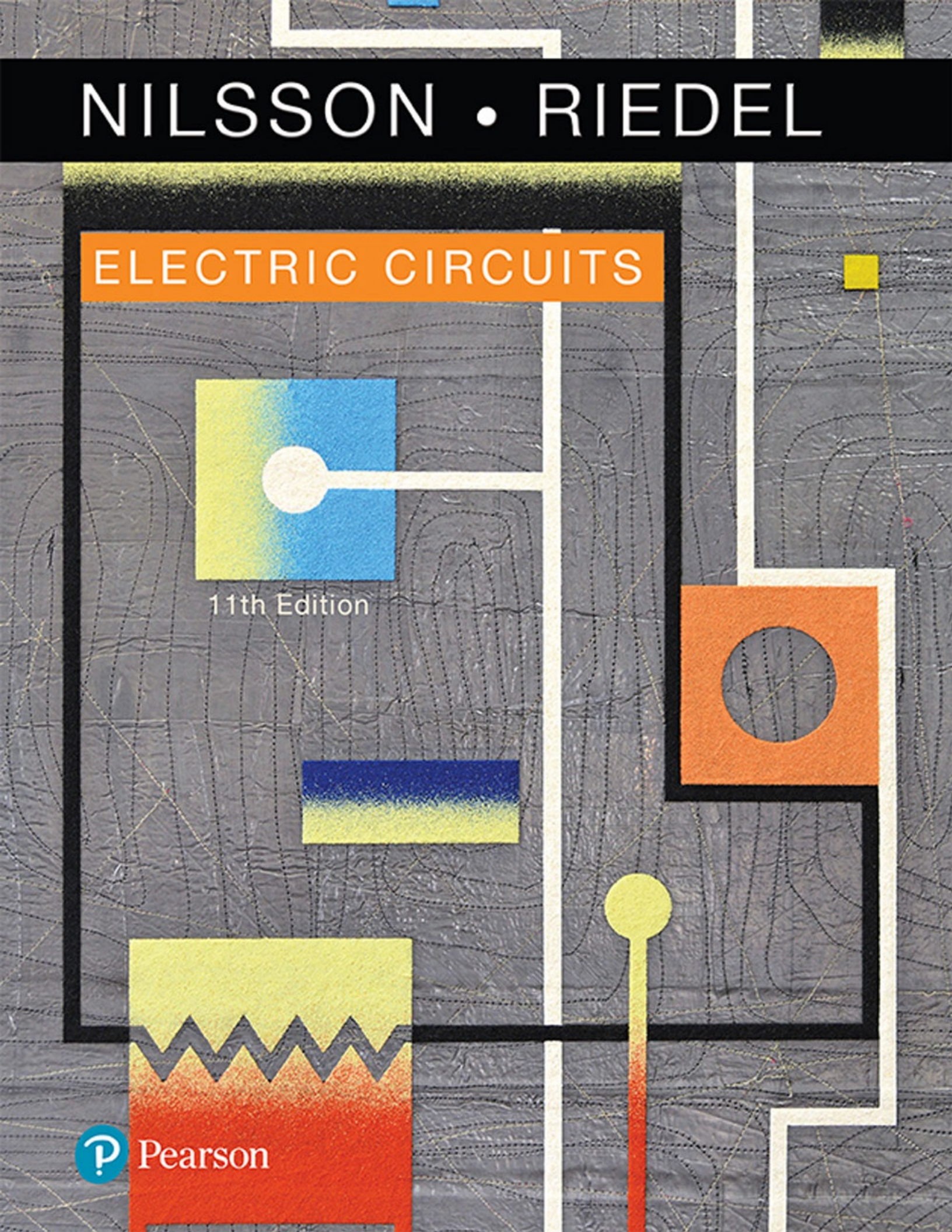
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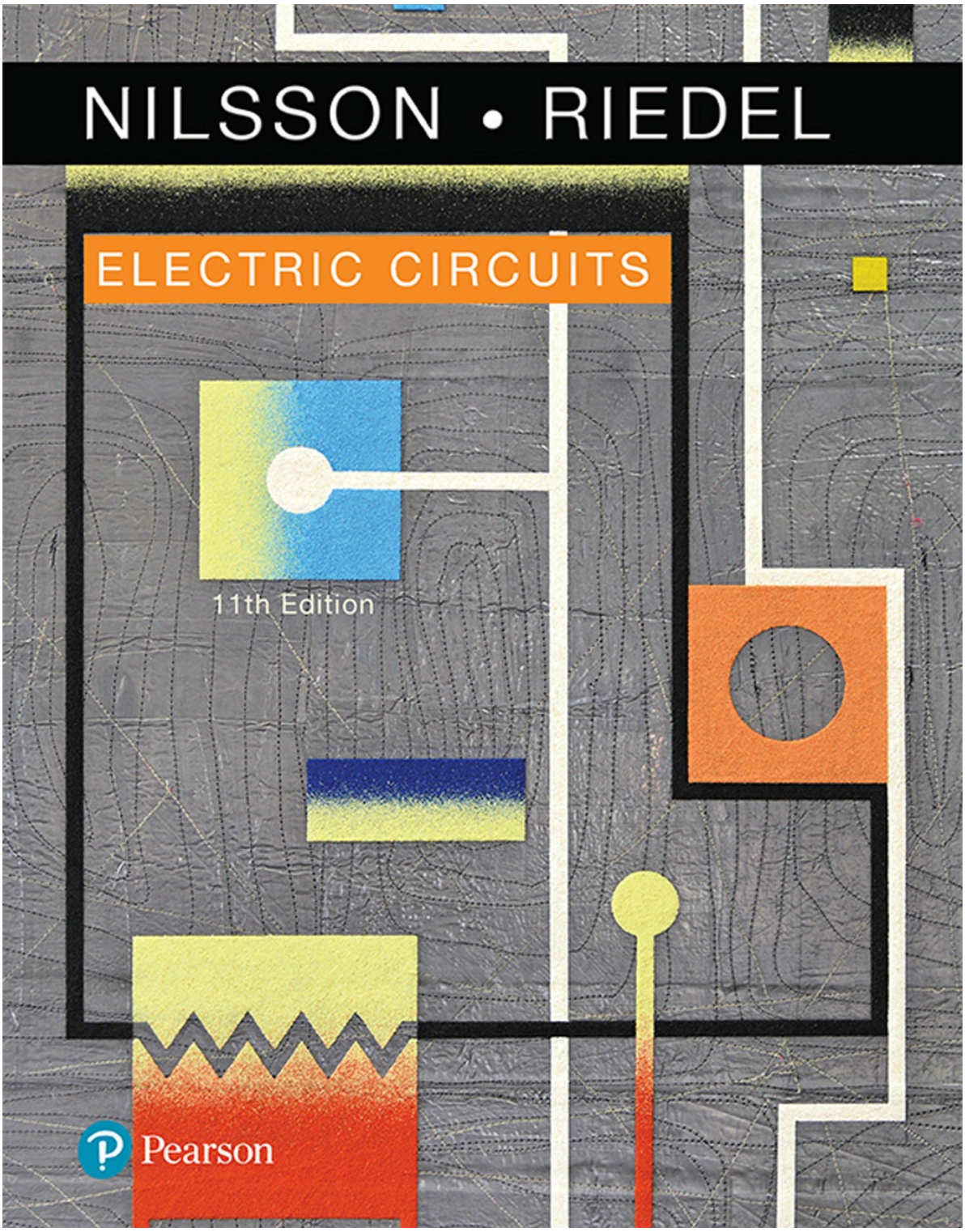
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Table 4.3 Steps in the Node-Voltage Method and the Mesh-Current Method

	Node-Voltage Method	Mesh-Current Method
Step 1 Identify nodes/meshes	Identify the essential nodes by circling them on the circuit diagram	Identify the meshes by drawing directed arrows inside each mesh
Step 2 Label node voltages/mesh currents Recognize special cases	Pick and label a reference node; then label the remaining essential node voltages • If a voltage source is the only component in a branch connecting the reference node and another essential node, label the essential node with the value of the voltage source • If a voltage source is the only component in a branch connecting two nonreference essential nodes, create a supernode that includes the voltage source and the two nodes on either side	Label each mesh current • If a current source is in a single mesh, label the mesh current with the value of the current source • If a current source is shared by two adjacent meshes, create a supermesh by combining the two adjacent meshes and temporarily eliminating the branch that contains the current source
Step 3 Write the equations	Write the following equations: • A KCL equation for any supernodes • A KCL equation for any remaining essential nodes where the voltage is unknown • A constraint equation for each dependent source that defines the controlling variable for the dependent source in terms of the node voltages • A constraint equation for each supernode that equates the difference between the two node voltages in the supernode to the voltage source in the supernode	Write the following equations: • A KVL equation for any supermeshes • A KVL equation for any remaining meshes where the current is unknown • A constraint equation for each dependent source that defines the controlling variable for the dependent source in terms of the mesh currents • A constraint equation for each supermesh that equates the difference between the two mesh currents in the supermesh to the current source eliminated to form the supermesh
Step 4 Solve the equations	Solve the equations to find the node voltages	Solve the equations to find the mesh currents
Step 5 Solve for other unknowns	Use the node voltage values to find any unknown voltages, currents, or powers	Use the mesh current values to find any unknown voltages, currents, or powers

Analyzing a Circuit With an Ideal

OP AMP

1. Check for a negative feedback path. If it exists, assume the op amp operates in its linear region.
2. Write a KCL equation at the inverting input terminal.
3. Solve the KCL equation and use the solution to find the op amp's output voltage.
4. Compare the op amp's output voltage to the power supply voltages to determine if the op amp is operating in its linear region or if it is saturated.

