FIFTH EDITION

MATERIALS SELECTION IN MECHANICAL DESIGN



MICHAEL F. ASHBY

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Materials Selection in Mechanical Design

FIFTH EDITION

Michael F. Ashby



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Preface

Mike Ashby, *Cambridge*

> *Materials, of themselves, affect us little; it is the way we use them which influences our lives.*
>
> *Epictetus, AD 50–100, Discourses Book 2, Chapter 5.*

Materials influenced lives 2000 years ago and continue to do so today. In Epictetus' day the number of engineering materials was small, today it is vast, more than 150,000. The opportunities for innovation that today's materials offer are equally immense. But advance is possible only if a procedure exists for making a rational choice from this great menu, and – if they are to be used – a way of identifying ways to shape, join and finish them. This book develops a systematic procedure for selecting materials and processes, leading to the subset that best matches the requirements of a design. It is unique in the methods it develops and in the way the information it contains is structured. The structure gives rapid access to data and allows the user great freedom in exploring the potential of choice. The approach lends itself to computer-aided design, guiding choice without the need for arbitrary judgements.

The approach emphasizes design with materials rather than materials 'science', although the underlying science is presented, wherever possible, to help with the structuring of criteria for selection. The first six chapters require little prior knowledge: a

grasp of materials and mechanics at the 1st-year level is enough. The chapters dealing with multiobjective selection, coselection of material and shape and the design of hybrid materials are a little more advanced but can be omitted on a first reading. The subsequent chapters on materials and the environment, material and industrial design and sustainable development, topics of current relevance, are introduced in ways that relate to the systematic methods developed earlier in the book.

This book interfaces, so to speak, with a set of others, all of which use the same underlying way of thinking. At the more elementary level, the text 'Materials, Engineering, Science, Processing and Design',[1] introducing materials from an engineering perspective, is aimed at 1st year undergraduates. At the more advanced levels, the texts 'Materials and the Environment',[2] 'Materials and Sustainable Development'[3] and 'Materials and Design'[4] enlarge on the economic, sustainability and industrial design aspects that appear in the last three chapters of this book.

Beyond this, the book is intended as a reference text of lasting value. The method, charts and tables of performance indices have application in real problems of materials and process selection. The tables of data and catalogue of 'useful solutions' (Appendices A and B) are particularly helpful in modelling – an essential ingredient of optimal design. The reader can use the book (and the software) at increasing levels of sophistication as his or her experience grows, starting with the material indices developed in the case studies of the text, and graduating to the modelling of new design problems, leading to new material indices and penalty functions, and new – and perhaps novel – choices of material. This continuing education aspect is helped by a list of references in a section titled Further Reading at the end of each chapter, and by exercises covering all aspects of the text.

How does the 5th edition differ from the 4th? The text and figures have been extensively revised and bought up to date; the more difficult concepts are explained with schematics and text that make them easier to grasp, and there are more than twice as many

examples and exercises, now placed at the ends of the relevant chapters. Material efficiency, criticality, circularity and sustainability are introduced and Appendix A, material property data, is updated using the most reliable available sources. The methods developed here are mirrored in the CES EduPack[5] Materials and Process Selection software, which, if available, forms a helpful adjunct to the text but is not necessary for its adoption.

Like any other book, the contents of this one are protected by copyright. Generally, it is an infringement to copy and distribute materials from a copyrighted source. But the best way to use the charts that are a central feature of the book is to have a clean copy on which students can draw, try out alternative selection criteria, write comments, and so forth; and presenting the conclusion of a selection exercise is often most easily done in the same way. Although the book itself is copyrighted, the instructor or reader is authorized to make unlimited copies of the charts and to reproduce these for teaching purposes, provided a reference to their source is attached to each.

June 2016

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Many colleagues have been generous in discussion, criticism and constructive suggestions. I particular wish to thank Professor Yves Bréchet of the University of Grenoble; Professor Anthony Evans of the University of California at Santa Barbara; Professor John Hutchinson of Harvard University; Dr. David Cebon; Professor Norman Fleck; Professor Ken Wallace; Dr. John Clarkson, Dr. Hugh Shercliff, of the Engineering Department, Cambridge University; Dr. Amal Esawi of the American University in Cairo, Egypt; Dr. Ulrike Wegst of the Thayer School of Engineering at Dartmouth; Dr. Paul Weaver of the Department of Aeronautical Engineering at the University of Bristol and Professor Michael Brown of the Cavendish Laboratory, Cambridge, UK. Equally valuable has been the contribution of my colleagues at Granta Design, Cambridge, who are

responsible for the development of the CES software that has been used to make the make many of the charts that are a feature of this book.

[1]Ashby, M. Shercliff, H. and Cebon, D. (2014) 3rd edition, *Materials: Engineering, Science, Processing and Design*, 2nd edition, Butterworth-Heinemann, Oxford, UK. ISBN-13: 978-0-08-097773-7, North American Edition: ISBN-13: 978-1-85617-743-6

[2]Ashby, M.F. (2013) 'Materials and the Environment – eco-informed material choice' 2nd edition, Butterworth Heinemann, Oxford, UK. ISBN 978-0-12-385971-6

[3]Ashby M.F. Ferrer-Balas, D. and Segalas Coral, J. (2016) 'Materials and Sustainable Development' Butterworth-Heinemann Ltd, Oxford. ISBN-10: 0081001762 ISBN-13: 978-0081001769

[4]Ashby, M.F. and Johnson K. (2014) 'Materials and design – the art and science of materials selection in product design', 3rd edition, Butterworth Heinemann, Oxford, UK. ISBN 978-0-08-098205-2

[5]The *CES EduPack materials and process selection platform*, is a product of Granta Design, www.grantadesign.com/education/

CHAPTER 1

Introduction

Materials and Design

Abstract

This book is about the role of materials in engineering design. The components that make up a product have shape; they have mass; they carry loads; they conduct heat and electricity; they are exposed to wear and to corrosive environments; they are made of one or more materials; and they must be manufactured. Chapter 1 introduces the design process and the role of materials in it.

Keywords

Design process; function; shape; concept; embodiment; market need; product specification

1.1 Introduction and Synopsis

'Design' is one of those words that can mean all things to all people. Every manufactured thing, from the most lyrical of ladies' hats to the greasiest of gearboxes, qualifies, in some sense or other, as a design. It can mean yet more. Nature, to some, is Divine Design; to others it is design by Natural Selection. The reader will agree that it is necessary to narrow the field, at least a little.

This book is about the role of materials in engineering design. The components that make up a product have shape; they have mass; they carry loads; they conduct heat and electricity; they are exposed to wear and to corrosive environments; they are made of one or more materials; and they must be manufactured. The book describes how these are related.

Materials have limited design since humans first made clothes, built shelters and waged wars. Today the range of materials and processes to shape manufactured items is expanding rapidly. This expansion creates new opportunities but it also brings problems. We now use more materials in greater quantities than ever before, and we use them in ways that – at present – are wasteful. The book develops strategies for seizing the opportunities and tackling the problems.

1.2 Materials in Design

The number of materials available to engineers is very great: 200,000 or more. Although standardisation strives to reduce the number, the continuing appearance of new materials with novel, exploitable, properties expands the options further. How, then, do engineers choose, from this vast menu, the material best suited to their purpose? Rely on experience? That's how it was done in the past, passing on this experience to apprentices who, later in their lives, might themselves assume the role of the in-house materials expert.

There is no question of the value of experience. But many things have changed in the world of engineering, and all of them work against the success of simple reliance on past practice. There is the drawn-out time scale of apprentice-based learning. There is job mobility, meaning that the expert who is here today may be gone tomorrow. And there is the rapid evolution of materials information, already mentioned. A strategy relying on experience is not in tune with the computer-based environment in which design, today, takes place. We need a *systematic* procedure – one with steps that can be taught quickly, is robust in the decisions it reaches, allows for computer implementation, and is compatible with modern design methods.

Design is the process of translating an idea into the detailed information from which a product can be manufactured. Each of its stages requires decisions about the materials of which the product is to be made and the process for making it. Cost enters, both in the choice of material and in the way the material is processed. Materials are derived from natural resources – ores for metals, oil for polymers, minerals for ceramics and glasses – and many of these resources are finite and all, in conversion to materials, need energy. Materials efficiency has now become as important as energy efficiency, with efforts increasingly directed towards a circular

materials economy, one that re-uses as much as possible, minimizing the draw-down from the natural resource base. And it must be recognised that good engineering design alone is not enough to sell products. In almost everything from home appliances to automobiles and aircraft, the form, texture, feel, colour, beauty and meaning of the product – the satisfaction it gives the person who owns or uses it – are important. This aspect, known confusingly as *industrial design*, is one that, if neglected, can lose markets. Good design works; excellent design also gives pleasure.

Design challenges, almost always, are open-ended. They do not have a unique 'correct' solution, though some solutions will clearly be better than others. They differ from the analytical problems used in teaching mechanics, or structures, or thermodynamics, which generally do have single, correct answers. So the first tool a designer needs is an open mind: the willingness to consider all possibilities. But a net cast widely draws in many different fish. A procedure is necessary for selecting the excellent from the merely good.

Our aim is to develop a methodology for selecting materials and processes that is *design-led;* that is, it uses, as inputs, the functional requirements of the design. To do so we must first look briefly at the design process itself. Like most technical fields it is encrusted with its own special jargon, some of it bordering on the incomprehensible. We don't need much, but we can't avoid it altogether. The next section introduces some of the words and phrases – the vocabulary – of design, the stages in its implementation, and the ways in which materials selection links with these.

1.3 The Design Process

The starting point of a design is a *market need;* the end point is the full *specification* of a solution that fills the need. It is essential to define the need precisely, that is, to formulate a *need statement* expressed as a set of *design requirements*. Writers on design emphasize that the need statement should be solution-neutral (i.e., it should not imply how the need will be met), to avoid narrow thinking constrained by preconceptions. Between the need statement and the product specification lie the set of stages shown in Fig. 1.1: the stages of *concept, embodiment* and *detailed design*, explained in a moment.

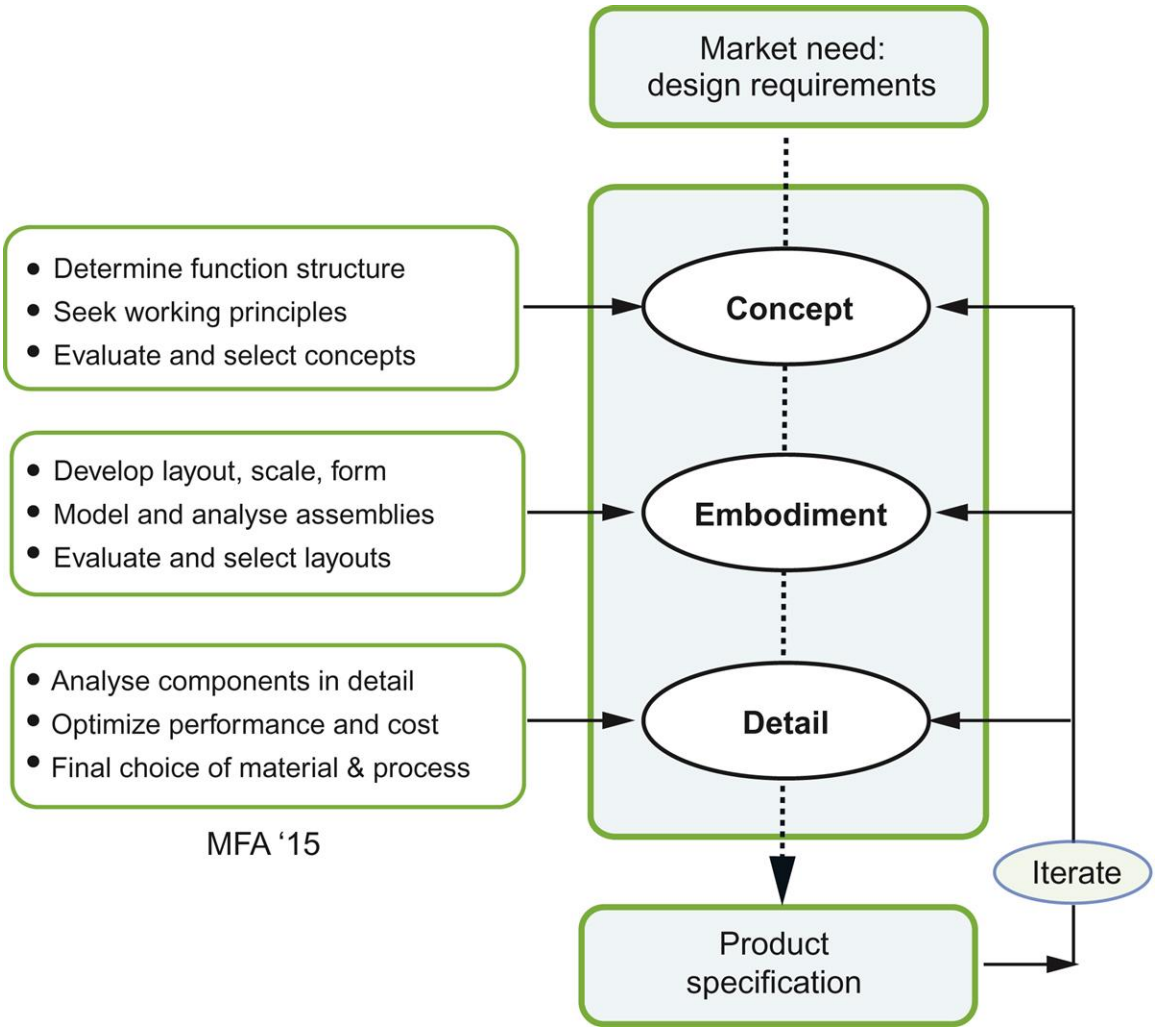


FIGURE 1.1 The design flow chart. The design proceeds from the identification of a market need, clarified as a set of design requirements, through concept, embodiment and detailed analysis to a product specification.

The product itself is called a *technical system*. A technical system consists of *subassemblies* and *components*, put together in a way that performs the required task, as in the breakdown of Fig. 1.2. It is like describing a cat (the system) as made up of one head, one body, one tail, four legs, etc. (the subassemblies), each composed of components – femurs, quadriceps, claws, fur. This decomposition is a useful way to analyse an existing design, but it is not of much help in the design process itself, that is, in devising new designs.

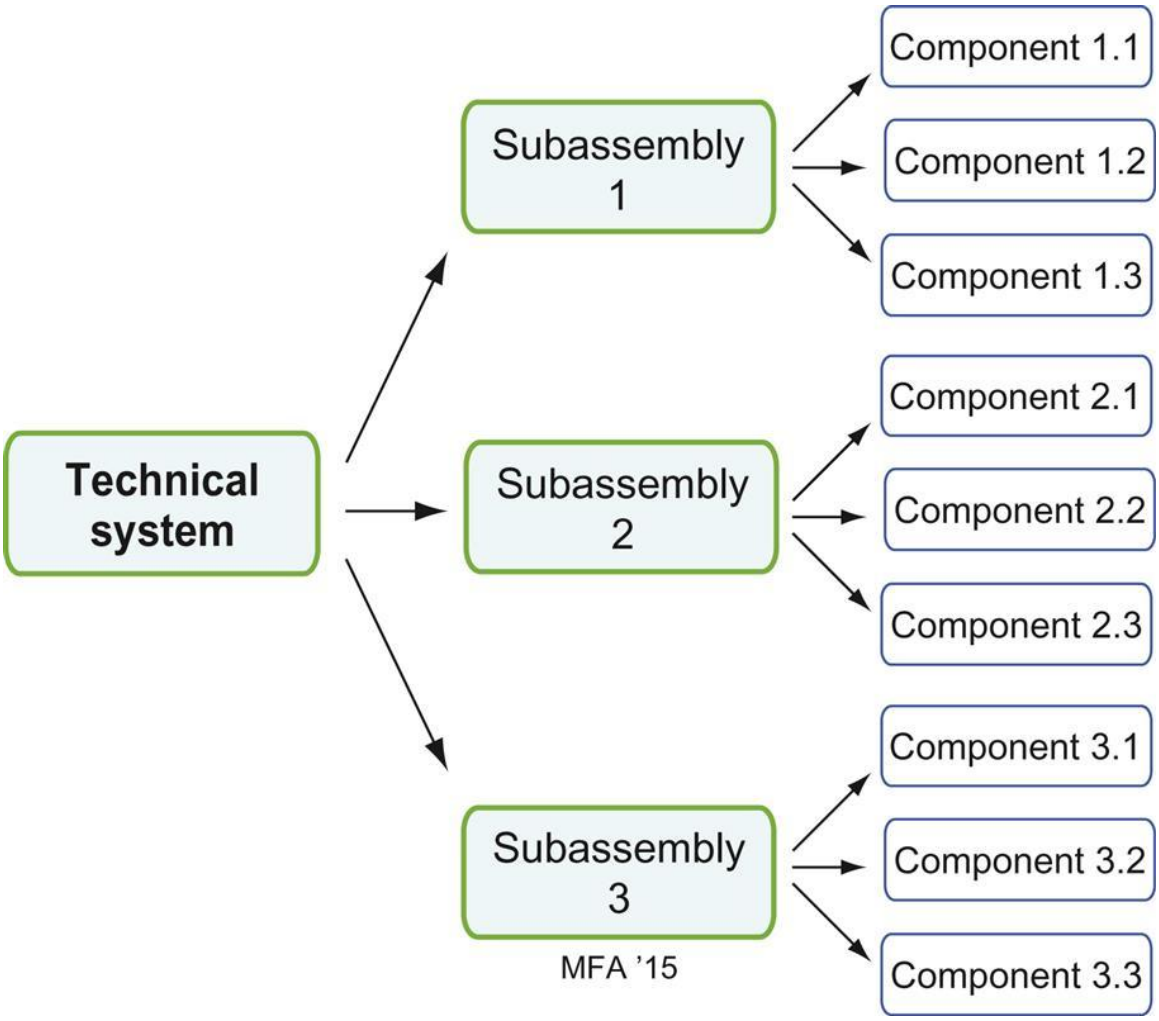


FIGURE 1.2 The analysis of a technical system as a breakdown into assemblies and components. Material and process selection is at the component level.

Better, for this purpose, is an approach based on the ideas of systems analysis. This approach thinks of the inputs, flows and outputs of information, energy and materials, as in Fig. 1.3. The design converts the inputs into the outputs. An electric motor converts electrical into mechanical energy; a forging press takes and reshapes material; a burglar alarm collects information and converts it to noise. In this approach, the system is broken down into connected subsystems each of which performs a specific function, as in Fig. 1.3. The resulting arrangement is called the *function-structure* or *function decomposition* of the system. It is like describing the cat as an appropriate linkage of a respiratory system, a cardio-vascular system, a nervous system, a digestive system and so on. Alternative designs link the unit functions in alternative ways, combine

functions, or split them. The function-structure gives a systematic way of assessing design options.

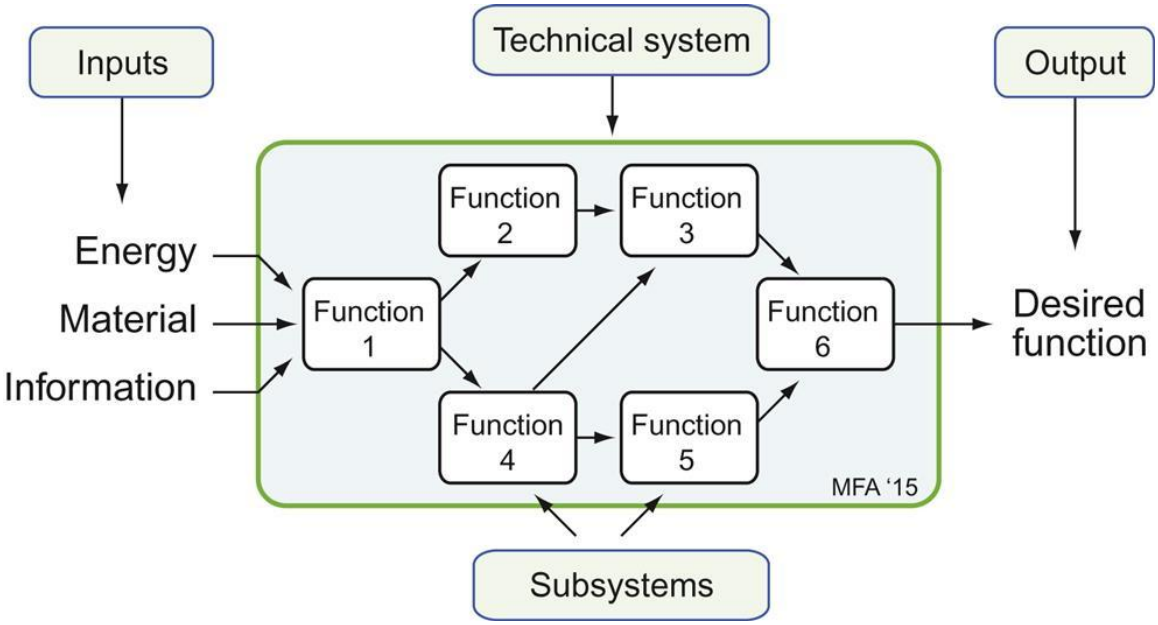


FIGURE 1.3 The function-structure is a systems approach to the analysis of a technical system, seen as transformation of energy, materials and information (signals). This approach, when elaborated, helps structure thinking about alternative designs.

Referring back to Fig. 1.1 the design proceeds by developing concepts to perform the functions in the function-structure, each based on a *working principle*. At this, the conceptual design stage, all options are open: the designer considers alternative concepts and the ways in which these might be separated or combined. The next stage, embodiment, takes the promising concepts and seeks to analyse their operation at an approximate level. This stage involves sizing the components and selecting materials that will perform properly in the ranges of stress, temperature and environment suggested by the design requirements, examining the implications for performance and cost. The embodiment stage ends with a feasible layout, which is then passed to the detailed design stage. Here specifications for each component are drawn up. Critical components may be subjected to precise mechanical or thermal analysis. Optimization methods are applied to components and groups of components to maximize performance. A final choice of geometry and material is made

and the methods of production are analysed and costed. The stage ends with a detailed production specification.

All that sounds well and good. If only it were so simple. The linear process suggested by Fig. 1.1 obscures the strong coupling between the three stages. The consequences of choices made at the concept or embodiment stages may not become apparent until the detail is examined. Iteration, looping back to explore alternatives, is an essential part of the design process. Think of each of the many possible choices that *could* be made as an array of blobs in design space as suggested by Fig. 1.4. Here C1, C2 … are possible concepts, and E1, E2, … and D1, D2, … are possible embodiments and detailed elaborations of them. The design process becomes one of creating paths, linking compatible blobs, until a connection is made from the top ('market need') to the bottom ('product specification'). Some trial paths have dead-ends, some loop back. It is like finding a track across difficult terrain – it may be necessary to go back many times if, in the end, one is to go forward. Once a path is found, it is always possible to make it look linear and logical (and many books do this), but the reality is more like Fig. 1.4 than Fig. 1.1. Thus a key part of design and of selecting materials is *flexibility*, the ability to explore alternatives quickly, keeping the big picture as well as the details in mind. Our focus in later chapters is on the selection of materials and processes, where the exact same need arises. This requires some kind of mappings of the 'universes' of materials and processes to allow quick surveys of alternatives while still providing detail when it is needed. The selection charts of Chapter 3, Materials Property Charts and the methods of Chapter 4, Materials Selection – the Basics help do this.

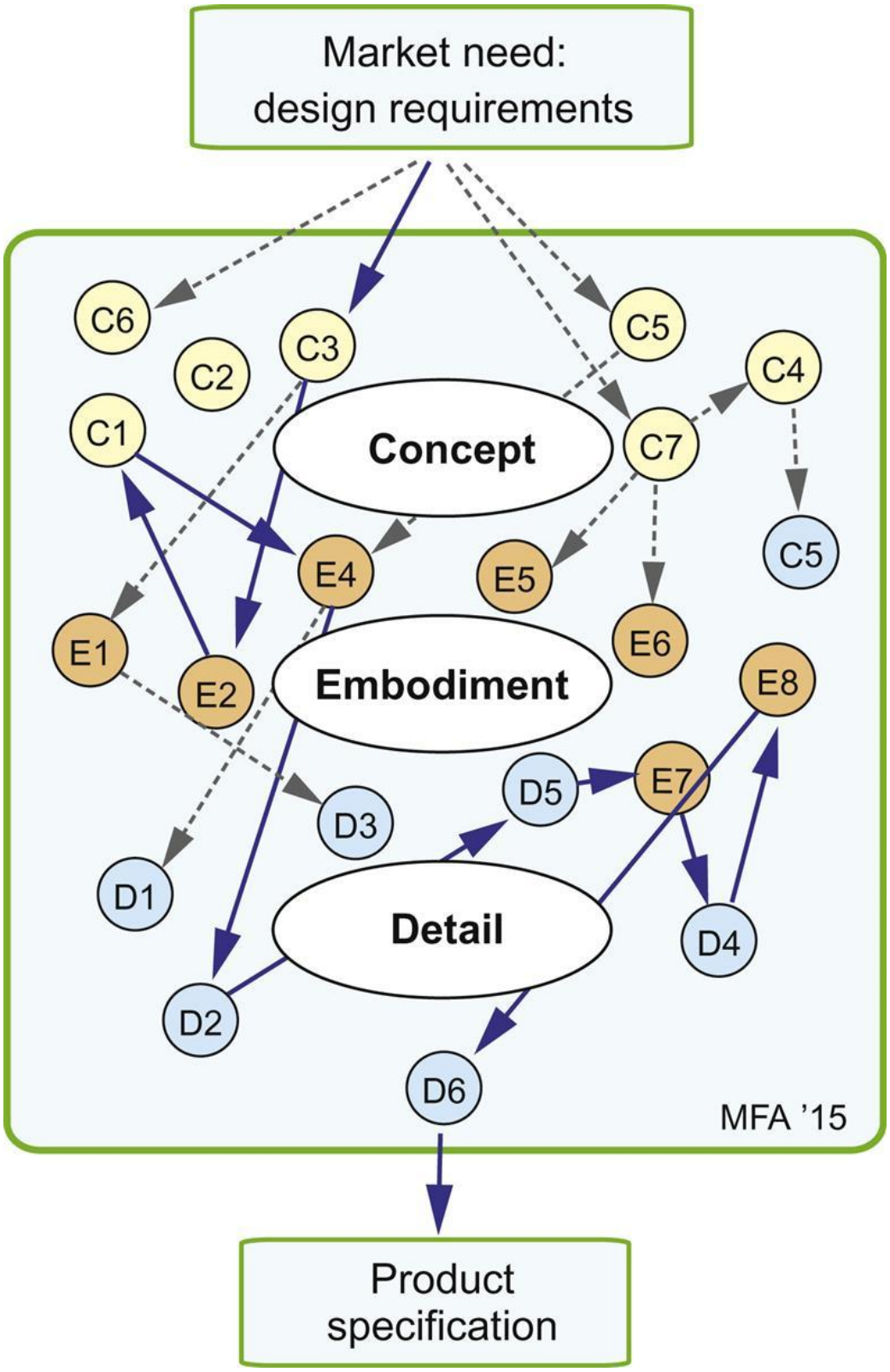


FIGURE 1.4 The convoluted path of design. Here the C-blobs represent concepts, the E-blobs embodiments of the C_s, and

the D-blobs detailed realisations of the E_s. The process is complete when a compatible path from 'Need' to 'Specification' can be identified. It is a devious path (the full blue line) with back-loops and dead-ends (the broken lines).

As described in the abstract, these ideas are not easy to grasp. An example will help – and it comes in Section 1.6. First, a look at types of design.

1.4 Types of Design

Original design starts from a new idea or working principle (the ball-point pen, the compact disc). New materials can offer new, unique combinations of properties that enable original design. Thus high-purity silicon enabled the transistor; high-purity glass, the optical fibre; high coercive-force magnets, the miniature earphone; solid-state lasers the compact disk. Sometimes the new material suggests the new product; however, sometimes the new product demands the development of a new material: nuclear technology drove the development of a series of new zirconium alloys and low-carbon stainless steels; space technology stimulated the development of light-weight composites; and gas turbine technology today drives development of high-temperature alloys and ceramic coatings. Original design sounds exciting, and it is. But most design is not like that.

Almost all design is *adaptive* or *developmental*. The starting point is an existing product or product-range. The motive for redesigning it may be to enhance performance, to reduce cost, or to adapt it to changing market conditions. Adaptive design takes an existing concept and seeks refinement of the working principle. It, too, is often made possible by developments in materials: polymers replacing metals in household appliances or carbon fibre replacing wood in sports equipment. The appliance and the sports-equipment markets are fast moving and competitive. These markets have frequently been won (and lost) by the way in which the manufacturer has adapted the product by exploiting new materials.

Finally, *variant design* involves a change of scale or dimension or detailing without change of function or the method of achieving it, for instance the scaling up of boilers or of pressure vessels. Change of scale or circumstances of use may require change of material: small boats are made

of fiberglass, large ships are made of steel; small boilers are made of copper, large ones of steel; subsonic planes are made of one alloy, supersonic of another; and for good reasons, detailed in later chapters.

1.5 Design Tools and Materials Data

To implement the steps of Fig. 1.1, use is made of *design tools*. They are shown as inputs, attached to the left of the main backbone of the design methodology in Fig. 1.5. The tools enable the modelling and optimization of a design, easing the routine aspects of each phase. Function-modellers suggest viable function structures. Configuration optimizers suggest or refine shapes. Geometric and 3-D solid modelling packages allow visualisation and create files that can be down-loaded to numerically controlled prototyping and manufacturing systems. Optimization, DFM, DFA[1] and cost-estimation software allows manufacturing aspects to be refined. Finite element (FE and CFD) packages allow precise mechanical and thermal analysis even when the geometry is complex, deformations are large and temperatures fluctuate. There is a natural progression in the use of the tools as the design evolves: approximate analysis and modelling at the conceptual stage; more sophisticated modelling and optimization at the embodiment stage; and precise ('exact' – but nothing is ever really that) analysis at the detailed design stage.

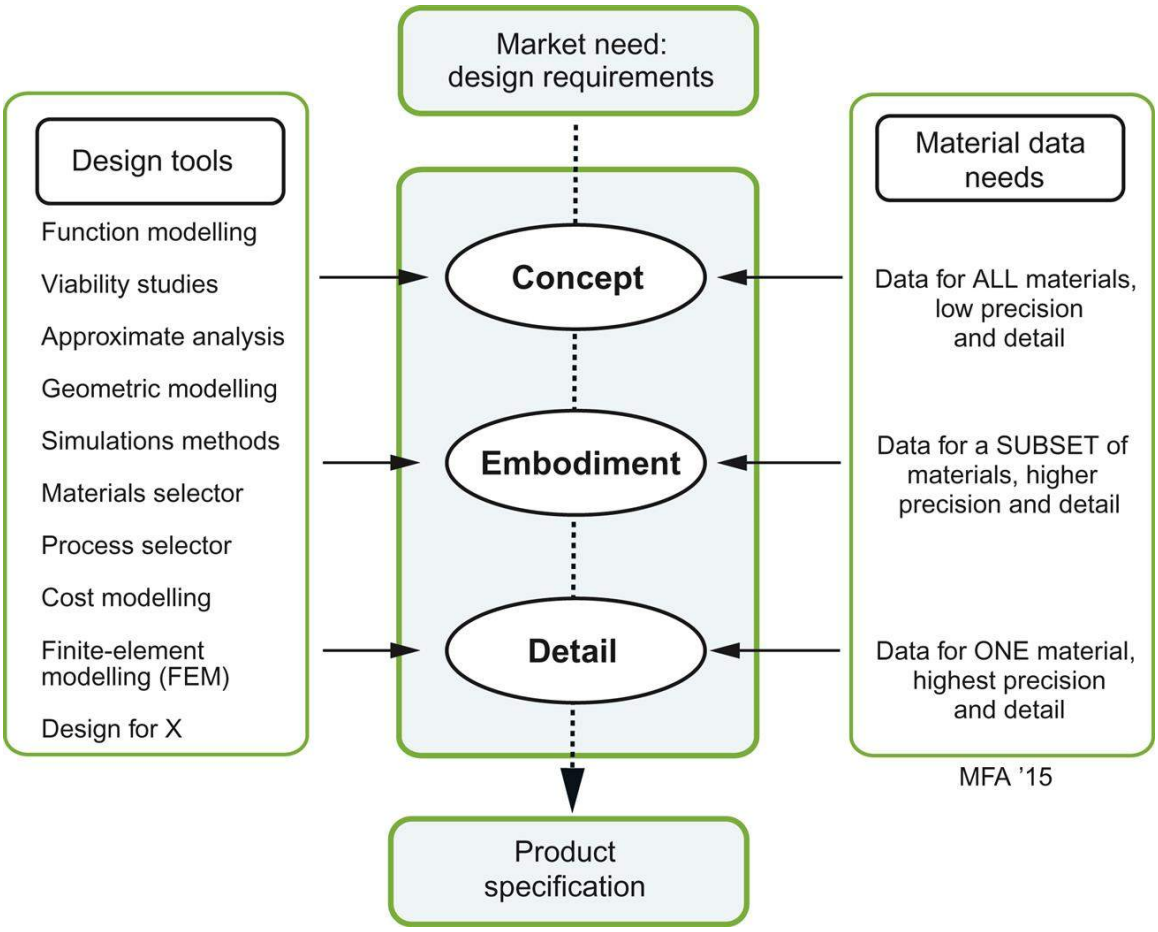


FIGURE 1.5 The design flow chart, showing how design tools and materials selection enter the procedure. Information about materials is needed at each stage, but at very different levels of breadth and precision. Iteration is part of the process.

Tools for material selection play a major part in each stage of the design. The nature of the data needed in the early stages differs greatly in its level of precision and breadth from that needed later on (Fig. 1.5, right hand side). At the concept-stage, the designer requires approximate property values, but for the widest possible range of materials. All options are open: a polymer may be the best choice for one concept, a metal for another, even though the function is the same. The problem, at this stage, is not precision and detail; it is breadth and speed of access: how can the vast range of data be presented to give the designer the greatest freedom in considering alternatives?

At the embodiment stage the landscape has narrowed. Here we need data for a subset of materials, but at a higher level of precision and detail. These are found in more specialised handbooks and software that deal

with a single class or subclass of materials – metals, or just aluminium alloys, for instance. The risk now is that of losing sight of the bigger spread of materials to which we must return if the details don't work out; it is easy to get trapped in a single line of thinking – a single set of 'connections' in the sense of Fig. 1.4 – when other connections offer a better solution.

The final stage, that of detailed design, requires a still higher level of precision and detail, but for only one or a very few materials. Such information is best found in the datasheets issued by the material producers themselves and in detailed databases for restricted material classes. A given material (polyethylene, for instance) has a range of properties that derive from differences in the ways different producers make it. At the detailed design stage, a supplier must be identified, and the properties of their product (which may differ from another supplier) used in the design calculations. Yet sometimes even this is not good enough. If the component is a critical one (meaning that its failure could, in some sense or another, be disastrous) then it is prudent to conduct in-house tests to measure the critical properties, using samples of the material that will be used to make the product itself.

The materials input does not end with the establishment of production. Products fail in service, and failures contain information. It is an imprudent manufacturer who does not collect and analyse data on failures. Often this points to the misuse of a material, one that redesign or re-selection can eliminate.

So material choice depends on function. But that is not the only constraint.

1.6 Function, Material, Shape and Process

The selection of material cannot be separated from those of process and shape. To make a shape, the material is subjected to processes that, collectively, we shall call *manufacture*: they include primary forming processes (like casting and forging), material removal processes (machining, drilling), joining processes (welding, for example) and finishing processes (such as painting or electro-plating). Function, material, shape and process interact (Fig. 1.6). Function, as we have just seen, influences material choice. Material choice influences that of processes through its ability to be cast or moulded or welded or heat-

treated. Process determines the shape, size, precision and, of course, cost. The interactions are two-way: specification of shape restricts the choice of material and process, but equally the specification of process limits the materials choice and the accessible shapes. The more sophisticated the design, the tighter the specifications and the greater the interactions.

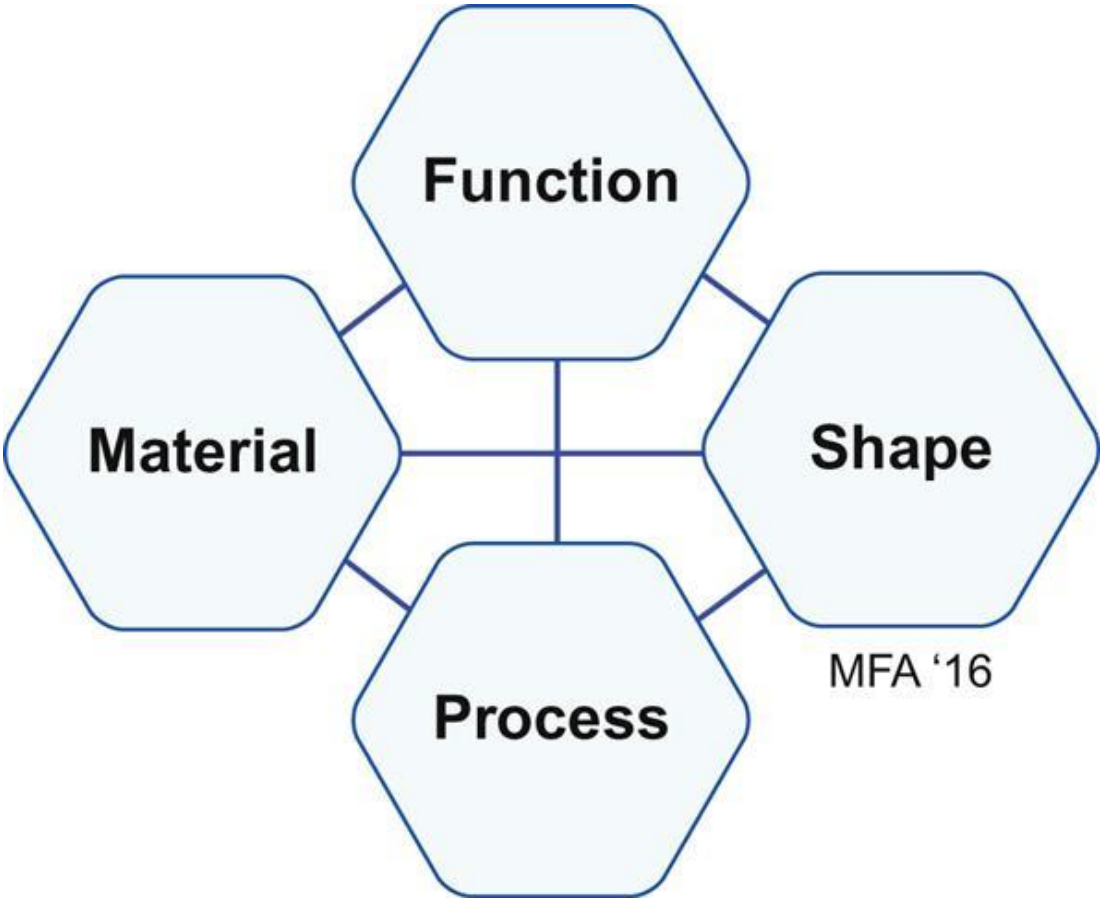


FIGURE 1.6 The central problem of materials the interaction between function, material, process and shape.

The interaction between function, material, shape and process lies at the heart of the material selection process. It is a theme we return to throughout the book, as we visit each of the hexagons of Fig. 1.6 in turn. But first a case study to illustrate the design process.

1.7 Case Study: Devices to Open Corked Bottles

Wine, like cheese, is one of humankind's improvements on nature. And ever since people have cared about wine, they have cared about cork to keep it safely sealed in flasks and bottles. 'Corticium … demovebit amphorae…'- 'Uncork the amphora…' sang Horace[2] (27 BC) to celebrate the anniversary of his miraculous escape from death by a falling tree. But how did he do it?

A corked bottle creates a market need: it is the need to gain access to the wine inside. We might state it thus, 'a device is required to pull corks from wine bottles'. But hold on. The need must be expressed in solution-neutral form, and this is not. The aim is to gain access to the wine; our statement implies that this will be done by removing the cork, and that it will be removed by pulling. There could be other ways. So we will try again: 'a device is required to allow access to wine in a corked bottle' (Fig. 1.7) and one might add, 'with convenience, at modest cost, and without contaminating the wine'.

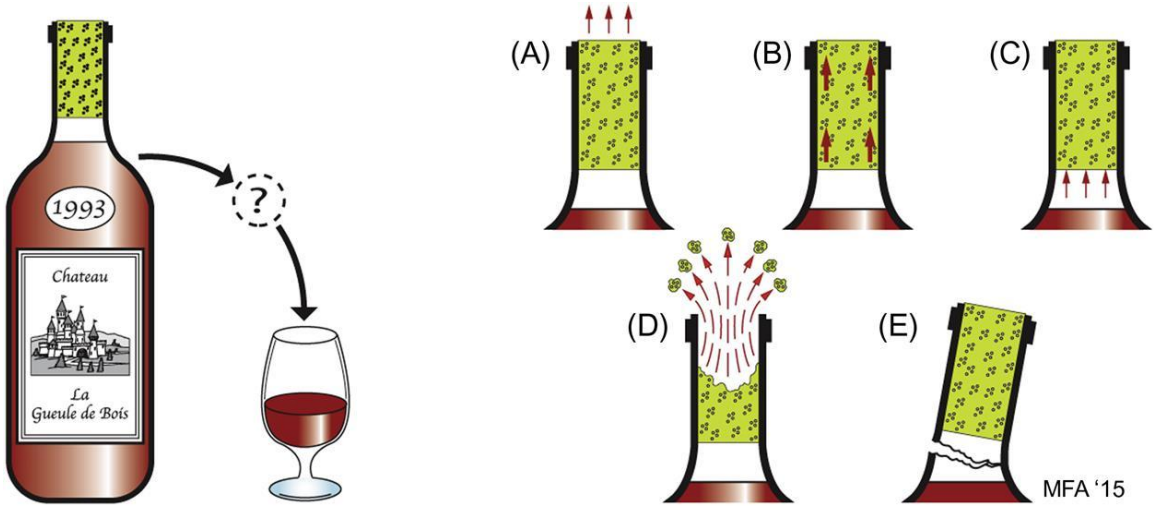


FIGURE 1.7 Left: the market need: a device is sought to allow access to wine contained in a corked bottle. Right: five possible concepts, illustrating physical principles, to fill the need. (A) axial traction, (B) shear traction, (C) pressure from below, (D) comminution and (E) desperation.

Five concepts for doing this are shown on the right of Fig. 1.7. In order, the devices act to remove the cork by axial traction (i.e., pulling); to remove it by applying shear tractions to its sides; to push it out from below; to pulverising it; and to by-pass it altogether by knocking the neck

off the bottle.[3] Numerous devices exist that use the first three of these concepts. The others are used too, though generally only in moments of desperation. We shall eliminate these on the grounds that they might contaminate the wine and examine the others more closely, exploring working principles.

Fig. 1.8 suggests embodiments for the surviving concepts. In the first, a screw is threaded into the cork to which an axial pull is applied; in the second, slender elastic blades inserted down the sides of the cork apply shear tractions when twisted and pulled; and in the third the cork is pierced by a hollow needle through which a gas is pumped to push the cork out. All have been developed to the detailed stage and entered production (Fig. 1.9).

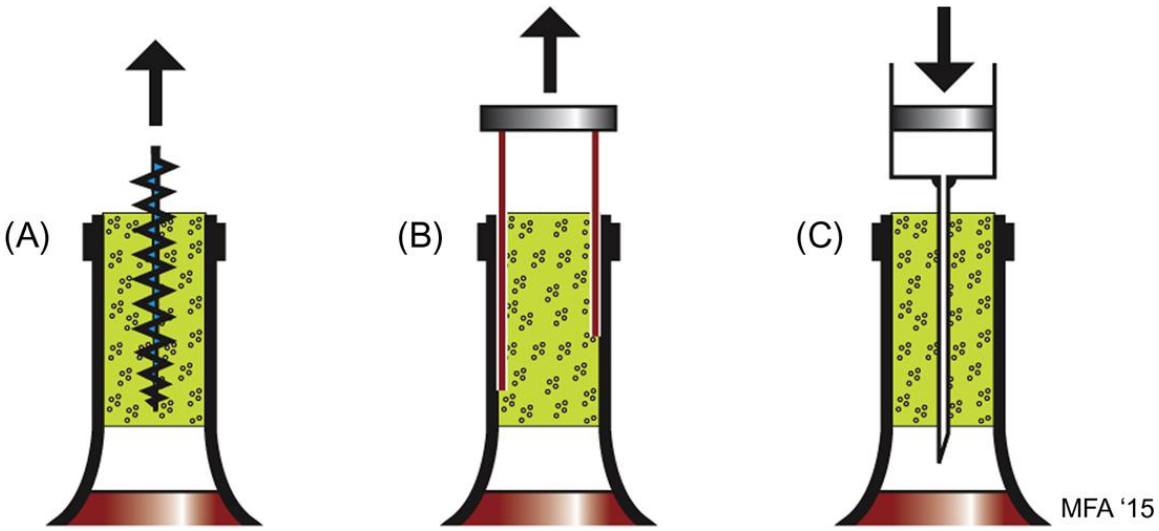


FIGURE 1.8 Working principles for implementing the first three concepts of Fig. 1.7.



FIGURE 1.9 Cork removers: clockwise from top left: levered pull, geared pull, shear traction (sounds weird but it works) and pressurized expulsion (now rare – the risks are too great).

Fig. 1.10, left, shows more detailed embodiments for devices based on just one concept – that of axial traction. The first is a direct pull; the other three use some sort of mechanical advantage – levered-pull, geared pull and spring-assisted pull. The embodiments identify the *functional requirements* of each component of the device, which might be expressed in statements like such as

- a cheap screw to transmit a prescribed load to the cork;
- a light lever (that is, a beam) to carry a prescribed bending moment;
- a slender elastic blade that will not buckle when driven between the cork and bottle neck;
- a thin, hollow needle, stiff, strong and sharp enough to penetrate a cork;

and so on. The functional requirements of each component are the inputs to the materials selection process. They lead directly to the *property limits* and *material indices* of Chapter 4, Materials Selection – the Basics. The procedure developed there takes requirements such as 'light, strong, beam' or 'slender, elastic, blade' and uses them to identify a subset of materials that will perform this function particularly well. The final choice of material and process forms part of the detailed stage of design (Fig. 1.10, right), which leads to a full specification to enable manufacture.

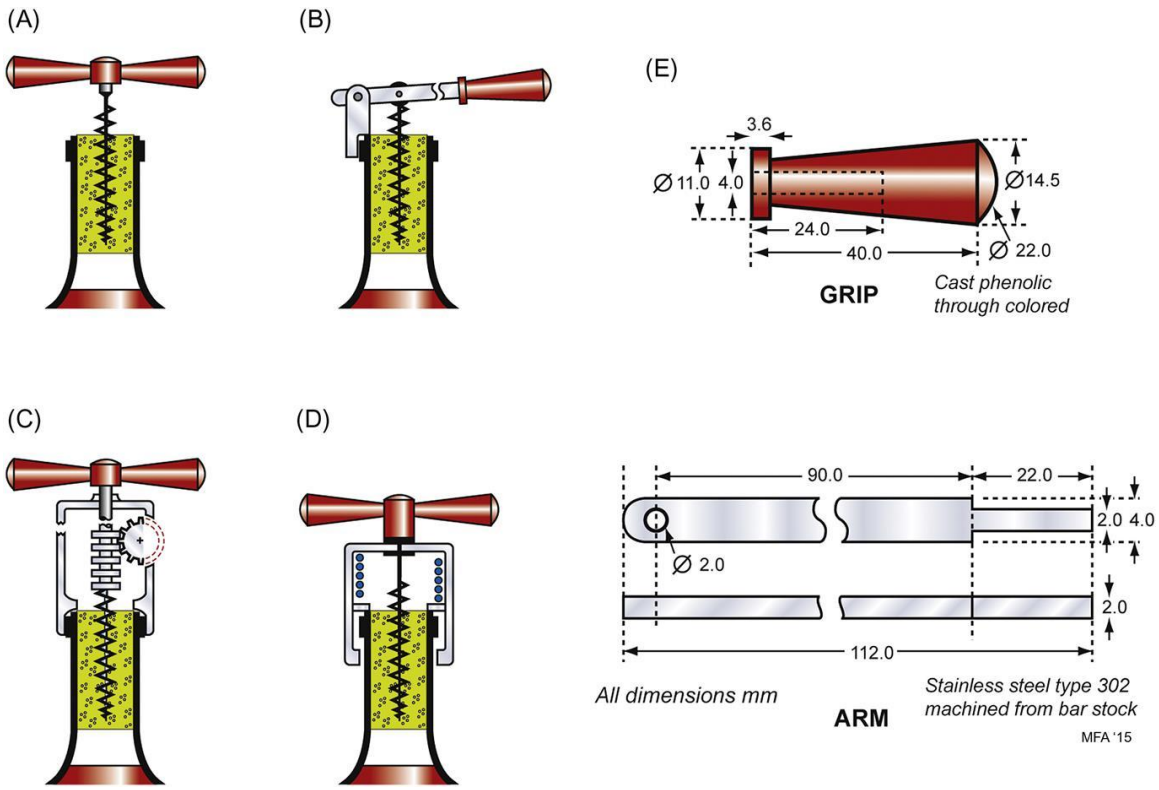


FIGURE 1.10 Left: embodiments (A) direct pull; (B) lever-assisted pull; (C) gear-assisted pull; (D) spring-assisted pull (a spring in the body is compressed as the screw is driven into the cork). Right: detailed design of the lever of embodiment (E), with material choice.

We conclude by returning to the idea of function-structure. That for the cork remover is sketched in the upper part of Fig. 1.11: generate a force, transmit a force, apply force to cork. The alternative designs differ in the working principle by which these functions are achieved, as indicated in the lower part of the figure. Others could be devised by making other links.

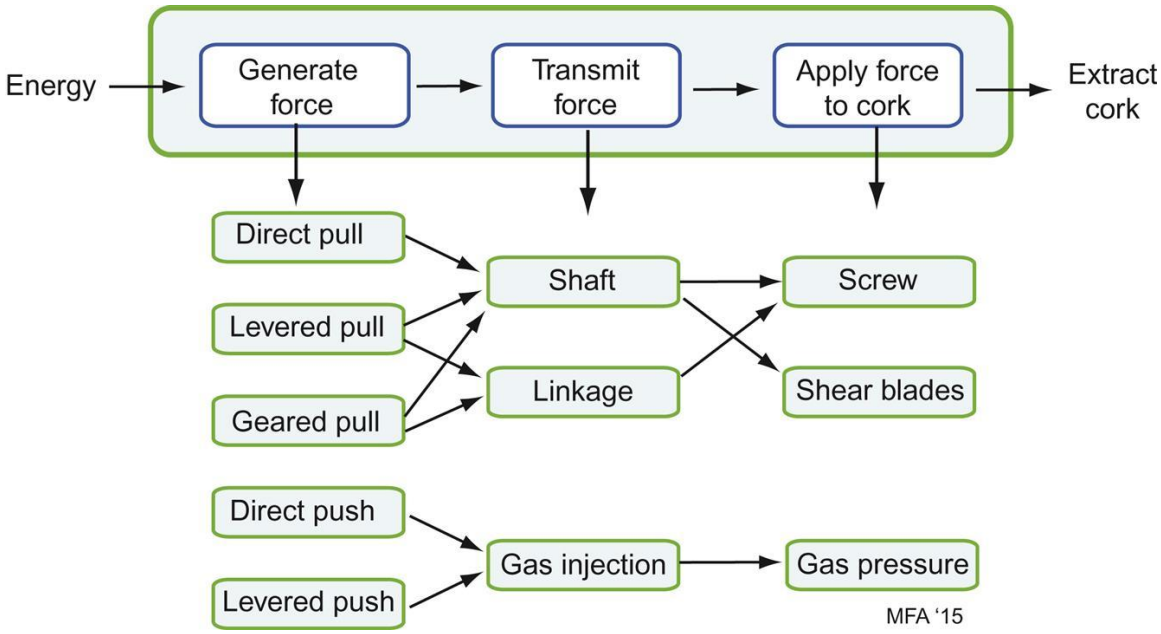


FIGURE 1.11 The function-structure and working principles of cork removers.

1.8 Summary and Conclusions

Design is an iterative process. The starting point is a market need captured in a set of design requirements. Concepts for a product that meet the need are devised. If initial estimates and exploration of alternatives suggest that a concept is viable, the design proceeds to the embodiment stage: working principles are selected, size and layout are decided, and initial estimates of performance and cost are made. If the outcome is successful, the designer continues to the detailed design stage: optimization of performance, full analysis of critical components, preparation of detailed production drawings (usually as CAD files), specification of tolerance, precision, assembly and finishing methods.

Materials selection enters at each stage, but at different levels of breadth and precision. At the conceptual stage all materials and processes are potential candidates, requiring a procedure that allows rapid access to data for a wide range of each, although without the need for great precision. The preliminary selection passes to the embodiment stage, the calculations and optimizations of which require information at a higher level of precision and detail. The design team then eliminates all but a small short-list candidate-materials and processes for the final, detailed

stage of the design. For these few, the need is for data of the highest quality.

Data exist that meet the needs of all these levels. Each level requires its own data-management scheme, described in the following chapters. The management system must be design led, yet must recognise the richness of choice and embrace the complex interaction between the material, its shape, the process by which it is given that shape, and the function it is required to perform. And it must allow rapid iteration – back-looping when a particular path proves to be unprofitable. Tools now exist to help with all of this. We will meet one – the CES EduPack materials and process selection platform – later in this book.

But given this complexity, why not opt for the safe bet: stick to what you used before? Many have chosen that option. Few are still in business.

1.9 Further Reading

A list of general readings is given in the following section; however, a chasm exists between books on design methodology and those on materials selection: each largely ignores the other. The book by French is remarkable for its insights, but the word 'material' does not appear in its index. Pahl and Beitz has near-biblical standing in the design camp, but is heavy going. Ullman and Cross take a more relaxed approach and are easier to digest. On the materials side, the books by Callister, Shackelford, Askeland, Budinski and Budinski and by Farag present the materials case well, but pay less attention to design. The best compromise, perhaps, is Dieter.

General Texts on Design Methodology

1. Cross N. *Engineering Design Methods* fourth ed. Chichester UK: Wiley; 2008; ISBN 978-0-470-51926-4. (*A durable text describing the design process, with emphasis on developing and evaluating alternative solutions*.).
2. Dieter GE, Schmidt LC. *Engineering Design* fifth ed. New York NY: McGraw Hill; 2012; ISBN 978-0073398143. (*A clear introduction from authors with a strong materials background*.).
3. Eggert RJ. *Engineering Design* second ed. Meridian, Idaho: High Peak Press; 2010; ISBN 978-0-615-31938-4. (*An affordable*

introduction to design for 1st and 2nd year undergraduate courses.).
4. French MJ. *Conceptual Design for Engineers* third ed. Berlin, Germany: Springer; 1998; ISBN 978-1852330279. (*The origin of the "Concept – Embodiment – Detail" block diagram of the design process. The book focuses on the concept stage, demonstrating how simple physical principles guide the development of solutions to design problems*.).
5. Pahl, G., Beitz, W. Feldhusen, J., Grote, K.-H., 2007. Engineering design, third ed. (translated by K. Wallace and L. Blessing), Springer Verlag, Berlin, Germany. ISBN 978-1-4471-6025-0. (The Bible – or perhaps more exactly the Old Testament – of the technical design field, developing formal methods in the rigorous German tradition).
6. Ullman DG. *The Mechanical Design Process* fourth ed. New York, NY: McGraw-Hill; 2009; ISBN 978-0072975741. (*An American view of design, developing ways in which an initially ill-defined problem is tackled in a series of steps, much in the way suggested by Figure 1.1 of the present text*.).
7. Ulrich KT. *Design – Creation of Artefacts in Society* Pennsylvania, PA: The University of Pennsylvania; 2011; ISBN 978-0-9836487-0-3. (*An excellent short introduction to the kind of structured reasoning that lies behind good product design. The text is available on* http://www.ulrichbook.org/).
8. Ulrich KT, Eppinger SD. *Product Design and Development* fifth ed. New York, NY: McGraw Hill; 2011; ISBN 978-0073404776. (*A readable, comprehensible text on product design, as taught at MIT. Many helpful examples but almost no mention of materials*.).

General Texts on Materials Selection in Design

1. Ashby M, Shercliff H, Cebon D. *Materials: Engineering, Science, Processing and Design* third ed. Oxford, UK: Butterworth-Heinemann; 2014; ISBN-13: 978-0-08-097773-7, North American Edition: ISBN-13: 978-1-85617-743-6. (*An introductory text introducing ideas that are developed more fully in the present text*.).
2. Askeland DR, Phulé PP, Wright WJ. *The Science and Engineering of Materials* sixth ed. Toronto, Canada: Thomson; 2010; ISBN 9780495296027. (*A well established materials text that deals well with the science of engineering materials*.).

3. Budinski KG, Budinski MK. *Engineering Materials, Properties and Selection* nineth ed. New York, NY: Prentice Hall; 2010; ISBN 978-0-13-712842-6. (*Like Askeland, this is a well established materials text that deals well with both material properties and processes*.).
4. Callister WD, Rethwisch DG. *Materials Science and Engineering: An Introduction* nineth ed. New York, NY: John Wiley & Sons; 2014; ISBN 978-1-118-54689-5. (*A well-established text taking a science-led approach to the presentation of materials teaching*.).
5. Dieter GE. *Engineering Design, a Materials and Processing Approach* third ed. New York, NY: McGraw-Hill; 1999; ISBN: 9-780-073-66136-0. (*A well-balanced and respected text focussing on the place of materials and processing in technical design*.).
6. Farag MM. *Materials and Process Selection for Engineering Design* third ed. London UK: CRC Press, Taylor and Francis; 2013; ISBN 9-781-4665640-9. (*A Materials-Science approach to the selection of materials*.).
7. Lewis G. *Selection of Engineering Materials* Englewood Cliffs, NJ: Prentice-Hall; 1990; ISBN 0-13-802190-2. (*A text on material selection for technical design, based largely on case studies*.).
8. Shackelford JF. *Introduction to Materials Science for Engineers* eighth ed. Englewood Cliffs, NJ: Prentice Hall; 2014; ISBN 978-0133789713. (*A well-established materials text with a materials and a design slant*.).

And on Corks and Corkscrews

1. McKearin H. On 'Stopping', Bottling and Binning. *Int Bottler Packer* 1973;47–54.
2. Perry E. *Corkscrews and Bottle Openers* Aylesbury, UK: Shire Publications Ltd; 1980.
3. Watney BM, Babbige HD. *Corkscrews* London, UK: Sotheby's Publications; 1981; ISBN 0-85667-113-4.

1.10 Exercises

E1.1. ***Concepts (working principles)***. Even if you shut the windows and doors dust settles in rooms like gentle snow. The need: a device to remove dust from household floors. Here are three established concepts:

- human-powered bellows pump with cloth filter;
- axial-flow electric fan with muslin filter; and
- centrifugal fan with centrifugal dust separation and collection.

What other concepts could fill this need? Nothing, at the concept stage, is too far-fetched; decisions about practicality and cost come later.

E1.2. ***Function structures***. Make a function-structure diagram for the Electrolux cleaner of Fig. E1.1.

E1.3. ***Concepts (working principles) for cooling power electronics***. Microchips, particularly those for power electronics, get hot. If they get too hot they cease to function. The need: a scheme for removing heat from power microchips. One such scheme is sketched in Fig. E1.2. Its working principle is that of thermal conduction to fins from which heat is removed by convection.

Four working principles are listed as follows: thermal conduction, convection by heat transfer to a fluid medium, evaporation exploiting the latent heat of evaporation of a fluid, and radiation, best achieved with a surface with high emissivity.

The best solutions may be found by combining two or more of these, such as conduction coupled with convection (as in the sketch – an often-used combination). Consider both the case of long term, continuous-operation heat sinks and the case of short-term, intermittent heat sinks (with pause between operations for system to return to ambient temperature). Suggest designs for efficient heat sinks for these two cases.

E1.4. ***Function structures***. Make a function-structure diagram for the heat sinks.

E1.5. ***Attaching paper sheets***. The need: documents consisting of several pages can become confused if the pages are loose. Use the internet to explore alternative concepts for attaching a small number of pages together in a way that still allows them to be read. Record, as far as possible, the materials of the attachment and the property that provides the attachment force.



FIGURE E1.1 Vacuum cleaners: a hand-powered cleaner from 1880, the Electrolux cylinder cleaner of 1960 and the Dyson centrifugal cleaner of 2010. Early cleaner courtesy Worcester News.

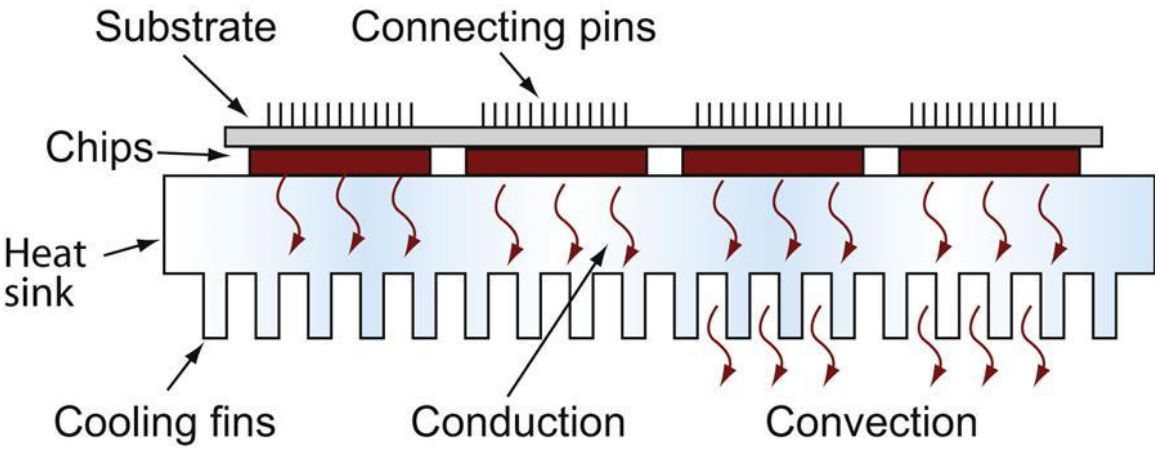


FIGURE E1.2 A heat sink.

[1]Design for Manufacture and Design for Assembly.

[2]Horace, Q. 27 BC, Odes, Book III, Ode 8, line 10.

[3]A Victorian invention for opening old port, the cork of which may become brittle with age and alcohol-absorption, involved ring-shaped tongs. The tongs were heated red on an open fire, then clamped onto the cold neck of the bottle. The thermal shock removed the neck cleanly and neatly.

CHAPTER 2

Engineering Materials and Their Properties

Abstract

This chapter introduces the families of materials – metals, polymers, ceramics and hybrids – and the information about them that is necessary for their selection. Selection is based on a search for a certain *profile of properties* – the profile that best meets the needs of the design. The chapter sets up the classification structure that is used throughout the book.

Keywords

Material families; metals; polymers; ceramics; glasses; composites; foams; mechanical properties; thermal properties; electrical properties; magnetic properties; environmental properties

2.1 Introduction and Synopsis

Materials, one might say, are the food of design. This chapter presents the menu: the materials shopping list. A successful product – one that performs well, is good value for money and gives pleasure to the user – uses the best materials for the job, and fully exploits their potential and characteristics. Brings out their flavour, so to speak.

The families of materials – metals, polymers, ceramics, and so forth – are introduced in Section 2.2. What do we need to know about them if we are to design with them? That is the subject of Section 2.3, in which distinctions are drawn between various types of materials information. However, it is not, in the end, a *material* that we seek; it is a certain *profile of*

properties – the profile that best meets the needs of the design. Properties are the currency of the materials world. They are the bargaining chips – the way you trade off one material against another. The properties important in thermo-mechanical design are defined briefly in Section 2.4. It makes boring reading. The reader who is confident in the definitions and units of moduli, strengths, damping capacities, thermal and electrical conductivities and the like, may wish to skip this, using it for reference, when needed, for the precise meaning and units of the data in the Property Charts that come later. Do not, however, skip Sections 2.2. It sets up the classification structure that is used throughout the book.

This chapter – and particularly the exercises at the end of it – serve another purpose. It is to introduce two of the appendices that you will find at the end of the book. The first is a compilation of material properties set out in a consistent way that makes it easy to use. The second is compilation of solutions to standard problems of mechanics and heat flow. Fluency in their use is a real asset; it gives the confidence that comes when, although you do not know the answer, you do know where to find it. We will draw on both of these appendices repeatedly throughout the chapters that follow.

The chapter ends, in the usual way, with a summary.

2.2 The Families of Engineering Materials

It is conventional to classify the materials of engineering into the six broad families shown in Fig. 2.1: metals, polymers, elastomers, ceramics, glasses and hybrids. The members of a family have certain features in common: similar properties, similar processing routes, and, often, similar applications.

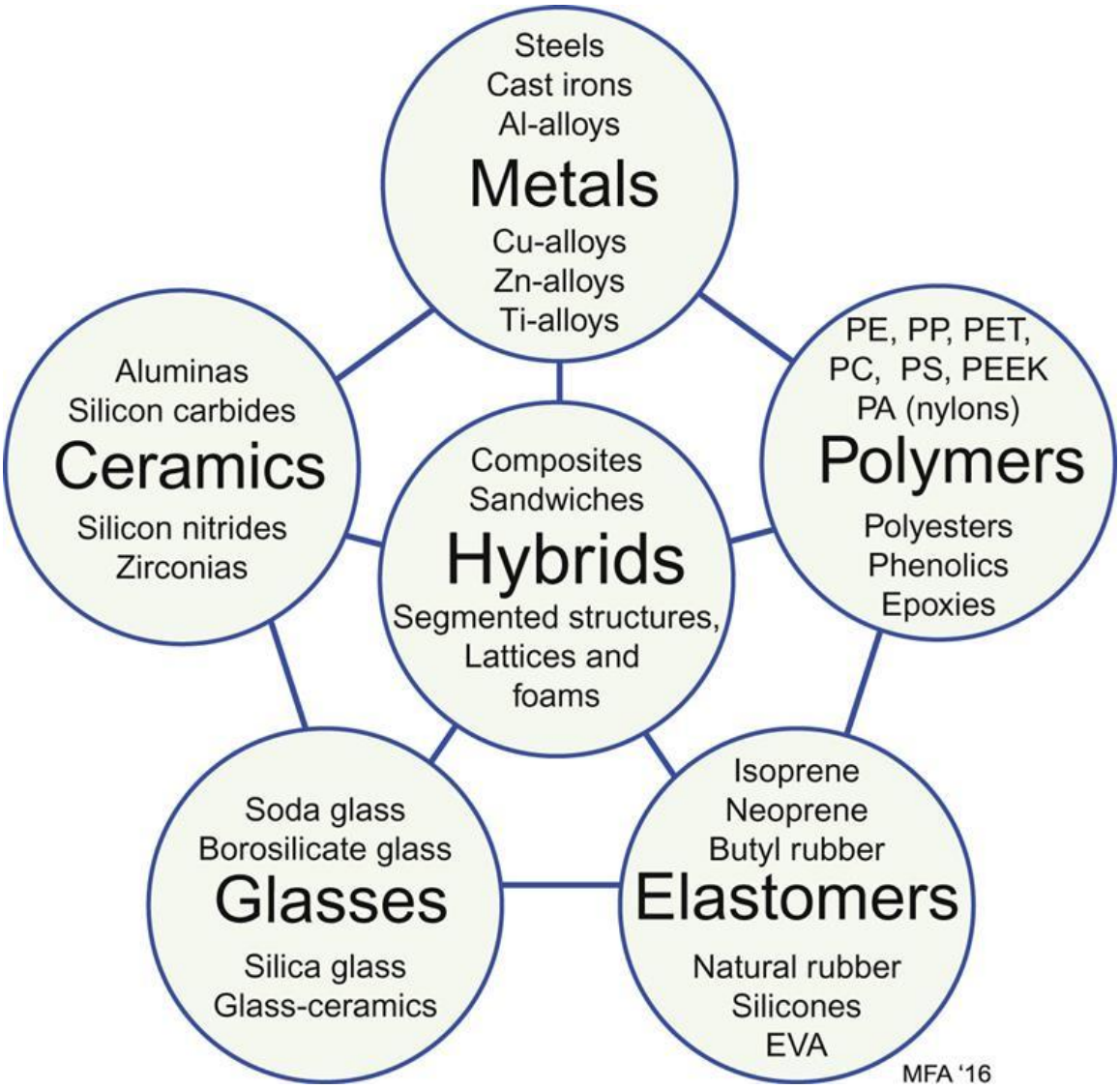


FIGURE 2.1 The menu of engineering materials. The basic families of metals, ceramics, glasses, polymers and elastomers can be combined in various geometries to create hybrids.

Metals encompass most of the elements of the periodic table. Metals are stiff – they have relatively high elastic moduli. They are good conductors of heat and electricity and, when pure, are soft and easily deformed. They can be made strong by alloying and by mechanical and heat treatment, but they remain ductile, allowing them to be formed by deformation processes. Certain high-strength alloys (spring steel, for instance) have ductilities as low as 1%, but even this is enough to ensure that the material yields before it fractures and that fracture, when it occurs, is of a tough,

ductile type. Partly because of their ductility, metals are prey to fatigue and of all material classes, they are the least resistant to corrosion.

Ceramics, too, have high moduli, but, unlike metals, they are brittle. Their 'strength' in tension means the brittle fracture strength; in compression, it is the brittle crushing strength, which is about 15 times greater. Because ceramics have no ductility, they have a low tolerance for stress concentrations (like holes or cracks) or for high contact stresses (at clamping points, for instance). Ductile materials accommodate stress concentrations by deforming in a way that redistributes the load more evenly, and because of this, they can be used under static loads within a small margin of their yield strength. Ceramics cannot. Brittle materials always have a wide scatter in strength and the strength itself depends on the volume of material under load and the time over which it is applied. So design with ceramics is more difficult than design with metals. Despite this, they have attractive features. They are stiff, hard and abrasion-resistant (hence their use for bearings and cutting tools); they retain their strength to high temperatures (they are used as thermal-barrier coatings on aircraft turbine blades); and they resist corrosion well (thus their use as toilets, basins and kitchen work-surfaces).

Glasses are noncrystalline ('amorphous') solids. The most common are the soda-lime and borosilicate glasses familiar as bottles and ovenware, but there are many more. Metals, too, can be made noncrystalline by cooling them sufficiently quickly. The lack of crystal structure suppresses plasticity, so, like ceramics, metallic glasses are hard, brittle and vulnerable to stress concentrations.

Polymers are at the other end of the spectrum. They have moduli that are low, roughly 50 times less than those of metals, but they can be strong – nearly as strong as metals. A consequence of this is that elastic deflections can be large. They creep, even at room temperature, meaning that a polymer component under load may, with time, acquire a permanent set. Moreover, their properties depend on temperature so that a polymer that is tough and flexible at 20°C may creep rapidly at the 100°C of boiling water yet be brittle at the 4°C of a household refrigerator. Few have useful strength above 200°C. Some polymers are partly crystalline; others are amorphous (noncrystalline) – the transparent ones are amorphous. If these aspects are allowed for in the design, the advantages of polymers can be exploited, and there are many. When combinations of properties, such as strength-per-unit-weight, are important, polymers can compete with metals. They are easy to shape. Complicated parts performing several

functions can be moulded from a polymer in a single operation. The large elastic deflections allow the design of polymer components that snap together, making assembly fast and cheap. By accurately sizing the mould and precolouring the polymer, no finishing operations are needed. Polymers resist corrosion (paints, for instance, are polymers) and have low coefficients of friction. Good design exploits these properties.

Elastomers are long-chain polymers above their glass-transition temperature, T_g. The covalent bonds that link the units of the polymer chain remain intact, but the weaker Van der Waals and hydrogen bonds that, below T_g, bind the chains to each other, have melted. This gives elastomers unique properties: their elastic moduli are as low as 10^{-3} GPa (10^5 times less than that typical of metals) and they are capable of enormous elastic extension. Their properties differ so much from those of other solids that special tests have evolved to characterize them. This creates a problem: if we wish to select materials by prescribing a desired attribute profile, as we do later in this book, then a prerequisite is a set of attributes common to all materials. To overcome this, we use a common set of properties in the early stages of design, and estimate approximate values for anomalies like elastomers. Specialised attributes, representative of one family only, are for use in the later stages.

Hybrids are combinations of two or more materials in a predetermined configuration and scale (the centre circle of Fig. 2.1). Their design is the subject of Chapter 12, Designing Hybrid Materials and Chapter 13, Hybrids: Case Studies. The best of them combine the attractive properties of the other families of materials while avoiding some of the drawbacks. The family of hybrids includes fibre and particulate composites; sandwich structures; lattice structures; foams; cables and laminates; and almost all the materials of nature – wood, bone, skin, leaf. Fibre-reinforced composites are, of course, the most familiar. Most of those at present available to the engineer have a polymer matrix reinforced by fibres of glass, carbon or Kevlar (an aramid). They are light, stiff and strong, and they can be tough. They, and other hybrids using a polymer as one component, cannot be used above 250°C because the polymer softens, but at room temperature their performance can be outstanding. Hybrid components are expensive and they are relatively difficult to form and join. So, despite their attractive properties, the designer will use them only when the added performance justifies the added cost. Today's emphasis

on vehicle fuel efficiency and performance provides increasing incentives to use them.

These, then, are the material families. What do we need to know about them?

2.3 Materials Information for Design

The engineer, in selecting materials for a developing design, needs data for their properties. Engineers are often conservative in their choice, reluctant to consider materials with which they are unfamiliar, and with good reason. Data for the old, well-tried materials are established, reliable, and easily found. Data for newer, emerging, materials may be incomplete or untrustworthy. Yet innovation is often made possible by new materials. So it is important to know how to judge data quality.

If you are going to design something, what sort of materials information do you need? Fig. 2.2 draws relevant distinctions. On the left a material is tested and the *raw data* are captured. Yet this raw data – unqualified numbers – are, for our purposes, useless. To make data useful requires statistical confidence. What is the mean value of the property when measured on a large batch of samples? What is the standard deviation? Given this information, it is possible to calculate *allowables*, values of properties that, with a given certainty (say, one part in 10^6) can be guaranteed. Material texts generally present test data; by contrast, data in most engineering handbooks are allowables. One can think of data with known precision and provenance as *information*. This information is generally reported as tables of numbers, as yes/no statements or as rankings: that is, it can be *structured*. Many attributes that can be structured are common to all materials; all, for instance, have a density, an elastic modulus, a strength, a thermal conductivity. Structured information can be stored in a database and – since all materials have values – it is the starting point for selecting between them. Data for material properties (Appendix A) are reported as ranges that reflect the differences in the way different producers make it.

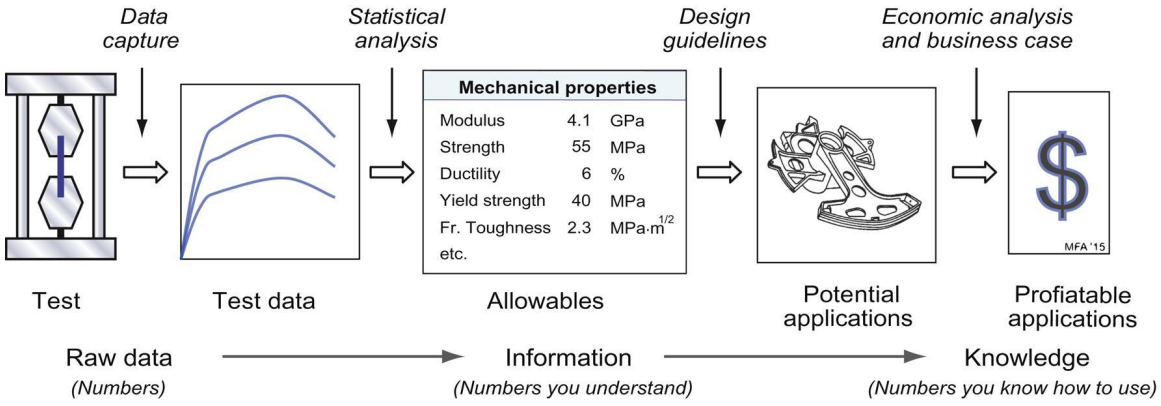


FIGURE 2.2 Types of material information.

This is a step forward, but it is not enough. To design with a material, you need to know its real character, its strengths and its weaknesses. How do you shape it? How join it? Who has used it before and for what? Did it fail? Why? This information is documented in handbooks as design guidelines, failure analyses and case studies. It consists largely as text, graphs and images, and while certain bits of it may be available for one material, for another they may not. It is messier, but it is essential in reaching a final selection. We refer to this supporting information as *documentation*. The image and text on the right of the ABS cover-record are examples of documentation.

There is more. Material uses are subject to standards and codes. These rarely refer to a single material but to classes or subclasses. For a material to be used in contact with food or drugs, it must carry FDA approval or the equivalent. Metals and composites for use in US military aircraft must have Military Specification qualification. The ISO 14040 set of standards stipulates eco-requirements required to meet ISO approval. And so forth. This, too, is a form of documentation.

There is yet more (Fig. 2.2, right-hand side). To succeed and be profitable in the market place, a product must be economically viable and compete successfully with the competition in terms of performance, consumer appeal and cost. All of these depend on material choice and the way the material is processed. Much can be said about this, but not here; for now the focus is on structured data and documentation.

That's the essential background. Now for the properties themselves.

2.4 Material Properties and Their Units

Material have properties. The property-set that characterizes a given material is its *property profile*. Property profiles are assembled by systematic testing. In this section we scan the nature of the tests and the definition and units of the properties. Units are given here in the SI system. Conversion factors to other systems are printed on the inside front and back cover of this book.

General Properties

Property	Symbol and Units
Density	ρ (kg/m^3)
Price	C_m ($/kg)

Density. The *density*, ρ (units: kg/m^3) is the mass per unit volume. We measure it today as Archimedes did: by weighing in air and in a fluid of known density.

Price. The price, C_m, (units: $/kg) of materials spans a wide range. Some cost as little as $0.2/kg, others more than $1000/kg (based on 2016 USD). Prices, of course, fluctuate, and they depend on the quantity you want and on your status as a 'preferred customer' with your chosen vendor. Despite this uncertainty, it is useful to have an approximate price in the early stages of material selection. You will find them in Appendix A.

Mechanical Properties

Property	Symbol and Units
Elastic moduli (Young's, shear, bulk)	E, G, K (GPa)
Yield strength	σ_y (MPa)
Tensile (ultimate) strength	σ_{ts} (MPa)
Compressive strength	σ_c (MPa)
Failure strength	σ_f (MPa)
Hardness	H (Vickers)
Elongation	ε (–)
Fatigue endurance limit	σ_e (MPa)
Fracture toughness	K_{1c} (MPa·m$^{1/2}$)
Toughness	G_{1c} (kJ/m^2)
Loss coefficient (damping capacity)	η (–)
Wear rate (archard) constant	K_A (MPa^{-1})

Elastic modulus. The elastic modulus E (units: GPa or GN/m^2) is the slope of the initial, linear-elastic, part of the stress-strain curve (Fig. 2.3). Young's modulus, E, describes response to tensile or compressive loading; the shear modulus, G, describes response to shear loading; and the bulk modulus, K, response to hydrostatic pressure. Poisson's ratio, ν, is the negative of the ratio of the lateral strain, ε_2, to the axial strain, ε_1, in axial loading:

$$\nu = -\frac{\varepsilon_2}{\varepsilon_1}$$

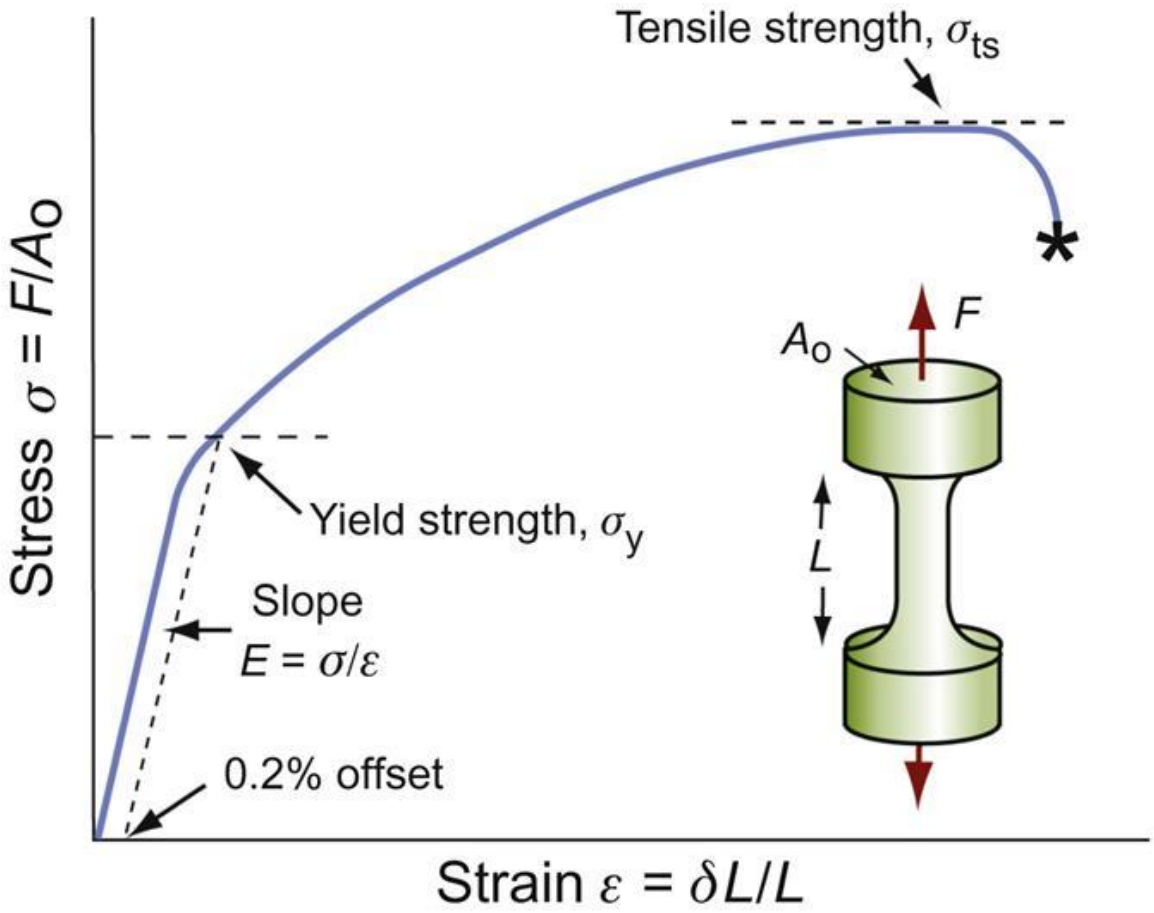


FIGURE 2.3 A stress-strain curve showing the modulus, E, the 0.2% yield strength, σ_y, and the ultimate tensile strength σ_{ts}.

In reality, moduli measured as slopes of stress-strain curves are inaccurate, often low by a factor of 2 or more, because of contributions to the strain from anelasticity, creep and other factors. Accurate moduli are measured dynamically by measuring the velocity of sound waves in the material or by exciting the natural vibrations of a beam or wire.

In an isotropic material, the moduli are related in the following ways:

$$E = \frac{3G}{1 + G/3K}; \quad G = \frac{E}{2(1+\nu)}; \quad K = \frac{E}{3(1-2\nu)} \tag{2.1}$$

Commonly $\nu \approx 1/3$ when

$$G \approx \frac{3}{8}E \quad \text{and} \quad K \approx E \tag{2.2a}$$

Elastomers are exceptional. For these $\nu \approx 1/2$ when

$$G \approx \frac{1}{3}E \quad \text{and} \quad K \gg E \tag{2.2b}$$

In this book we examine data for E; approximate values for the other moduli can be derived from Eqs (2.2a) and (2.2b) when needed.

Estimating Moduli

Young's modulus E for copper is 124 GPa.; its Poisson's ratio ν is 0.345. What is its shear modulus, G?

Answer

Inserting the values for E and ν in the central Eq. (2.1) gives G=46.1 GPa. The measured value is 45.6 GPa, a difference of only 1%.

Failure strength. The failure strength, σ_f, (units: MPa or MN/m^2) of a solid requires careful definition. For metals, we identify σ_f with the 0.2% offset yield strength σ_Y, that is, the stress at which the stress-strain curve for axial loading deviates by a strain of 0.2% from the linear-elastic line (Fig. 2.3). It is the same in tension and compression. For polymers, σ_f is identified as the stress at which the stress-strain curve becomes markedly nonlinear, at a strain, typically, of 1%. This may be caused by shear-yielding, the irreversible slipping of molecular chains, or it may be caused by crazing, the formation of low density, crack-like volumes that scatter light, making the polymer look white. Polymers are a little stronger in compression than in tension by a factor of about 1.2.

Strength, for ceramics and glasses, depends strongly on the mode of loading. In tension, 'strength' means the fracture strength, σ_t. In compression it means the crushing strength σ_c, which is much greater; typically

$$\sigma_c = 10 \text{ to } 15\ \sigma_t \tag{2.3}$$

When a material is difficult to grip, as is a ceramic, its strength can be measured in bending. The flexural strength or modulus of rupture, σ_{flex} (units: MPa) is the maximum surface stress in a bent beam at the instant of failure (Fig. 2.4). One might expect this to be the same as the strength measured in tension, but for ceramics it is greater by a factor of about 1.3 because the volume subjected to this maximum stress is small and the probability of a large flaw lying in it is small also. In simple tension all flaws see the same stress.

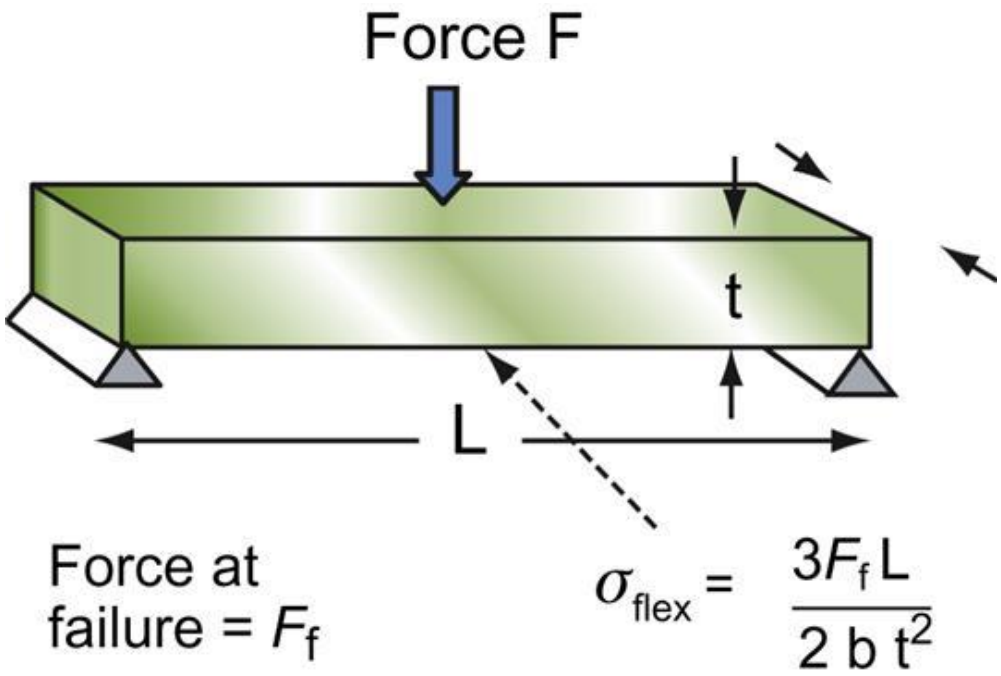


FIGURE 2.4 The modulus of rupture (MOR) is the surface stress at failure in bending.

The strength of a composite is best defined by a set deviation from linear-elastic behaviour; often an offset of 0.5% is taken. Composites that contain fibres, including natural composites such as wood, are a little weaker (up to 30%) in compression than tension because fibres buckle. In subsequent chapters, σ_f for composites means the tensile strength.

Strength, then, depends on material class and on mode of loading. Other modes of loading are possible: shear, for instance. Yield under multiaxial loads is related to that in simple tension by a *yield function*. For metals, the Von Mises' yield function is a good description:

$$(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2=2\sigma_f^2 \tag{2.4}$$

where σ_1, σ_2 and σ_3 are the principal stresses, positive when tensile; σ_1, by convention, is the greatest or most positive, σ_3 the smallest or least positive.

Sound travels through solids as an elastic wave. The longitudinal wave speed, v, is

$$v=\sqrt{E/\rho} \tag{2.5}$$

Using Yield Functions

A metal pipe of radius r and wall thickness t carries an internal pressure p. The pressure generates a circumferential wall stress of $\sigma_1 = p\,r/t$ and an axial wall stress of $\sigma_2 = p\,r/2t$. At what pressure will the pipe first yield?

Answer

Setting $\sigma_2=\sigma_1/2,\ \ \sigma_3=0$ and $\sigma_f=\sigma_Y$ in Eq. (2.4) gives the yield condition $\sigma_1=(2/\sqrt{3})\sigma_Y$. Thus the pressure p^* that just causes first yield is $p^*=\dfrac{2}{\sqrt{3}}\dfrac{t}{r}\sigma_Y$.

Tensile strength. The *tensile* (or *ultimate*) *strength* σ_{ts} (units: MPa) is the nominal stress at which a round bar of the material, loaded in tension, separates (Fig. 2.3). For brittle solids – ceramics, glasses and brittle polymers – it is the same as the failure strength in tension. For metals, ductile polymers and most composites, it is greater than the yield strength, σ_Y, by a factor of between 1.1 and 3 because of work hardening or, in the case of composites, because of load transfer to the reinforcement.

Fatigue strength. Cyclic loading can cause a crack to nucleate and grow in a material, culminating in fatigue failure. For many materials there exists a fatigue or *endurance limit*, σ_e (units: MPa), illustrated by the $\Delta\sigma-N_f$ curve of Fig. 2.5. It is the stress amplitude $\Delta\sigma$ below which fracture does not occur, or occurs only after a very large number ($N_f>10^7$) of cycles. The relationship of the stress range $\Delta\sigma_{\sigma m}$ for failure under a mean stress σ_m, to that for failure at zero mean stress $\Delta\sigma_{\sigma 0}$ is

$$\Delta\sigma_{\sigma m} = \Delta\sigma_{\sigma 0}\left(1 - \frac{\sigma_m}{\sigma_{ts}}\right) \tag{2.6}$$

where σ_{ts} is the tensile stress, giving a correction to the stress range. Increasing σ_m reduces the permissible range of superimposed cyclic stress.

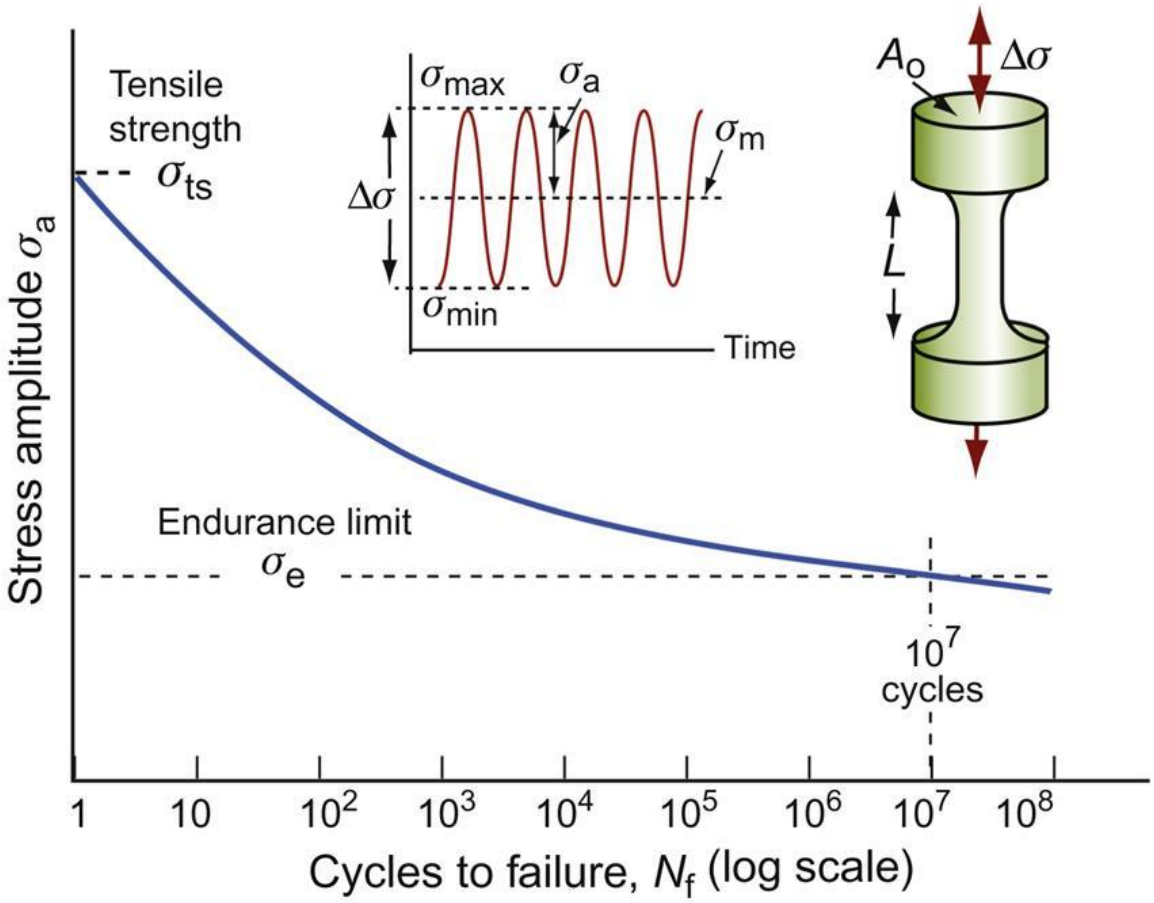


FIGURE 2.5 The endurance limit, σ_e, is the cyclic stress that causes failure in N_f=10^7 cycles.

Hardness. Tensile and compression tests are not always convenient: a large sample is needed and the test destroys it. The hardness test gives an approximate, nondestructive, measure of the strength. The *hardness*, *H*, (SI units: MPa) of a material is measured by pressing a pointed diamond or hardened steel ball into the surface of the material. The hardness is defined as the indenter force divided by the area of the indent projected normal to the direction of indentation. It is related to the quantity we have defined as σ_f by

$$H \approx 3\sigma_f \tag{2.7}$$

This, in the SI system, has units of MPa. Hardness is commonly reported in a bewildering array of other units, the most common of which is the Vickers H_v scale with units of kg/mm^2. It is related to H in the units used here by

$$H_v = \frac{H}{10} \tag{2.8}$$

A conversion chart for six hardness scales, relating them to yield strength (the last column), appears in Fig. 2.6.

Strength From Hardness

A steel has a hardness of 50 on the Rockwell C scale. Approximately what is its Vickers hardness and yield strength?

Answer

Fig. 2.6 shows that the Vickers hardness corresponding to a Rockwell C value of 50 is approximately $H_v = 500$ and the yield strength is approximately 1700 MPa.

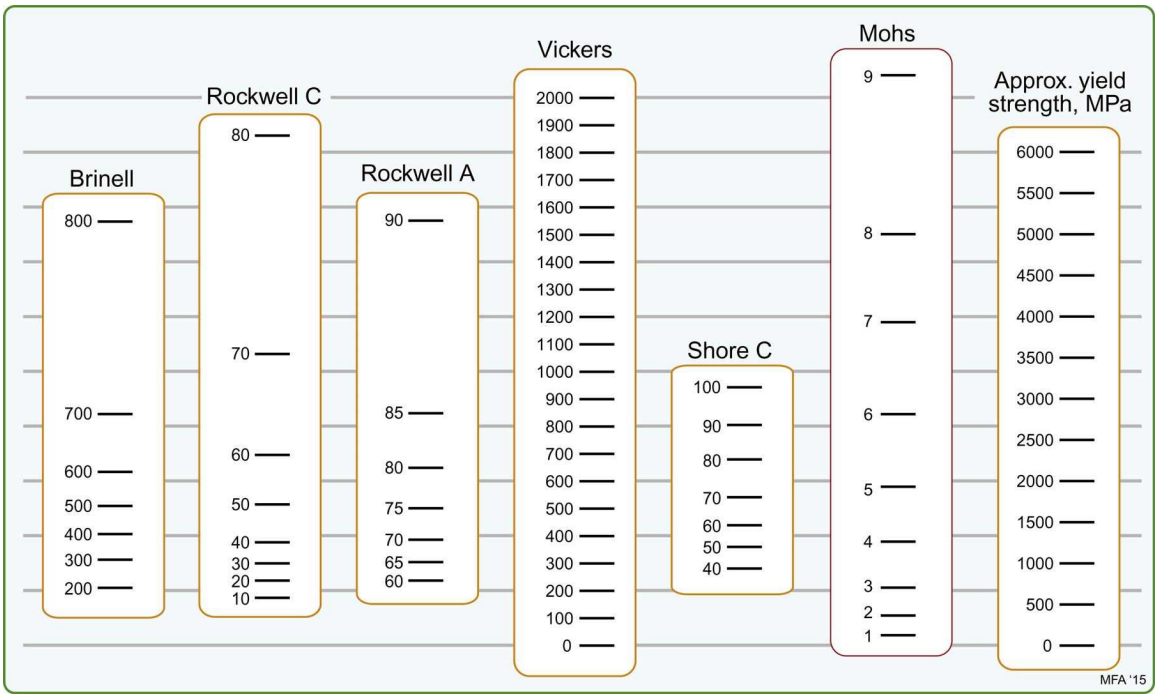


FIGURE 2.6 Commonly used scales of hardness related to each other and to the yield strength.

Toughness and fracture toughness. The *toughness*, G_{1c}, (units: kJ/m^2) and the *fracture toughness*, K_{1c}, (units: MPa m$^{1/2}$ or MN/m$^{1/2}$) measure the resistance of a material to the propagation of a crack. The fracture toughness is measured by loading a sample containing a deliberately introduced sharp crack of length 2c (Fig. 2.7), and then recording the tensile stress $\sigma*$ at which the crack propagates. The quantity K_{1c} is then calculated from

$$K_{1c} = Y\sigma * \sqrt{\pi c} \tag{2.9}$$

and the toughness from

$$G_{1c} = \frac{K_{1c}^2}{E(1+\nu)} \tag{2.10}$$

where Y is a geometric factor, near unity, that depends on details of the sample geometry, E is Young's modulus and ν is Poisson's ratio. Measured in this way K_{1c} and G_{1c} have well defined values for brittle materials (ceramics, glasses and many polymers). In ductile materials a plastic zone

develops at the crack tip, which introduces new features into the way in which cracks propagate that necessitate more involved characterisation. Values for K_{1c} and G_{1c} are, nonetheless, cited, and are useful as a way of ranking materials.

Using Fracture Toughness

A glass floor panel contains microcracks up to 2 microns in length. Glass has a fracture toughness of K_{1c}=0.6 MPa m$^{1/2}$. When the panel is walked upon, stresses as high as 30 MPa appear in it. Is it safe?

Answer

The stress required to make a 2 micron crack (so c=10^{-6} m) propagate in glass with a fracture toughness of K_{1c}=0.6 MPa m$^{1/2}$, using Eq. (2.9) with Y=1, is

$$\sigma_c = K_{1c}/\sqrt{\pi c} = 339 \text{ MPa}.$$

The panel is safe.

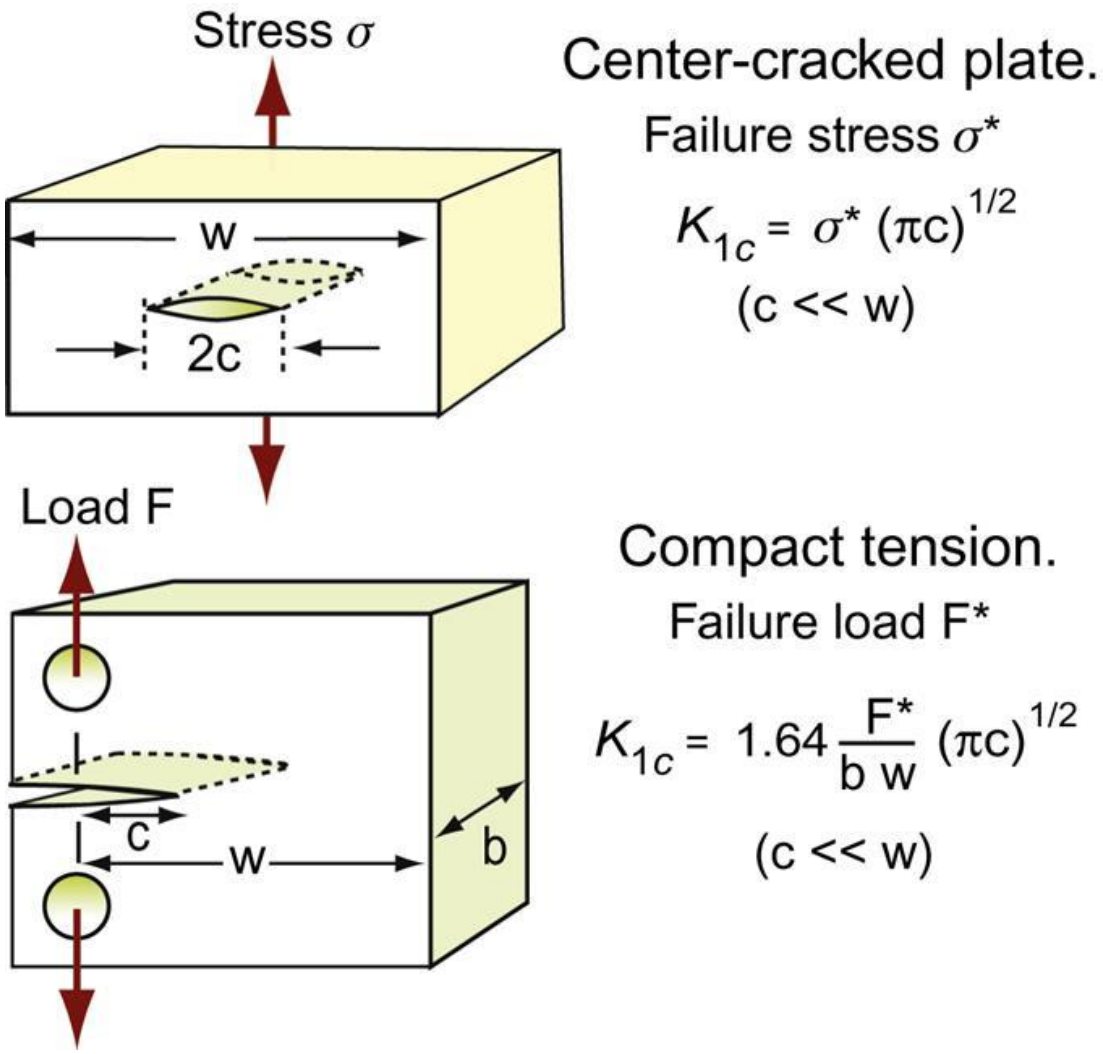


FIGURE 2.7 Measuring fracture toughness, K_{1c}.

Mechanical loss coefficient. The *loss coefficient* or *mechanical damping*, η (a dimensionless quantity) measures the degree to which a material dissipates vibrational energy. If a material is loaded elastically to a stress σ_{max}, it stores an elastic energy

$$U = \int_0^{\sigma_{max}} \sigma d\varepsilon \approx \frac{1}{2}\frac{\sigma_{max}^2}{E}$$

per unit volume. If it is loaded and then unloaded, it dissipates an energy

$$\Delta U = \oint \sigma \, d\varepsilon$$

The loss coefficient is

$$\eta = \frac{\Delta U}{2\pi U_{max}} \qquad \textbf{(2.11)}$$

where U_{max} is the stored elastic energy at peak stress. The value of η usually depends on the time-scale or frequency of cycling.

Other measures of damping include the *specific damping capacity*, $D = \Delta U/U$; the *log decrement*, Δ (the log of the ratio of successive amplitudes of natural vibrations); the *phase-lag*, δ, between stress and strain; and the *'Q'-factor* or *resonance factor*, Q. When damping is small (η<0.01) these measures are related by

$$\eta = \frac{D}{2\pi} = \frac{\Delta}{\pi} = \tan\delta = \frac{1}{Q} \qquad \textbf{(2.12)}$$

but when damping is large, they are no longer equivalent.

Using Loss Coefficients

A bell with a natural frequency of f=1000 Hz is made of a material with a loss coefficient $\eta = 0.01$. For how long will it ring after being struck? If the material is replaced by one of lower damping with $\eta = 10^{-4}$ how long will it ring? (Assume the ring has ended when the amplitude of oscillation A has fallen to one hundredth of its initial value).

Answer

Let A and $A+dA$ be amplitudes of successive cycles (dA is negative). Then

$$\mathrm{Log}\left(\frac{A}{A+dA}\right) = \Delta = \pi\eta \text{ from which } \frac{dA}{Adn} = \frac{1}{10^{\pi\eta}} - 1$$

Integrating over n cycles gives $\ln\frac{A}{A_0} = \left(\frac{1}{10^{\pi\eta}} - 1\right)n$ where A_0 is the initial amplitude. When A has fallen to $0.01A_0$, the term $\ln(A/A_0) = -4.6$, giving

$n = 4.6\left(\frac{10^{\pi\eta}}{10^{\pi\eta}-1}\right)$. Thus a bell with $\eta = 0.01$ will ring for n=66 cycles, giving a time n/f=66 milliseconds. A bell with $\eta = 10^{-4}$ will ring for $n = 6400$ cycles and a time of $n/f = 6.4$ seconds.

Wear rate. Wear, the loss of material when surfaces slide against each other, is a multibody problem – it depends on both the sliding material and the material on which it is slid. Nevertheless, wear can, to a degree, be quantified. When solids slide the volume of material lost from one surface, per unit distance slid, is called the wear rate, W (units: m^3/m, or m^2). The wear resistance of the surface is characterised by the *Archard wear constant*, K_A (units: 1/MPa) defined by the equation

$$\frac{W}{A} = K_A P \tag{2.13}$$

where A is the area of the slider surface and P the pressure (force per unit area) at the contacting surfaces.

Calculating Wear

A steel slider oscillates on a dry steel substrate at frequency f=0.2 Hz and an amplitude a=2 mm under a normal pressure P=2 MPa. The Archard wear constant for steel on steel is K_A=3×10^{-8} (MPa)$^{-1}$. By how much will the surface of the slider have been reduced in thickness after a time t=100 hours?

Answer

The distance slid in 100 hours is $d = 4\,a\,f\,t$ m. The thickness x removed from the slider over the time $t = 3.6 \times 10^5$ s is

$$x = \frac{\text{Volume removed}}{\text{Area } A} = 4\,a\,f\,t\,K_A\,P = 3.5 \times 10^{-5}\ \text{m} = 36\ \mu\text{m}$$

Thermal Properties

Property	Symbol and Units
Melting point	T_m (°C or K)
Glass temperature	T_g (°C or K)
Maximum service temperature	T_{max} (°C or K)
Minimum service temperature	T_{min} (°C or K)
Thermal conductivity	λ (W/m·K)
Specific heat	C_p (J/kg·K)
Thermal expansion coefficient	α (K^{-1})
Thermal shock resistance	ΔT_s (°C or K)

Melting point and glass temperature. Two temperatures, the *melting temperature*, T_m, and the *glass temperature*, T_g (units for both: K or °C) are fundamental because they relate directly to the strength of the bonds in the solid. Crystalline solids have a sharp melting point, T_m. Noncrystalline solids do not; the temperature T_g characterises the transition from true solid to very viscous liquid.

Use-temperature range. It is helpful, in engineering design, to define two further temperatures: the *maximum* and *minimum service temperature* T_{max} and T_{min} (both: K or °C). The first tells us the highest temperature at which the material can reasonably be used without oxidation, chemical change or excessive creep becoming a problem. The second is the temperature below which the material becomes brittle or otherwise unsafe to use.

Specific heat capacity. It costs energy to heat a material up. The *heat capacity* or *specific heat* (units J/kg·K) is the energy required to heat 1 kg of a material by 1 K. The measurement is usually made at constant pressure (atmospheric pressure) so it is given the symbol C_p. When dealing with gases, it is more usual to measure the heat capacity at constant volume (symbol C_v) and for gases this differs from C_p. For solids the difference is so slight that it can be ignored, and we shall do so here. The heat capacity is measured by calorimetry which is also the standard way of measuring the glass temperature T_g. A measured quantity of energy (usually, electrical energy) is pumped into a sample of material of known mass. The temperature rise is measured allowing the energy/kg·K to be calculated. Real calorimeters are more elaborate than this, but the principle is the same.

Using Specific Heat

How much energy is required to heat a 100 mm cube of copper from room temperature (20°C) to its melting point?

Answer

Data for melting point, T_m, specific heat, C_p and density ρ are listed in Appendix A. The values for copper are T_m=1082°C, C_p=380 J/kg K and ρ=8930 kg/m^3. The mass of copper in the cube is $\rho V = 8.93$ kg. The energy to heat it though ΔT=1062°C is

$$\rho V C_p \Delta T = 3.6 \text{ MJ}.$$

(For comparison, the energy in a litre of gasoline is 35 MJ).

Thermal conductivity. The rate at which heat is conducted through a solid at steady state (meaning that the temperature profile does not change with time) is measured by the *thermal conductivity*, λ (units: W/m·K). It is measured by recording the heat flux q (W/m^2) flowing through the material from a surface at higher temperature T_1 to a lower one at T_2 separated by a distance X. The conductivity is calculated from Fourier's law:

$$q = -\lambda \frac{dT}{dX} = \lambda \frac{(T_1 - T_2)}{X} \qquad \textbf{(2.14)}$$

The measurement is not, in practice, easy (particularly for materials with low conductivities), but reliable data are now generally available (Fig. 2.8).

Steady State Heat Flow

A heat exchanger has an exchange area of A=0.5 m^2. It passes heat from a fluid at temperature T_1=100°C to a second fluid at T_2=20°C. The exchange wall is made of copper sheet (thermal conductivity λ=350 W/m.K) with a thickness X=2 mm. How much energy flows from one fluid to the other in one hour?

Answer

The temperature gradient $dT/dX = 80/0.002 = 40,000°\text{C/m}$. The total energy Q that passes across the area A over a time t=3600 seconds is

$$Q = A\,t\,q = A\,t\,\lambda\frac{dT}{dX} = 2.5 \times 10^{10}\ \text{J} = 25\ \text{GJ}$$

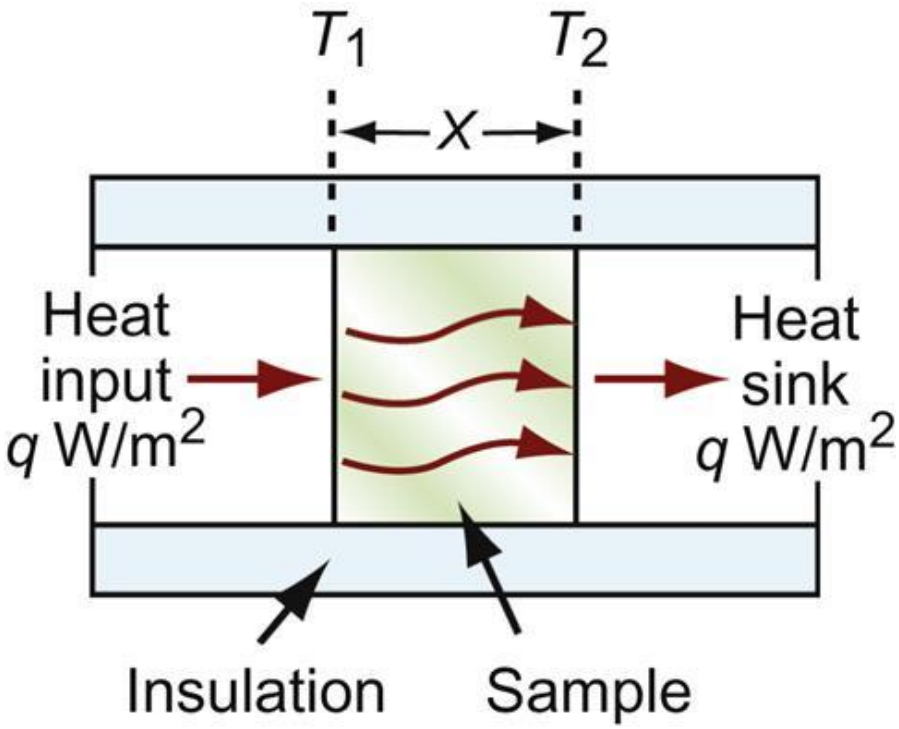


FIGURE 2.8 The thermal conductivity λ measures the flux of heat driven by a temperature gradient dT/dX.

Thermal diffusivity. When heat flow is transient, the flux depends instead on the *thermal diffusivity*, a (units: m^2/s), defined by

$$a = \frac{\lambda}{\rho C_p} \tag{2.15}$$

where ρ is the density and C_p is the heat capacity. The thermal diffusivity can be measured directly by measuring the decay of a temperature pulse when a heat source, applied to the material, is switched off; or it can be calculated from λ via Eq. (2.15). Solutions to problems of

transient heat flow (Appendix B, Section B15) typically have a time constant for a thermal front to advance by a distance x as

$$\tau \approx \frac{x^2}{2\alpha}$$

As a useful rule-of-thumb the distance x heat diffuses in a time t is approximately

$$x \approx \sqrt{2\alpha t} \tag{2.16}$$

Transient Heat Flow

You pour boiling water into a tea glass with a wall thickness x=3 mm. How many seconds do you have to carry it to the table before it becomes to hot to hold? (The thermal conductivity of glass λ=1.1 W/m K, its density is ρ=2450 kg/m^3 and its heat capacity C_p=800 J/kg K.)

Answer

Inserting the data into Eq. (2.15) gives a thermal diffusivity for glass of $a = 5.6 \times 10^{-7}$ m^2/s. Inserting this into Eq. (2.16) gives the approximate time

$$t \approx \frac{x^2}{2a} = 8 \text{ seconds.}$$

Thermal expansion coefficient. Most materials expand when they are heated. The thermal strain per degree of temperature change is measured by the *linear thermal expansion coefficient*, α (units: K^{-1} or, more conveniently, as 'μstrain/°C' or 10^{-6}°C^{-1}). If the material is thermally isotropic, the volume expansion, per degree, is 3α. If it is anisotropic, two or more coefficients are required, and the volume expansion becomes the sum of the principal thermal strains.

Thermal Stress

An aluminium pipe is rigidly clamped to the face of a concrete building. On a hot day the face of the building in direct sun rises to 80°C, and because the expansion of the aluminium is greater than that of concrete, stress appears in it. What is the value of the stress if the original clamping was done on a day when the temperature was 20°C?

Answer

The expansion coefficient of aluminium is $\alpha = 22.5 \times 10^{-6}/^{\circ}\mathrm{C}$, that of concrete is $\alpha = 9 \times 10^{-6}/^{\circ}\mathrm{C}$, using means of the ranges in Appendix A. The aluminium pipe is rigidly clamped, so the difference in thermal strain $\Delta\alpha\Delta T = 13.5 \times 10^{-6} \times 60 = 8.1 \times 10^{-4}$. This has to be accommodated by elastic compression of the aluminium (modulus $E = 75$ GPa from Appendix A), giving a stress $\Delta\, \alpha\, \Delta T\, E = 61$ MPa. This is enough to cause a soft aluminium to yield.

Thermal shock resistance. Sudden changes of temperature can cause materials to crack. *The thermal shock resistance*, ΔT_s, (units: K or °C) is the maximum temperature difference through which a material can be subjected suddenly without damage. It and the *creep resistance* are important in high-temperature design. Creep is the slow, time-dependent deformation that occurs when materials are loaded above about $\frac{1}{3}T_m$ or $\frac{2}{3}T_g$. Design against creep is a specialized subject. Here we rely instead on avoiding the use of a material above its maximum service temperature, $T_{\max}$, or, for polymers, the 'heat deflection temperature'.

Electrical Properties

Property	Symbol and Units
Electrical resistivity	ρ_e ($\Omega \cdot m$ or $\mu\Omega \cdot cm$)
Dielectric constant	ε_r (–)
Breakdown potential	V_b (10^6 V/m)
Dielectric loss factor	$\tan \delta$
Power factor	P (–)
Piezoelectric charge coefficient	d_{33} pC/N or pm/V
Piezoelectric voltage coefficient	g_{33} mV·m/N
Pyroelectric coefficient	γ μC/m²·K
Curie temperature (ferroelectric)	T_c °C

The *electrical resistivity*, ρ_e (SI units Ω m, but the unit μΩ cm is still widely used) is the resistance of a unit cube with unit potential difference between a pair of it faces. It has an immense range, from a little more than 10^{-8} in units of Ω m for good conductors (equivalent to 1 μΩ cm) to more than 10^{16} Ω m (10^{24} μΩ cm) for the best insulators. The electrical conductivity κ_e (units Siemens per meter (S/m) or $(\Omega\ m)^{-1}$) is simply the reciprocal of the resistivity expressed in Ω m.

Resistivity and Resistance

Tungsten has a conductivity of $\kappa_e = 8.3 \times 10^6$ Siemens. What is the resistance of a tungsten wire of radius $r = 100$ microns and length $L = 1$ m?

Answer

The resistivity of tungsten $\rho_e = 1/\kappa_e = 1.2 \times 10^{-7}\ \Omega\ m$. The resistance R of the wire is

$$R = \rho_e \frac{L}{\pi r^2} = 3.8 \text{ ohms}$$

Dielectric constant. When an insulator (or *dielectric*) is placed in an electric field, it becomes polarized and charges appear on its surfaces that tend to screen the interior from the electric field. The tendency to polarize is measured by the *dielectric constant*, ε_r, a dimensionless quantity. Its value for free space and, for practical purposes, for most gasses, is 1. Most solid insulators have values between 2 and 30, though low density foams approach the value 1 because they are largely air. Ferroelectric materials are exceptional; they have dielectric constants as high as 3000. More on them in a moment.

What does ε_r measure? Two parallel conducting plates separated by a dielectric of thickness t make a capacitor. Capacitors store charge. The charge Q (units: coulombs) is directly proportional to the voltage difference between the plates, V (volts):

$$Q = C\,V \tag{2.17}$$

where C (farads) is the capacitance. The capacitance of a parallel plate capacitor of area A, separated by empty space (or by air) is

$$C = \varepsilon_o \frac{A}{t} \tag{2.18}$$

where ε_o is the *permittivity of free* space (8.85×10^{-12} F/m, where F is farads). If the empty space is replaced by a dielectric, the capacitance increases because the dielectric polarizes. The field created by the polarization opposes the field $E = V/t$, which reduces the voltage difference V needed to support the charge. Thus the capacity of the condenser is increased to the new value

$$C = \varepsilon \frac{A}{t} \tag{2.19}$$

where ε is the *permittivity of the dielectric* with the same units as ε_o. It is usual to cite not this, but the *relative permittivity* or *dielectric constant*, ε_r:

$$\varepsilon_r = \frac{C_{with\ dielectric}}{C_{no\ dielectric}} = \frac{\varepsilon}{\varepsilon_o} \tag{2.20}$$

making the capacitance

$$C = \varepsilon_r\, \varepsilon_o \frac{A}{t} \tag{2.21}$$

When charged, the energy stored in a capacitor is

$$\frac{1}{2} QV = \frac{1}{2} CV^2 \tag{2.22}$$

This energy can be large: 'super-capacitors' with capacitances measured in farads store enough energy to power a car.

Stray Capacitance

The time constant τ for charging or discharging a capacitor C is

$$\tau = RC$$

where R is the resistance of the circuit. That means that stray capacitance in an electronic circuit (capacitance between neighbouring conducting lines of resistivity ρ_e mounted on a substrate of dielectric constant ε_r) slows its response. What material choices minimize this?

Answer

Choosing materials with low resistivity ρ_e for the conductors (to minimize R) and choosing insulators with low dielectric constant ε_r to separate them (to minimize C) minimizes τ.

Dielectric loss coefficient. Polarization involves the small displacement of charge (either of electrons or of ions or of molecules that carry a dipole moment) when an electric field is applied to the material. An oscillating field drives the charge between two alternative configurations. This charge-motion is like an electric current that – if there were no losses – would be 90° out of phase with the voltage. In real dielectrics this current

dissipates energy, just as a current in a resistor does, giving it a small phase shift, δ (Fig. 2.9). The *loss tangent*, $\tan\delta$, also called the *dissipation factor*, D, is the tangent of the loss angle. The *power factor*, P_f, is the sine of the loss angle. When δ is small, as it is for the materials of interest here, all three are essentially equivalent.

$$P_f \approx D \approx \tan\delta \approx \sin\delta \tag{2.23}$$

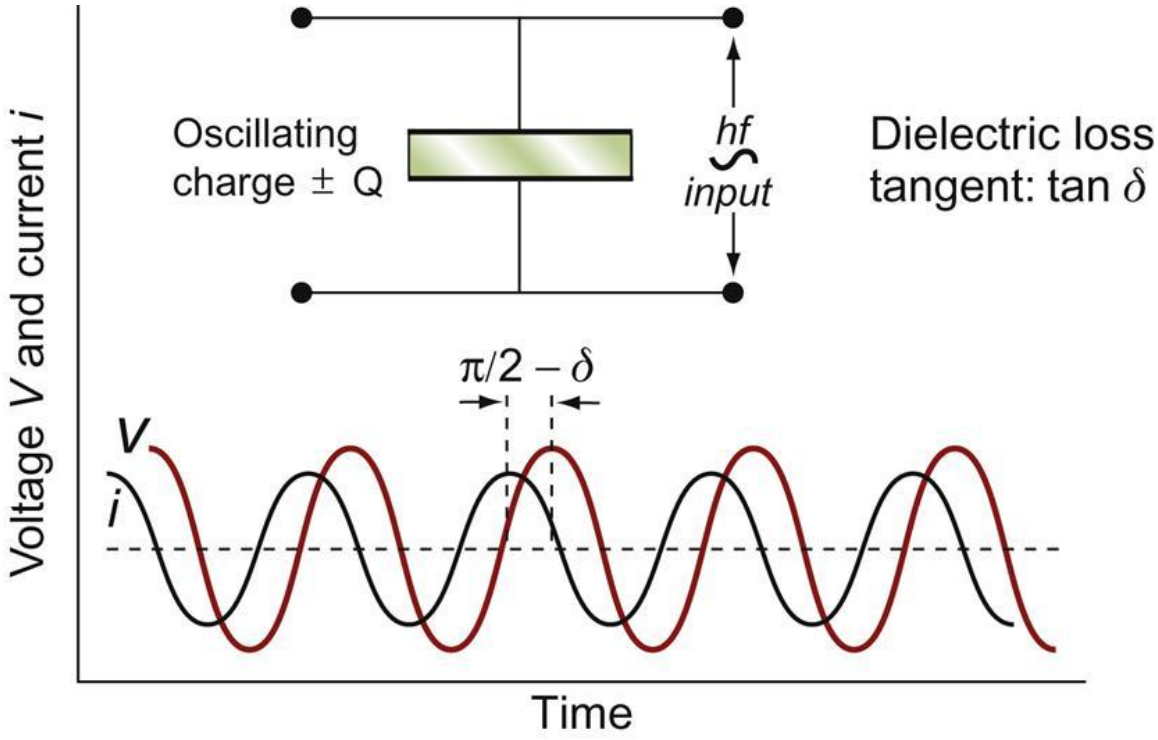


FIGURE 2.9 Dielectric loss, important in dielectric heating, as explained in the text.

More useful, for our purposes, is the *loss factor* L which is the loss tangent times the dielectric constant:

$$L = \varepsilon_r \tan\delta \tag{2.24}$$

The loss factor measures the energy dissipated by a dielectric when in an oscillating field. If you want to select materials to minimize or

maximize dielectric loss, then the measure you want is L.

When a dielectric material is placed in a cyclic electric field of amplitude E and frequency f, power P is dissipated and the field is correspondingly attenuated. The power dissipated per unit volume (W/m^3) is

$$P \approx fE^2\varepsilon \tan\delta = fE^2\varepsilon_o\varepsilon_r \tan\delta = fE^2\varepsilon_o L \tag{2.25}$$

where, as before, ε_r is the dielectric constant of the material and tan δ is its loss tangent. This power appears as heat; the higher the frequency or the field strength and the greater the loss factor $L = \varepsilon_r \tan\delta$, the greater are the heating and energy loss. This dielectric loss is exploited in processing, as, e.g., in the radio-frequency welding of polymers, and, of course, in the cooking of food.

Dielectric Heating

A nylon component is placed in a microwave cavity with a field strength $E = 10^4$ V/m and a frequency $f = 10^{10}$ Hz for a time $t = 100$ s. The dielectric loss factor for nylon is $L = 0.1$, its density is $\rho = 1130$ kg/m^3 and its heat capacity is $C_p = 1650$ J/kg·K. Assuming no heat loss, how hot will the component become?

Answer

The heat generated by the field is $Q = P\,t = f\,E^2\varepsilon_o\,L\,t = 8.85 \times 10^7$ J/m^3. The heat capacity of nylon per unit volume is $C_p\rho = 1.86 \times 10^6$ J/m^2·K. The temperature rise, ΔT is

$$\Delta T = \frac{f\,E^2\varepsilon_o L\,t}{C_p\rho} = 47.6°\text{C}$$

Breakdown potential. The *breakdown potential* (units: MV/m) is the electrical potential gradient at which an insulator breaks down and a damaging surge of current flows through it. It is measured by increasing a 60 Hz alternating potential applied across the faces of a plate of the material until breakdown occurs, typically at a potential gradient between 1 and 100 million volts per meter (units: MV/m).

Piezoelectric charge coefficient, d_{33} (pC/N). The piezoelectric effect appears in structures that lack a center of symmetry, such as the compounds barium titanate and lead zirconium titanante (PZT). The distortion of a piezoelectric crystal causes a relative shift between positive and negative ions, changing their dipole moment. The crystal acquires a polarization and a potential difference appears across its faces (Fig. 2.10). The potential difference can be large – large enough to trigger a spark discharge, an effect used in gas and cigarette lighters.

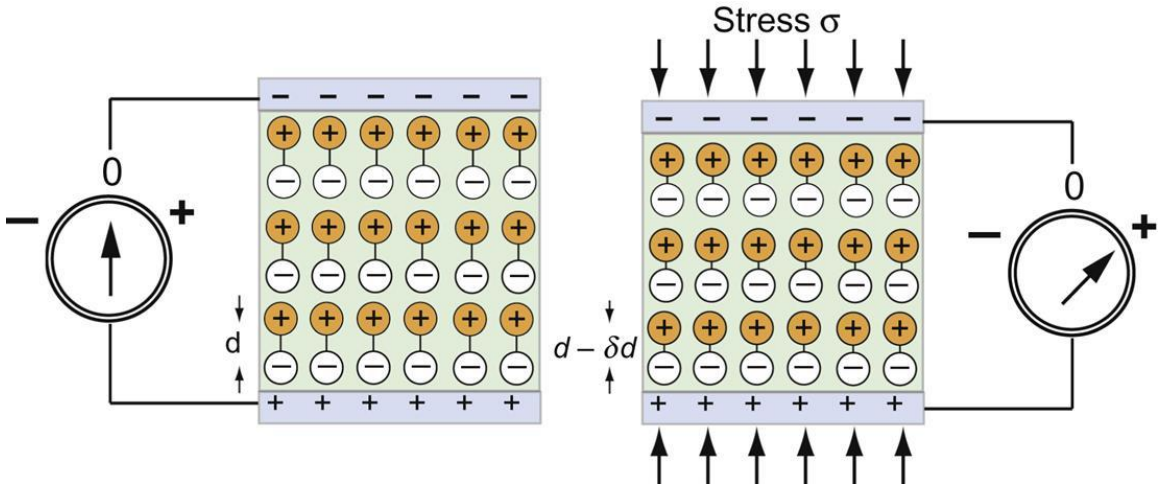


FIGURE 2.10 The piezoelectric effect.

The polarization P_3 that appears when a piezoelectric crystal is compressed is a linear function of the applied stress σ_3

$$P_3 = d_{33}\sigma_3 \tag{2.26}$$

where d_{33} is the *piezoelectric charge coefficient*.

Piezoelectric voltage coefficient, g_{33} (mV·m/N). The piezoelectric voltage coefficient, g_{33}, relates electric field E_3 to applied stress σ_3

$$E_3 = g_{33}\sigma_3 \tag{2.27}$$

Electric field is related to polarization by

$$E_3 = \frac{1}{\varepsilon_r \varepsilon_o} P_3 = \frac{d_{33}}{\varepsilon_r \varepsilon_o} \sigma_3,$$

meaning that

$$g_{33} = \frac{d_{33}}{\varepsilon_r \varepsilon_o} \tag{2.28}$$

where ε_r is the relative permittivity (dielectric constant) and ε_o the permittivity of vacuum (8.854×10^{-12} F/m).

Using Piezoelectric Properties

A disk of bismuth titanate is subjected to a compressive stress of 2 MPa. How large is the electric field that this stress will generate? Use the mean value of the piezoelectric voltage coefficient g_{33} (Appendix A, Table A8) to calculate it.

Answer

The mean value of the piezoelectric voltage coefficient for bismuth titanate is

$$13.5 \frac{\text{mV m}}{\text{N}} = 13.5 \times 10^{-3} \frac{V/m}{N/m^2}$$

Thus a stress of 2 MPa will create a field of
$13.5 \times 10^{-3} \times 2 \times 10^6 = 27{,}000 \times V/m$

Pyroelectric coefficient ($\mu C/m^2 \cdot K$). Below the ferroelectric Curie temperature, T_c, crystals with a unique direction carry a permanent dipole (even in the absence of an applied field) because positive and negative charges do not cancel along the unique direction. When the temperature is constant the charged faces of the crystal attract opposite charges from the surroundings, screening the charge. But when the temperature changes, the positive and negative charges along the unique direction move relative to one another, changing the polarization as in Fig. 2.11. The change in polarization ΔP caused by a change of temperature ΔT is given by

$$\Delta P = \gamma \, \Delta T \tag{2.29}$$

where γ is the pyroelectric coefficient. The change can be measured and used to actuate motion sensors and for infrared imaging.

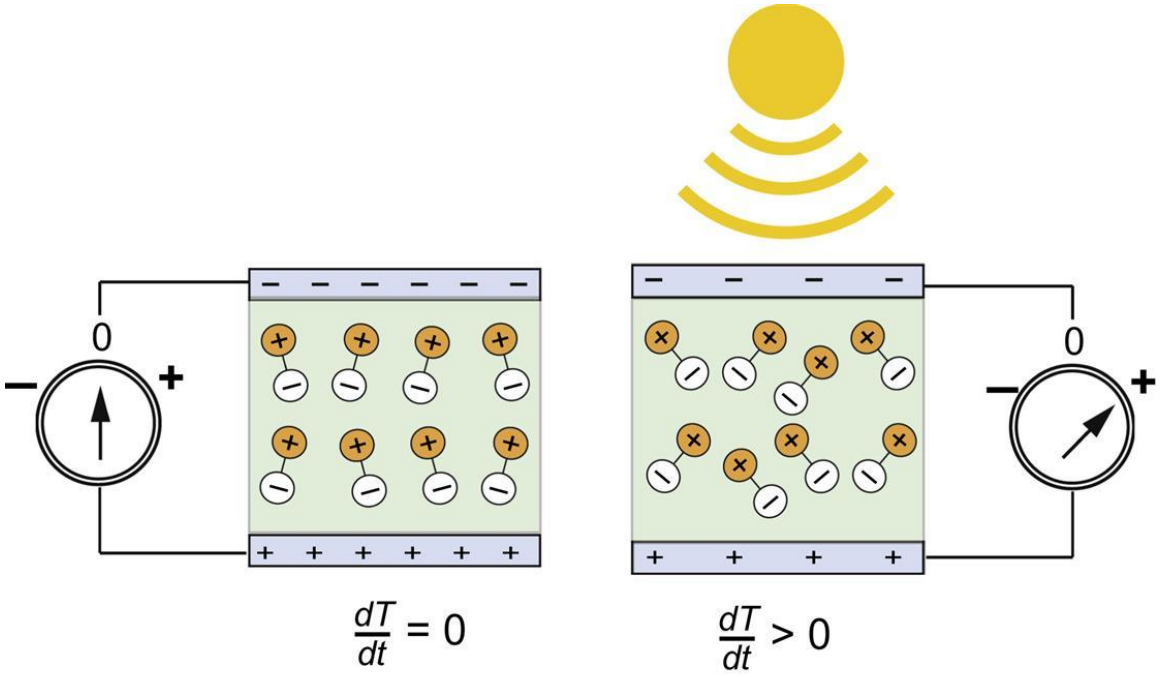


FIGURE 2.11 The pyroelectric effect.

Above the Curie temperature T_c the polarization is lost completely.

Magnetic Properties

Attribute	Unit
Remanent induction B_r	T
Coercive field H_c	A/m
Maximum energy product	MJ/m^3
Max permeability	No unit
Saturation magnetostriction	μstrain
Curie temperature (magnetic)	°C

Remanent induction (Tesla) and Coercive field (A/m). Nearly all materials respond to a magnetic field by becoming magnetized, but most are paramagnetic with a response so faint that it is of no practical use.

Ferromagnetic and *ferrimagnetic* materials (*ferrites* for short), however, contain atoms that have large magnetic moments with the ability to spontaneously magnetize – to align their moments in parallel – much as electric dipoles do in ferroelectric materials. These are of real practical use.

When a current i passes through a long, empty coil of n turns and length L, a magnetic field is generated. The magnitude of the field, H, is given by Ampère's law as

$$H = \frac{ni}{L}$$

and thus has units of amps/meter (A/m). The field induces a *magnetic induction* or *flux density*, B, which for vacuum or nonmagnetic materials is

$$B = \mu_o H \tag{2.30}$$

where μ_o is the *permeability of vacuum*, $\mu_o = 4\pi \times 10^{-7}$ henry/meter (H/m). The units of B are *tesla*, so a tesla is 1 HA/m^2.

If the space inside the coil of is filled with a material, as in Fig. 2.12, the induction within it changes. This is because its atoms respond to the field by forming little magnetic dipoles. The material acquires a macroscopic dipole moment or *magnetization*, M (its units are A/m, like H). The induction becomes

$$B = \mu_o(H + M)$$

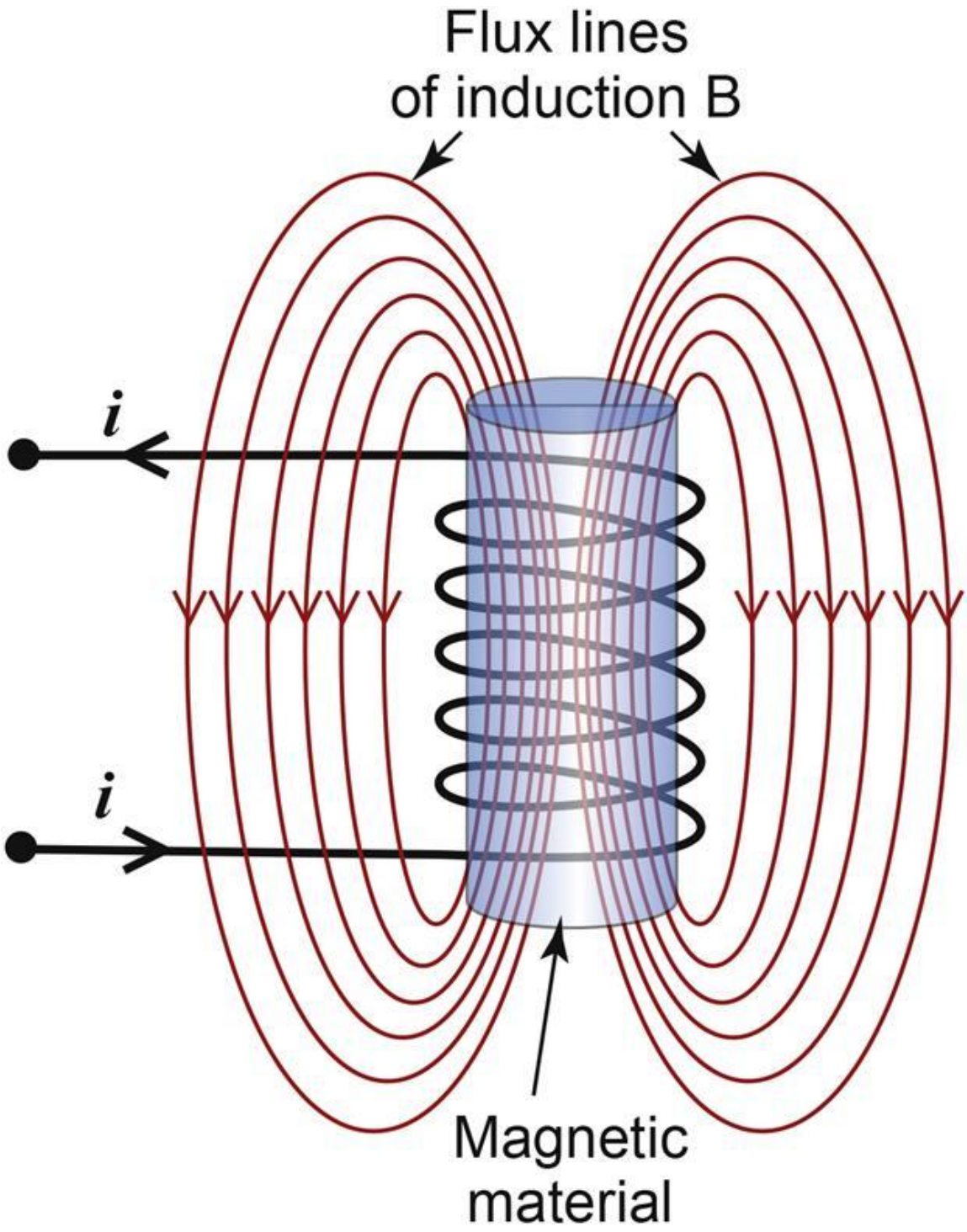


FIGURE 2.12 A magnetic material exposed to a field H becomes magnetized. The induction $B = \mu_o H$

If the material of the core is ferromagnetic, the response is a very strong one and it is nonlinear. It is usual to rewrite B in the form

$$B = \mu_R \mu_o H \tag{2.31}$$

where μ_R is the *relative permeability*. It is dimensionless. The *remanent induction* B_R is the induction that remains when the field H is removed. The *coercive field* H_c is the field required to fully magnetize the material, or (equivalently) to demagnetize it (Fig. 2.13).

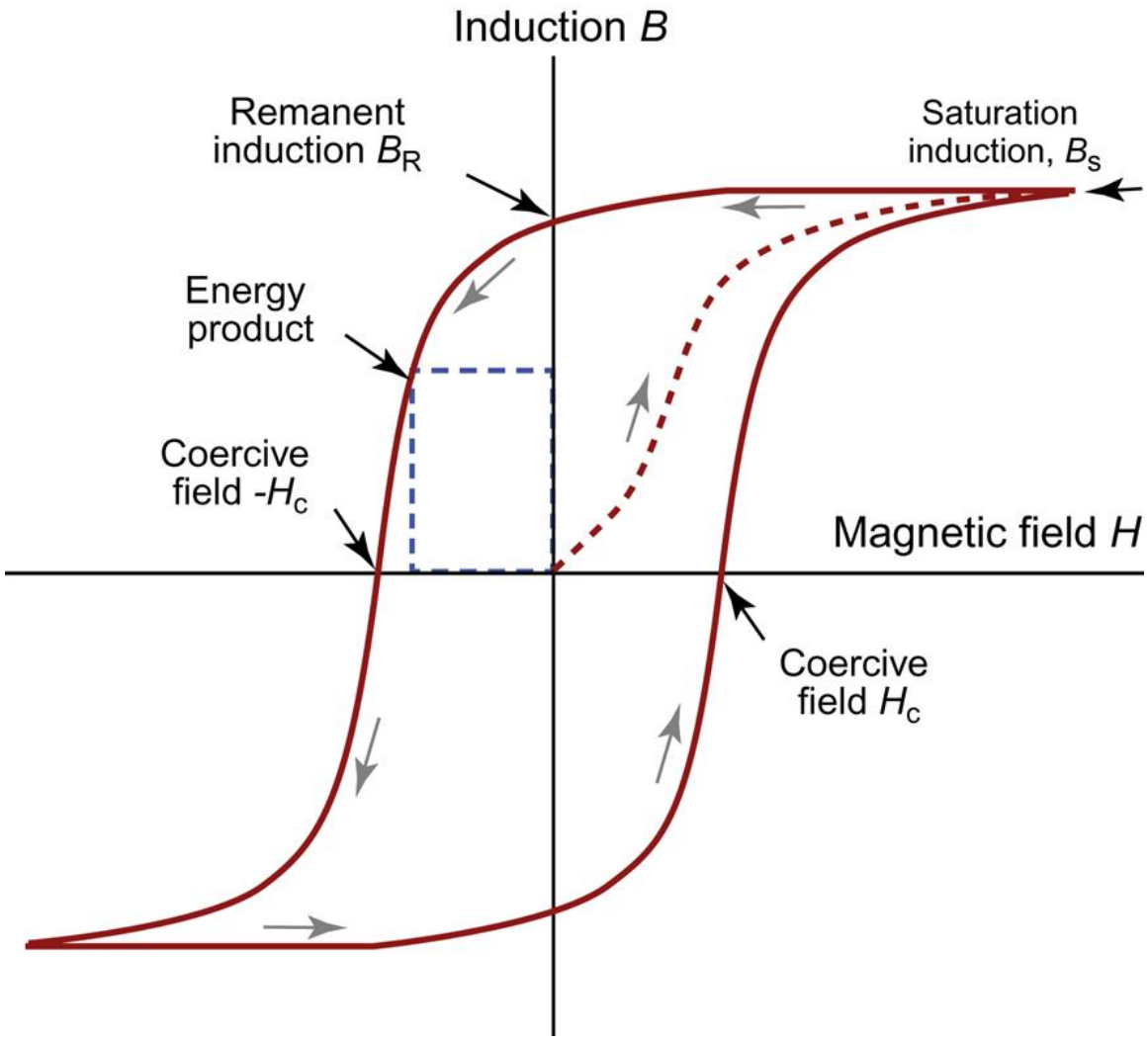


FIGURE 2.13 The B – H curve of a ferromagnetic material, showing the important properties.

Saturation magnetostriction (μstrain). Magnetostriction is strain caused by a magnetic field. The material is described as having positive magnetostriction if its length increases in the direction of the field and negative magnetostriction if it does the opposite. The *saturation*

magnetostriction is the maximum strain that can be induced in a material by applying a magnetic field.

Using Magnetostrictive Properties

A rod of galfenol (and alloy of gallium and iron, roughly $Ga_{0.2}Fe_{0.8}$), L=20 mm long, is surrounded by a coil of the same length with n=400 turns. By how much will the rod contract if a current of 0.1 amp is passed through the coil? Use the mean value of the saturation magnetostriction and coercive field for galfenol (Appendix A, Table A9) to find out.

Answer

The field created by a current of 0.1 amp passing through a coil

$$H = \frac{n\,i}{L} = \frac{400 \times 0.1}{2 \times 10^{-2}} = 2000\,A/m.$$

The coercive field for galfenol is 800 A/m. The field created by the coil far exceeds this field, thus saturating the rod and creating a strain of 150×10^{-6}. Thus the rod will contract by

$$\delta L = 150 \times 10^{-6} \times 20 \times 10^{-3} = 3 \times 10^{-6}\ \text{m} = 3\,\mu m$$

Optical Properties

Property	Symbol and Units
Refractive index	n (–)

All materials allow some passage of light, although for metals it is exceedingly small. The speed of light when in the material, c, is always less than that in vacuum, c_o. A consequence is that a beam of light striking the surface of a material at an angle of incidence, α, enters the material at an angle β, the angle of refraction. The *refractive index*, n (dimensionless) is

$$n = \frac{c}{c_o} = \frac{\sin\alpha}{\sin\beta} \qquad \textbf{(2.32)}$$

It is related to the dielectric constant, ε_r, at the same frequency by

$$n \approx \sqrt{\varepsilon_r}$$

The refractive index depends on wavelength and thus on the colour of the light. The denser the material, and the higher its dielectric constant, the greater is the refractive index. When $n = 1$, the entire incident intensity enters the material, but when $n > 1$, some is reflected. If the surface is smooth and polished, it is reflected as a beam; if rough, it is scattered. The percentage reflected, R, is related to the refractive index by

$$R = \left(\frac{n-1}{n+1}\right)^2 \times 100 \qquad \textbf{(2.33)}$$

As n increases, the value of R approaches 100%.

Environmental Properties

Property	Symbol and Units
Embodied energy	H_m (MJ/kg)
Carbon footprint	CO_2 (kg/kg)

Embodied energy and Carbon footprint. The *embodied energy* (units MJ/kg) is the energy required to make 1 kg of a material from its ores and feedstock. The associated CO_2 *footprint* (units: kg/kg) sounds like the mass of carbon dioxide released into the atmosphere during the production of 1 kg of material but it is usually larger than that because it includes the CO_2 equivalent (in terms of global warming) of the associated CO, NO_x, and SO_x emissions. These and other eco-attributes are the subject of Chapter 14, Materials and the environment.

2.5 Summary and Conclusions

There are six important families of materials for mechanical design: metals, ceramics, glasses, polymers, elastomers and hybrids that combine the properties of two or more of the others. Within a family there is certain commonality. Ceramics and glasses as a family are hard, brittle and corrosion resistant. Metals are ductile, tough and good thermal and electrical conductors. Polymers are light, easily shaped and electrical insulators. Elastomers can be stretched or sheared elastically to large strains. It is these differences that make the classification useful. But in design we wish to escape from the constraints of family, and think, instead, of the material name as an identifier for a certain property profile – one that will, in later chapters, be compared with a 'desired' profile of the properties defined in this chapter. It is this profile that guides material choice.

The final figure, Fig. 2.14, is an example of a data record for a material – here, ABS. Appendix A at the end of the book is a compilation of material properties. In Chapter 3, Materials Property Charts we develop a way of displaying these properties in a way that maximizes the information we can extract from them.

Acrylonitrile butadiene styrene (ABS)

General properties

Density	1100	kg/m^3
Price	2.6	USD/kg

Mechanical properties

Young's modulus	1.8	GPa
Yield strength	31	MPa
Tensile strength	39	MPa
Elongation	12	% strain
Hardness - Vickers	9.3	HV
Fatigue limit	16	MPa
Fracture toughness	2.3	$MPa.m^{1/2}$

Thermal properties

Glass temperature	106	°C
Thermal conductivity	0.25	W/m.°C
Specific heat capacity	1630	J/kg.°C
T-expansion coefficient	141	µstrain/°C

Electrical properties

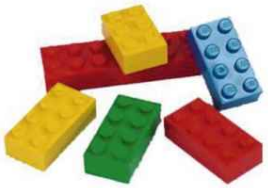
Electrical resistivity	9×10^{21}	µohm.cm
Dielectric constant	3.0	
Dissipation factor	0.004	
Dielectric strength	17	MV/m

Eco properties

Embodied energy	95	MJ/kg
Carbon footprint	3.8	kg/kg

The material

ABS is tough, resilient, and easily molded. The picture shows that ABS allows detailed moldings, accepts color well and is non-toxic and tough enough to survive the worst that children can do.



Typical uses

Safety helmets; camper tops; automotive instrument panels and other interior components; pipe fittings; home-security devices and housings for small appliances; communications equipment; business machines; plumbing hardware; automobile grilles; wheel covers; mirror housings; refrigerator liners; luggage shells; tote trays; mower shrouds; boat hulls

Tradenames

Claradex, Comalloy, Cycogel, Cycolac, Hanalac, Lastilac, Lupos, Lustran ABS, Magnum, Multibase, Novodur, Polyfabs, Polylac, Porene, Ronfalin, Sinkral, Terluran, Toyolac, Tufrex, Ultrastyr

FIGURE 2.14 Part of a record for ABS with structured and unstructured data.

2.6 Further Reading

Definitions of material properties can be found in numerous general texts on engineering materials, among them those listed here.

1. Ashby M, Shercliff H, Cebon D. *Materials: Engineering, Science, Processing and Design* third ed. Oxford, UK: Butterworth-Heinemann; 2014; ISBN-13: 978-0-08-097773-7, North American Edition: ISBN-13: 978-1-85617-743-6. (*An introductory text introducing ideas that are developed more fully in the present text*.).
2. Askeland DR, Phulé PP, Wright WJ. *The Science and Engineering of Materials* sixth ed. Toronto, Canada: Thomson; 2010; ISBN 9780495296027. (*A well established materials text that deals well with the science of engineering materials*.).

3. ASM Handbooks, Volume 8. *Mechanical Testing and Evaluation* Metals Park, Ohio: ASM International; 2004; (*An on-line, subscription-based resource, detailing testing procedures for metals and ceramics*.).
4. Budinski KG, Budinski MK. *Engineering Materials, Properties and Selection* ninth ed. New Jersey: Prentice Hall; 2010; ISBN 978-0-13-712842-6. (*Like Askeland, this is a well established materials text that deals well with both material properties and processes*.).
5. Callister WD, Rethwisch DG. *Materials Science and Engineering: An Introduction* ninth ed. New York, NY: John Wiley & Sons; 2014; ISBN 978-1-118-54689-5. (*A well-established text taking a science-led approach to the presentation of materials teaching*.).
6. Dieter GE. *Engineering Design, a Materials and Processing Approach* third ed. New York, NY: McGraw-Hill; 1999; ISBN: 9-780-073-66136-0. (*A well-balanced and respected text focussing on the place of materials and processing in technical design*.).
7. Farag MM. *Materials and Process Selection for Engineering Design* third ed. London UK: CRC Press, Taylor and Francis; 2013; ISBN 978146656409. (*A Materials-Science approach to the selection of materials*.).
8. Shackelford JF. *Introduction to Materials Science for Engineers* eighth ed. New Jersey: Prentice Hall; 2014; ISBN 978-0133789713. (*A well-established materials text with a design slant*.).

2.7 Exercises

These exercises introduce the reader to two useful resources: the data sheets of Appendix A *and the Solutions to Standard Problems of* Appendix B. *When drawing data from* Appendix A *use the mean values of the ranges listed there*.

E2.1. *Sound velocity*. The speed of sound in Pyrex (borosilicate) glass is 5610 m/s. Find the density of this glass from Appendix A and use it to estimate the modulus E of Pyrex.

E2.2. *Deflection of beams*. A cantilever beam has a length L=50 mm. It has a rectangular cross-section width b=5 mm and a thickness t=1 mm. It is made of an aluminium alloy. By how much will the end deflect under an end-load of F=5 N (roughly that exerted by the weight of 5 apples)? Use data from Appendix A4 for the (mean) value of Young's modulus of aluminium alloys, the equation for the elastic deflection of a cantilever

from Appendix B3 and for the second moment of a beam from Appendix B2 to find out.

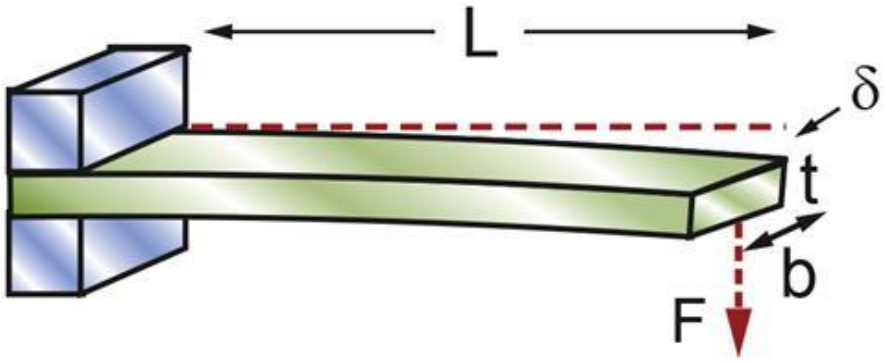


FIGURE E2.2

E2.3. *Deflection of beams*. The wings of a glider are each 4 m long. The load-bearing member is the wing spar, a tubular beam running the length of the wing. The wing spar in this glider has a diameter of 140 mm and a wall thickness of 6 mm. It is made of an aluminium alloy. In flight the wing spar is loaded in bending with (we will assume) a uniformly distributed force per unit length. By how much will the wing tip deflect in calm flight if the loaded glider weighs 1000 kg? Use data from Appendix A3 for the (mean) value of Young's modulus of aluminium alloys, the equation for the elastic deflection of a cantilever from Appendix B3 and for the second moment of a beam from Appendix B2 to find out.

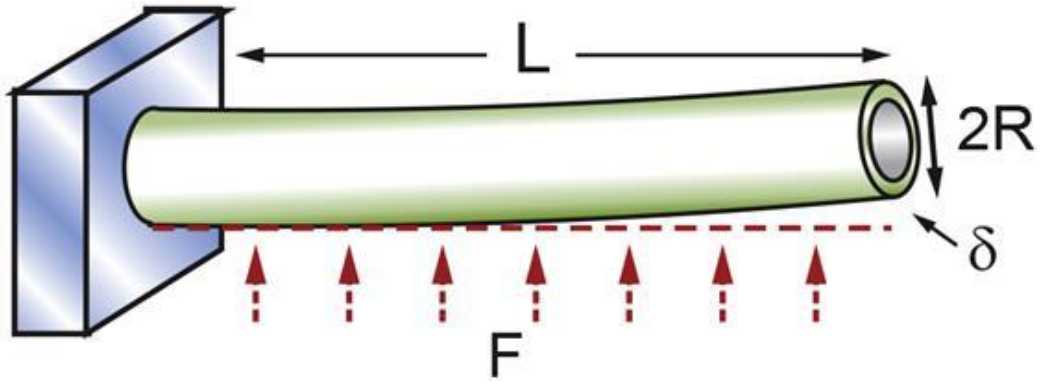


FIGURE E2.3

E2.4. *Vibration of beams*. A cantilever beam with length L=200 mm, width b=12 mm and thickness t=2 mm weighs 38 grams and has a natural

vibration frequency of f=42 hertz. What is the modulus of the material of the beam? You will find equations for natural frequencies of vibration and moments of area in Appendix B, Tables B12 and B2.

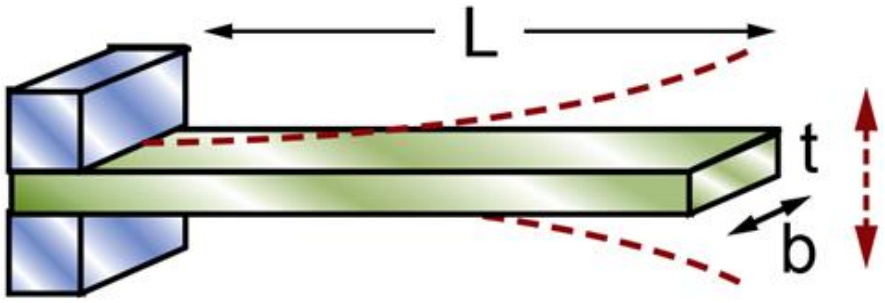


FIGURE E2.4

E2.5. *Forensic materials science*. Exercise E2.4 used a natural vibration frequency to measure the modulus of a beam. Use the information in E2.4 to calculate the density of the beam, and then use this and the result of the exercise (the modulus of the beam) as a forensic tool. Scan the modulus values in Table A3 of Appendix A seeking a match between the calculated modulus and those in the table. What subsets of materials match the calculated modulus? Of these, which subset also matches the calculated density?

E2.6. *Springs*. A spring, wound from stainless steel wire with a wire diameter d=1 mm, has n=20 turns of radius R=10 mm. How much will it extend when loaded with a mass M of 1 kg? Assume the shear modulus G of stainless steel to be 3/8E where E is Young's modulus, retrieve this from Appendix A3, and use the expression for the extension of springs from Appendix B6 to find out.

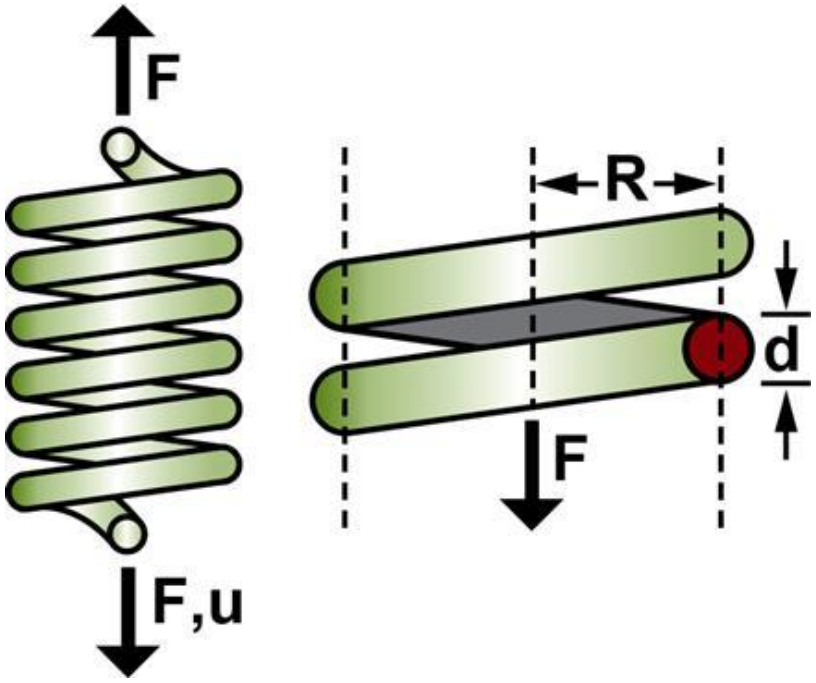


FIGURE E2.6

E2.7. *Torsion of tubes*. A thick-walled tube has an inner radius $r_i = 10$ mm and an outer radius $r_o = 15$ mm. It is made from polycarbonate, PC. What is the maximum torque T_f that the tube can carry without the onset of yield? Retrieve the (mean) yield strength σ_y of PC from Appendix A3, the expression for the torque at onset of yield from Appendix B6 and that for the polar moment of a thick-walled tube from Appendix B2 to find out.

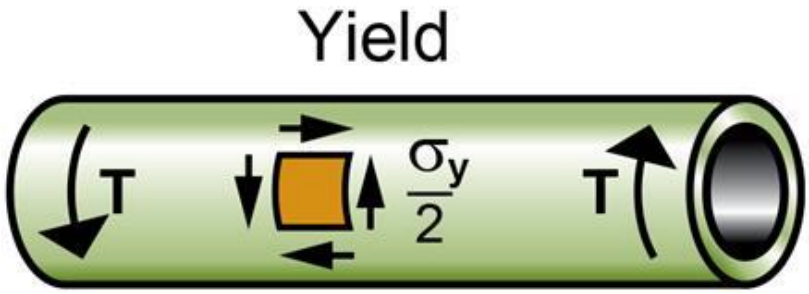


FIGURE E2.7

E2.8. *Stress concentrations*. A round bar, 20 mm in diameter, has a shallow circumferential notch with a depth $c = 1$ mm with a root radius $r = 10$ microns. The bar is made of a low carbon steel with a yield strength of $\sigma_y = 250$ MPa. It is loaded axially with a nominal stress, σ_{nom} (the axial

load divided by the un-notched area). At what value of σ_{nom} will yield first commence at the root of the notch? Use the stress concentration estimate of Appendix B9 to find out.

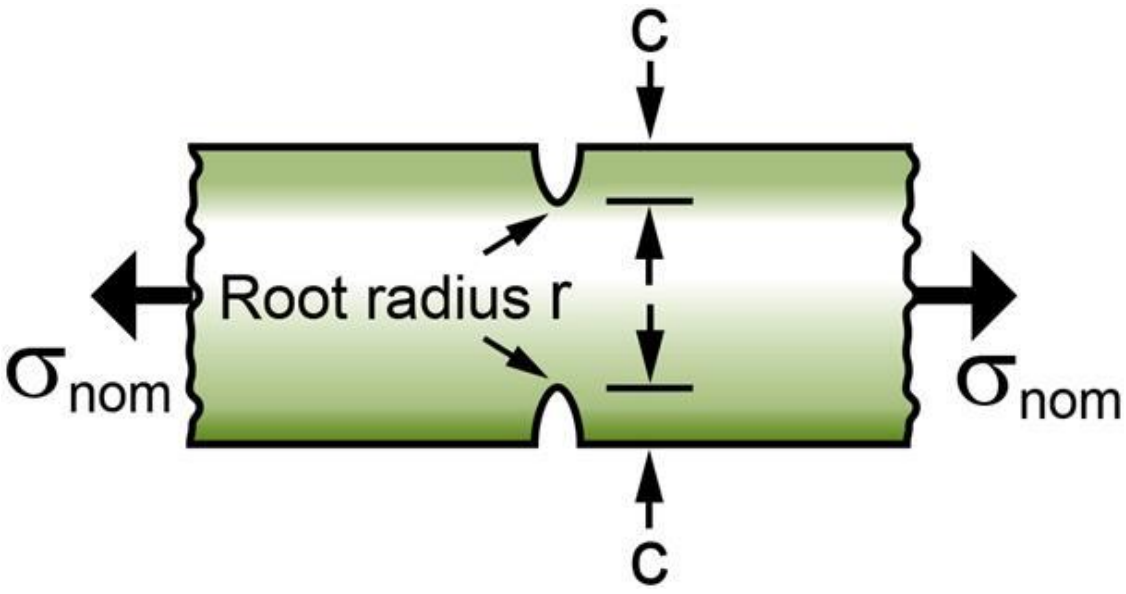


FIGURE E2.8

E2.9. *Thermal stress*. An acrylic (PMMA) window is clamped in a thick low carbon steel frame at $T = 20°C$. The temperature falls to $T = -20°C$, putting the window under tension because the thermal expansion coefficient of PMMA is larger than that of steel. If the window was stress-free at 20°C, what stress does it carry at −20°C? Use the result that the bi-axial stress caused by a bi-axial strain difference $\Delta\varepsilon$ is

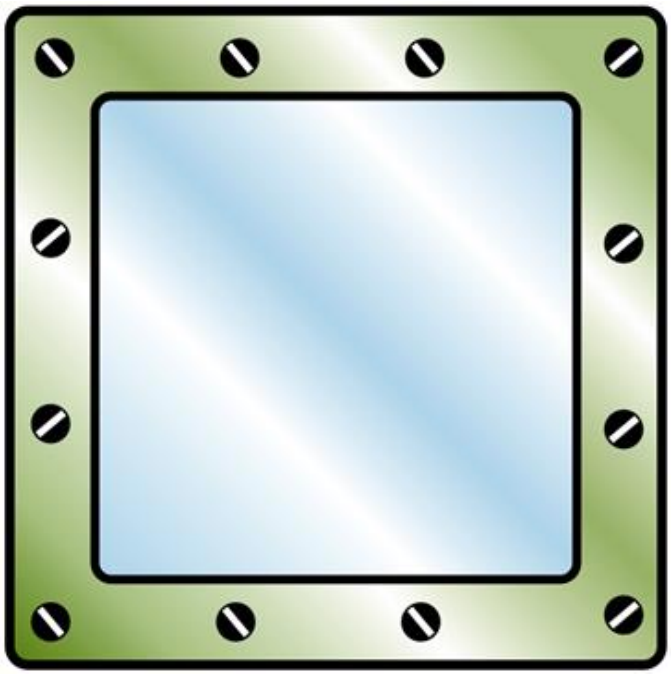


FIGURE E2.9

$$\sigma = \frac{E\Delta\varepsilon}{1-\nu}$$

where E is Young's modulus for PMMA and Poisson's ratio $\nu = 0.33$. You will find data for expansion coefficients in Table A6 and for moduli in Table A3, Appendix A. Use mean values.

E2.10. *Unstable cracks*. The PMMA window described in Exercise 2.9 has a contained crack of length $2a$=0.5 mm. If the maximum tensile stress that is anticipated in the window is σ=20 MPa, will the crack propagate? Choose an appropriate equation for crack propagation from Appendix B10 and data for the fracture toughness K_{1c} of PMMA from Appendix A6 to calculate the length of crack that is just unstable under this tensile stress.

E2.11. *Centrifugal stress*. A flywheel with a radius R=200 mm is designed to spin up to 8000 rpm. It is proposed to make it out of ductile (nodular) cast iron, but the casting shop can guarantee only that it will have no crack-like flaws greater than $2a$=2 mm in length. Use the expression for the maximum stress in a spinning disk in Appendix B7, that for the stress intensity at a small enclosed crack from Appendix B10 and data

for cast iron from Appendix A2 and A4 to establish if the flywheel is safe. Take Poisson's ratio ν for cast iron to be 0.33.

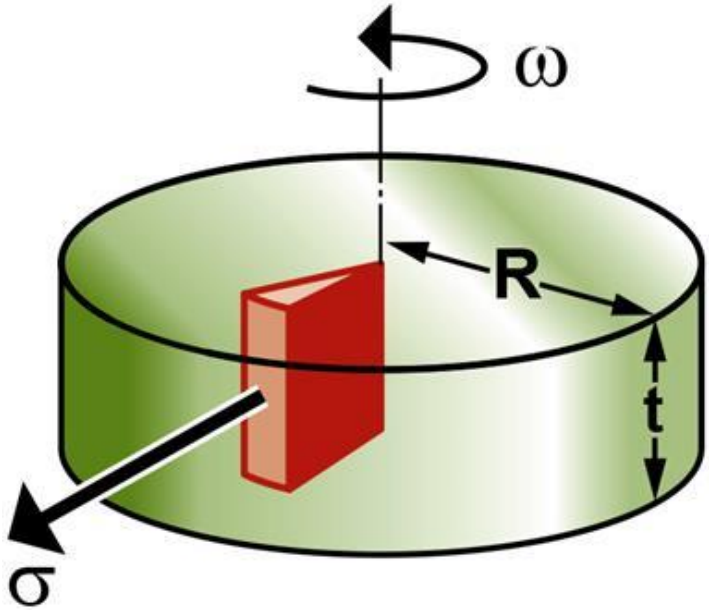


FIGURE E2.11

E2.12. *Blunting cracks*. Liberty ships, thousands of which were constructed during World War II, were welded rather than riveted. Many were mistakenly made from a steel that became brittle at the temperatures of the Atlantic in winter. They had square hatches with sharp corners from which a crack could start and propagate across the deck which, being welded, allowed a continuous path not interrupted by riveted plates. It is said that alert seamen, observing such a crack to start, would seize a power drill and drill a hole at the crack tips, effectively blunting it and reducing the stress concentration. If the crack when the seaman first saw it was 1 m long, and the largest drill he had was $2r = 25$ mm in diameter, what is the stress concentration at the end of the crack once the hole was drilled? You will find the equation for the stress at a distance r from the tip of a crack with a length $2a$ in Appendix B, Table B10.

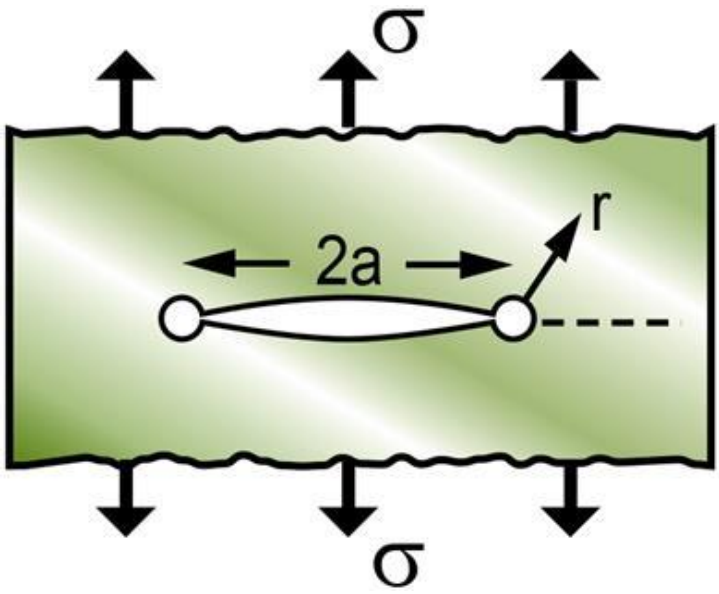


FIGURE E2.12

E2.13. *Contact forces*. A clamp has a hemispherical contact face of radius $R = 10\,\text{mm}$ to allow for misorientation. It is used to clamp a heavy lead archaeological treasure for X-ray examination. How much clamping force, F, can be applied without damage (meaning plastic deformation) to the treasured object? You will find the both the modulus E and yield strength σ_Y of lead alloys in Appendix A, Table A3 (use mean values), and the equation for force required to trigger plastic deformation beneath a spherical indenter in Appendix B, Table B8.

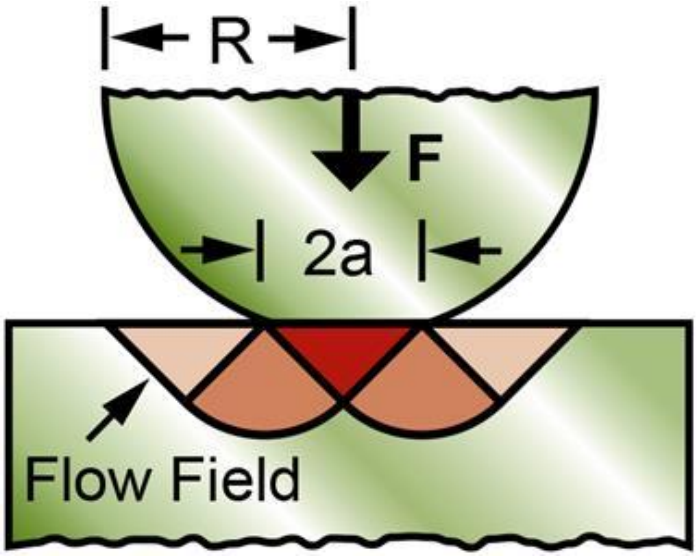


FIGURE E2.13

E2.14. *Estimating thermal properties*. You wish to measure, approximately, the thermal conductivity λ of polyethylene (PE). To do so you block one end of a PE pipe with a wall thickness of $x = 3$ mm and diameter of 30 mm and fill it with boiling water while clutching the outside with your other hand. You note that the outer surface of the pipe first becomes appreciably hot at a time $t \approx 18$ seconds after filling the inside with water. Use this information, plus data for specific heat C_p and density ρ of PE from Appendix A, Tables A2 and A5, to estimate λ for PE. How does your result compare with the listed value in Table A7?

E2.15. *Heat exchangers*. A power transistor with an area of A of 1 mm^2 dissipates 10 watts. It is attached to a heat sink 2 mm thick, effectively cooled to 30°C on the back face by forced airflow. If the heat sink is made of aluminium nitride, how hot will the microchip be at steady state? You will find equations of heat flow in Appendix B, Table B15, and the thermal properties of aluminium nitride in Appendix A, Tables A5 an A6.

E2.16. *Choosing dielectrics*. The capacitance C of a condenser with two plates each of area A separated by a dielectric of thickness t is

$$C = \varepsilon_r \varepsilon_o \frac{A}{t}$$

where ε_o is the permittivity of free space and ε_r is the dielectric constant of the material between the plates. Select a dielectric by scanning data in Appendix A, Tables A7 and Table A8, first to maximize C and second to minimize it, for a given A and t.

E2.17. *Piezoelectric actuators*. Piezoelectric materials respond to an electric field by change of shape. The strain ε induced by a field E is

$$\varepsilon = d_{33}E$$

where d_{33} is the piezoelectric charge coefficient, listed in Table A8. Its units are pm/V. A sample of soft PZT (lead zirconium titanate) is exposed to a field of 10 MV/m. How large is the resulting strain? You will find the piezoelectric charge coefficient for soft PZT in Table A8. Use a mean value.

E2.18. *Pyroelectric temperature sensing*. Remote, noncontact, temperature sensing is possible by focussing radiation from the source onto a pyroelectric material and by using the charge that appears on its surface as a measure of temperature. A sensor is required to measure temperature in the range 100–200°C. To work in this range the material must have a ferroelectric Curie temperature greater than 200°C, because otherwise it ceases to be pyroelectric in the sensing range. The signal is maximized by selecting a material with the largest possible pyroelectric coefficient. Use the data for pyroelectrics in Appendix A, Section A8, to make a selection.

E2.19. *Magnetostrictive actuation*. A magnetostrictive actuator has and active core of length L=20 mm. It is made of Terfenol D, a giant magnetostrictive alloy of terbium, dysprosium and iron. If a field sufficiently large to saturate the core is applied, by how much will the core change in length? Use the mean value of the data for Terfenol D listed in Appendix A, Section A9, to find out.

E2.20. *Material price*. It is proposed to replace the cast iron casing of a power tool with one with precisely the same dimension moulded from nylon. Will the material cost of the nylon casing be greater or less than that made of cast iron? Use data from Appendix A3 and A11 to find out.

E2.21. *Carbon footprint*. Plastics are sometimes portrayed as environmentally poor because most are synthesize from oil or natural gas, implying a larger embodied energy and carbon footprint than other materials. Is this an accurate portrait? Use the data from Appendix A, Table A10 and A2 to compare the carbon footprint of three of the materials most widely used in household products: polypropylene, aluminium and carbon steel. Use mean values of the values in the tables.

CHAPTER 3

Materials Property Charts

Abstract

Material property charts map materials onto material property space. The charts condense a large body of information into a compact, accessible form; they reveal correlations between material properties that aid in checking and estimating data; and in later chapters they become tools for materials selection, for exploring the effect of processing on properties, for demonstrating how shape can enhance structural efficiency, and for suggesting directions for further material development. This chapter explains how the charts work; the additional information, guiding selection, that can be included on them; and it presents a compilation of the most useful of the charts.

Keywords

Material property charts; guidelines for design; charts for mechanical; thermal; electrical; magnetic; environmental and economic properties

3.1 Introduction and Synopsis

There are an awful lot of materials and they have an awful lot of properties. We need a way to display and compare them. One property can be displayed as a ranked list or as a bar-chart. Yet it is seldom that the performance of a component depends on just one property. More often it is a combination of properties that matter: think, for instance, of the need for stiffness at low weight, or of strength combined with toughness, or of thermal conduction coupled with corrosion resistance. This suggests the

idea of mapping material properties, delineating the fields in property space occupied by each material class and the subfields occupied by individual materials.

The resulting *material property charts* are helpful in many ways. They condense a large body of information into a compact but accessible form; they reveal correlations between material properties that aid in checking and estimating data; and in later chapters they become tools for materials selection, for exploring the effect of processing on properties, for demonstrating how shape can enhance structural efficiency, and for suggesting directions for further material development.

The ideas behind material property charts are described briefly in Section 3.2. Section 3.3 is not so brief because it introduces the charts themselves. There is no need to read it all, but it is helpful to persist far enough to be able to read and interpret the charts fluently, and to understand the meaning of the design guidelines that appear on them. If, later, you use a particular chart, you should read the background to it given here to be sure of interpreting it correctly.

As explained in the preface to this book, the charts can be copied and distributed for teaching purposes without infringing copyright.[1]

3.2 Exploring Material Properties

The properties of materials have characteristic ranges of values. The span can be large: many have values that range over five or more decades. One way to display this is as a bar-chart like that of Fig. 3.1 for Young's modulus. Each bar describes one material; its length shows the range of modulus exhibited by that material in its various forms. The materials are segregated by class. Each class shows a characteristic range: metals and ceramics have high moduli; polymers have low; hybrids have a wide range, from low to high. The total range of modulus values is large – it spans a factor of about 10^6 – so logarithmic scales are used to display it.[2]

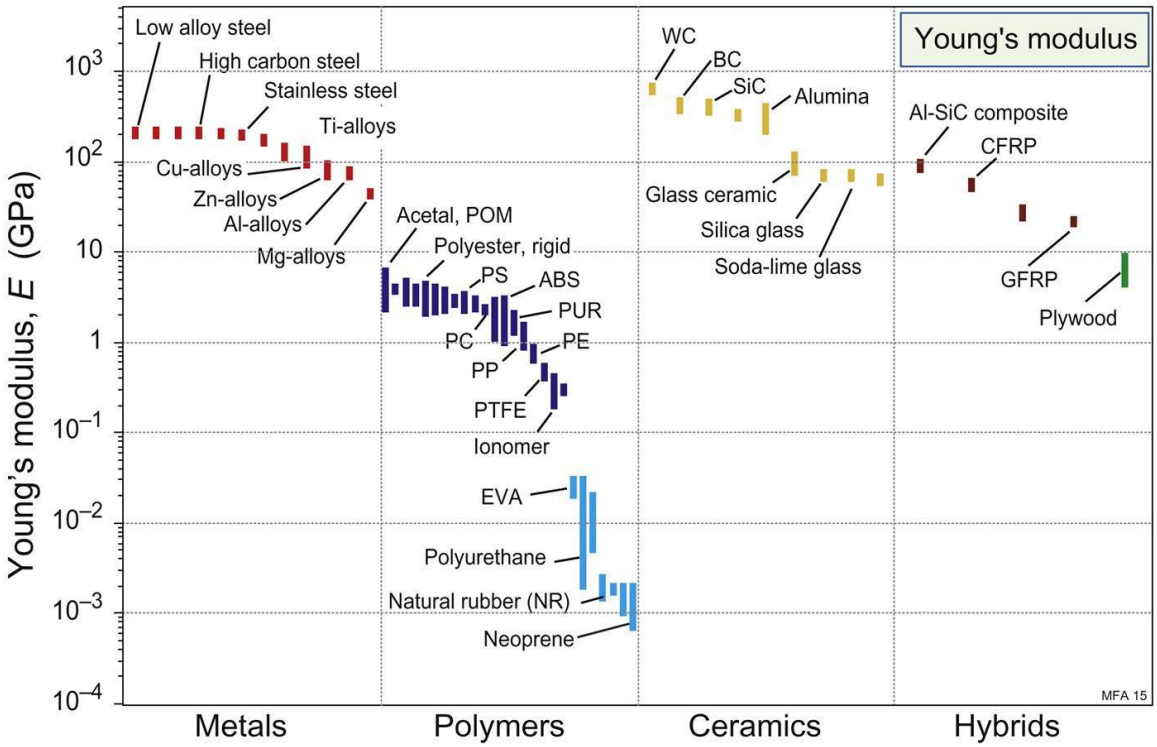


FIGURE 3.1 A bar-chart showing modulus for families of solid. Each bar shows the range of modulus offered by a material, some of which are labelled.

More information is displayed by an alternative plot, illustrated in the schematic of Fig. 3.2. Here, one property (the modulus, E, in this case) is plotted against another (the density, ρ). The range of the axes is chosen to include all materials, from the lightest, flimsiest foams to the stiffest, heaviest metals, and it is large, again requiring log scales. It is then found that data for a given family of materials (polymers for example) cluster together; the *subrange* associated with one material family is, in all cases, much smaller than the *full* range of that property. Data for one family can be enclosed in a property envelope – it is the envelopes that are shown on this schematic.

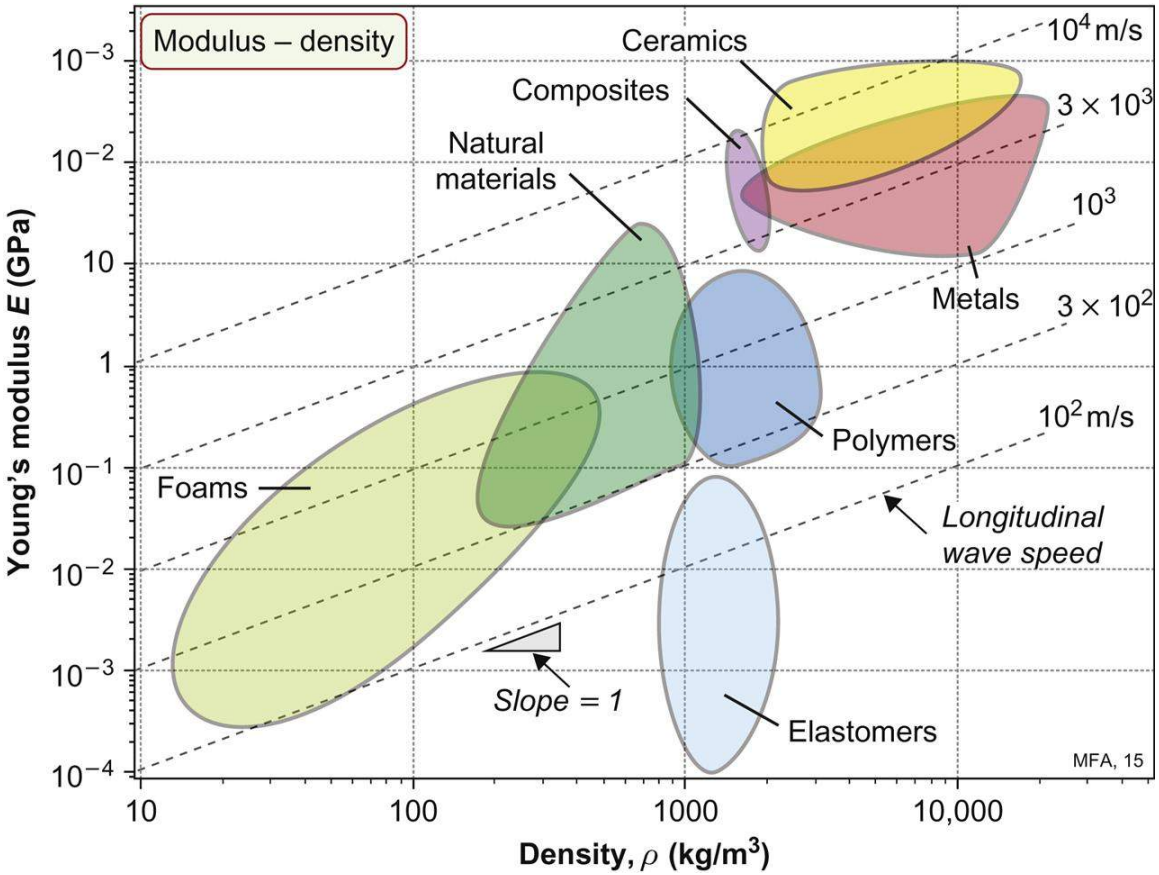


FIGURE 3.2 The idea of a Materials Property Chart: Young's modulus, E, is plotted against the density, ρ, on log scales. Each material class occupies a characteristic field. The contours show the longitudinal elastic wave speed $v = (E/\rho)^{1/2}$.

All this is simple enough – just a helpful way of displaying data. However, by choosing the axes and scales appropriately, more can be added. The speed of sound in a solid depends on E and ρ; the longitudinal wave speed v, for instance, is

$$v = \left(\frac{E}{\rho}\right)^{1/2}$$

or (taking logs)

$$\log E = \log \rho + 2 \log v$$

For a fixed value of v, this equation plots as a straight line of slope 1 on Fig. 3.2 (it is another of the advantages of log scales). This allows us to add *contours of constant wave velocity* to the chart: they are the family of parallel diagonal lines linking materials in which longitudinal waves travel with the same speed. All the charts allow additional fundamental relationships of this sort to be displayed – they are like height contours on a topological map. And there is more: design-optimizing parameters called *material indices* also plot as contours on to the charts. But that comes in Chapter 4, Materials Selection – the Basics.

Among the mechanical and thermal properties, there are 30 or so that are of primary importance, both in characterizing the material and in engineering design. They were listed in the tables of Chapter 2, Engineering Materials and Their Properties: they include density, moduli, strength, hardness, toughness, thermal and electrical conductivities, expansion coefficient and specific heat. The charts display data for these properties for the families and classes of materials listed in Table 3.1. The list is expanded from the original six of Figure 2.1 by distinguishing *composites* from *foams* and from *natural materials*, and by distinguishing the high strength *technical ceramics* (like silicon carbide) from the low strength, *nontechnical ceramics* (like concrete and brick). Within each family, data are plotted for a representative set of materials, chosen both to span the full range of behaviour for the class and to include the most common and most widely used members of it. In this way the envelope for a family encloses data not only for the materials listed in Table 3.1, but virtually all other members of the family as well.

Table 3.1

Material Families and Classes

Family	Classes	Short Name
Metals (The metals and alloys of engineering)	Aluminium alloys	Al alloys
	Copper alloys	Cu alloys
	Lead alloys	Lead alloys
	Magnesium alloys	Mg alloys
	Nickel alloys	Ni alloys
	Carbon steels	Steels
	Stainless steels	Stainless steels
	Tin alloys	Tin alloys
	Titanium alloys	Ti alloys
	Tungsten alloys	W alloys
	Lead alloys	Pb alloys
	Zinc alloys	Zn alloys
Ceramics, technical ceramics (Fine ceramics capable of load-bearing application)	Alumina	$Al_2 0_3$
	Aluminium nitride	AlN
	Boron carbide	B_4C
	Silicon carbide	SiC
	Silicon nitride	Si_3N_4
	Tungsten carbide	WC
Ceramics, nontechnical ceramics (porous ceramics of construction)	Brick	Brick
	Concrete	Concrete
Glasses	Soda-lime glass	Soda-lime glass
	Borosilicate glass	Borosilicate glass
	Silica glass	Silica glass
	Glass ceramic	Glass ceramic
Polymers (thermoplastics and thermosets)	Acrylonitrile butadiene styrene	ABS
	Cellulose acetate	CA
	Ionomers	Ionomers
	Epoxies	Epoxy
	Phenolics	Phenolics
	Polyamides (nylons)	PA
	Polycarbonate	PC
	Polyesters	Polyester
	Polyether ether ketone	PEEK
	Polyethylene	PE
	Polyethylene terephthalate	PET or PETE
	Polymethyl methacrylate	PMMA
	Polyoxymethylene (Acetal)	POM
	Polypropylene	PP
	Polystyrene	PS
	Polytetrafluorethylene	PTFE
	Polyvinylchloride	PVC
Elastomers (engineering rubbers, natural and synthetic)	Butyl rubber	Butyl rubber
	Ethylene vinyl acetate (EVA)	EVA
	Isoprene	Isoprene
	Natural rubber	Natural rubber
	Polychloroprene	Neoprene
	Polyurethane	PU
	Silicone elastomers	Silicones
Hybrids: composites	Carbon-fibre reinforced polymers	CFRP
	Glass-fibre reinforced polymers	GFRP
Hybrids: foams	Flexible polymer foams	Flexible foams
	Rigid polymer foams	Rigid foams
Hybrids: natural materials	Cork	Cork
	Bamboo	Bamboo
	Wood	Wood

The charts that follow show a *range* of values for each property of each material. Sometimes the range is narrow: the modulus of copper, for instance, varies by only a few percent about its mean value, influenced by purity, texture and such like. Sometimes, however, the range is wide: the strength of metals can vary by a factor of 100 or more, influenced by composition and the state of work hardening or heat treatment. Crystallinity and degree of cross-linking greatly influence the modulus of polymers. Porosity influences the strength of ceramics. These *process-sensitive* properties appear as elongated bubbles within the envelopes on the charts – they are explored in more depth in Chapter 6, Processes and their Effect on Properties.

3.3 The Material Property Charts

Mechanical Properties

The Modulus – Density Chart (Fig. 3.3)

Modulus and density are familiar properties. Steel is stiff, rubber is compliant: these are effects of modulus. Lead is heavy; cork is buoyant: these are effects of density. Fig. 3.3 shows the range of Young's modulus, E, and density, ρ, for engineering materials. Data for members of a particular family of material cluster together and can be enclosed by a coloured envelope. The same family envelopes appear on all the diagrams: they correspond to the main headings in Table 3.1.

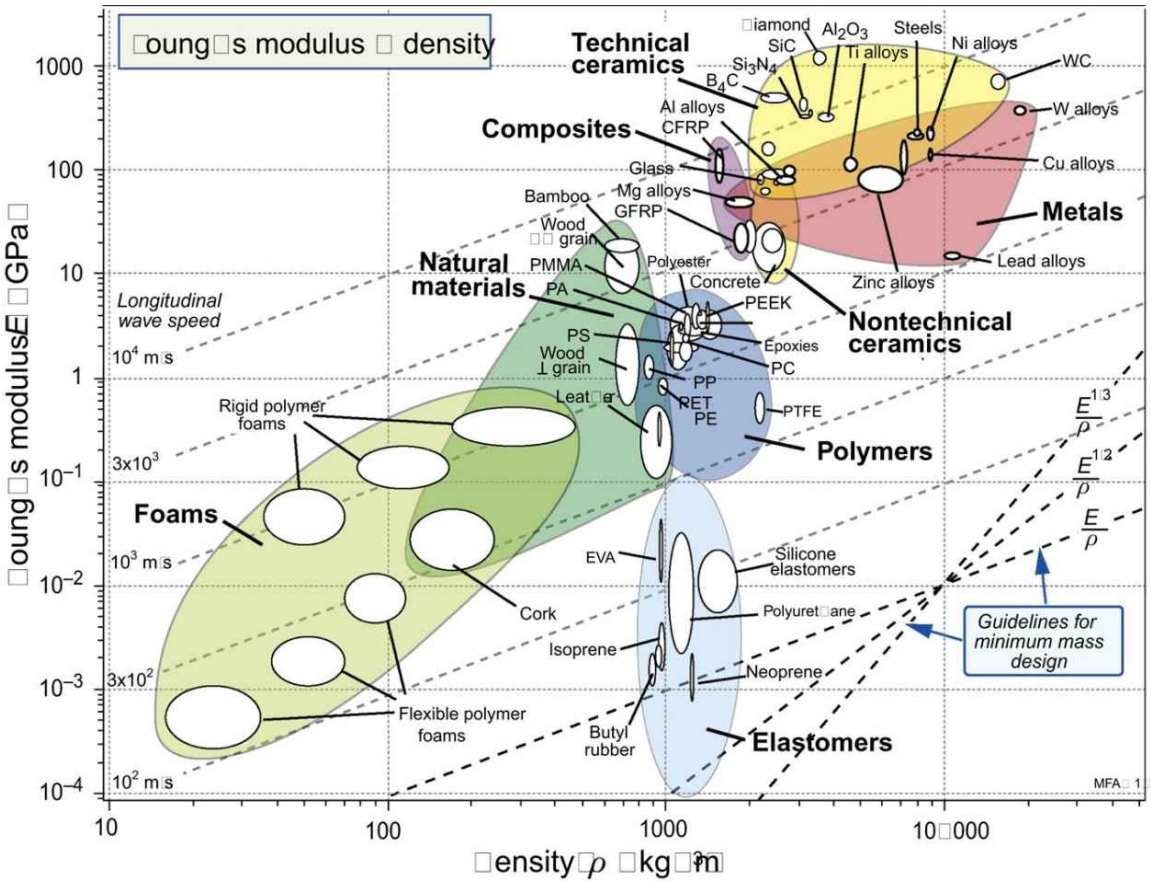


FIGURE 3.3 Young's modulus, E, plotted against density, ρ. The heavy envelopes enclose data for a given class of material. The diagonal contours show the longitudinal wave velocity. The guidelines of constant E/ρ, $E^{1/2}/\rho$ and $E^{1/3}/\rho$ allow selection of materials for minimum weight, deflection-limited, design.

The density of a solid depends on the atomic weight of its atoms or ions, their size, and the way they are packed. The size of atoms does not vary much: most have a volume within a factor of two of 2×10^{-29} m^3. Packing fractions do not vary much either – a factor of two, more or less. Close packing gives a packing fraction of 0.74; open networks such as that of the diamond cubic structure give about 0.34. The spread of density comes mainly from that of atomic weight, ranging from 1 for hydrogen to 238 for uranium. Metals are dense because they are made of heavy atoms, packed closely together; polymers have low densities because they are largely made of carbon (atomic weight: 12) and hydrogen (atomic weight: 1) in more open amorphous or crystalline packings. Ceramics, for the most part, have lower densities than metals because they contain light O, N or C

atoms. Even the lightest atoms, packed in the most open way, give solids with a density of around 1000 kg/m^3, the same as that of water. Materials with lower densities than this are *foams* – materials made up of cells containing a large fraction of pore space.

The *moduli* of most materials depend on two factors: bond stiffness, and the number of bonds per unit volume. A bond is like a spring, and, like a spring, it has a spring constant, S (units: N/m). Young's modulus, E, is roughly

$$E = \frac{S}{r_o} \tag{3.1}$$

where r_o is the 'atom size' (r_o^3 is the mean atomic or ionic volume). The wide range of moduli is largely caused by the range of values of S. The covalent bond is stiff (S=20–200 N/m); the metallic and the ionic a little less so (S=15–100 N/m). Diamond has a very high modulus because the carbon atom is small, giving a high bond density, and its atoms are linked by strong covalent springs (S=200 N/m). Metals have high moduli because close packing gives a high bond density and the bonds are strong, though not as strong as those of diamond. Polymers contain both strong diamond-like covalent bonds and weak hydrogen or Van der Waals bonds (S=0.5–2 N/m). It is the weak bonds that stretch when the polymer is deformed, giving low moduli.

But even large atoms ($r_o = 3 \times 10^{-10}$ m) bonded with the weakest bonds (S=0.5 N/m) have a modulus of roughly

$$E = \frac{0.5}{3 \times 10^{-10}} \approx 1 \text{ GPa} \tag{3.2}$$

This is the *lower limit* for true solids. The chart shows that many materials have moduli that are lower than this: they are either elastomers or foams. Elastomers have a low E because their weak secondary bonds have melted as their glass temperature, T_g, is below room temperature, leaving only the very weak 'entropic' restoring force associated with tangled, long-chain molecules. Foams have low moduli because the cell walls bend easily when the material is loaded. More on this in Chapter 10, Selection of Material and Shape.

The chart shows that the modulus of engineering materials spans seven decades,[3] from 0.0001 GPa (low density foams) to 1000 GPa (diamond). The density spans a factor of 2000, from less than 10 to 20,000 kg/m^3. Ceramics as a family are very stiff, metals a little less so – but none have a modulus less than 10 GPa. Polymers, by contrast, all cluster between 0.8 and 8 GPa.

The log scales allow more information to be displayed. As explained in the last section, the velocity of elastic waves in a material and the natural vibration frequencies of a component made of it are proportional to $(E/\rho)^{1/2}$. Contours of this quantity are plotted on the chart, labelled with the longitudinal wave speed. It varies from less than 50 m/s (soft elastomers) to a little more than 10^4 m/s (stiff ceramics). We note that aluminium and glass, because of their low densities, transmit waves quickly despite their low moduli. One might have expected the wave velocity in foams to be low because of the low modulus, but the low density almost compensates. The velocity in wood, across the grain, is low; but along the grain, it is high – roughly the same as steel – a fact made use of in the design of musical instruments.

The chart helps in the common problem of material selection for structures in which weight must be minimized. Guidelines corresponding to three common geometries of loading are drawn on Fig. 3.3. Their use to select materials for stiffness-limited design at minimum weight is described in Chapter 4, Materials Selection – the Basics and Chapter 5, Materials Selection – Case Studies.

Surveying Material Properties

Which class of metallic alloy is the least dense? Which the densest? Which the stiffest? Which the least stiff?

Answer

A glance at Fig. 3.3 reveals that the lightest class is that of magnesium alloys, the heaviest that of tungsten alloys; tungsten alloys are also the stiffest, lead alloys the least stiff.

All the charts that appear in this and subsequent chapters can be used for this fast access and comparisons.

Comparing Sound Velocities

A metal is needed in which longitudinal waves travel at 300 m/s. Use Fig. 3.3 to identify candidates.

Answer

The chart shows that zinc alloys, copper alloys and tungsten alloys all have a longitudinal wave speed close to 300 m/s.

The Strength–Density Chart (Fig. 3.4)

The modulus of a solid is a well-defined quantity with a sharp value. The strength is not. It is shown, plotted against density, ρ, in Fig. 3.4.

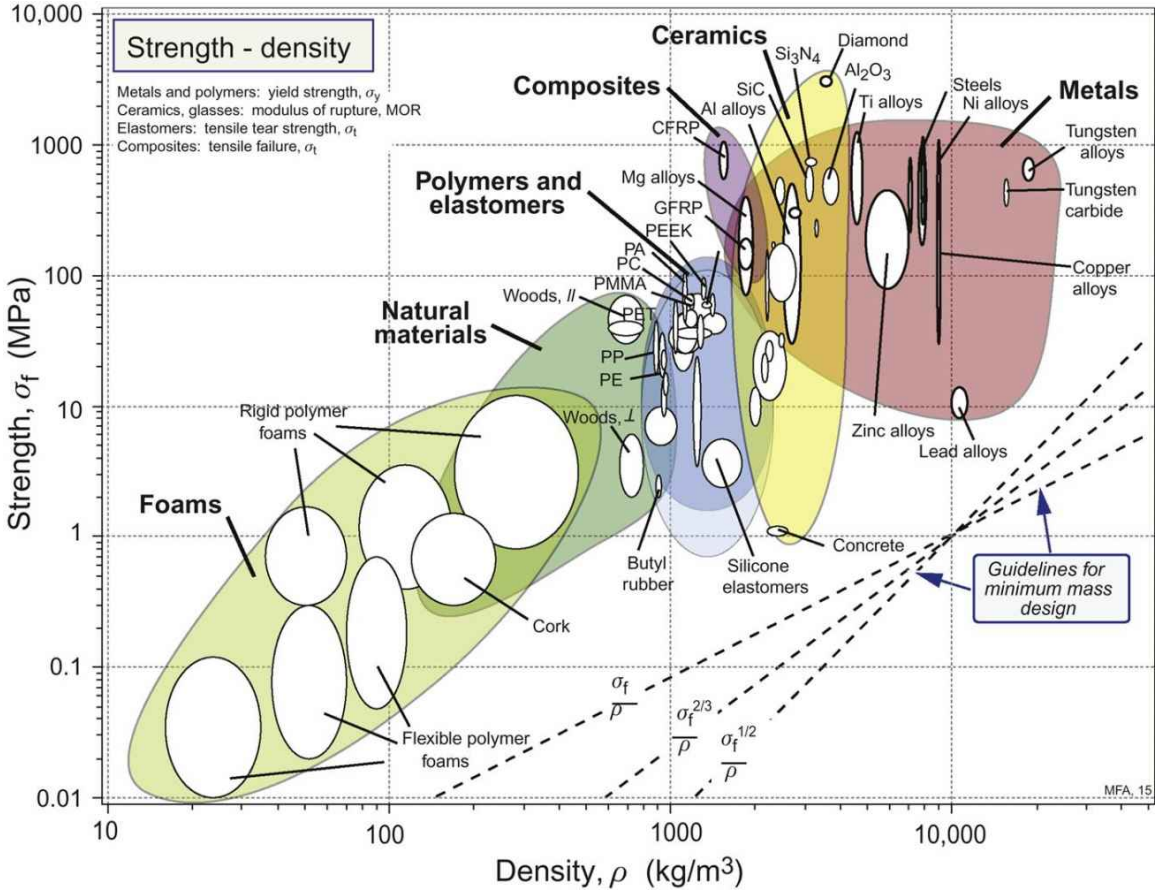


FIGURE 3.4 Strength, σ_f, plotted against density, ρ (yield strength for metals and polymers, compressive strength for ceramics, tear strength for elastomers and tensile strength for composites). The guidelines of constant σ_f/ρ, $\sigma_f^{2/3}/\rho$ and $\sigma_f^{1/2}/\rho$ are used in minimum weight, yield-limited, design.

The word 'strength' needs definition. For metals and polymers, it is the *yield strength*. The range of yield strength for a given material is large because it spans those that have been worked or hardened in some other way as well as those that have been softened by annealing. For brittle ceramics, the strength plotted here is the *modulus of rupture:* the flexural strength. It is slightly greater than the tensile strength, but much less than the compression strength, which, for ceramics is 10 to 15 times larger than the strength in tension. For elastomers, strength means the tensile *tear strength*. For composites, it is the *tensile failure strength* (the compressive strength can be less by up to 30% because of fibre buckling). We will use the symbol σ_f for all of these, despite the different failure mechanisms involved, to allow a first-order comparison.

The range of strength for engineering materials, like that of the modulus, spans many decades: from less than 0.01 MPa (foams, used in packaging and energy-absorbing systems) to 30,000 MPa (the strength of diamond, exploited in the diamond-anvil press). The single most important concept in understanding this wide range is that of the *lattice resistance* or *Peierls stress*. It is the intrinsic resistance of the structure to plastic shear. Plastic shear in a crystal involves the motion of dislocations. Pure metals are soft because the nonlocalized metallic bond does little to hinder dislocation motion, whereas ceramics are hard because their more localized covalent and ionic bonds (which must be broken and reformed when the structure is sheared) lock the dislocations in place. In noncrystalline solids we think instead of the energy barrier to the relative slippage of two segments of a polymer chain or to the shear of a small molecular cluster in a glass network. Thus if the unit step involves breaking strong bonds (as in an inorganic glass) the materials will be strong. If it only involves the rupture of weak bonds (the Van der Waals bonds in polymers for example), it will be weak. Materials that fail by fracture do so because the lattice resistance or its amorphous equivalent is so large that atomic separation (fracture) happens first.

The most striking feature of Fig. 3.4 is the extreme elongation of some of the material bubbles in the vertical (strength) direction. This is because materials with low lattice resistance can be strengthened by introducing obstacles to slip. In metals this is achieved by adding alloying elements, particles, grain boundaries and other dislocations ('work hardening'; and in polymers by cross-linking or by orienting the chains so that strong covalent bonds, as well as weak Van der Waals bonds, must be broken

when the material deforms. Chapter 6, Processes and their Effect on Properties goes into this in more detail.

An important use of the chart is in materials selection for light-weight strength-limited design. Guidelines are shown for materials selection in the minimum weight design of ties, columns, beams and plates, and for yield-limited design of moving components in which inertial forces are important. Their use is described in Chapters 4, Materials Selection – the Basics and Chapter 5, Materials Selection – Case Studies.

High Strength at Low Weight

Which material has the highest ratio of strength σ_f to density ρ? Use Fig. 3.4 to find out.

Answer

The materials with the largest values of σ_f/ρ are those towards the top left of the figure. The ratio plots as a line of slope 1 on the chart. There is a guideline with this slope among the three at the lower right. The materials with the highest ratio are the ones furthest above such a line. CFRP and diamond stand out as meeting this criterion.

The Modulus–Strength Chart (Fig. 3.5)

High tensile steel makes good springs. But so does rubber. How is it that two such different materials are both suited for the same task? This and other questions are answered by Fig. 3.5, one of the most useful of all the charts.

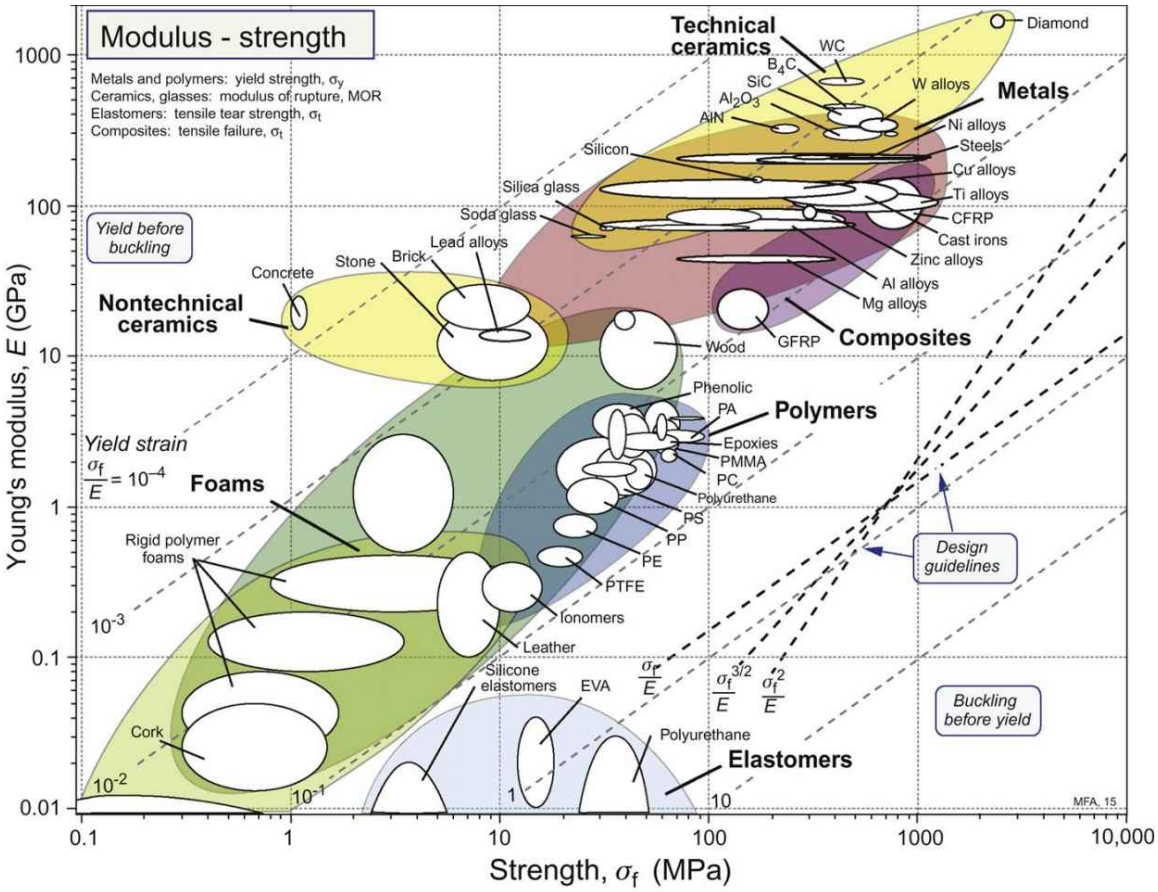


FIGURE 3.5 Young's modulus, E, plotted against strength σ_f. The design guidelines help with the selection of materials for springs, pivots, knife-edges, diaphragms and hinges; their use is described in Chapter 4, Materials Selection – the Basics and Chapter 5, Materials Selection – Case Studies.

It shows Young's modulus, E, plotted against strength, σ_f. The qualifications on 'strength' are the same as before. It means yield strength for metals and polymers, flexural strength (modulus of rupture) for ceramics, tear strength for elastomers, and tensile strength for composite and woods; the symbol σ_f is used for them all. Contours of *yield strain* or *fracture strain*, σ_f/E, (meaning the strain at which the material ceases to be linearly elastic) appear as a family of straight parallel lines.

Examine these first. Metals have a yield strain around 0.001 (0.1%). For polymers it is larger: between 0.01 and 0.1. Composites and woods lie on the 0.01 contour. Elastomers, because of their exceptionally low moduli, have values of σ_f/E larger than any other class of material: typically 1 to 10.

The distance over which interatomic forces act is small; a bond is broken if it is stretched to more than about 10% of its original length. So the force

F^* needed to break a bond is roughly

$$F^* \approx \frac{S\, r_o}{10} \tag{3.3}$$

where S, as before, is the bond stiffness. Then the failure strain of a solid should be roughly

$$\frac{\sigma_f}{E} \approx \frac{F^*}{r_o^2} \Big/ \left(\frac{S}{r_o}\right) = \frac{1}{10} \tag{3.4}$$

The chart shows that, for some polymers, the failure strain approaches this value. For most solids it is less, for two reasons.

First, nonlocalized bonds (those in which the cohesive energy derives from the interaction of one atom with large number of others, not just with its nearest neighbours) are not broken when the structure is sheared. The metallic bond, and the ionic bond for certain directions of shear, act in this way. Very pure metals, pure copper, e.g., yield at stresses as low as $E/10{,}000$, and strengthening mechanisms are needed to make them useful in engineering. The covalent bond *is* localized, and covalent solids, for this reason, have yield strength that, at low temperatures, are as high as $E/10$. It is hard to measure them (though it can sometimes be done by indentation) because of the second reason for weakness: they generally contain defects – concentrators of stress – from which fracture can propagate at stresses well below the 'ideal' $E/10$. Elastomers are anomalous (they have strengths of about E) because the modulus does not derive from bond stretching, but from the change in entropy of the tangled molecular chains when the material is deformed.

This has not yet explained how to choose good materials to make springs. This involves the design guidelines shown on the chart, but that has to wait until Chapter 5, Materials Selection – Case Studies, Section 5.7.

Strong Solids

Use the strength–density chart of Fig. 3.4 to identify three material classes with members with strengths exceeding 1000 MPa.

Answer

The material classes of steels, titanium alloys and CFRP all have members with strengths exceeding 1000 MPa.

The Specific Stiffness–Specific Strength Chart (Fig. 3.6)

Many designs, particularly those for things that move, call for stiffness and strength at minimum weight. To help with this, the data of the previous chart are replotted in Fig. 3.6 after dividing the values of modulus and strength by the density, so it shows E/ρ plotted against σ_f/ρ. These are measures of 'mechanical efficiency', meaning the use of the least mass of material to do the most structural work. They are not the only ones – we'll meet more in Chapter 4, Materials Selection – the Basics – but they are a good starting point.

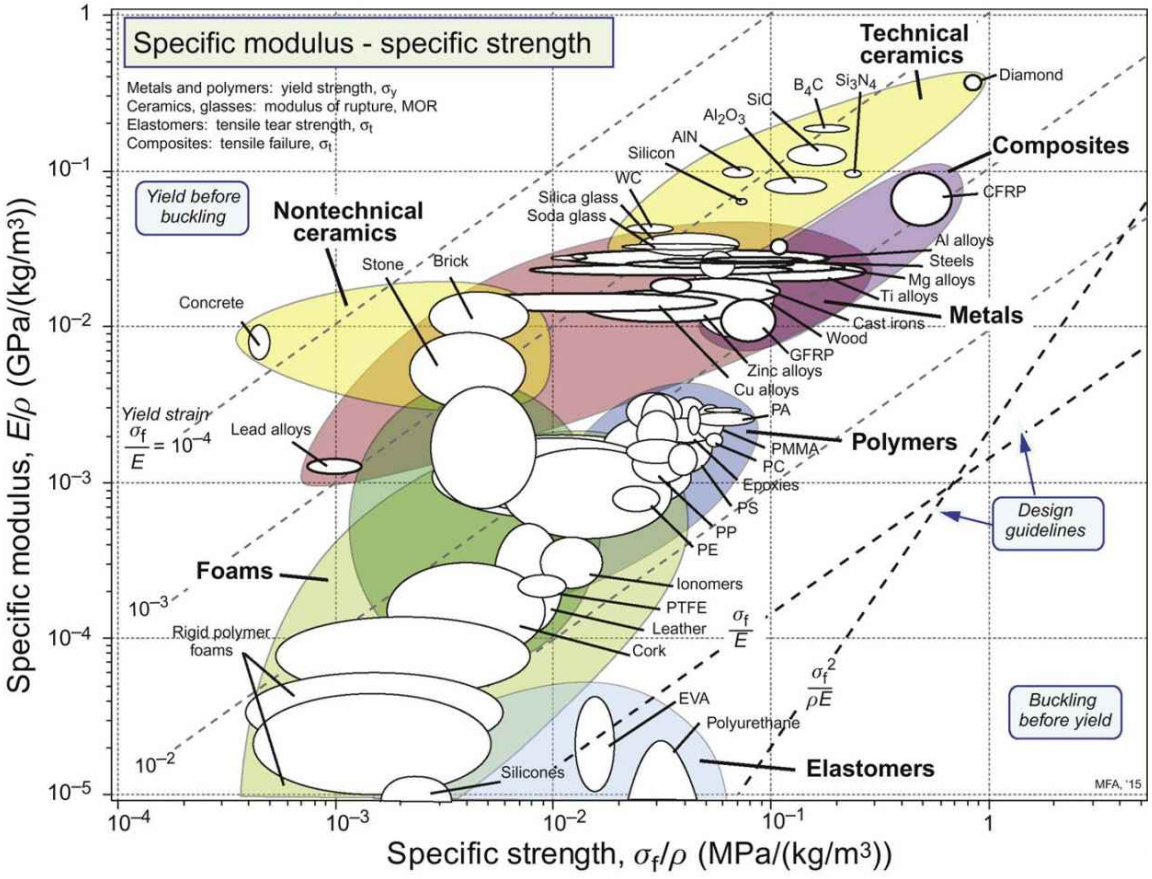


FIGURE 3.6 Specific modulus, E/ρ plotted against specific strength σ_f/ρ. The design guidelines help with the selection of materials for light-weight springs and energy-storage systems.

Composites, particularly CFRP, lie at the upper right. They emerge as the material class with the most attractive specific properties, one of the reasons for their increasing use in aerospace. Ceramics have exceptionally high stiffness per unit weight and their strength per unit weight is as good as metals but their brittleness excludes them from much structural use. Metals are penalized because of their relatively high densities. Polymers, because their densities are low, do better on this chart than on the last one.

The chart has application in selecting materials for light springs and energy-storage devices. But that too has to wait till Section 5.7.

High Strength at Low Weight

High quality mountain bikes are made of materials with particularly high values of the ratio σ_f/ρ, making them strong and light. Which metal class has the highest value of this ratio?

Answer

Fig. 3.6 shows that titanium alloys have the highest value.

The Fracture Toughness–Modulus Chart (Fig. 3.7)

Increasing the strength of a material is useful only as long as it remains plastic and does not become brittle; brittleness means a vulnerability to failure by fast fracture initiated from any tiny crack or defect it may contain. The resistance to the propagation of a crack is measured by the *fracture toughness*, K_{1c}, the units of which are MPa·m$^{1/2}$. It is plotted against modulus E in Fig. 3.7. Values range from less than 0.01 to over 100 MPa·m$^{1/2}$. At the lower end of this range are brittle materials like glass, which, when loaded, remain elastic until they fracture. For these, linear-elastic fracture mechanics works well, and the fracture toughness itself is a well-defined property. At the upper end lie the super-tough materials, all of which show substantial plasticity before they break. For these the values of K_{1c} are approximate, derived from critical J-integral (J_c) and critical crack-opening displacement (δ_c) measurements, by making the approximation $K_{1c} \cong (E J_c)^{1/2}$, for instance. They are helpful in providing a ranking of materials but should not be taken too literally. The figure shows one reason for the dominance of metals in engineering; they almost all have values of K_{1c} above 18 MPa·m$^{1/2}$, a value often quoted as a minimum for conventional design.

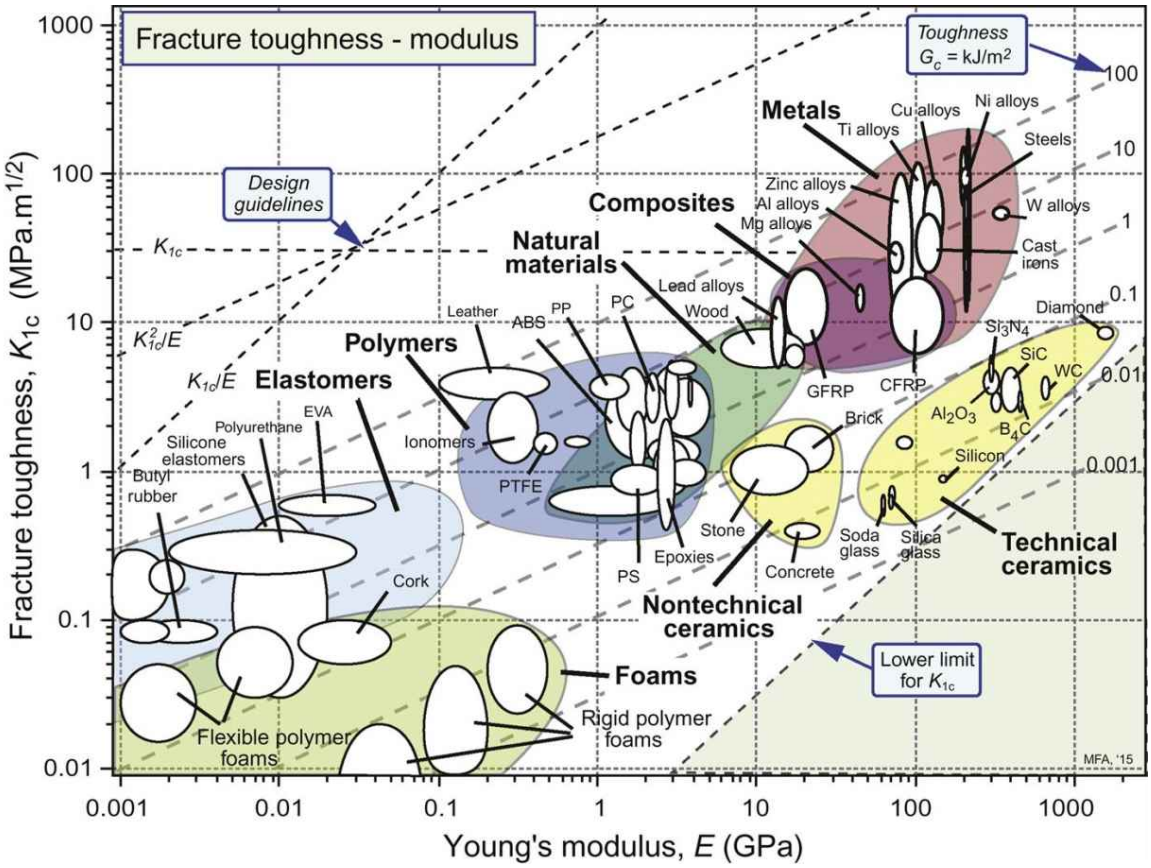


FIGURE 3.7 Fracture toughness, K_{1c}, plotted against Young's modulus, *E*. The family of lines are of constant K_{1c}^2/E (approximately G_{1c}, the fracture energy or toughness). These, and the guideline of constant K_{1c}/E, help in design against fracture. The shaded triangle shows the lower limit for K_{1c}.

As a general rule, the fracture toughness of polymers is about the same as that of ceramics and glasses. Yet polymers are widely used in engineering structures; ceramics, because they are 'brittle', are treated with much more caution. Fig. 3.7 helps resolve this apparent contradiction. Consider first the question of the *necessary condition for fracture*. It is that sufficient external work be done, or elastic energy released, to supply the surface energy, γ per unit area, of the two new surfaces that are created. We write this as

$$G \geq 2\gamma \qquad \textbf{(3.5)}$$

where G is the energy release rate. Using the standard relation $K = (E\,G)^{1/2}$ between G and stress intensity K, we find

$$K \geq (2\,E\,\gamma)^{1/2} \tag{3.6}$$

Now the surface energies, γ, of solid materials scale as their moduli. To an adequate approximation $\gamma \approx E\,r_o/20$ where r_o is the atom size, giving

$$K \geq E\left(\frac{r_o}{10}\right)^{1/2} \tag{3.7}$$

We identify the right-hand side of this equation with a lower limiting value of K_{1c}, when, taking r_o as 2×10^{-10} m, we find

$$\frac{(K_{1c})_{\min}}{E} = \left(\frac{r_o}{10}\right)^{1/2} \approx 4.5 \times 10^{-6}\ \mathrm{m}^{1/2} \tag{3.8}$$

This criterion is plotted on the chart as a shaded triangle at the lower right corner. The fracture toughness cannot lie in this triangle unless some other source of energy, such as a chemical reaction or the release of elastic energy stored in the special dislocation structures caused by fatigue loading, is available to help the crack propagate. Then it is given a new symbol such as $(K_1)_{scc}$ meaning 'the critical value of K_1 for stress-corrosion cracking' or $(\Delta K_1)_{threshold}$ meaning 'the minimum range of K_1 for fatigue-crack propagation'. We note that the most brittle ceramics lie close to the threshold. When they fracture, the energy absorbed is only slightly more than the surface energy. When metals, polymers and composites fracture, the energy absorbed is vastly greater, usually because of plasticity associated with crack propagation.

Plotted on Fig. 3.7 are contours of *toughness*, $G_c \approx K_{1c}^2/E$, a measure of the apparent fracture surface energy with the expected units of energy per unit area, here kJ/m^2. The true surface energies, γ, of solids lie in the range 10^{-4} to 10^{-3} kJ/m^2. The diagram shows that the values of the toughness start at 10^{-3} kJ/m^2 and range through almost five decades to over 100 kJ/m^2. It is here that the difference between polymers and ceramics emerges: the toughness qualities of ceramics (10^{-3}–10^{-1} kJ/m^2) are much lower than those

of polymers (10^{-1}–10 kJ/m^2). This is part of the reason polymers are more widely used in engineering than ceramics, a point developed further in Chapter 5, Materials Selection – Case Studies, Section 5.10.

Comparing Materials by Toughness

The fracture toughness K_{1c} of polypropylene (PP) is about 4 MPa·m$^{1/2}$. That of aluminium alloys is about 10 times larger. But in deflection-limited design, it is the toughness G_c that is the more important property. Use Fig. 3.7 to compare the two materials by toughness.

Answer

Aluminium and PP have almost exactly the same values of G_c – approximately 10 kJ/m^2.

The Fracture Toughness–Strength Chart (Fig. 3.8)

The stress concentration at the tip of a crack generates a *process zone:* a plastic zone in ductile solids, a zone of microcracking in ceramics, a zone of delamination, debonding and fibre pull-out in composites. Within the process zone, work is done against plastic and frictional forces; it is this that accounts for the difference between the measured fracture energy G_c and the true surface energy 2γ. The amount of energy dissipated must scale roughly with the strength of the material within the process zone, and with its size, d_γ. This size is found by equating the stress field of the crack ($\sigma = K/\sqrt{2\pi r}$) at $r = d_\gamma/2$ to the strength of the material, σ_f, giving

$$d_\gamma = \frac{K_{1c}^2}{\pi \sigma_f^2} \qquad \textbf{(3.9)}$$

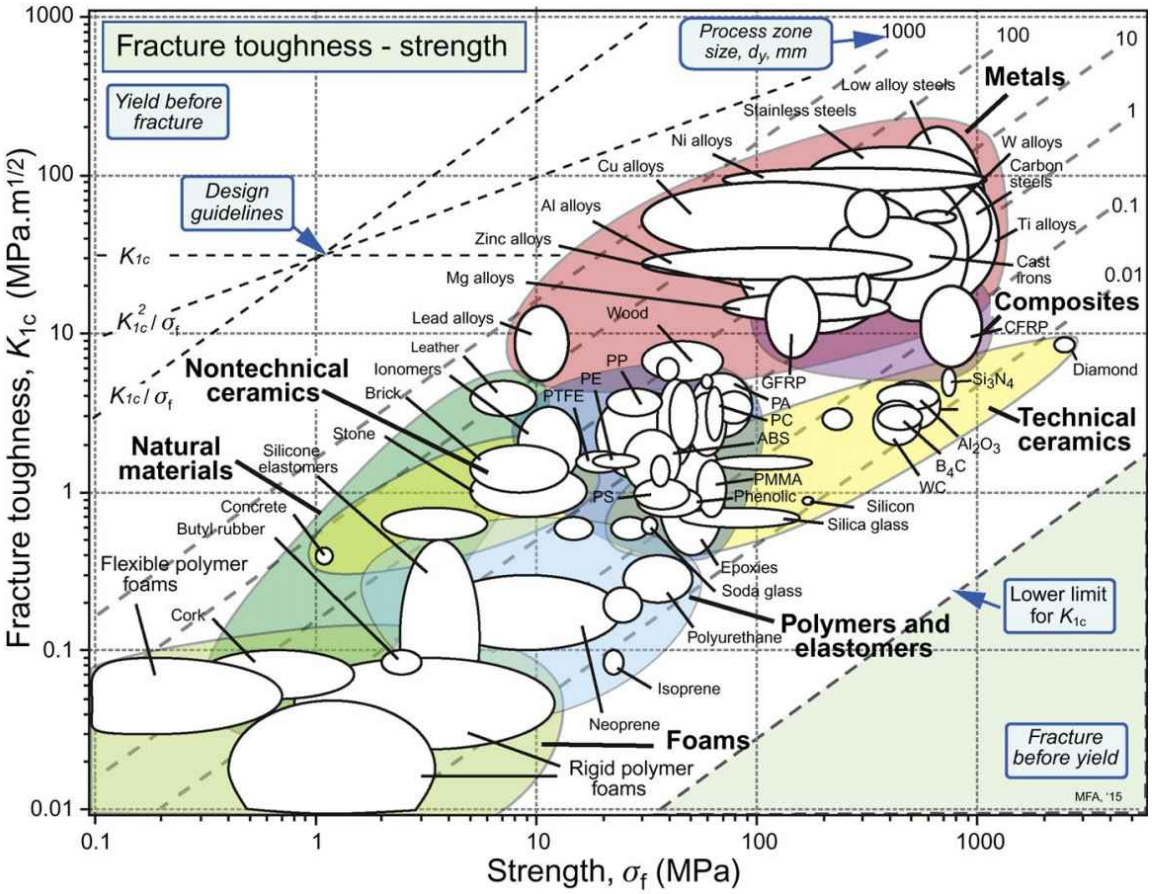


FIGURE 3.8 Fracture toughness, K_{1c}, plotted against strength, σ_f. The contours show the value of $K_{1c}^2/\pi\,\sigma_f^2$ – roughly, the diameter d_γ of the process zone at a crack tip. The design guidelines are used in selecting materials for damage tolerant design.

Fig. 3.8 shows fracture toughness plotted against strength. The size of the process zone, d_γ, is plotted on it as broken diagonal contours; it cannot be less than atomic dimensions, leading to the exclusion zone as a shaded triangle at the bottom right. The zone size of materials varies from near-atomic dimensions for very brittle ceramics and glasses to almost 1 metre for the most ductile of metals. At a constant zone size, fracture toughness tends to increase with strength, as expected. It is this that causes the data plotted in Fig. 3.8 to be clustered around the diagonal of the chart. Materials towards the bottom right have high strength and low toughness; they tend to *fracture before they yield*. Those towards the top left do the opposite: they *yield before they fracture*.

The diagram has application in selecting materials for the safe design of load-bearing structures. Examples are given in the case studies of Sections 5.10 and 5.11.

Valid Toughness Testing

A valid fracture toughness test requires a sample with dimension at least 10 times larger than the diameter of the process zone that forms at the crack tip. Use Fig. 3.8 to estimate the sample size needed for a valid test on ABS.

Answer

The process zone size for ABS is approximately 1 mm. A valid test requires a sample with dimensions exceeding 10 mm.

The Loss Coefficient–Modulus Chart (Fig. 3.9)

Bells, traditionally, are made of bronze. They can be made of glass; and they could (if you could afford it) be made of silicon carbide. Metals, glasses and ceramics all, under the right circumstances, have low intrinsic damping or 'internal friction', an important material property when structures vibrate. Intrinsic damping is measured by the *loss coefficient*, η, which is plotted in Fig. 3.9.

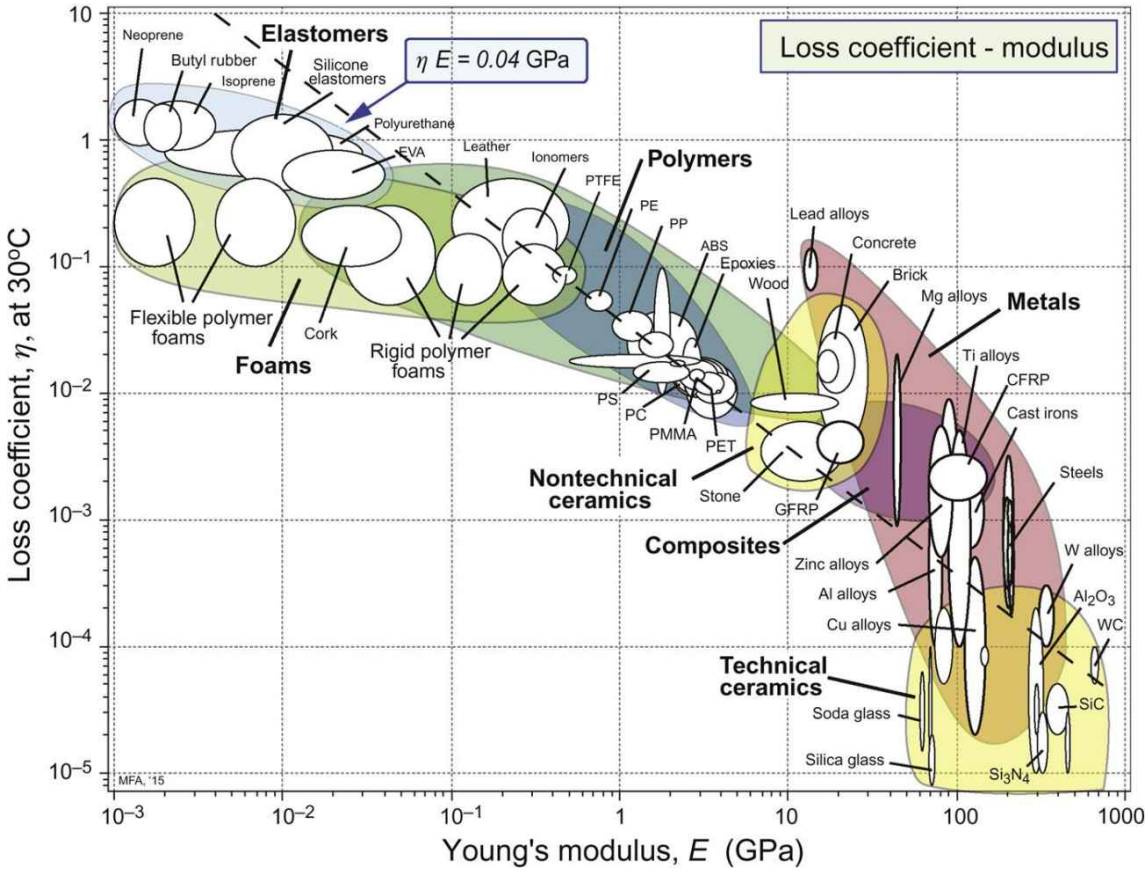


FIGURE 3.9 The loss coefficient, η, plotted against Young's modulus, E. The guideline corresponds to the condition $\eta = CE$

.

There are many mechanisms of intrinsic damping and hysteresis. Some (the 'damping' mechanisms) are associated with a process that has a specific time constant; then the energy loss is centred about a characteristic frequency. Others, the 'hysteresis' mechanisms, are associated with time-independent mechanisms; they absorb energy at all frequencies. In metals a large part of the loss is hysteretic, caused by dislocation movement: it is high in soft metals like lead and pure aluminium. Heavily alloyed metals like bronze and high-carbon steels have low loss because the solute pins the dislocations: these are the materials for bells. Exceptionally high loss is found in the Mn-Cu alloys, because of a strain-induced martensite transformation, and in magnesium, perhaps because of reversible twinning. The elongated bubbles for metals on Fig. 3.9 span the large range made accessible by alloying and work hardening. Engineering ceramics have low damping because the enormous lattice resistance pins dislocations in place at room temperature. Porous ceramics, on the other

hand, are filled with cracks, the surfaces of which rub, dissipating energy when the material is loaded. The high damping of flake-graphite cast irons has a similar origin. In polymers, chain segments rub against each other when loaded; the relative motion dissipates energy. The ease with which they slide depends on the ratio of the ambient temperature T, in this case room temperature, to the glass temperature, T_g, of the polymer. At low temperatures when $T/T_g < 1$, the secondary bonds are 'frozen', the modulus is high and the damping is relatively low. When $T/T_g > 1$, the secondary bonds have melted, allowing easy chain slippage; the modulus is low and the damping is high. This accounts for the obvious inverse dependence of η on E for polymers in Fig. 3.9; indeed, to a first approximation,

$$\eta = \frac{4 \times 10^{-2}}{E} \tag{3.10}$$

(with E in GPa) for polymers, woods and polymer-matrix composites.

Vibration Damping

A metal is sought for mountings to damp vibration of a small machine tool. Use Fig. 3.9 to find the metal with the greatest value of the damping coefficient η to use for the mountings.

Answer

Lead or lead alloys are the best choice.

Vibrational Heating

A centrifuge rotating at $f = 5000$ rpm is attached to a PTFE (Teflon) mounting. Poor balancing causes the centrifuge to vibrate, loading the PTFE up to a peak stress of $\sigma_{max} = 8$ MPa on each cycle. If a centrifuge-run lasts 2 minutes and no heat is lost from the PTFE, by how much will its temperature rise? Take the volume-specific heat for PTFE (which can be read from Fig. 3.11) to be $\rho C_p = 2 \times 10^6$ J/m^3.K, and retrieve the other material properties you need from Fig. 3.9.

Answer

Fig. 3.9 shows that the modulus of PTFE is $E = 0.4$ GPa and its loss coefficient is $\eta = 0.08$. The peak elastic energy stored in the PTFE in any one

cycle is

$$U_{\max} = \frac{\sigma_{\max}^2}{2E} = 80,000\ \mathrm{J/m^3}$$

of which $\Delta U = 2\pi\eta U_{\max}$ (Eq. 2.10) is lost on each cycle. Thus the energy dumped into the PTFE over 2 minutes is

$$U_{10\ \mathrm{mins}} = 2\pi\eta U_{\max}(2f) = 4\times 10^8\ \mathrm{J/m^3}$$

Dividing this by the volume-specific heat of PTFE reveals a temperature rise of 200°C. It will be necessary to ensure that heat can be conducted away from the mount to prevent it overheating.

Thermal Properties

The Thermal Conductivity–Electrical Resistivity Chart (Fig. 3.10)

The material property governing the flow of heat through a material at steady-state is the *thermal conductivity*, λ (units: W/m K). The valence electrons in metals are 'free', moving like a gas within the lattice of the metal. Each electron carries a kinetic energy $\frac{3}{2}kT$, where k is Boltzmann's constant. It is the transmission of this energy, via collisions, that conducts heat in metals. The thermal conductivity is described by

$$\lambda = \frac{1}{3}C_e\bar{c}\,\ell \qquad \textbf{(3.11)}$$

where C_e is the electron specific heat per unit volume, $\bar{c}$ is the electron velocity (2×10^5 m/s) and ℓ the electron mean free path, typically 10^{-7} m in pure metals. In heavily alloyed solid solutions (stainless steels, nickel-based superalloys and titanium alloys), the foreign atoms scatter electrons, reducing the mean free path to atomic dimensions ($\approx10^{-10}$ m), much reducing λ.

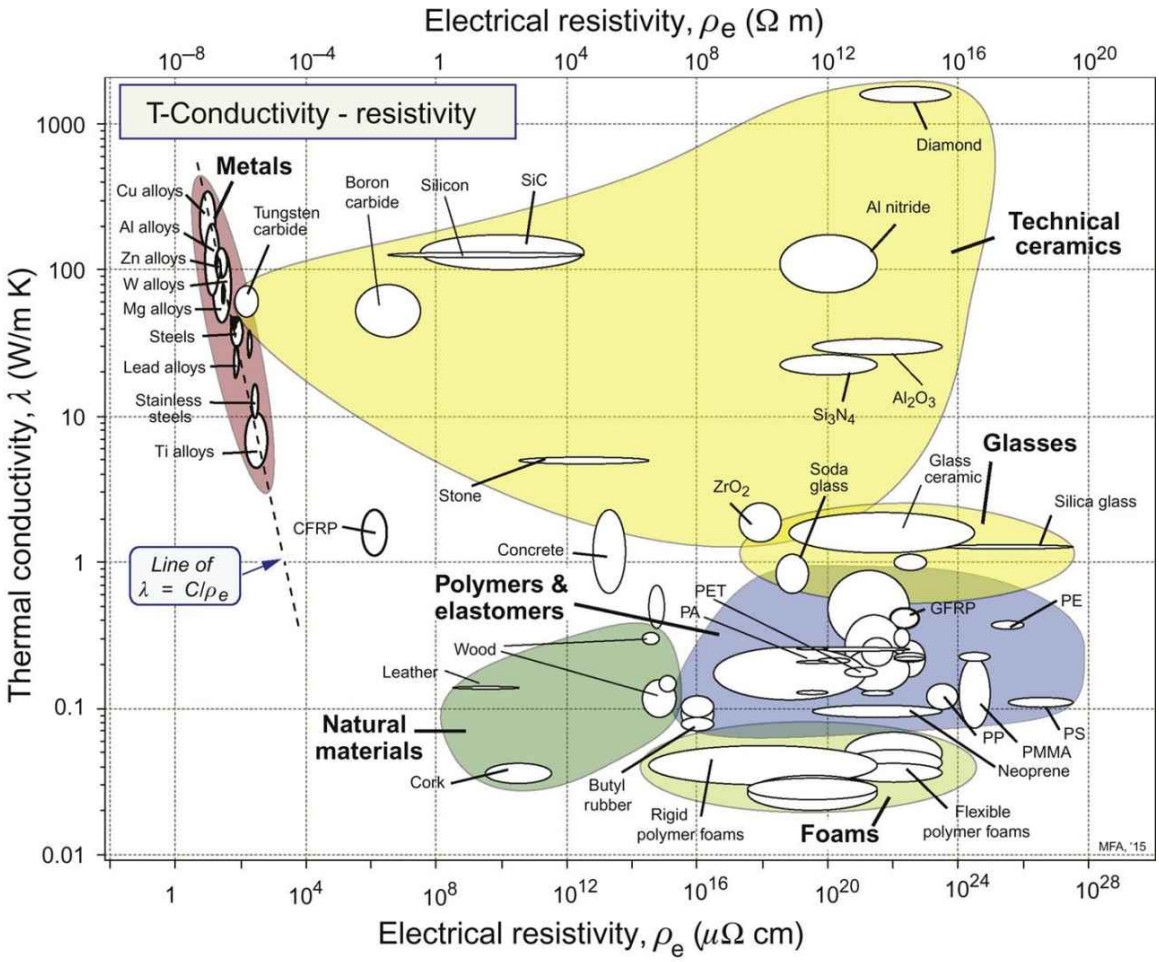


FIGURE 3.10 Thermal conductivity, λ, plotted against electrical resistivity, ρ_e. For metals the two are related.

These same electrons, when in a potential gradient, drift through the lattice, giving electrical conduction. The electrical conductivity, κ, is here measured by its reciprocal, the *resistivity* ρ_e (SI units: Ω·m, units of convenience μΩ·cm). The range of ρ_e is enormous: a factor of 10^{28}, far larger than that of any other property. As with heat, the conduction of electricity in metals is proportional to the density of carriers, the electrons, and their mean free path, leading to the Wiedemann–Franz relation

$$\lambda \propto \kappa = \frac{1}{\rho_e} \tag{3.12}$$

The quantities λ and ρ_e are the axes of Fig. 3.10. Data for metals appear at the top left. The broken line shows that, for metals, the Wiedemann–

Franz relation is well obeyed.

But what about the rest of the chart? Electrons do not contribute to thermal conduction in ceramics and polymers. Heat is carried instead by phonons – lattice vibrations of short wavelength. They are scattered by each other and by impurities, lattice defects and surfaces; it is these that determine the phonon mean free path, ℓ. The conductivity is still given by Eq. (3.11), which we write as

$$\lambda = \frac{1}{3}\rho C_p \bar{c}\, \ell \qquad \textbf{(3.13)}$$

but now $\bar{c}$ is the elastic wave speed (around 10^3 m/s – see Fig. 3.3), ρ is the density and C_p is the *specific heat per unit mass* (units: J/kg K). Phonon scattering is minimized if the crystal is particularly perfect and the temperature is well below its Debye temperature, as in diamond at room temperature. Then the phonon conductivity is high: it is for this reason that single crystal silicon carbide and aluminium nitride have thermal conductivities almost as high as that of copper and diamond has conductivities that exceed it. The low conductivity of glass is caused by its irregular aperiodic structure; the characteristic length of the molecular linkages (about 10^{-9} m) determines the mean free path. Polymers have low conductivities because the elastic wave speed $\bar{c}$ is low (Fig. 3.3), and the mean free path in the disordered structure is small. Highly porous materials like firebrick, cork and foams show the lowest thermal conductivities, limited by that of the gas in their cells.

