponds to I_F , and is obtained from

$$X_{dsat} = \frac{E_1(I_F)}{I_{3sc}(I_F)} = \frac{\overline{AA}''}{\overline{AA}'} \quad (6.48)$$

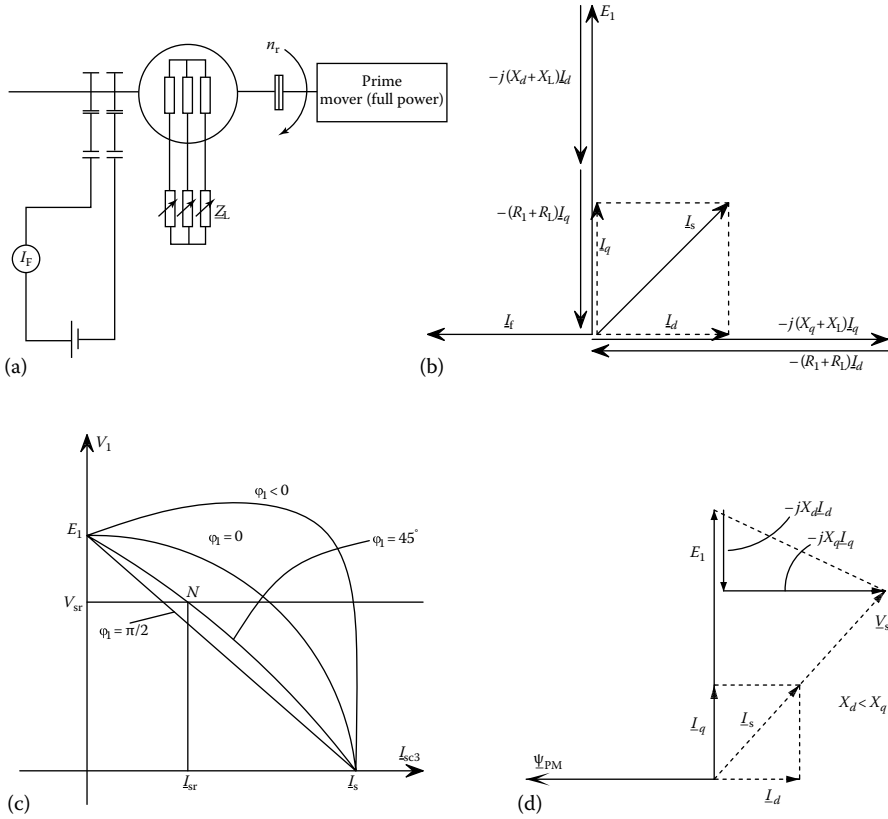
6.7.3 Load Curve: $V_s(I_s)$ /Lab 6.3

The load curve refers to terminal line (or phase) voltage, V_s , versus line current, I_s , for given speed, field current, and load power factor. For practical applications, the speed is kept constant through the speed controller (governor) of the prime mover, while the voltage is controlled to remain

**FIGURE 6.23**

Short-circuit curve: (a) test rig, (b) the curve, (c) phasor diagram, (d) flux density, and (e) short-circuit triangle.

within given limits of load variation through field current control. The load curves should only make sure that the design (tested) ASG will be capable to, say, provide the rated output voltage for the rated load at the lowest-settled lagging power factor level. For small- and medium-power SGs, a direct $R_L L_L$ ac load may be applied (as in Figure 6.24). Then the voltage equation


FIGURE 6.24

$V_s(I_s)$ load curves: (a) test rig, (b) phasor diagram, (c) the curves, and (d) PM-SG with $X_d < X_q$ and $E_1 = (V_s)_{I_n}$.

(in Equation 6.36) is completed with the load equation:

$$\underline{V}_s = R_L I_s + j\omega_r L_L I_s; \quad \cos \phi_L = \frac{R_L}{\sqrt{R_L^2 + \omega_r^2 L_L^2}} = \text{const.} \quad (6.49)$$

to obtain

$$\begin{aligned} E_1(I_F) &= I_q(R_s + R_L) + (X_d + X_L)(-I_d); & I_d \geq 0 \\ 0 &= (R_s + R_L)(-I_d) - (X_q + X_L)I_q; & I_q > 0 \end{aligned} \quad (6.50)$$

Assigning values to I_F , $\cos \phi_L$, and then to $Z_L = R_L / \cos \phi_L$, the values of I_d and I_q may be calculated from Equation 6.50. Then the active and reactive powers, P_s and Q_s , can be calculated, together with the terminal voltage:

$$V_s = \frac{R_L}{\cos \phi_L} I_s; \quad I_s = \sqrt{I_d^2 + I_q^2} \quad (6.51)$$

The no-load curve, $E_1(I_F)$, is used when I_F varies. It is also true that X_d (and even X_q) varies with magnetic saturation but this aspect is beyond our aim here. Load curves such as in Figure 6.24c are obtained.

The voltage regulation ΔV_s pointed out for the transformer or autonomous induction generator, is substantial for ASGs, because X_d and X_q are large:

$$\Delta V_s = \frac{E_1 - V_s}{E_1} = \frac{\text{no-load voltage} - \text{load voltage}}{\text{no-load voltage}} \quad (6.52)$$

If, for $\cos \varphi_{sc} = 0.707$ lagging minimum power factor, the rated voltage is still desired at the rated load current, it follows clearly from Figure 6.24 that

$$\frac{X_{d \text{ sat}}}{Z_n} = x_{d \text{ sat}} < 1; \quad Z_n = V_n / I_n \quad (6.53)$$

where x_d is the d -axis reactance in pu values. For $x_{d \text{ sat}} < 1$, large airgap values are required. But this means that a larger dc excitation mmf per pole would be required, and, thus, the excitation Joule losses would also be large. For PMSGs, the option of controlled field current is lost, but for interior PM pole rotors ($X_d < X_q$) with inversed saliency, it is possible, for the resistive load, to obtain zero voltage regulation at the rated load by design. For some applications this is enough, provided speed is maintained constant by the prime mover. SGs on automobiles are likely to operate either with diode rectifiers or with forced controlled rectifier loads. For such situations, the above considerations are only a basis for further developments [13].

Example 6.1 Autonomous SG

Let us consider an autonomous SG with $S_n = 1$ MVA, $2p_1 = 4$, $x_d = 0.6$ (pu), $x_q = 0.4$ (pu), and $V_{nl} = 6$ kV (star connection). Calculate the emf E_1 (per phase) at the rated current to preserve the rated voltage, for resistive ($\cos \varphi_L = 1$), resistive-inductive ($\cos \varphi_L = 0.707$), and resistive-capacitive ($\cos \varphi_L = 0.707$) loads.

Solution:

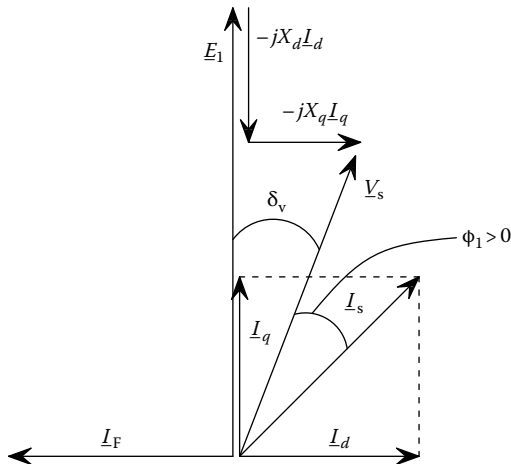
We have to go back to the general phasor diagram for nonunity power factor conditions and zero losses (Figure 6.25).

The rated current I_n is defined here for unity power factor (and zero losses):

$$I_n = \frac{S_n}{\sqrt{3}V_{nl}} = \frac{1 \times 10^6}{\sqrt{3} \times 6 \times 10^3} = 96.34 \text{ [A]} \quad (6.54)$$

From the rated current and $\cos \varphi_L = 1$, the resistive load R_L is

$$R_{\text{load}} = \frac{V_{nl} \cdot \cos \varphi_L}{\sqrt{3}I_n} = \frac{6000 \times 1}{\sqrt{3} \times 96.34} = 36 \text{ } [\Omega]; \quad Z_n = \frac{V_{nl} / \sqrt{3}}{I_n} = 36 \text{ } [\Omega] \quad (6.55)$$

**FIGURE 6.25**

Phasor diagram of SG for zero losses.

Then, from Equation 6.50, with $X_{\text{load}} = 0$, and Equation 6.54 we may calculate E_1 , I_d , and I_q :

$$\frac{(-I_d)}{I_q} = \frac{X_q}{R_{\text{load}}} = \frac{x_q}{R_{\text{load}}} \frac{V_{nl}/\sqrt{3}}{I_n} \quad (6.56)$$

But, from $I_n = \sqrt{I_d^2 + I_q^2}$; $I_n = I_q \sqrt{\left(\frac{I_d}{I_q}\right)^2 + 1}$

$$I_q = I_n \left/ \sqrt{\left(\frac{I_d}{I_q}\right)^2 + 1} \right. = \frac{96.34}{\sqrt{1 + 0.4^2}} = 89.45 \text{ [A]}$$

$$I_d = -0.4 \times I_q = -0.4 \times 89.45 = -35.78 \text{ [A]}$$

So, from Equation 6.50 the emf E_1 is

$$\begin{aligned} E_1 &= I_q \times R_L + X_d (-I_d) = 89.45 \times 36 + 0.6 \times 36 \times 35.78 \\ &= 3993 \text{ [V]} \end{aligned}$$

The rated phase voltage $V_{nph} = V_{nl}/\sqrt{3} = 6000/\sqrt{3} = 3468.2 \text{ [V]}$.

So the rated voltage regulation

$$(\Delta V)_{I_n, \cos \phi_L=1} = \frac{E_1 - V_{nph}}{E_1} = \frac{3993 - 3468.2}{3993} = 0.131 = 13.1 \%$$

For the other values of the load factor, the computation routine is the same as above and voltage regulation will be larger for the resistive-inductive load and negative for the resistive-capacitive load ($E_1 < V_{nph}$).

6.8 Synchronous Generators at Power Grid/Lab 6.4

Every SG, new or repaired, has to be connected to the power grid. The power grid is made of all SGs connected in parallel and the power lines, with step-up and step-down transformers, that deliver electric energy to various consumers.

Let us assume that the power grid has “infinite power” or zero internal (series) impedance, and thus its voltages, amplitudes, and phases do not change when an additional SG is connected. The connection of an SG to the power grid has to take place without large current (power) transients. To do so, the SG and power grid line voltages have to have the same sequences, amplitudes, and phases (frequencies also). This synchronization process is done today automatically through so-called digital synchronizers (Figure 6.26), which do coordinated voltage, frequency (speed), and phase control to minimize connection transients.

In a university lab, however, manual synchronization is performed, with two voltmeters and 3×2 light bulbs in series between the SG and the power grid. To adjust the SG voltages the field current, I_F , is modified, while frequency (phase)-refined adjustment is done until all light bulbs are dark. Then the power switch is closed. There will be some transients and the machine rotor oscillates a little and then settles down at synchronous speed. To deliver active power by the generator, the prime mover power is increased (hence water or fuel intake is increased). To deliver reactive power, the field current is increased.

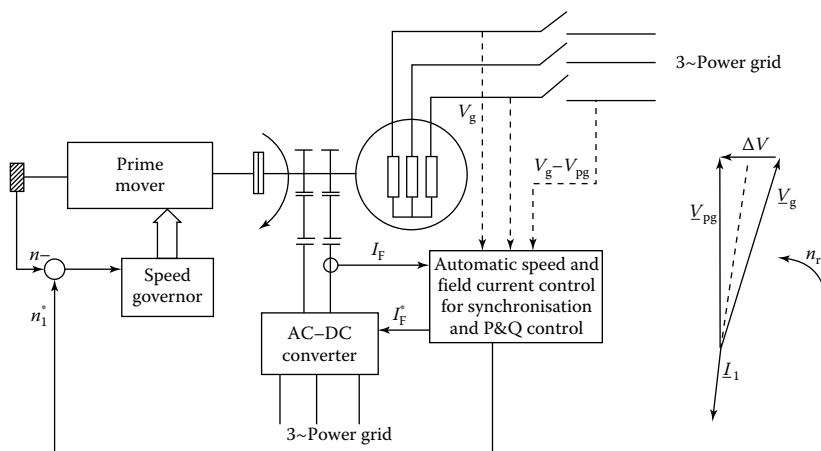


FIGURE 6.26

Connecting the SG to the power grid.

The operation at the power grid is characterized by three main curves :

- Active power/angle curve, $P_e(\delta_V)$
- V-shaped curves ($I_s(I_F), P_s = \text{const.}, V_s = \text{const.}, n = n_1 = \text{const.}$)
- Reactive power capability curves, $Q_s(P_s)$

6.8.1 Active Power/Angle Curves: $P_e(\delta_V)$

The voltage power angle, δ_V , is the phase shift angle between the emf, E_1 , and the phase voltage, V_s (see Figure 6.25). The δ_V concept (variable) can be used either for autonomous or for power grid operation, but the latter case is typical as it relates directly to the SG static and dynamic stabilities (still to be defined). The active and reactive powers have already been expressed in Equations 6.39 and 6.40 for a lossless SG. Here we add the I_d and I_q components expressions from Figure 6.25:

$$I_d = \frac{V_s \cos \delta_V - E_1}{X_d}; \quad I_q = \frac{V_s \sin \delta_V}{X_q} \quad (6.57)$$

From Equations 6.39 and 6.40, and Equation 6.57, the active (electromagnetic) and reactive, P_s and Q_s , powers are

$$P_s = \frac{3E_1 V_s \sin \delta_V}{X_d} + \frac{3}{2} V_s^2 \left(\frac{1}{X_q} - \frac{1}{X_d} \right) \sin 2\delta_V \quad (6.58)$$

$$Q_s = \frac{3E_1 V_s \cos \delta_V}{X_d} - 3V_s^2 \left(\frac{\cos^2 \delta_V}{X_d} + \frac{\sin^2 \delta_V}{X_q} \right) \quad (6.59)$$

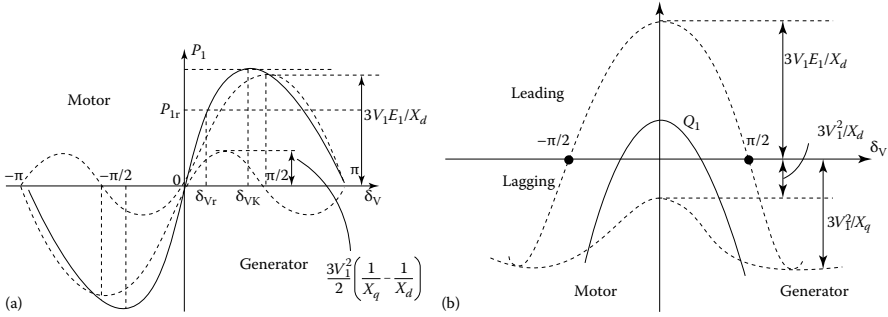
Note again that for PMSMs $E_1 = \omega_r \Psi_{PM}$ while for dc-excited SGs $E_1 = \omega_r M_{Fa} I_F$; $E_1 = 0$ for reluctance SMs.

The $P_s(\delta_V)$ and $Q_s(\delta_V)$ curves from Equations 6.58 and 6.59 are shown in Figure 6.27.

As the voltage equation's association of signs was taken for the generator mode, negative active power means the motor mode. Positive voltage power angle (from zero to $\oplus 180^\circ$) means generator operation and negative voltage power angle (from zero to $\ominus 180^\circ$) refers to motor operation. However, not all these zones provide stable operations.

A few remarks are in order :

- δ_V increases up to δ_{VK} , which corresponds to the maximum active power available (for given E_1 [field current] value).
- The reactive power (for constant E_1) changes from leading to lagging when δ_V increases either in motor or in generator modes.
- For the salient pole rotor, $\delta_{VK} < 90^\circ$.

**FIGURE 6.27**

(a) Active and (b) reactive powers, P_s and Q_s , vs. voltage power angle, δ_V .

- The rated (design) voltage power angle is chosen in the 22° – 30° range for nonsalient pole rotor SGs and 30° – 40° range for salient pole rotor SGs, because of the larger inertia in the latter.

6.8.2 V-Shaped Curves

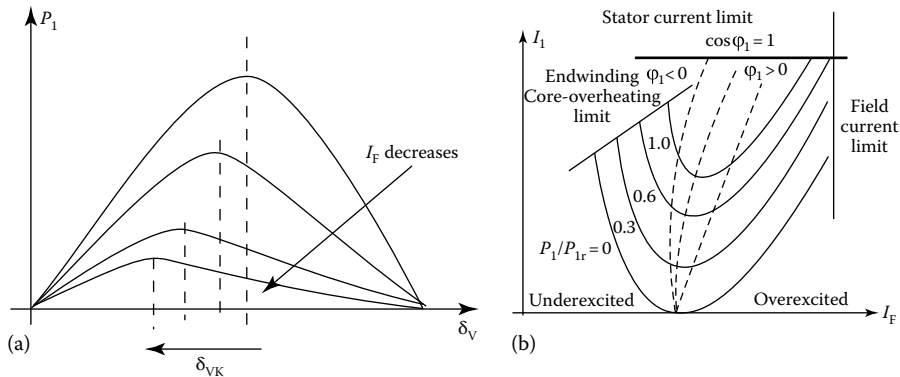
The V-shaped curves represent a family of I_s (I_F) curves drawn for constant terminal voltage (V_s), and speed ($\omega_r = \omega_1$), with active power P_s as a parameter. The computation of the V-shaped curves makes use of the P_s expression 6.58 and the no-load curve, E_1 (I_F), with known values of synchronous reactances, X_d and X_q . As the terminal voltage is kept constant, the total flux linkage, $\Psi_s \approx V_s / \omega_1$, is constant, and, thus, X_d and X_q do not vary much when I_F varies in steps, to build the V-shape curve.

The computation sequence is straightforward:

- Assign gradually decreasing values of I_F (from the maximum value allowed due to cooling reasons) and, for a given value of P_s , calculate the power angle, δ_V , from Equation 6.58 after “reading” E_1 from E_1 (I_F) curve, which is already known.
- Then, from Equation 6.57, with E_1 and δ_V known, I_d and I_q are calculated; finally the stator phase current is calculated using $I_s = \sqrt{I_d^2 + I_q^2}$.
- When decreasing I_F , at some point, $\delta_V = \delta_{VK}$; this is the last point on the V-shaped curves, to provide at least the static stability (Figure 6.28).

The minimum stator current (for given power and voltage) corresponds to unity power factor:

$$P_s = 3V_s I_s \cos \varphi_s \quad (6.60)$$

**FIGURE 6.28**

V-shaped curves: (a) $P_s (\delta_V)$ curves and (b) $I_s (I_F)$ curves.

With the machine underexcited (smaller I_F values) the power factor is lagging; it is leading for larger I_F values. Unity power factor corresponds to $Q_s = 0$ in Equation 6.59:

$$(E_1)_{Q_s=0} = V_s \left(\cos \delta_V + \frac{X_d \sin^2 \delta_V}{X_q \cos \delta_V} \right) \quad (6.61)$$

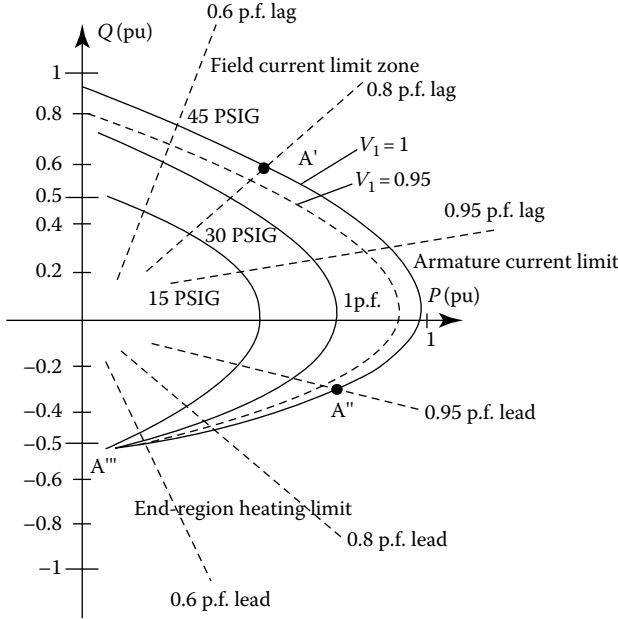
Maintaining unity power factor with increases in active power (and δ_V) corresponds to an increase in E_1 (or in I_F).

6.8.3 Reactive Power Capability Curves

The maximum limitation of I_F is due to thermal reasons. However, a machine's overall and hot spot temperatures depend on both currents: I_F and I_s . Also, I_s , I_F , and δ_V determine the stator flux, and thus the core losses. So, when the reactive power demand from an SG is increased, it implies an increase in the field current (I_F) and, thus, at some point, the stator current (I_s) and the active power have to be limited also.

The computation process for the V-shaped curves may be taken just one step further to calculate the reactive power from Equation 6.59, and then represent the $Q_s (P_s)$ curves (Figure 6.29).

For underexcited machines, $Q_s < 0$, but for this case, the d -axis reaction field of the stator (of I_d) is added to the d -axis field of excitation. In the end turns and end core zones, a large total ac magnetic field is produced, which creates additional, end-region, stator core losses that limit the maximum absorbed reactive power by the machine (Figure 6.29).

**FIGURE 6.29**

Reactive/active power limit curves for a hydrogen-cooled SG.

6.9 Basic Static- and Dynamic-Stability Concepts

We have inferred in the previous section that an SG can deliver active power up to the critical voltage power angle, $\delta_{VK} \leq \pm 90^\circ$, if its loading is going up or down very slowly.

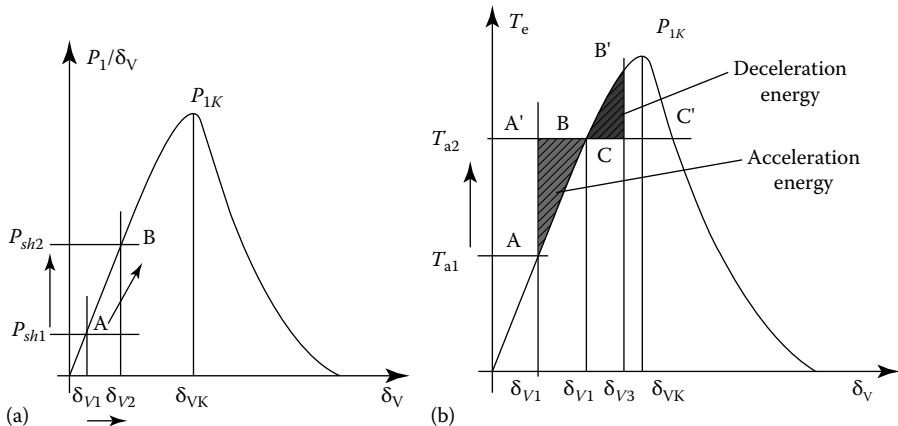
Static stability is the property of an SG to remain in synchronism in the presence of very slow load variations. It can be noticed that as long as the electric power, P_s , delivered by an SG increases with an increase in the prime mover output (mechanical power), the machine remains statically stable.

In other words, as long as $\partial P_s / \partial \delta_V > 0$ the machine is statically stable (Equation 6.58):

$$P_{ss} = \frac{\partial P_s}{\partial \delta_V} = 3E_1 V_s \cos \delta_V + 3V_s^2 \left(\frac{1}{X_q} - \frac{1}{X_d} \right) \cos 2\delta_V \quad (6.62)$$

P_{ss} is called the synchronizing power; as long as P_{ss} is positive the SM is statically stable ($P_{ss} = 0$ for δ_{VK} !). When the field current decreases (E_1 decreases), P_{ss} decreases, and thus δ_{VK} increases and reduces the statically stable region.

Dynamic stability is the property of an SM to remain in synchronism (with the power grid) for quick variations of the shaft (mechanical) power


FIGURE 6.30

Dynamic stability: (a) $P_s(\delta_V)$ and (b) the criterion of equal areas.

or of the electric power (load). In general, when the shaft inertia (of a prime mover and an electric generator) is large, the power angle (δ_V), speed (ω_r) transients for load variations are much slower than electric transients. So, for basic calculations, during sudden mechanical loads, the SG may still be considered at steady state with torque T_e :

$$T_e = \frac{P_s \times p_1}{\omega_r} = \frac{3p_1}{\omega_r} \left[\frac{E_1 V_s \sin \delta_V}{X_d} + \frac{V_s^2}{2} \left(\frac{1}{X_q} - \frac{1}{X_d} \right) \sin 2\delta_V \right] \quad (6.63)$$

Let us now consider the case of the nonsalient pole rotor SG to simplify T_e ($X_d = X_q$) (Figure 6.30).

Neglecting also the rotor cage torque during transients (which would otherwise be beneficial), the shaft motion equations during mechanical (shaft) load variation from P_{sh1} to P_{sh2} (Figure 6.30) are

$$\frac{J}{p_1} \frac{d\omega_r}{dt} = T_{\text{shaft}} - T_e; \quad \omega_r - \omega_{r0} = \frac{d\delta_V}{dt} \quad (6.64)$$

Multiplying Equation 6.64 by $d\delta_V/dt$, one obtains

$$d \left(\frac{J}{2p_1} \left(\frac{d\delta_V}{dt} \right)^2 \right) = (T_{\text{shaft}} - T_e) d\delta_V = \Delta T \cdot d\delta_V = dW \quad (6.65)$$

Equation 6.65 only illustrates that the kinetic energy variation of the mover (shaft) is translated into an acceleration area AA'B and a deceleration

area BB'C:

$$W_{AB} = \int_{\delta_{V1}}^{\delta_{V2}} (T_{\text{shaft}} - T_e) d\delta_V > 0 \quad (6.66)$$

$$W_{BB'} = \int_{\delta_{V2}}^{\delta_{V3}} (T_{\text{shaft}} - T_e) d\delta_V < 0 \quad (6.67)$$

Only when the two areas are equal to each other the SG will come back from point B' to B, after a few attenuated oscillations. Attenuation comes from the rotor-cage asynchronous torque neglected here. This is the so-called criterion of equal areas.

The maximum shaft power step up from zero that can be accepted (to preserve the return to synchronism), corresponds to the case when point A is in the origin and point B' is in C' (Figure 6.31a). On the other hand, a practical situation is related to the short-circuit clearing time from the on-load operation (Figure 6.31b).

During the short circuit, the electromagnetic torque is considered zero, so the machine tends to accelerate (δ_V increases). At point C (in Figure 6.31b), the short circuit is cleared, and, thus, the electromagnetic torque reoccurs and the machine decelerates.

Synchronism recovery takes place if

$$\text{Area (ABCD)} \leq \text{Area (CB'B'')} \quad (6.68)$$

Example 6.2 Short-Circuit Clearing Time, t_{sc}

Let us consider that a turbogenerator with $x_d = x_q = 1$ (pu), $S_n = 100$ MVA, $f_n = 50$ Hz, and $V_{nl} = 15$ kV operates at a power angle $\delta_V = 30^\circ$ and unity

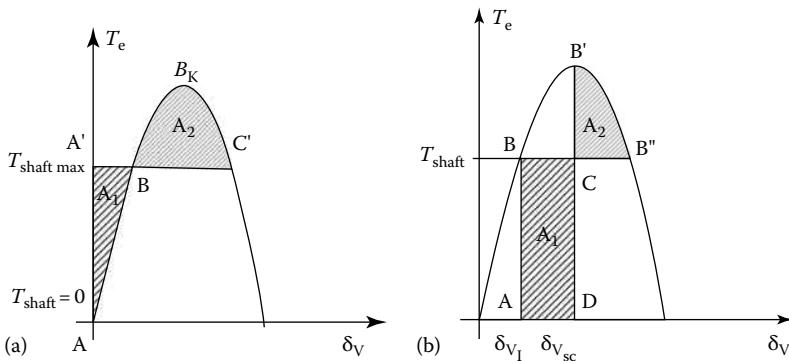


FIGURE 6.31

Dynamic-stability limits: (a) maximum allowable shaft torque step up from zero and (b) maximum short-circuit clearing time (angle $\delta_{V_{sc}} - \delta_{V1}$).

power factor. The inertia in seconds is $H = \frac{J}{S_n} \left(\frac{\omega_1}{p_1} \right)^2 = 4$ s. A sudden 3-phase short circuit occurs and its electric transients are so fast that they are neglected here. Calculate the short-circuit clearing angle δ_{sc} and time t_{sc} to save the synchronous operation after fault clearing.

Solution:

As shown above, criterion (6.68) is to be met to preserve synchronism after fault clearing.

We simply calculate and make the two areas in criterion (6.68) equal to each other.

$$\frac{T_{ek}}{2} (\delta_{V_{sc}} - \pi/6) = \int_{\delta_{V_{sc}}}^{\frac{5\pi}{6}} \left(T_{ek} \cdot \sin \delta_V - \frac{T_{ek}}{2} \right) d\delta_V \quad (6.69)$$

$$T_e = \frac{3p_1}{\omega_1} \frac{E_1 V_s \sin \delta_V}{X_d} = T_{ek} \sin \delta_V \quad (6.70)$$

for $\delta_V = \pi/6$; $T_{en} = T_{ek}/2$.

From Equation 6.69, $\delta_{V_{sc}} \approx 1.3955$ rad $\approx 80^\circ$.

Now we go back to the motion Equation 6.64:

$$\frac{J}{p_1} \frac{d^2 \delta_V}{dt^2} = T_{shaft\ n} = T_{e\ n} = S_n \frac{p_1}{\omega_1} \quad (6.71)$$

or

$$H \cdot \frac{1}{\omega_1} \frac{d^2 \delta_V}{dt^2} = 1 \quad (6.72)$$

and arrive at the solution:

$$\delta_V(t) = \pi \times 25 \times \frac{t^2}{2} + At + B \quad (6.73)$$

At $t = 0$, $\delta_V(0) = \pi/6$, and at $t = t_{sc}$, $\delta_V = \delta_{V_{sc}} = 1.3955$ rad; in addition, at $t = 0$, $\left(\frac{d\delta_V}{dt} \right)_{t=0} = 0$, because $(\omega_r)_{t=0} = \omega_{r_0} = \omega_1$. Finally, $A = 0$ and $B = \pi/6$. So the time t_{sc} for $\delta_{V_{sc}}$ is

$$\delta_{V_{sc}} = \frac{\pi \times 25 \times t_{sc}^2}{2} + \pi/6 = 1.3955 \text{ rad}$$

The short-circuit maximum clearing time to preserve synchronism is (from Equation 6.73)

$$t_{sc} = 0.14 \text{ s}$$

This subsecond value is not very far from industrial reality and shows how sensitive are SGs to transients because the rotor frequency is zero (dc or PM excitation in the rotor).

6.10 Unbalanced Load Steady State of SGs/Lab 6.5

SGs connected to the grid and in autonomous operation operate on unbalanced voltage power grids or unbalanced 3-phase current loads, respectively. In the general case, the ac stator 3-phase voltages and the currents are not balanced. To simplify the situation, let us consider the case of unbalanced currents:

$$I_{A,B,C}(t) = I_{A,B,C} \cos(\omega_1 t - \gamma_{A,B,C}); \quad I_A \neq I_B \neq I_C; \quad \gamma_A \neq \gamma_B \neq \gamma_C \neq 120^\circ \quad (6.74)$$

For constant (or absent) magnetic saturation, we may use the method of symmetrical components (Figure 6.32):

$$I_{A+} = \frac{1}{3} (I_A + a I_B + a^2 I_C); \quad a = e^{j\frac{2\pi}{3}} \quad (6.75)$$

$$I_{A-} = \frac{1}{3} (I_A + a^2 I_B + a I_C); \quad I_{A0} = \frac{1}{3} (I_A + I_B + I_C) \quad (6.76)$$

$$I_{B+} = a^2 I_{A+}; \quad I_{C+} = a I_{A+}; \quad I_{B-} = a I_{A-}; \quad I_{C-} = a^2 I_{A-} \quad (6.77)$$

For the direct component, the voltage Equation 6.35 is valid:

$$\underline{V}_{A+} = \underline{E}_{A+} - jX_d I_{dA+} - jX_q I_{qA+} \quad (6.78)$$

The dc excitation (or PM) rotor produces symmetric (balanced) emfs: $\underline{E}_{A+}$, $\underline{E}_{B+}$, and $\underline{E}_{C+}$. The inverse components of stator currents I_{A-} , I_{B-} , and I_{C-} produce an mmf that travels at opposite rotor speed ($-\omega_r$). Thus, the relative angular speed of inverse mmf is $2\omega_r$, and so is the frequency of its induced currents in the rotor cage and in the field circuit (if its supply source accepts ac currents). This is a kind of induction (asynchronous) machine behavior at slip S_- :

$$S_- = \frac{-\omega_r - \omega_r}{-\omega_r} = 2 \quad (6.79)$$

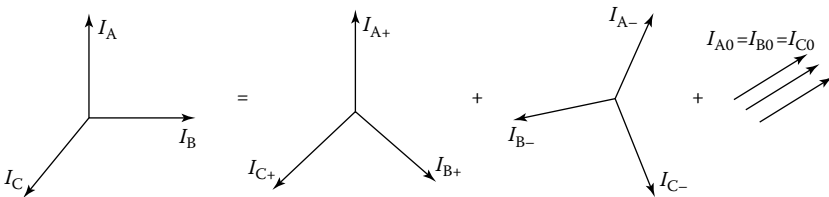


FIGURE 6.32

The symmetrical components of 3-phase ac currents.

Let us denote the equivalent negative sequence small impedance at $S = 2$, $\underline{Z}_-$. As $\underline{E}_{A-} = 0$ (symmetric emfs), the negative sequence equations are

$$\underline{I}_{A-}\underline{Z}_- + \underline{V}_{A-} = 0; \quad \underline{Z}_- = R_- + jX_- \quad (6.80)$$

The homopolar components $\underline{I}_{A0} = \underline{I}_{B0} = \underline{I}_{C0}$ produce a zero-traveling (fix) field because the 3-phase windings are spaced at 120° with each other. So they do not interact with the positive and negative sequence components. Their corresponding impedance is $Z_0 = R_0 + jX_0$:

$$X_0 \leq X_{sl}; \quad R_s < R_0 < R_s + R_{\text{irons}} \quad (6.81)$$

where

X_{sl} is the stator phase linkage reactance

R_{irons} is the stator core series equivalent resistance

In general, for a cage rotor dc-excited SM:

$$X_d > X_q > X_- > X_{sl} > X_0 \quad (6.82)$$

The stator phase equation for $\underline{I}_{A0}$ is similar to Equation 6.80:

$$jX_0 \underline{I}_{A0} + \underline{V}_{A0} = 0 \quad (6.83)$$

The total voltage of phase A is

$$\underline{V}_A = \underline{V}_{A+} + \underline{V}_{A-} + \underline{V}_0 \quad (6.84)$$

or

$$\underline{V}_A = \underline{E}_{A+} - jX_d \underline{I}_{dA+} - jX_q \underline{I}_{qA+} - \underline{Z}_- \underline{I}_{A-} - jX_0 \underline{I}_{A0} \quad (6.85)$$

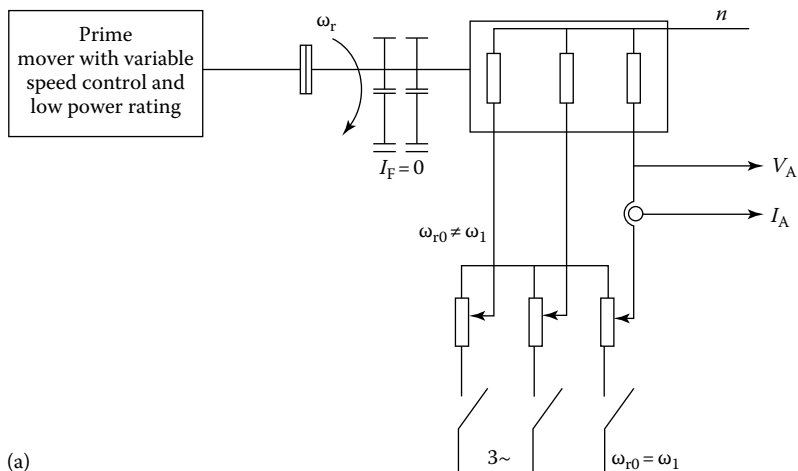
Similar equations hold for phases B and C. Once the phase load impedances are given and the machine parameters are known, the current asymmetric components may be calculated. The above machine parameters may be calculated (in the design stage) or measured.

6.10.1 Measuring X_d , X_q , Z_- , and X_0 /Lab

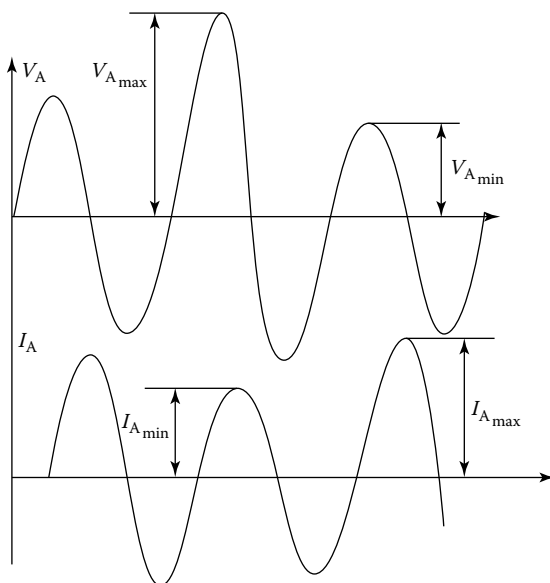
We will introduce here only some basic testing methods to calculate X_d , X_q , Z_- , and X_0 . To measure X_d and X_q (though not heavily saturated), the SG with open field circuit ($I_F = 0$), supplied with symmetric positive sequence voltages, is rotated around synchronous speed: $\omega_r \neq \omega_1$ (pole-slipping method), but

$$\omega_r = (1.01 - 1.02) \omega_1 \quad (6.86)$$

So, the currents induced in the rotor cage at $(0.01-0.02) \omega_1$ frequency are negligible. Recording phase A voltage and current, $V_A(t)$ and $I_A(t)$,



(a)



(b)

FIGURE 6.33

Pole-slipping method: (a) test arrangement and (b) voltage and current recordings.

(Figure 6.33), we notice that they pulsate with the slip (small) frequency of $(0.01 - 0.02) \omega_1$, due to the fact that $X_d \neq X_q$:

$$X_d \approx \frac{V_{Amax}}{I_{Amin}}; \quad X_q = \frac{V_{Amin}}{I_{Amax}} \quad (6.87)$$

The voltage amplitude pulsates because the supply transformer power is considered relatively small.

The negative sequence impedance, $\underline{Z}_-$, may be measured by driving the SG at synchronism ($\omega_r = \omega_1$) but supplying the stator with negative sequence ($-\omega_1$) small voltages (as Figure 6.33), and with the field circuit short-circuited.

Measuring power, current, and voltage on phase A: P_{A-} , I_{A-} , and V_{A-} , we calculate

$$|\underline{Z}_-| = \frac{V_{A-}}{I_{A-}}; \quad R_- = \frac{P_{A-}}{I_{A-}^2}; \quad X_- = \sqrt{|\underline{Z}_-|^2 - (R_-)^2} \quad (6.88)$$

A similar arrangement with all phases in series and ac supply, at zero speed (or at synchronous speed) (Figure 6.34) can be used to measure $\underline{Z}_0$ (the homopolar impedance):

$$\underline{Z}_0 = \frac{V_{A0}}{3I_{A0}}; \quad R_0 = \frac{P_0}{3I_{A0}^2}; \quad X_0 = \sqrt{|\underline{Z}_0|^2 - R_0^2} \quad (6.89)$$

The voltages V_{A-} and V_{A0} in the equations above should be made small to avoid large currents, and thus avoid machine overheating.

Note: For PM cageless rotors, the negative sequence impedance is equal to the positive one. The zero sequence (homopolar) impedance still has $X_0 \leq X_{sl}$.

Example 6.3 The Phase-to-Phase Short Circuit

Let us consider a lossless two-pole SG with $S_n = 100 \text{ kVA}$, $V_{nl} = 440 \text{ V}$ (star connection), $f_1 = 50 \text{ Hz}$, $x_d = x_q = 0.6 \text{ pu}$, $x_- = 0.20 \text{ pu}$, and

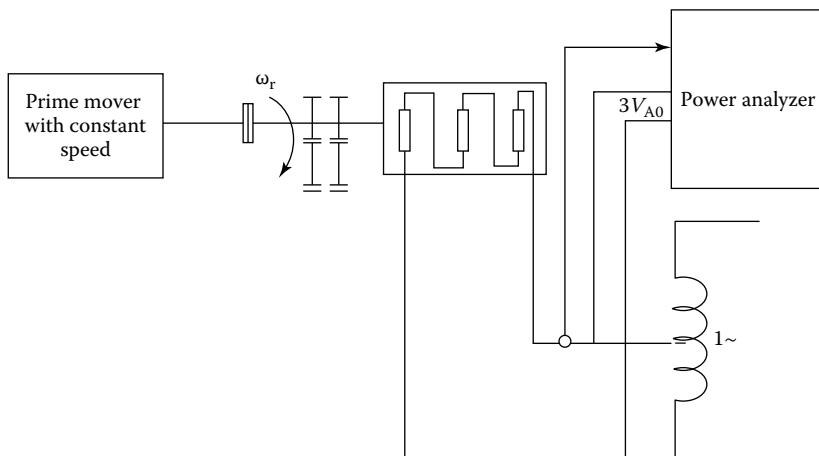


FIGURE 6.34

Homopolar impedance $\underline{Z}_0$ measurements.

$x_0 = 0.12$ pu that is connected in 3-phase, 2-phase, and 1-phase short circuits. Calculate the phase current RMS values in the three cases.

Solution:

As shown earlier in this chapter, the 3-phase short-circuit current, I_{3sc} , is

$$I_{3sc} = \frac{E_1}{X_d} \quad (6.90)$$

Rated short-circuit 3-phase current is obtained for $E_1 = V_{nl}/\sqrt{3}$.

First the reactance norm $X_n = \frac{V_{nl}/\sqrt{3}}{I_n}$, with $I_n = S_n/\sqrt{3}V_{nl} = 100 \times 10^3 / (440\sqrt{3}) = 131.37$ A, is $X_n = \frac{440}{\sqrt{3} \times 131.37} = 1.936 \Omega$.

$$I_{3sc} = \frac{V_{nl}/\sqrt{3}}{x_d \times X_n} = \frac{440}{\sqrt{3} \times 0.6 \times 1.936} = 218.95 \text{ A} > I_n$$

Here, we may also introduce the short-circuit ratio:

$$\frac{I_{3sc}}{I_n} = \frac{1}{x_d} = \frac{1}{0.6} = 1.66 \quad (6.91)$$

For a single-phase short circuit:

$$I_{A+} = I_{A-} = I_{A0} = \frac{I_{1sc}}{3}$$

So, from Equation 6.85, with $V_A = 0$:

$$I_{1sc} = \frac{3E_{A+}}{X_s + X_- + X_0} = \frac{3 \times 440/\sqrt{3}}{(0.6 + 0.2 + 0.1) \times 1.936} = 436.86 \text{ A}$$

For the phase-to-phase short circuit, the computation is a bit more complicated, but it starts from Equation 6.75 to Equation 6.77 for $I_A = 0$ and $I_B = -I_C = I_{2sc}$ ($V_B = V_C$):

$$I_{A+} = \frac{j}{\sqrt{3}} I_{2sc} = -I_{A-}; \quad I_{A0} = 0$$

$$\underline{V}_A = \underline{E}_{A+} - jX_+ I_{A+} - \underline{Z}_- I_{A-} = \underline{E}_{A+} - \frac{j}{\sqrt{3}} I_{2sc} (jX_+ - \underline{Z}_-)$$

$$V_B = a^2 \underline{V}_{A+} + a \underline{V}_{A-} = V_C = a \underline{V}_{A+} + a^2 \underline{V}_{A-} \quad (6.92)$$

Finally,

$$I_{2sc} = \frac{jE_{A+}\sqrt{3}}{(jX_+ + \underline{Z}_-)}; \quad \underline{V}_A = -2\underline{V}_B = \frac{2j}{\sqrt{3}} I_{2sc} \underline{Z}_- \quad (6.93)$$

Apparently, Equation 6.93, with V_A and I_{2sc} measured, directly yields $\underline{Z}_-$, while the first expression in Equation 6.93 would lead to the calculation of

X_+ . This may be feasible only if the fundamentals of the measured variables are extracted first. However, with $X_+ = X_d$ from the 3-phase short circuit, we may calculate $\underline{Z}_-$ from Equation 6.93.

In our example, $\underline{Z}_- = jX_-$, so, from Equation 6.93:

$$I_{2sc} = \frac{E_A \sqrt{3}}{X_+ + X_-} = \frac{440}{(0.6 + 0.2) \times 1.936} = 284.09 \text{ A} \quad (6.94)$$

Comparing I_{3sc} , I_{2sc} , and I_{1sc} , we may write

$$I_{3sc} < I_{2sc} < I_{1sc} \quad (6.95)$$

6.11 Large Synchronous Motors

As already alluded to, SMs may work also as motors, provided their voltage power angles become negative ($\underline{E}_1$ lagging $\underline{V}_s$) (Figure 6.35). The speed of the SM is constant with load (torque) if frequency $f_1 = p_1 n_1 = \text{const}$. So the grid-operated SM has to be started in the induction-motor mode with the field circuit connected to a resistor until stable asynchronous speed, $\omega_{\text{ras}} = (0.95 - 0.98) \omega_1$, is obtained. After that, the field circuit is disconnected from the resistor and connected to the dc source when, after a few oscillations, it synchronizes. For smaller PM cage rotor or reluctance cage rotor SMs, direct connection to the grid is performed, and, at least under light loads, these SMs should synchronize, but (again) using the induction-motor mode during acceleration.

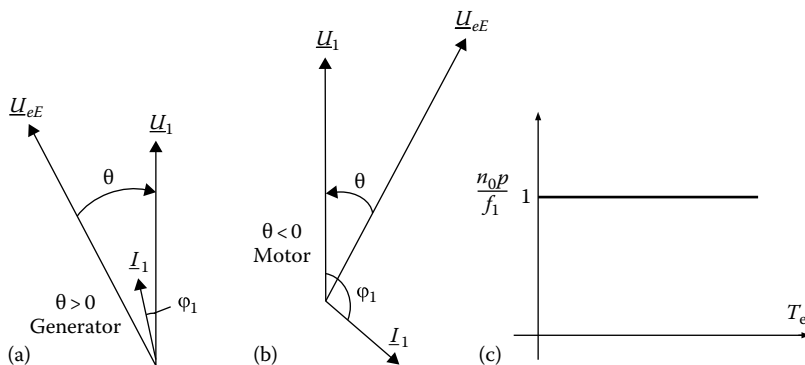


FIGURE 6.35

(a) SG, (b) motor, and (c) speed/torque curve.

6.11.1 Power Balance

To ease the mathematical expressions, so far we considered lossless SMs. In reality, all electric machines have losses, and even if their efficiencies are good, they have to be considered in the cooling system design. The difference between the SM electric input P_{1e} and its mechanical output P_{2m} is the summation of losses, $\sum p$:

$$\sum p = P_{1e} - P_{2m} = p_{\cos} + p_{\text{iron}} + p_s + p_{\text{mec}}; \quad p_{\cos} = 3R_s I_s^2 \quad (6.96)$$

The electric losses take place in stator windings ($p_{\cos}$) and in the stator iron core (p_{iron}), and there are also additional losses (p_s) due to stator space harmonics rotor-induced currents, etc. Iron losses depend approximately on the stator flux, Ψ_s :

$$|\underline{\Psi}_s| = |M_{Fa} I_F + L_d I_d + L_q I_q| \quad (6.97)$$

and frequency that, for power grid operation, are constant, as

$$V_s \approx \omega_r |\underline{\Psi}_s| \quad (6.98)$$

The field circuit losses, $p_{\text{exc}} = R_F I_F^2$, come frequently from a separate supply; this is not so for automotive alternators. For large motors, we still can neglect losses when calculating I_d and I_q currents:

$$I_d = \frac{V_s \cos \delta_V - E_1}{X_d}; \quad I_q = \frac{V_s \sin \delta_V}{X_q}; \quad I_s = \sqrt{I_d^2 + I_q^2} \quad (6.99)$$

Also, for the active and reactive powers, P_s and Q_s , Equations 6.58 and 6.59 still hold. So the power factor, $\cos \varphi_s$, is

$$\cos \varphi_s = \sqrt{1 - \frac{Q_s^2}{(3V_s I_s)^2}} \quad (6.100)$$

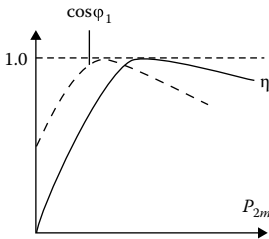


FIGURE 6.36

Efficiency, η , and power factor, $\cos \varphi_s$, versus output power, P_{2m} , for $I_F = \text{const.}$, $\omega_r = \text{const.}$, and $V_s = \text{const.}$

As already explained, for given field current, i_F , and speed, E_1 is given, and for given values of power angle (negative for motoring since Equation 6.36 are still used), we can calculate P_s , Q_s , $\cos \varphi_s$, I_s , and then all losses.

So the efficiency is

$$\eta = \frac{|P_s| - \sum p}{|P_s|} = \frac{P_{2m}}{|P_s|} \quad (6.101)$$

A qualitative graphic representation of η and $\cos \varphi_s$ versus P_{2m} (output [shaft] power) is shown in Figure 6.36.

The power factor starts by being leading at low load and then becomes lagging. By close-loop controlling of the field current, it is feasible to maintain a desired leading power factor angle from 5 (10)% to 100% of full load, in order to compensate the reactive power needed in the SM area.

6.12 PM Synchronous Motors: Steady State

The PMSMs may have a cage on their rotors and may be operated directly at the power grid, or the former may lack any cage on their rotors, and thus, by necessity, they have to be supplied from PWM frequency changers, as $f_1 = p_1 n_1$. In the latter case, they start and operate in synchronism as the stator currents frequency and phase angles are locked to the rotor position.

Here, we investigate only the steady state. The PMSMs are built for very small to large powers (torque) per unit, but, in general, both copper and iron losses, p_{co} and p_{iron} , are directly considered in the model. The efficiency is to be treated as in the previous section. We just take Equation 6.36, replace $M_{Fa} I_F$ by Ψ_{PMd} , and use the sink sign convention (IR-V):

$$(I_d + jI_q) R_s - \underline{V}_s = -j\Psi_{PMd}\omega_r - jX_d I_d - jX_q I_q \quad (6.102)$$

First we represent Equation 6.102 in the phasor diagram of Figure 6.37 when the power angle is positive for motoring and so is the electromagnetic power and torque.

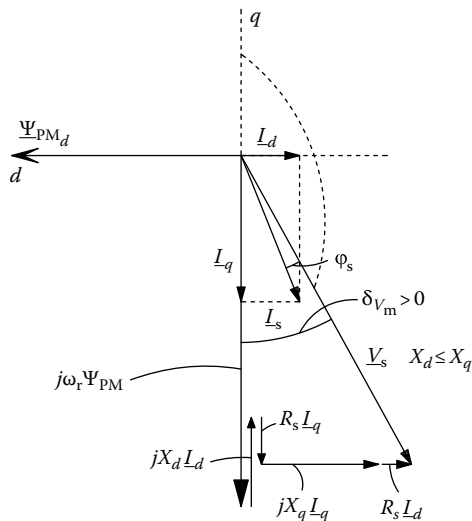


FIGURE 6.37
PMSM phasor diagram.

From Figure 6.37:

$$\begin{aligned} V_s \cos \delta_{V_m} - \omega_r \Psi_{PM} &= -R_s I_q + X_d I_d; & X_d &= \omega_r L_d; & I_q > 0 \\ V_s \sin \delta_{V_m} &= X_q I_q - R_s I_d; & X_q &= \omega_r L_q; & I_d < 0 \end{aligned} \quad (6.103)$$

From Equation 6.103, for V_s constant and δ_{V_m} given (as a parameter), we may calculate I_d , I_q , and then the electromagnetic torque (from Equation 6.42):

$$T_e = P_{elm} \frac{p_1}{\omega_r} = 3p_1 (\Psi_{PM} + (L_d - L_q) I_d) I_q \quad (6.104)$$

The core losses may be approximately considered proportional to V_s^2 irrespective of the frequency or the load, and, thus, they may be added when the efficiency is calculated.

Example 6.4 A PMSM with Interior PM Rotor

A PMSM with an interior PM rotor has the data: $V_{nl} = 380$ V (star connection), $p_1 = 2$, $f_1 = 50$ Hz, $X_d = 7.72 \Omega$, $X_q = 18.72 \Omega > X_d$, and $R_s = 1.32 \Omega$. Let us calculate I_d , I_q , I_s , T_e , P_e , and $\eta \cos \varphi_s$ for $\delta_{V_m} = 0^\circ, 30^\circ, 45^\circ, 60^\circ$, and 90° . Neglect all but copper losses in the stator.

Solution:

We directly apply Equations 6.103 and 6.104, the efficiency definition in section 6.11, and the power factor angle, φ_s , from the phasor diagram in Figure 6.37:

$$\varphi_s = \delta_V - \tan^{-1} \left(\frac{-I_d}{I_q} \right) \quad (6.105)$$

The computation routine is straightforward and the results are shown in Table 6.3.

The performance is unusually good ($\eta \cos \varphi_s$ product is high) despite high I_d (demagnetization) current levels. Also up to $\delta_{V_m} = 90^\circ$, the torque increases because the machine has reverse saliency ($X_d < X_q$), and the rated power grid operation at $\delta_{V_m} = 60^\circ - 80^\circ$ should be acceptable.

TABLE 6.3
PMSM Performance

δ_{V_m} [°]	I_d [A]	I_q [A]	$I_s = \sqrt{I_d^2 + I_q^2}$ [A]	T_e [Nm]	$P_e = \frac{\omega_r}{p_1} T_e$ [W]	$\eta \cos \varphi_s = \frac{P_e}{3V_s I_s}$
0	0	0	0	0	0	
30	-4.78	5.545	7.32	28.9	4537.0	0.941
45	-9.75	7.63	12.38	48.95	7685.7	0.943
60	-15.84	9.07	18.25	72.86	11139.0	0.952
90	-30.24	9.63	31.75	120.1	18857.0	0.902

Note: Recently, a few major companies replaced induction motors up to a few hundred kilowatt and speeds below 500 rpm ($2p_1 = 10, 12$) in variable speed drives for conveyors, etc., by PMSMs in order to increase $\eta \cos \varphi$, and thus cut the PWM frequency changer (converter) (kVA), and, hence, its costs and the energy bill.

Example 6.5 The Reluctance Synchronous Motor (RSM)

An RSM with a multiple flux barrier rotor has a rotor cage provided for grid-connected operation (Figure 6.38a). The phasor diagram (with $\Psi_{PM_d} = 0$ and $I_d > 0$) is shown in Figure 6.38b. Let us consider $V_{nph} = 220$ V, $I_n = 5$ A, $2p_1 = 4$ poles, $f_1 = 50$ Hz, $x_d = 2.5$ pu, $x_q = x_d/5$, and $r_s = 0.08$ (pu). Calculate I_d , I_q , I_s , T_e , P_e , $\eta \cos \varphi_s$, and $\cos \varphi_s$ for $\delta_V = 0^\circ, 10^\circ, 20^\circ$, and 30° .

Solution:

With

$$\begin{aligned} X_n &= V_{nph}/I_n = 220/5 = 44 \, \Omega \\ X_d &= x_d \cdot X_n = 2.5 \times 44 = 110 \, \Omega \\ X_q &= X_d/5 = 22 \, \Omega ; \quad R_s = r_s \cdot X_n = 0.08 \times 44 = 3.52 \, \Omega \end{aligned}$$

from the phasor diagram:

$$\begin{aligned} X_d I_d + R_s I_q &= V_s \cos \delta_{V_m}; \quad \varphi_s = \delta_{V_m} + \tan^{-1} \left(\frac{I_d}{I_q} \right) \\ X_q I_q - R_s I_d &= V_s \sin \delta_{V_m}; \quad I_s = \sqrt{I_d^2 + I_q^2} \end{aligned} \quad (6.106)$$

T_e is obtained from Equation 6.104, with $\Psi_{PM_d} = 0$:

$$T_e = 3p_1 (L_d - L_q) I_d I_q \quad (6.107)$$

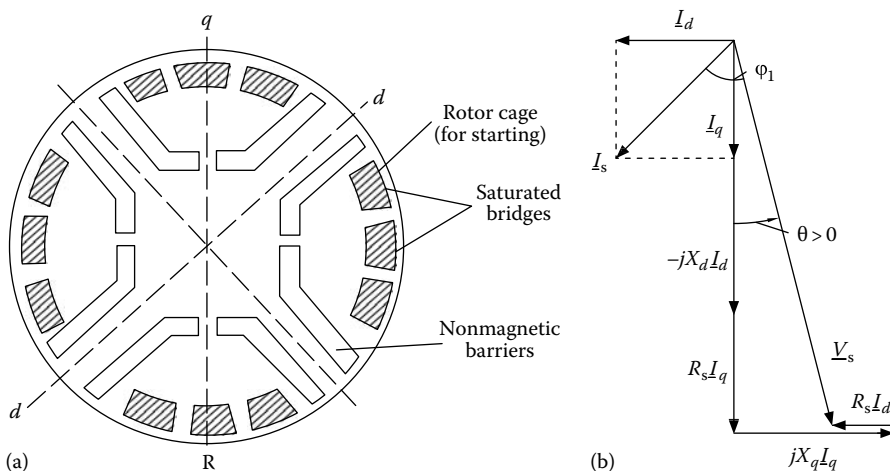


FIGURE 6.38

(a) An RSM with multiple flux barrier rotor and (b) phasor diagram.

TABLE 6.4

RSM Performance

δ_{V_m} [°]	I_d [A]	I_q [A]	I_s [A]	T_e [Nm]	P_e [W]	$\eta \cos \varphi_s$	$\cos \varphi_s$
0	1.987	0.318	2.01	1.06	166.7	0.1264	0.158
10	1.903	2.041	2.79	6.53	1025.2	0.5479	0.59
20	1.734	3.701	4.095	10.78	1692.1	0.628	0.693
30	1.563	5.25	5.696	13.80	2165.0	0.5866	0.67

$$\eta \approx \frac{T_e \cdot \omega_r / p_1}{T_e \cdot \omega_r / p_1 + 3R_s I_s^2} \quad (6.108)$$

The computing routine is straightforward and the results are as in Table 6.4.

As seen from Table 6.4, the power factor is below 0.7 and the $\eta \cos \varphi_s$ product is also notably smaller than for PMSMs. For RSMs, in power grid operation, the rated value of δ_{V_m} is $\delta_{V_{max}} = 20^\circ\text{--}30^\circ$. However, the cost of RSMs is lower, and, in some applications, RSMs may have a higher efficiency than IMs with the same stators. When part of a variable speed drive, and supplied at variable frequency (from zero), the rotor cage of RSM is eliminated and, in home appliance like applications, RSM might be preferred to IM or PMSM cost-wise.

6.13 Load Torque Pulsations Handling by Synchronous Motors/Generators

Diesel engine SGs and compressor loads for SMs are typical industrial situations where the shaft torque varies with the rotor position. It is also important to see how the SMs handle sudden load torque changes or asynchronous starting. At least for large machines (with large inertia), electric transients are usually faster than mechanical transients, and, thus, electric equations for the steady-state case can still be used. This is what we call here mechanical transients. For a generator:

$$\frac{J}{p_1} \frac{d\omega_r}{dt} = T_{\text{shaft}} - T_e + T_{\text{as}}; \quad \frac{d\delta_V}{dt} = \omega_r - \omega_1 \quad (6.109)$$

$$T_e \approx \frac{3p_1}{\omega_1} \left[\frac{V_s E_1 \sin \delta_V}{X_d} + \frac{V_s^2}{2} \left(\frac{1}{X_q} - \frac{1}{X_d} \right) \sin 2\delta_V \right] \quad (6.110)$$

T_{as} is the asynchronous torque due to the rotor cage and the rotor field circuit ac-induced currents (when $\omega_r \neq \omega_1$). Provided the initial values of

variables ω_r and δ_V are known and the input variables V_s , E_1 , and T_{shaft} evolution (in time or with speed) is given, any mechanical (slow) transient process can be solved through Equations 6.109 and 6.110 by numerical methods. However, linearization of Equations 6.109 and 6.110 can shed light on phenomena and offer a feeling of magnitudes which is essential for engineering insight. The asynchronous torque, around synchronism, varies almost linearly with slip speed, $\omega_2 = \omega_r - \omega_1$:

$$T_{\text{as}} = -K_a (\omega_r - \omega_1) = -K_a \frac{d\delta_V}{dt} \quad (6.111)$$

Positive T_{as} means motoring ($\omega_r < \omega_1$) and negative T_{as} corresponds to generating ($\omega_r > \omega_1$).

The synchronous torque T_e (Equation 6.110) may be linearized around an initial value, δ_{V_0} :

$$\begin{aligned} \delta_V &= \delta_{V_0} + \Delta\delta_V; & T_{\text{shaft}} &= T_{a_0} + \Delta T_a \\ T_e &= T_{e_0} + T_{\text{es}} \times \Delta\delta_V \\ T_{\text{es}} &= \left(\frac{\partial T_e}{\partial \delta_V} \right)_{\delta_{V_0}}; & T_{e_0} &= (T_e)_{\delta_{V_0}} = T_{a_0} \end{aligned} \quad (6.112)$$

From Equations 6.109 through 6.112:

$$\frac{J}{p_1} \frac{d^2(\Delta\delta_V)}{dt^2} + K_a \frac{d(\Delta\delta_V)}{dt} + T_{\text{es}} \Delta\delta_V = \Delta T_a \quad (6.113)$$

For small deviations, $\Delta\delta_V$, of power angle, δ_V , around δ_{V_0} , a second-order model for transients has been obtained. From it we distinguish the eigenvalues $\underline{\gamma}_{1,2}$:

$$\gamma_{1,2} = -\frac{K_a p_1}{2J} \pm \sqrt{\left(\frac{K_a p_1}{2J} \right)^2 - \omega_0^2}; \quad \omega_0 = \sqrt{\frac{p_1 T_{\text{es}}}{J}} \quad (6.114)$$

where ω_0 is the so-called proper (eigen) angular frequency of the generator/motor, and it acts when there is no cage on the rotor, $\left((\gamma_{1,2})_{K_a=0} = \pm j\omega_0 \right)$. It depends on the power angle (δ_{V_0}), the level of excitation (I_F), and machine inertia (J). It is in the range of a few hertz (3 to 1) Hz or less for the largest-power SGs. When an SM is connected to the grid, and it has a rotor cage, all terms in Equation 6.113 are active.

Now, if the shaft torque has pulsations (in diesel engines, ICEs, or compressor loads for SMs):

$$\Delta T_a = \sum T_{a_v} \sin(\Omega_v t - \Psi_v) \quad (6.115)$$

There is an amplification K_{mv} of rotor angle-oscillation amplitudes with respect to the case of autonomous generators ($T_{\text{es}} = 0$) with no cages ($K_a = 0$):

$$\frac{J}{p_1} \frac{d^2 \Delta \delta_V}{dt^2} = \sum T_{av} \sin (\Omega_v t - \Psi_v) \quad (6.116)$$

with the solution:

$$\Delta \delta_{Vva}(t) = -\frac{p_1}{J} \frac{T_{av}}{\Omega_v^2} \sin (\Omega_v t - \Psi_v) \quad (6.117)$$

When solving the Equation 6.113 we get

$$\Delta \delta_{Vv}(t) = \frac{T_{av} \sin (\Omega_v t - \Psi_v - \varphi_v)}{\sqrt{\left(\frac{J \Omega_v^2}{p_1} - T_{es}\right)^2 + (K_a \Omega_v)^2}}; \quad \varphi_v = \tan^{-1} \frac{K_a \Omega_v}{\frac{J}{p_1} \Omega_v^2 - T_{es}} \quad (6.118)$$

And K_{mv} (the mechanical resonance module) is

$$K_{mv} = \frac{(\Delta \delta_{Vv})_{\max}}{(\Delta \delta_{Vva})_{\max}} = \frac{1}{\sqrt{\left(1 - \frac{\omega_0^2}{\Omega_v^2}\right)^2 + K_{dv}^2}}; \quad K_{dv} = \frac{K_a p_1}{J \Omega_v} \quad (6.119)$$

where K_{dv} is the known damping coefficient in second-order systems.

A graphical representation of Equation 6.119 is shown in Figure 6.39. It is visible that for

$$1.25 > \frac{\omega_0}{\Omega_v} > 0.8 \quad (6.120)$$

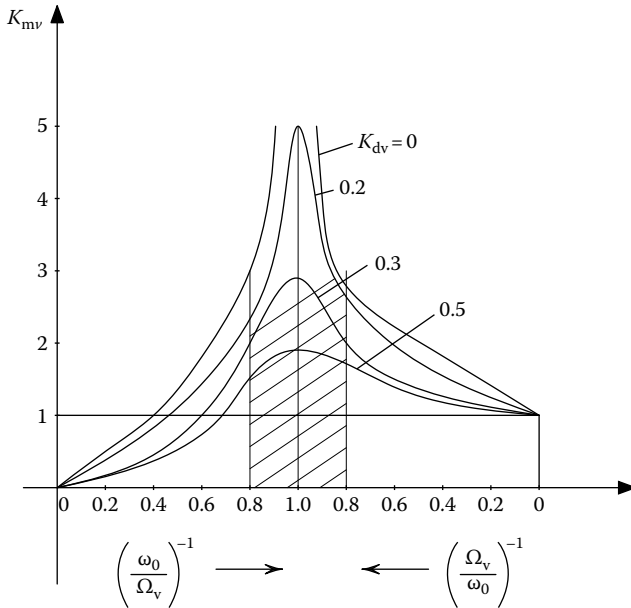


FIGURE 6.39

K_{mv} (mechanical resonance module).

the amplification of oscillations is large. A strong damper cage in the rotor ($K_a - \text{large}$) leads to a reduction in the amplitude of rotor-angle oscillations that is in accordance with diesel-engine or compressor drives. An inertial disk on the shaft is also beneficial. The power angle and speed oscillations lead also to stator current oscillations and input electric power oscillations, which have to be limited. Grid-connected SMs are in general-sensitive to shaft torque pulsations due to the danger of loosing synchronism.

6.14 Asynchronous Starting of SMs and Their Self-Synchronization to Power Grid

Asynchronous starting of grid-connected SMs is illustrated in Figure 6.40.

Large SMs are started as asynchronous motors by connecting the field circuit terminals to a $10 R_F$ resistance (R_F is the field circuit resistance) to limit the Georges' effect (asymmetrical circuit rotor effect [see Chapter 5]), and, thus, provide for smooth passing of the machine through 50% rated speed. Then, when speed stabilizes (at slip: $S = 0.03 - 0.02$), the field circuit is switched to the dc source. The field current reaches reasonable values in tens of milliseconds at best, but let us neglect this process and provide full I_F . When I_F occurs, the emf, E_1 , has a certain phase shift, δ_{V_0} , with respect to the terminal (power grid) voltage. It may be any value. The best situation, $\delta_{V_0} = 0$, and turning into motoring is illustrated in Figure 6.40b.

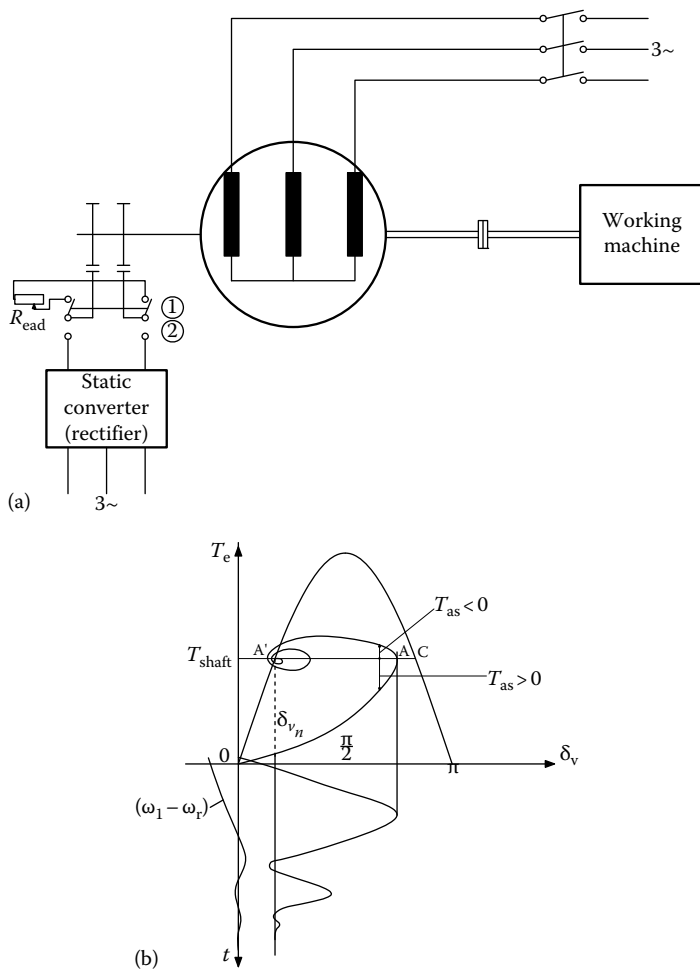
Equations below, rewritten for motoring, illustrate Figure 6.40b completely:

$$\begin{aligned} \frac{d\delta_{V_m}}{dt} &= \omega_1 - \omega_r; \quad T_{as} = K_a (\omega_1 - \omega_r) \\ \frac{J}{p_1} \frac{d\omega_r}{dt} &= -\frac{J}{p_1} \frac{d^2\delta_{V_m}}{dt^2} = T_e + T_{as} - T_{shaft} \end{aligned} \quad (6.121)$$

$$T_e = \frac{3p_1 V_s}{\omega_1} \left[\frac{E_1}{X_d} \sin \delta_{V_m} + \frac{V_s}{2} \left(\frac{1}{X_q} - \frac{1}{X_d} \right) \sin 2\delta_{V_m} \right] \quad (6.122)$$

with δ_{V_0} and ω_{r_0} as initial assigned values. The self-synchronization process may be solved by Equations 6.121 through standard numerical procedures with shaft torque (T_{shaft}) given as a function of speed (or even on rotor position). In this way, for heavy-starting applications (large coolant circulation pumps in nuclear electric power plants), the self-synchronization process may be simulated simply. It is also feasible to measure the ac field current during the asynchronous operation and start synchronization when the latter passes through zero, which corresponds to $\delta_{V_0} = 0$, because at low slip frequencies the field circuit looks strongly resistive. In this way safe (flawless) starting under load is ensured.

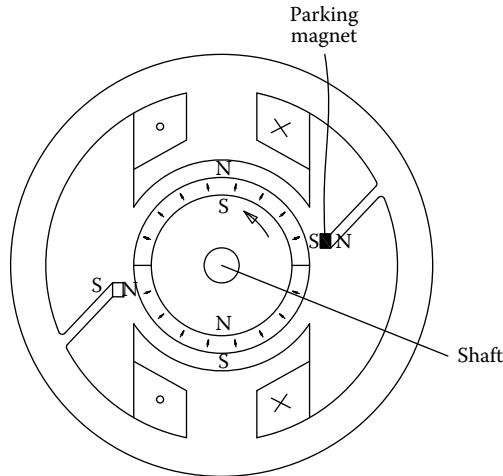
Note: Speed control by frequency control is mandatory with SMs. This aspect will be treated in detail in Chapter 9 dedicated to SM transients.

**FIGURE 6.40**

Asynchronous starting: (a) basic arrangement and (b) transients of self-synchronization from zero initial power angle.

6.15 Single-Phase and Split-Phase Capacitor PM Synchronous Motors

Single-phase PMSMs without a cage on the rotor and with a parking magnet for self-starting are supplied with a PWM inverter from zero frequency (Figure 6.41), and are typical for low-power, light-start,

**FIGURE 6.41**

Single-phase PMSM with parking magnet and PWM inverter frequency control (no cage on the rotor), $2p_1 = 2$.

variable-speed applications from sub watt to hundreds of watts and high speeds (30,000 rpm and more). On the other hand, if the stator is provided with two distributed windings at a space angle shift of 90° (like for the induction motor) while the rotor has PMs and a cage, the machine may be connected directly to the power grid. It starts as an induction machine motor and then eventually self-synchronizes on its own, up to a certain load torque level. This is the split-phase PMSM.

6.15.1 Steady State of Single-Phase Cageless-Rotor PMSMs

The single-phase (PWM inverter fed) PMSM (Figure 6.41) under steady state is rather easy to model as there are no windings on the rotor, and the parking magnet influence on steady state may be neglected. For a sinusoidal emf in the stator phase (produced by the PMs on the rotor) and sinusoidal terminal voltage, the machine voltage equation is straightforward:

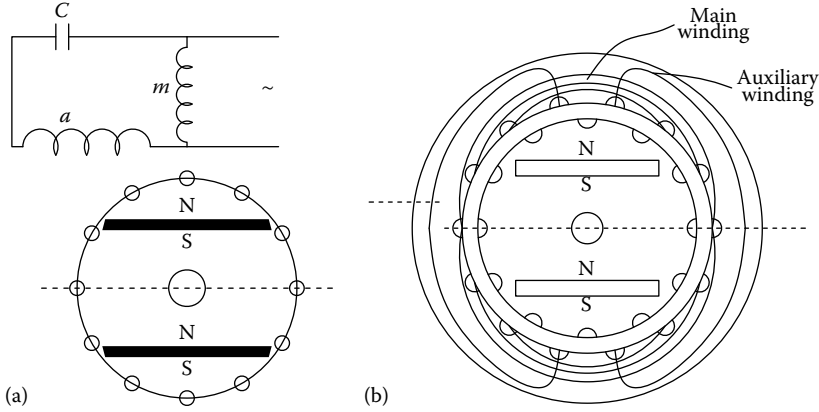
$$\underline{V}_s = R_s \underline{I}_s + \underline{E}_s + j\omega_1 L_s \underline{I}_s; \quad \underline{E}_s = j\omega_r \Psi_{PM}; \quad \omega_r = \omega_1 \quad (6.123)$$

Now with the sinusoidal emf, $\underline{E}_s$:

$$E_s(t) = E_{s1} \cos \omega_r t \quad (6.124)$$

With sinusoidal voltage $\underline{V}_s$, the current $\underline{I}_s$ is sinusoidal:

$$I_s(t) = I_{s1} \cos(\omega_r t - \gamma) \quad (6.125)$$

**FIGURE 6.42**

Grid-operated PMSM with two-pole split-phase capacitor (with PMs and a cage in the rotor).

The electromagnetic torque T_e —in the absence of rotor magnetic saliency—is

$$T_e = \frac{p_1 E_s(t) I_s(t)}{\omega_r} = \frac{p_1}{\omega_r} \frac{E_{s1} I_{s1}}{2} (\cos \gamma + \cos(2\omega_r t - \gamma)) \quad (6.126)$$

So the torque pulsation is 100%.

Now at zero stator current, there is a cogging torque due to slot openings in the stator. The stator has in fact two slots (for two poles) and $2p_1 = 2$ poles (Figure 6.42). So the number of cogging torque (T_{cogg}) periods is the LCM(2, 2) = 2:

$$T_{\text{cogg}} = T_{\text{cogg max}} \cdot \cos(2\omega_r t - \gamma_{\text{cogg}}) \quad (6.127)$$

If, by proper design, $\gamma_{\text{cogg}} = \gamma$ and

$$T_{\text{cogg max}} = \frac{p_1}{\omega_r} \frac{E_{s1} I_{s1}}{2} \quad (6.128)$$

the resultant torque becomes constant:

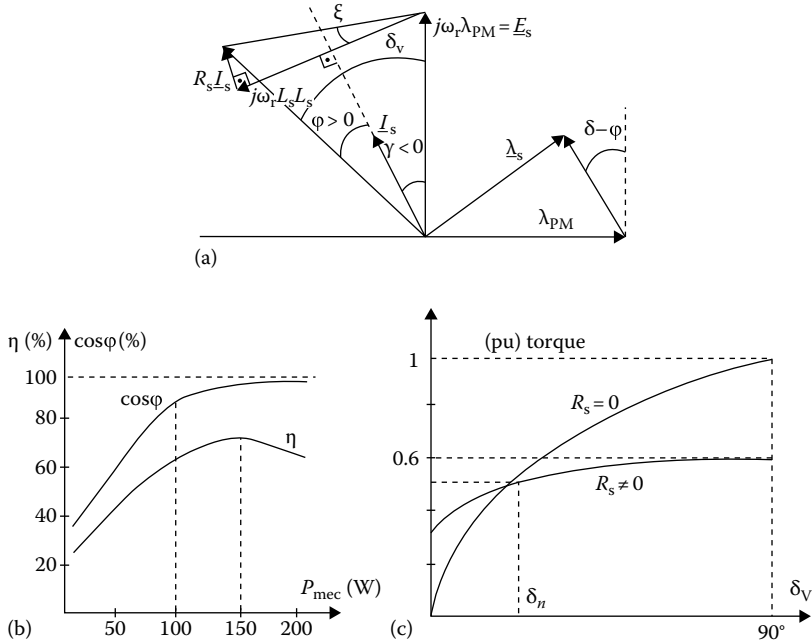
$$T_e + T_{\text{cogg}} = \frac{p_1}{\omega_r} \frac{E_{s1} I_{s1}}{2} \quad (6.129)$$

So the torque has lost its pulsations but only at, say, rated current.

The phasor diagram is straightforward (Figure 6.43a). The core losses, p_{iron} , are given by

$$p_{\text{iron}} \approx \frac{\omega_r \Psi_s^2}{R_{\text{iron}}}; \quad \Psi_s = \sqrt{\Psi_{\text{PM}}^2 + (L_s I_s)^2 - 2\Psi_{\text{PM}} L_s I_s \cos(\delta - \varphi)} \quad (6.130)$$

R_{iron} may be obtained from experiments or in the design stage.


FIGURE 6.43

(a) Phasor diagram of single-phase PMSM, (b) efficiency and power factor for constant voltage and frequency, and (c) pu torque versus power angle, δ_v .

The voltage power angle, δ_v , is defined as for 3-phase machines, and I_s is (from Figure 6.43a)

$$I_s = \frac{\sqrt{V_s^2 + E_s^2 - 2V_s E_s \cos \delta_v}}{Z} \quad (6.131)$$

$$Z = \sqrt{R_s^2 + \omega_1^2 L_s^2}; \quad \tan \xi = \frac{R_s}{\omega_1 L_s}; \quad \cos(\varphi + \xi) = \frac{E}{IZ} \sin \delta_v \quad (6.132)$$

The mechanical power, P_{2m} , is

$$P_{2m} = V_s I_s \cos \varphi - I_s^2 R_s - \frac{\omega_r^2 \Psi_s^2}{R_{iron}} - p_{mec} \quad (6.133)$$

where p_{mec} denotes the mechanical losses (in watt).

Efficiency η is

$$\eta = \frac{P_{2m}}{V_s I_s \cos \varphi} \quad (6.134)$$

The above model, with δ_v as a parameter, allows for calculating the stator current (I_s), $\cos \varphi$, T_e (average), efficiency, and the power factor.

Figure 6.43c shows the average torque T_e (in pu), η , and $\cos \varphi$ versus power angle (δ_V) for a 150 W machine with and without considering the stator winding losses (R_s).

The stator resistance limits notably the peak torque (Figure 6.43c). Higher efficiency (lower resistance, R_s) for this power range is feasible at the price of higher copper weight.

For a split-phase capacitor PMSM (or RSM) with cage rotor modeling for steady state, the symmetrical component method may be applied (as for the split-phase capacitor IM), but for $S = 0$ and by adding the emf $\underline{E}_s$ for the positive sequence component (see [14] for details).

Note: Test procedures for SMs.

The testing methods for synchronous (also for dc or induction) motors/generators may be classified as standard and research tests.

Tests of a more general nature are included under standards that are renewed periodically to reflect the progress in the art. The IEEE standards 115-1995 represent a comprehensive set of tests for SMs. They may be classified into

- Acceptance tests
- Steady-state performance tests
- Parameter estimation (for transients and control)

See [13] in Chapter 8 for a synthesis of IEEE standards 115-1995. Quite a few test procedures have been introduced earlier in this chapter under the logo “Lab.” Advanced testing for parameter identification will be treated in Chapter 9, dedicated to SM transients.

6.16 Preliminary Design Methodology of a 3-Phase PMSM by Example

General specifications:

- Base power $P_b = 100$ W
- Base speed $n_b = 1800$ rpm
- Maximum speed $n_{\max} = 3000$ rpm
- Power at maximum speed: P_b
- DC voltage $V_{dc} = 14$ V (automobile battery)
- Supply: PWM inverter; maximum line voltages as in Figure 6.44
- Star connection of stator phases

Additional specifications related to duty cycle, motor-cooling system, constraints related to volume, efficiency at base power, and cost of active materials in the motor may be added. Here a preliminary design methodology by example is introduced.

Maximum phase voltage:

According to Figure 6.44, the fundamental $V_{\text{ph max}}$ for the maximum phase voltage is

$$\begin{aligned} (V_{\text{ph max}})_{\text{RMS}} &= \frac{V_{\text{line max}}}{\sqrt{6}} = \frac{1}{\sqrt{6}} \frac{4}{\pi} \sin \frac{2\pi}{3} V_{\text{dc}} \\ &= \frac{\sqrt{2}}{\pi} V_{\text{dc}} = \frac{\sqrt{2}}{\pi} \times 14 = 6.28 \text{ V (RMS)} \end{aligned} \quad (6.135)$$

Interior stator diameter, D_{is} , and stator stack length, l_{stack} :

Here, the sizing of the machine is based on the tangential specific force $f_t = (0.2 - 1.2) \text{ N/cm}^2$ (for small torque [sub 1 Nm] levels).

The base torque T_{eb} , provided from $n_b = 0 \text{ rpm}$ to $n_b = 1800 \text{ rpm}$, is

$$T_{\text{eb}} \approx \frac{P_b}{2\pi n_b} = \frac{100}{2\pi \times 1800/60} = 0.5308 \text{ Nm} \quad (6.136)$$

The ratio of the stack length, l_{stack} , to the stator interior diameter, D_{is} , is $\lambda = l_{\text{stack}}/D_{\text{is}} = 0.3 - 3$. For $l_{\text{stack}}/D_{\text{is}} = 1.0$:

$$T_{\text{eb}} = \frac{D_{\text{is}}}{2} f_t \cdot \pi D_{\text{is}} \cdot \frac{l_{\text{stack}}}{D_{\text{is}}} \cdot D_{\text{is}}; \quad f_t = 1 \text{ N/cm}^2 \quad (6.137)$$

$$\begin{aligned} D_{\text{is}} &= \sqrt[3]{\frac{2T_{\text{eb}}}{\lambda \pi f_t}} = \sqrt[3]{\frac{2 \times 0.5308}{1 \times \pi \times 1 \times 10^4}} = 3.24 \times 10^{-2} \text{ m} \\ &= l_{\text{stack}} \end{aligned} \quad (6.138)$$

We have to choose now the number of poles. For maximum speed and $2p_1 = 4$, $f_{\text{max}} = p_1 n_{\text{max}} = 2 \times \frac{3000}{60} = 100 \text{ Hz}$. For this frequency, 0.5 mm thick laminated silicon steel can still be used for the magnetic cores of the stator (and the rotor).

To reduce the copper weight (and losses) and the fabrication costs (which are proportional to the number of coils in the stator winding), a six-slot stator and four-pole rotor configuration is used (Figure 6.45).

The PM span on the rotor, b_{PM} , is chosen equal to the stator slot pitch, τ_s , to reduce the cogging torque.

PM sizing and coil mmf $n_c I_b$ computation:

Let us consider that the PM flux linkage in the stator, 2 coils per phase, varies sinusoidally, with the maximum of Ψ_{PM} :

$$\Psi_{\text{PM max}} = B_{\text{gPM}} \times b_{\text{PM}} \times l_{\text{stack}} \times 2n_c \quad (6.139)$$

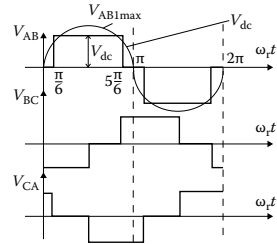


FIGURE 6.44

Ideal line voltages for six-pulse PWM inverter operation.

The torque for $I_d = 0$ and $I_q = I_b$ (RMS) is

$$T_{eb} = 3p_1 \frac{\Psi_{PM_{max}}}{\sqrt{2}} \times I_b \quad (6.143)$$

So, from Equations 6.142 and 6.143:

$$n_c I_b = \frac{0.5308 \times \sqrt{2}}{3 \times 2 \times 6.3874 \times 10^{-4}} = 195.28 \text{ A turns (RMS)}$$

Stator slot sizing:

There are two coils in each slot, and thus the active area of stator slot, A_{co} , is

$$A_{co} = \frac{2n_c I_b}{k_{fill} \times j_{cob}} = \frac{2 \times 195.28}{0.4 \times 6.5 \times 10^6} = 136.22 \times 10^{-6} \text{ m}^2 \quad (6.144)$$

where current density $j_{cob} = 6.5 \text{ A/mm}^2$ and slot filling factor $k_{fill} = 0.4$.
The stator tooth and top-slot width:

$$b_{t1} = b_{s1} \approx \tau_s/2 = \pi \times 3.24 \times 10^{-2}/12 = 8.478 \times 10^{-3} \text{ m} \quad (6.145)$$

Choosing the active slot height $h_{su} = 12 \times 10^{-3} \text{ m}$, the bottom width of slot b_{s2} is

$$\begin{aligned} b_{s2} &= \frac{\pi(D_{is} + 2h_{su})}{6} - b_{s1} = \frac{\pi(32.4 + 2 \times 12) 10^{-3}}{6} - 8.478 \times 10^{-3} \\ &= 21.04 \times 10^{-3} \text{ m} \end{aligned}$$

We may now check the active slot area A_{cof} :

$$\begin{aligned} A_{cof} &= \frac{(b_{s1} + b_{s2})}{2} \times h_{su} = \frac{(8.478 + 21.038) 10^{-3}}{2} \times 12 \times 10^{-3} \\ &= 147.58 \times 10^{-6} \text{ m}^2 \\ &> A_{co} \end{aligned} \quad (6.146)$$

So, the slot sizing holds.

Stator yoke height, h_{ys} , and outer diameter, D_{out} :

Let us accept $B_{ys} = 1.4 \text{ T}$ in the stator yoke. Then the yoke height, h_{ys} , is

$$h_{ys} = \frac{B_{gPM} \times b_{PM}}{2B_{ys}} = \frac{0.872 \times \pi \times 3.24 \times 10^{-2}}{2 \times 1.4 \times 6} = 5.2 \times 10^{-3} \text{ m} \quad (6.147)$$

So the outer stator diameter, D_{out} , is

$$\begin{aligned} D_{out} &= D_{is} + 2h_{su} + 2h_{sw} + 2h_{ys} = 3.24 \times 10^{-2} + 2 \times 1.055 \times 10^{-2} \\ &\quad + 2 \times 1.5 \times 10^{-3} + 2 \times 5.2 \times 10^{-3} = 66.8 \times 10^{-3} \text{ m} \end{aligned} \quad (6.148)$$

$D_{is}/D_{out} = (32.4 \times 10^{-3})/(66.8 \times 10^{-3}) = 0.485$, which is close to 0.5 that is considered close to the maximum efficiency design.

Machine parameters:

The phase resistance, R_s , is

$$R_s = \rho_{co} \frac{l_{\text{turn}} \times 2 \times n_c^2}{(n_c I_b / j_{co})} \quad (6.149)$$

The turn length, l_{turn} , is

$$l_{\text{turn}} \approx 2 (l_{\text{stack}} + 1.25\tau_s) = 2 (3.24 + 1.25 \times 1.695) \times 10^{-2} \\ \simeq 0.109 \text{ m}$$

$$R_s = \frac{2.3 \times 10^{-8} \times 0.109 \times n_c^2}{195.28 \times 10} \times (6.5 \times 10^6) = 1.669 \times 10^{-4} n_c^2 \quad (6.150)$$

The phase inductance comprises the main inductance L_m , the leakage inductance L_{sl} , and the coupling inductance L_{12} :

$$L_m \approx 2n_c^2 \mu_0 \frac{(\tau_s - b_{os})}{h_{PM} + g} I_{\text{stack}} = \frac{2 \times 1.256 \times 10^{-6} (16.95 - 2) 10^{-3} \cdot 3.24 \times 10^{-2}}{(2 + 0.5) \times 10^{-3}} \\ \times n_c^2 = 4.86 \times 10^{-7} n_c^2 \quad (6.151)$$

The phase mutual inductance $L_{12} \approx -L_m/3$ and the leakage inductance is approximated here by $L_{sl} = 0.3L_m$. So the synchronous inductance L_s is

$$L_s = L_m - L_{12} + L_{sl} = 4.86 \times 10^{-7} n_c^2 \left(\frac{4}{3} + 0.3 \right) = 7.92 \times 10^{-7} n_c^2 \quad (6.152)$$

We may now calculate the copper losses for base torque, T_{eb} :

$$P_{cob} = 3R_s I_b^2 = 3 \times 1.667 \times 10^{-4} (n_c I_b)^2 = 19 \text{ W} \quad (6.153)$$

Neglecting iron and mechanical losses, the efficiency at base power and speed would be

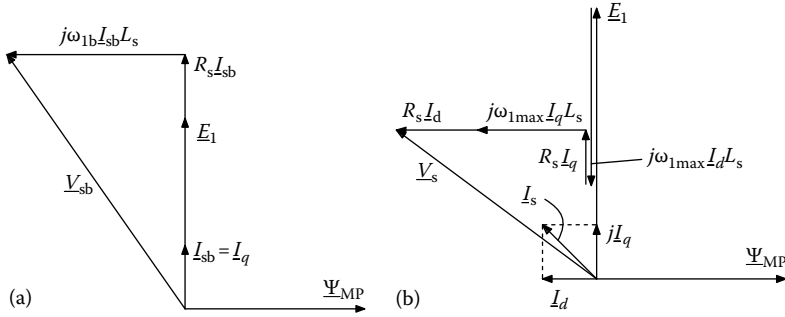
$$\eta_b = \frac{P_b}{P_b + P_{cob}} = \frac{100}{100 + 19} = 0.84 \quad (6.154)$$

Number of turns per coil, n_c :

Let us first draw the phasor diagram for base speed and torque (with pure I_q control [$I_d = 0$]) (Figure 6.46a). The frequency at base speed is $f_b = n_b \times p_1 = 1800 \times 2/60 = 60 \text{ Hz}$. The emf E_1 (RMS) is

$$E_1 = \frac{\omega_{1b} \Psi_{PM_{\text{max}}}}{\sqrt{2}} = 2\pi 60 \times \frac{6.3874 \times 10^{-4} \times n_c}{\sqrt{2}} = 0.24058 \times n_c \quad (6.155)$$

$$V_{ph_{\text{max}}} = \sqrt{(E_1 + R_s I_b)^2 + (\omega_{1b} L_s I_b)^2} = n_c \sqrt{0.273^2 + 0.05827^2} \\ = 6.28 \text{ V} \quad (6.156)$$

**FIGURE 6.46**

The phasor diagram: (a) at base speed (pure I_q control, $I_d = 0$) and (b) at maximum speed ($i_d < 0$).

So the number of turns, n_c , is

$$n_c = 22 \text{ turns/coil} \quad (6.157)$$

The RMS phase base current is

$$I_b = (n_c I_b) / n_c = 8.876 \text{ A} \quad (6.158)$$

The apparent input power, S_n , is

$$S_n = 3V_{ph\max} \times I_b = 3 \times 6.28 \times 8.876 = 167.23 \text{ VA} \quad (6.159)$$

So the base speed power factor, $\cos \phi_b$, is

$$\cos \phi_b = \frac{P_b}{\eta_b S_b} = \frac{100}{0.84 \times 167.23} = 0.712 \quad (6.160)$$

Maximum speed torque (P_b) capability verification:

To maintain the base power at 3000 rpm for which the frequency $f_{\max} = 3000 \times 2/60 = 100 \text{ Hz}$, we first calculate the required torque:

$$(T_e)_{n_{\max}} = \frac{P_b}{2\pi n_{\max}} = \frac{100}{2\pi \times 3000/60} = 0.318 \text{ Nm} \quad (6.161)$$

So the I_q current needed is

$$\frac{(I_q)_{n_{\max}}}{I_b} = \frac{(T_e)_{n_{\max}}}{T_{eb}} \quad (6.162)$$

So $(I_q)_{n_{\max}} = 8.876 \times \frac{0.318}{0.5308} = 5.3175 \text{ A}$.

We may now check what torque can be produced at rated current $I_b = 8.876 \text{ A}$. So I_d is

$$I_d = \sqrt{I_b^2 - (I_q)_{n_{\max}}^2} = \sqrt{8.876^2 - 5.3175^2} = 7.107 \text{ A} \quad (6.163)$$

So, from the phasor diagram in Figure 6.46b:

$$\begin{aligned}
 V_s &= \sqrt{(E_1 + R_s I_q - \omega_{1\max} L_s I_d)^2 + (R_s I_d + \omega_{1\max} \times L_s I_q)^2} \\
 &= \sqrt{\left(0.2045 \times \frac{3000}{1800} \times 22 + 1.669 \times 10^{-4} \times 22^2 \times 5.3175 - 2\pi 100 \times 7.92 \times 10^{-7} \times 22^2 \right.} \\
 &\quad \left. \times 7.107 \right)^2 + \left(1.669 \times 10^{-4} \times 22^2 \times 7.107 + 2\pi 100 \times 7.92 \times 10^{-7} \times 22^2 \times 5.3175 \right)^2} \\
 &= \sqrt{5.788^2 + (0.574 + 1.28)^2} = 6.077 < V_{\text{phmax}} = 6.28 \text{ V}
 \end{aligned}$$

So, with a strong negative (demagnetization) I_d , the machine can provide constant power, P_b , from 1800 to 3000 rpm at base current. Notice that at base speed (1800 rpm) $I_d = 0$.

6.17 Summary

- SMs have a 3 (2)-phase ac winding on the stator and a dc (or PM) or a variable reluctance rotor.
- As the rotor is dc and the stator is ac at frequency f_1 , the speed at steady state, $n_1 = f_1/p_1$, is constant with the load ($2p_1$ denotes the number of poles on the stator and on the rotor in general).
- SMs are used as generators in electric power systems or as autonomous sources for wind energy conversion, cogeneration, and stand-by power, or on automobiles, trucks, diesel–electric locomotives, vessels, and aircraft.
- SMs are used as motors from sub watt power (as single-phase machines) to 50 MW, 60 kV variable speed drives (for gas compressors).
- The PMSMs/PMSGs with variable frequency (speed) control via power electronics constitute the most dynamic chapter in industrial, transportation, and home appliance energy and motion control for energy savings and better productivity.
- Linear versions of SMs are applied for Maglev transport, industrial carriers, and, with oscillatory motion, for electromagnetic ICE valves, active suspension damping, and compressors drives of refrigerators.
- AC-distributed windings are typical (with q [slots/pole/phase] > 1), but nonoverlapping coil (concentrated) windings are used for PMSMs when $q \leq 1/2$ and the stator has N_s slots (teeth) and $2p_1$ poles ($N_s = 2p_1 \pm 2K$; $K = 1, 2, \dots$).

These kinds of PMSMs are characterized by lower copper losses, machine size, and cogging (zero current) torque pulsations.

- The number of cogging-torque periods per revolution is given by the LCM of N_s and $2p_1$: the larger the LCM the smaller the cogging torque.
- The stator emf, E_s , of dc excitation (or PMs) in the rotor is sinusoidal or trapezoidal (PM rotor with $q = 1$ stator windings or $q < 1/2$). A trapezoidal emf recommends rectangular (120° wide) ac current control with two phases active at any time (except for phase commutation intervals), and provides reasonable torque pulsations and simplified rotor-position-triggered control.
- During steady state, the stator currents do not induce any voltage (and currents) in the rotor (which may have dc [or PM] excitation and a cage in grid-operated applications) because the stator mmf travels at rotor speed.

Consequently, the stator (armature) winding current may be decomposed in two components: I_d and I_q on each phase. These components produce their airgap flux density aligned to the rotor pole (axis d) or to the interpole (axis q) to manifest themselves as two magnetization synchronous reactances, $X_{dm} > X_{qm}$; if we add the leakage reactances, the so-called synchronous reactances X_d and X_q are obtained. These are valid for steady-state symmetric stator currents.

So the voltage circuit (per phase) becomes straightforward:

$$\underline{I}_s R_s + \underline{V}_s = \underline{E}_s - jX_d \underline{I}_d - jX_q \underline{I}_q$$

For $X_d = X_q = X_s$ the nonsalient pole rotor configuration is obtained.

- The “internal” impedance of the SG is in fact related to X_d and X_q , which in PU terms is $x_s = X_s I_n / V_n = 0.5 - 1.5$ (pu) for dc-excited machines and smaller for PM pole rotor machines

$$(x_s < 0.5 \text{ pu}).$$

So voltage regulation is substantial in SGs for autonomous applications.

- Autonomous generators are characterized by no-load saturation, short-circuit, and external (load) curves.

The dc excitation circuit has been designed to maintain rated terminal voltage for rated current and the lowest-assigned power factor (above 0.707, lagging).

- Most SGs are connected to the power grid through a synchronization routine and under least-possible transients.

- Active power delivery by an SG is obtained by increasing the shaft power of the prime mover (turbine).
- Reactive power delivery/absorption is obtained through field current, I_F , control.
- The voltage power angle δ_V ($\underline{E}_s$, $\underline{V}_s$) is the crucial variable, and a statically stable operation is provided up to $|\delta_V| = \delta_{VK} \leq 90^\circ$ when the SG is tied to the power grid.
- Static stability is the property of an SM to remain in synchronism for slow shaft-power variations.
- PMSMs, with interior (salient) PM poles, are characterized by inverse saliency, $X_d < X_q$, and, thus, they may operate safely even close to $|\delta_V| = 90^\circ$.

The power angle $\delta_V < 0$ for motoring for the source (generator) association of signs.

- Grid operation is characterized by $P_s(\delta_V)$, $Q_s(\delta_V)$, $I_s(I_F)$ (V curves), and $Q_s(P_s)$ (reactive power envelopes).
- The electromagnetic torque, T_e , of SMs has an interaction term (between the dc excitation [or PM] field and the stator mmf) and a reluctance term ($X_d \neq X_q$).
- SGs may operate with unbalanced loads but they behave differently with respect to positive, negative, and zero (homopolar) stator currents sequences by $X_+ \rightarrow X_s > X_- > X_0$ reactances.

A limited degree of the negative current, $I_-/I_+ < 0.02 - 0.03$, is acceptable for SGs at power grids, to avoid overheating of rotor cages by induced currents at $2f_1$ frequency.

- Steady-state, 3-phase, 2-phase, and 1-phase short-circuit operations are not always dangerous for SMs but

$$I_{3sc} < I_{2sc} < I_{1sc}.$$

- The ratio $1/x_d = \frac{I_{3sc}}{I_n}$ is a specification parameter as it defines the maxim power capability of SMs.
- The SM has to be started as an induction motor, which then self-synchronizes at power grid. SMs may be operated at the leading power factor to compensate reactive power requirements in local power grids.
- SMs at power grid are characterized by efficiency and power factor versus output power, and by the V-shaped ($I_s(I_F)$) curves.
- PMSMs can operate at power grid or supplied by a PWM inverter at variable frequency (speed).

A very good product of the power factor and efficiency may be obtained with PMSMs even with a large number of pole pairs at a (low speed) medium-power range.

- Large anisotropy ($L_d/L_q > 3$)—multiple flux barrier rotor—motors may perform well at a low initial cost either at power grid (with a cage on the rotor) or fed from variable frequency PWM inverters, from 100 W to hundreds of kW.
- Single-phase PMSMs with $N_s/2p_1 = 1$, or with tooth-wound windings and no cage on the rotor, but with a rotor parking magnet (for starting) are used from subwatt to kilowatt power range in association with PWM inverters, for variable speeds.
- Alternatively, distributed ac stator 2-phase windings, cage rotor PMSMs in power grid direct connection may be used as split-phase capacitor motors for a strictly constant speed (with variable load), and better efficiency or a smaller motor volume.
- A few lab tests for SMs are introduced in this chapter and reference is made to the IEEE standard 115-1995 where a plethora of test procedures for acceptability, performance, and parameter estimation are described.
- A preliminary electromagnetic design methodology for a six-slot/four-pole, 3-phase PMSM for automotive applications is presented by a numerical example.
- Reference is made to other, special, configurations of SMs, rotary or linear, with pertinent literature suggested for further reading.

6.18 Proposed Problems

- 6.1** A salient pole rotor synchronous hydrogenerator that has $S_n = 72$ MVA, $V_{nline} = 13$ kV (star connection), $2p_1 = 90$ poles, $f_1 = 60$ Hz, and $q_1 = 3$ slots/pole/phase is considered lossless. The stator interior diameter $D_{is} = 13$ m, the stator stack length $l_{stack} = 1.4$ m, the air-gap under pole shoes $g = 20 \times 10^{-3}$ m, Carter coefficient $k_c = 1.15$, $\tau_p/\tau = 0.72$ (rotor pole shoe/pole pitch), and the saturation coefficient $k_s = 0.2$; it is a single-layer stator winding (diametrical span coils: $Y/\tau = 1$). It operates at the power grid.

Calculate

- a. The stator winding factor, k_{w1}
- b. The d and q magnetization reactances X_{dm} and X_{qm} if the number of turns per phase $W_s = p_1 \times q_1 \times n_c = 45 \times 3 \times 1 = 115$ turns/phase

- c. x_d and x_q in pu
- d. E_s , I_d , I_q , P_s , and Q_s for $\cos \varphi_1 = 1$ and $\delta_V = 30^\circ$
- e. The no-load airgap flux density, B_{gF} , and the corresponding excitation mmf per $W_F I_F$ required for it

Hints:

Use $k_{w1} = \sin \pi/6 (q \sin (\pi/6q))$, Equations 6.31 and 6.32,

$x_d = X_d \cdot V_{nph}/I_n$; $I_n = S_n / (\sqrt{3}V_{nl})$, Figure 6.19a, Equations 6.57 through 6.59, 6.15, and 6.6 and 6.7.

- 6.2** An aircraft lossless nonsalient pole rotor SG with the parameters $x_d = x_q = 0.6$ (pu), $S_n = 200$ KVA, $V_{nl} = 380$ V, $f_1 = 400$ Hz, and $2p_1 = 4$ operates at rated voltage on a balanced resistive load at rated current.

Calculate

- a. The rated phase current for star stator connection,
- b. X_d and X_q in Ω ,
- c. The load resistance, and
- d. E_1 , I_d , I_q , and δ_V (power angle).
- e. For rated current and 0.707 lagging power factor calculate I_d , I_q , I_s , and V_s (terminal voltage) for the same E_1 as above.
- f. Calculate the voltage regulation, ΔV_s , for (d) and (e).

Hints: See Equations 6.49 through 6.53 and Example 6.1.

- 6.3** For the SG in Problem 6.2, calculate the V-shaped curves (I_s (I_F)) if

$$E_1 = aI_F - bI_F^2 \quad (6.164)$$

and if E_1 is as in problem 6.2 for $I_F = 50$ A

and if E_1 is $E_1/2$ for $I_F = 15$ A.

Use $I_F = 10, 20, 30, 40, 50, 60$, and 70 A

Hints: Use (6.164) for the no-load saturation curve to find a and b and then, for $P_s/S_n = 0.0, 0.3, 0.6$, and 1.0 , calculate δ_V from Equations 6.57 and 6.58 for I_F given (E_1 from (6.164)) in 5 A steps from 10 to 70 A.

- 6.4** A nonsalient pole rotor synchronous autonomous generator with $S_n = 50$ KVA, $V_{nl} = 440$ V (star connection), $f_1 = 60$ Hz, $2p_1 = 2$, and $x_s = x_d = x_q = 0.6$ PU, $x_- = 0.2$ PU, and $x_0 = 0.15$ PU operates at rated current with only phase A connected to a resistive load ($I_B = I_C = 0$) for no-load phase voltage $E_1 = 300$ V (RMS).

Calculate

- The rated phase current I_n , X_s , X_+ , X_- , and X_0 in Ω
- I_{A+} , I_{A-} , and I_{A0}
- V_A (phase voltage for phase A)
- V_B and V_C (phase voltages on open phases B and C)

Hints: See Equations 6.75 through 6.80, Equation 6.85, and Equation 6.77.

- 6.5** A PMSM with surface PM poles is fed from a PWM inverter at variable frequency.

For $V_{nl} = 380$ V (star connection), $f_n = 50$ Hz, $X_d = X_q = 6 \Omega$, and $R_s = 1 \Omega$, calculate I_d , I_q , I_s , P_e , T_e , η , and $\cos \varphi$ for $\delta_V = 0^\circ, 15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ$, and 90° . Perform the same calculations for $V_{nl} = 100$ V and $f = 14$ Hz.

Hint: See Example 6.4.

- 6.6** A multiple flux barrier four-pole rotor PMSM has weak (ferrite) PMs in the flux barriers (Figure 6.47) and the parameters $L_d = 200$ mH, $L_q = 60$ mH, $\Psi_{PMq} = 0.215$ Wb, $2p_1 = 4$ poles, $(V_n)_{\text{phase}} = 220$ V, and zero losses. Calculate I_d , I_q , I_s , T_e , and $\cos \varphi$ versus $\delta_V = 0^\circ, 10^\circ, 30^\circ, 45^\circ, 60^\circ, 70^\circ, 80^\circ, 90^\circ$, and 100° , for $n = 1500$ rpm.

Hints: Notice that the PMs are placed in axis q , and thus the phasor diagram is different from that of Figure 6.37 (see Figure 6.47b).

So,

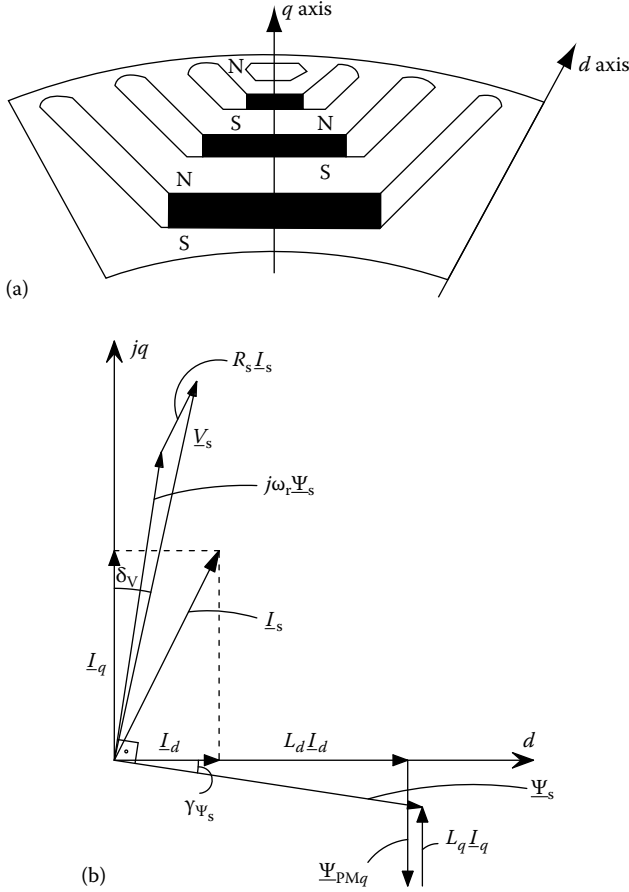
$$V_s \sin \delta_V = \omega_r L_q I_q - \Psi_{PMq} \omega_r ; R_s \approx 0$$

$$V_s \cos \delta_V = \omega_r L_d I_d$$

- 6.7** A 3-phase lossless PMSG with the data $E_1 = 250$ V(phase, RMS), $V_{nl} = 220\sqrt{3}$ V(star connection), $X_d = 10 \Omega$, $X_q = 20 \Omega$, $2p_1 = 4$, and $f_1 = 60$ Hz is driven at constant speed and balanced with a resistive load. Calculate the variation of voltage with power angle, δ_V , until the voltage regulation becomes zero again because of inverse saliency ($V_s = E_1$), and the corresponding delivered power.

Hints: Use Equation 6.50 and Figure 6.24b.

- 6.8** A large lossless SM has the data $P_n = 5$ MW, $f_1 = 60$ Hz, $(V_n)_{\text{phase}} = 2.2$ kV, $2p_1 = 2$ poles, $x_d = x_q = x_s = 0.6$ (pu), an inertia $H = \frac{1}{2} \left(\frac{\omega_1}{p_1} \right)^2 \times \frac{1}{p_n} = 10$ s, and $E_1 = 2.5$ kV (phase).

**FIGURE 6.47**

(a) PM-assisted RSM and (b) its phasor diagram.

Calculate

- The rated current, I_n , and X_s (in Ω)
- I_d , I_q , I_s , P_s , and Q_s at $\delta_V = 30^\circ$
- The electromagnetic torque (T_e) at $\delta_V = 30^\circ$
- The asynchronous torque coefficient K_a if at $S = 0.01$, the asynchronous torque $T_{as} = 0.3 \times (T_e)_{\delta_V=30^\circ}$
- The eigen frequency of the SG, ω_{0n} , and its variation when E_1 changes from 2.5 to 1.25 kV at $\delta_V = 30^\circ$
- The module of mechanical resonance, $K_{m\gamma}$, for $\omega_\gamma = 2\pi \times n_1/2$ and for ω_{0n} ; n_1 is the machine speed

Hints: See Section 6.13.

- 6.9 A single-phase $2p_1 = 2$ pole PMSM (driven by a PWM inverter and having a parking PM for safe starting) has the data $V_{sn} = 120 \text{ V}$, $f_1 = 60 \text{ Hz}$, the PM emf $E_s = 0.95V_{sn}$, $R_s = 3 \Omega$, and $L_s = 0.05 \text{ H}$.

Calculate

- The synchronous speed at $f_1 = 60 \text{ Hz}$
- The stator current at $\delta_V = 0^\circ, 15^\circ, 30^\circ, 45^\circ, 60^\circ$, and 90°
- The power factor and efficiency (only copper losses count for δ_V as under (b))
- The average torque versus $\delta_V\%$ if cogging torque is neglected

Hints: See in Section 6.15, Equations 6.123 through 6.134 and the computation routine described there.

- 6.10 Redo the preliminary 3-phase PMSM design in Section 6.16 for the specifications $P_b = 2000 \text{ W}$, $n_b = 1800 \text{ rpm}$, $n_{\max} = 3000 \text{ rpm}$ (at P_b), and $V_{dc} = 42 \text{ V}$.

Hints: Follow Section 6.16.

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Part II

Transients

Advanced Models for Electric Machines

7.1 Introduction

In Part I, only electrical machines under steady-state operation were treated. Under steady state, speed, terminal voltages, currents, amplitudes, and their frequency remain constant. In reality, all electric machines undergo speed, voltage, current, amplitude, and frequency variations under transients at (or when connected to) power grid and when associated with PWM static converters for variable speed close-loop control operation both in generator and motor modes.

Transients may also be preferred to estimate electric machine parameters, by the analysis of the corresponding machine response. This chapter introduces a few general electric machine models used to handle machine transients.

There are two main categories of machine models:

- Circuit models
- Field-circuit coupled models

Another classification of machine models is

- Fundamental frequency models
- Super high-frequency models: when stray capacitors inside the machine are considered—for instance, for switching frequency when a PWM static converter supplies the machine.

The main circuit models are

- Phase-coordinate circuit models
- Orthogonal (dq)—space phasor (complex variable)—models [1–6]

The magnetic field-to-circuit models have gained widespread acceptance:

- Analytical field models: In simplified form, they are used to derive circuit models [7,8] for steady-state operations as discussed in Part I.
- Finite element models [7,8]: Numerical integral field models, with field-to-circuit coupling to investigate electric machine operation

under steady state or transients while considering most (almost) secondary effects (such as slot openings, machine load saturation, skin effect). Part III of this book dwells extendedly on FEM as it is paramount in all design refinements of electric machines and drives.

- Multiple magnetic circuit models: The machine zones are divided into region permeances with the same flux density; they may be 3D and they consider the placement of windings in slots and their connections, slot openings, magnetic saturation, but, in a coarser way, with the advantage of at least 10 times lower computation time on similar CPUs [9].

Other models, such as spiral vector theory [10] have been proposed but have not gained widespread acceptance.

Of the above models we will discuss here thoroughly the physical orthogonal model and its spin-off, the space vector (complex variable) model, as they are applied extensively to investigate electric machine transients in power systems and in variable speed power electronics control of electric drives. The dq model may also be introduced mathematically, by a transformation of variables, or through a physical model (when the model may be built and tested).

We choose the physical model because it is believed to be more intuitive.

7.2 Orthogonal (dq) Physical Model

The dq physical model of electric machines is shown in Figure 7.1.

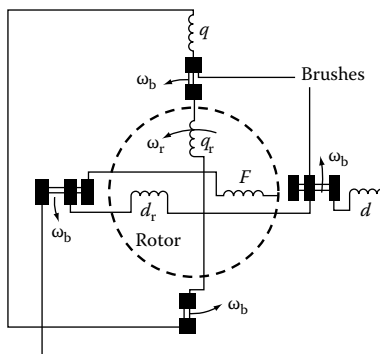


FIGURE 7.1

The dq physical model.

We call it a physical model because it can be actually built (in the United Kingdom it was used for teaching until recently).

The dq physical model as introduced here is provided with brush-commutator windings on the stator and the rotor with brushes along two orthogonal axes d and q . There are two such windings on the stator (d and q) and three such windings (d_r , F , along axis d , and q_r along axis q) on the rotor. The brushes of all windings are aligned to the orthogonal axes d and q . It is known that the armature field axis in brush-commutator machines falls along the brush axis for asymmetrical rotor coils.

It follows that the mmfs and the corresponding airgap fields of all windings are mostly along brush axes and are at a standstill during all operation modes.

On the other hand, the coils of stator windings are at a standstill while the rotor coils travel at speed ω_r , where the axes of their fields travel at brush speed ω_b . The brushes are rotated at ω_b by a small power drive.

We should mention here that in real brush-commutator windings, the airgap field is triangular rather than sinusoidal; only its fundamental is considered here. Let us observe also that for all orthogonal windings, self and mutual inductances are independent of rotor position if and only if the brushes are attached to the machine part (stator and rotor) with magnetic anisotropy (salient poles).

In this way we obtain a system of differential equations for the dq physical models that have constant coefficients (inductances).

This is an extraordinary simplification in treating transients of electric machines but we have to work out the equivalence between the dq physical model and the actual electric machine.

Let us now pursue the restrictions in terms of brush speed, ω_b , for synchronous brush-commutators and induction machines.

If the electric machine has a rotor magnetic saliency, such as in synchronous machines, it is mandatory to have $\omega_b = \omega_r$, in order to obtain rotor position independence of dq model parameters (inductances).

The dq model in Figure 7.1 with $\omega_b = \omega_r$ refers, in fact, directly to the synchronous machine. If the stator shows magnetic saliency—as in brush-commutator machines— $\omega_b = 0$ and all but the d -axis winding in the stator and the q_r winding in the rotor are eliminated.

For machines where the airgap is uniform (induction machines) for any speed of brushes (axes) ω_b , the dq-model inductances will be independent of rotor position. However, $\omega_b = 0$, ω_r , ω_1 are the main axes speed that are applied extensively.

The brush-commutators in the dq physical model (Figure 7.1) are the physical correspondent of coordinate (or variable) mathematical transformation that leads to same dq model.

When can the dq model can model properly actual 3(2)-phase synchronous and induction machines?

As is evident from Figure 7.1, the d and q windings, on the machine part that is moving in accordance with dq axes speed, ω_b , should be symmetric.

Also, the magnetic field (not necessarily the mmf) of all windings of the dq model (and of the real machine) should have a sinusoidal spatial distribution along the rotor periphery. Space harmonics of order ν of this field may be treated by adding additional dq windings with their dq axes moving at different speeds $\omega_{b\nu}$.

Can double magnetic saliency machines (switched reluctance machines) be treated via the dq model?

Yes, but only if they have no winding in the rotor and if their stator winding inductances vary sinusoidally with the rotor position (as in synchronous machines, but with almost zero (in 3-phase SRM) mutual inductances). In machines with PMs, the PM field should create sinusoidal emfs in the stator windings to make the dq model adequate for the scope.

So the dq model may be applied not only to distributed ac winding machines (synchronous and induction) but also to nonoverlapping coil stator winding machines (SRM and PMSM alike), provided there are no rotor windings and the stator winding inductances vary sinusoidally with the rotor position and so do, in time, the PM-produced emfs.

To document the above claims we will investigate the concept of transformer (pulsational) and motion-induced voltages.

7.3 Pulsational and Motion-Induced Voltages in dq Models

The total induced voltage in coupled electric circuits is generated by two types of voltages: one by pulsational (transformer) action, E_p , and the other by relative motion between conductors and magnetic fields, E_m . Let us consider the flux linkage of a winding $\Psi(\theta, t)$ as dependent on position and time.

The total emf E_t is

$$E_t = -\frac{d_s \Psi(\theta, t)}{dt} = E_p + E_m \quad (7.1)$$

$$E_p = -\frac{\partial \Psi(\theta, t)}{\partial t}; \quad E_m = -\frac{\partial \Psi(\theta, t)}{\partial \theta_r} \cdot \frac{d\theta_r}{dt} \quad (7.2)$$

Note: In linear electric machines, the rotor position angle, θ_r , is replaced by the linear positive, x .

Let us now assume that the flux linkage of the windings along the two axes varies sinusoidally with θ_r :

$$\Psi_d(\theta_r) = \Psi_m \sin \theta = \frac{\partial \Psi_q}{\partial \theta}; \quad \Psi_q(\theta_r) = -\Psi_m \cos \theta = -\frac{\partial \Psi_d}{\partial \theta} \quad (7.3)$$

Axis q in Figure 7.1 is ahead of axis d along the direction of rotation (of ω_r). Conditions (7.3) lead to the idea that the motion emf in one orthogonal axis is produced by the flux linkage in the other orthogonal axis.

However, both windings have the same maximum flux linkage, so they have to be symmetric as well. This condition is valid only if the dq axes are moving in accordance with those dq windings. It has already been discussed in Part I that only the motion emfs contribute to the torque production.

7.4 dq Model of DC Brush PM Motor ($\omega_b = 0$)

Let us reconsider here the case of the dc brush PM motor (Figure 7.2).

The dq model of this machine is obtained by eliminating all windings in Figure 7.1 but the q axis winding in the rotor (the armature winding) and by introducing a PM along axis d in the stator for the magnetic field production.

The magnetic saliency is on the stator (the PM in this case) and thus the brushes are fixed as they are in reality; thus the stator windings do not need to have a brush-commutator. We may now forget about the brushes (Figure 7.2b) by noticing that the axis of the rotor winding field is fixed along axis q . The PM induces an emf in the rotor by motion.

We end up with one voltage equation:

$$V_{qr} = R_a I_{qr} + \frac{\partial \Psi_{qr}}{\partial t} + \frac{d\Psi_{qr}}{d\theta_r} \cdot \frac{d\theta_r}{dt} \quad (7.4)$$

But $d\theta_r/dt$ refers to the relative motion between the conductors of the respective windings and the external field:

$$\frac{d\theta_r}{dt} = \omega_r - \omega_b = \omega_r \quad (7.5)$$

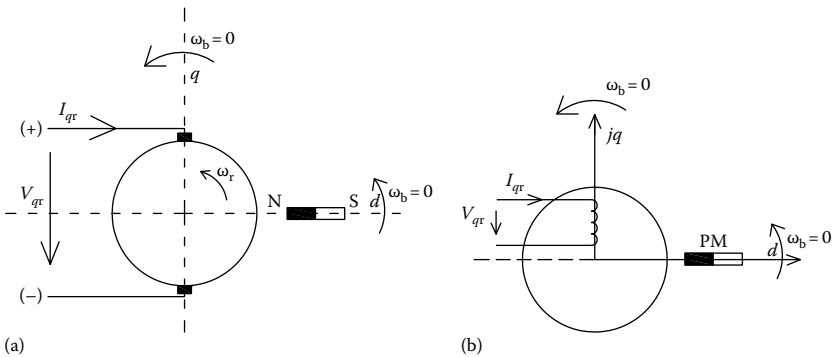


FIGURE 7.2

The dc brush PM motor (a) and its dq model (b).

Also (7.4)

$$\frac{\partial \Psi_{qr}}{\partial \theta_r} = \Psi_{dr} = \Psi_{PM}; \quad \Psi_Q = L_a I_{qr} \quad (7.6)$$

Let us note that Ψ_{dr} corresponds to the flux linkage of a fictitious rotor winding identical to q_r but placed along axis d_r . As this, in fact, does not exist, its flux linkage comes directly from the PM flux produced along axis d in the stator.

So Equation 7.4 becomes

$$V_{qr} = R_a I_{qr} + L_a \frac{di_{qr}}{dt} + \Psi_{PM} \omega_r \quad (7.7)$$

Let us now multiply Equation 7.7 by i_{qr} :

$$\begin{array}{llll} V_{qr} I_{qr} & = R_a I_{qr}^2 & + L_a I_a \frac{\partial I_{qr}}{\partial t} & + \Psi_{PM} \omega_r i_{qr} \\ \text{Input} & \text{Copper} & \text{Magnetic} & \text{Electromagnetic} \\ \text{power} & \text{power} & \text{energy} & \text{power } P_e \\ & \text{losses} & \text{variation} & \end{array} \quad (7.8)$$

Note that ω_r is the electrical rotor angular speed $\omega_r = p_1 \Omega_1 = p_1 2\pi n$; n is the rotor speed in rps; and p_1 is the pole pair of the motor.

From the electromagnetic power, expression P_e , the torque, T_e , is straightforward:

$$T_e = \frac{P_e}{\Omega_r} = p_1 \Psi_{PM} i_{qr} \quad (7.9)$$

In Equation 7.4 we had

$$T_e = \frac{p_1 N}{a} \cdot \frac{\Phi_p i_{q1}}{2\pi} = p_1 \Psi_{PM} i_{qr} \quad (7.10)$$

N is the total number of conductors in rotor slots, $2a$ is the current path count, and Φ_p is the pole flux.

So

$$\Psi_{PM} = \frac{N}{a} \cdot \frac{\Phi_p}{2\pi} \quad (7.11)$$

The dc brush machine is a simplified version of the dq model!

7.5 Basic dq Model of Synchronous Machines ($\omega_b = \omega_r$)

For synchronous machines (SMs), in general, the dq axes are fixed to the rotor ($\omega_b = \omega_r$), and thus there will be no motion-induced voltages in the rotor circuits, which are by construction asymmetric (Figure 7.3).

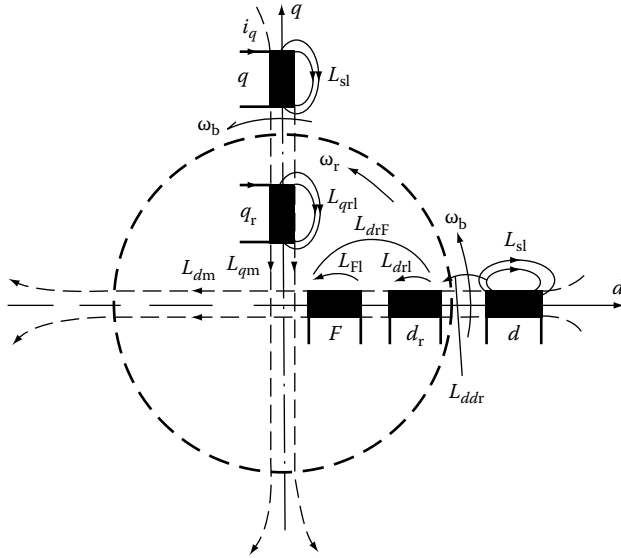


FIGURE 7.3
The dq model of SM.

The motion emfs in the stator are proportional to $(-\omega_b)$ as this is the speed of the stator coils with respect to the magnetic fields of dq axes.

So the stator equations are straightforward:

$$V_d = R_s I_d + \frac{\partial \Psi_d}{\partial t} - \omega_r \Psi_q; \quad \frac{\partial \Psi_d}{\partial \theta_{er}} = -\Psi_q; \quad \frac{d\theta_{er}}{dt} = \omega_r \quad (7.12)$$

$$V_q = R_s I_q + \frac{\partial \Psi_q}{\partial t} + \omega_r \Psi_d; \quad \frac{\partial \Psi_q}{\partial \theta_{er}} = \Psi_d \quad (7.13)$$

$$V_F = R_F I_F + \frac{\partial \Psi_F}{\partial t} \quad (7.14)$$

$$0 = R_{dr}I_{dr} + \frac{\partial \Psi_{dr}}{\partial t} \quad (7.15)$$

$$0 = R_{qr}I_{qr} + \frac{\partial \Psi_{qr}}{\partial t} \quad (7.16)$$

After multiplying Equation 7.12 by I_d and Equation 7.13 by I_q , and after adding the two we obtain

$$\begin{array}{ccccccc}
 V_d I_d + V_q I_q & = R_s (I_d^2 + I_q^2) & + I_d \frac{\partial \Psi_d}{\partial t} + I_q \frac{\partial \Psi_q}{\partial t} & + \omega_r (\Psi_d I_q - \Psi_q I_d) & \\
 \text{Input} & \text{Copper} & \text{Magnetic} & \text{Electromagnetic} & (7.17) \\
 \text{power} & \text{losses} & \text{energy} & \text{power} & \\
 & & \text{variation} & &
 \end{array}$$

But P_e is

$$P_e = T_e \frac{\omega_1}{p_1}; \quad T_e = p_1(\Psi_d I_q - \Psi_q I_d) \quad (7.18)$$

We may add the motion equations to complete the set:

$$\frac{J}{p_1} \frac{d\omega_r}{dt} = T_e - T_{\text{load}}; \quad \frac{d\theta_{er}}{dt} = \omega_r \quad (7.19)$$

We still have to add the flux linkage-currents relationships. Due to orthogonality of the axes, in the absence of magnetic saturation, there is no magnetic coupling between axes d and q and thus

$$\begin{aligned} \Psi_d &= L_{sl} I_d + \Psi_{dm}; & \Psi_{dr} &= L_{drl} I_{dr} + \Psi_{dm} + L_{drF}(I_{dr} + I_F) \\ \Psi_F &= L_{Fl} I_F + \Psi_{dm} + L_{drF}(I_{dr} + I_F); & \Psi_{dm} &= L_{dm}(I_d + I_{dr} + I_F) = L_{dm} I_{dm} \\ \Psi_q &= L_{sl} I_q + \Psi_{qm}; & \Psi_{qr} &= L_{qrl} I_{qr} + \Psi_{qm}; & \Psi_{qm} &= L_{qm}(I_q + I_{qr}) = L_{qm} I_{qm} \end{aligned} \quad (7.20)$$

From Equations 7.12 through 7.20 we decipher an eighth order system with the variables as six currents, ω_r , and θ_{er} and the inputs as V_d, V_q, V_F , and T_{load} . It is also feasible to use six flux variables with currents as dummy variables, etc.

7.6 Basic dq Model of Induction Machines

$$(\omega_b = 0, \omega_r, \omega_1)$$

Induction machines with symmetric slots and rotor windings are considered here. They have two stator and two rotor symmetric orthogonal windings (Figure 7.4). By now, we know from Chapter 5 that two symmetric

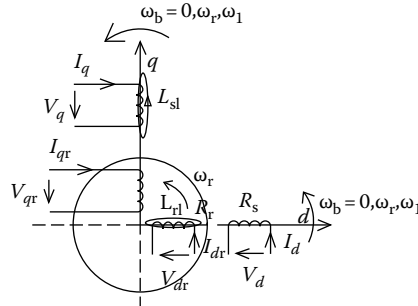


FIGURE 7.4

The basic dq model of IMs.

orthogonal windings are equivalent to three windings at 120° , as in induction machines.

Now the dq axes speed is without constraints, ω_b .

There are motion-induced voltages in the stator (by ω_b) and rotor (by $\omega_b - \omega_r$):

$$\begin{aligned} V_d &= R_s I_d + \frac{\partial \Psi_d}{\partial t} - \omega_b \Psi_q \\ V_q &= R_s I_q + \frac{\partial \Psi_q}{\partial t} + \omega_b \Psi_d \\ V_{dr} &= R_r I_{dr} + \frac{\partial \Psi_{dr}}{\partial t} - (\omega_b - \omega_r) \Psi_{qr} \\ V_{qr} &= R_r I_{qr} + \frac{\partial \Psi_{qr}}{\partial t} + (\omega_b - \omega_r) \Psi_{dr} \end{aligned} \quad (7.21)$$

$$\begin{aligned} T_e &= p_1 (\Psi_d I_q - \Psi_q I_d) \\ \frac{J}{p_1} \frac{d\omega_r}{dt} &= T_e - T_{\text{load}}; \quad \frac{d\theta_{er}}{dt} = \omega_r \end{aligned}$$

$$\begin{aligned} \Psi_d &= L_{sl} I_d + \Psi_{dm}; & \Psi_{dr} &= L_{drl} I_{dr} + \Psi_{dm}; & \Psi_{dm} &= L_m I_{dm}; & I_{dm} &= I_d + I_{dr} \\ \Psi_q &= L_{sl} I_q + \Psi_{qm}; & \Psi_{qr} &= L_{qrl} I_{qr} + \Psi_{qm}; & \Psi_{qm} &= L_m I_{qm}; & I_{qm} &= I_q + I_{qr} \end{aligned} \quad (7.22)$$

The torque may also be calculated from rotor equations as

$$T_e = -p_1 (\Psi_{dr} I_{qr} - \Psi_{qr} I_{dr}) \quad (7.23)$$

The sign $(-)$ appears as T_e in Equation 7.23 refers to the torque on the rotor. For the cage rotor, Equation 7.23 is obtained with $V_{dr} = V_{qr} = 0$, while for wound rotor $V_{dr} \neq V_{qr} \neq 0$.

7.7 Magnetic Saturation in dq Models

Flux-current relationships, Equations 7.20 and 7.22, exhibit magnetization inductances L_{dm} , L_{qm} , L_m and leakage inductances L_{sl} , L_{rl} . Magnetic saturation influences both, particularly main inductances, with the exception of the closed-rotor slot configurations or for very large stator (rotor) currents when leakage flux paths saturate as well. For inclusion of magnetic saturation in the dq model, presupposes to notice the type of magnetization in the stator and rotor of main electric machine cores (Table 7.1).

TABLE 7.1

Type of Magnetization	SM	IM	DC Brush Machine
dq model coordinates	$\omega_b = \omega_r$	$\omega_b = \omega_1$	$\omega_b = 0$
Rotor	DC	AC at $\omega_2 = \omega_1 - \omega_r$	AC at $\omega_2 = \omega_r$
Stator	AC at $\omega_1 = \omega_r$	AC at ω_1	DC

Consequently, magnetic saturation will manifest itself differently in the stator and rotor of various electric machines and so will core losses. However, in the dq model of all machines, as shown in Table 7.1, under steady state, voltages, currents, flux linkages are all dc quantities.

This is optimum for a control system design.

Among the many models that include magnetic saturation in the dq model of electric machines, we present here only the model of distinct and unique d, q magnetization curves, $\Psi_{dm}(i_m)$ and $\Psi_{qm}(i_m)$, as shown in Figure 7.5.

