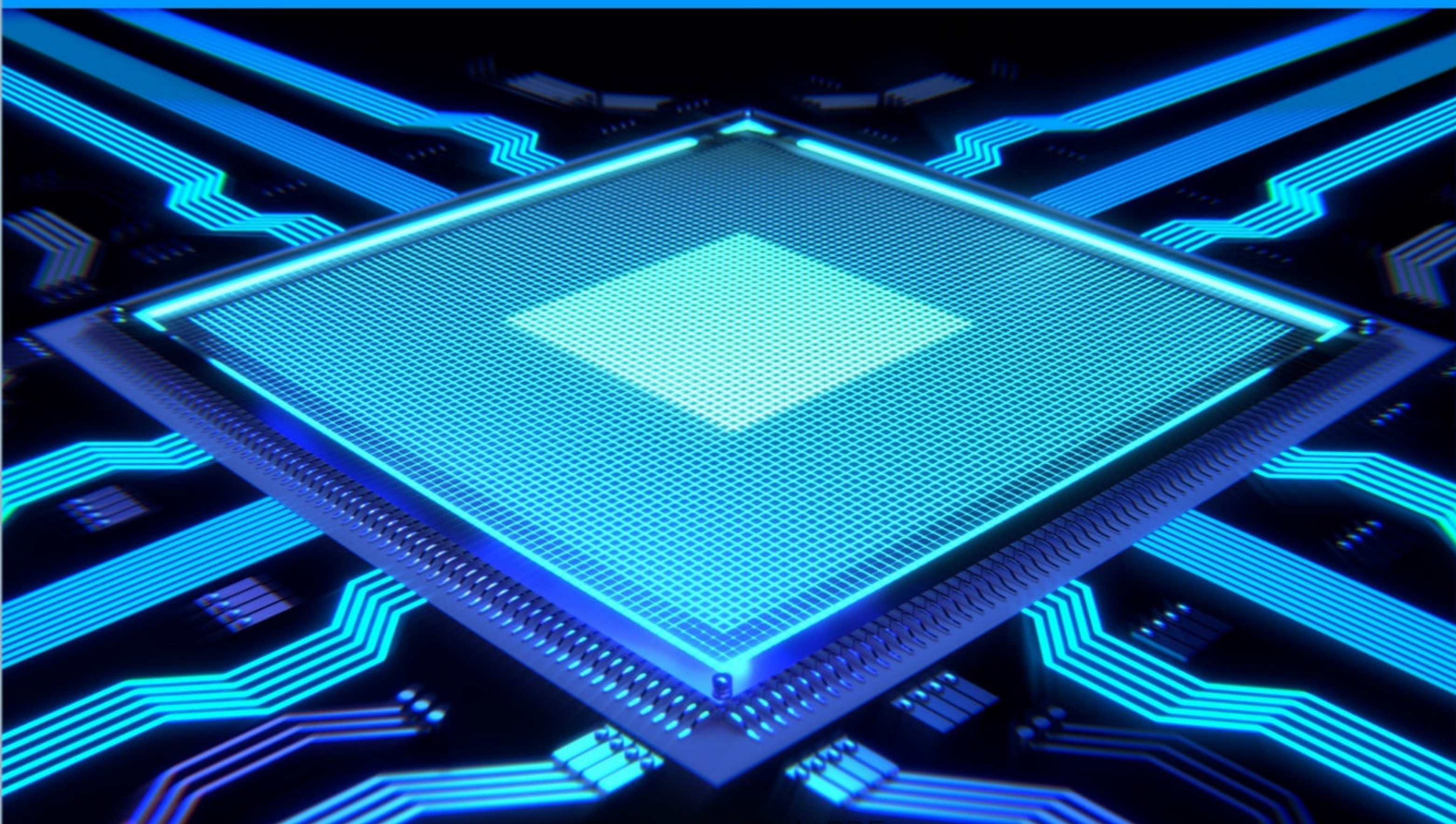
Electricity without the tears

COLM DURKAN



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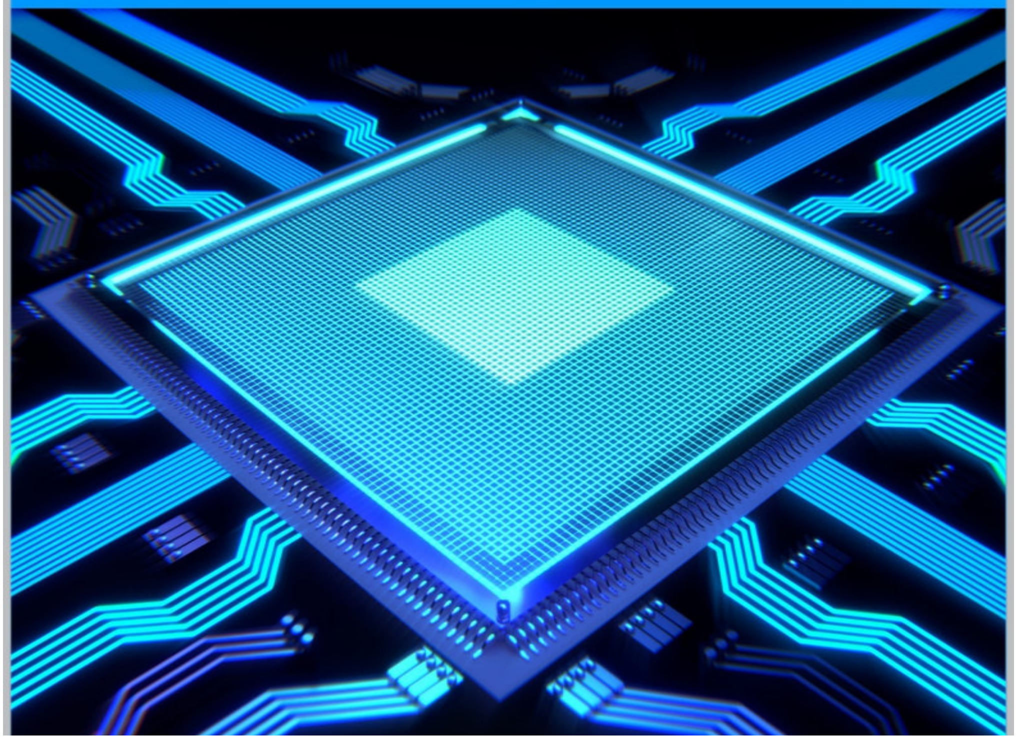

Electricity without the tears – updated version 2017

Colm Durkan

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Who is this booklet for?

This is particularly intended for those taking A-level or high-school Physics or equivalent, but in fact is useful for anybody who struggles with concepts in electricity, from UK year 9 (age 13-14) onwards.

Foreword

I am a Reader in Nanoscale Engineering at the University of Cambridge, and lecture first year engineering students on linear circuits – i.e. electrical stuff, as well as giving a course on Quantum Mechanics and Nanotechnology to Masters students. I have written this, which is the first of several intended booklets on various topics, based on the fact that I see so many very bright students struggling with what *should* be fairly straightforward topics. The reason: they never really understood it at school and didn't like it. There is a general perception that electricity is a difficult subject, as it is quite abstract – you can't see charges or fields, you can only see what they do. While this is true, it just means we have to work a little harder to explain things in terms that are familiar to us. The way this works is that you dip in and out of any given topic, and try a few of the questions.

Colm Durkan

Cambridge, UK, July 2017

Summary

The idea behind circuits is that we start with a power supply, e.g. a battery, and get the electric charge to go from one terminal to the other (by flowing through a circuit), and while it's at it, do something useful, e.g. generate heat or light, or move something, or power your smartphone or the device you are reading this on. If we want to figure out *how* this happens and be able to *predict* useful stuff, then we need to know how electricity works, which is what this booklet is all about.

Introduction, or why do we need to learn this stuff?

I should say BECAUSE IT'S INTERSTING, SO GIVE IT A CHANCE! Instead, I will be blunt and say BECAUSE IT'S ON YOUR EXAMS!!! I suppose it's a combination of both mixed together with my own personal belief that the more you understand about the world about you, the more rounded you are, and the more you have to offer. As a schoolchild, I remember whenever we did anything "electrical" in science and then in Physics, there would be a collective groan in the class. Nobody liked the subject (not even the teachers as far as I could tell), as we just didn't see why it was important, and it was difficult – you couldn't really see what was going on. So what if V = IR? Who cares? There are no cool experiments you can do, and what's worse, we really didn't understand any of it. At university, it was much the same. I did Physics, and while I loved electromagnetic theory (it is very neat and can explain a lot of everyday phenomena, and there's some pretty cool maths involved that I will make no apologies for liking), I detested circuit theory. It was just boring, and I wasn't interested in any of the circuits, so I couldn't remember them. I came to realise over time that actually this was OK, the circuits themselves aren't necessarily intrinsically interesting, but what they can be used for is! Thankfully I reached this epiphany before I started lecturing Cambridge Undergraduates on circuit theory – I was determined to show them that you don't have to be a geek to find this stuff interesting, or at the very least to have a basic understanding of how things work. At the end of the day, we wouldn't have smartphones (or any kind of phone), computers, electric heaters, lightbulbs, Xboxes, Wiis, TVs, microwaves……etc. etc. without understanding the basics of electricity. So why not give it a chance?

The circuits that we will look at are widely used

The structure of this e-book

This is an ebook about electricity, and tells you all you need to know pre-university. We start with the basics, i.e.

- what is electricity, it's place in the world,
- how electrical forces compare to other forces,
- different types of electricity,
- what we use it for,
- the rules that can be used to describe it.