Graphite and many intermetallic compounds such as WC and B_4C, like metals, have free electrons, but the number of carriers is smaller and the resistivity is higher than metals. Defects such as vacancies and impurity atoms in ionic solids create positive ions that require balancing electrons. These can jump from ion to ion, conducting charge, but little of it because the carrier density is low. Covalent solids and most polymers have no mobile electrons and are insulators ($\rho_e > 10^{12}\ \mu\Omega\cdot\text{cm}$) – they lie on the right-hand side of Fig. 3.10.

Under a sufficiently high potential gradient, anything will conduct. The gradient tears electrons free from even the most possessive atoms, accelerating them into collision with nearby atoms, knocking out more electrons and creating a cascade. The critical potential gradient is the

breakdown potential V_b (units: MV/m), defined in Chapter 2, Engineering Materials and Their Properties.

Conducting Heat But Not Electricity

Which materials are both good thermal conductors and good electrical insulators (an unusual combination)? Use Fig. 3.10 to find out.

Answer

The chart identifies diamond, aluminium nitride, alumina and silicon nitride as having these properties. They are the ones at the top right.

The Thermal Conductivity–Thermal Diffusivity Chart (Fig. 3.11)

Thermal conductivity, as we have said, governs the flow of heat through a material at steady-state. The property governing transient heat flow is the *thermal diffusivity*, a (units: m^2/s). The two are related by

$$a = \frac{\lambda}{\rho C_p} \tag{3.14}$$

where ρ in kg/m^3 is the density. The quantity ρC_p is the *volumetric specific heat* (units: $J/m^3 K$). Fig. 3.11 relates thermal conductivity, diffusivity and volumetric specific heat, at room temperature.

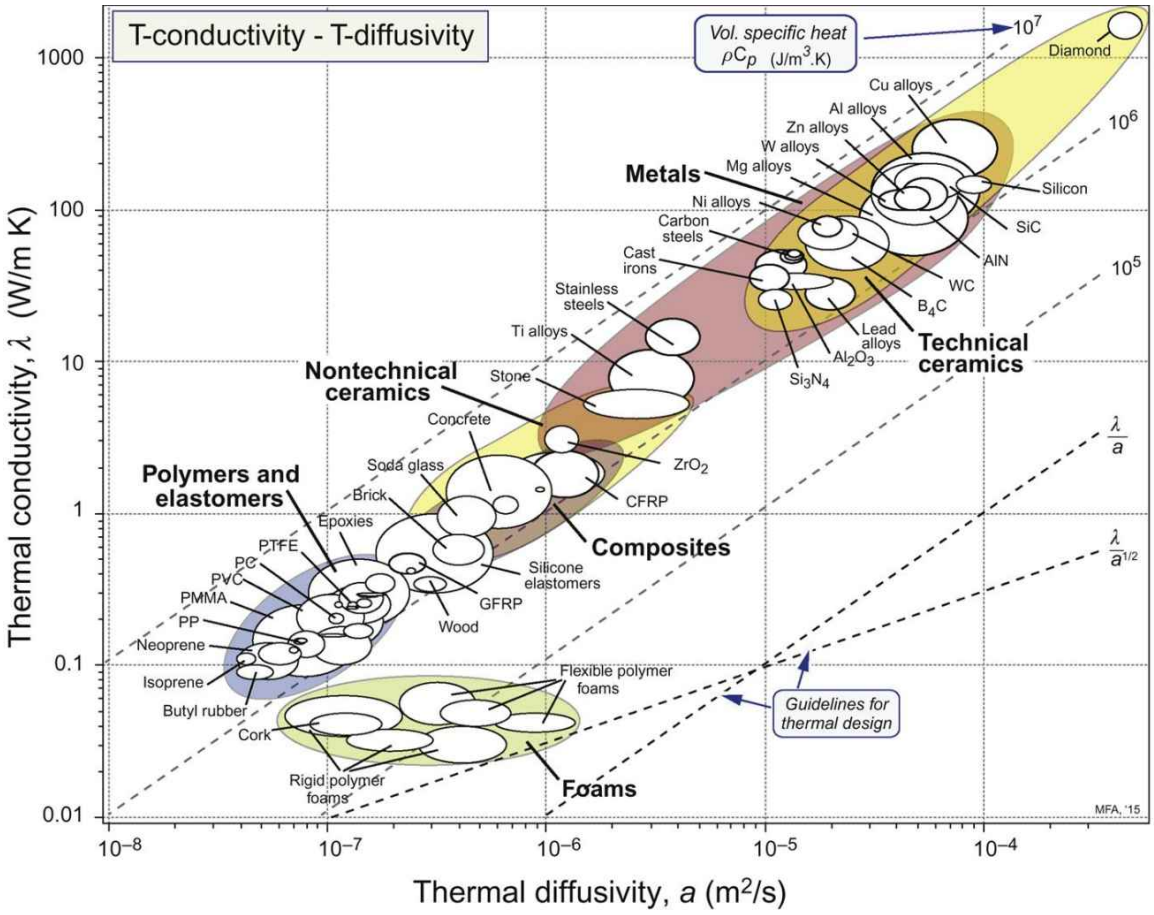


FIGURE 3.11 Thermal conductivity, λ, plotted against thermal diffusivity, a. The contours show the volume specific heat, ρC_p. All three properties vary with temperature; the data here are for room temperature.

The data span almost five decades in λ and a. Solid materials are strung out along the line of constant volume specific heat[4]

$$\rho C_p \approx 3 \times 10^6 \, J/m^3 \, \mathrm{K} \tag{3.15}$$

As a general rule, then,

$$\lambda = 3 \times 10^6 \, a \tag{3.16}$$

(λ in W/m K and a in m^2/s). Some materials deviate from this rule because they have lower than average volumetric specific heat. The largest

deviations are shown by porous solids: foams, low density firebrick, woods and the like. Their low density means that they contain fewer atoms per unit volume and, averaged over the volume of the structure, ρC_p is low. The result is that, although foams have low thermal *conductivities*, their thermal *diffusivities* are not necessarily low: they may not transmit much heat, but they reach a steady-state quickly. This is important in design, a point illustrated by the Case Study of Section 5.13.

Thermal Buffers

A good way to protect equipment from sudden temperature change is to encase it in a material with a very low thermal diffusivity (rather than conductivity) because an external temperature change then takes a long time to reach the inside. Use Fig. 3.11 to identify materials that could be good for this.

Answer

The chart identifies isoprene, neoprene and butyl rubber as potential candidates.

The Thermal Expansion–Thermal Conductivity Chart (Fig. 3.12)

Almost all solids expand on heating. The bond between a pair of atoms behaves like a linear-elastic spring when the relative displacement of the atoms is small, but when it is large, the spring is nonlinear. Most bonds become stiffer when the atoms are pushed together and less stiff when they are pulled apart. Such bonds are anharmonic. The thermal vibration of atoms, even at room temperature, involves large displacements, and as the temperature is raised, the anharmonicity of the bond pushes the atoms apart, increasing their mean spacing. The effect is measured by the linear *expansion coefficient*

$$\alpha = \frac{1}{\ell}\frac{d\ell}{dT} \tag{3.17}$$

where ℓ is a linear dimension of the body.

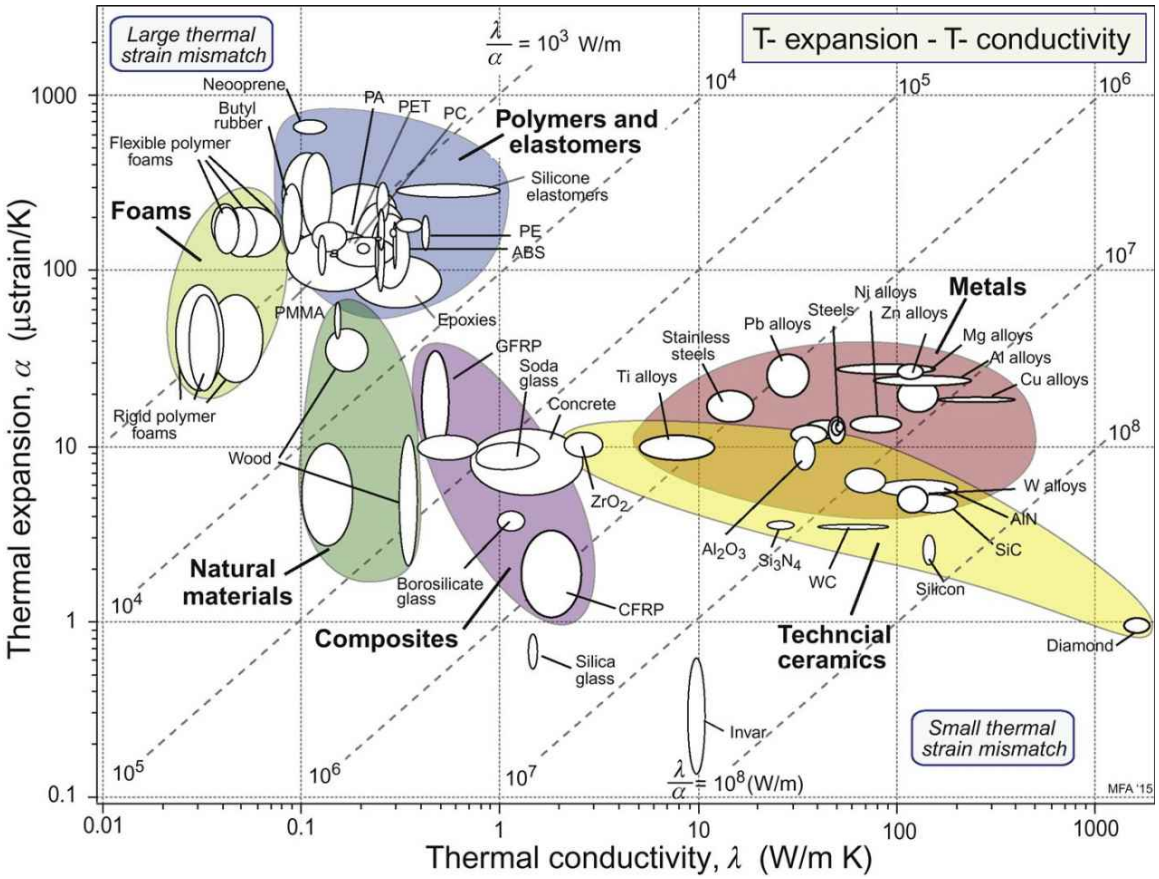


FIGURE 3.12 The linear expansion coefficient, α, plotted against the thermal conductivity, λ. The contours show the thermal distortion parameter λ/α. An extra material, the nickel alloy Invar, has been added to the chart; it is noted for its exceptionally low expansion at and near room temperature, useful in designing precision equipment that must not distort if the temperature changes.

The expansion coefficient is plotted against the thermal conductivity in Fig. 3.12. It shows that polymers have large values of α, roughly 10 times greater than those of metals and almost 100 times greater than ceramics. This is because the Van der Waals bonds of the polymer are very anharmonic. Diamond, silicon and silica glass (SiO_2) have covalent bonds that have low anharmonicity (that is, they are almost linear-elastic even at large strains), giving them low expansion coefficients. Composites, even though they have polymer matrices, can have low values of α because the reinforcing fibres, particularly carbon, expand very little.

The chart shows contours of λ/α, a quantity important in designing against thermal distortion. An extra material, Invar (a nickel alloy) has

been added to the chart because of its uniquely low expansion coefficient at and near room temperature, a consequence of a trade-off between normal expansion and a contraction associated with a magnetic transformation. An application that uses the chart is developed in Chapter 5, Materials Selection – Case Studies, Section 5.16.

Thermal Actuators

An actuator uses thermal expansion of its active element to generate the actuating force. Use Fig. 3.12 to identify the material with the largest expansion coefficient.

Answer

Neoprene, at the upper left of the chart, has a larger value of expansion coefficient than any other on the chart.

The Thermal Expansion–Modulus Chart (Fig. 3.13)

Thermal stress is the stress that appears in a body when it is heated or cooled but mechanically constrained, prevented it from expanding or contracting. Thermal stress depends on the expansion coefficient, α, of the material and on its modulus, E. A standard development of the theory of thermal expansion leads to the relationship

$$\alpha = \frac{\gamma_G \rho C_p}{3E} \qquad (3.18)$$

where γ_G is Gruneisen's constant. It has values between 0.4 and 4, but for most solids it is near 1. Since ρC_p is almost constant (Eq. 3.15), the equation tells us that α is proportional to $1/E$. Fig. 3.13 shows that this is broadly so. Ceramics, with the highest moduli, have the lowest coefficients of expansion; elastomers with the lowest moduli expand the most. Some materials with a low coordination number (silica, and some diamond cubic or zinc-blende structured materials) can absorb energy preferentially in transverse modes, leading to very small or negative value of γ_G and a low expansion coefficient (silica, SiO_2, is an example). Others, like Invar, contract as they lose their ferromagnetism when heated through the Curie temperature and over a narrow range of temperature they too show near-

zero expansion, which is useful in the manufacture of precision equipment and in glass-metal seals.

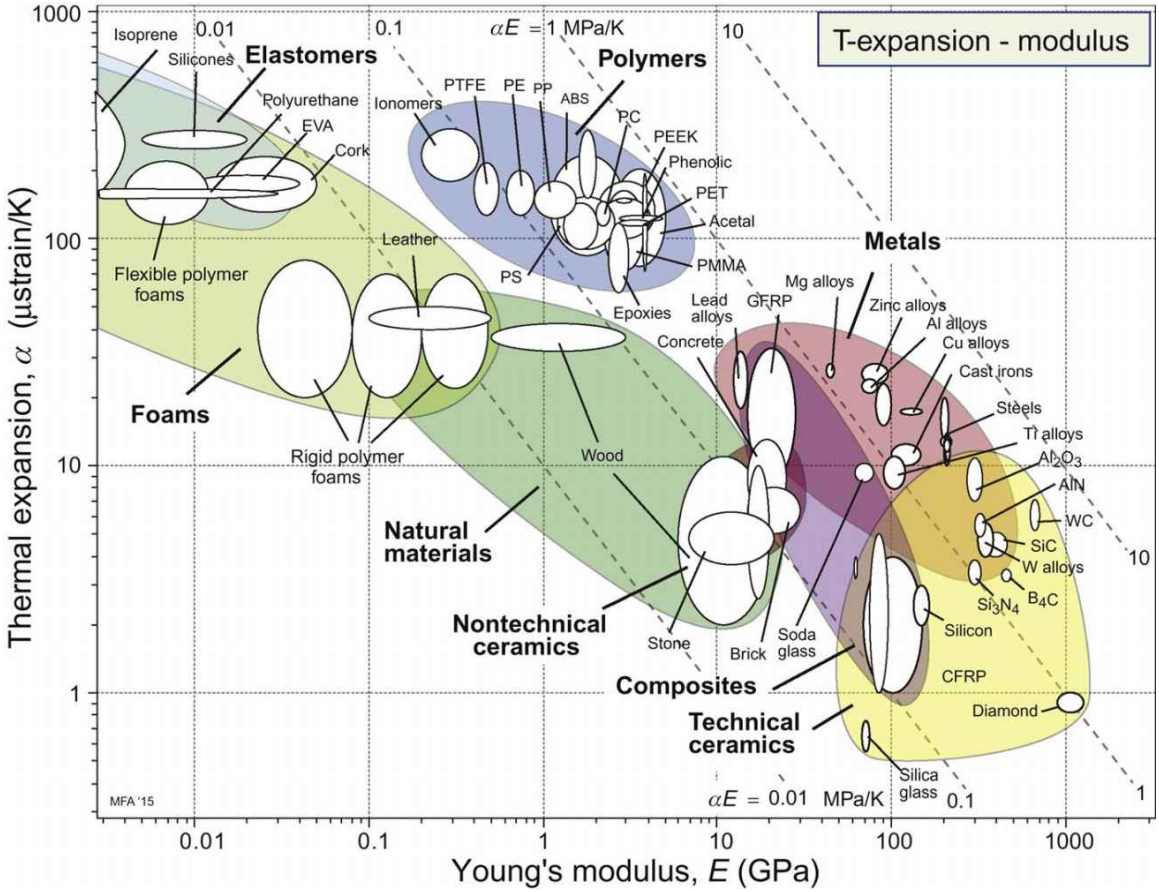


FIGURE 3.13 The linear expansion coefficient, α, plotted against Young's modulus, *E*. The contours show the thermal stress created by a temperature change of 1°C if the sample is axially constrained. A correction factor *C* is applied for biaxial or triaxial constraint (see text).

One more useful fact. The moduli of materials scale approximately with their melting point, T_m:

$$E \approx \frac{100\, k\, T_m}{\Omega} \tag{3.19}$$

where k is Boltzmann's constant and Ω the volume per atom in the structure. Substituting this and Eq. (3.15) for ρC_p into Eq. (3.18) for α gives

$$\alpha = \frac{\gamma_G}{100\, T_m} \tag{3.20}$$

where the expansion coefficient varies inversely with the melting point. Equivalently, the thermal strain for all solids, just before they melt, depends only on γ_G, making these strains roughly a constant at 1%. Eqs (3.18)– (3.20) are examples of property correlations, useful for estimating and checking material properties (Appendix A, Section A11).

Whenever the thermal expansion or contraction of a body is prevented, thermal stresses appear. If large enough, these stresses cause yielding, fracture, or elastic collapse (buckling). It is common to distinguish between thermal stress caused by external constraint (a rod, rigidly clamped at both ends, for example) and that which appears without external constraint because of temperature gradients in the body itself. All scale as the quantity $\alpha\, E$, shown as a set of diagonal contours in Fig. 3.13. More precisely, the stress $\Delta\sigma$ produced by a temperature change of 1°C in a constrained system, or the stress per °C caused by a sudden change of surface temperature in one that is not constrained, is given by

$$C\Delta\sigma = \alpha E \tag{3.21}$$

where C=1 for axial constraint, $(1-\nu)$ for biaxial constraint or normal quenching, and $(1-2\nu)$ for triaxial constraint, where ν is Poisson's ratio. These stresses are large: typically 1 MPa/K. They can cause a material to yield, or crack, or spall, or buckle when it is suddenly heated or cooled.

Thermal Stress

Approximately what stress will appear in a rod of steel, rigidly clamped at its ends, if it is heated by 100°C? Use Fig. 3.13 to find out.

Answer

The figure shows that, for steel, $\alpha E \approx 3$ MPa/K. Thus a temperature change of 100°C will create a stress of approximately 300 MPa.

The Maximum Service Temperature Chart (Fig. 3.14)

Temperature affects material performance in many ways. As the temperature is raised the material may creep, limiting its ability to carry

loads. It may degrade or decompose, changing its chemical structure in ways that make it unusable. And it may oxidise or interact in other ways with the environment in which it is used, leaving it unable to perform its function. The approximate temperature at which, for any one of these reasons, it is unsafe to use a material is called its *maximum service temperature* T_{max}. Fig. 3.14 show this quantity plotted as a bar-chart.

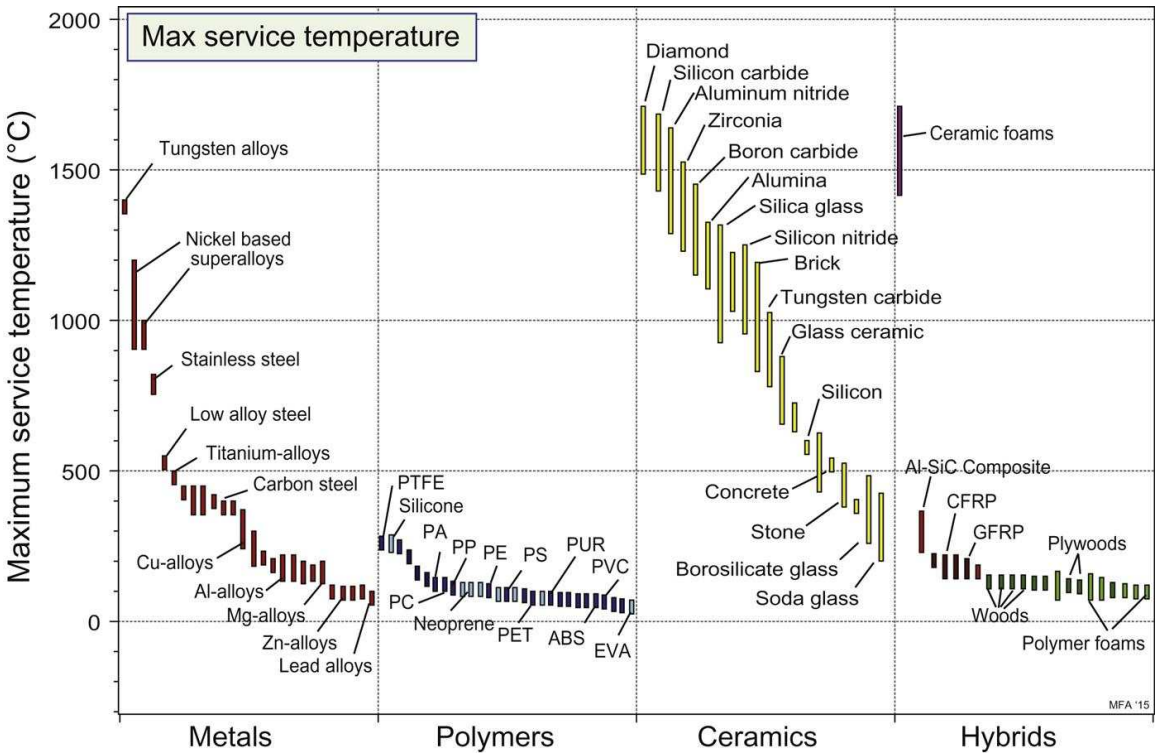


FIGURE 3.14 The maximum service temperature – the temperature above which materials become unusable.

The chart gives a birds-eye view of the regimes of temperature in which each material class is usable. Note that few polymers have can be used above 200°C, few metals above 800°C, and only ceramics offer strength above 1500°C.

Use Temperature for Stainless Steel

Stainless steel is proposed for use as part of a structure operating at 500°C. Is it safe to do so?

Answer

Fig. 3.14 shows that the maximum use temperature for stainless steel is in the range 700–1100°C. Use at 500°C appears to be practical.

Electrical Properties

The Dielectric Constant–Dielectric Strength Chart (Fig. 3.15)

The energy density (J/m^3) of a capacitor with a single dielectric layer increases with the square of the field E:

$$\text{Energy density} = \frac{1}{2}\varepsilon_r\,\varepsilon_o\,E^2$$

where ε_r is the relative permittivity (dielectric constant) and ε_o is the permittivity of vacuum (8.854×10^{-12} F/m). An upper limit for E is the dielectric strength, E_b, setting an upper limit for energy density:

$$\text{Maximum energy density} = \frac{1}{2}\,\varepsilon_r\,\varepsilon_o\,E_b^2 \qquad \textbf{(3.22)}$$

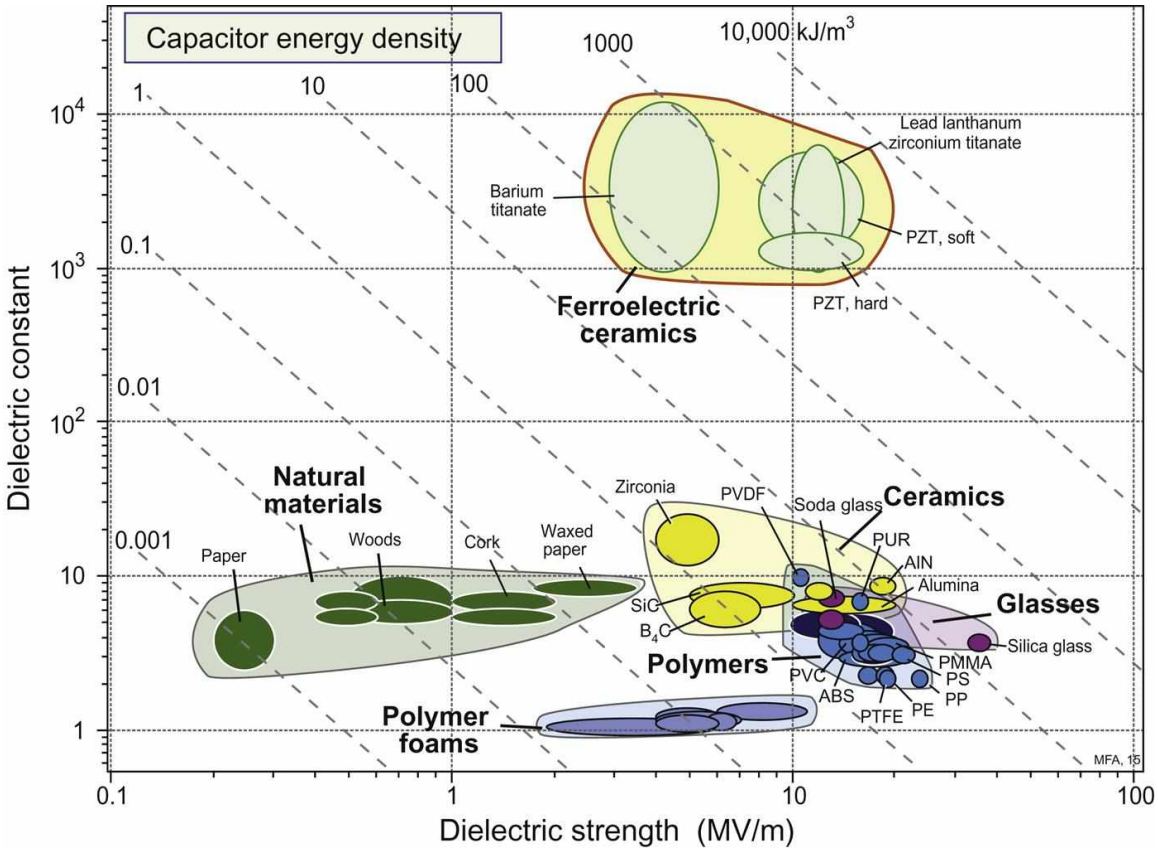


FIGURE 3.15 A plot showing the dielectric constant and strength of common materials. The ferroelectric ceramics have the highest energy density, shown as dashed lines.

Dielectric constant and dielectric strength are the axes of the chart shown as Fig. 3.15. The maximum energy density is plotted as a set of diagonal contours on the chart. Ordinary dielectrics like polyethylene, soda glass or alumina ceramic have dielectric constants between 3 and 10 and, at best, store 10 kJ/m^3. Ferroelectric ceramics like barium titanate or lead zirconium titanate (PZT) contain naturally occurring molecular dipoles which realign in the field, giving them much greater dielectric constants and energy-storing capacity. A typical barium titanate capacitor with $E_b = 4$ MV/m and $\varepsilon_r = 2500$, gives an energy density of 180 kJ/m^3. This is low compared with hydrocarbon fuels such as gasoline (energy density of about 30,000 kJ/m^3) but is comparable with the batteries (a nickel-cadmium battery has an energy density of approximately 1000 kJ/m^3). The energy in batteries, however, can be drawn down only at the rate that the chemical reactions releasing charge can take place, whereas the energy in a

capacitor is accessible instantaneously. The energy density in capacitors may be low, but the power density is high.

The Piezoelectric Constants Chart (Fig. 3.16)

The piezoelectric constants d_{33} and g_{33}, introduced in Chapter 2, Engineering Materials and Their Properties are plotted in Fig. 3.16. The best single measure of the strength of the piezoelectric effect is the electro-mechanical coupling constant, k_p. It is defined in terms of its square, k_p^2, as

$$k_p^2 = (\text{mechanical energy converted to electrical energy})/(\text{input mechanical energy})$$

or, equivalently (because the coupling is linear) as

$$k_p^2 = (\text{electrical energy converted to mechanical energy})/(\text{input electrical energy})$$

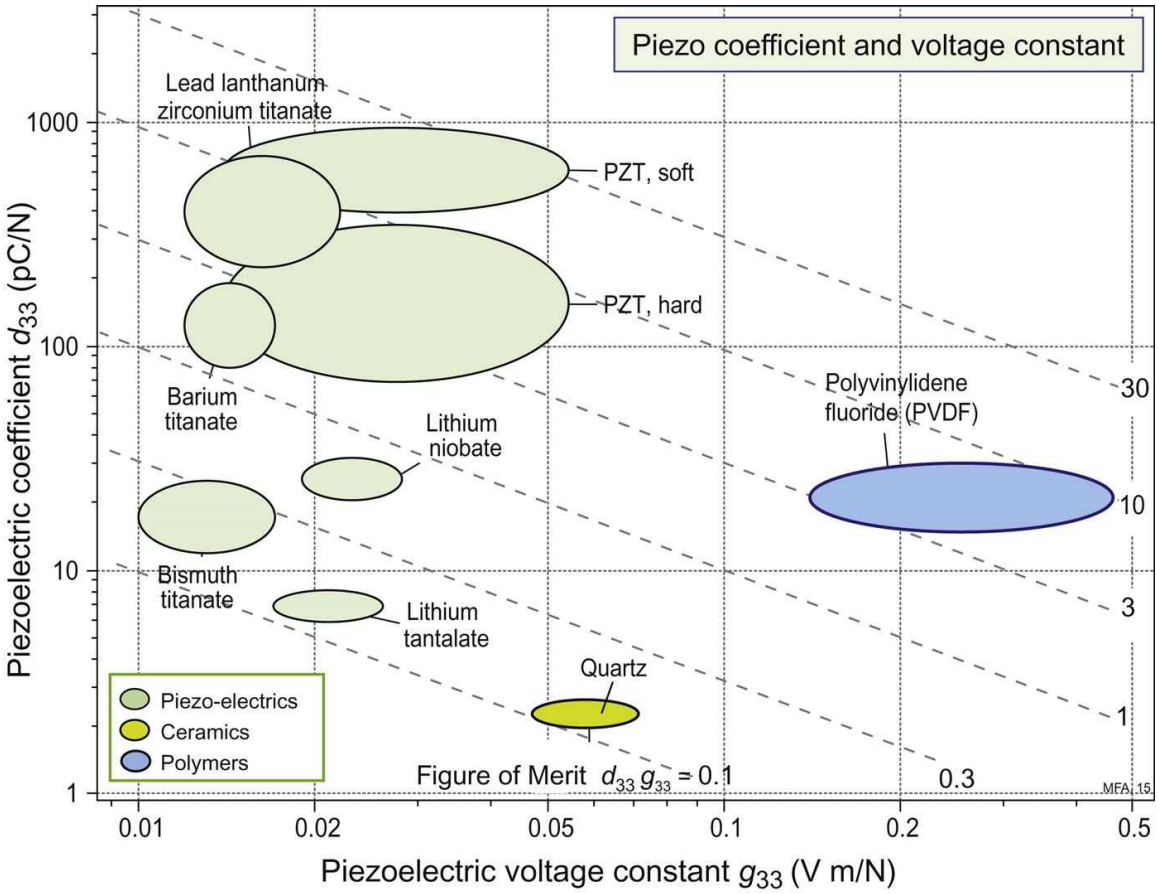


FIGURE 3.16 The piezoelectric constants d_{33} and g_{33} introduced in Chapter 2, Engineering Materials and Their Properties. The piezoelectric conversion efficiency $d_{33}\,g_{33}$ is shown as dashed lines.

If the elastic compliance of the material is S this ratio is

$$k_p^2 = \left[\frac{1}{2}\frac{(d_{33}E)^2}{S}\right] \Big/ \left[\frac{1}{2}\varepsilon_r\,\varepsilon_o\,E^2\right] = \frac{(d_{33})^2}{\varepsilon_r\,\varepsilon_o\,S} = \frac{d_{33}\,g_{33}}{S} \qquad \textbf{(3.23)}$$

High conversion efficiency is desirable. Thus the quantity $d_{33}\,g_{33}$ has the characteristics of a material index or figure of merit. It is shown as diagonal broken lines on Fig. 3.16. Since the energy conversion is never 100% efficient, $k_p < 1$. Values of k_p range from 0.1 for quartz to 0.7 for PZT.

Piezoelectric Accelerometers

Piezoelectric accelerometers rely on the inertia of a mass m to generate a force $F = ma$ when subjected to an acceleration or a deceleration a. If this force is applied to the end of a beam to which a thin layer of piezoelectric material is bonded, the piezoelectric layer is stretched or compressed. The resulting charge that appears on the piezoelectric can be detected and, if necessary, used to trigger a response if the acceleration or deceleration exceeds a critical value. For this, and piezoelectric material with a high piezoelectric charge coefficient d_{33} is desirable. Use Fig. 3.16 to identify the four best materials for this application.

Answer

Fig. 3.16 shows that the materials with the highest values of d_{33} are soft PZT and lead lanthanum zirconium titanate (PLZT).

The Thermal Conductivity–Dielectric Loss Factor Chart (Fig. 3.17)

The dielectric loss factor L and thermal conductivity λ are plotted in Fig. 3.17. Materials that are transparent to microwaves (radar uses microwave frequencies) lie at the left: polymer foams, certain polymers (PP, PE, PTFE) and some ceramics have particularly low values. Materials that absorb microwaves, becoming hot in the process, lie at the right: polymers containing a polar group, like nylon and polyurethane, and natural materials like woods (woods are dried by microwave heating). Water, which has a polar molecule, has a particularly large loss factor, which is why most foods can be cooked in a microwave oven.

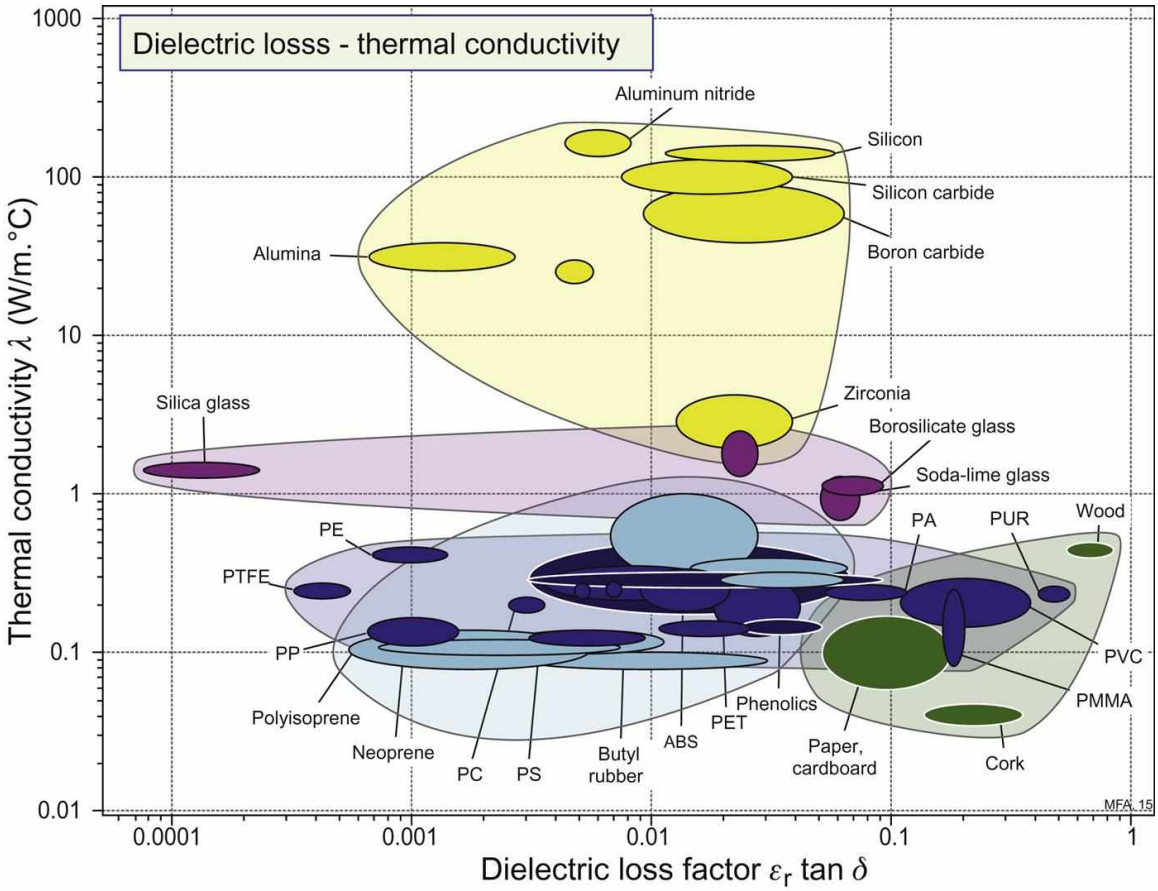


FIGURE 3.17 The thermal conductivity λ and the dielectric loss factor $\varepsilon_r \tan \delta$.

When a dielectric material is placed in a cyclic electric field of amplitude E and frequency f, power P is dissipated and the field is correspondingly attenuated. The power dissipated per unit volume, in W/m^3, is

$$P \approx f E^2 \varepsilon \tan \delta = f E^2 \varepsilon_o \varepsilon_r \tan \delta \qquad \textbf{(3.24)}$$

where, as before, ε_r is the dielectric constant of the material and $\tan \delta$ is its loss tangent. This power appears as heat and is generated uniformly (if the field is uniform) through the volume of the material. Thus the higher the frequency or the field strength and the greater the loss factor

$$L = \varepsilon_r \tan \delta$$

the greater is the heating and energy loss. This dielectric loss is exploited in processing – e.g., in radio frequency welding of polymers and the sintering of ceramic powders.

Microwave Heating

An oscillating voltage with an amplitude of 100 volts at a frequency $f = 60$ Hz appears across a nylon insulator of thickness $x = 1$ mm. Nylon has a dielectric loss factor $L \approx 0.1$.

1. How much power is dissipated per unit volume in the nylon? Assuming no heat loss, how fast will the temperature of the nylon rise? (Nylon has a volumetric specific heat C_p=3×10^6 J/m^3 K).
2. If the frequency was, instead, $f = 1$ GHz, what is the heating rate?

Answer

1. The amplitude of the field is $E = \frac{V}{x} = 10^5\ V/m$. The power dissipated is

$$P = f\,E^2 \varepsilon_o\, L = 0.53\ W/m^3$$

The rate of temperature rise is $\frac{dT}{dt} = \frac{P}{C_p} = 1.8 \times 10^{-7}\ ^\circ C/s$

2. If the frequency is raised to $f = 1$ GHz the heating rate becomes $\frac{dT}{dt} = 3^\circ C/s$.

Magnetic Properties

The Remanent Induction–Coercive Field Chart (Fig. 3.18)

Magnetic materials differ greatly in the shape and area of their hysteresis loop, introduced in Section 2.4. The greatest difference is that between *soft magnets*, which have low coercive fields and narrow hysteresis loops, and *hard magnets*, which have coercive fields that are many orders of magnitude higher (Fig. 3.18). The maximum energy product (BH_{max}, units J/m^3) is figure of merit for hard magnets – think of it as the energy required to demagnetize the magnet. More formally, it is the area of the largest area rectangle that can be inscribed inside the upper left quadrant of the B-H curve, shown on Figure 2.13. It depends on the shape of the B-H

curve but, for a given shape, it increases as the product Br.Hc, shown as the diagonals on Fig. 3.18.

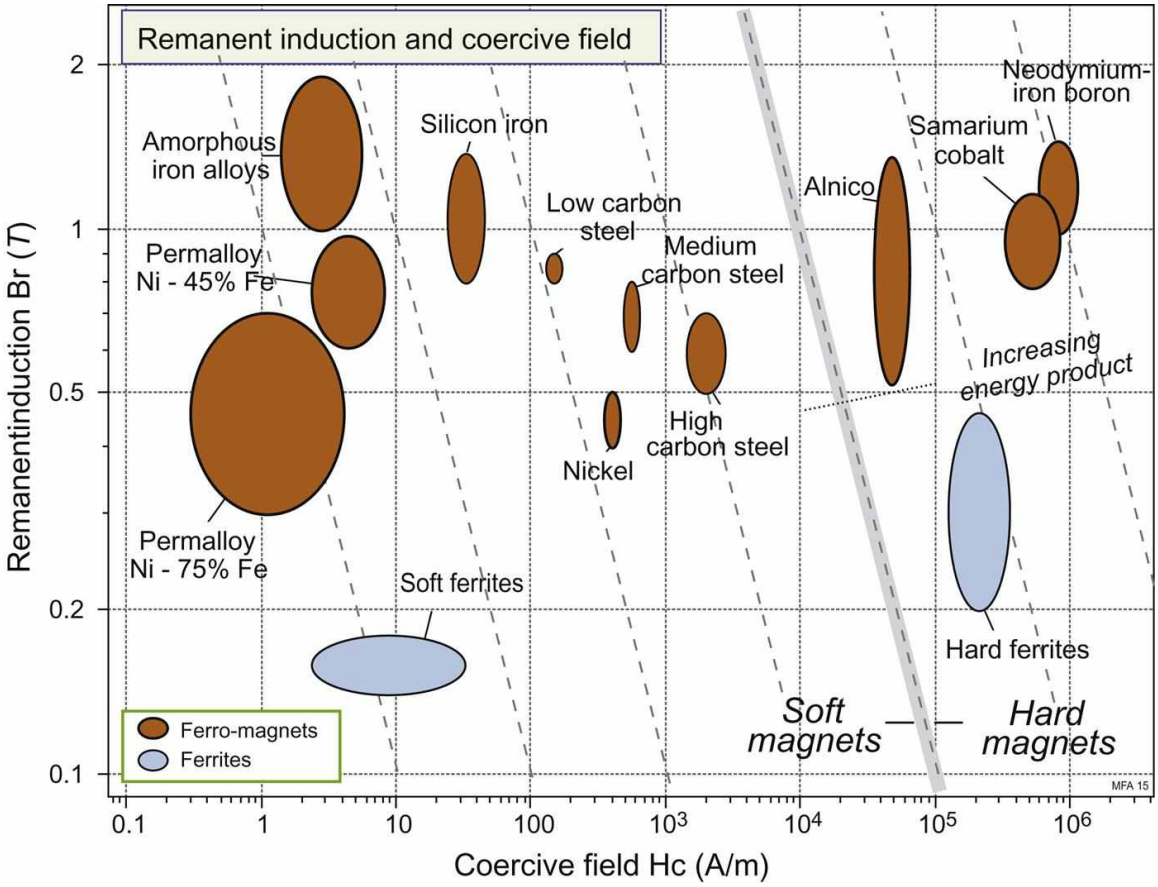


FIGURE 3.18 A chart of remanent induction and coercive field for common magnetic materials. There is a clear distinction between soft magnetic materials on the left and hard magnetic materials on the right.

Saturation Magnetostriction and Coercive Field (Fig. 3.19)

Magnetostriction is shape change caused by a magnetic field. The field exerts a force on magnetic domain walls, aligning the direction of magnetization parallel to the field. The realignment is associated with a strain – magnetostriction – positive if the material extends parallel to the field, negative if it shrinks. The saturation magnetostriction is the maximum strain when the domains are aligned completely. The electrical 'hum' of transformers is due to magnetostriction of the transformer core,

dissipating energy and reducing efficiency. To minimize this transformer cores are made from materials with low magnetostriction such as the alloy 81.5wt%Ni–18.5wt%Fe. Nickel has negative magnetostriction and iron has positive; the composition is chosen such that the two effects cancel each other out.

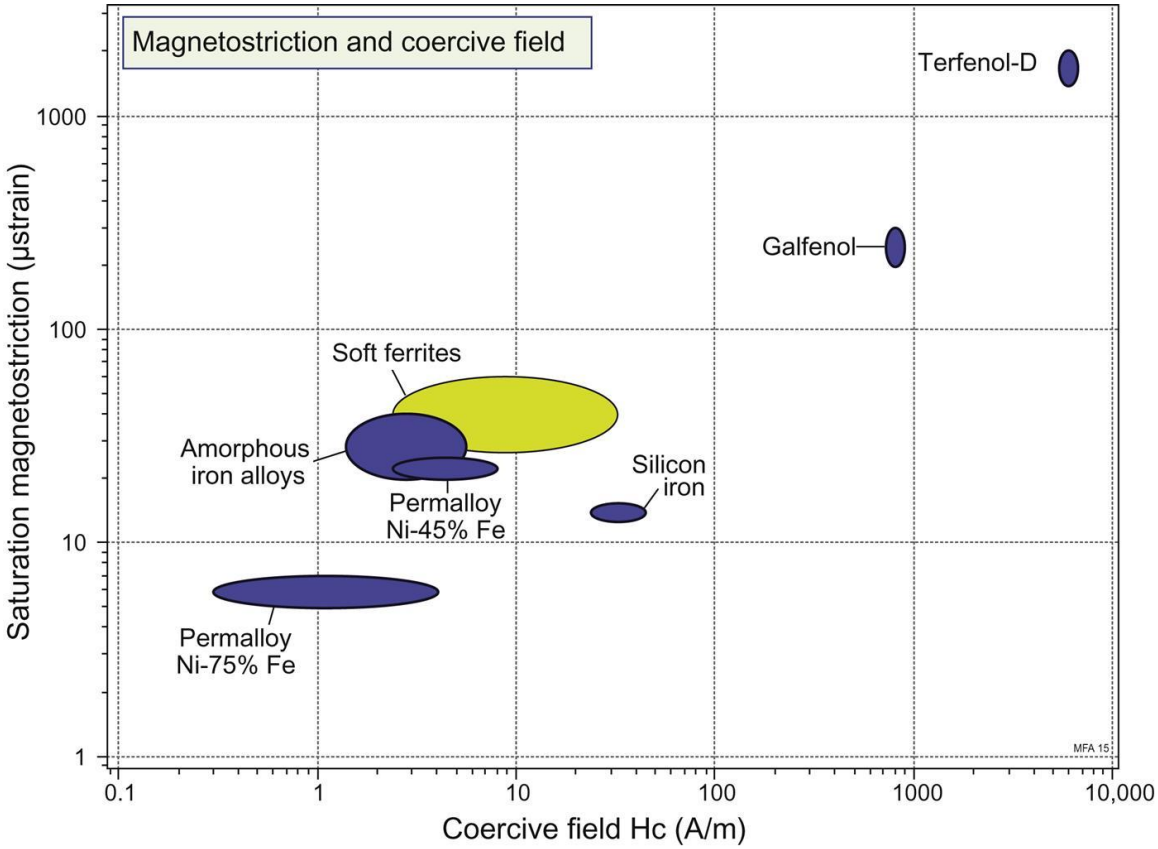


FIGURE 3.19 Saturation magnetostriction and coercive force for soft magnetic materials.

The ability to convert magnetic energy into kinetic energy and vice versa enables magnetic sensors and actuators, an effect exploited in sonar detection. For this application the best materials are so-called 'giant magnetostrictive' materials – those with extremely high magnetostriction – such as Terfenol-D (an alloy of terbium, dysprosium and iron) and Galfenol (an alloy of gallium and iron) (Fig. 3.19).

Environmental Properties

Embodied Energy Bar Charts (Figs. 3.20 and 3.21)

Extracting and refining material into usable form takes energy and releases emission such as CO_2, CH_4 and CO that act as greenhouse gases. The energy associated with the production of one kilogram of a material is the *embodied energy*, H_p (units MJ/kg). That required to make unit volume is $H_p\rho$ (units MJ/m^3) where ρ is the density of the material in kg/m^3. The bar charts of Figs. 3.20 and 3.21 show these two quantities for metals, polymers, ceramics and composites. On a 'per kg' basis (upper chart) steels and cast irons have the lowest energy; aluminium and the other light alloys carry the highest penalty. Polymers lie somewhere in the middle of this range. Concrete and woods have the lowest energy of all. If these same materials are compared on a 'per m^3' basis (lower chart) the picture changes. Now the commodity polymers such as PE, PP, PS and PVC carry a *lower* burden than any metal and even CFRP, with a big energy per unit weight, is now only half way up the range.

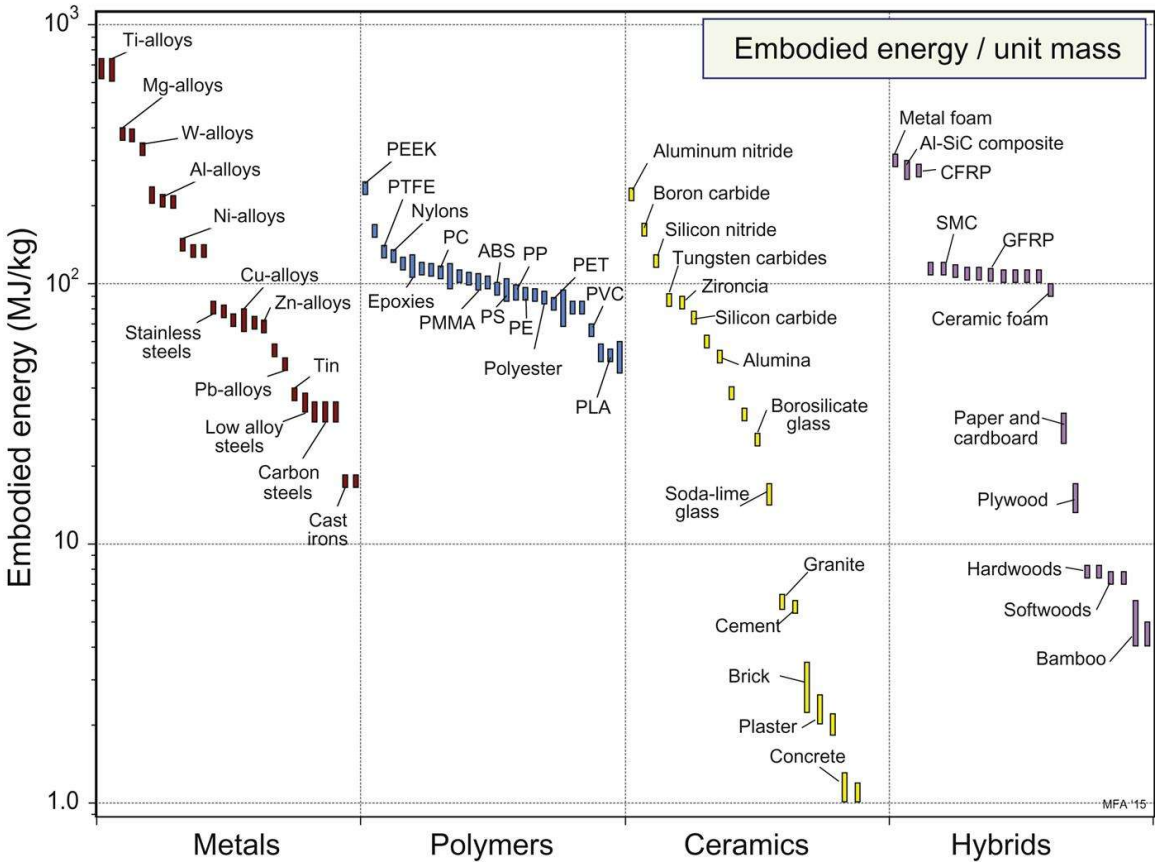


FIGURE 3.20 The embodied energies of materials per unit mass.

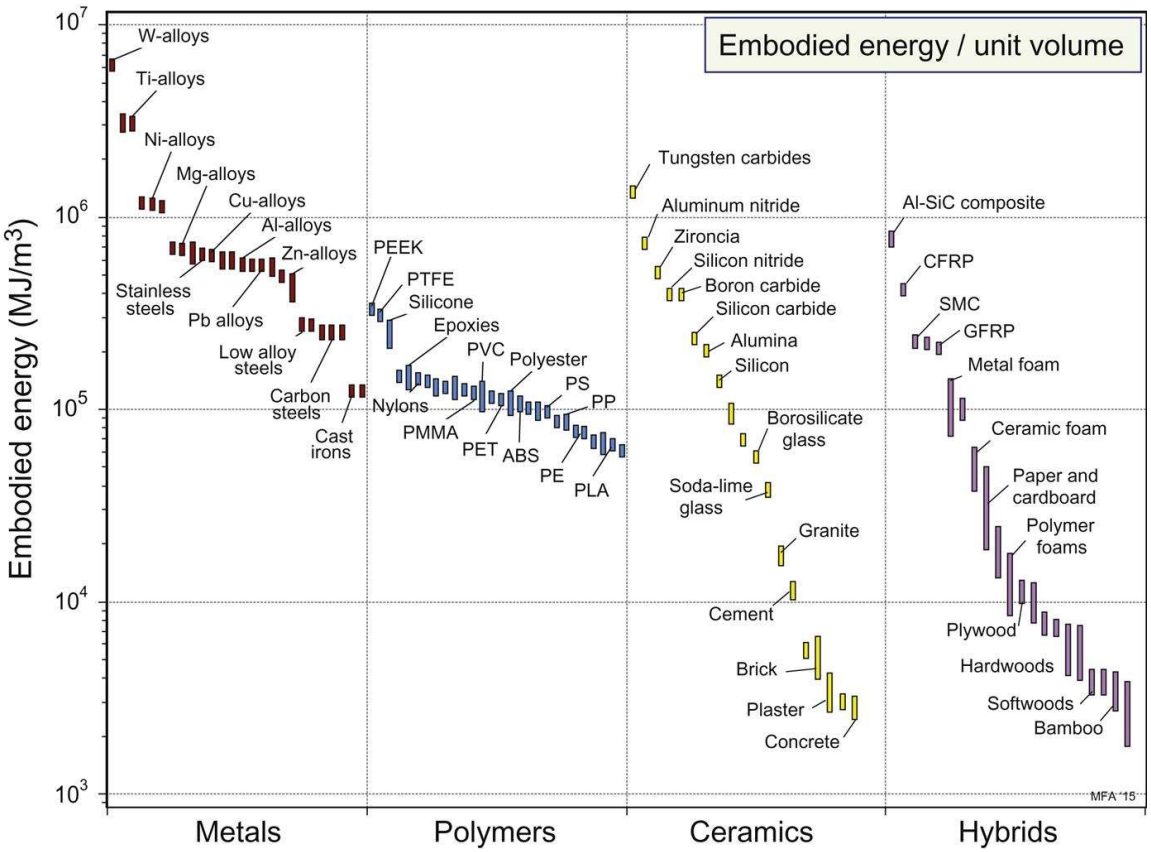


FIGURE 3.21 The embodied energy of materials per unit volume.

The Modulus–Embodied and Strength–Embodied Energy Charts (Figs. 3.22 and 3.23)

But is comparison 'per kg' or 'per m^3' the right way to do it? Rarely. To deal with environmental impact of materials properly we must seek to minimize the energy, the CO_2 burden or the eco-indicator value *per unit of function*. To do that we need charts plotting engineering properties and embodied energy together. Figs. 3.22 and 3.23 show two of these: modulus and strength plotted against embodied energy per unit volume. Materials that are stiff and strong and yet have a low embodied energy lie near the top left. To be more precise than this we need material indices, developed in the next chapter. The guidelines that you will see at the lower right on both charts show what they look like. Their use, with case studies, is detailed in Chapter 14, Materials and the Environment.

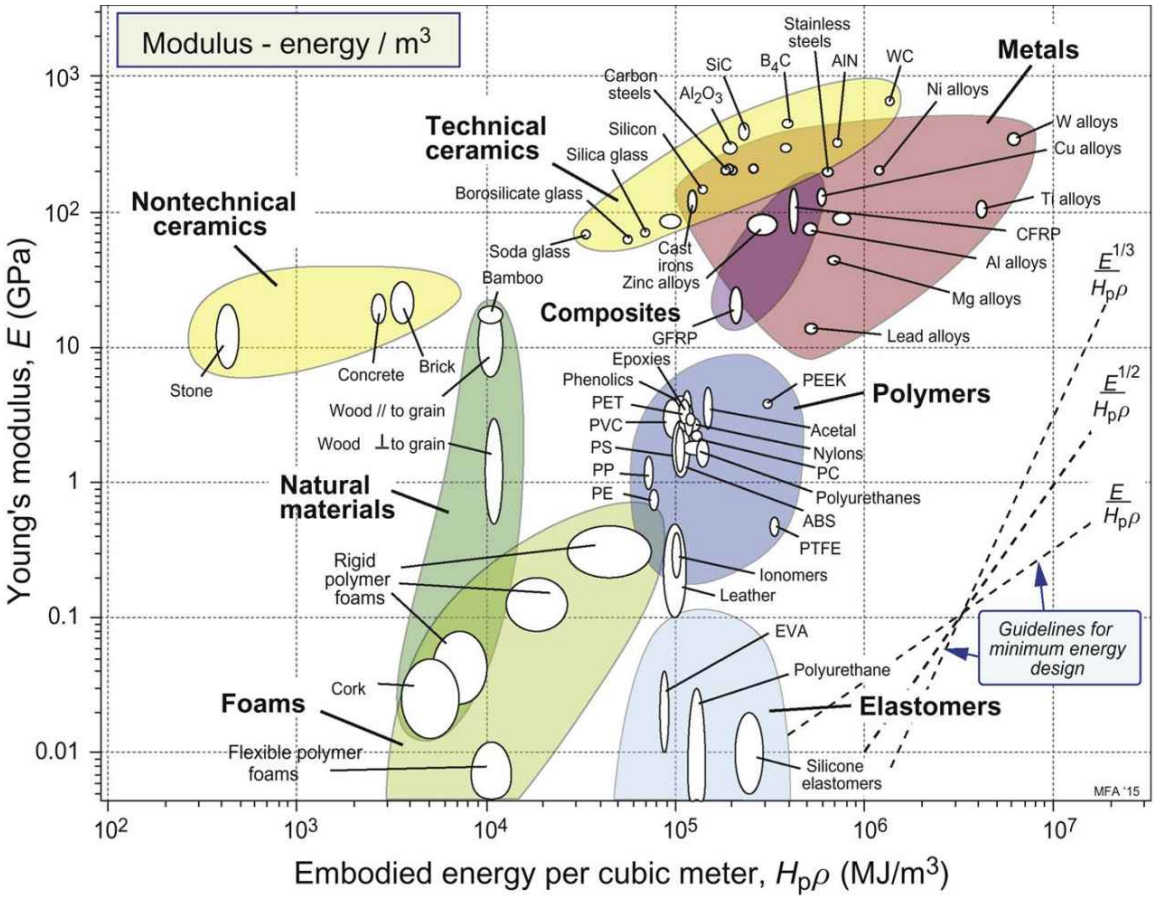


FIGURE 3.22 A selection chart for stiffness with minimum embodied energy.

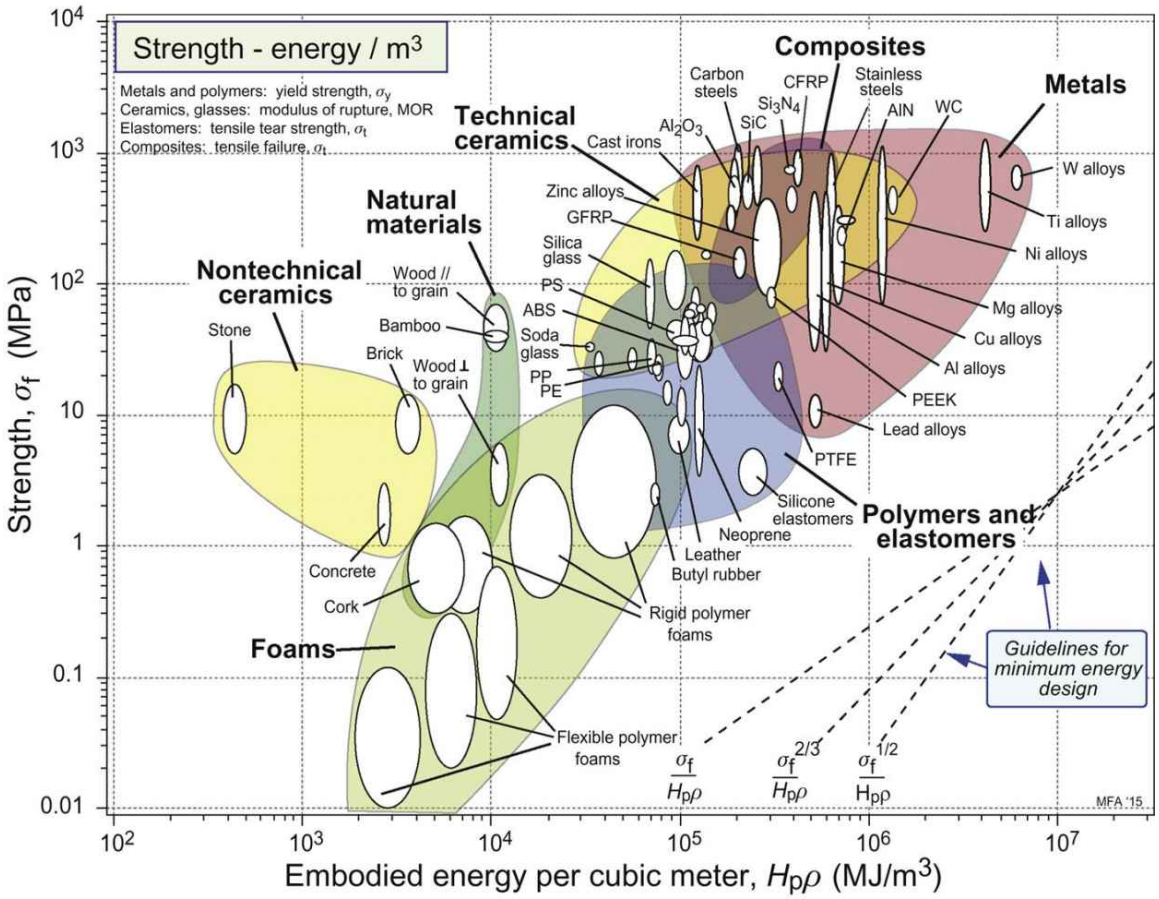


FIGURE 3.23 A selection chart for strength with minimum embodied energy.

The carbon footprint of a material is a measure of the greenhouse emission released to atmosphere associated with its production. The dominant such emission is CO_2 but others (CH_4, CO) also have a greenhouse effect. For this reason, the carbon footprint is measured in kg (CO_2 equivalent), meaning the mass of CO_2 that would have the same greenhouse effect as the actual emissions. Its value per unit mass of material produced has the units of kg/kg; that per unit volume of material has units kg/m^3.

Data for embodied energies and carbon footprints are much less precise than those for other engineering properties and they depend on where the material was made and how. For many materials data do not exist at all. For that reason it can be useful to have two very approximate 'rules of thumb' that allow values to be estimated when none are available. The first relates embodied energy to price (all quantities in 2016 USD).
Metals and Ceramics: Embodied energy in MJ/kg=26×Price in US$/kg

Oil-based Polymers: Embodied energy in MJ/kg=7×Price+65
The second relates carbon footprint to embodied energy.
Metals, Polymers and Ceramics: Carbon footprint in
kg/kg= 0.06×Embodied energy in MJ/kg

Estimating Embodied Energy and Carbon Footprint

A particular polymer blend has a price of US$4.5 per kg. What, very roughly might its embodied energy and carbon footprint be?

Answer

From the first for the rules of thumb above, the embodied energy is estimated as 96 MJ/kg. From the second, its carbon footprint is estimated as 5.8 kg/kg.

Economic Properties

Cost Bar Charts (Figs. 3.24 and 3.25)

Properties like modulus, strength or conductivity do not change with time. Cost is bothersome because it does. Supply, scarcity, speculation and inflation contribute to the considerable fluctuations in the cost-per-kilogram of a commodity like copper or silver. Data for cost-per-kg are tabulated for some materials in daily papers and trade journals; those for others are harder to come by. Approximate values for the cost of materials per kg, and their cost per m^3, are plotted in Figs. 3.24 and 3.25 in 2016 USD. Most commodity materials (glass, steel, aluminium and the common polymers) cost between US$0.5 and US$2.0 per kg. Because they have low densities, the cost/m^3 of commodity polymers is less than that of metals.

Polymer Prices

Which four polymers are the least expensive?

Answer

Figs. 3.24 and 3.25 shows that the four least expensive polymers, either by weight or by volume, are polypropylene (PP), polyethylene (PE), polyvinylchloride (PVC) and polyethylene terephthalate (PET). These are the commodity polymers – the ones used in the greatest quantities.

Relative Price

Are polypropylene and polyethylene more or less expensive than steel, per unit weight? What about per unit volume?

Answer

Figs. 3.24 and 3.25 reveals that the two polymers are more expensive than steel per unit weight, but because they have much lower densities, they are cheaper per unit volume. If we want a proper comparison what should be the basis of comparison? The answer, developed in later chapters, is: *per unit function*.

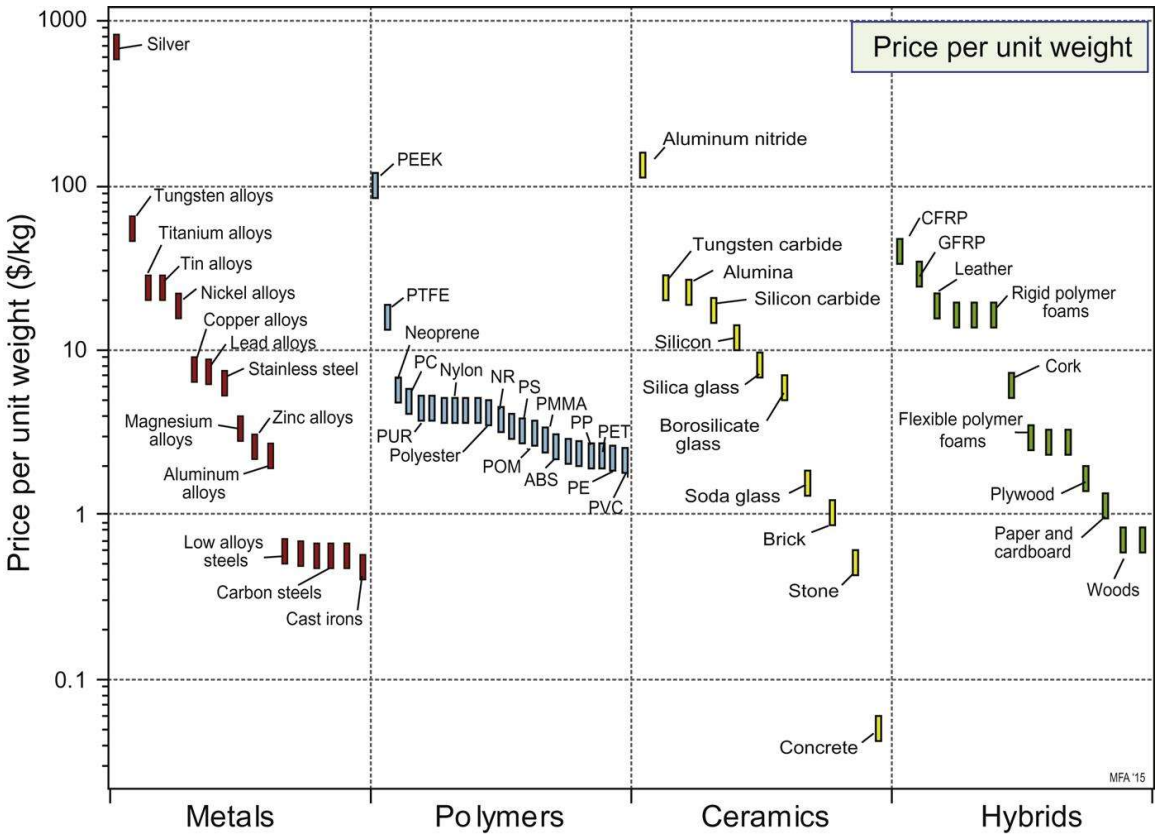


FIGURE 3.24 The approximate price/kg of materials. Commodity materials cost about $1/kg; special materials cost much more. All cost measured in 2016 USD.

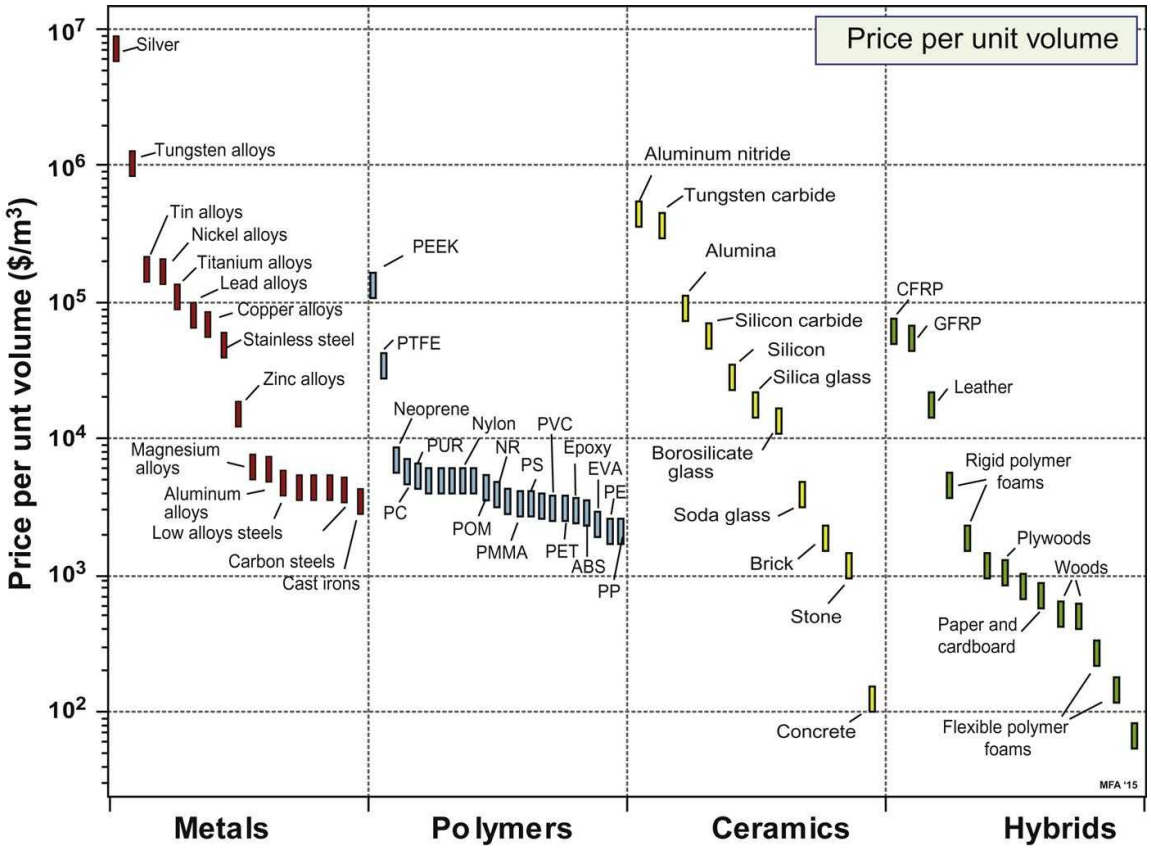


FIGURE 3.25 The approximate price/m^3 of materials. Polymers, because they have low densities, cost less per unit volume than most other materials.

The Modulus–Relative Cost and Strength–Relative Cost Charts (Figs. 3.26 and 3.27)

In design for minimum cost, material selection is guided by indices that involve modulus, strength and cost per unit volume. To make some correction for the influence of inflation and the units of currency in which cost is measured, we define a *relative cost per unit volume* $C_{v,R}$

$$C_{v,R} = \frac{Cost/kg \times Density\ of\ material}{Cost/kg \times Density\ of\ mild\ steel\ rod} \qquad \textbf{(3.25)}$$

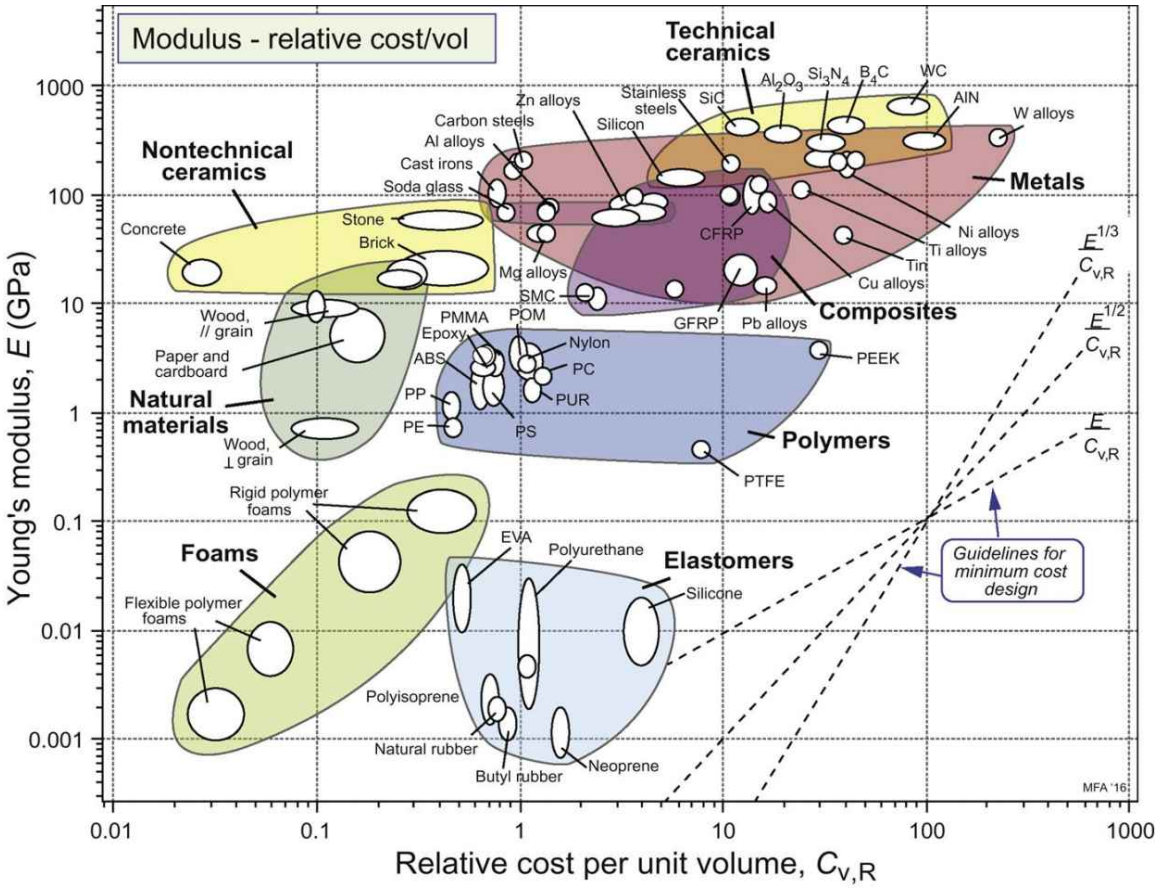


FIGURE 3.26 Young's modulus, E, plotted against relative cost per unit volume, $C_{v,R}$ The design guidelines help selection to maximize stiffness per unit cost.

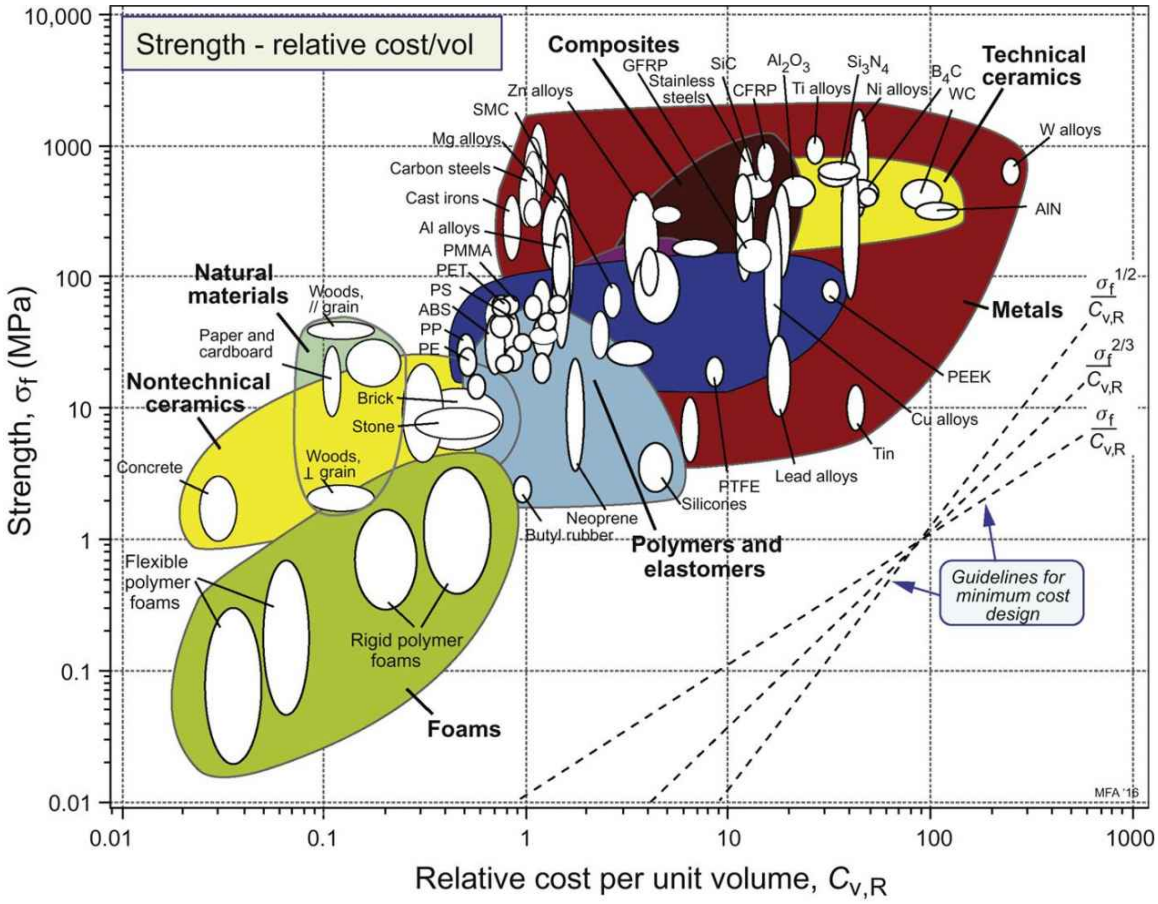


FIGURE 3.27 Strength, σ_f, plotted against relative cost per unit volume, $C_{v,R}$. The design guidelines help selection to maximize strength per unit cost.

At the time of writing, steel reinforcing rod costs about US$0.3/kg.

Fig. 3.26 shows the modulus E plotted against relative cost per unit volume $C_{v,R}$. Carbon steels lie on the contour $C_{v,R} = 1$. Cheap, stiff materials lie towards the top left. Guidelines for selecting materials that are stiff and cheap are plotted on the figure.

Cheap strong materials are selected using Fig. 3.27. This figure shows failure strength, defined as before, plotted against relative cost per unit volume, and carbon steel again lies on the $C_{v,R} = 1$ contour. Cheap, strong materials lie at the top left. The use of both charts is illustrated in Chapter 5, Materials Selection – Case Studies.

It must be emphasised that the data plotted here and on the chart of Fig. 3.26 *are less reliable than those of other charts and subject to unpredictable change*. Despite this dire warning, the two charts are genuinely useful. They allow selection of materials, using the criterion of 'function per unit cost'. An

example is given in Chapter 5, Materials Selection – Case Studies, Section 5.5.

3.4 Summary and Conclusions

The engineering properties of materials are usefully displayed as material selection charts. Twenty five of them are introduced in this chapter; more appear in later ones. The charts summarise material properties in a compact, easily accessible way, showing the range spanned by each material family and class. By choosing the axes in a sensible way, more information can be displayed. A chart of modulus E against density ρ reveals the longitudinal wave velocity $(E/\rho)^{1/2}$. A chart of fracture toughness K_{1c} against modulus E shows the toughness G_{1c}. A chart of thermal conductivity λ against diffusivity, a, also gives the volume specific heat ρC_v. A chart of strength, σ_f, against modulus, E, shows the energy-storing capacity σ_f^2/E, and there are many more.

The most striking feature of the charts is the way in which members of a material class cluster together. Despite the wide range of modulus and density of metals (as an example), they occupy a field that is distinct from that of polymers or that of ceramics or that of composites. The same is true of strength, toughness, thermal conductivity and the rest: the fields frequently overlap, but they always have a characteristic place within the whole picture.

The position of the fields and their relationship can be understood in simple physical terms: the nature of the bonding, the packing density, the lattice resistance and the vibrational modes of the structure, themselves a function of bonding and packing. It may seem odd that so little mention has been made of microstructure in determining properties. But the charts clearly show that the first-order difference between the properties of materials has its origins in the mass of the atoms, the nature of the interatomic forces and the geometry of packing. Alloying, heat treatment and mechanical working, to which we return in Chapter 6, Processes and their Effect on Properties all influence microstructure, and through this, properties, which give the elongated bubbles shown on many of the charts; but the magnitude of their effect is less than, by factors of 10, that of bonding and structure.

All the charts have one thing in common: parts of them are populated with materials and parts are not. Some parts are inaccessible for

fundamental reasons that relate to the size of atoms and the nature of the forces that bind their atoms together. But other parts are empty even though, in principle, they are accessible. If they were accessed, the new materials that lay there could allow novel design possibilities. Ways of doing this are explored further in Chapter 12, Designing Hybrid Materials and Chapter 13, Hybrids: Case Studies.

The charts have numerous applications. One is the checking and validation of data (Appendix A); here use is made both of the range covered by the envelope of material properties and of the numerous relations between them (like $E\,\Omega = 100\ \mathrm{k}\,\mathrm{T}_m$), described in Section 3.3. Another concerns the development of, and identification of uses for, new materials; materials that fill gaps in one or more of the charts generally offer some improved design potential. But most important of all, the charts form the basis for a procedure for materials selection. This advantage is developed in the following chapters.

3.5 Further Reading

The best general book on the physical origins of the mechanical properties of materials remains that by Cottrell (1964).

1. Ashby M, Shercliff H, Cebon D. *Materials: Engineering, Science, Processing and Design* third ed. Oxford, UK: Butterworth-Heinemann; 2014; ISBN-13: 978-0-08-097773-7, North American Edition: ISBN-13: 978-1-85617-743-6. (*An introductory text introducing ideas that are developed more fully in the present text*.).
2. Budinski KG, Budinski MK. *Engineering Materials, Properties and Selection* ninth ed. New Jersey: Prentice Hall; 2010; ISBN 978-0-13-712842-6. (*A well-established materials text that deals well with both material properties and processes*.).
3. Callister WD, Rethwisch DG. *Materials Science and Engineering: An Introduction* ninth ed. New York, NY: John Wiley & Sons; 2014; ISBN 978-1-118-54689-5. (*A well-established text taking a science-led approach to the presentation of materials teaching*.).
4. Cottrell AH. *Mechanical Properties of Matter* New York, NY: Wiley; 1964; Library of Congress Number 65-14262. (*An inspirational book, clear, full of insights and of simple derivations of the basic equations describing the mechanical behaviour of solids, liquids and gasses*.).

5. Shackelford JF. *Introduction to Materials Science for Engineers* eighth ed. New Jersey: Prentice Hall; 2014; ISBN 978-0133789713. (*A well-established materials text with a design slant*.).
6. Tabor D. *Properties of Matter* London, UK: Penguin Books; 1978; (*This text, like that of Cottrell, is notable for its clarity and physical insight*.).

3.6 Exercises

The 32 exercises in this section involve the simple use of the charts of this chapter, to find materials with give property profiles. They are answered by placing selection lines on the appropriate chart and reading off the materials that lie on the appropriate side of the lines. It is a good idea to present the results as a table. All can be solved by using the printed charts, which you are free to copy for your personal use.

If the CES Edu Materials Selection software is available, the same exercises can be solved by its use. This involves first creating the chart, then applying the appropriate box or line selections. The results, at Level 2, are the same as those read from the hard copy charts (most of which were made using the Level 2 database). The software allows more detailed charts using the Level 3 database.

E3.1. Use Fig. 3.3 to identify one metal and one polymer with a longitudinal wave speed close to 1000 m/s.

E3.2. A component is at present made from a brass, a copper alloy. Use the Young's modulus – Density ($E-\rho$) chart of Fig. 3.3 to suggest three other metals that, in the same shape, would be stiffer. 'Stiffer' means a higher value of Young's modulus.

E3.3. Use the Young's modulus – Density ($E-\rho$) chart of Fig. 3.3 to identify materials with both a modulus E>50 GPa and a density ρ<2000 kg/m^3.

E3.4. Use the Young's Modulus – Density ($E-\rho$) chart of Fig. 3.3 to find (a) metals that are stiffer and less dense than steels and (b) materials (not just metals) that are both stiffer and less dense than steel.

E3.5. Use the ($E-\rho$) chart of Fig. 3.3 to identify metals with both $E > 100$ GPa and $E/\rho > 0.02$ GPa/(kg/m^3).

E3.6. Use the E/ρ chart of Fig. 3.3 to identify materials with both E>100 GPa and $E^{1/3}/\rho > 0.003(\text{GPa})^{1/3}/(\text{kg/m}^3)$. Remember that, on taking logs, the index $M = E^{1/3}/\rho$ becomes

$$\log(E) = 3\log(\rho) + 3\log(M)$$

and that this plots as a line of slope 3 on the chart, passing through the point E=27 when $\rho = 1000$ in the units on the chart.

E3.7. Use the $E - \rho$ chart of Fig. 3.3 to establish whether woods have a higher specific stiffness E/ρ than epoxies.

E3.8. Do titanium alloys have a higher or lower specific strength (strength/density, σ_f/ρ) than the best steels? This is important when you want strength at low weight (landing gear of aircraft, mountain bikes). Use the σ_f/ρ chart of Fig. 3.4 to decide.

E3.9. The design of a cycle safety helmet requires a lining that will crush at a stress of 8–12 MPa and be as light as possible. Use the $\sigma_f - \rho$ chart of Fig. 3.4 to select an appropriate class of material to meet this need.

E3.10. Use the modulus–strength $E - \sigma_f$ chart of Fig. 3.5 to find materials that have E>10 GPa and $\sigma_f > 1000$ MPa.

E3.11. The undercarriage of a cargo plane requires a metal with a compressive strength above 200 MPa and a high bending strength per unit weight, requiring a high value of the index $\sigma_f^{2/3}/\rho$. What metals does the chart of Fig. 3.4 suggest?

E3.12. Transport systems require materials that are stiff and strong, but at the same time are light and have enough toughness to survive accidental overloads and stress concentrations. Use the Specific modulus E/ρ–Specific strength σ_f/ρ chart (Fig. 3.6) to identify materials that meet these criteria. Eliminate all ceramics on the grounds that they are brittle. Which material has the best combination of both specific modulus and specific strength? Which metal has the highest specific modulus? Which has the highest specific strength?

E3.13. Is the fracture toughness, K_{1c}, of the common polymers polycarbonate, ABS, or polystyrene larger or smaller than the engineering ceramic alumina, Al_2O_3? Are their toughness qualities $G_{1c} = K_{1c}^2/E$ larger or smaller? The K_{1c}–E chart, Fig. 3.7, will help.

E3.14. The bumpers of cars have to be tough and light. Compare the use of aluminium and of polypropylene for car bumpers. First read the approximate toughness, G_{1c} for the two materials from the chart of Fig. 3.7. Then divide each of these by the density of the material (read from any of the charts with density as an axis) to get a measure of toughness per unit mass. What conclusions can you draw?

E3.15. Use the fracture toughness–modulus chart (Fig. 3.7) to find materials that have a fracture toughness K_{1c} greater than 40 MPa·m$^{1/2}$ and a toughness $G_{1c} = K_{1c}^2/E$ (shown as contours on Fig. 3.7) greater than 10 kJ/m^3.

E3.16. The elastic deflection at fracture (the 'resilience') of an elastic-brittle solid is proportional to the failure strain, $\varepsilon_{fr} = \sigma_{fr}/E$, where σ_{fr} is the stress that will cause a crack to propagate:

$$\sigma_{fr} = \frac{K_{1c}}{\sqrt{\pi c}}$$

Here K_{1c} is the fracture toughness and c is the length of the longest crack the materials may contain. Thus

$$\varepsilon_{fr} = \frac{1}{\sqrt{\pi c}}\left(\frac{K_{1c}}{E}\right)$$

Materials that can deflect elastically without fracturing are therefore those with large values of K_{1c}/E. Use the K_{1c}–E chart of Fig. 3.7 to identify the class of materials with $K_{1c} > 1$ MPa m$^{1/2}$ and high values of K_{1c}/E. Position the K_{1c}/E criterion such that all metals are excluded.

E3.17. One criterion for design of a safe pressure vessel is that it should leak before it breaks: the leak can be detected and the pressure released. This is achieved by designing the vessel to tolerate a crack of length equal to the thickness t of the pressure vessel wall, without failing by fast fracture. The safe pressure p is then

$$p \leq \frac{4}{\pi}\frac{1}{R}\left(\frac{K_{1c}^2}{\sigma_f}\right)$$

where σ_f is the elastic limit, K_{1c} is the fracture toughness, R is the vessel radius. The pressure is maximized by choosing the material with the greatest value of

$$M = \frac{K_{1c}^2}{\sigma_y}$$

Use the $K_{1c} - \sigma_f$ chart of Fig. 3.8 to identify three alloys that have particularly high values of M.

E3.18. A material is required for the blade of a rotary lawnmower. Cost is a consideration. For safety reasons, the designer specified a minimum fracture toughness for the blade: it is $K_{1c} > 30$ MPa m$^{1/2}$. The other mechanical requirement is for high hardness, H, to minimize blade wear. Hardness, in applications like this one, is related to strength:

$$H \approx 3\,\sigma_y$$

where σ_f is the strength (Chapter 2, Engineering Materials and Their Properties gives a fuller definition). Use the $K_{1c} - \sigma_f$ chart of Fig. 3.8 to identify three materials that have $K_{1c} > 30$ MPa m$^{1/2}$ and the highest possible strength. To do this, position a 'K_{1c}' selection line at 30 MPa m$^{1/2}$ and then adjust a 'strength' selection line such that it just admits three candidates. Use the Cost chart of Fig. 3.19 to rank your selection by material cost, hence making a final selection.

E3.19. Bells ring because they have a low loss (or damping) coefficient, η; a high damping gives a dead sound. Use the Loss coefficient–Modulus ($\eta - E$) chart of Fig. 3.9 to identify material that should make good bells.

E3.20. A foundation is needed on which to mount vibration-sensitive measuring equipment. It is suggested that the foundation be made from concrete because of its ability to damp vibration. Is this assumption justified? Use Fig. 3.9 to find out.

E3.21. The track of a tracked vehicle is made up of hardened steel segments bonded to rubber connectors. When the vehicle is in motion the rubber is subjected to a tension-compression cycle as the track passes over the front and rear drive sprockets. If the vehicle travels at 20 kph, each unit of the rubber is stretched and compressed roughly 1 per second. The thermal conductivity of rubber is extremely low. If the maximum strain ε_{max} in the rubber is ± 0.4 and no heat is lost from it, how hot will the rubber get after 30 minutes? Take the volume specific heat $C_p\rho$ of the rubber used for the track to be 2.5×10^6 J/m^3 K, its modulus E is 2×10^{-3} GPa and its loss coefficient η is 0.5.

E3.22. Use the Loss coefficient–Modulus ($\eta - E$) chart (Fig. 3.9) to find metals with the highest possible damping.

E3.23. Use the Thermal conductivity–Electrical resistivity ($\lambda - \rho_e$) chart (Fig. 3.10) to find three materials other than diamond with high thermal

conductivity, λ, and high electrical resistivity, ρ_e.

E3.24. It is easier to measure the electrical resistivity of a metal than to measure its thermal conductivity. A new alloy has an electrical resistivity ρ_e of 55 μΩ cm. Use the Wiedemann–Franz relationship, plotted on Fig. 3.10, to estimate its thermal conductivity, λ.

E3.25. A novel strontium-silica glass has a thermal conductivity λ of 2.1 W/m K (exceptionally high for a glass). What, approximately, would you expect its thermal diffusivity a to be? Use the empirical relationship between λ and a to find out, then examine where it would lie on Fig. 3.11.

E3.26. A new design of frying pan is made from glass ceramic 9606, a material with excellent thermal shock resistance originally developed for missile radomes. The pan bottom is 4.2 mm thick. When placed on a gas cooker with a pat of butter in it, it is observed that it takes 7 seconds before the butter starts to melt. Use the rules of thumb of Chapter 2, Engineering Materials and Their Properties and this chapter 3 to estimate, first, the thermal diffusivity of glass ceramic 9606, and from this, its thermal conductivity.

E3.27. The window through which the beam emerges from a high-powered laser must obviously be transparent to light. Even then, some of the energy of the beam is absorbed in the window and can cause it to heat and crack. This problem is minimized by choosing a window material with a high thermal conductivity λ (to conduct the heat away) and a low expansion coefficient α (to reduce thermal strains), that is, by seeking a window material with a high value of

$$M = \lambda/\alpha$$

Use the $\alpha - \lambda$ chart of Fig. 3.12 to identify the best material for an ultra-high powered laser window.

E3.28. An external panel of a satellite is made of the Grade 9 titanium alloy Ti-3Al-2V, with a yield strength σ_Y of 550 MPa and Poisson's ratio ν of 0.36. The panel heats up when the face of the satellite is directly exposed to the sun and cools when the sun is obscured by Earth. The panel is rigidly constrained around its edges by attachment to the frame of the satellite, which is maintained at a constant temperature in order to stabilize the electrics it contains. If the temperature of the panel can vary

from −80°C to +120°C, is there any risk that the resulting thermal stress might cause the panel to yield? Read the value of the thermal stress parameter αE for titanium alloys from Fig. 3.13 and use it to find out. (Here α is the thermal expansion coefficient and E is Young's modulus.)

E3.29. Use the Maximum service temperature (T_{max}) chart (Fig. 3.14) to find polymers that can be used above 200°C.

E3.30. Which class of magnets has the highest energy product? Which has the lowest? Use the chart of Remanent induction and Coercive force, Fig. 3.18, to find out.

E3.31. If magnetostrictive strain is a linear function of the field H, what strain would appear in a Galfenol rod if it was subjected to a field of 100 A/m? Use data read from the chart of Fig. 3.19 to find out. 2.5×10^{-4} (0.025%). A field of 100 A/m might produce a strain of 1/8 of this, or about 0.003%.

E3.32.

a. Use the Young's modulus–Relative cost ($E - C_{v,R}$) chart (Fig. 3.26) to find the cheapest materials with a modulus, E, greater than 100 GPa.

b. Use the Strength–Relative cost ($\sigma_f - C_R$) chart (Fig. 3.27) to find the cheapest materials with a strength, σ_f, above 100 MPa.

[1]A set of the charts can be downloaded from www.grantadesign.com. All the charts shown in this chapter were created using Granta Design's CES EduPack Materials Selection software. With it you can make charts with any pair (or combination) of properties as axes.

[2]Most material properties are best viewed on log scales because the ranges are so large. Instead of incrementing by steps of 1, log scales increment by steps of a factor of 10.

[3]Very low density foams and gels (which can be thought of as molecular-scale, fluid-filled, foams) can have lower moduli than this. As an example, gelatin (as in Jello) has a modulus of about 10^{-5} GPa. Their strengths and fracture toughness, too, can be below the lower limit of the charts.

[4]This can be understood by noting that a solid containing N atoms has $3N$ vibrational modes. Each (in the classical approximation) absorbs thermal energy kT at the absolute temperature T, and the vibrational specific heat is $C_p \approx C_v = 3Nk$ (J/K) where k is Boltzmann's constant (1.34×10^{-23} J/K). The volume per atom, Ω, for almost all solids lies within a factor of two of 1.4×10^{-29} m^3; thus the volume of N atoms is ($N\Omega$) m^3. The volume specific heat is then (as the chart shows):

$$\rho C_v \cong 3Nk/N\Omega = \frac{3k}{\Omega} = 3\times10^6\ J/m^3K$$

CHAPTER 4

Materials Selection – The Basics

Abstract

The basic procedure for material selection, which establishes the link between material and function, involves

- *translation*, examining the design requirements to identify the constraints that they impose on material choice;
- *screening*, to eliminated the materials that cannot meet the constraints;
- *ranking*, ordering the candidates by their ability to maximize performance; and
- *documentation* to research the background of the most promising candidates.

Keywords

Material selection; translation; screening; ranking; documentation; material index; structural index; computer-based selection

4.1 Introduction and Synopsis

This chapter sets out the basic procedure for material selection, which establishes the link between material and function (Fig. 4.1). A material has *attributes*: its density, strength, cost, resistance to corrosion, and many more – as described in Chapter 2, Engineering Materials and Their Properties. A design demands a certain profile of these attributes: a low density, a high strength, a modest cost and resistance to sea water, perhaps. It is important to start with the full menu of materials as options; failure to do so may mean a missed opportunity.

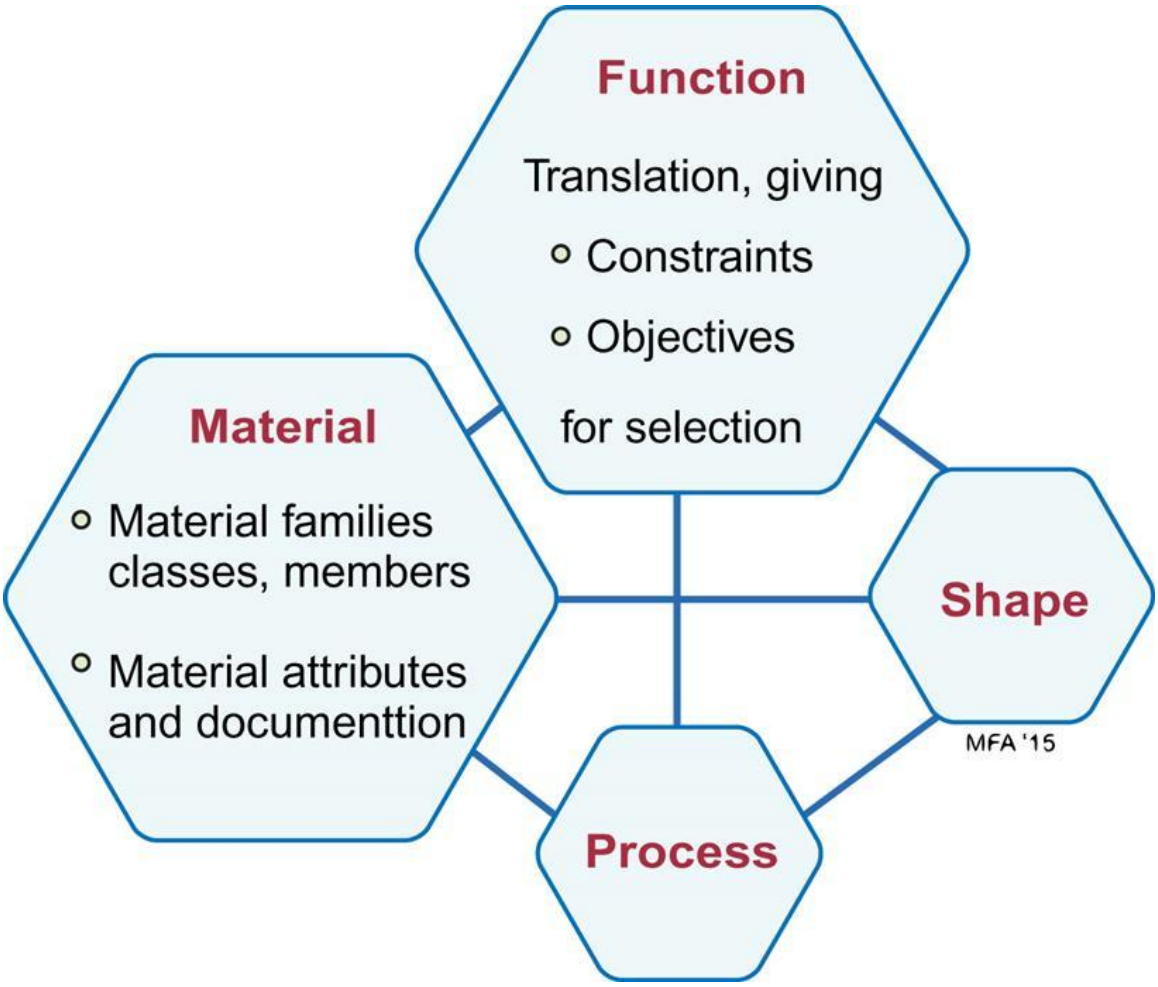


FIGURE 4.1 The intended function of a component sets the constraints and establishes the objectives for material selection. This chapter explains how this is done.

The task of selection, stated in two lines, is that of
1. identifying the profile of material attributes that best meets the requirements of the design, then.
2. comparing this desired profile with the attribute profiles of available engineering materials to find the best match.

The first step in tackling task is that of translation, examining the design requirements to identify the constraints that they impose on material choice. The immensely wide choice of materials is narrowed, first, by screening out the materials that cannot meet the constraints. Further narrowing is achieved by ranking the candidates by their ability to maximize performance. This chapter explains how to do these tasks.

The materials property charts introduced in Chapter 3, Materials Property Charts are designed for use with these criteria. Constraints and ranking criteria (objectives) can be plotted onto them, isolating the subset of materials that are the best choice for the design. The whole procedure can be implemented in software as a design tool, allowing computer-aided selection. The procedure is fast and makes for lateral ('what if…?') thinking. Examples of the method are given in the next and subsequent chapters.

4.2 The Selection Strategy

Material Attributes

If we are going to select materials we first need a *taxonomy* – a way of cataloguing materials and their attribute profiles so that we can find them when we want them. Fig. 4.2 illustrates how the 'universe' of materials is divided into families, classes, subclasses and members. Each member is characterised by a set of *attributes*: its properties. As an example, the Materials universe contains the family 'Metals', which in turn contains the class 'Aluminium alloys', the subclass '6000 series' and finally the particular member 'Alloy 6061'. It, and every other member of the universe, is characterized by a set of attributes that include its mechanical, thermal, electrical, optical, and chemical properties, its processing characteristics, its cost and availability, the environmental consequences of its use, and more. We call this attribute set its *property profile*. Selection involves seeking the best match between the property-profiles of the materials in the universe and that required by the design.

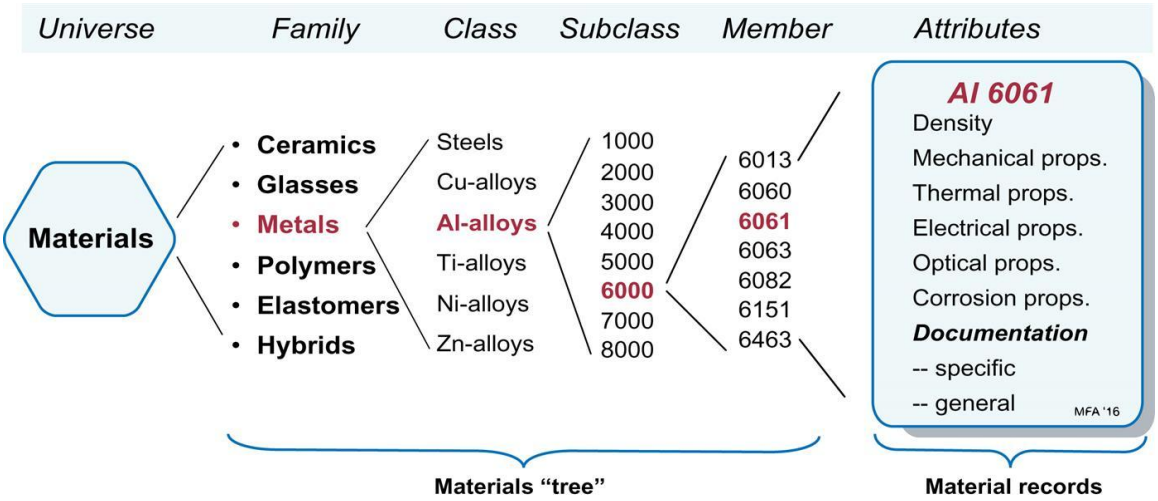


FIGURE 4.2 The taxonomy of the kingdom of materials and their attributes. Computer-based selection systems store data in a hierarchical structure like this.

Selection Strategies

Systematic selection requires a strategy. It is simpler to start with the selection of a product rather than with a material – the ideas are the same, but material selection has an added complication.

You need a new car. To meet your needs it must be a midsized, four-door, family saloon with a petrol engine delivering at least 150 horsepower – enough to tow your power boat. Given all of these, you wish it to cost as little to own and run as possible (Fig. 4.3, left-hand side). There are three constraints here, but they are not all of the same type.

- The requirements of *four-door family saloon* and *petrol power* are *simple constraints* – a car *must* have these to be a candidate.
- The requirement of *at least 150 hp* places a lower limit but no upper one on power; it is a *limit constraint* – any car with 150 hp or more is acceptable.

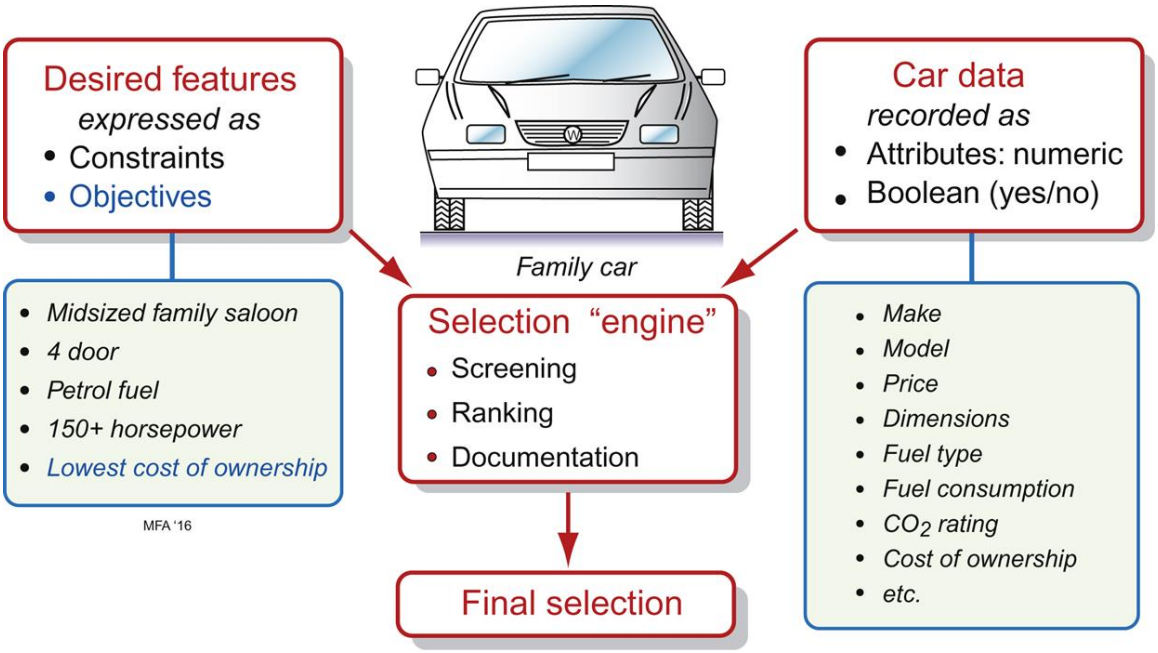


FIGURE 4.3 Choosing a car – and example of the selection strategy. Required features are constraints (in black on the left). They are used to screen out unsuitable cars. The survivors are ranked by the objective of minimizing cost of ownership (in blue on the left).

The wish for minimum cost of ownership is an *objective*, a criterion of excellence. The most desirable cars, from among those that meet the constraints, are those that minimize this objective.

To proceed you need information about available cars (Fig. 4.3, right-hand side). Car magazines, car makers' web sites and dealers list such information. It includes car type and size, number of doors, fuel type, engine power and price; car magazines go further and estimate the cost of ownership (meaning the sum of running costs, tax, insurance, servicing and depreciation), listing it as $/mile or €/km.

Now: decision time (Fig. 4.3, central box). The selection engine (you, in this example) uses the constraints to screen out, from all the available cars, those that are not four-door, petrol-powered, family saloons with at least 150 hp. Many cars meet these constraints; the list is still long. You need a way to order it so that the best choices are at the top. That is what the objective is for: it allows you to *rank* the surviving candidates by cost of ownership – those with the lowest values are ranked most highly. Rather than just choosing the one that is cheapest to won, it is better to keep the top three or four and seek further *documentation*, exploring their other

features in depth (delivery time, size of trunk, proximity of dealership, guarantee period …) weighing the small differences in cost against the desirability of these features. At that point you are in a position to make a final selection.

This seems like a straightforward strategy. What's the problem in applying it to materials? Selecting materials involves seeking the best match between design requirements and the properties of the materials that might be used to make it. Fig. 4.4 shows an example: the strategy applied to selecting materials for the protective visor of a safety helmet. On the left are the list of requirements that the material must meet, expressed as constraints and objectives. The constraints are ability to be moulded and, of course, transparency. The objective, if the visor is to protect the face, is that it must be as shatterproof as possible. On the right is the database of material attributes, drawn from compilations like that of Appendix A, the charts of this book, suppliers' data sheets, handbooks, web-based sources or software specifically designed for materials selection. The comparison engine – you again – applies the constraints on the left to the materials on the right, screening out materials that fail to meet them, and delivering a list of viable candidates, just as with cars. The list is then ranked by the fracture toughness. The three or so materials that meet the constraints and have the highest fracture toughness are then explored in depth by researching their background, the step labelled 'documentation'.

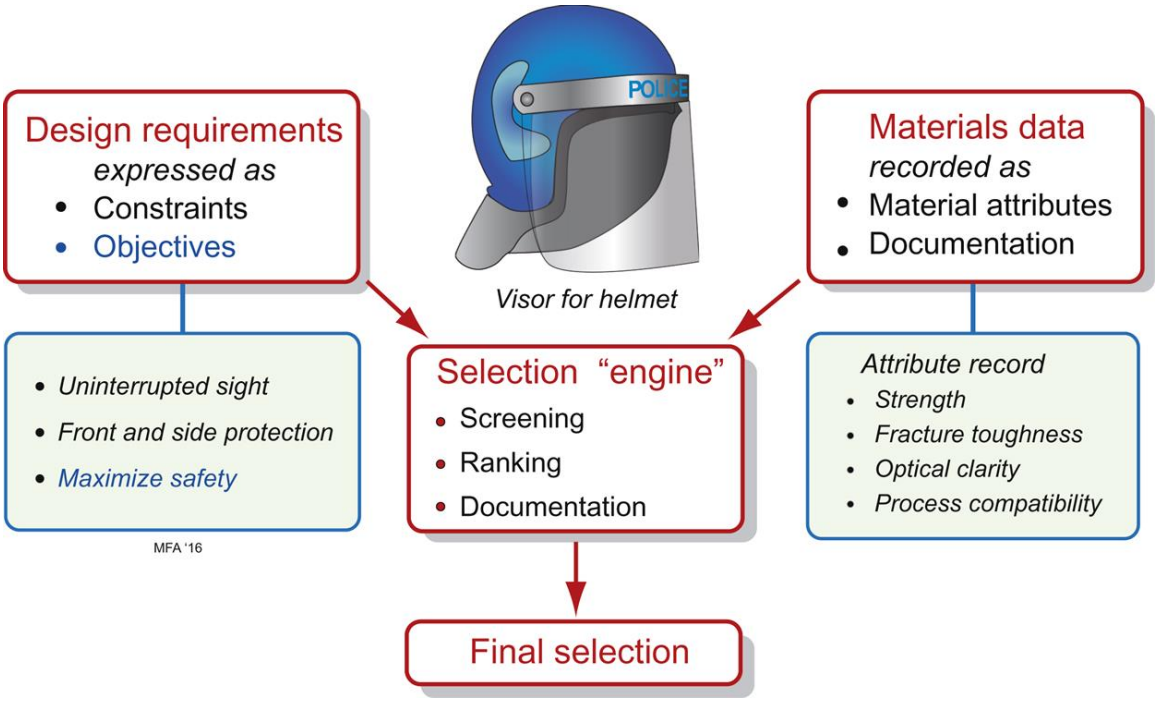


FIGURE 4.4 Choosing a material. Design requirements are first expressed as constraints and objectives. The constraints are used for screening. The survivors are ranked by the objective that the material of the visor be as shatterproof as possible.

There is, however, a complication. The requirements for the car were straightforward – doors, fuel type, power – all of these are explicitly listed by the manufacturer. The design requirements for a component of a product specify what it should *do* but not what *properties* its materials should have. There is no one-to-one correspondence between the requirements in the blue-shaded box on the left and the relevant attributes on the right, even though all of these attributes *are* relevant. So the first step in selecting materials is one of translation: converting the design requirements (often vague) into constraints and objectives that can be applied to the materials database. The next task is that of screening, as with cars, eliminating the materials that cannot meet the constraints. This is followed by the ranking step, ordering the survivors by their ability to meet a criterion of excellence, in this case maximizing impact resistance. The final task is to explore the most promising candidates in depth, examining how they are used at present, case-histories of failures, and how best to design with them, the step we called documentation.

Here is a second example. A material is sought for the hull of a portable canoe (Fig. 4.5). The material must be easy to shape and it must be stiff and strong enough to tolerate service loads. It must be durable. And if the canoe is be portable it must be as light as possible, and, to be competitive, the material of which it is made must be as cheap as possible. Here we have four constraints (mouldable, stiff, strong and durable) and two objectives (minimize mass, minimize material cost). All relate to the material attributes listed on the right of Fig. 4.5, but again there is no one-to-one correspondence. As before, the first task is to translate these requirements into constraints and objectives that can be directly applied to the screen and rank the available materials, to give a short list of possible candidates. The top-ranked candidates can then be explored in more depth, seeking documentation.

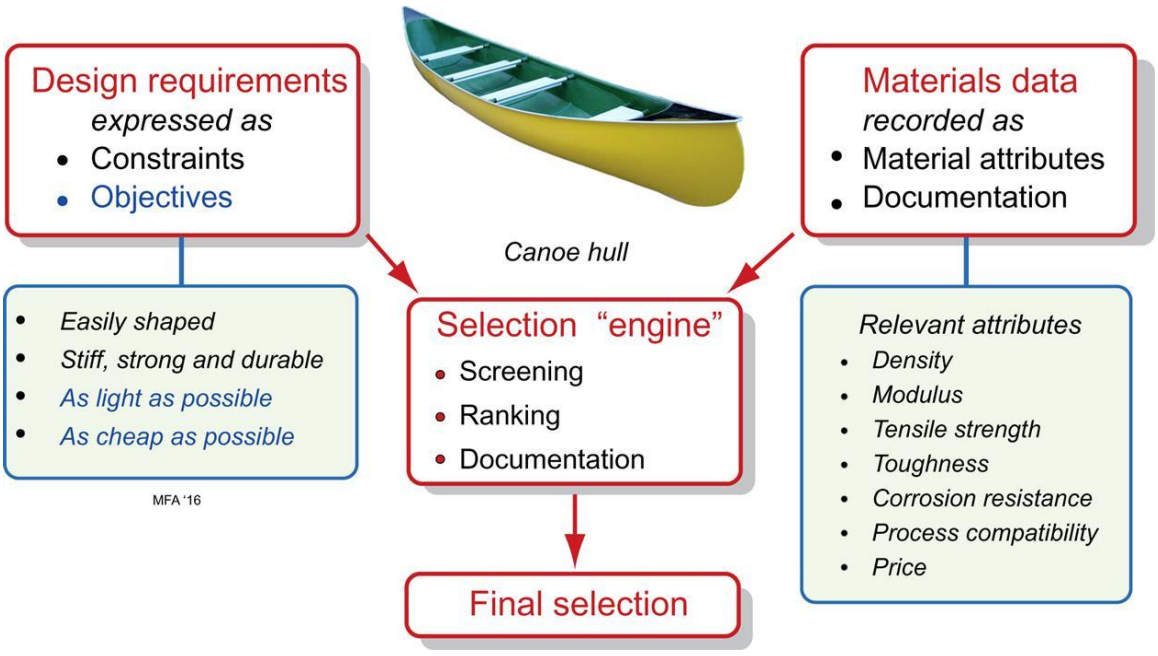


FIGURE 4.5 Choosing a material. Design requirements are first translated into constraints and objectives that can be used for screening and ranking. In this example there are four constraints and two objectives.

The steps are summarized in Fig. 4.6. Right now we'll take a closer look at each of them.

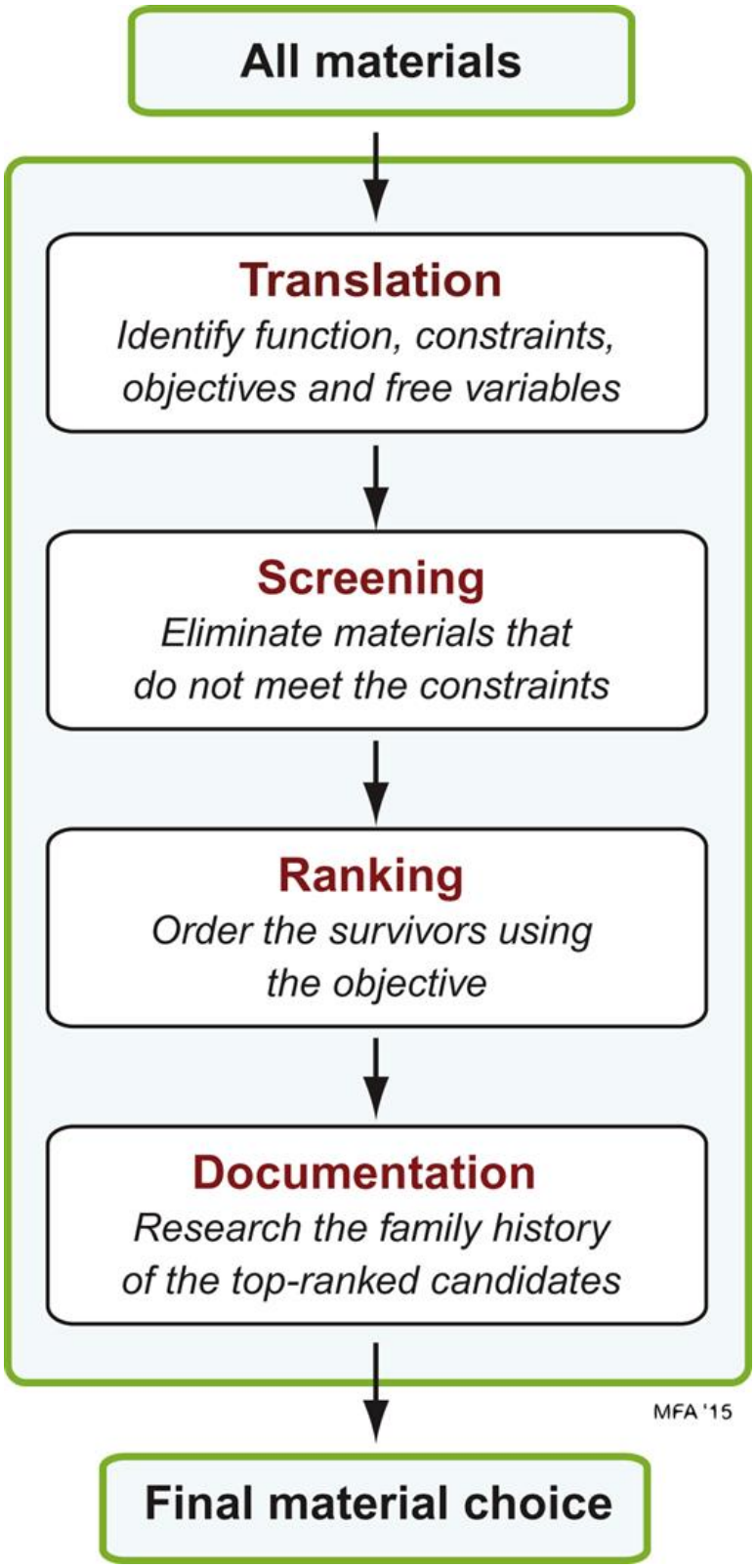


FIGURE 4.6 The strategy for materials selection. The four main steps – translation, screening, ranking and

documentation – are shown here.

Translation

How are the design requirements for a component (defining what it must do) translated into a prescription for material selection? Any engineering component has one or more *functions:* to support a load, to contain a pressure, to transmit heat, and so forth. This must be achieved subject to *constraints:* that certain dimensions are fixed, that the component must carry the design loads or pressures without failure, that it insulates or conducts, that it can function in a certain range of temperature and in a given environment, and many more. In designing the component, the designer has an *objective:* to make it as cheap as possible, perhaps, or as light, or as safe, or perhaps some combination of these. Certain parameters can be adjusted in order to optimize the objective – the designer is free to vary dimensions that have not been constrained by design requirements and, most importantly, free to choose the material for the component. We refer to these as *free variables*. Function, and constraints, objectives and free variables (Table 4.1) define the boundary conditions for selecting a material and – in the case of load-bearing components – a shape for its cross-section. The first step in relating design requirements to material properties is a clear statement of function, constraints, objectives and free variables.

Translation for the helmet visor of Fig. 4.4

To allow clear vision the visor must be optically transparent, the highest level of transparency. To protect the face from the front and from the sides it must be curved, requiring that the material can be moulded. We thus have two constraints: *optical transparency* and *ability to be moulded*.

Fracture of the visor would expose the face to damage: maximizing safety therefore translates in to maximizing resistance to fracture. The material property that measures resistance to fracture is the *fracture toughness*, K_{1c}. The objective is therefore to maximize K_{1c}.

Table 4.1

Function, Constraints, Objectives and Free Variables

Function	*What does the component do?*
Constraints[a]	*What nonnegotiable conditions must be met?* *What negotiable but desirable conditions…?*
Objective	*What is to be maximized or minimized?*
Free variables	*What parameters of the problem is the designer free to change?*

[a]It is sometimes useful to distinguish between 'hard' and 'soft' constraints. Stiffness and strength might be absolute requirements (hard constraints); colour might be negotiable (a soft constraint).

Screening: Attribute Limits

Unbiased selection requires that all materials are considered to be candidates until shown to be otherwise, using the steps in the boxes below 'Translate' in Fig. 4.6. The first of these, *screening*, eliminates candidates that cannot do the job at all because one or more of their attributes lies outside the limits set by the constraints. As examples, the requirement that 'the component must function in boiling water' or that 'the component must not contain toxic substances' imposes obvious limits on the material attributes of *melting point* and *toxicity* that successful candidates must meet. We refer to these as *attribute limits*. Applying these limits narrows the list of materials to a viable subset all of which meet the constraints.

Ranking: Material Indices

Attribute limits do not, however, help with ordering the candidates that remain. To do this we need optimisation criteria. They are found in the material indices, developed below, which measure how well a candidate that has passed the screening step can perform. Performance is sometimes limited by a single property, sometimes by a combination of them. Thus the best materials for buoyancy are those with the lowest density, ρ; those best for thermal insulation the ones with the smallest values of the thermal conductivity, λ. It is not always the smallest that we want; the best material for a heat exchanger, for instance, is that with the largest value of λ. Here maximizing or minimizing a single property maximizes performance. But – as we shall see – it is more usual that performance is limited not by one property but by a combination of them. Thus the best materials for a light

strong tie-rod or cable are those with the greatest value of the specific strength, σ_f/ρ, where σ_f is the tensile strength. The best materials for a spring are those with the greatest value of σ_f^2/E where E is Young's modulus. The property or property-group that maximizes the performance in a given design is called its *material index*. There are many such indices, each associated with maximizing some aspect of performance.[1] They provide *criteria of excellence* that allow ranking of materials by their ability to perform well in the given application. You will find a compilation of them in Appendix C.

To summarize: screening isolate candidates that could do the job; ranking identifies those among them that can do the job best.

Screening and ranking for the helmet visor

A search for transparent materials that can be moulded delivers the following list. The first four are thermoplastics, the last two, glasses. Fracture toughness values can be found in Appendix A, Table A4.

Material	Average Fracture Toughness K_{1c} MPa $m^{1/2}$
Polycarbonate (PC)	3.4
Polymethyl methacrylate (Acrylic, PMMA)	1.2
Cellulose acetate (CA)	1.1
Polystyrene (PS)	0.9
Soda-lime glass	0.6
Borosilicate glass	0.6

The constraints have reduced the number of viable materials to six candidates. When ranked by fracture toughness, the top-ranked candidates are PC, PMMA and CA.

Documentation

The outcome of the steps so far is a ranked short-list of candidates that meet the constraints and maximize or minimize the criterion of excellence, whichever is required. You could just choose the top-ranked candidate, but what secret vices might it have? What are its strengths and weaknesses? Does it have a good reputation? What, in a word, is its credit-rating? To proceed further we seek a detailed profile of each: its *documentation* (Fig. 4.6, bottom).

Documentation differs greatly from the structured property data used for screening. Typically, it is descriptive, graphical or pictorial: case studies of previous uses of the material, failure analyses and details of its corrosion, information of availability, pricing and the like. Such information is found in handbooks, suppliers' data sheets, in case studies of use and in failure analyses (Fig. 4.7). Documentation helps narrow the short-list to a final choice, allowing a definitive match to be made between design requirements and material attributes.

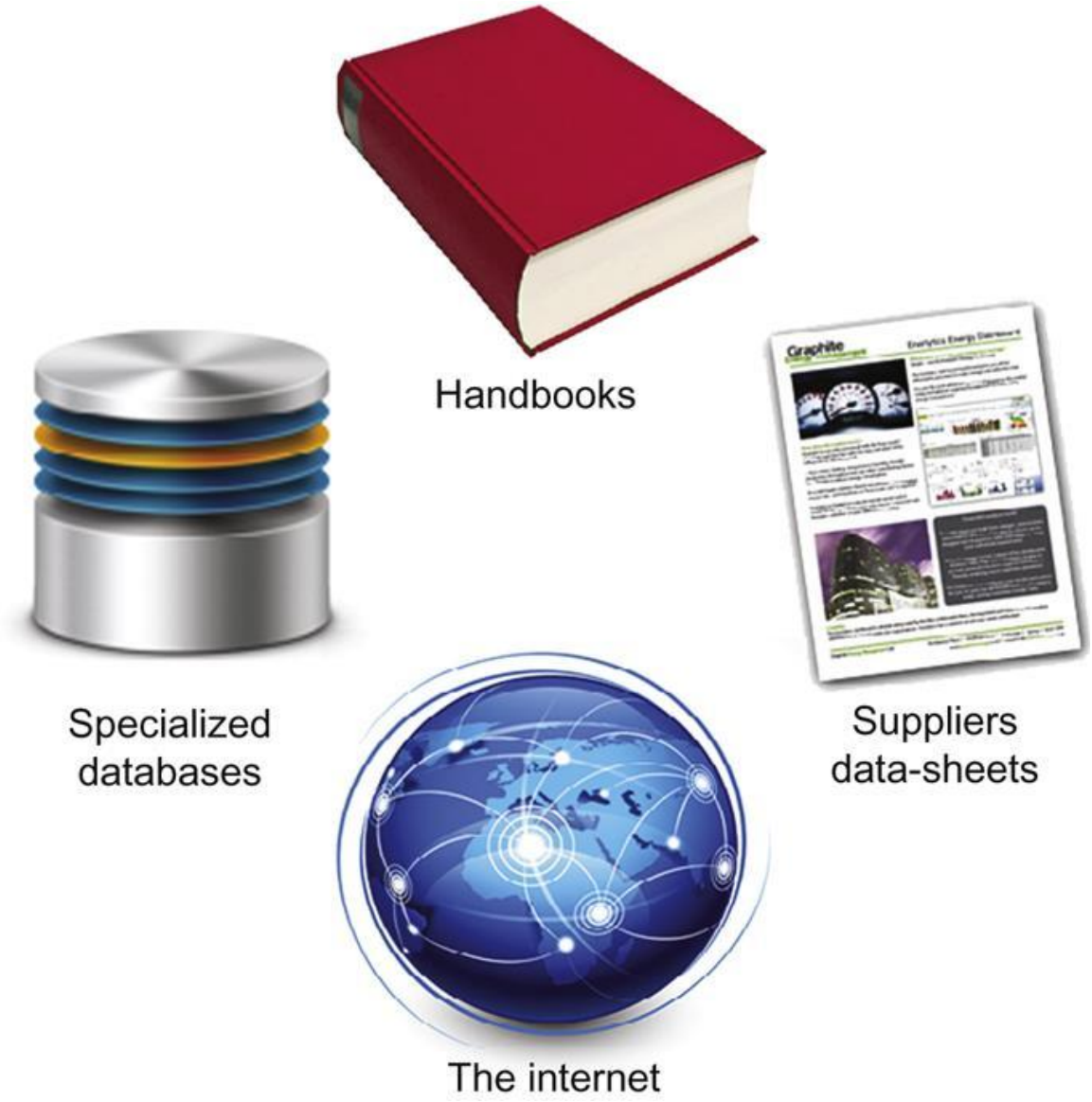


FIGURE 4.7 Sources of documentation: Handbooks, supplier's data-sheets, databases and the internet.

Why are all these steps necessary? Without screening and ranking, the candidate-pool is enormous and the volume of documentation overwhelming. Dipping into it, hoping to stumble on a good material, gets you nowhere. But once a small number of potential candidates have been identified by the screening-ranking steps, detailed documentation can be sought for these few alone, and the task becomes viable.

Documentation for materials for the helmet visor

At this point it helps to know how the three top-ranked candidates listed in the last examples-box are used. A quick web search reveals the typical uses of each.

Polycarbonate. Safety shields and goggles; lenses; light fittings; safety helmets; laminated sheet for bullet-proof glazing.

PMMA, Plexiglas. Lenses of all types; cockpit canopies and aircraft windows; containers; tool handles; safety spectacles; lighting, automotive tail lights.

Cellulose acetate. Spectacle frames; lenses; goggles; tool handles; covers for television screens; decorative trim and steering wheels for cars.

This is encouraging: all three materials have a history of use for goggles and protective screening. The one that ranked highest in our list – polycarbonate – has a history of use for protective helmets. We select this material, confident that, with its high fracture toughness, it is the best choice.

Local Conditions

The final choice between competing candidates will, often, depend on local conditions: in-house expertise or equipment, the availability of local suppliers, and so forth. A systematic procedure cannot help here – the decision must instead be based on local knowledge. This does not mean that the result of the systematic procedure is irrelevant. It is always important to know which material is best, even if, for local reasons, you decide not to use it.

We explore documentation more fully later. Here we focus on the derivation of property limits and indices.

4.3 Attribute Limits and Material Indices

Constraints set property limits. Objectives define material indices, for which we seek extreme values. When the objective is not coupled to a constraint, the material index is a simple material property. When, instead, they are coupled, the index becomes a group of properties like those cited above. Where do they come from? This section explains.

Think for a moment of the simplest of mechanical components. The loading on a component can generally be decomposed into some combination of axial tension, bending, torsion and compression. Almost

always, one mode of loading dominates. So common is this that the functional name given to the component describes the way it is loaded: *ties* carry tensile loads; *beams* and *panels* carry bending moments; *shafts* carry torques; and *columns* carry compressive axial loads. The words 'tie', 'beam', 'shaft' and 'column' each imply a function. Here we explore constraints, objectives and resulting material indices for some of these.

Minimum weight design is a good starting point. We consider the generic components and typical loadings shown in Fig. 4.8: ties, panels and beams.

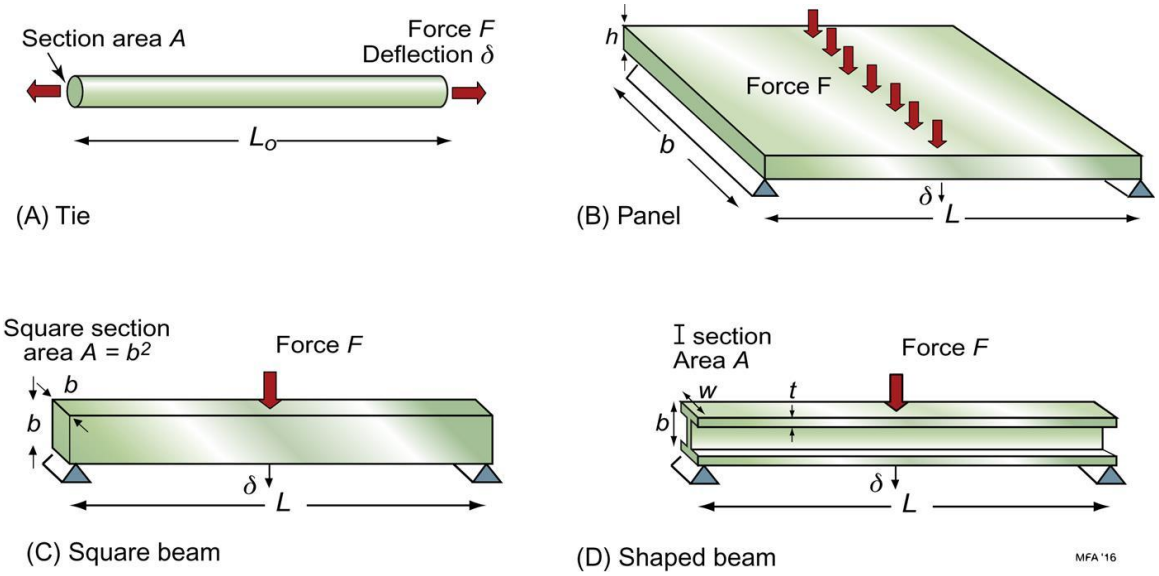


FIGURE 4.8 Generic components (A) a tie, a tensile component, (B) a panel, loaded in bending, and (C) and (D) beams, loaded in bending.

Minimizing mass: a light, strong tie. A design calls for a ties like those of the bi-plane on the cover picture of this chapter. It must carry a tensile force F^* without failure and be as light as possible (Fig. 4.8A). The length L is specified but the cross-section area A is not. Here, 'maximizing performance' means 'minimizing the mass while still carrying the load F^* safely'. The design requirements, translated, are listed in Table 4.2.

Table 4.2

Design Requirements for the Light Strong Tie

Function	■ *Tie-rod*
Constraints	■ *Length L is specified (a geometric constraint)* ■ *Tie must support axial tensile load* F^* *without failing (a functional constraint)*
Objective	■ *Minimize the mass m of the tie*
Free variables	■ *Cross-section area, A* ■ *Choice of material*

We first seek an equation describing the quantity to be maximized or minimized. Here it is the mass m of the tie, and it is a minimum that we seek. This equation, called *the objective function*, is

$$m = A\,L\,\rho \tag{4.1}$$

where A is the area of the cross-section and ρ is the density of the material of which it is made. The length L and force F are specified and are therefore fixed; the cross-section, A, is free. We can reduce the mass by reducing the cross-section, but there is a constraint: the section-area A must be sufficient to carry the tensile load F^*, requiring that

$$\frac{F^*}{A} \leq \sigma_f \tag{4.2}$$

where σ_f is the failure strength. Eliminating A between these two equations gives

$$m \geq (F^*)\ (L) \left(\frac{\rho}{\sigma_f}\right) \tag{4.3}$$

Material properties

Functional constraint

Geometric constraint

Note the form of this result. The first bracket contains the specified load F^*. The second bracket contains the specified geometry (the

length L of the tie). The last bracket contains the material properties. The lightest tie that will carry F^* safely[2] is that made of the material with the smallest value of ρ/σ_f. We could define this as the material index of the problem, seeking a minimum, but it is more usual, when dealing with specific properties, to express them in a form for which a maximum is sought. We therefore invert the material properties in Eq. (4.3) and define the material index M_t (subscript 't' for 'tie'), as

$$M_{t_1} = \frac{\sigma_f}{\rho} \tag{4.4}$$

The lightest tie-rod that will carry the load F^* without failing is that with the largest value of this index, the 'specific strength', σ_f/ρ, plotted in the chart of Figure 3.6. A similar calculation for a light *stiff* tie (one for which the stiffness S rather than the strength σ_f is specified) leads to the index

$$M_{t_2} = \frac{E}{\rho} \tag{4.5}$$

where E is Young's modulus. This time the index is the 'specific stiffness', E/ρ, also shown in Figure 3.6. The material index is a group (rather than just a single property) because minimizing the mass m – the objective – was coupled to one of the constraints, that of carrying the load F^* without failing or deflecting too much.

Note the procedure.

- Identify the objective (the quantity to be maximized or minimized), the constraints and the free variables.
- Write an equation for the objective (here, m). If it contains a free variable, then.
- Use a constraint (here, strength or stiffness) to eliminate the free variable.
- Read off the combination of material properties – the index M – for which an extreme value is sought.

It sounds straightforward, and it is so long as you are clear from the start what the constraints are, what you are trying to maximize or minimize, which parameters are specified and which are free.

That was easy. Now for some slightly more difficult (and important) situations.

Minimizing mass: a light, stiff panel. A *panel* is a flat slab, like a table top. Its length L and width is b are specified but its thickness h is free. It is loaded in bending by a central load F (Fig. 4.8B). The stiffness constraint requires that it must not deflect more than δ. The objective is to achieve this with minimum mass, m. Table 4.3 summarizes the design requirements.

Table 4.3
Design Requirements For a Light, Stiff Panel

Function	■ *Panel*
Constraints	■ *Bending stiffness S* specified* (a functional constraint) ■ *Length L and width b specified* (geometric constraints)
Objective	■ *Minimize the mass m of the panel*
Free variables	■ *Panel thickness, h* ■ *Choice of material*

The objective function for the mass of the panel is the same as that for the tie:

$$m = A\,L\,\rho = b\,h\,L\,\rho$$

Its bending stiffness S must be at least S^*:

$$S = \frac{C_1 EI}{L^3} \geq S^* \qquad \textbf{(4.6)}$$

Here C_1 is a constant that depends only on the distribution of the loads – we don't need its value (you can find it in Appendix B). The second moment of area, I, for a rectangular section, is

$$I = \frac{bh^3}{12} \qquad \textbf{(4.7)}$$

We can reduce the mass by reducing h, but only so far that the stiffness constraint is still met. Using the last two equations to eliminate h in the objective function gives

$$m = \left(\frac{12\,S^*}{C_1\,b}\right)^{1/3}\left(bL^2\right)\left(\frac{\rho}{E^{1/3}}\right) \quad \text{(4.8)}$$

Material properties

Functional constraint — Geometric constraints

The quantities S^*, L, b and C_1 are all specified; the only freedom of choice left is that of the material. The index is the group of material properties that appears in Eq. (4.8), which we invert such that a maximum is sought. Thus the best materials for a light, stiff panel are those with the greatest values of

$$M_{p_1} = \frac{E^{1/3}}{\rho} \quad \text{(4.9)}$$

Repeating the calculation with a constraint of strength rather than stiffness leads to the index

$$M_{p_2} = \frac{\sigma_y^{1/2}}{\rho} \quad \text{(4.10)}$$

These don't look much different from the previous indices, E/ρ and σ_y/ρ but it they are: they lead to different choices of material, as we shall see in a moment

Now for another bending problem, in which the freedom to choose shape is rather greater than for the panel.

Minimizing mass: a light, stiff beam. Beams come with many shapes of cross-section: solid rectangles, cylindrical tubes, I-sections and more. Some of these have too many free geometric variables to apply the method above directly. But if we constrain the shape to be *self-similar* (such that all dimensions of the cross-section change in proportion as we vary the overall size), the problem becomes tractable again. We therefore consider beams in two stages: first, to identify the optimum materials for a light,

stiff beam of a prescribed simple shape (a square section), and then second, we explore how much lighter it could be made, for the same stiffness, by using a more efficient shape.

Consider a beam of length L and square section $A = b \times b$ that may vary in size but not in shape – it remains square. The beam is loaded in bending by a central load F (Fig. 4.8C). It must not deflect more than δ under the load F (a stiffness constraint). The objective is to minimize the mass. Table 4.4 summarizes the design requirements.

Table 4.4

Design Requirements for a Light Stiff Beam

Function	■ *Beam*
Constraints	■ *Length L is specified* (a geometric constraint) ■ *Section shape square* (a geometric constraint) ■ *Beam must support a bending load F without deflecting too much, meaning that the bending stiffness S is specified as S** (a functional constraint)
Objective	■ *Minimize the mass m of the beam*
Free variables	■ *Cross-section area, A* ■ *Choice of material*

Proceeding as before, the objective function for the mass is

$$m = A\,L\,\rho = b^2\,L\,\rho \tag{4.11}$$

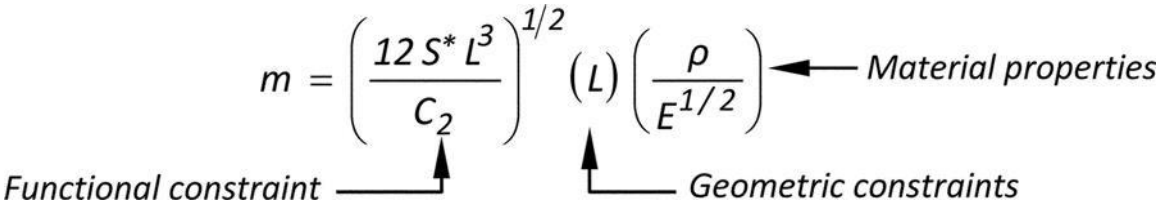
The bending stiffness S of the beam must be at least S^*

$$S = \frac{C_2\,EI}{L^3} \geq S^* \tag{4.12}$$

where C_2 is a constant (Appendix B). The second moment of area, I, for a square section beam is

$$I = \frac{b^4}{12} = \frac{A^2}{12} \tag{4.13}$$

For a given length L, the stiffness S^* is adjusted by adjusting the size of the square section. Eliminating b (or A) in the objective function for the mass gives



(4.14)

The quantities S^*, L and C_2 are all specified or constant – the best materials for a light, stiff beam are those with the largest values of the index M_{b_1} where

$$M_{b_1} = \frac{E^{1/2}}{\rho} \tag{4.15}$$

Repeating the calculation with a constraint of strength rather than stiffness leads to the index

$$M_{b_2} = \frac{\sigma_y^{2/3}}{\rho} \tag{4.16}$$

This analysis was for a square beam, but the result in fact holds for any shape, so long as the shape is held constant. This is a consequence of Eq. (4.13) – for a given shape, the second moment of area I can always be expressed as a constant times A^2, so changing the shape just changes the constant but not the resulting index.

As noted above, real beams have section shapes that improve their efficiency in bending, and require less material to get the same stiffness. By shaping the cross-section it is possible to increase I without changing A. This is achieved by locating the material of the beam as far from the neutral axis of bending as possible, as in thin-walled tubes or I-beams (Fig. 4.8D). Some materials are more amenable than others to being made into efficient shapes. Comparing materials on the basis of the index in M_b therefore requires some caution – materials with lower values of the index may 'catch up' by being made into more efficient shapes. To find out just how much, you will have to wait until Chapter 10, Selection of Material and Shape which discusses the selection of material and shape.

Minimizing material cost: cheap ties, panels and beams. When the objective is to minimize cost rather than mass the indices change but in a trivial way. If the material price is C_m \$/kg, the cost of the material to make a

component of mass m is just mC_m. The objective function for the material cost C of the tie, panel or beam then becomes

$$C = mC_m = A\,L\,C_m\,\rho \tag{4.17}$$

Proceeding as before then leads to indices that have the form of Eqs (4.4), (4.5), (4.9), (4.10), (4.15) and (4.16) with ρ replaced by $C_m\rho$. Thus the index guiding material choice for a tie of specified strength and minimum material cost is

$$M = \frac{\sigma_f}{C_m\rho} \tag{4.18}$$

where C_m is the material price per kg. That for a cheap stiff panel is

$$M_{p_1} = \frac{E^{1/3}}{C_m\rho} \tag{4.19}$$

and so forth. It must be remembered that the material cost is only part of the cost of a shaped component; there is also the manufacturing cost – the cost to shape, join and finish it. We return to that in Chapter 7, Processes Selection and Cost.

How General are Material Indices?

The components on the cover images of this chapter are labelled with function ('tie', 'beam' etc.) and with the index that guides the choice of material to make them. The biplane typifies lightweight design, meaning that its materials are chosen to carry the design loads at minimum mass. The airport structure uses very large quantities of materials: here the objective is to carry the design loads safely while minimizing the cost of the material. The guiding index for each structure is derived from a single objective, that of minimizing mass in one case, that of minimizing material cost in the other. In a more general sense each index is associated with a function, a constraint and an objective, which is a good way to catalogue them. Appendix C contains such a catalogue.

This is a good moment to describe the method in more general terms. *Structural elements* are components that perform a physical function: they carry loads, transmit heat, store energy and so on: in short, they satisfy *functional requirements*. Examples: a tie must carry a specified tensile load; a spring must provide a given restoring force or store a given energy, a heat exchanger must transmit heat a given heat flux, and so on.

The performance of a structural element is determined by three things: the functional requirements, the geometry and the properties of the material of which it is made.[3] The performance P of the element is described by an equation of the form

$$P = f\left[\begin{pmatrix}\text{Functional}\\ \text{requirements, } F\end{pmatrix}, \begin{pmatrix}\text{Geometric}\\ \text{parameters, } G\end{pmatrix}, \begin{pmatrix}\text{Material}\\ \text{properties, } M\end{pmatrix}\right]$$

or

$$P = f(F, G, M) \tag{4.20}$$

where P, the *performance metric*, describes some aspect of the performance of the component: its mass, or volume, or cost, or life for example; and '*f*' means '*a function of*'. *Optimum design* is the selection of the material and geometry that maximize or minimize P, according to its desirability or otherwise.

The three groups of parameters in Eq. (4.20) are said to be *separable* when the equation can be written

$$P = f_1(F) \cdot f_2(G) \cdot f_3(M) \tag{4.21}$$

where f_1, f_2 and f_3 are separate functions that are simply multiplied together. It turns out that, commonly, they are, and when this is so the optimum choice of material becomes independent of the details of the design; it is the same for *all* geometries, G, and for all values of the function requirement, F. Then the optimum subset of materials can be identified without solving the complete design problem, or even knowing all the details of F and G. This enables enormous simplification: the

performance for *all F* and *G* is maximized by maximizing $f_3\,(M)$, which is called the material efficiency coefficient, *or material index* for short. The remaining bit, $f_1\,(F)\cdot f_2\,(G)$, is related to *the structural efficiency coefficient*, or *structural index*. We don't need it now, but will examine it briefly in Section 4.6.

Each combination of function, objective and constraint leads to a material index (Fig. 4.9) and the index is characteristic of the combination, and thus of the function the component performs. The method is general, and, in later chapters, is applied to a wide range of problems.

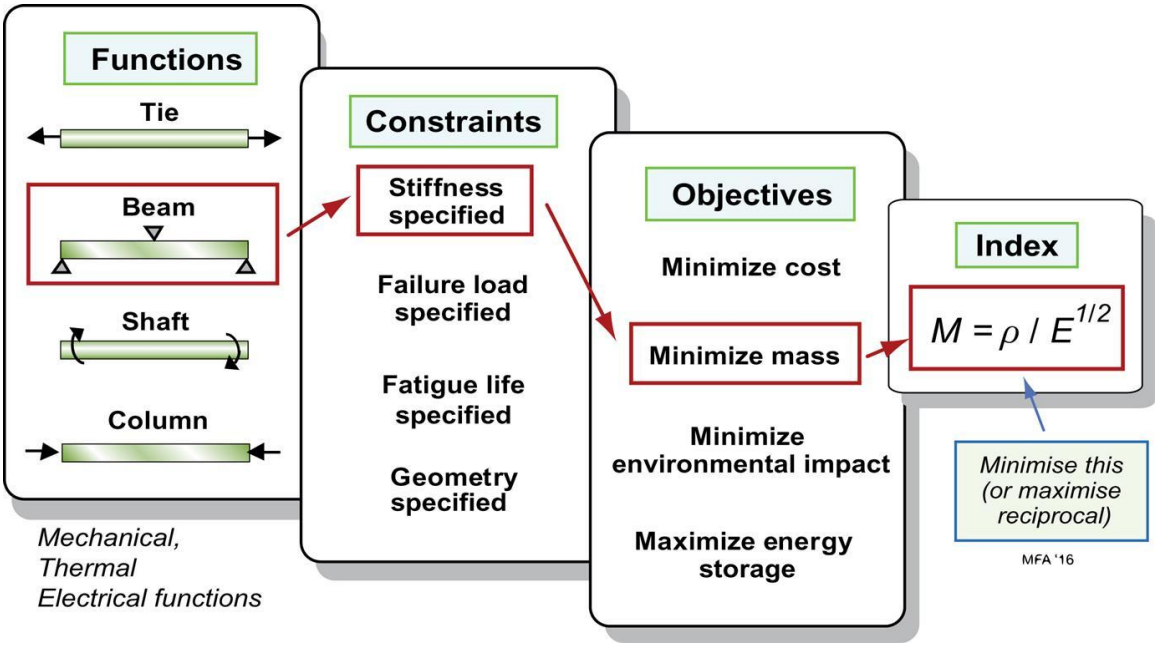


FIGURE 4.9 The specification of function, objective and constraint leads to a materials index. The combination in the highlighted boxes leads to the index $E^{1/2}/\rho$.

4.4 The Selection Procedure

We can now assemble the four steps into a systematic procedure.

1. *Translation and deriving the index*

Table 4.5 lists the steps. In a single sentence, identify the material attributes that are constrained by the design, decide what you will use as a criterion of excellence (to be minimized or maximized), substitute for any free

variables using one of the constraints, and read off the combination of material properties that optimize the criterion of excellence.

2. *Screening: applying attribute limits*

Any design imposes certain nonnegotiable demands ('constraints') on the material of which it is made. We have explained how these are translated into attribute limits. Attribute limits plot as horizontal or vertical lines on material selection charts, illustrated in Fig. 4.10. It shows a schematic E–ρ chart, in the manner of Chapter 3, Materials Property Charts. We suppose that the design imposes limits of E>10 GPa and $\rho < 3000\ \text{kg/m}^3$, shown on the figure. The optimizing search is restricted to the window boxed by the limits, labelled 'search region'.

Less quantifiable properties such as corrosion resistance, wear resistance or formability can all appear as limits, which take the form

$$A > A^*$$

or

$$A < A^* \qquad \textbf{(4.22)}$$

where A is an attribute (service temperature, for instance) and A^* is a critical value of that attribute, set by the design.

One should not be too hasty in applying attribute limits; it may be possible to engineer a route around them. A component that gets too hot can be cooled; one that corrodes can be coated with a protective film. Many designers apply 'precautionary' attribute limits for fracture toughness, K_{1c} and ductility ε_f insisting on materials with $K_{1c} > 15\ \text{MPa m}^{1/2}$ and $\varepsilon_f > 2\%$ in order to guarantee adequate tolerance to stress concentrations. There is some rationale in this, but applying them eliminates materials that the more innovative designer might use to good purpose. (The limits just cited for K_{1c} and ε_f eliminate all polymers and all ceramics, a rash step too early in the design.) At this stage, keep as many options open as possible.

3. *Ranking: indices on charts*

The next step is to seek, from the subset of materials that meet the property limits, those that maximize performance. We will use the design of light, stiff components as an example; the other material indices are used in a similar way.

Fig. 4.11 shows the E–ρ schematic again. Recall that the scales are logarithmic. This allows all three material indices E/ρ, $E^{1/2}/\rho$ and $E^{1/3}/\rho$ to be plotted onto the figure. The condition

$$\frac{E}{\rho} = C$$

becomes, on taking logs,

$$\text{Log}(E) = \text{Log}(\rho) + \text{Log}(C) \tag{4.23}$$

This equations describes a family of straight parallel lines of slope 1 on a plot of $\text{Log}(E)$ against $\text{Log}(\rho)$, with each line corresponding to a value of the constant C. The condition

$$\frac{E^{1/2}}{\rho} = C \tag{4.24}$$

when logged, becomes

$$\text{Log}(E) = 2\,\text{Log}(\rho) + 2\,\text{Log}(C) \tag{4.25}$$

This describes another set of parallel lines, this time with a slope of 2. The condition

$$\frac{E^{1/3}}{\rho} = C \tag{4.26}$$

gives yet another set, with slope 3. We shall refer to these lines as *selection guidelines*. They give the slope of the family of parallel lines belonging to that index. Where appropriate the charts of Chapter 3, Materials Property Charts show guidelines like these.

It is now easy to read off the subset of materials that optimally maximize performance for each loading geometry. All the materials that lie on a line of constant $E^{1/3}/\rho$ perform equally well as a light, stiff panel; those above the line are better, those below, worse. Fig. 4.12 shows a grid of lines corresponding to values of $E^{1/3}/\rho$ from 0.2 to 5 in units of $\text{GPa}^{1/3}/(\text{Mg m}^{-3})$. A material with $M=2$ in these units gives a panel that has one tenth the weight of one with $M=0.2$ that has the same stiffness. The subset of materials with particularly good values of the index is identified by picking a line that isolates a search area containing a reasonably small number of candidates, as shown schematically in Fig. 4.13 as a diagonal selection line. Attribute limits can be added, narrowing the search window. The one corresponding to $E>50$ GPa is shown as a horizontal line. The materials lying in the search region meet both criteria. Their number is expanded or contracted by moving the index line down or up.

4. *Documentation*

We now have a ranked short-list of potential candidate materials. The last step is to explore their character in depth. The list of constraints usually

contains some that cannot be expressed as simple attribute limits. Many of these relate to past experience with the material or to the behaviour of the material in a given environment or to the ways in which it can be shaped, joined or finished. Such information can be found in handbooks, manufacturer's data-sheets and computer-based sources.
And there will be constraints that, at this point, have been overlooked simply because they were not seen as such. Confidence is built by seeking case studies or failure analyses that document each candidate, building a dossier of its strengths and weaknesses, and ways in which these can be overcome. All of these come under the heading of documentation.

Table 4.5
Translation

Step	Action
1	*Define the design requirements:* a. *Function:* what does the component do? b. *Constraints:* essential requirements that must be met: stiffness, strength, corrosion resistance, forming characteristics…. c. *Objective:* what is to be maximized or minimized? d. *Free variables:* what are the unconstrained variables of the problem? *(Present the results as a table in the format of* Table 4.2–4.4*)*
2	*List the constraints* (no yield; no fracture; no buckling, etc.) and develop an equation for them if necessary.
3	*Develop an equation for the objective* in terms of the functional requirements, the geometry and the material properties (the *objective function*).
4	*Identify the free (unspecified) variables.*
5	*Substitute for the free variables* from the constraint equations into the objective function.
6	*Group the variables* into three groups: functional requirements, F, geometry, G, and material properties, M, thus Performance metric $P \leq f_1(F) \cdot f_2(G) \cdot f_3(M)$ or Performance metric $P \geq f_1(F) \cdot f_2(G) \cdot f_3(M)$
7	*Read off the material index*, expressed as a quantity M, that optimizes the performance metric P. M is the criterion of excellence.

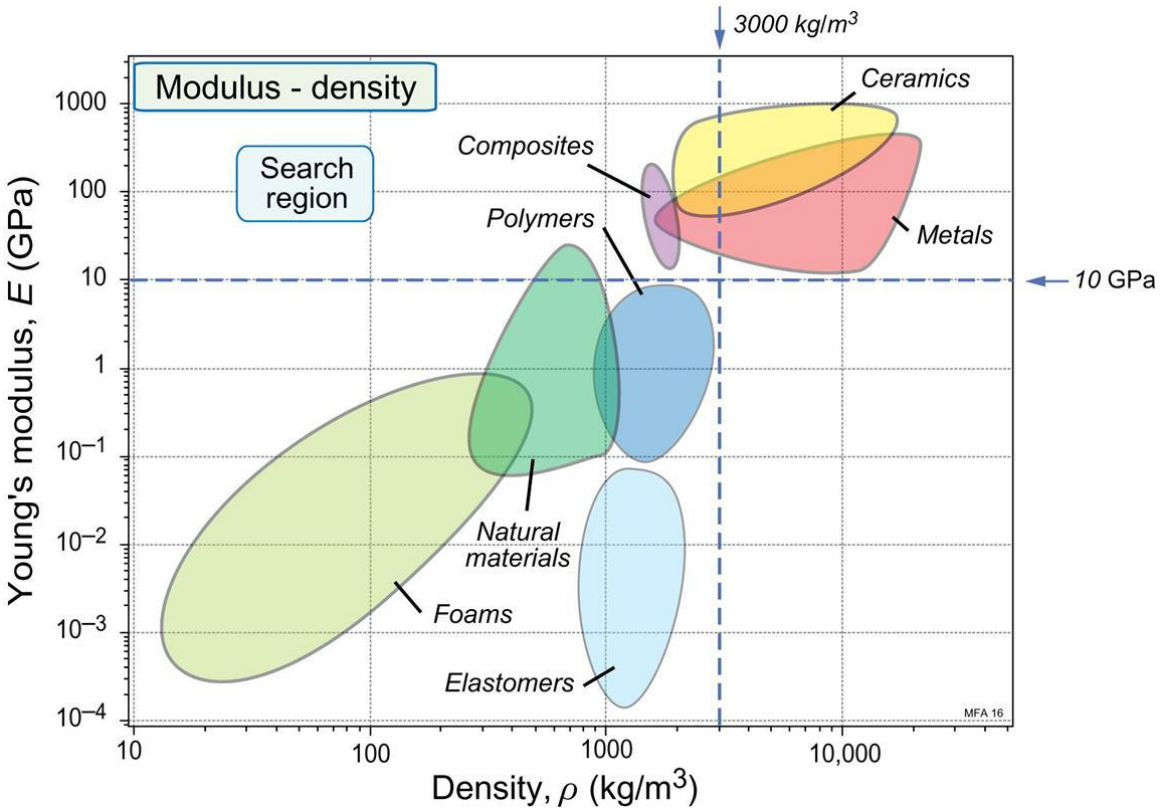


FIGURE 4.10 A schematic $E - \rho$ chart showing a lower limit for E and an upper limit for ρ.

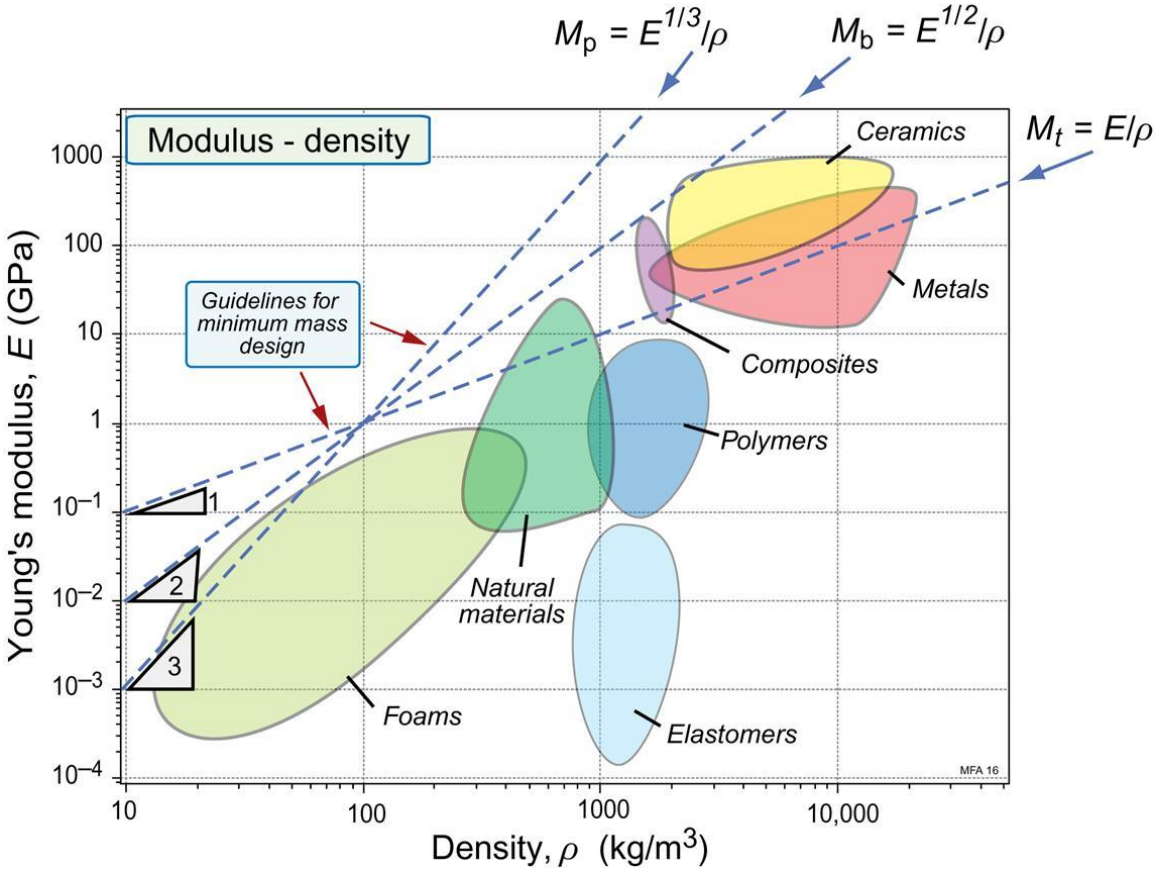


FIGURE 4.11 A schematic $E - \rho$ chart showing guide lines for the three material indices for stiff, lightweight design.

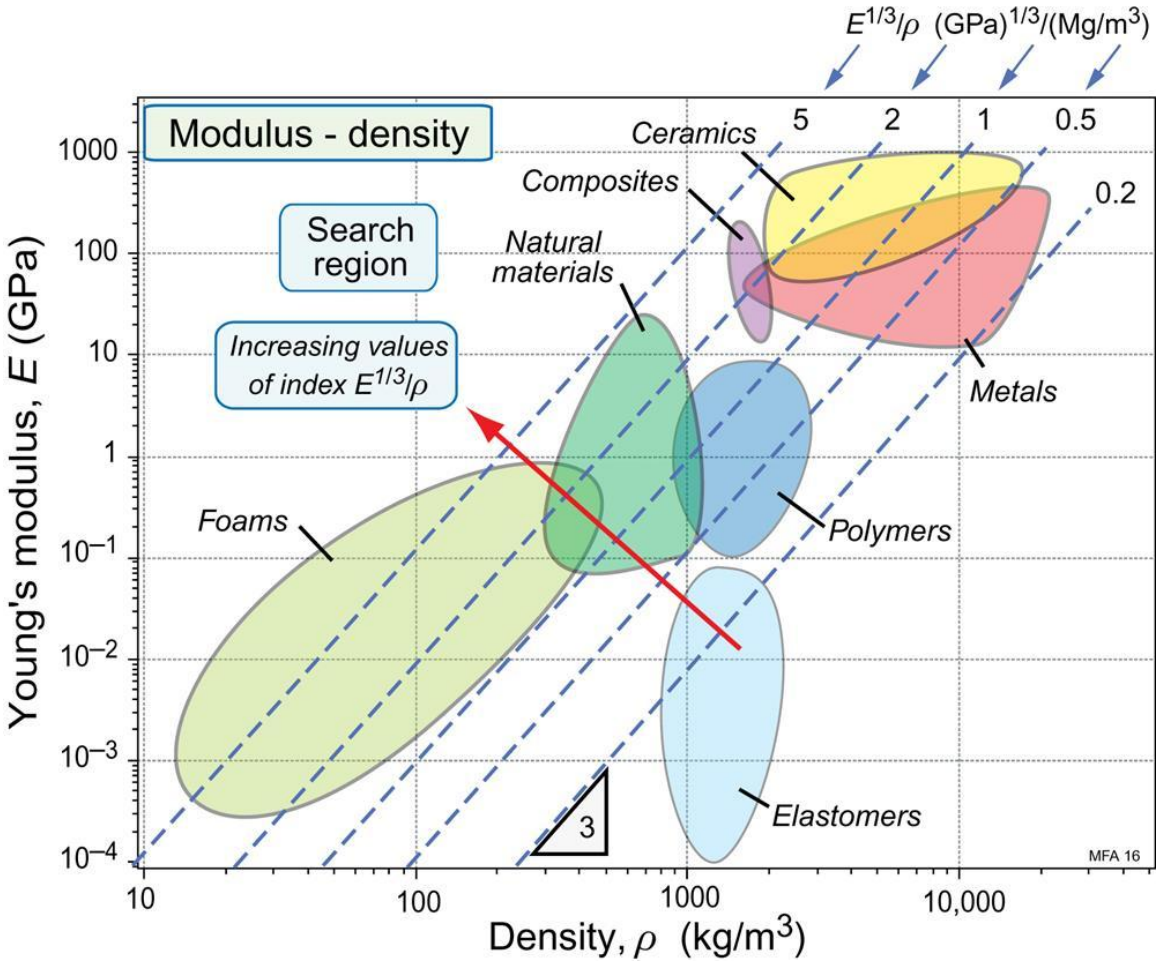


FIGURE 4.12 A schematic $E - \rho$ chart showing a grid of lines for the material index $M = E^{1/3}/\rho$. The units are $(\text{GPa})^{1/3}$ (Mg/m^3).

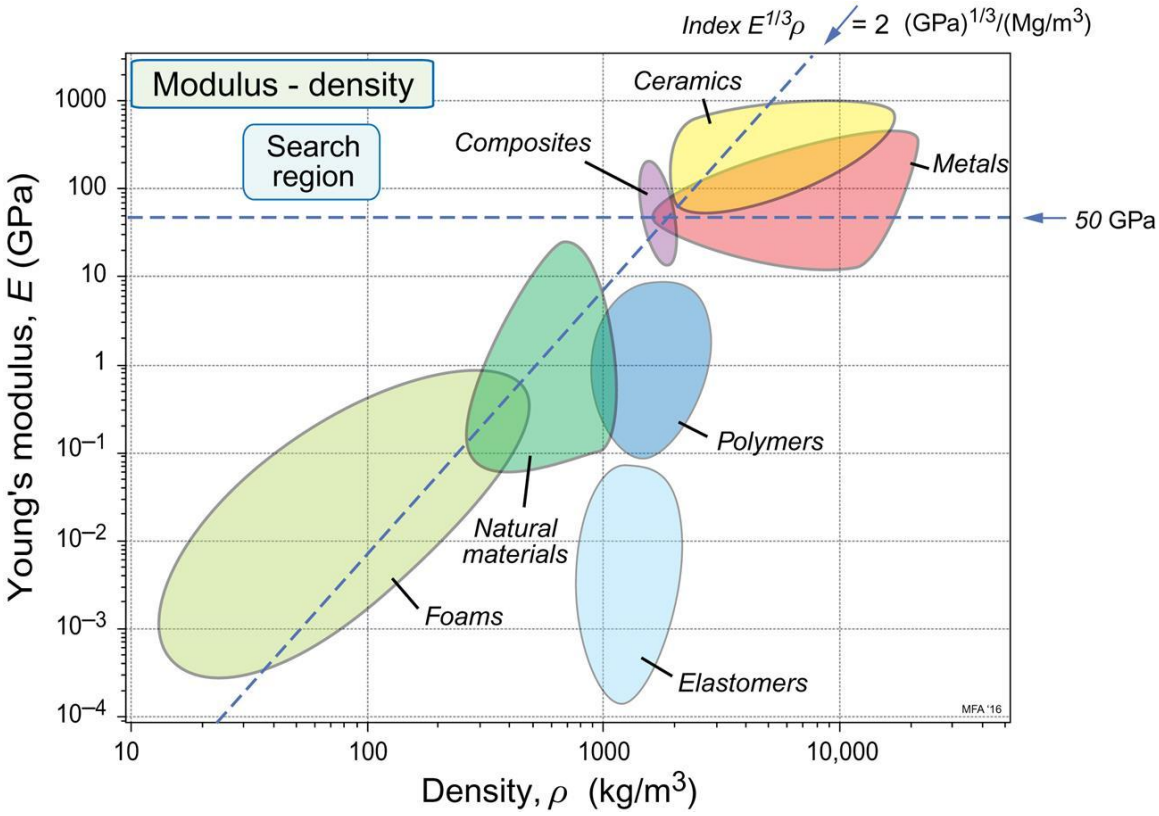


FIGURE 4.13 A selection based on the index $M = E^{1/3}/\rho > 2$ (GPa)$^{1/3}$ (kg/m^3) together with the property limit E>50 GPa. The materials contained in the search region become the candidates for the next stage of the selection process.

4.5 Computer-Aided Selection

The charts of Chapter 3, Materials Property Charts give an overview of material properties but the number of materials that can be shown on any one of them is obviously limited. Selection using them is practical when there are very few constraints. When there are many – as there usually are – it becomes cumbersome. Both problems are overcome by computer implementation of the methods.

The *CES* material and process selection software[4] is an example of such an implementation. A database contains records for materials, organised in the hierarchical manner shown in Fig. 4.2. Each record contains structured attribute data for a material, each stored as a range spanning its typical (or, often, the permitted) range. It also contains limited documentation in the form of text, images, and references to sources of

information about the material. The data are interrogated by a search engine that offers search interfaces shown schematically in Fig. 4.14. On the left is a simple query interface for screening on single properties (the cover-image of Chapter 3, Materials Property Charts is an example). The desired upper or lower limits for constrained attributes are entered, and the search engine rejects all materials with attributes that lie outside the limits. In the centre is shown a second way of interrogating the data: a bar chart like that shown earlier as Figure 3.1. It, and the bubble chart shown on the right, are ways both of applying constraints and of ranking. Used for ranking, a selection line or box is super-imposed on the charts with edges that lie at the constrained values of the attributes, eliminating the material in the shaded areas, and leaving the materials that meet all the constraints. If instead, ranking is sought (having already applied all necessary constraints) the line or box is positioned so that a few – say, three – materials are left in the selected area; these are the top-ranked candidates.

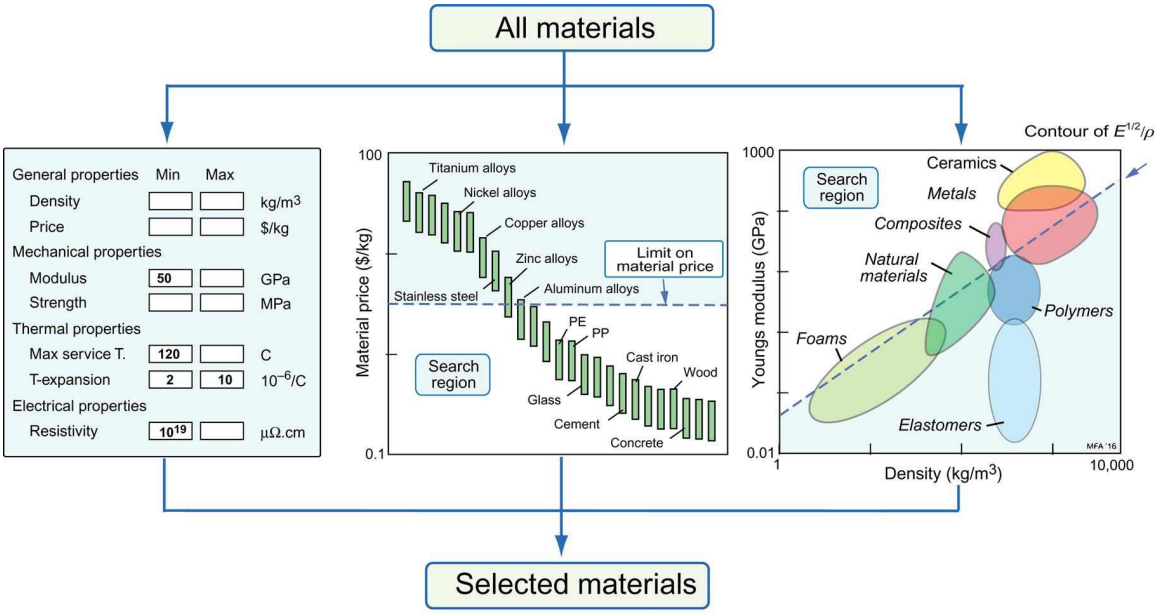


FIGURE 4.14 Computer-aided selection using the CES software. The schematic shows the three types of selection window. They can be used in any order and any combination. The selection engine isolates the subset of material that pass all the selection stages.

4.6 The Structural Index

Books on optimal design of structures (e.g., Shanley, 1960, see Further Reading) make the point that the efficiency of material usage in mechanically loaded components depends on the product of three factors: the material index, as defined here; a factor describing section shape, the subject of our Chapter 10, Selection of Material and Shape; and a structural index,[5] which contains elements of the G and F of Eq. (4.21). The subjects of this book – material and process selection – focuses on the material index and on shape; but we should examine the structural index briefly, partly to make the connection with the classical theory of optimal design, and partly because it becomes useful (even to us) when structures are scaled in size.

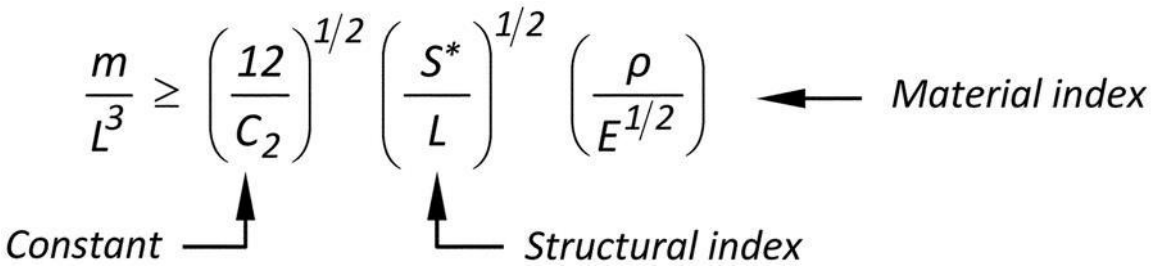
In design for minimum mass (Eqs 4.3, 4.8 and 4.14), a measure of the efficiency of the design is given by the quantity m/L^3. Eq. (4.3) for the light strong tie, when divided by L^3, becomes

$$\frac{m}{L^3} \geq \left(\frac{F^*}{L^2}\right)\left(\frac{\rho}{\sigma_f}\right) \quad \text{Material index}, \quad \text{Structural index} \tag{4.27}$$

Eq. (4.8) for the light stiff panel becomes

$$\frac{m}{L^3} \geq \left(\frac{12}{C_1}\right)^{1/3}\left(\frac{b^2 S^*}{L^3}\right)^{1/3}\left(\frac{\rho}{E^{1/3}}\right) \quad \text{Material index}, \quad \text{Constant}, \quad \text{Structural index} \tag{4.28}$$

and Eq. (4.14) for the light stiff beam becomes



(4.29)

This m/L^3 has the dimensions of density; the lower this pseudo-density the lighter is the structure for a given scale, and thus the greater is the structural efficiency. The first bracketed term on the right of Eqs (4.28) and (4.29) is a constant. The last bracketed term in all three equations is the material index. The remaining term, F^*/L^2 in Eqs (4.27), S^*b^2/L^3 in (4.28) and S^*/L in (4.29), is called the structural index. It has the dimensions of stress; it is a measure of the intensity of loading. Design proportions that are optimal, minimizing material usage, are optimal for structures of any size provided they all have the same structural index. The performance equations are written here in a way that isolates the structural index, a convention we shall follow in the case studies of Chapter 5, Materials Selection – Case Studies.

The structural index for a component of minimum cost is the same as that for one of minimum mass.

4.7 Summary and Conclusions

Material selection is tackled in four steps.

- Translation – reinterpreting the design requirements in terms of function, constraints, objectives and free variables.
- Screening – deriving attribute limits from the constraints and applying these to isolate a subset of viable materials.
- Ranking – ordering the viable candidates by the value of a material index, the criterion of excellence that maximizes or minimizes some measure of performance.
- Documentation – seeking further information about the top-ranked candidates, exploring aspects of their past history, their established uses, their behaviour in relevant environments, their availability and more until a sufficiently detailed picture is built up that a final choice can be made.

Hard-copy material charts allow a first go at the task and have the merit of maintaining breadth of vision: all material classes are in the frame, so to speak. But the number of materials is large, they have many properties and the number of combinations of these appearing in indices is very much larger. It is impractical to print charts to include them all. These problems are overcome by computer implementation, allowing freedom to explore the whole universe of materials and also providing detail when required.

The selection procedure described here is extended in Chapter 8, Multiple Constraints and Conflicting Objectives to deal with multiple constraints and objectives, and in Chapter 10, Selection of Material and Shape to include section shape. Before moving on to these, it is a good idea to consolidate the ideas so far by applying them in a number of Case Studies. They follow in Chapter 5, Materials Selection – Case Studies.

4.8 Further Reading

The books listed below discuss optimisation methods and their application in materials engineering.

1. Dieter GE. *Engineering Design, a Materials and Processing Approach* third ed. New York, NY: McGraw-Hill; 1999; ISBN: 9-780-073-66136-0. *(A well-balanced and respected text focussing on the place of materials and processing in technical design.)*.
2. Shanley FR. *Weight-Strength Analysis of Aircraft Structures* second ed. New York, NY: Dover Publications, Inc; 1960; Library of Congress Number 60-50107. *(A remarkable text, no longer in print, on the design of light-weight structures.)*.

4.9 Exercises

These exercises explore translation and the derivation and plotting of material indices.

Translation is the task of re-expressing design requirements in terms that enable material and process selection. Tackle the exercises by formulating the answers to the questions in this table. Don't try to model the behaviour at this point (that comes in later exercises). Just think out what the component does, and list the constraints that this imposes on material choice, including processing requirements.

Function	■ *What does component do?*
Constraints	■ *What essential conditions must be met?*
Objective	■ *What is to be maximized or minimized?*
Free variables	■ *What parameters of the problem is the designer free to change?*

Here it is important to recognize the distinction between constraints and objectives. As the table says, a constraint is an essential condition that must be met, usually expressed as a limit *on a material or process attribute. An objective is a quantity for which an* extremum *(a maximum or minimum) is sought, frequently cost, mass or volume, but there are others, several of which appear in the exercises below. Take the example of a bicycle frame. It must have a certain stiffness and strength. If it is not stiff and strong enough it will not work, but it is never required to have infinite stiffness or strength. Stiffness and strength are therefore constraints that become limits on modulus, elastic limit and shape. If the bicycle is for sprint racing, it should be as light as possible – if you could make it infinitely light, that would be best of all. Minimizing mass, here, is the objective, perhaps with an upper limit (a constraint) on cost. If instead it is a shopping bike to be sold through supermarkets it should be as cheap as possible – the cheaper it is, the more will be sold. This time minimizing cost is the objective, possible with an upper limit (a constraint) on mass. For most bikes, of course, minimizing mass and cost are both objectives, and then trade-off methods are needed. They come later. For now use judgement to choose the single most important objective and make all others into constraints.*

Two rules-of-thumb, useful in many 'translation' exercises. Many applications require sufficient fracture toughness for the component can survive mishandling and accidental impact during service; a totally brittle material (like un-toughened glass) is unsuitable. Then a necessary constraint is that of 'adequate toughness'. This is achieved by requiring that the fracture toughness $K_{1c} > 15\ \text{MPa m}^{1/2}$. *Other applications require some ductility, sufficient to allow stress redistribution under loading points, and some ability to bend or shape the material plastically. This is achieved by requiring that the (tensile) ductility* $\varepsilon_f > 2\%$.

(If the CES software is available it can be used to impose the constraints and to rank the survivors using the objective.)

E4.1. What is meant by an objective and what by a constraint in the requirements for a design? How do they differ? How are they used?

E4.2. What is meant by a free variable? Give examples of free variables.

E4.3. You are asked to design a cooking pan for camping. What constraints would you apply in selecting a material for the pan? What objectives

would you use to rank the materials that meet the constraints?



FIGURE E4.3 Credit the image to Vango.

E4.4. Bikes come in many forms, each aimed at a particular sector of the market:

- Sprint bikes
- Touring bikes
- Mountain bikes
- Shopping bikes
- Children's bikes
- Folding bikes

Use your judgement to identify the primary constraints and objective that you would apply in selecting a material for the frame for the differing bike types.

E4.5. A material is required for the windings of an electric heater for a sauna. The heating coils must be capable of temperatures up to 800°C. Think out what attributes a material must have if it is to be made into windings and function properly when exposed to air. List the function and the constraints; set the objective to 'minimize material price' and the free variables to 'choice of material'.



FIGURE E4.5 From: http://www.ebay.co.uk/itm/Electric-Sauna-Heater-Including-Rocks-PREMIUM-Quality-FREE-Gift-/261549166435

E4.6. A material is required to manufacture office scissors. Paper is an abrasive material, and scissors sometimes encounter hard obstacles like staples. List function and constraints, set the objective to 'minimize material price' and the free variables to 'choice of material'.



FIGURE E4.6

E4.7. A material is required for a heat exchanger for cooling a small marine engine, saline, water at 120°C (and thus under pressure). List function

and constraints, set the objective to ‘minimize material price’ and the free variables to ‘choice of material’.



FIGURE E4.7 From: https://en.wikipedia.org/wiki/Heat_exchanger#/media/File:Tubular_heat_exchanger.png

E4.8. A material is required for a disposable fork for a fast food chain that is conscious of its environmental image. List the objective and the constraints that you would see as important in this application.



FIGURE E4.8

E4.9. Formulate the constraints and objective you would associate with the choice of material to make the forks of a racing bicycle.

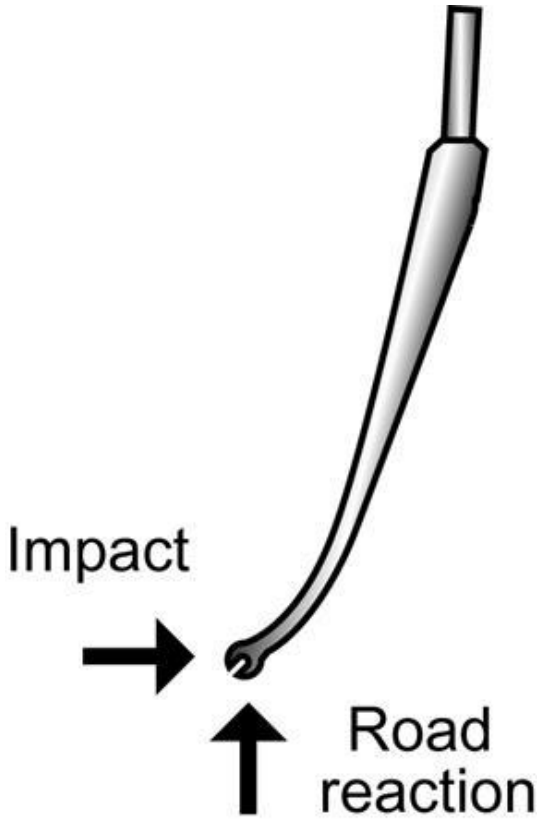


FIGURE E4.9

E4.10. The standard CD ('Jewel' case) cracks easily and, if broken, can scratch the CD. Jewel cases are made of injection moulded polystyrene, chosen because it is transparent, cheap and easy to injection mould. A material is sought to make CD cases that do not crack so easily. The case must still be transparent, able to be injection moulded, and able to compete with polystyrene in cost.

E4.11. Ultraprecise bearings that allow a rocking motion make use of knife-edges or pivots. As the bearing rocks, it rolls, translating sideways by a distance that depends on the radius of contact. The further it rolls, the less precise is its positioning, so the smaller the radius of contact R the better. But the smaller the radius of contact, the greater is the contact pressure (F/A). If this exceeds the hardness H of either face of the bearing, it will be damaged. Elastic deformation is bad too: it flattens the contact, increasing the contact area and the roll. Translate the requirements for the bearing, listing function, constraints, objective and free variable.

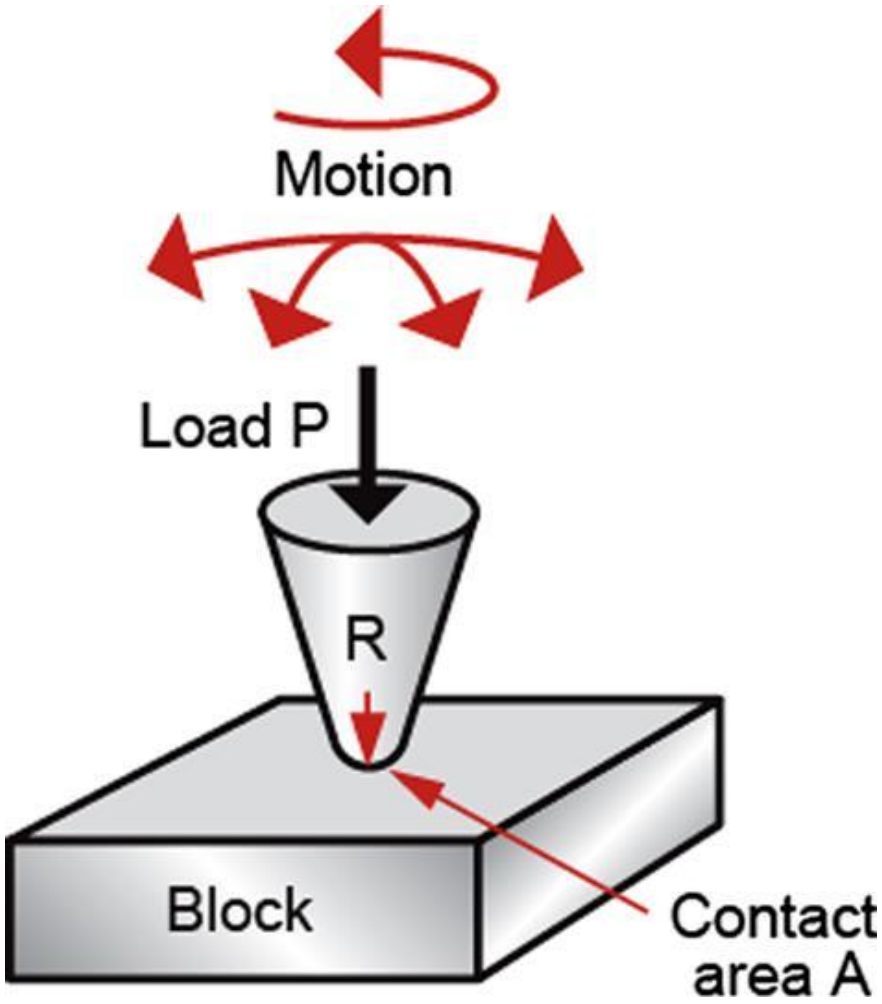


FIGURE E4.11

Deriving and using material indices. The exercises in this section give practice in deriving indices.
Start each by listing function, constraints, objectives and free variables; without having those straight, you will get in a mess. Then write down an equation for the objective. Consider whether it contains a free variable other than material choice; if it does, identify the constraint that limits it, substitute, and read off the material index.
If the CES Edu software is available, use it to apply the constraints and rank the survivors using the index (start with the Level 2 database). Are the results sensible? If not, what constraint has been overlooked or incorrectly formulated?

E4.12. *Material indices for elastic beams with differing constraints*. Start each of the four parts of this problem by listing the function, the objective and the constraints. You will need the equations for the deflection of a cantilever beam with a square cross-section $t \times t$, given in Appendix B,

Section B3. The two that matter are that for the deflection δ of a beam of length L under an end load F:

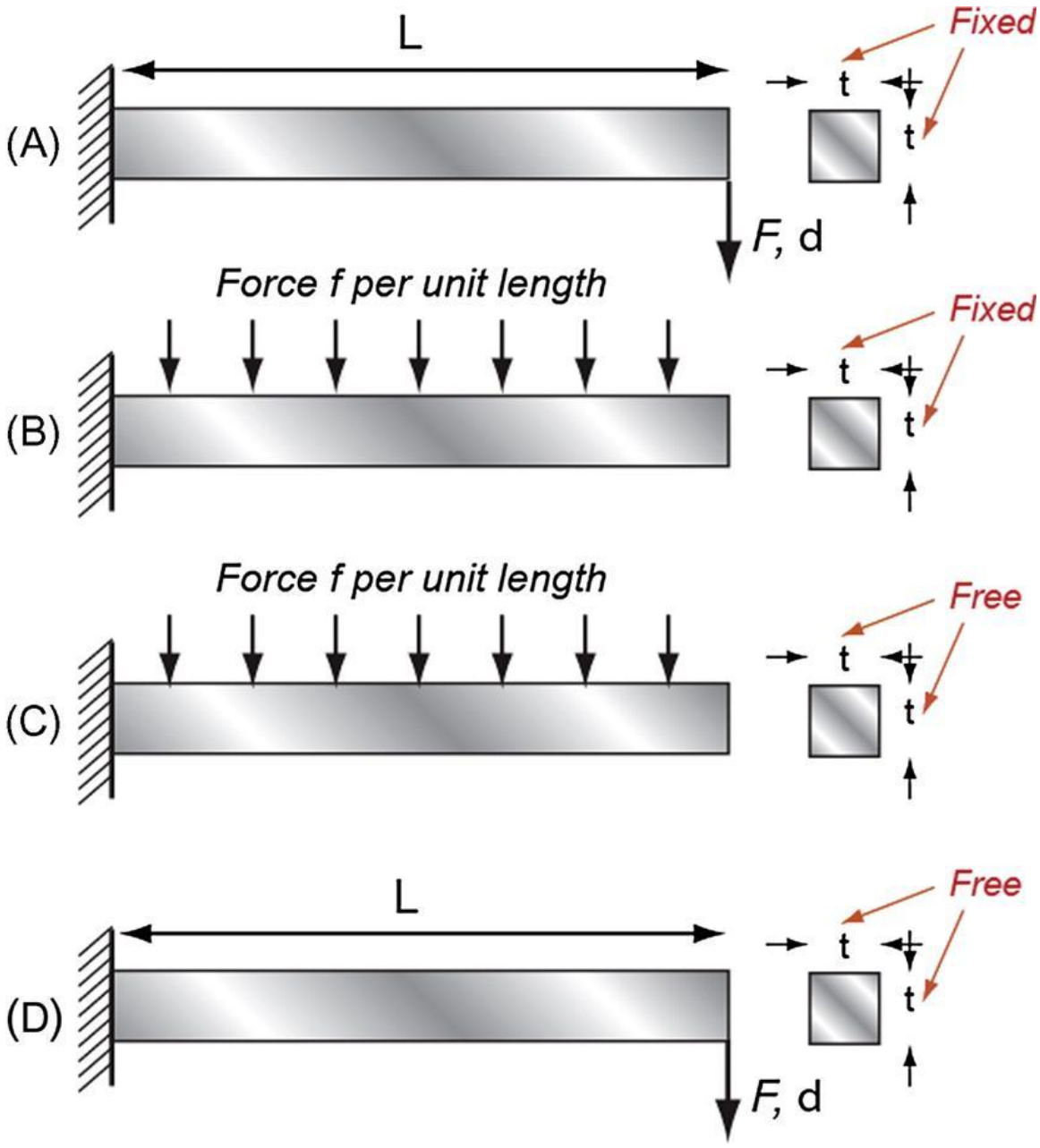


FIGURE E4.12

$$\delta = \frac{F\,L^3}{3EI}$$

and that for the deflection of a beam under a distributed load f per unit length:

$$\delta = \frac{1}{8}\frac{fL^4}{EI}$$

where $I = t^4/12$. For a self-loaded beam $f = \rho A g$ where ρ is the density of the material of the beam, A its cross-sectional area and g the acceleration due to gravity.

a. Show that the best material for a cantilever beam of given length L and given (i.e., fixed) square cross-section ($t \times t$), that will deflect least under a given end load F, is that with the largest value of the index $M=E$, where E is Young's modulus (neglect self-weight) Fig. E4.12A.
b. Show that the best material choice for a cantilever beam of given length L and with a given section ($t \times t$) that will deflect least under its own self-weight is that with the largest value of $M=E/\rho$, where ρ is the density Fig. E4.12B.
c. Show that the material index for the lightest cantilever beam of length L and square section (not given, i.e., the area is a free variable) that will not deflect by more than δ under its own weight is $M = E/\rho^2$ Fig. E4.12C.
d. Show that the lightest cantilever beam of length L and square section (area free) that will not deflect by more than δ under an end load F is that made of the material with the largest value of $M = E^{1/2}/\rho$ (neglect self-weight) Fig. E4.12D.

Plot each index on a copy of the ($E - \rho$) chart of Figure 3.3 indicating the search area for materials with the highest values of each.

E4.13. *Material index for a light, strong beam*. In stiffness limited applications, it is elastic deflection that is the active constraint: it limits performance. In strength limited applications, deflection is acceptable provided the component does not fail; strength is the active constraint. Derive the material index for selecting materials for a beam of length L, specified strength and minimum weight. For simplicity, assume the beam to have a solid square cross-section $t \times t$. You will need the equation for the failure load of a beam (Appendix B, Section B4). It is

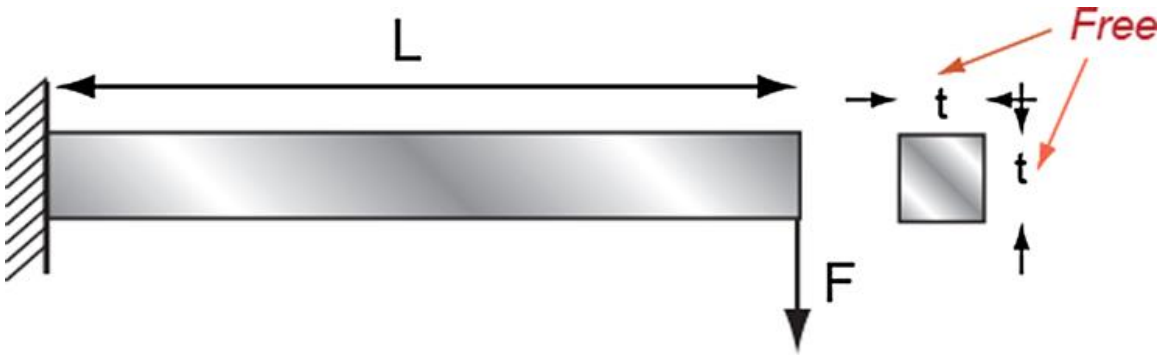


FIGURE E4.13

$$F_f = \frac{I\sigma_f}{y_m L}$$

where y_m is the distance between the neutral axis of the beam and its outer filament and $I = t^4/12 = A^2/12$ is the second moment of the cross-section. The table given in this exercise itemizes the design requirements.
Plot the criterion on a copy of the (σ_f — ρ) chart of Figure 3.4 and use it to identify promising candidate materials. (If you have access to the CES Edu software, make the chart and apply the index to find candidate materials.)

Function	■ Beam
Constraints	■ Length L is specified ■ Beam must support a bending load F without yield or fracture
Objective	■ Minimize the mass of the beam
Free variables	■ Cross-section area, A ■ Choice of material

E4.14. *Material index for a cheap, stiff column*. In the last two exercises the objective has been that of minimizing weight. There are many others. In the selection of a material for a spring, the objective is that of maximizing the elastic energy it can store. In seeking materials for thermal-efficient insulation for a furnace, the best are those with the lowest thermal conductivity and heat capacity. However, most common of all is the wish to minimize cost. So here is an example involving cost.

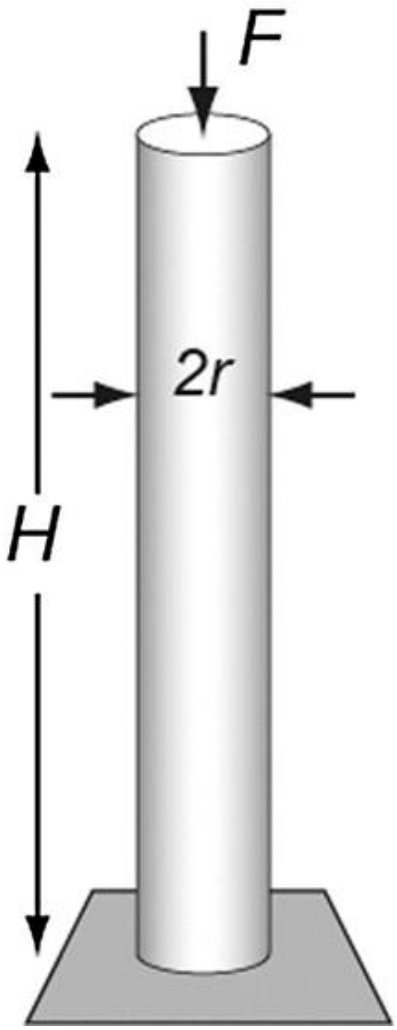


FIGURE E4.14

Columns support compressive loads: the legs of a table; the pillars of the Parthenon. Derive the index for selecting materials for the cheapest cylindrical column of specified height, H, that will safely support a load F without buckling elastically. You will need the equation for the load F_{crit} at which a slender column buckles. It is

$$F_{crit} = \frac{n^2 \pi^2 EI}{H^2}$$

where n is a constant that depends on the end constraints and $I = \pi r^4/4 = A^2/4\pi$ is the second moment of area of the column (see Appendix B for both). The table given here lists the requirements.

Function	■ Cylindrical column
Constraints	■ Length L is specified ■ Column must support a compressive load F without buckling
Objective	■ Minimize the material cost of the column
Free variables	■ Cross-section area, A ■ Choice of material

Plot the index you derive onto the modulus – relative cost chart of Figure 3.26 to find the cheapest candidates.

E4.15. *Indices for stiff plates and shell*. Aircraft and space structures make use of plates and shells. The index depends on the configuration. Here you are asked to derive the material index for

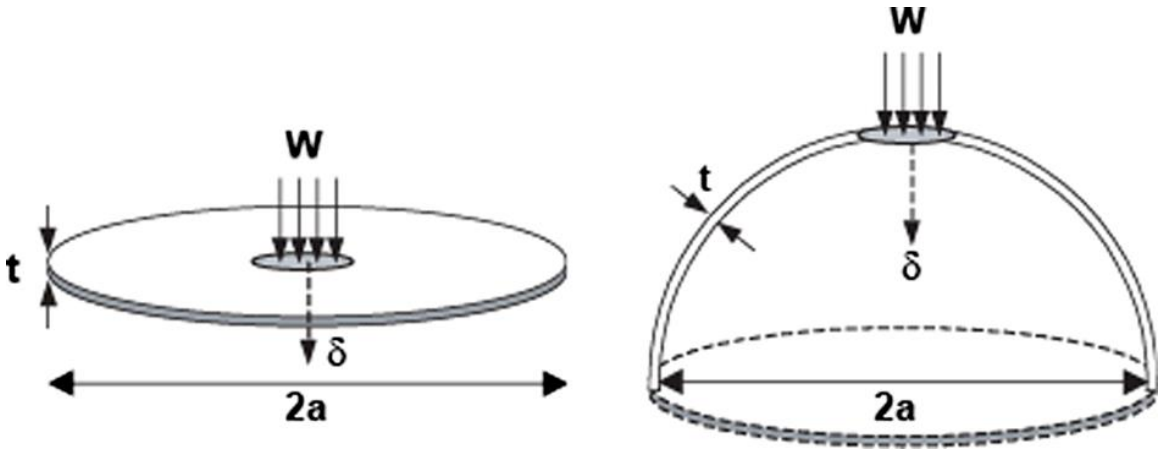


FIGURE E4.15

a. a circular plate of radius a carrying a central load W with a prescribed stiffness $S = W/\delta$ and of minimum mass and
b. a hemispherical shell of radius a carrying a central load W with a prescribed stiffness $S = W/\delta$ and of minimum mass, as shown in the figure.

Use the two results listed below for the midpoint deflection δ of a plate or spherical shell under a load W applied over a small central, circular area.

$$\text{Circular plate: } \delta = \frac{3}{4\pi}\frac{Wa^2}{Et^3}(1-\nu^2)\left(\frac{3+\nu}{1+\nu}\right)$$

$$\text{Hemispherical shell: } \delta = A\frac{Wa}{Et^2}(1-\nu^2)$$

in which $A \approx 0.35$ is a constant. Here E is Young's modulus, t is the thickness of the plate or shell and ν is Poisson's ratio. Poisson's ratio is almost the same for all structural materials and can be treated as a constant. The table included here summarizes the requirements.

Function	■ Stiff circular plate, or ■ Stiff hemispherical shell
Constraints	■ Stiffness S under central load W specified ■ Radius a of plate or shell specified
Objective	■ Minimize the mass of the plate or shell
Free variables	■ Plate or shell thickness, t ■ Choice of material

E4.16. A furnace is required to sinter powder-metal parts. It operates continuously at 650°C while the parts are fed through on a moving belt. You are asked to select a material for furnace insulation to minimize heat loss and thus to make the furnace as energy-efficient as possible. For reasons of space the insulation is limited to a maximum thickness of $x = 0.2$ m.

a. List the function, constraints, objective and free variable.

b. Derive a material index for selecting the material for the insulation.

E4.17. A storage heater captures heat over a period of time, and then releases it, usually to an air stream, when required. Those for domestic heating store solar energy or energy from cheap off-peak electricity and release it slowly during the cold part of the day. What is a good material for the core of a compact storage material capable of temperatures up to 80°C and a defined storage capacity? Interpret 'compact' to mean 'of minimum volume'.

a. List the function, constraints, objective and free variable.

b. Derive a material index for selecting the material for the heat storage material.

E4.18. Rework the storage heater of Exercise 4.17 to minimize the cost of the core instead of its volume.

a. List the function, constraints, objective and free variable.

b. Derive a material index for selecting the material for the heat storage material.

E4.19. A clamp is required for processing of electronic components. The clamp has a square cross-section of width x and given depth b. It is essential that the clamp have low thermal inertia so that it reaches temperature quickly. The time t it takes a component of thickness x to reach thermal equilibrium when the temperature is suddenly changed (a transient heat flow problem) is

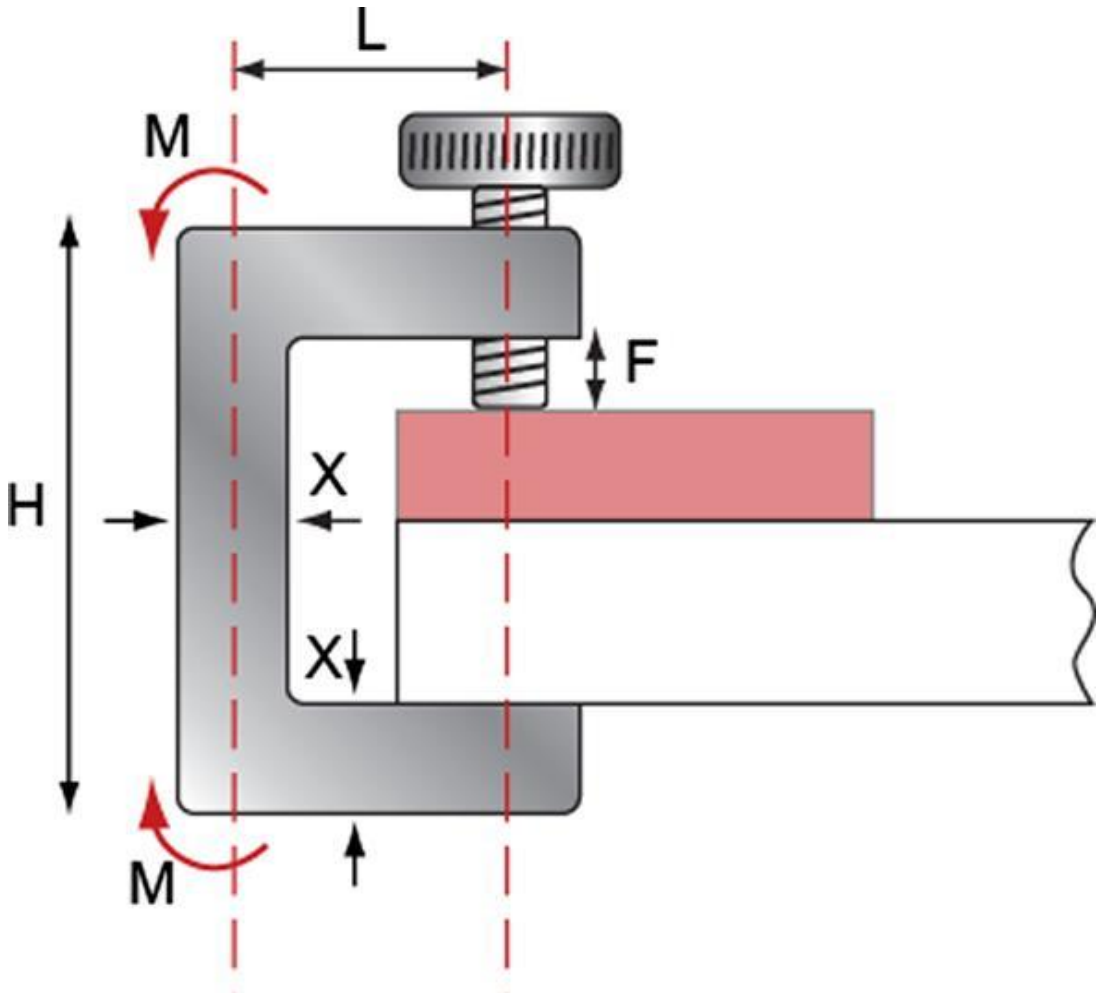


FIGURE E4.19

$$t \approx \frac{x^2}{2a}$$

where the thermal diffusivity $a = \lambda/\rho C_p$ and λ is the thermal conductivity, ρ the density and C_p the specific heat per unit mass.

The time to reach thermal equilibrium is reduced by making the section x thinner, but it must not be so thin that it fails in service. Use this constraint to eliminate x in the equation above, thereby deriving a material index for the clamp. Use the fact that the clamping force F creates a bending moment on the body of the clamp of $M = FL$, and that the peak stress in the body is given by (Appendix B, Section B15):

$$\sigma = \frac{x}{2}\frac{M}{I}$$

where $I = bx^3/12$ is the second moment of area of the body. The table below summarizes the requirements.

Function	■ C-clamp of low thermal inertia
Constraints	■ Depth b specified ■ Must carry clamping load F without failure
Objective	■ Minimize time to reach thermal equilibrium
Free variables	■ Width of clamp body, x ■ Choice of material

E4.20. *Springs for trucks*. In vehicle suspension design it is desirable to minimize the mass of all components. You have been asked to select a material and dimensions for a light spring to replace the steel leaf spring of an existing truck suspension. The existing leaf spring is a beam, shown schematically in the figure. The new spring must have the same length L and stiffness S as the existing one, and must deflect through a maximum safe displacement δ_{max} without failure. The width b and thickness t are free variables.

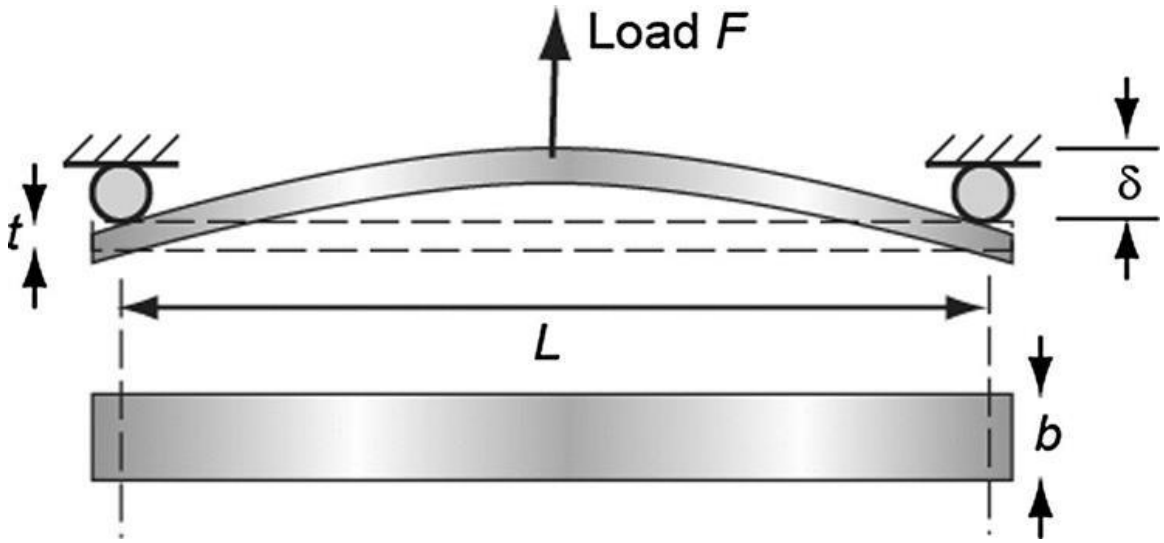


FIGURE E4.20

Derive a material index for the selection of a material for this application. Note that this is a problem with two free variables: b and t, and there are two constraints, one on safe deflection δ_{max} and the other on stiffness S. Use the two constraints to fix the values of the free variables. The table catalogues the requirements.

Function	■ Leaf spring for truck
Constraints	■ Length L specified ■ Stiffness S specified ■ Maximum displacement δ_{max} specified
Objective	■ Minimize the mass
Free variables	■ Spring thickness t ■ Spring width b ■ Choice of material

You will need the equation for the midpoint deflection of an elastic beam of length L

loaded in three-point bending by a central load F:

$$\delta = \frac{1}{48}\frac{FL^3}{EI}$$

and that for the deflection at which failure occurs

$$\delta_{\max} = \frac{1}{6}\frac{\sigma_f L^2}{tE}$$

where I is the second moment of area; for a beam of rectangular section, $I = bt^3/12$ and E and σ_f are the modulus and failure stress of the material of the beam. (See Appendix B.) Use the index you derive to rank the following candidate material for this application using data from Appendix A: High carbon steel, low alloy steel, titanium alloys and Carbon-fiber reinforced polymers (CFRP) (carbon fibre reinforced polymers).

E4.21. *Aperture grills for cathode ray tubes (CRT)*. Long ago in the days of cathode-ray technology there were two types of cathode ray tube. In the older technology, colour separation was achieved by using a *shadow mask*: a thin metal plate with a grid of holes that allow only the correct beam to strike a red, green or blue phosphor. A shadow mask can heat up and distort at high brightness levels ('doming'), causing the beams to miss their target and giving a blotchy image. To avoid this, shadow masks were made of Invar, a nickel alloy with a near-zero expansion coefficient between room temperature and 150°C. A consequence of shadow-mask technology was that the glass screen of the CRT curved inward on all four edges, increasing reflected glare.

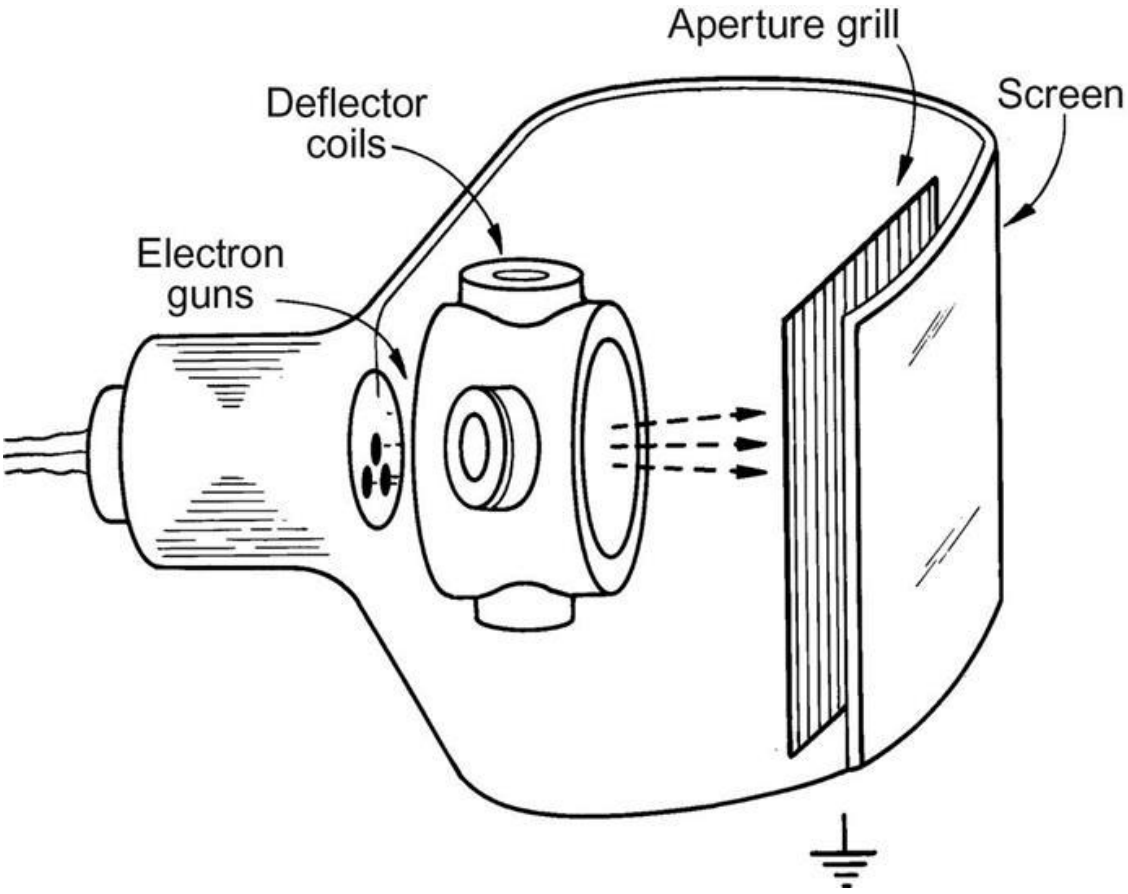


FIGURE E4.21

Sony's 'Trinitron' technology overcame this problem and allowed greater brightness by replacing the shadow mask by an *aperture grill* of fine vertical wires, each about 200 μm thick, that allows the intended beam to strike either the red, the green or the blue phosphor to create the image. The glass face of the Trinitron tube was curved in one plane only, reducing glare.

The wires of the aperture grill were pretensioned so that they remain taut even when hot – this tension allowed the greater brightness. What index guides the choice of material for the wires? The table summarizes the requirements.

Function	■ Aperture grill for CRT
Constraints	■ Wire thickness and spacing specified ■ Material must carry pretension without failure ■ Electrically conducting to prevent charging ■ Able to be drawn to wire
Objective	■ Maximize permitted temperature rise without loss of tension
Free variables	■ Choice of material

Use the index you derive to rank the following candidate material for this application using data from Appendix A: carbon steel, stainless steel, nickel-chromium alloys and tungsten. All are available as fine wire.

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[1]Maximizing performance often means minimizing something: cost is the obvious example; mass, in transport systems, is another. A low-cost or light component, here, improves performance.

[2]In reality a safety factor, S_f, is always included in such a calculation, such that Eq. (4.2) becomes $F/A = \sigma_f / S_f$. If the same safety factor is applied to each material, its value does not influence the choice. We omit it here for simplicity.

[3]In Chapter 10, Selection of Material and Shape we introduce a fourth: that of section shape.

[4]Granta Design Ltd., Cambridge UK (www.grantadesign.com).

[5]Also called the 'structural loading coefficient', the 'strain number' or the 'strain index'.

CHAPTER 5

Materials Selection – Case Studies

Abstract

Chapter 5 is a collection of case studies illustrating the selection methods of Chapter 4. Each is laid out in the following way:

- *The problem statement*, setting the scene,
- *The translation*, identifying function, constraints, objectives and free variables, from which emerge the attribute limits and material indices,
- *The selection*, in which the full menu of materials is reduced by screening and ranking to a short-list of viable candidates, and
- *The postscript*, allowing a commentary on results and philosophy.

Keywords

Oars; mirrors; tables; flywheels; springs; elastic hinges; pressure vessels; thermal walls; precision instruments; heat exchangers; radomes

5.1 Introduction and Synopsis

Here we have a collection of case studies illustrating the selection methods of Chapter 4. They are deliberately simplified to avoid obscuring the method under layers of detail. In most cases little is lost by this: the best choice of material for the simple example is the same as that for the more complex, for the reasons given in Chapter 4, Materials Selection – the Basics, Section 4.3.

Each case study is laid out in the following way:

- *The problem statement*, setting the scene,

- *The translation*, identifying function, constraints, objectives and free variables, from which emerge the attribute limits and material indices,
- *The selection*, in which the full menu of materials is reduced by screening and ranking to a short-list of viable candidates, and
- *The postscript*, allowing a commentary on results and philosophy.

The first few examples are straightforward, chosen to illustrate the method. Later examples are less obvious and require clear thinking to identify and distinguish objectives and constraints. Confusion here can lead to bizarre and misleading conclusions. Always apply common sense: does the selection include the traditional materials used for that application? Are some members of the subset obviously unsuitable? If they are, it is usually because a constraint has been overlooked or an objective misapplied. The answer is to rethink them.

Most of the case studies use the hardcopy charts of Chapter 3, Materials Property Charts; those at the end illustrate computer-based methods.

5.2 Materials for Oars

Credit for inventing the rowed boat seems to belong to the Egyptians. Boats with oars appear in carved relief on monuments built in Egypt between 3300 and 3000 BC. Boats, before steam power, could be propelled by poling, by sail or by oar. Oars gave more control than the other two, the military potential of which was well understood by the Romans, the Vikings and the Venetians.

Records of rowing races on the Thames in London extend back to 1716. Originally the competitors were watermen, rowing the ferries used to carry people and goods across the river. Gradually gentlemen became involved (notably the young gentlemen of Oxford and Cambridge University), sophisticating both the rules and the equipment. The real stimulus for development of boat and oar came in 1900 with the establishment of rowing as an Olympic sport. Since then both have drawn to the full on the craftsmanship and materials of their day. Consider, as an example, the oar.

The translation. Mechanically speaking, an oar is a beam, loaded in bending. It must be strong enough to carry the bending moment exerted by the rower without breaking, and it must have a stiffness to match the rower's own characteristics and give the right 'feel'. Meeting the strength

constraint is easy. Oars are designed on *stiffness*, that is, to give a specified elastic deflection under a given load.

The upper part of Fig. 5.1 shows an oar: a blade or 'spoon' is bonded to a shaft or 'loom' that carries a sleeve and collar to give positive location in the rowlock. The lower part of the figure shows how the oar stiffness is measured: a 10 kg weight is hung on the oar 2.05 m from the collar and the deflection δ at this point is measured. A soft oar will deflect nearly 50 mm; a hard one only 30. A rower, ordering an oar, will specify how hard it should be.

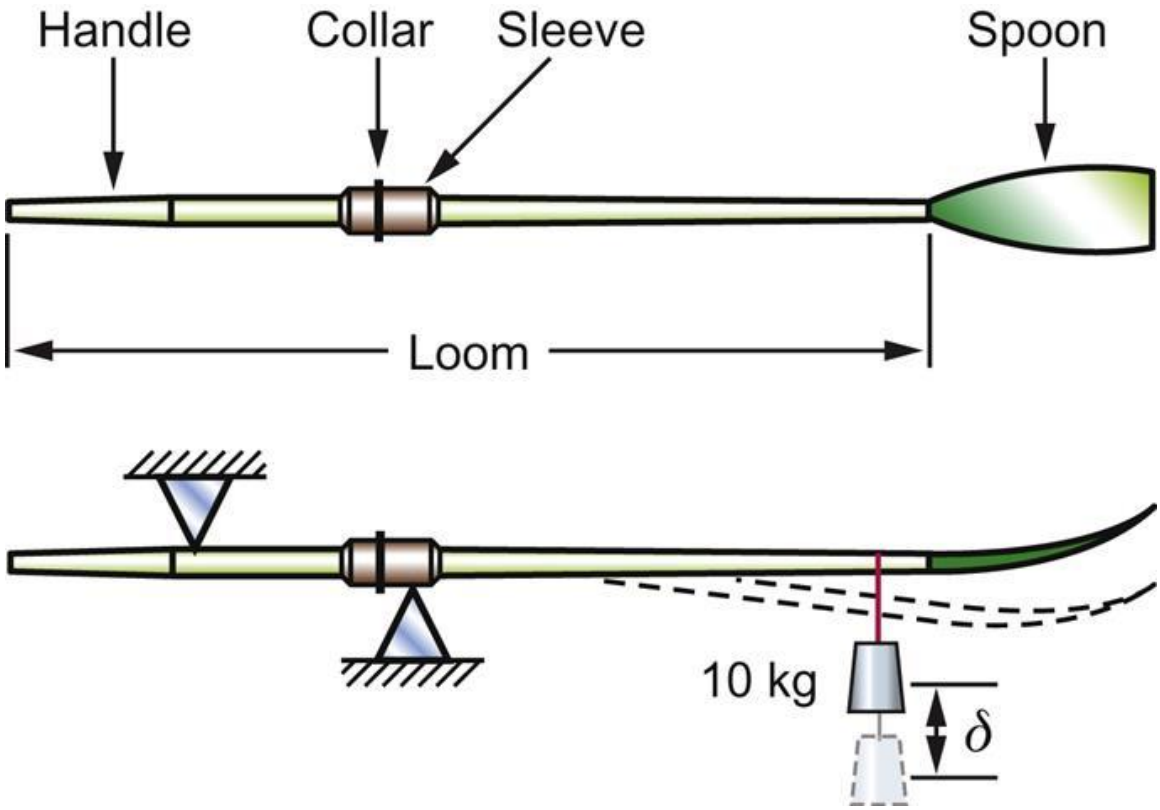


FIGURE 5.1 An oar. Oars are designed on stiffness, measured in the way shown in the lower figure, and they must be light.

The oar must also be light; extra weight increases the wetted area of the hull and the drag that goes with it. So there we have it: an oar is a beam of specified stiffness and minimum weight. The material index we want was derived in Chapter 4, Materials Selection – the Basics as Eq. (4.15). It is that for a light, stiff beam:

$$M = \frac{E^{1/2}}{\rho} \tag{5.1}$$

where E is Young's modulus and ρ is the density. There are other obvious constraints. Oars are dropped, and blades sometimes clash. The material must be tough enough to survive this, so brittle materials (those with a toughness G_{1c} less than 1 kJ/m^2) are unacceptable. Given these requirements, summarized in Table 5.1, what materials would you choose to make oars?

Table 5.1
Design Requirements for the Oar

Function	■ *Oar – meaning light, stiff beam*
Constraints	■ *Length L specified* ■ *Bending stiffness* S^* *specified* ■ *Toughness* $G_{1c} > 1\ \mathrm{kJ/m^2}$
Objective	■ *Minimize the mass* m
Free variables	■ *Shaft diameter* ■ *Choice of material*

The selection. Fig. 5.2 shows the appropriate chart: that in which Young's modulus, E, is plotted against density, ρ. The selection line for the index M has a slope of 2, as explained in Section 4.4, and it is positioned so that a small group of materials is left above it. They are the materials with the largest values of M, and it is these that are the best choice, provided they satisfy the other constraint (a simple attribute limit on toughness). They contain three classes of material: woods, carbon reinforced polymers, and certain ceramics (Table 5.2). Ceramics are brittle; the toughness–modulus chart of Figure 4.7 shows that all fail to meet that required by the design. The recommendation is clear. Make your oars out of wood or –better –out of carbon-fibre reinforced polymers (CFRP).

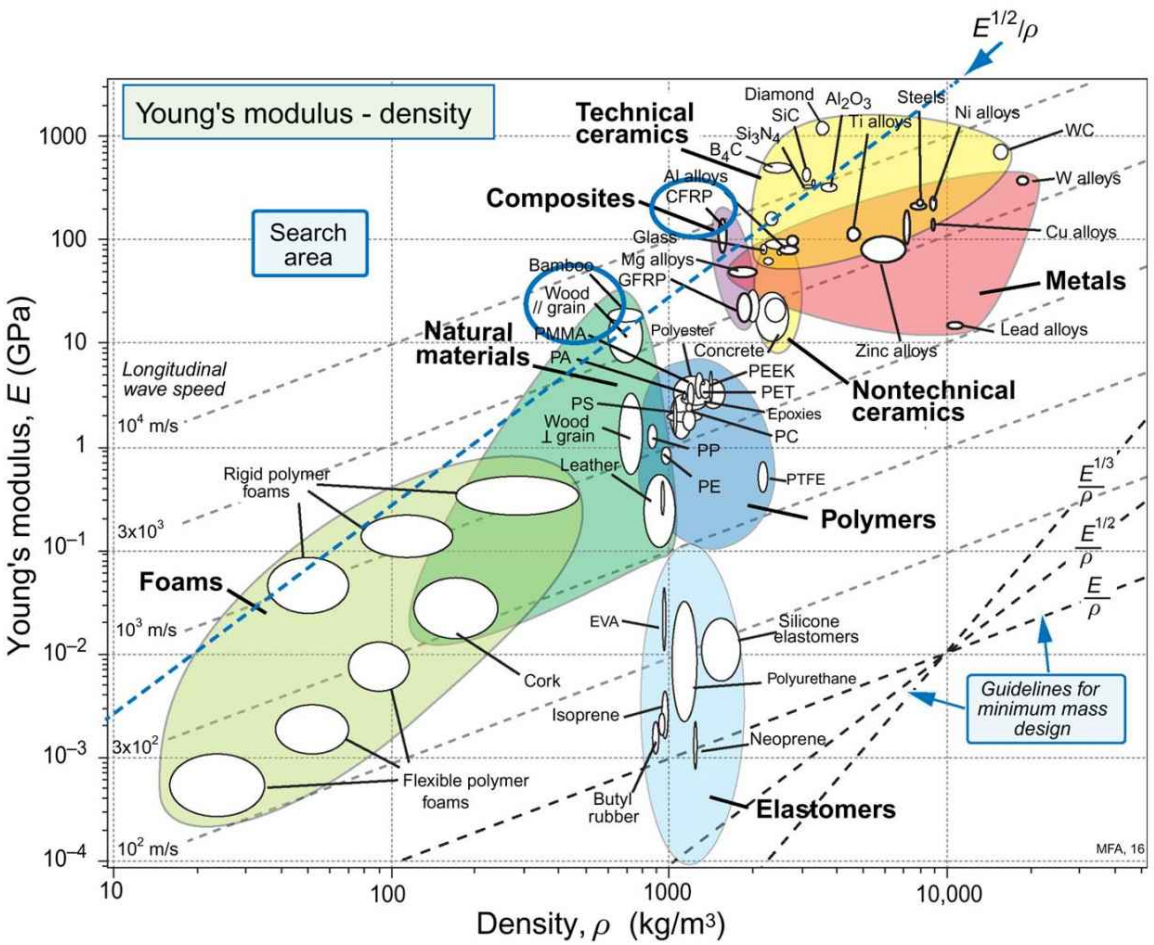


FIGURE 5.2 Materials for oars. CFRP is better than wood because the structure can be controlled.

Table 5.2
Material for Oars

Material	Index M $(GPa)^{1/2}/(Mg/m^3)$	Comment
Bamboo	4.0–4.5	The traditional material for oars for canoes
Woods	3.4–6.3	Cheap, traditional, but with natural variability
CFRP	5.3–7.9	As good as wood, more control of properties
Ceramics	4–8.9	Good M but toughness low and cost high

Postscript. Now we know what oars should be made of. What, in reality, is used? Racing oars and sculls are made either of wood or of a high-

performance composite: carbon-fibre reinforced epoxy.

Wooden oars are made today, as they were 100 years ago, by craftsmen working largely by hand. The shaft and blade are of Sitka spruce from the northern United States or Canada, the further north the better because the short growing season gives a finer grain. The wood is cut into strips, four of which are laminated together to average the stiffness and the blade is glued to the shaft. The rough oar is then shelved for some weeks to settle down, and finished by hand cutting and polishing. When finished, a spruce oar weighs between 4 and 4.3 kg.

Composite blades are a little lighter than wood for the same stiffness. The component parts are fabricated from a mixture of carbon and glass fibres in an epoxy matrix, assembled and glued. The advantage of composites lies partly in the saving of weight (typical weight: 3.9 kg) and partly in the greater control of performance: the shaft is moulded to give the stiffness specified by the purchaser. Until recently a CFRP oar cost more than a wooden one, but the price of carbon fibres has fallen sufficiently that the two now cost about the same.

Further Reading

Redgrave, S., 1992. "Complete Book of Rowing", Partridge Press, London.

Related Case Studies

5.3 'Mirrors for large telescopes'
5.4 'Table legs'
11.2 'Spars for man-powered planes'
11.3 'Forks for a racing bicycle'

5.3 Mirrors for Large Telescopes

There are some very large optical telescopes in the world. The newer ones employ complex and cunning tricks to maintain their precision as they track cross the sky – more on that in the postscript. But if you want a simple telescope, you make the reflector as a single rigid mirror. The largest such telescope is sited on Mount Semivodrike, near Zelenchukskaya, in the Caucasus Mountains of Russia. The mirror is 6 m (236 inches) in diameter. To be sufficiently rigid, the mirror is made of glass about 1 m thick and weighs 70 tonnes.

The total cost of a large (236-inch) telescope is, like the telescope itself, astronomical – about US$300 million. The mirror itself accounts for only

about 5% of this cost; the rest is that of the mechanism that holds, positions and moves it as it tracks across the sky. This mechanism must be stiff enough to position the mirror relative to the collecting system with a precision about equal to that of the wavelength of visible light. It might seem, at first sight, that doubling the mass m of the mirror would require that the sections of the support structure be doubled too, so as to keep the stresses (and hence the strains and displacements) the same; but the heavier structure then deflects under its own weight. In practice, the sections have to increase as m^2, and so does the cost.

A century ago, mirrors were made of speculum metal (density: about 8000 kg/m^3). Since then, they have been made of glass (density: 2300 kg/m^3), silvered on the front surface, so none of the optical properties of the glass are used. Glass is chosen for its mechanical properties only; the 70 tonnes of glass is just a very elaborate support for 100 nm (about 30 grams) of silver. Could one, by taking a radically new look at materials for mirrors, suggest possible routes to the construction of lighter, cheaper telescopes?

The translation. At its simplest, the mirror is a circular disk of diameter $2R$ and mean thickness t, simply supported at its periphery (Fig. 5.3). When horizontal, it will deflect under its own weight m; when vertical it will not deflect significantly. This distortion (which changes the focal length and introduces aberrations) must be small enough that it does not interfere with performance; in practice, this means that the deflection δ of the midpoint of the mirror must be less than the wavelength of light. Additional requirements are high dimensional stability (no creep), and low thermal expansion (Table 5.3).

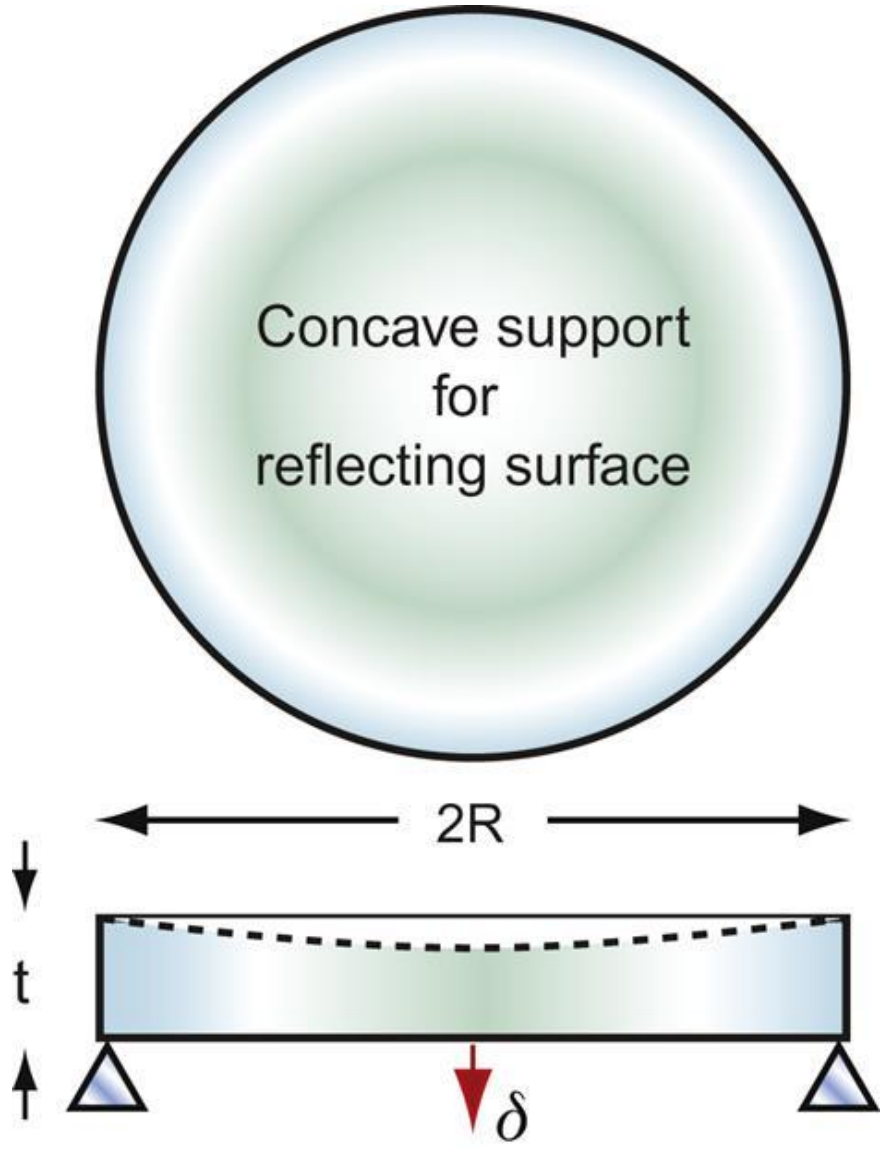


FIGURE 5.3 The mirror of a large optical telescope is modelled as a disk, simply supported at its periphery.

Table 5.3

Design Requirements for the Telescope Mirror

Function	■ *Precision mirror*
Constraints	■ *Radius R specified* ■ *Must not distort more than* δ *under self-weight* ■ *High dimensional stability: no creep, low thermal expansion*
Objective	■ *Minimize the mass, m*
Free variables	■ *Thickness of mirror, t* ■ *Choice of material*

The mass of the mirror (the property we wish to minimize) is

$$m = \pi R^2 t \rho \tag{5.2}$$

where ρ is the density of the material of the disk. The elastic deflection, δ, of the centre of a horizontal disk due to its own weight is given, for a material with Poisson's ratio of 0.3 (Appendix B: 'Useful Solutions'), by

$$\delta = \frac{3}{4\pi}\frac{mgR^2}{Et^3} \tag{5.3}$$

The quantity g in this equation is the acceleration due to gravity: 9.81 m/s^2; E, as before, is Young's modulus. We require that this deflection be less than (say) 10 μm. The diameter $2R$ of the disk is specified by the telescope design, but the thickness t is a free variable. Solving for t and substituting this into the first equation gives

$$m = \left(\frac{3g}{4\delta}\right)^{1/2} \pi R^4 \left[\frac{\rho}{E^{1/3}}\right]^{3/2} \tag{5.4}$$

The lightest mirror is the one with the greatest value of the material index

$$M = \frac{E^{1/3}}{\rho} \tag{5.5}$$

We treat the remaining constraints as attribute limits, requiring a melting point greater than 500°C to avoid creep, zero moisture take up, and a low thermal expansion coefficient (α<20×10^{-6}/K).

The selection. Here we have another example of elastic design for minimum weight. The appropriate chart is again that relating Young's modulus E and density ρ – but the line we now construct on it has a slope of 3, corresponding to the condition $M=E^{1/3}/\rho$=constant (Fig. 5.4). Glass lies at the value M=1.7 (GPa)$^{1/3}$·m^3/Mg. Materials that have larger values of M are better, those with lower, worse. Glass is much better than steel or speculum metal (that is why most mirrors are made of glass), but it is not as good as magnesium, several ceramics, carbon-fibre and glass-fibre reinforced polymers (GFRP), or – an unexpected finding – stiff foamed polymers. The short-list before applying the attribute limits is given in Table 5.4.

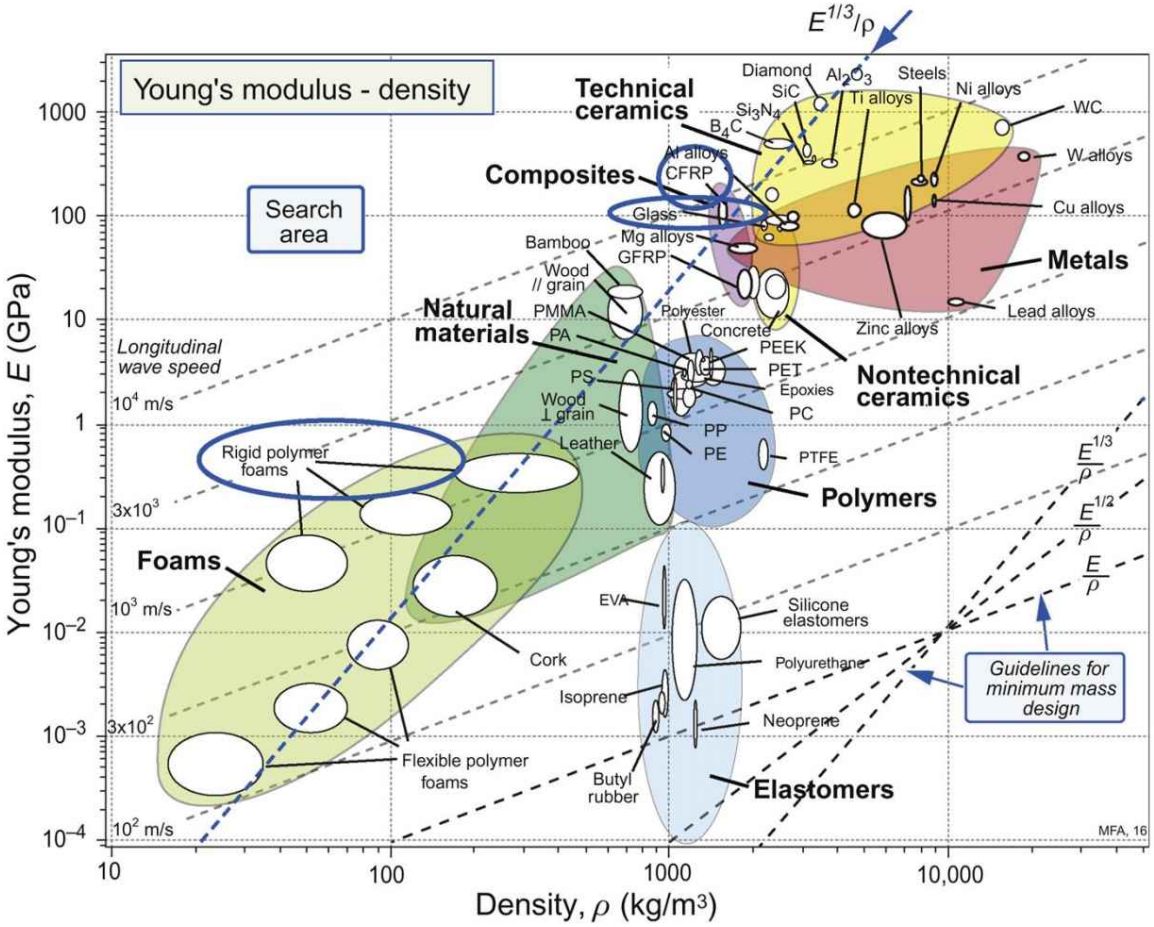


FIGURE 5.4 Materials for telescope mirrors. Glass is better than most metals, among which magnesium is a good choice. CFRP give, potentially, the lowest weight of all, but may lack adequate dimensional stability. Foamed glass is a possible candidate.

Table 5.4

Mirror Backing for 200-Inch (5.1 m) Telescope

Material	$M = E^{1/3}/\rho$ (GPa)$^{1/3}$·m^3/Mg	m (tonne) $2R$=5.1 m (from Eq. 5.4)	Comment
Steel (or speculum)	0.74	73.6	Very heavy. The original choice
GFRP	1.5	25.5	Not dimensionally stable enough – use for radio telescope
Al-alloys	1.6	23.1	Heavier than glass, and with high thermal expansion
Glass	1.7	21.6	The present choice
Mg-alloys	1.9	17.9	Lighter than glass but high thermal expansion
CFRP	3.0	9	Very light, but not dimensionally stable; use for radio telescopes
Foamed polystyrene	4.5	5	Very light, but dimensionally unstable. Foamed glass?

One must, of course, examine other aspects of this choice. The mass of the mirror, calculated from Eq. (5.4), is listed in the table. The CFRP mirror is less than half the weight of the glass one, and its support-structure could thus be as much as 4 times less expensive. The possible saving by using foam is even greater. But could they be made?