In other words, the main (magnetization) inductances, L_{dm} , L_{qm} , in axes d and q depend only on the total magnetization current, I_m :

$$I_m = \sqrt{I_{dm}^2 + I_{qm}^2}; \quad I_{dm} = I_d + I_{dr} + I_F; \quad I_{qm} = I_q + I_{qr} \quad (7.24)$$

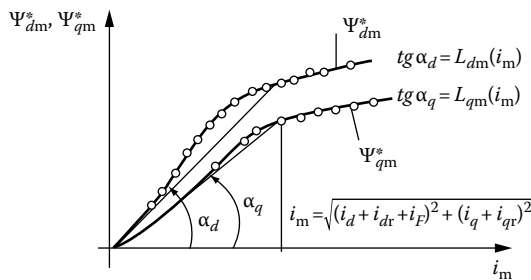
$$\Psi_{dm} = L_{dm}(I_m) \cdot I_{dm}; \quad \Psi_{qm} = L_{qm}(I_m) \cdot I_{qm} \quad (7.25)$$

with

$$L_{dm} = \frac{\Psi_{dm}^*(I_m)}{I_m}; \quad L_{qm} = \frac{\Psi_{qm}^*(I_m)}{I_m} \quad (7.26)$$

Ψ_{dm}^* , Ψ_{qm}^* are measured (or calculated) values.

L_{dm} , L_{qm} are related to the normal permeability, μ_n , in the core, as shown in Figure 7.5. They are valid for steady state (exactly so only in the dc rotor of SMs).

**FIGURE 7.5**

Distinct unique d,q magnetic curves.

For transients, the time derivatives of Ψ_{dm} and Ψ_{qm} are required. From Equations 7.24 through 7.26:

$$\frac{d\Psi_{dm}}{dt} = \frac{d\Psi_{dm}^*}{di_m} \cdot \frac{di_m}{dt} \cdot \frac{i_{dm}}{i_m} + \frac{\Psi_{dm}^*}{i_m^2} \left(i_m \frac{di_m}{dt} - i_{dm} \frac{di_m}{dt} \right) \quad (7.27)$$

$$\frac{d\Psi_{qm}}{dt} = \frac{d\Psi_{qm}^*}{di_m} \cdot \frac{di_m}{dt} \cdot \frac{i_{qm}}{i_m} + \frac{\Psi_{qm}^*}{i_m^2} \left(i_m \frac{di_{qm}}{dt} - i_{qm} \frac{di_m}{dt} \right) \quad (7.28)$$

$$\frac{di_m}{dt} = \frac{i_{qm}}{i_m} \cdot \frac{di_{qm}}{dt} + \frac{i_{dm}}{i_m} \cdot \frac{di_{dm}}{dt} \quad (7.29)$$

So, finally

$$\begin{aligned} \frac{d\Psi_{dm}}{dt} &= L_{ddm} \frac{di_{dm}}{dt} + L_{qdm} \frac{di_{qm}}{dt} \\ \frac{d\Psi_{qm}}{dt} &= L_{qdm} \frac{di_{qm}}{dt} + L_{qqm} \frac{di_{qm}}{dt} \end{aligned} \quad (7.30)$$

with

$$\begin{aligned} L_{ddm} &= L_{dmt} \frac{i_{dm}^2}{i_m^2} + L_{dm} \frac{i_{qm}^2}{i_m^2} \\ L_{qqm} &= L_{qmt} \frac{i_{qm}^2}{i_m^2} + L_{qm} \frac{i_{dm}^2}{i_m^2} \\ L_{dqm} &= (L_{dmt} - L_{dm}) \frac{i_{dm} i_{qm}}{i_m^2} = (L_{qmt} - L_{qm}) \frac{i_{dm} i_{qm}}{i_m^2} = L_{qdm} \\ L_{dmt} &= \frac{d\Psi_{dm}^*}{di_m}; \quad L_{qmt} = \frac{d\Psi_{qm}^*}{di_m} \end{aligned} \quad (7.31)$$

$L_{dqm} = L_{qdm}$; $(L_{dmt} - L_{dm} = L_{qmt} - L_{qm})$ because of reciprocity theorem. The above equations give rise to the following observations:

- The magnetization flux along each of the d and q axes is influenced by both currents i_d and i_q , both under steady state and transients; but, in this model, unique d and q magnetization curves exist. Only for the underexcited large synchronous machine, the model was found less acceptable, through the FEM [8].
- For transients ($d\Psi_{dm}/dt \neq 0$ and/or $d\Psi_{qm}/dt \neq 0$), there is a kind of magnetic coupling between the two orthogonal axes due to magnetic saturation.
- The magnetic coupling inductance ($L_{qdm} \neq 0$) occurs only in the absence of magnetic saturation ($L_{dm} = L_{dmt}$) or when either $i_d = 0$ or $i_q = 0$.
- For standstill operations at small ac p.u. currents, the transient inductances L_{dmt} , L_{qmt} are replaced by incremental inductances L_{dmi} , L_{qmi} :

$$L_{dmi} = \frac{\Delta\Psi_{dm}^*}{\Delta i}; \quad L_{qmi} = \frac{\Delta\Psi_{qm}^*}{\Delta i} \quad (7.32)$$

found from the local (small-amplitude) hysteresis cycle incremental permeabilities $\mu_i \approx (120 - 150)\mu_0$.

- Magnetic saturation is worth accounting for in modern, stressed-to-the-limit, electric machines.
- To consider magnetic saturation, we need to apply Equations 7.28 through 7.31 to the dq model of SM or IM and then introduce, perhaps, flux variables, but with Ψ_{dm} , Ψ_{qm} as stator variables instead of Ψ_d , Ψ_q , to handle the solution of the system of equations easily [11,12].

7.8 Frequency (Skin) Effect Consideration in dq Models

The skin (frequency) effect on rotor cage bars is known. It translates into an increase in rotor resistance R_r by $k_R > 1$ and a decrease in the rotor leakage inductance L_{rl} by $k_X < 1$, as the rotor frequency, ω_2 , increases. Large SGs exhibit strong skin effect in the solid iron rotor core (for nonsalient two pole configurations) or in the rotor damping cage.

The skin effect in the electric conductors is influenced by magnetic saturation also, but accounting for this influence is not considered here. We thus implicitly separate magnetic saturation from the skin effect.

To include the skin effect in the dq model, we introduce 2,3 fictitious cage constant R_{dr} , L_{ldr} parameters circuits in parallel such that their frequency response matches to an assigned global error the real machine with variable rotor parameters (Figure 7.6).

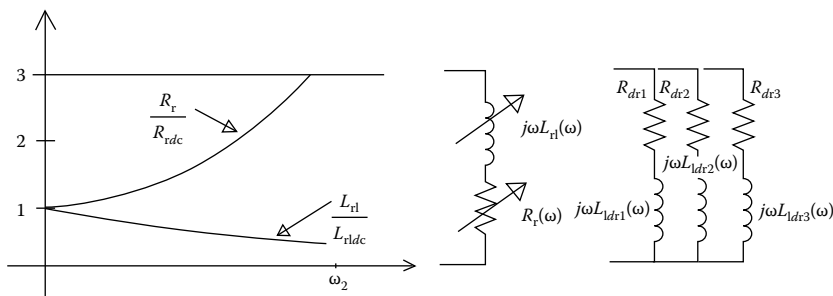


FIGURE 7.6

From single circuits with variable parameters to multiple circuits with constant parameters.

The multiple circuit parameters may be calculated by FEM or from the frequency response standstill test by regression methods.

In machines with strong core losses, the latter may be introduced in the dq model by adding two more orthogonal short-circuited stator windings (on the machine side with ac magnetization), whose leakage inductance may be neglected and whose resistances are calculated from measured or calculated iron losses in the machine [11].

Note: By including magnetic saturation, frequency effects, and core losses, the dq model has been enhanced considerably.

7.9 Equivalence between dq Models and AC Machines

The dq model is a fair representation of the actual ac machine if it behaves like the latter in terms of torque, power, speed, and losses.

Equivalence conditions are thus necessary.

Conservation of mmf fundamentals is an implicitly powerful equivalence condition. Let us project the A, B, C stator phase mmfs along the d and q axes (Figure 7.7):

$$\begin{aligned} F_d &= W_1 K_{w1} [i_A \cos(-\theta_{eb}) + i_B \cos(-\theta_{eb} + 2\pi/3) + i_C \cos(-\theta_{eb} - 2\pi/3)] \\ F_q &= W_1 K_{w1} [i_A \sin(-\theta_{eb}) + i_B \sin(-\theta_{eb} + 2\pi/3) + i_C \sin(-\theta_{eb} - 2\pi/3)] \end{aligned} \quad (7.33)$$

For

$$\begin{aligned} F_d &= W_d K_{wd} i_d \\ F_q &= W_d K_{wd} i_q \end{aligned} \quad (7.34)$$

In practice

$$\frac{W_1 K_{w1}}{W_d K_{wd}} = \frac{2}{3} \text{ or } \sqrt{\frac{2}{3}} \quad (7.35)$$

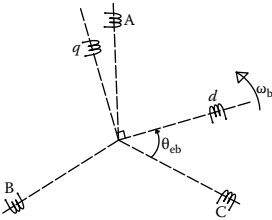


FIGURE 7.7
3-phase ac-dq mmf equivalence.

As can be shown easily, the $\sqrt{\frac{2}{3}}$ ratio conserves the power—from the dq model to the actual machine—while for the ratio $\frac{2}{3}$, the $\frac{3}{2}$ dq model power equals the actual active power of the machine; the same is true for torque equivalence.

Let us consider the $\frac{2}{3}$ ratio, which is extendedly used for electric machine control.

The equivalence between the 2-phase dq model and the 3-phase ac machine requires one more variable: the zero component, or V_0 , i_0 , Ψ_0 :

So, the so-called generalized Park transformation for the 3-phase stator, $S(\theta_{eb})$, is the same for currents, voltages, and flux linkages:

$$\begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} = |S_{dq0}| \cdot \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix}; \quad \theta_{eb} = \int \omega_b dt; \quad \frac{d\theta_{eb}}{dt} = \omega_b \quad (7.36)$$

$$|S_{dq0}| = \frac{2}{3} \begin{vmatrix} \cos(-\theta_{eb}) & \cos(-\theta_{eb} + 2\pi/3) & \cos(-\theta_{eb} - 2\pi/3) \\ \sin(-\theta_{eb}) & \sin(-\theta_{eb} + 2\pi/3) & \sin(-\theta_{eb} - 2\pi/3) \\ 1/2 & 1/2 & 1/2 \end{vmatrix} \quad (7.37)$$

The zero-sequence component, i_0 in Equation 7.36 is

$$i_0 = \frac{(i_A + i_B + i_C)}{3} \quad (7.38)$$

It does not contribute to the traveling field and thus interacts with the leakage inductance and resistance of stator phases:

$$V_{0s} \approx R_s + L_{sl} \frac{di_{0s}}{dt} \quad (7.39)$$

For symmetric steady state and transients $i_0 = 0$; it is also zero for a star connection anyway.

Note: The Park transformation may be extended to m phases ($m_1 = 6, 9, 12, \dots$) when the angle in Equation 7.37 is $2\pi/m_1$ and the number of lines is m_1 .

For every three symmetric phases, a zero sequence component is required for full (mathematical) equivalence, unless a separated star connection is applied for all three symmetric phases.

(For 2×3 phases with separate null points we will have only d_1q_1, d_2q_2).

The inverse matrix is

$$[S_{dq0}]^{-1} = \frac{3}{2} [S_{dq0}]^t \quad (7.40)$$

Let us calculate the power in the dq0 model:

$$P_{dq0} = [V_{dq0}]^t [I_{dq0}] = V_d I_d + V_q I_q + V_0 I_0 \quad (7.41)$$

It follows that

$$\begin{aligned} \frac{3}{2} P_{dq0} &= \frac{3}{2} [V_{ABC}]^T [S_{dq0}]^T [S_{dq0}] [I_{ABC}] = [V_{ABC}]^T [I_{ABC}] = P_{ABC} \\ &= V_a I_a + V_b I_b + V_c I_c \end{aligned} \quad (7.42)$$

This equivalence refers to the active instantaneous power and thus also to the instantaneous torque (see Ref. [5] for a detailed derivation of Equation 7.42):

$$\frac{3}{2}T_{\text{edq0}} = T_{\text{eABC}} = \frac{3}{2}p_1(\psi_d i_q - \psi_q i_d) \quad (7.43)$$

Note: At steady state in rotor coordinates ($\omega_b = \omega_r$) for SMs and synchronous coordinates ($\omega_b = \omega_1$) for IMs, voltages and currents are dc and thus the dq model does not show any reactive power.

However, it may be demonstrated that the reactive power, Q_{abc} , of the actual ac 3-phase machine is represented in the dq model by

$$Q_{\text{ABC}} = -\frac{3}{2}(V_d I_q - V_q I_d) \quad (7.44)$$

Also, stator copper losses are

$$p_{\text{copper}} = 3R_s(I_d^2 + I_q^2)/2 \quad (7.45)$$

R_s stays the same for the dq0 model as for the three ac phases.

The damping cage and the field winding in SMs are similar (orthogonal), both in the actual machine and in the dq0 model.

For the wound rotor induction machine, which has three phases in the rotor, a similar Park transformation as for the stator is applied, but, instead of θ_{es} , the d axis coordinate angle, θ_{ebr} , is

$$\theta_{\text{ebr}} = \int (\omega_b - \omega_r) dt; \quad \frac{d\theta_{\text{ebr}}}{dt} = (\omega_b - \omega_r) \quad (7.46)$$

because the rotor conductors rotate at $\omega_b - \omega_r$ with respect to the dq axes (coordinates) at a speed ω_b .

So far we have defined the voltage, current, and flux linkage equivalence between three ac phase windings, and the dq0 model with the power–torque losses relationships. The only point left out is the relationship between the three phase ac windings and the dq model inductances. This will be done in the following chapters. We will now introduce the space phasor (complex variable) model.

7.10 Space Phasor (Complex Variable) Model

The space phasor (or complex variable) model may be derived from the dq0 model because it is based on the same assumptions—but it may also be derived directly from the phase coordinate model [4,6].

Here we make use of the dq0 model because we have already derived it. For the space phasor model, we introduce the denotations in the stator:

$$\bar{I}_s = I_d + jI_q; \quad \bar{V}_s = V_d + jV_q; \quad \bar{\Psi}_s = \Psi_d + j\Psi_q \quad (7.47)$$

$\bar{I}_s$, $\bar{V}_s$, $\bar{\Psi}_s$ are called direct space phasors (vectors) of currents, voltages, and flux linkages.

At least for the flux linkage, $\bar{\Psi}_s$ refers to a space phasor whose position with respect to phase A in stator axis is that of the traveling magnetic field axis, which rotates at speed ω_b .

Making use of the I_d, I_q expressions in Equation 7.36 we obtain

$$I_d + jI_q = \bar{I}_s = \frac{2}{3} \left[I_A + I_B e^{j\frac{2\pi}{3}} + I_C e^{-j\frac{2\pi}{3}} \right] e^{-j\theta_{eb}}; \quad \frac{d\theta_{eb}}{dt} = \omega_b \quad (7.48)$$

For the rotor, $\theta_{ebr} = \int (\omega_b - \omega_r) dt$.

It is evident that the zero-sequence current is missing in Equation 7.48 and thus has to be added. Equation 7.48 illustrates, in fact, a rotational transformation by angle θ_{eb} :

$$\bar{I}_s = \bar{I}_s^s e^{-j\theta_{eb}} \quad (7.49)$$

where $\bar{I}_s^s$ is the space phasor in stator coordinates; if we take the transformation backward from Equation 7.49 we find

$$I_A(t) = \text{Re}(\bar{I}_s^s) + I_0 = \text{Re}(\bar{I}_s e^{j\theta_{eb}}) + I_0 \quad (7.50)$$

$$I_0 = \frac{1}{3}(I_A + I_B + I_C) \quad (7.51)$$

The stator voltage dq equations, Equations 7.12 through 7.14, and 7.6, are the same for SMs and IMs, for the dq axis rotating at ω_b and can simply be converted into the space phasor (complex variable) formulation:

$$\bar{V}_s = V_d + jV_q = R_s \bar{I}_s + \frac{d\Psi_s}{dt} + j\omega_b \Psi_s; \quad \bar{\Psi}_s = \Psi_d + j\Psi_q \quad (7.52)$$

For the wound rotor induction machine, the rotor voltage equation (Equation 7.6) is

$$\bar{V}_r = V_{dr} + jV_{qr} = R_r \bar{I}_r + \frac{d\Psi_r}{dt} + j(\omega_b - \omega_r) \Psi_r; \quad \bar{\Psi}_r = \Psi_{dr} + j\Psi_{qr} \quad (7.53)$$

The electromagnetic torque (Equation 7.44) is

$$T_e = \frac{3}{2} p_1 \text{Re} \left[j \bar{\Psi}_s \bar{I}_s^* \right] = -\frac{3}{2} p_1 \text{Re} \left[j \bar{\Psi}_r \bar{I}_r^* \right] \quad (7.54)$$

To illustrate phenomenologically the complex variable, let us follow the expression of the symmetric 3-phase sinusoidal currents, typical in ac machines:

$$I_{ABC} = I\sqrt{2} \cos \left[\omega_1 t - (i-1) \frac{2\pi}{3} \right]; \quad i = 1, 2, 3 \quad (7.55)$$

For stator coordinates ($\omega_b = 0$, $\theta_{eb} = \theta_{0s} = 0$) from Equations 7.48 and 7.55:

$$\bar{I}_s^s = \frac{3}{2} \left[i_A(t) + i_B(t)e^{j\frac{2\pi}{3}} + i_C(t)e^{-j\frac{2\pi}{3}} \right] = I\sqrt{2}[\cos \omega_1 t + j \sin \omega_1 t] \quad (7.56)$$

$\theta_{eb} = 0$ means that axis d is aligned to stator phase A axis.

For synchronous coordinates ($\omega_b = \omega_1$), with $\theta_{eb} = \omega_b t = \omega_1 t$ from Equation 7.49:

$$\bar{I}_s = \bar{I}_s^s e^{-j\omega_1 t} = I\sqrt{2}(\cos \omega_1 t + j \sin \omega_1 t)(\cos \omega_1 t - j \sin \omega_1 t) = I\sqrt{2} = I_d \quad (7.57)$$

So in the stator coordinates ($\omega_b = 0$) $\bar{I}_s^s$ refers to an ac vector whose position changes in time at ω_1 speed (Figure 7.8.a). On the other hand, in synchronous coordinates ($\omega_b = \omega_1$), we have a dc vector, along axis d (in our case, because the initial value of θ_{eb} was chosen as zero), which now rotates together with axis d at ω_1 speed (Figure 7.8.b).

Besides the abbreviation in writing the dq0 model equations, the space phasor (complex variable) formalism brings up some more concepts in the flux orientation control of electric machines.

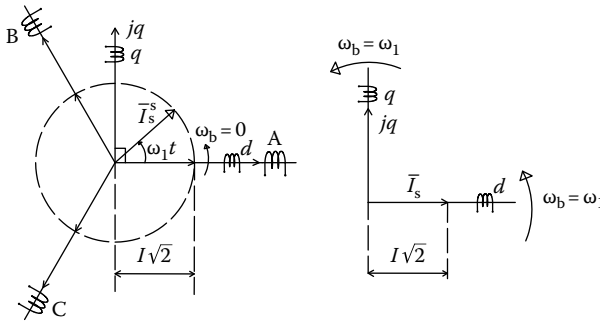


FIGURE 7.8

Steady-state ac current space phasor in stator coordinates (a) and synchronous coordinates (b).

7.11 High-Frequency Models for Electric Machines

Atmospheric (microsecond front) waves, commutation (tens of microsecond front) voltage pulses, and IGBT (MOSFET)-triggered pulses ($0.5\text{--}2\text{ }\mu\text{s}$ front waves) are so fast that the machine resistances and inductance influences are small, while the stray capacitors between turns, coils, and from windings to frame, similarly as for transformers (Figure 7.9), take over. Up to 400 Hz the R,L,E circuit model of electric machines is valid, while above 20 kHz, the high-frequency model is practical (Figure 7.9).

In between, (400 Hz–20 kHz) the electric machines have to be modeled correctly as they interact with the cables that carry the current from the power electronics supply to the motor. There we should use two types of voltage fast pulses to reach the machine terminals: differential mode (DM) and common mode (CM) pulses. The latter refers to the neutral potential pulsation with respect to the ground and the machine response through stray capacitors of windings to frame, and frame (bearing) to ground.

Very recently a universal, low-to-high frequency model for the IM has been introduced [13] (Figure 7.10). It is valid for all ac machines if the low-frequency section of the circuit model is the pertinent one (as developed in Part I).

Such a model is instrumental in explaining voltage tripling electromagnetic interference and bearing currents when EMs are associated with PWM converters for energy or motion modern control.

The bearing model in Figure 7.10 consists of common mode (windings to frame) capacitances: stator to frame, C_{sf} , rotor to frame, C_{rf} , stator to rotor, C_{sr} , and bearing balls race resistance, R_b , with a series/parallel combination of capacitance, C_b , and nonlinear impedance, Z , for random charging and discharging of the rotor shaft due to asperity point contacts puncturing the oil film. C_{sf-0} matches the low-frequency response.

The PWM converter intervenes here mainly as a neutral-to-ground zero-sequence voltage source, V_{nsg} . At low frequency, the voltage at machine terminals is distributed uniformly from the terminal point to the neutral point.

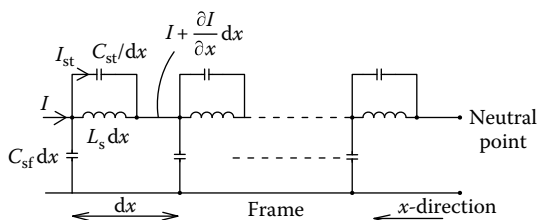
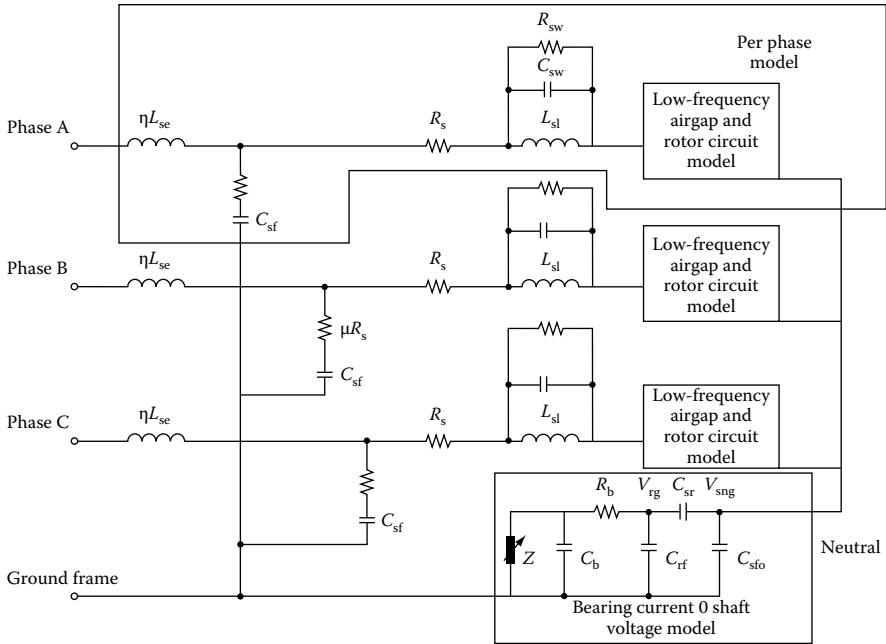


FIGURE 7.9

Distributed high-frequency parameter motor for ac stator windings.

**FIGURE 7.10**

Universal (low-to-high frequency) model of ac machines.

In the medium frequency (400–20 kHz) the impact of the airgap—in the rotor part of the low-frequency model—may be neglected. Three capacitors, C_{sf} , C_{sf0} , C_{sw} , are introduced in Figure 7.10 to match the mid-to-high frequency response:

- C_{sf} is the stator-to-frame capacitor of the first slot per phase
- C_{sf0} is the capacitance of stator-to-frame in the neutral point
- C_{sw} is the interturn capacitance per phase

The introduction of the universal model here was done for completeness and as a starting point when electromagnetic compatibility (EMC) aspects of PWM converter controlled generators and motors are of interest.

7.12 Summary

- Electric machines undergo transients when connected to/disconnected from the power source, for load variations, and during exposure to steep front voltage pulses from atmospheric sources, or from PWM converter for energy and motion control.

- The investigation of machine transients or of high-frequency behavior needs dedicated modeling that is both precise enough and practical in terms of CPU time.
- High-frequency models are treated separately as they include the electric machine–distributed stray capacitor network.
- Most transients, especially for energy and motion control, refer to low frequency and they are treated by circuit models among which the dq0 model and the space phasor (complex variable) model have gained widespread acceptance and are introduced here intuitively.

The machine circuit parameters for such models are either calculated in the design stage or measured by special tests.

- The dq0 (orthogonal axis) model's success is explained by its merit of exhibiting constant (rotor position independent) self and mutual inductances in contrast to the phase coordinate model. The latter will be introduced in the following chapters dedicated to ac machine transients.
- The dq0 physical model concept is introduced based on commutator orthogonal windings with all brushes aligned to dq axes that are solidary with the machine part that has magnetic (and winding) anisotropy. In this way motion-induced voltages are avoided in the anisotropic part of the machine, and the dq0 model shows constant inductances. The dq0 model is applicable to 2,3 or more phase ac machines.
- The zero-sequence current, which occurs for 3-phase ac machine modeling, besides the physical dq model, completes the equivalence. It is zero for star connection or for symmetric (balanced) operation. In the dq0 model, stator and rotor variables have the same frequency $\omega_1 - \omega_b$.
- For the modeling of 6-,9-,12-phase ac machines, 2(3,4) dq axes pairs plus 1(2,3) zero-sequence currents are required for full equivalence in terms of power, losses, flux linkages, and torques. The zero sequences act only on the stator (rotor) leakage inductances and resistances.
- For the synchronous machine dq0 model, the brush speed (dq axes speed) $\omega_b = \omega_r$ (rotor speed) because the rotor of the SM is dc (or PM) excited or of reluctance type ($L_{dm} \neq L_{qm}$). In this way, for a steady state, it is dc in the dq0 model, that is, the ideal case for control design.
- For the induction machine dq0 model, the brush speed (dq axes speed) is indifferent but $\omega_b = 0, \omega_r, \omega_1$ are most used. To get dc under steady state, $\omega_b = \omega_1$, that is, synchronous coordinates.

For wound rotor IMs with PWM converter frequency control in the rotor, the dq axes may also be attached to the rotor ($\omega_b = \omega_r$). Stator coordinates ($\omega_b = 0$) may be used for IMs to investigate soft-starter or stator–inverter-fed behavior.

- Magnetic crosscoupling saturation may be elegantly (though not locally) accounted for, in the $dq0$ model, by introducing transient inductances for single but distinct magnetization curves along axes d and q , with magnetization inductances, L_{dm} , L_{qm} , dependent only on the resultant magnetization current, I_m .
- Skin(frequency) influence on leakage inductances and resistances in the rotor of IMs and SMs may be handled in the $dq0$ model by adding additional fictitious constant parameter circuits in parallel. Three circuits in parallel suffice, in practice, for all SMs and IMs.
- The space phasor (complex variable) model is assembled from the dq model $\bar{V}_s = V_d + jV_q$, etc. It provides an abbreviated form of the equations and alludes to physical intuitional interpretations related to the traveling field concept. The zero-sequence current is again additional.
- The equivalence of ac machine stator or rotor symmetric 3-phase windings to the $dq0$ model is based here on the mmf equivalence, but it conserves power and torque to a 3/2 ratio. Reactive power of ac machines may also be calculated through the $dq0$ model, even with dc under steady state.
- The dc brush machine corresponds to a simplified case of the $dq0$ model by its own nature, due to stator and rotor magnetic field axes orthogonality, because of the placement of brushes in a neutral axis.
- As the voltage, current, flux linkage, power, torque equivalence between 3-phase ac machines and the $dq0$ model has been introduced here, only the parameter (inductances, resistances) correspondence is needed; this will be done in Chapters 9 and 10.
- We should point out that the main assumptions on which the $dq0$ model– m phase ac machines are based are
 - Nonsymmetric windings or magnetic anisotropies present either in the rotor or in the stator with dq axes attached to this part of the machine.
 - Distributed symmetric ac windings on the stator with sinusoidal armature flux density in the airgap: the salient pole rotor case is handled in this case by replacement with a thin fictitious (superconducting) flux barrier in axis d but for constant airgap.

- For nonoverlapping (concentrated) windings on the stator, but no windings on the rotor: the rotor may be anisotropic magnetically or may have PMs but the stator self and mutual inductances should vary sinusoidally with the rotor position. Sinusoidal inductance distributions are crucial for the dq0 model because of the dq axes coupling by motion emfs.
- For all other cases the phase coordinate models should be used from the start (nonsymmetric 3-phase windings on stator and rotor, etc.) as shown in subsequent chapters.
- Linear ac machines (LIMs and LSMs) follow the same pattern in modeling for transients but dynamic end effects (due to open magnetic circuit at stator (mover) ends, in the presence of Gauss flux law as constraint) make the modeling more complicated for high-speed applications and for a small number of poles on the mover part.

7.13 Proposed Problems

- 7.1 Does a separately excited dc brush motor qualify as a dq0 model? Demonstrate why and write the two voltage equations.

Hint: Check Section 7.4, add the excitation winding in place of PMs, observing zero motion emf in the latter.

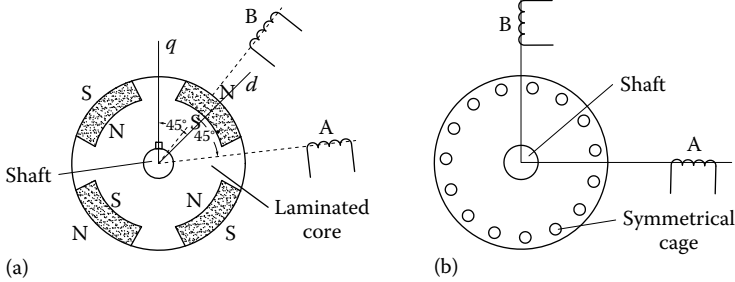
- 7.2 Based on the fact that the dq0 model of a distributed ac 3-phase SM with salient poles rotor ($L_{dm} > L_{qm}$) is strictly valid only if the latter has constant airgap, draw the pertinent d (or q) axis flux barrier rotor for $2p_1 = 2$ poles; for $L_{dm} < L_{qm}$ place PMs in the flux barrier.

Hints: Place a diametrical flux barrier in axis d for $L_{dm} > L_{qm}$ and in axis q for PM-filled barrier case and keep the rotor cylindrical.

- 7.3 Strip the 3-phase SM from the cage and dc excitation windings on the rotor and put PM d -axis poles instead; simplify the machine dq0 model equations and rewrite the flux–current relationship as well as the torque simplified equations.

Hints: Check Equations 7.12 through 7.20 and simplify them; replace $L_{dm}I_F$ by Ψ_{PM} .

- 7.4 Figure 7.11 shows a PM salient rotor pole ($L_{dm} < L_{qm}$) 2-phase SM machine. The stator windings do not have the same number of turns but use the same copper weight. Can this machine be treated for transients with the dq0 model (is the zero-sequence current nonzero)?

**FIGURE 7.11**

Salient pole PM rotor 2-phase SM (a) and cage rotor 2-phase IM (b).

In what coordinates ($\omega_b = ?$), if one winding in the stator is eliminated, can the machine still be treated by the dq model? Why is this so?

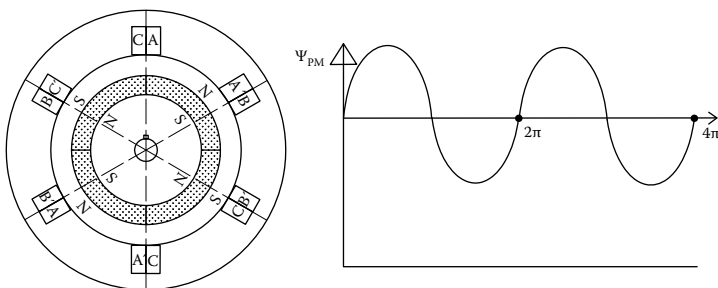
Hint: Two orthogonal stator windings with the same copper weight are basically symmetric.

- 7.5 Figure 7.11b shows the case of a 2-pole 2-phase IM whose stator windings are fully symmetric and so is the rotor cage. Can the dq0 model be applied to it? In what coordinates ($\omega_b = ?$)?

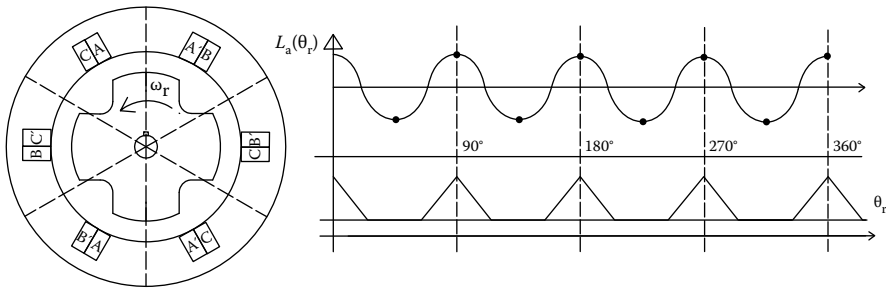
Hint: Two-phase symmetrical IM resembles the dq model without the zero-sequence component.

- 7.6 A 6-slot/4-pole 3-phase PMSM with wound coils as in Figure 7.12 has the PM flux linkage in the stator coils sinusoidal with the rotor position. Can the dq model be applied to this nonsalient pole (constant magnetic airgap) machine? In what coordinates ($\omega_b = ?$)?

Hints: The stator winding mmf has a $2p_1 = 4$ harmonic that makes this machine synchronous; the other stator mmf harmonics are added to stator leakage inductance as they do not produce nonzero average torque.

**FIGURE 7.12**

6-Slot/4-pole PMSM with sinusoidal PM flux linkage.

**FIGURE 7.13**

6-Slot/4-pole 3-phase switched reluctance machine.

- 7.7 A 3-phase 6-slot/4-pole switched reluctance machine (Figure 7.13) has zero mutual inductances but the self-inductances vary as

$$L_{A,B,C} = L_{se} + L_0 \cos \left(4\theta_r - (i-1) \frac{2\pi}{3} \right)$$

Can the dq0 model be used in this case? In what coordinates ($\omega_b = ?$)? Will it cause a problem if the actual current in the stator phases is sinusoidal in terms of torque production? With sinusoidal current will it be torque pulsations?

For the case of linear ramp and flat portion variation of L_{AB} with rotor position (Figure 7.13), may the dq0 model still be used? If not, why?

Hints: Notice that with sinusoidal inductances, the machine becomes a synchronous one with salient poles.

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Transients of Brush–Commutator DC Machines

8.1 Introduction

The dc brush–commutator—with PM stator—is still a favorite for low power levels in many applications such as small fans and automotive auxiliaries. The lower cost of PWM converters for unidirectional motion (in fact most) applications makes the dc brush PM small motors even more attractive.

On the other hand, dc excited brush–commutator machines are still used for traction in urban and interurban transportation. The dc generator mode is seldom used, despite its resistive only (small, in other words) voltage regulation, which is an advantage in standalone applications.

At low speeds (100 rpm or so) and with, say, 1 MW reversible drives in metallurgy, with a 3/1 starting to rated torque, the dc brush motor drive (with a dual converter for four quadrant operation) is still cost/performance-wise very competitive. The slotless rotor configuration provides for the fastest torque response to date.

Finally, the ac brush series (universal) motor is very much in use today, with voltage control only, for home and construction tool applications [1–4].

So, again, though the brush–commutator machines have been considered a dying breed, they seem to die very hard. This is one reason why their transients are treated in this chapter.

The second reason is that they represent the simplest second (third)-order system that can serve as a practical introduction to ac machine transients. The latter, when flux orientation control is used, undergo a forceful reduction in the model order to come closer to the dc brush machine, which provides naturally decoupled flux (excitation flux) and torque control.

8.2 Orthogonal (dq) Model of DC Brush Machines with Separate Excitation

In Section 7.4, we have derived the dc PM brush machine dq model. Here, by leaving only one winding on the stator (along axis d , the

field winding) and one along axis q in the rotor (the armature winding), we obtain the separately excited dc brush-commutator machines (Figure 8.1).

The dc excitation circuit in Figure 8.1 is separately supplied from a dc source, but by simple mathematical constraints, it may be shunted or connected in a series to the brushes. The commutation poles, if any, are lumped into the rotor armature winding.

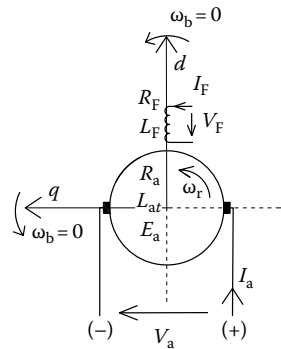


FIGURE 8.1
dq model of separately excited dc brush machine.

Shunt excitation: $V_F = V_a$

Series excitation: $I_F = I_a$; $V_F + V_a = V_{\text{source}}$

The field-circuit equation is straightforward as there is no motion-induced voltage in it ($\omega_b = 0$), and the orthogonality of the winding axes precludes any pulsational-induced voltage from armature winding:

$$R_F I_F - V_F = -L_{Ft} \frac{dI_F}{dt} \quad (8.1)$$

On the rotor, there is a motion-induced voltage and a pulsational-induced voltage:

$$R_a I_a - V_a = -L_a \frac{dI_a}{dt} - \omega_r \Psi_{dr}; \Psi_{dr} = L_{dm} I_F \quad (8.2)$$

Ψ_{d_r} is produced by a fictitious armature rotor winding placed in axis d_r . As this does not exist, its flux linkage is produced solely by the field winding from the stator. In other words, L_{dm} is reduced to the rotor, but L_F, R_F , and V_F are not.

In Chapter 4,

$$E_r = k_\Phi n \Phi_p; \quad k_\Phi = \frac{p_1 N}{a}; \quad \omega_r = (2\pi n)p_1 \quad (8.3)$$

where

n is the speed in rps

N is the total number of conductors in rotor slots

$2a$ is the current path count

p_1 is the pole pairs

If magnetic saturation is considered, the transient inductances in Equations 8.1 and 8.2, L_{Ft} , L_{at} , are

$$\begin{aligned} L_{Ft} &= L_F + \frac{\partial L_F}{\partial i_F} i_F \leq L_F \\ L_{at} &= L_a + \frac{\partial L_a}{\partial i_a} i_a \leq L_a \end{aligned} \quad (8.4)$$

The inductance L_{dm} (I_F) is the main inductance as it occurs in the motion-induced voltage.

The electromagnetic torque is simply calculated from P_e :

$$P_e = T_e 2\pi n = \omega_r \Psi_{dr} I_a = \omega_r L_{dm} I_F I_a \quad (8.5)$$

It is also possible that L_{dm} depends not only on I_F but also on I_a , due to crosscoupling magnetic saturation (Chapter 4).

These are, however, cases typical for high overloading (such as traction, metallurgical, handtool applications). In general, the $L_F(I_F)$, $L_{Ft}(I_F)$, $L_{at}(I_a)$, and $L_{dm}(I_F)$ functions are sufficient and may be obtained through adequate tests.

Adding the motion equations:

$$\frac{J}{p_1} \frac{d\omega_r}{dt} = T_e - T_{load}; \quad \frac{1}{p_1} \frac{d\omega_r}{dt} = \theta_r, \quad (8.6)$$

we obtain a fourth-order system (Equations 8.1, 8.2, and 8.6) with some parameters (inductances) dependent on some variables but independent of rotor position, as expected.

Denoting d/dt by s (Laplace operator), they lead to the structural diagram in Figure 8.2. The structural diagram depicts products of variables ($I_F I_a$, $I_F \omega_r$) and actual variable inductances. A nonlinear system is thus obtained.

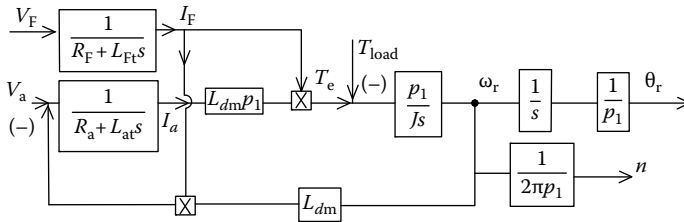


FIGURE 8.2

Structural diagram of separately excited dc brush machine.

To simplify our dealings with transients, we introduce the following three types:

- Electromagnetic transients ($n = \text{constant}$)
- Electromechanical transients (both electromagnetic and mechanical variables vary)
- Mechanical transients (only mechanical variables vary)

8.3 Electromagnetic (Fast) Transients

Let us consider a separately excited dc brush machine operating as a generator at constant speed ($\omega_r = \text{const}$).

From Equations 8.1 through 8.3, the load equations are loaded:

$$\begin{aligned} V_F &= R_F I_F + L_{Ft} \frac{di_F}{dt} \\ V_a &= R_a I_a + L_a \frac{di_a}{dt} + \omega_r L_{dm} I_F \\ V_a &= -R_{\text{load}} - L_{\text{load}} \frac{di_a}{dt} \end{aligned} \quad (8.7)$$

Or, in Laplace form ($d/dt \rightarrow s$):

$$\frac{\tilde{I}_F}{\tilde{V}_F} = \frac{1}{R_F + sL_{Ft}}; \tilde{I} = \frac{\tilde{V}_a}{R_a + sL_{at}} - \frac{\omega_r L_{dm} \tilde{V}_F}{(R_F + sL_{Ft})(R_a + sL_{at})} \quad (8.8)$$

$$\frac{\tilde{V}_a}{\tilde{V}_F} = \frac{\omega_r L_{dm} (R_{\text{load}} + sL_{\text{load}})}{(R_F + sL_{Ft})(R_{\text{load}} + sL_{\text{load}} + R_a + sL_{at})} \quad (8.9)$$

Besides R_{load} and L_{load} , an emf E_a may be added to simulate a dc brush motor load.

It is evident that the field current circuit introduces a delay in the output voltage $\tilde{V}_a$ by Equation 8.9. Also, a second-order system has been obtained from Equation 8.9, which is easy to handle, with constant inductances in the machine by the linear system routines.

Example 8.1 Sudden V_F Increase and Short-Circuit Transients

Let us consider a dc generator with separate excitation with the following data: $R_a = 0.1 \Omega$, $R_F = 1 \Omega$, $L_a = 0.5 \text{ mH}$, $L_F = 0.5 \text{ H}$, $V_{an} = 200 \text{ V}$, $I_{an} = -100 \text{ A}$, $I_{Fn} = 5 \text{ A}$, at $n = 1500 \text{ rpm}$, for resistive load R_{load} .

Calculate the output voltage transfer function (8.9) and the $V_a(t)$ and $I_a(t)$ after a 20% step increase in V_F . Also calculate the sudden short-circuit current variation.

Solution

The load resistive R_{load} is

$$R_{\text{load}} = \frac{-V_{an}}{I_{an}} = \frac{-200}{(-100)} = 2 \Omega \quad (8.10)$$

For Equation 8.9, we still need $\omega_r L_{dm}$:

$$\begin{aligned} V_{an} &= R_a I_{an} + \omega_r L_{dm} I_{Fn}; \quad 200 = 0.1 \times (-100) + \omega_r L_{dm} \times 5 \\ \omega_r L_{dm} &= 42 \Omega \end{aligned} \quad (8.11)$$

So Equation 8.9 becomes

$$\frac{\tilde{V}_a}{\tilde{V}_F} = \frac{42 \cdot 2}{(1 + 0.5s)(2 + 0.1 + 0.005s)}$$

For a 20% increase in V_F , we have

$$\tilde{V}_F = I_{Fn} R_F \frac{0.2}{s} = \frac{5 \times 2 \times 0.2}{s} = \frac{1}{s}$$

so

$$\tilde{V}_a = \frac{42 \cdot 2 \cdot 1}{s(1 + 0.5s)(2.1 + 0.005s)}$$

or

$$\Delta v_a(t) = 2(20 - 20.9e^{-2t} + 0.95e^{-420t})(V)$$

The load current variation $\Delta i_a(t)$ is

$$\Delta i_a(t) = -\frac{\Delta v_a(t)}{R_{\text{load}}} \quad (8.12)$$

As we can see, the output voltage and current variation (response) at a 20% increase in field circuit supply voltage are stable (attenuated) and nonperiodic. This is a special merit of the dc generator as a controlled power source. However, as the field circuit time constant $T_F = L_F/R_F$ is large, the response is slow. A large-voltage ceiling in V_F is required if a quick response is needed in the output voltage when V_F varies.

It is very similar to the case when the magnetization flux (current) varies in ac machines. This is how the flux orientation control of ac machines has come into play.

The sudden short circuit of the same dc generator is also an electromagnetic transient ($V_a = 0$, $I_F = I_{Fn}$):

$$\tilde{I}_{sc} = -\frac{\omega_r L_{dm} I_{Fn}}{R_a + sL_a} \quad (8.13)$$

with the solution

$$I_{sc}(t) = -\frac{\omega_r L_{dm} I_{Fn}}{R_a} + A e^{-\frac{t}{T_a}}; \quad T_a = L_a/R_a \quad (8.14)$$

Also, at $t = 0$ $I_{sc}(0) = -I_n$:

$$I_{sc}(t) = -2100 + 2000e^{-0.005t} \quad (8.15)$$

So the short-circuit transient is very fast because the armature electric time constant is small: $T_e = 5 \times 10^{-3}$ s; it is even less than 1 ms for slotless rotor windings. So the sudden short circuit of the dc brush machine (with separate or PM excitation) is very fast and dangerous to the machine, because the final current is only R_a resistance limited. Such an event has to be avoided by means of fast protection in all dc brush drives.

8.4 Electromechanical Transients

Most transients are electromechanical (both electrical and mechanical variables vary). To approach the problem gradually, let us first tackle constant excitation (or PM) flux transients.

8.4.1 Constant Excitation (PM) Flux, Ψ_{dr}

From Equations 8.1 through 8.6, we are now left with two equations, if speed control is targeted:

$$\begin{aligned} V_a &= R_a I_a + L_a \frac{di_a}{dt} + \omega_r \Psi_{dm}; \quad \Psi_{dm} = L_{dm} I_F = \Psi_{PM} \\ \frac{J}{p_1} \frac{d\omega}{dt} &= p_1 \Psi_{dm} I_a - T_{load} - B\omega_r \end{aligned} \quad (8.16)$$

In Laplace form, Equation 8.16 becomes

$$\begin{aligned} \tilde{V}_a &= (R_a + sL_a)\tilde{i}_a + \tilde{\omega}_r \Psi_{dm} \\ s\tilde{\omega}_r &= \frac{p_1^2}{J} \Psi_{dm} \tilde{i}_a - (\tilde{T}_{load} + B\tilde{\omega}_r) \frac{p_1}{J} \end{aligned} \quad (8.17)$$

We may now extract the open loop transfer functions for two cases:

$$\begin{aligned}
 & - \text{constant load torque } T_{\text{load}} = \text{const} \left(\tilde{T}_{\text{load}} = 0 \right) \\
 \tilde{i}_a &= \frac{\tilde{V}_a (Js + Bp_1)}{(R_a + sL_a)(Js + Bp_1) + p_1^2 \Psi_{dm}^2} = G_i(s) \tilde{V}_a; \quad \tilde{\omega}_r = -\frac{p_1 \Psi_{dm} \tilde{i}_a}{\frac{Js}{p_1} + B} = G_\omega(s) \tilde{i}_a
 \end{aligned} \tag{8.18}$$

$$- \text{constant voltage } V_a = \text{const} \left(\tilde{V}_a = 0 \right)$$

$$\begin{aligned}
 \tilde{i}_a &= \frac{\tilde{T}_{\text{load}} p_1 \Psi_{dm}}{(R_a + sL_a)(Js + Bp_1) + p_1^2 \Psi_{dm}^2} = G_{it}(s) \tilde{T}_{\text{load}}; \\
 \tilde{\omega}_r &= -\frac{(R_a + sL_a) \tilde{i}_a}{\Psi_{dm}} = G_{\omega t}(s) \tilde{i}_a
 \end{aligned} \tag{8.19}$$

Figure 8.3 illustrates the structural diagram corresponding to Equation 8.17.

A second-order system has been obtained. As analysis through linear systems routines is straightforward, the eigenvalues $s_{1,2}$ from Equation 8.17 are straightforward, and for $B = 0$ (zero friction torque):

$$s_{1,2} = \frac{-1 \pm \sqrt{1 - 4T_e/T_{em}}}{2T_e}; \quad B = 0; \quad T_{em} = \frac{JR_a}{(p_1 \Psi_{dm})^2}; \quad T_e = \frac{L_a}{R_a} \tag{8.20}$$

While T_e has already been defined as an electrical time constant, T_{em} is the electromechanical time constant. With $T_{em} > 4T_e$, the eigenvalues are real and negative, so the machine response is attenuated and nonperiodical. For $T_{em} < 4T_e$ (small inertia applications), the response is still attenuated but periodic. Again, the PM (constant flux) dc brush machine braves ideal transient behavior due to its inner feedback (speed, emf) loop (Figure 8.3).

Example 8.2 DC Brush PM Motor Transients

A PM dc brush motor has the following data: $P_n = 50 \text{ W}$, $V_n = 12 \text{ Vdc}$, $\eta_n = 0.9$, $R_a = 0.12 \Omega$, $T_e = 2 \text{ ms}$, $p_1 = 1$, and $n_n = 1500 \text{ rpm}$.

Calculate $\Psi_{dr} = \Psi_{PM}$ (PM flux linkage), rated electromagnetic torque, speed and current transients for sudden reduction of V_a from 12 to 10 Vdc,

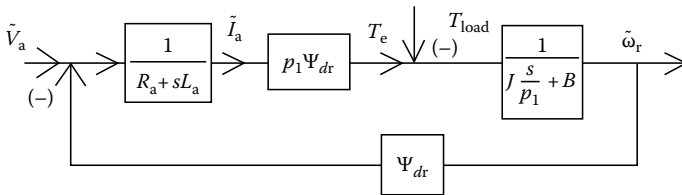


FIGURE 8.3

The structural diagram of a dc brush PM machine.

where the machine inertia $J = 2 \times 10^{-4} \text{ kg} \cdot \text{m}^2$ and $J' = 1 \times 10^{-3} \text{ kg} \cdot \text{m}^2$, and for constant load torque.

Solution

First, from Equation 8.16, with $d/dt = 0$

$$V_n = R_a I_n + \omega_{rn} \Psi_{PM}; \quad \omega_{rn} = 2\pi p_1 n_n$$

with

$$I_n = \frac{P_n}{\eta_n V_n} = \frac{50}{0.9 \times 12} = 4.63 \text{ A}$$

So

$$\Psi_{PM} = \frac{12 - 0.12 \times 4.63}{2\pi(1500/60)} = 0.0729 \text{ Wb}$$

$$T_{en} = p_1 \Psi_{PM} I_n = 1 \cdot 0.0729 \times 4.63 = 0.3375 \text{ N m} = T_{load}$$

From Equation 8.17, we can eliminate $\tilde{i}_a$ and switch back s to d/dt to obtain

$$T_{em} T_e \frac{d^2 \omega_r}{dt^2} + T_{em} \frac{d\omega_r}{dt} + \omega_r = \frac{V_a(t)}{\Psi_{PM}} - T_{load} \frac{R_a}{p_1 \Psi_{PM}^2} \quad (8.21)$$

With $T_e = 2 \times 10^{-3} \text{ s}$; T_{em} from Equation 8.20 is

$$T_{em} = \frac{J R_a}{(p_1 \Psi_{dm})^2} = \begin{cases} \frac{0.0002 \times 0.12}{0.0729^2} = 4.516 \times 10^{-3} \text{ s} < 4T_e \\ \frac{0.001 \times 0.12}{0.0729^2} = 22.58 \times 10^{-3} \text{ s} > 4T_e \end{cases}$$

So the eigenvalues are (Equation 8.20)

$$\gamma_{1,2} = \frac{-1 \pm \sqrt{1 - 4T_e/T_{em}}}{2T_e} = \begin{cases} \frac{-1 \pm j0.877}{4 \times 10^{-3}} \\ \frac{-1 \pm 0.803}{4 \times 10^{-3}} \end{cases}$$

The initial value of speed is the same for both inertia values: $n_n = 1500 \text{ rpm}$ ($(\omega_r)_{t_0} = 2\pi 1500/60 = 157 \text{ rad/s}$). Also, the final value of speed ω_{rf} is the same as the torque values:

$$(\omega_r)_{t=\infty} = \frac{(V_a)_{t=\infty}}{\Psi_{PM}} - \frac{T_{em} R_a}{p_1 \Psi_{PM}^2} = \frac{10}{0.0729} - \frac{0.3375 \times 0.12}{1 \times 0.0724^2} = 129.55 \text{ rad/s} \quad (8.22)$$

In addition,

$$\left(\frac{d\omega_r}{dt}\right)_{t=0} = 0 \quad (8.23)$$

The solution of Equation 8.21 for speed is now straightforward:

$$\omega_r(t) = (\omega_r)_{t=0} + Ae^{-250t} \cos(219.25t + \varphi) \quad (8.24)$$

$$\omega_r(t) = (\omega_r)_{t=0} + A_1e^{-49.25t} + A_2e^{-450t} \quad (8.25)$$

Based on boundary conditions (8.22) and (8.23), A , φ in Equation 8.24 and A_1 and A_2 in Equation 8.25 are easily found.

In any case, for the small inertia, the speed response is periodic and attenuated, while for the large inertia, it is nonperiodic and attenuated. Fast response open loop for low inertia implies speed response overshooting and attenuated oscillations (Figure 8.4).

The current response is obtained from Equation 8.17

$$i_a(t) = \left(\frac{J}{p_1} \frac{d\omega_r}{dt} + T_{\text{load}}\right) / (p_1 \Psi_{\text{PM}}) \quad (8.26)$$

Note: A similar problem could be solved for constant voltage but with a step increase (decrease) in load torque.

The only difference is that $(di_a/dt)_{t=0} = 0$ instead of $(d\omega_r/dt)_{t=0} = 0$, as it was for constant load torque.

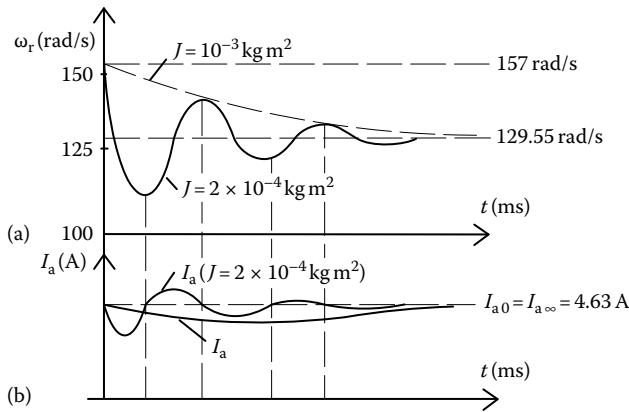


FIGURE 8.4

(a) Speed and (b) current response of a dc brush PM motor to sudden voltage reduction from 12 to 10 Vdc, for constant torque.

8.4.2 Variable Flux Transients

To extend the speed range above the rated (base) speed at the rated rotor voltage, flux weakening is required. So dc excitation is required. Let us consider here separate excitations.

Now, the transients model, ready for numerical solution, is collected from Equations 8.1, 8.2, and 8.6:

$$\frac{dI_F}{dt} = \frac{V_F - R_F I_F}{L_{Ft}}; \quad \frac{di_a}{dt} = \frac{V_a - R_a I_a - \omega_r L_{dm} I_F}{L_{at}} \quad (8.27)$$

$$\frac{d\omega_r}{dt} = \frac{p_1}{J} (p_1 L_{dm} I_F I_a - T_{load} - B\omega_r); \quad \frac{d\theta_r}{dt} = \frac{\omega_r}{p_1} \quad (8.28)$$

As we now have products of variables, *small deviation theory* is used to linearize the system:

$$\begin{aligned} V_F &= V_{F0} + \Delta V_F; & V_a &= V_0 + \Delta V_a; & T_{load} &= T_{L0} + \Delta T_L \\ I_F &= I_{F0} + \Delta I_F; & I_a &= I_0 + \Delta I_a, & \omega_r &= \omega_{r0} + \Delta \omega_r; & \theta_r &= \theta_{r0} + \Delta \theta_r \end{aligned} \quad (8.29)$$

For initial conditions ($t = 0$), with d/dt in Equations 8.27 and 8.28 as zero:

$$\begin{aligned} V_{F0} &= I_{F0} R_F; & V_0 &= R_a I_{a0} + \omega_{r0} L_{dm} I_{F0} \\ \theta_{r0} &= \theta_0; & T_{L0} + B\omega_{r0} &= p_1 L_{dm} I_{F0} I_{a0} \end{aligned} \quad (8.30)$$

For small deviations, Equations 8.27 and 8.28 with Equation 8.29 in matrix Laplace form are

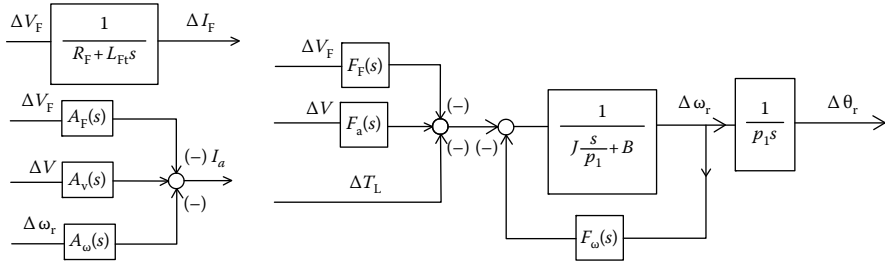
$$\begin{vmatrix} \Delta V_F \\ \Delta V_a \\ \Delta T_L \\ 0 \end{vmatrix} = \begin{vmatrix} R_F + sL_{Ft} & 0 & 0 & 0 \\ \omega_{r0} L_{dm} & R_a + sL_{at} & L_{dm} I_{F0} & 0 \\ p_1 L_{dm} I_0 & p_1 L_{dm} I_{F0} & -\frac{Js}{p_1} - B & 0 \\ 0 & 0 & -1/p_1 & 0 \end{vmatrix} \begin{vmatrix} \Delta I_F \\ \Delta I_a \\ \Delta \omega_r \\ \Delta \theta_r \end{vmatrix} \quad (8.31)$$

Various transfer functions between inputs ΔV_F , ΔV_a , and ΔT_L and output (variable) deviations ΔI_F , ΔI_a , $\Delta \omega_r$, and $\Delta \theta_r$ may be extracted from Equation 8.31. It is however evident that the eigenvalues are now obtained from the equation:

$$\Delta(s) = (R_F + sL_{Ft}) \left[(R_a + sL_a) \left(\frac{Js}{p_1} + B \right) + p_1^2 L_{dm}^2 I_{F0}^2 \right] = 0 \quad (8.32)$$

So the field circuit produces a separate (decoupled) negative real eigenvalue $\gamma_3 = -R_F/L_{Ft}$, while the other two, $\gamma_{1,2}$, are the same as for the constant flux transients (8.20), but calculated for the initial field current I_{F0} (flux).

The magnetic decoupling between the field circuit and the armature circuit (due to the orthogonal placing of the two windings) leads to this independent large time constant of the system.

**FIGURE 8.5**

Structural diagram for the separately excited dc brush motor.

Flux transients are slow, so flux weakening (reducing I_F) is accompanied by slower torque (I_a) current and torque transients.

This situation is very similar to flux weakening in flux orientation control of ac machines where the decoupling of field current and torque current components in ac-phase current is done mathematically, online, through DSPs.

This similarity has led to the introduction of the flux orientation (vector) control of ac machines that revolutionized motor/generator control through power electronics.

A rather elaborated structural diagram can be extracted from Equation 8.31 (Figure 8.5).

8.4.3 DC Brush Series Motor Transients

The generated scheme of the dc brush series machine is shown in Figure 8.6a. It has only one voltage equation and one motion equation (for speed control):

$$\begin{aligned} V_a &= (R_a + R_{Fs})I_a + (L_{at} + L_{Fst}(i_a))\frac{di_a}{dt} + \omega_r L_{dm}(i_a)I_a \\ T_e &= p_1 L_{dm}(i_a)I_a I_a \\ \frac{J}{p_1} \frac{d\omega_r}{dt} &= T_e - T_{load} \end{aligned} \quad (8.33)$$

$L_{dm}(i_a)$ reveals the fact that magnetic saturation is almost unavoidable, because the torque current i_a is also the field current (or it is proportional to it).

Small deviation theory is required for linearization and then for control design:

$$V_a = V_0 + \Delta V; \quad \omega_r = \omega_{r0} + \Delta\omega_r; \quad I_a = I_0 + \Delta I_a; \quad T_L = T_{L0} + \Delta T_L \quad (8.34)$$

Also for initial situation at $(d/dt = 0)$:

$$V_0 = (R_a + R_{Fs}) I_0 + \omega_{r0} L_{dm} I_0; \quad p_1 L_{dm} I_0^2 = T_{L0} \quad (8.35)$$

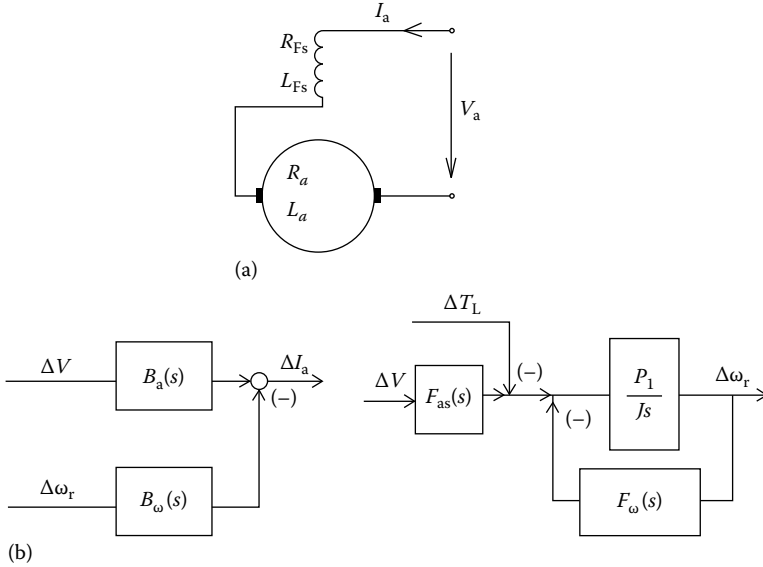


FIGURE 8.6
DC brush series motor: (a) general scheme and (b) structural diagrams.

From Equations 8.33 through 8.35 the small deviation model arises:

$$\begin{vmatrix} \Delta V \\ \Delta T_L \end{vmatrix} = \begin{vmatrix} R_a + R_{Fs} + \omega_{r0}L_{dm} + s(L_{at} + L_{Fst}) & L_{dm}I_0 \\ 2p_1L_{dm}I_0 & -\frac{Js}{p_1} \end{vmatrix} \cdot \begin{vmatrix} \Delta I_a \\ \Delta \omega_r \end{vmatrix} \quad (8.36)$$

The eigenvalues are extracted from the characteristic equation of Equation 8.36:

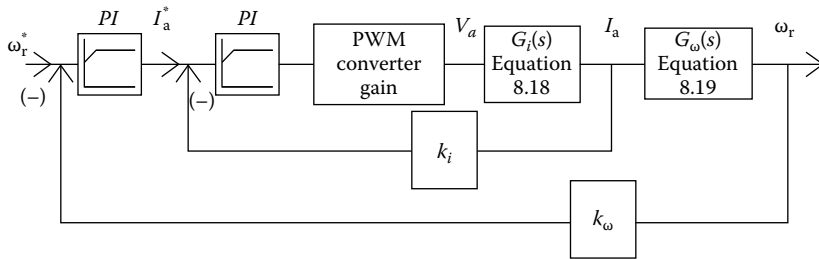
$$\Delta(s) = (R_a + R_{Fs} + \omega_{r0}L_{dm} + s(L_{at} + L_{Fst}))Js + 2(p_1L_{dm}I_0) = 0 \quad (8.37)$$

Comparing with $\Delta(s)$ of separate excitation dc brush machines Equation 8.32, the equivalent electrical time constant is

$$T_{es} = (L_{at} + L_{Fst})/(R_a + R_{Fs} + \omega_{r0}L_{dm}) \quad (8.38)$$

So the equivalent electrical time constant, T_{es} , decreases with speed only if magnetic saturation decreases with speed (because I_a decreases with speed). In any case, it appears that $T_{es} < T_e = L_a/R_a$, so a quicker response is expected.

The structural diagrams of Equation 8.36 are shown in Figure 8.6b

**FIGURE 8.7**

Cascaded basic close speed control with inner current loop for a dc brush PM motor.

8.5 Basic Closed-Loop Control of DC Brush PM Motor

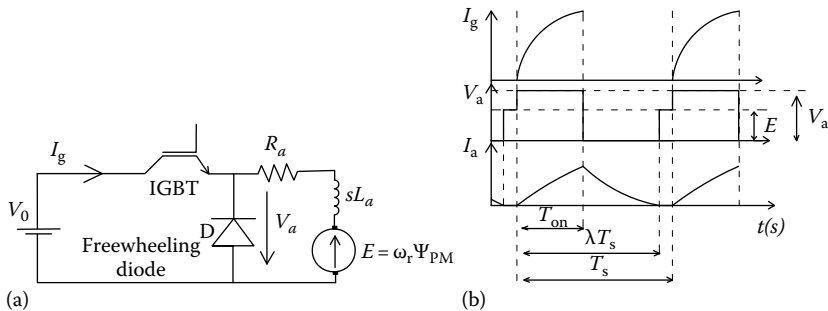
Based on Equations 8.18 and 8.19 we may introduce speed or torque basic close-loop control to be explored more fully in electric drives. Here, we only illustrate speed control dc brush PM motor transfer functions (Figure 8.7) based on Equations 8.18 and 8.19.

K_i and K_ω are the gains of the current and speed sensors. There are two close loops, one fast (current loop) and one slow (speed loop). Voltage change in PWM converter (dc-dc converter, in general) is fast because of the large switching frequency and thus the converter is modeled as a constant gain. This is acceptable as long as the current is continuous. Discontinuous current leads to sluggish speed response and hence should be avoided.

8.6 DC-DC Converter-Fed DC Brush PM Motor

To accomplish speed control, the average voltage, V_{av} , has to be changed in a dc-dc converter. Here, only the single quadrant dc-dc converter is considered (Figure 8.8a). Let us consider the case of discontinuous current to explore its consequences. Figure 8.8b shows the source IGBT current, I_g , terminal voltage, $V_a(t)$, and motor discontinuous current, $i_a(t)$.

The reverse of the switching period ($1/T_s$) is larger than 250 Hz; in many cases, it is in the kilohertz range. A constant frequency, (T_s), pulse width modulation is used to modify the average voltage, V_{av} , applied to the motor. Speed pulsations are considered negligible here due to a much larger electromechanical time constant, T_{em} .

**FIGURE 8.8**

Basic dc-dc converter-driven dc brush PM motor scheme (a) and current and voltage waveforms (b).

The voltage equation for the IGBT “on” and for the diode D “on” is

$$R_a I_a + L_a \frac{di_a}{dt} + \omega_r \Psi_{PM} = V_0; \quad 0 \leq t < T_{on} \quad (8.39)$$

$$R_a I_a + L_a \frac{di_a}{dt} + \omega_r \Psi_{PM} = 0; \quad T_{on} \leq t < T_s \quad (8.40)$$

$$T_e = p_1 \Psi_{PM} i_a$$

For a discontinuous current ($\lambda < 1$), the solutions of i_a in Equations 8.39 and 8.40 are straightforward:

$$\begin{aligned} I_a(t) &= \frac{1}{R_a} (V_0 - \omega_r \Psi_{PM}) + C_1 e^{-t/T_e}; \quad 0 \leq t < T_{on} \\ &= -\frac{\omega_r \Psi_{PM}}{R_a} + C_2 e^{-(t-\lambda T_s)/T_e}; \quad T_{on} \leq t < \lambda T_s = 0; \\ T_e &= L_a/R_a; \quad \lambda T_s \leq t < T_s \end{aligned} \quad (8.41)$$

The continuity of current at $t = 0$, T_{on} , and T_s yields the three unknowns C_1 , C_2 , and λ :

$$\begin{aligned} C_1 &= -\frac{1}{R_a} (V_0 - \omega_r \Psi_{PM}); \quad C_2 = \omega_r \Psi_{PM}/R_a \\ \lambda T_s &= T_{on} + T_e \ln \left[\frac{V_a}{\omega_r \Psi_{PM}} \left(1 - e^{-T_{on}/T_e} \right) + e^{-T_{on}/T_e} \right] \end{aligned} \quad (8.42)$$

for the continuous current $\lambda = 1$ in Equation 8.42. Then, from the same continuity conditions we find, at $t = 0$, T_{on} , and T_s , new conditions for C_1 and C_2 :

$$C_2 = C_1 + V_0/R_a; \quad C_1 e^{-T_{on}/T_e} + V_a/R_a = C_2 e^{-(T_s - T_{on})/T_e} \quad (8.43)$$

The average values of the armature voltage, V_{av} , for the two cases is

$$\begin{aligned} V_{av} &= \frac{T_{on}}{T_s} V_0 = \alpha V_0; \quad \text{for } \lambda = 1 \text{ (continuous current)} \\ V_{av} &= \frac{T_{on}}{T_s} V_0 + (1 - \lambda) \omega_r \Psi_{PM}; \quad \text{(discontinuous current)} \end{aligned} \quad (8.44)$$

The average motor current, I_{av} , and torque, T_{av} , are

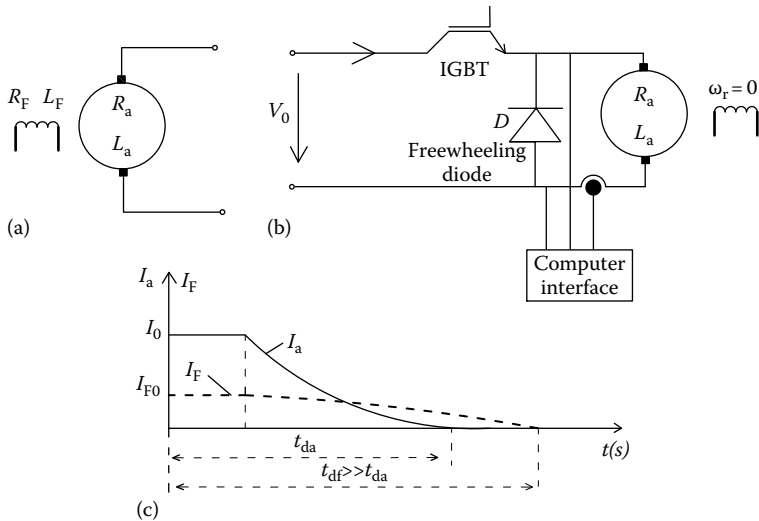
$$\begin{aligned} I_{av} &= \frac{1}{T} \int_0^T I_a(t) dt \\ T_{av} &= p_1 \Psi_{PM} I_{av} \end{aligned} \quad (8.45)$$

So the average gain of the dc–dc converter is larger for a discontinuous current. To offset the armature effect of the discontinuous current on the dynamic behavior of the motor, the latter situation is first detected and then the modulation index (gain), α^* , is increased artificially by $\Delta\alpha$ from a lookup table so as to maintain the average voltage that would exist for the continuous current at $\alpha^* + \Delta\alpha = \alpha_a$. For control purposes, the dc–dc converter can then be approximated by a sample followed by a zero-order hold, with the α_a gain. Controlled rectifiers and 2,4 quadrant buck/boost dc–dc converters are also used to control dc brush motors, but this subject is not discussed here [5,6].

8.7 Parameters from Test Data/Lab 8.1

In the preceding sections, the parameters involved in the transient equations of dc brush machines were explained. In this section, we briefly discuss how these parameters can be determined from experiments. Machines of medium (traction) or large (metallurgy) powers carry large armature currents. In some cases, measures are taken by compensating windings to cancel the armature reaction field, but magnetic saturation due to field current, I_F , still exists and varies with I_F . On the other hand, low power machines do not have interpoles and compensating windings and, in such cases, large armature currents may influence the excitation field, and thus the level of magnetic saturation. PM dc brush motors with PM stator poles, with armature windings in slots or “in air,” do not exhibit notable or variable magnetic saturation.

Only one basic test for parameter estimation is used here: current decay standstill tests. Standstill tests do not imply the coupling of another machine, and the energy consumption for them is small. Standstill tests include step and frequency response standstill tests. We present here only step response (current decay) tests.

**FIGURE 8.9**

(a) Separately excited dc brush machine, (b) fed at standstill from a dc–dc converter, and (c) current decay after IGBT turn off.

Let us consider the machine shown in Figure 8.9a that is supplied through a dc–dc converter at an average initial current, I_0 , in the armature circuit (Figure 8.9b). Then the IGBT is opened and the current freewheels through the diode until it reaches, say, $0.01i_0$, after time t_{da} . The freewheeling diode voltage, V_d , and the current, I_a , are acquired through a computer interface.

The machine voltage equation is

$$R_a i_a + L_a \frac{di_a}{dt} + V_d(t) = 0; \quad (i_a)_{t=0} = I_0 \quad (8.46)$$

Considering the final current as zero (in reality, $0.01i_0$), the integration of Equation 8.46 leads to

$$L_a i_0 = \int_0^{t_{da}} V_d(t) dt + \int_0^{t_{da}} R_a i_a(t) dt \quad (8.47)$$

Now, if we have already measured the armature resistance, R_a , by using the initial diode voltage, V_{d0} , and initial current, i_0 , i.e., $R_a = V_{d0}/i_0$, Equation 8.47 yields the armature inductance, L_a , that corresponds to i_0 . A few different values of i_0 may be chosen to check the eventual variation of L_a with i_0 and to adopt an average value from measurements. The same test may be performed for the field circuit, with an open armature circuit, to obtain $L_F(I_{F0})$.

Additionally, the current decay test in one axis (d or q) may be done in the presence of a dc constant current in the other axis, to check the

so-called cross-coupling saturation effect. The inclusion of a freewheeling diode voltage, V_d , is important, especially in high-rated current or low-voltage machines.

Note: To determine the $L_{dm}i_F$ or Ψ_{PM} excitation flux in an emf, E , a no-load motor test may be run, with known i_a , V_a , and ω_r and neglected or known brush voltage drop, ΔV_{brush} :

$$E = \omega_r L_{dm} i_F \approx V_a - R_a I_a - \Delta V_{brush} \quad (8.48)$$

The inertia, J , can be found from a free deceleration test with $I_F = 0$, and measured $\omega_r(t)$:

$$\frac{J}{p_1} \frac{d\omega_r}{dt} = -p_{mec} \frac{p_1}{\omega_r} \quad (8.49)$$

with p_{mec} segregated from the motor no-load tests at constant speed but gradually smaller voltage, V_a , and field current, I_F . For a dc brush PM machine, this method does not work because core losses may not be separated from mechanical losses. In this case, J may be measured by the pendulum method. Complete testing of dc brush machines is described in standards such as those of NEMA, IEEE, and IEC.

8.8 Summary

- Single dc excitation, or PM, brush machines with one stator and one wound rotor with fixed brushes (or fixed magnetic fields, or stator coordinates) fit into the simplified dq model.
- Single dc excitation may be separate, shunt, or series type, but the structural diagram of the dc brush machine shows two electrical time constants and variable product nonlinearities.
- The field circuit is decoupled from the armature circuit.
- The order of the separately excited dc brush machine set of equations is four, with I_F , I_a , ω_r , and θ_r as variables and V_a , V_F , and T_{load} as inputs.
- For electromagnetic (fast) transients, the speed may be considered constant.
- A second-order system is obtained for a dc generator supplying an R_L and L_L load.

The excitation circuit introduces its large time constant to delay the V_a response to V_F variations, but the voltage regulation is very small ($R_a I_a$).

- Sudden short circuit at terminals is an electromagnetic (fast) transient, which is also dangerous above 5% of rated speed, at rated field current, I_{Fn} (or PM).
- For constant flux linkage (or PM) electromechanical transients, the order of the dc brush machine system is two, with i_a and ω_r as variables and V_a and T_{load} as inputs.

With constant parameters (inductances), the second-order system is linear, and thus easy to investigate.

- The two eigenvalues always have negative real parts, so the response is always stable, but it may be oscillatory if $4T_e > T_{em}$, where T_e is the electrical constant and T_{em} is the electromechanical time constant.
- For variable flux electromechanical transients, one more large real and negative eigenvalue is added, $\gamma_3 = -L_F/R_F$, to the case of constant flux transients.

This is why, for fast response it is desirable to keep the flux constant.

- The dc brush series motor electromechanical transients can be described, after linearization, as a second-order system, but with an electric time constant that decreases with speed ω_r .
- When dc–dc converter fed, the dc brush machine perceives a variable average voltage, V_{av} , and may operate under a continuous or discontinuous current mode. The discontinuous mode has to be avoided to escape sluggish control, especially at low speeds.
- Close-loop control of dc brush motor for variable speed is only introduced here along with the dc current decay standstill tests for parameter identification.

8.9 Proposed Problems

- 8.1 A separately excited dc generator running at constant speed supplies a load with $R_l = 1 \Omega$, $L = 1 \text{ H}$. The armature resistance is $R_a = 0.1 \Omega$, and $L_a = 0$. The field circuit, characterized by $R_F = 50 \Omega$ and $L_F = 5 \text{ H}$, is suddenly connected to a 120 V dc source. For the given speed, $E/i_F = \omega_r L_{dm} = 40 \text{ V/A}$, determine the buildup of the armature current.

Hint: Check Example 8.1.

- 8.2 A dc brush PM motor having an armature resistance, $R_a = 0.12 \Omega$ and $L_a = 0$, $2p_1 = 2$, is started on a load, $T_L = 0.2 + 10^{-3} \times \omega_r$, from rest, by connecting it to a 12 V dc source. The torque is constant, $p_1 \Psi_{PM} = 0.073 \text{ N m/A}$, and the inertia, $J = 2 \times 10^{-4} \text{ kg m}^2$.

Calculate the speed, $\omega_r(t)$, armature current, $i_a(t)$, and torque, $T_e(t)$, during the starting process.

Hint: Use Equation 8.21, with $T_e = 0$, and consider $V_a = 12 \text{ V dc} = \text{constant}$.

- 8.3 The motor in Problem 8.2 operates at 1500 rpm at steady state. Subsequently, the load torque is decreased stepwise by 20%. Calculate the steady state armature current and then the speed, ω_r , and torque, T_e , during a stepwise torque decrease.

Hint: Check Example 8.2 and notice the new boundary condition $(di_a/dt)_{t=0} = 0$.

- 8.4 A series dc brush motor has the data $R_a = 2R_F = 1 \Omega$, $L_a = 10^{-2} \text{ H}$, $L_{Fs} = 0.5 \text{ H}$, $p_1 = 2$ and operates at $n_n = 1500 \text{ rpm}$ from $V_{dc} = 500 \text{ V}$ and at $I_a = 100 \text{ A}$. All but rotor windings losses are neglected. After calculating the rated emf, E , L_{dm} ($\omega_r L_{dm} I_{an} = E$), and the steady state, T_e , determine the machine eigenvalues $\gamma_{1,2}$ for $n_n = 1500 \text{ rpm}$ and for $n = 750 \text{ rpm}$. Calculate the current and speed transients at constant voltage ($\Delta V = 0$) for a 20% increase in the load torque at 1500 rpm.

Hint: Check Section 8.4 and Equations 8.35 through 8.37.

- 8.5 A dc brush PM machine with $R_a = 1 \Omega$, $L_a/R_a = 5 \times 10^{-3} \text{ s}$, $p_1 = 1$, and emf $= 0.05 \text{ V/rad/s}$ operates at $\omega_r = 120 \text{ rad/s}$. The motor is fed from a dc-dc converter with the switching period $T_s = 10^{-3} \text{ s}$ and $V_a = 12 \text{ V dc}$. Assuming constant speed and instantaneous dc-dc converter commutation, determine the armature current profile.

Hint: Check Section 8.6.

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Synchronous Machine Transients

9.1 Introduction

In Chapter 6, we investigated the steady state of synchronous machines, when the currents and voltages have constant amplitude, and the frequency is constant and equal to the electric speed, $\omega_1 = \omega_{ro}$. Also, the voltage power angle, δ_v , and the torque were constant.

During transient processes such as connection (or disconnection) from (to) the power grid, or when the load varies or the SM is fed through PWM state converters for energy or motion control at variable speed, all or most electric (amplitude of voltages and currents) or mechanical (power angle, δ_v , torque, T_e , speed, ω_r) variables vary in time.

During transients, the steady-state model of the SM is not operational. On the other hand, the dq (complex variable) model—described in Chapter 7—is quite suitable for the modeling of transients of ac machines, especially for synchronous machines.

The phase coordinate model is introduced here first and then the parameter equivalence with the dq model is worked out. The modeling of transients is treated in general for the dq model of SM, and then typical transients are dealt with. Transients can be classified as

- Electromagnetic transients
- Electromechanical transients: small deviation and large deviation theories
- Electromagnetic and electromechanical transients for controlled flux
- Variable speed SMs, for modern drives and generator control

Modeling of transients of a split-phase capacitor PMSM (or reluctance SM) is described in detail. Also, rectangular current-controlled PMSM and switched reluctance motor modeling is illustrated in detail.

Finally, standstill current decay and frequency response tests for the SM parameter estimation is described extensively, as it is already a part of recent IEEE standards.

9.2 Phase Inductances of SMs

We start with the salient pole SM (Figure 9.1a and b). The distributed ac windings are considered to have inductances that vary sinusoidally with the rotor position (Figure 9.1c).

The flux barrier in Figure 9.1b, provides constant airgap, which, for a sinusoidal ideal winding distribution, produces sinusoidal airgap flux density. This is a physical justification for the main condition: airgap flux density sinusoidal distribution for the equivalence of dq model and the actual machine. *Note:* The case of rotor winding loss in a constant airgap PMSM with nonoverlapping coil ac windings ($q < 0.5$ slots pole phase) and sinusoidal emf can also be treated by the dq0 model. In this case, the synchronous inductances, $L_d = L_q = L_s$, are constant.

The phase A inductance (Figure 9.1c) is expressed as

$$L_{AA, BB, CC}^{\theta_{er}} = L_{sl} + L_0 + L_2 \cos \left(2\theta_{er} + (i-1)\frac{2\pi}{3} \right); \quad i = 1, 2, 3 \quad (9.1)$$

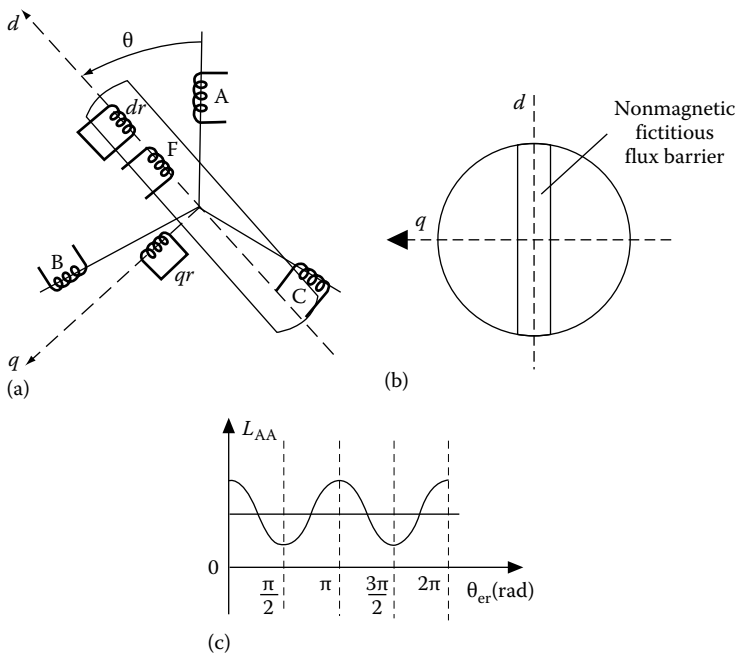


FIGURE 9.1

The phase circuits and inductances of SMs: (a) phase circuits, (b) two-pole salient rotor with constant airgap and one flux barrier, and (c) phase A self-inductance vs. rotor position, θ_{er} (electric angle).

For $\theta_{er} = 0$, and $\pi, 2\pi$, L_{AA} is maximum, and L_{sl} is the leakage inductance. Also, the mutual stator inductances, $L_{AB,BC,CA}$, are

$$L_{BC,CA,AB}^{\theta_{er}} = M_0 + L_2 \cos \left(2\theta_{er} + (i-1) \frac{2\pi}{3} \right); \quad i = 1, 2, 3 \quad (9.2)$$

For a symmetric 3-phase distributed winding ($q \geq 2$), the constant component of mutual inductance, M_0 , is

$$M_0 = L_0 \cos \left(\frac{2\pi}{3} \right) = -\frac{L_0}{2} \quad (9.3)$$

The stator/rotor mutual inductances are straightforward, because of the permitted sinusoidal distribution of windings and the constant airgap:

$$L_{A,B,C,F} = M_F \cos \left(\theta_{er} + (i-1) \frac{2\pi}{3} \right); \quad i = 1, 2, 3 \quad (9.4)$$

$$L_{A,B,C,dr} = M_{dr} \cos \left(\theta_{er} + (i-1) \frac{2\pi}{3} \right) \quad (9.5)$$

$$L_{A,B,C,qr} = -M_{qr} \sin \left(\theta_{er} + (i-1) \frac{2\pi}{3} \right) \quad (9.6)$$

$L_{sl} + L_0, L_2, M_F$ can be determined using standstill tests when a single phase (A) is ac fed at a low frequency (to neglect the core losses) and when we measure V_{A0}, V_{B0} , and V_{F0} for two distinct rotor positions, $\theta_{er} = 0_{(axis d)}$, and $\theta_{er} = \frac{\pi}{2} (axis q)$:

$$I_{A0} \approx \frac{V_{A0}}{\omega L_{AA}}, \quad L_{AB} = \frac{V_{B0}}{\omega I_{A0}}, \quad L_{AF} = \frac{V_{Fe}}{\omega I_{A0}} \quad (9.7)$$

It is true that these measurements do not necessarily reflect the actual magnetic saturation in the machine at the rated speed and load, but they serve to enforce the above assumptions.

Moreover, FEM may be used to calculate the above-mentioned inductance coefficients. Determination of M_{dr} and M_{qr} require more elaborated tests, explained in paragraph 9.19 of this chapter.

9.3 Phase Coordinate Model

In a matrix form, the stator phase coordinate circuit model for the 3-phase SM is

$$\left| I_{ABCFdrqr} \right| \times \left| R_{ABCFdrqr} \right| - \left| V_{ABCFdrqr} \right| = -\frac{d}{dt} \left\{ \left| L_{ABCFdrqr}^{\theta_{er}} \right| \times \left| I_{ABCFdrqr} \right| \right\} \quad (9.8)$$

with

$$|V_{\text{ABCFdrqr}}| = |V_A, V_B, V_C, V_F, 0, 0|^T \quad (9.9)$$

$$|I_{\text{ABCFdrqr}}| = |I_A, I_B, I_C, I_F^r, I_{dr}^r, I_{qr}^r|^T \quad (9.10)$$

$$|R_{\text{ABCFdrqr}}| = \text{Diag} \left[R_s, R_s, R_s, R_F^r, R_{dr}^r, R_{qr}^r \right] \quad (9.11)$$

$$|L_{\text{ABCFdrqr}}^{(\theta_{er})}| = \begin{pmatrix} L_{AA}(\theta_{er}) & L_{AB}(\theta_{er}) & L_{CA}(\theta_{er}) & L_{AF}(\theta_{er}) & L_{Adr}(\theta_{er}) & L_{Aqr}(\theta_{er}) \\ L_{AB}(\theta_{er}) & L_{BB}(\theta_{er}) & L_{BC}(\theta_{er}) & L_{BF}(\theta_{er}) & L_{Bdr}(\theta_{er}) & L_{Bqr}(\theta_{er}) \\ L_{CA}(\theta_{er}) & L_{BC}(\theta_{er}) & L_{CC}(\theta_{er}) & L_{CF}(\theta_{er}) & L_{Cdr}(\theta_{er}) & L_{Cqr}(\theta_{er}) \\ L_{AF}(\theta_{er}) & L_{BF}(\theta_{er}) & L_{CF}(\theta_{er}) & L_F^r & L_{Fdr}^r & 0 \\ L_{Adr}(\theta_{er}) & L_{Bdr}(\theta_{er}) & L_{Cdr}(\theta_{er}) & L_{Fdr}^r & L_{dr}^r & 0 \\ L_{Aqr}(\theta_{er}) & L_{Bqr}(\theta_{er}) & L_{Cqr}(\theta_{er}) & 0 & 0 & L_{qr}^r \end{pmatrix} \quad (9.12)$$

Only the rotor self-inductances, $L_F^r, L_{dr}^r, L_{qr}^r$, and the mutual inductance, L_{Fdr} , are independent of the rotor position in Equation 9.12. For the non-salient pole rotor SMs with dc electromagnetic or PM excitation, $L_2 = 0$ and, thus, L_{AA}, L_{BB}, L_{CC} , and $L_{BC,CA,AB}$ are also constant. Still, the stator/rotor mutual inductances are dependent on the rotor position. (The PM excitation may be seen as a constant, i_F , dc excitation circuit.)

Multiplying Equation 9.8 by $[I]^T$, we obtain

$$\begin{aligned} [I]^T \times [V] &= [I]^T [I] [I] + \frac{d}{dt} \left(\frac{1}{2} [I]^T |L(\theta_{er})| [I] \right) + \frac{1}{2} [I]^T \\ &\quad \times \left| \frac{\partial L(\theta_{er})}{\partial \theta_{er}} \right| \times [I] \times \frac{d\theta_{er}}{dt} \end{aligned} \quad (9.13)$$

In the absence of iron losses, the last term in Equation 9.13 is the electromagnetic power, P_e

$$P_e = T_e \times \frac{d\theta_{er}}{p_1 dt} = \frac{1}{2} [I]^T \left| \frac{\partial L(\theta_{er})}{\partial \theta_{er}} \right| [I] \times \frac{d\theta_{er}}{dt} \quad (9.14)$$

So, the electromagnetic torque is

$$T_e = \frac{p_1}{2} [I]^T \left| \frac{\partial L(\theta_{er})}{\partial \theta_{er}} \right| [I] \quad (9.15)$$

The motions equations are then added:

$$\frac{J}{p_1} \frac{d\omega_r}{dt} = T_e - T_{\text{load}}; \quad \frac{d\theta_{er}}{dt} = \omega_r \quad (9.16)$$

The SM model described above exhibits an eighth order, with six currents, ω_r, θ_{er} as variables, four voltages, and load torque, T_{load} , as inputs.

Such a high-order system with products of variables and variable coefficients (position dependent inductances) can be solved only numerically, requiring large CPU time. It is also very difficult to apply it to a system control.

The phase coordinate model may be used in special cases, such as for uniform airgap SMs without a rotor cage, the so-called brushless dc PM motors supplied with rectangular dc currents through PWM converters in variable frequency (speed) drives.

9.4 dq0 Model—Relationships of 3-Phase SM Parameters

The dq0 model for the SM, in rotor coordinates, has been introduced in Chapter 7, together with the voltage, current, and flux equivalence relationships:

$$\frac{d\psi_d}{dt} = V_d - R_s I_d + \omega_r \psi_q; \quad \frac{d\psi_q}{dt} = V_q - R_s I_q - \omega_r \psi_d \quad (9.17)$$

$$\frac{d\psi_F^r}{dt} = V_F^r - R_F^r I_F^r; \quad \frac{d\psi_{dr}^r}{dt} = -R_{dr}^r I_{dr}^r; \quad \frac{d\psi_{qr}^r}{dt} = -R_{qr}^r I_{qr}^r \quad (9.18)$$

$$\frac{d\psi_0}{dt} = V_0 - R_s I_0; \quad \frac{d\omega_r}{dt} = \frac{p_1}{J} \left(\frac{3p_1}{2} (\psi_d i_q - \psi_q i_d T_{\text{load}}) \right) \quad (9.19)$$

and

$$\begin{vmatrix} i_d \\ i_q \\ i_0 \end{vmatrix} = |S_{dq0}(\theta_{er})| \begin{vmatrix} I_A \\ I_B \\ I_C \end{vmatrix}; \quad \theta_{er} = \int \omega_r dt + \theta_0 \quad (9.20)$$

$$S_{dq0}(\theta_{er}) = \frac{2}{3} \begin{vmatrix} \cos(-\theta_{er}) & \cos\left(-\theta_{er} + \frac{2\pi}{3}\right) & \cos\left(-\theta_{er} - \frac{2\pi}{3}\right) \\ \sin(-\theta_{er}) & \sin\left(-\theta_{er} + \frac{2\pi}{3}\right) & \sin\left(-\theta_{er} - \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{vmatrix} \quad (9.21)$$

Equivalence 9.20 is also valid for voltages V_d , V_q , and V_0 and for flux linkages ψ_d , ψ_q , and ψ_0 :

$$\begin{vmatrix} \psi_d \\ \psi_q \\ 0 \end{vmatrix} = |S_{dq0}(\theta_{er})| \begin{vmatrix} \psi_A \\ \psi_B \\ \psi_C \end{vmatrix} \quad (9.22)$$

The rotor variables of the SM need not be changed for the dq0 model because the rotor windings (F , d_r , q_r) are orthogonal by nature.

From Equation 9.13 and inductance matrix 9.12, the stator flux linkages, ψ_A, ψ_B, ψ_C are

$$\begin{aligned}\psi_A &= L_{AA}I_A + L_{AB}I_B + L_{CA}I_C + L_{AF}^r I_F^r + L_{Adr}^r I_{dr}^r + L_{Aqr}^r I_{qr}^r \\ \psi_B &= L_{AB}I_A + L_{BB}I_B + L_{CB}I_C + L_{BF}^r I_F^r + L_{Bdr}^r I_{dr}^r + L_{Bqr}^r I_{qr}^r \\ \psi_C &= L_{CA}I_A + L_{CB}I_B + L_{CC}I_C + L_{CF}^r I_F^r + L_{Cdr}^r I_{dr}^r + L_{Cqr}^r I_{qr}^r\end{aligned}\quad (9.23)$$

In Equation 9.23, the inverse Park transformation is used, in order to eliminate stator currents:

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \frac{3}{2} [S_{dq0}]^T \begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix}\quad (9.24)$$

Finally, we obtain

$$\begin{aligned}\psi_d &= L_{sl}I_d + \frac{3}{2}(L_0 + L_2)I_d + \frac{3}{2}M_F I_F^r + \frac{3}{2}M_{dr} I_{dr}^r \\ \psi_q &= L_{sl}I_q + \frac{3}{2}(L_0 - L_2)I_q + \frac{3}{2}M_{qr} I_{qr}^r\end{aligned}\quad (9.25)$$

But for the dq0 model with the rotor reduced to the stator (Chapter 7), ψ_d , and ψ_q are

$$\begin{aligned}\psi_d &= L_{sl}I_d + L_{dm}(I_d + I_F + I_{dr}) \\ \psi_q &= L_{sl}I_q + L_{qm}(I_q + I_{qr})\end{aligned}\quad (9.26)$$

The parameter equivalence between Equations 9.25 and 9.26 is straightforward:

$$L_{dm} = \frac{3}{2}(L_0 + L_2); \quad L_{qm} = \frac{3}{2}(L_0 - L_2)\quad (9.27)$$

$$I_F = I_F^r K_F; \quad K_F = \frac{3}{2} \frac{M_F}{L_{dm}}; \quad I_{dr} = I_{dr}^r K_{dr}; \quad K_{dr} = \frac{3}{2} \frac{M_{dr}}{L_{dm}}\quad (9.28)$$

$$I_{qr} = I_{qr}^r K_{qr}; \quad K_{qr} = \frac{3}{2} \frac{M_{qr}}{L_{qm}}\quad (9.29)$$

From the conservation of rotor power and losses

$$V_F = \frac{V_F^r}{K_F}; \quad R_F = R_F^r \frac{1}{K_F^2}; \quad R_{dr} = R_{dr}^r \frac{1}{K_{dr}^2}; \quad R_{qr} = R_{qr}^r \frac{1}{K_{qr}^2}\quad (9.30)$$

Also, from the conservation of the leakage magnetic field energy

$$L_{Fl} = L_{Fl}^r \frac{1}{K_F^2}; \quad L_{drl} = L_{drl}^r \frac{1}{K_{dr}^2}; \quad L_{qrl} = L_{qrl}^r \frac{1}{K_{qr}^2}; \quad L_{Fdr} = L_{Fqr}^r \frac{1}{K_F K_{dr}}\quad (9.31)$$

The stator resistance, R_s , and phase leakage inductance, L_{sl} , remain in the dq0 model with their values in the real three-phase machine. L_{Fl} , L_{drl} , L_{qrl} , and L_{Fdr} are rotor leakage inductances reduced to the stator. Now L_{dm} , and

L_{qm} are the cyclic magnetization inductances expressed in Chapter 6 for the steady and state.

For the PMSM without a rotor cage, the dq0 model greatly simplifies to

$$\frac{d\psi_d}{dt} = V_d - R_s I_d + \omega_r \psi_q; \quad \psi_d = \psi_{PM} + L_d I_d \quad (9.32)$$

$$\frac{d\psi_q}{dt} = V_q - R_s I_q - \omega_r \psi_d; \quad \psi_q = L_q I_q; \quad L_d < L_q \quad (9.33)$$

The torque Equation 9.19 is

$$T_e = \frac{3}{2} p_1 (\psi_{PM} + (L_d - L_q) I_d) I_q \quad (9.34)$$

For the 3-phase reluctance synchronous motor, $\psi_{PM} = 0$ and $L_d > L_q$.

9.5 Structural Diagram of the SM dq0 Model

The dq0 model of the SM Equations 9.17 through 9.19 and the rotor flux/current relationships

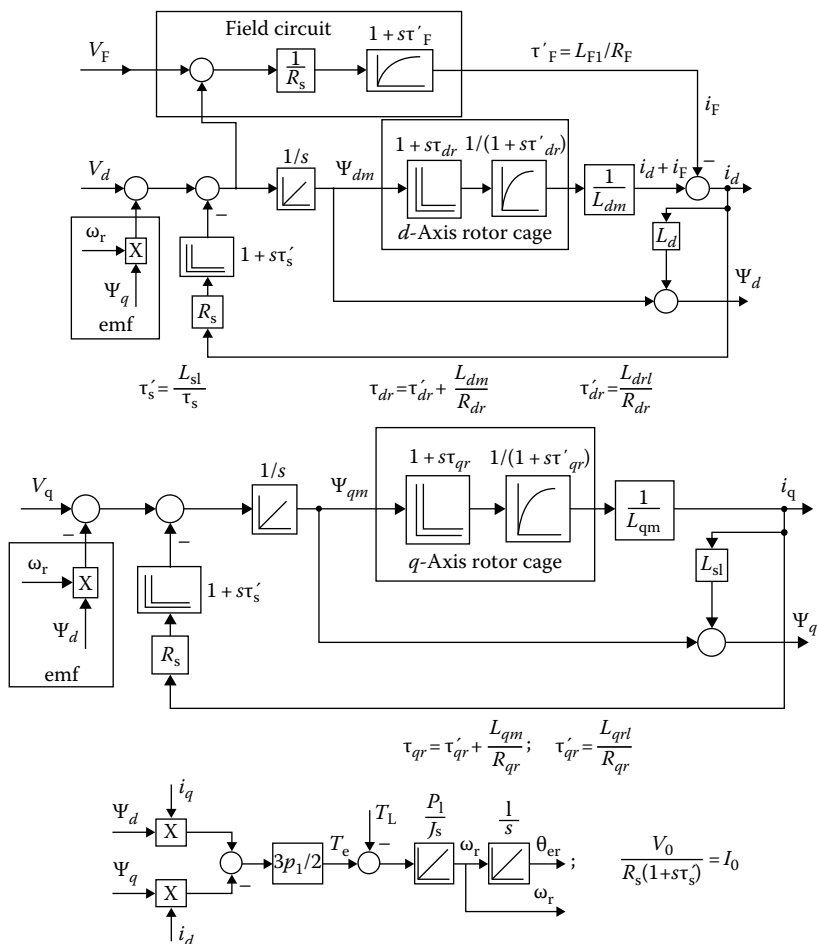
$$\begin{aligned} \psi_F &= L_{FI} I_F + \psi_{dm} \\ \psi_{dr} &= L_{drl} I_{dr} + \psi_{dm} \\ \psi_{qr} &= L_{qrl} I_{qr} + \psi_{qm} \end{aligned} \quad (9.35)$$

may be put under the Laplace form $\left(\frac{d}{dt} \rightarrow s\right)$ as

$$\begin{aligned} (V_F - s\psi_{dm}) \times \frac{1}{R_F(1 + s\tau'_F)} &= I_F \\ V_d - R_s(1 + s\tau'_s)I_d + \omega_r \psi_q &= s\psi_{dm} \\ I_d + I_F &= \frac{\psi_{dm}(1 + s\tau'_{dr})}{L_{dm}(1 + s\tau'_{dr})} \\ V_q - R_s(1 + s\tau'_s)I_q - \omega_r \psi_d &= s\psi_{qm} \\ \frac{\psi_{qm}(1 + s\tau'_{qr})}{L_{qm}(1 + s\tau'_{qr})} &= i_q \\ \tau'_s &= L_{sl}/R_s; \quad \tau_{dr} = (L_{drl} + L_{dm})/R_{dr}; \quad \tau_{qr} = (L_{qrl} + L_{qm})/R_{qr} \\ \tau'_{dr} &= L_{drl}/R_{dr}; \quad \tau'_{qr} = L_{qrl}/R_{qr}; \quad \tau'_F = L_{FI}/R_F \end{aligned} \quad (9.36)$$

The above-mentioned equations are illustrated in the form of a structural diagram in Figure 9.2.

We may identify small (milliseconds to tens of milliseconds) time constants, τ'_s , τ'_F , τ'_{dr} , and τ'_{qr} and two large (hundreds of milliseconds) time constants τ_{dr} , and τ_{qr} .

**FIGURE 9.2**

Structural diagram of the dq0 model of SMs: (a) axis d , (b) axis q , and (c) motion equations.

For the steady state, at $\omega_b = \omega_r$ (rotor coordinates), $s = 0$ and thus the structural diagram loses the influence of all time constants. In the absence of d , q axes rotor cages, τ_{dr} , τ'_{dr} , τ_{qr} , and τ'_{qr} are eliminated from the structural diagram all together.

Now if the machine has PMs or a variable reluctance rotor and no cage on the rotor, the structural diagram simplifies notably (Figure 9.3).

Once the motion emfs, $\omega_r \psi_d$, and $\omega_r \psi_q$, are added (in axis d) or subtracted (in axis q) to the respective voltages, V_d , V_q , only the stator leakage (small) time constants remain active.

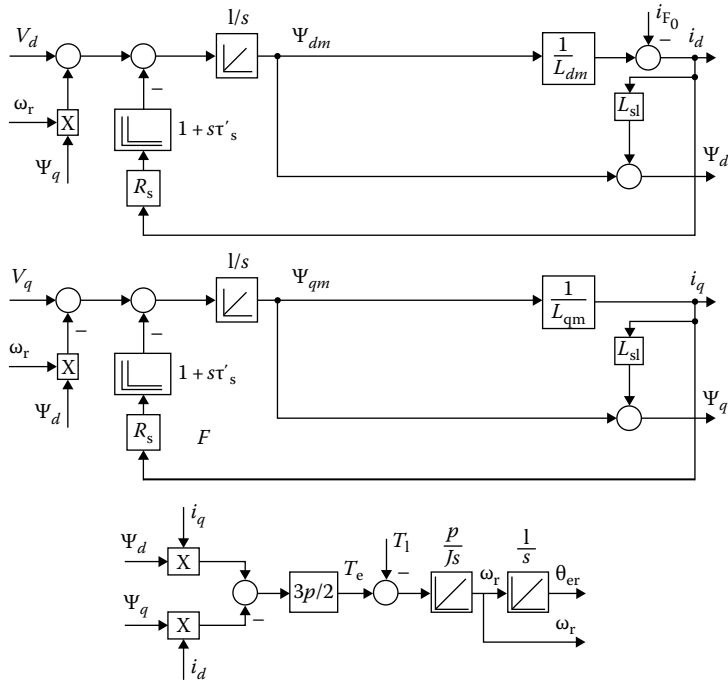


FIGURE 9.3

Structural diagram for the PM and reluctance SMs (without damper cage): (a) axis d , (b) axis q , and (c) motion equations.

This was how dc current control in PMSMs originated. The same equations (Equation 9.31) may be arranged into the well-known equivalent circuits for transients (Figure 9.4). For the PM machine, the dc excitation circuit is replaced by a constant current source, $i_{F0} = \psi_{PM}/L_{dm}$.

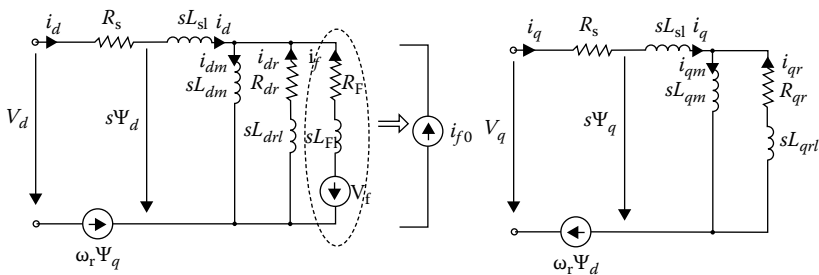


FIGURE 9.4

SM dq model equivalent circuits for transients.

Again, a steady state is obtained for $s = 0$; iron losses are not yet considered. They do not have much influence other than during the first 3–5 ms into the transients [1]. However, in large iron core loss machines, they might be included as resistances in parallel with the motion emfs, $\omega_r \psi_d$ and $\omega_r \psi_q$. At zero speed, $\omega_r \psi_d = 0$ and $\omega_r \psi_q = 0$, and the equivalent circuits become very instrumental for the parameter estimation from the standstill current decay and frequency response tests described in paragraph 9.19 of this chapter.

9.6 pu dq0 Model of SMs

The pu (per unit) system is used to norm the system of equations by expressing voltages, currents, flux linkages, resistances, inductances, torque, and power in relative units.

Such a denotation limits all numerical values to, say, 30, at most; it also brings more generality to the results and leads the way . . . toward a global standardization.

To build a pu system, base voltages are required. There is more than one way of doing this.

One method that is widely accepted is described here:

- $V_{no}\sqrt{2}$ —base voltage (peak value of phase voltage)
- $I_{no}\sqrt{2}$ —base current (peak value of phase current)
- $X_{no} = V_{no}/I_{no} = \omega_{10}L_{no}$ —base inductance (reactance at base frequency, ω_{10})
- $\psi_{no} = V_{no}\sqrt{2}/\omega_{10}$ —base flux linkage
- $P_{no} = 3V_{no}I_{no}$ —base power
- $T_{no} = P_{no} \times p_1/\omega_{10}$ —base torque
- $H = \frac{J\omega_{10}^2}{2p_1^2P_{no}}$ —inertia in seconds

Now if time is measured in seconds, then $\frac{d}{dt} \rightarrow \frac{1}{\omega_{10}} \frac{d}{dt}$ in the original model (Equations 9.17 through 9.19):

$$\begin{aligned}
 \frac{1}{\omega_{10}} \frac{d\psi_d}{dt} &= V_d - r_s i_d + \omega_r \psi_q; & \frac{1}{\omega_{10}} \frac{d\psi_{qr}}{dt} &= V_q - r_s i_q - \omega_r \psi_d \\
 \frac{1}{\omega_{10}} \frac{d\psi_F}{dt} &= V_F - r_F i_F; & \frac{1}{\omega_{10}} \frac{d\psi_{dr}}{dt} &= -r_{dr} i_{dr}; & \frac{1}{\omega_{10}} \frac{d\psi_{qr}}{dt} &= -r_{qr} i_{qr} \\
 \frac{d\omega_r}{dt} &= \frac{1}{2H} (\psi_d i_d - \psi_q i_q - t_{shaft}); & t_e &= \psi_d i_q - \psi_q i_d; & \frac{1}{\omega_{10}} \frac{d\theta_{er}}{dt} &= \omega_r
 \end{aligned}
 \tag{9.37}$$

The flux/current relationships in the pu system remain practically the same as for the actual dq variables, but the difference is that lower case letters are often used (Equation 9.20):

$$\begin{aligned}\psi_d &= l_{sl}i_d + l_{dm}(i_d + i_{dr} + i_F) \\ \psi_q &= l_{sl}i_q + l_{qm}(i_q + i_{qr}) \\ \psi_F &= l_{Fl}i_F + l_{dm}(i_d + i_{dr} + i_F) + l_{Fdr}(i_{dr} + i_F) \\ \psi_{dr} &= l_{dr}i_{dr} + l_{dm}(i_d + i_{dr} + i_F) + l_{Fdr}(i_{dr} + i_F) \\ \psi_{qr} &= l_{qr}i_{qr} + l_{qm}(i_q + i_{qr})\end{aligned}\quad (9.38)$$

The mutual inductance, l_{Fdr} , allows for an additional leakage flux coupling between the d axis cage and the field circuit.

Example 9.1 PU Parameters in Ohms

A large synchronous motor operated at the power grid has the design data: $P_n = 1850$ kW, efficiency $\eta_n = 0.983$, $\cos \varphi_n = 0.9$, $f_1 = 50$ Hz, $V_{nl} = 10.0$ KV, $n_1 = 100$ rpm, $l_{dm} = 0.6$ (pu), $l_{qm} = 0.4$ (pu), $l_{sl} = 0.1$ (pu), $l_{dr} = 0.11$ (pu), $r_{dr} = 0.05$ (pu), $l_{qr} = 0.025$ (pu), $r_s = 0.01$ (pu), $l_{Fl} = 0.17$ (pu), $r_F = 0.016$ (pu), and $l_{Fdr} = -0.037$ (pu).

Let us consider that the field current, i_F , for the case in point (rated power) is $i_F = 2.5$ (pu) and $H = 2$ s. Calculate the rated current, number of pole pairs, p_1 , base torque, base reactances, all resistances and inductances in ohms, inertia J (in kg m²), i_F in amperes, and V_F in volts. Also, calculate the time constants, $\tau'_F, \tau'_s, \tau'_{dr}, \tau'_{qr}, \tau_{qr}, \tau_{qr'}$, in seconds.

Solution:

According to the efficiency, η_1 , and power factor definitions, the rated current I_n is simply

$$I_n = \frac{P_n}{\sqrt{3}V_{nl}\eta_n \cos \varphi_n} = \frac{1800 \times 10^3}{10\sqrt{3} \times 10^3 \times 0.9 \times 0.983} = 1176 \text{ A} \quad (9.39)$$

The number of poles

$$2p_1 = \frac{2f_1}{n_1} = \frac{2 \times 50}{(100/60)} = 60$$

The base torque

$$T_{no} = P_{no} \times p_1 / \omega_{10} = 1800 \times 10^3 \times 30 / (2\pi 50) = 171.970 \text{ N m} = 171.970 \text{ kN m}$$

The base reactance

$$X_n = \frac{V_n I}{I_n \sqrt{3}} = \frac{10 \times 10^3}{\sqrt{3} \times 1176} = 4.915 \Omega$$

The actual inertia

$$J = 2H \left(\frac{p_1}{\omega_{10}} \right)^2 P_{n0} = 2 \times \left(\frac{30}{314} \right)^2 \times 1,800,000 = 78.87 \times 10^3 \text{ kg m}^2$$

The field current i_F in amperes

$$i_F(A) = i_F(\text{pu}) \times I_n = 2.5 \times 1176 = 2940 \text{ A}$$

The field circuit voltage V_F (reduced to the stator)

$$V_F(V) = R_F(\Omega) i_F(A) = r_F(\text{pu}) X_n I_F(A) = 0.016 \times 4.915 \times 2940 = 231.2 \text{ V}$$

All resistances and reactances may be calculated as

$$R_s(\Omega) = r_s(\text{pu}) \times X_n(\Omega) = 0.01 \times 4.915 = 0.04915 \Omega$$

$$X_{sl}(\Omega) = l_{sl}(\text{pu}) \times X_n(\Omega) = 0.1 \times 4.915 = 0.4915 \Omega$$

The time constants in seconds are

$$\tau'_F = \frac{l_{Fl}}{r_F} \frac{1}{\omega_{10}} = \frac{0.17}{0.016} \times \frac{1}{314} = 3.38 \times 10^{-2} \text{ s}$$

$$\tau'_s = \frac{l_{sl}}{r_s} \frac{1}{\omega_{10}} = \frac{0.1}{0.01} \times \frac{1}{314} = 3.185 \times 10^{-2} \text{ s}$$

$$\tau_{dr} = \frac{l_{drl} + l_{dm}}{r_{dr}} \frac{1}{\omega_{10}} = \frac{0.11 + 0.6}{0.05} \times \frac{1}{314} = 4.522 \times 10^{-2} \text{ s}$$

$$\tau'_{dr} = \frac{l_{drl}}{r_{dr}} \frac{1}{\omega_{10}} = \frac{0.1}{0.05} \times \frac{1}{314} = 6.369 \times 10^{-3} \text{ s}$$

$$\tau_{qr} = \frac{l_{qr} + l_{qrl}}{r_{qr}} \frac{1}{\omega_{10}} = \frac{0.4 + 0.025}{0.03} \times \frac{1}{314} = 4.51 \times 10^{-2} \text{ s}$$

$$\tau'_{qr} = \frac{l_{qrl}}{r_{qr}} \frac{1}{\omega_{10}} = \frac{0.025}{0.03} \times \frac{1}{314} = 2.654 \times 10^{-3} \text{ s}$$

9.7 Balanced Steady State via the dq0 Model

A balanced steady state means, for the ideal grid-connected SM, symmetric sinusoidal phase voltages and currents:

$$\begin{aligned} V_{A,B,C} &= V_0 \sqrt{2} \times \cos \left(\omega_1 t - \left(i - 1 \frac{2\pi}{3} \right) \right); \quad i = 1, 2, 3 \\ I_{A,B,C} &= I_0 \sqrt{2} \times \cos \left(\omega_1 t - (i - 1) \frac{2\pi}{3} - \varphi_1 \right); \quad i = 1, 2, 3 \end{aligned} \quad (9.40)$$

Applying the Park transformation (Equation 9.21) in rotor coordinates ($\theta_{er} = \omega_1 t + \theta_0$) to A, B, C voltages and currents in Equation 9.40 yields

$$\begin{aligned} V_{d0} &= V_0 \sqrt{2} \cos(\theta_0); \quad I_{d0} = I_0 \sqrt{2} \cos(\theta_0 - \varphi_1) \\ V_{q0} &= -V_0 \sqrt{2} \sin(\theta_0); \quad I_{q0} = -I_0 \sqrt{2} \sin(\theta_0 - \varphi_1) \end{aligned} \quad (9.41)$$

A positive φ_1 means a lagging power factor (motor association of voltage/current signs). As the stator voltages and currents in the dq model, for the balanced steady state, are dc, the field current is dc and $d/dt = 0$ and, thus, $i_{dr} = i_{qr} = 0$ (no damper cage currents):

$$V_{F0} = R_F I_{F0}; \quad i_{dr0} = i_{qr0} = 0 \quad (9.42)$$

The torque, T_e , (Equation 9.34) is simplified to

$$T_e' = \frac{3}{2}p_1(\psi_d i_q - \psi_q i_d) = \frac{3}{2}p_1(L_{dm} i_{F0} + (L_d - L_q) i_{d0}) i_{q0} \quad (9.43)$$

The dq stator equations for the steady state ($d/dt = 0$) may be written in a space phasor form as

$$\begin{aligned}\bar{V}_{s0} &= R_s \bar{i}_{s0} + j\omega_r \bar{\Psi}_{s0}; & \bar{\Psi}_{s0} &= \psi_{d0} + j\psi_{q0}; & \bar{i}_{s0} &= i_{d0} + ji_{q0} \\ \psi_{d0} &= L_{dm} i_{F0} + L_d i_d; & \psi_{q0} &= L_{qr} i_{q0}\end{aligned}\quad (9.44)$$

So we can represent Equation 9.44 in a space phasor (vector) diagram as shown in Figure 9.5.

The space phasor (vector) diagram reproduces all phase shift angles of the phasor diagram of the SM described in Chapter 6. Here, the angles for the phase phasors are “space” angles whereas those for the phase phasors described in Chapter 6 were “time” angles.

The relationships between θ_0 in the space phasor diagram (dq model) and the voltage power angle, δ_v , (Chapter 6) is

$$\theta_0 = -\left(\frac{\pi}{2} + \delta_v\right); \quad \delta_v > 0 \quad (9.45)$$

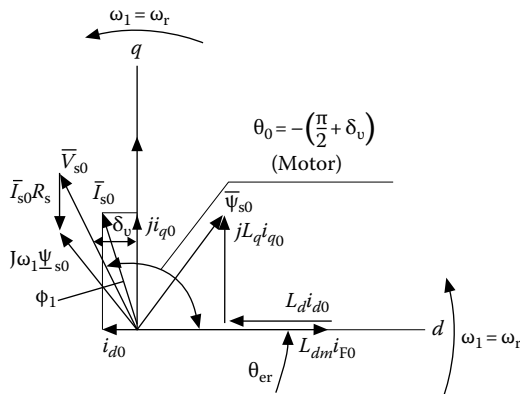


FIGURE 9.5

Space phasor diagram of MS for the balanced steady state.

for a motor operation and

$$\theta_0 = -\left(\frac{\pi}{2} + \delta_v\right); \quad \delta_v < 0 \quad (9.46)$$

for a generator operation mode.

Example 9.2 Balanced Steady-State Operation with dq0 Model

The SM in Example 9.1 operates as a motor at an unity power factor and at $\delta_v = 30^\circ$.

Calculate

- The emf, E_s (no-load voltage)
- V_{d0} , i_{d0} , V_{q0} , i_{q0} , V_{s0} , and i_{s0}
- Steady-state short circuit current and braking torque

To solve the problem, we make use of the space phasor diagram in Figure 9.5 where $\varphi_1 = 0$ and $\delta_v = 30^\circ (\frac{\pi}{6})$.

From Equations 9.41 and 9.45

$$\begin{aligned} V_{d0} &= V_0 \sqrt{2} \times \cos\left(-\frac{\pi}{2} - \delta_v\right) = -V_0 \sqrt{2} \sin \delta_v = -5780 \sqrt{2} \frac{1}{2} = -4074 \text{ V} \\ I_{d0} &= I_0 \sqrt{2} \times \cos\left(-\frac{\pi}{2} - \delta_v\right) = -I_0 \sqrt{2} \sin \delta_v \end{aligned} \quad (9.47)$$

$$\begin{aligned} V_{q0} &= -V_0 \sqrt{2} \times \sin\left(-\frac{\pi}{2} - \delta_v\right) = -V_0 \sqrt{2} \cos \delta_v = 5780 \sqrt{2} \frac{\sqrt{3}}{2} = 7091.77 \text{ V} \\ I_{q0} &= -I_0 \sqrt{2} \times \sin\left(-\frac{\pi}{2} - \delta_v\right) = I_0 \sqrt{2} \cos \delta_v \end{aligned} \quad (9.48)$$

Note: For the generator, φ_1 in Equation 9.45 would have been $\varphi_1 = \pi$. From Example 9.1, $V_0 = \frac{V_{nl}}{\sqrt{3}} = \frac{10 \times 10^3}{\sqrt{3}} = 5780 \text{ V}$

The space phasor diagram (Figure 9.5) with $\varphi_1 = 0$ provides

$$\begin{aligned} V_{d0} &= -\omega_1 \psi_{q0} + R_s i_{d0} = -\omega_1 L_q i_{q0} + R_s i_{d0} \\ V_{q0} &= \omega_1 \psi_{d0} + R_s i_{q0} = \omega_1 (L_d i_{d0} + L_{dm} i_{F0}) + R_s i_{q0} \end{aligned} \quad (9.49)$$

From Equations 9.47 in 9.53, we can compute the two remaining unknowns, I_0 and i_F , if the machine inductances (reactances) are known (Example 9.1).

$$\begin{aligned} X_d &= \omega_1 L_d = (l_{dm} + l_{sl}) X_n = (0.6 + 0.1) 4.915 = 3.44 \Omega \\ X_q &= \omega_1 L_q = (l_{qm} + l_{sl}) X_n = (0.4 + 0.1) 4.915 = 2.4575 \Omega \\ \omega_1 L_{dm} &= l_{dm} X_n = 0.6 \times 4.915 = 2.95 \Omega \\ R_s &= 0.04915 \Omega \end{aligned}$$

A second-order equation has to be solved for this purpose. But to simplify the solution we may neglect the effect of the stator resistance ($r_s = 0.01(\text{pu})$) here.

In this case

$$i_{q0} = \frac{-V_{d0}}{X_q} = \frac{V_0 \sqrt{2} \sin \delta_v}{X_q} = \frac{5780 \times \sqrt{2} \times 0.5}{2.4575} = 1658 \text{ A}$$

$$i_{d0} = -i_{q0} \times \tan \delta_v = -1658 \times \frac{1}{\sqrt{3}} = -958 \text{ A}$$

The stator current phasor is $i_{s0} = I_0 \sqrt{2} = i_{q0} / \cos \delta_v = 1658 / 0.867 = 1912 \text{ A}$ (peak phase value; 1356.2 A (RMS phase value)).

$$i_{F0} \approx \frac{(V_{q0} - X_d i_{d0})}{X_{dm}} = \frac{5780 \sqrt{2} \times \left(\frac{\sqrt{3}}{2}\right) - 3.44(-958)}{2.95} = 3506.8 \text{ A}$$

The no-load voltage is $E_0 = X_{dm} i_{F0} = 3506.80 \times 2.95 = 10,345 \text{ V}$ (peak phase value; 7336 V (RMS, phase value)) For the short circuit, we just put $V_{d0} = V_{q0} = 0$ in the steady-state Equation 9.49 and i_{dsc} and i_{qsc} :

$$\begin{aligned} 0 &= -X_q i_{qsc} + R_s i_{dsc} \\ 0 &= X_d i_{dsc} + X_{dm} i_F + R_s i_{qsc} \end{aligned} \quad (9.50)$$

We retain the stator resistance in Equation 9.50 for generality. In the kilo watt range, for example PMSMs, neglecting R_s would lead to ignoring the braking torque during shortcircuit, with small l_d , l_q , and l_{qm} (which would become 0.5 pu). This would mean ignoring 30%–60% rated torque braking.

The solution of Equation 9.49 is straightforward:

$$i_{qsc} = i_{dsc} \times \frac{R_s}{X_q}; \quad i_{dsc} = \frac{-X_{dm} i_{F0}}{X_d + R_s^2 / X_q} \quad (9.51)$$

And the torque, from Equation 9.43, or from stator copper losses, is

$$T_{esc3} = -\frac{3}{2} R_s (i_{dsc}^2 + i_{qsc}^2) \frac{p_1}{\omega_1} \quad (9.52)$$

$$i_{dsc} = -\frac{10345}{3.44 + 0.04915^2 / 2.4575} = -3006.4 \text{ A}$$

$$i_{qsc} = -3006.4 \times \frac{0.04915}{2.4575} = -60.128 \text{ A}$$

From Equation 9.52, the braking torque is $T_{esc3} = -\frac{3}{2} 0.04915 (3006.4^2 + 60.128^2) \frac{30}{314} = -63,666 \text{ N m} = -63.666 \text{ kN m}$ in pu terms (see Example 9.1; with $T_{en} = 171.90 \text{ kN m}$) $t_{esc3} = T_{esc3} / T_{en} = -63.666 / 171.90 = -0.37$ (pu).

It should be noticed that even at the short circuit, the SM works as a generator, so $T_{esc3} < 0$, as both i_{dsc} and i_{qsc} are negative in the dq model.

9.8 Laplace Parameters for Electromagnetic Transients

From the equivalent circuits for transients (Figure 9.4), the flux current relationships in Laplace terms may be extracted, after elimination of the rotor cage currents

$$\begin{aligned}\psi_d(s) &= L_d(s)i_d(s) + G(s)V_F(s) \\ \psi_q(s) &= L_q(s)i_q(s)\end{aligned}\quad (9.53)$$

with

$$\begin{aligned}L_d(s) &= L_d \frac{(1 + s\tau'_d)(1 + s\tau''_d)}{(1 + s\tau'_{d0})(1 + s\tau''_{d0})} \\ L_q(s) &= L_q \frac{(1 + s\tau''_q)}{(1 + s\tau''_{q0})} \\ G(s) &= \frac{L_{dm}}{R_F} \frac{1 + s\tau'_{dr}}{(1 + s\tau'_{d0})(1 + s\tau''_{d0})}\end{aligned}\quad (9.54)$$

where $L_d(s)$, and $L_q(s)$, $G(s)$ are the so-called operational (Laplace) parameters for SMs. Their form is the same in pu, where only $l_d(s)$, $l_q(s)$, and $g(s)$ would appear in Equation 9.54 instead of $L_d(s)$, $L_q(s)$ and $G(s)$, because the time constants are still measured in seconds:

$$\begin{aligned}\tau'_d &= \left(L_{Fl} + L_{Fdr} + \frac{L_{dm}L_{sl}}{L_{dm} + L_{sl}} \right) \frac{1}{R_F} \\ \tau'_{d0} &= (L_{dm} + L_{Fdr} + L_{Fl}) \frac{1}{R_F} \\ \tau''_d &= \frac{1}{R_{dr}} \left(L_{dr} + \frac{L_{dm}L_{Fdr}L_{Fl} + L_{dm}L_{sl}L_{Fl} + L_{sl}L_{Fdr}L_{Fl}}{L_{dm}L_{Fl} + L_{Fl}L_{sl} + L_{dm}L_{Fl} + L_{se}L_{Fdr} + L_{sl}L_{dm}} \right) \\ \tau''_{d0} &= \frac{1}{R_{dr}} \left(L_{dr} + \frac{L_{Fl}(L_{dm} + L_{Fdr})}{L_{Fl} + L_{dm} + L_{Fdr}} \right) \\ \tau''_{q0} &= \frac{1}{R_{qr}} (L_{qrl} + L_{qm}); \quad \tau'_{dr} = L_{dr}/R_{dr} \\ \tau''_q &= \frac{1}{R_{qr}} \left(L_{qrl} + \frac{L_{qm}L_{sl}}{L_{qm} + L_{sl}} \right)\end{aligned}\quad (9.55)$$

It should be noticed that the Laplace parameters do not contain any information related to machine speed; this is because the dq0 model of SM is used in rotor coordinates. The initial and final values of $L_d(s)$ and $L_q(s)$ corresponding to *subtransient* L''_d and L''_q , and *synchronous inductances* L_d and L_q are

$$L_d'' = \lim_{s \rightarrow \infty (t \rightarrow 0)} L_d(s) = L_d \frac{T_d' T_d''}{T_{d0}' T_{d0}''} < L_d \quad (9.56)$$

$$L_q'' = \lim_{s \rightarrow \infty (t \rightarrow 0)} L_q(s) = L_q \frac{T_q''}{T_{q0}''} < L_q$$

$$L_d = \lim_{s \rightarrow 0 (t \rightarrow \infty)} L_d(s)$$

$$L_q = \lim_{s \rightarrow 0 (t \rightarrow \infty)} L_q(s)$$

In the absence of the damper cage along the d , the so-called transient inductance axis, $L_{d'}$ is defined as

$$L_d'' < L_{d'} = \lim_{s \rightarrow \infty, T_d'' = T_{d0}''} L_d(s) = L_d \frac{T_d'}{T_{d0}'} < L_d \quad (9.57)$$

L_d'' and L_q'' reflect the SM's initial reaction to transients, based on the flux conservation law. Additional (transient) currents occur in the rotor to conserve the initial value of flux that "looses" the "support" of initial stator currents, which change quickly but not instantaneously. It is expected that after a sudden short circuit at SM terminals, the currents in the stator and rotor change dramatically. Indeed, the sudden short-circuit current is large in SMs with a strong (low resistance) rotor cage. For PMSMs without any rotor cage, only synchronous inductances exist, and thus the transients at a sudden short circuit are lower and slower.

As even with the dq0 model the study of transients in general is faced with difficult mathematical hurdles, the transients are approximated according to three categories:

1. Electromagnetic transients
2. Electromechanical transients
3. Mechanical transients

9.9 Electromagnetic Transients at Constant Speed

During fast transients, the speed can be approximated as constant. These are considered electromagnetic transients. The stator voltage build-up in a synchronous generator at no load is such a typical transient, but the sudden 3-phase scurtcircuit is the most important one recognized by industry.

The investigation of transients for constant machine inductances (magnetic saturation level does not vary) in time can be undertaken using Equation 9.17 with $d/dt = s$, and with the Laplace definition of parameters Equations 9.53 and 9.54

$$\begin{aligned} V_d(s) &= R_s i_d(s) + s[L_d(s)i_d(s) + G(s)V_F(s)] - \omega_r L_q(s)i_q(s) \\ V_q(s) &= R_s i_q(s) + sL_q(s)i_q(s) + \omega_r[L_d(s)i_d(s) + G(s)V_F(s)] \end{aligned} \quad (9.58)$$

We stress that Equations 9.58 are written in Laplace form, and thus deal only with the deviations of variables $i_d(s)$ and $i_q(s)$ and of inputs $V_d(s)$, $V_q(s)$ and $V_F(s)$ with respect to their initial values (at $t = 0$). The rotor speed, ω_r , is considered constant.

Example 9.3 Voltage Build-Up

Let us consider the case of an SM, at no load and at speed ω_r , whose full-field circuit voltage is applied suddenly. Calculate the stator voltage components and phase voltage build-up expressions in pu for $l_{dm} = 1.2$ pu, $l_{fl} = 0.2$ pu, $V_{F0} = 0.005833$ pu, $r_F = 0.01$ pu, and $\omega_r = 1$ pu ($\omega_{10} = 377$ rad/s)

Solution:

For a voltage build-up, the field circuit pu voltage step up $V_{F0}(s)$ in the Laplace form is

$$V_F(s) = \frac{V_{F0}}{s} \omega_{10} \quad (9.59)$$

It should be noticed that in pu terms, s is replaced by s/ω_{10} (in our case $\omega_{10} = 377$ rad/s (60 Hz)). The rated speed is inferred as $\omega_r = 1$ pu.

For no load, $i_d(s) = 0$, $i_q(s) = 0$ and thus, from Equation 9.58

$$\begin{aligned} V_d(s) &= g(s) \frac{s}{\omega_{10}} V_F(s) \\ V_q(s) &= \omega_r g(s) V_F(s) \end{aligned} \quad (9.60)$$

But from Equation 9.54, with l_{dm}/r_F instead of L_{dm}/R_F , in the absence of rotor cage $\tau_d'' = 0$, $\tau_{dr}' = 0$, the stator/field winding transfer function $g(s)$ is

$$g(s) = \frac{l_{dm}}{r_F} \frac{1}{\left(1 + (l_{dm} + l_{fl}) \frac{s}{r_F \omega_{10}}\right)} \quad (9.61)$$

With Equations 9.59 and 9.61, Equations 9.60 have these straightforward solutions:

$$\begin{aligned} V_d(t) &= \frac{V_{F0} l_{dm}}{l_{dm} + l_{fl}} e^{-\frac{t}{\tau_{d0}'}}; \quad \tau_{d0}' = \frac{l_{dm} + l_{fl}}{r_F \omega_{10}} \\ V_q(t) &= \omega_r \frac{l_{dm} V_{F0}}{r_F} (1 - e^{-t/\tau_{d0}'}) \end{aligned} \quad (9.62)$$

With $V_{F0} = 0.005833$ pu, $r_F = 0.01$ pu, $l_{dm} = 1.2$ pu, $l_{Fl} = 0.2$ pu, $\omega_r = 1$ pu, and $\omega_{10} = 377$ rad/s, Equations 9.62 become

$$\begin{aligned} V_d(t) &= 0.05e^{-2.7 \times t}; \quad \text{in pu} \\ V_q(t) &= 0.70(1 - e^{-2.7 \times t}); \quad \text{in pu} \end{aligned}$$

The stator phase voltage, $V_A(t)$, is obtained through the inverse Park transformation:

$$V_A(t) = V_d(t) \cos(\omega_{10}t) - V_q \sin(\omega_{10}t) \quad (9.63)$$

For $\omega_r \neq 1$ pu in Equation 9.63, it would be $(\omega_r \times \omega_{10}t)$ instead of $(\omega_{10}t)$ that reflects the actual frequency of the stator voltage which is equal to the actual electric speed, ω_r , in radians per second.