That's really what you need to know – the rules, where they come from and how to use them to solve problems. You probably won't be surprised if I say that the rules that electricity obey are (i) conservation of energy and (ii) conservation of charge. That's it! Of course, to make those statements in this way isn't helpful, we need to re-state them in a way that it relevant to circuits, which we will get to later.

An analogy to electricity – water…

As this is somewhat of an abstract subject, a good analogy might help you to visualize what's going on. I find a very helpful one is to think of electric current in a wire, which is just the flow of electric charge, as being like water flowing through a pipe. A resistor is something that reduces the current that flows in a circuit, and the analogy is a kink in the water pipe. The voltage that drives the current in the first place is like the water pressure in the tap the pipe is connected to.

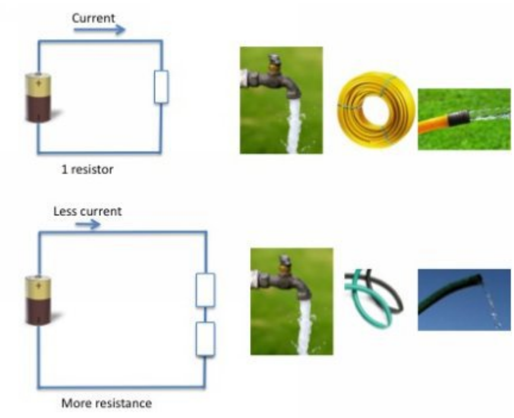


Figure 1. Analogy of water flow to current flow. Water comes out of a tap under pressure, which is analogous to the potential difference of a battery; the hose pipe is analogous to a wire in that water flows in it in a similar way to how current flows through a wire; resistance is equivalent to having any impediment to the water flow, eg friction in the hose, or a kink. The more kinks, the less water gets through, in the same way as the higher the resistance of a circuit, the less current flows.

The basics

What is electricity?

Electricity is anything to do with electric charges, how they interact with each other, what happens when they move and the fields they produce. We use the energy associated with moving charges to do work, which is a complicated way of saying that we use electricity to do useful things, such as create light and heat our homes, and power appliances and devices. So, pretty much everything then.

Fig. 2. Some examples of electricity at use. Clockwise, starting top left: a battery, a lightbulb, a power transmission cable, an electric car, an electric heater. All images courtesy of Wikipedia.

Here's a bit of science history – people have been aware of the concept of electricity and magnetism for millennia, starting with the Chinese who discovered the magnetic properties of lodestone (One of the magnetic forms of Iron Oxide, known as magnetite) and used it to make compasses back in the 7th century, B.C, and wrote about it. The Egyptians almost a thousand years before then made various objects (even some of which were discovered in Tutankhamun's tomb) out of Iron. Not only this, but to make it even more magical, the Iron they used came from meteorites – the name for it then was "metal from the sky". However, as they did not write about it having special properties other than having fallen from the sky, we can't be sure that they were aware of its magnetic properties, although it would be very surprising if they were not. The discovery of electric charge took a bit longer, and the first recorded mention of it was by the ancient Greeks in around 600 BC, when Thales noted that when amber was rubbed against fur, it could attract other light objects such as hair. For the next 2400 years, little changed until the dawn of the scientific age and the invention of the first battery by Allesandro Volta in 1792. Since then, our understanding of the world around us has deepened significantly, and we have a better handle on how things work. We do not necessarily know *why* many things are the way they are – that's for philosophers to argue about endlessly! Everything that we know about electricity and magnetism was distilled down to 4 elegant equations in the early 1860s by Maxwell. I have reproduced the equations below which are in the differential form. As you can see, these don't look particularly easy to use or understand unless you have spent a few years learning about calculus, so instead we have a lot of "rules of thumb" that we use to explain how things work.

1. $\nabla \cdot \mathbf{D} = \rho_V$
2. $\nabla \cdot \mathbf{B} = 0$
3. $\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$
4. $\nabla \times \mathbf{H} = \dfrac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$

Maxwell's equations. These describe everything you would ever need to know about electromagnetism…..

These equations relate the electric field, ***E*** to the magnetic flux density ***B*** , the magnetic field strength, ***H*** to the electric displacement ***D*** and the current density, ***J*** , and the electric displacement to the charge density, □ $_V$. Now you see why we try to simplify things a bit in this field (no pun intended. Well, maybe a little one!). Let us embark on our exploration of electricity and magnetism by thinking about the nature of electric charge.

What are electric charges?

Subatomic particles (that make up atoms, which in turn make up almost everything around us) have been found to exhibit many properties (often given unusual names by bored physicists such as flavor, strangeness, spin) one of which is charge. There are two types of charge, which we call positive (+) or negative (-). Like charges repel and opposite charges attract, hence the old cliché. Positive charges reside in protons and negative charges reside in electrons. Electrons are 1,837 times lighter than protons, so are able to move much faster, and when they move, we call this an *electric* current. If protons or charged atoms move, we call it an *ionic* current.

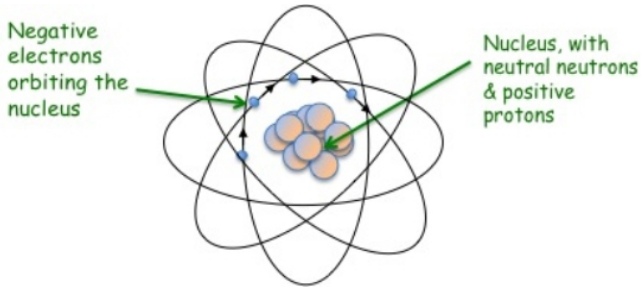


Fig. 3. The structure of an atom. The nucleus resides at the centre, and electrons orbit it. The electrons are held there by the electric force between them and the protons in the nucleus. The nucleus is held together by something we call the *Strong nuclear force.* Unfortunately, atoms don't really look like as simple as this, as quantum mechanics shows, but nonetheless, this is a useful picture to get an idea. This looks like the moon or satellites orbiting the earth, or the earth orbiting the sun.

A crucial discovery made in the 20th century is that charge is *quantized* or *discrete* , which is a fancy way of saying that charge can only take certain values, and it turns out that the minimum or *fundamental* charge is the amount found on electrons and protons, which as we will see later, is 1.6×10^{-19} Coulombs. This is an important point – I am saying that the electric charge on any object will always be a multiple of this amount, and can *never* be anything else.

Why are some things charged?

Materials become charged if for some reason the electrons and protons in the atoms within that material become separated. This can happen due to chemical reactions (e.g. in a battery) or friction – van de Graaf generator, lightning due to clouds essentially rubbing against each other, static shocks you get when wearing a fleece or if your hair is particularly dry. Some animals have cells that generate a voltage by chemically separating charges, and use this to protect themselves or stun their prey. The best example of this is the electric eel. In fact, the ancient Egyptians discovered them, as they were common in the Nile around 5,000 years ago, and obtained a basic working knowledge of electricity by studying them. The Greeks and Romans took it a bit further, and used eels to administer electric shocks to treat a variety of maladies, including headaches. I don't know about you, but I'd rather take a paracetemol…….

Fig. 4. An electric eel ▪ a perfect example of nature's own battery.

What are the forces between charges like?

The electric force between two charges, let us say two electrons, each with a charge *e* , was shown by Coulomb (in 1785) to be $e^2/4\pi\varepsilon_0 r^2$ where ε_0 is a fundamental constant called the permittivity of free space, and has the value 8.8542×10^{-12} m^{-3} kg^{-1} s^4 A^2 . So, if we take an electron and a proton 0.05 nanometres apart (i.e. the average distance between the electron and proton in a Hydrogen atom), the electrical force between them will be 9.2×10^{-8} N (N is the symbol for the *Netwon* , the unit of force). Just to compare, I weigh 75 kg (if I have been scoffing stuff I shouldn't), which means that I apply a force of roughly 735 N on the floor. Given that the mass of the electron is around 10^{-30} kg, on a per unit mass basis, the force inside the atom is enormous compared to my weight. For a further comparison, the gravitational force between the electron and proton is approximately 42 *orders of magnitude* weaker. If any of you have read *The Hitchhiker's guide to the galaxy* , you may remember that the answer to life, the universe and everything was 42. I went to hear Douglas Adams give a talk to the student's union when I was an undergraduate and asked him if he chose that number for the reason above. He enigmatically answered "you might be onto something there". Back to the topic at hand, therein lies the clue as to what holds things together – electrical forces. Beyond a certain mass, gravity will take over, which is why stars and planets are mostly round, and asteroids below a critical size (a few hundred km) are randomly shaped. However, for us and the things we build and things around us, they are held together by electric forces.

Fig. 5. Two asteroids of different sizes, showing that, although things are held together by electric forces, this gets superceded above a certain size, when gravity takes over. Left: 243 Ida, a 53 km long asteroid, right: Vesta 4, a 500 km diameter asteroid. Images courtesy of Wikipedia.

How do charges exert forces on each other?

This is a tricky one, and to answer it in a meaningful way, we need to consider the idea of *action at a distance* , which caused no end of angst at the beginning of the twentieth century. In fact, Einstein was so perplexed by this that he came up with the theory of Relativity in an attempt to make sense of it all. Basically, we know that the fastest anything can travel (apart from in Star Trek…..) is light speed or 299,792,458 m/s. What that means then, is that if we have some charges, the forces they exert on each other cannot travel any faster than that. In other words, when you move a charge, there will be a time lag before any other charges feel a change in the force from/on it. It is the same with gravity, and the way a gravitational disturbance travels from one place to another is as a gravitational wave (Which have been detected since I wrote the first version of this e-book in 2013). With electricity, the disturbance created by a charge is what we call an *electric field* (given the symbol ***E***), and this travels around also as a wave, that we call electromagnetic radiation, examples of which are light, microwaves and radio waves. This is very complicated, and is at the heart of quantum mechanics, and what I am saying is that when charges interact with each other, they do so by exchanging packets of electromagnetic radiation (photons). The strength of the electric field, E, is defined as the force per unit charge. In other words, a charge Q Coulombs in an electric field E, feels a force, $F = QE$.