Some of the choices – polystyrene foam or CFRP – may at first seem impractical. But the potential cost-saving (the factor of 16) is so vast that they are worth examining. There are ways of casting a thin film of silicone rubber or of epoxy onto the surface of the mirror backing (the polystyrene or the CFRP) to give an optically smooth surface that could be silvered. The most obvious obstacle is the lack of stability of polymers – they change dimensions with age, humidity, temperature and so on. But glass itself can be foamed to give a material that is denser than polystyrene foam but much less dense than solid glass. Foamed glass has the same chemical and environmental stability as solid glass. It could provide a route to large cheap mirrors.

Postscript. There are, of course, other things you can do. The stringent design criterion ($\delta < 10\ \mu m$) can be partially overcome by engineering design without reference to the material choice. The 8.2 m Japanese telescope on Mauna Kea, Hawaii, and the Very Large Telescope at Cerro Paranal Silla in Chile each have a thin glass reflector supported by an array of hydraulic or piezoelectric jacks that exert distributed forces over its back surface, controlled to vary with the attitude of the mirror. The Keck telescope, also on Mauna Kea, is segmented, each segment independently positioned to give optical focus. But the limitations of this sort of mechanical system still require that the mirror meet a stiffness target. While stiffness at minimum

weight is the design requirement, the material selection criteria remain unchanged.

Radio telescopes do not have to be quite as precisely dimensioned as optical ones because they detect radiation with a longer wavelength, about 0.25 mm rather than the 0.02 mm of visible light waves. But they are much bigger (60 metres rather than 6) and they suffer from similar distortional problems. A recent 45 m radio telescope built for the University of Tokyo has a parabolic reflector made up of 6000 CFRP panels, each servo controlled to compensate for macro-distortion. Radio telescopes are now routinely made from CFRP, for exactly the reasons we deduced.

Related Case Studies

5.16 'Minimizing distortion in precision devices'

5.4 Materials for Table Legs

Luigi Tavolino, furniture designer, conceives of a light-weight table of daring simplicity: a flat sheet of toughened glass supported on slender, unbraced, cylindrical legs (Fig. 5.5). The legs must be solid (to make them thin) and as light as possible (to make the table easier to move). They must support the table top and whatever is placed upon it without buckling. What materials could one recommend (Table 5.5)?

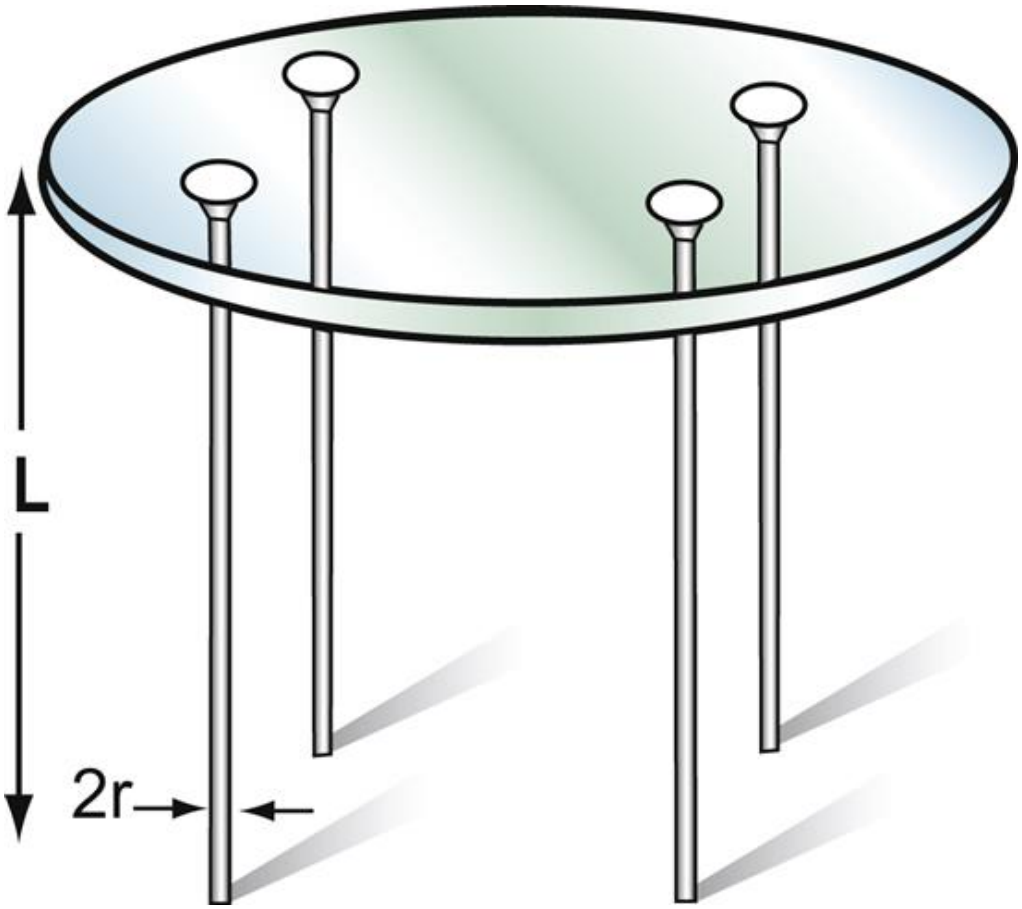


FIGURE 5.5 A light-weight table with slender cylindrical legs. Lightness and slenderness are independent objectives.

Table 5.5

Design Requirements for Table Legs

Function	■ *Column (supporting compressive loads)*
Constraints	■ *Length L specified* ■ *Must not buckle under design loads* ■ *Must not fracture if accidentally struck*
Objectives	■ *Minimize the mass, m* ■ *Maximize slenderness*
Free variables	■ *Diameter of legs, 2r* ■ *Choice of material*

The translation. This is a problem with two objectives:[1] weight is to be minimized, and slenderness maximized. There is one constraint: resistance to buckling. Consider minimizing weight first.

The leg is a slender column of material of density ρ and modulus E. Its length, L, and the maximum load, F, it must carry are determined by the design: they are fixed. The radius r of a leg is a free variable. We wish to minimize the mass m of the leg, given by the objective function

$$m = \pi r^2 L \rho \tag{5.6}$$

subject to the constraint that it supports a load P without buckling. The elastic buckling load F_{crit} of a column of length L and radius r (see Appendix B, 'Useful Solutions') is

$$F_{crit} = \frac{\pi^2 EI}{L^2} = \frac{\pi^3 E r^4}{4L^2} \tag{5.7}$$

using $I = \pi r^4/4$ where I is the second moment of the area of the column. The load F must not exceed F_{crit}. Solving for the free variable, r, and substituting it into the equation for m gives

$$m \geq \left(\frac{4F}{\pi}\right)^{1/2} (L)^2 \left[\frac{\rho}{E^{1/2}}\right] \tag{5.8}$$

The material properties are grouped together in the last pair of brackets. The weight is minimized by selecting the subset of materials with the greatest value of the material index

$$M_1 = \frac{E^{1/2}}{\rho}$$

(a result we could have taken directly from Appendix C, 'Material indices').

Now slenderness. Inverting Eq. (5.7) with F_{crit} set equal to F gives an equation for the thinnest leg that will not buckle:

$$r \geq \left(\frac{4F}{\pi^3}\right)^{1/4} (L)^{1/2} \left[\frac{1}{E}\right]^{1/4} \tag{5.9}$$

The thinnest leg is that made of the material with the largest value of the material index

$$M_2 = E$$

The selection. We seek the subset of materials that have high values of $E^{1/2}/\rho$ and E. We need the $E - \rho$ chart again (Fig. 5.6). A guideline of slope 2 is drawn on the diagram; it defines the slope of the grid of lines for values of $E^{1/2}/\rho$. The guideline is displaced upwards (retaining the slope) until a reasonably small subset of materials is isolated above it; it is shown at the position M_1=5 GPa$^{1/2}$/(Mg/m^3). Materials above this line have higher values of M_1. They are identified on the figure: *woods* (the traditional material for table legs), *composites* (particularly CFRP) and certain *engineering ceramics*. Polymers are out: they are not stiff enough; metals too: they are too heavy (even magnesium alloys, which are the lightest). The choice is further narrowed by the requirement that, for slenderness, E must be large. A horizontal line on the diagram links materials with equal values of E; those above are stiffer. Fig. 5.6 shows that placing this line at M_1=100 GPa

eliminates woods and GFRP. If the legs must be really thin, then the short-list is reduced to CFRP and ceramics: they give legs that weigh the same as the wooden ones but are barely half as thick. Ceramics, we know, are brittle: they have low values of fracture toughness. Table legs are exposed to abuse – they get knocked and kicked; common sense suggest that an additional constraint is needed, that of adequate toughness. This can be done using the chart of Figure 4.7; it eliminates ceramics, leaving CFRP. The cost of CFRP may cause Snr. Tavolino to reconsider his design, but that is another matter: he did not mention cost in his original specification.

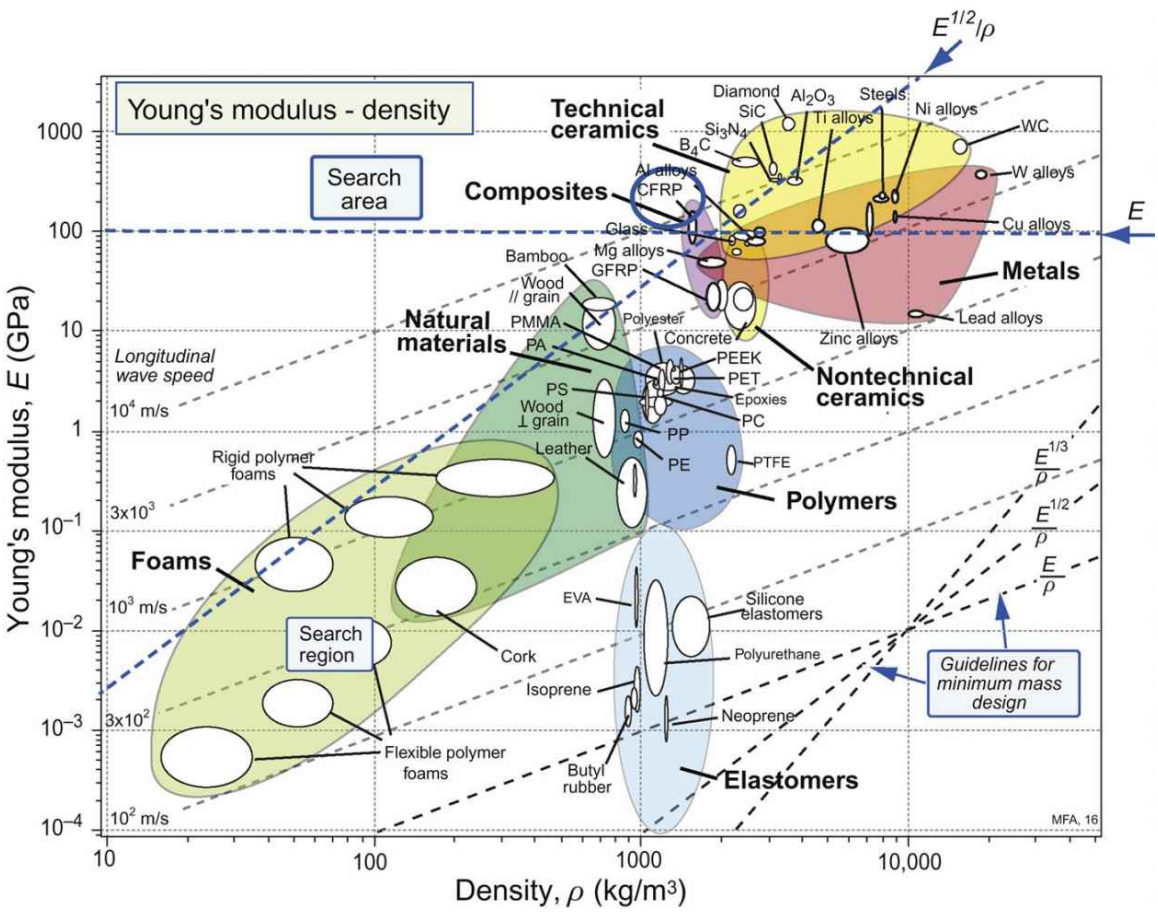


FIGURE 5.6 Materials for light, slender legs. Wood is a good choice; so is a composite such as CFRP, which, having a higher modulus than wood, gives a column that is both light and slender. Ceramics meet the stated design goals, but are brittle.

It is a good idea to lay out the results as a table, showing not only the materials that are best, but those that are second-best – they may, when other considerations are involved, become the best choice. Table 5.6 shows the way to do it.

Table 5.6
Materials for Table Legs

Material	Typical M_1 ($GPa^{1/2} \cdot m^3/Mg$)	Typical M_2 GPa	Comment
GFRP	2.5	20	Cheaper than CFRP, but lower M_1 and M_2
Woods	4.5	10	Outstanding M_1; poor M_2
			Cheap, traditional, reliable
Ceramics	6.3	300	Outstanding M_1 and M_2. Eliminated by brittleness
CFRP	6.6	100	Outstanding M_1 and M_2, but expensive

Postscript. Tubular legs, the reader will say, must be lighter than solid ones. True; but they will also be fatter. So it depends on the relative importance Snr. Tavolino attaches to his two objectives – lightness and slenderness – and only he can decide that. If he can be persuaded to live with fat legs, tubing can be considered – and the material choice may be different. Materials selection when section shape is a variable comes in Chapter 10, Selection of Material and Shape.

Ceramic legs were eliminated because of low toughness. If (improbably) the goal was to design a light, slender-legged table for use at high temperatures, ceramics should be reconsidered. The brittleness problem can be bypassed by protecting the legs from abuse, or by prestressing them in compression.

Related Case Studies

5.2 'Materials for oars'
5.3 'Mirrors for large telescopes'
9.5 'Conflicting objectives: table legs again'
11.2 'Spars for man-powered planes'
11.3 'Forks for a racing bicycle'
11.5 'Table legs yet again: thin or light?'

5.5 Cost: Structural Materials for Buildings

The most expensive thing that most people buy is the house they live in. Roughly half the cost of a house is the cost of the materials of which it is made, and they are used in large quantities (family house: around 200 tonnes; large apartment block: around 20,000 tonnes). The materials are used in four ways: structurally to hold the building up; as cladding, to keep the weather out; as 'internals', to insulate against heat and sound, and as services to provide heating, cooling, power, water and drainage.

Consider the selection of materials for the structure (Fig. 5.7). They must be stiff, strong and cheap. Stiff, so that the building does not flex too much under wind and internal loads; strong, so that there is no risk of it collapsing. And cheap, because such a lot of material is used. The structural frame of a building is rarely exposed to the environment, and is not, in general, visible, so criteria of corrosion resistance or appearance are not important here. The design goal is simple: strength and stiffness at minimum cost. To be more specific: consider the selection of material for floor joists. Table 5.7 summarizes the requirements.

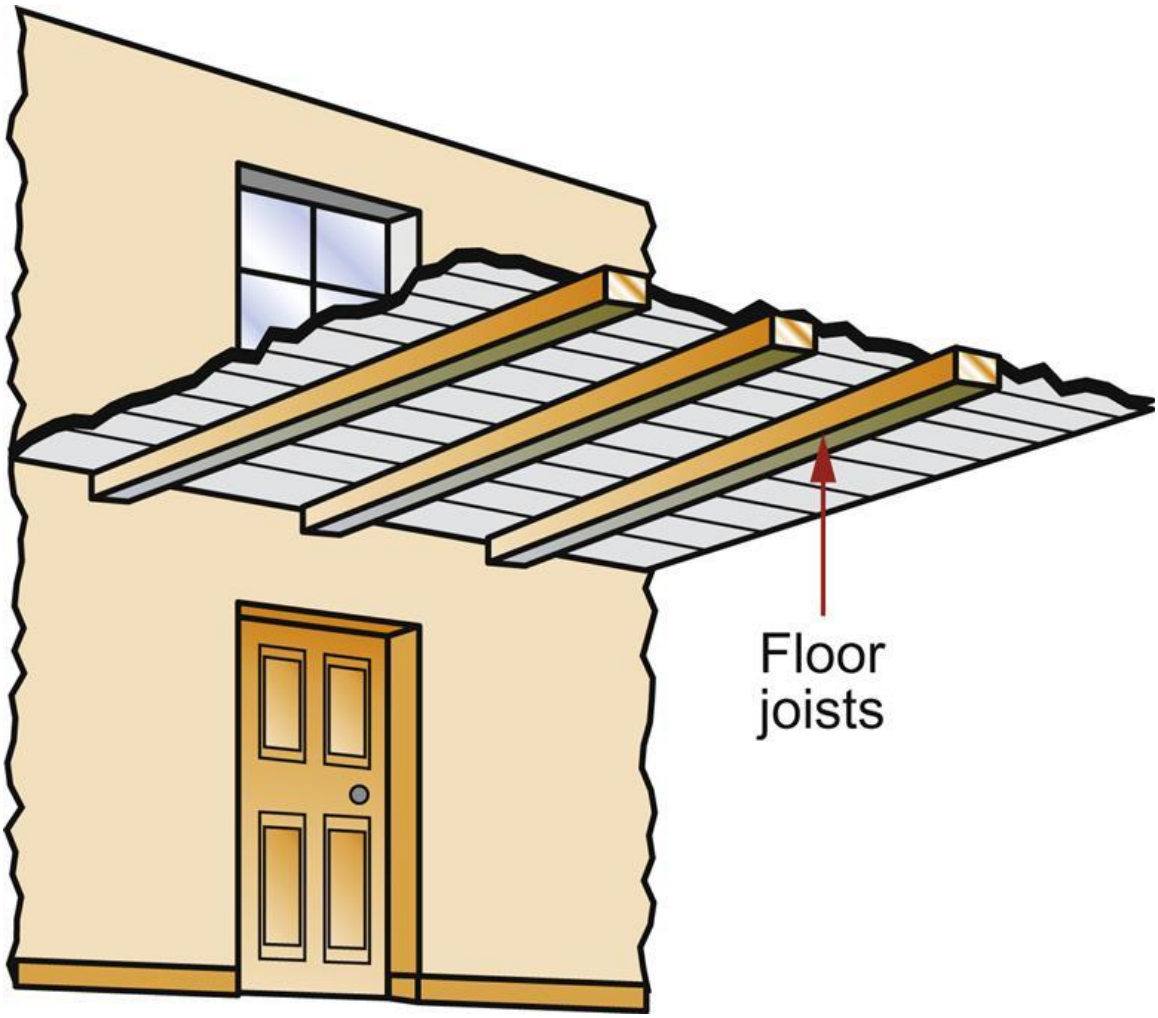


FIGURE 5.7 The materials of a building provide structure, externals, internals and services. The selection criteria depend on the function.

Table 5.7
Design Requirements for Floor Beams

Function	■ *Floor beam*
Constraints	■ *Length L specified* ■ *Stiffness: must not deflect too much under design loads* ■ *Strength: must not fail under design loads*
Objectives	■ *Minimize the cost, C*
Free variables	■ *Cross-section area of beam, A* ■ *Choice of material*

The translation. Floor joists are beams; they are loaded in bending. The material index for a stiff beam of minimum mass, m, was developed in Chapter 4, Materials Selection – the Basics (Eqs 4.11–4.15). The cost C of the beam is just its mass, m, times the cost per kg, C_m, of the material of which it is made:

$$C = mC_m = AL\rho C_m \tag{5.10}$$

which becomes the objective function of the problem. Proceeding as in Chapter 4, Materials Selection – the Basics, we find the index for a stiff beam of minimum cost to be:

$$M_1 = \frac{E^{1/2}}{\rho C_m}$$

– it is the index for a light stiff beam with ρ replaced by ρC_m.

The index when strength rather than stiffness is the constraint was not derived earlier. Here it is. The objective function is still Eq. (5.10), but the constraint is now that of strength: the beam must support load F without failing. The failure load of a beam (Appendix B, Section B4) is:

$$F_f = C_2 \frac{I\sigma_f}{y_m L} \tag{5.11}$$

where C_2 is a constant, σ_f is the failure strength of the material of the beam and y_m is the distance between the neutral axis of the beam and its outer filament. We consider a rectangular beam of depth d and width b. We assume the proportions of the beam are fixed so that $d = \alpha b$ where α is the aspect ratio, typically 2 for wood beams. Using this and $I = bd^3/12$ to eliminate A in Eq. (5.10) gives the cost of the beam that will just support the load F_f:

$$C = \left(\frac{6\sqrt{\alpha}}{C_2}\frac{F_f}{L^2}\right)^{2/3}(L^3)\left[\frac{\rho C_m}{\sigma_f^{2/3}}\right] \tag{5.12}$$

The mass is minimized by selecting materials with the largest values of the index

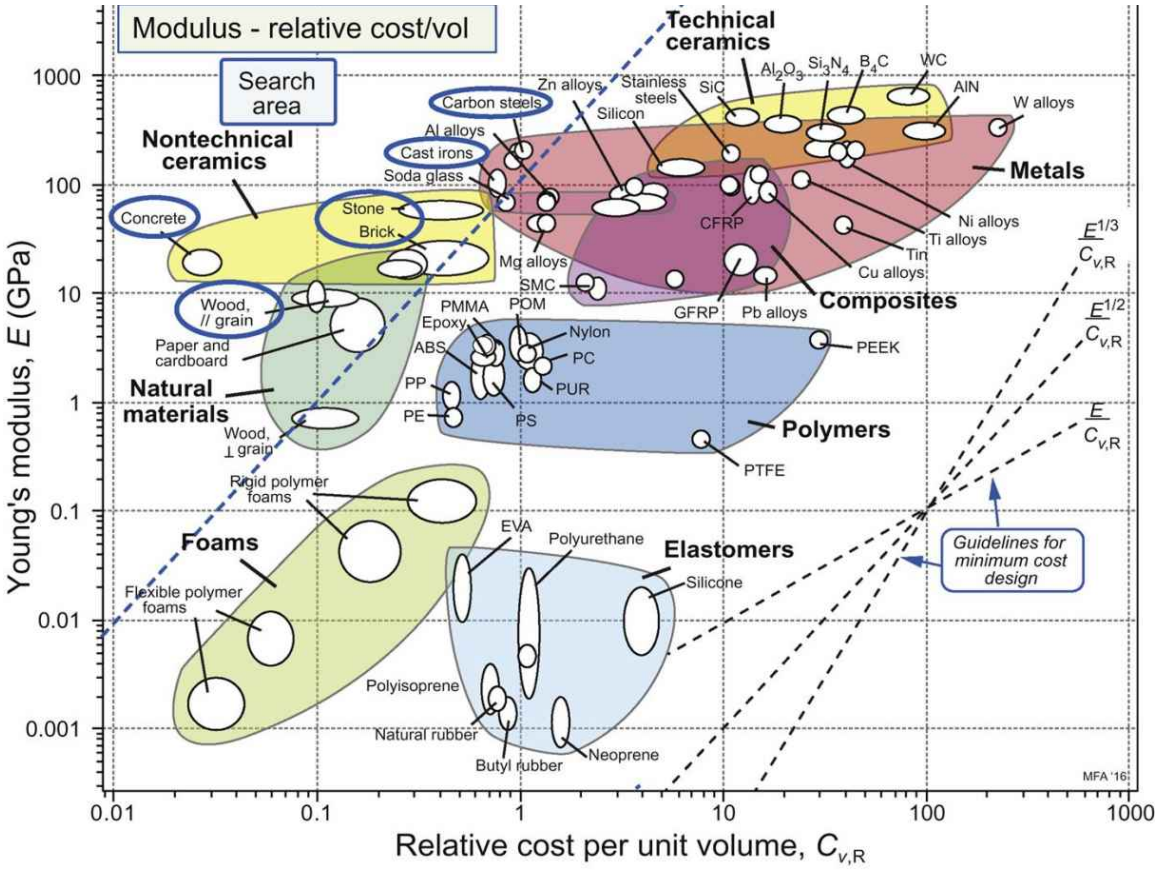
$$M_2 = \frac{\sigma_f^{2/3}}{\rho C_m}$$

As we found with the stiffness index, this is the index for a light strong beam with ρ replaced by ρC_m.

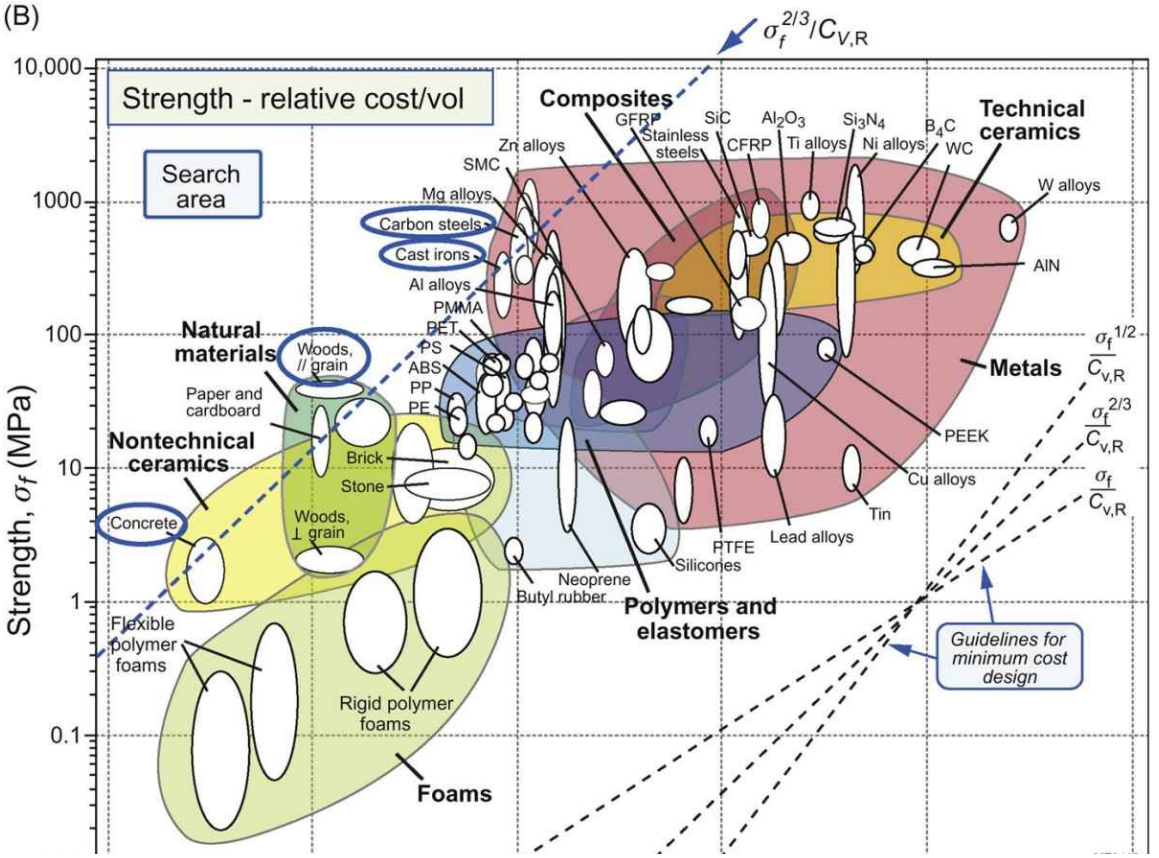
As explained in Chapter 3, Materials Property Charts, cost/kg, C_m, causes trouble because it fluctuates over time and is measured in different currencies in different countries. To bypass the problem the charts we replace ρC_m by a relative cost per unit volume, $C_{v,R}$: the cost per unit volume of the material normalized by that for mild steel. The ranking of materials is unchanged, so all the selection methods can be applied just as before.

The selection. Stiffness is considered first. Fig. 5.8A shows the relevant chart: modulus E against relative cost per unit volume, $C_{v,R}$. The selection line has the appropriate slope for M_1; it isolates concrete, stone, brick, woods, cast irons and carbon steels. Fig. 5.8B shows strength against relative cost. The selection line $-M_2$ this time – gives almost the same selection. They are listed, with values, in the table. They are exactly the materials with which buildings have been, and are, made (Table 5.8).


(A)
$E^{1/2}$/ $C_{V,R}$


Modulus - relative cost/vol
Search area
Technical ceramics
Nontechnical ceramics
Metals
Composites
Polymers
Natural materials
Foams
Elastomers
Guidelines for minimum cost design
Young's modulus, E (GPa)
Relative cost per unit volume, $C_{v,R}$
MFA '16

(B)


$\sigma_f^{2/3}/C_{V,R}$
Strength - relative cost/vol
Search area
Composites
Technical ceramics
Metals
Natural materials
Nontechnical ceramics
Polymers and elastomers
Foams
Guidelines for minimum cost design
Strength, σ_f (MPa)

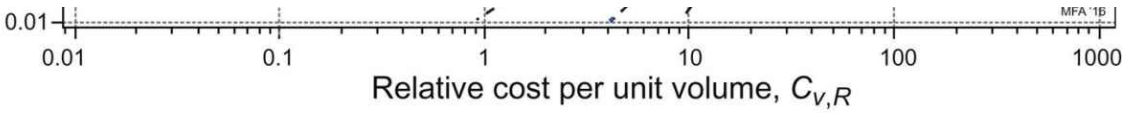


FIGURE 5.8 (A) The selection of cheap, stiff materials for the structural frames of buildings and (B) the selection of cheap, strong materials for the structural frames of buildings.

Table 5.8
Structural Materials for Buildings

Material	M_1 (GPa$^{1/2}$/(kg/m^3))	M_2 (MPa$^{2/3}$/(kg/m^3))	Comment
Concrete	160	14	Use in compression only
Brick	12	12	
Stone	9.3	12	
Woods	21	90	Can support bending and tension as well as compression, which allows greater freedom of shape
Cast iron	17	90	
Steel	14	45	

Postscript. Concrete, stone and brick have strength only in compression; the form of the building must use them in this way (columns, arches). Wood, steel and reinforced concrete have strength in bending and tension as well as compression; steel, additionally, can be given efficient shapes (I-sections, box sections, tubes, discussed in Chapter 10, Selection of Material and Shape). The form of the building made from these has much greater freedom.

It is sometimes suggested that architects live in the past; that in the 21st century they should be building with carbon CFRP and fibreglass (GFRP) composites, aluminium and titanium alloys and stainless steel. Some do, but the last two figures give an idea of the penalty involved: the material cost of achieving the same stiffness and strength is between 5 and 50 times greater.

Further Reading

Cowan, H.J., Smith, P.R., 1988. The Science and Technology of Building Materials, Van Nostrand-Reinhold, NY.

Doran, D.K., 1992. The Construction Reference Book, Butterworth Heinemann, Oxford, UK.

Related Case Studies

5.2 'Materials for oars'
5.4 'Materials for table legs'
9.2 'Floor joists again'

11.4 ‘Floor joists: wood, bamboo or steel?’

5.6 Materials for Flywheels

Flywheels store energy. Small ones – the sort found in children’s toys – are made of lead. Old steam engines and modern automobiles have flywheels too; they are made of cast iron. Flywheels are increasingly used for power storage and regenerative braking (KER) systems for vehicles, some made of high-strength steel, some of composites. Lead, cast iron, steel, composites – there is a strange diversity here. What *is* the best choice of material for a flywheel?

An efficient flywheel stores as much *energy per unit weight* as possible. As the flywheel is spun up, increasing its angular velocity ω, it stores more energy. But if the centrifugal stress exceeds the tensile strength of the flywheel, it flies apart. So strength sets an upper limit on the energy that can be stored.

The flywheel of a child’s toy is not efficient in this sense. Its angular velocity is limited by the pulling-power of the child and never remotely approaches the burst velocity. In this case, and for the flywheel of an automobile engine, we wish to maximize the *energy stored* at a given *angular velocity* in a flywheel with an outer radius, R, that is constrained by the size of the cavity in which it must sit.

The objective and constraints in flywheel design thus depend on its purpose. The two alternative sets of design requirements are listed in Tables 5.9(a) and (b).

Table 5.9

***(a)* Design Requirements for Maximum Energy Flywheel**

Function	■ *Flywheel for energy storage*
Constraints	■ *Outer radius, R, fixed* ■ *Must not burst* ■ *Adequate toughness to give crack tolerance*
Objectives	■ *Maximize kinetic energy per unit mass*
Free variables	■ *Choice of material*
***(b)* Design requirements for fixed velocity**	
Function	■ *Flywheel for child's toy*
Constraints	■ *Outer radius, R, fixed*
Objectives	■ *Maximize kinetic energy per unit volume at fixed angular velocity*
Free variables	■ *Choice of material*

The translation. An efficient flywheel of the first type stores as much energy per unit weight as possible, without failing. Think of it as a solid disk of radius R and thickness t, rotating with angular velocity ω (Fig. 5.9). The energy U stored in the flywheel (Appendix B, 'Spinning disks') is

$$U = \frac{1}{2} J\omega^2 \qquad \textbf{(5.13)}$$

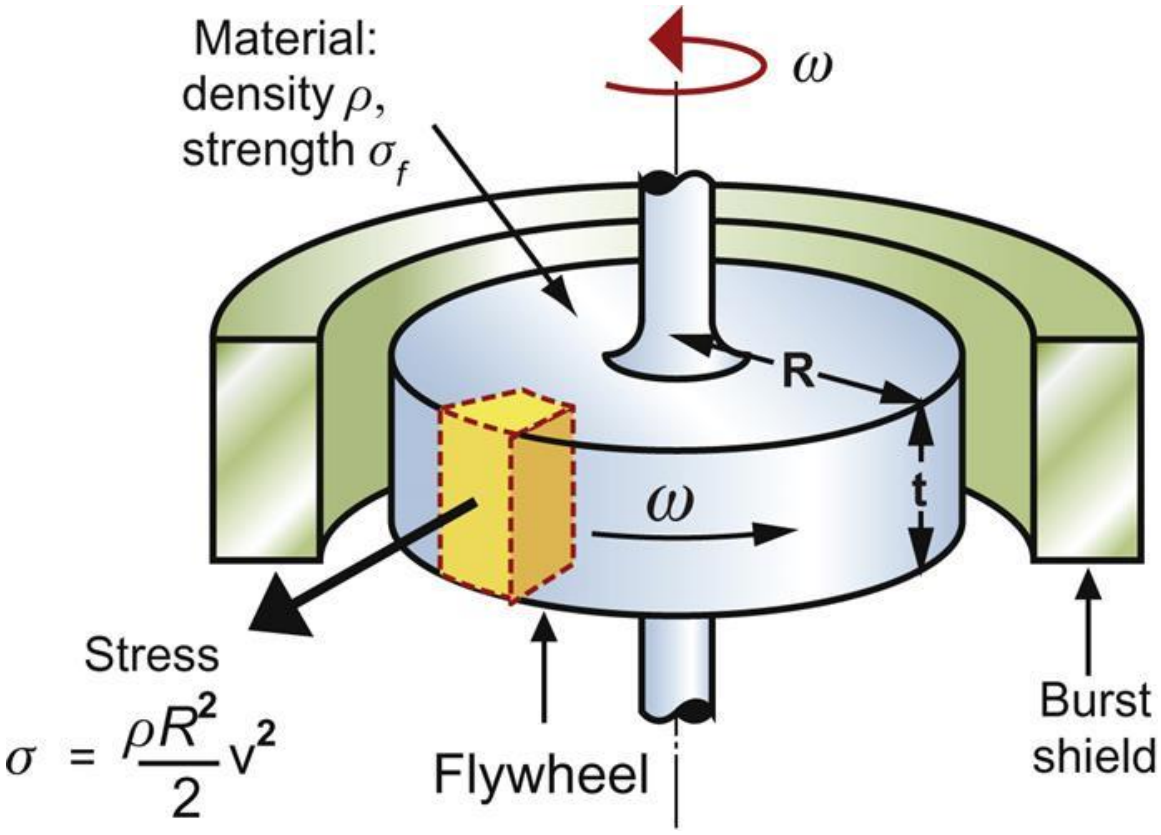


FIGURE 5.9 A flywheel. The maximum kinetic energy it can store is limited by its strength.

Here $J = \frac{\pi}{2}\rho R^4 t$ is the polar moment of inertia of the disk and ρ the density of the material of which it is made, giving

$$U = \frac{\pi}{4}\rho R^4 t\omega^2 \tag{5.14}$$

The mass of the disk is

$$m = \pi R^2 t\rho \tag{5.15}$$

The quantity to be maximized is the kinetic energy per unit mass, which is the ratio of the last two equations:

$$\frac{U}{m} = \frac{1}{4}R^2\omega^2 \tag{5.16}$$

As the flywheel is spun up, the energy stored in it increases, but so does the centrifugal stress. The maximum principal stress in a spinning disk of uniform thickness (Appendix B) is

$$\sigma_{\max} = \left(\frac{3+\upsilon}{8}\right)\rho R^2\omega^2 \approx \frac{1}{2}\rho R^2\omega^2 \tag{5.17}$$

where υ is Poisson's ratio ($\upsilon \approx 1/3$). This stress must not exceed the failure stress σ_f (with an appropriate factor of safety, here omitted). This sets an upper limit to the product of the angular velocity, ω, and the disk radius, R. Eliminating $R\,\omega$ between the last two equations gives

$$\frac{U}{m} = \frac{1}{2}\left(\frac{\sigma_f}{\rho}\right) \tag{5.18}$$

The best materials for high-performance flywheels are those with high values of the material index

$$M_1 = \frac{\sigma_f}{\rho} \tag{5.19}$$

which has units of kJ/kg.

And now the other sort of flywheel – that of the child's toy. Here we seek the material that stores the most energy per unit volume V at constant velocity, ω. The energy per unit volume at a given ω is (from Eq. 5.2):

$$\frac{U}{V} = \frac{1}{4}\rho R^2\omega^2.$$

Both R and ω are fixed by the design, so the best material is now that with the greatest value of

$$M_2 = \rho \tag{5.20}$$

The selection. Fig. 5.10 shows the strength – density chart. Values of M_1 correspond to a grid of lines of slope 1. One such is plotted as a diagonal line at the value $M_1 = 200\ \text{kJ/kg}$. Candidate materials with high values of M_1 lie in Search area 1 towards the top left. The best choices are unexpected ones: composites, particularly CFRP, high-strength titanium alloys and some ceramics, but these are ruled out by their low toughness.

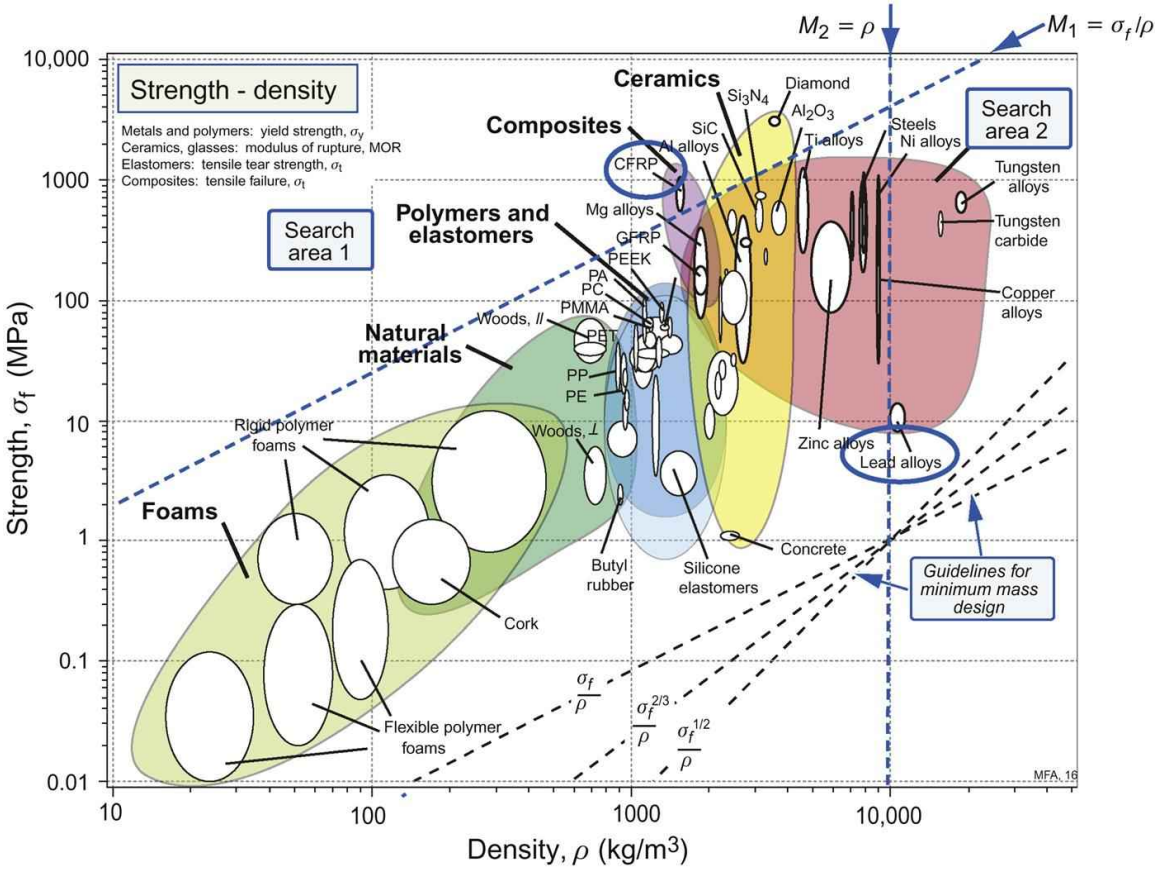


FIGURE 5.10 Materials for flywheels. Composites are the best choices. Lead and cast iron, traditional for flywheels, are good when performance is limited by rotational velocity, not strength.

But what of the lead flywheels of children's toys? There could hardly be two more different materials than CFRP and lead: the one, strong and light, the other, soft and heavy. Why lead? It is because, in the child's toy, the constraint is different. Even a super child cannot spin the flywheel of his or her toy up to its burst velocity. The angular velocity ω is limited instead by the drive mechanism (pull-string, friction drive). Then as we

have seen, the best material is that with the largest density. The second selection line on Fig. 5.10 shows the index M_2 at the value 10,000 kg/m^3. We seek materials in Search area 2 to the right of this line. Lead is good. Tungsten is better, but more expensive. Cast iron is less good, but cheaper. Gold, platinum and uranium (not shown on the chart) are the best of all, but may be thought unsuitable for other reasons.

Postscript. A CFRP rotor is able to store around 200 kJ/kg. A lead flywheel, by contrast, can store only 1 kJ/kg before disintegration; a cast iron flywheel, about 30. All these are small compared with the energy density in gasoline: roughly 20,000 kJ/kg. Even so, the energy density in the flywheel is considerable; its sudden release in a failure could be catastrophic. The disk must be surrounded by a burst-shield and precise quality control in manufacture is essential to avoid out-of-balance forces. This has been achieved in a number of composite energy-storage flywheels intended for use in trucks and buses and as an energy reservoir for smoothing wind power generation.

And now a digression: the electric car. Hybrid petrol-electric cars are already on the roads, using advanced battery technology to store energy. But batteries have their problems: the energy density they can contain is low (see Table 5.10); their weight limits both the range and the performance of the car. It is practical to build flywheels with an energy density about equal to that of the best batteries. Consideration is now being given to a flywheel for electric vehicles. A pair of counter-rotating CFRP disks are housed in a steel burst-shield. Magnets embedded in the disks pass near coils in the housing, which induces a current and allows power to be drawn to the electric motor that drives the wheels. Such a flywheel could, it is estimated, give an electric car an adequate range, at a cost competitive with the gasoline engine and with none of the local pollution.

Table 5.10

Energy Density of Energy Storage Systems

Source	Energy Density kJ/kg	Comment
Gasoline	20,000	Oxidation of hydrocarbon – mass of oxygen not included
Rocket fuel	5000	Less than hydrocarbons because oxidizing agent forms part of fuel
Flywheels	Up to 200	Attractive, but not yet proven
Lithium-ion battery	Up to 350	Attractive but expensive and with limited
Nickel-cadmium battery	120–200	life
Lead-acid battery	50–80	Large weight for acceptable range
Super capacitors	10–20	Capacitors release energy very quickly – so power delivery is high
Springs, rubber bands	Up to 5	Much less efficient method of energy storage than flywheel

Further Reading

Christensen, R.M., 1979. Mechanics of Composite Materials, Wiley Interscience, N.Y., p. 213 et seq.

Lewis, G., 1990. Selection of Engineering Materials, Prentice Hall, NJ, Part 1, p. 1.

Medlicott, P.A.C., Potter, K.D., 1986. The development of a composite flywheel for vehicle applications. In: "High Tech – the way into the nineties", edited by Brunsch, K., Golden, H-D. and Horkert, C-M. Elsevier, Amsterdam, p. 29.

Related Case Studies

5.7 'Materials for springs'
5.11 'Safe pressure vessels'
9.3 'Con-rods for high-performance engines'

5.7 Materials for Springs

Springs come in many shapes (Fig. 5.11) and have many purposes: think of tensile springs (a rubber band, for example), leaf springs, helical springs, spiral springs, torsion bars. Regardless of their shape or use, the best

material for a spring of minimum volume is that with the greatest value of σ_f^2/E, and for minimum weight it is that with the greatest value of $\sigma_f^2/\rho E$ (derived below). We use springs as a way of introducing the use of two of the most useful of the charts: Young's modulus E plotted against strength σ_f, and specific modulus E/ρ plotted against specific strength σ_f/ρ (Figures 3.5 and 3.6) (Table 5.11).

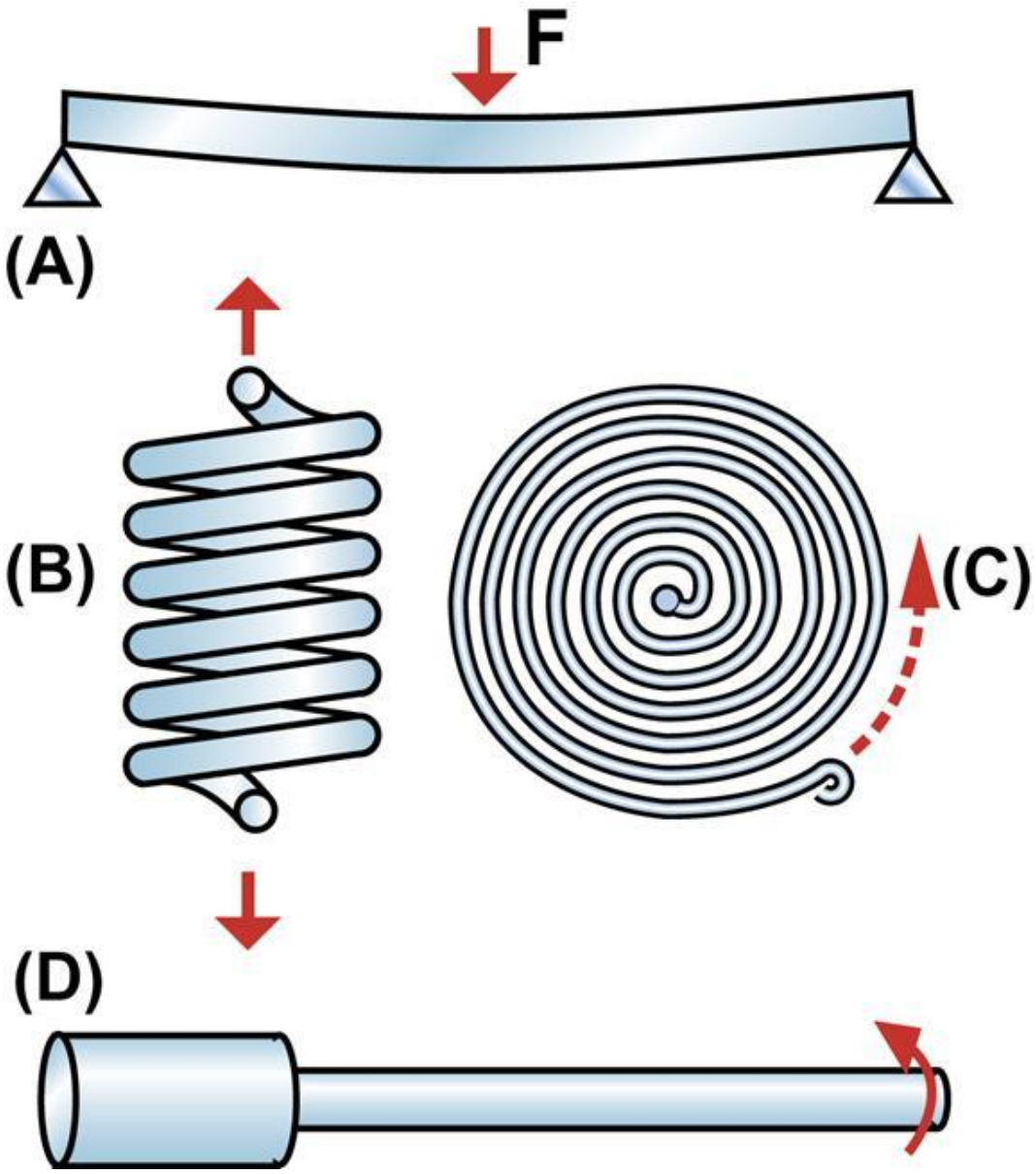


FIGURE 5.11 Springs store energy. What is the best choice of material to make them?

Table 5.11

Design Requirements for Springs

Function	■ *Elastic spring*
Constraint	■ *No failure, meaning* $\sigma < \sigma_f$ *throughout the spring*
Objectives	■ *Maximum stored elastic energy per unit volume, or* ■ *Maximum stored elastic energy per unit weight*
Free variables	■ *Choice of material*

The translation. The primary function of a spring is to store elastic energy and – when required – release it again. The elastic energy stored per unit volume in a material stressed uniformly to a stress σ is

$$W_v = \frac{1}{2}\frac{\sigma^2}{E} \tag{5.21}$$

where E is Young's modulus. We wish to maximize W_v. The spring will be damaged if the stress σ exceeds the yield stress or failure stress σ_f; the constraint is $\sigma < \sigma_f$. Thus the maximum energy density is

$$W_v = \frac{1}{2}\frac{\sigma_f^2}{E} \tag{5.22}$$

Torsion bars and leaf springs are less efficient than axial springs because much of the material is not fully loaded: the material at the neutral axis, for instance, is not loaded at all. For leaf springs

$$W_v = \frac{1}{4}\frac{\sigma_f^2}{E}$$

and for torsion bars

$$W_v = \frac{1}{3}\frac{\sigma_f^2}{E}$$

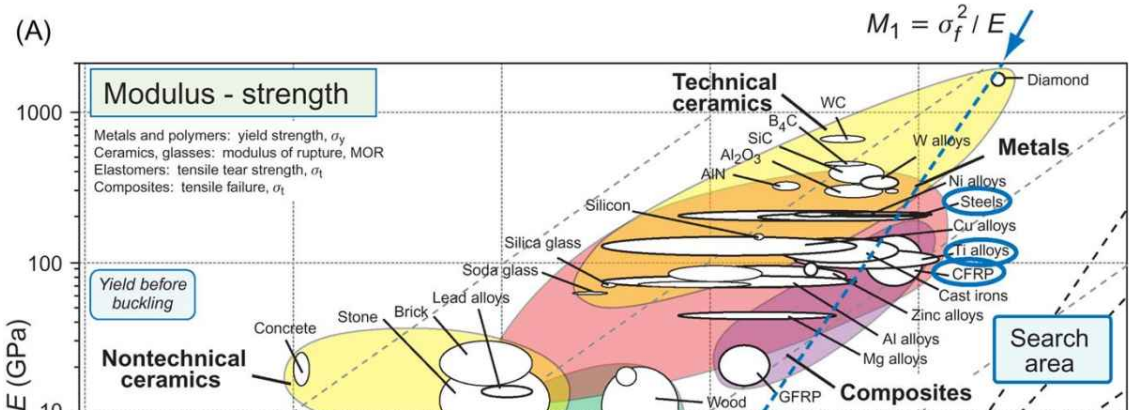
But – as these results show – this has no influence on the choice of material. The best stuff for a spring regardless of its shape is that with the biggest value of

$$M_1 = \frac{\sigma_f^2}{E} \tag{5.23}$$

If weight, rather than volume, matters, we must divide this by the density ρ (giving energy stored per unit weight), and seek materials with high values of

$$M_2 = \frac{\sigma_f^2}{\rho E} \tag{5.24}$$

The selection. The choice of materials for springs of minimum volume is shown in Fig. 5.12A. A family lines of slope 2 link materials with equal values of $M_1 = \sigma_f^2/E$; those with the highest values of M_1 lie towards the lower right. The heavy line is one of the family; it is positioned at 3 MJ/m^3 isolating the materials in the search area. The best choices are a *high-strength steel* lying near the top end of the line. Other materials are suggested too: *CFRP* (now used for truck springs), *titanium alloys* (good but expensive) and *nylon, PA* (children's toys often have nylon springs), and, of course, *elastomers*. Note how the procedure has identified a candidate from almost every class of materials: metals, polymers, elastomers and composites. They are listed, with commentary, in Table 5.12(a).


(A)
$M_1 = \sigma_f^2 / E$
Modulus - strength
Metals and polymers: yield strength, σ_y
Ceramics, glasses: modulus of rupture, MOR
Elastomers: tensile tear strength, σ_t
Composites: tensile failure, σ_t
Yield before buckling
1000
100
E (GPa)
Technical ceramics
WC
B_4C
SiC
Al_2O_3
AlN
Silicon
Silica glass
Soda glass
Lead alloys
Brick
Stone
Concrete
Nontechnical ceramics
Diamond
W alloys
Metals
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Mg alloys
Search area
Wood
GFRP
Composites

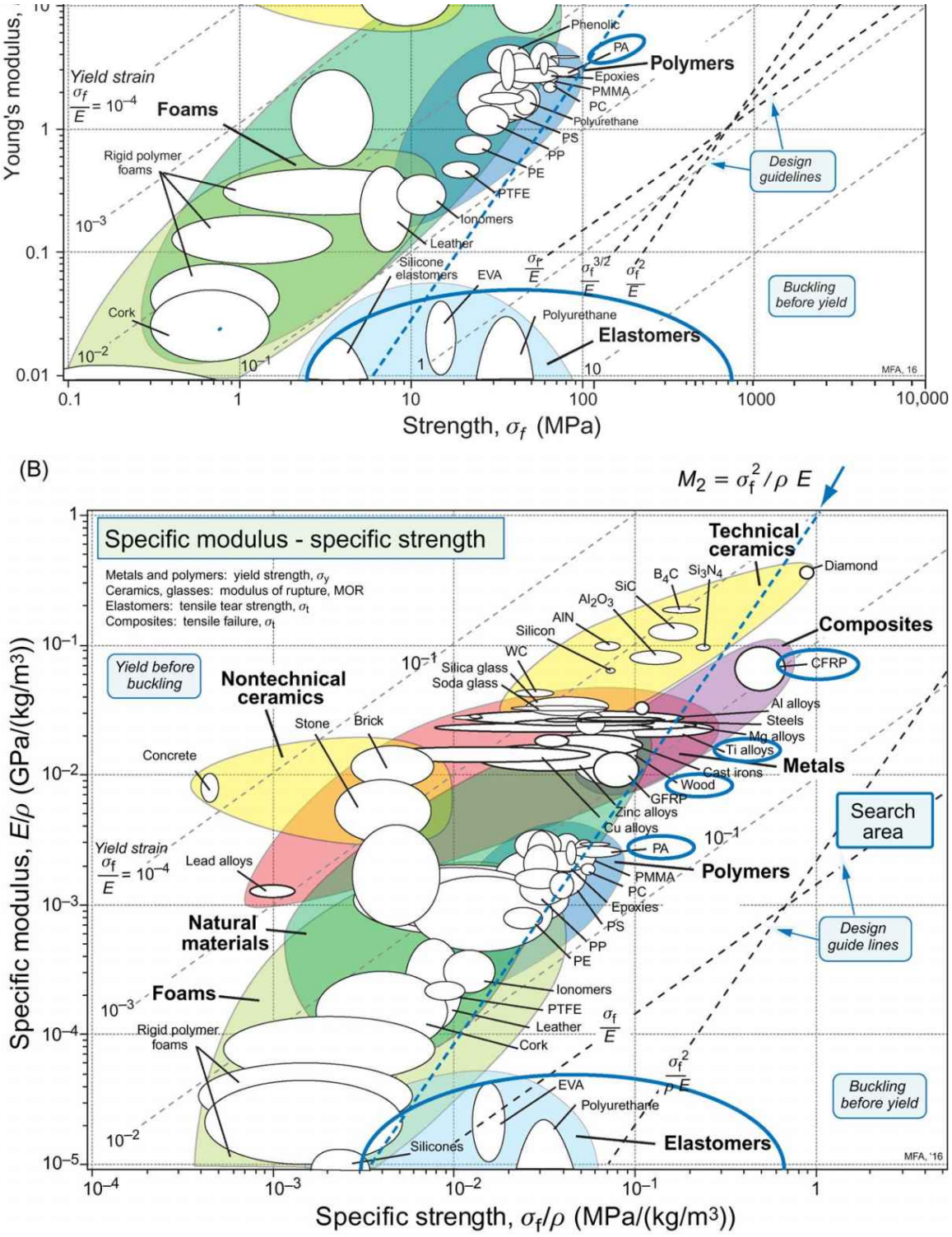


FIGURE 5.12 (A) Materials for small springs. High-strength ('spring') steel, glass, CFRP and GFRP all, make good springs. Elastomers excel. (B) Materials for light springs. Metals are disadvantaged by their high densities. Composites and elastomers are excellent.

Table 5.12

***(a)* Materials for Efficient Small Springs**

Material	$M_1 = \sigma_f^2/E$ (MJ/m³)	Comment
Ti-alloys	4–12	Expensive, corrosion resistant.
CFRP	6–10	Comparable in performance with steel; expensive.
Spring steel	3–7	The traditional choice: easily formed and heat treated.
Nylon	1.5–2.5	Cheap and easily shaped, but high loss factor.
Rubber	20–50	Better than spring steel; but high loss factor.

(b) Materials for Efficient Light Springs		
Material	$M_1 = \sigma_f^2/\rho E$ (kJ/kg)	Comment
Ti-alloys	0.9–2.6	Better than steel; corrosion resistant; expensive.
CFRP	3.9–6.5	Better than steel; expensive.
GFRP	1.0–1.8	Better than spring steel; less expensive than CFRP.
Spring steel	0.4–0.9	Poor, because of high density.
Wood	0.3–0.7	On a weight basis, wood makes good springs.
Nylon	1.3–2.1	As good as steel, but with a high loss factor.
Rubber	18–45	Outstanding; 20 times better than spring steel; but with high loss factor.

Materials selection for light springs is shown in Fig. 5.12B. A family of lines of slope 2 link materials with equal values of

$$M_2 = \left(\frac{\sigma_f}{\rho}\right)^2 / \left(\frac{E}{\rho}\right) = \frac{\sigma_f^2}{E\rho} \qquad (5.25)$$

One is shown at the value $M_2 = 1$ kJ/kg. Metals, because of their high density, are less attractive than composites, and much less attractive than elastomers. (You can store roughly 18 times more elastic energy, per unit weight, in a rubber band than in the best spring steel.) Candidates are listed in Table 5.12(b). Wood – the traditional material for archery bows, now appears.

Postscript. Many additional considerations enter the choice of a material for a spring. Springs for vehicle suspensions must resist fatigue and corrosion; engine valve springs must cope with elevated temperatures. A subtler property is the loss coefficient, shown in Figure 3.9. Polymers have a relatively high loss factor and dissipate energy when they vibrate;

metals, if strongly hardened, do not. Polymers, because they creep, are unsuitable for springs that carry a steady load for long periods of time, though they are still perfectly good for catches and locating springs that spend most of their time unstressed.

Further Reading

Boiton, R.G., 1963. "The mechanics of instrumentation", Proc. I. Mech. E. vol 177, No 10, 269–288.

Hayes, M., 1990. "Materials update 2: springs", Engineering, May, p. 42.

Related Case Studies

5.8 'Elastic hinges and couplings'
11.7 'Shapes that flex: leaf and strand structures'
11.8 'Ultra-efficient springs'
13.7 'Connectors that don't relax their grip'

5.8 Elastic Hinges and Couplings

Nature makes much use of elastic (or 'natural') hinges: skin, muscle, cartilage all allow large, recoverable deflections. Designers, too, make use of *flexure* and *torsion hinges*: ligaments that connect or transmit load between components while allowing limited relative movement between them by deflecting elastically (Fig. 5.13 and Table 5.13). Which materials make good hinges?

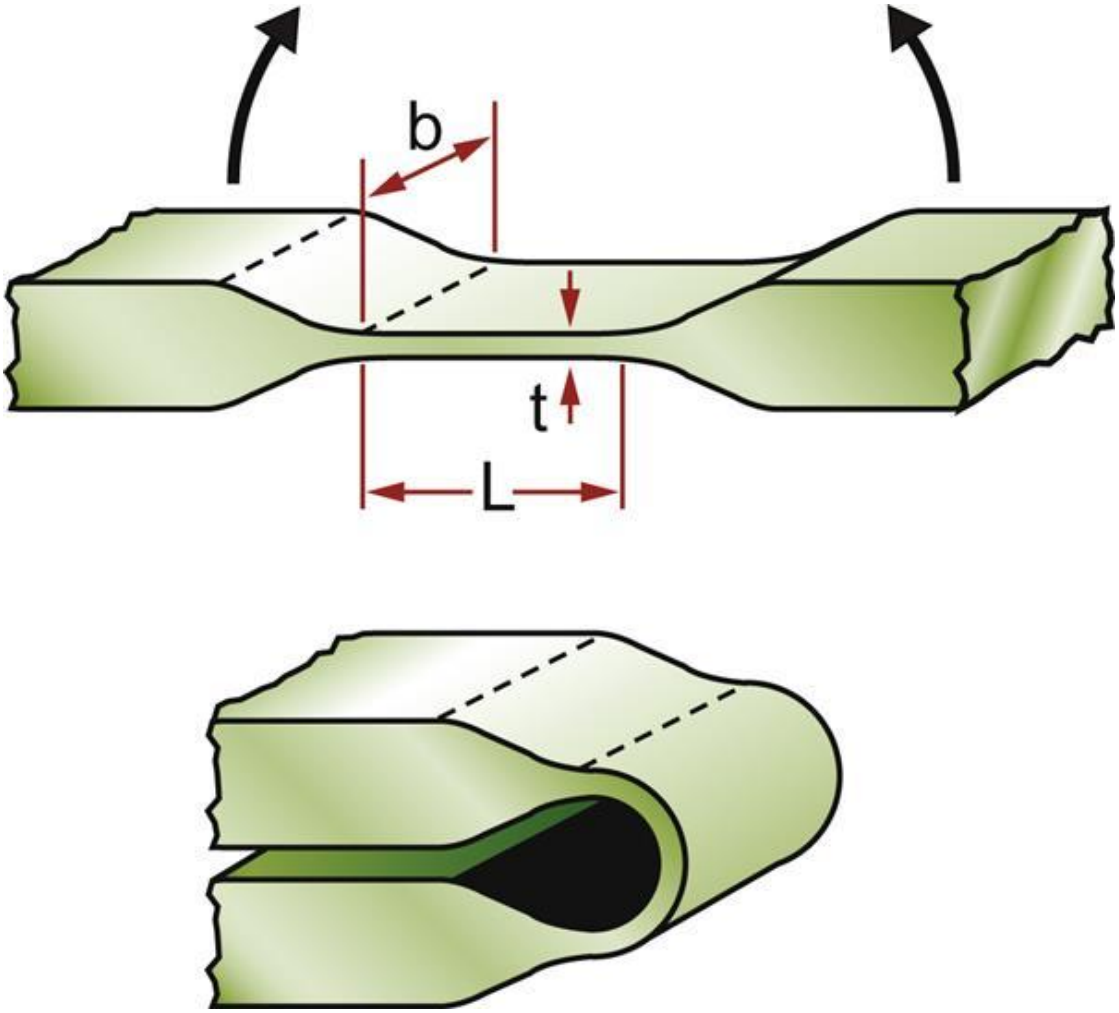


FIGURE 5.13 Elastic or 'natural' hinges. The ligaments must bend repeatedly without failing.

Table 5.13

Design Requirements for Elastic Hinges

Function	■ *Elastic hinge*
Constraint	■ *No failure, meaning* $\sigma < \sigma_f$ *throughout the hinge*
Objectives	■ *Maximize elastic flexure*
Free variables	■ *Choice of material*

The translation. Consider the hinge for the lid of a box. The box, lid and hinge are to be moulded as a single unit. The hinge is a thin ligament that

flexes elastically as the box is closed, as in the figure, but it carries no significant axial loads. Then the best material is the one that (for given ligament dimensions) bends to the smallest radius without yielding or failing. When a ligament of thickness t is bent elastically to a radius R, the surface strain is

$$\varepsilon = \frac{t}{2R} \tag{5.26}$$

and – since the hinge is elastic – the maximum stress is

$$\sigma = E\varepsilon = E\frac{t}{2R} \tag{5.27}$$

This must not exceed the yield or failure strength σ_f. Thus the minimum radius to which the ligament can be bent without damage is

$$R \geq \frac{t}{2}\left[\frac{E}{\sigma_f}\right] \tag{5.28}$$

The best material is the one that can be bent to the smallest radius, i.e., the one with the greatest value of the index

$$M = \frac{\sigma_f}{E} \tag{5.29}$$

The selection. We need the $\sigma_f - E$ chart again (Fig. 5.14). Candidates are identified by using the guideline of slope 1; a line is shown at the position $M = \sigma_f/E = 2 \times 10^{-2}$. The best choices for the hinge lie to the right of this line: they are all polymeric materials. The short-list (Table 5.14) includes polyethylene, polypropylene, nylon, and, best of all, elastomers, though these may be too flexible for the body of the box itself. Cheap products with this sort of elastic hinge are generally moulded from polyethylene, polypropylene or nylon. Spring steel and other metallic spring materials (like phosphor bronze) are possibilities: they combine usable σ_f/E with high E, giving flexibility with good positional stability (as in the suspensions of relays). The table gives further details.

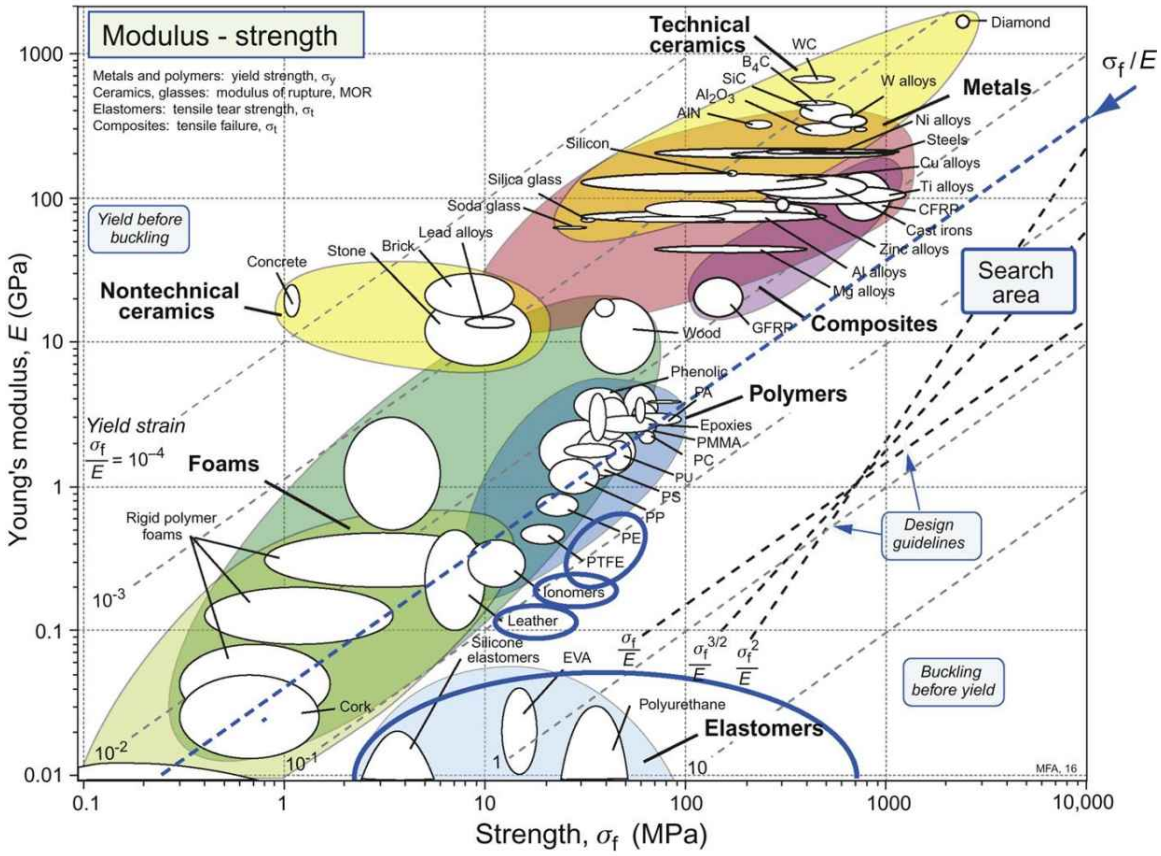


FIGURE 5.14 Materials for elastic hinges. Elastomers are best, but may not be rigid enough to meet other design needs. Then polymers such as nylon, PTFE and PE are better. Spring steel is not as good, but much stronger.

Table 5.14
Materials for Elastic Hinges

Material	M ($\times 10^{-3}$)	Comment
Polyethylene	32	Widely used for cheap hinged bottle caps, etc.
Polypropylene	30	Stiffer than polyethylene. Easily moulded
Nylon	30	Stiffer than polyethylene. Easily moulded
PTFE	35	Very durable; more expensive than PE, PP, etc.
Elastomers	100–1000	Outstanding, but low modulus
High-strength copper alloys	4	M not as good as polymers. Use when high tensile stiffness is required
Spring steel	6	

Postscript. Polymers give more design freedom than metals. The elastic hinge is one example of this because it can reduce the box, hinge and lid (3 components plus the fasteners needed to join them) to a single box-hinge-lid unit, moulded in one operation. Their spring-like properties allow snap-together, easily joined parts. Another example is the elastomeric coupling – a flexible universal joint, allowing high angular, parallel and axial flexibility with good shock absorption characteristics. Elastomeric hinges offer many opportunities to be exploited in engineering design.

Related Case Studies

5.7 'Materials for springs'
5.9 'Materials for seals'
5.10 'Deflection-limited design with brittle polymers'
11.7 'Shapes that flex: leaf and strand structures'

5.9 Materials for Seals

A reusable elastic seal consists of a cylinder of material compressed between two flat surfaces (Fig. 5.15). The seal must form the largest possible contact width, b, while keeping the contact stress, σ, sufficiently low that it does not damage the flat surfaces, and the seal itself must remain elastic so that it can be reused many times. What materials make good seals? Elastomers – everyone know that. But let us do the job properly; there may be more to be learnt. We build the selection around the requirements of Table 5.15.

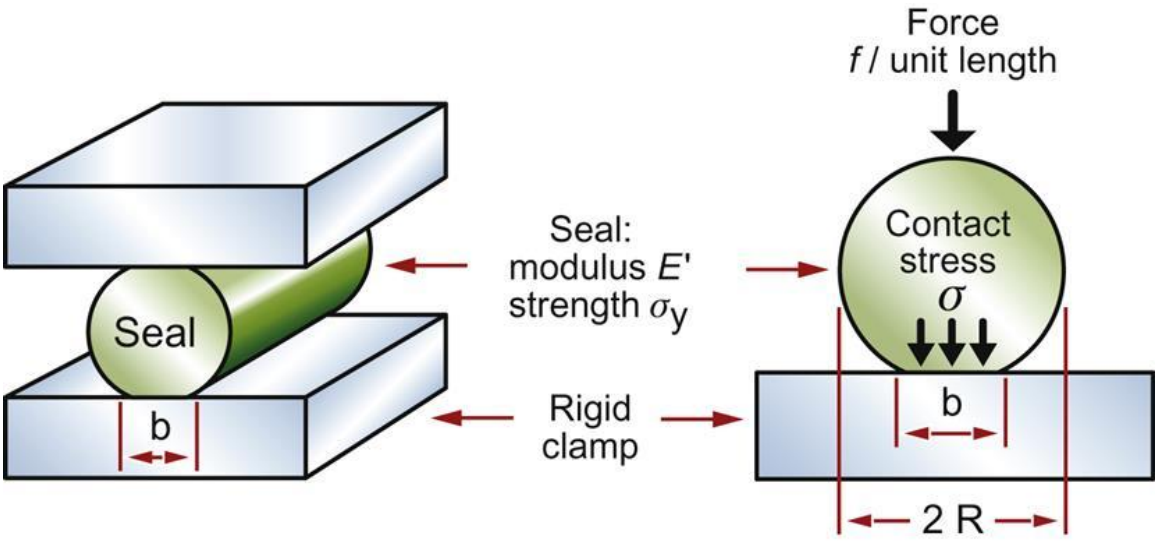


FIGURE 5.15 An elastic seal. A good seal gives a large conforming contact area without imposing damaging loads on itself or on the surfaces with which it mates.

Table 5.15

Design Requirements for Elastic Seals

Function	■ *Elastic seal*
Constraint	■ *Limit on contact pressure* ■ *Low cost*
Objectives	■ *Maximum conformability to surface*
Free variables	■ *Choice of material*

The translation. A cylinder of diameter $2R$ and modulus E, pressed on to a rigid flat surface by a force f per unit length, forms an elastic contact of width b (Appendix B, 'Useful Solutions') where

$$b \approx 2.3\left(\frac{fR}{E}\right)^{1/2} \qquad \textbf{(5.30)}$$

This is the quantity to be maximized: the objective function. The contact stress, both in the seal and in the surface, is adequately approximated (Appendix B) by

$$\sigma = 0.57\left(\frac{fE}{R}\right)^{1/2} \tag{5.31}$$

The constraint: the seal must remain elastic, i.e., σ must be less than the yield or failure strength, σ_f, of the material of which it is made. Combining the last two equations with this condition gives

$$b \leq 4.0R\left(\frac{\sigma_f}{E}\right). \tag{5.32}$$

The contact width is maximized by maximizing the index

$$M_1 = \frac{\sigma_f}{E}$$

It is also required that the contact stress σ be kept low to avoid damage to the flat surfaces. Its value when the maximum contact force is applied (to give the biggest width) is simply σ_f, the failure strength of the seal. Suppose the flat surfaces are damaged by a stress of greater than 100 MPa. The contact pressure is kept below this by requiring that Polytetrafluorethylene (PTFE), ionomers, leather and cork. The candidates are listed in Table 5.16 with commentary. The value of $M_2 = 100$ MPa admits all elastomers as candidates. If M_2 were reduced to 10 MPa, elastomers, leather and cork remain (Fig. 5.16).

Table 5.16
Materials for Reusable Seals

Material	$M_1 = \frac{\sigma_f}{E}$	Comment
Elastomeric EVA	0.7–1	The natural choice; poor resistance to heat and to some solvents.
Polyurethanes	2–5	Widely used for seals.
Silicone rubbers	0.2–0.5	Higher temperature capability than carbon-chain elastomers, chemically inert.
PTFE	0.05–0.1	Expensive but chemically stable and with high-temperature capability.
Polyethylenes	0.02–0.05	Cheap but liable to take a permanent set.
Polypropylenes	0.2–0.04	Cheap but liable to take a permanent set.
Nylons	0.02–0.03	Near upper limit on contact pressure.
Cork	0.03–0.06	Low contact stress, chemically stable.
Polymer foams	up to 0.03	Very low contact pressure; delicate seals.

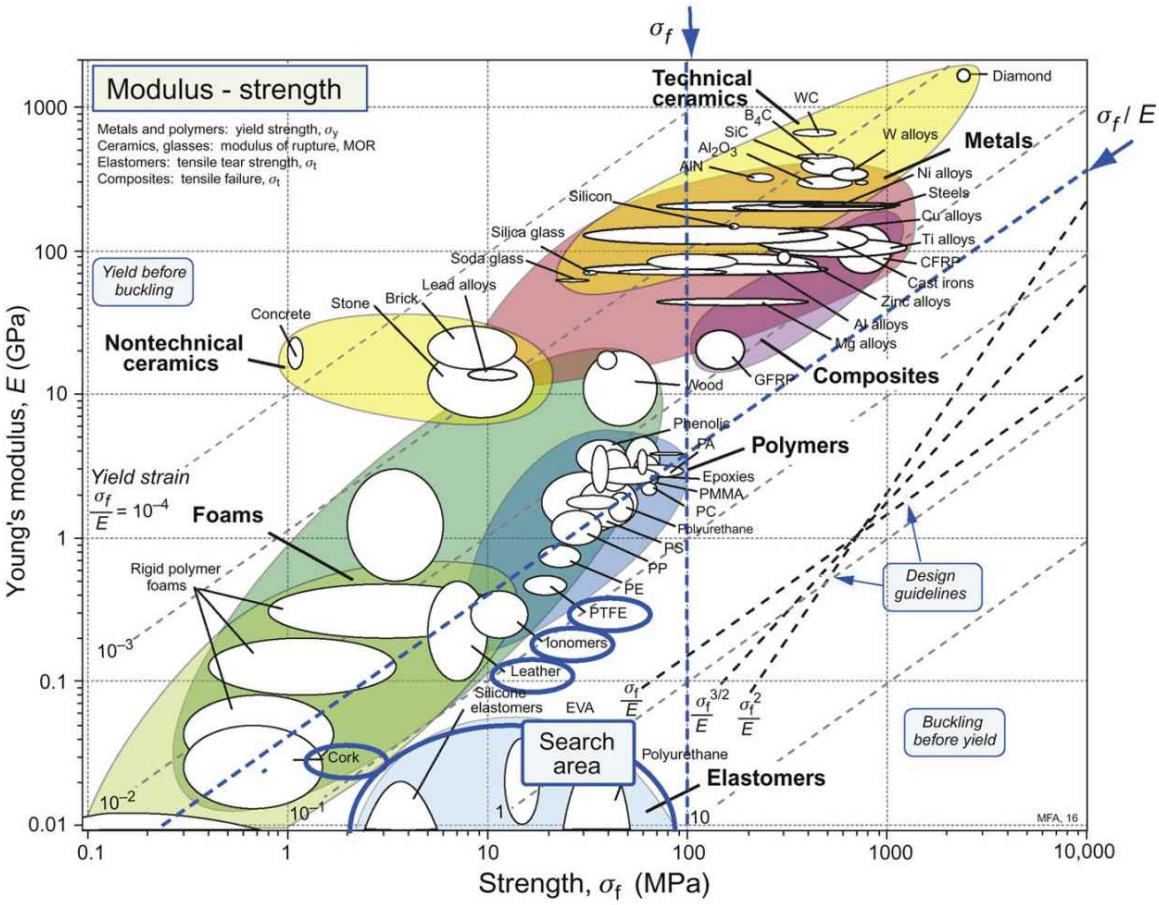


FIGURE 5.16 Materials for elastic seals. Elastomers, compliant polymers and foams make good seals.

Postscript. The analysis highlights the functions that seals must perform: large contact area, limited contact pressure, environmental stability. Elastomers maximize the contact area; foams and cork minimize the contact pressure; PTFE and silicone rubbers best resist heat and organic solvents. The final choice depends on the conditions under which the seal will be used.

Related Case Studies

5.7 'Materials for springs'

5.8 'Elastic hinges and couplings'

5.10 Deflection-Limited Design with Brittle Polymers

The resistance of a material to the propagation of a crack is measured by its plane-strain fracture toughness value, K_{1c}. Among mechanical engineers there is a rule-of-thumb: avoid materials with $K_{1c} < 15$ MPa m$^{1/2}$. Almost all metals pass: they have values of K_{1c} in the range of 20–100 in these units. White cast iron and some powder-metallurgy products fail; they have values as low as 10 MPa·m$^{1/2}$. Ordinary engineering ceramics have values in the range 1–6 MPa·m$^{1/2}$, and mechanical engineers view them with deep suspicion. But engineering polymers are even less tough, with K_{1c} in the range 0.5–3 MPa·m$^{1/2}$ and yet engineers use them all the time. What is going on here?

When a brittle material is deformed, it deflects elastically until it fractures. The stress at which this happens is

$$\sigma_f = \frac{CK_{1c}}{\sqrt{\pi a_c}} \qquad \textbf{(5.33)}$$

where K_c is an appropriate fracture toughness, a_c is the length of the largest crack contained in the material and C is a constant that depends on geometry, but is usually about 1. In a *load-limited* design – a tension member of a bridge, say – the part will fail in a brittle way if the stress exceeds that given by Eq. (5.33). Here, obviously, we want materials with high values of K_{1c}.

But not all designs are load-limited; some are *energy-limited*, others are *deflection limited*. When this happens, the criterion for selection changes.

Consider, then, the three scenarios created by the three alternative constraints of Table 5.17.

Table 5.17
Design Requirements

Function	■ *Resist brittle fracture*
Constraint	■ *Design load specified or* ■ *Design energy specified or* ■ *Design deflection specified*
Objectives	■ *Minimize volume (mass, cost)*
Free variables	■ *Choice of material*

The translation. In load-limited design the component must carry a specified load or pressure without fracturing. It is usual to identify K_c with the plane-strain fracture toughness, K_{1c}, corresponding to the most highly constrained cracking conditions, because this is conservative. Then, as Eq. (5.33) shows, the best choice of materials for minimum volume design are those with high values of

$$M_1 = K_{1c} \tag{5.34}$$

For load-limited design using thin sheet, a plane-stress fracture toughness may be more appropriate; and for multilayer materials, it may be an interface fracture toughness that matters. The point, though, is clear enough: the best materials for load-limited design are those with large values of the appropriate K_c.

But, as we have said, not all design is load-limited. Springs, and containment systems for turbines and flywheels are *energy*-limited. Take the spring (Fig. 5.11) as an example. The elastic energy per unit volume stored in it is the integral over the volume of

$$U_e = \frac{1}{2}\sigma\varepsilon = \frac{1}{2}\frac{\sigma^2}{E}$$

The stress is limited by the fracture stress of Eq. (5.33) so that – if 'failure' means 'fracture' – the maximum energy the spring can store is

$$U_e^{\max} = \frac{C^2}{2\pi\alpha_c}\left(\frac{K_{1c}^2}{E}\right)$$

For a given initial flaw size, energy is maximized by choosing materials with large values of

$$M_2 = \frac{K_{1c}^2}{E} \approx G_c \tag{5.35}$$

where G_c is the toughness (usual units: kJ/m^2).

There is a third scenario: that of *displacement*-limited design (Fig. 5.17). Snap-on bottle tops, snap-together fasteners and such like are displacement-limited: they must allow sufficient elastic displacement to permit the snap-action without failure, which requires a large failure strain ε_f. The strain is related to the stress by Hooke's law $\varepsilon = \sigma/E$ and the stress is limited by the fracture Eq. (5.33). Thus the failure strain is

$$\varepsilon_f = \frac{C}{\sqrt{\pi a_c}}\frac{K_{1c}}{E}$$

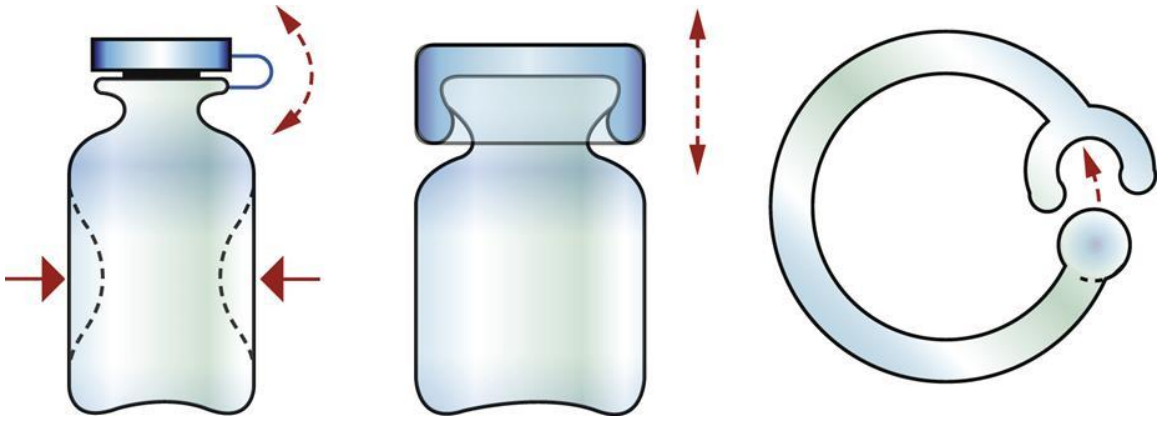


FIGURE 5.17 Polymers, having low moduli, frequently require deflection-limited design methods.

The best materials for displacement-limited design are those with large values of

$$M_3 = \frac{K_{1c}}{E}$$

The selection. Fig. 5.18 shows the chart of fracture toughness, K_{1c}, plotted against modulus E. It allows materials to be compared by values of fracture toughness, M_1, by toughness, M_2, and by values of the deflection-limited index M_3. As the engineer's rule-of-thumb demands, almost all metals have values of K_{1c} that lie above the 15 MPa·m$^{1/2}$ acceptance level for load-limited design, shown as a horizontal selection line on the figure. Polymers and ceramics do not.

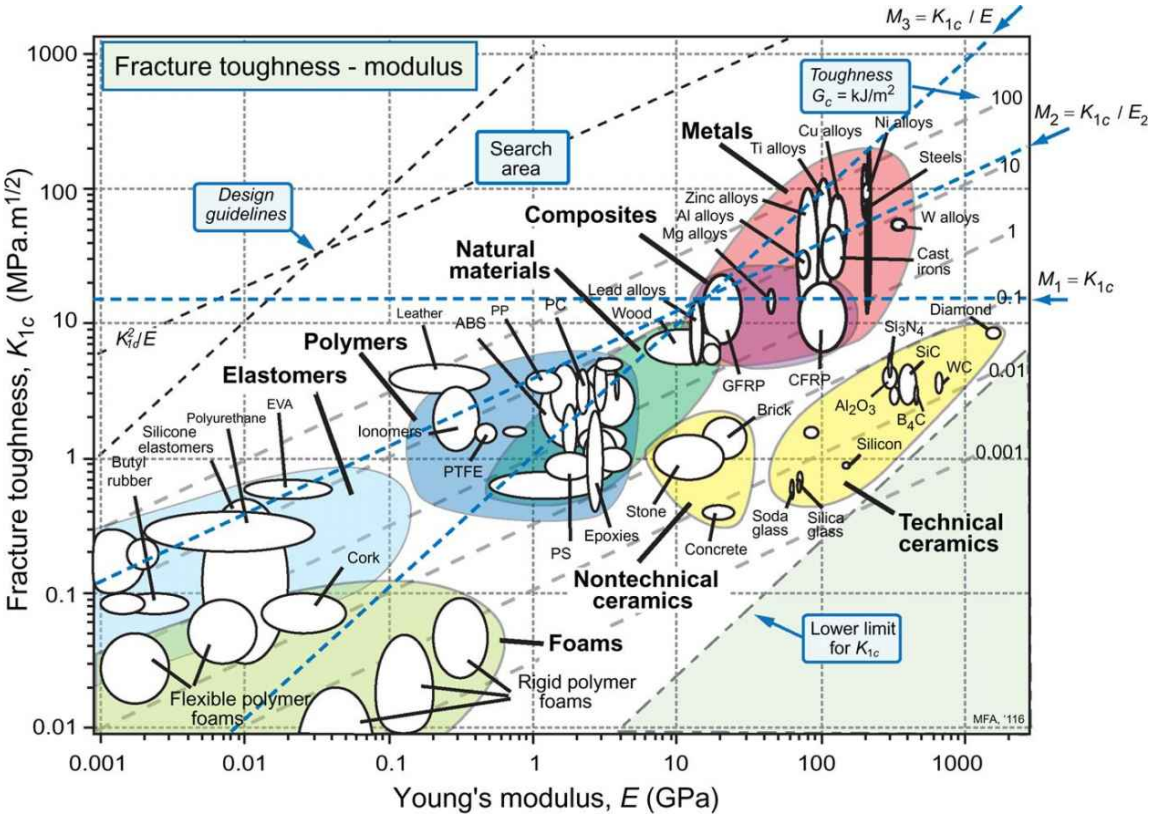


FIGURE 5.18 The selection of materials for load, deflection and energy-limited design. In deflection-limited design, polymers are as good as metals, despite having very low values of the fracture toughness.

The line showing M_2 on Fig. 5.18 is placed at the value 1 kJ/m^2. Materials with values of M_2 greater than this have a degree of shock resistance with which engineers feel comfortable (another rule-of-thumb). Metals, composites and some polymers qualify; ceramics do not. When we come to deflection-limited design, the picture changes again. The line shows the index $M_3 = K_{1c}/E$ at the value 10^{-3} m$^{1/2}$. It illustrates why polymers find such wide application: when the design is deflection-limited, polymers – particularly polypropylene, ABS and nylons – are better than the best metals (Table 5.18).

Table 5.18

Materials Fracture-Limited Design

Design Type and Rule-of-Thumb Material	
Load-limited design $K_{1c} > 15\ \text{Mpa m}^{1/2}$	Metals, polymer-matrix composites
Energy-limited design $J_c > 1\ \text{kJ/m}^2$	Metals, composites and some polymers
Displacement-limited design $K_{1c}/E > 10^{-3}\text{m}^{1/2}$	Polymers, elastomers and the toughest metals

Postscript. The figure gives further insights. The mechanical engineers' love of metals (and, more recently, of composites) is inspired not merely by the appeal of their K_{1c} values. They are good by all three criteria (K_{1c}, K_{1c}^2/E and K_{1c}/E). Polymers have good values of K_{1c}/E and are acceptable by K_{1c}^2/E. Ceramics are poor by all three criteria. Herein lie the deeper roots of the engineers' distrust of ceramics.

Further Reading

Background in fracture mechanics and safety criteria can be found in the following.

Brock, D., 1984. "Elementary engineering fracture mechanics", Martinus Nijoff, Boston.

Hellan, K., 1985. "Introduction to fracture mechanics", McGraw-Hill.

Hertzberg, R.W., (1989) "Deformation and fracture mechanics of engineering materials", Wiley, NY.

Related Case Studies

5.7 'Materials for springs'
5.8 'Elastic hinges and couplings'
5.11 'Safe pressure vessels'

5.11 Safe Pressure Vessels

Pressure vessels, from the simplest aerosol-can to the biggest boiler, are designed for safety, to yield or leak before they break. The details of this design method vary. Small pressure vessels are usually designed to allow general yield at a pressure still too low to cause any crack the vessel may

contain to propagate ('yield-before-break'); the distortion caused by yielding is easy to detect and the pressure can be released safely. With large pressure vessels this may not be possible. Instead, safe design is achieved by ensuring that the smallest crack that will propagate unstably has a length greater than the thickness of the vessel wall ('leak-before-break'). The leak is easily detected and it releases pressure gradually and thus safely (Table 5.19). The two criteria lead to different material indices. What are they?

Table 5.19

Design Requirements for Safe Pressure Vessels

Function	■ *Pressure vessel (contain pressure p safely)*
Constraint	■ *Radius R specified*
Objectives	1. *Maximize safety using yield-before-break criterion, or* 2. *Maximize safety using leak-before-break criterion*
Free variables	■ *Choice of material*

The translation. The stress in the wall of a thin-walled spherical pressure vessel of radius R (Fig. 5.19) is

$$\sigma = \frac{pR}{2t} \qquad \textbf{(5.36)}$$

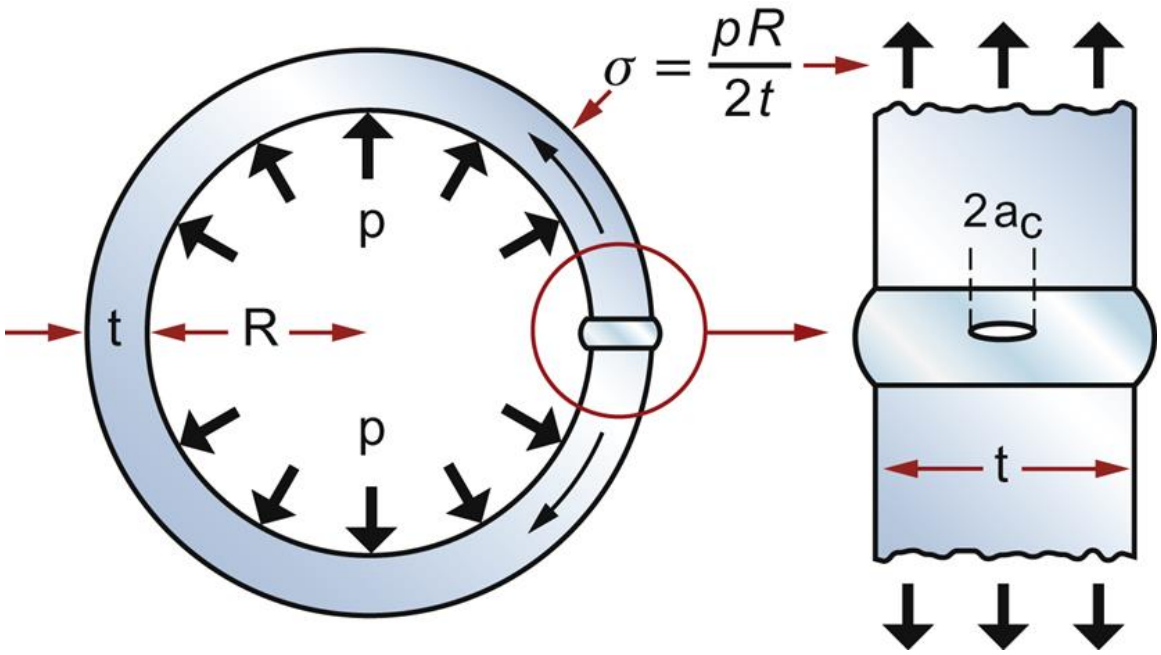


FIGURE 5.19 A pressure vessel containing a flaw.

In pressure vessel design, the wall thickness, t, is chosen so that, at the working pressure p, this stress is less than the yield strength σ_f of the wall (with a safety factor, of course). A small pressure vessel can be examined ultrasonically, or by X-ray methods, or proof tested, to establish that it contains no crack or flaw of diameter greater than $2a_c^*$; then the stress required to make the crack propagate[2] is

$$\sigma = \frac{CK_{1c}}{\sqrt{\pi a_c^*}} \tag{5.37}$$

where C is a constant near unity and K_{1c} is the plane-strain fracture toughness. Safety can be achieved by ensuring that the working stress is less than this, giving

$$p \leq \frac{2t}{R}\frac{K_{1c}}{\sqrt{\pi a_c^*}} \tag{5.38}$$

The largest pressure (for a given R, t and a_c^*) is carried by the material with the greatest value of

$$M_1 = K_{1c} \tag{5.39}$$

But this design is not fail-safe. If the inspection is faulty, or if, for some other reason a crack of length greater than a_c^* appears, catastrophe follows. Greater security is obtained by requiring that the crack will not propagate even if the stress reaches the general yield stress – for then the vessel will deform stably in a way that can be detected. This condition is expressed by setting σ equal to the yield stress σ_Y giving

$$\pi a_c \leq C^2 \left[\frac{K_{1c}}{\sigma_Y}\right]^2$$

The tolerable crack size, and thus the integrity of the vessel, is maximized by choosing a material with the largest value of

$$M_2 = \frac{K_{1c}}{\sigma_Y} \tag{5.40}$$

Large pressure vessels cannot always be X-rayed or sonically tested; and proof testing them may be impractical. Further, cracks can grow slowly because of corrosion or cyclic loading, so that a single examination at the beginning of service life is not sufficient. Then safety can be ensured by arranged that a crack just large enough to penetrate both the inner and the outer surface of the vessel is still stable, because the leak caused by the crack can be detected. This is achieved by setting $a_c^* = t/2$. Safety is ensured if the stress is always less than or equal to

$$\sigma = \frac{CK_{1c}}{\sqrt{\pi t/2}} \tag{5.41}$$

The wall thickness t of the pressure vessel was, of course, designed to contain the pressure p without yielding. From Eq. (5.36), this means that

$$t \geq \frac{pR}{2\sigma_Y} \tag{5.42}$$

Substituting this into the previous equation (with $\sigma = \sigma_f$) gives

$$p \leq \frac{4C^2}{\pi R}\left(\frac{K_{1c}^2}{\sigma_y}\right) \tag{5.43}$$

The maximum pressure is carried most safely by the material with the greatest value of

$$M_3 = \frac{K_{1c}^2}{\sigma_y} \tag{5.44}$$

Both M_2 and M_3 could be made large by making the yield strength of the wall, σ_y, very small: lead, for instance, has high values of both, but you would not choose it for a pressure vessel. That is because the vessel wall must also kept thin, both for economy of material, and to keep it light. The thinnest wall, from Eq. (5.42), is that with the largest yield strength, σ_y. Thus we wish also to maximize

$$M_4 = \sigma_y \tag{5.45}$$

narrowing further the choice of material.

The selection. These selection criteria are explored by using the chart shown in Fig. 5.20: the fracture toughness, K_{1c}, plotted against elastic limit σ_f (meaning yield strength for metals and polymers). The indices M_1, M_2, M_3 and M_4 appear as lines of slope 0, 1, 1/2 and as lines that are vertical. Take 'yield-before-break' as an example. A diagonal line corresponding to a constant value of $M_2 = K_{1c}/\sigma_y$ links materials with equal performance; those above the line are better. The line shown in the figure at a value of M_1 corresponding to a process zone of size 10 mm, excludes everything but the toughest steels, copper, aluminium and titanium alloys, though some polymers – PP, PE and PET for instance – nearly make it (pressurised lemonade and beer containers are made of these polymers) (Table 5.20).

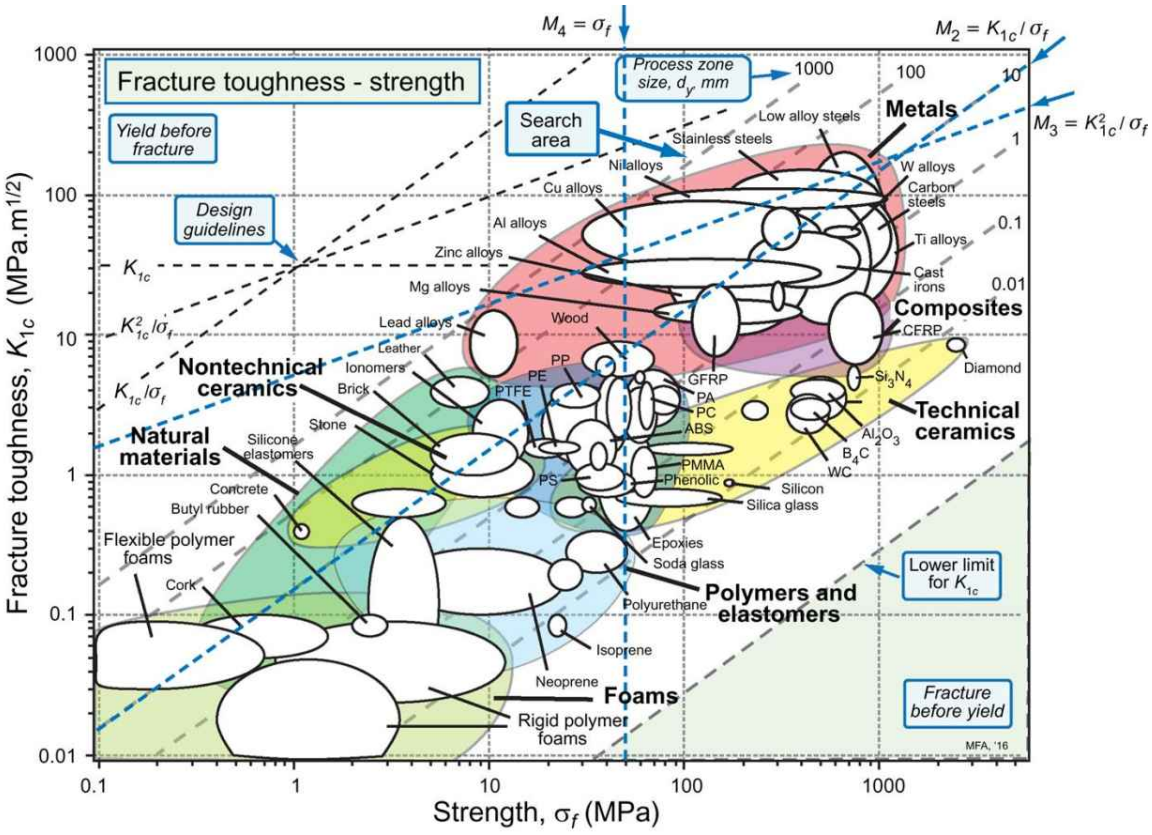


FIGURE 5.20 Materials for pressure vessels. Steel, copper alloys and aluminium alloys best satisfy the 'yield-before-break' criterion. In addition, a high yield strength allows a high working pressure. The materials in the search areas triangle are the best choice. The leak-before-break criterion leads to essentially the same selection.

Table 5.20
Materials for Safe Pressure Vessels

Material	$M_2 = K_{1c}/\sigma_y$ ($m^{1/2}$)	$M_4 = \sigma_y$ (MPa)	Comment
Stainless steels	0.35	300	Nuclear pressure vessels are made of grade 316 stainless steel.
Low alloys steels	0.2	800	These are standard in this application.
Copper	0.5	200	Hard drawn copper is used for small boilers and pressure vessels.
Aluminium alloys	0.15	200	Pressure tanks of rockets are aluminium.
Titanium alloys	0.13	800	Good for light pressure vessels, but expensive.

The leak-before-break criterion

$$M_3 = \frac{K_{Ic}^2}{\sigma_y}$$

favours low alloy steel, stainless and carbon steels more strongly. Polymers not longer qualify as candidates.

Postscript. Large pressure vessels are made of steel or – in the case of submarines – of titanium. Those for models – a model steam engine, for instance – are made of copper. Copper is chosen, even though it is more expensive, because of its greater resistance to corrosion. Corrosion rates do not scale with size. The loss of 0.1 mm through corrosion is not serious in a pressure vessel that is 10 mm thick; but if it is only 1 mm thick it becomes a concern.

Boiler failures used to be common place – there are even songs about it. Now they are rare, though when safety margins are pared to a minimum (rockets, new aircraft designs) pressure vessels still occasionally fail. This (relative) success is one of the major contributions of fracture mechanics to engineering practice.

Further Reading

Background in fracture mechanics and safety criteria can be found in the following:

Brock, D., 1984. 'Elementary engineering fracture mechanics', Martinus Nijoff, Boston.

Hellan, K., 1985. 'Introduction to fracture mechanics', McGraw-Hill.

Hertzberg, R.W., 1989. 'Deformation and fracture mechanics of engineering materials', Wiley, NY.

Related Case Studies

5.6 'Materials for flywheels'
5.9 'Deflection-limited design with brittle polymers'
9.11 'Multiple constraints: light pressure vessels'

5.12 Stiff, High Damping Materials for Shaker Tables

Shakers, if you live in Pennsylvania, are the members of an obscure and declining religious sect, noted for their austere wooden furniture, tables among them. To those who live elsewhere they are devices for vibration-testing (Fig. 5.21). This second sort of shaker consists of an electromagnetic actuator driving a table, at frequencies up to 1000 Hz, to which the test

object (a space probe, an automobile, an aircraft component or the like) is clamped. The shaker applies a spectrum of vibration frequencies, f, and amplitudes, A, to the test-object to explore its response.

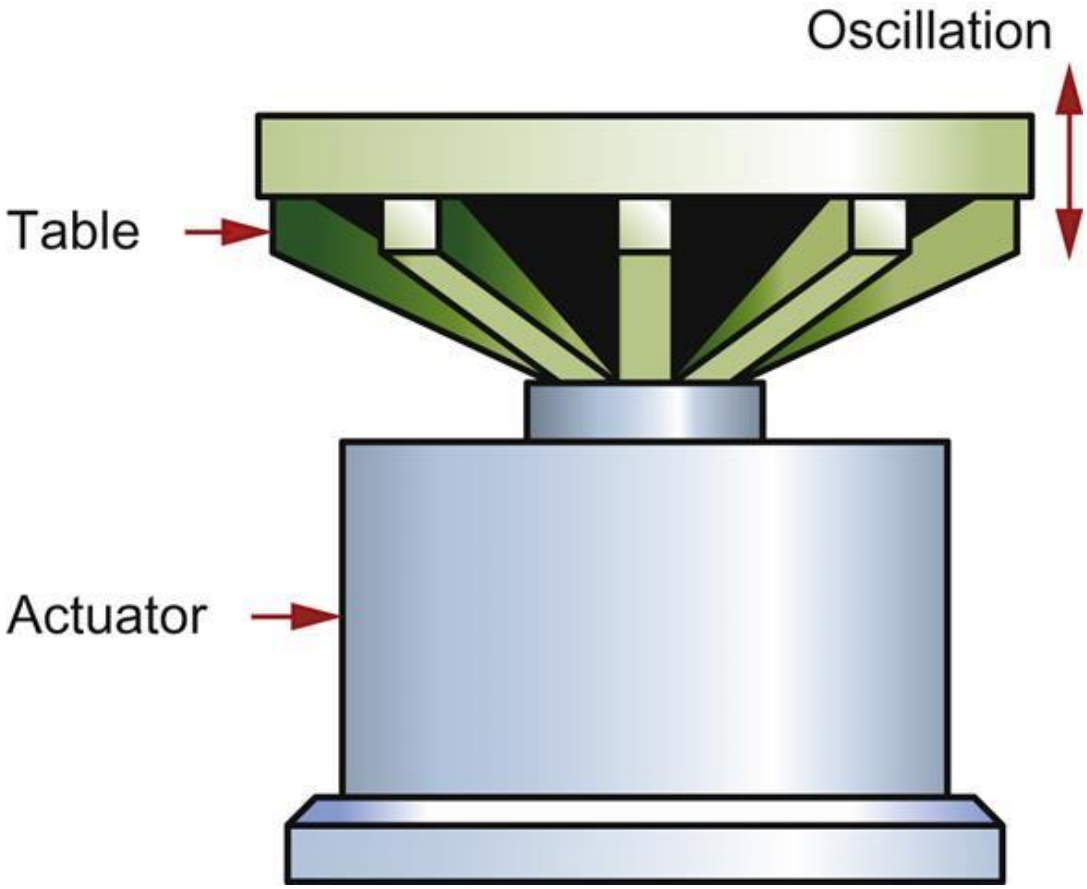


FIGURE 5.21 A shaker table. It must be stiff but have high intrinsic 'damping' or loss coefficient.

A big table operating at high frequency dissipates a great deal of power. The primary objective is to minimize this, but subject to a number of constraints itemised in Table 5.21. What materials make good shaker tables?

Table 5.21

Design Requirements for Shaker Tables

Function	■ *Table for vibration tester ('shaker table')*
Constraint	■ *Radius, R, specified* ■ *Must be stiff enough to avoid distortion by clamping forces* ■ *Natural frequencies above maximum operating frequency (to avoid resonance)* ■ *High damping to suppress resonance and natural vibrations* ■ *Tough enough to withstand mishandling and shock*
Objectives	■ *Minimize power consumption*
Free variables	■ *Choice of material* • *Table thickness, t*

The translation. The power p (Watts) consumed by a dissipative vibrating system with a sinusoidal input is

$$p = C_1 mA^2\omega^3 \tag{5.46}$$

where m is the mass of the table, A is the amplitude of vibration, ω is the frequency (rad/s) and the constant $C_1 \approx 1$, provided the operating frequency ω is significantly less than the resonant frequency of the table. The amplitude A and the frequency ω are prescribed. To minimize the power lost in shaking the table itself, we must minimize its mass m. We idealize the table as a disk of given radius, R. Its thickness, t, is a free variable. Its mass is

$$m = \pi R^2 t\rho \tag{5.47}$$

where ρ is the density of the material of which it is made. The thickness influences the bending stiffness of the table – and this is important both to prevent the table flexing too much under clamping loads and because it determines its natural vibration frequencies. The bending stiffness, S, is

$$S = \frac{C_2 EI}{R^3}$$

where C_2 is a constant. The second moment of the section, I, is proportional to t^3R. Thus, for a given stiffness S and radius R,

$$t = C_3\left(\frac{SR^2}{E}\right)^{1/3}$$

where C_3 is another constant. The thinnest table is the one made from the material with the greatest value of

$$M_1 = E$$

Inserting this expression for t into Eq. (5.47) we obtain

$$m = C_3\pi R^{8/3}S^{1/3}\left(\frac{\rho}{E^{1/3}}\right). \quad \textbf{(5.48)}$$

The mass of the table, for a given stiffness and minimum vibration frequency, is therefore minimized by selecting materials with high values of

$$M_2 = \frac{E^{1/3}}{\rho}$$

There are three further requirements. The first is that of high mechanical damping, measured by the loss coefficient, η, to suppress resonance. The second that the yield strength and fracture toughness K_{1c} of the table be sufficient to withstand mishandling and clamping forces. And it must to be too thick.

The selection. If we had a chart with $E^{1/3}/\rho$ on one axis and η on the other, we could read off the materials with high values of both. Computer-based methods, illustrated in later case studies, allow charts with any desired combination of properties as axes. But for now we will stick with the charts of Chapter 3, Materials Property Charts, requiring a 2-step selection. Fig. 5.4 (back in Section 5.3) is an $E-\rho$ chart with a contour of $E^{1/3}/\rho$ already plotted on it. Materials with high values lie above or just below it. They are listed in the first column of Table 5.22. We now turn to the $\eta - E$

chart, reproduced as Fig. 5.22. Materials with high values of M_1 lie to the right of the vertical line (here set at 30 GPa); those with $\eta > 0.001$ lie above the horizontal one. The search region contains a number of the candidates in the table: CFRP and alloys of magnesium, aluminium and titanium. All are possible candidates. Table 5.22 compares their properties.

Table 5.22

Materials for Shaker Tables

Material	$M_1 = E^{1/3}/\rho$ GPa$^{1/3}$/(Mg/m^3)	Loss coeff, η	Fr. Toughness K_{1c} MPa m$^{1/2}$	Comment
Mg-alloys	Up to 2.3	Up to 0.03	15	The best combination of properties.
Al-alloys	Up to 1.7	Up to 0.002	30	Less damping than Mg or Ti.
Ti-alloys	Up to 1.1	Up to 0.003	60	Good damping but expensive.
CFRP	Up to 3.4	Up to 0.003	15	Less damping than Mg-alloys, but possible.
Various ceramics	Up to 3.0	About 0.0002	3	Ruled out by low damping and toughness.
Various foams	Up to 10	Up to 0.5	0.1	Lacks strength and toughness to carry service loads.

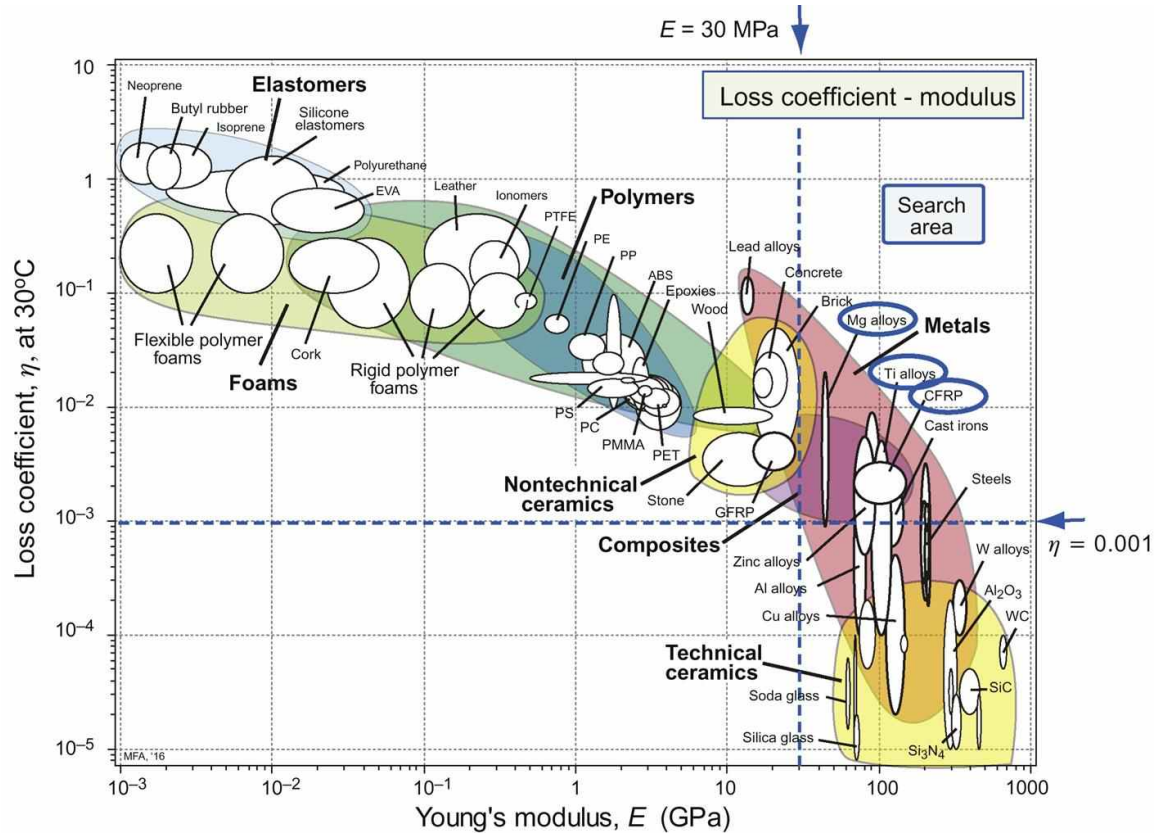


FIGURE 5.22 Selection of materials for the shaker table. Magnesium alloys, cast irons, GFRP, concrete and the special high-damping Mn–Cu alloys are candidates.

Postscript. Stiffness, high natural frequencies and damping are qualities often sought in engineering applications such as engine and machine-tool mountings, supports for precision instruments and building foundations. The shaker table found its solution (in real life as well as this case study) in the choice of a cast magnesium alloy.

Sometimes a solution is possible by combining materials (more on this in Chapter 12, Designing Hybrid Materials). The loss coefficient chart shows that polymers and elastomers have high damping. Sheet steel panels, prone to vibration, can be damped by coating one surface with a polymer, a technique exploited in automobiles, machine tools and typewriters (before they became obsolete). Aluminium structures can be stiffened (raising natural frequencies) by bonding carbon-fibre to them: an approach sometimes use in aircraft design. And structures loaded in bending or torsion can be made lighter, for the same stiffness (again increasing natural frequencies), by shaping them efficiently: by attaching ribs to their underside, for instance. Shaker tables – even the austere wooden tables of the Pennsylvania Shakers – exploit shape in this way.

Further Reading

Tustin, W., Mercado, R., 1984. "Random vibrations in perspective", Tustin Institute of Technology Inc, Santa Barbara, CA, USA.

Cebon, D., Ashby, M.F., 1994. "Materials selection for precision instruments", Meas. Sci. and Technol. 5 pp 296–306.

Related Case Studies

5.4 'Materials for table legs'
5.7 'Materials for springs'
5.16 'Minimizing distortion in precision devices'

5.13 Insulation for Short-Term Isothermal Containers

Each member of the crew of a military aircraft carries, for emergencies, a radio beacon. If forced to eject, the crew member could find himself in trying circumstances – in water at 4°C, for example (much of the Earth's surface is ocean with a mean temperature of roughly this). The beacon guides friendly rescue services, minimizing exposure time.

But microelectronic metabolisms (like those of humans) are upset by low temperatures. In the case of the beacon, it is its transmission frequency that starts to drift. The design specification for the egg-shaped package

containing the electronics (Fig. 5.23) requires that, when the temperature of the outer surface is changed by 30°C, the temperature of the inner surface should not change significantly for an hour. To keep the device small, the wall thickness is limited to a thickness w of 20 mm. What is the best material for the package? A Dewar system is out – it is too fragile.

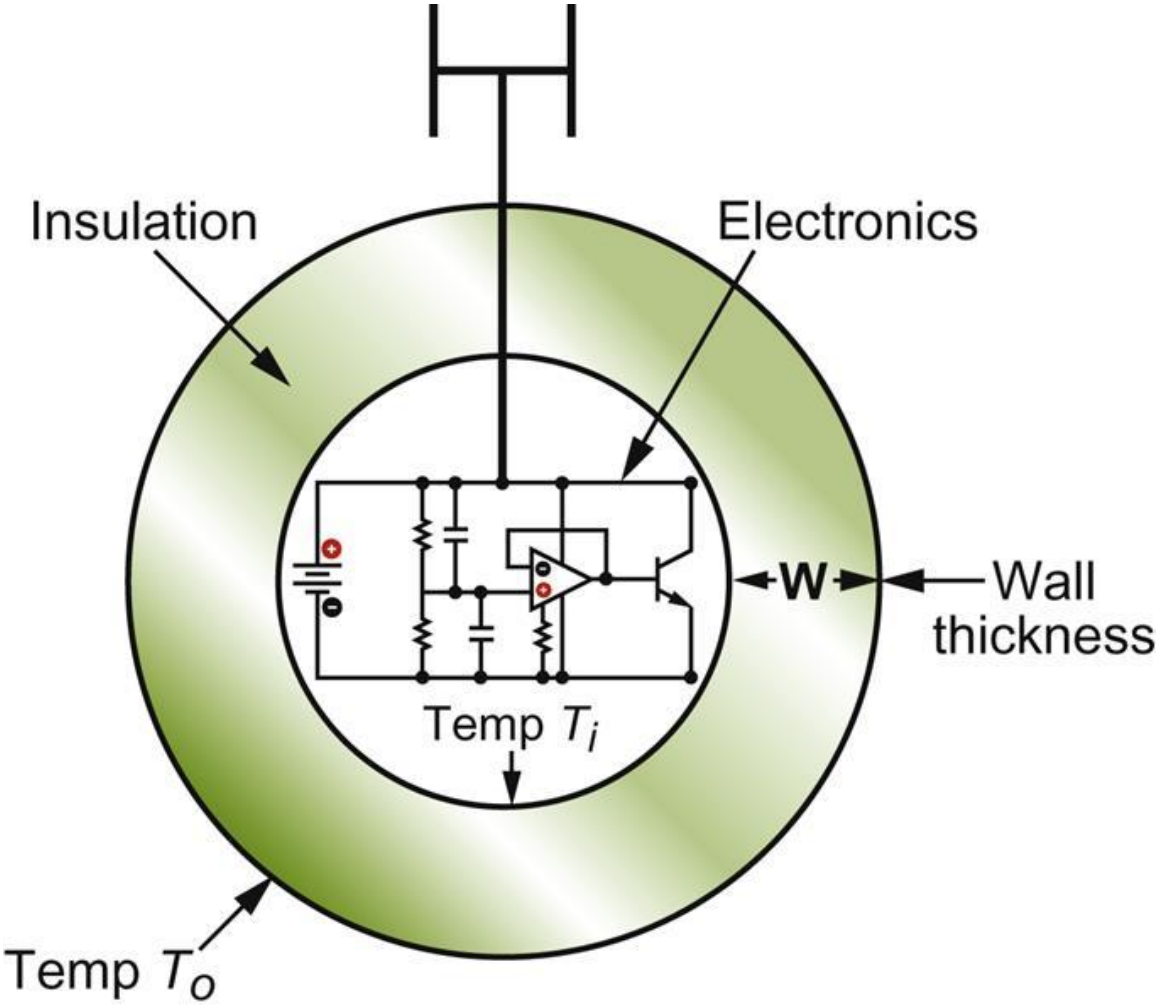


FIGURE 5.23 An isothermal container.

A foam of some sort, you might think. But here is a case in which intuition leads you astray. So let us formulate the design requirements (Table 5.23) and do the job properly.

Table 5.23

Design Requirements for Short-Term Insulation

Function	■ *Short-term thermal insulation*
Constraint	■ *Wall thickness must not exceed w*
Objectives	■ *Maximize time t before internal temperature changes when external temperature suddenly drops*
Free variables	■ *Choice of material*

The translation. We model the container as a wall of thickness w, thermal conductivity λ. The heat flux q through the wall, once a steady state has been established, is given by Fick's first law (Appendix B):

$$q = -\lambda \frac{dT}{dx} = \lambda \frac{(T_i - T_o)}{w} \tag{5.49}$$

where T_o is the temperature of the outer surface, T_i is that of the inner one and dT/dx is the temperature gradient (Fig. 5.23). The only free variable here is the thermal conductivity, λ. The flux is minimized by choosing a wall material with the lowest possible value of λ. The $\lambda - \alpha$ chart (Fig. 5.24) shows that this is, indeed, a foam.

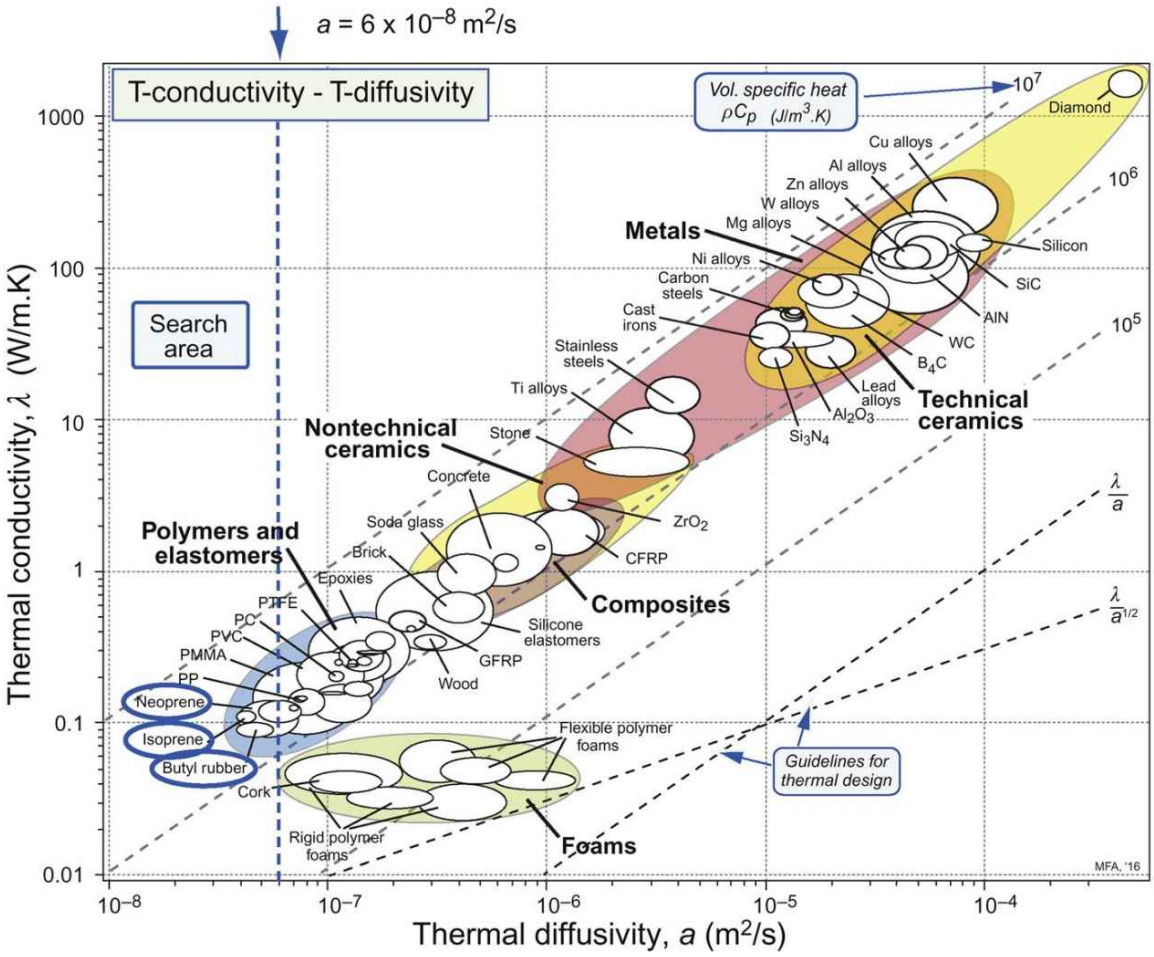


FIGURE 5.24 Materials for short-term isothermal containers. Elastomers are good; foams are not.

But we have answered the wrong question. The design brief was not to minimize the *heat flux* through the wall, but to maximize the *time* before the temperature of the inner wall changed appreciably. When the surface temperature of a body is suddenly changed, a temperature wave, so to speak, propagates inwards. The distance x it penetrates in time t is approximately $\sqrt{2at}$ (Appendix B). Here a is the thermal diffusivity, defined by

$$a = \frac{\lambda}{\rho C_p} \tag{5.50}$$

where ρ is the density and C_p is the specific heat. Equating this to the wall thickness w gives

$$t \approx \frac{w^2}{2a} \qquad (5.51)$$

The time is maximized by choosing the smallest value of the thermal diffusivity, a, not the conductivity λ.

The selection. Fig. 5.24 shows that the thermal diffusivities of foams are not particularly low; it is because they have so little mass and thus heat capacity. The diffusivity of heat in a solid polymer or elastomer is much lower because their specific heats are exceptionally large. A package made of solid rubber, neoprene or isoprene, would – if of the same thickness – give the beacon a life 10 times greater than one made of (say) a polystyrene foam – though of course it would be heavier. Table 5.24 summarizes the conclusions. The reader can confirm, using Eq. (5.51), that 22 mm of neoprene (a=5×10^{-8} m^2/s, read from Fig. 5.24) will allow a time interval of more than 1 hour after an external temperature change before the internal temperature shifts much.

Table 5.24
Materials for Short-Term Thermal Insulation

Material	Comment
Elastomers: Butyl rubber, neoprene and isoprene are examples	Best choice for short-term insulation.
Commodity polymers: polyethylene and polypropylene	Cheaper than elastomers, but somewhat less good for short-term insulation.
Polymer foams	Much less good than elastomers for short-term insulation; best choice for long-term insulation at steady state.

Postscript. One can do better than this. The trick is to exploit other ways of absorbing heat. If a liquid – a low-melting wax, for instance – can be found that solidifies at a temperature equal to the minimum desired operating temperature for the transmitter (T_i), it can be used as a 'latent heat sink'. Channels in the package are filled with the liquid; the inner temperature can only fall below the desired operating temperature when all the liquid has solidified. The latent heat of solidification must be supplied to do this, giving the package a large (apparent) specific heat, and thus an exceptionally low diffusivity for heat at the temperature T_i.

The same idea is used, in reverse, in 'freezer packs' that solidify when placed in the freezer compartment of a refrigerator and remain cold (by melting, at 4°C) when packed around warm beer cans in a portable cooler.

Further Reading

Holman, J.P., 1981. Heat transfer, fifth ed., McGraw-Hill, NY, USA.

Related Case Studies

5.14 'Energy-efficient kiln walls'
5.15 'Materials for passive solar heating'

5.14 Energy-Efficient Kiln Walls

The energy cost of one firing cycle of a large pottery kiln (Fig. 5.25) is considerable. Part is the cost of the energy that is lost by conduction through the kiln walls; it is reduced by choosing a wall material with a low thermal conductivity, λ, and by making the wall thick. The rest is the cost of the energy used to raise the kiln to its operating temperature; it is reduced by choosing a wall material with a low heat capacity, C_p, and by making the wall thin. Is there a material index that captures these apparently conflicting design goals? And if so, what is a good choice of material for kiln walls? The choice is based on the requirements of Table 5.25.

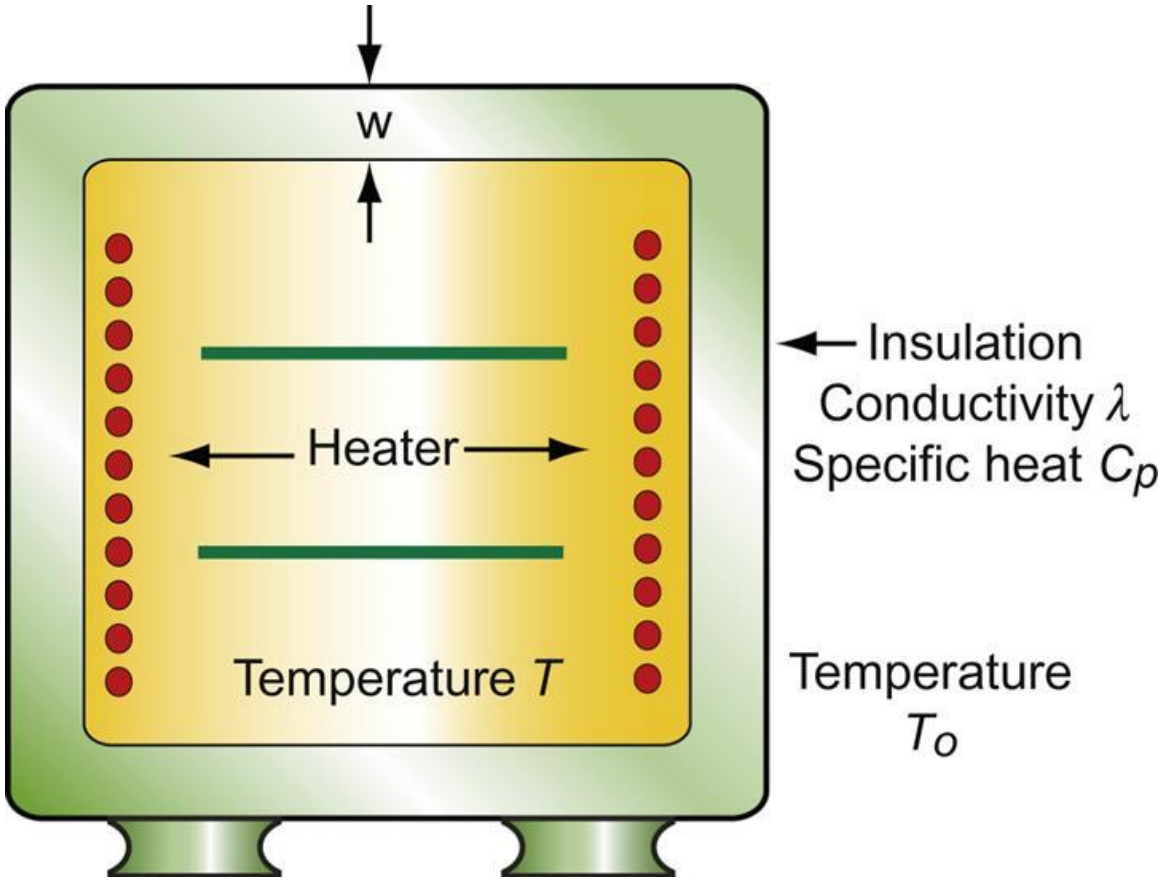


FIGURE 5.25 A kiln. The kiln wall has at thermal mass $C_p\rho$ per unit volume.

Table 5.25

Design Requirements for Kiln Walls

Function	■ *Thermal insulation for kiln (cyclic heating and cooling)*
Constraint	■ *Maximum operating temperature 1000°C* ■ *Possible limit on kiln wall thickness for space reasons*
Objectives	■ *Minimize energy consumed in firing cycle*
Free variables	■ *Kiln wall thickness, w* ■ *Choice of material*

The translation. When a kiln is fired, the internal temperature rises quickly from ambient, T_o, to the operating temperature, T_i, where it is held for the firing time t. The energy consumed in the firing time has, as we

have said, two contributions. The first is the heat conducted out: at steady state the heat loss by conduction, Q_1, per unit area, is given by the first law of heat flow (Appendix B). If held for time t it is

$$Q_1 = -\lambda \frac{dT}{dx} t = \lambda \frac{(T_i - T_o)}{w} t \tag{5.52}$$

Here dT/dx is the temperature gradient and w is the insulation wall thickness. The second contribution is the heat absorbed by the kiln wall in raising it to T_i, and this can be considerable. Per unit area, it is

$$Q_2 = C_p \rho w \left(\frac{T_i - T_o}{2} \right) \tag{5.53}$$

where C_p is the specific heat (J/kg) of the wall material and ρ is its density. The total energy consumed per unit area is the sum of these two:

$$Q = Q_1 + Q_2 = \frac{\lambda (T_i + T_o) t}{w} + \frac{C_p \rho w (T_i - T_o)}{2} \tag{5.54}$$

A wall that is too thin loses much energy by conduction, but absorbs little energy in heating the wall itself. One that is too thick does the opposite. There is an optimum thickness, which we find by differentiating Eq. (5.54) with respect to wall thickness w and equating the result to zero, giving:

$$w = \left(\frac{2\lambda t}{C_p \rho} \right)^{1/2} = (2at)^{1/2}, \tag{5.55}$$

where $a = \lambda / \rho C_p$ is the thermal diffusivity. The quantity $(2at)^{1/2}$ has dimensions of length and is a measure of the distance heat can diffuse in time t. Eq. (5.55) says that the most energy-efficient kiln wall is one that only gets really hot on the outside as the firing cycle approaches completion. Substituting Eq. (5.55) back into Eq. (5.54) to eliminate w gives:

$$Q = (T_i - T_o)(2t)^{1/2}(\lambda C_p \rho)^{1/2}$$

Q is minimized by choosing a material with a low value of the quantity $(\lambda C_p \rho)^{1/2}$, that is, by maximizing

$$M = (\lambda C_p \rho)^{-1/2} = \frac{a^{1/2}}{\lambda} \tag{5.56}$$

By eliminating the wall thickness w we have lost track of it. It could, for some materials, be excessively large. Before accepting a candidate material we must check, by evaluating Eq. (5.55) how thick the wall made from it will be.

The selection. Fig. 5.26 shows the $\lambda - a$ chart, here with additional refractories and foams, with a selection line corresponding to $M = a^{1/2}/\lambda$ plotted on it: material below the line are a better choice than those above. To keep the wall thin, we also require a low value of thermal diffusivity, a, and the vertical line limits choice to materials with $a < 3 \times 10^{-6} \mathrm{m}^2/\mathrm{s}$. Polymer foams, cork and solid polymers are good, but only if the internal temperature is less than 150°C. Real pottery kilns operate near 1000°C, requiring materials with a maximum service temperature above this value. The figure suggests refractory brick, aerated concrete and vermiculite, all of which have a maximum operating temperature above 1000°C (Table 5.26). here the limitation of the hardcopy charts becomes apparent: there is not enough room to show specialized materials such as refractory bricks and concretes. The limitation is overcome by the computer-based methods mentioned in Chapter 4, Materials Selection – the Basics, allowing a search over a much larger number of materials.

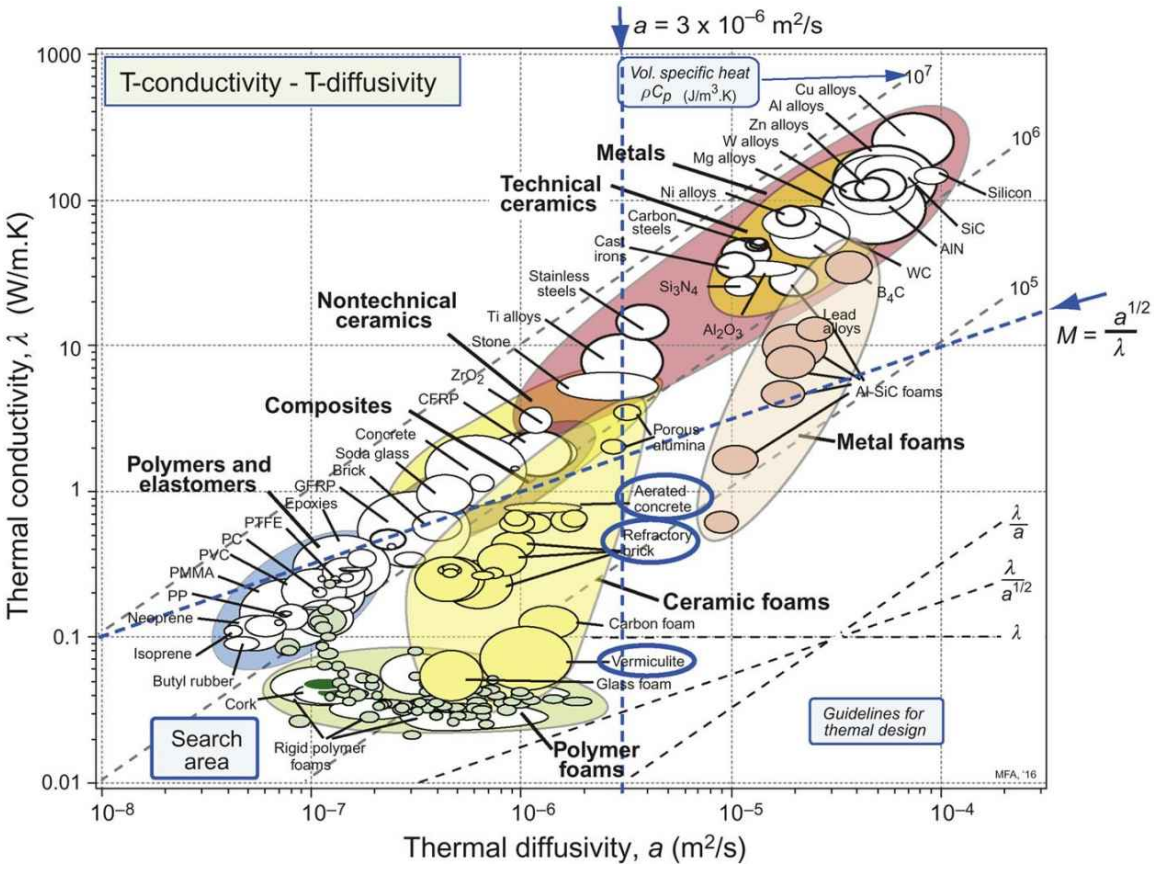


FIGURE 5.26 Materials for kiln walls. Low density, porous or foam-like ceramics are the best choice.

Table 5.26
Materials for Energy-Efficient Kilns

Material	$M = a^{1/2}/\lambda$ ($m^2K/W.s^{1/2}$)	Thickness w (mm)	Comment
Refractory brick	5×10^{-3}	100	The obvious choice: the lower the density, the better the performance. Special refractory bricks have values of M as high as 3×10^{-3}.
Aerated concrete	2×10^{-3}	110	High-temperature concrete can withstand temperatures up to 1000°C.
Glass and carbon foams	Up to 10^{-2}	140	Both offer exceptional thermal insulation, but are limited to temperatures below 800°C.
Woods	2×10^{-3}	60	The boiler of Stevenson's 'Rocket' steam engine was insulated with wood.
Solid elastomers and solid polymers	2×10^{-3}–3×10^{-3} 2×10^{-3}	50	Good values of material index. Useful if the wall must be very thin. Limited to temperatures below 150°C.
Polymer foam, cork	3×10^{-3}–6×10^{-2}	50–140	The highest value of M – hence their use in house insulation. Limited to temperatures below 150°C.

Having chosen a material, the acceptable wall thickness is calculated from Eq. (5.55). It is listed, for a firing time of 3 hours (approximately 10^4 seconds) in Table 5.26.

Postscript. It is not generally appreciated that, in an efficiently-designed kiln, as much energy goes in heating up the kiln itself as is lost by thermal conduction to the outside environment. It is a mistake to make kiln walls

too thick; a little is saved in reduced conduction-loss, but more is lost in the greater heat capacity of the kiln itself.

That, too, is the reason that foams are good: they have a low thermal conductivity *and* a low heat capacity. Centrally heated houses in which the heat is turned off at night suffer a cycle like that of the kiln. Here (because T_i is lower) the best choice is a polymeric foam, cork or fibreglass (which has thermal properties like those of foams). But, as this case study shows, turning the heat off at night doesn't save you as much as you think, because you have to supply the heat capacity of the walls in the morning.

Further Reading

Holman, J.P., 1981. "Heat transfer" 5th Edition, McGraw-Hill, NY, USA.

Related Case Studies

5.13 'Insulation for short-term isothermal containers'
5.15 'Materials for passive solar heating'

5.15 Materials for Passive Solar Heating

There are a number of schemes for capturing solar energy for home heating: roof-mounted solar panels, liquid filled heat exchangers and thermal storage systems. The simplest of these is the heat-storing wall: a thick wall, the outer surface of which is heated by exposure to direct sunshine during the day and from which heat is extracted at night by blowing air over its inner surface (Fig. 5.27). An essential of such a scheme is that the time constant for heat flow through the wall be about 12 hours; then the wall first warms on the inner surface roughly 12 hours after the sun first warms the outer one, giving out at night what it took in during the day. We will suppose that, for architectural reasons, the wall must not be more than ½ m thick. What materials maximize the thermal energy captured by the wall while retaining a heat-diffusion time of up to 12 hours? Table 5.27 summarizes the requirements.

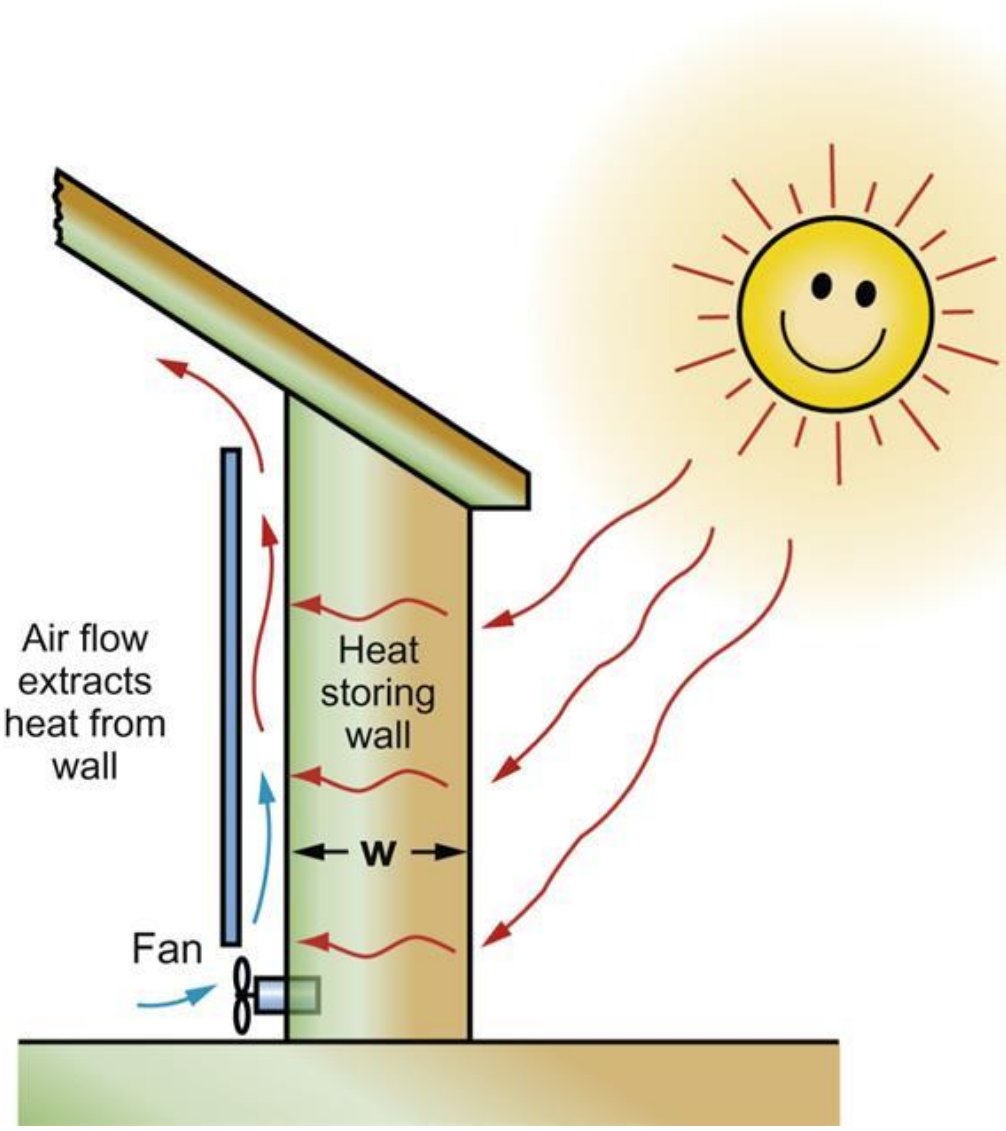


FIGURE 5.27 A heat-storing wall.

Table 5.27

Design Requirements for Passive Solar Heating

Function	■ *Heat-storing medium*
Constraint	■ *Heat-diffusion time through wall t≈12 hours* ■ *Wall thickness ≤0.5 m* ■ *Adequate working temperature* T_{max} *>100°C*
Objective	■ *Maximize thermal energy stored per unit material cost*
Free variables	■ *Wall thickness w* ■ *Choice of material*

The translation. The heat content, Q, per unit area of wall, when heated through a temperature interval ΔT gives the objective function

$$Q = w\rho C_p \Delta T \tag{5.57}$$

where w is the wall thickness, and ρC_p is the volumetric specific heat (the density ρ times the specific heat C_p). The 12-hour time constant is a constraint. It is adequately estimated by the approximation used earlier for the heat-diffusion distance in time t (see Appendix B, 'Useful Solutions')

$$w = \sqrt{2at} \tag{5.58}$$

where a is the thermal diffusivity. Eliminating the free variable w gives

$$Q = \sqrt{2t}\Delta T a^{1/2} \rho C_p \tag{5.59}$$

or, using the fact that $a = \lambda/\rho C_p$ where λ is the thermal conductivity,

$$Q = \sqrt{2t}\Delta T \left(\frac{\lambda}{a^{1/2}}\right)$$

The heat capacity of the wall is maximized by choosing material with a high value of

$$M = \frac{\lambda}{a^{1/2}} \tag{5.60}$$

– it is the reciprocal of the index of the previous case study. The restriction on thickness w requires (from Eq. 5.58) that

$$a \leq \frac{w^2}{2t}$$

with $w \leq 0.5$ m and t=12 hours (4×10^4 s), we obtain an attribute limit (Table 5.28).

$$a \leq 3 \times 10^{-6} \mathrm{m}^2/s \tag{5.61}$$

Table 5.28

Materials for Passive Solar Heat-Storage

Material	$M_1 = \lambda/a^{1/2}$ $(\mathrm{W \cdot s^{1/2}/m^2 \cdot K})$	Approx. Cost $/m³ (2016 USD)	Comment
Adobe	1.0×10^3	0–200	Cheap if you do it yourself; less so if professional.
Concrete	2.2×10^3	200	A good choice – good performance at minimum cost.
Stone	3.5×10^3	1400	Better performance than concrete because specific heat is greater, but more expensive.
Brick	10^3	1400	Less good than concrete.
Glass	1.6×10^3	10,000	Useful – part of the wall could be glass.
Titanium	4.6×10^3	200,000	An unexpected, but valid, selection. Expensive.

The selection. Fig. 5.28 shows thermal conductivity λ plotted against thermal diffusivity a with M and the limit on a plotted on it. It identifies the group of materials, listed in the table, that maximize M_1 while meeting the constraint on wall thickness. Solids are good; porous materials and foams (often used in walls) are not.

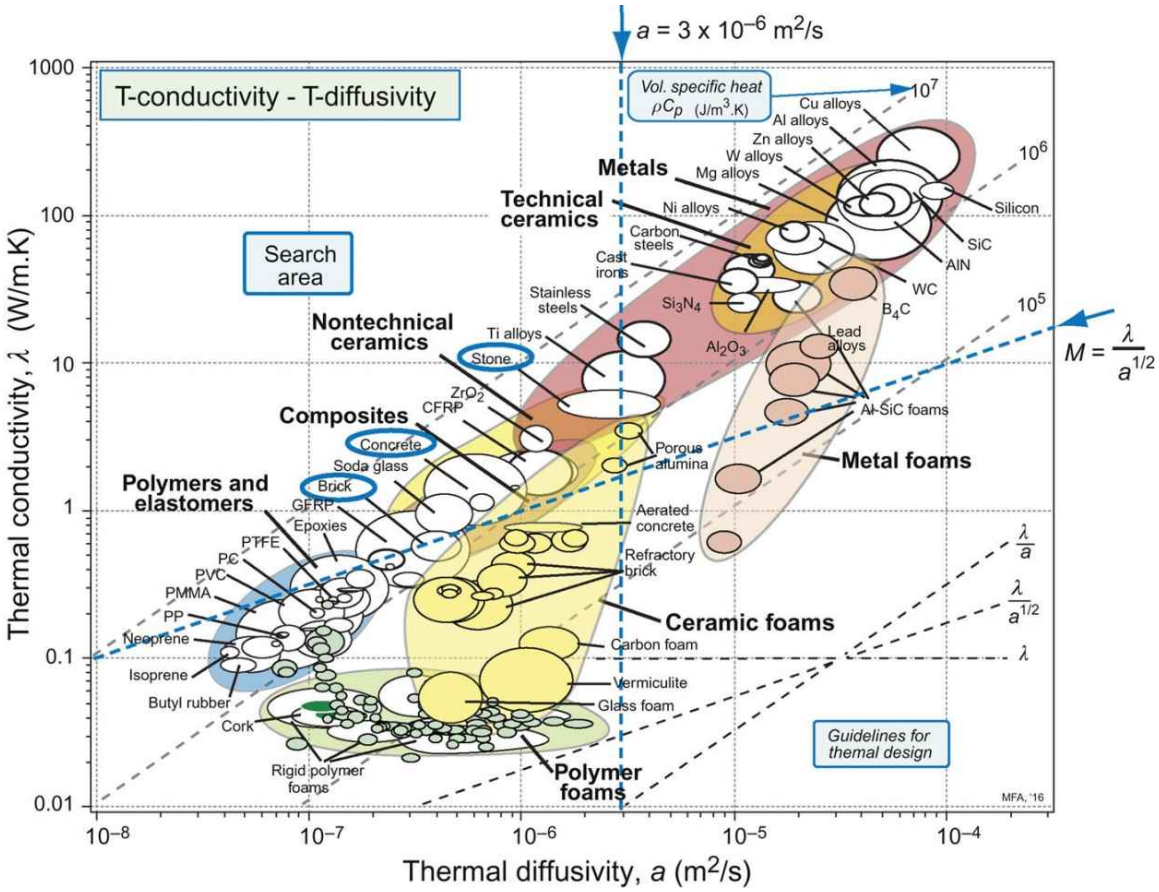


FIGURE 5.28 Materials for heat-storing walls. Adobe (rammed earth, concrete and stone are practical choices; brick is not as good.

Postscript. All this is fine, but what of cost? If this scheme is to be used for housing, cost is an important consideration. The approximate cost per unit volume, read from Table A2 of Appendix A, are listed in the table – it points to the selection of concrete, with stone and brick as alternatives.

Related Case Studies

5.13 'Insulation for short-term isothermal containers'

5.14 'Energy-efficient kiln walls'

5.16 Materials to Minimize Thermal Distortion in Precision Devices

The precision of a measuring device, like a submicrometre displacement gauge, is limited by its stiffness and by the dimensional change caused by temperature gradients. Compensation for elastic deflection can be

arranged; and corrections to cope with thermal expansion are possible too – provided the device is at a uniform temperature. *Thermal gradients* are the real problem: they cause a change of shape – that is, a distortion – for which compensation is not possible. Sensitivity to vibration is also a problem: natural excitation introduces noise and thus imprecision into the measurement. So it is permissible to allow expansion in precision instrument design, provided distortion does not occur (See Further Reading, Chetwynd, 1987). Elastic deflection is allowed, provided natural vibration frequencies are high.

What, then, are good materials for precision devices? Table 5.29 lists the requirements.

Table 5.29
Design Requirements for Precision Devices

Function	■ *Force loop (frame) for precision device*
Constraint	■ *Must tolerate heat flux* ■ *Must tolerate vibration*
Objectives	■ *Minimize distortion to maximize positional accuracy*
Free variables	■ *Choice of material*

The translation. Fig. 5.29 is a schematic of such a device. It consists of a probe, a force loop, an actuator and a sensor. We want a material for the force loop. It will, in general, support heat sources: the fingers of the operator of the device in the figure, or, more usually, electrical or electronic components that release heat. The relevant material index is found by considering the simple case of one-dimensional heat flow through a rod insulated except at its ends, one of which is at ambient and the other connected to the heat source. In the steady state, Fourier's law is

$$q = -\lambda \frac{dT}{dx} \tag{5.62}$$

where q is heat flux per unit area, λ is the thermal conductivity and $\frac{dT}{dx}$ is the resulting temperature gradient. The strain is related to temperature by

$$\varepsilon = \alpha(T - T_o) \tag{5.63}$$

where α is the thermal expansion coefficient and T_o is ambient temperature. The distortion is proportional to the gradient of the strain:

$$\frac{d\varepsilon}{dx} = \frac{\alpha dT}{dx} = \left(\frac{\alpha}{\lambda}\right)q$$

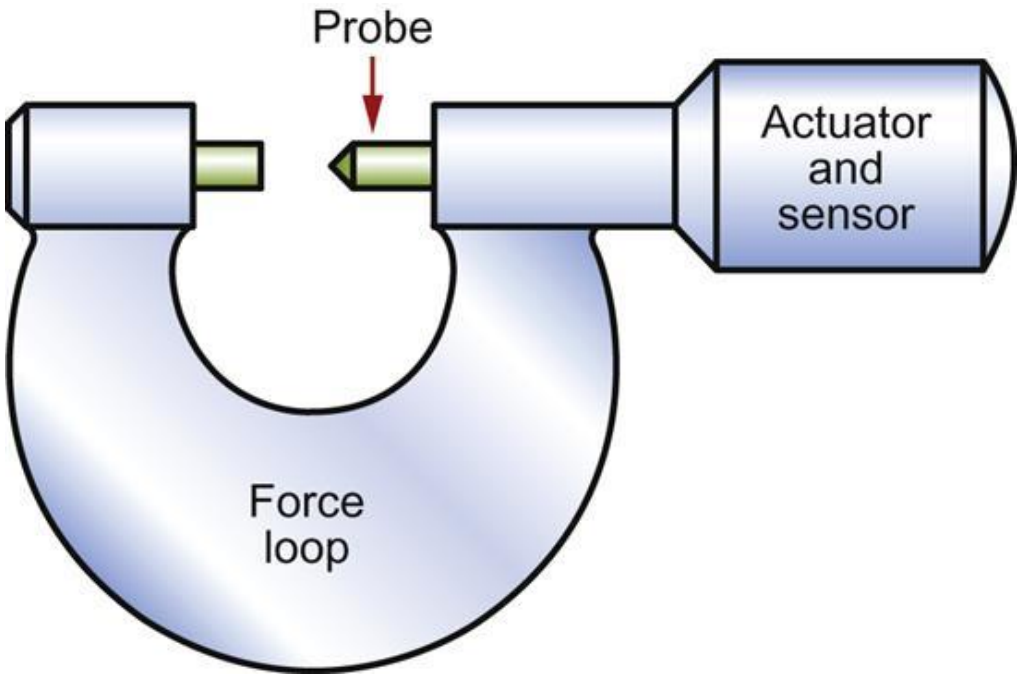


FIGURE 5.29 A schematic of a precision measuring device.

Thus for a given geometry and heat flow, the distortion $d\varepsilon/dx$ is minimized by selecting materials with large values of the index

$$M_1 = \frac{\lambda}{\alpha} \tag{5.64}$$

The other problem is vibration. The sensitivity to external excitation is minimized by making the natural frequencies of the device as high as possible. The flexural vibrations have the lowest frequencies; they are proportional to

$$M_2 = \frac{E^{1/2}}{\rho}$$

A high value of this index will minimize the problem.

The selection. Fig. 5.30 reproduces the chart of expansion coefficient, α, and thermal conductivity, λ. Contours show constant values of the quantity λ/α. A search region is isolated by the line $\lambda/\alpha = 10^7\ W/m$, giving the short-list of Table 5.30. Values of $M_2 = E^{1/2}/\rho$ read from the $E - \rho$ chart of Figure 3.3 are included in the table. Among metals, copper, tungsten and the special nickel alloy Invar have the best values of M_1 but are disadvantaged by having high densities and thus poor values of M_2. The best choice is silicon, available in large sections, with high purity. Silicon carbide is an alternative.

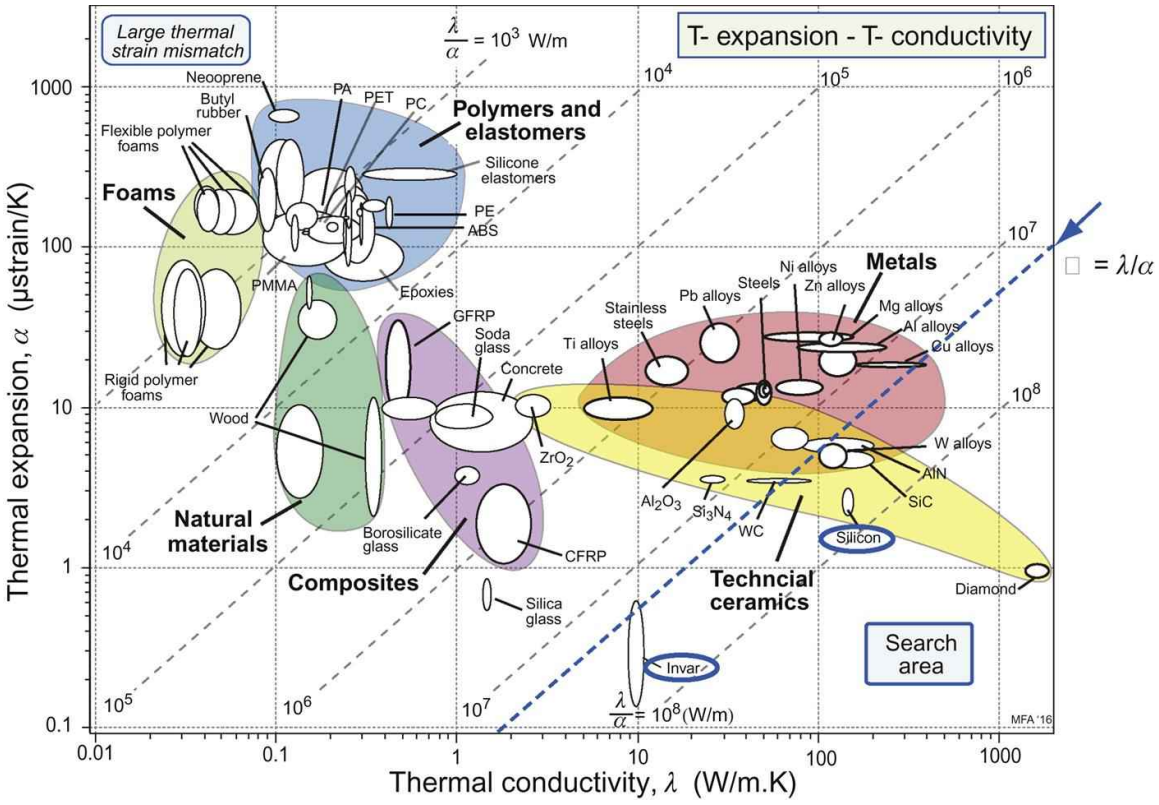


FIGURE 5.30 Materials for precision measuring devices. Metals are less good than ceramics because they have lower vibration frequencies. Silicon may be the best choice.

Table 5.30

Materials to Minimize Thermal Distortion

Material	$M_1 = \lambda/\alpha$ (W/m)	$M_2 = E^{1/2}/\rho$ (GPa$^{1/2}$/(Mg/m^3))	Comment
Silicon	6×10^7	5.2	Excellent M_1 and M_2
Silicon carbide	3×10^7	6.4	Excellent M_1 and M_2 but more difficult to shape than silicon.
Copper	2×10^7	1.3	High density gives poor value of M_2.
Tungsten	3×10^7	1.1	Better than copper, silver or gold, but less good than silicon or SiC.
Aluminium alloys	10^7	3.3	The cheapest and most easily shaped choice.

Postscript. Nanoscale measuring and imaging systems suffer from the problem analysed here. The atomic-force microscope and the scanning–tunnelling microscope both rely on a probe, supported on a force loop, typically with a piezoelectric actuator and electronics to sense the proximity of the probe to the test surface. Closer to home, the mechanism of a video recorder and that of a hard disk drive qualify as precision instruments; both have a sensor (the read head) attached, with associated electronics, to a force loop. The materials identified in this case study are the best choice for the force loop.

Further Reading

Chetwynd, D.G. 1987. Precision engineering, **9**, (1), 3.

Cebon, D. and Ashby, MF (1994) Meas. Sci. and Technol., **5**, 296.

Related Case Studies

5.3 ‘Mirrors for large telescopes’

5.13 ‘Insulation for short-term isothermal containers’

5.17 Materials for Heat Exchangers

This and the next two case studies illustrate how the CES software, described in Section 4.5, can be used to explore material selection in greater depth.

Heat exchangers take heat from one fluid and pass it to a second (Fig. 5.31). The fire-tube array of a steam engine is a heat exchanger, taking heat from the hot combustion gases of the firebox and transmitting it to the water in the boiler. The network of finned tubes in an air conditioner is a heat exchanger, taking heat from the air of the room and dumping it into the working fluid of the conditioner. A key element in all heat exchangers is the tube wall or membrane that separates the two fluids. It is required to transmit heat, and there is frequently a pressure difference across it that can be large.

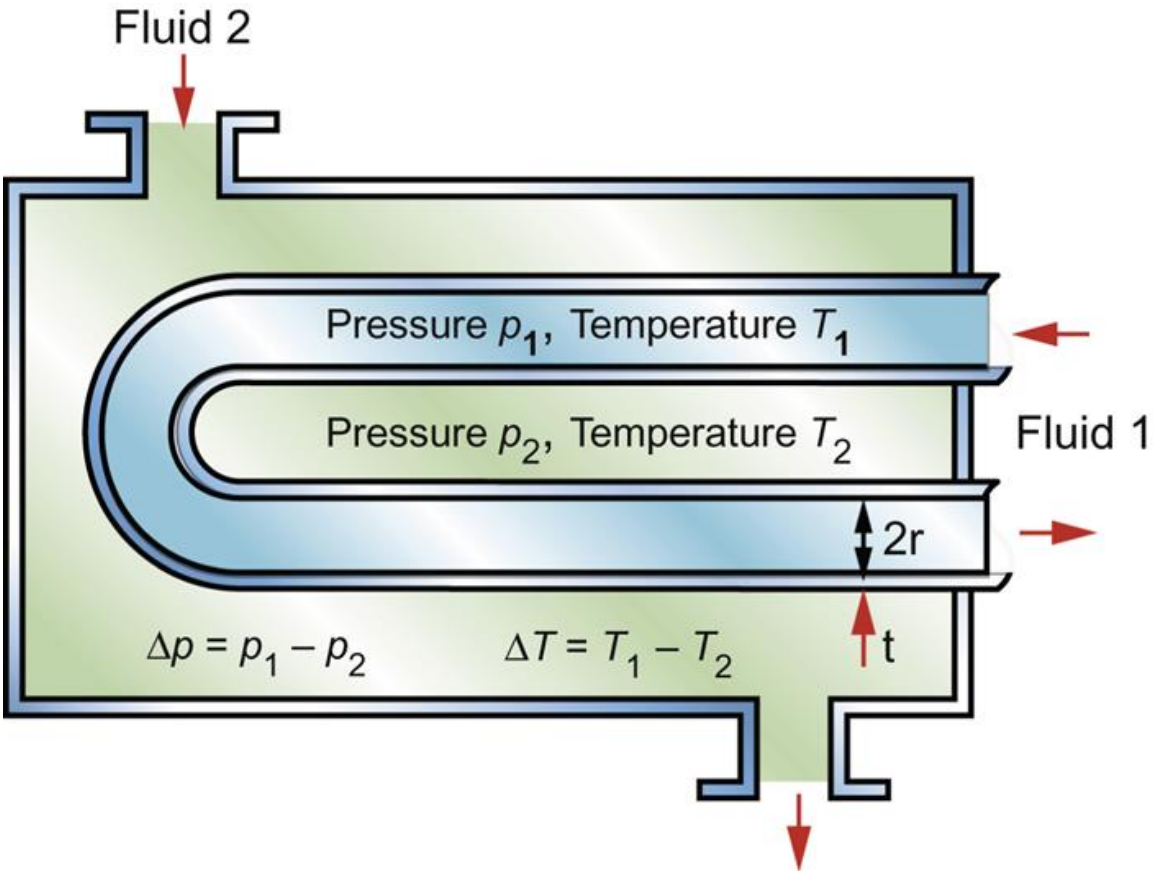


FIGURE 5.31 A heat exchanger. There is a pressure difference Δp and a temperature difference ΔT across the tube wall.

What are the best materials for making heat exchangers? Or, to be specific, what are the best materials for a conduction-limited exchanger with substantial pressure difference between the two fluids, one of them containing chloride ions (sea water)? Table 5.31 summarizes these requirements.

Table 5.31

Design Requirements for a Heat Exchanger

Function	■ *Heat exchanger*
Constraints	■ *Support pressure difference* Δp ■ *Withstand chloride ions* ■ *Operating temperature up to 150 °C* ■ *Modest cost*
Objectives	■ *Maximize heat flow per unit area (minimum volume exchanger) or* ■ *Maximize heat flow per unit mass (minimum mass exchanger)*
Free variables	■ *Tube-wall thickness, t* ■ *Choice of material*

The translation. First, a little background on heat flow. Heat transfer from one fluid, through a membrane to a second fluid, involves *convective* transfer from fluid 1 into the tube wall, *conduction* through the wall, and *convection* again to transfer it into fluid 2. The heat flux into the tube wall by convection (W/m^2) is described by the heat transfer equation:

$$q = h_1 \Delta T_1 \tag{5.65}$$

in which h_1 is the heat transfer coefficient and ΔT_1 is the temperature drop across the surface from fluid 1 into the wall. Conduction is described by the conduction (or Fourier) equation, which, for one-dimensional heat-flow takes the form:

$$q = \lambda \frac{\Delta T}{t} \tag{5.66}$$

where λ is the thermal conductivity of the wall (thickness t) and ΔT is the temperature difference across it. It is helpful to think of the *thermal resistance* at surface 1 as $1/h_1$; that of surface 2 is $1/h_2$; and that of the wall itself is t/λ. Then continuity of heat flux requires that the total resistance $1/U$ is

$$\frac{1}{U} = \frac{1}{h_1} + \frac{t}{\lambda} + \frac{1}{h_2} \tag{5.67}$$

where U is called the 'total heat transfer coefficient'. The heat flux from fluid 1 to fluid 2 is then given by

$$q = U(T_1 - T_2) \tag{5.68}$$

where $(T_1 - T_2)$ is the difference in temperature between the two working fluids.

When one of the fluids is a gas – as in an air conditioner – convective heat transfer at the tube surfaces contributes most of the resistance, and then fins are used to increase the surface area across which heat can be transferred. But when both working fluids are liquid, convective heat transfer is rapid and conduction through the wall dominates the thermal resistance; $1/h_1$ and $1/h_2$ are negligible compared with t/λ. In this case, simple tube or plate elements are used, making their wall as thin as possible to minimize t/λ. We will consider the second case: conduction-limited heat transfer. Then the heat flow is adequately described by Eq. (5.66).

Consider, then, a heat exchanger with n tubes of length L, each of radius r and wall thickness t. Our aim is to select a material to maximize the total heat flow:

$$Q = qA = \frac{A\lambda}{t}\Delta T, \tag{5.69}$$

where $A = 2\pi rLn$ is the total surface are of tubing,

This is the objective function. The constraint is that the wall thickness must be sufficient to support the pressure Δp between the inside and outside, as in Fig. 5.31. This requires that the stress in the wall remain below the elastic limit, σ_y, of the material of which the tube is made (multiplied by a safety factor – which we can leave out):

$$\sigma = \frac{\Delta pr}{t} < \sigma_y \tag{5.70}$$

This constrains the minimum value of t. Eliminating t between Eqs (5.69) and (5.70) gives

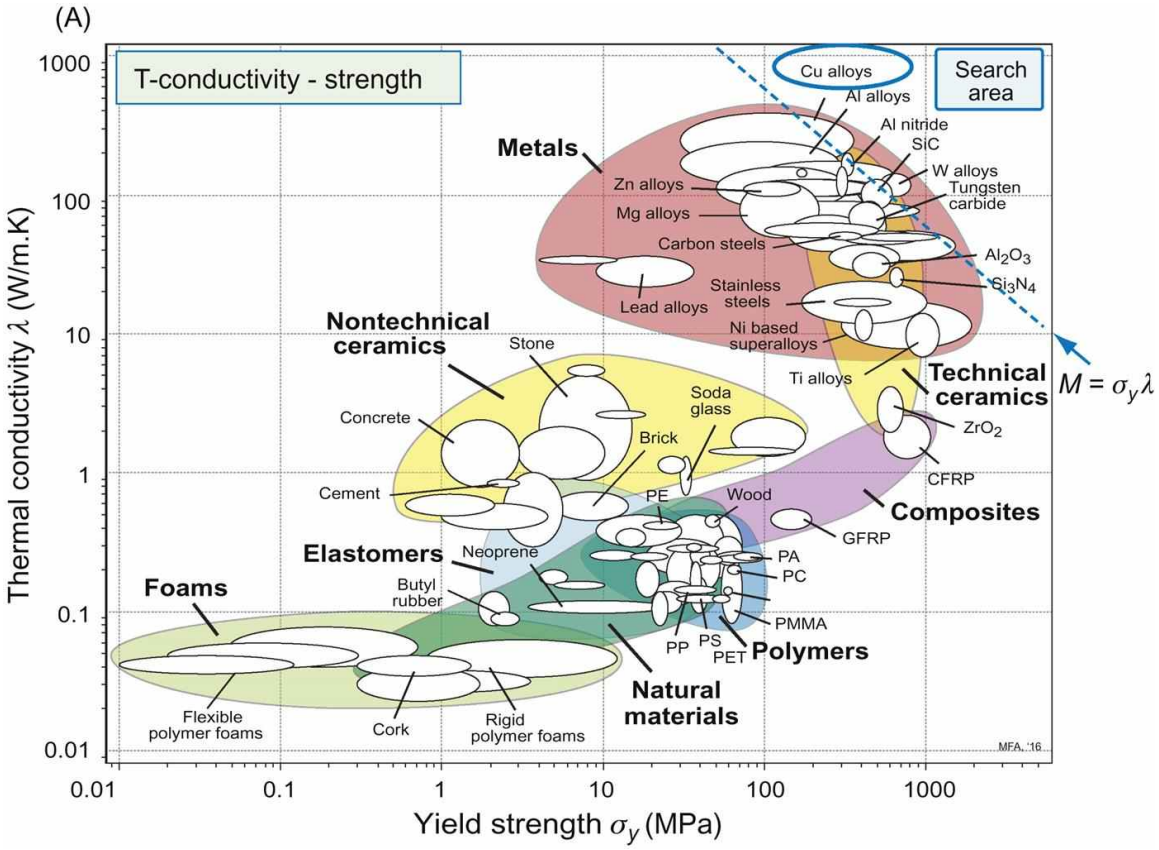
$$Q = \frac{A\Delta T}{r\Delta p}(\lambda\sigma_y). \tag{5.71}$$

The heat flow per unit area of tube wall, Q/A, is maximized by maximizing

$$M_1 = \lambda\sigma_y \tag{5.72}$$

Three further considerations enter the selection. It is essential to choose a material that can withstand corrosion in the working fluids, which we take to be sea water. Its maximum service temperature must be adequately above the temperature of the hotter working fluid and the material must have sufficient ductility to be drawn to tube or rolled to sheet.

The selection. A preliminary search for materials with large values of M_1, using the $\lambda - \sigma_f$ chart of Fig. 5.32A, suggests *wrought copper alloys* as one possibility. We turn then computer-based methods[3] for help. We apply limits of 150°C on maximum service temperature, 30% on elongation, a material cost of less than \$6 (2016 USD)/kg, a rating of 'very good' resistance to sea water and a restriction of the search to copper alloys. With these in place we construct a new chart (Fig. 5.32B) of σ_y versus λ enabling $M_1 = \sigma_y\lambda$ to be maximized again. The materials with large M_1 are listed in Table 5.32.


(A)
T-conductivity - strength
Search area
Cu alloys
Al alloys
Al nitride
SiC
W alloys
Tungsten carbide
Metals
Zn alloys
Mg alloys
Carbon steels
Lead alloys
Stainless steels
Ni based superalloys
Ti alloys
Technical ceramics
Al_2O_3
Si_3N_4
ZrO_2
CFRP
Nontechnical ceramics
Stone
Concrete
Brick
Soda glass
Cement
Composites
Wood
GFRP
Elastomers
Neoprene
Butyl rubber
PE
PA
PC
PMMA
PP
PS
PET
Polymers
Foams
Natural materials
Flexible polymer foams
Cork
Rigid polymer foams
$M = \sigma_y \lambda$
Thermal conductivity λ (W/m.K)
Yield strength σ_y (MPa)
MFA, '16

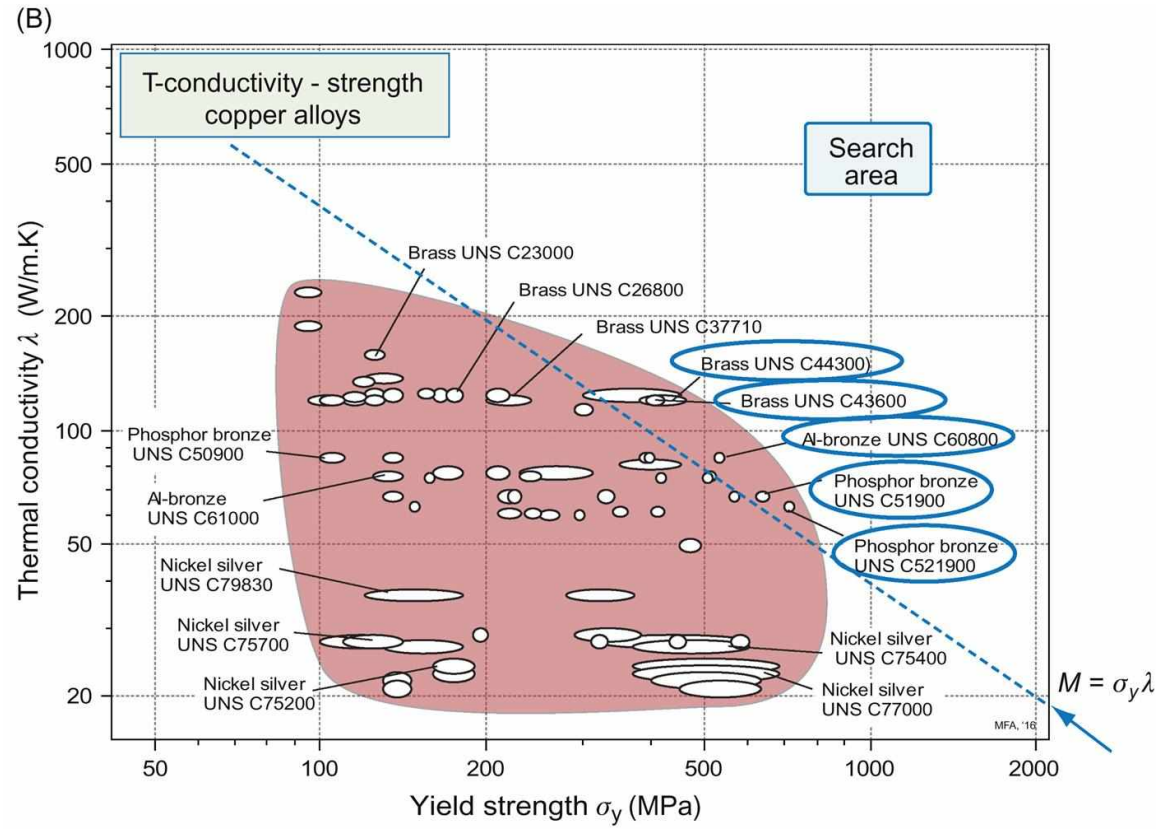

(B)
T-conductivity - strength copper alloys
Search area
Brass UNS C23000
Brass UNS C26800
Brass UNS C37710
Brass UNS C44300
Brass UNS C43600
Al-bronze UNS C60800
Phosphor bronze UNS C51900
Phosphor bronze UNS C521900
Phosphor bronze UNS C50900
Al-bronze UNS C61000
Nickel silver UNS C79830
Nickel silver UNS C75700
Nickel silver UNS C75200
Nickel silver UNS C75400
Nickel silver UNS C77000
$M = \sigma_y \lambda$
Thermal conductivity λ (W/m.K)
Yield strength σ_y (MPa)
MFA, '16

FIGURE 5.32 (A). A chart of yield strength (elastic limit) σ_y against thermal conductivity, λ showing the index M_1. (B). A more specialized chart showing copper alloys.

Table 5.32
Materials for Heat Exchangers

Material	Index M $W \cdot MN/m^3 \cdot K$	Comment
Brasses, naval brass	5×10^4	Liable to dezincification.
Phosphor bronzes	4×10^4	Cheap, but not as corrosion resistant as aluminium bronzes.
Aluminium-bronzes, wrought	3.8×10^4	An economical and practical choice.
Nickel-iron-aluminium-bronzes	2.5×10^4	More corrosion resistant, but more expensive.
Silicon bronze	2.2×10^4	Less good than aluminium bronze.

Postscript. Conduction may limit heat flow in theory, but unspeakable things go on inside heat exchangers. Sea water seethes with biofouling organisms that attach themselves to tube walls and thrive there, like barnacles on a boat, creating a layer of high thermal resistance and impeding fluid flow. A search for documentation reveals that some materials are more resistant to biofouling than others; copper-nickel alloys are particularly good, probably because the organisms dislike copper salts, even in very low concentrations. Otherwise the problem must be tackled by adding chemical inhibitors to the fluids, or by scraping – the traditional winter pastime of boat owners.

It is sometimes important to minimize the weight of heat exchangers. Repeating the calculation to seek materials the maximum value of Q/m (where m is the mass of the tubes) gives, instead of M_1, the index

$$M_2 = \frac{\lambda \sigma_y^{\,2}}{\rho} \qquad \textbf{(5.73)}$$

where ρ is the density of the material of which the tubes are made. (The strength σ_y is now raised to the power of 2 because the weight depends on wall thickness as well as density, and wall thickness varies as $1/\sigma_y$ (Eq.

5.67)). Similarly, the cheapest heat exchangers are those made of the material with the greatest value of

$$M_3 = \frac{\lambda {\sigma_y}^2}{C_m \rho} \qquad \textbf{(5.74)}$$

where C_m is the cost per kg of the material. In both cases aluminium alloys score highly because they are both light and cheap. The selections are not shown but can readily be explored using the CES system.

Further Reading

JP Holman, 'Heat transfer' 5th edition, (1981), McGraw-Hill, NY, USA.

Related Case Studies

5.11 'Safe Pressure vessels'
5.16 'Minimizing distortion in precision devices'
5.18 'Heat sinks for hot microchips'

5.18 Heat Sinks for Hot Microchips

A microchip may only consume milliwatts, but this energy is dissipated in a tiny volume. The power is low but the *power-density* is high. As chips shrink and clock-speeds grow, heating becomes a problem. The chip of today's PCs already reaches 85°C, requiring forced-air cooling. Multiple-chip modules pack hundreds of chips on to a single substrate. Heating is kept under control by attaching the chip to a heat sink (Fig. 5.33), taking pains to ensure good thermal contact between the chip and the sink. The heat sink now becomes a critical component, limiting further development of the electronics. How can its performance be maximized?

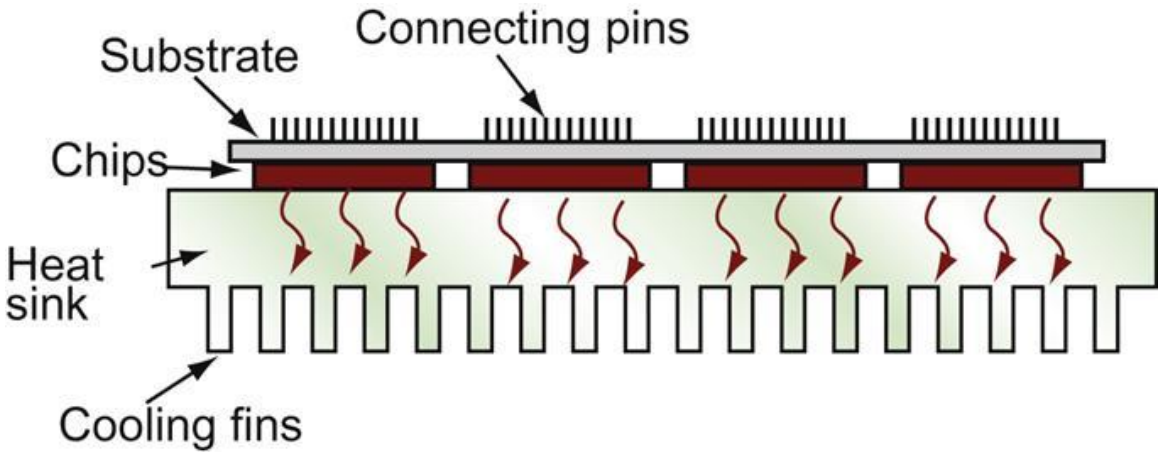


FIGURE 5.33 A heat sink for power microelectronics.

To prevent electrical coupling and stray capacitance between chip and heat sink, the heat sink must be a good electrical insulator, meaning a resistivity, $\rho_e > 10^{18}\ \mu\Omega\ \text{cm}$. But to drain heat away from the chip as fast as possible, it must also have the highest possible thermal conductivity, λ. The translation step is summarized in Table 5.33, where we assume that all dimensions are constrained by other aspects of the design.

Table 5.33

Function, Constraints, Objective and Free Variable for the Heat Sink

Function	■ *Heat sink*
Constraints	■ *Material must be 'good insulator', or* $\rho_e > 10^{18}\ \mu\Omega\ \text{cm}$ ■ *Maximum service temperature* >150°C ■ *All dimensions are specified*
Objective	■ *Maximize thermal conductivity,* λ
Free variable	■ *Choice of material*

The translation. Resistivity is treated as a *constraint*, a go/no–go criterion. Materials that fail to qualify as 'good insulator' or have a resistivity greater than the value listed in the table are screened out. The thermal conductivity is treated as an *objective*: of the materials that meet the constraint, we seek those with the largest values of λ and rank them by this – it becomes the material index for the design. If we assume that all dimensions are fixed by the design, there remains only one free variable in

seeking to maximize heat flow: the choice of material. The procedure, then, is to *screen* on resistivity, then *rank* on conductivity.

The selection. The steps can be implemented using the $\lambda - \rho_e$ chart of Figure 3.10, reproduced as Fig. 5.34. Draw a vertical line at $\rho_e = 10^{18}\ \mu\Omega\ \text{cm}$, then pick off the materials that both lie above this line and have the highest λ. The result: aluminium nitride, AlN, alumina, Al_2O_3, silicon nitride, Si_3N_4 and diamond, which we eliminate on grounds of cost. Of the remaining three, aluminium nitride has the highest thermal conductivity.

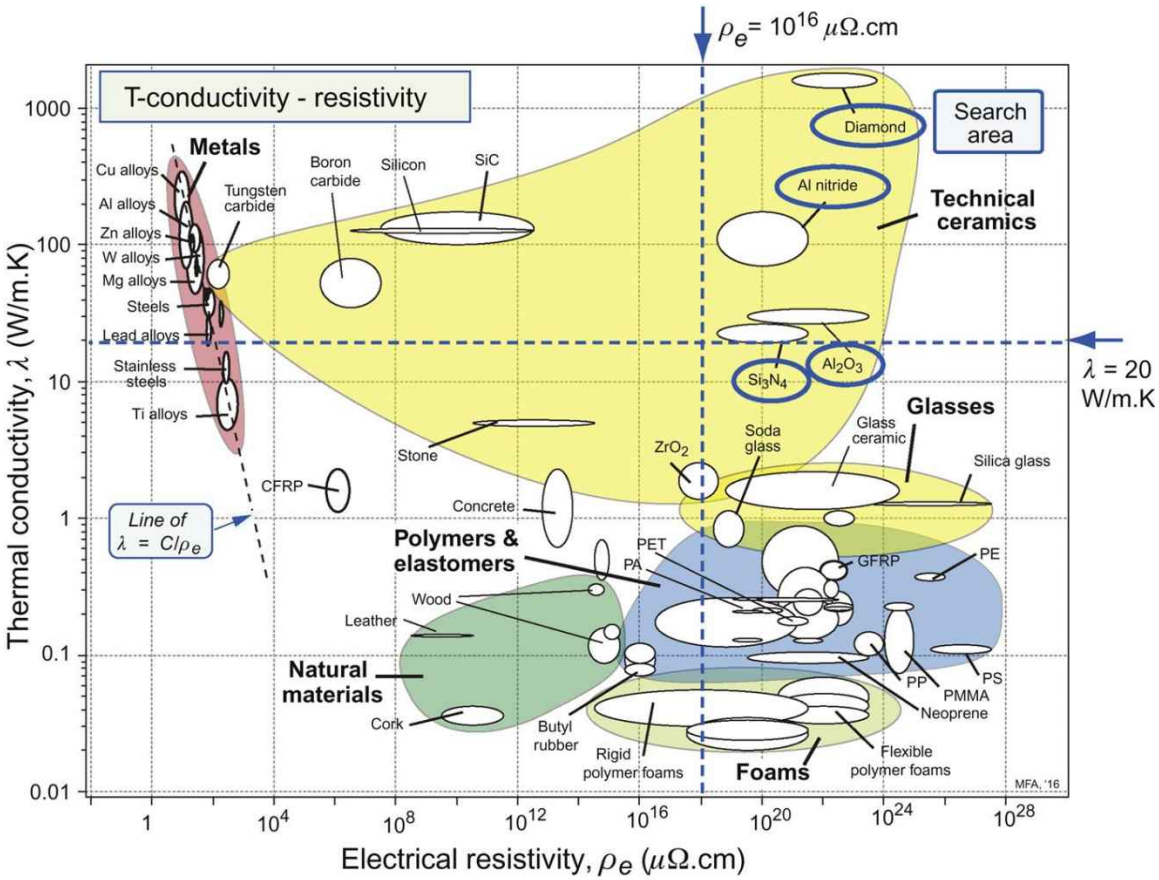


FIGURE 5.34 The $\lambda - \rho_e$ chart with the attribute limit $\rho_e > 10^{18}\ \mu\Omega\ \text{cm}$ and the index λ plotted on it. The selection is refined by raising the position of the λ selection line.

Postscript. A quick search for documentation, seeking 'Applications of aluminium nitride', yields the following text. Heat sinks are specifically mentioned. The method has led us quickly to a reliable choice.

Aluminium nitride

Typical uses. Substrates and heat sinks for microcircuits, chip carriers, electronic components; windows, heaters, chucks, clamp rings, gas distribution plates.

5.19 Materials for Radomes

When the BBC, The British Broadcasting Corporation, derives its income from the licence fee paid by owners of television receivers. Failure to pay the fee deprives the BBC of its income – hence the sophisticated detection scheme. When they want to catch you watching television without a licence, they park outside your house an unmarked vehicle equipped to detect high frequency radiation. The vehicle looks normal enough, but it differs from the norm in one important respect: the body-skin is not made of pressed steel, but of a material transparent to microwaves. The requirements of the body are much the same as those for the protective dome enclosing the delicate detectors that pick up high frequency signals from space; or those that protect the radar equipment in ships, aircraft and spacecraft. What are the best materials to make them?

The function of a radome is to shield a microwave antenna from the adverse effects of the environment, while having as little effect as possible on the electrical performance. When detecting incoming signals that are weak to begin with, even a small attenuation of the signal as it passes through the radome decreases the sensitivity of the system. Yet the radome must withstand structural loads, loads caused by pressure difference between the inside and outside of the dome, and – in the case of supersonic flight – high temperatures. Table 5.34 summarizes the design requirements.

Table 5.34

Design Requirements for a Radome

Function	■ *Radome*
Constraints	■ *Support pressure difference* Δp ■ *Tolerate temperature up to* T_{max}
Objectives	■ *Minimize dielectric loss in transmission of microwaves*
Free variables	■ *Thickness of skin, t* ■ *Choice of material*

The translation. Fig. 5.35 shows an idealised radome. It is a hemispherical skin of microwave-transparent material of radius R and thickness t, supporting a pressure difference Δp between its inner and outer surfaces. The two critical material properties in determining radome performance are the dielectric constant, ε_r, and the electric loss tangent $\tan\delta$. Losses are of two types: *reflection* and *absorption*. The fraction of the signal that is reflected is related to the dielectric constant ε_r; the higher the frequency, the higher the reflected fraction. Air has a dielectric constant of 1; a radome with the same dielectric constant, if it were possible, would not reflect any radiation ('stealth' technology seeks to achieve this).

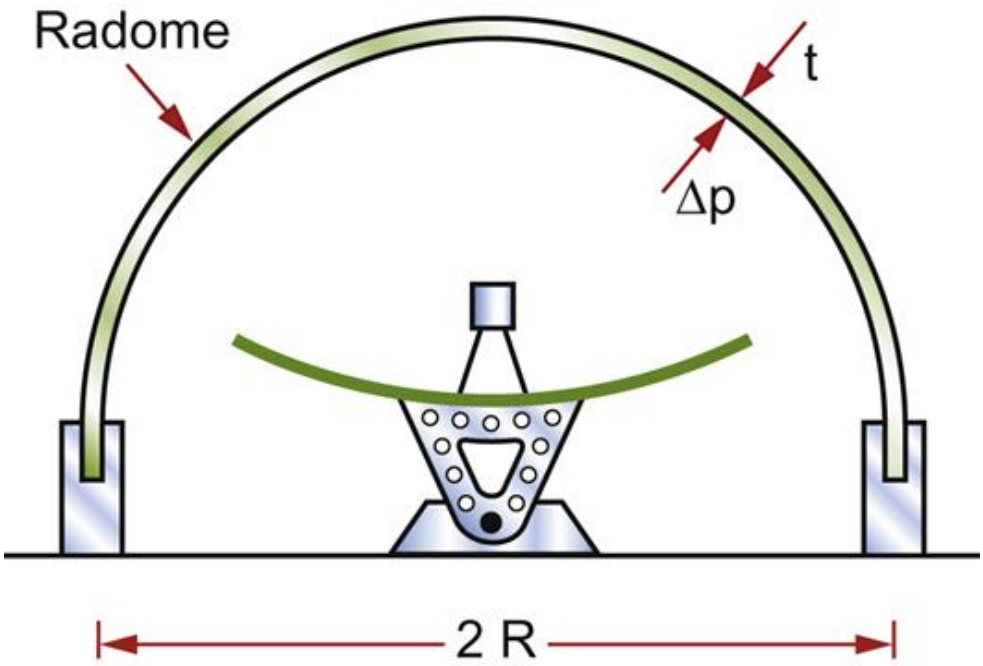


FIGURE 5.35 A radome. It must be transparent to microwaves yet support a pressure difference.

The second, and often more important loss is that due to absorption as the signal passes through the skin of the radome. When an electromagnetic wave of frequency f (cycles/sec) passes through a dielectric with loss tangent $\tan\delta$, the fractional *power loss* in passing through a thickness dt is

$$\left|\frac{dU}{U_o}\right| = \frac{fA^2\varepsilon_o}{2}(\varepsilon_r \tan\delta)dt \qquad \textbf{(5.75)}$$

where A is the electric amplitude of the wave and ε_o the permittivity of vacuum. For a thin shell (thickness t) the loss per unit area is thus

$$\left|\frac{\Delta U}{U_o}\right| = \frac{fA^2\varepsilon_o t}{2}(\varepsilon_r \tan\delta). \qquad \textbf{(5.76)}$$

This is the quantity we wish to minimize – the objective function – and this is achieved by making the skin as thin as possible. But the need to support a pressure difference Δp imposes a constraint. If the radome is spherical in shape, the pressure difference creates a stress

$$\sigma = \frac{\Delta p R}{2t} \tag{5.77}$$

in the skin. If it is to support Δp, this stress must be less than the failure stress σ_f of the material of which it is made, imposing a constraint on the thickness:

$$t \geq \frac{\Delta p R}{2\sigma_f}.$$

Substituting this into the Eq. (5.76) gives

$$\left|\frac{\Delta U}{U}\right| = \frac{f A^2 \varepsilon_o \Delta p R}{4} \left(\frac{\varepsilon_r \tan \delta}{\sigma_f}\right). \tag{5.78}$$

The power loss is minimized by maximizing the index

$$M = \frac{\sigma_f}{\varepsilon_r \tan \delta} \tag{5.79}$$

The selection. A preliminary survey using the strength – dielectric loss chart of Fig. 5.36 shows that polymers have attractive values of M but have poor strength. Some ceramics have excellent values of M and are stable to high temperatures. We need to explore these two classes of material in more depth. Appropriate charts are shown in Figs. 5.37A and B, the first for polymers, both plain and glass-reinforced, the second for ceramics. The axes are σ_f and $\varepsilon \tan \delta$. Both have a selection line of slope 1 showing the index M. The selection is summarised in Table 5.35. The materials of the first row – Teflon PTFE, polyethylene and polypropylene – maximize M. If greater strength or impact resistance is required, the fibre reinforced polymers of the second row are the best choice. When, additionally, high temperatures are involved, the ceramics listed in the third row become candidates.

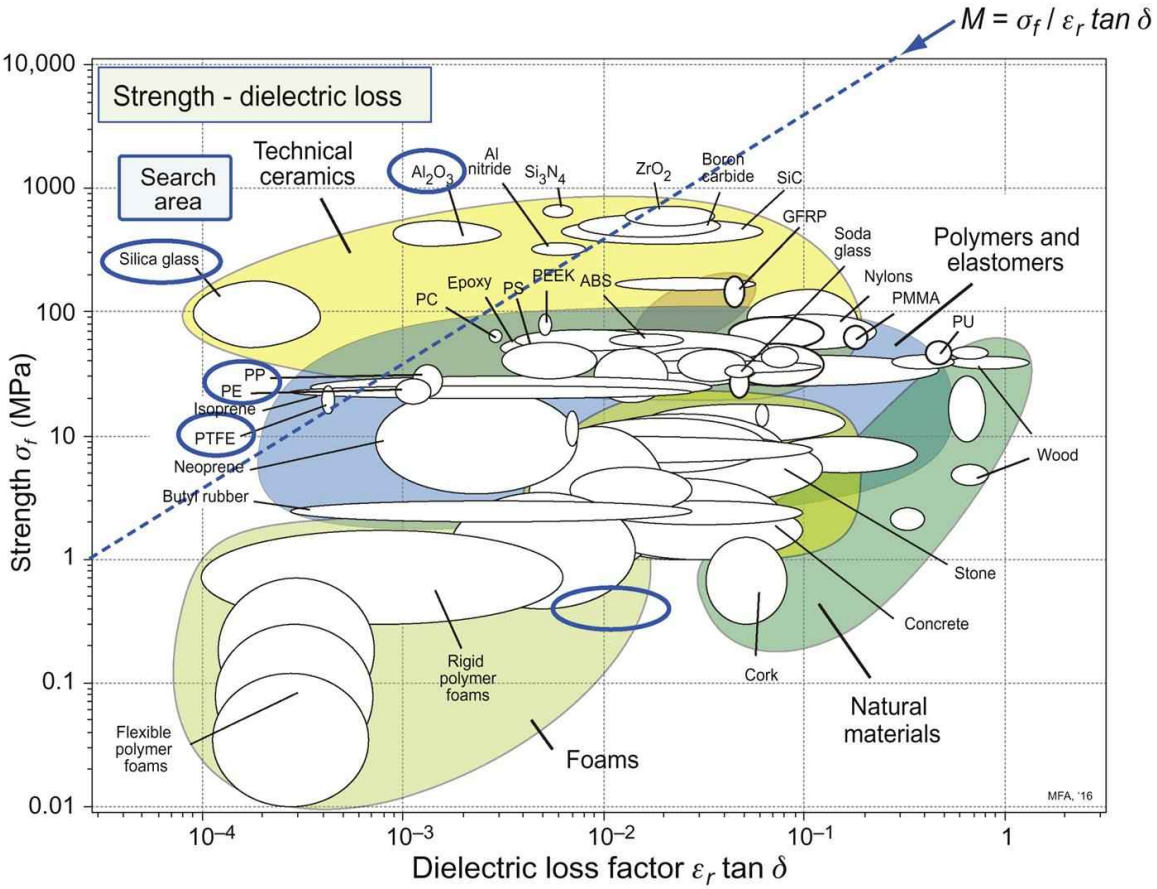
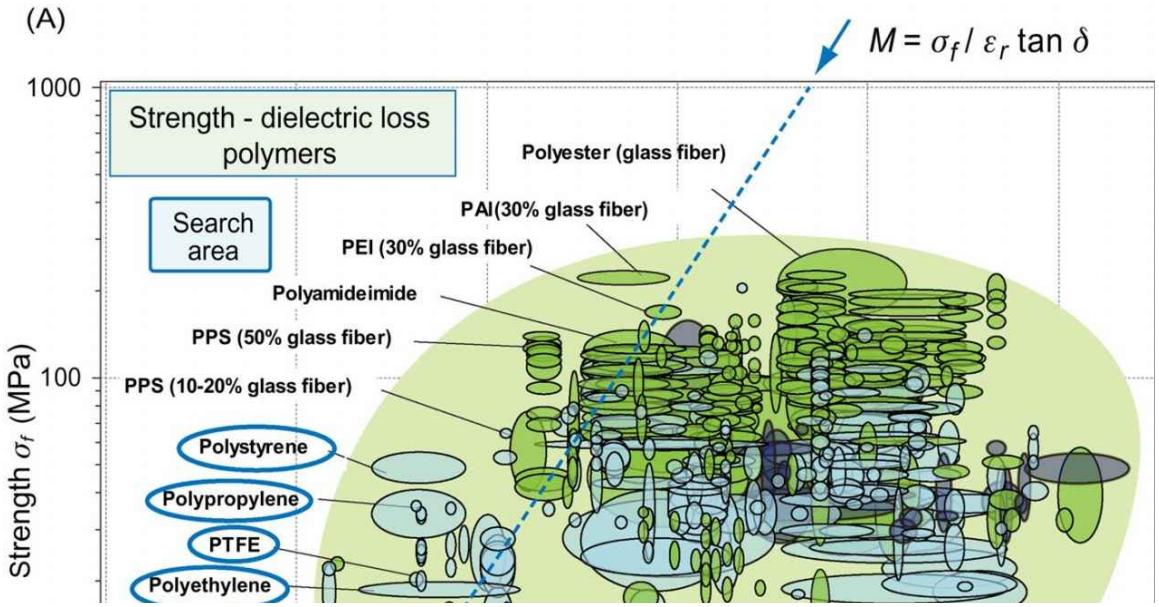


FIGURE 5.36 The elastic limit, σ_f, plotted against the loss factor, $\varepsilon_r \tan \delta$, showing the index M.


(A)
$M = \sigma_f / \varepsilon_r \tan \delta$
1000
100
Strength σ_f (MPa)
Strength - dielectric loss polymers
Search area
Polyester (glass fiber)
PAI(30% glass fiber)
PEI (30% glass fiber)
Polyamideimide
PPS (50% glass fiber)
PPS (10-20% glass fiber)
Polystyrene
Polypropylene
PTFE
Polyethylene

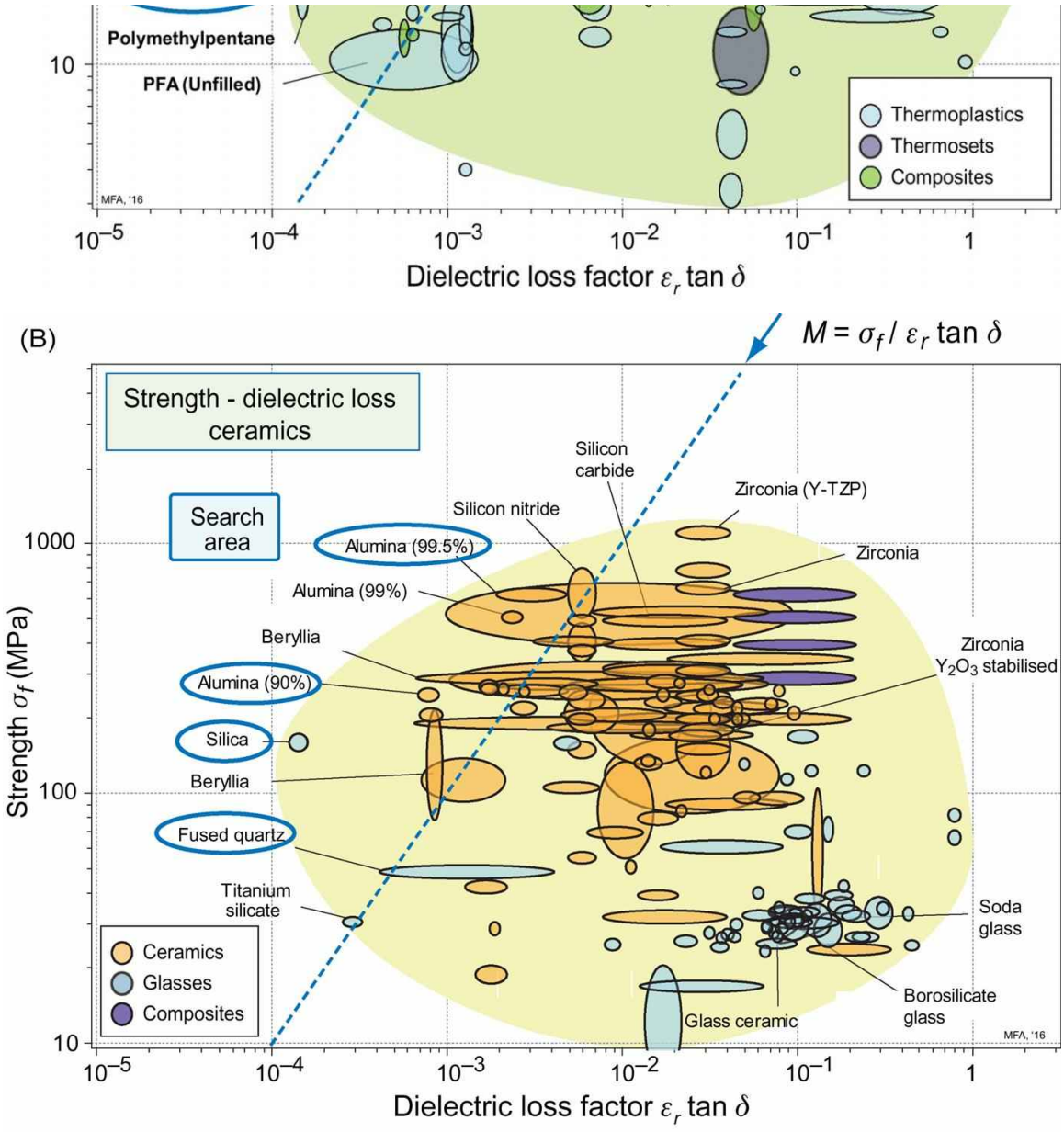


FIGURE 5.37 The elastic limit, σ_f, plotted against the power factor, $\varepsilon_r \tan \delta$, in detail, (A) for polymers, filled polymers and composites and (B) for ceramics.

Table 5.35

Materials for Radomes

Material	Comment
PTFE, polyethylenes, polypropylenes, polystyrene and polyphenylene sulphide (PPS)	Minimum dielectric loss, but limited to near room temperature.
Glass-reinforced polyester, PTFE, polyethylenes and polypropylenes, polyamideimide	Slightly greater loss, but greater strength and temperature resistance.
Silica, alumina, beryllia, silicon carbide	The choice for re-entry vehicles and rockets where heating is great.

Postscript. What are real radomes made of? Among polymers, PTFE and polycarbonate are the most common. Both are very flexible. Where structural rigidity is required (as in the BBC van) GFRP (epoxy or polyester reinforced with woven glass cloth) are used, though with some loss of performance. When performance is at a premium, glass-reinforced PTFE is used instead. For skin-heating up to 300°C, polyimides meet the requirements; beyond that temperature it has to be ceramics. Silica (SiO_2), alumina (Al_2O_3), beryllia (BeO) and silicon nitride (Si_3N_4) are all employed. The choices we have identified are all there.

Further Reading

Huddleston, G.K. and Bassett, H.L., in 'Antenna engineering handbook', 2nd edition, Johnson, R.C. and Jasik, H. (editors). McGraw-Hill, NY, Chapter 44.

Lewis, C.F. 'Materials keep a low profile', Mechanical engineer, June 1988, p. 37–41.

Related Case Studies

5.11 'Safe pressure vessels'
9.2 'Multiple constraints: light pressure vessels'

5.20 Summary and Conclusions

The case studies of this chapter illustrate how the choice of material is narrowed from the initial, broad, menu to a small subset that can examined in depth. Most designs make certain nonnegotiable demands on a material: it must withstand a temperature greater than T, it must resist corrosive fluid F, and so forth. These constraints narrow the choice to a

few broad classes of material. The choice is narrowed further by seeking the combination of properties that maximize performance (combinations like $E^{1/2}/\rho$) or maximize safety (combinations like K_{1c}/σ_f) or conduction or insulation (like $a^{1/2}/\lambda$). These, plus economics, isolate a small subset of materials for further consideration.

The final choice between these will depend on more detailed information on their properties, considerations of manufacture, economics and aesthetics. These are discussed in the chapters that follow.

Exercises. This chapter is one of case studies. Exercises relating to the methods they use are at the end of Chapter 4, Materials Selection – the Basics.

[1]Formal methods for dealing with multiple objectives are developed in Chapter 8.

[2]If the wall is sufficiently thin, and close to general yield, it will fail in a plane-stress mode. Then the relevant fracture toughness is that for plane stress, not the smaller value for plane strain.

[3]The CES Edu system set at Level 3 (www.Grantadesign.com).

CHAPTER 6

Processes and Their Effect on Properties

Abstract

A *process* is a method of shaping, joining or finishing a material. The choice, for a given component, depends on the material of which it is to be made, on its size, shape and required precision, and on how many are to be made.
This chapter develops the classification scheme for processes, dividing them into *families* and members each with a unique set of *attributes:* the materials they can handle, the shapes they can make and the precision with which they can do it. Processing can be used to control properties. Process–property interactions are explored, using charts to plot process–property profiles.

Keywords

Shaping; joining; surface finishing; casting; moulding; powder methods; composite forming; additive manufacture; heat treatment; laser treatment; work hardening; alloy hardening; age hardening; fillers; foaming

6.1 Introduction and Synopsis

A *process* is a method of shaping, joining or finishing a material. *Sand casting*, *injection moulding*, *fusion welding* and *electropolishing* are all processes; there are hundreds of them. The choice, for a given component, depends on the material of which it is to be made, on its size, shape and required precision, and on how many are to be made – in short, on the *design requirements* (Fig. 6.1).

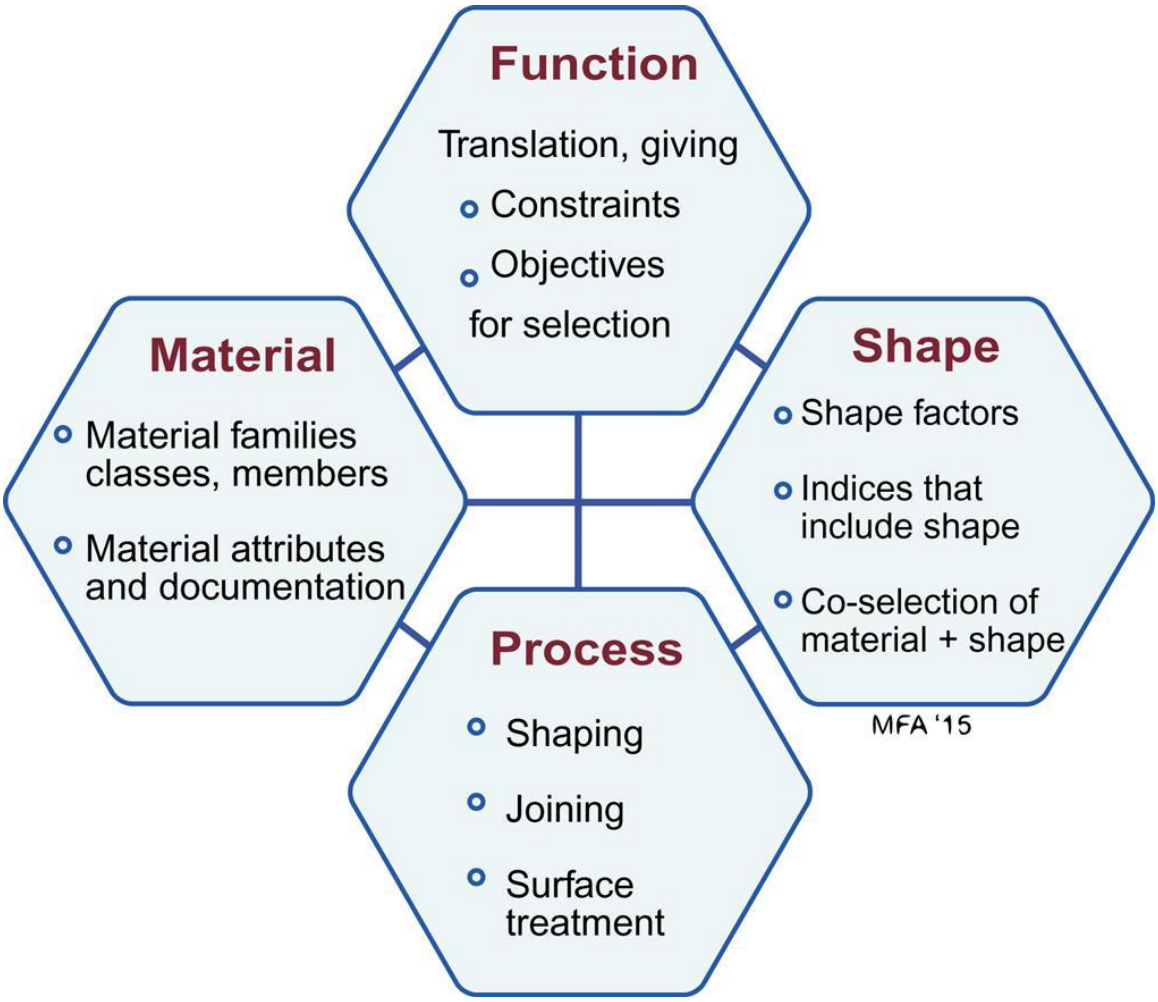


FIGURE 6.1 Processing selection depends on material and shape.

Processing has much in common with cooking. Sounds trivial but it's not. Cooking involves blending, rolling, stretching, casting, heat treatment, foaming . . . and for all of these the key control variables are time, temperature and pressure. To continue this analogy into the chapter would get tedious so we'll leave it there. But don't forget the parallel: time, temperature, pressure.

To select processes we must first classify them. Section 6.2 develops the classification. It is used to structure Section 6.3 in which process *families* and their *attributes* are described: the materials they can handle, the shapes they can make and the precision with which they can do it.

Processing has dual functions. The obvious one is that of shaping, joining and finishing. The less obvious one is that of property control. Metals are strengthened by rolling and forging; steels are heat treated to

enhance hardness and toughness; polymers are drawn to increase modulus and strength; and ceramics are hot pressed, again to increase strength. Process–property interactions are explored more closely in Section 6.4. The chapter ends, as always, with a summary and annotated recommendations for further reading and suggested exercises.

The four-way links between process, material, shape and function of Fig. 6.1 become the framework for *process selection*, the subject of Chapter 7, Processes Selection and Cost.

6.2 Classifying Processes

Fig. 6.2 is a classification of manufacturing processes. *Primary processes* create shapes. The first row lists six classes of primary shaping processes: casting, moulding, deformation, powder methods, composite forming and additive manufacture (AM). *Secondary processes* modify shapes or enhance properties; here they are shown as 'machining', which adds features to an already shaped body and 'heat treatment', which enhances surface or bulk properties. Below these comes *joining* and *finishing*.

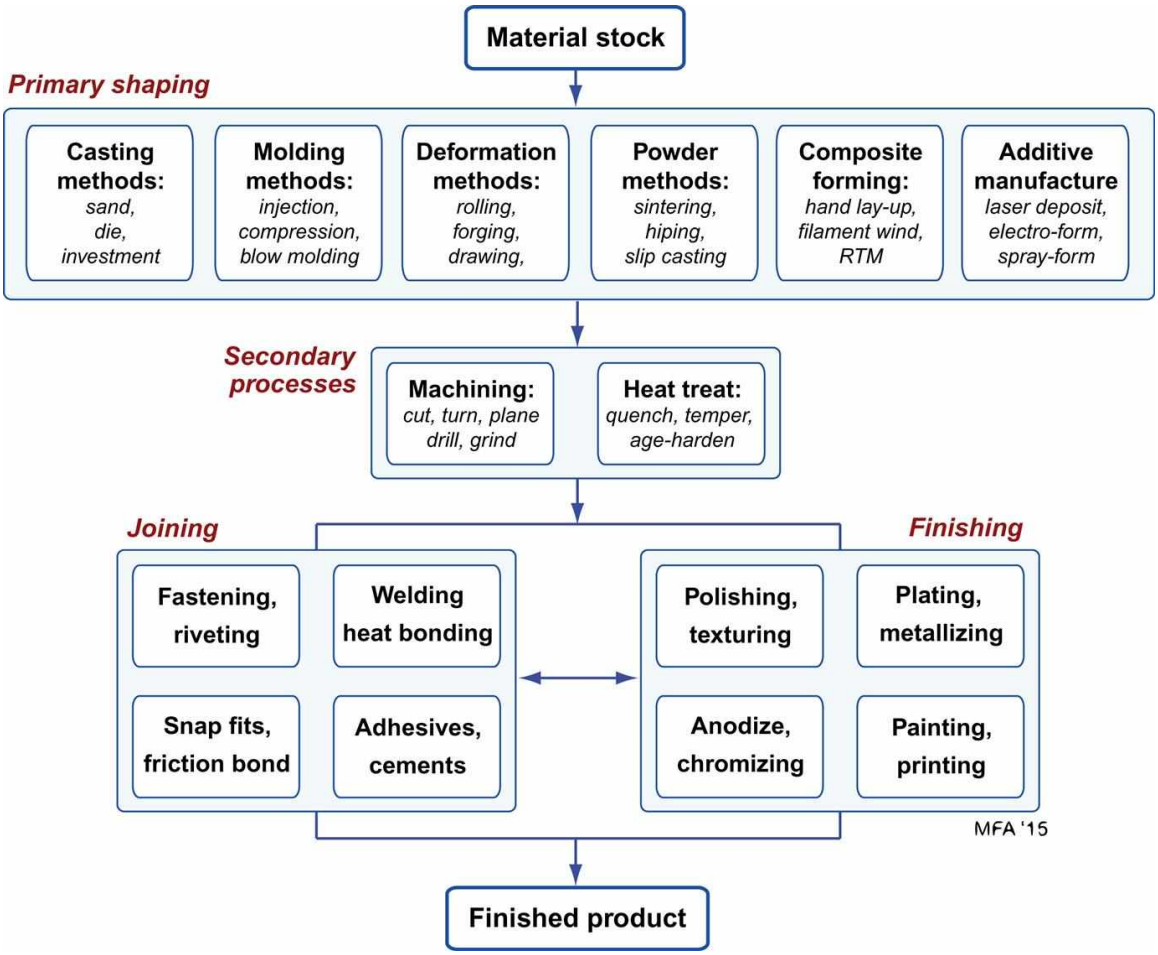


FIGURE 6.2 The classes of process. The first row contains the family of shaping processes; below lie the secondary processes of machining and heat treatment, followed by the families of joining and finishing processes.

Fig. 6.2 looks like as a flow chart: a progression through a manufacturing sequence, but it should not be treated too literally: the order of the steps can be varied to suit the needs of the design. The point it makes is that there are three broad process families: those of shaping, joining and finishing. The attributes of one family differ so greatly from those of another that, in assembling and structuring data for them, they must be treated separately.

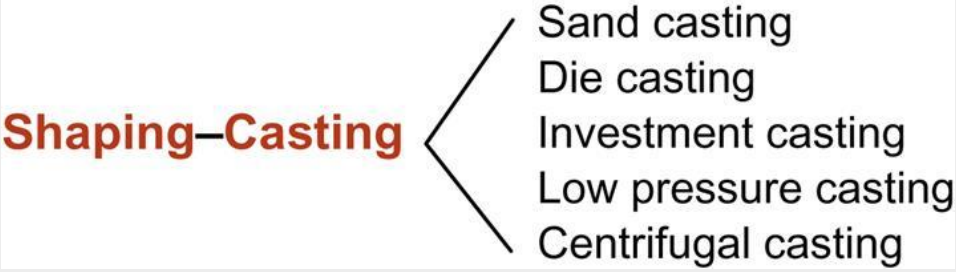
To organize processes in more detail, we need a hierarchical classification like that used for materials in Chapter 3, Materials Property Charts. Fig. 6.3 shows part of it. At the left are the families of shaping, joining and finishing. The shaping family is expanded to show classes: casting, deformation, moulding etc. One of these – moulding – is again

expanded to show its members: rotation moulding, blow moulding, injection moulding and so forth. Each of these processes has certain attributes: the materials it can handle, the shapes it can makes, their size, precision, and an economic production run, meaning the minimum number of units that must be made for the process to become economic.

Expanding Branches of the Shaping Tree

Expand the casting family at about the same level of detail as that used for moulding in Fig. 6.3.

The Result



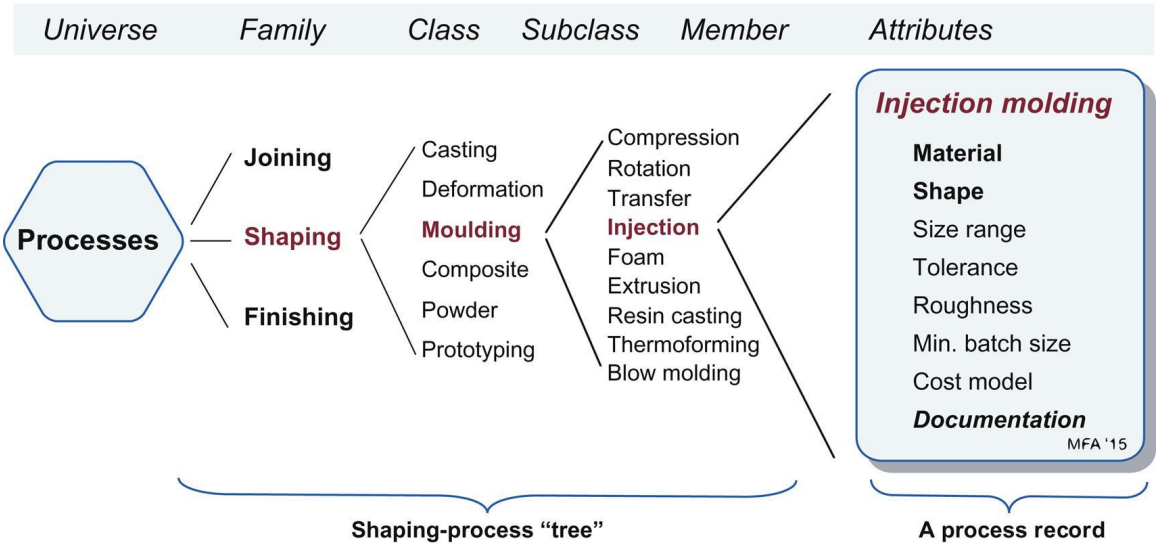


FIGURE 6.3 The processes tree with part of the *shaping* branch expanded. Each member is characterized by a set of attributes.

The other two families are partly expanded in Fig. 6.4. The joining family contains three broad classes: adhesives, welding and fasteners. In this figure, one of them – welding – is expanded to show its members. As before, each member has attributes. The first is the materials it can join. After that the attribute list differs from that for shaping. Here the geometry of the joint and the way it will be loaded are important, as are requirements that the joint can or cannot be disassembled, be watertight, be electrically conducting and the like.

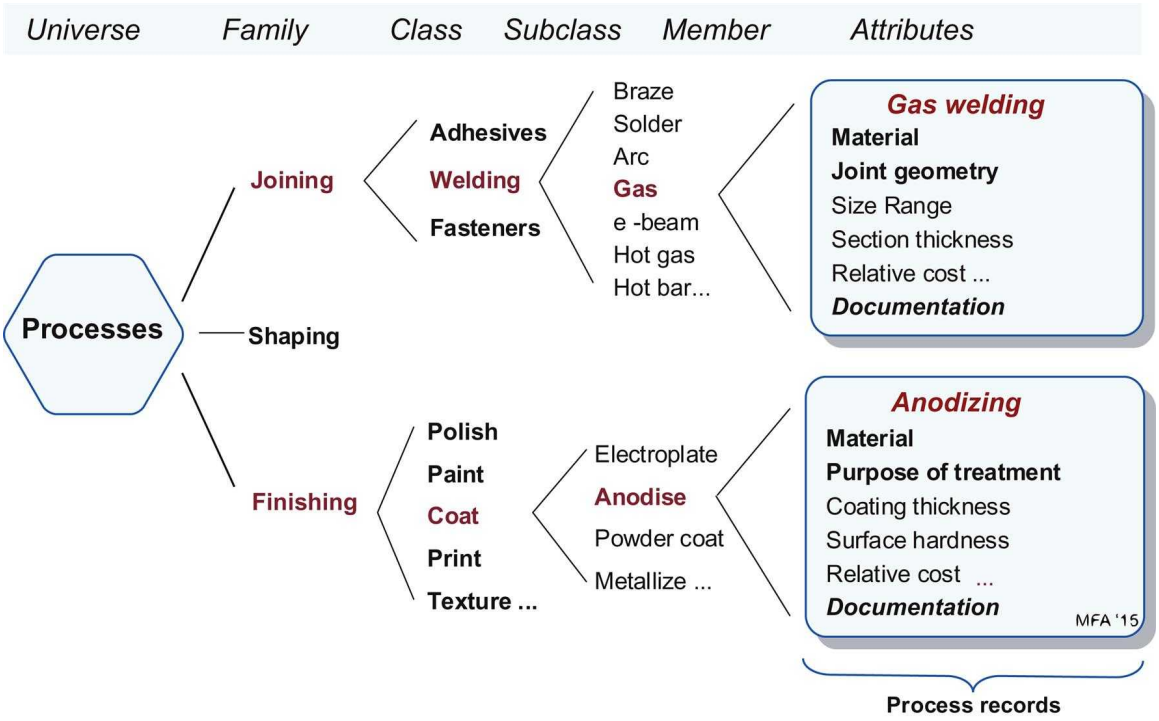


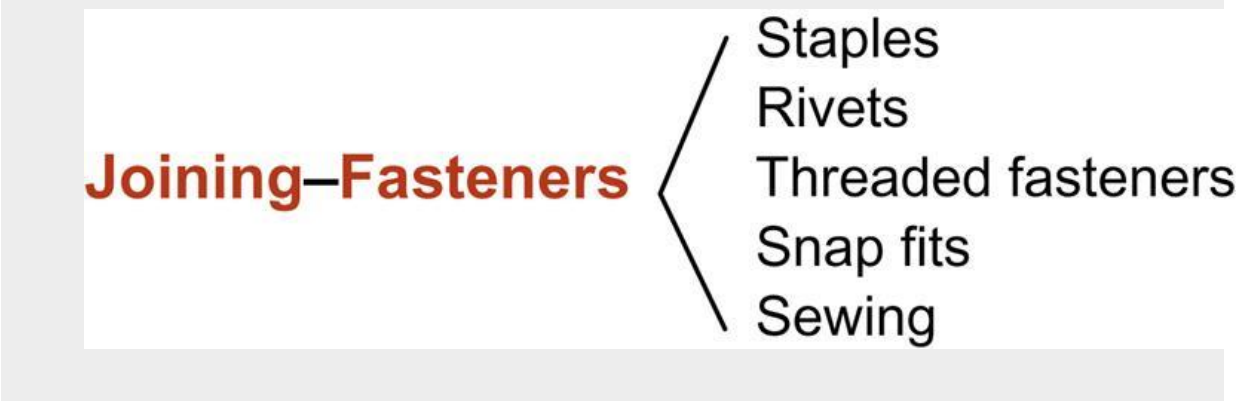
FIGURE 6.4 The process tree again, with the branches of *joining* and *finishing* partly expanded.

The lower part of the figure expands the family of finishing. Some of the classes it contains are shown; one – coating – is expanded to show some of its members. As with shaping and joining, the material to be coated is an important attribute but the others differ. Most important is the purpose of the treatment (protection, surface hardening, decoration…), followed by properties of the coating itself.

Expanding Branches of the Joining Tree

Expand the fasteners family at about the same level of detail as that used for moulding in Fig. 6.3.

The Result



With this background we can embark on a lightning tour of processes. It will be kept as concise as possible; details can be found in the numerous books listed in the suggested further reading (Section 6.6).

6.3 The Processes: Shaping, Joining, Finishing

Shaping Processes

In *casting* (Fig. 6.5), a liquid is poured or forced into a mould where it solidifies by cooling. Casting is distinguished from moulding, which comes next, by the low viscosity of the liquid: it fills the mould by flow under its own weight (as in gravity sand and investment casting) or under a modest pressure (as in die casting and pressure sand casting). Sand moulds for one-off castings are cheap; metal dies for die-casting can be expensive. Between these extremes lie a number of other casting methods: shell, investment, plaster mould and so forth.

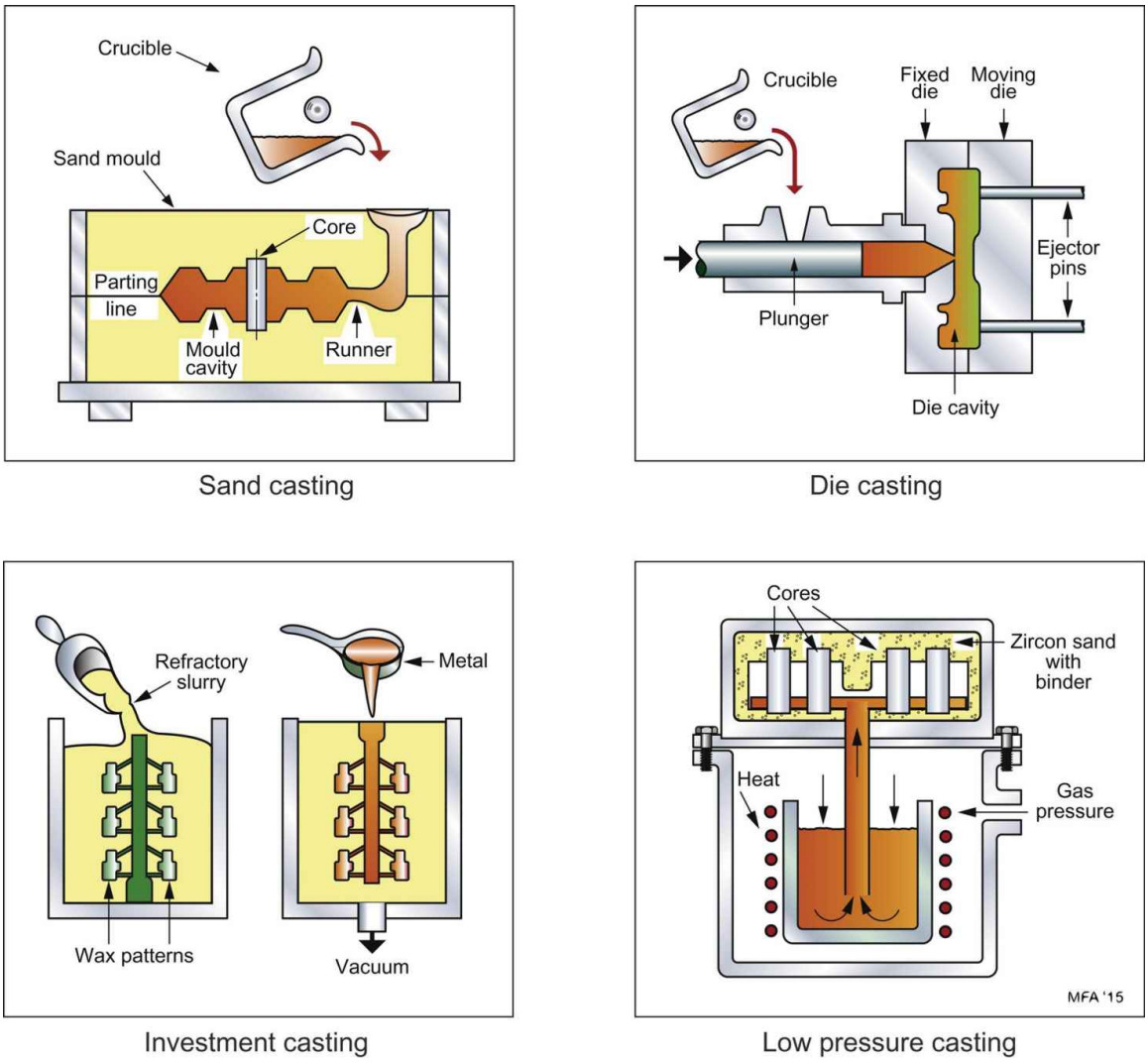


FIGURE 6.5 Casting processes. In *sand casting*, liquid metal is poured into a split sand mould. In *die casting*, liquid is forced under pressure into a metal mould. In *investment casting*, a wax pattern is embedded in refractory, melted out, and the cavity filled with metal. In *pressure casting*, a mould or die is filled from below, giving control of atmosphere and of the flow of metal into the die.

Cast shapes must be designed for easy flow of liquid to all parts of the mould and for progressive solidification that does not trap pockets of liquid in a solid shell giving shrinkage cavities. Whenever possible, section thicknesses are kept uniform (the thickness of adjoining sections should not differ by more than a factor of 2). The shape is designed so that the pattern and the finished casting can be removed from the mould. Keyed-in shapes are avoided because they lead to 'hot tearing' (a tensile creep-

fracture) as the solid cools and shrinks. The tolerance and surface finish of a casting vary from poor (for sand casting) to excellent (for precision die-castings). Tolerance and surface finish are quantified in Chapter 7, Processes Selection and Cost.

When metal is poured into a mould, the flow is turbulent, trapping surface oxide and debris within the casting, giving casting defects. These are avoided by filling the mould from below in such a way that flow is laminar, driven by a vacuum or gas pressure as shown in Fig. 6.5.

Moulding (Fig. 6.6) is casting adapted for materials that are very viscous when molten, particularly thermoplastics and glasses. The hot, viscous fluid is pressed or injected into a die under considerable pressure, where it cools and sets. The die must withstand repeated application of pressure, temperature and the wear involved in separating and removing the part, and therefore is expensive. Elaborate shapes can be moulded, but at the penalty of complexity in die shape and in the way it separates to allow removal. The moulds for thermo-forming, by contrast, are cheap. Variants of the process use gas pressure or vacuum to press a heated polymer sheet onto a single-part mould. Blow moulding, too, uses a gas pressure to expand a polymer or glass blank into a split outer-die. It is a rapid, low-cost process well suited for mass-production of cheap parts like bottles. Polymers, like metals, can be extruded; most rods, tubes and other prismatic sections are made in this way. In film moulding a polymer blank is expanded until it is like an enormous soap bubble that is then cut and flattened.

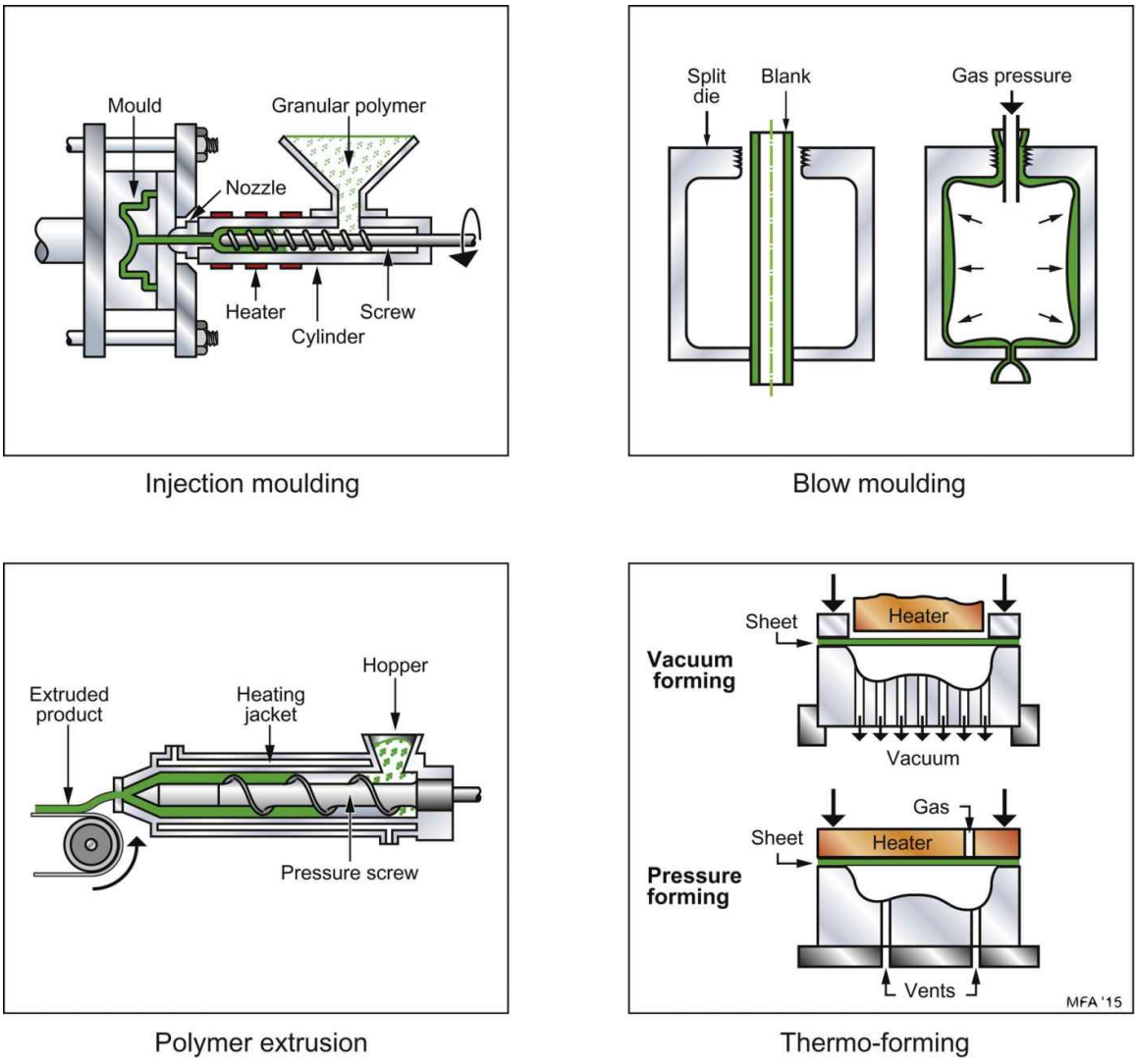


FIGURE 6.6 Moulding processes. In *injection moulding*, a granular polymer is heated, compressed and sheared by a screw feeder, forcing it into the mould cavity. In *blow moulding*, a tubular blank of hot polymer or glass is expanded by gas pressure against the inner wall of a split die. In *polymer extrusion*, shaped sections are formed by extrusion through a shaped die. In *thermo-forming*, a sheet of thermoplastic is heated and deformed into a female die by vacuum or gas pressure.

Deformation processing (Fig. 6.7) can be hot, warm or cold – cold, that is, relative to the melting point T_m of the material being processed. Extrusion, hot forging and hot rolling ($T > 0.55\,T_m$) have much in common with moulding, though the material is a true solid not a viscous liquid. The high temperature lowers the yield strength and allows simultaneous

recrystallisation, both of which lower the forming pressures. Warm working ($0.35\ T_m < T < 0.55\ T_m$) allows recovery but not recrystallisation. Cold forging, rolling and drawing ($T < 0.35\ T_m$) exploit work hardening to increase the strength of the final product, but at the penalty of higher forming pressures.

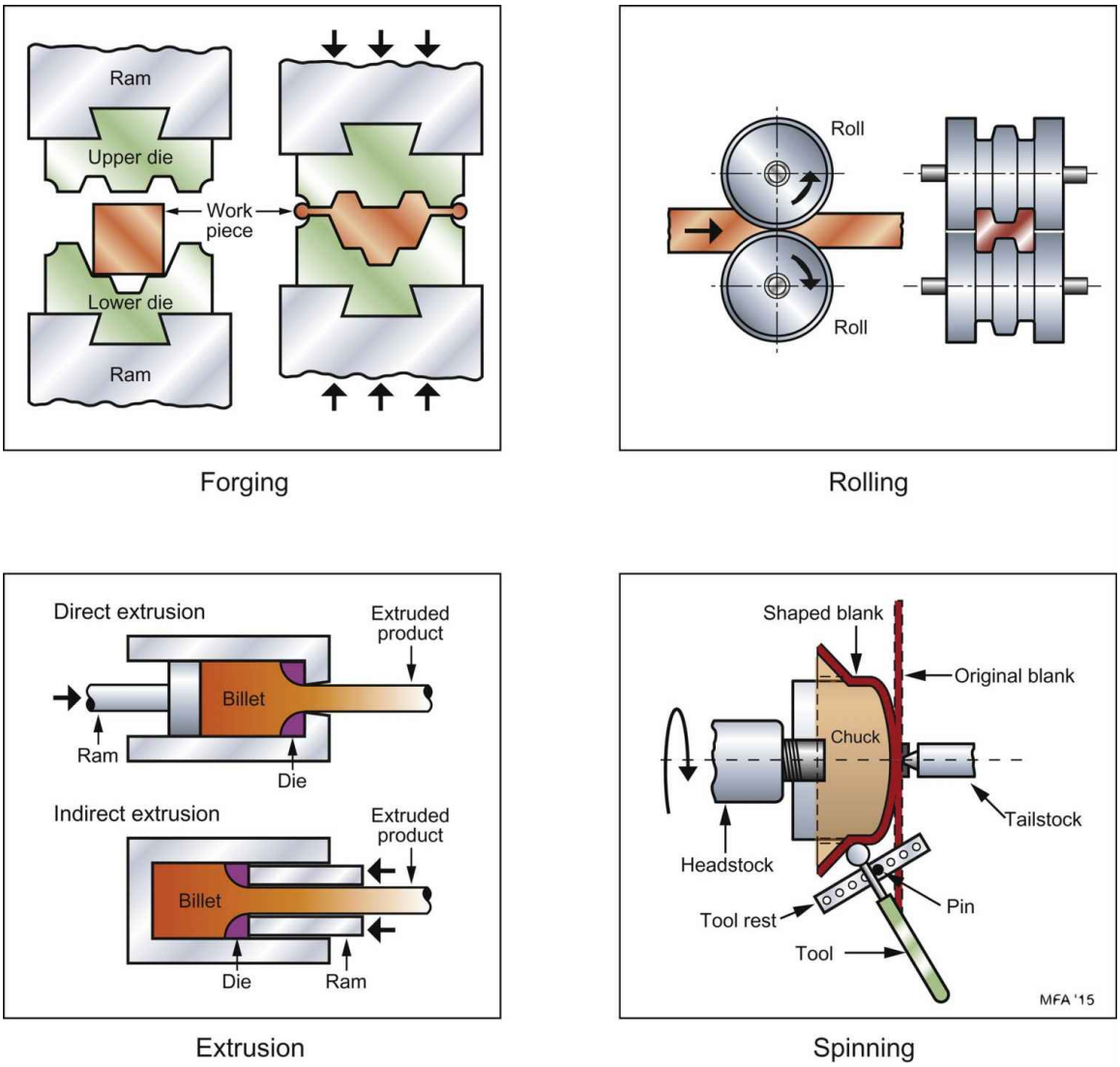


FIGURE 6.7 Deformation processes. In *forging*, a slug of metal is shaped between two dies held in the jaws of a press. In *rolling*, a billet or bar is reduced in section by compressive deformation between the rolls. In *extrusion*, metal is forced to flow through a die aperture to give a continuous prismatic shape – think of red-hot toothpaste. In *spinning*, a spinning disk of ductile metal is shaped over a wooden pattern by repeated sweeps of the smooth, rounded, tool.