9.10 Sudden 3-Phase Short Circuit from a Generator at No Load/Lab 9.1

For a generator at no load (at steady state), we have

$$I_{d0} = I_{q0} = 0, \quad V_{d0} = 0, \quad (\delta_{V0} = 0), \quad V_{q0} = \omega_r L_{dm} i_{F0}, \quad \theta_0 = -\pi/2 \quad (9.64)$$

Also

$$V_{F0} = R_F i_{F0} \quad \psi_{q0} = 0, \quad \psi_{d0} = L_{dm} i_{F0}$$

As already $V_{d0} = 0$, to simulate a sudden short circuit, a step voltage, $-V_{q0}$, is applied to the q axis of Equation 9.58:

$$\begin{bmatrix} 0 \\ -\frac{V_{q0}}{s} \end{bmatrix} = \begin{bmatrix} R_s + sL_d(s) & -L_q(s)\omega_r \\ L_d(s)\omega_r & R_s + sL_q(s) \end{bmatrix} \begin{bmatrix} i_d(s) \\ i_q(s) \end{bmatrix} \quad (9.65)$$

Note that we are again operating with actual (and not pu) variables (we switch back and forth to make the reader get used to both systems).

Solving for $i_d(s)$ and $i_q(s)$ in Equation 9.65, after neglecting terms containing R_s'' and using the approximation

$$\frac{R_s}{2} \left(\frac{1}{L_d(s)} + \frac{1}{L_q(s)} \right) \approx \frac{R_s}{2} \left(\frac{1}{L_d''} + \frac{1}{L_q''} \right) = 1/\tau_a \quad (9.66)$$

we obtain

$$i_d(s) = -\frac{V_{q0}\omega_r^2}{s(s^2 + \frac{2s}{\tau_a} + \omega_r^2)} \frac{1}{L_d(s)}; \quad i_q(s) = -\frac{V_{q0}\omega_r}{(s^2 + \frac{2s}{\tau_a} + \omega_r^2)} \frac{1}{L_q(s)} \quad (9.67)$$

And finally, after considerable analytical work, with $L_d(s)$ and $L_q(s)$ of Equation 9.54,

$$i_d(t) = -\frac{V_{q0}}{\omega_r} \left[\frac{1}{L_d} + \left(\frac{1}{L'_d} - \frac{1}{L_d} \right) e^{-t/\tau'_d} + \left(\frac{1}{L''_d} - \frac{1}{L'_d} \right) e^{-t/\tau''_d} - \frac{1}{L''_d} e^{-t/\tau_a} \cos(\omega_r t) \right]$$

$$i_q(t) \approx -\frac{V_{q0}}{\omega_r L''_q} l \sin(\omega_r t) \quad (9.68)$$

The sign (−) in $i_q(t)$ is related to the fact that under short circuit the machine operates as a generator, and dq0 model equations are given for the motor mode. The phase A current is obtained again using the inverse Park transformation (Equation 9.18):

$$i_A(t) = i_d \cos(\omega_r t + \gamma_0) - i_q \sin(\omega_r t + \gamma_0)$$

$$= -\frac{V_{q0}}{\omega_r} \left(\left(\frac{1}{L_d} + \left(\frac{1}{L'_d} - \frac{1}{L_d} \right) e^{-t/\tau'_d} + \left(\frac{1}{L''_d} - \frac{1}{L'_d} \right) e^{-t/\tau''_d} \right) \cos(\omega_r t + \gamma_0) \right.$$

$$\left. - \frac{1}{2} \left(\frac{1}{L''_d} + \frac{1}{L'_q} \right) e^{-t/\tau_a} - \frac{1}{2} \left(\frac{1}{L''_d} - \frac{1}{L'_q} \right) e^{-t/\tau_a} \cos(2\omega_r t + \gamma) \right) \quad (9.69)$$

Ignoring stator resistance leads to the elimination of the periodic components of the frequency, ω_r , in $i_d(t)$ and $i_q(t)$, while in $i_A(t)$, only the frequency term, ω_r , remains with two time constants, τ'_d and τ''_d . The field current transients are related to the stator current by

$$i_F(s) = -sG(s)i_d(s)$$

$$i_F(s) = i_{F0} + i_{F0} \frac{L_d - L'_d}{L_d} \left[e^{-t/\tau'_d} - \left(1 - \frac{\tau'_{dr}}{\tau''_d} \right) e^{-t/\tau''_d} - \frac{\tau'_{dr}}{\tau''_d} e^{-t/\tau_a} \cos(\omega_r t) \right] \quad (9.70)$$

The peak short-circuit current, $i_{A\max}$, can be obtained from Equation 9.69 for $\partial i_A / \partial t = 0$, but approximately it is

$$i_{A\max} \approx \frac{V_{q0}}{\omega_r L''_d} \times 1.8 = \frac{V_0 \sqrt{2}}{\omega_r L''_d} \times 1.8 = I_{sc3} \times \frac{L_d}{L''_d} \times 1.8 \times \sqrt{2} \approx (8 - 20) I_{sc3} \quad (9.71)$$

I_{sc3} is the RMS phase current for the steady-state short circuit.

The flux linkages $\psi_d(s)$ and $\psi_q(s)$ are

$$\psi_d(s) = L_d(s)i_d(s); \quad \psi_q(s) = L_q(s)i_q(s) \quad (9.72)$$

and finally for $(1/\tau_a \ll \omega_r)$

$$\begin{aligned}\psi_d(t) &= \psi_{d0} + \psi_d(t) = \frac{V_{q0}}{\omega_r} e^{-t/\tau_a} \cos(\omega_r t) \\ \psi_q(t) &= 0 + \psi_q(t) = -\frac{V_{q0}}{\omega_r} e^{-t/\tau_a} \sin(\omega_r t)\end{aligned}\quad (9.73)$$

From this approximation, it appears that flux linkages vary only if $R_s \neq 0$, and their variation is fast.

The electromagnetic torque during shortcircuit is

$$T_{esc}(t) = \frac{3}{2} p_1 [\psi_d(t) i_q(t) - \psi_q(t) i_d(t)] \quad (9.74)$$

Note: It is evident that during the sudden shortcircuit, the magnetic saturation level decreases continuously from no-load saturated to steady-state nonsaturated magnetic conditions. Despite this situation, the complicated mathematical description ignores this, just to obtain analytical solutions valuable for intuitional interpretation. A qualitative view of $i_d(t)$, $i_q(t)$, $i_F(t)$, $i_A(t)$ is shown in Figure 9.6. Typical values of transient parameters in pu and time constants (in seconds) are given in Table 9.1.

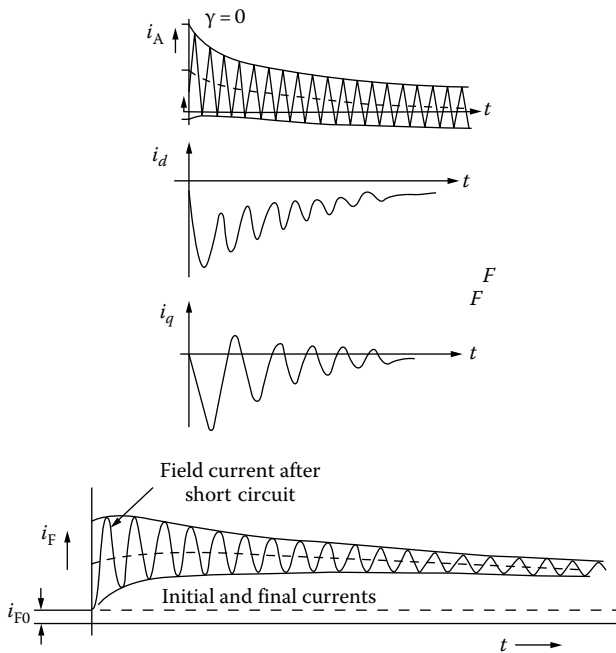


FIGURE 9.6

Sudden 3-phase short-circuit currents in SMs.

TABLE 9.1

Typical SG Parameter Values

Parameter	Two-Pole Turbogenerator	Hydrogenerators
l_d (pu)	0.9–1.5	0.6–1.5
l_q (pu)	0.85–1.45	0.4–1.0
l'_d (pu)	0.12–0.2	0.2–0.5
l'_q (pu)	0.07–0.14	0.13–0.35
l_{Fdr1} (pu)	–0.05 to +0.05	–0.05 to +0.05
l_0 (pu)	0.02–0.08	0.02–0.2
l_{sl} (pu)	0.07–0.14	0.15–0.2
r_s (pu)	0.0015–0.005	0.002–0.02
τ_{d0} (s)	2.8–6.2	1.5–9.5
τ_d (s)	0.35–0.9	0.5–3.3
τ''_d (s)	0.02–0.05	0.01–0.05
τ''_{d0} (s)	0.02–0.15	0.01–0.15
τ''_q (s)	0.015–0.04	0.02–0.06
τ''_{q0} (s)	0.04–0.08	0.05–0.09
l''_q (pu)	0.2	–0.45

The sudden short circuit may serve to identify SM time constants in axis d , while the peak short-circuit current is useful in designing the stator end connections mechanically, against largest electrodynamic forces at i_{Amax} .

9.11 Asynchronous Running of SMs at a Given Speed

Synchronous motors with dc excitation and starting/damper cage windings, when operated at a constant frequency and voltage power grid, are often started in the asynchronous mode, with the excitation winding connected first to a resistor, $R_x \approx 10R_F$. Then, when the speed stabilizes at a certain value, $\omega_r = \omega_1(1 - S)$, below the synchronous speed, $\omega_1 = \omega_{r0}$, the dc field circuit is commutated to the dc source; after a few oscillations, the SM eventually synchronizes. This is an electromechanical transient that is discussed later in the chapter. Here, we treat the average asynchronous torque at various slips. The dq voltages, V_d and V_q , can be expressed in relation to the stator voltages:

$$V_{A,B,C}(t) = V_0\sqrt{2}\cos(\omega_1 t + \delta_v) \quad (9.75)$$

$$\begin{aligned} V_d + jV_q &= \frac{2}{3} \left(V_A(t) + V_B(t)e^{j\frac{2\pi}{3}} + V_C(t)e^{-j\frac{2\pi}{3}} \right) e^{-j\omega_r t} \\ &= V_0\sqrt{2}[\cos((\omega_1 - \omega_r)t + \delta_v) - j\sin((\omega_1 - \omega_r)t + \delta_v)] \end{aligned} \quad (9.76)$$

$$\omega_1 - \omega_r = S\omega_1; \quad \omega_r = \omega_1(1 - S) \quad (9.77)$$

We may now introduce the complex expression of V_d and V_q separately:

$$\underline{V}_d(jS\omega_1) = V_0\sqrt{2}e^{j(S\omega_1 t + \delta_v)}; \quad \underline{V}_q = jV_0\sqrt{2}e^{j(S\omega_1 t + \delta_v)} \quad (9.78)$$

Now, with $s = jS\omega_1$ in the Laplace form, the stator equations with $V_F(jS\omega_1) = 0$ but R_F replaced by $(R_F + R_x)$ in $L_d(jS\omega_1)$, become

$$\begin{aligned} \underline{V}_d(jS\omega_1) &= (R_s + jS\omega_1 L_d(jS\omega_1))\underline{I}_d(jS\omega_1) - \omega_1(1 - S)L_q(jS\omega_1)\underline{I}_q(jS\omega_1) \\ \underline{V}_q(jS\omega_1) &= (R_s + jS\omega_1 L_q(jS\omega_1))\underline{I}_q(jS\omega_1) + \omega_1(1 - S)L_d(jS\omega_1)\underline{I}_d(jS\omega_1) \end{aligned} \quad (9.79)$$

From Equation 9.54, the complex inductances, $L_d(jS\omega_1)$ and $L_q(jS\omega_1)$, are

$$L_d(jS\omega_1) = \underline{L}_d = \frac{(1 + jS\omega_1\tau'_d)(1 + jS\omega_1\tau''_d)}{(1 + jS\omega_1\tau'_{d0})(1 + jS\omega_1\tau''_{d0})} \underline{L}_d; \quad R_F \rightarrow R_F + R_x \quad (9.80)$$

$$L_q(jS\omega_1) = \underline{L}_q = \frac{(1 + jS\omega_1\tau'_q)}{(1 + jS\omega_1\tau'_{q0})} \underline{L}_q \quad (9.81)$$

The ac field current, $I_F(jS\omega_1)$, is Equation 9.70:

$$\underline{I}_F(jS\omega_1) = -jS\omega_1 G(jS\omega_1) \underline{I}_d(jS\omega_1) \quad (9.82)$$

The average torque T_e is simply

$$T_{\text{eav}} = \frac{3}{2} p_1 \text{Re}(\underline{\psi}_d(jS\omega_1) \underline{I}_q^*(jS\omega_1) - \underline{\psi}_q(jS\omega_1) \underline{I}_d^*(jS\omega_1)) \quad (9.83)$$

with

$$\underline{\psi}_d(jS\omega_1) = \underline{L}_d \underline{I}_d; \quad \underline{\psi}_q(jS\omega_1) = \underline{L}_q \underline{I}_q \quad (9.84)$$

We may thus calculate $\underline{I}_d(jS\omega_1)$, $\underline{I}_q(jS\omega_1)$ and $I_F(jS\omega_1)$, T_e from Equations 9.79 through 9.84. For zero stator resistance, ($R_s = 0$), from Equation 9.79

$$\underline{I}_d \approx \frac{\underline{U}_d}{j\omega_1 \underline{L}_d}; \quad \underline{I}_q \approx \frac{\underline{U}_q}{j\omega_1 \underline{L}_q} \quad (9.85)$$

So the average torque is

$$T_{\text{eav}} = \frac{3}{2} p_1 \frac{(V_0\sqrt{2})^2}{\omega_1} \text{Re} \left[\frac{1}{j\omega_1 \underline{L}_d^*(jS\omega_1)} + \frac{1}{j\omega_1 \underline{L}_q^*(jS\omega_1)} \right] \quad (9.86)$$

Now, the currents, $i_d(t)$, and $i_q(t)$, are finally

$$\begin{aligned} i_d(t) &= \text{Re} \left[\underline{I}_d e^{j(S\omega_1 t + \delta_v)} \right] \\ i_q(t) &= \text{Re} \left[\underline{I}_q e^{j(S\omega_1 t + \delta_v)} \right] \\ \psi_d(t) &= \text{Re} \left[\underline{\psi}_d e^{j(S\omega_1 t + \delta_v)} \right] \\ \psi_q(t) &= \text{Re} \left[\underline{\psi}_q e^{j(S\omega_1 t + \delta_v)} \right] \end{aligned} \quad (9.87)$$

and

$$i_A(t) = i_d(t) \cos(\omega_1(1 - S)t) - i_q(t) \sin(\omega_1(1 - S)t) \quad (9.88)$$

In this interpretation, $i_d(t)$ and $i_q(t)$ will experience only slip frequency, while the stator current, $i_A(t)$, will have a fundamental frequency, ω_1 , and, if $R_s \neq 0$, the additional frequency will be $\omega_1(1 - 2S) = \omega'_1$.

The asynchronous torque will experience pulsations at a frequency of $2S\omega_1$ that may be as high as 50% of the rated synchronous torque. These are mainly due to the asymmetry of the rotor windings (and due to magnetic anisotropy along the d and q axes). The instantaneous torque in the general torque formula is

$$T_e = \frac{3}{2} p_1 (\psi_d(t) i_q(t) - \psi_q(t) i_d(t)) \quad (9.89)$$

The average asynchronous torque versus slip, S , for a SM in the per unit system is as follows: $V_0 \sqrt{2} = 1$ pu, $l_{sl} = 0.15$ pu, $l_{dm} = 1.0$ pu, $l_{Fl} = 0.3$ pu, $l_{qm} = 0.6$ pu, $l_{qrl} = 0.12$ pu, $L_{drl} = 0.2$ pu, $r_s = 0.012$ pu, $r_{dr} = 0.03$ pu, $r_{qr} = 0.04$ pu, $r_F = 0.03$ pu ($r_F + r_x = 10r_F$), as shown in Figure 9.7.

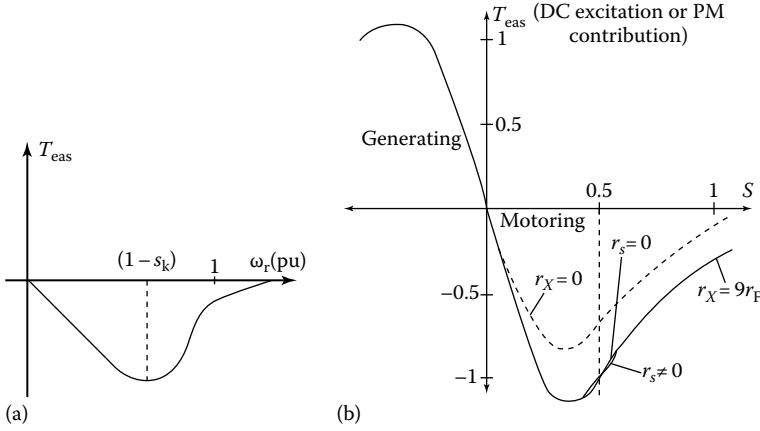
It should be noted that when $R_x = 10R_F$ is connected to the field circuit, more asynchronous torque is obtained (Figure 9.7b).

Example 9.4 DC Field Current (or PM) Rotor-Induced Asynchronous Stator Losses

If the dc excitation is nonzero (or PMs) on the rotor, then it produces additional (asynchronous) losses in the stator windings, at speed frequency $\omega'_1 = \omega_1(1 - S)$. It is required to derive the expression of this torque and calculate the speed where its maximum occurs for the SM with data above in the text.

Solution:

In rotor coordinates, the dq currents, I'_d and I'_q , are dc and the stator windings can be considered shortcircuited (because the power grid internal impedance is zero; infinite-power grid). Also, the rotor currents are $I_{dr} = I_{qr} = 0$ and $I_F = I_{F0}$: $V_F(s) \rightarrow V_{F0} = R_F i_{F0}$. So, with $\underline{V}_d = \underline{V}_q = 0$, $s \rightarrow 0$; $V_F \rightarrow R_F i_{F0}$

**FIGURE 9.7**

(a) Asynchronous average torque in pu vs. slip, S , of an SM and (b) dc (or PM) rotor field asynchronous torque.

Equations 9.79 degenerate into steady-state Equation 9.90:

$$\begin{aligned} 0 &= R_s I'_d - \omega_1 (1 - S) L_q I'_q \\ 0 &= (1 - S) \omega_1 (L_d I'_d + L_{dm} I_{F0}) + R_s I'_q \end{aligned} \quad (9.90)$$

The solutions of Equation 9.90 are straightforward:

$$\begin{aligned} I'_d &= \frac{-L_{dm} L_q I_{F0} \omega_1^2 (1 - S)^2}{R_s^2 + (1 - S)^2 \omega_1^2 L_d L_q} \\ I'_q &= \frac{-L_{dm} I_{F0} \omega_1 (1 - S) R_s}{R_s^2 + (1 - S)^2 \omega_1^2 L_d L_q} \end{aligned} \quad (9.91)$$

The torque corresponds to the stator winding losses, W'_{co} :

$$W'_{\text{co}} = \frac{3}{2} R_s (I_d'^2 + I_q'^2) = -T_{\text{edc}} \frac{\omega_1 (1 - S)}{p_1} \quad (9.92)$$

$$T_{\text{edc}} = -\frac{3}{2} p_1 R_s (L_{dm} I_{F0}) 2 \omega_1 (1 - S) \frac{[R_s^2 + I_q'^2 \omega_1^2 (1 - S)^2]}{[R_s^2 + L_d L_q \omega_1^2 (1 - S)^2]^2} \quad (9.93)$$

The maximum torque occurs at S'_k :

$$S'_k \approx 1 - \sqrt{\frac{2 L_d L_q \omega_1^2 - R_s^2}{2 L_q^2 \omega_1^2 + L_d L_q \omega_1^2}} \quad (9.94)$$

for the above data given in pu

$$S'_k \approx 1 - \sqrt{\frac{2l_d l_q - r_s^2}{2l_q^2 + l_d l_q}} \approx 1 - \sqrt{\frac{2 \times 1 \times 0.6 - 0.012^2}{2 \times 0.6^2 + 1 \times 0.6}} \approx 0.28 \quad (9.95)$$

So at 72% of rated speed, the torque, T_{edc} , Figure 9.7b, is maximum. For $l_d = l_q$ and $r_s = 0$, S'_k is $(1 - \sqrt{2/3}) = 0.1875$. For PMSM, $L_{dm} i_{F0} = \psi_{\text{PM}}$, which is the PM flux linkage in the dq model.

This torque is dependent on stator resistance, and thus it is important in low power machines (PMSMs). It should be noticed that the operating mode is, in fact, as a generator in short circuit at speed $\omega_r = \omega_1 (1 - S)$. The results from Example 9.3 can be used to explain this case. This point, though redundant, is reiterated here for the benefit of the reader. When self-starting of a PMSM at power grid takes place, this torque may decay or even hamper the process of self-synchronization, especially for a large torque load. Also, in case of a PWM converter supply failure, this torque continuously brakes the work machine and in some applications, such as car steering by a wire, it becomes an additional design constraint.

9.12 Reduced-Order dq0 Models for Electromechanical Transients

For power systems stability and control investigation, where many synchronous generators (SGs) work in parallel, in the point of common connection (interest), the modeling of SGs has to be detailed, while, for those at a distance, simplified models may be used, to save computing time.

A few such approximations have gained wide acceptance and are thus presented here:

9.12.1 Neglecting Fast Stator Electrical Transients

Neglecting the pulsational stator voltages in Equation 9.17, we obtain

$$\left(\frac{d\psi_d}{dt} \right)_{\omega_1 = \omega_{r0}} = \left(\frac{d\psi_q}{dt} \right)_{\omega_1 = \omega_{r0}} \quad (9.96)$$

Now we are left with two options: to consider that the speed does not vary, or if the speed varies ($\omega_r \neq \omega_1$), the machine inductances become dependent on rotor position.

In essence, the approximation is better if in the motion-induced voltages we use ω_1 instead of ω_r , [2], but keep the speed varying (not much) through

the motion equation:

$$\begin{aligned} V_d &= R_s I_d - \omega_1 \psi_q; & V_q &= R_s I_q + \omega_1 \psi_d \\ \frac{d\psi_F}{dt} &= V_F - R_F I_F, & \frac{d\psi_{dr}}{dt} &= -R_{dr} I_{dr}, & \frac{d\psi_{qr}}{dt} &= -R_{qr} I_{qr} \end{aligned} \quad (9.97)$$

$$\frac{d\omega_r}{dt} = \frac{p_1}{J} [p_1 (\psi_d I_q - \psi_q I_d) - T_{\text{load}}]; \quad \frac{d\theta_{er}}{dt} = \omega_r \quad (9.98)$$

To conclude, neglecting stator transients means disregarding the attenuated components of the frequency, ω_1 and $2\omega_1$, in the transient currents and torque.

9.12.2 Neglecting Stator and Rotor Cage Transients

Now, in addition, the rotor cage transients are neglected:

$$\frac{d\psi_{dr}}{dt} = \frac{d\psi_{qr}}{dt} = 0; \quad I_{dr} = I_{qr} = 0 \quad (9.99)$$

So the dq model loses two more orders:

$$V_d = R_s I_d - \omega_r \psi_q; \quad V_q = R_s I_q + \omega_r \psi_d \quad (9.100)$$

$$\frac{d\psi_F}{dt} = V_F - R_F I_F \quad (9.101)$$

The motion Equations 9.80 still hold. Only the field current transients are considered, and here the transient inductance, L'_d , comes into play.

9.12.3 Simplified (Third-Order) dq Model Adaptation for SM Voltage Control

This model starts with the third-order model mentioned above (Equations 9.97 through 9.101) and, in order to prepare the model for field voltage regulation, the field current, I_F , is eliminated and thus a new transient emf, e'_q , (in pu) is defined for the generator mode:

$$e'_q = \omega_r \frac{l_{dm}}{l_F} \psi_F; \quad l_F = l_{dm} + l_{Fl} \quad (9.102)$$

The stator equation along the q axis (Equation 9.100) becomes

$$V_q = -r_s i_q - x'_d i_d - e'_q; \quad x'_d = \omega_r \left(l_d - \frac{l_{dm}^2}{l_F} \right) \quad (9.103)$$

Now Equation 9.101 may be rearranged as

$$\dot{e}'_q = \frac{V_F - e'_q + i_d (x_d - x'_d)}{\tau'_{d0}}; \quad \tau'_{d0} = \frac{l_F}{r_F \omega_{10}} \quad (9.104)$$

The torque is

$$T_e = \frac{3}{2} p_1 (\psi_d i_q - \psi_q i_d) = \frac{3}{2} p_1 [L_{dm} i_F i_q + \psi_{PMq} i_d + (L_d - L_q) i_d i_q]$$

Hence the torque has four terms, suggesting that more torque may be produced per given winding loss or per stator current. However, the torque is limited by magnetic saturation for a given cooling system and admissible temperatures. To limit the stator current for maximum torque, it appears that by setting $i_d = 0$ and $L_q i_q - \psi_{PM} = 0$, the torque remains with the first term:

$$(T_e)_{i_d=0, \psi_q=0} = \frac{3}{2} L_{dm} i_F i_q; \quad i_q = \frac{\psi_{PMq}}{L_q} = \text{constant} \quad (9.107)$$

But, for the steady state, if $i_d = 0$ and $\psi_q = 0$, the stator voltage Sol current equation becomes

$$\bar{I}_s = 0 + j i_q; \quad V_s = R_s i_s + \omega_r L_{dm} i_F \quad (9.108)$$

Hence the power factor is implicitly unity, and Equation 9.108 shows that for the stator, only the resistive voltage drop occurs. This is exactly the situation in a separately excited dc brush machine, which has minimum (resistive) voltage regulation. The situation is brought about by controlling $i_d = 0$ and $i_q = \psi_{PM}/L_q = \text{const}$. By reducing i_F in these conditions, the speed may be increased theoretically to infinity at constant power, if mechanical losses are neglected. A wide but constant power speed range with the least overrating of SM and its PWM converter is thus obtained. This is typical in starters/alternators and for most autonomous synchronous generators.

This is just one example of the many innovative SM configurations that can bring extraordinary control for new, challenging applications at small and medium powers. It may be argued that the rotor is not very rugged mechanically, but for every merit, most often, there also is a drawback.

9.13 Small-Deviation Electromechanical Transients (in PU)

The small deviation theory investigates small transients around an initial steady-state point (situation). From the space phasor diagram (Figure 9.5) and Example 9.2, we continue with the steady-state conditions here (for motoring $\delta_v > 0$):

$$\begin{aligned} V_d &= -V\sqrt{2} \sin(\delta_v) = -\omega_r L_q I_q + R_s i_d \\ T_{e0} &= \frac{3}{2} p_1 (L_{dm} i_{F0} + (L_d - L_q) i_{d0}) i_{q0} \\ V_q &= V\sqrt{2} \cos(\delta_v) = \omega_r (L_d i_d + L_{dm} i_F) + R_s i_q \end{aligned} \quad (9.109)$$

with $V_d = V_{d0}$, $V_q = V_{q0}$, $i_d = i_{d0}$, $i_q = i_{q0}$, $\delta_v = \delta_{v0}$ (initial voltage power angle), and $T_{e0} = T_{L0}$.

For small transients, we need to use the general Equation 9.62, but for small deviations (from Equation 9.109):

$$\begin{aligned} V_d &= V_{d0} + \Delta V_d; & V_q &= V_{q0} + \Delta V_q; & \delta_v &= \delta_{v0} + \Delta \delta_v \\ \Delta V_d &= -\Delta V \sqrt{2} \sin \delta_{v0} - V_0 \sqrt{2} \cos \delta_{v0} \Delta \delta_v \\ \Delta V_q &= \Delta V \sqrt{2} \cos \delta_{v0} - V_0 \sqrt{2} \sin \delta_{v0} \Delta \delta_v \end{aligned} \quad (9.110)$$

Also

$$\begin{aligned} V_F &= V_{F0} + \Delta V_F; & T_L &= T_{L0} + \Delta T_L; & T_e &= T_{e0} + \Delta T_e \\ \omega_r &= \omega_{r0} + \Delta \omega_r; & \omega_1 &= \omega_{r0} + \Delta \omega_1 \end{aligned} \quad (9.111)$$

The flux/current relationships are also linearized:

$$\begin{aligned} \Delta \psi_d &= L_{sl} \Delta i_d + L_{dm} \Delta i_{dm}; & \Delta i_{dm} &= \Delta i_d + \Delta i_F + \Delta i_{dr} \\ \Delta \psi_q &= L_{sl} \Delta i_q + L_{qm} \Delta i_{qm}; & \Delta i_{qm} &= \Delta i_q + \Delta i_{qr} \\ \Delta \psi_F &\approx L_{Fl} \Delta i_F + L_{dm} \Delta i_{dm}; & \Delta \psi_{dr} &= L_{drl} \Delta i_{dr} + L_{dm} \Delta i_{dm} \\ \Delta \psi_{qr} &= L_{qrl} \Delta i_{qr} + L_{qm} \Delta i_{qm} \end{aligned} \quad (9.112)$$

Now, the dq0 model of Equations 9.17 through 9.19 is linearized to obtain

$$\begin{aligned} \frac{d\Delta \psi_d}{dt} &= \Delta V_d - R_s \Delta i_d + \omega_{r0} \Delta \psi_q + \Delta \omega_r \psi_{q0} \\ \frac{d\Delta \psi_q}{dt} &= \Delta V_q - R_s \Delta i_q - \omega_{r0} \Delta \psi_d - \Delta \omega_r \psi_{d0} \\ \frac{d\Delta \psi_F}{dt} &= \Delta V_F - R_F \Delta i_F; & \frac{d\Delta \psi_{dr}}{dt} &= -R_{dr} \Delta i_{dr} \\ \frac{d\Delta \psi_{qr}}{dt} &= -R_{qr} \Delta i_{qr} \\ \Delta T_e &= \frac{3}{2} p_1 (\Delta \psi_d i_q + \psi_{d0} \Delta i_q - \Delta \psi_q i_{d0} - \psi_{q0} \Delta i_d) \end{aligned} \quad (9.113)$$

with $\psi_{d0} = L_d i_{d0} + L_{dm} i_{F0}$; $\psi_{q0} = L_q i_{q0}$; $i_{dm0} = i_{d0} + i_{F0}$; $i_{qm0} = i_{q0}$

$$\frac{d}{dt} \Delta \delta_v = \Delta \omega_1 - \Delta \omega_r; \quad \frac{J}{p_1} \frac{d}{dt} \Delta \omega_r = \Delta T_e - \Delta T_L \quad (9.114)$$

The variation of stator frequency, $\Delta \omega_1$, which is in fact that of the power supply, is introduced here as an additional variable, but it may be considered zero ($\Delta \omega_1 = 0$) to simplify the computational process. The above model can be arranged into a matrix format

$$[L][\dot{\Delta X}] = [R][\Delta X] + B[\Delta U] \quad (9.115)$$

with

$$\Delta U = [\Delta V, \Delta V, \Delta V_F, 0, 0, \Delta T_L, \Delta \omega_1]^T \quad (9.116)$$

$$\Delta X = [\Delta i_d, \Delta i_q, \Delta i_F, \Delta i_{dr}, \Delta i_{qr}, \Delta \omega_r, \Delta \delta_v]^T \quad (9.117)$$

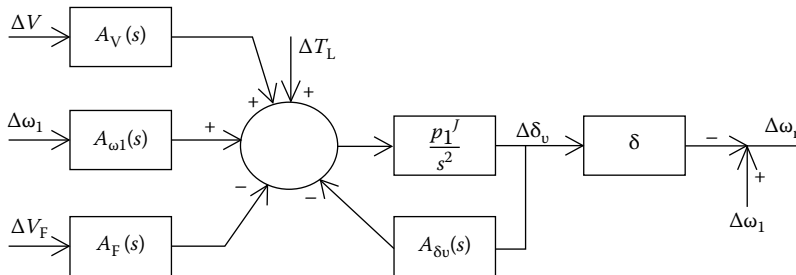
$$[L] = \begin{bmatrix} L_{sl} + L_{dm} & 0 & L_{dm} & L_{dm} & 0 & 0 & 0 \\ 0 & L_{sl} + L_{qm} & 0 & 0 & L_{qm} & 0 & 0 \\ L_{dm} & 0 & L_{Fl} + L_{dm} & L_{dm} & 0 & 0 & 0 \\ L_{dm} & 0 & L_{dm} & L_{drl} + L_{dm} & 0 & 0 & 0 \\ 0 & L_{qm} & 0 & 0 & L_{qrl} + L_{qm} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (9.118)$$

$$|B| = \begin{vmatrix} -\sqrt{2} \sin(\delta_{v0}) \\ \sqrt{2} \cos(\delta_{v0}) \\ 1 \\ 0 \\ 0 \\ -1 \\ 1 \end{vmatrix} \quad (9.119)$$

$$|R| = [|C| + |D|] \quad (9.120)$$

$$|C| = \begin{vmatrix} -R_s & \omega_{r0}(L_{sl} + L_{qm}) & 0 \\ -\omega_{r0}(L_{sl} + L_{dm}) & -R_s & -\omega_{r0}L_{dm} \\ 0 & 0 & -R_F \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{3}{2}p_1[(L_{sl} + L_{dm})i_{q0} - \psi_{q0}] & -\frac{3}{2}p_1[(L_{sl} + L_{qm})i_{d0} + \psi_{d0}] & -\frac{3}{2}p_1L_{dm}i_{q0} \\ 0 & 0 & 0 \end{vmatrix} \quad (9.121)$$

$$|D| = \begin{vmatrix} 0 & \omega_{r0}L_{qm} & \psi_{q0} & -V_0\sqrt{2} \cos(\delta_{v0}) \\ -\omega_{r0}L_{dm} & 0 & -\psi_{dr0} & -V_0\sqrt{2} \sin(\delta_{v0}) \\ 0 & 0 & 0 & 0 \\ -R_{dr} & 0 & 0 & 0 \\ 0 & -R_{qr} & 0 & 0 \\ \frac{3}{2}p_1L_{dm}i_{q0} & -\frac{3}{2}L_{dm}p_1i_{d0} & 0 & 0 \\ 0 & 0 & -1 & 0 \end{vmatrix} \quad (9.122)$$


FIGURE 9.9

Structural diagram of SM for small deviation transients.

The derivation of the matrices of $[L]_{7 \times 7}$ and $[R]_{7 \times 7}$ from Equations 9.113 and 9.114 is straightforward. If the current deviations are replaced by flux deviations, $\Delta\psi_d$, $\Delta\psi_q$, $\Delta\psi_F$, $\Delta\psi_{dr}$, $\Delta\psi_{qr}$, the matrix $[L]$ becomes a 6×1 simple, single column matrix, which is easy to solve numerically.

Once the terms of Equation 9.115 are known, with $d/dt \rightarrow s$ the, linear system theory can be used to investigate the small deviation transients. One particular transfer function, $\Delta\omega_r(s)$, is of particular interest for the motor mode:

$$\frac{J}{p_1} s^2 \Delta\delta_v = -A_{\delta_v}(s) \Delta\delta_v + A_{\omega_1}(s) \Delta\omega_1 - A_F(s) \Delta V_F + A_V(s) \Delta V + \Delta T_L \quad (9.123)$$

The coefficients, $A_{\delta_v}(s)$, $A_{\omega_1}(s)$, $A_F(s)$, and $A_V(s)$ also stem from the above equations. Equation 9.123 leads to the structural diagram shown in Figure 9.9, which reveals the complexity involved in control for a multiple input system.

Example 9.6 Investigation of Response to Forced Torque Pulsations by Small Deviation Theory

Solution:

$$\Delta T_L = \sum \Delta T_{L\nu} \cos(\omega_\nu t + r_\nu) \quad (9.124)$$

However, for a first approximation, all other inputs are zero ($\Delta V = 0$, $\Delta\omega_1 = 0$, $\Delta V_F = 0$).

Equation 9.123 degenerates to

$$\frac{J}{p_1} s^2 (\Delta\delta_v) = -A_{\delta_v}(s) \Delta\delta_v + \Delta T_L; \quad s \Delta\delta_v = -\Delta\omega_r \quad (9.125)$$

We now divide the first right-hand term of Equation 9.125 into two terms, by separating its real and imaginary parts:

$$\left[\frac{J}{p_1} \omega_\nu^2 + A_{\delta_{\nu i}}(\omega_\nu) j \omega_\nu + A_{\delta_{\nu r}} \right] \Delta\delta_v = \Delta T_{L\nu} \quad (9.126)$$

Equation 9.126 is written in complex number terms as it refers to one load torque harmonic pulsation. The fact that the coefficients $A_{\delta_{vi}}$ and $A_{\delta_{vr}}$ vary with the torque pulsation frequency allows us to investigate with better precision the transient response of SM derived by a prime mover or by driving a load machine with mechanical torque pulsations.

9.14 Large-Deviation Electromechanical Transients

In transients with large variations of variables, the complete dq model of SM has to be used. Typical examples are the asynchronous starting and self-synchronization of SMs with dc rotor or PM excitation. In both cases, the rotor is provided with a cage on the rotor.

9.14.1 Asynchronous Starting and Self-Synchronization of DC-Excited SMs/Lab 9.2

The stator voltages are considered symmetric:

$$V_{A,B,C,F} = V\sqrt{2} \cos\left(\omega_1 t - \left(i - 1\frac{2\pi}{3}\right)\right); \quad i = 1, 2, 3 \quad (9.127)$$

Using the Park transformation with $\theta = \int (\omega_r dt + \theta_0)$, we obtain, in rotor coordinates,

$$\begin{aligned} V_d(t) &= V\sqrt{2} \cos(\omega_1 t - \theta); & \frac{d\theta}{dt} &= \omega_r \\ V_q(t) &= -V\sqrt{2} \sin(\omega_1 t - \theta) \end{aligned} \quad (9.128)$$

As expected, the rotor speed varies continuously during asynchronous motor acceleration. The dq model in pu of Equations 9.6 and 9.38 holds.

For the dc-excited SM, the field circuit is connected to a resistance R_x and thus $V_F = -R_x i_F$ in the dq model (Figure 9.10). The angle θ_0 refers to the initial (at start) phase of the stator voltages. After the acceleration process settles at a speed $\omega_{ra} < \omega_1$ ($\omega_{ra}/\omega_1 \approx 0.95 - 0.98$), at a certain chosen moment, the field circuit is disconnected from the resistance, R_x , and is connected to the dc excitation source. A transient process occurs, where both the asynchronous and synchronous torques interact and the SM eventually synchronizes. The success of the synchronization depends on the load torque, and mainly on the initial voltage power angle, δ_v , when the dc voltage is connected to the rotor. A zero value of $\delta_v = -\theta_0 - \frac{\pi}{2} = 0$, is considered optimal because $\omega_r < \omega_1$, and thus inevitably the rotor trails the stator field and the synchronous torque is motoring from the start, toward a more probable synchronization. As the rotor slip (S) is small, the frequency of field current is small ($f_F = Sf_1$), and thus its circuit is almost resistive. The zero crossing of the slip frequency field current corresponds to zero emf. This means,

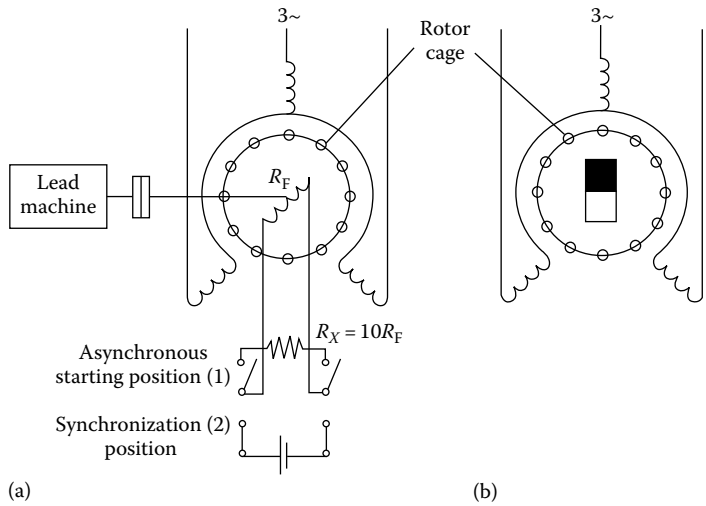


FIGURE 9.10
SM self-synchronization with (a) dc excitation and (b) PM rotor.

however, that there is maximum stator flux in the latter, which means a zero power angle when the dc voltage is applied to the rotor. This can serve as a means for repetitious starting, necessary in many applications.

The asynchronous starting and synchronization in (pu) for the large SM in Example 9.1 are shown in Figure 9.11 (for torque and speed).

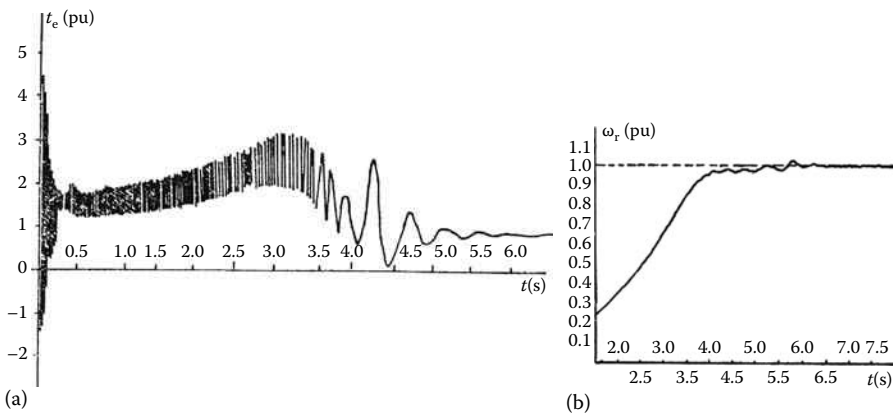


FIGURE 9.11
Asynchronous starting and synchronization of a 1800 KW SM: (a) Electromagnetic torque transients and (b) speed transients.

At $t = 4.15$ s, the dc voltage is connected to the rotor and the SM synchronizes at 6 s under a load of 0.8 pu. Notable torque pulsations are visible and the peak torque goes over 4.5 pu during starting.

9.14.2 Asynchronous Self-Starting of PMSMs to Power Grid

For the asynchronous self-starting of PMSM, the complete dq model is considered. But the latter gets simplified because the field circuit is eliminated and $L_{dm}i_F = \psi_{PM}$. The dq voltages in Equation 9.76 still hold. The dq model, Equations 9.32 through 9.34, is added here for completeness:

$$\begin{aligned}\frac{d\psi_d}{dt} &= V_d - R_s i_d + \omega_r \psi_q; & \psi_d &= L_{sl} i_d + L_{dm}(i_d + i_{dr}) + \psi_{PMd} \\ \frac{d\psi_q}{dt} &= V_q - R_s i_q - \omega_r \psi_d; & \psi_q &= L_{sl} i_q + L_{dm}(i_q + i_{qr}) \\ \frac{d\psi_{dr}}{dt} &= -R_{dr} i_{dr}; & \frac{d\psi_{qr}}{dt} &= -R_{qr} i_{qr}; & \psi_{dr} &= L_{dr} i_{dr} + \psi_{PMd} + L_{dm} i_d \\ \psi_{qr} &= L_{qr} i_{qr} + L_{qm}(i_q + i_{qr}); & T_e &= \frac{3}{2} p_1 (\psi_d i_q - \psi_q i_d)\end{aligned}\quad (9.129)$$

$$\frac{J}{p_1} \frac{d\omega_r}{dt} = T_e - T_L; \quad \frac{d\theta}{dt} = \omega_r \quad (9.130)$$

9.14.3 Line-to-Line and Line-to-Neutral Faults

Line-to-line and line-to-neutral faults (Figure 9.12a and b) are kind of extreme transients in the sense that their short-circuit current peaks are very large for PMSMs without a rotor cage. They have to be known for the proper design of the power source (for example, a PWM converter) protection system.

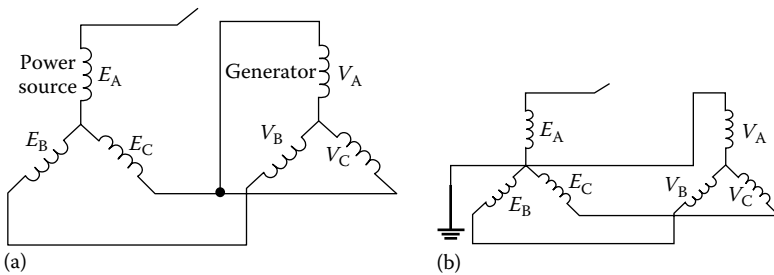


FIGURE 9.12

(a) Line-to-line and (b) line-to-neutral faults.

We suppose that the power source (grid) is of infinite power and thus its voltages are

$$E_{A,B,C}(t) = V\sqrt{2}\cos\left(\omega_1 t - (i-1)\frac{2\pi}{3}\right); \quad i = 1, 2, 3 \quad (9.131)$$

From Figure 9.12a

$$\begin{aligned} V_B - V_C &= E_B - E_C \\ V_A &= V_C; \quad I_A + I_B + I_C = 0, \quad \text{so} \quad V_A + V_B + V_C = 0 \end{aligned} \quad (9.132)$$

Consequently,

$$V_C(t) = V_A(t) = \frac{1}{3}(E_C - E_B); \quad V_B(t) = -2V_C(t) \quad (9.133)$$

for the line-to-line shortcircuit and

$$\begin{aligned} V_B - V_C &= E_B - E_C \\ V_C - V_A &= E_C; \quad V_A + V_B + V_C = 0 \end{aligned} \quad (9.134)$$

for the line-to-neutral fault (Figure 9.12b).

Hence

$$V_A = -\frac{(E_C + E_B)}{3}; \quad V_B = V_A + E_B; \quad V_C = V_A + E_C \quad (9.135)$$

Practically, $V_A(t)$, $V_B(t)$, and $V_C(t)$ can be expressed as functions of time. With the Park transformation

$$V_d + jV_q = \frac{2}{3} \left(V_A(t) + V_B(t)e^{j\frac{2\pi}{3}} + V_C(t)e^{-j\frac{2\pi}{3}} \right) e^{-j\theta} \quad (9.136)$$

and $d\theta/dt = \omega_r$, all we need is ready to apply the dq model and find all variables during such severe faults.

9.15 Transients for Controlled Flux and Sinusoidal Current SMs

Sinusoidal current control, at a constant d axis (ψ_d) or total stator flux (ψ_s), called flux orientation or vector control, presupposes voltage-source PWM converters in variable speed electric drives.

In such cases, no damper cage is placed on the rotor to reduce current ripples (because of large L_d and L_q values) and to avoid additional rotor losses. *Note:* There is a scalar control (V/f or i/f) where the frequency is ramped up and the amplitude of the voltage rises proportionally with frequency:

$$V_1 = V_0 + kf_1 \quad (9.137)$$

In such cases, stabilizing loops are added to preserve synchronism dynamically, and thus a damper cage is beneficial on the rotor. The complete dq model for transients as described earlier in this chapter is to be used.

9.15.1 Constant d -Axis (ψ_d) Flux Transients in Cageless SMs

PM action corresponds to a fictitious constant field current rotor winding. Taking up the dq model in Equations 9.17 through 9.19 and simplifying it for the cageless rotor with constant field current and constant i_{d0} , and $\psi_{d0} = \text{const}$:

$$V_d = R_s i_{d0} - \omega_r L_q i_q; \quad \psi_{d0} = L_d i_{d0} + \psi_{PMd}$$

$$L_q \frac{di_q}{dt} = V_q - R_s i_q - \omega_r \psi_{d0} \quad (9.138)$$

$$\frac{J_1}{p_1} \frac{d\omega_r}{dt} = (T_e - T_L - B\omega_r)$$

$$T_e = \frac{3}{2} p_1 (\psi_{d0} i_q - L_q i_q i_{d0}) = \frac{3}{2} p_1 (\psi_{d0} - L_q i_{d0}) i_q \quad (9.139)$$

For the steady state $d/dt = 0$

$$V_{d0} = R_s i_{d0} - \omega_r L_q i_{q0}; \quad V_{d0}^2 + V_{q0}^2 = V_s^2$$

$$V_{q0} = R_s i_{q0} + \omega_r \psi_{d0} \quad (9.140)$$

Now, neglecting R_s Equations 9.140 become

$$V_s^2 = \omega_r^2 (L_q^2 i_{q0}^2 + \psi_{d0}^2) \quad (9.141)$$

Replacing i_{q0} with T_e (from Equation 9.139), Equation 9.141 becomes

$$V_s = \omega_r \sqrt{L_q^2 \frac{4}{9} \frac{T_e^2}{p_1^2 (\psi_{d0} - L_q i_{d0})^2} + \psi_{d0}^2} \quad (9.142)$$

The no-load ideal speed $(\omega_{r0})_{T_e=0}$ is

$$\omega_{r0} = \frac{V_s}{\psi_{d0}} = \frac{V_s}{\psi_{PMd} + L_d i_{d0}} \quad (9.143)$$

For the infinite ideal no-load speed

$$i_{d0} = -\frac{\psi_{PMd}}{L_d} \quad (9.144)$$

This equality has been proved to be a key design condition where a wide speed range but constant electromagnetic power is required. It goes without saying that the speed, ω_r , has to vary in tact with the stator frequency, $\omega_1 = \omega_{r0}$.

The above equations allow us to calculate the speed versus torque for a given value of voltage, V_s (Figure 9.13).

The model for transients where only one electrical time constant T_{eq} occurs is

$$i_q(1 + sT_{eq}) = (V_q - \omega_r \psi_{d0})/R_s; \quad T_{eq} = L_q/R_s \quad (9.145)$$

$$\omega_r(1 + sT_m) = \frac{1}{B} \left[\frac{3}{2} p_1 (\psi_{d0} - L_q i_{d0}) i_q - T_L \right] \quad (9.146)$$

$$T_m = \frac{J_1}{p_1 B} \quad (9.147)$$

T_m is the mechanical time constant related to the friction torque component, proportional to speed. The structural diagram built after Equations 9.145 and 9.146 Figure 9.13b is similar to that of the PM dc brush rotor (Chapter 8).

The V_d voltage is calculated a posteriori with the ω_r and i_q variables determined from Equations 9.145 through 9.147. As expected, the evolution of transients is now much simpler.

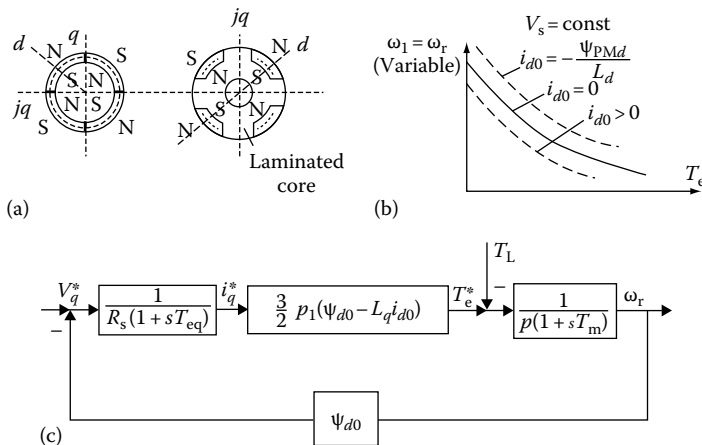


FIGURE 9.13

(a) PMSMs and (b) their speed/torque, for constant ψ_{d0} and (c) voltage structural diagram.

We may eliminate i_q from Equations 9.145 and 9.146:

$$\begin{aligned} \tilde{\omega}_r \left[s^2 T_m T_{eq} R_s B + s(T_m + T_{eq}) R_s B + R_s B + \frac{3}{2} p_1 (\psi_{d0} - L_q i_{d0}) \psi_{d0} \right] \\ = \frac{3}{2} p_1 (\psi_{d0} - L_q i_{d0}) \tilde{V}_q - \tilde{T}_L R_s (1 + s T_{eq}) \end{aligned} \quad (9.148)$$

$$i_q = (V_q - \tilde{\omega}_r \psi_{d0}) / R_s / (1 + s T_{eq}) \quad (9.149)$$

A second-order system similar to that for the dc brush PM was obtained.

Similar transients are expected for $\tilde{V}_q$ and $\tilde{T}_L$ inputs. The voltage, V_d , is just calculated a posteriori from Equation 9.120:

$$\tilde{V}_d = R_s i_{d0} - \tilde{\omega}_r L_q i_q \quad (9.150)$$

Note: All the above simplifications are valid for constant i_{d0} and PM excitation.

Example 9.7 Consider a PMSM with the following data: $\psi_{PMd} = 1$ Wb, $R_s = 1 \Omega$, $L_d = 0.05$ H, $L_q = 0.10$ H, $p_1 = 2$, $B = 0.01$ Nms, and $T_m = 0.3$ s.

Calculate the electrical time constant, T_{eq} , inertia, J , the current, i_{d0} , for infinite zero-torque speed and the eigenvalues for the speed response.

Solution:

From Equations 9.145 through 9.147

$$T_{eq} = L_q / R_s = 0.1 / 0.1 = 0.1 \text{ s}$$

$$J = T_m p_1 B = 0.3 \times 2 \times 0.01 = 6 \times 10^{-3} [\text{kg m}^2]$$

The value of i_{d0} for infinite zero-torque speed (Equation 9.144) is

$$i_{d0} = -\frac{-\psi_{PMd}}{L_d} = -\frac{1}{0.05} = -20 \text{ A}$$

The eigenvalues for the speed response (Equation 9.148) correspond to the characteristic equation:

$$\begin{aligned} s^2 (0.3 \times 0.1 \times 1 \times 0.01) + s(0.3 + 0.1) \times 1 \times 0.01 + 1 + 0.01 \\ + \frac{3}{2} \times 2(0 - 0.1 \times 20) \times 0 = 0 \\ 0 = 10^{-2} + s^2 \times 3 \times 10^{-4} + s \times 4 \times 10^{-3}; \\ s_{12} = \frac{-2 \times 10^{-3} \pm \sqrt{4 \times 10^{-6} - 10^{-2} \times 3 \times 10^{-4}}}{3 \times 10^{-4}} \\ = -3.33, \text{ respectively, } -10.0 \end{aligned} \quad (9.151)$$

Consequently, the speed response should be stable and aperiodic. For $i_d = i_{d0} = -\psi_{PM} / L_q (\psi_{d0} = 0)$, Equation 9.149 gets simplified to

$$(\tilde{i}_q)_{\psi_{d0}=0} = \frac{\tilde{V}_q}{R_s (1 + s T_{eq})} \quad (9.152)$$

9.15.2 Vector Control of PMSMs at Constant ψ_{d0} ($i_{d0} = \text{const}$)

On the basis of Equations 9.15.1 and 9.139, we can introduce the vector current control of PMSMs (Figure 9.14). The reference value of i_d is considered zero up to base speed, ω_b , and then is made negative for flux weakening above ω_b . Then, the reference torque, T_e^* , is “delivered” by the speed regulator. From the torque expression, with T_e^* , and i_{d0}^* known, the reference torque current, i_q^* , is calculated. Then, with i_d^* and i_q^* known, using the Park

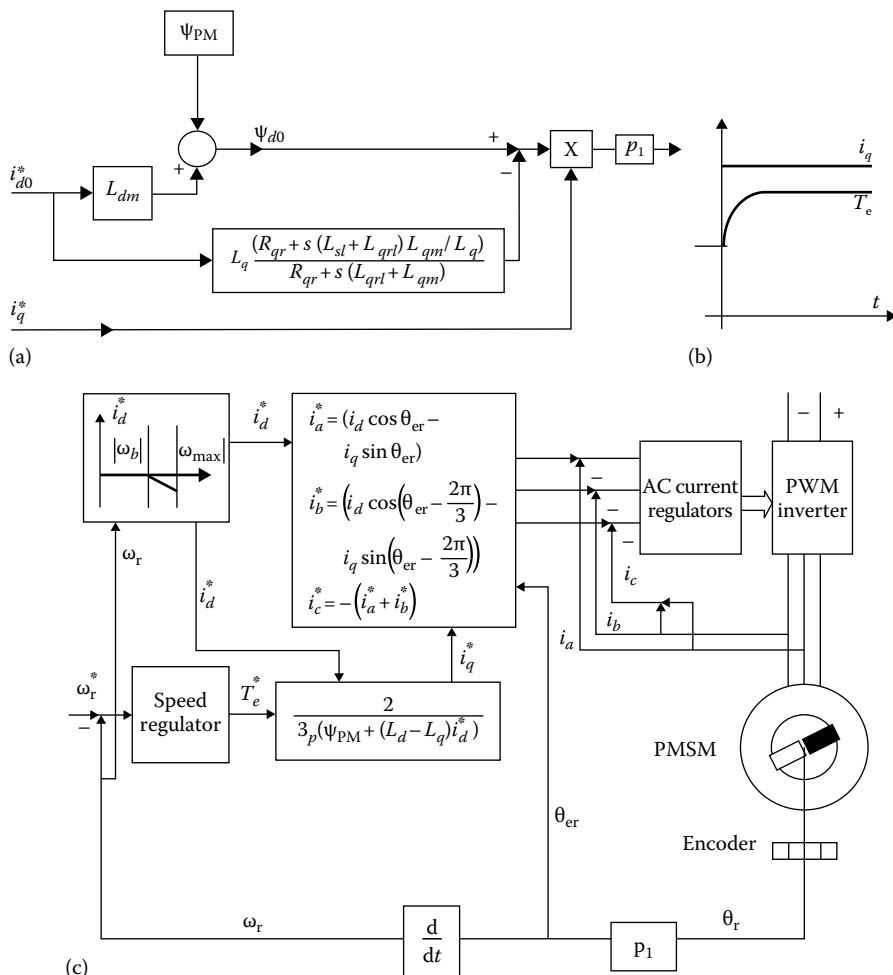


FIGURE 9.14

Vector control of PMSM with constant i_{d0} . (a) PMSM structural diagram at i_{d0} , (b) torque response to sudden i_q^* increase, and (c) basic vector control scheme.

transformation from rotor to stator coordinates (θ_{er}), the reference ac phase currents, $i_a^*(t)$, $i_b^*(t)$, and $i_c^*(t)$, are calculated and then regulated via ac current regulators of various configurations.

The current regulators provide the PWM the means to produce the required ac voltages that supply the motor through the PWM inverter. This is how variable speed with high performance is obtained.

9.15.3 Constant Stator Flux Transients in Cageless SMs at $\cos \psi_1 = 1$

Constant stator total flux conditions, $|\psi_s| = |\psi_d + j\psi_q|$, may be maintained when the machine is loaded, only for dc excitation on the rotor. Also, for this case, operation at unity power factor is typical. Representative applications use large SMs supplied from a PWM (two or three, multi, level) voltage source converter variable speed drives (up to 50 MW, 60 kV gas compressor drive units).

Again, we start with the stator equation, but for the unity power factor and constant stator flux

$$V_d + jV_q = V_s = R_s i_s + \omega_r \psi_s \quad (9.153)$$

$$\psi_d = L_d i_d + L_{dm} i_F; \quad \psi_q = L_q i_q$$

$$(L_{Fl} + L_{dm}) \frac{di_F}{dt} + L_{dm} \frac{di_d}{dt} = V_F - R_F i_F \quad (9.154)$$

$$\psi_d^2 + \psi_q^2 = \psi_s^2 \quad (9.155)$$

The motion equation is

$$\frac{J}{p_1} \frac{d\omega_r}{dt} = T_e - T_L; \quad T_e = \frac{3}{2} p_1 \psi_s i_s \quad (9.156)$$

The space phasor diagram is shown in Figure 9.15b. The voltage power angle, δ_v , is equal to the current angle, δ_i , and flux angle, δ_{ψ_s} , due to the unity power factor. The conditions for the unity power factor are

$$\sin(\delta_v) = \frac{L_q i_q}{\psi_s} = -\frac{i_d}{i_s}; \quad i_d < 0 \quad (9.157)$$

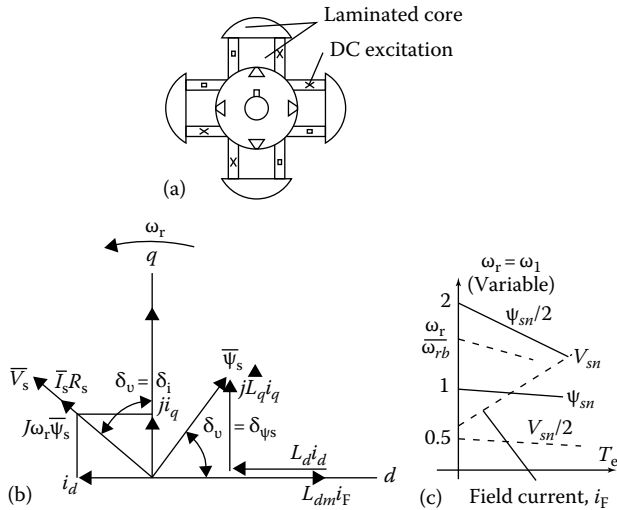
And thus

$$\tan \delta_v = L_q i_s / \psi_s \quad (9.158)$$

$$L_{dm} i_F = \psi_s \cos \delta_v - L_d i_d; \quad i_d < 0 \quad (9.159)$$

So, for a given stator flux, ψ_s^* , and torque, T_e^* , from Equation 9.156

$$i_s^* = \frac{2}{3} \frac{T_e^*}{p_1 \psi_s^*} \quad (9.160)$$


FIGURE 9.15

(a) DC-excited cageless rotor SM, (b) its space phasor diagram at $\cos \varphi_1$, and (c) speed/torque curves.

From Equation 9.158 δ_v^* is calculated, and then

$$i_d = -i_s^* \sin \delta_v^*, \quad i_q = i_s^* \cos \delta_v^* \quad (9.161)$$

Finally, from Equation 9.159, I_F^* is calculated. This sequential approach is useful for achieving variable speed drive control. It is worth noting that for the steady state, Equation 9.153 can be written as

$$V_s = R_s \frac{2}{3} \frac{T_e^*}{p_1 \psi_s^*} + \omega_r \psi_s^* \quad (9.162)$$

Equation 9.162 suggests a linear speed/torque curve. This is similar to the constant excitation flux dc brush machines. It is implicit that the frequency, ω_1 , varies in fact with ω_r .

The speed may be controlled by

- Voltage control, V_s , up to base speed, ω_{rb}
- Flux weakening (ψ_s decreases) above ω_{rb}

Example 9.8 A constant ψ_s , $\cos \psi_s = 1$, variable-speed SM at 1800 kW, where $V_{nl} = 4.2$ kV, $f_0 = 60$ Hz, $2p_1 = 60$ with dc excitation and a cageless rotor $x_d = 0.6$ pu, $x_{dm} = 0.5$ pu, $x_q = 0.4$ pu, $R_s = 0.01$ pu, operates at the unity power factor from 10% to 100% rated current.

Calculate

- Rated current, I_n , with efficiency $\eta_r = 0.985$, unity power factor, rated speed, and the required field current, i_{Fn}
- No-load ideal speed for full voltage and 50% voltage for i_{Fn} and for full voltage and $i_{Fn}/2$ (magnetic saturation is neglected)
- $\omega_r (T_e)$ and $i_F (T_e)$, for $V_{nl}/2$ for 25%, 50%, and 200% I_n

Solution:

- The rated current, I_n , is calculated from the definition of efficiency:

$$I_n = \frac{P_n}{\eta_n \sqrt{3} V_{nl} \cos \varphi_n} = \frac{1800 \times 10^3}{0.985 \times \sqrt{3} \times 4.2 \times 10^3 \times 1.0} = 251.50 \text{ A}$$

$$I_{sn} = I_n \sqrt{2} = 251.50 \sqrt{2} = 354.62 \text{ A}$$

The voltage Equation 9.153 yields:

$$V_{sn} = \frac{V_{nl}}{\sqrt{3}} \sqrt{2} = R_s i_{sn} + 2\pi f_b \psi_{sn}$$

$$\psi_{sn} = \frac{4.2 \times 10^3 \sqrt{2/3} - 0.01 \frac{4.2 \times 10^3}{251.50 \sqrt{3}} \times 251.50 \sqrt{2}}{2\pi 60} = 8.9938 \text{ Wb}$$

The electromagnetic torque is

$$T_{en} = \frac{P_n}{\frac{\omega_{rb}}{p_1}} = \frac{1800 \times 10^3 \times 30}{2\pi 60} = 143.312 \times 10^3 \text{ N m}$$

Tan δ_v Equation 9.158 yields

$$\tan \delta_{v_n} = L_q \frac{i_{sn}}{\psi_{sn}} = \frac{0.4}{\psi_{sn}} \frac{V_{nl}}{\sqrt{3}} \frac{I_n \sqrt{2}}{I_n \omega_{rb}} = \frac{0.4 \times 4200 \sqrt{2/3}}{8.9938 \times 120 \times \pi} = 0.40; \quad \delta_{v_n} = 22^\circ \text{ (A)}$$

So the rated voltage (flux, current) power angle is 22° , which is quite a realistic value.

Now from Equation 9.161

$$i_{dn} = -I_{sn} \sin \delta_{v_n} = -354.62 \times \sin 22^\circ = -132.84 \text{ A} \quad \text{(B)}$$

So the required field current, i_{Fn} , Equation 9.159 is

$$i_F = \frac{\psi_s \cos \delta_v - L_d i_d}{L_{dm}} = \frac{8.9938 \times 0.927}{0.5 \times \frac{4200}{\sqrt{3}} 25 / 0.5 \times 120 \pi}$$

$$- \frac{0.6}{0.5} (-132.84) = 650.876 + 159.41 = 810.284 \text{ A} \quad \text{(C)}$$

Note that the field current is reduced to the stator.

- b. The no-load ideal speed (Equation 9.162) ω_{r0} is

$$\omega_{r0} = \frac{V_s}{\psi_s} = \frac{4200 \times \sqrt{2/3}}{8.9438} = 380.609 \text{ rad/s}$$

This is slightly larger than the rated speed, $\omega_{rb} = 2\pi 60 = 376.8 \text{ rad/s}$. For half the voltage, $(V_{sn}/2)$, the ideal no-load speed is halved to 190.304 rad/s . On the other hand, for $i_{Fn}/2$, $\psi_s = \psi_{sn}/2$ and thus the no-load ideal speed is doubled, $2 \times 380.99 \text{ rad/s}$.

- c. To calculate the field current for $i_s = (25\%, 50\%) I_{sn}$, and the unity power factor and full flux, we just have to repeat the above-mentioned process, starting with $\tan \delta_v$, then i_d , then i_F from Equations A through C and the torque, $T_e = \frac{3}{2} p_1 \psi_{sn} i_s$ from Equation 9.160

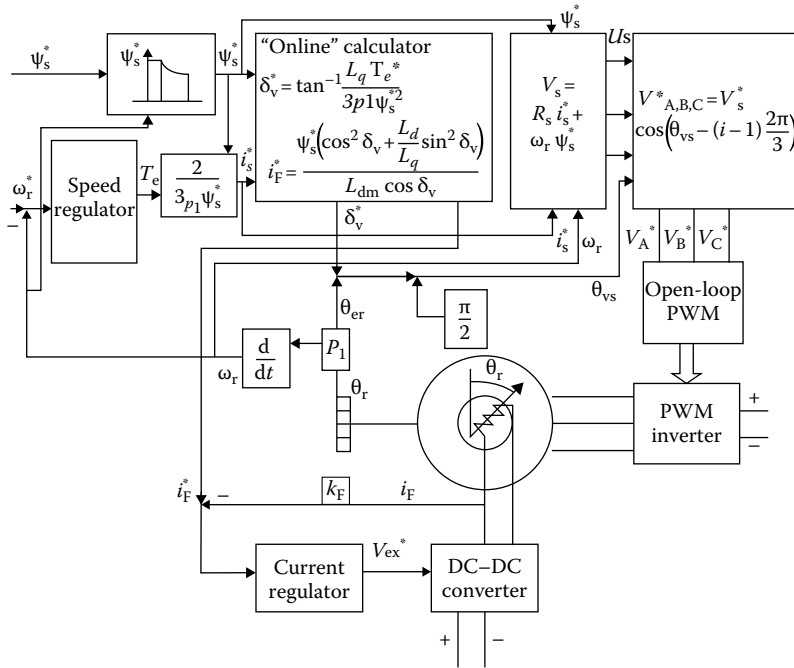
As seen in Figure 9.15, the speed decreases a little with torque, denoting stable behavior while the excitation (field) current, to maintain the unity power factor, increases with torque, as expected.

9.15.4 Vector Control of SMs with Constant Flux (ψ_s) and $\cos \varphi_s = 1$

A basic vector control scheme for constant ψ_s and the unity power factor is shown in Figure 9.16

- The reference stator flux, ψ_s^* , is set constant up to base speed (when full voltage is attained), and then decreased inversely proportional to speed.
- The reference torque, T_e^* , is the output of the speed regulator and it also has to be limited (reduced) above base speed.
- On the basis of known ψ_s^* , and T_e^* on line, i_s^* , δ_v^* , and i_F^* are calculated as in Equations 9.157 through 9.161; then V_s is calculated as in Equation 9.153 and the voltage angle $\theta_{vs} = (\frac{\pi}{2} + \delta_v + \theta_{er})$ in Equation 9.24 is used for the Park transformation to yield $V_A^*(t)$, $V_B^*(t)$, and $V_C^*(t)$, which are then open-loop PWM-“fabricated” in the inverter.
- Simultaneously, the reference field current, i_F^* , (after reduction to the rotor via k_F) is close-loop regulated through a dc–dc (or ac–dc) PWM converter.
- For an easy correction of parameter mismatch, the power factor angle is measured and the reference current, i_F^* , is corrected with the output of an additional PI regulator of φ_s (set to zero, $\varphi_s^* = 0$).

The absence of stator current regulators means that other safe current protection means have to be used. Figure 9.16 shows an example of the usefulness of transients modeling for the design of variable speed motor/generator control (more on electric drives can be found in [5]).



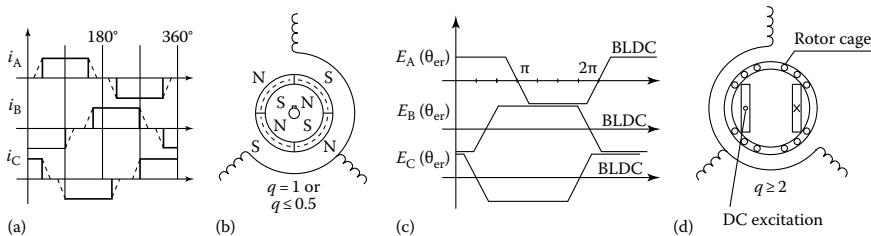
Basic vector control of SMs with constant flux, ψ_s , and $\cos \varphi_s = 1$.

Rectangular current control of SMs is used in two extreme cases:

- Low-power cageless rotor PMSM with a proximity Hall sensor and PWM voltage source inverters for variable speed and for lower costs (a BLDC motor) (Figure 9.17a through d)
- High-power cage rotor dc excitation SMs with a proximity Hall sensors and current source inverters for lower costs (Figure 9.17a and b)

As both are widely used in industry, we tackle the SM transients for both, which will serve as a foundation for further studies in electric drives.

The PMSM with a cageless rotor, trapezoidal emf—with $q = 1$ or $q < 0.5$ slots/pole/phase stator windings—controlled with ideal rectangular (in factor


FIGURE 9.17

(a) Rectangular currents, (b) surface PM rotor (BLDC) motor, (c) emfs for BLDC motor, and (d) dc-excited cage rotor SM.

trapezoidal) ac currents, in tact with rotor position, three proximity Hall sensors (120° apart), and supplied by a PWM voltage source inverter, is called the brushless dc motor (or BLDC motor).

Most BLDC motors have surface PMs on the rotor and thus $L_d = L_q = L_s$, independent of the rotor position. But the PM-produced emfs in the stator phase are rather trapezoidal (ideally rectangular and upto 180° wide, both positive and negative), Figure 9.17c.

The emfs may be decomposed into odd harmonics as

$$\begin{aligned}
 E_A(t) &= \sum_{\nu=1,3,5,\dots} \omega_r \psi_{PM\nu} \cos(\nu\theta_{er}) \\
 E_B(t) &= \sum_{\nu=1,3,5,\dots} \omega_r \psi_{PM\nu} \cos\left(\nu\left(\theta_{er} - \frac{2\pi}{3}\right)\right) \\
 E_C(t) &= \sum_{\nu=1,3,5,\dots} \omega_r \psi_{PM\nu} \cos\left(\nu\left(\theta_{er} + \frac{2\pi}{3}\right)\right)
 \end{aligned} \quad (9.163)$$

For practical purposes, the 1st, 3rd, 5th harmonics suffice.

As surface PM pole rotors are considered, the cyclic inductance, L_s , is

$$L_s = L_{sl} + \frac{4}{3}L_g; \quad L_{AB} = -L_g/3 \quad (9.164)$$

So the phase coordinate model is straightforward:

$$\begin{bmatrix} L_s & 0 & 0 \\ 0 & L_s & 0 \\ 0 & 0 & L_s \end{bmatrix} \frac{d}{dt} \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} V_A(t) \\ V_B(t) \\ V_C(t) \end{bmatrix} - \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} - \begin{bmatrix} E_A(t) \\ E_B(t) \\ E_C(t) \end{bmatrix} \quad (9.165)$$

The torque is

$$T_e = \frac{E_A(t)i_A(t) + E_B(t)i_B(t) + E_C(t)i_C(t)}{(\omega_r/p_1)} \quad (9.166)$$

$$\frac{J}{p_1} \frac{d\omega_r}{dt} = T_e - T_{load}; \quad \frac{d\theta_{er}}{dt} = \omega_r \quad (9.167)$$

If the voltage waveforms are applied in tact to rotor position (θ_{er}) by a PWM and with phase commutation every 60° (electrical), the machine behaves much like a dc machine.

While Equations 9.163 through 9.167 constitute the complete model, for a steady state, ignoring the commutation of phases, the 120° wide, smooth current blocks lead to a 2-phase operation of the machine (Figure 9.17):

$$V_{dc} = V_A - V_B = 2L_s \left(\frac{di_a}{dt} \right)_{=0} + 2R_s i_{A0} + E_A - E_B \quad (9.168)$$

Consider one voltage pulse, $V_A - V_B = V_{dc}$, and constant (180° wide) emf “blocks,” $E_A - E_B = 2E$:

$$V_{dc} = 2R_s i_{A0} + 2E \quad (9.169)$$

From Equation 9.169, the electromagnetic torque is

$$T_e = \frac{2E i_{A0}}{\omega_r / p_1} \quad (9.170)$$

$$E = \omega_r \psi_{PM} \quad (9.171)$$

Hence

$$V_{dc} = 2R_s i_{A0} + 2\omega_r \psi_{PM} \quad (9.172)$$

$$V_{dc} = R_s T_e / (\psi_{PM} p_1) + 2\omega_r \psi_{PM} \quad (9.173)$$

But this is again similar to the dc brush PM machine with a known linear speed/torque curve. As the phase currents are “in phase” with the phase emfs, during commutation of phases (from AB, AC, CB to BA), the current in phase B will go to zero and the current in phase C will take its place. The process evolves with the participation of the capacitance, C_{dc} (Figure 9.18).

To keep the current wavefrom flat, chopping is required and possibly up to the base speed where a full V_{dc} voltage is applied. The fact that the

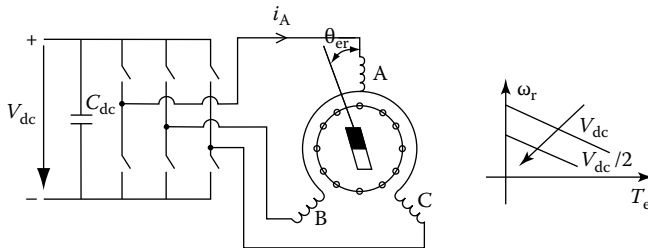


FIGURE 9.18

Brushless dc PM motor with a voltage source PWM inverter.

position sensor is required only to produce six signals (shifted by $\pi/3$ electric radians) leads to low-cost proximity Hall sensors. For more on brushless dc motor control, the reader can refer to [6].

9.16.2 DC-Excited Cage Rotor SM Model for Rectangular Current Control

As the SM has a cage rotor, with a dc excitation circuit, the complete dq model may be used to simulate any transients, including those with rectangular current control. It is supplied by a current source inverter (Figure 9.19a).

However, to simplify the treatment, we may assume that for the commutation of phases (for rectangular current control), a load (emf) commutation is used. This means leading power factor for the fundamental component of

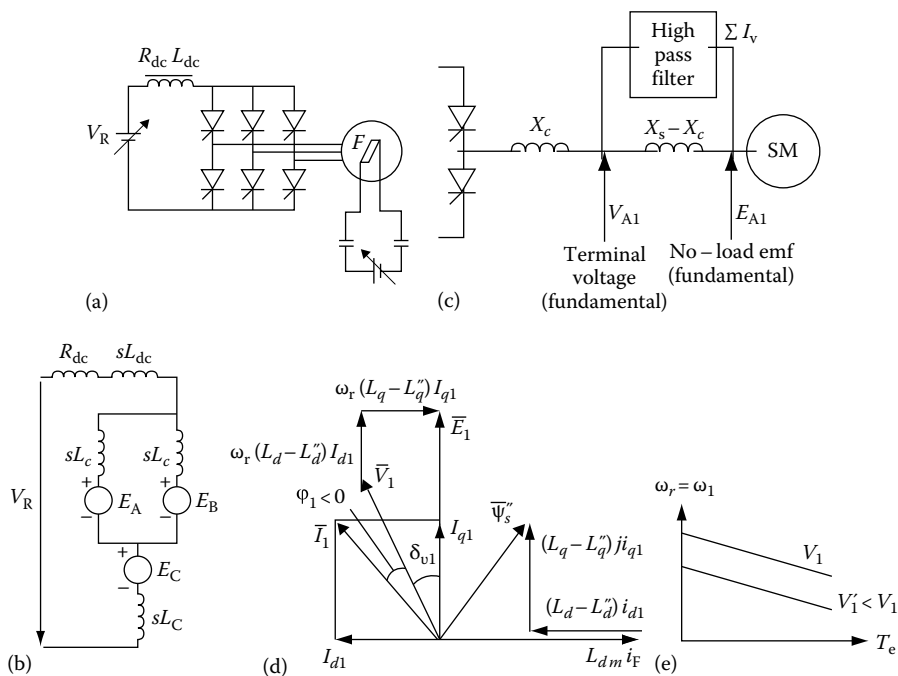


FIGURE 9.19

DC-excited cage rotor SM with rectangular current control: (a) the current source inverter plus SM, (b) equivalent circuit during "load" commutation of phases (from AC' to BC'), (c) steady-state equivalent circuit, (d) vector diagram for the fundamental components, and (e) speed (torque) linear dependence, with position control and leading power factor, $\varphi_i = -(8 - 12)^\circ$.

rectangular current, or machine over-excitation. Moreover, the commutation inductance is $L_c = (L_d'' + L_q'')/2$, where L_d'' and L_q'' are subtransient inductances as defined in previous sections, and which are smaller as the rotor cage gets stronger.

The SM may be thus modeled by L_c for the phase commutation (transients) and by $L_s - L_c$ for the steady state (Figure 9.19b).

The commutation process is more subtle but the in essence, the smaller L_c , the larger the load (phase) current that can be commutated within the 15–30° (electrical) available for it. We may treat the steady state approximately for the fundamental component of current by assuming a constant flux behind subtransient reactance (inductance), ψ_s'' , and a leading power factor angle, Figure 9.19:

$$\begin{aligned} V_{s1} &\approx R_s i_{s1} \cos \varphi_1 + \omega_r \psi_s'' \\ \psi_{d1} &= L_{dm} i_F + (L_d - L_d'') i_{d1}; \quad \psi_s'' = \sqrt{\psi_{d1}^2 + \psi_{q1}^2} \\ \psi_{q1} &= (L_q - L_q'') i_{q1}; \quad T_e = \frac{3}{2} p_1 \psi_s'' i_{s1} \cos \varphi_1 \end{aligned} \quad (9.174)$$

From the vector diagram in Figure 9.19d

$$\begin{aligned} \tan(\delta_{v1} - \varphi_1) &= -I_d/I_q \\ \tan \delta_{v1} &= \psi_{d1}/\psi_{q1} \end{aligned} \quad (9.175)$$

For given ψ_s^* and T_e^* , and $\varphi_1^* < 0$, we may calculate using Equations 9.174 and 9.175, as for the case of $\cos \varphi_1 = 1$, i_{s1}^* , δ_{v1}^* , I_{d1}^* , I_{q1}^* , and I_F^* . As expected, i_F^* increases with torque, as for $\cos \varphi_1 = 1$, but this time only for larger i_F^* values.

The voltage Equation 9.174 again leads to a stable (linear) speed/torque. For more on the current source inverter SM drives, the reader is directed to [5].

9.17 Switched Reluctance Machine Modeling for Transients

SRMs are doubly salient, simply excited electric machines with passive rotors [7,8]. Their nonoverlapping (tooth wound) coils (phases) are turned on sequentially (in relation to rotor position) to produce torque, through dc voltage pulses that produce unipolar phase currents. Three or more phase SRMs may be started from any position but single-phase SRMs need a self-starting artifact, a parking PM, stepped rotor airgap, an energy drain cage additional winding, short-circuit coil (shaded poles) on rotor, or a zone of easy saturation (with slots) on rotor poles.

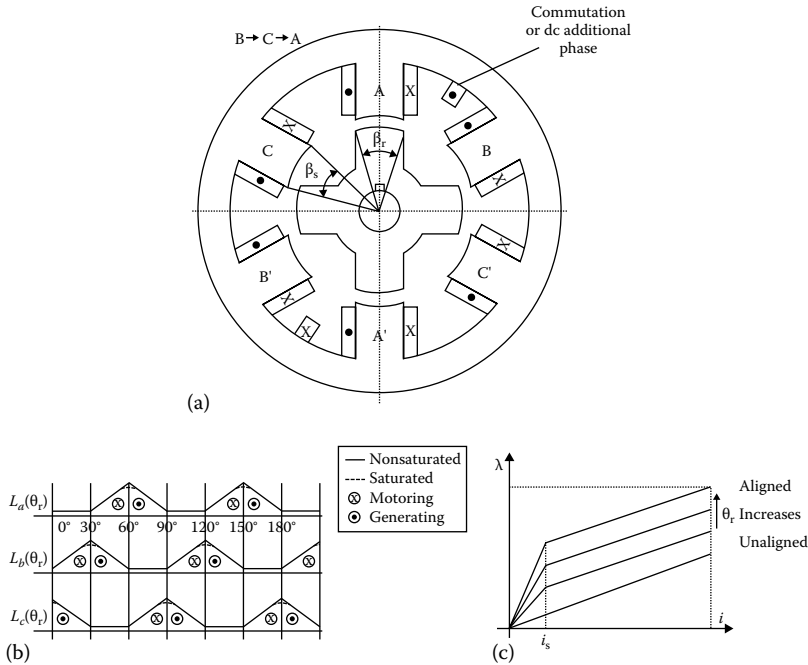


FIGURE 9.20 (a) Three-phase 6/4 SRM, (b) inductances, and (c) simplified flux/current/position curves.

Most SRMs lack mutual flux between phases, which makes them more fault tolerant, at the price of losing an important torque component.

A typical 3-phase 6/4 SRM is shown in Figure 9.20a; its phase inductances vary with the rotor position as shown in Figure 9.20b. The simplified flux/current/position family of curves in Figure 9.20c reveals the potential source of torque production.

The flux/current/position curves obtained by FEM or by tests may be approximated by various methods to analytical expressions. For the linear dependencies shown in Figure 9.19c:

$$\begin{aligned} \psi &\approx \left(L_u + \frac{K_s(\theta_r - \theta_0)}{i_s} \right) i; \quad \text{for } i \leq i_s \\ \psi &\approx L_u i + K_s(\theta_r - \theta_0); \quad \text{for } i \geq i_s. \end{aligned} \tag{9.176}$$

θ_r varies only from the unaligned to the aligned position. As there is no coupling between phases and the machine has double saliency, phase

coordinates are to be used:

$$V_{A,B,C} = R_s i_{A,B,C} + \frac{d_s \psi_{A,B,C}(i, \theta_r)}{dt} \quad (9.177)$$

$$T_{e,A,B,C} = \left(\frac{\partial W_m \text{ coenergy}}{\partial \theta_r} \right)_{i=\text{const}} \quad (9.178)$$

We may write Equation 9.177 as

$$V_{A,B,C} = R_s i_{A,B,C} + \frac{\partial \psi_{A,B,C}(i, \theta_r)}{\partial i} \frac{di}{dt} + \frac{\partial \psi_{A,B,C}(i, \theta_r)}{\partial \theta_r} \frac{d\theta_r}{dt} \quad (9.179)$$

$$J \frac{d\Omega_r}{dt} = T_e - T_{\text{load}} - B\Omega_r; \quad \frac{d\theta_r}{dt} = \Omega_r \quad (9.180)$$

$E_i = \frac{\partial \psi_{A,B,C}(i, \theta_r)}{\partial \theta_r}$ looks like an emf but it is a pseudo-emf because the torque expression:

$$T_e = \frac{1}{2} \sum \frac{E_i i_i}{\Omega_r} = \frac{1}{2} \sum i_i^2 \frac{\partial L_i}{\partial \theta_r}; \quad \Omega_r = \frac{d\theta_r}{dt} \quad (9.181)$$

is valid only for $L_i(\theta_r) = \psi_i/L_i$, that is, for the linear case (no magnetic saturation). E_i changes sign with the rotor position (it is $\oplus$ for a rising inductance slope and $\ominus$ for a decaying slope) and thus with positive current, positive (motoring) and negative torques are obtained.

There are N_r (rotor pole number) energy cycles per phase per revolution. So, the SRM operates like a SM with N_r pole pairs. The average torque T_{eav} is

$$T_{\text{eav}} = m N_r (T_{\text{eav}})_{\text{cycle}} \quad (9.182)$$

where m is the number of stator phases.

A small signal model of SRM per phase may be developed for the linear case and,

$$\begin{aligned} i &= i_0 + \Delta i; & \Omega_r &= \Omega_{r0} + \Delta \Omega_r \\ V &= V_0 + \Delta V; & T_L &= T_{L0} + \Delta T_L \end{aligned} \quad (9.183)$$

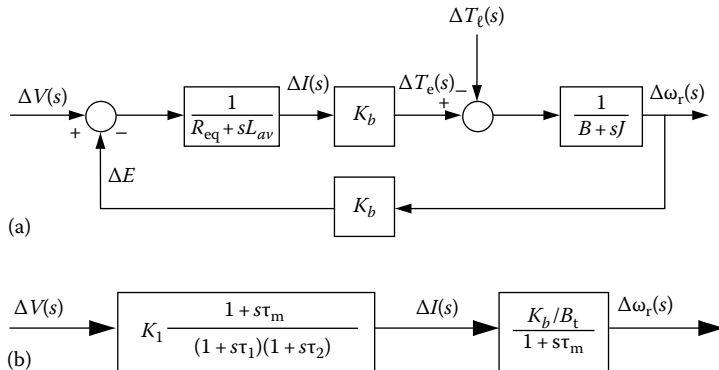
by using Equations 9.177 through 9.180:

$$\left(s + \frac{1}{\tau_e} \right) \Delta i + \frac{K_b}{L_{\text{av}}} \Delta \Omega_r = \frac{\Delta V}{L_{\text{av}}} \quad (9.184)$$

$$-\frac{1}{J} K_b \Delta i + \left(s + \frac{1}{\tau_m} \right) \Delta \Omega_r = -\frac{\Delta T_L}{J} \quad (9.185)$$

with

$$\begin{aligned} R_e &= R_s + \frac{\partial L}{\partial \theta_r} \omega_{r0}; & T_{\text{eq}} &= \frac{L_{\text{av}}}{R_e}; & L_{\text{av}} &\approx \frac{L_{\text{max}} + L_{\text{min}}}{2} \\ K_b &= \frac{dL}{d\theta_r} i_0; & \Delta E &= K_b \Delta \Omega_r; & E &\approx K_b \Omega_r \end{aligned} \quad (9.186)$$

**FIGURE 9.21**

SRM: (a) structural diagram and (b) its reduced form.

Equations 9.184 and 9.185 lead to the linearized structural diagram of Figure 9.21a with its reduced form ready for control, shown in Figure 9.21b.

In deriving the small signal model, L was considered constant as L_{av} , where its derivative with the rotor position was still calculated. The structural diagram is very similar to that of the dc brush series machine where K_b varies with the initial current, i_0 , and $\partial L/\partial \theta_r$ (with rotor position) and R_e varies with $dL/d\theta_r$ and Ω_r . $T_m = J/B$ is the mechanical time constant. The eigenvalues of the linear model, τ_1 and τ_2 , dependent on i_0 , Ω_{r0} , and $\partial L/\partial \theta_r$ can be extracted from Equations 9.184 and 9.185 and both of them have a negative real part, so that the response is stable; however, it may be periodic or aperiodic as for dc the brush series machines.

One major difference with respect to the dc brush series machine is that this one is motoring for $\partial L/\partial \theta_r > 0$ and generating for $\partial L/\partial \theta_r < 0$. Numerous applications for SRMs have been proposed, but only a few have reached the market. Their ruggedness to temperature and chemically aggressive environments makes them favorites for niche applications. For more on SRMs and drives, refer [9].

Example 9.9 A 3-phase 6/4 SRM has the phase inductance (Figure 9.20b) with $L_{\min} = 2\text{mH}$ and $L_{\max} = 10\text{mH}$ and the phase resistance $R_s = 1.0\Omega$ is operated at $n = 3000\text{rpm}$ and controlled with an ideal constant current, I_0 , over the entire $\theta_{\text{dwell}} = 30^\circ$ that corresponds to the stator and rotor pole spans, with an average voltage, $V_0 = 36V_{\text{dc}}$.

Calculate:

- The constant current, I_0 max., phase flux linkage and average torque, $T_{e0} = T_{L0}$, at $\omega_r = \omega_{r0} = 2\pi 9000/60 = 300\pi\text{rad/s}$
- For a constant load torque and 10% increase in average voltage V_0 ($\Delta V = +0.1V_0$), calculate the eigenvalues and current and speed

transients using the small signal approach. $\tau_m = J/B = 0.1 \text{ s}$, $B = 3.33 \times 10^{(-4)} \text{ N m s}$

Solution:

- a. We take Equation 9.180 and integrate it over time, t_{on} , corresponding to 30° :

$$t_{\text{on}} = \frac{\pi/6}{2\pi n} = \frac{\pi/6}{2\pi 3000/60} = 1.666 \times 10^{-3} \text{ s}$$

and obtain

$$V_0 t_{\text{on}} = R_s I_0 t_{\text{on}} + (L_{\text{max}} - L_{\text{min}}) I_0$$

$$I_0 = \frac{36 \times 1.666 \times 10^{-3}}{(10 - 2) \times 10^{-3} + 1 \times 1.666 \times 10^{-3}} = 6.2048 \text{ A}$$

The maximum flux linkage is $\psi_{\text{max}} = L_{\text{max}} I_0 = 10 \times 10^{-3} \times 6.2048 = 0.062048 \text{ Wb}$.

The average torque may be calculated from the power balance per cycle, considering that there is ideally a single phase working at any given time, $T_{\text{eav}} 2\pi n = (V_{\text{dc0}} - R_s I_0) I_0$.

$$T_{\text{eav0}} = \frac{(36 - 1 \times 6.2048) \times 6.2048}{2\pi(3000/60)} = 0.588 \text{ N m}$$

The load torque is equal to the motor torque for the initialization of transients:

$$T_{\text{eav0}} = T_{L0} + B\omega_{r0} = T_{L0} + 3.33 \times 10^{-4} \times 2\pi 50; \quad T_{L0} = 0.483 \text{ N m}$$

- b. The inductance varies from L_{min} to L_{max} , linearly from $\theta_r = 0$ to $\theta_r = \pi/6$, so

$$L(\theta_r) = L_{\text{min}} + (L_{\text{max}} - L_{\text{min}}) \frac{\theta_r}{\pi/6} H; \quad 0 \leq \theta_r \leq \pi/6$$

Consequently

$$K_b = \frac{\partial L}{\partial \theta_r} i_0 = \left[(L_{\text{max}} - L_{\text{min}}) \frac{6}{\pi} \right] i_0 = \left[(10 - 2) \frac{6}{\pi} \right] 10^{-3} \times 6.2048$$

$$= 0.09485 \text{ Wb}$$

From Equation 9.186

$$R_e = R_s + \frac{\partial L}{\partial \theta_r} \omega_{r0} = 1.0 + 15.2866 \times 10^{-3} \times 300\pi = 15.4 \Omega$$

The inertia, J , is

$$J = T_m B = 0.33 \times 10^{-4} \text{ kg m}^2$$

$$\tau_e = L_{av}/R_e = \frac{(2 + 10) \times 10^{-3}}{2 \times 15.4} = 0.3895 \times 10^{-3} \text{ s}$$

Hence the Equations 9.184 and 9.185 become (for $\Delta V = +3.6 \text{ V}$, $\Delta T_L = 0$)

$$\left(s + \frac{1}{0.3895 \times 10^{-3}}\right) \Delta i(s) + \frac{0.09485}{6 \times 10^{-3}} \Delta \Omega_r(s) - \frac{3.6}{s \times 6 \times 10^{-3}} \\ - \frac{0.09485}{0.33 \times 10^{-4}} \Delta i(s) + \left(s + \frac{1}{0.1}\right) \Delta \Omega_r(s) = 0$$

The eigenvalues are obtained from the characteristic equation:

$$(s + 2567)(s + 10) + \frac{0.09485^2}{0.333 \times 10^{-4} \cdot 6 \times 10^{-3}} = s^2 + 2557.7 + 4.756 \times 10^4 = 0$$

$$s_{1,2} = \frac{-2557.7 \pm \sqrt{2557.7^2 - 4 \times 4.756 \times 10^4}}{2 \cdot 2539.8} = -18.6, \text{ respectively } -2558.4$$

Now, the solution for speed transients is straightforward:

$$\omega_r(t) = \omega_{r0} + \Delta \omega_r = \omega_{r0} + A_1 e^{-18.6t} + A_2 e^{-2558.4t} + (\omega_{r \text{ final}} - \omega_{r0})$$

The final speed value, for constant torque (constant current, i_0 , and thus same flux variation), is straightforward from the voltage equation integrated over the new time, t'_{on} , which corresponds to a $\pi/6$ rotor angle rotation:

$$(V_0 + \Delta V)t'_{\text{on}} = R_s I_0 t'_{\text{on}} + (L_{\text{max}} - L_{\text{min}})I_0$$

$$t'_{\text{on}} = \frac{(10 - 2)10^{-3} \times 6.2048}{36 + 3.6 - 1 \times 6.2048} = 1.4864 \times 10^{-3} \text{ s}$$

So, the speed, $\omega_{r \text{ final}}$, is

$$\omega_{r \text{ final}} = \omega_{r0} \frac{t_{\text{on}}}{t'_{\text{on}}} = 100\pi \times \frac{1.666 \times 10^{-3}}{1.4864 \times 10^{-3}} = 351.94 \text{ rad/s}$$

Also at $t = 0$, $\omega_r = \omega_{r0} = 314 \text{ rad/s}$ and $(d\omega_r/dt)_{t=0} = 0$. Thus, both constants, A_1 and A_2 , can be found:

$$A_1 + A_2 = \omega_{r0} - \omega_{r \text{ final}} = 314 - 351.94 = -37.94$$

$$-18.6A_1 - 2558.4A_2 = 0$$

$$A_1 = -38.218 \quad A_2 = 0.2779$$

The current transients start and end at $I_0 = 6.2048 \text{ A}$ and follow Equation 9.185 with $\Delta T_L = 0$:

$$i = i_0 + \Delta i = i_0 + (\Delta \omega_r(t)) \times \frac{J}{\tau_m K_B} + \frac{J}{K_B} \times \frac{d(\Delta \omega_r(t))}{dt}$$

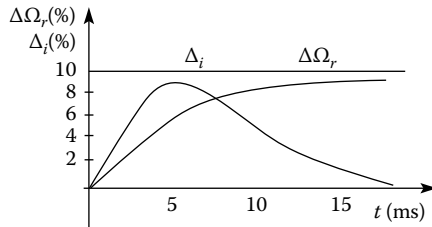


FIGURE 9.22

Speed, $\Delta\omega_r$, and current, Δi , small transients at a step rise of 10% voltage for a constant load torque.

A graphical representation of speed $\Delta\Omega_r(t)$ and current $\Delta i(t)$ variations (in %) is shown qualitatively in Figure 9.22.

Note: The response is stable as expected, but it is also aperiodic, because the inertia and mechanical time constant, τ_m , are large with respect to the equivalent electrical time constant, τ_e .

9.18 Split-Phase Cage Rotor SMs

A split-phase cage rotor SM (Figure 9.23) is used for small power single-phase grid connected (constant frequency (speed), constant voltage) domestic applications, as better efficiency can be obtained than with split-phase IMs.

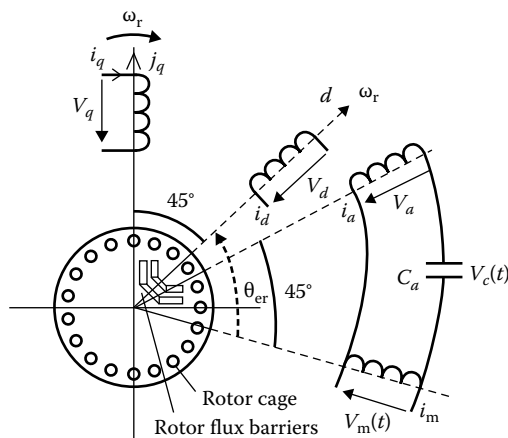


FIGURE 9.23

Split-phase cage rotor.

For the four-pole machine shown in Figure 9.23, the geometrical axes that correspond to the electrical orthogonal axes are $\pi/4$ apart. PMs may or may not be placed in the rotor flux barriers. Strong PMs are preferable for the steady state but they produce a large braking torque during self-starting, which hampers self-starting and synchronization unless a larger starting capacitor is used.

Self-starting transients are essential for split-phase SMs, even for synchronous reluctance, ($L_d < L_q$), with or without weak PMs in the d -axis (in our case).

The machine has magnetic saliency on the rotor, so if the dq model is to be used, it can be applied to the rotor. But, in this case, the stator main and auxiliary windings should be symmetric (equivalent) in the sense that when the auxiliary winding is reduced to the main winding its resistance, R'_a , and leakage inductance, L'_{al} , should be equal to R_m and L_{ml} , respectively

$$\begin{aligned} R_m &= R_a \left(\frac{W_m K_{W_m}}{W_a K_{W_a}} \right)^2 = R_s; \quad a = \frac{W_a K_{W_a}}{W_m K_{W_m}} \\ L_{ml} &= L_{al} \left(\frac{W_m K_{W_m}}{W_a K_{W_a}} \right)^2 = L_{sl} \end{aligned} \quad (9.187)$$

This equivalence is met using the same copper weight in the main and auxiliary windings. If this condition is not met, phase coordinates should be used from the start.

Let us suppose that Equations 9.187 hold. In this case, the dq model may be used here directly:

$$\begin{aligned} \frac{d\lambda_d}{dt} &= -i_d R_s + V_d + \omega_r \lambda_q \\ \frac{d\lambda_q}{dt} &= -i_q R_s + V_q - \omega_r \lambda_d \\ \frac{d\lambda_{dr}}{dt} &= -i_{dr} R_{rd}; \quad \frac{d\lambda_{qr}}{dt} = -i_{qr} R_{rq}; \\ T_e &= p_1 (\lambda_d i_q - \lambda_q i_d) \end{aligned} \quad (9.188)$$

$$\begin{aligned} \lambda_d &= L_{sl} i_d + L_{dm} (i_d + i_{dr}) + \lambda_{PM} \\ \lambda_q &= L_{sl} i_q + L_{qm} (i_q + i_{qr}) \\ \lambda_{dr} &= L_{dr} i_{dr} + L_{dm} (i_d + i_{dr}) + \lambda_{PM} \\ \lambda_{qr} &= L_{qr} i_{qr} + L_{qm} (i_q + i_{qr}) \end{aligned}$$

with $L_d < L_q$

$$\frac{J}{p_1} \frac{d\omega_r}{dt} = T_e - T_{load}; \quad \frac{d\theta_{er}}{dt} = \omega_r \quad (9.189)$$

and θ_{er} is the rotor position in electrical degrees.

The equivalence, in terms of voltages and stator currents, between the real machine and the dq model is

$$\begin{aligned}
 V_m(t) &= V\sqrt{2}\cos(\omega_1 t + \gamma) \\
 V'_a(t) &= \frac{V_m(t) - V_c(t)}{a}; \quad \frac{dV_c}{dt} = \frac{i_a}{C_a} \\
 V_d &= V_m(t)\cos\theta_{er} + V'_a(t)\sin\theta_{er} \\
 V_q &= -V_m(t)\sin\theta_{er} + V'_a(t)\cos\theta_{er} \\
 I_m &= I_d\cos\theta_{er} - I_q\sin\theta_{er} \\
 I'_a &= I_d\sin\theta_{er} + I_q\cos\theta_{er} \\
 I_a &= I'_a/a
 \end{aligned} \tag{9.190}$$

Magnetic saturation may be included in a simplified manner only in the q axis by the $L_{qm}(i_{qm})$ function, $i_{qm} = i_q + i_{qr}$. It is all that is needed if the state variables are $\psi_d, \psi_q, \psi_{dr}, \psi_{qr}, V_c, \omega_r$, and θ_{er} .

9.19 Standstill Testing for SM Parameters/Lab 9.3

By SM parameters we mean

- d and q axes magnetization curve families: $\psi_{dm}(i_{dm}, i_{qm})$, and $\psi_{qm}(i_{dm}, i_{qm})$
- Transient and subtransient inductances and time constants: l_d, l'_d, l''_d , and l''_q (in pu), and $\tau'_{d0}, \tau''_{d0}, \tau'_{q0}, \tau'_d, \tau''_d, \tau'_q$, and τ_{dr} in seconds
- Stator leakage inductance and resistance: R_s and L_{sl}
- Field circuit leakage inductance and resistance: R_F and L_{Fl}
- Rotor to stator field winding reduction factor: $K_{iF} = i_f/i_F^*$
- Rotor inertia: H (in seconds)

A few remarks are in order:

- The above-mentioned complete set of parameters is typical for single cage (per axis) and field circuit rotor SMs. For the skin effect affected SMs (large power, or solid rotor), one more cage is added along the d axis and two more such circuits along the q axis, in order to correctly model the machine for transients in the 0.001–100 Hz frequency spectrum.
- PMSMs, or dc-excited SMs without a cage on the rotor, lack all sub (or sub-sub) transient inductances and time constants. So the parameter identification is significantly simpler.

- Though there are standard tests for parameter identification in the running machine, ($\omega_r \neq 0$), such as sudden short-circuit current waveforms processing, only standstill tests are introduced here, because they are recent, comprehensive, require less hardware and testing time, and make full use of the dq model.

9.19.1 Saturated Steady-State Parameters, L_{dm} and L_{qm} , from Current Decay Tests at Standstill

In what follows, we adopt the concept of unique magnetization curves, $\psi_{dm}^*(i_m)$ and $\psi_{qm}^*(i_m)$, as presented in [1,9,10] and the total magnetization current, $i_m = \sqrt{i_{dm}^2 + i_{qm}^2}$. Current decay tests at standstill are done in the d,q axes and in the field circuit, from various initial values of currents: i_{d0} , i_{q0} , and i_{F0} . The test arrangements for current (flux) decay tests for the axes d,q are shown in Figure 9.24. The placing of rotor in the d axis ($L_d > L_q$) may be achieved easily when the dc source is connected to the arrangement in d axis (Figure 9.24a).

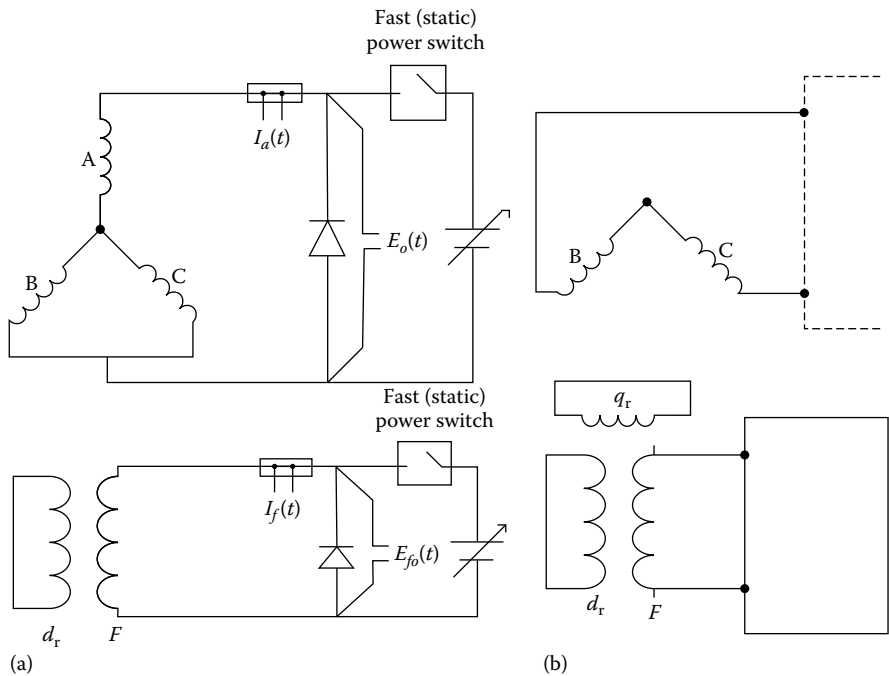


FIGURE 9.24
Current decay tests in the (a) d axis and (b) q axis.

Note: In PMs with rotor flux barriers, finding the location of the rotor in the d axis and the polarity of the PM needs special care (as $L_d < L_q$).

After a certain dc current is installed in the stator (or field winding), the stator is disconnected and the stator current decays through the free wheeling diode (as done for the dc brush machine; see Chapter 8) At zero speed ($\omega_r = 0$), the dq model of stator equations is

$$i_d R_s - V_d = -\frac{d\psi_d}{dt}; \quad i_q R_s - V_q = -\frac{d\psi_q}{dt} \quad (9.191)$$

but with the Park transformation (θ_{er}) for the d axis

$$i_d = \frac{2}{3} \left(i_A + i_B \cos \frac{2\pi}{3} + i_C \cos \left(-\frac{2\pi}{3} \right) \right) = i_A \quad (9.192)$$

$$i_q = \frac{2}{3} \left(0 + i_B \sin \frac{2\pi}{3} + i_C \sin \left(-\frac{2\pi}{3} \right) \right) = 0; \quad (i_B = i_C) \quad (9.193)$$

The flux in the q axis is zero (for Figure 9.24a). After shortcircuiting the stator over the freewheeling diode:

$$R_s \int_0^{\infty} i_d(t) dt + \frac{2}{3} \int_0^{\infty} V_{diode}(t) dt = (\psi_d)_{\text{initial}} - (\psi_d)_{\text{final}} \quad (9.194)$$

The final flux linkage is solely produced by the field current, i_{F0} , if any:

$$\psi_d \text{ final} = L_{dm} i_{F0} \quad (9.195)$$

Current decay tests may be run on the field circuit from some initial value of i_{F0} with the stator open, but this time the stator voltage $V_{ABC}(t) = V_{diode}(t) = 3/2 V_d(t)$ is recorded:

$$\psi_d \text{ initial}(i_{F0}) = \int_0^{\infty} \frac{2}{3} V_{ABC}(t) dt; \quad i_A = 0 \quad (9.196)$$

Running the d axis stator (for zero i_F) and then the field circuit (stator open) current decay tests, we can infer from Equations 9.194 and 9.196:

$$(\psi_{\text{initial}})_{i_F=0} = (L_{dm} + L_{sl}) i_{d0} = \psi_{dm}(i_{d0}) + L_{sl} i_{d0} \quad (9.197)$$

$\psi_{dm}(i_{F0})$ and $\psi_{dm}(i_{d0})$, with L_{sl} given, are thus calculated. For equal values of fluxes, we can calculate the field current reduction ratio:

$$K_F = \frac{i_{d0}}{i_{F0}} \quad (9.198)$$

We now return to Equation 9.194, with $(\psi_d)_{\text{final}}$ from the field current decay curve as

$$\psi_d \text{ initial}(i_{dm0} = i_{d0} + i_{F0}) = L_{sl} i_{d0} + \psi_{dm}(i_{dm0}) \quad (9.199)$$

But $i_{dm0} = i_{m0}$ in our case because $i_{q0} = 0$, and thus the test yields the $\psi_{dm}(i_{dm})$ saturation curve.

For the q axis (Figure 9.23b)

$$i_d = 0; \quad i_A = 0; \quad i_B = -i_C; \quad (9.200)$$

Also

$$\begin{aligned} i_q &= \frac{2}{3} \left(i_B \sin \frac{2\pi}{3} + i_C \sin \left(-\frac{2\pi}{3} \right) \right) = \frac{2}{\sqrt{3}} i_B \\ V_q &= \frac{2}{3} (V_B - V_C) \sin \frac{2\pi}{3} = \frac{V_B - V_C}{\sqrt{3}} \end{aligned} \quad (9.201)$$

As done for the d axis, the current decay test yields:

$$\psi_q(i_{m0}) = \psi_{q \text{ initial}}(i_{q0}, i_{F0}) = \frac{2}{\sqrt{3}} \int i_B R_s dt + \frac{1}{\sqrt{3}} \int_0^\infty V_{\text{diode}} dt \quad (9.202)$$

The final flux in the q axis is zero even if $i_{F0} \neq 0$ in the field circuit, to secure a desired level of magnetic (corresponding) saturation. Now the initial magnetization current i_{m0} is

$$i_{m0} = \sqrt{i_{q0}^2 + i_{F0}^2} \quad (9.203)$$

The tests can be performed with zero or nonzero dc field current to check the crosscoupling magnetic saturation importance. If the field current shows transients during the q axis decay test, this is a sign of a significant cross-coupling saturation effect.

Note: Similar tests may be run in any rotor position with Equations 9.172 through 9.179 used to calculate simultaneously the $\psi_{dm}(i_m)$ and $\psi_{qm}(i_m)$ curves but, in this case, with nonzero i_{F0} , the magnetization current, i_{m0} , is

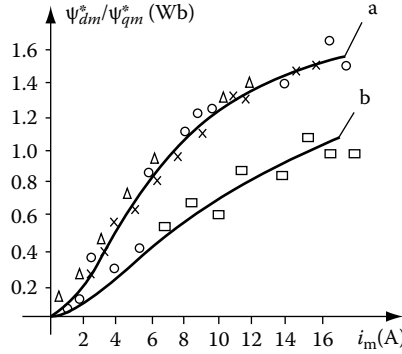
$$i_{m0} = \sqrt{(i_{d0} + i_{F0})^2 + i_{q0}^2} \quad (9.204)$$

The machine magnetization inductances, $L_{dm}(i_{m0})$ and $L_{qm}(i_{m0})$, are

$$L_{dm}^*(i_{m0}) = \frac{\psi_{dm}^*(i_{m0})}{i_m}; \quad L_{qm}^*(i_{m0}) = \frac{\psi_{qm}^*(i_{m0})}{i_{m0}} \quad (9.205)$$

These inductances pertain to the unique d and q axes magnetization curves [9]. Typically, such $\psi_{dm}^*(i_m)$ curves for a 3 KW SM are shown in Figure 9.25 where $\psi_{dm}^*(i_{F0})$ was also calculated from a no-load running generator test, to verify the results obtained.

Note: The dc current decay tests transients may be used also to identify all operational parameters of $L_d(s)$, $L_q(s)$ and $G(s)$ using curve fitting methods [11,12].

**FIGURE 9.25**

Current decay tests in (a) axis d and (b) axis q .

9.19.2 Single Frequency Test for Subtransient Inductances, L_d'' and L_q''

The subtransient inductances, L_d'' and L_q'' (or reactances, in pu, x_d'' and x_q''), refer to fast transients. At standstill, if we supply the machine, at frequency ω_1 , from a single phase transformer as shown in Figure 9.24a and b (without the free wheeling diode), and measure voltages E_{ABC} , I_A , P_d and E_{BC} , I_B , P_q we obtain

$$Z_d'' = \frac{2}{3} \frac{E_{ABC}}{I_A}; \quad R_d'' = \frac{2}{3} \frac{P_d}{I^2} \quad (9.206)$$

$$Z_q'' = \frac{1}{2} \frac{E_{BC}}{I_B}; \quad R_q'' = \frac{P_q}{2I_B^2} \quad (9.207)$$

$$X_d'' = \sqrt{(Z_d'')^2 - (R_d'')^2}; \quad L_d'' = X_d''/\omega_1$$

$$X_q'' = \sqrt{(Z_q'')^2 - (R_q'')^2}; \quad L_q'' = X_q''/\omega_1 \quad (9.208)$$

The $\ominus$ sequence parameters, R_- , X_- can be adapted as

$$R_- = (R_d'' + R_q'')/2 \quad \text{or} \quad R_- = \sqrt{R_d'' R_q''} \quad (9.209)$$

$$X_- = (X_d'' + X_q'')/2 \quad \text{or} \quad X_- = \sqrt{X_d'' X_q''} \quad (9.210)$$

Note: In reality for the $\ominus$ sequence, the rotor experiences $2f_1$ frequency (and not f_1 frequency). So we may increase the standstill frequency to $2f_1$ and repeat the tests to see the relative significance of the frequency (skin) effects in the rotor.

9.19.3 Standstill Frequency Response Tests

The standstill frequency response tests (SSFRS) refer to arrangements in Figure 9.24 (the freewheeling diode is eliminated), where the machine is fed

through almost sinusoidal voltages of variable frequency, from 0.001 (0.01) to 100 Hz for SGs at the power grid. The machine voltage, current, and its phase angle, for the d and q axes tests, and closed or open field windings, are done for each frequency; (8–10) value per decade are acquired. The tests are done at 10% of the rated current, as the duration of tests is lengthy and machine overheating has to be avoided.

The current/voltage dq–ABC relationships remain the same as for current decay tests.

But the d–q relationships based on Equation 9.158, and $\omega_r = 0$ and $V_F(s) = 0$, are

$$\underline{Z}_d(j\omega) = \frac{V_d(j\omega)}{I_d(j\omega)} = R_s + j\omega L_d(j\omega); \quad \underline{V}_d(j\omega) = \frac{2}{3} V_{ABC}$$

$$I_d(i\omega) = \underline{I}_A \quad (9.211)$$

$$\underline{Z}_q(j\omega) = \frac{V_q(j\omega)}{I_q(j\omega)} = R_s + j\omega L_q(j\omega); \quad \underline{V}_q(j\omega) = \frac{(V_B - V_C)}{\sqrt{3}}$$

$$I_q(j\omega) = \underline{I}_B \frac{2}{\sqrt{3}} \quad (9.212)$$

where $L_d(j\omega)$ and $L_q(j\omega)$ are of the form Equation 9.58:

$$L_{d,q}(j\omega) = L_{d,q}(0) \frac{(1 + j\omega T'''_{d,q})(1 + j\omega T''_{d,q})(1 + j\omega T'_{d,q})}{(1 + j\omega T'''_{d,q0})(1 + j\omega T''_{d,q0})(1 + j\omega T'_{d,q0})} \quad (9.213)$$

In Equation 9.213, three rotor circuits have been considered along each axis (sufficient for large power SGs with a solid rotor).

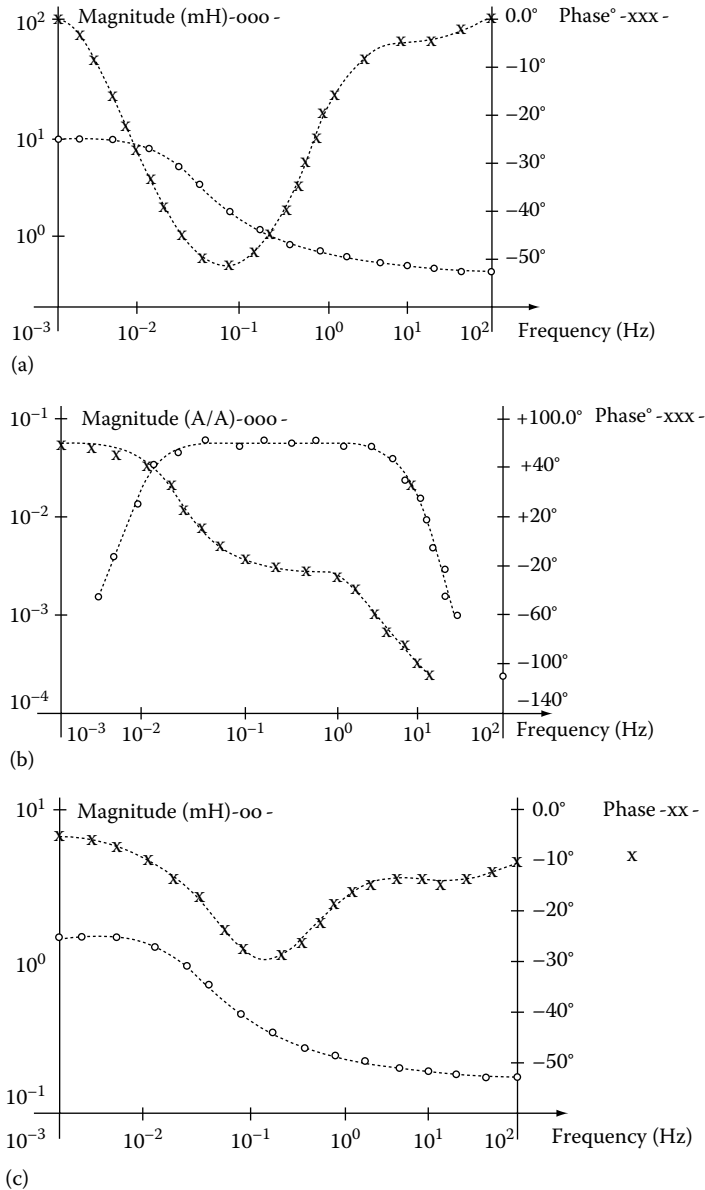
Also, from

$$-j\omega G(j\omega) = \frac{I_F(j\omega)}{\underline{I}_d(j\omega)} \quad (9.214)$$

with $G(i\omega)$ as in Equation 9.54 for short-circuited field winding and acquired $i_F(j\omega)$:

$$G(j\omega) = \frac{L_{dm}(0)}{R_F} \frac{(1 + pT_{dr1})(1 + pT_{dr2})}{(1 + j\omega T'''_{do})(1 + j\omega T''_{do})(1 + j\omega T'_{do})} \quad (9.215)$$

Typical experimental results are shown in Figure 9.26. Once $L_d(j\omega)$, $L_q(j\omega)$, and $j\omega G(j\omega)$ are obtained experimentally, finding the time constants in their expressions Equations 9.213 through 9.215 become a problem for curve fitting [12], with some methods requiring the calculation of gradients and others avoiding them, such as pattern search (IEEE-standard 115-1995). Straightforward analytical expressions of SSFR parameters are given in [13]. For the phenomenology of multiple rotor circuits models, an intuitive method to identify the time constants based on finding the phase response extremes' frequencies is presented in [14]. For a complete description of SM testing, see the IEEE standard 115-1995, while for PMSM consult [15].

**FIGURE 9.26**

Typical (a) $L_d(j\omega)$, (b) $j\omega G(j\omega)$, and (c) $L_q(j\omega)$.

9.20 Linear Synchronous Motor Transients

As discussed in Chapter 6, an LSM may be built with active guideway, and dc excitation, or superconducting dc excitation, or PM excitation on

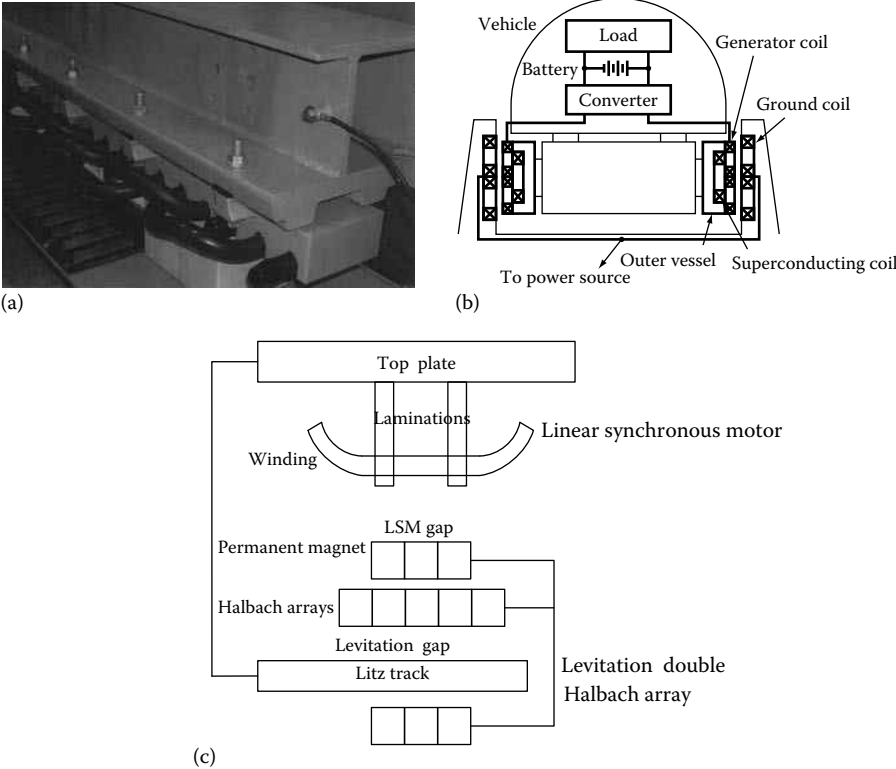
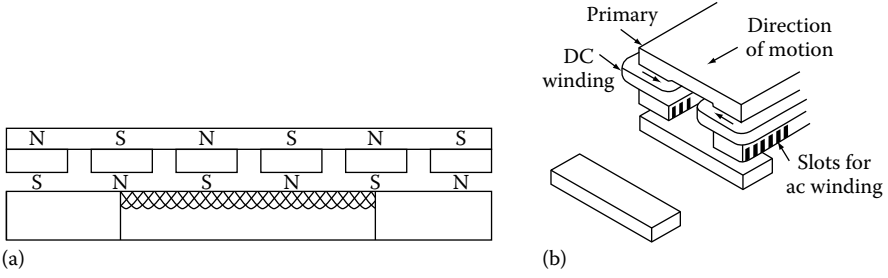


FIGURE 9.27 Active guideway LSMs with (a) dc excitation, (b) dc superconducting excitation, and (c) PM excitation.

board of the mover (Figure 9.27a through c). For limited travel, the PMs are placed along the track and the 3 phase winding is placed on board of mover (Figure 9.28a). Finally, it is possible to have both PMs (or dc excitation) and the ac windings on the mover, with a salient homopolar pole core structure along the track (Figure 9.28b).

But whatever the configuration, if the number of poles exceeds $2p_1 \geq 6$, the end effects due to the openness of the magnetic circuit along the direction of motion are negligible; the dq model typical for rotary SMs applies here but with three differences in variables:

- Synchronous (rotor) speed, ω_r rad/s, turns into linear synchronous (field) speed, $U_s = 2\tau f_1$ (m/s); U_m is the mover actual speed in m/s.
- Electromagnetic torque, T_e , (Nm) is replaced by thrust F_e (N).
- Rotor angle variable, θ_r , (rad) changes into the linear variable, $X(m)$; the angle θ_{er} in the inductance expression changes to $x\pi/\tau$.

**FIGURE 9.28**

LSM with (a) PM guideway and (b) dc excitation and ac winding on the mover.

The electromagnetic power, P_{elm} , changes from $P_{elm} = T_e \omega_r / p_1$ to $P_{elm} = F_e \cdot 2\tau f_1$. Consequently, the dq model of LSMs is

$$\begin{aligned} \bar{V}_s &= R_s \bar{I}_s + jU_m \bar{\Psi}_s \frac{\pi}{\tau} + \frac{d\bar{\Psi}_s}{dt}; & V_F &= R_F i_F + \frac{d\psi_F}{dt} \\ \bar{\Psi}_s &= \psi_d + j\psi_q \\ \psi_d &= L_{dm}(g)i_F + L_d(g)i_d; & \psi_F &= L_F(g)i_F + L_{dm}(g)i_d; & \psi_q &= L_q(g)i_q \end{aligned} \quad (9.216)$$

g -the magnetic (or mechanical) airgap

$$F_e = \frac{3}{2} \frac{\pi}{\tau} (\psi_d i_q - \psi_q i_d); \quad M_m dU_m/dt = F_e - F_{load}; \quad dx/dt = U_m$$

$$\begin{vmatrix} V_d \\ V_q \\ V_0 \end{vmatrix} = \frac{2}{3} \begin{vmatrix} \cos\left(-\frac{\pi}{\tau}x\right) & \cos\left(-\frac{\pi}{\tau}x + \frac{2\pi}{3}\right) & \cos\left(-\frac{\pi}{\tau}x - \frac{2\pi}{3}\right) \\ \sin\left(-\frac{\pi}{\tau}x\right) & \sin\left(-\frac{\pi}{\tau}x + \frac{2\pi}{3}\right) & \sin\left(-\frac{\pi}{\tau}x - \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{vmatrix} \quad (9.217)$$

The sign $(-\frac{\pi}{\tau}x)$ in Equation 9.217 is valid for the active guideway LSM, but it turns to $(+\frac{\pi}{\tau}x)$ for the passive guideway LSM, where the armature (ac) winding is on board of the mover. This is so because the coordinate system of the dq model is attached to the dc-excited or passive (magnetically anisotropic) part of the LSM.

Also, the dq model is valid for distributed ($q \geq 2$) ac armature windings with uniform slots, and for fractionary ($q \leq 0.5$) tooth wound coil windings where the PM-produced emf varies sinusoidally with the mover position, x , as for rotary SMs. Linear switched reluctance or stepper motors require, in general, phase coordinate models as do their rotary counterparts.

The LSM may be controlled, as rotary machines, for constant ψ_{d0} or constant ψ_s (unity power factor is possible only with dc excitation LSM). In

addition, LSM may be controlled for integrated propulsion and suspension control (for Maglevs). In the latter case, the stator (or airgap flux) is controlled in order to dynamically keep the airgap, g , constant (within $\pm 20\%$ – 25%).

The normal force, F_n , which produces suspension is

$$F_{na} \approx \frac{3}{2} \left(i_d \frac{\partial \psi_d}{\partial g} + i_q \frac{\partial \psi_q}{\partial g} \right) \quad (9.218)$$

The state feedback or variable structure control has been proved adequate for the active suspension control.

Note: For the dc excitation LSM, suspension control is performed through the field current, while i_d and i_q control is done through propulsion control, which is much faster than the former. For more on LSM control, see [16–19].

9.21 Summary

- By SM transients, we mean the slow to fast variation of SM voltages, currents, flux magnitudes, and/or frequency in time.
- The two reaction steady-state models of SM (Chapter 6) are not applicable to SM transients.
- The phase coordinate model of SM shows variable self- and mutual inductances with the rotor position, and thus is computer-time prohibitive in its solving of transients. It is, however, not practical for the fast computation of transients and for control design.
- The dq0 model of SM in rotor coordinates is quite suitable in modeling transients as its inductances are independent of the rotor position. It is also widely used for SM control in modern, variable speed drives.
- The dq0 model applies, in general, only to distributed ($q \geq 2$) ac windings and to constant airgap configurations. To create magnetic saliency, a flux barrier along the q axis is provided.
- The dq0 model may also be applied (with caution) to $q \leq 0.5$ (slots/pole/phase) PMSMs (with tooth-wound coils) where the emf is sinusoidal in time and the rotor is winding-less.
- For specially shaped rotor/stator poles, even for a switched reluctance machine ($q \leq 0.5$) with double magnetic saliency, and thus inductances with sinusoidal variation of the rotor position (and no rotor windings), the dq0 model can be applied with caution.

- The dq0 model of SM with a single-cage rotor and dc excitation represents an eighth-order system.
- The pu dq0 model equations use $\frac{1}{\omega_{10}} d/dt$ instead of d/dt and have all parameters (resistance and inductances) in p.u., but the inertia, H , and various time constants are in seconds.
- As expected, in rotor coordinates, the dq0 model variables for the balanced steady state of the SM are all dc.
- For the symmetric stator, the space phasor form of equations is not only feasible, but also useful.
- For an ideal no-load generator in the dq model, not only $i_{d0} = i_{q0} = 0$ but also $V_{d0} = 0$ and $V_{q0} = V_0\sqrt{2}$ (V_0 is the RMS phase voltage).
- For a steady-state 3-phase short circuit, the machine is not saturated and the current is in general smaller than $3I_{\text{rated}}$, even for a cageless rotor PMSM. Due to stator resistances, there is a sizable short-circuit braking torque, which may be a design constraint for some fault-tolerant applications.
- Transients at constant speed are purely electromagnetic; for constant SM parameters (inductances), their operational parameters are defined as $L_d(s)$ and $L_q(s)$, and $sG(s)$, if the SM changes its inductances along the d and q axes during transients. The initial values, L_d'' and L_q'' , are called subtransient inductances, and L_d , the transient inductance, corresponds to later transients when there are no longer any rotor cage effects. Finally, at steady state, the SM shows L_d and L_q . The operational parameters also contain a few time constants, depending on the number of the d , q rotor axes circuits used to simulate frequency (skin) effects.
- The 3-phase sudden short-circuit current waveform may be processed so that the d axis parameters, L_d'' , L_d' , T_d'' , T_d' , T_{d0}'' , and T_{d0}' , can be determined by curve fitting methods.
- Asynchronous running can also be treated in Laplace formulation with $s = jS\omega_1$, (S is the slip), to calculate asynchronous torque versus speed; this information is useful in estimating the asynchronous starting capabilities of SMs at power grid.
- Simplified dq0 models include neglecting stator and/or also rotor transients, when the SM model with dc excitation retains a third order.
- For electromechanical transients, the speed also varies and the dq model equations are linearized. They have ΔV , $\Delta\omega_1$, ΔV_F , and ΔT_L as inputs and Δi_d , Δi_q , Δi_{dr} , Δi_{qr} , Δi_F , $\Delta\omega_r$, and $\Delta\delta_v$ as variables.