Fig. 6. Left: Two charges, showing electric field lines between them. If another charge was placed near these charges, it would follow the field lines. Right: two charges, showing exchange of a photon ▪ this is the underlying interaction between charged particles, it is just that the idea of an electric field is easier to manage. Field lines are not real structures: they just show the path that a test charge would take if placed in an electric field.

Different types of electricity

Electricity is as defined above, and from a circuit point of view, there are three different regimes: (i) when charges are not moving (Electrostatics), (ii) when they are moving steadily in one direction through a wire (DC) and (iii) when they are moving back and forth in a wire (AC). DC electricity is what you get from batteries, and is what is generated in nature (i.e. by plants & animals), whereas AC is what you get from the mains, and is generated by moving magnets inside coils – the basis of electric motors and generators.

What do we use electricity for?

This is a bit of a no-brainer, as we use it for almost all aspects of our lives. A better question would be ***how** do we use electricity?* As soon as you have two or more charges exerting forces on each other, they are capable of doing mechanical work. For example, in a battery, there are two terminals at different voltage, which is generated by a chemical reaction. This means that there is an electric field between the terminals, so the charge (electrons) with the higher potential wants to flow to the terminal where it will have lower potential. As electrons are negatively charged, this means they want to flow from the negative terminal to the positive one. This process is exactly the same as a ball (or, in fact, anything) rolling down a hill – it does so to reduce its potential energy. The ball of mass m at a height h has a potential energy mgh , so wants to reduce h . Why does this happen? We don't know *why* things want to minimise their energy, we just know they *do* . However, we also know that energy is conserved (this is the so-called first law of thermodynamics), so where does the excess energy go when the ball flows down the hill? The answer is that it is converted into another form of energy, in this case Kinetic energy, or energy of motion. This moving ball, if it hits something, can impart some momentum to it, so it can do mechanical work on it. This is also true of charge moving from a battery. The way they are constructed, it is impossible for the charge to move from one terminal to the other inside a battery, but as soon as the terminals are connected via a circuit, the charge can move through that circuit to get from one terminal to the other. The point is that as the electrons move from the negative terminal to the positive terminal, they convert their energy from potential to kinetic. Where does this kinetic energy go you might ask? Well, it goes into doing work in the components in the circuit, for example resistors heat up (the amount of energy dumped in resistors is equal to the energy difference between the terminals), motors move, etc. In a nutshell, we use electric currents to do something useful!

The rules

Physics is full of rules and laws that often seem baffling, abstract or counter-intuitive, and the worst offender here is definitely circuit theory. The most important rules are

- conservation of energy and
- conservation of charge

You will already be aware of these rules in other contexts, and when talking about circuits, they are known as Kirchoff's first and second laws, respectively.

Important quantities

These are Voltage (V), current (I), resistance (R) and power, (P). We had better define these below and then we can consider them each in a little more detail as we go on.

Voltage is a measure of energy, specifically the electrical potential difference between two points, and is measured in Volts (V). A difference in electrical potential between two points gives rise to an electric field, E, which is the gradient of the potential, i.e. $E = dV/dx$ or for simple cases, V/x. Any charge, Q in this field experiences a force, $F = QE$. If the charge is free to move, it will do so – this is what electric current is. As the voltage difference between two points is what *pushes* current around a circuit, it is often called electromotive force, or EMF. In many countries, the symbol for volts is U rather than V, as voltage is a measure of energy, and in physics, the letter for energy is usually U. As electric field is the gradient of the voltage, i.e. $E = -\frac{dV}{dx}$, (the minus sign is often forgotten about and just tells us that a positive charge will move from a point of high voltage to a point of low voltage and vice versa for negative charges – akin to saying that a ball will roll down a hill rather than up it) we can say that the voltage is the integral of the electric field, or

$V = -\int_R^d E dx$, where R is a reference point where the potential is zero. This is usually taken as $x = \infty$, as we know that voltage and electric field decrease with increasing distance from a charge, and if you are infinitely far away, they drop to zero. It is easier that this in real circuits, as we usually have a ground or earth point, and that is our reference, and we don't need to use calculus to figure out what is happening.

One Volt is defined as the potential difference across a conductor when a current of 1 Ampere dissipates 1 Watt (W) of power. A charge of 1*C* moving between two points with a potential difference of 1V will gain 1 Joule of energy. Other names for voltage in common use are electromotive force (*EMF*), and potential difference (pd).

Current is the flow of electric charge, usually electrons, and is measured in Amperes, or Amps (*A*).

One Amp is defined as 1 Coulomb (*C*) of charge flowing through a conductor per second. The amount of charge on each electron is 1.6×10^{-19} C. Therefore, 1*A* is the same as $1/1.6 \times 10^{-19} = 6.3 \times 10^{18}$ electrons per second. Current is defined as flowing from the terminal of high potential to the terminal of low potential, so flows from the + to the – terminal of a battery. However, this is very confusing, as actually, the current is due to the flow of electrons which are negatively charged, so they flow from the – terminal to the + terminal – in the opposite direction. The *current* is assumed to be due to the flow of positive charges (even though it's not….), and this is a convention that I am afraid we are stuck with. It came about as when people were discovering electricity, they assumed (as you do) that electrons were positively charged, and all circuit analysis is based on this. Then JJ Thomson went and mucked it all up by showing that electrons are actually negatively charged. For this, he won the Nobel prize in Physics and had a road named after him in Cambridge – I have the honor of having my lab situated there, as is the Cavendish lab (the Physics Department of the University of Cambridge).

Resistance is a measure of how much current flows between two points for a given voltage between those points, and is measured in Ohms (□). The greater the resistance, the lower the current. The resistance of an object depends on its geometry and a material property called *resistivity* . So, resistors control the amount of current flowing in a circuit, mostly by controlling the number of electrons that flow, rather than by changing their speed, which mainly depends on the voltage across the resistor.

Power is measured in Watts (W), and is the amount of energy used (or work done, same thing) per second, so Watts have the units of Joules/second. Given that volts are Joules/Coulomb and current is Coulombs/second, the product of both is

$$VI = \frac{Joules}{Coulomb} \cdot \frac{Coulomb}{second} = \frac{Joules}{second} = \text{power, } P$$

A bit about electrostatics & electric fields

Consider a charge, on it's own, with charge Q_1, sitting out in space, with nothing nearby. We say that the electric field spreads out evenly (*isotropically* or *uniformly*) in all directions, following this relationship with distance, r, away from the charge:

$$E = \frac{Q_1}{4\pi\varepsilon_0 r^2}$$

A second charge, Q_2 placed at a distance r from the first one feels this field, so experiences a force $F = EQ_2$, or:

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$$

Which is known as *Coulomb's law*. If there is a material in between the charges, it will act to reduce the electric field. To see why, have a look at the typical arrangement of atoms in a material, as shown in Figure 7. Atoms are arranged at regular positions, known as a *lattice*, with the electrons either shared directly between specific atoms (in bonds – as is the case in most insulators and semiconductors), or across the material as a whole (in metals).

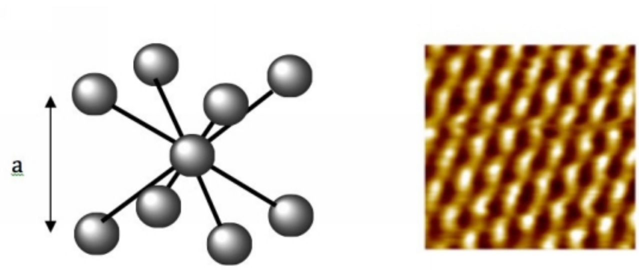


Figure 7. left: schematic of atoms in a material, the atoms are represented by spheres and the bonds between them by lines. This is the imaginatively named "ball and stick model" of atomic structure; Right: surface of a material showing how regular the positions of atoms are. Image taken with one of the author's Scanning Tunnelling Microscopes (STM).

Now let's look at the arrangement of the *charges* in a material. The atoms tend to be arranged in a regular pattern known as a *lattice* . The valence electrons are involved in bonding and are usually fixed, but there are some electrons that can either escape a bond (just due to heat – everything has a small amount of thermal energy), or in the case of a metal, there are plenty of electrons free to move around (roughly one per atom). If a metal is heated up, the atoms start to jiggle around, and this has the effect on increasing the electrical resistance. A semiconductor on the other hand is a little different. There is a component of resistance that increases in the same way as in a metal, but this is completely blown away by the fact that in a semiconductor, any increase in temperature causes some electrons to become free of the bonds, and the availability of more electrons causes the resistance to go down. When an electric field is applied to a material, the electrons try to move against it, and how much they move depends on how free they are. In an insulator, they are caught up in bonds so are not at all free. In that case, then, the atoms stretch as shown in Fig. 8. This slight stretch creates a small electric field within the material in the opposite direction to the field we applied.