Forged parts are designed to avoid sudden changes in thickness and sharp radii of curvature since both require large local strains that can cause the material to tear or to fold back on itself ('lapping'). Hot forging of metals allows larger changes of shape but generally gives a poorer surface and tolerance because of oxidation and warpage. Cold forging gives

greater precision and finish, but forging pressures are higher and the deformations are limited by work hardening.

Powder methods (Fig. 6.8) create the shapes by compressing fine particles of the material in a die, then heating ('sintering') them. The powder can be cold-pressed and then sintered at up to 0.8 T_m to give diffusion bonding; it can be pressed in a heated die ('die pressing'); or the powder can be heated in a thin-walled preform under a gas pressure ('hot isostatic pressing' or 'HIPing'). Metals that are too high melting to be cast and too strong to be deformed can be made (by chemical methods) into powders and then shaped in this way. But the processes are not limited to 'difficult' materials; almost any material can be shaped by subjecting it in powder form to pressure and heat.

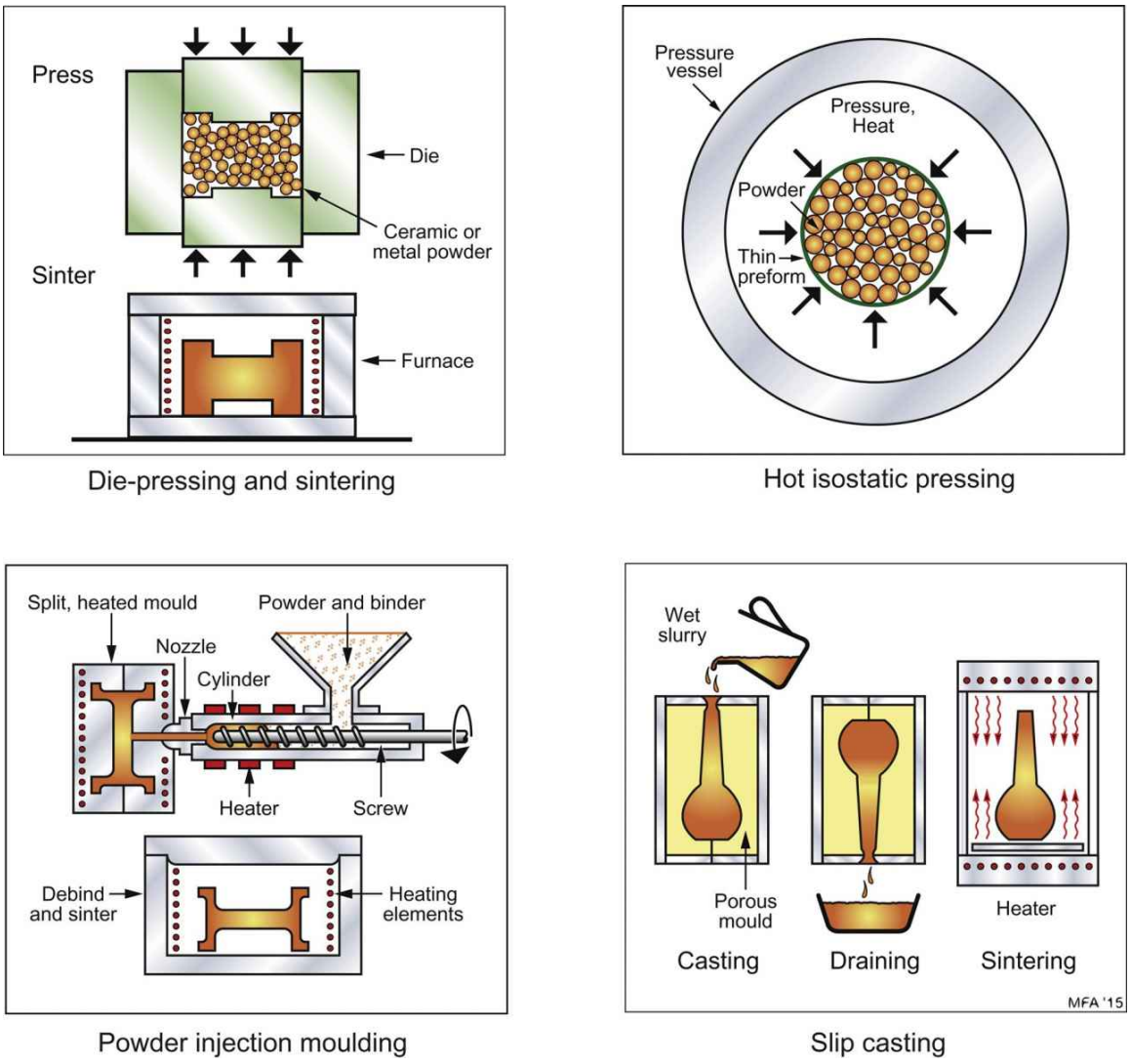


FIGURE 6.8 Powder processing. In *die pressing* and *sintering* the powder is compacted in a die and the green compact is then fired to give a more or less dense product. In *hot isostatic pressing*, powder in a thin preform is heated and compressed by an external gas pressure. In *powder injection moulding*, powder and binder are forced into a die to give a green blank that is then fired. In *slip casting*, a water-based powder slurry is poured into a porous plaster mould that absorbs the water, leaving a powder shell that is subsequently fired.

Ceramics, difficult to cast and impossible to deform, are routinely shaped by powder methods. In slip casting, a water-based powder slurry is poured into a plaster mould. The porous mould wall absorbs water, leaving a semi-dry skin of slurry over its inner wall. The remaining liquid is drained out, and the dried slurry shell is fired to give a ceramic body. In

powder injection moulding (the way sparkplug insulators are made) a ceramic powder in a polymer binder is moulded in the conventional way, and then the moulded part is fired, burning of the binder and sintering the powder.

Powder processing is most widely used for small metallic parts like gears and bearings for cars and appliances. It is economic in its use of material, it allows parts to be fabricated from materials that cannot be cast, deformed or machined, and it can give a product that requires little or no finishing. Pressure is not transmitted uniformly through a bed of powder because of die-wall friction. A consequence is that the length of a die-pressed powder part should not exceed 2.5 times its diameter. Sections must be nearly uniform because the powder will not flow easily around corners. In addition, the shape must be simple and easily extracted from the die.

Composite forming methods (Fig. 6.9) make polymer-matrix composites reinforced with continuous or chopped fibres. Large components are made by filament winding or by laying-up preimpregnated mats of carbon, glass or Kevlar fibre ('prepreg') to the required thickness, then pressing and curing. Parts of the process can be automated, but it remains a slow manufacturing route; and, if the component is a critical one, extensive ultrasonic testing may be necessary to confirm its integrity. Higher integrity is given by vacuum-bag or pressure-bag moulding, which squeezes bubbles out of the matrix before it polymerizes. Lay-up methods are best suited to a small number of high-performance, tailor-made, components. More routine components (car bumpers, tennis racquets) are made from chopped-fibre composites by pressing and heating a 'dough' of resin premixed with chopped glass or carbon fibres, known as bulk moulding compound or sheet moulding compound or by injection moulding a rather more fluid mixture into a die. The flow pattern is critical in aligning the fibres, so that the designer must work closely with the manufacturer to exploit the composite properties fully.

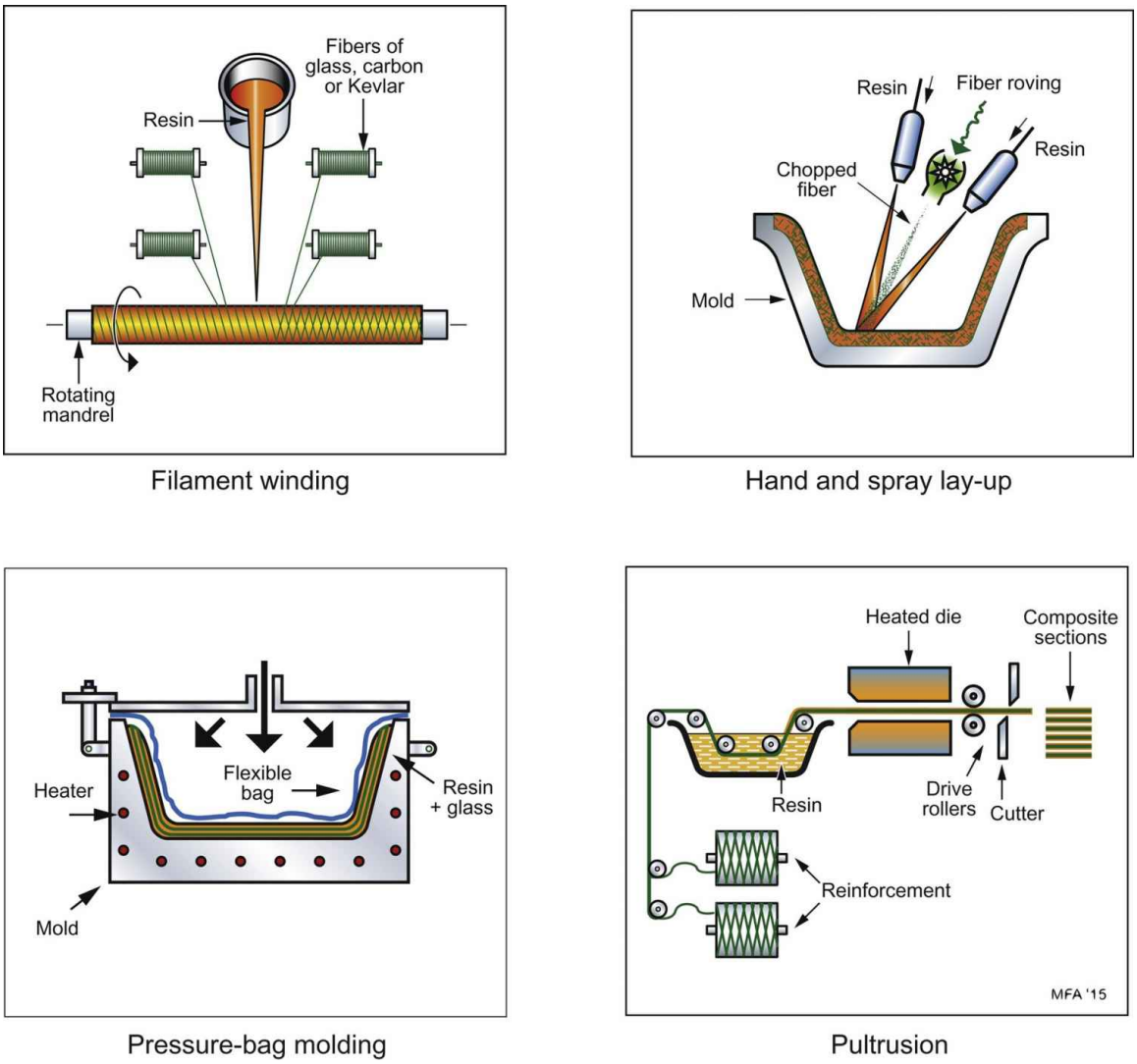


FIGURE 6.9 Composite forming methods. In *filament winding*, fibres of glass, Kevlar or carbon are wound onto a former and impregnated with a resin-hardener mix. In *roll* and *spray lay-up*, fibre reinforcement is laid up in a mould onto which the resin-hardener mix is rolled or sprayed. In *vacuum-bag* and *pressure-bag moulding*, fibre mats impregnated with resin are compressed and heated to cause polymerization. In *pultrusion*, fibres are fed through a resin bath into a heated die to form continuous prismatic sections.

AM (Fig. 6.10) uses computer-controlled tools to build shapes layer by layer. All AM methods can create shapes of great complexity without the need for dies or moulds. The precision and surface roughness, at present, are limited to ±0.1 mm at best and the process is slow (typically 1 to 20 hours per part) but the flexibility of choice of shape is enormous.

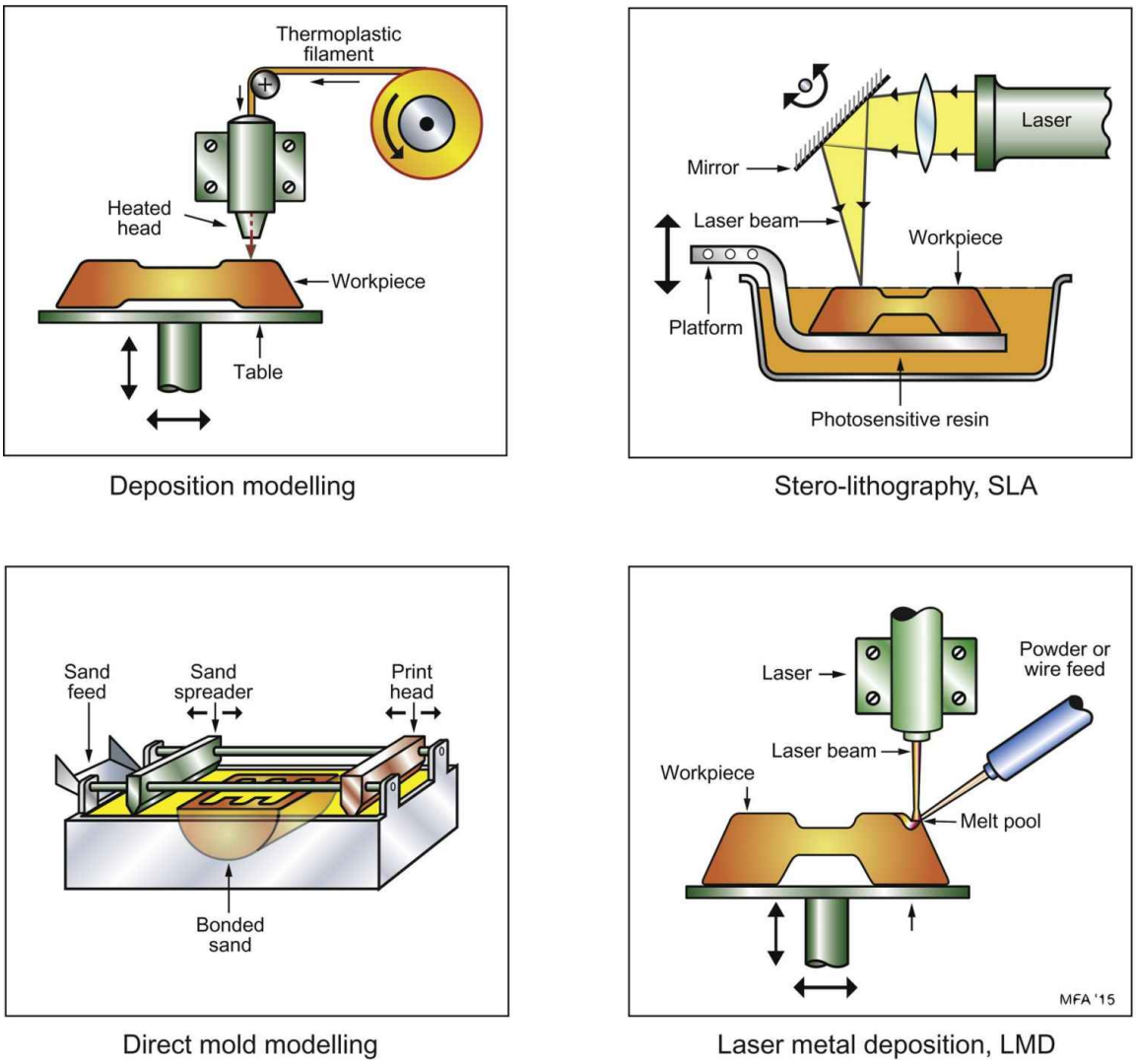


FIGURE 6.10 Additive manufacture. In *deposition modelling* and *ballistic particle manufacture* (BPM), a solid body is created by the layer-by-layer deposition of polymer droplets. In *stereo-lithography* (SLA), a solid shape is created layer by layer by laser-induced polymerisation of a resin. In *direct mould modelling*, a sand mould is built up layer by layer by selective spraying of a binder from a scanning print head. In *laser metal deposition*, metal powder is fed onto the part surfaces where it is selectively sintered by a laser beam.

There are at least six broad classes.

- *Polymer extrusion*. The shape is built up from a thermoplastic fed to a single scanning head that heats and extrudes it like a thin layer of toothpaste ('Fused Deposition Modelling' or FDM), squirts it as tiny droplets ('Ballistic Particle Manufacture', BPM), or ejects it in a patterned

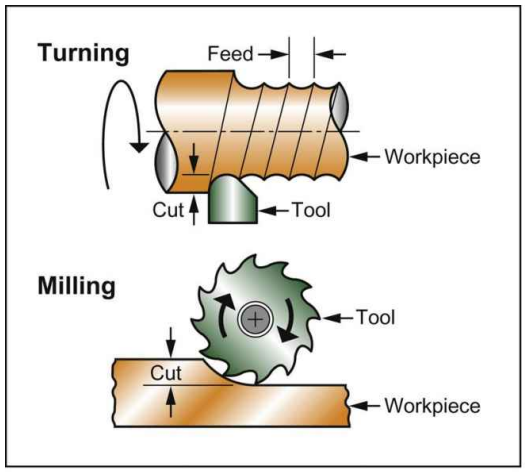
array like a bubble-jet printer ('3-D printing'). The process can be modified to extrude a ceramic slurry that is then fired to give complex ceramic parts.

- *Photo-polymerization: s*canned laser-induced polymerisation of a photosensitive monomer (Stereo-lithography or SLA). After each scan, the workpiece is incrementally lowered, allowing fresh monomer to cover the surface.
- *Sheet laminating* uses a laser beam to cut bondable polymer or paper sheet to give a paper-thin single layer. Each layer is then heat-bonded to the one below.
- *Selected laser sintering* (SLS) allows components to be fabricated directly in thermoplastic, metal or ceramic. A laser scans a bed of particles, sintering a thin surface layer where the beam strikes. A new layer of particles is swept across the surface and the laser sintering step is repeated, building up a 3-dimensional body.
- *Spray deposition*: a stream of molten metal particles is produced by feeding a metal wire into a plasma torch; the computer-controlled stream is directed onto a substrate on which it solidifies and which is subsequently removed.
- *Bonded sand moulding* offers the ability to make large complex metal parts easily. Here a multi-jet print head squirts a binder onto a bed of loose casting sand, building up the mould shape much as SLS does, but more quickly. When complete the mould is lifted from the remaining loose sand and used in a conventional casting process.

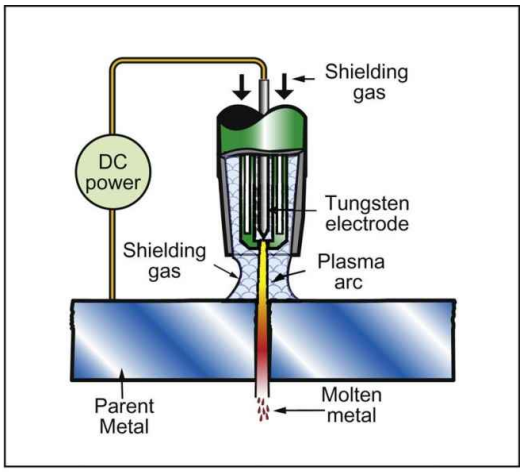
AM is used as a visualization tool for product design: the aesthetics of an object may be evident only when viewed as a prototype. It is used to validate complex assemblies, ensuring that parts fit, can be assembled and are accessible. It is used for pattern making: the prototype becomes the master from which moulds for conventional processing, such as casting, can be made. Architects use it to create models of proposed structures so that clients can better judge how they will look and function. In medical practice it allows the making of custom-designed implants and prostheses for bone and tissue replacement. It can be scaled-up to construct small buildings by the layered-deposition of cement or concrete. Furthermore, it allows remote manufacture of replacement parts for equipment that requires servicing in inaccessible locations (in space, for example).

Machining (Fig. 6.11). Most engineering components, whether made of metal, polymer or ceramic, are subjected to some kind of machining during manufacture. To make this possible they should be designed to

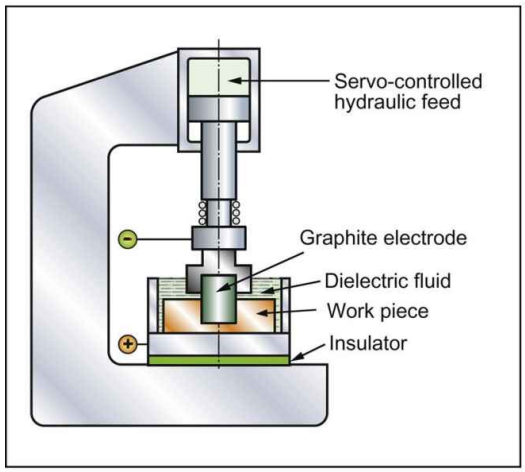
make them easy to grip and to keep the symmetry high: symmetric shapes need fewer operations. Metals differ greatly in their *machinability*, a measure of the ease with which they can be cut cleanly and without excessive tool wear. Machinability is easy to detect but hard to predict – leaded-brass, AISI 1137 carbon steels and AISI 303 stainless steels, e.g., have high machinability but AISI 302 and 316 stainless steels do not.



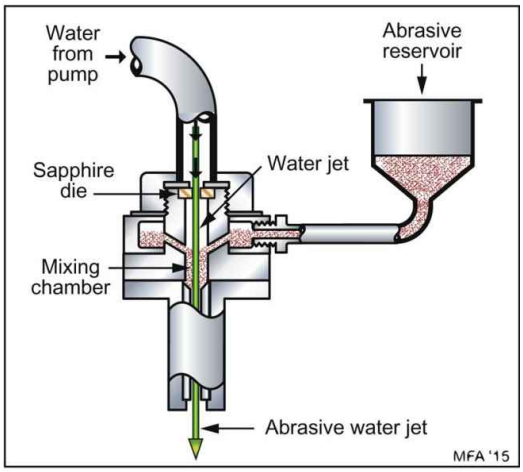
Turning and milling



Plasma (flame) cutting



Electro-discharge machining



Water-jet cutting

FIGURE 6.11 Machining operations. In *turning* and *milling*, the sharp, hardened tip of a tool cuts a chip from the workpiece surface. In *plasma cutting* a high-energy beam of ionized gas selectively melts a slot thorough a conducting workpiece. In *electro-discharge machining*, electric discharge between a graphite electrode and the workpiece, submerged in a dielectric such as paraffin, erodes the workpiece to the desired shape. In *water-jet cutting*, an abrasive entrained in a high-speed water-jet erodes the material in its path.

Most polymers are moulded to a final shape. When necessary they can be machined but their low moduli mean that they deflect during the machining operation and their low thermal conductivity can make them melt and stick to the tool, limiting precision. Ceramics and glasses can be ground and lapped to high tolerance and finish (think of the mirrors of

telescopes). There are many 'special' machining techniques with particular applications; they include electro-discharge machining, ultrasonic cutting, chemical milling, cutting by water-jets, sand-jets, electron beams and laser beams.

Sheet metal forming involves punching, bending and stretching. Holes cannot be punched to a diameter less than the thickness of the sheet although they can be cut with a laser beam. The minimum radius to which a sheet can be bent, its *formability*, is sometimes expressed in multiples of the sheet thickness t: a value of 1 is good; a value of 4 is average. Radii of bending are best made as large as possible, and never less than t. The formability also determines the amount the sheet can be stretched or drawn without necking and failing. The *forming limit diagram* gives more precise information: it shows the combination of principal strains in the plane of the sheet that will cause failure (2-way stretching causes failure sooner than stretching in a single direction). The part is designed so that the strains do not exceed this limit.

Machining is often a secondary operation applied to castings, mouldings or powder products to increase finish and tolerance. Higher finish and tolerance means higher cost; over-specifying either is a mistake.

Joining Processes

Joining (Fig. 6.12) is made possible by a number of techniques. Almost any material can be joined with adhesives, though ensuring a sound, durable bond can be difficult. Bolting, riveting, stapling and snap fitting are commonly used to join polymers and metals. They can be disassembled if necessary, useful in recycling.

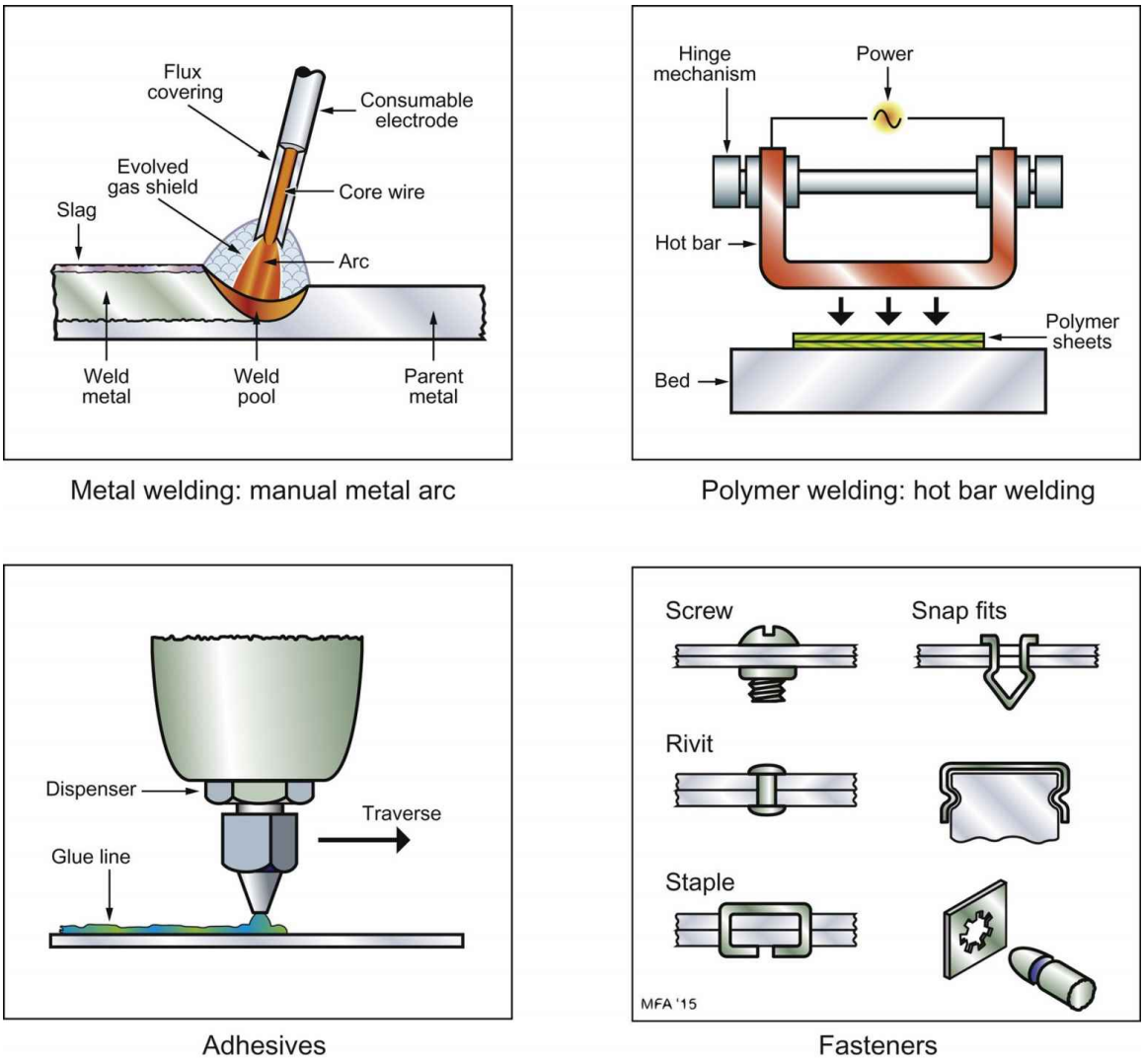


FIGURE 6.12 Joining operations. In *metal fusion welding*, metal is melted, and more added from a filler rod, to give a bond or coating. In *thermoplastic polymer welding*, heat is applied to the polymer components, which are simultaneously pressed together to form a bond. In *adhesive bonding*, a film of adhesive is applied to one surface, which is then pressed onto the mating one. *Fastening* is achieved by bolting, riveting, stapling, push-through and push-on snap fastener.

Welding is widely used to bond metals and polymer; specialized techniques have evolved to deal with each class. A gas or plasma torch, and electric arc or a laser beam melts the material locally, allowing bonding by fusion. Friction welding and friction-stir welding rely on the heat and deformation generated by friction and plastic deformation to create a bond between different metals. Soldering and brazing, too, can

join different materials; they use a lower melting solder or braze to avoid the need to melt the underlying metal. Ceramics can be diffusion-bonded to themselves, to glasses and to metals.

If components are to be welded, the material of which they are made must have high *weldability*. Like machinability, weldability measures a mysterious mix of basic properties. A low thermal conductivity allows welding with low heat input but can lead to greater distortion on cooling. Low thermal expansion gives small thermal strains with less risk of distortion. A solid solution is better than a heat-treatable alloy because over-ageing and softening can occur in the heat-affected zone on either side of the weld. Welding always leaves internal stresses that are roughly equal to the yield strength of the parent material. They can be relaxed by postweld heat treatment but this is expensive, so it is better to minimize their effect by good design. To achieve this, parts to be welded are made of equal thickness whenever possible and the welds are located where stress or deflection is least critical.

Finishing Processes

Finishing involves treatments applied to the surface of the component or assembly. Some aim to improve surface properties, others to enhance appearance.

Finishing treatments to improve surface properties (Fig. 6.13). Grinding, lapping and polishing increase precision and smoothness, particularly important for bearing surfaces. Electroplating deposits a thin metal layer onto the surface to give resistance to corrosion and abrasion. Anodising, phosphating and chromating create a thin layer of oxide, phosphate or chromate on the surface, imparting corrosion resistance. Plating and painting are both made easier by a simple part shape with largely convex surfaces: channels, crevices and slots are difficult to reach.

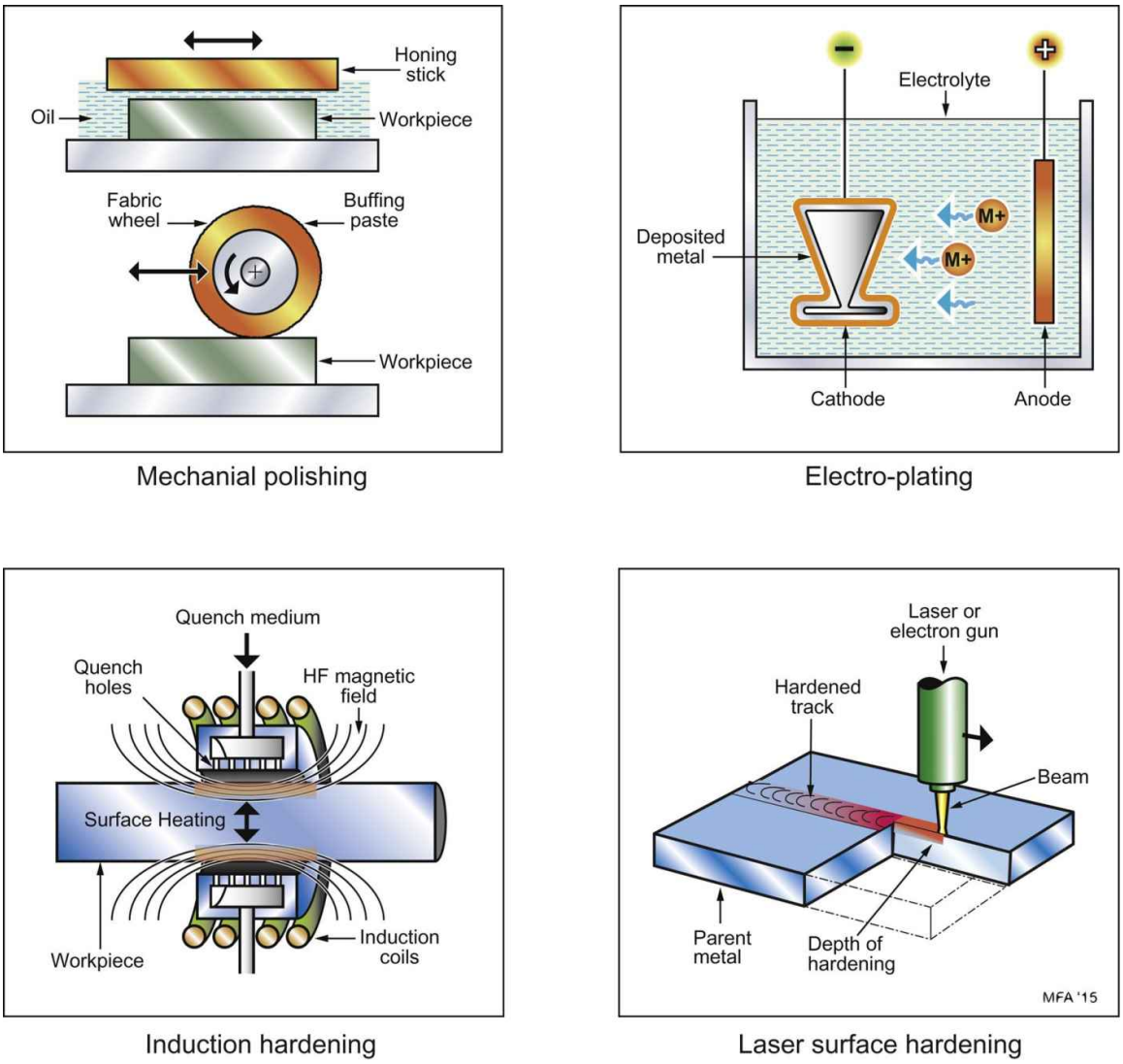


FIGURE 6.13 Finishing processes to protect and enhance properties. *Mechanical polishing* reduces surface roughness and can increase precision by removing material with finely ground abrasives. In *electroplating*, metal is electro-deposited onto a conducting workpiece. In *induction hardening*, a surface layer of the workpiece heated, and then quenched, to create a hard surface layer. In *laser surface hardening* a surface track is melted then cooled rapidly by conduction to give hard tracks.

Heat treatment is a necessary part of the processing of many materials. Age-hardening alloys of aluminium, titanium and nickel derive their strength from a precipitate produced by a controlled heat treatment: quenching from a high temperature followed by ageing at a lower one. The hardness and toughness of steels is controlled in a similar way: by

quenching from the 'austenitizing' temperature (about 800°C) and tempering. The treatment can be applied to the entire component, as in bulk carburizing, or just to a surface layer, as in flame hardening, induction hardening and laser surface hardening.

Quenching – very rapid cooling – is a savage procedure; the sudden thermal contraction associated with it can produce stresses large enough to distort or crack the component. The stresses are caused by a nonuniform temperature distribution, and this, in turn, is related to the geometry of the component. To avoid damaging stresses, the section thickness should be as uniform as possible, and nowhere so large that the quench-rate falls below the critical value required for successful heat treatment. Stress concentrations should be avoided because they are a source of quench cracks. Materials that have been moulded or deformed may contain internal stresses that can be removed, at least partially, by stress-relief anneals – another sort of heat treatment.

Finishing treatments that enhance aesthetics (Fig. 6.14). The processes just described can be used to enhance the visual and tactile attributes of a material: electroplating and anodising are examples. There are many more, of which painting is the most widely used. Organic-solvent based paints give durable coatings with high finish, but the solvent poses environmental problems. Water-based paints overcome these problems but they dry more slowly and the resulting paint film is less perfect. In polymer powder-coating and polymer powder-spraying a film of a thermoplastic – nylon, polypropylene or polyethylene – is deposited on the surface, giving a protective layer that can be brightly coloured. In screen printing an oil-based ink is squeegeed through a mesh on which a blocking film holds back the ink where it is not wanted; full colour printing requires the successive use of up to four screens. Increasingly, screen printing is replaced by wide-bed bubble-jet printing because of its flexibility. Curved surfaces require the use of pad printing, in which a pattern, etched onto a metal 'cliche', is inked and picked up on a soft rubber pad. The pad is pressed onto the product, depositing the pattern on its surface; the compliant rubber conforms to the curvature of the surface.

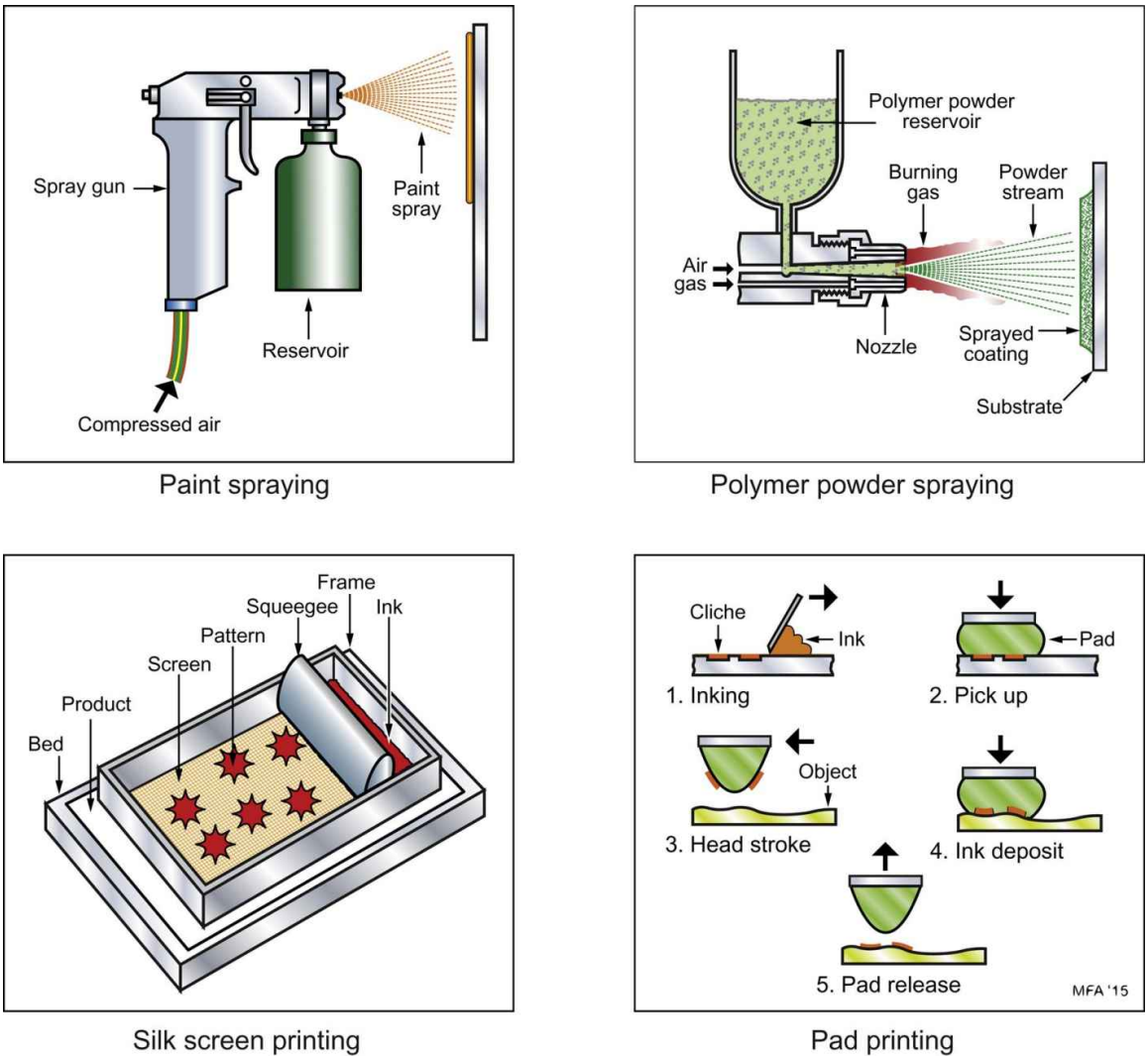


FIGURE 6.14 Finishing processes to enhance appearance. In *paint spraying*, a pigment in an organic or water-based solvent is sprayed onto the surface to be decorated. In *polymer powder-coating* a layer of thermoplastic is deposited on the surface by direct spraying in a gas flame, or by immersing the hot workpiece in a bed of powder. In *silk-screen printing*, ink is wiped onto the surface through a screen onto which a blocking-pattern has been deposited, allowing ink to pass in selected areas only. In *pad printing*, an inked pattern is picked up on a rubber pad and applied to the surface, which can be curved or irregular.

That's enough about the processes themselves; for more detail the reader will have to consult the suggested further reading in Section 6.6 or the CES EduPack, which contains schematics and attributes for some 245

shaping, joining and finishing processes. Now a look at what processing does to properties.

6.4 Process–Property Trajectories

The extent of the material bubbles on the property charts of Chapter 3, Materials Property Charts gives an idea of the degree to which properties can be manipulated by processing. Take a look at Fig. 6.15; it is a copy of the modulus-strength chart. Metals have long thin bubbles because the processes used to manipulate their properties – alloying, heat treatment and mechanical deformation – have almost no effect on the modulus but increase strength enormously. Polymers, by contrast, have almost round bubbles: the processes used to manipulate their properties – cross-linking, blending and filling with particulates – change both modulus and strength. The rather different shapes of the bubbles for composites and for foams reflect the way the properties of the first depend on fibre content and orientation, and those of the second on the extent of foaming, measured by the porosity or relative density. The strength, particularly, of ceramics depends on porosity, another aspect of microstructure that is directly influenced by processing.

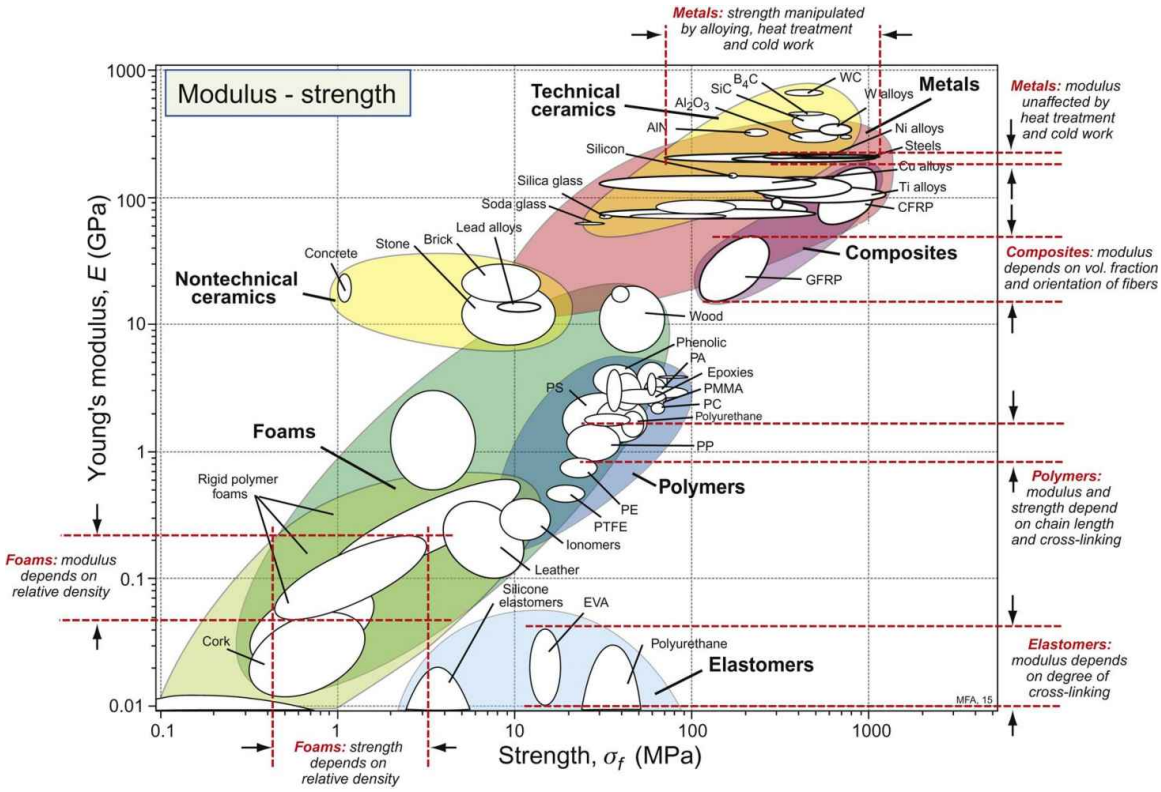
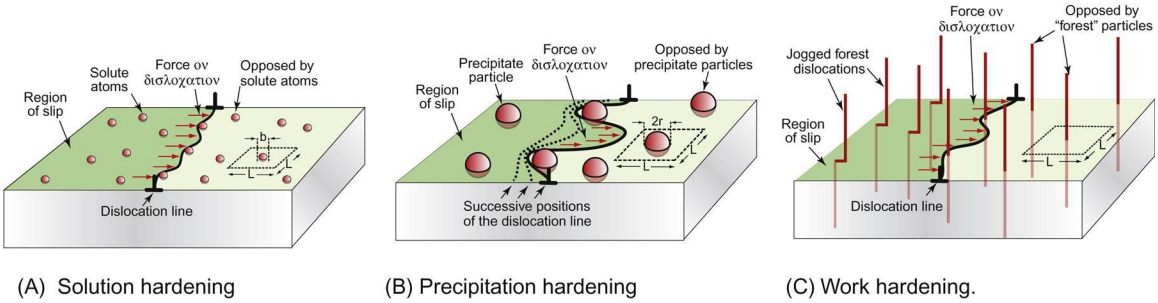


FIGURE 6.15 The extent of the material bubbles on the property charts gives an idea of the degree to which properties can be manipulated by processing.

The effect of processing on properties is nicely illustrated by plotting *process trajectories* onto material property charts, which shows how properties evolve with alloying, blending, heat treatment, working, or foaming. In this section we explore 10 such process–trajectory charts.

Strength and ductility of alloys. If you want to manipulate the mechanical properties of metals there are three basic ways to do it: by solution hardening, by precipitation hardening and by work hardening (Fig. 6.16).



(A) Solution hardening (B) Precipitation hardening (C) Work hardening.

FIGURE 6.16 Strengthening mechanisms in metals. Deformation involves dislocation motion. Solute, precipitates and 'forest' dislocations introduced by working obstruct the motion, which increases the strength but influences other properties also. (A) Solution hardening, (B) Precipitation hardening, (C) Work hardening.

Of all the properties that materials scientists and engineers have sought to manipulate, the strength of metals and alloys is probably the most explored. Fig. 6.17 illustrates the large gains in strength of copper alloys that these mechanisms allow. Solution hardening (green bubbles) lifts the strength of copper from 50 MPa to 200 MPa. Work hardening (blue) pushes it further to 400 MPa, but at a penalty: there is a loss of ductility, measured by the elongation ε_f, small for solution hardening, large for work hardening. Each process has created a distinctive property trajectory.

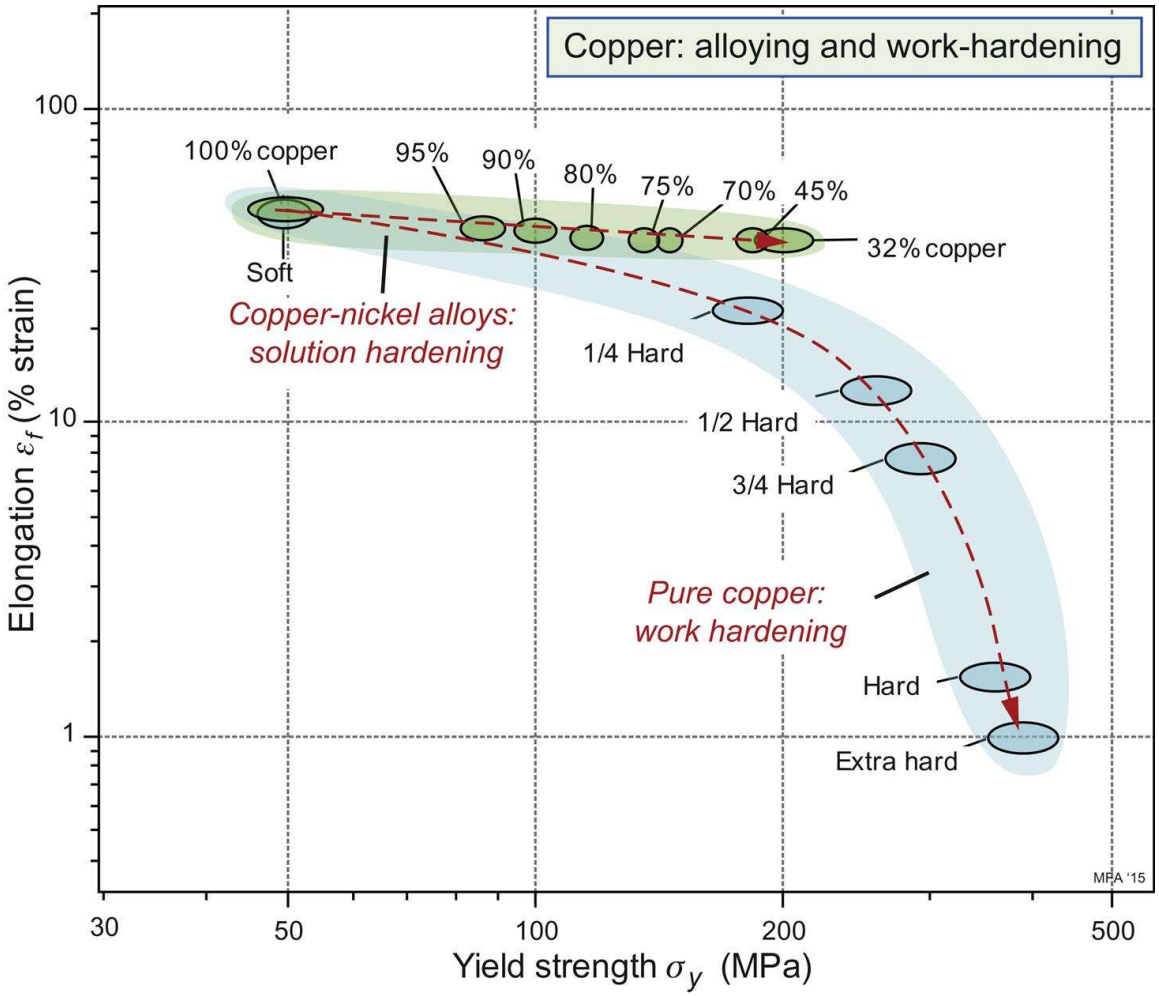


FIGURE 6.17 The effect of alloy hardening and work hardening on the strength and elongation of copper.

Elongation is not the only property to suffer when increased strength is the objective. Fig. 6.18 shows the trade-off between thermal conductivity and strength (the electrical conductivity behaves is a similar way). Here an extra trajectory has been added to show the combined effect of alloying to create a precipitation hardened alloy coupled with work hardening. Solution hardening greatly reduces the conductivity because solute atoms scatter electrons and phonons effectively. Work hardening, by contrast, reduces conductivity only slightly; that is why heat exchangers use almost pure copper or aluminium. Precipitation hardening gives the best compromise between very high strength and good conductivity.

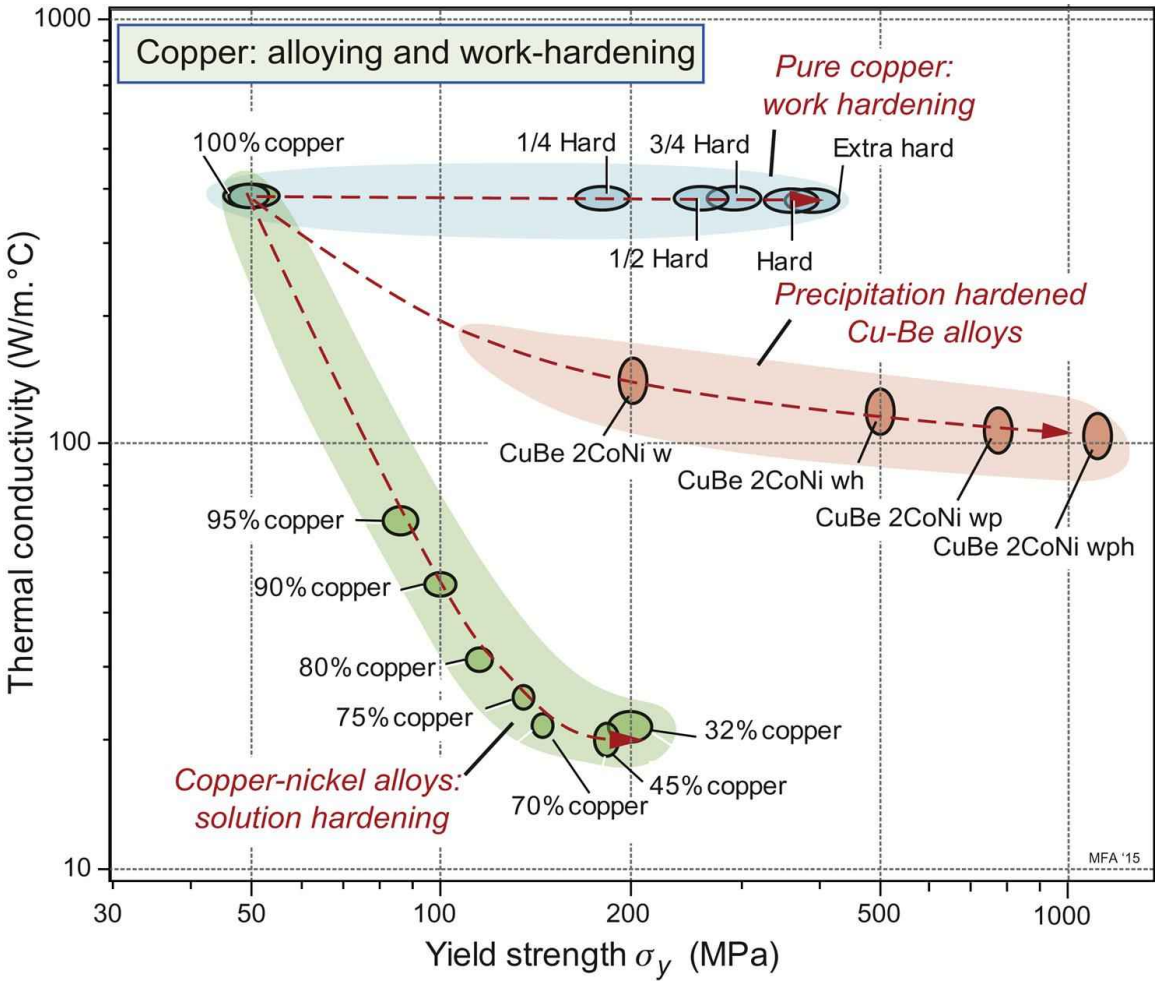


FIGURE 6.18 The effect of solution hardening, precipitation hardening and work hardening on the strength, and thermal conductivity of copper. The symbol 'w' means solution heat treated; 'wh' means solution heat treated and work hardened; 'wp' means solution heat treated and aged to allow precipitation; 'whp' means solution treated, work hardened and aged to allow precipitation.

Strength, ductility thermal and electrical conductivity are *structure-sensitive properties* – they depend on composition and microstructure, and these, in turn, are controlled by processing. Hardness, fatigue strength, and fracture toughness, too, are structure-sensitive properties. Much processing is fine tuned to produce particular combinations of these.

The two most important properties for mechanical integrity are strength and fracture toughness. Fig. 6.19 show the trajectory of these two properties with alloying and heat treatment for the most important class of light alloys: those based on aluminium. Age-hardening alloys (orange)

rely on precipitation hardening; they have the most attractive combinations of toughness and strength. The nonage-hardening grades (light green) rely on work hardening and solution hardening. They are slightly less tough than the age-hardening grades. The loss of toughness as the strength is increased is most extreme for the cast grades (blue).

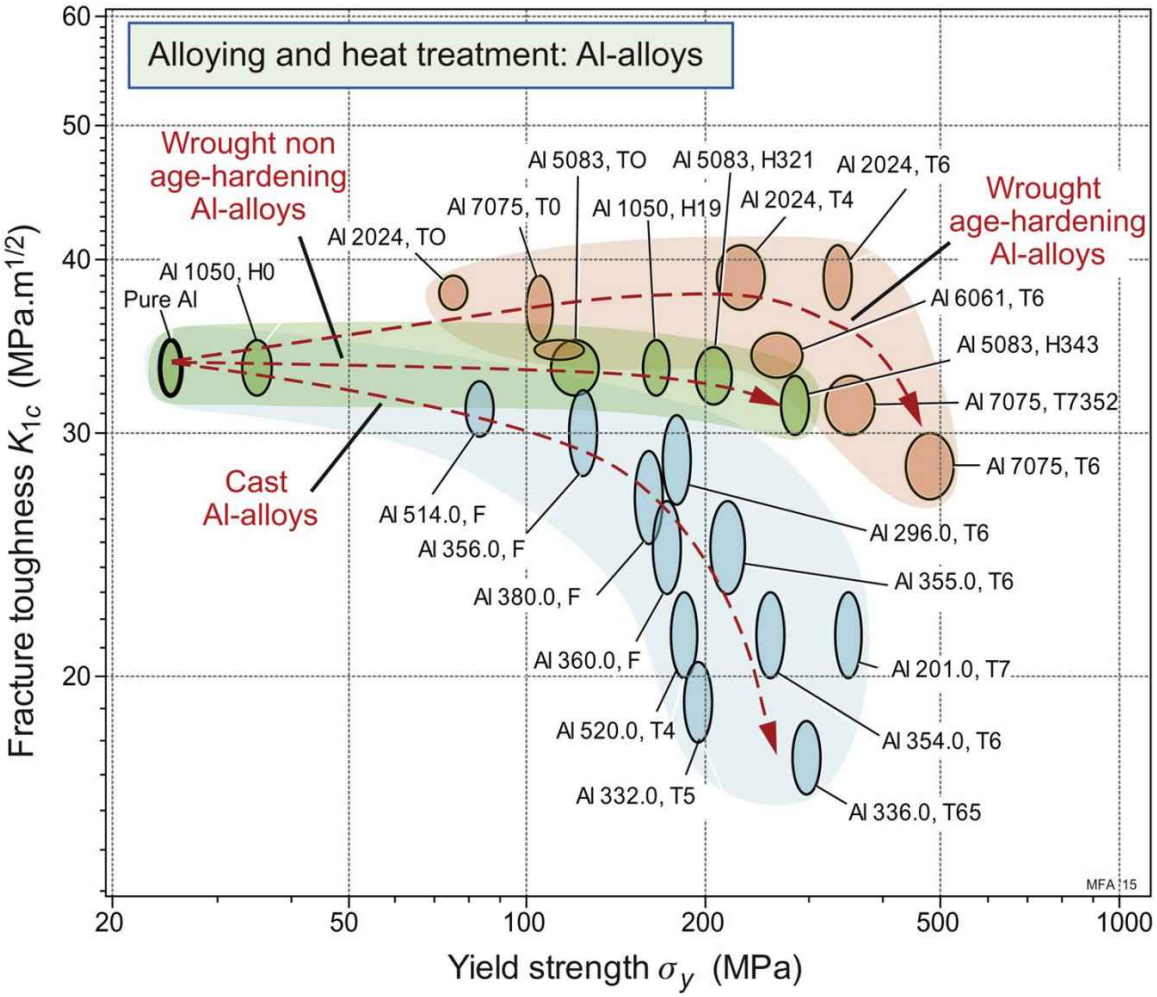


FIGURE 6.19 The trajectories of toughness and strength for classes of aluminium alloys. All three of the mechanisms of Fig. 6.16 are involved.

Heat treatment of steels. This toughness–strength trade-off is central to the selection of steels for mechanical applications. Fig. 6.20 shows the property trajectory as the carbon content is increased from near zero (pure iron) to 0.5% (high carbon steel) and then to 2% or more (cast irons).

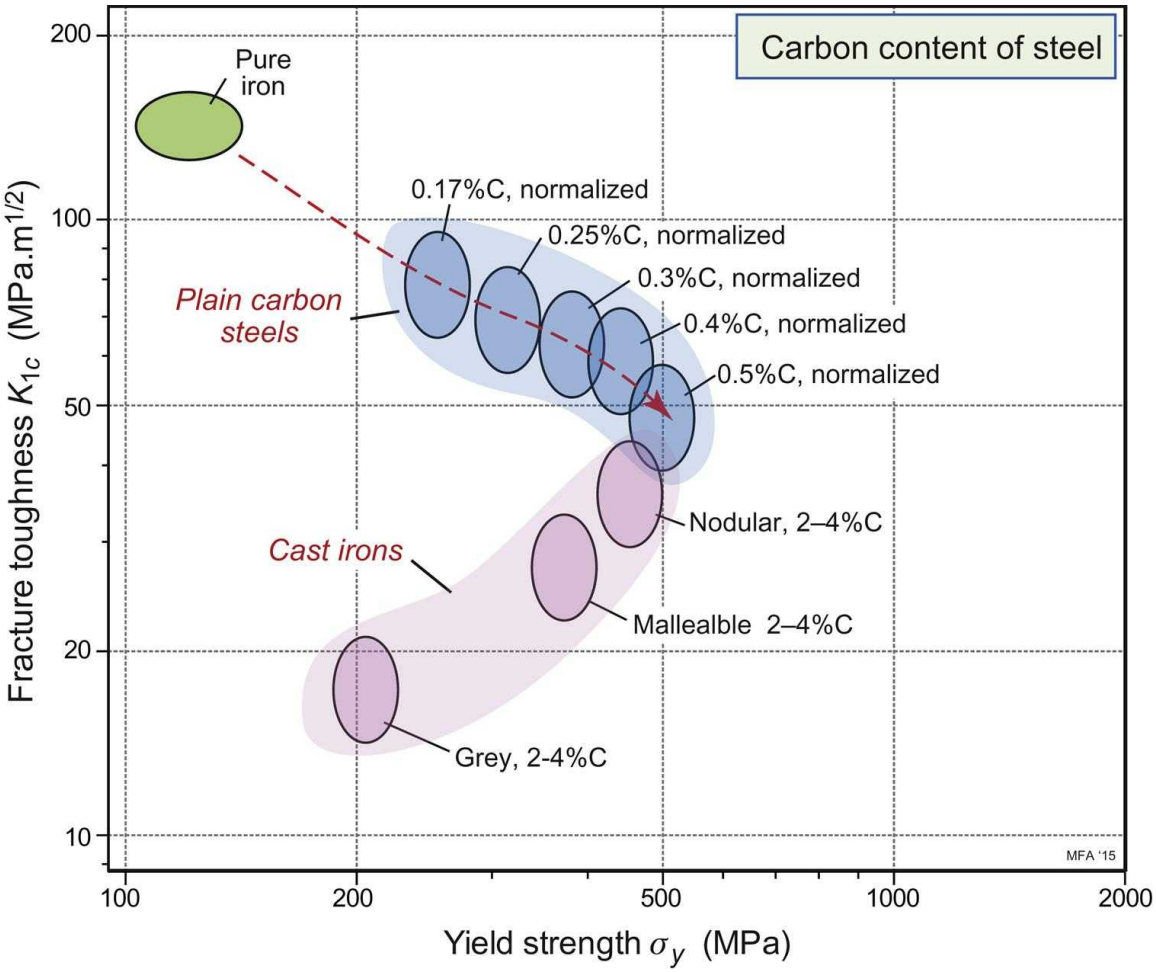


FIGURE 6.20 The toughness–strength trajectory for iron carbon alloys as the carbon content is increased.

The properties of steels can be controlled to a remarkable degree by heat treatment. When a low, medium or high carbon steel is held in the austenitic region of the phase diagram (Fig. 6.21) the carbon dissolves. If it is then quenched rapidly to room temperature, the carbon is retained in solution in a distorted, martensitic, structure that is very hard but very brittle. The structure and properties can subsequently be fine tuned by tempering at a controlled temperature and time.

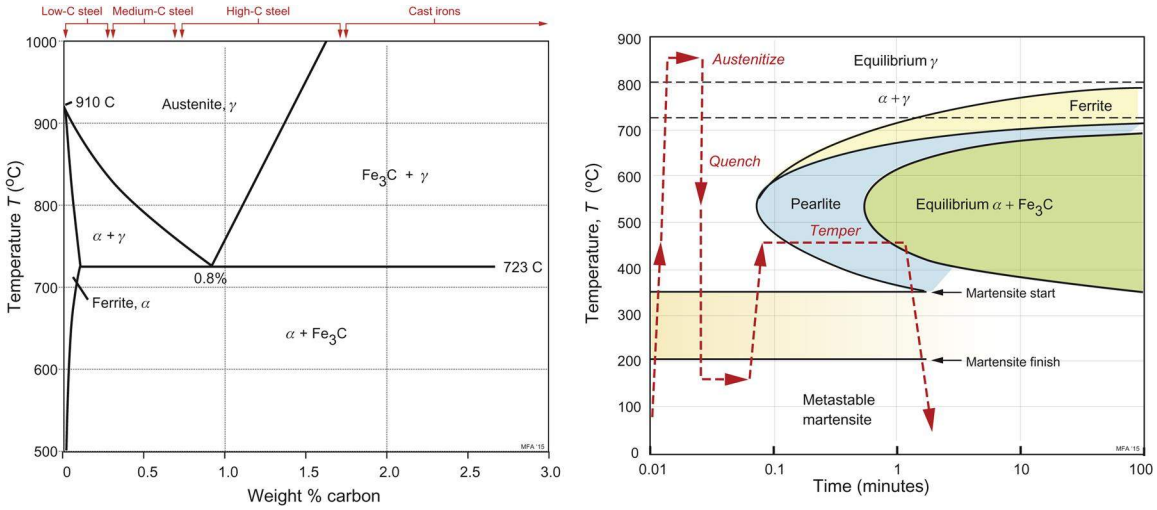


FIGURE 6.21 (Left) the iron-carbon phase diagram showing the composition ranges for carbon steels and cast irons. (Right) a schematic Time-Temperature-Transformation curve for a medium carbon steel, showing the process route.

Fig. 6.22 shows the resulting property trajectory, in this case for a medium carbon steel. The steel is austenitized (heated into the face-centered cubic austenite field) to dissolve all the carbon it contains, and then quenched in water or oil which causes it to transform to hard, brittle martensite. Martensite has high hardness but is also brittle – meaning that its fracture toughness K_{1c} is so low that it is almost useless as a structural material. Tempering reduces the hardness and yield strength but restores toughness to a degree that depends on the tempering temperature and time. The desired property set is obtained by controlling these.

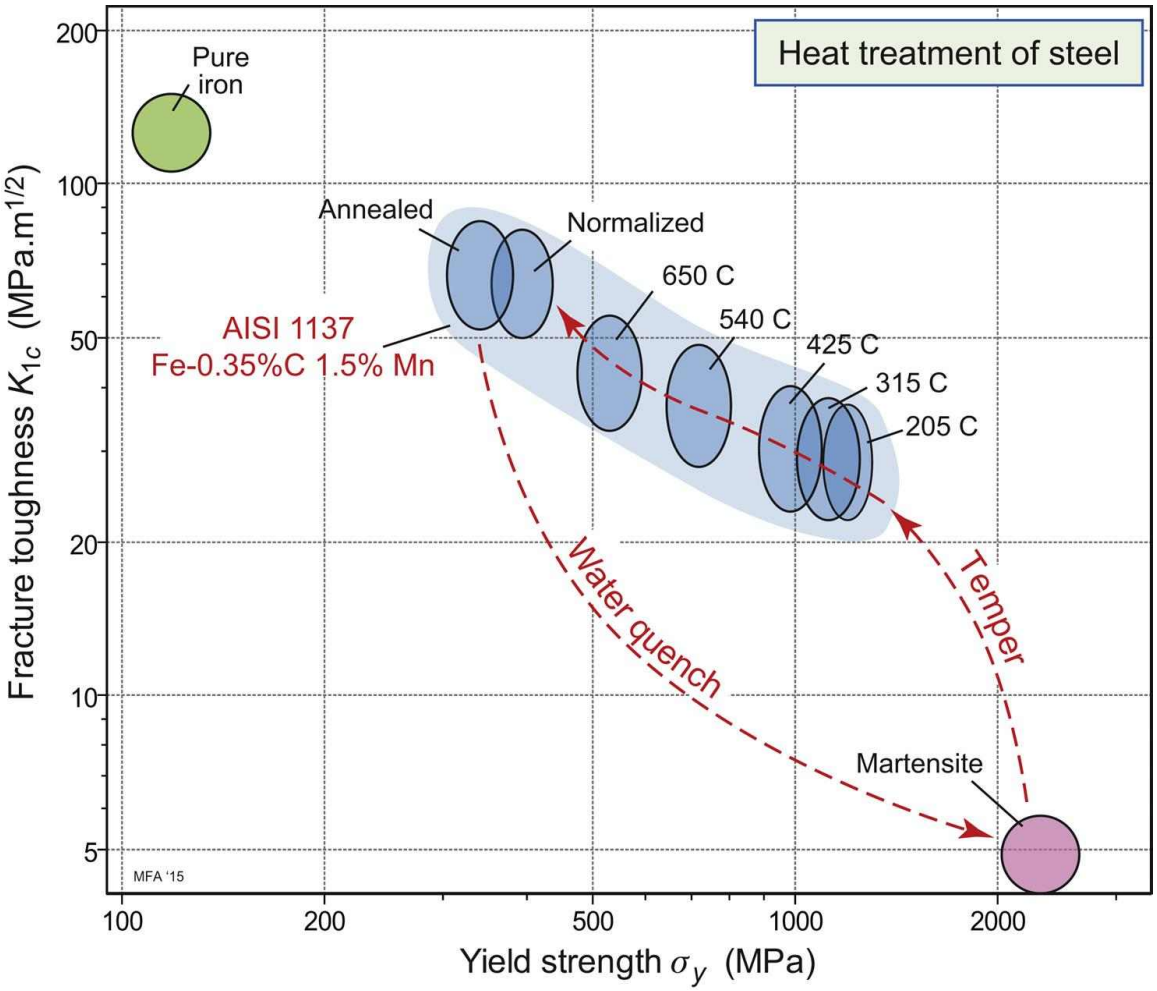


FIGURE 6.22 The toughness–strength trajectory for the tempering of a medium carbon steel.

Perhaps the most striking example of the effect of processing on properties is that of the stainless steels. Fig. 6.23 shows the toughness–strength trajectories for three of the most important: the austenitic, the precipitation hardened and the martensitic stainless steels. The alloys shown here combine all three of the mechanisms of Fig. 6.16 to achieve an unusual combination of toughness and strength.

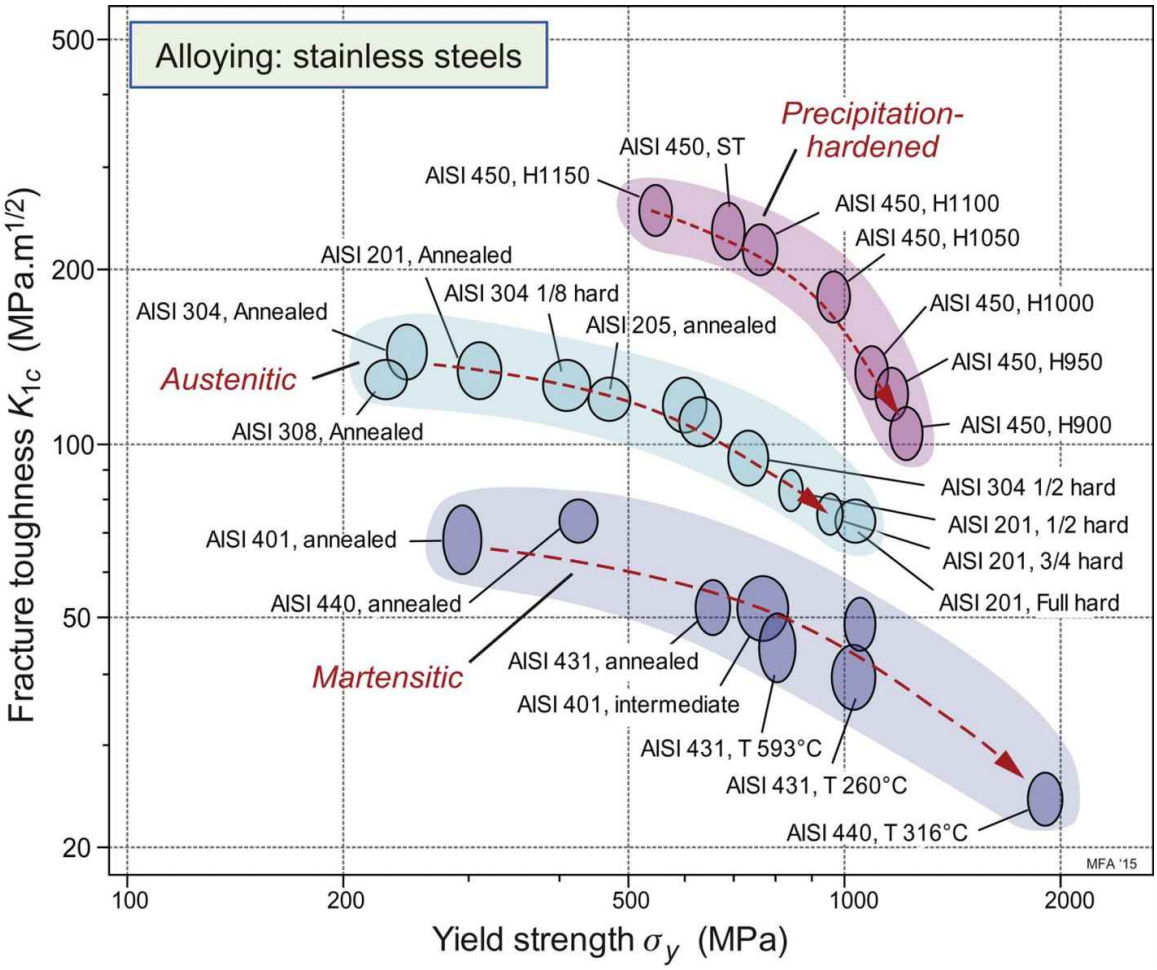


FIGURE 6.23 Stainless steels. The extent of alloying, heat treatment and work hardening increase from left to right.

Polymer processing. The bonds that link atoms along a polymer chain are strong, but the Van der Waals and hydrogen bonds between chains are weak (Fig. 6.24). The low elastic moduli reflect the low stiffness of the inter-chain bonds. Polymer strength, too, reflects the nature of the bonds. Polymers deform by the relative slippage between polymer chains. If this only involves the rupture of weak bonds, the strength is low. The modulus, the strength and the fracture toughness depend on degree of crystallinity and cross-linking and can all be manipulated by *blending*, by *drawing*, and by *reinforcement* with particulate, fibre or fabric fillers.

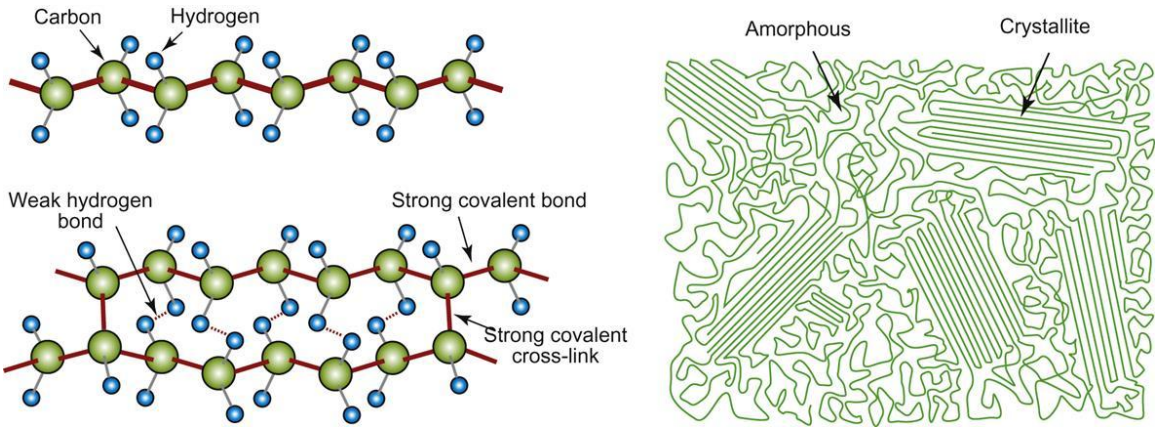


FIGURE 6.24 Polymer structures.

A blend is a mixture of two polymers, stirred together in a sort of industrial food-mixer. The resulting strength and modulus are just the average of those of the components, weighted by volume fraction (the rule of mixtures). If one of these is a low molecular weight hydrocarbon, it acts as a plasticizer, reducing the modulus and giving the blend a leather-like flexibility.

Drawing is the deliberate use of the molecule-aligning effect of stretching to greatly increase stiffness and strength in the direction of stretch. Fishing line is drawn nylon, Mylar film is a polyester with molecules aligned parallel to the film, and geotextiles, used to restrain earth banks, are made from drawn polyethylene. Cross-linking creates strong bonds between molecules that were previously linked by weak Van der Waals forces. Vulcanised rubber is rubber that has been cross linked, and the superior strength of epoxies derives from cross linking.

Reinforcement is possible with particles of cheap fillers – silica (sand), calcium carbonate (chalk), talc or wood dust. Far more effective is reinforcement with fibres – usually glass or carbon – either continuous or chopped. Figs. 6.25 and 6.26 shows how these influence the modulus E, strength σ_Y and fracture toughness K_{1c} of polypropylene, PP. Blending or copolymerization with impact-modifying elastomers such as ethylene propylene rubber (EPR) or ethylene propylene diene monomer (EDPM) reduces the modulus but increases the fracture toughness K_{1c} and toughness G_c. Filling with cheap powdered glass, talk or calcium carbonate more than doubles the modulus but at the expense of some loss of toughness. Plasticizing (blending with low molecular weight polymers) lowers the modulus even more dramatically. Between them, these

processes can change the polymer modulus by a factor of 100, the toughness by a factor of 10.

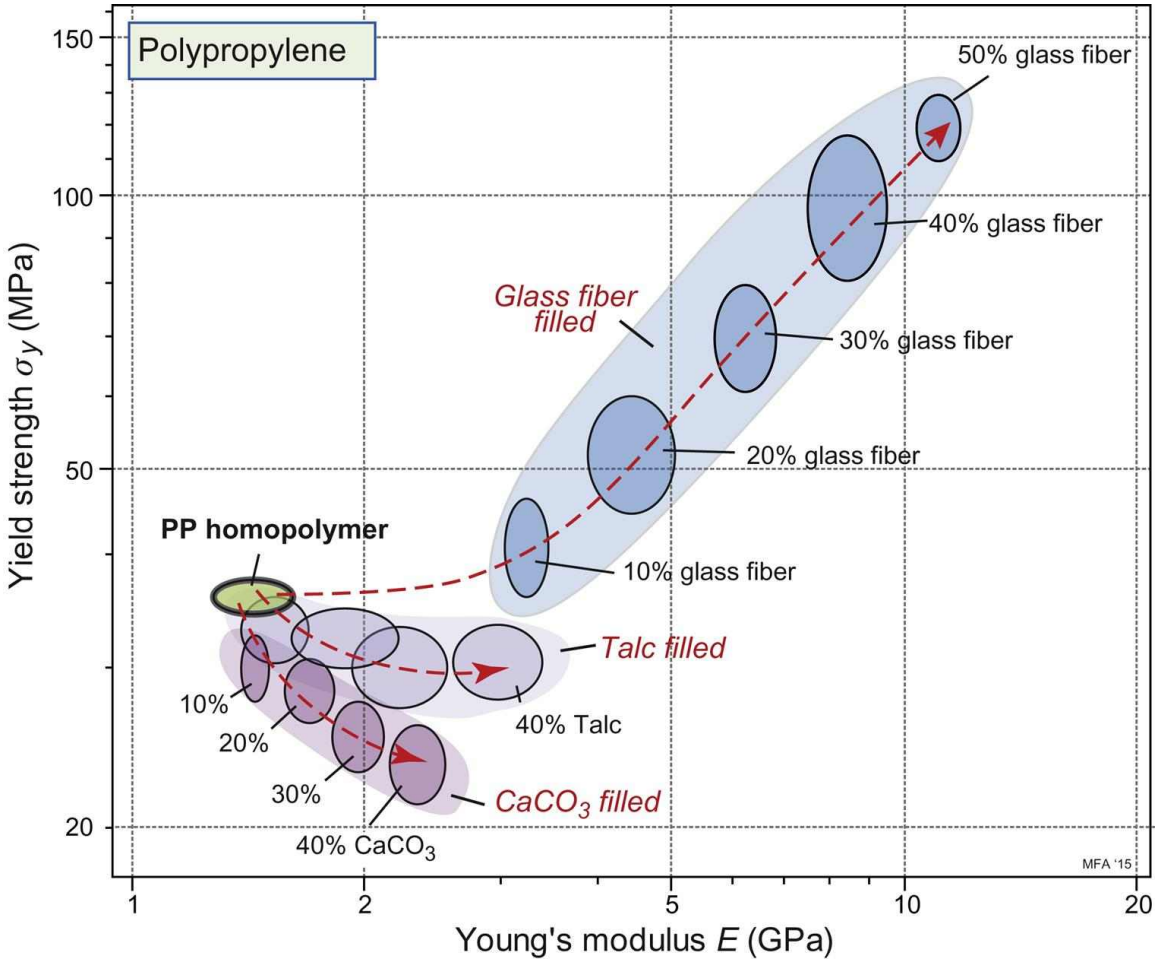


FIGURE 6.25 Strength and modulus of filled polypropylene.

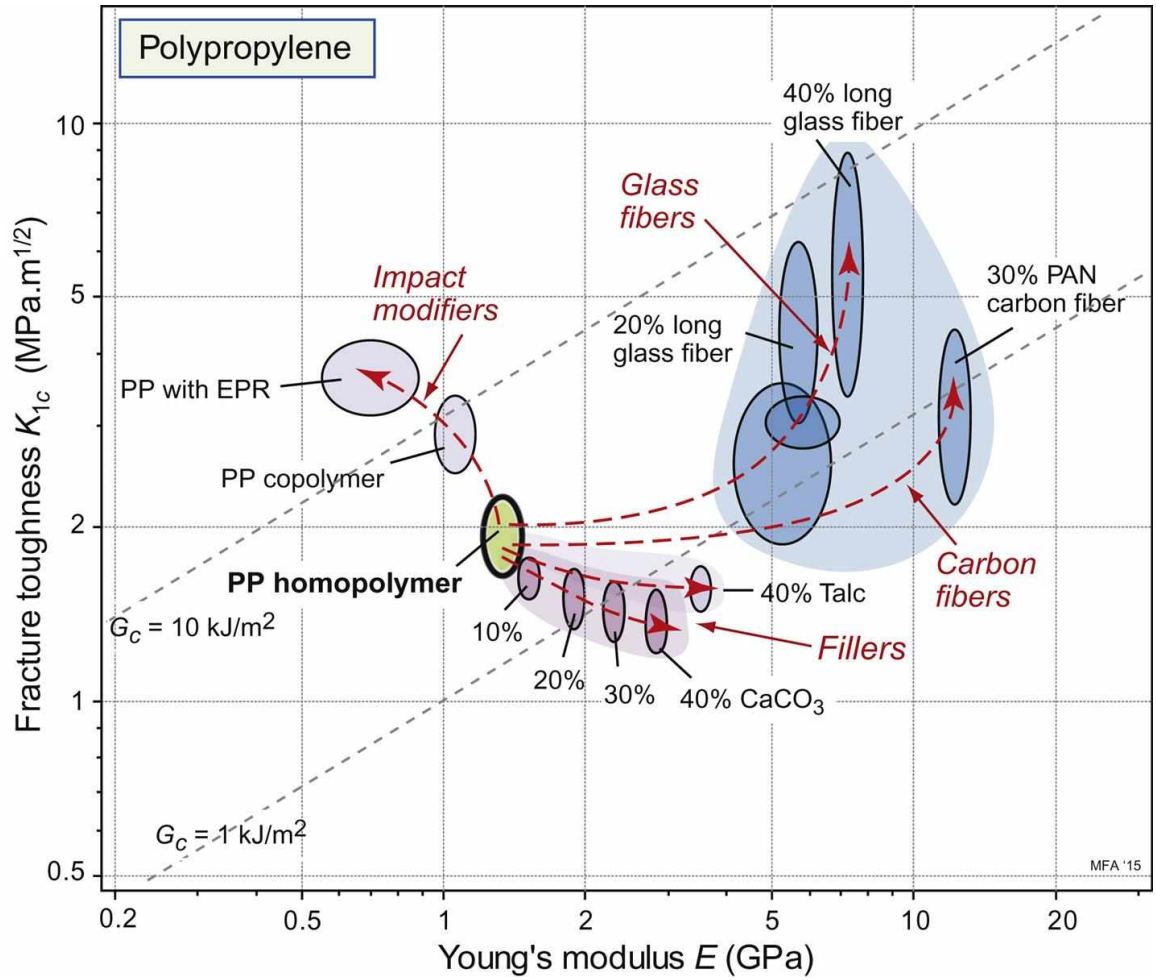


FIGURE 6.26 The effect of fillers, impact modifiers and fibres on the strength and toughness of polypropylene.

Consolidation of ceramics. Most ceramics are made by the firing or sintering of a powder. The powder is press to an initial 'green' relative density of about 0.7 (or 30% porosity). It is then heated, sometimes under pressure, to the sintering temperature where it is held for a fixed time, then allowed to cool. During sintering diffusion allows necks to grow between the powder particles, the residual porosity spheroidizes and shrinks until – if the sintering is done under vacuum – it disappears and full density is reached (Fig. 6.27).

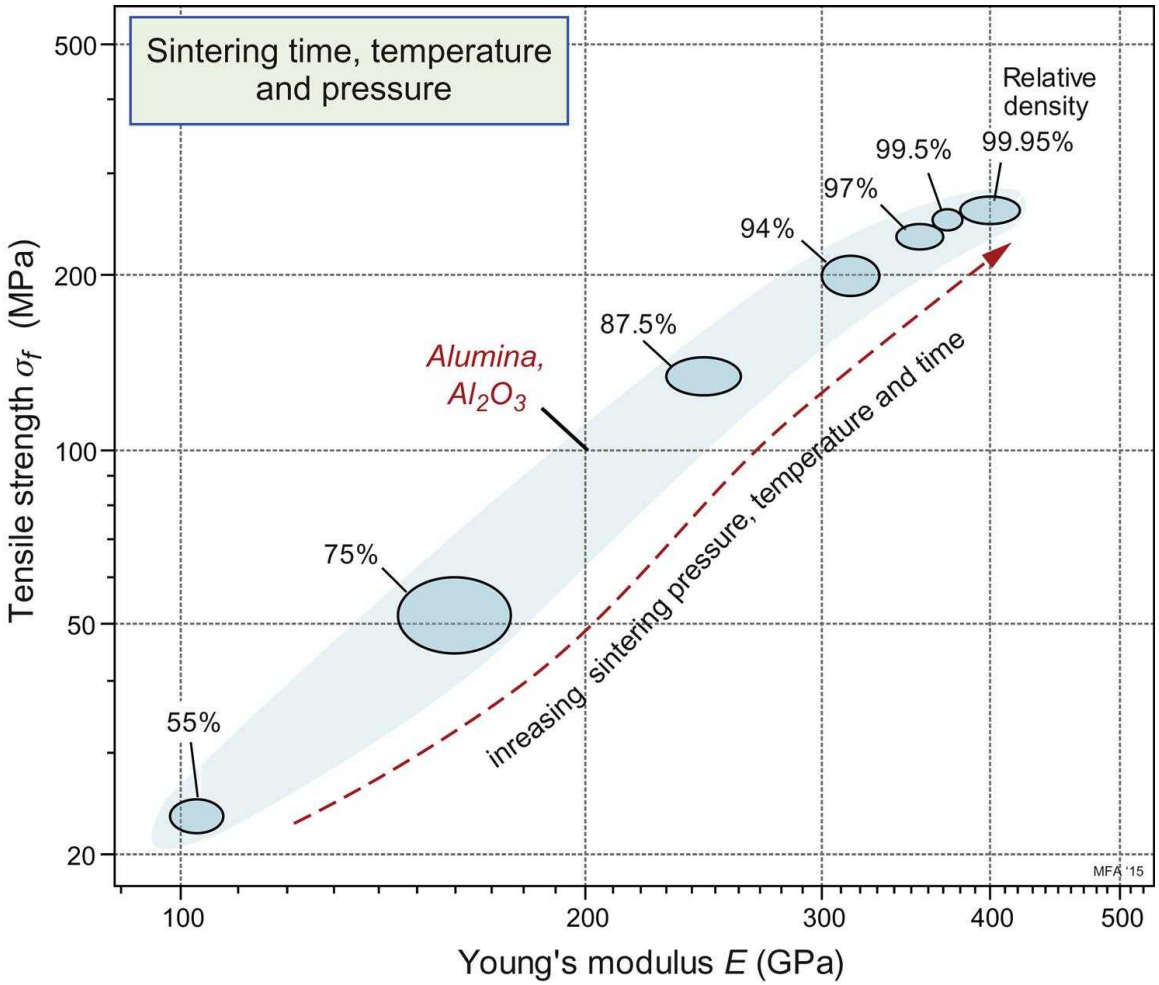


FIGURE 6.27 The effect of sintering time and temperature on porosity and properties.

Residual porosity affects almost all properties of a ceramic. Fig. 6.28 illustrates how the modulus and strength of alumina increase as the porosity is reduced. Low porosity, clearly, is desirable but there is a penalty: the sintering times are long and the temperatures are high (of order 1000°C) and that means cost.

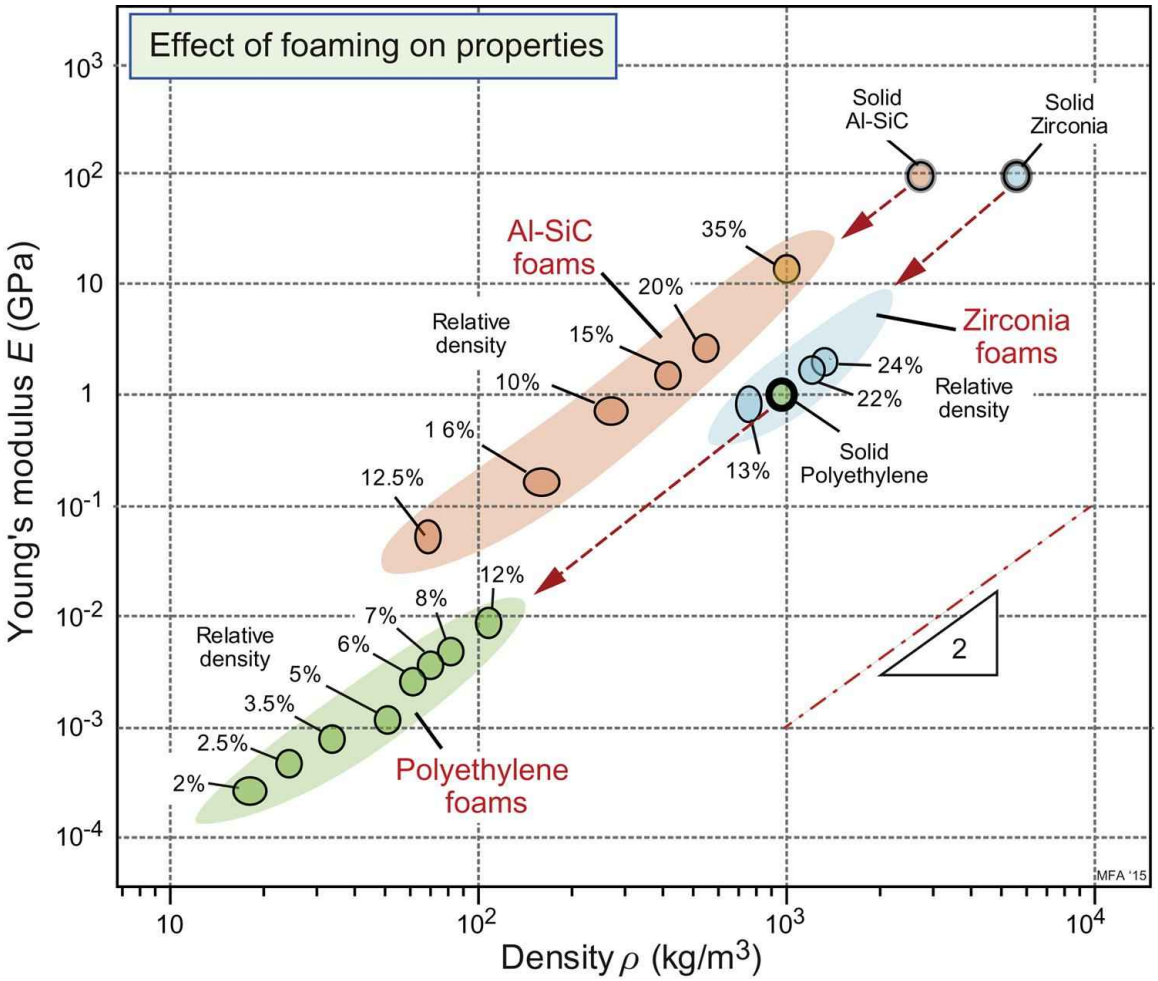


FIGURE 6.28 The effect of foaming the Young's modulus of metals, polymers and ceramics.

Effect of foaming on properties. Polymers, metals, ceramics and glasses can all be foamed, reducing their densities to as little as 1% of that of the solid of which they are made. You might expect that modulus, strength, thermal and electrical conductivity would decrease in proportion to the drop in density but that is not the case – the dependence, usually, is a stronger one. Fig. 6.28 is an example: the modulus decreases as the square of the density. Circles with bold outlines pin-point the modulus and density of a solid polymer (polyethylene), a solid metal (aluminium) and a solid ceramic (zirconia). The red arrows show the trajectory that each follows as it is foamed. We return to foams in Chapter 12, Designing Hybrid Materials, where the origin of this behaviour is analysed.

6.5 Summary and Conclusions

A wide range of shaping, joining and finishing processes is available to the design engineer. Each has certain characteristics, which, taken together, suit it to the processing of certain materials in certain shapes, but disqualify it for others. Processes are the culinary tools needed to cook up materials with the flavours you want. The process parameters are those familiar in the kitchen: composition, deformation, temperature and time. Applying a process and stepping through the allowed process parameter range causes properties to track along particular paths – we've called them property trajectories. Ten such trajectories are shown in the chapter. Win-win trajectories show properties evolving in mutually supportive ways, but this is rare. Almost always the gain in one property is associated with the drop in another and a compromise must be sought. These 'trade-off' trajectories are the most interesting ones, pointing to the optimal combination of process parameters to create a desired property set. The exercises that follow include examples of this optimization.

6.6 Further Reading

Processing is a complex subject. Texts about Processing tend to resemble, at one extreme, chain-store catalogs of alternative choices or, at the other, instructor manuals for car maintenance. Some of those listed below are a bit like that (useful none the less), others take a broader stance, integrating the basic science with the applied science.

1. Ashby M, Shercliff H, Cebon D. Materials: Engineering, Science, Processing and Design second ed. Oxford, UK: Butterworth-Heinemann; 2014; ISBN-13: 978-0-08-097773-7, North American Edition: ISBN-13: 978-1-85617-743-6. (*An introductory text introducing ideas that are developed more fully in the present text*.).
2. ASM Handbook Series. *Volume 4 "Heat Treatment", Volume 5 "Surface Engineering", Volume 6 "Welding, Brazing and Soldering", Volume 7 "Powder Metal Technologies, Volume 14 "Forming and Forging", Volume 15 "Casting" and Volume 16 "Machining"* Metals Park, Ohio: ASM International; 1971–2015; (A comprehensive set of handbooks on processing, occasionally updated, and now available on-line at www.asminternational.org/hbk/index.jsp).

3. Bralla JG. *Design for Manufacturability Handbook* second ed. New York, NY: McGraw-Hill; 1998; ISBN 0-07-007139-X. (*Turgid reading, but a rich mine of information about manufacturing processes*.).
4. Budinski KG, Budinski MK. *Engineering Materials, Properties and Selection* ninth ed. New Jersey: Prentice Hall; 2010; ISBN 978-0-13-712842-6. (*A well established text on the processing and use of engineering materials, now in its 9th edition*.).
5. Campbell J. *Casting* Oxford, UK: Butterworth Heinemann; 1991; ISBN 0-7506-1696-2. (*The fundamental science and technology of casting processes*.).
6. Dieter GE. *Engineering Design, a Materials and Processing Approach* second ed. New York, NY: McGraw-Hill; 1991; ISBN 0-07-100829-2. (*A well-balanced and respected text focussing on the place of materials and processing in technical design*.).
7. Dieter GE, Schmidt LC. *Engineering Design* fourth ed. New York, NY: McGraw Hill; 2009; ISBN 978-0-07-283703-2. (*Prof. Dieter is a pioneer in presenting design from a materials perspective. The book contains a notable chapter on conceptualization*.).
8. Esawi A, Ashby MF. Computer-based selection of manufacturing processes: methods, software and case studies. *Proc Inst Mech Eng.* 1998;212:595–610 (*A paper describing the development and use of the CES database for process selection*.).
9. Grainger S, Blunt J. *Engineering Coatings, Design and Application* Cambridge UK: Abington Publishing; 1998; ISBN 1-85573-369-2. (*A handbook of surface treatment processes to improve surface durability – generally meaning surface hardness*.).
10. Granta Design, 2015. "The CES Edu system" and other teaching resources, accessed from. <Grantadesign.com/education/>.
11. Houldcroft PT. *Which Process?* Cambridge UK: Abington Publishing; 1990; ISBN 1-85573-008-1. (*The title of this useful book is misleading – it deals only with a subset of joining process: the welding of steels. But here it is good, matching the process to the design requirements*.).
12. Kalpakjian S, Schmidt SR. *Manufacturing Processes for Engineering Materials* fifth ed. New Jersey: Prentice Hall; 2008; ISBN 978-0-13-227271-1. (*A comprehensive and widely used text on material processing*.).
13. Kalpakjian S, Schmidt SR. *Manufacturing Engineering and Technology* seventh ed. New Jersey: Prentice Hall; 2013; ISBN-13:

9780133128741. (*A comprehensive and widely used text on material processing*.).
14. Lascoe OD. *Handbook of Fabrication Processes* Metals Park, Ohio: ASM International; 1988; ISBN 0-87170-302-5. (*A reference source for fabrication processes*.).
15. Shackelford JF. *Introduction to Materials Science for Engineers* eighth ed. New Jersey: Prentice Hall; 2014; ISBN 978-0133789713. (*A well-established materials text with a design slant*.).
16. Swift KG, Booker JD. *Process Selection, From Design to Manufacture* second ed. London, UK: Arnold; 2003; ISBN 9780750654371 (*Details of 48 processes in a standard format, structured to guide process selection*.).
17. Tempelman E, Shercliff H, van Eyben BN. *Manufacturing and Design* Oxford: Butterworth Heinemann; 2014; ISBN978-0-08-099922-7. (*A text exploring manufacturing principles relating function, quality and cost*.).
18. Thompson R. *Manufacturing Processes for Design Professionals* Thames and Hudson 2015; ISBN-13: 978-0500513750. (*A book with a structured approach to process methods, remarkable for the quality and diversity of its illustrations*.).
19. Wise RJ. *Thermal Welding of Polymers* Cambridge UK: Abington Publishing; 1999; ISBN 1-85573-495-8. (*An introduction to the thermal welding of thermoplastics*.).

6.7 Exercises

E6.1. *Exploring processes*. This chapter introduced the families of processes with schematics of some of them. Good images of many processes can be found on the internet. Use the internet to find images for the following:

a. Plastic film blowing
b. Filament winding
c. Additive manufacture
d. Metal spinning
e. Spot welding
f. Polymer extrusion

E6.2. *Making property trajectories (1)*. The table lists properties of copper–nickel alloys from pure copper to pure nickel. Plot a trajectory of

thermal conductivity against copper content. How do you explain the shape of the trajectory?

Copper–Nickel Alloy	Yield Strength (MPa)	Fracture Toughness (MPa·m$^{1/2}$)	T-conductivity (W/m·K)
100% copper	49.7	102	381
95% copper	85.9	85.8	22.8
90% copper	99.3	76.8	20.2
80% copper	115	67.5	22.4
75% copper	132	67.9	25.4
70% copper	145	70.4	31.4
45% copper	187	72.9	47.3
32% copper	199	79.4	66.1
0% copper			88.7

E6.3. *Making property trajectories (2)*. The table in Exercise E6.2 lists properties of copper–nickel alloys from pure copper to pure nickel. Plot trajectory of fracture toughness against yield strength.

E6.4. *Using property trajectories*. An aluminium alloy is sought for a demanding application. For safety reasons the designer has specified a minimum fracture toughness of 30 MPa·m$^{1/2}$ and the highest possible strength. Use the property trajectory of Fig. 6.19 to select an alloy and a heat treatment that meets the requirements. Use the internet to find out more about the alloy and the heat treatment that you select.

E6.5. *Choosing optimal processing conditions*. An approximate measure of the energy-absorbing capability of a metal is the product $\sigma_y \varepsilon_f$ of its yield strength σ_y and its elongation to fracture ε_f. These two properties for copper and copper–nickel alloys are the axes of Fig. 6.17. Plot contours of $\sigma_y \varepsilon_f$ onto a copy of this figure and use them to identify the optimal processing to maximize energy absorption capacity.

E6.6. The CES EduPack ’16 software contains sets of records for making process–property charts like those shown in Section 6.4 of this chapter. If this tool is available, use these record sets to plot the process–property trajectory for

a. Thermal conductivity versus electrical resistivity for solution hardened copper as a function of solute concentration.
b. Vickers hardness versus elongation for medium carbon steel as function of tempering temperature.
c. Heat exchangers require materials with high thermal conductivity and – if pressurized – high strength. Explore the process–property

trajectory for the thermal conductivity versus yield strength for aluminium grades to find the best combination for heat exchangers.

d. Do fillers change the thermal properties of polymers? Explore the process-property trajectories for the thermal conductivity and the expansion coefficient for filled polypropylene to find out.

CHAPTER 7

Processes Selection and Cost

Abstract

Process selection parallels that for materials, seeking first the subset that can handle the material and produce the shape required by the design. When alternatives are viable, criteria of cost and quality allow them to be ranked. The final choice is based on *documentation*: guidelines, case histories and examples of process routes used for related products. This chapter explains and illustrates the method and develops a cost model that allows ranking.

Keywords

Selection strategies; process selection matrices; process limitations; flow in casting; forging pressure; tolerance; roughness; adhesives; welding; brazing; heat-affected zone; cost modelling

7.1 Introduction and Synopsis

Chapter 6, Processes and Their Effect on Properties, explored processes and process–property interactions. Now is the time to explore *process selection* – finding the best match between process attributes and design requirements. Material properties limit the choice of process. Ductile materials can be forged, rolled and drawn; those that are brittle my have to be shaped using powder methods. Materials that melt at modest temperatures to low-viscosity liquids can be cast; those that do not have to be processed by other routes. Shape, too, influences the choice of process. Slender shapes can be made easily by rolling or drawing; casting slender shapes is more difficult. Hollow shapes can be made by casting or moulding but not by forging or swaging.

Like the other aspects of design, process selection is an iterative procedure. The first iteration gives one or more possible processes routes. The design must

then be rethought to adapt it, as far as possible, to ease of manufacture by the most promising of these routes. When alternatives are viable, criteria of cost and quality allow them to be ranked. The final choice is based on *documentation*: guidelines, case histories and examples of process routes used for related products. Documentation helps in another way: that of dealing with the coupling between process and material properties.

The chapter ends, as always, with a summary, further reading and exercises.

7.2 Process Selection: The Strategy

The strategy for selecting processes parallels that for materials. Fig. 7.1 lists the now familiar steps: *translation, screening, ranking* and *documentation*.

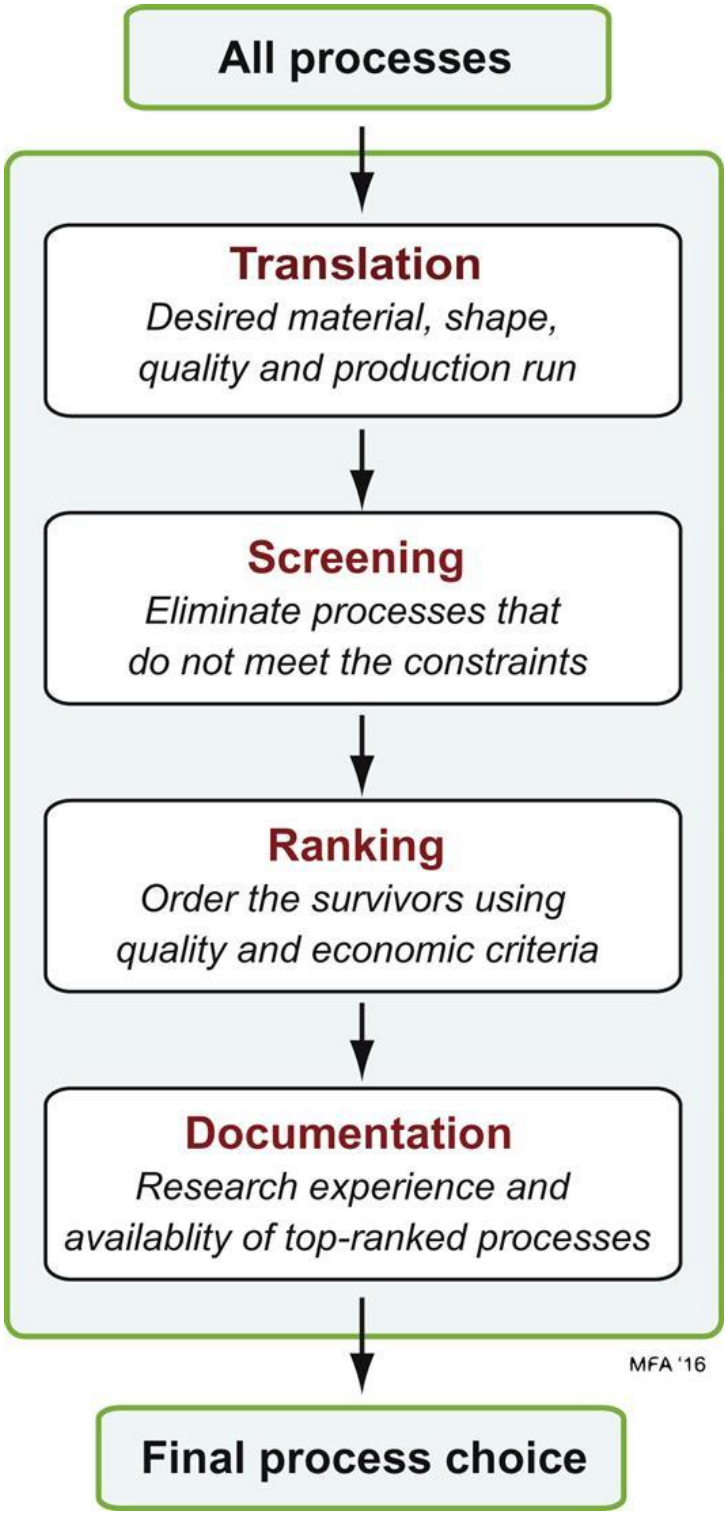


FIGURE 7.1 A flow chart of the procedure for process selection.

Translation. When design is performance driven, the *function* of a component dictates the choice of material and shape; these in turn constrain the choice of processes. When, instead, design is cost driven, processes are chosen for their

economy and production speed, and the material is chosen to be compatible with the process. It is helpful to think of two types of constraint: *technical* – can the process do the job at all? – and *quality* – can it do so sufficiently well? One technical constraint is always there: it is the compatibility of material and process. Quality constraints include achieving the desired precision, surface finish and properties (remember that processing changes properties) while avoiding defects. The usual *objective* in processing is to minimize cost. The *free variables* are largely limited to choosing the process itself and its operating parameters (such as strain, temperatures and time). Table 7.1 summarizes the outcome of the translation stage.

Example: Shaping a Steel Connecting Rod

The connecting rod shown in Fig. 7.2 is to be made of a medium carbon steel. The minimum section is around 8 mm, and an estimate of the volume gives a mass around 0.35 kg. Dimensional precision is important to ensure clearances at both the little and the big end. The con-rod carries cyclic loads with the consequent risk of fatigue failure. A modest batch size of 10,000 is required. Control of properties and of defects is dominated by the need to avoid fatigue crack initiation. Table 7.2 summarises.

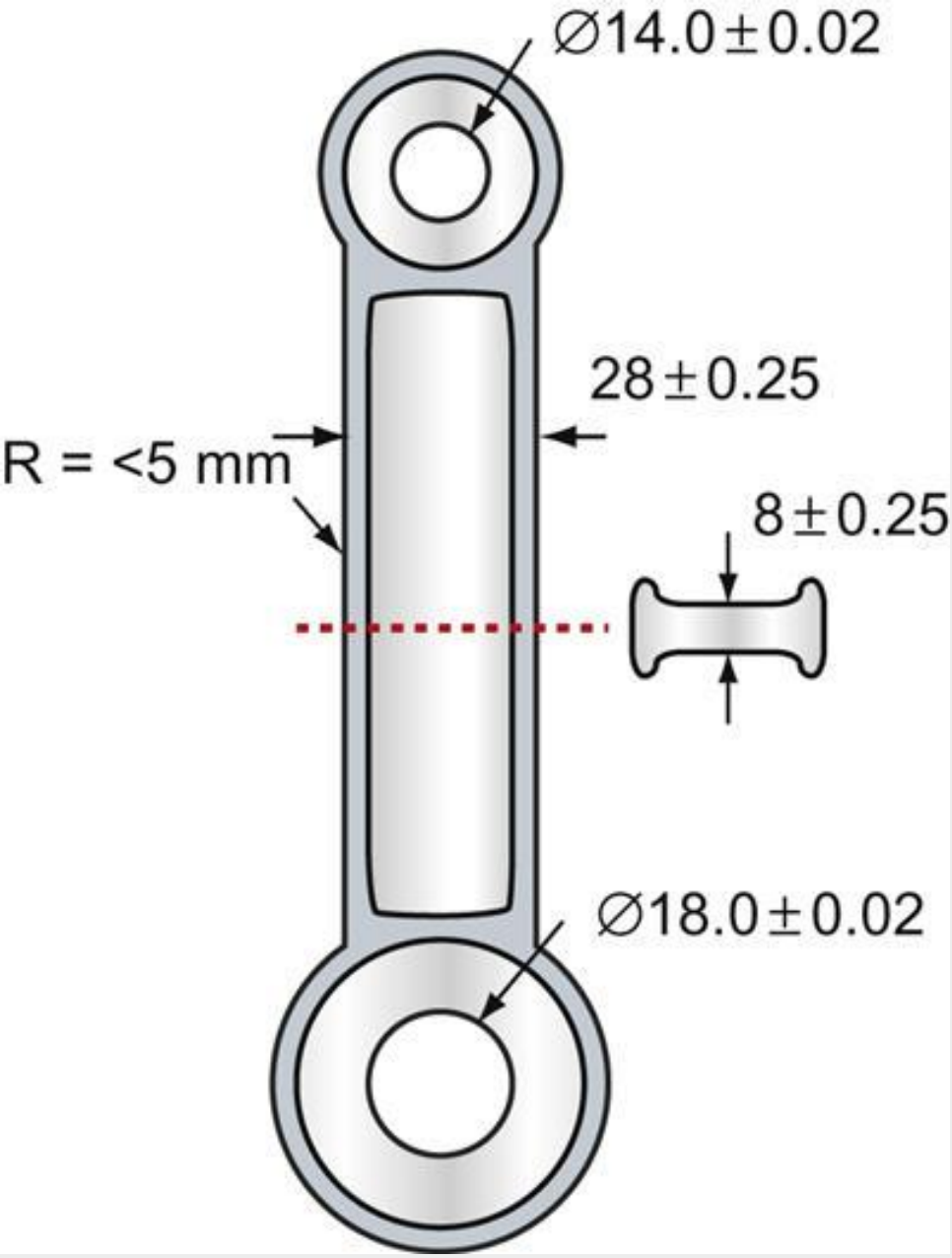


FIGURE 7.2 A con-rod.

Table 7.2

Translation for Shaping a Steel Connecting Rod

Function	■ *Connecting rod ('con rod')*	
Objective	■ *Minimize cost*	
Constraints	■ *Material: medium carbon steel* ■ *Shape: 3D solid* ■ *Estimated mass: 0.3–0.4 kg* ■ *Minimum section: 8 mm*	*Technical constraints*
	■ *Tolerance: < 0.25 mm (surface)* ■ *Tolerance: < 0.02 mm (bores)* ■ *Roughness: < 5 μm*	*Quality constraints*
	■ *Batch size: 10,000*	*Economic constraint*
Free variable	■ *Choice of shaping process, and process operating conditions*	

Table 7.1

Translation of Process Requirements

Function	■ *What must the process do? (Shape? Join? Finish?)*
Constraints	■ *What technical limits must be met? (Material and shape compatibility)* ■ *What quality limits must be met? (Precision, avoidance of defects …)*
Objectives	■ *What is to be maximized or minimized? (Cost? Quality?)*
Free variables	■ *Choice of process and process operating conditions*

Screening. The screening step applies the constraints and eliminates processes that cannot meet them. Key constraints are

- The material to be processed
- The desired shape
- The desired batch size (the number of units to be made)
- The quality level and sensitivity to defects

Matrices that help apply these constraints come in Section 7.3. Requirements such as 'a 3-D solid shape made of magnesium in a batch of 5000' quickly narrows the choice of process; these can then be explored in more detail to establish their feasibility.

Ranking. Ranking, as before, is based on one or more *objectives*, the most obvious of which is that of minimizing cost. In certain demanding applications it may be replaced by the objective of maximizing quality regardless of cost, though more usually it is a trade-off between the two that is sought.

Documentation. Screening and ranking do not cope adequately with the less tractable issues of quality and productivity; they are best explored through a

search for documentation: design guidelines, best practice guides, case studies and failure analyses. A given process has an optimum window of operating conditions under which it works best. Failure to operate within this window can lead to manufacturing defects such as porosity, cracking or residual stress. This in turn leads to scrap and lost productivity, and, if passed on to the user, may cause premature failure. Documentation is an essential step in assuring that this does not happen.

7.3 Implementing the Strategy: Selection Matrices

A process is characterized by a set of attributes: the materials it can handle, the shapes it can make, its cost. These can be displayed as matrices and bar charts to provide the tools we need for screening. The hard-copy versions, shown here, are necessarily simplified, and illustrate only a limited number of processes and attributes. Computer implementations described in Section 7.5 allow exploration of a much larger number of both.

Material–process compatibility. Fig. 7.3 is a material–process compatibility matrix. Shaping processes are at the top, with compatible combinations marked by coloured dots that identify the material family. Using the matrix for screening is straightforward – specify the material and read off the processes, or (conversely) specify the process and read off the materials. The diagonal spread of the dots across the matrix reveals that each material family – metals, polymers, and so on – has its own set of processes. There are some overlaps: powder methods are compatible with both metals and ceramics, moulding with both polymers and glasses. Machining (when used for shaping) is compatible with almost all families. Joining processes using adhesives and fasteners are very versatile because they can be used with almost any material; welding methods are less flexible because they are material-specific. Finishing processes add quality (precision machining for example) or have a decorative function (painting, for instance).

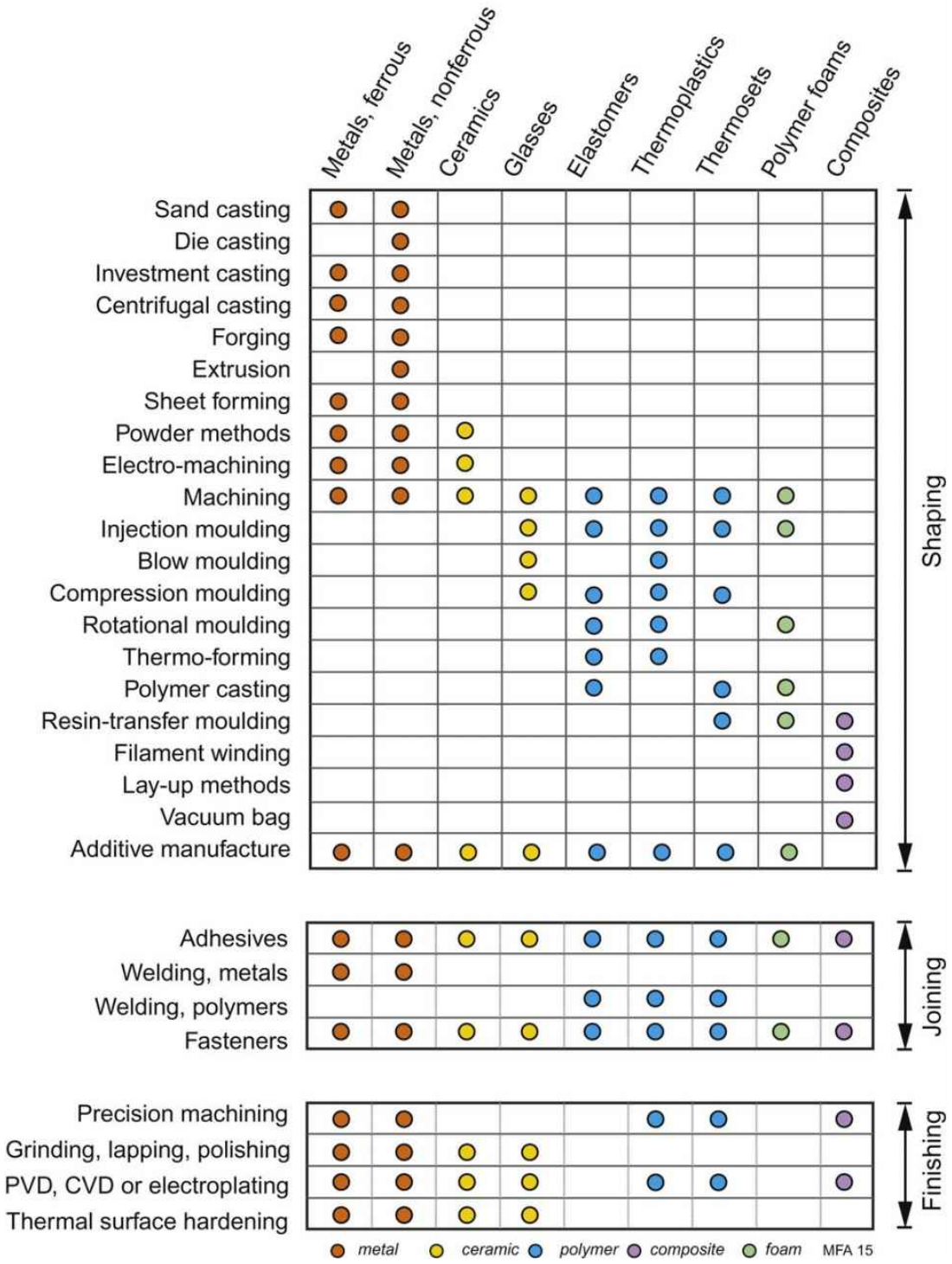


FIGURE 7.3 The process – material matrix. A coloured dot indicates that the two are compatible.

Process-shape compatibility. Shape is the most difficult attribute to characterize. Many processes involve rotation or translation of a tool or of the material, directing our thinking towards axial symmetry, translational symmetry,

uniformity of section and such like. Turning creates axisymmetric (or circular) shapes; extrusion, drawing and rolling make prismatic shapes, both circular and noncircular. Sheet-forming processes make flat shapes (stamping) or dished shapes (drawing or folding). Certain processes can make arbitrary 3-dimensional shapes, and among these some can make hollow shapes whereas others cannot. The upper part of Fig. 7.4 illustrates this classification scheme. The prismatic shapes on the left, made by rolling, extrusion or drawing, can be made in continuous lengths. The other shapes cannot; they are discrete, and the processes that make them are called batch processes. Continuous processes are well suited to long, prismatic products such as railway track or standard stock such as plate and channel sections. Cylindrical rolls produce sheets. Shaped rolls make more complex profiles – rail track is one. Extrusion is a particularly versatile process, allowing complex prismatic profiles with internal channels and ribs to be made in one step.

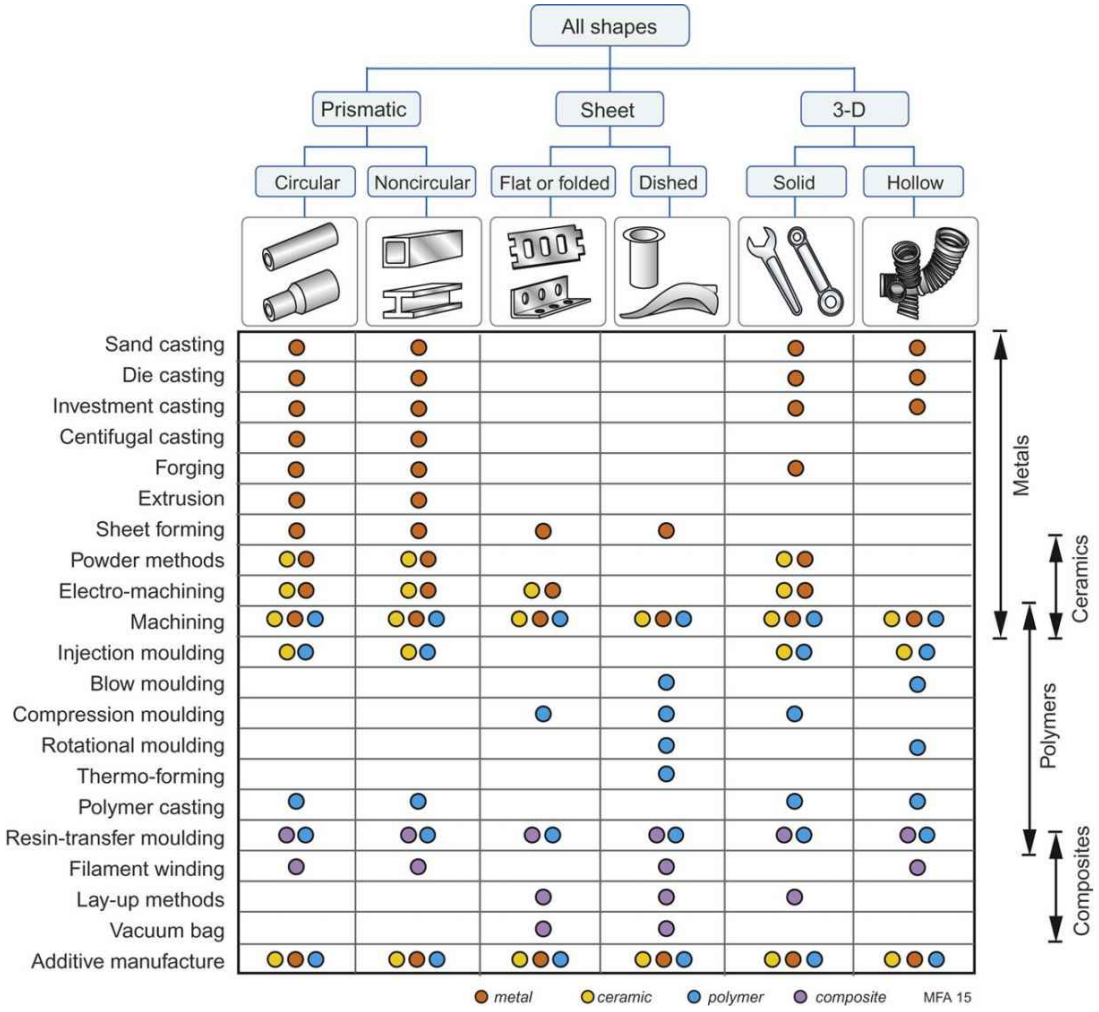


FIGURE 7.4 The process-shape matrix. Information about material compatibility is included at the extreme right.

The process-shape matrix of Fig. 7.4 displays the links between the two. If the process cannot make the shape we want, it may be possible to combine it with a secondary process to give a process-chain that adds the additional features: casting followed by machining or forging followed by welding are examples.

Economic batch size. Economic batch size is best introduced with an example. If you want to sharpen a pencil, you can do it with a knife. If, instead, you want to sharpen a thousand pencils, it would pay to buy an electric sharpener. If you want to sharpen a million, you might wish to equip yourself with an automatic feeding, gripping and sharpening system. To cope with pencils of different length and diameter, you could go further and devise an 'intelligent' computer-controlled system with auto-feed that can sense pencil dimensions and adjust sharpening pressure. The choice of process, then, depends on the

number of pencils you wish to sharpen, that is, on the *batch size*. The best choice is that one that costs least per pencil sharpened.

Fig. 7.5 is a schematic of how the cost of sharpening a pencil varies with batch size. A knife does not cost much but it is slow, so the labour cost is high. The other processes involve progressively greater capital investment but do the job more quickly, reducing labour costs. The balance between capital cost and rate gives the shape of the curves. In this figure the best choice is the lowest curve – a knife for up to 100 pencils; an electric sharpener for 100 to 10,000 an automatic system for 10,000 to 1 million, and so on.

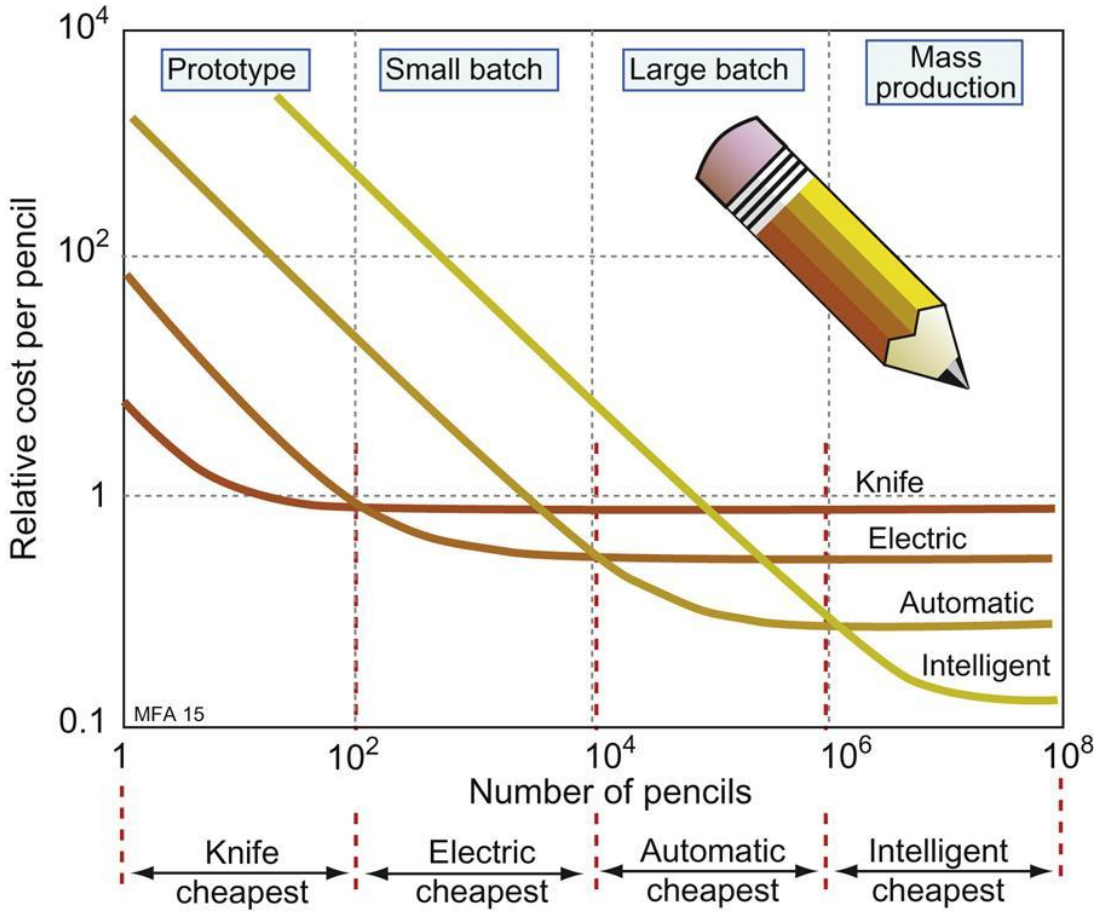


FIGURE 7.5 The cost of sharpening a pencil plotted against batch size for four processes.

Each process has a range of *economic batch size*. A process with an economic batch size with the range B_1–B_2 is one that is found by experience to be competitive in cost when the desired number of units of output lies within that range; below or above it some other process is generally cheaper. The economic

batch size is commonly cited for processes (Fig. 7.6). The easy way to introduce economy into the selection is to rank candidate processes by economic batch size and retain those that are economic in the range you want. A cost model developed in Section 7.5 gives greater insight.

Example: Forming a Fan

The automobile cooling fan shown as Fig. 7.7 is to be made from Nylon 66. The desired batch size is 25,000. Surface smoothness is important to allow a smooth air flow and to reduce noise. What shaping processes meet these constraints? Use the matrices of Figs. 7.3, 7.4 and 7.6 to identify potential candidates.

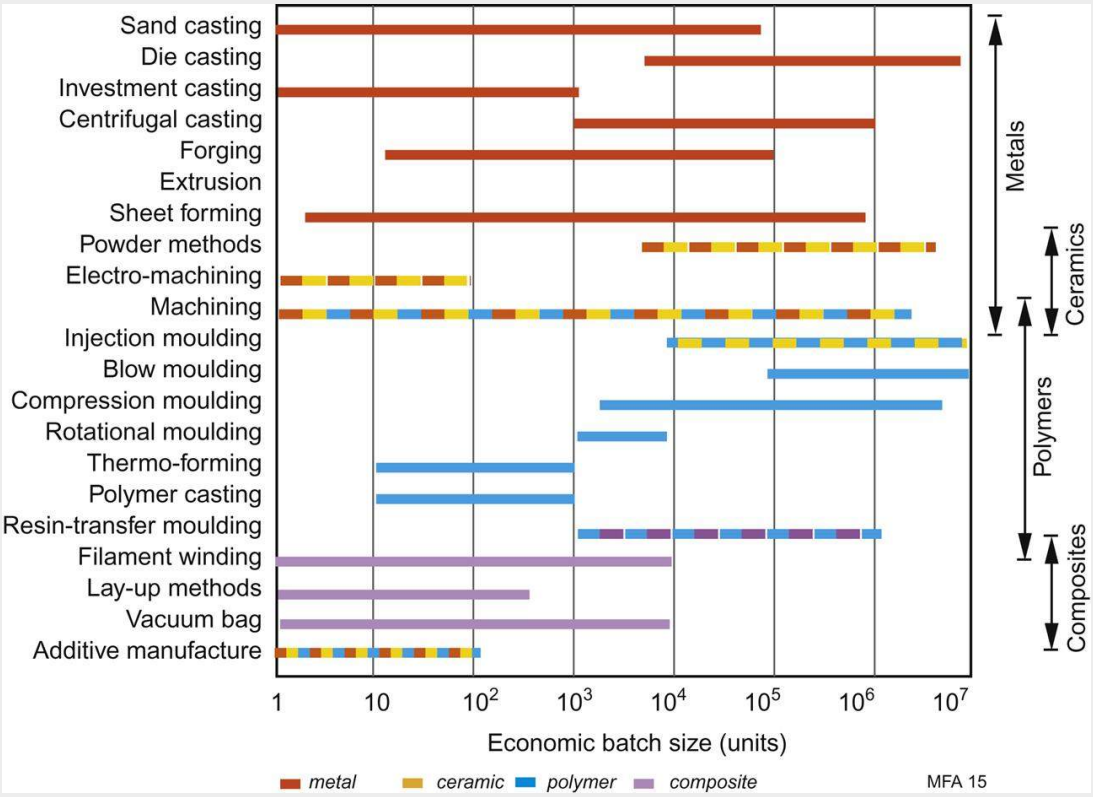


FIGURE 7.6 The economic batch-size chart.

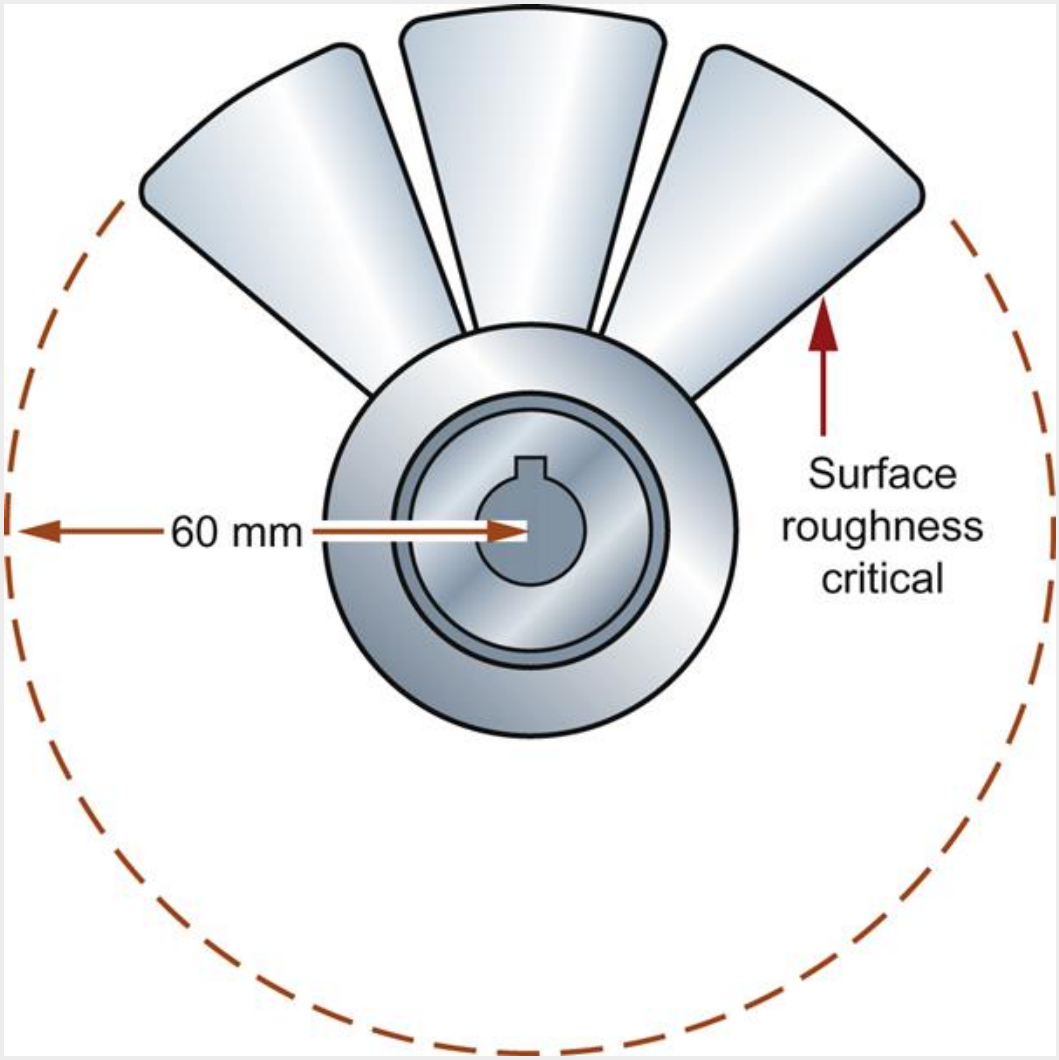


FIGURE 7.7 A cooling fan.

Method

The material is a thermoplastic. The shape is a 3-D solid. The batch size is 50,000. Table 7.3 shows processes that survive the principal constraints. Of the nine processes listed here as capable of shaping thermoplastics, just two meet the primary constraints. The last column comments on issues of quality. Cost modelling is needed to rank them.

Table 7.3

Translation for Shaping the Fan

Compatible With Polymers (Fig. 7.3)	Compatibility With Shape (Fig. 7.4)	Compatibility With Batch Size (Fig. 7.6)	Quality Comment
Machining		*Fails*	
Injection moulding			Meets requirement for finish and tolerance
Blow moulding	*Fails*		
Compression moulding	*Fails*		
Rotation moulding	*Fails*		
Thermo-forming	*Fails*		
Polymer casting		*Fails*	
Resin transfer moulding			Meets requirements for finish and tolerance
Additive manufacture		*Fails*	

7.4 Limitations and Quality

Limitations and Quality (a) Shape

Scale: size and section thickness. There are limits to the size of component and section thickness that a process can make.

Lower limits. Casting and moulding both rely on material flow in the liquid or semiliquid state. Lower limits on section thickness are imposed by the physics of flow. Viscosity and surface tension oppose flow through narrow channels, and heat loss from the large surface area of thin sections cools the flowing material, raising the viscosity before the channel is filled (Fig. 7.8). Pure metals solidify at a fixed temperature, with a step increase in viscosity, but for alloys solidification happens over a range of temperature, known as the 'mushy zone,' in which the alloy is part liquid, part solid. The width of this zone can vary from a few degrees centigrade to several hundred – so metal flow in castings depends on alloy composition. In general, higher pressure die casting and moulding methods enable thinner sections to be made, but the equipment costs more and the faster, more turbulent flow can entrap porosity and damage the mould.

Flow in Thin Channels

A mould for casting an intricate aluminium casting has some channel-like features with a width of only $10\,\mu m$. Will an over pressure of 1 atmosphere (0.1 MPa) be sufficient to overcome surface tension and allow the feature to be filled? The surface tension γ of liquid aluminium is 1.1 J/m^2.

Answer

The pressure required to overcome surface tension and force the metal into a parallel-sided channel of width $2x$ is $p = \gamma/x$. Thus the narrowest channel that

can be filled with an overpressure of 1 atmosphere is

$$2x = \frac{2\gamma}{p} = \frac{2.2}{0.1 \times 10^6} = 2.2 \times 10^{-5} = 22\ \mu\text{m} \qquad \textbf{(7.1)}$$

An over pressure of 1 atmosphere is not sufficient to fill the channel. An overpressure of 5 atmospheres would do it comfortably.

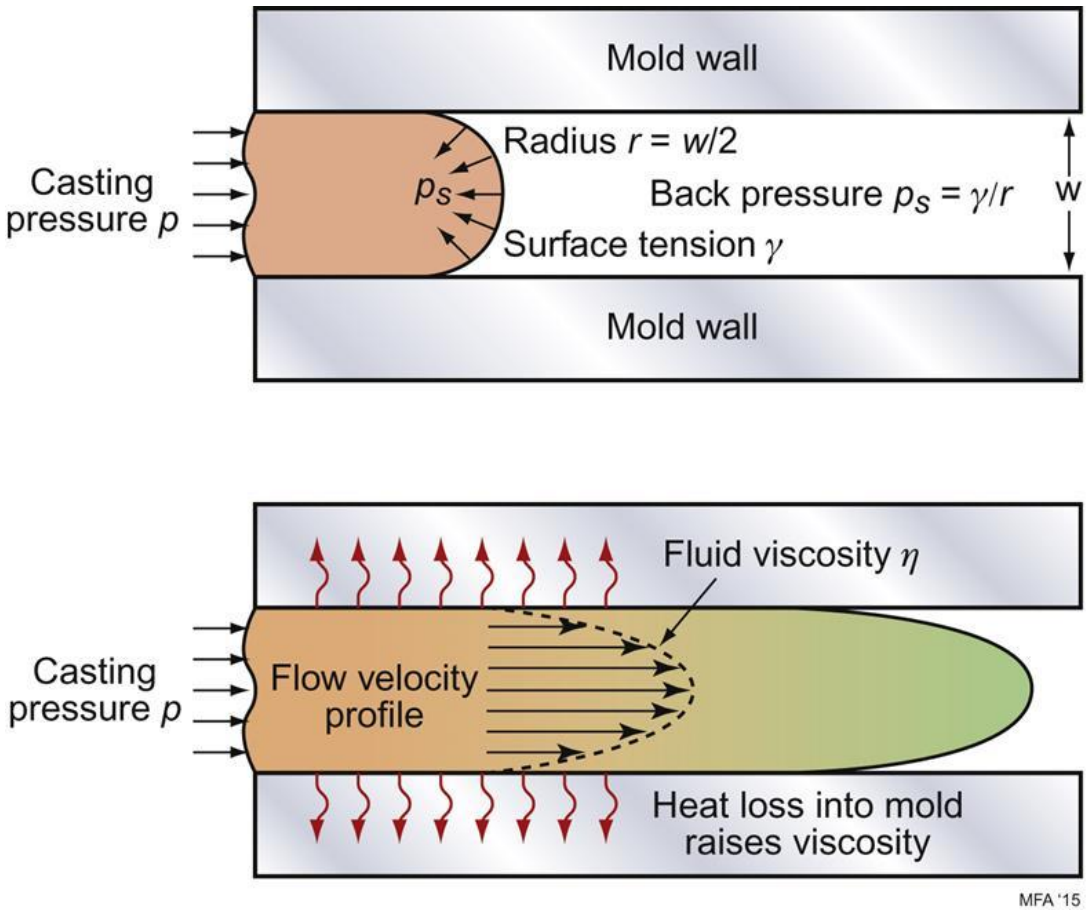


FIGURE 7.8 Flow of liquid metal or polymer into thin sections is opposed by surface tension as at (a) and by viscous forces.

Deformation processing – hot or cold rolling, forging or extrusion – also involves flow. The thinness of the section that can be forged, rolled or extruded is limited by plastic flow in much the same way that thinness in casting is limited by viscosity. Fig. 7.9 illustrates the problem. Friction changes the pressure distribution on the die and under the rolls. When they are well lubricated, as in (A), the loading is almost uniaxial and the material flows when

the pressure p equals the uniaxial yield stress σ_Y of the material. Without lubrication the spreading of the material is opposed by friction at metal-die interface and the pressure ramps up towards the middle giving a 'friction hill' shown at (B).

Forming Pressure in Forging

What limits the thinness of a forged metal part?

Answer

Consider the equilibrium of forces on the element of material in Fig. 7.9 that lies between x and $x + dx$. The horizontal forces are in balance when

$$h\, d\sigma_1 = 2\, \mu\, p\, dx$$

If the yield condition is $\sigma_1 - \sigma_2 = \sigma_Y$ where σ_Y is the yield strength and we identify σ_2 with $-p$ so that $dp = -d\sigma_1$ we find

$$\frac{dp}{p} = -\left(\frac{2\mu}{h}\right) dx \tag{7.2}$$

Integrating this gives the forming pressure across the forging:

$$p(x) = \sigma_Y \exp\left(\frac{2\mu}{h}(w - x)\right) \tag{7.3}$$

(because $\sigma_1 = 0$ at $x = w$). Thus the forming pressure rises exponentially across the compression surface, growing rapidly as the height h decreases. The greater the aspect ratio of the section (width/thickness), the higher the maximum pressure needed to continue deformation.

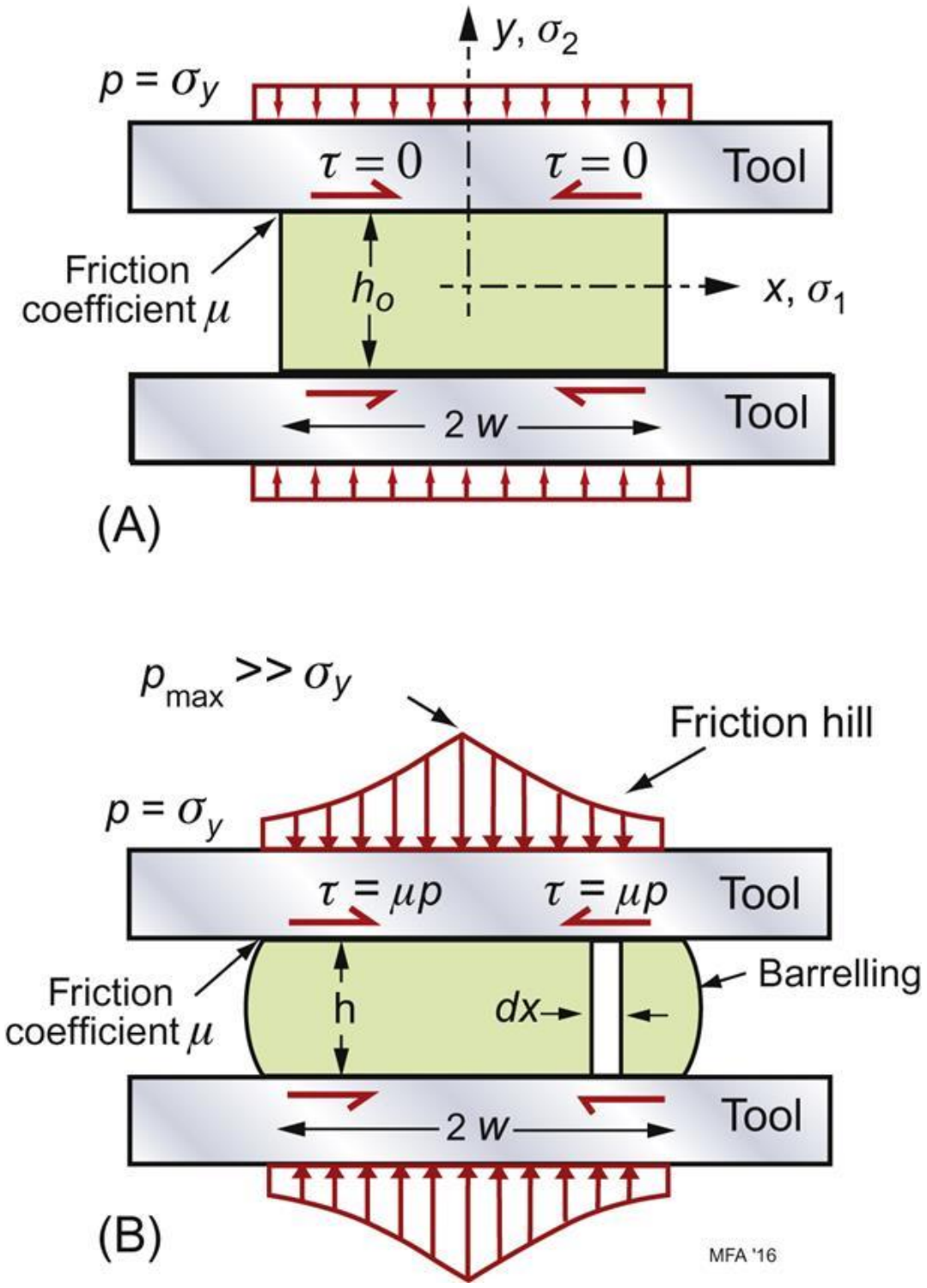


FIGURE 7.9 Friction in forging: (A) no friction; (B) with friction.

Friction also limits aspect ratio in powder processing. The externally applied pressure is diminished by die wall friction (Fig. 7.10) with the result that, if the

aspect ratio is too great, there is insufficient pressure to compact powder in the centre of the product.

Limiting Aspect Ratio for Powder Pressing

How large is the pressure drop in a cylindrical powder mass like that of Fig. 7.10 caused by die wall friction if the coefficient of friction on the die wall is $\mu = 0.5$?

Answer

The frictional force opposing sliding in the green band of thickness dx is $2\pi r\ \mu\ p\ dx$. where p is the pressure at a distance x below the die face. Dividing this by the cross-sectional area of the powder pack, πr^2 gives the pressure drop

$$dp = -\frac{2}{r}\mu\, p\, dx$$

Integrating from $x = 0$ where $p = p_o$ to $x = x$ where $p = p(x)$ gives

$$p(x) = p_o \exp -\frac{2\mu x}{r} \tag{7.4}$$

With a coefficient of friction of $\mu = 0.5$, the pressure drops to half its remote value p_o at a depth-to-radius ratio of only

$$\frac{x}{r} = -\frac{1}{2\mu}\ln\left(\frac{1}{2}\right) = 0.69$$

The answer it to lubricate the mould, reducing μ.

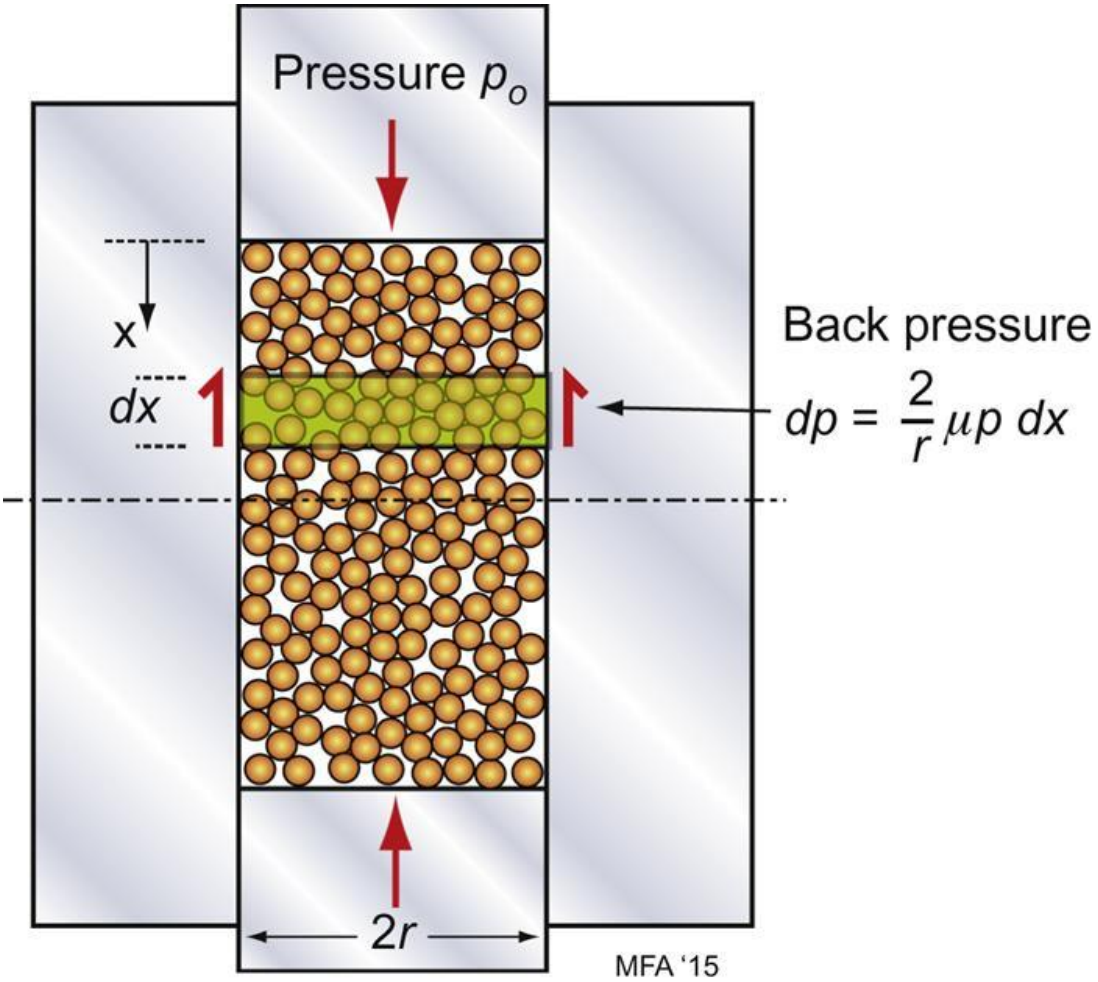


FIGURE 7.10 The height-to-width aspect ratio in powder forging is limited by die wall friction.

Upper limits. Upper limits to size and section in casting and moulding are set by problems of heat flow and shrinkage.

Cooling rates during solidification and during heat treatments have a profound effect on microstructural scale and on mechanical properties. Cooling rates in sand casting may be limited by heat transfer at mould surfaces but cooling rates in cold metal moulds or during the quenching after heat treatment are limited by the dimensions of the component itself. When a liquid metal is poured into a cold mould heat is lost by conduction into the mould walls. The mass of the mould is generally greater than that of the casting, making this a transient heat-flow problem. All transient heat-flow problems have a characteristic decay time of

$$t^* = \frac{x^{*2}}{\beta a} \tag{7.5}$$

where β is constant, a is the thermal diffusivity (m^2/s) and $x^*(m)$ is a characteristic length (Appendix B, Section B15). We identify x^* with the length $x^* = V/A$ where V is the volume of the casting and A is its surface area (Fig. 7.11). The decay time then becomes

$$t^* = \frac{1}{\beta a}\left(\frac{V}{A}\right)^2 \tag{7.6}$$

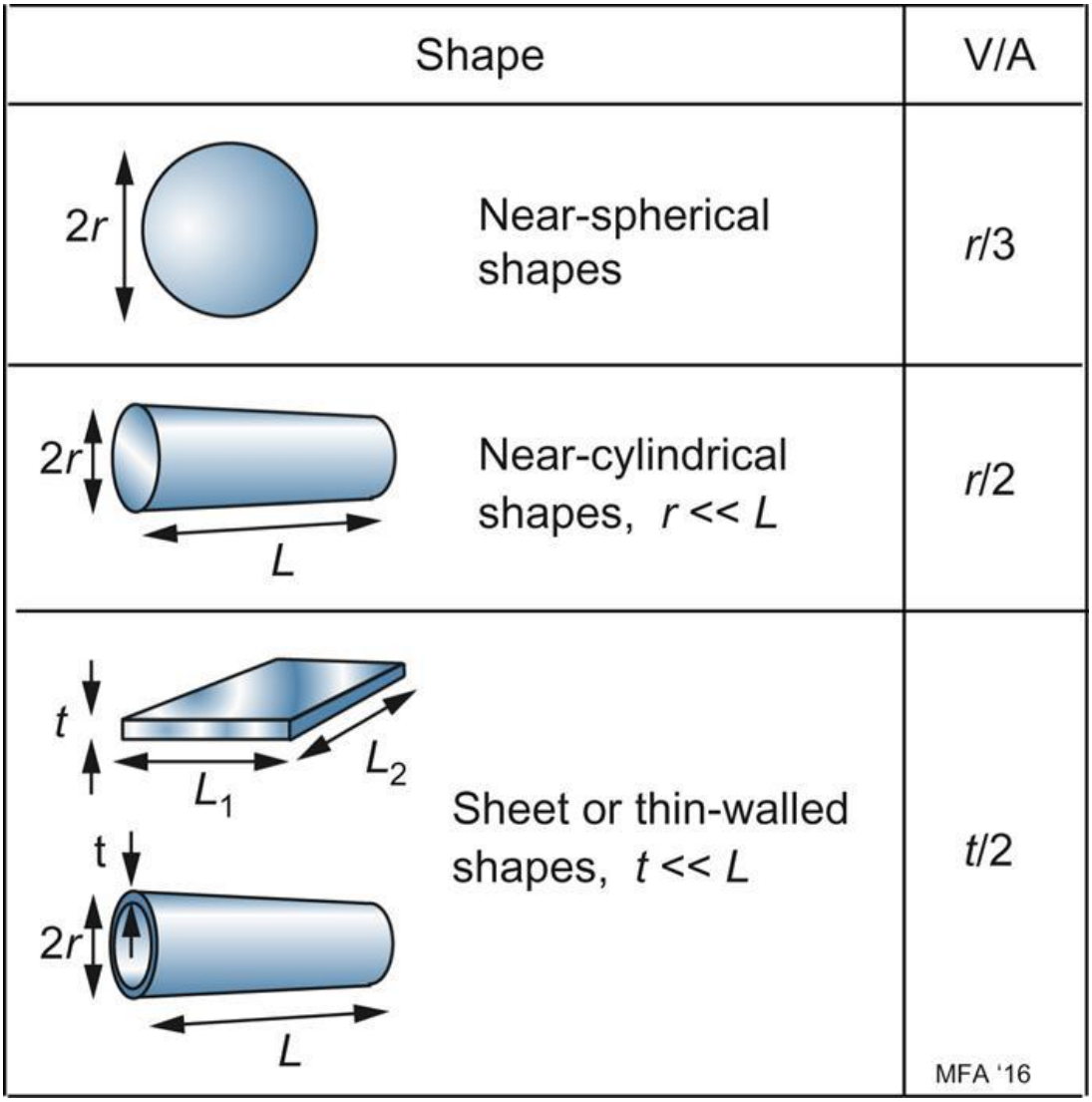


FIGURE 7.11 The length V/A for generic shapes.

Thus the time constant depends on size (for a given shape, V/A is proportional to size) and to the thermal properties of the mould. The thermal diffusivity of a sand mould is about 0.4×10^{-6} m^2/s, while that for a steel mould is 14×10^{-6} m^2/s, 35 times larger. Cooling rates, dT/dt, are inversely proportional to cooling times, so we expect

$$\frac{dT}{dt} \propto \beta a \left(\frac{A}{V}\right)^2 \tag{7.7}$$

The larger the component, the slower the rate at which it can be cooled and the coarser its microstructure will be.

Metals shrink when they solidify, shrinking most if they crystallize to a close-packed structure – typically the volume change is about 4%. The casting contracts further as it cools from the melting point to room temperature, ΔT_{MP-RT}. The linear thermal contraction over this temperature interval is almost the same for all metals: $\alpha\Delta T_{MP-RT} \approx 0.014 = 1.4\%$ so there is a further shrinkage of about 4%. Both types of shrinkage can cause casting defects. Solidification shrinkage leads to porosity if the mould is not designed to allow liquid to flow in to compensate for the volume change. Thermal contraction leads to hot tearing (creep fracture) if the shape of the casting keys it into the mould in a way that constrains it against shrinkage. Polymers, too, change volume as they a cooled from the moulding temperature. Typically the in-mould shrinkage is 0.5% and the subsequent thermal contraction from the moulding temperature to room temperature is a further 3%.

Tolerance and roughness. The precision and surface finish of a component are aspects of its quality. They are measured by the *tolerance*, T, and the *surface roughness*, R. When the dimensions of a component are specified the surface quality is specified as well, though not necessarily over the entire surface. Surface quality is critical in contacting surfaces such as the faces of flanges that must mate to form a seal or sliders running in grooves. It is also important for resistance to fatigue crack initiation and for aesthetic reasons. The tolerance T on a dimension y is specified as $y = 100 \pm 0.1$ mm, or as $y = 50^{+0.01}_{-0.001}$ mm, indicating that there is more freedom to oversize than to undersize. Surface roughness, R, is specified as an upper limit, e.g., $R < 100\ \mu$m.

Surface roughness R is a measure of the irregularities of the surface (Fig. 7.12). It is defined as the root mean square (RMS) amplitude of the surface profile:

$$R^2 = \frac{1}{L}\int_0^L y^2(x)dx \tag{7.8}$$

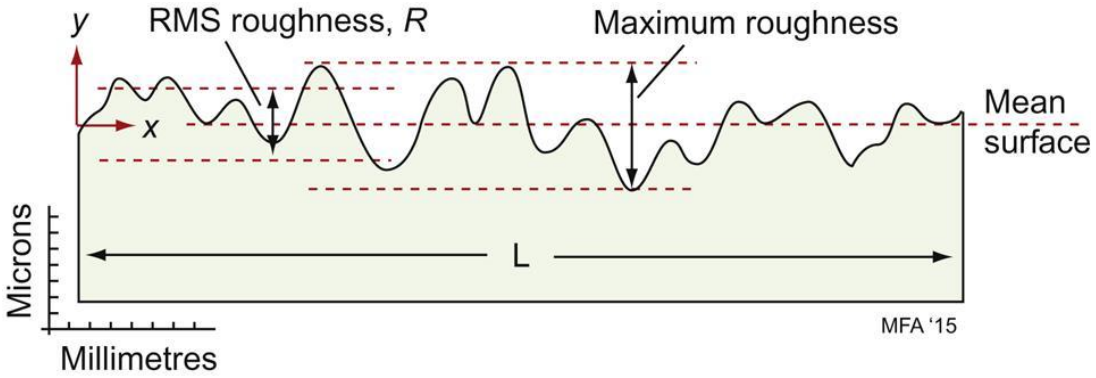


FIGURE 7.12 Surface roughness, measured by the (RMS) root mean square roughness, *R*.

It is measured by dragging a light, sharp stylus over the surface in the *x*-direction while recording the vertical profile $y(x)$, like playing a gramophone record. *Optical profilometry*, which is faster and more accurate, uses laser-interferometry to map surface irregularity.

The tolerance *T* is obviously greater than 2*R;* indeed, because *R* is the RMS roughness, the peak roughness, and hence absolute lower limit for tolerance, is more like 5*R*. Real processes give tolerances that range from 10*R* to 1000*R*.

Typical surface roughness values are listed in Table 7.4. Sand casting gives a rough surface; casting into metal dies gives a smoother one but none do better than *T*=0.1 mm and $R = 0.5\ \mu\text{m}$. Machining, capable of high dimensional accuracy and surface finish, is used after casting or deformation processing to bring the tolerance or finish up to the desired level, creating a *process chain*. Metals and ceramics can be surface ground and lapped to a high precision and smoothness: a large telescope reflector has a tolerance approaching $5\ \mu\text{m}$ and a roughness of about 1/100 of this over a dimension of a meter or more. But precision and finish carry a cost: processing costs increase exponentially as the requirements for both are made more severe.

Table 7.4
Levels of Finish

Finish, μm	Process	Typical Application
R=0.01	Lapping	Mirrors
R=0.1	Precision grind or lap	High quality bearings
R=0.2–0.5	Precision grinding	Cylinders, pistons, cams, bearings
R=0.5–2	Precision machining	Gears, ordinary machine parts
R=2–10	Machining	Lightly loaded bearings, noncritical components
R=3–100	Unfinished castings	Nonbearing surfaces
R=100–1000	Additive manufacture	Prototypes, low-volume production

Moulded polymers inherit the finish of the moulds used to shape them and thus can be very smooth; machining to improve the finish is rarely necessary. Tolerances better than ± 0.2 mm are seldom possible because internal stresses left by moulding cause distortion and because polymers creep in service.

Limitations and Quality (b) Joining

Material compatibility. Processes for joining metals, polymers, ceramics and glasses differ. A given adhesive will bond to some materials but not to others; welding methods for polymers differ from those for welding metals; and ceramics, which cannot be welded, are joined instead by diffusion bonding or glaze bonding. The material–process matrix (Fig. 7.3) includes four classes of joining process.

When the joint is between dissimilar materials, the process must be compatible with both. Adhesives and fasteners allow joints between different materials; many welding processes do not. If dissimilar metals are joined in such a way that they are in electrical contact, a corrosion couple appears if the joint is wet. This can be avoided by inserting an insulating layer between the surfaces. Thermal-expansion mismatch gives internal stresses in the joint if the temperature changes, with risk of distortion or damage. Identifying good practice in joining dissimilar materials is part of the documentation step.

Joint geometry and mode of loading. The geometry of the joint and the way it is loaded (Fig. 7.13) influences process choice. Adhesive joints support shear but are poor in peeling – think of stripping off adhesive tape. Adhesives need a large working area – lap joints work well, butt joints do not. Rivets and staples, too, are well adapted for shear loading of lap joints but are less good in tension. Welds and threaded fasteners are more adaptable, but here, too, matching choice of process to geometry and loading is important.

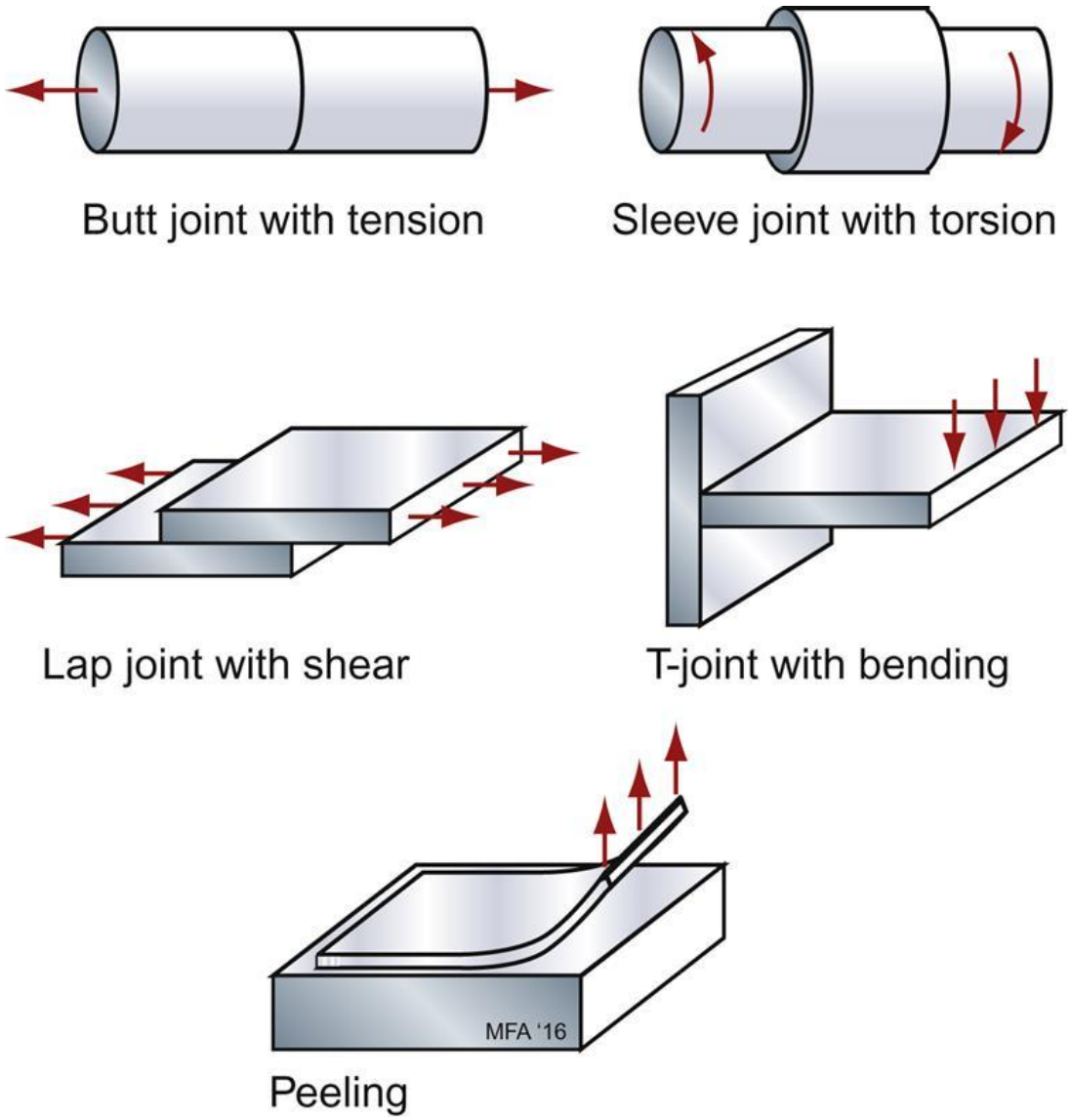


FIGURE 7.13 Joint geometries and modes of loading.

Secondary functions of joints. A joint may serve as a seal, and be required to exclude gases or liquids. It may be required to conduct or insulate against the conduction of heat or electricity, or to operate at an elevated operating temperature. It may be permanent or require disassembly for maintenance or to allow recycling: threaded fasteners and adhesives that can be loosened with solvents or heat permit this.

Adhesives. All early civilizations seem to have known how to make glues from starch, animal gelatins, bitumen or gum arabic. Synthetic adhesives appeared around 1900. Subsequent advances in polymer chemistry have extended their range and performance into every industrial sector. *Structural adhesives* are those that are used to perform some mechanical function, though

they may have a secondary role as a sealant. Many are rigid, which gives a stiff bond; *flexible adhesives* are generally less strong but allow small relative displacements across the joint. Within each group, it is usual to use a classification based on chemistry (Table 7.5).

Table 7.5

Adhesives and What They Stick To

	Metals	Wood	Polymers	Elastomers	Ceramics	Composites	Textiles
Metals	Acr. Aner. Cyan. EP PU Phen. Sil						
Wood	Acr. EP Phen. Hot-melt	EP. Phen. PVA					
Polymers	Acr. Cyan. EP Phen	EP PU Phen. PVA	Acr. Cyan. EP Phen				
Elastomers	Cyan. EP Sil	Acr. Phen. Sil	Cyan. EP Phen. Sil	PU Sil			
Ceramic	Acr. Cyan. EP Ceram	Cyan. EP PVA Ceram	Acr. EP PU PVA Ceram	Acr. EP PU PVA Sil	Acr. Cyan. EP Ceram		
Composites	Acr. Cyan. EP Imide	Acr. Cyan. EP PVA	Acr. EP PVA Sil	EP PU Sil	Cyan. EP Sil	EP Imide PES Phen	
Textiles	PU Hot-melt	Acr. PVA Hot-melt	Acr. PVA	Acr. PU PVA	Acr. PU PVA Hot-melt	Acr. PU PVA	PVA Hot-melt

Acrylic=Acr.; Epoxy=EP; Polyurethane=PU; Silicone=Sil.; Anaerobic=Aner.; Imide=Imide; Phenolic=Phen.; Thermoplastic=Hot-melt; Cyanoacrylate=Cyan.; Polyester=PES; Polyvinyl acetate=PVA; Ceramic-based=Ceram.

There are five general mechanisms for getting two bodies to stick together; the same mechanisms are responsible for the bonding of a paint and other coating to a surface (Fig. 7.14). Surface roughening by sand blasting or chemical etching creates cavities into which an adhesive, paint or coating cures and sets, giving mechanical interlocking. Van der Waals adhesion relies on the attractive interaction between dipoles in nonmetallic surfaces giving a weak but reversible bond (it's how flies walk on ceilings). Ionic adhesion occurs between surfaces with polar groups; it is of little help in bonding most thermoplastics because they lack ionic functionality. Molecular entanglement works well for coatings or adhesives on amorphous plastics such as PC, ABS and PVC but is less effective with semi-crystalline thermoplastics. Chemical adhesion is the formation of chemical bonds between the surfaces, stimulated by a chemical adhesion promoter. The reactive 'head' of the promoter forms a strong chemical bond with one surface, while its functional 'tail' interacts and bonds with the adhesive (epoxy, for example) or paint coating. Silanes (SiH_4) or phosphonates (containing C-$PO(OR)_2$ groups) are used for metallic, ceramic or glass surfaces but they don't work for thermoplastics. For these, highly reactive carbene (R=C:) treatments are used.

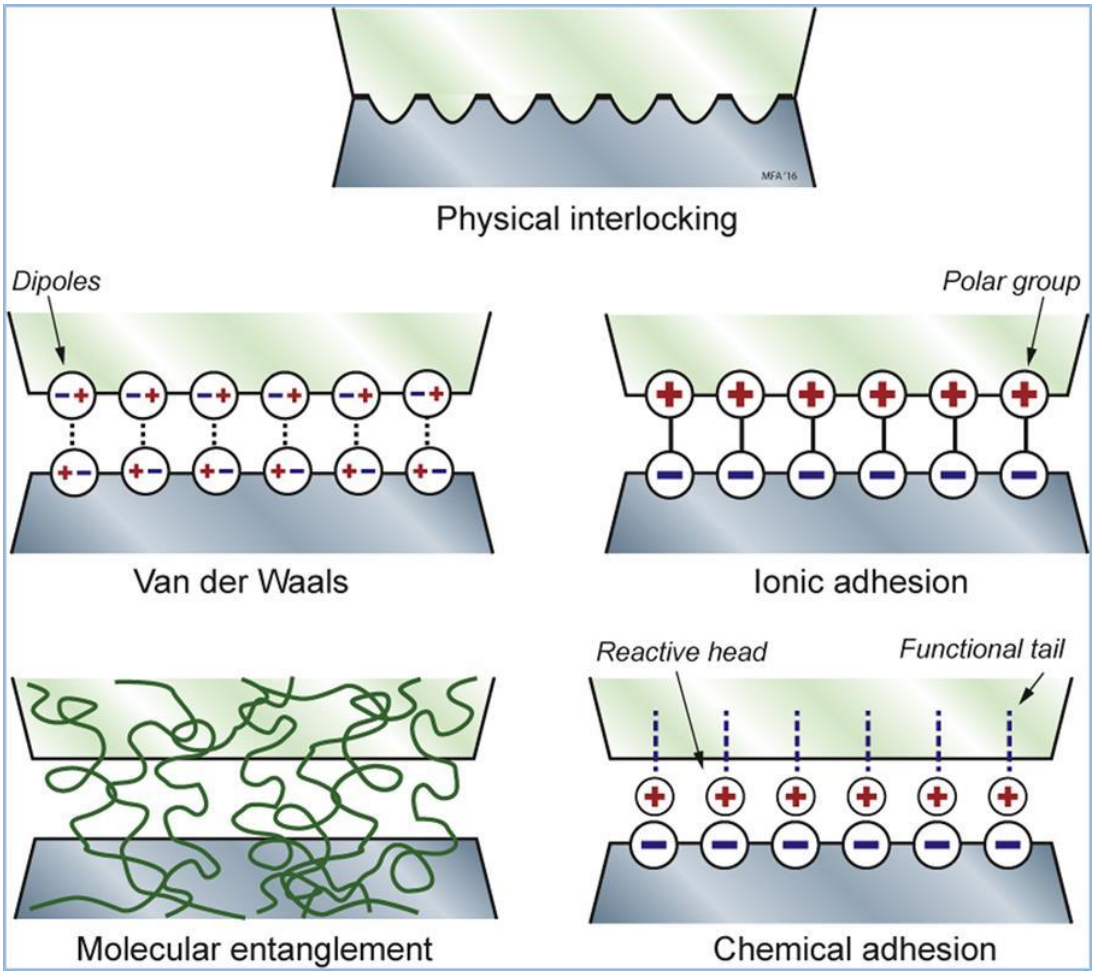


FIGURE 7.14 Mechanisms of adhesion.

Adhesive allow great design freedom as almost any material or combination of materials can be adhesively bonded: they can be of very different thickness (thin foils can be bonded to massive sections) and the processing temperatures are low, seldom exceeding 180°C. The flexibility of some adhesives tolerates differential thermal expansion on either side of the joint and allow joints that are impermeable to water and air. Adhesive joints are usually lighter than the equivalent mechanical fastener and for that reason they are increasingly used in automotive applications, particularly for joining fibre and fabric-reinforced thermoplastics. The main disadvantages are the limited service temperatures (most adhesives are unstable above 190°C, though some are usable up to 260°C), the uncertain long-term stability and the unpleasant solvents that some contain.

Welding, brazing and soldering. Welding is the joining of two similar metals or plastics by locally melting them or heating them to a temperature at which they diffusion-bond. Brazing and soldering are the bonding of metals (or metallized

ceramic or glass) with a lower-melting alloy: a *braze* (typically a copper-zinc or copper-tin alloy melting at 650–800°C) or a *solder* (previously a lead-tin alloy melting at 180–250°C, now replaced by lead-free tin-bismuth or tin-silver-copper alloys).

All of these processes use a moving heat source to raise the temperature locally. The thermal field of a moving heat source (Fig. 7.15) is documented in Appendix B, Section B16: the temperature rises quickly to a peak T_p then cools more slowly as the source moves on and heat is lost by conduction. The temperature rise is large enough to change the properties of the material on either side of the weld, an effect of particular significance in the welding of steels.

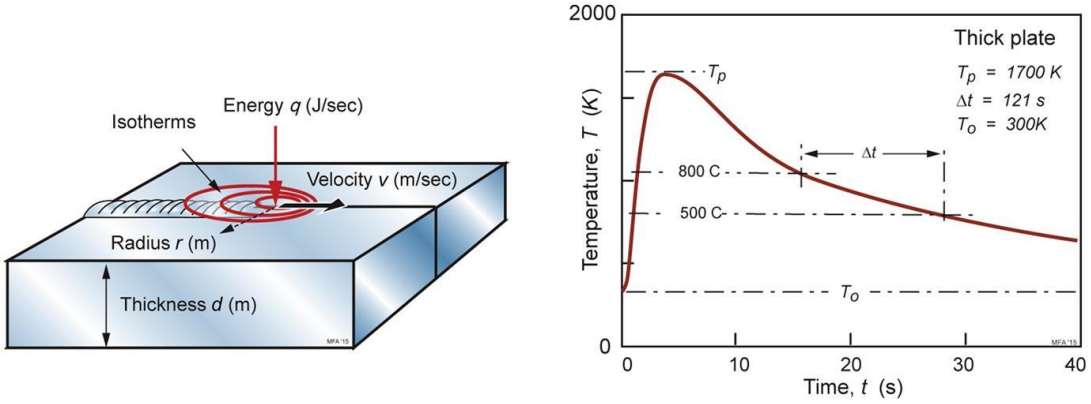


FIGURE 7.15 The parameters of the weld model and a typical thermal profile.

Fig. 7.16 shows this heat-affected zone and its relationship to the iron–carbon phase diagram. At the centre is the melt zone with a columnar structure that forms as the melt solidifies. On either side is a zone of recrystallization with a band of intense grain coarsening. Beyond that the temperatures are too low to permit recrystallization but high enough to produce a softened tempered zone. Understanding this structure and its effect on local properties is central to managing welding properly.

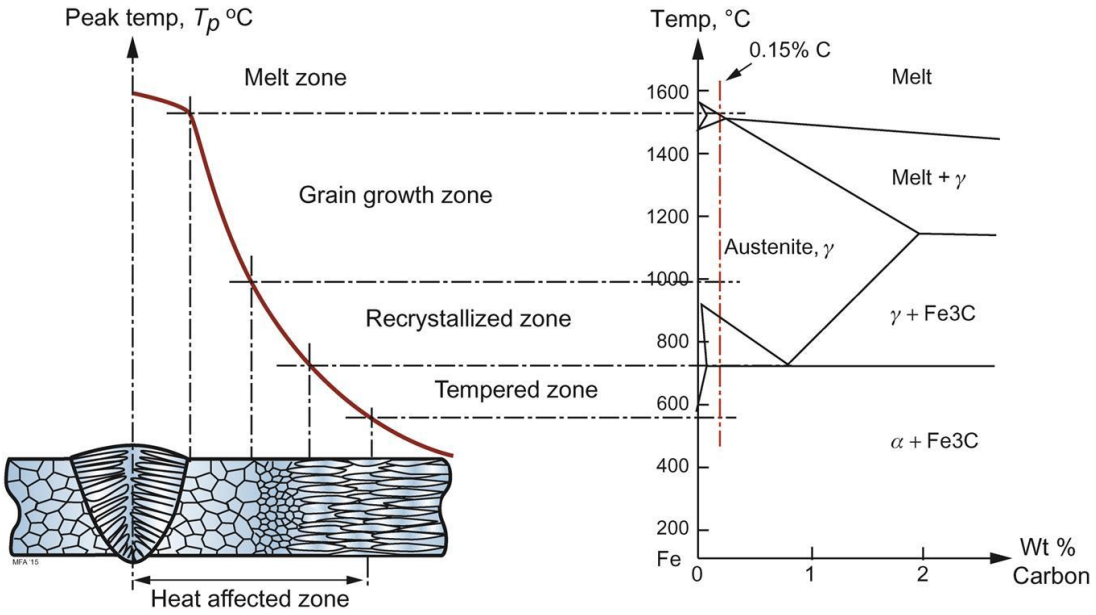


FIGURE 7.16 The heat-affected zone in a low-carbon steel plate.

Limitations and Quality (c) Surface Treatment

Material compatibility. Some surface treatments are specific to metals, some to polymers. Some work for all. Material–process compatibility for surface treatments is shown at the bottom of the matrix in Fig. 7.3.

The purpose of the surface treatment. All surface treatment adds cost but the benefit can be large. Table 7.6 illustrates the diversity of functions that surface treatments can provide. Some protect, some enhance performance, and still others are primarily aesthetic. Protecting a component surface extends product life and reduces maintenance. Coatings on cutting tools enable faster cutting speeds and greater productivity. And surface hardening processes may enable the substrate alloy to be replaced with a cheaper material – e.g., using a plain carbon steel with a hard carburized surface or a coating of hard titanium nitride (TiN), instead of using a more expensive alloy steel.

Table 7.6
Providing Surface Functionality

Functions Provided by Surface Treatments	
■ Corrosion protection, aqueous environments	■ Thermal insulation
■ Corrosion protection, gas environments	■ Electrical insulation
■ Wear resistance	■ Magnetic response
■ Friction control	■ Decoration
■ Fatigue resistance	■ Colour
■ Thermal conduction	■ Reflectivity

Steel surfaces are hardened by immersing the part in a carburizing bath at 800–950°C (where the steel has the fcc austenitic structure) followed by quenching to give a surface layer of martensite. Nitriding is done at lower temperatures (around 550°C, when the steel has a bcc ferritic structure), causing nitrogen to diffuse into the surface where it reacts with alloy elements to give a fine precipitate of nitrides. The depth of the hardened surface layer, x^*, depends on the heating time and temperature in the way described in Appendix B, Sections B14 and B15. If the carbon or nitrogen source is of high activity, it is approximated by

$$x^* \approx \sqrt{2Dt} \tag{7.9}$$

where

$$D = D_o \exp -\left(\frac{Q_d}{RT}\right) \tag{7.10}$$

is the diffusion coefficient (m^2/s) of carbon or nitrogen in iron and t (s) is time. The preexponential factor D_o and activation energy Q_d for the diffusion of carbon and nitrogen in iron are listed in Table 7.7. R is the gas constant, 8.314 J/mol.K.

Table 7.7

Diffusion Data for Carbon and Nitrogen in Iron

System	Preexponential, D_o (m^2/s)	Activation Energy Q_d (kJ/mol)
Carbon in bcc (ferritic) iron	1.1×10^{-6}	87.4
Carbon in fcc (austenitic) iron	7×10^{-6}	131
Nitrogen in bcc (ferritic) iron	4.7×10^{-7}	76.5
Nitrogen in fcc (austenitic) iron	3.4×10^{-7}	145

Carburizing and nitriding are immersion processes, heating the entire component. While the aim is to harden a surface layer, the bulk properties are inevitably affected too. Localised processes, by contrast, change the surface properties only. In flame hardening and laser surface hardening a torch or laser beam is scanned across the surface, moving at a rate such that the heat penetrates only a small distance into the sample. The thermal field created by a moving heat source has already illustrated in connection with welding. Exactly the same models (Appendix B, Section B16) apply to these localised surface hardening techniques.

Example

A carburizing treatment for an austenitic steel at 800°C produces a carburized layer 32 μm deep in 30 minutes. How deep will the layer be if the treatment is changed to 15 minutes at 850°C?

Answer

The ratio of the new depth (temperature T_2, time t_2) to the old one (temperature T_1, time t_1) is

$$\frac{x_2}{x_1} = \frac{\sqrt{2\,D_2 t_2}}{\sqrt{2\,D_1 t_1}} = \sqrt{\frac{1}{2}\frac{\exp-\left(\frac{131,000}{8.314\times1123}\right)}{\exp-\left(\frac{131,000}{8.314\times1073}\right)}} = 0.96$$

The two depths are almost the same. The small ($\approx 5\%$) increase in the temperature has resulted in a large (almost 50%) change in the time to reach the same carburized depth.

Secondary compatibilities. Some surface treatments, such as anodizing, leave the dimensions, precision and roughness of the surface unchanged. Electro- and vapour-deposited coatings change the dimensions a little, but may still leave a perfectly smooth surface. Polymer powder coating builds up a relatively thick smooth layer; others, like weld-deposition create a deliberately thick layer with a rough surface, requiring refinishing. 'Line-of-sight' deposition processes coat only the surface at which they are directed, leaving inaccessible areas uncoated; others, with what is called 'throwing power', coat flat, curved and re-entrant surfaces equally well. Many surface-treatment processes require heat. These can only be used on materials that can tolerate the rise in temperature. Some paints are applied cold, but many require a bake at up to 150°C.

7.5 Ranking: Process Cost

Part of the cost of a component is that of the material of which it is made. The rest is the cost of manufacture, that is, of forming it to a shape, and of joining and finishing it. Before turning to details, there are four common-sense rules for minimizing cost.

Keep things standard. If someone already makes the part you want, it will probably be cheaper to buy it than to make it. If nobody does, then it is cheaper to design the part to be made from standard stock (sheet, rod, tube) than from nonstandard shapes or from special castings or forgings. Use standard materials, and as few of them as possible because it reduces inventory costs and simplifies recycling.

Keep things simple. If a part has to be machined, it will have to be clamped; the cost increases with the number of times it will have to be re-jigged or re-oriented, particularly if special tools are necessary. If a part is to be welded or brazed, the welder must be able to see it and to reach it with the weld torch. If it is to be cast or moulded or forged, recall that high (and expensive) pressures are required to make fluids flow into narrow channels, and that re-entrant shapes greatly complicate mould and die design.

Aim for easy assembly. Assembly takes time, and time means cost. If the overhead rate is a mere $60 per hour, every minute of assembly time adds another $1 to the cost (2016 USD). Design for assembly (DFA) addresses this problem with a set of common-sense criteria and rules. Briefly, there are three:

- Minimize part count;
- Design parts to be self-aligning on assembly; and
- Use joining methods that are fast. Snap-fits and spot welds are faster than threaded fasteners or, usually, adhesives.

Do not specify more performance than is needed. Performance must be paid for. High strength metals are more heavily alloyed with expensive additions; high performance polymers are chemically more complex; high performance ceramics require greater quality control in their manufacture. All of these increase material costs. In addition, high strength materials are hard to fabricate. The forming pressures (whether for a metal or a polymer) are higher; tool wear is greater; ductility is usually less so that deformation processing can be difficult or impossible. This can mean that new processing routes may be needed: investment casting or powder forming instead of conventional casting and mechanical working; more expensive moulding equipment operating at higher temperatures and pressures, and so on. The better performance of the high strength material must be paid for, not only in greater material cost but also in the higher cost of processing. Finally, there are the questions of tolerance and roughness. Cost rises exponentially with demands for precision and surface finish. The message is clear. Performance costs money. Do not over-specify it.

To make further progress, we must examine the contributions to process costs and their origins.

Cost Modelling

Earlier in this chapter we introduce 'economic batch size' as a simple economic criterion for screening processes. A cost model, which we now explore, gives deeper insight.

The manufacture of a component consumes resources (cover image of this chapter, summarized in Fig. 7.17), each of which has an associated cost. The final cost is the sum of those of the resources it consumes. They are detailed in Table 7.8. Thus the cost of producing a component entails the cost C_m (\$/kg) of the materials and feedstocks from which it is made. It involves the cost of dedicated tooling, C_t (\$), and that of the capital equipment, C_c (\$), in which the tooling will be used. It requires time, chargeable at an overhead rate $\dot{C}_{oh}$ (thus with units of \$/hr), in which we include the cost of labour, administration and general plant costs. It requires energy, which is sometimes charged against a process step if it is very energy intensive but more usually is treated as part of the overhead and lumped into $\dot{C}_{oh}$, as we shall do here. Finally there is the cost of information, meaning that of research and development, royalty or licence fees; this, too, we view as a cost per unit time and lump it into the overhead.

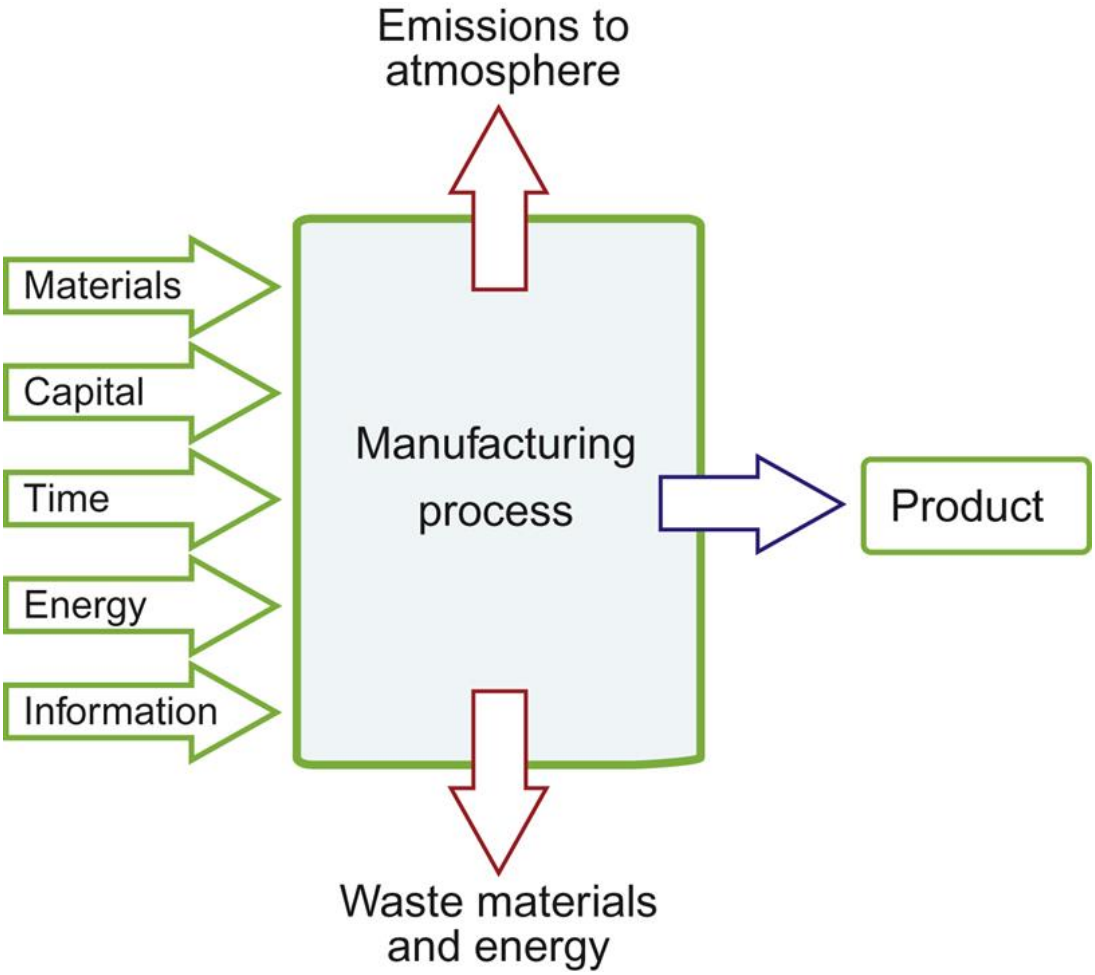


FIGURE 7.17 The inputs to a cost model.

Table 7.8
Symbols, Definitions and Units

Resource		Symbol	Unit
Materials:	Including consumables	C_m	\$/kg
Capital:	Cost of tooling	C_t	\$
	Cost of equipment	C_c	\$
Time:	Overhead rate, including labour, administration, rent …	$\dot{C}_{oh}$	\$/hr
Energy:	Cost of energy	$\dot{C}_e$	\$/hr
Information:	R & D or royalty payments	$\dot{C}_i$	\$/year

Think now of the manufacture of a component (the 'unit of output') weighing m kg, made of a material costing C_m \$/kg. The first contribution to the

unit cost is that of the material mC_m magnified by the factor 1/(1-*f*) where *f* is the scrap fraction – the fraction of the starting material that ends up as sprues, risers, turnings, rejects or waste:

$$C_1 = \frac{mC_m}{(1-f)} \tag{7.11}$$

The cost C_t of a set of tooling – dies, moulds, fixtures and jigs – is what is called a *dedicated cost*: one that must be wholly assigned to the production run of this single component. It is written off against the numerical size *n* of the production run. Tooling wears out. If the run is a long one, replacement will be necessary. Thus tooling cost per unit takes the form

$$C_2 = \frac{C_t}{n}\left\{Int\left(\frac{n}{n_t} + 0.51\right)\right\} \tag{7.12}$$

where n_t is the number of units that a set of tooling can make before it has to be replaced, and '*Int*' is the integer function. The term in curly brackets simply increments the tooling cost by that of one tool-set every time *n* exceeds n_t.

The capital cost of equipment, C_c, by contrast, is rarely dedicated. A given piece of equipment – a powder press, e.g., – can be used to make many different components by installing different die-sets or tooling. It is usual to convert the capital cost of *nondedicated* equipment, and the cost of borrowing the capital itself, into an overhead by dividing it by a capital write-off time, t_{wo}, (5 years, say) over which it is to be recovered. The quantity C_c/t_{wo} is then a cost per hour – provided the equipment is used continuously. That is rarely the case, so the term is modified by dividing it by a load factor, *L* – the fraction of time for which the equipment is productive. The cost per unit is then this hourly cost divided by the rate $\dot{n}$ at which units are produced:

$$C_3 = \frac{1}{\dot{n}}\left(\frac{C_c}{L\,t_{wo}}\right) \tag{7.13}$$

Finally there is the overhead rate $\dot{C}_{oh}$. It becomes a cost per unit when divided by the production rate $\dot{n}$ units per hour (averaged over the production run to allow for downtime):

$$C_4 = \frac{\dot{C}_{oh}}{\dot{n}} \tag{7.14}$$

The total shaping cost per part, C, is the sum of these four terms, taking the form:

$$C = \frac{mC_m}{(1-f)} + \frac{C_t}{n}\left\{Int\left(\frac{n}{n_t} + 0.51\right)\right\} + \frac{1}{\dot{n}}\left(\frac{C_c}{L\, t_{wo}} + \dot{C}_{oh}\right) \tag{7.15}$$

The equation shows that the cost has three essential contributions – a material cost per unit of production that is independent of batch size and rate, a dedicated cost per unit of production that varies as the reciprocal of the production volume ($1/n$), and a gross overhead per unit of production that varies as the reciprocal of the production rate ($1/\dot{n}$). The equation describes a set of curves relating cost C to batch size n, one for each process. Each has the shape of the pencil-sharpening curves of Fig. 7.5.

Example: Using the Cost Model

The design requirements for a steel connecting rod were listed in Section 7.2 as an example of translation. Applying the constraints on material, shape and batch size using the compatibility matrices of Figs. 7.3, 7.4 and 7.6 narrows the choice to the following

- Sand casting and its variants
- Forging
- Powder methods

} with subsequent machining of bores

Rank these alternatives by cost for a batch size of 100 and one of 10,000.

Answer

Table 7.9 lists values for the parameters in the cost model for three processes: sand-casting (here, low-pressure sand casting), forging and powder methods. We assume that all three use the same amount of material. Fig. 7.18 **shows the computed relative cost per part against batch size for the three processes**.

Table 7.9

Process Cost Model Input Data for Shaping Methods to Make the Con Rod

Parameters	Sand Casting	Forging	Powder Methods
Material, $mC_m/(1-f)$ *($)*	1	1	1
Basic overhead $\dot{C}_{oh}$ *($ per hour)*	100	100	100
Capital write-off time t_{wo} *(years)*	5	5	5
Load factor	0.5	0.5	0.5
Dedicated tooling cost, $C_{t\,(\$)}$	2000	8500	5000
Capital cost, $C_{c\,(\$)}$	37,000	550,000	140,000
Production rate, $\dot{n}$ *(per hour)*	16	220	95

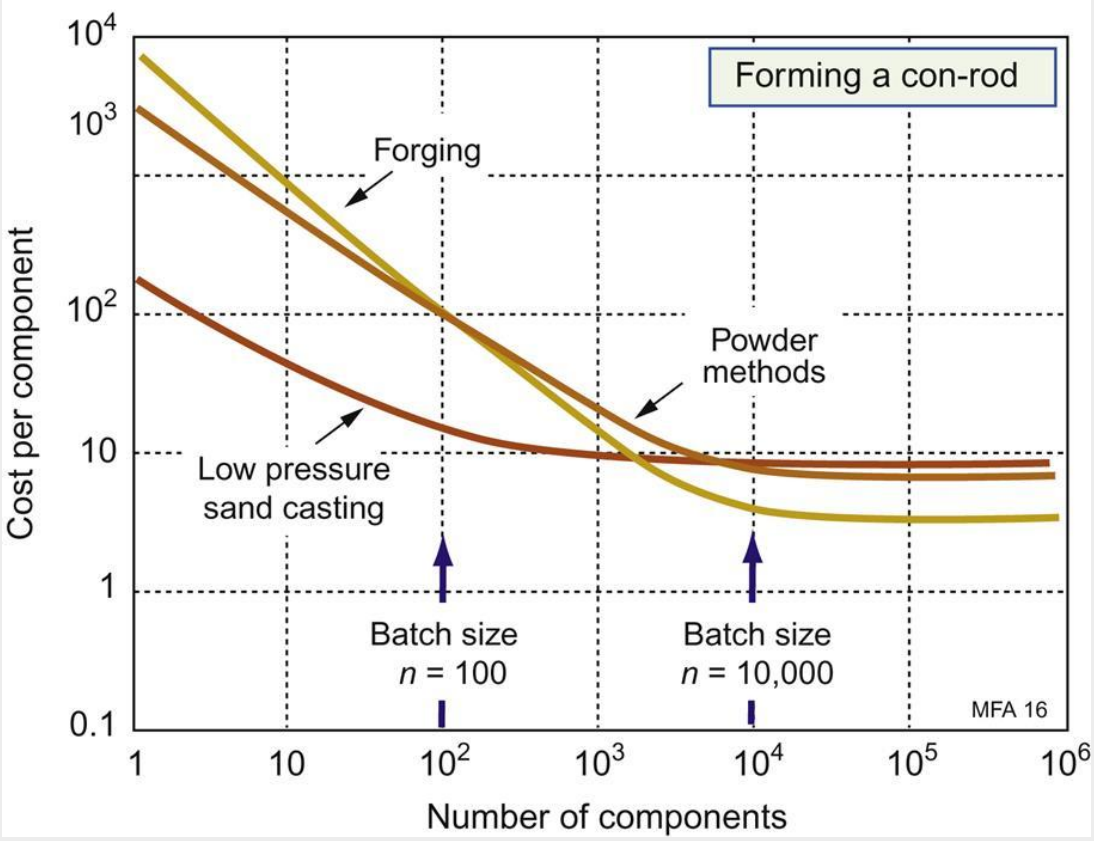


FIGURE 7.18 Unit cost as a function of batch size.

If only 100 con rods are planned, sand casting is the cheapest option because the tooling cost (the sand mould) is cheap. But sand casting is slow. If 10,000 units are planned, forging is a cheaper option because it is fast, reducing the overhead (labour) cost per unit.

Technical cost modelling. Eq. (7.15) is the first step in modelling cost. Greater predictive power is possible with technical cost models that exploit understanding of the way in which the design, the process and cost interact. The capital cost of equipment depends on size and degree of automation. Tooling cost and production rate depend on complexity. These and many other dependencies can be captured in theoretical or empirical formulae or look-up tables that can be built into the cost model, giving more resolution in ranking competing processes. For more advanced analyses the reader is referred to the literature listed as further reading at the end of this chapter.

7.6 Computer-Aided Process Selection

If process attributes are stored in a database with an appropriate user interface, selection charts can be created and selection boxes manipulated with much greater freedom. The CES platform, mentioned earlier, is an example of such a system. The database contains records, each describing the attributes of a single process. Table 7.10 shows part of a typical record: it is that for injection moulding. A schematic indicates how the process works; it is supported by a short description. This is followed by a list of attributes: the materials it can handle, shapes it can make, the economic batch size, and the attributes relating to shape and physical characteristics, and default values for the parameters of the cost model (which can be overwritten). It ends with brief documentation in the form of guidelines, technical notes and typical uses. The numeric attributes are stored as ranges, indicating the range of capability of the process. Each record is linked to records for the materials with which it is compatible, allowing choice of material to be used as a screening criterion, like the material compatibility matrix of Fig. 7.3 but with greater resolution. A short list of candidates is extracted in two steps: screening to eliminate processes that cannot meet the design specification and ranking using the cost model where this is available.

Table 7.10

Part of a Record for a Process

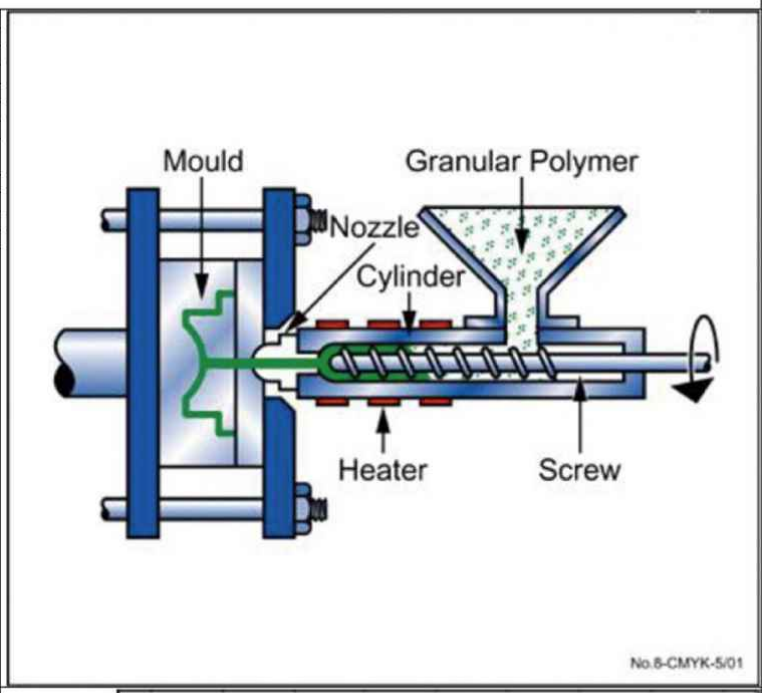
Injection moulding

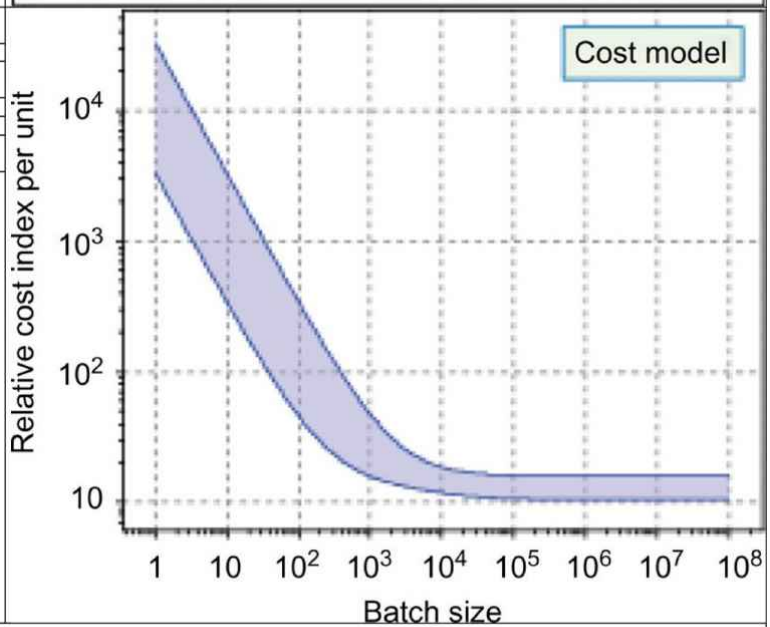
The process. The most widely used process for shaping thermoplastics is the reciprocating screw injection moulding machine, shown in the schematic. Polymer granules are fed into a spiral press where they are heated, mixed and softened to a dough-like consistency that is forced through one or more channels ('sprues') into the die. The polymer solidifies under pressure and the component is then ejected.

Thermoplastics, thermosets and elastomers can also be injection moulded. Coinjection allows moulding of components with different materials, colours and features. Injection foam moulding allows economical production of large moulded components by using inert gas or chemical blowing agents to make components that have a solid skin and a cellular inner structure.

Material compatibility

Links to:	Thermoplastics
Shape compatibility	
Circular prismatic	✓
Noncircular prismatic	✓
Solid 3-D	✓
Hollow 3-D	✓
Economic compatibility	
Economic batch size (units)	10^4–10^6
Physical and quality attributes	
Mass range	0.01–25 kg
Range of section thickness	0.4–6.3 mm
Tolerance	0.2–1 mm
Roughness	0.2–1.6 μm
Surface roughness (A=v. smooth)	A
Cost modelling	
Capital cost	3×10^4–7×10^5 (USD)
Material utilisation fraction	0.6–0.9
Production rate (units)	60–1000/hr
Tooling cost	3000–30,000 USD
Tool life (units)	10^4–10^6





Documentation

Design guidelines. Injection moulding is the best way to mass-produce small, precise, polymer components with complex shapes. The surface finish is good; texture and pattern can be easily altered in the tool, and fine detail reproduces well. Decorative labels can be moulded onto the surface of the component. The only finishing operation is the removal of the sprue.

Technical notes. Most thermoplastics can be injection moulded, although those with high melting temperatures (e.g., PTFE) are difficult. Thermoplastic based composites (short fibre and particulate filled) can be processed providing the filler-loading is not too large. Large changes in section area are not recommended. Small re-entrant angles and complex shapes are possible, though some features (e.g., undercuts, screw threads, inserts) may result in increased tooling costs.

Typical uses. Housings, containers, covers, knobs, tool handles, plumbing fittings, lenses, toys etc.

More information about computer aided selection can be found in the sources listed under Granta Design (2016) in Further Reading.

7.7 Summary and Conclusions

A wide range of shaping, joining and finishing processes is available to the design engineer. Each has certain characteristics, which, taken together, suit it to the processing of certain materials in certain shapes, but disqualify it for others. The structured, systematic approach of this chapter guides the user to processes that are compatible with the material, shape and production scale required by the design.

The method parallels that for selection of material, using process selection matrices and charts to implement the procedure. A component design dictates a certain, known, combination of process attributes and selection charts and identifies the subset of processes able to provide them. The method lends itself to computer implementation, which allows selection from a large portfolio of processes by screening on attributes and ranking by economic criteria.

There is, of course, much more to process selection than this. It is to be seen, rather, as a first systematic step, replacing a total reliance on local experience and past practice. The narrowing of choice helps considerably: it is now much easier to identify the right source for more expert knowledge and to ask of it the right questions. But the final choice still depends on local economic and organisational factors that can only be decided on a case-by-case basis.

7.8 Further reading

These are exercises in Process Selection using the Selection matrices, the simple models, and the more systematic methods developed in the text.

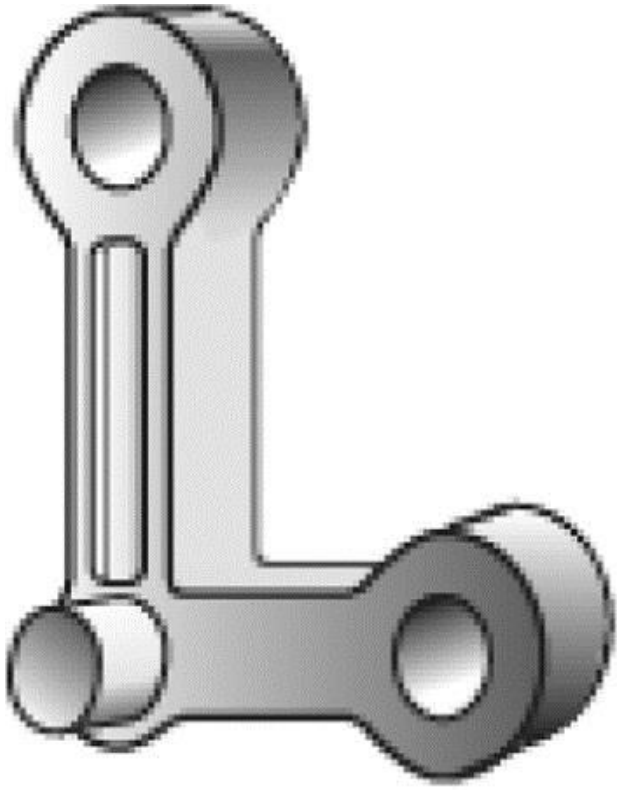
1. ASM Handbook Series. *Volume 4 "Heat Treatment", Volume 5 "Surface Engineering", Volume 6 "Welding, Brazing and Soldering", Volume 7 "Powder Metal Technologies, Volume 14 "Forming and Forging", Volume 15 "Casting" and Volume 16 "Machining"* Metals Park, Ohio: ASM International; 1971–2016; (A comprehensive set of handbooks on processing, occasionally updated, and now available on-line at <www.asminternational.org/hbk/index.jsp>).
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712842-6. *(A well established text on the processing and use of engineering materials, now in its 9th edition.)*.
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10. Granta Design, 2016. "The CES EduPack" and Teaching Resources http://www.grantadesign.com/education/teaching/.
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7.9 Exercises

E7.1. *Selecting shaping processes*. A process is sought to make the rocker arm shown here. It is to be made from and aluminium–silicon alloy. The shape is '3-D solid'. The expected production run (batch size) is 2000. It weight about 1 kg. Surface roughness and tolerance are not critical because the bores and the boss will be machined to final size.



A rocker arm

FIGURE E7.1

E7.2. A camera body is shown on the right. The design requires that it be made of a magnesium alloy and that the initial production run will be 10,000 units. You are asked to suggest processes by which the shape could be made. Surface finish and tolerance must be high. Use the matrices of Figs. 7.3, 7.4 and 7.6 to identify potential candidates. Lay out the results in a table like that in the in-text example.

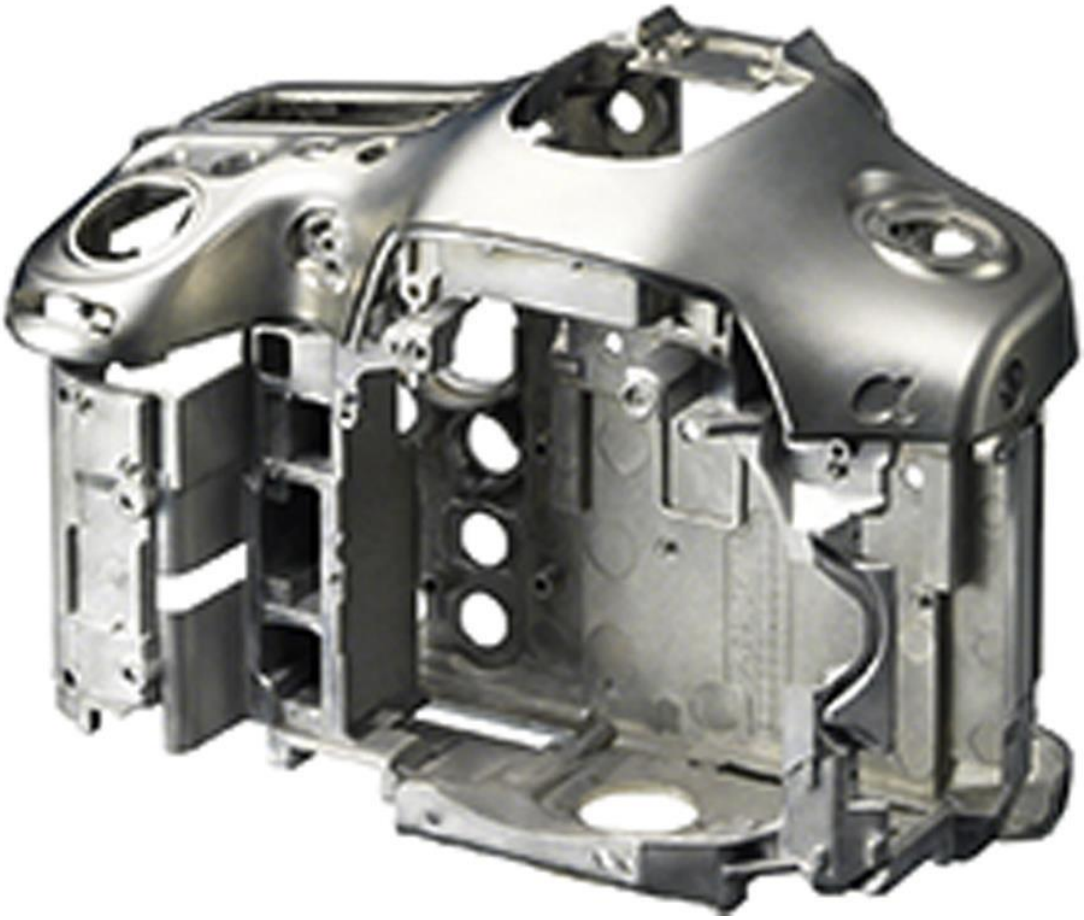


FIGURE E7.2 https://en.wikipedia.org/wiki/Magnesium_alloy#/media/File:Samsung_NX1_chassis.jpg

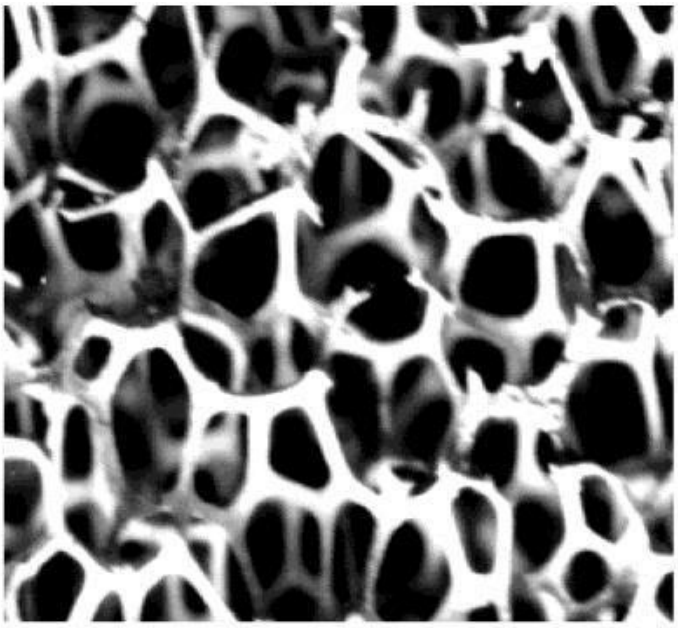
E7.3. The car roof box shown in the image is to be made of glass-fibre reinforced polyester in a batch of 10,000. It is made up of two separate shell-like components – an upper and a lower half. What processes are available to shape them? Use the matrices of Figs. 7.3, 7.4 and 7.6 to identify potential candidates.



A roof box

FIGURE E7.3 From: https://commons.wikimedia.org/wiki/File:Thule_Dynamic_roof_box_%2811726731603%29.jpg

E7.4. Metal foams can be made by first making an open-cell polymer foam, then embedding this in plaster, burning out the polymer and finally forcing metal under pressure into the resulting mould to replicate the foam. If the cell walls of the polymer foam are cylindrical with a diameter of is 0.5 mm, what pressure will be needed to replicated it using a zinc die-casting alloy? The surface tension of the zinc alloy is 0.76 J/m^2.



A metal foam made by replicating a polymer foam.

FIGURE E7.4

E7.5. A copper ingot is compressed between two anvils. If the yield strength σ_Y of copper is 80 MPa and the coefficient of friction μ at the interface between the copper and the anvil is 0.1, what is the maximum forming pressure at the centre of the ingot when the aspect ratio $2w/h$ is 20:1?

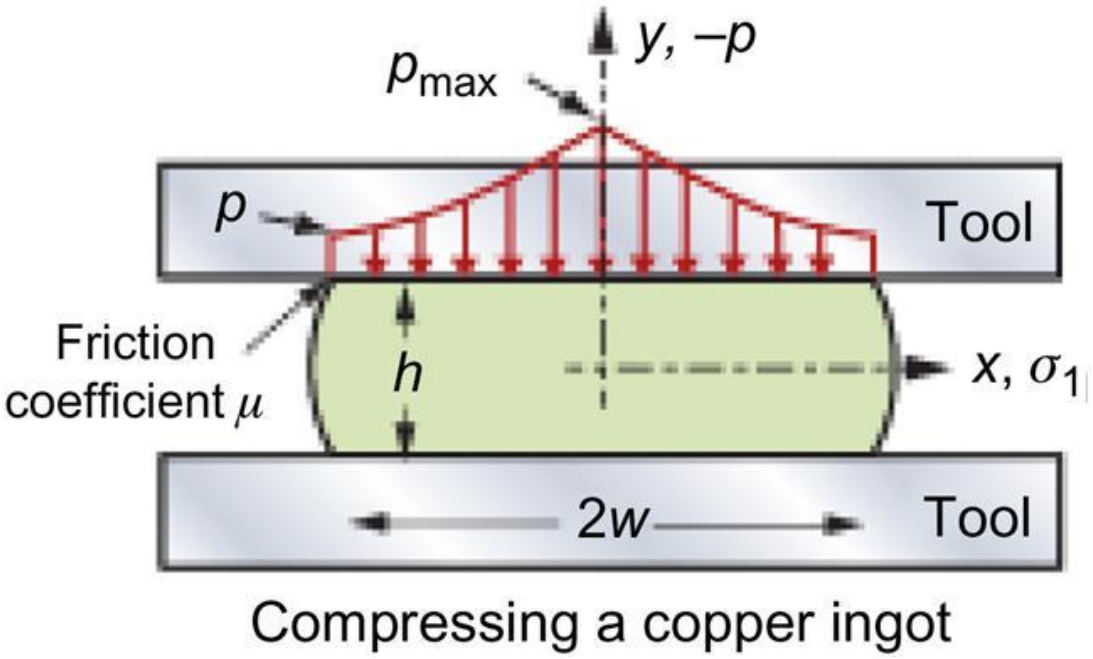


FIGURE E7.5

E7.6. A cylindrical magnet is to be made from Alnico (an iron-aluminium-nickel-cobalt alloy) by powder compaction and sintering. The aspect ratio of the cylinder, $h/2r$, is 2. The coefficient of friction μ of the powder with the die wall, if unlubricated, is 0.5. By what factor will the compaction pressure have fallen at the mid plane of the cylinder because of die wall friction? If, instead, a lubricant is mixed with the powder (it will burn out during sintering), which reduces the coefficient of friction to 0.05, by how much will the compaction pressure have fallen at the mid plane?

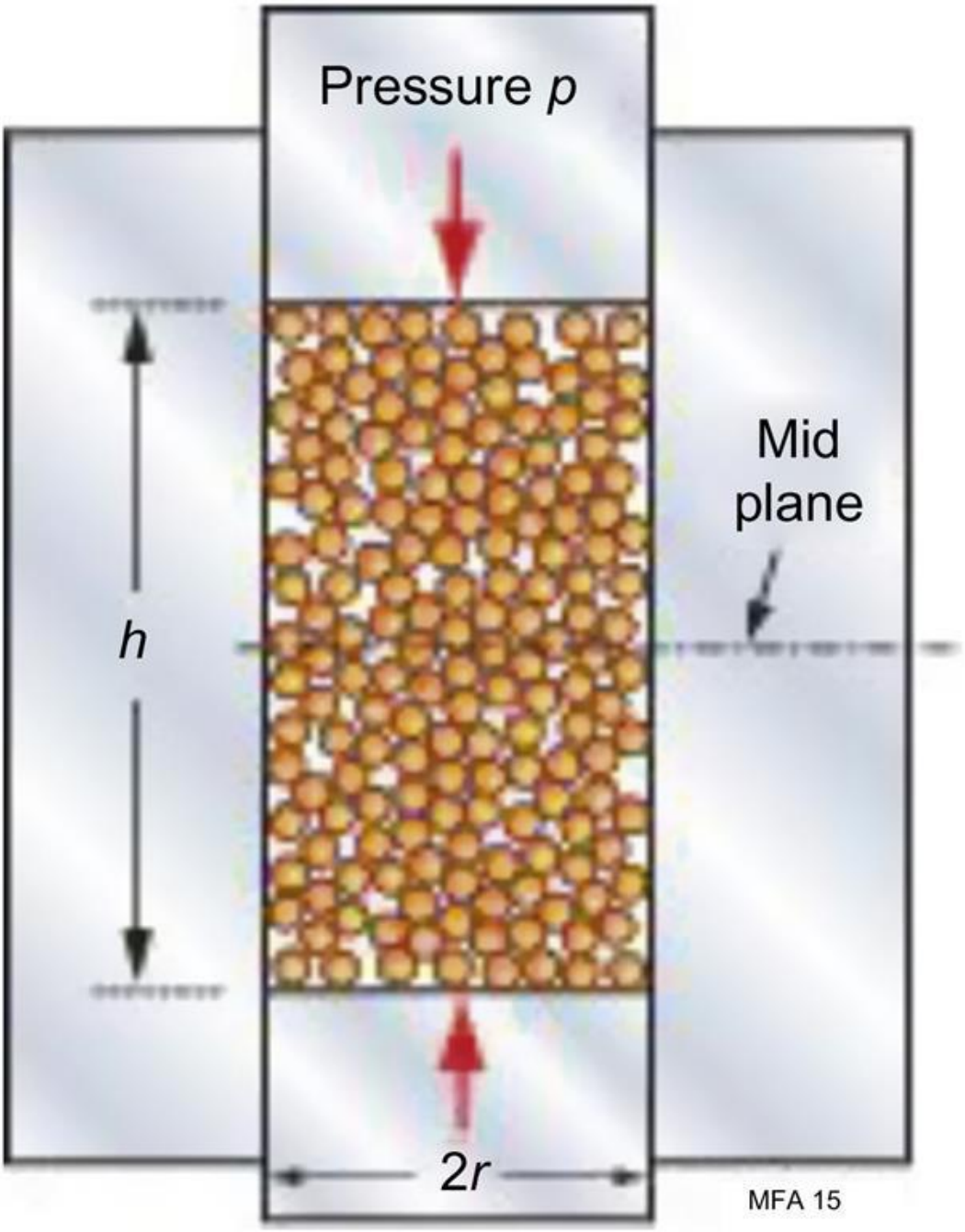
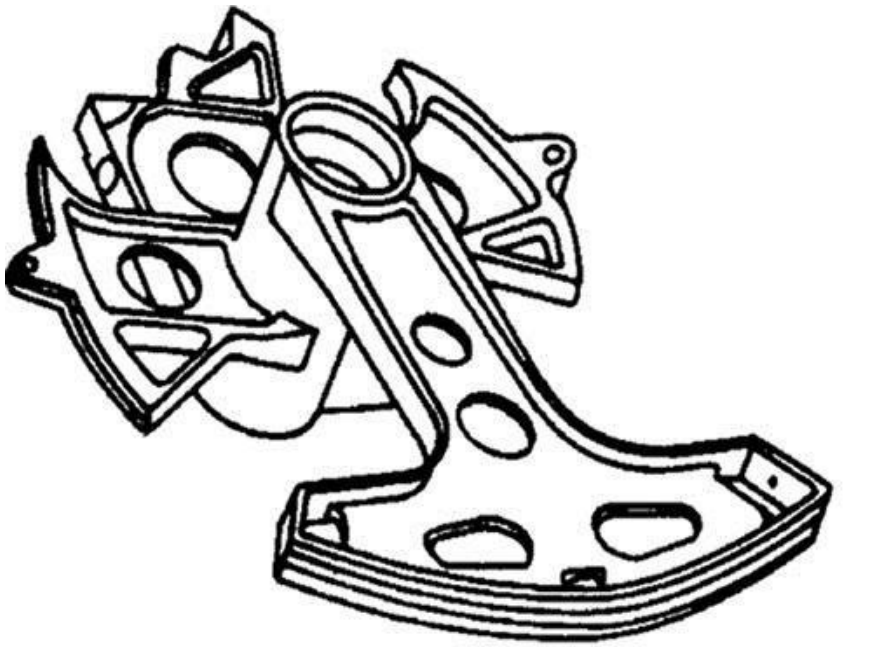


FIGURE E7.6

E7.7. *Elevator control quadrant*. The quadrant sketched here is part of the control system for the wing-elevator of a commercial aircraft. It is to be made of a light alloy (aluminium or magnesium) with the shape shown in the figure. It weighs about 5 kg. The minimum section thickness is 5 mm, and – apart from the bearing surfaces – the requirements on surface finish and precision

are not strict: surface finish ≤10 μm and precision ≤0.5 mm. The bearing surfaces require a surface finish ≤1 μm and a precision ≤0.05 mm. A production run of 100 is planned.



Elevator control quadrant

FIGURE E7.7

E7.8. *Casing for an electric plug*. The electric plug is perhaps the commonest of electrical products. It has a number of components: the casing, the pins, connectors, a cable clamp, fasteners, and, in some plugs, a fuse. The task is to investigate processes for shaping the two-part insulating casing, which, for safety reasons, is to be made of a thermosetting plastic. Each part weighs about 30 grams and is to be made in one step from with a planned batch size of 2×10^6. The required tolerance of 0.3 mm and surface roughness of 1 μm must be achieved without using secondary operations.



Electric plug

FIGURE E7.8

a. Itemize the function and constraints, leave the objective blank and enter 'Choice of process' for the free variable.
b. Use the charts of Figs. 7.3, 7.4 and 7.6 to identify processes to shape the casing

E7.9. *Ceramic valves for taps*. Few things are more irritating than a dripping tap. Taps drip because the rubber washer is worn or the brass seat is pitted by corrosion, or both. Ceramics have good wear resistance, and they have excellent corrosion resistance in both pure and salt water. Many household taps now use ceramic valves.

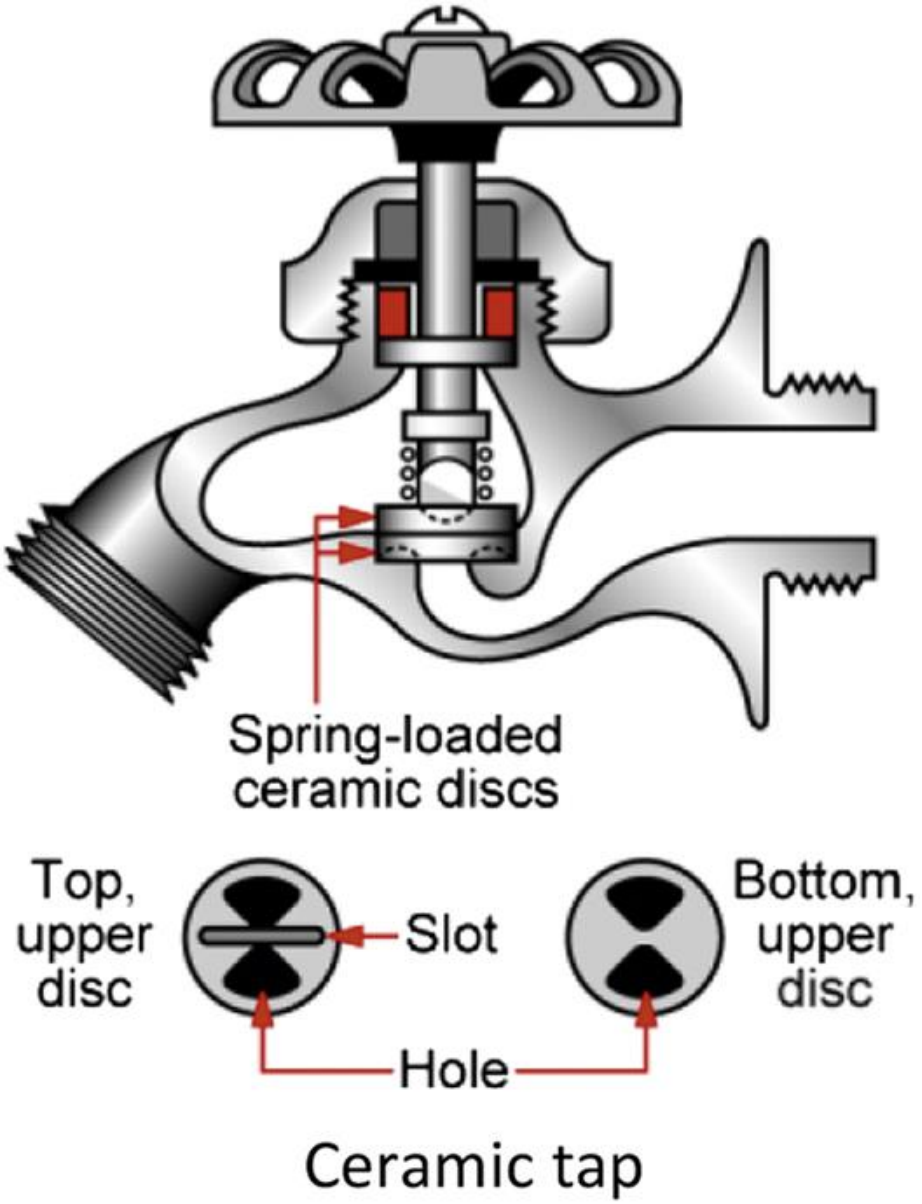


FIGURE E7.9

The sketch shows how they work. A ceramic valve consists of two disks mounted one above the other, spring-loaded so that their faces are in contact. Each disk has a diameter of 20 mm, a thickness of 3 mm and weighs about 10 grams. In order to seal well, the mating surfaces of the two disks must be flat and smooth, requiring high levels of precision and surface finish; typically tolerance <0.02 mm and surface roughness <0.1 μm. The outer face of each has a slot that registers it and allows the upper disc to be rotated through 90° (1/4 turn). In the 'off' position the holes in the upper disc are blanked off by the solid part of the lower one; in the 'on' position the holes are aligned. A production run of 10^5–10^6 is envisaged. The task is to select a process to make the ceramic disks.

a. List the function and constraints, leave the objective blank and enter 'Choice of process' for the free variable.
b. Use the charts of Figs. 7.3, 7.4 and 7.6 to identify processes to shape the casing.

E7.10. *Shaping plastic bottles*. Polyethylene bottles are used to contain fluids as various as milk and engine oil. A typical polyethylene bottle weighs about 30 grams and has a wall thickness of about 0.8 mm. The shape is 3-D hollow. The batch size is large (1,000,000 bottles). What process could be used to make them?

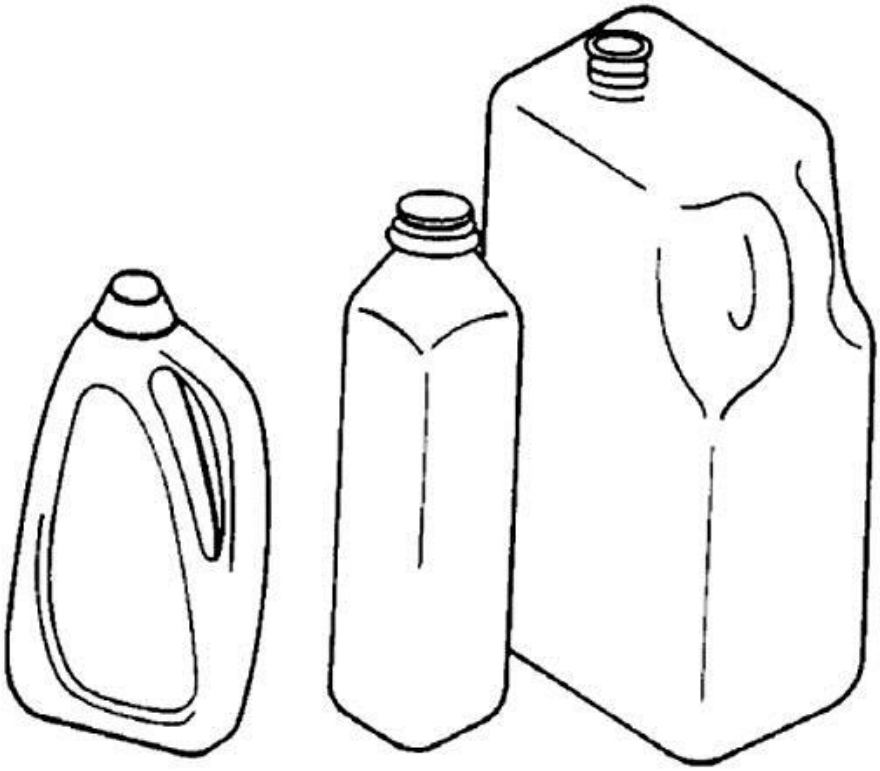


FIGURE E7.10

a. List the function and constraints, leave the objective blank and enter 'Choice of process' for the free variable.
b. Use the charts of Figs. 7.3, 7.4 and 7.6 to identify processes to shape the casing.

E7.11. *Car hood (bonnet)*. As weight-saving assumes greater importance in automobile design, the replacement of steel parts with polymer-composite substitutes becomes increasingly attractive. Weight can be saved by replacing a steel hood with one made from a thermosetting composites. The weight of the hood depends on the car model: a typical composite hood weighs is 8–10 kg. The shape is a dished-sheet and the requirements on tolerance and

roughness are 1 mm and 2 μm, respectively. A production run of 100,000 is envisaged.

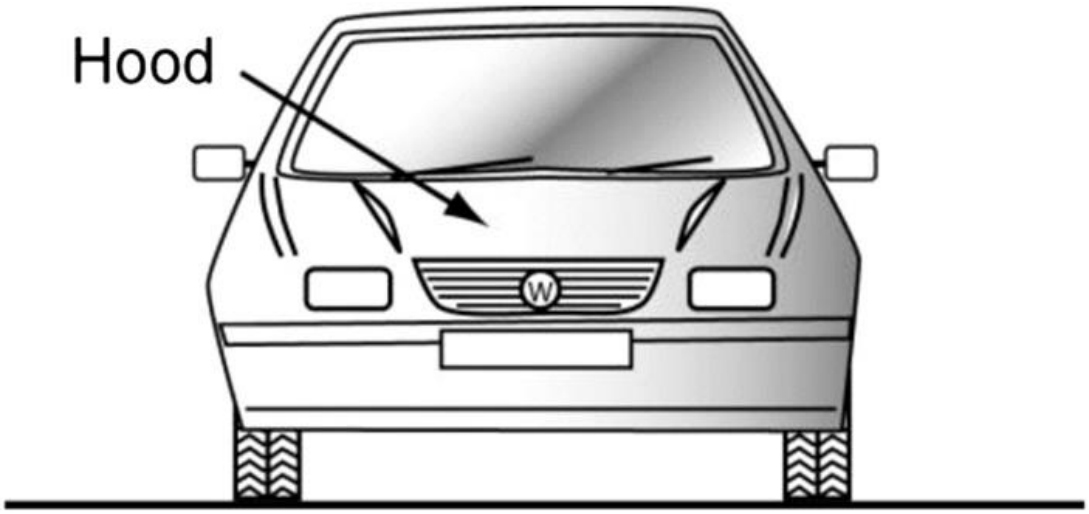


FIGURE E7.11

a. List the function and constraints, leave the objective blank and enter 'Choice of process' for the free variable.
b. Use the charts of Figs. 7.3, 7.4 and 7.6 to identify processes to shape the casing.

E7.12. *Selecting joining processes*. *This exercise and the next require the use of the CES EduPack Materials Selection software*.

a. Use CES to select a joining process to meet the following requirements.

Function	■ *Create a permanent butt joint between steel plates*	
Constraints	■ *Material class: carbon steel* ■ *Joint geometry: butt joint* ■ *Section thickness: 8 mm* ■ *Permanent* ■ *Watertight*	*Technical constraints*
Objective	–	
Free variables	■ *Choice of process*	

b. Use CES to select a joining process to meet the following requirements.

Function	■ *Create a watertight, demountable lap joint between glass and polymer*	
Constraints	■ *Material class: glass and polymers* ■ *Joint geometry: lap joint* ■ *Section thickness: 4 mm* ■ *Demountable* ■ *Watertight*	*Technical constraints*
Objective	–	
Free variables	■ *Choice of process*	

E7.13. Selecting surface-treatment processes. This exercise, like the last, requires the use of the CES EduPack Materials Selection software.

a. Use CES to select a surface-treatment process to meet the following requirements.

Function	■ *Increase the surface hardness and wear resistance of a high carbon steel component*	
Constraints	■ *Material class: carbon steel* ■ *Increase surface hardness and wear resistance*	*Technical constraints*
Objective	–	
Free variables	■ *Choice of process*	

b. Use CES to select a surface-treatment process to meet the following requirements.

Function	■ *Apply colour and pattern to the curved surface of a polymer moulding*	
Constraints	■ *Material class: thermoplastic* ■ *Purpose of treatment: aesthetics, color* ■ *Curved surface coverage: good or very good*	*Technical constraints*
Objective	–	
Free variables	■ *Choice of process*	

E7.14. *Cost*. A method is sought to mould the wide-necked containers show in the image. It suggested that they might be blow-moulded, injection moulded or rotation moulded. The table lists approximate values for the parameters that enter the cost model. Use these to assess the cheapest process choice.



FIGURE E7.14

Parameters	Blow Moudling	Injection Moudling	Rotation Moudling
Material cost, $mC_m/(1-f)$ *($)*	0.1	0.1	0.1
Basic overhead $\dot{C}_{oh}$ *($ per hour)*	100	100	100
Capital write-off time t_{wo} *(years)*	5	5	5
Load factor	0.5	0.5	0.5
Dedicated tooling cost, C_t *($)*	3000	10,000	5000
Capital cost, C_c *($)*	16,000	160,000	6800
Production rate, $\dot{n}$ *(per hour)*	270	240	8

CHAPTER 8

Multiple Constraints and Conflicting Objectives

Abstract

Commonly the selection of a material involves satisfying several constraints. Dealing with this is straightforward: apply the constraints in sequence, rejecting at each step the materials that fail to meet them. Commonly, too, it involves more than one objective, and here the conflict is more severe: the choice of materials that best meets one objective will not usually be that which best meets the others. Resolving this requires trade-off methods, combining the objectives into a single penalty function for which a minimum is sought.

Keywords

Multiple constraints; conflicting objectives; trade-off methods; penalty functions; exchange constants; weight factors; pressure vessels; connecting rods; high-field magnets

8.1 Introduction and Synopsis

Most decisions in life involve trade-offs. Sometimes the trade-off is to cope with conflicting constraints: I must pay this bill but I must also pay that one – you pay the one that is most pressing. At other times the trade-off is to balance divergent objectives: I want to be rich but I also want to be happy – and resolving this is harder because wealth is rarely measured in the same units as happiness.

So it is with selecting materials and processes. The selection must satisfy several, often conflicting, constraints. In the design of an aircraft wing-spar, weight must be minimized, with constraints on stiffness, fatigue strength, toughness and geometry. In the design of a disposable hot-drink cup, cost is what matters; it must be minimized subject to constraints on stiffness, strength and thermal conductivity, though painful experience suggests that designers sometimes neglect the last. These two examples each have one design objective (minimizing weight in the first, minimizing cost in the second) with many constraints, a situation we have already met in Chapter 4, Materials Selection – The Basics. Its solution is straightforward: apply the constraints in sequence, rejecting at each step the materials that fail to meet them. The survivors are viable candidates. Rank them by their ability to meet the single objective, and then explore documentation for the top-ranked candidates. Usually this does the job, but sometimes there is an extra twist. It is described in Section 8.2.

A second class of problem involves more than one objective, and here the conflict is more severe. Nature being what it is, the choice of materials that best meets one objective will not usually be that which best meets the others. The designer charged with selecting a material for a wing-spar that must be both light *and* cheap faces an obvious difficulty: the lightest materials are not be the cheapest, and vice versa. To make any progress, the designer needs a way of trading weight against cost, and this is a problem we have not encountered till now.

There are a number of quick though subjective ways of dealing with multiple constraints and conflicting objectives: the *method of weight-factors* and methods employing *fuzzy logic* – they are discussed in an appendix at the end of this chapter. They are a good way of getting into the problem, so to speak, but they rely heavily on personal judgment; their subjective nature must be recognized. Subjectivity is eliminated by employing the *active constraint method* to resolve multiple constraints (Section 8.2), and by combining conflicting objectives into a single *penalty function* (Section 8.3). These are standard tools of multicriteria optimization. To use them, we must adopt, in this chapter, the convention that all objectives are expressed as quantities to be *minimized*; without it the penalty-function method does not work.

So now the important stuff. Fig. 8.1 is the road map. We start on the top path and work down.