- Speed response to load torque pulsations can easily be calculated by the small deviation theory.
- Large deviation transients make full use of the dq model in rotor coordinates. Asynchronous starting and self-synchronization and line-to-line and line-to-neutral faults constitute large transients. The results are useful for a refined machine and protection design.
- Constant d axis flux, ψ_{d0}^* , and stator flux, ψ_s^* , control are used in variable speed (and frequency) control.
- The dq model reveals for constant, ψ_{d0}^* , frequency (speed) control—applicable especially to PMSMs—a decreasing speed with torque characteristic. The ideal no-load speed is $\omega_{r0} = V_s/\psi_{d0}$; $\psi_{d0} = \psi_{PM} + L_d i_{d0}$. For $i_{d0} = -\psi_{PM}/L_d$, $\omega_{r0} \rightarrow \infty$, it is ideal for constant power wide-speed range, needed for many applications.
- For constant stator flux, ψ_s^* , (and unity power factor), the speed/torque curve is a descending straight line as in separately excited dc motors. The field current, i_F , for $\cos \varphi_1 = 1$, increases also with torque, as expected. The case is typical for the voltage-source PWM inverter-fed SM drives. No damper cage is provided on the rotor.
- Two basic vector control drives schemes are introduced.
- For rectangular current, 2-phase conducting, controlled PMSMs (brushless dc motors) and cage rotor dc-excited rotor SM (supplied from current source inverters for variable speed), the basic transient equations and speed/torque curves are derived. Again, linear descending speed versus torque curves are obtained. These two situations correspond to the two typical variable speed SM drives described.
- Switched reluctance motors have double saliency, and thus the phase coordinate model is used. It is not a traveling field machine but the frequency of current pulses, f_1 , into a stator phase is directly related to the number of rotor salient poles, N_r , and speed, n : $f_1 = nN_r$.
- A small deviation model of SRM reveals a transient behavior very similar to the dc brush series motor. Due its ruggedness, the SRM is suitable in thermally or chemically aggressive environments. Stepper motors without or with PMs (hybrid) operate as an SRM but are supplied open loop (by frequency ramping), so as not to loose steps and applied in harsh environments.
- Split-phase cage rotor SMs with magnetic rotor saliency ($L_d > L_q$) or with PMs ($L_d \leq L_q$) may be used for home appliances, to increase efficiency and reduce motor size for direct power grid operation. The

dq model in rotor coordinates holds only if the two orthogonal stator windings (main and auxiliary), are equivalent (made of same copper weight). For auxiliary-start-only-windings, this is not the case, in general, and thus the phase coordinates are required.

- Testing for SM parameters is essential for control design. Stand-still flux d current/decay tests and frequency response (SSFRs) are described in detail to exemplify the practicality of dq model; for complete SM testing, see the IEEE standard 115-1995.
- Linear progressive motion synchronous motors are very similar to rotary ones in terms of topology and modeling for transients (see [16,17]), with some end effects to be considered at high speed or, for $2p_1 = 2, 4$ poles, motor applications.
- Linear oscillatory motion PMSMs are being used for small refrigerator compressor drives. They work at resonance (electrical frequency, f_e , is equal to the mechanical proper frequency, f_m ($f_e = f_m$)) (see [16–19]).

9.22 Proposed Problems

- 9.1** Draw the space phasor diagrams for SM motoring and generating at unity power factor.

Hint: Check Section 9.7, Figure 9.5, and Equations 9.45 and 9.46.

- 9.2** For the large SM in Example 9.1, calculate the inductances $l_d'', l_d', l_q'', l_{d0}'', \tau_{d0}'', \tau_d'', \tau_d', \tau_q'',$ and τ_{q0}'' .

Hint: Check Equation 9.55.

- 9.3** For the MS in Example 9.1, calculate peak value of $i_d(t)$, $i_F(t)$, $i_A(t)$ during sudden 3-phase short circuit from no load. Consider that $V_{q0} = 1.2(V_{nl}/\sqrt{3})\sqrt{2}$ and $i_{F0} = 2I_n$.

Hint: See Equations 9.68 through 9.70.

- 9.4** The machine in Example 9.5 with the parameters, $\psi_{PMd} = 1.0$ Wb, $L_d = 0.05$ H, $L_{dm} = 0.9 L_d$, $L_q = 0.1$ H, $R_s = 1 \Omega$, $p_1 = 2$, operates at the unity power factor at $V_{s0} = V_0\sqrt{2} = 18\sqrt{2}V$. Calculate the corresponding speed, ω_{rb} , rated stator current, torque, and electromagnetic power. For $3\omega_{rb}$, calculate the torque and electromagnetic power.

Hint: Check Example 9.5.

- 9.5** Find the eigenvalues of SM for the small deviation dq model. The initial situation corresponds to the following data: $L_d = L_q = 0.1$ H,

$L_{sl} = 0.1L_d = L_{Fl} = L_{drl} = L_{qrl}$, $\delta_{v0} = 30^\circ$; $\omega_{r0} = 314 \text{ rad/s}$,
 $R_s = 1 \Omega = R_F$; $R_{dr} = R_{qr} = 3R_s$, $V_{s0} = V_0\sqrt{2} = 220\sqrt{2} \text{ V}$, $i_{F0} = 2i_{d0}$;
 $J = 1 \text{ kg m}^2$, $p_1 = 2$.

Hint: Calculate first i_{d0} , i_{F0} , i_{q0} , ψ_{d0} , and ψ_{q0} .

- 9.6** A PMSM with $\psi_{PMd} = 2 \text{ Wb}$, $R_s = 2 \Omega$, $L_d = L_q = 0.2 \text{ H}$, $p_1 = 4$, $B = 5 \times 10^{-3} \text{ N m s}$, and $T_m = 0.4 \text{ s}$ is supplied from $V_s = V_0\sqrt{2} = 120\sqrt{2} \text{ V}$ at $n = 1500 \text{ rpm}$.

Calculate

- The current, i_q , torque, efficiency, and power factor, for $i_{d0} = 0$.
- For constant speed and $i_{d0} = 0$, at 10% increase in the load torque, calculate the speed and current, i_q , transient.

- 9.7** A cage rotor large SM with dc excitation is controlled by a PWM voltage source inverter.

The motor data are as follows $P_n = 5 \text{ MW}$, $V_{s0} = 4200\sqrt{2/3} \text{ V}$, $n_b = 10 \text{ rpm}$, $f_b = 5 \text{ Hz}$, $\eta = 0.985$, $\cos \varphi_1 = 1$, (only copper losses are considered), $x_d = x_q = 0.65 \text{ pu}$, $X_{dm} = 0.55 \text{ pu}$.

Calculate

- The voltage power angle δ_v , i_{d0} , i_{q0} , and i_{F0} if $E_o = 1.2V_{s0}$
- Same deliverable as in (a) but for $n_{\max} = 1.6 n_b$

Hint: Follow closely Example 9.6.

- 9.8** Draw the speed/torque curve for the rectangular current control of brushless dc motor with $V_{dc} = 42 \text{ V}$, $R_s = 1 \Omega$, and $\psi_{PM} = 0.1 \text{ Wb}$ (the emf is considered flat and 180° wide for both polarities)

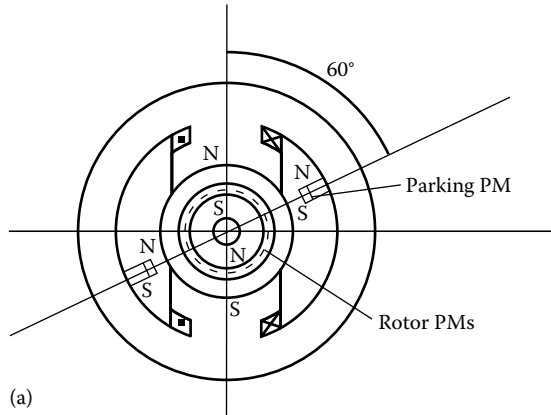
Hint: Use Equations 9.169 through 9.173.

- 9.9** A cage dc-excited rotor, large SM has the following data: $V_{s1} = 4200\sqrt{2/3} \text{ V}$, $I_{s1} = 1000 \text{ A}$, $f_b = 60 \text{ Hz}$, $n_b = 1800 \text{ rpm}$ has the data $x_d = 1.3$, $x_q = 0.6 \text{ (pu)}$. $L_d'' = 0.3 L_d$, $L_q'' = 0.3 L_q$, $r_s = 0.01 \text{ (pu)}$, $\varphi_1 = -10^\circ$ (leading power factor) operates with rectangular current control from a current-source inverter at I_{s1} .

Calculate

- Torque, T_{em} , voltage power angle δ_{v1} , I_{d1} , I_{q1} , and I_F
- No-load ideal speed
- For same voltage at $n_{\max} = 2n_b$, $\varphi_1 = -10^\circ$, I_{s1} , calculate again δ_{v1} , I_{d1} , I_{q1} , I_F

Hint: Check Section 9.10 and Equations 9.174 and 9.175.

**FIGURE 9.29**

Single phase PMSM.

- 9.10** A cageless surface PM two-pole rotor single-phase SM (Figure 9.29) has a parking PM which provides a $\theta_{r\text{initial}} = 60^\circ$ away from the stator pole alignment of rotor poles, for self-starting. The emf varies sinusoidally with rotor position, θ_r , and the main PM cogging torque (at zero) varies sinusoidally with $2\theta_r$. Write the phase coordinate model of the machine. Write a MATLAB[®]-Simulink code to simulate the machine transients. Include pertinent data for a small (100 W, 3600 rpm) motor and apply the code for the voltages $V(t) = V\sqrt{2}\cos(\omega_r t + \gamma)$, and $V = V_0 + K_r\omega_r$, where ω_r is the current speed. Vary γ and discuss the results.

Hint: Use phase coordinates, with constant stator inductances and sinusoidal emf.

- 9.11** A small excursion single-sided linear PMSM carrier with active guideway is controlled at zero i_d and develops, thrust $F_{xn} = 1$ kN, for a pole pitch $\tau = 0.06$ m at $U_m = 1.56$ m/s, $2p_1 = 8$, thrust density is 2.5 N/cm²; and the airgap $g = 1$ mm; surface PMs, $h_{PM} = 3.0$ mm, thick.

The PM span is equal to the pole pitch and the PM airgap flux density $B_{gPM} = B_r \times h_{PM}/(h_{PM} + g) = 0.8$ T, $L_{sl} = 0.35L_{dm}$, ($L_{dm} = L_{qm}$), number of phase turns $W_s K_{W_s} = 750$ turns/phase.

Calculate (for steady state)

- a. PM flux linkage, $\psi_{PMd} = W_s K_{W_s} \frac{2}{\pi} B_{gPM} \tau l_i$, where l_i is the stack length to be calculated from the thrust density

b. the magnetization inductance

$$L_m = \frac{6\mu_0 (W_s K_{W_s})^2}{\pi^2 p (g + h_{PM})} \tau l_i \quad (9.219)$$

- c. With $\psi_{PMd} = L_m i_{F0}$, calculate i_{F0} corresponding to the PMs' excitation
- d. Stator current, i_q , for rated thrust F_{xn} , and $i_d = 0$
- e. Values of $\psi_d = \psi_{PM}$, ψ_q , f_1 , ω_1 (stator frequency), and V_s (stator voltage) for zero resistance losses
- f. Normal attraction force, F_{na} , both from Equation 9.218 and $F_{na} \approx \frac{B_{gPM}^2}{2\mu_0} 2p_1 \tau l_i$, and the ratio F_{na}/F_{xn} .

Hints: Check Section 9.20 (selective results $W_s K_{W_s} = 750$, $f_1 = 13$ Hz, $V_s \approx 130\sqrt{2}$ V (peak phase voltage)), $F_n \approx 10$ kN (10/1 normal (suspension) to propulsion force).

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Transients of Induction Machines

An induction machine (IM) is built with 1-, 2-, 3- (or 6-) phase ac primary stator windings and a cage or wound rotor. In the latter case, the rotor winding is of a 3-phase ac type, and is connected through copper sliprings and brushes to an external source (PWM converter) that provides variable voltage and frequency. The so-called wound rotor or doubly fed induction machine is thus obtained. Also linear induction motors (LIMS) with aluminum-slab-on-iron or ladder secondary, or with two (3)-phase ac secondary winding on board of mover, have also been built for niche applications.

All the above-mentioned classes of IMs that are discussed in this chapter can be modeled for transients.

The 3-phase cage-secondary (rotor) IMs require a separate discussion. We start with the phase variable model of an IM with 3-phase windings both on the stator and on the rotor (the rotor cage, when healthy, is equivalent to a symmetric 3-phase winding).

10.1 Three-Phase Variable Model

Let us consider a symmetric 3-phase winding stator and rotor IM (Figure 10.1).

The machine equations (Figure 10.1) for the phase variables (coordinates) may be directly written in a matrix form:

$$[I_{ABCabc}] [R_{ABCabc}] - [V_{ABCabc}] = -\frac{d}{dt} [\Psi_{ABCabc}^{(\theta_{er}, t)}] \quad (10.1)$$

with

$$[\Psi_{ABCabc}^{(\theta_{er}, t)}] = [L_{ABCabc}^{(\theta_{er})}] [I_{ABCabc}] \quad (10.2)$$

The resistance matrix is diagonal:

$$[R_{ABCabc}] = \text{Diag} [R_s \quad R_s \quad R_s \quad R_r \quad R_r \quad R_r] \quad (10.3)$$

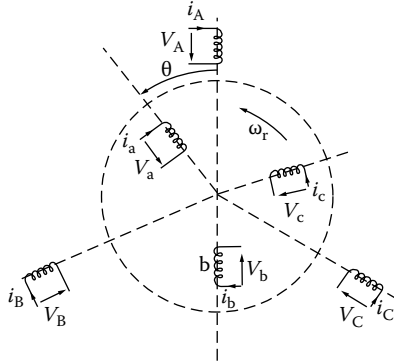


FIGURE 10.1
Three-phase IM.

but the inductance matrix is 6×6 and the stator/rotor mutual inductances vary with the electrical rotor position. The stator-only and rotor-only inductances do not vary with the rotor position (slot openings are neglected). For distributed ac windings ($q \geq 2$), the variation of the mutual inductances with the electrical rotor position is sinusoidal. So, the inductance matrix, $L_{ABCabc}^{(\theta_{er})}$, is straightforward:

$$L_{ABCabc}^{(\theta_{er})} = \begin{bmatrix} L_{sl} + L_{os} & -L_{os}/2 & -L_{os}/2 & L_{sr} \cos \theta_{er} & L_{sr} \cos (\theta_{er} + 2\pi/3) & L_{sr} \cos (\theta_{er} - 2\pi/3) \\ -L_{os}/2 & L_{sl} + L_{os} & -L_{os}/2 & L_{sr} \cos (\theta_{er} - 2\pi/3) & L_{sr} \cos \theta_{er} & L_{sr} \cos (\theta_{er} + 2\pi/3) \\ -L_{os}/2 & -L_{os}/2 & L_{sl} + L_{os} & L_{sr} \cos (\theta_{er} + 2\pi/3) & L_{sr} \cos (\theta_{er} - 2\pi/3) & L_{sr} \cos \theta_{er} \\ L_{sr} \cos \theta_{er} & L_{sr} \cos (\theta_{er} - 2\pi/3) & L_{sr} \cos (\theta_{er} + 2\pi/3) & L_{rl} + L_{or} & -L_{or}/2 & -L_{or}/2 \\ L_{sr} \cos (\theta_{er} + 2\pi/3) & L_{sr} \cos \theta_{er} & L_{sr} \cos (\theta_{er} - 2\pi/3) & -L_{or}/2 & L_{rl} + L_{or} & -L_{or}/2 \\ L_{sr} \cos (\theta_{er} - 2\pi/3) & L_{sr} \cos (\theta_{er} + 2\pi/3) & L_{sr} \cos \theta_{er} & -L_{or}/2 & -L_{or}/2 & L_{rl} + L_{or} \end{bmatrix} \quad (10.4)$$

with

$$\frac{L_{or}}{L_{sr}} = \frac{L_{sr}}{L_{os}}; \quad L_{os} \cos 2\pi/3 = -L_{os}/2; \quad L_{or} \cos 2\pi/3 = -L_{or}/2 \quad (10.5)$$

From Equations 10.1 and 10.2, and after multiplication with $[I_{ABCabc}]^T$, we obtain

$$\begin{aligned}
 [V_{ABCabc}][I_{ABCabc}]^T &= \underbrace{[I_{ABCabc}][I_{ABCabc}]^T[R_{ABCabc}]}_{\text{winding losses}} \\
 &+ \underbrace{\frac{d}{dt} \left\{ \frac{[I_{ABCabc}] \left[L_{ABCabc}^{(\theta_{er})} \right] [I_{ABCabc}]^T}{2} \right\}}_{\frac{\partial W_{mag}}{\partial t}} \\
 &+ \underbrace{\frac{1}{2} [I_{ABCabc}] \left[\frac{\partial L_{ABCabc}^{(\theta_{er})}}{\partial \theta_{er}} \right] [I_{ABCabc}]^T \frac{d\theta_{er}}{dt}}_{P_{elm} = \frac{T_e}{p_1} \frac{d\theta_{er}}{dt}}
 \end{aligned} \quad (10.6)$$

The instantaneous torque, T_e , “springs” from the electromagnetic power, P_{elm} :

$$T_e = \frac{P_{elm}}{p_1 \frac{d\theta_{er}}{dt}} = \frac{p_1}{2} [I_{ABCabc}] \left[\frac{\partial L_{ABCabc}^{(\theta_{er})}}{\partial \theta_{er}} \right] [I_{ABCabc}] \quad (10.7)$$

The motion equations complete the phase variable model:

$$\frac{J}{p_1} \frac{d\omega_r}{dt} = T_e - T_{load}; \quad \frac{d\theta_{er}}{dt} = \omega_r; \quad \omega_r = 2\pi p_1 n \quad (10.8)$$

An eighth-order nonlinear system with variable coefficients (through $[L^{(\theta_{er})}_{ABCabc}]$) has been obtained. It is evident that such a system is difficult to handle as it requires a large CPU time in digital simulations. The dq model, in its space phasor form, as derived in Chapter 7, is the key to a practical approach to IM transients.

10.2 dq (Space Phasor) Model of IMs

The dq model of an IM with single rotor circuits per the d , q axes—from Chapter 7 (Figure 10.2)—is given here in short:

$$\begin{aligned}
 i_d R_s - V_d &= -\frac{\partial \Psi_d}{\partial t} + \omega_b \Psi_q; & i_q R_s - V_q &= -\frac{\partial \Psi_q}{\partial t} - \omega_b \Psi_d \\
 i_{dr} R_r - V_{dr} &= -\frac{\partial \Psi_{dr}}{\partial t} + (\omega_b - \omega_r) \Psi_{qr}; & i_{qr} R_r - V_{qr} &= -\frac{\partial \Psi_{qr}}{\partial t} - (\omega_b - \omega_r) \Psi_{dr}
 \end{aligned} \quad (10.9)$$

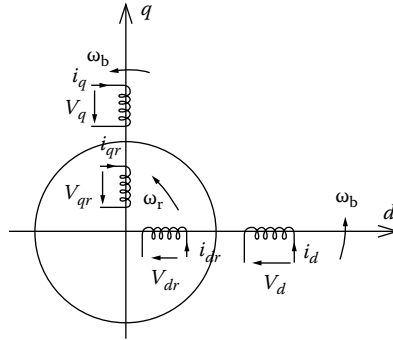


FIGURE 10.2
dq model of IM.

$$T_e = \frac{3}{2} p_1 (\Psi_d i_q - \Psi_q i_d); \quad \frac{J}{p_1} \frac{d\omega_r}{dt} = T_e - T_{\text{load}}; \quad \frac{d\theta_b}{dt} = \omega_b \quad (10.10)$$

$$P_s = \frac{3}{2} (V_d i_d + V_q i_q); \quad Q_s = \frac{3}{2} (V_d i_q - V_q i_d) \quad (10.11)$$

P_s , Q_s are the active and reactive input powers.

As described in Equations 10.10 and 10.11, the torque and power equivalence of the dq model with the 3-phase IM has been included. In a space phasor notation

$$\begin{aligned} \bar{V}_s &= V_d + jV_q; & \bar{V}_r &= V_{dr} + jV_{qr} \\ \bar{\Psi}_s &= \Psi_d + j\Psi_q; & \bar{\Psi}_r &= \Psi_{dr} + j\Psi_{qr} \end{aligned} \quad (10.12)$$

$$\bar{i}_r = i_{dr} + ji_{qr}$$

$$\begin{aligned} \bar{i}_s R_s - \bar{V}_s &= -\frac{\partial \bar{\Psi}_s}{\partial t} - j\omega_b \bar{\Psi}_s \\ \bar{i}'_r R_r - \bar{V}'_r &= -\frac{\partial \bar{\Psi}'_r}{\partial t} - j(\omega_b - \omega_r) \bar{\Psi}'_r \end{aligned} \quad (10.13)$$

with

$$\begin{aligned} T_e &= \frac{3}{2} p_1 \text{Real} [j\bar{\Psi}_s \bar{i}'_s^*] \\ \bar{\Psi}_s &= L_{sl} \bar{i}_s + L_m (\bar{i}_s + \bar{i}'_r) \\ \bar{\Psi}'_r &= L'_{rl} \bar{i}'_r + L_m (\bar{i}_s + \bar{i}'_r) \end{aligned} \quad (10.14)$$

It is evident that in the space phasor (dq) model, the rotor has been reduced to the stator (because we directly add $\bar{i}_s$ and $\bar{i}'_r$).

10.3 Three-Phase IM–dq Model Relationships

The space phasor transformation of currents (Chapter 7) is

$$\begin{aligned}\bar{i}_s &= \frac{2}{3} \left[i_A(t) + i_B(t) e^{j\frac{2\pi}{3}} + i_C(t) e^{-j\frac{2\pi}{3}} \right] e^{-j\theta_b} \\ \bar{i}_r &= \frac{2}{3} \left[i_a(t) + i_b(t) e^{j\frac{2\pi}{3}} + i_c(t) e^{-j\frac{2\pi}{3}} \right] e^{-j(\theta_b - \theta_{er})}; \quad \frac{d\theta_{er}}{dt} = \omega_r\end{aligned}\quad (10.15)$$

$$i_{0s} = \frac{1}{3} (i_A + i_B + i_C); \quad i_{0r} = \frac{1}{3} (i_a + i_b + i_c) \quad (10.16)$$

The same transforms, Equations 10.15 and 10.16, hold for flux linkages, $\bar{\psi}_s$ and $\bar{\psi}_r$. If we apply them, and then use the inductance matrix $[L_{ABCabc}^{(\theta_{er})}]$, Equation 10.4, we finally obtain

$$\begin{aligned}\bar{\psi}_s &= L_{sl}\bar{i}_s + \frac{3}{2}L_{0s}(\bar{i}_s + \bar{i}_r); \quad \bar{i}_r' = \bar{i}_r \times L_{sr}/L_{0s} \\ \bar{\psi}_r' &= L_{rl}'\bar{i}_r' + \frac{3}{2}L_{0s}(\bar{i}_s + \bar{i}_r'); \quad \bar{\psi}_r' = \bar{\psi}_r \times L_{sr}/L_{0r} = \bar{\psi}_r \times L_{0s}/L_{sr} \\ \bar{i}_r' &= \bar{i}_r \times L_{0r}/L_{sr}; \quad L_{rl}' = L_{rl}(L_{sr}/L_{0r})^2; \quad R_r' = R_r(L_{sr}/L_{0r})^2 \\ \psi_{0s} &= L_{sl}i_{0s}; \quad \psi_{0r} = L_{rl}i_{0r}; \quad V_{0s} = R_{0s}i_{0s} + L_{sl}\frac{di_{0s}}{dt}; \quad V_{0r}' = R_r'i_{0r}' + L_{rl}'\frac{di_{0r}'}{dt}\end{aligned}\quad (10.17)$$

Notice again that $L_{0r}/L_{sr} = L_{sr}/L_{0s}$.

In Equation 10.17 the rotor has been already reduced to the stator. Comparing Equation 10.14 with Equation 10.17 we see that

$$L_m = \frac{3}{2}L_{0s} \quad (10.18)$$

One can notice that L_m is the cyclic magnetization inductance of IM, as defined in Chapter 5. As the leakage inductances, L_{sl} , L_{rl}' remain the same through the 3-phase IM–dq model transition, so do the phase resistances, R_s , R_r' . The winding losses are

$$P_{\cos} = \frac{3}{2}R_s |\bar{i}_s|^2; \quad P_{\text{cor}} = \frac{3}{2}R_r' |\bar{i}_r'|^2 \quad (10.19)$$

As expected, the resistance and inductance relationships between the 3-phase IM and its dq (space phasor) model are very simple, because of the “blessing” of the sinus and cosinus in the inductance expressions. Note that the zero sequence (V_{0s} , i_{0s} , V_{0r}' , i_{0r}') in Equation 10.17 completes the IM–dq model equivalences.

10.4 Magnetic Saturation and Skin Effects in the dq Model

In Chapter 5, we introduced the unique magnetization curve of the IM as

$$\psi_m(i_m) = L_m(i_m) i_m; \quad \bar{i}_m = \bar{i}_s + \bar{i}_r' \quad (10.20)$$

where i_m is the total (magnetization) current:

$$i_m = \sqrt{i_{dm}^2 + i_{qm}^2}; \quad i_{dm} = i_d + i_{dr}'; \quad i_{qm} = i_q + i_{qr}' \quad (10.21)$$

The stator and rotor space phasors, $\bar{\psi}_s, \bar{\psi}_r'$, may be written with an airgap (main) flux space phasor, $\bar{\psi}_m$, as the common term:

$$\bar{\psi}_m = L_m \bar{i}_m; \quad \bar{\psi}_s = L_{sl} \bar{i}_s + \bar{\psi}_m; \quad \bar{\psi}_r' = L_{rl}' \bar{i}_r' + \bar{\psi}_m \quad (10.22)$$

The time derivative of the main flux, $\bar{\psi}_m$, is straightforward:

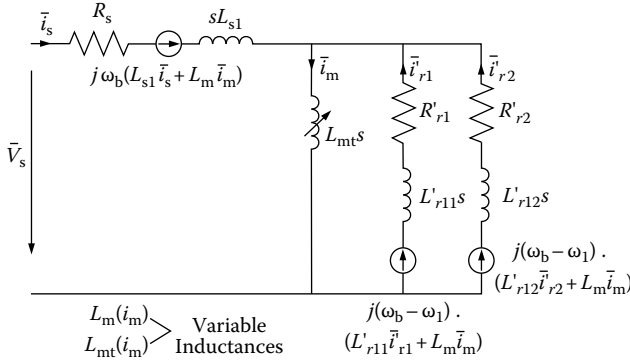
$$\begin{aligned} \frac{d\bar{\psi}_m}{dt} &= L_m \frac{d\bar{i}_m}{dt} + \frac{\partial L_m}{\partial i_m} i_m \frac{d\bar{i}_m}{dt} = \left(L_m + \frac{\partial L_m}{\partial i_m} i_m \right) \frac{d\bar{i}_m}{dt} \\ &= L_{mt} \frac{d\bar{i}_m}{dt}; \quad L_{mt} = \frac{d\psi_m}{di_m} \end{aligned} \quad (10.23)$$

Consequently, the transient magnetization inductance, L_{mt} , occurs in the main flux time derivative, but in the flux, Ψ_m , Equation 10.22, the normal inductance, L_m , holds. Both L_{mt} and L_m depend on the magnetization current.

Note: If only flux variables are used, with currents as dummy variables, L_{mt} is not required. Unfortunately, in IM control, often currents are controlled, and thus L_{mt} occurs inevitably.

The rotor skin effect can be simulated by multiple fictitious cages arranged in parallel. Let us consider two cages in the rotor:

$$\begin{aligned} \bar{V}_s &= R_s \bar{i}_s + L_{sl} \frac{d\bar{i}_s}{dt} + \frac{d\bar{\Psi}_m}{dt} + j\omega_b (L_{sl} \bar{i}_s + \bar{\Psi}_m) \\ 0 &= R_{r1}' \bar{i}_{r1}' + L_{rl1}' \frac{d\bar{i}_{r1}'}{dt} + \frac{d\bar{\Psi}_m}{dt} + j(\omega_b - \omega_r) (L_{rl1}' \bar{i}_{r1}' + \bar{\Psi}_m) \\ 0 &= R_{r2}' \bar{i}_{r2}' + L_{rl2}' \frac{d\bar{i}_{r2}'}{dt} + \frac{d\bar{\Psi}_m}{dt} + j(\omega_b - \omega_r) (L_{rl2}' \bar{i}_{r2}' + \bar{\Psi}_m) \\ \frac{d\bar{\Psi}_m}{dt} &= L_{mt}(i_m) \frac{d\bar{i}_m}{dt}; \quad \bar{i}_m = \bar{i}_s + \bar{i}_{r1}' + \bar{i}_{r2}'; \quad \bar{\Psi}_m = L_m(i_m) \bar{i}_m \\ T_e &= \frac{3}{2} p_1 \text{Re} \left(j \bar{\Psi}_s \bar{i}_s^* \right) = p_1 L_m \text{Re} \left[j \bar{i}_s \left(\bar{i}_{r1}' + \bar{i}_{r2}' \right) \right] \end{aligned} \quad (10.24)$$

**FIGURE 10.3**

Space phasor equivalent circuit of IM with saturation and skin effect.

With $d\bar{\Psi}_m/dt$ as the common voltage, Equation 10.24 suggests a circuit equivalent to that shown in Figure 10.3, though in reality there may be a leakage coupling between the two virtual or real cages. Its influence is lumped into the fictitious cage parameters. There are motion emfs in the stator and in the rotor as ω_b occurs generally. For $\omega_b = \omega_r$ (rotor coordinates), the rotor emf disappears while for $\omega_b = 0$ (stator coordinates) the emf disappears in the stator. For synchronous coordinates ($\omega_b = \omega$), the emf appears both in the stator and in the rotor.

Note: The fictitious cage parameters may be found from the standstill frequency response (described later in this chapter), as done for the SM.

10.5 Space Phasor Model Steady State: Cage and Wound Rotor IMs

For a steady state, let us consider sinusoidal voltages, constant speed, and load torque:

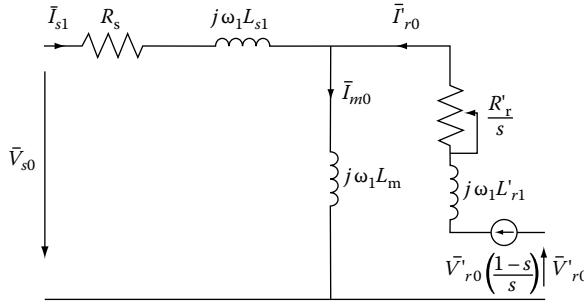
$$V_{A,B,C}^{(t)} = V\sqrt{2} \cos\left(\omega_1 t - (i-1) \frac{2\pi}{3}\right); \quad i = 1, 2, 3 \quad (10.25)$$

From the space phasor transformation (Equation 10.15), with $\theta_b = \omega_1 t$, for synchronous coordinates ($\omega_b = \omega_1$):

$$\bar{V}_s = \frac{3}{2} \left[V_A(t) + V_B(t) e^{j\frac{2\pi}{3}} + V_C(t) e^{-j\frac{2\pi}{3}} \right] e^{-j\omega_1 t} \quad (10.26)$$

Making use of Equation 10.25 in Equation 10.16 yields

$$\bar{V}_s = V\sqrt{2} = V_d + jV_q \Rightarrow V_d = V\sqrt{2}; \quad V_q = 0 \quad (10.27)$$

**FIGURE 10.4**

The space phasor circuit of the IM under a steady state.

The voltage space phasor is a dc quantity and so are the currents and flux linkages of the space phasor model under steady-state and synchronous coordinates, both in the stator and the rotor. With $d/dt = 0$ (in general, $\frac{d}{dt} = j(\omega_1 - \omega_b)$), Equation 10.13 becomes

$$\overline{V_{s0}} = R_s \overline{i_{s0}} + j\omega_1 \overline{\Psi_{s0}}; \quad \overline{\Psi_{s0}} = L_{sl} \overline{i_{s0}} + \overline{\Psi_{m0}}; \quad \overline{\Psi_{m0}} = L_m (\overline{i_{s0}} + \overline{i'_{r0}}) \quad (10.28)$$

$$\overline{V'_{r0}} = R'_r \overline{i'_{r0}} + jS\omega_1 \overline{\Psi_{r0}}; \quad \overline{\Psi'_{r0}} = L'_r \overline{i'_{r0}} + \overline{\Psi_{m0}}; \quad S = (\omega_1 - \omega_r)/\omega_1 \quad (10.29)$$

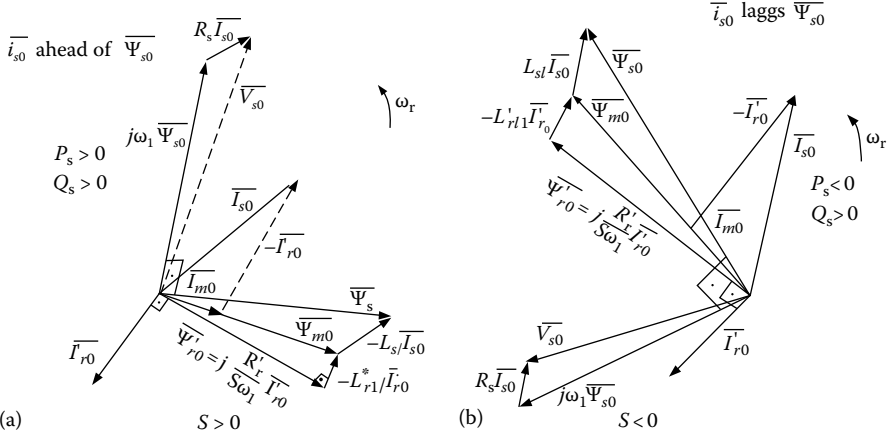
$$T_e = \frac{3}{2} p_1 \operatorname{Re} \left(j \overline{\Psi_{s0}} \overline{i'_{r0}}^* \right) \quad (10.30)$$

where S is the slip, already defined in Chapter 5.

The space phasor equivalent circuit for the steady state (as suggested by Equations 10.28 and 10.29) is shown in Figure 10.4. It is similar to that of one phase in terms of the phasors (at frequency ω_1) described in Chapter 5.

It should be noticed that the rotor voltage, $\overline{V'_{r0}}$, is also dc (in synchronous coordinates). But the phase (position) of $\overline{V'_{r0}}$ with respect to $\overline{V_{s0}}$ is variable with respect to the load in the motor or the generator operation. Let us now draw the space phasor diagrams of the IM, first for the cage rotor and then for the motor and generator operation modes (Figure 10.5a and b).

We should notice that at steady state, for the cage rotor IM, the rotor flux and current space phasors, $\overline{\Psi'_{r0}}, \overline{i'_{r0}}$, are orthogonal. The same is true even for transients, but with a constant rotor flux, $\Psi_r = \Psi_{r0} = \text{const}$. This is the so-called rotor flux orientation (vector) control. The slip is positive for motoring ($S > 0$) and negative for generating ($S < 0$), but in both cases the stator flux space phasor amplitude is largest, $\Psi_{s0} > \Psi_{m0} > \Psi_{r0}$, a clear sign that the machine magnetization arises from the power supply in both cases (from an external source, anyway). For the wound rotor (doubly fed) induction machine, we have to define first the no-load ideal speed, ω_{r0} (slip S_0), which

**FIGURE 10.5**

Space phasor diagram of cage rotor IM: (a) motor mode and (b) generator mode.

corresponds to zero rotor current in Equation 10.14:

$$\overline{V}_{r0} = jS_0\omega_1\overline{\Psi}'_{r0}; \quad \omega_{r0} = \omega_1(1 - S_0) \quad (10.31)$$

So the ideal no-load speed may occur for positive (subsynchronous) or negative (hypersynchronous) S_0 :

$$\begin{aligned} \omega_{r0} &< \omega_1 & \text{for } S_0 > 0 \\ \omega_{r0} &> \omega_1 & \text{for } S_0 < 0 \end{aligned} \quad (10.32)$$

The sign and relative value of S_0 depends on the amplitude and phase of $\overline{V}_{r0}$ (with respect to $\overline{\Psi}'_{r0}$ or $\overline{V}_{s0}$). Both motoring and generating is possible around S_0 . It is feasible to magnetize the machine from the rotor or from the stator sources or from both. The active and reactive powers of the stator (Equation 10.18) and the rotor are

$$\begin{aligned} P_s &= \frac{3}{2} \text{Re}(\overline{V}_{s0}\overline{i}_{s0}^*) = \frac{3}{2} (V_{d0}i_{d0} + V_{q0}i_{q0}) \\ Q_s &= \frac{3}{2} \text{Im}(\overline{V}_{s0}\overline{i}_{s0}^*) = -\frac{3}{2} (V_{d0}i_{q0} - V_{q0}i_{d0}) \\ P_r &= \frac{3}{2} \text{Re}(\overline{V}_{r0}\overline{i}'_{r0}) = \frac{3}{2} (V'_{dr0}i'_{qr0} + V'_{qr0}i'_{dr0}) \\ (Q_r)'_{S\omega_1} &= \frac{3}{2} \text{Im}(\overline{V}_{r0}\overline{i}'_{r0}) = -\frac{3}{2} (V'_{dr0}i'_{qr0} - V'_{qr0}i'_{dr0}) \end{aligned}$$

Q_r^r is considered at the slip frequency ($S\omega_1$) in synchronous coordinates; in other words, Q_r^r/S is the reactive power of the rotor source seen at the stator frequency. So the reactive power produced at the $S\omega_1$ frequency is “amplified” $\frac{1}{S}$ times when observed at the ω_1 frequency; this is based on the fact that the corresponding magnetic energy is conserved.

Let us consider the case of $S < 0$ (supersynchronous) operation at the unity rotor power factor in the motor and generator modes (Figure 10.6) based (still) on Equations 10.28 through 10.30.

The unity rotor power factor means a minimum kVA of the rotor power source; it also means that the machine magnetization is done from the stator ($\Psi_{s0} > \Psi'_{r0}$). For the unity power factor in the stator, the magnetization arises from the rotor power source, and thus $\Psi'_{r0} > \Psi_{s0}$.

Example 10.1 Space-Phasor Steady State of the Cage Rotor IM

Let us consider a 3-phase cage rotor IM with the following data: $P_n = 1.5$ kW, $\eta_n = 0.9$, $\cos \Psi_n = 0.9$, $f_1 = 60$ Hz, $V_{s0} = 120\sqrt{2}$ (pu), $r_s = 0.04$ (pu), $r'_r = 0.03$ (pu), $l_{sl} = l'_{rl} = 0.1$ (pu), $2p_1 = 4$ poles, $l_m = 3$ (pu), and operating at a frequency, $f_1 = 60$ Hz, and, slip $S = 0.03$. Calculate R_s , i_n , L_{sl} , L'_{rl} , L_m in Henry, $\bar{I}_{s0}$, $\bar{I}'_{r0}$, $\bar{\Psi}'_{r0}$, $\bar{\Psi}_{s0}$, and T_e (torque), and speed, ω_r (and n in rps).

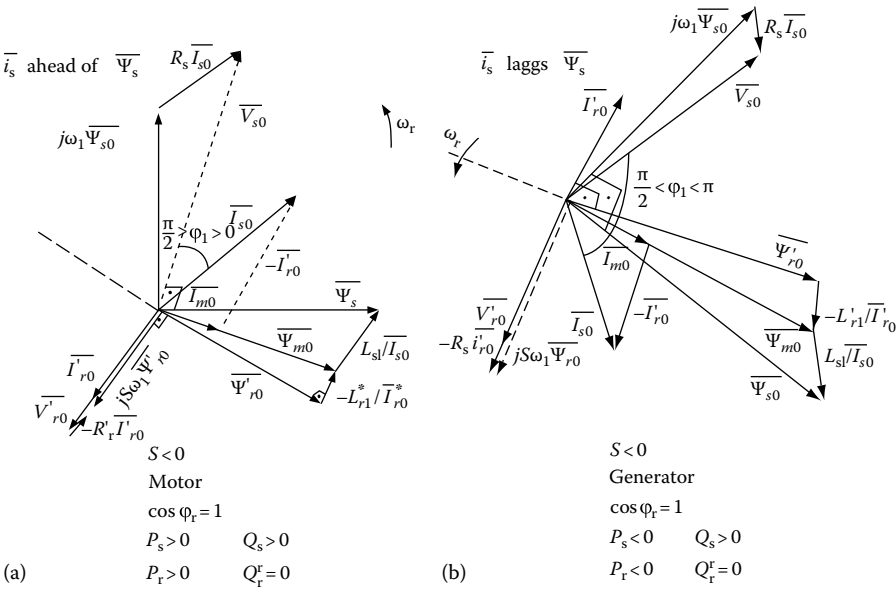


FIGURE 10.6 Space phasor diagram of doubly fed IM at $S < 0$: (a) motoring and (b) generating at the unity rotor power factor.

Solution:

The pu norm reactance, X_n , is (as for the SM)

$$X_n = \frac{V_{nph}\sqrt{2}}{I_{nph}\sqrt{2}}; \quad I_{nph} = \frac{P_n}{3V_{nph}\eta_n \cos \phi_n} = \frac{1500}{3 \times 120 \times 0.9 \times 0.9} = 5.144 \text{ A}$$

$$X_n = \frac{120\sqrt{2}}{5.144\sqrt{2}} = 23.328 \Omega; \quad R_s = r_s \times X_n = 0.04 \times 23.328 = 0.933 \Omega,$$

$$\begin{aligned} R'_r &= 0.03 \times 23.328 = 0.6998 \Omega, L_{sl} = L'_{rl} = 0.1 \times \frac{X_n}{2\pi f_1} \\ &= \frac{0.1 \times 23.328}{2\pi \times 60} = 6.19 \text{ mH} \end{aligned}$$

$$L_m = 3 \times \frac{X_n}{2\pi f_1} = 0.1857 \text{ H}$$

From Equations 10.28 and 10.29

$$\begin{aligned} 120\sqrt{2} &= [0.933 + j2\pi 60 (0.00619 + 0.1857)] \overline{i_{s0}} + j2\pi 60 \times 0.1857 \overline{i'_{r0}} \\ 0 &= [0.6998 + j0.03 \times 2\pi 60 (0.00619 + 0.1857)] \overline{i'_{r0}} + j \times 0.03 \\ &\quad \times 2\pi 60 \times 0.1857 \times \overline{i_{s0}} \end{aligned}$$

Both $\overline{i_{s0}}$ and $\overline{i'_{r0}}$ can be easily calculated from the above equations:

$$\overline{i'_{r0}} = -6.5596 + j1.593$$

$$\overline{i_{s0}} = 6.393 - j3.362$$

$$\overline{V_{s0}} = 120\sqrt{2}$$

So, the flux linkages, $\overline{\Psi}_{s0}$ and $\overline{\Psi}'_{r0}$, are

$$\overline{\Psi}_{s0} = \frac{\overline{V_{s0}} - R_s \overline{i_{s0}}}{j\omega_1} = \frac{120\sqrt{2} - 0.933 (6.393 - j3.362)}{j2\pi 60}$$

$$\overline{\Psi}'_{r0} = \frac{-R'_r \overline{i'_{r0}}}{jS\omega_1} = \frac{-0.6998 \times (-6.5596 + j1.159)}{j \times 0.03 \times 2\pi \times 60}$$

$$\overline{\Psi}_{s0} = -j0.4332 + 0.008$$

$$\overline{\Psi}'_{r0} = -j0.40608 - 0.0717$$

It is evident that $\Psi_{s0} > \Psi'_{r0}$ (in amplitude) and is leading (motoring).

The torque, T_e , is

$$\begin{aligned} T_e &= \frac{3}{2} p_1 \operatorname{Re} \left(j \bar{\Psi}_{s0} \vec{i}_{s0}^* \right) = \frac{3}{2} \times 2 \times \operatorname{Re} [(0.008 - j0.4332) (6.343 + j3.362)] \\ &= 4.522 \text{ Nm} \end{aligned}$$

Example 10.2 Doubly Fed IM Steady State

Let us consider a doubly fed IM with the following parameters: $R_s = R'_r = 0.018 \, \Omega$, $X_{sl} = X_{rl} = 0.18 \, \Omega$, $X_m = 14.4 \, \Omega$ at $f_n = 50 \text{ Hz}$, $2p_1 = 4$, $V_{sn \text{ line}} = 6000 \text{ V}$, $I_{1n/\text{phase}} = 1204 \text{ A (RMS)}$, and having a star connection. The ratio of rotor to stator turns is $K_{rs} = 4/1$, while the slip is $S = -0.25$. For $\cos \Psi_s = 1$ (stator unity power factor), calculate the rotor active and reactive powers, P_r^r and Q_r^r and the total active delivered power by the generator $P_{\text{gen}} = P_s + P_r^r$.

Solution:

With Equation 10.28 at the unity stator power factor, $V_{s0} = R_s i_{s0} + j\omega_1 \bar{\Psi}_{s0}$, and i_{s0} is negative because the generator mode is considered here.

$$\begin{aligned} \frac{6000}{\sqrt{2}} \times \sqrt{2} &= 0.018 \times (-1204) + j2\pi \times 50 \bar{\Psi}_{s0} \\ \Psi_{s0} &= -j15.64 \text{ Wb} \end{aligned}$$

But,

$$\begin{aligned} \bar{\Psi}'_{r0} &= \bar{\Psi}_{s0} \frac{L_{rl} + L_m}{L_m} - L_{sc} i_{s0} = -j15.64 \times \frac{(14.4 + 0.18)}{14.4} - \frac{0.36}{314} \times (-1204) \\ &= -j15.8355 + 1.380 \end{aligned}$$

Hence, the rotor current is

$$\begin{aligned} I'_{r0} &= \frac{\bar{\Psi}'_{r0} - L_m i_s}{L_m + L'_{rl}} = \frac{-j15.835 + 1.380 - 14.4 \times (-1024) / 314}{(14.58/314)} \\ &= \frac{-j15.835 + 56.49}{4.6433 \times 10^{-2}} = -j341.03 + 1219.29 \\ I'_{r0} &= 1266 \text{ A} > I_{s0} \end{aligned}$$

because the machine is magnetized from the rotor.

The rotor voltage is obtained from Equation 10.19:

$$\begin{aligned}\bar{V}'_{r0} &= R'_r \bar{I}'_{r0} + jS\omega_1 \bar{\Psi}'_{r0} \\ &= 0.018 \times (-j341.28 + 1219) + j(-0.25) 2\pi 50 \times (-j15.8255 + 1.380) \\ &= -1221 - j114.468\end{aligned}$$

The stator active power, P_s ($\cos \Psi_s = 1$) is

$$\begin{aligned}P_s &= \frac{3}{2} V_{s0} i_{s0} = \frac{3}{2} \times \frac{6000\sqrt{2}}{\sqrt{3}} (-1024) = -8.8316 \times 10^6 \text{ W} = -8.8316 \text{ MW} \\ S_r^r &= P_r^r + jQ_r^r = \frac{3}{2} (\bar{V}'_{r0} \bar{i}'_{r0}^*) \\ &= \frac{3}{2} (-1221 - j114.246) (1219.29 + j341.03) \\ &= -2.174 \text{ MW} + j0.55566 \text{ MVAR}\end{aligned}$$

So, the total delivered active power of the doubly fed IG, P_{gen} , is

$$P_{\text{gen}} = P_s + P_r^r = (-88316 - 2174) \text{ MW} \approx -11 \text{ MW}$$

The reactive power at the stator would be

$$Q_r = Q_r^r / |S| = \frac{0.55566}{0.25} \text{ MVAR} = 2.2226419 \text{ MVAR}$$

But for the design of the rotor side converter Q_r^r and P_r^r are paramount. As $K_{rs} = L_{0r}/L_{sr} = 4/1$, it means that the actual rotor voltage V_{r0} is four times larger than V'_{r0} while the current I_{r0} is four times smaller than I'_{r0} . The DFIG is dominant in modern wind generation systems at variable speeds.

10.6 Electromagnetic Transients

As for synchronous machines, there are fast transients in cage IMs during which the motor speed may be considered constant. Such transients are called electromagnetic transients.

For fast transients, it seems appropriate to use rotor coordinates ($\omega_b = \omega_r$) in the dq (space phasor) model. Let us consider the IM with a

dual rotor cage (to account for the rotor skin effect). There are no motion emfs in the rotor and, for pu equations, d/dt is replaced by s/ω_{10} , as for the SMs:

$$\begin{aligned}\overline{V_s} &= \overline{i_s} r_s + \frac{s}{\omega_{10}} \overline{\Psi_s} + j\omega_{r0} \overline{\Psi_s}; & \overline{\Psi_s} &= l_{sl} \overline{i_{sl}} + l_m (\overline{i_s} + \overline{i'_{r1}} + \overline{i'_{r2}}) \\ 0 &= \overline{i'_{r1}} r'_{r1} + \frac{s}{\omega_{10}} \overline{\Psi'_{r1}}; & \overline{\Psi'_{r1}} &= l'_{r1l} \overline{i'_{r1}} + l_m (\overline{i_s} + \overline{i'_{r1}} + \overline{i'_{r2}}) \\ 0 &= \overline{i'_{r2}} r'_{r2} + \frac{s}{\omega_{10}} \overline{\Psi'_{r2}}; & \overline{\Psi'_{r2}} &= l'_{r2l} \overline{i'_{r2}} + l_m (\overline{i_s} + \overline{i'_{r1}} + \overline{i'_{r2}}) \\ T_e &= Re \left[j l_m \left(\overline{i_s} \times \left(\overline{i'_{r1}}^* + \overline{i'_{r2}}^* \right) \right) \right]\end{aligned}\quad (10.33)$$

where ω_{10} is the norm angular frequency in radians per second.

We may eliminate $\overline{i'_{r1}}$ and $\overline{i'_{r2}}$ from Equation 10.33 and obtain the stator flux, $\overline{\Psi_s}$, in the Laplace form:

$$\begin{aligned}\overline{i_s} &= l(s) \overline{i_s}(s) \\ l(s) &= (l_{sl} + l_{ml}) \frac{(1 + s\tau'')(1 + s\tau')}{(1 + s\tau'_0)(1 + s\tau'_0)}\end{aligned}\quad (10.34)$$

The operational inductance, $l(s)$, is similar to that of an SM with two rotor circuits in parallel, with the time constants in seconds:

$$\begin{aligned}\tau'' &= \frac{1}{r'_{r1} \omega_{10}} \left(l'_{r1l} + \frac{l_m l'_{r2l} l_{sl}}{l_m l'_{r2l} + l_m l_{sl} + l'_{r2l} l_{sl}} \right) \\ \tau'_0 &= \frac{1}{r'_{r1} \omega_{10}} \left(l'_{r1l} + \frac{l'_{r2l} l_m}{l'_{r2l} + l_m} \right) \\ \tau' &= \frac{1}{r'_{r2} \omega_{10}} \left(l'_{r2l} + \frac{l_m l_{sl}}{l_m + l_{sl}} \right) \\ \tau'_0 &= \frac{1}{r'_{r2} \omega_{10}} (l'_{r2l} + l_m)\end{aligned}\quad (10.35)$$

Again, subtransient, l'' , transient, l' , and synchronous, l_s , inductances are defined:

$$\begin{aligned}l'' &= \lim_{\substack{s \rightarrow \infty \\ (t \rightarrow 0)}} = (l_{sl} + l_m) \frac{\tau' \tau''}{\tau'_0 \tau'_0} \\ l' &= \lim_{\substack{s \rightarrow \infty \\ \tau'' = \tau'_0 = 0}} = (l_{sl} + l_m) \frac{\tau'}{\tau'_0} \\ l_s &= \lim_{\substack{s \rightarrow 0 \\ (t \rightarrow \infty)}} (l(1)) = l_{sl} + l_m\end{aligned}\quad (10.36)$$

10.7 Three-Phase Sudden Short Circuit/Lab 10.1

A 3-phase sudden short circuit may occur accidentally at large IMs terminals and produce significant faults in the local power grid.

Alternatively, as for the SM, the sudden short circuit may be produced right after disconnection from the power grid (less than 1 ms later) and then, with the stator current acquired, machine parameters can be identified by curve fitting. We maintain P.U. variables as this case is of primary interest to power system analysis.

To simplify the mathematics, let us consider that the machine runs at ideal no load ($\omega_r = \omega_{r0} = \omega_1$) when the sudden short circuit occurs.

So, the initial current, i_{s0} , is

$$\bar{i}_{s0} = \frac{\bar{v}_{s0}}{r_s + j\omega_1 l_s} \quad (10.37)$$

Let us consider the machine phase voltages as

$$V_{ABC} = v\sqrt{2} \cos \left(\omega_1 t + \Psi_0 - (i-1) \frac{2\pi}{3} \right); \quad i = 1, 2, 3 \quad (10.38)$$

$$\bar{v}_{s0} = \frac{2}{3} \frac{1}{V\sqrt{2}} \left(V_A + V_B e^{j\frac{2\pi}{3}} + V_C e^{-j\frac{2\pi}{3}} \right) e^{-j\omega_1 t} = v_{s0} e^{j\Psi_0} \quad (10.39)$$

To short-circuit the machine, $\bar{v}_s = -\bar{v}_{s0}$ has to be applied. But the machine voltage equation (from Equations 10.34 and 10.35) is

$$\bar{i}'_s(s) = \frac{\bar{v}_s(s)}{r_s + \left(\frac{s}{\omega_{10}} + j\omega_1 \right) l(s)} \quad (10.40)$$

with

$$\bar{v}_s(s) = -\frac{v_{s0} e^{j\Psi_0}}{\left(\frac{s}{\omega_{10}} \right)} \quad (10.41)$$

From Equations 10.40 and 10.41

$$\bar{i}'_s(s) = \frac{-v_{s0} e^{j\Psi_0}}{\frac{s}{\omega_{10}} \left(\frac{r_s}{l(s)} + \frac{s}{\omega_{10}} + j\omega_1 \right) l(s)} \quad (10.42)$$

Approximating

$$\tau_a = \frac{l(s)}{\omega_{10} r_s} \approx \frac{l''}{\omega_{10} r_s} \quad (10.43)$$

yields

$$i'_s(s) \approx -v_{s0} e^{j\Psi_0} \left[\frac{\omega_{10}}{\left(\frac{s}{\omega_{10}} + \frac{1}{\tau_a \omega_{10}} + j\omega_1\right) \left(s + \frac{1}{\tau'}\right)} \left(\frac{1}{l'} - \frac{1}{l_s}\right) + \frac{1}{\frac{s}{\omega_{10}} \left(\frac{s}{\omega_1} + \frac{1}{\tau_a \omega_{10}} + j\omega_1\right)} \frac{1}{l_s} + \frac{\omega_{10}}{\left(\frac{s}{\omega_{10}} + \frac{1}{\tau_a \omega_{10}} + j\omega_1\right) \left(s + \frac{1}{\tau''}\right)} \left(\frac{1}{l''} - \frac{1}{l'_s}\right) \right] \quad (10.44)$$

Finally, the resultant current space vector, $\bar{i}_s(t)$, is

$$\bar{i}_s(t) = \bar{i}_{s0} + \bar{i}'_s(t) = -\frac{v_{s0} e^{j\Psi_0}}{\omega_1} \left[\left(e^{-\left(\frac{1}{\tau_a \omega_{10}} + j\omega_1\right)t\omega_{10}} - e^{-\frac{t}{\tau'}} \right) \left(\frac{1}{l'} - \frac{1}{l_s}\right) + \left(e^{-\left(\frac{1}{\tau_a \omega_{10}} + j\omega_1\right)t\omega_{10}} - e^{-\frac{t}{\tau''}} \right) \left(\frac{1}{l''} - \frac{1}{l'_s}\right) + \left(e^{-\left(\frac{1}{\tau_a \omega_{10}} + j\omega_1\right)t\omega_{10}} - 1 \right) \frac{1}{l_s} \right] \quad (10.45)$$

The phase A current, $i_A(t)$, becomes

$$i_A(t) = \operatorname{Re} \left(\bar{i}_s(t) e^{j\omega_1 \omega_{10} t} \right) = \frac{v_{s0}}{\omega_1 l''} e^{-\frac{t}{\tau_a}} \sin \Psi_0 - \left[\left(\frac{1}{\omega_1 l'} - \frac{1}{\omega_1 l_s} \right) e^{-\frac{t}{\tau'}} + \left(\frac{1}{\omega_1 l''} - \frac{1}{\omega_1 l'_s} \right) e^{-\frac{t}{\tau''}} \right] v_{s0} \sin(\omega_1 \omega_{10} t + \Psi_0) \quad (10.46)$$

A few remarks are in order:

- ω_{10} is the norm in radians per second but ω_1 is the angular frequency in (pu) with respect to ω_{10} .
- Short-circuit current shows a large but fast decaying, nonperiodic component and a sinusoidal component whose amplitude attenuates with two time constants, τ' and τ'' .
- Maximum peak value of $i_A(t)$ occurs apparently for $\Psi_0 = \pi/2$.
- By curve fitting, with known $i_A(t)$, τ_a , τ' , τ'' , l'' , l' , and l_s , can be identified. The similitude with the SM is clearly visible but the steady-state short-circuit current is, as expected, zero, for the IM.

Example 10.3 Sudden Short Circuit

Calculate and represent graphically (in pu) the $\bar{i}_s(I_d, I_q)$ and $i_A(t)$ for the 3-phase sudden short circuit of the IM with the following data: $v_{s0} = 1$ pu, $l'' = 0.20$ (pu), $l' = 0.35$ (pu), $l_s = 4.0$ (pu), $\tau_a = 0.05$ s, $\tau' = 0.1$ s, $\tau'' = 0.05$ s,

$\Psi_0 = \pi/6$, $\omega_1 = 1.0$ (pu), and $\omega_{10} = 2\pi 50$ rad/s. Also derive the expression of torque during the short circuit.

Solution:

We go straight to Equation 10.46 to obtain

$$i_A(t) = \frac{1 \times e^{-t/0.05}}{1 \times 0.2} \cos \frac{\pi}{6} - \left[\left(\frac{1}{1 \times 0.35} - \frac{1}{1 \times 4.0} \right) e^{-t/0.1} + \left(\frac{1}{1 \times 0.2} - \frac{1}{1 \times 0.35} \right) e^{-t/0.03} \right] \cdot 1 \times \sin \left(2\pi 50t + \frac{\pi}{6} \right) \quad (10.47)$$

The space phasor, $\vec{i}_s$, in synchronous coordinates (Equation 10.45) is

$$\vec{i}_s = i_d + j i_q$$

Its representation as $I_d(I_q)$ is shown in Figure 10.7a $i_A(t)$ in Figure 10.7b.

To calculate the flux space phasor, $\Psi_s(s)$, we use Equation 10.34

$$\bar{\Psi}_s(s) = l_s \vec{i}_{s0} + \frac{(-v_{s0}) e^{j\Psi_0}}{\frac{s}{\omega_{10}} \left(\frac{s}{\omega_{10}} + \frac{1}{\tau_a \omega_{10}} + j\omega_1 \right)} \quad (10.48)$$

and thus

$$\bar{\Psi}_s(t) \approx l_s \vec{i}_{s0} - \frac{v_{s0} e^{j\Psi_0}}{\frac{1}{\tau_a \omega_{10}} + j\omega_1} \left(-e^{-\left(\frac{1}{\tau_a \omega_{10}} + j\omega_1 \right) t \omega_{10}} + 1 \right) \quad (10.49)$$

The flux transients are described by only one (small) time constant, τ_a .

The torque, t_e , during the short-circuit period is

$$t_e(t) = \text{Re} \left(j \bar{\Psi}_s(t) \vec{i}_s^*(t) \right) \quad (10.50)$$

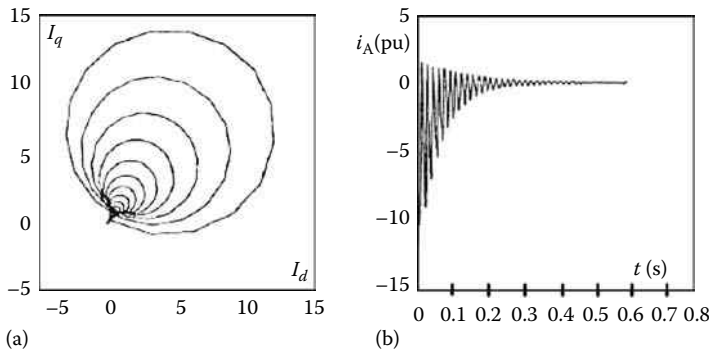


FIGURE 10.7

Sudden short-circuit current (a) $I_d(I_q)$ and (b) $i_A(t)$ in pu.

with $\bar{i}_s(t)$ obtained from Equation 10.45. It turns out that the peak torque can reach values of 5–6 (pu). Finally, after the transient process, both the flux and the torque go to zero, as expected.

10.7.1 Transient Current at Zero Speed

A large inertia-load IM may be considered at standstill in the first two to four voltage periods after connection to the power grid. In this case, $\omega_{r0} = \omega_b = 0$, and $\dot{i}_{s0} = 0$ (at $t = 0$). According to Equations 10.34 and 10.41, for $\omega_r = 0$

$$i_s(s) = \frac{\bar{v}_s(s)}{r_s + \frac{s}{\omega_{10}} l(s)}; \quad \Psi_s(s) = l(s) i_s(s); \quad t_e(t) = \text{Re}(\bar{\Psi}_s t_s^*) \quad (10.51)$$

with

$$\bar{v}_s(t) = v_{s0} e^{j(\Psi_0 + \omega_{10} t)} \quad (10.52)$$

During such a process, the current peak value can surpass 10 (pu). For such high values of current, the leakage flux paths saturate, and thus the peak current values increase further. Consequently, in industry, either leakage flux path magnetic saturation is considered (by reducing l_{sl} , l'_{rl1} , l_{rl2}) or experiments at full voltage are required to calculate (measure) correctly the peak starting current and torque, which are so important in safety-critical starting applications (such as in heat-exchanger pump motors in nuclear power plants).

10.8 Small-Deviation Electromechanical Transients

In induction motors, quite frequently, there are small variations in load torque, supply frequency, and voltage amplitude. Also, linearization of the IM model is required for control design.

For linearization, we consider the dq model (Equations 10.9 through 10.11) of a single-cage IM with

$$i_d = i_{d0} + \Delta i_d; \quad i_q = i_{q0} + \Delta i_q \quad (10.53)$$

to obtain

$$|A|s|\Delta X| + B|\Delta X| = |C||\Delta V| + |D|\Delta T_{\text{load}} + |E|\Delta\omega_1 \quad (10.54)$$

with

$$\begin{aligned}
 |\Delta X| &= |\Delta i_d \quad \Delta i_q \quad \Delta i'_{dr} \quad \Delta i'_{qr} \quad \Delta \omega_r|^T \\
 |\Delta V| &= |\Delta V_d \quad \Delta V_q \quad 0 \quad 0 \quad 0|^T \\
 C &= [1 \quad 1 \quad 0 \quad 0 \quad 0] \\
 D &= [0 \quad 0 \quad 0 \quad 0 \quad -1]^T \\
 E &= \begin{bmatrix} L_s I_{q0} + L_m I_{qr0} - (L_s I_{d0} + L_m I_{dr0}) & L_m I_{q0} + L'_r I_{qr0} \\ - (L_m I_{dr0} + L'_r I_{dr0}) & 0 \end{bmatrix}^T
 \end{aligned} \tag{10.55}$$

$$|A| = \begin{vmatrix} L_s & 0 & L_m & 0 & 0 \\ 0 & L_s & 0 & L_m & 0 \\ L_m & 0 & L'_r & 0 & 0 \\ 0 & L_m & 0 & L'_r & 0 \\ 0 & 0 & 0 & 0 & -\frac{J}{p_1} \end{vmatrix} \tag{10.56}$$

$$|B| = \begin{vmatrix} R_s & -\omega_{10} L_s & 0 & -\omega_{10} L_m & 0 \\ \omega_{10} L_s & R_s & \omega_{10} L_m & 0 & 0 \\ 0 & -\omega_{20} L_m & R'_r & -\omega_{20} L'_r & - (L_m i_{q0} + L'_r i'_{qr0}) \\ \omega_{20} L_m & 0 & \omega_{20} L'_r & R'_r & - (L_m i_{d0} + L'_r i'_{dr0}) \\ -\frac{3}{2} p_1 L_m i'_{qr0} & \frac{3}{2} p_1 L_m i'_{dr0} & \frac{3}{2} p_1 L_m i_{d0} & -\frac{3}{2} p_1 L_m i_{q0} & 0 \end{vmatrix} \tag{10.57}$$

where

ω_{10} is the initial frequency of the stator (synchronous coordinates)

$\omega_{20} = \omega_{10} - \omega_{r0}$ is the initial rotor slip frequency

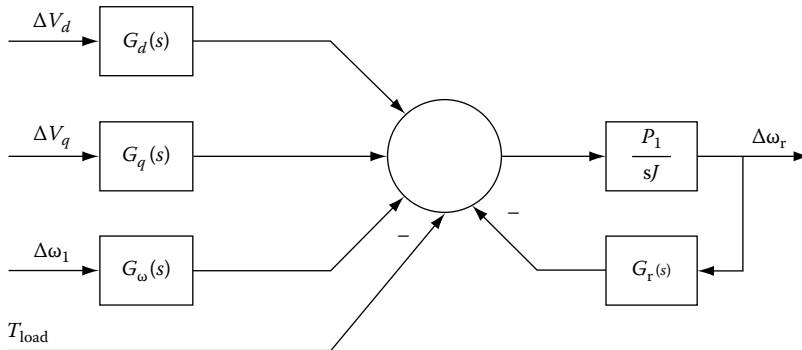
ω_{r0} initial speed

$L_s = L_{sl} + L_m$ and $L'_r = L'_{rl} + L_m$.

For the initial, steady state is the situation; the dq model equation yields

$$\begin{vmatrix} V_{d0} \\ V_{q0} \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} R_s & -\omega_{10} L_s & 0 & -\omega_{10} L_m \\ \omega_{10} L_s & R_s & \omega_{10} L_m & 0 \\ 0 & -\omega_{20} L_m & R'_r & -\omega_{20} L'_r \\ \omega_{20} L_m & 0 & \omega_{20} L'_r & R'_r \end{vmatrix} \tag{10.58}$$

Equations 10.55 refers to a fifth-order system with ΔV_d , ΔV_q , $\Delta \omega_1$, and ΔT_{load} as the inputs and Δi_d , Δi_q , $\Delta i'_{dr}$, $\Delta i'_{qr}$, and $\Delta \omega_r$ as the variables. This can be treated with the methods typical for the linear systems.

**FIGURE 10.8**

IM small deviation speed transfer function.

If we manage to eliminate the current variables from Equation 10.55, we end up with a small perturbation speed transfer function of the form

$$\frac{J}{p_1} s \Delta \omega_r = -G_r(s) \Delta \omega_r + G_d(s) \Delta V_d + G_q(s) \Delta V_q - \Delta T_{\text{load}} + G_\omega(s) \Delta \omega_1 \quad (10.59)$$

A structural diagram, as in Figure 10.8, illustrates Equation 10.59.

To apply the structural diagram to the actual machine, we have to add

$$V_d + jV_q = \frac{2}{3} \left(V_A + V_B e^{j\frac{2\pi}{3}} + V_C e^{-j\frac{2\pi}{3}} \right) e^{-j\theta_b}$$

$$\frac{d\theta_b}{dt} = \omega_1 = \omega_{10} + \Delta \omega_1 \quad (10.60)$$

It should be noticed that even after linearization the fifth (sixth)-order of the system does not permit simple analytical solutions of transients; however, the stability analysis is facilitated by the possibility of using the linear system theories heritage (eigenvalue theory, etc.).

10.9 Large-Deviation Electromechanical Transients/Lab 10.2

Large voltage sags, starting by direct connection to the grid, large load-torque perturbations, accidental disconnection and reconnection to the power grid, and load dumping for autonomous generator mode represent large deviation transients; for these cases, the complete dq model should be used.

It is preferable to use flux-linkage variables for the numerical solution of the dq model equations; in stator coordinates, we have

$$\begin{aligned}
 \frac{d\Psi_d}{dt} &= V_d - R_s i_d; \quad i_d = C_{11}\Psi_d - C_{12}\Psi'_{dr}; \quad C_{11} = \frac{L'_r}{\sigma L_m^2} \\
 \frac{d\Psi_q}{dt} &= V_q - R_s i_q; \quad i_q = C_{11}\Psi_q - C_{12}\Psi'_{qr}; \quad C_{12} = \frac{1}{\sigma L_m} \\
 \frac{d\Psi'_{dr}}{dt} &= -R_r i'_{dr} - \omega_r \Psi'_{qr}; \quad i'_{dr} = -C_{12}\Psi_d + C_{22}\Psi'_{dr}; \quad C_{22} = \frac{L_s}{\sigma L_m^2} \quad (10.61) \\
 \frac{d\Psi'_{qr}}{dt} &= -R_r i'_{qr} + \omega_r \Psi'_{dr}; \quad i'_{qr} = -C_{12}\Psi_q + C_{22}\Psi'_{qr}; \quad \sigma = \frac{L_s L_r}{L_m^2} - 1 > 0 \\
 \frac{d\omega_r}{dt} &= \frac{p_1}{J} [T_e - T_{\text{load}}]; \quad T_e = \frac{3}{2} p_1 (\Psi_d i_q - \Psi_q i_d)
 \end{aligned}$$

As long as the stator phase voltages are given as a function of time, in stator coordinates

$$\begin{aligned}
 V_{A,B,C}^{(t)} &= V_1 \sqrt{2} \cos \left(\omega_1 t + \varphi_0 - (i-1) \frac{2\pi}{3} \right) \\
 V_d + jV_q &= \frac{2}{3} \left(V_A + V_B e^{j\frac{2\pi}{3}} + V_C e^{-j\frac{2\pi}{3}} \right) \\
 &= V_1 \sqrt{2} \cos(\omega_1 t + \varphi_0) - jV_1 \sqrt{2} \sin(\omega_1 t + \varphi_0) \quad (10.62)
 \end{aligned}$$

Now the frequency, ω_1 , might also be variable, together with the voltage amplitude, V_1 , as long as the voltages are symmetric (balanced). An additional rotor cage means two more equations in Equation 10.62, to account for the skin effect in the rotor cage. Numerical methods, such as the Runge–Kutta–Gill method, are now available as toolboxes in MATLAB®/Simulink® that can be used to solve Equation 10.61.

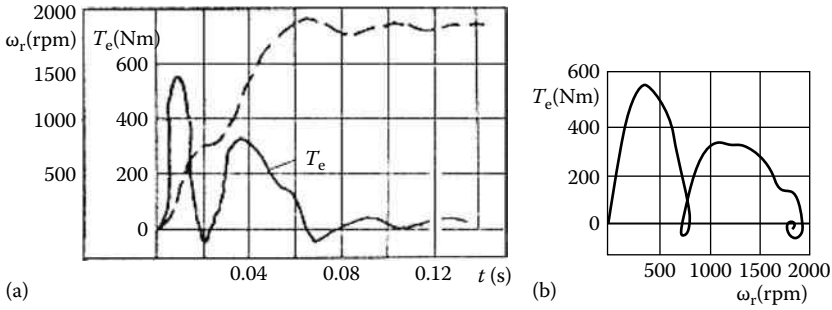
Example 10.4 Starting Transients

Let us consider an IM with the following data: $R_s = 0.063 \Omega$, $R'_r = 0.083 \Omega$, $p_1 = 2$ pole pairs, $L_m = 29 \text{ mH}$, $L_s = L'_r = 30.4 \text{ mH}$, $T_{\text{load}} = 6 \text{ Nm}$ (low torque load), $\frac{J}{p_1} = 0.06 \text{ kg m}^2$, $V_1 = 180/\sqrt{2} \text{ V(RMS)}$, $f_1 = 60 \text{ Hz}$. Find the speed and torque versus speed during starting by direct connection to the power source.

Solution:

The above model with $\varphi_0 = 0$ is solved numerically for T_e , ω_r , Ψ_d , and Ψ_q , and then for i_d and i_q . Finally, to find the phase current (in stator coordinates)

$$i_A(t) = i_d(t) \quad (10.63)$$

**FIGURE 10.9**

(a) IM speed and torque build up and (b) transient torque vs. speed

The results are shown in Figure 10.9a and b. Observe that all the initial values of the variables are zero in this case.

Due to the reduced inertia and small load, transients in torque and speed are significant. The difference of the transient torque/speed curve (Figure 10.9.b) from the smooth, single maximum point, steady-state, torque/speed curve of IM is an indication of fast mechanical and torque transients. Speed oscillations are also typical for such cases.

10.10 Reduced-Order dq Model in Multimachine Transients

When a large group of IMs are simulated for transients, the calculation of the fifth order of the dq model may be too time consuming. For synchronous coordinates ($\omega_b = \omega_1$), it is then tempting to neglect the stator transients:

$$\begin{aligned} \frac{d\Psi_d}{dt} &= \frac{d\Psi_q}{dt} = 0 \\ V_d &= R_s i_d - \omega_1 \Psi_q; \quad V_q = R_s i_q + \omega_1 \Psi_d \\ \frac{d\Psi'_{dr}}{dt} &= -R_r i'_{dr} + (\omega_1 - \omega_r) \Psi'_{qr}; \quad \frac{d\Psi'_{qr}}{dt} = -R_r i'_{qr} - (\omega_1 - \omega_r) \Psi'_{dr} \quad (10.64) \\ \frac{d\omega_r}{dt} &= \frac{p_1}{J} \left[\frac{3}{2} p_1 (\Psi_d i_q - \Psi_q i_d) - T_{load} \right] \end{aligned}$$

A third-order system has been obtained. However, the fast transients due to flux and current amplitude variations are ignored, even in torque. But, the speed response tends to be usable. Other simplified (modified first order) models, adequate for small or large IMs, have been proposed [1,2], but they have to be used cautiously. A good part of industrial electric loads is represented by induction motors, from a few kilowatts to 20–30 MW/unit. A local (industrial) transformer supplies, in general, a group of such IMs,

through pertinent power switches; random load perturbations and turning on and off may occur together with bus transfer. During the time interval between the turn off from one power grid and turn on of the emergency power grid, the group of IMs, with a common reactance feeder and with a parallel capacitive bank (for power factor compensation), exhibits residual voltage, which dies slowly until reconnection takes place. The attenuating rotor currents of IMs produce stator emfs depending on their speed (inertia) and parameters such that some may act as motors and some as generators, until their stored magnetic energy runs out.

To treat such complex phenomena the complete fifth-order dq model is the obvious choice. But, neglecting stator transients greatly simplifies the problem. During turn off (Figure 10.10)

$$\sum_{j=1}^n \bar{i}_{sj} + \bar{i}_c = 0; \quad \frac{d\bar{V}_s}{dt} = \frac{\bar{i}_c}{C} \quad (10.65)$$

where i_c is the capacitor current.

A unique synchronous reference system is used as the stator frequency for the whole group. The third-order model (Equation 10.64) allows to calculate the residual stator voltages during an IM turn off. It has been shown that the third-order model can predict the residual stator voltage well but experiments show a sudden jump in this voltage right after turn off. A possible explanation of this anomaly might be the influence of magnetic saturation, which maintains the self-excitation in some IMs with larger inertia for some time; after this, de-excitation of those that act as motors might take place. Magnetic saturation should be considered in dq model mandatorily.

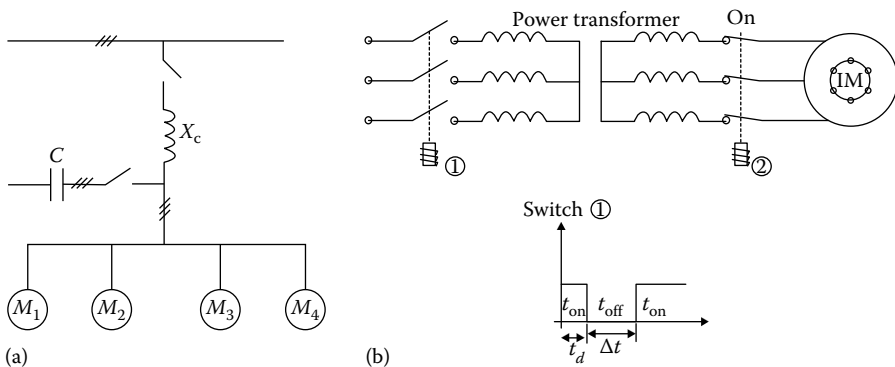


FIGURE 10.10

Grid connection: (a) Group pf IMs with common feeder and (b) transformer feeds an IM.

10.10.1 Other Severe Transients

There are many other severe electromechanical transients that lead to very large peak currents and torque, related to turn on and turn off and reconnection on-the-fly of IMs. The case of turn off and turn on of the power switch in the primary of a transformer that supplies an IM (Figure 10.10b) seems to produce occasionally very large transients (up to 25 pu torque for 6 kW IMs and of only 12 pu torque for 400 kW IMs [3]). For a certain turn off interval (30–40 ms), peak torques of 35 pu may occur. This explains premature aging and mechanical defects in IMs after such situations. Only the complete fifth-order dq model can predict transients as those mentioned here; again magnetic saturation of main and leakage flux paths have to be considered.

Elastical couplings of large IMs may produce severe torsional torque transients [4, Chapter 13] while series capacitors, to avoid voltage sags during their direct turn on starting, may produce synchronous resonance [4]. Such transients may again be treated using the dq (space phasor) model of IM.

10.11 m/N_r Actual Winding Modeling of IMs with Cage Faults

By an m/N_r winding model, we mean an IM with m stator windings and N_r actual (rotor slot count) windings [5]. Each rotor bar and end-ring segment is modeled as such. Stator winding faults such as local short circuits or open coils lead to dedicated self- and mutual inductance expressions as defined using the winding function method [6,7].

For symmetrical stators, self- or mutual (stator to bar loop circuits) inductances have simpler expressions [4]. There are N_r loops (bars) in the rotor plus one more equation for the end ring current. So the model has $m + N_r + 1 + 1$ equations; the last one is the motion equation. However, it is not easy to define practical expressions for stator-to-rotor loop mutual inductances, while measuring them is not practical (or is it?). Field distribution methods may be used to solve the problem.

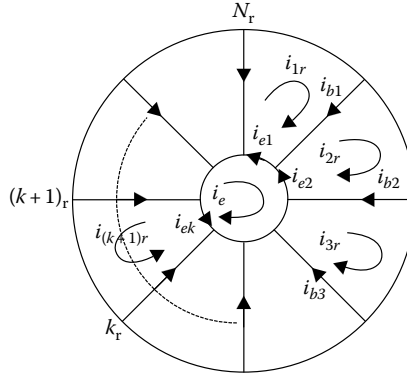
For the symmetrical stator, the phase equations are

$$V_{A,B,C} = i_{A,B,C}R_s + \frac{d\Psi_{A,B,C}}{dt} \quad (10.66)$$

The rotor cage structure and unknowns are illustrated in Figure 10.11.

For the k_r^{th} rotor loop (in rotor coordinates)

$$0 = 2 \left(R_b + \frac{R_e}{N_r} \right) i_{kr} + \frac{d\Psi_{kr}}{dt} - R_b (i_{k-1,r} + i_{k+1,r}) \quad (10.67)$$

**FIGURE 10.11**

Rotor cage with rotor loop currents.

For one end ring segment (R_e, L_e -parameters of one end ring)

$$0 = R_e i_e + L_e \frac{di_e}{dt} - \sum_1^{N_r} \left(\frac{R_e}{N_r} i_{kr} + \frac{L_e}{N_r} \frac{di_{kr}}{dt} \right) \quad (10.68)$$

For a healthy cage, the end ring total current is $i_e = 0$. The bar/ring currents are related by

$$i_{bk} = i_{k,r} - i_{k+1,r}; \quad i_{ek} = i_{k,r} - i_e \quad (10.69)$$

This explains why only $N_r + 1$ independent variables remain.
The stator self- and mutual phase inductances are

$$L_{AA} = L_{BB} = L_{CC} = L_{sl} + L_{0s}; \quad L_{AB} = L_{BC} = L_{CA} = -\frac{L_{0s}}{2}$$

and (Chapter 5)

$$L_{0s} = \frac{4\mu_0 (w_s k_{ws})^2 \tau l_i}{\pi^2 k_c g (1 + k_s) p_1} \quad (10.70)$$

where

k_s is the saturation coefficient

L_{sl} is the leakage stator inductance

τ is the pole pitch

g is the airgap

p_1 is the pole pair
 k_c Carter coefficient
 w_s is the turns per phase
 k_{ws} is the winding factor
 l_i is the stack length

The self-inductance of a rotor loop (based on its area), L_{k_r, k_r} , is

$$L_{k_r, k_r} = \frac{2\mu_0 (N_r - 1) p_1 \tau l_i}{N_r^2 k_c g (1 + k_s)} \quad (10.71)$$

An average mutual inductance between two rotor loops is

$$L_{k, k_r + 1} = -\frac{2\mu_0 p_1 \tau L}{N_r^2 k_c g (1 + k_s)} \quad (10.72)$$

The stator-to-rotor loop mutual inductances are

$$\begin{aligned}
 L_{A_{k_r}}(\theta_{er}) &= L_{sr} \cos[\theta_{er} + (k_r - 1)\alpha] \\
 L_{B_{k_r}}(\theta_{er}) &= L_{sr} \cos\left[\theta_{er} + (k_r - 1)\alpha - \frac{2\pi}{3}\right] \\
 L_{C_{k_r}}(\theta_{er}) &= L_{sr} \cos\left[\theta_{er} + (k_r - 1)\alpha + \frac{2\pi}{3}\right] \\
 \alpha &= p_1 \frac{2\pi}{N_r}; \quad \theta_{er} = p_1 \theta_r; \quad L_{sr} = -\frac{(w_s k_{ws}) \mu_0 2p_1 \tau l_i}{4p_1 k_c g (1 + k_s)} \sin \frac{\alpha}{2}
 \end{aligned} \quad (10.73)$$

Adding up Equations 10.67 through 10.73 in a matrix form, we obtain

$$\begin{aligned}
 [V] &= [R_s][i] + |L'(\theta_{er})| \frac{d}{dt}[i] + \left| \frac{\partial L'(\theta_{er})}{\partial \theta_{er}} \right| [i] \frac{d\theta_{er}}{dt} \\
 [V] &= [V_A \quad V_B \quad V_C \quad 0 \quad 0 \quad \dots \quad 0]^T \\
 [I] &= [i_A \quad i_B \quad i_C \quad i_{1r} \quad i_{2r} \quad \dots \quad i_{N_r} \quad i_e]
 \end{aligned} \quad (10.74)$$

$[L'(\theta_{er})]$ is an $(m + N_r + 1) \times (m + N_r + 1)$ matrix and so is $[R]$; $[R]$ is much sparser, however. Their components are easy to add up using Equations 10.67 through 10.74. The motion equations can be added with torque, T_e :

$$\begin{aligned}
 T_e &= p_1 L_{sr} \left[\left(i_A - \frac{1}{2} i_B - \frac{1}{2} i_C \right) \sum_1^{N_r} i_{k_r} \sin(\theta_{er} + (k_r - 1)\alpha) \right. \\
 &\quad \left. + \frac{\sqrt{3}}{2} (i_B - i_C) \sum_1^{N_r} i_{k_r} \cos(\theta_{er} + (k_r - 1)\alpha) \right]
 \end{aligned} \quad (10.75)$$

We end up with an extended phase variable model with many rotor-position-dependent inductances. The case of faulty rotor bars or end ring segments is handled by increasing their respective resistances 10^3 – 10^4 times. For an IM with the following data, $R_s = 10 \Omega$, $R_b = R_e = 155 \mu\Omega$, $L_{sl} = 35 \text{ mH}$, $L_{0s} = 378 \text{ mH}$, $w_s = 340$ turns/phase, $k_{ws} = 1$, $N_r = 30$ rotor slots, $p_1 = 2$ pole pairs, $L_e = L_b = 0.1 \mu\text{H}$, $L_{sr} = 0.873 \text{ mH}$, $\tau = 62.8 \text{ mm}$ (pole pitch), $L_i = 66 \text{ mm}$ (stack length), $g = 0.37 \text{ mm}$ (airgap), $J = 5.4 \times 10^{-3} \text{ kgm}^2$, $f = 50 \text{ Hz}$, $V_{\text{line}} = 380 \text{ V}$ (star connection), $P_n = 736 \text{ W}$, and $I_{0n} = 21 \text{ A}$, broken bars situations have been simulated [8].

Numerical results for a broken bar no. 2 ($R_{b2} = 200R_b$), with the machine under a load, $T_L = 3.5 \text{ Nm}$ are shown in Figure 10.12a through c for the speed, torque and respective bar current [7]. The rotor bar no. 2 breaks at $t_0 = 2 \text{ s}$.

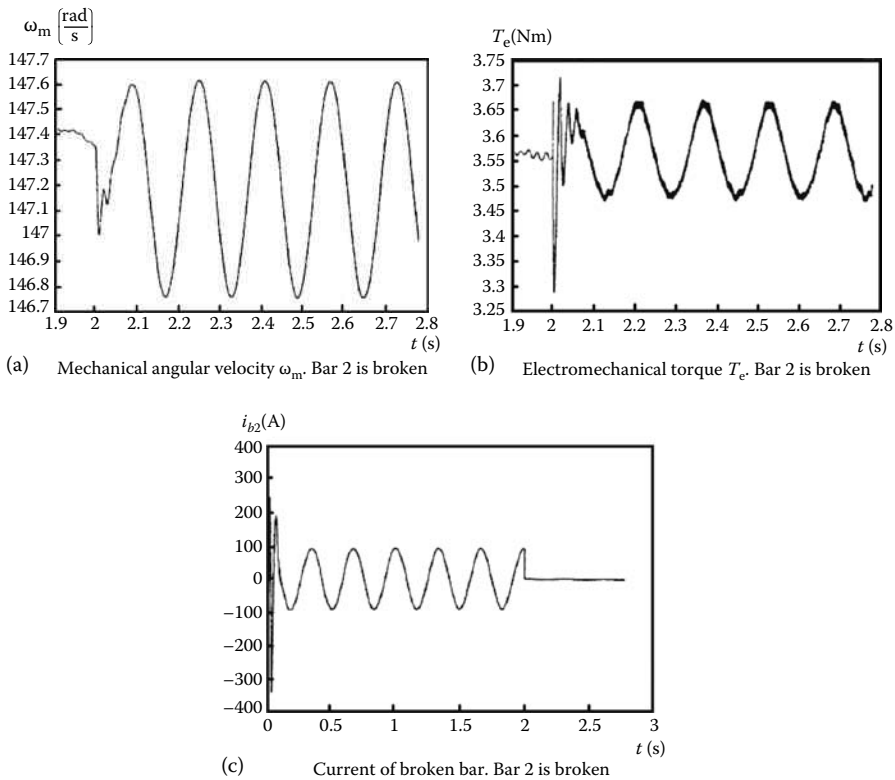


FIGURE 10.12

Rotor bar no. 2 broken at 3 Nm load torque: (a) speed, (b) torque, and (c) broken bar current.

Slight pulsations in speed and more notable pulsations in torque occur. More broken bars, or end ring segments, produce notable speed and torque pulsations. They also produce $2Sf_1$ frequency pulsations in the stator current. All this information can be processed to diagnose broken bars or end ring segments.

In the above model, the inter-bar electrical resistance to the slot wall has been considered infinite, so there are no inter-bar currents. This is hardly true. Inter-bar rotor currents tend to diminish broken bar effects [9]. As expected, coupled finite-element-circuit 3D methods can also be used to simulate complex transients as those aforementioned [10], which is true, but they take more CPU time [11].

10.12 Transients for Controlled Magnetic Flux and Variable Frequency

Due to its ruggedness, the cage rotor IM is very suitable for variable speed drives or as electric generator at variable speed. The speed, ω_r , of the IM is

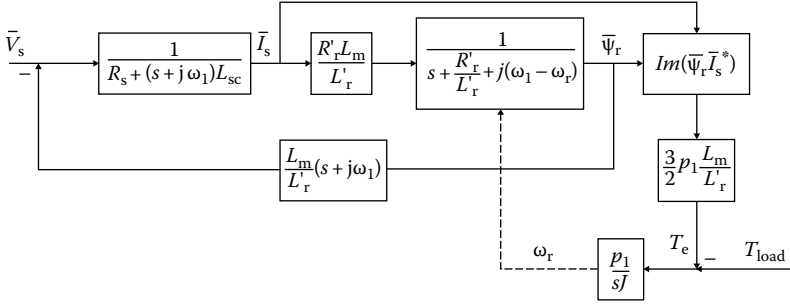
$$\omega_r = \omega_1 (1 - S) = \omega_1 - \omega_2; \quad S = \frac{\text{rotor cage losses}}{\text{electromagnetic power}} \quad (10.76)$$

It is evident that during operation at variable speed, ω_r , the smaller the slip S (in cage rotor IMs), the lower the rotor cage losses. So the smaller the slip frequency (rotor currents frequency), $\omega_2 = S\omega_1$, the better; $\omega_2 > 0$ for motoring and $\omega_2 < 0$ for generating. In addition, quick responses from the torque (in motors) and the active power (in generators) are needed. In such situations, magnetic flux constancy is beneficial since it also keeps magnetic saturation under control.

Frequency (speed) control at a constant flux (Ψ_s , Ψ_m , or Ψ'_r) thus becomes imperative. Two main speed (power) control strategies have become dominant: rotor flux and stator flux orientation (or vector) control. Investigating the IM transients for vector control follows. The space phasor model in synchronous coordinates is most suitable for the scope of this scheme. Let us reproduce the IM equations with the stator current, $\bar{i}_s$, and the rotor flux, $\bar{\Psi}'_r$, as variables.

10.12.1 Complex Eigenvalues of IM Space Phasor Model

The voltage equations of an IM in the space phasor form Equations 10.13 and 10.14 may be written with the stator current, $\bar{i}_s$, and the rotor flux, $\bar{\Psi}'_r$, as complex-state variables, in general coordinates (Figure 10.13):

**FIGURE 10.13**

IM structural diagram with $\bar{i}_s$ and $\bar{\Psi}_r$ as complex variables in synchronous coordinates.

$$[R_s + (s + j\omega_b)L_{sc}]\bar{i}_{sc} + (s + j\omega_b)\frac{L_m}{L'_r}\bar{\Psi}_r = \bar{V}_s;$$

$$S = 1 - \frac{\omega_r}{\omega_b}; \quad -\frac{R'_r}{L'_r}\bar{i}_s + \left[s + \frac{R'_r}{L'_r} + j(\omega_b - \omega_r)\right]\frac{\bar{\Psi}_r}{L_m} = \frac{\bar{V}'_r}{L_m} \quad (10.77)$$

For given slip S , to investigate Equation 10.77 means to treat electromagnetic (constant speed) transients. As we can see from Equation 10.77, the space phasor formalism implies only a second-order characteristic equation with two complex eigenvalues:

$$(R_s + (s + j\omega_b)L_{sc})\left(s + \frac{R'_r}{L'_r} + j(\omega_b - \omega_r)\right)\frac{1}{L_m} + \frac{R'_r}{L'_r}(s + j\omega_b)\frac{L_m}{L'_r} = 0 \quad (10.78)$$

The complex eigenvalues, $s_{1,2}$, as solutions of Equation 10.78 depend on the slip, S , the frequency, ω_1 , and machine parameters. So, direct analytical solutions exist for constant slip, S , and ω_1 transients, when the voltages, $\bar{V}_s$ and $\bar{V}'_r$, vary in amplitude or phase. The stator current, $\bar{i}_s$, and the rotor flux, $\bar{\Psi}'_r$, are of the form

$$\bar{i}_s = \underline{A} + \underline{B}e^{s_1 t} + \underline{C}e^{s_2 t} \quad (10.79)$$

with $s_{1,2}$ from

$$s^2 L_{sc} + s \left[R_s + R'_r \left(\frac{L_m}{L'_r} \right)^2 + L_{sc} \left(\frac{R'_r}{L'_r} + j(\omega_b - \omega_r) + j\omega_b \right) \right]$$

$$+ R_s \left[\frac{R'_r}{L'_r} + j(\omega_b - \omega_r) \right] + j\omega_b L_{sc} \left[\frac{R'_r}{L'_r} + j(\omega_b - \omega_r) \right] + R'_r \frac{L_m}{L'^2_r} j\omega_b = 0 \quad (10.80)$$

A simple numerical computation is required to solve Equation 10.80.

10.13 Cage Rotor Constant Stator Flux Transients and Vector Control Basics

A constant rotor flux means that $s\bar{\Psi}_r = 0$ (in synchronous coordinates), in Equation 10.78:

$$[R_s + (s + j\omega_1)L_{sc}]\bar{i}_s + j\omega_1 \frac{L_m}{L'_r} \bar{\Psi}'_r = \bar{V}_s \quad (10.81)$$

$$-\frac{R'_r}{L'_r} \bar{i}_s + \left[\frac{R'_r}{L'_r} + j(\omega_1 - \omega_r) \right] \frac{\bar{\Psi}'_r}{L_m} = 0$$

It is now evident that only one complex eigenvalue remains. The fifth-order system is further reduced:

$$[R_s + (s + j\omega_1)L_{sc}] \left[\frac{R'_r}{L'_r} + j(\omega_1 - \omega_r) \right] \frac{1}{L_m} + j \frac{R'_r}{L'_r} \omega_1 \frac{L_m}{L'_r} = 0 \quad (10.82)$$

Consequently, s is

$$\underline{s} = - \left\{ \frac{\omega_1 \frac{L_m^2}{L'_r} (\omega_1 - \omega_r) \frac{L'_r}{R'_r}}{L_{sc} \left[1 + (\omega_1 - \omega_r)^2 \frac{L_r'^2}{R_r'^2} \right]} + \frac{R_s}{L_{sc}} \right\} - j\omega_1 \left[\frac{\frac{L_m^2}{L'_r L_{sc}} \frac{1}{\left(1 + (\omega_1 - \omega_r)^2 \frac{L_r'^2}{R_r'^2} \right)}}{1} + 1 \right] \quad (10.83)$$

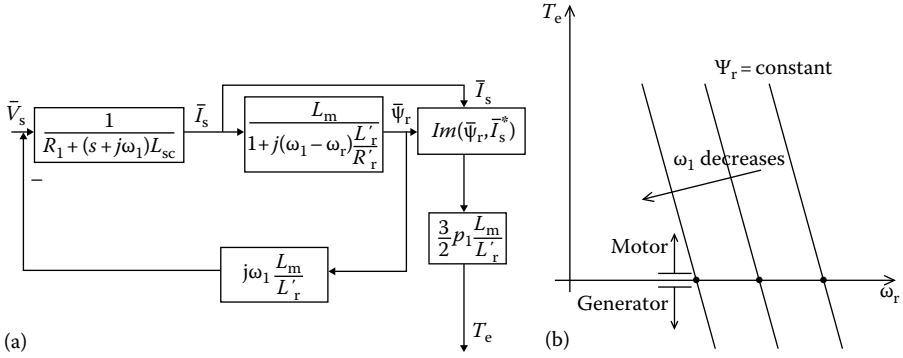
As expected, at $\omega_r = 0$

$$(s)_{\omega_r=0} \approx - \frac{R_s + R_r \left(L_m / L'_r \right)^2}{L_{sc}} - j\omega_1 \quad (10.84)$$

Even now $\underline{s}$ depends on speed and machine parameters but $\text{Re}(\underline{s}) > 0$ at any speed in the motoring ($\omega_1 - \omega_r > 0$), which means stable behavior, and this explains partly the commercialization of rotor flux orientation control. At small speeds and small frequencies, ω_1 , and the generation mode, the real part of $\underline{s}$ may become positive and instability may occur, and, hence, special measures, through closed-loop control, would be required to mend the trouble. The machine model structural diagram gets simplified considerably, to that Figure 10.14, using Equation 10.81.

The second equation in Equations 10.81 suggests that the stator current is related to the constant (imposed) rotor flux, $\Psi_r^* = \Psi_{dr}^*$:

$$\bar{i}_s = I_M + jI_T; \quad I_M = \frac{\Psi_r^*}{L_m}; \quad I_T = I_M S \omega_1 \frac{L'_r}{R'_r}; \quad S = 1 - \frac{\omega_r}{\omega_1} \quad (10.85)$$

**FIGURE 10.14**

IM structural diagram (a) with constant rotor flux in synchronous coordinates ($\omega_b = \omega_1$; steady state is dc) and (b) steady-state torque/speed straight line.

where

I_M is the flux component

I_T is the torque component

The rotor current is such that

$$\Psi'_{qr} = 0 = L'_r I'_{qr} + L_m I_T \quad (10.86)$$

Hence

$$\bar{I}'_r = j I'_{qr} = -j \frac{L_m}{L'_r} I_T; \quad L_{sc} = L_s - \frac{L_m^2}{L_r} \quad (10.87)$$

And therefore

$$\bar{\Psi}_s = L_s \bar{I}_s + L_m \bar{I}'_r = L_s I_M + j L_{sc} I_T \quad (10.88)$$

These are considerable simplifications in building the space phasor model of the machine, all due to the constant rotor flux constraint. The torque expression becomes

$$T_e = \frac{3}{2} p_1 \text{Re} (j \Psi_s \bar{I}_s^*) = \frac{3}{2} p_1 (L_s - L_{sc}) I_M I_T = \frac{3}{2} p_1 \frac{\Psi_r^2}{R'_r} (\omega_1 - \omega_r) \quad (10.89)$$

Under steady state ($s = 0$ in Equation 10.81)

$$\bar{V}_s = R_s \bar{I}_s + j \omega_1 \bar{\Psi}_s \quad (10.90)$$

The above developments lead to the following remarks:

- For a constant rotor flux, the order of the space phasor model is reduced to a single complex eigenvalue for a constant speed.
- The stator current space phasor, $\bar{I}_s$, can be divided into two orthogonal (d , and q) components: I_M (flux component) and I_T (torque component). For a constant I_M , I_T may be varied by varying the frequency ω_1 , for given speed. Thus, the torque may be modified in a decoupled way with respect to the flux. This is the essence of vector control; it is similar to a dc brush machine with a separate excitation control.
- For a constant rotor flux, the torque expression (Equation 10.89) reveals that the IM operates as a reluctance SM where L_d is L_s and L_q is $L_{sc} \ll L_s$. A large apparent magnetic saliency is obtained because the d axis of the dq model falls along the rotor flux axis and the rotor current space phasor lies along the q axis, to cancel the rotor flux component along the q axis, that is, rotor flux orientation.

To illustrate the usefulness of the above equations, a basic (indirect) vector control scheme is presented in Figure 10.15.

We should recognize Equation 10.85 in Figure 10.15. The rotor flux speed, ω_1^* , is calculated based on the rotor speed, ω_r , and the slip speed, $(S\omega_1)^*$. Then the rotor flux angle, Θ_{Ψ_r} , is calculated by integration of speed ω_1^* with time. The initial value of Θ_{Ψ_r} is irrelevant due to a constant airgap of the IM. Then the Park (or space vector) inverse transformation leads to reference phase currents. AC current controllers are used to PWM control of the PWM inverter and realize the desired (reference) stator currents. It is implicitly admitted that the ac current controllers are capable of “executing” almost instantly the reference currents and that the machine parameters are constant

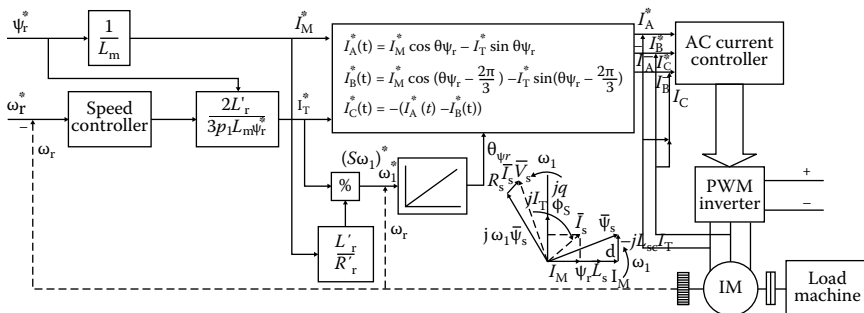


FIGURE 10.15

Basic (indirect) vector control of IM for constant rotor flux.

and known. As shown by Equation 10.83, there is a small time constant in the current ($\bar{I}_s$) response even at a constant rotor flux. For more on rotor flux (vector) control of IMs, see [12, Chapter 10].

Example 10.5 Steady State of the Constant Rotor Flux IM

A cage rotor induction motor is controlled at a variable frequency and a constant rotor flux. Its parameters are $R_s = R'_r = 0.1 \Omega$, $L_s = L'_r = 0.093 \text{ H}$, $L_m = 0.09 \text{ H}$, and $2p_1 = 4$, and it operates at $n = 600 \text{ rpm}$ with $I_M = 20 \text{ A}$ (flux current) and $I_T = \pm 50 \text{ A}$ (torque current), in the space phasor formulation.

Calculate

- Developed electromagnetic torque,
- Slip frequency, $S\omega_1$, ω_1 ,
- Stator phase voltage, current, and power factor, $\cos \varphi_s$, and
- For a small flux current, $I'_M = 5 \text{ A}$, and $I_T = 50 \text{ A}$ at the same voltage as above, calculate $(S\omega_1)'$, ω'_1 , ω'_r , T'_e , and $\cos \varphi'_s$.

Solution:

- According to Equation 10.89 the electromagnetic torque, T_e , is

$$T_e = \frac{3}{2}p_1 (L_s - L_{sc}) I_M I_T = \frac{3}{2} \times 2 \times (0.093 - 0.006) \times 20 \times (\pm 50) = \pm 26.13 \text{ Nm}$$

with the $\oplus$ sign for motoring and the $\ominus$ sign for generating.

- The slip frequency, $S\omega_1$, from Equation 10.85 is

$$S\omega_1 = \frac{I_T}{I_M (L'_r/R'_r)} = \frac{50 \times 0.1}{20 \times 0.093} = 2.688 \text{ rad/s}$$

The electrical rotor speed, ω_r , is

$$\omega_r = 2\pi p_1 n = 2\pi \times 2 \times 600/60 = 125.6 \text{ rad/s}$$

So, for motoring

$$\omega_1 = S\omega_1 + \omega_r = 2.688 + 125.6 = 128.288 \text{ rad/s}$$

$$f_1 = 20.42 \text{ Hz}$$

$$\Psi_r = L_m I_M = 0.09 \times 20 = 1.8 \text{ Wb}$$

c. From Equation 10.90:

$$V_d = R_s I_M - \omega_1 L_{sc} I_T = 0.1 \times 20 - 128.288 \times 0.006 \times 50 = -36.5 \text{ V}$$

$$V_q = R_s I_T + \omega_1 L_s I_M = 0.1 \times 50 + 128.288 \times 0.093 \times 20 = 243.6 \text{ V}$$

So, the phase voltage and current are

$$(V_{\text{phase}})_{\text{RMS}} = \sqrt{\frac{(V_d^2 + V_q^2)}{2}} = \sqrt{\frac{(-36.5)^2 + (243.6)^2}{2}} = 174.185 \text{ V}$$

$$(I_{\text{phase}})_{\text{RMS}} = \sqrt{\frac{(I_M^2 + I_T^2)}{2}} = \sqrt{\frac{(-36.5)^2 + (243.6)^2}{2}} = 38.08 \text{ A}$$

From the space phasor diagram in Figure 10.17

$$\begin{aligned} \varphi_s &= -\tan^{-1}\left(\frac{V_d}{V_q}\right) + \tan^{-1}\left(\frac{I_M}{I_T}\right) \\ &= -\tan^{-1}\left(\frac{-36.5}{243.6}\right) + \tan^{-1}\left(\frac{20}{50}\right) = 30.32^\circ \end{aligned}$$

$$\cos \varphi_s \approx 0.865$$

d. We have to calculate again the slip frequency:

$$(S\omega_1)' = \frac{I'_T}{I'_M (L'_r/R'_r)} = \frac{50 \times 0.1}{5 \times 0.093} = 10.75 \text{ rad/s}$$

$$\Psi'_r = L_m I'_M = 0.09 \times 5 = 0.45 \text{ Wb}$$

The voltage equations are again

$$V_d = R_s I'_M - \omega'_1 L_{sc} I'_T = 0.1 \times 5 - \omega'_1 \times 0.006 \times 50$$

$$V_q = R_s I_T + \omega'_1 L_s I_M = 0.1 \times 50 + \omega'_1 \times 0.093 \times 5$$

To a first approximation, the first terms may be neglected ($R_s \approx 0$):

$$V_d^2 + V_q^2 = (\omega'_1)^2 \left[(L_{sc} I'_T)^2 + (L_s I'_M)^2 \right]$$

$$(-36.5)^2 + (243.6)^2 = (\omega'_1)^2 \left[(0.006 \times 50)^2 + (0.093 \times 5)^2 \right]$$

$$\omega'_1 = \frac{246.32}{0.5533} = 445.12 \text{ rad/s}$$

The speed is

$$\omega'_r = \omega'_1 - (S\omega_1)' = 445.12 - 10.75 = 434.37 \text{ rad/s}$$

$$n = \frac{\omega'_r}{2\pi p_1} = \frac{434.37}{2\pi \times 2} = 34.58 \text{ rps} = 2075 \text{ rpm}$$

A more than three times increase in speed has been obtained for the same voltage by reducing the rotor flux four times.

The new torque value, T'_e , is

$$T'_e = \frac{3}{2}p_1 (L_s - L_{sc}) I'_M I'_T = \frac{3}{2} \times 2 \times (0.093 - 0.006) \times 5 \times 50 = 65.25 \text{ Nm}$$

The actual voltage components are

$$V'_d = 0.5 - 0.3 \times 445.12 = -133 \text{ V}$$

$$V'_q = 5 + 0.465 \times 445.12 = 212 \text{ V}$$

The electromagnetic powers, for the two speeds (600 and 2075 rpm) and the same phase voltage, but different frequencies (20.42 and 70.87 Hz, respectively), and also a 4/1 rotor flux weakening, are

$$P_{\text{elm}} = T_e \frac{\omega_r}{p_1} = 261.3 \times \frac{125.6}{2} = 16.409 \text{ W}$$

$$P'_{\text{elm}} = T'_e \frac{\omega'_r}{p_1} = 65.25 \times \frac{434.37}{2} = 14.171 \text{ W}$$

That is an almost constant power for a 3.458/1 speed range.

10.13.1 Cage-Rotor Constant Stator Flux Transients and Vector Control Basics

For a constant stator flux, let us consider stator coordinates ($\omega_b = 0$) and replace $\bar{\Psi}'_r$ by $\bar{\Psi}_s$ in Equation 10.77; a constant flux in stator coordinates means a constant amplitude of $\bar{\Psi}_s$ at the input voltage frequency, ω_1 .

$$R_s \bar{I}_s + s \bar{\Psi}_s = \bar{V}_s$$

$$\left[\frac{L_s}{L_m} R'_r + (s - j\omega_r) L_{sc} \right] \bar{I}_s - \bar{\Psi}_s \left[\frac{R_r}{L_m} + \frac{L'_r}{L_m} (s - j\omega_r) \right] = 0 \quad (10.91)$$

For a constant stator flux magnitude (in stator coordinates):

$$\bar{\Psi}_s = \bar{\Psi}_{s0} e^{j\omega_1 t} \quad (10.92)$$

So, $s\bar{\Psi}_s$ becomes $j\omega_1\bar{\Psi}_s$ in Equation 10.91:

$$R_s\bar{I}_s + j\omega_1\bar{\Psi}_{s0} = \bar{V}_s$$

$$\left[\frac{L_s}{L_m}R'_r + (s - j\omega_r)L_{sc} \right] \bar{I}_s - \bar{\Psi}_{s0} \left[\frac{R_r}{L_m} + \frac{L'_r}{L_m}j(\omega_1 - \omega_r) \right] = 0 \quad (10.93)$$

This is again a single complex eigenvalue system with the characteristic equation:

$$R_s \left[\frac{R_r}{L_m} + \frac{L'_r}{L_m}j(\omega_1 - \omega_r) \right] + \left[\frac{L_s}{L_m}R'_r + (s - j\omega_r)L_{sc} \right] j\omega_1 = 0 \quad (10.94)$$

and

$$\underline{s} = - \left[\frac{L_s R'_r}{L_m L_{sc}} + \frac{R_s L'_r (\omega_1 - \omega_r)}{\omega_1 L_{sc} L_m} \right] + j \left(\omega_r + \frac{R_s R_r}{\omega_1 L_{sc} L_m} \right) \quad (10.95)$$

Again, as for a constant rotor flux, the real part of the eigenvalue ($Re(\underline{s})$) for current transients is negative for $0 < S < 1$ ($0 < \omega_r < \omega_1$). With $R_s \approx R'_r$, it follows that even for the generator mode ($\omega_r > \omega_1$) but with $|S| \ll 1$, the real part of $\underline{s}$ remains negative. This is the case in most situations with a variable frequency control of IMs. The situation is more delicate with large values of slip, S , typical at very low speeds.

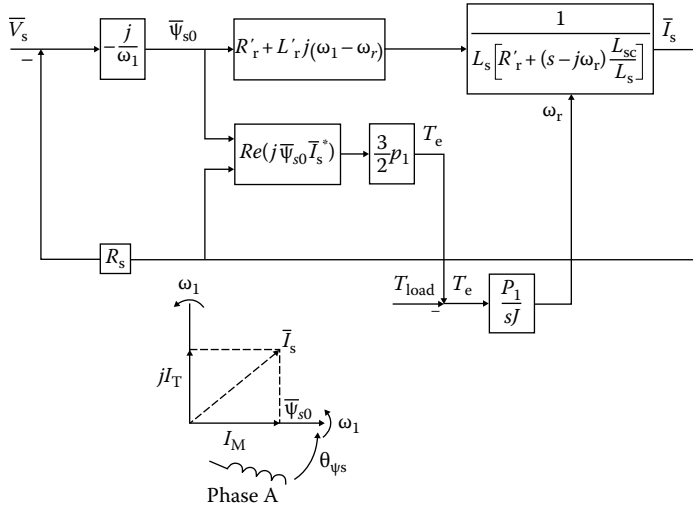
This phenomenon, typical for both the constant rotor or the constant stator flux control, in the generator/motor mode, has been observed with variable speed drives at very low speeds. The closed-loop control should solve this matter.

The structural diagram of an IM, corresponding to Equation 10.93, is illustrated in Figure 10.16.

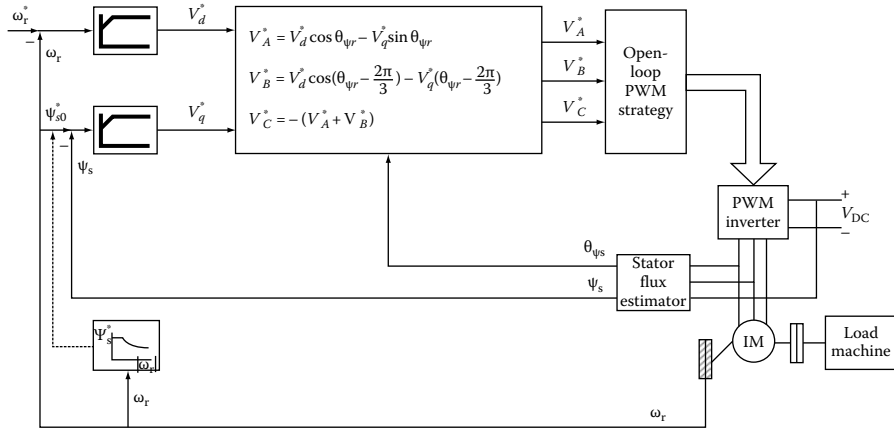
The IM control at a constant stator flux should take advantage of the stator equation (Equations 10.93) in stator coordinates (which is rather simple), to estimate the stator flux amplitude and its position when a closed-loop regulates the stator flux amplitude (Figure 10.17).

The basic (principle) stator flux orientation (vector) control scheme in Figure 10.17 can be characterized by

- A stator flux estimator in stator coordinates (Equation 10.93).
- A flux amplitude closed-loop regulator.
- A voltage vector rotator in stator flux coordinates.
- The ac voltages, V_A^* , V_B^* , and V_C^* are “reproduced” by an open-loop PWM (in the PWM inverter) strategy, characteristic to either large power or super high-speed IM drives where the switching frequency (f_{sw}) ratio, f_{sw}/f_1 , in the static inverter is low (less than 20–30).

**FIGURE 10.16**

Structural diagram of IM for constant stator flux amplitude, Ψ_s , in stator coordinates ($\omega_b = 0$).

**FIGURE 10.17**

Basic direct stator flux orientation (vector) control of IM.

- The apparent absence of any current controller is compensated by the flux closed-loop control and by the limiters at the output of both regulators.

Estimators for the stator (or rotor) flux, eventually also of speed, are problems beyond our scope here but common in modern electric drives or generator controls [12].

10.13.2 Constant Rotor Flux Transients and Vector Control Principles of Doubly Fed IMs

The constant rotor or stator flux transients of doubly fed IMs are characterized by the same eigenvalues as for cage rotor IMs. Equations 10.81, with $s\Psi_r'$ in synchronous coordinates as zero, for a constant rotor flux, are still used:

$$\begin{aligned} (R_s + (s + j\omega_1)L_{sc})\bar{I}_s + j\omega_1 \frac{L_m}{L_r'}\Psi_r' &= \bar{V}_s \\ -\frac{L_m R_r'}{L_r'}\bar{I}_s + \left[\frac{R_r'}{L_r'} + j(\omega_1 - \omega_r) \right] \Psi_r' &= \bar{V}_r' = V_{dr}' + jV_{qr}' \end{aligned} \quad (10.96)$$

The torque, T_e , is

$$T_e = \frac{3}{2}p_1 \frac{L_m}{L_r'} \Psi_r' I_{qs} \quad (10.97)$$

In rotor flux coordinates,

$$\Psi_{dr}' = \Psi_r'; \quad \Psi_{qr}' = L_r' I_r' + L_m I_{qs0} = 0$$

This time the rotor equation is used for control as $\bar{V}_s$ is imposed both in terms of amplitude and frequency (phase).

$$\begin{aligned} V_{dr}' &= \frac{R_r'}{L_r'} \Psi_r' - \frac{L_m}{L_r'} R_r' I_{ds}; \quad \Psi_r' = \Psi_{dr}' = L_m I_{ds} \quad (I_{dr}' = 0) \\ V_{qr}' &= (\omega_1 - \omega_r) \Psi_r' - \frac{L_m}{L_r'} R_r' I_{qs} \end{aligned} \quad (10.98)$$

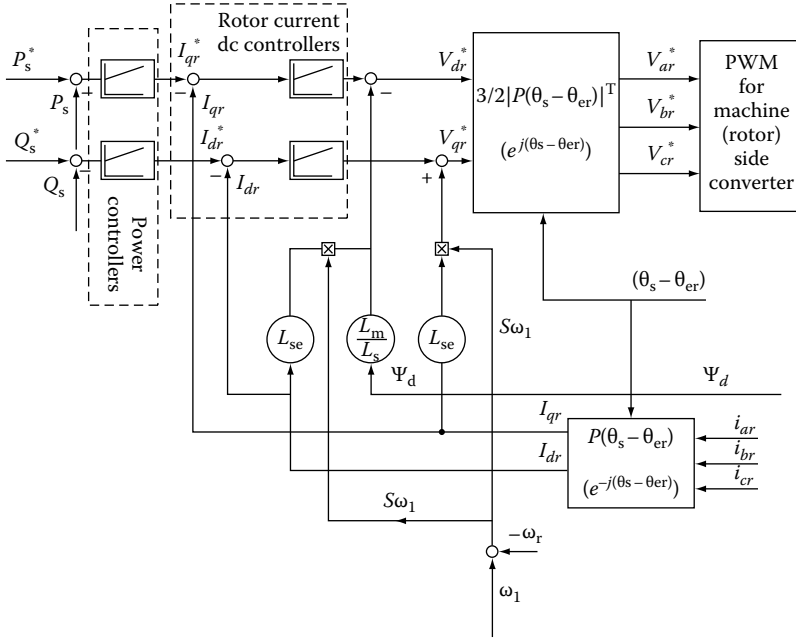
The reactive power in the rotor, Q_r^r , is

$$Q_r^r = \frac{3}{2}p_1 S \omega_1 \Psi_r I_{ds} \quad (10.99)$$

Also the rotor active power, P_r^r , is

$$P_r^r = \frac{3}{2} \left(V_{dr}' I_{dr}' + V_{qr}' I_{qr}' \right) \approx \frac{3}{2}p_1 S \omega_1 \Psi_r I_{qs} \quad (10.100)$$

To regulate the rotor (and stator) active power as a generator (or speed as a motor), we need to control I_{qs} , that is, V_{qr}' . For rotor reactive power control, I_{ds} has to be controlled via V_{dr}' . A basic vector control scheme is shown in Figure 10.18, but with P_s and Q_s (stator powers) instead of P_r and Q_r closed-loop controllers. We call this scheme primitive as though, in principle, it is implementable as it is. Prescribing the rotor voltages is not easy as they tend to be small around the standard synchronous speed (zero slip: $\omega_r = \omega_1$). So, the rotor current closed-loop control should be more robust. But this is

**FIGURE 10.18**

Primitive direct vector control of doubly fed induction generator in rotor coordinates, for controlled rotor flux.

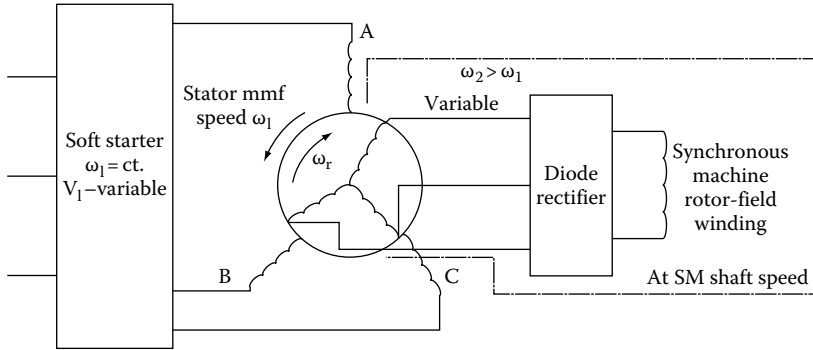
beyond our scope here. See [13, Chapter 2] for more on the DFIM as a variable speed generator.

10.14 Doubly Fed IM as a Brushless Exciter for SMs

The doubly fed IM may be ac-fed in the stator at constant frequency, ω_1 , but for variable voltage (by using a thyristor Soft starter). The rotor is rotated at speed $-\omega_r$, opposite to the stator mmf traveling speed, ω_1 . So, the rotor emf has the slip frequency ω_2 (Figure 10.19).

$$\omega_2 = \omega_1 + \omega_r > \omega_1 \quad (10.101)$$

At zero speed, the rotor emf is produced solely by “transformer” action. When ω_r increases, the rotor emf is more and more produced by motion. If $\omega_r = 4\omega_1$ (this situation occurs when the number of pole pairs of the DFIM, p_1 , is four times larger than that of the SM (p_{1SG}), which is excited through the diode rectifier placed also on the rotor), up to 80% of the dc excitation power is produced by motion (from the mechanical shaft power) and 20% by

**FIGURE 10.19**

Doubly fed IM as brushless exciter for SMs.

the “transformer” action. The space phasor equations in stator coordinates are again

$$\begin{aligned}\bar{V}_s &= R_s \bar{I}_s + \frac{d\bar{\Psi}_s}{dt} \\ \bar{V}'_r &= -R'_r \bar{I}'_r + \frac{d\bar{\Psi}'_r}{dt} + j\omega_r \bar{\Psi}'_r\end{aligned}\quad (10.102)$$

For steady-state and sinusoidal rotor currents (the diode rectifier at a high frequency, ω_2 , is characterized often by such situations), $d/dt = j\omega_1$ (stator coordinates).

$$\begin{aligned}\bar{V}_s^s &= R_s \bar{I}_s^s + j\omega_1 \bar{\Psi}_s^s; \quad \bar{\Psi}_s^s = L_s \bar{I}_s + L_m \bar{I}_r^s \\ \bar{V}_r^{s|} &= -R'_r \bar{I}_r^{s|} + j(\omega_1 + \omega_r) \bar{\Psi}_r^{s|}; \quad \bar{\Psi}_r^{s|} = L'_r \bar{I}_r^s + L_m \bar{I}_s\end{aligned}\quad (10.103)$$

$\bar{V}'_r$, as in Equation 10.103, has the frequency ω_1 , but in reality (in rotor coordinates) $\bar{V}_r^{r|}$ is

$$\bar{V}_r^{r|} = \bar{V}_r^{s|} e^{j\omega_r t} = \bar{V}_r^{s|} e^{j(\omega_1 + \omega_r)t} \quad (10.104)$$

and thus has the frequency: $\omega_2 = \omega_1 + \omega_r$. In synchronous coordinates, however, it is all dc in steady state. But the steady-state equations (Equation 10.103) keep the same aspect in all coordinates.

We can solve Equation 10.103 to get $\bar{I}_s$ and $\bar{I}'_r$. Let us consider a pure 3-phase resistive load of this particular generator (the ideal diode rectifier can be assimilated to the unity power factor load):

$$\bar{V}'_r = \bar{I}'_r R_{\text{load}} \quad (10.105)$$

So

$$\bar{I}_s = \frac{jI'_r (R'_r + j\omega_2 L'_r + R_{\text{load}})}{\omega_2 L_m} \quad (10.106)$$

$$\bar{I}'_r = \frac{\bar{V}_s}{\frac{j(R'_r + j\omega_2 L'_r + R_{\text{load}})(R_s + j\omega_1 L_s)}{\omega_2 L_m} + j\omega_1 L_m} \quad (10.107)$$

The stator apparent power is

$$\bar{S}_s = P_s + jQ_s = \frac{3}{2} (\bar{V}_s I_s^*) \quad (10.108)$$

while the rotor power, P_r , is

$$P_r = \frac{3}{2} \text{Re} (\bar{V}'_r \bar{I}'_r^*) = \frac{3}{2} R_{\text{load}} (I'_r)^2 \quad (10.109)$$

Example 10.6 Doubly Fed IM (DFIM) as Brushless SM Exciter

Let us consider a DFIM used to excite a large synchronous generator (SG). Its parameters are $R_s = R'_r = 3.815 \Omega$, $L_{sl} = L'_{rl} = 9.45 \times 10^{-5} \text{ H}$, $L_m = 2.02 \times 10^{-3} \text{ H}$, stator frequency $f_1 = 60 \text{ Hz}$, and rotor speed $n = 1800 \text{ rpm}$ (4-pole SG). The number of DFIM pole pairs is $p_1 = 6$. The ratio of the rotor/stator turns is $a_{rs} = 1$ and $V_{\text{snl}} = 440 \text{ V}$ (line voltage, RMS).

Calculate

- The rotor frequency and the ideal no-load actual rotor voltage, V'_{r0} , at $n = 1800 \text{ rpm}$ and $n = 0 \text{ rpm}$
- R_{load} , V'_r , and P_r (all in the rotor) at zero speed. For $I'_r = 1000 \text{ A}$ (phase RMS)
- The stator voltage V_s , I_s , P_s , Q_s , and P_r for the same R_{load} and current $I'_r = 1000 \text{ A}$ but at $n = 1800 \text{ rpm}$

Solution:

- The rotor frequency, ω_2 , is, from Equation 10.102,

$$\omega_2 = \omega_1 \left(1 + \frac{p_1}{p_{1\text{SG}}} \right) = \omega_1 \left(1 + \frac{6}{2} \right) = 4\omega_1$$

Consequently, $f_{2n} = 4f_1 = 240 \text{ Hz}$.

The rotor ideal no-load voltage, V'_{r0} , is

$$(V'_{r0})_{n=1800 \text{ rpm}} = V_s a_{rs} \frac{\omega_2}{\omega_1} = 1 \times 440 \times \frac{4}{1} = 1760 \text{ V (line, RMS)}$$

At zero speed

$$(V'_{rol})_n = 0 \text{ rpm} = V_s a_{rs} \frac{\omega_1}{\omega_1} = 1 \times 440 \times \frac{1}{1} = 440 \text{ V (line, RMS)}$$

$$(V'_{ro})_{\text{phase, RMS}} = \frac{440}{\sqrt{3}} \approx 254 \text{ V}$$

- b. For $n = 0$ (zero speed), from Equation 10.107, with $\omega_2 = \omega_1$, $I'_r = 1000 \sqrt{2} \text{ A}$, and $V_s = (440/\sqrt{3}) \sqrt{2} \text{ V}$, we find the load resistance $R_{\text{load}} = 0.226 \Omega/\text{phase}$. The rotor phase voltage is

$$(V'_{ro})_{\text{phase, RMS}} = R_{\text{load}} (I'_r)_{\text{phase, RMS}} = 0.226 \times 1000 = 226 \text{ V}$$

So the voltage regulation, at zero speed, is

$$\Delta V = \frac{V_s - V_r}{V_s} = \frac{254 - 226}{254} = 0.1102 = 11.02\%$$

The rotor (output) power at zero speed, $(P_r)_{n=0}$, is

$$(P_r)_{n=0} = 3V_r I_r = 3 \times 226 \times 1000 = 678 \text{ kW}$$

- c. For $n = 1800 \text{ rpm}$, again from Equations 10.106 and 10.107, but with $\omega_2 = 4\omega_1$, we now calculate $\bar{V}_s$, as R_{load} and I'_r are given:

$$(\bar{V}_s)_{\text{phase, RMS}} = -64.06 - j72.1; \quad V_s = 96.8 \text{ (RMS per phase)}$$

The stator current, I_s , (from Equation 10.106) is

$$\bar{I}_s = -1046 + j75.3; \quad I_s = 1049.3 \text{ A} > I_r = 1000 \text{ A}$$

because machine magnetization is done from the stator.

The stator powers (Equation 10.108) are

$$P_s + jQ_s = 184.752 \text{ kW} + j240.751 \text{ kVAR}$$

The delivered rotor power is the same as for zero speed, $P_r = 678 \text{ kW}$. So, the difference in powers comes from the shaft (from the mechanical power of the SG).

Discussion:

- At standstill, all output rotor power comes from the stator and the DFIM operates as a transformer with the rectifier load. The required stator power and the voltage are maximum.

- As the speed increases, less and less active power is required from the stator as more and more of output (rotor) power is extracted from the shaft mechanical (SG) power.
- The DFIM may serve as a brushless exciter for SMs (SGs) from zero speed and is less sensitive to SG (SM) terminal voltage sags (due to grid faults), because most power is produced mechanically.
- The voltage regulation is small because the “internal reactance” of the DFIM is the shortcircuit reactance, $\omega_1 L_{sc}$. Only an ac Variac (thyristor Soft starter) is required to control the SG (SM) field current.

With

$$\bar{\Psi}'_r = \frac{L_m}{L_s} \bar{\Psi}_s + L_{sc} \bar{I}'_r \quad (10.110)$$

the rotor voltage equation (Equation 10.103) becomes

$$\bar{V}'_r = j\omega_2 \frac{L_m}{L_s} \bar{\Psi}_s - (R'_r + j\omega_2 L_{sc}) \bar{I}'_r = \bar{E}'_r - Z_{DFIM} \bar{I}'_r \quad (10.111)$$

For $R_s \approx 0$

$$\bar{E}'_r = j\omega_2 L_m \bar{\Psi}_s \approx \bar{V}_s \frac{L_m}{L_s} \frac{\omega_2}{\omega_1} \quad (10.112)$$

So the rotor emf, $\bar{E}'_r$, varies with the stator voltage, V_s , and with the rotor frequency, ω_2 (or speed ω_r). The short-circuit reactance, $\omega_1 L_{sc}$, is evident.

Voltage regulation is much smaller than that in an inverted-configuration synchronous auxiliary generator on a shaft as an SG dc exciter (stator dc excitation with a 3-phase rotor winding and a diode rectifier), which extracts practically all output power mechanically, and thus may not operate from zero speed. Zero speed operation is required in large variable speed synchronous motor drives.

In view of the above merits, no wonder why the DFIM is used as a brushless exciter for large-power synchronous machines by major global manufacturers in this field.

10.15 Parameter Estimation in Standstill Tests/Lab 10.3

By parameters we mean

- The magnetization curve, $\Psi_m^*(I_m)$, with the magnetization inductance, $L_m(I_m) = \Psi_m^*(I_m) / I_m$, and the transient magnetization inductance $L_{mt}(I_m) = d\Psi_m^*(I_m) / dI_m$; I_m is the magnetization current.

- The resistances and leakage inductances of the stator, R_s and L_{sl} , and of single (or double, or triple) rotor circuits, R'_{r1} , R'_{r2} , R'_{r3} , L'_{rl1} , L'_{rl2} , and L'_{rl3} , reduced to the stator. These parameters are required in the investigation of steady state and transient performance and for control or monitoring of the system design.

While the main flux path magnetic saturation appears in no-load to load operation, the leakage flux path saturation occurs at overcurrents (above 2–3 I_{rated}), unless closed slots are used on the rotor, in order to reduce noise, vibration, and stray-load losses.

Testing of 3-phase IMs is highly standardized (see IEC-34 standard series, NEMA 1961–1993 standards for large IMs). Only standstill flux decay and frequency response tests for parameter identification are detailed here.

10.15.1 Standstill Flux Decay for Magnetization Curve Identification: $\Psi_m^*(I_m)$

At standstill, the rotor cage IM is dc supplied at an initial value, I_{A0} , with phase A in series with phases B and C in parallel (Figure 10.20).

The arrangement in Figure 10.20 signifies a few important constraints:

$$\begin{aligned}
 I_B &= I_C = -I_A/2; \quad I_A + I_B + I_C = 0 \\
 V_B &= V_C; \quad V_A = -2V_B, \text{ because } V_A + V_B + V_C = 0 \\
 V_{ABC} &= V_A - V_B = \frac{3}{2} V_A(t)
 \end{aligned} \tag{10.113}$$

The current and voltage space phasors in stator coordinates are

$$\begin{aligned}
 \bar{I}_s(t) &= \frac{2}{3} [I_A + I_B e^{j2\pi/3} + I_C e^{-j2\pi/3}] = I_A(t) \\
 \bar{V}_s(t) &= \frac{2}{3} [V_A + V_B e^{j2\pi/3} + V_C e^{-j2\pi/3}] \\
 &= V_A(t) = \frac{2}{3} V_{ABC}^{(t)}
 \end{aligned} \tag{10.114}$$

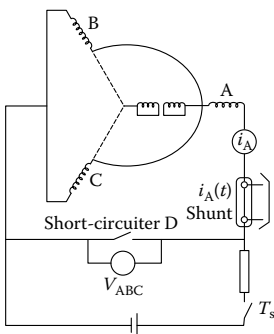


FIGURE 10.20
Standstill flux decay test of IM.

Now, once the dc current, I_{A0} , is “installed,” the switch, T_s , is turned off and the stator current continues to flow until extinction (decay) through the freewheeling diode D (as for the SM tests). The stator equation after T_s is turned off is

$$-\frac{2}{3} V_{diode}^{(+)} = \bar{I}_s R_s + L_{sl} \frac{d\bar{I}_s}{dt} + \frac{d\bar{\Psi}_m}{dt} \tag{10.115}$$

The rotor equations for a dual cage rotor configuration are

$$\begin{aligned} 0 &= \bar{I}'_{r1} R'_{r1} + L'_{r1l} \frac{d\bar{I}'_{r1}}{dt} + \frac{d\bar{\Psi}_m}{dt} \\ 0 &= \bar{I}'_{r2} R'_{r2} + L'_{r2l} \frac{d\bar{I}'_{r2}}{dt} + \frac{d\bar{\Psi}_m}{dt} \end{aligned} \quad (10.116)$$

We may use these equations in two ways:

- By integrating the stator equation (only) to find the magnetization curve, $\Psi_m(I_m)$, $I_m = I_{A0}$:

$$\Psi_m(I_m) = L_m(I_m) I_m = R_s \int I_A(t) dt + \frac{2}{3} \int V_{\text{diode}}^{(+)} dt - L_{sl} I_m \quad (10.117)$$

The leakage inductance, L_{sl} , has to be already known from the design or from the stalled standard frequency ac test.