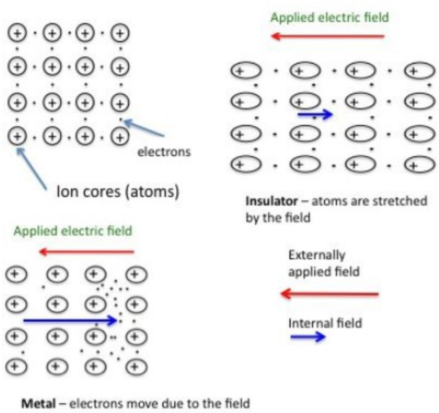


Figure 8. left: schematic of atoms in a material, the positively charged atomic cores are represented by circles and the bonding electrons are represented by dots. Right: How a material responds to an electric field. The atoms become stretched by a fraction of a percent (not to scale), resulting in an internal electric field (blue arrow). The sum of these two (which is really a difference as they are in opposite directions) is the resultant field. The ratio between the applied field, E_{app} and the resultant field, E is the relative permittivity, ε_r. In a metal (bottom right), the electrons are free to move, giving rise to a much larger ε_r.

The resultant electric field is the difference between the applied electric field and the internal electric field, and in the ideal case of a perfect metal or a superconductor, the internal field exactly equals the applied field, with no net field. The ratio of the applied field to the net field is called the relative permittivity or the dielectric constant, which are the square of the refractive index. For a perfect metal, it is infinite, and for air/free space, it is 1.

So what? Well, if we are looking at an electric field, or thinking about the force on a charge in an electric field, then we need to take into account whether there is a material present, as materials always have the net effect of reducing electric fields. Coulomb's law for the force between two charges Q_1 and Q_2 a distance r apart in the presence of a material with relative permittivity ε_r is

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0\varepsilon_r r^2}$$

This is particularly important when we consider capacitors, which we should do now briefly.

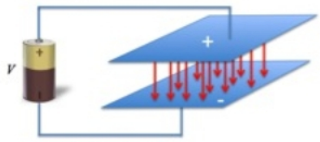


Figure 9. A capacitor connected to a battery of voltage *V*. The plates are separated by a distance *d*. The top plate is positively charged and the bottom plate is negatively charged. There is an electric field between the plates of magnitude *V/d*. The electric fields *outside* the plates cancel each other out.

A capacitor is a component that stores electric charge. The two plates have an equal but opposite charge, and there is an electric field between them. For the type of capacitor shown in Figure 9, known as a *parallel-plate capacitor*, the electric field is uniform and is given by $E = V/d$. By definition, the capacitance is $C = Q/V$, where Q is the amount of charge stored on each plate. As $V = Ed$, the capacitance is $C = Q/Ed$, and we can reason that if we fill the space between the plates with a material of relative permittivity ε_r, then it becomes $C = Q/(Ed / \varepsilon_r) = \varepsilon_r Q/Ed$, or a factor of ε_r larger that it was before it was filled. We will come back to capacitors later on, but the most important aspect of them is that they store charge for use later in a circuit, and when they release it, this takes some time, and this is useful for controlling the speed at which a circuit does something.

Electronic Components

There are a vast number of different electronic components in everyday use, but we can classify them into two basic classes: those that have linear current-voltage characteristics (i.e. they follow Ohm's law, which we will look at shortly) and those that do not. Let us very quickly look at what these components are and their main functions.

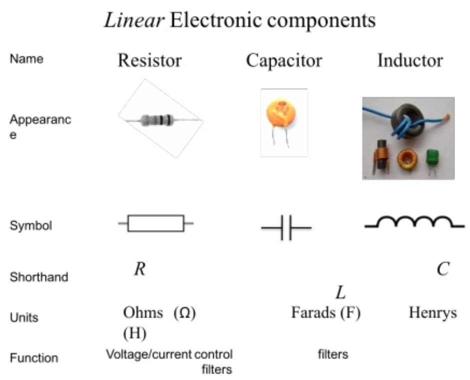


Figure 10. Electronic compo

Ohm's law

Everybody's heard of Ohm's law, and knows that $V=IR$. Most people however, are not sure how to use it, so let us start at the beginning. At the dawn of the 18th century, Alessandro Volta invented the battery, enabling people to perform experiments with electricity in a way that was impossible until then, as the only other way of generating electricity was by using friction-based machines (there is some historical evidence of batteries in the middle east over a thousand years before that, but it is not clear what they were used for – look up the *Baghdad battery* if you're interested). For his trouble, the unit of potential difference was named after him. For the next 20 years, various advances were made, particularly with electromagnetism, culminating with Faraday's invention of the electric motor in 1821. In 1826, having played with batteries and various bits of metal and other materials, Georg Ohm reported the following, now famous observation:

For a constant temperature, the current, I, flowing through a conductor is directly proportional to the potential difference, V, between its ends.

In other words, $V \propto I$. The constant of proportionality is called the resistance, R, so we can write

$$V = IR$$

Which is known around the world as Ohm's law.

To illustrate this, consider the circuit below: we have a battery with V Volts across its terminals, and we connect this to a resistor of R Ohms . A current, I, will flow through the circuit.

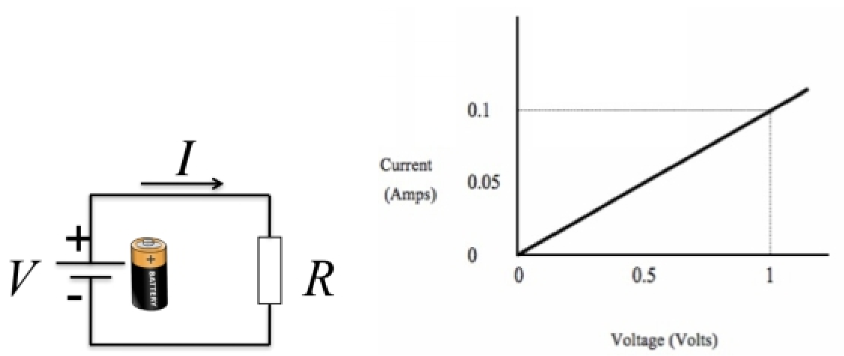


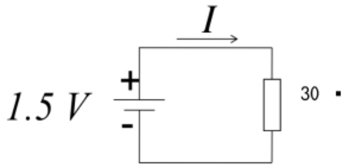
Figure 11. Illustration of Ohm's law. A battery with voltage *V* is connected to a circuit consisting of a resistor, *R*. A current *I* flows.

The figure on the right shows the current versus voltage for a 10 Ohm resistor : $I = V/R$. so 1 V will give a current of 1/10 A.

Now, Ohm's law is more subtle than it seems, as we can apply it individually to each component in a circuit, or to a circuit as a whole. Which approach we take depends on what we are trying to work out, as we will see by example.

Time to put this all to the test now. Let's look at a few examples of Ohm's law, starting with abstract bookwork, ending with something potentially useful.

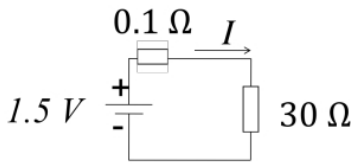
Example 1. Find the current through a 30 Ohm resistor for a 1.5V battery.



Solution . $V = IR$. Rearranging gives us that $I = V/R$.

Therefore $I = \frac{V}{R} = \frac{1.5V}{30\Omega}$. = 0.05 A = 50 mA

Example 2. Find the current through a 30 Ohm resistor for a 1.5V battery with an internal resistance of 0.1 Ohm.

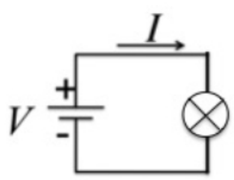


Solution . $V = IR$. Rearranging gives us that $I = V/R$. The only difference is that now R = 32Ohm □ 0.1Ohm □

This gives us $I = \frac{V}{R} = \frac{1.5V}{30.1\Omega}$ = 49.83 mA

Batteries and power sources *always* have some internal resistance, but it is usually very low as in the above example, and especially if it is much smaller than the resistance of everything else in the circuit, then it makes very little difference. The reduction in current when we take the internal resistance into account is (50-49.83)/50 which is less than 0.7%, i.e. it is negligible.

Example 3. A light bulb is connected to a 9V battery. It draws 1A of current. Calculate (i) the resistance of the bulb and (ii) the amount of power dissipated in the bulb. Where does this power come from?



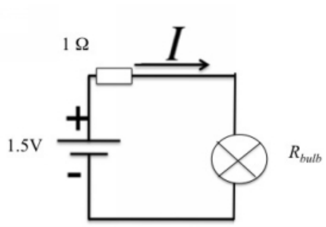
Solution . (i) $V = IR$. Rearranging gives us that $R = V/I$.

This gives us $R = \frac{9V}{1A} = 9W$

(ii) Power, $P = VI = 9V x 1A = 9W$.

The power comes from the potential difference between the battery's terminals.

Example 4. A light bulb is connected to a 9V battery with an internal resistance of 1 Ohm. It draws 0.9A of current. Calculate (i) the resistance of the bulb, (ii) the amount of power dissipated in the bulb, (iii) the amount of power dissipated in the battery and (iv) the total amount of power dissipated.



Solution . (i) The resistances of the battery and the bulb are in series, so the total resistance of the circuit is the resistance of the bulb plus the internal resistance of the battery, or $R_{total} = R_{bulb}$ + 1 Ohm. From Ohm's law, we can then say that $R_{total} = \frac{9V}{0.9A} = 10W$, which means that R_{bulb} = 9 Ohm.

(ii) All components have the same current flowing through them, so the power dissipated in the bulb is given by V_{bulb} xI . Now, at the moment, we don't know how much voltage is dropped across the bulb, but we do know that of the 9V we started with, some is dropped across the internal resistance of the battery, and the rest is dropped across the bulb. From Ohm's law, $V_{bulb} = I$ xR_{bulb} . We can combine all of this to give us:

$$P_{bulb} = V_{bulb} \text{ x } I = (I \text{ x } Rbulb) \text{ x } I = I^2 \text{ x } R_{bulb} .$$

Plugging in the numbers from above, we find the amount of power dissipated in the bulb is $(0.9A)^2 \times 9W$ = 7.29W.