General Method for Natural and Step Response of RL and RC Circuits

1. **Identify the variable x(t),** which is the inductor current for RL circuits and capacitor voltage for RC circuits.
2. **Calculate the initial value X0**, by analyzing the circuit to find x(t) for $t<0$
3. **Calculate the time constant** τ; for RL circuits $\tau=L/R$ and for RC circuits $\tau=RC$, where R is the equivalent resistance connected to the inductor or capacitor for $t\geq0$
4. **Calculate the final value Xf**, by analyzing the circuit to find x(t) as $t\rightarrow\infty$; for the natural response, $Xf=0$
5. **Write the equation for x(t),** $x(t)=Xf+(X0-Xf)\ e^{-t/\tau}$, for $t\geq0$.

6. **Calculate other quantities of interest** using x(t).

Natural Response of a Parallel *RLC* Circuit

1. **Determine the initial capacitor voltage (V0) and inductor current (I0)** from the circuit.
2. **Determine the values of** α **and** ω0 using the equations in Table 8.1.
3. **If** α2>ω02, **the response is overdamped** and v(t)=A1es1t+A2es2t,t≥0
4. **If the response is overdamped, calculate s1 and s2** using the equations in Table 8.1.
5. **If the response is overdamped, calculate A1 and A2** by simultaneously solving Eqs. 8.10 and 8.11.
6. **Write the equation for v(t) from Step 3** using the results from Steps 4 and 5; find any desired branch currents.

Table 8.2 Equations for analyzing the natural response of parallel RLC circuits

Characteristic equation	$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$
Neper, resonant, and damped frequencies	$\alpha = \frac{1}{2RC} \quad \omega_0 = \sqrt{\frac{1}{LC}} \quad \omega_d = \sqrt{\omega_0^2 - \alpha^2}$
Roots of the characteristic equation	$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}, \quad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$
$\alpha^2 > \omega_0^2$: overdamped	$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}, \text{t} \geq 0$
	$v(0^+) = A_1 + A_2 = V_0$
	$\frac{dv(0^+)}{dt} = s_1 A_1 + s_2 A_2 = \frac{1}{C}\left(\frac{-V_0}{R} - I_0\right)$
$\alpha^2 < \omega_0^2$: underdamped	$v(t) = B_1 e^{-\alpha t}\cos \omega_d t + B_2 e^{-\alpha t}\sin \omega_d t, \text{t} \geq 0$
	$v(0^+) = B_1 = V_0$
	$\frac{dv(0^+)}{dt} = -\alpha B_1 + \omega_d B_2 = \frac{1}{C}\left(\frac{-V_0}{R} - I_0\right)$
$\alpha^2 = \omega_0^2$: critically damped	$v(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}, \text{t} \geq 0$
	$v(0^+) = D_2 = V_0$
	$\frac{dv(0^+)}{dt} = D_1 - \alpha D_2 = \frac{1}{C}\left(\frac{-V_0}{R} - I_0\right)$

(Note that the equations in the last three rows assume that the reference direction for the current in every component is in the direction of the reference voltage drop across that component.)

Step Response of a Parallel *RLC*

Circuit

1. **Determine the initial capacitor voltage (V0), the initial inductor current (I0), and the final inductor current (If)** from the circuit.

2. **Determine the values of** α **and** $\omega 0$ using the equations in Table 8.3.

3. **If** $\alpha 2 > \omega 02$, **the response is overdamped** and $iL(t) = If + A'1es1t + A'2es2t$, $t \geq 0+$;

   If $\alpha 2 > \omega 02$ **the response is underdamped** and $iL(t) = If + B'1e^{-\alpha t}\cos\omega dt + B'2e^{-\alpha t}\sin\omega dt$, $t \geq 0+$;

   If $\alpha 2 = \omega 02$, **the response is critically damped** and $iL(t) = If + D'1te^{-\alpha t} + D'2e^{-\alpha t}$, $t \geq 0+$

4. **If the response is overdamped, calculate s1 and s2** using the equations in Table 8.3;

   If the response is underdamped, calculate ωd using the equation in Table 8.3.

5. **If the response is overdamped, calculate** $A1'$ **and** $A2'$ by simultaneously solving the equations in Table 8.3;

   If the response is underdamped, calculate $B1'$ **and** $B2'$ by simultaneously solving the equations in Table 8.3;

   If the response is critically damped, calculate $D1'$ **and** $D2'$ by simultaneously solving the equations in Table 8.3.

6. **Write the equation for iL(t) from Step 3** using the results from Steps 4 and 5; find the inductor voltage and any desired branch currents.

Table 8.3 Equations for

analyzing the step response of parallel RLC circuits

Characteristic equation	$s^2 + \frac{1}{RC}s + \frac{1}{LC} = \frac{I}{LC}$
Neper, resonant, and damped frequencies	$\alpha = \frac{1}{2RC} \quad \omega_0 = \sqrt{\frac{1}{LC}} \quad \omega_d = \sqrt{\omega_0^2 - \alpha^2}$
Roots of the characteristic equation	$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}, \quad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$
$\alpha^2 > \omega_0^2$: overdamped	$i_L(t) = I_f + A_1' e^{s_1 t} + A_2' e^{s_2 t}, \quad t \geq 0$
	$i_L(0^+) = I_f + A_1' + A_2' = I_0$
	$\frac{di_L(0^+)}{dt} = s_1 A_1' + s_2 A_2' = \frac{V_0}{L}$
$\alpha^2 < \omega_0^2$: underdamped	$i_L(t) = I_f + B_1' e^{-\alpha t} \cos \omega_d t + B_2' e^{-\alpha t} \sin \omega_d t, \quad t \geq 0$
	$i_L(0^+) = I_f + B_1' = I_0$
	$\frac{di_L(0^+)}{dt} = -\alpha B_1' + \omega_d B_2' = \frac{V_0}{L}$
$\alpha^2 = \omega_0^2$: critically damped	$i_L(t) = I_f + D_1' t e^{-\alpha t} + D_2' e^{-\alpha t}, \quad t \geq 0$
	$i_L(0^+) = I_f + D_2' = I_0$
	$\frac{di_L(0^+)}{dt} = D_1' - \alpha D_2' = \frac{V_0}{L}$

(Note that the equations in the last three rows assume that the reference direction for the current in every component is in the direction of the reference voltage drop across that component.)

Electric Circuits

Eleventh Edition

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Courtesy of Anna Nilsson

In Memoriam

We remember our beloved author, James W. Nilsson, for his lasting legacy to the electrical and computer engineering field.

The first edition of *Electric Circuits* was published in 1983. As this book evolved over the years to better meet the needs of both students and their instructors, the underlying teaching methodologies Jim established remain relevant, even in the Eleventh Edition.

Jim earned his bachelor's degree at the University of Iowa (1948), and his master's degree (1952) and Ph.D. (1958) at Iowa State University. He joined the ISU faculty in 1948 and taught electrical engineering there for 39 years.

He became an IEEE fellow in 1990 and earned the prestigious IEEE Undergraduate Teaching Award in 1992.