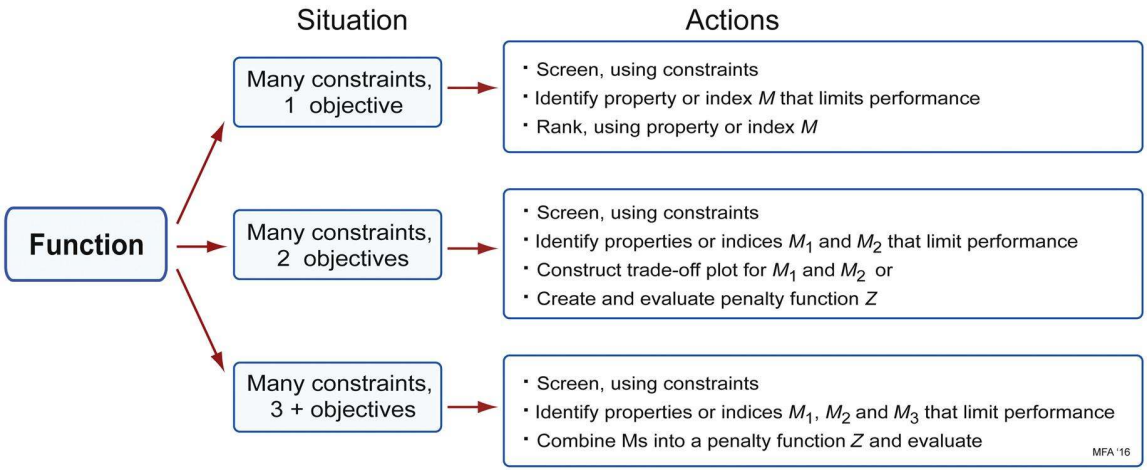


FIGURE 8.1 Strategies for tackling selection with multiple constraints and conflicting objectives.

8.2 Selection with Multiple Constraints

Nearly all material-selection problems are *over-constrained*, meaning that there are more constraints than free variables. We've already seen multiple constraints in Chapter 4, Materials Selection – The Basics and Chapter 5, Materials Selection – Case Studies. Recapitulating, they are tackled by identifying the constraints and the objective imposed by the design requirements, and applying the following steps.

- *Screen*, using each constraint in turn, rejecting all materials that fail to meet any constraint.
- *Rank*, using the performance metric describing the objective (often mass, volume or cost) or simply by the value of the material index that appears in the equation for the metric.
- *Seek documentation* for the top-ranked candidates and use this to make the final choice.

Steps 1 and 2 are illustrated in Fig. 8.2, which we think of as the central methodology. The box on the left represents screening by imposing constraints on properties, on requirements such as corrosion resistance or on the ability to be processed in a certain way. That on the right – here a bar chart for cost for the surviving candidates – indicates how the survivors with the lowest cost are found and are ranked. All very simple.

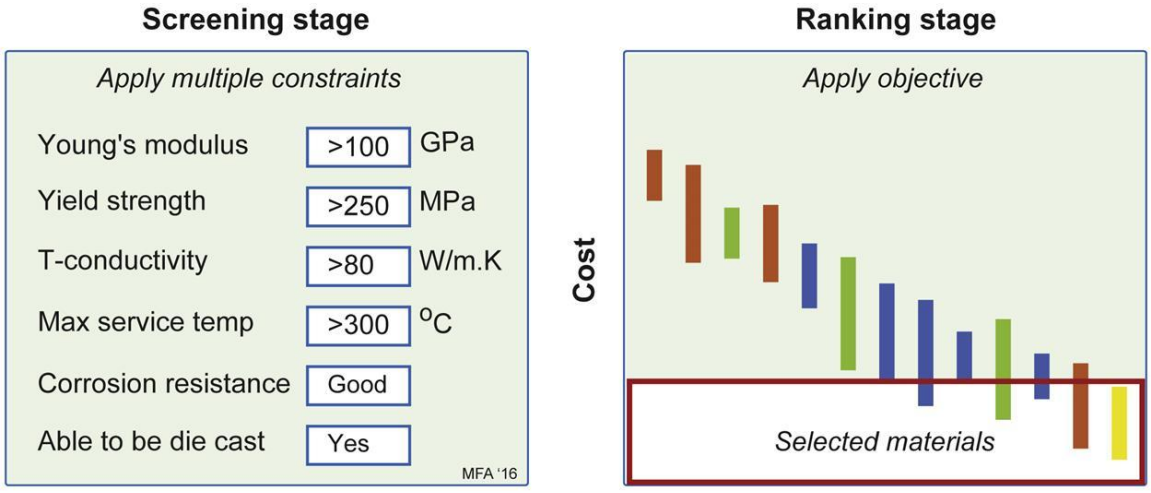


FIGURE 8.2 Selection with multiple constraints (left) and a single objective. Screen, using the constraints, rank using the objective.

But not so fast. There is one little twist. It concerns the special case of a single objective that can be limited by more than one constraint. As an example, the requirements for a tie-rod of minimum mass might specify both stiffness *and* strength, leading to two independent equations for the mass. Following exactly the steps of Chapter 4, Materials Selection – The Basics, Eq. (4.3), the situation is described by the chain of reasoning shown in Fig. 8.3.

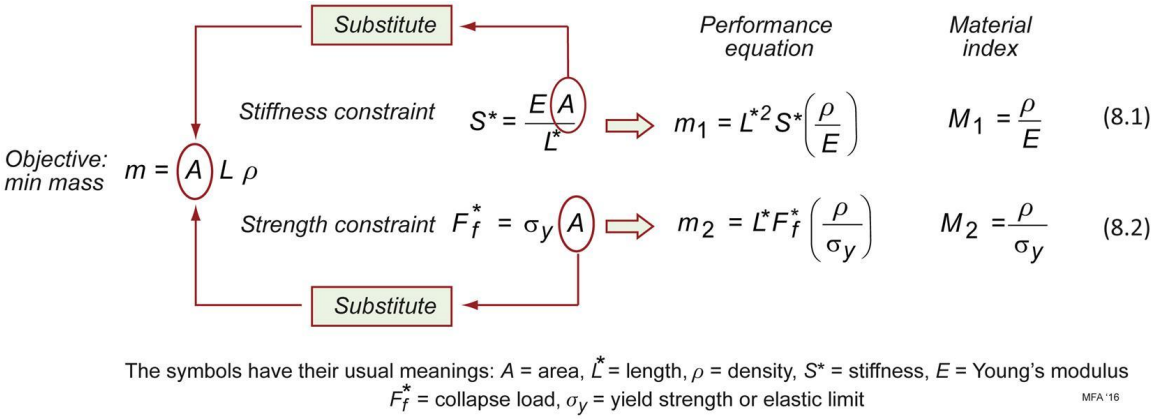


FIGURE 8.3 One objective (here, minimizing mass) with two constraints leads to two performance equations, each with its own index *M*.

If stiffness is the dominant constraint the mass of the rod is m_1; if it is strength the mass is m_2. If the tie is to meet the requirements on both, its mass has to be the greater of m_1 and m_2. Writing

$$\tilde{m} = \max(m_1, m_2) \tag{8.3}$$

We search for the material that offers the smallest value of $\tilde{m}$. This is an example of a 'min–max' problem, not uncommon in the world of optimization. We seek the smallest value (min) of a metric that is the larger (max) of two or more alternatives.

The analytical method. Powerful methods exist for solving min–max problems when the metric (here, mass) is a continuous function of the control variables (the things on the right of the two performance equations in Fig. 8.3). But here one of the control variables is the choice of material, and we are dealing with a population of materials each of which has its own unique values of material properties. The problem is discreet, not continuous.

One way to tackle it is to evaluate both m_1 and m_2 for each member of the population and assign the larger of the two to each member, and then rank the members by the assigned value, seeking a minimum. Here is an example.

Example

A material is required for a light tie of specified length L, stiffness S and collapse load F_f with the values

$$L^* = 1\text{ m} \quad S^* = 3 \times 10^7\text{ N/m} \quad F_f^* = 10^5\text{ N}$$

Answer

Substituting these values and the material properties shown in the table below into Eqs (8.1) and (8.2) of Fig. 8.3 gives the values for m_1 and m_2 shown in the table. The last column shows $\tilde{m}$ calculated from Eq. (8.1). The smallest value underscored. For these design requirements Ti-6-4 is

emphatically the best choice: it allows the lightest tie that satisfies both constraints.

Selection of a material for a light, stiff, strong tie

Material	ρ (kg/m^3)	E (GPa)	σ_y (MPa)	m_1 (kg)	m_2 (kg)	$\tilde{m}$ (kg)
1020 Steel	7850	200	320	1.12	2.45	2.45
6061 Al	2700	70	120	1.16	2.25	2.25
Ti-6-4	4400	115	950	1.15	0.46	**1.15**

If the constraints are now changed to

$$L^* = 3\ \text{m} \quad S^* = 10^8\ \text{N/m} \quad F_f^* = 3 \times 10^4\ \text{N}$$

the selection changes. Now steel is the best choice: it gives the lightest tie that satisfies all the constraints. Try it.

When there are 3000 rather than 3 materials from which to choose, simple computer codes can be used to sort and rank them. But this numerical approach lacks visual immediacy and the stimulus for creative thinking that a more graphical method allows. We describe graphical methods next.

The graphical method. Suppose, for a population of materials, that we plot m_1 against m_2 as suggested by Fig. 8.4, left. Each bubble represents a material. All the variables in both equations for m_1 and m_2 are specified except the material, so the only difference between one bubble and another is the material. We wish to minimize mass, so the best choices lie somewhere near the bottom left. But where, exactly? If stiffness is paramount and strength is unimportant the choice must surely differ from that if the opposite were true. The line $m_1 = m_2$ separates the chart into two regions. In one, $m_1 > m_2$ and constraint 1 (stiffness) is dominant. In the other, $m_2 > m_1$ and constraint 2 (strength) dominates. In region 1 our objective is to minimize m_1, since it is the larger of the two; in region 2, the opposite is true. This defines a box-shaped selection envelope (shown in black) with its corner on the $m_1 = m_2$ line. The nearer the box is pulled to the bottom left, the smaller is $\tilde{m}$. The best choice is the last material left in the box.

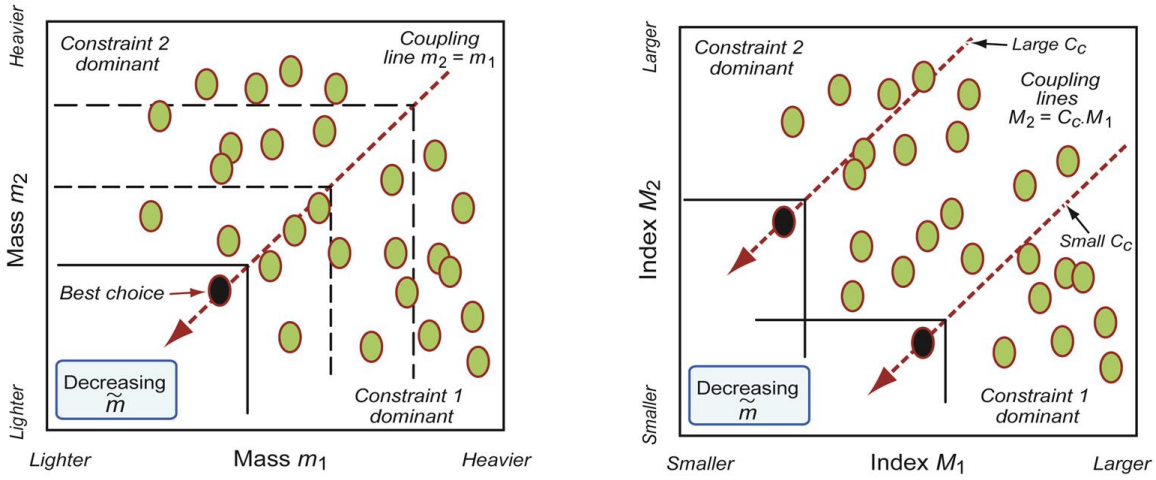


FIGURE 8.4 The graphical approach to min–max problems. Left: coupled selection using performance metrics (here, mass *m*). Right: a more general approach: coupled selection using material indices *M* and a coupling constant C_c. The axes are logarithmic.

This explains the idea, but there is a better way to implement it. Fig. 8.4, left, with m_1 and m_2 as axes, is specific to single values of L^*, S^* and F_f^*; if these change we need a new chart. Suppose, instead, that we plot the material indices $M_1 = \rho/E$ and $M_2 = \rho/\sigma_y$ that are contained in the performance equations, as shown on the right of Fig. 8.4. Each bubble still represents a material, but now its position depends only on material properties, not on the values of L^*, S^* and F_f^*. The condition $m_1 = m_2$, substituting from Eqs (8.1) and (8.2), yields the relationship

$$M_2 = \left(\frac{L^* S^*}{F_f^*}\right) M_1 \tag{8.4}$$

or, on logarithmic scales

$$Log(M_2) = Log(M_1) + \log\left(\frac{L^* S^*}{F_f^*}\right) \tag{8.5}$$

This describes a line of slope 1, in a position that depends on the value of $L^* S^*/F_f^*$. We refer to this line as the *coupling line*, and to $L^* S^*/F_f^*$ as the *coupling constant*, symbol C_c. The selection strategy remains the same: a

box, with its corner on the coupling line, is pulled down towards the bottom left. But the chart is now more general, covering all values of L^*, S^* and F_f^*. Changing either one of these, or the geometry of the component (here described by L^*) moves the coupling line and changes the selection.

Worked examples are given in Chapter 9, Multiple Constraints and Conflicting Objectives – Case Studies.

8.3 Conflicting Objectives

Real-life materials selection almost always requires that a compromise be reached between conflicting objectives. Three objectives crop up all the time. They are these.

- Minimizing *mass* – a common target in designing things that move or have to be moved, or oscillate, or must response quickly to a limited force (think of aerospace and ground transport systems).
- Minimizing *volume* – because less material is used and because space is increasingly precious (think of the drive for ever thinner mobile phones, portable computers, MP3 players… and the need to pack more and more functionality into a fixed volume).
- Minimizing *cost* – profitability depends on the difference between cost and value (more on this in Chapter 13, Hybrids: Case Studies and Chapter 16, Sustainable Response to Forces for Change); the most obvious way to increase profitability is to reduce cost.

To this we must now add a fourth objective;

- Minimizing *environmental impact* – the damage to our surroundings caused by material production, product manufacture and product use (Chapter 14, Materials and the Environment).

There are, of course, other objectives, specific to particular applications. Some are just one of the four above expressed in different words. The objective of maximizing *power-to-weight ratio* translates into minimizing mass for a given power output. Maximizing *energy storage* in a spring, battery or flywheel means minimizing the volume for a given stored energy. Some objectives can be quantified in engineering terms, such as maximizing reliability. Others cannot, such as maximizing consumer appeal – an amalgam of performance, styling, image and marketing (Chapter 15, Materials and Industrial Design).

So we have four common objectives, each characterized by a performance metric P_i. At least two are involved in the design of almost

any product. Conflict arises because the choice that optimizes one objective will not, in general, do the same for the others; then the best choice is a compromise, optimizing none, but pushing all as close to their optima as their interdependence allows. And this highlights the central problem: how is mass to be compared with cost, or volume with environmental impact? Unlike the performance equations of Fig. 8.3, each is measured in different units. We need strategies to deal with this. They come in a moment. First, some definitions.

Trade-off strategies. Consider the choice of material to minimize both cost (performance metric P_1) and mass (performance metric P_2) while also meeting a set of constraints such as a required maximum service temperature and corrosion resistance in a certain environment. Following the standard terminology of optimizations theory, we define a *solution* as a viable choice of material, meeting all the constraints but not necessarily optimal by either of the objectives. Fig. 8.5, left, is a plot of P_1 against P_2 for alternative solutions, each bubble describing one of them. The solutions that minimize P_1 do not minimize P_2, and vice versa. Some solutions, such as that at **A**, are far from optimal – all the solutions in the box attached to **A** have lower values of both P_1 and P_2. Solutions like **A** are said to be *dominated* by others. Solutions like those at **B** have the characteristic that no other solutions exist with lower values of both P_1 and P_2. These are said to be *nondominated* solutions. The line or surface on which they lie is called the nondominated or optimal *trade-off surface*. The values of P_1 and P_2 corresponding to the nondominated set of solutions are called the *Pareto set*.

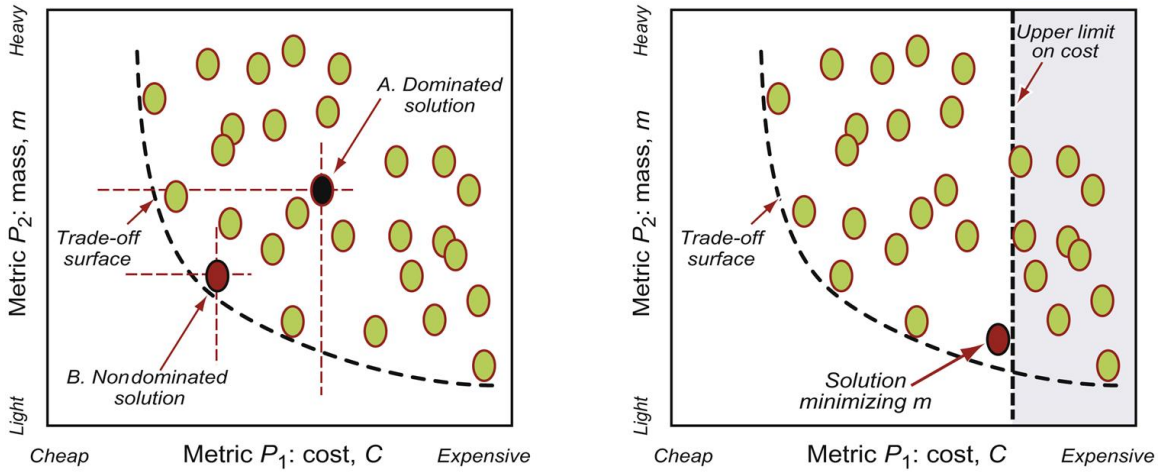


FIGURE 8.5 (Left) A trade-off plot to seek materials that cominimizes mass and cost. Each bubble is a *solution* – a material choice the meets all constraints. (Right) The trade-off plot with a simple constraint imposed on cost.

There are three strategies for progressing further. The solutions on or near the trade-off surface offer the best compromise; the rest can be rejected. Often, this is enough to identify a short-list, using intuition to rank them, for which documentation can then be sought (Strategy 1). Alternatively (Strategy 2) one objective can be reformulated as a constraint, as illustrated on the right of Fig. 8.5. Here an upper-limit is set on cost; the solution that minimizes the other constraint can then be read off. But this is cheating: it is not a true optimization. To achieve that, we need Strategy 3: that of *penalty functions*.

Penalty functions. The trade-off surface identifies the subset of solutions that offer the best compromises between the objectives. Ultimately, though, we want a single solution. One ways to do this is to aggregate the various objectives into a single compound objective formulated such that its minimum defines the most preferable solution. To do this we define a locally linear *penalty function Z*, also called a *value function* or *utility function:*

$$Z = \alpha_1 P_1 + \alpha_2 P_2 + \alpha_3 P_3 \ldots \quad \textbf{(8.6)}$$

The best choice is the material with the smallest value of Z. The α s are called *exchange constants* (or, equivalently, utility constants or scaling

constants); they convert the units of performance into those of Z, which is usually that of currency ($). The exchange constants are defined by

$$\alpha_i = \left(\frac{\partial Z}{\partial P_i}\right)_{P_j, j \neq i} \tag{8.7}$$

The method allows a local minimum to be found. The exchange constants measure the increment in penalty for a unit increment in a given performance metric, all others being held constant. If, e.g., the metric P_2 is mass m, then α_2 is the change in Z associated with unit increase in m. When the search space is large, it is necessary to recognize that the values of the exchange constants α_i may themselves depend on the values of the performance metrics P_i.

Frequently one of the objectives to be minimized is cost, C, so that $P_1 = C$. Then it makes sense to measure penalty, Z, in the same units, those of currency. With this choice, unit change in C gives unit change in Z, with the result that $\alpha_1 = 1$ and Eq. (8.6) becomes

$$Z = C + \alpha_2 P_2 + \alpha_3 P_3 \ldots \tag{8.8}$$

Consider now the earlier example in which P_1=cost C, and P_2=mass m, so that

$$Z = C + \alpha m \tag{8.9}$$

α is now the value ($) of a unit decrease in mass (kg) or equivalently the penalty ($) of a unit increase in mass (kg) and so its units are $/kg. Rearranging Eq. (8.9) gives

$$m = -\frac{1}{\alpha} C + \frac{1}{\alpha} Z \tag{8.10}$$

This α is the change in Z associated with unit increase in m. Eq. (8.10) defines a linear relationship between m and C. This plots as a family of

parallel penalty-lines each for a given value of Z, as shown in Fig. 8.6, left. The slope of the lines is the reciprocal of the exchange constant, $-1/\alpha$. The value of Z decreases towards the bottom left: the best choices lie there. The optimum solution is the one nearest the point at which a penalty line is tangential to the trade-off surface because it is the one with the smallest value of Z. Narrowing the choice to just one candidate at this stage is not sensible – we do not yet know what the search for documentation will reveal. Instead, we choose the subset of solutions that lie closest to the tangent point.

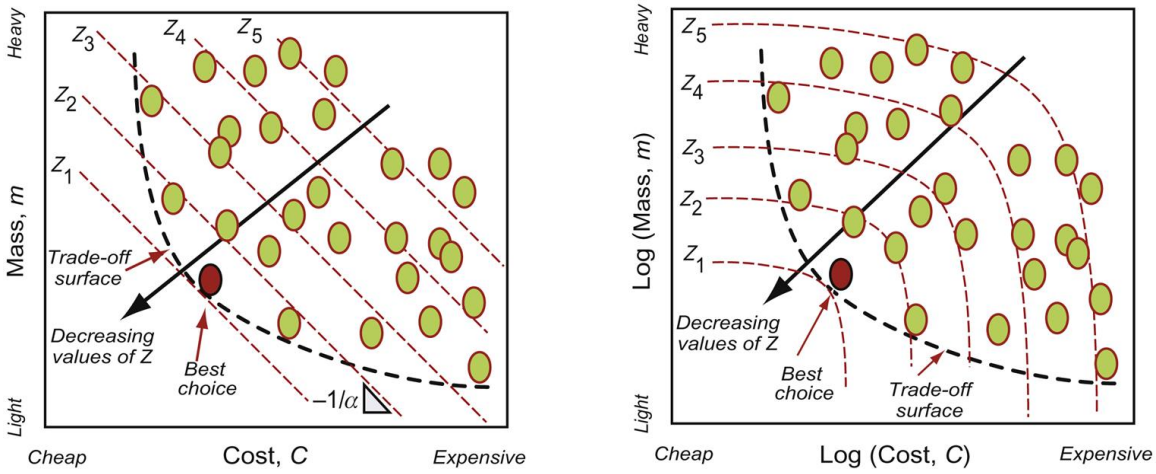


FIGURE 8.6 Left: the penalty function Z superimposed on the trade-off plot. The contour that is tangent to the trade-off surface identifies the optimum solution. Right: the same, plotted on logarithmic scales; the linear relation now plots as curved lines.

One little quirk. Almost all the material-selection charts use logarithmic scales, for very good reasons (Chapter 3, Materials Property Charts). A linear relationship, plotted on log scales, appears as a curve, as shown in Fig. 8.6, right. But the procedure remains the same: the best candidates are those nearest the point at which one of these curves just touches the trade-off surface.

Example: Using Exchange Constant

The exchange constant for weight saving in a small truck is α=\$12/kg, meaning that the value of weight reduction over the life of the vehicle is

$12 for each kilogram saved. A maker of such vehicles offers three models. The first uses steel panels for the body work. The second uses aluminium, costs $2500 more but weighs 300 kg less. The third offers carbon-fibre panelling, costs $8000 more and weighs 500 kg less. Which is the best buy?

Answer

The penalty functions for the steel (1) and aluminium (2) vehicles are

$$Z_1 = C_1 + \alpha m_1$$

and

$$Z_2 = C_2 + \alpha m_2$$

The aluminium vehicle is attractive only if its value of Z is lower than that of the steel one. Writing

$$\begin{aligned}\Delta Z = Z_2 - Z_1 &= C_2 - C_1 + \alpha(m_2 - m_1)\\ &= 2500 - 12 \times 300 = -\$1100\end{aligned}$$

The aluminium-panelled vehicle offers a life-saving of $1100 – it is a good buy. Repeating the comparison for the composite-panelled vehicle give a value of $\Delta Z = +\$2000$. It is not a good buy.

Relative penalty functions. When, as is often the case, we seek a better material for an *existing* application, it is more helpful to compare the new material choice with the existing one. To do this we define a *relative* penalty function

$$Z^* = \frac{C}{C_o} + \alpha^* \frac{m}{m_o} \qquad \textbf{(8.11)}$$

in which the subscript 'o' means properties of the existing material and the asterisk * on Z^* and α^* is a reminder that both are now dimensionless.

The relative exchange constant α^* measures the fractional gain in value for a given fractional gain in performance. Thus $\alpha^* = 1$ means that, at constant Z,

$$\frac{\Delta C}{C_o} = -\frac{\Delta m}{m_o}$$

and that halving the mass is perceived to be worth twice the cost.

Fig. 8.7 shows the relative trade-off plot, here on linear scales. The axes are C/C_o and m/m_o. The material currently used in the application appears at the coordinates (1,1). Solutions in quadrant **A** are both lighter and cheaper than the existing material, those in quadrant **B** are cheaper but heavier, those in **C** are lighter but more expensive, and those in **D** are uninteresting. Contours of Z^* can be plotted onto the figure. The contour that is tangent to the relative trade-off surface again identifies the optimum search area. As before, when logarithmic scales are used the contours of Z^* become curves. Some of the case studies of Chapter 9, Multiple Constraints and Conflicting Objectives – Case Studies make use of relative penalty functions.

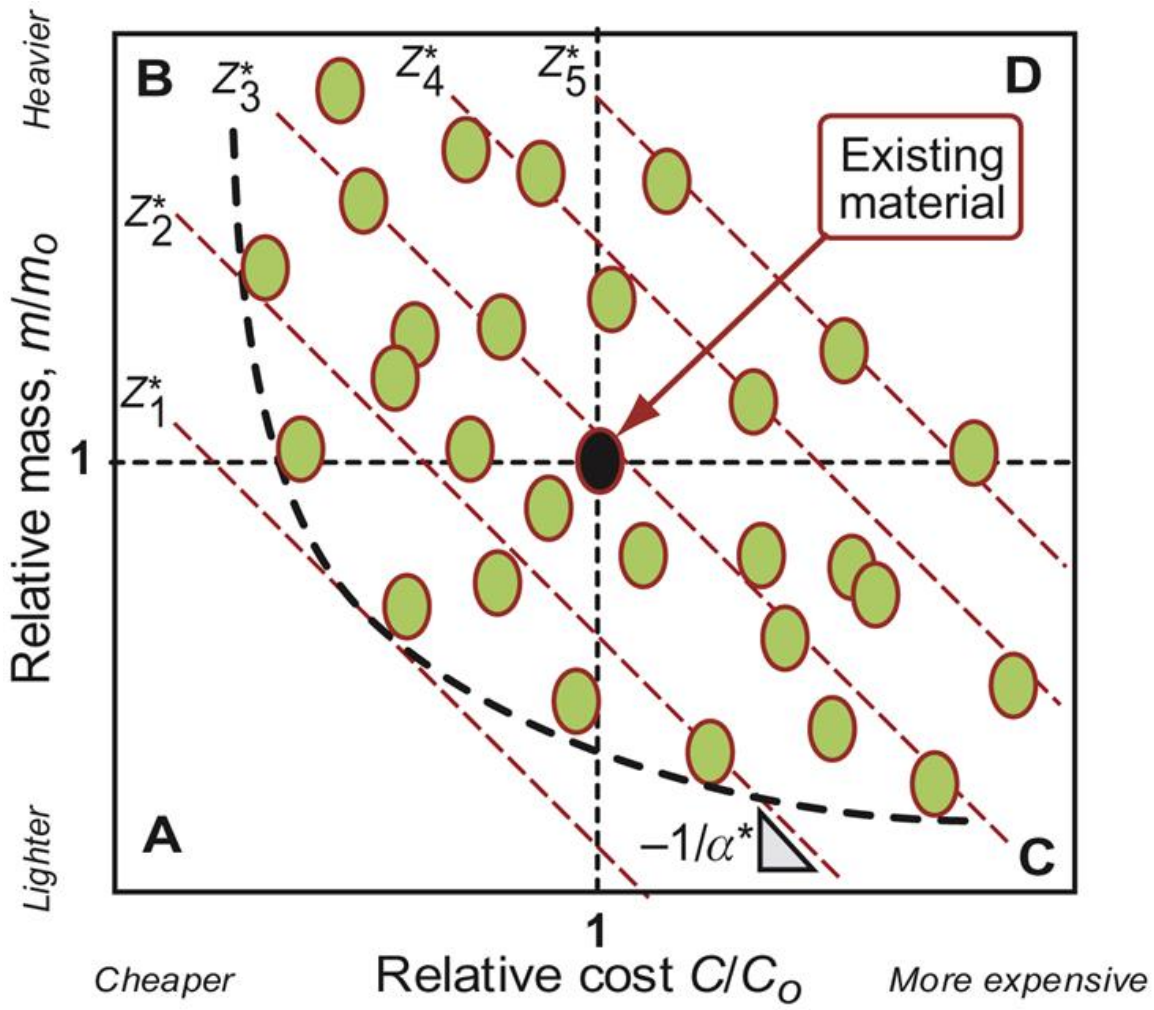


FIGURE 8.7 A relative trade-off plot, useful when exploring substitution of an existing material with the aim of reducing mass, or cost, or both. The existing material sits at the coordinates (1,1). Solutions in sector A are both lighter and cheaper.

So if values for the exchange constants are known, a completely systematic selection is possible. But that is a big 'if'. It is discussed next.

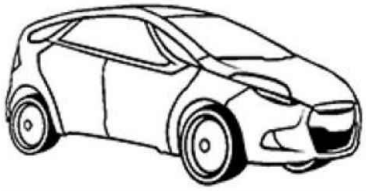
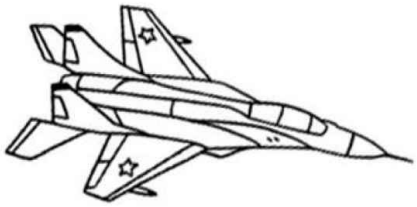
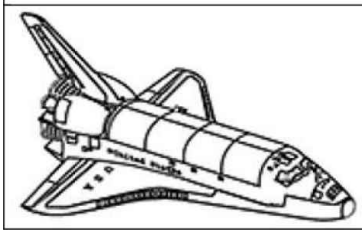
Values for the exchange constants, α. An exchange constant is a measure of the penalty of unit increase in a performance metric, or – more easily understood – it is the value or 'utility' of a unit decrease in the metric. Its magnitude and sign depend on the application. Thus the utility of weight saving in a family car is small though significant; in aerospace it is much larger. The utility of heat transfer in house insulation is directly related to the cost of the energy used to heat the house; that in a heat-exchanger for power electronics can be much higher because it increases electrical

performance. The utility can be real, meaning that it measures a true saving of cost. But it can, sometimes, be perceived, meaning that the consumer, influenced by advertising, image or fashion, will pay more or less than the true value of these metrics.

In many engineering applications the exchange constants can be derived approximately from technical models for the life-cost of a system. Thus the utility of weight saving in transport systems is derived from the value of the fuel saved or that of the increased payload, evaluated over the life of the system. Table 8.1 gives approximate values for α. The most striking thing about them is the enormous range: the exchange constant depends strongly on the application in which the material will be used. It is this that lies behind the difficulty in adopting aluminium alloys for cars despite their universal use in aircraft, the much greater use of titanium alloys in military than in civil aircraft, and the restriction of beryllium to use in space vehicles.

Table 8.1

Exchange Constants α for the Mass – Cost Trade-Off for Transport Systems

Sector: Transport Systems		Basis of Estimate	Exchange Constant, α (US$/kg)	Main Structural Materials Today
	Family car	Fuel saving	4–6	Steel Aluminium
	Truck, bus	Increased payload Fuel saving	20–40	Aluminium Steel
	Civil aircraft	Increased payload Fuel saving	200–500	Aluminium (Composite)
	Military aircraft	Performance Increased payload	500–1000	Titanium Composite
	Space vehicle	Increased payload	5000–10000	Advanced composite Ceramic

Exchange constants can be estimated approximately in various ways. The cost of launching a payload into space lies in the range $3000 to $10,000/kg; a reduction of 1 kg in the weight of the launch structure would allow a corresponding increase in payload, giving the ranges of α shown in the table. Similar arguments based on increased payload or decreased fuel consumption give the values shown for civil aircraft, commercial trucks and automobiles. The values change with time, reflecting changes in fuel costs, legislation to increase fuel economy and such like. Special circumstances can change them dramatically – an aero-engine builder who

has guaranteed a certain power/weight ratio for an engine, may be willing to pay more than $1000 to save a kilogram if it is the only way in which the guarantee can be met.

These values for the exchange constant are based on engineering criteria. More difficult to assess are those based on perceived value. That for the weight/cost trade-off for a bicycle is an example. To the enthusiast, a lighter bike is a better bike. Fig. 8.8 shows just how much cyclists value reduction in weight. It is a trade-off plot of mass and cost of bicycles, using data from bike magazines. The tangent to the trade-of line at any point gives a measure of the exchange constant: it ranges from $20/kg to $2000/kg, depending on the mass. Does it make sense for the ordinary cyclist to pay $2000 to reduce the mass of the bike by 1 kg when, by dieting, he could reduce the mass of the system (himself plus the bike) by more without spending a penny? Possibly. But mostly it is perceived value. Advertising aims to increase the perceived value of a product, increasing its value without increasing its cost. It influences the exchange constants for family cars and it is the motive behind the use of titanium for watches, carbon fibre for spectacle frames and exotic materials in much sports equipment even when cheaper materials are just as good. For these, the value of α is harder to pin down.

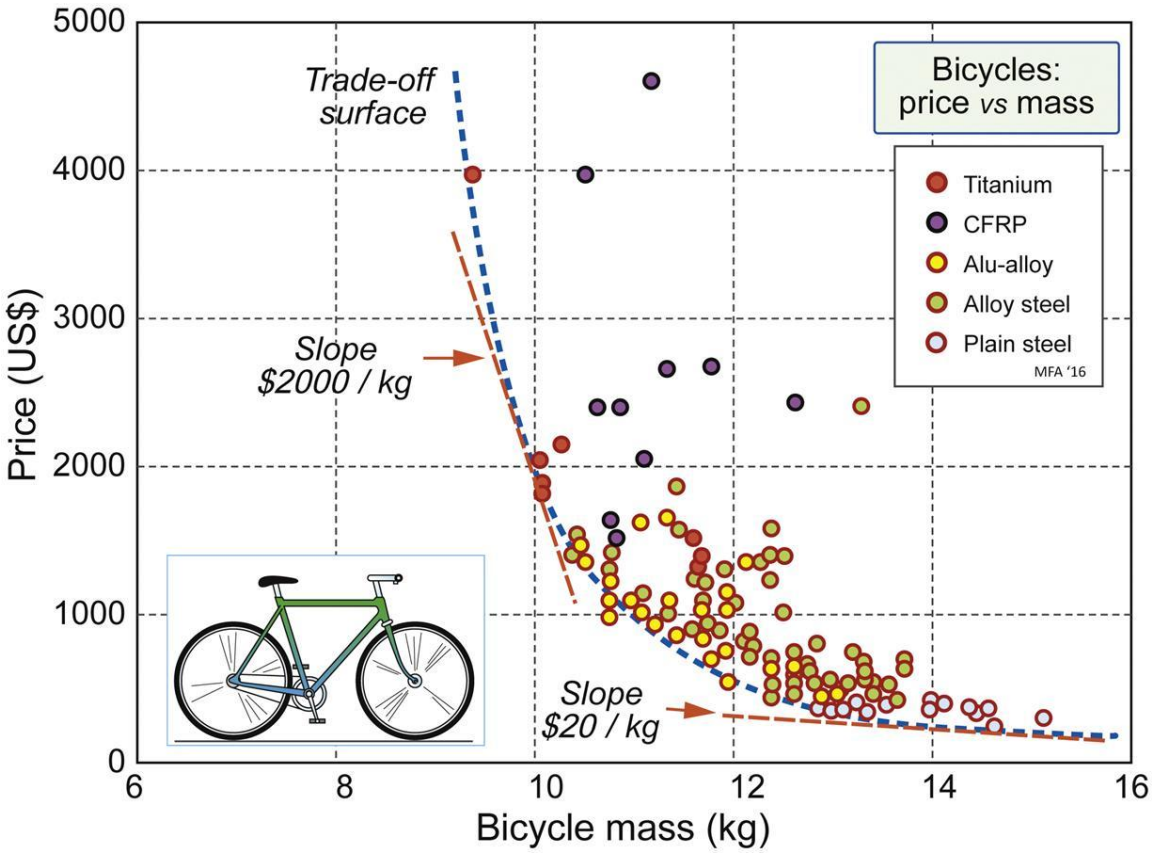


FIGURE 8.8 A cost – mass trade-off plot for bicycle frames. Solutions are colour-coded with the material. The tangent to the trade-off surface at any point gives an estimate of the exchange constant. It depends on the application: to a consumer seeking a cheap bike for shopping the value of weight saving is low ($20/kg). To an enthusiast who want performance, it can be high ($2000/kg).

There are other circumstances in which establishing the exchange constant can be difficult. What, e.g., is the value of α for environmental impact – the damage to the environment caused by manufacture, or use, or disposal of a product? Minimizing environmental impact has now become an important objective, like minimizing cost. But how much is unit decrease in eco-impact worth? Until an exchange constant is agreed or imposed, it is difficult for the designer to respond.

Things, though, are not quite as difficult as they at first appear. Useful engineering decisions can be reached even when exchange constants are imprecisely known, as explained in the next section.

Calculating Exchange Constants

In Britain a typical family car covers 150,000 km over its life and consumes on average 5.6 L of petrol per 100 km. Assume petrol in Britain currently costs £1.1 (US$1.6) per litre, so the life cost for fuel is £9240 (US$12,860). The typical car, when loaded, weighs 1400 kg. If the petrol consumption is directly proportional to the weight, what is the value of reducing the weight of the car by 1 kg?

Answer

The reduction in life fuel-cost if the weight is reduced by 1 kg is

$$\alpha = \frac{9240}{1400} = £6.6/\text{kg}\ (\$9.5/\text{kg})$$

How do exchange constants influence choice? The discreteness of the search space for material selection means that a given solution on the trade-off surface is optimal for a certain range of values of α; outside this range another solution becomes the optimal choice. The range can be large, so any value of exchange constant within the range leads to the same choice of material. This is illustrated in Fig. 8.9. For simplicity the solutions have been moved so that, in this figure, only three are potentially optimal. For $\alpha \leq 0.1$ (so that $1/\alpha \geq 10$) solution A is the optimum; for $0.1 < \alpha < 10$, solution B is the best choice; and for $\alpha \geq 10$ it is solution C. This information is captured in the bar on the right of the figure showing the range of values of α, subdivided at the points at which a change of optimum occurs and labelled with the solution that is optimal in each range.

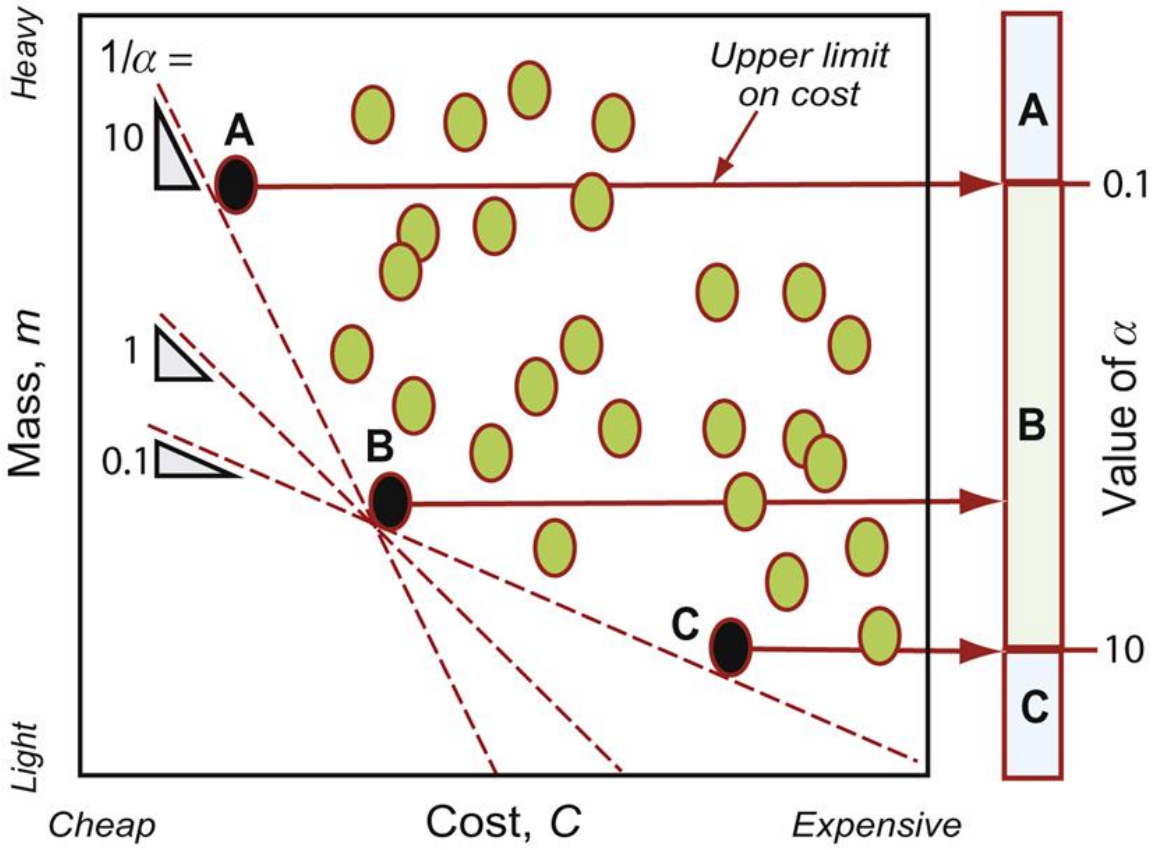


FIGURE 8.9 It is often the case that a single material (or subset of materials) is optimal over a wide range of values of the exchange constant. Then approximate values for exchange constants are sufficient to reach precise conclusions about the choice of materials.

8.4 Summary and Conclusions

The method of material indices developed in Chapter 4, Materials Selection – The Basics, allows a simple, transparent procedure for selecting materials to minimize a single objective while meeting a set of simple constraints. But things are rarely that simple – different measures of performance compete, and a compromise has to be found between them.

Judgement can be used to rank the importance of the competing constraints and objectives. Weight factors or fuzzy logic, described in the Appendix (Section 8.6), put the judgement on a more formal footing, but can also obscure its consequences. When possible, judgement should be replaced by analysis. For multiple constraints, this is done by identifying

the active constraint and basing the design on this. The procedure can be made graphical by deriving coupling equations that link the material indices, and then simple constructions on material-selection charts with the indices as axes identify unambiguously the subset of materials that maximize performance while meeting all the constraints. Compound objectives require the formulation of a penalty function, Z, containing one or more exchange constants; α_i. This allows all objectives to be expressed in the same units (usually cost) and aggregated. Minimizing Z identifies the optimum choice in a systematic and rigorous way.

When multiple constraints operate, or a compound objective is involved, the best choice of material is far from obvious. It is here that the methods developed here have real power. Chapter 9, Multiple Constraints and Conflicting Objectives – Case Studies gives examples.

8.5 Further reading

The texts below describe the history and the application of multiobjective optimization methods, with particular application to the selection of materials.

1. Ashby MF. Multi-objective optimization in material design and selection. *Acta Mater.* 2000;48:359–369 *(An exploration of the use of trade-off surfaces and utility functions for material selection.)*.
2. Bader MG. *Proc of ICCM-11, Gold Coast, Australia, Vol 1: "Composites Applications and Design"* London: ICCM; 1977; (*An example of trade-off methods applied to the choice of composite systems.)*.
3. Bourell DL. Decision matrices in materials selection. In: Dieter GE, ed. *ASM Handbook, Vol 20, "Materials Selection and Design"*. Materials Park, Ohio: ASM International; 1997;291–296. ISBN 0-87170-386-6. (*An introduction to the use of weight-factors and decision matrices*.).
4. Clark JP, Roth R, Field FR. Techno-economic issues in material science. In: Dieter GE, ed. *ASM Handbook, Vol 20, "Materials Selection and Design"*. Materials Park, Ohio: ASM International; 1997;255–265. ISBN 0-87170-386-6. (*The authors explore methods of cost and utility analysis, and of environmental issues, in materials selection*.).
5. Dieter GE. *Engineering Design, a Materials and Processing Approach* Third ed. New York, NY: McGraw-Hill; 2000;150–153 and 255-257.

ISBN 0-07-366136-8. (*A well-balanced and respected text, now in its 3rd edition, focussing on the role of materials and processing in technical design*.).
6. Field FR, de Neufville R. Material selection – maximizing overall utility. *Met Mater.* 1988;June:378–382 (*A summary of utility analysis applied to material selection in the automobile industry*.).
7. Goicoechea A, Hansen DR, Druckstein L. *Multi-Objective Decision Analysis with Engineering and Business Applications* New York, NY: Wiley; 1982; (*A good starting point for the theory of multi-objective decision-making*.).
8. Keeney RL, Raiffa H. *Decisions with Multiple Objectives: Preferences and Value Tradeoffs* Second ed. Cambridge, UK: Cambridge University Press; 1993; ISBN 0-521-43883-7. (*A notably readable introduction to methods of decision-making with multiple, competing objectives*.).
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8.6 Appendix: Weight Factors and Fuzzy Methods

Suppose you want a component with a required stiffness (constraint 1) and strength (constraint 2) and it must be as light as possible (one objective). You could choose materials with high modulus E for stiffness, and then the subset of these that have high elastic limits σ_Y for strength, and the subset of those that have low density ρ for light weight. Then again, if you wanted a material with a required stiffness (one constraint) that was simultaneously as light (objective 1) and as cheap (objective 2) as possible, you could apply the constraint and then find the subset of survivors that were light and the subset of *those* survivors that were cheap. Some selection systems work that way, but it is not a good idea because there is no guidance in deciding relative importance of the limits on stiffness, strength, weight and cost. This is not a trivial difficulty: it is exactly this relative importance that makes aluminium the prime structural material for aerospace and steel that for ground-based structures.

These problems of relative importance are old: engineers have sought methods to overcome them for at least a century. The traditional approach is that of assigning *weight-factors* to each constraint and objective and using them to guide choice in ways that are summarized below. The up-side: experienced engineers can be good at assessing relative weights. The down-side: the method relies on judgement. In assessing weights judgements can differ, and there are subtler problems, one of them discussed below. For this reason, this chapter has focussed on systematic methods. But one should know about the traditional methods because they are still widely used.

The method of weight-factors. Weight-factors seek to quantify judgement. The method works like this. The key properties or indices are identified and their values M_i are tabulated for promising candidates. Since their absolute values can differ widely and depend on the units in which they are measured, each is first scaled by dividing it by the largest index of its group, $(M_i)_{\max}$, so that the largest, after scaling, has the value 1. Each is then multiplied by a weight factor, w_i, with a value between 0 and 1, expressing its relative importance for the performance of the component. This give a weighted index W_i:

$$W_i = w_i \frac{M_i}{(M_i)_{\max}} \tag{8.12}$$

For properties that are not readily expressed as numerical values, such as weldability or wear resistance, rankings such as *A* to *E* are expressed instead by a numeric rating, *A*=5 (very good) to *E*=1 (very bad), then dividing by the highest rating value as before. For properties that are to be minimized, like corrosion rate, the scaling uses the minimum value $(M_i)_{\min}$, expressed in the form

$$W_i = w_i \frac{(M_i)_{\min}}{M_i} \tag{8.13}$$

The weight-factors w_i are chosen such that they add up to 1, that is: $w_i<1$ and $\Sigma w_i=1$. There are numerous schemes for assigning their values (see Further Reading); all require, in varying degrees, the use of judgement. The property judged to be most important is given the largest w, the second most important, the second largest and so on. The W_i's are calculated from Eqs (8.12) and (8.13) and summed. The best choice is the material with the largest value of the sum

$$W = \Sigma_i W_i \tag{8.14}$$

Sounds simple, but there are problems, some obvious (like that of subjectivity in assigning the weights), some more subtle. Here is an example.

Example

A material is sought to make a light component (low ρ) that must be strong (high σ_y). Table 8.2 gives values for four possible candidates. Weight, in our judgement, is more important than strength, so we assign it the weight factor

$$W_1 = 0.75$$

Table 8.2
Example of Use of Weight Factors

Material	ρ (Mg/m^3)	σ_y (MPa)	W (inc. Be)	W (excl. Be)
1020 Steel	7.85	320	0.27	0.34
6061 Al (T4)	2.7	120	0.55	**0.78**
Ti-6Al-4V	4.4	950	0.57	0.71
Beryllium	1.86	170	**0.79**	–

That for strength is then

$$W_2 = 0.25$$

Normalize the index values (as in Eqs 8.10 and 8.11) and sum them (Eq. 8.12) to give *W*. The second last column of the table shows the result: beryllium (bold, underscored) wins easily; Ti-6Al-4V comes second, 6061 aluminium third. But observe what happens if beryllium (which is very expensive and can be toxic) is omitted from the selection, leaving only the first three materials. The same procedure now leads to the values of *W* in the last column: 6061 aluminium (bold, underscored) wins, Ti-6Al-4V is second. Removing one, nonviable, material from the selection has reversed the ranking of those that remain. Even if the weight factors could be chosen with accuracy, this dependence of the outcome on the population from which the choice is disturbing. The method is inherently unstable, sensitive to irrelevant alternatives.

Fuzzy logic. Fuzzy logic takes weight-factors one step further. Fig. 8.10 at the upper left, shows the probability *P(R)* of a material having a property with a value *R* in a given range. Here the property has a well-defined range for each of the four materials **A, B, C** and **D** (the values are *crisp* in the terminology of the field). The selection criterion, shown at the top right, identifies the range of *R* that is sought for the properties, and it is *fuzzy*, that is to say, it has a well-defined *core* defining the ideal range sought for the property, with a wider *base*, extending the range to include boundary regions in which the value of the property is allowable, but with

decreasing acceptability as the edges of the base are approached. This defines the probability $S(R)$ of a choice being a successful one.

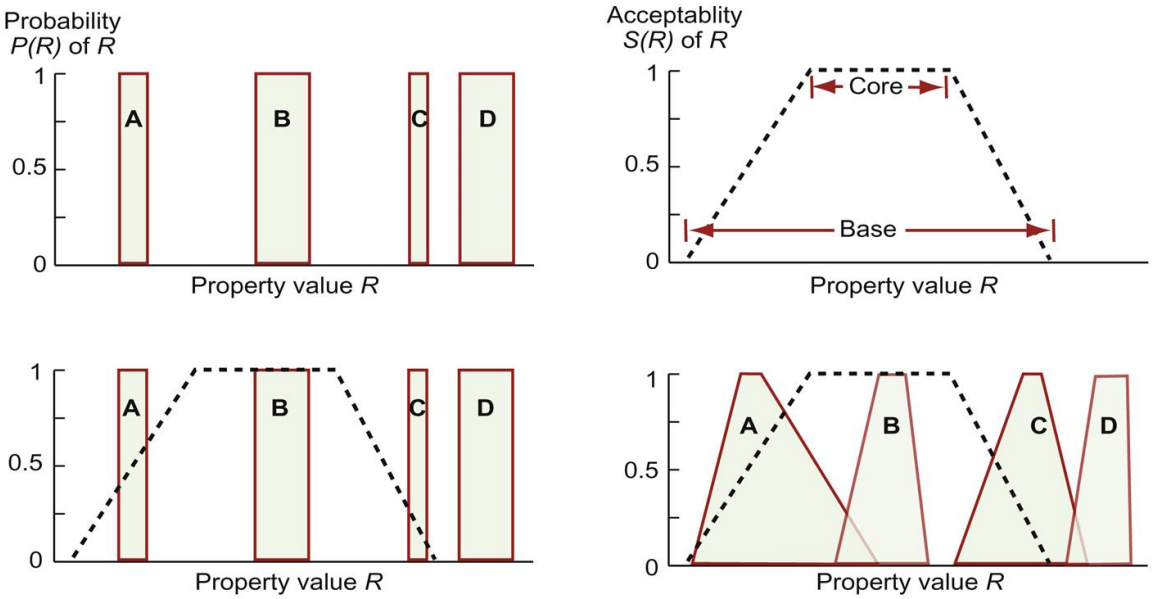


FIGURE 8.10 Fuzzy selection methods. Sharply-defined properties and a fuzzy selection criterion, shown in the top row, are combined to give weight-factors for each material, centre. The properties themselves can be given fuzzy ranges, as shown at the lower right.

The superposition of the two figures, shown at the lower left of Fig. 8.10 illustrates a single selection stage. Desirability is measured by the product $P(R)\cdot S(R)$. Here material **B** is fully acceptable – it acquires a weight of 1. Material **A** is acceptable but with a lower weight, here 0.5; **C** is acceptable with a weight of roughly 0.25, and **D** is unacceptable – it has a weight of 0. At the end of the first selection stage, each material in the database has one weight factor associated with it. The procedure is repeated for successive stages, which could include indices derived from other constraints and objectives. The weights for each material are aggregated – by multiplying them together, for instance – to give each a super-weight with a value between 0 (totally unacceptable) to 1 (fully acceptable by all criteria). The method can be refined further by giving fuzzy boundaries to the material properties or indices as well as to the selection criteria, as illustrated in the lower right of Fig. 8.10. Techniques exist to choose the positions of the

cores and the bases, but despite the sophistication the basic problem remains: the selection of the ranges S(R) is a matter of judgement.

Weight factors and fuzzy methods all have merit when more rigorous analysis is impractical. They can be a good first step. But if you really want to identify the best material for a complex design, you need the methods of Sections 8.2 and 8.3.

8.7 Exercises

Over-constrained problems are normal in materials selection. Often it is just a case of applying each constraint in turn, retaining only those solutions that meet them all. But when constraints are used to eliminate free variables in an objective function, (as discussed in Section 8.2 of the text) the 'active constraint' method must be used. The first three exercises in this section illustrate problems with multiple constraints. Those that follow concern multiple objectives and trade-off methods and penalty functions (Section 8.3 of the text).

E8.1. *Multiple constraints: a light, stiff, strong tie* (Fig. E8.1).

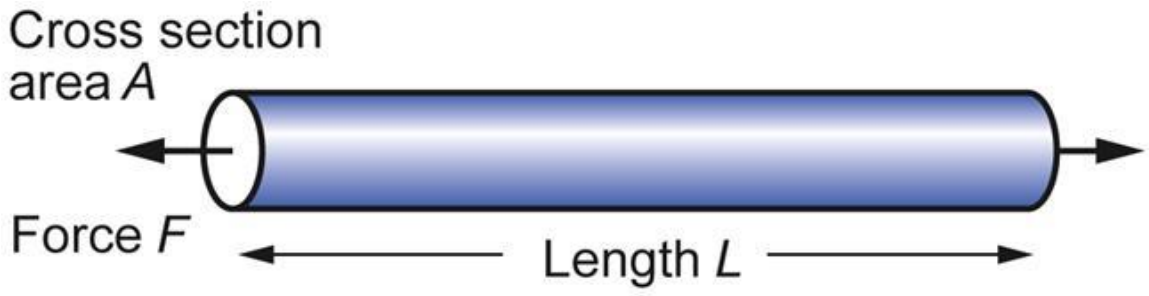


FIGURE E8.1

A tie, of length L loaded in tension, is to support a load F, at minimum weight without failing (implying a constraint on strength) or extending elastically by more than δ (implying a constraint on stiffness, F/δ). The table summarizes the requirements.

Function	■ *Tie rod*
Constraints	■ *Must not fail by yielding under force F* ■ *Must have specified stiffness, F/δ* ■ *Length L and axial load F specified*
Objective	■ *Minimize mass m*
Free variables	■ *Section area A* ■ *Choice of material*

a. Follow the method of this chapter to establish two performance equations for the mass, one for each constraint, from which two material indices and one coupling equation linking them are derived. Show that the two indices and the coupling equation are (in order):

$$M_1 = \frac{\rho}{E} \quad \text{and} \quad M_2 = \frac{\rho}{\sigma_y} \quad \text{and} \quad \left(\frac{\rho}{\sigma_y}\right) = \frac{L}{\delta}\left(\frac{\rho}{E}\right)$$

b. Use these and the material chart of Fig. E8.2, which has the indices as axes, to identify candidate materials for the tie (1) when the permitted elastic strain is $\delta/L=10^{-3}$ and (2) when $\delta/L=10^{-2}$.

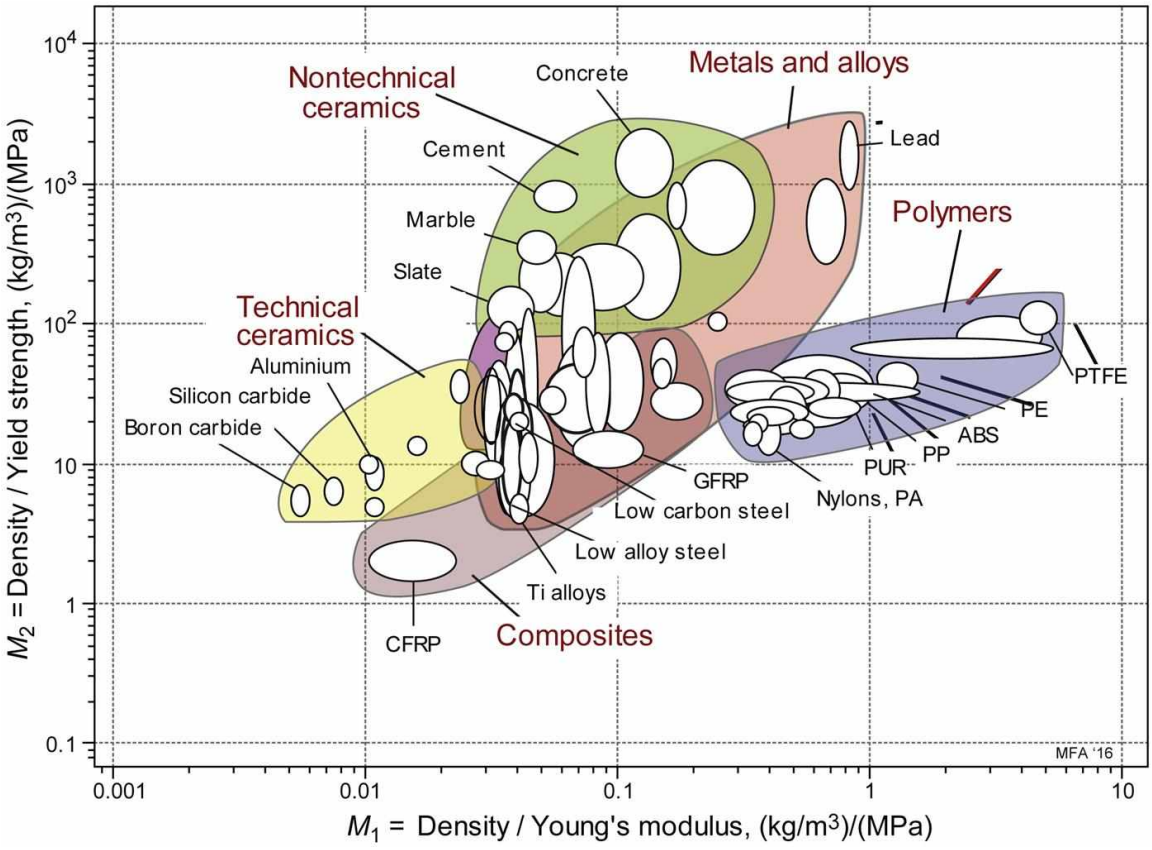


FIGURE E8.2

E8.2. *Multiple constraints: a cheap column that must not buckle or crush* (Fig. E8.3). The best choice of material for a light strong column depends on its aspect ratio: the ratio of its height H to its diameter D. This is because short, fat columns fail by crushing; tall slender columns buckle instead. Derive two performance equations for the material cost of a column of solid circular section and specified height H, designed to support a load F large compared to its self-load, the first using the constraints that the column must not crush, the second that it must not buckle. The table summarizes the needs.

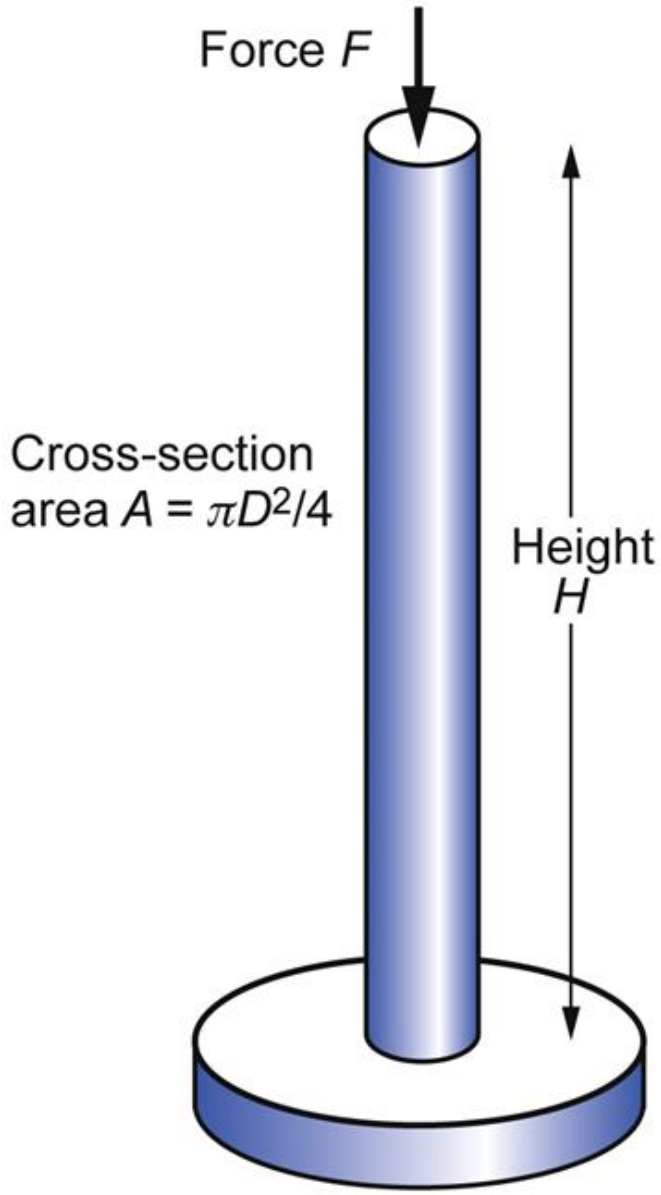


FIGURE E8.3

Function	■ *Column*
Constraints	■ *Must not fail by compressive crushing* ■ *Must not buckle* ■ *Height H and compressive load F specified.*
Objective	■ *Minimize material cost C*
Free variables	■ *Diameter D* ■ *Choice of material*

a. Proceed as follows

1. Write an expression for the material cost of the column – its mass times its cost per unit mass, C_m.
2. Express the two constraints as equations and use them to substitute for the free variable, D, to find the cost of the column that will just support the load without failing by either mechanism
3. Identify the material indices M_1 and M_2 that enter the two equations for the mass, showing that they are

$$M_1 = \left(\frac{C_m \rho}{\sigma_c}\right) \quad \text{and} \quad M_2 = \left[\frac{C_m \rho}{E^{1/2}}\right]$$

where C_m is the material cost per kg, ρ the material density, σ_c its crushing strength and E its modulus.

b. Data for six possible candidates for the column are listed in the table with this problem. Use these to identify candidate materials when F=10^5 N and H=3 m. Ceramics are admissible here, because they have high strength in compression.

Data for candidate materials for the column

Material	Density ρ (kg/m^3)	Cost/kg C_m ($/kg)	Modulus E (MPa)	Compression strength σ_c (MPa)
Wood (spruce)	700	0.5	10,000	25
Brick	2100	0.35	22,000	95
Granite	2600	0.6	20,000	150
Concrete	2300	0.08	20,000	13
Cast iron	7150	0.25	130,000	200
Structural steel	7850	0.4	210,000	300
Al-alloy 6061	2700	1.2	69,000	150

E8.3. Fig. E8.4 shows a material chart with the two indices of Exercise E8.2 as axes. Identify and plot coupling lines for selecting materials for a column with F=10^6 N and H=3 m (the same conditions as above), and for a second column with F=10^3 N and H=20 m.

E8.4. *Approximate values for exchange constants (1)*. In the US a typical family car covers 120,000 miles over its life and delivers, on average 25 miles per US gallon (6.6 miles per litre). Petrol in the US costs $0.63 per litre, so the life cost for fuel is $11,455. The typical family car, when loaded, weighs 2315 kg. If the petrol consumption is directly proportional to the weight, what is the value of reducing the weight of the car by 1 kg (Fig. E8.5)?

E8.5. *Approximate values for exchange constants (2)*. The makers of the car shown in Fig. 8.5 plan to market it in Europe. If the car covers 120,000 miles (193,000 km) over its life and consumes on average 9.5 litres per 100 km. Petrol in the Europe costs €1.3 per litre ($1.5 per litre), so the life cost for fuel is $27,500. The car, when loaded, weighs 2315 kg. If the petrol consumption is directly proportional to the weight, what is the value of reducing the weight of the car by 1 kg?

E8.6. *Approximate values for exchange constants (3)*. A Routemaster – a London bus – weighs 12 tonnes when empty, 18 tonnes with full. It does, on average, 6.6 miles (11 km) per imperial gallon of diesel, which cost £5 per imperial gallon. The bus travels an average of 57,000 km per year and has a life of 20 years. Assuming that the average load factor is 50% (meaning that the bus weighs 15 tonnes on average) and that the fuel consumption is directly proportional to the weight, what is the value of weight saving for a Routemaster? (Data for Routemaster from London Transport.) (Fig. E8.6).

E8.7. *Conflicting objectives: an air cylinder for a truck* (Fig. E8.7). Trucks rely on compressed air for braking and other power-actuated systems. The air is stored in one or a cluster of cylindrical pressure tanks like that

shown here (length L, diameter $2R$, hemispherical ends). Most are made of low carbon steel, and they are heavy. The task: to explore the potential of alternative materials for lighter air tanks, recognizing that there must be a trade-off between mass and cost – if it is too expensive, the truck owner will not want it even if it *is* lighter. The table summarizes the design requirements.

Function	■ *Air cylinder for truck*
Constraints	■ *Must not fail by yielding* ■ *Diameter 2R and length L specified so the ratio Q=2R/L is fixed.*
Objectives	■ *Minimize mass m* ■ *Minimize material cost C*
Free variables	■ *Wall thickness, t* ■ *Choice of material*

Show that the mass and material cost of the tank described in relative to one made of low carbon steel are given by

$$\frac{m}{m_o} = \left(\frac{\rho}{\sigma_y}\right)\left(\frac{\sigma_{y,o}}{\rho_o}\right) \quad \text{and} \quad \frac{C}{C_o} = \left(\frac{C_m \rho}{\sigma_y}\right)\left(\frac{\sigma_{y,o}}{C_{m,o}\rho_o}\right)$$

where ρ is the density, σ_y the yield strength and C_m the cost per kg of the material, and the subscript 'o' indicates values for mild steel.

E8.8. *Conflicting objectives: an air cylinder for a truck – material choice*. Explore the trade-off between relative cost and relative mass for the air cylinder of Exercise E8.7, considering the replacement of a mild steel tank with one made, first, of low alloy steel, and, second, one made of filament-wound CFRP. The relevant material properties in the table below. Define a relative penalty function

$$Z^* = \alpha^* \frac{m}{m_o} + \frac{C}{C_o}$$

where α^* is a relative exchange constant, and evaluate Z^* for α^*=1 and for α^*=100.

Material	Density ρ (kg/m^3)	Yield strength σ_c (MPa)	Price per/kg C_m ($/kg)
Mild steel	7850	314	0.55
Low alloy steel	7850	775	0.85
CFRP	1550	760	42.1

E8.9. *Conflicting objectives: an air cylinder for a truck – the graphical method*. Fig. E8.8, below, is a chart with axes of m/m_o and C/C_o derived in Exercise E8.7 Mild steel (here labelled 'Low carbon steel') lies at the coordinates (1,1). Sketch a trade-off surface – remember that it is simply a lower envelope to the data, with no mathematical function associated with it. Then plot contours of Z^* (see Exercise E8.8) that are approximately tangent to the trade-off surface for α^*=1 and for α^*=100. They are plots of the linear relationship

$$\alpha^* \frac{m}{m_o} = -\frac{1}{\alpha^*}\frac{C}{C_o}$$

but appear as curves because of the log scales. What selections do these suggest?

E8.10. *Conflicting objectives: low-cost insulating walls for freezers and refrigerator trucks* (Fig. E8.9). Freezers and refrigerated trucks have panel-walls that provide thermal insulation, and at the same time are stiff, strong and light (stiffness to suppress vibration, strength to tolerate rough usage). To achieve this, the panels are usually of sandwich construction, with two skins of steel, aluminium or GFRP (providing the strength) separated by, and bonded to, a low density insulating core. In choosing the core we seek to minimize thermal conductivity, λ, and at the same time to maximize stiffness because this allows thinner steel faces, and thus a lighter panel, while still maintaining the overall panel stiffness. The table summarizes the design requirements.

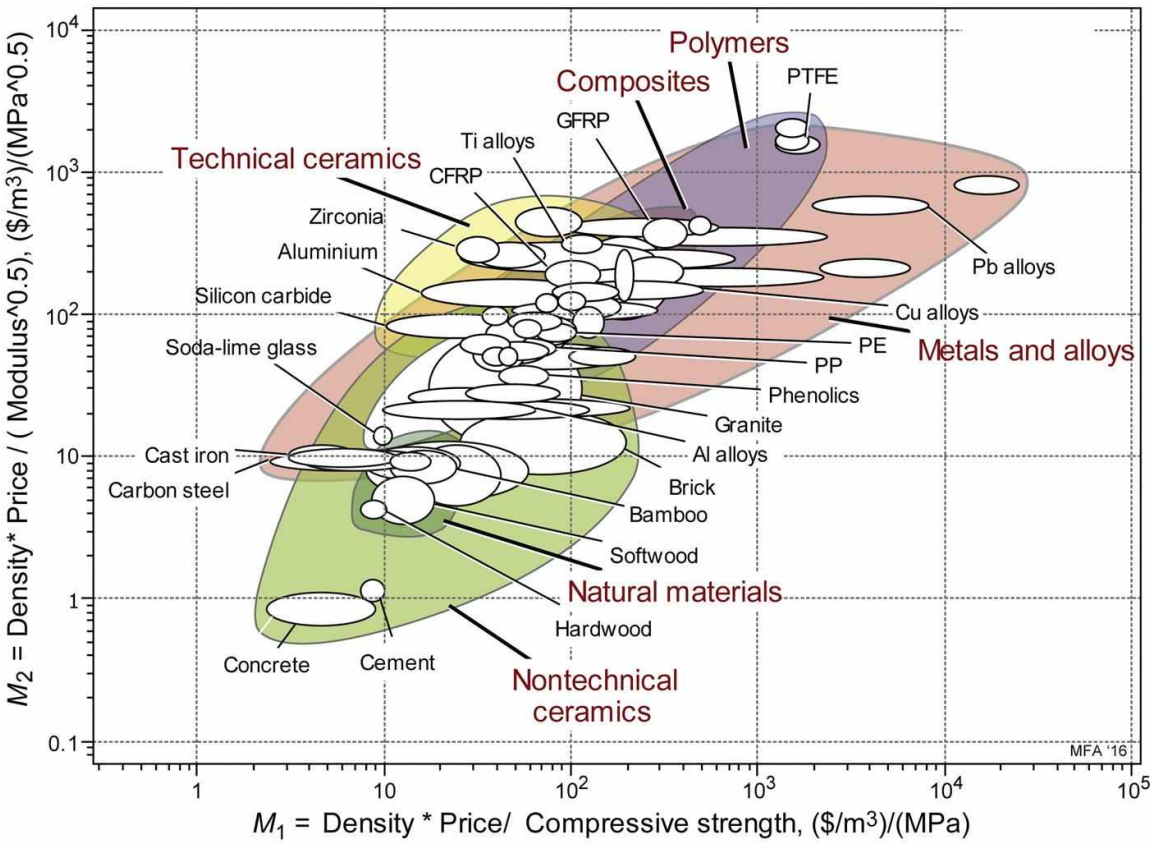


FIGURE E8.4



FIGURE E8.5 A family car. From:

https://en.wikipedia.org/wiki/Car_classification#/media/File:2010_Jaguar_XJ_(X351)_5.0_L_sedan_(2016-01-03)_01.jpg



FIGURE E8.6 A Routemaster bus. From:
https://en.wikipedia.org/wiki/Double-decker_bus#/media/File:Routemaster.JPG

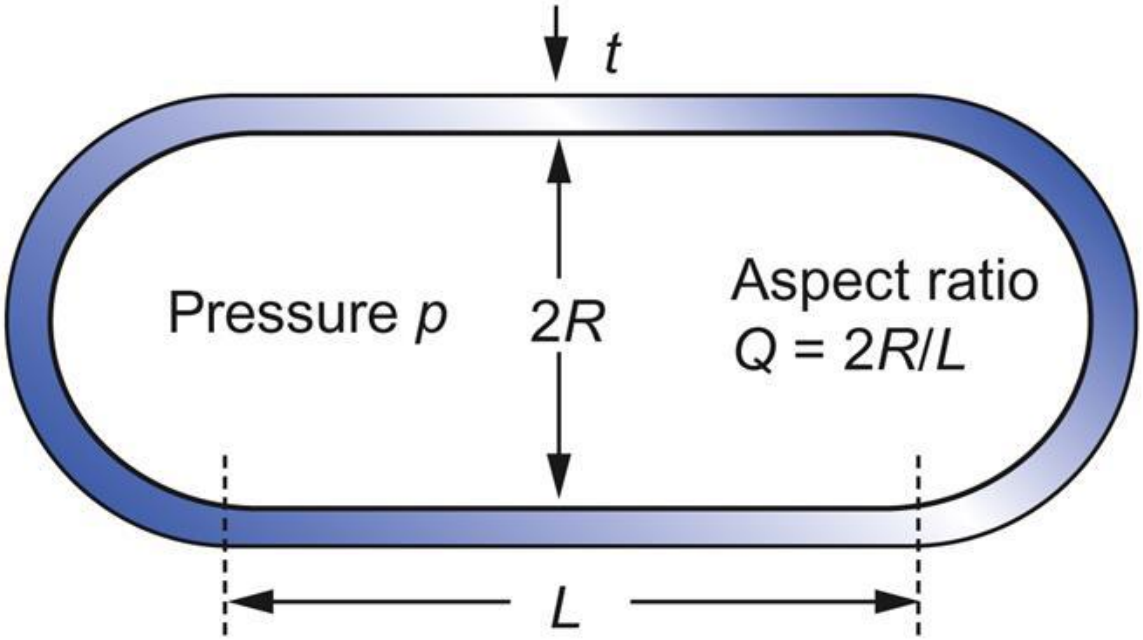


FIGURE E8.7

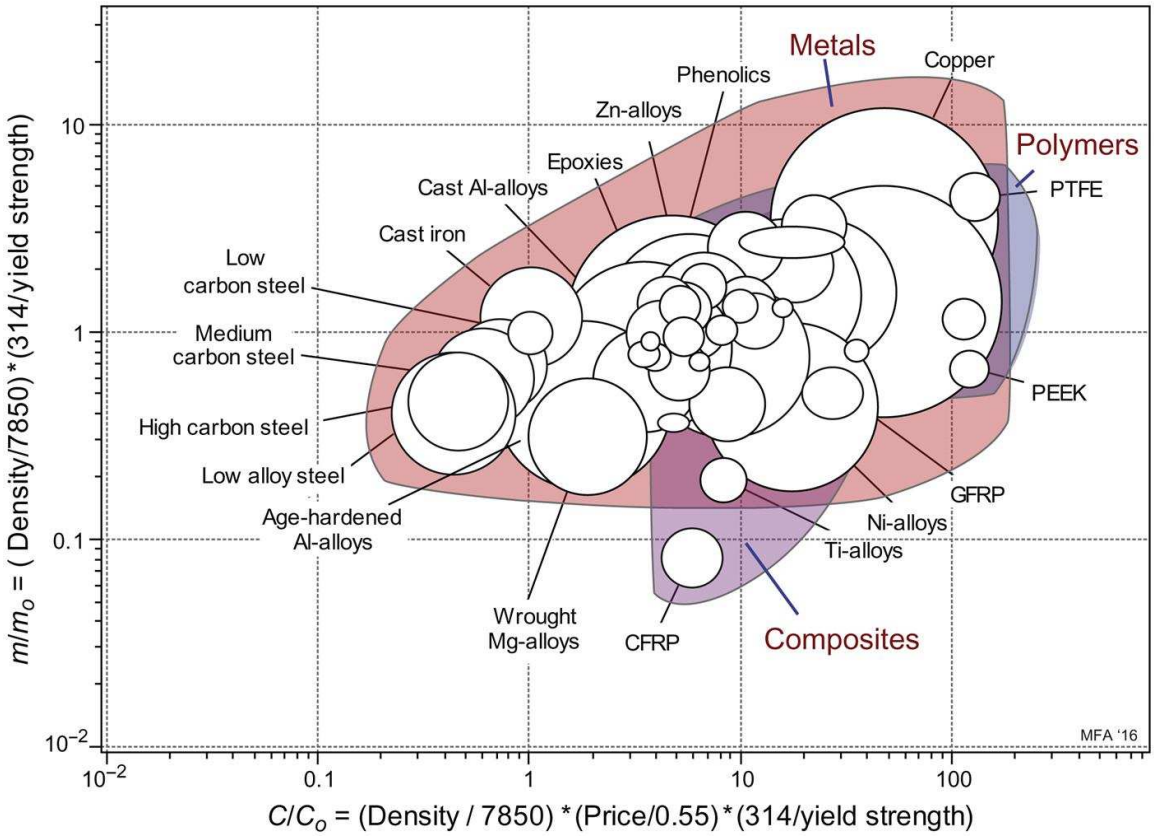

Metals
Copper
Phenolics
Zn-alloys
Epoxies
Cast Al-alloys
Cast iron
Low carbon steel
Medium carbon steel
High carbon steel
Low alloy steel
Age-hardened Al-alloys
Wrought Mg-alloys
CFRP
Polymers
PTFE
PEEK
GFRP
Ni-alloys
Ti-alloys
Composites
MFA '16
10
1
0.1
10^{-2}
10^{-2}
0.1
1
10
100
m/m_o = (Density/7850) * (314/yield strength)
C/C_o = (Density / 7850) * (Price/0.55) * (314/yield strength)

FIGURE E8.8



FIGURE E8.9 A refrigerator truck. From:
https://en.wikipedia.org/wiki/Truck#/media/File:Isuzuelf6.jpg

Function	■ *Foam for panel-wall insulation*
Constraint	■ *Panel wall thickness specified.*
Objectives	■ *Minimize foam thermal conductivity,* λ ■ *Maximize foam stiffness, meaning Young's modulus, E*
Free variables	■ *Choice of material*

Fig. E8.10 shows the thermal conductivity λ of foams plotted against their elastic compliance $1/E$ (the reciprocal of their Young's moduli E, since we must express the objectives in a form that requires minimization). The numbers in brackets are the densities of the foams in Mg/m^3. The foams with the lowest thermal conductivity are the least stiff; the stiffest have the highest conductivity. Explain the reasoning you would use to select a foam for the truck panel using a penalty function.

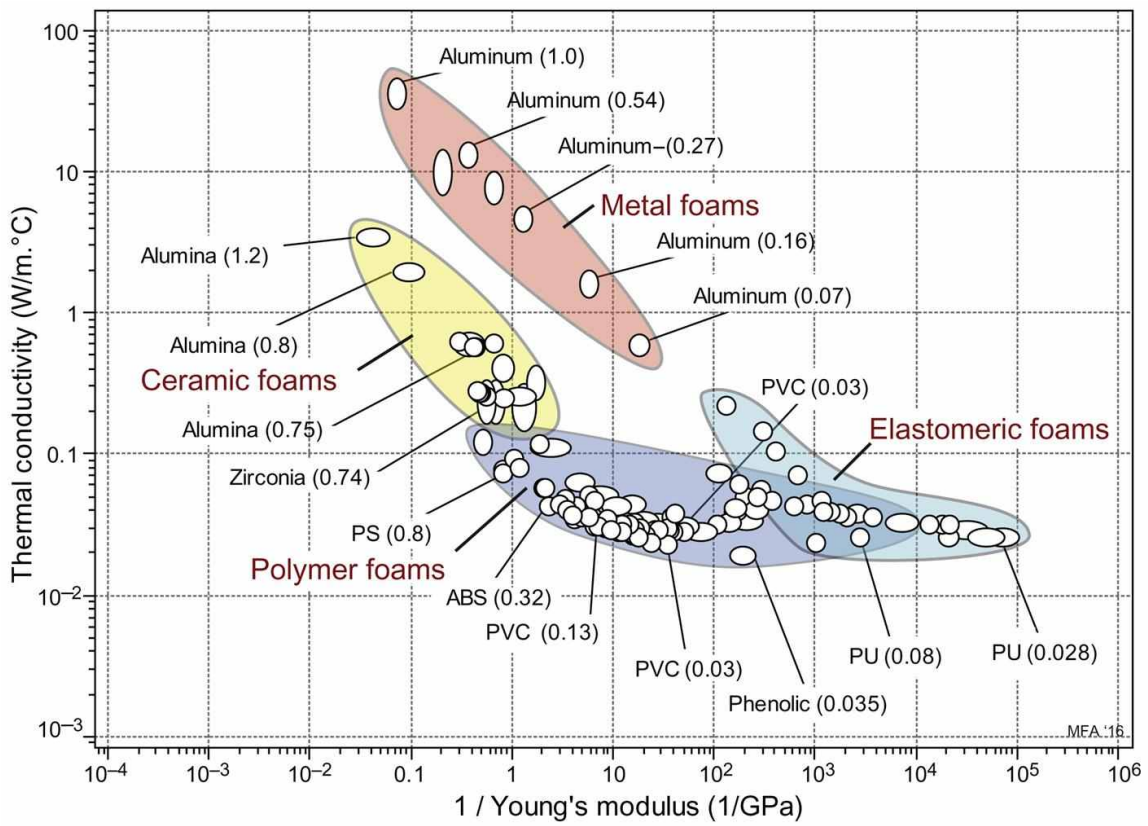

Thermal conductivity (W/m.°C)
1 / Young's modulus (1/GPa)
Aluminum (1.0)
Aluminum (0.54)
Aluminum–(0.27)
Metal foams
Aluminum (0.16)
Aluminum (0.07)
Alumina (1.2)
Alumina (0.8)
Ceramic foams
Alumina (0.75)
Zirconia (0.74)
PVC (0.03)
Elastomeric foams
PS (0.8)
Polymer foams
ABS (0.32)
PVC (0.13)
PVC (0.03)
Phenolic (0.035)
PU (0.08)
PU (0.028)
MFA 16

FIGURE E8.10

CHAPTER 9

Multiple Constraints and Conflicting Objectives – Case Studies

Abstract

Commonly the selection of a material involves several constraints. Dealing with this is straightforward: apply the constraints in sequence and reject at each step the materials that fail to meet them. Commonly, too, it involves more than one objective, and here the conflict is more severe: the choice of materials that best meets one objective will not usually be that which best meets the others. Resolving this requires trade-off methods, and combining the objectives into a single penalty function for which a minimum is sought.

Keywords

Multiple constraints; conflicting objectives; trade-off methods; penalty functions; exchange constants; weight factors; pressure vessels; connecting rods; high-field magnets

9.1 Introduction and Synopsis

These case studies illustrate how the techniques described in the last chapter work. The charts now become more complicated: they have combinations of properties as axes and some of the indices are those

derived in earlier chapters, some are new. All are made with the same CES computer-based system (see www.Grantadesign.com) that was used to make all the other charts in this book. However, the charts are deliberately simplified to avoid clouding the illustration with unnecessary detail. This simplification is rarely as limiting as it may at first appear: the choice of material is determined primarily by the physical principles of the problem, not by details of geometry. The principles remain the same when much of the detail is removed so that the selection is largely independent of these. The methods developed in Chapter 8, Multiple Constraints and Conflicting Objectives are so widely useful that they appear in the case studies of later chapters as well as this one. A reference is made to related case studies at the end of each section.

We start with three examples of coupled constraints using the methods of Section 8.2. We then explore four examples of conflicting objectives with the methods of Section 8.3.

9.2 Multiple Constraints: Light Pressure Vessels

When a pressure vessel has to be mobile, its weight becomes important. Aircraft bodies, rocket casings and liquid-natural gas containers are examples; they must be light and at the same time they must be safe. What is the best material to make a light pressure vessel? It must contain a pressure p without failing by yield or by fast fracture. In addition, it must be as light as possible (Table 9.1). This is a 'min-max' problem of the sort described in Section 8.2. Pressure vessels were the subject of Section 5.11. We draw on some of the results developed there to tackle this new problem (Fig. 9.1).

Table 9.1

Requirements for a Light Pressure Vessel

Function	■ *Pressure vessel*
Constraints	■ *Diameter 2R and pressure difference* Δp *specified* ■ *Must not fail by yielding* ■ *Must not fail by fast fracture*
Objective	■ *Minimize mass m*
Free variables	■ *Wall thickness, t* ■ *Choice of material*

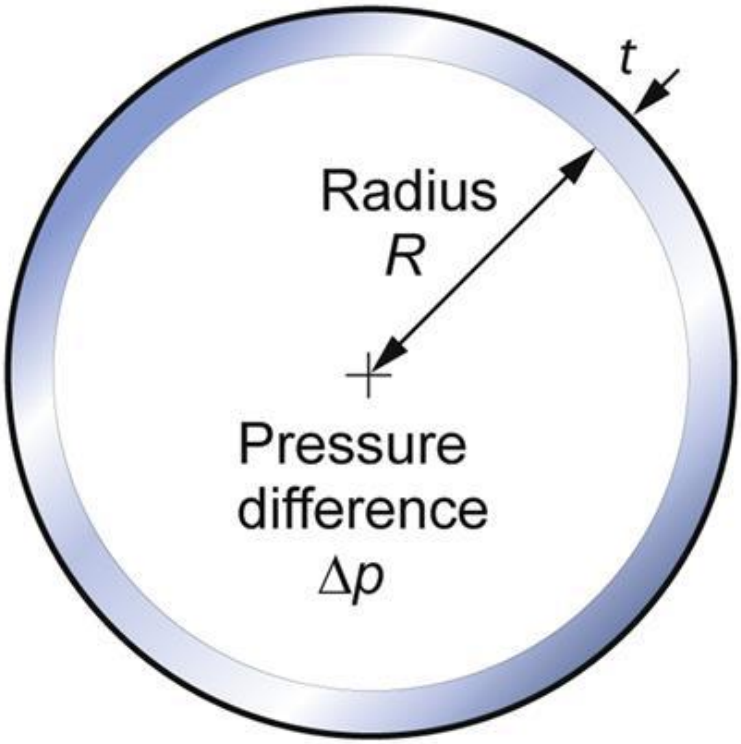


FIGURE 9.1 A spherical pressure vessel.

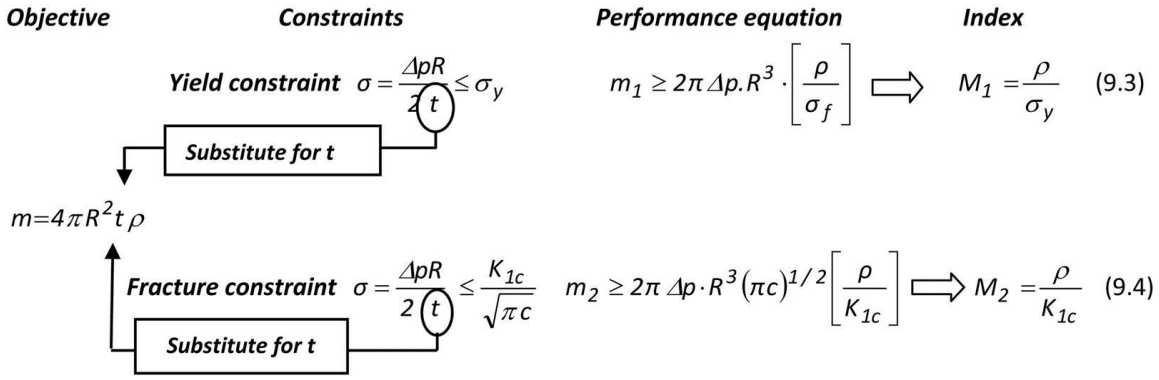
The translation. Assume, for simplicity, that the pressure vessel is spherical, of specified radius R, and that the wall thickness, t (the free variable) is small compared with R. Then the tensile stress in the wall is

$$\sigma = \frac{\Delta pR}{2t} \tag{9.1}$$

The objective function is the mass of the pressure vessel:

$$m = 4\pi R^2 t\rho \tag{9.2}$$

The first constraint is that the vessel should not yield – that is, that the tensile stress in the wall should not exceed σ_y. The second is that it should not fail by fast fracture; this requires that the stress in the wall should be less than $K_{1c}/\sqrt{\pi c}$, where K_{1c} is the fracture toughness of the material of which the pressure vessel is made and c is the length of the longest crack that the wall might contain, typically the detection limit of the nondestructive test used to monitor the pressure vessel. Using each constraint in turn to substitute for the free variable t in the objective function leads to the performance equations and indices laid out below.



The coupling equation is found by equating m_1 to m_2, giving a relationship between M_1 and M_2:

$$M_1 = (\pi c)^{1/2} M_2 \tag{9.5}$$

The position of the coupling line depends on the detection limit, c_1 for cracks, through the term $(\pi c)^{1/2}$.

Fig. 9.2 shows the appropriate chart with two coupling lines, one for c=10 mm and the other for c=10 μm. The resulting selection is summarized

in Table 9.2.

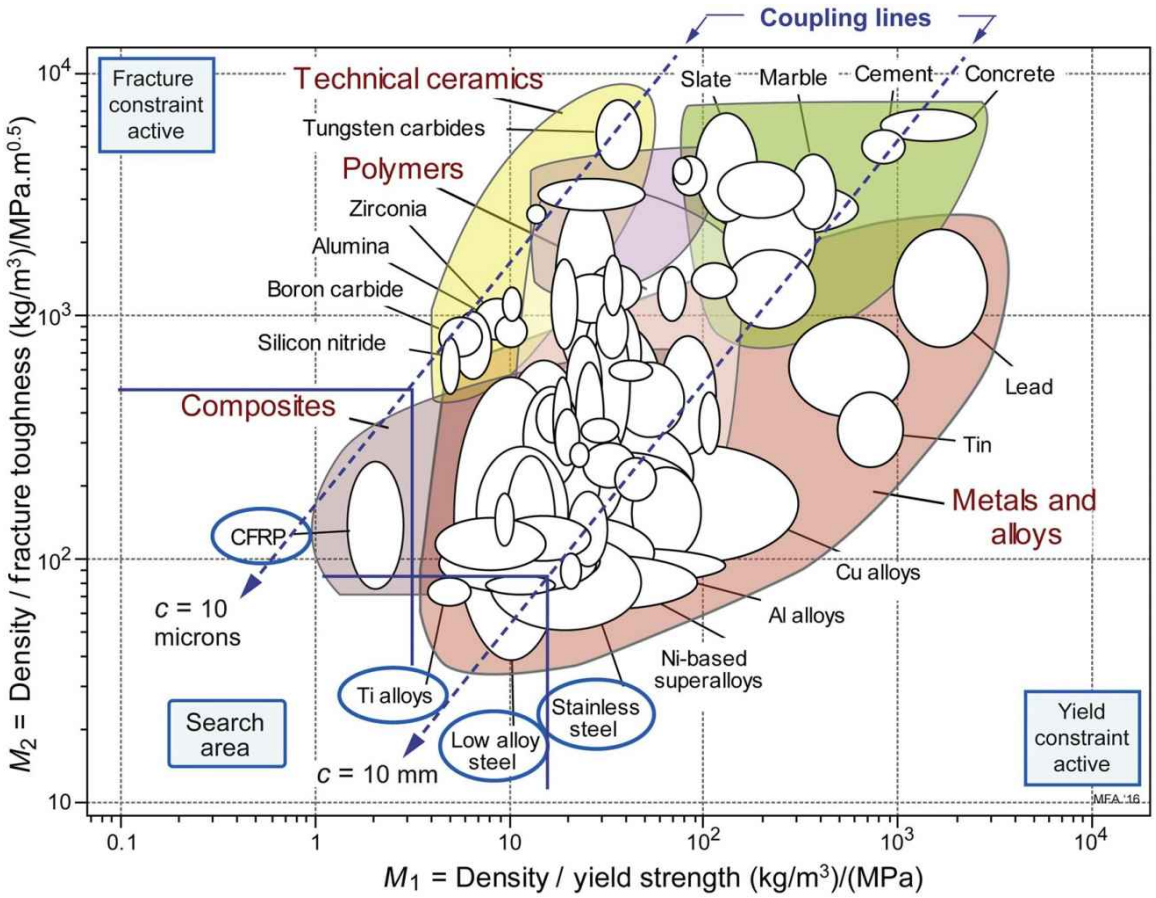


FIGURE 9.2 Over-constrained design leads to two or more performance indices linked by a coupling equation. The diagonal lines show the coupling equation for two values of crack length, c.

Table 9.2
The Selection

Coupling Condition	Material Choice	Comment
Crack length $c \leq 10$ mm ($\sqrt{\pi c} = 0.18$)	Titanium alloys Aluminium alloys Stainless steel	These are the standard materials for pressure vessels. Steels appear, despite their high density, because their toughness and strength are so high
Crack length $c \leq 10$ μm ($\sqrt{\pi c} = 0.006$)	CFRP Silicon carbide Silicon nitride Alumina	Ceramics, potentially, are attractive structural materials, but the difficult of fabricating and maintaining them with no flaws greater than 5 μm is enormous

In large engineering structures it is difficult to ensure that there are no cracks of length greater than 1 mm; therefore, the tough engineering alloys based on steel, aluminium and titanium are the safe choice. In the field of MEMS (micro electro-mechanical systems), in which films of micron-thickness are deposited on substrates, etched to shapes and then loaded in various ways, it is possible – even with brittle ceramics – to make components with no flaws greater than 1 μm in size. In this regime, ceramics become viable candidates.

Related Case Studies

9.3 'Con-rods for high-performance engines'
9.4 'Multiple constraints: windings for high-field magnets'
11.3 'Forks for racing bicycles'
11.5 'Table legs yet again: thin or light?'

9.3 Multiple Constraints: Con-Rods for High-Performance Engines

The connecting rod of a high-performance engine, compressor or pump is a critical component: if it fails, catastrophe follows. Yet to minimize inertial forces and bearing loads it must weigh as little as possible, implying the use of light, strong materials, stressed near their limits. When minimizing cost is the objective, con-rods are frequently made of cast iron because it is so cheap. But what are the best materials for con-rods when the objective is to maximize performance? Table 9.3 summarizes the design requirements for a connecting rod of minimum weight with two constraints: that it must carry a peak load F without failing either by fatigue or by buckling elastically. For simplicity, we assume that the shaft has a rectangular section $A=bw$ (Fig. 9.3).

Table 9.3

The Design Requirements: Connecting Rods

Function	■ *Connecting rod for reciprocating engine or pump*
Constraints	■ *Must not fail by high-cycle fatigue* ■ *Must not fail by elastic buckling* ■ *Stroke, and thus con-rod length L, specified*
Objective	■ *Minimize mass*
Free variables	■ *Cross section A* ■ *Choice of material*

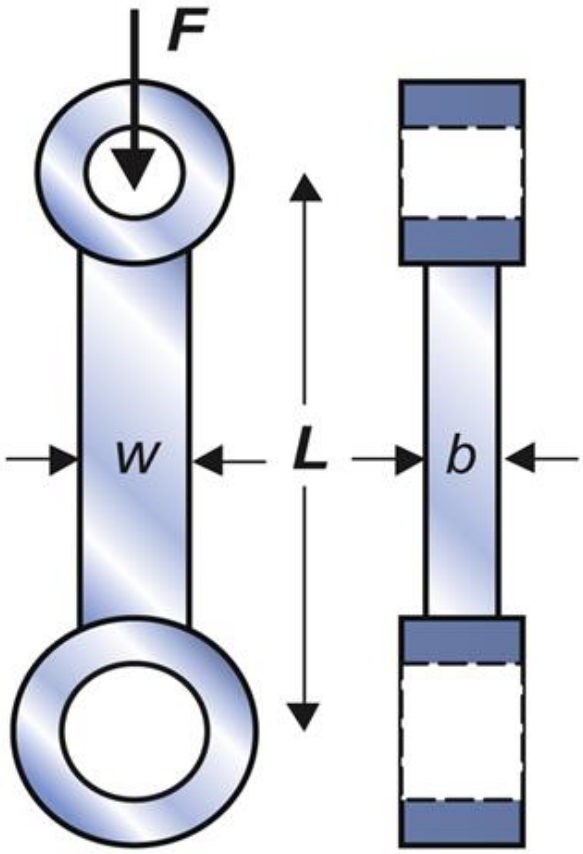


FIGURE 9.3 A con rod.

The translation. This, like the last, is a 'min-max' problem. The objective function is an equation for the mass that we approximate as

$$m = \beta AL\rho \tag{9.6}$$

where L is the length of the con-rod and ρ the density of the material of which it is made, A the cross section of the shaft and β a constant multiplier to allow for the mass of the bearing housings.

The fatigue constraint requires that

$$\frac{F}{A} \leq \sigma_e \tag{9.7}$$

where σ_e is the endurance limit of the material of which the con-rod is made.

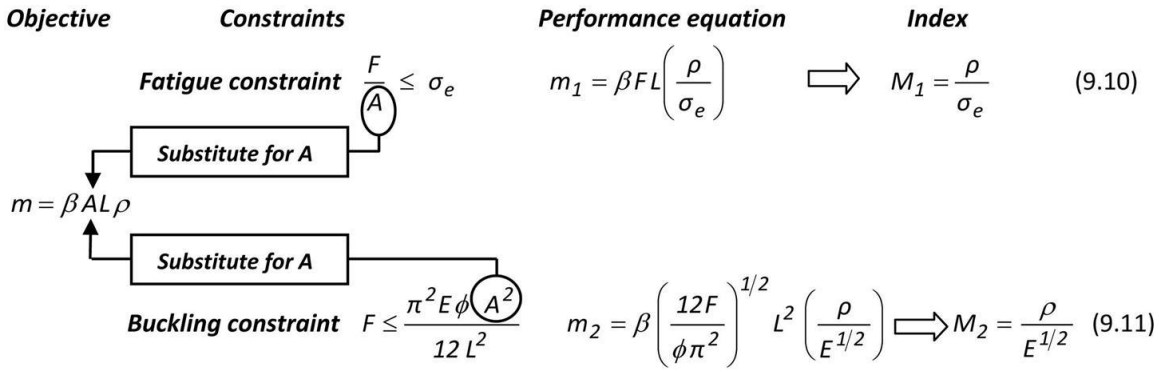
The buckling constraint requires that the peak compressive load F does not exceed the Euler buckling load:

$$F \leq \frac{\pi^2 EI}{L^2} \tag{9.8}$$

with $I = \frac{b^3 w}{12}$ (Appendix B). Writing $b = \phi w$ and $bw = A$ where ϕ is a dimensionless 'shape-constant' characterising the proportions of the cross-section gives

$$F \leq \frac{\pi^2 E \phi A^2}{12L^2} \tag{9.9}$$

Assembling the constraints and using them to substitute for the cross-section area A in the objective function gives the following sequence:



The con-rod, to be safe, must meet both constraints. For a given length, L, the active constraint is the one leading to the largest value of the mass, m. Fig. 9.4 shows the way in which m varies with L (a sketch of performance Eqs (9.10) and (9.11)), for a single material. Short con-rods are liable to fatigue failure, long ones are prone to buckle.

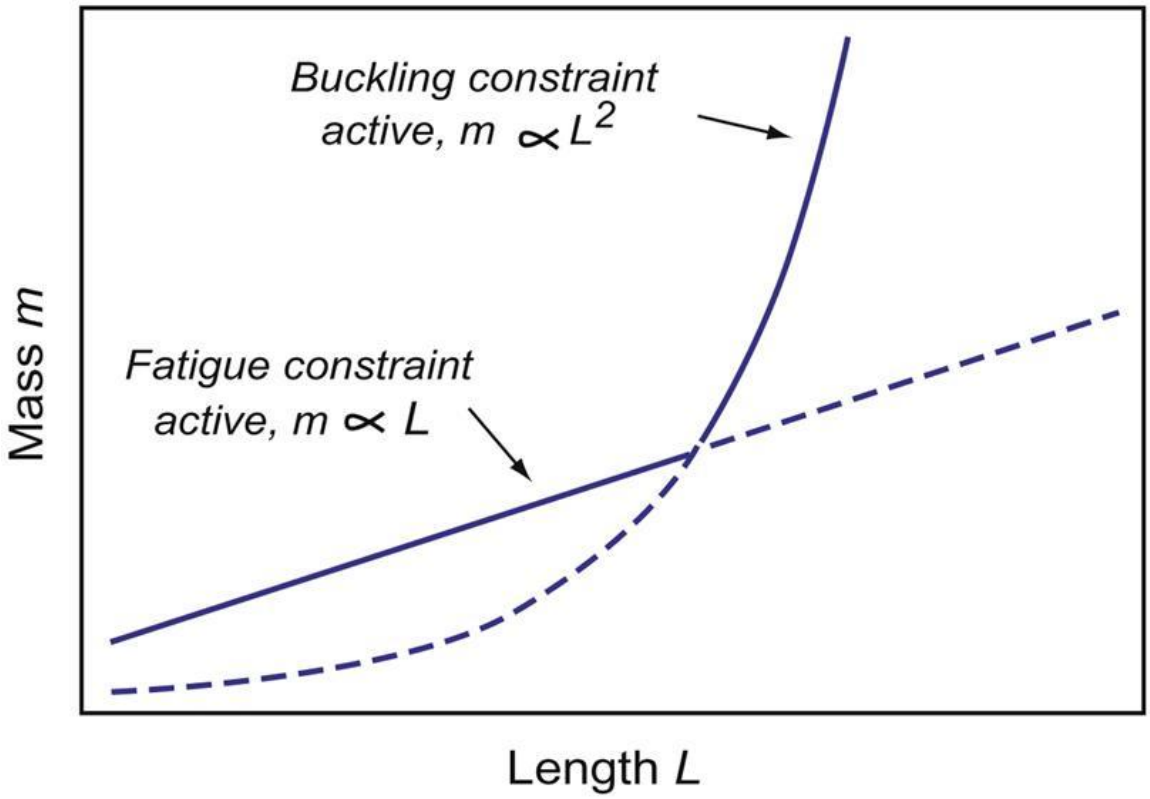


FIGURE 9.4 The equations for the mass of the con-rod are shown schematically as a function of L.

The selection. Consider first the selection of a material for the con-rod from among the limited list of Table 9.4. The specifications are

$$L = 200 \text{ mm} \quad F = 50 \text{ kN}$$
$$\phi = 0.8 \quad \beta = 1.5$$

Table 9.4

Selection of a Material for the Con-Rod

Material	ρ (kg/m^3)	E (GPa)	σ_e (MPa)	m_1 (kg)	m_2 (kg)	$\tilde{m} = \max(m_1, m_2)$ kg
Nodular cast iron	7150	178	250	**0.43**	0.22	0.43
HSLA steel 4140 (o.q. T-315)	7850	210	590	0.20	**0.28**	0.28
Al S355.0 casting alloy	2700	70	95	**0.39**	0.14	0.39
Duralcan Al-SiC(p) composite	2880	110	230	**0.18**	0.12	0.18
Titanium 6Al 4V	4400	115	530	0.12	**0.17**	**0.17**

The table lists the mass m_1 of a rod that will just meet the fatigue constraint, and the mass m_2 that will just meet that on buckling (Eqs (9.10) and (9.11)); the "max" value of each pair is underscored. For three of the materials the active constraint is that of fatigue; for two it is that of buckling. The quantity $\tilde{m}$ in the last column of the table is the larger of m_1 and m_2 for each material; it is the lowest mass that meets both constraints. The material offering the lightest rod of that with the smallest value of $\tilde{m}$; this "min" value is underscored. Here it is the titanium alloy Ti 6Al 4V. The metal-matrix composite Duralcan 6061 – 20% SiC is a close second. Both weigh less than half as much as a cast-iron rod.

Well, that is one way to use the method, but it is not the best. First, it assumes some 'preselection' procedure has been used to obtain the materials listed in the table without explaining how this is to be done; and second, the results apply only to the values of F and L listed above – change these, and the selection changes. If we wish to escape these restrictions, the graphical method is the way to do it.

The mass of the rod that will survive both fatigue and buckling is the larger of the two masses m_1 and m_2 (Eqs (9.9) and (9.12)). Setting them equal gives the equation of the coupling line (defined in Section 9.2):

$$M_2 = \left[\left(\frac{\phi\pi^2}{12}\cdot\frac{F}{L^2}\right)^{1/2}\right]\cdot M_1 \qquad \textbf{(9.12)}$$

The quantity in square brackets is the coupling constant C_c. It contains the quantity F/L^2 – the 'structural loading coefficient' of Section 4.6.

Materials with the optimum combination of M_1 and M_2 are identified by creating a chart with these indices as axes. Fig. 9.5 illustrates this, using a database of light alloys, but including cast iron for comparison. Coupling lines for two values of F/L^2 are plotted on it, taking $\phi = 0.7$. Two extreme selections are shown, one isolating the best subset when the structural loading coefficient F/L^2 is high, the other when it is low. Beryllium and its alloys emerge as the best choice for all values of C_c within this range. Leaving them aside, the best choice when F/L^2 is large (F/L^2=5 MPa) are titanium alloys such as Ti 6Al 4V. For the low value (F/L^2=0.05 MPa), magnesium alloys such as AZ61 offer lighter solutions than aluminium or titanium. Table 9.5 lists the conclusions.

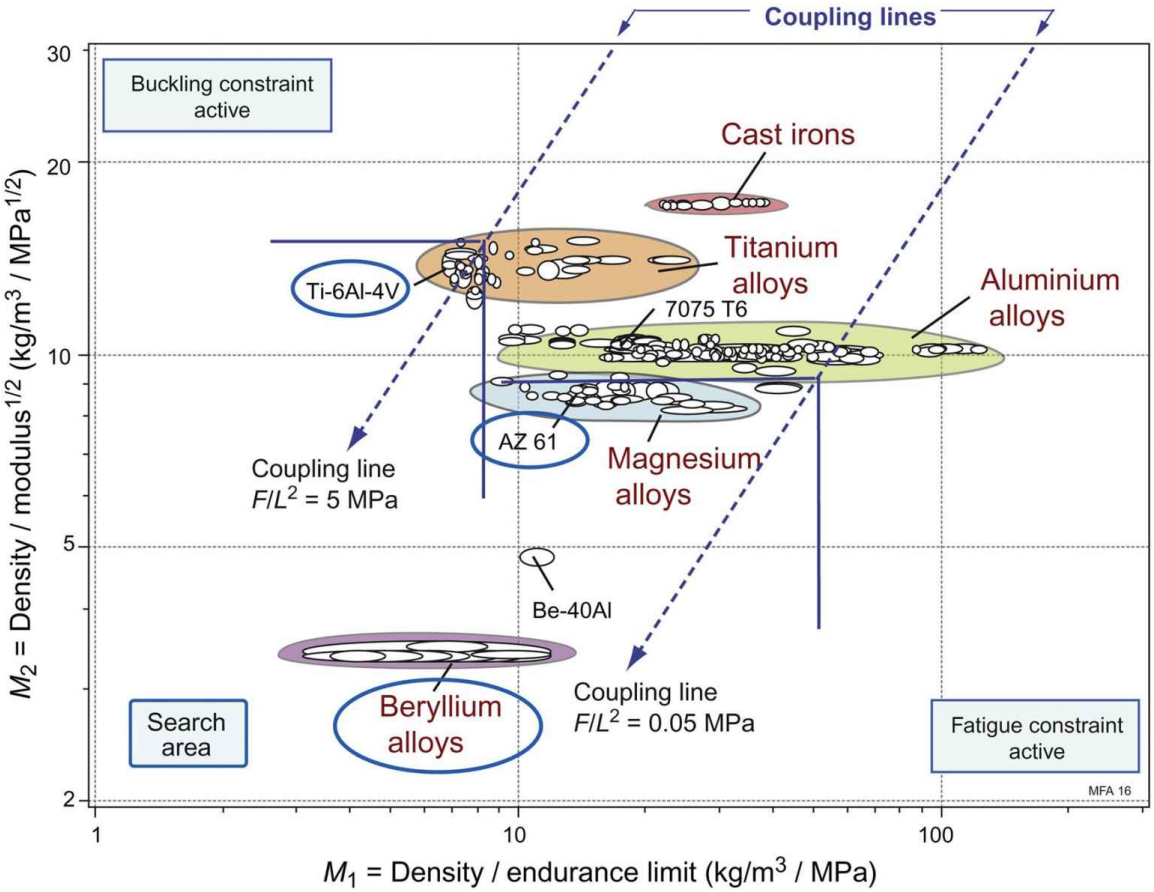


FIGURE 9.5 The coupled-constraint construction for the con-rod. The diagonal broken lines show the coupling equation for two extreme values of F/L^2. The box-shaped search areas are shown.

Table 9.5
Materials for High-Performance Con-Rods

Material	Comment
Magnesium alloys	AZ61 and related alloys offer good all-round performance
Titanium alloys	Ti-6-4 is the best choice for high F/L^2
Beryllium alloys	The ultimate choice, but difficult to process and very expensive
Aluminium alloys	Cheaper than titanium or magnesium, but lower performance

Postscript. Con-rods have been made from all the materials in the table: aluminium and magnesium in road cars, titanium and (rarely) beryllium in racing engines. Had we included CFRP in the selection, we would have found that it, too, performs well by the criteria we have used. This

conclusion has been reached by others, who have tried to do something about it: at least three designs of CFRP con-rods have been prototyped. It is not easy to design a CFRP con-rod. It is essential to use continuous fibres, which must be wound in such a way as to enclose both the shaft and the bearing housings, and the shaft must have a high proportion of fibres that lie parallel to the direction in which F acts. You might, as a challenge, think how you would do it.

Related Case Studies

5.4 'Materials for table legs'
9.2 'Multiple constraints: light pressure vessels'
9.4 'Multiple constraints: windings for high-field magnets'
11.3 'Forks for racing bicycles'
11.5 'Table legs yet again: thin or light?'

9.4 Multiple Constraints: Windings for High-Field Magnets

Physicists, for reasons of their own, like to see what happens to things in high-magnetic fields. 'High' means 50 Tesla or more. The only way to get such fields is the old-fashioned way: dump a huge current through a wire-wound coil like that shown schematically in Fig. 9.6. Neither permanent magnets (practical limit: 1.5 T) nor superconducting coils (present limit: 25 T) can achieve such high fields. The current generates a field-pulse that lasts as long as the current flows. The upper limits on the field and its duration are set by the material of the coil itself: if the field is too high, magnetic forces blow the coil apart; if the pulse is too long, it melts. So choosing the right material for the coil is critical. What should it be? The answer depends on the pulse length.

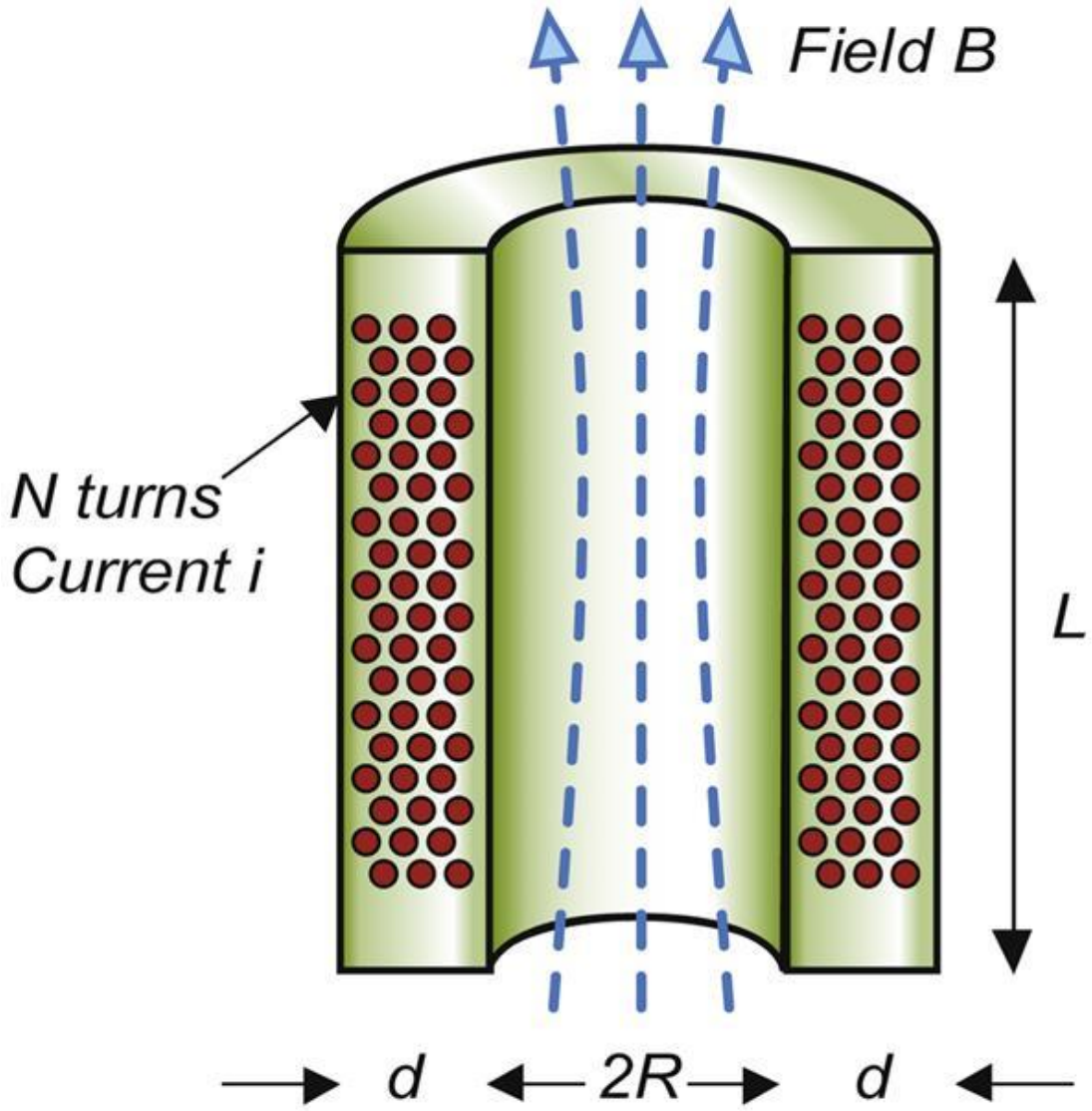


FIGURE 9.6 Windings for high-powered magnets.

Pulsed fields are classified according to their duration and strength as in Table 9.6. The requirements for the survival of the magnet producing them are summarized in Table 9.7. There is one objective – to maximize the field – with two constraints deriving from the requirement of survivability: that the windings are strong enough to withstand the radial force on them caused by the field, and that they do not heat up too much.

Table 9.6

Duration and Strengths of Pulsed Fields

Classification	Duration	Field Strength
Continuous	1 s–∞	<30 T
Long	100 ms–1 s	30–60 T
Standard	10–100 ms	40–70 T
Short	10–1000 μs	70–80 T
Ultra-short	0.1–10 μs	>100 T

Table 9.7

Design Requirements for the Windings

Function	■ *Magnet windings*
Constraints	■ *No mechanical failure* ■ *Temperature rise <100°C* ■ *Radius R and length L of coil specified*
Objective	■ *Maximize magnetic field*
Free variables	■ *Choice of material for the winding*

The translation. Detailed modelling gets a little complicated, so let us start with some intelligent guesses. First, if the windings must carry load (the first constraint) they must be strong – the higher the strength, the greater the field they can tolerate. So we want materials with a high elastic limit, σ_Y. Second, a current i flowing for a time t_p through a coil of resistance R_e dissipates $i^2 R_e t_p$ joules of energy, and if this takes place in a volume V, the temperature rise is

$$\Delta T = \frac{i^2 R_e t_p}{V C_p \rho} \tag{9.13}$$

where C_p is the specific heat of the material and ρ is its density. So to maximize the current (and thus the field B) we need materials with low values of $R_e/C_p\rho$ or, because resistance R_e is proportional to resistivity ρ_e for a fixed coil geometry, materials with low $\rho_e/C_p\rho$.

Both guesses are correct. More detailed modelling is possible, but we already have what we need to select materials. We search for material with high σ_y – or rather, with low $M_1 = 1/\sigma_y$ (since we must express objectives in a form to be minimized) – and low $M_2 = \rho_e/C_p\rho$. The two are plotted in Fig. 9.7 for some 1200 metals and alloys. The materials with the best combination of indices lie along the lower envelope of the populated region. Strength is the dominant constraint when pulses are short, which requires materials with low M_1; those near **A** are the best choice. Heating is the dominant constraint when pulses are long, and materials near **B**, with low M_2, are the answer.

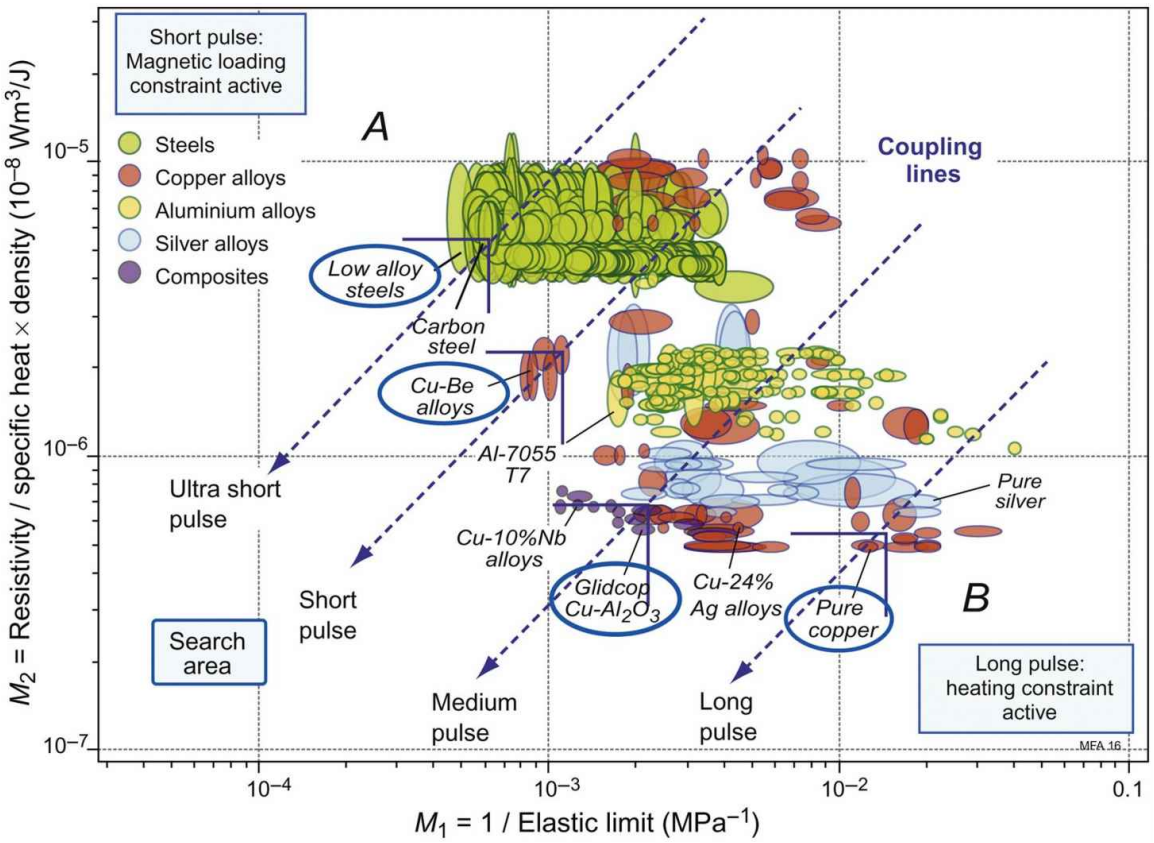


FIGURE 9.7 A chart for choosing materials for the windings of high-powered magnets or electric motors. The coupling lines and boxes identify the best choice for a given pulse duration.

Coupling lines are lines along which $M_2 = M_1$, so they have a slope of 1 when plotted on the chart. Four are shown, with attached boxes to capture the best choices. The results are summarized in Table 9.8.

Table 9.8

Materials for High-Field Magnet Windings

Material	Comment
Continuous and long pulse High conductivity coppers Pure silver	Best choice for low field, long pulse magnets (heat limited)
Short pulse Copper-AL_2O_3 composites (Glidcop) H-C copper cadmium alloys H-C copper zirconium alloys H-C copper chromium alloys Drawn copper-niobium composites	Best choice for high field, short pulse magnets (heat and strength limited)
Ultra-short pulse, ultra high field Copper-beryllium-cobalt alloys High-strength, low-alloy steels	Best choice for high field, short pulse magnets (strength limited)

Postscript. The case study, as developed here, is an oversimplification. Today's magnet design is very sophisticated, and involves nested sets of electro- and superconducting magnets (up to 9 deep), with geometry the most important variable. But the selection scheme for coil materials has validity: when pulses are long, resistivity is the primary consideration; when they are very short, it is strength, and the best choice for each is that developed here. Similar considerations enter the selection of materials for very high-speed motors, for bus-bars and for relays.

Further Reading

Herlach, F., 1988. The technology of pulsed high-field magnets. IEEE Trans. Magn. 24, 1049.

Wood, J.T., Embury, J.D., Ashby, M.F., 1995. An approach to material selection for high field magnet design. Acta Metal. et Mater. 43, 212.

Related Case Studies

9.3 'Multiple constraints: con-rods for high-performance engines'

9.5 Conflicting Objectives: Table Legs Again

We now turn from coupled constraints to conflicting objectives, applying the methods of Chapter 8, Multiple Constraints and Conflicting Objectives, Section 8.3. We start with a simple example, revisiting the selection of materials for slender table legs.

The translation. The design requirements set down by Snr. Tavolino for his table (Chapter 5, Materials Selection – Case Studies, Table 5.5) involved two objectives: that the leg be as light *and* as thin as possible. The mass m of a leg (Eq. 5.8) is proportional to

$$M_1 = \frac{\rho}{E^{1/2}}$$

The thickness $2r$ (Eq. 5.9) scales with

$$M_2 = \frac{1}{E}$$

Snr. Tavolino has two objectives: he wishes to minimize both.

The selection. The proper way to tackle multiobjective problems like this one is to make a trade-off plot. Fig. 9.8 is an example: M_1 is on the vertical axis, M_2 is on the horizontal one. For clarity only ferrous alloys, light alloys, composites and woods appear on the plot – it is a set that includes almost all materials that could be thought of as candidates. The classes cluster very tightly because both modulus and density have narrow ranges.

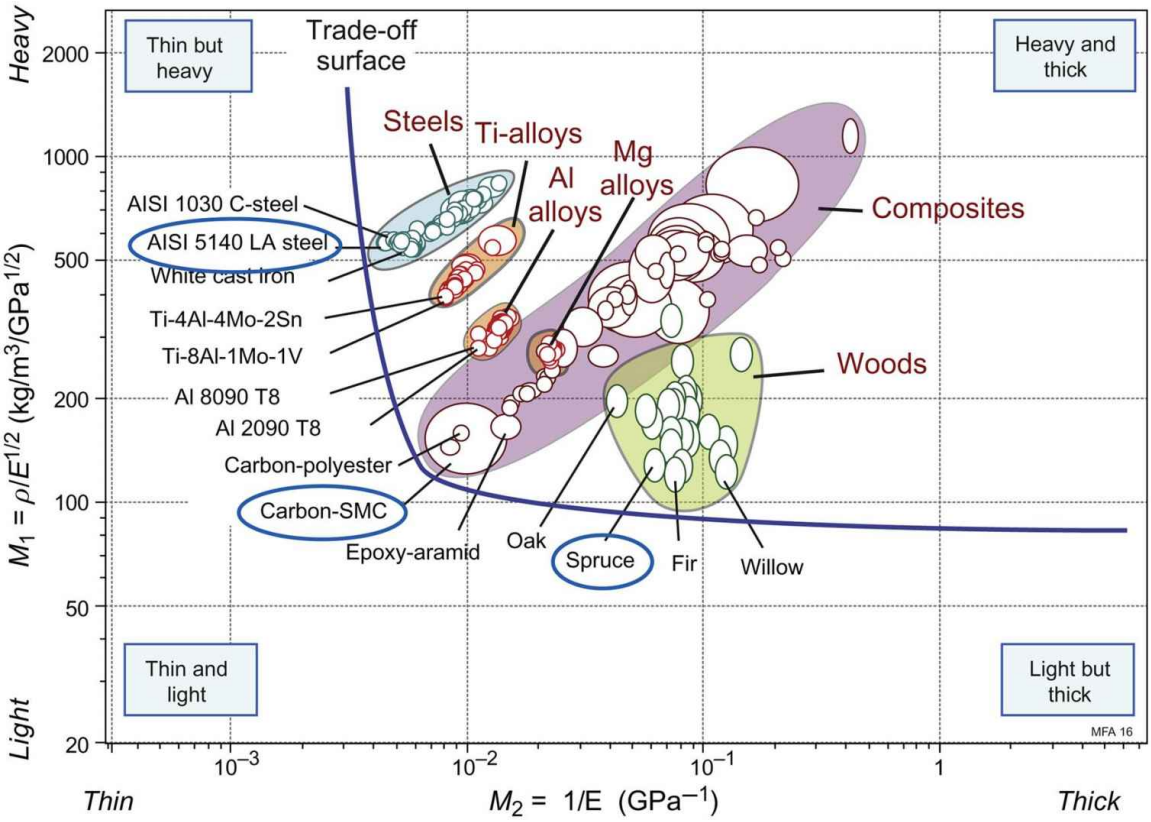


FIGURE 9.8 The trade-off plot for the table leg. Materials that lie near the trade-off surface are identified.

Legs made of spruce or fir are potentially lighter than those made of any other material. Composites offer legs that are almost as light, and much thinner. But, interestingly, they do not offer the thinnest of all – steel does better because of its high modulus. Light alloys allow legs that are lighter than steel, but not nearly as thin. Overall, composites offer the best compromise – very low weight and attractive slenderness.

Postscript. But (as we asked in Section 5.4), wouldn't a tubular leg be lighter? The answer will have to wait a little longer – until Chapter 11, Material and Shape: Case Studies, Section 11.5.

Related Case Studies

6.3 'Materials for table legs'

11.5 'Table legs yet again'

9.6 Conflicting Objectives: Wafer-Thin Casings for Must-Have Electronics

Slenderness in consumer electronics – portable computers, mobile phones, personal organizers and MP3 players – is a major design driver. The ideal is a device that you can slip into a shirt pocket and not even notice that it is there. The casing has to be stiff and strong enough to protect the electronics – the display, particularly – from damage. Casings used to be made of moulded ABS or PC. To be stiff enough, an ABS case has to be at least 2 mm thick, too much for today's designs in which thinness and lightness are very highly valued. But the consequences of making the casing too thin are severe: mobile phones are sat on, and portable computers end up under piles of books. If the casing is insufficiently stiff, it flexes and damages the screen. The challenge: to identify materials for casings that are at least as stiff as a 2 mm ABS case but are thinner and lighter. We must recognize that the thinnest may not be the lightest, and vice versa. A trade-off will be needed. Table 9.9 summarizes the requirements.

Table 9.9

The Design Requirements: Casing for Portable Electronics

Function	■ *Light, thin (cheap) casing*
Constraints	■ *Bending stiffness, S*, specified* ■ *Dimensions L and W specified*
Objectives	■ *Minimize thickness of casing* ■ *Minimize mass of casing* ■ *(Minimize material cost)*
Free variables	■ *Thickness t of casing wall* ■ *Choice of material*

The translation. We idealize the loading on a panel of the casing in the way shown in the Fig. 9.9. External loads cause it to bend. The bending stiffness is

$$S = \frac{48EI}{L^3}$$

with

$$I = \frac{Wt^3}{12} \tag{9.14}$$

where E is Young's modulus, I the second moment of the area of the panel, and the dimensions L, W and t are shown on the figure. The stiffness S must equal or exceed a design requirement S^* if the panel is to perform its function properly. Combining the two equations and solving for the thickness t gives

$$t \geq \left(\frac{S^* L^3}{4EW}\right)^{1/3} \tag{9.15}$$

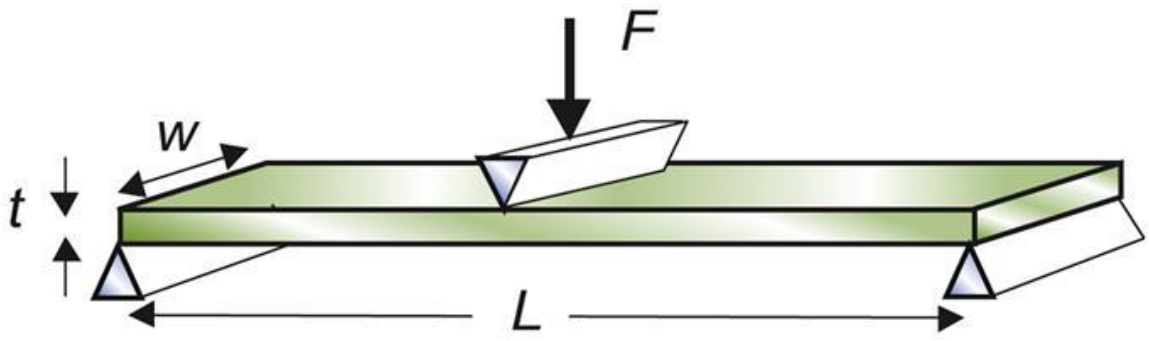


FIGURE 9.9 The casing can be idealized as a panel of dimensions $L \times W$, and thickness t, loaded in bending.

The thinnest panel is that made from the material with the smallest value of the index

$$M_1 = \frac{1}{E^{1/3}}$$

The mass of the panel per unit area, m_a, is just ρt, where ρ is its density – the lightest panel is that made from the material with the smallest value of

$$M_2 = \frac{\rho}{E^{1/3}} \tag{9.16}$$

We use the existing ABS panel, stiffness S^*, as the standard for comparison. If ABS has a modulus E_o and a density ρ_o, then a panel made from any other material (modulus E, density ρ) will, according to Eq. (9.15) have a thickness t relative to that of the ABS panel, t_o, given by

$$\frac{t}{t_o} = \left(\frac{E_o}{E}\right)^{1/3} \tag{9.17}$$

and a relative mass per unit area

$$\frac{m_a}{m_{a,o}} = \left(\frac{\rho}{E^{1/3}}\right)\left(\frac{E_o^{1/3}}{\rho_o}\right) \tag{9.18}$$

We wish to explore the trade-off between t/t_o and $m_a/m_{a,o}$ for possible solutions.

The selection. Fig. 9.10 shows the necessary plot, here limited to a few material classes for simplicity. It is divided into four sectors with ABS at the centre at the co-ordinates (1,1). The solutions in sector **A** are both thinner and lighter than ABS, some by a factor of 2. Those in sector **B** and **C** are better by one metric but worse by the other. Those in sector **D** are worse by both. To narrow in on an optimal choice we sketch in a trade-off surface, shown as the blue line. The solutions nearest to this surface are, in terms of one metric or the other, good choices. Intuition guides us to those in or near Sector **A**.

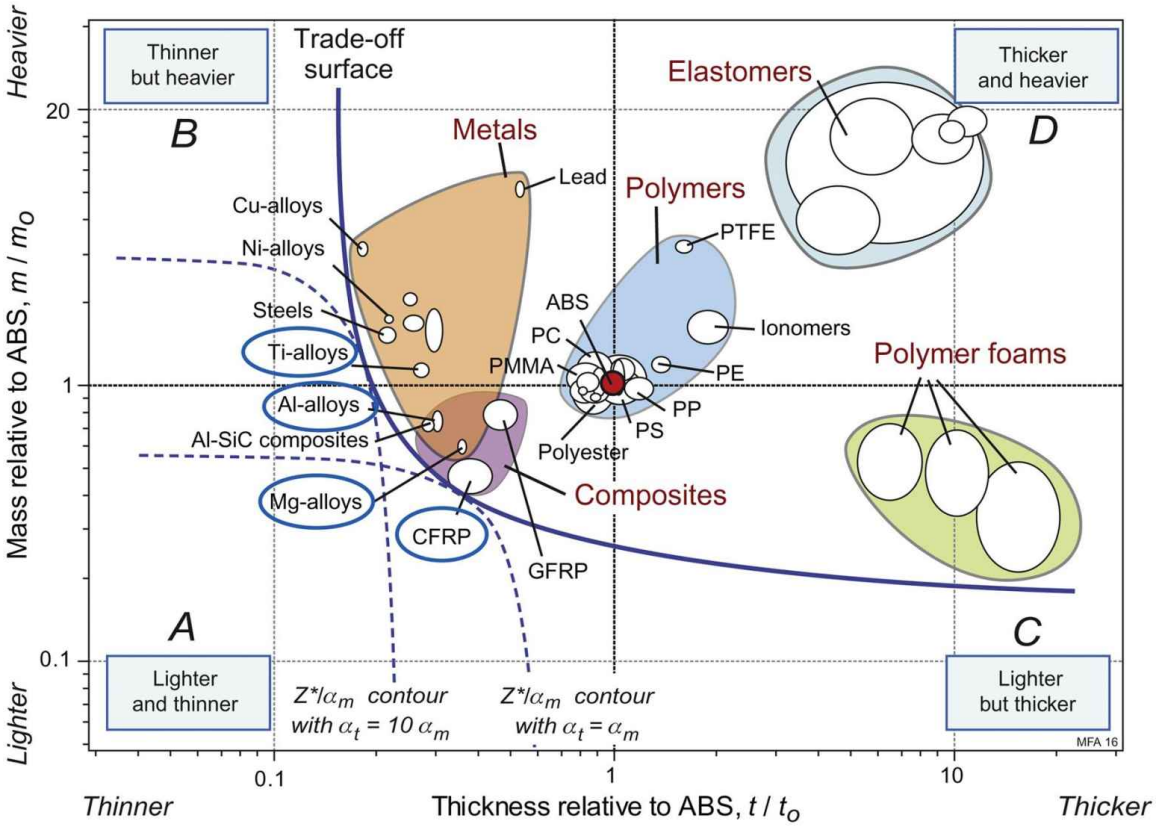


FIGURE 9.10 The relative thickness and mass of casings made from alternative materials. Those near the trade-off surface are identified.

This is already enough to suggest choices that offer savings in thickness and in weight. If we want to go further we must formulate a *relative penalty function*. Define Z^*, measured in units of currency, as

$$Z^* = \alpha_t^* \frac{t}{t_o} + \alpha_m^* \frac{m_a}{m_{a,o}} \qquad \textbf{(9.19)}$$

The exchange constant α_t^* measures the decrease in penalty – or gain in value – for a fractional decrease in thickness and α_m^* is the gain in value for a fractional decrease in mass. As an example, set $\alpha_t^* = \alpha_m^*$, meaning that we value both equally. Then solutions with equal penalty Z^* are those on the contour

$$\frac{Z^*}{\alpha_m^*} = \frac{t}{t_o} + \frac{m_a}{m_{a,o}} \qquad (9.20)$$

where the first term on the right is given by Eq. (9.17) and the second by Eq. (9.18). This is plotted for a selection of metals, polymers and composites in Fig. 9.11. ABS lies near the middle of the polymer group. CFRP, GFRP, titanium, aluminium and magnesium all offer casings with lower (better) values of Z^*.

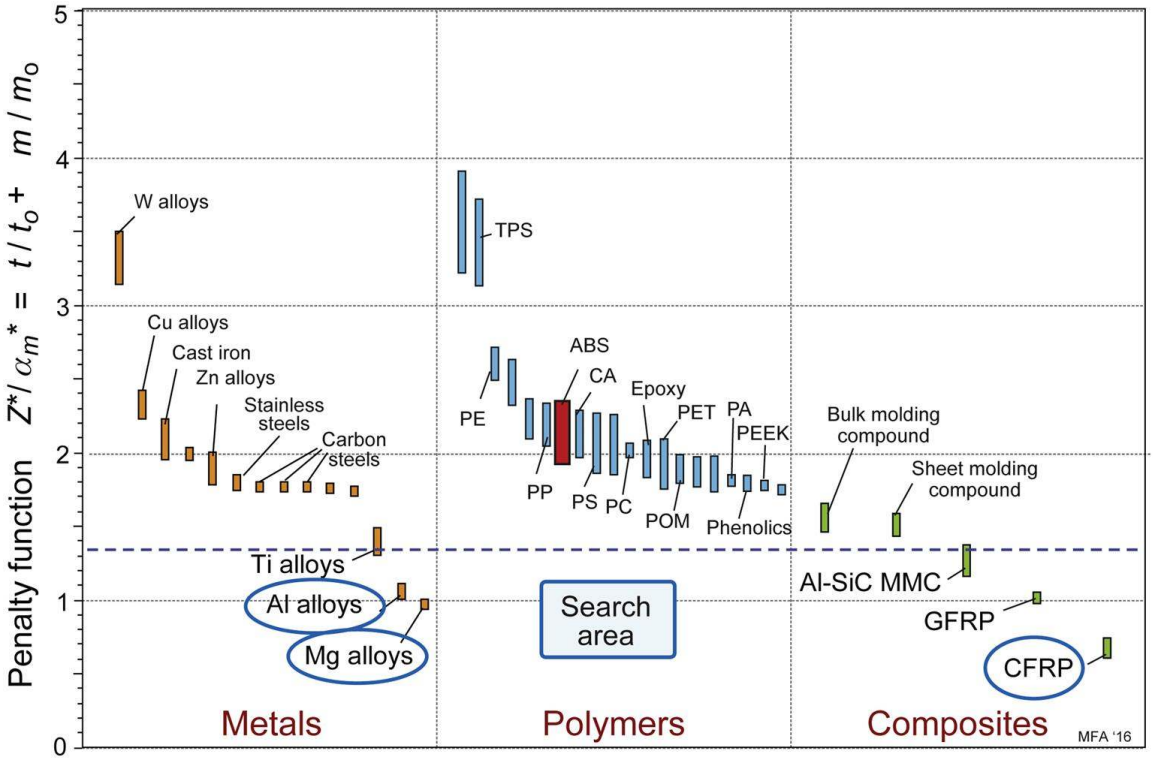


FIGURE 9.11 The penalty function Z^*/α_m^* when $\alpha_t^* = \alpha_m^*$. ABS lies near the middle of the polymer column. Materials below it have a lower penalty – they are better choices.

The problem with this plot is that it is specific to a single value of the ratio α_t^*/α_m^*. If the relative importance of thinness and lightness are changed, the ranking changes too. We need a more general method. It is provided by constructing penalty contours on the trade-off plot. Two are shown as blue lines on Fig. 9.10. The linear relationship of Eq. (9.20) plots as a family of curves (not straight lines because of the log scales) with Z^*/α_m^* decreasing towards the bottom left. The absolute value of Z^*/α_m^* does not matter – all we need it for is to identify the point at which a contour is

tangent to the trade-off surface as shown in Fig. 9.10. The solutions nearest this point are the optimum choices: CFRP, magnesium alloys, and Al-SiC composites. If, instead, we set $\alpha_t^* = 10\ \alpha_m^*$, meaning that thinness is much more highly valued than lightness, the contour moves to the second position shown in Fig. 9.10. Now titanium, and even steel (stainless, to avoid magnetic coupling), become attractive candidates.

Postscript. Back in 1997 when extreme thinness and lightness first became major design drivers, the conclusions reached here were new. At that time almost all casings for hand-held electronics were made of ABS, polycarbonate, or, occasionally, steel. Now, some 20 years later, examples of aluminium, magnesium, titanium and CFRP casings can be found in currently marketed products. The value of the case study (which dates from 1996) is as an illustration of the way in which systematic methods can be applied to multiobjective selection.

Related Case Studies

9.8 'Conflicting objectives: materials for a disk-brake calliper'

9.7 Conflicting Objectives: Cost-Effective Bumpers

The bumper of a car is design to support bending moments (Fig. 9.12). Its function in an accident is to transfer load from the point of impact to crush-elements that support or absorb it. To do this the material of the bumper must have high strength, σ_Y. The bumper adds to the mass of the vehicle and thus to its fuel consumption. It also has a cost. The task is to select a material for a light, strong bumper (Table 9.10). It is not straightforward: the lightest material is unlikely to be the cheapest; nor the cheapest the lightest. We will need trade-off methods.

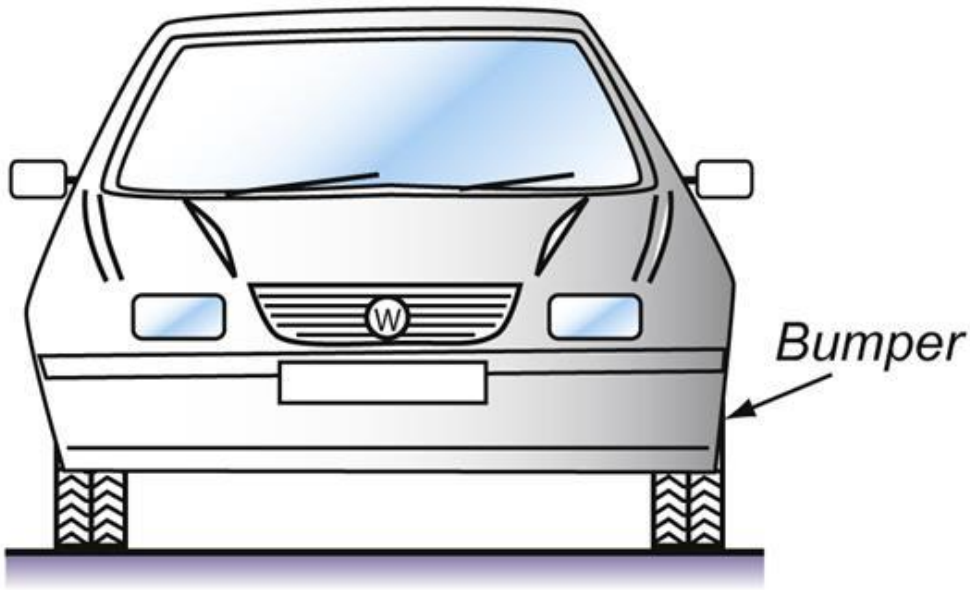


FIGURE 9.12 Car bumper.

Table 9.10
Design Requirements for Crash Barriers

Function	■ *Bumper: transmit impact load to absorbing elements at sides*
Constraint	■ *High strength* ■ *Adequate fracture toughness*
Objectives	■ *Minimize mass for a given bending strength* ■ *Minimize cost for a given bending strength*
Free variables	■ *Choice of material*

We know from Chapter 4, Materials Selection – The Basics, that the mass per unit of bending strength, for a given geometry, is proportional to the index

$$M_1 = \frac{\rho}{\sigma_y^{2/3}} \tag{9.21}$$

where ρ is the material density. The cost of the material is proportional to this times the cost C_m per unit mass:

$$M_2 = \frac{C_m \rho}{\sigma_y^{2/3}} \tag{9.22}$$

To proceed further we define a penalty function

$$Z = \frac{C_m \rho}{\sigma_y^{2/3}} + \alpha \frac{\rho}{\sigma_y^{2/3}} = \frac{\rho}{\sigma_y^{2/3}} (C_m + \alpha) \tag{9.23}$$

in which α is the exchange constant, which, as we saw in Table 8.1, depends on the transport system. For a given value of α the best choice is the material with the lowest value of Z.

This penalty function of Eq. (9.23) is plotted in Fig. 9.13 for three values of α, retaining in each case only the materials with the lowest Z. When $\alpha = 1$, carbon and low-alloy steels are the best choices. When $\alpha = 10$ light alloys based on aluminium and magnesium win. When $\alpha = 100$, CFRP becomes a more attractive choice than any other material.

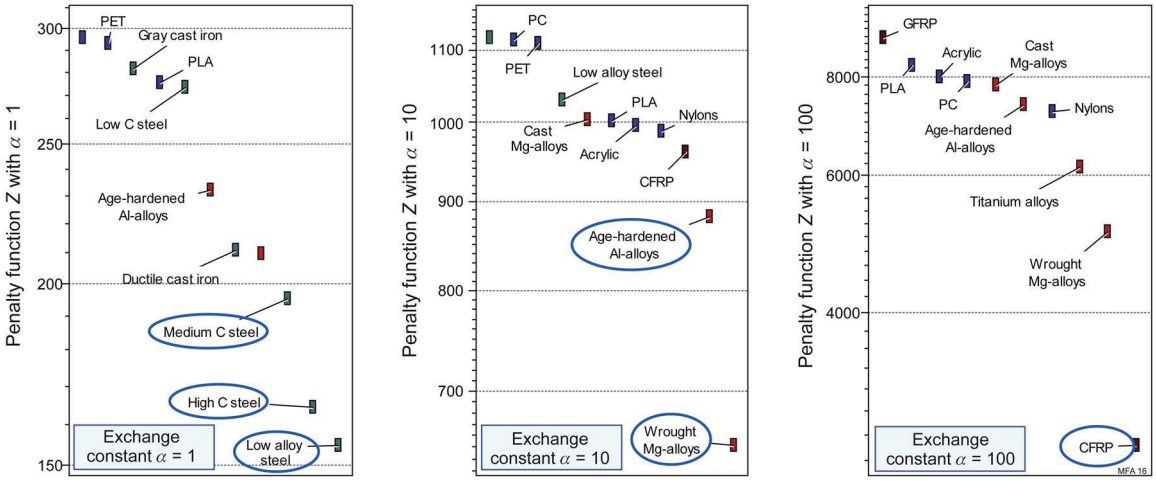


FIGURE 9.13 The choice of material for the bumper depends on the value of the exchange constant, α.

Postscript. There are, of course, other components of cost besides that of the material: the cost of shaping and finishing the bumper and of the attachments needed to fix it to the vehicle all have a cost. But the case study nicely illustrates the dramatic way in which the exchange constant of Table 8.1 influences material cost.

Related Case Studies

9.5 ‘Conflicting objectives: table legs again’
9.6 ‘Conflicting objectives: wafer-thin casings for must-have electronics’
9.8 ‘Conflicting objectives: materials for a disk-brake callipers’

9.8 Conflicting Objectives: Materials for a Disk-Brake Caliper

It is unusual – very unusual – to ask whether cost is important in selecting a material and to get the answer ‘NO’. But it does sometimes happen, notably when the material is to perform a critical function in space (beryllium for structural components, iridium for radiation screening), in medical procedures (think of gold tooth fillings) and in equipment for highly competitive sports (one racing motorcycle had a solid silver cylinder head because of its high thermal conductivity). Here is another example – materials for the brake callipers of a Formula 1 racing car.

The translation. The brake calliper can be idealized as two beams of length L, depth b and thickness h, locked together at their ends (cover picture of the chapter and Fig. 9.14). Each beam is loaded in bending when the braking pressure is applied, and because braking generates heat, it gets hot. The lower schematic represents one of the beams. Its length L and depth b are set by the design. The beam stiffness S is critical: if it is inadequate the calliper will flex, impairing braking efficiency and allowing vibration. Its ability to transmit heat, too, is critical since part of the heat generated in braking must be conducted out through the calliper. Table 9.11 summarizes the requirements.

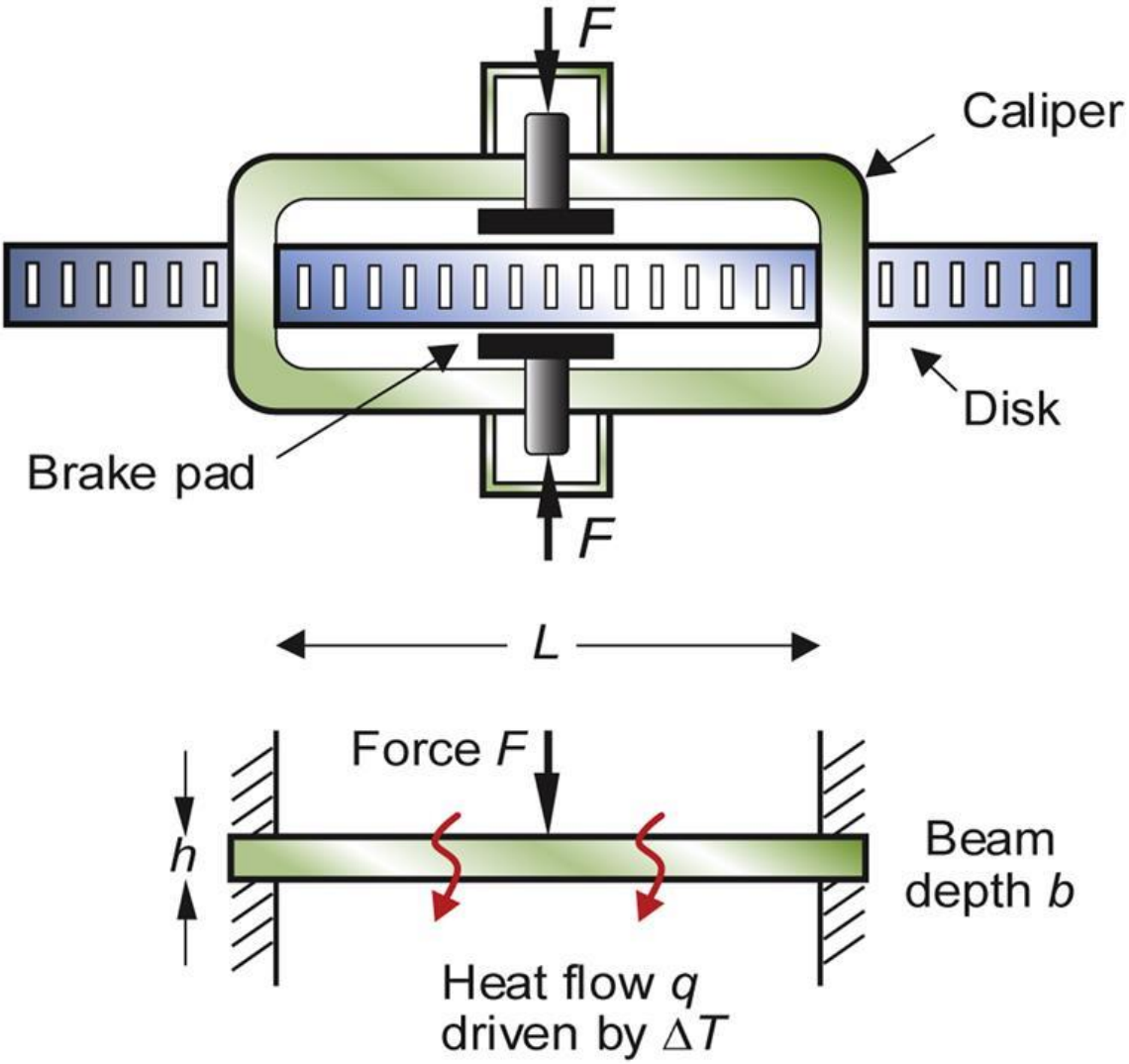


FIGURE 9.14 A schematic of a brake calliper. The long arms of the calliper are loaded in bending and must conduct heat well to prevent overheating.

Table 9.11

The Design Requirements: Brake Calliper

Function	■ *Brake calliper*
Constraints	■ *Bending stiffness, S*, specified* ■ *Dimensions L and b specified*
Objectives	■ *Minimize mass of calliper* ■ *Maximize heat transfer through calliper*
Free variables	■ *Thickness h of calliper wall* ■ *Choice of material*

The mass of the calliper scales with that of one of the beams. Its mass per unit area is simply

$$m_a = h\rho \quad \text{(units: kg/m}^2\text{)} \tag{9.24}$$

where ρ is the density of the material of which it is made. Heat transfer q depends on the thermal conductivity λ of the material of the beam; the heat flux per unit area is

$$q_a = \lambda \frac{\Delta T}{h} \quad \text{(units: Watts/m}^2\text{)} \tag{9.25}$$

where ΔT is the temperature difference between the surfaces.

The quantities L, b and ΔT are specified. The only free variable is the thickness h. But there is a constraint: the calliper must be stiff enough to ensure that it does not flex or vibrate excessively. To achieve this we require that

$$S = \frac{C_1 EI}{L^3} = \frac{C_1 Ebh^3}{12L^3} \geq S^* \quad \text{(units: N/m)} \tag{9.26}$$

where S^* is the desired stiffness, E is Young's modulus, C_1 is a constant that depends on the distribution of load and $I = bh^3/12$ is the second moment of the area of the beam. Thus

$$h \geq \left(\frac{12S^*}{C_1 bE}\right)^{1/3} L \tag{9.27}$$

Inserting this in Eqs (9.24) and (9.25) gives equations for the mass m_a of the arm and the heat q_a transferred through it, per unit area:

$$m_a \geq \left(\frac{12S^*}{C_1 b}\right)^{1/3} L\left(\frac{\rho}{E^{1/3}}\right) \quad \text{(units: kg/m}^2\text{)} \tag{9.28}$$

$$q_a = \frac{\Delta T}{L}\left(\frac{C_1 b}{12S^*}\right)^{1/3} (\lambda E^{1/3}) \quad \text{(units: W/m}^2\text{)} \tag{9.29}$$

The first equation contains the material index

$$M_1 = \frac{\rho}{E^{1/3}}$$

the second (expressed such that a minimum is sought), contains the index

$$M_2 = \frac{1}{\lambda E^{1/3}}$$

The standard material for a brake calliper is ductile cast iron – it is cheap and stiff, but it is also heavy and a relatively poor conductor. We use this as a standard for comparison, normalizing Eqs (9.28) and (9.29) by the values for cast iron (density ρ_o, modulus E_o and conductivity λ_o), giving

$$\frac{m_a}{m_{a,o}} = \left(\frac{\rho}{E^{1/2}}\right)\left(\frac{E_o^{1/3}}{\rho_o}\right) \tag{9.30}$$

and

$$\frac{q_{a,o}}{q_a} = \frac{\lambda_o E_o^{1/3}}{\lambda E^{1/3}} \tag{9.31}$$

Fig. 9.15 shows a chart with these as axes. It is divided into four quadrants, centred on cast iron at the point (1,1). Each bubble describes a material. Those in the lower left are better than cast iron by both objectives; an aluminium calliper, e.g., has half the weight and offers twice the heat transfer. The ultimate choice is beryllium or its alloy Be 40% Al.

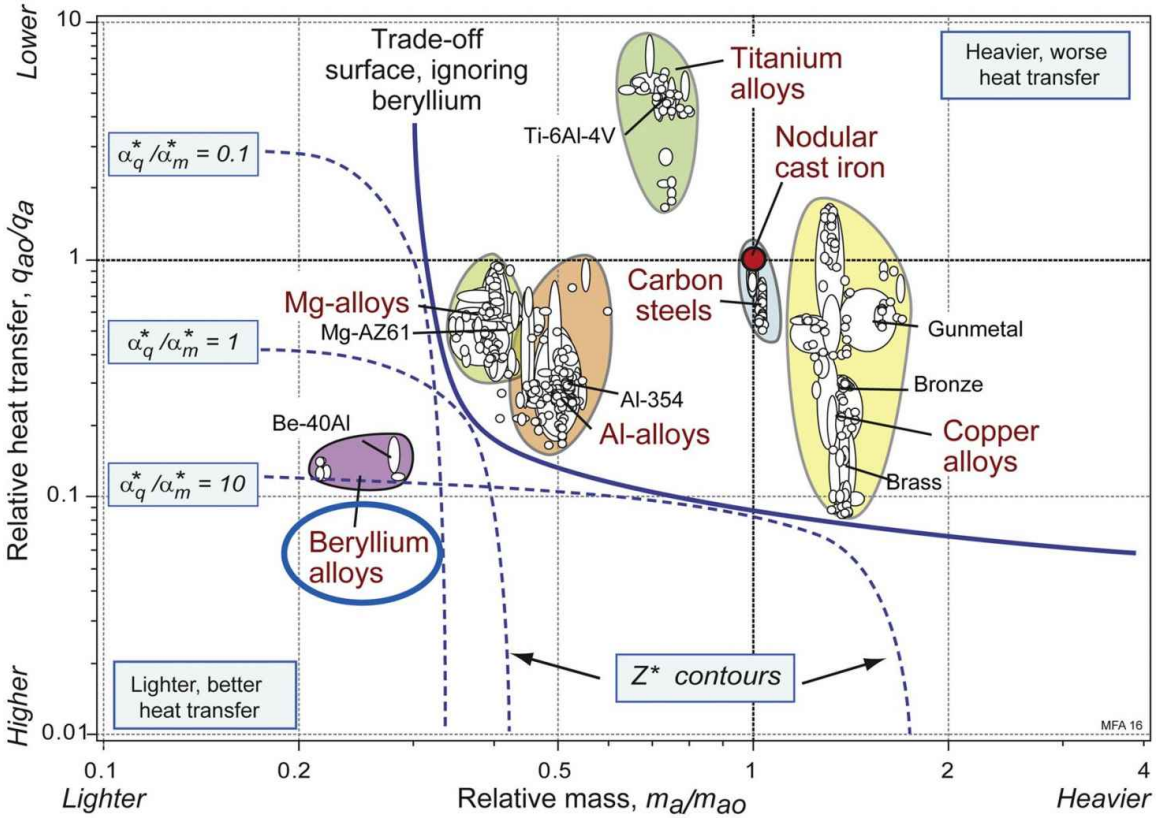


FIGURE 9.15 A chart with Eqs (9.30) and (9.31) as axes. Beryllium and its alloys minimize mass and maximize heat transfer. But if these exotics are excluded, the choice becomes dependent on the ratio α_q^*/α_m^*.

To go further we formulate the relative penalty function

$$Z^* = \alpha_m^* \left(\frac{m_a}{m_{a,o}}\right) + \alpha_q^* \left(\frac{q_{a,o}}{q_a}\right) \quad \textbf{(9.32)}$$

in which the terms in brackets are given by Eqs (9.30) and (9.31) and the exchange constants α_m^* and α_q^* measure the relative value of a fractional saving of weight or increase in heat transfer relative to cast iron. The penalty function in plotted in Fig. 9.15 for three values of the ratio α_q^*/α_m^* of the exchange constants. Each is tangent to a trade-off surface that excludes the 'exotic' beryllium alloys, which otherwise dominate the selection for all values. For $\alpha_q^*/\alpha_m^* = 0.1$, meaning mass reduction is of prime importance, magnesium alloys are the best choice. If mass reduction and heat transfer are given equal weight ($\alpha_q^*/\alpha_m^* = 1$) aluminium alloys become a good choice. If heat transfer is the overriding consideration, ($\alpha_q^*/\alpha_m^* = 10$) alloys based on copper win. But if you really want the best it has to be beryllium.

Postscript. Ferrari Racing did, at one time, commission beryllium brake callipers. Today restrictions on materials, imposed to make Formula 1 more competitive, outlaw their use.

Related Case Studies

5.16 'Minimizing thermal distortion in precision devices'
9.5 'Conflicting objectives: table legs again'
9.7 'Conflicting objectives: wafer-thin casings for must-have electronics'
9.8 'Conflicting objectives: cost-effective bumpers

9.9 Summary and Conclusions

Most designs are over constrained: they must simultaneously meet several competing, and often conflicting, requirements. But although they conflict, an optimum selection is still possible. The 'active constraint' method, developed in Chapter 8, Multiple Constraints and Conflicting Objectives, allows the selection of materials that optimally meet two or more constraints. It is illustrated here by three case studies, two of them mechanical, one electro-mechanical.

Greater challenges arise when the design must meet two or more conflicting objectives (such as minimizing mass, volume, cost and environmental impact). Here we need a way to express all the objectives in the same units, a 'common currency', so to speak. The conversion factors are called the 'exchange constants'. Establishing the value of the exchange constant is an important step in solving the problem. With it, a penalty

function Z can be constructed that combines the objectives. Materials that minimize Z meet all the objectives in a properly balanced way. The most obvious common currency is cost itself, requiring an 'exchange rate' to be established between cost and the other objectives. This can be done for mass, and – at least in principle – for other objectives too. The method is illustrated by four further case studies.

Exercises. This chapter is one of case studies. Exercises relating to the methods they use are at the end of Chapter 8, Multiple Constraints and Conflicting Objectives.

CHAPTER 10

Selection of Material and Shape

Abstract

Shape can be used to increase the mechanical efficiency of a material. Shaped sections carry bending, torsional and axial-compressive loads more efficiently than solid sections. 'Shaped' means that the cross-section is formed to a tube, a box-section, an I-section or the like. 'Efficient' means that, for given loading conditions, the section uses as little material as possible. Tubes, boxes and I-sections are simple shapes. Even greater efficiencies are possible with sandwich panels (thin load-bearing skins bonded to a foam or honeycomb interior) and with more elaborate structures (the Warren truss, for instance). This pair of chapters extends selection methods so as to include shape.

Keywords

Shaped sections; shape factors; shape efficiency; material-shape combinations; limits to shape factors; microscopic shape; man-powered planes; racing bikes; ultra-efficient springs

10.1 Introduction and Synopsis

Pause now for a moment and reflect on how shape is used to modify the ways in which materials behave. A material has a modulus and a strength, but it can be made stiffer and stronger when loaded in bending or torsion by shaping it into an I-beam or a hollow tube. It can be made less stiff by flattening it into a flat leaf or winding it, in the form of wire, into a helix. Thinned shapes help dissipate heat; cellular shapes help conserve it. There are shapes to maximize electrical capacitance, to conserve magnetic field,

shapes that control optical reflection, diffraction and refraction, shapes to reflect a sound and shapes to absorb it. Shape is even used to change the way a material feels, making it smooth or rough, slippery or grippy. And of course, it is shape that distinguishes the Venus de Milo from the marble block from which she was carved. It is a rich subject.

Here we explore one part of it – the way shape can be used to increase the mechanical efficiency of a material. Shaped sections carry bending, torsional and axial-compressive loads more efficiently than solid sections. By 'shaped' we mean that the cross-section is formed to a tube, a box-section, an I-section or the like. By 'efficient' we mean that, for given loading conditions, the section uses as little material as possible. Tubes, boxes and I-sections will be referred to as 'simple shapes'. Even greater efficiencies are possible with sandwich panels (thin load-bearing skins bonded to a foam or honeycomb interior) and with more elaborate structures (the Warren truss, for instance).

This chapter extends selection methods so as to include shape (Fig. 10.1). Often this is not needed: in the Case Studies of Chapter 6, Processes and Their Effect on Properties, shape either did not enter at all, or, when it did, it was not a variable (that is, we compared different materials with the same shape). But when two different materials are available, each with its own section shape, the more general problem arises: how to choose the best combination? Take the example of a bicycle: its forks are loaded in bending. They could, say, be made of steel or of wood – early bikes *were* made of wood. But steel is available as thin-walled tube, whereas the wood is not; wood components, usually, are solid. A solid wood bicycle is certainly lighter for the same stiffness than a solid steel one, but is it lighter than one made of steel tubing? Might a magnesium I-section be lighter still? How, in short, is one to choose the best combination of material and shape?

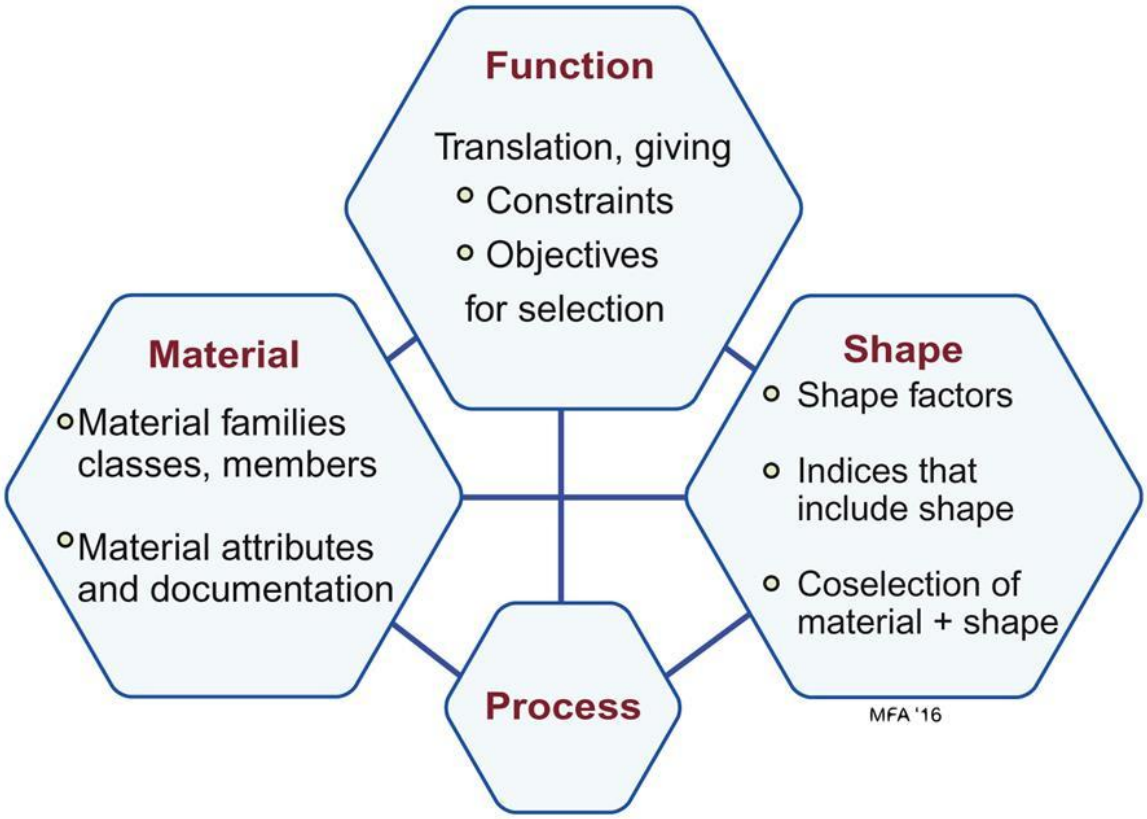


FIGURE 10.1 Section shape is important for certain modes of loading. When shape is a variable, a new term, the shape factor ϕ, appears in some of the material indices.

A procedure for answering these and related questions is developed in this chapter. It involves the definition of *shape factors*. A *material* can be thought of as having properties but no shape. A *structure* is a material made into a shape (Fig. 10.2). Shape factors are measures of the efficiency of material usage. They enable the definition of material indices like those of Chapter 4, Materials Selection – the Basics, but that now include shape. When shape is constant, the indices reduce exactly to those of Chapter 4, Materials Selection – the Basics; but when shape is a variable, the shape factor appears in the expressions for the indices. They let you compare shaped materials and guide the choice of the best combination of material and shape. The symbols used in the development are listed, for convenience, in Table 10.1. Don't be put off by them; the ideas are not difficult.

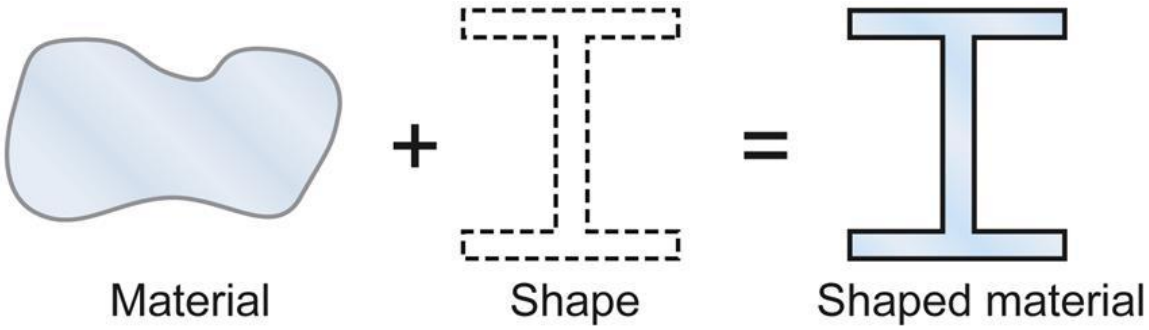


FIGURE 10.2 Mechanical efficiency is obtained by combining material with macroscopic shape. The shape is characterized by a dimensionless shape factor, ϕ.

Table 10.1
Definition of Symbols

Symbol	Definition
M	Moment (Nm)
F	Force (N)
E	Young's modulus of the material of the section (GPa)
G	Shear modulus of the material of the section (GPa)
σ_f	Yield or failure strength of the material of the section (MPa)
ρ	Density of the material of the section (kg/m^3)
m_l	Mass per unit length of the section (kg/m)
A	Cross-sectional area of section (m^2)
I	Second moment of area of the section (m^4)
I_o	Second moment of area of the square reference section (m^4)
Z	Section modulus of the section (m^3)
Z_o	Section modulus of the square reference section (m^3)
K	Torsional moment of area (m^4)
K_o	Torsional moment of area for square reference section (m^4)
Q	Torsional section modulus (m^3)
Q_o	Torsional section modulus for square reference section (m^3)
ϕ_B^e	Macro shape factor for elastic bending deflection (–)
ϕ_B^f	Macro shape factor for onset of plasticity or failure in bending (–)
ϕ_T^e	Macro shape factor for elastic torsional deflection (–)
ϕ_T^f	Macro shape factor for onset of plasticity or failure in torsion (–)
ψ_B^e	Micro shape factor for elastic bending deflection (–)
ψ_B^f	Micro shape factor for onset of plasticity or failure in bending (–)
ψ_T^e	Micro shape factor for elastic torsional deflection (–)
ψ_T^f	Micro shape factor for onset of plasticity or failure in torsion (–)
S_B	Bending stiffness (N/m)
S_T	Torsional stiffness (N/m)
(EI)	Essential term in bending stiffness (N·m^2)
$(Z\sigma_f)$	Essential term in bending strength (N·m)
t	Web and flange thickness (m)
c	Web height (m)
d	Section height ($2t$+c) of sandwich (m)
b	Section (flange) width (m)
L	Section length (m)

10.2 Shape Factors

The loads on a component can be decomposed into those that are axial, those that exert bending moments and those that exert torques. One of these usually dominates to such an extent that structural elements are specially designed to carry it, and these have common names. Thus *ties* carry tensile loads; *beams* carry bending moments; *shafts* carry torques; *columns* carry compressive axial loads. Fig. 10.3 shows these modes of loading applied to shapes that resist them well. The point it makes is that the best material-and-shape combination depends on the mode of loading. In what follows, we separate the modes, dealing with each separately.

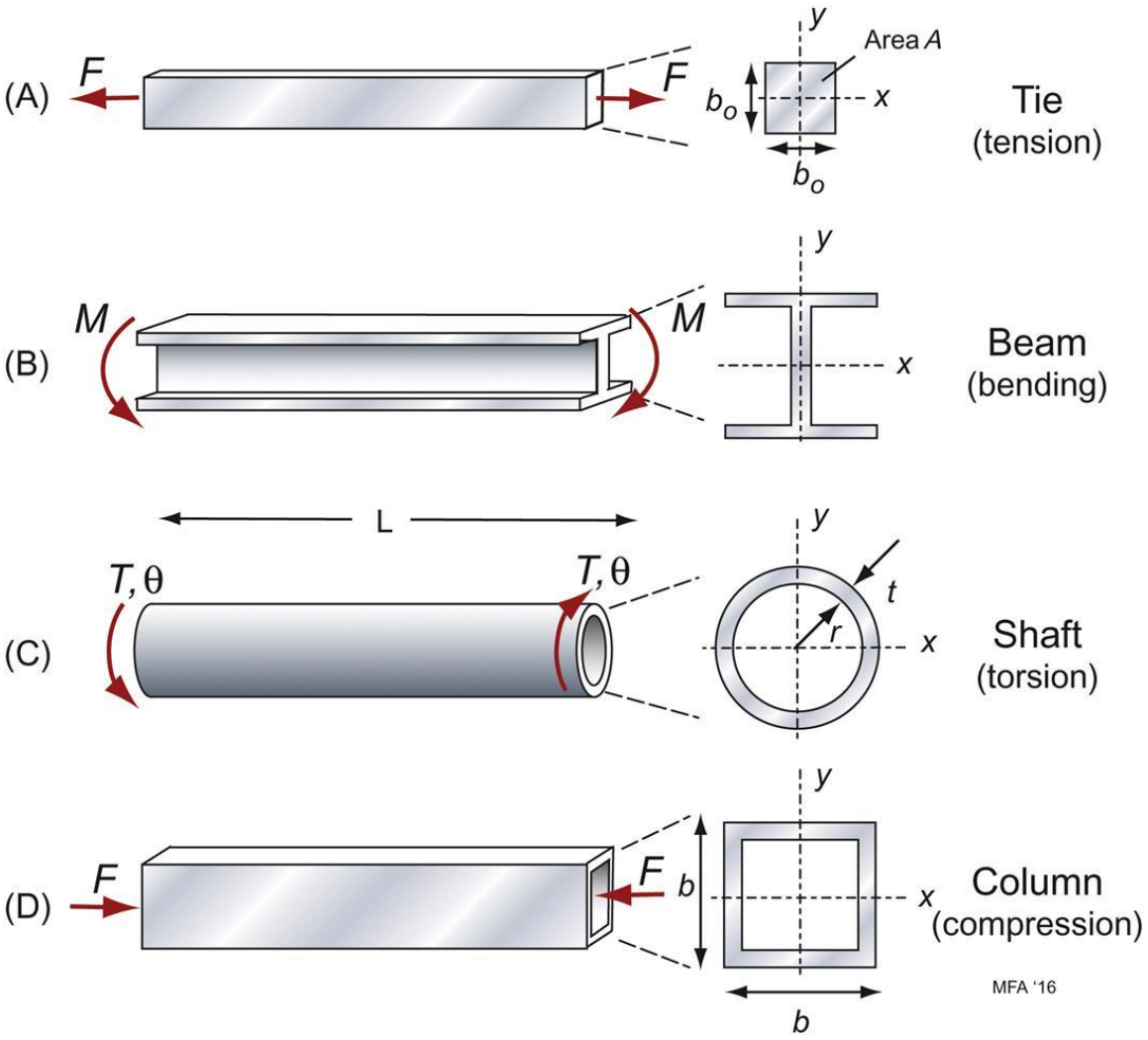


FIGURE 10.3 Common modes of loading and the section-shapes that are chosen to support them: (A) axial tension (B) bending (C) torsion and (D) axial compression, which can lead to buckling.

In axial tension, the area of the cross-section is important but its shape is not: all sections with the same area will carry the same load. Not so in bending: beams with hollow-box or I-sections are better than solid sections of the same cross-sectional area. Torsion too, has its efficient shapes: circular tubes, for instance, are more efficient than either solid sections or I-sections. To characterize this we need a metric – a way of measuring the structural efficiency of a section shape, independent of the material of which it is made. An obvious one is that given by the ratio ϕ (phi) of the stiffness or strength of the shaped section to that of a 'neutral' reference shape, which we take to be that of a solid square section with the same

cross-sectional area A, and thus the same mass per unit length, m_l, as the shaped section (Fig. 10.4).

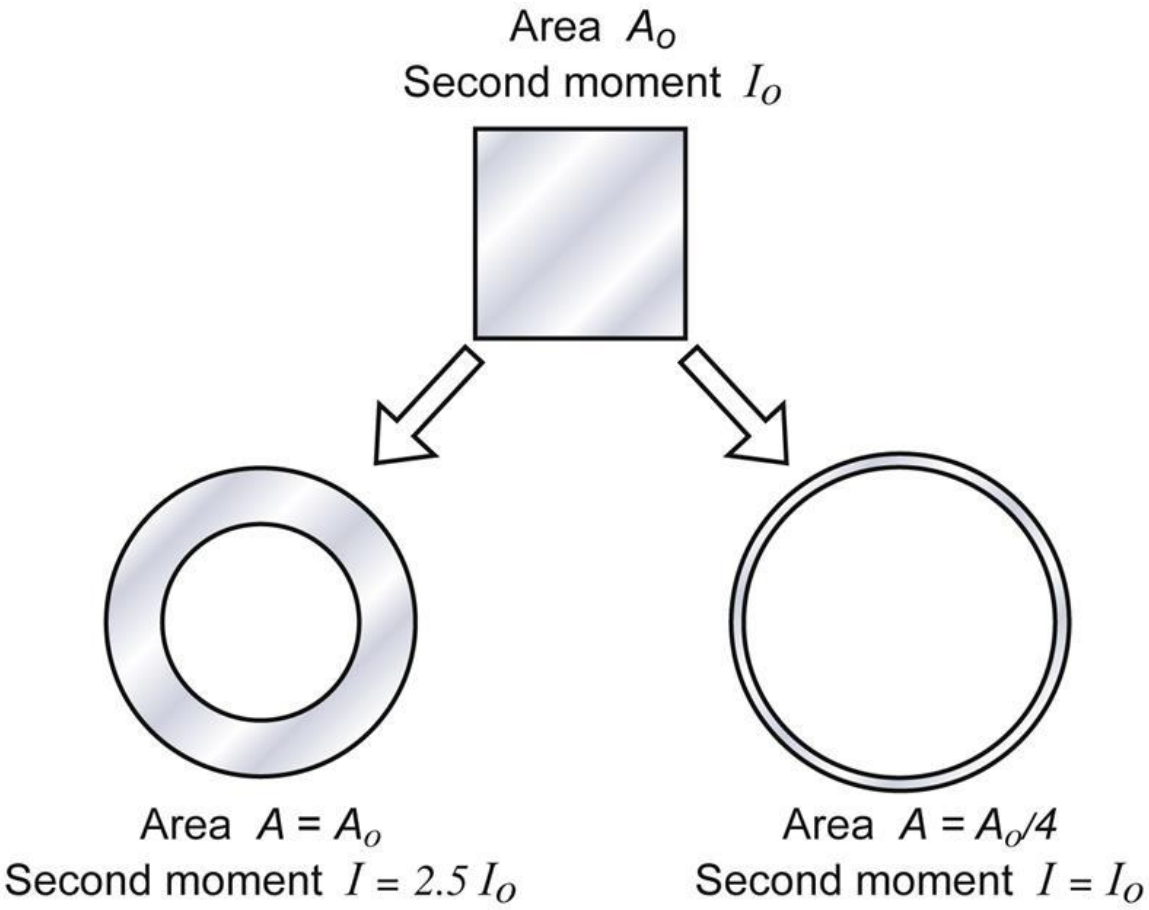


FIGURE 10.4 The effect of section shape on bending stiffness EI: a square-section beam compared, left, with a tube of the same area (but 2.5 times stiffer) and, right, a tube with the same stiffness (but 4 times lighter).

Elastic bending of beams (Fig. 10.3B). The bending stiffness S of a beam is proportional to the product EI

$$S \propto \frac{EI}{L^3}$$

Here E is Young's modulus and I is the second moment of area of the beam of length L about the axis of bending (the x axis):

$$I = \int_{\text{section}} y^2 dA \tag{10.1}$$

where y is measured normal to the bending axis and dA is the differential element of area at y. Values of the moment I and of the area A for common sections are listed in the first two columns of Table 10.2. Those for the more complex shapes are approximate, but completely adequate for present needs. The second moment of area, I_o, for a reference beam of square section with edge length b_o and section area $A = b_o^2$ is simply

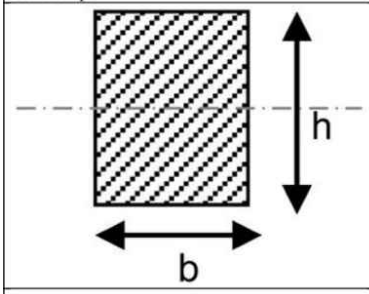
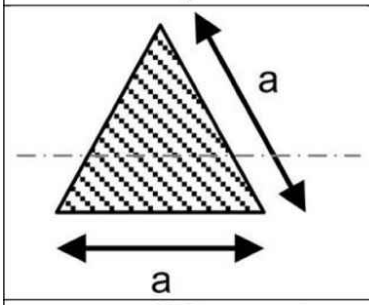
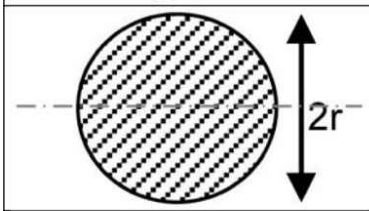
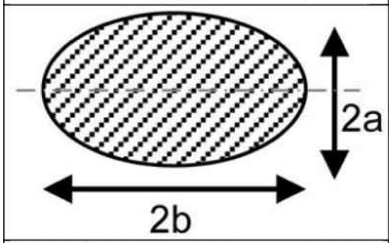
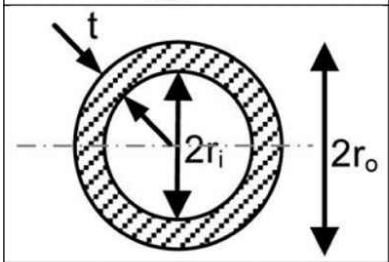
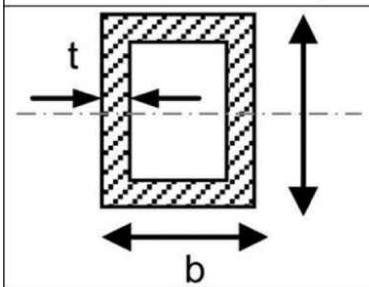
$$I_o = \frac{b_o^4}{12} = \frac{A^2}{12} \tag{10.2}$$

(Here and elsewhere the subscript 'o' refers to the solid square reference section.) The bending stiffness of the shaped section differs from that of a square one with the same area A by the factor ϕ_B^e where

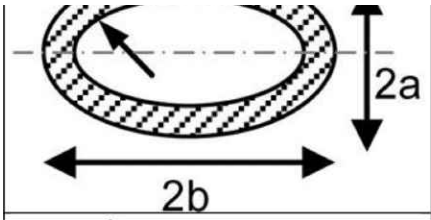
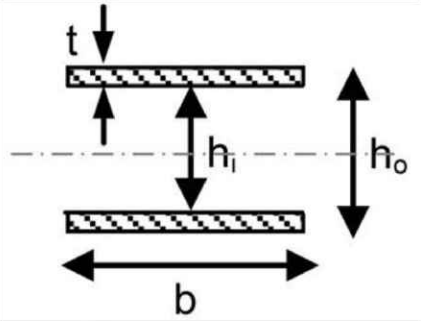
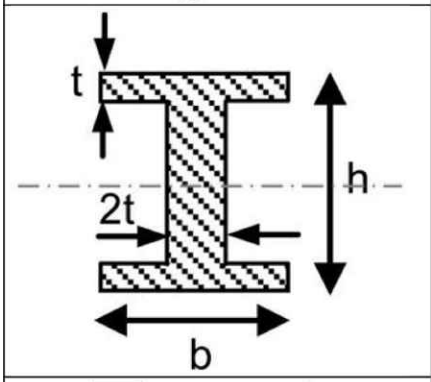
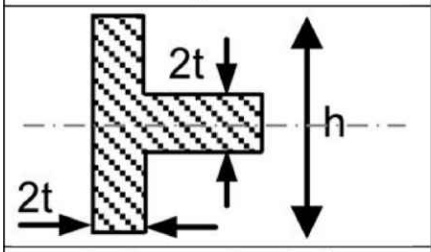
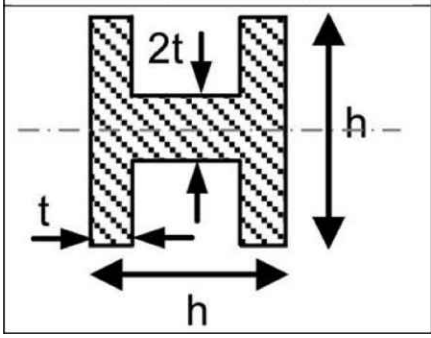
$$\phi_B^e = \frac{S}{S_o} = \frac{EI}{EI_o} = \frac{12I}{A^2} \tag{10.3}$$

Table 10.2

Moments of Sections (With Units)

Section Shape	Area A (m²)	Moment I, (m⁴)	Moment K, (m⁴)	Moment Z, (m³)	Moment Q, (m³)
	bh	$\frac{b\,h^3}{12}$	$\frac{b\,h^3}{3}\left(1-0.58\frac{b}{h}\right)(h>b)$	$\frac{b\,h^2}{6}$	$\frac{b^2\,h^2}{(3h+1.8b)}$ $(h>b)$
	$\frac{\sqrt{3}}{4}a^2$	$\frac{a^4}{32\sqrt{3}}$	$\frac{\sqrt{3}a^4}{80}$	$\frac{a^3}{32}$	$\frac{a^3}{20}$
	$\pi\,r^2$	$\frac{\pi}{4}r^4$	$\frac{\pi}{2}r^4$	$\frac{\pi}{4}r^3$	$\frac{\pi}{2}r^3$
	πab	$\frac{\pi}{4}a^3b$	$\frac{\pi\,a^3b^3}{(a^2+b^2)}$	$\frac{\pi}{4}a^2b$	$\frac{\pi}{2}a^2b$ $(a<b)$
	$\pi(r_o^2-r_i^2)$ $\approx 2\,\pi rt$	$\frac{\pi}{4}(r_o^4-r_i^4)$ $\approx \pi r^3 t$	$\frac{\pi}{2}(r_o^4-r_i^4)$ $\approx 2\pi r^3 t$	$\frac{\pi}{4\,r_o}(r_o^4-r_i^4)$ $\approx \pi r^2 t$	$\frac{\pi}{2\,r_o}(r_o^4-r_i^4)$ $\approx 2\pi r^2 t$
	$2t(h+b)$ $(h,b>>t)$	$\frac{1}{6}h^3t\left(1+3\frac{b}{h}\right)$	$\frac{2b^2h^2}{(h+b)}\left(1-\frac{t}{h}\right)^4$	$\frac{1}{3}h^2t\left(1+3\frac{b}{h}\right)$	$2tbh\left(1-\frac{t}{h}\right)^2$

Section Shape	Area A (m²)	Moment I, (m⁴)	Moment K, (m⁴)	Moment Z, (m³)	Moment Q, (m³)
	$\pi(a+b)t$ $(a,b>>t)$	$\frac{\pi}{4}a^3t\left(1+\frac{3b}{a}\right)$	$\frac{4\pi(ab)^{5/2}t}{(a^2+b^2)}$	$\frac{\pi}{4}a^2\,t(1+\frac{3b}{a})$	$2\pi t(a^3b)^{1/2}$ $(b>a)$

					
	$b(h_o - h_i)$ $\approx 2bt$ $(h, b >> t)$	$\frac{b}{12}(h_o^3 - h_i^3)$ $\approx \frac{1}{2} b t h_o^2$	–	$\frac{b}{6\,h_o}(h_o^3 - h_i^3)$ $\approx b\,t\,h_o$	–
	$2\,t(h+b)$ $(h, b >> t)$	$\frac{1}{6}h^3 t\left(1+3\frac{b}{h}\right)$	$\frac{2}{3}b\,t^3\left(1+4\frac{h}{b}\right)$	$\frac{1}{3}h^2 t\left(1+3\frac{b}{h}\right)$	$\frac{2}{3}b\,t^2\left(1+4\frac{h}{b}\right)$
	$2t(h+b)$ $(h, b >> t)$	$\frac{t}{6}(h^3 + 4bt^2)$	$\frac{t^3}{3}(8b + h)$	$\frac{t}{3h}(h^3 + 4bt^2)$	$\frac{t^2}{3}(8b + h)$
	$2t(h+b)$ $(h, b >> t)$	$\frac{t}{6}(h^3 + 4bt^2)$	$\frac{2}{3}ht^3\left(1+4\frac{b}{h}\right)$	$\frac{t}{3h}(h^3 + 4\,b\,t^2)$	$\frac{2}{3}ht^2\left(1+4\frac{b}{h}\right)$

We call ϕ_B^e the *shape factor for elastic bending*. Note that it is dimensionless – I has dimensions of (length)^4 and so does A^2. It depends only on shape, not on scale: big and small beams have the same value of ϕ_B^e if their section shapes are the same.[1] This is shown in Fig. 10.5. The three members of

each horizontal group differ in scale but have the same shape factor – each member is a magnified or shrunken version of its neighbours.

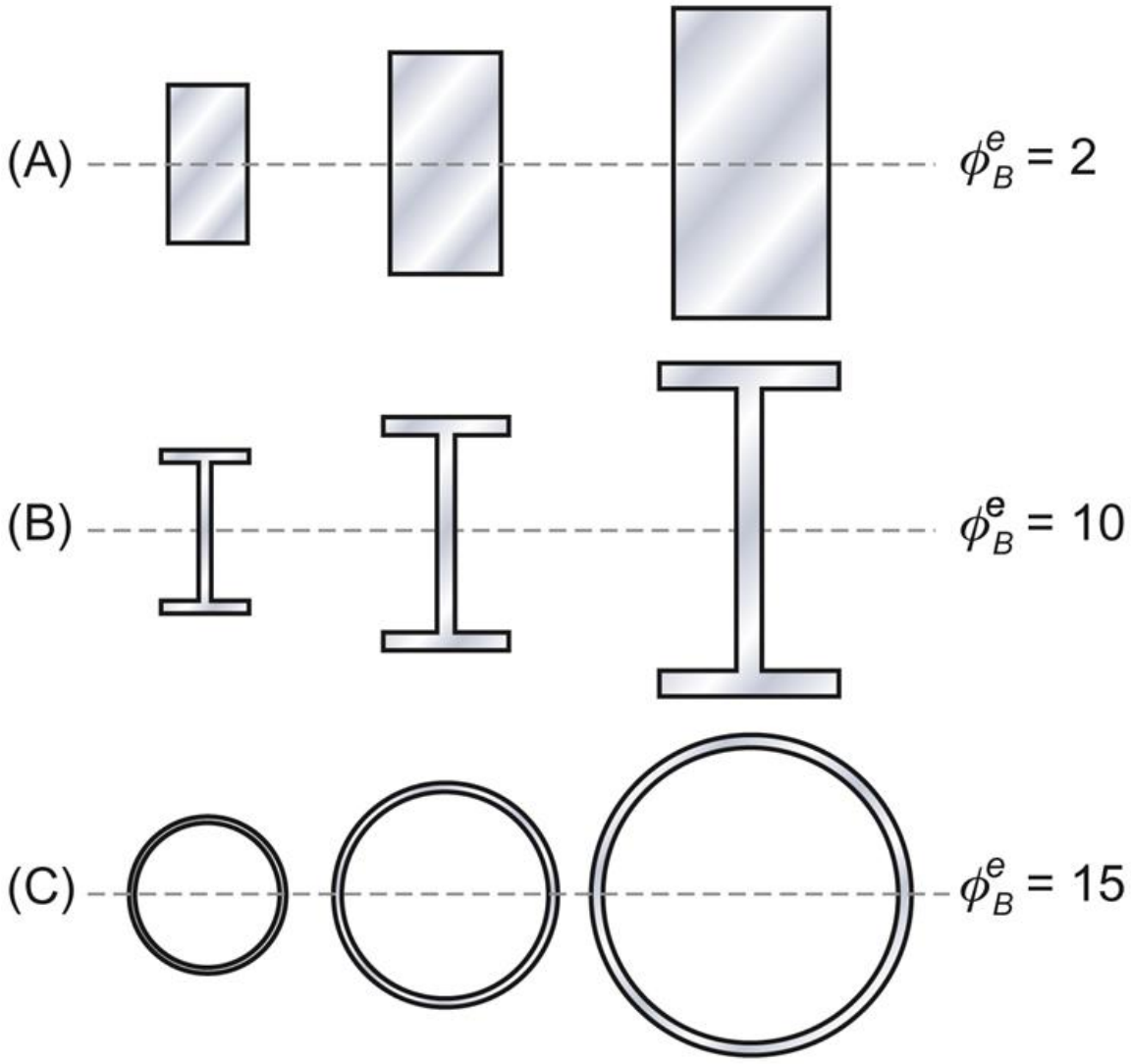


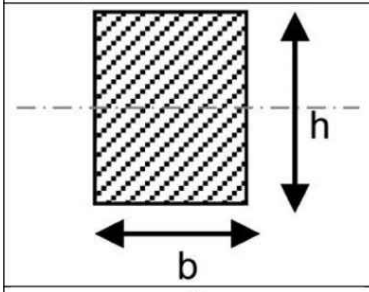
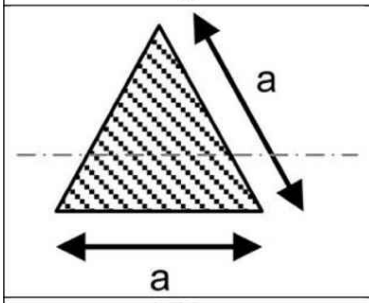
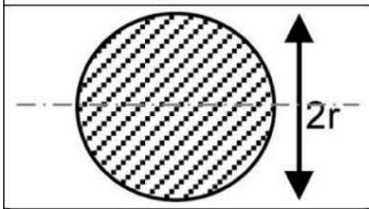
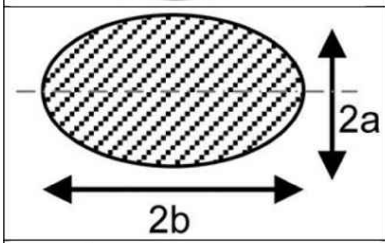
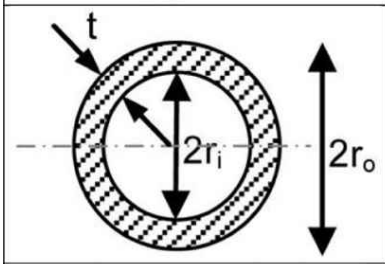
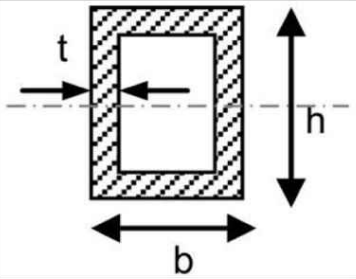
FIGURE 10.5 (A) A set of rectangular sections with $\phi_B^e = 2$; (B) a set of I-sections with $\phi_B^e = 10$; and (C) a set of tubes with $\phi_B^e = 15$. Members of a set differ in size but not in shape.

Shape-efficiency factors ϕ_B^e for common shapes in bending, calculated from the expressions for A and I in Table 10.2, are listed in the first column of Table 10.3. Solid equiaxed sections (circles, squares, hexagons, octagons) all have values very close to 1 – for practical purposes they can be set equal to 1. But if the section is elongated, or hollow, or of I-section, things change; a thin-walled tube or a slender I-beam can have a value of ϕ_B^e of 50

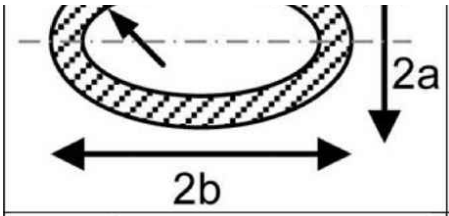
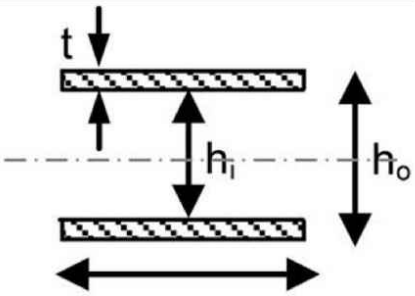
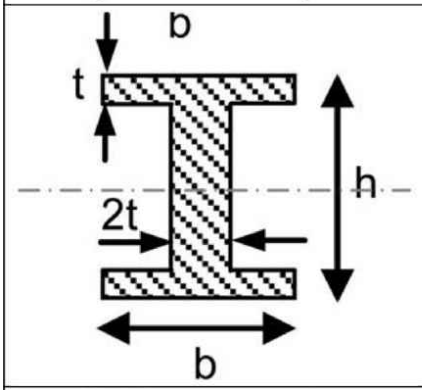
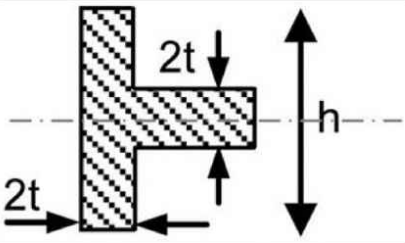
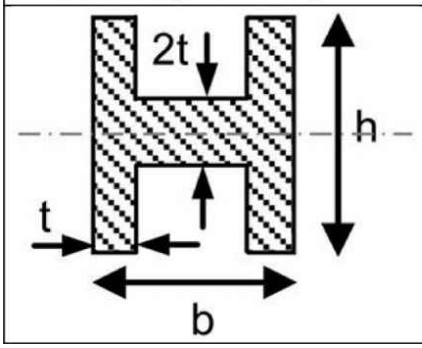
or more. A beam with $\phi_B^e = 50$ is 50 times stiffer in bending than a solid beam of the same weight.

Table 10.3

Shape Efficiency Factors

Section Shape	Bending Factor ϕ_B^e	Torsional Factor ϕ_T^e	Bending Factor ϕ_B^f	Torsional Factor ϕ_T^f
	$\frac{h}{b}$	$2.38\frac{h}{b}\left(1-0.58\frac{b}{h}\right)$ $(h>b)$	$\left(\frac{h}{b}\right)^{0.5}$	$1.6\sqrt{\frac{b}{h}}\frac{1}{\left(1+0.6\frac{b}{h}\right)}$ $(h>b)$
	$\frac{2}{\sqrt{3}}=1.15$	0.832	$\frac{3^{1/4}}{2}=0.658$	0.83
	$\frac{3}{\pi}=0.955$	1.14	$\frac{3}{2\sqrt{\pi}}=0.846$	1.35
	$\frac{3}{\pi}\frac{a}{b}$	$\frac{2.28ab}{(a^2+b^2)}$	$\frac{3}{2\sqrt{\pi}}\sqrt{\frac{a}{b}}$	$1.35\sqrt{\frac{a}{b}}$ $(a<b)$
	$\frac{3}{\pi}\left(\frac{r}{t}\right)$ $(r>>t)$	$1.14\left(\frac{r}{t}\right)$	$\frac{3}{\sqrt{2}\pi}\sqrt{\frac{r}{t}}$	$1.91\sqrt{\frac{r}{t}}$
	$\frac{1}{2}\frac{h}{t}\frac{(1+3b/h)}{(1+b/h)^2}$ $(h,b>>t)$	$\frac{3.57b^2(1-\frac{t}{b})^4}{th\left(1+\frac{b}{h}\right)^3}$	$\frac{1}{\sqrt{2}}\sqrt{\frac{h}{t}}\frac{\left(1+\frac{3b}{h}\right)}{\left(1+\frac{b}{h}\right)^{3/2}}$	$3.39\sqrt{\frac{h^2}{b\,t}}\frac{1}{\left(1+\frac{b}{h}\right)^{3/2}}$

Section Shape	Bending Factor ϕ_B^e	Torsional Factor ϕ_T^e	Bending Factor ϕ_B^f	Torsional Factor ϕ_T^f
	$\frac{3}{\pi}\frac{a}{t}\frac{(1+3b/a)}{(1+b/a)^2}$ $(a,b>>t)$	$\frac{9.12(a\ b)^{5/2}}{t(a^2+b^2)(a+b)^2}$	$\frac{3}{2\sqrt{\pi}}\sqrt{\frac{a}{t}}\frac{\left(1+\frac{3b}{a}\right)}{\left(1+\frac{b}{a}\right)^{3/2}}$	$5.41\sqrt{\frac{a}{t}}\frac{1}{\left(1+\frac{a}{b}\right)^{3/2}}$

				
	$\frac{3}{2}\frac{h_o^2}{bt}$ $(h,b >> t)$	–	$\frac{3}{\sqrt{2}}\frac{h_o}{\sqrt{b\,t}}$	–
	$\frac{1}{2}\frac{h}{t}\frac{(1+3b/h)}{(1+b/h)^2}$ $(h,b >> t)$	$1.19\left(\frac{t}{b}\right)\frac{\left(1+\frac{4b}{h}\right)}{\left(1+\frac{b}{h}\right)^2}$	$\frac{1}{\sqrt{2}}\sqrt{\frac{h}{t}}\frac{\left(1+\frac{3b}{h}\right)}{\left(1+\frac{b}{h}\right)^{3/2}}$	$1.13\sqrt{\frac{t}{b}}\frac{\left(1+\frac{4b}{h}\right)}{\left(1+\frac{b}{h}\right)^{3/2}}$
	$\frac{1}{2}\frac{h}{t}\frac{(1+4bt^2/h^3)}{(1+b/h)^2}$ $(h,b >> t)$	$0.595\left(\frac{t}{h}\right)\frac{\left(1+\frac{4b}{h}\right)}{\left(1+\frac{b}{h}\right)^2}$	$\frac{3}{4}\sqrt{\frac{h}{t}}\frac{\left(1+\frac{4bt^2}{h^3}\right)}{\left(1+\frac{b}{h}\right)^{3/2}}$	$0.565\sqrt{\frac{t}{h}}\frac{\left(1+\frac{4b}{h}\right)}{\left(1+\frac{b}{h}\right)^{3/2}}$
	$\frac{1}{2}\frac{h}{t}\frac{(1+4bt^2/h^3)}{(1+b/h)^2}$ $(h,b >> t)$	$1.19\left(\frac{t}{h}\right)\frac{\left(1+\frac{4b}{h}\right)}{\left(1+\frac{b}{h}\right)^2}$	$\frac{3}{4}\sqrt{\frac{h}{t}}\frac{\left(1+\frac{4bt^2}{h^3}\right)}{\left(1+\frac{b}{h}\right)^{3/2}}$	$1.13\sqrt{\frac{t}{h}}\frac{\left(1+\frac{4b}{h}\right)}{\left(1+\frac{b}{h}\right)^{3/2}}$

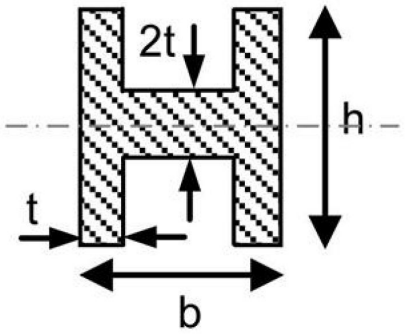
Section Shape	Bending Factor ϕ_B^e	Torsional Factor ϕ_T^e	Bending Factor ϕ_B^f	Torsional Factor ϕ_T^f
	$\frac{1}{2}\frac{h}{t}\frac{(1+4bt^2/h^3)}{(1+b/h)^2}$ $(h,b >> t)$	$1.19\left(\frac{t}{h}\right)\frac{\left(1+\frac{4b}{h}\right)}{\left(1+\frac{b}{h}\right)^2}$	$\frac{3}{4}\sqrt{\frac{h}{t}}\frac{\left(1+\frac{4bt^2}{h^3}\right)}{\left(1+\frac{b}{h}\right)^{3/2}}$	$1.13\sqrt{\frac{t}{h}}\frac{\left(1+\frac{4b}{h}\right)}{\left(1+\frac{b}{h}\right)^{3/2}}$

Fig. 10.6 is a plot of I against A for values of ϕ_B^e (Eq. 10.3). The contour for $\phi_B^e = 1$ describes the square-section reference beam. Those for $\phi_B^e = 10$ and $\phi_B^e = 100$ describe more efficient shapes, as suggested by the icons at the bottom left, in each of which the axis of bending is horizontal. But it is not always high stiffness that is wanted. Springs, cradles, suspensions, cables and other structures that must flex yet have high tensile strength rely on having a low bending stiffness. Then we want low shape efficiency. It is achieved by spreading the material in a plane containing the axis of bending to form sheets or wires, as suggested by the contours for $\phi_B^e = 0.1$ and 0.01.

Calculating Bending Shape Factors

A tube has a radius $r = 10$ mm and a wall thickness $t = 1$ mm. How much stiffer is it in bending than a solid cylinder of the same mass per unit length, m_1?

Answer

The difference is the ratio of the two shape factors. The shape factor for the tube, from Table 10.3, is $\phi_B^e = \frac{3}{\pi}\left(\frac{r}{t}\right) = 9.55.$ That for a solid circular section is $\phi_B^e = \frac{3}{\pi} = 0.955.$ The tube is stiffer then the solid section by a factor of 10.

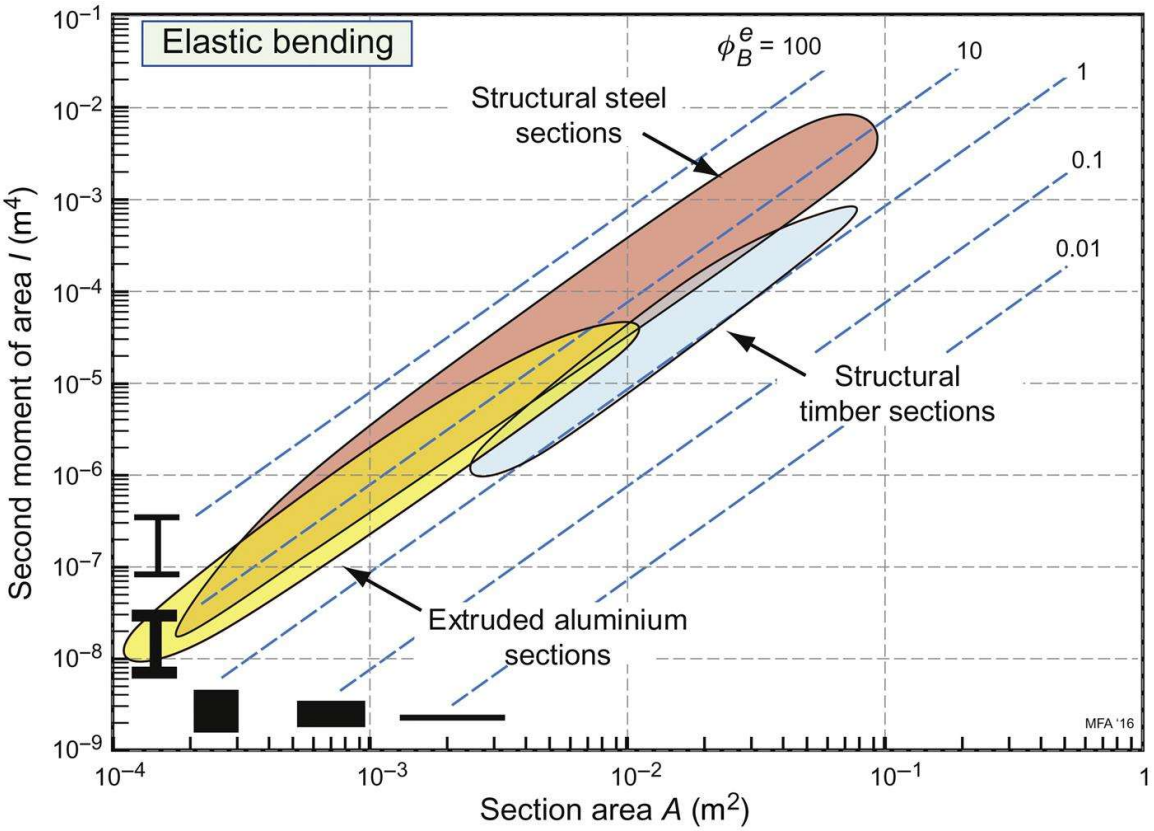


FIGURE 10.6 The second moment of area, I, plotted against section area A. Efficient structures have high values of ϕ_B^e; inefficient structures (ones that bend easily) have low values. Real structural sections have values that lie in the shaded zones. The upper limit for shape efficiency ϕ_B^e depends on the material.

Elastic twisting of shafts (Fig. 10.3C). Shapes that resist bending well may not be so good when twisted. The steps in finding a shape factor for torsion exactly parallel those for bending. The stiffness of a shaft – the torque T divided by the angle of twist, θ (Fig. 10.3C) – is proportional to GK, where G is its shear modulus and K its torsional moment of area. For circular sections K is identical with the polar moment of area, J:

$$J = \int_{\text{section}} r^2 dA \qquad \textbf{(10.4)}$$

where dA is the differential element of area at the radial distance r, measured from the centre of the section. For noncircular sections, K is less

than J; it is defined such that the angle of twist θ is related to the torque T by

$$S_T = \frac{T}{\theta} = \frac{KG}{L} \tag{10.5}$$

where L is length of the shaft and G the shear modulus of the material of which it is made. Approximate expressions for K are listed in Table 10.2.

The shape factor for elastic twisting is defined, as before, by the ratio of the torsional stiffness of the shaped section, S_T, to that, S_{To}, of a solid square shaft of the same length L and cross-section A, which, using Eq. (10.5), is:

$$\phi_T^e = \frac{S_T}{S_{To}} = \frac{K}{K_o} \tag{10.6}$$

The torsional constant K_o for a solid square section (Table 10.2, top row with b=h) is

$$K_o = 0.14\,A^2$$

giving

$$\phi_T^e = 7.14\frac{K}{A^2} \tag{10.7}$$

It, too, has the value 1 for a solid square section, and has values near 1 for any solid, equiaxed section; but for thin-walled shapes, particularly tubes, it can be large. As before, sections with the same value of ϕ_T^e differ in size but not shape. Values derived from the expressions for K and A in Table 10.2 are listed in Table 10.3.

Failure in bending. Plasticity starts when the stress, somewhere, first reaches the yield strength, σ_Y; fracture occurs when this stress first exceeds the fracture strength, σ_{fr}; and fatigue failure if it exceeds the endurance limit σ_e. Any one of these constitutes failure. As in earlier chapters, we use the symbol σ_f for the failure stress, meaning 'the local stress that will first cause yielding or fracture or fatigue failure'.

In bending, the stress σ is largest at the point y_m in the surface of the beam that lies furthest from the neutral axis. Its value is:

$$\sigma = \frac{My_m}{I} = \frac{M}{Z} \tag{10.8}$$

where M is the bending moment. Failure occurs when this stress first exceeds σ_f. Thus in problems of failure of beams, shape enters through the *section modulus*, $Z = I/y_m$. The strength-efficiency of the shaped beam, ϕ_B^f, is measured by the ratio Z/Z_o, where Z_o is the section modulus of a reference beam of square section with the same cross-sectional area, A:

$$Z_o = \frac{b_o^3}{6} = \frac{A^{3/2}}{6} \tag{10.9}$$

Thus:

$$\phi_B^f = \frac{Z}{Z_o} = \frac{6Z}{A^{3/2}} \tag{10.10}$$

Like the other shape-efficiency factor, it is dimensionless and therefore independent of scale. As before, $\phi_B^f = 1$ describes the square-section reference beam. Table 10.3 gives expressions for ϕ_B^f for other shapes derived from the values of the section modulus, Z, in Table 10.2. A beam with a failure shape-efficiency factor of 10 is 10 times stronger in bending than a solid square section of the same weight. Fig. 10.7 is a plot of Z against A for values of ϕ_B^f (Eq. (10.10)). The other contours describe shapes that are more or less efficient, as suggested by the icons.

Calculating Failure Shape Factors for Bending

A beam has a square box section with a height $h = 100$ mm, a width $b = 100$ mm and a wall thickness $t = 5$ mm. What is the value of its shape factor ϕ_B^f?

Answer

The shape factor for the box section, from Table 10.3, is
$\phi_B^f = \frac{1}{\sqrt{2}}\sqrt{\frac{h}{t}\frac{\left(1+\frac{3b}{h}\right)}{\left(1+\frac{b}{h}\right)^{3/2}}} = 4.47.$

The box section is stronger than a solid square-section beam of the same mass per unit length by a factor of 4.5.

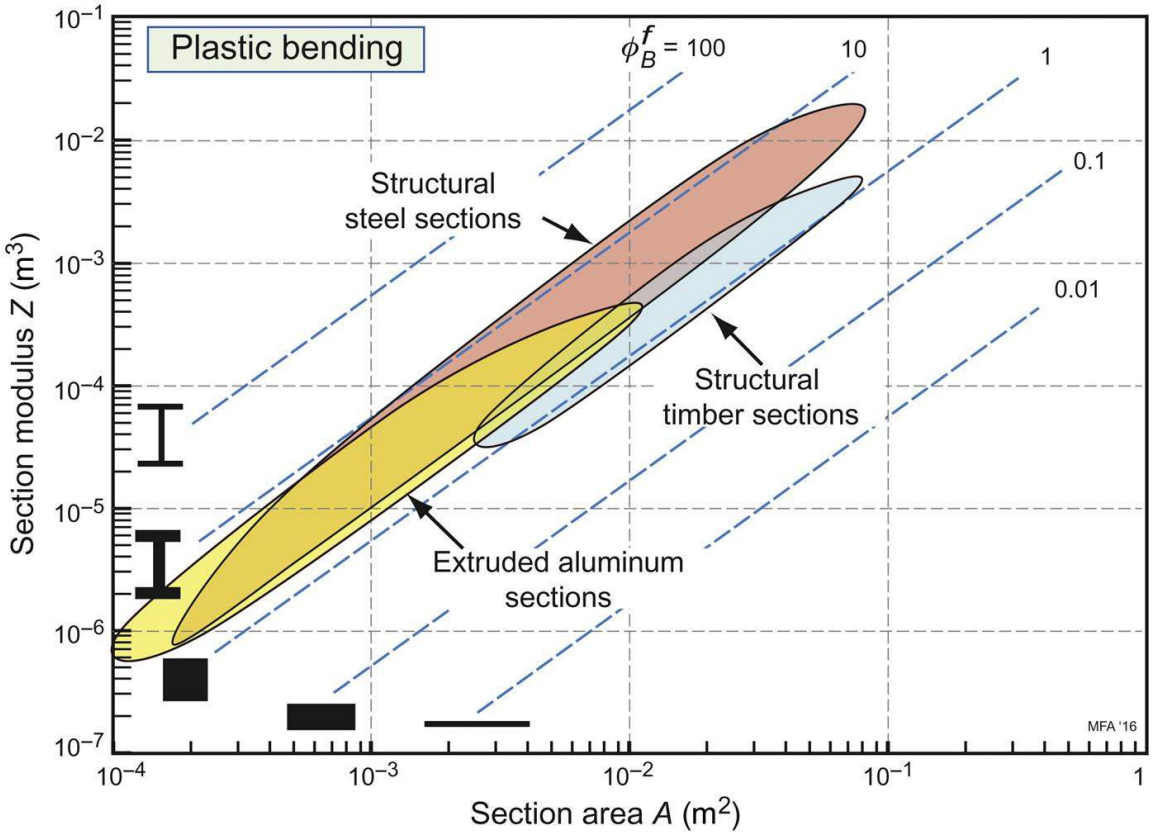


FIGURE 10.7 The section modulus, Z, plotted against section area A. Efficient structures have high values of the ratio ϕ_B^f; inefficient structures (ones that bend easily) have low values. Real structural sections have values of Z and A that lie in the shaded zones. The upper limit for ϕ_B^f again depends on the material.

Failure in torsion. In torsion the problem is more complicated. For circular rods or tubes subjected to a torque T (as in Fig. 10.3C) the shear stress τ is a maximum at the outer surface, at the radial distance r_m from the axis of bending:

$$\tau = \frac{Tr_m}{J} \tag{10.11}$$

The quantity J/r_m in twisting has the same character as I/y_m in bending. For noncircular sections with ends that are free to warp, the maximum surface stress is given instead by

$$\tau = \frac{T}{Q} \tag{10.12}$$

where Q, with units of m^3 now plays the role in torsion that Z plays in bending. This allows the definition of a shape factor, ϕ_T^f, for failure in torsion, following the same pattern as before:

$$\phi_T^f = \frac{Q}{Q_o} = 4.8\frac{Q}{A^{3/2}} \tag{10.13}$$

Values of Q and ϕ_T^f are listed in Tables 10.2 and 10.3. Shafts with a solid equiaxed sections all have values of ϕ_T^f close to 1.

Fully plastic bending or twisting (such that the yield strength is exceeded throughout the section) involve a further pair of shape factors. Generally speaking, shapes that resist the onset of plasticity well are resistant to full plasticity also, so ϕ_B^{pl} does not differ much from ϕ_B^f. New shape factors for these are not, at this stage, necessary.

Axial loading: column buckling. A column of length L, loaded in compression, buckles elastically when the load exceeds the Euler load

$$F_c = \frac{n^2\pi^2 EI_{\min}}{L^2} \tag{10.14}$$

where n is a constant that depends on the end-constraints. The resistance to buckling, then, depends on the smallest second moment of area, I_{min}, and the appropriate shape factor (ϕ_B^e) is the same as that for elastic bending (Eq. (10.3)) with I replaced by $I_{\min}$,

Using Shape to Supress Buckling

A slender, solid, cylindrical column of height L supports a load F. If overloaded, the column fails by elastic buckling. By how much is the load-bearing capacity increased if the solid cylinder is replaced by a hollow circular tube of the same cross-section, A?

Answer

Replacing $I_{\min}$ in Eq. (10.14) by $\phi_B^e A^2/12$ from Eq. (10.3) gives:

$$F_c = \frac{n^2\pi^2 A^2}{12L^2} E\phi_B^e$$

The failure load is increased by the ratio of the shape factor for the tube, $\phi_B^e = 3r/\pi t$, to that of the solid cylinder, $\phi_{B,Solid}^e = 3/\pi$ (Table 10.3). Thus

$$\frac{F_c}{F_{c,Solid}} = \frac{\phi_B^e}{\phi_{B,Solid}^e} = \frac{r}{t}$$

10.3 Limits to Shape Efficiency

The conclusions so far: if you wish to make stiff, strong structures that are efficient (using as little material as possible) choose the shape with the largest possible shape factor, ϕ. All true, but there are limits. We examine these next.

Empirical limits. There are practical limits for the slenderness of sections, and these determine, for a given material, the maximum attainable efficiencies. These limits may be imposed by manufacturing constraints: the difficulty or expense of making an efficient shape may simply be too great. More often they are imposed by the properties of the material itself. We explore these limits in two ways. The first is empirical: by examining the shapes in which real materials – steel, aluminium etc. – are actually made, recording the limiting efficiency of available sections. The second is by the analysis of the mechanical stability of shaped sections.

Standard sections for beams, shafts, and columns are generally prismatic. Prismatic shapes are easily made by rolling, extrusion, drawing, pultrusion or sawing (see the cover picture of this chapter). The section may be solid, closed-hollow (like a tube or box) or open-hollow (an I-, U- or L-section, for instance). Each class of shape can be made in a range of

materials. Some are available as standard, off-the-shelf, sections, notably structural steel, extruded aluminium alloy, pultruded GFRP (glass fibber reinforced polyester or epoxy), and structural timber. Fig. 10.8 shows values for I and A (the same axes as Fig. 10.6) for 1880 standard sections made from these four materials, with contours of the shape factor ϕ_B^e superimposed. Some of the sections have $\phi_B^e \approx 1$; they are the ones with solid cylindrical or square sections. More interesting is that none have values of ϕ_B^e greater than about 65; there is an *upper limit* for the shape. A similar plot for Z and A (the axes of Fig. 10.7) indicates an upper limit for ϕ_B^f of about 15. When these data are segregated by material[2] it is found that each material has its own upper limit of shape, and that they differ considerably. Similar limits hold for the torsional shape factors. They are listed in Table 10.4 and were plotted as shaded bands on Figs. 10.6 and 10.7.

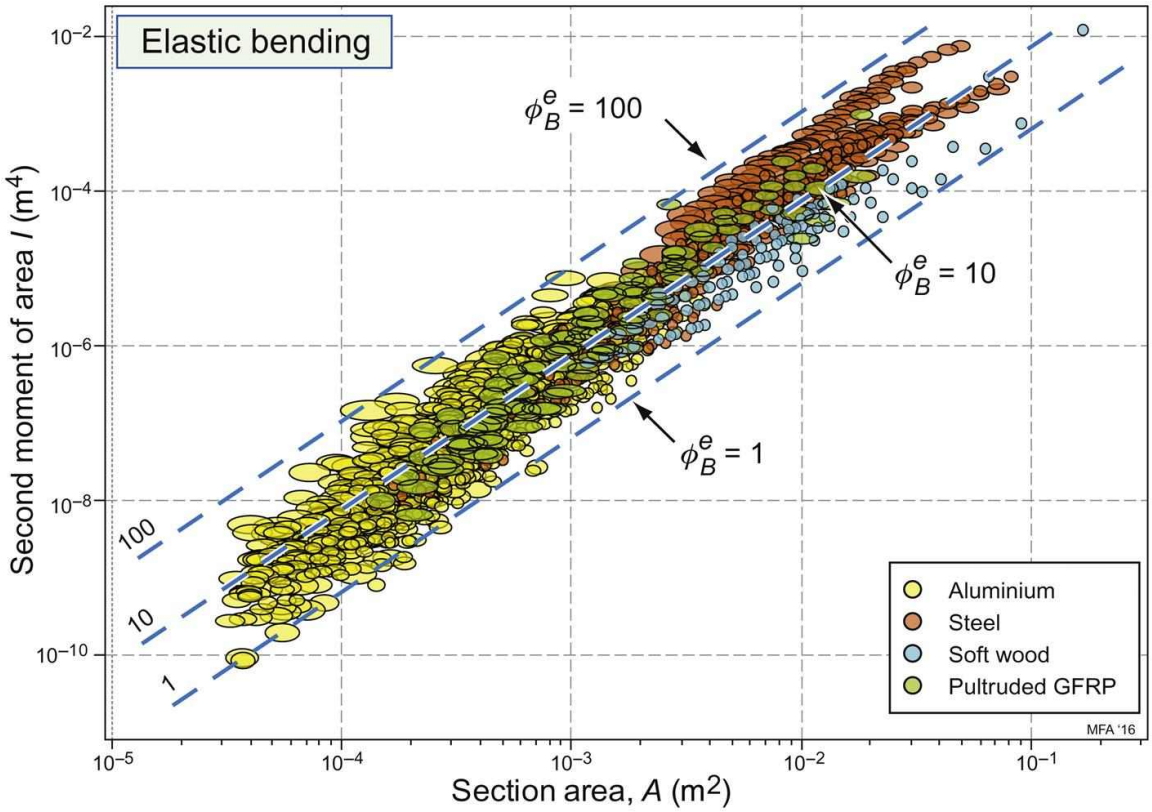


FIGURE 10.8 Log (I) plotted against log (A) for standard sections of steel, aluminium, pultruded GFRP and wood. Contours of ϕ_B^e are shown, illustrating that there is an upper limit. A similar plot for log (Z) against log (A) reveals an upper limit for ϕ_B^f.

Table 10.4

Empirical Upper Limits for the Shape Factors ϕ_B^e, ϕ_T^e, ϕ_B^f and ϕ_T^f

Material	$(\phi_B^e)_{max}$	$(\phi_T^e)_{max}$	$(\phi_B^f)_{max}$	$(\phi_T^f)_{max}$
Structural steel	65	25	13	7
6061 aluminium alloy	44	31	10	8
GFRP and CFRP	39	26	9	7
Polymers (e.g., nylons)	12	8	5	4
Woods (solid sections)	5	1	3	1
Elastomers	<6	3	–	–

The upper limits for shape efficiency are important. They are central to the design of structures that are light, or for which, for other reasons (cost, perhaps) the material content should be minimized. Two questions then arise. What sets the upper limit on shape efficiency? And why does the limit depend on material? One explanation is simply the difficulty of making them. Steel, e.g., can be drawn to thin-walled tubes or formed (by rolling, folding or welding) into efficient I-sections; shape factors as high as 50 are common. Wood cannot so easily be shaped; plywood technology could, in principle, be used to make thin tubes or I-sections, but in practice a wood-shape with a value of ϕ_B^e greater than 5 is rare. Composites, too, can be limited by the present difficulty in making them into thin-walled prismatic shapes, though the technology for doing this now exists.

But there is a more fundamental constraint on shape efficiency. It has to do with local buckling.

Limits imposed by local buckling. When efficient shapes *can* be fabricated, the limits of the efficiency are set by the competition between failure modes. Inefficient sections fail in a simple way: they yield, they fracture, or they suffer large-scale buckling. In seeking greater efficiency, a shape is chosen that raises the load required for the simple failure modes, but in doing so the structure is pushed nearer the load at which new modes – particularly those involving *local buckling* – become dominant.

It is a characteristic of shapes that approach their limiting efficiency that two or more failure modes occur at almost the same load. Why? Here is a simple explanation. If failure by one mechanism occurs at a lower load

than all others, the section shape can be adjusted to suppress it; but this pushes the load upwards until another mechanism becomes dominant. If the shape is described by a single variable (ϕ) then when two mechanisms occur at the same load you have to stop – no further shape adjustment can improve things. Adding webs, ribs or other stiffeners gives additional variables, allowing shape to be optimised further, but we shall not pursue that here.

Here's a simple example to illustrate this limit imposed by local buckling. Think of a drinking straw – it is a thin-walled hollow tube about 5 mm in diameter. It is made of polystyrene, but there is not much of it. If the straw were collapsed into a solid cylinder, the cylinder would have a diameter of less than 1 mm, and because polystyrene has a low modulus, it would have a low bending stiffness. If bent sufficiently it would fail by plastic yielding, and if bent further, by fracturing. Now restore it to its straw-shape and bend it. It is much stiffer than before, but as it bends it becomes ovoid and then fails suddenly and unpredictably by kinking – a form of local buckling (try it).

A fuller analysis[3] indicates that the maximum practical shape efficiency – when not limited by manufacturing constraints – is indeed dictated by the onset of local buckling. A thick-walled tube, loaded in bending, yields before it buckles locally. It can be made more efficient by increasing its diameter and reducing its wall thickness, increasing I and Z, but this reduces the load at which the increasingly slender walls of the section start to buckle like the drinking straw. When the load for local buckling falls below that for yield, failure occurs by local buckling and this is undesirable because buckling is defect-dependent and causes sudden, unpredictable, collapse. Other section-shapes – boxes, I-sections and the like – also have their own local buckling modes when thin. The implication drawn from detailed versions of plots like that of Fig. 10.8 is that real sections are designed to avoid local buckling, setting the upper limit on shape efficiency. Not surprisingly, the limit depends on the material – those with low strength and high modulus yield easily but do not easily buckle, and vice versa. A rough rule-of-thumb follows from this: it is that

$$(\phi_B^e)_{\max} \approx 2.3\left(\frac{E}{\sigma_f}\right)^{1/2} \qquad \textbf{(10.15a)}$$

and (a consequence of the fact that $I/Zh \approx 0.5$ where h is the depth of the section)

$$(\phi_B^f)_{\max} \approx \sqrt{(\phi_B^e)_{\max}} \qquad \textbf{(10.15b)}$$

allowing approximate estimates for the maximum shape efficiency of materials.

Higher efficiencies are possible when precise loading conditions are known, allowing customised application of stiffeners and webs to suppress local buckling. This allows a further increase in the ϕ values until new, localised, buckling modes appear. These, too, can be suppressed by a further hierarchy of structuring; ultimately, the ϕ values are limited only by manufacturing constraints. But this is getting more sophisticated than we need for a general selection of material and shape. Eqs (10.15a and b) will do all we need.

10.4 Exploring Material-Shape Combinations

Stiffness-limited design. The $E - \rho$ material property chart (Figure 3.3) displays the stiffness-related property (the modulus) of materials. The shape-efficiency chart of Fig. 10.6 captures information about the influence of shape on bending stiffness. By linking them[4] the performance of the section can be explored. In Fig. 10.9 the two charts are placed in opposite corners of a square. The material property chart (here much simplified, with only a few materials) is at the top left. The shape-efficiency chart is at the lower right, with its axes interchanged so that I is along the bottom and A is up the side; shaded bands on it show the populated areas, derived from plots like that of Fig. 10.8. The remaining two quadrants automatically form two more charts, each sharing axes with the first two. That at the upper right has axes of E and I; the diagonal lines show the bending stiffness of the section, EI. That at the lower left has axes of A and ρ; the diagonal contours show the metric of performance: the mass per unit length, $m_l = \rho A$, of the section.

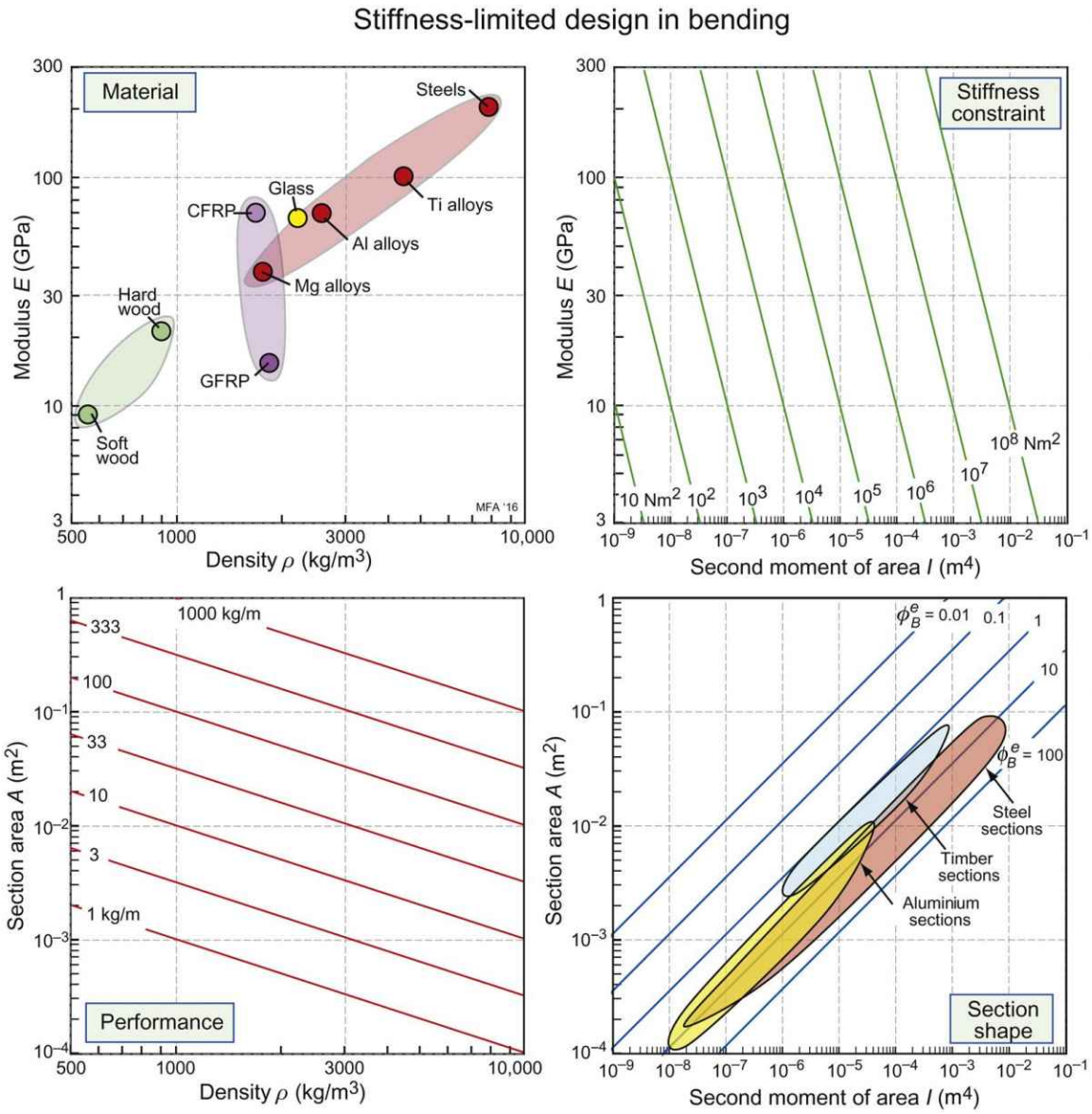


FIGURE 10.9 The 4-quadrant chart assembly for exploring structural sections for stiffness-limited design. Each chart shares its axes with its neighbours.

This set of charts enables the assessment and comparison of stiffness-limited sections. It can be used in several ways, of which the following, shown in Fig. 10.10, is typical.

- Chose a material for the section and mark its modulus E and density ρ onto the **material property chart** in the first quadrant of the figure.
- Choose the desired section stiffness (EI); it is a constraint that must be met by the section. Extend a horizontal line from the value of E for the

material to the appropriate contour in the **stiffness constraint chart** in the second quadrant.

- Drop a vertical from this point onto the **section shape chart** in the third quadrant to the line describing the shape factor ϕ_B^e for the section. Values of I and A outside the shaded bands are forbidden.
- Extend a horizontal line from this point to the **performance chart** in the final quadrant. Drop a vertical from the material density ρ in the material chart. The intersection shows the mass per unit length of the section.

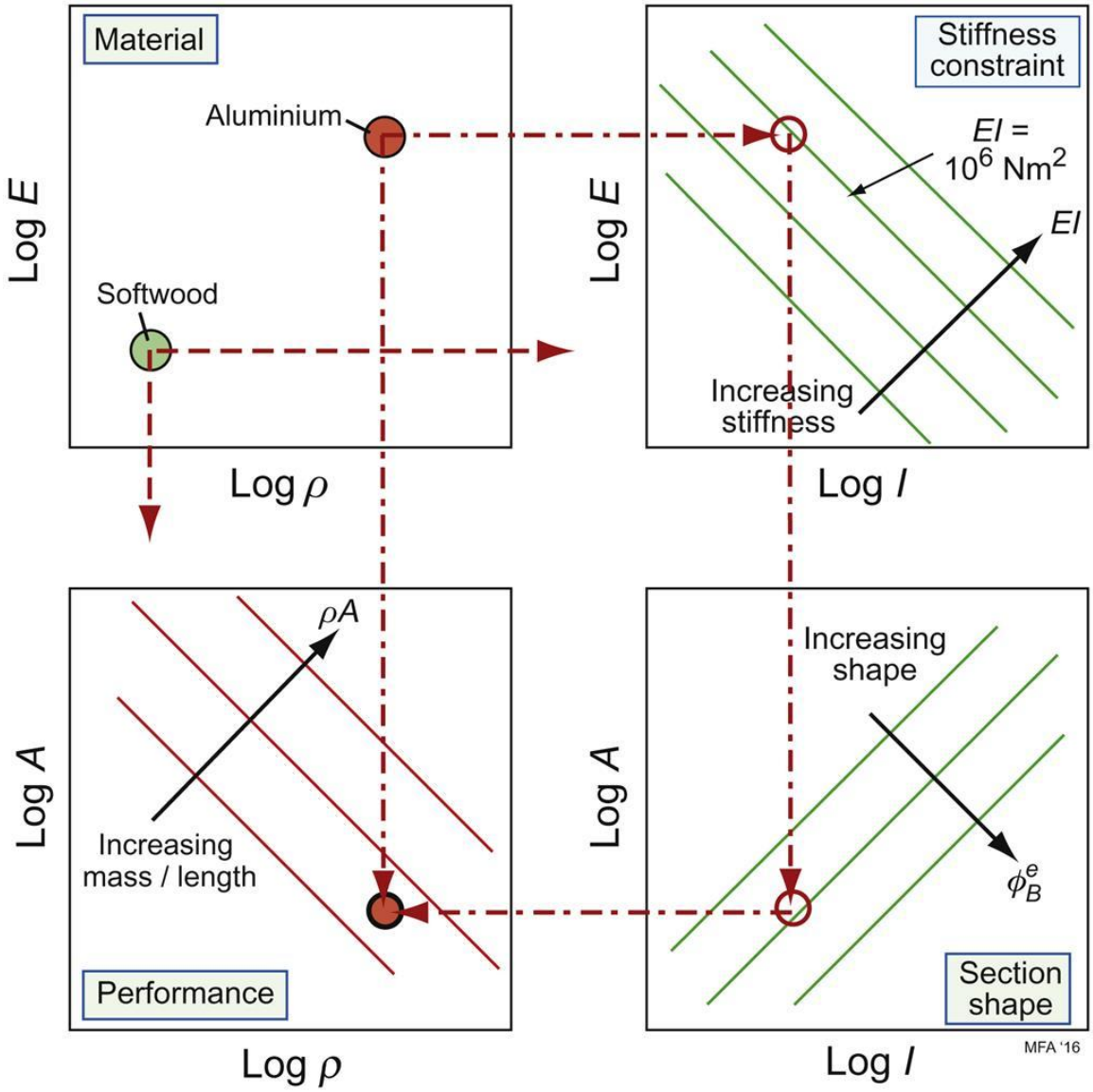


FIGURE 10.10 A schematic showing how the 4-quadrant chart is used.

Fig. 10.11 compares the mass of rolled steel and extruded aluminium sections both with $\phi_B^e = 10$, and timber sections with $\phi_B^e = 2$, with the constraint of a bending stiffness of $EI = 10^6\ \mathrm{N \cdot m^2}$. The extruded aluminium section gives the lightest beam. Remarkably, an efficiently shaped steel beam is nearly as light – for a given bending stiffness – as is one made of timber, even though the density of steel is 12 times greater than that of wood. It is because of the higher shape factor possible with steel.

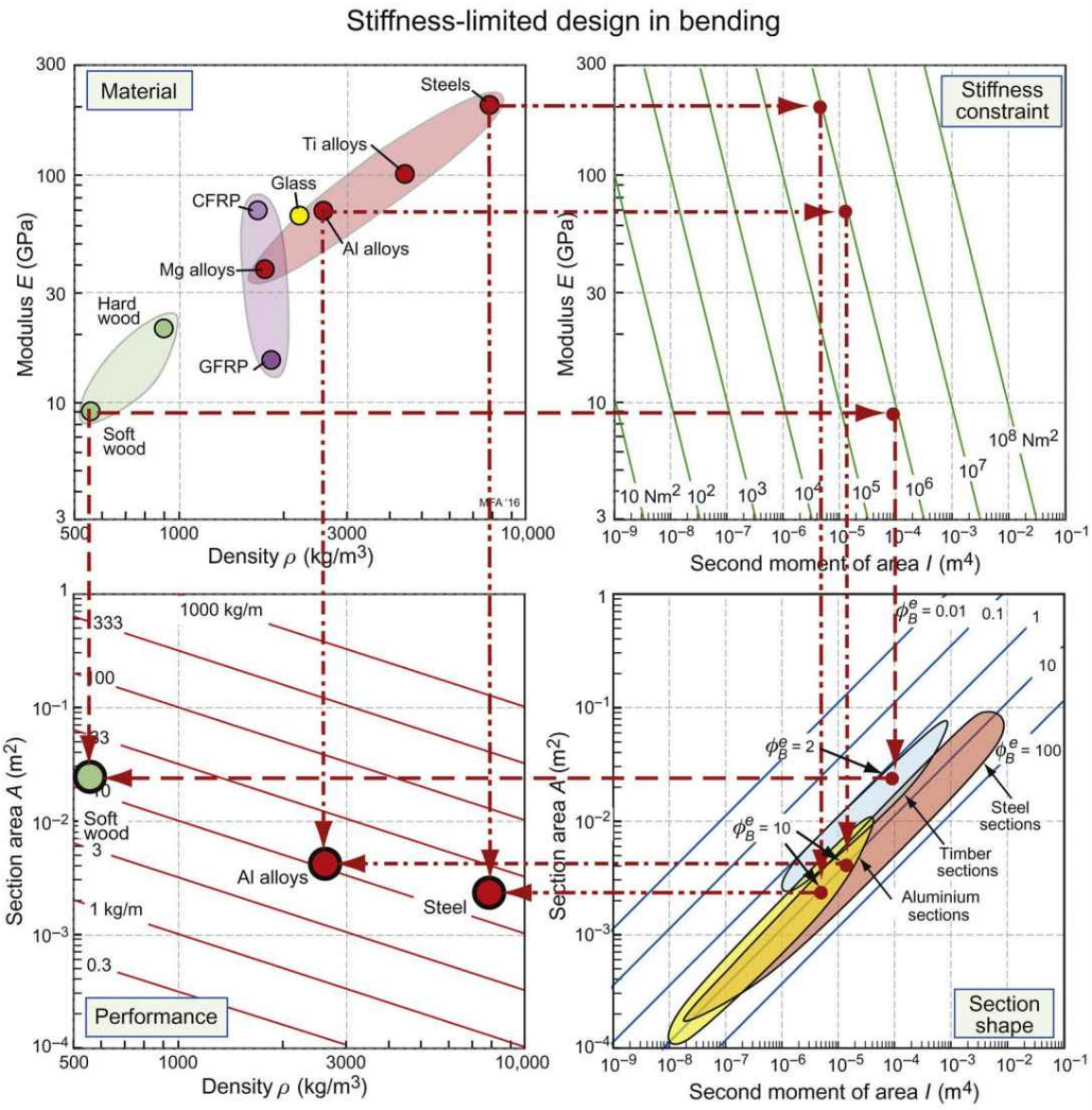


FIGURE 10.11 A comparison of steel, aluminium and wood section for stiffness-limited design with EI=10^6 Nm^2. Aluminium gives section with a mass of 10 kg/m; steel is almost three times heavier.