(iii) Using the same idea as in part (ii), we can say that

$$P_{battery} = V_{battery} \times I = (I \times R_{battery}) \times I = I^2 \times R_{battery}$$

Now using the numbers from above, we find the amount of power dissipated in the battery is $(0.9A)^2 \times 1W$ = 0.81W.

(iv) The total amount of power dissipated is due to both the bulb and the internal resistance of the battery, so is just the sum of the two numbers above, i.e.

$$P_{total} = P_{battery} + P_{bulb} = I^2 R_{battery} + I^2 R_{bulb} = I^2 (R_{battery} + R_{bulb})$$

$= (0.9A)^2 \square\ (9 \square + 8 \square) = 8.1W$

Is this a lot? Well, it's not much, given that a typical kettle is 1.5 kW (1,500 Watts). We also note that the effect of the internal resistance of the battery is that the overall amount of current is reduced, and as a consequence, so is the power that is dissipated in the circuit.

Related issues

1. When we talk about power being *dissipated* in a resistor, what do we mean by that? Well, the word dissipated normally means used up, or lost. In physics it has a more specific meaning which is the loss of energy through conversion to heat. Therefore, when we pass a current through anything with resistance, it heats up, and in line with conservation of energy, the amount of energy "lost" in the resistor is the same as the potential difference between the terminals of the battery.

2. Take the light bulb example from above. We found that around 8-9W of power was dissipated in it. If it's a normal, filament type

(incandescent) light bulb, then only around 10% of that is converted into light, the rest into heat. This is what has driven the development of energy-saving bulbs which have an efficiency of up to around 80%, so don't get hot. Given that lighting accounts for around 25% of a typical household's electricity bill, in the long run, it makes sense to spend a bit extra on energy-saving bulbs.

Different ways of connecting resistors

There are two ways in which two or more resistors can be connected to a power supply – in ***series*** or in ***parallel*** .

Resistors in series

When we connect resistors in series, the total resistance is just the sum of the individual resistors. When dealing with circuits, we will always only think about what happens in the steady state, i.e. when the currents have been flowing for some time and reached equilibrium (I say this because when you turn on a battery and current flows, it doesn't know what is in the circuit until it encounters it, but once it has gone around once, things settle down).

The total resistance of resistors in series is additive – the more resistors we put in, the *higher* the resistance, and the *lower* the current will be for a given supply voltage. As there is only one path for current to take and charge is conserved, the same current flows through each resistor, each of which may have a different voltage dropped across it (which we can find using Ohm's law).

We can reason this out by taking a battery with voltage V Volts, and connecting it to a number of circuits with different numbers of resistors, and let's see what Ohm's law tells us.

First, consider a circuit with a resistor, R_1 : for a current, I_1 , the voltage across the resistor will be $I_1 R_1$.

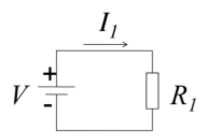


Figure 12. A single resistor of value R_1 with a current I_1 through it will develop a voltage $V = I_1R_1$ across it.

Let's start adding resistors in series, one at a time. For the case of 2 resistors, R_1 & R_2, we will call the current I_2, so the voltage across the two resistors is

$I_2(R_1 + R_2)$.

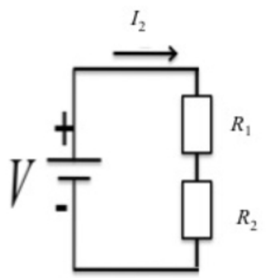


Figure 13. Two resistors of value R_1 and R_2 with a current I_2 through them will each develop a voltage $V_1 = I_2R_1$ and $V_2 = I_2R_2$ across them. The total voltage is $V = V_1 + V_2$.

Now consider a large number, N of resistors, with a current I_N, the voltage is now

$I_N(R_1 + R_2 + R_3 + \ldots + R_N)$.

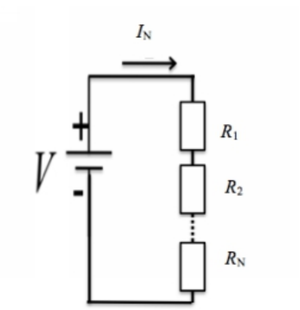


Figure 14. N resistors of value R_1, R_2 ⋯.R_N, with a current I_N through them will develop voltages $V_1 = I_2R_1$, $V_2 = I_2R_2$ etc; up to $V_N = I_NR_N$ across them. The total voltage is $V = V_1 + V_2 + \cdots.. V_N$.

Where is this heading?

Well, we had the same voltage, V, in each circuit, which means the voltages above must all equal each other, or

$$I_1 R_1 = I_2 (R_1 + R_2) = I_N (R_1 + R_2 + R_3 + \ldots + R_N)$$

We are trying to find the resistance of each circuit, to see what effect series connection has, and it is just given by V/I (Remember, Ohm's law says $V = IR$, which can be rearranged as $R = V/I$).

So, the resistances just add up! To get to grips with this, imagine the water analogy – put one kink in the hosepipe, and the flow slows down. Put a second kink in, and it will slow down even more.

Resistors in Parallel

When we connect resistors in parallel, it's a bit more complicated and the resistance is ***not*** additive. The more resistors we add in parallel, the *lower* the resistance gets, and the *higher* the current will be that is taken from the battery (in electricity, we use the word *drawn* to mean current taken from a power source such as a battery). It is almost the opposite to what happens in series. The reason for the difference is that every time you add another resistor in parallel with the others, you are creating a new path for current to flow through, and as it is in parallel, it is independent of the existing current paths. Let us look at some examples to see if we can make sense of it.

First, consider one resistor, this is exactly the same as above, so the resistance is just R_1.

Secondly, add another resistor in parallel. You can see from the figure that both resistors have the same voltage across them, which is just V Volts. We will call I_1 the current through R_1 and I_2 the current through R_2. As both of these currents are coming from the same battery, the **total** current coming from the battery is $I_1 + I_2$.

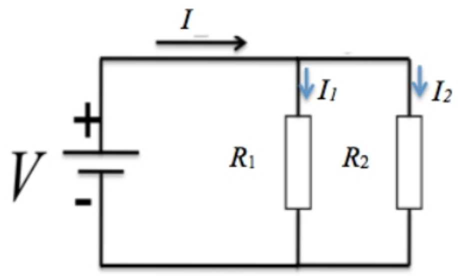


Figure 15. 2 resistors of value R_1 and R_2 in parallel, with currents I_1 and I_2 through them will develop voltages $V = I_1R_1$ and I_2R_2.

We can also use Ohm's law to say that

$I_1 = V/R_1$ and $I_2 = V/R_2$, so the total current, $I = I_1 + I_2 = V/R_1 + V/R_2$

$= V(1/R_1 + 1/R_2)$

As $I = V/R$, we can rearrange to get

$$I = \frac{V}{R} = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) => \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Cross-multiplying, we find that $R = \frac{R_1R_2}{R_2 + R_2}$, which is $\frac{\text{Product}}{\text{sum}}$

Now consider a large number of resistors in parallel, as shown in Figure 16 below:

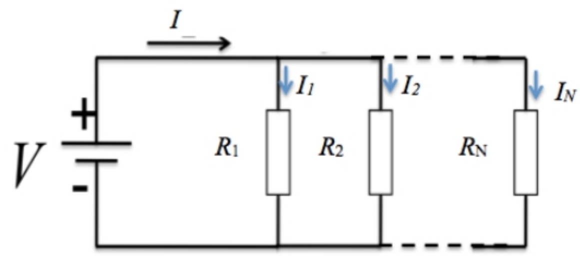


Figure 16. Multiple resistors of value R_1 and R_2 ….R_N in parallel, with currents I_1 and I_2 ….. I_N through them will develop voltages $V = I_1R_1$, I_2R_2 and ….. I_NR_N.

As before, $I_1 = V/R_1$, $I_2 = V/R_2$, and so on up to $I_N = V/R_N$.

The total current is the sum of the individual currents, or

$$I = I_1 + I_2 + \ldots.. + I_N$$

Using Ohm's law as before, we can write this as

$$I = V/R = V\,(1/R_1 + 1/R_2 + \ldots\ldots + 1/R_N)$$

So,

$$1/R = (1/R_1 + 1/R_2 + \ldots\ldots + 1/R_N)$$

This is where the water analogy really helps: each time we add a new resistor in parallel, it's like adding another hose, which draws more water (current) from the reservoir, even though the amount of water (current) carried by each hose depends on it's geometry (resistance) and the height of the reservoir (voltage), and the hoses are independent of each other.

Let us look at a few examples where this knowledge might come in useful.

Example 5 . We are going to power two light bulbs using a 9V battery. The battery has an internal resistance of 0.2 Ohm and the light bulbs, when they are on, have a resistance of 13 Ohm and 7 Ohm. They are connected in series initially and then in parallel.

In each case, determine

(i) which bulb will be brighter, and

(ii) the total power dissipated in the two bulbs.

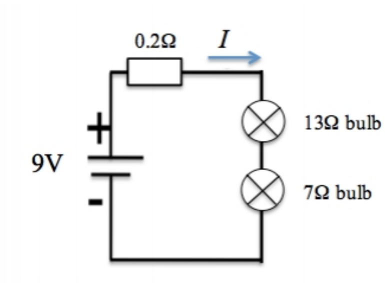
Solution .