For Anna

Brief Contents

1. List of Examples xii
2. List of Tables xvi
3. List of Analysis Methods xvii
4. Preface xx

1. Chapter 1 Circuit Variables 2
2. Chapter 2 Circuit Elements 26
3. Chapter 3 Simple Resistive Circuits 58
4. Chapter 4 Techniques of Circuit Analysis 92
5. Chapter 5 The Operational Amplifier 150
6. Chapter 6 Inductance, Capacitance, and Mutual Inductance 182
7. Chapter 7 Response of First-Order RL and RC Circuits 220
8. Chapter 8 Natural and Step Responses of RLC Circuits 272
9. Chapter 9 Sinusoidal Steady-State Analysis 318
10. Chapter 10 Sinusoidal Steady-State Power Calculations 374
11. Chapter 11 Balanced Three-Phase Circuits 412
12. Chapter 12 Introduction to the Laplace Transform 444
13. Chapter 13 The Laplace Transform in Circuit Analysis 482
14. Chapter 14 Introduction to Frequency Selective Circuits 536

15. Chapter 15 Active Filter Circuits 572
16. Chapter 16 Fourier Series 618
17. Chapter 17 The Fourier Transform 660
18. Chapter 18 Two-Port Circuits 692

1. Appendix A The Solution of Linear Simultaneous Equations 718
2. Appendix B Complex Numbers 727
3. Appendix C More on Magnetically Coupled Coils and Ideal Transformers 733
4. Appendix D The Decibel 741
5. Appendix E Bode Diagrams 743
6. Appendix F An Abbreviated Table of Trigonometric Identities 757
7. Appendix G An Abbreviated Table of Integrals 758
8. Appendix H Common Standard Component Values 760
9. Answers to Selected Problems 761
10. Index 771

Contents

1. List of Examples xii
2. List of Tables xvi
3. List of Analysis Methods xvii
4. Preface xx

1. Chapter 1 Circuit Variables 2
    1. Practical Perspective: Balancing Power 3
    1. 1.1 Electrical Engineering: An Overview 4
    2. 1.2 The International System of Units 9
    3. 1.3 Circuit Analysis: An Overview 11
    4. 1.4 Voltage and Current 12
    5. 1.5 The Ideal Basic Circuit Element 14
    6. 1.6 Power and Energy 15
    1. Practical Perspective: Balancing Power 18
    2. Summary 19
    3. Problems 20
2. Chapter 2 Circuit Elements 26
    1. Practical Perspective: Heating with Electric Radiators 27
    1. 2.1 Voltage and Current Sources 28

2. 2.2 Electrical Resistance (Ohm’s Law) 32
3. 2.3 Constructing a Circuit Model 36
4. 2.4 Kirchhoff’s Laws 39
5. 2.5 Analyzing a Circuit Containing Dependent Sources 45

1. Practical Perspective: Heating with Electric Radiators 48
2. Summary 50
3. Problems 50

3. Chapter 3 Simple Resistive Circuits 58

1. Practical Perspective: Resistive Touch Screens 59

1. 3.1 Resistors in Series 60
2. 3.2 Resistors in Parallel 61
3. 3.3 The Voltage-Divider and Current-Divider Circuits 64
4. 3.4 Voltage Division and Current Division 68
5. 3.5 Measuring Voltage and Current 70
6. 3.6 Measuring Resistance—The Wheatstone Bridge 73
7. 3.7 Delta-to-Wye (Pi-to-Tee) Equivalent Circuits 75

1. Practical Perspective: Resistive Touch Screens 78
2. Summary 79
3. Problems 80

4. Chapter 4 Techniques of Circuit Analysis 92

1. Practical Perspective: Circuits with Realistic Resistors 93

1. 4.1 Terminology 94
2. 4.2 Introduction to the Node-Voltage Method 96
3. 4.3 The Node-Voltage Method and Dependent Sources 98
4. 4.4 The Node-Voltage Method: Some Special Cases 100
5. 4.5 Introduction to the Mesh-Current Method 104
6. 4.6 The Mesh-Current Method and Dependent Sources 107
7. 4.7 The Mesh-Current Method: Some Special Cases 108
8. 4.8 The Node-Voltage Method Versus the Mesh-Current Method 112
9. 4.9 Source Transformations 115
10. 4.10 Thévenin and Norton Equivalents 118
11. 4.11 More on Deriving the Thévenin Equivalent 123
12. 4.12 Maximum Power Transfer 126
13. 4.13 Superposition 129

1. Practical Perspective: Circuits with Realistic Resistors 131
2. Summary 134
3. Problems 136

5. Chapter 5 The Operational Amplifier 150

1. Practical Perspective: Strain Gages 151

1. 5.1 Operational Amplifier Terminals 152

2. 5.2 Terminal Voltages and Currents 152
3. 5.3 The Inverting-Amplifier Circuit 156
4. 5.4 The Summing-Amplifier Circuit 158
5. 5.5 The Noninverting-Amplifier Circuit 160
6. 5.6 The Difference-Amplifier Circuit 162
7. 5.7 A More Realistic Model for the Operational Amplifier 167

1. Practical Perspective: Strain Gages 171
2. Summary 172
3. Problems 173

6. Chapter 6 Inductance, Capacitance, and Mutual Inductance 182

1. Practical Perspective: Capacitive Touch Screens 183

1. 6.1 The Inductor 184
2. 6.2 The Capacitor 189
3. 6.3 Series-Parallel Combinations of Inductance and Capacitance 194
4. 6.4 Mutual Inductance 199
5. 6.5 A Closer Look at Mutual Inductance 203

1. Practical Perspective: Capacitive Touch Screens 209
2. Summary 211
3. Problems 212

7. Chapter 7 Response of First-Order RL and RC Circuits 220

1. Practical Perspective: Artificial Pacemaker 221

1. 7.1 The Natural Response of an RL Circuit 222
2. 7.2 The Natural Response of an RC Circuit 228
3. 7.3 The Step Response of RL and RC Circuits 233
4. 7.4 A General Solution for Step and Natural Responses 241
5. 7.5 Sequential Switching 246
6. 7.6 Unbounded Response 250
7. 7.7 The Integrating Amplifier 252

1. Practical Perspective: Artificial Pacemaker 255
2. Summary 256
3. Problems 256

8. Chapter 8 Natural and Step Responses of RLC Circuits 272

1. Practical Perspective: Clock for Computer Timing 273

1. 8.1 Introduction to the Natural Response of a Parallel RLC Circuit 274
2. 8.2 The Forms of the Natural Response of a Parallel RLC Circuit 278
3. 8.3 The Step Response of a Parallel RLC Circuit 289
4. 8.4 The Natural and Step Response of a Series RLC Circuit 296
5. 8.5 A Circuit with Two Integrating Amplifiers 303

1. Practical Perspective: Clock for Computer Timing 308

2. Summary 309
3. Problems 310

9. Chapter 9 Sinusoidal Steady-State Analysis 318
    1. Practical Perspective: A Household Distribution Circuit 319
    1. 9.1 The Sinusoidal Source 320
    2. 9.2 The Sinusoidal Response 323
    3. 9.3 The Phasor 324
    4. 9.4 The Passive Circuit Elements in the Frequency Domain 327
    5. 9.5 Kirchhoff's Laws in the Frequency Domain 332
    6. 9.6 Series, Parallel, and Delta-to-Wye Simplifications 333
    7. 9.7 Source Transformations and Thévenin–Norton Equivalent Circuits 340
    8. 9.8 The Node-Voltage Method 344
    9. 9.9 The Mesh-Current Method 345
    10. 9.10 The Transformer 347
    11. 9.11 The Ideal Transformer 351
    12. 9.12 Phasor Diagrams 357
    1. Practical Perspective: A Household Distribution Circuit 359
    2. Summary 361
    3. Problems 362

10. Chapter 10 Sinusoidal Steady-State Power Calculations 374

1. Practical Perspective: Vampire Power 375

1. 10.1 Instantaneous Power 376
2. 10.2 Average and Reactive Power 377
3. 10.3 The rms Value and Power Calculations 382
4. 10.4 Complex Power 384
5. 10.5 Power Calculations 386
6. 10.6 Maximum Power Transfer 393

1. Practical Perspective: Vampire Power 399
2. Summary 401
3. Problems 401

11. Chapter 11 Balanced Three-Phase Circuits 412

    1. Practical Perspective: Transmission and Distribution of Electric Power 413

    1. 11.1 Balanced Three-Phase Voltages 414
    2. 11.2 Three-Phase Voltage Sources 415
    3. 11.3 Analysis of the Wye-Wye Circuit 416
    4. 11.4 Analysis of the Wye-Delta Circuit 422
    5. 11.5 Power Calculations in Balanced Three-Phase Circuits 425
    6. 11.6 Measuring Average Power in Three-Phase Circuits 430