Strength-limited design. The reasoning follows a similar path. In Fig. 10.12 the strength-density ($\sigma_f - \rho$) **material property chart** is placed at the top left. The *Z*-*A* **shape chart** (Fig. 10.7 with the axes interchanged) is at the bottom right. As before, the remaining two quadrants generate two more charts. The **strength constraint chart** at the upper right has axes of σ_f and Z; the diagonal lines show the bending strength of the section, $Z\sigma_f$. The **performance chart** at the lower left has the same axes as before – the

quantities A and ρ – and the diagonal contours again show the metric of performance: the mass per unit length, $m_l = \rho A$, of the section.

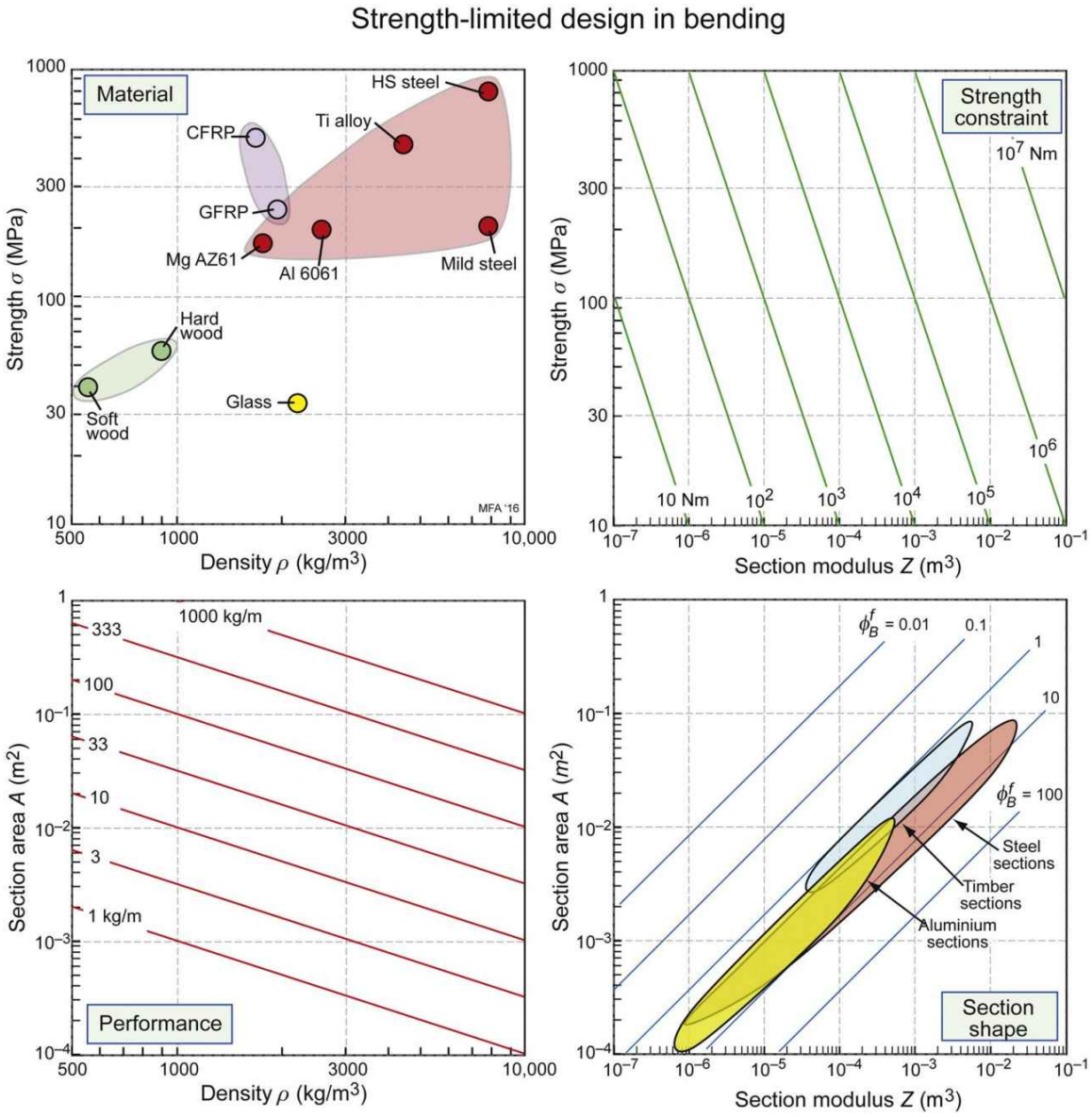


FIGURE 10.12 The 4-quadrant chart assembly for exploring structural sections for strength-limited design. Like those for stiffness, each chart shares its axes with its neighbours.

It is used in the same way as that for stiffness-limited design. Try it for steel with $\phi_B^f = 15$, aluminium with $\phi_B^f = 10$ and GFRP with $\phi_B^f = 5$, for a required failure moment of $Z\sigma_f = 10^4\ \mathrm{N \cdot m}$.

You will find that GFRP offers the lightest solution.

10.5 Material Indices That Include Shape

The chart-arrays of Figs. 10.9 and 10.12 link material, shape, constraint and performance objective in a graphical, though rather clumsy, way. There is a neater way – and this is to build them into the material indices of Chapter 4, Materials Selection – the Basics. Remember that most indices don't need this refinement – the performance they characterize does not depend on shape. Stiffness and strength-limited design do. The indices for these can be adapted to include the relevant shape factor, such that they characterize material-shape combinations.

The method is illustrated below for minimum weight design. It can be adapted to other objectives in obvious ways. It follows the derivations of Chapter 4, Materials Selection – the Basics, with one extra step to bring in the shape.

Elastic bending of beams. Consider the selection of a material for a beam of specified bending stiffness S_B^* and length L (the constraints), to have minimum mass, m (the objective). The mass m of a beam of length L and section area A is given, as before, by

$$m = AL\rho \tag{10.16}$$

Its bending stiffness is

$$S_B = C_1 \frac{EI}{L^3} \tag{10.17}$$

where C_1 is a constant that depends only on the way the loads are distributed on the beam. Replacing I by $\phi_B^e A^2/12$ (Eq. 10.3) gives

$$S_B = \frac{C_1}{12}\frac{E}{L^3}\phi_B^e A^2 \tag{10.18}$$

Using this to eliminate A in Eq. (10.16) and inserting the target stiffness S_B^* gives the mass of the beam.

$$m = \left(\frac{12S_B^*}{C_1}\right)^{1/2} L^{5/2} \left[\frac{\rho}{(\phi_B^e E)^{1/2}}\right] \tag{10.19}$$

This is identical to Eq. 4.14 but for one thing: the modulus E in the earlier derivation is replaced by $\phi_B^e E$ in this one. Everything in the equation is specified except the term in square brackets, and this depends only on material and shape. For beams with the *same* shape (and thus the same value of ϕ_B^e) the best choice is the material with the greatest value of $E^{1/2}/\rho$ – the result derived in Chapter 4, Materials Selection – the Basics. But if we want the lightest material-shape combination, it's the one with the greatest value of the index

$$M_1 = \frac{(\phi_B^e E)^{1/2}}{\rho} \tag{10.20}$$

This index allows material-shape combinations to be ranked. Here is an example.

Comparing Different Materials With Different Shapes, Stiffness

A shaped material is required for a stiff beam of minimum mass. Four materials are available with the properties and typical shapes listed in Table 10.5. Which material-shape combination has the lowest mass for a given stiffness?

Table 10.5

The Selection of Material and Shape for a Light, Stiff, Beam

Material	$\rho(\mathrm{Mg/m^3})$	E (GPa)	ϕ_B^e	$E^{1/2}/\rho$	$(\phi_B^e E)^{1/2}/\rho$
1020 Steel	7.85	205	20	1.8	8.2
6061-T4 Al	2.7	70	15	3.1	**12.0**
GFRP (isotropic)	1.75	28	8	2.9	8.5
Wood (oak)	0.9	13.5	2	**4.1**	5.8

Answer

The second to last column of the table shows the simple 'fixed shape' index $E^{1/2}/\rho$: wood has the greatest value, more than twice that of steel. But when each material is shaped efficiently (last column) wood has the *lowest* value of M_1 – even steel is better; the aluminium alloy wins, surpassing steel and GFRP.

Elastic twisting of shafts. The procedure for elastic twisting of shafts is similar. A shaft of section A and length L is subjected to a torque T. It twists through an angle θ. We want the torsional stiffness, $S_T = T/\theta$ to meet a specified target, S_T^*, at minimum mass. The torsional stiffness is

$$S_T = \frac{KG}{L} \tag{10.21}$$

where G is the shear modulus. Replacing K by ϕ_T^e using Eq. (10.7) gives

$$S_T = \frac{G}{7.14L}\phi_T^e A^2 \tag{10.22}$$

Using this to eliminate A in Eq. (10.16) and inserting the target stiffness S_T^* gives

$$m = \left(7.14\frac{S_T^*}{L^3}\right)^{1/2} L^{3/2}\left[\frac{\rho}{(\phi_T^e G)^{1/2}}\right] \tag{10.23}$$

The best material-and-shape combination is that with the greatest value of $\frac{(\phi_T^e\, G)^{1/2}}{\rho}$. The shear modulus, G, is closely related to Young's modulus E. For practical purposes we approximate G by $3/8E$, then the index becomes

$$M_2 = \frac{(\phi_T^e E)^{1/2}}{\rho} \tag{10.24}$$

For shafts of the same shape, this reduces to $E^{1/2}/\rho$ again. When shafts differ in both material and shape, the material index (10.24) is the one to use.

Eqs. (10.19) and (10.23) give a way of calculating shape factors for complex structures like bridges and trusses. Inverting 10.19, e.g., gives

$$\phi_B^e = \frac{12 S_B}{C_1}\frac{L^5}{m^2}\left[\frac{\rho^2}{E}\right] \tag{10.25}$$

Thus if the mass of the structure, its length and its bending stiffness are known (as they are for large bridge-spans) and the density and modulus of the material of which it is made are known, the shape factor can be calculated. Inserting data for existing bridges gives values between 50 and 200. These are larger than the maximum values in Table 10.4. It is because the bridges are 'structured-structures' with two or more levels of structure. The high values of ϕ are examples of the way in which shape efficiency can be increased by a hierarchy of structuring.

Back Calculating Shape Factors From Known Mass and Stiffness

A hollow tubular aluminium extrusion of complex ribbed shape has a mass per unit length $m_l = 0.3$ kg/m. A length $L = 1$ m of the extrusion, loaded in 3-point bending by a central load $W = 10$ kg suffers a midpoint deflection $\delta = 2$ mm. What is the shape factor ϕ_B^e of the section? (For aluminium $E = 70$ GPa, $\rho = 2700$ kg/m^3 and for 3-point bending, $C_1 = 48$ (Appendix A).

Answer

The force exerted by the load W is $F = Wg = 98.1$ N. The stiffness of the beam is $S_B = F/\delta = 4.9 \times 10^4$ N/m. Inserting the data into Eq. (10.25) gives $\phi_B^e = 13.2$.

Failure of beams and shafts. The procedure is the same. A beam of length L, loaded in bending, must support a specified load F without failing and be as light as possible. When section shape is a variable the best choice is found as follows. Failure occurs if the bending moment exceeds

$$M = Z\sigma_f$$

where Z is the section modulus and σ_f is the stress at which failure occurs. Replacing Z by $\frac{1}{6}\phi_B^f A^{3/2}$ (Eq. (10.10)) gives

$$M = \frac{\sigma_f}{6}\phi_B^f A^{3/2} \tag{10.26}$$

Substituting this into Eq. (10.16) for the mass of the beam gives

$$m = (6\ M)^{2/3} L \left[\frac{\rho^{3/2}}{\phi_B^f \sigma_f}\right]^{2/3} \tag{10.27}$$

The best material and shape combination is that with the greatest value of the index

$$M_3 = \frac{(\phi_B^f \sigma_f)^{2/3}}{\rho} \tag{10.28}$$

A similar analysis for torsional failure gives

$$M_4 = \frac{(\phi_T^f\ \sigma_f)^{2/3}}{\rho} \tag{10.29}$$

At constant shape both indices reduce to the familiar ${\sigma_f}^{2/3}/\rho$ of Chapter 4, Materials Selection – the Basics; but when shape as well as material are to be compared, the full index must be used.

Comparing Different Materials With Different Shapes, Strength

A shaped material is required for a strong beam of minimum mass. Four materials are available with the properties and shapes listed in Table 10.6. Which combination has the lowest mass for a given bending strength?

Table 10.6

The Selection of Material and Shape for a Light, Strong, Beam

Material	$\rho(\mathrm{Mg/m^3})$	$\sigma_f(\mathrm{MPa})$	ϕ_B^f	$\sigma_f^{2/3}/\rho$	$(\phi_B^f \sigma_f)^{2/3}/\rho$
1020 Steel, normalized	7.85	330	5	6.1	17.8
6061-T4 Al	2.7	110	4	8.5	**21.4**
GFRP SMC (isotropic)	2.0	80	3	9.3	19.3
Wood (oak), along grain	0.9	50	1.5	**15**	19.7

Answer

The second to last column of the table shows the simple 'fixed shape' index $\sigma_f^{2/3}/\rho$: wood has the greatest value, more than three times that of steel. But when each material is shaped (last column) the aluminium alloy wins, surpassing steel and GFRP.

10.6 Graphical Coselecting Using Indices

Shaped materials can be plotted onto material property charts. All the selection criteria still apply. Here is how it works.

The material index for elastic bending (Eq. (10.20)) can be rewritten as

$$M_1 = \frac{(\phi_B^e E)^{1/2}}{\rho} = \frac{(E/\phi_B^e)^{1/2}}{\rho/\phi_B^e} = \frac{E^{*1/2}}{\rho^*} \tag{10.30}$$

The equation says that a material with modulus E and density ρ, when structured, can be thought of as a new material with a modulus and density of

$$E^* = \frac{E}{\phi_B^e} \quad \text{and} \quad \rho^* = \frac{\rho}{\phi_B^e} \tag{10.31}$$

The $E - \rho$ chart is shown schematically in Fig. 10.13. The 'new' material properties E^* and ρ^* can be plotted onto it. Introducing shape ($\phi_B^e = 10$ for example) moves the material **M** to the lower left along a line of slope 1, from the position E, ρ to the position $E/10, \rho/10$ as shown in the figure. The

selection criteria are plotted onto the figure as before: a constant value of the index of $E^{1/2}/\rho$ for instance, plots as a straight line of slope 2, shown for one value of $E^{1/2}/\rho$ as a blue line. The introduction of shape has moved the material from a position below this line to one above; its performance has improved. Elastic twisting of shafts is treated in the same way.

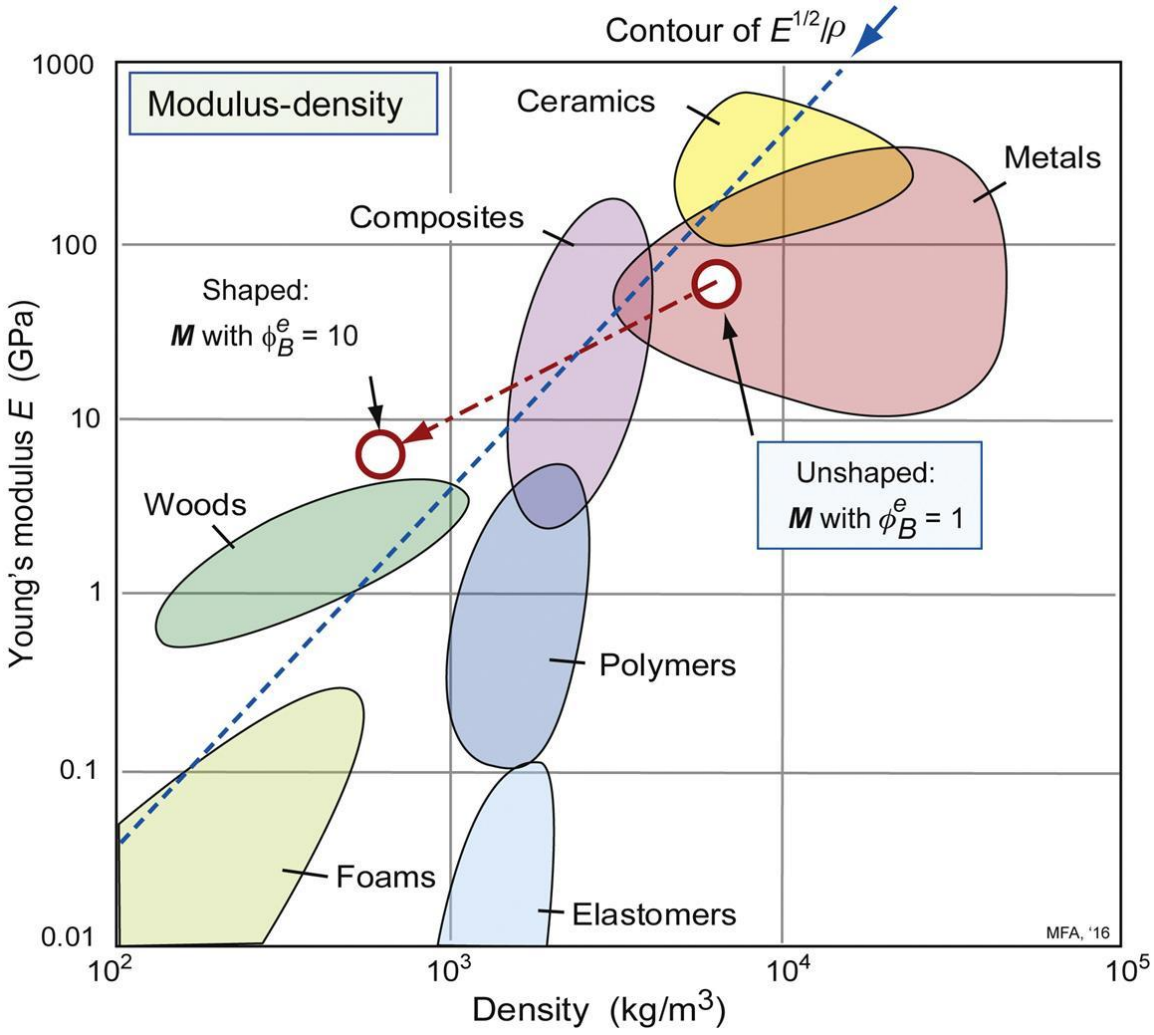


FIGURE 10.13 The structured material behaves like a new material with a modulus $E^* = E/\phi_B^e$ and a density $\rho^* = \rho/\phi_B^e$, moving it from a position below the broken selection line to one above.

Materials selection based on strength (rather than stiffness) at minimum weight uses a similar procedure. The material index for failure in bending (Eq. 10.28), can be rewritten as follows

$$M_3 = \frac{(\phi_B^f \sigma_f)^{\frac{2}{3}}}{\rho} = \frac{(\sigma_f/(\phi_B^f)^2)^{\frac{2}{3}}}{\rho/(\phi_B^f)^2} = \frac{\sigma_f^{*\,2/3}}{\rho^*} \qquad \textbf{(10.32)}$$

The material with strength σ_f and density ρ, when shaped, behaves in bending like a new material of strength and density

$$\sigma_f^* = \frac{\sigma_f}{(\phi_B^f)^2} \quad \text{and} \quad \rho^* = \frac{\rho}{(\phi_B^f)^2} \qquad \textbf{(10.33)}$$

The rest will be obvious. Introducing shape ($\phi_B^f = 3$, say) moves a material **M** along a line of slope 1, taking it, in the schematic, from a position σ_f, ρ below the material index line (the broken line) to the position $\sigma_f/9, \rho/9$ that lies above it. The performance has again improved. Torsional failure is analysed by using ϕ_T^f in place of ϕ_B^f.

The value of this approach is that the plots have retained their generality. It allows selection by any of the earlier criteria, correctly identifying materials for the lightest ties, beams or panels.

10.7 Architectured Materials: Microscopic Shape

Survival, in nature, is closely linked to structural efficiency. The tree that captures the most sunlight is the one that, with a given resource of cellulose, grows the tallest. The creature that and wins the most fights or – if prey rather than predator – runs the fastest is the one that, with a given allocation of hydroxyapatite develops the strongest, lightest bone structure. Structural efficiency means survival. It is worth asking how nature does it.

Microscopic shape. The shapes listed in Tables 10.2 and 10.3 achieve efficiency through their *macroscopic* shape. Structural efficiency can be achieved in another way: through shape on a small scale; *microscopic* or 'microstructural' shape (Fig. 10.14). Wood is an example. The solid component of wood (a composite of cellulose, lignin and other polymers) is shaped into small prismatic cells, dispersing the solid further from the axis of bending or twisting of the branch or trunk of the tree, increasing both stiffness and strength. The added efficiency is characterized by a set of *microscopic shape factors*, ψ *(psi)*, with definitions exactly like those of ϕ.

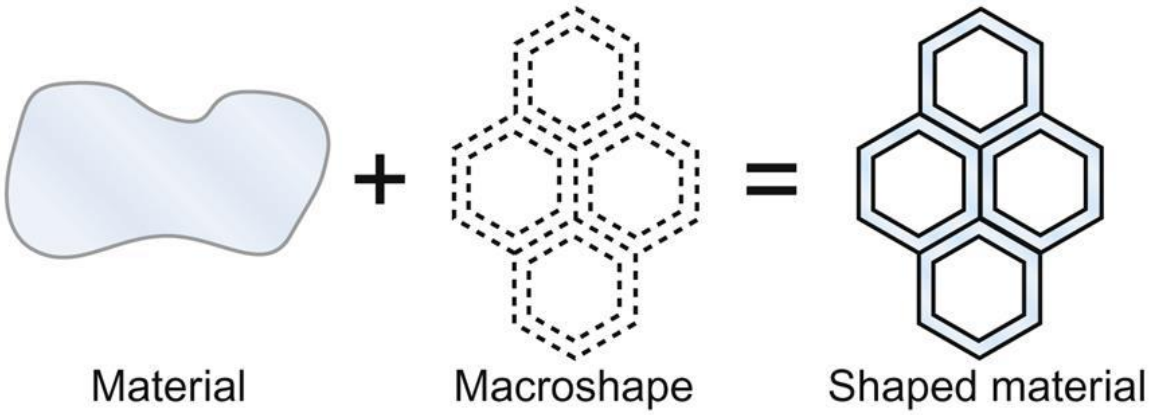


FIGURE 10.14 Mechanical efficiency can be obtained by combining material with microscopic, or internal, shape, which repeats itself to give an extensive structure. The shape is characterised by microscopic shape factors, ψ.

The characteristic of microscopic shape is that the structure repeats itself: it is *extensive*. The microstructured solid can be thought of as a 'material' in its own right: it has a modulus, a density, a strength, and so forth. Shapes can be cut from it. Provided the shapes are large compared with the size of the cells (so they contain many cells) they inherit its properties. It is possible, for instance, to fabricate an I-section from wood, and such a section has both macroscopic shape (as defined earlier) as well as microscopic shape as suggested by Fig. 10.15. It will be shown in a moment that the total shape factor for a wooden I-beam is the *product* of the shape factor for the wood structure and that for the I-beam, and this can be large.

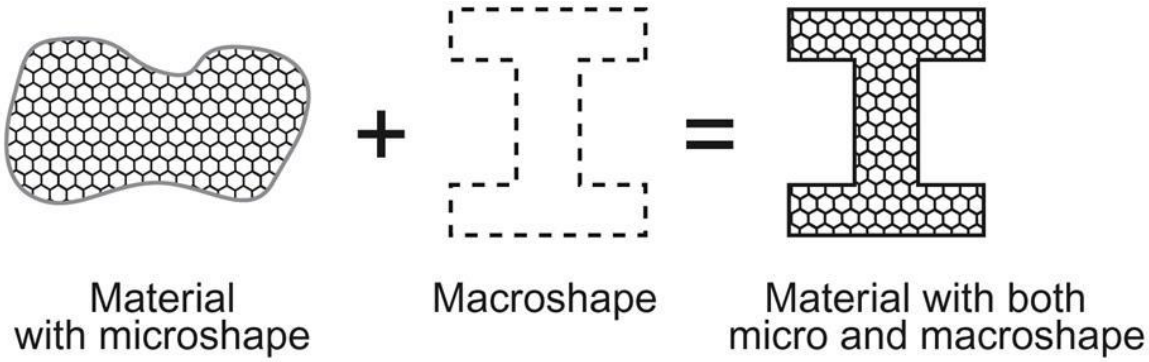


FIGURE 10.15 Microstructural shape can be combined with macroscopic shape to give efficient structures. The overall shape factor is the product of the microscopic and macroscopic shape factors.

Many natural materials have microscopic shape. Wood is just one example. Bone, the stalks and leaves of plants and the cuttlebone of a squid all have structures that give high stiffness at low weight. It is harder to think of manufactured examples, though it would appear possible to make them. Fig. 10.16 shows four extensive structures with microscopic shape, all of them found in nature. The first is a wood-like structure of hexagonal-prismatic cells; it is isotropic in the plane of the section when the cells are regular hexagons. The second is an array of fibres separated by a foamed matrix typical of palm wood, and it too is isotropic in-plane. The third is an axisymmetric structure of concentric cylindrical shells separated by a foamed matrix, like the stem of some plants. And the fourth is a layered structure, a sort of multiple sandwich panel, like the shell of the cuttlefish.

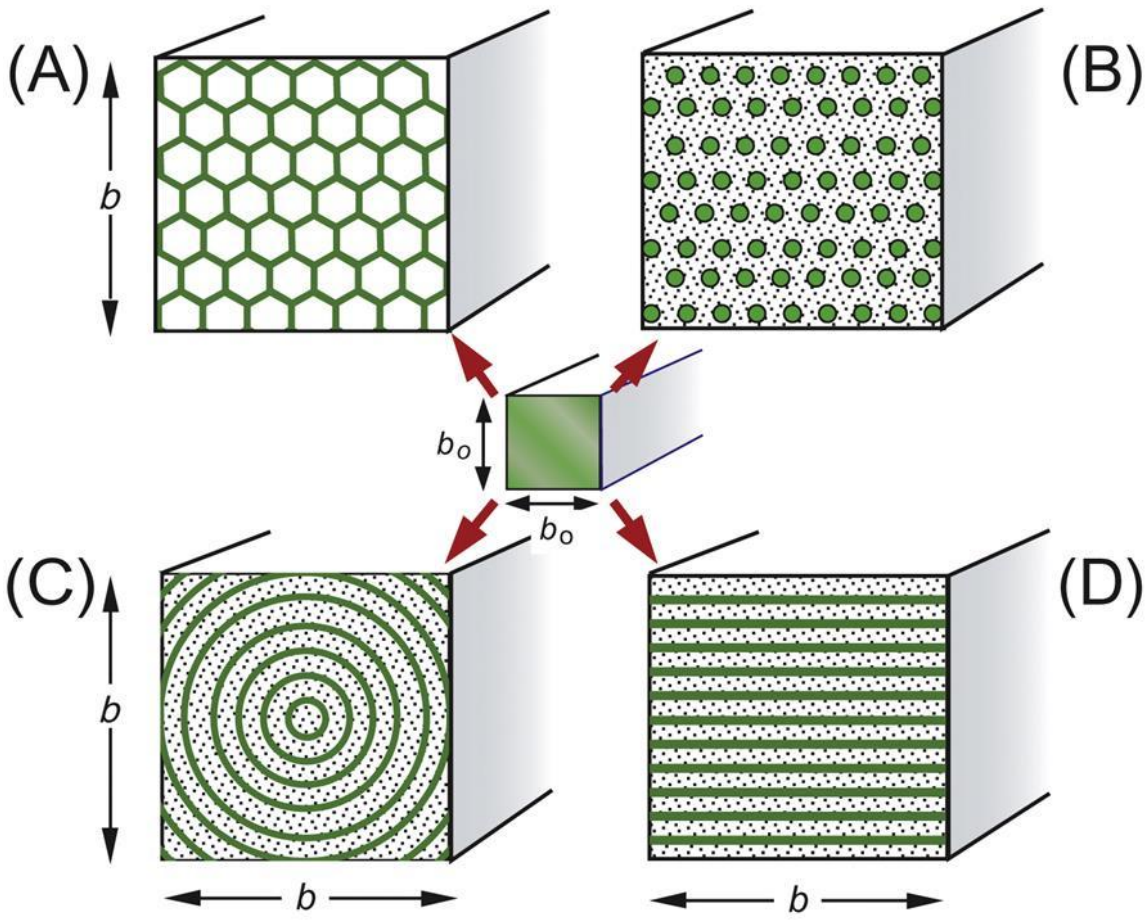


FIGURE 10.16 Four extensive microstructured materials which are mechanically efficient: (A) prismatic cells, (B) fibres embedded in a foamed matrix, (C) concentric cylindrical shells with foam between, and (D) parallel plates separated by foamed spacers.

Microscopic shape factors. Consider the gain in bending stiffness when a solid square beam like that shown as a solid square of side b_o in Fig. 10.16 is expanded, at constant mass, to a larger square section with any one of the structures that surround it in the figure. The bending stiffness S_s of the original solid beam is proportional to the product of its modulus E_s and its second moment of area I_s:

$$S_s \propto E_s I_s \tag{10.34}$$

where the subscript '*s*' means 'a property of the solid beam' and $I_s = b_o^4/12$. When the beam is expanded at constant mass its density falls from ρ_s to ρ and its edge-length increases from b_o to b where

$$b = \left(\frac{\rho_s}{\rho}\right)^{1/2} b_o \tag{10.35}$$

with the result that its second moment of area increases from I_s to

$$I = \frac{b^4}{12} = \frac{1}{12}\left(\frac{\rho_s}{\rho}\right)^2 b_o^4 = \left(\frac{\rho_s}{\rho}\right)^2 I_s \tag{10.36}$$

If the cells, fibres or rings in Fig. 10.16A, B and C extend parallel to the axis of the beam, the modulus parallel to this axis falls from that of the solid, E_s, to

$$E = \left(\frac{\rho}{\rho_s}\right) E_s \tag{10.37}$$

The bending stiffness of the expanded beam scales as *EI*, so that it is stiffer than the original solid one by the factor

$$\psi_B^e = \frac{S}{S_s} = \frac{EI}{E_s I_s} = \frac{\rho_s}{\rho} \tag{10.38}$$

We refer to ψ_B^e as the *microscopic shape factor for elastic bending*. That for prismatic structures like those of Fig. 10.16A is simply the reciprocal of the relative density, ρ/ρ_s. Note that, in the limit of a solid (when $\rho = \rho_s$), ψ_B^e takes the value 1, as it obviously should. A similar analysis for failure in bending gives the shape factor

$$\psi_T^e = \frac{\rho_s}{\rho} \tag{10.39}$$

Torsion, as always, is more difficult. When the structure of Fig. 10.16C, which has circular symmetry, is twisted, its rings act like concentric tubes and for these

$$\psi_T^e = \frac{\rho_s}{\rho} \quad \text{and} \quad \psi_B^f = \left(\frac{\rho_s}{\rho}\right)^{1/2} \tag{10.40}$$

The other structures have lower torsion stiffness and strength (and thus lower microshape factors) for the same reason that I-sections, good in bending, perform poorly in torsion.

Prismatic versus Foam-Like Micro-Shape

What is the gain in bending stiffness EI if a beam, initially with a solid cross-section, is expanded to create a prismatic structure like those of Fig. 10.16? If, instead, it is expanded to create a foam-like structure for which $E = \left(\frac{\rho}{\rho_s}\right)^2 E_s$, what is the gain in bending stiffness?

Answer

The shape factor ψ_B^e of Eq. (10.38) measures the ratio of the stiffness of a prismatic microstructured beam to that of a solid beam of the same mass. The gain in EI scales as $\psi_B^e = \rho_s/\rho$. Repeating the derivation using the expression for foam modulus in terms of relative density given above, we find $\psi_B^e = 1$. Foaming gives no gain in bending stiffness.

Structuring, then, converts a solid with modulus E_s and strength $\sigma_{f,s}$ into a new solid with properties E and σ_f. If this new solid is now shaped efficiently (into a tube, say, or and I-section) its bending stiffness increases by a further factor of ϕ_B^e. Then the stiffness of the beam, expressed in terms of that of the solid of which it is made, is

$$S = \psi_B^e \phi_B^e S_s \tag{10.41}$$

that is, the shape factors simply multiply. The same is true for strength.

This is an example of structural hierarchy and the benefits it brings. It is possible to extend it further: the individual cell walls or layers could, for instance, be structured, giving a third multiplier to the overall shape

factor, and these units, too could be structured. Nature does this to good effect, but for man-made structures there are difficulties. There is the obvious difficulty of making hierarchically-structured shapes. And there is the less obvious one of structural resilience. If the structure is optimised at all levels of structure, then a failure of a member at any one level can trigger failure at the level above, causing a cascade that ends with the failure of the structure as a whole. The more complex the structure, the harder it becomes to ensure the integrity at all levels. This might be overcome by incorporating redundancy (or a safety factor) at each level, but this implies a cumulative loss of efficiency. Two levels of structure are usually as far as it is practical to go.

As pointed out earlier, a microstructured material can be thought of as a new material. It has a density, a strength, a thermal conductivity, and so on; difficulties arise only if the sample size is comparable to the cell size, when 'properties' become size dependent. This means that microstructured materials can be plotted on the material charts – indeed, wood appears on them already – and that all the selection criteria developed in Chapter 4, Materials Selection – the Basics apply, unchanged, to the microstructured materials. This line of thinking is developed further in Chapter 13, Hybrids: Case Studies, which includes material charts for a range of natural materials.

10.8 Summary and Conclusions

The designer has two groups of variables with which to optimise the performance of a load-bearing component: the material properties and the shape of the section. They are not independent. The best choice of material, in a given application, depends on the shapes in which it is available or to which it could potentially be formed.

The contribution of shape is isolated by defining four shape factors. The first, ϕ_B^e, is for the elastic bending and buckling of beams; the second, ϕ_T^e, is for the elastic twisting of shafts; the third, ϕ_B^f is for the plastic failure of beams loading in bending; and the last, ϕ_T^f, is for the plastic failure of twisted shafts (Table 10.7). The shape factors are dimensionless numbers that characterize the efficiency of use of the material in each mode of loading. They are defined such that all four have the value 1 for a solid square section. With this definition, all equiaxed solid sections (solid cylinders, and hexagonal and other polygonal sections) have shape factors

close to 1. Efficient shapes that disperse the material far from the axis of bending or twisting (I-beams, hollow tubes, hollow box-sections, etc.) have shape factors that are much larger. They are assembled for common shapes in Table 10.3.

Table 10.7
Definitions of Shape Factors[a]

Design Constraint[a]	Bending	Torsion
Stiffness	$\phi_B^e = \frac{12I}{A^2}$	$\phi_T^e = \frac{7.14K}{A^2}$
Strength	$\phi_B^f = \frac{6Z}{A^{3/2}}$	$\phi_T^f = \frac{4.8Q}{A^{3/2}}$

[a]A; I, K, Z and Q are defined in the text and tabulated in Table 10.2.

The shapes to which a material can, in practice, be made are limited by manufacturing constraints and by the restriction that the section should yield before it suffers local buckling. These limits can be plotted onto a 'shape chart' which, when combined with a material property chart in a four-chart array (Figs. 10.9 and 10.12) allow the potential of alternative material-shape combinations to be explored.

While this is educational, there is a more efficient alternative: that of developing indices that include the shape factors. The best material-shape combination for a light beam with a prescribed bending stiffness is that which maximizes the material index

$$M_1 = \frac{(E\phi_B^e)^{1/2}}{\rho}$$

The material-shape combination for a light beam with a prescribed strength is that which maximizes the material index

$$M_3 = \frac{(\phi_B^f \sigma_f)^{2/3}}{\rho}$$

These allow shaped sections to be plotted onto property charts. They are used for selection in exactly the same way as the indices of Chapter 4, Materials Selection – the Basics.

Similar combinations involving ϕ_T^e and ϕ_T^f gives the lightest stiff or strong shaft. Here, the criterion of 'performance' was that of meeting a design specification at minimum weight. Other such material-shape combinations maximize other performance criteria: minimizing cost rather than weight, e.g., or maximizing energy storage.

The procedure for selecting material-shape combinations is best illustrated by examples. These, can be found in the exercise at the end of this chapter and the case studies of Chapter 11, Material and Shape: Case Studies.

10.9 Further Reading

.

1. Ashby MF. Material and shape. *Acta Metall Mater.* 1991;39:1025–1039 (*The paper in which the ideas on which this chapter is based were first developed*).
2. Birmingham RW, Jobling B. *Material selection: comparative procedures and the significance of form*. *International Conference on Lightweight Materials in Naval Architecture* London: The Royal Institution of Naval Architects; 1996; (*The article in which the 4-quadrant charts like those of Figures 10.12 and 10.15 were introduced*.).
3. Gerard G. *Minimum Weight Analysis of Compression Structures* New York, NY: New York University Press; 1956; Library of Congress Catalog Number 55-10052. (*This and the book by Shanley, cited below, establish the principles of minimum weight design. Both, unfortunately, are out of print but can be found in libraries*.).
4. Gere JM, Timoshenko SP. *Mechanics of Materials* London, U.K.: Wadsworth International; 1985; (*An introduction to the mechanics of elastic solids*.).
5. Shanley FR. *Weight-Strength Analysis of Aircraft Structures* second ed. New York, NY: Dover Publications; 1960; Library of Congress Catalog Number 60-501011. (*This and the book by Gerard, cited above, establish the principles of minimum weight design. Both, unfortunately, are out of print but can be found in libraries*.).

6. Timoshenko SP, Gere JM. *Theory of Elastic Stability* London, U.K.: McGraw-Hill Koga Kusha Ltd; 1961; Library of Congress Catalog Number 59-8568. (*The definitive text on elastic buckling*.).
7. Weaver PM, Ashby MF. *Material Limits for Shape Efficiency*. *Prog Mat Sci.* Vol. 41 Oxford, UK: Elsevier; 1998;61–128 (*A review of the shape-efficiency of standard sections, and the analyses leading to the results used in this chapter for the maximum practical shape factors for bending and twisting*.).
8. Young WC. *Roark's Formulas for Stress and Strain* sixth ed. New York, NY: McGraw-Hill; 1989; ISBN 0-07-100373-8. (*A sort of Yellow Pages of formulae for stress and strain, cataloging the solutions to thousands of standard mechanics problems*.).

10.10 Exercises

The examples in this section relate to the analysis of material and shape of Chapter 9, Multiple Constraints and Conflicting Objectives – Case Studies and this chapter. *They cover the derivation of shape factors, of indices that combine material and shape, and the use of the 4-quadrant chart arrays to explore material and shape combinations. For this last purpose it is useful to have clean copies of the chart arrays of* Figs. 10.9 and 10.12. *Like the material property charts, they can be copied for educational purposes without restriction of copyright*.

E10.1. *Stiffness shape factors for tubes (*Fig. E10.1*)*. Evaluate the shape factor ϕ_B^e for stiffness-limited design in bending of a square box section of outer edge-length h=100 mm and wall thickness t=3 mm. Is this box shape more efficient than tube made of the same material with diameter $2r$=100 mm and wall thickness t=3.82 mm (giving it the same mass per unit length, m/L)? Treat both as thin-walled shapes.
Use the expressions in Table 10.3 of the text for the shape factor ϕ_B^e.

E10.2. *Strength shape factors for tubes* (Fig. E10.1 again). Evaluate the shape factor ϕ_B^f for strength-limited design in bending of a square box section of outer edge-length h=100 mm and wall thickness t=3 mm. Is this box shape more efficient than tube made of the same material with diameter $2r$=100 mm and wall thickness t=3.82 mm (giving it the same mass per unit length, m/L)? Treat both as thin-walled shapes.
Use the expressions given in Table 10.3 of the text for the shape factors ϕ_B^f.

E10.3. *Deriving shape factors for stiffness-limited design* (Fig. E10.2). Derive the expression for the shape-efficiency factor ϕ_B^e for stiffness-limited design for a circular tube with outer radius $5t$ and wall thickness t, loaded in bending (Fig. E10.2). Do not assume that the thin-wall approximation is valid.

E10.4. *Deriving shape factors for stiffness-limited design* (Fig. E10.3). Derive the expression for the shape-efficiency factor ϕ_B^e for stiffness-limited design for a channel section of thickness t, overall flange width $5t$ and overall depth $10t$, bent about its major axis (the dash-dot line in Fig. E10.3). Do not assume that the thin-wall approximation is valid.

E10.5. *Deriving shape factors for stiffness-limited design* (Fig. E10.4). Derive the expression for the shape-efficiency factor ϕ_B^e for stiffness-limited design for a square box section of wall thickness t, and height and width h_1=$10t$ bent about its major axis (the dash-dot line in Fig. E10.4). Do not assume that the thin-wall approximation is valid.

E10.6. *Deriving shape factors for strength-limited design*. Determine the value of the shape-efficiency factor ϕ_B^f for strength-limited design in bending using the dimensions shown on the diagrams

a. For the tube-section shown in Fig. E10.5(A)
b. For the box section shown in Fig. E10.5(B) and
c. For the channel section shown in Fig. E10.5(C).

The expression for ϕ_B^f for the first two sections can be read from Table 10.3. You will have to derive the expression for the third.

E10.7. *Deriving indices that include shape*. A beam of length L, loaded in bending, must support a specified bending moment M without failing and be as light as possible. Show that to minimize the mass of the beam per unit length, m/L, one should select a material and a section-shape to maximize the quantity

$$M = \frac{(\phi_B^f \sigma_f)^{2/3}}{\rho}$$

where σ_f is the failure stress and ρ the density of the material of the beam, and ϕ_B^f is the shape-efficiency factor for failure in bending.

E10.8. *Calculating shape factors from stiffness data*. The elastic shape factor measures the gain in stiffness by shaping, relative to a solid square section of the same cross-section area and thus mass per unit length. Shape factors can be determined by experiment by measuring the stiffness and mass of a structure and using these to calculate ϕ_B^e by inverting Eq. (10.19) of the text and solving for ϕ_B^e. Apply this approach to calculate the shape factor ϕ_B^e from the following experimental data, measured on an aluminium alloy beam (Fig. E10.6) loaded in 3-point bending (for which C_1=48 – see Appendix B, Section B3) by using the data shown in the table.

Attribute	**Value**
Bending stiffness S_B	7.2×10^5 N/m
Mass/unit length m/L	1 kg/m
Beam length L	1 m
Beam material	6061 aluminium alloy
Material density ρ	2670 kg/m^3
Material modulus E	69 GPa

E10.9. *Calculating shape factors from stiffness data*. A steel truss bridge shown in Fig. E10.7 has a span L and is simply supported at both ends. It weighs m tonnes. As a rule of thumb, bridges are designed with a stiffness S_B such that the central deflection δ of a span under its self-weight is less than 1/300 of the length L (thus $S_B \geq 300\ mg/L$ where g is the acceleration due to gravity, 9.81 m/s^2). Use this information to calculate a minimum value for the shape factor ϕ_B^e of the three steel truss bridge spans listed in the table. Take the density ρ of steel to be 7900 kg/m^3 and its modulus E to be 205 GPa. The constant C_1=384/5=76.8 for uniformly distributed load (Appendix B, Section B3).

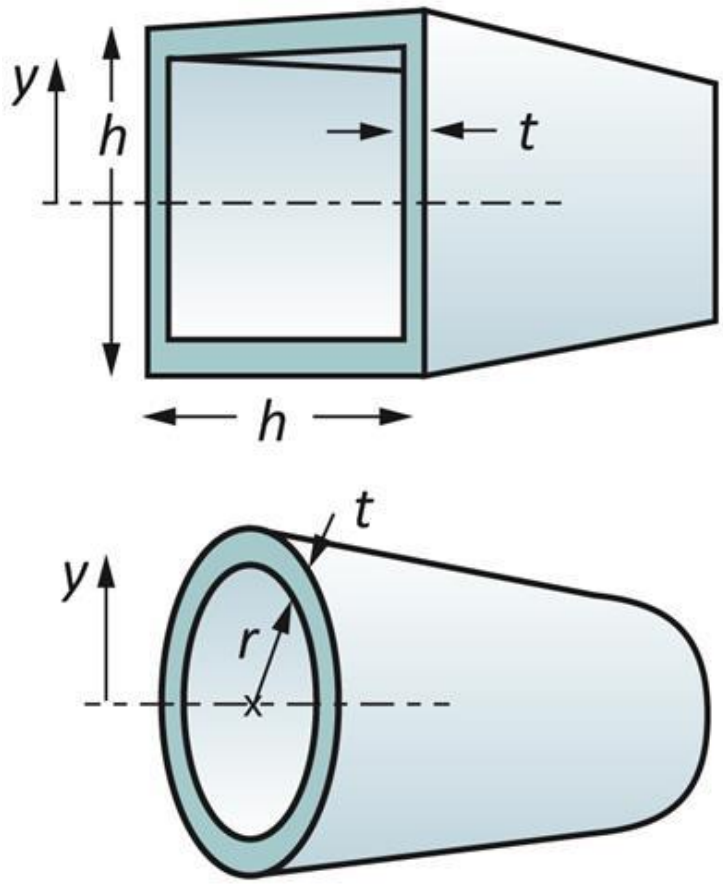


FIGURE E10.1

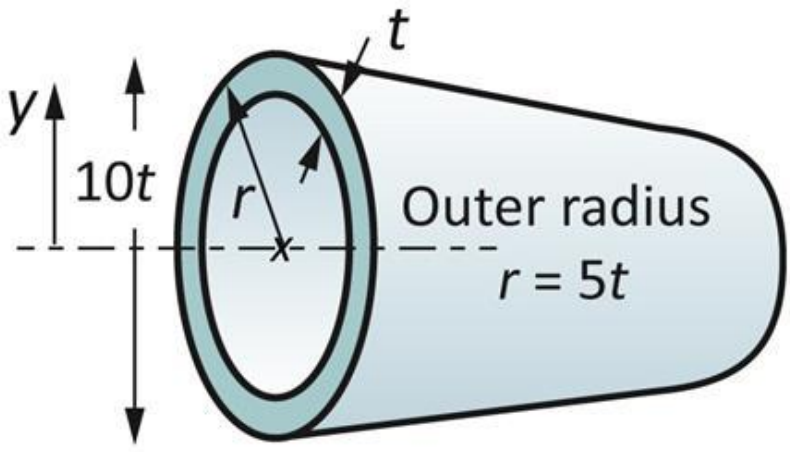


FIGURE E10.2

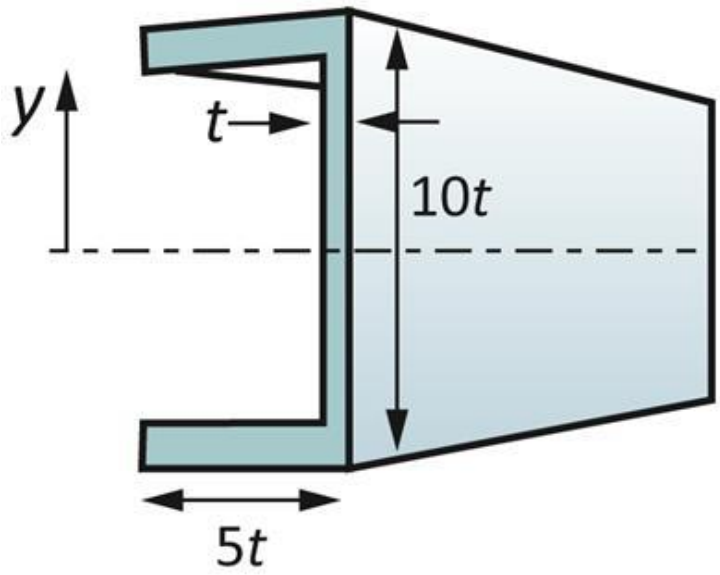


FIGURE E10.3

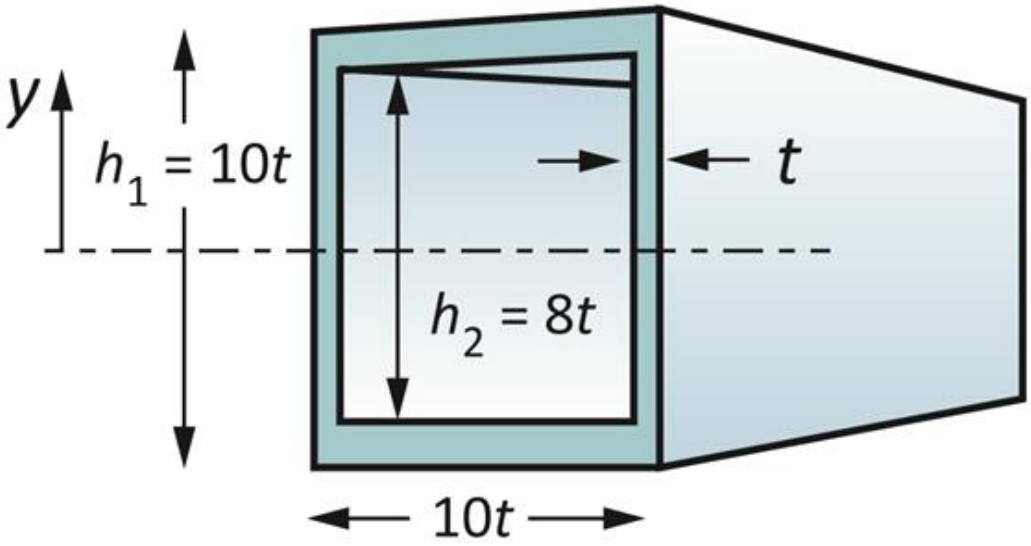


FIGURE E10.4

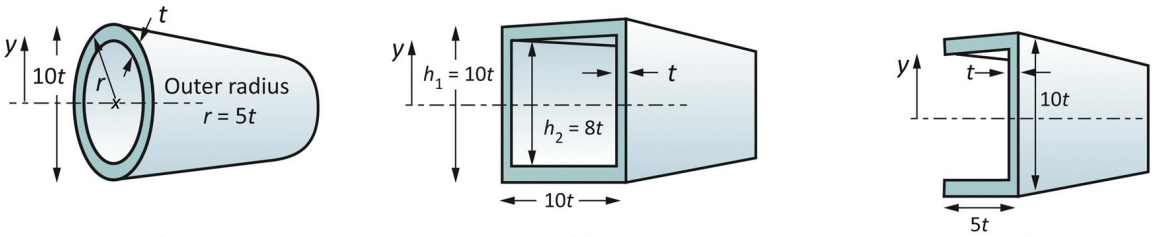


FIGURE E10.5

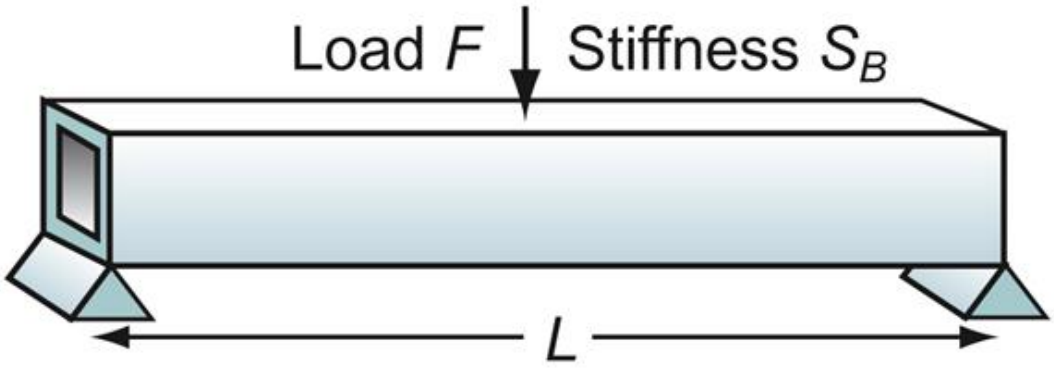


FIGURE E10.6

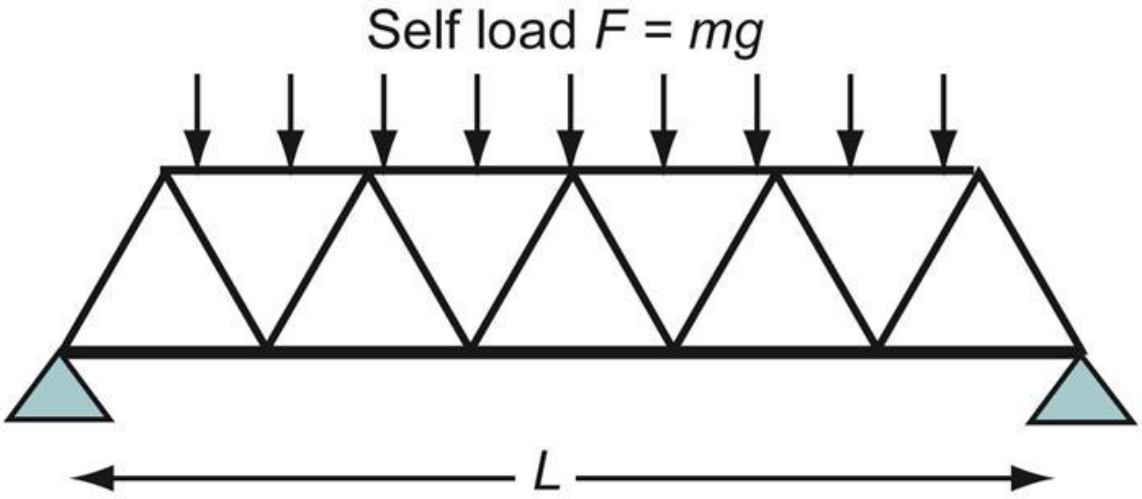


FIGURE E10.7

Bridge and Construction Date[a]	Span L (m)	Mass m (tonnes)
Royal Albert bridge, Tamar, Saltash UK (1857)	139	1060
Carquinez Strait bridge, California (1927)	132	650
Chesapeake Bay bridge, Maryland USA (1952)	146	850

[a]Bridge Engineering Handbook, 2nd edition, (2014), W-F Chen and L. Duan (Eds), CRC Press, London.

E10.10. *Deriving indices for bending*. A beam, loaded in bending, must support a specified bending moment M^* without failing and be as light as possible. Section shape is a variable, and 'failure' here means the first onset of plasticity. Derive the material index. The table summarizes the requirements.

Function	• *Light weight beam*
Constraints	• *Specified failure moment M** • *Length L specified*
Objective	• *Minimize mass m*
Free variables	• *Choice of material* • *Section shape and scale*

E10.11. *Deriving indices for torsion* (Fig. E10.8). A shaft of length L, loaded in torsion, must support a specified torque T^* without failing and be as cheap as possible. Section shape is a variable and 'failure' again means the first onset of plasticity. Derive the material index. The table summarizes the requirements.

Function	• *Cheap shaft*
Constraints	• *Specified failure torque T** • *Length L specified*
Objective	• *Minimize material cost C*
Free variables	• *Choice of material* • *Section shape and scale*

E10.12. *Material and shape to suppress buckling (*Fig. E10.9*)*. The figure shows a concept for a lightweight display stand. The stalk must support a mass m of 100 kg, to be placed on its upper surface at a height h, without failing by elastic buckling. It is to be made of stock tubing and must be as light as possible. Use the methods of this chapter to derive a material index for the tubular material of the stalk of the stand that includes the shape of the section, described by the shape factor

$$\phi_B^e = \frac{12I}{A^2}$$

where I is the second moment of area and A is the section area of the stalk. The table summarizes the requirements.

Function	• *Light weight column*
Constraints	• *Specified buckling load F* • *Height h specified*
Objective	• *Minimize mass m*
Free variables	• *Choice of material* • *Section shape and scale*

E10.13. *Selection of material and shape to suppress buckling*. Cylindrical tubing is available from stock in the following materials and sizes. Use this information and the material index to identify the best stock material for the stalk of the stand shown in Fig. E10.8.

Material	**Modulus *E* (GPa)**	**Tube Radius *r***	**Wall Thickness/Tube Radius, *t/r***
Aluminium alloys	69	25 mm	0.07 to 0.25
Steel	210	30 mm	0.045 to 0.1
Copper alloys	120	20 mm	0.075 to 0.1
Polycarbonate (PC)	3.0	20 mm	0.15 to 0.3
Various woods	7–12	40 mm	Solid circular sections only

E10.14. *Microscopic shape: tube arrays (*Fig. E10.10*)*. Calculate the gain in bending efficiency, ψ_B^e, when a solid is formed into small, thin-walled tubes of radius r and wall thickness t that are then assembled and bonded into a large array, part of which is shown in the figure. Let the solid of which the tubes are made have modulus E_s and density ρ_s. Express the result in terms of r and t.

E10.15. *The structural efficiency of foamed panels (*Fig. E10.11*)*. Calculate the change in structural efficiency for both bending stiffness and strength when a solid flat panel of unit area and thickness t is foamed to give a

foam panel of unit area and thickness h, at constant mass. The modulus E and strength σ_f of foams scale with relative density ρ/ρ_s as

$$E = \left(\frac{\rho}{\rho_s}\right)^2 E_s \quad \text{and} \quad \sigma_f = \left(\frac{\rho}{\rho_s}\right)^{3/2} \sigma_{f,s}$$

where E, σ_f and ρ are the modulus, strength and density of the foam and E_s, $\sigma_{f,s}$ and ρ_s those of the solid panel.

E10.16. *Use of the four segment chart for stiffness-limited design*. Use the 4-segment chart for stiffness-limited design of Fig. 10.9 of the text to compare the mass per unit length, m/L, of a section with $EI = 10^5$ Nm^2 made from

a. structural steel with a shape factor ϕ_B^e of 20, modulus E=210 GPa and density $\rho = 7900$ kg/m^3

b. carbon fiber reinforced plastic (CFRP) with a shape factor ϕ_B^e of 10, modulus E=70 GPa and density $\rho = 1600$ kg/m^3, and

E10.17. *Numerical comparison for stiffness-limited design*. Show, by direct calculation, that the conclusion of Exercise 10.16 – that the CFRP beam with $EI = 10^5$ Nm^2 and $\phi_B^e = 10$ weighs less than the steel beam with the same $\phi_B^e = 20$ – is consistent with the idea that to minimize mass for a given stiffness one should maximize $\sqrt{E^*}/\rho^*$ with $E^* = E/\phi_B^e$ and $\rho^* = \rho/\phi_B^e$.

E10.18. *Use of the four segment chart for strength*. Use the 4-segment chart for strength-limited design of Fig. 10.12 of the text to compare the mass per unit length, m/L, of a section with $Z\sigma_f = 10^4$ Nm (where Z is the section modulus) made from

a. 6061 grade aluminium alloy with a shape factor ϕ_B^f of 3, strength $\sigma_f = 200$ MPa and density $\rho = 2700$ kg/m^3, and

b. titanium alloy with a shape factor ϕ_B^f of 10, strength $\sigma_f = 480$ MPa and density $\rho = 4420$ kg/m^3.

E10.19. *Numerical comparison for stiffness-limited design*. Show, by direct calculation, that the conclusion of Exercise E10.18 – that the titanium alloy beam with $Z\sigma_f = 10^4$ Nm and ϕ_B^f of 10 is much lighter than the 6061 aluminium alloy beam of the same strength and ϕ_B^f – is consistent with the idea that to minimize mass for a given stiffness one should maximize $\frac{(\phi_B^f \sigma_f)^{2/3}}{\rho}$.

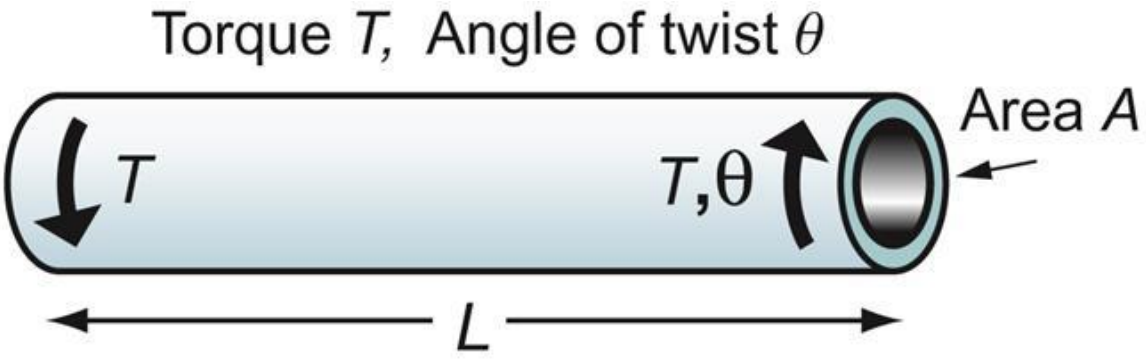


FIGURE E10.8

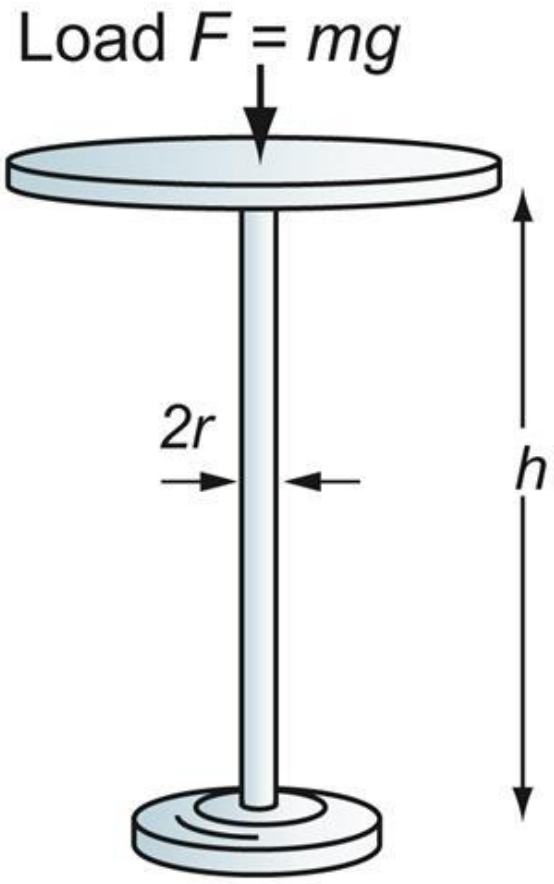


FIGURE E10.9

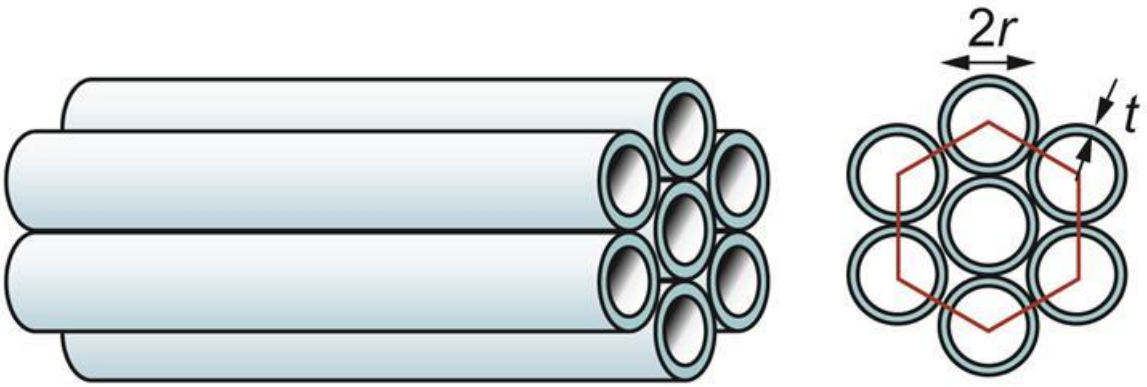


FIGURE E10.10

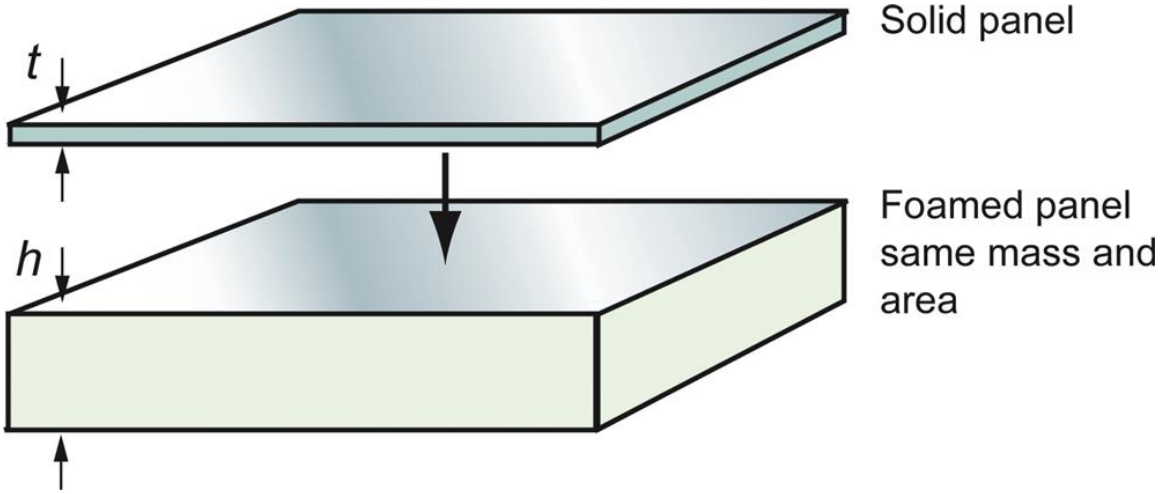


FIGURE E10.11

[1]This elastic shape-efficiency factor is related to the radius of gyration, R_g, by $\phi_B^e = 12R_g^2/A$. It is related to the 'shape parameter', k_1, of Shanley (1960) by $\phi_B^e = 12k_1$.

[2]Birmingham and Jobling, 1996, and Weaver and Ashby, 1997, in Further Reading.

[3]See Gerard (1956) and Weaver and Ashby (1997) in Further reading.

[4]Birmingham (1996).

CHAPTER 11

Material and Shape

Case Studies

Abstract

Shape can be used to increase the mechanical efficiency of a material. Shaped sections carry bending, torsional and axial-compressive loads more efficiently than solid sections. 'Shaped' means that the cross-section is formed to a tube, a box-section, an I-section or the like. 'Efficient' means that, for given loading conditions, the section uses as little material as possible. Tubes, boxes and I-sections are simple shapes. Even greater efficiencies are possible with sandwich panels (thin load-bearing skins bonded to a foam or honeycomb interior) and with more elaborate structures (the Warren truss, for instance). Chapters 10 and 11 extends selection methods so as to include shape.

Keywords

Shaped sections; shape factors; shape efficiency; material-shape combinations; limits to shape factors; microscopic shape; human-powered planes; racing bikes; ultra-efficient springs

11.1 Introduction and Synopsis

This chapter, like Chapter 5, Materials Selection – Case Studies and Chapter 9, Multiple Constraints and Conflicting Objectives – Case Studies, is a collection of case studies. They illustrate the use of shape factors, of the 4-quadrant chart construction and of material indices that include shape. They are necessary for the restricted class of problems in which section

shape directly influences performance, that is, when the prime function of a component is to carry loads that cause it to bend, twist or buckle.

Indices that include shape provide a tool for optimizing the coselection of material-and-shape. The important ones are summarised in Table 11.1. The selection procedure is, first, to identify candidate materials and the section shapes in which each is available or could be made. The relevant material properties and shape factors for each are tabulated and the relevant index is evaluated. The best material-and-shape combination is that with the greatest value of the index. The same information can be plotted onto material selection charts, allowing a graphical solution to the problem – one that often suggests further possibilities.

Table 11.1

Indices With Shape: Stiffness- and Strength-Limited Design at Minimum Weight

Component Shape, Loading and Constraints	Stiffness-Limited Design[a]	Strength-Limited Design[a]
Tie (tensile member) Load, stiffness and length specified, section-area free	$\frac{E}{\rho}$	$\frac{\sigma_f}{\rho}$
Beam (loaded in bending) Loaded externally or by self; weight, stiffness, strength and length specified; section area and shape free	$\frac{(\phi_B^e E)^{1/2}}{\rho}$	$\frac{(\phi_B^f \sigma_f)^{2/3}}{\rho}$
Torsion bar or tube Loaded externally, stiffness, strength and length specified, section area and shape free	$\frac{(\phi_T^e E)^{1/2}}{\rho}$	$\frac{(\phi_T^f \sigma_f)^{2/3}}{\rho}$
Column (compression strut) Collapse load by buckling or plastic crushing and length specified, section area and shape free	$\frac{(\phi_B^e E)^{1/2}}{\rho}$	$\frac{\sigma_f}{\rho}$

[a]The shape factors ϕ_B^e and ϕ_B^f are for bending, ϕ_T^e and ϕ_T^f are for torsion. For minimum cost design, replace ρ by $C_m\rho$ in the indices.

The method has other uses. It gives insight into the way in which natural materials – many of which are very efficient – have evolved. Bamboo is an example: it has both internal or microscopic shape and a tubular, macroscopic shape, giving it very attractive properties. This and other aspects are brought out in the case studies that follow.

11.2 Spars for Human-Powered Planes

Much engineering design involves a complex trade-off between performance and cost. But in designing a spar for a human-powered plane the objective is simple: the spar must be as light as possible and still be stiff enough to maintain the aerodynamic efficiency of the wings. Strength, reliability, even cost, hardly matter when records are to be broken. The

plane (Fig. 11.1) has two main spars: the transverse spar supporting the wings, and the longitudinal spar carrying the tail assembly. Both are loaded primarily in bending (torsion cannot, in reality, be neglected, although we shall do so here).

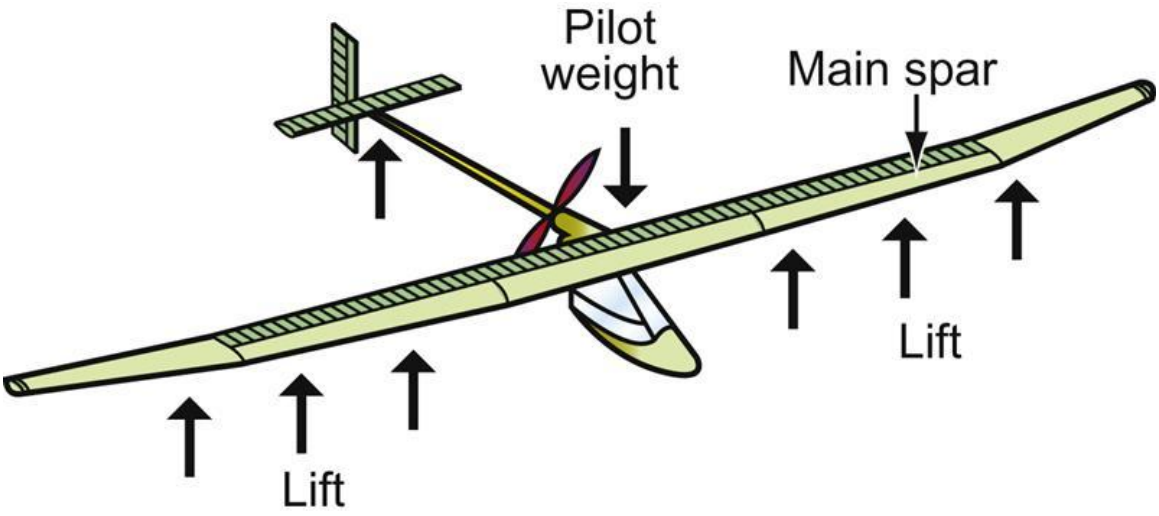


FIGURE 11.1 The loading on a human-powered plane is carried by two spars, one spanning the wings and the other linking the wings to the tail. Both are designed for stiffness at minimum weight.

Some 60 human-powered planes have flown successfully. Planes of the first generation were built of balsa wood, spruce and silk. The second generation relied on aluminium tubing for the load-bearing structure. The electric powered microlight planes pictured on the cover of the chapter have tubular aluminium frames. The present, third, generation uses carbon-fibre/epoxy spars, moulded to appropriate shapes. How has this evolution come about? And how much further can it go? (Table 11.2).

Table 11.2

Design Requirements for Wing Spars

Function	■ *Wing spar*
Constraints	■ *Specified stiffness* ■ *Length specified*
Objective	■ *Minimum mass*
Free variables	■ *Choice of material* ■ *Section shape and scale*

The translation and the selection. We seek a material-and-shape combination that minimizes mass for a given bending stiffness. The measure of performance, read from Table 11.1, is

$$M_1 = \frac{(\phi_B^e E)^{1/2}}{\rho} \tag{11.1}$$

Data for five materials are assembled in the upper part of Table 11.3. If all have the same shape, M_1 reduces to the familiar $E^{1/2}/\rho$ and the ranking is that of the fourth column. Balsa and spruce are extraordinarily efficient; that is why model aircraft builders use them now and the builders of real aircraft relied so heavily on them in the past. Solid CFRP comes close. Steel and aluminium are far behind.

Table 11.3
Materials for Wing Spars

Material	Modulus E (GPa)	Density $\rho(\mathrm{kg}/m^3)$	Index[a] $E^{1/2}/\rho_{(\mathrm{GPa}^{1/2}/\mathrm{Mg/m3})}$	Shape Factor ϕ_B^e	Index[a] M_1 $(\phi_B^e E)^{1/2}/\rho_{(\mathrm{GPa}^{1/2}/\mathrm{Mg/m3})}$
Balsa	4.6	210	**10**	2	14
Spruce	10.3	450	8	2	11
Steel	205	7850	1.8	25	9
Al 7075 T6	70	2700	3	25	15
CFRP	115	1550	7	10	**22**
Beryllium	300	1840	9.3	15	**36**
Be 38%Al (AlBeMet 162)	185	2100	6.5	15	25
Borosilicate glass	63	2200	3.6	10	11

[a]The values of the index are based on mean values of the material properties. Best choices are shown in bold, underscored.

Now add shape. Achievable shape factors for the five materials appear in column 5 of the table; they are typical of commercially available sections and are well below the maximum for each material. The effect of shaping the section to a rectangle for the woods, to a box-section for aluminium and CFRP, gives the results in the last column. Aluminium is now marginally better than the woods; CFRP is best of all.

The same information is shown graphically in Fig. 11.2, using the method of Section 10.6. Each shape is treated as a new material with modulus $E^* = E/\phi_B^e$ and $\rho^* = \rho/\phi_B^e$. The values of E^* and ρ^* are plotted on the chart. The superiority of both the aluminium tubing with $\phi_B^e = 25$ and the CFRP box-sections with $\phi_B^e = 10$ are clearly demonstrated.

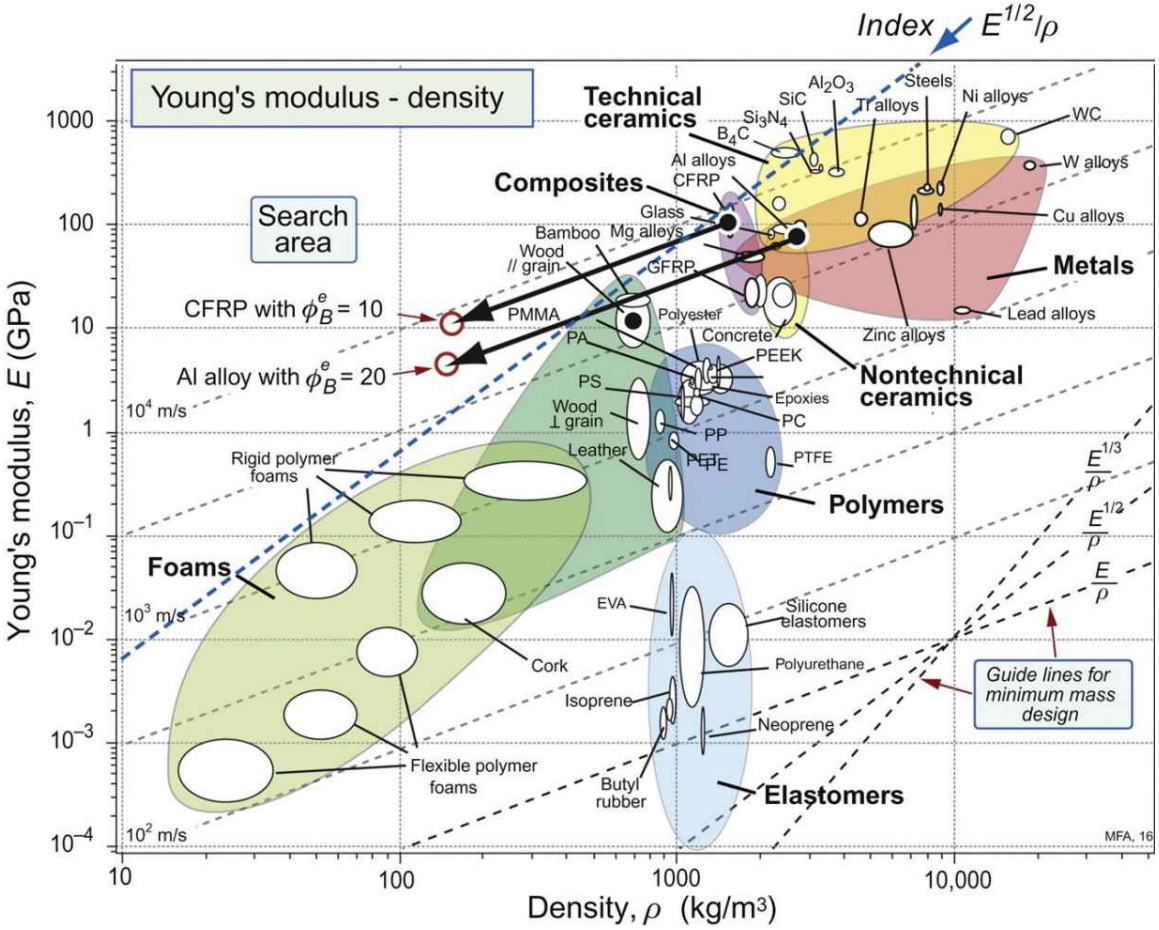


FIGURE 11.2 The materials-and-shapes for wing-spars, plotted on the modulus–density chart. A spar made of CFRP with a shape factor of 10 outperforms spars made of aluminium with a shape factor of 20, and wood with shape factor close to 1.

Postscript. Why is wood so good? With no shape it does as well as heavily shaped steel. It is because wood *is* shaped: its cellular structure gives it internal microshape, increasing the performance of the material in bending; it is nature's answer to the I-beam. Advances in the technology of drawing thin-walled aluminium tubes allowed a shape factor that cannot be reproduced in wood, giving aluminium a performance edge – a fact that did not escape the designers of the second generation of human-powered planes. There is a limit of course: tubes that are too thin will kink (as described in Chapter 10, Selection of Material and Shape), setting an upper limit to the shape factor for aluminium at about 40. Further advance required a new material with lower density and higher modulus, conditions met by CFRP. Add shape to it and CFRP outperforms them all.

Can we do better? Not easily, but if it is really worth it, maybe. Chapter 12, Designing Hybrid Materials, develops methods for designing material combinations that outperform anything that one material could do by itself, but we will leave these for later. What can be done with a single material? If we rank materials by $E^{1/2}/\rho$ we get a list headed by diamond, boron, and – ah! – beryllium. Beryllium has an exceptionally low density, a high modulus, and it can be shaped but its toxicity and expense inhibit its use in most ordinary engineering applications. Many ceramics have very high values of $E^{1/2}/\rho$ but they are rarely practical choices because of their brittleness and the difficulty of forming them into any useful shape. But above these, in the middle of the ceramics and far above magnesium and aluminium alloys, is … glass. A glider with glass wing-spars? Sounds crazy, but think for a moment. Toughed glass, bullet-proof glass, glass floors, glass stairs; glass can be used as a structural material. And it is easy to shape. It is not even that expensive.

So suppose beryllium or glass were given shape – could either one outperform CFRP? Again, not easily, but maybe. Here we have to guess. In theory (Eq. 10.15a) beryllium could be given a shape factor of 60, glass of 30. Being more realistic, factors of 15 and 10 are possible. The lower part of the table shows what all this means. Beryllium outperforms CFRP, and by a large margin. Glass doesn't. Well, it was a thought. For more, see Chapter 12, Designing Hybrid Materials.

Further Reading

There is a large literature on human-powered flight. For instance, try the following.

Bliesner, W., 1991. The design and construction details of the marathon eagle. In: "Technology for Human Powered Aircraft." Proceedings of the Human-Powered Aircraft Group Half Day Conference. The Royal Aeronautical Society, London. (*Details of another attempt to build a record-breaking plane*.).

Drela, M., Langford, J.D., 1985. Man-powered flight. Sci. Am. 122. (*A concise history of man powered flight up to 1985*.).

Grosser, M., 1981. Gossamer Odyssey. Dover Publications, New York, NY. ISBN 0-486-26645-1. (*Accounts of the Gossamer Condor and Albatross, attempts to capture the world record for man-powered flight*.).

Nadel, E.R., Bussolari, S.R., 1988. The Daedalus project: physiological problems and solutions. Am. Sci., July–August. (*An account of the Daedalus project, a competition to for man-powered flight from Crete and mainland Greece – the mythical route of Daedalus and his father*.).

Sherwin, K., 1971. Man powered flight. Model & Allied publications, Argus Books Ltd., Kings Langley.
Sherwin, K., 1976. To fly like a bird. Bailey Brothers & Swinfen Ltd. ISBN 561 00283 5.
Related Case Studies
5.5 'Cost: structural materials for buildings'
11.3 'Forks for a racing bicycle'
11.4 'Floor joists: wood, bamboo or steel?'

11.3 Forks for a Racing Bicycle

The first consideration in bicycle design (Fig. 11.3) is strength. Stiffness matters, of course, but the initial design criterion is that the frame and forks should not collapse in normal use. The loading on the forks is predominantly *bending*. If the bicycle is for racing, then the mass is a primary consideration: the forks should be as light as possible. What is the best combination of material and shape for light bicycle forks? (Table 11.4).

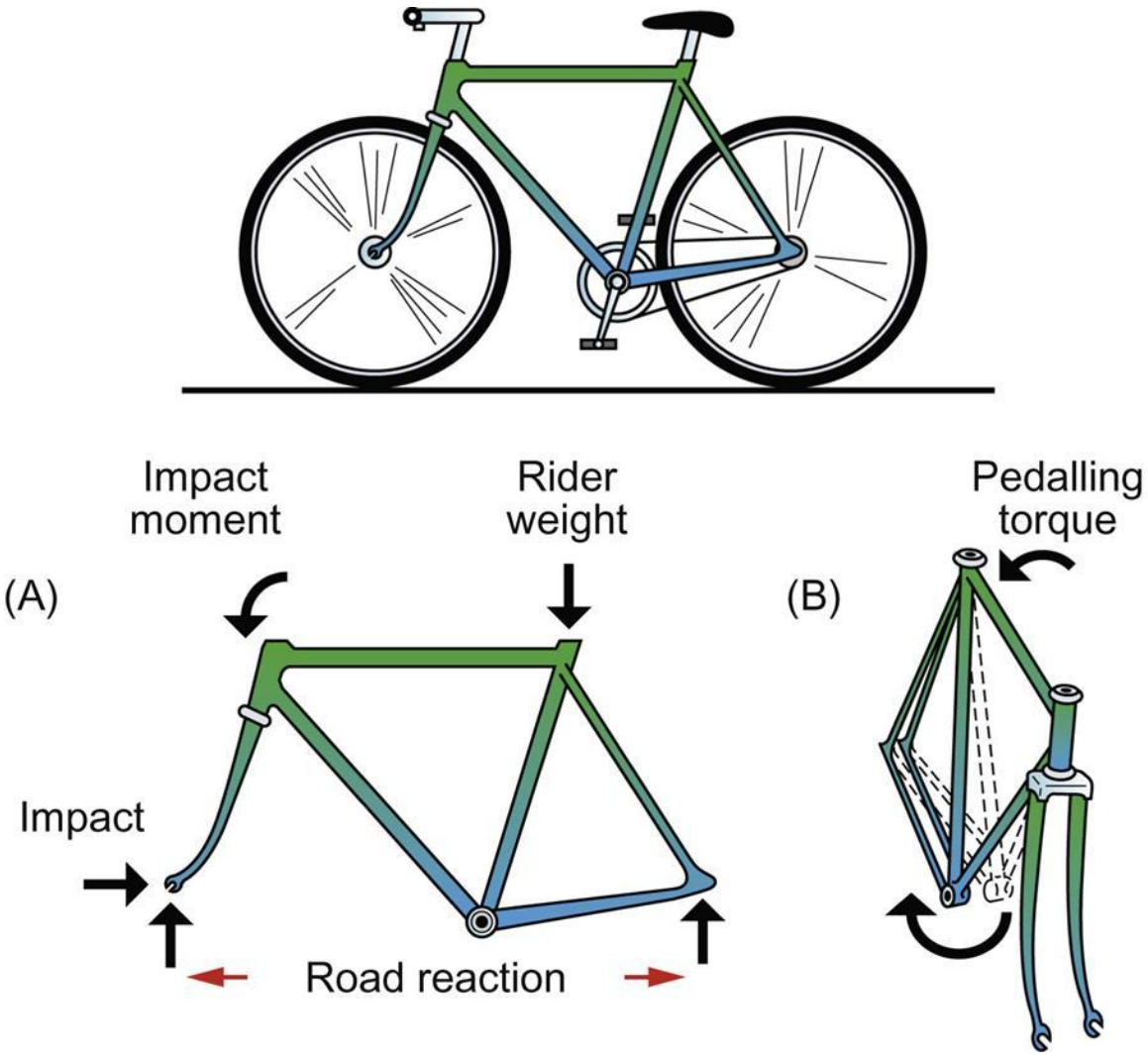


FIGURE 11.3 The bicycle (A) the forks and (B) the frame are loaded in bending. The lightest forks that will not collapse plastically under a specified design load are those made of the material and shape with the greatest value of $(\phi_B^f \sigma_f)^{2/3}/\rho$.

Table 11.4

Design Requirements for Bicycle Forks

Function	■ *Bicycle forks*
Constraints	■ *Must not fail under design loads – a strength constraint* ■ *Length specified*
Objective	■ *Minimize mass*
Free variables	■ *Choice of material* ■ *Section shape*

The translation and the selection. We model the forks as beams of length L that must carry a maximum load P (both fixed by the design) without plastic collapse or fracture. The forks are tubular, of radius r and fixed wall-thickness, t. We seek the best material and shape combination for a light, strong beam. Further details of load and geometry are unnecessary: the best material and shape, read from Table 11.1, is that with the greatest value of

$$M_2 = \frac{(\phi_B^f \sigma_f)^{2/3}}{\rho} \qquad \textbf{(11.2)}$$

Table 11.5 lists seven candidate materials with their properties. If the forks were solid, meaning that $\phi_B^f = 1$, spruce wins (see the second to last column of the table). Bamboo is special because it grows as a hollow tube with a macroscopic shape factor ϕ_B^f of about 2.2, giving it a bending strength that is much higher than solid spruce (last column). When shape is added to the other materials, however, the ranking changes. The shape factors listed in the table are achievable using normal production methods. Steel is good. Titanium 6-4 is better. Best of all is CFRP. Magnesium, despite its low density, is poor in strength-limited applications.

Table 11.5
Material for Bicycle Forks

Material	Strength σ_f(MPa)	Density ρ(kg/m^3)	Shape Factor ϕ_B^f	Index[a] $\sigma_f^{2/3}/\rho$ ((MPa)$^{2/3}$/Mg/m^3)	Index M_2[a] $(\phi_B^f\sigma_f)^{2/3}/\rho$ ((MPa)$^{2/3}$/Mg/m^3)
Spruce (Norwegian)	75	450	1.5	**39**	51
Bamboo	70	700	2.2	24	41
Steel (Reynolds 531)	880	7850	7.5	12	46
Alu (6061-T6)	250	2700	5.9	15	49
Titanium 6%Al-4%V	950	4420	5.9	22	72
Magnesium AZ 61	165	1810	4.25	17	45
CFRP	375	1550	4.25	33	**87**

[a]The values of the indices are based on mean values of the material properties. Best choices are shown in bold, underscored.

Postscript. Bicycles have been made of all seven of the materials listed in the table – you can still buy bicycles made of six of them. Early bicycles were made of wood; present-day racing bicycles of steel, aluminium or CFRP, sometimes interleaving the carbon fibres with layers of glass or Kevlar to improve the fracture-resistance. Mountain bicycles, for which strength and impact resistance are particularly important and have steel or titanium forks.

The reader may be perturbed by the cavalier manner in which theory for a straight beam with an end load acting normal to it is applied to a curved beam loaded at an acute angle. No alarm is necessary. When (as explained in Chapter 4, Materials Selection – the Basics, Section 4.4) the variables describing the functional requirements (F), the geometry (G) and the materials (M) in the performance equation are separable, the details of loading and geometry affect the terms F and G but not M. This is an example: beam curvature and angle of application of load do not change the material index, which depends only on the design requirement of strength in bending at minimum weight.

Further Reading

Oliver, T., 1992. Touring bikes, a practical guide. Crowood Press, Wiltshire, UK. ISBN 1-85223-339-7. (*A good source of information about bike materials and construction, with tables of data for the steels used in tube sets*.).

Sharp, A., 1979. Bicycles and tricycles, an elementary treatise on their design and construction. The MIT Press, Cambridge, MA. ISBN 0-262-69066-7. (*A far from elementary treatise, despite its title, first published in 1977. It is the place to look if you need the mechanics of bicycles*.).

Watson, R., Gray, M., 1978. The Penguin book of the bicycle. Penguin Books, Harmondesworth, UK. ISBN 0-1400-4297-0. (*Watson and Gray*

describe the history and use of bicycles. Not much on design, mechanics or materials.).

Whitt, F.R., Wilson, D.G., 1982. Bicycling Science, second ed., The MIT Press, Cambridge, MA. ISBN 0-262-73060-X. (*A book by two MIT professorial bike enthusiasts, more easily digestible than Sharp, and with a good chapter on materials*.).

Wilson, D.G., 1986. A short history of human powered vehicles. Am. Sci., 74, p. 350. (*Typical Scientific American article: good content, balance and presentation. A good starting point*.).

Bike magazines such as *Mountain Bike, Which Bike?* and *Cycling and Mountain Biking* carry extensive tables of available bikes and their characteristics – type, maker, cost, weight, gear-set etc.

Related Case Studies

5.5 'Cost: structural materials for buildings'
11.2 'Spars for man powered planes'
11.4 'Floor joists: wood, bamboo or steel?'

11.4 Floor Joists: Wood, Bamboo or Steel?

Floors are supported on joists: beams that span the space between the walls (Figure 5.7 and Fig. 11.4). Let us suppose that a joist is required to support a specified bending load (the 'floor loading') without sagging excessively or failing; and it must be cheap. Traditionally, joists in the United States and Europe are made of wood with a rectangular section of aspect ratio 2:1, giving an elastic shape factor (Table 10.3) of $\phi_B^e = 2$. In Asian countries bamboo, with a 'natural' shape factor for elastic bending of about $\phi_B^e = 3.2$, replaces wood in smaller buildings. But as wood becomes scarcer and buildings larger, steel replaces wood and bamboo as the primary structural material. Standard steel I-section joists have shape factors in the range $5 < \phi_B^e < 25$ (special I-sections can have much larger values). Are steel I-joists a cheaper choice than wooden ones? Table 11.6 summarises the design requirements.

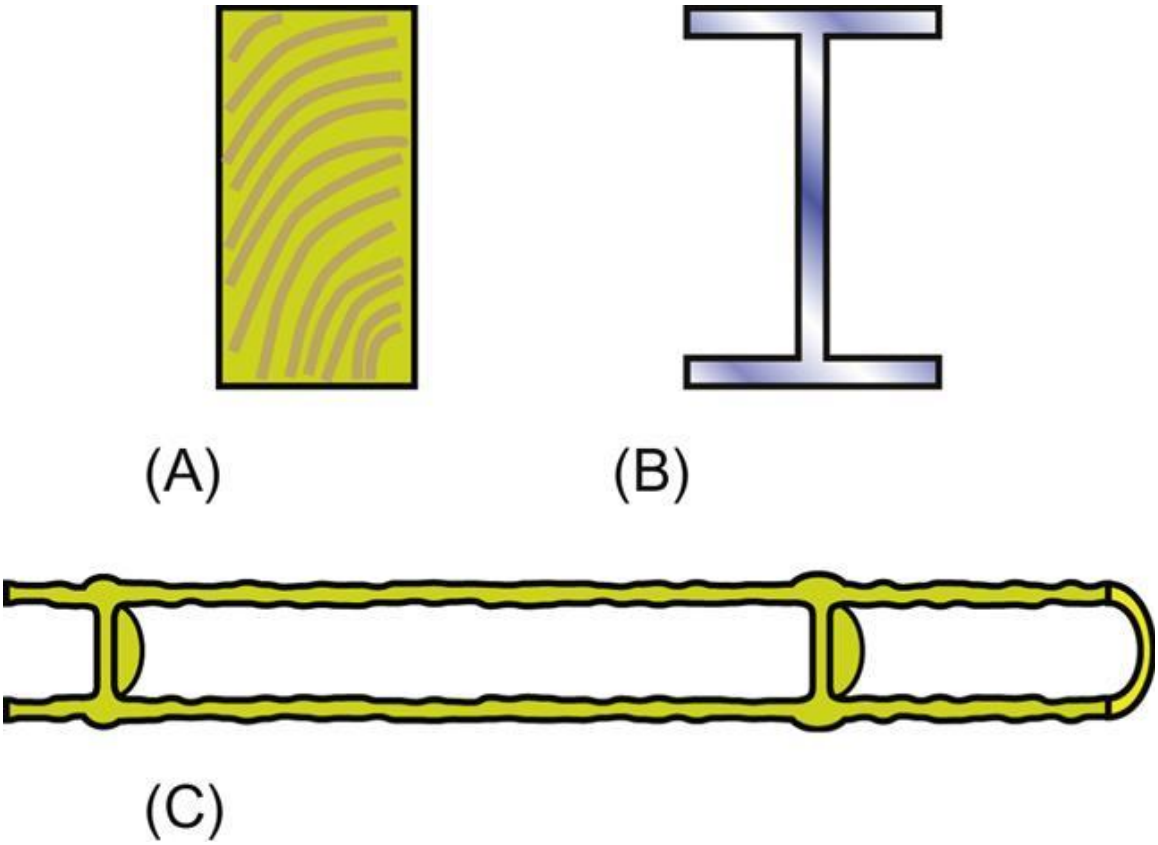


FIGURE 11.4 The cross-sections of (A) a wooden beam ($\phi_B^e = 2$), (B) a steel I-beam ($\phi_B^e = 10$), and (C) a bamboo ('natural' shape factors of $\phi_B^e = 3.2$). The values of ϕ_B^e are calculated from the ratios of dimensions of each beam, using the formulas of Table 10.3.

Table 11.6

Design Requirements for Floor Joists

Function	■ *Floor joist*
Constraints	■ *Length specified* ■ *Stiffness specified* ■ *Strength specified*
Objective	■ *Minimum material cost*
Free variables	■ *Choice of material* ■ *Section shape*

The translation and the selection. Consider stiffness first. The cheapest beam, for a given stiffness, is that with the largest value of the index (read from Table 11.1 with ρ replaced by $C_m\rho$ to minimize cost):

$$M_1 = \frac{(\phi_B^e E)^{1/2}}{C_m \rho} \tag{11.3}$$

Data for the modulus E, the density ρ, the material cost C_m and the shape factor ϕ_B^e are listed in Table 11.7, together with the values of the index M_1 with and without shape. The steel beam with $\phi_B^e = 25$ has a slightly larger value M_1 than wood, meaning that it is a little cheaper for the same stiffness.

Table 11.7

(A) Material properties for Floor Joists. (B) Indices for Floor Joists

(A)				
Material	Density $\rho(\mathrm{kg}/m^3)$	Cost C_m US$/kg	Flexural Modulus E (GPa)	Flexural Strength σ_f (MPa)
Wood (Pine)	490	1.0	9.5	41
Bamboo	700	1.9	17	42
Steel (1020)	7850	0.65	205	355

(B)						
Material	Shape Factor ϕ_B^e	Shape Factor ϕ_B^f	Index $E^{1/2}/C_m\rho$	Index[a] M_1 $(\phi_B^e E)^{1/2}/C_m\rho$	Index $\sigma_f^{2/3}/C_m\rho$	Index[a] M_2 $(\phi_B^f \sigma_f)^{2/3}/C_m\rho$
Wood (Pine)	2	1.4	**6.3**	8.9	**24**	**30**
Bamboo	3.2	2	3.1	5.5	9	14
Steel (1020)	15	4	2.8	**11**	9.8	25

[a]The values of the indices are based on means of the material properties. The units of the indices for elastic deflection are $(\mathrm{GPa})^{1/2}/(\mathrm{k\$/m^3})$; those for failure are $(\mathrm{MPa})^{2/3}/(\mathrm{k\$/m^3})$. Best choices are shown in bold, underscored.

But what about strength? The best choice for a light beam of specified strength is that which maximizes the material index:

$$M_2 = \frac{(\phi_B^f \sigma_f)^{2/3}}{C_m \rho} \qquad \textbf{(11.4)}$$

The values of failure strength σ_f, shape factor ϕ_B^f and index M_2 are also given in the table. Wood performs better than even the most efficient steel I-beam.

Postscript. So the conclusion, as far as performance per unit material cost is concerned, is that there is not much reason to choose between the standard wood and the standard steel sections used for joists. As a general statement, this is no surprise – if one were much cheaper than the other, the other would no longer exist. But – looking a little deeper – wood dominates in certain market sectors, bamboo in others, and steel in still others. Why?

Wood and bamboo are indigenous to some countries and grow locally; steel has to come further, with associated transport costs. Assembling wood structures is easier than those of steel; it is more forgiving of mismatches of dimensions, it can be trimmed on site, you can hammer nails into it anywhere. It is a user-friendly material.

But wood is a variable material, and, like us, is vulnerable to the ravishes of time, prey to fungi, insects and small mammals with sharp teeth. The problems they create in a small building – a family home, say – are easily overcome, but in a large commercial building – an office block, for instance – they create greater risks and are harder to fix. Here, steel wins.

Further Reading

Cowan, H.J., Smith, P.R., 1988. The science and technology of building materials. Van Nostrand Reinhold, NY. ISBN 0-442-21799-4. (*A broad survey of materials for the structure, cladding, insulation and interior surfacing of buildings – and excellent introduction to the subject*.).

Farrelly, D., 1984. The Book of Bamboo. Sierra Club Books, San Francisco. ISBN 0-87156-825-X. (*An introduction to bamboo and its many varieties*.).

Janssen, J.J., 1995. Building with Bamboo: A Handbook. Practical Action. ISBN 1-85339-203-0. (*Bamboo remains a building material of great importance, as well as the material of flooring, matting and basket-making. The techniques of building with bamboo have a long history, documented here*.).

Related Case Studies

5.5 'Cost: structural materials for buildings'
11.2 'Spars for human-powered planes'
11.3 'Forks for a racing bicycle'

11.5 Table Legs Yet Again: Thin or Light?

Luigi Tavolino's table (Section 5.4 and Figure 5.5) is a great success. He decides to develop a range of less-expensive tubular-legged furniture. Some are to have thin legs, others have to be light. He needs a more general way of finding the material and shape that will work best.

The translation and the selection. Tubes can be made with almost any radius r and wall thickness t, though (as we know from Chapter 10, Selection of Material and Shape) it is impractical to make them with r/t too large because they buckle locally, and that is bad. Luigi chooses GFRP as the material for his legs, and for this material the maximum available shape factor is $\phi_B^e = 10$ corresponding to a value of r/t=11.5 (Table 10.3). The cross-sectional area A and the second moment of area I of a thin-walled tube are

$$A = 2\pi rt$$

and

$$I = \pi r^3 t$$

from which

$$r = \left(\frac{2I}{A}\right)^{1/2} \tag{11.5}$$

This allows contours of constant r to be plotted on the $A - I$ quadrant of the four-quadrant diagram shown in Fig. 11.5 (they form a family of lines of slope 1), which can now be used to select either to minimize mass or to minimize tube radius.

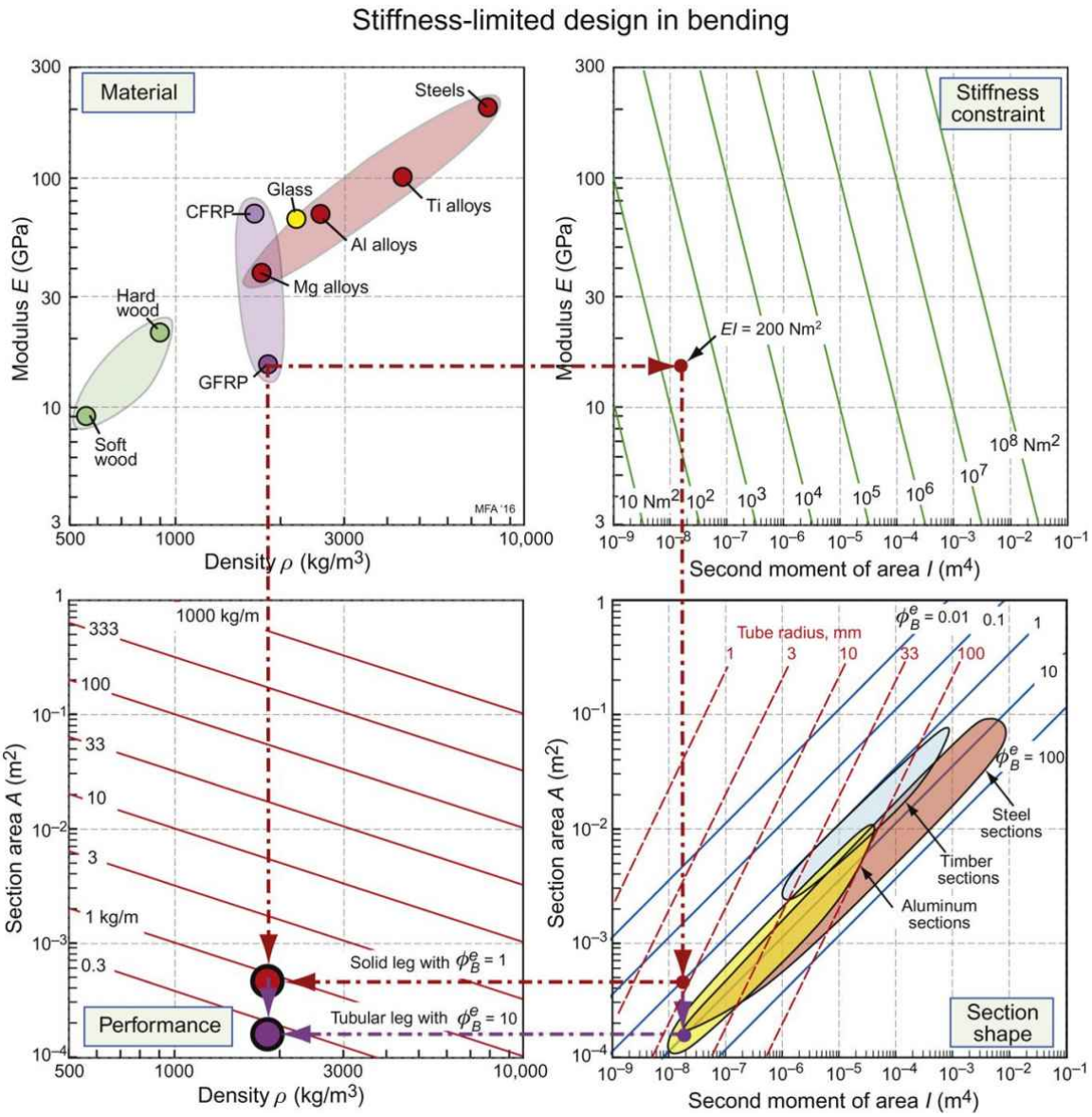


FIGURE 11.5 The chart-assembly for exploring stiffness-limited design with tubes. The *I-A* quadrant now has contours of tube radius *r*. The thinnest GFRP leg is one with $\phi_B^e = 1$. The lightest one is that with $\phi_B^e = 10$, the maximum value for GFRP.

Luigi might use it in the following way. He designs a table with cylindrical, unbraced, legs each with length L=1 m. For safety, each leg must support 50 kg without buckling, requiring that

$$F_{buckling} = \frac{\pi^2}{4}\frac{EI}{\ell^3} \geq 500 \text{ N}$$

from which

$$EI \geq 200 \text{ N}\cdot\text{m}^2.$$

Luigi plots now plots a rectangular selection path on the four-quadrant diagram in the way shown in Fig. 11.5. The lightest leg is that with the largest allowable value of ϕ_B^e (10 in the case of GFRP) but this gives a fat leg, one about 15 mm in radius, as read from the radius contours. The thinnest leg is a solid one, and that has $\phi_B^e = 1$. It is much thinner – only 8 mm in radius – but it is also almost three times heavier.

Postscript. So Luigi will have to compromise: fat and light or thin and heavy – or somewhere in between. Or he could use another material. The strength of the four-quadrant method is that he can explore other materials with ease. Repeating the construction for aluminium, putting an upper limit on ϕ_B^e of, say, 15, or for CFRP with the same upper limit as GFRP is only a moment's work.

Related Case Studies

5.4 'Materials for table legs'

9.5 'Conflicting objectives: table legs again'

11.6 Increasing the Stiffness of Steel Sheet

How could you make steel sheet stiffer? There are many reasons you might wish to do so. The most obvious are to enable stiffness-limited sheet structures to be lighter than they are now; to allow panels to carry larger in-plane compressive loads without buckling; and to raise the natural vibration frequencies of sheet structures. Bending stiffness is proportional to $E{\cdot}I$ (E is Young's modulus, I is the second moment of area of the sheet, equal to $t^3/12$ per unit width where t is the sheet thickness). There is nothing much you can do to change the modulus of steel, which is always close to 210 GPa. The answer is to add a bit of shape, increasing I. So consider the design brief of Table 11.8.

Table 11.8

Design Requirements for Stiffened Steel Sheet

Function	■ *Steel sheet for stiffness-limited structures*
Constraints	■ *Profile limited to a maximum deviation ±5 times the sheet thickness from flatness* ■ *Cheap to manufacture*
Objective	■ *Maximize bending stiffness of sheet*
Free variables	■ *Section profile*

The translation and the selection. The age-old way to make sheet steel stiffer is to corrugate it, giving it a roughly sinusoidal profile. The corrugations increase the second moment of area of the sheet about an axis normal to the corrugations themselves. The resistance to bending in one direction is thereby increased, but in the cross-direction it is not changed at all.

Corrugations are the clue but – to be useful – they must stiffen the sheet in all directions, not just one. A hexagonal grid of dimples (Fig. 11.6) achieves this. Now there isn't any direction of bending that is not dimpled. The dimples need not be hexagons; any pattern arranged in such a way that you cannot draw a straight line across it without intersecting dimples will do. Hexagons are probably about the best.

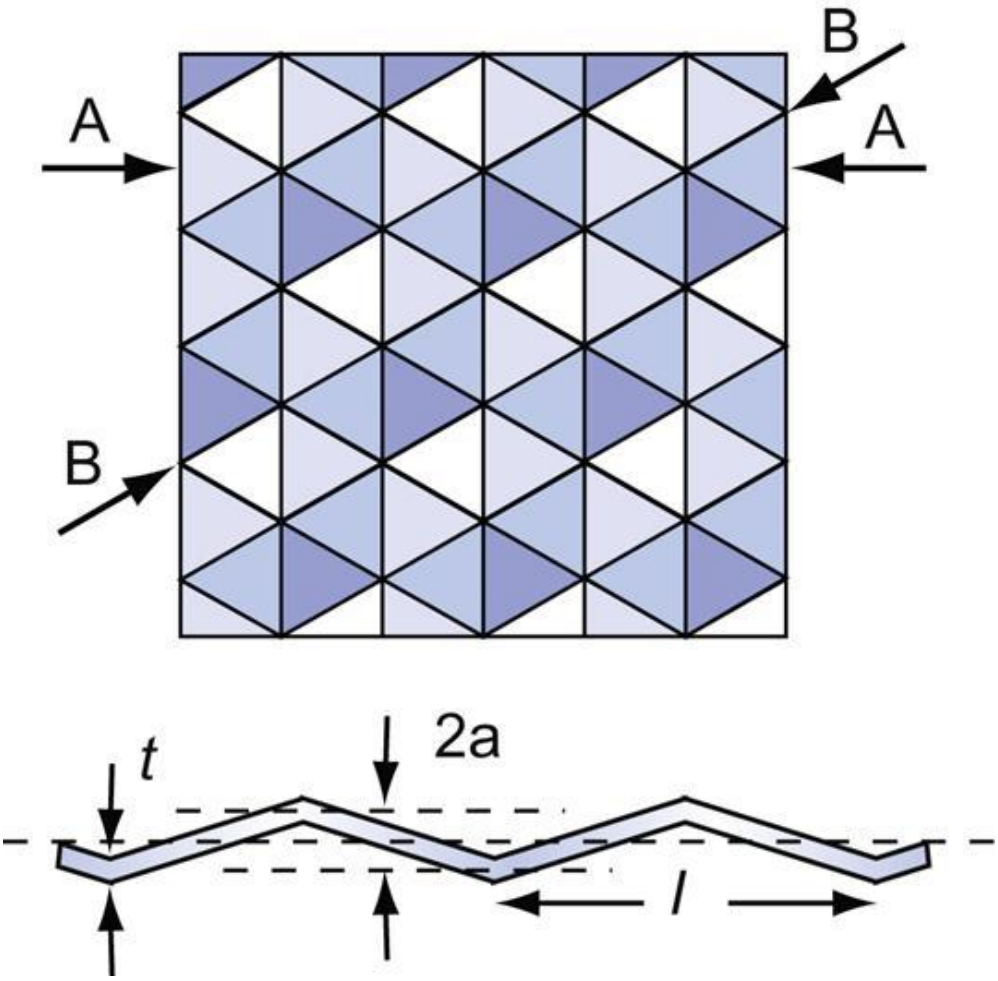


FIGURE 11.6 A sheet with a profile of adjacent hexagonal dimples.

Consider an idealised cross-section as in the lower part of Fig. 11.6, which shows the section A-A, enlarged. As before, we define the shape factor as the ratio of the stiffness of the dimpled sheet to that of the flat sheet from which it originated. The second moment of area of the flat sheet per unit width is

$$I_o = \frac{t^3}{12} \tag{11.6}$$

That of the dimpled sheet with amplitude a is

$$I \approx \frac{1}{12}(2a+t)^2 t \qquad \textbf{(11.7)}$$

giving a shape factor, defined as before as the ratio of the stiffness of the sheet before and after corrugating

$$\phi_B^e = \frac{I}{I_o} \approx \frac{(2a+t)^2}{t^2} \qquad \textbf{(11.8)}$$

Note that the shape factor has the value unity when the amplitude is zero but increases as the amplitude increases. The equivalent shape factor for failure in bending is

$$\phi_B^f = \frac{Z}{Z_o} \approx \frac{(2a+t)}{t} \qquad \textbf{(11.9)}$$

These equations predict large gains in stiffness and strength. The reality is a little less rosy. This is because, while all cross-section of the sheet are dimpled, only those that cut through the peaks of the dimples have an amplitude equal to the peak height (all others have less) and, even among these, only some have adjacent dimples; the section B-B, for example does not. Despite this, and limits set by the onset of local buckling, the gain is real.

Postscript. Dimpling can be applied to most rolled-sheet products. It is done by making the final roll-pass through mating rolls with meshing dimples, adding little to the cost. It is most commonly applied to sheet steel. Here it finds applications in the automobile industry for bumper armatures, seat frames, side impact bars, which allows weight-saving without compromising mechanical performance. Stiffening sheet also raises its natural vibration frequencies, making them harder to excite, thus helping to suppress vibration in panels.

But a final word of warning: stiffening the sheet may change its failure mechanism. Flat sheet yields when bent; dimpled sheet, if thin, could fail by a local buckling mode. It is this that ultimately limits the useful extent of dimpling.

Further Reading

Fletcher, M., 1998. Cold-rolled dimples gauge strength. Eureka, p. 212. (*A brief account of the shaping of steel panels to improve bending stiffness and*

strength.).

Related Case Studies

11.7 ‘Shapes that flex: leaf and strand structures’

11.7 Shapes that Flex: Leaf and Strand Structures

Flexible cables, leaf and helical springs, and flexural hinges require low, not high, structural efficiency. Here the requirement is for low flexural or torsional stiffness about one or two axes while retaining high stiffness and strength in other directions. The simplest example is the torsion bar (Fig. 11.7A), with a shape factor $\phi_T^e \approx 1$. The single-leaf spring (Fig. 11.7B) allows much lower values of ϕ_B^e ($\phi_B^e = h/w$; the dimensions are defined on the figure).

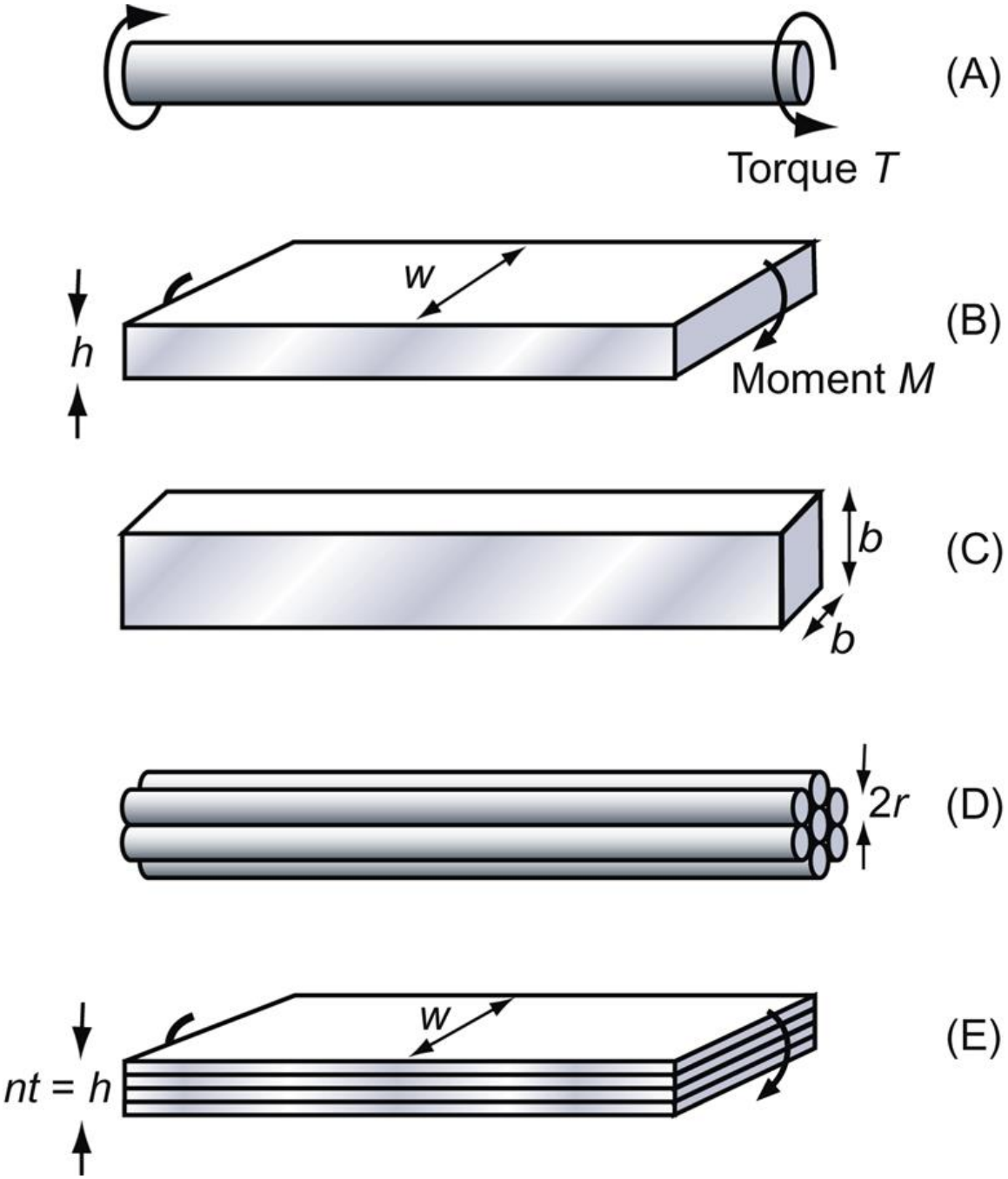


FIGURE 11.7 Shapes that twist or bend easily but retain axial strength and stiffness. (A) a torsion bar, (B) a thin plate, (C) a slender rod, (D) a multi-strand cable, and (E) a leaf spring.

Multistrand cables and multileaf assemblies do much better. Consider the change in efficiency when a solid square beam of section $A=b^2$ is subdivided into n cylindrical strands each of radius r (Figure 11.7C,D), such that

$$n\pi r^2 = b^2 \tag{11.10}$$

The axial stiffness of the original bar is proportional to EA, and its bending stiffness, S_o, to EI_o where E is the modulus of the material and I_o the second moments of area. When made into a cable of n parallel cylindrical strands, the axial stiffness is unchanged, but the bending stiffness S falls to a value proportional to nEI, where I is the second moment of a single strand. Thus

$$S \propto nE\frac{\pi r^4}{4} = \frac{1}{4\pi}E\frac{b^4}{n} \tag{11.11}$$

Thus the structural efficiency of the cable is

$$\phi_B^e = \frac{S}{S_o} = \frac{nEI}{EI_o} = \frac{3}{n\pi} \tag{11.12}$$

The number of strands, n, can be very large, allowing the flexural stiffness to be adjusted over a wide range while leaving the axial stiffness unchanged.

Multileaf assemblies allow even more dramatic anisotropy. If the thick leaf shown at (b) in the figure is divided into the stack of thin ones of the same width shown at (e), the bending stiffness changes from

$$S_o = Ewh^3/12$$

to

$$S = nEwt^3/12$$

where t=h/n. The shape efficiency of the stack is

$$\phi_B^e = \frac{S}{S_o} = \frac{nt^3}{h^3} = \frac{1}{n^2} \tag{11.13}$$

Thus a stack of 10 layers is 100 times less stiff than a simple leaf of the same total thickness, while the in-plane stiffness is unchanged.

Postscript. When a segmented structure is bent, the segments slide against each other. The analysis we have just done assumed that there was no friction opposing the sliding. A large friction prevents the sliding, and, until sliding starts, the bending stiffness is the same as that of an unsegmented bar or plate. Leaf springs are lubricated, or are interleaved with Teflon shims, to make sliding easy.

Related Case Studies

11.6 'Increasing the stiffness of steel sheet'

11.8 'Ultra-efficient springs'

11.8 Ultra-Efficient Springs

Springs store energy. We deduced in Section 5.7 that they are best made of a material with a high value of σ_f^2/E, or, if mass is more important than volume, then of $\sigma_f^2/\rho E$. Springs can be made more efficient still by shaping their section. Just how much more is revealed below.

We take as a measure of performance the energy stored per unit volume of solid of which the spring is made; we wish to maximize this energy. Energy per unit weight and per unit cost are maximized by similar procedures (Table 11.9).

Table 11.9

Design Requirements for Ultra-Efficient Springs

Function	■ *Material-efficient spring*
Constraint	■ *Must remain elastic under design loads*
Objective	■ *Maximum stored energy per unit volume (or mass, or cost)*
Free variables	■ *Choice of material* ■ *Section shape*

The translation and the selection. Consider a single-leaf spring first (Fig. 11.8A). A leaf spring is an elastically bent beam. The energy stored in a bent beam, loaded by a force F, is

$$U = \frac{1}{2}\frac{F^2}{S_B} \tag{11.14}$$

where S_B, the bending stiffness of the spring, is given by Eq. (10.17), or, after replacing I by ϕ_B^e, by Eq. (10.18), which, repeated, is:

$$S_B = \frac{C_1}{12L^3} E\phi_B^e A^2 \tag{11.15}$$

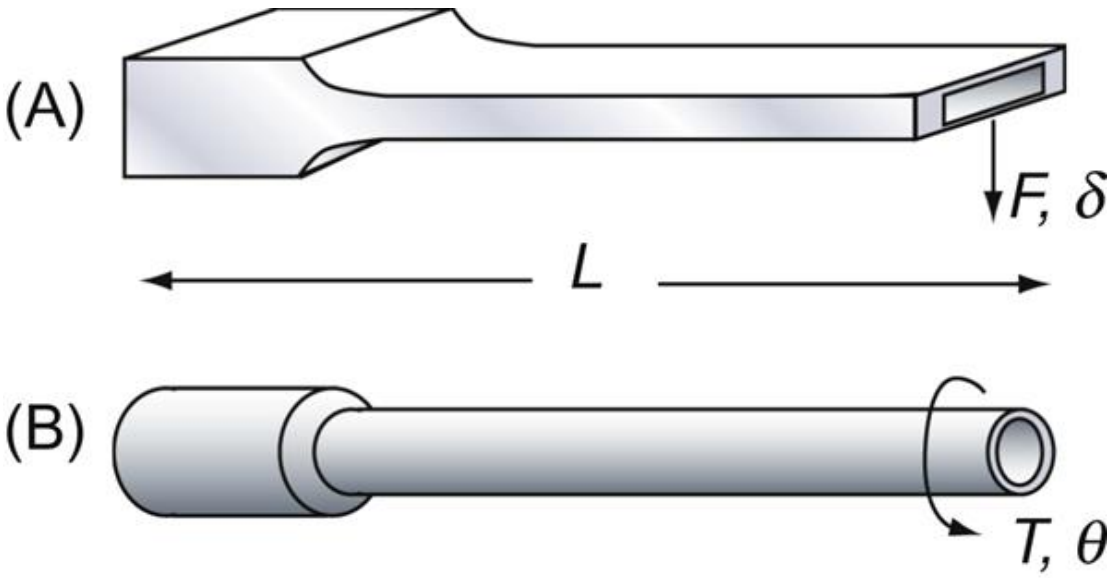


FIGURE 11.8 Hollow springs use material more efficiently than solid springs. (A) a hollow leaf spring, (B) a hollow torsion bar.

The force F in Eq. (11.14) is limited by the onset of yield; its maximum value is

$$F_f = C_2 Z\frac{\sigma_f}{L} = \frac{C_2}{6L}\sigma_f \phi_B^f A^{3/2} \tag{11.16}$$

(The constants C_1 and C_2 are tabulated in Appendix B, Section B3 and B4). Assembling these gives the maximum energy the spring can store:

$$\frac{U_{\max}}{V} = \frac{{C_2}^2}{6C_1}\left(\frac{(\phi_B^f\,\sigma_f)^2}{\phi_B^e E}\right) \tag{11.17}$$

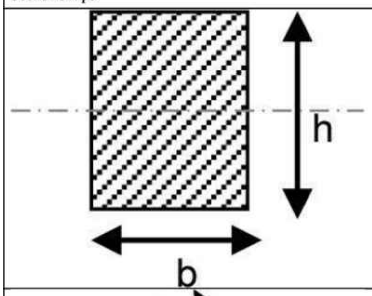
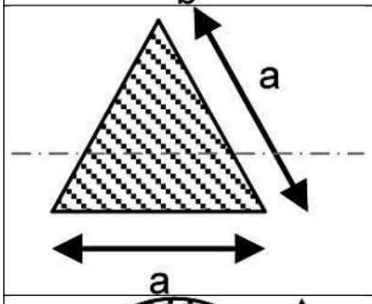
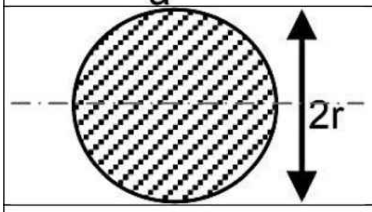
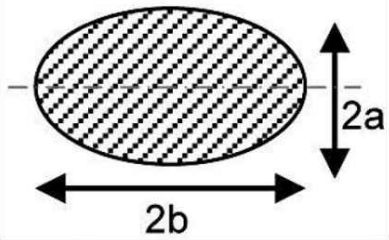
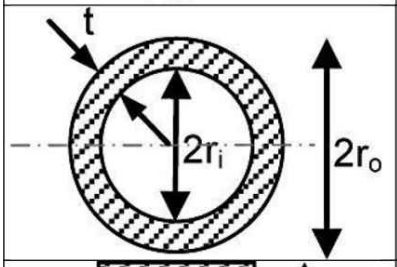
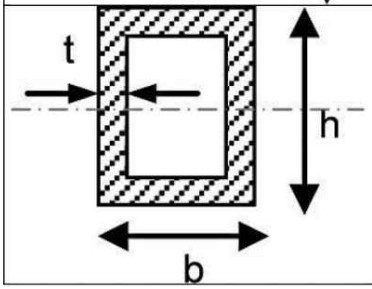
where $V = AL$ is the volume of solid in the spring. The best material and shape for the spring – the one that uses the least material – is that with the greatest value of the quantity

$$M_1 = \frac{(\phi_B^f \sigma_f)^2}{\phi_B^e E} \tag{11.18}$$

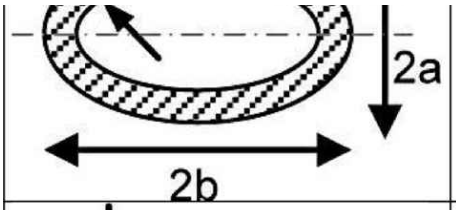
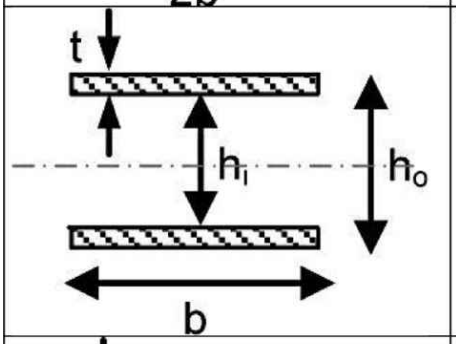
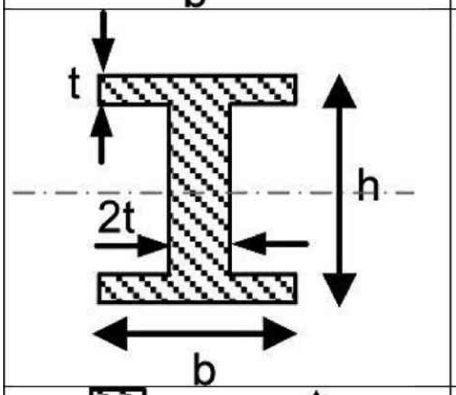
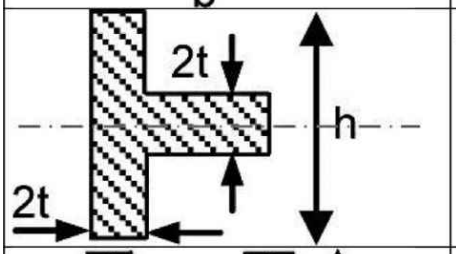
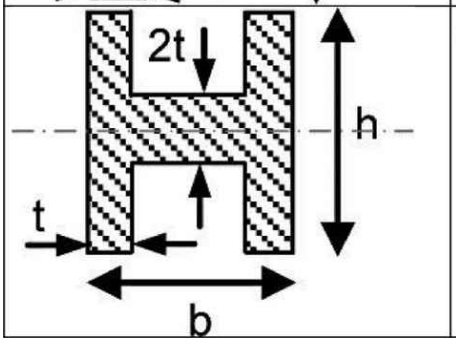
For a fixed section shape, the ratio involving the two ϕs is a constant, and then the best choice of material is that with the greatest value of σ_f^2/E – the same result as before. When shape is a variable, the most efficient shapes are those with large $(\phi_B^f)^2/\phi_B^e$. Values for these ratios are tabulated for common section shapes in Table 11.10; hollow box and elliptical sections are up to 3 times more efficient than solid shapes.

Table 11.10

Shape Factors for the Efficiency of Springs

Section Shape	$(\phi_B^f)^2/\phi_B^e$	$(\phi_T^f)^2/\phi_T^e$
	1	$1.08\frac{b^2}{h^2}\frac{1}{\left(1+0.6\frac{b}{h}\right)^2\left(1-0.58\frac{b}{h}\right)}$
	0.38	0.83
	0.75	1.6
	0.75	$0.8\left(1+\frac{a^2}{b^2}\right)\quad (a<b)$
	1.5	3.2
	$\frac{(1+3\,b/h)}{(1+b/h)}\quad (h,b>>t)$	$3.32\frac{1}{(1-t/h)^4}\quad (h,b>>t)$

Section Shape	$(\phi_B^f)^2/\phi_B^e$	$(\phi_T^f)^2/\phi_T^e$
	$\frac{3}{4}\frac{(1+3\,b/a)}{(1+b/a)}\quad (a,b>>t)$	$3.2\frac{(1+a^2/b^2)}{(1+a/b)}\left(\frac{b}{a}\right)^{3/2}\quad (a,b>>t)$

		
	3	–
	$\frac{(1+3\,b/h)}{(1+b/h)}$ $(h,b>>t)$	$1.07\frac{(1+4\,h/b)}{(1+h/b)}$ $(h,b>>t)$
	$1.13\frac{(1+4\,bt^2/h^3)}{(1+b/h)}$ $(h>>t)$	$0.54\frac{(1+8\,b/h)}{(1+b/h)}$ $(h,b>>t)$
	$1.13\frac{(1+4\,bt^2/h^3)}{(1+b/h)}$ $(h>>t)$	$1.07\frac{(1+4\,h/b)}{(1+h/b)}$ $(h,b>>t)$

Torsion bars and helical springs are loaded in torsion (Fig. 11.8B). A similar calculation gives

$$\frac{U_{\max}}{V} = \frac{1}{6.5}\frac{(\phi_T^f \sigma_f)^2}{\phi_T^e G} \tag{11.19}$$

The most efficient material and shape for a torsional spring is that with the largest value of

$$M_2 = \frac{(\phi_T^f \sigma_f)^2}{\phi_T^e E} \tag{11.20}$$

(where G has been replaced by $3E/8$). The criteria are the same: when shape is not a variable, the best torsion-bar materials are those with high values of σ_f^2/E. Table 11.10 shows that the best shapes are hollow tubes, with a ratio of $(\phi_T^f)^2/\phi_T^e$ that is twice that of a solid cylinder; all other shapes are less efficient. Springs that store the maximum energy per unit weight (instead of unit volume) are selected with indices given by replacing E by $E\rho$ in Eqs (11.18) and (11.20). For maximum energy per unit cost, replace E by $EC_m\rho$ where C_m is the material cost per kg.

Postscript. Hollow springs are common in vibrating and oscillating devices and for instruments in which inertial forces must be minimized. The hollow elliptical section is widely used for springs loaded in bending; the hollow tube for those loaded in torsion. More about this problem can be found in the classic paper by Boiten (1963).

Further Reading

Boiten, R.G., 1963. Mechanics of Instrumentation. Proc. I. Mech. E. 177, p. 269. (*A definitive analysis of the mechanical design of precision instruments*.).

Related Case Studies

5.7 'Materials for springs'
11.7 'Shapes that flex: leaf and strand structures'

11.9 Summary and Conclusions

In designing components that are loaded such that they bend, twist or buckle, the designer has two groups of variables with which to optimize performance: the choice of *material* and of *section shape*. The best choice of material depends on the shapes in which it is available, or to which it could potentially be formed. The procedures developed in Chapter 10,

Selection of Material and Shape, gives a method for optimizing the coupled choice of material and shape.

Their use is illustrated in this chapter. Often the designer has available certain stock materials in certain shapes. Then that with the greatest value of the appropriate material index (of which a number were listed in Table 11.1) maximizes performance. Sometimes sections can be specially designed; then material properties and design loads determine a maximum practical value for the shape factor, above which local buckling leads to failure. Again, the procedure gives an optimal choice of material and shape.

Further gains in efficiency are possible by combining microscopic with macroscopic shape – something we return to in a later chapter.

CHAPTER 12

Designing Hybrid Materials

Abstract

Hybrid materials are combinations of two or more materials assembled in such a way as to have attributes not offered by either one alone. Particulate and fibrous composites are examples of one type of hybrid, but there are many others: sandwich structures, lattice structures, segmented structures and more. This chapter explores ways of designing hybrid materials, emphasizing the choice of the components, their configuration, their relative volume fraction and their scale. The approach we adopt is to use bounding methods to estimate the properties of each configuration. With these, the properties of a given pair of materials in a given configuration can be calculated. These can then be plotted on material selection charts, which become tools for selecting both configuration and material.

Keywords

Holes in material-property space; composites; sandwich structures; segmented structures; cellular structures; lattice structures; multilayers; natural materials

12.1 Introduction and Synopsis

Why do horse breeders cross a horse with a donkey, delivering a mule? Why do farmers prefer hybrid corn to the natural strain? Mules, after all, are best known for their stubbornness, and – like hybrid corn – they cannot reproduce, so you have to start again for each generation. So – why? Because, although they have some attributes that are less desirable

than their forebears, they have others – hardiness, strength, resistance to disease – that are better. The botanical phrase 'hybrid vigor' sums it up.

So let us explore the idea of hybrid materials – combinations of two or more materials assembled in such a way as to have attributes not offered by either one alone (Fig. 12.1, central circle). Like the mule, we may find that some attributes are less good (the cost, or the ability to be recycled, for example), but if particular attributes that matter to us are better, something is achieved. Particulate and fibrous composites are examples of one type of hybrid, but there are many others: sandwich structures, lattice structures, segmented structures and more. Here we explore ways of designing hybrid materials, emphasizing the choice of the components, their configuration, their relative volume fraction and their scale (Table 12.1). The new variables expand design space, allowing the creation of new 'materials' with specific property profiles.

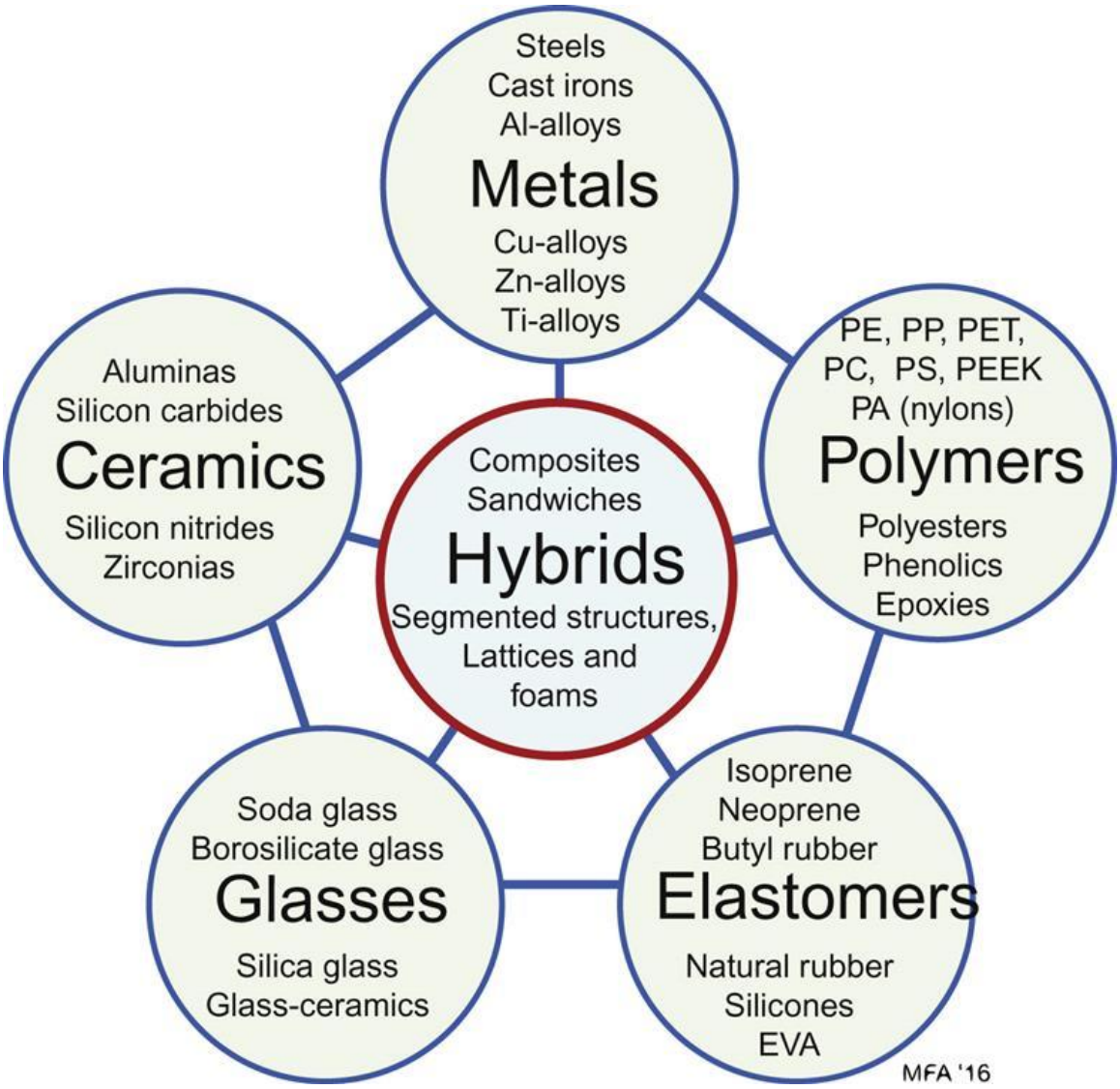


FIGURE 12.1 Hybrid materials combine the properties of two (or more) monolithic materials, or of one material and space.

Table 12.1

Ingredients of Hybrid Design

Components	*The choice of materials to be combined*
Configuration	*The shape and connectivity of the components*
Relative volumes	*The volume fraction of each component*
Scale	*The length-scale of the structural unit*

And that highlights one of the challenges. How are we to compare a hybrid like a sandwich with monolithic materials like – say –

polycarbonate or titanium? To do this we must think of the sandwich not only as a hybrid with faces on one material bonded to a core of another, but as a 'material' in its own right, with its own set of 'effective properties'; it is these that allow the comparison.

The approach adopted here is one of breadth rather than precision. The aim is to assemble methods to allow the properties of alternative hybrids to be scanned and compared, seeking those that best meet a given set of design requirements. Once materials and configuration have been chosen, standard methods – optimization routines, finite-element analyses – can be used to refine them. But what the standard methods are *not* good at is the quick scan of alternative combinations. That is where the approximate methods developed below, in which material and configuration become the variables, pay off.

The word 'configuration' requires elaboration. Fig. 12.2 shows four different configurations of bridge. In the first all members are loaded in compression. In the second, members carry both tension and compression, depending on how the bridge is loaded. In the third and fourth, the suspension cables are loaded purely in tension. Any one of these can be optimized, but no amount of optimization will cause one to evolve into another because this involves a discrete jump in configuration, each characterized by its own set of variables.[1]

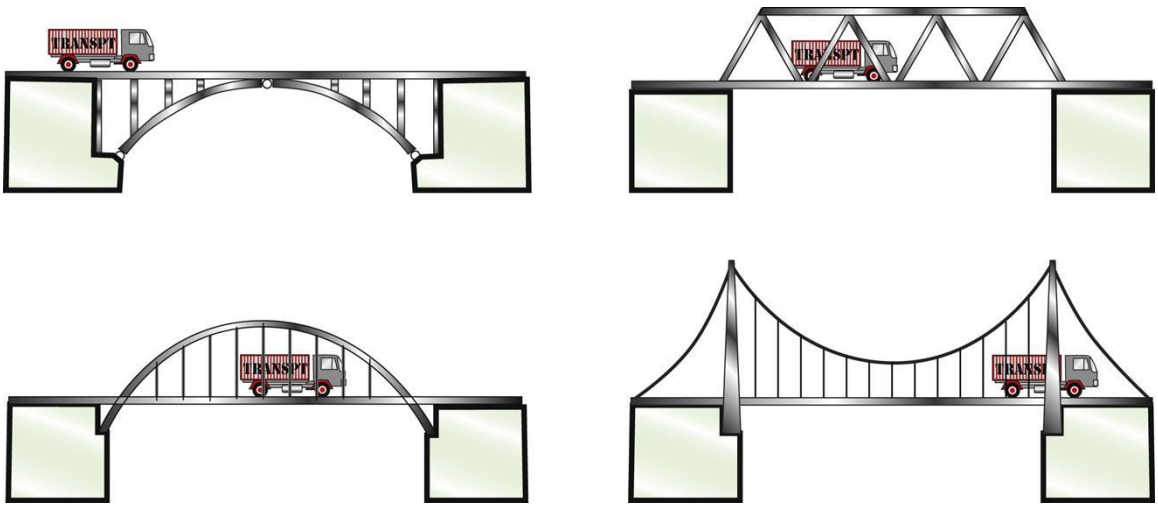


FIGURE 12.2 Four configurations for a bridge. The design variables describing the performance of each differ.

Hybrid design has the same feature: the classes of hybrid are distinguished by their configuration. Here we focus on five classes, each with a number of discrete members. The cover images of this chapter suggest what they look like. To avoid a mouthful of words every time we refer to one, we use the shorthand: *composite, sandwich, multilayer, cellular* and *segmented* structures. *Composites* combine two solid components, one (the reinforcement) as fibres or particles, contained in the other (the matrix). Their properties are some average of those of the components, and, on a scale large compared to that of the reinforcement, the composite behaves as it were a homogeneous materials. *Sandwiches* have outer faces of one material supported by a core of another, usually a low density material – a configuration that gives greater flexural stiffness per unit weight than that offered by either component alone. A *multilayer* is a generalization of sandwich with successive layers of different materials of differing thicknesses, like the cladding of a modern building. *Cellular structures* are combinations of material and space (which can, of course, be filled with another material). *Segmented structures* are materials subdivided in one, two or three dimensions; the subdivisions both lower the stiffness, and, by dividing the material into discrete units, impart damage tolerance.

The approach we adopt is to use bounding methods to estimate the properties of each configuration. With these estimations, the properties of a given pair of materials in a given configuration can be calculated. These properties can then be plotted on material selection charts, which become tools for selecting both configuration and material. But first: what are we trying to achieve? What will be our criteria of excellence?

12.2 Holes in Material–Property Space

All the charts of Chapter 3, Materials Property Charts have one thing in common: parts of them are populated with materials and parts are not – there are *holes* (Fig. 12.3). Some parts are inaccessible for fundamental reasons that relate to the size of atoms and the nature of the forces that bind them together. But other parts are empty even though, in principle, they could be filled.

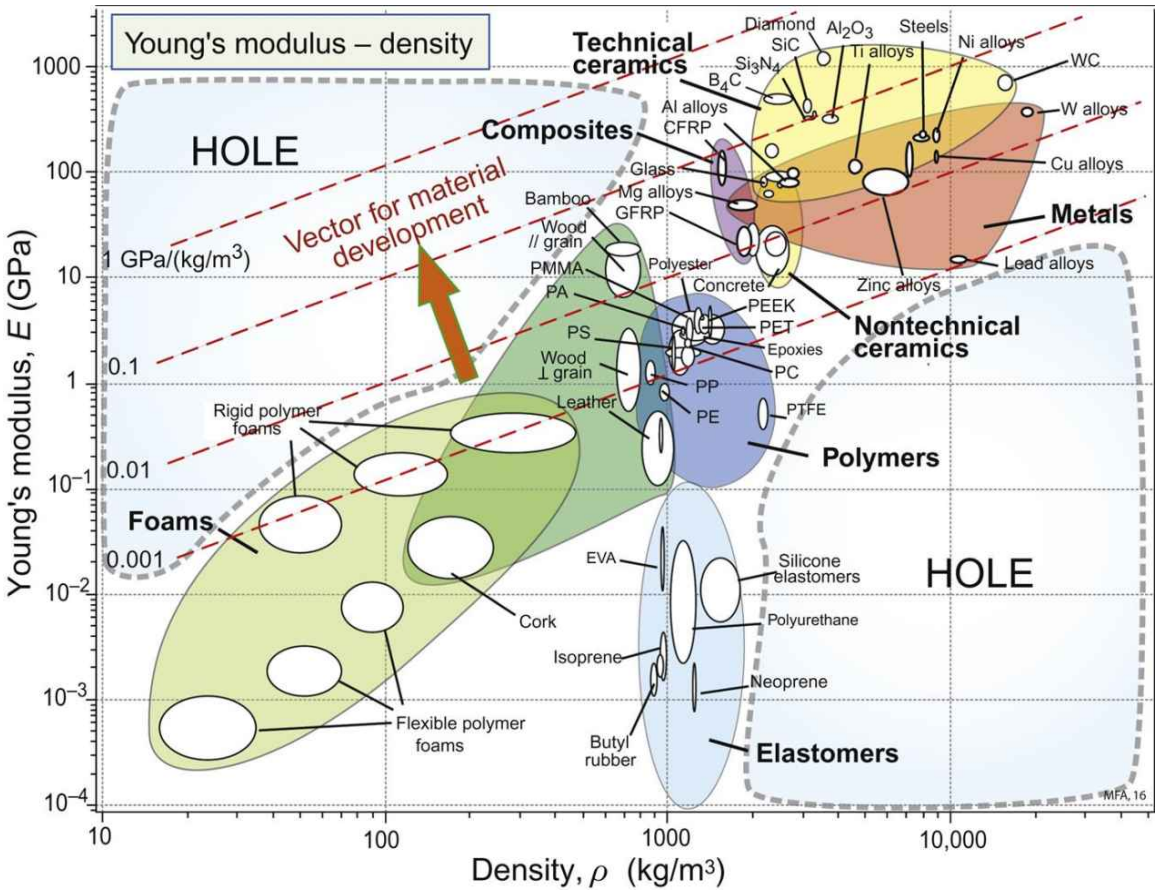


FIGURE 12.3 Holes in modulus–density space, with contours of specific modulus, E/ρ. Material development that extended the occupied territory in the direction of the arrow (the 'vector for development') allows components with greater stiffness-to-weight than any current material allows.

Is anything to be gained by developing materials (or material combinations) that lie in these holes? The material indices of Chapter 4, Materials Selection – The Basics show where this is profitable. A grid of lines of one index – E/ρ– is plotted on Fig. 12.3. If the filled areas can be expanded in the direction of the arrow (i.e., to greater values of E/ρ) it will enable lighter, stiffer structures to be made. The arrow lies normal to the index lines. It defines a *vector for material development*.

One approach to filling holes – the long-established one – is that of developing new alloy compositions, new polymer chemistries and new compositions of glass and ceramic that extend the populated areas of the property charts. But developing new materials can be an expensive and uncertain process, and the gains tend to be incremental rather than step-like. An alternative is to combine two or more existing materials so as to

allow a superposition of their properties – in short, to create hybrids. The spectacular success of carbon and glass-fibre reinforced composites at one extreme, and of foamed materials at another (hybrids of material and space) in filling previously empty areas of the property charts is encouragement enough to explore ways in which such hybrids can be designed.

12.3 Key Concepts for Hybrid Design

If you mix two materials, why aren't the properties of the resulting hybrid simply the sum of those of the ingredients each weighted by it volume fraction? Some properties do follow this 'rule of mixtures': the density, the specific heat and the dielectric constant behave in exactly this way (Figure 2.4, central box). But others combine in other ways. Depending on the configuration of the materials and the way they are combined, we may find any one of the following (Fig. 12.4).

- The *'Greatest of both' scenario*. The ideal, often, is the creation of a hybrid with the best properties of both components. There are examples, most commonly when a bulk property of one material is combined with the surface properties of another. Zinc coated steel has the strength and toughness of steel with the corrosion resistance of zinc. Glazed pottery exploits the formability and low cost of clay with the permeability and durability of glass.
- The *'Harmonic mean' scenario*. Sometimes we have to live with a lesser compromise, typified by the stiffness of particulate composites, in which the hybrid properties fall below those of a rule of mixtures, lying closer to the harmonic than the arithmetic mean of the properties. Although the gains are less spectacular, they are still useful.
- The *'Least of both'* or *'Weakest-link'* scenario. Sometimes it is not the greatest but the least of the properties that we seek. Fire-sprinkler systems, e.g., use a wax-metal hybrid designed to fail, releasing the spray, when the melting point of the lower-melting material (the wax) is exceeded.

  And, occasionally,
- The *'Bonus card'* scenario in which the hybrid acquires a property value that exceeds that of either of its components – the toughness of composites, frequently, is greater than that of either of its components.

- The *'Deliberate flaw'* scenario in which the hybrid property falls below that of either component, achieved, e.g., by processing in such a way as to embed a distribution of cracks into the hybrid.

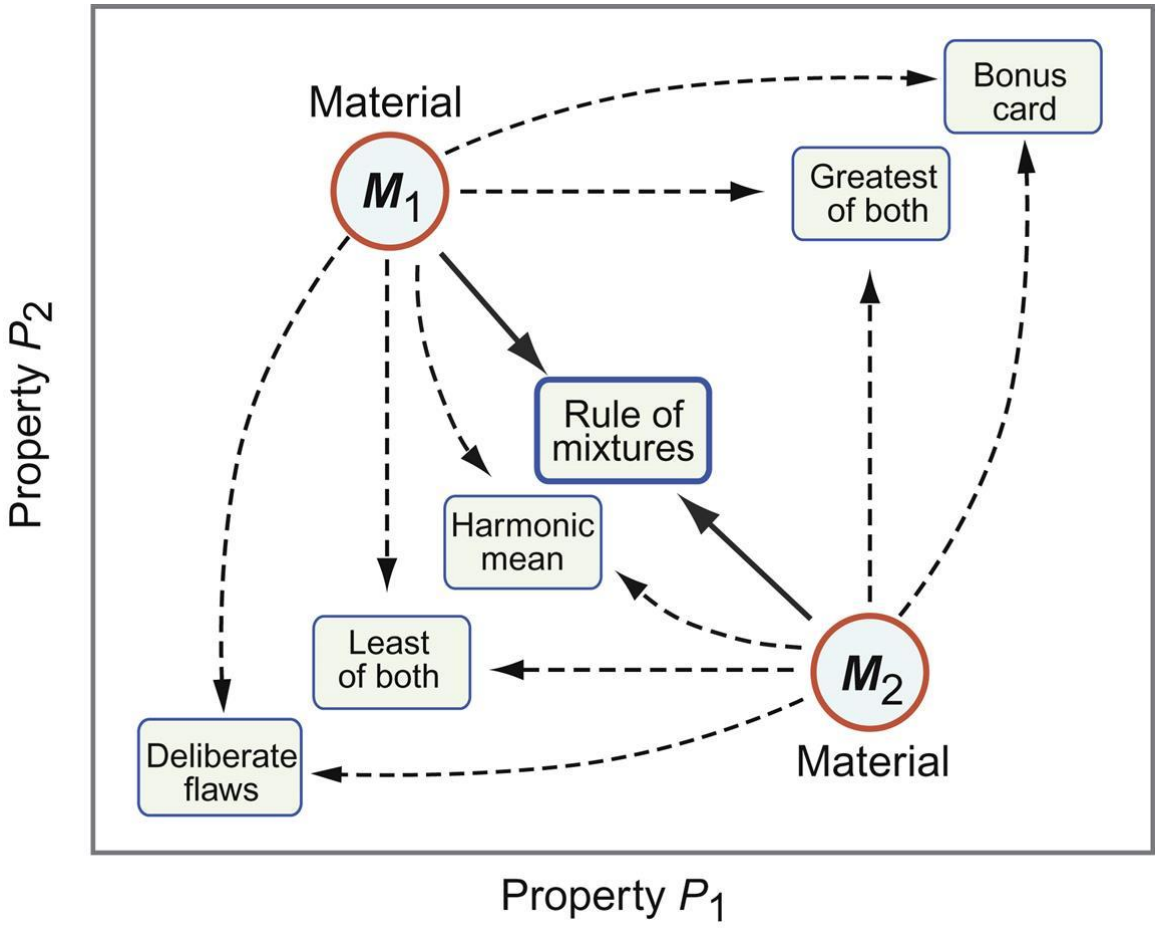


FIGURE 12.4 Hybridization scenarios.

When is a hybrid a 'material'? There is a certain duality about the way in which hybrids are thought about and discussed. Some, like filled polymers, composites or wood are treated as materials in their own right, each characterized by its own set of material properties. Others – like galvanized steel – are seen as one material (steel) to which a coating of a second (zinc) has been applied, even though if could be regarded as a new material with the strength of steel but the surface properties of zinc ('stinc', perhaps?). Sandwich panels illustrate the duality, sometimes viewed as two sheets of face-material separated by a core material, and sometimes – to allow comparison with bulk materials – as a 'material' with their own

density, flexural stiffness and strength. To call any one of these a 'material' and characterize it as such is a useful shorthand, allowing designers to use existing methods when designing with them. But if we are to design the hybrid itself, we must deconstruct it, and think of it as a combination of materials (or of material and space) in a chosen configuration.

We explore specific hybrid systems in subsequent sections of this chapter. Before doing so, it is helpful to examine some underlying concepts.

Packing of Rods and Spheres

Many hybrids consist of discrete particles of one phase (the 'reinforcement') dispersed in a second continuous phase (the 'matrix'). The properties of the resulting hybrid depend on the volume fraction occupied by the reinforcement (the 'packing fraction') and the coordination number (number of contacts per particle, charmingly called the 'kissing number' in early literature). There are limits to the volume fraction of rods or spheres that can be packed into given space. Parallel rods, randomly packed, will spontaneously 'crystallize' into a close-packed array if compressed, occupying a maximum volume fraction of 0.91. Spheres, too, can be close packed, occupying a maximum volume fraction of 0.74, but they do not do so spontaneously. Spheres of a single size poured into a jar initially form a noncrystalline 'loose random packed' array occupying a volume fraction of about 0.6; 'loose' means that the array can be sheared without change of volume. But if the loose random packed spheres are vibrated in the earth's gravitational field, the array densifies to a 'dense random packed' array occupying a volume fraction of 0.64 that dilates if sheared. If the restriction of a single size of rod or sphere is relaxed, higher packing fractions become possible. Table 12.2 summarizes.

Table 12.2

Packing Fraction and Coordination of Mono-Dispersed Rods and Spheres

Type of Packing	Packing Fraction	Coordination Number
Parallel rods	0.91	6
Diamond cubic array	0.34	4
Loose random packed spheres	0.56–0.6	4 to 6
Dense random packed spheres	0.6–0.64	6 to 8
Spheres in a body-centred cubic array	0.68	8
Spheres in a face-centred cubic array	0.74	12

Load Transfer Across Interfaces

Particles or fibres of glass and ceramic are stiff and strong but they lack toughness and ductility. Load transfer allows some of their desirable properties to be imparted to a matrix of lower strength and stiffness. This property sharing relies on load transfer (illustrated by Fig. 12.5 and the accompanying example) and load transfer in turn relies on the interface between the particle or fibre (the 'inclusion') and the material in which it is embedded (the 'matrix'). If inclusion and matrix are strongly bonded or the coefficient of friction between them is high, deformation of the matrix transfers load onto the inclusion. The load needed to deform the hybrid is the volume average of the load carried by the inclusion and matrix, so by transferring load onto the inclusion, the strength of the hybrid is increased.

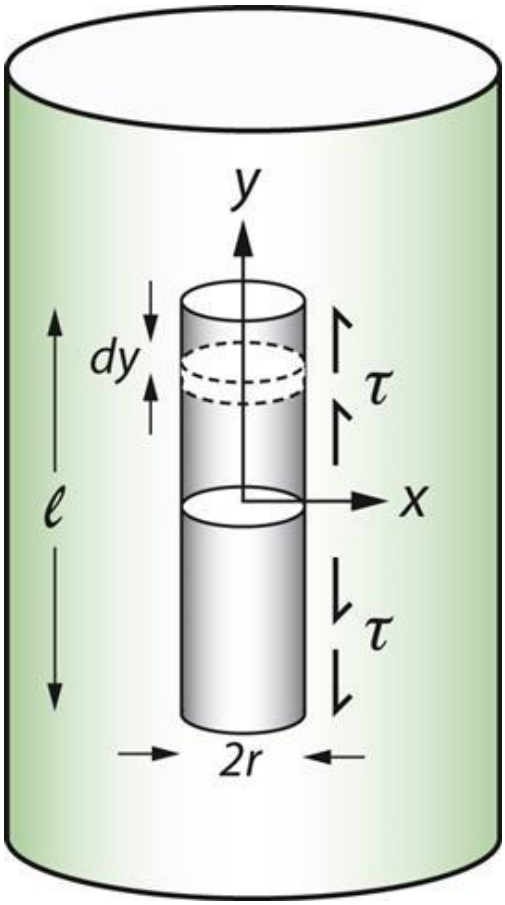


FIGURE 12.5 A hybrid unit cell.

A number of hybrids rely on load transfer. They include fibre-reinforced composites, sandwich structures and structural multilayers.

Calculating Load Transfer

Fig. 12.5 shows a unit of a hybrid consisting of short fibres of length ℓ and radius r embedded in a matrix. The shear strength of the interface at which they meet is τ. How much load can be transferred from the matrix to the fibre?

The transfer of load F across a segment of fibre of length dy is

$$dF = 2\pi r \tau \, dy$$

Integrating from the centre of the fibre to its tip gives the load $F = \pi r \ell \tau$ at the centre section; dividing this by the cross section of the fibre gives

the peak stress in the fibre

$$\sigma_{fiber} = \frac{F}{\pi r^2} = \frac{\ell}{r}\tau$$

If we identify τ with the shear strength of the matrix itself and approximate $\tau \approx \sigma_m/2$ where σ_m is the tensile strength of the matrix, we find

$$\sigma_{fiber} = \frac{\ell}{2r}\sigma_m \qquad \textbf{(12.1)}$$

The stress in the fibre is magnified by the aspect ratio ℓ/r of the fibre.

Bend and Stretch-Dominated Structures

One class of hybrid is that created by combining a solid with space, which makes a porous or lattice-like material. If the connectivity is low, such that network is open, the members of the lattice bend when the structure is loaded (Fig. 12.6A and B). If, instead the connectivity is higher, such that the lattice is triangulated, its bending is suppressed and the members stretch or are compressed when the structure is loaded (Fig. 12.6C). Bend-dominated structures have low moduli and strengths. Stretch-dominated structures are much stiffer and stronger. The distinction is a crucial in modelling foams and lattices.

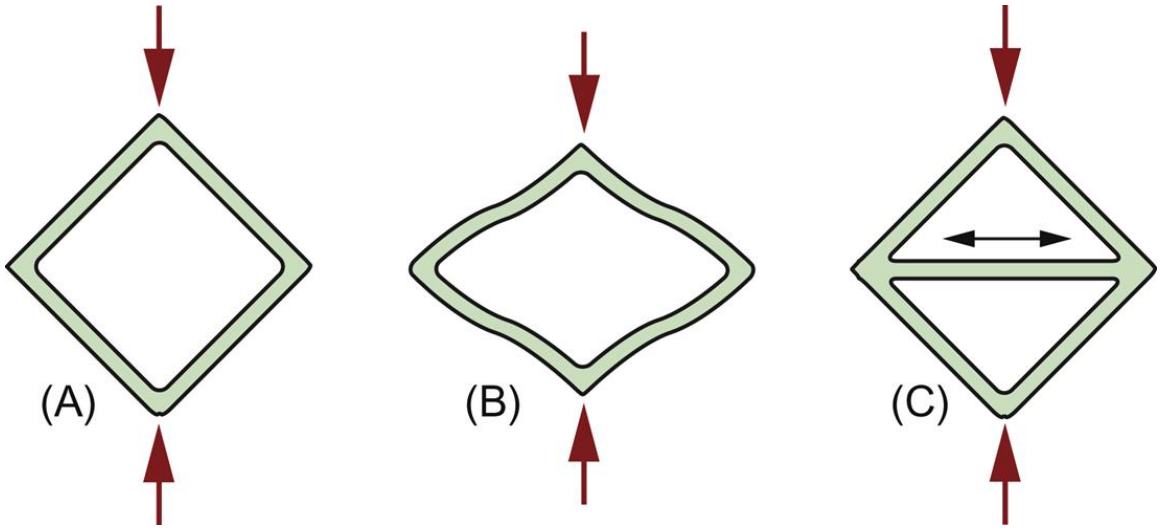


FIGURE 12.6 Bend and stretch dominated structures (A) undeformed, (B) deformation by bending and (C) deformation involving stretching.

To understand these more fully we need one of those simple yet profound laws: the Maxwell stability criterion. The condition that a pin-jointed frame (meaning one that is hinged at its corners) made up of b struts and j frictionless joints (Fig. 12.7) to be both *statically* and *kinematically determinate* (meaning that it is rigid and does not fold up when loaded) in 2 dimensions, is:

$$M = b - 2j + 3 \geq 0 \qquad \textbf{(12.2)}$$

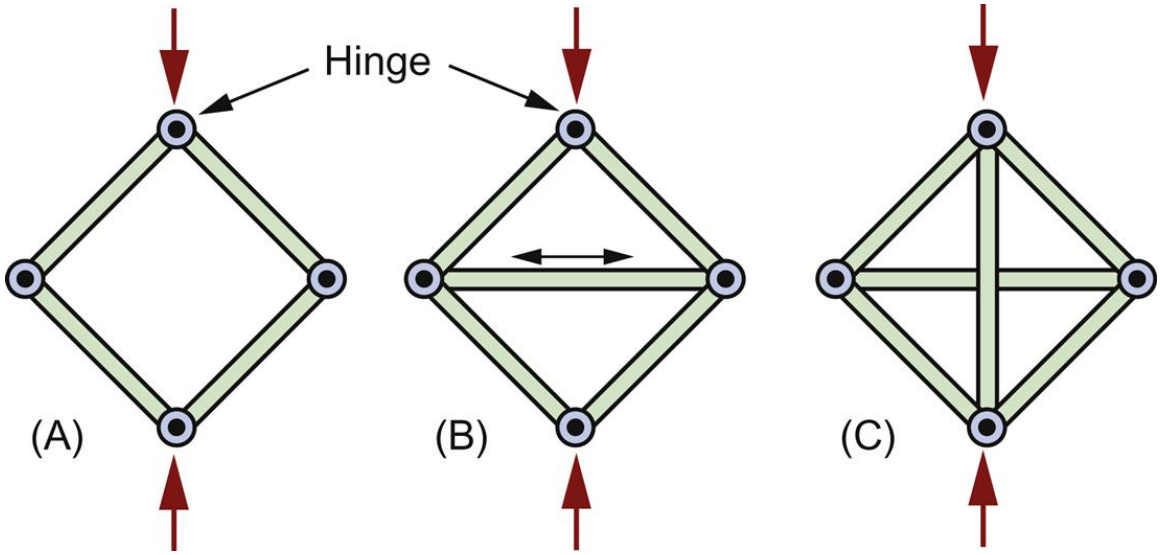


FIGURE 12.7 The pin-jointed frame at (A) is a mechanism; that at (B) is an optimally triangulated structure; that at (C) is over-constrained.

In 3 dimensions the equivalent condition is

$$M = b - 3j + 6 \geq 0 \qquad \textbf{(12.3)}$$

If $M<0$, the frame is a *mechanism* (Fig. 12.7A). It has no stiffness or strength; it collapses if loaded. If its joints are locked, preventing rotation, the bars of the frame *bend* when the structure is loaded, like those in Fig. 12.6B. If, instead, $M \geq 0$ the frame ceases to be a mechanism. The pin-jointed, triangulated, frame at Fig. 12.7B is stiff when loaded because the transverse bar carries tension, preventing collapse. Locking the hinges now makes little difference because slender structures are much stiffer when stretched than when bent. The frame at C is over-constrained. If the horizontal bar is tightened, the vertical one is put in tension even when there are no external loads. Over-constrained structures are less desirable because of this capacity to carry self-stresses, which can be damaging.

There is an underlying principle here: *stretch-dominated structures have high structural efficiency; bending-dominated structures have low*.

Connectivity

A 3-D frame is made up of b=120 struts pin-jointed at j=36 joints. Is it a mechanism or a rigid structure?

The Maxwell sum for a 3-D frame requires that, for rigidity,

$$M = b - 3j + 6 \geq 0$$

Inserting the data we find $M = 18$. The frame is rigid. It is also over-constrained (it has more struts that are necessary for rigidity), and so may carry self-stresses.

Percolation

The transport properties of hybrids – electrical conductivity for example – depend on connectivity. Think of mixing conducting and insulating spheres of the same size by shaking them up in a jar. If there are only a few conducting spheres, they do not form a connecting path and the array as a whole is an insulator. If each conducting sphere contacts just one other, there is still no connecting path. If, on average, each touches two, there is still no path. Adding more conducting spheres gives larger clusters, but they can still be discrete. For bulk conduction we need connectivity: the array first becomes a conductor when a single trail of contacts links one surface to the other, that is, when the volume fraction f of conducting spheres reaches the *percolation threshold*, f_c.

Percolation problems are easy to describe but difficult to solve. Research since 1960 has provided approximate solutions to most of the percolation problems associated with the design of hybrids (see Further Reading for a review). For simple cubic packing f_c=0.248, for close packing f_c=0.180. For a random array, sketched on the left of Fig. 12.8, it is somewhere in between – approximately 0.2. These results are for infinite, or at least very large, arrays. Experiments generally give values in the range 0.19–0.22, with some variability because of the finite size of the samples.

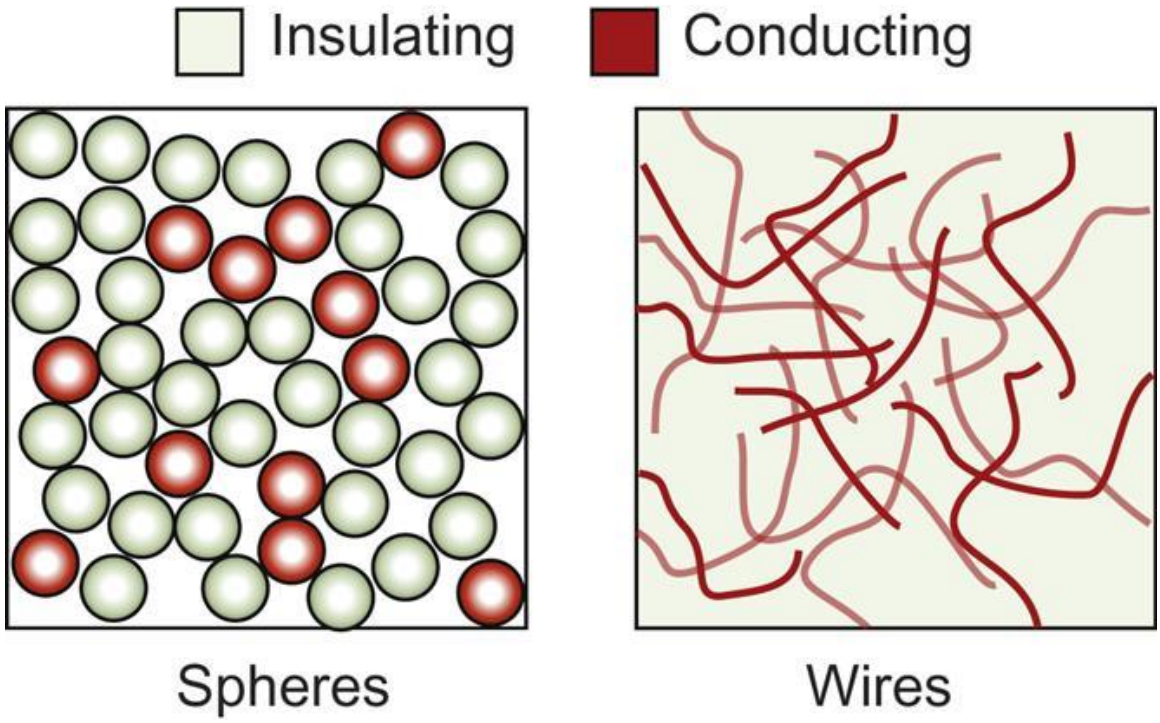


FIGURE 12.8 Randomly packed spheres and distributed wires.

Making the spheres smaller smears the transition out. The percolation threshold is still 0.2, but the first connecting path is now thin and extremely devious – it is the only one out of the vast number of almost complete paths that actually connects. Increase the volume fraction and the number of conducting paths increases initially as $(f - f_c)^2$, then linearly, reverting to a rule of mixtures. If the particles are very small, as much as 40% may be needed to give good conduction. A loading of 40% is a lot, enough to have a large effect on the other properties of the hybrid. Getting conduction with a much lower loading would be attractive.

Shape gives a way out. If the spheres are replaced by slender fibres, they touch more easily and the percolation threshold falls (Fig. 12.8, right hand side. If the aspect ratio of the fibres is $\beta = L/d$ (where L is the fibre length, d the diameter) then, empirically, the percolation threshold falls from f_c to roughly $f_c/\beta^{1/2}$, so and aspect ratio of 25 reduces f_c, the volume fraction of conducting phase, by a factor of 5.

The concept of percolation is a necessary tool in designing hybrids. Electrical conductivity is just one example. Another is the passage of liquids through foams or porous media – no connected channels, and no fluid flows; just one (out of a million possibilities) and there is a leak. Add

a few more connections and there is a flood. Percolation ideas are particularly important in understanding the transport properties of hybrids – properties that determine the flow of electricity or heat; of fluid; or of flow by diffusion – especially when the differences in properties of the components are extreme because it is then that connected paths the greatest effect.

Percolation

A volume fraction f=0.05 of fine silver wires with an aspect ratio of β=36 is randomly dispersed in an acrylic matrix. Will the silver-acrylic hybrid be electrically conducting?

The criterion for percolation, meaning a connecting path through contacting silver wires, is

$$f \geq \frac{f_c}{\beta^{1/2}} = \frac{0.2}{6} = 0.033$$

The 5% of silver wires is sufficient to make the hybrid a conductor.

Segmentation

Shape can be used to reduce flexural stiffness and strength as well as increase them. Springs, suspensions, flexible cables and other structures that must flex use shape to give a low bending stiffness. This is achieved by shaping the material into strands or leaves, as explained in Chapter 11, Material and Shape: Case Studies, Section 11.7. The slender strands or leaves bend easily but do not stretch when the section is bent: an n-strand cable is less stiff by a factor of $3/\pi n$ than the solid reference section; an n-leaf panel by a factor $1/n^2$.

Subdivision can be used in another way: to impart *damage tolerance*. A glass window, hit by a projectile, will shatter. One made of small glass bricks, laid as bricks usually are, will lose a brick or two but not shatter totally; it is damage tolerant. By subdividing and separating the material, a crack in one segment does not penetrate into its neighbours, allowing local but not global failure. That is the principle of 'topological toughening'. Builders in stone and brick have exploited the idea for thousands of years: stone and brick are brittle, but buildings made of them – even those made without cement ('dry-stone building') – survive ground movement, even

small earthquakes, through their ability to deform with some local failure but without total collapse.

Taking the simplest view, two things are necessary for topological damage tolerance: discreteness of the structural units, and an interlocking of the units in such a way that the array as a whole can carry load. Brick-like arrangements (Fig. 12.9A) carry large loads and are damage tolerant in compression and shear, but disintegrate under tension. Strand and layer like structures are damage tolerant in tension because if one strand fails the crack does not penetrate its neighbours – the principle of multistrand ropes and cables. The jigsaw puzzle configuration (Fig. 12.9B) carries in-plane tension, compression and shear, but at the cost of introducing a stress concentration factor of about $\sqrt{R/r}$, where R is the approximate radius of a unit and r that of the interlock. The sources listed in Further Reading explore a particular set of topologies that rely on compressive or rigid boundary conditions to create continuous layers that tolerate out-of-plane forces and bending moments, as illustrated in Fig. 12.9C. This is done by creating interlocking units with nonplanar surfaces that have curvature both in the plane of the array and normal to it. Provided the array is constrained at its periphery, the nesting shapes limit the relative motion of the units, locking them together. Topological interlocking of this sort allows the formation of continuous layers that can be used for ceramic claddings or linings to give surface protection. And of course the units need not be made of one material alone. Since the only requirement is that of interlocking shape, the segments can be made of different materials. Just as a builder constructing a brick wall can insert porous bricks for ventilation and transparent bricks to admit light, the designer of a segmented hybrid can add functionality through the choice of material for the units.

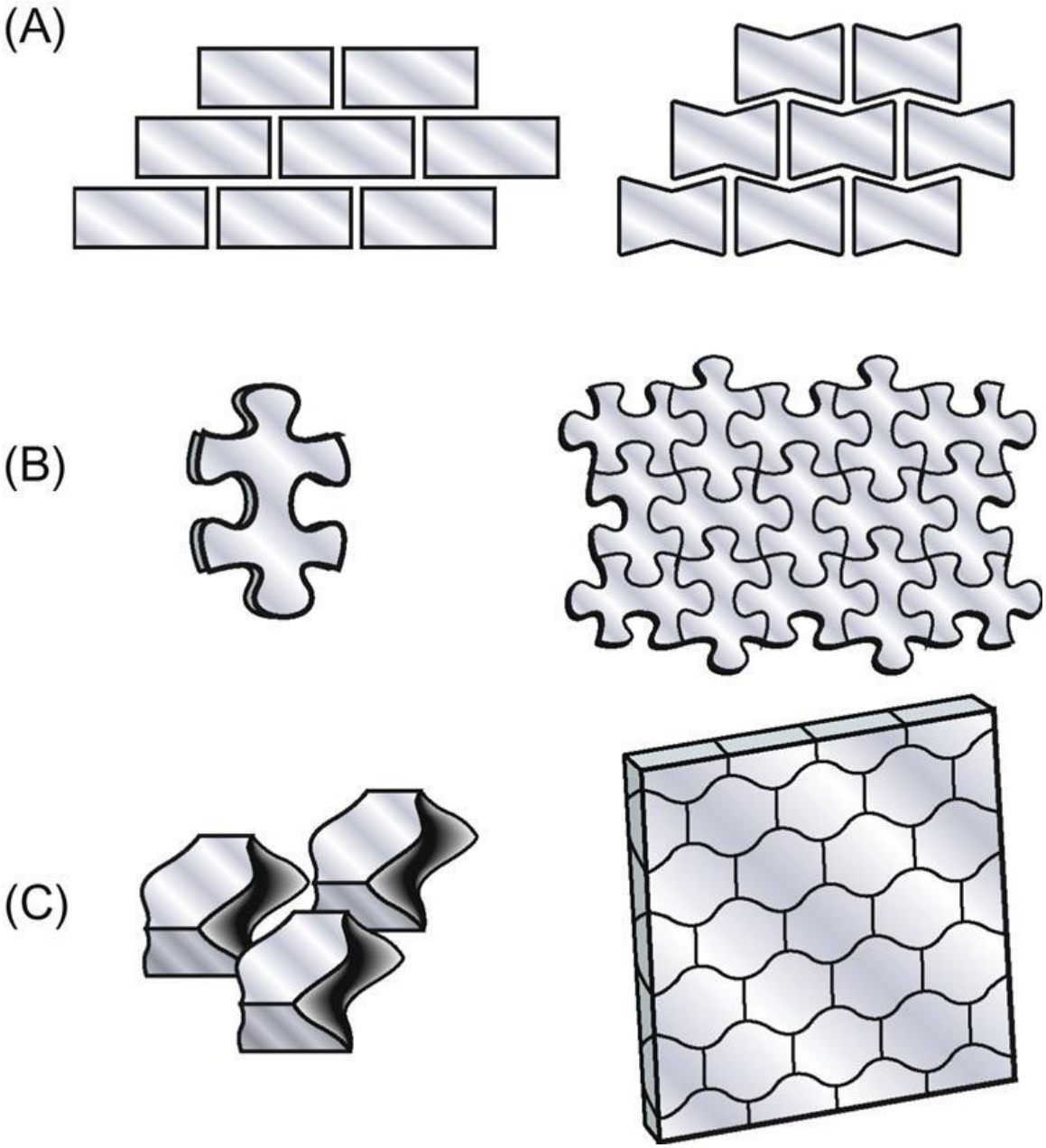


FIGURE 12.9 Examples of topological interlocking: discrete, unbonded or weakly-bonded structures that carry load (A) brick-like assemblies, (B) segments that interlock in 2-dimensions and (C) segments that interlock in 3-dimensions.

Modelling Methods: Bounds and Limits

On a macroscopic scale – one which is large compared to that of the components – a hybrid behaves like a homogeneous solid with its own set

of mechanical, thermal and electrical properties. Calculating these precisely can be done, but is difficult. It is much easier to bracket them by *bounds* or *limits*: upper and lower values between which the properties lie. The term 'bound' means a rigorous boundary, one which the value of the property *cannot* – subject to certain assumptions – exceed or fall below. It is not always possible to derive bounds, in which case the best that can be done is to derive 'limits' outside which it is *unlikely* that the value of the property will lie. The important point is that the bounds or limits bracket the properties of *all* the configurations of matrix and reinforcement shown in Fig. 12.10: by using these four configurations we escape from the need to model individual geometries. In the following sections we explore five classes of hybrid – composites, cellular structures, sandwiches and multilayers, and segmented structures, developing bounds and limits for their properties[2] and examining which can fill holes in material-property space.

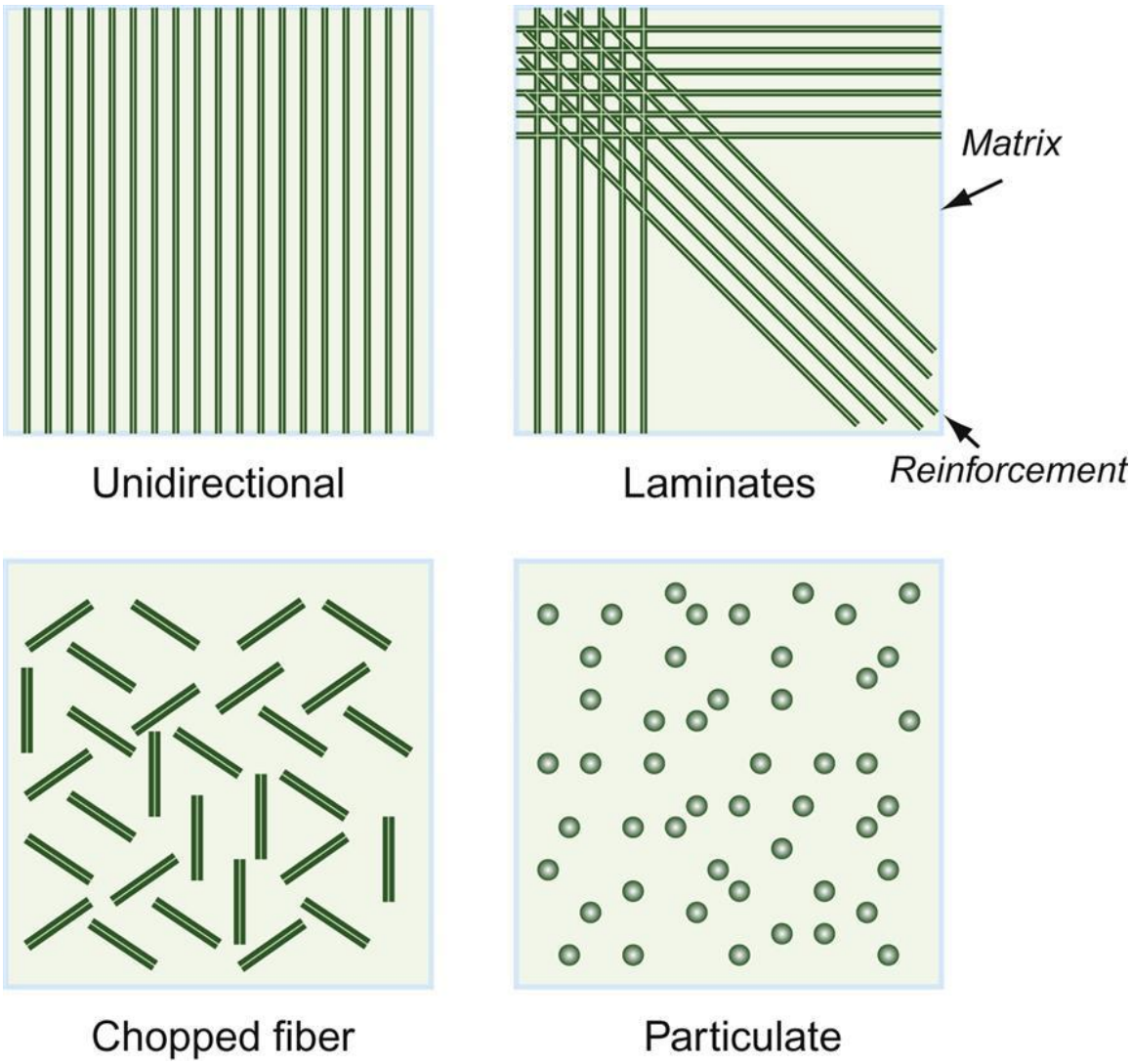


FIGURE 12.10 Schematic of hybrids of the composite type: unidirectional fibrous, laminated fibre, chopped fibre and particulate composites. Bounds and limits, described in the text, bracket the properties of all of these.

The derivations get a bit tedious; if you want to see what can be done with them move on to Chapter 13, Hybrids: Case Studies, which contains case-studies. If later you want to know the background or are simply interested in the methods, return to this one.

12.4 Composites

Aircraft engineers, automobile makers, and designers of sports equipment all have one thing in common: they all want materials that are stiff, strong,

tough and light. The single-material choices that best achieve this are the *light alloys*: alloys based on magnesium, aluminium and titanium. Much research aims at improving their properties. But they are not all that light – polymers have much lower densities. Nor are they all that stiff – ceramics are much stiffer and, especially in the form of small particles or thin fibres, much stronger. These facts are exploited in the family of structural hybrids that we usually refer to as *particulate and fibrous composites*.

Any two materials can, in principle, be combined to make a composite and they can be mixed in many geometries (Fig. 12.10). In this section we restrict the discussion to *fully dense, strongly bonded, composites* such that there is no tendency for the components to separate at their interfaces when the composite is loaded, and to those in which the scale of the reinforcement is large compared to that of the atom or molecule size and the dislocation spacing, allowing the use of continuum methods.

Density. When a volume fraction f of a reinforcement r (density ρ_r) is mixed with a volume fraction $(1-f)$ of a matrix m (density ρ_m) to form a composite with no residual porosity, the composite density $\tilde{\rho}$ is given exactly by a rule of mixtures (an arithmetic mean, weighted by volume fraction):

$$\tilde{\rho} = f\rho_r + (1-f)\rho_m. \qquad \textbf{(12.4)}$$

The geometry or shape of the reinforcement does not matter except in determining the maximum packing-fraction of reinforcement and thus the upper limit for f.

Modulus. The modulus of a composite is bracketed by the well-known Voigt and Reuss bounds. The upper bound, $\tilde{E}_u$, is obtained by postulating that, on loading, the two components suffer the same strain; the stress is then the volume average of the local stresses and the composite modulus follows a rule of mixtures:

$$\tilde{E}_u = f\,E_r + (1-f)E_m. \qquad \textbf{(12.5a)}$$

Here E_r is the Young's modulus of the reinforcement and E_m that of the matrix. The lower bound, $\tilde{E}_L$, is found by postulating instead that the two

components carry the same stress; the strain is the volume average of the local strains and the composite modulus is

$$\tilde{E}_L = \frac{E_m E_r}{f E_m + (1-f) E_r} \qquad \textbf{(12.5b)}$$

Narrower bounds are possible but the simple ones are adequate to illustrate the method.

To see how the bounds work, consider the following example.

Composite Design for Stiffness at Minimum Mass

A beam is at present made of an aluminium alloy. Beryllium is both lighter and less dense than aluminium; the ceramic alumina (Al_2O_3) is also stiffer but denser. Can hybrids of aluminium with either of these offer improved performance, measured by the criterion of excellence $E^{1/2}/\rho$ derived in Chapter 4, Materials Selection – The Basics?

The three materials are plotted on a segment of the $E-\rho$ property space in Fig. 12.11. Composites made by mixing these materials have densities given exactly by Eq. (12.4) and moduli that are bracketed by the bounds of Eqs (12.5a) and (12.5b). Both of these moduli depend on volume fraction of reinforcement, and through this, on density. Upper and lower bounds for the modulus–density relationship can thus be plotted onto the $E-\rho$ chart using volume fraction f as a parameter, as shown in Fig. 12.11. Any composite made by combining aluminium with alumina will have a modulus contained in the envelope for Al–Al_2O_3; the same for Al–Be. Fibrous reinforcement gives a modulus in a direction parallel to the fibres near the upper bound; particulate reinforcement or transversely loaded fibres give moduli near the lower one.

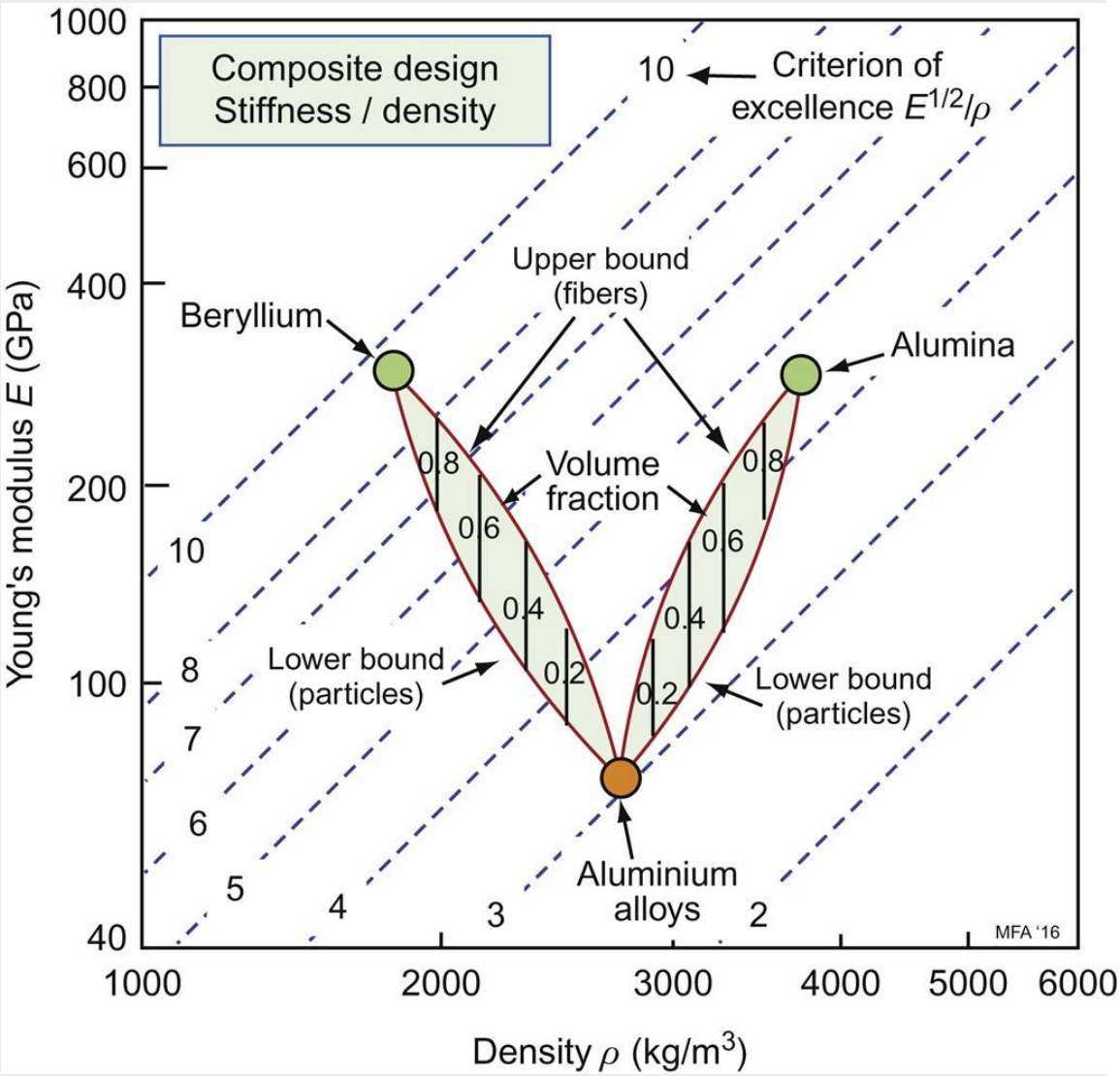


FIGURE 12.11 Part of the E-ρ property chart, showing aluminium alloys, beryllium and alumina (Al_2O_3). Bounds for the moduli of hybrids made by mixing aluminium with the other two are shown. The diagonal contours plot the criterion of excellence for a light, stiff beam, $E^{1/2}/\rho$.

The criterion of excellence, $E^{1/2}/\rho$, is plotted as a grid on the figure. The bound envelope for Al–Be composites extends almost normal to the grid, while that for Al–Al_2O_3 lies at a shallow angle to it. Beryllium fibres improve performance (as measured by $E^{1/2}/\rho$) roughly four times as much as alumina fibres do, for the same volume fraction. The difference for particulate reinforcement is even more dramatic. The lower bound for Al–Be lies normal to the contours: 30% of particulate beryllium increases

$E^{1/2}/\rho$ by a factor of 1.5. The lower bound for Al–Al_2O_3 is, initially, parallel to the $E^{1/2}/\rho$ grid: 30% of particulate Al_2O_3 gives almost no gain. The underlying reason is that both beryllium and Al_2O_3 increases the modulus, but only beryllium decreases the density; the criterion of excellence is more sensitive to density than to modulus.

The commercial alloy AlBeMet (62% Be, 38% Al) exploits this idea. The two metals are mutually insoluble, creating a 2-phase composite of Al and Be with $E^{1/2}/\rho = 6.5$ compared with 3.1 for Al alone.

Creating Anisotropy

The elastic and plastic properties of bulk monolithic solids are frequently anisotropic, but weakly so – the properties do not depend strongly on direction. Hybridization gives a way of creating controlled anisotropy, and it can be large. We have already seen an example in Fig. 12.11, which shows the upper and lower bounds for the moduli of composites. The longitudinal properties of unidirectional long-fibre composites lie near the upper bound, the transverse properties near the lower one. The vertical width of the band in between them measures the anisotropy. A unidirectional continuous-fibre composite has a maximum anisotropy ratio R_a given by the ratio of the bounds – in this example

$$R_a = \frac{\tilde{E}_u}{\tilde{E}_L} = f^2 + (1-f)^2 + f(1-f)\left(\frac{E_m}{E_r} + \frac{E_r}{E_m}\right)$$

In Fig. 12.11 the maximum R_a is only 1.5. A more dramatic example involving thermal properties is given in Chapter 13, Hybrids: Case Studies.

Strength. Of all the bounds and limits described in this chapter, those for strength are the least satisfactory. The nonlinearity of the problem, the multitude of failure mechanisms[3] and the sensitivity of strength and toughness to impurities and processing defects makes accurate modelling difficult. The literature contains many calculations for special cases: reinforcement by unidirectional fibres, or by a dilute dispersion of spheres. We wish to avoid models which require detailed knowledge of how a particular architecture behaves, and seek less restrictive limits.

As the load on a continuous fibre composite is increased, load is redistributed between the components until one suffers general yield or fracture (Fig. 12.12A). Beyond this point the composite has suffered permanent deformation or damage but can still carry load; final failure requires yielding or fracture of both. The composite is strongest if both reach their failure state simultaneously. Thus the upper bound for a continuous fibre ply like that labelled 'Unidirectional' in Fig. 12.10 loaded parallel to the fibres (the axial strength in tension, subscript a) is a rule of mixtures

$$(\tilde{\sigma}_f)_{u,a} = f(\sigma_f)_r + (1-f)(\sigma_f)_m \qquad \textbf{(12.6a)}$$

where $(\sigma_f)_m$ is the strength of the matrix and $(\sigma_f)_r$ is that of the reinforcement. If one fails before the other, the load is carried by the survivor. Thus a lower bound for strength in tension is given by

$$(\tilde{\sigma}_f)_{L,a} = \textit{Greater of}(f(\sigma_f)_r,\ (1-f)(\sigma_f)_m) \qquad \textbf{(12.6b)}$$

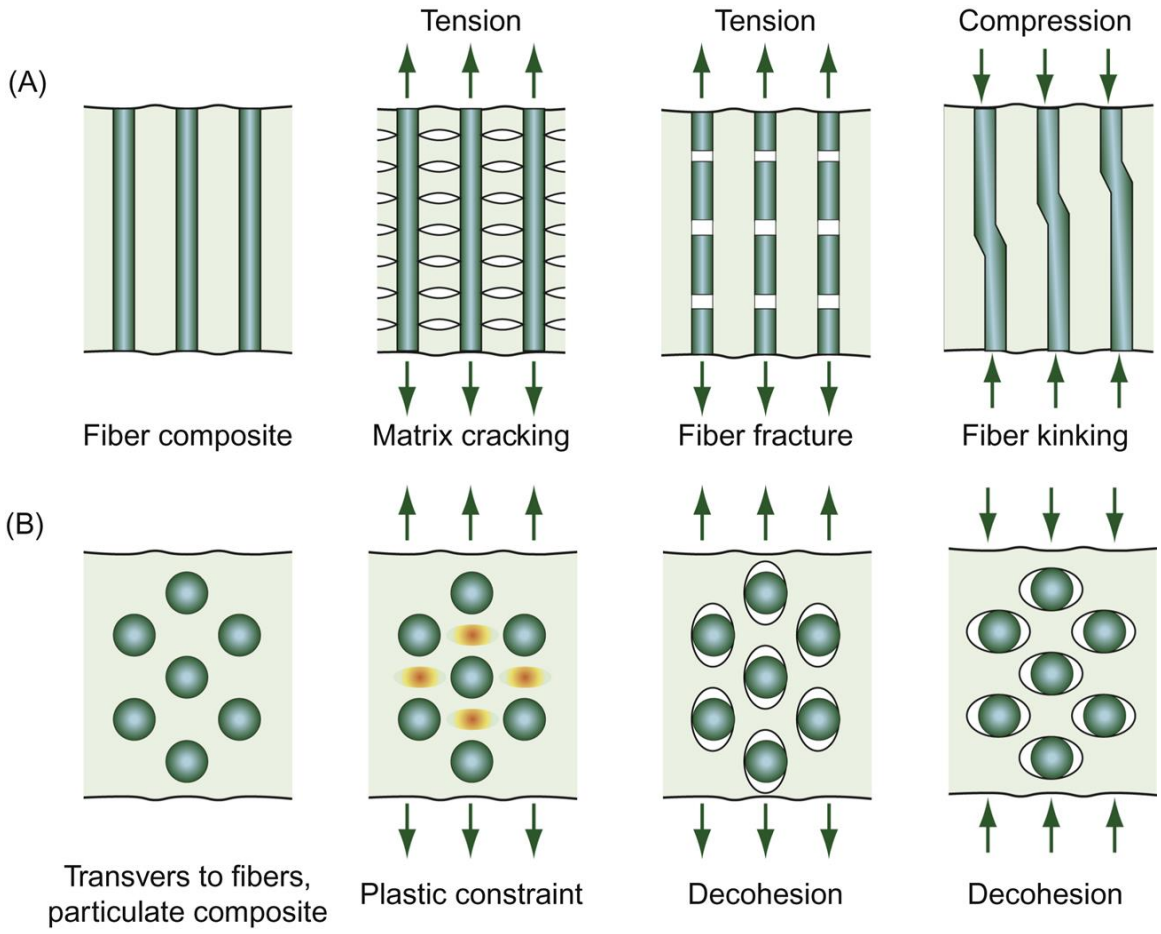


FIGURE 12.12 Failure modes in composites (A) fibrous composites and (B) particulate composites.

The transverse strength (Fig. 12.12B) is more difficult. It depends on interface bond strength, fibre distribution, stress concentrations and voids. In general the transverse strength is less than that of the unreinforced matrix, and the strain to failure, too, is less. In a continuous, ductile, matrix containing strongly bonded, nondeforming particles or fibres, the flow in the matrix is constrained. The constraint increases the stress required for flow in the matrix, giving an upper limiting tensile strength

$$\text{Lesser of} \begin{cases} (\tilde{\sigma}_f)_{u,t} \approx (\sigma_f)_m \left(\dfrac{1}{1 - f^{1/2}} \right) \\ (\tilde{\sigma}_f)_{u,t} \approx (\sigma_f)_r \end{cases} \tag{12.7a}$$

More usually the transverse strength is lower than that of the matrix alone because of stress concentration and debonding at the fibre–matrix interface. Hull and Clyne (1996) gives the approximate lower limit for tensile strength as

$$(\tilde{\sigma}_f)_{L,t} \approx (\sigma_f)_m (1 - f^{1/2}) \tag{12.7b}$$

The two pairs of limits allow the potential of a given choice of reinforcement and matrix to be explored. Fig. 12.13 shows the limits for axial and transverse strength of an epoxy-glass composite ply.

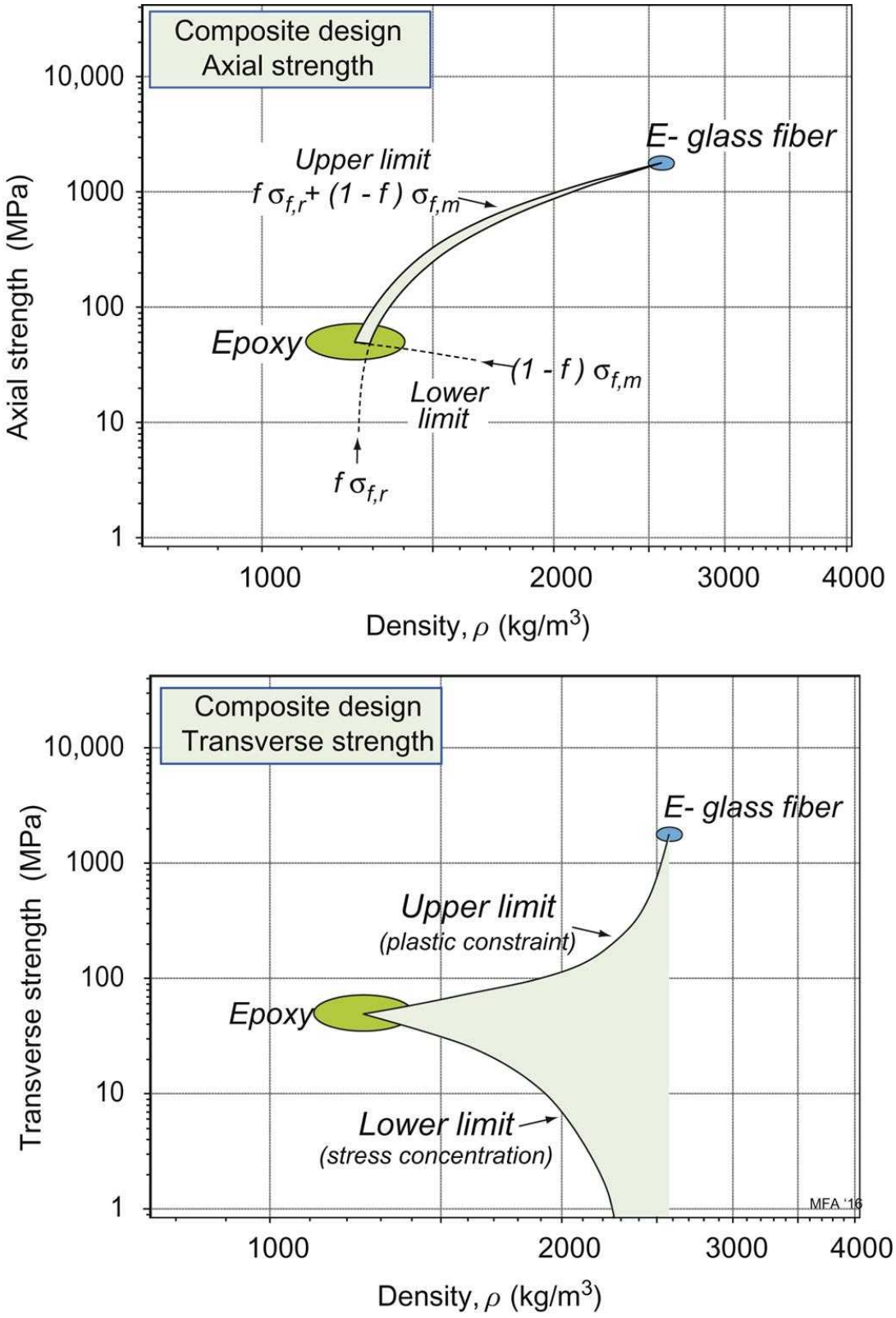


FIGURE 12.13 The limits for axial and transverse strength of a composite ply.

Continuous fibre composites can fail in compression by fibre kinking (Fig. 12.12A, extreme right). The kinking is resisted by the shear strength of the matrix, approximately $(\sigma_f)_m/2$. This leads an axial compressive stress for buckling of fibres of

$$(\tilde{\sigma}_c)_{u,a} = \frac{1}{\vartheta}\frac{(\sigma_f)_m}{2} \approx 14(\sigma_f)_m \qquad \textbf{(12.8)}$$

Here ϑ is the initial misalignment of the fibres from the axis of compression, in radians. Experiments show that a typical value, in carefully aligned composites, is $\vartheta \approx 0.035$, giving the final value shown on the right of the equation. We identify the upper bound with the lesser of this and Eq. (12.6a). When, instead, misalignment is severe, meaning $\vartheta \approx 1$, the strength falls greatly. We identify the lower bound for compressive failure with that of the matrix, $(\sigma_f)_m$.

Specific heat. The specific heats of solids at constant pressure, C_p, are almost the same as those at constant volume, C_v. If they were identical, the heat capacity per unit volume of a composite would, like the density, be given exactly by a rule-of-mixtures

$$\tilde{\rho}\,\tilde{C}_p = f\rho_r(C_p)_r + (1-f)\rho_m(C_p)_m \qquad \textbf{(12.9)}$$

where $(C_p)_r$ is the specific heat of the reinforcement and $(C_p)_m$ is that of the matrix (the densities enter because the units of C_p are J/kg·K). A slight difference appears because thermal expansion generates a misfit between the components when the composite is heated; the misfit creates local pressures on the components and thus changes the specific heat. The effect is very small and need not concern us further.

Thermal expansion coefficient. The thermal expansion of a composite can, in some directions, be greater than that of either component, in others, less. This is because an elastic constant – Poisson's ratio – couples the principal elastic strains; if the matrix is prevented from expanding in one direction (by embedded fibres, for instance) then it expands more in the transverse directions. For simplicity we shall use the approximate lower bound

$$\tilde{\alpha}_L = \frac{E_r \alpha_r f + E_m \alpha_m (1-f)}{E_r f + E_m (1-f)} \qquad \textbf{(12.10)}$$

(it reduces to the rule of mixtures when the moduli are the same) and the upper bound

$$\tilde{\alpha}_u = f\alpha_r(1+\nu_r) + (1-f)\alpha_m(1+\nu_m) - \alpha_L[f\nu_r + (1-f)\nu_m] \qquad \textbf{(12.11)}$$

where α_r and α_m are the two expansion coefficients and ν_r and ν_m the Poisson's ratios.

Thermal conductivity. The thermal conductivity determines heat flow at steady rate. A composite of two materials, bonded to give good thermal contact, has a thermal conductivity λ that lies between those of the individual components, λ_m and λ_r. Not surprisingly, a composite containing parallel continuous fibres has a conductivity, parallel to the fibres, given by a rule-of-mixtures

$$\tilde{\lambda}_u = f\lambda_r + (1-f)\lambda_m \qquad \textbf{(12.12)}$$

This is an upper bound: in any other direction the conductivity is lower. The transverse conductivity of a parallel-fibre composite (again assuming good bonding and thermal contact) lies near the lower bound first derived by Maxwell

$$\tilde{\lambda}_L = \lambda_m \left(\frac{\lambda_r + 2\lambda_m - 2f(\lambda_m - \lambda_r)}{\lambda_r + 2\lambda_m + f(\lambda_m - \lambda_r)} \right) \qquad \textbf{(12.13)}$$

Particulate composites, too, have a conductivity near this lower bound. Poor interface conductivity can make λ drop below it. Debonding or an interfacial layer between reinforcement and matrix can cause this; so, too, can a large difference of modulus between reinforcement and matrix (because this reflects phonons, creating an interface impedance) or a structural scale that is shorter than the phonon wavelengths.

Thermal diffusivity. The thermal diffusivity

$$a = \frac{\lambda}{\rho\, C_p}$$

determines heat flow when conditions are transient, that is, when the temperature field changes with time. It is formed from three of the earlier properties: λ, ρ and C_p. The second and third of these are given exactly by Eqs (12.4) and (12.9), allowing the diffusivity to be expressed as

$$\tilde{a} = \frac{\tilde{\lambda}}{f\rho_r(C_p)_r + (1-f)\rho_m(C_p)_m} \qquad \textbf{(12.14)}$$

Its upper and lower bounds are found by substituting those for $\tilde{\lambda}$ (Eqs (12.12) and (12.13) into this equation.

Composite Design for Controlled Thermal Response

Thermo-mechanical design involves the specific heat, C_p, the thermal expansion, α, the conductivity, λ, and the diffusivity, a. These composite properties are bounded by Eqs (12.9)– (12.14). They are involved in a number of indices. One is the criterion for minimizing thermal distortion derived in Chapter 5, Materials Selection – Case Studies, Section 5.16: it is that of maximising the index λ/α.

Fig. 12.14 shows a small part of the $\alpha - \lambda$ materials selection chart, with a grid showing the criterion of excellence, λ/α, superimposed on it. Three materials are shown: aluminium, boron nitride (BN) and silicon carbide (SiC). The thermal properties of Al–BN and Al–SiC composites are bracketed by envelopes calculated from the bounding equations. (Both α and λ have upper and lower bounds so there are four possible combinations for each material pair. Those shown in the figure are the outermost pair of the four.) The plot reveals immediately that SiC reinforcement in aluminium increases performance (as measured by λ/α); reinforcement with BN decreases it.

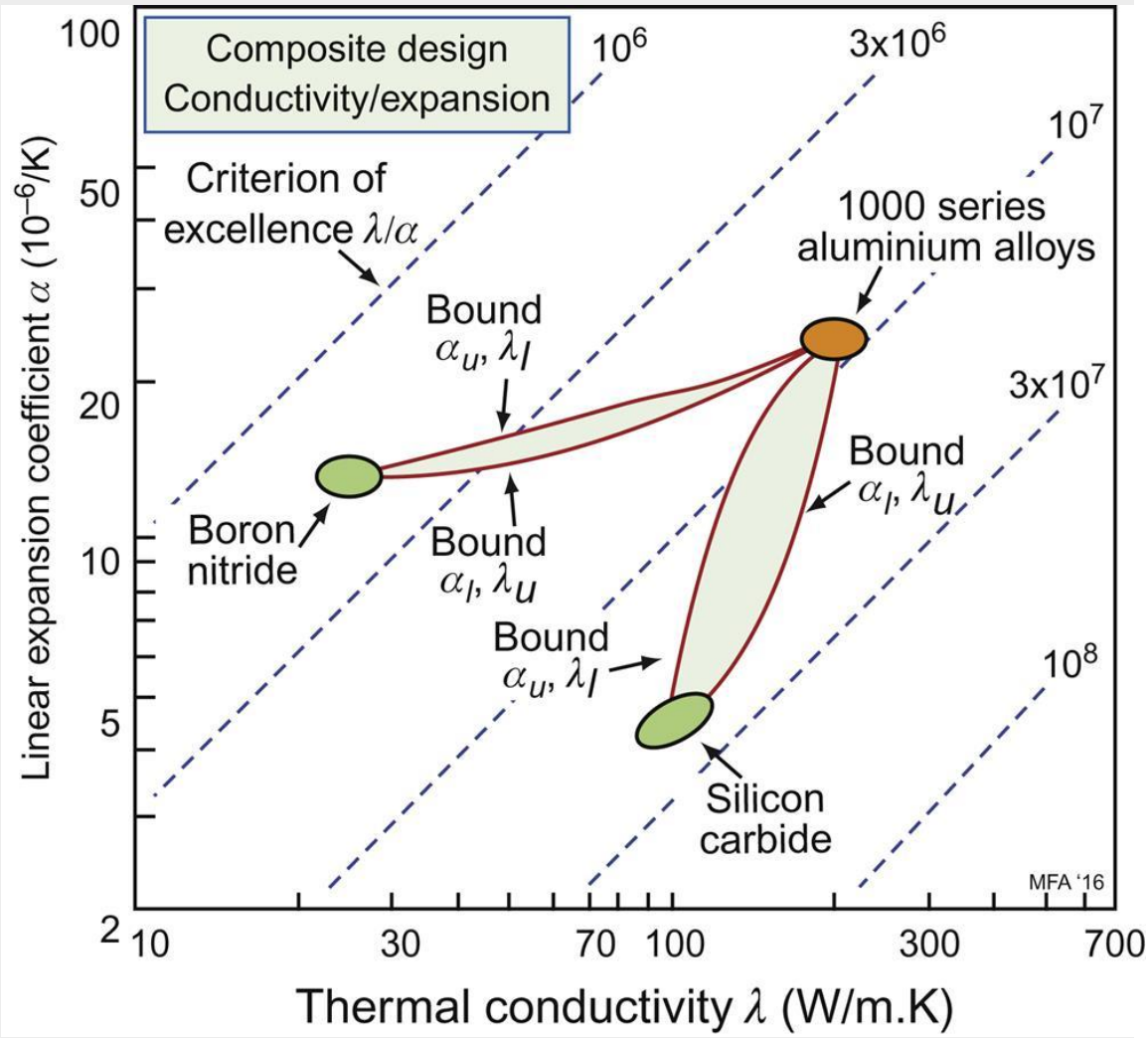


FIGURE 12.14 A part of expansion-coefficient/conductivity space showing aluminium alloys, boron nitride and silicon carbide. The properties of Al-BN and Al-SiC composites are bracketed by the bounds of Eqs (12.10)– (12.13). The Al-SiC composites enhance performance, the Al-BN composites reduce it.

Dielectric constant. The dielectric constant $\tilde{\varepsilon}_r$ is given by a rule of mixtures

$$\tilde{\varepsilon}_r = f\,\varepsilon_{r,r} + (1-f)\varepsilon_{r,m} \qquad \textbf{(12.15)}$$

where $\varepsilon_{r,r}$ is the dielectric constant of the reinforcement and $\varepsilon_{r,m}$ that of the matrix.

Electrical conductivity and percolation. When the electrical conductivities κ of the components of a composite are of comparable magnitude, bounds for the electrical conductivity are given by those for thermal conductivity with λ replaced by the κ. When, instead, they differ by many orders of magnitude (a metallic powder dispersed in an insulating polymer, for instance) questions of percolation arise, as discussed in Section 12.3.

Filling property space with composites. We end this section with two figures illustrating how the development of composites has filled-out the occupancy of material property space. The first, Fig. 12.15, is a modulus-density ($E-\rho$) section. The areas filled by unreinforced metals and polymers appear as pale red and pale blue envelopes; members are labelled in grey (they are taken from the original $E-\rho$ chart of Figure 3.3). Polymer matrix composites occupy the bolder purple zone, metal matrix composites the darker red one. Both extend into areas that were previously empty. Using any one of the indices for light, stiff structures (E/ρ, $E^{1/2}/\rho$ and $E^{1/3}/\rho$) as a criterion of excellence, we find that composites offer performance that was not previously attainable.

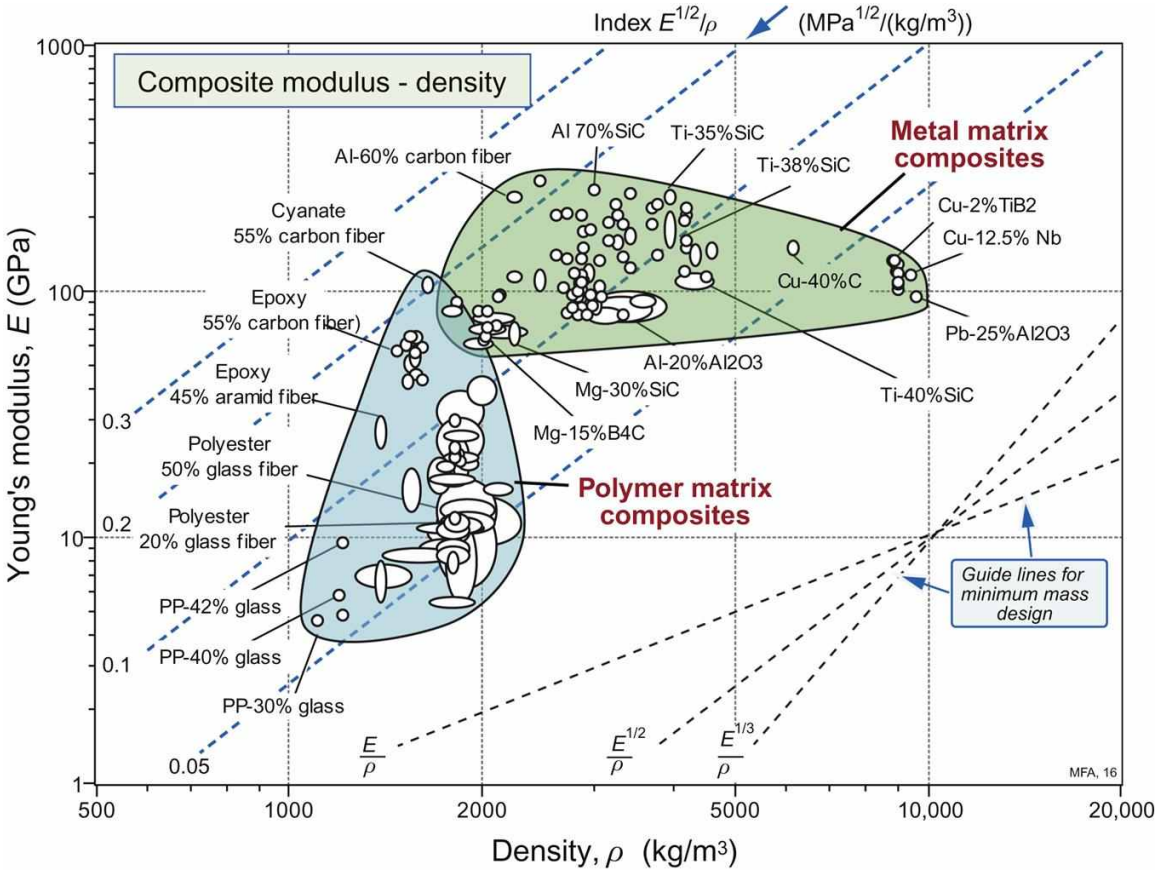


FIGURE 12.15 Polymer (PMC) and metal (MMC) matrix composites expand the occupied area of modulus-density space. Data from the CES EduPack 2016.

Fig. 12.16 paints a similar picture for the strength-density ($\sigma_y - \rho$) section. The colour coding is the same as that of the previous figure. Composites again expand the populated area in a direction that, using the indices for light strong structures (σ_y/ρ, $\sigma_f^{2/3}/\rho$ and $\sigma_f^{1/2}/\rho$) as a criterion, offers enhanced performance.

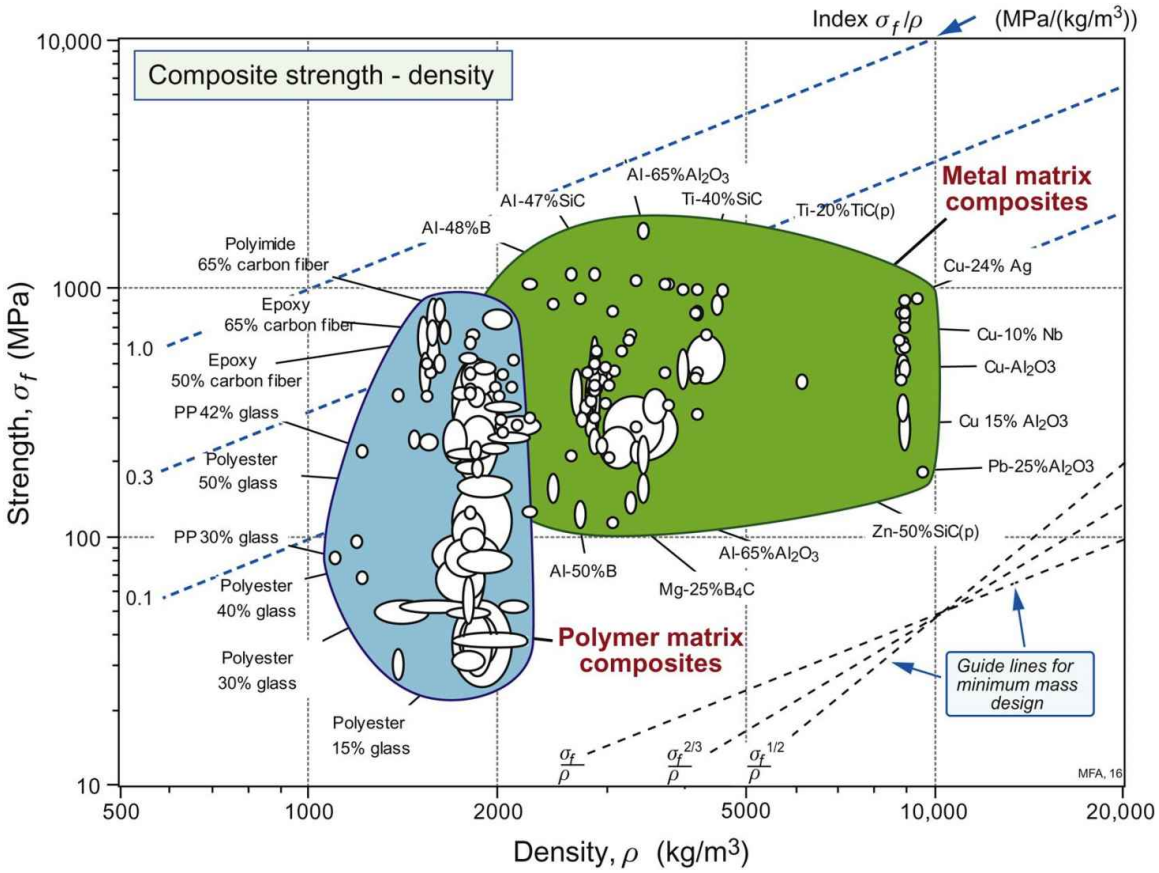


FIGURE 12.16 Polymer (PMC) and metal (MMC) matrix composites also expand the occupied area of strength-density space. Data from the CES EduPack 2016.

12.5 Cellular Structures: Foams and Lattices

Cellular structures – foams and lattices – are hybrids of a solid and a gas. The properties of the gas might at first sight seem irrelevant, but this is not so. The thermal conductivity of low-density foams of the sort used for insulation is determined by the conductivity of the gas contained in its pores; and the dielectric constant and breakdown potential, and even the compressibility, can depend on the gas properties.

There are two distinct species of cellular solid. The distinction is most obvious in their mechanical properties. The first, typified by foams, are *bending-dominated structures*; the second, typified by triangulated lattice structures, are *stretch dominated* – a distinction explained in Section 12.3. To

give an idea of the difference: a foam with a relative density of 0.1 (meaning that the solid cell walls occupy 10% of the volume) is less stiff by a factor of 10 than a triangulated lattice of the same relative density. The word 'configuration' has special relevance here.

Foams: Bending-Dominated Structures

Foams are cellular solids made by expanding polymers, metals, ceramics or glasses with a foaming agent – a generic term for one of many ways of introducing gas, much as yeast does in bread-making. Fig. 12.17 shows an idealized cell of a low-density foam. It consists of solid cell walls or edges surrounding a void space containing a gas or fluid. Cellular solids are characterized by their *relative density*, which for the structure shown here (with $t \ll L$) is

$$\frac{\tilde{\rho}}{\rho_s} \approx \left(\frac{t}{L}\right)^2 \qquad \textbf{(12.16)}$$

where $\tilde{\rho}$ is the density of the foam, ρ_s is the density of the solid of which it is made, L is the cell size, and t is the thickness of the cell edges.

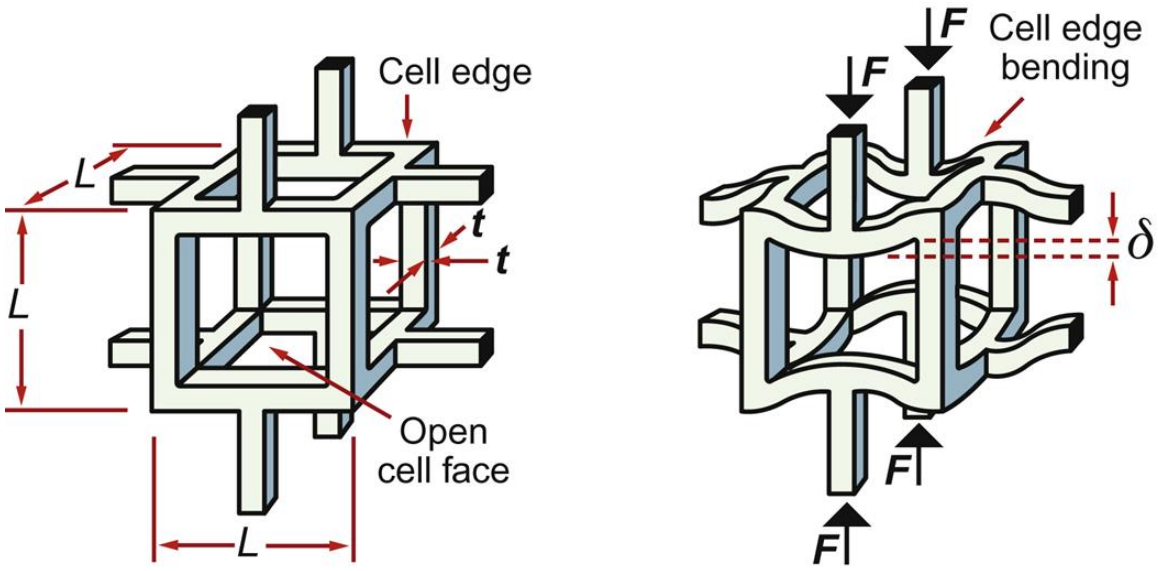


FIGURE 12.17 A cell in a low density foam. When the foam is loaded, the cell edges bend.

Foams have the characteristic that, when loaded, the cell walls *bend*, with consequences that we now analyse.

Mechanical properties. The compressive stress–strain curve of bending-dominated foams looks like Fig. 12.18. The material is linear elastic, with modulus $\tilde{E}$ up to its elastic limit, at which point the cell edges yield, buckle or fracture. The foam continues to collapse at a nearly constant stress (the 'plateau stress', $\tilde{\sigma}_{pl}$) until opposite sides of the cells impinge (the 'densification strain' $\tilde{\varepsilon}_d$), when the stress rises rapidly.

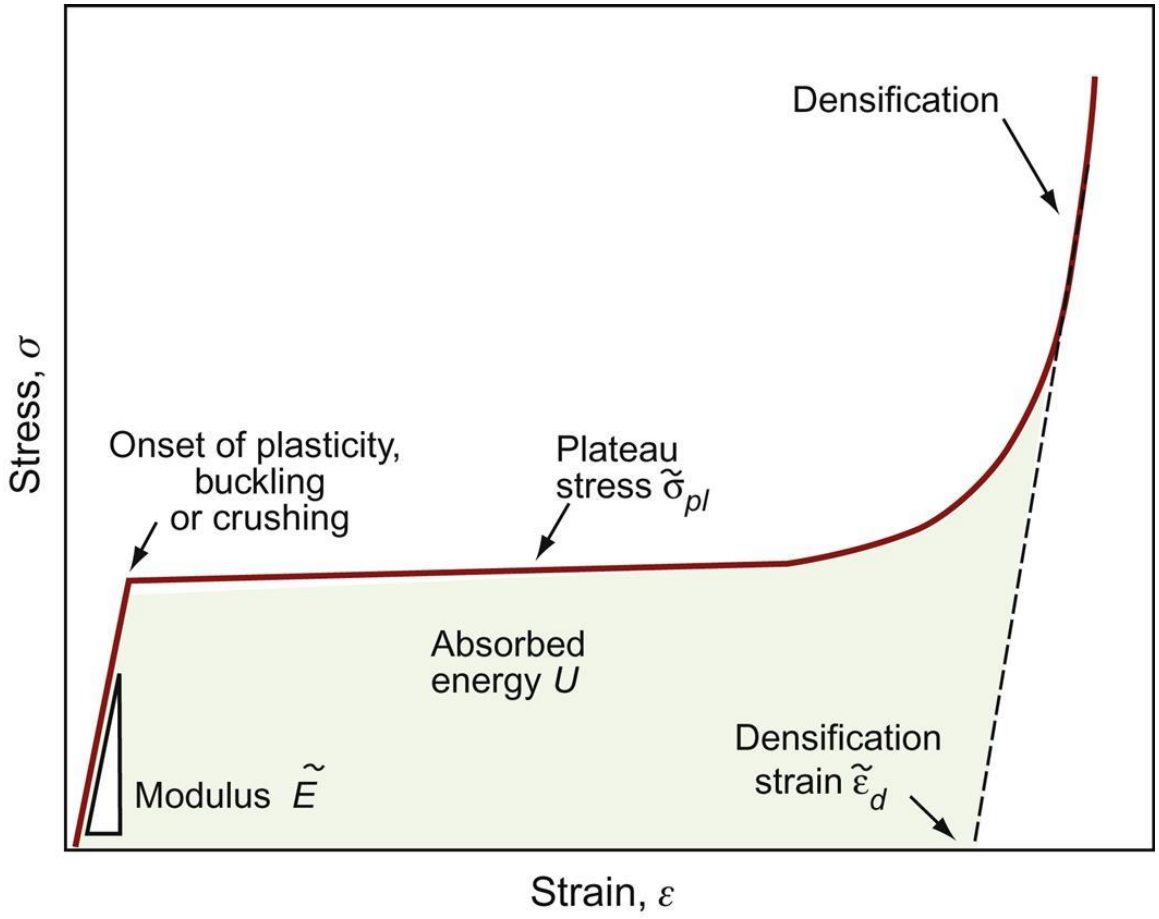


FIGURE 12.18 The plateau stress is determined by buckling, plastic bending or fracturing of the cell walls.

A remote compressive stress σ exerts a force $F \propto \sigma L^2$ on the cell edges, causing them to bend and leading to a bending deflection δ, as shown in Fig. 12.17. For the open-celled structure shown in the figure, the bending deflection (Appendix B3) scales as

$$\delta \propto \frac{FL^3}{E_s I} \tag{12.17}$$

where E_s is the modulus of the solid of which the foam is made and $I = \frac{t^4}{12}$ is the second moment of area of the cell edge of square cross section, t x t. The compressive strain suffered by the cell as a whole is then $\varepsilon = 2\delta/L$. Assembling these results gives the modulus $\tilde{E} = \sigma/\varepsilon$ of the foam as

$$\frac{\tilde{E}}{E_s} \propto \left(\frac{\tilde{\rho}}{\rho_s}\right)^2 \quad \text{(bending-dominated behavior)} \tag{12.18}$$

Since $\tilde{E} = E_s$ when $\tilde{\rho} = \rho_s$, we expect the constant of proportionality to be close to unity – a speculation confirmed both by experiment and by numerical simulation. The quadratic dependence means that a small decrease in relative density causes a large drop in modulus (Fig. 12.19, left).

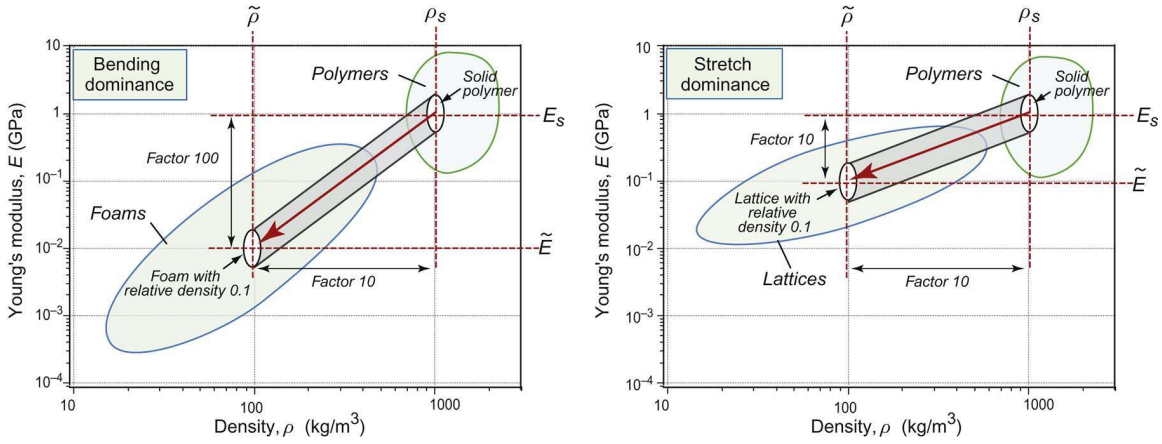


FIGURE 12.19 Left: foaming creates bending dominated structures with lower modulus and density. Right: lattices that are stretch-dominated have moduli that are much greater than those of foams of the same density.

A similar approach can be used to model the plateau stress of the foam. The cell walls yield, as shown in Fig. 12.20A when the force exerted on them exceeds their fully plastic moment (Appendix A4)

$$M_f = \frac{\sigma_s\, t^3}{4} \tag{12.19}$$

where σ_s is the yield strength of the solid of which the foam is made. This moment is related to the remote stress by $M \propto FL \propto \sigma L^3$. Assembling these results gives the failure strength $\tilde{\sigma}_{pl}$

$$\frac{\tilde{\sigma}_{pl}}{\sigma_{f,s}} = C\left(\frac{\tilde{\rho}}{\rho_s}\right)^{3/2} \quad \text{(bending-dominated behavior)} \tag{12.20}$$

where the constant of proportionality, $C \approx 0.3$, has been established both by experiment and by numerical computation.

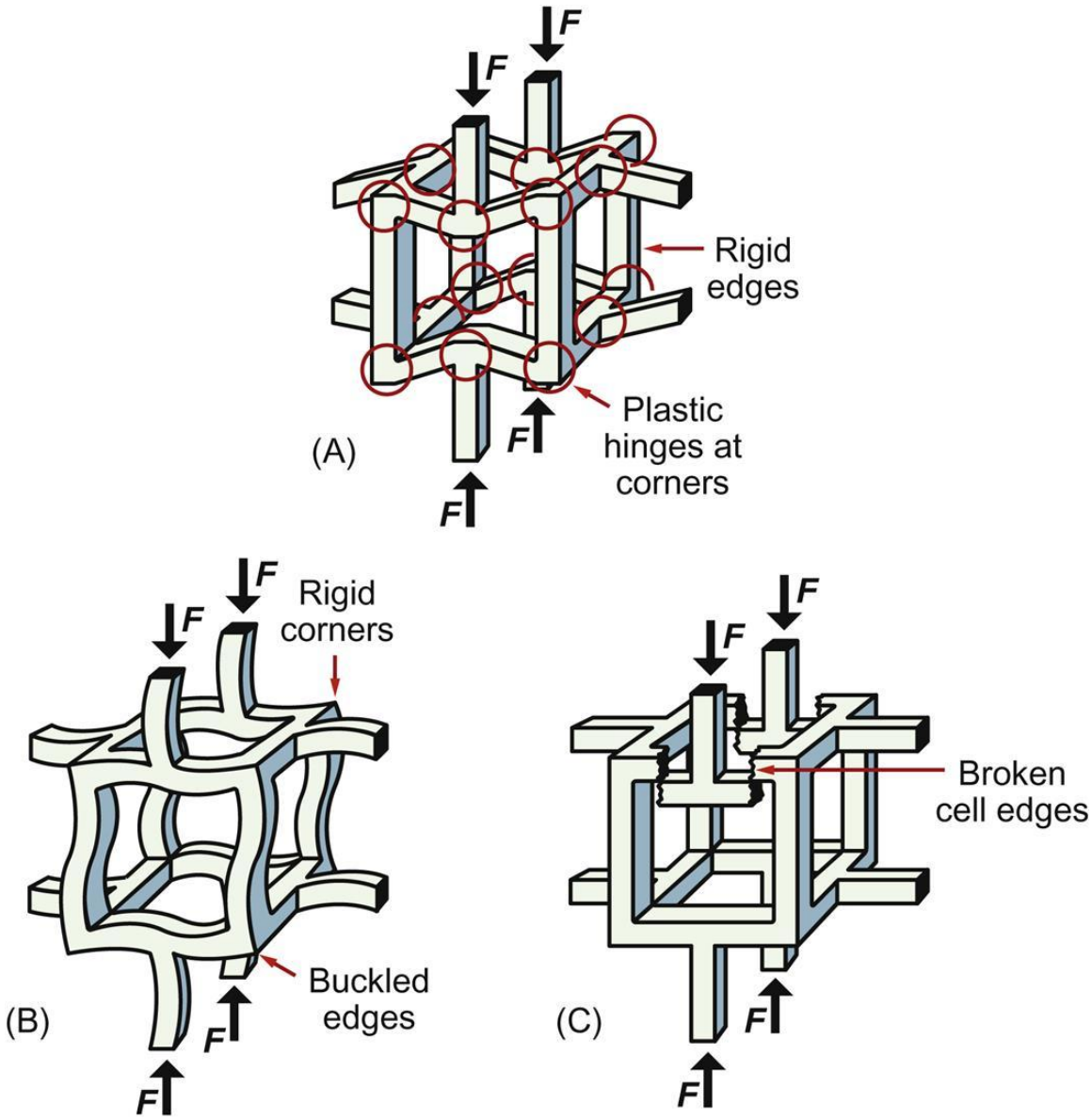


FIGURE 12.20 Collapse of foams. (A) When a foam made of a plastic materials loaded beyond its elastic limit, the cell edges bend plastically. (B) An elastomeric foam, by contrast, collapses by the elastic buckling of its cell edges. (C) A brittle foam collapses by the successive fracturing of cell edges.

Elastomeric foams collapse not by yielding but by elastic bucking; brittle foams by cell-wall fracture (Figs. 12.20B and C). As with plastic collapse, simple scaling laws describe this behaviour well. Collapse by buckling (Appendix B5) occurs when the stress exceeds $\tilde{\sigma}_{el}$, given by

$$\frac{\tilde{\sigma}_{el}}{E_s} \approx 0.05\left(\frac{\tilde{\rho}}{\rho_s}\right)^2 \tag{12.21}$$

and by cell wall fracture (Appendix B4 again) when it exceeds $\tilde{\sigma}_{cr}$, where

$$\frac{\tilde{\sigma}_{cr}}{\sigma_{cr,s}} \approx 0.3\left(\frac{\tilde{\rho}}{\rho_s}\right)^{3/2} \tag{12.22}$$

and where $\sigma_{cr,s}$ is the flexural strength of the cell-wall material. Densification, when the stress rises rapidly, is a purely geometric effect: the opposite sides of the cells are forced into contact and further bending or buckling is not possible. It is found to occur at a strain $\tilde{\varepsilon}_d$ (the densification strain) of

$$\tilde{\varepsilon}_d \approx 1 - 1.4\left(\frac{\tilde{\rho}}{\rho_s}\right) \tag{12.23}$$

Foams are often used for cushioning, packaging or to protect against impact. The useful energy that a foam can absorb per unit volume is approximated by

$$\tilde{U} \approx \tilde{\sigma}_{pl}\,\tilde{\varepsilon}_d \tag{12.24}$$

where $\tilde{\sigma}_{pl}$ is the plateau stress – the yield, buckling or fracturing strength of the foam, whichever is least.

This behaviour is not confined to open-cell foams with the structure idealized as in Fig. 12.17. Most closed-cell foams also follow these scaling laws, at first sight an unexpected result because the cell faces must carry membrane stresses when the foam is loaded and these should lead to a linear dependence of both stiffness and strength on relative density. The explanation lies in the fact that the cell faces are very thin; they buckle or rupture at stresses so low that their contribution to stiffness and strength is small, leaving the cell edges to carry most of the load.

Thermal properties. The specific heat of foams, when expressed in units of J/m^3·K, is given by a rule of mixtures, summing the contributions from the solid and the gas. The thermal expansion coefficient of an open-cell foam is the same as that of the solid from which it is made. The same is true of

rigid closed-cell foams but not necessarily of low density elastomeric foams because the expansion of the gas within the cells can expand the foam itself, which gives an apparently higher coefficient.

The cells in most foams are sufficiently small that convection of the gas within them is completely suppressed. The thermal conductivity of the foam is thus the sum of that conducted through the cell walls and that through the still air (or other gas) they contain. To an adequate approximation

$$\tilde{\lambda} = \frac{1}{3}\left(\left(\frac{\tilde{\rho}}{\rho_s}\right) + 2\left(\frac{\tilde{\rho}}{\rho_s}\right)^{3/2}\right)\lambda_s + \left(1 - \left(\frac{\tilde{\rho}}{\rho_s}\right)\right)\lambda_g \qquad \textbf{(12.25)}$$

where λ_s is the conductivity of the solid and λ_g that of the gas (for dry air it is 0.025 W/m·K). The term associated with the gas is important: blowing agents for foams intended for thermal insulation are chosen to have a low value of λ_g.

Estimating the Properties of Bending-Dominated Foams

A polyethylene foam has a density ρ of 150 kg/m^3. The density, modulus, strength and thermal conductivity of the polyethylene of which it is made are listed in the table. What would you expect the same properties of the foam to be?

High molecular weight PE, solid	Density ρ_s (kg/m^3)	Young's modulus E_s (GPa)	Flexural strength $\sigma_{f,s}$ (MPa)	Thermal conductivity λ_s (W/m·K)
	950	0.94	33	0.195

Answer

The relative density of the foam is $\tilde{\rho}/\rho_s = 0.16$. Using Eqs (12.18), (12.20) and (12.25) we find

High molecular weight PE foam	Young's modulus $\tilde{E}$ (GPa)	Flexural strength $\tilde{\sigma}_{pl}$ (MPa)	Thermal conductivity $\tilde{\lambda}$ (W/m·K)
	0.024	0.63	0.04

Electrical properties. Insulating foams are attractive for their low dielectric constant, $\tilde{\varepsilon}_r$, falling towards 1 (the value for air or vacuum) as the relative

density decreases:

$$\tilde{\varepsilon}_r = 1 + (\varepsilon_{r,s} - 1)\left(\frac{\tilde{\rho}}{\rho_s}\right) \tag{12.26}$$

where $\varepsilon_{r,s}$ is the dielectric constant of the solid of which the foam is made. The electrical conductivity follows the same scaling law as the thermal conductivity.

Lattice: Stretch-Dominated Structures

If conventional foams have low stiffness because the configuration of their cell edges allows them to bend, might it not be possible to devise other configurations in which the cell edges were made to stretch instead? This thinking leads to the idea of *microtruss lattice-structures* (Fig. 12.21).