(i) In each case, the bulb that will be brighter is the one with the most current through it or the most voltage across it. Remember, Power (which determines the brightness) is $P = VI = V^2/R = I^2 R$.

In series, both bulbs have the same *current* through them, so the brighter one will be the one with the largest voltage across it. This will be the one with the largest resistance - the 13 Ohm one.

In parallel, both bulbs have the same *voltage* , so the brighter one will be the one with the largest current through it. This will be the one with the smallest resistance – the 7 Ohm one.

(ii) In series, the circuit looks like:



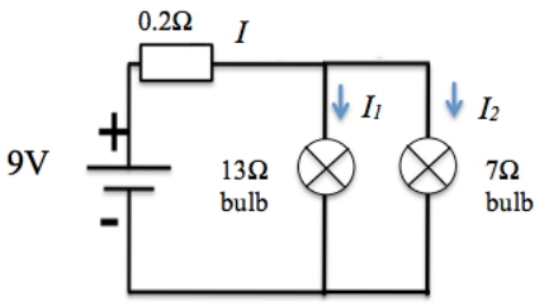
The current, I = V/R = 9V/20.2 □ = 0.445 A

The power dissipated in each bulb is I^2R , so the total power in the two bulbs is

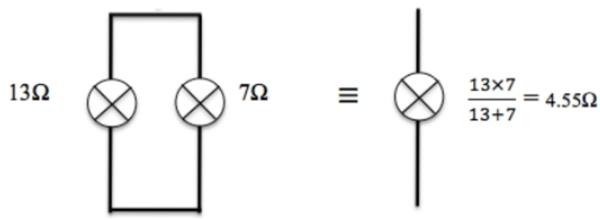
$$P = I^2(R_1 + R_2)$$

$= 0.445^2 \times (13 + 7) = 3.96$ W

In parallel, the circuit looks like:



The two bulbs can be represented by a single resistor that has the same resistance as the parallel combination of the two, i.e.



Therefore, the total current being drawn from the battery is the total voltage divided by the total resistance, which is the sum of the battery's internal resistance and the resistance of the parallel combination of bulbs, or

$$I = \frac{9V}{4.55\Omega + 0.2\Omega} = 1.89A$$

Then the power dissipated in the bulbs is $I^2R = 16.25\ W$, or over 4 times as much as when they were connected in series!

Example 6 . We are going to power two light bulbs using a 9V battery, as shown in Fig. 17. The battery has negligible internal resistance and the light bulbs *A* and *B* are rated as 9V 1W and 9V 3W, respectively. The battery has a capacity of 500 mAh.

Calculate:

(i) The resistance of each bulb,

(ii) the current passing through each bulb

(iii) the current drawn from the battery

(iv) how long the bulbs can be left on for before the battery runs out of charge

Solution .

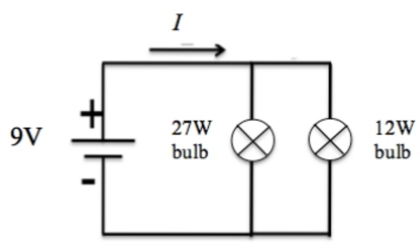


Figure 17. Battery powering two bulbs, for example 6.

We know the power and voltage for each bulb, so we can work out the resistance and current by using the fact that Power, $P = \frac{V^2}{R}$:

	Bulb 1	Bulb 2
Power	27W	12W
Resistance ($R = \frac{V^2}{P}$)	$\frac{9^2}{1} = 81\Omega$	$\frac{9^2}{3} = 27\Omega$
Current ($I = \frac{V}{R}$)	$\frac{9}{81} = \frac{1}{9}$ A	$\frac{9}{27} = \frac{1}{3}$ A

The current drawn from the battery is the sum of the currents passing through each resistor, so is $\frac{1}{9}$ A + $\frac{1}{3}$ A, or $\frac{4}{9}$ A.

The time the bulbs will remain lit for is given by the following relationship:

$$\text{Time} = \frac{\text{Battery capacity}}{\text{Current}} = \frac{0.5\ \text{Ah}}{\frac{4}{9}\text{A}} = 0.115\ \text{h} = 67.5\ \text{minutes}$$

Example 7 . Fig 18 shows two resistors connected in series with a 12V battery which has negligible internal resistance. The series resistance of the two resistors together is 120 Ohm, and the voltmeter reads a voltage of 3V.

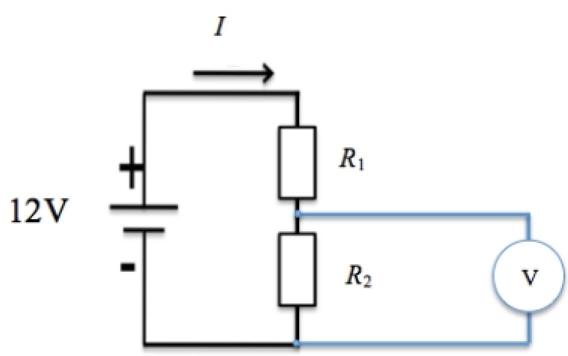
Calculate

(i) The values of R_1 and R_2 ,

(ii) The current drawn from the battery,

(iii) The amount of charge passing through the circuit per minute,

(iv) The number of electrons passing through the circuit per second



Voltmeter

Figure 18. Circuit for Example 7.

Solution .

$R_1 + R_2 = 120$ Ohm

(i) The voltage across R_2 = 3V, which is ¼ of the total voltage, so R_2 is ¼ of the total resistance. This means R_2 = 30 Ohm and R_1 = 120 Ohm □

(ii) The current coming from the battery is the total voltage divided by the total resistance, or $I = \frac{V}{R_1 + R_2}$ = 12/120 = 0.1A

(iii) The amount of charge passing through the circuit per minute is (the amount of charge passing through the circuit per second) × 60. This is 0.1 Coulombs per second times 60 seconds per minute, which is 6 Coulombs per minute.

(iv) We have 6 Coulombs per minute, or 0.1 Coulombs per second. Given that each electron has a charge of $1.6x10^{-19}$ Coulombs, this means each Coulomb corresponds to $\frac{1}{1.6 \times 10^{-19}}$ electrons, which is 6.25×10^{18}, so a current of 0.1 A consists of a flow of 0.625×10^{18} or 6.25×10^{17} electrons per second.

Example 8 . Fig 19 shows a resistor connected in series with a thermistor and a 12V battery that has negligible internal resistance. The combined series resistance of the resistor and thermistor at room temperature is 240 Ohm, and the voltmeter reads a voltage of 5V.

(i) Calculate the resistance of the thermistor.

(ii) The resistance of the thermistor decreases by 4% when the temperature in Celsius increases by 10% near room temperature. If the temperature increases from 20^o C to 21^o C, calculate the change in voltage the voltmeter will measure.

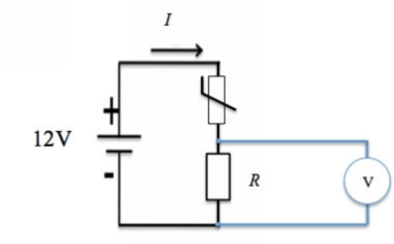


Figure 19. Circuit for example 8.

(i) The total resistance of the circuit is 240 Ohm. The voltage across the resistor is 5V which is 5/12 of the total voltage. Therefore, the resistor has 5/12 of the total resistance of the circuit, which is 5/12 of 240 Ohm, or 100 Ohm. This means the thermistor has a resistance of 140 Ohm.

(ii) The temperature increases by 1° C, which is 1/20 or a 5% increase in temperature. Therefore, the thermistor resistance will decrease by 2%, to bring it down to 140x0.98= 137.2 Ohm . The new total resistance is

100 Ohm + 137.2 Ohm = 237.2 Ohm □ The proportion of the total voltage dropped across the resistor is now $\frac{100}{237.2}$ = 0.4216, so the new voltage across the resistor is 0.4216x12V = 5.06V. Therefore, the change in voltage measured across the resistor is 60mV.

Batteries – the basics

Given that we rely so heavily on portable electronic devices, it is worthwhile gaining some insight into the basic principles behind the operation of batteries. As was mentioned earlier, the forerunner to the battery (the Galvanic cell) was invented in 1792 by the Italian scientist Allesandro Volta, after whom the *Volt* was named. A battery is something that uses a chemical reaction to create ions and electrons, so in other words it turns stored chemical energy into electrical energy. The idea is illustrated in Figure 20 below. We have to make use of some chemical information to understand this. In order to make a battery, we need two metals (*electrodes*) at different positions in the *electrochemical series* , placed in an *electrolyte* (a material that ions can flow through, and this can be either a liquid or a solid). The most common materials used in the fabrication of batteries are zinc (Zn) and copper (Cu). In the example shown here, we take a piece of Zn and a piece of Cu, immersed in dilute sulphuric acid ($H_2 SO_4$).

When we add an acid to water, it splits up into ions, in this case Hydrogen (H^+) & sulphate ($S0_4{}^{2-}$). As Zn is higher in the electrochemical series than H and Cu, it attracts the electrons in the sulphate ions and forms zinc sulphate and gains a net negative charge (2 electrons per sulphate ion), becoming the *anode* . At the same time, the hydrogen ions attract electrons from the Cu and make it positive (the *cathode*), while turning 2 hydrogen ions into H_2 gas which forms bubbles at the surface of the Cu electrode. This charge imbalance between the Zn and Cu gives rise to a potential difference between them, of the order 1.1 V.