    1. Practical Perspective: Transmission and Distribution of Electric Power 433

2. Summary 435
3. Problems 436

12. Chapter 12 Introduction to the Laplace Transform 444
    1. Practical Perspective: Transient Effects 445
    1. 12.1 Definition of the Laplace Transform 446
    2. 12.2 The Step Function 447
    3. 12.3 The Impulse Function 449
    4. 12.4 Functional Transforms 452
    5. 12.5 Operational Transforms 453
    6. 12.6 Applying the Laplace Transform 458
    7. 12.7 Inverse Transforms 460
    8. 12.8 Poles and Zeros of F(s) 470
    9. 12.9 Initial-and Final-Value Theorems 472
    1. Practical Perspective: Transient Effects 474
    2. Summary 476
    3. Problems 477
13. Chapter 13 The Laplace Transform in Circuit Analysis 482
    1. Practical Perspective: Surge Suppressors 483
    1. 13.1 Circuit Elements in the s Domain 484
    2. 13.2 Circuit Analysis in the s Domain 486

3. 13.3 Applications 488
4. 13.4 The Transfer Function 500
5. 13.5 The Transfer Function in Partial Fraction Expansions 502
6. 13.6 The Transfer Function and the Convolution Integral 505
7. 13.7 The Transfer Function and the Steady-State Sinusoidal Response 511
8. 13.8 The Impulse Function in Circuit Analysis 514

1. Practical Perspective: Surge Suppressors 520
2. Summary 521
3. Problems 522

14. Chapter 14 Introduction to Frequency Selective Circuits 536

1. Practical Perspective: Pushbutton Telephone Circuits 537

1. 14.1 Some Preliminaries 538
2. 14.2 Low-Pass Filters 539
3. 14.3 High-Pass Filters 545
4. 14.4 Bandpass Filters 550
5. 14.5 Bandreject Filters 560

1. Practical Perspective: Pushbutton Telephone Circuits 564
2. Summary 564
3. Problems 565

15. Chapter 15 Active Filter Circuits 572

1. Practical Perspective: Bass Volume Control 573

1. 15.1 First-Order Low-Pass and High-Pass Filters 574
2. 15.2 Scaling 577
3. 15.3 Op Amp Bandpass and Bandreject Filters 580
4. 15.4 Higher-Order Op Amp Filters 587
5. 15.5 Narrowband Bandpass and Bandreject Filters 600

1. Practical Perspective: Bass Volume Control 605
2. Summary 608
3. Problems 609

16. Chapter 16 Fourier Series 618

1. Practical Perspective: Active High-Q Filters 619

1. 16.1 Fourier Series Analysis: An Overview 621
2. 16.2 The Fourier Coefficients 622
3. 16.3 The Effect of Symmetry on the Fourier Coefficients 625
4. 16.4 An Alternative Trigonometric Form of the Fourier Series 631
5. 16.5 An Application 633
6. 16.6 Average-Power Calculations with Periodic Functions 639
7. 16.7 The rms Value of a Periodic Function 641
8. 16.8 The Exponential Form of the Fourier Series 642
9. 16.9 Amplitude and Phase Spectra 645

1. Practical Perspective: Active High-Q Filters 647
2. Summary 649
3. Problems 650

17. Chapter 17 The Fourier Transform 660
    1. Practical Perspective: Filtering Digital Signals 661
    1. 17.1 The Derivation of the Fourier Transform 662
    2. 17.2 The Convergence of the Fourier Integral 664
    3. 17.3 Using Laplace Transforms to Find Fourier Transforms 666
    4. 17.4 Fourier Transforms in the Limit 668
    5. 17.5 Some Mathematical Properties 671
    6. 17.6 Operational Transforms 672
    7. 17.7 Circuit Applications 677
    8. 17.8 Parseval's Theorem 679
    1. Practical Perspective: Filtering Digital Signals 685
    2. Summary 686
    3. Problems 686
18. Chapter 18 Two-Port Circuits 692
    1. Practical Perspective: Characterizing an Unknown Circuit 693
    1. 18.1 The Terminal Equations 694
    2. 18.2 The Two-Port Parameters 695

3. 18.3 Analysis of the Terminated Two-Port Circuit 703
4. 18.4 Interconnected Two-Port Circuits 708
1. Practical Perspective: Characterizing an Unknown Circuit 711
2. Summary 712
3. Problems 713

1. Appendix A The Solution of Linear Simultaneous Equations 718
    1. A.1 Preliminary Steps 718
    2. A.2 Calculator and Computer Methods 719
    3. A.3 Paper-and-Pencil Methods 721
    4. A.4 Applications 723
2. Appendix B Complex Numbers 727
    1. B.1 Notation 727
    2. B.2 The Graphical Representation of a Complex Number 728
    3. B.3 Arithmetic Operations 729
    4. B.4 Useful Identities 730
    5. B.5 The Integer Power of a Complex Number 731
    6. B.6 The Roots of a Complex Number 731
3. Appendix C More on Magnetically Coupled Coils and Ideal Transformers 733
    1. C.1 Equivalent Circuits for Magnetically Coupled Coils 733
    2. C.2 The Need for Ideal Transformers in the Equivalent Circuits 737

4. Appendix D The Decibel 741
5. Appendix E Bode Diagrams 743
    1. E.1 Real, First-Order Poles and Zeros 743
    2. E.2 Straight-Line Amplitude Plots 744
    3. E.3 More Accurate Amplitude Plots 747
    4. E.4 Straight-Line Phase Angle Plots 748
    5. E.5 Bode Diagrams: Complex Poles and Zeros 750
    6. E.6 Straight-Line Amplitude Plots for Complex Poles 751
    7. E.7 Correcting Straight-Line Amplitude Plots for Complex Poles 752
    8. E.8 Phase Angle Plots for Complex Poles 754
6. Appendix F An Abbreviated Table of Trigonometric Identities 757
7. Appendix G An Abbreviated Table of Integrals 758
8. Appendix H Common Standard Component Values 760
9. Answers to Selected Problems 761
10. Index 771

List of Examples

1. Chapter 1
    1. 1.1 Using SI Units and Prefixes for Powers of 10 11
    2. 1.2 Relating Current and Charge 15
    3. 1.3 Using the Passive Sign Convention 17
    4. 1.4 Relating Voltage, Current, Power, and Energy 17
2. Chapter 2
    1. 2.1 Testing Interconnections of Ideal Sources 30
    2. 2.2 Testing Interconnections of Ideal Independent and Dependent Sources 31
    3. 2.3 Calculating Voltage, Current, and Power for a Simple Resistive Circuit 34
    4. 2.4 Constructing a Circuit Model of a Flashlight 36
    5. 2.5 Constructing a Circuit Model Based on Terminal Measurements 38
    6. 2.6 Using Kirchhoff’s Current Law 41
    7. 2.7 Using Kirchhoff’s Voltage Law 42
    8. 2.8 Applying Ohm’s Law and Kirchhoff’s Laws to Find an Unknown Current 42
    9. 2.9 Constructing a Circuit Model Based on Terminal Measurements 43

10. 2.10 Analyzing a Circuit with a Dependent Source 45
11. 2.11 Applying Ohm’s Law and Kirchhoff’s Laws to Find an Unknown Voltage 46
12. 2.12 Applying Ohm’s Law and Kirchhoff’s Law in an Amplifier Circuit 47

3. Chapter 3
    1. 3.1 Applying Series-Parallel Simplification 62
    2. 3.2 Solving a Circuit Using Series-Parallel Simplification 63
    3. 3.3 Designing a Simple Voltage Divider 65
    4. 3.4 Adding a Resistive Load to a Voltage Divider 65
    5. 3.5 The Effect of Resistor Tolerance on the Voltage-Divider Circuit 66
    6. 3.6 Designing a Current-Divider Circuit 67
    7. 3.7 Using Voltage Division and Current Division to Solve a Circuit 69
    8. 3.8 Using a d’Arsonval Ammeter 71
    9. 3.9 Using a d’Arsonval Voltmeter 72
    10. 3.10 Using a Wheatstone Bridge to Measure Resistance 75
    11. 3.11 Applying a Delta-to-Wye Transform 77
4. Chapter 4
    1. 4.1 Identifying Node, Branch, Mesh, and Loop in a Circuit 94
    2. 4.2 Using Essential Nodes and Essential Branches to Write