By gradually increasing the initial current, I_{A0} , value, the entire magnetization curve can be obtained. To avoid errors caused by temperature in R_s , the latter may be calculated as $R_s = (2V_{ABC0})/(3I_{A0})$, before each current decay test (which should last 1–2 s).

Results such as those in Figure 10.21 are obtained.

- By curve fitting, the magnetization curve may be approximated to a differentiable function, and then, from here, $L_{mt}(I_m)$ is calculated as

$$L_{mt}(I_m) = \frac{d\Psi_m^*}{dI_m} \quad (10.118)$$

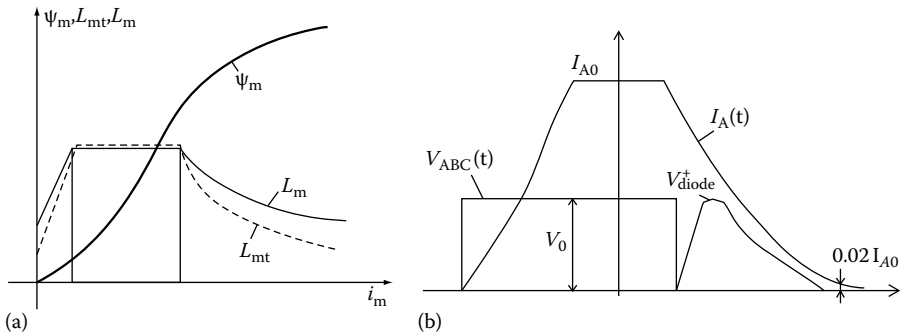


FIGURE 10.21

IM flux decay test outputs: (a) The magnetization curve, $\Psi_m^*(I_m)$, and normal and transient magnetization inductances, L_m and L_{mt} , and (b) current, $I_A(t)$, and V_{diode}^+ during the flux decay test.

The magnetization curve may be obtained from no-load motor tests (as described in Chapter 5) at variable input voltages, but it requires more time and resources to do it. All comparisons between results showed that the standstill flux decay tests produce reliable results for in-speed operations.

As a significant degree of magnetic saturation is allowed by design, to cut machine size, and in the stand-alone induction generator mode (self-excited directly by a parallel-series capacitor or through a PWM inverter), the magnetization curve is a crucial parameter.

10.15.2 Identification of Resistances and Leakage Inductances for Standstill Flux Decay Tests

Returning to Equations 10.115 and 10.116 and with $\bar{I}_s(t) = I_A(t)$, we can process the acquired descending curve and identify, by curve fitting, R'_{r1} , R'_{r2} , L'_{rl1} , and L'_{rl2} (for a dual cage rotor model) (Figure 10.21b). With $d/dt \rightarrow s$ we get

$$\underline{Z}_s \approx \frac{\bar{V}_s(s)}{\bar{I}_s(s)} = -\frac{2}{3} \frac{\bar{V}_{ABC}(s)}{\bar{I}_A(s)} = R_s + s(L_{sl} + L_{mt}) \frac{(1 + s\tau'')(1 + s\tau')}{(1 + s\tau_0'')(1 + s\tau_0')} \quad (10.119)$$

For a step voltage application (which is synonymous with freewheeling through a diode)

$$\bar{V}_s(s) = -\frac{2}{3} V_{ABC}(s) = -\frac{2}{3} \frac{V_0}{s} \quad (10.120)$$

So

$$\bar{I}_s(s) = -\frac{2V_0}{3s\underline{Z}_s(s)}; \quad L_s = L_{sl} + L_m \quad (10.121)$$

But the time variation of $\bar{I}_s(t)$ can be derived directly from Section 10.7.1 on a sudden short circuit, with $\omega_1 = 0$, $\Psi_0 = 0$, and initial current $I_{s0} = I_{A0} = V_{s0}/R_s$. If, at this time, $\tau_a \approx L_s/R_s$, then, approximately,

$$\begin{aligned} I_s(t) \approx & \frac{V_{s0}}{R_s} - V_{s0} \left[\frac{\tau' \tau_a}{\tau_a - \tau'} \left(e^{-t/\tau_a} - e^{-t/\tau'} \right) \left(\frac{1}{L'} - \frac{1}{L_s} \right) \right. \\ & + \frac{\tau'' \tau_a}{\tau_a - \tau''} \left(e^{-t/\tau_a} - e^{-t/\tau''} \right) \left(\frac{1}{L''} - \frac{1}{L'} \right) \\ & \left. + \frac{\tau_a}{L_s} (1 - e^{-t/\tau_a}) \right] \end{aligned} \quad (10.122)$$

In this case, the initial value of current is $I_{A0} = (I_s)_{t=0} = V_{s0}/R_s$ while the final value is zero, as expected. Curve-fitting regression methods are applied to find the values of L' , L'' , and L_s , and τ' and τ'' . If the initial current, I_{A0} , is smaller than the rated magnetization current, magnetic saturation is not relevant, and, thus, a nonsaturated value of synchronous inductance, $L_s = L_{sl} + L_m$, is used, as known from design (or from the no-load current value). Due to the special mix of frequencies in step-down to zero voltage pulses, it is apparent that the frequency content of rotor currents in real running conditions is not well matched in the flux decay tests.

This is how standstill frequency response (SSFR) tests have come into play.

10.15.3 Standstill Frequency Response Tests

SSFR tests for IMs are similar to those for SM, but they are conducted only once, at random rotor position. The experimental arrangement of Figure 10.20 still holds but, this time, a wide variable frequency sinusoidal voltage source is required (from 0.01 to 100 Hz, which will suffice for the most practical cases). This test will be done for each frequency separately, maintaining it for a few periods and measuring the voltage, current amplitudes, and their phase shift angle, $\underline{Z}(s)$, in Equation 10.119, that now becomes $\underline{Z}(j\omega)$:

$$\underline{Z}(j\omega) = R_s + j\omega (L_{sl} + L_{mt}) \frac{(1 + j\omega\tau'')(1 + j\omega\tau')}{(1 + j\omega\tau_0'')(1 + j\omega\tau_0')} \quad (10.123)$$

where

$$|\underline{Z}(j\omega)| = \frac{2}{3} \frac{|V_{ABC}|_{\text{RMS}}}{|I_{ABC}|_{\text{RMS}}}; \quad \text{Arg}(\underline{Z}(j\omega)) = \alpha(V_{ABC}, I_{ABC}) \quad (10.124)$$

Typical results from such tests in pu are given in Figure 10.22. For the data in Figure 10.22, using only the curve fitting of the amplitude yields results such as

$$L(s) = 3 \frac{(1 + s \times 0.0125)(1 + s \times 0.318)}{(1 + s \times 0.0267)(1 + s \times 1.073)} \quad (10.125)$$

It is also feasible to use only $\text{Arg}(\underline{Z}(j\omega))$ information and identify the above time constants [14]. The $L(s)$ has two poles and two zeros: $1/\tau_0''$, $1/\tau_0'$, $1/\tau'$, and $1/\tau''$; also $\tau_0'' > \tau''$ and $\tau_0' > \tau'$. Let us denote $\alpha = \tau'/\tau_0' < 1$; this is a lag circuit. The maximum phase lag, φ'_c , (Figure 10.22) is

$$\sin \varphi'_c \approx \frac{\alpha - 1}{\alpha + 1} \quad (10.126)$$

The gain change due to the respective zero/pole pair, with α from Figure 10.22, is

$$\text{gain change} = -20 \log \alpha \quad (10.127)$$

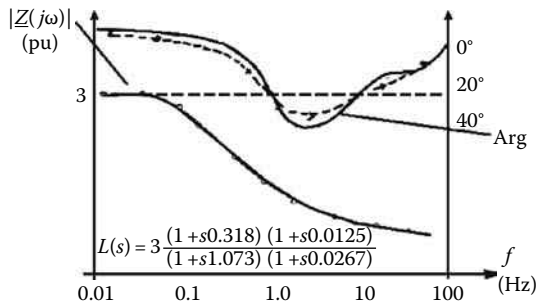


FIGURE 10.22
SSFR of a two-cage rotor IM.

So

$$\tau' = \frac{\tau'_0}{\alpha} \text{ and } \tau'_0 = \frac{\sqrt{\alpha}}{2\pi f'_c}. \quad (10.128)$$

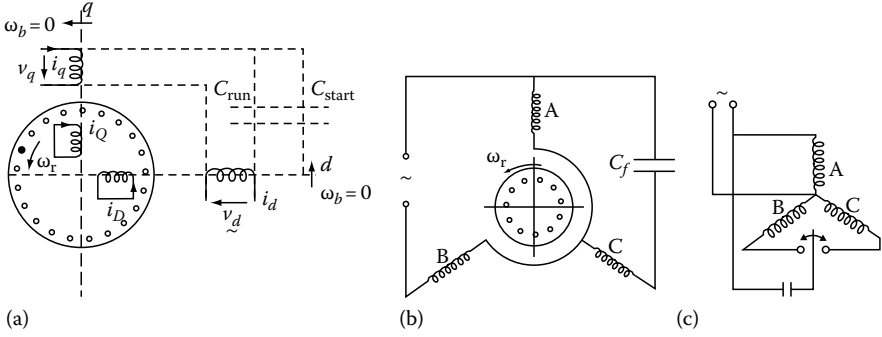
After the first zero/pole pair, τ' , τ'_0 have been calculated. They are introduced in Equation 10.125, and then $\text{Arg}(\underline{Z}(j\omega))$ is recalculated versus frequency, and, thus, a new maximum argument is obtained at φ'' (at frequency f'_c) and the computation in Equations 10.127 and 10.128 is done again to find τ'' and τ''_0 . This process is continued until no more maxima of $\text{Arg}(\underline{Z}(j\omega))$ occurs.

Additional steps to improve precision in finding f'_c and f''_c , and φ' and φ'' may be taken [14].

It is evident from Figure 10.22 that the two $\text{Arg}(\underline{Z}(j\omega))$ maxima suggest a fair double-cage representation. For more on 3-phase IM testing, see [4, Chapter 22].

10.16 Split-Phase Capacitor IM Transients/Lab 10.4

The split-phase IM is still used at a remarkable performance (efficiency) in driving pumps or compressors for household appliances, in the range of 100 W or more. Many other domestic tools, such as cloth washers, drillers, and sawyers, make use of split-phase capacitor IMs in the range of hundreds of watts to 1 kW. They use a starting resistor, R_{start} , or a capacitor, C_{start} , in the auxiliary phase. The latter may be kept on during the on-load operation, to increase efficiency, but with a smaller running capacitor, C_{run} ($C_{\text{run}} < C_{\text{start}}$) (Figure 10.23a). Yet another application may use a 3-phase winding for uni-directional or reversible motion (Figure 10.23b and c).

**FIGURE 10.23**

Split-phase capacitor IMs with (a) auxiliary phase, (b) 3-phase, unidirectional, and (c) 3-phase, bidirectional.

For some cloth-washing machines, there is a 3-phase 12-pole winding used for washing (at a low speed) and a separate 2-pole orthogonal winding for spinning (at a high speed). Switching back and forth for the washing mode and for the drying mode implies important transients.

The 3-phase connections (Figure 10.23b and c) may be reduced to orthogonal windings (Figure 10.23a) [4, Chapter 23]. So, here, only orthogonal windings are treated.

For a better starting performance, 120° space-angle-shifted main and auxiliary windings may be used, but they also may be reduced to two orthogonal ones [4, Chapter 23].

Finally, the general case, when the main and auxiliary windings are orthogonal, but they use different number of slots and copper weights, should be treated in phase variables.

10.16.1 Phase Variable Model

In this case, the rotor is modeled by orthogonal windings, d_r and q_r :

$$|I_{m,a,d_r,q_r}| |R_{m,a,d_r,q_r}| - |V_{m,a,d_r,q_r}| = -\frac{d}{dt} |\Psi_{m,a,d_r,q_r}| \quad (10.129)$$

$$\begin{aligned} |I_{m,a,d_r,q_r}| &= |I_m, I_a, I_{d_r}, I_{q_r}|^T \\ |I_{m,a,d_r,q_r}| &= \text{Diag}(R_m, R_a, R_r, R_r) \\ |V_{m,a,d_r,q_r}| &= |V_m, V_a, 0, 0| \end{aligned} \quad (10.130)$$

$$\begin{bmatrix} \Psi_m \\ \Psi_a \\ \Psi_{dr} \\ \Psi_{qr} \end{bmatrix} = \begin{bmatrix} L_{ml} + L_m^m & 0 & L_{mr} \cos \theta_{er} & -L_{mr} \sin \theta_{er} \\ 0 & L_{al} + L_m^a & L_{ar} \sin \theta_{er} & L_{ar} \cos \theta_{er} \\ L_{mr} \cos \theta_{er} & L_{ar} \sin \theta_{er} & L'_{rl} + L_m^m & 0 \\ -L_{mr} \sin \theta_{er} & L_{ar} \cos \theta_{er} & 0 & L'_{rl} + L_m^m \end{bmatrix} \cdot \begin{bmatrix} I_m \\ I_a \\ I_{dr} \\ I_{qr} \end{bmatrix} \quad (10.131)$$

$$T_e = p_1 [I] \frac{\partial |L(\theta_{er})|}{\partial \theta_{er}} [I]^T \quad (10.132)$$

$$\frac{J}{p_1} \frac{d\omega_r}{dt} = T_e - T_{load}; \quad \frac{d\theta_{er}}{dt} = \omega_r; \quad \omega_r = 2\pi p n \quad (10.133)$$

We also should use the constraints

$$\begin{aligned} V_m(t) &= V_m \sqrt{2} \cos(\omega_1 t + \gamma_0) \\ V_m(t) &= V_a(t) + V_C(t) \end{aligned} \quad (10.134)$$

and

$$\frac{dV_C}{dt} = \frac{1}{C} I_a \quad (10.135)$$

When a resistor is added to the auxiliary winding, V_C is replaced by $R_{start} I_a$ and Equation 10.135 is eliminated. The system has a seventh order, with many variable coefficients, and may be solved by numerical methods, albeit within a large CPU time.

This general model gets simplified if the auxiliary winding and the main winding make use of the same copper weight when

$$\begin{aligned} R_a &= R_m a^2; \quad a = w_a k_{wa} / (w_m k_{wm}) \\ L_{al} &= L_{ml} a^2 \\ L_{ar} &= L_{mr} a \\ L_m^a &= L_m^m a^2 \end{aligned} \quad (10.136)$$

In such a case, the dq model in any coordinates may be used because the stator windings become symmetric under conditions 10.136, despite different number of turns.

10.16.2 dq Model

Let us use stator coordinates and consider the d axis along the m (main) winding axis and the q axis along the auxiliary winding axis. There is no need to reduce the auxiliary winding to the main winding as long as Equation 10.136 is satisfied.

The dq model equations are straightforward as the stator windings are already orthogonal, and, thus, no Park transformation is needed (for the phase machine connection in Figure 10.23b the latter is instrumental [4, Chapter 23]:

$$\begin{aligned}
 I_d R_m - V_d &= -\frac{d\Psi_d}{dt}; & I_q R_a - V_q &= -\frac{d\Psi_q}{dt} \\
 I_{dr} R_r &= -\frac{d\Psi_{dr}}{dt} - \omega_r \Psi_{qr}; & I_{qr} R_r &= -\frac{d\Psi_{qr}}{dt} + \omega_r \Psi_{dr} \\
 \Psi_d &= L_{ml} I_d + L_{dm} (I_{dr} + I_d) \\
 \Psi_{dr} &= L'_{rl} I_{dr} + L_{dm} (I_{dr} + I_d) \\
 \Psi_q &= L_{al} I_q + L_{qm} \left(I_q + \frac{1}{a} I_{qr} \right) \\
 \Psi_{qr} &= L'_{rl} I_{qr} + L_{qm} (a I_q + I_{qr})
 \end{aligned} \tag{10.137}$$

Again

$$\frac{L_{qm}}{L_{dm}} = a^2 \text{ and } L_{dm} = L_m^m; \quad I_m = I_d; \quad I_a = I_q \tag{10.138}$$

$$V_d = V_m; \quad V_q = V_a = V_m - V_C(t); \quad \frac{dV_C}{dt} = \frac{I_q}{C} \tag{10.139}$$

$$T_e = -p_1 (\Psi_{dr} I_{qr} - \Psi_{qr} I_{dr}) = p_1 L_{dm} (a I_q I_{dr} - I_d I_{qr}) \tag{10.140}$$

Adding the motion equations (Equation 10.133), the complete model is obtained. The system's order is now six (θ_{er} (the rotor position) is irrelevant here).

Note: Again, the dq model is valid only if conditions 10.136 are satisfied.

For steady state, the $\pm$ space phasor model has already been used in Chapter 5, and hence it is not repeated here. For more on the subject, see [4, Chapters 24 through 28].

Magnetic saturation can be handled simply in the dq model by the $L_m^m(I_m)$ function, with fluxes as variables. Magnetic saturation causes, under steady state, nonsinusoidal stator currents [4, Chapter 25].

10.17 Linear Induction Motor Transients

LIMs are used today in a wide variety of applications, such as urban people movers and propulsion of vehicles on wheels (UTDC in Canada), and with active magnetic suspension by controlled dc-fed electromagnets on board (Figure 10.24) (Japan, Korea).

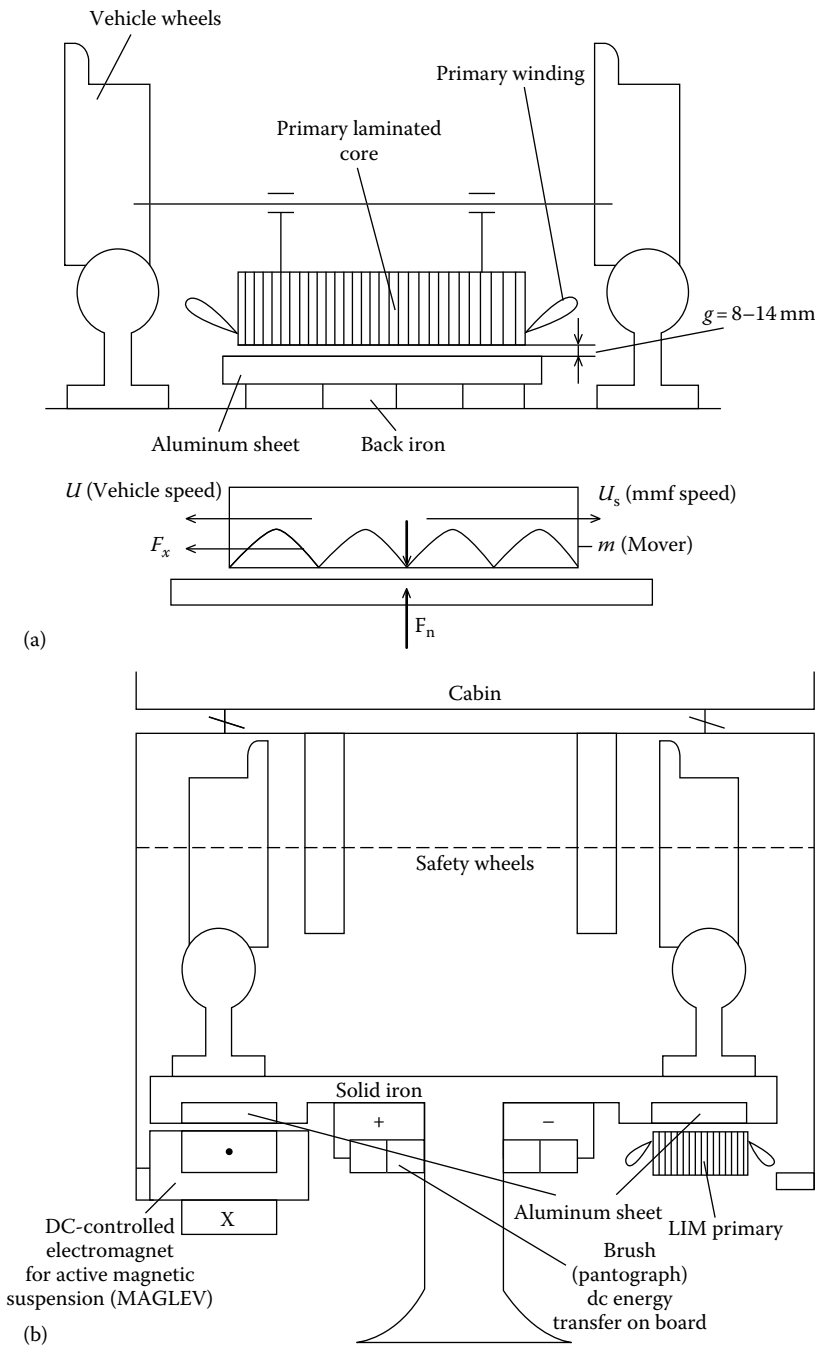


FIGURE 10.24 Single-sided LIM for people movers: (a) Vehicle on wheels and (b) MAGLEV.

Along each side of the MAGLEV in Figure 10.24b, there are LIMs interleaved with dc electromagnets, for suspension control. In this application, both use the same solid iron track (beam) as a back iron core [17].

It should be noticed also that if the normal (vertical) force of the LIM is of attraction type, then it helps the dc-controlled electromagnets in producing active magnetic suspension Figure 10.24b. This is the case in well-designed (for good efficiency) LIMs [15, Chapter 3].

The LIM flux control may be performed to produce certain dynamic properties in propulsion, while adding a 20%–30% to suspension via attraction force up to the base speed, U_b , above which flux weakening might be mandatory for propulsion control.

To a first approximation, we may ignore the frequency and saturation effects in the secondary (solid back iron plus aluminum sheet track). We also ignore the longitudinal end effect, though, even for urban people movers (at 20–40 m/s peak speed), the latter is already affecting performance by reducing thrust (propulsion), efficiency, and power factor (Chapter 5). It is possible to use correction coefficients to reduce F_x and L_m and increase R_r (all variable with slip), to account for the longitudinal end effect. The dq model is thus straightforward (as for LSMs in Chapter 9), but now, in synchronous coordinates,

$$\begin{aligned}\bar{V}_s &= R_s \bar{I}_s + j U_s \pi \bar{\Psi}_s / \tau + d\bar{\Psi}_s / dt; \quad U_s \text{—synchronous speed (m/s)} \\ 0 &= R_r \bar{I}'_r + d\bar{\Psi}'_r / dt + j(U_s - U) \pi \bar{\Psi}'_r / \tau; \quad U \text{—speed (m/s)} \\ F_x &= \frac{3}{2} \frac{\pi}{\tau} \operatorname{Re}(j \bar{\Psi}_s \bar{I}_s^*)\end{aligned}\tag{10.141}$$

with

$$\bar{\Psi}_s = L_s \bar{I}_s + L_m \bar{I}'_r; \quad \bar{\Psi}'_r = L'_r \bar{I}'_r + L_m \bar{I}_s\tag{10.142}$$

Also, approximately, $L'_{rl} \ll L_{sl}$, so it may be neglected, and thus

$$\bar{\Psi}'_r = \bar{\Psi}_m = L_m (\bar{I}'_r + \bar{I}_s) = L_m \bar{I}_m\tag{10.143}$$

So, the secondary flux, $\bar{\Psi}'_r$, is identical to the airgap flux, and, thus, its closed-loop control might lead to the normal (suspension) force, F_x , control.

We should be careful when applying Park transformation to primary variables, because the primary variable is the mover:

$$\bar{I}_s = \frac{2}{3} (I_A + I_B e^{j\frac{2\pi}{3}} + I_C e^{-j\frac{2\pi}{3}}) e^{-j\Theta_s}; \quad \frac{d\Theta_s}{dt} = -\frac{\pi}{\tau} x\tag{10.144}$$

where x is the linear motion variable.

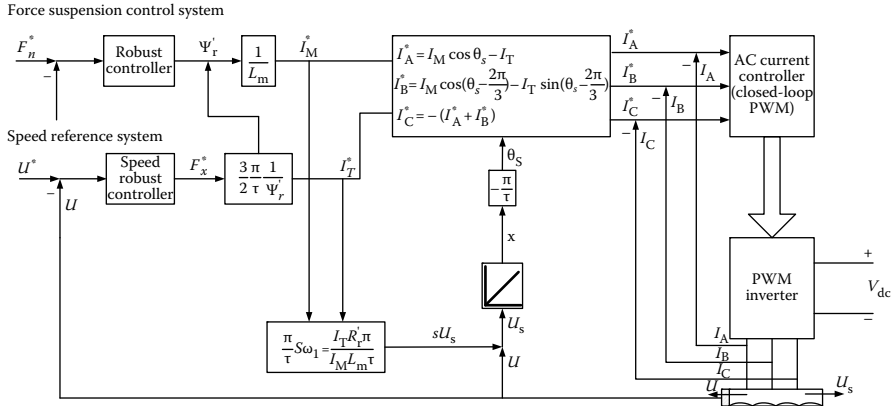


FIGURE 10.25
Vector control of LIM.

To a first approximation, the normal (suspension) attraction force, F_n is

$$F_n \approx \frac{3}{2} I_m^2 \frac{\partial L_m}{\partial g} \quad (10.145)$$

Vector control (for rotor flux orientation) may be used for propulsion, while the normal force, F_n , may be estimated and then controlled as desired, to reduce noise and vibration, and help active magnetic suspension (Figure 10.25).

Let us recall that not only we ignored the frequency effects in the secondary (upon R'_r) and magnetic saturation in the solid back iron (on R'_l and L_m), but also that L_m (magnetization inductance) varies with the airgap due to track irregularities and that a dynamic error of 20%–25% is allowed in the magnetic suspension airgap (height) control, in order to limit peak kVAs in the PWM converters used to control the dc electromagnets, for MAGLEVs.

There are two ways to handle such a situation:

- Better modeling
- More robust propulsion and suspension control

Both are worthy of investigation but the second looks more practical and might deserve the first shot. However, 3D FE circuit models or a ladder secondary circuit model having a large number of bars (as in this chapter: the mN_r model, Section 10.11) have been tried to simulate transient performance, including end effects, for transients. For control system design, simpler models to account for longitudinal end effect, frequency and saturation effects in secondary and more practical airgap [16], secondary flux, Ψ'_r , and speed, U , estimators are all still due.

10.18 Summary

- When connected (or reconnected “on the fly”) to the power grid, under various grid faults, after load torque perturbations or voltage sags, or under PWM converter supplies, the induction motor voltages and current amplitude and frequency, and speed vary in time. That is, they undergo transients.
- The phase variable model, with stator/rotor circuit mutual inductances, variable with the rotor position, is the most general circuit model applicable to IM transients. Unfortunately, it implies, for numerical solutions, large CPU time.
- For 3-phase symmetric stator and rotor windings and uniform air-gap (zero rotor eccentricity), the phase variable IM model may be transformed simply into the dq0 model. The parameter equivalence is very simple: the phase resistances and leakage inductances remain unchanged and the magnetization inductance, L_m , is the cyclic one ($L_m = 1.5L_{11m}$).
- The zero sequence stator and rotor current equations relate only to stator (or rotor) resistances and leakage inductances. They do not contribute to torque but produce additional losses. For a star connection, they are zero.
- For 6-phase IMs, two dq0 models are required.
- Magnetic saturation can be simply introduced in the dq model by $L_m(I_m) = \Psi_m/I_m$ and $L_{mt} = d\Psi_m/dI_m$ as functions of the resultant (magnetization) current, I_m .
- The rotor frequency (skin) effects may be introduced in the dq0 model through additional (fictitious) rotor cage constant parameter circuits in parallel. Core loss windings may be also introduced in the dq model [18].
- Steady state of space phasor model in general coordinates, ω_b , means a frequency of $\omega_1 - \omega_b$ for all variables (ω_1 stator voltage frequency). In synchronous coordinates $\omega_b = \omega_1$, the variables are dc in the steady state.
- Under steady state, for cage rotor conditions, the rotor flux linkage, $\bar{\Psi}'_{r0}$, and rotor current, $\bar{I}'_{r0}$, space phasors are orthogonal.
- Even during transients, if the amplitude of the rotor flux remains constant, $\bar{\Psi}'_r$ and $\bar{I}'_r$ remain orthogonal.
- For the motor mode, the stator space phasor, $\bar{I}_s$, is ahead of $\bar{\Psi}_s$ in the direction of motion; the opposite is true for the generator mode.

- For a zero rotor current in the doubly fed IM ($V_r' \neq 0$), the ideal no-load speed is obtained at the slip, $S_0 \neq 0$, which is positive or negative; consequently, motor and generator operations are feasible both for $S < 0$ and $S > 0$ when the rotor is fed at a frequency, $\omega_2 = S\omega_1$, through a bidirectional PWM ac-ac converter.
- In a doubly fed IM (DFIM), the machine magnetization can be done from the rotor source or from the stator source. Note that in the DFIM, the stator frequency and voltage are, in general, constant, while V_r' and its frequency, ω_2 , in the rotor are variable such that $\omega_2 = \omega_1 - \omega_r$; it may be supposed that the DFIM operates as a synchronous machine with ac rotor excitation; the truth is there is an additional power component in the stator and rotor that is asynchronous [13].
- For a DFIM in subsynchronous operation ($S > 0$), the power enters on one side and exits through the other for motor and generator operations.
- In the DFIM supersynchronous mode ($S < 0$), the power enters or exits from both sides for motor and generator operations. For a limited speed range, $|S_{\max}| < 0.25$, the rotor side PWM bidirectional converter is sized at $|S_{\max}|P_n$, and thus it costs less. This is applied in most modern wind generator systems.
- Electromagnetic transients mean constant speed transients. For constant parameters, operational (Laplace) parameters may be defined for rotor coordinates in the dq model, as done for synchronous machines.
- The 3-phase sudden short circuit may be approached in Laplace formulation. For the two rotor circuit model, the short-circuit current transients exhibit three time constants: one due to the stator and two due to the rotor circuits. It may be used to identify, by curve fitting, the machine inductances, L'' , L' , and L_s and the time constants τ'' , τ_0'' , τ' , τ_0' , and τ_a .
- The small deviation theory is used to linearize the dq model of IM for electromechanical transients (ω_r is also variable); a fifth-order system is obtained and its eigenvalues may be used to check the stability conditions.
- For large deviation transients, the direct usage of the dq model (with magnetic saturation and the rotor frequency effect included, if necessary) is required. Direct connection to the power grid or large load torque perturbations are typical of large deviation transients. For small inertia, the transient torque/speed curves are far away from steady-state curves, exhibiting up to 10/1 current peaks and 5–6 torque peaks, and speed and current oscillations.

- For multimachine transients, reduced order dq model is required. Ignoring the stator transients ($d\Psi_d/dt = d\Psi_q/dt = 0$) in synchronous coordinates, $\omega_b = \omega_1$, is typical.
- Basic vector control schemes are given to illustrate the principles and relate to modern “electric drives,” which is, anyway, a separate subject.
- Doubly fed IMs can also be controlled, as a generator/motor, for a limited speed range, by vector control in the rotor, but regulating the stator active and reactive powers separately. This possibility is also described in this chapter.
- The model and performance of a DFIM as a brushless exciter to SMs, with rotor output voltage at a frequency, $\omega_2 = |\omega_1| + |\omega_r| > |\omega_1|$, with most part of output power extracted from shaft (mechanical) power, is introduced; the output is controlled from the stator by a Soft starter (Variac) at constant ω_1 and variable voltage (decreasing with speed). This is a widely used industrial solution, as its costs are reasonable, it works from zero speed, has small voltage regulation, and fast SM field current response due to the large stator voltage ceiling at the rated SM speed.
- Standstill tests to identify IM parameters are presented in detail as they are not standard in many places. Flux decay tests are used to determine both the magnetization curve and the machine time constants (by curve fitting).
- Standstill frequency response (SSFR) tests from 0.01 to 100 Hz (in general) are used to determine the operational impedance, $\underline{Z}(j\omega)$, amplitude, and phase angle. By curve fitting of $|\underline{Z}(j\omega)|$, the machine time constants, visible in the operational inductance, are determined. Alternatively, the maxima of the $\underline{Z}(j\omega)$ argument (angle) frequencies $f'_c, f''_c, \dots$ and values $\varphi', \varphi'', \dots$ lead to a very simple calculation routine of time constant pairs, $\tau' < \tau'_0$ and $\tau'' < \tau''_0$.
- Disconnection from the power grid and immediate reconnection (before the residual stator voltage has dropped notably) in the primary of the feeding local grid transformer have shown very high transients in torque and current, especially in small IMs. Such events may endanger the integrity of IMs and should be avoided by adequate protection mechanisms.
- To investigate broken rotor bars or end-ring segments, each rotor loop (bar) has to be modeled, with $m + N_r + 1 + 1$ (m —stator phases, N_r —rotor bars, 1—ring current I_e ($I_e = 0$ for healthy rings), 1—for motion equation) equations (variables). The various stator/rotor loop coupling inductances are defined analytically and they depend on the rotor position.

- The non-infinite rotor bar to the core wall resistance leads to inter-bar currents that tend to reduce broken bar effects, and thus make the former's diagnosis more difficult.
- Controlled rotor and stator fluxes are typical in variable speed motor and generator control via PWM inverter supplies.
- If the space phasor model is used (with single rotor circuit model) the stator current, $\bar{I}_s$, and rotor, $\bar{\Psi}'_r$, or stator, $\bar{\Psi}_s$, flux space phasors are used as variables, at constant speed (slip) electromagnetic transients, only two complex eigenvalues occur. Their real part depends on the speed (slip) and their imaginary part depends on the coordinate system's speed, ω_b . It means that all electromagnetic transients can be treated with analytical solutions in space phasors.
- For a constant stator and rotor flux, only one complex eigenvalue remains for constant speed (slip) transients. This is a formidable simplification that leads, for constant rotor flux, to a straight line torque/speed curve as in separately excited dc brush machine. The stator current may be decomposed into two components, one for flux and one for torque which can be controlled separately. This type of vector control is characterized by fast torque control for a wide speed range and good performance.
- Split-phase capacitor IMs are still extensively used in home appliances as only a single-phase power supply is available. A phase variable model for two orthogonal (but essentially different in slot occupancy and copper weight) stator windings (the main and the auxiliary (starting) windings) and a cage rotor is introduced. When the copper weight of the main and auxiliary windings are the same, the dq model may be used in any coordinate to investigate transients. Still, a six-order system is obtained.
- A linear induction motor (three phase) with an aluminum sheet on solid iron track is used in urban (suburban) transportation, on wheels and in MAGLEVs (with active magnetic suspension). LIMs have secondary skin and saturation effects and the longitudinal end effect (Chapter 5). If they are neglected, the dq model can be used with thrust, F_x , in place of torque, and linear speed, U (in m/s), instead of angular rotor speed, ω_r (in rad/s). The single-sided LIM develops a normal (suspension) force, which is of attraction type at a small slip frequency, $S\omega_1$, in well designed LIMs. This normal force may provide 20%–25% of total suspension (vehicle weight) force if the thrust is capable of 1 m/s^2 vehicle acceleration. Vector control is thus feasible. The constant rotor flux reference is related to suspension control by additional dc electromagnets. As there are quite a few LIMs and dc electromagnets on a MAGLEV vehicle, a complex system has to

be modeled and controlled for good steady state and dynamic performance.

- Power electronics frequency control has transformed the IM from the work horse to the race horse of industry. This trend is here apparently to stay.

10.19 Proposed Problems

- 10.1** Write the phase variable equations of a two orthogonal phase stator cage rotor IM.

Hints: Check Section 10.1 but notice the $\sin \theta_{er}$ and $\cos \theta_{er}$ dependence of the mutual stator/rotor inductances.

- 10.2** The magnetization inductance, L_m , of an IM has the dependence on the magnetization current, I_m , as

$$L_m(I_m) = \frac{L_{m0}}{a + bI_m}$$

Calculate the transient inductance function, $L_{mt}(I_m)$.

Hints: Check Equation 10.24 for the L_{mt} expression and use it.

- 10.3** A cage rotor induction motor has the following parameters: $R_s = 1.0 \Omega$, $R'_r = 0.7 \Omega$, $L_m = 0.2 \text{ H}$, $L_{sl} = L'_{rl} = 6 \text{ mH}$, and $p_1 = 2$ pole pairs. It operates at 1800 rpm with $S = 0.02$ and rotor flux, $\Psi'_{r0} = 1 \text{ Wb}$.

Calculate

- Rotor current, $\tilde{I}'_{r0}$, and ω_1 (stator frequency)
- Magnetization, $\bar{\Psi}_m$, and stator flux, $\bar{\Psi}_{s0}$
- Stator current, $\bar{I}_{s0}$
- The stator voltage phasor, $\bar{V}_{s0}$

Hints: Check Example 10.1, but start with Equation 10.28, then Equation 10.29, etc.

- 10.4** A doubly fed IM has the following parameters: $R_s = R_r = 0.02 \Omega$, $X_{sl} = X'_{rl} = 0.2 \Omega$, $X_m = 15 \Omega$ at $f_{1n} = 60 \text{ Hz}$, $2p_1 = 4$, and operates at a slip $S = +0.25$ as a motor, with the stator power factor in the stator $\cos \phi_s = 0.93$ (lagging) at a stator phase current $I_{nph} = 1200 \text{ A}$ and $V_{line} = 4200 \text{ V}$ (line voltage, RMS).

Calculate

- Stator flux vector, $\bar{\Psi}_{s0}$
- Rotor flux vector, $\bar{\Psi}'_{r0}$
- Rotor voltage vector, $\bar{V}'_{r0}$
- Stator and rotor active and reactive powers, P_s , Q_s , P_r , and Q_r

Hints: Check Example 10.2 and notice the non-zero stator current phase angle, φ_s , with respect to voltage.

- 10.5** For the single cage rotor IM with the data as given in Example 10.3 at zero speed, calculate the flux, $\Psi_s(t)$, and $T_e(t)$ transients and represent them in graphs, for a direct connection to the power grid, to $V_{en} = 220$ V (RMS)-star connection.

Hints: Use directly Equations 10.49 through 10.51.

- 10.6** The single-cage rotor IM in Problem 10.3 operates at steady state and a slip frequency $\omega_{20} = 2\pi$ rad/s, $\omega_{10} = 2\pi \times 60$ rad/s, and voltage $V_1 = 380$ V (line voltage).

- Calculate the space phasors, $\bar{I}_{s0}$, $\bar{I}'_{r0}$ (I_{d0} , I_{q0} , I'_{dr0} , I'_{qr0}), and the torque, $T_{e0} = T_{L0}$ (Example 10.1).
- Calculate the eigenvalues of the linearized dq model system around the steady-state point of (a) based on the equation:

$$|A|s|\Delta X| + |B||\Delta X| = 0$$

with $|A|$ and $|B|$ from Equations 10.57 and 10.58.

- 10.7** Based on theory and the motor data given in Section 10.11, write a MATLAB Simulink program and simulate 1, 2, 3, 4 broken bars, plotting stator current, broken bars currents, and torque versus time.

- 10.8** Calculate, based on Equation 10.80, the complex eigenvalues of IM for $R_s = R_r = 0.1 \Omega$, $L_s = L'_r = 32$ mH, $L_m = 30$ mH, for $S = 1, 0.02, -0.02$, at a frequency $\omega_1 = 2\pi \times 60$ rad/s, in stator coordinates ($\omega_b = 0$) and synchronous coordinates ($\omega_b = \omega_1$). Discuss the results.

- 10.9** Calculate the eigenvalues, $\underline{s}$, for the IM in Problem 10.8, for constant rotor flux, at a low frequency, $\omega_1 = 2\pi \times 1.2$ rad/s, and $s = \pm 0.7$ (low speed) and discuss the results.

Hints: Use Equation 10.83 in synchronous coordinates.

- 10.10** A cage rotor induction motor operates at constant rotor flux $R_r = R'_r = 0.1 \Omega$, $L_s = L'_r = 0.093$ H, $L_m = 0.09$ H, $2p_1 = 4$, $n = 120$ rpm with $I_m = 20$ A and slip frequency $S\omega_1 = \pm 2\pi \times 1$ rad/s.

Calculate

- a. Developed torque
- b. Frequency, ω_1
- c. Stator phase voltage, current, and power factor

Hints: Follow closely Example 10.5.

- 10.11** Find from Equation 10.95, the eigenvalues, s , for the IM in Problem 10.8 for a constant stator flux at low frequency $\omega_1 = 2\pi \times 1.2$ rad/s and $S = \pm 0.7$ (low speed full torque) and synchronous coordinates. Discuss the results.
- 10.12** The doubly fed IM in Problem 10.4 operates at a constant rotor flux $\Psi'_r = 10$ Wb and $S = -0.25$, as a motor for $T_e = 42.3$ kNm, $\omega_1 = 2\pi \times 50$ rad/s, and at unity stator power factor.

Calculate

- a. Stator current I_{d0} and I_{q0}
- b. Rotor current space phasors, $I'_r = jI'_{qr}$; $\bar{\Psi}'_r = \Psi'_{dr0} = L_m I_{ds0}$
- c. Rotor voltage components, V_{dr} and V_{qr}
- d. Rotor active and reactive power, P'_r and Q'_r
- e. Stator voltage, $\bar{V}_s$ (phase voltage, RMS), stator powers, P_s and Q_s , and total power, $P_{\text{tot}} = P'_r + P_s$
- f. Angle between $\bar{V}_s$ and $\bar{V}'_r$

Hints: See Example 10.2 and Section 10.13 and notice the unity power factor in the stator, when calculating the stator voltage, V_s .

- 10.13** The SSFR (standstill frequency response) of an induction machine impedance (in pu) shows an amplitude of 3 pu inductance ($l(s)$) at zero frequency. Its phase has two maxima, one of $\varphi'_c = -30^\circ$ at $f'_C = 7$ Hz and the other of $\varphi''_c = -15^\circ$ at $f''_C = 60$ Hz. Making use of the phase method, identify the time constants, τ' , τ'_0 and τ'' , τ''_0 , and finally write $l(s)$ (in pu) (Equation 10.123) with $r_s = 0.02$ (in pu) and compare with the results in Figure 10.22.

Hints: Check and use Equations 10.125 through 10.128 for the case in point.

- 10.14** A split-phase induction motor has two orthogonal windings that use the same copper weight (they fulfill Equation 10.136) and has the

following parameters: $R_m = 20 \Omega$, $L_{ml} = 0.2 \text{ H}$, $L_{dm} = 10L_{ml}$; $p_1 = 1$ pole pair, $a = w_a k_{wa}/w_m k_{wm} = 1.2$, and $J = 10^{-4} \text{ kgm}^2$.

Calculate

- Auxiliary winding parameters, R_a , L_{al} , and L_{qm}
- Write a MATLAB code to investigate starting and other transients using the dq model for a resistance, R_{start} , for start and a capacitor for running. The switching between the two is instantaneous and takes place at a desired moment in time. Also, provide for the possibility to introduce load torque variations in time and with speed. Debug and run the program for $R_{start} = 3R_a$ and $C_{run} = 4 \times 10^{-6} \text{ F}$ and $V_m = 120\sqrt{2}\cos(2\pi 60t)$.

Hints: Use Equations 10.136 through 10.139.

- 10.15** A 3-phase single-sided linear induction motor for an urban MAGLEV has the following parameters: pole pitch $\tau = 0.25 \text{ m}$ (number of poles $2p_1 = 8$). The longitudinal end effect and secondary frequency effects are neglected, rated airgap the $g = 10 \text{ mm}$, aluminum sheet thickness, $h_{Al} = 5 \text{ mm}$, and the back iron permeabilities (both in the primary and in the track) are infinite; it operates at constant rotor flux ($L'_{rl} = 0$): $L_m = L'_r$. The LIM operates at a speed of $U = 30 \text{ m/s}$ and at a slip $S = 0.1$, $\Psi'_r = \Psi_m = 1.5 \text{ Wb}$, $I_T/I_M = 1.6/1$, base thrust, $F_{xn} = 12 \text{ kN}$; and $R_s = R_r/2.5$.

Calculate

- Primary frequency, $f_1(\omega_1)$
- Secondary resistance, R'_r , stator resistance, R_s , and magnetization inductance, L_m : $L_{sl} = 0.35L_m$
- Normal (attraction) force considering that L_m varies inversely proportional to $(g + h_{Al})$
- The stator flux components in rotor flux orientation and finally the stator phase voltage, current, $\cos \varphi_s$, efficiency (only winding losses are considered)

Hints: Follow Equations 10.140 through 10.144 for F_x and F_n expressions and then Example 10.5 for the rest of the questions. Expected results: $f_s = 66.66 \text{ Hz}$, $I_M = 265 \text{ A}$, $R_r = 0.148 \Omega$, $L_m = 5.625 \times 10^{-3} \text{ H}$, $V_s \approx 920 \text{ V}$ (phase peak value), $\cos \varphi_s \approx 0.61$, $\eta \approx 0.85$, $F_n = 39.8 \text{ kN}$ (this is slightly more than three times F_{xn}), $P_{elm} = F_{xn}U = 360 \text{ kW}$.

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Part III

FEM Analysis and Optimal Design

Essentials of Finite Element Method in Electromagnetics

11.1 Vectorial Fields

Though apparently a rather abstract concept, “field,” a form of matter, plays a key role in explaining electric and magnetic phenomena with deep implications on the design of electric machines [1–17]. Mathematically, there are scalar fields such as those of temperatures, pressures, and current densities, and vectorial fields such as electric fields, magnetic fields, and mechanical stresses fields in solid bodies. For scalar fields, a scalar is assigned to each point in space. For vectorial fields, a vector is linked to every point in space. The relations between scalar and magnetic fields are based on the fundamental laws of electromagnetics, known as Maxwell equations. But first the main properties of operations with vectors are introduced.

11.1.1 Coordinate Systems

In *Cartesian coordinates* (Figure 11.1), vector $\vec{A}$ is defined by its projections along the orthogonal coordinate axes:

$$\vec{A} = A_x \cdot \vec{u}_x + A_y \cdot \vec{u}_y + A_z \cdot \vec{u}_z \quad (11.1)$$

where $\vec{u}_x$, $\vec{u}_y$, and $\vec{u}_z$ are unitary vectors aligned to the orthogonal axes x , y , and z .

For *cylindrical coordinates* (Figure 11.2), the application point of vector, $P(r, \theta, z)$, is defined by the cylinder radius, r , its angle, θ , with axis x , and the height, z . The unitary vectors along the coordinate axes are $\vec{u}_r$, $\vec{u}_\theta$, and $\vec{u}_z$. The transformation matrix between Cartesian and cylindrical coordinates is given by Equation 11.2:

$$\begin{pmatrix} A_r \\ A_\theta \\ A_z \end{pmatrix} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} \quad (11.2)$$

The cylindrical coordinates are useful where the investigated field shows a cylindrical symmetry as in the case of a radial electric field produced by a straight-line conductor, which is electrostatically loaded ($\bar{E}_r \neq 0$, $\bar{E}_\theta = 0$, and $\bar{E}_z = 0$), or the magnetic field of the same conductor flowed by current ($\bar{B}_r = 0$, $\bar{B}_\theta \neq 0$, and $\bar{B}_z = 0$). For the points on the z -axis of cylindrical coordinates, the unitary vectors, $\bar{u}_\theta$ and $\bar{u}_z$, are not defined.

Spherical coordinates represent a set of curve-line coordinates used to naturally describe the position on a sphere. The coordinates are given by r , the distance to the origin; θ the azimuthal angle (the angle between the position vector projection in xoy plan and the positive semiaxis, “ x ”); and ϕ , that is, the zenithal angle (the angle of the position vector with the positive semiaxis, “ z ”). The unitary vectors of vector $\bar{A}$ (Figure 11.3) are $\bar{u}_r$, $\bar{u}_\theta$, and $\bar{u}_\phi$, and their orientation dependence on the application point coordinates is portrayed in Equation 11.3:

$$\begin{pmatrix} \bar{u}_r \\ \bar{u}_\theta \\ \bar{u}_\phi \end{pmatrix} = \begin{pmatrix} \cos(\theta) \sin(\phi) & \sin(\theta) \sin(\phi) & \cos(\phi) \\ -\sin(\theta) & \cos(\theta) & 0 \\ \cos(\theta) \cos(\phi) & \sin(\theta) \cos(\phi) & -\sin(\phi) \end{pmatrix} \cdot \begin{pmatrix} \bar{u}_x \\ \bar{u}_y \\ \bar{u}_z \end{pmatrix} \quad (11.3)$$

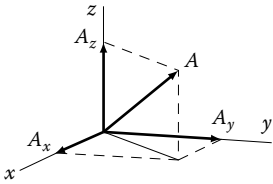


FIGURE 11.1
Cartesian coordinates.

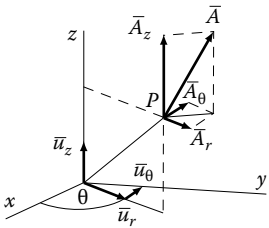


FIGURE 11.2
Cylindrical coordinates.

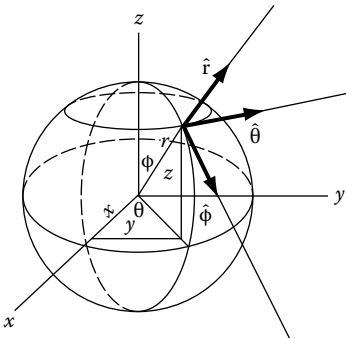


FIGURE 11.3
Spherical coordinates.

The unitary vectors, $\bar{u}_r$, $\bar{u}_\theta$, and $\bar{u}_\phi$, are not univoquely defined at the point of origin. The transformation matrix between Cartesian and spherical coordinates is given by Equation 11.4:

$$\begin{pmatrix} A_r \\ A_\theta \\ A_\phi \end{pmatrix} = \begin{pmatrix} \cos(\theta) \sin(\phi) & -\sin(\theta) & \cos(\theta) \cos(\phi) \\ \sin(\theta) \sin(\phi) & \cos(\theta) & \sin(\theta) \cos(\phi) \\ \cos(\phi) & 0 & -\sin(\phi) \end{pmatrix} \cdot \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} \quad (11.4)$$

The spherical coordinates are very useful for fields having spherical symmetry, such as the electrostatic field produced by a point-shaped charge where only the radial component of the field is nonzero ($\bar{E}_r \neq 0$, $\bar{E}_\theta = 0$, and $\bar{E}_\phi = 0$).

11.1.2 Operations with Vectors

Let us consider two vectors $\bar{A}$ and $\bar{B}$ with amplitudes $|A|$ and $|B|$, and phase shift α . Their scalar product is defined as

$$\bar{A} \cdot \bar{B} = |A| \cdot |B| \cdot \cos(\alpha) \quad (11.5)$$

In Cartesian coordinates the scalar product is

$$\bar{A} \cdot \bar{B} = A_x B_x + A_y B_y + A_z B_z \quad (11.6)$$

The vectorial product is

$$\bar{A} \times \bar{B} = |A| \cdot |B| \cdot \sin(\alpha) \cdot \bar{n}_{AB} \quad (11.7)$$

where $\bar{n}_{AB}$ is the unitary vector that is normal to the plane of vectors $\bar{A}$ and $\bar{B}$, with the direction given by the right-hand rule.

In Cartesian coordinates the vectorial product is

$$\begin{aligned} \bar{A} \times \bar{B} &= \begin{vmatrix} \bar{u}_x & \bar{u}_y & \bar{u}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \\ &= (A_y B_z - A_z B_y) \bar{u}_x + (A_z B_x - A_x B_z) \bar{u}_y + (A_x B_y - A_y B_x) \bar{u}_z \end{aligned} \quad (11.8)$$

There are some properties of operations with vectors such as

1. Absolute value:

$$\bar{A} \cdot \bar{A} = |A|^2 \quad (11.9)$$

2. Commutation of scalar product:

$$\bar{A} \cdot \bar{B} = \bar{B} \cdot \bar{A} \quad (11.10)$$

3. Anticommutation of vectorial product:

$$\bar{A} \times \bar{B} = -\bar{B} \times \bar{A} \quad (11.11)$$

4. Distributivity of scalar product for addition:

$$\bar{A} \cdot (\bar{B} + \bar{C}) = \bar{A} \cdot \bar{B} + \bar{A} \cdot \bar{C} \quad (11.12)$$

5. Distributivity of vector product for addition:

$$\bar{A} \times (\bar{B} + \bar{C}) = \bar{A} \times \bar{B} + \bar{A} \times \bar{C} \quad (11.13)$$

6. Distributivity of double vector product:

$$\bar{A} \times (\bar{B} \times \bar{C}) = (\bar{A} \cdot \bar{C})\bar{B} - (\bar{A} \cdot \bar{B})\bar{C} \quad (11.14)$$

11.1.3 Line and Surface (Flux) Integrals of a Vectorial Field

By the *line integral* of a vectorial field we mean the integral of the scalar product between the respective vector and the unitary vector that is tangent to that line (curve) at every point:

$$L_{12} = \int_c \bar{A} \cdot \bar{dl} \quad (11.15)$$

In Cartesian coordinates the line integral is

$$L_{12} = \int_c \bar{A} \cdot \bar{dl} = \int_{P1}^{P2} (A_x dx + A_y dy + A_z dz) = \int_{x1}^{x2} A_x dx + \int_{y1}^{y2} A_y dy + \int_{z1}^{z2} A_z dz \quad (11.16)$$

The line integral of a vectorial field has a well-defined physical meaning. For a field of forces it is the mechanical work, while for the magnetic field it is the number of ampere turns (magnetomotive force) required to produce the field between the two points. If the result L_{12} does not depend on the shape

of the line, but relies on the initial and final points, then the field is called *conservative*. A field is conservative when and only when its integral on any closed line (curvature) is zero. The flux of a vectorial field through a surface S is given by the surface integral of the scalar product between vector $\vec{A}$ and its unitary vector normal (at 90°) to the surface S :

$$\phi = \int_S \vec{A} \cdot d\vec{a} = \int_S (\vec{A} \cdot \vec{n}) da \quad (11.17)$$

The expression of ϕ in Cartesian coordinates is

$$\phi = \int_S \vec{A} \cdot d\vec{a} = \int_S A_x dydz + \int_S A_y dx dz + \int_S A_z dx dy \quad (11.18)$$

For a fluid, the surface integral of its speed vector represents the volumic flow rate through that surface.

11.1.4 Differential Operations

The *gradient* of a scalar field, φ , is a vectorial field, whose vectors show at every point in space the direction along which the variation of the scalar field is maximum, $\vec{u}_{\max}$; its amplitude is equal to the scalar field derivative along that direction:

$$\text{grad}(\vartheta) = \nabla \vartheta = \max \left(\frac{\partial \vartheta}{\partial l} \right) \vec{u}_{\max} \quad (11.19)$$

In Cartesian coordinates the gradient is

$$\text{grad}(\vartheta) = \nabla \varphi = \frac{\partial \vartheta}{\partial x} \vec{u}_x + \frac{\partial \vartheta}{\partial y} \vec{u}_y + \frac{\partial \vartheta}{\partial z} \vec{u}_z \quad (11.20)$$

The *surface gradient* is defined for surfaces along which the scalar field is discontinuous with the normal direction, $\vec{n}_\Sigma$, to the surface and the amplitude equal to the difference between scalar field values on the two faces of the surface:

$$\text{grad}_\Sigma (\vartheta) = (\varphi_2 - \varphi_1) \vec{n}_\Sigma \quad (11.21)$$

The gradient of a *scalar field* is always a conservative field. The “*rotor*” (vortex) of a vectorial field is defined as the limit of the ratio between a closed surface integral of a *vectorial product* between the respective vector and the normal to the surface unit vector, to the volume closed by the surface when the latter tends to zero:

$$\text{rot}(\vec{A}) = \nabla \times \vec{A} = \lim_{V \rightarrow 0} \frac{1}{V} \oint_\Sigma \vec{n} \times \vec{A} dS \quad (11.22)$$

The projection of the rotor of a vectorial field on a given direction, $\bar{n}$, is equal to the limit of the field integral along a closed line (curve) of the surface closed by the respective line when the latter tends to zero; the closed line (curve) is situated in a plane defined by its normal, $\bar{n}$:

$$\bar{n} \cdot \text{rot}(\bar{A}) = \lim_{S \rightarrow 0} \frac{1}{S} \oint_C \bar{A} \cdot d\bar{l} \quad (11.23)$$

In Cartesian coordinates the rotor expression is

$$\begin{aligned} (\bar{A}) = \nabla \times \bar{A} &= \begin{vmatrix} \bar{u}_x & \bar{u}_y & \bar{u}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} \\ &= \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) \bar{u}_x + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) \bar{u}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \bar{u}_z \end{aligned} \quad (11.24)$$

If the field is not continuous through a surface, then its surface rotor is defined as the vectorial product between the normal to the surface, $\bar{n}_\Sigma$, and the difference between vector values on the two sides of the discontinuity surface:

$$\text{rot}_\Sigma(\bar{A}) = \bar{n}_\Sigma \times (\bar{A}_2 - \bar{A}_1) \quad (11.25)$$

The “divergence” of a field is a scalar (associated to every point) that shows the tendency of the field to spring from ($\text{div}(\bar{A}) > 0$) or to ($\text{div}(\bar{A}) < 0$) convergence to a respective point. The divergence of a field is the limit of the ratio between the field flux through a closed surface and the volume closed by that surface when the latter tends to zero:

$$\text{div}(\bar{A}) = \nabla \cdot \bar{A} = \lim_{V \rightarrow 0} \frac{1}{V} \oint_\Sigma \bar{A} \cdot d\bar{S} \quad (11.26)$$

In Cartesian coordinates the divergence is

$$\text{div}(\bar{A}) = \nabla \cdot \bar{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \quad (11.27)$$

If the field is discontinuous on the surface, a surface divergence is defined as

$$\text{div}_\Sigma(\bar{A}) = (\bar{A}_2 - \bar{A}_1) \cdot \bar{n} \quad (11.28)$$

The fields whose divergence is zero at every point are called *solenoidal fields*; the flux density field is a solenoidal field.

The *Laplacian* is a second-order derivative operator, denoted by Δ (or ∇^2). The fields whose Laplacian is zero at every point are called *harmonic fields*.

The *Laplacian of a scalar field* is also a scalar field equal to the divergence of its gradient:

$$\nabla^2 \varphi = \nabla \cdot (\nabla \varphi) = \text{div}(\text{grad}(\varphi)) \quad (11.29)$$

In Cartesian coordinates the Laplacian of a scalar field is

$$\nabla^2 \varphi = \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} \quad (11.30)$$

The *Laplacian of a vectorial field* is

$$\nabla^2 \vec{A} = \nabla(\nabla \cdot \vec{A}) - \nabla \times (\nabla \times \vec{A}) = \text{grad}(\text{div}(\vec{A})) - \text{rot}(\text{rot}(\vec{A})) \quad (11.31)$$

The Laplacian of a vectorial field in Cartesian coordinates is written as

$$\begin{aligned} \nabla^2 \vec{A} &= \nabla^2 A_x \vec{u}_x + \nabla^2 A_y \vec{u}_y + \nabla^2 A_z \vec{u}_z \\ &= \left(\frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_x}{\partial y^2} + \frac{\partial^2 A_x}{\partial z^2} \right) \vec{u}_x + \left(\frac{\partial^2 A_y}{\partial x^2} + \frac{\partial^2 A_y}{\partial y^2} + \frac{\partial^2 A_y}{\partial z^2} \right) \vec{u}_y \\ &\quad + \left(\frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_z}{\partial y^2} + \frac{\partial^2 A_z}{\partial z^2} \right) \vec{u}_z \end{aligned} \quad (11.32)$$

11.1.5 Integral Identities

The *first theorem of the gradient* says that the line integral of a field of gradients between points P_1 and P_2 equals the difference between the scalar field values φ_1 and φ_2 at the two points:

$$\varphi_{12} = \int_{P_1}^{P_2} \text{grad}(\varphi) \cdot \vec{dl} = \varphi_2 - \varphi_1 \quad (11.33)$$

The *second theorem of the gradient* says that the volume integral of a field of gradients is equal to the scalar field integral at the origin over the surface that closes the respective volume:

$$\int_V \text{grad}(\varphi) \, dV = \oint_{\Sigma} \varphi \, \vec{dS} \quad (11.34)$$

This theorem of the gradient is applied particularly in electrostatics.

The *rotor (Kelvin–Stokes) theorem* states that the closed-line integral of a vectorial field is equal to the flux produced by the rotor of the respective vectorial field through a surface S bordered by the respective closed line:

$$\oint_c \vec{A} \cdot \vec{dl} = \int_S \text{rot}(\vec{A}) \cdot \vec{dS} \quad (11.35)$$

The *divergence* (Ostrogradsky–Gauss) *theorem* says that the flux of a vectorial field through a closed surface is equal to the integral of its divergence on the volume closed by the respective closed surface:

$$\int_{\Sigma} \bar{A} \cdot d\bar{S} = \int_V \text{div}(\bar{A}) dV \quad (11.36)$$

This theorem is particularly important in formulating the laws of electric and magnetic fluxes.

The *first theorem of Green* is the equivalent of the integrals by parts for scalar and vectorial fields, respectively; the *second Green's theorem* is a direct application of the first theorem for symmetric expressions.

The first theorem of Green for scalar fields is

$$\int_{\tau} U \text{div}(k \text{grad}(V)) + k \text{grad}(U) \cdot \text{grad}(V) d\tau = \oint_S k U \frac{\partial V}{\partial n} d\bar{S} \quad (11.37)$$

The second theorem of Green is

$$\int_{\tau} U \text{div}(k \text{grad}(V)) - V \text{div}(k \text{grad}(U)) d\tau = \oint_S k \left(U \frac{\partial V}{\partial n} - V \frac{\partial U}{\partial n} \right) d\bar{S} \quad (11.38)$$

For vectorial fields, the first theorem of Green is

$$\int_{\tau} k \text{rot}(\bar{A}) \cdot \text{rot}(\bar{B}) - \bar{A} \cdot \text{rot}(k \text{rot}(\bar{B})) d\tau = \oint_S k \bar{A} \times \text{rot}(\bar{B}) \cdot d\bar{S} \quad (11.39)$$

while the second one is

$$\int_{\tau} \bar{B} \cdot \text{rot}(k \text{rot}(\bar{A})) - \bar{A} \cdot \text{rot}(k \text{rot}(\bar{B})) d\tau = \oint_S k [\bar{A} \times \text{rot}(\bar{B}) - \bar{B} \times \text{rot}(\bar{A})] \cdot d\bar{S} \quad (11.40)$$

The Green's theorems are used in the finite element method (FEM) to determine the expressions of the coefficients of the algebraic equation system that substitute the partial derivative field equations with boundary conditions.

11.1.6 Differential Identities

The differential operators for the product of a constant k and a scalar field U , and, respectively, a vectorial field, $\bar{A}$, produce results identical with the product of the constant k and the respective field operator:

$$\text{grad}(kU) = k \text{grad}(U) \quad (11.41)$$

$$\text{rot}(k\bar{A}) = k \text{rot}(\bar{A}) \quad (11.42)$$

$$\text{div}(k\bar{A}) = k \text{div}(\bar{A}) \quad (11.43)$$

$$\nabla^2(k\bar{A}) = k \nabla^2(\bar{A}) \quad (11.44)$$

The differential operators for the summation of two scalar fields, U and V , and, respectively, two vectorial fields, $\vec{A}$ and $\vec{B}$, are equal to the summation operators applied separately to the two terms:

$$\text{grad}(U + V) = \text{grad}(U) + \text{grad}(V) \quad (11.45)$$

$$\text{rot}(\vec{A} + \vec{B}) = \text{rot}(\vec{A}) + \text{rot}(\vec{B}) \quad (11.46)$$

$$\text{div}(\vec{A} + \vec{B}) = \text{div}(\vec{A}) + \text{div}(\vec{B}) \quad (11.47)$$

$$\nabla^2(U + V) = \nabla^2(U) + \nabla^2(V) \quad (11.48)$$

For the product of two fields, these rules do not generally apply. For the gradient, however,

$$\text{grad}(UV) = U\text{grad}(V) + V\text{grad}(U) \quad (11.49)$$

$$\text{rot}(U\vec{A}) = U\text{rot}(\vec{A}) + \text{grad}(U) \times \vec{A} \quad (11.50)$$

$$\text{div}(U\vec{A}) = U \text{div}(\vec{A}) + \text{grad}(U) \cdot \vec{A} \quad (11.51)$$

$$\text{div}(\vec{A} \times \vec{B}) = -\vec{A} \cdot \text{rot}(\vec{B}) + \text{rot}(\vec{A}) \cdot \vec{B} \quad (11.52)$$

$$\nabla^2(UV) = U\nabla^2V + V\nabla^2U + 2 \text{grad}(U) \cdot \text{grad}(V) \quad (11.53)$$

The gradient of composed fields is

$$\text{grad}(U(V)) = \frac{\partial U}{\partial V} \text{grad}(V) \quad (11.54)$$

The rotor of the fields of gradients is always zero:

$$\text{rot}(\text{grad}(U)) = 0 \quad (11.55)$$

The divergence of a field of rotors is also always zero:

$$\text{div}(\text{rot}(\vec{A})) = 0 \quad (11.56)$$

The last two identities bear a special importance on electromagnetic fields, by allowing to define the scalar and vectorial magnetic potential concepts.

11.2 Electromagnetic Fields

11.2.1 Electrostatic Fields

Electrostatics investigates the electric field produced by electric charges in the absence of magnetic field variations.

The electric field, $\vec{E}$, is nonrotor type and is called electrostatic:

$$\text{rot}(\vec{E}) = 0 \quad (11.57)$$

Consequently, the electrostatic field is a field of gradients, derived from a scalar field called the electrostatic potential, V :

$$\vec{E} = -\text{grad}(V) \quad (11.58)$$

The displacement vector, $\vec{D} = \epsilon \vec{E}$, through a closed surface, Σ , is equal to the electric charge within the closed surface (the Gauss law):

$$\oint_{\Sigma} \epsilon \vec{E} dS = \int_V \rho dv \quad (11.59)$$

The local form of the Gauss law is

$$\text{div}(\epsilon \vec{E}) = \rho \quad (11.60)$$

By substituting Equation 11.58 in Equation 11.60, the electric potential (V) equation is obtained:

$$\text{div}(\epsilon \text{ grad}(V)) = -\rho \quad (11.61)$$

Making use of differential identities in Section 11.1.6 yields

$$\epsilon \nabla^2 V + \nabla \epsilon \cdot \nabla V = -\rho \quad (11.62)$$

In most cases, the electric permittivity of a medium is constant, and thus $\nabla \epsilon$ in Equation 11.62 is zero, and, thus, the Poisson equation is obtained:

$$\nabla^2 V = -\frac{\rho}{\epsilon_0} \quad (11.63)$$

In charge-less domains ($\rho = 0$), the Laplace equation holds:

$$\nabla^2 V = 0 \quad (11.64)$$

11.2.2 Fields of Current Densities

The motion of electric charges is described by the current density vector, $\vec{J}$:

$$\vec{J} = q n \vec{V}_d = \rho \vec{V}_d \quad (11.65)$$

where

q is the particle charge

n is the particle count per unit volume

$\vec{V}_d$ is the average (drift) speed

The electric current through a surface is given by the surface integral of the current density over the surface:

$$I = \int_S \vec{J} \cdot d\vec{S} \quad (11.66)$$

The total electric current through a closed surface is equal to the time variation of electric charge within the closed surface.

The local form of the law of electric charge conservation is expressed by the continuity equation:

$$\text{div}(\bar{J}) = -\frac{\partial \rho}{\partial t} \quad (11.67)$$

It is possible to define the total current density as the summation of conduction and displacement components:

$$\bar{J}_{\text{tot}} = \bar{J} + \frac{\partial \bar{D}}{\partial t} \quad (11.68)$$

The continuity equation of the total current density thus becomes

$$\text{div}(\bar{J}_{\text{tot}}) = 0 \quad (11.69)$$

so the total current density is a solenoidal field.

The relationship between the current density, $\bar{J}$, and the electric field, $\bar{E}$, in conductors is

$$\bar{J} = \sigma \bar{E} \quad (11.70)$$

where σ is the electric conductivity.

11.2.3 Magnetic Fields

The magnetic field is a component of the electromagnetic field that exerts forces on electric charges in motion. The magnetic field is characterized by two vectorial variables (concepts) that are the magnetic field, $\bar{H}$, and the magnetic flux density, $\bar{B}$:

$$\bar{B} = \mu \bar{H} \quad (11.71)$$

where μ is the permeability of the medium in henry per meter (H/m).

The flux density field is solenoidal:

$$\text{div}(\bar{B}) = 0 \quad (11.72)$$

So the magnetic field may be derived from a vector potential, $\bar{A}$:

$$\bar{B} = \text{rot}(\bar{A}) \quad (11.73)$$

The magnetic field is produced by electric charges in motion, and the relationship between the forces of the magnetic field and the respective field is given by *Ampere's law*. Ampere's law states that the closed-line integral of

the magnetic field is equal to the electric current through the surface that subtends the closed line.

$$\oint_{\Gamma} \vec{H} \cdot d\vec{l} = i \quad (11.74)$$

The local form of Ampere's law is written as

$$\text{rot}(\vec{H}) = \vec{J}_{\text{tot}} \quad (11.75)$$

The relationship between the magnetic potential, A , and the magnetic field sources is

$$\text{rot}\left(\frac{1}{\mu}\text{rot}(\vec{A})\right) = \vec{J}_{\text{tot}} \quad (11.76)$$

11.2.4 Electromagnetic Fields: Maxwell Equations

The electromagnetic field is a physical field produced by electric charges in motion, which influences the behavior of electric charges. It may be seen as a combination of the electric and the magnetic fields. Maxwell has put together in a set of equations the electric charges, the field production, and the interaction:

$$\begin{aligned} \text{div}(\vec{E}) &= \frac{\rho}{\varepsilon_0} \\ \text{div}(\vec{B}) &= 0 \\ \text{rot}(\vec{E}) &= -\frac{\partial \vec{B}}{\partial t} \\ \text{rot}(\vec{B}) &= \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \end{aligned} \quad (11.77)$$

where

ε_0 is the vacuum permittivity

μ_0 is the vacuum permeability

These equations are valid also for other mediums with constants ϵ and μ . For the domains free of electric charges ($\rho = 0$) and without electric current ($J = 0$), the equations of electromagnetic waves are obtained:

$$\nabla^2 \vec{E} = \frac{1}{c^2} \cdot \frac{\partial^2 \vec{E}}{\partial t^2} \quad (11.78)$$

$$\nabla^2 \vec{B} = \frac{1}{c^2} \cdot \frac{\partial^2 \vec{B}}{\partial t^2} \quad (11.79)$$

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \quad (11.80)$$

where c is the speed of light in vacuum.

The waves equations (Equation 11.78) are useful in designing antennas, and wave guides in telecommunications, and in investigating the electromagnetic compatibility of electric devices.

11.3 Visualization of Fields

A few methods for representing and visualizing fields have been introduced here in order to facilitate the understanding of the intricate physical phenomena related to fields. For scalar fields, the direct methods of the equipotential surface (equipotential field lines in 2D space) are used, in general. As an example, Figure 11.4 represents the electric (scalar) potential produced by two straight conductors of infinite lengths charged electrostatically with opposite polarities. A similar method uses colored maps to represent 2D scalar fields or sections in 3D space. To every point in plane a distinct color is assigned. To visualize vectorial fields, the vectors of the field are shown by a few points. The orientation of the arrows coincides with the orientation of the vectors of the field and their length is proportional to their amplitude (see, e.g., Figure 11.5).

A widespread method for the visualization of vectorial fields consists of showing the field lines (paths). The concept of field line (path) was introduced by Faraday. The field line (path) is an imaginary line that is tangent at every point to the field vector at that point.

Figure 11.6 illustrates the magnetic field paths produced by two (opposite polarity) parallel electric currents of conductors of infinite lengths. The geometric description of field lines means describing that field. For magnetic (solenoidal) fields, the field lines are dense in the zones with an intense (strong) field; valuable information on field amplitude distribution is

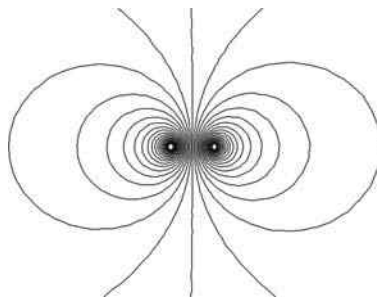


FIGURE 11.4

Electric potential of two electrostatically charged conductors.

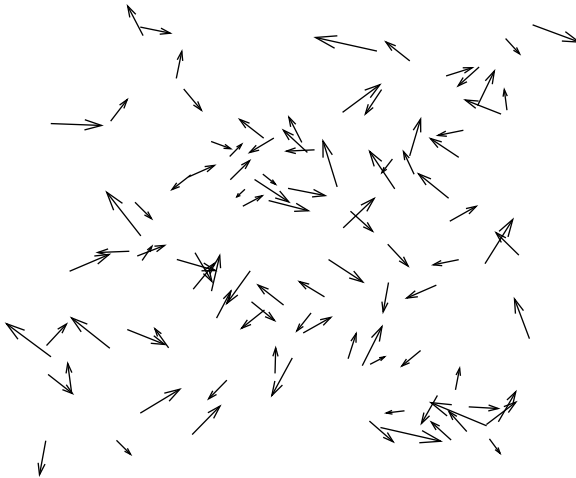


FIGURE 11.5
Speeds field of chaotic particles.

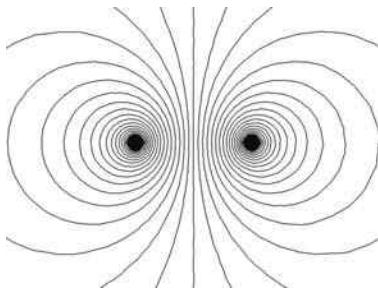
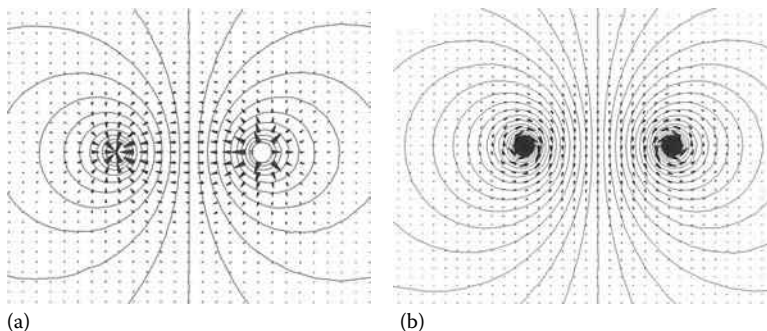


FIGURE 11.6
Magnetic field lines of two parallel conductors with currents of opposite polarity.

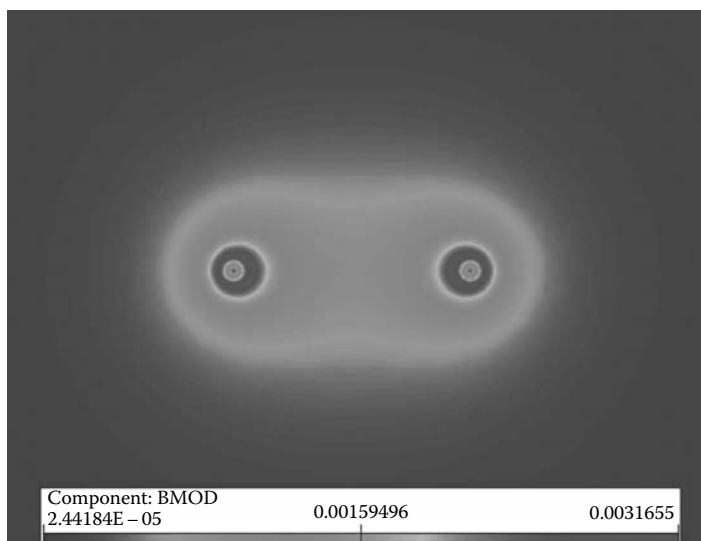
obtained in this way. For nonsolenoidal fields, there are points where field lines appear or disappear. For 2D-domain magnetic fields, the representation of lines is performed by drawing the magnetic potential contour lines (the components perpendicular to the plane of symmetry). When the scalar potential concept is used to solve the field problem, in order to represent the magnetic field lines, a scalar variable equivalent to the normal component of the magnetic potential is defined. For most commercial FEM softwares, it is difficult to do such a thing. We may, however, represent the field by arrows associated with field vectors, together with the equipotential lines of the applied field potential. Figure 11.7a and b illustrates comparatively the

**FIGURE 11.7**

Comparative representation of (a) the electrostatic field (field of gradients) and (b) the magnetostatic field (field of rotors).

field vectors and the equipotential lines for an electrostatic and a magnetostatic field, respectively.

The vectorial fields may be represented by equipotential lines or the color map, for the vectors' amplitudes and projections along a given direction. The distribution of the magnetic flux density amplitude produced by two currents (Figure 11.8) implies the need to add a legend for the color/amplitude correspondence.

**FIGURE 11.8**

Visualization of vectorial fields by the color map method.

11.4 Boundary Conditions

Differential and partial derivative equations accept a unique solution only if correct initial boundary conditions are given.

11.4.1 Dirichlet’s Boundary Conditions

Dirichlet’s boundary conditions imply that the magnetic potential of the investigated field is specified along the boundary.

$$\Phi = \Phi_f \tag{11.81}$$

where Φ is the generic vector potential.

If $\Phi_f = 0$ on the boundary, homogenous Dirichlet’s conditions are obtained. For a unique (single) field solution, Φ_f has to be specified at least at one point on the boundary. For such a magnetic field, the field lines are tangent to the surface, while for an electrostatic field, they are normal to that surface (boundary) (Figure 11.9).

11.4.2 Neumann’s Boundary Conditions

Neumann’s boundary conditions specify the field potential derivative along the normal to the surface of the boundary. If this derivative is equal to zero, Neumann’s conditions are called homogenous:

$$\frac{\partial \Phi}{\partial n} = 0 \tag{11.82}$$

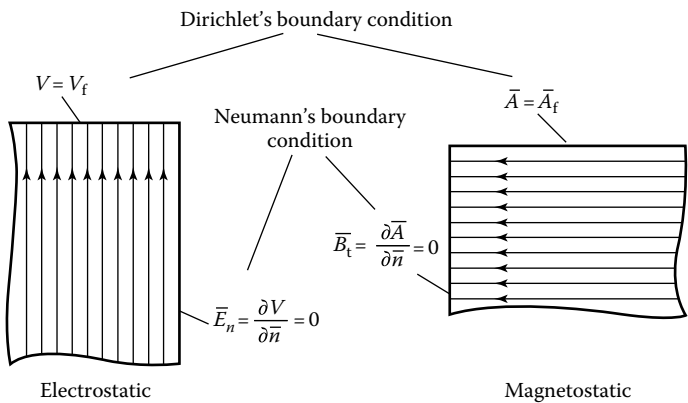


FIGURE 11.9
Dirichlet’s and Neumann’s boundary conditions.

For such a magnetic field, the field lines are normal to the surface, while for an electrostatic field, they are tangent to the surface (boundary) (Figure 11.9).

11.4.3 Mixed Robin's Boundary Conditions

$$\frac{\partial \Phi}{\partial n} + k\Phi = \Phi_g \quad (11.83)$$

If the composite magnetic potential, Φ_g , is zero we again have homogeneous conditions. Robin's boundary conditions allow for defining boundary impedances through which the effect of fields outside the boundary is considered.

11.4.4 Periodic Boundary Conditions

Periodic boundary conditions allow to reduce the computation effort (zone) by the use of symmetry. The field potential on the two boundaries Γ_1 and Γ_2 is the same for homologous points as in Figure 11.10b (even symmetry), or has equal and opposite values as in Figure 11.10c (odd symmetry):

$$\Phi_{\Gamma_1} = \pm \Phi_{\Gamma_2} \quad (11.84)$$

The symmetry conditions may be imposed only for identical boundaries (those that may be overlapped by translation or rotation). The shape of the boundaries may be quite intricate (Figure 11.11), which allows for the utilization of symmetric boundary conditions even if a part of the geometry moves.

11.4.5 Open Boundaries

Open boundaries are used in the absence of natural boundaries, such as in the case of core-loss solenoidal coils, when the magnetic potential decreases to zero at infinity. Three main solutions for such problems are recommended in the literature:

11.4.5.1 Problem Truncation

The method of problem truncation implies the selection of an arbitrary boundary sufficiently away from the zone of interest. The distance to the arbitrary boundary has to be at least five times longer than the radius of the zone of interest, to secure good precision results [6]. The main disadvantage of this method consists in the large number of discretization elements outside the zone of interest, which leads to a larger computation effort (time).

11.4.5.2 Asymptotical Boundary Conditions

Outside the external boundaries there is no field source, and thus the field potential converges to zero when the radius goes to infinity. For 2D magnetic

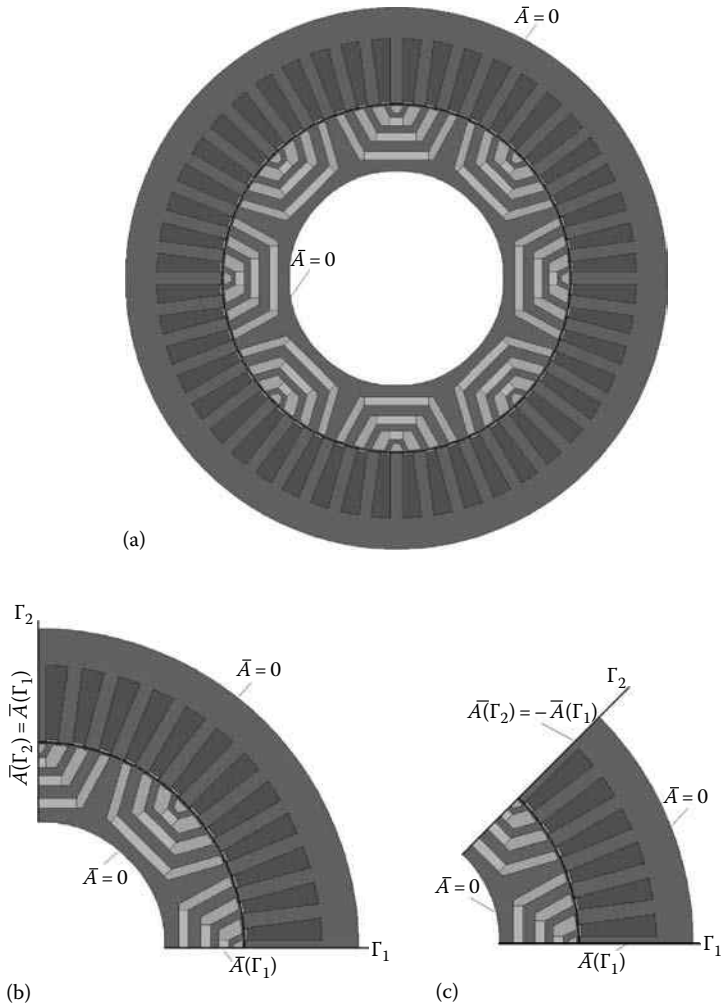
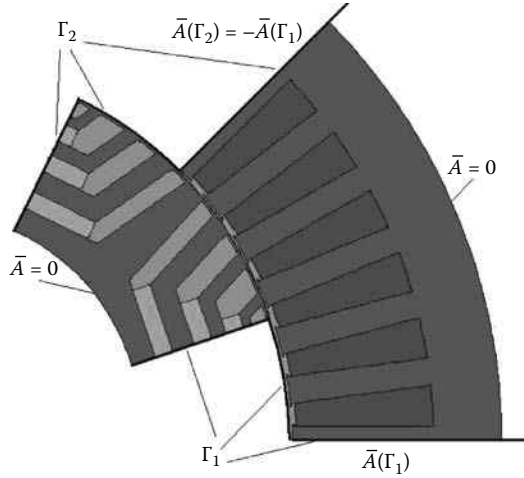


FIGURE 11.10 Symmetric boundary conditions for (a) full motor, (b) even symmetry, and (c) odd symmetry.

field problems, admitting the potential A on the circular boundary to be dependent on an angle θ , an analytical solution exists:

$$A(r, \theta) = \sum_{k=1}^{\infty} \frac{a_k}{r^k} \cos(k\theta + \alpha_k) \tag{11.85}$$

where a_k and α_k are coefficients calculated such that the analytical solution (valid outside the boundary), on the boundary, coincides (fits) with the finite element solution inside the boundary.


FIGURE 11.11

Symmetrical boundary conditions and rotor displacement.

As high harmonics attenuate rapidly, only one term, n , may be retained:

$$A(r, \theta) \approx \frac{a_n}{r^n} \cos(n\theta + \alpha_n) \quad (11.86)$$

The derivative of Equation 11.86 along the radius r (normal to the boundary) yields

$$\frac{\partial A}{\partial r} + \frac{n}{r}A = 0 \quad (11.87)$$

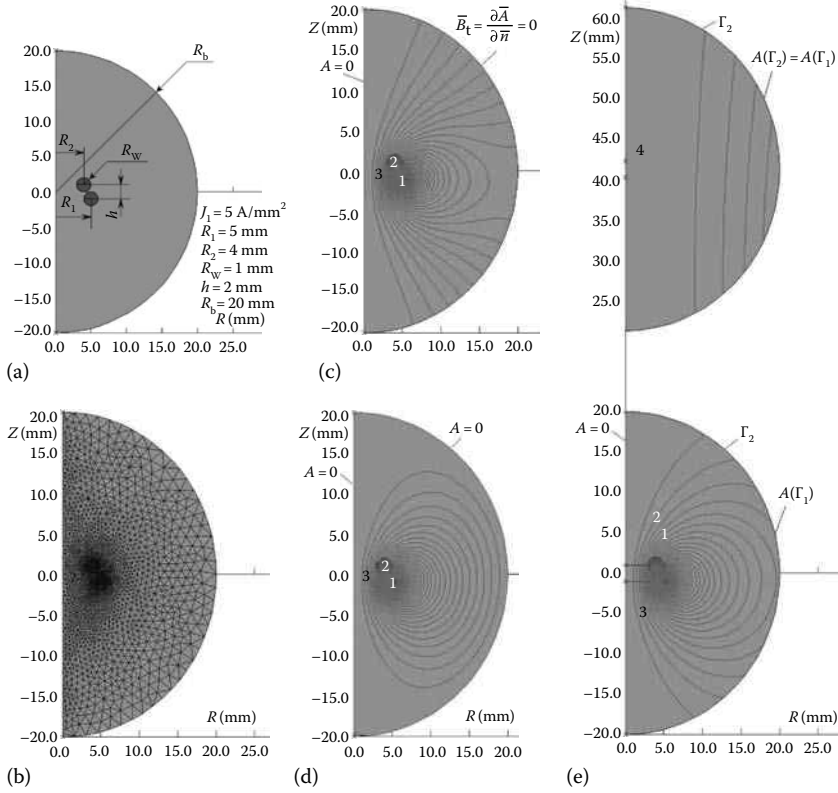
It should be noticed that Equation 11.87 is equivalent to Robin's condition (Equation 11.83) where

$$k = \frac{n}{r}; \quad \Phi_g = 0 \quad (11.88)$$

By imposing mixed boundary conditions the precision of field solution increases, without additional computation effort [6].

11.4.5.3 Kelvin Transform

For remote field regions, the field is rather homogenous, because in general there are no field sources there and the medium is air or vacuum. In such cases, the magnetic potential observes Laplace's equation. The Kelvin transform (Equation 11.89) maps the points, r , outside the circle of radius r_0 into

**FIGURE 11.12**

Boundary conditions when the field extends to infinity. (a) Dual conductor geometry, (b) discretization mesh, (c) magnetic field lines for Neumann's boundary conditions, and (d) Dirichlet's boundary conditions.

points inside the circle, R (Figure 11.12a), while the Laplace equation (Equation 11.90) takes a similar form with respect to r or R :

$$R = \frac{r_0^2}{r} \quad (11.89)$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial A}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 A}{\partial \theta^2} = 0 \quad (11.90)$$

By the r to R variable change, the boundary at infinity is mapped to the center of the circle, r_0 , and thus

$$A_R(R = 0) = 0 \quad (11.91)$$

Symmetrical conditions are imposed on the boundaries of the investigated and created domains:

$$\frac{\partial A}{\partial r} = -\frac{\partial A}{\partial R} \quad (11.92)$$

Figure 11.12 illustrates the single-turn coil current produced magnetic field. Cylindrical symmetry of the problem leads to the use of cylindrical coordinates in solving the magnetic field problem. As the FEM can investigate finite domains, the investigated infinite domain is mapped into a sphere of radius R_b (Figure 11.2a), which is four times larger than the coil radius. Neumann's boundary conditions (Figure 11.12d) apply for infinite permeability outside the sphere. Dirichlet's boundary conditions (Figure 11.12d) hold for zero permeability (superconducting material) outside the sphere R_b . By using the Kelvin transform, the exterior of sphere R_b is reduced to a sphere with the same radius as the investigated domain, but without field sources. By imposing symmetrical conditions (Figure 11.2e) on the two spheres, the correct solution of the field problem is obtained. The field lines are quite different near the boundary for the three situations. However, as will be shown in Section 11.6, the self and mutual inductances of coils 1 and 2 are about the same, mainly because the domain radius ratio $R_b/r_0 = 4$.

11.5 Finite Element Method

To simplify the mathematics, the system of differential equations, which describes a phenomenon, is written in the operational form:

$$L\Phi(P, t) = f(P, t) \quad (11.93)$$

where L is a generic operator that symbolizes the system of equations with partial derivatives, ϕ is a space, and function P and time t represent

The FEM solves approximately partial derivative and integral equations [3–5]. With this method, the field domain is divided into a finite number of simple elements (triangles, plane rectangles, tetrahedrons, prisms, and parallelepipeds) and the solution of the field problem is approximated by simple functions.

$$\Phi^*(P, t) = \sum_j^N \Phi_j v_j(P, t) \quad (11.94)$$

where v_j are interpolations or expansions of basic functions and ϕ_j are coefficients still to be determined. Physically, ϕ_j represents the magnetic potential in the nodes of the discretization mesh.

The equivalent system of equations should be stable, in the sense that the errors from the input data and those from intermediate calculations should not accumulate and produce gross (useless) results.

11.5.1 Residuum (Galerkin's) Method

Galerkin's method operates directly with differential equations. The exact solution ϕ , which satisfies Equation 11.93, is approximated by a simple function ϕ^* , which satisfies the relationship:

$$L\Phi^* - f = r \quad (11.95)$$

The smaller the r the better the approximation. A globally good approximation over the entire domain is targeted. The residuum, r , is integrated over the entire domain, after multiplication by a weighting function, w_i , and this integral is forced to zero:

$$R_i = \int_D w_i (L\Phi^* - f) d\tau = 0 \quad (11.96)$$

The most used residuum method is Galerkin's method where the weighting functions are chosen equal to the expansion functions, v_j (Equation 11.94):

$$R_i = \int_D \left(v_i \left(\sum_{j=1}^N \Phi_j v_j \right) - v_i f \right) d\tau \quad (11.97)$$

By forcing all terms in Equation 11.97 to zero, a system of matrix equations is obtained:

$$S \cdot \Phi = T \quad (11.98)$$

where

S is the system (or stiffness) matrix, because the FEM was first used in mechanical engineering

ϕ is the column matrix of the vector potential in the mesh nodes

T is a column matrix, whose terms depend on function f

$$t_i = \int_D v_i f d\tau \quad (11.99)$$

The elements of the stiffness matrix, S , depend on the interpolation functions, v_i and v_j :

$$S_{ij} = \frac{1}{2} \int_D (v_i L v_j + v_j L v_i) d\tau \quad (11.100)$$

The matrix S is a sparse matrix. Additionally, if Equation 11.97 holds, the matrix S is symmetric:

$$\begin{aligned}\langle L\Phi, \varphi \rangle &= \langle \Phi, L\varphi \rangle \\ S_{ij} &= S_{ji} = \int_D v_i L v_j d\tau\end{aligned}\quad (11.101)$$

In such a case, the algebraic system of equations by Galerkin and variational methods are identical.

11.5.2 Variational (Rayleigh–Ritz) Method

Starting again from the system of differential equations, a functional $F(\phi)$ is defined such that its minimum with respect to ϕ is the solution of the system [1];

$$F(\Phi) = \frac{1}{2} \langle L\Phi, \Phi \rangle - \frac{1}{2} \langle \Phi, f \rangle - \frac{1}{2} \langle f, \Phi \rangle \quad (11.102)$$

To define the functional, the inner product of two functions is expressed as the integral of one function multiplied by the complex conjugate of the second function:

$$\langle \Phi, \varphi \rangle = \int_D \Phi \bar{\varphi} d\tau \quad (11.103)$$

The function ϕ is substituted with ϕ^* in Equation 11.94. The ϕ_j coefficients are calculated by zeroing the derivative of F , with respect to the former:

$$\frac{\delta F}{\delta \Phi_j} = 0; i = 1, 2, 3, \dots, N \quad (11.104)$$

For the variational method, condition (11.101) has to hold. In such conditions, the function that minimizes F is a system (Equation 11.93) solution [1]. The stiffness matrix is always symmetric and the system of algebraic equations is identical to that of Galerkin's method. When the boundary conditions are nonhomogenous, Equation 11.101 is not satisfied. In such a case, the functional definition has to consider the boundary nonhomogenities:

$$F(\Phi) = \frac{1}{2} \langle L\Phi, \Phi \rangle - \frac{1}{2} \langle L\Phi, \Psi \rangle + \frac{1}{2} \langle \Phi, L\Psi \rangle - \frac{1}{2} \langle \Phi, f \rangle - \frac{1}{2} \langle f, \Phi \rangle \quad (11.105)$$

where Ψ is any particular function that satisfies the nonhomogeneous boundary conditions.

11.5.3 Stages in Finite Element Method Application

11.5.3.1 Domain Discretization

The investigated field domain is divided into N elements. If the domain is 1D, the respective curve is divided into linear segments. For 2D problems, the

domain is a surface and every subdomain is a polygon, in general, a triangle or a rectangle. Finally, for 3D problems, the volumic domain is divided into tetrahedrons and prisms with triangular and parallelepipedic bases.

The precision of this method strongly depends on the number of elements, N , and on how the mesh was built.

11.5.3.2 Choosing Interpolation Functions

The interpolation functions approximate the unknown function in every subdomain (finite element). The general form of the interpolation is given by Equation 11.94, where P represents the coordinates of a point in a subdomain. The first- and second-order polynomials are, in general, used as interpolation functions.

11.5.3.3 Formulation of Algebraic System Equations

The residuum or variational methods yield the algebraic system equations of Equation 11.98, where the unknown vector refers to the scalar or the vector potential in the nodes of the mesh.

11.5.3.4 Solving Algebraic Equations

When the material in the field domain is linear, the coefficients of the algebraic equations are constant, and the Gauss–Seidel method is used often to solve the algebraic equations. If the material (iron core) is nonlinear, the coefficients in the stiffness matrix are not constant anymore, and the Newton and Raphson method [2] is often used to solve the algebraic equations.

11.6 2D FEM

Many field problems in electrical engineering can be solved two dimensionally by observing the parallel plane symmetry in rotary electrical machines, in electrostatic fields around in power transmission lines, or the axial symmetry in solenoidal coils and other tubular, linear electric actuators. For electric machines with plane symmetry, the magnetic fluxes are calculated for unit depth (stack length) and then multiplied by the stack length to find the actual magnetic flux in the machine. It is also necessary to calculate separately the coils' end-connection inductances by analytical or equivalent 2D FEM methods. The axial variation of the magnetic field is neglected and so is the effect of the various frame parts. With the exception of small-stack-length electric machines, the 2D FEM gives satisfactory results in the presence of the plane symmetry. For 2D $(x, y; r, \theta)$ magnetic fields, the magnetic potential, A , has

only one nonzero component (along the z axis) thereby reducing the computation effort by one order of magnitude.

In what follows, the derivation of the algebraic equations for the magnetostatic 2D fields is pursued.

Equation 11.96, for $\bar{A} \rightarrow A_z$, can be written as

$$\frac{1}{\mu} \frac{\partial^2 A_z}{\partial x^2} + \frac{1}{\mu} \frac{\partial^2 A_z}{\partial y^2} = -J_z \quad (11.106)$$

The first-order interpolation function for the steady state is

$$A(x, y) = a + bx + cy \quad (11.107)$$

where a , b , and c are constants whose values may be determined by imposing the value of A_z in the nodes of the discretization mesh:

$$\begin{pmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} \quad (11.108)$$

The rotor of A_z inside a finite element does not depend on the chosen point.

$$\text{rot}(\bar{A}) = \text{rot}(0, 0, A_z) = (c, -b, 0) \quad (11.109)$$

The expression of a functional whose steady-state solution is the field solution from the m mesh elements becomes

$$\begin{aligned} F_m &= \int_{Q_m} \left(\frac{1}{2} \bar{B} \cdot \bar{H} - \bar{J} \cdot \bar{A} \right) dS = \frac{1}{2\mu} \int_{Q_m} (\text{rot}(\bar{A}) \cdot \text{rot}(\bar{A})) dS - J_z \int_{Q_m} A_z dS \\ &= \frac{1}{2\mu} Q_m (b^2 + c^2) - J_z Q_m (a + bx_m + cy_m) \end{aligned} \quad (11.110)$$

where

x_m and y_m are centroid coordinates of triangle m

Q_m is the area of subdomain m

Coefficients a , b , and c depend on the coordinates of the nodes of the domain m and on the magnetic potential in these nodes; consequently, in matrix form, Equation 11.105 becomes

$$F_m = \frac{1}{2} A_{123}^t S_m A_{123}^t - A_{123}^t T_m \quad (11.111)$$

where

A_{123} is the column vector of A_z in the nodes of element m

S_m is the stiffness matrix

T_m is the column vector electric currents imposed as the field source

The elements of these matrices, s_i and t_i , are

$$s_{ij} = \frac{q_i q_j + r_i r_j}{4\mu Q_m} \quad (11.112)$$

$$t_i = -\frac{Q_m}{3} J_z \quad (11.113)$$

where q_1, q_2 , and q_3 and r_1, r_2 , and r_3 [1] are obtained by solving Equation 11.108:

$$\begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \quad (11.114)$$

$$\begin{pmatrix} r_1 \\ r_2 \\ r_3 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \quad (11.115)$$

The functional over the entire field domain is

$$F = \sum_{m=1}^M F_m = \frac{1}{2} A^T S A - A T \quad (11.116)$$

with the condition

$$\frac{\partial F}{\partial A_i} = 0 \quad \text{for } i = 1, 2, 3, \dots, N \quad (11.117)$$

The algebraic equations system is thus obtained, and is ready to be solved:

$$S A - T = 0 \quad (11.118)$$

11.7 Analysis with FEM

After the field solution is obtained, it has to be verified. This is done first by visualizing the field lines. This way all analytical symmetries are verified. The magnetic field lines never meet or intersect each other. If it happens, it may be that the graphic resolution is too small or the field solution is wrong. The magnetic flux lines are always closed. Sometimes they close through the boundary and then part of them is not visible. The total electric current through the area closed by a flux line should be nonzero with the exception that the flux lines cross regions with permanent magnetization. For a parallel plane symmetric geometry, the magnetic flux through any surface that intersects two flux lines is the same irrespective of the surface shape

of points of intersection, and is equal to the difference between the magnetic potential attributed to the two field lines.

Visualization of a magnetic flux density map provides valuable information, in the sense that the designer may reduce or enlarge some portions of the magnetic circuit. Also, the consolidation elements, ventilation channels, will be placed in regions with very low flux density. For rotary and linear electric machines, the distribution of the radial component of airgap flux density along the rotor periphery is very important in predicting torque pulsations, vibration, and noise. The total magnetic flux in a coil with a magnetic core is made of main and leakage components. Using FEM the total flux in the coil may be calculated by applying Equation (11.35). For an infinitely thin-side coil with plane symmetry, the total flux is

$$\phi_b = (A_2 - A_1) l_z \quad (11.119)$$

where

A_1 and A_2 are the magnetic potentials in the points in the plane intersected by the coil sides
 l_z is the coil length

If the magnetic vector potential is not constant along the transverse cross-section (because of leakage flux of coils placed in electric machine slots), an average flux per coil, ϕ_b , is calculated:

$$\phi_b = \left(\frac{1}{S_2} \int_{S_2} A dS - \frac{1}{S_1} \int_{S_1} A dS \right) l_z \quad (11.120)$$

By using Equation 11.120, the distribution factor of coils in slots can be considered, coil by coil.

The current density in the coil wires is considered constant along the cross-section and so no skin or proximity effect is considered.

The self-inductance of a coil may be calculated either from the magnetic field energy, E_m , or from the coil flux with current, I_b :

$$L_b = \frac{2E_m}{I_b^2} \quad (11.121)$$

$$L_b = \frac{\phi_b}{I_b} \quad (11.122)$$

The two formulae have to produce the same result.

With the flux density, $\vec{B}$, and the magnetic field, $\vec{H}$, expressed as functions of the magnetic potential, $\vec{A}$, and applying Green's theorem (Equation 11.39),

the magnetic energy, E_m , formula becomes

$$2E_m = \int_V \vec{B} \cdot \vec{H} dv = \int_V \frac{1}{\mu} \text{rot}(\vec{A}) \cdot \text{rot}(\vec{A}) dv = \int_V \vec{A} \cdot \text{rot}\left(\frac{1}{\mu} \text{rot}(\vec{A})\right) dv + \oint_S \frac{1}{\mu} \vec{A} \times \text{rot}(\vec{A}) \cdot \vec{ds} \quad (11.123)$$

Substituting Equations 11.73 and 11.76 in Equation 11.123, E_m becomes

$$2E_m = \int_V \vec{A} \cdot \vec{J} dv + \oint_S \vec{A} \times \vec{H} \cdot \vec{ds} \quad (11.124)$$

If all boundary conditions are homogenous, then the second term in Equation 11.124 is zero and

$$2E_m = \int_V \vec{A} \cdot \vec{J} dv \quad (11.125)$$

Using E_m from Equation 11.125 and ϕ_b from Equation 11.120 in Equations 11.121 and 11.122, for a geometry with plane parallel symmetry and no skin effect, the two formulae of L_b become equivalent.

For coils made of massive conductors (with skin effect), the coil flux, ϕ_b , becomes

$$\phi_b = \frac{1}{I_b} \left(\int_{S_2} AJ dS - \int_{S_1} AJ dS \right) l_z \quad (11.126)$$

For the mutual L_{12} inductance between coils 1 and 2, the definition in relation to mutual flux, ϕ_{12} , and current solely in the first coil, I_1 , is used:

$$L_{12} = \frac{\phi_{12}}{I_1} \quad (11.127)$$

Let us now illustrate the computation of magnetic energy, E_m , and flux, ϕ_i , of the coil in Figure 11.12.

The computation of E_m from the volume integral is reduced to a plane surface integral because of symmetry around the rotation axis:

$$E_m(B, H) = \int_S \pi r B H ds \quad (11.128)$$

$$E_m(A, J) = \int_S \pi r A J ds \quad (11.129)$$

The computation of magnetic flux linkage in turns 1 and 2 is performed by integrating the magnetic potential over their surfaces:

$$\phi_i = \frac{1}{S_i} \int_{S_i} 2\pi r A ds \quad (11.130)$$

TABLE 11.1

Energy and Inductances

Boundary Type	$E_m(BH)$ [μJ]	$E_m(AJ)$ [μJ]	Φ_1 [nWb]	Φ_2 [nWb]	$L_{11}(E_m)$ [nH]	$L_{11}(\Phi)$ [nH]	L_{12} [nH]
Neumann	1.49	1.5086	192.07	78.7989	12.329	12.278	5.037
Dirichlet	1.4613	1.4796	188.395	76.4771	12.093	12.043	4.889
Kelvin transform	1.4804	1.4963	190.517	77.8844	12.229	12.179	4.979

The results of these calculations are shown in Table 11.1.

Observations: Due to discretization errors, the values in Table 11.1 are not identical; the same is true for the self-inductance of turn 1 calculated from E and ϕ , but the errors are acceptable due to the large distances from the boundary. The energy and inductances are larger for Neumann's conditions (infinite permeability beyond the border) and smallest for Dirichlet's conditions. Reducing the domain (boundary) radius, R_b , leads to larger errors.

11.7.1 Electromagnetic Forces

An important objective of FEM analysis is the computation of electromagnetic forces. There are three main methods to calculate them, as follows.

11.7.1.1 Integration of Lorenz Force

By this method the force exerted on conductors with electric currents is calculated:

$$\vec{F} = \int_V \vec{J} \times \vec{B} \, dv \quad (11.131)$$

For problems with a plane or axial symmetry, the volume integral turns into a surface integral with the force vector situated in the symmetry plane:

$$\vec{F} = I_z \int_S \vec{J} \times \vec{B} \, ds \quad (11.132)$$

$$\vec{F} = \int_S 2\pi r \vec{J} \times \vec{B} \, ds \quad (11.133)$$

For cylindrical symmetry, the resultant force has only one nonzero component along the axis of symmetry. Some commercial FEM softwares (such as "Vector Field") calculate two force components even for a cylindrical symmetry. A knowledge of radial force is required to verify the mechanical stress of the electrical conductors of the coil. The Lorenz force procedure is precise, but can be applied only for electric conductors in air. In most electrical machines, however, the coils are placed in slots where the actual force

is exerted on the slot walls and not on the electric conductors, and thus the Lorenz force procedure is inoperable.

11.7.1.2 Maxwell Tensor Method

The resultant field force upon the objects found within a closed surface is calculated by integrating the Maxwell stress tensor (Figure 11.13) along the closed surface:

$$\bar{F} = \oint_{\Sigma} d\bar{F} = \mu_0 \oint_{\Sigma} \left((\bar{H} \cdot \bar{n})\bar{H} - \frac{1}{2}H^2\bar{n} \right) ds \quad (11.134)$$

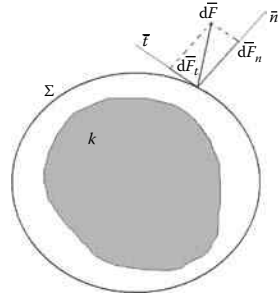


FIGURE 11.13
Integration of Maxwell tensor.

The normal and tangential forces to the surface, dF_n and dF_t , per unit area are

$$dF_t = \mu_0 H_t H_n ds \quad (11.135)$$

$$dF_n = \frac{1}{2} \mu_0 (H_n^2 - H_t^2) ds \quad (11.136)$$

To reduce computation errors, by refined discretization of the domain with 60° (equilateral) triangles with their nodes in linear medium, is required.

11.7.1.3 Virtual Work Method

The field force computation is based on the total energy conservation principle

$$dw_{\text{mec}} + dw_m = dw_{\text{el}} \quad (11.137)$$

where

dw_{mec} is the mechanical work

dw_m is the magnetic energy increment

dw_{el} is the received electric energy from outside (a source)

A coenergy of the system, w'_m , is defined as the difference between the received electric energy, w_{el} , and the energy stored in the magnetic field, w_m :

$$w'_m = w_{\text{el}} - w_m \quad (11.138)$$

The projection of this force along the direction of motion is calculated as the magnetic coenergy derivative with aspect to the motion variable:

$$\bar{F} \cdot \bar{l}_{\delta l} = \frac{\delta w'_m}{\delta l} \quad (11.139)$$

This method implies the computation of magnetic field and magnetic coenergy in two adjacent positions. As the solution to the field problem is also numerical, the numerical derivative will amplify the computation errors (Figure 11.14).

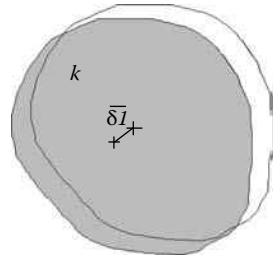


FIGURE 11.14
Virtual displacements.

11.7.2 Loss Computation

Losses in a conductive medium with electric currents are Joule losses, P_{co} :

$$P_{co} = \int_V \rho j^2 dv \quad (11.140)$$

For a magnetostatic regime, the current density is constant and the Joule losses imply only current and resistance calculations. For alternative or transient current operation modes in a massive conductive medium, the current density distribution depends on the solution of the field problem, and thus the volume integral in Equation 11.140 is required.

11.7.2.1 Iron Losses

The iron loss computation is based on the volume integral of the iron loss volumic density (W/m^3) over the magnetic core volume; alternatively, the Steinmetz approximation formulae are used. Finally, the complex permeability concept in the field solution is utilized.

$$P_{iron} = \int_V p_{B_s f_s} \left(\frac{B_m}{B_s} \right)^{\alpha_B} \left(\frac{f}{f_s} \right)^{\alpha_f} \gamma_{Fe} dv \quad (11.141)$$

where

$p_{B_s f_s}$ are the specific (volumic) losses at a given flux density B_s and frequency f_s , respectively

B_m is the maximum ac flux density in volume element dv

f is the working frequency

$\alpha_B = (1.6 - 2.2)$ is the exponent for iron loss dependence on B_m

$\alpha_f = (1.5 - 1.7)$ is the exponent for iron loss dependence on frequency

γ_{Fe} is the mass density

For iron core coils or single-phase electric transformers, the maximum flux density in every point is related to the maximum magnetization current. For polyphase rotary machines (even single-phase), the maximum flux density cannot be determined by solving the field problem at a single moment in time, but for more than one moment within a period.

Even in this case, the calculated loss value is not exact because there are higher time harmonics in magnetic flux due to space and time flux density

harmonics. If the flux density pulsates, is traveling (circular), or elliptical in type, in various points in the magnetic core, the computation of iron loss becomes more complex. Finally, the mechanical machining of magnetic silicon-iron sheets (laminations) in electrical machines adds additional core losses; experimental work is necessary to validate almost any iron loss computation method.

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12

FEM in Electric Machines: Electromagnetic Analysis

12.1 Single-Phase Linear PM Motors

The FEM analysis of electric machines consists of three major stages:

1. Preprocessor stage—This stage provides a description of the field problem associated with the studied machine and its operation regimes. It contains several steps such as choosing the problem symmetry, drawing the machine geometry embedded drawing, choosing the frontier conditions, choosing the magnetic field source, generating mesh, choosing the type of the solved problem (magnetostatic, ac problem, transients problem, rotating machine problem, linear machine problem, etc.), choosing the settings for the solver (maximum number of iterations, mesh refinement, solution global error, etc.), and creating an input file for the solver.
2. Solving the field problem.
3. Postprocessor stage—In this stage, the field distribution can be shown and the circuit equivalent parameters can be computed.

A single-phase linear PM motor in tubular construction is presented in Figure 12.1. It was proposed [1] as a thermal engine valve actuator.

It contains

- A dual part somaloy passive mover
- A tubular PM made of 6–12 sectors to reduce the eddy current losses in the PMs due to the ac mmf of twin coil currents
- A stainless steel (shaft) coupled to the thermal engine valve

The electromechanical device also contains two mechanical springs that are not shown in Figure 12.1. The two mechanical springs act in opposite directions and their forces are in equilibrium when the mover is in the middle position. The magnetic force at zero current acts the mover so as to increase its displacement (Figure 12.2). This force has an unstable equilibrium point

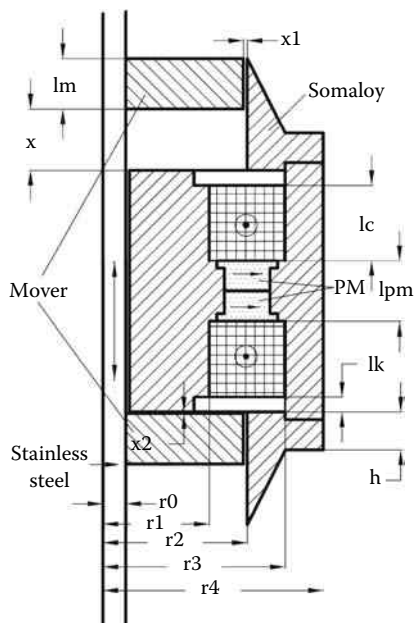


FIGURE 12.1
Electro-valve linear PM actuator.

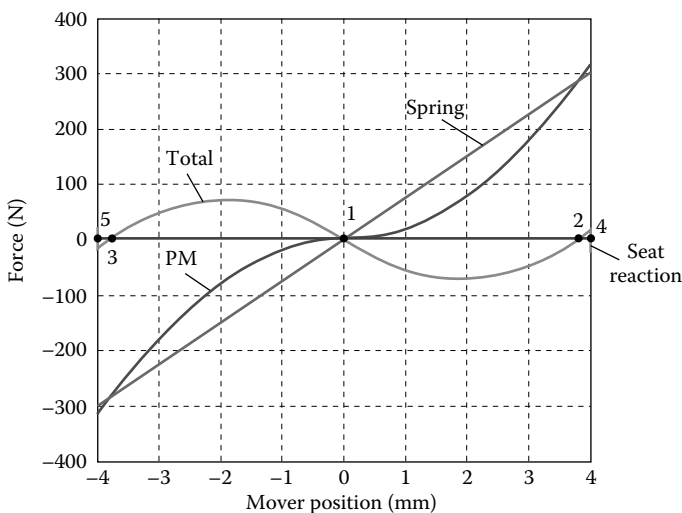


FIGURE 12.2
Mechanical spring and permanent magnet thrust.

when the mover is in the middle position. The magnetic field produced by coil currents breaks the force equilibrium and orients the mover in a certain direction according to the current polarity. The magnetic force at zero current is larger than the spring force when the mover is in the extreme position and holds the mover in that position, which produces a good reduction in copper losses. Changing the coil currents decreases the electromagnetic force and the springs drive the mover through the equilibrium point.

The main geometric dimensions of the linear actuator of our case study, Figure 12.1, are given in Table 12.1.

The FEM analysis was performed using the Vector Field computer software. The FEM analysis methodology is similar to Flux2D from Cedrat, FEMM, or other commercial softwares for magnetostatic FEMs.

12.1.1 Preprocessor Stage

The FEM analysis was performed in magnetostatic mode for several mover positions. The axis symmetry with modified vector potential, “r A,” was chosen due to the tubular construction of the analyzed device. To introduce the device geometry, it is adequate to use the GUI interface [2,3], or to use “comi scripts” in the Vector Field or “lua scripts” in the FEMM. For a simple problem, when is not necessary to solve the problem repetitively for other geometric dimensions, it is preferable to use the GUI interface. When it is necessary to solve a similar problem repetitively, in order to find better performance or to compute circuit parameters versus mover position, then it is feasible to write one or more scripts in order to build an automatic parameterized drawing. In this case, it is necessary to compute the electro-valve thrust versus the mover position, the coil linkage flux versus the position,

TABLE 12.1

Electro-Valve Main Dimensions

No.	Dimensions	Value	Units	Observation
1	r0	3	mm	Mover rod radius
2	r1	14	mm	Outer radius of inner stator core
3	r2	19	mm	Inner radius of stator pole
4	r3	24	mm	Inner radius of stator outer core
5	r4	29	mm	Outer radius of stator core
6	xm	4	mm	Mover displacement magnitude
7	x1	0.5	mm	Radial airgap
8	x2	0.2	mm	Minimum values of axial airgap
9	lm	6.7	mm	Mover pole length
10	lc	10	mm	Coil height
11	lk	2	mm	Distance between coil and stator pole
12	h1	5	mm	Stator core overlength
13	lpm	8	mm	PM height

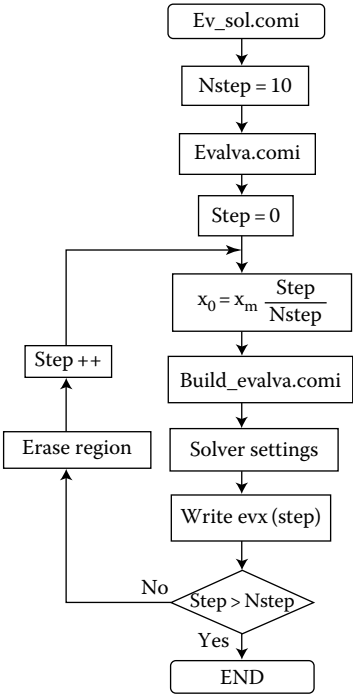
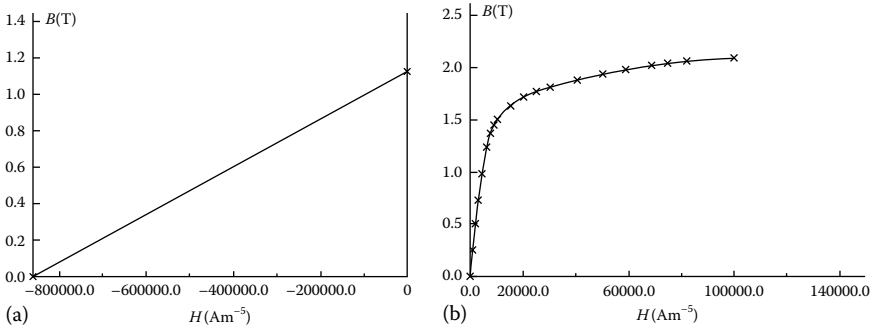


FIGURE 12.3
Block diagram of ev_sol script.

and the coil inductance versus mover position. Consequently, it is necessary to solve a similar problem several times. The computer code is more flexible if it is divided into three scripts: one script contains only the device dimensions—the input script; the second script contains the geometry drawing description; and the third script is used to set the solution parameters. For our example, this script is called “ev_sol.comi” and its block diagram is shown in Figure 12.3. In this script the number of mover positions (Nstep) is set. The “evalva.comi” script is called upon to set the main geometric dimension as is presented in Table 12.1. The mover position is set according to the current step and then the “build_evalva.comi” script is called upon to build the electro-valve drawing according to the current mover position. Solver specifications such as tolerance and the maximum number of iterations are set. The “scale factor” is also chosen. For every mover position it is possible to solve the problem for several currents in the coils. In this case, the following scale factor was chosen: 0, 0.25, 0.5, 0.75, 1.25, 1.5, 1.75, 2, −0.25, −0.5, −0.75, −1, −1.25, −1.5, −1.75, −2. A unitary factor scale is chosen implicitly. The solution will be computed for rated currents and also for rated currents multiplied with a given scale factor. Choosing several scale factors is useful to check the influence of saturation on thrust, linkage flux, and inductance.

**FIGURE 12.4**

Magnetization features. (a) PM demagnetization curve and (b) Somaloy 550—magnetization curve.

A large number of scale factors increases the computation time. A zero value scale factor implies a solution where the field is produced only by the permanent magnet. Different behaviors are expected for positive and negative currents due to the prepolarization of the PM magnetic circuit.

Every mover position is saved as an independent magnetostatic problem. If the step does not reach the maximum number of steps, the regions are erased, the step is increased, and a new problem is generated. The script is finished when the maximum number of steps is reached. The last problem regions are not erased, so they remain on the display. In this way, the user can check the region drawing and the mesh. The geometric dimensions unit is set in millimeters and the current density unit in Ampere per square millimeter in the input script, "evalva.comi." The magnetic nonlinear material is also set in this file by associating the material number with a text file containing the magnetization curve. The permanent magnet features are given through its demagnetization curve (Figure 12.4).

Before starting the geometry drawing it is important to observe if there is geometric symmetry and also if a complex geometry could be decomposed in simple and identical shape components. In this example, the mover rod is made of stainless steel. Considering its permeability to be equal to the permeability of vacuum it could be assimilated with vacuum in magneto-static mode, and, consequently, it could be omitted from the drawing. There is no symmetry in the field due to the mover position and the PM and current magnetic field over position, so the entire field problem will be analyzed. The stator has symmetry around the x -axis and thus it is enough to draw only the stator parts placed over the x -axis. The stator component placed under the x -axis will be constructed using copy command.

The command "Draw" is used in Vector Field in order to construct the geometry. It is a complex command that is used to assign the material features, the number of subdivisions for each line, the frontier condition, and other such requirements [2].

The electro-valve could be decomposed in the following parts:

- Mover magnetic disks
- Stator inner cylinder
- Permanent magnet
- Coils
- Outer stator part
- Background

The mover disk in RZ plane projection is a simple rectangle and, consequently, a quadrilateral shape with regular subdivision (H shape) could be chosen [2, pp. 2–48]. The upper disk corners are computed:

$$\begin{aligned} X_{12} &= r_2 - x_1 \\ X_{34} &= r_0 \end{aligned} \quad (12.1)$$

$$\begin{aligned} Y_{14} &= x_0 + \frac{l_{PM}}{2} + l_c + l_k + x_m \\ Y_{23} &= x_0 + \frac{l_{PM}}{2} + l_c + l_k + x_m + l_m \end{aligned} \quad (12.2)$$

where x_0 is the mover position. The down mover disk corner coordinate is computed in a similar way.

$$\begin{aligned} Y_{14} &= x_0 - \left(\frac{l_{PM}}{2} + l_c + l_k + x_m \right) \\ Y_{23} &= x_0 - \left(\frac{l_{PM}}{2} + l_c + l_k + x_m + l_m \right) \end{aligned} \quad (12.3)$$

The construction of different parts of the electro-valve by decomposing them in simple geometry parts is shown in Figure 12.5.

An example of a computer code in order to draw the mover disk is as follows:

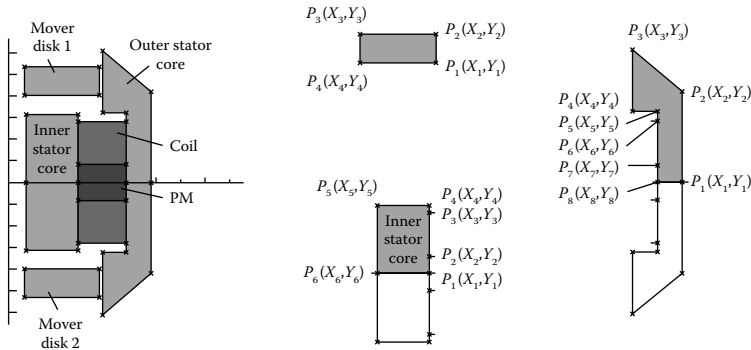
/Mover magnetic piece

DRAW SHAP=H, MATE=5, PHAS=0, DENS=0,

X12=#r2-#x1, X34=#r0,

Y14=#x0+#lpm/2+#lc+#lk+#xm, Y23=#x0+#lpm/2+#lc+#lk+#xm+#lm,

N1=#nmz, N2=#nmr, F1=NO, F2=NO, F3=NO, F4=NO

**FIGURE 12.5**

Building the electro-valve geometry.

The “Draw” command is followed by choosing the shape type, the material number, and, finally, the current density.

The borderline between PM regions and coil regions should meet the inner stator core in a mesh node. In order for this to be ensured, it is good to add one point to the borderline on the inner stator when the external medium is discontinuous. In this way, we avoid the preprocessor interrogation when a PM or a coil region is built. An unexpected interrogation (add new points?) from the preprocessor when a script is running is a source of error. Finally, the inner stator core (half of this) is described by six points, despite its rectangular shape. The “Poly” shape is chosen in order to describe the inner and outer stator cores. The first corner, P_1 , of the inner stator core is set by its Cartesian coordinates (X_1 and Y_1). The second point, P_2 , could be given by a translation along Y-axis with half of the permanent magnet length. The preprocessor allows for setting the polygon corner using Cartesian or polar coordinates, as well as translations along the coordinate axis.

A background object that fills all free space between material objects is added. When the interaction forces are computed using the Maxwell tensor, it is better to design the integration path through finite elements that have all nodes in a linear medium without currents, in order to reduce the integration error. This condition could be achieved if the mesh has at least three layers in the airgap. In order to enforce three mesh layers in the airgap we can define an extra region along the airgap with air features. In our case, such a region was defined around the mover disk, placed in such a manner that it does not reach this region when it is moved from one extreme position to the other. In the generated mesh, Figure 12.6, there are three layers in the airgap. Also, the regular mesh of “H” region with mover disks, coils, and permanent magnets can be observed in Figure 12.6.

The preprocessor stage is over after the problem description is written for every mover position. The solver can be run interactively or by using a

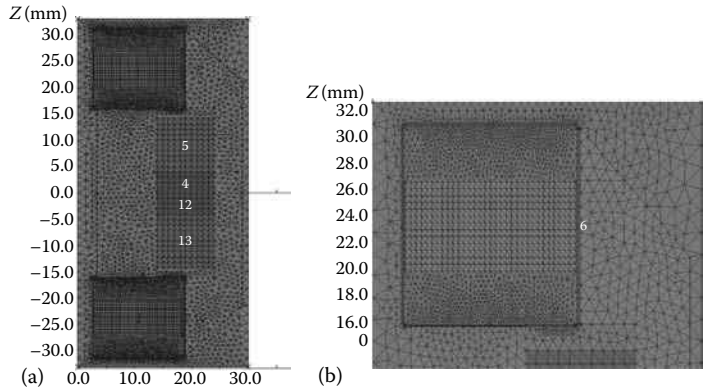


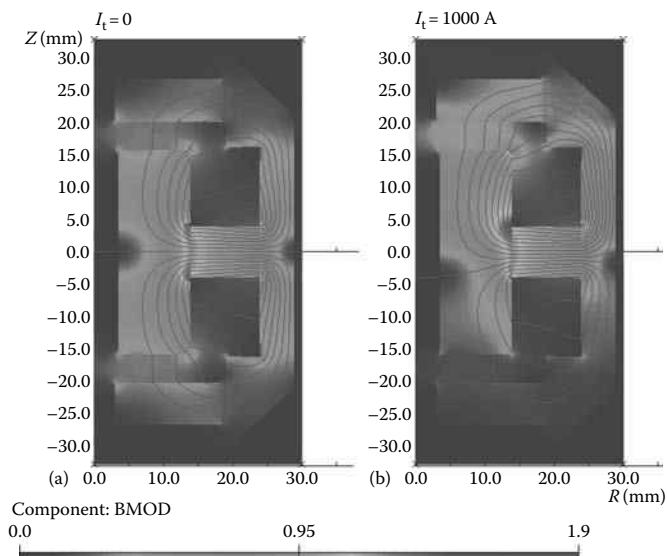
FIGURE 12.6
Electro-valve mesh. (a) Full problem and (b) details around mover.

batch process. When there are several problems to be solved (in our case, one problem for every mover position), it is preferable to add all problems to a batch query and then start the batch process; however, this takes some time. In our electro-valve, for 11 mover positions and 18 currents for each position, 2 h and 40 min on a Pentium 4 at 2.4 GHz with 512 MB of RAM memory were needed.

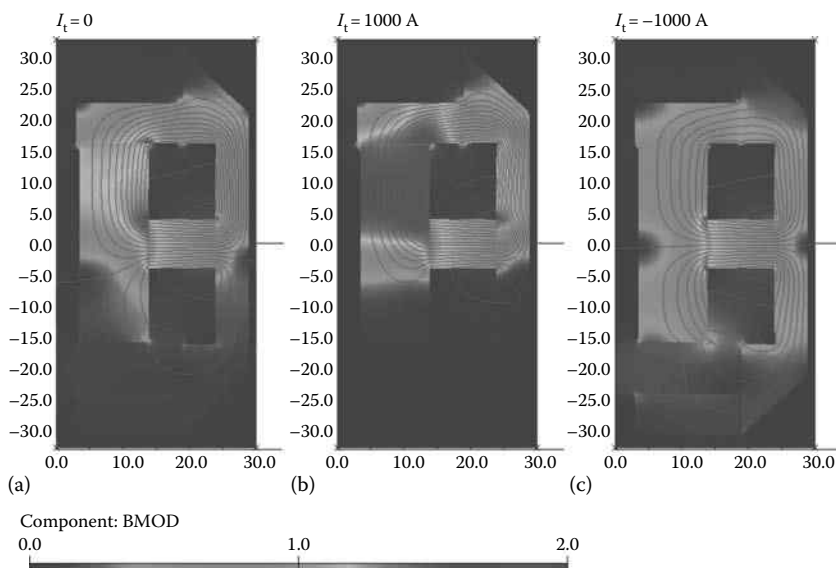
12.1.2 Postprocessor Stage

The main reason of the postprocessor stage is to extract the performance parameters from the finite element solutions. The results file will be loaded into the postprocessor. The magnetic field lines can be shown immediately. This feature is a common option included in the menu list for most commercial FEM softwares. In Vector Field, we have to select a “Component” that is equal to “POT” and then in the submenu “Contour Plot” we have to choose “Execute.” It is possible to set the number of lines, the plotting style, the label type, and whether the drawing needs to be refreshed or not [3]. The flux density module could also be represented directly as a color map. It is recommended to choose a filled zone style with a large number of lines in order to represent the flux density module, and a contour line with a small number of lines in order to represent the field lines. Using the “refresh” switch, the field line could be superimposed on the flux density map as shown in Figures 12.7 and 12.8.

When the mover is in the central position and the current is zero, the field distribution (lines and flux density module) should be symmetric in relation to the X-axis as shown in Figure 12.7a. If the magnetic field from FEM is not symmetric when the geometry and magnetic field source are symmetrical,

**FIGURE 12.7**

Flux density and field lines for equilibrium mover position.

**FIGURE 12.8**

Flux density and field line for mover position $x_0 = -4$ mm.

then the solution is wrong, and, before computing any performance parameter, it is necessary to find the mistake. We have to check the border conditions, the region descriptions, the current and PM values, and the phase.

The flux density module map shows (Figure 12.7), in our case, local and global saturation. When the mover is in the middle position there is a small region with local saturation. The total current is defined as the product between the number of turns and current in each turn, or as an equivalent current density integral over the conductor area. In our example, the equivalent current density is 5 A/mm^2 , which means about 10 A/mm^2 on the net conductor area if the filling factor is considered to be around 0.5. This is an acceptable value for the peak current density. It could be doubled, for short-lived transients or when considering a very efficient cooling system. The current field adds to the permanent magnet field in some regions and subtracts in other regions. The field distributions become unsymmetrical but without notable global saturation when the mover is in the central position. When the mover is in its maximum displacement, Figure 12.8, the permanent magnet field is asymmetrically distributed. A positive current creates a notable saturation in the upper part of the magnetic circuit.

The magnetic core saturation also depends on the material features. A magnetic flux density of 2 T indicates a heavy saturation for Somaloy as it is used for electro-valve core but it could indicate a moderate saturation for Hiperco laminations. The saturation level could be better appreciated if the magnetic relative permeability is presented as shown in Figure 12.9.

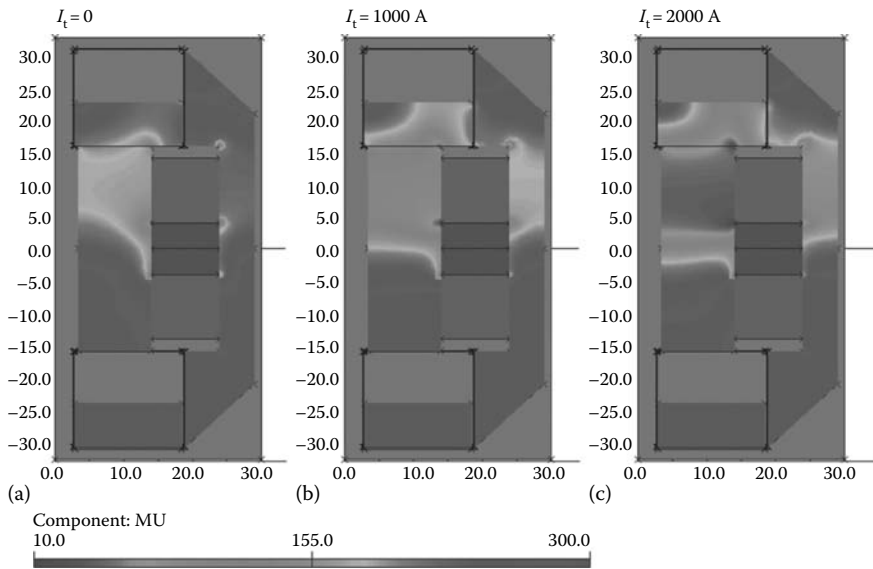


FIGURE 12.9
Relative magnetic permeability in the magnetic core.

The magnetic permeability decreases in the region where the current field is added to the permanent magnet field; for rated current ($I_t = 1000$), heavy saturation occurs and the permeability is reduced at half of its maximum value (Figure 12.9b). Increasing further the coils current at 2000 A will reduce magnetic permeability by 10 times from its maximum value, Figure 12.9c. The magnetic permeability map is presented only for the soft magnetic core (Somaloy). This is an important option that could be used in order to avoid an unusual representation scale.