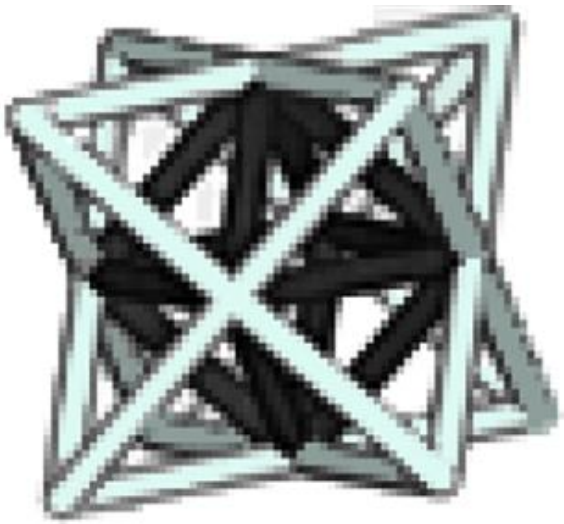


FIGURE 12.21 The unit cell of a micro-truss.

Mechanical properties. These criteria give a basis for the design of efficient microlattice structures. For the cellular structure of Fig. 12.17, Maxwell's $M<0$, and bending dominates. For the structure shown in Fig. 12.21, however, $M>0$ and it behaves as an almost isotropic, stretch-dominated structure. On average one third of its bars carry tension when the structure is loaded in simple tension. Thus

$$\frac{\tilde{E}}{E_s} \approx \frac{1}{3}\left(\frac{\tilde{\rho}}{\rho_s}\right) \quad \text{(isotropic stretch-dominated behavior)} \tag{12.27}$$

The modulus is linear, not quadratic, in density (Fig. 12.19, right hand side), giving a much stiffer structure for the same density. Collapse occurs when the cell edges yield, giving the collapse stress

$$\frac{\tilde{\sigma}}{\sigma_{f,s}} \approx \frac{1}{3}\left(\frac{\tilde{\rho}}{\rho_s}\right) \quad \text{(isotropic stretch-dominated behavior)} \tag{12.28}$$

This is an upper bound since it assumes that the struts yield in tension or compression when the structure is loaded. If the struts are slender, they may buckle before they yield. Then the 'strength', like that of a buckling foam (Eq. 12.21), is

$$\frac{\tilde{\sigma}_{el}}{E_s} \approx 0.2\left(\frac{\tilde{\rho}}{\rho_s}\right)^2 \tag{12.29}$$

Thermal and electrical properties. The bending/stretching distinction influences mechanical properties profoundly, but has no effect on thermal or electrical properties, which are adequately described by the equations listed above for foams.

Estimating the Properties of Stretch-Dominated Lattices

A polyethylene lattice has a density ρ of 150 kg/m^3. The density, modulus, strength and thermal conductivity of the polyethylene are the same as those listed for the last example. What would you expect the properties of the lattice to be?

Answer

The relative density of the lattice is $\tilde{\rho}/\rho_s = 0.16$. Using Eqs (12.27), (12.28) and (12.25) we find

High molecular weight PE lattice	Young's modulus $\tilde{E}$ (GPa)	Flexural strength $\tilde{\sigma}$ (MPa)	Thermal conductivity $\tilde{\lambda}$ (W/m·K)
	0.05	1.8	0.04

Filling property space with cellular structures. All the charts of Chapter 4, Materials Selection – The Basics have an envelope labelled 'Foams', indicating where the properties commercial polymer foams lie. It is highlighted in Fig. 12.22 which again shows the modulus–density ($E - \rho$) section through material property space. The polymer foam envelope is extended along a line of slope 2, as predicted by Eq. (12.18). Lattice structures, by contrast, extend along a line of slope 1, as Eq. (12.27) predicts. Both fill areas of $E - \rho$ space that are not filled by solid materials. Lattice structures push the filled area to higher values of the indices $E^{1/2}/\rho$ and $E^{1/3}/\rho$.

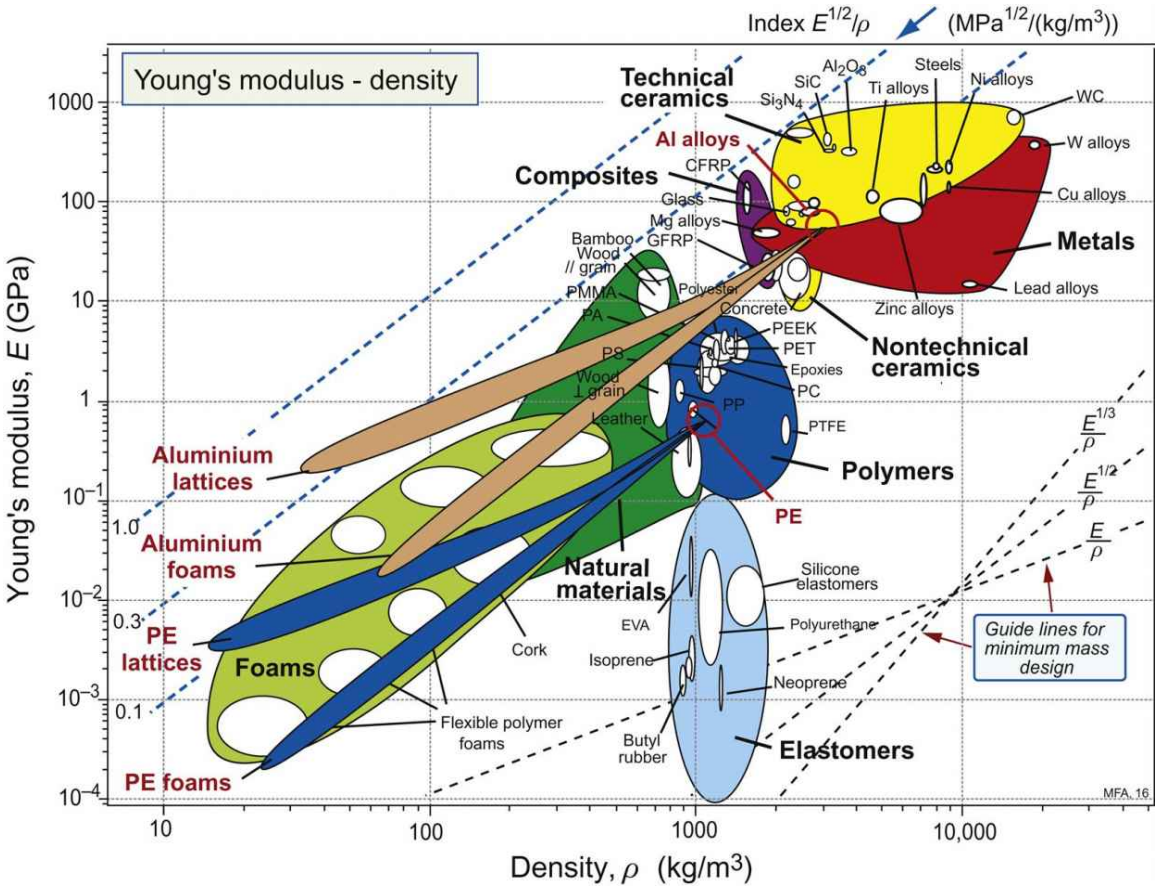


FIGURE 12.22 Foams and microtruss structures are hybrids of material and space. Foams are usually bending-dominated and lie along a line of slope 2 on this chart. Microtruss structures are stretch-dominated and lie on a line of slope 1. Both extend the occupied area of this chart by many decades.

12.6 Sandwich Structures and Multilayers

A sandwich panel epitomizes the concept of a hybrid. It combines two materials in a specified geometry and scale, configured such that one forms the faces, the other the core, to give a structure of high bending stiffness and strength at low weight (Fig. 12.23). The separation of the faces by the core increases the moment of inertia of the section, I, and its section modulus, Z, producing a structure that resists bending and buckling loads well. Sandwiches are used where weight-saving is critical: in aircraft, trains, trucks and cars, in portable structures, and in sports equipment. Nature, too, makes use of sandwich designs: sections through the human skull, the wing of a bird and the stalk and leaves of many plants show a low-density foam-like core separating solid faces.[4]

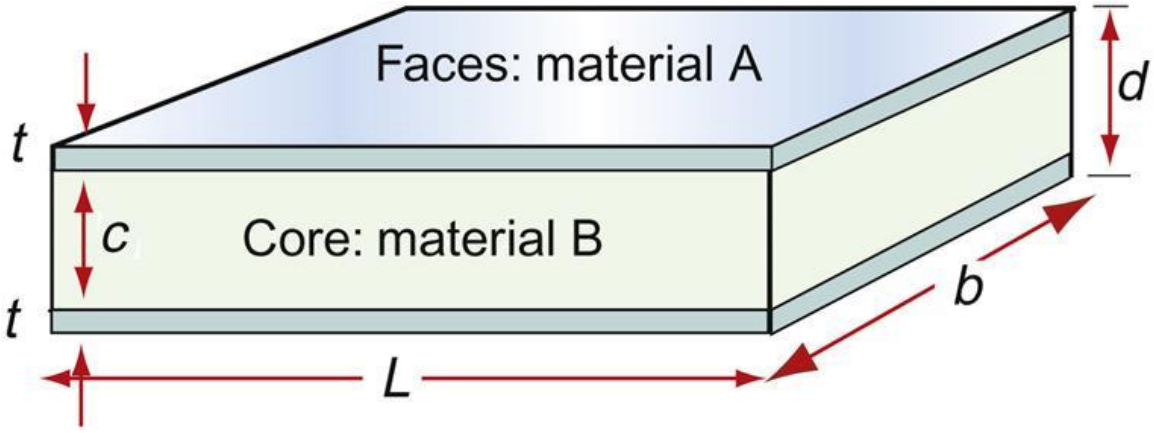


FIGURE 12.23 A sandwich panel.

The faces, each of thickness t, carry most of the load, so they must be stiff and strong; and they form the exterior surfaces of the panel so they must also tolerate the environment in which it operates. The core, of thickness c, occupies most of the volume, it must be light, and be stiff and strong enough to carry the shear stresses necessary to make the whole panel behave as a load bearing unit (if the core is much thicker than the faces these stresses are small).

A sandwich as a 'material'. So far we have spoken of the sandwich as a *structure*: faces of material A supported on a core of material B, each with its own density, modulus and strength. But we can also think of the panel as a *material* with its own set of properties, and this is useful because it

allows comparison with more conventional materials. To do this we calculate *equivalent material properties* for the sandwich, identifying them, as with composites, by a tilde (e.g., $\tilde{\rho}, \tilde{E}$). The quantities $\tilde{\rho}$ and $\tilde{E}$ can be plotted on the modulus-density chart, allowing a direct comparison with all the other materials on the chart. All the constructions using material indices apply unchanged. The symbols the bounds and limits derived below involve a number of symbols. They are defined in Table 12.3.

Table 12.3

The Symbols

Symbol	Meaning and Usual Units
t, c, d	Face thickness, core thickness and overall panel thickness (m)
L, b	Panel length and width (m)
m_a	Mass per unit area of the panel (kg/m^2)
$f=2t/d$	Relative volumes occupied by the faces
$(1-f)=c/d$	Relative volume occupied by the core
I	Second moment of area (m^4)
ρ_f, ρ_c	Densities of face and core material (kg/m^3)
$\tilde{\rho}$	Equivalent density of panel (kg/m^3)
E_f	Young's modulus of the faces (GN/m^2)
E_c, G_c	Young's modulus and shear modulus of the core (GN/m^2)
$\tilde{E}_{in-plane}$, $\tilde{E}_{flex}$	Equivalent in-plane and flexural modulus of panel (GN/m^2)
σ_f,	Yield strength of faces (MN/m^2)
σ_c, τ_c	Yield strength and shear-yield strength of core (MN/m^2)
$\tilde{\sigma}_{in-plane}$	Equivalent in-plane strength of panel (MN/m^2)
$\tilde{\sigma}_{flex\ 1}$, $\tilde{\sigma}_{flex\ 2}$, $\tilde{\sigma}_{flex\ 3}$	Equivalent flexural strength of panel, depending on mechanism of failure (MN/m^2)

Finding Equivalent Properties of Structured Materials by Experiment

Consider a sandwich structure with solid skins separated by a cellular core. The panel has a equivalent density equal to its mass divided by volume, m_a/d, where m_a is its mass per unit area and $d = 2t + c$ is its overall thickness. It has a flexural stiffness EI, measured by loading the panel in bending and recording the deflection. We define an equivalent homogeneous material with $\tilde{\rho} = \rho$ and $\tilde{E}\tilde{I} = EI$, where $\tilde{I} = bd^3/12$ (Appendix B2) is the second moment of area for a *homogeneous* panel with the same dimensions as the real one. The equivalent density and modulus are then

$$\tilde{\rho} = \frac{m_a}{d} \tag{12.30}$$

$$\tilde{E} = \frac{12EI}{bd^3} \tag{12.31}$$

Loading the panel to failure allows the failure moment, M_f, to be measured experimentally. It is then possible to define an equivalent flexural strength via $\tilde{Z}\,\tilde{\sigma}_{flex} = M_f$, where $\tilde{Z} = b\,d^2/4$ is the (fully plastic) section modulus of the panel (Appendix B3). The equivalent flexural strength is then

$$\tilde{\sigma}_{flex} = \frac{4M_f}{b\,d^2} \tag{12.32}$$

A brief example will illustrate the method.

Tests on a carbon-aramid sandwich panel used as flooring in Boeing aircraft gave the results in the following table.

Face material	0.25 mm carbon/phenolic
Core material	3.2 mm cell, 147 kg/m^3, aramid honeycomb
Panel weight per unit area, m_a	2.69 kg/m^2
Panel length, L	510 mm
Panel width, b	51 mm
Panel thickness, d	10.0 mm
Flexural stiffness EI	122 N·m^2
Failure moment M_f	196 Nm

The equivalent density, from Eq. (12.13) is

$$\tilde{\rho} = \frac{m_a}{d} = 269\ kg/m^3$$

The equivalent modulus $\tilde{E}$ from Eq. (12.14) is

$$\tilde{E} = \frac{12\,EI}{b\,d^3} = 28.8\ GPa$$

The equivalent flexural strength $\overleftrightarrow{\sigma}_{flex}$ from Eq. (12.15) is

$$\tilde{\sigma}_{flex} = \frac{4M_f}{b\,d^2} = 154\ MPa$$

Equivalent Properties of Sandwich Structures by Analysis

In this section we develop equations for the stiffness and strength of sandwich panels and express them as properties of an equivalent homogeneous material.

Equivalent density. The equivalent density of the sandwich (its mass divided by its volume) is

$$\tilde{\rho} = f\rho_f + (1-f)\rho_c \qquad \textbf{(12.33)}$$

Here f is the volume fraction occupied by the faces: $f = 2t/d$.

Mechanical properties. Sandwich panels are designed to be stiff and strong in bending. In thinking of the panel as a 'material' we must therefore distinguish the in-plane modulus and strength from those in bending. The effective in-plane modulus, $\tilde{E}_{in-plane}$ and strength $\tilde{\sigma}_{in-plane}$ are given, to an adequate approximation, by the rule of mixtures.

Equivalent flexural modulus. The flexural properties are quite different. The flexural compliance (the reciprocal of the stiffness) has two contributions, one from the bending of the panel as a whole, the other from the shear of the core (Fig. 12.24). They add. The bending stiffness is

$$EI = \frac{b}{12}(d^3 - c^3)E_f + \frac{bc^3}{12}E_c$$

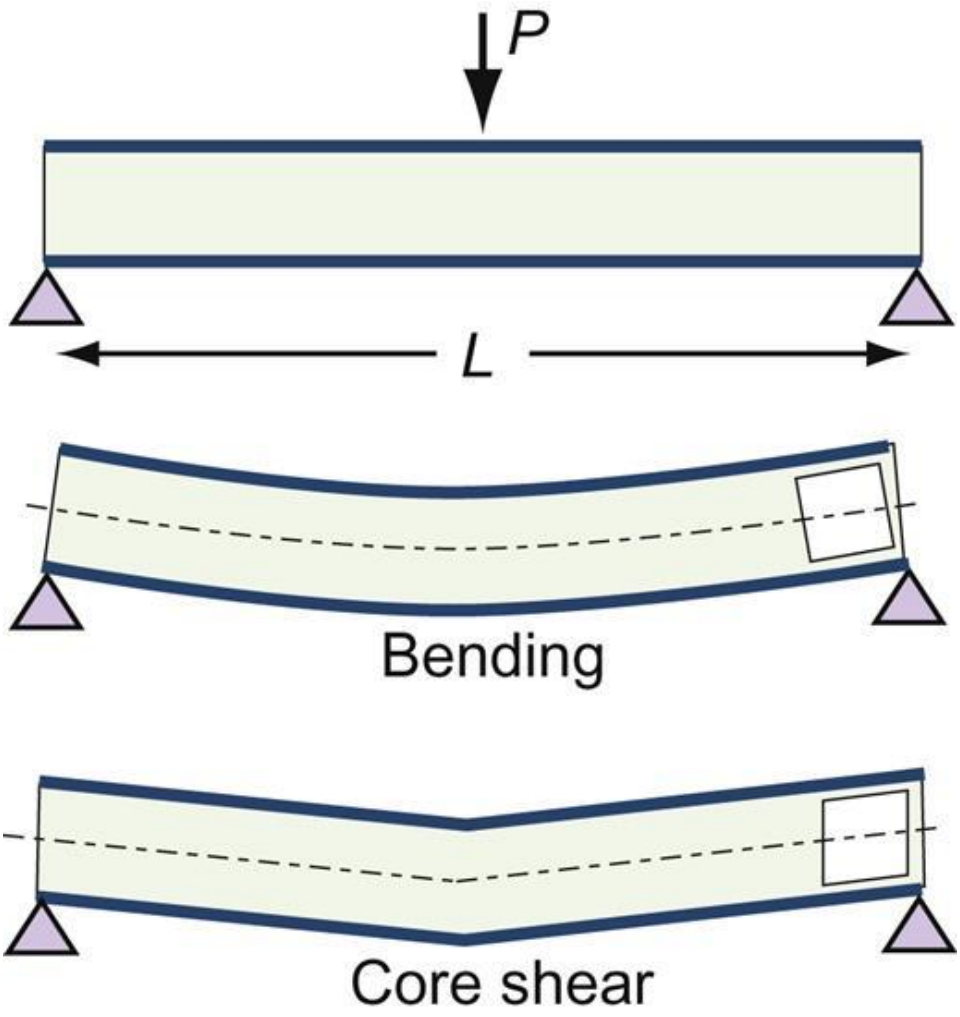


FIGURE 12.24 Sandwich panel flexural stiffness. There are contributions from bending and from core shear.

The shear stiffness is

$$AG = \frac{bd^2}{c} G_c$$

Here the dimensions, d, c, t and L are identified in Fig. 12.23, E_f is Young's modulus of the face sheets and G_c is the shear modulus of the core and A is the area of its cross-section.

Summing deflections gives

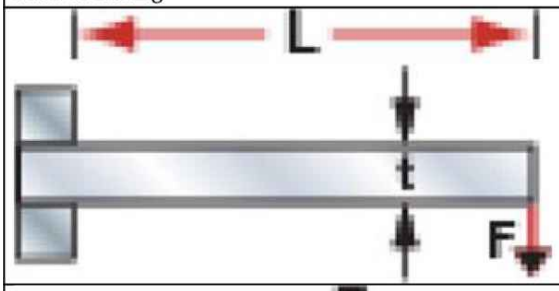
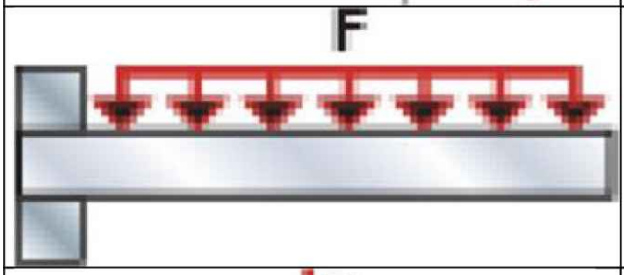
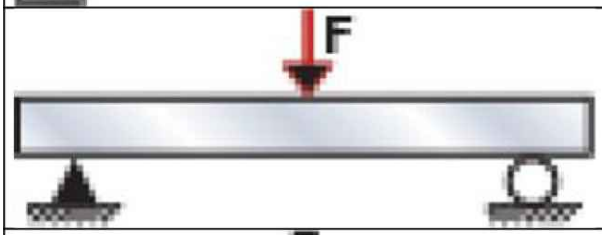
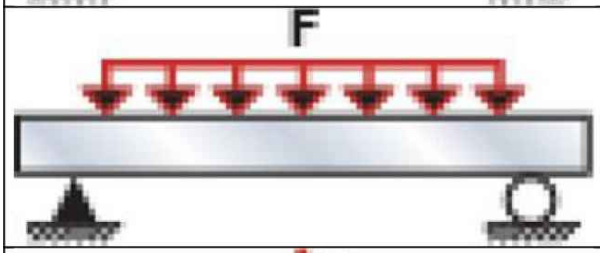
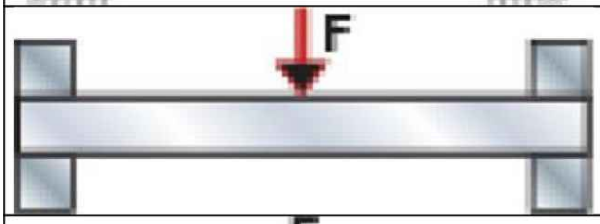
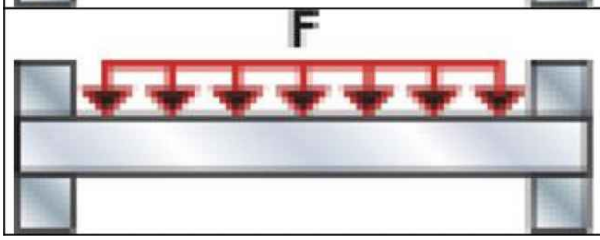
$$\delta = \frac{12PL^3}{B_1 b\{(d^3 - c^3)E_f + c^3 E_c\}} + \frac{PLc}{B_2 d^2 b G_c} \tag{12.34}$$

The load configuration determines the values of the constants B_1 and B_2, as summarized in Table 12.4. Comparison with $\delta = \frac{12PL^3}{\tilde{E}d^3 b}$ for the 'equivalent' material gives

$$\frac{1}{\tilde{E}_{flex}} = \frac{1}{12} \frac{1}{E_f \left\{ (1-(1-f)^3) + \frac{E_c}{E_f}(1-f)^3 \right\}} + \frac{B_1}{B_2} \left(\frac{d}{L}\right)^2 \frac{(1-f)}{G_c} \tag{12.35}$$

Table 12.4

Constants to Describe Modes of Loading

Mode of Loading	Description	B_1	B_2	B_3	B_4
	Cantilever, end load	3	1	1	1
	Cantilever, uniformly distributed load	8	2	2	1
	Three point bend, central load	48	4	4	2
	Three point bend, uniformly distributed load	384/5	8	8	2
	Ends built in, central load	192	4	8	2
	Ends built in, uniformly distributed load	384	8	12	2

Note that, except for the bending-to-shear balancing term $(d/L)^2$ the equivalent property is scale-independent (as a material property should be); the only variable is the relative thickness of faces and core, f. The bending stiffness (EI) is recovered by forming the $\tilde{E}\tilde{I}$ where $\tilde{I}$ is the second moment of a homogeneous panel ($\tilde{I} = bd^3/12$).

Equivalent flexural strength. Sandwich panels can fail in many different ways (Fig. 12.25). The failure mechanisms compete, meaning that the one that happens at the lowest load dominates. We calculate an *equivalent flexural strength* for each mode, then seek the lowest.

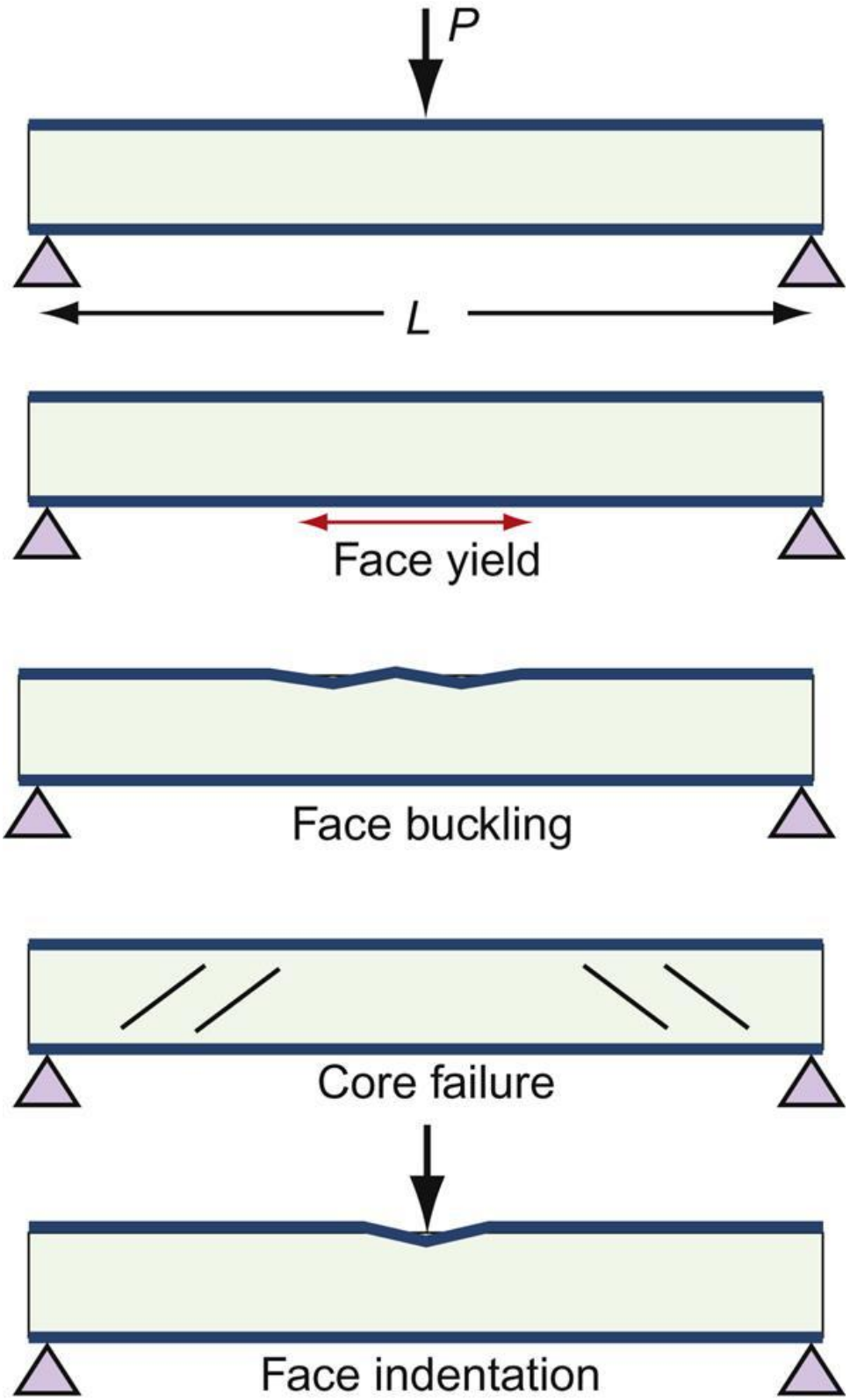


FIGURE 12.25 Failure modes of sandwich panels in flexure.

Face yield. The fully plastic moment of the sandwich is

$$M_f = \frac{b}{4}\{(d^2 - c^2)\sigma_f + c^2\sigma_c\}$$

Using the fact that $c/d = (1-f)$, Eq. (12.15) gives the equivalent failure strength when face yielding is the dominant failure mode as

$$\tilde{\sigma}_{flex\,1} = (1-(1-f)^2)\sigma_f + (1-f)^2\sigma_c \qquad \textbf{(12.36)}$$

which, again, is independent of scale.

Face buckling (Fig. 12.26). In flexure, one face of the sandwich is in compression. If it buckles, the sandwich fails. The face-stress at which this happens[5] is

$$\sigma_b = 0.57(E_f E_c^2)^{1/3} \qquad \textbf{(12.37)}$$

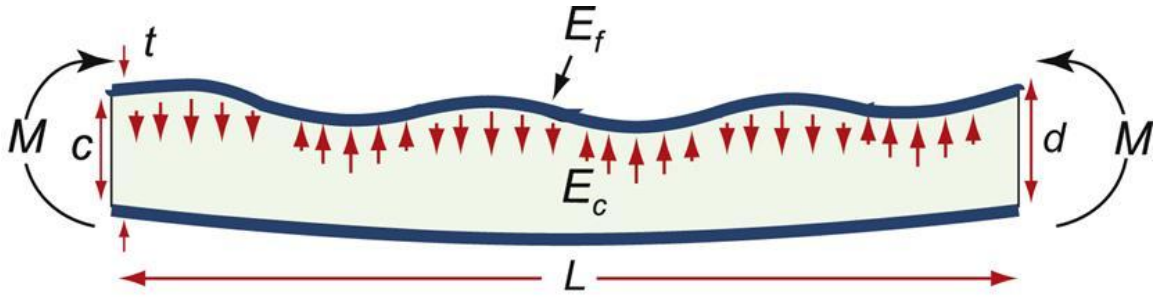


FIGURE 12.26 Face wrinkling.

Buckling is a problem only when faces are thin and the core offers little support. The failure moment M_f is then well approximated by

$$M_f = 2\sigma_b tbc = 1.14(E_f E_c^2)^{1/3} tbc$$

which, via Eq. (12.32) gives

$$\tilde{\sigma}_{flex\,2} = = 2.28 f(1-f)(E_f E_c^2)^{1/3} \tag{12.38}$$

Core shear (Fig. 12.27). Failure by core shear[1] occurs at the load

$$P_f = B_4 bc\left(\tau_c + \frac{t^2}{cL}\sigma_f\right)$$

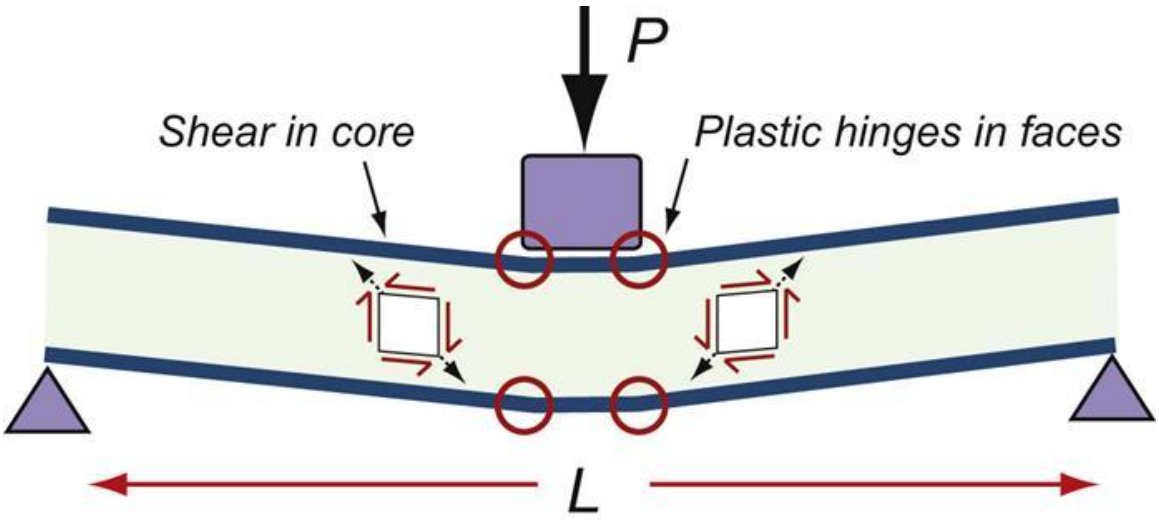


FIGURE 12.27 Core shear.

Here the first term results from shear in the core, the second from the formation of plastic hinges in the faces. Equating to

$$P_f = \frac{B_3 bd^2}{4L}\tilde{\sigma}_3$$

gives the equivalent strength when failure is by shear:

$$\tilde{\sigma}_{flex\,3} = \frac{B_4}{B_3}\left\{4\frac{L}{d}(1-f)\tau_c + f^2\sigma_f\right\} \tag{12.39}$$

The load configuration determines the constant values B_3 and B_4, as summarized in Table 12.4. When the core material is roughly isotropic (as

foams are) τ_c can be replaced by $\sigma_c/2$. When it is not (an example is that of a honeycomb core), τ_c must be retained.

Indentation. The indentation pressure $p_{ind} = P/a$ is

$$\frac{F}{ab} = p_{ind} = \frac{2t}{a}(\sigma_y^f \sigma_y^c)^{1/2} + \sigma_y^c \tag{12.40}$$

from which we find the minimum face thickness to avoid indentation (Ashby et al., 2000).

The efficiency of sandwich structures. Sandwiches are compared with monolithic materials in the way illustrated by Figs. 12.28 and 12.29. In the first of these shows the equivalent density $\tilde{\rho}$ and equivalent flexural modulus, $\tilde{E}$, (Eqs 12.33 and 12.35) for sandwiches, using the data in Table 12.4. Here CFRP face sheets are combined with a high performance foam core in different ratios, stepping through values of $2t/d$, for a chosen ratio of d/L to give the trajectory shown. Its doubly-curved shape arises because of the interplay of bending and shear modes of deformation. Contours show values of the index for a light, stiff, panel:

$$M_3 = \frac{E^{1/3}}{\rho}$$

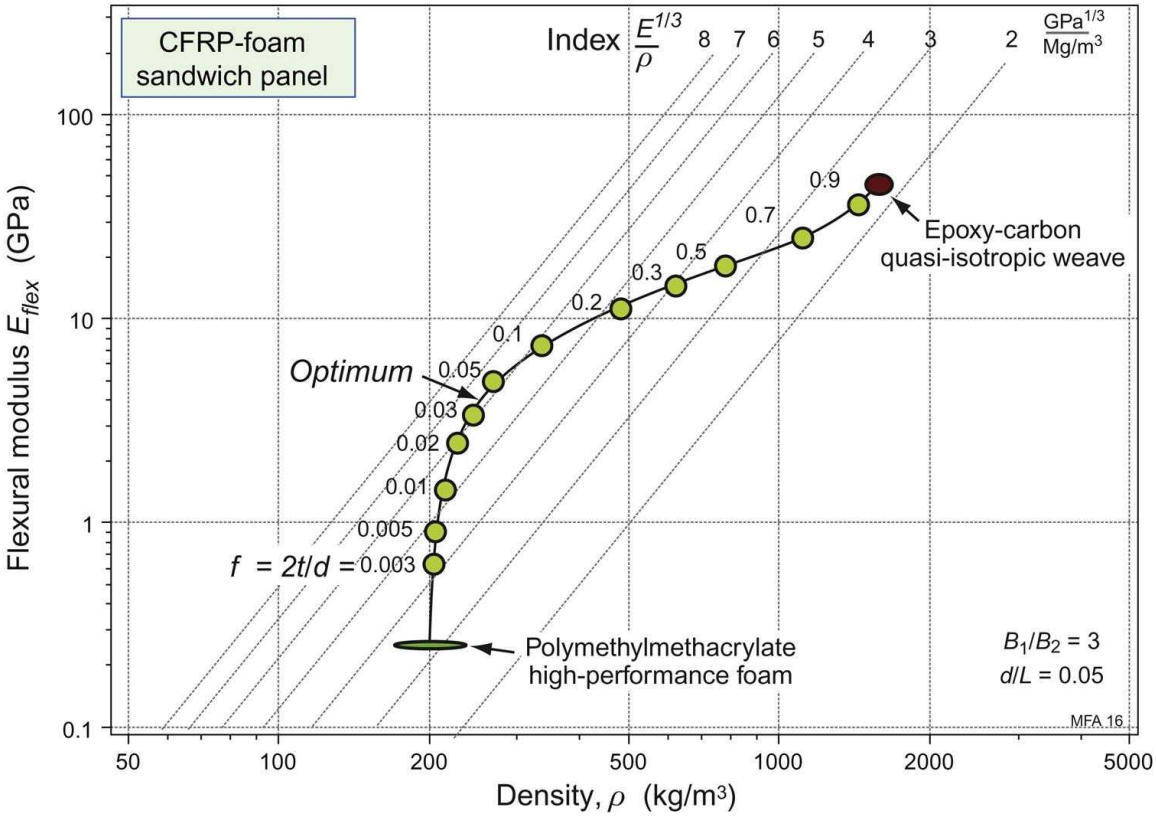


FIGURE 12.28 The equivalent modulus and density of a CFRP – foam sandwich is compared with that of monolithic materials. The contours of the index $E^{1/3}/\rho$ allow optimization of the proportions of the sandwich.

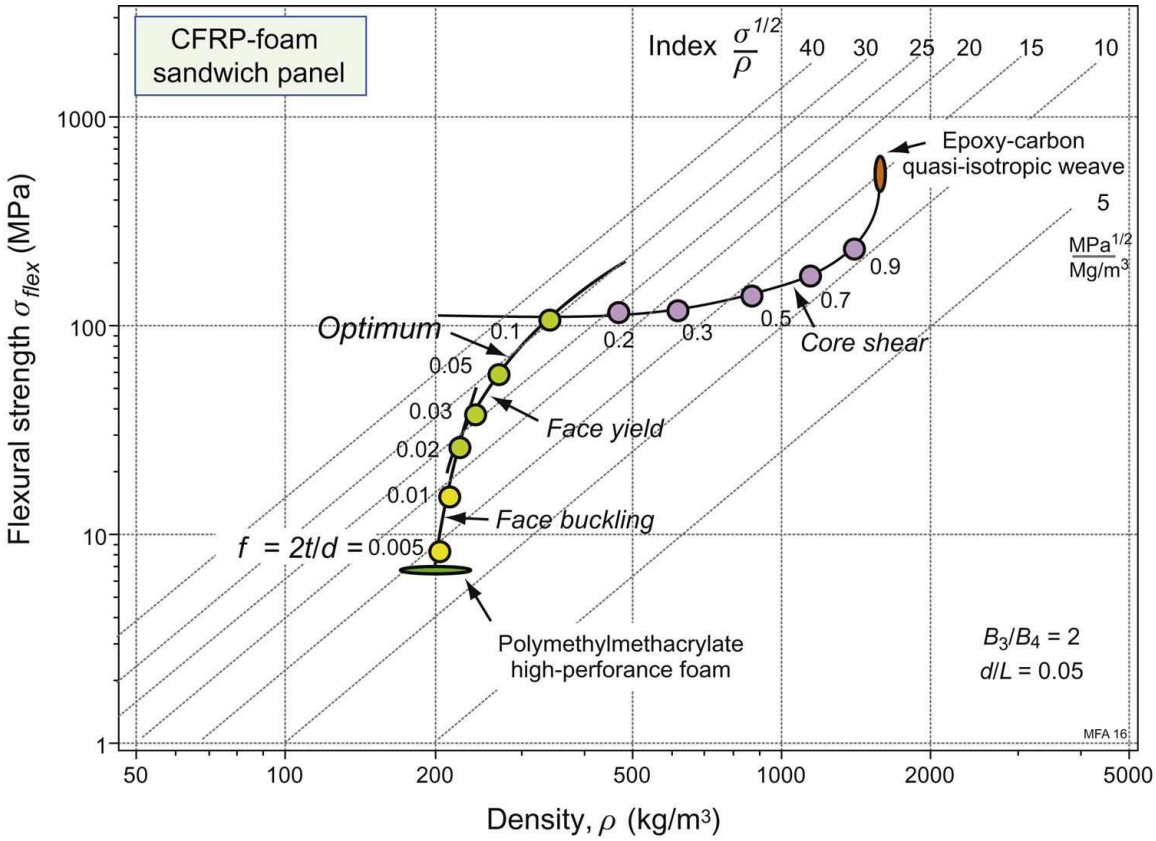


FIGURE 12.29 The equivalent strength and density of a CFRP – foam sandwich is compared with that of monolithic materials. The envelope shows the least strong of the competing failure modes. The contours of the index $\sigma^{1/2}/\rho$ allow optimization of the proportions of the sandwich. Indentation is included by imposing a minimum on the thickness ratio $2t/d$.

The optimum panel, from a stiffness per unit weight perspective, is the one at which a contour is tangent to the trajectory. Fig. 12.28 shows that this occurs a $f \approx 0.04$ giving a panel that is 2.8 times lighter than a solid CFRP panel of the same stiffness (or $(2.8)^3 = 22$ times stiffer for the same mass) (Table 12.5).

Table 12.5

Data for Face and Core of the Sandwich

Face and Core Material	Density ρ kg/m³	Modulus E (GPa)	Strength σ_f (MPa)
Carbon-epoxy quasi-isotropic weave	1570	46	550
Polymethacrylimide high performance foam	200	0.255	6.8

Strength (Fig. 12.29) is handled in a similar way, but here there is the problem of competing mechanisms. We take the equivalent failure strength to be the least of $\tilde{\sigma}_{flex1}$, $\tilde{\sigma}_{flex2}$ and $\tilde{\sigma}_{flex3}$ (Eqs 12.36, 12.38 and 12.39), thereby properly accounting for the competition between them. For the conditions chosen here face buckling dominates for $f < 0.025$, face yield dominates from $f = 0.025$ to $f = 0.1$, when a switch to core shear takes place. The envelope shows the achievable strength of CFRP - foam sandwich structures, and allows direct comparison with monolithic materials. Contours show the index for light strong structures:

$$M_6 = \frac{\sigma_f^{1/2}}{\rho}$$

which measures material efficiency when flexural strength is the main requirement. The optimum lies just below $f = 0.1$ at which the panel is 2.0 times lighter than a solid CFRP panel of the same strength (or $2.0^2 = 4.0$ times stronger for the same mass).

Indentation was not included in this competition because it is a local mechanism – it depends on the area of contact (or of impact) with the indenter, often an event for which the panel was not primarily designed. Protection is possible by estimating a 'worst case' for the indentation load and area and calculating the value t/a required to withstand it. This is done using Eq. (12.40) to calculate a lower safe limit for t/a, which is then applied as a constraint to the trajectory.

Thermal properties. Thermal properties are treated in a similar way. The specific heat, C_p follows a rule of mixtures, Eq. (12.9). The in-plane thermal conductivity $\lambda_{//}$, too, follows such a rule – Eq. (12.12). The through-thickness conductivity, $\lambda_{\perp}$, is given by the harmonic mean

$$\tilde{\lambda}_{\perp} = \left(\frac{f}{\lambda_f} + \frac{(1-f)}{\lambda_c}\right)^{-1} \quad \textbf{(12.41)}$$

Thermal expansion in-plane is complicated by the fact that faces and core have different expansion coefficients, but being bonded together, they are forced to suffer the same strain. This constraint leads to an in-plane expansion coefficient of

$$\tilde{\alpha}_{//} = \frac{f\,E_f\,\alpha_f + (1-f)E_c\,\alpha_c}{f\,E_f + (1-f)E_c} \quad \textbf{(12.42)}$$

The through-thickness coefficient is simpler; it is given by the weighted mean

$$\tilde{\alpha}_{\perp} = f\,\alpha_f + (1-f)\alpha_c \quad \textbf{(12.43)}$$

Through-thickness thermal diffusivity is not a single-valued quantity, but depends on time. At short times heat does not penetrate the core and the diffusivity is that of the face, but at longer times the diffusivity tends to the value given by the ratio $\tilde{\lambda}/\tilde{\rho}\,\tilde{C}_p$.

Electrical properties. The dielectric constant of a sandwich, as with composites, is given by a rule of mixtures, Eq. (12.15) with $f = 2t/d$. Polymer foams have very low dielectric constants, so sandwiches with GFRP faces and polymer foam cores allow the construction of stiff, strong shells with exceptionally low dielectric loss. In-plane electrical conductivity, too, follows a rule of mixtures. Through-thickness electrical conductivity, like that of heat, is described by the harmonic mean (the equivalent of Eq. (12.41).

Multi-layers. A sandwich is an example of a multi-layer structure. Its bending stiffness and strength are relatively easy to analyze because there are only three layers and the structure is symmetric about its mid-plane. More generally multilayers have more than three layers and are not symmetric, so that the neutral axis of bending no longer lies in the mid-plane. The mathematical framework for dealing with this is given in the Appendix to this chapter. They do not lend themselves to simple analytic solutions but are easily solved numerically. They are implemented in the Synthesizer tool of the CES EduPack, which allows the properties of user-

defined multilayers to be plotted on material property charts like those of this chapter.

Filling property space with sandwich structures. We end this section, as we did the last, with two figures illustrating how sandwich structures can expand the occupancy of material property space. The first, Fig. 12.30, is a flexural modulus–density ($E - \rho$) section. The areas filled by metals, polymers ceramics, composites and foams appear as pale envelopes; members are labelled in grey. The stiffness and density of the CFRP-foam sandwiches of Fig. 12.28 are superimposed. Those with $0.01 < f < 0.2$ extend into an area that were previously empty. Using the index $E^{1/3}/\rho$ for a light, stiff panel as a criterion of excellence, we find that sandwiches offer performance that was not previously attainable.

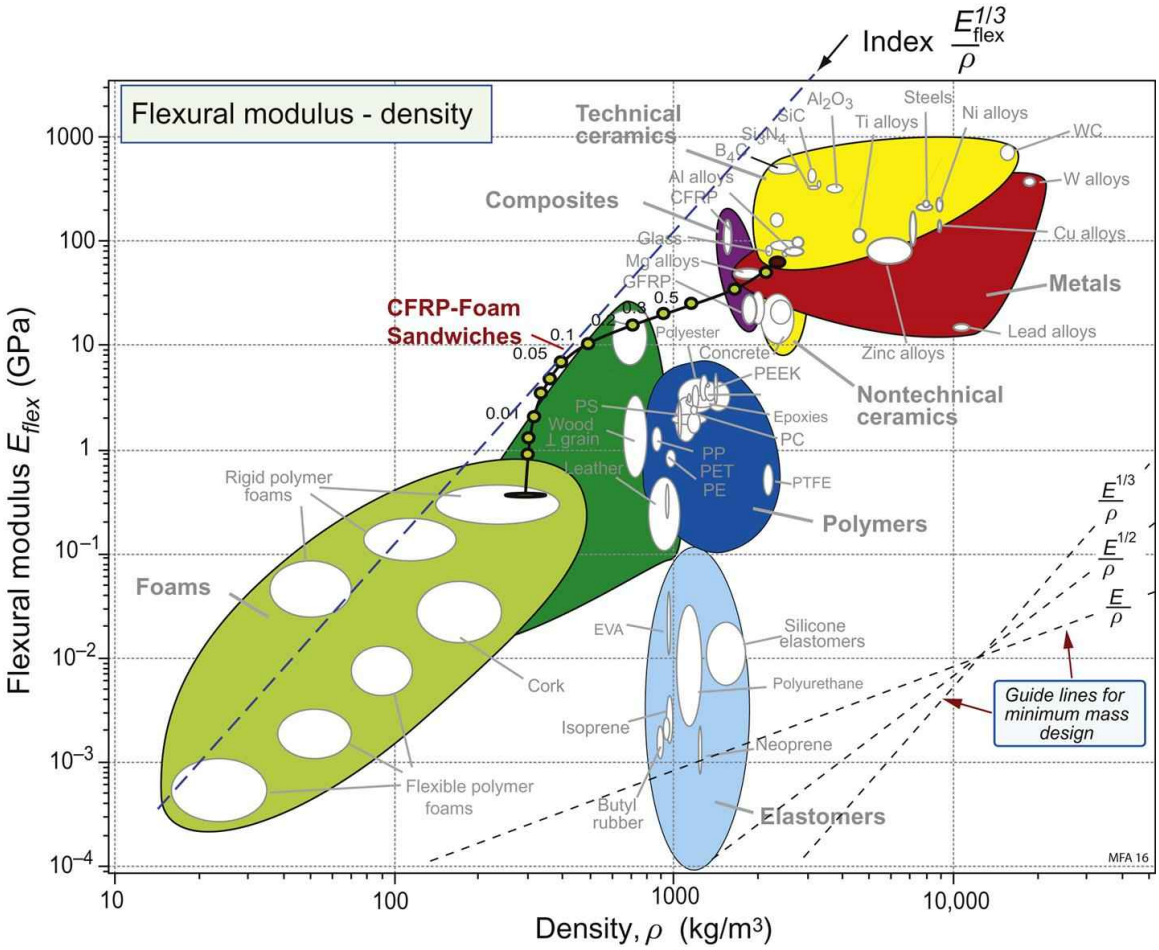


FIGURE 12.30 The sandwich data of Fig. 12.28 superimposed on a modulus density chart, showing the exceptional value of the flexural stiffness index $E^{1/3}/\rho$.

Fig. 12.31 tells a similar story for the strength–density ($\sigma_f - \rho$) section. The colour coding is the same as that of the previous figure. The strength–density trajectory of Fig. 12.29 is overlaid. Sandwiches again expand the populated area in a direction that, using the index for a light strong panel ($\sigma_f^{1/2}/\rho$) as a criterion, offers enhanced performance.

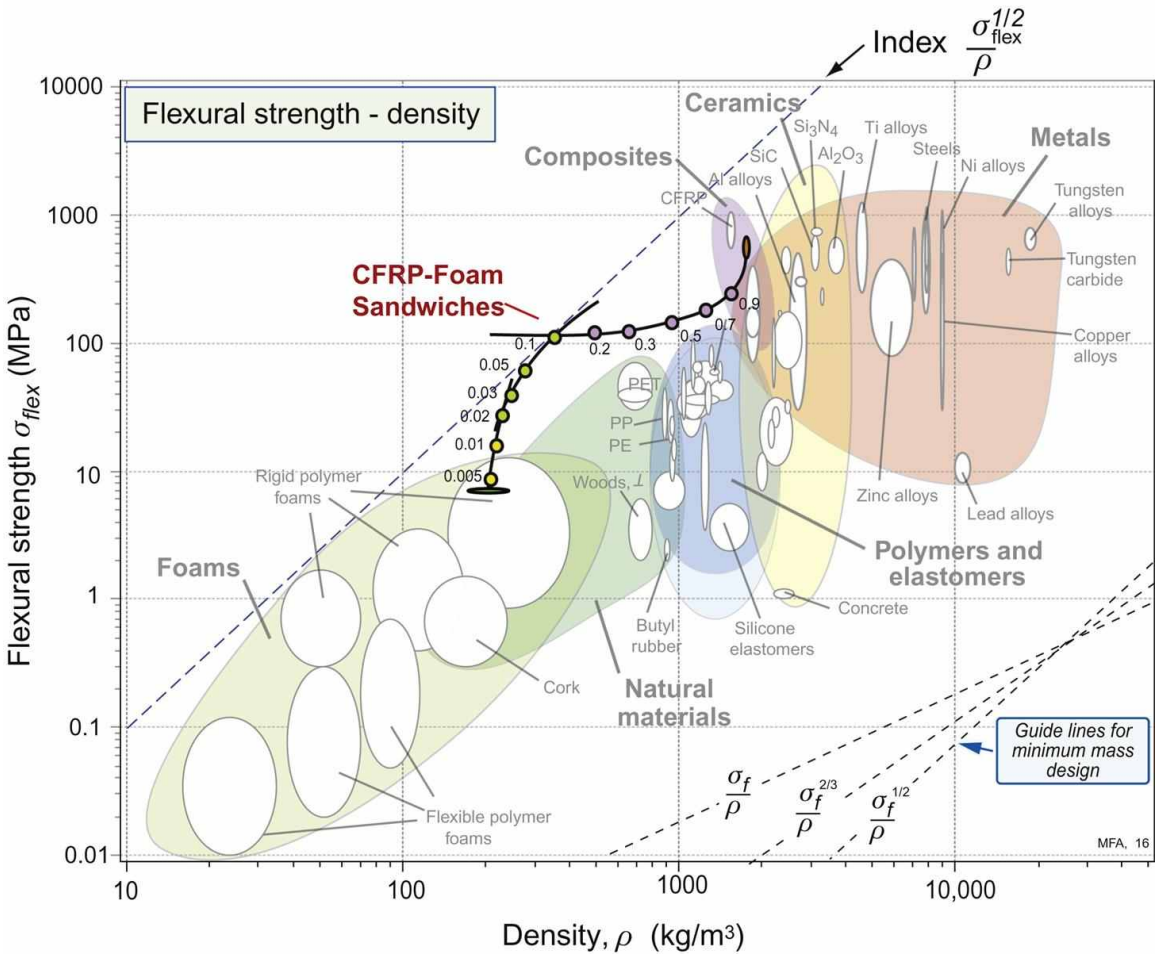


FIGURE 12.31 The sandwich data of Fig. 12.29 superimposed on a strength-density chart, showing the exceptional value of the flexural stiffness index $\sigma^{1/2}/\rho$.

12.7 Segmented Structures

Subdivision as a Design Variable

Jig-saw puzzles, Lego assemblies and Rubik-cubes are examples of segmented structures, introduced in Section 12.3. At first sight load-bearing structures assembled from discrete, interlocking but otherwise unbonded segments do not sound promising. But, as explained earlier, segmentation gives damage-tolerance; it allows disassembly and the ability to mix segments made from different materials, creates structures with properties that cannot be achieved in other ways.

12.8 Summary and Conclusions

The properties of engineering materials can be thought of as defining the axes of a multidimensional space with each property as a dimension. Chapter 3, Materials Property Charts, showed how this space can be mapped. The maps reveal that some areas of property-space are occupied, others are empty – there are holes. The holes can sometimes be filled by making hybrids: combinations of two (or more) materials in a chosen configuration and scale. Design requirements isolate a small box in a multidimensional material-property space. If it is occupied by materials, the requirements can be met. But if it hits a hole in any one of the dimensions, we need a hybrid. A number of families of configuration exist, each offering different combinations of functionality. Each configuration is characterized by a set of bounds, bracketing its effective properties. The methods developed in this chapter provide tools for exploring alternative material-configuration combinations. Chapter 13, Hybrids: Case Studies gives examples of their use.

12.9 Further Reading

This chapter draws on many diverse sources. To make Further Reading more accessible, the references are grouped by topic.

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Cellular Structures

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3. Gibson LJ, Ashby MF, Harley B. *Cellular Bio-Materials* Cambridge, UK: Cambridge University Press; 2010; (*An analysis of the functions of cellular solids in nature*.).

Sandwich Structures

1. Allen HG. *Analysis and Design of Structural Sandwich Panels* Oxford, UK: Pergamon Press; 1969; (*The bible: the book that established the principles of sandwich design*.).
2. Ashby MF, Evans AG, Fleck NA, Gibson LJ, Hutchinson JW, Wadley HNG. *Metal Foams: a Design Guide* Oxford: Butterworth Heinemann; 2000; ISBN 0-7506-7219-6. (A text establishing the experimental and theoretical basis of the properties of metal foams, with data for real foams and examples of their applications.).
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12.10 Appendix: The Stiffness and Strength for Multilayers[6]

Basic equations. Consider a multilayer with n strongly bonded layers. It is bent to a curvature K, positive when concave upwards. We assume that plane sections remain plane. Define coordinate y such that (Fig. 12.32)

$$y_o = 0,$$

and

$$y_i = \sum_{j=1}^{i} h_j$$

with

$$i = 1, n$$

The strain at y is $\varepsilon = \varepsilon_o - K\, y$
The stress in ith layer is $\sigma = \overline{E}_i\, \varepsilon$
where

$$\bar{E}_i = E_i/(1-\nu_i^2)$$

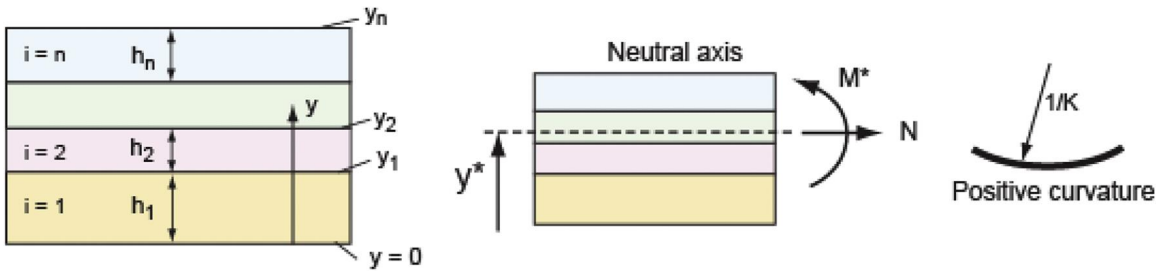


FIGURE 12.32 The multilayer, the loading and the curvature. The key step is to calculate the position of the neutral axis.

Define N=Force per unit depth and M=Moment per unit depth about y=0. Then

$$N = \int \sigma \, dy = \int \bar{E}_i(\varepsilon_o - Ky)dy = \varepsilon_o \sum_{i=1}^{n} \bar{E}_i h_i - K \sum_{i=1}^{n} \frac{1}{2}\bar{E}_i(y_i^2 - y_{i-1}^2)$$

and

$$M = -\int \sigma \cdot y \, dy = -\int \bar{E}_i(\varepsilon_o - Ky)\cdot y \, dy = -\varepsilon_o \sum_{i=1}^{n} \frac{1}{2}\bar{E}_i(y_i^2 - y_{i-1}^2) + K \sum_{i=1}^{n} \frac{1}{3}\bar{E}_i(y_i^3 - y_{i-1}^3)$$

Let

$$C_{11} = \sum_{i=1}^{n} \bar{E}_i h_i \quad C_{12} = C_{21} = -\sum_{i=1}^{n} \frac{1}{2}\bar{E}_i(y_i^2 - y_{i-1}^2) \quad C_{12} = C_{21} = -\sum_{i=1}^{n} \frac{1}{2}\bar{E}_i(y_i^2 - y_{i-1}^2)$$

Then $N = C_{11}\varepsilon_o + C_{12}K$ and $M = C_{12}\,\varepsilon_o + C_{22}\,K$
Identify the neutral bending axis

$$N = C_{11}\,\varepsilon_o + C_{12}K$$

$$M^* = M + N\,y^*$$

So $M^* = (C_{12} + C_{11}\,y^*)\varepsilon_o + (C_{22} + C_{12}\,y^*)K$
Let $\varepsilon_o^* = \varepsilon_o - K\,y^*$ from which $\varepsilon_o = \varepsilon_o^* + K\,y^*$
and

$$N = C_{11}\,\varepsilon_o^* + (C_{12} + C_{11}\,y^*)K$$

$$M^* = (C_{12} + C_{11}\,y^*)\varepsilon_o^* + (C_{22} + 2\,C_{12}\,y^* + C_{11}\,y^{*2})K$$

In simple bending, $N = 0$, giving $y^* = -\dfrac{C_{12}}{C_{11}}$.
The key results so far are assembled in the box below.

$$\left.\begin{aligned} &N = C_{11}\,\varepsilon_o^* \\ &M^* = BK \\ &B = \left[C_{22} - \frac{2C_{12}^2}{C_{11}} + \frac{C_{12}^2}{C_{11}}\right] = \left[C_{22} - \frac{C_{12}^2}{C_{11}}\right] \end{aligned}\right\} \quad \textbf{(12.44)}$$

Stiffness of multilayer beams and panels. The key equation is

$$\frac{M}{I} = \frac{\tilde{E}}{R} = \tilde{E}\,K$$

where $I = \dfrac{b\,h^3}{12}$ and $\tilde{E}$ is the effective flexural modulus. We found the relationship between M and K on the previous page to be (Fig. 12.32)

$$M^* = B\,K \quad \text{with} \quad B = \left[C_{22} - \frac{C_{12}^2}{C_{11}}\right]$$

and

$$C_{11} = \sum_{i=1}^{n} \bar{E}_i h_i \quad C_{12} = C_{21} = -\sum_{i=1}^{n} \frac{1}{2}\bar{E}_i(y_i^2 - y_{i-1}^2) \quad C_{22} = \sum_{i=1}^{n} \frac{1}{3}\bar{E}_i(y_i^3 - y_{i-1}^3)$$

Thus

$$\tilde{E} = \frac{B}{I} = \frac{12\,B}{b\,h^3}.$$

The beam stiffness is then

$$S = \frac{C_1\,\tilde{E}\,I}{L^3}$$

with C_1 given by the values on Table 3 of Appendix 2, 'Useful Solutions'.

Strength of multilayer beams and panels (Fig. 12.33). We define 'Failure' as the yielding of a layer anywhere in the beam. If plane sections remain plane the strain at any point y in the multilayer when it is bent to a curvature K is

$$\varepsilon = (y - y^*)K$$

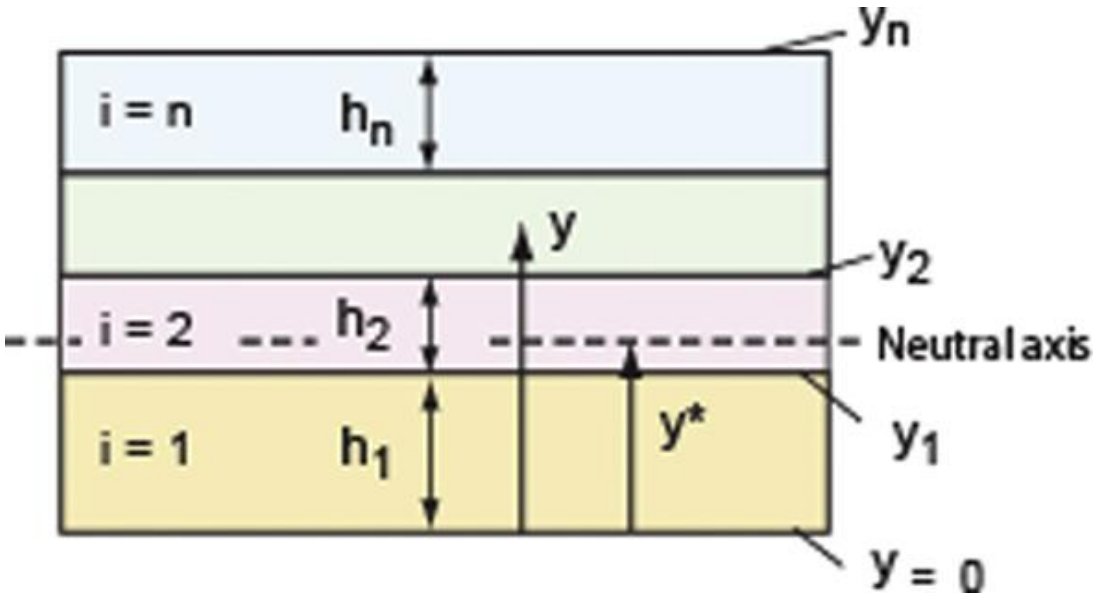


FIGURE 12.33 Calculating first yield.

The centre of layer i lies at a distance

$$y_i - y^* = \sum_{j=1}^{i-1} h_j + \frac{1}{2}h_i - y^*$$

and the stress in that layer is

$$\sigma_i = E_i(y_i - y^*)K = E_i K\left(\sum_{j=1}^{i-1} h_j + \frac{1}{2}h_i - y^*\right)$$

The yield condition in layer i is $\sigma_i \geq \sigma_{y,i}$ where $\sigma_{y,i}$ is the yield strength of layer i. The maximum curvature before one layer yields is thus

$$K_{\max} = \textit{Least of}\left\{\frac{\sigma_{y,i}}{E_i(y_i - y^*)}\right\}_{i=1-n} \quad \text{with} \quad y_i - y^* = \sum_{j=1}^{i-1} h_j + \frac{1}{2}h_i - y^*$$

The failure moment $M_f = \tilde{E}\, I\, K_{\max}$. The failure loads are given by $F = \frac{C_2\, M_f}{L}$ with values of C_2 given by Table 4 of Appendix 2, 'Useful Solutions'.

12.11 Exercises

The examples in this section relate to the design of hybrid material described in Chapter 12 and Chapter 13, Hybrids: Case Studies. *The first four illustrate the use of the key concepts of* Section 12.3. *The next four involve the use of bounds for evaluating the potential of composite systems and of hybrid design to fill holes in property space. The remaining exercises make use of the charts for natural materials and raise a challenge: to explore the potential of hybridizing two very different materials.*

E12.1. *Packing fraction for nondirectional bonding*. The bcc and diamond cubic structures of Table 12.1 are stable only when the bonds linking the atoms depend on direction. When bonding is nondirectional (when spheres are allowed to pack in the earths gravitational field, for example) packing can be loose-random, dense-random or close packed. Plot the packing fraction P against coordination number C for nondirectional packings. Is there a correlation?

E12.2. *Chopped fibre composites*. Nylon 66 has a tensile strength σ_m of 55 MPa. It is to be reinforced with 12 micron diameter chopped S-glass fibres with a tensile strength $\sigma_{ts,f}$ of 4750 MPa. What is the maximum length ℓ of fibre for useful strengthening?

E12.3. *Rigid lattices* (Fig. E12.1). The unit cell of the lattice shown on the right has:

12 bars that are unique to the cell
24 bars that are shared between 2 cells } thus 24 bars, b, per cell

6 joints each shared between 2 cells
8 joints each shared between 4 cell } thus 6 joints, j, per cell

Apply Maxwell's stability criterion to decide if this structure is

a. A mechanism
b. An optimally rigid structure, free of self-stresses
c. An over-constrained structure, with potential for self-stress.

E12.4. *Conducting polymers*. Conducting carbon nanotubes with an aspect ratio of β=200 are dispersed in a polylactide matrix, a nonconducting biopolymer. If the dispersion is random (not an easy thing to achieve) what volume fraction of nanotubes would be required to first allow conduction?

E12.5. *Concepts for light, stiff composites (1)* (Fig. E12.2). The chart for exploring stiff composites with light alloy or polymer matrices is shown below. A construction like that of Fig. 12.11 of the text allows the potential of any given matrix-reinforcement combination to be assessed. Four matrix materials are shown, highlighted in red. The materials shown in green are available as fibres (f), whiskers (w) or particles (p). The criteria of excellence (the indices E/ρ, $E^{1/2}/\rho$ and $E^{1/3}/\rho$ for light, stiff structures) are shown; they increase in value towards the top left. Use the chart to compare the performance of a titanium-matrix composite reinforced with (a) zirconium carbide, ZrC, (b) Saffil alumina fibres and (c) Nicalon silicon carbide fibres. Keep it simple: use Eqs (12.4) and (12.5) to calculate the density and upper and lower bounds for the modulus at a volume fraction of f=0.5 and plot these points. Then sketch arcs of circles from the matrix to the reinforcement to pass through them. In making your judgement, assume that f=0.5 is the maximum practical reinforcement level.

E12.6. *Concepts for light, stiff composites (2)*. Use the Modulus – Density chart for light alloys and fibres shown as Fig. E12.2 to explore the relative potential of Magnesium – E-glass-fibre composites and Magnesium – Beryllium composites for light, stiff structures.

E12.7. *Concepts for composites with tailored thermal properties* (Fig. E12.3). The chart below allows exploration of components for composites with desired combinations of thermal conductivity and expansion, using light alloy or polymer matrices. Three possible matrices –aluminium, magnesium and titanium – are shown in red. The materials shown in green are available as fibres (f), whiskers (w) and particles (p). A construction like that of Fig. 12.14 allows the potential of any given matrix-reinforcement combination to be assessed. One criterion of excellence (the index for materials to minimize thermal distortion, λ/α) is shown; it increases in value towards the bottom right. Use the chart to compare the performance of a magnesium AZ63 alloy-matrix composite reinforced with (a) alloy steel fibres, (b) silicon carbide fibres, SiC (f) and (c) diamond-structured carbon particles. Keep it simple: use Eqs (12.10)–(12.13) to calculate the upper and lower bounds for α and λ at a volume

fraction of f=0.5 and plot these points. Then sketch curves linking matrix to reinforcement to pass through the outermost of the points. In making your judgement, assume that f=0.5 is the maximum practical reinforcement level.

E12.8. *Hybrids with exceptional combinations of stiffness and damping*. The loss-coefficient–modulus ($\eta - E$) chart of Figure 3.9 is populated only along one diagonal band. (The loss coefficient η measures the fraction of the elastic energy that is dissipated during a load-unload cycle.) Monolithic materials with low E have high η, those with high E have low η. The challenge here is to devise hybrids to fill the holes, with the following applications in mind.

a. Sheet steel (as used in car body panels, for instance) is prone to lightly damped flexural vibration. Devise a hybrid sheet that combines the high stiffness, E, of steel with high loss coefficient, η.
b. High loss coefficient means that energy is dissipated on mechanical cycling. This energy appears as heat, sometimes with undesirable consequences. Devise a hybrid with low modulus E and low loss coefficient, η.

E12.9. *Sandwich panels*. An aircraft quality sandwich panel has the characteristics listed in the table.

a. Use the data and Eqs (12.30)– (12.32) of the text to calculate the equivalent density, flexural modulus and strength.
b. Plot these on a copy of a modulus–density, a Strength–Density and Modulus–Strength chart (Figures 3.3, 3.4 and 3.5). Do the flexural properties of the panel lie in a region of property space not filled by monolithic materials?

Data for glass fibre/aluminium honeycomb sandwich panel

Face material	0.38 mm glass fiber/epoxy
Core material	3.2 mm cell, 97.8 kg/m^3, 5052 Alu honeycomb
Panel weight per unit area, m_a	2.65 kg/m^2
Panel length, L	510 mm
Panel width, b	51 mm
Panel thickness, d	10.0 mm
Flexural stiffness EI	67 N·m^2
Failure moment M_f	160 Nm

E12.10. *Natural hybrids that are light and stiff*. Plot aluminium alloys, steels, CFRP and GFRP onto a copy of the $E - \rho$ chart for natural materials (Figure 13.15 of Chapter 13, Hybrids: Case Studies), where E is Young's modulus and ρ is the density. How do they compare, using the flexural stiffness index $E^{1/2}/\rho$ as criterion of excellence? The table lists the necessary data.

Material	Young's Modulus E (GPa)	Density ρ (kg/m^3)
Aluminium alloy	74	2700
Steel	210	7850
CFRP	100	1550
GFRP	21	1850

E12.11. *Natural hybrids that are light and strong*. Plot aluminium alloys, steels, CFRP and GFRP onto a copy of the strength-density chart for natural materials of Figure 13.16 of Chapter 13, Hybrids: Case Studies, using the flexural strength index $\sigma_f^{2/3}/\rho$ (where σ_f is the failure stress) as criteria of excellence. The table lists the necessary data.

Material	Density ρ (kg/m^3)	Strength, σ_f (MPa)
Aluminium alloy	2700	335
Steel	7850	700
CFRP	1550	760
GFRP	1850	182

E12.12. *Natural hybrids that act as springs*. The table lists the moduli and strengths of spring materials. Plot these onto a copy of the $E - \sigma_f$ chart for natural materials of Figure 13.15 of Chapter 13, Hybrids: Case Studies, and compare their energy-storing performance with that of natural materials, using the σ_f^2/E as the criterion of choice. Here σ_f is the failure stress, E is Young's modulus and ρ is the density.

Material	Modulus (GPa)	Strength (MPa)	Density (kg/m^3)
Spring steel	206	1100	7850
Copper-2% Beryllium	130	980	8250
CFRP filament wound	68	760	1580

E12.13. *Finding a substitute for bone*. Find an engineering material that most closely resembles the longitudinal strength of compact bone (*cortical bone (L)*)in its strength and density characteristics by plotting data for this material, read from Figure 13.15 of Chapter 13, Hybrids: Case Studies, onto a copy of the strength–density chart for engineering materials (Figure 3.4).

E12.14. *Creativity: what could you do with X?* The same materials appear on all the charts of Chapter 3, Materials Property Charts. These can be used as the starting point for 'what if…?' exercises. As a challenge, use any chart or combination of charts to explore what might be possible by hybridizing any pair of the materials listed below, in any configuration you care to choose.

a. Cement
b. Wood
c. Polypropylene
d. Steel
e. Copper.

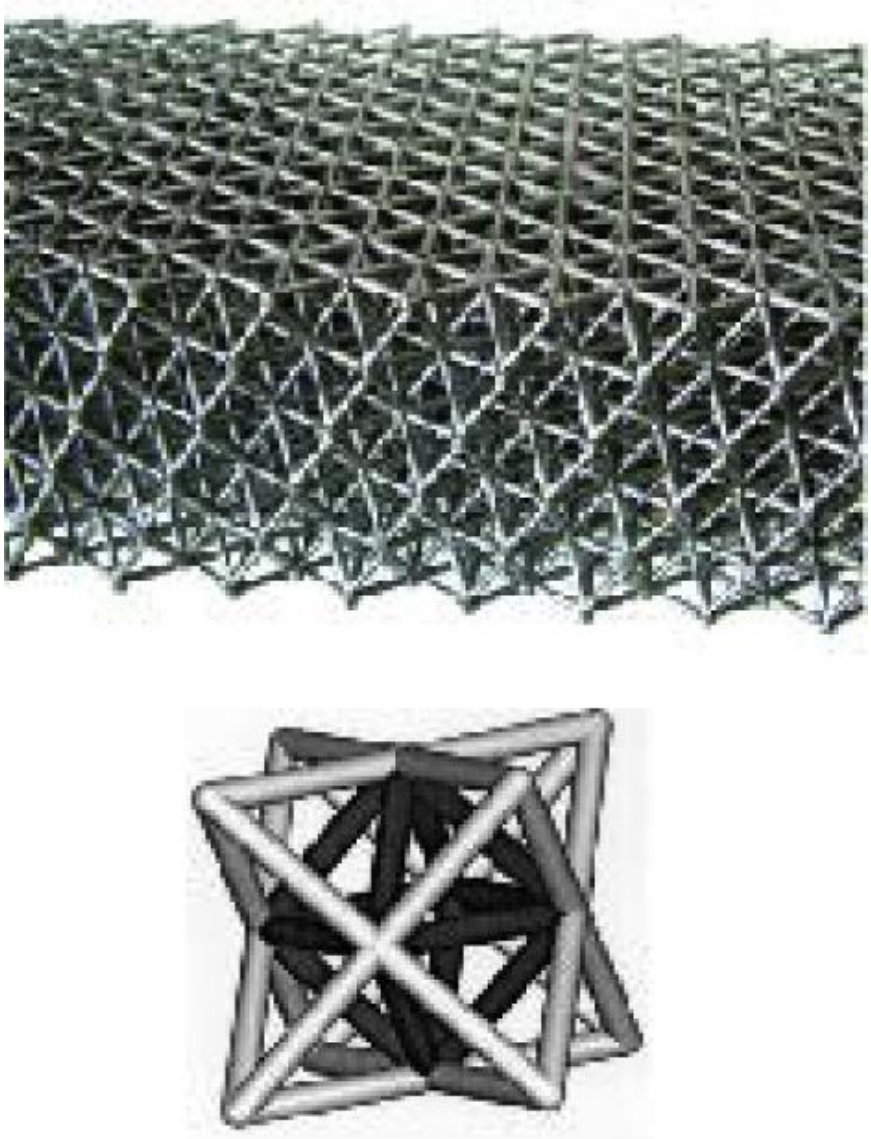


FIGURE E12.1 The lattice and its unit cell sandwich.

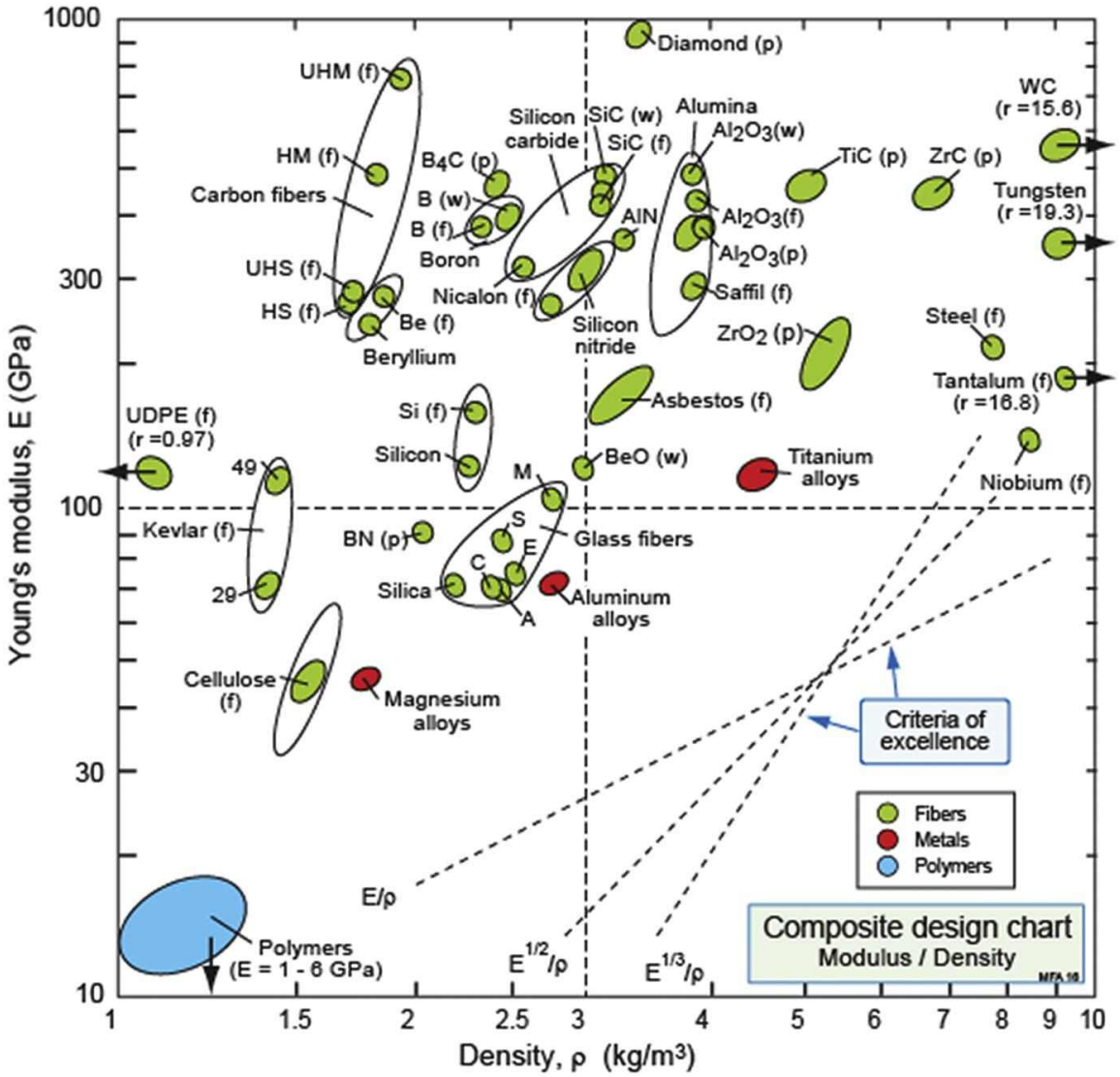


FIGURE E12.2 The moduli and densities of light alloys and fibres.

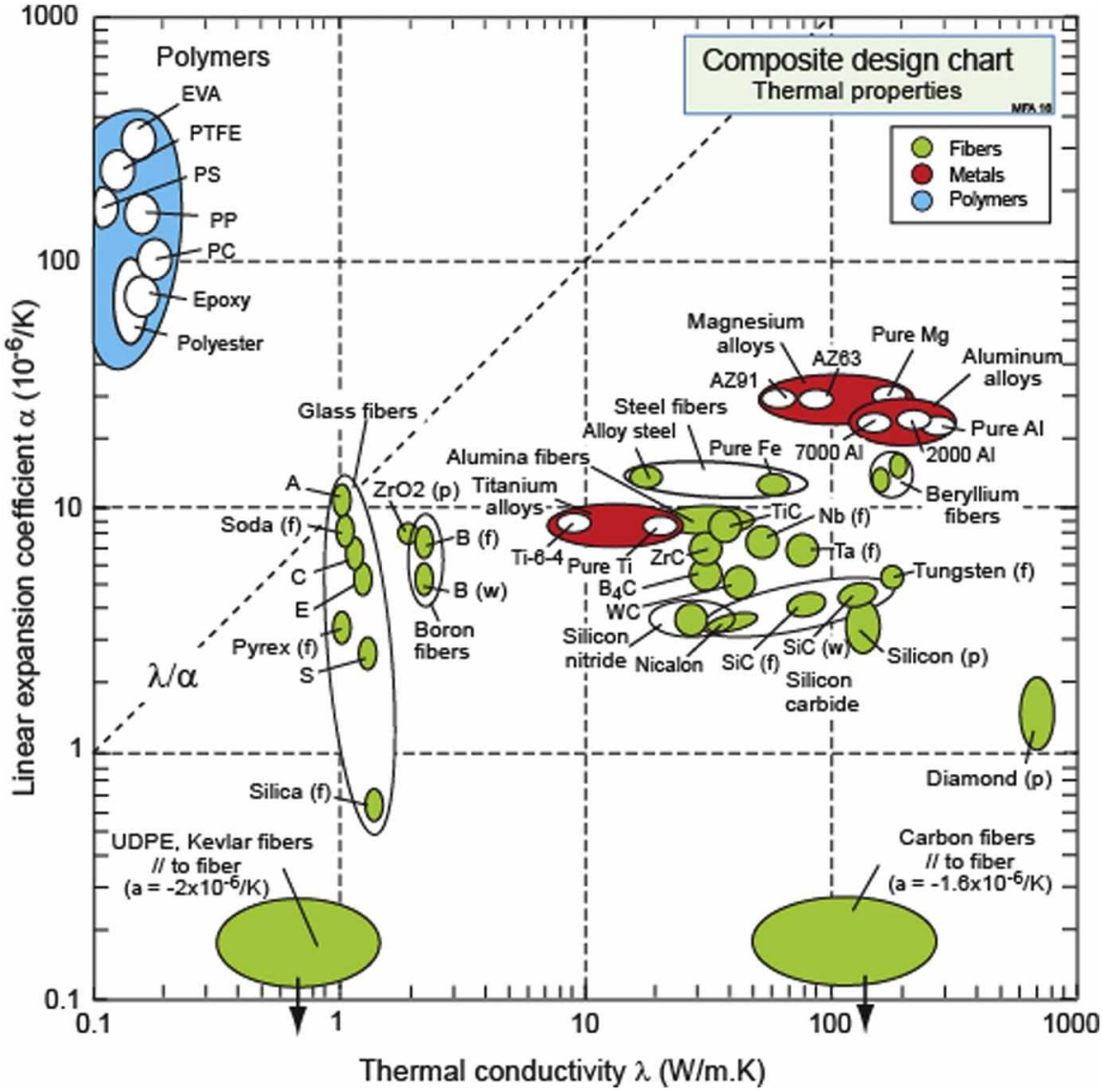


FIGURE E12.3 Expansion and conductivity for fibres, light alloys and polymers.

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[1]Numerical tools now exist that allow a degree of *topological optimization*, meaning the development of a configuration. They work like this. Start with an envelope – a set of boundaries – and fill it with a homogenous 'material' with a relative density initially set at 0.5, with properties that depend linearly on relative density. Impose constraints, meaning the mechanical, thermal and other loads the structure must support, give it a criterion of excellence and let it condense out into regions of relative density 1 and regions where it is zero, retaining only changes that increase the measure of excellence. The method is computationally intensive but has had some success in suggesting configurations that use a material efficiently (see Further Reading for more information).

[2]These are implemented in the Synthesizer tool of the CES EduPack and Selector: www.grantadesign.com.

[3]The treatment here is the simplest that allow the method to be demonstrated. The full complexity of the modelling of composite failure is documented in the texts listed under Further Reading.

[4]The books by Allen (1969) and Zenkert (1995) give good introductions to the design of sandwich panels for engineering applications. That by Gibson et al. (2010) does the same for their use in nature.

[5]Derivations of this and the other equations cited here can be found in Gibson and Ashby (1997) in Further Reading at the end of this chapter.

[6]My thanks to Prof. J.W. Hutchinson for the framework of this derivation. The equations are implemented in the CES EduPack Synthesizer tool.

CHAPTER 13

Hybrids

Case Studies

Abstract

Hybrid materials are combinations of two or more materials assembled in such a way as to have attributes not offered by either one alone. Particulate and fibrous composites are examples of one type of hybrid, but there are many others: sandwich structures, lattice structures, segmented structures and more. This chapter explores ways of designing hybrid materials, emphasizing the choice of the components, their configuration, their relative volume fraction and their scale. The approach we adopt is to use bounding methods to estimate the properties of each configuration. With these, the properties of a given pair of materials in a given configuration can be calculated. These can then be plotted on material selection charts, which become tools for selecting both configuration and material.

Keywords

Holes in material-property space; composites; sandwich structures; segmented structures; cellular structures; lattice structures; multilayers; natural materials

13.1 Introduction and Synopsis

Chapter 12, Designing Hybrid Materials explored hybrids of four broad types: composites, foams and lattices, sandwiches and multilayers and segmented structures. Each is associated with a set of models that allow its properties to be estimated. In this chapter we illustrate their use to design hybrids to fill specified needs – needs that cannot be filled by single

material choices. The cover picture is a reminder of the widespread use of hybrids to maximize performance: the yacht is almost entirely made of hybrid materials, listed at the foot of the cover page.

13.2 Designing Metal Matrix Composites

The most common state of loading in structures is that of bending. One measure of excellence in designing materials to carry bending moments at minimum weight is the index $E^{1/2}/\rho$, where E is Young's modulus and ρ the density. Alloys of aluminium and of magnesium rank highly by this criterion; titanium alloys and steels are less desirable. How could the performance of magnesium (the best of the lot) be enhanced further? Table 13.1 summarizes the challenge.

Table 13.1

Design Requirements for the Material of the Panel

Function	■ *Light stiff beam*
Constraints	■ *Magnesium matrix*
Objective	■ *Maximize stiffness to weight in bending (index* $E^{1/2}/\rho$*)*
Free variables	■ *Choice of reinforcement and volume fraction*

The method. Fig. 13.1 is a chart of E and ρ for light alloys and fibres of polyethylene, carbon, silicon carbide and alumina.[1] Magnesium alloys appear at the extreme left of the 'Light alloys' envelope. The criterion of excellence $E^{1/2}/\rho$ is shown as a set of diagonal contours, increasing towards the top left. Magnesium ranks slightly higher than the other alloy classes.

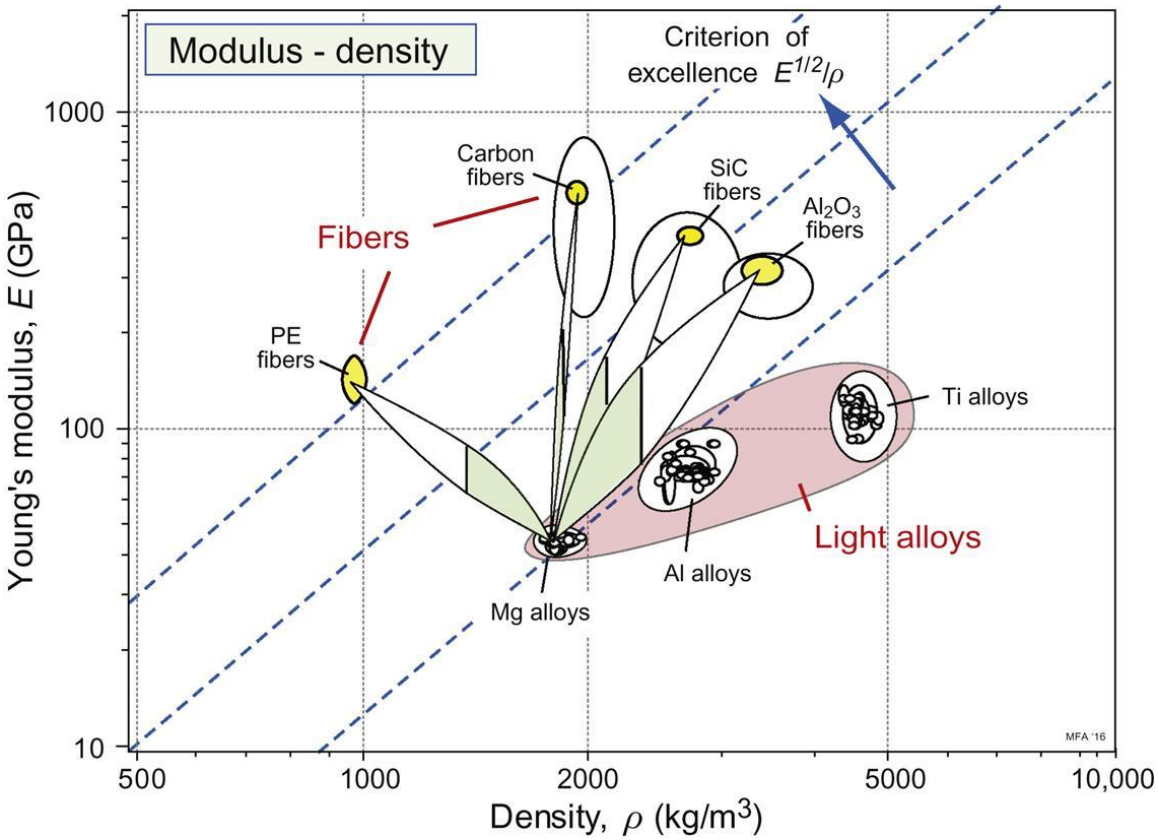


FIGURE 13.1 Possible magnesium-matrix composites. The lozenges show the areas bracketed by the upper and lower bounds of Table 13.2. The green-shaded areas within them extend up to a volume fraction of 0.5.

The four upper envelopes show the attainable performance for four magnesium-based composites based on the bounds listed in the Table 13.2. The upper limit for the volume fraction, which we will set at 0.5, is shown as a vertical bar within each envelope. Only the green-shaded part of the envelope below the bar is accessible. The combinations with the highest values of the criterion of excellence offer the greatest gain in stiffness per unit weight.

Table 13.2

Superposition Rules for Composite Density and Modulus[a]

Property	Lower Bound	Upper Bound
Density	$\tilde{\rho} = f\rho_r + (1-f)\rho_m$ *(exact)*	
Modulus	$\tilde{E}_L = \frac{E_m E_r}{fE_m + (1-f)E_r}$	$\tilde{E}_u = fE_r + (1-f)E_m$

[a]Subscripts *m* and *r* mean 'matrix' and 'reinforcement'; *f*=volume fraction.

The results. The greatest promise is shown by the composites of magnesium with drawn polyethylene (PE) or carbon fibres; magnesium-SiC is less attractive. Magnesium-Al_2O_3 offers almost no gain at all – the envelope extends almost parallel to a contour of the criterion of excellence. The method allows the potential of alternatives to be explored rapidly.

Postscript. Magnesium-PE composites look good promising, but there remains the challenge of actually making them. Polyethylene fibres are already used in ropes and cables because of their high stiffness and strength and low weight. However, they are destroyed by temperatures much above 120°C, so casting or sintering of the magnesium around the fibres is not an option. One possibility is to use drawn PE sheet rather than fibres, and fabricate a multilayer laminate by bonding the PE sheets between sheets of magnesium. A second is to explore ternary composites: dispersing magnesium powder in an epoxy and using this mix as the matrix to contain the PE fibres, for example. Otherwise we must fall back on magnesium-carbon, an attractive option.

Related Case Studies

5.12 'Stiff, high damping materials for shaker tables'
13.3 'Natural-fibre composites'

13.3 Natural Fiber Composites

Carmakers express an interest in composites that contain natural rather than artificial fibres, perhaps contained in a biopolymer matrix. The idea is to use them for nonstructural panels: the inner door facings and seat backs, for instance. In applications like these bending stiffness at low weight is wanted so the criterion of excellence is the index for light-stiff panels, $E^{1/3}/\rho$. There are many natural fibres and many potential matrices.

Before embarking on an expensive development program, it is useful to explore which combinations might be the most attractive. What's the best way to do this? (Table 13.3).

Table 13.3

Design Requirements for the Material of the Panel

Function	■ *Light, stiff panels using natural materials*
Constraints	■ *Cost and properties comparable with GFRP*
Objective	■ *Maximize stiffness to weight in bending (index* $E^{1/3}/\rho$*)*
Free variables	■ *Choice of reinforcement, matrix and volume fraction*

The method. The approach developed in Section 12.3 and used in Case Study 13.2 works equally well for natural materials. Fig. 13.2 shows the moduli and densities of ten natural fibres and nine possible matrix materials, four of them biopolymers (Table 13.4). The bounds allow construction of the envelope composites based on a pair of these. The most promising is shown on the figure: it is polyformaldehyde (PF) reinforced with ramie fibres; the polyformaldehyde could be replace by PLA with little loss of performance. Other combinations are much less promising: thermoplastic starch (TPS) reinforced with sisal, for instance, gives composite properties that are less useful than PF or PLA in the unreinforced state. A second example – PF reinforced with cotton has a trajectory that is almost parallel to the criterion of excellence contour – offers no real gain in performance.

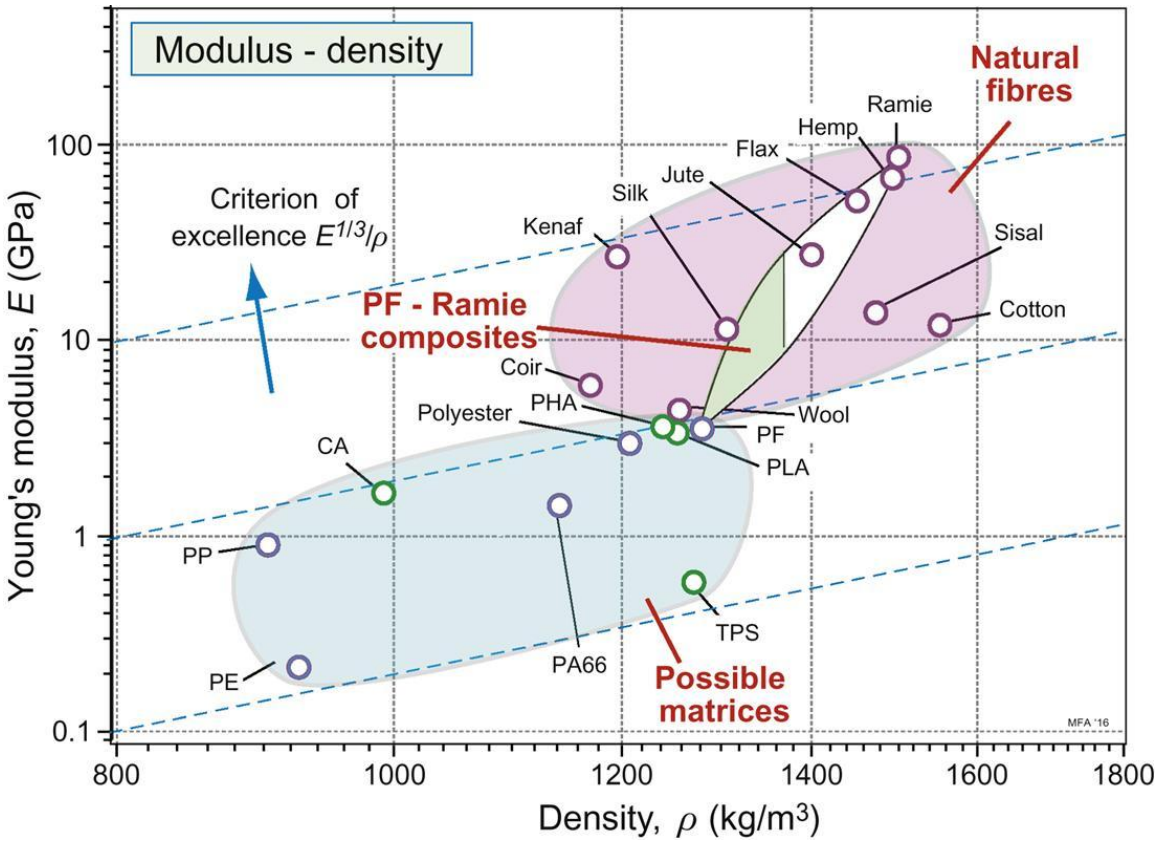


FIGURE 13.2 Exploring possible natural fibre composites using the criterion of excellence for light, stiff panels, $E^{1/3}/\rho$. TPS–Sisal composite offer very little. P –Ramie composites, enclosed by the envelope, offer much more. The green shaded area within it extends up to a volume fraction of 0.5.

Table 13.4

Short Names for the Matrix Materials

PE	*Polyethylene*	*Commodity oil-derived polymers*
PP	*Polypropylene*	
PA66	*Nylon 66*	
PF	*Polyformaldehyde*	
Polyester	*Thermosetting polyester*	
CA	*Cellulose acetate*	*Biopolymers*
TPS	*Thermplastic starch*	
PLA	*Polylactide*	
PHA	*Polyhydroxyalkanoates*	

Postscript. The search for natural-fibre base composites for cars is not new: Henry Ford prototyped a hemp-based composite for body panels as long ago as 1942. It didn't catch on, in part because the word 'hemp' has undesirable associations despite the fact that the species used in the composite is not the same as the fragrant weed.

Related Case Studies

13.2 'Designing metal matrix composites'

13.4 Materials for Long-Span Power Cables

In the design of suspended power cables, the objectives are to minimize the electrical resistance, but at the same time to maximize the strength, because this allows the greatest span. This is an example of multiobjective optimization, discussed in Chapter 8, Multiple Constraints and Conflicting Objectives. As explained there, each objective is expressed, by convention, such that a minimum for it is sought. Table 13.5 summarizes the design requirements.

Table 13.5

Materials for Long-Span Cables

Function	■ *Conduct electrical power with fewest supporting towers*
Constraints	■ *Minimize electrical resistance per unit length of cable* ■ *Maximise tensile load-bearing capacity* ■ *Affordable*
Free variables	■ *Choice of matrix, reinforcement, configuration and volume fraction*

The method and results. We seek materials with the lowest values of resistivity, ρ_e and the reciprocal of yield strength, $1/\sigma_y$. Fig. 13.3 shows the result: materials that best meet the design requirements lie towards the bottom left. But here there is a hole: all 1700 metals and alloys plotted here have properties that lie within the green envelope, none below. Those with the lowest resistance – copper, aluminium, and certain of their alloys – are not very strong; those that are strongest – drawn carbon and low-alloy steel – do not conduct very well.

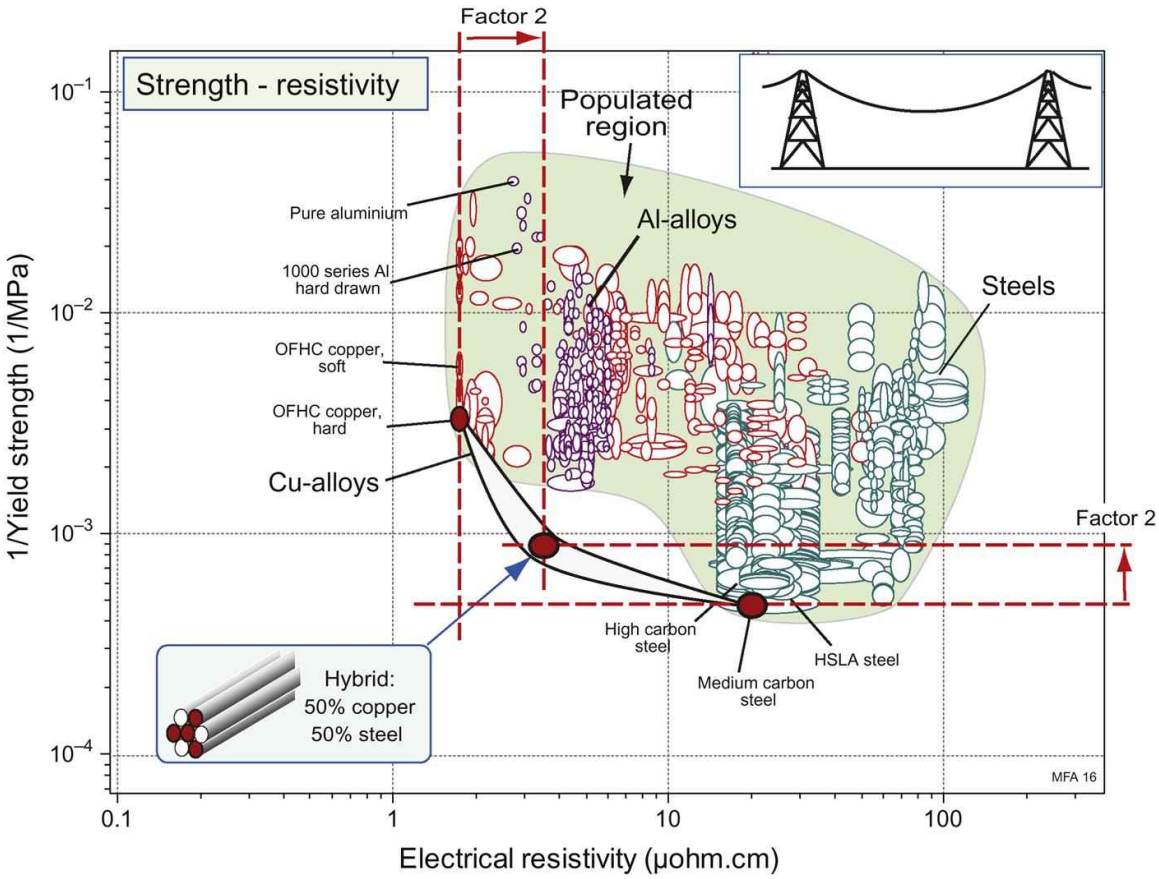


FIGURE 13.3 The resistivity and reciprocal of tensile strength for 1700 metals and alloys. The construction is for a hybrid of hard-drawn OFHC copper and drawn medium carbon steel, but the figure itself allows many hybrids to be investigated.

Now consider a cable made by interleaving strands of copper and steel such that each occupies half the cross-section. If the steel carries no current and the copper no load (the most pessimistic scenario) the performance of the cable will lie at the point marked by the arrow on the figure – it has twice the resistivity of the copper and half the strength of the steel. It lies in a part of property space that was empty, offering performance that was not previously possible. Other ratios of copper-to-steel fill other parts of the space; by varying the ratio the shaded envelope on the figure is covered. Similar hybrids of aluminium and steel fill a different area, as is easily seen by repeating the construction using '1000 series Al alloys' in place of 'OFHC copper, hard' in the combination. Their combinations of ρ_e and σ_y are less acceptable, but they are lighter and cheaper and for this reason are widely used.

Postscript. Similar hybrids of aluminium and steel fill a different area, as is easily seen by repeating the construction using '1000 series Al alloys' in place of 'OFHC copper, hard' in the combination. Their combinations of ρ_e and σ_γ are less acceptable, but they are lighter and cheaper and for this reason are widely used.

Related Case Studies

13.9 'Conductors that don't lose their grip'

13.5 Conducting Elastomers

A material is required for demountable seals in specialized equipment. The material must conform to the curved surfaces between which it is clamped, it must be electrically conducting to prevent charge build-up, and should be mouldable. Table 13.6 summarizes the requirements.

Table 13.6

Materials for Flexible Conductors

Function	■ *Flexible conducting solid*
Constraints	■ *Low Young's modulus to allow conformation* ■ *Low resistivity to permit conduction* (ρ_e<1000 μΩ·cm) ■ *Able to be moulded*
Free variables	■ *Choice of matrix, reinforcement, configuration and volume fraction*

The method and results. Metals, carbon and some carbides and intermetallics are good conductors, but they are stiff and cannot be moulded (Fig. 13.4). Elastomers can be moulded and are flexible, but they do not conduct. How are they to be combined? Metal coating of polymers is workable if the product will be used in a protected environment, but coatings are easily damaged. If a robust, flexible, product is needed, bulk rather than surface conduction is essential. How is it to be done? The answer in one word: percolation.

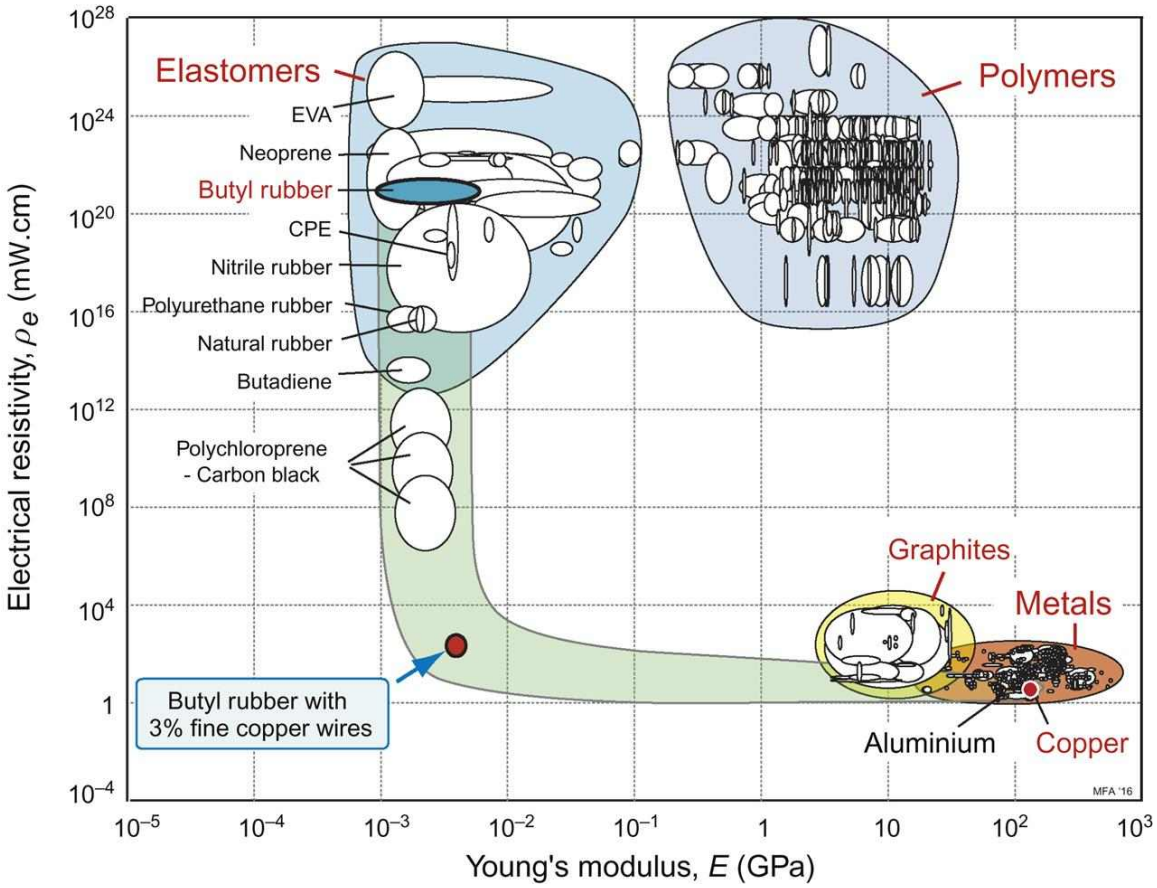


FIGURE 13.4 When conducting particles or fibres are mixed into an insulating elastomer, a hole in material-property space is filled. Carbon-filled butyl rubbers lie in this part of the space.

Elastomers can be made conducting by dispersing particles or wires of a conducting material in them. Electrical conductivity requires a conducting path. As explained in Section 12.3, the percolation threshold at which a mix of conducting and insulating spheres first conducts is about 0.2. For conducting wires the threshold is lower – 0.02 if the aspect ratio of the wires is 100:1. Above the percolation threshold the conductivity tends towards a rule of mixtures (Eq. 12.12 with thermal conductivity replaced by electrical conductivity):

$$\tilde{\kappa}_1 = f\kappa_{cu} + (1-f)\kappa_{PE} \qquad \textbf{(13.1)}$$

There is a large hole in the lower left of Fig. 13.4 where materials with the property combination we want would lie. Polychloroprene–carbon

black (graphite) hybrids edge down into this region, but because the carbon is particulate in shape and extremely fine in scale it takes a great deal to give significant conduction. The pale-green L-shaped envelope encloses the properties of hybrids of rubber with chopped copper wires. The resistivity drops sharply at the percolation threshold, which, for wires, falls to a few percent. The material retains the flexibility of the elastomer but behaves as a bulk conductor.

Postscript. Conducting elastomers, exploiting these ideas, are widely available. Such materials find application in antistatic clothing and mats, as pressure sensing elements, even as solder-less connections.

Related Case Studies

13.5 'Extreme combinations of thermal and electrical conductivity'

13.6 Extreme Combinations of Thermal and Electrical Conduction

Materials that are good electrical conductors are good thermal conductors too. Copper, for example, excels at both. Most polymers, by contrast, are electrical insulators (meaning that their conductivity is so low that for practical purposes they do not conduct at all), and as solids go, they are also poor thermal conductors – polyethylene is an example. Thus the 'high-high' and the 'low-low' combinations of conduction can be met by monolithic materials and there are plenty of them. The 'high-low' and 'low-high' combinations are a different matter: nature gives us very few of either one. The challenge is summarized in Table 13.7: using only copper and polyethylene, devise hybrid materials that achieve both of these combinations. Table 13.8 gives data for both.

Table 13.7

Requirements for the Conducting Hybrids of Copper and Polyethylene

Function	■ *Extreme conduction combinations*
Constraints	■ *Materials: copper and polyethylene*
Objective	■ *Maximize difference between electrical and thermal conductivities*
Configuration	■ *Free choice*
Free variables	■ *Configuration and relative volume fractions of the two materials.*

Table 13.8

Data for Copper and Polyethylene

Material	Electrical Conductivity (1/μΩ·cm)	Thermal Conductivity (W/m·K)
High conductivity copper	0.6	395
High density polyethylene	1×10^{-25}	0.16

The methods and results. Fig. 13.5 shows two possible hybrid configurations that differ greatly in volume fraction of conducting phase (colored red). The first is a tangle of fine copper wires embedded in a polyethylene matrix. As we saw in Section 12.3, electrical conductivity requires percolation; above the percolation threshold the conductivity tends to a rule of mixtures (Eq. 13.1). The thermal conductivity for a random array like this will lie near the lower bound (Eq. 12.13):

$$\tilde{\lambda}_1 = \lambda_{PE}\left(\frac{\lambda_{Cu} + 2\lambda_{PE} - 2f(\lambda_{PE} - \lambda_{Cu})}{\lambda_{Cu} + 2\lambda_{PE} + f(\lambda_{PE} - \lambda_{Cu})}\right) \tag{13.2}$$

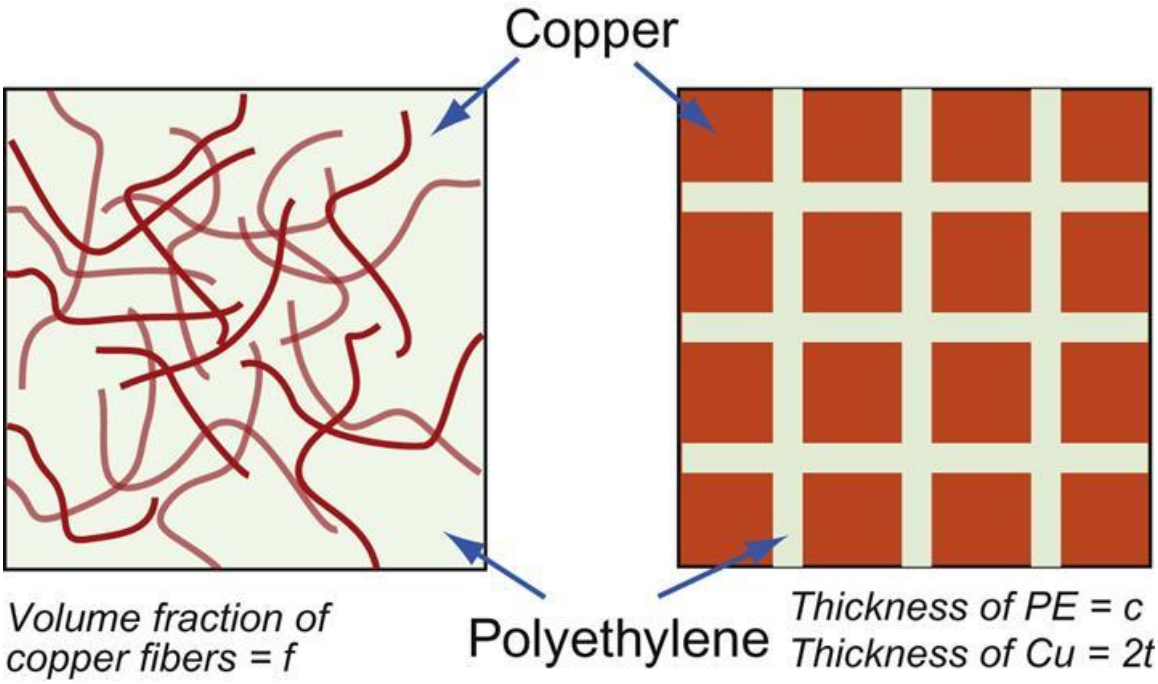


FIGURE 13.5 Two alternative configurations of copper and polyethylene.

The composite properties $\tilde{\kappa}_1$ and $\tilde{\lambda}_1$ are plotted in Fig. 13.6, stepping through values of f, giving the upper curve.

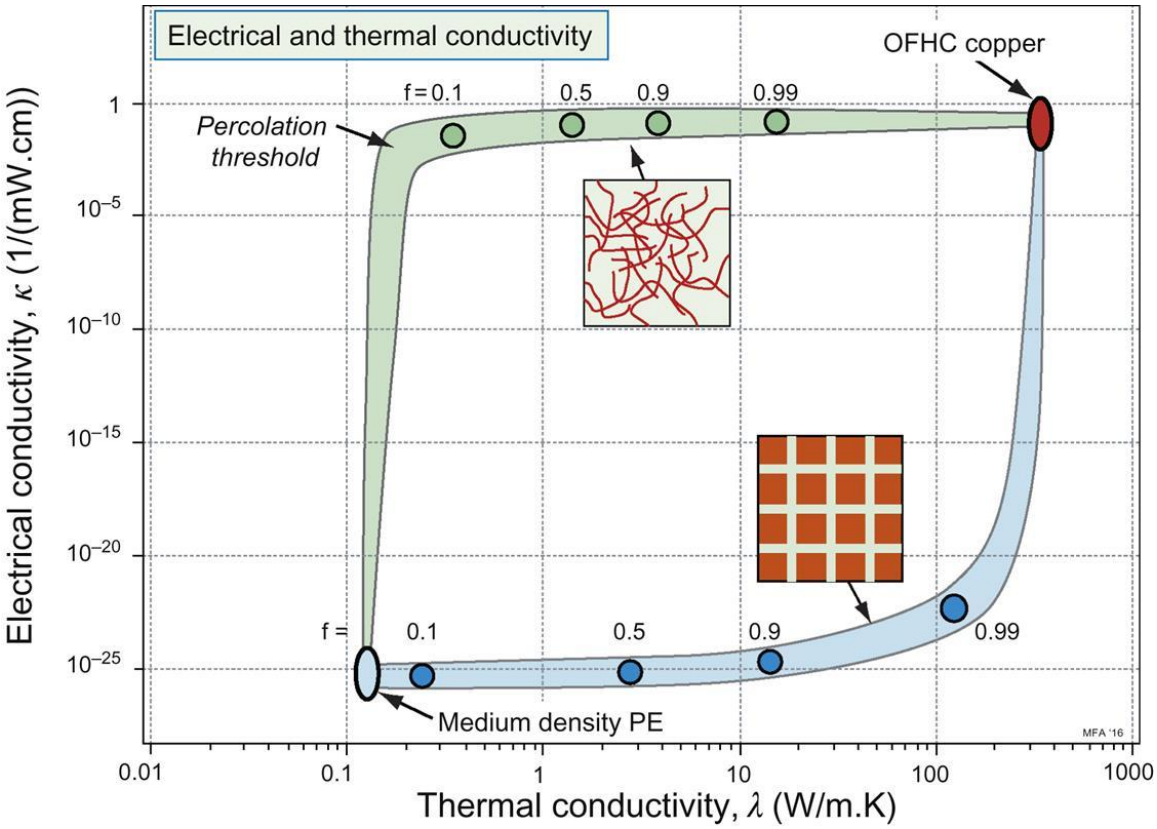


FIGURE 13.6 Two alternative hybrid-configurations of copper and polyethylene give very different combinations of thermal and electrical conductivity, and create new 'materials' with properties that are not found in homogeneous materials.

The second hybrid is a multilayered composite with three orthogonal families of PE sheet separating blocks of copper. We draw again on the bounds of Section 12.4. When the PE layers are thin, the through-thickness thermal resistances add; the same is true of the electrical resistances. This means that both the electrical and thermal conductivities are given by the harmonic means:

$$\tilde{\kappa}_2 = \left(\frac{f}{\kappa_{Cu}} + \frac{(1-f)}{\kappa_{PE}} \right)^{-1} \tag{13.3}$$

$$\tilde{\lambda}_2 = \left(\frac{f}{\lambda_{Cu}} + \frac{(1-f)}{\lambda_{PE}} \right)^{-1} \tag{13.4}$$

The composite properties $\tilde{\kappa}_2$ and $\tilde{\lambda}_2$ are also plotted in Fig. 13.6, giving the lower curve. It shape is a consequence of the very different ranges of values of the two properties: a factor of 1000 for λ, a factor of 10^{25} for κ. The two hybrids differ dramatically in their behavior. The hybridization has allowed the creation of 'materials' with extreme combinations of conductivities.

Postscript. Hybrids of the first type are widely used to give electrical screening to computer and TV cabinets. Those of the second type are less common, but could find application as heat sinks for power electronics in which large scale electrical conduction would lead to coupling and eddy current losses. How would you make it? Perhaps by thermally bonding a stack of copper sheets interleaved with polyethylene film, dicing the stack normal to the layers and re-stacking the slices with PE film interlayers, finally dicing and re-stacking a third time to give the last set of layers.

Related Case Studies

5.17 'Materials for heat exchangers'
13.5 'Conducting elastomers'
13.9 'Connectors that don't relax their grip'

13.7 Refrigerator Walls

The panels of a refrigerator or freezer like that in Fig. 13.7 perform two primary functions. The first is to insulate, and for this the through-thickness thermal conductivity must be minimized. The second is mechanical: the walls provide stiffness and strength and support the shelves on which the contents rest. For a given thickness of panel the first is achieved by minimizing $\tilde{\lambda}$ where $\tilde{\lambda}$ the appropriate thermal conductivity. The second is achieved by seeking materials or hybrids that maximize $\tilde{E}_{flex}$ where $\tilde{E}_{flex}$ is the flexural modulus. Table 13.9 summarizes the requirements.



FIGURE 13.7 A freezer. The panels must insulate and be stiff and strong in bending.

Table 13.9
Design Requirements for the Insulating Panel

Function	■ *Insulating panel*
Constraints	■ *Sufficient stiffness to suppress vibration and support internal loads* ■ *Low cost* ■ *Protect against environment* ■ *Not too thick*
Objective	■ *Minimize heat transfer through thickness*
Free variables	■ *Material for faces and core; their relative thicknesses*

The method and results. We select a hybrid of type 'sandwich' and explore how the performance of various combinations of face and core compare with each other and with monolithic materials. The quantity $\tilde{E}_{flex}$ for the sandwich is given by Eq. (12.35) and its through-thickness thermal conductivity is given by Eq. (12.41). Simplified equations describing them are assembled in Table 13.10, in which t is the face sheet thickness, d the panel thickness, E_f the modulus of the face material, λ_f and λ_c the conductivities of face and core, and K_s is the knock-down factor to allow for core shear, ideally equal to 1 (no shear) but potentially as low as 0.5.

Table 13.10

Superposition Rules for Sandwich Stiffness and Conductivity

Property	Lower Bound	Upper Bound
Flexural modulus	$\tilde{E}_{flex} = \left(1 - \left(1 - \frac{2t}{d}\right)^3\right) E_f K_s$	$\tilde{E}_{flex} = \left(1 - \left(1 - \frac{2t}{d}\right)^3\right) E_f$
Through-thickness conductivity	$\tilde{\lambda}_\perp = \left(\frac{2t/d}{\lambda_f} + \frac{(1-2t/d)}{\lambda_c}\right)^{-1}$ *(exact)*	

Fig. 13.8 shows the appropriate plot, using the thermal conductivity $\tilde{\lambda}_\perp$ and the flexural compliance $1/\tilde{E}_{flex}$ instead of its inverse so that a minimum is sought for both quantities. The trade-off surface shown in blue identifies the monolithic materials that offer the best combinations of stiffness and insulation. The approximate performance of a sandwich hybrid with mild steel faces and a core of rigid foamed PVC is plotted using the equations in the table for four values skin-to-core thickness; all other values are contained in the blue band. The panel offers combinations of stiffness and insulation that that are far (by a factor of 10) better than the best monolithic metals, composites, polymers or foams. Other combinations of face and core (aluminium or SMC faces with foamed polystyrene core, for example) can be evaluated using the same chart.

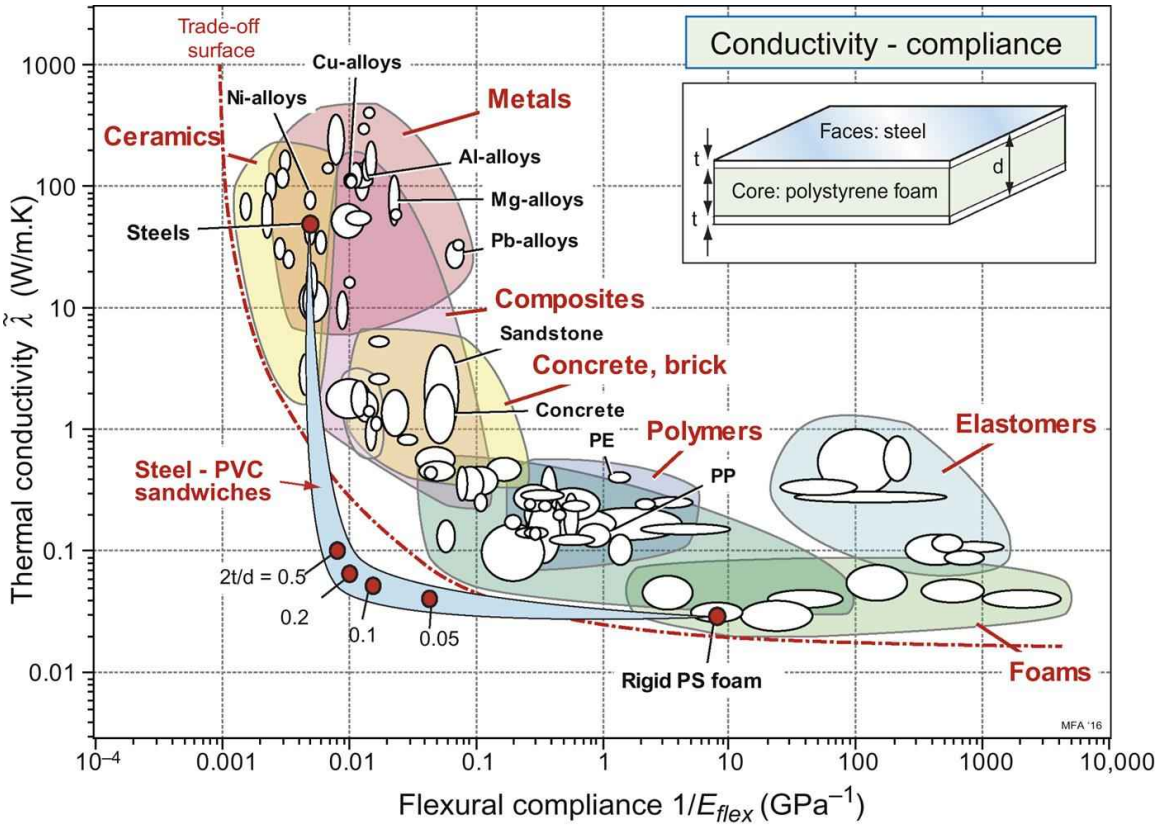


FIGURE 13.8 The broken line tracks the performance of steel-polystyrene foam bonded sandwiches. The thermal performance is plotted on the vertical axis, the mechanical performance on the horizontal one. Both are to be minimized.

Postscript. Adhesive technology has advanced rapidly over the last 2 decades. Adhesives are now available to bond almost any two materials, and to do so with high bond-strength (though some adhesives are expensive). Fabricating the sandwich should not be a problem.

Related Case Studies

5.14 'Energy-efficient kiln walls'

13.9 'Materials for microwave-transparent enclosures'

13.8 Materials for Microwave-Transparent Enclosures

Microwave-transparent radomes were introduced in Chapter 5, Materials Selection – Case Studies, Section 5.19. The radome is a thin panel or shell, requiring flexural stiffness and strength, but also requiring as low a

dielectric constant, ε_r, as possible. Could hybrids offer better performance than monolithic materials?

The method and results. Sandwich structures offer flexural stiffness and strength, and allow some control of electrical properties. We therefore explore these, seeking to meet the requirements of Table 13.11.

Table 13.11

Requirements for Low Dielectric Constant Radome Skin

Function	■ *Material for protection of microwave detector*
Constraints	■ *Must meet constraints on flexural strength* σ_{flex}
Objective	■ *Minimize dielectric constant* ε_r
Free variables	■ *Choice of material for face and core* ■ *Relative thickness of the two materials*

Fig. 13.9 shows the flexural strength, σ_{flex}, of foams, polymers and ceramics, plotted against the dielectric constant ε_r. Many polymers have dielectric constants between 2 and 5. The dielectric constant of the foam falls linearly with the relative density, approaching 1 at low densities (Eq. 12.26):

$$\tilde{\varepsilon}_r = 1 + (\varepsilon_{r,s} - 1)\left(\frac{\tilde{\rho}}{\rho_s}\right) \tag{13.5}$$

where $\varepsilon_{r,s}$ is the dielectric constant of the solid of which the foam is made and $\tilde{\rho}/\rho_s$ is its relative density. Foams, however, are not very strong. Glass fiber reinforced polyester (GFRP), with a dielectric constant of 5, is much stronger. Based on a survey of possible faces and cores from among those plotted in Fig. 13.9, we choose to explore a sandwich with faces of GFRP and a core of a low/medium density (LD/MD) rigid polymer foam. The flexural strength of the sandwich, provided it is properly manufactured and the core material has sufficient strength, is (Eq. 12.36)

$$(\tilde{\sigma}_{flex})_U = (1 - (1-f)^2)\sigma_f + (1-f)^2\sigma_c \tag{13.6}$$

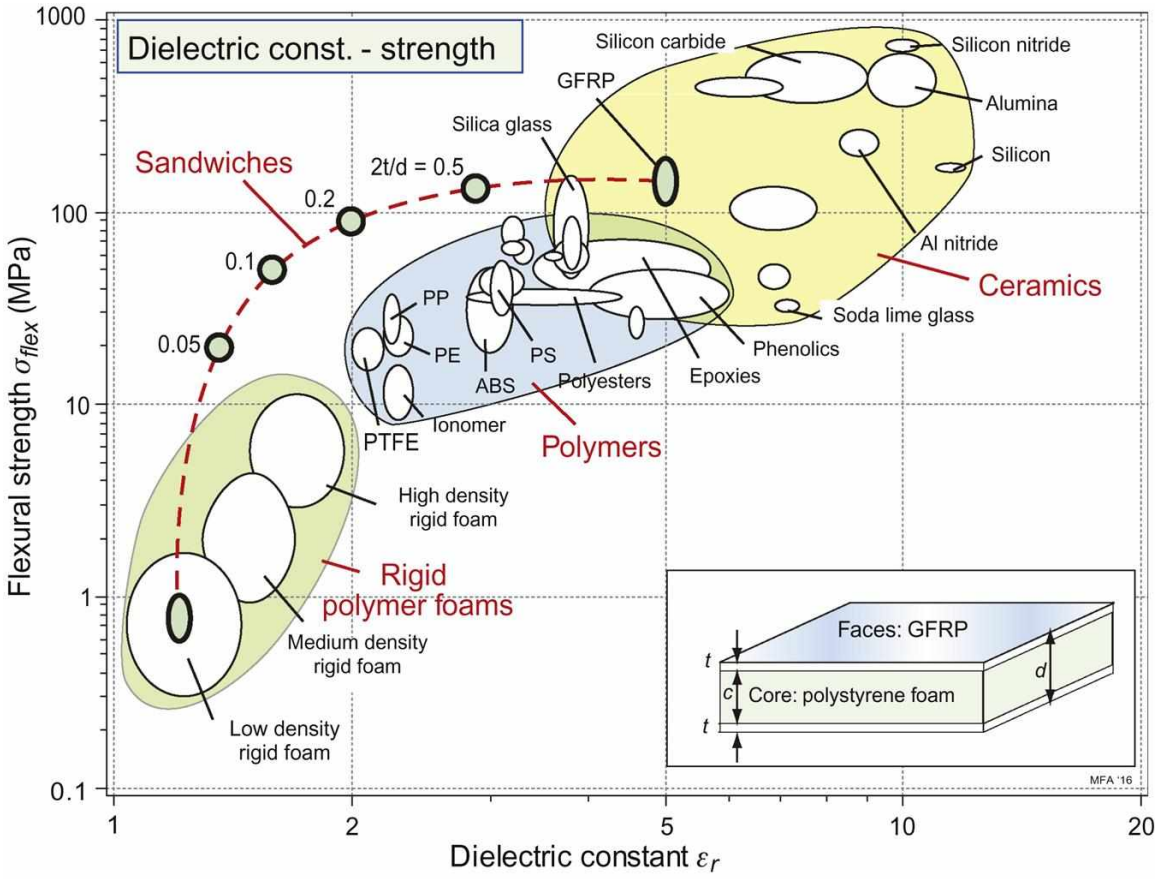


FIGURE 13.9 A plot of flexural modulus and dielectric constant for low dielectric constant materials. The trajectory shows the possibilities offered by hybrids of GFRP and polymer foam.

The dielectric constant for the panel (Eq. (12.15) with f=$2t/d$) is

$$\tilde{\varepsilon}_r = f\varepsilon_f + (1-f)\varepsilon_c \qquad \textbf{(13.7)}$$

The simplest way to explore these is to plot the two, using $f = 2t/d$ as a parameter to link them. Fig. 13.9 shows the results. The sandwich allows the creation of a set of materials with combinations of flexural strength and dielectric constant that outperform all the homogeneous materials that appear on the figure – indeed it outperforms even the best composites (not shown here). The figure is used by identifying the desired flexural strength, σ_{flex}, and reading off the values of $2t/d$ and dielectric constant.

Further Reading

Huddleston, G.K., Bassett, H.L., In: Johnson, R.D., Jasik, H. (Eds.), Antenna Engineering Handbook, second ed., McGraw-Hill, New York, NY (Chapter 44).

Lewis, C.F., 1998. Materials keep a low profile. Mech. Eng. 37–41.

Related Case Studies

5.19 'Materials for radomes'

13.9 Connectors that Don't Relax Their Grip

There are kilometres of wiring in a car. The transition to drive-by-wire control systems will increase this further. Wires have ends; they don't do much unless the ends are connected to something. The connectors are the problem: they loosen with time until, eventually, connection is lost.

Carmakers, responding to market forces, now design cars to run for at least 300,000 kilometres and last, on average, 10 years. The electrical system is expected to operate *without servicing* for the lifetime of the car. Its integrity is vital: you would not be happy in a drive-by-wire car with loose connectors. With increasing instrumentation on engine and exhaust systems, many of the connectors get hot; some have to maintain good electrical contact at temperatures up to 200°C.

The primary material choice for connectors, today, is a copper–beryllium alloy, Cu 2% Be: it has excellent conductivity and the high strength needed to act as a spring to give the required clamping force at the connection. But the maximum long-term service temperature of copper–beryllium alloys is only about 130°C. At higher temperatures, creep relaxation causes the connector to loose its grip. The challenge: to suggest a way to solve this problem (Table 13.12).

Table 13.12
Design Requirements for the Connector

Function	■ *Hybrid conductor for electrical connector*
Constraints	■ *Provide good electrical connection* ■ *Maintain clamping force at 200°C for life of vehicle*
Objective	■ *Minimize cost*
Free variables	■ *Material 1 and 2; their relative thicknesses*

The method and results. The answer is to separate the functions, select the best material for each, check for compatibility, and combine them. So here goes.

- Function 1: conduct electricity. Copper excels at this; no other affordable material is as good. Its alloys (among them, copper–beryllium) are stronger, but at the cost of some loss of conductivity and a great increase in price. We choose copper to provide the conduction.
- Function 2: provide clamping force for the lifetime of the vehicle. The material chosen to fill function 2 will have to be bonded to the copper, and if the combination is not to distort when heated, it must have the same expansion coefficient.

Figs. 13.10 and 13.11 guide the choice. The first shows the maximum service temperature and expansion coefficient for copper, Cu 2% Be, and a range of steels. The box encloses materials with the same expansion coefficient as copper. Type 302 and 304 austenitic stainless steels match copper in expansion coefficient, and can be used at much higher temperatures. But do they make good springs? And are they affordable? The second chart answers these questions. Good materials for springs (Chapter 5, Materials Selection – Case Studies, Section 5.7) are those with high values of σ_f^2/E – this appears as one axis of the chart. The other axis is approximate price/kg. The chart shows that both 302 and 304 stainless steels, in the wrought condition, are almost as good as Cu 2% Be as a spring, and considerably cheaper.

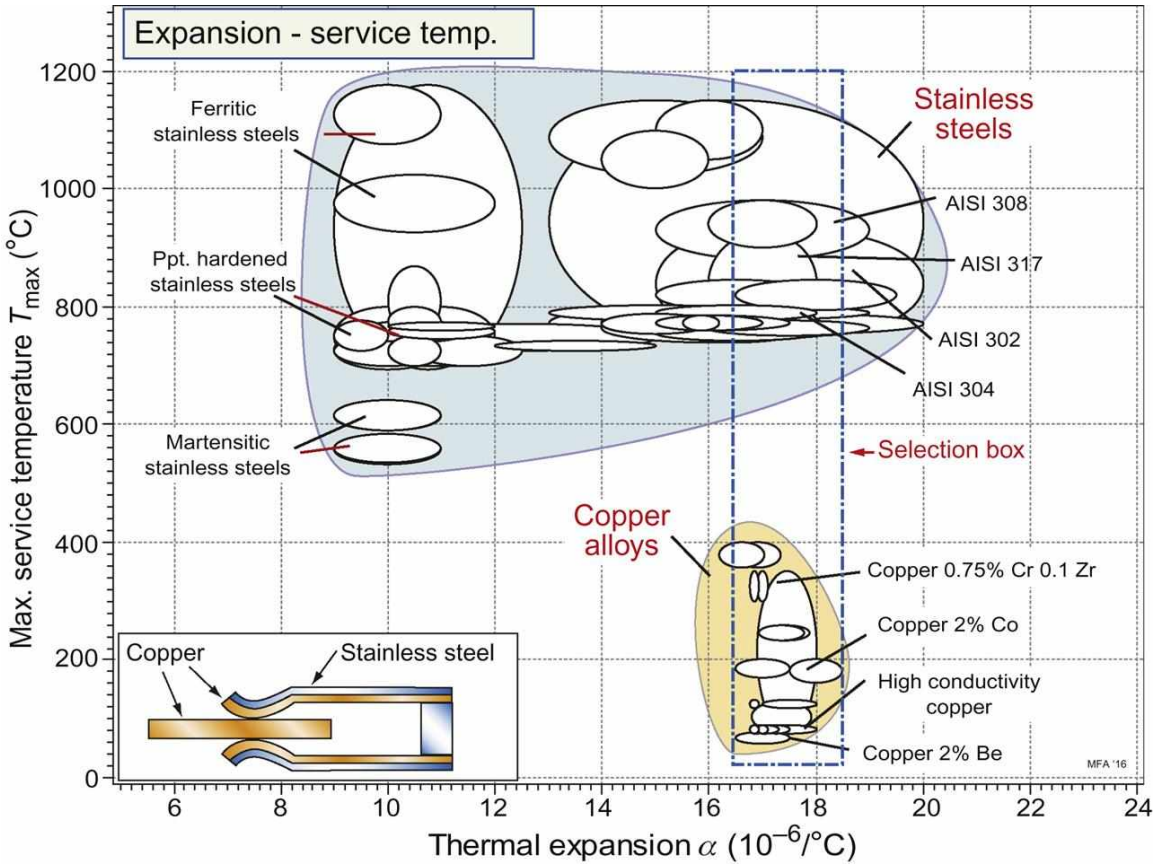


FIGURE 13.10 A hybrid connector. We seek materials with matching thermal expansion, one of which retains strength and stiffness well above 200°C. Copper is chosen for Material 1 because of its excellent electrical conductivity. Austenitic stainless steel is a good choice for Material 2.

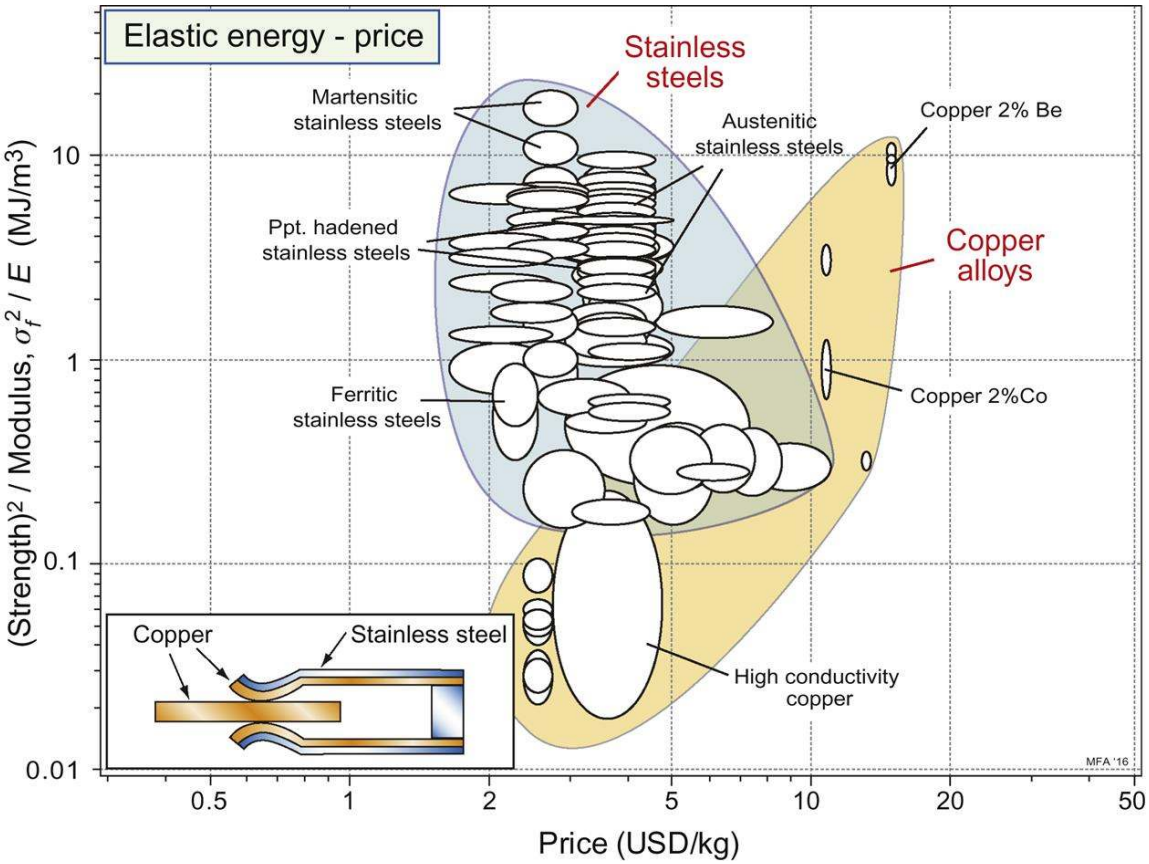


FIGURE 13.11 The connector has two jobs – to conduct and to exert a clamping force that does not relax. High conductivity copper and austenitic stainless steel are both much cheaper than Cu 2% Be.

The proposed solution, then, is a hybrid of copper and type 302 stainless steel, roll-bonded to form a bi-layer like that shown in the inset on both figures. Further detailed design will, of course, be needed to establish the thicknesses of each layer, the best degree of cold work, the formability and the resistance to the environment in which it will be used. But the method has guided us to a sensible concept, quickly and efficiently.

Related Case Studies

5.7 'Materials for springs'

11.8 'Ultra-efficient springs'

13.10 Exploiting Anisotropy: Heat-Spreading Surfaces

A saucepan made from a single material, when heated on an open flame, develops hot spots that can locally burn its contents. That is because the saucepan is thin; heat is transmitted through the thickness more quickly than it can be spread transversely to bring the entire pan surface to a uniform temperature. The metals of which saucepans are usually made – cast iron, aluminium, stainless steel or copper – have thermal conductivities that are isotropic, the same in all directions. What we clearly want is a thermal conductivity that is higher in the transverse direction than in the through-thickness direction. A bi-layer (or multilayer) hybrid can achieve this. Table 13.13 summarizes the situation.

Table 13.13

Design Requirements for the Panel

Function	■ *Heat-spreading surface*
Constraints	■ *Temperature up to 200°C without distortion*
Objective	■ *Maximize thermal anisotropy while maintaining good conduction*
Free variables	■ *Choice of materials and their relative thicknesses*

The method and results. Heat transmitted in the plane of a bi-layer (Fig. 13.10, insert) has two parallel paths; the total heat flux is a sum of that in each of the paths. If it is made of a layer of material 1 with thickness t_1 and conductivity λ_1, bonded to a layer of material 2 with thickness t_2 and conductivity λ_2, the conductivity parallel to the layers is

$$\tilde{\lambda}_{//} = f\lambda_1 + (1-f)\lambda_2 \qquad \textbf{(13.8)}$$

Eq. (12.12) with $f=t_1/(t_1+t_2)$. Perpendicular to the layers the conductivity is

$$\tilde{\lambda}_{\perp} = \left(\frac{f}{\lambda_1} + \frac{(1-f)}{\lambda_2}\right)^{-1} \qquad \textbf{(13.9)}$$

(the harmonic mean of Eq. (12.41)).

If the bi-layer is made of materials that differ greatly in thermal expansion coefficient, the pan will distort when heated. We therefor seek a

pair of materials with almost the same expansion coefficient, but that differ as much as possible in conductivity in order to maximize the difference between Eqs (13.8) and (13.9).

The results. Copper is an excellent thermal conductor. We saw, in Case Study 13.9, that the thermal expansion of copper matches that of 302 or 304 stainless steel. Their thermal conductivities, however, differ. Fig. 13.12 shows the conductivities $\tilde{\lambda}_{\perp}$ and $\tilde{\lambda}_{//}$ of the bi-layer as a function of the relative thickness f for a bi-layer of copper (λ_1=390 W/m·K) and austenitic stainless steel (λ_2=16 W/m·K). The maximum separation between $\tilde{\lambda}_{\perp}$ and $\tilde{\lambda}_{//}$ occurs broadly at f=0.5 (each material occupies about half the thickness) when the ratio of the two conductivities (the anisotropy ratio) is 6.7. The hybrid has extended the occupied area of property space along an unusual dimension – that of thermal anisotropy.

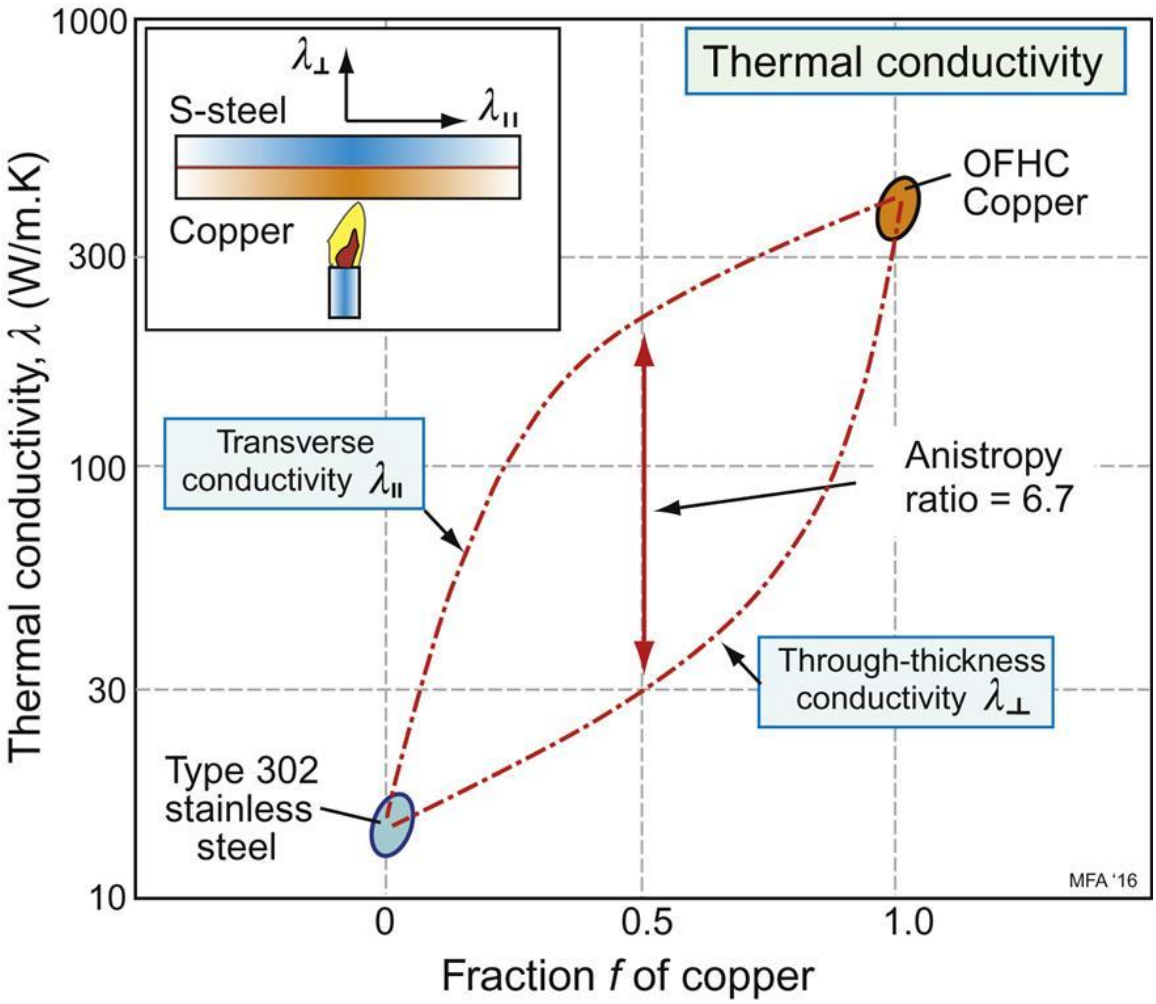


FIGURE 13.12 Creating anisotropy. A bi-layer of copper and stainless steel creates a 'material' with good conductivity and an anisotropy ratio greater than 6.

Related Case Studies

5.15 'Materials for heat-storing walls'
13.9 'Connectors that don't relax their grip'

13.11 The Mechanical Efficiency of Natural Materials

'As a general principle natural selection is continually trying to economise every part of the organisation', wrote Charles Darwin some 150 years ago. Natural materials are remarkably efficient. By efficient we mean that they fulfil the complex requirements posed by the way plants and animals function and

that they do so using as little material as possible. Many of these requirements are mechanical in nature: the need to support static and dynamic loads created by the mass of the organism or by wind loading, the need to store and release elastic energy, the need to flex through large angles, and the need to resist buckling and fracture.

Virtually all natural materials are hybrids. They consist of a relatively small number of polymeric and ceramic components or building blocks that often are composites themselves. Plant cell walls, for instance, combine cellulose, hemicellulose and pectin, and can be lignified; animal tissue consists largely of collagen, elastin, keratin, chitin and minerals such as salts of calcium or silica. From these limited 'ingredients' nature fabricates a remarkable range of structured hybrids. Wood, bamboo and palm consist of cellulose fibres in a lignin/hemicellulose matrix, shaped to hollow prismatic cells of varying wall thickness. Hair, nail, horn, wool, reptilian scales and hooves are made of keratin while insect cuticle contains chitin in a matrix of protein. The dominant ingredient of mollusc shell is calcium carbonate, bonded with a few percent of protein. Dentine, bone and antler are formed of 'bricks' of hydroxyapatite cemented together with collagen. Collagen is the basic structural element for soft and hard tissues in animals, such as tendon, ligament, skin, blood vessels, muscle and cartilage, often used in ways that exploit shape.

From a mechanical point of view, there is nothing very special about the individual building blocks. Cellulose fibres have Young's moduli that are about the same as that of nylon fishing-line, but a lot less than steel; and the lignin-hemicellulose matrix in which they are embedded has properties very similar to that of epoxy. Hydroxyapatite has a fracture toughness comparable with that of manufactured ceramics, that is to say: it is brittle. It is thus the *configuration* of the components that give rise to the striking efficiency of natural materials.

The complex hierarchies of configurations that make up wood and bone are illustrated in Figs. 13.13 and 13.14. The stiffness, strength and toughness of wood derive largely from those of the cellulose molecule, shown on the left of Fig. 13.13. Crystalline microfibrils are built up of aligned molecules some 30–60 nm in length. These form the reinforcing fibers of lamellae, the matrix of which is amorphous lignin and hemicellulose. Stacks of lamellae in the four-layer pattern and fiber orientation shown on the right become the structural material of the cell wall. The fibrils of the primary cell wall interweave randomly, as in cotton wool. In subsequent layers the fibrils are parallel and closely packed. The

outer S_1 layer has lamellae with alternate right and left-handed spiral winding of fibrils. Beneath it, the thicker S_2 layer has fibrils that are more nearly oriented along the axis of the cell. The innermost S_3 layer has a lay-up like that of S_1. The cell as a whole is bonded to its neighbors by the middle lamella (not shown), a lignin-pectin complex devoid of cellulose.

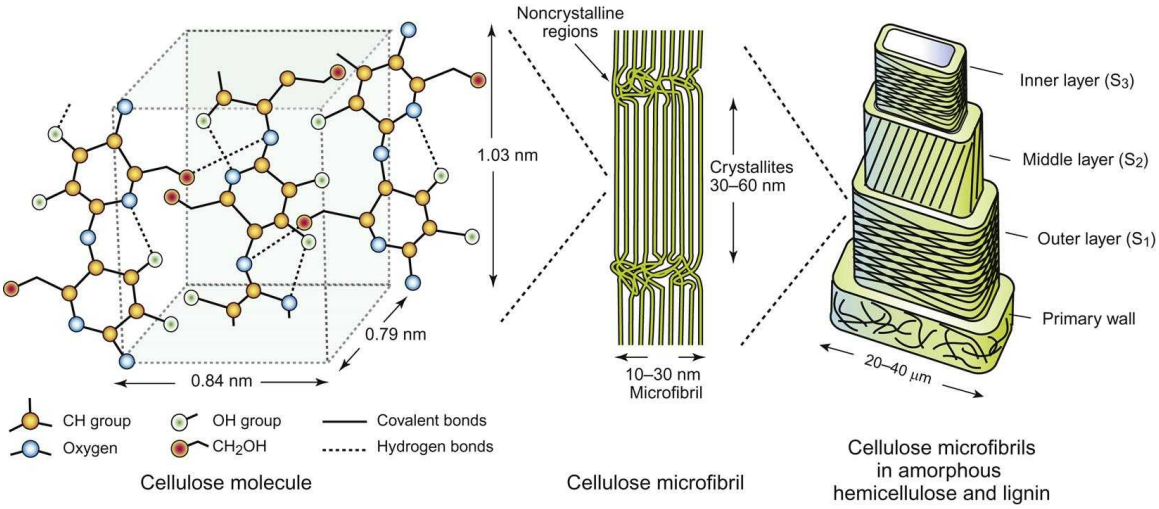


FIGURE 13.13 The hierarchical structure of wood.

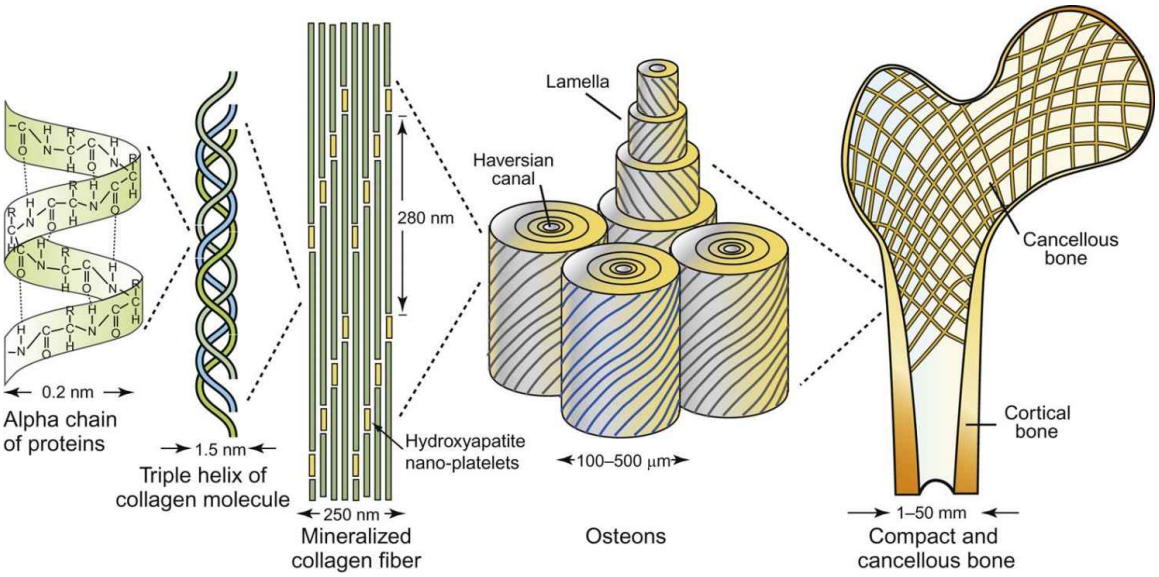


FIGURE 13.14 The hierarchical structure of bone.

There is an interesting parallel between the hierarchical structure of bone and that of wood, despite the great differences in their molecular chemistry (Fig. 13.14). The starting point here is the triple-helical structure of the collagen molecule, shown on the left. But – unlike wood – this becomes the matrix, not the reinforcement of the mineralized tissue of bone. Hydroxyapatite nanoplatelets deposit in the nascent tissue and increase in volume fraction over time to give the mature osteons, with an ordered lay-up of highly mineralized fibers with the strength and stiffness to support structural loads that bone must carry in a mature organism. At the most macro scale, there is an outer shell of fully dense compact bone containing reinforcement of highly porous trabecular (or cancellous) bone.

Like engineering materials, the building blocks of natural materials can be grouped into classes: *bio-ceramics* (calcite, aragonite, hydroxyapatite), *bio-polymers* (the organic building blocks: polysaccharides cellulose and the proteins chitin, collagen, silk and keratin) and *natural elastomers* (elastin, resilin, abductin, skin, artery and cartilage). These combine to give a range of hybrids, among them composites and sandwiches (bone, antler, enamel, dentine, shell, cuttle and coral), cellular structures (natural cellular materials like wood, cork, palm, bamboo, trabecular bone) and segmented structures (scales, hair). Their properties, like those of engineering materials, can be explored and compared using material property charts. We conclude this chapter by examining charts for natural hybrids and the building blocks from which they are made.

The Young's modulus– density chart. Fig. 13.15 shows data for Young's modulus, E, and density, ρ. Those for the classes of natural materials are circumscribed by large envelopes; class members are shown as smaller bubbles within them. Data for woods, palms, corks, trabecular (foam-like) bone and corals are enclosed in subsidiary envelopes. Natural fibres (silk, flax, jute and the like) have their own envelope, as do mineralized tissue (bone, shell etc.) and soft tissue (ligament, cartilage and so forth). Many are anisotropic. When they are, the moduli parallel (symbol $||$) and perpendicular (symbol $\perp$) to the fibre orientation or grain have been plotted separately. Trabecular bone exhibits a particularly wide range of densities; for these, bubbles for high density (HD), MD and LD are plotted. The familiar stiffness guidelines E/ρ, $E^{1/2}/\rho$ and $E^{1/3}/\rho$ are shown, each representing the material index for a particular mode of loading.

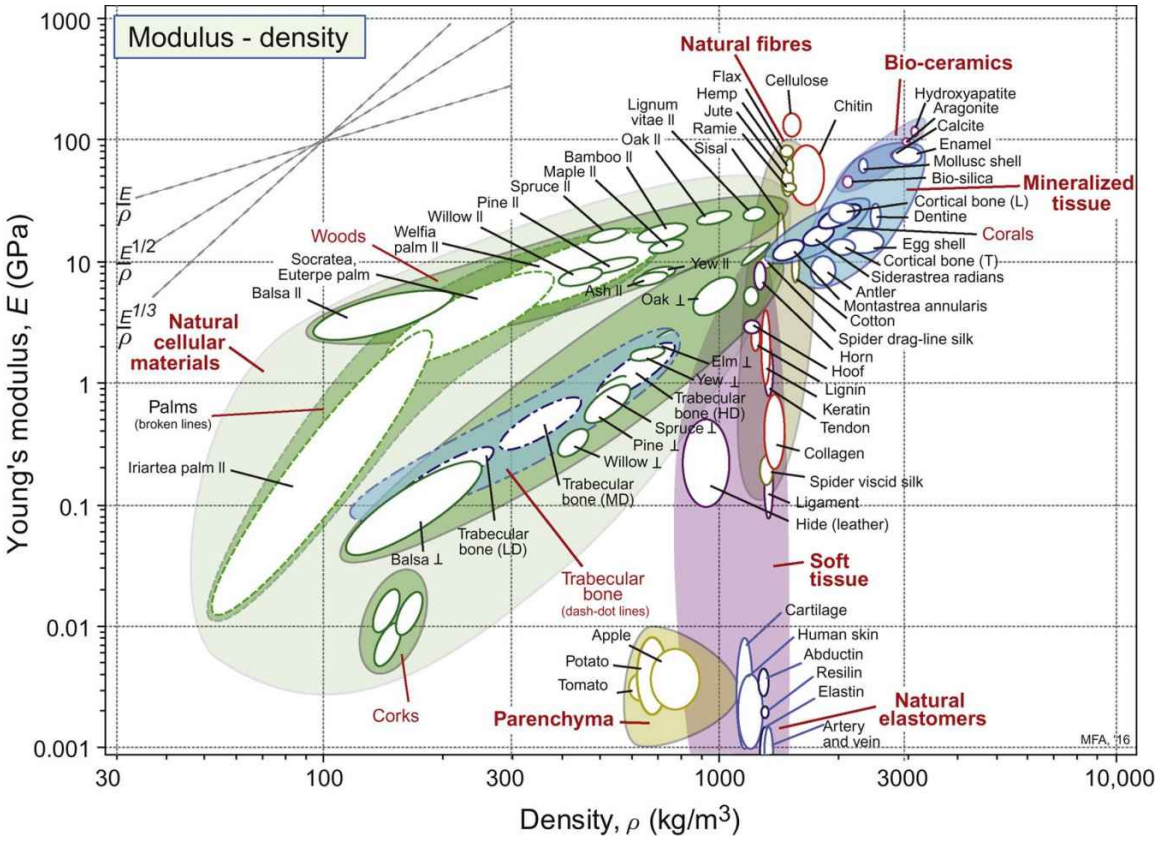


FIGURE 13.15 A material property chart for natural materials, plotting Young's modulus against density. Guidelines identify structurally efficient materials that are light and stiff.

The natural polymer with the highest efficiency in tension, measured by the index E/ρ, is cellulose; it exceeds that of steel by a factor of about 2.6. The high values for the fibres flax, hemp and cotton derive from this. Wood, palm and bamboo are particularly efficient in bending and resistant to buckling, as indicated by the high values of the flexure index $E^{1/2}/\rho$ when loaded parallel to the grain. That for balsa wood, for example, can be 5 times greater than that of steel.

The tensile strength–density chart. Data for the strength, σ_f, and density, ρ, of natural materials are shown in Fig. 13.16. The colour coding and envelope scheme parallels that of Fig. 13.15. For natural ceramics the tensile strength is identified with the flexural strength (modulus of rupture) in beam bending, symbol. For natural polymers and elastomers, the strengths are tensile strengths. In addition, for natural cellular materials, the tensile stress is either the plateau stress or the flexural strength, symbol (T), depending on the nature of the material. Where they

differ, the strengths parallel (symbol ||) and perpendicular (symbol ⊥) to the fibre orientation or grain have been plotted separately.

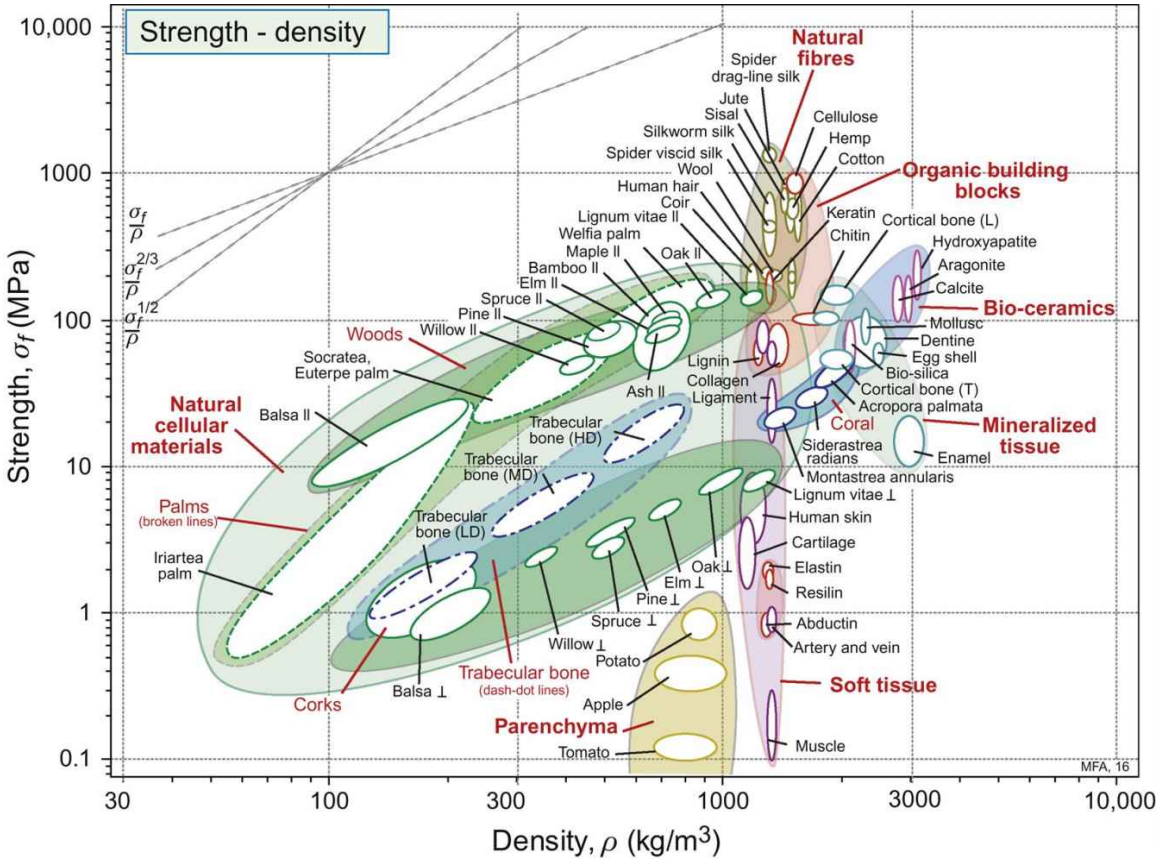


FIGURE 13.16 A material property chart for natural materials, plotting strength against density. Guidelines identify structurally efficient materials that are light and strong.

Evolution to provide tensile strength would – we anticipate – lead to materials with high values of σ_f/ρ. Where strength in bending or buckling is required we expect to find materials with high $\sigma_f^{2/3}/\rho$. Silk and cellulose have the highest values of σ_f/ρ; that of silk is even higher than that of carbon fibres. The fibres of flax, hemp and cotton, too, have high values of this index. Bamboo, palm and wood have high values of $\sigma_f^{2/3}/\rho$ giving resistance to flexural failure.

The Young's modulus–strength chart. Data for the strength σ_f and modulus E of natural materials are shown in Fig. 13.17. Two combinations are significant. Materials with large values of σ_f^2/E store elastic energy and

make good springs; those with large values of σ_f/E have exceptional resilience. Both criteria are plotted on the figure. Silks (including the silks of spider's webs) stand out as exceptionally efficient, having values of σ_f^2/E that exceed those of spring steel or rubber. High values of the other index, σ_f/E, mean that a material allows large, recoverable deflections and, for this reason, make good elastic hinges. Palm (coconut timber) has a higher value of this index than wood, allowing palms to flex in a high wind. Nature makes much use of these: skin, leather and cartilage are all required to act as flexural and torsional hinges.

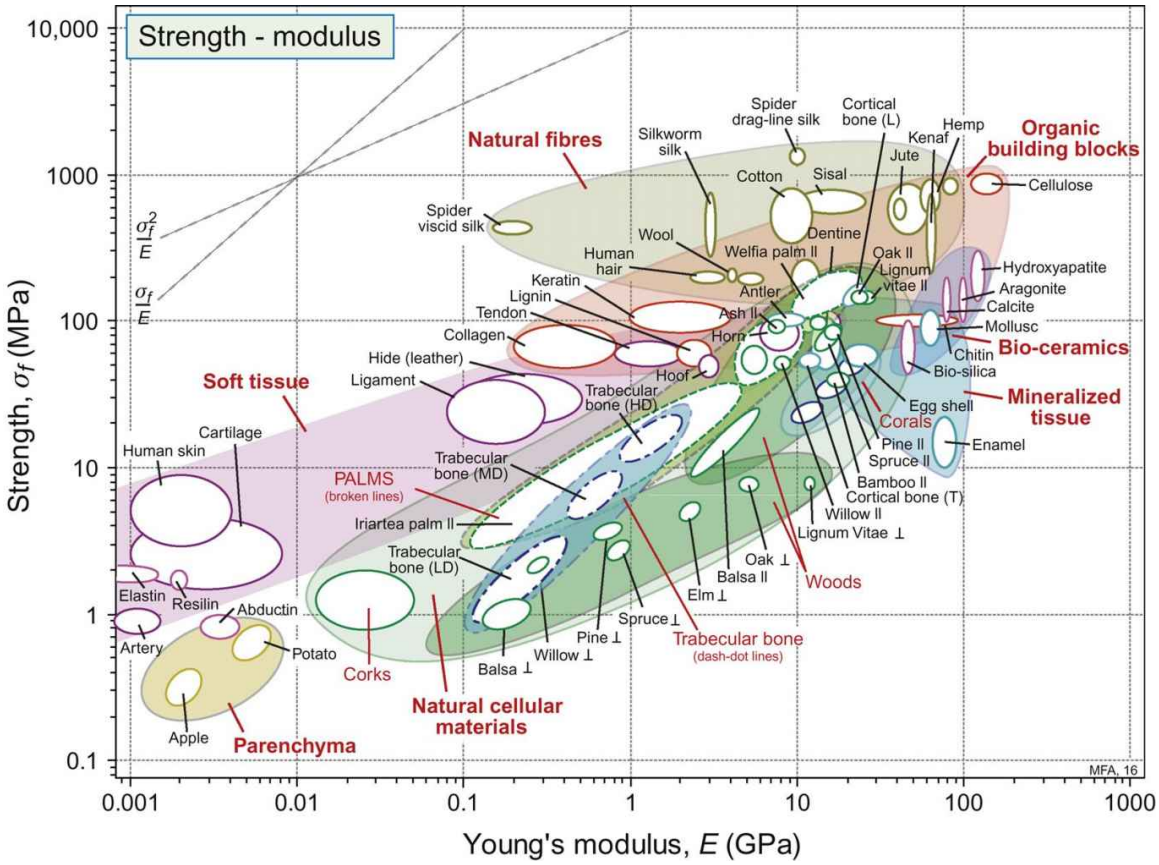


FIGURE 13.17 A material property chart for natural materials, plotting Young's modulus against strength. Guidelines identify materials that store the most elastic energy per unit volume and that make good elastic hinges.

The toughness–Young's modulus chart. The toughness of a material measures its resistance to the propagation of a crack. The limited data for the toughness, J_c, and Young's modulus, E, of natural materials are shown

in Fig. 13.18. When the component is required to absorb a given *impact energy* without failing, the best material are those with the largest value of J_c. These materials lie at the top of Fig. 13.18: antler, hoof, horn, bamboo and woods stand out. When instead a component containing a crack must carry a given *load* without failing, the safest choice of material is that with the largest values of the fracture toughness $K_{1c} \approx (EJ_c)^{1/2}$. Diagonal contours sloping from the upper left to lower right on Fig. 13.18, show values of this index. Mollusc shell and tooth enamel stand out.

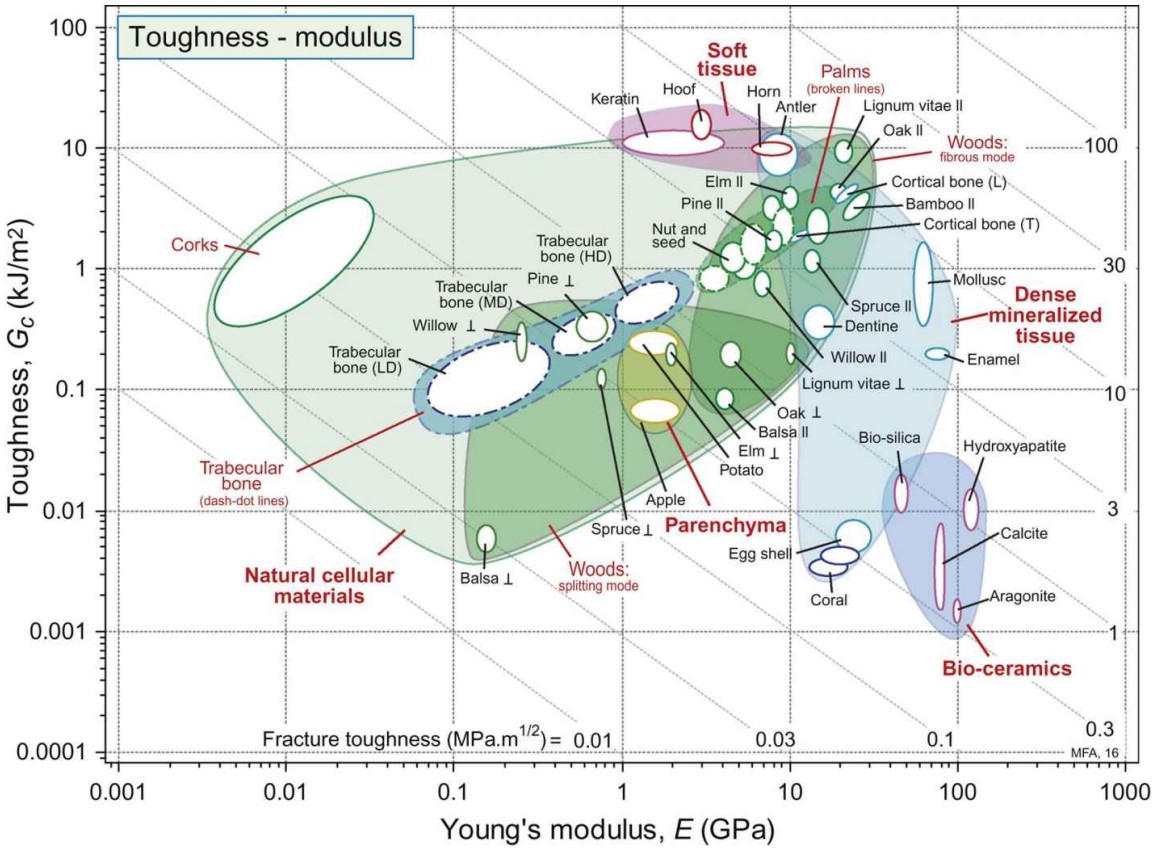


FIGURE 13.18 A material property chart for natural materials, plotting toughness against Young's modulus. Guidelines show fracture toughness $(EJ_c)^{1/2}$ (MPa)$^{1/2}$.

Many engineering materials – steels, aluminium, alloys – have values of J_c and K_{1c} that are much higher than those of the best natural materials. However, the toughness values of natural ceramics like nacre, dentine, cortical (dense) bone and enamel are an order of magnitude higher than those of conventional engineering ceramics like alumina. Their toughness

derives from their segmented structure: platelets of ceramics such as calcite, hydroxyapatite or aragonite, bonded by a small volume fraction of polymer, usually collagen; it increases with decreasing mineral content and increasing collagen content.

Postscript. The charts bear out Darwin's idea that natural materials evolve to make the most of what is available to them – they are efficient, in the sense used earlier. But there is more to it than that. For deeper insight, see the books listed in Further Reading.

Further Reading: Natural Materials

There is a growing literature surveying the mechanical and thermal properties of natural materials. The books listed below provide a good, broad introduction.

1. Beukers A, van Hinte E. *Lightness The Inevitable Renaissance of Minimum Energy Structures* Rotterdam: 010 Publishers; 1998; ISBN 9064503346.
2. Currey JD, Wainwright SA, Biggs WD. *Mechanical Design in Organisms* Princton: Princeton University Press; 1982; ISBN 0691083088.
3. Gibson LJ, Ashby MF. *Cellular Solids* Second ed. Cambridge, UK: Cambridge University Press; 1997; ISBN 0-521-49560-1. (*A broad-based introduction to the structure and properties of foams and cellular solids of all types*.).
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5. McMahon T, Bonner J. *On Size and Life* New York, NY: American Books; 1983; ISBN 0716750007.
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8. Vincent JFV. *Structural Biomaterials* Revised ed. Princeton: Princeton University Press; 1990; ISBN: 0691025134.
9. Vincent JFV, Currey JD. *The mechanical properties of biological materials*. *Proceedings of the Symposia of the Society for Experimental*

Biology. vol. 34 Cambridge: Cambridge University Press for the Society for Experimental Biology; 1980.
10. Wegst UGK, Ashby MF. The mechanical efficiency of natural materials. *Philos Mag.* 2004;84:2167–2181.

[1]Spectra PE, VHM carbon, Nicalon SiC and Nextel Al_2O_3.

CHAPTER 14

Materials and the Environment

Abstract

The manufacture and use of products, with their associated consumption of materials and energy, has an impact on the environment in which we live. *Design for the environment* is generally interpreted as the effort to adjust our present design methods to correct known, measurable, environmental degradation; the time-scale of this thinking is 10 years or so, an average product's expected life. *Sustainable development* is the longer view: adaptation to a lifestyle that meets present needs without compromising the needs of future generations. The time-scale here is less clear – it is measured in decades or centuries – and the adaptation required is much greater. This chapter focuses on the role of materials and processes in achieving design for the environment. We return to the broader issue of sustainability in Chapter 16, Sustainable Response to Forces for Change.

Keywords

Material life-cycle; eco-attributes of materials; embodied energy; carbon footprint; life-cycle assessment; eco-audits; energy fingerprints; transport; end of life; recycling; reconditioning; reuse; combustions; Landfill

14.1 Introduction and Synopsis

All human activity has some impact on the environment in which we live. The environment has some capacity to cope with this, so a certain level of impact can be absorbed without lasting damage. But it is clear that current human activities exceed this threshold with increasing frequency,

diminishing the quality of the world in which we now live and threatening the wellbeing of future generations. The manufacture and use of products, with their associated consumption of materials and energy, are among the culprits. The position is dramatized by the following statement: at a global growth rate of 3% per year we will mine, process and dispose of more 'stuff' in the next 25 years than in the entire industrial history of mankind. *Design for the environment* is generally interpreted as the effort to adjust our present design methods to correct known, measurable, environmental degradation. The time-scale of this thinking is 10 years or so, an average product's expected life. *Sustainable development* is the longer view: that of adaptation to a lifestyle that meets present needs without compromising the needs of future generations. The time-scale here is less clear – it is measured in decades or centuries – and the adaptation required is much greater. This chapter focuses on the role of materials and processes in achieving design for the environment. We return to the broader issue of sustainability in Chapter 16, Sustainable Response to Forces for Change.

14.2 The Material Life-Cycle

The nature of the problem is brought into focus by examining the materials life-cycle, sketched in Fig. 14.1. Ore and feedstock, most of them nonrenewable, are processed to give materials; these are manufactured into products that are used, and, at the end of their lives, retired, a fraction perhaps entering a recycling loop, the rest committed to incineration or landfill. Energy and materials are consumed at each point in this cycle (we shall call them 'phases'), with an associated penalty of CO_2 and other emissions – heat, gaseous, liquid and solid waste.

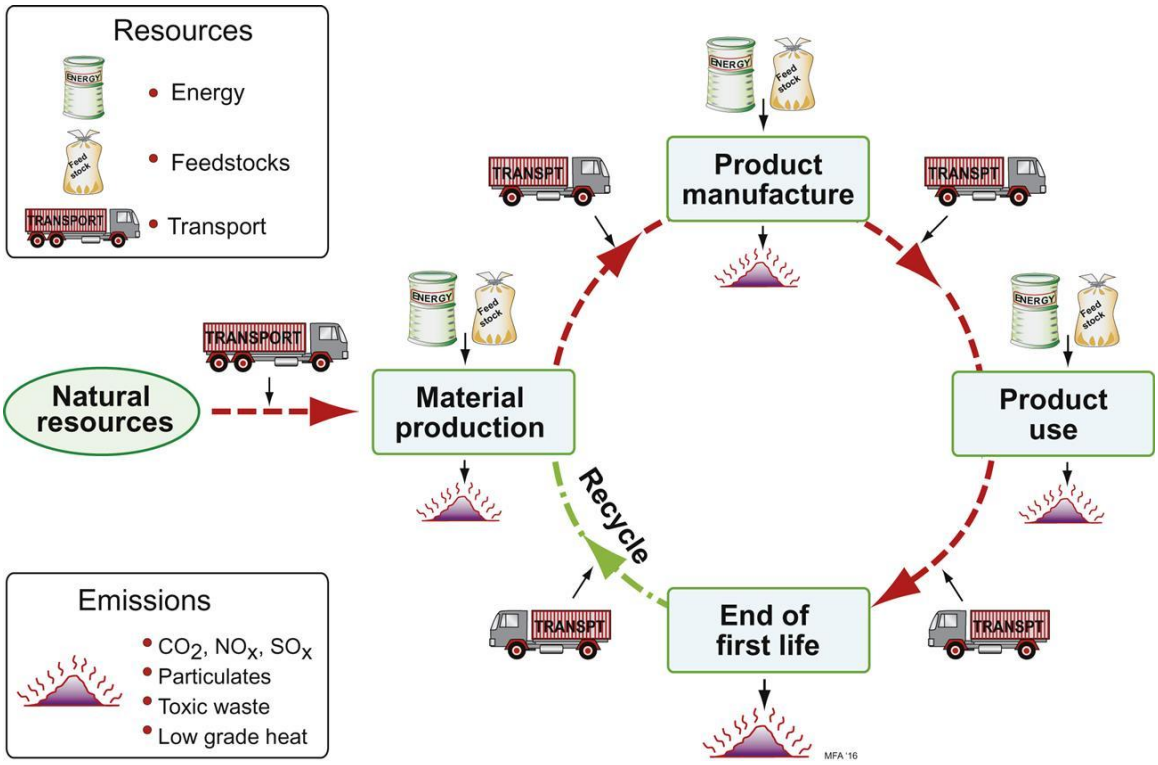


FIGURE 14.1 The material life-cycle.

The reasoning of this chapter goes like this. First explore the resource consumption and emissions associated with each of the phases of life. Identify the phase that is guilty of making the greatest impact – because reducing this will be the most effective way to reduce the total. Then use the material selection methods developed earlier in this book to seek a substitute or a redesign of a more far-reaching kind to reduce the impact of the culprit.

Eco-systems can be complex; changing one part can have unanticipated effects on other parts. So before we start on the main argument, here is a quick reminder of the interactivity of complex systems.

14.3 Material and Energy-Consuming Systems

It would seem that the obvious ways to conserve materials is to make products smaller, make them last longer, and recycle them when they finally reach the end of their lives. But the seemingly obvious can sometimes be deceptive. Materials and energy form part of a complex and

highly interactive system, of which Fig. 14.2 is a cartoon. Here primary catalysts of consumption such as *increasing affluence, population growth, new technology* and *planned obsolescence* influence aspects of product use and through these, the consumption of materials and energy and the by-products that these produce. The connecting lines indicate influences; a green line suggests positive, broadly desirable influence; a red line suggests negative, undesirable influence and red-green suggests that the driver has the capacity for both positive and negative influence.

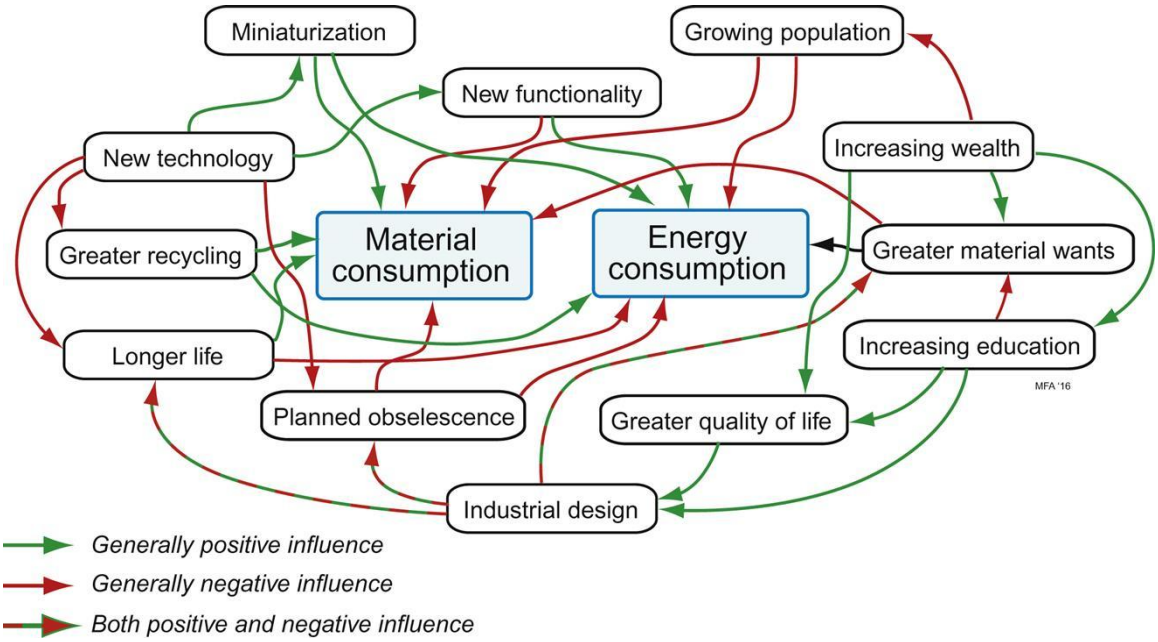


FIGURE 14.2 The influences on consumption of materials and energy. It is essential to see eco-design as a systems problem, not solved by simple choosing 'good' and avoiding 'bad' materials, but rather that of matching the material to the system requirements.

The diagram brings out the complexity. Follow, for instance the lines of influence of *new technology* and its consequences. It offers more material and energy-efficient products, but by also offering new functionality it creates obsolescence and the desire to replace a product that has useful life left in it. Electronic products are prime examples of this: most are discarded while still functional. And observe, even at this simple level, the consequences of *longer life* – a seemingly obvious measure. It can certainly help to conserve materials (a positive influence) but, in an era in which

new technology delivers more energy-efficient products (particularly true of cars, electronics and household appliances today), extending the life of old products can have a negative influence on energy consumption and emissions.

As a final example, consider the bivalent influence of industrial design – the subject of Chapter 15, Materials and Industrial Design. The lasting designs of the past are evidence of its ability to create products that are treasured and conserved. But today it is frequently used as a potent tool to stimulate consumption by deliberate obsolescence, creating the perception that 'new' is desirable and that even slightly 'old' is unappealing.

14.4 The Eco-Attributes of Materials

Material Production: Energy and Emissions

Most of the energy consumed in the phases of Fig. 14.1 is at present derived from fossil fuels. Some is consumed in that state – as gas, oil, coal or coke. Much is first converted to electricity at an average US and European conversion efficiency of about 38%. Not all electricity comes from fossil fuels – there are contributions from nuclear, hydro, solar, wind and geothermal sources. But with the exception of Norway (70% hydro) and France (80% nuclear) the predominant energy sources are fossil hydrocarbons.

The energy consumed in making one kilogram of material is called its *embodied energy*. Some of the energy is stored in the created material and can be recovered, in one sense or another, at the end of life. Polymers made from oil (as most are) contain energy in another sense – that of the oil that enters the production as a primary feedstock. Natural materials such as wood, similarly, contain 'intrinsic' or 'contained' energy, this time derived from solar radiation absorbed during growth. Views differ on whether the intrinsic energy should be included in the embodied energy or not. There is a sense in which not only polymers and woods, but also metals, carry intrinsic energy that could – by chemical reaction or by burning the metal in the form of finely-divided powder – be recovered, so omitting it when reporting production energy for polymers but including it for metals seems inconsistent. For this reason we include intrinsic energy from nonrenewable resources in reporting embodied energies, which generally lie in the range 25–250 MJ/kg (Fig. 3.20) though a few are much higher. The existence of intrinsic energy has another consequence: the

energy to recycle a material is sometimes much less than that required for its first production because the intrinsic energy is retained. Typical values for recycling energy lie in the range 10–100 MJ/kg.

Material production is associated with *undesired gas emissions*, among which CO_2, NO_x, SO_x and CH_4 cause general concern (global warning, acidification, ozone-layer depletion). The quantities can be large – each kilogram of virgin aluminium produced by using electrical power from fossil fuels creates some 9 kg of CO_2, 40 g of NO_x and 90 g of SO_x. Material production is generally associated with other undesirable outputs, particularly toxic wastes and particulates, but these can, in principle, be dealt with at source.

Approximate data for embodied energy and CO_2 footprint of materials are listed in Appendix A, Section A10.

Material Processing Energies and Emissions

Many processes depend on casting, evaporation or deformation. It is helpful to have a feel for the approximate magnitudes of energies required by these. The use of primary energy (thought of as the equivalent quantity of oil) for processing materials involves several energy conversion steps, each with a conversion efficiency that is less than 100%. Many processes use electric energy, generated from primary energy with a conversion efficiency of order 38%. The use of this for electro-heating or for electro-processing (like electro-forming) involves further losses. It is possible, as we do below, to estimate the energy to melt, mould, vaporize or deform a material, but to express this in terms of primary energy it must be divided by the conversion efficiencies of each conversion step. For the sake of example we set this overall efficiency at a (realistic) 15%.

Melting. To melt a material, it must first be raised to its melting point, requiring a minimum input of the heat $C_p(T_m - T_o)$, and then caused to melt, requiring the latent heat of melting, L_m

$$H_{\min} = C_p(T_m - T_o) + L_m \qquad \textbf{(14.1)}$$

where $H_{\min}$ is the minimum energy per kilogram for melting, C_p is the specific heat, T_m is the melting point, and T_o is the ambient temperature. A close correlation exists between L_m and $C_p\,T_m$

$$L_m \approx 0.4 C_p T_m \tag{14.2}$$

and for most metals and alloys $T_m >> T_o$ giving

$$H_{\min} \approx 1.4 C_p T_m \tag{14.3}$$

Assuming a process efficiency of 15%, the estimated energy to melt one kilogram, H_m^*, is

$$H_m^* \approx 8.4 C_p T_m \tag{14.4}$$

the asterisk recalling that it is an estimate. For metals and alloys, the quantity H_m^* lies in the range 1–8 MJ/kg.

Vaporisation. As a rule of thumb the latent heat of vaporisation, L_v, is larger than that for melting, L_m, by a factor of 24 ± 5, and the boiling point T_b is larger than the melting point, T_m, by a factor 2.1 ± 0.5. Using the same assumptions as before, we find an estimate for the energy to evaporate 1 kg of material (as in physical vapor deposition (PVD) processing) to be

$$H_v^* \approx 76 C_p T_m \tag{14.5}$$

again assuming an efficiency of 15%. For metals and alloys, the quantity H_v^* lies in the range 6–60 MJ/kg.

Deformation. Deformation processes like rolling or forging generally involve large strains. Assuming an average flow-strength of $(\sigma_\gamma + \sigma_{uts})/2$, a strain of order 1 and an efficiency factor of 15% we find, for the work of deformation per kg to be

$$W_D^* \approx 3 \ (\sigma_\gamma + \sigma_{uts}) \tag{14.6}$$

where σ_γ is the yield strength and σ_{uts} is the tensile strength. For metals and alloys, the quantity W_D^* lies in the range 0.05–2 MJ/kg.

We conclude that casting or deformation require processing energies that are small compared to the production energy of the material being

processed, but the larger energies required for vapour-phase processing may become comparable with those for material production.

End of Life

Fig. 14.3 introduces the options: *landfill, combustion for heat recovery, recycling, re-engineering* and *re-use*.

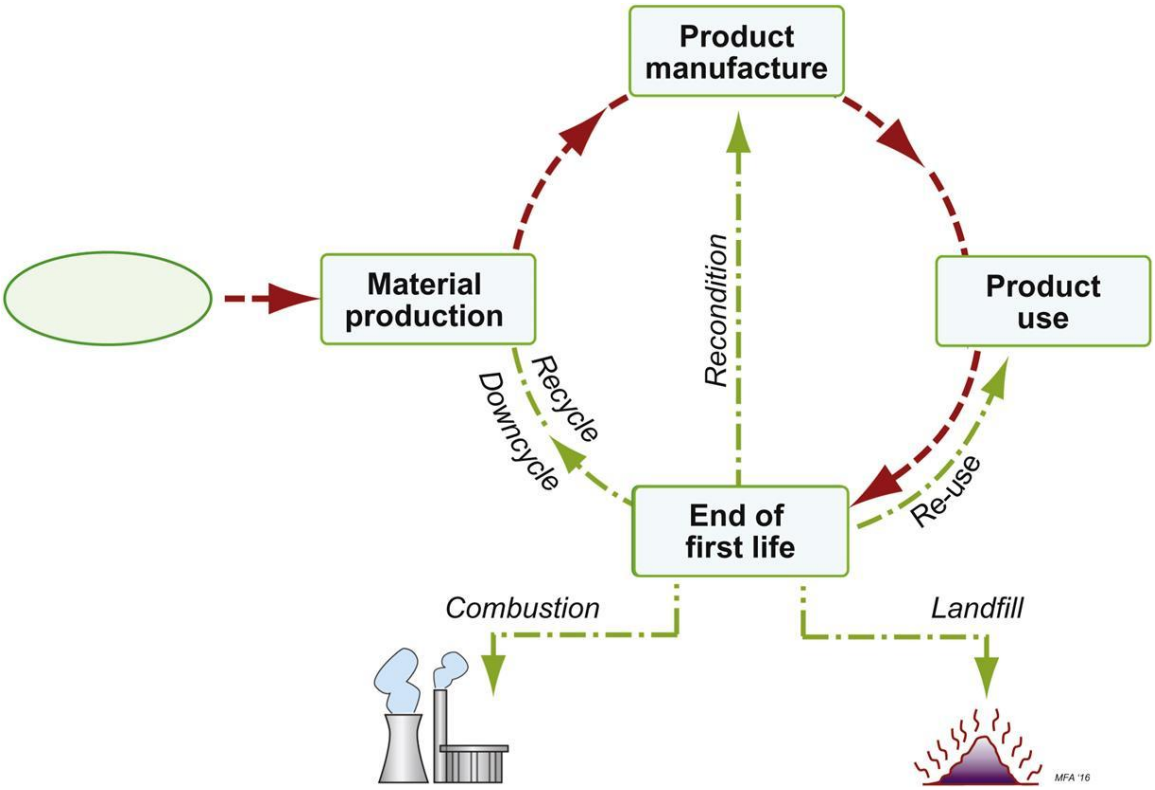


FIGURE 14.3 End of life options are shown in green: landfill, combustion with energy recovery, recycling, reconditioning and reuse.

Landfill. Much of what we now reject is committed to landfill. Already there is a problem – the land available to 'fill' in this way is already, in some European countries, almost full. Administrations react by charging a landfill tax – currently somewhere near €50 per tonne and rising, seeking to divert waste into the other channels of Fig. 14.3.

Combustion for heat recovery. Heat recovery allows some of the energy stored in the material to be retrieved by controlled combustion, re-capturing the heat. But this is not as easy as it sounds. First there is the

need for a primary sorting to separate combustibles from noncombustible material. Then the combustion must be carried out under conditions that do not generate toxic fumes or residues, requiring high temperatures, sophisticated control and expensive equipment. The energy recovery is imperfect partly because it is incomplete and partly because the incoming waste carries a moisture content that has to be boiled off.

Recycling requires energy, and this energy carries its burden of gases. But the *recycle energy* is generally small compared to the initial production energy, making recycling – when it is possible at all – an energy-efficient propositions. It may not, however, be one that is cost-efficient; that depends on the degree to which the material has become dispersed. In-house scrap, generated at the point of production or manufacture, is localised and is already recycled efficiently (near 100% recovery). Widely distributed 'scrap' – material contained in discarded products – is a much more expensive proposition to collect, separate and clean. Many materials cannot recycled, although they may still be reused in a lower-grade activity; continuous-fibre composites, for instance, cannot be re-separated economically into fibre and polymer in order to recycle them, though they can be chopped and used as fillers. Most other materials require an input of virgin material to avoid build-up of uncontrollable impurities. Thus the fraction of a material production that can ultimately re-enter the cycle of Fig. 14.3 depends both on the material itself and on the product into which it has been incorporated.

Reconditioning is the re-engineering or upgrading of the product or of its recoverable components. Certain criteria must be met to make it practical. One is that the design of the product is fixed or that the technology on which it is based is evolving so slowly that there remains a market for the restored product. Examples are car and truck engines, agricultural machinery, housing, office space, road and rail infrastructure.

Re-use. Re-use is the redistribution of the product to a consumer sector that is willing to accept it in its used state, perhaps to reuse for its original purpose (a second hand car, for instance), perhaps to adapt to another (converting the car to a hot-rod, or a bus into a mobile home). Charity shops pass on clothing, books and objects, acquired from those for whom they had become waste, and sell them on to others who perceive them to have value. Reuse is the most benign of the end-of-life scenarios.

Reusable versus Disposable Glasses

The environment-aware organizers of a major sports event plan to replace the disposable polystyrene glasses used in their refreshment area with reusable polycarbonate glasses which will be collected, washed and reused. The material polycarbonate has a higher embodied energy and water footprint than polystyrene (see the table below, data from Appendix A, Table A10) and reusable glasses are sturdier and thus heavier than disposable ones.



Material	Embodied Energy* (MJ/kg)	Water Footprint (l/kg)
Polystyrene	96	140
Polycarbonate	105	175

The act of washing the glasses also consumes energy and water. Here are the details:

- Polystyrene (PS) disposable cup weighs 0.04 kg and is used only once
- The Polycarbonate (PC) reusable cup weighs 0.1 kg and is used n times
- A dishwasher holds 40 glasses, consumes 1 kW·h (3.6 MJ) and 18 L of water per run

How many times (n) do the reusable glasses have to be reused before they become more energy and water-efficient than the disposable ones?

Answer

The energy and water footprint per glass per use for the disposable glass is just the energy H_m and water W_m per kg of the material multiplied by the mass m of the glass itself, thus 3.8 MJ and 5.6 L. The energy for the reusable glass, per use, is

$$\text{Energy per use} = \frac{1}{n}(m \cdot H_m + n \cdot H_{\text{wash}})$$

where $H_{\text{wash}} = \frac{3.6}{40} 0.09 \text{ MJ}$. Equating the energy per use of the reusable glass to that for the one-off disposable one (3.8 MJ), solving for n and rounding to the nearest whole number gives

$$n = \frac{m \cdot H_m}{(3.8 - H_{\text{wash}})} = 3$$

An analogous analysis for water gives a break-even reuse number of 2.

Postscript. It looks as if reusable glasses are a good choice from an environmental viewpoint. The use of reusable glasses was tried at the Barcelona Olympics in 1992. It did not work because too few glasses were returned: too many rendered unusable, could not be retrieved or were stolen. Environmental problems can have a human element.

14.5 Life-Cycle Assessment, Eco-Audits and Energy Fingerprints

The starting point for the life-cycle of a material is its creation from ore and feedstock. The materials are manufactured into products that are distributed and used. Products (like us) have a finite life, at the end of which they become scrap. The materials they contain, however, are still there; some (unlike us) can be reused, reconditioned or recycled to provide further useful service.

Life-cycle assessment (LCA) traces this progression, documenting the resources consumed and the emissions excreted during each phase of life. The output is a sort of biography, documenting where the materials have been, what they have done and the consequences of this for their surroundings. In particular it documents the energy consumed and the

carbon emissions to atmosphere (the *carbon footprint*) caused by the product over its life.

Responsible design, today, aims to provide safe, affordable services while minimizing the drain on resources and the release of unwanted emissions. To do this, the designer needs feedback on the eco-profile of the design (or re-design) as it progresses. To be useful this feedback must be fast, allowing quick 'what if?' exploration of the consequences of alternative choices of material, use pattern and end-of-life choice. A full LCA is not well adapted for this task – it is slow and expensive. *Streamlined LCA* and the *eco-audit methods* have evolved to fill the gap.

These methods are approximate but the distinctions that they draw can be sufficiently sharp to distinguish the most damaging phases of product life and the changes that can improve the eco-performance and those that cannot. The output is an *energy-fingerprint:* the characterizing pattern of energy consumption across the phases of life. The carbon emissions (and those of sulphur, and nitrogen) do not scale exactly with energy, but because so much energy of the energy we use is derived from hydrocarbon fuels, the energy fingerprints are a sort of proxy for the others.

Fig. 14.4 introduces the output of an eco-audit. Each bar describes, for a given product, the energy consumed in each phase of the life-cycle: material production, product manufacture, product use, product disposal at end of life and the transport in between. The carbon and other emissions follow a similar pattern, not exactly, but fairly closely. There are two bars for end-of-life, a complication explained in a moment. The total life energy is the sum of the five left-hand bars, as shown on the figure.

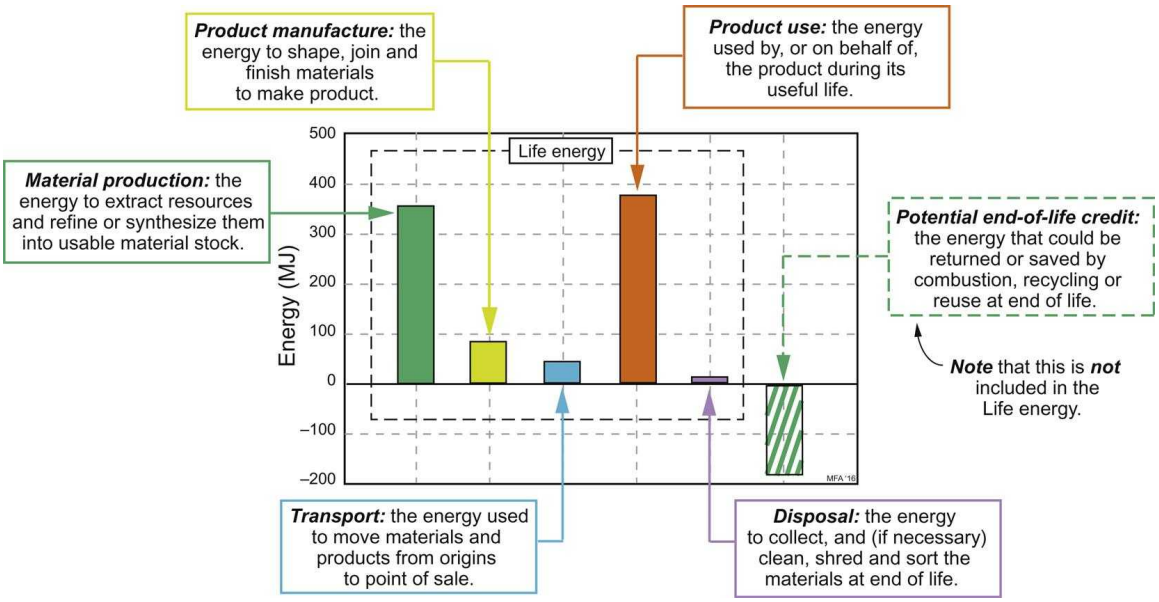


FIGURE 14.4 The output of an eco-audit: the energy fingerprint of a product.

The first, green, bar shows the mass-weighted sum of the embodied energies of the materials in the product. Allowance is made for the inclusion of recycled material. The second, yellow, bar is an estimate of the energy to manufacture the product. Transport is involved – the third, blue bar sums the energy involved in this. The fourth, red, bar describes the energy consumed by, or on behalf of, the product during use. Some products (photocopiers, for example) consume both energy and materials (paper, toner) during use. These are treated here as an energy contribution to the use phase. The fifth, purple bar, almost always very small, is the energy to collect the product at end of life, clean, shred and sort its materials. The sum of these five bars gives an estimate of the life energy.

There is one more bar. A designer wishing to minimize the eco-impact of a product may wish to use recycled materials, thereby lowering the life energy. The eco-audit gives credit for this, which appears as a reduced height of the green materials bar. The same designer may also wish to structure the design so that it is easy to dismantle for recycling and use materials that recycle well. That cannot be included as a credit in the life energy because credit has already been awarded for use of recycled materials at start of life and because it remains a future option that may not be adopted. Yet the eco-audit should signal the energy return that is enabled by allowing recycling. That is what the last bar, hatched in green, describes.

Fig. 14.5 show what real eco-audits look like for a range of products. There are six families, each with two examples. The first pair shows passive products, those that require little or no energy during use. For these it is the embodied energies of their materials the make the largest contribution. The next two pairs show products that require power to function. For these, it is the use phase that dominates.

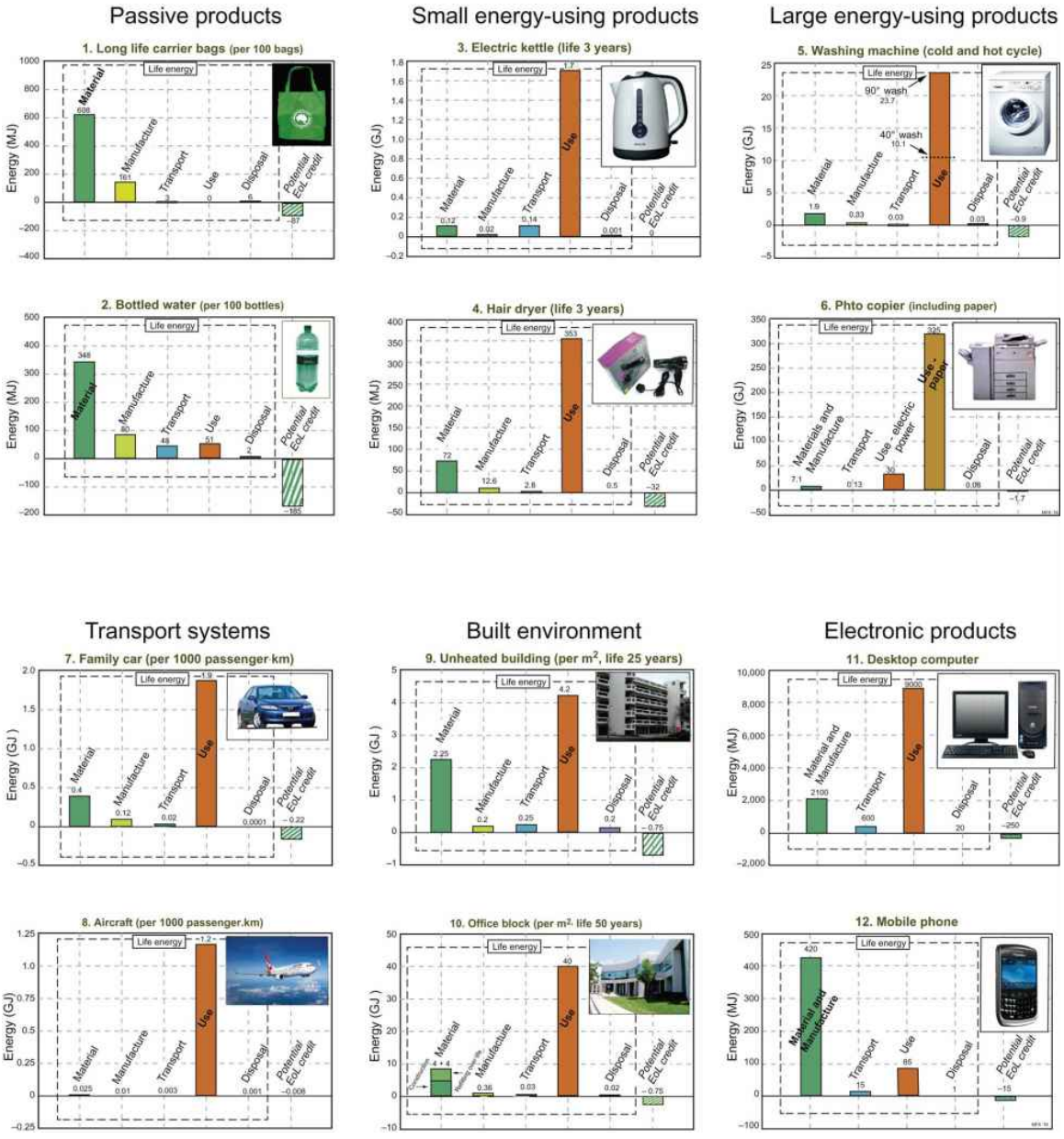


FIGURE 14.5 Approximate energy fingerprints of products. They are grouped in pairs with characteristics in common.

The fourth pair of Fig. 14.5 – transport systems – shows an interesting contrast. The red use-phase bar for aircraft (also trains, shipping) consumes far more energy that all the other phases of life together. For cars, the distribution is less skewed: the material energy is now a larger fraction of the life energy, though still less than the use energy. The increasing use of composites to reduce vehicle weight reduces the use energy, but at the penalty of increasing the material energy. The next pair describes buildings. The relative heights of the bars depend on the life of the building: the longer it lasts, the larger the use energy becomes. The assumed lives are indicated. The material energy, too, increases with life because of maintenance, redecoration and refitting. Even with these uncertainties, it is the use energy that dominates in today's building. The last group in Fig. 14.5 – electronic products – also has some surprises. The materials and manufacture of electronic products is energy intensive and their lives are relatively short, so much so that the energy of materials and manufacture can exceed that of use.

14.6 Eco-Selection

To select materials to minimize the impact on the environment we must first ask – as we did in Section 14.2 – which phase of the life cycle of the product under consideration makes the largest contribution? The answer guides the choice of strategy to ameliorate it (Fig. 14.6). The strategies are described below.

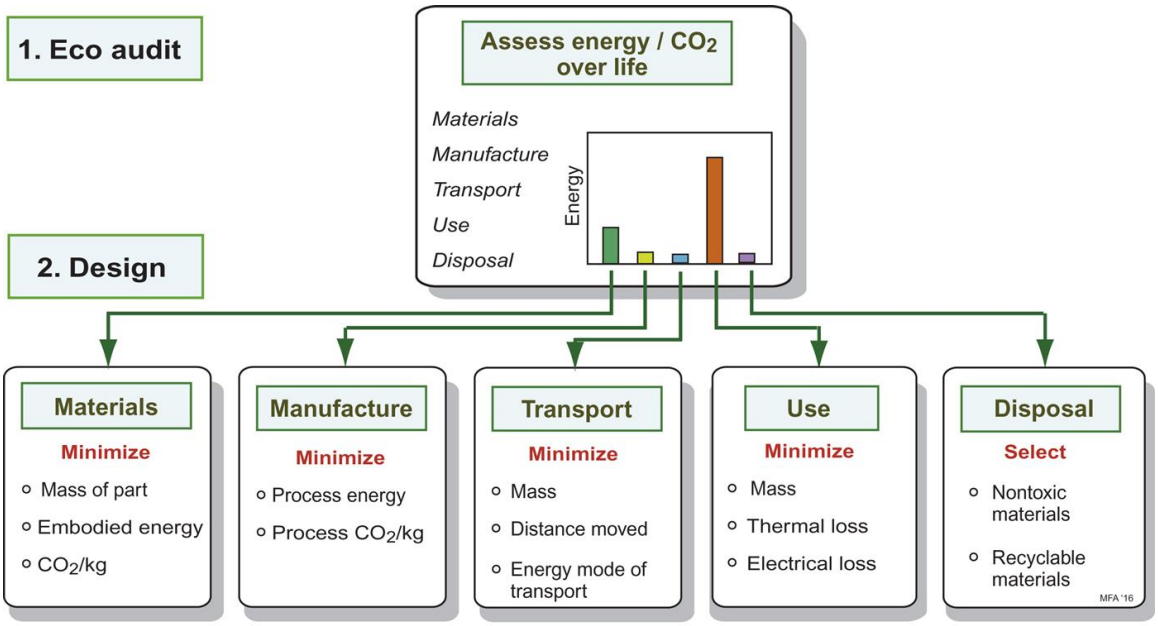


FIGURE 14.6 Design for the environment starts with an analysis of the phase of life to be targeted, guiding the choice of tactic to minimize eco-impact.

The material production phase. If material production is the dominant phase of life this becomes the first target. Drink containers provide an example: they consume materials and energy during material extraction and production but, apart from transport and refrigeration, they do not consume much thereafter. We use the embodied energy of the material as the measure and as a proxy for CO_2, NO_x and SO_x emissions. The energy associated with the production of one kilogram of a material is H_p, that per unit volume is $H_p\rho$ where ρ is the density of the material. The bar charts of Figs. 3.20 and 3.21 showed these two quantities for materials. On a 'per kg' basis glass, the material of the first container, carries the lowest penalty. Steel is slightly higher. Polymers are higher still. Aluminium carries the highest energy penalty of all. But if these same materials are compared on a 'per m^3' basis the conclusions change: glass is still the lowest, but now commodity polymers such as polyethylene (PE) and polypropylene (PP) carry a *lower* burden than steel. But is comparison 'per kg' or 'per m^3' the right way to do it? Rarely. To deal with environmental impact at the production phase properly we must seek to minimize the energy or the CO_2 footprint *per unit of function*.

Performance indices that include energy content are derived in the same way as those for weight or cost (see chapter: Materials Selection – The Basics). As an example consider the selection of a material for a beam that must meet a stiffness constraint at minimum energy content. Repeating the derivations of Chapter 4, Materials Selection – The Basics but with the objective of minimizing embodied energy rather than mass leads to performance equations and material indices that are simply those of Chapter 4, Materials Selection – The Basics with ρ replaced by $H_p\rho$. Thus the best materials to minimize embodied energy of a beam of specified stiffness and length are those with large values of the index

$$M_1 = \frac{E^{1/2}}{H_p\rho} \tag{14.7}$$

where E is the modulus of the material of the beam. The stiff tie of minimum energy content is best made of a material of high $E/H_p\rho$; the stiff panel of a material with high $E^{1/3}/H_p\rho$ and so on.

Strength works the same way. The best materials for a beam of specified bending strength and minimum embodied energy are those with large values of

$$M_3 = \frac{\sigma_f^{2/3}}{H_p\rho} \tag{14.8}$$

where σ_f is the failure strength of the beam-material. Other indices follow in a similar way.

Figs. 14.7 and 14.8 are a pair of charts for selection to minimize embodied energy H_p per unit of function (charts for CO_2 burden look very similar). The first show modulus E plotted against $H_p\rho$; the guidelines give the slopes for three of the commonest performance indices. The second shows strength σ_f plotted against $H_p\rho$; again, guidelines give the slopes. The two charts are used in exactly the same way as the $E - \rho$ and $\sigma_f - \rho$ charts for minimum mass design.

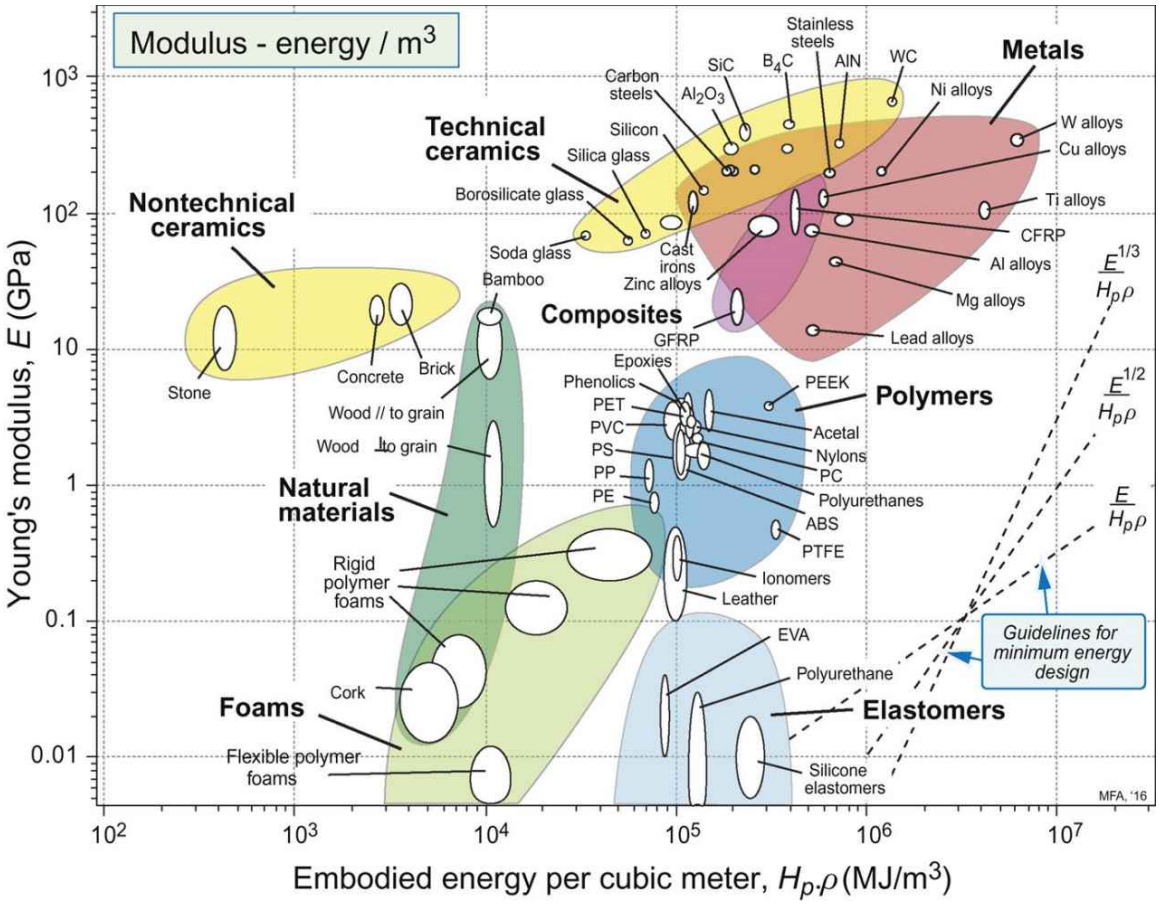


FIGURE 14.7 A selection chart for stiffness with minimum production energy. It is used in the same way as Fig. 3.3.

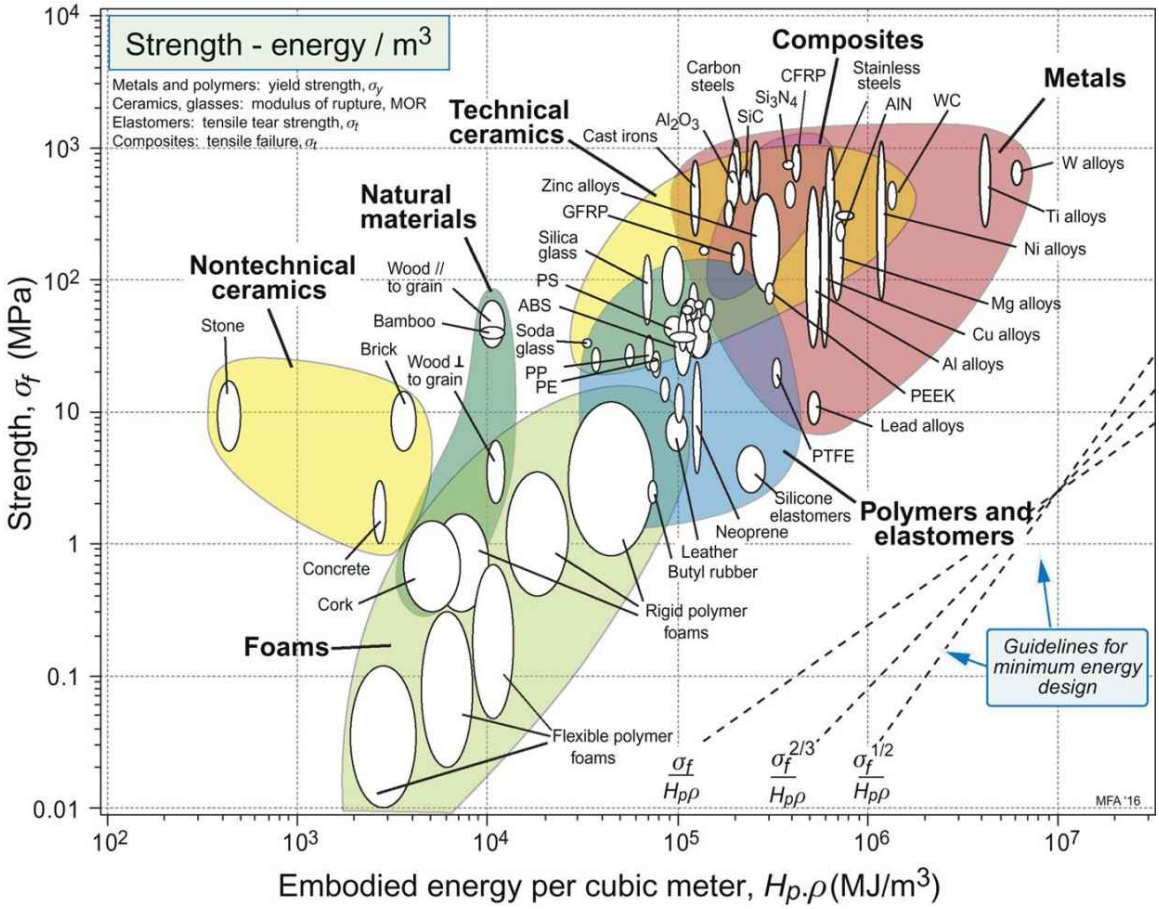


FIGURE 14.8 A selection chart for strength with minimum production energy. It is used in the same way as Fig. 3.4.

Most polymers are derived from oil. This leads to statements that they are energy-intensive and therefore a bad choice. The two charts show that, per unit of function in bending (the commonest mode of loading), most polymers carry a lower energy penalty than primary aluminium, magnesium or titanium, and that several are competitive on and energy basis with steel.

Relative Contributions to Embodied Energy of Products

The core of a catalytic converter for cars has a core of alumina (ceramic) honeycomb carrying 5 g of platinum. The active components are contained in a stainless steel casing. Which component contributes most to the embodied energy of the product?

Component	Mass (kg)	Embodied Energy (MJ/kg)	Energy Contribution to Product (MJ)
Alumina core	0.7	52	36
Platinum catalyst	0.005	2.9×10^5	1450
Stainless steel casing	1.2	68	81

Answer

The relevant data are shown in the table above. The platinum catalyst accounts for only 0.3% of the mass of the converter but 93% of its embodied energy.

The product-manufacture phase. Shaping materials, as we have seen, requires energy. Certainly it is important to save energy in production. But higher priority often attaches to the *local* impact of emissions and toxic waste during manufacture and this depends crucially on local circumstances. Paper-making (to take an example) uses very large quantities of water. Historically the waste water was heavily polluted with alkalis and particulates, devastating the river systems into which it was dumped. Today, the best paper mills discharge water that is as clean as it was when it entered the plant. Clean manufacture is the issue here.

The use phase. The eco-impact of the use phase of energy-consuming products has nothing to do with the energy content of the materials themselves – indeed, minimizing this may frequently have the opposite effect on use energy. Use energy depends on mechanical, thermal and electrical efficiencies; it is minimized by maximizing these efficiencies. Fuel efficiency in transport systems (measured, say, by MJ/km) correlates closely with the mass of the vehicle itself; the objective then becomes that of minimizing mass. Energy efficiency in refrigeration or heating systems is achieved by minimizing the heat flux into or out of the system; the objective is then that of minimizing thermal conductivity or thermal inertia. Energy efficiency in electrical generation, transmission and conversion is maximized by minimizing the ohmic losses in the conductor; here the objective is to minimize electrical resistance while meeting necessary constraints on strength, cost etc. Selection to meet these objectives is exactly what the previous chapters of this book were about.

Carbon Footprint and Cost of Use Phase

A small car weighs 1300 kg; it covers 180,000 miles over its life at an average fuel economy of 35 miles per US gallon. The car carries a spare wheel and tyre weighing 13 kg, which the owner has never had occasion to use. If fuel consumption scales linearly with vehicle weight, what has been the carbon and cost penalty of carrying the spare wheel over the vehicle life? (Gasoline cost an average of $2.1 per gallon in the United States in 2016. Burning one US gallon of gasoline releases 11 kg of carbon dioxide.)

Answer

The car burns 180,000/35=5140 gallons of petrol costing $10,800 over its life and releases 56,540 kg of carbon dioxide. Increasing its weight by 1% (the contribution of the spare wheel) thus carries the penalty of 565 kg of carbon dioxide and costs about $110 over life. In Europe the carbon penalty is the same but the cost is about twice as much.

The product disposal phase. The environmental consequences of the final phase of product life have many aspects. Increasingly, legislation dictates disposal procedures, take-back and recycle requirements, and – through landfill taxes and subsidised recycling – deploys market forces to influence the end-of-life choice.

Transport. Globalization means that materials and products are transported over large distances before they reach the consumer. This is often seen as misguided – better, it is argued, to use indigenous materials and to manufacture locally. Economists argue otherwise: the lower labour costs in emerging economies more that offsets the cost of transporting materials and products between them and developed the countries where they are sold and used. Table 14.1 gives an idea of the energy and carbon footprint of various transport modes.

Estimating Energies and Carbon Footprints for Transport

Television sets are manufactured in Shanghai, China, and transported to Le Havre, France. What is the carbon footprint per set, (shipping mass 21 kg per unit) if sent by sea freight? By air freight?

Answer

The distance from Shanghai to Le Havre by sea via the Cape of Good Hope is 13,800 km. The carbon footprint of this transport leg is $0.21 \times 0.015 \times 13{,}800 = 43.5$ kg CO_2.

The distance from Shanghai to Le Havre by air over Russia is 5714 km. The carbon footprint of this transport leg is $0.21 \times 0.8 \times 5714 = 960$ kg CO_2, about 20 times more than by sea.

Table 14.1

Approximate Energy and Carbon Footprint for Transport

Transport Type and Fuel	Energy (MJ/tonne·km)	Carbon Footprint (kg CO_2/tonne·km)
Shipping – diesel	0.16	0.015
Rail – diesel	0.25	0.019
Large truck – diesel	0.71	0.05
Small truck – diesel	1.5	0.11
Family car – diesel	1.4–2.0	0.1–0.14
Family car – gasoline	2.2–3.0	0.14–0.19
Family car – hybrid	1.55	0.10
Aircraft – kerosene	19–15	0.6–1.0

14.7 Case Studies: Drink Containers and Crash Barriers

Drink containers. The drink containers of Fig. 14.9 are made from five different materials. All five can be recycled. They are examples of products for which the first phase of life – material production – consumes the most energy and generates the most emissions. Eco-design focusses on reducing the impact of this phase. Which is the best choice? Table 14.2 summarises the requirements.

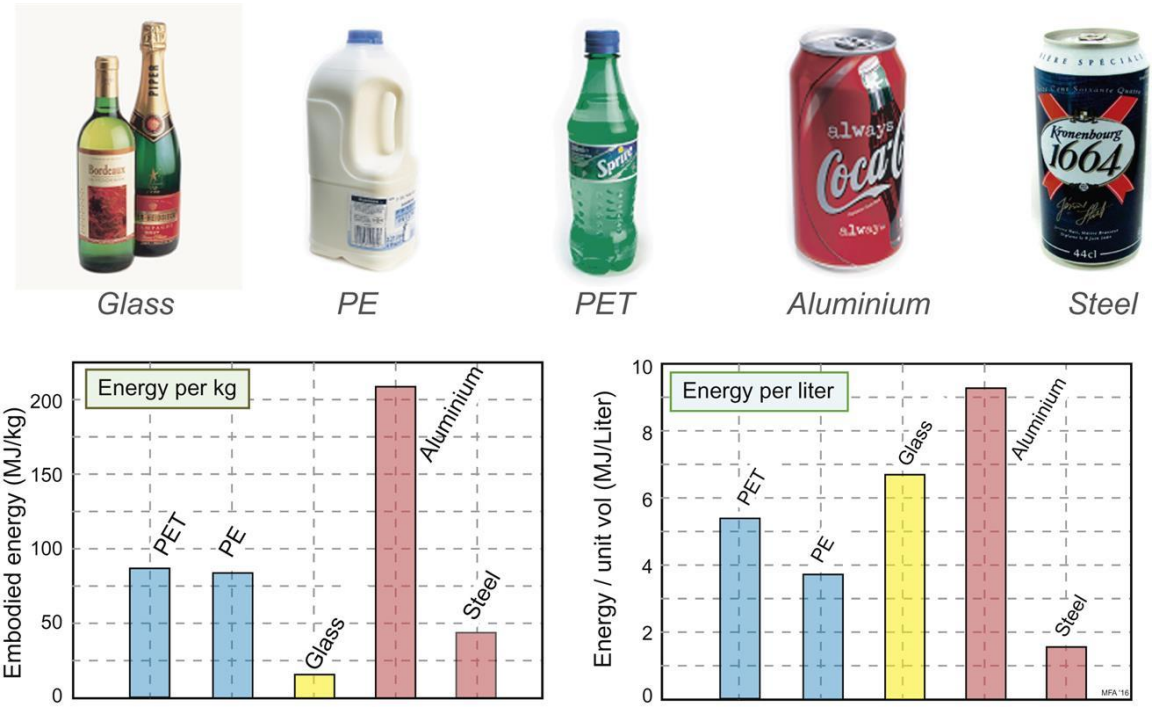


FIGURE 14.9 Liquid containers: glass, polyethylene, PET, aluminium and steel. All can be recycled. Which carries the lowest energy penalty and carbon footprint?

Table 14.2

Design Requirements for the Containers

Function	• *Container for cold drink*
Constraints	• *Must be recyclable*
Objective	• *Minimize embodied energy per unit capacity*
Free variables	• *Choice of material*

The materials of each container, their embodied energies per kg, and the container mass and volume are listed in Table 14.3. From these is calculated the energy per litre of contained fluid (last column). The steel container (value shown bold, underscored) has the lowest energy penalty per unit volume, glass and aluminium the highest.

Table 14.3

Details of the Containers

Container Type	Material	Embodied Energy of Material (MJ/kg)	Mass (g)	Mass/Litre, (kg/L)	Energy/Litre (MJ/L)
PET 400 mL bottle	PET	84	25	62	5.4
PE 1 L milk bottle	High density PE	81	38	38	3.2
Glass 750 mL bottle	Soda glass	15.5	325	433	8.2
Al 440 mL can	5000 series Al alloy	210	20	45	9.0
Steel 440 mL can	Plain carbon steel	32	45	102	2.4

Crash barriers. Barriers to protect driver and passengers of road vehicles are of two types: those that are static – the central divider of a freeway, for instance – and those that move – the bumper of the vehicle itself (Fig. 14.10). The static type lines tens of thousands of miles of road. Once in place they consume no energy, create no CO_2 and last a long time. The dominant phases of their life in the sense of the life-cycle of Fig. 14.1 are those of material production and manufacture. The bumper, by contrast, is part of the vehicle; it adds to its weight and thus to its fuel consumption. The dominant phase here is that of use. This means that, if eco-design is the objective, the criteria for selecting materials for the two sorts of barrier will differ. We take as a criterion for the first that of maximizing the energy that the barrier can absorb per unit of production energy; for the second we take absorbed energy per unit mass. Table 14.4 summarizes.

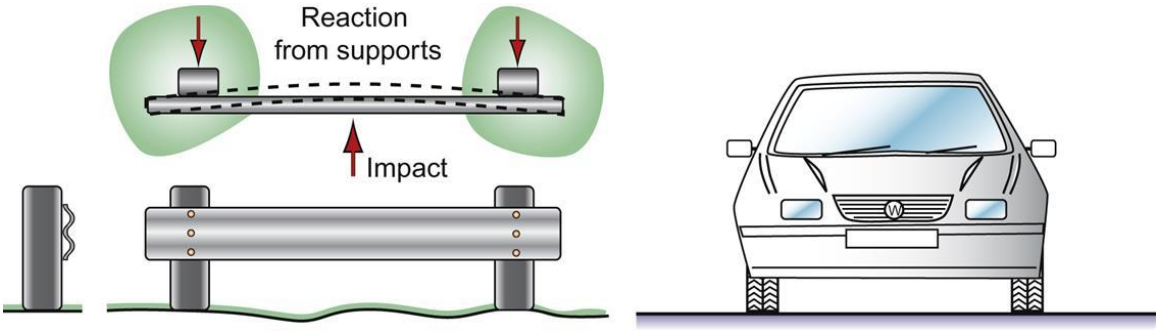


FIGURE 14.10 Two crash barriers, one static, the other – the bumper – attached to something that moves. Different eco-criteria are needed to select materials for each.

Table 14.4

Design Requirements for the Crash Barriers

Function	• *Energy-absorbing crash barriers*
Constraints	• *Must be recyclable*
Objective	• *Maximize energy absorbed per unit production energy, or* • *Maximize energy absorbed per unit mass*
Free variables	• *Choice of material*

In an impact the barrier is loaded in bending (Fig. 14.10). Its function is to transfer load from the point of impact to the support structure where reaction from the foundation or from crush-elements in the vehicle support or absorb it. To do this the material of the barrier must have high strength, σ_f, be adequately tough and able to be recycled. That for the static barrier must meet these constraints with *minimum embodied energy* as the objective because this will reduce the overall life energy most effectively. We know from Section 14.6 this means materials with large values of the index

$$M_3 = \frac{\sigma_f^{2/3}}{H_p \rho} \tag{14.9}$$

where σ_f is the yield strength, ρ the density and H_p the embodied energy per kg of material. For the car bumper it is mass, not embodied energy that is the problem. If we change the objective to that of *minimum mass*, we require materials with high values of the index

$$M_4 = \frac{\sigma_f^{2/3}}{\rho} \tag{14.10}$$

These indices can be plotted onto the charts of Figs. 14.8 and 3.4, enabling a selection. We leave that as one of the exercises in order to show here an alternative: simply plotting the index itself as a bar-chart. Figs. 14.11 and 14.12 show the result for metals, polymers and polymer-matrix composites. The first guides the selection for static barriers. It shows that

embodied energy (for a given load bearing capacity) is minimized by making the barrier from carbon steel or cast iron, wood or bamboo; nothing else comes close. The second, Fig. 14.12, guides selection for the mobile barrier. Here carbon-fibre reinforced polymers (CFRP) (continuous fibre carbon-epoxy, for instance) excels in its strength per unit weight, but it is not recyclable. Heavier, but recyclable, are alloys of magnesium, titanium and aluminium. Polymers, which rank poorly on Fig. 14.11, now become candidates – even without reinforcement, polymers can be as good as steel because their density is so low.

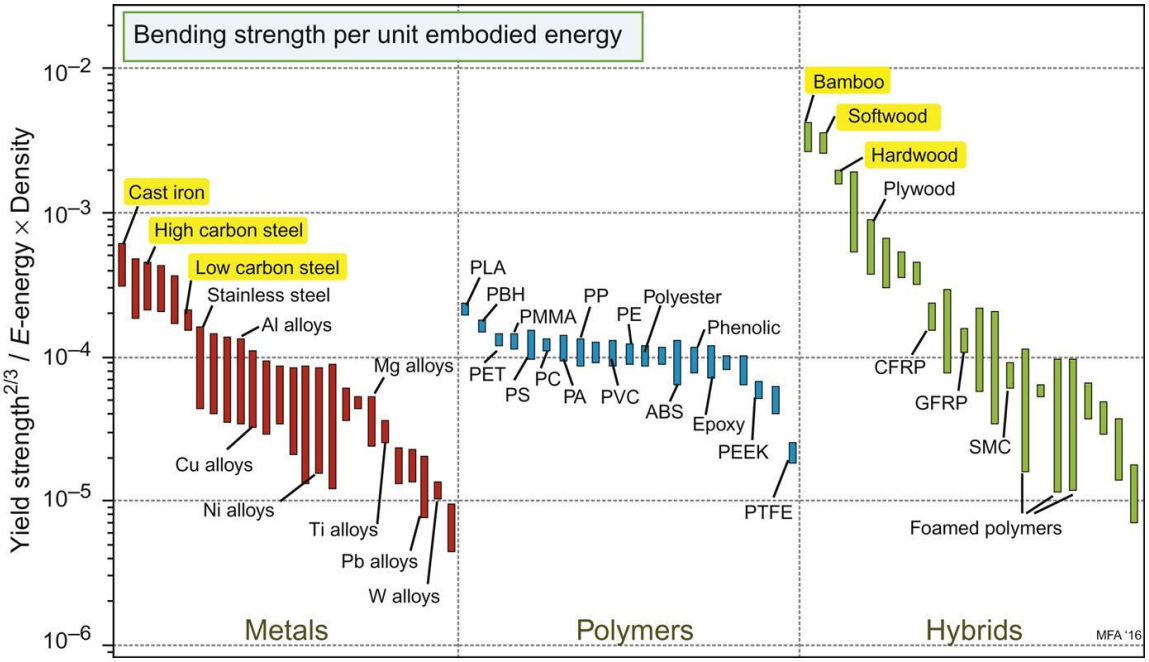


FIGURE 14.11 Material choice for the static barrier (the units are $(MPa)^{2/3}/(MJ/m^3)$). Cast irons, carbon steels, low alloy steels or wood are the best choices.

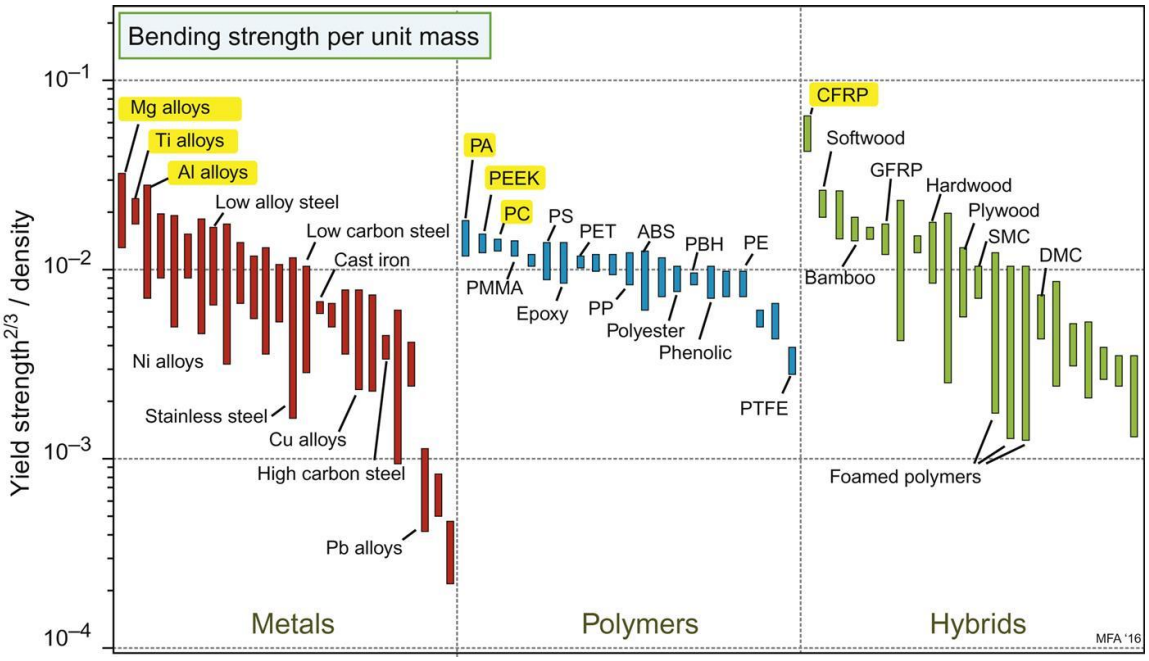


FIGURE 14.12 Material choice for the mobile barrier (the units are $(MPa)^{2/3}/(kg/m^3)$). CFRP and light alloys offer the best performance; nylon and polycarbonate perform as well as steel.

Postscript. Metal crash barriers have a profile like that shown on the left of Fig. 14.10. The curvature increases the second moment of area of the cross-section, and through this, the bending stiffness and strength. This is an example of combining material choice and section shape (see chapter: Selection of Material and Shape) to optimize a design.

14.8 Summary and Conclusions

Rational selection of materials to meet environmental objectives starts by identifying the phase of product life that causes greatest concern: production, manufacture, use or disposal. Dealing with all of these requires data not only for the obvious eco-attributes (energy, CO_2 and other emissions, toxicity, ability to be recycled and the like) but also data for mechanical, thermal, electrical and chemical properties. Thus if material production is the phase of concern, selection is based on minimizing production energy or the associated emissions (CO_2 production for example). But if it is the use phase that is of concern, selection is based instead on lightness of weight, excellence as a thermal

insulator, or excellence as an electrical conductor (while meeting other constraints on stiffness, strength, cost etc.). This chapter develops methods to deal with these. The methods are most effective when implemented in software. The CES EduPack system, described in Chapter 4, Materials Selection – The Basics and Chapter 5, Materials Selection – Case Studies contains eco-data for materials and includes an eco-audit tool to analyse product life in the manner of Figs. 14.4 and 14.5.

If you found this chapter interesting and would like to read more, you will find the ideas it contains developed more fully in the first book listed under Further Reading.

14.9 Further Reading

Texts on Design for the Environment and for Sustainable Development differ in their approach. Some perceive wilful neglect and impending doom on every side, others take a more detached and analytical approach. Those listed below aim at cool, balanced assessment.