Simultaneous Equations 95

3. 4.3 Using the Node-Voltage Method 97
4. 4.4 Using the Node-Voltage Method with Dependent Sources 99
5. 4.5 Node-Voltage Analysis of the Amplifier Circuit 102
6. 4.6 Using the Mesh-Current Method 106
7. 4.7 Using the Mesh-Current Method with Dependent Sources 107
8. 4.8 A Special Case in the Mesh-Current Method 108
9. 4.9 Mesh-Current Analysis of the Amplifier Circuit 111
10. 4.10 Understanding the Node-Voltage Method Versus Mesh-Current Method 113
11. 4.11 Comparing the Node-Voltage and Mesh-Current Methods 114
12. 4.12 Using Source Transformations to Solve a Circuit 116
13. 4.13 Using Special Source Transformation Techniques 117
14. 4.14 Finding a Thévenin Equivalent 120
15. 4.15 Finding a Norton Equivalent 121
16. 4.16 Finding the Thévenin Equivalent of a Circuit with a Dependent Source 122
17. 4.17 Finding the Thévenin Equivalent Resistance Directly from the Circuit 123
18. 4.18 Finding the Thévenin Equivalent Resistance Using a Test Source 124
19. 4.19 Finding the Thévenin Equivalent of a Circuit with Dependent

Sources and Resistors 124

20. 4.20 Using a Thévenin Equivalent to Analyze the Amplifier Circuit 125
21. 4.21 Calculating the Condition for Maximum Power Transfer 127
22. 4.22 Using Superposition to Solve a Circuit 129
23. 4.23 Using Superposition to Solve a Circuit with Dependent Sources 130

5. Chapter 5
    1. 5.1 Analyzing an Op Amp Circuit 155
    2. 5.2 Designing an Inverting Amplifier 157
    3. 5.3 Designing a Summing Amplifier 159
    4. 5.4 Designing a Noninverting Amplifier 161
    5. 5.5 Designing a Difference Amplifier 163
    6. 5.6 Calculating the CMRR 167
    7. 5.7 Analyzing a Noninverting-Amplifier Circuit using a Realistic Op Amp Model 169
6. Chapter 6
    1. 6.1 Determining the Voltage, Given the Current, at the Terminals of an Inductor 184
    2. 6.2 Determining the Current, Given the Voltage, at the Terminals of an Inductor 186
    3. 6.3 Determining the Current, Voltage, Power, and Energy for an Inductor 187

4. 6.4 Determining Current, Voltage, Power, and Energy for a Capacitor 191

5. 6.5 Finding v, p, and w Induced by a Triangular Current Pulse for a Capacitor 192

6. 6.6 Finding the Equivalent Inductance 196

7. 6.7 Finding the Equivalent Capacitance 197

8. 6.8 Finding Mesh-Current Equations for a Circuit with Magnetically Coupled Coils 201

9. 6.9 Calculating the Coupling Coefficient and Stored Energy for Magnetically Coupled Coils 209

7. Chapter 7

    1. 7.1 Determining the Natural Response of an RL Circuit 224

    2. 7.2 Determining the Natural Response of an RL Circuit with Parallel Inductors 227

    3. 7.3 Determining the Natural Response of an RC Circuit 230

    4. 7.4 Determining the Natural Response of an RC Circuit with Series Capacitors 231

    5. 7.5 Determining the Step Response of an RL Circuit 234

    6. 7.6 Determining the Step Response of an RC Circuit 239

    7. 7.7 Using the General Solution Method to Find an RL Circuit's Natural Response 242

    8. 7.8 Using the General Solution Method to Find an RC Circuit's Step Response 243

    9. 7.9 Using the General Solution Method to Find an RL Circuit's

Step Response 244

10. 7.10 Determining the Step Response of a Circuit with Magnetically Coupled Coils 245
11. 7.11 Analyzing an RL Circuit that has Sequential Switching 247
12. 7.12 Analyzing an RC Circuit that has Sequential Switching 249
13. 7.13 Finding the Unbounded Response in an RC Circuit 251
14. 7.14 Analyzing an Integrating Amplifier 253
15. 7.15 Analyzing an Integrating Amplifier that has Sequential Switching 253

8. Chapter 8

    1. 8.1 Finding the Roots of the Characteristic Equation of a Parallel RLC Circuit 277
    2. 8.2 Finding the Overdamped Natural Response of a Parallel RLC Circuit 280
    3. 8.3 Calculating Branch Currents in the Natural Response of a Parallel RLC Circuit 281
    4. 8.4 Finding the Underdamped Natural Response of a Parallel RLC Circuit 284
    5. 8.5 Finding the Critically Damped Natural Response of a Parallel RLC Circuit 288
    6. 8.6 Finding the Overdamped Step Response of a Parallel RLC Circuit 293
    7. 8.7 Finding the Underdamped Step Response of a Parallel RLC Circuit 294

8. 8.8 Finding the Critically Damped Step Response of a Parallel RLC Circuit 294
9. 8.9 Comparing the Three-Step Response Forms 295
10. 8.10 Finding Step Response of a Parallel RLC Circuit with Initial Stored Energy 295
11. 8.11 Finding the Natural Response of a Series RLC Circuit 302
12. 8.12 Finding the Step Response of a Series RLC Circuit 302
13. 8.13 Analyzing Two Cascaded Integrating Amplifiers 305
14. 8.14 Analyzing Two Cascaded Integrating Amplifiers with Feedback Resistors 307

9. Chapter 9
    1. 9.1 Finding the Characteristics of a Sinusoidal Current 321
    2. 9.2 Finding the Characteristics of a Sinusoidal Voltage 322
    3. 9.3 Translating a Sine Expression to a Cosine Expression 322
    4. 9.4 Calculating the rms Value of a Triangular Waveform 322
    5. 9.5 Adding Cosines Using Phasors 326
    6. 9.6 Calculating Component Voltages Using Phasor Techniques 331
    7. 9.7 Using KVL in the Frequency Domain 333
    8. 9.8 Combining Impedances in Series 334
    9. 9.9 Combining Impedances in Series and in Parallel 337
    10. 9.10 Using a Delta-to-Wye Transform in the Frequency Domain 339

11. 9.11 Performing Source Transformations in the Frequency Domain 341
12. 9.12 Finding a Thévenin Equivalent in the Frequency Domain 342
13. 9.13 Using the Node-Voltage Method in the Frequency Domain 344
14. 9.14 Using the Mesh-Current Method in the Frequency Domain 346
15. 9.15 Analyzing a Linear Transformer in the Frequency Domain 349
16. 9.16 Analyzing an Ideal Transformer Circuit in the Frequency Domain 355
17. 9.17 Using Phasor Diagrams to Analyze a Circuit 357
18. 9.18 Using Phasor Diagrams to Analyze Capacitive Loading Effects 358

10. Chapter 10
    1. 10.1 Calculating Average and Reactive Power 380
    2. 10.2 Making Power Calculations Involving Household Appliances 382
    3. 10.3 Determining Average Power Delivered to a Resistor by a Sinusoidal Voltage 384
    4. 10.4 Calculating Complex Power 385
    5. 10.5 Calculating Power Using Phasor Voltage and Current 387
    6. 10.6 Calculating Average and Reactive Power 389
    7. 10.7 Calculating Power in Parallel Loads 390

8. 10.8 Balancing Power Delivered with Power Absorbed in an AC Circuit 391
9. 10.9 Determining Maximum Power Transfer without Load Restrictions 395
10. 10.10 Determining Maximum Power Transfer with Load Impedance Restriction 396
11. 10.11 Finding Maximum Power Transfer with Impedance Angle Restrictions 396
12. 10.12 Finding Maximum Power Transfer in a Circuit with an Ideal Transformer 397

11. Chapter 11
    1. 11.1 Analyzing a Wye-Wye Circuit 420
    2. 11.2 Analyzing a Wye-Delta Circuit 423
    3. 11.3 Calculating Power in a Three-Phase Wye-Wye Circuit 428
    4. 11.4 Calculating Power in a Three-Phase Wye-Delta Circuit 428
    5. 11.5 Calculating Three-Phase Power with an Unspecified Load 429
    6. 11.6 Computing Wattmeter Readings in Three-Phase Circuits 432
12. Chapter 12
    1. 12.1 Using Step Functions to Represent a Function of Finite Duration 448
    2. 12.2 Using Laplace Transforms to Predict a Circuit’s Response 460
    3. 12.3 Finding the Inverse Laplace Transform when F(s) has Distinct Real Roots 462

4. 12.4 Finding the Inverse Laplace Transform when F(s) has Distinct Complex Roots 465
5. 12.5 Finding the Inverse Laplace Transform when F(s) has Repeated Real Roots 467
6. 12.6 Finding the Inverse Laplace Transform when F(s) has Repeated Complex Roots 468
7. 12.7 Finding the Inverse Laplace Transform of an Improper Rational Function 470
8. 12.8 Finding and Plotting the Poles and Zeros of an s-Domain Function 471
9. 12.9 Applying the Initial-and Final-Value Theorems 474