The most important performance computed by the FEM is the coil linkage flux and the electromagnetic force. The linkage flux is computed by applying relation (11.129) on the coil surface. The integral is computed by choosing an adequate menu in the Vector Field software. The user has to divide the integral values given by the software with the coil surface, and it is also necessary to take into account the length unity. For example, if the length unity is in millimeters, then the flux from Equation 11.129 is in mWb. The linkage flux is computed for several mover positions and several currents for each mover position. In this way, it is possible to compute inductance versus current to prove the presence of magnetic saturation. The data extraction could be mechanized by using a “comi” script in Vector Field or “lua” script in the FEMM software [4,5]. The desired data is stored in a table and then graphic representation and other data processing can be done in a dedicated software, developed, for example, in MATLAB®.

The total linkage flux versus mover position and coil current is shown in Figure 12.10. The linkage flux is produced by the permanent magnet and the coil current.

$$\psi(x_0, i_c) = \psi_{PM}(x_0, i_c) + L(x_0, i_c) \cdot i_c \quad (12.4)$$

The superposition principle is not perfectly valid in the nonlinear magnetic circuit and even then only an approximate value of linkage inductance can

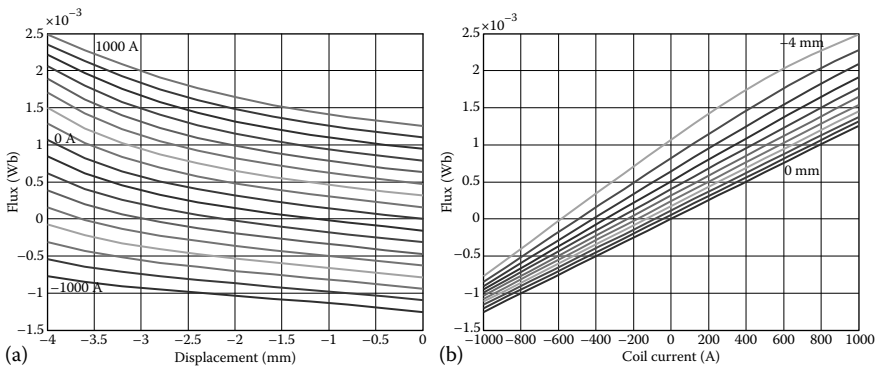


FIGURE 12.10

Linkage total flux for two coils in series and 1 turn/coil. (a) Linkage flux vs. displacement and (b) linkage vs. total current.

be computed.

$$L(x_0, i_c) = \frac{\psi(x_0, i_c) - \psi_{PM}(x_0, i_c)}{i_c} \cong \frac{\psi(x_0, i_c) - \psi_{PM}(x_0, 0)}{i_c} \quad (12.5)$$

This value is useful to compute the linkage flux from the current and permanent magnet components, but, for emf voltage computation, it is better to use the differential inductance approximation:

$$L_d(x_0, i_c) = \frac{\partial \psi(x_0, i_c)}{\partial i_c} \cong \frac{\psi(x_0, i_2) - \psi(x_0, i_1)}{i_2 - i_1} \quad (12.6)$$

The static and differential linkage inductance approximations are shown in Figure 12.11.

The real differential inductance should be equal or smaller than static inductance, but, in Figure 12.11, a small violation of this rule is noticed in some points due to approximations. However, the static and dynamic inductance values are close to each other for our case study.

The thrust on the mover is computed by the integration of the Maxwell tensor. A family of thrust curves versus mover position and several currents are shown in Figure 12.12. The electromagnetic force could be computed for every part of the device using this method. The contribution of the upper mover disk to the total mover thrust is shown in Figure 12.13. It could be remarked that the thrust on the upper disk is always negative. The thrust is always positive on the lower mover disk. The electromagnetic thrust along the free direction is small, so the mechanical spring is the main contributor in releasing the mover and in accelerating it in the opposite direction. When the mover passes through the middle position, it can have large electromagnetic acceleration, but, if any mechanical shock is to be avoided, the motion should be stopped when it is on that part of its trajectory. Unfortunately, the braking

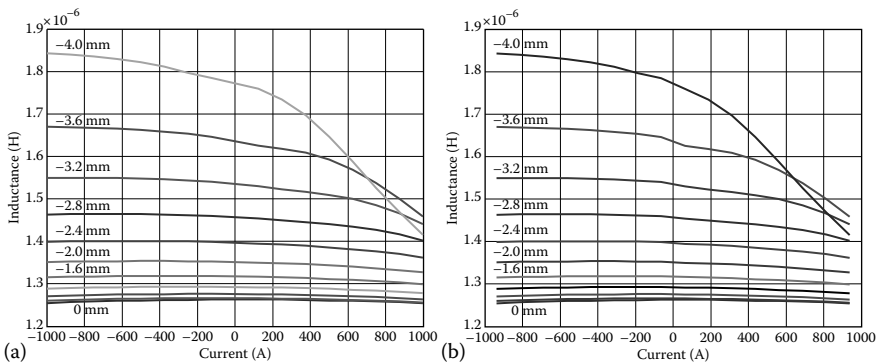


FIGURE 12.11

Linkage inductance for two coils in series and 1 turn/coil. (a) Linkage inductance and (b) differential linkage inductance.

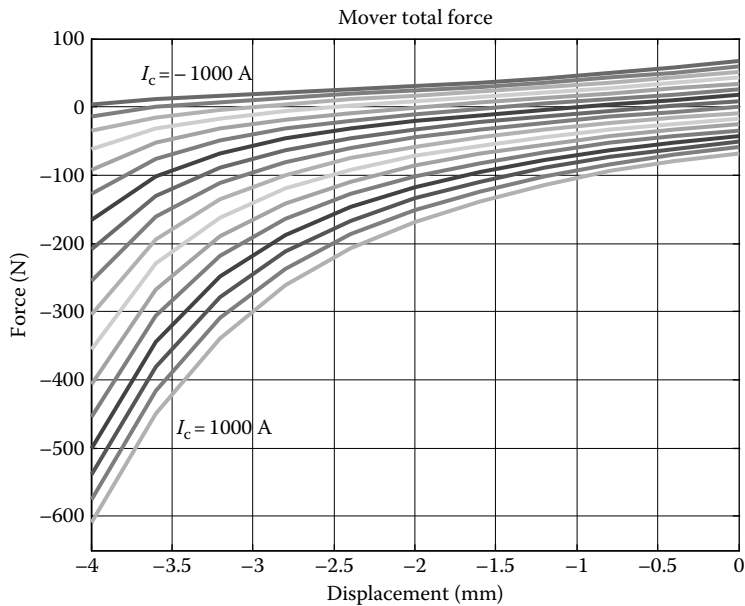


FIGURE 12.12
Mover thrust vs. position.

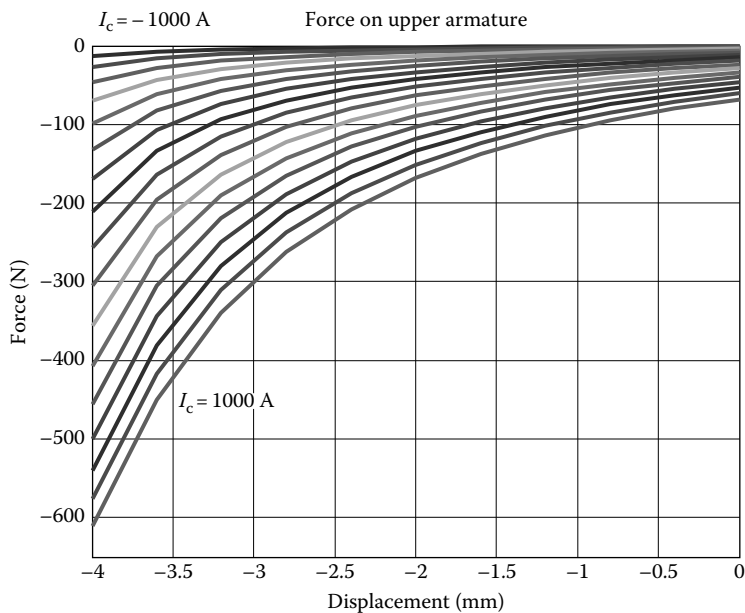


FIGURE 12.13
Contribution of the upper mover disk to the mover thrust.

thrust is poor for that part of the trajectory and the spring has to do again the main braking job. In conclusion, the described device has rather poor motor features but it is excellent to use as an electromechanical latch.

The axial force on the coils versus current is shown in Figure 12.14 for different mover displacements. This force is used for the mechanical design of the coils.

A global characterization of the electro-valve is given by its electro-magnetic energy, Figure 12.15, computed with Equation 11.128 for total

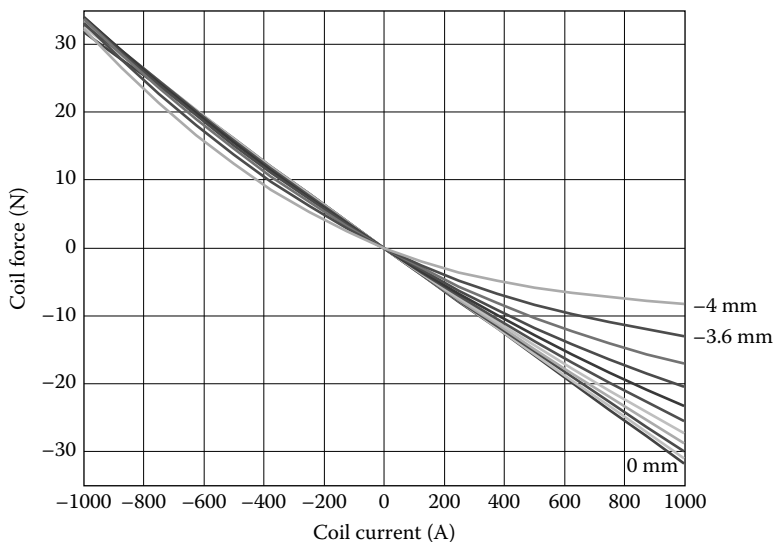


FIGURE 12.14
Resultant axial forces on the coils.

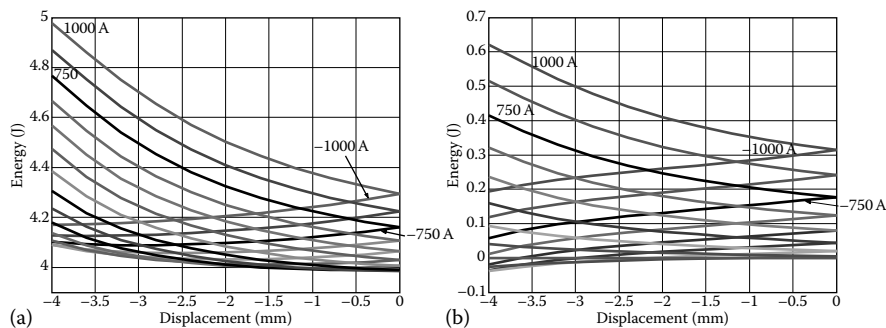


FIGURE 12.15
Magnetic field energy vs. mover position and stator current. (a) Total magnetic field energy and (b) magnetic field energy from electric circuit.

field-stored energy (permanent magnet and current source), and with relation (11.127) for only field energy stored from current sources in the presence of PM.

12.1.3 Summary

The magnetostatic field solution by the FEM for a tubular PM linear motor as presented here emphasizes the fundamentals of the FEM application to electric machines, allowing also for thrust and inductance calculations to prepare for high-precision circuit models needed for the investigation of dynamics and control of electric machines.

12.2 Rotary PMSMs (6/4)

The rotary permanent magnet synchronous motor (PMSM) with surface permanent magnet 4-pole rotor and six coils in the stator, Figure 12.16, is used to illustrate the finite element methodology applied on rotary machines without current in the rotor. This motor is used in order to improve the motor efficiency at reasonable manufacturing costs in home appliances and automotive industries. The configuration with three stator coils per two rotor poles is probably less costly but finite element analysis has shown large radial forces, especially when the motor is loaded.

The finite element analysis is illustrated on a 200 W, 1500 rpm motor and 300 V dc link voltage. The stator lamination, Figure 12.17 main geometrical dimensions are

- D_{si} - inner stator diameter
- D_{so} - outer stator diameter
- w_{sp} - stator pole width
- h_{sc} - stator yoke width
- a_{sp} - relative stator pole span angle
- h_{s3} and h_{s4} - the heights of the stator slot closure
- R_1 - stator pole shoe curvature
- h_{s1} - stator teeth height

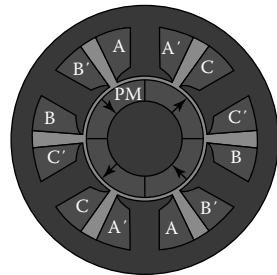


FIGURE 12.16 The 6-slot/4-pole PMS with surface PM rotor.

The rotor geometrical dimensions, shown in Figure 12.18 are

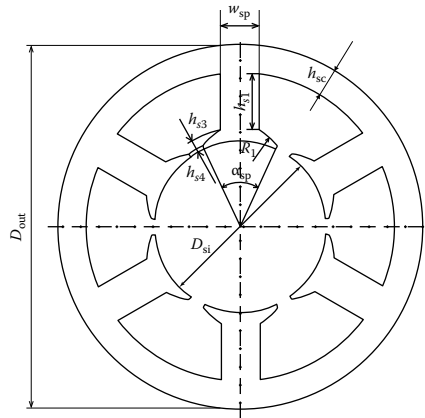


FIGURE 12.17
Stator main dimensions.

- D_{ro} - rotor outer diameter
- D_{ri} - rotor inner diameter
- h_{PM} - PM height

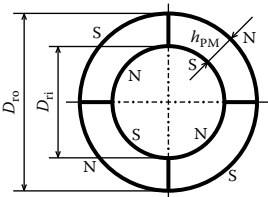


FIGURE 12.18
Rotor main dimensions.

The stator core stack length, “lstack” and airgap length, g , are also important geometric dimensions. The main geometric dimensions used in the following simulations are given in Table 12.2.

12.2.1 BLDC Motor: Preprocessor Stage

When a PMSM is controlled with trapezoidal (rather than sinusoidal) currents and two active phases (of three), it is called brushless dc (BLDC) motor.

The BLDC motor has a plane parallel symmetry if the coil end connections are neglected, and, consequently, the BLDC motor magnetic field is solved as an “ xy symmetric problem.” Suitable length (millimeter) and current density (Ampere per square millimeters) units are chosen. The main geometric dimensions are declared and initialized as user-defined variables in order to mechanize the machine drawing and problem description. The machine drawing is divided into elementary parts and then the “copy” and “rotate” technique is used in order to construct the entire cross section. The ideal 6/4 pole machine has a symmetry over 180° . If we are not interested to study the rotor/stator eccentricity, then it is enough to solve the magnetic field problem only for one half of the cross section machine as shown in Figure 12.19. The machine rotor could be automatically rotated using the rotating machine module from Vector Field.

TABLE 12.2
The Main PMSM Geometric Dimensions

Data Name	Initial Data	Units
D_{out}	68	mm
D_{si}	30	mm
h_{s4}	0.6	mm
h_{s3}	1.4	mm
h_{sc}	4.5	mm
w_{sp}	7	mm
l_{stack}	50	mm
α_{sp}	51	°
h_{PM}	3	mm
g	0.5	mm

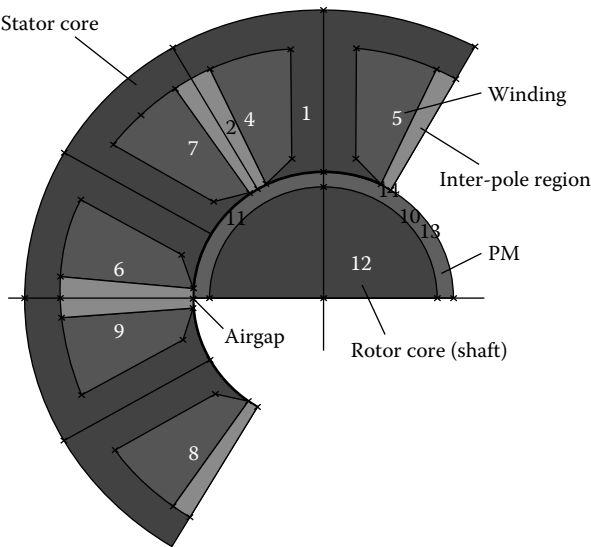


FIGURE 12.19
The BLDC motor problem description using symmetry.

The complex shape of the stator core is drawn in a simple manner as shown in Figure 12.20. Nine points are necessary to draw the stator core element and only four points to draw the coil side. The points' coordinates are computed from the input geometry dimensions. The user could choose Cartesian or polar coordinates for every point. For the first point, P_1 , it is simpler to use polar coordinates:

$$P_1 (R_P, P_p) = P_1 \left(\frac{D_{si}}{2}, 90^\circ \right) \tag{12.7}$$

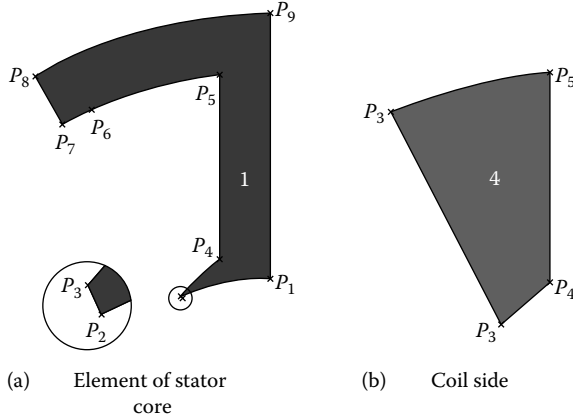


FIGURE 12.20
Stator core and coil elementary shape.

The coordinates of the second type point are also given in polar coordinates:

$$P_2 (R_P, P_P) = P_1 \left(\frac{D_{si}}{2} + \Delta\delta, 90^\circ + \alpha_{sp} \right) \quad (12.8)$$

where $\Delta\delta$ is the airgap variation in order to consider geometry with nonuniform airgap. In our case, the airgap is, however, uniform and thus $\Delta\delta=0$. The line curvature specification between points P_1 and P_2 is equal to the inverse of the stator inner radius if the length of the airgap is uniform. The line curvature that links points P_1 and P_2 , is set at the same time with the P_2 coordinate (in the Vector Field software). It is not necessary to compute the coordinates for all points that describe the geometry elements. In our example, it is not necessary to compute the P_3 coordinates because it could be introduced as a radius variation with “sh4” length. The straight movement is obtained for zero curvature specification. The point P_4 is given through Cartesian coordinates:

$$P_4 (X_P, Y_P) = P_4 \left(-\frac{w_{sp}}{2}, y_4 \right) \quad (12.9)$$

$$y_4 = \sqrt{\left(\frac{D_{si}}{2} + sh4 + sh3 \right)^2 - \left(\frac{w_{sp}}{2} \right)^2} \quad (12.10)$$

The point, P_5 , is introduced as an abscise variation with “sh1” length. The points P_6 and P_7 are introduced in polar coordinates:

$$P_6(R_P, P_P) = P_6\left(\frac{D_{s0}}{2} - h_{sc}, 90^\circ + \alpha_{sp}\right) \quad (12.11)$$

$$P_7(R_P, P_P) = P_7\left(\frac{D_{s0}}{2} - h_{sc}, 90^\circ + \frac{180^\circ}{N_{sp}}\right) \quad (12.12)$$

Point P_8 is introduced as a radius variation with “hsc,” while point P_9 is introduced as a polar angle variation with $180^\circ/N_{sc}$ angle in negative direction (clockwise). After the elementary geometry of stator core presented in Figure 12.20 is built, it could be replicated, mirrored, and rotated in order to obtain the stator core drawing as shown in Figure 12.19.

The inter-pole region and an airgap layer close to the stator are built in the same way as the stator element. The inter-pole region and airgap layer close to the stator are obtained by the “replicate” command at the same time with the stator. The first coil side is built in the same way as the stator core elements, but the other coil sides are obtained by the “copy” command instead of the “replicate” command because it is necessary to set different values of current density for each coil side and this is possible only if they represent different regions [2,3].

The rotor is built in the same way as the stator. At first, the permanent magnet region is built. The second permanent region is a copy of the first permanent magnet region in order to avoid opposite polarization settings. The permanent magnet polarization direction can be set later when all geometry is built. The rotor core under first pole is drawn directly, while the whole desired rotor core is made using “replication” of the first part. An airgap layer close to the rotor is built.

The computation of force through the Maxwell tensor is much more accurate if the integration path passes only through linear elements that have all nodes in the same medium. In order to achieve better accuracy for force computation, a third airgap layer bordering the stator airgap layer is defined.

The rotating machine module of Vector Field software allows transients regime with rotation of a part of a machine. A special region, “rotating machine airgap” should be introduced in the description of the problem. The region parameters are the average radius and the symmetry. The circle with the given radius must not pass through any region, so it must be outside of any point on the rotor, and inside of any point on the stator. As a convention, the inner part of this special region rotates. The outer rotor machine could be also studied, despite inner regions rotation, because the electromechanical phenomena are governed by relative position and relative speed. The “symmetry” parameter is an integer value and its absolute value is specifying the number of rotational symmetries of the model. In our case, the symmetry is 2 because half of the machine is simulated and the sign is “plus” because the periodicity conditions are positive (a pair of poles is studied). If the whole machine is modeled, the symmetry value must be 1. It is essential that the value of this parameter matches the symmetry

of the model correctly. The outside edge of the rotor and the inside edge of the stator are recommended to have a constant radius and the subdivisions on rotor and stator sides of the gap elements have a similar size. Using the additional airgap layers near the stator and rotor, which in the model are part of the stator and rotor, we can adjust the rotor outer and stator inner radius and make them constant, even when the real airgap is variable, and the number of inner stator subdivisions could also be set equal to the “outer rotor.” Finally, the airgap contains four layers as shown in Figure 12.21.

The radial magnetization of permanent magnet means that the magnetic polarization is a function of position and it could be set only after mesh setting, when the coordinate X and Y variables are available. The PM polarization is introduced as an extra condition as shown in the following example:

```

EXTRA
REG 10 C=PHASE, F=atan2d(Y; X)
REG 11 C=PHASE, F=180+atan2d(Y; X)
Quit

```

The current density for each conductor region is also set after regions meshing. At this stage, the current density could be computed from total current per coil and coil cross area, which is a region-intrinsic parameter after the region is meshed. A circuit label number is also set for each conductor at this stage. This label number will be used when the current through the coil is computed from an external circuit or when it is given from an

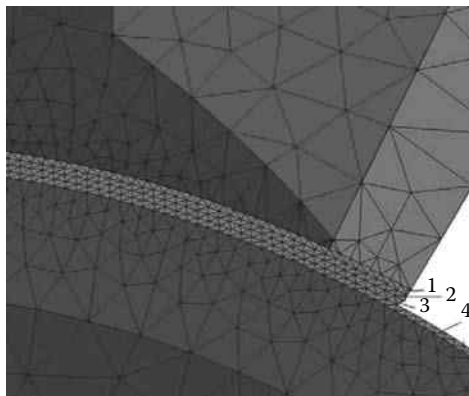


FIGURE 12.21

Airgap mash layers: 1—layer close to the stator, 2—layer containing the integration path of Maxwell tensor, 3—rotating airgap layer, and 4—layer close to the rotor.

external driven function. By using an external driven function, it is possible to change the phase currents according to the rotor position and simulate the synchronous running of the machine. The current density and circuit label setting, are shown in the following example where the region number is from Figure 12.19.

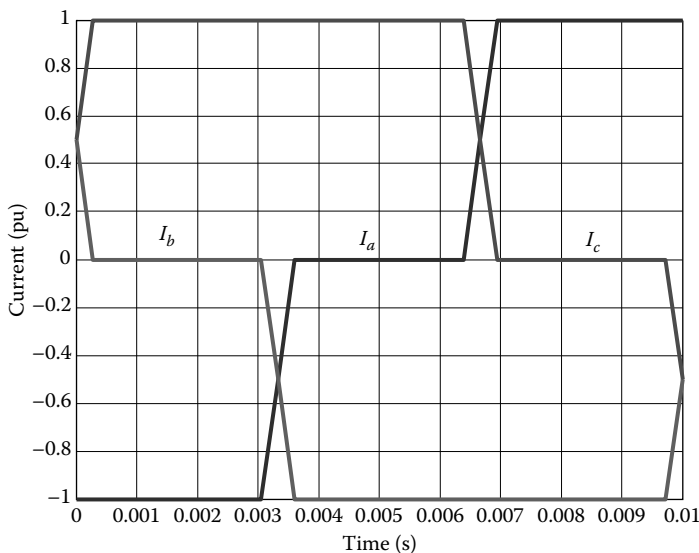
```
MODI REG1=4 DENS=-#It/area, N=1
MODI REG1=5 DENS=#It/area, N=1
MODI REG1=6 DENS=-#It/area, N=2
MODI REG1=7 DENS=#It/area, N=2
MODI REG1=8 DENS=-#It/area, N=3
MODI REG1=9 DENS=#It/area, N=3
```

The rotor speed could be set as a constant or as a variable speed, when it is given as a table in a file. Also, it is possible to compute the rotor speed directly by the finite element software using the mechanical equation:

$$\frac{1}{J} \cdot \frac{d\Omega}{dt} = T_{em} - \text{sign}(\Omega) \cdot T_f - T_l - k_t \Omega$$

The inertia moment, J , the friction torque, T_f , the load torque, T_l , and the speed-varying torque coefficient, k_t , are set by the user while the electromechanical torque, T_{em} , is computed from the FEM software. The machine length should be set by the user at the same time with previous parameters. A more complex mechanical equation (with load torque function of rotor position or as an explicit function of time) is possible using a command file. A “logfile” is created when the solver runs for the rotating machine and the desired solution parameters could be stored for every rotor position. An adaptive or fixed-time step could be used. The entire field solution is stored for a given time. There is a simple way to transform the desired rotor position in a time moment when constant speed is used. The stator currents could be maintained at a constant when torque and flux versus internal angle will be extracted in the postprocessor, or when their values are correlated with rotor position using the “drive functions” when the torque pulsation and linkage flux versus time is studied at constant internal angle. The “drive function” could be an elementary function such as sine, cosine, step, or exponential function but they could also be read from a table. In our example, the phase currents have a trapezoidal wave shape as in Figure 12.22 and the current waveforms are given as a table versus time.

The time dependence of the current density is computed by multiplying the current density from each region with the drive function at the current time. In our example, the drive function is given in per unit and its maximum value is unity. In this way, the user definition for the drive function is compatible with embedded drive function (dc, sine, cosine, ramp, exponential).

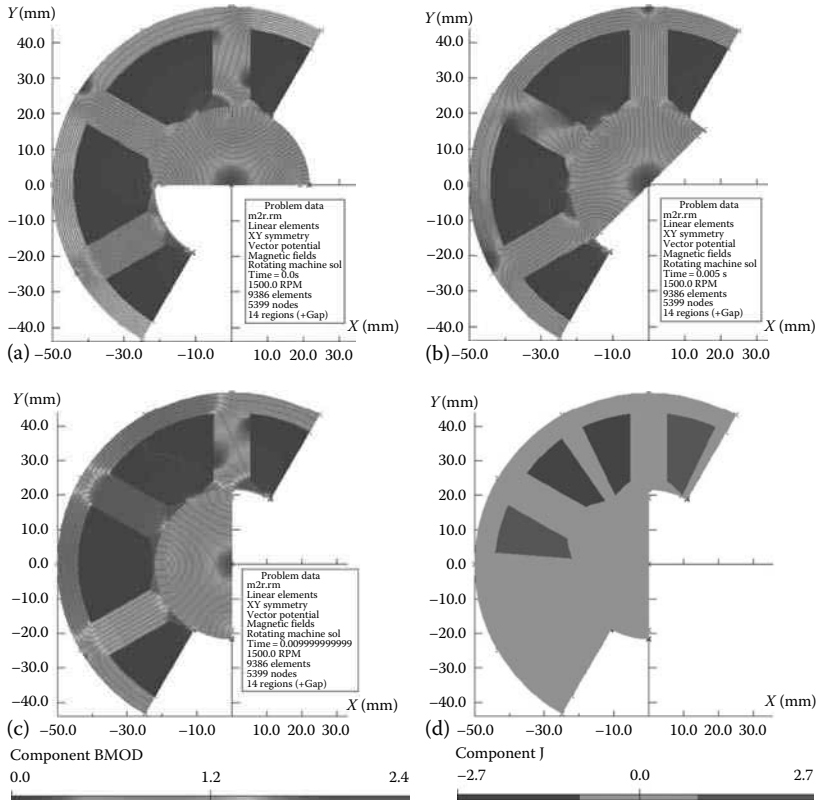
**FIGURE 12.22**

Drive function for trapezoidal current wave shape.

12.2.2 BLDC Motor Analysis: Postprocessor Stage

Machine analysis is performed in postprocessors, where field solutions from FEMs are used, to compute the machine circuit parameters, torque capability, and magnetic saturation level. At first, it is necessary to check the FEM solution by observing the magnetic field lines. In the first simulation of the BLDC motor, the machine rotor was rotating at constant speed (1500 rpm) while the current was maintained constant: negative in the first coil, positive in the second coil, and zero in the third coil as shown in the Figure 12.23d. The magnetic field solutions were stored for 37 rotor positions (time moments) while a quarter mechanical rotor revolution was done. The magnetic field lines and flux density map for three rotor positions (first, median, and the last) are shown in Figure 12.23. Using the problem periodicity, only half the machine was simulated. In this way, the computation effort was dramatically reduced. The solution storage memory was reduced by two times and the computation time was reduced by more than two times. The magnetic field lines are closed through expected paths in all three cases and we can conclude that the periodicity conditions are appropriate.

The magnetic field of the coil is added to the PM field, increasing or decreasing the total magnetic field. For some rotor positions, as in Figure 12.23c, the magnetic fields adding is predominant and resulting field is increased for a large region of the stator core. The current density presented

**FIGURE 12.23**

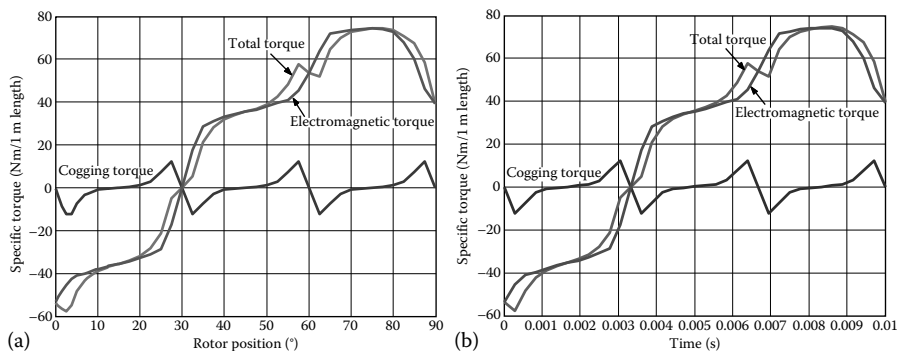
BLDC motor—magnetic field lines: (a) $t = 0$ s, (b) $t = 0.005$ s, (c) $t = 0.01$ s, (d) current distribution.

in Figure 12.23d is an average value over all conductor regions. The real current density is computed taking into account the slot-filling factor. Considering a usual filling factor $k_{\text{fill}} = 0.4$, the real current density is computed as

$$J_{\text{cu}} = \frac{J_{\text{reg}}}{k_{\text{fill}}} = \frac{2.7}{0.4} = 6.75 \text{ A/mm}^2 \quad (12.13)$$

This is an acceptable value of current density at peak torque (about two times larger than rated torque).

The electromechanical torque is returned from the solver in the “log-file” (for “Vector Fields”) but it is also computed at the postprocessor stage by integrating the Maxwell tensor. The symmetry parameter is automatically considered for the torque from “log-file,” but, when the torque is computed from the Maxwell tensor integration, only the interaction through the integration path is considered, and, consequently, in order to obtain total

**FIGURE 12.24**

Specific torque for 1 m machine length for 529 A turns/coil. (a) Specific torque—computed from energy and (b) specific torque—from Maxwell tensor.

machine torque, it is necessary to multiply the integration results with the symmetry parameter. In our example, the integration path is a circle arc with R_{torq} radius and an angle between 60° and 240° , where the R_{torq} radius is

$$R_{\text{torq}} = \frac{D_{\text{si}}}{2} - 0.375 \cdot g \quad (12.14)$$

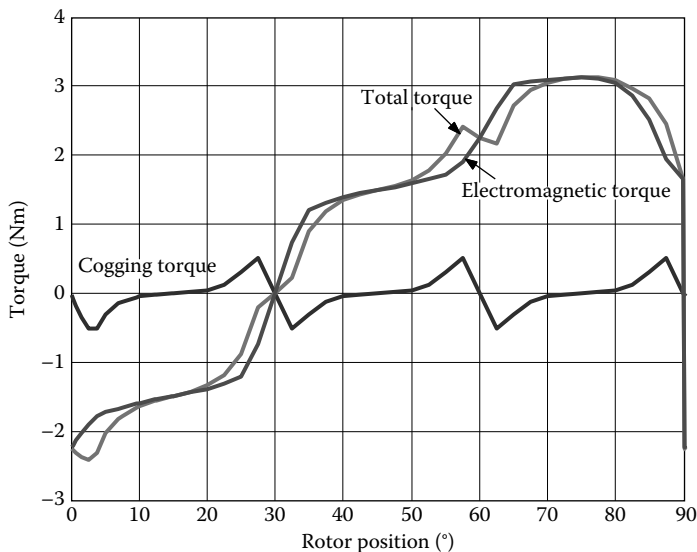
The machine core length is not considered in 2D FEM, and the computed torque is in fact a specific torque for a machine length of 1 m as shown in Figure 12.24. The torque could be presented versus either the rotor position or time. However, these are equivalent at constant speed.

It could be noted that there are no notable differences between the torque computed from magnetic energy variation and that computed from Maxwell tensor integration (Figure 12.24).

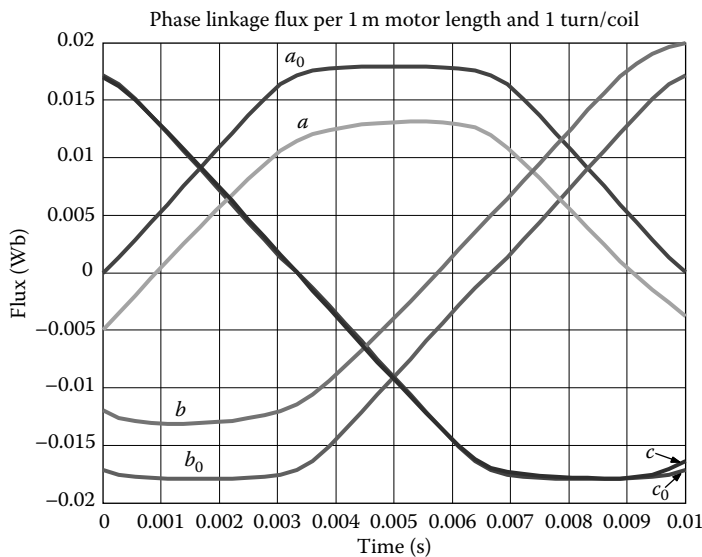
The machine torque is computed by multiplying the specific torque from the FEM with the machine length. Figure 12.25 shows the machine torque versus the rotor position.

The electromagnetic torque has maximum values between 60° and 90° of rotor mechanical position; consequently, these angles will be used to switch the phase currents in such a way as to repeat the field configuration from 60° to 90° for all rotor positions (the drive function from Figure 12.22 was chosen in this way).

The phase flux linkage is computed as the average of the vector potential difference between the coil sides (Equation 11.119). The computed flux linkage is for a coil with a single turn and a core length of 1 m. The coil flux linkage is obtained by multiplying the FEM value with the core length and with the number of turns per coil. The flux linkage in the phase with zero current (phase c) is equal to the permanent magnet flux linkage for all rotor

**FIGURE 12.25**

Machine torque versus rotor position for 529 A turns/coil.

**FIGURE 12.26**

Flux linkage.

positions, Figure 12.26, despite the magnetic phase coupling and core saturation. This feature could be used in control when the voltage of the idle phase is measured.

Observing the flux density distribution (absolute value) from Figure 12.23 it is difficult to choose a value for iron loss computation. The space average of the square flux density on the stator core could be a proper value for iron loss computation:

$$B_{sav} = \sqrt{\frac{1}{S} \int_S B^2 ds} \tag{12.15}$$

The space average values of the flux density are variable in time, Figure 12.27. For some rotor positions, the total space average flux density is larger than the permanent magnet average flux density while for other rotor positions it is smaller.

The space average flux density depends monotonically on the current for a single ac current excitation device and its maximum is reached when the current reaches its maximum. This value should be a proper value for iron loss computation but this is so only for single-phase transformers. The same value is obtained if the rms value of the flux density is multiplied with the crest factor, which is $\sqrt{2}$ for sinusoidal variations:

$$B_{pk} = \sqrt{\frac{2}{T} \int T B_{sav}(t) dt} \tag{12.16}$$

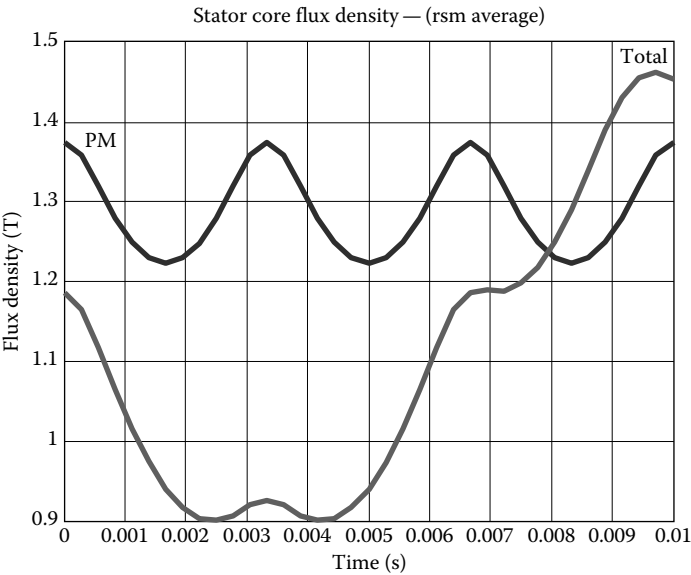


FIGURE 12.27
Stator core flux density.

Assuming that there is a region with pure rotational magnetic field with the flux density magnitude equal to B_{sav} , the iron losses for the rotating field double the iron losses for the pulsation field of the same magnitude. This implies that the equivalent flux density from Equation 12.16 could also be used to compute iron losses for pure rotating fields. In conclusion, we assume that the peak flux density from Equation 12.16 could be used to compute the iron losses for complex field variations because it works for extreme situations, i.e., pulsation fields and rotational fields.

The stator currents commutation according to the drive function as shown in Figure 12.22. The 6/4 poles BLDC machine was chosen because it is assumed that radial forces are zero when the rotor eccentricity is zero, whereas the 3/2 poles machine has important radial forces. The entire machine was studied in order to compute radial forces. The following results are presented for zero eccentricity but it is possible to consider the eccentricity effect on the radial forces, torque pulsations, and flux linkage. All regions that are inside the “rotating airgap” rotate around the origin of the coordinate axis. We can produce an eccentricity by moving the rotor regions or the stator regions along the radius. The airgap layers will be resized according to the new geometry configuration. A full mechanical revolution is necessary to study the eccentricity behavior.

In the following example, the radial forces are computed at zero eccentricity, thus, again, it is enough to study only a quarter of the mechanical revolution but for the entire machine geometry. The map of the flux density (absolute values) and magnetic field lines are presented in Figure 12.28 at zero current and in Figure 12.29 at rated currents. Two significant moments

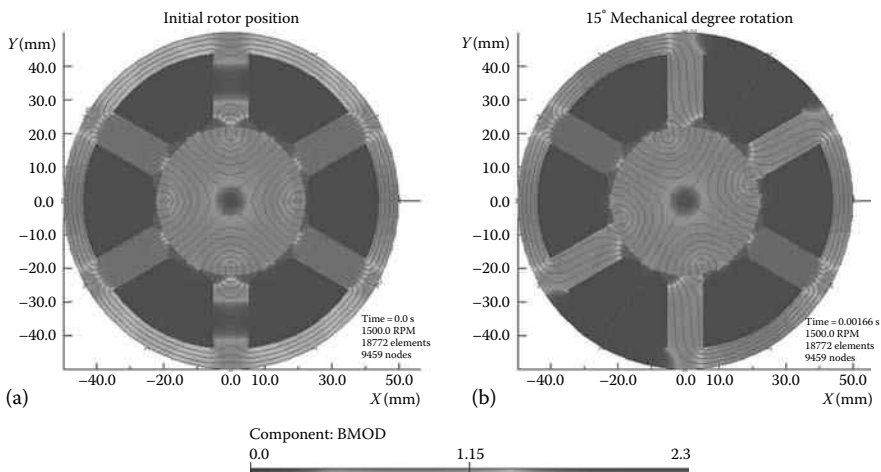


FIGURE 12.28

The flux density map and magnetic field lines produced only by PMs.

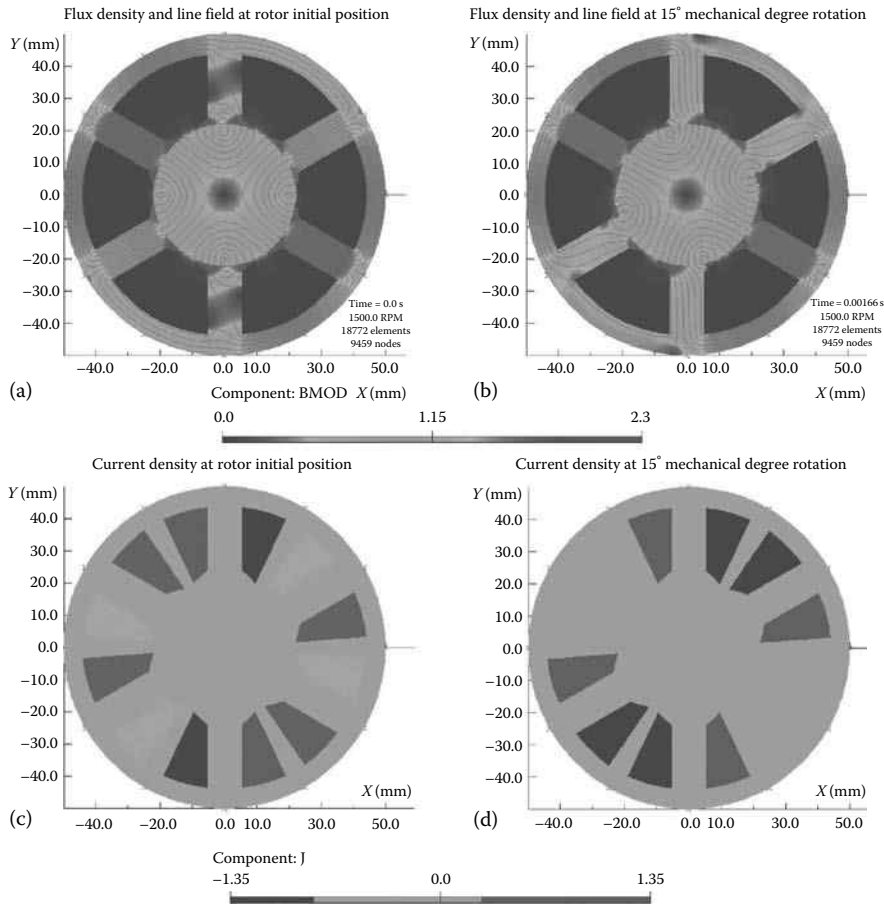


FIGURE 12.29
The flux density map and magnetic field lines at rated current.

were chosen to present the flux density and magnetic field lines: the current commutation moment that happens at rotor initial position (Figure 12.29a) and the moment of maximum torque that occurs after 30° electrical degrees, which means a mechanical rotation of 15°.

The current density distribution shown in Figure 12.29a proves that at zero rotor position, the current in phase b is equal to the current in phase c as it was set by the drive function (Figure 12.22) and then the current in phase b becomes zero. The flux density map and magnetic field lines give general information about field configurations for different rotor positions with and without coil currents but it is difficult to appreciate, only from the pictures, the exact field values in the points of interest. We can see that the maximum flux density value in the teeth corner is close to 2.3 T when only

PMs produce magnetic fields and it increases to 2.35 T when the coils are supplied with rated current, but it is difficult to appreciate the flux density in the airgap and along the magnetic core path. It is possible to pick the flux density in several points but more organized information is available using the flux density graphs along the path of interest. All classical machines design is based on the airgap flux density and its distribution. The radial component of the flux density along the airgap is presented in Figure 12.30 for initial rotor position without current, curve “p0”; with current curve “ip0”; and for 15° mechanical rotation without current, curve “p 15” and with current curve “ip 15.” There are similar ways to get the flux density graph along a path, in different commercial softwares. It is only necessary to choose the represented component and to define the path. Sometimes, it is useful to save the desired graph values in a table and then use this off line for more complex analyses, such as harmonics extraction and solution comparisons. The fundamental of the flux density distribution moves with the rotor while the stator slot opening effects remain fixed to the stator coordinate as shown in Figure 12.30.

The flux density along the stator teeth is also an important value in the classical design. Using the field distribution map we can choose one or more paths to represent the field density. In our example, the phase b teeth seem to be heavily saturated and a path along the b teeth axis, as shown in Figure 12.31 (Path 1), was chosen to represent the flux density. The tangential component of the flux density along Path 1 is shown in Figure 12.32. The normal

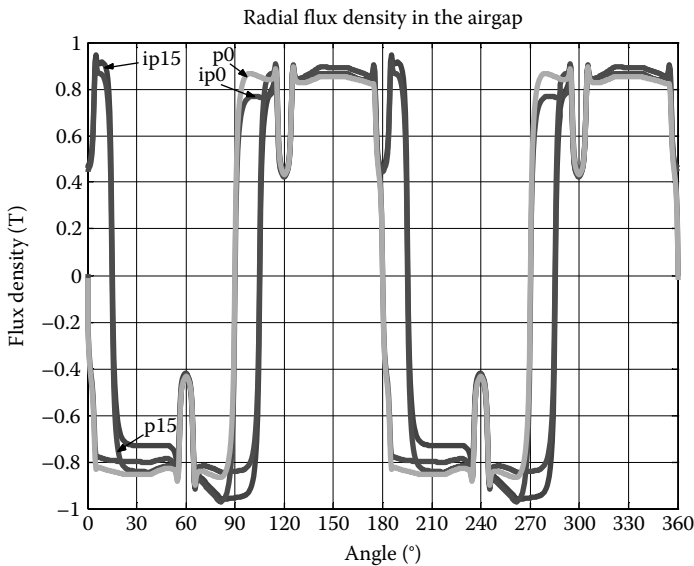


FIGURE 12.30

Radial component of the flux density in the middle of the airgap ($R = 21.75$ mm).

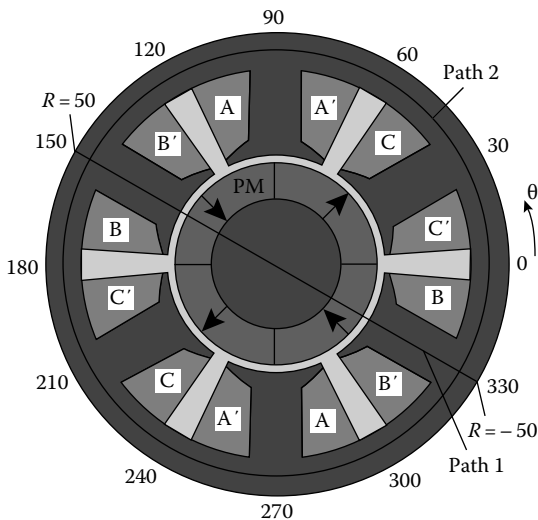


FIGURE 12.31
Path definition for flux density representation.

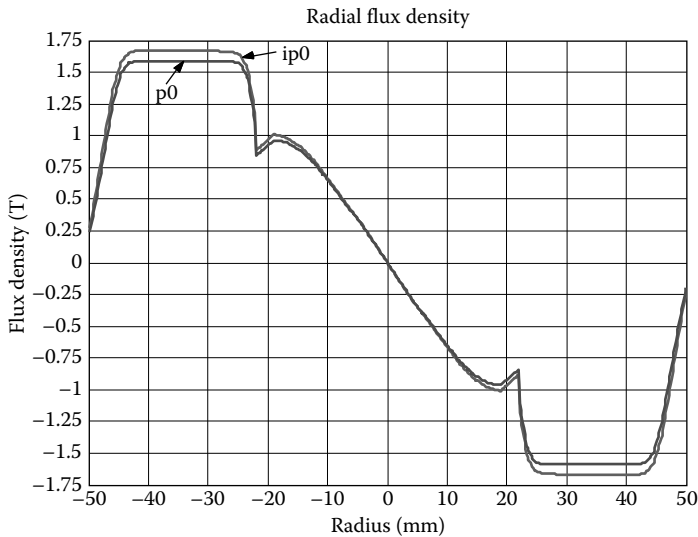


FIGURE 12.32
Radial flux density along b axis (Path 1 in Figure 12.31) for initial rotor position.

flux component could be also presented but it will be small due to the magnetic field symmetry along the b phase axis for the chosen rotor position. The absolute value of the flux density is practically equal to the tangential value in this situation. The equivalent radial flux density in the stator teeth is, in Figure 12.32, an average between that at the inner stator yoke radius and that at the inner stator radius. The curve “p0” shows the flux density along Path 1, for initial rotor position, produced only by the permanent magnets and curve ip0 shows the flux density produced by the permanent magnets and currents. A slight increase in the flux density can be seen when the currents are present. There is no notable difference between radial flux density with and without current, when the rotor mechanical position is 15° . These curves are similar to the ip0 curve, and, consequently, they are not presented in Figure 12.32.

The tangential flux density along the stator yoke is also an important parameter in the machine classical design where an mmf drop is computed for each machine part. The radial flux density component in the yoke does not contribute directly to the mmf drop on the yoke but it could increase the absolute flux density value and thus increase the saturation level. The tangential flux density and the absolute flux density value along the yoke path (Path 2 in Figure 12.31) are presented in accordance to the position angle in Figure 12.33. The Path 2 is a circle placed in the middle of the yoke. The flux density is shown at initial rotor position without current (curve p0) and with current (curve ip0) and for 15° rotor position, without current (curve p15) and in the presence of current (curve ip15). The large influence of the coil currents' presence can be seen in some yoke regions. The radial component of the flux density produces visible effects on the flux density magnitude, which is different from the tangential component in some zones. The peak value, which is considered in classical design, is only slightly changed by the

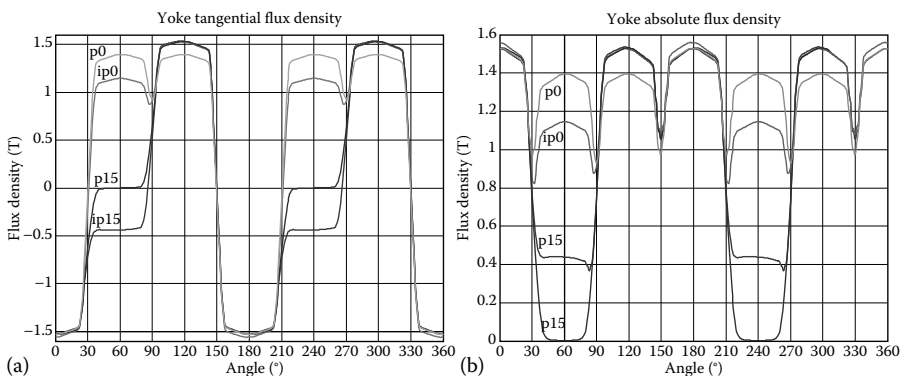
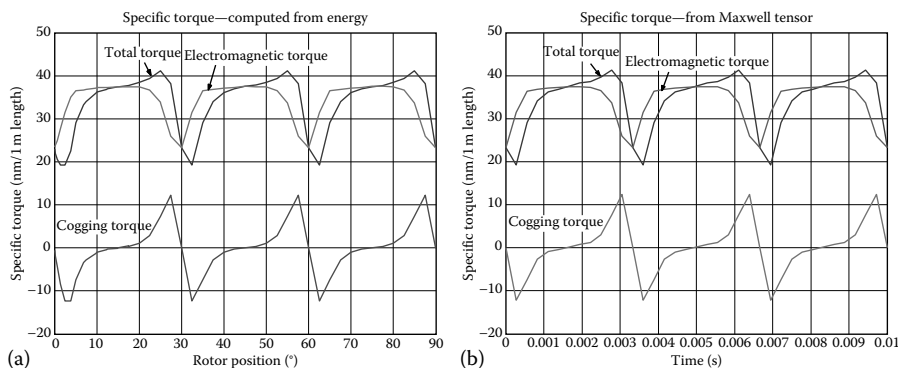


FIGURE 12.33

(a) Tangential and (b) absolute flux density along the yoke path.

**FIGURE 12.34**

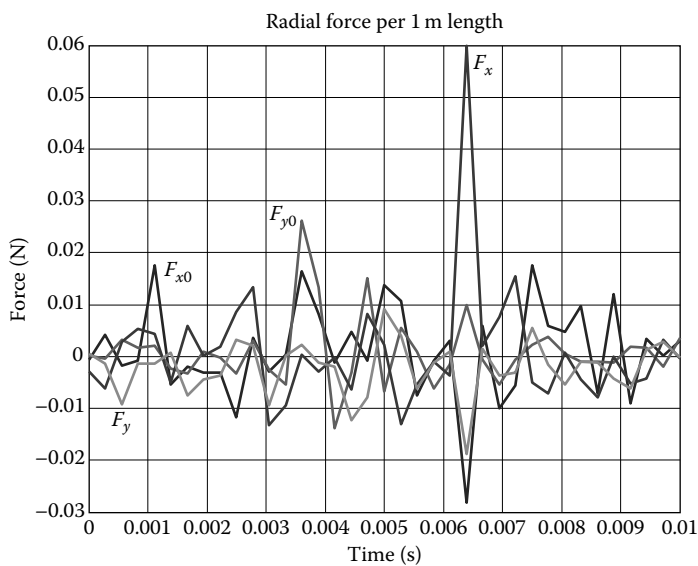
Specific torque versus rotor position in synchronous operation at 265 A turns/coil.

radial component of the yoke flux density. The flux density along the yoke path moves along with the PM rotor but the six-teeth stator structure has an important effect on the flux density distribution in the stator yoke.

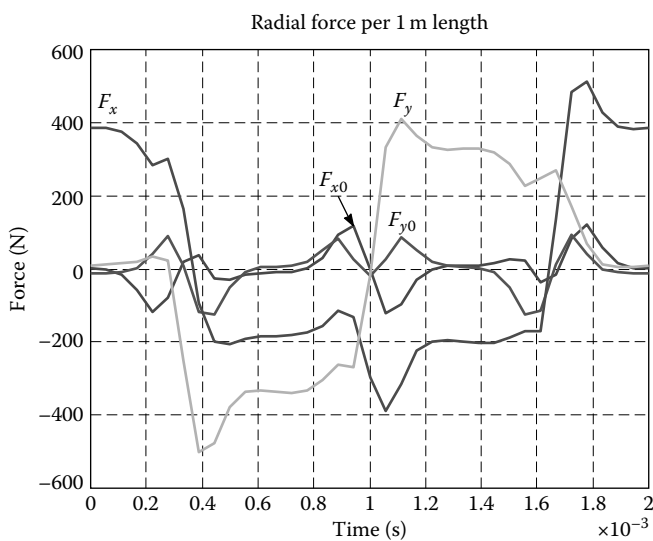
The specific torque per unit core length computed from energy variation is in good agreement with the specific torque computed by the Maxwell tensor integration, Figure 12.34. A large torque pulsation with 12 pulses per complete revolution can be seen. The torque produced from PM interaction with stator current (electromagnetic torque) at 265 A turns/coil is around half the peak torque in Figure 12.25 for the doubled mmf. This means a negligible reduction of the peak torque due to the magnetic cross saturation.

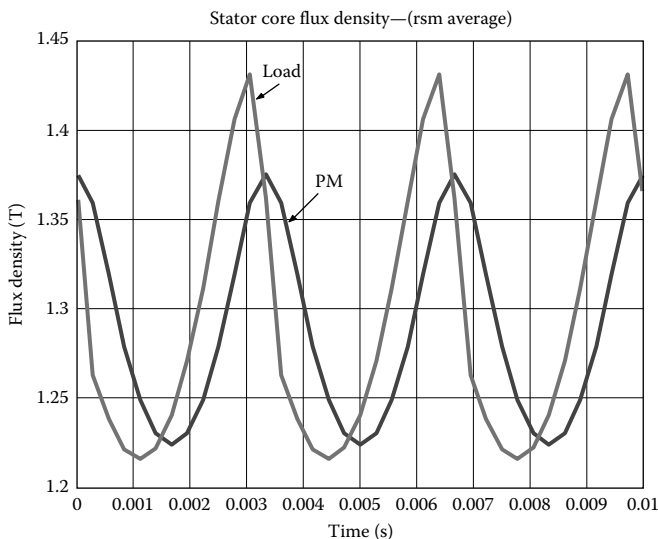
The specific radial force components on the rotor are negligible under no load and also in the load regime as shown in Figure 12.35 where curves F_{x0} and F_{y0} are the no-load-specific forces while F_x and F_y are the radial force components when the machine is loaded. The radial forces are presented versus time and 0.01 s corresponds to a 90° rotor movement. The specific radial forces presented in Figure 12.35 represent numerical computational errors because the magnetic force on the half rotor is around 5000 N/m. The peak value of uncompensated radial forces from Figure 12.35, 0.06 N/m, shows that the accuracy of radial forces computation is good. For comparison, Figure 12.36 shows the specific radial force of a 3/2 teeth/pole BLDC machine with a smaller inner diameter (15 mm, while in our example the stator inner diameter is 44 mm). The specific radial forces are around 400 N/m, which is 10,000 larger than in the 6/4 configuration. This large radial force produces motor vibration, noise, and premature bearing wears.

The space average flux density versus time is shown in Figure 12.37 for no-load and load regime. The equivalent peak flux density values for core

**FIGURE 12.35**

Radial forces of the 6/4 BLDC motor.

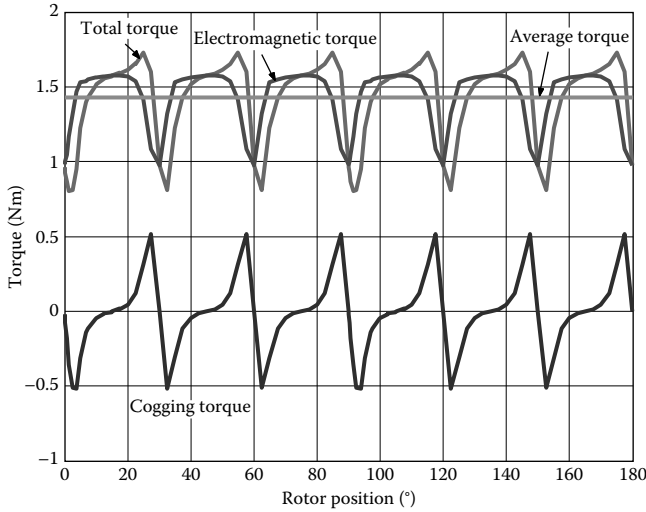
**FIGURE 12.36**Radial forces of a BLDC machine with 3/2 stator teeth/rotor pole with $D_{si} = 15$ mm.

**FIGURE 12.37**

Space average flux density in synchronous operation.

losses computation, calculated with Equation 12.16, are $B_{pk0} = 1.8286$ T for no load and $B_{pk} = 1.8374$ T for rated load. The core losses increasing from no load to rated load is small (about 1% of rated power if the core losses are considered in direct ratio with the flux density magnitude). The high order flux density harmonics increase the core losses much more.

The maximum flux density in the teeth from Figure 12.32 is 1.67 T, and in the yoke, from Figure 12.33, is 1.53 T. The peak flux density from FEM is with 14.73% larger-than-average value used in classical design if the square averaging between teeth and yoke flux density is considered for a quick comparison. This increase in the flux density is the effect of nonhomogeneous flux density distributions and local saturation and it increases the iron losses by 31.63%, if we consider only the fundamentals. The average flux variation is presented for half the fundamental period; this means that the flux pulsation from Figure 12.37 is dominated by the sixth harmonic whose magnitude is 0.103 T. The frequency is six times larger than the fundamental frequency and the iron losses will be 11.39% higher than the iron losses produced by a fundamental field. These losses represent 15% when compared with classical computational methods. Finally, the iron losses are 46% larger when compared with classical computational methods even when the flux density from the classical method is based on FEM. The classical design error is usually corrected by a large manufacturing factor, which increases the computed iron losses by 1.5–2.5 times, to be in agreement with test results.

**FIGURE 12.38**

Motor torque pulsations at 265 A turns/coil.

The circuit parameters and mechanical features are computed using the finite element results. The motor electromagnetic torque is computed by multiplying the specific torque from Figure 12.34 with the core axial length. The torque for a complete electrical period could be computed using the periodicity feature. The six torque pulses per electrical period are obtained as shown in Figure 12.38. Several methods were proposed to reduce the torque pulsations, but only PM step skewing yields acceptable results in terms of total torque and torque pulsation reduction [8]. The average value of the electromagnetic torque at a given current magnitude is computed and then it is used for adjusting the final current magnitude in order to produce the rated torque.

In our case, a linear variation of torque with the current magnitude was proven up to the peak torque, which is around two times larger than the rated torque. In this case, a linear relation is used to adjust the rated current magnitude:

$$I_{tn} = I_{t1} \frac{T_n}{T_{n1}} \quad (12.17)$$

where

I_{tn} is the rated value of total coil current

I_{t1} is the total coil current used in the model

T_n is the rated torque

T_{n1} is the average torque from simulations

If the motor torque is not in direct ratio to the stator coil current magnitude due to cross saturation or due to inductance variation, then several finite element runs will be performed at different currents and the rated current will be adjusted by a linear or quadratic interpolation.

The average length of one coil turn is computed keeping in mind the core length and coil end connection length. At every moment, excepting the commutation moments, two phases are in conduction and the current is at its magnitude. The copper losses are computed as

$$P_{co} = 2\rho_{tc} \frac{l_c}{k_{fill}A_c} I_{tn}^2 \quad (12.18)$$

where

ρ_{tc} is the copper resistivity at working coil Tc temperature

l_c is the coil turn average length

A_c is the coil region surface—could be computed from FEM by surface integral

k_{fill} is the coil region filling factor

After the computation of copper losses, the one turn equivalent winding resistance is computed as

$$R_{1turn} = \frac{P_{co}}{2I_{tn}^2} \quad (12.19)$$

The iron losses could be computed based on the peak flux density using the Steinmetz method or a more complex [10] method. In this example, a simple method based on a single losses coefficient is used:

$$P_{Fe} = p_{50Hz1T} \left(B_{pk}^2 \left(\frac{f_{1n}}{50} \right)^2 + B_{1h}^2 \left(\frac{6f_{1n}}{50} \right)^2 \right) \cdot m_{core} \quad (12.20)$$

Then the electric efficiency is computed.

The synchronous inductance for the equivalent one turn winding is computed as the ratio between the variation of one coil winding flux and the total current.

The induced voltage in one turn equivalent winding is computed directly by multiplying the core length with the linkage flux derivative, which is available in the rotating machine module of “Vector Fields”. The motor control keeps the phase currents in phase with the PM-induced voltage. The phase currents are considered to be constant except for commutation moments. This means that the flux-linkage time derivative does not depend on the currents when two motor phases work together. The numbers of turns in series per phase windings is computed as

$$N_1 = \text{round} \left(\frac{V_{dc}}{2(V_{pk1turn} + R_{1turn}I_{tn})} \right) \quad (12.21)$$

The real phase current and linkage flux are simple to compute since the number of turns per phase winding are known:

$$I_n = \frac{I_{tn}}{N_1}$$

$$I_a = \frac{I_{ta}}{N_1} \quad (12.22)$$

$$\psi_a = N_1 \cdot I_{\text{stack}} \cdot \psi_{\text{as1turn}}$$

where ψ_{as1turn} is the specific flux linkage per phase (a single turn and 1 m core length). Similarly, the current and flux linkage are computed for all phases. The phase current waveform versus rotor position is shown in Figure 12.39 and the flux variation is shown in Figure 12.40. The curves f_a , f_b , and f_c are the phase flux linkage at rated load, and f_{a0} , f_{b0} , and f_{c0} are the no-load phase flux linkages, respectively (produced only by PMs).

The real values of the resistance and inductance are finally computed as

$$R_s = N^2 \cdot R_{1\text{turn}} \quad (12.23)$$

$$L_s = N^2 \cdot L_{1\text{turn}}$$

The computed inductance does not include the end coil–connections inductance. This value should be computed analytically or from a separate finite element method.

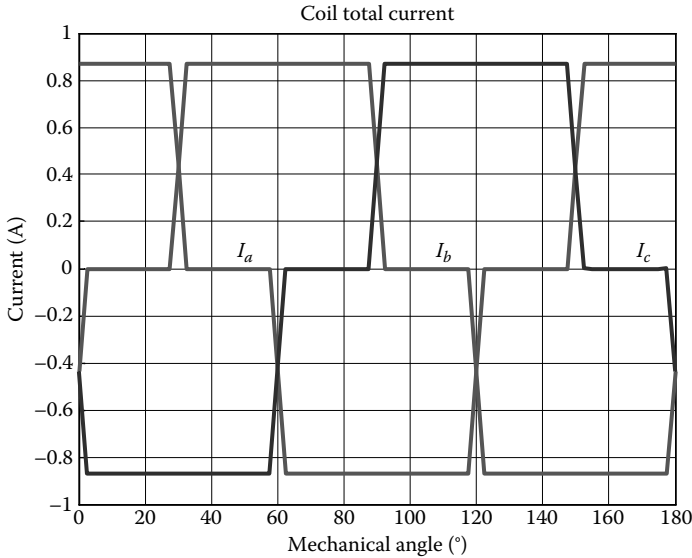


FIGURE 12.39

Phase current waveforms (304 turns/coil).

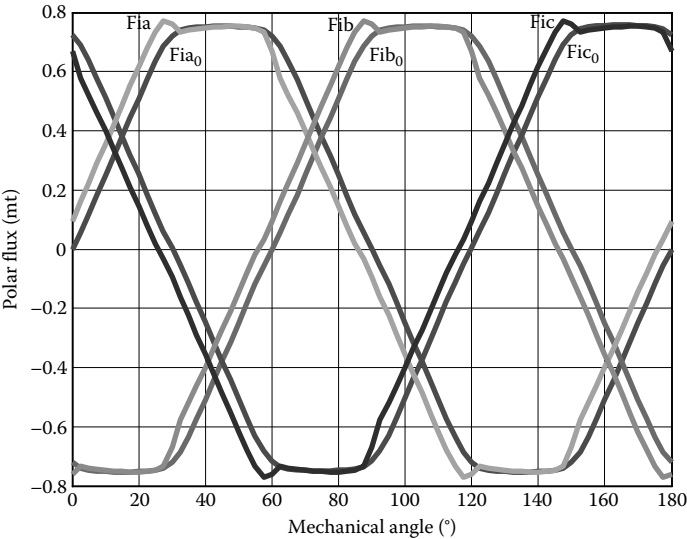


FIGURE 12.40
Phase flux linkage.

The coil conductor's cross section is

$$q_{cu} = \frac{k_{fill} \cdot A_c}{a_1 \cdot N_1} \tag{12.24}$$

where a_1 is the number of parallel paths and A_c is slot area/coil.

12.2.3 Summary

This paragraph shows how the magnetostatic FEM solution for magnetic field distribution allows for a rather comprehensive analysis and design of a brushless dc motor (PMSM with trapezoidal currents); core loss calculations are also included.

Moreover, FEM investigations with coupled field circuit solution are already available for a rather complete dynamics analysis of controlled PMSMs (with sinusoidal or trapezoidal current control). But this is beyond our scope here.

FEM analysis of internal PM rotor and distributed stator windings is presented in [10].

12.3 The 3-Phase Induction Machines

The first step in modeling an induction machine using the finite element method is to find a periodicity symmetry in order to reduce the computation

effort. The maximum value of periodicity symmetry is equal to the larger common divisor of the pole number, stator slot number, and rotor slot number. The entire induction machine is divided in a number of sectors equal to the periodicity symmetry and only one part of this will be modeled using the finite element method. If the studied part contains an odd number of poles, then the symmetry is considered negative (odd) because the magnetic vectors' potential in the paired points along the cutting border has the same value but an opposite sign. If the studied part contains an even number of poles, then the periodicity symmetry is positive (even) and the magnetic vector potential in the paired points along the cutting border has the same value. The required geometric dimensions could be divided in main induction machine dimensions and slots geometric dimensions. The main induction machine dimensions are internal stator diameter— D_{si} , outer stator diameter— D_{so} , inner rotor diameter— D_{ro} , airgap length— g , and core length. The core length does not appear in the two-dimensional finite element model but it is used to compute the main induction machine parameters and performance from the finite element analysis. The number of poles, p_p , the number of stator slots, N_{ss} , and the number of rotor slots, N_{rs} , are also main parameters for building induction machine geometry. The stator slot and rotor slot dimensions depend on the slot type and here we will present the slots geometric dimensions for a few particular cases. A cross section of induction machines indicating the geometric main dimensions is shown in Figure 12.41, and the main dimension values are given in Table 12.3.

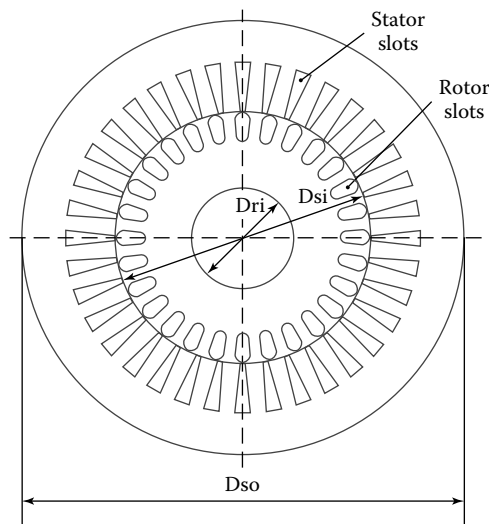


FIGURE 12.41

Cross section of induction machine geometry.

TABLE 12.3
Example of Induction Motors Main Dimensions (22 kW)

No.	Parameter	Value	Measure Unit	Observation
1	Dsi	155	mm	Stator inner diameter
2	Dso	250	mm	Stator outer diameter
3	Dri	50	mm	Rotor inner diameter
4	lc	210	mm	Core length
5	g	0.6	mm	Airgap thickness
6	Poles	2		Number of poles
7	Nss	36		Number of stator slots
8	Nrs	28		Number of rotor slots

In this example, the largest common divisor of poles, stators slots, and rotor slots is two, and, consequently, only half of the motor is modeled using the finite element method. The model contains a single pole and the symmetry is negative (odd).

A large number of stator and rotor slot geometries were presented in Chapter 5. In our study, we choose the stator slot type presented in Figure 12.42 and the rotor slot type presented in Figure 12.43. Their geometry dimensions are given in Tables 12.4 and 12.5, respectively.

The stator slot has a trapezoidal form for small machines, while the body of the stator teeth is rectangular. If the constant width of the stator teeth is given, then the width of the slots is computed using a geometry constraint. When the stator winding has two layers, it is necessary to divide the main height of the slot in two parts S_1 , and S_2 , which has the same cross area.

For the rotor slots, Figure 12.43, all geometric dimensions could be independent for a general case. If a rotor slot has the upper and bottom curvature tangent to the rotor teeth lateral wall, then the curvature radii, R_{r1} and R_{r2} , and heights, h_{r2} and h_{r4} (curvature parts height), are computed from slot widths w_{r1} and w_{r2} , and main part height, h_{r3} :

$$\alpha_{rs} = \operatorname{atan}\left(\frac{w_{r1} - w_{r2}}{2h_{r3}}\right) \tag{12.25}$$

$$R_{r1} = \frac{w_{r1}}{2 \cos(\alpha_{rs})} \tag{12.26}$$

$$R_{r2} = \frac{w_{r2}}{2 \cos(\alpha_{rs})} \tag{12.27}$$

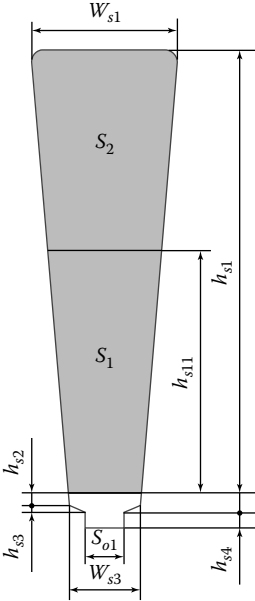


FIGURE 12.42
Stator slot geometry.

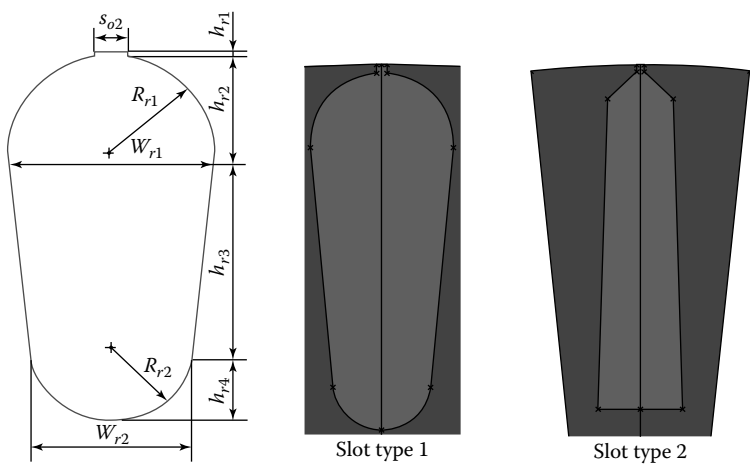


FIGURE 12.43 Rotor slot geometry.

TABLE 12.4
Stator Slot Dimensions

No.	Parameters	Value	Units	Observation
1	hs1	20.35	mm	Coils height
2	hs11	11.2	mm	First layer height
3	hs2	0.65	mm	Figure 14.42
4	hs3	1.2	mm	Figure 14.42
5	hs4	0.8	mm	Figure 14.42
6	so1	3.9	mm	Slot opening
7	sbt	6.5	mm	Stator teeth width
8	kfill	0.4		Slot filing factor

TABLE 12.5
Rotor Slot Dimensions

No.	Parameters	Slot Type 1	Slot Type 2	Units
1	hr1	0.5	0.5	mm
2	hr2	4.31	2	mm
3	hr3	16	23	mm
4	hr4	2.54	0	mm
5	so2	0.6	0.6	mm
6	wr1	8	5.2	mm
7	wr2	5.5	6.7	mm
8	1/Rr1	0.2492	0	1/mm
9	1/Rr2	0.3625	0	1/mm

$$h_{r2} = \sqrt{R_{r1}^2 + \frac{s_{o2}^2}{4}} + R_{r1} \sin(\alpha_{rs}) \quad (12.28)$$

$$h_{r4} = R_{r2} (1 - \sin(\alpha_{rs})) \quad (12.29)$$

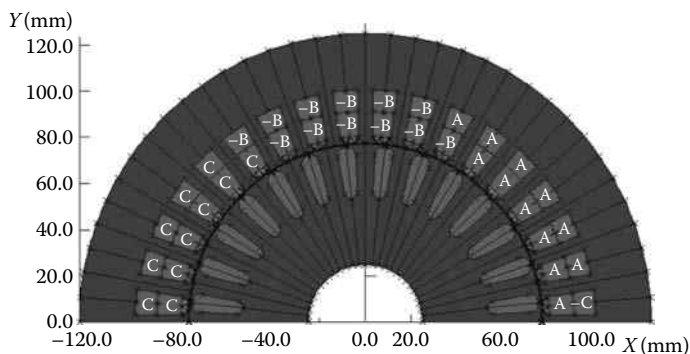
If the rotor teeth have plane parallel walls (constant width), then the slots wall angle is equal to half the angle between the two adjacent teeth. The slot width and slot height are not independent in this case and Equation 12.25 could be used to compute one of the dimensions when the other two are given.

The “I” comments in Table 12.5 indicate input data, while the, “C” comments indicate computed data in order to satisfy geometry restrictions.

In the rotor slot construction, the curvature of the slots wall (inverse of the radius R_{r1} and R_{r2}) is used. Setting the curvature to zero and $h_{r2} = 0$, $h_{r4} = 0$, the trapezoidal slot shape is obtained as a particular case. The induction machine geometry description is similar to the PMSM geometry description and it will not be presented in detail. In the rotor with general slot geometry, it is simple to introduce the drawing and the teeth drawing considering that the slots axes, are parallel to the x -axis of the Cartesian coordinate system. After this, the slots and teeth drawings are rotated in a clockwise direction with angle α_{rs} in order to reach the desired position. When only a part of the induction motor is studied, it is better to cut the studied part along a tooth axis and not along a slot axis. The induction machine study using the finite element method is more complex than that of PMSM because the rotor currents are produced by induced voltage and they are not directly controlled by the user. Fortunately, the behavior of the induction machine is determined by a circuit model with good accuracy and there is a simple method to extract the circuit parameters from the finite element. The machine is modeled in FEM for two simple cases: ideal no load, and, separately, for the rotor bar skin effect.

12.3.1 Induction Machines: Ideal No Load

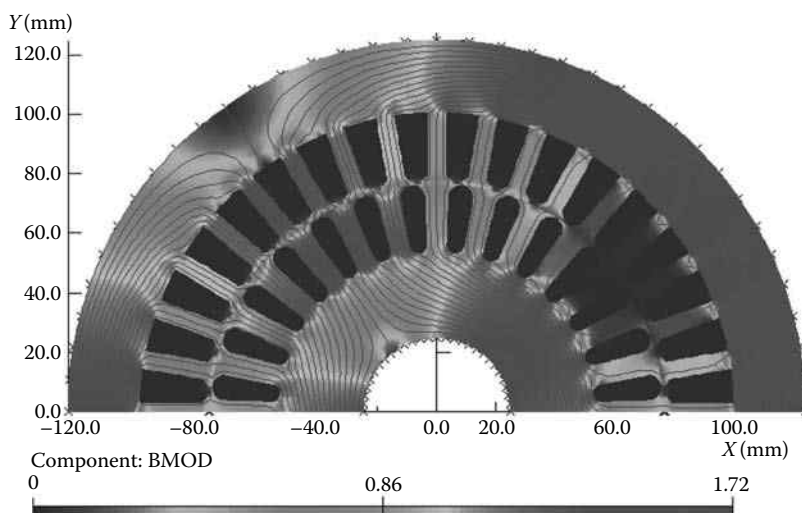
The rotor current could be considered to be equal to zero for the ideal no-load regime of the induction machine. The stator current is assumed to have a sinusoidal time variation. Assuming the stator current magnitude and initial phase values the current is computed in each phase. Even in this ideal condition (sinusoidal stator current and synchronous rotor speed), the rotor bar currents are not equal to zero due to the space harmonics. The rotor current produced by space harmonics could be neglected since the main objective of no-load induction machine simulation by finite element is to compute the magnetization inductance and stator leakage inductance, and to observe the magnetic saturation level in different regions.

**FIGURE 12.44**

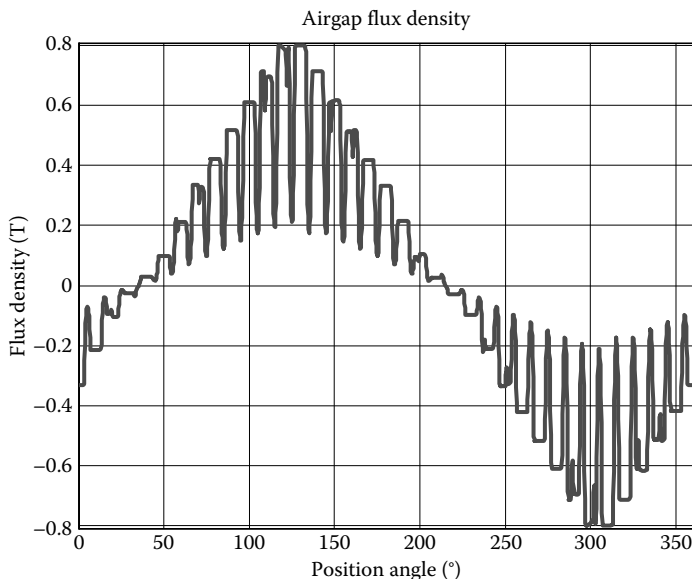
Induction machine phases distribution in slots.

The phase current distribution in the stator slots considering a two-layer winding is shown in Figure 12.44 for a coil pitch that is shorter, with one slot pitch than a pole pitch. The negative sign in front of the phase name shows the negative current for that coil side. Also, it can be seen from Figure 12.44 that the stator and rotor drawing composition from elementary parts are as described for PMSM.

The magnetic field lines and flux density map are shown in Figure 12.45 for the stator current equal to the rated value of the magnetization current. The radial component of the flux density in the airgap versus position is shown in Figure 12.46. The influence of stator and rotor slot opening

**FIGURE 12.45**

Flux density map and magnetic field lines at $I_m = 10.67$ A, 5 turns/coil.

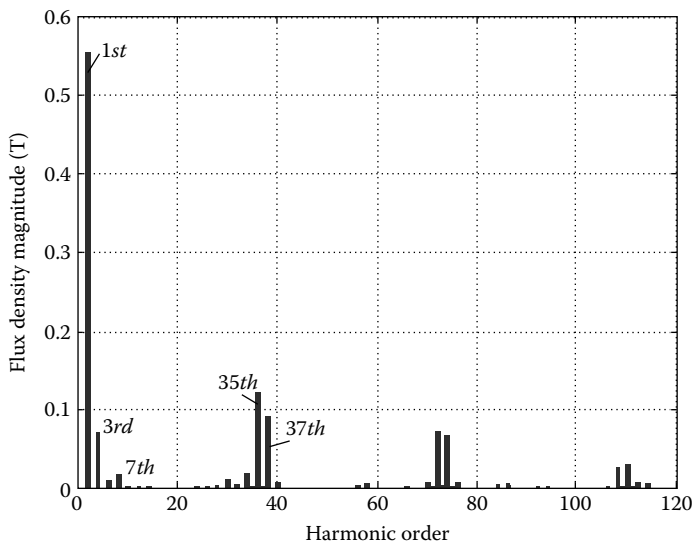
**FIGURE 12.46**

Airgap flux density distribution at $I_m = 10.67$ A.

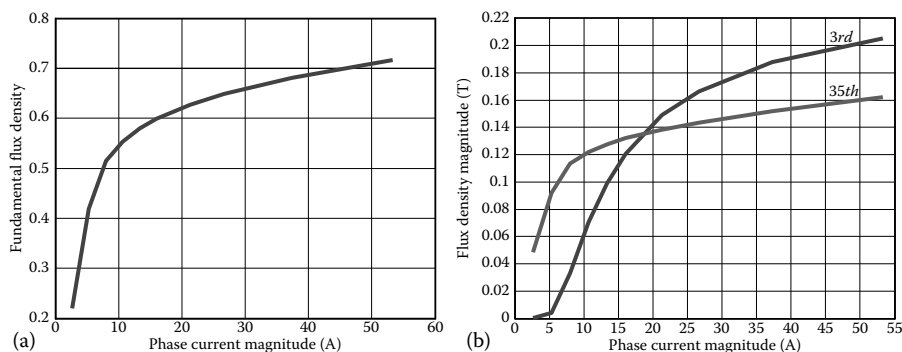
on the flux density waveform, and also the saturation effect, are visible in Figure 12.46.

The airgap flux density spectrum is shown in Figure 12.47 for a magnetization current equal to 10.67 A (peak value). The harmonics spectrum of airgap flux density depends on the magnetic saturation level. The fundamental and most important harmonic magnitudes versus phase current are shown in Figure 12.48. The magnetic saturation occurs around 9 A, and, after this value, the fundamental flux density magnitude increases slowly while the third harmonic, which has small values for non saturation current, increases rapidly. When the magnetic saturation is very deep, the third harmonic growing trends are reduced, but, however, it is more than 25% of the fundamental magnitude. The first stator slot harmonic (35) is one of most important harmonics and it increases with the magnetization current in a similar way to the fundamental component. The rotor slot harmonics are also present in the airgap flux density but in this example they have small magnitudes because the rotor slot openings are small. The rotor teeth corners saturation is expected for a large current in the rotor (starting current) and then the equivalent rotor slot opening increases. The rotor slot harmonics increase for large currents but they still remain smaller than stator slot harmonics.

The fundamental linkage flux, Figure 12.49, which represents the magnetization flux or main flux, is computed with the analytical Equation 12.30 where the first harmonic fundamental of airgap flux density comes from

**FIGURE 12.47**

Airgap flux density spectrum at $I_m = 10.67$ A.

**FIGURE 12.48**

Fundamental and representative harmonics of airgap flux density magnitude versus magnetization current.

finite elements. The main phase (magnetization) inductance, Figure 12.50, is computed dividing the first harmonic flux magnitude to the magnetization current magnitude (Equation 12.31):

$$\Psi_{m1} = \frac{2}{\pi} k_{w1} N_1 \tau l_c B_{ag1} \quad (12.30)$$

$$L_m = \frac{\Psi_{m1}}{I_m} \quad (12.31)$$

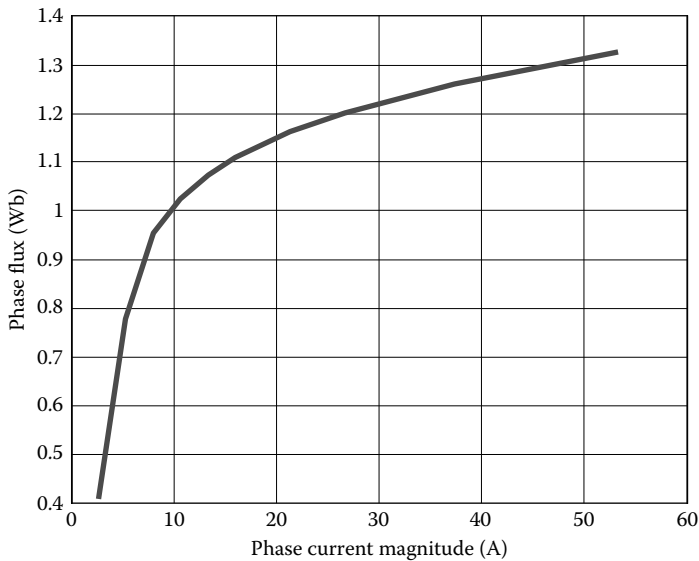


FIGURE 12.49
Main phase flux versus magnetization current (5 turns/coil).

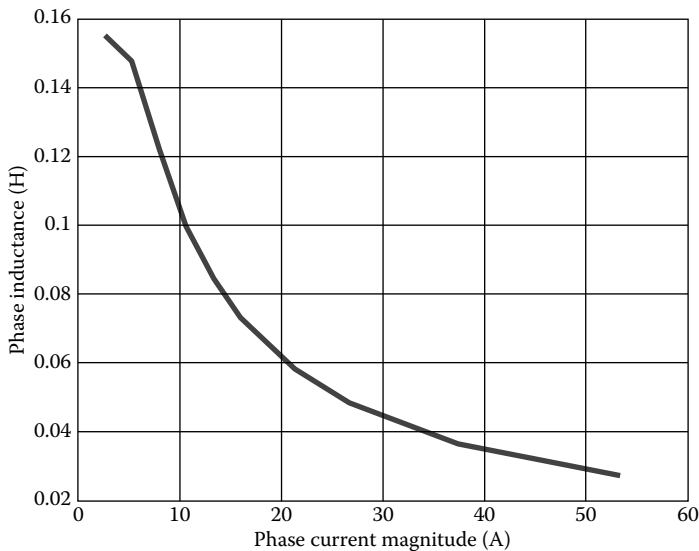


FIGURE 12.50
Magnetization inductance versus current (5 turns/coil).

where

k_{w1} is the winding factor for the first harmonic

N_1 is the number of turns in series per phase

τ is the pole pitch

l_c is the core length

B_{ag1} is the magnitude of the first harmonic of airgap flux density

The airgap flux density was computed with symmetric currents in all phases.

The total phase flux could be computed by the integration of magnetic vector potential, Equation 11.119, for each phase. In our case, the “phase a” current is equal to current magnitude and the linkage total flux also reaches its magnitude. We can observe that this value is smaller than the magnitude of the main flux. The total linkage flux contains the main flux plus airgap and leakage flux. When the third harmonic alone is larger than 25% of the fundamental harmonic for the main magnetic flux, it is possible that the maximum value of the total flux is smaller than the magnitude of the first harmonic of the magnetization flux. For such large saturation levels it is not possible to compute the leakage flux as difference between the maximum value of the total flux and the magnitude of the main flux (first harmonic). The leakage flux will be computed as a sum of the leakage flux in different parts of the machine. From the two-dimensional finite element model, it is possible to compute the airgap leakage flux and slots leakage flux. The airgap leakage flux is related with space high order harmonics (Equation 12.33) while the slots leakage flux is related with the magnetic field energy stored in the stator slots. The end connection inductance is computed using analytical models [11].

$$L_{\sigma 1} = L_{\sigma h} + L_{\sigma s1} + L_{\sigma el} \quad (12.32)$$

$$\Psi_{agh} = \frac{2}{\pi} N_1 \tau l_c \sqrt{\sum_{v=3}^{v_{\max}} \left(\frac{k_{wv}}{v} B_{agv} \right)^2} \quad (12.33)$$

$$L_{\sigma h} = \frac{\Psi_{agh}}{I_m} = \sqrt{\sum_{v=3}^{v_{\max}} \left(\frac{k_{wv} B_{agv}}{v k_{w1} B_{ag1}} \right)^2} \cdot L_m \quad (12.34)$$

where

k_{wv} is the winding factor for the v th harmonic

B_{agv} is the magnitude of the v th airgap flux density harmonic

$v_{\max}$ is the largest harmonic that is considered

This number should be smaller than half the number of flux density samples per period.

It is difficult to segregate the magnetic energy produced by a phase current in each slot when short winding steps are used. This is simple to compute in FEMs from the magnetic energy stored in all the slots, in the coil parts, and in the free (wedge) parts:

$$E_{ms} = \frac{1}{2} L_{\sigma s1} (i_a^2 + i_b^2 + i_c^2) = \frac{3}{4} L_{\sigma s1} I_m^2 \quad (12.35)$$

The slot's magnetic energy is computed only for the simulated part of the machine, E_{mss} . In order to compute the stored energy in all slots, we have to multiply the FEM value with the symmetry number, n_s , which is equal to two in our example because half the motor is modeled. The slot leakage inductance is

$$L_{\sigma s1} = \frac{4}{3} n_s \frac{E_{mss}}{I_m^2} \quad (12.36)$$

A part of the main magnetic field enters into the opening of slots and increases the slot energy producing large errors on leakage slots inductance. This error could be reduced if the leakage inductance is computed for large currents (rated currents).

The total linkage flux per phase is computed as the magnetization flux plus the leakage flux:

$$\Psi_1 = \Psi_{m1} (I_m) + L_{\sigma 1} I_m \quad (12.37)$$

The stator resistance is also computed as

$$R_s = \rho_{tc} \frac{N_1 l_{turn}}{q_{co}} \quad (12.38)$$

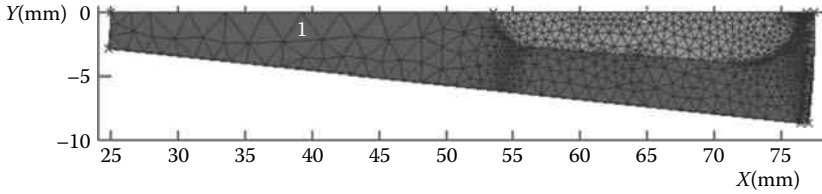
The required voltage versus current is

$$V_1 = \sqrt{(\omega_1 \Psi_1)^2 + (R_s I_m)^2}$$

The no-load ideal current at rated voltage is computed by interpolation and in our example it is $I_{m0} = 9.82$ A.

12.3.2 Rotor Bar Skin Effect

The skin effect is studied on a single rotor bar. In fact, due to the symmetry, the problem could be reduced to half bar as shown in Figure 12.51. The “steady-state harmonic (ac)” module is used in “Vector Fields” in order to study the eddy current in the rotor bar. The discrete element dimensions should be small enough in regions where the magnetic field or current density has large variations. The automatic mesh refinement is chosen, and, in our example, the number of elements was increased three times (from 938

**FIGURE 12.51**

The rotor bar geometry for skin effect study.

elements to 2658 elements, and from 522 nodes to 1424 nodes). This is a reasonable (quite small) number of nodes for a FEM problem but if the entire machine is simulated instead of only half of the rotor bar and half of the rotor teeth, then the number of nodes would increase seriously (about 50 times). The ac magnetic problem could be solved for several frequencies and we can then know the rotor parameter (resistance and leakage inductance) at different values of rotor slip.

The elements are very small in regions where the current density or the magnetic flux density show large variations and they are larger in the other regions. This means an efficient distribution of nodes and a efficient use of the computation resources.

The magnetic field and current distribution is shown in Figure 12.52 at 50 and 10 Hz, respectively. The current distribution map and field lines could be presented at different times (phase), Figure 12.52, but they could also be presented by the magnitude values as in Figure 12.52 or by the time average values for each point. The magnitude mode representation gives the maximum value of the current density or of the magnetic potential for every point. Usually, the current density magnitude or the vector magnetic potential magnitude does not reach all points at the same time and thus the current distribution and the magnetic potential lines may not satisfy the Maxwell equation in the magnitude or average representation mode.

From Figure 12.53 we can see that current density is not in phase all through. A current density distribution along the rotor slot axis is presented in Figure 12.54 at different times (total current phase). Now it is clear that current distribution in the slots depends on the time moment. The current density phase angle along the rotor slot axis is presented in Figure 12.55 for different current frequencies. The current density phase angle is related to the total current phase through the slot.

A large difference of current density phase angle between the top and the root of the rotor slot is observed at 50 Hz current frequency. Small differences in current density phase due to skin effect are observed even at small frequencies (1 Hz respectively 0.5 Hz) where the skin effect on current density magnitude is practically negligible. A comparison of current density distributions along the rotor slot axis is presented in Figure 12.56. At

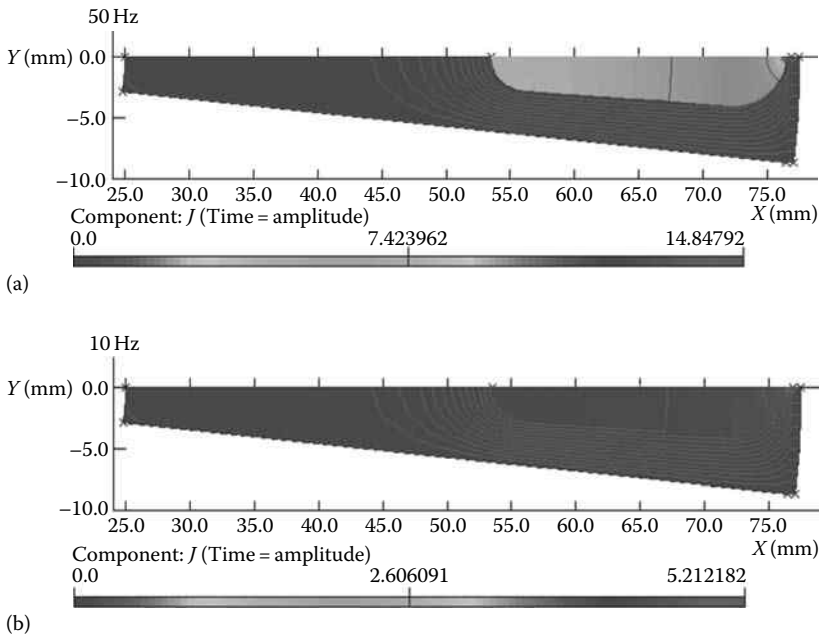


FIGURE 12.52
Current density distribution comparison: (a) at 50 Hz, (b) at 10 Hz.

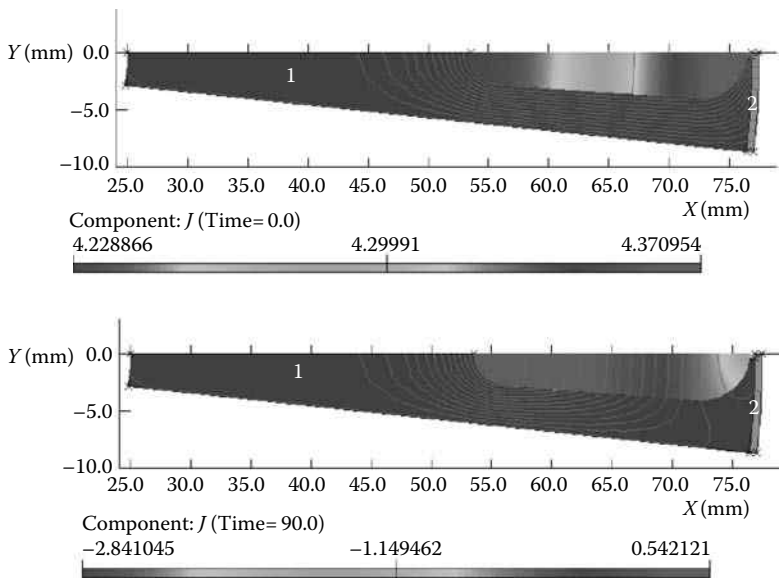
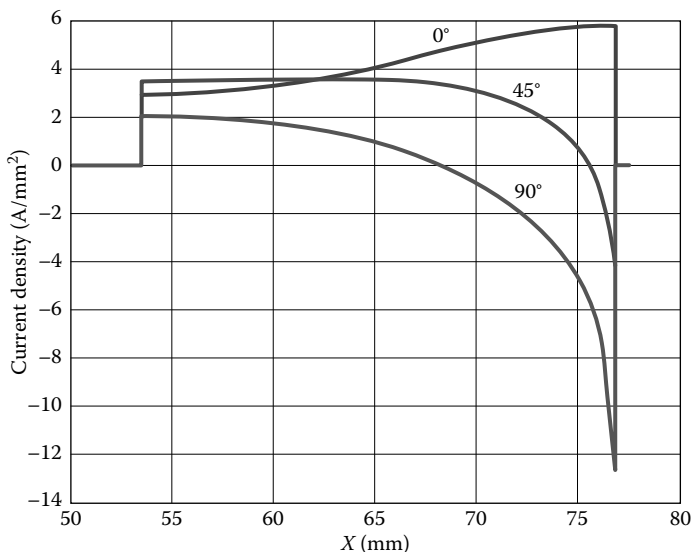
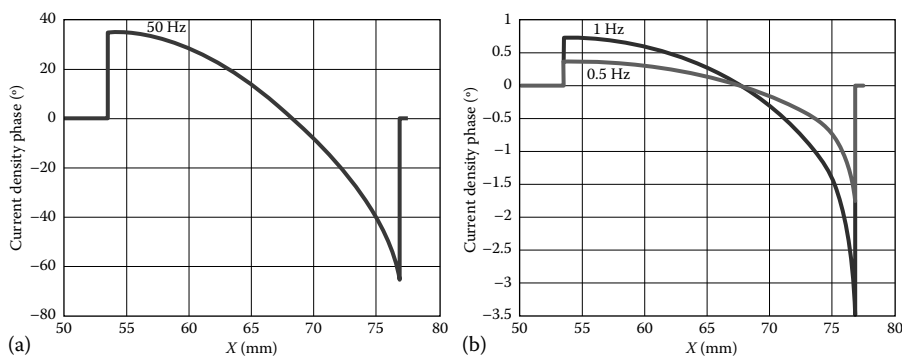


FIGURE 12.53
Current density and field line at two times.

**FIGURE 12.54**

Current density distribution at different times.

**FIGURE 12.55**

Current density phase angle along the slot axis.

low frequency, the current density is uniformly distributed on the slot surface and it has small values, while for industrial frequency, the nonuniform current density distribution produces a large current density peak at the slot opening.

The large current density increases the Joule-Lentz losses that are computed using specific losses integration on the slot volume.

$$p_{co}(t) = \int_V \rho (J(t))^2 dV \quad (12.39)$$

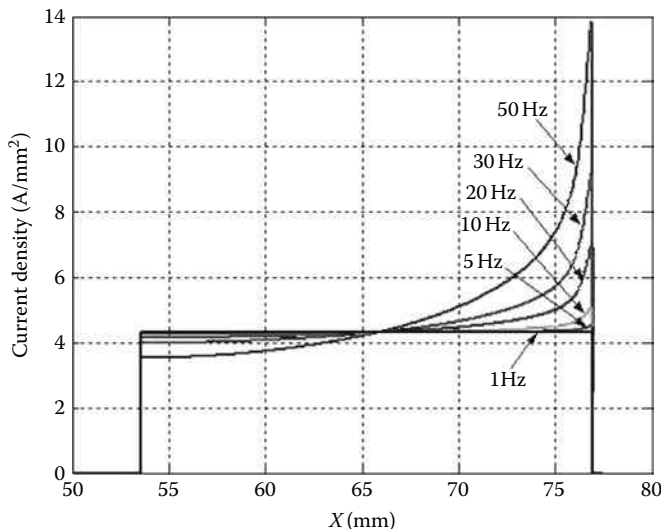


FIGURE 12.56
Current density magnitude along slot axes.

$$p_{co} = \frac{1}{T} \int_{t_1}^{t_1+T} p_{co}(t) dt = \frac{1}{2} \rho l_c \int_S J_m^2 dS \quad (12.40)$$

Some commercial FEM softwares compute the coil losses directly per unity length or for certain core lengths. An equivalent rotor-winding resistance considers the skin effect in the circuit parameter. The rotor bar resistance is computed from average bar losses:

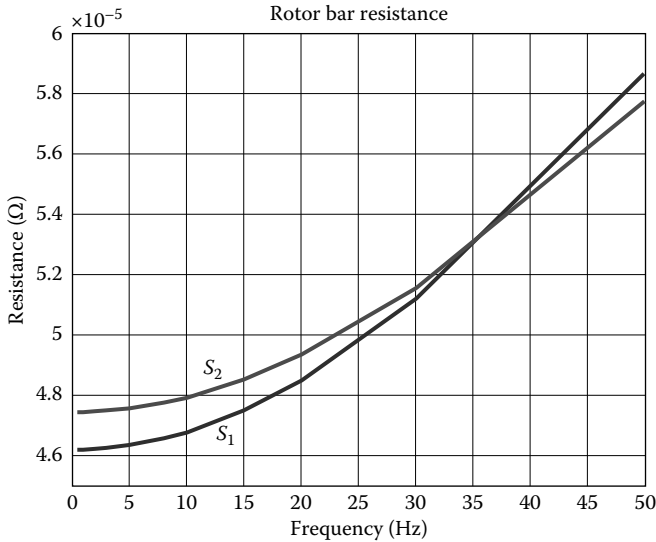
$$R_b = \frac{p_{coh}}{2I_{bh}^2} l_c \quad (12.41)$$

where

p_{coh} is the average losses in half slot
 I_{bh} is the total current on half bar

The equivalent resistance versus frequency is shown in Figure 12.57 for two slot types. The slot shapes are presented in Figure 12.43 and their dimensions are provided in Table 12.5.

Nonuniform current density distribution also decreases the slot leakage inductance, compared to dc leakage inductance, because a small current in the lower part of the slot produces a small field in that slot area. The slot leakage inductance is computed from the slot magnetic stored energy as

**FIGURE 12.57**

Rotor bar resistance versus frequency: s_1 —slot type 1; s_2 —slot type 2 (Figure 12.43).

for steady state. The stator slot inductance versus frequency is shown in Figure 12.58 for the two same slot types. Small oscillations of the rotor slot inductance with frequency at low frequency reflect the accuracy limitation of the numerical solution. The slots leakage inductance increases slightly when the frequency is increased for the second slot type (Figure 12.58 curve s_2). This is also produced by the accuracy limitation of the numerical solution. However, these errors are small and they have no practical importance.

The skin effect factor, which is the ratio between the actual value of the resistance or inductance and their dc values, reflects the skin effect on the circuit parameter better. The resistance skin effect factor, Figure 12.59, is always larger or equal to unity while the inductance skin effect factor, Figure 12.60, is always smaller or equal to unity. The single reason of this rule violation, in Figure 12.60 is the computation error. However, these errors are small, less than 0.1%, and they could be neglected.

The finite element analysis showed that the skin effect at 50 Hz is larger for slot type s_1 , which means larger torque starts at direct connections. The saturation of the rotor teeth root is smaller for slots s_1 and also mechanical resistance of rotor root is better because they have a larger base and a smaller stress concentrator due to the large adaptation radius. In conclusion, slot s_1 is better than slot s_2 .

The total rotor resistance and leakage inductance also contain the short-circuit ring resistance and its inductance. The short-circuit ring parameters as well as the winding stator end-connections inductance cannot be computed

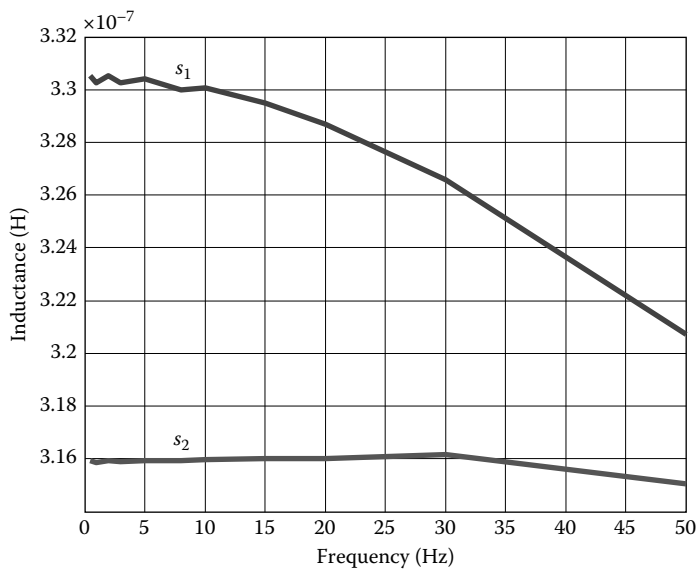


FIGURE 12.58
Rotor bar inductance versus frequency: s_1 —slot type 1; s_2 —slot type 2.

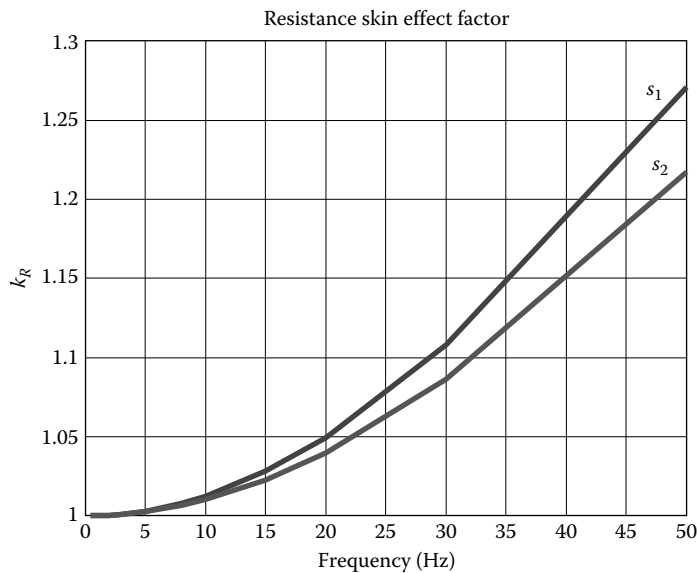
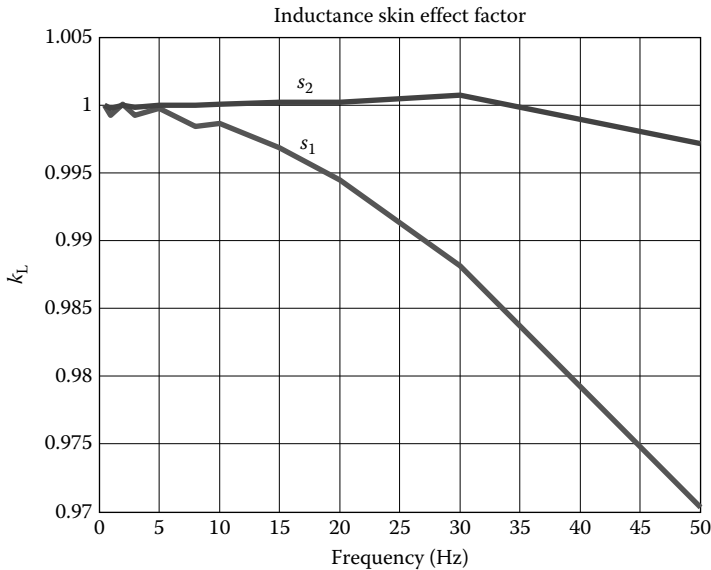


FIGURE 12.59
Resistance skin effect factor: s_1 —slot type 1; s_2 —slot type 2.

**FIGURE 12.60**

Inductance skin effect factor: s_1 —slot type 1; s_2 —slot type 2.

directly using two-dimensional finite element methods. Analytical methods [11] could be used to compute the ring end parameters. The skin effect in the short-circuited end ring is small and the equivalent skin effect of the rotor winding is smaller than that computed for the rotor bar.

12.3.3 Summary

This paragraph shows how to use the FEM in order to compute the circuit parameters of induction machines. The magnetostatic model is used to determine the magnetization inductance and the stator slot leakage inductance. The ac model is used to determine the rotor slot leakage inductance and the skin effect factor of similar ac models could be used to find the skin effect in the stator winding. The circuit parameters are not constant: the magnetization inductance depends on the magnetization current while the bar resistance and rotor leakage inductance depend on the frequency. An exhaustive finite element model of induction machines should consider stator windings and rotor bars connected with the external circuit. The torque versus position in mechanical steady-state regime could also be computed. Torque pulsations and radial forces could also be computed in that case, but the computation effort is much larger. Numerous applications request only circuit parameters, which could also be simply computed from magnetostatic and steady-state ac simulations.

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13

Optimal Design of Electric Machines: The Basics

13.1 Electric Machine Design Problem

The design of electric machines consists of two distinct stages: the dimensioning calculus stage and the verification calculus stage.

Dimensioning includes the choosing of materials, the topology, and the calculus of all geometrical parameters starting from the machine design specifications. Because the relationships between the machine performance indices and the geometrical dimensions are rather involved (and nonlinear), it is impossible to produce a set of inversion functions to distinguish the latter from the former:

$$p_i = f_i(X) \quad i = 1, n \quad (13.1)$$

where

p_i is the performance index from the design specifications

X is the geometric variable and material properties vector

f_i is the involved (nonlinear) function

In general, there is more than one variable vector X that satisfies p_i . Also, for some design specifications there may be no design solutions (e.g., unity power factor for an induction motor) with a certain type of electric machine. By classical nonoptimal design, a possible solution X is found that satisfies most if not all design specifications. The optimal design selects, after mastering a good part of all feasible variable values, the solution that satisfies a given optimum criterion (e.g., minimum initial cost, weight, or maximum efficiency).

As the inverses of functions f_i are not admissible to determine the vector of geometric variables, the electric and magnetic stresses are chosen (current density, A/m²; linear current loading, A turns/m; flux density, T; shear rotor stress N/m²; etc.) based on past experiences. Also some detailed slot geometry parameters, such as slot opening at the airgap or slot wedge thickness, is given. Then, using the laws of electricity (Maxwell equation) and geometrical

relationships, all geometrical parameters are calculated. As the electric and magnetic stresses have been chosen arbitrarily, there is no guaranty that the geometrical variable vector, X , will satisfy the design specifications, p_i . Consequently, the verification of performance is mandatory as part of the design process.

The verification calculus includes

- Electromagnetic verifications
- Thermal verifications
- Mechanical verifications

In general, to fulfill the performance p_i , the dimensioning calculus has to be redone a few times. So the geometrical dimensioning is found iteratively even within a classical design. The number of iterations depends on the strategy chosen to change the electric/magnetic stresses after each verification calculus routine. To reduce the number of iterations, a correlation between electric/magnetic stress changes and geometrical parameter variations has to be established. However, the number of iterations remains high, except for a highly experienced designer who “feels the way the design goes.”

The stages of a classical design approach are shown in Figure 13.1. The dimensioning and verification calculi are computerized but the selection of electric/magnetic stresses and of initial geometric parameters is still done by the designer. If the design specifications are too restrictive, finding a solution can either be very tedious or even impossible. On the contrary, for low-level specification machines, configurations of lower cost or of good performance may be obtained at the same cost.

The stages of an optimal design are shown in Figure 13.2. The optimal design specifications contain, in general, minimal performance requirements and an objective function. The initial electromagnetic stresses and geometrical parameters are either specified by the designer or randomly generated through a dedicated routine within a specified domain. The dimensioning and parameter calculi are done as in classical designs, but, additionally, the objective function is also calculated. Then the electromagnetic, thermal, and mechanical verification calculi are performed, and for every performance index that is not suitable, a penalty is added to the objective function. If the penalty coefficients are high enough, then their requirements will be satisfied in the final design solution. Within this book the thermal model is largely simplified while mechanically, only the minimum shaft diameter is verified.

The detection of the minimum objective function value and the selection of the new values of electromagnetic stresses and initial geometrical variables before every iteration cycle are heavily dependent on the objective function optimum search method, as shown in the following text.

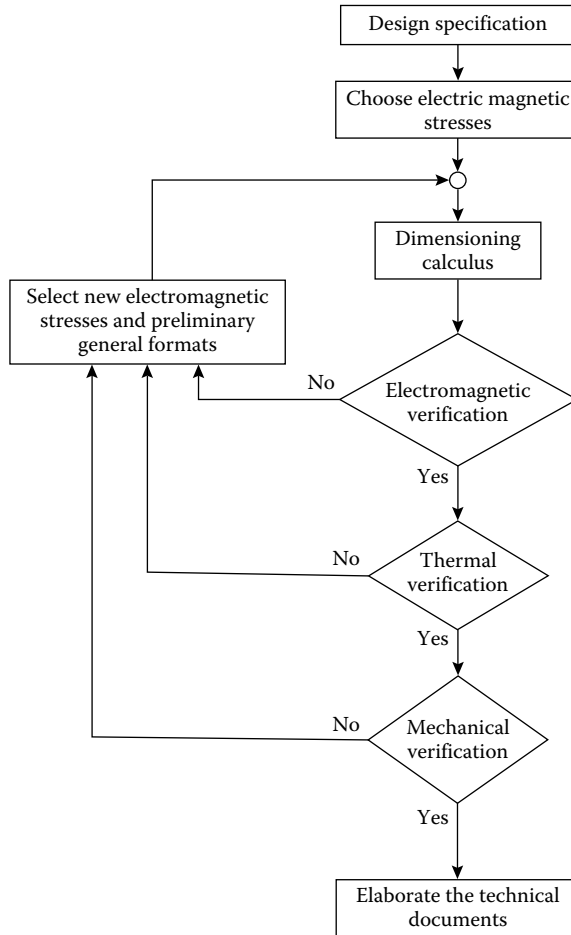


FIGURE 13.1
Classical design flowchart.

13.2 Optimization Methods

There are many definitions for optimization from the viewpoints of mathematics, science, and economics. A concise definition is given in [1]: "Optimization is the act to obtain the best results under given circumstance."

For electrical machines, the optimization could be applied in the machine (drive) design process in order to obtain the best machine (drive) performance according to a given criteria with minimum production cost, or it

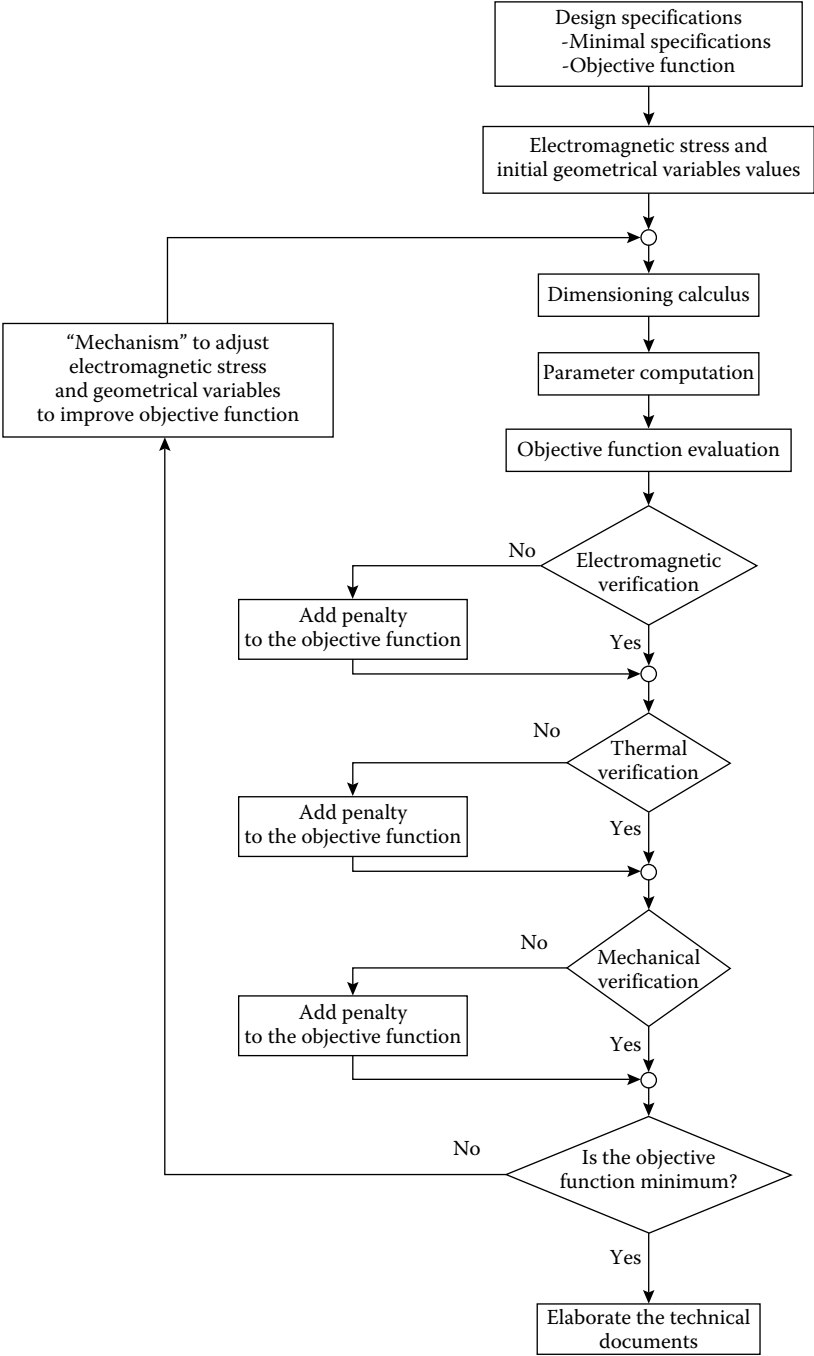


FIGURE 13.2
Optimal design flowchart.

can be used in the drive/generator control. The first problem, designing and manufacturing the machine, is a large-scale optimization process. The optimization process has sense only associated with an optimization criterion. The optimization criterion for electric machine designs could be to reduce the production cost and to improve the electric machine performance, or to do both simultaneously. The production cost has a complex structure containing

- Materials used to manufacture the electric machine, which could be divided into active materials (e.g., stator and rotor coils, magnetic soft material cores, and permanent magnets) and passive materials (e.g., bearings, shafts, frames, ventilation (cooling) systems, winding terminals, and terminal boxes)
- Materials and energy used during electrical machine manufacturing
- Production facilities costs
- Designing, including optimization costs
- Promoting costs, marketing costs, etc.

The production cost optimization is a complex problem that entails managerial and engineering decisions. This book deals only with the active material cost optimization, which is a small but important brick in the global cost reduction. The influence of production facilities on active material costs is considered through technological limitations and technological factors that are constant inputs in active material optimization algorithms. For example, the slot-filling factor, which plays an important role in machine performance such as torque per volume (torque per unity mass), depends on the windings technology. The slot-filling factor is usually smaller than 0.35 for automatic winding machines for a series production but it could rise up to 0.7 for a highly skilled handworker. Increasing the winding slot-filling factor reduces the cost of the active material but it needs better production facilities. The optimization process produces new geometrical dimensions of magnetic core laminations. Introducing a new lamination geometry in production would be costly because it would require a new punching machine. It will be necessary to study how much the new machines would cost and how much time it would take to make returns on the investment. The optimal laminations dimension with the same criterion (let as consider, for simplicity, only the active material costs) depends on the prices of the active materials (i.e., copper, iron, aluminum, and permanent magnet the prices offer). The geometric dimensions of magnetic laminations are changing with time due to the high dynamics of the changing prices of materials. The new optimized machine will be introduced in production only when the active material cost reduction is large enough to compensate for the new facilities cost, for a sustained period of investment recovery. A market price prediction is also useful for this purpose.

Often the minimum cost of a machine does not imply an optimum machine for the user. Users have other criteria, such as good efficiency and small initial price, by which to evaluate the optimum design of an electric machine. These two requirements may seem contradictory but there is an optimum somewhere between maximum efficiency and smaller machine cost. In many applications, the reliability of electric machines is more important than its cost.

The optimal design in mathematical formulation consists in minimization of objective function, f_{ob} , which depends on optimization variable vector, X , and a set of constraints, ($g_i(X)$ —inequality constraint and $h_i(X)$ —equality constraint). Usually the optimization variable domain is lower and upper bounded:

$$\begin{aligned} &\text{find } \bar{X} \text{ to minimize } f_{ob}(\bar{X}) \\ &g_i(\bar{X}) \leq 0, \quad 1 \leq i \leq m \\ &h_i(\bar{X}) = 0, \quad 1 \leq i \leq k \\ &x_{i\min} \leq x_i \leq x_{i\max}, \quad 1 \leq i \leq n \end{aligned} \tag{13.2}$$

In electric machine design, the objective function could be the cost of active materials, the weight of active materials, the machine energy efficiency, or a combination of these. The usual constraints are machines over-temperature smaller than an acceptable temperature value, torque or power larger than the required value, starting current smaller than a maximum acceptable value, torque pulsation smaller than an acceptable value, and technological limitations. The objective function, f_{ob} , and constraint functions, g_i and h_i , are complex functions and do not usually have analytical expressions for their derivatives. Thus, analytical methods such as augmented Lagrange multipliers method cannot be applied. The equality constraints could be eliminated by choosing adequate optimization variables. Sometimes the objective function becomes more complex with the new optimization variable. There are several numerical methods [1] to solve the optimization problem described in Equation 13.2. The main drawback of numerical methods is the algorithm convergence to a local optimum, which could be far away from a global optimum. The optimization algorithm could fail in a local minimum even if the objective function is a unimodal function (i.e, a function that has a single minimum), due to the constraints. The probability of finding a global minimum increases by starting the optimization algorithm from several initial points. Another approach uses evolutionary algorithms, such as the genetic algorithm, which already works with a population of design vectors. The simulated annealing algorithm also has a better probability to reach the global optimum because it permits, sometimes, movement against the objective function gradient according to the virtual temperature.

A large class of optimization algorithms deals with search methods. Many search algorithms have been developed to solve the unconstrained optimization problem. The same methods could also be applied for the constraint problem, but it is necessary to check if the optimization vector belongs to the feasible domain (i.e., whether all constraints are satisfied). The penalty function is another category of method that transforms the constraint problem into an equivalent unconstrained problem. The equivalent objective function becomes

$$F(\bar{X}) = f(\bar{X}) + r_k \sum_{i=1}^m G_i(g_i(\bar{X})) \quad (13.3)$$

There are two categories of penalty functions, G_i : interior penalty functions and exterior penalty functions [1]. If the interior penalty functions are used, the optimal solutions locus changes but it remains in the feasible domain. When the r_k constant decreases, the vector that minimizes the equivalent function, $f(X)$, becomes closer to the vector that minimizes the initial objective function, $f(X)$. Extrapolating the series of vector $X(r_k)$ to $r_k = 0$ it is possible to obtain the vector that minimizes the initial objective function. The usual internal penalty functions are

$$\begin{aligned} G_i &= -\frac{1}{g_i(\bar{X})} \\ G_i &= \log(-g_i(\bar{X})) \end{aligned} \quad (13.4)$$

If the exterior locus penalty function is used, then the vector locus which minimizes the equivalent objective function coincides with the vector which minimizes the objective function $f(X)$, when it is placed inside the feasible domain but it is placed outside of feasible domain when objective function, $f(X)$ has its minimum outside of feasible domain. The outside vector locus is moved closer to the feasible domain when the penalty factor, r_k , increases. When the penalty factor tends to infinity the solution locus tends to the feasible domain border. The usual external functions are

$$\begin{aligned} G_i &= \max(0, g_i(\bar{X})) \\ G_i &= \left\{ \max(0, g_i(\bar{X})) \right\}^2 \end{aligned} \quad (13.5)$$

In this book, the modified Hooke–Jeeves algorithm is used to illustrate the optimal design for induction machines and permanent magnet synchronous machines. The external penalty function based on physical interpretation is used to transform the constraint system into an equivalent unconstrained system. It is well known that increasing the winding temperature over an admissible temperature reduces the life of the machine, which, in terms of optimization, means increasing cost in direct ratio with reducing life.

Considering the over-temperature constraint as in Equation 13.6, where θ is the machine temperature depending on the optimization variable and θ_{ad} is the maximum admissible temperature, the penalty function is computed considering the initial cost of the machine, $f_i(X)$, and the gain factor, k_t , which considers, for example, the number of times the machine life is reduced when the over-temperature increases with a given percent over the admissible temperature.

$$g_\theta(\bar{X}) = \frac{\theta(\bar{X}) - \theta_{ad}}{\theta_{ad}} \quad (13.6)$$

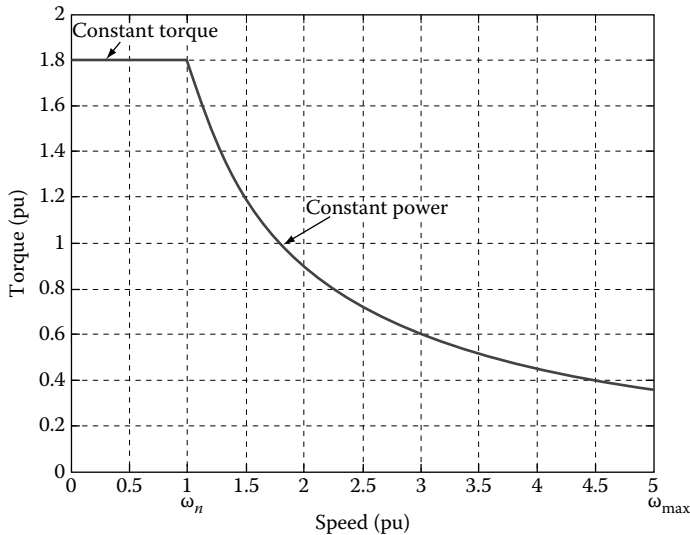
$$G_\theta = k_t \cdot \max(0, g_\theta(\bar{X})) \cdot f_i(\bar{X}) \quad (13.7)$$

The same judgment could also be considered for the demagnetization constraint of permanent magnet machines. There, a safety factor was considered to avoid the permanent magnet demagnetization for uncertain over-currents and uncertainties in material features. When the safety factor reduces, the probability of demagnetization increases and the machine life is reduced. Again, a penalty function could be built, considering the initial machine cost. Using the phenomenological penalty function, it will not be necessary to build the optimal solutions series for larger and larger penalty factor r_k as has been described for classical external penalty functions, in order to reach the feasible border. In fact, the feasible domain border is not a fixed border. The design optimization is usually done considering the rated load, though, usually, the machine runs at different loads. For some applications the probability of load density is known and it could be used for optimal design. Also, the material features and their price evolution are known as their average and variance; thus, in fact, they are stochastic variables. The stochastic optimization methods are able to optimize the machine design using stochastic variables.

Today many electrical machines are designed for variable speed and one frequently constraint condition is to produce larger power or torque than minimum given power or minimum given torque versus speed as it is shown in Figure 13.3. This is a parametric constraint. Its mathematical formulation is given in Equation 13.8 where $T_{\max}(X, n)$ is maximum the torque available for the optimization variable vector, X , at speed n , while $T_d(n)$ is the minimum required torque at speed n :

$$-T_{\max}(\bar{X}, n) + T_d(n) \leq 0, \quad n_{\min} \leq n \leq n_{\max} \quad (13.8)$$

A simple method to solve the parametric constraint is by replacing it with several constraints. Generally, this method does not work efficiently and a large number of cases are necessary to replace the parametric constraint. Fortunately, for this particular case (Figure 13.3), only two or three particular constraints are necessary to replace the parametric constraint. This will

**FIGURE 13.3**

Parametric constraint—minimum torque versus speed for 5/1 constant speed and 1.8 torque overloading capability.

start by introducing torque constraint at rated speed and maximum speed. After optimal design, the parametric constraint will be checked and if the constraint requirements are satisfied for rated and maximum speeds but they are not satisfied for a region between these points, then it is necessary to add extra points to this region and repeat the design optimization. If there is a too restrictive constraint, such as maximum machine external diameter or machine length, maximum allowed current, and maximum permanent magnet emf voltage, then the designed machine does not satisfy the requirement torque even at base (rated) speed. It is then necessary to relax some of the constraints.