1. Ashby MF. *Materials and the Environment – Eco-Informed Material Choice* second ed. Oxford: Butterworth Heinemann; 2012; ISBN 978-0-12-385971-6. (*A teaching text that provides the resources – background, methods, data – to enable environmental issues relating to materials to be explored in depth*.).
2. Ashby MF, Ferrer-Balas D, Segalas Coral J. *Materials and Sustainable Development* Oxford: Butterworth-Heinemann Ltd; 2016; ISBN-10:0081001762 ISBN-13:978-0-08-100176-9. (*An introduction to the concept of sustainable development with case studies and exercises*.).
3. CESEdu. *The Cambridge Engineering Selector* Cambridge: Granta Design; 2016; (*The material selection platform now has an optional eco-design module*. <www.grantadesign.com>).
4. Fuad-Luke A. *The Eco-Design Handbook* London: Thames and Hudson; 2002; ISBN 0-500-28343-5. (*A remarkable source-book of examples, ideas and materials of eco-design*.).
5. Goedkoop MJ, Demmers M, Collignon MX. *Eco-Indicator '95, Manual* Amersfort: Pré Consultants, and the Netherlands Agency for Energy and the Environment; 1995; ISBN 90-72130-80-4.
6. Goedkoop M, Effting S, Collignon M. *The Eco-Indicator 99: A Damage Oriented Method for Life Cycle Impact Assessment* Manual for

Designers 2000; (*PRe Consultants market a leading life-cycle analysis tool and are proponents of the eco-indicator method*. <http://www.pre.nl>).
7. ISO 14001, 1996. and ISO 14040 (1997, 1998, 1999) International organisation for standardisation (ISO). Environmental Management System-Specification with Guidance for Use, Geneva.
8. Lovins LH, von Weizsäcker E, Lovins AB. *Factor Four: Doubling Wealth, Halving Resource Use* London: Earthscan publications; 1998; ISBN 1-853834-068. (*An influential book arguing that resource productivity can and should grow fourfold, that is, the wealth extracted from one unit of natural resources can quadruple allowing the world to live twice as well yet use half as much.*).
9. MacKay DJC. *Sustainable Energy – Without the Hot Air* Cambridge: Department of Physics, Cambridge University; 2008; <www.withouthotair.com/> (*MacKay brings a welcome dose of common sense into the discussion of energy sources and use. Fresh air replacing hot air.*).
10. Mackenzie D. *Green Design: Design for the Environment* second ed. London: Lawrence King Publishing; 1997; ISBN 1-85669-096-2. (*A lavishly produced compilation of case studies of eco design in architecture, packaging, and product design.*).
11. Schmidt-Bleek F. *How Much Environment Does the Human Being Need – Factor 10 – The Measure for an Ecological Economy* Munich: Deutscher Taschenbuchverlag; 1997; (*The author argues that true sustainability requires a reduction in energy and resource consumption by the developed nations by a facto of 10.*).
12. Wenzel H, Hauschild M, Alting L. *Environmental Assessment of Products*. vol. 1 London: Chapman and Hall; 1997; (*Professor Alting leads Danish research in eco-design.*).

14.10 Exercises

E14.1. *Carbon footprint and embodied energy of materials*. Make a bar chart of CO_2 footprint divided by embodied energy, using data from the data sheets of Appendix A, Table A10, for

a. Cement
b. Low carbon steel
c. Copper

d. Polyvinylchloride (PVC)
e. Aluminium alloys and
f. Softwood

Which material has the highest ratio? Why? Note the median value of this ratio for most of the materials – it is almost constant and is useful when estimating carbon footprints from embodied energies and vice versa.

E14.2. *Embodied energies*.

a. Window frames are made from extruded aluminium. It is argued that making them instead from extruded PVC would give a product with a lower embodied energy and carbon footprint. If the section shape and thickness of the aluminium and the PVC windows are the same and both are made from virgin material, is the claim justified? You will find embodied energies and densities for the two materials in the data sheets of Appendix A, Tables A2 and A10. Use the mean values of the ranges given there.
b. Products like window frames would more usually be made from recycled aluminium. If the embodied energy of recycled aluminium is 30 MJ/kg, does the conclusion change?

E14.3. *Embodied energy*.

a. A range of office furniture includes a chunky hardwood table weighing 18 kg and a much lighter table with a 3.0 kg virgin aluminium frame and a 3.0 kg glass top. Which of the two tables has the lower embodied energy? Use data from the data sheets of Appendix A to find out.
b. If the alumina-glass table is made from recycled aluminium with an embodied energy of 30 MJ/kg, does the conclusion change?

E14.4. *Getting a feel for embodied energies*. Aluminium is made by the electrolysis of bauxite. The embodied energy of virgin aluminium is approximately (all embodied energies are approximate) 210 MJ/kg. What else could you do with this much electrical energy? Explore

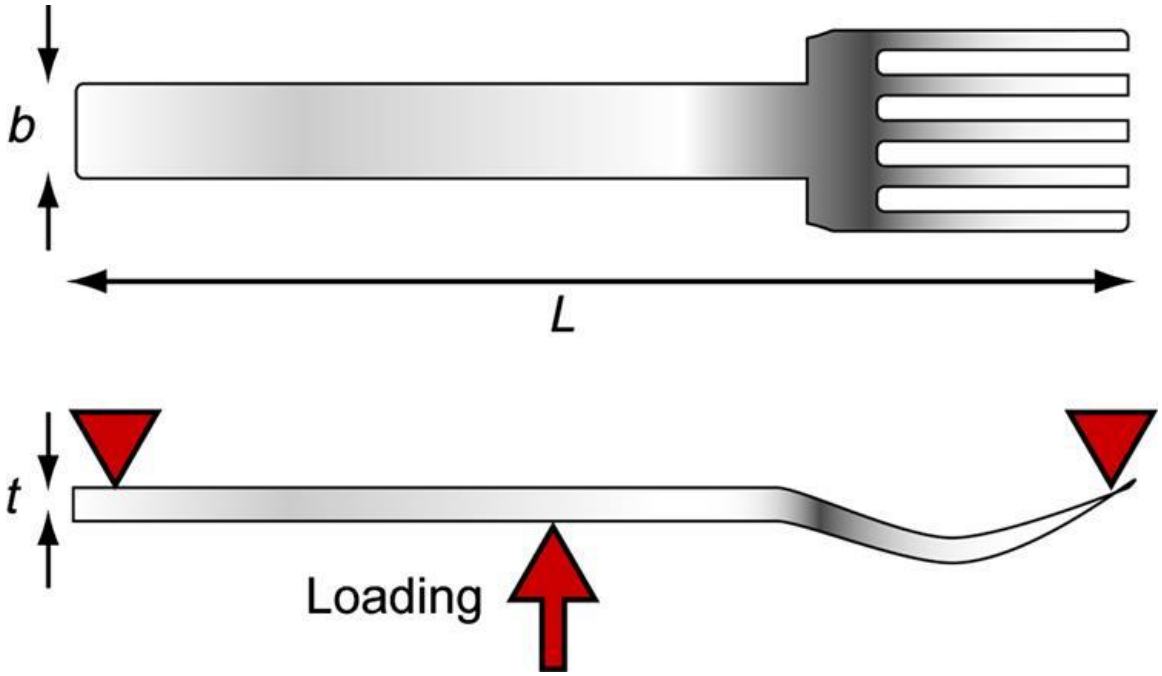
a. How much water would it boil?
b. How far would it drive an electric car if the average at-wheel energy to propel a small car is 0.6 MJ/km?

E14.5. *Minimizing embodied energy*. Use the $E - H_p\rho$ chart of Fig. 14.7 to find the polymer with a modulus E greater than 1 GPa and the lowest embodied energy per unit volume.

E14.6. *Comparing products by embodied energy*. A maker of polypropylene (PP) garden furniture is concerned that the competition is stealing market share by claiming that the 'traditional' material for garden furniture, cast iron, is less energy and CO_2 intensive than the PP. A typical PP chair weighs 1.6 kg; one made of cast iron weighs 8.5 kg.
   a. Use the data for these two materials in Appendix A, Table A10, to find out who is right – are the differences significant if the data for embodied energy are only accurate to ±25%?
   b. If the PP chair lasts 5 years and the cast iron chair lasts 25 years, does the conclusion change?

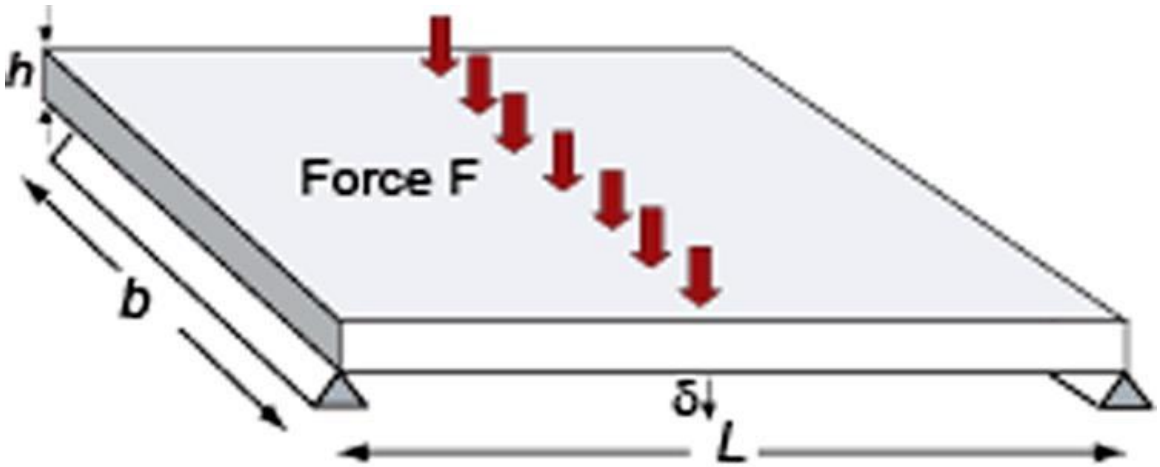
E14.7. *Materials with low embodied energy*. Identical casings for a power tool could be die-cast in aluminium or moulded in acrylonitrile butadiene styrene or polyester glass-fibre reinforced polymers (GFRP). Use the embodied-energy per unit volume bar chart of Figure 3.21 to decide which choice minimizes the embodied energy of the casing, assuming the same volume of material is used for each casing.

E14.8. *Disposable knives and forks*. Disposable knives and forks are ordered by an environmentally-conscious pizza house. The shape of each (and thus the length, width and profile) are fixed, but the thickness is free: it is chosen to give enough bending-stiffness to cut and impale the pizza without excessive flexure. The pizzeria proprietor wishes to enhance the greenness of his image by minimizing the energy content of his throw-away tableware, which could be moulded from polystyrene (PS) or stamped from aluminium sheet.



Establish an appropriate material index for selecting materials for energy-economic forks. Model the eating implement as a beam of fixed length L and width w, but with a thickness t that is free, loaded in bending, as in the figure. The objective-function is the volume of material in the fork times its energy content, $H_p\rho$, per unit volume (H_p is the embodied energy per kg, and ρ the density). The limit on flexure imposes a stiffness constraint (Appendix B, Section B3). Use this information to develop the index.
Flexure, in cutlery, is an inconvenience. Failure – whether by plastic deformation or by fracture – is more serious: it causes loss of function; it might even cause hunger. Repeat the analysis, deriving an index when a required strength is the constraint.

E14.9. *Materials for low embodied energy structures*. Show that the index for selecting materials for a strong panel with the dimensions shown in the figure, loaded in bending, with minimum embodied energy content is that with the largest value of



$$M = \frac{\sigma_y^{1/2}}{H_p \rho}$$

where H_p is the embodied energy of the material, ρ its density and σ_y its yield strength. To do so, rework the panel derivation in Chapter 4, Materials Selection – The Basics (Eq. 4.9) replacing the stiffness constraint with a constraint on failure load F requiring that it exceed a chosen value F^* where

$$F = C_2 \frac{I \sigma_y}{hL} > F^*$$

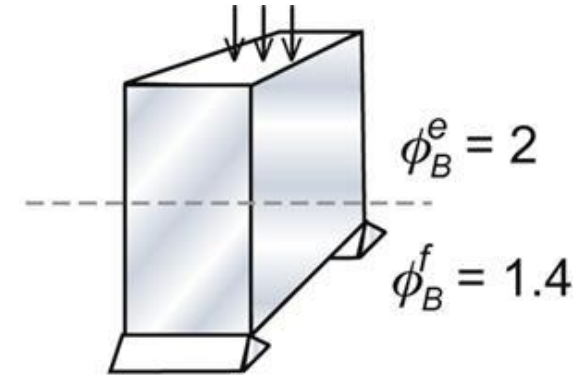
where C_2 is a constant and I is the second moment of area of the panel, $I = \frac{bh^3}{12}$.

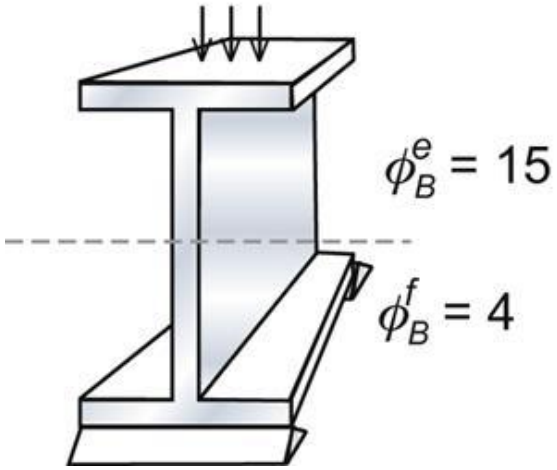
E14.10. *Indices for selection to minimize embodied energy*. Use the indices for the crash barriers (Eqs 14.9 and 14.10) with the charts for strength and density (Fig. 3.4) and strength and embodied energy (Fig. 14.8) to select materials for each of the barriers. Position your selection line to include one metal for each. Reject ceramics and glass on the grounds of brittleness. List what you find for each barrier.

E14.11. *Energy-efficient materials for transport*. The makers of a small electric car wish to make bumpers out of a moulded thermoplastic. Which index is the one to guide this selection if the aim is to maximize the range for a given battery storage capacity? Plot it on the appropriate chart selected form the text and make a selection.

E14.12. *Energy-efficient floor joists: embodied energy and shape*. Floor joists are beams loaded in bending. They can be made of wood, of steel, or of

steel-reinforced concrete, with the shape factors listed below. For a given bending stiffness and strength, which of these carries the lowest production-energy burden? The relevant data, drawn from the tables of Appendix A, are listed.



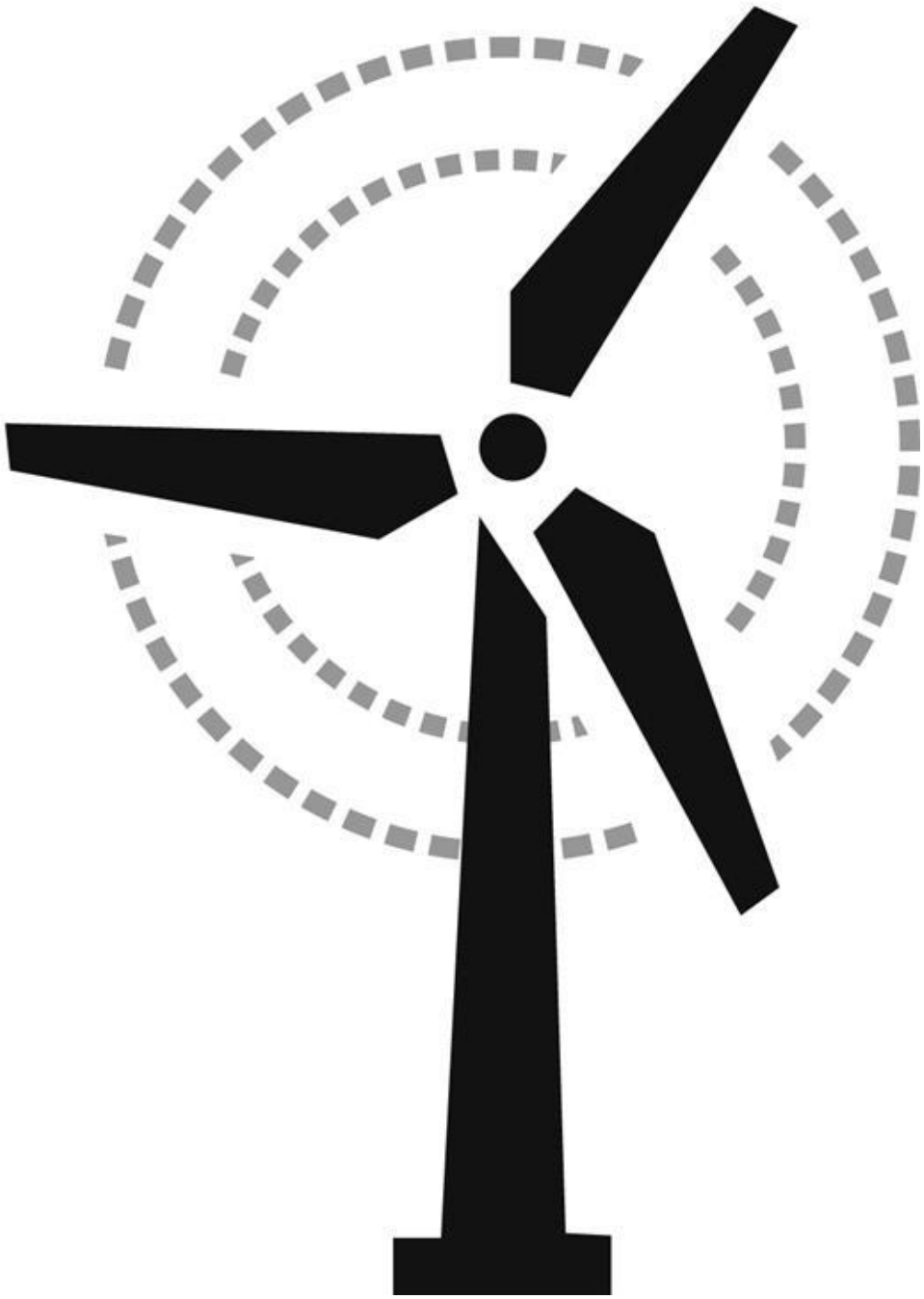


a. Start with stiffness. Locate from Eq. (10.20) of the text the material index for stiffness-limited, shaped beams of minimum mass. Adapt this to make the index for stiffness-limited, shaped beams of embodied energy by multiplying density ρ by the embodied energy/kg, H_p. Use the modified index to rank the three beams.
b. Repeat the procedure, this time for strength, creating the appropriate index for strength-limited shaped beams at minimum energy content by adapting Eq. (10.28).

What do you conclude about the relative energy-penalty of design with wood and with steel?

Material	Density ρ (kg/m^3)	Modulus E (GPa)	Strength σ_f (MPa)	Energy H_p (MJ/kg)	Shape Factor ϕ_B^e	Shape Factor ϕ_B^f
Soft wood	700	10	40	7.5	2	1.4
Concrete	2900	35	10	5	2	1.4
Steel	7900	210	200	30	15	4

E14.13. *Power systems: energy pay-back time*. The table below shows data for the main materials used in a wind turbine: the approximate mass used per kW installed and the embodied energy per kg of these materials.



Bill of Materials	Mass Per Installed Unit of Power Capacity (kg/kW)	Embodied Energy of Materials (MJ/kg)
Aluminium	2	200
CFRP	7.5	480
Concrete	500	1.8
Copper	1.5	65
GFRP	7.5	160
Steel	120	25
Neodymium (magnets)	0.012	85
Plastics	5	80

a. Use these data to estimate the total embodied energy of the materials in a 1 MW wind turbine.
b. The capacity factor for a typical land-based wind turbine is 20%. (Capacity factor is the equivalent fraction of the time that the turbine is operating at its rated capacity; thus a 1 MW turbine produces, on average, the equivalent of 0.2 MW of electrical power). How long will it take a 1 MW turbine to return the embodied energy of the materials of which it is made?

E14.14. *Global population*. The global population, P, is 7.5 billion, rising exponentially with a growth rate $\alpha = 1.13\%$ per year. At this growth rate, how long will it take to double to 15 billion?

E14.15. *Global biopolymer population*. Global biopolymer population, P, is currently 1 million tonnes per year and is said to be rising exponentially with a growth rate $\alpha = 12\%$ per year. At this growth rate, how long will it take to double?

E14.16. *Cumulative consumption*. The opening paragraph of this chapter contained the statement that at a global growth rate of 3% per year we will mine, process, use and dispose of more 'stuff' in the next 5 years than in the entire industrial history of humankind. Prove it, assuming that the industrial history of humankind started with the industrial revolution, 300 years ago.

CHAPTER 15

Materials and Industrial Design

Abstract

Good design works. Excellent design also gives pleasure. Pleasure derives from form, colour, texture, feel, and the associations that these invoke. Pleasing design says something about itself, and generally speaking, honest statements are more satisfying than deception, though eccentric or humorous designs can be appealing too. Materials play a central role in design. A major reason for introducing new materials is the greater freedom of design that they allow. This chapter introduces some of the ideas of industrial design, emphasising the role of materials. It ends with case studies.

Keywords

Industrial design; functionality; usability and satisfaction; product character; product personality; aesthetics; associations; perceptions; materials and the senses

15.1 Introduction and Synopsis

Good design works. Excellent design also gives pleasure.

Pleasure derives from form, colour, texture, feel, and the associations that these invoke. Pleasing design says something about itself, and generally speaking, honest statements are more satisfying than deception, though eccentric or humorous designs can be appealing too.

Materials play a central role in this. A major reason for introducing new materials is the greater freedom of design that they allow. Metals, in the past century, allowed structures that could not have been built before: cast

iron, the Crystal Palace; wrought iron, the Eiffel Tower; drawn steel cable, the Golden Gate Bridge, all undeniably beautiful. Polymers lend themselves to bright colours, satisfying textures and great freedom of form. They have opened new styles of design, of which some of the best examples are found in household appliances: food mixers, hair dryers, mobile phones, MP3 players and vacuum cleaners make extensive and imaginative use of materials to allow styling, weight, feel and form that give pleasure.

Those who concern themselves with this aesthetic dimension of engineering are known, rather confusingly, as 'industrial designers'. This chapter introduces some of the ideas of industrial design, emphasising the role of materials. It ends with case studies. But first a word of caution.

Previous chapters have dealt with systematic ways of choosing material and processes. 'Systematic' means that if *you* do it and *I* do it, following the same procedure, we will get the same result, and that the result, next year, will be the same as it is today. Industrial design is not, in this sense, systematic. Success, here, involves sensitivity to fashion, culture and educational background and is influenced (manipulated, even) by advertising and promotion. The views of this chapter are partly those of writers who seem to me to say sensible things about industrial design and partly my own. You may not agree with them, but if they make you think about designing to give pleasure, the chapter has done its job.

15.2 The Requirements Pyramid

The pen with which I am writing this chapter cost $5 (Fig. 15.1, upper image). If you go to the right shop you can find a pen that costs well over $1000 (lower image). Does it write 200 times better than mine? Unlikely – mine writes perfectly well. Yet there is a market for such pens. Why?



FIGURE 15.1 Pens, inexpensive and expensive.

A product has a *cost* – the outlay in manufacture, distribution and marketing it. It has a *price* – the sum at which it is offered to the consumer. And it has a *value* – a measure of what the consumer thinks it is worth. The expensive pens command the price they do because the consumer perceives their value to justify it.

What determines value? Three things. *Functionality*, provided by sound technical design, clearly plays a role. The requirements pyramid of Fig. 15.2 has this as its base: the product must work properly, be safe and be economical. Functionality alone is not enough: the product must be easy to understand and operate, and these are questions of *usability*, the second tier of the figure. The third, completing the pyramid, is the requirement that the product gives *satisfaction*: that it enhances the life of those who own or use it. The lower part of the pyramid tends to be called 'Technical design', the upper part, 'Industrial design', which suggests they are separate activities. It is better to think of think of all three tiers as part of a single process that we shall call 'Product design'.

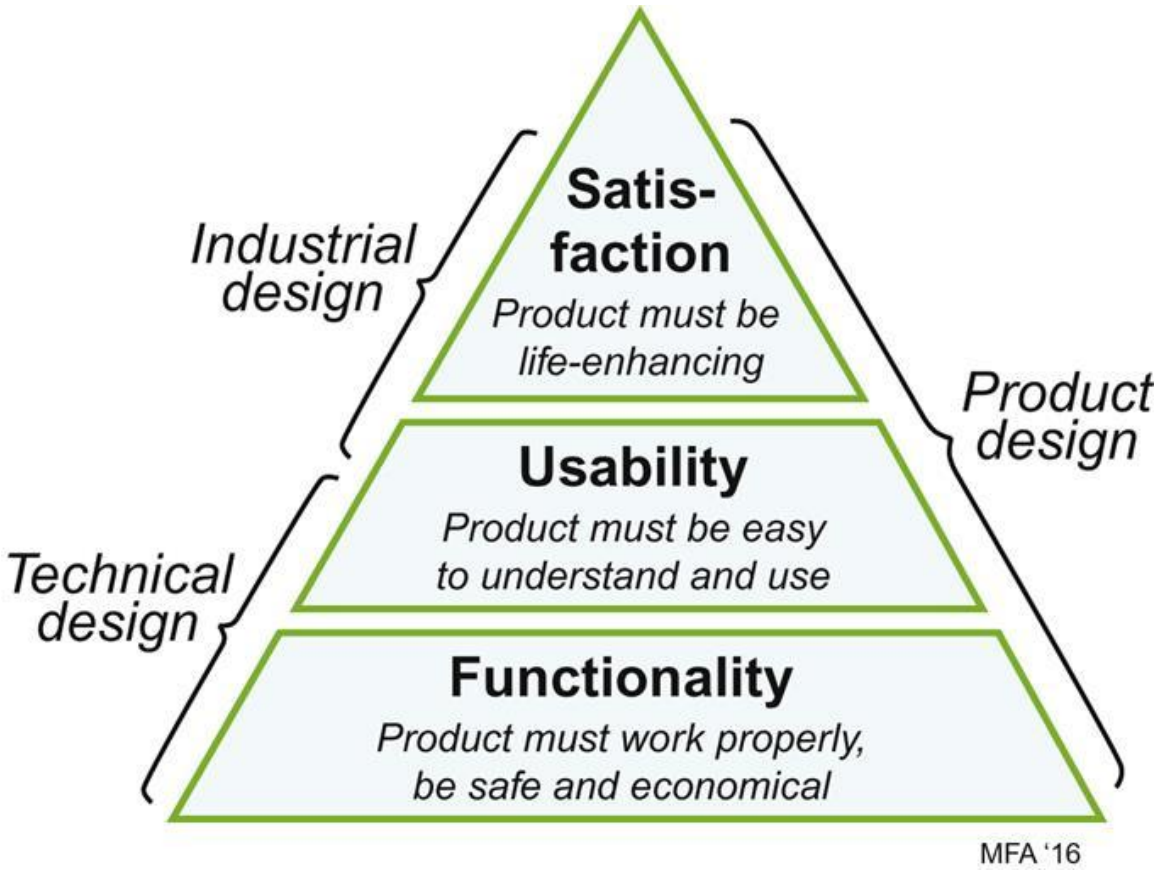


FIGURE 15.2 The requirements pyramid.

The value of a product is a measure of the degree to which it meets (or exceeds) the expectation of the consumer in all three of these – functionality, usability and satisfaction. Think of this as the character of the product. It is very like human character. An admirable character is one who functions well, interacts effectively and is rewarding company. An unappealing character is one that does none of these. An odious character is one that does one or more of them in a way so unattractive that you cannot bear to be near them.

Products are the same. All the pens in Fig. 15.1 function well and are easy to use. The huge difference in price implies that the lower pair provide a degree of satisfaction not offered by the upper ones. The most obvious difference between them is in the materials of which they are made – the upper pair of moulded acrylic, the lower pair of gold, silver and enamel. Acrylic is the material of toothbrush handles, something you throw away after use. Gold and silver are the materials of precious jewellery; they have associations of craftsmanship, of heirlooms passed from one generation to the next. Well, that's part of the difference, but

there is more. To find it we need to address a key question – what creates product character?

15.3 Product Character

Fig. 15.3 shows a way of dissecting product character. It is a map of the ideas we are going to explore; like all maps there is a lot of detail, but we need it to find our way. In the centre is information about the PRODUCT itself: the basic design requirements, its function, its features. The way these are thought through and developed is conditioned by the *context*, shown in the circle above it. The context is set by the answers to the questions in the uppermost box: *Who? Where? When? Why?* Consider the first of these: *Who?* A designer seeking to create a product attractive to homemakers will make choices that differ from those for a product intended for children, or for elderly people, or for athletes. *Where*? A product for use in the home requires a different choice of material and form than one to be used – say – in a school or hospital. *When?* A product intended for occasional use is designed in a different way than one that is used all the time; one for formal occasions differs from one for informal use. *Why?* A product that is primarily utilitarian involves different design decisions than one that is largely a lifestyle statement. The context influences and conditions all the decisions that the designer takes in finding a solution. It sets the *mood*.[1]

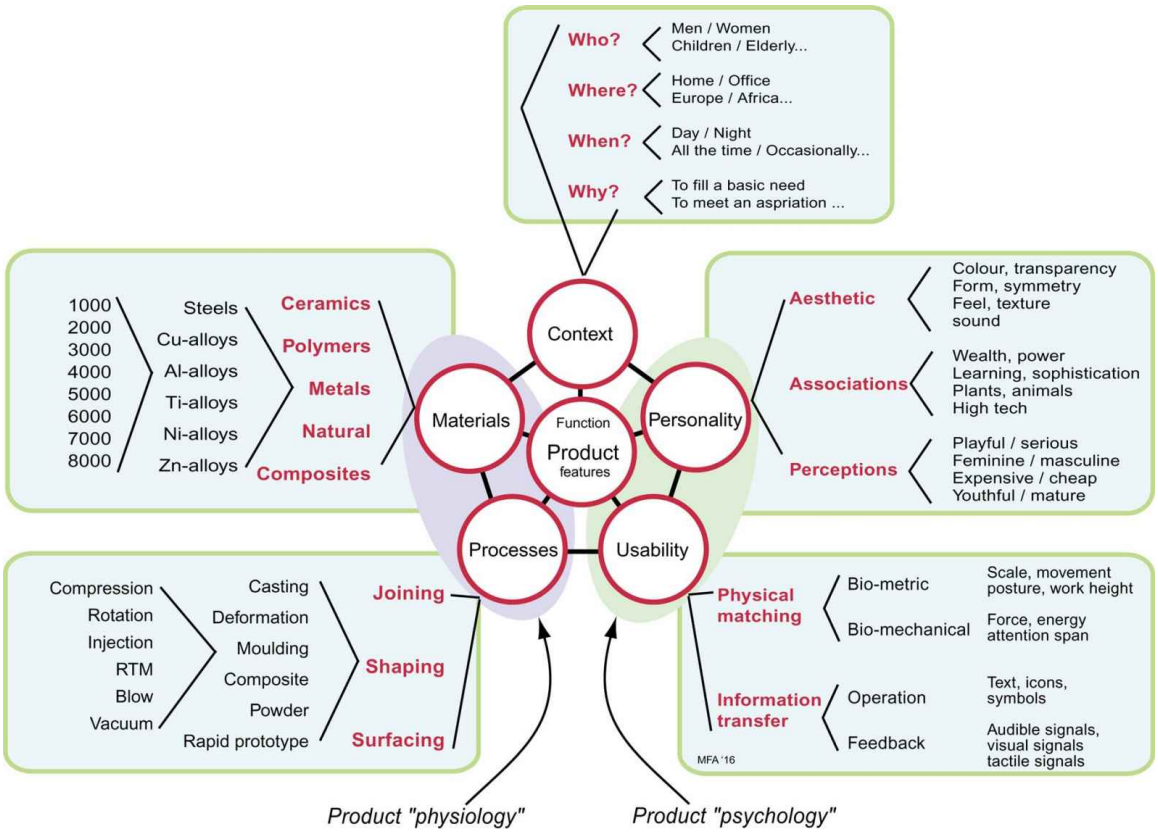


FIGURE 15.3 The breakdown of product character. *Context* defines the intentions or 'mood'; *materials* and *processes* create the flesh and bones; *usability* determines how the user and the product interact, and the aesthetics, associations and perceptions of the product create its *personality*.

On the left of PRODUCT lie packages of information about the MATERIALS and the PROCESSES used to shape, join and finish it. Each illustrates the menu, so to speak, from which the choices can be made. They are the families, classes and members that we first met in Chapter 4, Materials Selection – The Basics. Choosing these to provide functionality – the bottom of the pyramid – was the subject of this book so far. Material and process give the product its tangible form, its flesh and bones so to speak; they create the *product physiology*.

On the right of Fig. 15.3 are two further packages of information. The lower one – USABILITY – characterizes the ways in which the product communicates with the user: the interaction with their sensory, cognitive and motor functions. Product success requires a mode of operation that, as far as possible, is intuitive and an interface that communicates the state of the product and its response to user actions by visible, acoustic or tactile

response. It is remarkable how many products fail in this, and by doing so, exclude many of their potential users. There is, today, an awareness of aspect of design, giving rise to research on *inclusive design*: design to make products that can be used by a larger spectrum of the population.

One package remains: the one labelled PERSONALITY. Product personality derives from *aesthetics*, *associations* and *perceptions*, three words that need explanation.

Anaesthetics dull the senses. Aesthetics do the opposite: they stimulate the five senses: sight, hearing, touch, taste, smell, and via them, the brain. The first row of the personality box elaborates: we are concerned here with colour, form, texture, feel, smell and sound. Think of a new car: its styling, its smell the sound its doors make on closing. They are no accident. Car makers spend millions to make them as they are.

A product also has associations – the second row of the box. Associations are the things the product reminds you of, the things it suggests. The Land Rover and other SUVs have forms and (often) colours that mimic those of military vehicles. The streamlining of US cars of the 1960s and 1970s carried associations of aerospace. It may be an accident that the VW Beetle has a form that suggests the insect, but the others are no accident; they were deliberately chosen by the designer to appeal to the consumer group (the *Who?*) at which the product was aimed.

Finally, the most abstract quality of all, perceptions. Perceptions are the reactions the product induces in an observer, the way it makes you *feel*. Here there is room for disagreement; the perceptions of a product change with time depends on the culture and background of the observer. Yet in the final analysis it is the perception that causes the consumer, when choosing between a multitude of similar models, to prefer one above the others; it creates the 'must have' feeling (see the cover image of this chapter). Table 15.1 lists some perceptions with their opposites, in order to sharpen the meaning. They derive from product reviews and magazines specialising in product design; they are a part of a vocabulary, one that is used to communicate views about product character.

Table 15.1

Some Perceived Attributes of Products

Perceptions With Opposites	
Aggressive – passive	Extravagant – restrained
Cheap – expensive	Feminine – masculine
Classic – trendy	Formal – informal
Clinical – friendly	Hand-crafted – mass produced
Clever – silly	Honest – deceptive
Common – exclusive	Humorous – serious
Decorated – plain	Informal – formal
Delicate – rugged	Irritating – loveable
Disposable – lasting	Lasting – disposable
Dull – sexy	Mature – youthful
Elegant – clumsy	Retro – futuristic

15.4 Using Materials and Processes to Create Product Personality

Do materials, of themselves, have a personality? There is a school of thinking that holds as a central tenant that materials must be used 'honestly'. By this they mean that deception and disguise are unacceptable – each material must be used in ways that expose its intrinsic qualities and natural appearance. It has its roots in the tradition of craftsmanship – the potters' use of clays and glazes, the carpenters' use of woods and tools, the skills of silversmiths and glassmakers in crafting beautiful objects that exploit the unique qualities of the materials with which they work.

This is a view to be respected but it is not the only one. Design integrity is a quality that consumers value, but they also value other qualities: humour, sympathy, surprise, provocation, even shock. You don't have to look far to find a product that has one of these, and often it is achieved by using materials in ways that deceive. Polymers are frequently used in this way – their adaptability invites it. And, of course, it is partly a question of definition – if you say that a characterizing attribute of polymers is their ability to mimic other materials, then using them in this way *is* honest.

Materials and the senses: aesthetic attributes. Aesthetic attributes are those that relate to the senses: touch, sight, hearing, taste and smell (Table 15.2). Almost everyone would agree that metals feel 'cold'; that cork feels 'warm'; that a wine glass, when struck, 'rings'; that a pewter mug sounds 'dull', even 'dead'. A polystyrene water glass can look indistinguishable from one made of glass, but pick it up and it feels lighter, less cold, less rigid; tap it and it does not sound the same. The impression it leaves is so different from glass that, in an expensive restaurant, it would be completely unacceptable. Materials, then, have certain characterizing aesthetic attributes. Let us see if we can pin these down.

Table 15.2

Some Aesthetic Attributes of Materials

Sense	Attribute	Sense	Attribute
Touch	Warm Cold Soft Hard Flexible Stiff	**Hearing**	Muffled Dull Sharp Resonant Ringing High-pitched Low pitched
Sight	Optically clear Transparent Translucent Opaque Reflective Glossy Matte Textured	**Taste, Smell**	Bitter Sweet Tart Rancid

Touch: soft-hard/warm-cold. Steel is 'hard'; so is glass; diamond is harder the either of them. Hard materials do not scratch easily; indeed, they can be used to scratch other materials. They generally accept a high polish, resist wear and are durable. The impression that a material is hard is directly related to its Vickers hardness H. Here is an example of a sensory attribute that relates directly to a technical one.

'Soft' sounds like the opposite of 'hard' but it has more to do with modulus E than with hardness H. A soft material deflects when handled, it gives a little, it is squashy, but when it is released it returns to its original shape (Fig. 15.4). Elastomers (rubbers) feel soft; so do polymer foams. Both have moduli that are 100 to 10,000 lower than ordinary 'hard' solids; it is this that makes them feel soft. 'Soft to hard' is used as one axis of Fig. 15.5. It uses the quantity $\sqrt{EH}$ as a measure, scaled to a range of 1–10.

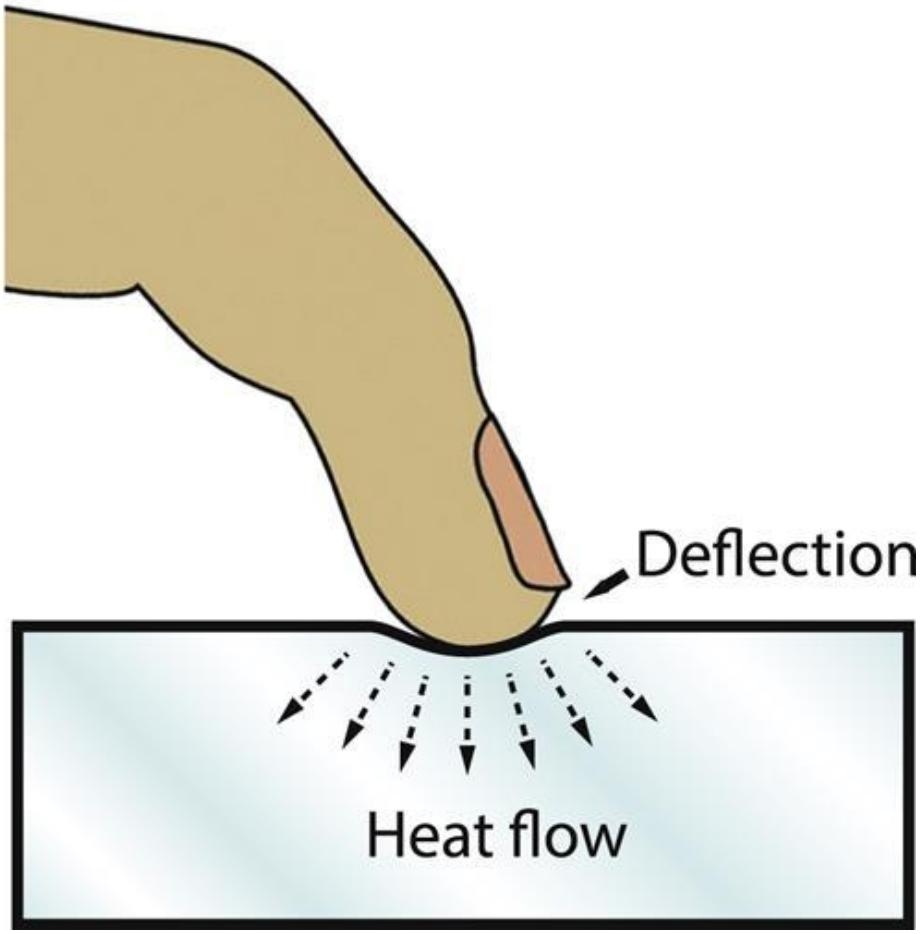


FIGURE 15.4 Tactile properties.

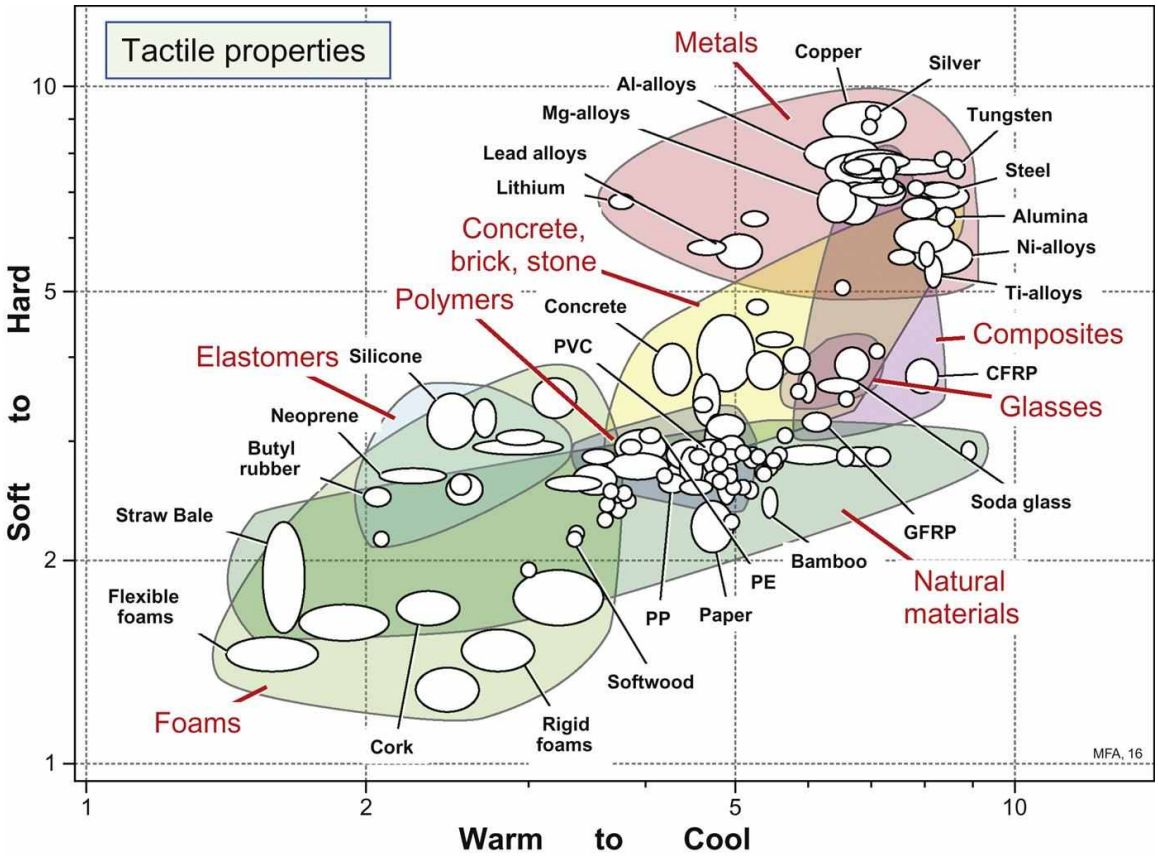


FIGURE 15.5 Tactile qualities of materials. Foams and many natural materials are soft and warm; metals, ceramics and glasses are hard and cold. Polymers lie in between.

A material feels 'cold' to the touch if it conducts heat away from the finger quickly; it is 'warm' if it does not (Fig. 15.4 again). This has something to do with its thermal conductivity λ but there is more to it than that – it depends also on its specific heat C_p. Heat flows from the finger into the surface such that after time t a depth x of material has been warmed significantly while its remoter part has not. Solutions to transient heat-flow problems of this class (Appendix B, Section B15) all have the feature that

$$x \approx \sqrt{at}$$

where a is the thermal diffusivity of the material

$$a = \frac{\lambda}{\rho C_p}$$

and where λ is the thermal conductivity, C_p is the specific heat and ρ, is the density. The quantity of heat that has left each unit area of finger in time t scales as

$$Q = x\rho C_\rho = \sqrt{\rho\lambda C_p}$$

If Q is small the material feels warm; if large, it feels cold. Thus a measure of the perceived coldness or warmth of a material is the quantity $\sqrt{\lambda C_p \rho}$. It is shown as the other axis of Fig. 15.5, which nicely displays the tactile properties of materials. Polymer foams and low-density woods are warm and soft; so are balsa and cork. Ceramics and metals are cold and hard; so is glass. Polymers and composites lie in between.

Hearing: pitch and brightness. The frequency of sound (pitch) emitted when an object is struck relates to its material properties. A measure of this pitch, $\sqrt{E/\rho}$, is used as one axis of Fig. 15.6. Frequency is not the only aspect of acoustic response – the other has to do with the damping or loss coefficient, η. A highly damped material sounds dull and muffled; one with low damping rings. Acoustic brightness – the inverse of damping – is used as the other axis of Fig. 15.6. It groups materials that have similar acoustic response.

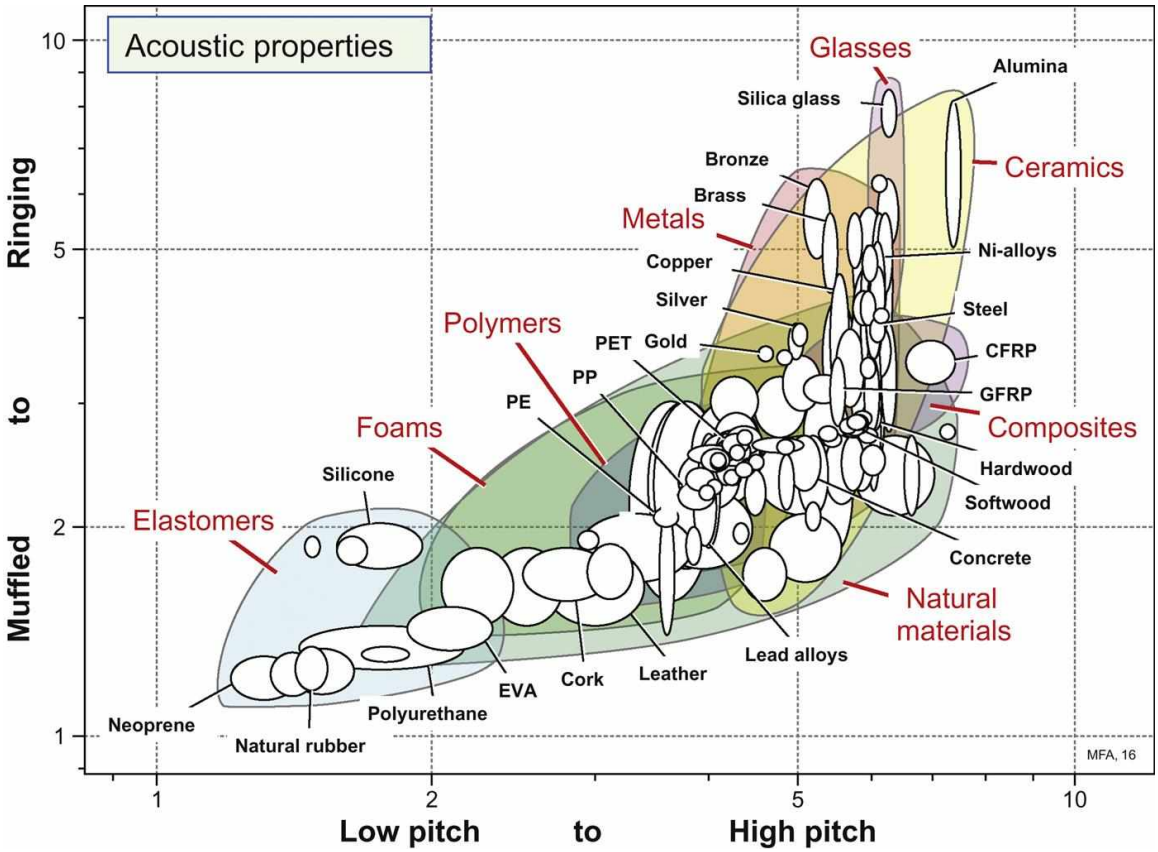


FIGURE 15.6 Acoustic properties of materials. A wine glass 'rings' because glass in an acoustically bright material with a high natural pitch; a plastic glass emits a dull 'ping' because polymers are much less bright and – in the same shape – vibrate at a lower frequency. Materials at the top right make good bells; those at the bottom left are good for damping sound.

Bronze, glass and steel ring when struck, and the sound they emit has – on a relative scale – a high pitch; they are used to make bells. Alumina, on this ranking, has bell-like qualities. Rubber, foams and many polymers sound dull and they vibrate at lower frequencies than metals; they are used for sound damping. Lead, too, is dull and low-pitched; it is used to clad buildings for acoustic insulation.

Robustness: Abrasion Resistance and Resilience. Abrasion resistance is resistance to scratching; it is directly related to the material property 'hardness', H. Resilience is ability to withstand misuse, particularly impact. That is related the ability of a material to absorb energy when struck – a resilient object survives a hammer blow, a brittle one shatters. It is measured by the technical attribute, 'Toughness', defined by

$$G_{1C} = \frac{K_{1C}^2}{E}$$

Abrasion resistance and resilience are used as the axes of Fig. 15.7.

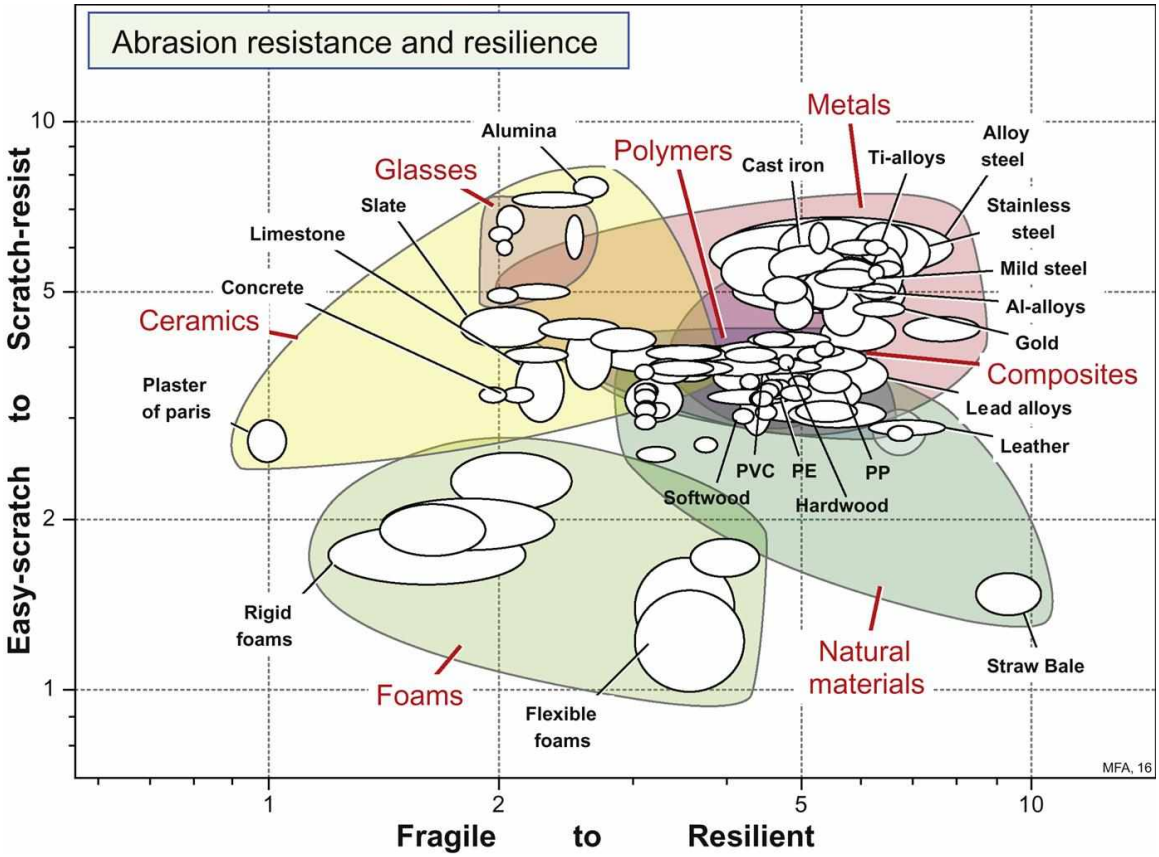


FIGURE 15.7 Scratch resistance and resilience. Ceramics are scratch resistant but they are not resilient. Polymers are resilient but the scratch easily. Metals – particularly steels – are scratch resistant and resilient.

Sight: transparency, colour, reflectivity. Metals are opaque because they contain free electrons. Most ceramics do not contain free electrons but are still either opaque or translucent because they are polycrystalline and the crystals scatter light. Glasses and single crystals of some ceramics are transparent. Polymers have the greatest diversity of optical properties, ranging from transparency to complete opacity. Transparency is commonly described by a four-level ranking that uses easily understood everyday words: 'opaque', 'translucent', 'transparent' and 'water-clear'.

Colour can be quantified by analysing spectra but this – from a design standpoint – doesn't help much. A more effective method is one of colour matching, using colour charts such as those provided by Pantone[2]; once a match is found it can be described by the code that each colour carries. Finally there is reflectivity, an attribute that depends partly on material and partly on the state of its surface. Like transparency, it is commonly described by a ranking: dead matte, eggshell, semi-gloss, gloss and mirror.

The three charts we have examined show that each material class has a certain recognisable aesthetic character. Ceramics are hard, cold, high-pitched and acoustically bright. Metals, too, are relatively hard and cold but although some, like bronze, ring when struck, others – like lead – are dull. Polymers and foams are most nearly like natural materials – warm, soft, low-pitched and muffled, though some have outstanding optical clarity and almost all can be coloured. But their low hardness means that they scratch easily, and lose their gloss.

A product acquires some of the attributes of the material from which it is made, an effect that designers recognise and use when seeking to create a personality. A stainless steel fascia, whether it be in a car or on a hi-fi system, has a different personality than one of carbon-fibre reinforced polymers (CFRP), of polished wood or of leather, and that in part is because the product has acquired some of the aesthetic qualities of the material of which it is made.

Materials and the Mind: Associations and Perceptions

So a material certainly has aesthetic qualities – but can it be said to have a personality? At first sight, no – it only acquires one when used in a product. Like an actor, it can assume many different personalities, depending on the role it is asked to play. Wood in fine furniture suggests craftsmanship, but in a packing case, cheap utility. Glass in the lens of a camera has associations of precision engineering, but in a beer bottle, that of disposable packaging. Even gold, so often associated with wealth and power, has different associations when used in microcircuits: that of technical functionality.

But wait. The object in Fig. 15.8 has its own sombre association. It appears to be made of polished hardwood – the traditional material for such things. If you had to choose one, you would probably not think

polished hardwood inappropriate. But suppose I told you it was made of *polystyrene foam* – would you feel the same? Suddenly it becomes like a bin, a wastebasket, inappropriate for its dignified purpose. Materials, it seems, *do* have personality.



FIGURE 15.8 A coffin.

Expression through material. Think of wood. It is a natural material with a grain that gives it a surface texture, pattern, colour and feel that other materials do not have. It is tactile – it is perceived as warmer than many other materials, and seemingly softer. It is associated with characteristic sounds and smells. It has a tradition; it carries associations of craftsmanship. No two pieces are exactly alike; the woodworker selects the piece on which to work because of its grain and texture. Wood enhances value: the interior of cheap cars is plastic, that of expensive ones is burr-walnut and mahogany. And it ages well, acquiring additional character with time; objects made of wood are valued more highly when they are old than when they are new. There is more to this than just aesthetics; there are the makings of a personality, to be brought out by the designer, certainly, but there none the less.

And metals … Metals are cold, clean, and precise. They ring when struck. They reflect light – particularly when polished. They are accepted and trusted: machined metal looks strong, and its very nature suggests it has been *engineered*. Metals are associated with robustness, reliability, permanence. Their strength allows slender structures – the cathedral-like space of railway stations or the span of bridges. They can be worked into

flowing forms like intricate lace or cast into solid shapes with elaborate, complex detail. The history of humans and of metals is intertwined – the titles 'bronze age' and 'iron age' tell you how important metals were – and their qualities are so sharply defined that that they have become ways of describing human qualities – an iron will, a silvery voice, a golden touch, a leaden look. And, like wood, metals can age well, acquiring a patina with age that makes them more attractive than when newly made – the bronze of sculptures, the pewter of mugs, the lead of roofs.

Ceramics and glass? They have an exceptionally long tradition – think of Greek pottery and Roman glass. They accept almost any colour. Their total resistance to scratching, abrasion, discoloration and corrosion gives them a certain immortality, threatened only by their brittleness. They are – or were – the materials of great craft-based industries: the glass of Venice, the porcelain of Meissen, the pottery of Wedgwood, valued at certain times more highly than silver. But at the same time they can be robust and functional – think of floor tiles and car windscreens. The transparency of glass gives it an ephemeral quality – sometimes you see it, sometimes you don't. It interacts with light, transmitting it, refracting it, reflecting it. And ceramics today have additional associations – those of advanced technology: kitchen stove-tops, high pressure/high temperature valves, space shuttle tiles … materials for extreme conditions.

And finally polymers. 'A cheap, plastic imitation', it used to be said – and that is a hard reputation to live down. It derives from the early use of plastics to simulate the colour and gloss of Japanese handmade pottery, much valued in Europe. Commodity polymers *are* cheap. They are easily coloured and moulded (that is why they are called 'plastic'), making imitation easy. Unlike ceramics, their gloss is easily damaged and their colours fade – they do not age gracefully. You can see where the reputation came from. But is it justified? No other class of material can take on as many characters as polymers: coloured, they look like ceramics; printed, they can look like wood or textile; metalized, they look exactly like metal. They can be as transparent as glass or as opaque as lead, as flexible as rubber or, when fibre-reinforced, as stiff as aluminium. Plastics emulate precious stones in jewellery, glass in drinking glasses and glazing, wood in counter tops, velvet and fur in clothing, and even grass. But despite this chameleon-like behaviour they *do* have a certain personality: they feel warm – much warmer than metal or glass. They are adaptable – that is part of their special character; and they lend themselves, particularly, to brightly coloured, light-hearted, even humorous, design.

But their very cheapness creates problems as well as benefits: our streets, county side and rivers are littered with discarded plastic packaging that decays only very slowly.

15.5 Case Studies: Analysing Product Personality

The ways in which material, processes, usability and personality combine to create a product character tuned to the context or 'mood' are best illustrated by examples. Fig. 15.9 shows the first. The lamp in the upper image is designed for the office. It is angular, functional, creamy-grey, and it is heavy. Its form and colour echo those of computer consoles and keyboards, creating associations of contemporary office technology. Its weight conveys a sense of stability, robustness, efficiency and fitness for task – but for tasks in the workplace, not in the bedroom. Materials and processes have been chosen to reinforce these associations and perceptions. The enamelled frame is pressed and folded sheet steel, the base weight is cast iron, the reflector is stainless steel set in a high-impact ABS enclosure.





FIGURE 15.9 Lamps. The two have similar technical ratings but differ completely in their personalities.

The lamp in the lower part of Fig. 15.9 has a similar technical rating of that above; and it has the same functionality and usability. But there the resemblance ends. This product is not designed for the busy executive but for children (and adults that still enjoy being children), to be used the playroom or bedroom. It has a contoured form, contrasting translucent colours, and it is light. It is made of coloured acrylic in translucent grades so that the outside of the lamp glows warmly when it is lit. Its form is partly derived from nature, partly from cartoons and comic strips, giving it a light-hearted character. I perceive it as playful, funny, cheerful and clever – but also as eccentric and easily damaged. You may perceive it in other ways – perception is a personal thing; it depends where you are coming from. Skilled designers manipulate perception to appeal to the user group that they wish to attract.

Fig. 15.10 shows a second example. Here are two contrasting ways of presenting electronic home entertainment systems. Above: a music centre aimed at successful professionals with disposable income, comfortable with (or addicted to) advanced technology for whom only the best is good enough. The linear form, the use of primitives (rectangles, circles, cylinders, cones) and the matt silver and black proclaim that this product has not just been *made*, it has been *Designed* (big *D*). The formal geometry and finish suggest precision instruments, telescopes, electron microscopes and the shapes resemble those of organ pipes (hence associations of music, of culture). The perception is that of cutting-edge technology, a symbol of discriminating taste. The form has much to do with this associations and perceptions, but so too do the materials: brushed aluminium, stainless steel and black enamel – these are not materials you choose for a cuddly toy.



FIGURE 15.10 Consumer electronics. The product in the upper image is aimed at a different consumer group than

those below.

Fig. 15.10 presents electronics in another way. This is a company that has retained market share, even increased it, by *not* changing, at least as far as appearance is concerned (I had a radio 50 years ago that looked exactly like this). The context? Clearly, the home, perhaps aimed at consumers who are uncomfortable with modern technology (though the electronics in these radios is modern enough) or who simply feel that it clashes with the domestic environment. Each radio has a simple form, it is pastel-coloured, it feels soft and warm. It is the materials that make the difference: these products are clothed in suede or leather in six or more colours. The combination of form and material create associations of comfortable furniture, leather purses and handbags, (hence, luxury, comfort, style), the past (hence, stability) and perceptions of solid craftsmanship, reliability, retro-appeal, traditional but durable design.

So there is a character hidden in a material even before it has been made into a recognizable form – a sort of embedded personality, a shy one, not always obvious, easily concealed or disguised, but one that, when appropriately manipulated, imparts its qualities to the design. It is for this reason that certain materials are so closely linked to certain *design styles*. S*tyle* is short-hand for a manner of design with a shared set of aesthetics, associations and perceptions. The Early Industrial style (1800–90)[3] embraced the technologies of the Industrial Revolution, using cast iron and steel, often elaborately decorated to give it a historical façade. The Arts and Crafts movement (1860–1910) rejected this, choosing instead natural materials and fabrics to create products with the character of traditional hand-crafted quality. Art Nouveau (1890–1918), by contrast, exploited the fluid shapes and durability made possible by wrought iron and cast bronze, the warmth and textures of hard wood and the transparency of glass to create products of flowing, organic character. The Art Deco movement (1918–35) extended the range of materials to include for the first time plastics (Bakelite and Catalin) allowing production both of luxury products for the rich and also mass-produced products for a wider market. The simplicity and explicit character of Bauhaus designs (1919–33) is most clearly expressed by the use of chromed steel tubing, glass and moulded plywood. Plastics first reach maturity in product design in the cheeky iconoclastic character of the Pop-Art style (1940–60). Since then the range of materials has continued to increase, but their role in helping to mould product character remains.

15.6 Summary and Conclusions

What do we learn? The element of satisfaction is central to contemporary product design. It is achieved through an integration of good technical design to provide functionality, proper consideration of the needs of the user and imaginative industrial design to create a product that will appeal to the consumers at whom it is aimed.

Materials play a central role in this. Functionality is dependant on the choice of proper material and process to meet the technical requirements of the design safely and economically. Usability depends on the visual and tactile properties of materials to convey information and respond to user actions. Above all, the aesthetics, associations and perceptions of the product are strongly influenced by the choice of the material and its processing, imbuing the product with a personality that, to a greater or lesser extent, reflects that of the material itself.

Consumers look for more than functionality in the products they purchase. In the sophisticated markets of developed nations, the 'consumer durable' is a thing of the past. The challenge for the designer no longer lies in meeting the functional requirements alone, but in doing so in a way that also satisfies aesthetic and emotional needs. The product must carry the image and convey the meaning that the consumer seeks: timeless elegance, perhaps; or racy newness. One Japanese manufacturer goes so far as to say: '*Desire* replaces *need* as the engine of design'.

Not everyone, perhaps, would wish to accept that. So we end with simpler words – the same ones with which we started. Good design works. Excellent design also gives pleasure. The imaginative use of materials provides it.

15.7 Further Reading

If you found this chapter interesting and would like to read more, you will find the ideas it contains developed more fully in the first book listed under Further Reading.

1. Ashby MF, Johnson K. *Materials and Design – The Art and Science of Materials Selection in Product Design* third ed. Oxford: Butterworth Heinemann; 2014; ISBN 978-0-08-098205-2. (*A text that develops further the ideas outlined in this chapter*.).

2. Clark P, Freeman J. *Design, a Crash Course* New York, NY: The Ivy Press Ltd, Watson-Guptil Publications, BPI Communications Inc.; 2000; ISBN 0-8230-0983-1. (*An entertainingly-written scoot through the history of product design from 5000BC to the present day*.).
3. Coates D. *Watches Tell more than the Time* New York, NY: McGraw Hill; 2003; ISBN 0-07-136243-6. (*An analysis of the aesthetics, associations and perceptions of products past and present with examples, many drawn from automobile design*.).
4. Dormer P. *Design Since 1945* London: Thames and Hudson; 1993; ISBN 0-500-20269-9. (*A well-illustrated and inexpensive paperback documenting the influence of industrial design in furniture, appliances and textiles – a history of contemporary design that complements the wider-ranging history of Haufe (1998), q.v.*).
5. Figuerola M, Lai Q, Ashby M. *The CES EduPack Products, Materials and Processes Database – a White Paper* Cambridge: Granta Design; 2016; (www.grantadesign.com/education). (*The White Paper introduces a database of products, each linked to the materials and processes used to make it, designed to support the teaching of product design*.).
6. Forty A. *Objects of Desire – Design in Society Since 1750* London: Thames and Hudson; 1986; ISBN 0-500-27412-6. (*A refreshing survey of the design history of printed fabrics, domestic products, office equipment and transport system. The book is mercifully free of eulogies about designers, and focuses on what industrial design does, rather than who did it. The black and white illustrations are disappointing, mostly drawn from the late 19th or early 20th centuries, with few examples of contemporary design*.).
7. Haufe T. *Design, a Concise History* London (originally in German): Laurence King Publishing; 1998; ISBN 1-85669-134-9. (*An inexpensive soft-cover publication. Probably the best introduction to industrial design for students (and anyone else). Concise, comprehensive, clear and with intelligible layout and good, if small, color illustrations*.).
8. Jordan PS. *Designing Pleasurable Products* London: Taylor and Francis; 2000; ISBN 0-748-40844-4. (*Jordan, Manager of Aesthetic Research and Philips Design, argues that products today must function properly, must be usable, and must also give pleasure. Much of the book is a description of market-research methods for eliciting reactions to products from users*.).

9. Julier G. *Encyclopedia of 20th Century Design and Designers* London: Thames & Hudson; 1993; ISBN 0-500-20261-3. (*A brief summary of design history with good pictures and discussions of the evolution of product designs*.).
10. Karana E, Pedgley O, Rognoli V. *Materials Experiences – Fundamentals of Materials and Design* Oxford: Butterworth Heinemann; 2014; ISBN 978-0-08-099359-1. (*A compilation of essays and interviews by leading industrial designers and materials experts*.).
11. Manzini E. *The Material of Invention* London: The Design Council; 1989; ISBN 0-85072-247-0 (*Intriguing descriptions of the role of material in design and in inventions. The translation from Italian to English provides interesting – and often inspiring – commentary and vocabulary that is rarely used in traditional writings about materials*.).
12. McDermott C. *The Product Book* London: D & AD in association with Rotovison; 1999; (*50 essays by respected designers who describe their definition of design, the role of their respective companies and their approach to product design*.).
13. Norman DA. *The Design of Everyday Things* New York, NY: Doubleday; 1988; ISBN 0-385-26774-6. (*A book that provides insight into the design of products with particular emphasis on ergonomics and ease of use*.).
14. Vezzoli C, Manzini E. *Design for Environmental Sustainability* London: Springer-Verlag; 2008; ISBN 978-1-84800-162-6. (*A book co-authored by the writer of "The Materials of Invention" describing tools and strategies for integrating environmental requirements into product development*.).

15.8 Exercises

E15.1. *Exploring context*. The images show two contrasting coffee machines. Analyse the context for which these were designed by responding to the five W questions Who? What? Where? When? Why?

(A)



L'Express coffee maker

(B)



Lacimbala coffee machine

FIGURE E15.1 (A) From: https://en.wikipedia.org/wiki/Moka_pot#/media/File:Moka_Express_sideview.png; (B) From: https://en.wikipedia.org/wiki/Coffee_preparation#/media/File:LaCimbali_Junior_2.jpg

E15.2. *Value and price (1)*. Today (2016) bagless vacuum cleaners vary in price by almost a factor of 10 an Argos VC 403 is priced at £38 ($53), a Dyson DC23 is priced at £333 ($466). There is a market for both, so a market sector must exist that perceives the value of the more expensive cleaner to be greater than its high price. How has Dyson created this perception of value? Examine these (or two equivalent) products on the internet and analyse how he has done it.

(A)



Panasonic bagless vacuum cleaner

(B)



Dyson bagless vacuum cleaner

FIGURE E15.2 (A) From:
https://commons.wikimedia.org/wiki/File:Panasonic_MCE8013_vacuum_cleaner_close.jpg; (B) From:

https://en.wikipedia.org/wiki/List_of_Dyson_products#/media/File:%D0%9F%D1%8B%D0%BB%D0%B5%D1%81%D0%BE%D1%81_Dyson_%D0%BC%D0%BE%D0%B4%D0%B5%D0%BB%D1%8C_DC32.jpg

E15.3. *Value and price (2)*. Watches vary in price by a factor of over 10,000. I have a very cheap watch which I like because it has a stopwatch and is waterproof, so I can swim with it. It is extremely accurate and if I lose it a new one will cost me £5.80 (about $9) from Amazon. This watch is shown in the figure that accompanies this exercise, along with a Rolex, currently priced at $11,870 but that is nothing compared with the Sky Moon Tourbillon 5002 P from Patek Philippe which will set you back $1.5 million.

How do the makers of really expensive watches create the value that their products apparently have, at least to some people? Their advertisements in glossy magazines and their web sites give away a lot. Research these to reveal how materials, associations, brand image status can create value.



FIGURE E15.3 From: https://en.wikipedia.org/wiki/Rolex#/media/File:RolexDaytona.jpg; From: https://en.wikipedia.org/wiki/Casio#/media/File:Casio_F-91W_5051.jpg

E15.4. *Value and price (3)*. Carry out a comparison and analysis like that of the previous exercise for

a. *Suitcases (luggage)*

b. *Refrigerators*

What, for a given case or fridge capacity, is the range of price? What aspect of product character gives the most expensive of the products its high value? The prices, sizes and other information you need is readily available in Consumer Magazines or on product-comparison websites.

E15.5. *Associations*. Write down the first association that comes to your mind when you glance at each of the images of cars shown below. How has the designer suggested this association?

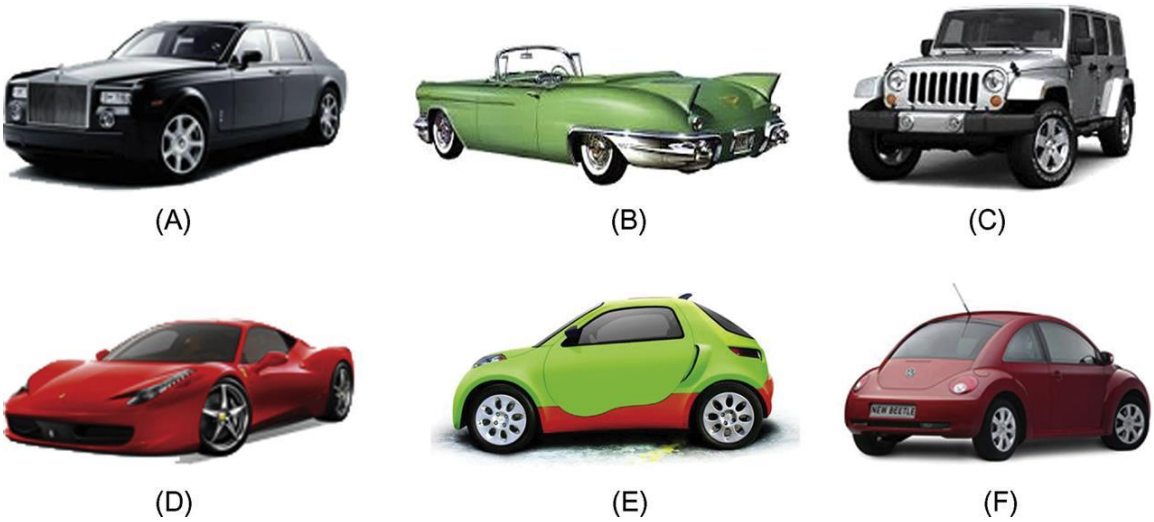


FIGURE E15.5 (A) https://en.wikipedia.org/wiki/Rolls-Royce_Motor_Cars#/media/File:Rolls-Royce_Ghost_-_Flickr_-_Alexandre_Pr%C3%A9vot_(3)_(cropped).jpg; (B) From: https://en.wikipedia.org/wiki/Cadillac_de_Ville_series#/media/File:Cad59.jpg; (C) From: https://de.wikipedia.org/wiki/Jeep_Wrangler#/media/File:Jeep_Wrangler_Sport_(JK)_%E2%80%93_Frontansicht,_16._April_2011,_D%C3% BCsseldorf.jpg; (D) From: https://en.wikipedia.org/wiki/Ferrari#/media/File:F12pebblebeach.jpg; (E) From: https://commons.wikimedia.org/wiki/File:YouDrive_carsharing_car_in_Moscow.jpg; (F) From: https://en.wikipedia.org/wiki/Compact_car#/media/File:Citro%C3%ABn-C4_Mk2_Front-view.JPG

E15.6. *Perceptions*. Look at the six car-images again and ponder on them for a moment. How do you perceive each one? How do they make you feel? If you had a choice of any one of these, which would you choose? Why?

E15.7. *Associations of materials*. What do people think of when they see something made of gold? They might – because of its expense – associate it with wealth and luxury; or perhaps – because it is a visible symbol of riches – with extravagance and power; or perhaps – because of its total resistance to chemical attack – with stability and permanence. What associations would you attach to one or more of the following materials? Ask: 'What do you think of when you see something made of…?'

Machined steel	Lead	Polished cherry wood
Stainless steel	Polyethylene	Plastic with a simulated wood surface
Rusted steel	Nylon	Marble
Brushed aluminium	Diamond	Glass

E15.8. *Capturing associations*. Here is a little design exercise. You have acquired the rights to market a new biopolymer – one made from a fast-growing, prolific, crop. What are you going to call it? The name should carry the right associations. Define the problem (what?). Explore

alternatives (how?). Select a solution (which?).
Here is a try.

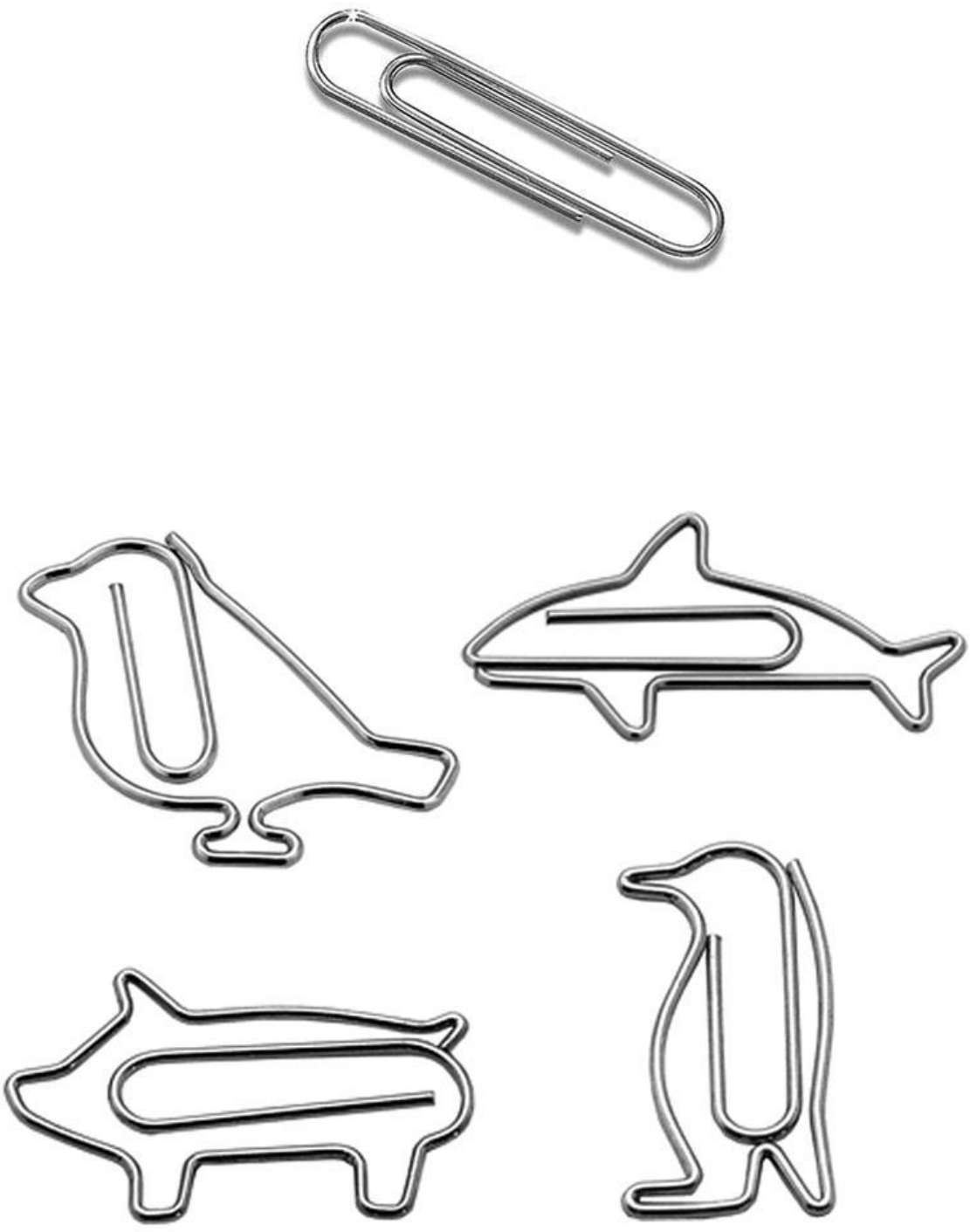
a. *Challenge?* Name for new biopolymer
b. *What?* An evocative name. Why? To convey that it is bio, green. Why? To sell it to an increasingly eco-concerned public.
c. *How?* Some synthesis of green words or notions? Green. Renewable. Responsible. Benevolent. Saving the future. Cool. Next generation. OK, here are some trys: *Respoxy. Biofutene Gen4biocool. Benegen*.
d. *Which? Respoxy* (definite no). *Biofutene* (I don't think so – too obvious), *Gen4biocool* (kind of nice but too many syllables). *Benegen* (maybe – might have to go with that).

Not a great success. Now you have a go. It has to be a name not already in use (Google it to find out).

E15.9. *Products with long design-lives*. Many very simple products have had (and continue to have) very long lives. The table lists some of these. Research the history of one of them, seeking the context (Who, What, Where, When, Why) the inventor had in mind, and the present-day view of their utility, aesthetics and associations. Can you identify what has made it so durable?

Product	Materials	Product	Materials
Bar codes (1966)	*Printing ink*	Zippers (1913)	*POM*
Sticking plaster (early 20th century)	*Woven fabric or plastic with adhesive*	Velcro (1948)	*Nylon – Polyester*
Coffee filters (1908)	*Crepe paper*	Corkscrew (1795)	*Steel*
Snap fasteners (1850)	*Spring steel*	Egg boxes (1930s)	*Paper pulp*
Ball point pen 1888	*Textured tungsten carbide*	Paper clip (1899)	*Mild steel*
Tin can (1813)	*Mild steel/tin*	Postit notes (1977)	*Adhesive*
Umbrella (2000 BC)	*Bamboo/silk*	Clothes pegs (1832)	*Wood/polymer*
Rubber band (1845)	*Natural rubber Latex*	Tea bags (1904)	*Paper*
Cats eyes (road reflectors) (1933)	*Glass, cast iron*	Adhesive tape (1930)	*Cellulose acetate /rubber based adhesive*
Tungsten filament light bulb (1906)	*Tungsten, glass*	Baby dummy (about 1850)	*Rubber*
Carabiner (1910)	*Steel/aluminium*	Shipping container (1956)	*Steel*
Bubble wrap (1960)	*PE*	Lego (1949)	*ABS*
Pencil (1565)	*Graphite*	Kleenex (1924)	*Paper*

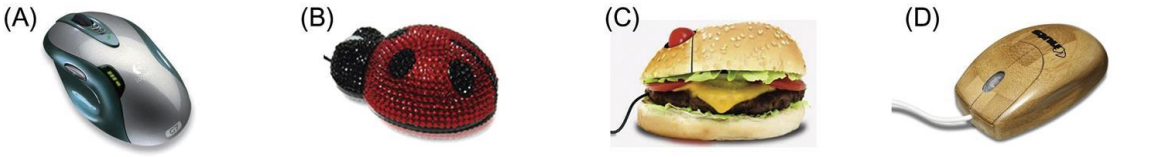
E15.10. *Humour*. A paperclip (invented 1899) is one of the simplest of products with one of the longest lives. Every office has them. They don't normally arouse interest or amusement. Here is an example of a re-design that provides these missing elements.
Find a really simple everyday-product (perhaps one from the table in the previous exercise) and re-design it to introduce an element of humour.



A paperclip and variations on the theme of paperclips

FIGURE E15.10

E15.11. *Niche design*. Each of these mice is designed to fill a particular niche in the mouse-market. Apply the method of product analysis outlined in the previous exercise to explain the designer's choices of materials, form aesthetics and associations.



Logitec mouse, $23 *Ladybird mouse $14.50* *Hamburger mouse $20* *Bamboo mouse, $50 up*

FIGURE E15.11 (A) From: https://en.wikipedia.org/wiki/Logitech#/media/File:Logitech_G7bc.JPG; (B) From: https://commons.wikimedia.org/wiki/File:AppleWirelessMouse.JPG; (C) From: https://commons.wikimedia.org/wiki/File:Wireless-trackman-mouse.jpg; (D) From: https://commons.wikimedia.org/wiki/File:MS-Arc-Mouse.jpg

E15.12. *Analysing product character*. Examine a product and ask the following:

a. What does the product do?
b. Who will use it? Where? When? Why?
c. What are their aspirations? How do they see themselves?
d. What aesthetics has the designer used? Why?
e. What associations? How did the designer create them? Why?
f. What perceptions? What is your reaction to the product? What made you feel that way?
g. How (intentionally or unintentionally) did the designer create that perception?
h. And finally: What was the designer trying to say?

[1]Many designers, working on a project, assemble a *mood-board* with images of the sort of people for whom the product is intended, the surroundings in which they suppose it will be used, and other products that the intended user group might own, seeking to capture the flavour of their lifestyle.

[2]Pantone (www.pantone.com.) provide detailed advice on colour selection, including colour-matching charts and good descriptions of the associations and perceptions of colour.

[3]The dates are, of course, approximate. Design styles do not switch on an off on specific dates, they emerge as a development of, or reaction to, earlier styles with which they often coexist, and they merge into the styles that follow.

CHAPTER 16

Sustainable Response to Forces for Change

Abstract

The evolving circumstances of the world in which we live change the boundary conditions for design and, with them those for selecting materials and processes. These changes are driven by a number of forces. There is the *market-pull:* the demand from industry for materials that are lighter, stiffer, stronger, tougher, cheaper, more tolerant of extremes of temperature and environment and that offer greater functionality. There is the *science-push*: the curiosity-driven researches of materials experts in the laboratories of universities, industries and government. There is the driving force of what might be called *mega-projects*: alternative energy generation and storage, response to the threats posed by terrorism; autonomous high-speed transport and the increasing the functionality and interactivity of household products. The challenge is to adapt to these forces for change in ways that are sustainable.

Keywords

Market-pull; science-push; critical elements; population; natural capital; manufactured capital; human and social capital; linear materials economy; circular materials economy; sustainable development

16.1 Introduction and Synopsis

If there were no forces for change, everything would stay the same. But we live in a dynamic, not a static, world, with the consequence that exactly the opposite is true: things are changing faster now than ever before. The evolving circumstances of the world in which we live change the

boundary conditions for design and with them those for selecting materials and processes.

These changes are driven by a number of forces. There is the *market-pull*: the demand from industry for materials that are lighter, stiffer, stronger, tougher, cheaper, more tolerant of extremes of temperature and environment and offering greater functionality. Then there is the *science-push*: the curiosity-driven researches of materials experts in the laboratories of universities, industries and government. There is the driving force of what might be called *mega-projects*: historically, the Manhattan Project, the development of nuclear power, the space-race and various defence programmes. Today one might think of alternative energy generation and storage, clean high-speed transport, response to threats posed by terrorism, autonomous systems and the interactivity of products. There is legislation regulating product safety, there is the increased emphasis on liability established by recent legal precedent and there is the awareness that we have to manage our adaptation to these forces for change in ways that are sustainable. Quite a list.[1]

This chapter examines forces for change and the directions in which they push materials and their deployment, ending with the ultimate challenge: that of sustainable development. Fig. 16.1 set sets the scene.

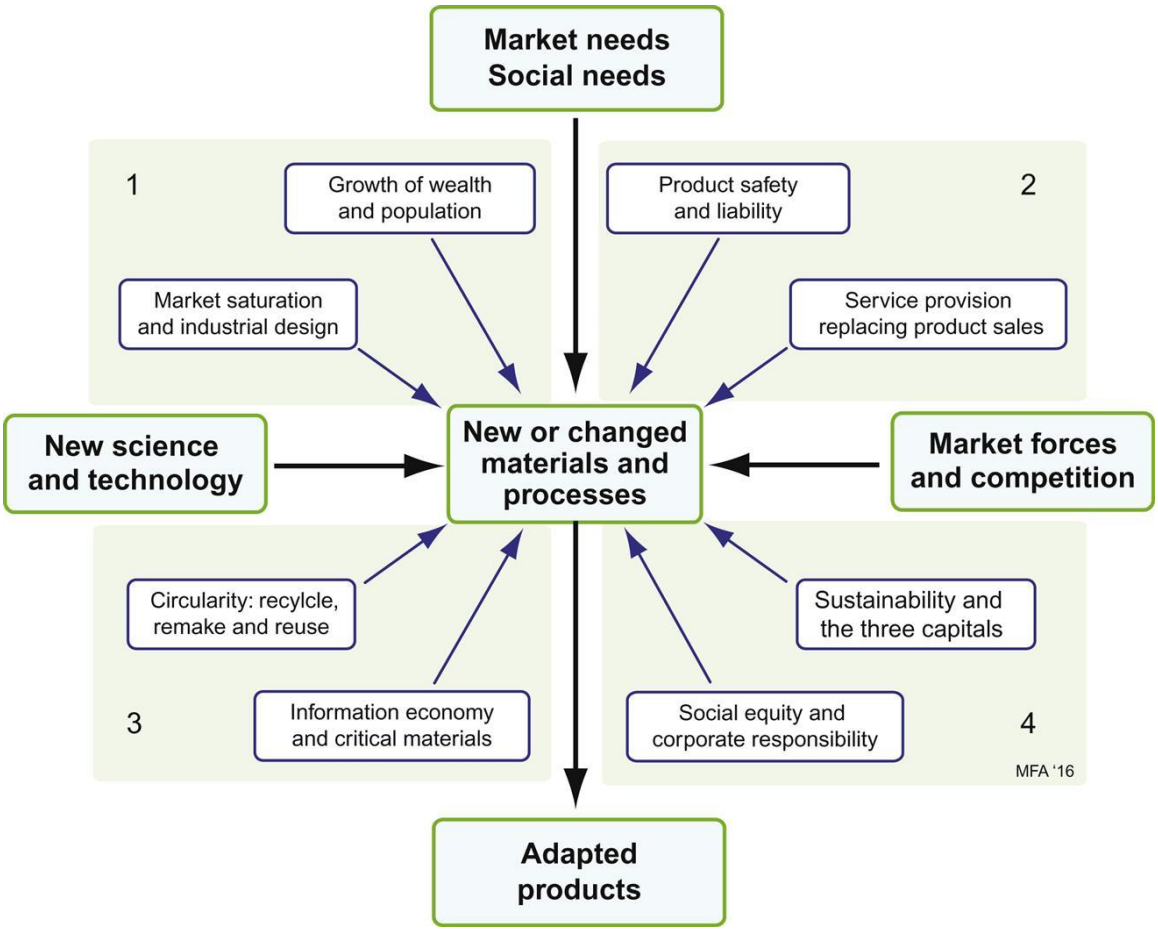


FIGURE 16.1 Some forces for change. Each exerts pressure to change the choice of material and process and stimulates efforts to develop new ones.

16.2 Market-Pull and Science-Push

Market Forces and Competition

The end-users of materials are the manufacturing industries. They decide which material they will purchase and they adapt their designs to make best use of them. Their decisions are based on the nature of their products. Materials for large civil structures (which can weigh 100,000 tonnes or more) must be cheap and reliable; availability and economy are the overriding consideration. By contrast, the materials for high-tech products (sports equipment, military hardware, space projects, biomedical applications) are chosen for their performance; cost plays a less important

role. The material cost of an artificial heart valve, for instance, is almost irrelevant.

The market price of a product has several contributions. There is the cost of the materials of which the product is made, the cost of manufacture and marketing, the cost of the research and development that went into its design (see chapter: Processes Selection and Cost). There is, too, the margin created by the perceived value associated with desirability, advertising, fashion and rarity (see chapter: Materials and Industrial Design). When the material costs are a large part of the market value (50%, say) – that is, when the value added to the material is small – the manufacturer seeks to economise on material usage to increase profit or market share. When, by contrast, material costs are a tiny fraction of the market value (1%, say), the manufacturer seeks the materials that will most improve the performance of the produce with little concern for their cost.

With this background, examine Figs. 16.2 and 16.3. The vertical axis of the first is the price per unit weight ($/kg, 2016 USD) of materials; that of the second is the same thing for products. It gives a common measure by which materials and products can be compared. The measure is a crude one but has the great merit that it is unambiguous, easily determined, and bears some relationship to value-added. A product with a price/kg that is only two or three times that of the materials of which it is made is material-intensive and is sensitive to material costs; one with a price/kg that is 100 times that of the its materials is insensitive to material costs, and is probably performance-driven rather than cost-driven. On this scale the price per kg of a contact lens differs from that of a glass bottle by a factor of 10^5 even though both can be made of almost the same glass; the cost per kg of a heart valve differs from that of a plastic bottle by a similar factor, even though both are made of polyethylene. There is obviously something to be learned here.

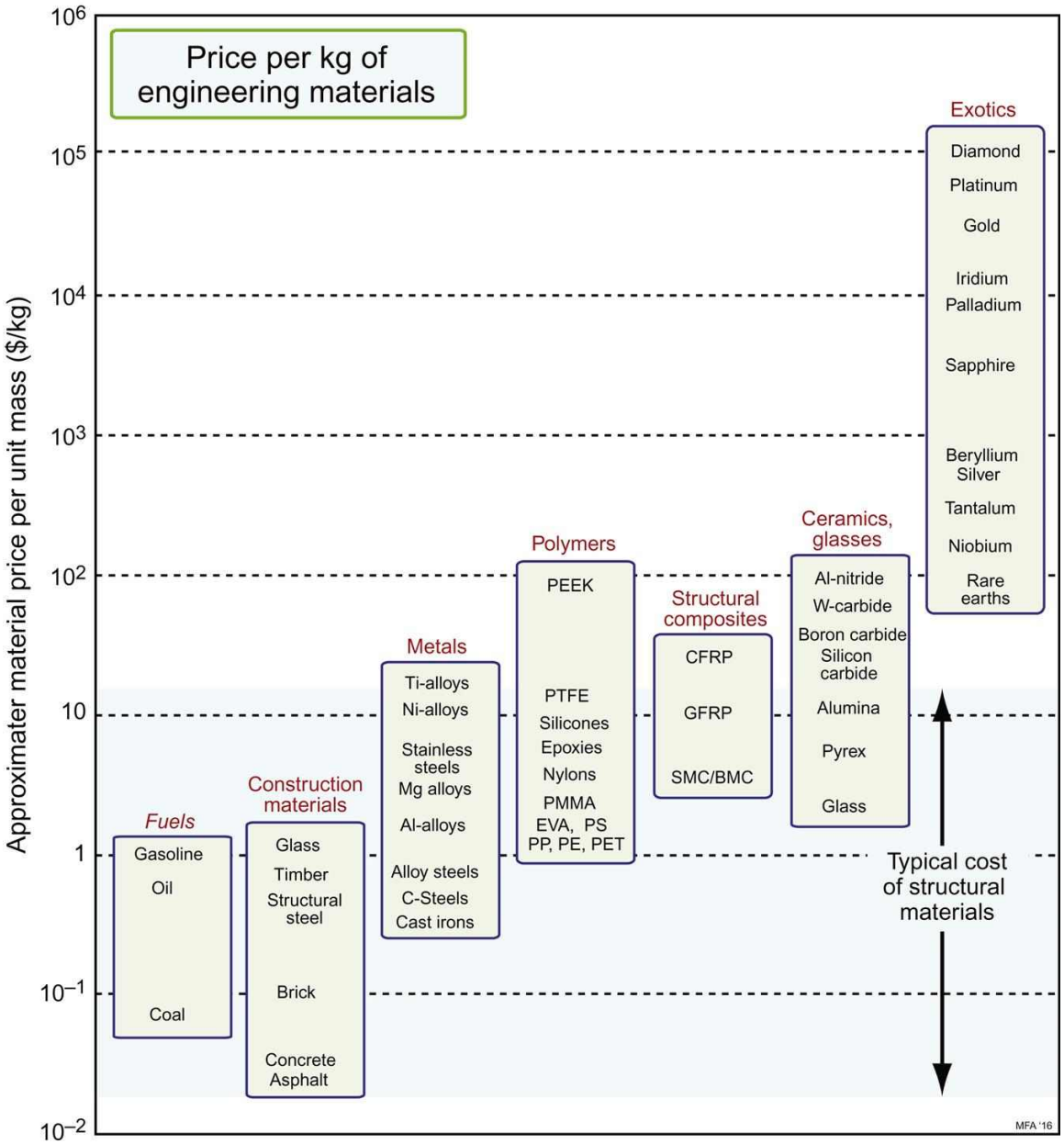


FIGURE 16.2 The price per unit weight diagram for materials. The shaded band spans the range in which the most widely used commodity materials of manufacture and construction lie.

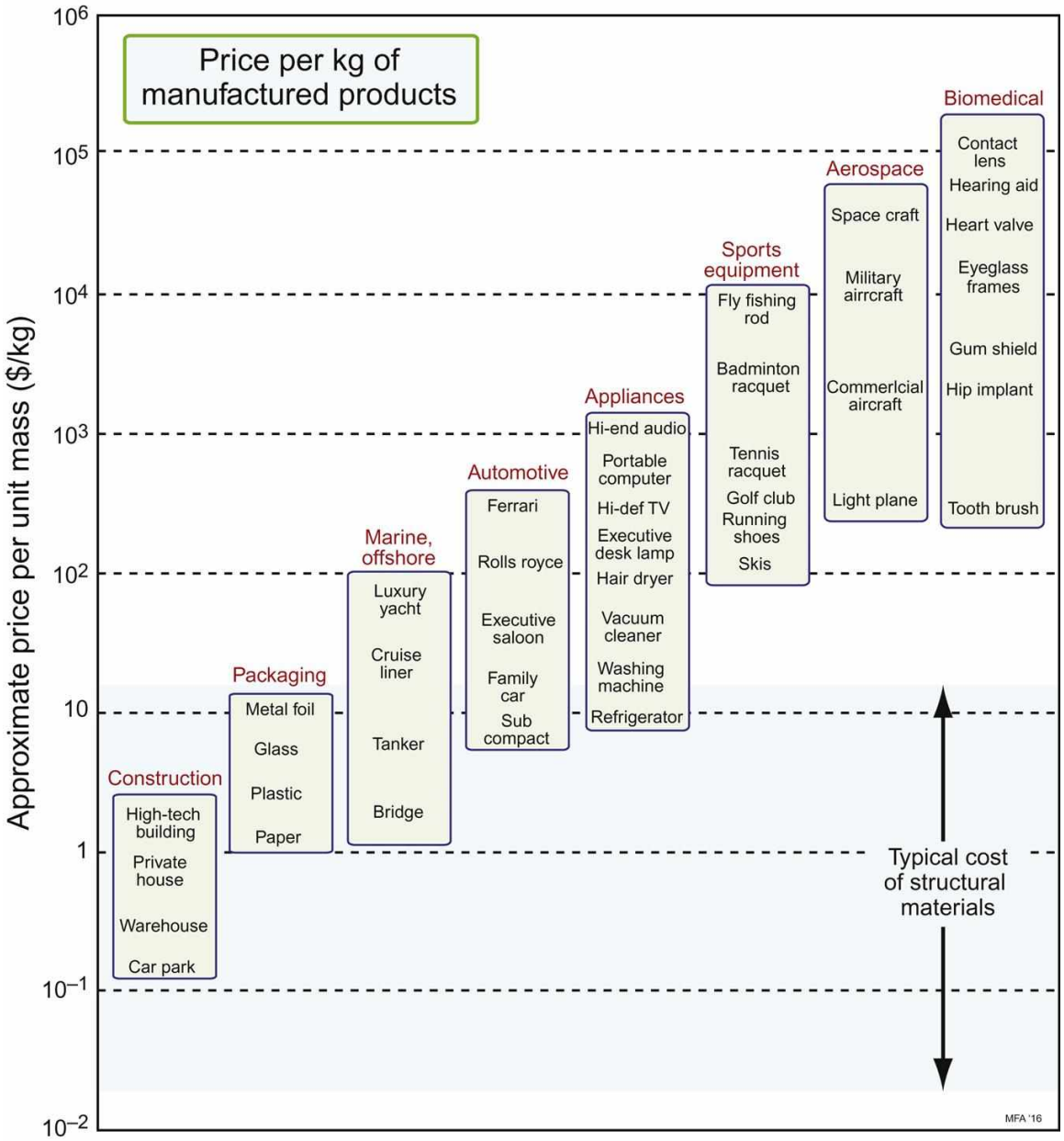


FIGURE 16.3 The price per unit weight diagram for products. The shaded band spans the range of most of the materials of which products are made. Products in the shaded band are material-intensive; those above it are not.

The bulk 'commodity' materials of construction and manufacture lie in the shaded band of Fig. 16.2; they all cost between \$0.05 and \$20/kg. Construction materials like concrete, brick, timber and structural steel lie at the lower end; high-tech materials like titanium alloys lie at the upper. Polymers span a similar range: polyethylene at the bottom, Polyetheretherkeytone (PEEK) near the top. Composites lie in a similar

range with polyester sheet-moulding compounds at the lower end and carbon fibre reinforced epoxies at the top. Engineering ceramics, at present, lie higher still, though this is changing as production increases. Only the low-volume 'exotic' materials lie much above the shaded band.

The price per kg of products (Fig. 16.3) shows a different distribution. Eight market sectors are shown, covering much of the manufacturing industry. The shaded band on this figure spans the cost of commodity materials, exactly as on the previous figure. Sectors and their products within the shaded band have the characteristic that material cost is a major fraction of product price: about 50% in civil construction, large marine structures and some consumer packaging, falling to perhaps 20% as the top of the band is approached (a low-cost family car). The value added in converting material to product in these sectors is relatively low, but the market volume is large. The driver here is for improved processing of conventional materials to reduce cost.

The products in the upper half of the diagram are technically more sophisticated. The materials of which they are made account for less than 10% – sometimes less than 1% – of the price of the product. The value added to the material during manufacture is high. Designers in these sectors have greater freedom in their choice of material and there is a readier acceptance of new materials with attractive property-profiles. The market-pull here is for performance, with cost as a secondary consideration. These smaller volume, higher value-added sectors drive the development of new or improved materials that are lighter, or stiffer, or stronger, or tougher, or expand less, or conduct better – or all of these at once.

The sectors have been ordered to form an ascending sequence, prompting the question: what does the horizontal axis measure? Many factors are involved here, one of which can be identified as 'information content'. The technical knowledge involved in the manufacture of a contact lens or a heart valve is clearly greater than that to make a water-glass or a plastic bottle. The sectors on the left make few demands on the materials they employ; those on the right push materials to their limits, and at the same time demand the highest reliability. These features make them information-intensive. But there are also other factors: market size, competition (or lack of it), perceived value, advertising, fashion and taste. For this reason the diagram should not be over-interpreted: it is a help in structuring information, but it is not a quantitative tool.

New Science: Curiosity-Driven Research

Curiosity may kill cats, but it is the life-blood of innovative engineering. Technically advanced countries sustain the flow of new ideas by supporting research in universities, government laboratories and industrial research laboratories. Some of the scientists and engineers working in these institutions are given the freedom to pursue ideas that may have no immediate economic objective but that can evolve into the materials and manufacturing methods of future decades. Numerous now-commercial materials started in this way. Aluminium, in the time of Napoleon III, was a scientific wonder – he commissioned a set of aluminium spoons for which he paid more than those of solid silver. Titanium, more recently, has had a similar history. Amorphous (i.e., noncrystalline) metals, now important in transformer technology and in read-heads of disc-drives, were, for years, of only academic interest. Semiconductors and superconductors did not appear in response to market forces; it took long-term curiosity-driven research, rewarded in both cases by Nobel prizes, to reveal the principles on which they are based. Polyethylene was discovered by chemists studying the effect of pressure on chemical reactions, not by the sales or marketing departments of multi-national corporations. Even the sticky back of Post-it notes was an inspired application of what at first seemed a failure. History is dotted with examples of materials and processes that have emerged from inquisitiveness, accident and happenstance.

New materials continue to emerge. *Monolithic ceramics*, now produced in commercial quantities, offer high hardness, chemical stability, wear resistance and resistance to extreme temperatures. Their use as substrates for microcircuits is established; their use in wear-resistant applications is growing, and their use in heat engines is being explored. The emphasis in the development of *composite materials* is shifting towards those that can support loads at higher temperatures. *Metal-matrix composites* (example: aluminium containing particles or fibres of silicon-carbide) and *intermetallic-matrix composites* (titanium-aluminide or molybdenum-disilicide containing silicon-carbide, for instance) can do this. So, potentially, can *ceramic-matrix composites* (alumina with silicon carbide fibres) though the extreme brittleness of these materials requires new design techniques. *Metallic foams*, up to 90% less dense than the parent metal, promise light, stiff sandwich structures competing with composites.

Aerogels, ultra-low density foams, provide exceptionally low thermal conductivities.

New *bio-materials*, designed to be implanted in the human body, have structures onto which growing tissue will bond without rejection. New *polymers* that can be used at temperatures up to 350°C allow plastics to replace metals in even more applications – the inlet manifold of the automobile engine is an example. New *elastomers* are flexible but strong and tough; they allow better seals, elastic hinges, and resilient coatings. Techniques for producing *functionally-graded materials* can give tailored gradients of composition and structure through a component so that it could be corrosion resistant on the outer surface, tough in the middle and hard on the inner surface. *Nano-structured materials* promise unique mechanical, electrical, magnetic and optical properties. *'Intelligent' materials* that can sense and report their condition (via embedded sensors) allow safety margins to be reduced. *Self-healing materials* have the capacity to repair service damage without human intervention.

Developments in *additive manufacture* now permit single, complex, parts to be made quickly without dies or moulds from a wide range of materials. *Micron-scale fabrication methods* create miniature electro-mechanical systems. New techniques of *surface engineering* allow the alloying, coating or heat treating of a thin surface layer of a component, modifying its properties to enhance its performance. They include laser hardening, coatings of well-adhering polymers and ceramics, ion implantation, and even the deposition of ultra-hard carbon films with a structure and properties like those of diamond. New *adhesives* displace rivets and spot-welds; the glue-bonded automobile is a real possibility. And new techniques of *process modelling and control* allow much tighter control of composition and structure in manufacture, reducing cost and increasing reliability and safety.

All these and many more are now realities. They have the potential to enable new designs and to stimulate redesign of products that already have a market, increasing their market share. The designer must stay alert.

16.3 Growing Population and Wealth, and Market Saturation

The global population continues to grow in number and in wealth. Thus far the growth of the world's product-producing capacity has grown faster

still, with the result that, in developed and developing countries, product markets are saturated. If you want a product – a mobile phone, a refrigerator, a car – you don't have to put your name on a waiting list (as was the case within living memory throughout Europe). Instead an eager sales-person guides you through an array of near-identical products with near-identical prices to find the one that appeals to you the most.

This wide choice has certain consequences. One is the continuing growth in consumption of energy and material resources and the impact these have on the environment – the subject of Chapter 14, Materials and the Environment. Another arises because, in a saturated market, the designer must seek new ways to attract the consumer. The more traditional reliance on engineering qualities to sell a product are replaced (or augmented) by visual qualities, associations and carefully managed perceptions created by industrial design. This, too, influences material and process choice, and it, too, was the subject of an earlier chapter – the one before this one.

16.4 Product Liability and Service Provision

Legislation now requires that, if a product has a fault, it must be recalled and the fault rectified. Product recalls are exceedingly expensive and they damage the company's image. The higher up the chart of Fig. 16.3 we go, the more catastrophic the consequences of a fault become. When you buy a packet of six ball-point pens, it is no great tragedy if one doesn't write properly. The probability ratio 1:6, in a Gaussian distribution, is one standard deviation or '1-sigma'. But would you fly in a plane designed on a '1-sigma' basis? Safety critical systems today, are designed on a '6-sigma' reliability rating, meaning that the probability that the component or assembly fails to meet specification is less than one part in 10^9.

This influences the ways in which materials are produced and processed. Reliability and reproducibility require sophisticated process control, monitoring and verification. Clean steels (steels with greatly reduced inclusion-account), aluminium alloys with tight composition control, real-time feedback control of processing, new nondestructive testing for quality and integrity, and random sample testing all help ensure that quality is maintained.

This pressure on manufacturers to assume full-life responsibility for their products causes them to consider taking maintenance and replacement out of the hands of the consumer and doing it themselves. In this way they can monitor product use and replace it, taking back the original, not when it is worn out but when it is optimal to recondition and re-deploy it. The producers no longer sell a product; they sell a service. In common with many others, my university does not own copying machines, though we have many. Instead it has a contract with a provider to guarantee a certain copying-service – the provision of 'copied pages per week', you might call it.

This changes the economic boundary conditions for the designer. The objective is no longer that of building copiers with the most features at the lowest price, but rather those that will, over the long term, provide copies at the lowest cost per page. The design priority becomes ease of replacement, reconditioning and reuse, plus standardisation of materials and components between models. The copier is not an isolated example. Aircraft engine makers now adopt a strategy of providing 'power by the hour': they own the engines, monitor their use, and replace them on the plane at the point at which the trade-off between cost, reliability and power-providing capacity is optimal.

Legislation exerts other forces for change. Manufacturers must now register the restricted substances, of which there are a large number, that are contained in the products they make or are used in its manufacture. If the level of use exceeds a threshold, the material must be replaced. The list includes the metals cadmium, lead and mercury, along with a large number of the compounds and chemicals, many of them used in material processing. So the restrictions exert forces for change both in material choice and in the way that it is shaped, joined and finished.

16.5 The Information Economy, Critical Materials and Circularity

The information economy. Information is now so central to the way we live that governments regard provision of internet access to be as important as the provision of fresh water and electrical power. Digital communication is not just central to the way we communicate with each other but also the way we bank, navigate and defend ourselves; it is the way products interact with each other and with their environment, the basis of robotics

and of autonomous transport, and much more. A modern economy is relies heavily on the elements that enable electronic communication.

Of the 110 elements of the periodic table some 90 are usable, most of them metals. Half a century ago we used relatively few of them. The phone of the 1960s contained just nine metals for the frame, conductors, magnets, springs and contacts. A contemporary smartphone contains about 53 (Fig. 16.4). Even seemingly unsophisticated structural materials – steels, aluminium alloys – now depend for their properties on a greatly expanded portfolio of elements (Table 16.1).

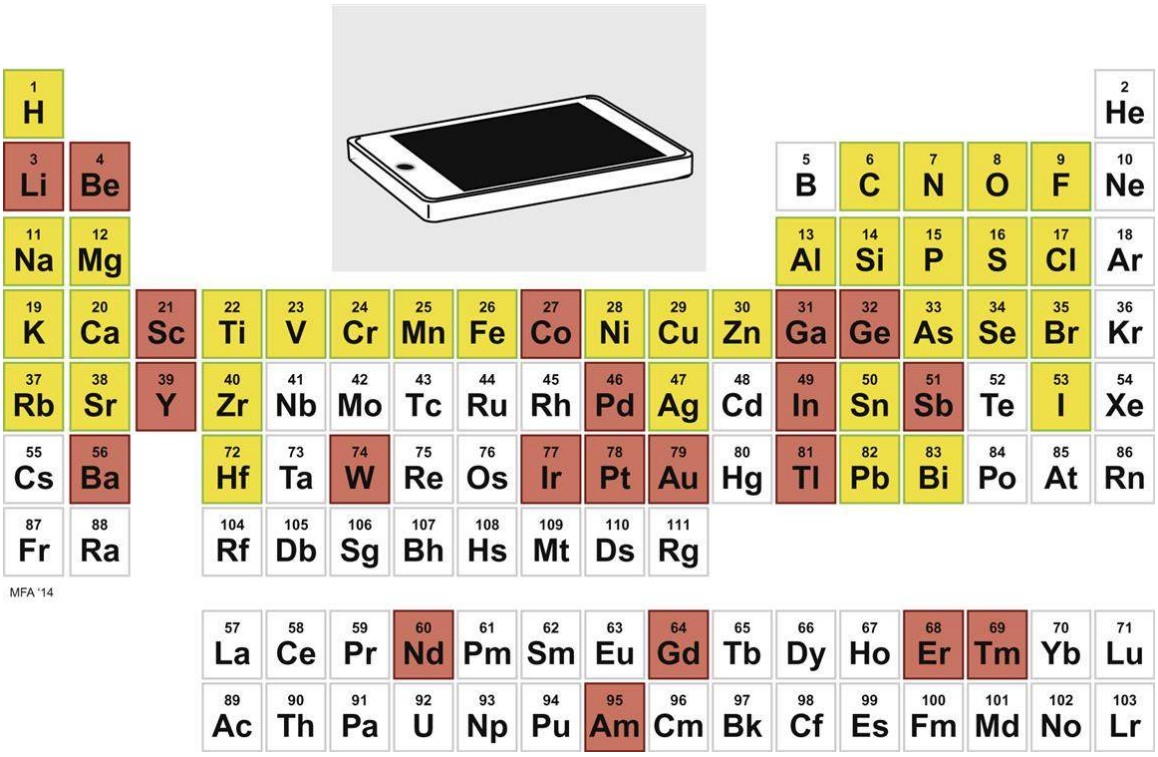


FIGURE 16.4 The elements in a smart phone are coloured in on a periodic table. Those coloured red are critical elements.

Table 16.1

The Increasing Diversity of Elements used in Structural Materials Over the Past 75 Years

Alloys and Devices	Changing Demand for Elements in Structural Materials Over Time	
	75 years ago	Today
Iron-based alloys[a]	Fe, C	Al, Co, Cr, Fe, Mn, Mo, Nb, Ni, Si, Ta, Ti, V, W
Aluminium alloys[a]	Al, Cu, Si	Al, Be, Ce, Cr, Cu, Fe, Li, Mg, Mn, Si, I, V, Zn, Zr
Nickel alloys[a]	Ni, Cr	Al, B, Be, C, Co, Cr, Cu, Fe, Mo, Ni, Si, Ta, Ti, W, Zr
Copper alloys[a]	Cu, Sn, Zn	Al, Be, Cd, Co, Cu, Fe, Mn, Nb, P, Pb, Si, Sn, Zn
Magnetic materials[a]	Fe, Ni, Si	Al, B, Co, Cr, Cu, Dy, Fe, Nd, Ni, Pt, Si, Sm, V, W

[a]Data from the composition fields of records in the CES EduPack '14 Level 3 database, Granta Design (2016).

Critical materials. Materials today are sourced and traded globally. The ores and feedstock needed to make some of them are widely available, allowing a free market to operate smoothly. Others, however, are extracted from lean ores that are geographically restricted in their distribution. The nations from which they come may limit supply for economic or political reasons; and political instability or conflict can interrupt supply completely. The supply-chain of cobalt, for instance, was disrupted recently by war in the Democratic Republic of the Congo, and that of rare-earth metals has not operated smoothly in the recent past because of monopoly constraints that led to shortages and unstable pricing.

Materials that are seen as essential for national security or industrial health are classified as 'critical' if access to them could be restricted. Governments draw up lists of these critical materials and develop strategies to ensure supply by negotiating exclusive supply agreements, seeking new sources, or stockpiling. These lists are not identical because 'criticality' depends to some extent on the needs and domestic resources of the nation, but they have a great deal in common. Those that appear in US lists are shown in Table 16.2. The lists act as warnings, alerting manufacturers of potential supply risk.

Table 16.2

Critical Materials[a]

Antimony	Beryllium	Bismuth	Cerium
Chromium	Cobalt	Dysprosium	Erbium
Europium	Gallium	Germanium	Indium
Iridium	Lanthanum	Lithium	Manganese
Neodymium	Osmium	Palladium	Platinum
Praseodymium	Rhodium	Ruthenium	Samarium
Scandium	Tantalum	Tellurium	Terbium
Thulium	Tin	Tungsten	Yttrium

[a]Drawn from US lists. A European list looks very similar.

The circular materials economy. Before the Industrial Revolution materials were expensive and labour was cheap. The high value of materials relative to labour ensured that the products made from them were maintained, repaired and upgraded. Material efficiency was normal practice.

Since the Industrial Revolution, large-scale mining and global trade have allowed material costs to fall. At the same time labour costs rose with the result that manufacturing methods (mass production, robotic assembly and the like) evolved to minimise the use of labour. The availability and low cost of materials encouraged industry to adopt an increasingly linear, open-loop approach to material resources, summarized as: *take – make – use – dispose* (Fig. 16.5).

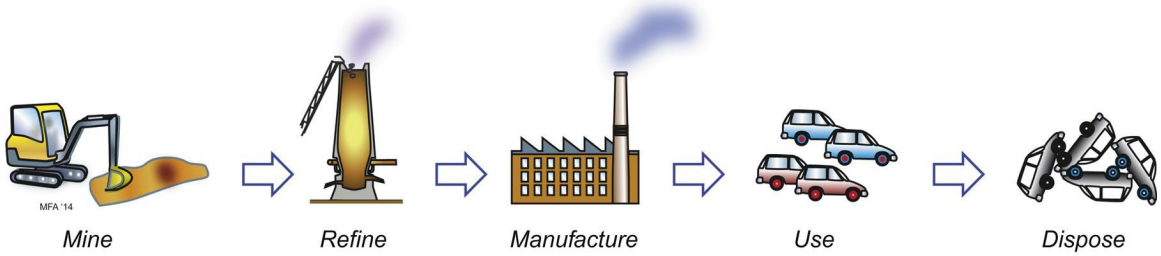


FIGURE 16.5 The linear materials economy: *take – make – use – dispose*.

Some 80% of material usage still follows such a path but there is an increasing awareness that this cannot go on. The global consumption of materials, at present about 77 billion tonnes per year, is expected to rise to 100 billion by 2030.[2] The drain of natural resources needed to meet this consumption cannot be sustained, creating drivers to conserve materials rather than reject them. One way to do this is to establish a more circular materials economy. Materials in a circular economy are seen, not as disposable commodities, but as valued assets to be tracked and conserved for reuse, in rather the same way that financial capital is invested, recovered as revenues and re-invested. Fig. 16.6 introduces the idea. Materials are produced and manufactured into products that enter service where they remain for their design life. In the circular model, disposal at end of life as landfill or waste is not an option. Instead the product is reused in a less demanding way, or reconditioned to give it a second lease of life, or dismantled into its component materials for recycling. All three of these options retain the materials of the product as *active stock* (the upper green box on the figure). It may be impractical to recycle perishable materials; those that are bio-degradable can be composted, returning them to the bio-sphere; those that are combustible can be incinerated with energy recovery; only the remaining residue goes to landfill. The more material that can be retained within the green 'Active stock' box in the upper part of Fig. 14.6, the less needs to be added through primary production, shown on the left. Indeed if design improvements use materials more efficiently it may be possible to function, at least for a while, with no primary production at all.

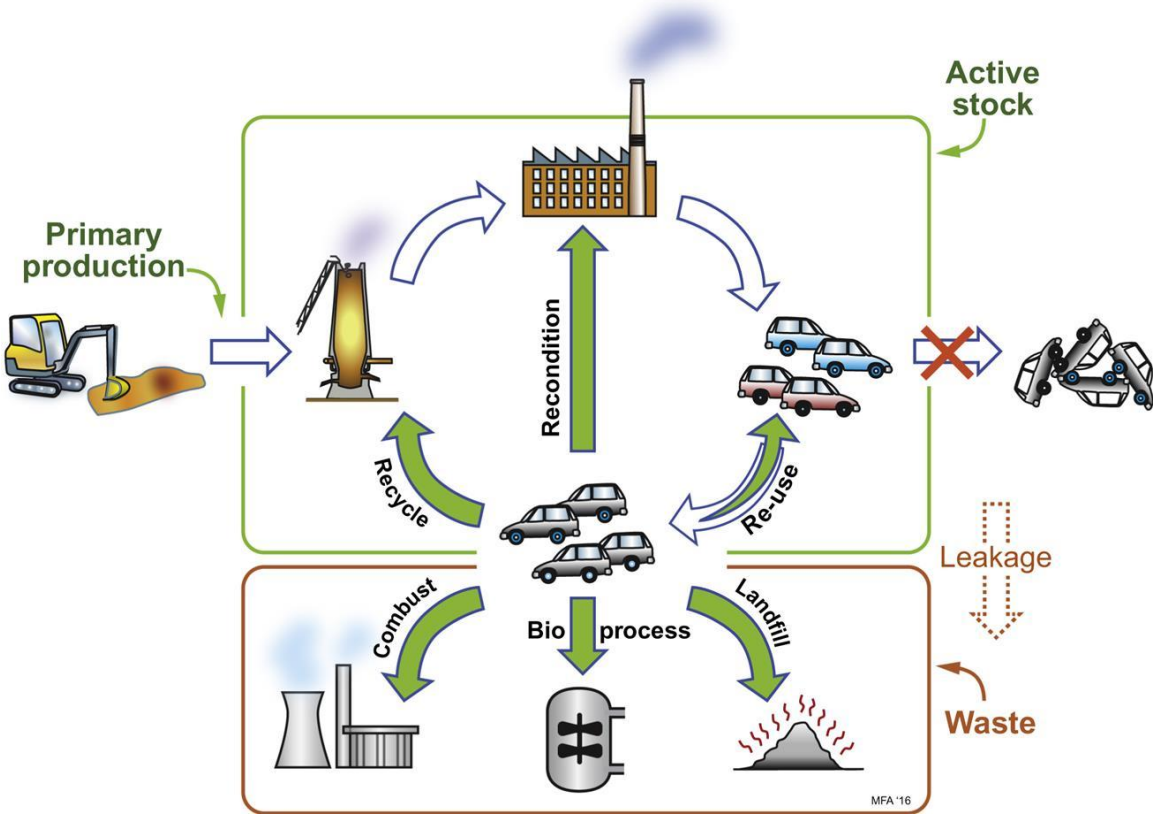


FIGURE 16.6 The circular materials economy. The aim is to retain materials in the 'Active stock' box by reuse, reconditioning and recycling, minimizing leakage into the 'Waste' box.

The *circular economy* means more than just efficient recycling. Taken literally, it means relying on renewable energy, tracking materials through the economy so that their locations are known and using them in designs that allow their reuse with as little reprocessing as possible. The concept goes beyond the mechanics of production and consumption of goods, moving from the idea of *consuming* of materials to one of *using* them, somewhat in the way properly-managed land is used for agriculture without consuming it. This implies a different approach to design, one that retains or regenerates materials during several manufacturing cycles[3] and reduces demand on the use of critical materials. It is an important part of a resource-efficient, low-carbon economy, reducing costs and supply risks, and generating value.[4]

That is the ideal. Ideals have value; they set a target, something to be worked towards even if perfect fulfilment is not possible.

16.6 Response to Forces for Change: Sustainable Development

Governments, nongovernment organizations and individuals respond to forces for change by … well … proposing to change something. When considering a response to one of the many forces for change we need a way to assess the degree to which it contributes to a more sustainable world. We need to ask: is this a *sustainable development*?

But what is a 'sustainable development'? It has become a buzz-phrase, something warm and comfortable but often meaning little more than 'environmentally desirable'; and it has become a favoured way of relabelling something that is already happening to give it the ring of responsibility. We need to do better than that. Here is a short answer: a sustainable development is one that provides needed products or services in ways that reduce the drain on natural resources, is legal, economically viable, acceptable to all stakeholders and equitable both within and between generations. To get further we need the three Capitals[5] (Fig. 16.7).

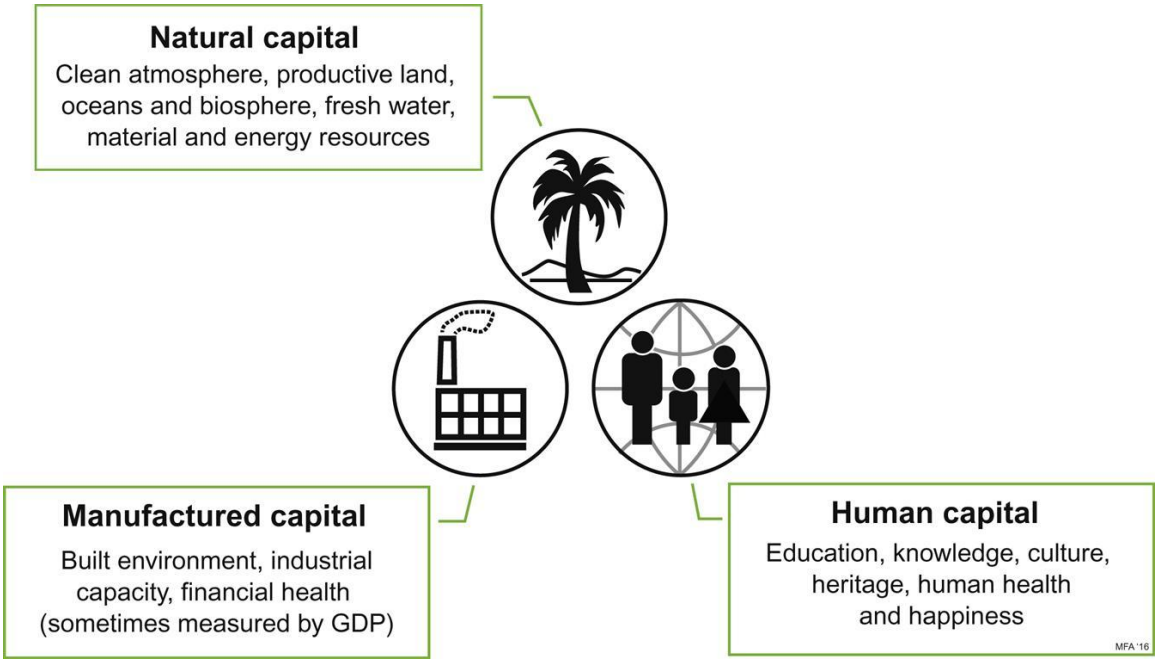


FIGURE 16.7 The three Capitals.

Three essential 'Capitals' underpin society as we know it today. We met the first – Natural Capital – in Chapter 14, Materials and the Environment.

The ultimate goal of eco-design is that of preserving a clean atmosphere, productive land and sea, fresh water humans, livestock and agriculture and conservation of material and energy resources. Eco-design is a component of sustainable design but it is not the only component. Manufactured Capital provides the essential infrastructure for society: housing, transport, the manufacturing base and the financial services that allow the exchange of allow goods and services. None of this can be managed without Human Capital, a measure not only of education and creativity but also of ability to pass knowledge, culture and heritage from one generation to the next and to do so ways that promote fulfillment and happiness. Each capital is like a bank balance on which we draw and into which contributions can be made, but they cannot be thought about as three independent accounts. The Capitals interact; they are components of a complex and incompletely understood system.

The merit of the Three Capitals concept is that it provides yardsticks against which a proposed sustainable development can be judged. A set of checklists (Table 16.3(a), (b) and (c)) suggest questions to expose the impact of a proposed development on each Capital. Not all can be answered – some are more relevant for some projects, others for others. But the attempt to answer them provides the background that allows the ultimate merit of the proposal to be debated.

Table 16.3

(A), (B) and (C): Check Lists for the Three Capitals

(A) Natural Capital
• Does the project reduce dependence on finite resources? • Does it reduce emissions to air, water and land? • How are biodiversity and ecosystems affected? • Does it cause irreversible change? • Is a rebound effect possible? (Greater efficiency causing increased consumption.)
(B) Manufactured Capital
• What will the project cost? What revenue will it generate? • Will it increase industrial capacity? • How will existing institutions be affected? • Does it increase employment and livelihood? • Is it creating new opportunities for development or innovations?
(C) Human Capital
• Have stakeholders been properly consulted? Have their concerns been addressed? • How are human health, education and skills affected by the articulation? • Will the project contribute to human happiness and well-being? • Does it increase knowledge? • Is it culturally acceptable? Does it affect cultural identity or heritage? • Does it promote equality? • Is it consistent with principles of freedom of information and speech, good governance and democracy?

It is here that values, culture, beliefs and ethics enter. Think of each Capital as representing the world-view associated with a particular set of values. An environmentalist might argue that the impact on Natural Capital ranked most highly: after all, the natural environment is the support system of all life. Humanists might see the sharing of knowledge, understanding, reason, humanity and happiness as the central pillars of a civilized society and feel that any adverse impact on Human Capital was unacceptable. To an economist, economic stability and growth of Manufactured Capital could seem to be the first priority, arguing that these provide the resources needed to protect the environment, enable innovation and support a vibrant society. Each of these groups recognizes the cases made by the others; indeed they have many concerns in common. But their final judgment will be influenced by their underlying beliefs and values, cultural, religious and political. It is no surprise that one set of facts can be interpreted in more than one way. A balanced view is best formed by debating the facts from the perspective of each of each of these value-sets in turn, seeking to identify what is of value and what is unacceptable to each. There is no completely 'right' answer to questions of

sustainability development; instead there is a thoughtful, well-researched response that recognizes the many conflicting facets and seeks the most productive compromise.

These ideas are pursued more fully in the text which appears as the second entry under Further Reading.

16.7 Summary and Conclusions

Powerful forces drive the development of new and improved materials, encourage substitution, and modify the way in which materials are produced and used. Market forces and military prerogatives, historically the most influential, remain the strongest. The ingenuity of research scientists, too, drives change by revealing a remarkable spectrum of new materials with exciting possibilities, though the time it takes to develop and commercialise them is long: typically 12 years from laboratory to market place.

Today additional new drivers influence material development and use. Increasing wealth creates markets for ever more sophisticated products. The trend to products that are smaller in size, lighter in weight and have greater functionality makes increasing demands on the mechanical properties of the materials used to make them. Greater insistence on product reliability and safety, with the manufacture held liable for failures or malfunctions, requires materials with consistently reproducible properties and processes that are tightly controlled. Concern for the impact of industrial growth on the natural environment introduces the new objective of selecting materials in such a way as to minimize it.

Responses to forces for change, today, aim at developments that are ‘sustainable’. This includes concern for the natural environment, but there is more to it than that. A development that ignores the needs, culture and wishes of the people affected by it is exposed to opposition and disruption. One that is not financially viable, equally, risks failure. Issues of sustainability are not simple: they include legal, social and environmental issues that interact in complex ways. Examining proposed developments through the lens of the Three Capitals in the way described in the chapter gives a pathway to anticipate problems and an opportunity to reflect on how these might be overcome.

16.8 Further Reading

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3. Ashby MF, Ferrer-Balas D, Segalas Coral J. *Materials and Sustainable Development* Oxford: Butterworth-Heinemann Ltd; 2016; ISBN-10: 0081001762 ISBN-13: 978-0081001769. (*An introduction to the concept of sustainable development with case studies and exercises*.).
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material evolution.).

[1]If you want a vision of the Grand Challenges we face, here is one current in 2016: that of the US National Academy of Engineering http://www.engineeringchallenges.org/14373/GrandChallengesBlog/8275.aspx. Many of them involve materials. But be aware that the vision will have evolved by the time you read this.

[2]http://www.foe.co.uk/sites/default/files/downloads/overconsumption.pdf.

[3]http://www.ellenmacarthurfoundation.org/.

[4]https://www.innovateuk.org/competition-display-page/-/asset_publisher/RqEt2AKmEBhi/content/resource-efficiency-new-designs-for-a-circular-economy.

[5]Often misleadingly simplified to 'Planet – People – Profit'.

APPENDIX A

Data for Engineering Materials

This appendix lists the names and typical applications common engineering materials, together with data for their properties (Table A.1–A.10).

Table A.1
Material Names and Applications

Polymers		Applications
Elastomers		
Butyl rubber		Tyres, seals, antivibration mountings, electrical insulation, tubing
Ethylene vinyl acetate	EVA	Bags, films, packaging, gloves, insulation, running shoes
Isoprene	IR	Tyres, inner tubes, insulation, tubing, shoes
Natural rubber	NR	Gloves, tyres, electrical insulation, tubing
Polychloroprene (Neoprene)	CR	Wetsuits, O-rings and seals, footwear
Polyurethane elastomers	el-PU	Packaging, hoses, adhesives, fabric coating
Silicone elastomers		Electrical insulation, electronic encapsulation, medical implants
Thermoplastics		
Acrylonitrile butadiene styrene	ABS	Communication appliances, automotive interiors, luggage, toys, boats
Cellulose polymers	CA	Tool and cutlery handles, decorative trim, pens
Ionomer	I	Packaging, golf balls, blister packs, bottles
Polyamides (Nylons)	PA	Gears, bearings, plumbing, packaging, bottles, fabrics, textiles, ropes
Polycarbonate	PC	Safety goggles, shields, helmets, light fittings, medical components
Polyetheretherketone	PEEK	Electrical connectors, racing car parts, fibre composites
Polyethylene	PE	Packaging, bags, squeeze tubes, toys, artificial joints
Polyethylene terephthalate	PET	Blow moulded bottles, film, audio/video tape, sails
Polymethyl methacrylate	PMMA	Aircraft windows, lenses, reflectors, lights, compact discs
Polyoxymethylene (Acetal)	POM	Zips, domestic and appliance parts, handles
Polypropylene	PP	Ropes, garden furniture, pipes, kettles, electrical insulation, astroturf
Polystyrene	PS	Toys, packaging, cutlery, audio cassette/CD cases
Polyurethane thermoplastics	tp-PU	Cushioning, seating, shoe soles, hoses, car bumpers, insulation
Polyvinylchloride	PVC	Pipes, gutters, window frames, packaging
Polytetrafluoroethylene (Teflon)	PTFE	Nonstick coatings, bearings, skis, electrical insulation, tape
Thermosets		
Epoxies	EP	Adhesives, fibre composites, electronic encapsulation
Phenolics	PHEN	Electrical plugs, sockets, cookware, handles, adhesives
Polyester	PEST	Furniture, boats, sports goods
Polymer Foams		
Flexible polymer foam		Packaging, buoyancy, cushioning, sponges, sleeping mats
Rigid polymer foam		Thermal insulation, sandwich panels, packaging, buoyancy

Natural Materials	Applications
Cork	Corks and bungs, seals, floats, packaging, flooring
Leather	Shoes, clothing, bags, drive-belts
Wood	Construction, flooring, doors, furniture, packaging, sports goods

Composites	Applications
Aluminium/Silicon carbide	Automotive parts, sports goods
CFRP	Lightweight structural parts (aerospace, bike frames, sports goods, boat hulls and oars, springs)
GFRP	Boat hulls, automotive parts, chemical plant

Ceramics	Applications
Glasses	

Glasses	
Borosilicate glass	Ovenware, laboratory ware, headlights
Glass ceramic	Cookware, lasers, telescope mirrors
Silica glass	High performance windows, crucibles, high temperature applications
Soda-Lime glass	Windows, bottles, tubing, light bulbs, pottery glazes
Technical	
Alumina	Cutting tools, spark plugs, microcircuit substrates, valves
Aluminium nitride	Microcircuit substrates and heat sinks
Boron carbide	Lightweight armour, nozzles, dies, precision tool parts
Silicon	Microcircuits, semiconductors, precision instruments, IR windows, MEMS
Silicon carbide	High temperature equipment, abrasive polishing grits, bearings, armour
Silicon nitride	Bearings, cutting tools, dies, engine parts
Tungsten carbide	Cutting tools, drills, abrasives
Nontechnical	
Brick	Buildings
Concrete	General civil engineering construction
Stone	Buildings, architecture, sculpture

Table A.2
Density and Price

Metals		Density (kg/m³)	Price (2016 US$/kg)
Ferrous metals	Cast iron, ductile (nodular)	7100–7300	0.54–0.6
	Cast iron, grey	7100–7300	0.45–0.5
	High carbon steel	7800–7900	0.52–0.58
	Low alloy steel	7800–7900	0.56–0.62
	Low carbon steel	7800–7900	0.52–0.58
	Medium carbon steel	7800–7900	0.52–0.58
	Stainless steel	7600–8100	5.9–6.5
Nonferrous	Aluminium alloys	2500–2900	2.1–2.3
	Copper alloys	8900	7.1–7.8
	Lead alloys	10,000–11,000	6.9–7.6
	Magnesium alloys	1700–2000	3.1–3.4
	Nickel alloys	8800–9000	17–19
	Silver	10,000–10,100	650–710
	Tin	7300	23–25
	Titanium alloys	4400–4800	22–25
	Tungsten alloys	10,800–20,000	51–57
	Zinc alloys	5000–7000	2.4–2.6

Polymers		Density (kg/m³)	Price (2016 US$/kg)
	Polyurethane (PUR)	1100–1200	4.1–4.6
	Polyvinylchloride (PVC)	1300–1600	2–2.2
Thermosets	Epoxies	1100–1400	2.2–2.5
	Phenolics	1200–1300	1.7–1.9
	Polyester	1000–1400	3.8–4.3
Foams	Flexible polymer foam (LD)	38–70	2.6–2.9
	Flexible polymer foam (MD)	70–120	2.8–3.1
	Rigid polymer foam (HD)	170–470	15–17
	Rigid polymer foam (LD)	36–70	15–17

Composites		Density (kg/m³)	Price (2016 US$/kg)
	CFRP, epoxy matrix (isotropic)	1500–1600	37–42
	GFRP, epoxy matrix (isotropic)	1800–2000	24–34



Table A.3
Young's Modulus and Yield Strength

Metals		Young's Modulus (GPa)	Yield Strength (MPa)
Ferrous metals	Cast iron, ductile (nodular)	170–180	250–680
	Cast iron, grey	80–140	140–420
	High carbon steel	200–220	400–1200
	Low alloy steel	210–220	400–1500
	Low carbon steel	200–220	250–400
	Medium carbon steel	200–220	310–900
	Stainless steel	190–210	170–100
Nonferrous	Aluminium alloys	68–82	30–500
	Copper alloys	110–150	30–500
	Lead alloys	13–15	8–14
	Magnesium alloys	42–47	70–400
	Nickel alloys	190–220	70–1100
	Silver	69–73	190–300
	Tin	41–45	7–15
	Titanium alloys	90–120	250–1,200
	Tungsten alloys	310–380	530–800
	Zinc alloys	68–95	80–450

Polymers		Young's Modulus (GPa)	Yield Strength (MPa)
	Polyurethane (tpPUR)	1.3–2.1	40–54
	Polyvinylchloride (tpPVC)	2.1–4.1	35–52
Thermosets	Epoxies	2.4–3.1	36–72
	Phenolics	2.8–4.8	28–50
	Polyester	2.1–4.4	33–40
Foams	Flexible polymer foam (LD)	0.001–0.003	0.02–0.3
	Flexible polymer foam (MD)	0.004–0.012	0.048–0.7
	Rigid polymer foam (HD)	0.2–0.48	0.8–12
	Rigid polymer foam (LD)	0.023–0.080	0.3–1.7

Composites		Young's Modulus (GPa)	Yield Strength (MPa)
	CFRP, epoxy matrix (isotropic)	69–150	550–1100
	GFRP, epoxy matrix (isotropic)	15–28	110–190

Table A.4

Tensile Strength and Fracture Toughness

Metals		Tensile Strength (MPa)	Fracture Toughness (MPa·m$^{1/2}$)
Ferrous metals	Cast iron, ductile (nodular)	410–830	22–54
	Cast iron, grey	140–450	10–24
	High carbon steel	550–1600	27–92
	Low alloy steel	550–1800	14–200
	Low carbon steel	350–580	41–82
	Medium carbon steel	410–1200	12–92
	Stainless steel	480–2200	62–150
Nonferrous	Aluminium alloys	58–550	22–35
	Copper alloys	100–550	30–90
	Lead alloys	12–20	5–15
	Magnesium alloys	190–480	12–18
	Nickel alloys	350–1200	80–110
	Silver	260–340	40–60
	Tin	11–18	15–30
	Titanium alloys	300–1600	14–120
	Tungsten alloys	720–300	50–60
	Zinc alloys	140–520	10–100

Polymers		Tensile Strength (MPa)	Fracture Toughness (MPa·m$^{1/2}$)
	Polyurethane (tpPUR)	31–62	1.8–5
	Polyvinylchloride (tpPVC)	41–65	1.5–5.1
Thermosets	Epoxies	45–90	0.4–2.2
	Phenolics	35–62	0.79–1.2
	Polyester	41–90	1.1–1.7
Foams	Flexible polymer foam (LD)	0.24–2.4	0.015–0.05
	Flexible polymer foam (MD)	0.43–3	0.03–0.09
	Rigid polymer foam (HD)	1.2–12	0.024–0.09
	Rigid polymer foam (LD)	0.45–2.3	0.002–0.02

Composites		Tensile Strength (MPa)	Fracture Toughness (MPa·m$^{1/2}$)
	CFRP, epoxy matrix (isotropic)	550–1100	6.1–20
	GFRP, epoxy matrix (isotropic)	140–240	7–23

Table A.5

Melting Temperature, Glass Temperature and Specific Heat

Metals		T_m or T_g (°C)	Specific Heat (J/kg·C)
Ferrous metals	Cast iron, ductile (nodular)	1100–1200	460–500
	Cast iron, grey	1100–1400	430–500
	High carbon steel	1300–1500	440–510
	Low alloy steel	1400–1500	410–530
	Low carbon steel	1470–1500	460–510
	Medium carbon steel	1400–1500	440–520
	Stainless steel	1350–1400	450–530
Nonferrous	Aluminium alloys	470–680	860–990
	Copper alloys	980–1100	370–390
	Lead alloys	320–330	120–150
	Magnesium alloys	450–650	960–1100
	Nickel alloys	1400–1500	450–460
	Silver	960–970	230–240
	Tin	230	220–230
	Titanium alloys	1500–1700	520–600
	Tungsten alloys	3200–3400	130–140
	Zinc alloys	370–490	410–540

Polymers		T_m or T_g (°C)	Specific Heat (J/kg·C)
	Polyurethane (tpPUR)	60–90	1600
	Polyvinylchloride (tpPVC)	75–100	1400
Thermosets	Epoxies	67–170	1500–200
	Phenolics	170–270	1500
	Polyester	150–210	1500–1600
Foams	Flexible polymer foam (LD)	−110–−13	1800–2300
	Flexible polymer foam (MD)	−110–−13	1800–2300
	Rigid polymer foam (HD)	67–170	1000–1900
	Rigid polymer foam (LD)	67–170	1100–1900

Composites		T_m or T_g (°C)	Specific Heat (J/kg·C)
	CFRP, epoxy matrix (isotropic)	100–180	900–100

	GFRP, epoxy matrix (isotropic)	150–200	100–1200

Table A.6

Thermal Conductivity and Thermal Expansion

Metals		Thermal Conductivity (W/m/K)	Thermal Expansion (10^{-6}°C)
Ferrous metals	Cast iron, ductile (nodular)	29–44	10–13
	Cast iron, grey	40–72	11–13
	High carbon steel	47–53	11–14
	Low alloy steel	34–55	11–14
	Low carbon steel	49–54	12–13
	Medium carbon steel	45–55	10–14
	Stainless steel	12–24	13–20
Nonferrous	Aluminium alloys	76–240	21–24
	Copper alloys	160–390	17–18
	Lead alloys	22–36	18–32
	Magnesium alloys	50–160	25–28
	Nickel alloys	67–91	12–14
	Silver	420	20
	Tin	60–62	23–24
	Titanium alloys	7–14	7.9–11
	Tungsten alloys	100–140	4–5.6
	Zinc alloys	100–140	23–28

Polymers		Thermal Conductivity (W/m/K)	Thermal Expansion (10^{-6}°C)
	Polyurethane (tpPUR)	0.23–0.24	90–140
	Polyvinylchloride (tpPVC)	0.15–0.29	100–150
Thermosets	Epoxies	0.18–0.5	58–120
	Phenolics	0.14–0.15	120
	Polyester	0.29–0.3	99–180
Foams	Flexible polymer foam (LD)	0.040–0.059	120–220
	Flexible polymer foam (MD)	0.041–0.078	120–220
	Rigid polymer foam (HD)	0.034–0.063	22–70
	Rigid polymer foam (LD)	0.023–0.040	20–80

Composites		Thermal Conductivity (W/m/K)	Thermal Conductivity (10^{-6}°C)
	CFRP, epoxy matrix (isotropic)	1.3–2.6	1–4
	GFRP, epoxy matrix (isotropic)	0.4–0.55	8.6–33

Table A.7
Electrical Resistivity and Dielectric Constant

Metals		Resistivity (μohm·cm)	Dielectric Constant (–)
Ferrous metals	Cast iron, ductile (nodular)	49–56	
	Cast iron, grey	62–86	
	High carbon steel	17–20	
	Low alloy steel	15–35	
	Low carbon steel	15–20	
	Medium carbon steel	15–22	
	Stainless steel	64–110	
Nonferrous	Aluminium alloys	3.8–6	
	Copper alloys	1.7–5	
	Lead alloys	15–22	
	Magnesium alloys	5.5–15	
	Nickel alloys	8–10	
	Silver	1.7–1.8	
	Tin	10–12	
	Titanium alloys	100–170	
	Tungsten alloys	10–14	
	Zinc alloys	5.4–7.2	

Polymers		Resistivity (μohm·cm)	Dielectric Constant (–)
	Polyurethane (tpPUR)	3.3×10^{18}8–30×10^{18}	6.6–7.1
	Polyvinylchloride (tpPVC)	100×10^{18}–10×10^{21}	3.1–4.4
Thermosets	Epoxies	100×10^{18}–6×10^{21}	3.4–5.7
	Phenolics	3.3×10^{18}–30×10^{18}	4–6
	Polyester	3.3×10^{18}–30×10^{18}	2.8–3.3
Foams	Flexible polymer foam (LD)	100×10^{18}–100×10^{21}	1.1–1.2
	Flexible polymer foam (MD)	100×10^{18}–100×10^{21}	1.2–1.3
	Rigid polymer foam (HD)	10×10^{15}–100×10^{18}	1.2–1.5
	Rigid polymer foam (LD)	100×10^{15}–1×10^{21}	1–1.1

Composites		Resistivity (μohm·cm)	Dielectric Constant (–)
	CFRP, epoxy matrix (isotropic)	170×10^{3}–950×10^{3}	
	GFRP, epoxy matrix (isotropic)	2.4×10^{21}–19×10^{21}	4.9–5.2

Table A.8

Piezoelectric, Pyroelectric and Ferroelectric Materials

Ferroelectric Materials		Dielectric Constant (–)	Breakdown Potential (MV/m)
	Barium titanate	3400	4.2
	PLZT	2500	12
	PZT, hard	1300	11
	PZT, soft	2700	11

Table A.9

Magnetic, Magnetostrictive and Magnetocaloric Materials

Hard Magnetic Materials		Remanent Induction (T)	Coercive Force (A/m)	Max. Energy Product (MJ/m^3)
	Alnico	0.52–1.4	3600–6300	9600–60,000
	Ferrites, hard	0.2–0.46	13,000–35,000	6400–41,000
	Neodymium iron boron	0.98–1.5	60,000–1,100,000	190,000–180,000
	Samarium cobalt	0.78–1.2	33,000–82,000	110,000–250,000

Table A.10

Embodied Energy, Carbon Footprint and Water Demand

Metals		Energy (MJ/kg)	Carbon (kg/kg)	Water (L/kg)
Ferrous metals	Cast iron, ductile (nodular)	18–22	1.7–1.8	43–47
	Cast iron, grey	17–21	1.7–1.8	42–46
	High carbon steel	25–28	1.7–1.9	44–48
	Low alloy steel	29–32	1.9–2.1	48–53
	Low carbon steel	25–28	1.7–1.9	43–48
	Medium carbon steel	25–28	1.7–1.9	44–48
	Stainless steel	80–89	4.7–5.2	130–140
Nonferrous	Aluminium alloys	200–220	12–13	1100–1300
	Copper alloys	57–63	3.5–3.9	290–320
	Lead alloys	66–73	4.2–4.6	2400–2700
	Magnesium alloys	290–320	34–37	930–1000
	Nickel alloys	160–180	11–12	220–250
	Silver	1400–1600	95–110	1200–3500
	Tin	220–240	13–14	10,000–12,000
	Titanium alloys	650–720	44–49	190–210
	Tungsten alloys	510–560	33–36	150–160
	Zinc alloys	57–63	3.9–4.3	400–440

Polymers		Energy (MJ/kg)	Carbon (kg/kg)	Water (L/kg)
	Polyvinylchloride (tpPVC)	55–61	2.4–2.6	200–220
Thermosets	Epoxies	130–140	6.8–7.6	27–29
	Phenolics	75–83	3.4–3.8	49–54
	Polyester	68–75	2.8–3.1	190–210
Foams	Flexible polymer foam (LD)	100–110	4.3–4.7	220–240
	Flexible polymer foam (MD)	100–110	3.4–3.8	170–180
	Rigid polymer foam (HD)	97–110	3.7–4.1	440–480
	Rigid polymer foam (LD)	97–110	3.7–4.1	440–480

Composites		Energy (MJ/kg)	Carbon (kg/kg)	Water (L/kg)
	CFRP, epoxy matrix (isotropic)	450–500	33–36	1300–1500
	GFRP, epoxy matrix (isotropic)	150–170	9.5–11	150–170

A.1 Ways of Checking and Estimating Data

The value of a database of material properties depends on its precision and its completeness – in short, on its quality. One way of maintaining or enhancing its quality is to subject its contents to validating procedures. The property range checks and dimensionless correlations described below provide powerful tools for doing this. The same procedures fill a second function: that of providing estimates for missing data, essential when no direct measurements are available.

Property ranges. Each property of a given class of materials has a characteristic *range*. A convenient way of presenting the information is as a table in which a low (L) and a high (H) value are stored, identified by the material family and class. An example listing Young's modulus, E, is shown in Table A.11, in which E_L is the lower limit and E_H the upper one.

Table A.11

Ranges of Young's Modulus *E* for Broad Material Classes

Material Class	E_L (GPa)	E_H (GPa)
All Solids	0.00001	1000
Classes of Solid		
Metals: ferrous	70	220
Metals: nonferrous	4.6	570
Technical ceramics[a]	91	1000
Glasses	47	83
Polymers: thermoplastic	0.1	4.1
Polymers: thermosets	2.5	10
Polymers: elastomers	0.0005	0.1
Polymeric foams	0.00001	2
Composites: metal-matrix	81	180
Composites: polymer-matrix	2.5	240
Woods: parallel to grain	1.8	34
Woods: perpendicular to grain	0.1	18

[a]Technical ceramics are dense, monolithic ceramics such as SiC, Al_2O_3, ZrO_2 etc.

All properties have characteristic ranges like these. The range becomes narrower if the classes are made more restrictive. For purposes of checking and estimation, described in a moment, it is helpful to break down the family of *metals* into classes of cast irons, steels, aluminium alloys, magnesium alloys, titanium alloys, copper alloys and so on. Similar subdivisions for polymers (thermoplastics, thermosets, elastomers) and for ceramics and glasses (engineering ceramics, whiteware, silicate glasses, minerals) increase resolution here also.

Correlations between material properties. Materials that are stiff have high melting points. Solids with low densities have high specific heats. Metals with high thermal conductivities have high electrical conductivities. These rules-of-thumb describe correlations between two or more material properties that can be expressed more quantitatively as limits for the values of *dimensionless property groups*. They take the form

$$C_L < P_1 P_2^{\,n} < C_H \tag{A.1}$$

or

$$C_L < P_1 P_2^{\,n} P_3^{\,m} < C_H \tag{A.2}$$

(or larger groupings) where P_1, P_2, P_3 are material properties, n and m are powers (usually −1, −1/2, +1/2 or +1), and C_L and C_H are dimensionless constants – the lower and upper limits between which the values of the property-group lies. The correlations exert tight constraints on the data, giving the "patterns" of property envelopes that appear on the material selection charts. An example is the relationship between expansion coefficient, α (units: K^{-1}), and the melting point, T_m (units: K) or, for amorphous materials, the glass temperature T_g:

$$C_L \leq \alpha T_m \leq C_H \tag{A.3a}$$

$$C_L \leq \alpha T_g \leq C_H \tag{A.3b}$$

a correlation with the form of Eq. (A.1). Values for the dimensionless limits C_L and C_H for this group are listed in Table A.12 for a number of material classes. The values span a factor to 2 to 10 rather than the factor 10 to 100 of the property ranges. There are many such correlations. They form the basis of a hierarchical data checking and estimating scheme (one used in preparing the charts in this book), described next.

Table A.12

Limits for the Group αT_m and αT_g for Broad Material Classes[a]

Correlation[a] $C_L<\alpha T_m<C_H$	C_L ($\times 10^{-3}$)	C_H ($\times 10^{-3}$)
All Solids	0.1	56
Classes of Solid		
Metals: ferrous	13	27
Metals: nonferrous	2	21
Technical ceramics[a]	6	24
Glasses	0.3	3
Polymers: thermoplastic	18	35
Polymers: thermosets	11	41
Polymers: elastomers	35	56
Polymeric foams	16	37
Composites: metal-matrix	10	20
Composites: polymer-matrix	0.1	10
Woods: parallel to grain	2	4
Woods: perpendicular to grain	6	17

[a]For amorphous solids the melting point T_m is replaced by the glass temperature T_g.

Data checking. Data checks proceed in three steps. Each datum is first associated with a material class, or, at a higher level of checking, with a subclass. This identifies the values of the property range and correlation limits against which it will be checked. The datum is then compared with the range limits L and H for that class and property. If it lies within the range limits, it is accepted; if it does not, it is flagged for checking.

Why bother with such low-level stuff? Because it provides a sanity-check. The commonest error in handbooks and other compilations of material or process properties is that of a value that is expressed in the wrong units, or is, for less obvious reasons, in error by one or more orders of magnitude (slipped decimal point, for instance). Range checks catch errors of this sort. If a demonstration of this is needed, it can be found by applying them to the contents of almost any standard reference data-books; none among those we have tried has passed without errors.

In the third step, each of the dimensionless groups of properties like that of Table A.12 is formed in turn, and compared with the range bracketed by the limits C_L and C_H. If the value lies within its correlation limits, it is accepted; if not, it is checked. Correlation checks are more discerning than range checks and catch subtler errors, allowing the quality of data to be enhanced further.

Data estimation. The relationships have another, equally useful, function. There remain gaps in our knowledge of material properties. The fracture toughness of many materials has not yet been measured, nor has the electric breakdown potential; even moduli are not always known. The absence of a datum for a material would falsely eliminate it from a screening exercise that used that property, even though the material might be a viable candidate. This difficulty is avoided by using the correlation and range limits to estimate a value for the missing datum, adding a flag to alert the user that it is an estimate.

In estimating property values, the procedure used for checking is reversed: the dimensionless groups are used first because they are the more accurate. They can be surprisingly good. As an example, consider estimating the expansion coefficient, α of polycarbonate from its glass temperature T_g. Inverting Eq. (A.3b) gives the estimation rule:

$$\frac{C_L}{T_g} \leq \alpha \leq \frac{C_H}{T_g} \qquad \textbf{(A.4)}$$

Inserting values of C_L and C_H from Table A.12, and the value T_g=420 K for a particular sample of polycarbonate gives the mean estimate

$$\overline{\alpha} = 63 \times 10^{-3}\ K^{-1} \qquad \textbf{(A.5)}$$

The reported value for polycarbonate is

$$\alpha = 54 - 62 \times 10^{-3} K^{-1}$$

The estimate is within 9% of the mean of the measured values, perfectly adequate for screening purposes. However, that it is an estimate must not be forgotten: if thermal expansion is crucial to the design, better data or direct measurements are essential.

Only when the potential of the correlations is exhausted are the property ranges invoked. They provide a crude first-estimate of the value of the missing property, far less accurate than that of the correlations, but still useful in providing guide values for screening.

Further Reading

1. Ashby MF. "Checks and estimates for material properties" Cambridge University Engineering Department. *Proc R Soc A*. 1998;454:1301–1321 (*This and the next reference detail the data-checking and estimation methods, listing the correlations*.).
2. Bassetti D, Brechet Y, Ashby MF. Estimates for material properties: the method of multiple correlations. *Proc R Soc A*. 1998;454:1323–1336 (*This and the previous reference detail the data-checking and estimation methods, with examples*.).

APPENDIX B

Useful Solutions for Standard Problems

Introduction and Synopsis

Modelling is a key part of design. In the early stage, approximate modelling establishes whether the concept will work at all, and identifies the combination of material properties that maximize performance. At the embodiment stage, more accurate modelling brackets values for the forces, the displacements, the velocities, the heat fluxes and the dimensions of the components. And in the final stage, modelling gives precise values for stresses, strains and failure probability in key components; power, speed, efficiency and so forth.

Many components with simple geometries and loads have been modelled already. Many, more complex, components can be modelled approximately by idealising them as one of these. There is no need to reinvent the beam or the column or the pressure vessel; their behaviour under all common types of loading has already been analysed. The important thing is to know that the results exist and where to find them.

This appendix summarises the results of modelling a number of standard problems. Their usefulness cannot be overstated. Many problems of conceptual design can be treated, with adequate precision, by patching together the solutions given here; and even the detailed analysis of noncritical components can often be tackled in the same way. Even when this approximate approach is not sufficiently accurate, the insight it gives is valuable.

This appendix contains 16 double-page sections that list, with a short commentary, results for constitutive equations; for the loading of beams,

columns and torsion bars; for contact stresses, cracks and other stress concentrations; for pressure vessels, vibrating beams and plates; and for the flow of heat and matter. They are drawn from numerous sources, listed under Further Reading in Section B.17.

B.1 Constitutive Equations for Mechanical Response

The behaviour of a component when it is loaded depends on the *mechanism* by which it deforms. A beam loaded in bending may deflect elastically; it may yield plastically; it may deform by creep; and it may fracture in a brittle or in a ductile way. The equation that describes the material response is known as a *constitutive equation*. Each mechanism is characterised by a different constitutive equation. The constitutive equation contains one or more than one *material property*: Young's modulus, E, and Poisson's ratio, ν, are the material properties that enter the constitutive equation for linear-elastic deformation; the yield strength, σ_Y, is the material property that enters the constitutive equation for plastic flow; creep constants, $\dot{\varepsilon}_0$, σ_0 and n enter the equation for creep; and the fracture toughness, K_{1c}, enters that for brittle fracture.

The common constitutive equations for mechanical deformation are listed on the facing page. In each case the equation for uniaxial loading by a tensile stress σ is given first; below it is the equation for multiaxial loading by principal stresses σ_0, σ_2 and σ_3, always chosen so that σ_1 is the most tensile and σ_3 the most compressive (or least tensile) stress. They are the basic equations that determine mechanical response.

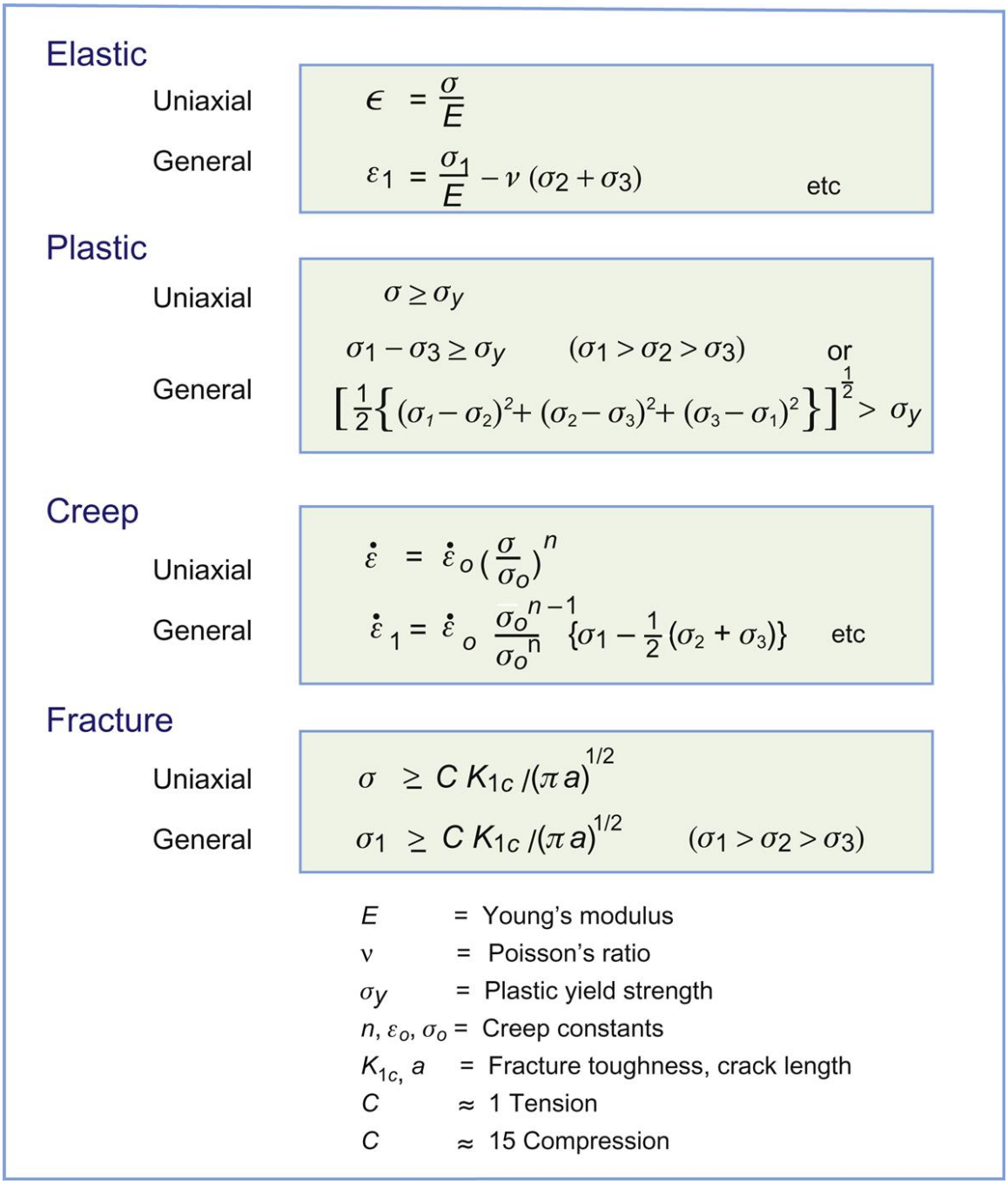


FIGURE B.1 Mechanical constitutive equations.

B.2 Moments of Sections

A beam of uniform section, loaded in simple tension by a force F, carries a stress

$$\sigma = F/A$$

where A is the area of the section. Its response is calculated from the appropriate constitutive equation. Here the important characteristic of the section is its area, A. For other modes of loading, higher moments of the area are involved. Those for various common sections are given on the facing page. They are defined as follows.

The second moment I measures the resistance of the section to bending about a horizontal axis (shown as a broken line). It is

$$I = \int_{\text{section}} y^2 b(y) dy$$

where y is measured vertically and $b(y)$ is the width of the section at y. The moment K measures the resistance of the section to twisting. It is equal to the polar moment J for circular sections, where

$$J = \int_{\text{section}} 2\pi r^3 dr$$

where r is measured radially from the centre of the circular section. For noncircular sections, K is less than J.

The section modulus $Z=I/y_m$ (where y_m is the normal distance from the neutral axis of bending to the outer surface of the beam) measures the surface stress generated by a given bending moment, M:

$$\sigma = \frac{My_m}{I} = \frac{M}{Z}$$

Finally, the moment Z_p, defined by

$$Z_p = \int_{\text{section}} yb(y) dy$$

measures the resistance of the beam to fully-plastic bending. The fully plastic moment for a beam in bending is

$$M_p = Z_p \sigma_y$$

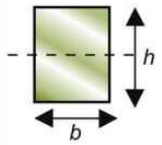
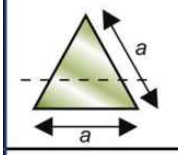
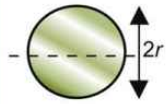
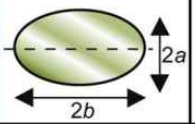
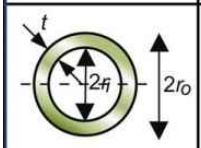
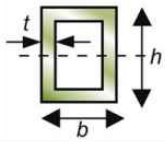
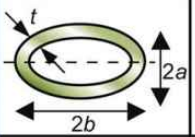
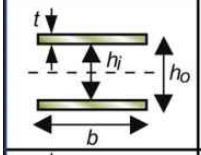
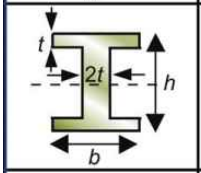
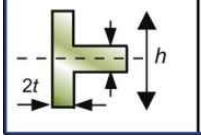
Section shape	Area A m^2	Moment I m^4	Moment K m^4	Moment Z m^3	Moment Z_p m^3
	bh	$\frac{bh^3}{12}$	$\frac{bh^3}{3}(1-0.58\frac{b}{h})$ $(h>b)$	$\frac{bh^2}{6}$	$\frac{bh^2}{4}$
	$\frac{\sqrt{3}}{4}a^2$	$\frac{a^4}{32\sqrt{3}}$	$\frac{\sqrt{3}a^4}{80}$	$\frac{a^3}{32}$	$\frac{3a^3}{64}$
	πr^2	$\frac{\pi}{4}r^4$	$\frac{\pi}{2}r^4$	$\frac{\pi}{4}r^3$	$\frac{\pi}{3}r^3$
	πab	$\frac{\pi}{4}a^3b$	$\frac{\pi a^3b^3}{(a^2+b^2)}$	$\frac{\pi}{4}a^2b$	$\frac{\pi}{3}a^2b$
	$\pi(r_o^2-r_i^2)$ $\approx 2\pi rt$	$\frac{\pi}{4}(r_o^4-r_i^4)$ $\approx \pi r^3 t$	$\frac{\pi}{2}(r_o^4-r_i^4)$ $\approx 2\pi r^3 t$	$\frac{\pi}{4r_o}(r_o^4-r_i^4)$ $\approx \pi r^2 t$	$\frac{\pi}{3}(r_o^3-r_i^3)$ $\approx \pi r^2 t$
	$2t(h+b)$ $(h,b>>t)$	$\frac{1}{6}h^3t(1+3\frac{b}{h})$	$\frac{2tb^2h^2}{(h+b)}(1-\frac{t}{h})^4$	$\frac{1}{3}h^2t(1+3\frac{b}{h})$	$bht(1+\frac{h}{2b})$
	$\pi(a+b)t$ $(a,b>>t)$	$\frac{\pi}{4}a^3t(1+\frac{3b}{a})$	$\frac{4\pi(ab)^{5/2}t}{(a^2+b^2)}$	$\frac{\pi}{4}a^2t(1+\frac{3b}{a})$	$\pi abt(2+\frac{a}{b})$
	$b(h_o-h_i)$ $\approx 2bt$ $(h,b>>t)$	$\frac{b}{12}(h_o^3-h_i^3)$ $\approx \frac{1}{2}bth_o^2$	--	$\frac{b}{6h_o}(h_o^3-h_i^3)$ $\approx bth_o$	$\frac{b}{4}(h_o^2-h_i^2)$ $\approx bth_o$
	$2t(h+b)$ $(h,b>>t)$	$\frac{1}{6}h^3t(1+3\frac{b}{h})$	$\frac{2}{3}bt^3(1+4\frac{h}{b})$	$\frac{1}{3}h^2t(1+3\frac{b}{h})$	$bht(1+\frac{h}{2b})$
	$2t(h+b)$ $(h,b>>t)$	$\frac{t}{6}(h^3+4bt^2)$	$\frac{t^3}{3}(8b+h)$	$\frac{t}{3h}(h^3+4bt^2)$	$\frac{th^2}{2}\{1+\frac{2t(b-2t)}{h^2}\}$

FIGURE B.2 Moments of sections.

B.3 Elastic Bending of Beams

When a beam is loaded by a force F or moments M, the initially straight axis is deformed into a curve. If the beam is uniform in section and properties, long in relation to its depth and nowhere stressed beyond the elastic limit, the deflection δ, and the angle of rotation, θ, can be calculated using elastic beam theory (see Further Reading in Section B.17). The basic differential equation describing the curvature of the beam at a point x along its length is

$$EI\frac{d^2y}{dx^2} = M$$

where y is the lateral deflection, and M is the bending moment at the point x on the beam. E is Young's modulus and I is the second moment of area (Section B.2). When M is constant this becomes

$$\frac{M}{I} = E\left(\frac{1}{R} - \frac{1}{R_o}\right)$$

where R_o is the radius of curvature before applying the moment and R the radius after it is applied. Deflections δ and rotations θ are found by integrating these equations along the beam. The stiffness of the beam is defined by

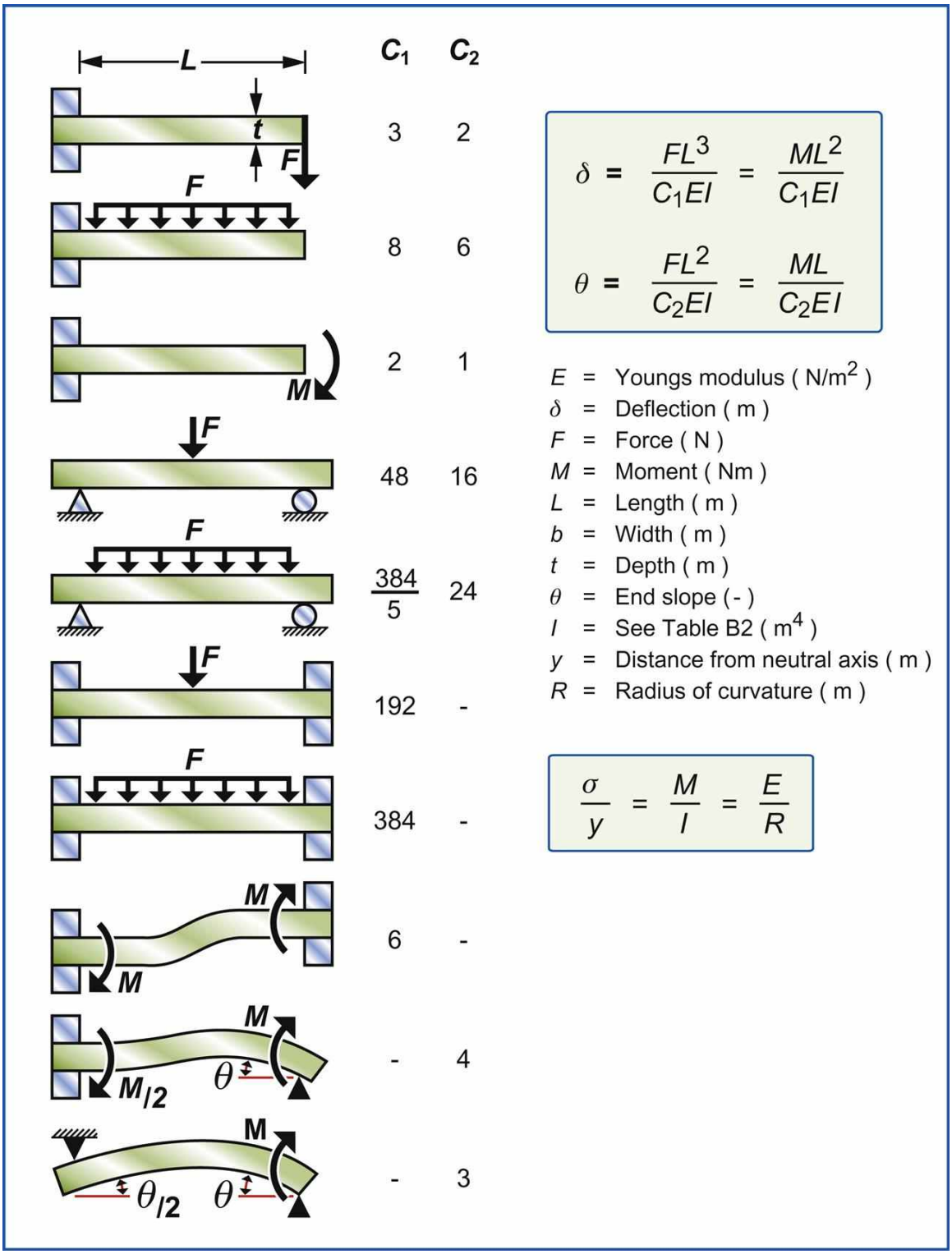
$$S = \frac{F}{\delta} = \frac{C_1 EI}{L^3}$$

It depends on Young's modulus, E, for the material of the beam, on its length, L, and on the second moment of its section, I. The end-slope of the beam, θ, is given by

$$\theta = \frac{FL^2}{C_2 EI}$$

Equations for the deflection, δ, and end slope, θ, of beams, for various common modes of loading are shown on the facing page together with

values of C_1 and C_2.



C_1	C_2
3	2
8	6
2	1
48	16
$\frac{384}{5}$	24
192	-
384	-
6	-
-	4
-	3

$$\delta = \frac{FL^3}{C_1 EI} = \frac{ML^2}{C_1 EI}$$

$$\theta = \frac{FL^2}{C_2 EI} = \frac{ML}{C_2 EI}$$

E = Youngs modulus (N/m^2)
δ = Deflection (m)
F = Force (N)
M = Moment (Nm)
L = Length (m)
b = Width (m)
t = Depth (m)
θ = End slope (-)
I = See Table B2 (m^4)
y = Distance from neutral axis (m)
R = Radius of curvature (m)

$$\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R}$$

FIGURE B.3 Deflection of beams.

B.4 Failure of Beams and Panels

The longitudinal (or 'fibre') stress σ at a point y from the neutral axis of a uniform beam loaded elastically in bending by a moment M is

$$\frac{\sigma}{y} = \frac{M}{I} = E\left(\frac{1}{R} - \frac{1}{R_o}\right)$$

where I is the second moment of area (Section B.2), E is Young's modulus, R_o is the radius of curvature before applying the moment and R is the radius after it is applied. The tensile stress in the outer fibre of such a beam is

$$\sigma = \frac{My_m}{I} = \frac{M}{Z}$$

where y_m is the perpendicular distance from the neutral axis to the outer surface of the beam. If this stress reaches the yield strength σ_Y of the material of the beam, small zones of plasticity appear at the surface (top diagram, facing page). The beam is no longer elastic, and, in this sense, has failed. If, instead, the maximum fibre stress reaches the brittle fracture strength, σ_f (the 'modulus of rupture', often shortened to MOR) of the material of the beam, a crack nucleates at the surface and propagates inwards (second diagram); in this case, the beam has certainly failed. A third criterion for failure is often important: that the plastic zones penetrate through the section of the beam, linking to form a plastic hinge (third diagram).

The failure moments and failure loads, for each of these three types of failure, and for each of several geometries of loading, are given on the diagram. The formulae labelled 'onset' refer to the first two failure modes; those labelled 'full plasticity' refer to the third. Two new functions of section shape are involved. Onset of failure involves the quantity Z; full plasticity involves the quantity H. Both are listed in the table of Section B.2, and defined in the text that accompanies it.

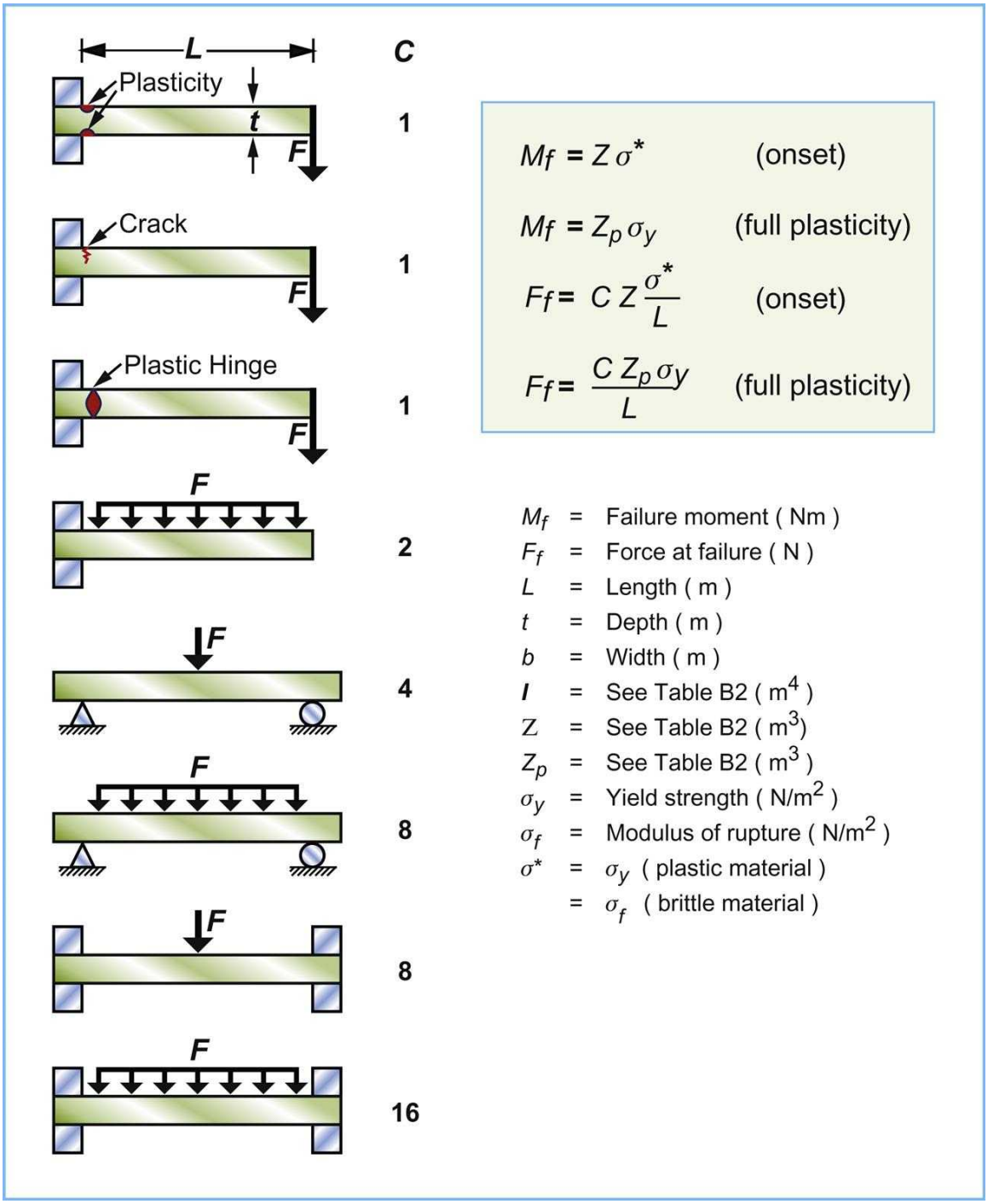


FIGURE B.4 Failure of beams.

B.5 Buckling of Columns, Plates and Shells

If sufficiently slender, an elastic column, loaded in compression, fails by elastic buckling at a critical load, F_{crit}. This load is determined by the end constraints, of which four extreme cases are illustrated on the facing page: an end may be constrained in a position and direction; it may be free to rotate but not translate (or 'sway'); it may sway without rotation; and it may both sway and rotate. Pairs of these constraints applied to the ends of column lead to the five cases shown opposite. Each is characterised by a value of the constant n that is equal to the number of half-wavelengths of the buckled shape.

The addition of the bending moment M reduces the buckling load by the amount shown in the second box. A negative value of F_{crit} means that a tensile force is necessary to prevent buckling.

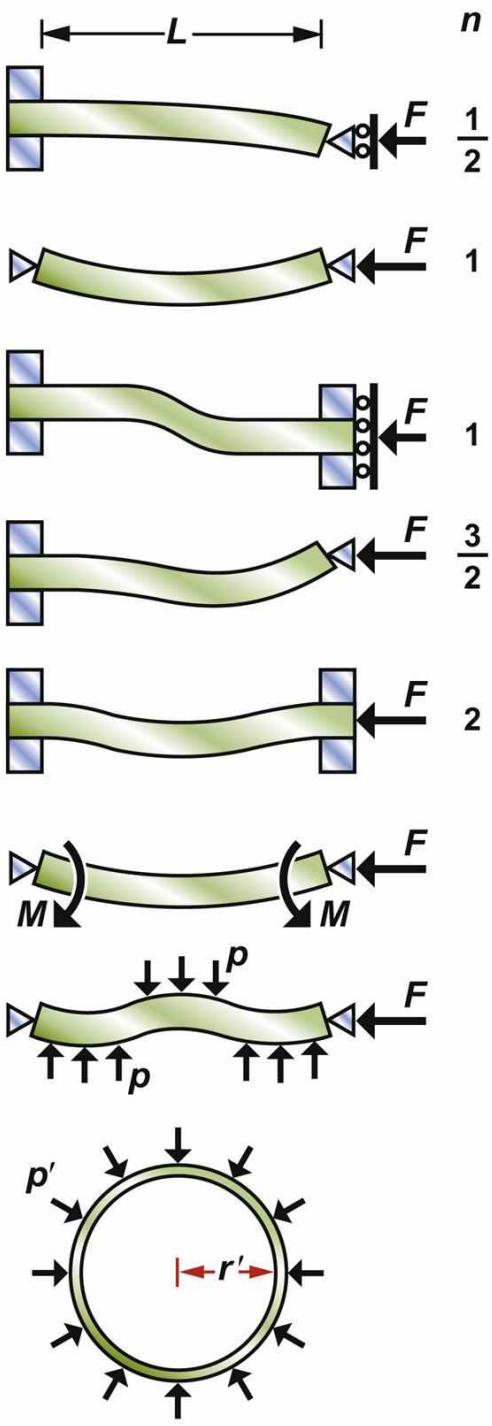
An elastic foundation is one that exerts a lateral restoring pressure, p, proportional to the deflection

$$p = ky$$

where k is the foundation stiffness per unit depth and y the local lateral deflection. Its effect is to increase F_{crit}, by the amount shown in the third box.

A thin-walled elastic tube will buckle inwards under an external pressure p', given in the last box. Here I refers to the second moment of area of a section of the tube wall cut parallel to the tube axis.

Thin or slender shapes may buckle locally before they yield or fracture. The shape consideration sets a practical limit to the thinness of tube walls and webs.



$$F_{crit} = \frac{n^2 \pi^2 E I}{L^2}$$

$$\text{or} \quad \frac{F_{crit}}{A} = \frac{n^2 \pi^2 E}{(L/r)^2}$$

F = Force (N)
M = Moment (Nm)
E = Youngs modulus (N/m^3)
L = Length (m)
A = Section area (m^2)
I = See Table B2 (m^4)
r = Gyration radius $(I/A)^{1/2}$ (m)
k = Foundation stiffness (N/m^2)
n = Half-wavelengths in buckled shape
p' = Pressure (N/m^2)

$$F_{crit} = \frac{\pi^2 E I}{L^2} - \frac{M^2}{4 E I}$$

$$F_{crit} = \frac{n^2 \pi^2 E I}{L^2} - \frac{k L^2}{n^2}$$

$$p'_{crit} = \frac{3 E I}{(r')^3}$$

FIGURE B.5 Buckling of columns, plates and shells.

B.6 Torsion of Shafts

A torque T applied to the ends of an isotropic bar of uniform section, and acting in the plane normal to the axis of the bar, produces an angle of twist θ. The twist is related to the torque by the first equation on the facing page, in which G is the shear modulus. For round bars and tubes of circular section, the factor K is equal to J, the polar moment of inertia of the section, defined in Section B.2. For any other section shape, K is less than J. Values of K are given in Section B.2.

If the bar ceases to deform elastically, it is said to have failed. This will happen if the maximum surface stress exceeds either the yield strength σ_y of the material or the stress at which it fractures, σ_{fr}. For circular sections, the shear stress at any point a distance r from the axis of rotation is

$$\tau = \frac{Tr}{K} = \frac{G\theta r}{K}$$

The maximum shear stress, τ_{max}, and the maximum tensile stress, σ_{max}, are at the surface and have the values

$$\tau_{max} = \sigma_{max} = \frac{Td_o}{2K} = \frac{G\theta d_o}{2L}$$

If τ_{max} exceeds $\sigma_y/2$ (using a Tresca yield criterion), or if σ_{max} exceeds σ_{fr}, the bar fails, as shown on the figure. The maximum surface stress for the solid ellipsoidal, square, rectangular and triangular sections is at the points on the surface closest to the centroid of the section (the midpoints of the longer sides). It can be estimated approximately by inscribing the largest circle that can be contained within the section and calculating the surface stress for a circular bar of that diameter. More complex section shapes require special consideration, and, if thin, may additionally fail by buckling.

Helical springs are a special case of torsional deformation. The extension of a helical spring of n turns of radius R, under a force F, and the failure force F_{crit}, are given on the facing page.

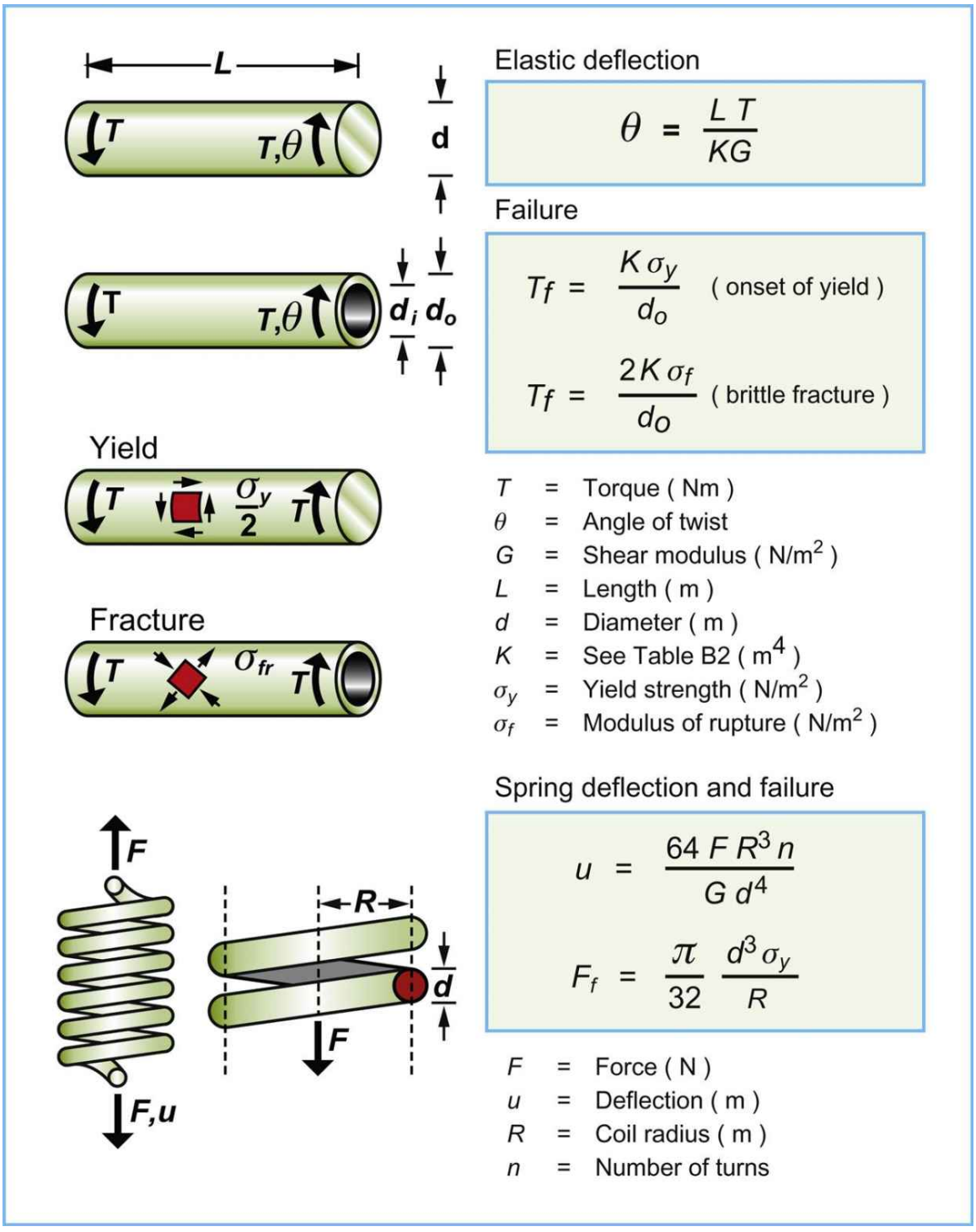
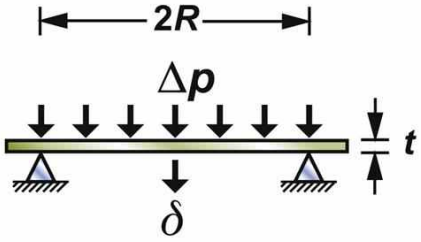


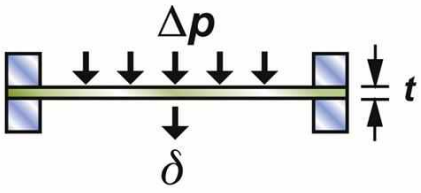
FIGURE B.6 Torsion of shafts.

B.7 Static and Spinning Disks

A thin disk deflects when a pressure difference Δp is applied across its two surfaces. The deflection causes stresses to appear in the disk. The first box on the facing page gives deflection and maximum stress (important in predicting failure) when the edges of the disk are simply supported. The second gives the same quantities when the edges are clamped. The results for a thin horizontal disk deflecting under its own weight are found by replacing Δp by the mass-per-unit-area, $\rho g t$, of the disk (here ρ is the density of the material of the disk and g is the acceleration due to gravity). Thick disks are more complicated; for those, see Further Reading.

Spinning disks, rings and cylinders store kinetic energy. Centrifugal forces generate stresses in the disk. The two boxes list the kinetic energy and the maximum stress σ_{max} in disks and rings rotating at an angular velocity ω (radians/sec). The maximum rotation rate and energy are limited by the burst-strength of the disk. They are found by equating the maximum stress in the disk to the strength of the material.





Simply supported

$$\delta = \frac{3}{4}(1-\nu^2)\frac{\Delta p\, R^4}{E\, t^3}$$

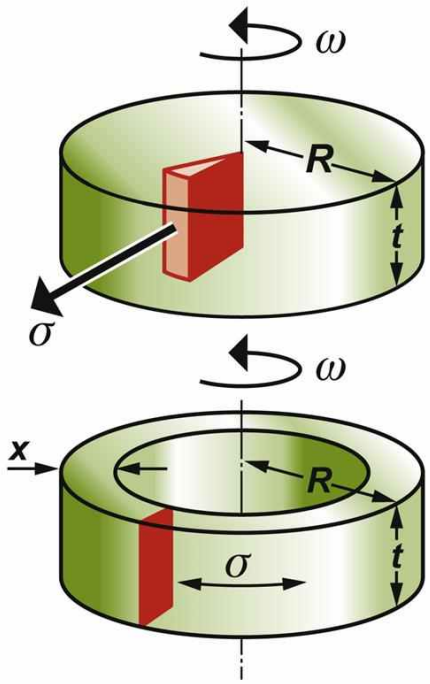
$$\sigma_{max} = \frac{3}{8}(3+\nu)\frac{\Delta p\, R^2}{t^2}$$

Clamped

$$\delta = \frac{3}{16}(1-\nu^2)\frac{\Delta p\, R^4}{E\, t^3}$$

$$\sigma_{max} = \frac{3}{8}(1+\nu)\frac{\Delta p\, R^2}{t^2}$$

δ = Deflection (m)
E = Young's modulus (N/m^2)
Δp = Pressure difference (N/m^2)
ν = Poisson's ratio



Disk

$$U = \frac{\pi}{4}\rho\, t\, \omega^2 R^4$$

$$\sigma_{max} = \frac{1}{8}(3+\nu)\rho\, \omega^2 R^2$$

Ring

$$U = \pi \rho\, t\, \omega^2 R^3 x$$

$$\sigma_{max} = \rho\, \omega^2 R^2$$

U = Energy (J)
ω = Angular velocity (rad/s)
ρ = Density (kg/m^3)

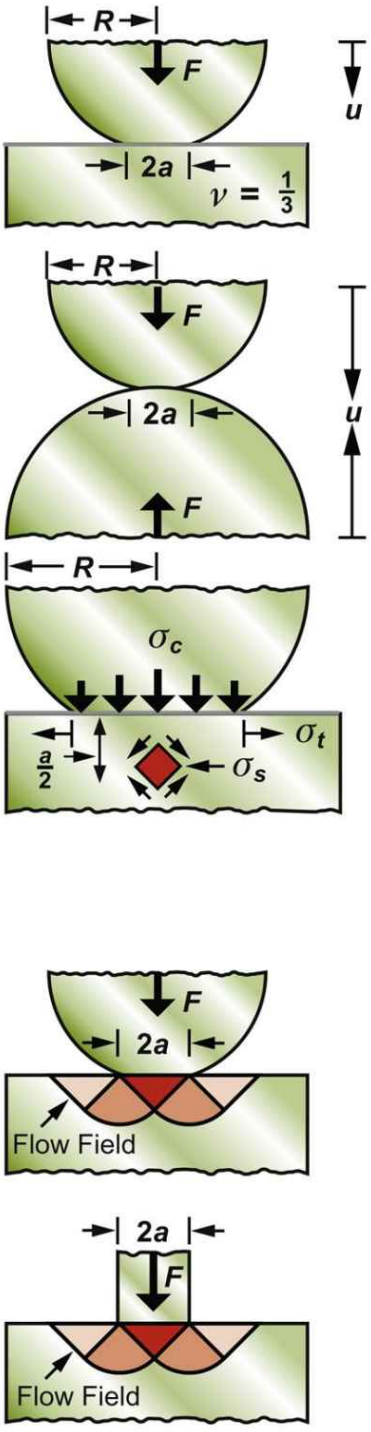
FIGURE B.7 Static and spinning disks.

B.8 Contact Stresses

When surfaces are placed in contact they touch at one or a few discrete points. If the surfaces are loaded, the contacts flatten elastically and the contact areas grow until failure of some sort occurs: failure by crushing (caused by the compressive stress, σ_c), tensile fracture (caused by the tensile stress, σ_t) or yielding (caused by the shear stress σ_s). The boxes on the facing page summarise the important results for the radius a of the contact zone, the centre-to-centre displacement u and the peak values of σ_c, σ_t and σ_s.

The first box shows results for a sphere on a flat, when both have the same moduli and Poisson's ratio has the value 1/3. Results for the more general problem (the 'Hertzian indentation' problem) are shown in the second box: two elastic spheres (radii R_1 and R_2, moduli and Poisson's ratios E_1, v_1 and E_2, v_2) are pressed together by a force F.

If the shear stress σ_s exceeds the shear yield strength $\sigma_Y/2$, a plastic zone appears beneath the centre of the contact at a depth of about $a/2$ and spreads to form the fully-plastic field shown in the two lower figures. When this state is reached, the contact pressure is approximately three times the yield stress, as shown in the bottom box.



$$a = 0.7 \left(\frac{F R}{E}\right)^{\frac{1}{3}}$$

$$u = 1.0 \left(\frac{F^2}{E^2 R}\right)^{\frac{1}{3}}$$

$\nu = 0.33$

$$a = \left(\frac{3}{4} \frac{F}{E^*} \frac{R_1 R_2}{(R_1 + R_2)}\right)^{\frac{1}{3}}$$

$$u = \left(\frac{9}{16} \frac{F^2}{(E^*)^2} \frac{(R_1 + R_2)}{R_1 R_2}\right)^{\frac{1}{3}}$$

$$(\sigma_c)_{max} = \frac{3F}{2\pi a^2}$$

$$(\sigma_s)_{max} = \frac{F}{2\pi a^2}$$

$$(\sigma_t)_{max} = \frac{F}{6\pi a^2}$$

R_1, R = Radii of spheres (m)
E_1, E_2 = Moduli of spheres (N/m^2)
ν_1, ν_2 = Poisson's ratio
F = Load (N)
a = Radius of contact (m)
u = Displacement (m)
σ = Stresses (N/m^2)
σ_y = Yield stress (N/m^2)

$$E^* = \left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}\right)^{-1}$$

$$\frac{F}{\pi a^2} = 3\sigma_y$$

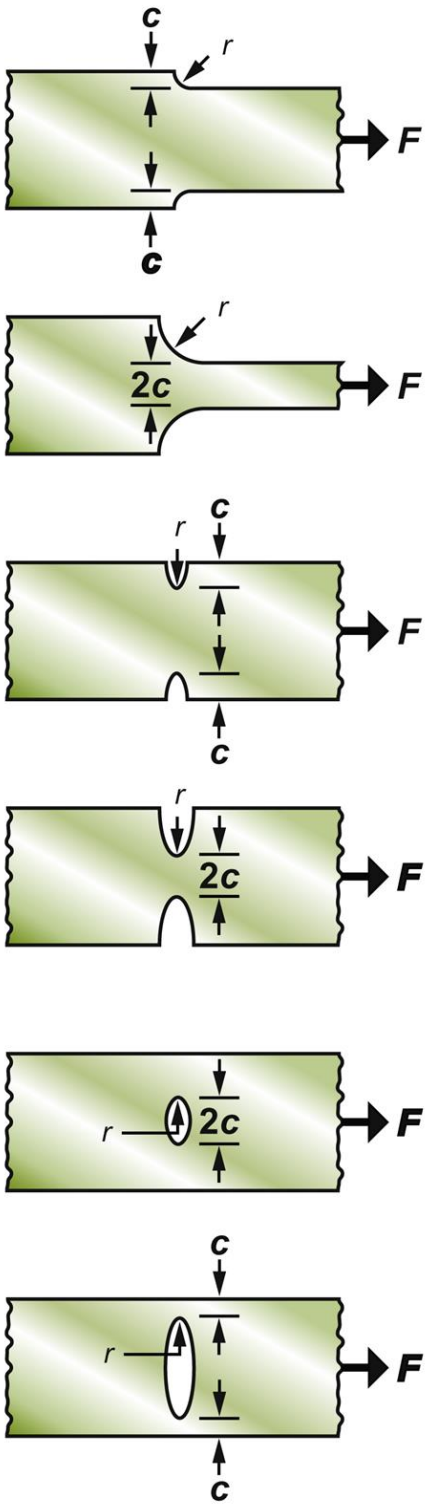
FIGURE B.8 Contact stresses.

B.9 Estimates for Stress Concentrations

Stresses and strains are concentrated at holes, slots or changes of section in elastic bodies. Plastic flow, fracture and fatigue cracking start at these places. The local stresses at the stress concentrations can be computed numerically, but this is often unnecessary. Instead, they can be estimated using the equation shown on the facing page.

The stress concentration caused by a change in section dies away at distances of the order of the characteristic dimension of the section-change (defined more fully below), an example of St Venant's principle at work. This means that the maximum local stresses in a structure can be found by determining the nominal stress distribution, neglecting local discontinuities (such as holes or grooves), and then multiplying the nominal stress by a stress concentration factor. Elastic stress concentration factors are given approximately by the equation. In it, σ_{nom} is defined as the load divided by the minimum cross-section of the part, ρ is the minimum radius of curvature of the stress-concentrating groove or hole, and c is the characteristic dimension: either the half-thickness of the remaining ligament, the half-length of a contained crack, the length of an edge-crack or the height of a shoulder, whichever is *least*. The drawings show examples of each such situation. The factor α is roughly 2 for tension, but is nearer 1/2 for torsion and bending. Though inexact, the equation is an adequate working approximation for many design problems.

The maximum stress is limited by plastic flow or fracture. When plastic flow starts, the strain concentration grows rapidly while the stress concentration remains constant. The strain concentration becomes the more important quantity, and may not die out rapidly with distance (St Venant's principle no longer applies).



$$\frac{\sigma_{max}}{\sigma_{nom}} = 1 + \alpha\left(\frac{c}{r}\right)^{\frac{1}{2}}$$

F	=	Force (N)
A_{min}	=	Minimum section (m^2)
σ_{nom}	=	F/A_{min} (N/m^2)
r	=	Radius of curvature (m)
c	=	Characteristic length (m)
α	≈	0.5 (Torsion)
α	≈	2.0 (Tension)

FIGURE B.9 Stress concentrations.

B.10 Sharp Cracks

Sharp cracks (i.e., stress concentrations with a tip radius of curvature of atomic dimensions) concentrate stress in an elastic body more acutely than rounded stress concentrations do. To a first approximation, the local stress falls off as $1/r^{1/2}$ with radial distance r from the crack tip. A tensile stress σ applied normal to the plane of a crack of length $2a$ contained in an infinite plate (as in the top figure on the facing page) gives rise to a local stress field σ_ℓ that is tensile in the plane containing the crack and given by

$$\sigma_\ell = \frac{C\sigma\sqrt{\pi a}}{\sqrt{2\pi r}}$$

where r is measured from the crack tip in the plane $\theta = 0$ and C is a constant. The mode 1 *stress intensity factor* K_I, is defined as

$$K_1 = C\sigma\sqrt{\pi a}$$

Values of the constant C for various modes of loading are given on the figure. The stress σ for point loads and moments is given by the equations at the bottom. The crack propagates when $K_1 > K_{1c}$, the *fracture toughness*.

When the crack length is very small compared with all specimen dimensions and compared with the distance over which the applied stress varies, C is equal to 1 for a contained crack and 1.1 for an edge crack. As the crack extends in a uniformly loaded component, it interacts with the free surfaces, giving the correction factors shown opposite. If, in addition, the stress field is nonuniform (as it is in an elastically bent beam), C differs from 1; two examples are given on the figure. The factors, C, given here, are approximate only, good when the crack is short but not when the crack tips are very close to the boundaries of the sample. They are adequate for most design calculations. More accurate approximations and other less common loading geometries can be found in the references listed in Further Reading.

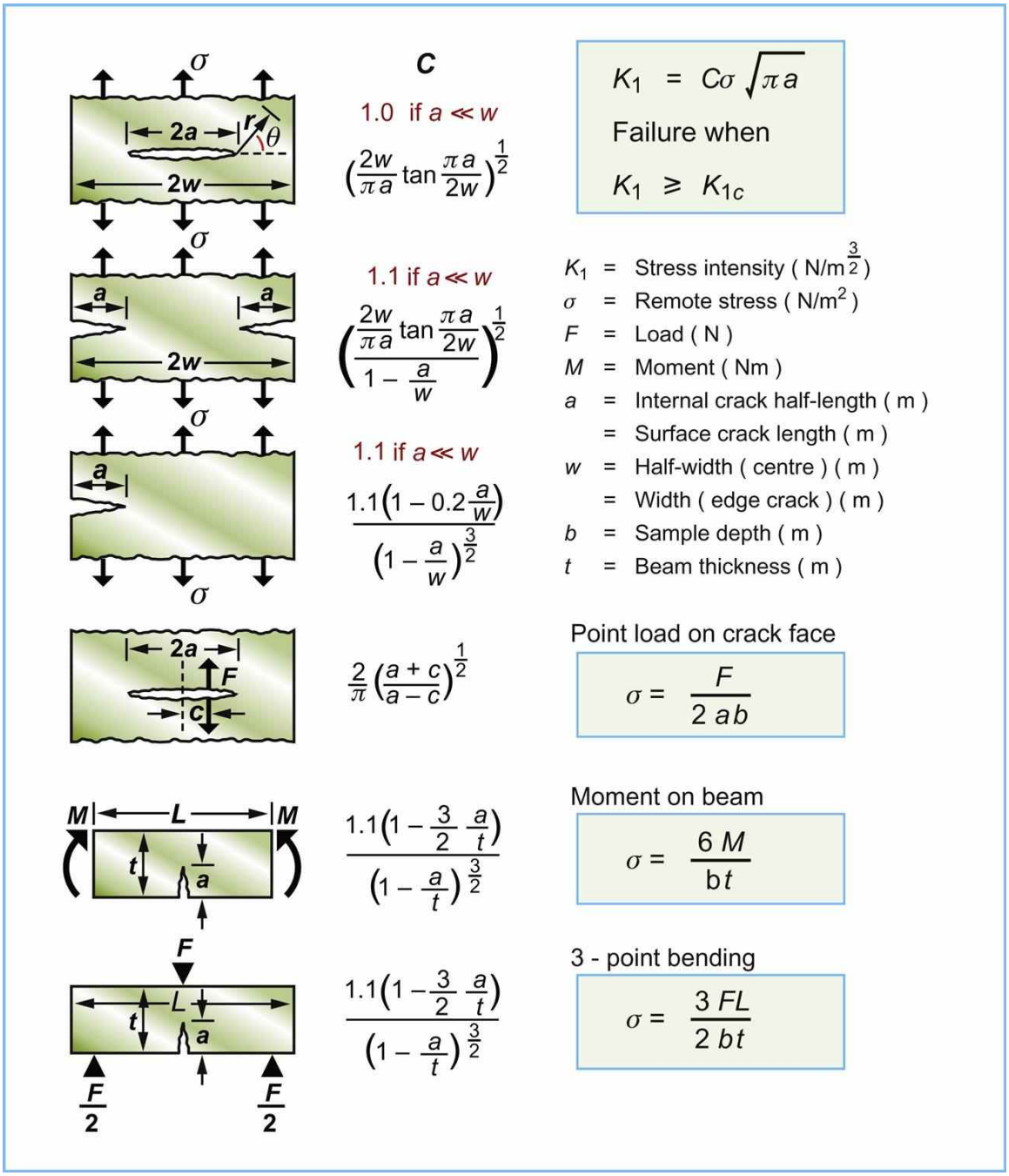


FIGURE B.10 Sharp cracks.

B.11 Pressure Vessels

Thin-walled pressure vessels are treated as membranes. The approximation is reasonable when $t<b/4$. The stresses in the wall are given on the facing page; they do not vary significantly with radial distance, r.

Those in the plane tangent to the skin, σ_θ and σ_z for the cylinder and σ_θ and σ_ϕ for the sphere, are just equal to the internal pressure amplified by the ratio b/t or $b/2t$, depending on geometry. The radial stress σ_r is equal to the mean of the internal and external stress, $p/2$ in this case. The equations describe the stresses when an external pressure p_e is superimposed if p is replaced by $(p-p_e)$.

In thick-walled vessels, the stresses vary with radial distance r from the inner to the outer surfaces, and are greatest at the inner surface. The equations can be adapted for the case of both internal and external pressures by noting that when the internal and external pressures are equal, the state of stress in the wall is

$$\sigma_\theta = \sigma_r = -p \quad \text{(cylinder)}$$

or

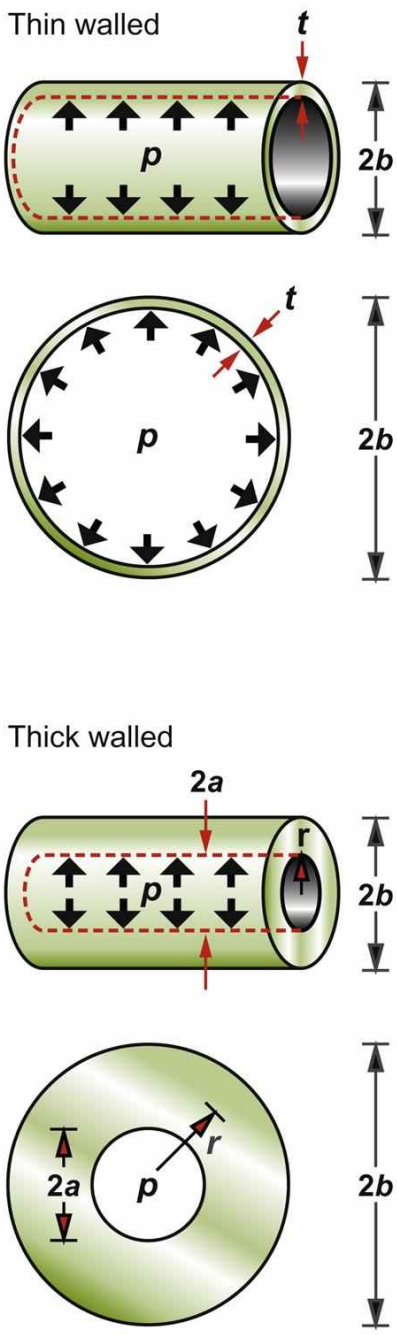
$$\sigma_\theta = \sigma_\phi = \sigma_r = -p \quad \text{(sphere)}$$

allowing the term involving the external pressure to be evaluated. It is not valid to just replace p by $(p-p_e)$.

Pressure vessels fail by yielding when the Von Mises equivalent stress first exceed the yield strength, σ_Y. They fail by fracture if the largest tensile stress exceeds the fracture stress σ_{fr}, where

$$\sigma_{fr} = \frac{CK_{1c}}{\sqrt{\pi a}}$$

and K_{1c} is the fracture toughness, a the half-crack length and C a constant given in Section B.10.



Cylinder

$$\sigma_\theta = \frac{p\,b}{t}$$

$$\sigma_r = -\frac{p}{2}$$

$$\sigma_z = \frac{p\,b}{2\,t} \quad \text{(Closed ends)}$$

Sphere

$$\sigma_\theta = \sigma_\phi = \frac{p\,b}{2\,t}$$

$$\sigma_r = -\frac{p}{2}$$

p = Pressure (N/m^2)
t = Wall thickness (m)
a = Inner radius (m)
b = Outer radius (m)
r = Radial coordinate (m)

Cylinder

$$\sigma_\theta = \frac{p\,a^2}{r^2}\left(\frac{b^2 - r^2}{b^2 - a^2}\right)$$

$$\sigma_r = -\frac{p\,a^2}{r^2}\left(\frac{b^2 + r^2}{b^2 - a^2}\right)$$

Sphere

$$\sigma_\theta = \sigma_\phi = \frac{p\,a^3}{2\,r^3}\left(\frac{b^3 + 2\,r^3}{b^3 - a^3}\right)$$

$$\sigma_r = -\frac{p\,a^3}{r^3}\left(\frac{b^3 - r^3}{b^3 - a^3}\right)$$

FIGURE B.11 Pressure vessels.

B.12 Vibrating Beams, Tubes and Disks

Any undamped system vibrating at one of its natural frequencies can be reduced to the simple problem of a mass m attached to a spring of stiffness K. The lowest natural frequency of such a system is

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$$

Specific cases require specific values for m and K. They can often be estimated with sufficient accuracy to be useful in approximate modelling. Higher natural frequencies are simple multiples of the lowest.

The first box on the facing page gives the lowest natural frequencies of the flexural modes of uniform beams with various end-constraints. As an example, the first can be estimated by assuming that the effective mass of the beam is one quarter of its real mass, so that

$$m = \frac{m_o L}{4}$$

where m_o is the mass per unit length of the beam and that K is the bending stiffness (given by F/δ from Section B.3); the estimate differs from the exact value by 2%. Vibrations of a tube have a similar form, using I and m_o for the tube. Circumferential vibrations can be found approximately by 'unwrapping' the tube and treating it as a vibrating plate, simply supported at two of its four edges.

The second box gives the lowest natural frequencies for flat circular disks with simply supported and clamped edges. Disks with doubly curved faces are stiffer and have higher natural frequencies.

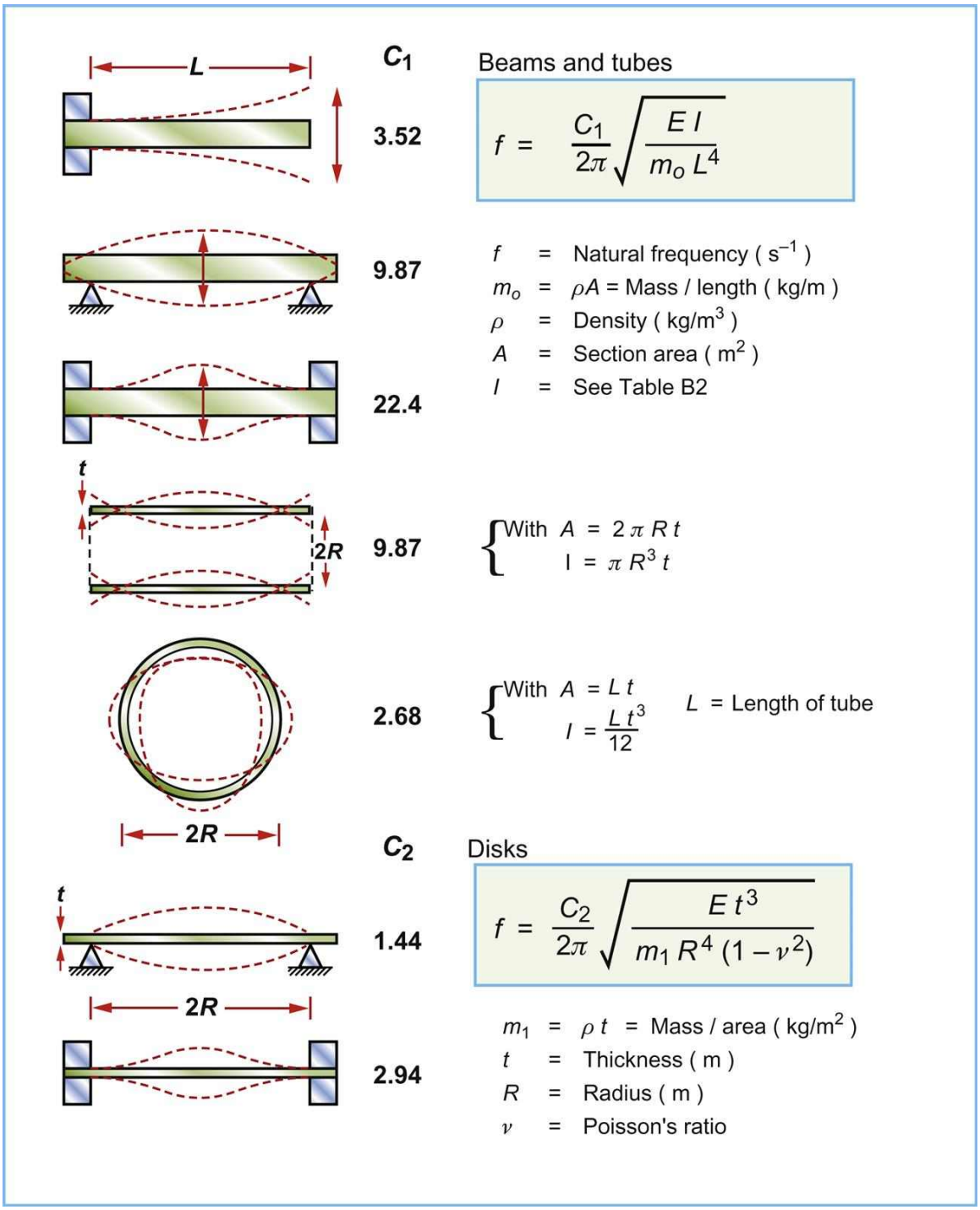


FIGURE B.12 Vibrating beams, tubes and disks.

B.13 Creep and Creep Fracture

At temperatures above 1/3 T_m (where T_m is the absolute melting point), materials creep when loaded. It is convenient to characterise the creep of a material by its behaviour under a tensile stress σ at a temperature T_m. Under these conditions the steady-state tensile strain rate $\dot{\varepsilon}_{ss}$ is often found to vary as a power of the stress and exponentially with temperature:

$$\dot{\varepsilon}_{ss} = A\left(\frac{\sigma}{\sigma_o}\right)^n \exp -\frac{Q}{RT}$$

where Q is an activation energy, A is a kinetic constant and R is the gas constant. At constant temperature this becomes

$$\dot{\varepsilon}_{ss} = \dot{\varepsilon}_o\left(\frac{\sigma}{\sigma_o}\right)^n$$

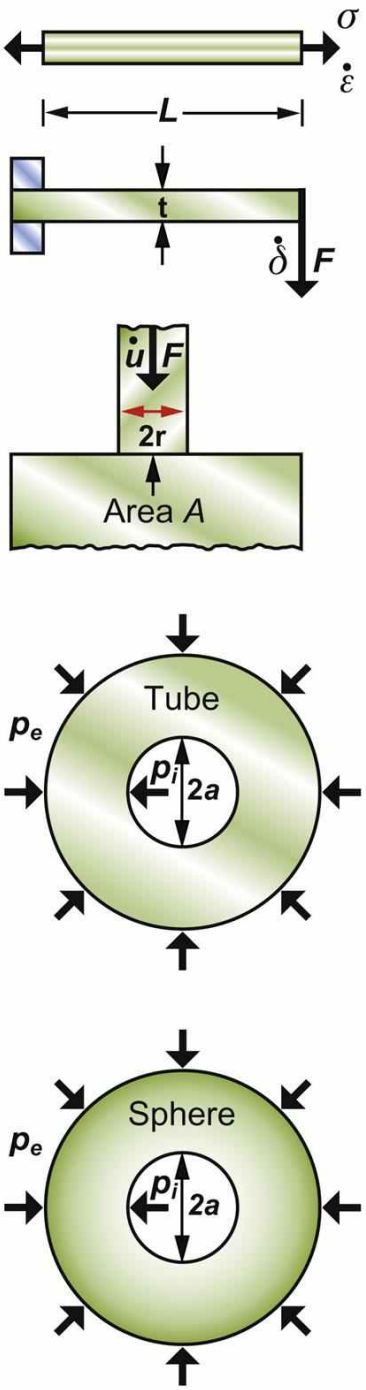
where $\dot{\varepsilon}_o(\mathrm{s}^{-1})$, $\sigma_o(\mathrm{N/m}^2)$ and n are creep constants.

The behaviour of creeping components is summarised on the facing page that gives the deflection rate of a beam, the displacement rate of an indenter and the change in relative density of cylindrical and spherical pressure vessels in terms of the tensile creep constants.

Prolonged creep causes the accumulation of creep damage that ultimately leads, after a time t_f, to fracture. To a useful approximation

$$t_f\dot{\varepsilon}_{ss} = C$$

where C is a constant characteristic of the material. Creep-ductile materials have values of C between 0.1 and 0.5; creep-brittle materials have values of C as low as 0.01.



$$\dot{\varepsilon} = \dot{\varepsilon}_o \left(\frac{\sigma}{\sigma_o}\right)^n$$

$$\dot{\delta} = \frac{2\dot{\varepsilon}_o}{n+2}\left(\frac{(2n+1)}{n}\frac{F}{s_o}\frac{}{b\,t}\right)^n \left(\frac{L}{t}\right)^{n+1} L$$

$$\dot{u} = C_1 \dot{\varepsilon}_o \sqrt{A}\left(\frac{C_2 F}{\sigma_o A}\right)^n$$

σ	=	Stress (N/m^2)
F	=	Force (N)
$\dot{\delta}, \dot{u}$	=	Displacement rates (m/s)
$n, \dot{\varepsilon}_o, \sigma_o$	=	Creep constants
L, b, t	=	Beam dimensions (m)
a, b	=	Radii of pressure vessels (m)
ρ	=	Relative density, $\frac{b^3 - a^3}{b^3}$
$\dot{\rho}$	=	Rate of change of ρ
C_1, C_2	=	Constants

$$\dot{\rho} = 2\dot{\varepsilon}_o \frac{\rho(1-\rho)}{(1-(1-\rho)^{1/n})^n}\left(\frac{2}{n}\frac{(p_e - p_i)}{\sigma_o}\right)^n$$

$$\dot{\rho} = \frac{3}{2}\dot{\varepsilon}_o \frac{\rho(1-\rho)}{(1-(1-\rho)^{1/n})^n}\left(\frac{3}{2n}\frac{(p_e - p_i)}{\sigma_o}\right)^n$$

FIGURE B.13 Creep.

B.14 Flow of Heat and Matter

Heat flow can be limited by conduction, by convection or by radiation. The constitutive equations for each are listed on the facing page. The first equation is Fourier's first law, describing steady-state heat flow; it contains the thermal conductivity, λ. The second is Fourier's second law, which treats transient heat-flow problems; it contains the thermal diffusivity, a, defined by

$$a = \frac{\lambda}{\rho C_p}$$

where ρ is the density and C_p the specific heat at constant pressure. Solutions to these two differential equations are given in Section B.15.

The third equation describes convective heat transfer. It, rather than conduction, limits heat flow when the Biot number

$$B_i = \frac{hs}{\lambda} < 1$$

where h is the heat-transfer coefficient and s is a characteristic dimension of the sample. When, instead, B_i>l, heat flow is limited by conduction. The final equation is the Stefan-Boltzmann law for radiative heat transfer. The emissivity, ε, is unity for blackbodies, less for all other surfaces.

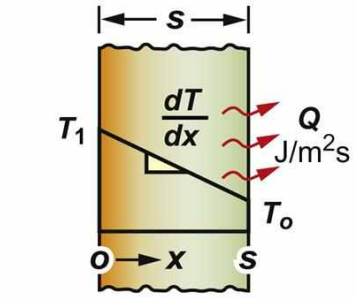
Diffusion of matter follows a pair of differential equations with the same form as Fourier's two laws, and with similar solutions. They are commonly written

$$J = -D\nabla C = -D\frac{dC}{dx} \quad \text{(steady state)}$$

and

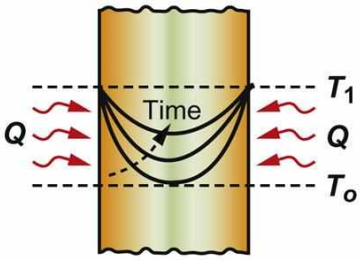
$$\frac{\partial C}{\partial t} = D\nabla^2 C = D\frac{\partial^2 C}{\partial x^2} \quad \text{(time-dependent flow)}$$

where J is the flux, C is the concentration, x is the distance and t is time. Solutions are given in Section B.15.



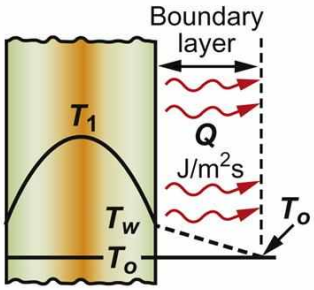
$$Q = -\lambda \nabla . T = -\lambda \frac{dT}{dx}$$

Q = Heat flux (J/m^2s)
T = Temperature (K)
x = Distance (m)
λ = Thermal conductivity (W/mK)



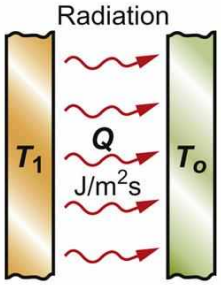
$$\frac{\partial T}{\partial t} = a \nabla^2 T = a \frac{\partial^2 T}{\partial x^2}$$

t = Time (s)
ρ = Density (kg/m^3)
C_p = Specific heat (J/kg.K)
a = Thermal diffusivity, $\frac{\lambda}{\rho C_p}$ (m^2/s)



$$Q = h (T_w - T_o)$$

T_w = Surface temperature (K)
T = Fluid temperature (K)
h = Heat transfer coefficient (W/m^2K)
= 5–50 $W/m^2.K$ in air
= 1000–5000 $W/m^2.K$ in water



$$Q = \varepsilon \sigma (T_1^4 - T_o^4)$$

ε = Emissivity (1 for black body)
σ = Stefan's constant
= 5.67×10^{-8} $W/m^2.K^4$

FIGURE B.14 Constitutive equations for heat flow.

B.15 Solutions for Diffusion Equations

Solutions exist for the diffusion equations for a number of standard geometries. They are worth knowing because many real problems can be approximated by one of these.

At steady state the temperature or concentration profile does not change with time. This is expressed by the boxed equations within the first box at the top of the facing page. Solutions for these are given below for uniaxial flow, radial flow in a cylinder and radial flow in a sphere. The solutions are fitted to individual cases by matching the constants A and B to the boundary conditions. Solutions for matter flow are found by replacing temperature, T, by concentration, C, and conductivity, λ, by diffusion coefficient, D.

The box within the second large box summarises the governing equations for time-dependent flow, assuming that the diffusivity (a or D) is not a function of position. Solutions for the temperature or concentration profiles, $T(x,t)$ or $C(x,t)$, are given below. The first equation gives the 'thin-film' solution: a thin slab at temperature T_1, or concentration C_1 is sandwiched between two semi-infinite blocks at T_o or C_o, at t=0, and flow allowed. The second result is for two semi-infinite blocks, initially at T_1 and T_o, (or C_1 or C_o) brought together at t=0. The last is for a T or C profile that is sinusoidal, of wavelength $\lambda/2\pi$ and amplitude A at t=0.

Note that all transient problems end up with a characteristic time constant t^* with

$$t^* = \frac{x^2}{\beta a} \quad \text{or} \quad \frac{x^2}{\beta D}$$

where x is a dimension of the specimen; or a characteristic length x^* with

$$x^* = \sqrt{\beta a t} \quad \text{or} \quad \sqrt{\beta D t}$$

where t is the time scale of observation, with $1 < \beta < 4$, depending on geometry.

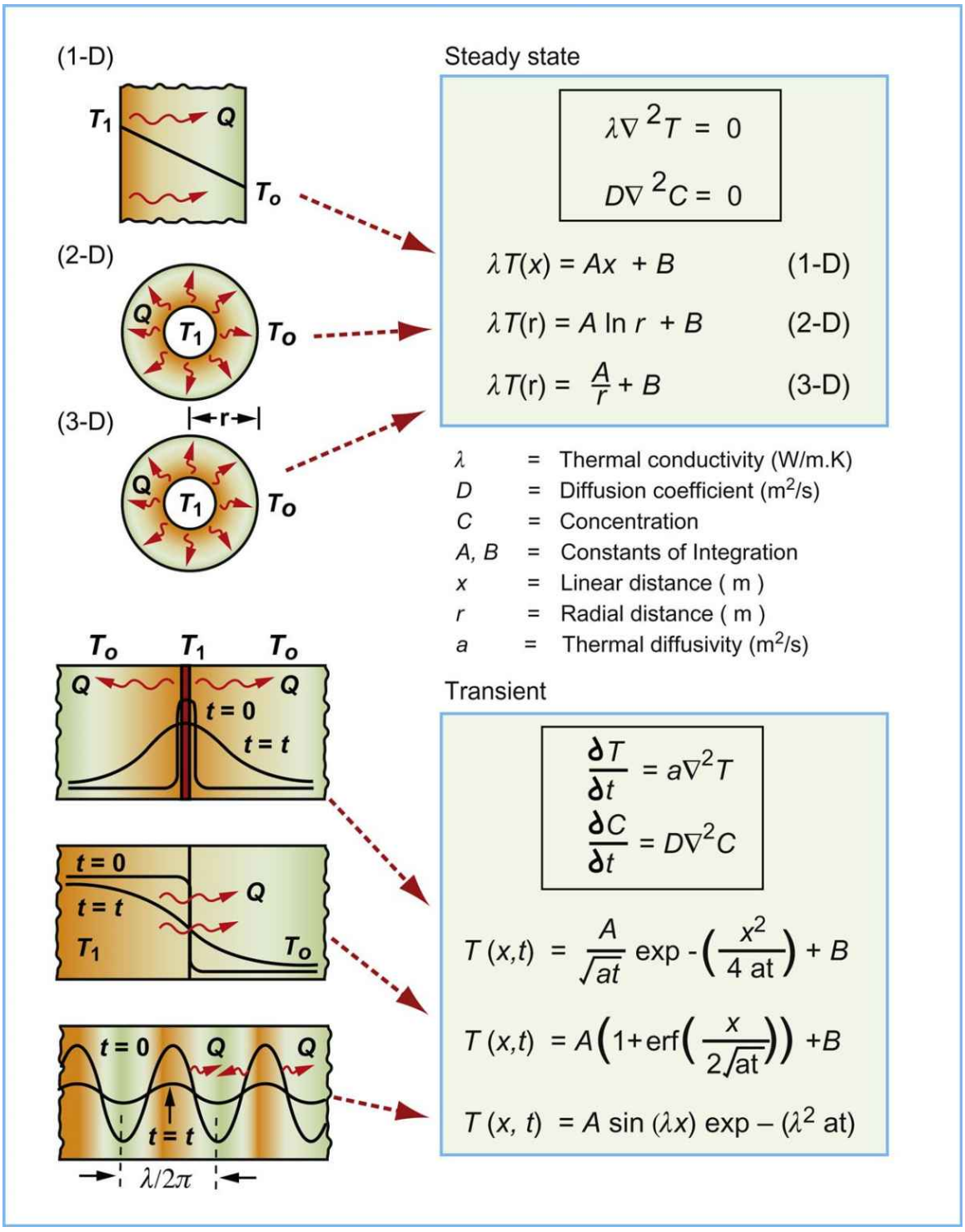


FIGURE B.15 Solutions for diffusion equations.

B.16 Thermal Field for Moving Heat Source

When a plate is exposed to a heat source q moving at velocity, v, a thermal field is created. The field tracks along with the moving heat source such the temperature T at a distance r from the path of the source first rises, then decays back to ambient temperature T_o. In welding, the peak temperature T_p reached in the cycle exceeds the melting point; in brazing and soldering it only reaches the melting point of the braze or solder.

Three characteristics of the thermal field are useful: the temperature T as a function of position r and time t; the peak temperature T_p reached at the point r during the thermal cycle; and (for steels) the time interval Δt to cool from 800 to 500°C because this determines the structure of the weld at that point in carbon steels. For a thin plate (one with a thickness d that is small compared to the width of the thermal field) these quantities are given by

$$T = T_o + \frac{q/v}{d(4\pi\lambda\rho C_p t)^{1/2}} \exp - \left(\frac{r^2}{4at}\right); \quad T_p = T_o + \left(\frac{2}{\pi e}\right)^{1/2} \frac{q/v}{2d\rho C_p r}$$

and

$$\Delta t = \frac{(q/vd)^2}{4.6 \times 10^6\, \lambda\rho C_p}$$

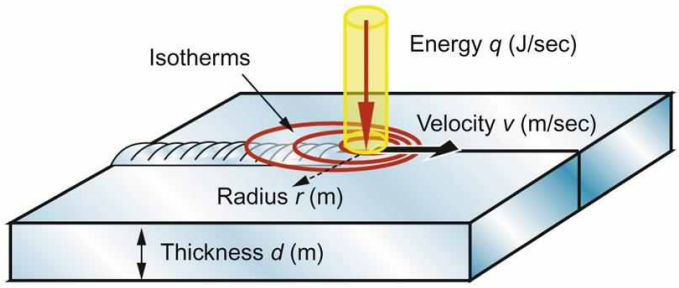
For a thick plate (one with a thickness that is large compared to the width of the thermal field) the corresponding equations are

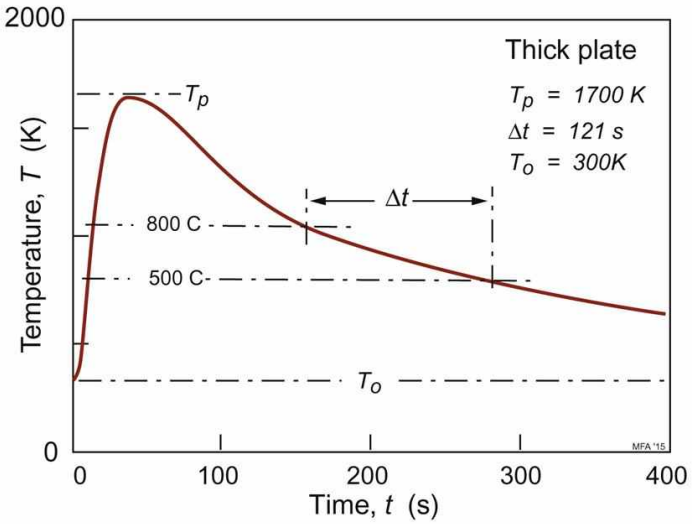
$$T = T_o + \frac{q/v}{2\pi\lambda t} \exp - \left(\frac{r^2}{4at}\right); \quad T_p = T_o + \left(\frac{2}{\pi e}\right) \frac{q/v}{\rho C_p r^2}$$

and

$$\Delta t = \frac{(q/v)}{1.2 \times 10^3\, \lambda}$$

Symbols are defined on the facing page.





q	=	Energy input from beam (J/sec)
v	=	Beam tracking velocity (m/s)
r	=	Lateral distance from beam center (m)
d	=	Plate thickness (m)
T	=	Temperature (K)
T_p	=	Peak temperature (K)
T_o	=	Temperature remote from source (K)
t	=	Time (s)
Δt	=	Time for temperature to fall from 800 to 600 C (s)
λ	=	Thermal conductivity (W/m.K)
ρ	=	Density (kg/m^3)
C_p	=	Specific heat (J/kg.K)
a	=	$\lambda/\rho C_p$ = Thermal diffusivity (m^2/s)
e	=	Base of natural logarithms (2.718)

FIGURE B.16 Temperature field of moving heat source.

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APPENDIX C

Material Indices

C.1 Introduction and Synopsis

The performance, *P*, of a component is characterized by a performance equation. The performance equation contains groups of material properties. These groups are the material indices. Sometimes the 'group' is a single property; thus, if the performance of a beam is measured by its stiffness, the performance equation contains only one property, the elastic modulus *E*. It is the material index for this problem. More commonly, the performance equation contains a group of two or more properties. Familiar examples are the specific stiffness, E/ρ, and the specific strength, σ_y/ρ, (where σ_y is the yield strength or elastic limit, and ρ is the density), but there are many others. They are a key to the optimal selection of materials. Details of the method, with numerous examples are given in Chapter 4, Materials Selection – The Basics, Chapter 5, Materials Selection – Case Studies, Chapter 8, Multiple Constraints and Conflicting Objectives and Chapater 9, Multiple Constraints and Conflicting Objectives – Case Studies. This appendix compiles indices for a range of common applications.

C.2 Uses of Material Indices

Material selection. Components have functions: to carry loads safely, to transmit heat, to store energy, to insulate, and so forth. Each function has an associated material index. Materials with high values of the

appropriate index maximize that aspect of the performance of the component. For reasons given in Chapter 5, Materials Selection – Case Studies, the material index is generally independent of the details of the design. Thus the indices for beams in the tables that follow are independent of the detailed shape of the beam; that for minimizing thermal distortion of precision instruments is independent of the configuration of the instrument, and so forth. This gives them great generality.

Material deployment or substitution. A new material may have potential application in functions for which its indices have unusually high or low values. Fruitful applications for a new material can be identified by evaluating its indices and comparing them with those of existing, established materials. Similar reasoning points the way to identifying viable substitutes for an incumbent material in an established application.

How to read the tables. The indices listed in the Tables C.1–C.7 are, for the most part, based on the objective of minimizing mass. To minimize material cost, use the index for minimum mass, but replace the density ρ by the cost per unit volume, $C_m\rho$, where C_m is the cost per kg. To minimize material embodied energy or CO_2 burden, replace ρ by $H_p \cdot \rho$ or by CO_2 ρ where H_p is the production energy per kg and CO_2 is the CO_2 footprint per kg (Appendix A, Table A.10).

Table C.1

Stiffness-Limited Design at Minimum Mass

Function and Constraints	Maximize
TIE (tensile strut)	
Stiffness, length specified; section area free	E/ρ
SHAFT (loaded in torsion)	
Stiffness, length, shape specified, section area free	$G^{1/2}/\rho$
Stiffness, length, outer radius specified; wall thickness free	G/ρ
Stiffness, length, wall-thickness specified; outer radius free	$G^{1/3}/\rho$
BEAM (loaded in bending)	
Stiffness, length, shape specified; section area free	$E^{1/2}/\rho$
Stiffness, length, height specified; width free	E/ρ
Stiffness, length, width specified; height free	$E^{1/3}/\rho$
COLUMN (compression strut, failure by elastic buckling)	
Buckling load, length, shape specified; section area free	$E^{1/2}/\rho$
PANEL (flat plate, loaded in bending)	
Stiffness, length, width specified, thickness free	$E^{1/3}/\rho$
PLATE (flat plate, compressed in-plane, buckling failure)	
Collapse load, length and width specified, thickness free	$E^{1/3}/\rho$
CYLINDER WITH INTERNAL PRESSURE	
Elastic distortion, pressure and radius specified; wall thickness free	E/ρ
SPHERICAL SHELL WITH INTERNAL PRESSURE	
Elastic distortion, pressure and radius specified, wall thickness free	$E/(1-\nu)\rho$

Table C.2

Strength-Limited Design at Minimum Mass

Function and Constraints	Maximize
TIE (tensile strut)	
Stiffness, length specified; section area free	σ_f/ρ
SHAFT (loaded in torsion)	
Load, length, shape specified, section area free	$\sigma_f^{2/3}/\rho$
Load, length, outer radius specified; wall thickness free	σ_f/ρ
Load, length, wall-thickness specified; outer radius free	$\sigma_f^{1/2}/\rho$
BEAM (loaded in bending)	
Load, length, shape specified; section area free	$\sigma_f^{2/3}/\rho$
Load length, height specified; width free	σ_f/ρ
Load, length, width specified; height free	$\sigma_f^{1/2}/\rho$
COLUMN (compression strut)	
Load, length, shape specified; section area free	σ_f/ρ
PANEL (flat plate, loaded in bending)	
Stiffness, length, width specified, thickness free	$\sigma_f^{1/2}/\rho$
PLATE (flat plate, compressed in-plane, buckling failure)	
Collapse load, length and width specified, thickness free	$\sigma_f^{1/2}/\rho$
CYLINDER WITH INTERNAL PRESSURE	
Elastic distortion, pressure and radius specified; wall thickness free	σ_f/ρ
SPHERICAL SHELL WITH INTERNAL PRESSURE	
Elastic distortion, pressure and radius specified, wall thickness free	σ_f/ρ
FLYWHEELS, ROTATING DISKS	
Maximum energy storage per unit volume; given velocity	ρ
Maximum energy storage per unit mass; no failure	σ_f/ρ

Table C.3

Strength-Limited Design: Springs, Hinges

Function and Constraints	Maximize
SPRINGS	
Maximum stored elastic energy per unit volume; no failure	σ_f^2/E
Maximum stored elastic energy per unit mass; no failure	$\sigma_f^2/E\rho$
ELASTIC HINGES	
Radius of bend to be minimized (max flexibility without failure)	σ_f/E
KNIFE EDGES, PIVOTS	
Minimum contact area, maximum bearing load	σ_f^3/E^2 and H
COMPRESSION SEALS AND GASKETS	
Maximum conformability; limit on contact pressure	$\sigma_f^{3/2}/E$ and $1/E$
DIAPHRAGMS	
Maximum deflection under specified pressure or force	$\sigma_f^{3/2}/E$
ROTATING DRUMS AND CENTRIFUGES	
Maximum angular velocity; radius fixed; wall thickness free	σ_f/ρ

Table C.4

Vibration-Limited Design

Function and Constraints	Maximize
TIES, COLUMNS	
Maximum longitudinal vibration frequencies	E/ρ
BEAMS, all dimensions prescribed	
Maximum flexural vibration frequencies	E/ρ
BEAMS, length and stiffness prescribed	
Maximum flexural vibration frequencies	$E^{1/2}/\rho$
PANELS, all dimensions prescribed	
Maximum flexural vibration frequencies	E/ρ
PANELS, length, width and stiffness prescribed	
Maximum flexural vibration frequencies	$E^{1/3}/\rho$
TIES, COLUMNS, BEAMS, PANELS, stiffness prescribed	
Minimum longitudinal excitation from external drivers, ties	$\eta E/\rho$
Minimum flexural excitation from external drivers, beams	$\eta E^{1/2}/\rho$
Minimum flexural excitation from external drivers, panels	$\eta E^{1/3}/\rho$

Table C.5

Damage-Tolerant Design

Function and Constraints	Maximize
TIES (tensile member)	
Maximum flaw tolerance and strength, load-controlled design	K_{1c} and σ_f
Maximum flaw tolerance and strength, displacement-control	K_{1c}/E and σ_f
Maximum flaw tolerance and strength, energy-control	K_{1c}^2/E and σ_f
SHAFTS (loaded in torsion)	
Maximum flaw tolerance and strength, load-controlled design	K_{1c} and σ_f
Maximum flaw tolerance and strength, displacement-control	K_{1c}/E and σ_f
Maximum flaw tolerance and strength, energy-control	K_{1c}^2/E and σ_f
BEAMS (loaded in bending)	
Maximum flaw tolerance and strength, load-controlled design	K_{1c} and σ_f
Maximum flaw tolerance and strength, displacement-control	K_{1c}/E and σ_f
Maximum flaw tolerance and strength, energy-control	K_{1c}^2/E and σ_f
PRESSURE VESSEL	
Yield-before-break	K_{1c}/σ_f
Leak-before-break	K_{1c}^2/σ_f

Table C.6

Thermal and Thermo-Mechanical Design

Function and Constraints	Maximize
THERMAL INSULATION MATERIALS	
Minimum heat flux at steady state; thickness specified	$1/\lambda$
Minimum temp rise in specified time; thickness specified	$1/a = \rho C_p/\lambda$
Minimize total energy consumed in thermal cycle (kilns, etc.)	$\sqrt{a}/\lambda = \sqrt{1/\lambda \rho C_p}$
THERMAL STORAGE MATERIALS	
Maximum energy stored/unit material cost (storage heaters)	C_p/C_m
Maximize energy stored for given temperature rise and time	$\lambda/\sqrt{a} = \sqrt{\lambda \rho C_p}$
PRECISION DEVICES	
Minimize thermal distortion for given heat flux	λ/α
THERMAL SHOCK RESISTANCE	
Maximum change in surface temperature; no failure	$\sigma_f/E\alpha$
HEAT SINKS	
Maximum heat flux per unit volume; expansion limited	$\lambda/\Delta\alpha$
Maximum heat flux per unit mass; expansion limited	$\lambda/\rho\,\Delta\alpha$
HEAT EXCHANGERS (pressure-limited)	
Maximum heat flux per unit area; no failure under Δp	$\lambda\,\sigma_f$
Maximum heat flux per unit mass; no failure under Δp	$\lambda\,\sigma_f/\rho$

Table C.7

Electro-Mechanical Design

Function and Constraints	Maximize
BUS BARS	
Minimum life-cost; high current conductor	$1/\rho_e\ \rho\ C_m$
ELECTRO-MAGNET WINDINGS	
Maximum short-pulse field; no mechanical failure	σ_f
Maximize field and pulse-length, limit on temperature rise	$C_p\rho/\rho_e$
WINDINGS, HIGH-SPEED ELECTRIC MOTORS	
Maximum rotational speed; no fatigue failure	σ_e/ρ_e
Minimum ohmic losses; no fatigue failure	$1/\rho_e$
RELAY ARMS	
Minimum response time; no fatigue failure	$\sigma_e/E\ \rho_e$
Minimum ohmic losses; no fatigue failure	$\sigma_e^2/E\ \rho_e$

The symbols. The symbols used in Tables C.1–C.7 are defined below

Class	Property	Symbol (Units)
General	Density	ρ (kg/m^3)
	Price	C_m (\$/kg)
Mechanical	Elastic moduli (Young's, shear, bulk)	E, G, K (GPa)
	Poisson's ratio	ν (–)
	Failure strength (yield, fracture)	σ_f (MPa)
	Fatigue strength	σ_e (MPa)
	Hardness	H (Vickers)
	Fracture toughness	K_{1c} (MPa·m$^{1/2}$)
	Loss coefficient (damping capacity)	η (–)
Thermal	Thermal conductivity	λ (W/m·K)
	Thermal diffusivity	a (m^2/s)
	Specific heat	C_p (J/kg·K)
	Thermal expansion coefficient	α (K^{-1})
	Difference in thermal conductivity	$\Delta\alpha$ (K^{-1})
Electrical	Electrical resistivity	ρ_e (μΩ·cm)
Eco-properties	Energy/kg to produce material	E_m (MJ/kg)
	CO_2/kg footprint of material production	CO_2 (kg/kg)

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