13. Chapter 13

1. 13.1 Transforming a Circuit into the s Domain 488
2. 13.2 The Natural Response of an RC Circuit 489
3. 13.3 The Step Response of an RLC Circuit 489
4. 13.4 Analyzing a Circuit with a Sinusoidal Source 491
5. 13.5 Analyzing a Circuit with Multiple Meshes 493
6. 13.6 Creating a Thévenin Equivalent in the s Domain 495
7. 13.7 Analyzing a Circuit with Mutual Inductance 497
8. 13.8 Applying Superposition in the s Domain 499
9. 13.9 Deriving the Transfer Function of a Circuit 501
10. 13.10 Analyzing the Transfer Function of a Circuit 503

11. 13.11 Using the Convolution Integral to Find an Output Signal 509
12. 13.12 Using the Transfer Function to Find the Steady-State Sinusoidal Response 513
13. 13.13 A Series Inductor Circuit with an Impulsive Response 515
14. 13.14 A Circuit with Both Internally Generated and Externally Applied Impulses 518

14. Chapter 14
    1. 14.1 Designing a Low-Pass Filter 543
    2. 14.2 Designing a Series RC Low-Pass Filter 544
    3. 14.3 Designing a Series RL High-Pass Filter 547
    4. 14.4 Loading the Series RL High-Pass Filter 548
    5. 14.5 Designing a Bandpass Filter 555
    6. 14.6 Designing a Parallel RLC Bandpass Filter 555
    7. 14.7 Determining Effect of a Nonideal Voltage Source on a RLC Bandpass Filter 557
    8. 14.8 Designing a Series RLC Bandreject Filter 562

15. Chapter 15
    1. 15.1 Designing a Low-Pass Op Amp Filter 575
    2. 15.2 Designing a High-Pass Op Amp Filter 576
    3. 15.3 Scaling a Series RLC Filter 578
    4. 15.4 Scaling a Prototype Low-Pass Op Amp Filter 579

5. 15.5 Designing a Broadband Bandpass Op Amp Filter 583
6. 15.6 Designing a Broadband Bandreject Op Amp Filter 586
7. 15.7 Designing a Fourth-Order Low-Pass Active Filter 589
8. 15.8 Calculating Butterworth Transfer Functions 592
9. 15.9 Designing a Fourth-Order Low-Pass Butterworth Filter 594
10. 15.10 Determining the Order of a Butterworth Filter 597
11. 15.11 An Alternate Approach to Determining the Order of a Butterworth Filter 597
12. 15.12 Designing a Butterworth Bandpass Filter 599
13. 15.13 Designing a High-Q Bandpass Filter 602
14. 15.14 Designing a High-Q Bandreject Filter 604

16. Chapter 16

    1. 16.1 Finding the Fourier Series of a Triangular Waveform 623
    2. 16.2 Finding the Fourier Series of a Periodic Function with Symmetry 630
    3. 16.3 Calculating Forms of the Trigonometric Fourier Series for Periodic Voltage 632
    4. 16.4 Finding the Response of an RLC Circuit to a Square-Wave Voltage 637
    5. 16.5 Calculating Average Power for a Circuit with a Periodic Voltage Source 640
    6. 16.6 Estimating the rms Value of a Periodic Function 642

7. 16.7 Finding the Exponential Form of the Fourier Series 644
8. 16.8 Plotting the Amplitude and Phase Spectra for a Periodic Voltage 646

17. Chapter 17
    1. 17.1 Finding the Fourier Transform of a Constant 665
    2. 17.2 Finding the Fourier Transform from the Laplace Transform 667
    3. 17.3 Deriving an Operational Fourier Transform 675
    4. 17.4 Using the Fourier Transform to Find the Transient Response 677
    5. 17.5 Using the Fourier Transform to Find the Sinusoidal Steady-State Response 678
    6. 17.6 Applying Parseval's Theorem 681
    7. 17.7 Applying Parseval's Theorem to an Ideal Bandpass Filter 682
    8. 17.8 Applying Parseval's Theorem to a Low-Pass Filter 683
    9. 17.9 Calculating Energy Contained in a Rectangular Voltage Pulse 684
18. Chapter 18
    1. 18.1 Finding the z Parameters of a Two-Port Circuit 696
    2. 18.2 Finding the a Parameters from Measurements 697
    3. 18.3 Finding h Parameters from Measurements and Table 18.1 700
    4. 18.4 Determining Whether a Circuit Is Reciprocal and Symmetric 701

5. 18.5 Analyzing a Terminated Two-Port Circuit 707
6. 18.6 Analyzing Cascaded Two-Port Circuits 710

List of Tables

1. 1.1 The International System of Units (SI) 10
2. 1.2 Derived Units in SI 10
3. 1.3 Standardized Prefixes to Signify Powers of 10 10
4. 1.4 Interpretation of Reference Directions in Fig. 1.5 14
5. 1.5 Voltage and Current Values for the Circuit in Fig. 1.7 19
6. 4.1 Terms for Describing Circuits 95
7. 4.2 PSpice Sensitivity Analysis Results 133
8. 4.3 Steps in the Node-Voltage Method and the Mesh-Current Method 135
9. 6.1 Inductor and Capacitor Duality 198
10. 7.1 Value of $e^{-t/\tau}$ For t Equal to Integral Multiples of τ 226
11. 8.1 Natural-Response Parameters of the Parallel RLC Circuit 276
12. 8.2 Equations for Analyzing the Natural Response of Parallel RLC Circuits 288
13. 8.3 Equations for Analyzing the Step Response of Parallel RLC Circuits 293
14. 8.4 Equations for Analyzing the Natural Response of Series RLC Circuits 299
15. 8.5 Equations for Analyzing the Step Response of Series RLC Circuits 301

16. 9.1 Impedance and Reactance Values 331
17. 9.2 Admittance and Susceptance Values 336
18. 9.3 Impedance and Related Values 361
19. 10.1 Annual Energy Requirements of Electric Household Appliances 381
20. 10.2 Three Power Quantities and Their Units 385
21. 10.3 Average Power Consumption of Common Electrical Devices 399
22. 12.1 An Abbreviated List of Laplace Transform Pairs 453
23. 12.2 An Abbreviated List of Operational Transforms 458
24. 12.3 Four Useful Transform Pairs 469
25. 13.1 Summary of the s-Domain Equivalent Circuits 486
26. 14.1 Input and Output Voltage Magnitudes for Several Frequencies 543
27. 15.1 Normalized (so that ωc=1 rad/s) Butterworth Polynomials up to the Eighth Order 593
28. 17.1 Fourier Transforms of Elementary Functions 670
29. 17.2 Operational Transforms 675
30. 18.1 Parameter Conversion Table 699
31. 18.2 Two-Port Parameter Relationships for Reciprocal Circuits 701
32. 18.3 Terminated Two-Port Equations 704

List of Analysis Methods

1. Analysis Method 4.1: The Basic Version of the Node-Voltage Method 97
2. Analysis Method 4.2: Modified Step 3 for the Node-Voltage Method 99
3. Analysis Method 4.3: Complete Form of the Node-Voltage Method 102
4. Analysis Method 4.4: The Basic Version of the Mesh-Current Method 105
5. Analysis Method 4.5: Modified Step 3 for the Mesh-Current Method 107
6. Analysis Method 4.6: Complete Form of the Mesh-Current Method 110
7. Analysis Method 5.1: Analyzing an Ideal Op Amp Circuit with a Negative Feedback Path 154
8. Analysis Method 7.1: Finding the RL Natural Response 224
9. Analysis Method 7.2: Finding the RC Natural Response 230
10. Analysis Method 7.3: Finding the RL Step Response 234
11. Analysis Method 7.4: Finding the RC Step Response 238
12. Analysis Method 7.5: Finding the RL and RC Natural and Step Response 242
13. Analysis Method 8.1: The Natural Response of an Overdamped Parallel Rlc Circuit 280
14. Analysis Method 8.2: The Natural Response of an Overdamped or Underdamped Parallel RLC Circuit 283

15. Analysis Method 8.3: The Natural Response of Parallel RLC Circuits 287

16. Analysis Method 8.4: The Step Response of Parallel RLC Circuits 292

17. Analysis Method 8.5: The Natural Response of Series RLC Circuits 299

18. Analysis Method 8.6: The Step Response of Series RLC Circuits 301

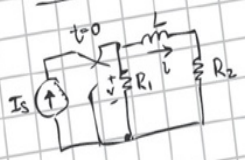
19. Analysis Method 13.1: Laplace-Transform Circuit Analysis Method 487

Combine this...