13.3 Optimum Current Control

The torque of electrical machines is produced by a pair of magnetization and torque currents. One of the optimum control problems is to minimize the motor losses or the current. A case study is presented in the following text that considers the steady-state rotor field control of an induction motor. The optimization vector has two variables, magnetization current and torque current, that are considered as phase current components. There are also two

constraints: an equality constraint given through the torque equation and an inequality constraint, which states that the required stator voltage should be smaller than or equal to the available voltage. The stator voltage depends on the speed of the motor, so the second constraint is a parametric constraint. For small enough speeds, the voltage constraint does not influence the optimum current solution and in this case the optimum problem becomes

$$i_1 = \sqrt{i_M^2 + i_T^2} \quad (13.9)$$

$$T = 3p_1 \frac{L_m}{L_r} i_M i_T = 3p_1 L_{mr} i_M i_T \quad (13.10)$$

We have here an optimization problem with two variables and an equality constraint. It is possible to eliminate one variable using the equality constraints. Usually the magnetization inductance depends on the magnetization current but it could be considered independent of the torque current. From this reason, it is simple to compute the torque current from Equation 13.10 and replace its values in Equation 13.9.

$$i_1 = \sqrt{i_M^2 + \left(\frac{T}{3p_1 L_{mr}} \right)^2 \frac{1}{i_M^2}} \quad (13.11)$$

A single variable (without constraint) optimum problem is thus obtained. Differential calculi will be used to find the solution but, before that, we can change the objective function from Equation 13.11 by a simple function that has the same extreme points:

$$f_{obj} = i_M^2 + \left(\frac{T}{3p_1 L_{mr}} \right)^2 \frac{1}{i_M^2} \quad (13.12)$$

The minimum current occurs for the solution of equation:

$$\frac{\partial f_{obj}}{\partial i_M} = 0 \quad (13.13)$$

If the magnetization inductance is constant, then Equation 13.13 becomes Equation 13.14 and it admits the analytical solution 13.15:

$$2i_M - 2 \left(\frac{T}{3p_1 L_{mr}} \right)^2 \frac{1}{i_M^3} = 0 \quad (13.14)$$

$$i_M = \sqrt{\frac{T}{3p_1 L_{mr}}} \quad (13.15)$$

The torque current is computed from the constraint equation where the magnetization current is replaced with its optimal value. Finally, an important

result should be noted: the minimum current for a constant magnetization inductance is obtained when the magnetization current is equal to the torque current.

$$i_T = \frac{T}{3p_1 L_{mr} i_M} = \sqrt{\frac{T}{3p_1 L_{mr}}} = i_M \quad (13.16)$$

The magnetization inductance is not constant in the real motor, so it is important to study the motor behavior when it is controlled with the magnetization and torque currents according to the optimization principle at constant inductance. Numerical results will be presented for an induction motor with the following parameters: rated power $P_n = 1.1$ kW, phase rated voltage $V_n = 220$ V, phase rated current $I_n = 2.77$ A, rated frequency $f_n = 50$ Hz, rated speed $n = 1410$ rpm, stator phase resistance $R_1 = 5.31 \, \Omega$, equivalent rotor resistance $R_2 = 5.64 \, \Omega$, short-circuit inductance $L_{sc} =$ at rated current and with the magnetization inductance versus magnetization linkage flux (peak value) from Figure 13.4. The phase current versus torque is presented in Figure 13.5, where the motor is controlled at constant rated magnetization current and according to the optimum criterion of magnetization current equal to the torque current ($i_M = i_T$, $L_{mr} = \text{constant}$). Also, $i_M = i_T$ for the real motor with magnetization inductance dependent on magnetization current produces largely different (larger) currents. We should note that for small

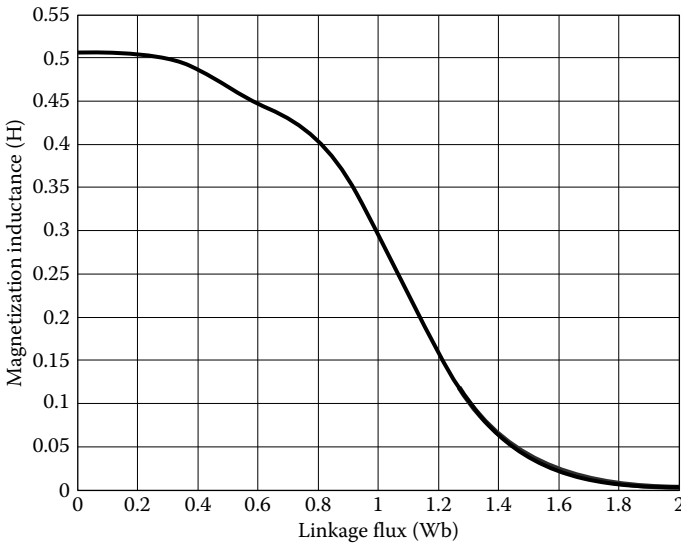
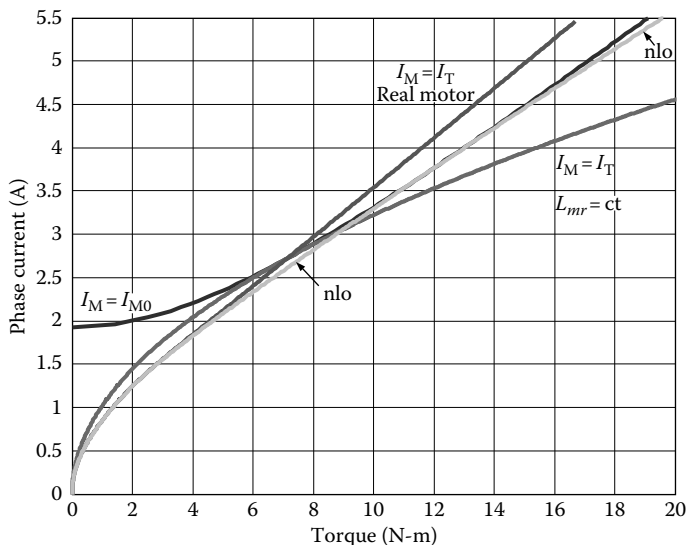


FIGURE 13.4

Magnetization inductance vs. linkage flux.

**FIGURE 13.5**

Phase current (rms value) vs. torque ($Tn = 7.45$, $I_{M0} = 1.93$ A, $L_{rnn} = 0.32$ H).

torques there is a reduction in current even if the model is not exact but, for large torques, the phase current considering the optimization criteria is larger than for $i_M = \text{constant}$ strategy. The optimal control current (curve “nlo”) is somewhere between with good current prediction at low loads as expected. An important conclusion should be noted: when the process model is not accurate enough, the optimal solution could sometimes be worse than the nonoptimal solution. It is important to verify if each variable is reflected correctly in the objective function with the available model. The optimization principle shows better performance than the performance at constant magnetization current for torques that are smaller than rated torques but we should not jump to such conclusions because the studied motors are small motors with magnetization currents around 69% of the rated current. The positive effect of large magnetization currents is the motor capacity to produce 2.5 times larger torques than rated torques with only 2 times larger currents. At the rated load, the magnetization current is almost equal to the torque current.

The derivative of the objective function solution 13.12 is also the optimization key, if the dependence of magnetization inductance on magnetization current is considered, but in this case we get the following nonlinear equation:

$$i_M^4 = \left(1 + \frac{i_M}{L_{mr}} \cdot \frac{\partial L_{mr}}{\partial i_M}\right) \left(\frac{T}{3p_1 L_{mr}}\right)^2 \quad (13.17)$$

The L_{mr} inductance depends nonlinearly on the magnetization current and in many cases only a table with inductance values at several currents is available from measurements or from a numerical model. The available values are not quite exact because they are affected by measurement errors or by numerical errors. Some methods to filter these errors use an analytical approximation of the inductance versus current based on least square error minimization. For example, the magnetization inductance curve presented in Figure 13.4 is based on several measured no-load current versus flux linkage and then an analytical approximation is found

$$L_m = c_0 + \sum_{i=1}^5 c_i e^{-2i\psi_m^2} \quad (13.18)$$

where the series coefficients are $c_0 = 0.002093$ H, $c_1 = 3.3311$ H, $c_2 = -10.6093$ H, $c_3 = 17.1183$ H, $c_4 = -13.2716$ H, $c_5 = 3.93556$ H. It is possible to use many or fewer terms in the series but the coefficients should be recalculated again, because base functions of the exponential series are not orthogonal. Equation 13.17 becomes transcendental and only a numerical solution is possible. The iterative algorithm could be avoided if the torque versus magnetization current is computed

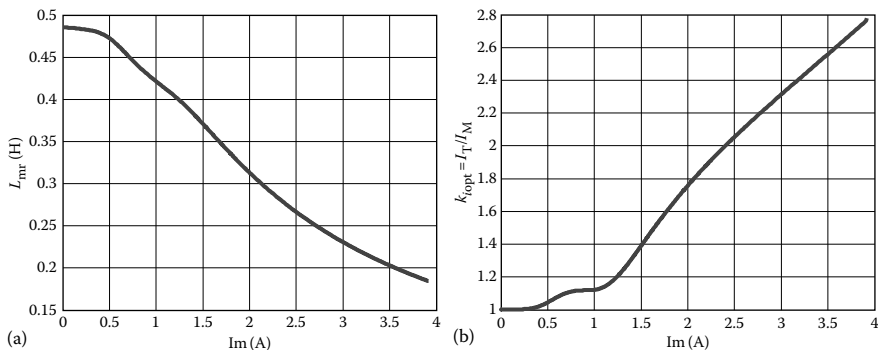
$$T = 3p_1 L_{mr} i_M^2 \frac{1}{\sqrt{1 + \frac{i_M}{L_{mr}} \cdot \frac{\partial L_{mr}}{\partial i_M}}} \quad (13.19)$$

The torque current is computed from the torque equation. The ratio between optimal torque current and magnetization current, k_{iopt} , depends only on the magnetization current and on the machine construction. This nonlinear function could be stored in a table and then it could be used in the optimal control:

$$i_T = \frac{1}{\sqrt{1 + \frac{i_M}{L_{mr}} \cdot \frac{\partial L_{mr}}{\partial i_M}}} i_M = k_{iopt} i_M \quad (13.20)$$

An example is given in Figure 13.6 where, based on Figure 13.4, $L_m(i_m)$ and k_{iopt} are given. Returning to Figure 13.5 we can notice that the stator current for nonlinear optimization (nlo) is close to constant inductance IM control, which suggests that our machine is already saturated.

The energy performance using optimal current control is improving at low torque, (Figure 13.5) but the dynamic performance is poor when the machine is in flux weakening. The time constant to install the magnetic field in the motor is around 10 times larger than the time constant to install the torque. The minimum current optimization can be applied only when there is no sudden variation in the load torque and where fast torque response is not necessary. The stator current minimization reduces the losses but it is not equivalent to the minimum total losses optimization, which has the

**FIGURE 13.6**

The magnetization inductance and the optimal ratio, k_{iopt} , between torque and magnetization current.

following objective function:

$$\begin{aligned}
 p_{\text{loss}} &= R_1 (I_T^2 + I_M^2) + R_2 I_T^2 + \frac{(\omega_1 L_m)^2}{R_{Fe}} I_M^2 \\
 &= \left(R_1 + \frac{(\omega_1 L_m)^2}{R_{Fe}} \right) I_M^2 + (R_1 + R_2) I_T^2
 \end{aligned} \tag{13.21}$$

Decreasing the magnetization current at low load torque in order to reduce the stator current reduces the stator copper losses but the torque component increases and the rotor cage losses also increase. The iron losses decrease with the magnetization current but this also depends on the speed. If the speed is small, the iron losses are negligible. A usual minimum losses optimum criterion gives a little larger magnetization current than the minimum current criterion. This text should serve only a basis for evolved optimal design methods as those discussed in the following.

13.4 Modified Hooke–Jeeves Optimization Algorithm

The Hooke–Jeeves optimization algorithm [2] is a pattern search algorithm that uses two kinds of moves: exploratory moves and pattern moves. In the following text, a modified version of Hooke–Jeeves algorithms is presented in order to allow constrained system optimization using external penalty functions.

The optimization algorithm contains the following steps:

1. Choose the optimized variables and the constant geometrical dimensions. The optimized variables are grouped in a vector. In our case

(a PM brushless motor),

$$\bar{X} = (\text{poles } D_{\text{ext}} w_{\text{st}} w_c h_c h_{\text{sy}} l_{\text{pm}})^T$$

where

poles is the number of poles per primitive machine

D_{ext} is the stator core outer diameter

w_{st} is the stator teeth width

w_c is the coil width

h_c is the coil height in the stator slot

h_{sy} is the axial stator yoke length

l_{pm} is the permanent magnet radial length

2. Find the technological limitations and geometrical constraints, and choose the optimized variable range $\bar{X}_{\min}$ and $\bar{X}_{\max}$. In a mathematical formulation,

$$\bar{X}_{\min} \leq \bar{X} \leq \bar{X}_{\max} \quad (13.22)$$

$$\bar{g}_1(\bar{X}) = \bar{0} \quad (13.23)$$

$$\bar{g}_2(\bar{X}) \leq \bar{0} \quad (13.24)$$

where $\bar{g}_1 : \Re^n \rightarrow \Re^p$ and $\bar{g}_2 : \Re^n \rightarrow \Re^q$ are vector functions defined on variable space, considering “ p ” equality constraints and “ q ” inequality constraints.

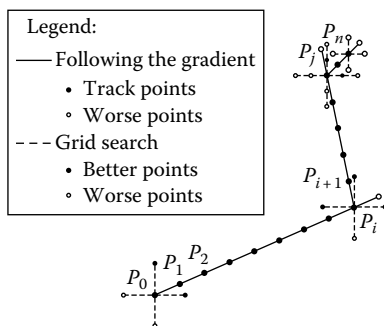
3. Choose the objective scalar function, $f_1(\bar{X})$, $f_1 : \Re^n \rightarrow \Re$. The constraints could be included in the objective function considering the penalty function $f_p(\bar{X})$ and $f_p : \Re^{p+q} \rightarrow \Re_+$:

$$f_p = \sum_{i=1}^{p+q} f_{pi}(g_i(\bar{X}))$$

$$\text{with } f_{pi} = \begin{cases} 0 & \text{if } 1 \leq i \leq p \quad \text{and} \quad g_i(\bar{X}) = 0 \\ 0 & \text{if } p+1 \leq i \quad \text{and} \quad g_i(\bar{X}) \leq 0 \\ \text{monotonic positive } \forall & \text{otherwise} \end{cases} \quad (13.25)$$

Finally, the objective function is

$$f(\bar{X}) = f_1(\bar{X}) + f_p(\bar{X}) \quad (13.26)$$

**FIGURE 13.7**

Search and gradient movement considering a 2D optimization problem.

4. Choose the initial values of optimized variable vector, $\bar{X}_0$, the initial step vector, $\bar{dX}_0$, the minimum step vector, $\bar{dX}_{\min}$, and the step actualization ratio, r , with $0 < r < 1$.
5. Compute all necessary geometrical dimensions, the motor's performance from analytical models, and then evaluate the objective function f_0 .
6. Make a research movement along each optimized variable in positive and negative directions (local grid search) using the initial step as has been shown for a two-dimensional vector in Figure 13.7. Compute the objective function and then compute the objective function gradient:

$$\bar{h} = (h_1, h_2, \dots, h_n) = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \dots, \frac{\partial f}{\partial x_n} \right) \quad (13.27)$$

The partial derivative is computed numerically using function evaluation in three points along the k direction.

$$\frac{\partial f}{\partial x_k} = \begin{cases} \frac{f_k - f_0}{dx_k} & \text{if } f_{-k} \geq f_0 > f_k \\ \frac{f_0 - f_{-k}}{dx_k} & \text{if } f_{-k} < f_0 \leq f_k \\ \frac{f_k - f_{-k}}{2dx_k} & \text{if } f_{-k} < f_0 > f_k \\ 0 & \text{if } f_{-k} > f_0 < f_k \end{cases} \quad (13.28)$$

where f_k is the objective function evaluation in point $\bar{X}_k$ obtained from $\bar{X}_0$ by moving along the k direction with dx_k , and respectively $-dx_k$ for f_{-k} . The point $\bar{X}_0$ is placed on the slope in the first and second case from Equation 13.28. The $\bar{X}_0$ is the worse point in case 3, but, is the better point in case 4. In the last two cases, the partial derivative is not

mathematically correct but it provides information to leave the worse point or to stay near the better point. The step along gradient is computed as

$$\bar{\Delta} = (\Delta_1, \Delta_2, \dots, \Delta_n) = \left(\frac{h_1 \cdot dx_1}{\|\bar{h}\|}, \frac{h_2 \cdot dx_2}{\|\bar{h}\|}, \dots, \frac{h_n \cdot dx_n}{\|\bar{h}\|} \right) \quad (13.29)$$

7. Make an optimized variable vector movement with step $\bar{\Delta}$ until the objective function decreases. Point P_i is reached in the two-dimensional example from Figure 13.7.
8. Repeat the research movement (step "e") in order to find a new gradient direction and then repeat the gradient movement (step "f") until the research movement is not able to find better points around the current point. This condition is equivalent to $\|\bar{h}\| = 0$, point P_j in Figure 13.7.
9. Reduce the variation step by ratio r and repeat the previous steps until the minimum of the variable variation is reached and the gradient norm vanishes. When the research movement cannot find better points even using the smallest variable variation, then a minimum objective function is reached with the given resolution. This is probably not the global minimum. The algorithm should be run several times with different initial values of optimized variables in order to increase the probability to find the global minimum. The optimization algorithm may start with a 1.6 mm variation step for all variables and this is then reduced by ratio 2 until it reaches the minimum value equal by 0.1 mm. The minimum step value is usually chosen around the technological permissible variation. There is no reason to reduce the minimum step more than the permissible variation. The variation step should be a vector for a large machine because the ratio between the largest dimension and the smaller dimension of a variable could be hundreds or thousands.

The objective function could be considered as the initial cost of the machine plus the cost of the loss penalty plus penalty of the constraint function:

$$f = c_i (\bar{X}) + c_e (\bar{X}) + c_p (\bar{X}) \quad (13.30)$$

where

c_i is the initial cost dependence

c_e is the energy losses cost

c_p , is the penalty cost

All of them must be in the same units; thus, for example, consider doubling the initial cost for every 10°C larger over-temperature than the

acceptable maximum temperature or increase the cost in direct ratio to over-temperature when this becomes larger than the acceptable maximum.

Usually, the initial cost is the active material costs of the machine, and, for some applications, the required power electronics cost could be added:

$$C_i = c_{Cu}m_{Co} + c_{lam}m_{sFe} + c_{Fe}m_{rFe} + c_{PM}m_{PM} + c_a m_t + c_{pe} \frac{P_n}{pf} \quad (13.31)$$

where

- C_{Co} is the copper price, say, \$10/kg
- C_{lam} is the lamination price, say \$5/kg
- C_{Fe} is the rotor iron price, say, \$5/kg
- C_{PM} is the permanent magnet price, say, \$50/kg
- C_a is the extra price per total mass
- m_t, m_{c0} are the windings mass
- m_{sFe} is the stator lamination mass
- m_{rFe} is the rotor iron mass
- m_{PM} is the permanent magnet mass

The power electronics cost depends on the peak apparent power (in direct ratio to active power, P_n , and inverse ratio to power factor, pf) and power electronics price per VA - c_{pe} , say, \$0.025/VA for medium power converters. The machine design converges to an optimal power factor when the power electronics cost is added to the objective function because the power electronics cost depends to the apparent power and this is minimized when the power factor tends to unity. The grid connection equipment cost and reactive energy penalty cost could be used instead of the power electronics cost in order to optimize the power factor for grid-connected machines.

The energy efficiency optimization is introduced by considering the energy loss during the expected run time of the machine.

The energy losses cost expression is

$$c_e = P_n \left(\frac{1}{\eta(\bar{X})} - 1 \right) \cdot t_l \cdot p_e \quad (13.32)$$

where

- P_n is the machine-rated power
- η_n is the rated efficiency
- t_l is the operation time
- energy price is, say, \$0.1/kWh

More sophisticated energy losses cost could be implemented, taking into account the loading factor, k_j , variation in time, and efficiency at that

loading factor, η_j :

$$c_e = P_n \cdot t_l \cdot p_e \cdot \sum_j \left(\frac{1}{\eta_j(\bar{X})} - 1 \right) p_{kj} k_j$$

$$\sum_j p_{kj} = 1 \quad (13.33)$$

The sum of probabilities to run at different loads, p_{kj} , should be equal by unity.

From many numerical examples, it could be observed that the loss energy cost is much larger than the initial cost. The optimization process is dominated by efficiency improvement rather than by initial cost reduction in these cases. The manufacturers and also many costumers are interested in the initial cost reduction with a minimum acceptable efficiency, given by governmental regulations or through free markets. It is then feasible to modify the objective function from Equation 13.32 according to this requirement:

$$c_e = \begin{cases} P_n \left(\frac{1}{\eta(\bar{X})} - \frac{1}{\eta_{\min}} \right) \cdot t_l \cdot p_e & \text{if } \eta(\bar{X}) < \eta_{\min} \\ 0 & \text{if } \eta(\bar{X}) \geq \eta_{\min} \end{cases} \quad (13.34)$$

Constraint penalty cost is composed from penalty cost of the over-temperature and penalty cost of the permanent magnet demagnetization due to peak or direct start current:

$$c_p = c_{pt} + c_{pdm} \quad (13.35)$$

The penalty cost for an over-temperature that is larger than the acceptable maximum is

$$c_{pt} = c_{pts} + c_{ptr} \quad (13.36)$$

$$c_{pts} = \max(0, T_w - T_{wad}) \cdot k_{ts} \cdot c_i \quad (13.37)$$

$$c_{ptr} = \max(0, T_r - T_{rad}) \cdot k_{tr} \cdot c_i \quad (13.38)$$

where

c_{pts} is the penalty cost for stator (winding) over-temperature

c_{ptr} is the penalty cost for rotor (PM) over-temperature

T_w is the winding temperature

T_{wad} is the maximum admissible winding temperature

K_t is the a proportional constant

c_i is the initial cost from Equation 13.31

T_r is the rotor temperature

T_{rad} is the maximum admissible rotor temperature

The penalty cost for permanent magnet demagnetization was computed considering that the “PM” is demagnetized when the total flux through the permanent magnet becomes negative due to the stator demagnetization current component:

$$c_{\text{pdm}} = \max \left(0, -\frac{\min(\Phi_{\text{PMdm}})}{\Phi_{\text{PM0}}} \right) \cdot k_{\text{dm}} \cdot c_i \quad (13.39)$$

where

Φ_{PMdm} is the total flux trough PM at maximum demagnetization current equal to the rated peak current multiplied by a demagnetization safety factor

Φ_{PM0} is the total flux trough PM at no load

k_{dm} is the proportional constant

13.5 Electric Machine Design Using Genetic Algorithms

After a pattern search optimization algorithm such as that of Hooke–Jeeves we introduce here an evolutionary one.

The genetic algorithms consider a population of candidate solutions evolution under specific selection rules to a state that minimizes the cost function [3–5]. The genetic algorithms require a method to code the optimization variables, the objective function, and a set of selection rules for the evolution of the population’s members. Developing a genetic algorithm for optimal design of electrical machines involves the following steps:

- Choosing the optimization variables, their minimum and maximum values, and their resolution. Some of the optimization variables should be integer numbers. In this case, the resolution is equal by unity. It is not necessary to provide an initial value for the optimization vector.
- Choose a genetic codification of the optimization vector. There are two methods for the codification of the optimization variables as a genetic code: the binary code and the continuous (real) code.
- Choose the objective function and develop an analytical model in order to evaluate the objective function starting from the optimization variable vector. It is possible also to use FEM in order to compute the machine behavior, but, in this case, it is necessary to have a full parametric model to fully mechanize FEM (problem generation, solve the problem, and interpret the results). When such models are developed, it is necessary to evaluate thousands of candidate solutions. This method will be feasible by using parallel computations. However, the model should be accurate enough over the entire domain.

- Choosing a selection algorithm in order to produce members for the next generation. Also, it is necessary to choose some criterion to stop the algorithm.

The number of bits required for the genetic code for every variable, n_{bit} , for binary codification is computed considering the number of distinct values for each variable, n_{vx} , and this depends on the variable range and its resolution:

$$n_{\text{vx}}(i) = \text{ceil} \left(\frac{X_{\text{max}}(i) - X_{\text{min}}(i)}{r_x(i)} \right) \quad (13.40)$$

$$n_{\text{bit}}(i) = 1 + \text{floor}(\log_2(n_{\text{vx}}(i))) \quad (13.41)$$

where “ceil” is the round function to the nearest integer through plus infinity and “floor” is the round function through minus infinity.

For an induction machine with 16 optimization variables, it is necessary between two bits (number of slots per pole per phase is an integer number between 2 and 4 for small machines with four pairs of poles) and seven bits to code the variable that needs good resolution. In this particular example, the total number of bits required to store the whole vector information is 86, which means 2^{86} cases (about 7.73×10^{25}). Now, here is clear evidence that there is no chance for exhaustive search of an optimum solution.

The optimization vector competes from the genetic binary code with relation (13.42), and with relation (13.43) for continuous codification where g_{xb} is a integer between zero and n_{vx} , and g_{xc} is a real number between zero and unity:

$$\bar{X} = \bar{X}_{\text{min}} + \bar{g}_{xb} \cdot \bar{r}_x \quad (13.42)$$

$$\bar{X} = \bar{X}_{\text{min}} + \bar{g}_{xc} \cdot (\bar{X}_{\text{max}} - \bar{X}_{\text{min}}) \quad (13.43)$$

Many of the codes do not satisfy a minimum request condition, as is the case for geometric constraints, and they are not viable codes. The critical geometric dimensions are computed immediately and a member discards it if it does not observe the minimal requirement. Another random code will be generated until the initial population is complete. Usually every variable is represented separately on the maximum number of bits. It is possible to store the entire information on a single “long integer” and execute the genetic operation simultaneously by the “all variables code” but the gain will not be important because the objective function evaluation is a very difficult operation.

The optimization design goal is to minimize the objective function while the genetic algorithm maximizes the fitness function. The genetic code is like an ascending order objective function. Thus, the better code is the first code. The Monte-Carlo roulette selection method selects the members to provide

genetic codes for the next generation with a probability in direct ratio to the fitness function. A proper fitness function could be computed by dividing unity to the objective function. Then a per unit fitness function, f_f , is computed

$$f_g(\bar{X}_i) = \frac{1}{f(\bar{X}_i)} \quad (13.44)$$

$$f_r(\bar{X}_i) = \frac{f_g(\bar{X}_i)}{\sum_i^{ps} f_g(\bar{X}_i)} \quad (13.45)$$

$$r_k = \sum_{i=1}^k f_r(\bar{X}_i) \quad (13.46)$$

A random number, p , between zero and unity is generated. The index, k , of the smaller rank, r_k , larger than the random number, p , is chosen to be the first parent. For the second parents another random number is generated and it is chosen using the same method. In this way, more adapted members have a chance to become parents and transmit the genetic information to the next generation. The first populations are very nonhomogeneous populations, and, using only this method to transmit genetic code through to the next generation, produces a rapid convergence where the generality is lost. In order to avoid a rapid convergence to a local solution, the fitness function of the members that are already parents is multiplied by an exclusion factor. In this way the chance to become parents again is reduced. Two offsprings are produced by crossovers before they go to the next generation. Randomly, a part of the offspring suffers genetic mutation before it goes to the next generation. The members of the old generation are ranked again and then the process to produce new members is continued until a new, complete, generation is produced. The new generation will take the place of the old generation and the algorithm is repeated until the given number of generations is reached. The algorithm could be stopped before the maximum number of generations is reached if the fitness function difference between the best and the worst member become too small. This means that all population members are “close relatives” and probably many of them are perfect twins. It is possible to avoid twins in the population by discarding every new offspring that is identical with any already existing member from the next generation. The described principle could be applied for binary codification as well as for continuous codification.

The crossover is a technique to obtain two offspring from two parents by recombining the genetic code, as shown in Figure 13.8, for the binary codification of a few components of optimization vectors of induction machines such as specific linear current density (elsp—specific linear electric load),

	The linear electric load						The airgap flux density				Core length per polar pitch				The stator current density								
	Cutting points																						
	msp ↓ lsp						msp ↓ lsp				msp ↓ lsp				msp ↓ lsp								
Parent 1	0	1	0	0	1	0	1	0	0	0	0	1	1	0	0	0	0	0	1	1	0	0	
Parent 2	1	0	0	1	1	0	0	0	0	1	0	0	0	1	0	0	1	0	1	0	0	0	
Offspring 1	0	1	0	1	1	0	0	1	0	1	0	0	0	0	1	0	0	0	0	0	1	0	0
Offspring 2	1	0	0	1	0	0	1	0	0	0	0	1	1	0	0	0	1	0	1	1	1	0	0

FIGURE 13.8

Offspring generation using crossover.

airgap flux density (Bagsp), the ratio between machine length and pole pitch (lcpertau), and the stator current density (Js). In this example, 8 bits are used to codify the specific electric load, 6 bits to codify the specific flux density in the airgap, 5 bits to specify the ratio between core length and pole pitch, and 7 bits to specify the stator flux density. The whole information is stored on 86 bits. There are many possibilities for cutting and recombining the genetic code either as a single cutting or as multiple cuttings that could be in the same position or in variable positions. In this example, multiple cuttings, were chosen one for each piece of information coded and variable position cutting. The parts of the code available after cutting each variable region are assembled randomly in order to avoid sending the most significant code part (msp) from “parent 1” to “offspring 1” and least significant part (lsp) from “parent 2” to “offspring 1.”

Mutation is another technique to produce offspring from a single parent. The parent code is copied identically to the offspring with an exception of one or two variables where the parent code is replaced with a random code as is observed in Figure 13.9 where a mutation occurs for “core length per pole pitch” variable.

A part of offspring code produced by crossover for continuous coding is computed by Equation 13.46 and the other part by Equation 13.47 where i

	The linear electric load	The airgap flux density	Core length per polar pitch	The stator current density
Parent	0 1 0 1 0 0 1 0	1 0 0 0 0 1	1 0 0 0 0	0 0 0 1 1 0 0
			↓ Mutation	
Offspring	0 1 0 1 0 0 1 0	1 0 0 0 0 1	0 1 0 0 1	0 0 0 1 1 0 0

FIGURE 13.9

Offspring generation using mutation.

and j are the optimization variable indexes. Applying Equation 13.47 or 13.48 is an arbitrary choice, to avoid a specific pattern for the crossover process.

$$\begin{aligned} g_{o1}(i) &= g_{p1}(i) + \alpha (g_{p2}(i) - g_{p1}(i)) \\ g_{o2}(i) &= g_{p2}(i) + \alpha (g_{p1}(i) - g_{p2}(i)) \end{aligned} \quad (13.47)$$

$$\begin{aligned} g_{o1}(j) &= g_{p2}(j) + \alpha (g_{p1}(j) - g_{p2}(j)) \\ g_{o2}(j) &= g_{p1}(j) + \alpha (g_{p2}(j) - g_{p1}(j)) \end{aligned} \quad (13.48)$$

The offspring optimization variables are always between the parent variables while for binary coding this happens only if the Gray code is used.

Examples of optimal design using genetic algorithms will be presented in the next chapters for induction machines and permanent magnet synchronous machines. In the examples provided there; the following parameters are used to control the genetic algorithm: population size; number the of generation; elitism, that is, the number of the best members that are sent directly in the next generation; mutation rate, that is, the probability to have a mutation for each variable genetic code; and the excluding factor, that is, the rate of fitness function reduction for the parents after mating.

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14

Optimization Design of Surface PMSMs

14.1 Design Theme

In general, the following specifications make up the design theme of a variable speed PMSM:

- Base continuous power, P_b
- Base speed, n_b
- Maximum voltage, V_n
- Overload factor, k_1
- Maximum speed, $n_{\max}$
- Power at maximum speed, $P_{\max}$
- Number of phases, m

Also a few constraints are added:

- Efficiency at P_b and n_b
- Material insulation class (allowed temperature)
- Protection degree against alien bodies
- Total initial cost of active materials

Flux weakening ($n_{\max}/n_b > 1.5$) is not very feasible with surface PMSMs (SPMSMs) (unless their synchronous inductance, L_s , is large in pu, such as for tooth-wound stator windings), and, thus, $n_{\max} = n_b$.

14.2 Electric and Magnetic Loadings

The first stage in choosing the electric and magnetic loadings consists of general or optimization design of electric machines. For SPMSMs, these loadings are described as follows [1–6]:

- Specific electric loading, J_l (A/m), represents the total effective value of ampere turns in stator slots per stator periphery length. J_l relates to thermal loading and torque density. Large values of J_l lead to large torque density, and consequently to smaller machine size, which may end up in machine overheating and lower efficiency. When high torque density is target in the design, the specific tangential force, f_{tsp} , concept may be preferred. It is measured in N/cm² and varies from 0.1 N/cm² in micromotors to 10 N/cm² in larger torque density designs. J_l and f_{tp} are complementary concepts.
- PM airgap flux density, B_{ag} (T), varies from 0.2 T in micromotors to 1 T in large torque density designs. Together B_{ag} , J_l , and f_{tp} determine the volume of the machine for given base torque (T_{eb}).
- Stator tooth flux density, B_{st} (T), determines the degree of magnetic saturation in the machine; it varies from 1.2 to 1.8 T, in general, for silicon-laminated stator cores. As the machine's magnetic airgap (which includes the surface PM thickness) is large, the influence of the magnetic saturation of the teeth is rather small, and, thus, a large value of B_{st} may be chosen; in contrast, if B_{st} is large (and the fundamental frequency, f_{1b} , too is large) the core losses would also be large. Smaller values of B_{st} lead to wider teeth, and thus thinner slots, that is, larger design current densities, and, consequently, larger copper losses. To prevent this, deeper slots are adopted, but then the machine's external diameter (and volume) is increased. Moreover, copper losses may not decrease notably by deepening the slot, as the average diameter (and length) of end connection of coils increases. So the design should start with applying moderate to large values of B_{st} , and then reducing or increasing them according to the performance of the machine based on these values, in comparison to its performance based on the design theme.
- Stator yoke flux density, B_{sy} (T), is chosen as a compromise between the level of magnetic saturation and the limitations due to core losses. Small values of B_{sy} may lead to a larger machine size and weight, especially if the number of poles is small ($2p = 2, 4$).
- Current density, J_s (A/mm²), determines the copper losses and the copper volume. Corroborated with thin and deep slots that have small values of J_s (2–3.5 A/mm²) may lead to a high leakage inductance and machine volume (and weight). On the other hand, high J_s values (>8 A/mm²), in general, not only imply forced cooling but also lead to lower efficiencies while reducing the machine volume.

- Rotor yoke flux density, $B_{ry}(T)$, is important in the machine with a large number of poles (and a large diameter) when the PMs are not any more placed directly on the shaft.

14.3 Choosing a Few Dimensioning Factors

- Machine shape factor, γ_c : The ratio between the axial (stack) length of the machine, l_{stack} , and the pole pitch, τ ($\gamma_c = l_{stack}/\tau$, usually between 0.15 and 3).
- PM pole span factor, γ_{PM} : The ratio between the PM width and the pole pitch. This factor influences the PM total weight, the emf space harmonics content, and, to some extent, the cogging torque.
- Number of slots per pole per phase, q_1 .
- Number of winding layers, n_L ($n_L = 1$ for single-layer windings and $n_L = 2$ for two-layer windings).
- Stator winding current path count, a_1 , is always a divisor of the number of poles for two-layers windings and of the number of pole pairs for single-layer windings. To avoid the circulation current between current paths, due to inherent machine symmetry imperfection, $a_1 = 1$ whenever possible, with the exception of low-voltage, large-current (automotive) applications.
- Elementary conductors in parallel: For large currents (or for large fundamental frequency) the coil turns are made of multiple elementary conductors in parallel, with the same degree of transposition, to reduce skin effects.
- Coil span, x_s , is the distance between the forward and return sides of the coil; it may be measured not only in mm but also in number of slot pitches: between $2q_1$ and $3q_1$ for 3-phase distributed windings, but below 0.5 for tooth-wound windings.
- Slot opening, s_o (see Figure 14.4): The minimum value of s_o is limited by the possibility to introduce the coils, turn by turn, in the slot and by the increase of slot leakage inductance and PM flux fringing. Its maximum value is limited by the PM flux reduction, cogging torque increase, and torque ripple.
- Tooth top height, h_{s4} : The minimum value of h_{s4} is limited by technological (and magnetic saturation) constraints and its maximum value is limited by the increase the slot leakage inductance. (In high-speed

PMSMs, with $f_{1b} > 1$ kHz, h_{s4} may be increased, to increase machine inductance L_s , and thus decrease current ripple).

14.4 A Few Technological Constraints

- Laminated-core-filling factor, $k_{stk} = 0.8\text{--}0.95$, is the ratio of the lamination's height to the total laminated core length (lamination's insulation coating means $k_{stk} < 1$).
- Slot-copper-filling factor, $k_{sf} = 0.33\text{--}0.7$, has lower values corresponding to semi-closed slots with coils introduced, turn by turn, in the slot, while larger values corresponding to open slots and premade coils made of conductors having rectangular cross sections.
- Minimum airgap, g_{min} , from mechanical considerations in super high-speed machines is increased to reduce PM eddy current losses due to stator mmf space harmonics; a resin coating of PMs is also added for mechanical rigidity.
- Overload demagnetization safety factor, k_{sPM} : The maximum overload stator mmf that avoids PM demagnetization in the worst load scenario is limited by this factor.
- Slot geometry type: If more than one slot shape is used, a design expression for the chosen shape should be provided, and the computer code calls for the specific geometrical and specific magnetic permeance expressions on a menu basis.
- Angle of slot wedge, α_w .
- Slot insulation thickness.
- Straight-turn end connection, l_{f1} , that exits the slot.
- Difference between PM, l_{PM} , and stator stack length, l_{stack} ($l_{PM} - l_{stack} = (1 - 2)(g + h_{PM})$), where h_{PM} is the PM thickness (along the magnetization direction), in order to avoid axial forces on the bearings.
- Minimum shaft diameter, based on the maximum envisaged torque.

14.5 Choosing Magnetic Materials

We refer here to permanent magnets and magnetic cores. For PMs, we choose the remnant flux density, B_r , the coercive field, H_c , and the maximum

allowed rotor (PM) temperature, T_r . The choice of B_r depends on the airgap flux density: $B_{ag} < B_r$ for SPMSMs. The coercive field (force), H_c , depends on B_r and on the recoil permeability: $\mu_{rec} \approx (1.05-1.3)$ for NeFeB, SmCo5, and ferrite PMs ($B_r = (0.3-0.4)$ T); $\mu_{rec} \approx 1.05$ and $B_r = 0.6-0.8$ T for bonded PMs; and $B_r = 1.1-1.37$ T for sintered PMs at 20°C.

$$H_c \approx \frac{B_r}{\mu_{rec(pu)} \mu_0} \text{ (A/m)} \quad (14.1)$$

The PM characteristics are obtained from the manufacturer's catalogues and they include

- B_r at 20°C
- H_c at 20°C
- B_r and H_c temperature coefficients
- Energy density
- Interior coercive fields (force)
- PM magnetization field, $H_c \times (1.8-3)$ and $B_r \times (1.8-2.5)$
- Maximum operation temperature
- Electric resistivity of PM materials, to calculate the PM eddy current losses due to stator mmf space and time harmonics: much higher for bonded NeFeB than for sintered NeFeB
- Mass density (kg/m^3)

For up to 150 Hz maximum fundamental frequency, general silicon-laminated cores made of 0.5 mm (0.65 mm for increased productivity) unoriented grain laminations are used. For fundamental frequencies above 150 Hz, thinner laminations (0.2 mm or less, but with stacking factor $k_{sk} > 0.8$) are to be used to reduce eddy current losses. The magnetization curve characterizes the silicon lamination (Figure 14.1).

Using soft magnetic materials with higher saturation flux densities ($B_{sat} = 2.35$ T at $H_{sat} = 10^4$ A/m for frequencies 50 Hz) leads to more compact PMSMs at a slightly higher initial cost. (Hyperco50 is more expensive than M19 silicon steel.) Other important characteristics of soft magnetic materials are the core losses that depend on the flux density amplitude and frequency. The core losses are segregated into eddy current and hysteresis losses. Some manufacturers offer core losses for a few frequencies (Figure 14.2). M19 and SURA are two widely used trade names for silicon laminations. Hyperco50 is a trade name for a high saturation flux density laminated material with up to 50% cobalt and moderate losses at frequencies below 500 Hz. Somaloy is the trade name of a soft material composite for $f_B > 500$ Hz (in general) and

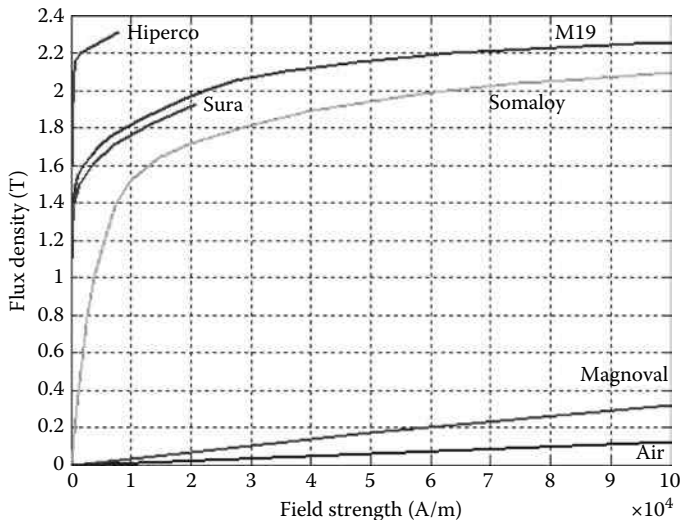


FIGURE 14.1
Magnetization curves for a few typical soft magnetic materials.

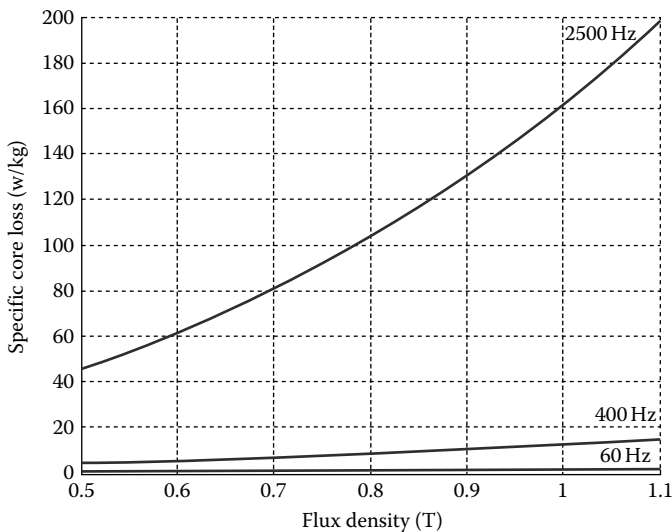


FIGURE 14.2
SURA007 iron losses vs. flux density.

(or) where three-dimensional ac magnetic field lines have to be accommodated. Thin laminations such as SURA007 [6] (0.18 mm) have shown acceptable losses at 2500 Hz (Figure 14.2). Magnoval is a sintered material with low relative magnetic permeability ($\mu_{\text{rec}} = (3\text{--}5)\mu_0$) used for slot wedges

to decrease PM flux density pulsations, and cogging and total torque pulsations, at the cost of larger leakage and synchronous inductances. Magnoval is also good to increase the machine inductance in high-frequency ($f_{1b} > 1 \text{ kHz}$) machines to reduce current ripple.

A good approximation of core power losses (in W/kg) for the practical frequency and the flux density is, still, the Steinmetz formula:

$$dP_{\text{tot}} = k_h B_m^2 f + \pi^2 \frac{\sigma d^2}{6} (B_m f)^2 + 8.67 k_e (B_m f)^{2/3} \quad (14.2)$$

14.6 Dimensioning Methodology

First we calculate the machine size constant, C_0 , and the stator bore diameter, D_{si} , based on some already chosen parameters:

$$C_0 = \frac{\pi^2}{\sqrt{2}} B_{\text{ag}} J_1 k_w \quad (14.3)$$

where

B_{ag} is the PM airgap flux density (T)

J_1 is the linear specific electric loading ($A_{\text{turns}}/\text{m}$)

k_w is the winding factor, which includes the zone factor, k_{ws} , and the chording factor, k_{chs}

y_s is the coil span in slot pitch count

$$k_w = k_{\text{ws}} \cdot k_{\text{chs}}; \quad k_{\text{ws}} = \frac{\sin\left(\frac{\pi}{6}\right)}{q_1 \sin\left(\frac{\pi}{q_1 6}\right)}; \quad k_{\text{chs}} = \sin\left(\frac{y_1}{q_1 m} \cdot \frac{\pi}{2}\right) \quad (14.4)$$

Equation 14.4 is strictly valid for integer q_1 . D_{si} in mm is

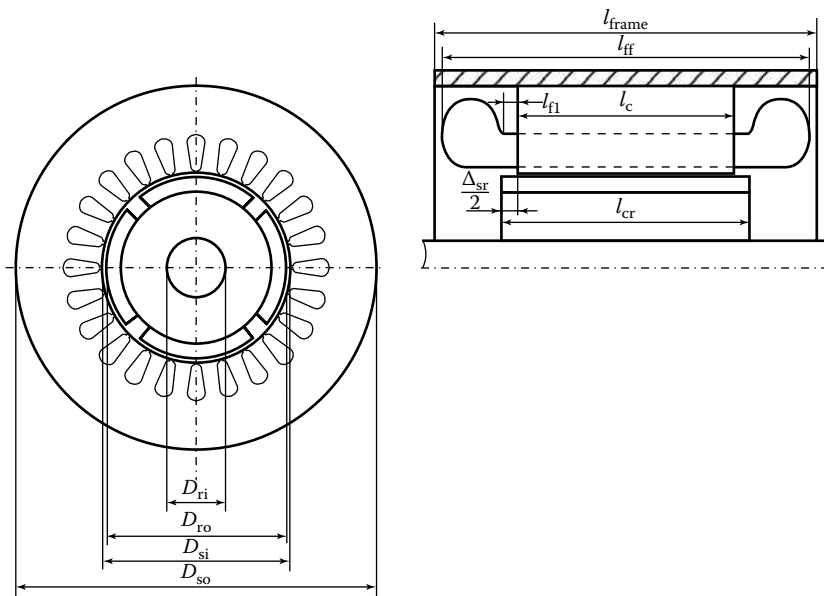
$$D_{\text{si}} = 1000 \cdot \sqrt[3]{\frac{60 p_s}{\pi n_n} \cdot \frac{P_n}{\lambda_c C_0}} \quad (14.5)$$

where

p_s is the number of rotor poles ($p_s = 2p_1$), which is either given or calculated for the base speed and chosen frequency

P_b is the base/rated electromagnetic power

It is safe to use Equation 14.5 mainly if the reaction airgap flux density of the stator mmf is small (less than 25%) with respect to the PM airgap flux

**FIGURE 14.3**

The main geometrical dimensions of a PMSM.

density, B_{ag} . An alternative equation uses the electromagnetic torque, T_{eb} , and the tangential specific force, f_{tsp} (in N/cm²):

$$D_{si} \text{ (mm)} = 100 \cdot \sqrt[3]{\frac{2p_s}{10\pi^2} \cdot \frac{T_{eb}}{\lambda_c f_{tsp}}} \quad (14.6)$$

Figure 14.3 shows the main geometrical parameters of a PMSM.

Once D_{si} is established, the pole pitch, τ , and the core length stack, l_c , are calculated:

$$\tau_p = \pi \frac{D_{si}}{p_s} \quad (14.7)$$

$$l_c = \lambda_c \tau_p \quad (14.8)$$

Based on given B_{ag} (0.2 – 1 T) and λ_{PM} (0.6 – 0.9), the PM flux per pole, Φ_{PM} , is

$$\Phi_{PM} = \frac{2}{\pi} B_{ag} \tau l_c \sin\left(\lambda_{PM} \frac{\pi}{2}\right) \quad (14.9)$$

Now the stator and the rotor yoke thicknesses, h_{sy} and h_{ry} , may be calculated:

$$h_{sy} = \frac{B_{ag}}{B_{sy}} \cdot \frac{\tau_p}{\pi} \quad (14.10)$$

$$h_{ry} = \frac{B_{ag}}{B_{ry}} \cdot \frac{\tau_p}{\pi} \quad (14.11)$$

The number of stator slots, N_{ss} , the stator slot pitch, τ_{ss} , and the teeth width, w_{st} , are

$$N_{ss} = q_q \cdot m \cdot p_s \quad (14.12)$$

$$\tau_{ss} = \pi \frac{D_{si}}{N_{ss}} \quad (14.13)$$

$$w_{st} = \tau_{ss} \frac{B_{ag}}{B_{st}} \quad (14.14)$$

The geometrical stator parameters in Figure 14.4 may all be calculated. But before that, the slot geometrical angle, α_s , and the angle of tooth slice, α_{st} are determined:

$$\alpha_s = \frac{2\pi}{N_{ss}}; \quad \alpha'_s = \frac{\pi}{N_{ss}} \quad (14.15)$$

$$\alpha_{s0} = 2a \sin\left(\frac{s_0}{D_{si}}\right) \quad (14.16)$$

$$\alpha_{st} = \alpha_s - \alpha_{s0} \quad (14.17)$$

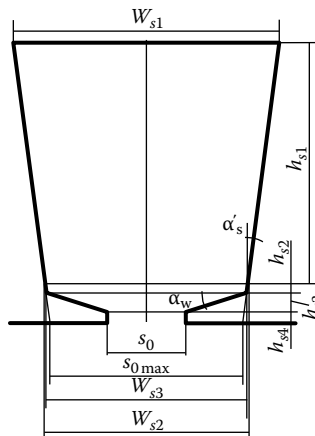


FIGURE 14.4

Stator slot geometry.

We verify now if the tooth top (slice) is larger than the tooth width, w_{st} (Equation 14.12); if not, the slot opening, s_0 , is reduced accordingly. At the limit, the slot may remain open when the central angle for a tooth is

$$\alpha_{stmin} = 2a \sin\left(\frac{w_{st}}{D_{si}}\right) \quad (14.18)$$

For open slots,

$$s_{0max} = D_{si} \sin(\alpha'_s - \alpha_{stmin}/2) \quad (14.19)$$

The main slot geometrical parameters (Figure 14.4) are

$$h_{s3} = \frac{\frac{s_{0max} - s_0}{2} + h_{s4} \cdot tg(\alpha'_s)}{\frac{1}{tg(\alpha'_w)} - tg(\alpha'_s)} \quad (14.20)$$

$$w_{s3} = s_{0max} + 2(h_{s4} + h_{s3}) tg(\alpha'_s) \quad (14.21)$$

$$w_{s2} = w_{s3} + 2h_{s2} tg(\alpha'_s) \quad (14.22)$$

where h_{s2} is the slot classic insulation thickness.

The copper area in a slot, S_{Cu} , and the required slot area, S_{slot} , are

$$S_{Cu} = \frac{I_{ts}}{J_s} \quad (14.23)$$

$$S_{slot} = \frac{S_{Cu}}{K_{sf}} \quad (14.24)$$

where I_{ts} is the total RMS ampere turn/slot, which is to be calculated from either Equation 14.23 or Equation 14.24:

$$I_{ts} = \frac{\pi D_{si}}{J_l N_{ss}} \quad (14.25)$$

$$I_{ts} = \frac{T_{em}}{\sqrt{2} p_1 \Phi_p N_{ss}} \quad (14.26)$$

Knowing the active slot area, S_{slot} , its main dimensions are

$$h_{s1} = \frac{-w_{s2} + \sqrt{w_{s2}^2 + 4S_{slot} \tan(\alpha'_s)}}{2 \tan(\alpha'_s)} \quad (14.27)$$

$$w_{s1} = w_{s2} + 2h_{s1} \tan(\alpha'_s) \quad (14.28)$$

$$h_s = h_{s4} + h_{s3} + h_{s2} + h_{s1} \quad (14.29)$$

Then the outer stator diameter, D_{s0} , is approximated to an integer in millimeters:

$$D_{s0} = \text{round} (D_{si} + 2h_s + 2h_{sy}) \quad (14.30)$$

Now the stator yoke thickness, h_{sy} , is

$$h_{sy} = \frac{D_{s0} - D_{si}}{2} - h_s \quad (14.31)$$

The end-connection coil length, l_f , the total half-turn length, l_{mc} , and the axial length of the stator between end connections, l_{ff} , are

$$l_f = \frac{\pi}{2} \tau_p \frac{y_1}{q_1 m} \left(1 + \frac{h_s}{D_{si}} \right) + 2l_{f1} \quad (14.32)$$

$$l_{mc} = l_c + l_f \quad (14.33)$$

$$l_{ff} = l_c + 2l_{f1} + \tau_p \frac{y_1}{q_1 m} \left(1 + \frac{h_s}{D_{si}} \right) \quad (14.34)$$

The motor frame length can be calculated using l_{mc} .

14.6.1 Rotor Sizing

The PM thickness, h_{PM1} , is supposed to produce a certain PM flux density in the airgap, B_{ag0} , and also to avoid demagnetization under a small overload:

$$h_{PM1} = \frac{B_{ag0} \delta_{\min} k_s}{\mu_0 H_c \left(1 - \frac{B_{ag0}}{B_r} k_s \right)} \quad (14.35)$$

$$h_{PM2} = 1000 \frac{I_{ts} N_{ss}}{p_1 \sqrt{2}} k_1 k_{\sigma PM} H_c \quad (14.36)$$

where k_s is the saturation factor.

In both cases, h_{PM} is in millimeters and the largest of h_{PM1} and h_{PM2} is chosen and rounded to tens of millimeters. If h_{PM2} is larger than h_{PM1} , the airgap, g , has to be recalculated:

$$g = \mu_0 H_c h_{PM} \frac{1 - \frac{B_{ag0}}{B_r} k_s}{B_{ag0} k_s} \quad (14.37)$$

The gap g is rounded to a multiple of 0.05 mm:

$$g = 0.05 \cdot \text{round}(20g) \quad (14.38)$$

The external and interior diameters of the rotor, D_{r0} and D_{ri} , and the core length, l_{cr} , are

$$D_{r0} = D_{si} - 2\delta \quad (14.39)$$

$$D_{ri} = D_{r0} - 2(h_{PM} + h_{ry}) \quad (14.40)$$

$$l_{cr} = l_c + \Delta_{sr} \quad (14.41)$$

The interior rotor diameter, D_{ri} , is rounded to an integer in millimeter, and, thus, the rotor yoke thickness is slightly adjusted. If D_{ri} is smaller than the shaft diameter, either the dimensioning methodology is redone for a larger D_{si} (by reducing B_{ag} , J_l , and f_{tsp}) or shape factor, λ_c , or the PMs are placed on the shaft (for a massive rotor yoke).

14.6.2 PM Flux Computation

Failing to realize the required PM flux implies large current (and copper) losses for the base speed (power) torque. For the preliminary design, we added a saturation factor, k_{sat} , and presupposed that it is feasible. This is needed to recalculate the PM flux with better precision, as the PMs have a linear demagnetization curve with $\mu_{rec} = (1.05-1.1)\mu_0$. Consequently, the Carter coefficient has to be calculated for the equivalent airgap, g_e :

$$g_e = g + \frac{\bar{B}_r}{\mu_0 H_c} h_{PM} \quad (14.42)$$

$$k_c = \frac{\tau_s}{\tau_s - \gamma_s s_0} \quad (14.43)$$

where

$$\gamma_s = \frac{\left(\frac{s_0}{g_e}\right)^2}{5 + \frac{s_0}{g_e}} \quad (14.44)$$

Due to the nonlinearity of magnetic core materials, the PM airgap flux density has to be calculated iteratively after we use k_s for saturation from the preliminary design.

$$B_{ag0} = \mu_0 H_c \frac{h_{PM}}{g_e k_C k_s} \quad (14.45)$$

The PM flux per pole is calculated from Equation 14.9 and then we can recalculate B_{st} , B_{sy} , and B_{ry} from Equations 14.46 through 14.48.

$$B_{st} = B_{ag} \frac{\tau_s}{w_{st}} \quad (14.46)$$

$$B_{sy} = \frac{\Phi_p}{2l_c h_{sy}} 10^6 \quad (14.47)$$

$$B_{ry} = \frac{\Phi_p}{2l_c h_{ry}} 10^6 \quad (14.48)$$

Making use of the core magnetization curve through interpolation, the magnetic field in the stator teeth, H_{st} , in the stator yoke, H_{sy} , and in the rotor yoke, H_{ry} , are calculated. Based on this, the magnetization voltages, V_{mst} , V_{msy} , and V_{mry} , in the stator teeth, the yoke, and the rotor yoke are calculated:

$$V_{mst} = 0.001 H_{st} h_s \quad (14.49)$$

$$V_{msy} = 0.001 H_{sy} \frac{x}{p_s C_x} \quad (14.50)$$

$$V_{mry} = 0.001 H_{ry} \frac{y}{p_s C_y} \quad (14.51)$$

where

D_x and D_y are, respectively, the average diameters around which the average flux path closes in the two yokes

C_x and C_y are, respectively, two path length reduction coefficients that account approximately for the fact that nonuniform local saturation takes place in the stator and rotor yokes

$$D_x = D_{s0} - \frac{4}{3} h_{sy} \quad (14.52)$$

$$D_y = D_{ro} - 2h_{PM} - \frac{2}{3} h_{ry} \quad (14.53)$$

C_x and C_y depend on the flux density distribution and on the number of poles of the rotor. These may be calculated through interpolation from the C curves in Figure 14.5, obtained empirically or, better, by the FEM.

The magnetic voltage along the airgap, V_{mg} , is

$$V_{mg} = 0.001 g_e k_C \frac{B_{ag}}{\mu_0} \quad (14.54)$$

So the total mmf, V_m , required to produce B_{ag0} in the airgap is

$$V_m = V_{mst} + V_{msy} + V_{mry} + V_{mg} \quad (14.55)$$

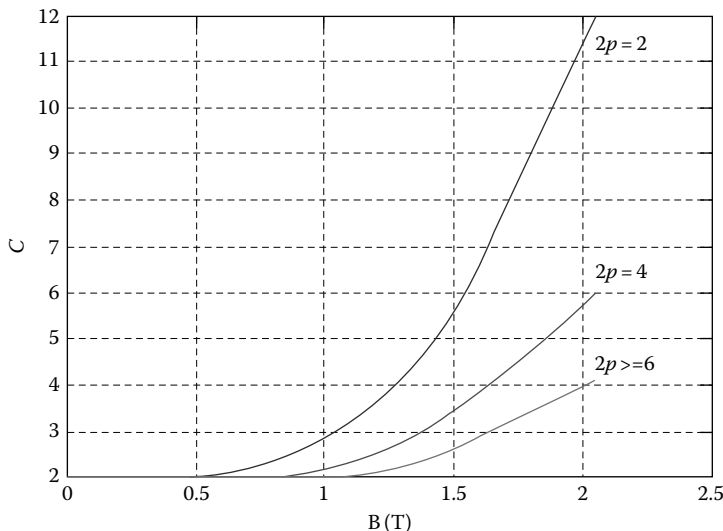


FIGURE 14.5
Flux path length reduction coefficient, C .

The new value of the saturation factor, k_{sat} , is calculated using

$$k_s = \frac{V_m}{V_{\text{mg}}} \quad (14.56)$$

Based on this new value of k_s , the computation cycle of B_{ag0} is redone until an error of less than 1% between two successive values of k_s is obtained. High precision is required in the optimal design, but then a mix of an analytical and an FEM calculation has to be converged. To speed up the convergence of k_s (or B_{ag}) a relaxation coefficient may be used.

$$B_{ag,k} = (1 - r)B_{ag,k-1} + r \cdot B_{ag,\text{new}} \quad (14.57)$$

where

$B_{ag,k-1}$ is the old value of the airgap flux density

$B_{ag,\text{new}}$ is the value calculated with the k_{sat} value calculated from Equation 14.56

$B_{ag,k}$ the value of B_{ag} used in the next iteration cycle

A value of $r = 0.2$ – 0.3 should produce good convergence for saturated machines but slow convergence for less-saturated machines. The convergence problem may occur for very low saturation degrees, but then the trouble may be avoided by assigning a reasonable (safe) value to K_s from the start ($k_s = 1.05$ – 1.1). After B_{ag} is calculated with the desired precision, the

PM flux per pole, Φ_p , is recalculated and, based on this, the PM flux linkage for a single-turn coil, Ψ_{PM1} , is computed:

$$\Psi_{PM1} = q_1 \frac{n_L p_1}{2a_1} \Phi_p k_w \quad (14.58)$$

Now the equivalent current through the one-turn coil winding is

$$i_{q1} = \frac{2T_n}{3p_1 \Psi_{PM1}} \quad (14.59)$$

The stator phase resistance for the one-turn coil winding, R_{s1} , is

$$R_{s1} = 1000\rho \frac{n_{cs} l_{mc}}{a_1 S_{Cu}} \quad (14.60)$$

where

ρ is the copper electric resistivity at the working temperature, T_{w1} (Equation 14.61)

n_{cs} is the number of coils in series

$$\rho = \rho_{20} (1 + (T_{w1} - 20)\alpha_{Cu}) \quad (14.61)$$

where

ρ_{20} is the copper electric resistivity at 20 °C

α_{Cu} is the resistivity coefficient with temperature

The number of conductors in series per phase for a single-turn coil is

$$n_{cs} = \frac{2q_1 p_1}{n_L a_1} \quad (14.62)$$

where

a_1 is the number of current paths

n_L is the number of winding layers ($n_L = 1, 2$)

The cyclic magnetization inductance of the single-turn coil winding is

$$L_{m1} = 2m\mu_0 \left(\frac{n_{cs} f_w}{2} \right) l_c \tau \quad (14.63)$$

The saturation factor, k_s , has been calculated on no load. For heavily saturated machines, k_s has to be calculated including the influence of the stator

mmf. Equation 14.63 is valid for $q_1 > 1$. The leakage inductance is calculated using geometrical slot- and end-connection permeances, λ_{ss} and λ_{s0} .

$$k_2 = \begin{cases} \frac{3 \left(2 - \frac{y_1}{mq_1} \right) + 1}{4} & \text{if } \frac{y_1}{mq_1} > 1 \\ \frac{1 + 3 \frac{y_1}{mq_1}}{4} & \text{if } \frac{2}{3} < \frac{y_1}{mq_1} < 1 \\ \frac{6 \frac{y_1}{mq_1} - 1}{4} & \text{if } \frac{y_1}{mq_1} < \frac{2}{3} \end{cases} \quad (14.64)$$

$$k_1 = \frac{1 + 3k_2}{4} \quad (14.65)$$

$$\lambda_{ss} = k_1 \frac{h_{s1}}{b_1} + k_2 \left(\frac{h_{s2}}{b_2} + \frac{h_{s3}}{b_3} + \frac{h_{s4}}{s_0} \right) \quad (14.66)$$

where

b_1 is the active slot average width

b_2 and b_3 are the widths corresponding to the trapezoidal-shape zone free of conductors

$$b_1 = \frac{(w_{s1} + w_{s2})^2}{0.25(3w_{s1} + w_{s2}) + 0.5w_{s1}^2 \frac{w_{s2} - 3w_{s1}}{(w_{s1} - w_{s2})^2} + \frac{w_{s1}^4}{(w_{s1} - w_{s2})^3} \log \left(\frac{w_{s1}}{w_{s2}} \right)} \quad (14.67)$$

$$b_2 = \frac{w_{s2} - w_{s3}}{\log \left(\frac{w_{s2}}{w_{s3}} \right)} \quad (14.68)$$

$$b_3 = \frac{w_{s2} - s_0}{\log \left(\frac{w_{s3}}{s_0} \right)} \quad (14.69)$$

The analytical computation of the geometrical end-connection permeance, λ_{s0} , is difficult, but there are quite a few empirical approximate expressions that are widely used.

$$\lambda_{s0} = 0.34q_1 \frac{l_f - 0.64(D_{si} + h_s) \frac{y_1}{2p_1mq_1}}{l_c} \quad (14.70)$$

This expression is mainly valid for distributed windings ($q_1 > 1$), since the tooth-wound coil, λ_{s0} , is larger as the conductors are closer to the core but the end connections are shorter.

Finally, the leakage inductance, $L_{\sigma 1}$, per phase for a single-turn coil is

$$L_{\sigma 1} = 2\mu_0 \left(\frac{n_{cs}}{2} \right)^2 \frac{\lambda_{ss} + \lambda_{s0}}{p_1 q_1} \cdot \frac{l_c}{1000} \quad (14.71)$$

The total cyclic inductance, L_{s1} , (when all three currents are present) is

$$L_{s1} = L_{m1} + L_{\sigma 1} \quad (14.72)$$

Considering the total current in axis q and the parameters of the single-turn coil, we may calculate the number of turns per coil, s_{b1} , for operating at given voltage and speed (frequency):

$$s_{b1} = \frac{V_n}{\sqrt{(\omega_1 \Psi_{PM1} + R_{s1} I_{q1})^2 + (\omega_1 L_{s1} I_{q1})^2}} \quad (14.73)$$

The number of turns per coil, s_{b1} , has to be an integer, so it has to be rounded to s_b . Consequently, the machine length has to be recalculated:

$$l_c = \frac{s_{b1}}{s_b} l'_c \quad (14.74)$$

where l'_c is the old machine length (Equation 14.8). The l_c value also is rounded to an integer in millimeters. The machine parameters, dependent on the machine length, such as R_s , Ψ_{PM1} , and L_{s1} , are recomputed. However, at least for $n_{s1} > 20$ turns, the use of Equation 14.74 only, is feasible.

Now we calculate the conductor area:

$$q_{Cu} = \frac{S_{Cu}}{s_b} \quad (14.75)$$

To reduce the skin effect, the use of elementary conductors (n_{ce}) of diameter d_{ce} in parallel may be required.

$$d_{ce} = 2 \sqrt{\frac{q_{Cu}}{\pi n_{ce}}} \quad (14.76)$$

After norming d_{ce} (according to standards), the total final conductor area is recalculated. All machine parameters, N_1 (number of turns), Ψ_{PM} , R_s , L_m , and $L_{\sigma i}$ are recalculated for the final values of s_b and l_c calculated above:

$$N_1 = s_b q_1 \frac{n_L p_1}{a_1} \quad (14.77)$$

$$\Psi_{PM} = N_1 \Phi_p k_w \quad (14.78)$$

$$I_n = \frac{2\sqrt{2}}{m} \cdot \frac{T_{nem}}{p_1 \Psi_{PM}} \quad (14.79)$$

$$R_s = 1000\rho \frac{2N_1 l_{mc}}{a_1 q_{Cu}} \quad (14.80)$$

$$L_m = 2m\mu_0 \frac{(N_1 k_w)^2 l_c \tau_p}{\pi^2 p_1 a_1 k_C k_s g_e} \quad (14.81)$$

$$L_{\sigma 1} = 2\mu_0 \frac{N_1^2}{p_1 a_1 q_1} (\lambda_{ss} + \lambda_{s0}) \cdot \frac{l_c}{1000} \quad (14.82)$$

14.6.3 Weights of Active Materials

The weight of windings:

$$m_{cu} = 2ma_1 N_1 l_{mc} q_{Cu} \gamma_{Cu} 10^{-9} \quad (14.83)$$

The PM weight:

$$m_{PM} = \pi \lambda_{PM} (D_{r0} - h_{PM}) h_{PM} l_{cr} \gamma_{Fe} 10^{-9} \quad (14.84)$$

The rotor yoke weight:

$$m_{rFe} = \pi (D_{ri} + h_{ry}) h_{ry} l_{cr} \gamma_{Fe} 10^{-9} \quad (14.85)$$

The stator yoke weight:

$$m_{sy} = \pi (D_{s0} - h_{sy}) h_{sy} l_c k_{stk} \gamma_{Fe} \quad (14.86)$$

The stator teeth weight:

$$m_{sth} = N_{ss} w_{st} h_s l_c k_{stk} \gamma_{Fe} 10^{-9} \quad (14.87)$$

The stator core weight:

$$m_{sFe} = m_{sy} + m_{sth} \quad (14.88)$$

The active material weight:

$$m_{tot} = m_{Fe} + m_{PM} + m_{sFe} + m_{Cu} \quad (14.89)$$

As the laminations are stamped from squares, their weight, m_{Fe} , is in fact

$$m_{uFe} = D_{s0}^2 l_c k_{stk} \gamma_{Fe} 10^{-9} \quad (14.90)$$

This weight has to be used when the critical cost of the materials of the machine is calculated.

The rotor inertia, J_s , is

$$J_s = m_{PM} \frac{D_{r0}^2 + D_{r1}^2}{8} 10^{-6} + m_{ry} \frac{D_{r1}^2 + D_{ri}^2}{8} 10^{-6} \quad (14.91)$$

The shaft inertia is added to the value from Equation 14.91.

14.6.4 Losses

Winding losses:

$$P_{Cu} = mR_s I_n^2 \quad (14.92)$$

Core losses:

The core losses are calculated separately for the stator teeth and the stator yoke. Based on available core-loss data, we may calculate hysteresis and eddy current losses either separately or together [4]:

$$p_{ht} = \frac{1}{\gamma_{Fe}} k_h f_n^{efh} B_{st}^{eBh} \quad (14.93)$$

$$p_{hy} = \frac{1}{\gamma_{Fe}} k_h f_n^{efh} B_{sy}^{eBh} \quad (14.94)$$

$$p_{et} = \frac{\pi^2}{6\rho_{Fe}\gamma_{Fe}} f_n^{efe} B_{st}^{eBe} \delta_t^{egt} \quad (14.95)$$

$$p_{ey} = \frac{\pi^2}{6\rho_{Fe}\gamma_{Fe}} f_n^{efe} B_{sy}^{eBe} \delta_t^{egt} \quad (14.96)$$

$$p_{Fe_t} = p_{ht} + p_{et} \quad (14.97)$$

$$p_{Fe_y} = p_{hy} + p_{ey} \quad (14.98)$$

$$p_{Fe_t} = p_{Fe_t} m_{sth} \quad (14.99)$$

$$p_{Fe_y} = p_{Fe_y} m_{sy} \quad (14.100)$$

$$p_{Fe} = p_{Fe_t} + p_{Fe_y} \quad (14.101)$$

where

efh is the frequency exponent for hysteresis losses

eBh is the flux density exponent for hysteresis losses

efe is the frequency exponent for eddy current losses

eBe is the flux density exponent for eddy current losses

egt is the thickness exponent

The segregation of yoke, p_{Fe_y} , and teeth, p_{Fe_t} , core losses allows to follow their evolution, and, thus, facilitates corrective measures to reduce them in the critical zone.

14.6.5 Thermal Verification

To roughly verify the winding over temperature of the machine, first the total frame area, A_{fr} , for heat transfer is calculated:

$$A_{fr} = 10^{-6}(\pi k_f D_{s0} l_f + \frac{\pi^2}{2} D_{s0}^2) \quad (14.102)$$

where k_f is the cooling surface increase ratio due to fins.

Then adding an equivalent heat transfer coefficient, α_t (in W/m^2C), say from 14 (for unventilated frames) to 100 (for water-cooled frame jackets), the winding over temperature, T_{w1} , is

$$T_w = \frac{P_{Cu} + P_{Fe}}{\alpha_t A_{fr}} \quad (14.103)$$

If the over temperature is high, while α_t has a realistic value in accordance with the adopted cooling system, then the machine design is redone from the start, with a smaller J_1 (or f_{tsp}) and/or longer λ_c (longer core length).

14.6.6 Machine Characteristics

In general, the machine characteristics for a PMSM include

- Base speed, n_b and maximum speed, n_{max}
- Continuous torque at base speed and base (maximum) voltage
- Peak torque at base speed
- Torque at maximum speed
- EMF at maximum speed
- Efficiency and power factor (or stator current) versus load and speed for given maximum fundamental inverter voltage, V_1

More on realistic modeling of SPMSMs is available in Refs. [2–4].

14.7 Optimal Design with Genetic Algorithms

For the optimization design of SPMSMs with genetic algorithms (GA), we introduce the following optimization variables:

- Linear electric loading, J_1 (A/m)
- Airgap flux density, B_{ag} (T)
- Stator teeth flux density, B_{st} (T)

- Stator yoke flux density, B_{sy} (T)
- Rotor yoke flux density, B_{ry} (T)
- Stator current density, J_s (A/mm²)
- Machine shape factor, $\lambda_c = l_c/\tau$
- Slots per pole per phase, q_1
- Slot opening, s_o (mm)
- Tooth top height, h_{s4} (mm)
- Coil span, y_1 (mm)
- PM pole span coefficient, λ_{PM}

These 12 variables are grouped in a vector X_0 . The variable vector X is not directly initialized by the designer, but he/she will attribute minimum and maximum values, $X_{\min}$ and $X_{\max}$, together with the corresponding resolution ΔX . Also, all minimum values of geometrical variables that are imposed for technological or for mechanical reasons are grouped in $G_{\min}$:

- Outer rotor diameter
- Interior rotor diameter
- Minimum active slot height
- Minimum total slot height
- Minimum slot width
- Minimum active slot area

The chosen GA operates the binary coding of optimization variables. As some of the optimization variables, such as the number of stator slots per pole per phase, q_1 , and the coil span in slot pitches, are integers while others are rational numbers with extended dynamics, they are represented with different numbers of bits. Now, with the variable domain, $X_{\max} - X_{\min}$, and the representative resolution, the required number of bits, N_{bit} , is

$$N_{\text{bit}} = 1 + \text{int} \left(\log_2 \left(\frac{X_{\max} - X_{\min}}{\Delta X} \right) \right) \quad (14.104)$$

To set the GA (detailed in Chapter 13) we give the population size, n_p , the number of individuals (in a generation), the number of generations, n_g , the elitism factor, k_{elit} , the mutation rate, r_m , and the occupation factor, k_{ex} , of those individuals who take part in producing genetic material for the next generation.

14.7.1 Objective (Fitting) Function

We have chosen a complex objective (fitting) function here: the initial cost of the motor plus the loss cost for the machine's active life plus additional framing and transportation costs due to its weight. The initial cost, C_i , includes only the costs of active materials, as the total manufacturing and selling costs depend highly on the fabrication technology and cost management of the particular manufacturer:

$$C_i = m_{cu}p_w + m_{Feu}p_{lam} + m_{PM}p_{PM} \quad (14.105)$$

where p_w , p_{lam} , and p_{PM} are the copper, the lamination, and the PM unitary prices, say in USD(or EU)/kg.

The cost of losses, C_E , is

$$C_E = P_N \left(1 - \frac{1}{\eta_N}\right) n_{hy}n_y p_E \quad (14.106)$$

where

n_{hy} is the annual operational hours

n_y is the operational years

p_E is the energy cost (USD(EU)/kWh

As a motor very rarely operates all the time at the rated power and speed, we may introduce an ideal operation cycle characterized by probability α_i to operate at different power loads, P_i , and efficiencies, η_i :

$$C_E = n_{hy}n_y p_E \sum_{i=1}^n \alpha_i P_i \left(1 - \frac{1}{\eta_i}\right) \quad (14.107)$$

where $\sum \alpha_i = 1, i = \overline{1, n}$.

Though C_E is in general larger than C_i , the value of C_E is diminished in the design as many buyers cannot afford a higher initial cost; in such cases, in fact, a kind of optimal efficiency in terms of the initial/total cost is adopted. Now the cost of losses, C_E , is

$$C_E = n_{hy}n_y p_E \sum_{i=1}^n \alpha_i P_i \quad (14.108)$$

where $P_i = \begin{cases} P_i \left(\frac{1}{\eta_{oi}} - \frac{1}{\eta_i}\right) & \text{if } \eta_i < \eta_{oi} \\ 0 & \text{if } \eta_i \geq \eta_{oi} \end{cases}$

In aircraft applications, low weight is a critical performance index. A similar situation occurs in wind generators when low-cost but poor-performance materials lead to a larger machine weight, which implies a heavy nacelle and tower, and their extra costs may offset the reduction in the generator initial

costs. In order to force the algorithm to reduce the motor (generator) weight, we may add some additional costs, C_m , proportional to the motor weight, m_t :

$$C_m = m_t p_m \quad (14.109)$$

Also, surpassing the maximum allowable winding (as well as the core and the PM) temperature, leads to immature aging of the equipment. To avoid this, with the optimization algorithm we introduce a penalty cost for the over temperature, C_{temp} , in the objective function. This penalty cost, C_{temp} , may vary linearly or exponentially with the over temperature:

$$C_{temp} = \begin{cases} k_T(T - T_{max})C_i & \text{if } T > T_{max} \\ 0 & \text{if } T \leq T_{max} \end{cases} \quad (14.110)$$

We may add penalty costs for any other technical constraints. For example, we may add a penalty for a lower power factor, which is related to the converter kVA rating costs, as also for the outer diameter or for the machine axial length, etc. The larger the specific costs of these constraints, the higher the probability that the optimization design will observe them.

Finally, the total cost, C_t , is

$$C_t = C_i + C_E + C_m + C_p \quad (14.111)$$

14.7.2 PMSM Optimization Design Using Genetic Algorithms: A Case Study

An exercise design of a 3-phase, four-pole synchronous motor with a surface PM rotor for a rated power of 2.2 kW and base frequency of 50 Hz is presented. The maximum and minimum values of optimization variables are presented in Table 14.1. The technological constraints used for this exercise design can be found in the input file (pmm1.m) of the PMSM optimal design MATLAB code, which is stored on the CD (which is attached to the book) in the “PMSMdesign” folder. The total cost, C_t , represents the objective function.

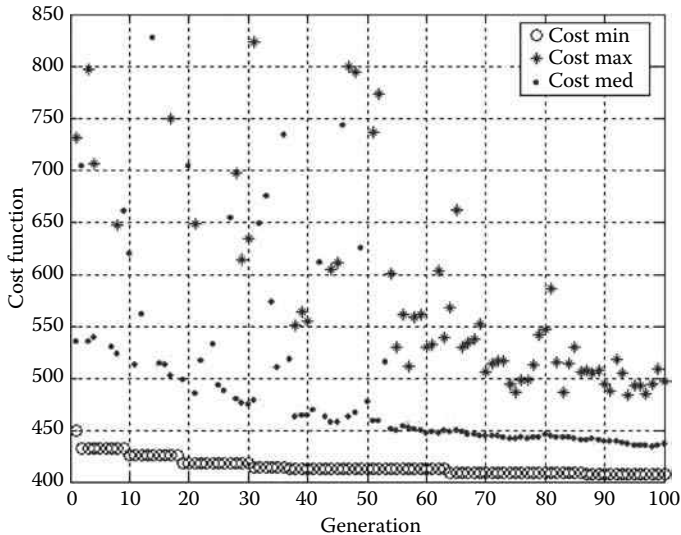
The evolution of the genetic algorithm is illustrated in the following figures. Every generation has a member that is best fitted for the required optimum criterion, which minimizes the cost function. The generation itself is characterized by the average cost function. For each generation, there is also a least-adapted member. Following the evolution of these three values, we may appreciate the GA convergence. At the beginning, there is a large difference between the cost function associated to the best member and the average value of the cost function. The value of the cost function of the least-adapted member is larger than 10 times the average value. A large gap between the minimum and the maximum cost function for members over a

TABLE 14.1
Optimization Variable Bounds

Optimization Variable	Minimum Values	Maximum Values	Units	Comments
J_1	15	30	kA/M	Specific electric load specifications
B_{agsp}	0.45	0.75	T	Airgap flux density
B_{st}	1	2	T	Stator tooth flux density
B_{sy}	0.9	1.9	T	Stator yoke flux density
B_{ry}	0.9	2.1	T	Rotor yoke flux density
J_s	3	8	A/mm ²	Stator current density
$l_{cpertau}$	0.5	3		Core stack length per pole pitch
q_1	2	4		Stator slots per pole per phase
so	1	5	mm	Stator slot opening width
$sh4$	0.5	2	mm	Stator tooth pole tip height
$cSpan$	0.66	1		Coil span per pole pitch
$Alpm$	0.5	1		Permanent magnet angle per pole angle

generation shows a population with a good evolution perspective. The statistics values (minimum, average, and maximum) of the cost function tend to decrease during the evolution. The minimum value of the cost function is decreasing monotonically along the generations while the average and the maximum values sometimes are increasing, especially for the first generation (Figure 14.6), where a large variation of the maximum cost can be observed. In time, the members of the population begin to hold similar codes, and the variation of the maximum and the minimum cost function from one generation to the next diminishes. Gradually, the minimum value of the objective function remains unchanged for many generations and the cost variation is small even if it exists.

When the members were chosen randomly, in pairs, to generate an offspring, a minimum difference in the cost function was imposed, in order to avoid a premature evolution. A premature evolution has produced numerous twins in the population, and the maximum and average costs collapsed

**FIGURE 14.6**

Minimum, maximum, and average cost function evolutions.

to the minimum cost. The main method, crossover, used to generate an offspring is not able to bring about any improvement in this state. Then a new genetic code is produced only by mutation, but if the new members have cost functions higher than those of the majority of the population members, which are very similar, the new genetic code will be eliminated in a few generations.

The main geometric dimensions and the slot geometry evolution of the best member along generations are presented in Figure 14.7. It can be observed that geometric dimensions take discrete values and the same dimension can be arrived again after several generations.

The power losses and the energy efficiency evolution of the best member are shown in Figure 14.8. The mechanical power loss is set constant as 0.5% of the rated power in the input power. A small efficiency improvement (around 1%) is observed in the first ten generations. The best-candidate solution from the first generation has already a good efficiency because it represents the best solution of a population of 150 members.

The machine components' weights and their cost evolutions are presented in Figure 14.9. The laminations are punched from rectangular sheets and the minimum rectangular (square) surface that encloses the stator lamination is considered when computing the weight of used iron. When the rotor core is made of laminations, then it is punched from the same sheet as the stator lamination at no extra cost for the material. A solid rotor core is

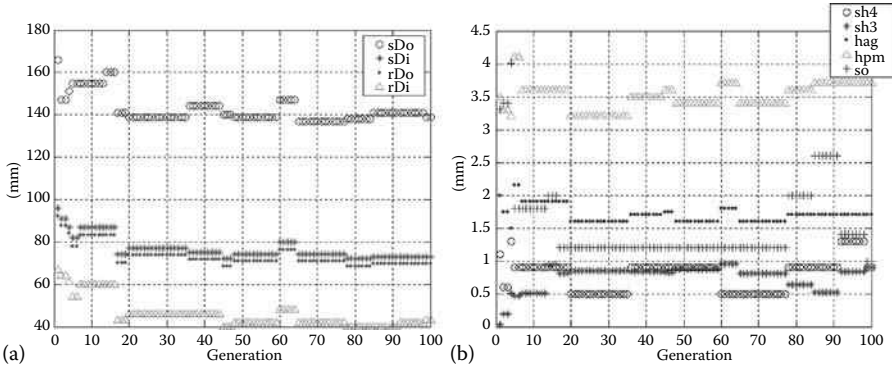


FIGURE 14.7
(a) Main geometric and (b) slot dimensions evolution of the best member.

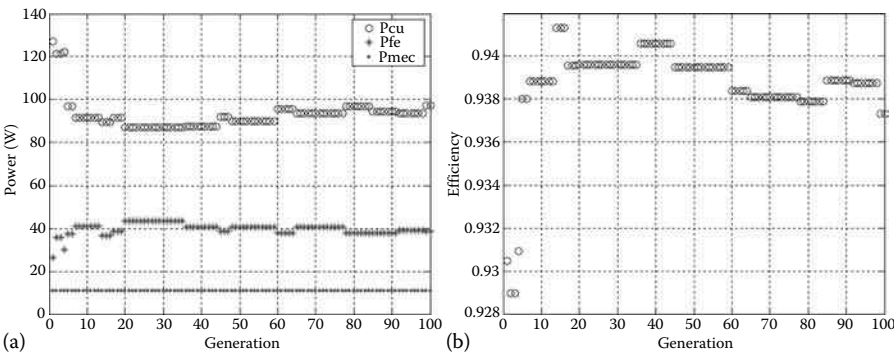


FIGURE 14.8
(a) Power losses and (b) efficiency evolution of the best member.

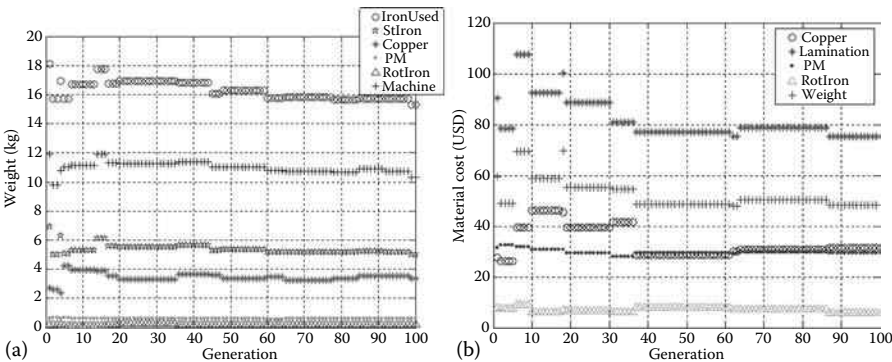
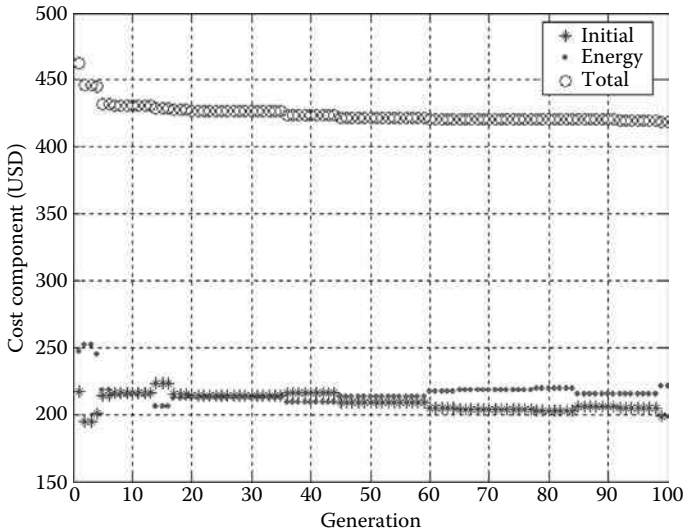


FIGURE 14.9
(a) The components' weights and (b) their costs for the best-candidate solution.

**FIGURE 14.10**

Evolution of the cost (objective) function of the best-candidate solution.

used in many cases for small PMSMs, and, in this case, the cost of the rotor core material is added.

The initial cost and energy cost evolutions for the best-candidate solution are shown in Figure 14.10. We can observe that their values are about the same, which is a well-known optimization “thumb rule.” The sum of the initial cost and the energy cost is equal to the total cost, which is the objective function plotted in Figure 14.10 in a better scale. Usually, the over temperature penalty cost is equal to zero for the best-candidate solution.

The optimization variable evolutions for the best-candidate solution are presented in the following figures. The maximum and minimum values of the optimization variables as well as their resolutions are given in the input file. The number of discrete levels of each variable is usually not equal to powers of two. The required number of bits (Equation 14.102), necessary to cover the discrete levels of the optimization variable, could code a larger number of levels that is required from the initial file specification. Using the described crossover method, it is possible to generate an offspring that has one or more optimization variables larger than the upper bounds. One solution is to discard all offsprings that have their optimization variables outside the bounds and try again to generate viable offsprings. Another solution is to let the optimization variable be larger than the upper limits and let the optimization algorithm eliminate infeasible candidates. This method is applied when an optimization variable larger than the upper bounds has a physical meaning such as for electric and magnetic loads. One more method to solve this problem is to decrease the resolution in such a way that all binary

codes are used to represent variables between given bounds. When a variable value out of its bounds is a nonsense, then a mathematical transformation is applied to force the optimization variable back between its bounds. For example, the ratio between the PM width and the pole pitch larger than one is a nonsense. The following mathematical transformation is applied to avoid this nonsense:

$$\lambda_{PM} = \begin{cases} \lambda_{PM} & \text{if } \lambda_{PM} \leq 1 \\ 2 - \lambda_{PM} & \text{if } \lambda_{PM} > 1 \end{cases} \tag{14.112}$$

Some features may have two distinct codes. For example, the code that is producing the number 1.1, is forced to produce a relative PM width equal to 0.9, and there exists also the code that produces directly 0.9.

The optimal value of linear electric load is larger than 40 kA/m (Figure 14.11), while the maximum value in the input file was set at 30 kA/m. The optimal value of the airgap flux also tends to its upper bound (Figure 14.12). Larger limits of these optimization variables will be set if there are no other restrictions.

The upper limit of the current density is large enough in our design, and, thus, there is no optimization value of the best-candidate solution larger than the upper limit (Figure 14.13). The codification method allows for larger current densities because the required domain needs 50 samples to be covered, and 6 bits are enough to code 64 samples.

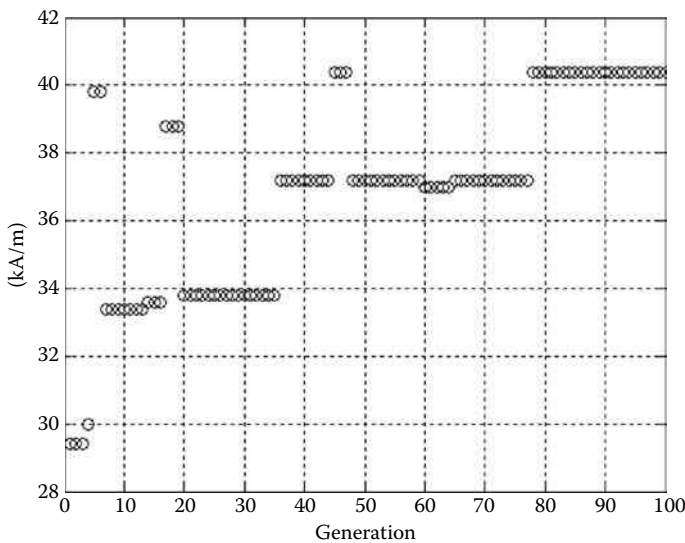


FIGURE 14.11
Specific electric load evolution of the best-candidate solution.

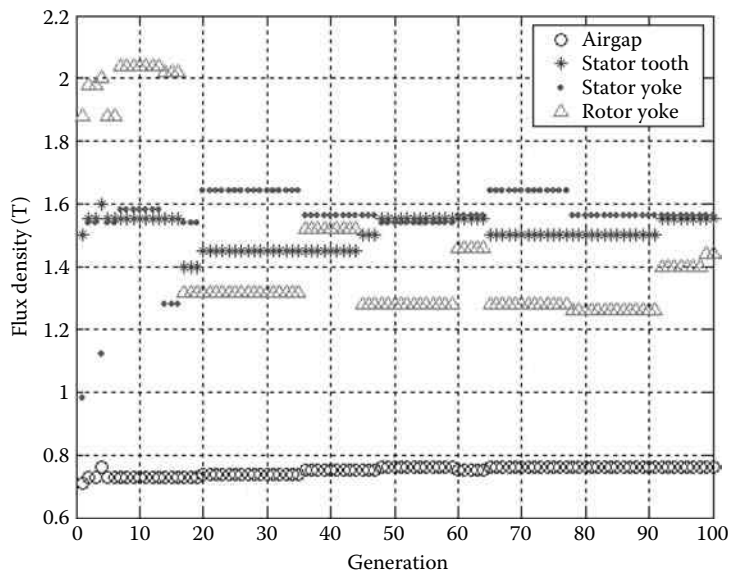


FIGURE 14.12

Flux density evolutions of the best-candidate solution.

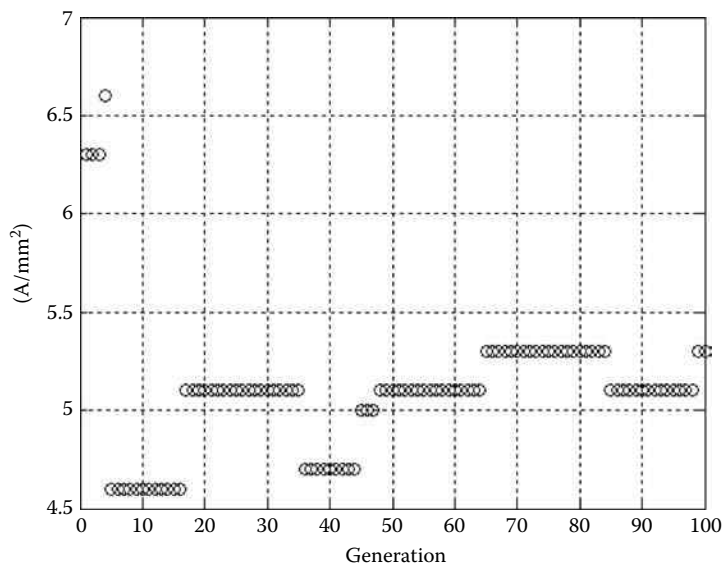


FIGURE 14.13

Current density evolution of the best-candidate solution.

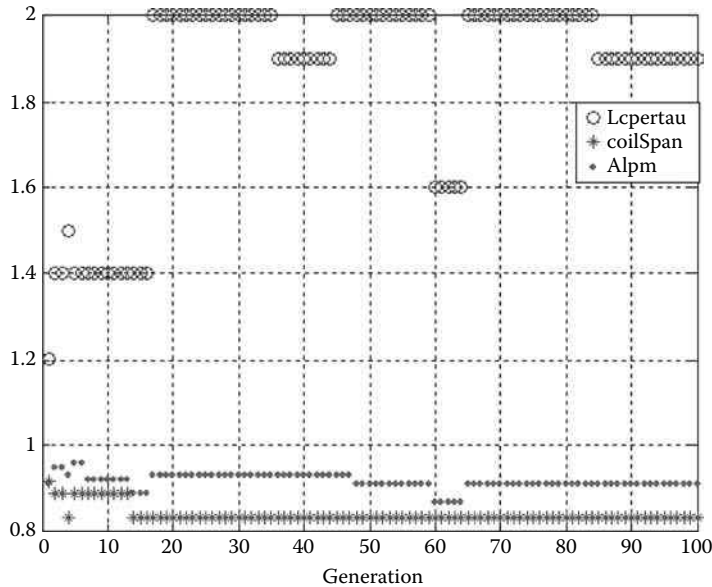


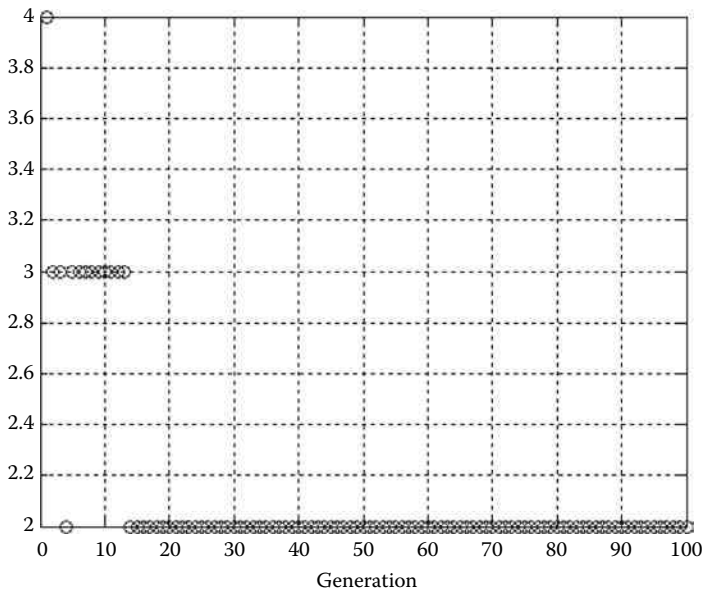
FIGURE 14.14 Evolutions of the ratios of dimensions of the best-candidate solution: core length per pole pitch (lcpertau), coil span per pole pitch (coilSpan), and PM width per pole pitch (Alpm) (all in pu).

The ratio of the core length to the pole pitch, the ratio of the coil span to the pole pitch, and the ratio of the PM width to the pole pitch are presented in Figure 14.14 for the best-candidate solution from each generation. The ratio of the coil span to the pole pitch given by the GA is rounded to the nearest rational number, which has a denominator equal to the number of slots per pole.

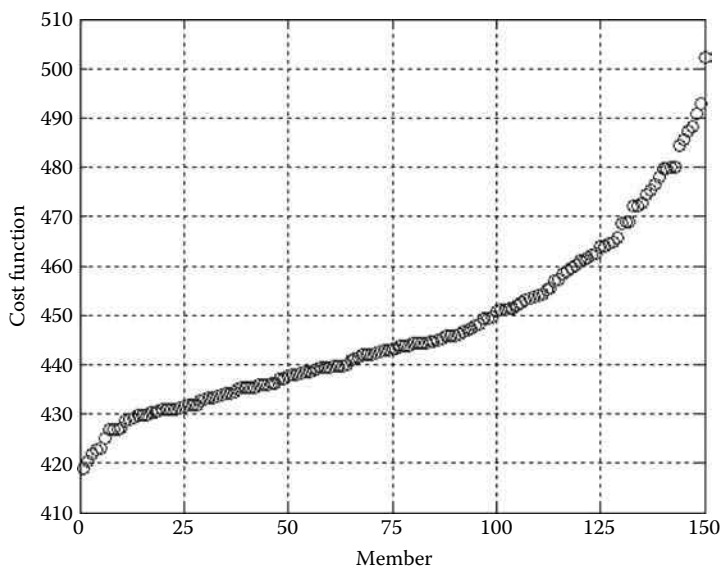
The number of stator slots per pole per phase is always an integer and is shown in Figure 14.15 for the best-candidate solution from each generation.

All members of the last generation are obtained through a long selection process and it would be interesting to analyze the distribution of the optimization variables. At first sight the members look close to the objective function (Figure 14.16). We may observe that not all possible levels are taken (Figures 14.17 through 14.20), and that there is no unambiguous rule on what values could minimize the optimization function.

We are not able to assert that always the value of linear electric load that characterizes the best solution (40.4 kA/m) will produce good results (Figure 14.17). We can state that probably a linear electric load smaller than 25 kA/m is too small to get an acceptable motor cost. Good solutions are possible with an airgap flux density around 0.75 T (Figure 14.18),

**FIGURE 14.15**

Evolution of the slots per pole per phase of the best-candidate solution.

**FIGURE 14.16**

The cost function over the last generation.

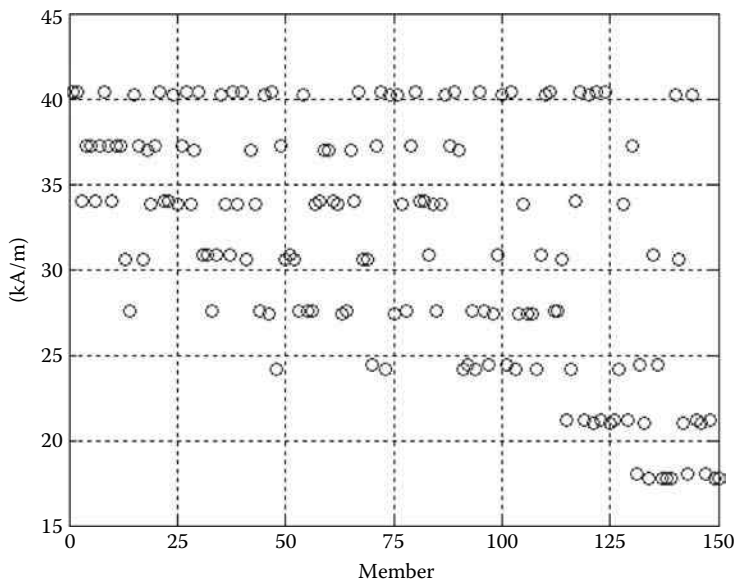


FIGURE 14.17
The specific electric load over the last generation.

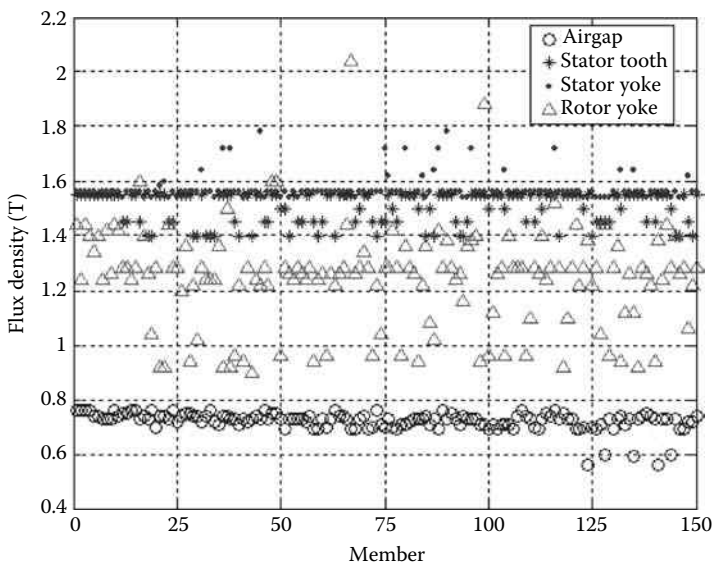


FIGURE 14.18
Flux densities over the last generation.

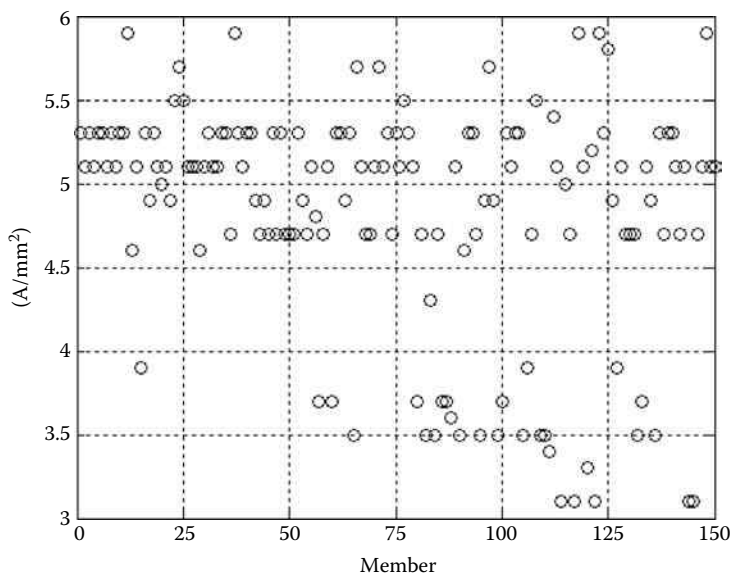


FIGURE 14.19
Current density over the last generation.

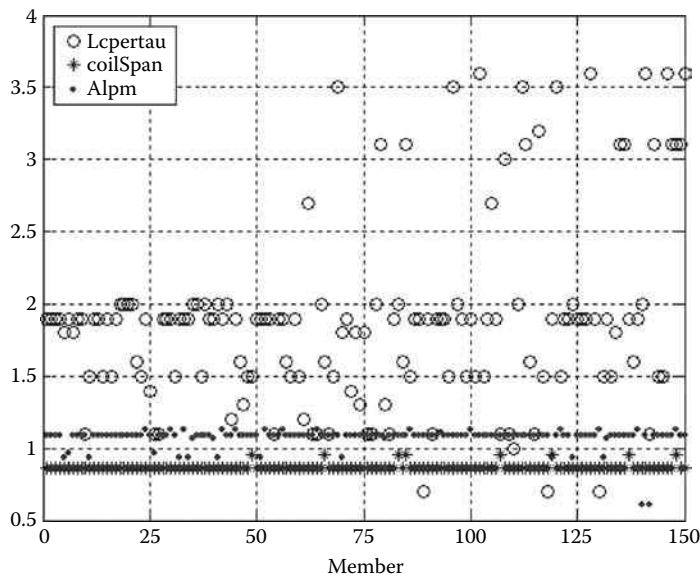


FIGURE 14.20
Ratio of dimensions over the last generation.

respectively, with the stator yoke and stator tooth flux densities around 1.55 T. The rotor yoke flux density has a small influence on the objective function (Figure 14.18). It is not possible, probably, to minimize the objective function when current densities become smaller than 4 A/mm². About the ratio-type optimization variables, we can state that a coil span per pole pitch equal to 0.83 and the PM width to pole pitch of around 0.91 could produce good solutions (Figure 14.20). A ratio of the core length to the pole pitch larger than 2.5 probably always produces bad results (Figure 14.20).

There are only two values selected for the number of slots per pole per phase, and we can remark that in the last generation two slots per pole per phase had higher occurrence frequencies than three slots per pole per phase, especially for high-performance members (Figure 14.21). The best-candidate solution for many populations has two slots per pole per phase, so, probably, this value will minimize the objective function.

Further improvement of the GA is possible by reducing the search area and even by setting some variables constant and then eliminating these from the optimization variable vector.

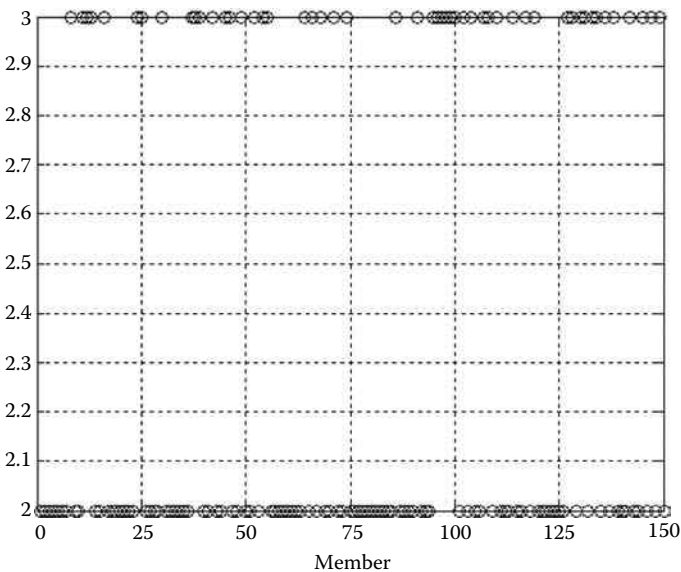


FIGURE 14.21
Slots per pole per phase over the last generation.

14.8 Optimal Design of PMSMs Using Hooke–Jeeves Method

The same optimization variable vector and objective function, as those presented for the GA method, are used also to implement the Hooke–Jeeves optimization algorithm (presented in detail in Chapter 13). The geometric dimensions as well as the optimization variables and cost function modifications look now as continuous functions, compared to the GA functions, where these functions look rather discontinuous. The evolutions of the main geometric and slot dimensions are presented in Figure 14.22. The power losses and efficiency evolutions are shown in Figure 14.23, while the components' weights and costs are visible in Figure 14.24. Cost function, specific (linear) electric load flux density, and current density evolutions are shown in Figures 14.25 through 14.28.

The ratio of the PM width to the pole pitch and the ratio of the coil span to the pole pitch are going to the same values as in the GA (Figure 14.29), while the core length per pole pitch ratio is going to a smaller value. The algorithm starts with three slots per pole per phase and after three steps it changes to two slots per pole per phase and remains at that value (Figure 14.30).

Comparative results of the optimal design using GAs and the Hooke–Jeeves method respectively, are presented in Table 14.2. The total cost is about the same for the two methods but the simulation time is about 75 times larger for GAs than for the Hooke–Jeeves algorithm. The GA chooses some optimization variables that are values outside the bounds, while the Hooke–Jeeves optimization variables remain strictly within the specified bounds. The motor efficiency designed by the Hooke–Jeeves algorithm is larger than

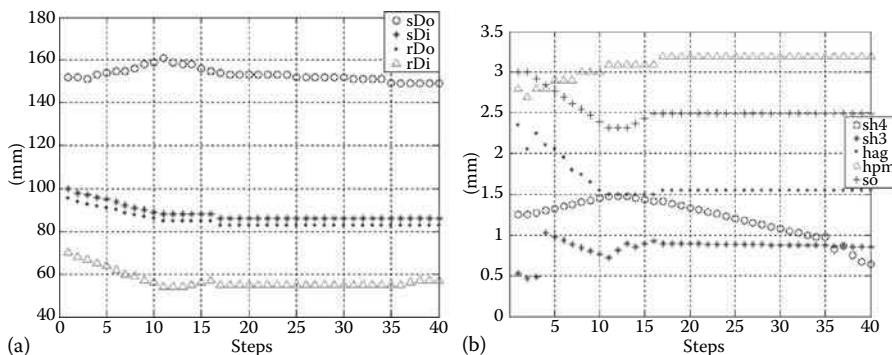


FIGURE 14.22

Geometric and slot dimensions evolution. (a) Main dimension and (b) slot and PM dimension.

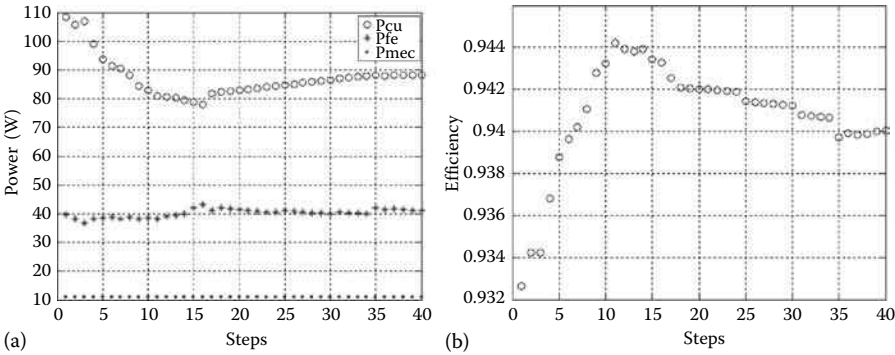


FIGURE 14.23
(a) Power losses and (b) efficiency evolutions.

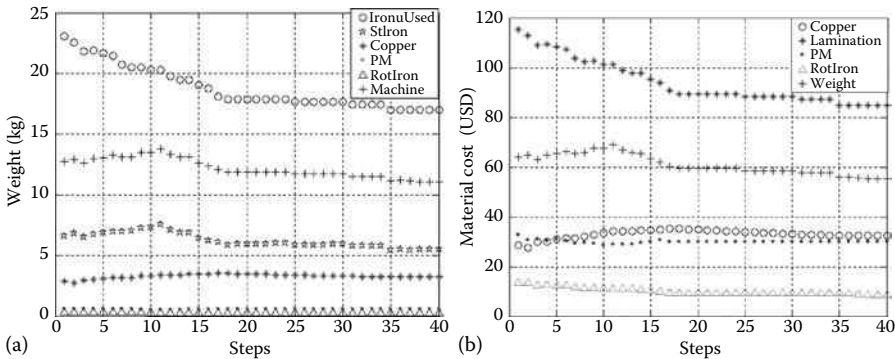


FIGURE 14.24
(a) Components' weights and (b) cost evolutions.

the motor efficiency designed by the GA, but the initial cost is smaller for the GA and, probably, this is a consequence of a larger linear electric load allowed for the GA. The first variation step for each variable in the research process of the Hooke–Jeeves algorithm was 10% of the entire domain. Such a large initial step may help the algorithm examine a large area, and, thus, increase the chances to pass over local minima.

14.9 Conclusion

This chapter may be considered a solid introduction to the analytical optimization design of PMSMs. The results of the full analytical model should help to grasp better the principles and some subtleties.

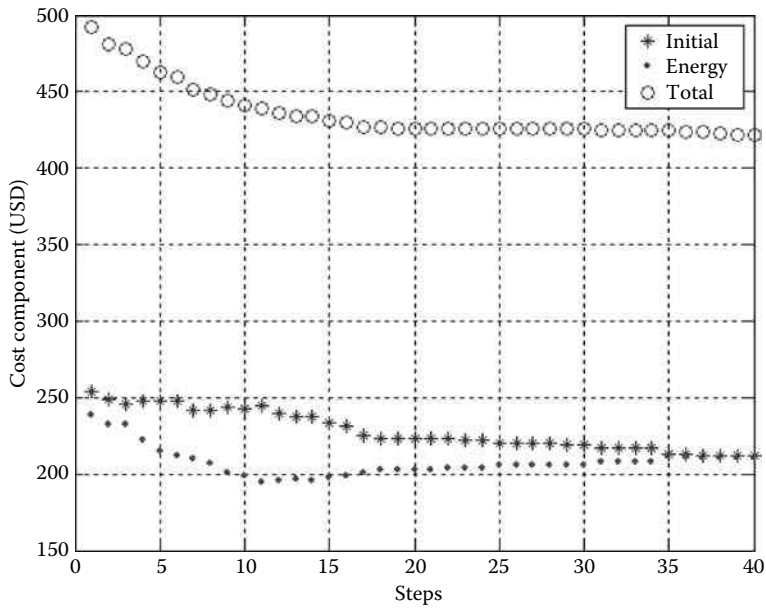


FIGURE 14.25
Cost (objective) function evolution.

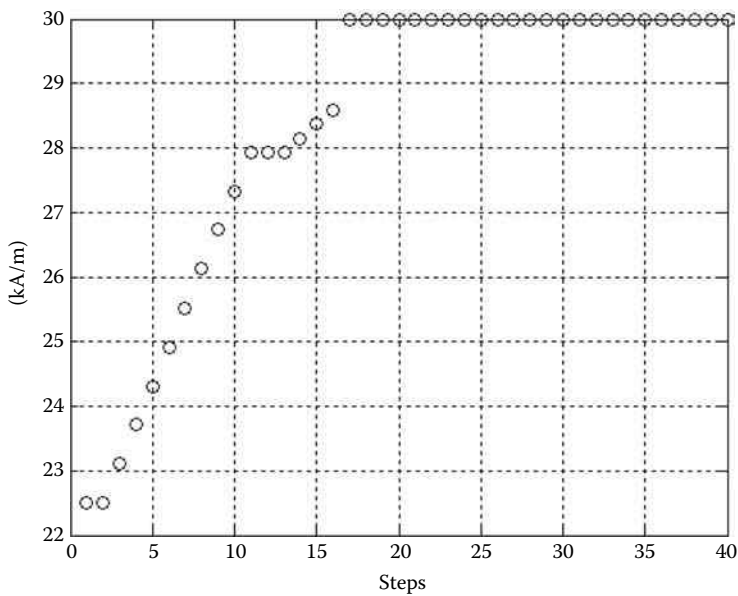


FIGURE 14.26
Specific load evolution.

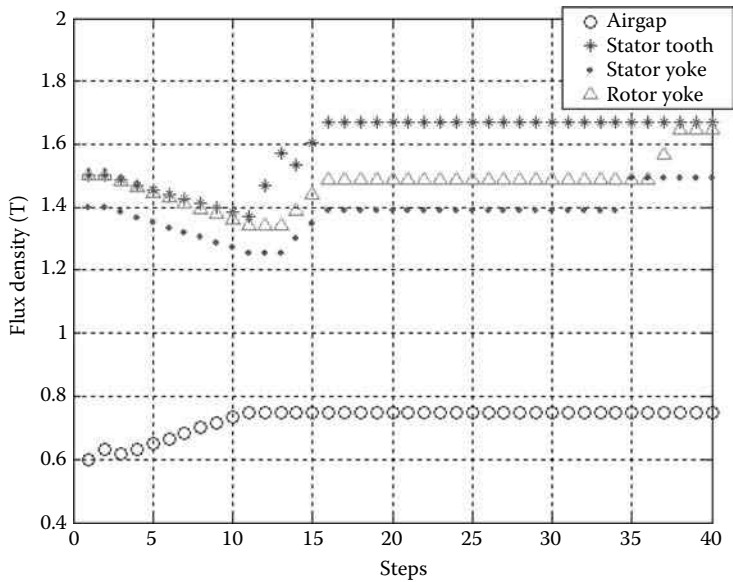


FIGURE 14.27
Flux density evolution.

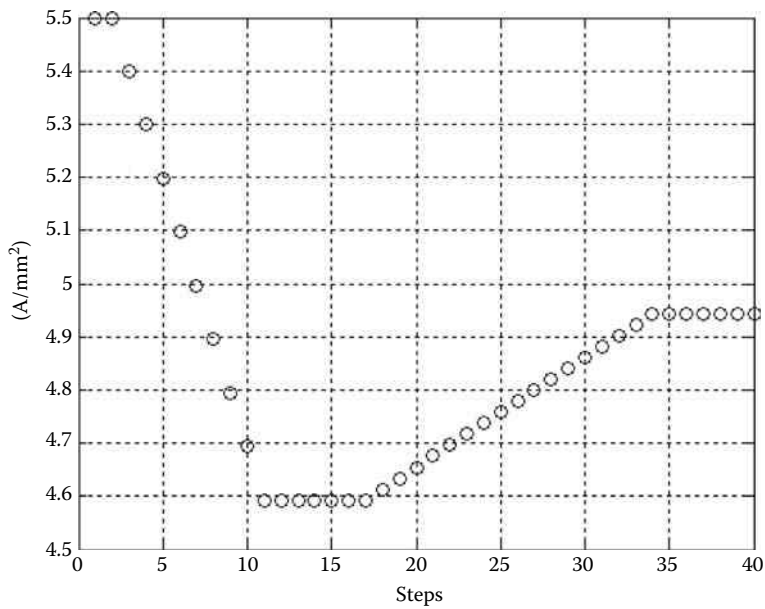


FIGURE 14.28
Current density evolution.

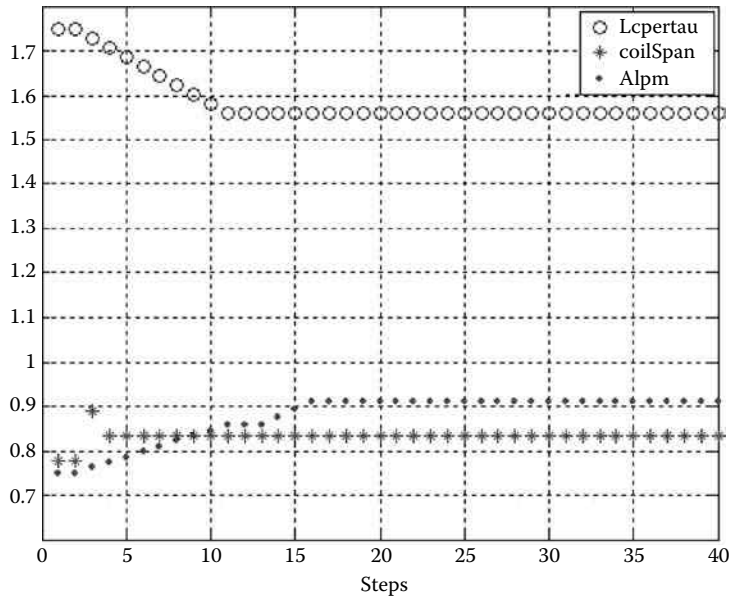


FIGURE 14.29
Dimensions ratios evolution.

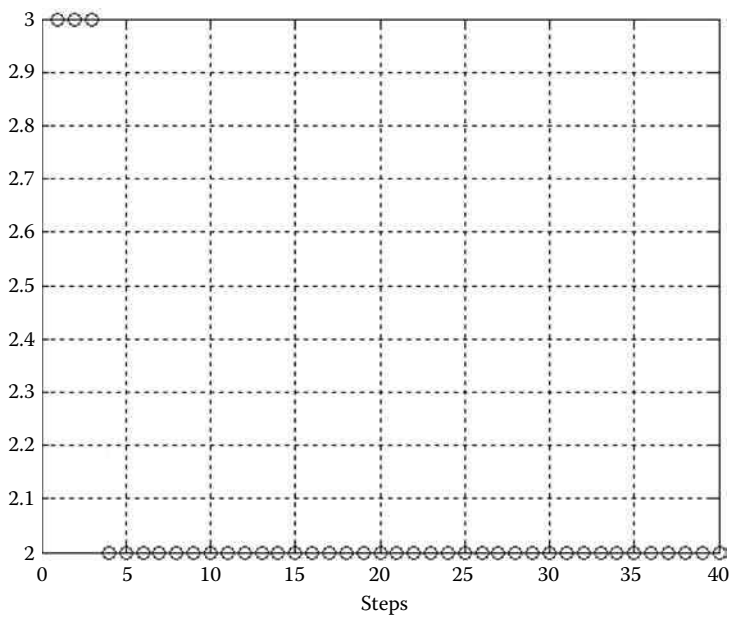


FIGURE 14.30
Slots per pole per phase evolution.

TABLE 14.2
Comparisons of Optimization Results

Parameter	GA	HG	Units	Comments
sDo	139	149	mm	Stator external diameter
sDi	73	86	mm	Stator inner diameter
sh4	0.9	0.6	mm	Stator tooth pole tip height
so	1	2.5	mm	Stator slot opening width
sh3	0.9	0.9	mm	Height of wedge for stator slots
shOA	24.2	20.7	mm	Total height of stator slot
sh1	21.7	18.5	mm	Main (coil) height of the stator slots
shy	8.8	10.8	mm	Stator yoke height
swt	4.7	5.1	mm	Stator teeth width
N ₁	336	312		Numbers of turns per phase
rDo	69.6	82.9	mm	Rotor outer diameter
rDi	43	57	mm	Rotor inner diameter
hpm	3.7	3.2	mm	Permanent magnet height (thickness)
hag	1.7	1.55	mm	Airgap
J _l	40.4	30	kA/m	Linear electric load
Bagsp	0.76	0.75	T	Airgap flux density
sBtp	1.55	1.67	T	Stator tooth flux density
sByp	1.56	1.49	T	Stator yoke flux density
rByp	1.44	1.65	T	Rotor yoke flux density
J _s	5.3	4.94	A/mm ²	Stator current density
Lcpertau	1.9	1.6		Ratio of core stack length to pole pitch
q ₁	2	2		Stator slots per pole per phase
cSpan	0.83	0.83		Ratio of coil span to pole pitch
alpm	0.91	0.91		Ratio of PM width to pole pitch
WeightStCu	3.365	3.255	kg	Coil weight
WeightIronUsed	15.268	16.961	kg	Used iron weight
WeightStIron	5.003	5.509	kg	Stator lamination weight
WeightPM	0.595	0.603	kg	PM weight
WeightRtIron	1.372	1.712	kg	Rotor iron weight
WeightMot	10.336	11.079	kg	Motor total weight
Iqn	3.9	3.81	A	Rated current
Vfn	220	220	V	Rated voltage
Pcu	97.39	88.15	W	Copper losses
Pfe	38.76	41.2	W	Iron losses
Pmec	11	11	W	Mechanical losses

TABLE 14.2 (continued)

Comparisons of Optimization Results

Parameter	GA	HG	Units	Comments
Etan	93.73	94		Efficiency
Cu_c	33.649	32.547	USD	Copper cost
lam_c	76.339	84.806	USD	Used laminations cost
PM_c	29.772	30.173	USD	Permanent magnet cost
rotIron_c	6.8634	8.5575	USD	Rotor solid iron cost
pmw_c	51.679	55.395	USD	Penalty cost of active material weight
i_{cost}	198.30	211.48	USD	Initial material cost
energy_c	220.73	210.52	USD	Lost-energy cost during 10 years, 1500 h/year
t_{cost}	419.03	422.00	USD	Total cost (objective function)
sim_{time}	340.86	4.516	s	Simulation time on Pentium IV-2.6 GHz

Though magnetic saturation was considered in the analytical model, the cogging torque was not taken into account. Total torque pulsations, mainly due to magnetic saturation, or due to static or dynamic eccentricities, were not considered either.

Tying up analytical optimization design methodologies (as in this chapter) and integrating them into a finite element software, for validation and exploration of the cogging torque, total torque ripple, etc., are, perhaps, the next steps. Interested readers are invited to follow them up on their own.

References

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3. W. Ouyang, D. Zarko, and T.A. Lipo, Permanent magnet machine design practice and optimization, *Industry Applications Conference, 41st IAS Annual Meeting. Conference Record of the 2006 IEEE*, Tampa, FL, Vol. 4, pp. 1905–1911, 2006.

4. C.B. Rasmusen, Modeling and simulation of surface mounted PM motors, PhD thesis, Institute of Energy Technology, Aalborg University, Aalborg, Denmark, 1996.
5. IEC/EN 60034.
6. Thin non-oriented Electrical Steel—SURA 007, Products Information, Surahammars Bruk, Sweden.

15.1 Realistic Analytical Model for Induction Machine Design

In this chapter, we introduce a rather complete analytical realistic model of the induction machine and use it in an optimal design computer code based on the GA and the Hooke–Jeeves algorithm via a numerical example.

The general design of induction motor is similar to the general design of the PMSM, with minor differences. For induction machine design, there are specific design themes and a set of chosen variables and technological constraints [1–10].

15.1.1 Design Theme

In the design theme of the induction machine, the following parameters are always given:

- Base continuous power, P_b , or rated power, P_n
- Base speed, n_b
- Rated voltage, V_n
- Number of phases, m
- Overload factor, k_l

When the motor is designed to run without power electronics, the minimum starting torque and the maximum direct grid connection starting current are required in the specifications list.

For variable speed applications, the maximum speed, $n_{\max}$, and the power at maximum speed, $P_{\max}$, are added to the specifications.

Other frequent constraints are

- Efficiency at P_b and n_b
- Power factor at rated power and speed, pf_{spa}
- Material insulation class (allowed temperature)

- Protection degree against alien bodies
- Total initial cost of active materials

15.1.2 Design Variables

The design variables are chosen and iteratively changed directly by the designer or by an optimization algorithm in order to achieve the design theme requirements. The main design variables are

- Linear current density (electric linear load), J_l (kA/m)
- Airgap flux density, B_{agsp} (T)
- Machine shape factor—ratio of core length stack to pole pitch, λ_c
- Stator current density, J_s (A/mm²)
- Rotor current density, J_r (A/mm²)
- Stator teeth flux density, B_{st} (T)
- Stator yoke flux density, B_{sy} (T)
- Rotor teeth flux density, B_{rt} (T)
- Rotor yoke flux density, B_{ry} (T)
- Number of stator slots per pole per phase, q_1
- Stator slot opening width, s_{os} (mm)
- Rotor slot opening width, s_{or} (mm)
- Stator tooth top height, h_{s4} (mm)
- Rotor tooth top height, h_{r1} (mm)
- Stator coil span, $cSpan$

The following parameters are chosen according to the available technology and motor size:

- Current parallel path count, a_1
- Number of winding layers, n_L
- Airgap, g
- Difference between rotor core length and stator core length, dl_r
- Straight-turn end connection length, l_{f1} , that exits the slot
- Axial distance between end-coil connection and frame, l_{f2}
- Core-stacking factor, k_{stk}

- Slot-filling factor, k_{sf}
- Ratio between rotor end ring and rotor bar current densities, $k_{jendr2jr}$
- Rotor slot slant in number of rotor slot pitches, r_{ss}
- Angle of slot wedge, a_w
- Slot insulation thickness, $slotInsulThick$
- Thickness of slot closure (wedge), $slotClosureThick$
- Number of elementary conductors per turn, n_{ce}
- Assumed average temperature of the stator winding, T_{w1}
- Assumed average temperature of the rotor winding (cage), T_{w2}
- Maximum admissible winding temperature, $T_{w_{max}}$ (it depends on the insulation class)
- Maximum temperature of the cooling fluid (or ambient), T_{amb}
- Thermal transmission coefficient, α_t
- Increasing factor of cooling surface from frame fins and cooling fluid velocity, k_{ff}
- Iron loss factor (the iron loss is larger due to field nonuniformity), k_{pfe}

The soft magnetic material for stator and rotor magnetic cores and winding material (copper or aluminum for the rotor) are also chosen at this time. The mechanical losses estimation is also performed.

For the induction machine, the minimum mechanical feasible airgap is used. For the computation to start, the following values could be chosen:

$$g = \begin{cases} 0.1 + 0.2\sqrt[3]{P_n} & \text{if } p_1 = 1 \\ 0.1 + 0.1\sqrt[3]{P_n} & \text{if } p_1 \geq 2 \end{cases} \quad (15.1)$$

The airgap, g , is obtain in millimeters and the rated power, P_n , is given in kilowatts in Equation 15.1.

15.1.3 Induction Machine Dimensioning

The induction machine design starts with several preparation calculations, such as grid apparent power S_n , using chosen variables and constraints. Then, the machine constant and stator inner diameter are computed:

$$S_n = \frac{P_n}{\eta_{spec} \cdot pf_{spec}} \quad (15.2)$$

The winding factor k_w is computed from the zone factor, f_{ws} , and the chording factor, k_{chs} , using the same relation 14.4 as for the permanent machine (see also Ref. [3]):

$$C_0 = \frac{\pi^2}{\sqrt{2}} B_{agsp} I_l k_w \quad (15.3)$$

$$D_{si} = 1000 \cdot \sqrt[3]{\frac{60 p_s}{\pi n_n} \cdot \frac{S_n}{\lambda_c C_0}}; \quad (\text{mm}) \quad (15.4)$$

The pole pitch, τ , and stator core length, l_{c1} , are computed in the same way as for PMSM (Equations 14.7 and 14.8) using the inner diameter, D_{si} .

The main geometric dimensions are shown in Figure 15.1. The stator slots geometry is shown in Figure 12.42 and the rotor slots geometry is shown in Figure 12.43.

The stator and rotor yoke widths are

$$h_{sy} = \frac{\tau_p B_{agsp}}{\pi B_{sy}} \quad (15.5)$$

$$h_{ry} = \frac{\tau_p B_{agsp}}{\pi B_{ry}} \quad (15.6)$$

The number of stator slots and the stator slots pitch and tooth width are

$$N_{ss} = q_1 m \cdot p_1 \quad (15.7)$$

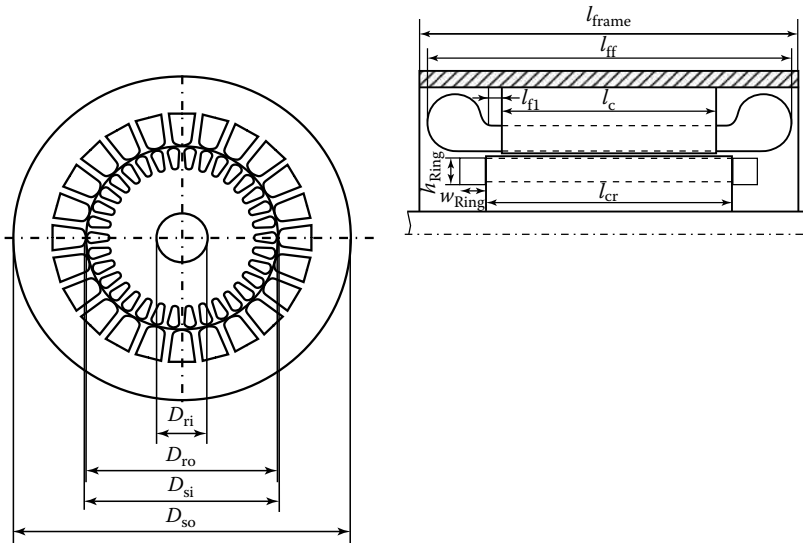


FIGURE 15.1

Induction machine geometry.

$$\tau_{ss} = \frac{\pi D_{si}}{N_{ss}} \quad (15.8)$$

$$w_{st} = \frac{B_{agsp}}{B_{st}} \tau_{ss} \quad (15.9)$$

Now, the polar flux, Ψ_p , and the first approximation of the number of turns per phase, N_1 , and the number of turns per coil, s_{b1} , are computed:

$$\Psi_p = \frac{2}{\pi} B_{agsp} \tau_p l_c \quad (15.10)$$

$$N_1 = \frac{V_{fn}}{\pi \sqrt{2} f_n f_w \Psi_p} \quad (15.11)$$

$$s_{b1} = \frac{N_1 a_1}{p_1 q_1 n_L} \quad (15.12)$$

The number of turns per coil should be an integer number so the computed value, s_{b1} , is rounded to the nearest integer, s_b . If the nearest integer is zero, then one turn per coil will be used. The number of turns per phase, N_1 , and the machine core length, l_c , will be recomputed:

$$N_1 = \frac{s_b q_1 p_1 n_L}{a_1} \quad (15.13)$$

$$l_c = \frac{s_{b1}}{s_b} l_{c1} \quad (15.14)$$

Now, the stator slot dimensions are computed in the same way as for PMSMs (Equations 14.15 through 14.22).

15.1.3.1 Rotor Design

The number of rotor slots is chosen according to the number of poles and the number of stator slots (slots per pole per phase) in such a way as to avoid large synchronous parasitic torques and radial forces. Table 15.1 gives the most used pairs of stator and rotor counts.

Now, the rotor outer diameter, D_{ro} , is computed followed by the rotor slots pitch, τ_{rs} , and the rotor tooth width, w_{rt} :

$$D_{ro} = D_{si} - 2g \quad (15.15)$$

$$\tau_{rs} = \frac{\pi D_{ro}}{N_{rs}} \quad (15.16)$$

$$w_{rt} = \frac{B_{agsp}}{B_{rt}} \tau_{rs} \quad (15.17)$$

The rotor slot opening is compared with the maximum feasible rotor slot opening, and if it is larger then it will be adjusted to a maximum feasible value.

TABLE 15.1

Stator and Rotor Number Turns

Pole Pairs	q_1	Number of Rotor Slots
1	4	18 20 22 28 30 33 34
	6	25 27 28 29 30 43
2	2	16 18 20 30 33 34 35 36
	3	24 28 30 32 34 45 48
	4	30 36 40 44 57 59
	5	36 42 48 50 70 72 74
	6	42 48 54 56 60 61 62 68 76 82 86 90
3	2	20 22 28 44 47 49
	3	34 36 38 40 44 46
	4	44 46 50 60 61 62 82 83
4	2	26 30 34 35 36 38 58
	3	42 46 48 50 52 56 60
6	2	69 75 80
	2.5	86 87 93 94

The rotor slots geometry is computed next. At first the current, I_{rb} , through the rotor bars and then the requested area, A_{rb} , for the rotor bar are calculated. The coefficient 0.95 accounts for the fact that a part of the stator current is the magnetization current:

$$I_{rb} = 0.95 \frac{\pi D_{si} I}{N_{rt}} \quad (15.18)$$

$$A_{rb} = \frac{I_{rb}}{J_r} \quad (15.19)$$

The rotor tooth is assumed to have parallel walls.