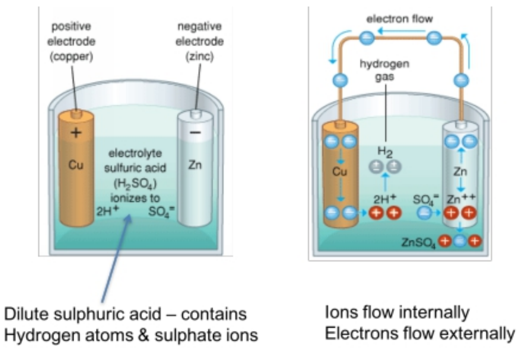


Figure 20. Inside a *Galvanic cell* ▪ the first kind of battery

As soon as an external circuit is connected to the Cu and Zn electrodes, electrons can flow through it and there you go - we have a battery-powered circuit! Of course, we do not actually use Galvanic cells any more, we mostly use *alkaline* batteries. These account for 60-80% of battery sales worldwide, a market in excess of 20 billion batteries per year. We will not go into the internal workings of alkaline batteries apart from mentioning that they have a solid paste electrolyte of potassium hydroxide (KOH) and generate a potential difference of 1.43 V. The anode is still made from Zn, but the cathode is made from Manganese dioxide (MnO_2). The important parameters of batteries are their voltage, their internal resistance (which determines the maximum current they can deliver to a circuit) and their *capacity* - a measure of how long they can deliver a given current for. This is measured in milli-Amp hours, abbreviated to mAh. As an example, the battery in the iPhone 7 plus has a capacity of 2,900 mAh. This means it can deliver a current of 2,900 mA for 1 hour, or 290 mA for 10 hours etc. As a battery gets used, the voltage starts to drop, but the main issue is that the internal resistance starts to increase and the chemicals inside have all reacted, so need to be replenished. For typical alkaline batteries, this is a one-off reaction, but for rechargeable batteries, it is sufficient to charge them back up by passing a current in the reverse direction – this replaces all of the electrons that have been moved by the chemical reactions inside. A comparison between the different battery types is shown in Figure 21 below.

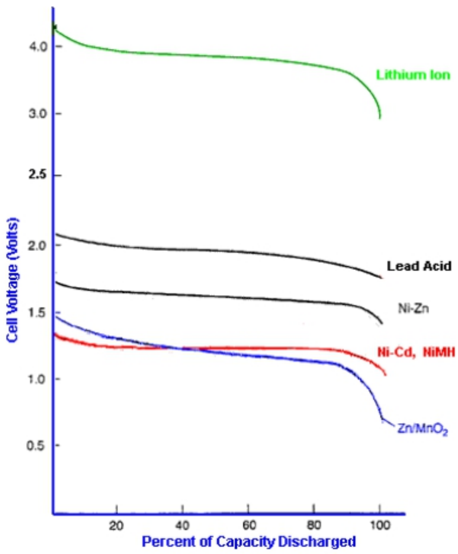


Figure 21. A comparison between different battery types and how their output voltage changes as they discharge. Data courtesy of Energizer.

People are often uncertain of the difference between a battery and a capacitor as at a superficial level at least, they do seem to do something similar – they store electrical energy. The key difference between them is that a battery stores the energy chemically and generates free charge as and when needed (i.e. when a current is drawn), and can usually do so for some time, whereas a capacitor stores charge, and when it is connected in a circuit, it usually discharges very quickly. Before we look at capacitors in a little more detail, let us wrap things up for batteries by looking at an example – an iPhone battery.

Example 9 . We have a battery in a mobile phone. It's voltage is 3.82V, and its capacity is 1,810 mAh. Calculate (a) the total energy supplied by the battery and (b) how long it should take to charge using a 5 W supply.

Solution .

(a)Energy = *Power* × *Time* = *VIt* = Voltage × Capacity

= 3.82 V × 1.81 A × 1 hour × 3600 seconds/hour

= 24891 Joules = 24.891 kiloJoules (kJ)

(b)Charging time = *Energy/Power* = 24.891 kJ/5W

= 4978 s = 1 hour, 23 minutes

Capacitors

Capacitors are extremely useful components, and a typical circuit will contain at least one for some reason or another. I stated earlier that capacitors are used as filters. What does this mean? Well, most electronic signals of interest are ac, and have a range of frequencies. It turns out that the way capacitors work depends on the frequency, and they are very good at not letting unwanted dc signals pass through. By appropriate selection of resistors and capacitors, we can control which frequencies of our electronic signal go where in a circuit. For now, though, we are only interested in dc circuits, so let us think about what capacitors do under dc conditions.

The best starting place is to consider a battery on its own first. Let there be a potential difference of V Volts between its terminals. Let us now connect some wires to these terminals. There is still the same potential difference between these, and each wire will be at the same potential as the terminal it is connected to. Finally, let's connect each of these wires to a metal plate. Now the plates will be at the same potential as the terminals they are connected to, and there will be a potential difference between them, of V volts. This is a capacitor. The effect of this potential difference is that the plates (and the wires joining them to the terminals) become charged. The electrons in the – terminal are trying to get to the + terminal, so they travel as far as they can to attempt to do that, and go onto the – plate. This plate therefore becomes negatively charged which creates an electric field. The other plate feels this electric field, and some electrons in it are repelled, and travel into the + terminal of the battery. What we are left with then is two plates, each with a net charge. The charges are equal and opposite, and there is an electric field between the plates. The relationship between the voltage between the plates and the amount of charge on them is called *Capacitance* , which is defined as $C = Q/V$.

How can we work out the capacitance of a capacitor? Well, we need to work out the relationship between the voltage across the plates of a capacitor and the charge on those plates. If we consider a single plate, it can be shown (using something called *Gauss's law* , which is beyond the scope of the A-levels) that if it has a charge Q , and an area A , then as long as the plate is large enough that we can neglect edge effects, then the electric field coming from that plate is uniform (i.e. the same everywhere), and of strength $\frac{Q}{2\varepsilon_0 A}$. This is illustrated in Figure 22. In a capacitor, there are two plates whose electric fields add together in the middle (and they cancel outside), to produce an electric field $\frac{Q}{\varepsilon_0 A}$. The next thing is, how do we relate electric field to voltage?

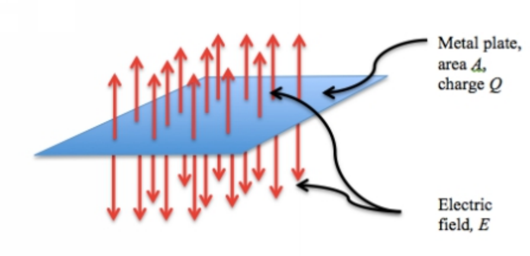


Figure 22. The electric field coming from a charged plate.

Well, we saw earlier that $V = -\int_R^d E dx$. From figure 9, where there are two plates, let us say, a distance d, apart, then we know one plate is at 0V and the other is at V Volts. Therefore, we can rewrite the integral as:

$$V = -\int_0^d E dx = -\int_0^d \frac{Q}{\varepsilon_0 A} dx = -\frac{Qd}{\varepsilon_0 A}$$

As capacitance is $C = Q/V$ *(the sign of the voltage doesn't matter, as one plate is positive & the other is negative- which is which is irrelevant)* , we can rearrange to get:

$$C = \frac{\varepsilon_0 A}{d}$$

If we fill the plates with a material of relative permittivity $\square_r$, then the electric field is weakened by that factor, and the capacitance is accordingly *increased* to:

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

Discharging/charging a capacitor

We would like to get an idea of what happens when we discharge a capacitor. To do that, consider a capacitor that we have charged up, and we connect to a resistor as shown in figure 23. As the capacitor is charged, it will have a potential difference, *V* across the plates. So will the resistor, as they are in parallel with each other. As soon as the circuit is made, this potential difference between the plates will cause a current to flow, essentially from one plate to the other, through the resistor. In this way, the energy stored in the capacitor is released in dissipating power (heat) in the resistor. Current is the flow of charge, and is often written as $I = Q/t$. However, this only works if the flow is *steady* , which is definitely not the case when it is charging or discharging, so it is better to define it at any instant as the rate of change of charge, i.e.

$$I = -\frac{dQ}{dt}$$

The minus sign is just there to show that as the capacitor discharges, the current will decrease.

We also know, from Ohm's law, that the current through the resistor is

$$I = \frac{V}{R}$$

As we are dealing with a capacitor, where $C = Q/V$, a constant, we have that

$$I = -\frac{dQ}{dt} = -\frac{d(CV)}{dt} = -C\frac{dV}{dt}$$

Both expressions for current must be the same, as they are the same current! Therefore,

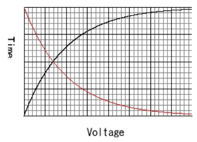
$$\frac{V}{R} = -C\frac{dV}{dt}$$

and

$$\frac{1}{V}dV = -\frac{1}{RC}dt$$

Integrating both sides, we find $\ln\left(\frac{V}{V_0}\right) = -\frac{t}{RC}$ or $V = V_0 e^{-\frac{t}{RC}}$

Likewise, when charging a capacitor using a battery of voltage V_0, the voltage across the capacitor is described as $V = V_0\left(1 - e^{-\frac{t}{RC}}\right)$



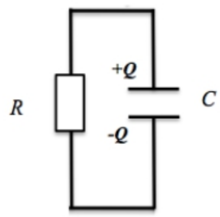


Figure 23. **Left:** A capacitor being discharged through a resistor.
Right: charging and discharging a capacitor - voltage versus time.

As you can see from these expressions, there is a characteristic *time constant*, *RC*, which is the time taken for the voltage to drop (rise) to 1/e of it's maximum value when discharging (charging).