PART A

Given:
$I_s = 51.0\ mA$
$R_1 = 54.0\ k\Omega$
$R_2 = 51.0\ k\Omega$
$L = 51.0\ mH$

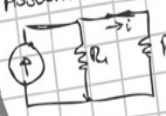
Find:
Initial current $i(0^-)$ before break switch



$I_s = i$

$\boxed{i = 51.0\ mA}$

Assume when circuit is in steady state, inductor acts as a short $\omega \to \infty$



Use Kirchhoff's current law

$I_s = i + i_R$

$I_s = i + \frac{V}{R}$

$V = iR$

$i = \frac{V}{R}$

$I_s = i + \frac{iR_1}{R_2}$

$I_s = i\left(1 + \frac{R_1}{R_2}\right)$

$\frac{I_s}{\left(1 + \frac{R_1}{R_2}\right)} = i$

$i = \frac{51.0\ mA}{\left(1 + \frac{54.1\ k\Omega}{51\ k\Omega}\right)} = \boxed{24.77\ mA}$

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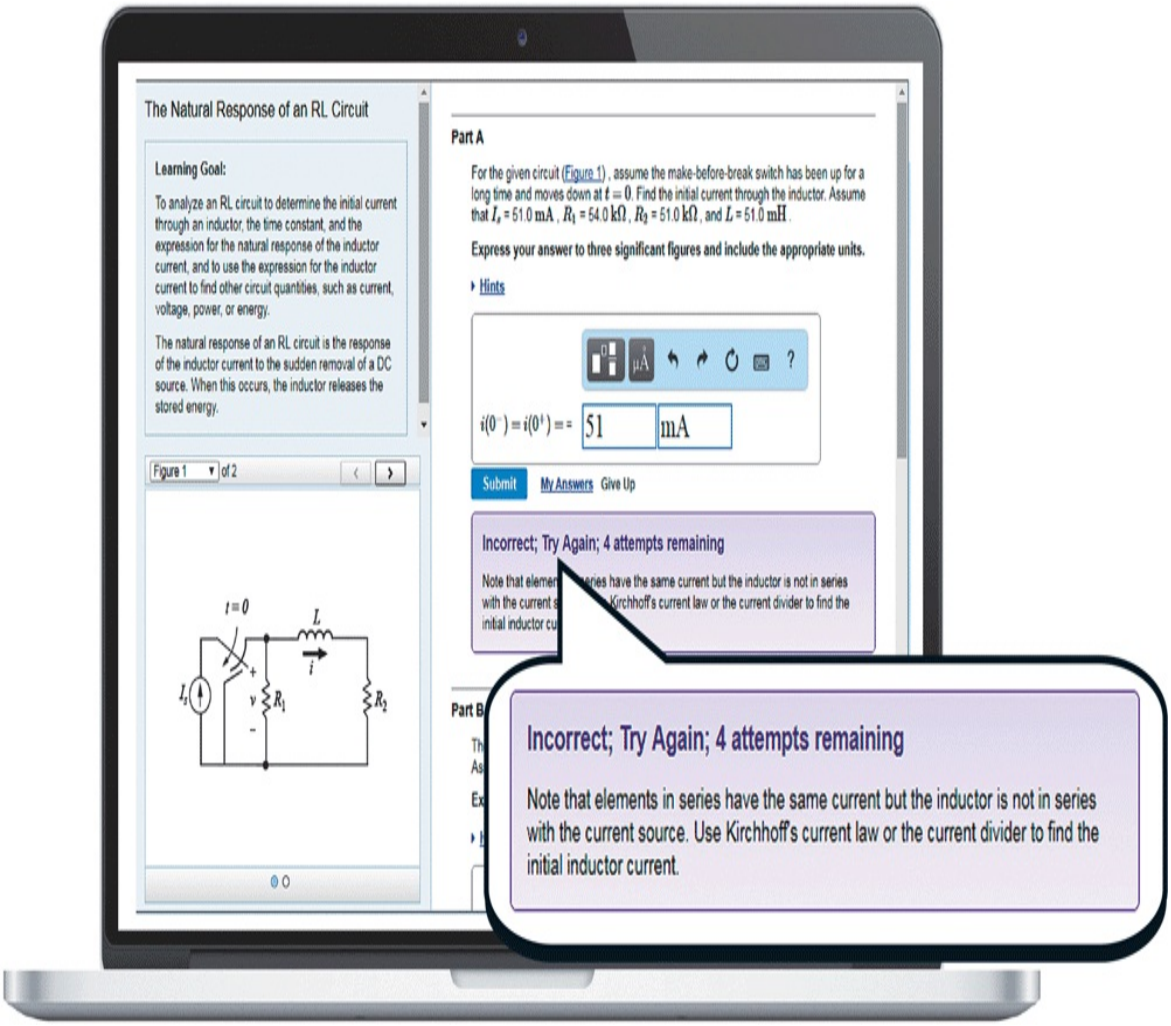

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Preface

The Eleventh Edition of *Electric Circuits* represents the most extensive revision to the text since the Fifth Edition, published in 1996. Every sentence, paragraph, subsection, and chapter has been examined to improve clarity, readability, and pedagogy. Yet the fundamental goals of the text are unchanged. These goals are:

- To build new concepts and ideas on concepts previously presented. This challenges students to see the explicit connections among the many circuit analysis tools and methods.

- To develop problem-solving skills that rely on a solid conceptual foundation. This challenges students to examine many different approaches to solving a problem before writing a single equation.

- To introduce realistic engineering experiences at every opportunity. This challenges students to develop the insights of a practicing engineer and exposes them to practice of engineering.

Why This Edition?

The Eleventh Edition of *Electric Circuits* incorporates the following new and revised elements:

- Analysis Methods – This new feature identifies the steps needed to apply a particular circuit analysis technique. Many students struggle just to get started when analyzing a circuit, and the analysis methods will reduce that struggle. Some of the analysis methods that are used most often can be found inside the book’s covers for easy reference.

- Examples – Many students rely on examples when developing and refining their problem-solving skills. We identified many places in the text that needed additional examples, and as a result the number of

examples has increased by nearly 35% to 200.

- End-of-chapter problems – Problem solving is fundamental to the study of circuit analysis. Having a wide variety of problems to assign and work is a key to success in any circuits course. Therefore, some existing end-of-chapter problems were revised, and some new end-of-chapter problems were added. Approximately 30% of the problems in the Eleventh Edition were rewritten.

- Fundamental equations and concepts – These important elements in the text were previously identified with margin notes. In this edition, the margin notes have been replaced by a second-color background, enlarged fonts, and a descriptive title for each fundamental equation and concept. In additional, many equation numbers have been eliminated to make it easier to distinguish fundamental equations from the many other equations in the text.

- Circuit simulation software – The PSpice® and Multisim® manuals have been revised to include screenshots from the most recent versions of these software simulation applications. Each manual presents the simulation material in the same order as the material is encountered in the text. These manuals include example simulations of circuits from the text. Icons identify end-of-chapter problems that are good candidates for simulation using either PSpice or Multisim.

- Solving simultaneous equations – Most circuit analysis techniques in this text eventually require you to solve two or more simultaneous linear algebraic equations. Appendix A has been extensively revised and includes examples of paper-and-pencil techniques, calculator techniques, and computer software techniques.

- Student workbook – Students who could benefit from additional examples and practice problems can use the Student Workbook, which has been revised for the Eleventh Edition of the text. This workbook has examples and problems covering the following material: balancing power, simple resistive circuits, node voltage method, mesh current method, Thévenin and Norton equivalents, op amp circuits, first-order circuits, second-order circuits, AC steady-state analysis, and Laplace

transform circuit analysis.

- The Student Workbook now includes access to Video Solutions, complete, step-by-step solution walkthroughs to representative homework problems.

- Learning Catalytics, a “bring your own device” student engagement, assessment, and classroom intelligence system is available with the Eleventh Edition. With Learning Catalytics you can:

    - Use open-ended questions to get into the minds of students to understand what they do or don’t know and adjust lectures accordingly.

    - Use a wide variety of question types to sketch a graph, annotate a circuit diagram, compose numeric or algebraic answers, and more.

    - Access rich analytics to understand student performance.

    - Use pre-built questions or add your own to make Learning Catalytics fit your course exactly.

- Pearson Mastering Engineering is an online tutorial and assessment program that provides students with personalized feedback and hints and instructors with diagnostics to track students’ progress. With the Eleventh Edition, Mastering Engineering will offer new enhanced end-of-chapter problems with hints and feedback, Coaching Activities, and Adaptive Follow-Up assignments. Visit www.masteringengineering.com for more information.

Hallmark Features

Analysis Methods

Students encountering circuit analysis for the first time can benefit from step-

by-step directions that lead them to a problem’s solution. We have compiled these directions in a collection of analysis methods, and revised many of the examples in the text to employ these analysis methods.