The radius, R_{r1} , of the circular upper part of rotor slot is

$$R_{r1} = \left(\frac{a}{\cos\left(\frac{\alpha_{rs}}{2}\right)} - \sqrt{\left(a \cdot \tan\left(\frac{\alpha_{rs}}{2}\right)\right)^2 - \left(\frac{o_{rs}}{2}\right)^2} \right) \tan\left(\frac{\alpha_{rs}}{2}\right) \quad (15.20)$$

where

$$a = D_{r0} - h_{r1} - \frac{\sqrt{D_{r0}^2 - o_{rs}^2}}{2} - \frac{w - rt}{2 \sin\left(\frac{\alpha_{rs}}{2}\right)} \quad (15.21)$$

If the required rotor bar area, A_{rb} , is larger than area of a circle with R_{r1} radius, then the rotor bare slots dimensions are computed as

$$w_{r1} = 2R_{r1} \cos\left(\frac{\alpha_{rs}}{2}\right) \quad (15.22)$$

$$h_{r2} = \sqrt{R_{r1}^2 - \left(\frac{o_{rs}}{2}\right)^2} \quad (15.23)$$

The required and the maximum possible area of the trapezoidal part of the rotor slot are now computed:

$$A_{r2} = A_{rb} - o_{rs}h_{r1} - \frac{1}{2} \left(\pi + \alpha_{rs} - 2a \sin \left(\frac{o_{rs}}{2R_{r1}} \right) + \sin(\alpha_{rs}) \right) R_{r1}^2 + \frac{1}{2} o_{r1}h_{r2} \quad (15.24)$$

$$A_{r2 \max} = \frac{w_{r1}^2}{4 \tan \left(\frac{\alpha_{rs}}{2} \right)} \quad (15.25)$$

A maximum available area smaller than the required area means a smaller diameter, or a smaller rotor current density or rotor tooth flux density. When the computation is done automatically using a computer code, it is necessary to solve this problem in the same way, and, in our example, the required area is reduced to the available area. This implies an increase in the rotor current density. A warning message will be prompted on the display.

The height of the trapezoidal part of the rotor slots, h_{r3} , is

$$h_{r3} = \frac{w_{r1}}{4 \tan \left(\frac{\alpha_{rs}}{2} \right)} \left(1 - \sqrt{1 - \frac{\frac{A_{r2}}{1-a} - a}{\frac{A_{r2 \max}}{1-a}}} \right) \quad (15.26)$$

where

$$a = \frac{\pi - \alpha_{rs} - \sin(\alpha_{rs})}{2 \left(\cos \left(\frac{\alpha_{rs}}{2} \right) \right)^2} \tan \left(\frac{\alpha_{rs}}{2} \right) \quad (15.27)$$

The other rotor slot dimensions are

$$w_{r2} = w_{r1} - 2h_{r3} \tan \left(\frac{\alpha_{rs}}{2} \right) \quad (15.28)$$

$$R_{r2} = \frac{w_{r2}}{2 \cos \left(\frac{\alpha_{rs}}{2} \right)} \quad (15.29)$$

$$h_{r4} = R_{r2} \left(1 - \sin \left(\frac{\alpha_{rs}}{2} \right) \right) \quad (15.30)$$

$$h_{rOA} = h_{r1} + h_{r2} + h_{r3} + h_{r4} \quad (15.31)$$

If the required rotor bar area is smaller than the area of a circle with R_{r1} radius, then the rotor bare slots become a circle with radius

$$R_{r1} = R_{r2} = \sqrt{\frac{A_{rb} - o_{rs}h_{r1}}{\pi}} \quad (15.32)$$

Now, the rotor inner diameter is computed:

$$D_{ri} = D_{r0} - 2(h_{rOA} + h_{ry}) \quad (15.33)$$

The rotor inner diameter is rounded to an integer value in millimeters.

15.1.3.2 Stator Slot Dimensions

The stator slot area, A_{Cu} , to host the bare coils, the slot area, A_{ss} , the slot useful height, h_{s1} , the total height, h_{sOA} , and the slot width, w_{s1} , are

$$A_{Cu} = \frac{\pi D_{si} I_l}{N_{ss} J_s} \quad (15.34)$$

$$A_{ss} = \frac{A_{Cu}}{k_{sf}} \quad (15.35)$$

$$h_{s1} = \frac{-w_{s2} + \sqrt{w_{s2}^2 + 4A_{ss} \tan\left(\frac{\alpha_{ss}}{2}\right)}}{2 \tan\left(\frac{\alpha_{ss}}{2}\right)} \quad (15.36)$$

$$h_{sOA} = h_{s1} + h_{s2} + h_{s3} + h_{s4} \quad (15.37)$$

$$w_{s1} = w_{s3} + 2 \left(h_{s1} + h_{s2} \tan\left(\frac{\alpha_{ss}}{2}\right) \right) \quad (15.38)$$

The stator outer diameter becomes

$$D_{s0} = D_{si} + 2(h_{sOA} + h_{sy}) \quad (15.39)$$

In the previous geometric computation we met different mathematical operations including square root extraction and subtraction, and, sometimes for a bad correlation of input parameters, it could be possible to obtain a negative or quite complex value, which does not quite make sense. If the computation steps are made one by one, it is possible to stop the computation and choose other initial variables immediately when such a useless value is observed. The same path could also be followed when the computation is done by a computer code but the computer code is simpler when the useless geometric values are verified in groups. At this stage, it is possible to verify if R_{r1} , h_{r2} , and h_{r3} are real and D_{ri} , $sTeethAlpha$, h_{s3} , w_{s3} , R_{r1} , h_{r2} , h_{r1} , R_{r2} , w_{r2} are larger than some minimum values. If these verifications fail, new variables are produced by genetic optimization algorithms and the previous step is repeated until all parameters pass the test. For the Hooke–Jeeves algorithm, the highest value is attributed to the objective function (all parameter computation is skipped). It is important to have compatible initial values of optimization variables for the Hooke–Jeeves algorithm; otherwise, the computation code result will be an error message. All stator and rotor geometric dimensions are computed after all checked values pass the test.

15.1.3.3 Winding End-Connection Length

The end-connection length of the stator winding, l_f , is computed considering the end coil as a semicircle with an average diameter equal to the coil average opening. The right parts of the coil end connection, l_{f1} , are added.

$$l_f = \frac{\pi}{2} k_y \tau_p \left(1 + \frac{h_{sOA}}{D_{si}} \right) + 2l_{f1} \quad (15.40)$$

The wound stator length, l_{ff} , is approximated

$$l_{ff} = l_c + l_{ff} + k_y + \tau_p \left(1 + \frac{h_{sOA}}{D_{si}} \right) \quad (15.41)$$

The rotor end-connection axial length computed considering its height, which is equal to the total height rotor slot of the, h_{rOA} , the ring current, I_{er} , and the current density in the rotor ring (in direct ratio with the rotor bar current density, J_r , with the ratio factor, k_{jr}).

$$I_{er} = \frac{I_{rb}}{2 \sin \left(\frac{p\pi}{2N_{rs}} \right)} \quad (15.42)$$

$$A_{Ring} = \frac{I_{er}}{k_{jr} J_r} \quad (15.43)$$

$$h_{Ring} \approx h_{rOA} \quad (15.44)$$

$$w_{Ring} = \frac{A_{Ring}}{h_{Ring}} \quad (15.45)$$

The inertia of the rotor without shaft is

$$J_{ir} = \frac{1}{8} \left((D_{ri}^2 + D_{rc}^2) m_{ry} + (D_{rc}^2 + D_{ro}^2) (m_{rt} + m_{rb}) + ((D_{ri} - 2h_{r1})^2 + (D_{ri} - 2h_{r1} - 2h_{Ring})^2) m_{ring} \right) \quad (15.46)$$

15.1.4 Induction Machine Parameters

The stator windings resistance is

$$R_{s0} = \frac{s_b q_1 p n_L (L_c + l_{soh})}{A_{sb} a_1^2} \rho_{Tw} \quad (15.47)$$

The rotor resistance is computed separately for the rotor bar and short-circuit ring and then it is reduced to the stator winding:

$$R_{rb} = \rho_{2Tw} \frac{l_{rc}}{A_{rslot}}; \quad \rho_{2Tw} - \text{rotor resistivity} \quad (15.48)$$

$$R_{Ring} = k_{rRing} \cdot \rho_{2Tw} \frac{\pi D_{mRing}}{A_{Ring} N_{rs}} \quad (15.49)$$

where

D_{mRing} is the average diameter of the short-circuit ring

k_{rRing} is the reduction factor of the rotor ring to the rotor bar

The expression of the rotor ring resistance was computed considering the equivalent losses.

$$k_{rRing} = \frac{1}{2 \sin \left(\frac{p_1 \pi}{N_{rs}} \right)} \quad (15.50)$$

The dimension of the cross area of the short-circuit ring is comparable with the rotor slot pitch. The short-circuit ring is rather a massive conductor and its real resistance is not equal to the computed resistance. In some books, this difference is included in the reduction factor, which is considered an empirical coefficient.

Finally, the rotor equivalent resistance is

$$R_r = (R_{rb} + R_{rRing}) \left(N_1 \frac{k_w}{k_{wr}} \right)^2 \frac{m}{N_{rs}} \quad (15.51)$$

Usually, the rotor bars are skewed in order to reduce the torque pulsations and noise. The rotor winding factor, f_{wr} , is computed from the skewing ratio, r_{sk} (rotor slot skewing divided by rotor slot pitch), given by the designer as an initial parameter:

$$f_{wr} = \frac{2\tau_p \sin \left(r_{sk} \frac{\pi\tau_{rs}}{2\tau_p} \right)}{r_{sk}\pi\tau_{rs}} \quad (15.52)$$

The magnetization inductance depends on the magnetization current. It is simple to compute the magnetization current versus the magnetization flux considering the magnetic curve of the magnetic core. At the beginning, one value (or several values for a curve) of the airgap flux density is considered. The flux density in the stator tooth, stator yoke, rotor tooth, and rotor yoke are computed using the magnetic tube circuit rule. The magnetic field density using the magnetization curve is computed by interpolation and then the magneto-motive force necessary to produce that field is

$$V_m = V_{mag} + V_{mst} + V_{msy} + V_{mrt} + V_{mry} \quad (15.53)$$

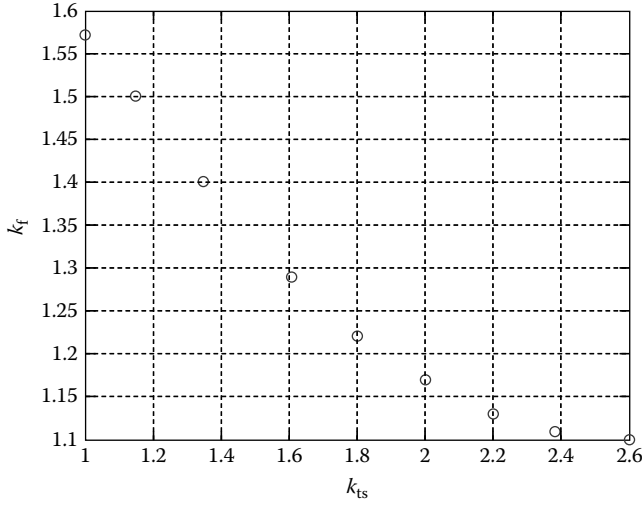
$$V_{mag} = \frac{B_{ag}}{\mu_0} k_C g \quad (15.54)$$

where k_C is the Carter factor and it is given as product of Carter factors k_{C1} and k_{C2} computed separately for the stator and the rotor as in Equation 14.43. The magneto-motive force for the stator and the rotor teeth and yokes are also computed in a similar way as for PMSM.

The polar flux, Φ , is given by the Equation 15.56 where k_f is the shape factor of the airgap flux density distribution. The shape factor is equal to $\pi/2$ for unsaturated core and it decreases as is shown in Figure 15.2 when the magnetic core is saturated. The shape factor, k_f , is shown versus stator tooth saturation factor, k_{ts} :

$$k_{ts} = 1 + \frac{V_{mst} + V_{mrt}}{V_{mag}} \quad (15.55)$$

$$\Phi = \frac{\tau_p}{k_f} l_c B_{ag} \quad (15.56)$$

**FIGURE 15.2**

The shape form factor of the flux density distribution vs. stator tooth saturation factor k_{ts} .

The phase airgap these linkage Ψ_m , magnetization current I_m , and inductance L_m are:

$$\Psi_m = N_1 k_w \Phi \quad (15.57)$$

$$I_m = \frac{\pi p_1 V_m}{m \cdot N_1 f_w} \quad (15.58)$$

$$L_m = \frac{\Psi_m}{I_m} \quad (15.59)$$

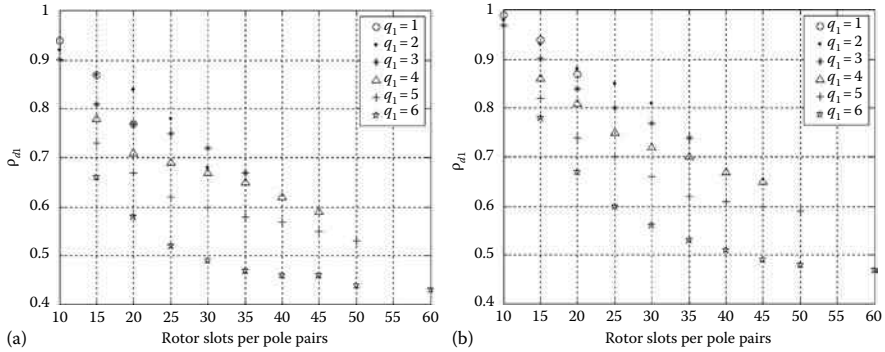
It is also possible to compute nonsaturated values of the cyclic magnetization inductance, L_{m0} :

$$L_{m0} = 2\mu_0 m \frac{(N_1 k_w)^2}{\pi^2} \frac{\tau_p l_c}{p_1 k_C g} \quad (15.60)$$

The stator leakage inductance is computed considering the slot, end connection, and differential permeances. If the stator slots have the same shape form and the same end-connection distributions as for the PMSM stator, then the same expressions are used for stator slots permeance (Equations 14.63 through 14.69), respectively Equation 14.70, for end connection.

The stator differential permeance is [1]:

$$\lambda_{sd} = 0.3 \rho_{d1} k_{01} \sigma_{d1} (q_1 k_w)^2 \frac{\tau_{ss}}{k_C g} \quad (15.61)$$

**FIGURE 15.3**

The ρ_{d1} factor. (a) No rotor bar slant and (b) rotor bar slant equal with rotor slot pitch.

where

$$k_{01} = 1 - 0.033 \frac{o_{ss}^2}{g\tau_{ss}} \quad (15.62)$$

$$\sigma_{d1} = \left(\frac{\sin\left(\frac{\pi}{6q_1}\right)}{\sin\left(y_c \frac{\pi}{2}\right)} \right)^2 \sum_{u=6k \pm 1, k \geq 1} \left(\frac{\sin\left(v y_c \frac{\pi}{2}\right)}{\sin\left(v \frac{\pi}{6q_1}\right)} \right)^2 \quad (15.63)$$

And ρ_{d1} is given in Figure 15.3a when the rotor bars slant is zero and in Figure 15.3b, when the rotor bar slant is equal to the rotor slot pitch.

Finally, the stator leakage inductance, $L_{s\sigma}$, is

$$L_{s\sigma} = 2\mu_0 \frac{N_1^2}{p_1 q_1} (\lambda_{ss} + \lambda_{sd} + \lambda_{s0}) l_c \quad (15.64)$$

The rotor slot permeance for the current rotor slot shape, Figure 15.1, is

$$\lambda_{rs} = \frac{\pi}{6} + \frac{2h_{r3}}{3(w_{r1} + w_{r2})} + \frac{h_{r1}}{o_{rs}} \quad (15.65)$$

The rotor end ring permeance is

$$\lambda_{r0} = k_{r\text{Ring}}^2 \cdot \frac{D_{w\text{Ring}}}{l_{rc} \cdot N_{rs}} \cdot \log \left(4.7 \frac{D_{w\text{Ring}}}{h_{\text{Ring}} + 2w_{\text{Ring}}} \right) \quad (15.66)$$

The rotor differential permeance is [1]

$$\lambda_{rd} = 0.3k_{02} \cdot \tau_{d2} \cdot \left(\frac{N_{rs}}{6p_1} \cdot f_{wr} \right)^2 \cdot \frac{\tau_{rs}}{k_c \cdot g} \quad (15.67)$$

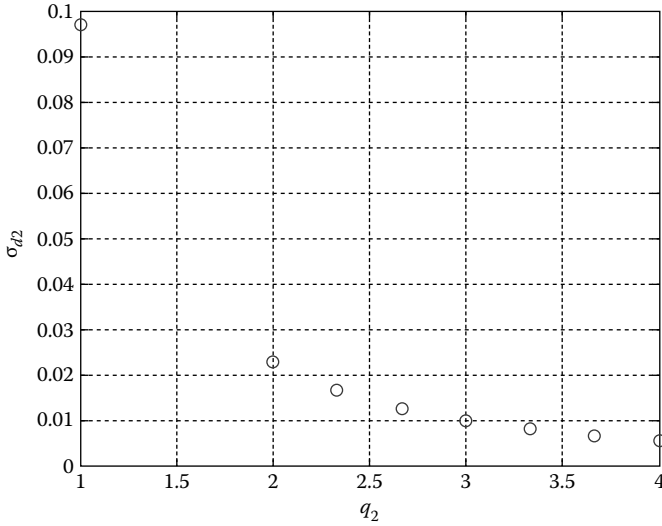


FIGURE 15.4
The σ_{d2} factor.

where

$$k_{02} = 1 - 0.033 \cdot \frac{\sigma_{rs}^2}{g \cdot \tau_{rs}} \quad (15.68)$$

The σ_{d2} coefficient is given in Figure 15.4 versus q_2 , the average number of slots per pole and stator phase in the rotor:

$$q_2 = \frac{N_{rs}}{6p_1} \quad (15.69)$$

Finally, the rotor leakage inductance is

$$L_{r\sigma} = 4\mu_0 m \frac{(N_1 \cdot k_w)^2}{N_{rs}} \cdot (\lambda_{rs} + \lambda_{rd} + \lambda_{r0}) l_{rc} \quad (15.70)$$

The machine performance is computed using the steady-state circuit model (see Section 5.26).

15.2 Induction Motor Optimal Design Using Genetic Algorithms

After the rather complete model of induction machine was introduced earlier, an exercise design of 3-phase two-pole induction motor with short-circuit rotor at rated power 22 kW and 50 Hz base frequency is presented

TABLE 15.2
Optimization Variables

Optimization Variable	Minimum Values	Maximum Values	GA	GA1	HG	Units
J_l	12	40	28	24.2	27.31	kA/m
B_{agsp}	0.4	0.85	0.63	0.64	0.63	T
B_{st}	1.2	2.1	2.46	1.66	2.1	T
B_{sy}	1	2	1.74	1.62	1.7	T
B_{rt}	1.2	2.1	2.34	2.42	1.75	T
B_{ry}	1	2	1.84	1.88	1.56	T
J_s	2.5	8	2.8	3.2	3.52	A/mm ²
J_r	2	7.5	2.2	2	3.62	A/mm ²
sh_4	0.3	2	0.8	2.1	1.1	mm
s_{os}	0.5	2	1.4	0.8	1.2	mm
rh_1	0.3	2	0.5	0.9	1.2	mm
s_{or}	0.3	2	1.2	0.3	1.1	mm
Lcpertau	0.5	2	0.9	1	1	
q_1	2	6	5	5	4	
cSpan	0.66	1	0.73	0.8	0.83	
rSlots			18	18	18	

here. The total motor cost is the objective function. The maximum and the minimum of optimization variables are presented in Table 15.2. The technological constraints used for this exercise design could be found in the input file (im1.m) of the induction machine optimal design MATLAB code, which is stored on the CD-ROM attached to the book, in the “IMdesign” folder. Table 15.2 also contains the final values of optimization design exercise for three cases: GA—genetic algorithm with 100 members on the population and 200 generations evolution; GA1—genetic algorithm with 30 members of the population and 50 generations evolution; and the Hooke–Jeeves algorithm (HG). The optimization objective cost function is the motor life cost, which includes the initial active material cost and the energy loss cost.

The main optimization variables, specific electric load, J_l , and airgap flux density, B_{agsp} , have similar values for the three compared cases. For GA1, the specific electric load is smaller but 50 generations is too small a number to find the optimum motor. The optimization variable code used in genetic algorithms allow larger values than maximum bounds as can be observed for rotor and stator yoke flux densities. The specific flux densities in the stator and rotor teeth seem to be too large for the usual magnetic core like “M19.” In fact, the real flux density is smaller because, at such large saturations, the shape form distribution of the airgap flux density decreases from the ideal value 1.57 up to 1.1 for high saturation. The same polar flux produced for ideal (sinusoidal) flux distribution with a peak of 2.5 T flux density in the teeth needs less than 1.8 T in the teeth. Every generation is characterized by the average, the minimum, and the maximum values of the cost

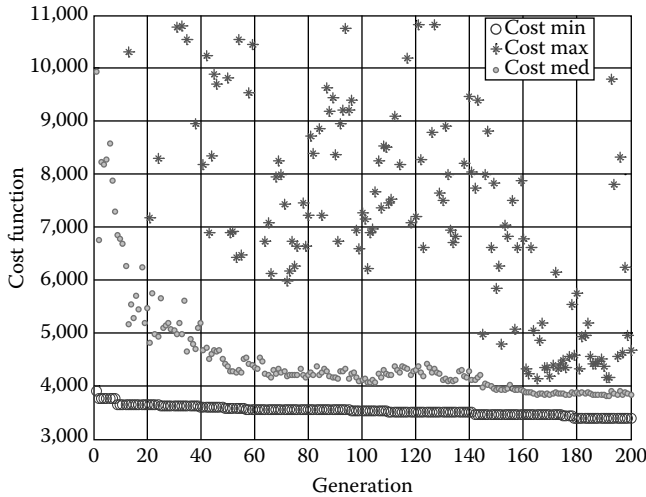


FIGURE 15.5

Minimum, maximum, and average cost function evolutions.

function. Their evolution is shown in Figure 15.5. The minimum value, which belongs to the best machine from the population, decreases monotonically. The average value of the cost function also tends to decrease.

The evolution of representative optimization variables of the best member along the 200 generations is shown in Figures 15.6 through 15.11. The linear electric load and the current density in the stator and rotor have several discrete values but we cannot observe any such tendency when the performance of the best member is improved, Figure 15.6. Moreover, for the

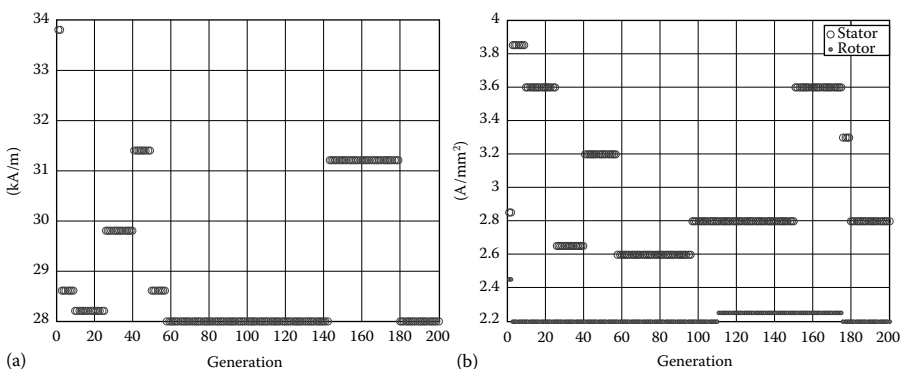


FIGURE 15.6

(a) Linear electric load and (b) current density evolutions.

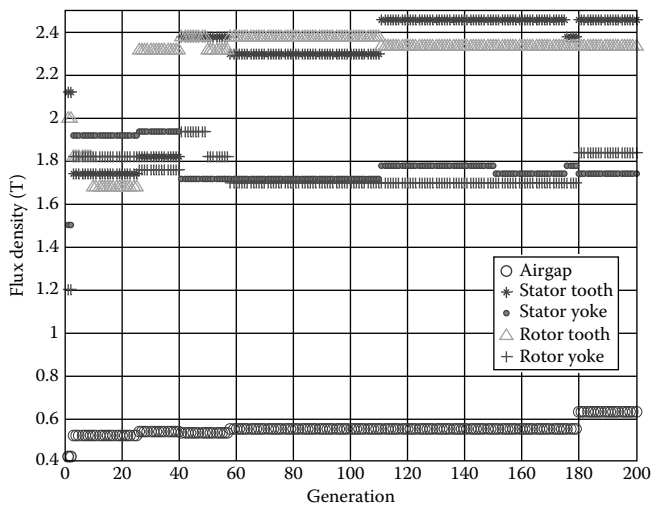


FIGURE 15.7
Airgap and magnetic core flux density evolutions.

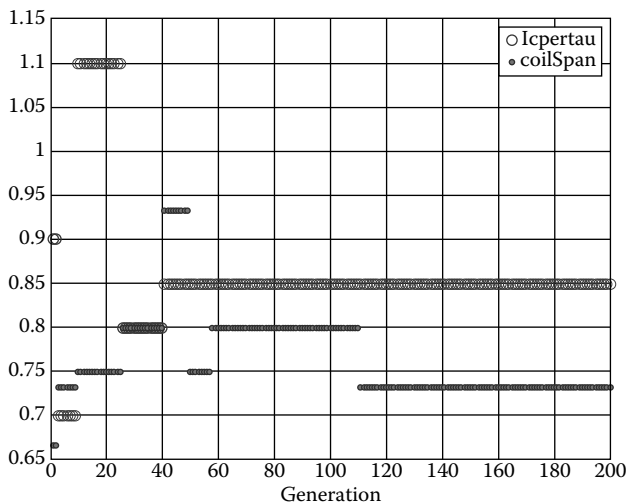
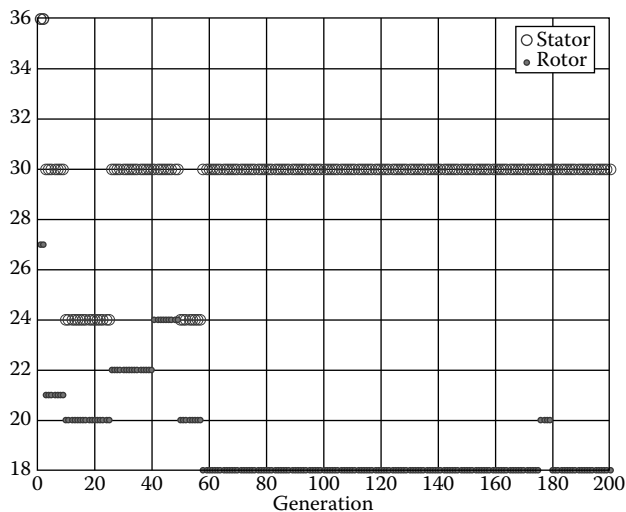


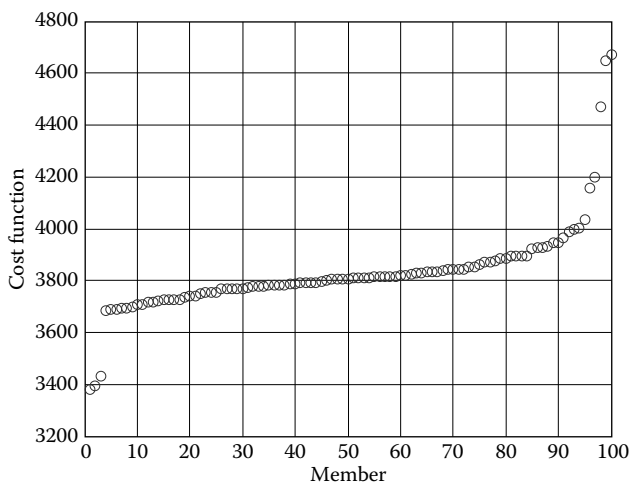
FIGURE 15.8
Dimensions of ratio evolutions of the best-candidate solutions: core length per pole pitch ($I_{cpertau}$) and coil span per coil pitch ($coilSpan$).

rotor current only three discrete values appear but the first one shows up only at the first generation.

The airgap flux density also shows discrete values but it increases slowly from the first generation to the last generation, Figure 15.7. There seems to be

**FIGURE 15.9**

Number of stator and rotor slots evolutions.

**FIGURE 15.10**

The cost function over the last generation.

a good correlation between the decreasing objective function and the increasing airgap flux density. The flux density in the stator and the rotor tooth also increase when the best-candidate solution improves.

The ratio between core length and pole pitch, respectively, the coil span and polar pitch, shows a random variation, with several discrete values,

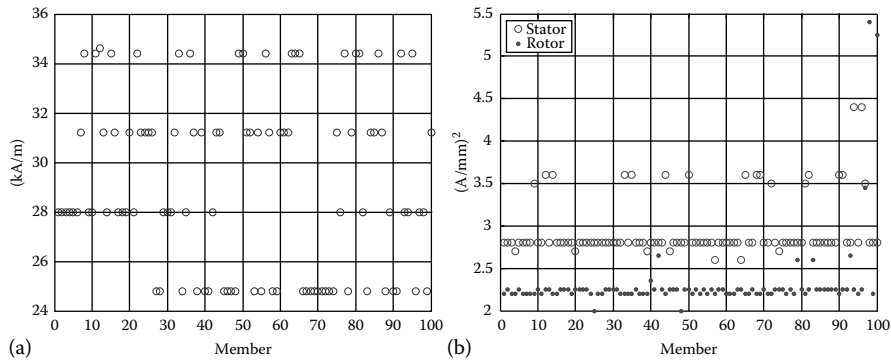


FIGURE 15.11
(a) Specific electric load and (b) current density over the last generation.

Figure 15.8, and, then, for the last generation, they remain unchanged. The numbers of stator and rotor slots, Figure 15.9, also have only a few discrete values and they remain unchanged for the last generations.

The cost function for the members of the last generation and the optimization variable distribution could show the evolution potential of the genetic algorithm if it would be continuous. In our case, we can observe in Figure 15.10 that the cost function still has a large variation. The members were sorted in the order of increasing cost function. We can observe that there is no correlation between the specific electric load and the cost function, Figure 15.11. Only several discrete values of the specific electric load were retained in the last generation, but their distribution still covers a large region. The stator current density distribution contains only a few values and their distribution is not correlated with the cost function. They are placed at the lowest limit of the current density. The rotor current, though, is an exception; for the three members, the rotor current density is larger but the cost function also has larger values. The airgap flux density shows a correlation with the cost function for the last generation, Figure 15.12. The largest values correspond to small values of the cost function. There are only a few discrete values of the airgap flux density and the cover range is small. If the global minimum of the objective function is placed at a larger value of the airgap flux density, then further evolution for this member could be very slow because only a good mutation could produce larger airgap flux. We can see also that a large teeth flux density could produce a better objective function.

The ratio between the core length and the pole pitch, and the ratio between the coil span and the pole pitch are reduced to a few values and their distribution seems to be uncorrelated with the cost function, Figure 15.13. For the number of stator slots, only two from five possible values were retained

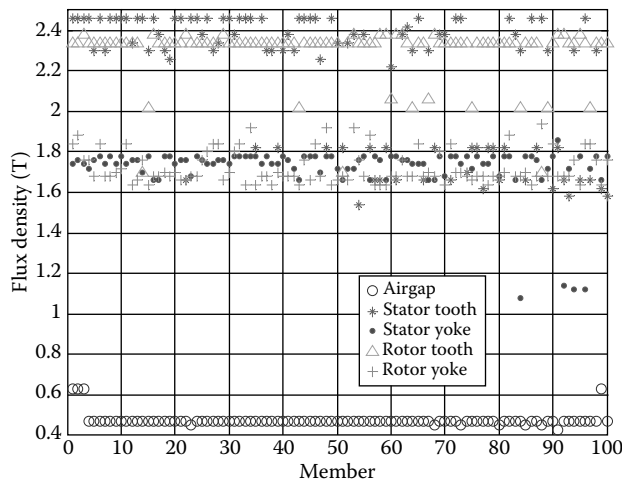


FIGURE 15.12
Flux densities over the last generation.

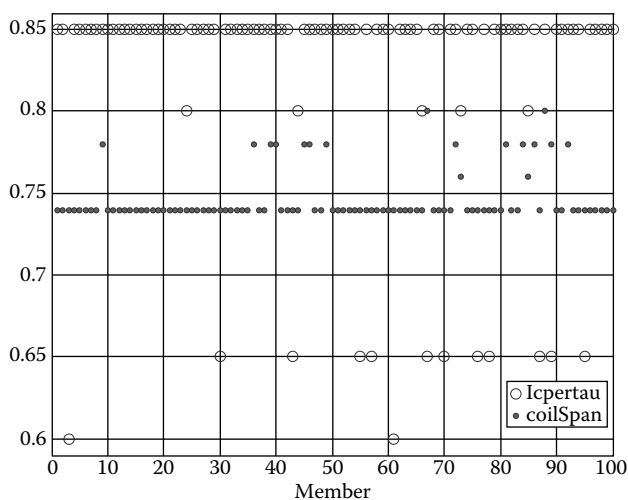


FIGURE 15.13
Dimension ratios over the last generation.

as is shown in Figure 15.14. These are combined with only three values for the rotor slots number. There is no visible correlation between the number of the slots and the cost function. However, we can conclude that 24 or 30 slots in the stator are better than 18, which probably is too small a number, and 36 which probably is too large a number, because these have no selection in the last generation, which has better performance than in the first generation.

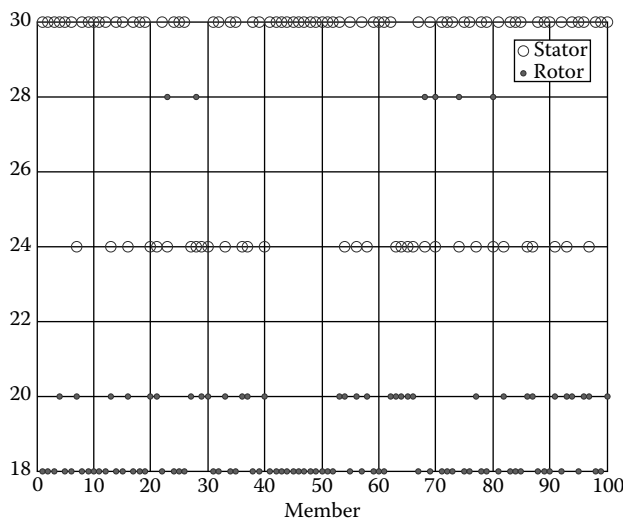


FIGURE 15.14
Stator and rotor slot numbers over the last generation.

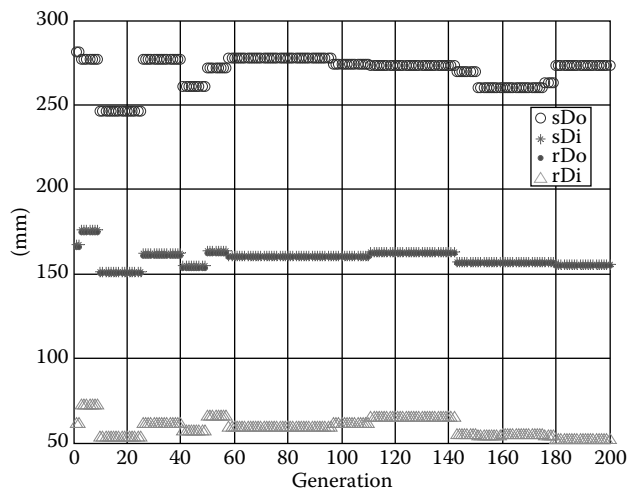
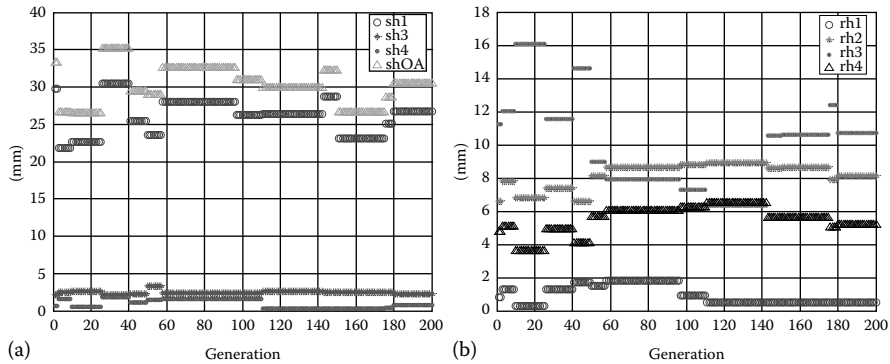
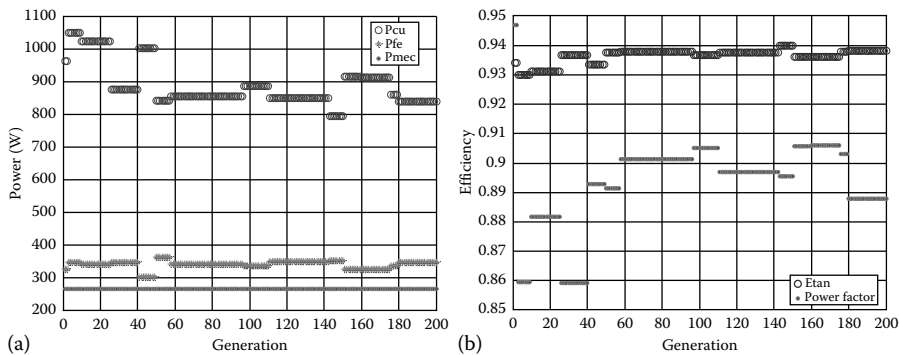


FIGURE 15.15
Rotor and stator diameters evolution.

The rotor and stator diameter evolution is shown in Figure 15.15. There are several discrete values that cover a small variation domain. The stator and rotor slots dimension variation is shown in Figure 15.16. The variation domain of the slot height is moderate compared with its values.


FIGURE 15.16

(a) Stator and (b) rotor slot dimensions evolutions.


FIGURE 15.17

(a) Power losses power factor and (b) efficiency evolution of the best member.

The power losses and efficiency evolutions are shown in Figure 15.17. The copper losses show a tendency to decrease, while the motor efficiency shows a tendency to increase. The variation is discrete and not monotonous. The power factor evolution is presented in Figure 15.17. This also shows a non-monotonic tendency to increase.

The motor and its stator and rotor weights evolution are shown in Figure 15.18. The weight of the windings and magnetic core components evolution are shown in Figure 15.19. We cannot observe any direct correlation between the weight evolution and the cost function evolution, which decreases monotonically, as shown in Figure 15.20.

The components of the cost function such as copper, laminations, cage, and undesired loss energy are also shown in Figure 15.20. The initial cost

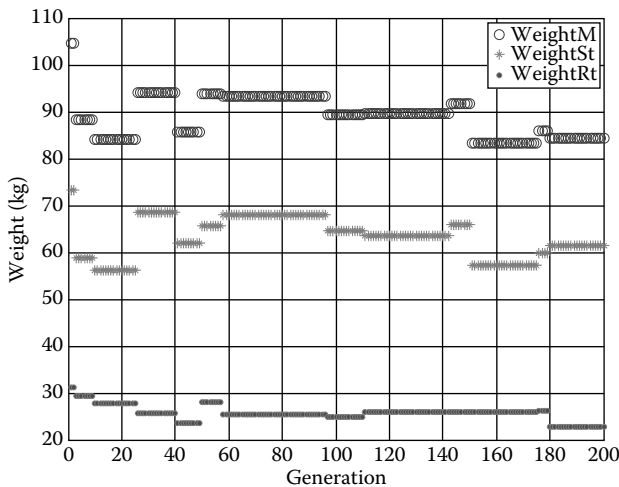


FIGURE 15.18
Motor weight evolution.

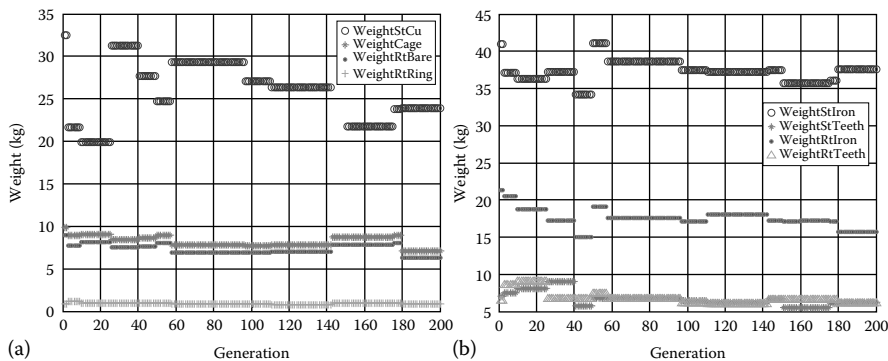
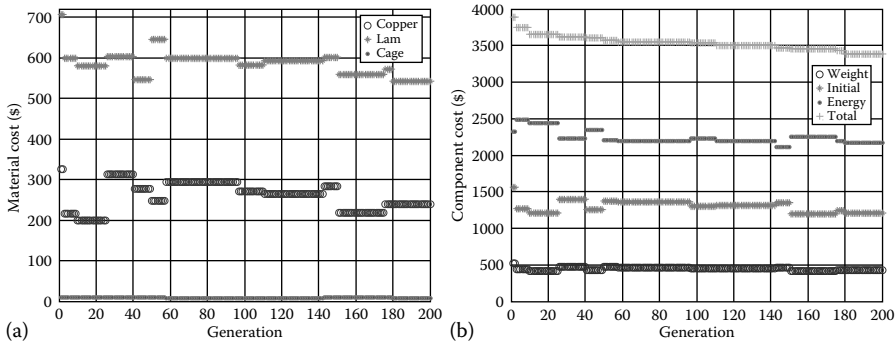


FIGURE 15.19
Components of motor weight evolution (GA).

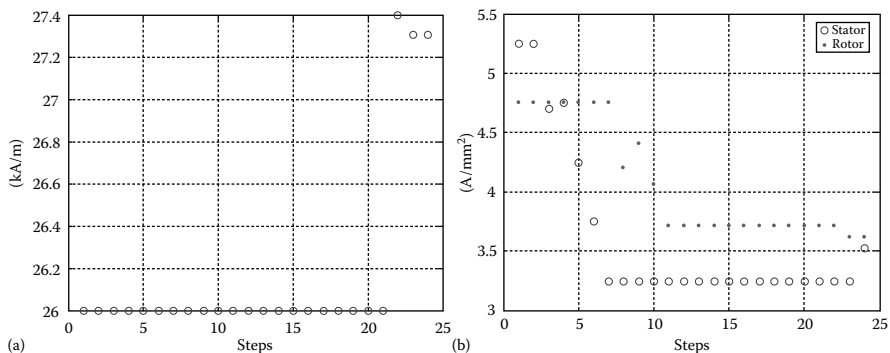
contains the cost of active materials such as copper, lamination, and cage as the weight cost. The total cost (objective function) contains the initial cost and the energy loss cost. It may also contain a penalty cost for an over-temperature larger than the maximum admissible, and a starting current penalty cost if the latter is larger than the maximum allowed (in our case, six times larger than the rated current). Note that no penalty cost is present in Figure 15.20, which shows the best member from every generation.

**FIGURE 15.20**

Objective function and their component evolution (GA).

15.3 Induction Motor Optimal Design Using Hooke–Jeeves Algorithm

The same optimization variable vector and objective function, total cost C_E , as that presented for genetic algorithm method is also used to implement the Hooke–Jeeves optimization algorithm. The specific electric load has a small step variation during the optimization process, Figure 15.21. The final value is close to the final value obtained from the genetic algorithm method. It can also be seen from this figure that stator and rotor current densities decrease.

**FIGURE 15.21**

Specification of (a) electric load and (b) current density evolution (HJ).

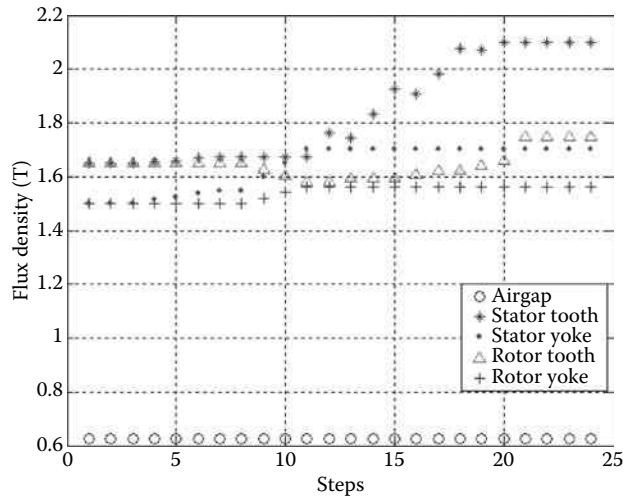


FIGURE 15.22
Flux density evolution (HJ).

The airgap flux density remains constant during the optimization process, Figure 15.22, but its initial value is around the optimum value from the genetic algorithm optimization. The stator tooth flux increases toward its maximum value. The other core flux densities also have a tendency to increase, but they do not reach the upper bounds.

The ratio between the core length and the pole pitch decreases by about 25%, while the coil span ratio remains constant, Figure 15.23. The stator slots number remains constant at 24 slots, Figure 15.24, the same as that which appears in the last generation of the genetic algorithm optimization. The number of rotor slots decreases at the first optimization step from 34 to 18 slots, which was also found as optimum with the genetic algorithm.

The rotor and stator diameter evolutions are shown in Figure 15.25. The stator inner diameter, sDi , and the rotor outer diameter, rDo , increase a little, while the stator outer diameter, sDo , increases a little more. Therefore, the stator slot height also increases, Figure 15.26.

The copper losses decrease while the iron losses increase a little, so the efficiency, η_{tan} , increases, Figure 15.27. The power factor shown in the Figure 15.27 also decreases to acceptable values.

The motor weight increases by about 10 kg due to the increase in stator weight, Figure 15.28. The increase in stator weight is produced especially by additional stator windings (copper), Figure 15.29.

The objective function decreases consistently, Figure 15.30. The first decreasing step is produced by the elimination of penalty cost. The current density and the machine diameter are not changed between the first and the

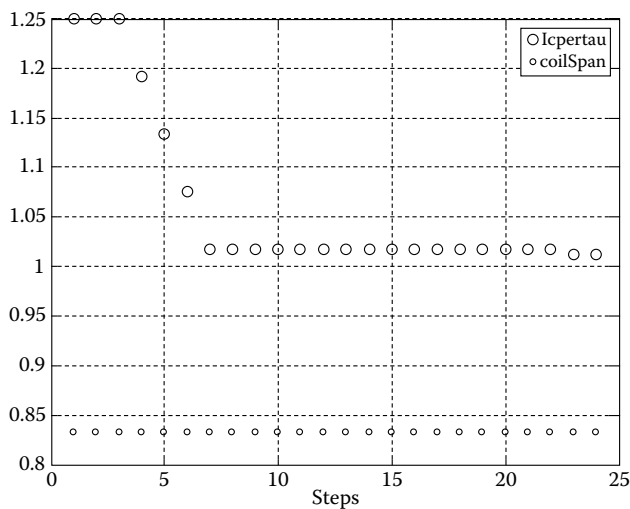


FIGURE 15.23
Dimension ratios evolution (HJ).

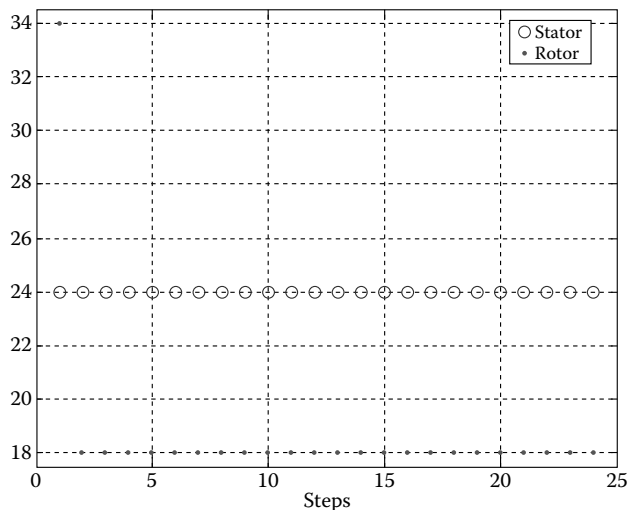


FIGURE 15.24
Stator and rotor slots evolution (HJ).

second solution, and the stator teeth top, $hs4$, increases significantly; thus we can conclude that the start current penalty has been eliminated. Thereby the cost function continues to decrease due to efficiency improvement despite the initial increase.

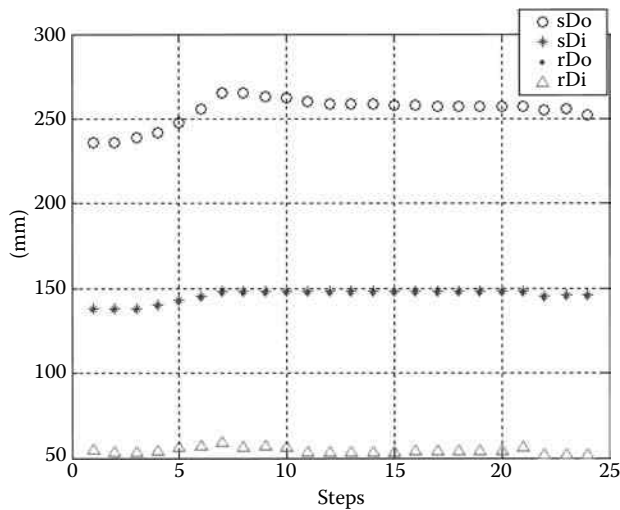


FIGURE 15.25
Rotor and stator diameters evolution (HJ).

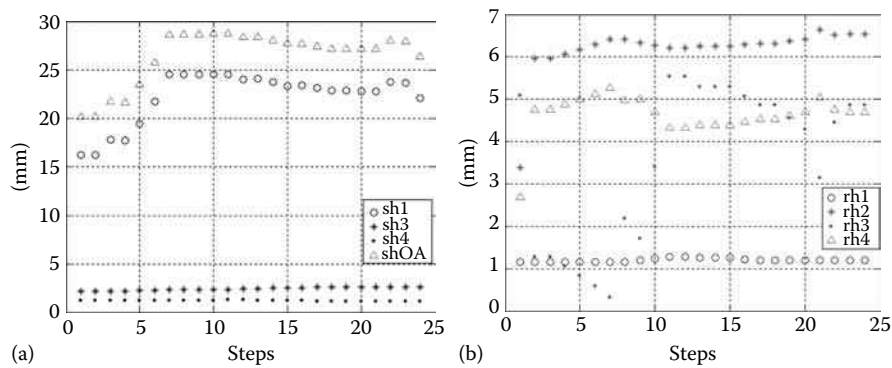


FIGURE 15.26
(a) Stator and (b) rotor slots dimension evolutions (HJ).

15.4 Machine Performance

The machine performance was computed using the circuit model but the magnetization inductance was computed considering magnetic saturation. The performance characteristics of the motor designed with genetic algorithm, GA, are compared with the motor performance designed by the Hooke–Jeeves optimal algorithm, HG.

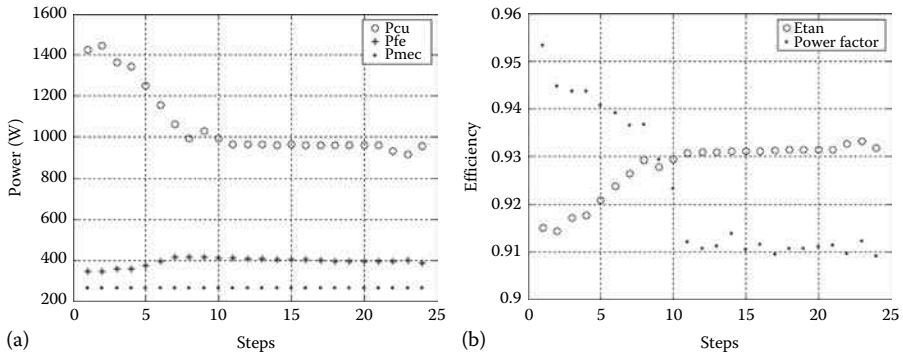


FIGURE 15.27
Losses and efficiency evolution (HJ).

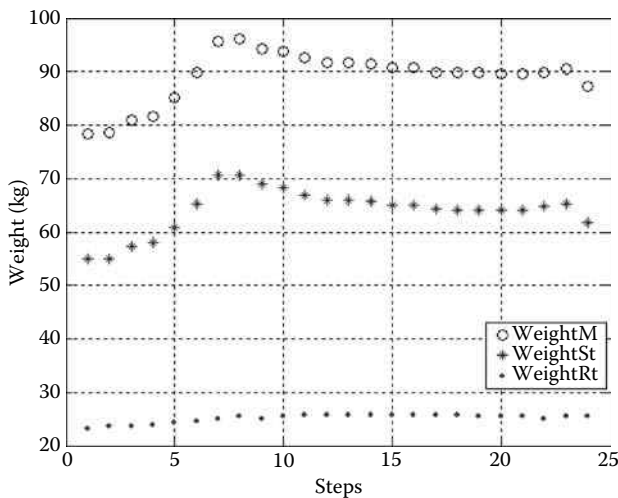


FIGURE 15.28
Motor weight evolution (HJ).

The linkage flux versus the magnetization current is presented in Figure 15.31 for both motors. It can be seen that at small currents (5 A), the linkage flux of GA motors is a little larger than HG machines while at medium and large currents, the flux of HG motor is larger than for GA motors. Therefore, HG motor is more saturated than the GA motor. The same conclusion can also be drawn from magnetization inductance versus magnetization current, Figure 15.32, and from airgap flux density versus magnetization current, Figure 15.33. The saturation factors versus the magnetization current are shown in Figure 15.34. The stator yoke has the largest

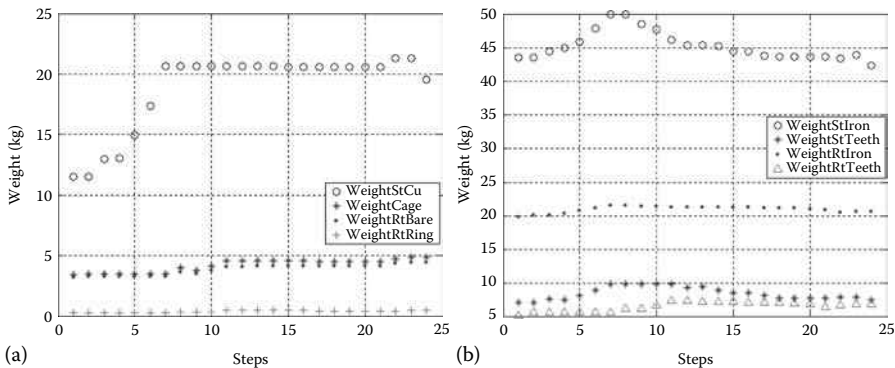


FIGURE 15.29
Components of motor weight evolution (HJ).

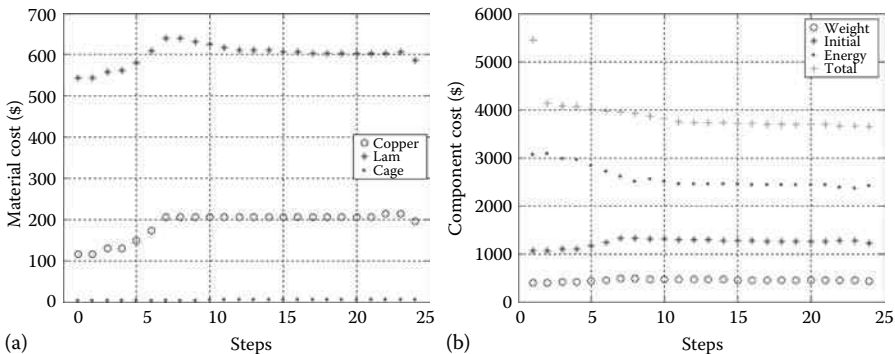


FIGURE 15.30
The objective (cost) function and their components evolutions.

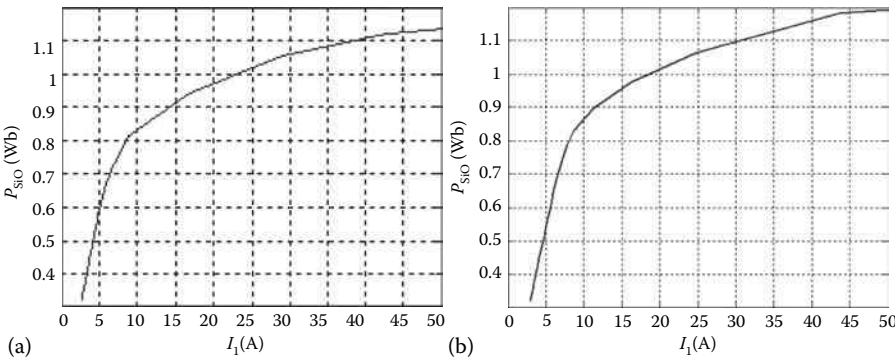


FIGURE 15.31
Magnetization linkage flux vs. current: GA (left) and HG (right).

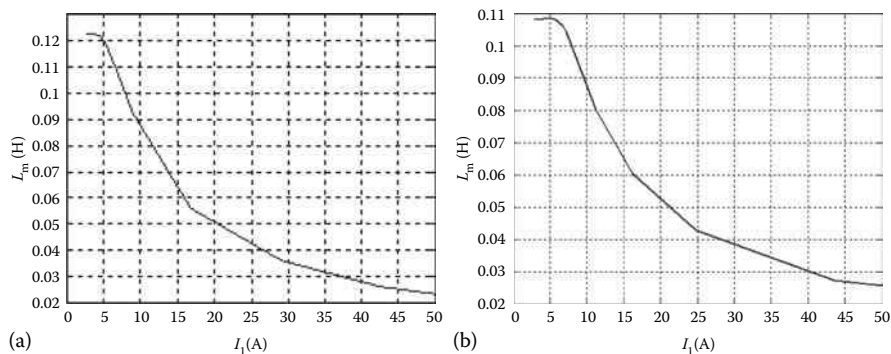


FIGURE 15.32
Magnetization inductance: GA (left) and HG (right).

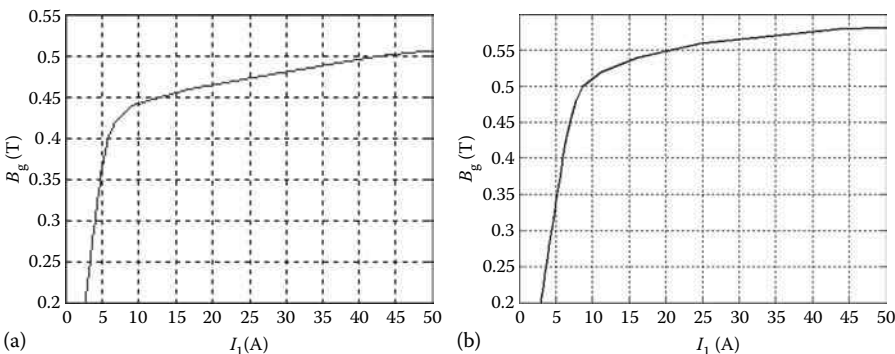


FIGURE 15.33
Airgap flux density vs. magnetization current: GA (left) and HG (right).

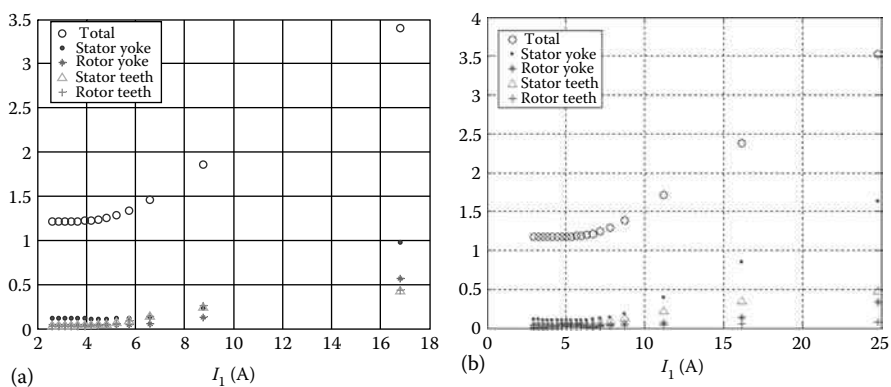


FIGURE 15.34
Saturation factors vs. magnetization current: GA (left) and HG (right).

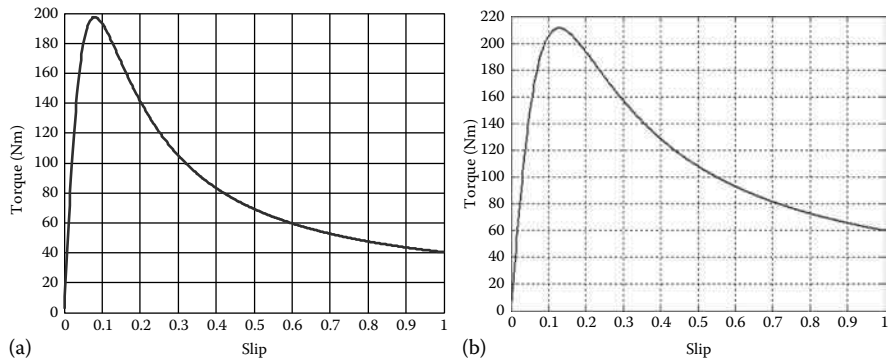


FIGURE 15.35
Torque vs. rotor slip: GA (left) and HG (right).

contribution to the saturation factor despite the large specific flux density in the stator and the rotor teeth. This is the same as in two-poles machines that have a long magnetic line path through the stator yoke.

The torque versus slip for both machines is shown in Figure 15.35. The GA motor has a smaller peak torque and a smaller critical slip than the HG machine. Also, the starting torque of the GA motor is smaller than that of the HG motor. The smaller rotor resistance of the GA motor (Table 15.3) explains these features.

The starting current of the HG motor is a little smaller than for a GA motor, Figure 15.36.

The acceptable ratio of the starting current to the rated current was obtained by imposing a penalty cost for the motors with the starting current larger than allowed for. Without this penalty, the optimization algorithm produces motors with a starting current up to 12 times larger than rated currents. Such kinds of motors are not suitable to be directly connected to the grid. However, they have a large peak torque, which implies a large speed range at constant power, in variable speed drive.

The input and output powers versus the slip are shown in Figure 15.37, while the efficiency and power factors versus the slip are shown in Figure 15.38.

The current versus torque for both motors is shown in Figure 15.39, while the efficiency and power factors versus torque are shown in Figure 15.40.

The main dimensions and parameters of the designed motors are presented in Table 15.3.

The optimization steps of the Hooke–Jeeves algorithm are smaller and the required computation time is also much smaller as is evident in Table 15.3. It is only 5.73 s for the Hooke–Jeeves algorithm, while for the genetic algorithm with a population of 100 members and 200 generations, the simulation time reaches 286 s. The Hooke–Jeeves algorithm falls into a local minimum

TABLE 15.3

Parameters of Optimized Induction Motors

Parameter	GA1	GA2	HG	Units	Comments
D_{so}	273	269	252	mm	Stator outer diameter
D_{si}	156	152.2	146	mm	Stator inner diameter
D_{ri}	52	52	52	mm	Rotor inner diameter
L_c	192.2	223.5	244.5	mm	Stator core length
g	0.65	0.65	0.65	mm	Airgap
h_{sOA}	30.5	28.2	26.4	mm	Stator slot height
h_{s1}	26.7	23.5	22.1	mm	Stator slot main height
h_{s3}	2.3	2	2.6	mm	Stator slot wedge height
h_{sy}	28.1	30.2	26.6	mm	Stator yoke height
w_{st}	4.2	6.1	5.7	mm	Stator teeth width
$N1$	60	50	48		Number of turns per phase
h_{rOA}	24.5	23.7	17.3	mm	Rotor slot height
w_{r1}	16.1	15.7	12.9	mm	Rotor slot width
w_{r2}	12.3	12.3	11.2	mm	Rotor slot width
h_{ry}	26.7	25.9	29.2	mm	Rotor yoke height
b_{rt}	7.5	7.2	9.3	mm	Rotor teeth width
WeightStCu	23.868	19.234	19.536	kg	Stator winding weight
WeightIronUsed	108.391	122.337	117.475	kg	Processed iron weight
WeightStIron	37.58	47.671	42.308	kg	Stator iron weight
WeightRtIron	9600.13	10.722	13.76	kg	Rotor iron weight
WeightCage	7.161	7.58	4.861	kg	Rotor cage weight
WeightM	84.31	92.07	87.261	kg	Motor weight (active material)
I_{1n}	39.49	40.24	38.58	A	Rated current (rms)
V_{fn}	220	220	220	V	Phase rated voltage (rms)
P_{cu}	837.56	845.16	956.75	W	Copper losses
P_{fe}	346.54	382.59	386.81	W	Iron losses
P_{mec}	264	264	264	W	Mechanical losses (1.2% of P_n)
E_{tan}	93.82	93.65	93.19	%	Rated efficiency
$\cos\phi_{in}$	0.89	0.87	0.91		Rated power factor
sR	0.113	0.116	0.112	Ω	Stator dc resistance
rR	0.08	0.071	0.12	Ω	Rotor dc resistance
RFe	372	332	340	Ω	Equivalent iron losses resistance
$Lm0_{sat}$	53.17	54.959	62.07	mH	Magnetization inductance at rated voltage

(continued)

TABLE 15.3 (continued)
Parameters of Optimized Induction Motors

Parameter	GA1	GA2	HG	Units	Comments
Lsl	1.632	1.774	1.449	mH	Stator leakage inductance
Lrl	1.544	2.287	1.537	mH	Rotor leakage inductance
rJ	0.075	0.079	0.074	kgm ²	Rotor inertia
Cu _c	238.677	192.338	195.363	USD	Stator winding cost
lam _c	541.957	611.685	587.374	USD	Iron cost
cage _c	7.161	7.58	4.861	USD	Processed iron cost
pmw _c	421.552	460.352	436.304	USD	Cage cost
i _{cost}	1209.348	1271.955	1223.903	USD	Weight penalty cost
energy _c	2172.148	2237.62	2411.335	USD	Initial cost
t _{cost}	3381.496	3509.575	3635.238	USD	Total cost (objective function)
sim _{time}	286.047	23.343	5.735	s	Simulation time (Intel dual core)

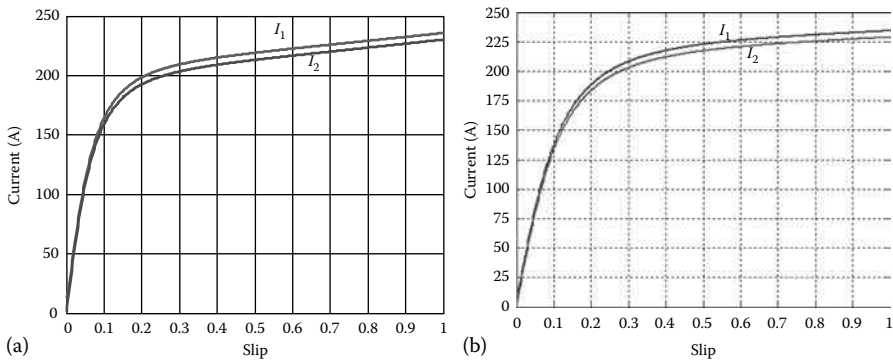


FIGURE 15.36
Stator and rotor current: GA (left) and HG (right).

with the objective function equal by \$3635 while the genetic algorithm finds an objective function of \$3381, which is about 5% smaller. The initial cost seems to be similar for both HG and GA. Using a small number of population members (30) and a small number of generations, the optimization time with GA is reduced but the objective function is comparable with the Hooke–Jeeves method. Starting HJ method with a few random initial variable vectors should increase probability for a global optimum, with still notably lower computation time than for GA.

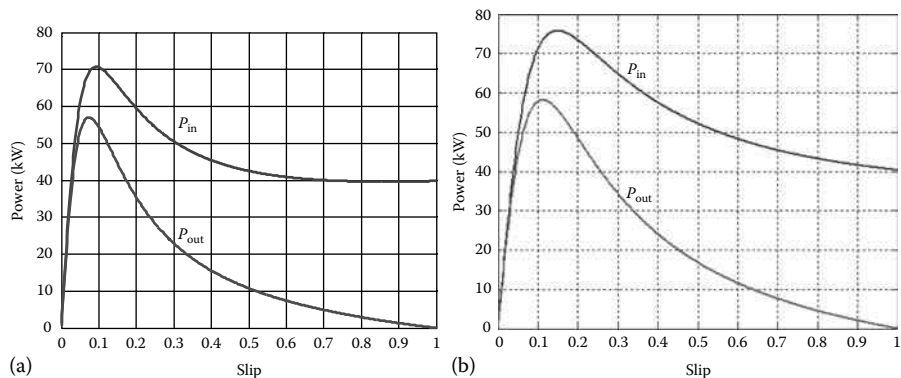


FIGURE 15.37
Input and output powers: GA (left) and HG (right).

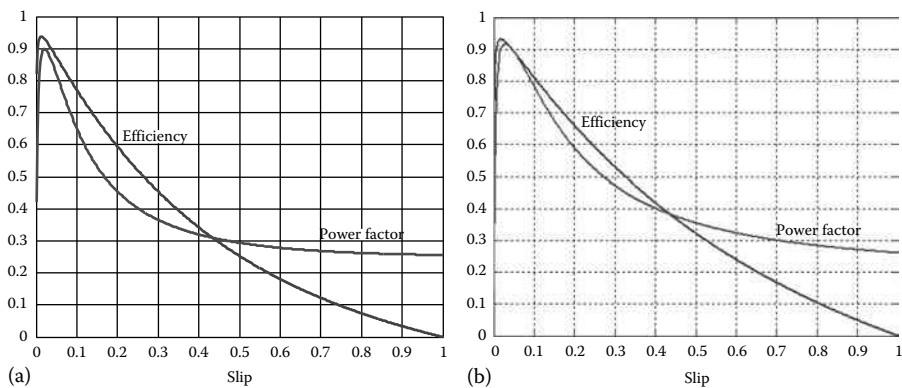


FIGURE 15.38
Efficiency and power factor vs. slip: GA (left) and HG (right).

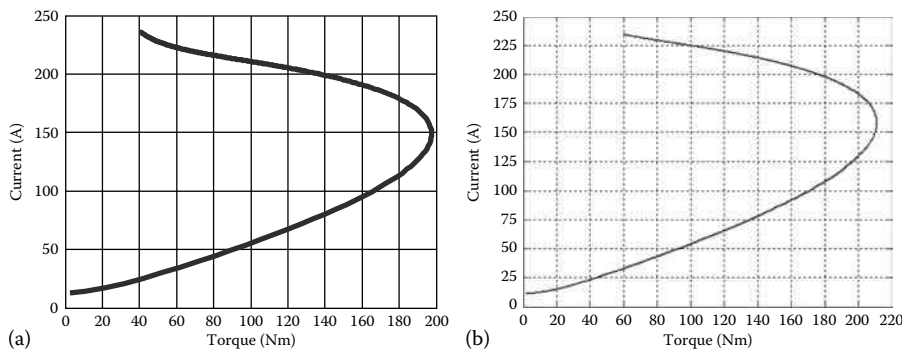
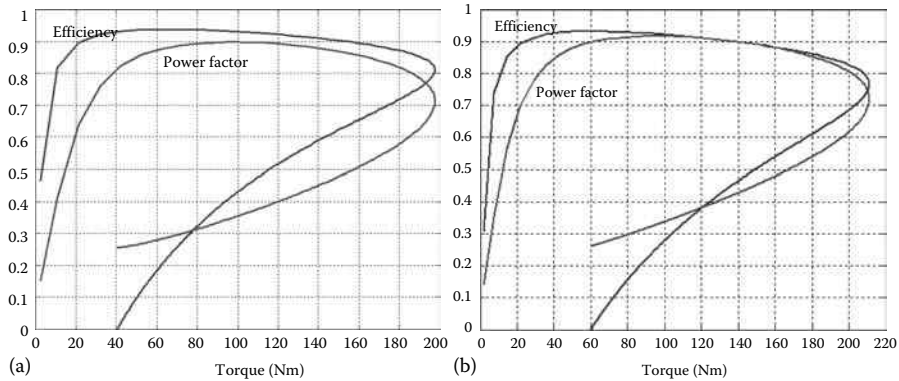


FIGURE 15.39
Current vs. torque: GA (left) and HG (right).

**FIGURE 15.40**

Efficiency and power factor vs. torque: GA (left) and HG (right).

15.5 Conclusion

To summarize, this chapter has proceeded in the following manner:

- A rather complete analytical model of IM, including magnetic saturation, was provided; for alternative IM models see Refs. [4,6]
- The analytical model was then incorporated into an optimal design computer code based on GA, and, respectively, the Hooke-Jeeves algorithm
- For the total motor cost as the cost function, the same machine was optimally designed and the results were thoroughly illustrated and discussed
- Both optimal design algorithms produced good results but the HG algorithm required 50 times less time for computation
- To avoid falling into local optimum, the HG algorithm has to be started from a few (5–8) initial sets of variables
- After optimal design, the use of FEM would be in place for trial verifications
- Integration of the optimal design codes into a complete design code, as house based on GA and HG, is yet to follow
- For more on IM optimization design see *IEEE Trans.* vol. EC & MAG.

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Index

A

AC distributed windings

cage rotor windings

airgap flux density, 223–224

bar and ring resistances, 223

chording and distribution factor, 224

end ring and cage-bar currents, 223

factor components, 223–224

rotor cage geometry, 222–223

skewing factor, 223

total winding factor, 224–225

MMF space harmonics, integer q

chording factor, 212

linear current density, 213

mmf distribution, 212

peak value, 213

pole pitch and slot copper area, 214

winding distribution factor, 212

pole count changing ac 3-phase winding, 220–221

practical one-layer ac 3-phase winding

bar (single turn) coils, 216

concentric coil phase belt, 216–217

identical (chain) coil phase, 216–217

layout, lap and wave bar coil, 218–219

self-induced emf angle shift, 216

single-layer 24-slot, 4-pole winding, 217–218

slot-emf phasor star, 218–219

primitive single-layer winding, 209–211

primitive two-layer 3-phase winding, 211–212

traveling magnetomotive force (MMF)

airgap flux density, 207

brush-commutator windings, 208

ideal multiphase MMF, 207–208

integer, 208

lap and wave single-turn (bar) coils, 208–209

number of slots, 208

sinusoidal currents and spatial distribution, 207

two-phase ac windings, 221–222

Active power, 332–333

Active rotor electric machines

dc rotor and ac stator currents, 128

dc stator and ac rotor, 129–131

magnetic conergy, 127

primitive active rotor single-phase electric machine, 127

rotor and stator ac currents, 128–129

m/Nr Actual winding modeling, IM

bar/ring currents, 519

inter-bar rotor currents, 522

matrix form, 520

motion equations, 520–521

rotor cage, 518–519

stator self- and mutual phase inductances, 519–520

stator-to-rotor loop, 520

symmetrical stator, 518

torque, speed and broken bar current, 521

Airgap flux density

armature reaction, 160–164

leakage factor, 161

leakage flux, 160–161

linear curve, 163

magnetic field, 163

mmf, 161–163

- radial/parallel magnetization, 160
- stator diameter, 164
- dimensioning methodology, 681–682
- electric machine design, 673
- electromagnetic force (EMF), 170
- stator excitation MMF, 154–155
- Ampere's law
 - electric conductors and skin effects, 49
 - flux linkages and inductances, 60
 - magnetic core ac coil, 32–33
 - magnetic field, 571–572
 - no-load magnetization curve, 156
- Analytical field models, 375
- Anti-overvoltage electrostatic transients, 102
- Autonomous induction generator, 245–247
- Autonomous synchronous generators
 - characteristic curves, 325–326
 - load curve
 - terminal line voltage *vs.* line current, 326
 - voltage equation, 327–328
 - voltage regulation, 329
 - $V_s(I_s)$ load curves, 327–328
 - no-load saturation curve, 325–326
 - phasor diagram, 329–330
 - rated current, 329
 - short-circuit curve, 326–327
 - test rig, 325
- Autotransformers, 103–104, 258
- B**
- Boost dc-dc converter, 106–107
- Brush-commutator machines
 - ac brush series universal motor
 - automotive small dc brush PM motor, 186–187
 - electromagnetic torque, 185
 - handheld universal contemporary motor, 186–187
 - instantaneous steady-state torque, 185
 - machine voltage equation, 184
 - pulsation self-induced voltage, 185
 - speed control, 186
 - torque speed curve, 186–187
 - airgap flux density
 - armature reaction, 160–164
 - stator excitation MMF, 154–155
 - armature windings
 - airgap windings, 152
 - coil span, 146
 - double winding, 147
 - phase shift angle, 147–148
 - simple lap windings,
 - $N_s = 16$, $2p_1 = 4$, 148–150
 - simple wave windings,
 - $N_s = 9$, $2p_1 = 2$, 150–152
 - brush gear and positioning, 152–153
- commutation process
 - ac excitation brush-commutation winding, 166–168
 - brush-segment contact resistance, 164
 - commutating coil circuit equation, 165
 - definition, 164
 - linear and resistive commutation, 166
 - rotor coil current reversal, 164–165
- dc brush motor/generator
 - efficiency, 173
 - electromagnetic power, 171
 - electromagnetic torque, 171–172
 - excitation circuit equation, 172
 - excitation losses, 173
 - separate excitation, 171
 - transient inductance, 172
- dc brush PM motor
 - artificial loading, 190
 - closed-loop control, 413
 - cogging torque, 189–190
 - countercurrent braking, 176–180
 - dc-dc converter, 413–415
 - efficiency, 190
 - iron losses, 188
 - pole flux, 189
 - rotor parameter derivation, 180–181
 - speed control methods, 175–176
 - steady-state, 173–175
 - twin motors, 188–189
- dc brush series motor
 - characteristic curves, 182–183
 - electromagnetic power, 183

- equivalent circuit, 181–182
 - self-excitation regenerative braking, 181
 - starting and speed control, 183–184
 - voltage equation and torque, 182
 - electromagnetic force (EMF)
 - armature reaction, 168
 - brush emf, 170
 - compensation winding, 168–169
 - motion emf, 168
 - resultant nonuniform flux density, 168–169
 - electromagnetic transients
 - flux orientation control, 405
 - load equations, 404
 - sudden short circuit, 405–406
 - electromechanical transients
 - constant excitation (PM) flux, 406–407
 - dc brush series motor transients, 411–412
 - variable flux transients, 410–411
 - equivalent circuit and excitation connection, 170–171
 - no-load magnetization curve
 - Ampere's law, 156
 - current density, 160
 - definition, 155
 - magnetic flux line path, 155–156
 - pole body mmf, 158
 - rotor tooth mmf, 156–157
 - rotor yoke, 157
 - stator back iron, 158
 - stator pole shoe and body, 157–158
 - torque, 159
 - orthogonal (dq) model
 - dc excitation circuit, 402
 - electromagnetic torque, 403
 - field-circuit equation, 402
 - motion-induced voltage, 402–403
 - pulsational-induced voltage, 402
 - stator and rotor winding, 401–402
 - structural diagram, 403
 - transients, 404
 - parameter estimation
 - cross-coupling saturation effect, 417
 - dc-dc converter, 416
 - excitation flux, 417
 - freewheeling diode voltage, 416–417
 - machine voltage equation, 416
 - standstill tests, 415
 - preliminary design
 - coil wire diameter, 194
 - copper losses, 195
 - current density, 193
 - electromagnetic power and efficiency, 195
 - PM stator geometry, 192–193
 - rotor diameter, 192
 - slot active area, 193–194
 - slot height left, 193
 - specifications, 191
 - stack length, 192
 - torque, 191
 - turns per rotor periphery, 193–194
 - silver-copper segments, 152
 - stator and rotor construction elements
 - automobile engine-starter motors, 145
 - dc/ac electromagnetic excitation stator, 145
 - PM dc brush machines, 143–145
 - universal motor, 145–146
 - testing
 - artificial loading, 190
 - cogging torque, 189–190
 - efficiency, 188, 190
 - IEEE/IEC Standards, 187
 - iron losses, 188
 - pole flux, 189
 - twin motors, 188–189
 - Brushless dc motor, 465–468
- ## C
- Cage rotor constant stator flux
 - constant rotor flux, 524–525
 - developments, 526
 - doubly fed IMs, 532–533
 - fifth-order system, 524
 - IM structural diagram, 524–525
 - indirect vector control, 526–567
 - steady state, 525, 527–529
 - synchronous coordinates, 524

- torque expression, 525
- vector control basics, 529–531
- Cage rotor windings
 - airgap flux density, 223–224
 - applications and topologies, 203–204
 - bar and ring resistances, 223
 - chording and distribution factor, 224
 - end ring and cage-bar currents, 223
 - factor components, 223–224
 - rotor cage geometry, 222–223
 - skewing factor, 223
 - total winding factor, 224–225
- Capacitor start 2-phase winding, 221–222
- Cartesian coordinates, 561
- Circuit models, 375
- Common mode (CM) pulses, 392
- Cyclic inductance, 466
- Cylindrical coordinates, 561–562
- D**
- DC brush PM motor
 - artificial loading, 190
 - closed-loop control, 413
 - cogging torque, 189–190
 - countercurrent braking, 176–180
 - dc-dc converter
 - continuous and discontinuous current, 414–415
 - current and voltage waveforms, 413–414
 - modulation index, 415
 - speed pulsation, 413
 - voltage equation, 414
 - efficiency, 190
 - iron losses, 188
 - pole flux, 189
 - rotor parameter derivation, 180–181
 - speed control methods, 175–176
 - steady-state
 - current/speed curve, 173
 - ideal no-load speed, 174–175
 - short-circuit torque, 175
 - speed/torque curve, 174
 - transients, 407–409
 - twin motors, 188–189
- DC brush series motor
 - characteristic curves, 182–183
 - electromagnetic power, 183
 - equivalent circuit, 181–182
 - self-excitation regenerative braking, 181
 - starting and speed control, 183–184
 - voltage equation and torque, 182
- DC-excited cageless rotor, 461–462
- DC-excited cage rotor, rectangular
 - current control, 468–469
- Differential mode (DM) pulses, 392
- Dirichlet's boundary conditions, 576
- Distributed power systems, 23–24
- Doubly fed induction machine (DFIM)
 - brushless SM exciter, 533–537
 - slip frequency (ω_2), 533
 - space phasor equations, 534
 - stator apparent power, 535
 - transformer action, 533–534
- Dq0 model
 - balanced steady state
 - braking torque, 435
 - lagging power factor, 433
 - Park transformation, 432
 - sinusoidal phase voltages and currents, 432
 - space phasor diagram, 433–434
 - torque, 433
 - parameter equivalence, 426
 - per unit (pu) system
 - base voltages, 430
 - flux/current relationships, 431
 - parameters, Ohms, 431–432
 - reduced-order model,
 - electromechanical transients
 - biaxial excitation generator, automobiles, 448–449
 - fast stator electrical transients, 446–447
 - stator and rotor cage transients, 447
 - third-order dq model, SM voltage control, 447–448
 - stator flux linkages, 426
 - structural diagram
 - axis, 427–428
 - equivalent circuits, transients, 429
 - Laplace form, 427
 - PM and reluctance SMs, 428–429
 - voltage, current, and flux equivalence relationship, 425

E

Efficiency, 345

Electric machines

 automotive electric motor
 applications, 23–24

 electric energy

 coupled electric and magnetic
 circuits, 5–6

 electric motor drive applications, 5
 Faraday's law, 5–6

 generator/motor operation mode,
 3–4

 measurement unit, 3

 electromagnetic torque, 7

 finite element-coupled circuit models,
 23

 fixed magnetic field machines

 brush-commutator machines, 8–9

 magnetic flux density, 7–8

 speed and power limitations, 8

 stator, 7

 high-frequency electric transformers,
 24

 IEEE and IEC standards, 23

 losses and efficiency

 acceleration-deceleration

 sequences, 14

 definition, 13–14

 electromagnetic torque, 15

 mechanical and electrical power,
 14

 unity and lagging power factor, 15

 magnetic composite soft materials, 24

 methods of analysis, 21–22

 MOSFET and IGBT, 23

 moving field electric machines

 induction machine (IM), 9–11

 synchronous machine (SM), 11–13

 nameplate ratings, 19–21

 physical limitations and ratings

 electromagnetic rotor shear stress,
 17–18

 electromagnetic torque, 18

 global cost function, 18–19

 insulation materials, 17

 power grid, 24

 types of transformers, 6

Electric transformers

 autotransformers, 103–104

cores

 single-phase distribution
 transformer, 54

 single-phase power transformer,
 52–53

 stacking pattern, 54

 three-phase power transformer,
 53–54

electric conductors and skin effects

 ac resistance factor, 50–51

 Ampere's and Faraday's laws, 49

 copper conductor, 45

 critical conductor height, 51

 dc current conductors, 45

 laminated magnetic core vicinity,
 48

 magnetic field, 49–50

 Poynting vector, 50

 Roebel bar, 51–52

 slot leakage and ac resistance, 50

instrument transformers, 102–103

magnetic core ac coil

 airgap, 32

 emf ratio, 35

 equivalent magnetic circuit, 33–34

 Faraday's law, 34–35

 flux (Gauss) law, 33

 inductance, 34

 magnetic circuit (Ampere's) law,
 32–33

 magnetic core, 33

 magnetic permeability, 35

 phasor diagram, 35–36

 soft magnetic materials, 33

magnetization curve and hysteresis
 cycle

 differential permeability, 40–41

 frequency and graphical

 representation, 39

 incremental permeability, 40–41

 magnetic permeability, 38, 40

 normal permeability, 40–41

no-load steady state ($I_2 = 0$)

 core loss, 68

 equivalent circuit, 67

 magnetic saturation, 70–71

 no-load magnetization curve, 69

 no-load power, 68

- phasor diagram and power breakdown, 67
- permanent magnets, 41–42
- power electronics
 - boost dc-dc converter, 106–107
 - contact-less battery charging system, 107–108
 - galvanic separation, 106
 - series-connected transformer, 107
 - static-power
 - semiconductor-controlled switches, 106
 - two stage ac-ac power electronics converter, 107
- preliminary transformer design
 - active material weight, 113
 - deliverables, 109
 - equivalent circuit, 113
 - losses and efficiency, 112
 - magnetic circuit sizing, 109–110
 - no-load current, 112
 - specifications, 108–109
 - windings sizing, 110–112
- single-phase transformer, 31–32
 - circuit equations, core losses, 64–65
 - flux linkages and leakage inductances, 59–63
 - steady-state operation on load, 74–77
- soft magnetic material losses
 - B–H curve, 45–46
 - core loss, 45, 47
 - eddy current losses, 43–44
 - hysteresis losses, 42–43
- steady state and equivalent circuit
 - complex variables, 65
 - core loss series resistance, 66
 - input voltage, 65
 - load equation, 66
 - primary and secondary winding losses, 67
- steady-state short-circuit mode, 71–73
- three-phase transformers
 - general equations, 84–86
 - no-load current asymmetry, 82–83
 - paralleling, 91–92
 - phase connections, 78–81
 - unbalanced load steady state, 87–89
- Y primary connection, 3-limb core, 83–84
- transients
 - anti-overvoltage electrostatic transients, 102
 - electromagnetic (R,L) transients, 94–95
 - electrostatic (C,R) ultrafast transients, 99–102
 - forces, peak short-circuit current, 97–98
 - inrush current transients, 95–96
 - no load sudden short circuit ($V'_2 = 0$), 96–97
- windings
 - alternate windings, 57
 - concentric and biconcentric cylindrical windings, 57
 - conductor strand, insulation paper lapping, 54–55
 - continuously transposed cable, 54–55
 - corrugated tank, 58
 - disc windings, 55–56
 - foil windings, 56
 - layer and helical windings, 55–56
 - oil circulation, 57
 - transformer tank and oil conservator, 57–58
 - turn-ratio limited range regulation, 56
 - types, 55
- Electromagnetic compatibility (EMC), 393
- Electromagnetic (active) power, 323
- Electromagnetics
 - boundary conditions
 - asymptotic analysis, 577–579
 - Dirichlet boundary condition, 576
 - Kelvin transform, 579–581
 - mixed Robin boundary condition, 577
 - Neumann boundary condition, 576–577
 - periodic boundary conditions, 577
 - problem truncation, 577
 - current densities, 570–571
 - electrostatic fields, 569–570
 - finite element method (FEM)

- algebraic system equations, 584
- analysis, 586–592
- 2D FEM, 584–586
- differential equations, 581
- domain discretization, 583–584
- interpolation functions, 584
- residuum (Galerkin's) method, 582–583
- solving algebraic equations, 584
- variational (Rayleigh–Ritz) method, 583
- magnetic fields, 571–572
- Maxwell equations, 572–573
- 3-phase induction machines
 - cross section, 633
 - finite element method, 632–633
 - ideal no-load regime, 636–642
 - main dimension values, 633–634
 - rotor bar skin effect, 642–649
 - rotor slot geometry, 634–636
 - stator slot geometry, 634–635
- rotary PMSMs (6/4)
 - postprocessor stage, BLDC motor, 616–632
 - preprocessor stage, BLDC motor, 610–616
 - 6-slot/4-pole PMS, 609
 - stator main dimensions, 609–610
- single-phase linear PM motors
 - FEM analysis, 595
 - mechanical springs, 595–597
 - postprocessor stage, 602–609
 - preprocessor stage, 597–602
 - thermal engine valve actuator, 595–596
- vectorial fields
 - coordinate systems, 561–563
 - differential identities, 568–569
 - differential operations, 565–567
 - integral identities, 567–568
 - line and surface (flux) integrals, 564–565
 - operations, 563–564
- visualization
 - chaotic particles, 573–574
 - comparative representation, 574–575
 - electric (scalar) potential, 573
 - magnetic field lines, 573–574
 - vectorial fields, 575
 - widespread method, 573
- Electromagnetic torque, 323–324, 463
- Electromagnetic (R,L) transients, 94–95
- Electro-magneto-mechanical energy
 - conversion, 122
- Electromechanical time constant, 257
- Electrostatic (C,R) ultrafast transients
 - distributed capacitor model, 100–101
 - microsecond front voltage
 - propagation, 99–100
 - resonance phenomena, 102
 - single-layer winding, 99
 - wave equation, 101
 - zero initial transformer voltage, 100
- End-connection leakage inductance, 229
- Energy conversion
 - active rotor electric machines
 - dc rotor and ac stator currents, 128
 - dc stator and ac rotor, 129–131
 - magnetic conergy, 127
 - primitive active rotor single-phase electric machine, 127
 - rotor and stator ac currents, 128–129
 - coupling magnetic field, 121–122
 - electromagnetic torque
 - cogging torque, 123–124
 - Faraday's law, 122
 - mechanical energy conversion, 122–123
 - stored magnetic energy, 123
 - electromagnetomotive force (emf), 122
 - energy losses, 121
 - fixed magnetic field electric machines, 131–132
 - linear electric machines
 - linear compressor coil-mover PM linear motor, 138–139
 - linear oscillatory motor/generator, 135
 - loudspeaker/microphone, 136–138
 - 3-phase linear induction machine, 134
 - single-phase synchronous tubular configuration, 135
 - three-phase linear flat synchronous machine, 134–135

- passive rotor electric machines
 - coenergy, 124, 126
 - constant instantaneous torque, 126
 - elementary 3-phase reluctance machine, 125
 - magnetic saliency, 124
 - reluctance synchronous machine, 126
 - self-/mutual-inductance, 124
 - single-phase reluctance machine, 125
 - stepper reluctance motor, 126–127
 - switched reluctance machines, 125–126
 - traveling field electric machines, 132–133
- Equivalent bar resistance, 288
- F**
- Faraday's law
 - electric conductors and skin effects, 49
 - electromagnetic induction, 5–6
 - electromagnetic torque, 122
 - magnetic core ac coil, 34–35
- Field-circuit coupled models, 375
- Field circuit losses, 345
- Finite element method (FEM)
 - analysis
 - coil flux, 587
 - electromagnetic forces, 589–591
 - energy and inductances, 589
 - Green's theorem, 587–588
 - loss computation, 591–592
 - magnetic flux lines, 586–587
 - massive conductors, 588
 - 2D FEM
 - electric machines, 584–585
 - magnetostatic 2D fields, 585
 - matrices, 586
 - steady-state solution, 585
 - differential equations, 581
 - 3-phase induction machines
 - cross section, 633
 - ideal no-load regime, 636–642
 - main dimension values, 633–634
 - periodicity symmetry, 632–633
 - rotor bar skin effect, 642–649
 - rotor slot geometry, 634–636
 - stator slot geometry, 634–635
 - residuum (Galerkin's) method, 582–583
 - rotary PMSMs (6/4)
 - postprocessor stage (BLDC motor analysis), 616–632
 - preprocessor stage (BLDC motor), 610–616
 - 6-slot/4-pole PMS, 609
 - stator main dimensions, 609–610
 - single-phase linear PM motors
 - FEM analysis, 595
 - mechanical springs, 595–597
 - postprocessor stage, 602–609
 - preprocessor stage, 597–602
 - thermal engine valve actuator, 595–596
 - stages
 - algebraic system equations, 584
 - domain discretization, 583–584
 - interpolation functions, 584
 - solving algebraic equations, 584
 - variational (Rayleigh–Ritz) method, 583
- Finite element models, 375–376
- Fixed magnetic field machines
 - brush-commutator machines, 8–9, 131–132
 - magnetic flux density, 7–8
 - speed and power limitations, 8
 - stator, 7
- Flux barrier, 422
- Four-pole machine, 475–476
- Freewheeling diode voltage, 416–417
- Fundamental frequency models, 375
- G**
- Galerkin's method, 582–583
- Gauss law, 33, 570
- Genetic algorithms (GA)
 - induction machines
 - airgap and magnetic core flux density, 732
 - cost function evolutions, 730–731
 - dimension ratios, 734–735
 - flux densities, 734–735
 - last generation, 733–734

- linear electric load and current
 - density, 731
 - motor weight evolution, 737–738
 - objective function, 737–739
 - optimization variables, 729–730
 - power losses and efficiency
 - evolutions, 737
 - ratio evolutions, 732–733
 - rotor and stator diameters, 736
 - specific electric load, 734
 - stator and rotor slots, 733–734, 736
 - optimal design
 - airgap flux density (B_{gap}), 673
 - crossover, offspring generation, 672–674
 - fitness function, 672
 - Gray code, 674
 - Monte-Carlo roulette method, 671–672
 - mutation, offspring generation, 673
 - round function, 671
 - steps, 670–671
 - surface PMSMs (SPMSMs)
 - case study, optimization design, 697–708
 - objective (fitting) function, 696–697
 - optimization variables, 694–695
 - technological/mechanical reasons, 695
 - Green's theorem, 568
- H**
- High-frequency models, 392–393
 - Hooke–Jeeves optimization algorithm
 - induction machines
 - components, 740, 744
 - dimension ratios, 740–741
 - electric load and current density
 - evolution, 739
 - flux density, 740
 - losses and efficiency, 740, 743
 - motor weight evolution, 740, 743
 - objective (cost) function, 740, 744
 - rotor and stator diameters, 740, 742
 - stator and rotor slots, 740–741
 - optimal design
 - constraint function, 667
 - energy losses cost expression, 668
 - load factor, 668–669
 - peak/direct start current, 669
 - penalty cost, 669–670
 - power electronics cost, 667–668
 - stator demagnetization, 670
 - steps, 664–667
 - surface PMSMs (SPMSMs)
 - components' weights and costs, 709–710
 - cost function evolution, 709, 711
 - current density evolution, 709, 712
 - dimensions ratios, 709, 713
 - flux density evolution, 709, 712
 - geometric and slot dimensions, 709
 - optimization results, 709, 714–715
 - power losses and efficiency
 - evolutions, 709–710
 - slots per pole per phase, 709, 713
 - specific load evolution, 709, 711
 - Hybrid electric vehicle (HEV)
 - actuators, 24
- I**
- Induction generator, 244–245
 - Induction machines (IMs)
 - AC distributed windings
 - cage rotor windings, 222–225
 - MMF space harmonics, integer q , 211–216
 - pole count changing ac 3-phase winding, 220–221
 - practical one-layer ac 3-phase winding, 216–219
 - primitive single-layer winding, 209–211
 - primitive two-layer 3-phase winding, 211
 - traveling magnetomotive force (MMF), 206–209
 - two-phase ac windings, 221–222
 - m/N_r actual winding modeling
 - bar/ring currents, 519
 - inter-bar rotor currents, 522
 - matrix form, 520
 - motion equations, 520–521
 - rotor cage, 518–519
 - stator self- and mutual phase inductances, 519–520
 - stator-to-rotor loop, 520

- symmetrical stator, 518
 - torque, speed and broken bar current, 521
- applications and topologies, 203
- autonomous generator mode, 245–247
- cage rotor constant stator flux
 - constant rotor flux, 524–525
 - developments, 526
 - doubly fed IMs, 532–533
 - fifth-order system, 524
 - IM structural diagram, 524–525
 - indirect vector control, 526–567
 - steady state, 525, 527–529
 - synchronous coordinates, 524
 - torque expression, 525
 - vector control basics, 529–531
- capacitor split-phase induction motors
 - dual capacitor IM, 271
 - equivalent impedances, 271–272
 - source voltage, 271
 - symmetrical components theory, 271–273
 - torque, 272
- construction elements
 - ball bearings, 205
 - cage rotor, 3-phase, 204–205
 - single-phase supply capacitor, 204–205
 - slots, 204–205
 - wound rotor, 3-phase, 204
- controlled magnetic flux and variable frequency
 - cage rotor IM, 522
 - complex eigenvalues, 522–523
 - IM structural diagram, 523
- deep-bar and dual-cage rotors, 252–253
- doubly fed IM (DFIM)
 - brushless SM exciter, 533–537
 - slip frequency (ω_2), 533
 - space phasor equations, 534
 - stator apparent power, 535
 - subsynchronous operation, 550
 - supersynchronous mode, 550
 - transformer action, 533–534
- dq (space phasor) model, 497–498
- electromagnetic torque and motor characteristics
 - electromagnetic power, 251
 - rated current, 250
 - rated slip, 250–251
 - rated speed, 250
 - rotor and stator current, 248
 - rotor cage losses, 249
 - stator resistance, 251
 - steady-state characteristics, 249–250
 - torque *vs.* slip *S*, 248–249
- electromagnetic transients, 507–508
- genetic algorithms
 - airgap and magnetic core flux density, 732
 - cost function evolutions, 730–731
 - dimension ratios, 734–735
 - flux densities, 734–735
 - last generation, 733–734
 - linear electric load and current density, 731
 - motor weight evolution, 737–738
 - objective function, 737–739
 - optimization variables, 729–730
 - power losses and efficiency evolutions, 737
 - ratio evolutions, 732–733
 - rotor and stator diameters, 736
 - specific electric load, 734
 - stator and rotor slots, 733–734, 736
- Hooke–Jeeves optimization algorithm
 - components, 740, 744
 - dimension ratios, 740–741
 - electric load and current density evolution, 739
 - flux density, 740
 - losses and efficiency, 740, 743
 - motor weight evolution, 740, 743
 - objective (cost) function, 740, 744
 - rotor and stator diameters, 740, 742
 - stator and rotor slots, 740–741
- ideal no-load operation
 - active power, 236
 - equivalent circuit, 235–237
 - iron losses, 237
 - phasor diagram, 236–237
 - supersynchronous operation, 236
- large-deviation electromechanical transients
 - dq model equations, 514–515

- Runge–Kutta–Gill method, 515
- starting transients, 515–516
- stator coordinates, 515
- leakage inductance
 - cage rotors, end rings, 229
 - differential leakage stator inductance, 227
 - end-connection leakage inductance, 229
 - flux lines classification, 227
 - rectangular slot leakage field, 228
 - slot geometrical permeance coefficient, 228–229
- linear induction motor (LIM)
 - airgap flux density, 278–279
 - dc electromagnets, 548
 - double-layer, full-pitch windings, 276–277
 - dynamic end effect, 279–280
 - end effect thrust, 279
 - equivalent goodness factor, 278
 - flat geometry, 275–276
 - MAGLEV, 546–547
 - Park transformation, 547
 - primary length, 279
 - secondary current density path, 277
 - single layer windings, 276
 - single-sided LIM, 545–546
 - static end effect, 277
 - synchronous coordinates, 547
 - transverse edge effect, 277–278
 - urban (suburban) transportation, 552–553
 - vector control, 548
- on load operation, 243–244
- machine performance
 - airgap flux density *vs.* magnetization current, 743, 745
 - circuit model, 742
 - current *vs.* torque, 746, 749
 - input and output powers, 746, 749
 - linkage flux *vs.* current, 743–744
 - magnetization inductance, 743, 745
 - parameters, 746–748
 - power factor *vs.* slip, 746, 749
 - power factor *vs.* torque, 746, 750
 - saturation factors *vs.* magnetization current, 743, 745
 - stator and rotor current, 746, 748
 - torque *vs.* rotor slip, 746
 - magnetic saturation and skin effects, 500–501
- main inductance
 - Carter coefficient, 225
 - cyclic main (magnetization) inductance, 226
 - magnetic saturation factor, 225
 - magnetization characteristics, 226
- no-load motor operation, 241–243
- one rotor phase, 269–270
- one stator phase
 - copper losses, 268–269
 - rotor and no load current, 267
 - symmetric steady state torque, 268
 - unbalanced voltage components, 266
- parasitic (space harmonics) torques
 - asynchronous parasitic torques, 254–255
 - mmf space harmonics, 254
 - slot skewing, 255
 - stator and rotor harmonic relationship, 253
 - synchronous parasitic torques, 255–256
- 3-phase induction machine, circuit equations
 - complex number variables, 233
 - direct and inverse sequence currents, 232
 - electromagnetic power, 233
 - electromagnetic torque, 233–234
 - flux linkages, 231–233
 - reduced rotor windings, 230–231
 - rotor and stator equation, 233
 - symmetric steady state, 234–236
- power grid generation, 244–245
- preliminary electromagnetic design
 - breakdown slip and torque, 291
 - efficiency and power factor, 294
 - electric circuit, 287
 - magnetic circuit, 283–286
 - magnetization reactance, X_m , and core losses, p_{iron} , 291–293
 - main specifications, 282
 - no-load and rated currents, 293–294

- parameters, 287–290
- starting current and torque, 290–291
- realistic analytical model
 - design theme, 717–718
 - design variables, 718–719
 - dimensions, 719–725
 - parameters, 725–729
- reduced-order dq model
 - fifth-order dq model, 517
 - grid connection, 517
 - other severe transients, 518
 - stator transients, 516
 - third-order system, 516–517
- regenerative and virtual load testing
 - artificial loading, mechanical force, 281
 - frequency mixing method, 282
- rotor cage reduction, stator, 229–230
- small-deviation electromechanical transients
 - dq model equation yields, 513
 - IM small deviation, 514
 - single-cage IM, 512–513
 - speed transfer function, 514
- space phasor model steady state
 - active and reactive powers, 503–504
 - cage rotor IM, 502–503
 - doubly fed IM steady state, 506–507
 - 3-phase cage rotor IM, 504–506
 - steady state, 501–502
 - synchronous coordinates, 501
- speed control methods
 - breakdown torque, 263–264
 - critical rotor frequency, 263
 - critical slip and efficiency, 262
 - pole count changes, 260
 - short-circuit reactance, 262
 - soft starter, 260
 - torque–speed curves, 263–264
 - variable frequency and voltage, 260
 - wound rotor, 261
- split-phase capacitor
 - auxiliary phase, 542
 - dq model, 544–545
 - 3-phase-unidirectional and bidirectional, 542–543
 - phase variable model, 543–544
 - single-phase power supply, 552
- standstill tests
 - magnetization curve identification, 538–540
 - operational impedance, 551
 - parameters, 537–538
 - resistances and leakage inductances, 540–541
 - standstill frequency response (SSFR) tests, 541–542
- starting methods
 - additional rotor resistance, 258–259
 - direct starting (cage rotor), 256–257
 - reduced stator voltages, 257–258
- three-phase–dq model relationships, 499
- three-phase sudden short circuit
 - current space vector, 510
 - equations, 509–510
 - example, 510–512
 - machine phase voltages, 509
 - remarks, 510
 - transient current, 512
- three-phase variable model
 - inductance matrix, 496
 - machine equations, 495
 - phase variable model, 497
 - resistance matrix, 495–496
- unbalanced supply voltages, 264–265
- wound rotor reduction, stator, 230
- zero speed operation
 - 3-phase zero-speed testing, 238–239
 - rotor (slip) frequency, 238
 - short-circuit current *vs.* voltage, 240
 - short circuit impedance, 238
 - single-phase zero-speed testing, 239–240
 - starting current, 238
 - starting torque, 239
- Inrush current transients, 95–96
- Institute of Electrical and Electronics Engineers (IEEE), 6
- Interaction torque, 127

International Electrotechnical

Commission (IEC), 6

Iron loss computation, 591–592

Iron loss resistance, 293

K

Kelvin–Stokes theorem, 567

Kelvin transform, 579–581

L

Lagging power factor, 15

Laplace form, 404, 406, 410, 427

Laplacian, vectorial field, 566–567

Large-deviation electromechanical
transients

asynchronous starting

PMSMs, 455

self-synchronization, dc rotor,
453–455

line-to-line and line-to-neutral faults,
455–456

Linear compressor coil-mover PM linear
motor, 138–139

Linear electric machines

linear compressor coil-mover PM
linear motor, 138–139

linear oscillatory motor/generator,
135

loudspeaker/microphone

BIL force, 137

mechanical resonance conditions,
137–138

steady-state harmonic motion, 138

tubular configuration, 136

3-phase linear induction machine, 134

single-phase synchronous tubular
configuration, 135

three-phase linear flat synchronous
machine, 134–135

Linear induction motor (LIM)

airgap flux density, 278–279

dc electromagnets, 548

double-layer, full-pitch windings,
276–277

dynamic end effect, 279–280

end effect thrust, 279

equivalent goodness factor, 278

flat geometry, 275–276

MAGLEV, 546–547

Park transformation, 547

primary length, 279

secondary current density path, 277

single layer windings, 276

single-sided LIM, 545–546

static end effect, 277

synchronous coordinates, 547

transverse edge effect, 277–278

urban (suburban) transportation,
552–553

vector control, 548

Linear synchronous motor transients

active guideway, 484

active suspension control, 486

dc excitation and ac winding, 484–485

dq model, 485

normal force, 486

PM guideway, 484–485

Load torque pulsations

asynchronous torque, 350

damping coefficient, 351

mechanical resonance module, 351

proper (eigen) angular frequency, 350

Lorenz force integration, 589–590

M

Magnetic core ac coil

airgap, 32

emf ratio, 35

equivalent magnetic circuit, 33–34

Faraday's law, 34–35

flux (Gauss) law, 33

inductance, 34

magnetic circuit (Ampere's) law,
32–33

magnetic core, 33

magnetic permeability, 35

phasor diagram, 35–36

soft magnetic materials, 33

Magnetic field distribution, 21

Magnetic field-to-circuit models, 375–376

Magnetic materials

magnetization curve and hysteresis
cycle

differential permeability, 40–41

frequency, 39

graphical representation, 39

incremental permeability, 40–41

- magnetic permeability, 38, 40
 - normal permeability, 40–41
- permanent magnets, 41–42
- soft magnetic material losses
 - B–H curve, 45–46
 - core loss, 45, 47
 - eddy current losses, 43–44
 - hysteresis losses, 42–43
- Maxwell equation, 572–573
- Motion equation, 461
- Moving field electric machines
 - induction machine (IM)
 - asynchronous machine, 10
 - 3-phase cage rotor, 10–11
 - standard ideal no-load speed, 9
 - wound rotor, 9–10
 - synchronous machine (SM)
 - anisotropic rotor, 12
 - heteropolar magnetic flux density, 11
 - self-synchronization, 12
 - switched reluctance and stepper motors, 12–13
- Multiple magnetic circuit models, 376
- Mutual inductance, 423
- N**
- Neumann's boundary conditions, 576–577
- O**
- Operational (Laplace) parameters, 436
- Optimal design
 - case study, GA
 - best-candidate solution, 701
 - cost function, 704–705
 - current density, 704, 707
 - current density evolution, 702–703
 - dimension ratios, 704
 - electric load evolution, 702
 - flux density evolutions, 702–703
 - genetic algorithm, 697–698
 - geometric dimensions, 699
 - high-performance members, 708
 - last generation, flux densities, 704, 706
 - machine components' weights, 699–700
 - mathematical transformation, 702
 - maximum and average costs, 698–699
 - optimization variable bounds, 697–698
 - rotor yoke flux density, 708
 - slots per pole per phase, 704–705
 - specific electric load, 704, 706
- classical design flowchart, 652–653
- dimensioning calculus stage, 651
- flowchart, 652, 654
- genetic algorithms
 - airgap flux density (Bagsp), 673
 - crossover, offspring generation, 672–674
 - fitness function, 672
 - Gray code, 674
 - Monte-Carlo roulette method, 671–672
 - mutation, offspring generation, 673
 - round function, 671
 - steps, 670–671
- Hooke–Jeeves optimization algorithm
 - constraint function, 667
 - energy losses cost expression, 668
 - load factor, 668–669
 - peak/direct start current, 669
 - penalty cost, 669–670
 - power electronics cost, 667–668
 - stator demagnetization, 670
 - steps, 664–667
- induction machines (IM)
 - genetic algorithms, 729–737
 - Hooke–Jeeves optimization algorithm, 739–744
 - machine performance, 742–750
 - realistic analytical model, 717–729
- optimization methods
 - algorithms, 656–657
 - definitions, 653
 - disadvantages, 656
 - equivalent objective function, 657
 - machine (drive) design process, 653, 655
 - parametric constraint, 658–659
 - penalty and external functions, 657
 - production cost, 655
 - temperature constraint, 658

- torque *vs.* speed, 658–659
 - variable domain, 656
 - optimum current control
 - analytical solution, 660
 - inductance *vs.* linkage flux, 661
 - magnetization current, 660–661, 663–664
 - no-load current *vs.* flux linkage, 663
 - nonlinear equation, 662
 - phase current *vs.* torque, 661–662
 - torque equation, 659–660
 - torque *vs.* magnetization current, 663
 - surface PMSMs (SPMSMs)
 - design theme, 675
 - dimensioning factors, 677–678
 - dimensioning methodology, 681–694
 - electric and magnetic loadings, 675–677
 - genetic algorithms (GA), 694–708
 - magnetic materials, 678–681
 - technological constraints, 678
 - verification calculus stage, 651–652
 - Orthogonal (dq) physical model, 375
 - DC brush PM motor ($\omega_b = 0$), 379–380
 - dc excitation circuit, 402
 - electromagnetic torque, 403
 - equivalence conditions
 - active and reactive power, 389
 - generalized Park transformation, 388
 - instantaneous torque, 389
 - inverse matrix, 388
 - 3-phase ac-dq mmf equivalence, 387
 - stator copper losses, 389
 - torque equivalence, 387
 - zero-sequence component, 387–388
 - field-circuit equation, 402
 - frequency (skin) effect consideration, 386–387
 - induction machines ($\omega_b = 0, \omega_r, \omega_1$), 382–383
 - magnetic saturation
 - d,q magnetic curves, 384
 - hysteresis cycle incremental permeability, 386
 - incremental inductance, 385–386
 - magnetic coupling inductance, 385
 - magnetization and leakage inductances, 383
 - stator and rotor magnetization, 383–384
 - total magnetization current, 384
 - transient inductance, 385
 - motion-induced voltage, 402–403
 - pulsational and motion-induced voltages, 378–379
 - pulsational-induced voltage, 402
 - stator and rotor winding, 401–402
 - structural diagram, 403
 - synchronous machines ($\omega_b = \omega_r$)
 - construction asymmetric, 380–381
 - flux linkage-current, 382
 - motion emf, 381
 - motion equation, 382
 - stator equation, 381
 - transients, 404
 - Ostrogradsky–Gauss theorem, 568
- P**
- Park transformation, 432
 - Passive rotor electric machines
 - coenergy, 124, 126
 - constant instantaneous torque, 126
 - elementary 3-phase reluctance machine, 125
 - magnetic saliency, 124
 - reluctance synchronous machine, 126
 - self-/mutual-inductance, 124
 - single-phase reluctance machine, 125
 - stepper reluctance motor, 126–127
 - switched reluctance machines, 125–126
 - Permanent magnet (PM) plunger solenoids, 135
 - Permanent magnet synchronous generators (PMSGs), 304–305
 - 3-Phase cage rotor, 10–11
 - Phase coordinate circuit models, 375
 - Phase coordinate model

- electromagnetic torque and power, 424
- matrix form, 423–424
- uses, 425
- 3-Phase generator step-up transformer, 6–7
- 3-Phase induction machines
 - cross section, 633
 - finite element method, 632–633
 - ideal no-load regime
 - airgap flux density distribution, 637–638
 - analytical models, 641
 - flux density map and magnetic field lines, 637
 - fundamental and representative harmonics, 638–639
 - harmonic magnitudes *vs.* phase current, 638
 - IM phases distribution, 636–637
 - magnetization inductance *vs.* current, 639–640
 - main phase flux *vs.* magnetization, 638, 640
 - slot leakage inductance, 642
 - voltage *vs.* current, 642
 - main dimension values, 633–634
 - rotor bar skin effect
 - average bar losses, 646
 - current density, 643, 645
 - geometry, 642–643
 - inductance factor, 647, 649
 - inductance *vs.* frequency, 647–648
 - Joule–Lentz losses, 645
 - magnetic field and current distribution, 643–644
 - resistance factor, 647–648
 - resistance *vs.* frequency, 646–647
 - rotor slot axis, 643, 645–646
 - steady-state harmonic (ac) module, 642
 - rotor slot geometry, 634–636
 - stator slot geometry, 634–635
- Phase leakage inductance, 228
- 3-Phase linear induction machine, 134
- 3-Phase short circuit, no load generator
 - electromagnetic torque, 441
 - field current transients, 440
 - flux linkages, 440–441
 - peak short-circuit current, 440
 - phase A current, 440
 - qualitative view, 441
 - SG parameter values, 441–442
 - step voltage, 439
- Phase voltage unbalance index, 265
- PM flux computation
 - airgap flux density, 686
 - Carter coefficient, 686
 - cyclic inductance, 691
 - flux path length reduction, 687–688
 - geometrical slot-and end-connection, 690
 - machine parameters, 691–692
 - magnetic voltage, 687–688
 - relaxation coefficient, 688
 - single-turn coil, 691
 - stator and rotor yokes, 687
 - stator phase resistance, 689
- PM synchronous and switched reluctance motors, 24
- PM synchronous motors
 - electromagnetic torque, 347
 - interior PM rotor, 347–348
 - phasor diagram, 346
 - reluctance synchronous motor (RSM), 348–349
- Pole changing winding, 220
- Pole pitch, 207
- Pole-slipping method, 340–341
- Power balance, motoring, 243
- Poynting vector, 50
- Preliminary electromagnetic design
 - breakdown slip and torque, 291
 - efficiency and power factor, 294
 - electric circuit, 287
 - magnetic circuit
 - airgap flux density, 283
 - rated torque, 284
 - rotor diameter, 284–285
 - rotor yoke, 285
 - stator and rotor slot areas ratio, 286
 - stator back iron (yoke) height, 285
 - stator stack length, 284
 - tangential specific force, 283
 - tooth/slot pitch ratio, 285
 - top and bottom widths, 285–286

- magnetization reactance, X_m , and
 - core losses, p_{iron}
 - Carter coefficient, 291
 - magnetic saturation, 291–292
 - magnetization reactance, 292
 - stator wire diameter, 292
 - stator yoke weight, 292–293
- main specifications, 282
- no-load and rated currents, 293–294
- parameters
 - coil turn length and end ring area, 288
 - leakage reactances, 289–290
 - length, end ring segment, 288
 - reduction factor, 289
 - rotor aluminum bar resistance, 288
 - stator resistance, 287
- starting current and torque, 290–291
- Preliminary transformer design
 - active material weight, 113
 - deliverables, 109
 - equivalent circuit, 113
 - losses and efficiency, 112
 - magnetic circuit sizing, 109–110
 - no-load current, 112
 - specifications, 108–109
 - windings sizing
 - filling factor, 110
 - leakage reactance, 111
 - number of turns, 110–111
 - primary and secondary resistances, 111
 - primary emf, 110
 - radial width, 111
 - short-circuit rated voltage, 111–112
- Primary winding, 203
- Primitive active rotor single-phase electric machine, 127
- Pulse-width-modulated (PWM) converter, 392–393
- Pulse width modulation, 413
- R**
- Rayleigh–Ritz method, 583
- Reactive power, 332–333
- Realistic analytical models
 - design theme, 717–718
 - design variables, 718–719
- induction machine dimensioning
 - geometry, 720
 - machine core length, 721
 - rotor design, 721–723
 - stator and rotor yoke, 720
 - stator slot dimensions, 724
 - winding end-connection length, 724–725
 - winding factor, 720
- parameters
 - cyclic magnetization inductance, 727
 - equivalent resistance, 726
 - flux distribution *vs.* tooth saturation factor, 726–727
 - magneto-motive force, 726
 - pd1 factors, 728
 - rotor differential permeance, 728–729
 - rotor ring resistance, 725
 - $\sigma d2$ factors, 729
 - stator windings, 725
- Reluctance synchronous machine (RSM), 11–12, 348–349
- Robin’s boundary conditions, 577
- Roebel bar, 51–52
- Rotary permanent magnet synchronous machines (PMSMs) (6/4)
 - geometric dimensions, 610–611
- postprocessor stage
 - BLDC motor–magnetic field lines, 616–617
 - copper losses, 630
 - current density distribution, 622
 - drive function, 622–623
 - flux linkage, 619
 - initial rotor position, 623–624
 - iron loss computation, 620
 - linear variation, 629
 - linkage flux and real phase current, 631
 - machine torque *vs.* rotor position, 618–619
 - magnetic field lines and flux density map, 621–622
 - Maxwell tensor integration, 617–618
 - motor torque pulsations, 629
 - path definition, 623–624

- phase current waveforms, 631
 - phase flux linkage, 618–619, 631–632
 - radial component, 623
 - radial forces, 626–627
 - real current density, 617
 - space average flux density, 626, 628
 - specific torque *vs.* rotor position, 626
 - stator core flux density, 620
 - Steinmetz method, 630
 - tangential flux density, 625
 - preprocessor stage
 - airgap mash layers, 614
 - BLDC motor magnetic field, 610–611
 - Cartesian coordinates, 612
 - drive functions, 615–616
 - magnetic polarization, 614
 - mechanical equation, 615
 - polar coordinates, 611–613
 - replicate command, 613
 - rotating machine airgap, 613–614
 - stator core and coil elementary shape, 611–612
 - trapezoidal current wave shape, 615–616
 - 6-slot/4-pole PMS, 609
 - stator main dimensions, 609–610
 - Rotor leakage inductance, 229
 - Runge–Kutta–Gill method, 515
- S**
- Secondary winding, 203
 - Shaft power, 294
 - Single-phase and split-phase capacitor PM
 - classification, 357
 - cogging torque (T_{cogg}) periods, 355
 - efficiency, 356
 - electromagnetic torque, 355
 - parking magnet, PWM inverter, 353–354
 - phasor diagram, 355–356
 - pu torque *vs.* power angle, 356–357
 - two-pole split-phase capacitor, 355
 - voltage equation, 354
 - Single-phase linear PM motors
 - FEM analysis, 595
 - mechanical springs, 595–597
 - postprocessor stage
 - axial force, 608
 - equilibrium mover position, 602–603
 - FEM softwares, 602
 - flux density module map, 603–604
 - global characterization, 608–609
 - linkage flux *vs.* mover position, 605
 - magnetic core, 604
 - magnetic field energy *vs.* mover position, 608–609
 - MATLAB®, 605
 - mover position, 602–603
 - mover thrust *vs.* position, 606–607
 - real differential inductance, 606
 - thrust curves *vs.* mover position, 606
 - upper mover disk, 606–607
 - preprocessor stage
 - build_evalva.comi script, 598
 - computer code, 600
 - down mover disk corner coordinate, 600
 - Draw command, 599, 601
 - electro-valve geometry, 600–601
 - electro-valve main dimensions, 597–598
 - electro-valve mesh node, 601–602
 - ev_slol script, 598
 - GUI interface, 597
 - magnetization features, 599
 - Maxwell tensor, 601
 - thermal engine valve actuator, 595–596
 - Single-phase transformer, 31–32
 - circuit equations, core losses, 64–65
 - flux linkages and leakage inductances
 - alternate windings, 62–63
 - Ampere’s law, 60
 - cylindrical windings, 61–62
 - definition, 60
 - primary and secondary mmfs, 59
 - resultant magnetic reluctance, 60
 - steady-state operation on load
 - active and reactive power break down, 74

- dual voltage single-phase residential transformer, 77
 - efficiency *vs.* load factor, 74–75
 - equivalent circuit and phasor diagram, 75–76
 - voltage regulation *vs.* load factor, 76
- Skewing leakage inductance, 229
- Skin effect parameter, 290
- Small-deviation electromechanical transients
 - flux/current relationships, 450
 - forced torque pulsations, 452–453
 - matrix format, 450–451
 - steady-state conditions, 449
 - structural diagram and transfer function, 452
- Soft magnetic material losses
 - B–H curve, 45–46
 - core loss, 45, 47
 - eddy current losses, 43–44
 - hysteresis losses, 42–43
- Soft starter, 258
- Space phasor (complex variable) model
 - dq model, IM, 497–498
 - electromagnetic torque, 390
 - phase coordinate model, 389
 - rotor voltage equation, 390
 - stator and synchronous coordinates, 391
- Space phasor model steady state
 - active and reactive powers, 503–504
 - cage rotor IM, 502–503
 - doubly fed IM steady state, 506–507
 - 3-phase cage rotor IM, 504–506
 - steady state, 501–502
 - synchronous coordinates, 501
- Spherical coordinates, 562–563
- Split-phase cage rotor
 - efficiency, 475
 - four-pole machine, 475–476
 - magnetic saliency, 476
 - magnetic saturation, 477
- Star-delta switch, 258
- Stator copper losses, 257
- Stator teeth weight, 292–293
- Stator winding factor, 287
- Stator (armature) windings
 - factors, concentrated windings, 310–311
 - four-pole PMSM windings, 308, 310
 - LCM, N_s and $2p_1$, 310, 312
 - single-phase tooth-wound machines, 312–313
 - switched reluctance motor (SRM), 310–312
 - TF-PMSMs, 312–313
 - types, 306–307
- Super high-frequency models, 375
- Surface permanent magnet synchronous machines (SPMSMs)
 - design theme, 675
 - dimensioning factors, 677–678
 - dimensioning methodology
 - active material weights, 692
 - airgap flux density, 681–682
 - geometrical dimensions, 682
 - losses, 693
 - machine characteristics, 694
 - parameters, 681
 - PM flux computation, 686–692
 - RMS ampere turn/slot, 684
 - rotor sizing, 685–686
 - slot geometrical parameters, 683–684
 - stator slot geometry, 683
 - stator yoke thickness, 685
 - thermal verification, 694
 - electric and magnetic loadings, 675–677
 - genetic algorithms (GA)
 - case study, optimization design, 697–708
 - objective (fitting) function, 696–697
 - optimization variables, 694–695
 - technological/mechanical reasons, 695
- Hooke-Jeeves method, 709–715
- magnetic materials
 - magnetization curves, 679–680
 - PM characteristics, 678–679
 - SURA007 iron losses *vs.* flux density, 680–681
- technological constraints, 678
- Switched reluctance machine modeling
 - average torque, 471, 473
 - constant load torque, 475

- flux/current/position curves, 470
- inertia, 474
- linearized structural diagram, 472
- 3-phase 6/4 SRM, 470
- speed, 474
- torque expression, 471
- Synchronous inductances, 436–437
- Synchronous machines, steady state
 - applications and topologies
 - automobile alternators, 304
 - automotive electric power
 - actuators, 304–305
 - hard disks, 304, 306
 - linear SMs, 305, 308
 - open cross section, 306, 309
 - permanent magnet synchronous generators (PMSGs), 304–305
 - power systems, 303–304
 - single-phase micro-PMSMs, 304–306
 - special configuration PMSMs, 305, 307
 - stepper machines, 303
 - switched reluctance machines, 303
 - typical slot shapes, windings, 306, 309
 - armature reaction and magnetization reactances
 - d and q axes, 320–321
 - magnetization inductances, 322
 - nonsinusoidal airgap flux densities, 320–321
 - synchronous inductances, 322
 - asynchronous starting and self-synchronization, 352–353
 - autonomous synchronous generators
 - load curve, 326–329
 - no-load saturation curve, 325–326
 - short-circuit curve, 326–327
 - basic static- and dynamic-stability concepts
 - attenuation, 337
 - short-circuit clearing time, 337–338
 - synchronizing power, 335
 - torque, 336
 - large synchronous motors
 - power balance, 345–346
 - speed/torque curve, 344
 - load torque pulsations
 - asynchronous torque, 350
 - damping coefficient, 351
 - mechanical resonance module, 351
 - proper (eigen) angular frequency, 350
 - PM synchronous motors
 - electromagnetic torque, 347
 - interior PM rotor, 347–348
 - phasor diagram, 346
 - reluctance synchronous motor (RSM), 348–349
 - preliminary design methodology
 - base current, 362
 - general specifications, 357–358
 - interior stator diameter and stator stack length, 358
 - machine parameters, 361
 - maximum phase voltage, 358
 - maximum speed torque, 362–363
 - number of turns, 362
 - phasor diagram, 361–362
 - PM flux, 358–359
 - stator slot size, 360
 - stator yoke height and outer diameter, 360
 - rotors
 - PM, airgap flux density, 317–318
 - salient and unsalient poles, 314–316
 - stator phase winding, 317
 - single-phase and split-phase capacitor PM
 - classification, 357
 - cogging torque (T_{cogg}) periods, 355
 - efficiency, 356
 - electromagnetic torque, 355
 - parking magnet, PWM inverter, 353–354
 - phasor diagram, 355–356
 - pu torque *vs.* power angle, 356–357
 - two-pole split-phase capacitor, 355
 - voltage equation, 354
 - stator (armature) windings
 - factors, concentrated windings, 310–311
 - four-pole PMSM windings, 308, 310
 - LCM, N_s and $2p_1$, 310, 312

- single-phase tooth-wound machines, 312–313
- SRM, 310–312
- TF-PMSMs, 312–313
- types, 306–307
- synchronous generators, power grid
 - active power/angle curves, 332–333
 - digital synchronizers, 331
 - field current, 331
 - reactive power capability curves, 334–335
 - V-shaped curves, 333–334
- synchronous reactances, 364
- two-reaction principle, generator mode
 - emfs and current phasors, 318–319
 - load, 318–319
 - mutual inductance, 320
 - phasor diagram, 319–320
- unbalanced 3-phase current loads
 - homopolar impedance measurement, 342
 - impedance, 340
 - phase-to-phase short circuit, 342–344
 - pole-slipping method, 340–341
 - slip, 339
 - symmetrical components, 339
 - total voltage, 340
- uses, 363
- Synchronous machine transients
 - asynchronous mode
 - ac field current, 443
 - average torque, 443–444
 - dc field current, 444–446
 - stator current, 444
 - stator voltages, 442
 - torque *vs.* slip, 444–445
 - controlled flux and sinusoidal current
 - cageless rotor, constant d -axis, 457–459
 - constant flux, vector control, 464–465
 - constant stator total flux
 - conditions, cageless SMs, 461–462
 - flux orientation/vector control, 456
 - vector control, PMSMs, 460–461
- dq0 model
 - balanced steady state, 432–435
 - parameter equivalence, 426
 - per unit (pu) system, 430–432
 - reduced-order model,
 - electromechanical transients, 446–449
 - stator flux linkages, 426
 - structural diagram, 427–430
 - voltage, current, and flux equivalence relationship, 425
- electromagnetic transients
 - constant speed, 437–438
 - Laplace parameters, 436–437
- large-deviation electromechanical transients
 - asynchronous self-starting, PMSMs, 455
 - asynchronous starting and self-synchronization, dc rotor, 453–455
 - line-to-line and line-to-neutral faults, 455–456
- linear synchronous motor transients
 - active guideway, 484
 - active suspension control, 486
 - dc excitation and ac winding, 484–485
 - dq model, 485
 - normal force, 486
 - PM guideway, 484–485
- parameters, standstill testing
 - current decay tests, 478, 480–481
 - experimental results, 481–483
 - field current reduction ratio, 479
 - final flux linkage, 479
 - initial magnetization current, 480
 - magnetization inductances, 480
 - parameter identification, 477–478
 - standstill frequency response tests, 481–483
 - subtransient inductances,
 - frequency test, 481
- phase coordinate model
 - electromagnetic torque and power, 424
 - matrix form, 423–424
 - uses, 425

- phase inductances, 422–423
 - 3-phase short circuit, no load
 - generator
 - electromagnetic torque, 441
 - field current transients, 440
 - flux linkages, 440–441
 - peak short-circuit current, 440
 - phase A current, 440
 - qualitative view, 441
 - SG parameter values, 441–442
 - step voltage, 439
 - rectangular current and controlled flux transients, 465–469
 - rectangular current control
 - brushless dc motor transients, 465–468
 - dc-excited cage rotor, 468–469
 - small-deviation electromechanical transients
 - flux/current relationships, 450
 - forced torque pulsations, 452–453
 - matrix format, 450–451
 - steady-state conditions, 449
 - structural diagram and transfer function, 452
 - split-phase cage rotor
 - efficiency, 475
 - four-pole machine, 475–476
 - magnetic saliency, 476
 - magnetic saturation, 477
 - switched reluctance machine
 - modeling
 - average torque, 471, 473
 - constant load torque, 475
 - flux/current/position curves, 470
 - inertia, 474
 - linearized structural diagram, 472
 - 3-phase 6/4 SRM, 470
 - speed, 474
 - torque expression, 471
- T**
- Three-phase linear flat synchronous machine, 134–135
 - Three-phase transformers
 - general equations
 - inductance measurement, 85–86
 - 3-limb core transformer, 84–85
 - magnetic coupling and phase coordinates, 85
 - no-load current asymmetry, 82–83
 - paralleling, 91–92
 - phase connections
 - connection schemes, 80–81
 - delta and star connections, 79–80
 - primary line voltage, 81
 - primary/secondary 3-phase windings, 78
 - stator connection, 79
 - zig-zag (Z) connections, 79–80
 - unbalanced load steady state
 - direct and inverse components, 88
 - inverse transformation, 87
 - secondary neutral point potential, 89
 - symmetrical components
 - superposition, 87
 - zero sequence current, 88
 - zero sequence impedance, 88–89
 - Y primary connection, 3-limb core, 83–84
- Transient inductance axis, 437
- Traveling magnetomotive force (MMF)
 - airgap flux density, 207
 - brush-commutator windings, 208
 - ideal multiphase MMF, 207–208
 - integer, 208
 - lap and wave single-turn (bar) coils, 208–209
 - number of slots, 208
 - sinusoidal currents and spatial distribution, 207
- Two-phase induction motor, 221
- U**
- Unbalanced 3-phase current loads
 - homopolar impedance measurement, 342
 - impedance, 340
 - phase-to-phase short circuit, 342–344
 - pole-slipping method, 340–341
 - slip, 339
 - symmetrical components, 339
 - total voltage, 340
 - Unity power factor, 15

V

Virtual work method, 590–591
Voltage and current measurement
 transformers, 6
Voltage build-up, 438–439
V-shaped curves, 333–334

W

Wound rotor, 203–204

Z

Zig-zag leakage inductance, 228

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