For Example, if we have a 1 microfarad capacitor together with a 1 kiloOhm resistor, the time constant is $10^{-6} \times 10^{3} = 10^{-3}$ seconds = 1 ms.

We will now think about how to consider circuits where there are more than one capacitor. As with resistors, there are two ways we can connect capacitors - in series and in parallel.

Capacitors in series

When we connect capacitors in series, mathematically, this is the same as combining resistors in parallel. To see why, imagine we have two capacitors in series as shown in Fig. 24. As the right hand plate of capacitor 1 is connected to the left hand plate of capacitor 2, they must have the same amount charge, although of opposite sign. Why? Remember that the plates of a capacitor are not physically connected to each other, as they are separated by an air gap, so no current can flow between them. As soon as we connect the first capacitor to the negative terminal of our battery, that plate of the capacitor becomes negatively charged, by an amount $-Q$. This induces a charge $+Q$ on the adjacent plate. However, where does this charge come from, as this second plate is connected to the second capacitor? The charge $+Q$ has come from the left plate of the second capacitor, but as it isn't charged to start with, that plate becomes negatively charged by an amount $-Q$ (i.e. the difference between the zero charge we started with and the $+Q$ we removed). Finally, the right hand plate of the second capacitor becomes positively charged by an amount $+Q$.

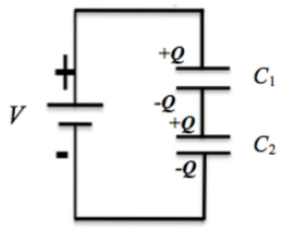


Figure 24. Two capacitors in series. DC current cannot flow through a capacitor (apart from initially when it charges up), so we are considering what happens once the circuit has been on for a while. The top plate of capacitor 1 is connected to the positive terminal of the battery, so is positively charged. We are calling the charge on it $+Q$. This induces an equal but opposite charge on the other plate (as the top plate, being charged, creates an electric field pointing downwards which attracts negative charge onto the bottom plate), $-Q$. This charge has to come from somewhere, so when we extract an amount of charge $-Q$ from the lower part of the circuit, we are left with $+Q$ on the top plate of capacitor 2. This induces a charge $-Q$ on the bottom plate of capacitor 2.

This leads us to two useful bits of information:

(i) The capacitors have the same charge, Q

(ii) In series, the sum of voltages across the capacitors equals the total voltage.

Now, the voltage across capacitor 1 is

$$V_1 = \frac{Q}{C_1}$$

and across capacitor 2 is

$V_2 = \frac{Q}{C_2}$

We know that $V = V_1 + V_2$, so

$$V = \frac{Q}{C_1} + \frac{Q}{C_2} = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

As $C = Q/V$, we can rearrange to find:

$$C = \frac{Q}{V} = \frac{1}{\left(\frac{1}{C_1} + \frac{1}{C_2}\right)}$$

or, in a more familiar format: $\frac{1}{C} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)$, a bit like the formula for resistors in parallel.

Capacitors in parallel

When we connect capacitors in parallel, mathematically, this is the same as combining resistors in series. To see why, imagine we have two capacitors in parallel as shown in Fig. 25. Both capacitors have the same voltage. The two capacitors have capacitance C_1, C_2, and charge on each plate of $\pm Q_1$ & $\pm Q_2$.

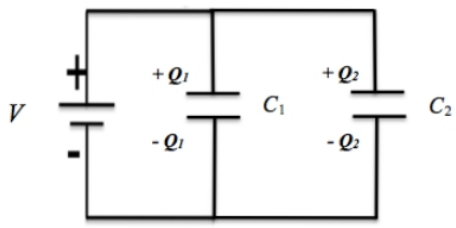


Figure 25. Two capacitors in parallel. The top plates of each capacitor are connected to the positive terminal of the battery, and are therefore at the same potential. The bottom plates are connected to the negative terminal, and are also at the same potential. The potential difference between the top and bottom plates is *V*, and if the two capacitors have different capacitance, then they will have a different amount of charge (Remember, Capacitance, $C = Q/V$).

As the top plates are connected together, they must be at the same voltage but there is no reason why they should have the same charge on them. The total charge on them is $Q_1 + Q_2$. The bottom plates have a total charge of $-(Q_1 + Q_2)$. Therefore, we can say, from the definition of capacitance, that

$$C_{total} = \frac{Q_{total}}{V} = \frac{Q_1 + Q_2}{V}$$

However, this can be written as $C_{total} = \frac{Q_1}{V} + \frac{Q_2}{V} = C_1 + C_2$

So, in parallel, capacitors add up, just like resistors in series.

Example 9 . We have two capacitors with capacitance of 1μF and 3μF, respectively. They are connected to a 1.5V battery, first in parallel and then in series.

Calculate the magnitude of charge stored in the combined capacitance.

Solution :

(i) In parallel, the total capacitance is $C = C_1 + C_2$ = 1μF + 3μF = 4μF.

$C = Q/V$, from which we have $Q = CV$. Therefore, Q = 4μF/1.5V = 2.67μC.

Of course, the *total* amount of charge stored is zero, as there is an equal amount of positive and negative charge, so we are just talking about the amount of positive or negative charge.

(ii) In series, the total capacitance is $C = \frac{C_1 C_2}{C_1 + C_2}$ $= \frac{1\mu F \times 3\mu F}{1\mu F + 3\mu F}$ = 0.75 μ F. Therefore, Q = 0.75 μ F/1.5V = 0.5 μ C.

A few words about ac

We know that the voltage we get from the mains is ac. What does that mean exactly? Well, it means that the value of voltage varies over time, and follows a sinusoidal pattern with a frequency of 50Hz (i.e. one full cycle of the pattern takes 1/50 s = 20ms). You might think this is a bit odd, as surely we need a constant voltage for things to work? As it happens, think about what most appliances do with that ac voltage – if it is for heating or lighting, it is varying too quickly for us to notice. For other applications, like a TV or computer, or charger, the appliance takes that ac voltage and converts it into a dc one. The leading question is, why is electricity generated as ac, and then why is it transmitted as ac? The answer boils down to cost and complexity. The way we generate electricity is by electromagnetic induction (remember Faraday's law which says emf = $-\frac{d\varphi}{dt}$ where φ is magnetic flux?). This involves spinning a magnet (attached to a *rotor* , which is attached to a turbine in a power station) inside a coil (which we call the *stator*). This naturally results in a sinusoidal emf being generated in the coil, which is our generated electricity. We then build lots of these machines, and make them big enough that they can generate a lot of power (typically several megawatts each) and connect them together (in parallel), and hey presto- we have a national grid, and we can charge unsuspecting folk an arm and a leg for it! We then transmit this ac voltage across the country, and it gets used. Unfortunately, the power cables lose quite a bit of power (by Joule heating), so what we do to get over this is step up the voltage for transmission, using transformers (no, not autobots….). This might seem counter-intuitive, as surely if we have a higher voltage, then we will also have a higher current through the power cables, in line with Ohm's law?

However, remember that energy is conserved, and what we generate is a certain amount of power. If we transform the voltage to a higher value, then as $P = VI$, the current must decrease. The voltage is stepped up to several tens of kV, and as this is very dangerous, the cables are either buried or carried high up on pylons. At the local distribution end, sub-stations are used as places where this high voltage is stepped back down to domestic values (240V).

The convention in electricity when we are talking about ac voltages and currents, is not to talk about their peak (maximum) values, but something known as their root mean square (rms) value. The reasons for this are beyond the scope of this booklet, but it is basically to do with the fact that the mean value of any sinusoid is zero, and when talking about power, which depends on V^2 or I^2, the mean value of that turns out to be $\frac{1}{\sqrt{2}}$ of the peak value, and this is what we use in any calculations of power. We should look at how we describe ac waveforms, as highlighted in Fig. 26.

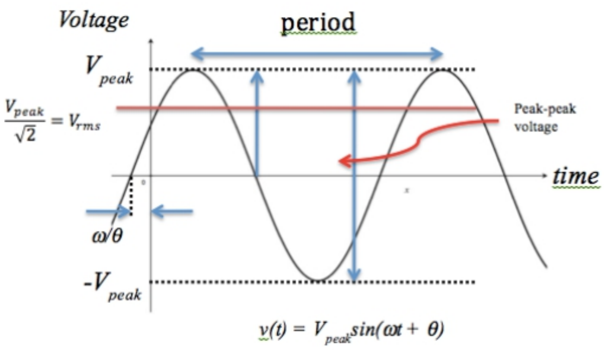


Figure 26. A plot of an ac waveform, of the type used to describe the voltage from the mains electricity. The frequency is 1/period.

To describe the mains, we have V_{rms} = 240V, which means the peak voltage is $240 \times \sqrt{2}$ V = 339.4 V. The frequency is 50Hz, and as the power is proportional to V^2 or I^2, which themselves will vary as $\sin^2$ (□ t), the actual rate at which power is dissipated is 100Hz, i.e. if you look at a lightbulb in slow motion, you will find that the brightness varies cyclically (sinusoidally), with 10ms between successive bright times. The Human eye cannot pick this up, so we just see the average brightness. If the mains was slowed down to 10Hz, we would notice a flicker.

So, we have presented electricity (mostly dc) in a way that we hope will be more accessible to you and easier to understand without being patronizing. The examples given are along the lines of past A-level Physics questions. This is not intended to be a stand-alone booklet, but is aimed at complementing what you already have. Should you have any suggestions for useful additions to the book, or questions you can't figure out, you can email me at cjdurkan@gmail.com .

Good luck!

Colm Durkan,

Cambridge,

July 2017