Chapter Problems

Users of *Electric Circuits* have consistently rated the Chapter Problems as one of the book's most attractive features. In the Eleventh Edition, there are 1185 end-of-chapter problems with approximately 30% that have been revised from the previous edition. Problems are organized at the end of each chapter by section.

Practical Perspectives

The Eleventh Edition continues using Practical Perspectives to introduce the chapter. They provide real-world circuit examples, taken from real-world devices. Every chapter begins by describing a practical application of the material that follows. After presenting that material, the chapter revisits the Practical Perspective, performing a quantitative circuit analysis using the newly introduced chapter material. A special icon identifies end-of-chapter problems directly related to the Practical Perspective application. These problems provide additional opportunities for solving real-world problems using the chapter material.

Assessment Problems

Each chapter begins with a set of chapter objectives. At key points in the chapter, you are asked to stop and assess your mastery of a particular objective by solving one or more assessment problems. The answers to all of the assessment problems are given at the conclusion of each problem, so you can check your work. If you are able to solve the assessment problems for a given objective, you have mastered that objective. If you need more practice, several end-of-chapter problems that relate to the objective are suggested at the conclusion of the assessment problems.

Examples

Every chapter includes many examples that illustrate the concepts presented in the text in the form of a numeric example. There are now nearly 200 examples in this text, an increase of about 35% when compared to the previous edition. The examples illustrate the application of a particular concept, often employ an Analysis Method, and exemplify good problem-solving skills.

Fundamental Equations and Concepts

Throughout the text, you will see fundamental equations and concepts set apart from the main text. This is done to help you focus on some of the key principles in electric circuits and to help you navigate through the important topics.

Integration of Computer Tools

Computer tools can assist students in the learning process by providing a visual representation of a circuit's behavior, validating a calculated solution, reducing the computational burden of more complex circuits, and iterating toward a desired solution using parameter variation. This computational support is often invaluable in the design process. The Eleventh Edition supports PSpice and Multisim, both popular computer tools for circuit simulation and analysis. Chapter problems suited for exploration with PSpice and Multisim are marked accordingly.

Design Emphasis

The Eleventh Edition continues to support the emphasis on the design of circuits in many ways. First, many of the Practical Perspective discussions focus on the design aspects of the circuits. The accompanying Chapter Problems continue the discussion of the design issues in these practical examples. Second, design-oriented Chapter Problems have been labeled explicitly, enabling students and instructors to identify those problems with a design focus. Third, the identification of problems suited to exploration with PSpice or Multisim suggests design opportunities using these software tools. Fourth, some problems in nearly every chapter focus on the use of realistic component values in achieving a desired circuit design. Once such a problem has been analyzed, the student can proceed to a laboratory to build and test the circuit, comparing the analysis with the measured performance of the actual circuit.

Accuracy

All text and problems in the Eleventh Edition have undergone our strict hallmark accuracy checking process, to ensure the most error-free book possible.

Resources For Students

Mastering Engineering. Mastering Engineering provides tutorial homework problems designed to emulate the instructor's office hour environment, guiding students through engineering concepts with self-paced individualized coaching. These in-depth tutorial homework problems provide students with feedback specific to their errors and optional hints that break problems down into simpler steps. Visit www.masteringengineering.com for more information.

Learning Catalytics. Learning Catalytics is an interactive student response tool that encourages team-based learning by using student's smartphones, tablets, or laptops to engage them in interactive tasks and thinking. Visit www.learningcatalytics.com for more information.

Student Workbook. This resource teaches students techniques for solving problems presented in the text. Organized by concepts, this is a valuable problem-solving resource for all levels of students. The Student Workbook now includes access to Video Solutions, complete, step-by-step solution walkthroughs to representative homework problems.

Introduction to Multisim and Introduction to PSpice Manuals—Updated for the Eleventh Edition, these manuals are excellent resources for those wishing to integrate PSpice or Multisim into their classes.

Resources for Instructors

All instructor resources are available for download at www.pearsonhighered.com. If you are in need of a login and password for this site, please contact your local Pearson representative.

Instructor Solutions Manual—Fully worked-out solutions to Assessment Problems and end-of-chapter problems.

PowerPoint lecture images—All figures from the text are available in PowerPoint for your lecture needs. An additional set of full lecture slides with embedded assessment questions are available upon request.

MasteringEngineering. This online tutorial and assessment program allows you to integrate dynamic homework with automated grading and personalized feedback. MasteringEngineering allows you to easily track the performance of your entire class on an assignment-by-assignment basis, or the detailed work of an individual student. For more information visit www.masteringengineering.com.

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Prerequisites

In writing the first 12 chapters of the text, we have assumed that the reader has taken a course in elementary differential and integral calculus. We have also assumed that the reader has had an introductory physics course, at either the high school or university level, that introduces the concepts of energy, power, electric charge, electric current, electric potential, and electromagnetic fields. In writing the final six chapters, we have assumed the student has had, or is enrolled in, an introductory course in differential equations.

Course Options

The text has been designed for use in a one-semester, two-semester, or a three-quarter sequence.

- Single-semester course: After covering Chapters 1–4 and Chapters 6–10 (omitting Sections 7.7 and 8.5) the instructor can develop the desired emphasis by covering Chapter 5 (operational amplifiers), Chapter 11 (three-phase circuits), Chapters 13 and 14 (Laplace methods), or Chapter 18 (Two-Port Circuits).

- Two-semester sequence: Assuming three lectures per week, cover the first nine chapters during the first semester, leaving Chapters 10–18 for the second semester.

- Academic quarter schedule: Cover Chapters 1–6 in the first quarter, Chapters 7–12 in the second quarter, and Chapters 13–18 in the third quarter.

Note that the introduction to operational amplifier circuits in Chapter 5 can be omitted with minimal effect on the remaining material. If Chapter 5 is omitted, you should also omit Section 7.7, Section 8.5, Chapter 15, and those assessment problems and end-of-chapter problems that pertain to operational

amplifiers.

There are several appendixes at the end of the book to help readers make effective use of their mathematical background. Appendix A presents several different methods for solving simultaneous linear equations; complex numbers are reviewed in Appendix B; Appendix C contains additional material on magnetically coupled coils and ideal transformers; Appendix D contains a brief discussion of the decibel; Appendix E is dedicated to Bode diagrams; Appendix F is devoted to an abbreviated table of trigonometric identities that are useful in circuit analysis; and an abbreviated table of useful integrals is given in Appendix G. Appendix H provides tables of common standard component values for resistors, inductors, and capacitors, to be used in solving many end-of-chapter problems. Selected Answers provides answers to selected end-of-chapter problems.

Acknowledgments

I will be forever grateful to Jim Nilsson for giving me the opportunity to collaborate with him on this textbook. I started by revising the PSpice supplement for the Third Edition, and became a co-author of the Fifth Edition. Jim was a patient and gracious mentor, and I learned so much from him about teaching and writing and hard work. It is a great honor to be associated with him through this textbook, and to impact the education of the thousands of students who use this text.

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I am very grateful for the many instructors and students who have done formal reviews of the text or offered positive feedback and suggestions for

improvement more informally. I am pleased to receive email from instructors and students who use the book, even when they are pointing out an error I failed to catch in the review process. I have been contacted by people who use our text from all over the world, and I thank all of you for taking the time to do so. I use as many of your suggestions as possible to continue to improve the content, the pedagogy, and the presentation in this text. I am privileged to have the opportunity to impact the educational experience of the many thousands of future engineers who will use this text.

Susan A. Riedel

Electric Circuits

Eleventh Edition

Chapter 1 Circuit Variables

Chapter Contents

1. 1.1 Electrical Engineering: An Overview
2. 1.2 The International System of Units
3. 1.3 Circuit Analysis: An Overview
4. 1.4 Voltage and Current
5. 1.5 The Ideal Basic Circuit Element
6. 1.6 Power and Energy

Chapter Objectives

1. Understand and be able to use SI units and the standard prefixes for powers of 10.

2. Know and be able to use the definitions of *voltage* and *current*.

3. Know and be able to use the definitions of *power* and *energy*.

4. Be able to use the passive sign convention to calculate the power for an ideal basic circuit element given its voltage and current.

Electrical engineering is an exciting and challenging profession for anyone who has a genuine interest in, and aptitude for, applied science and mathematics. Electrical engineers play a dominant role in developing systems that change the way people live and work. Satellite communication links, cell phones, computers, televisions, diagnostic and surgical medical equipment, robots, and aircraft represent systems that define a modern technological society. As an electrical engineer, you can participate in this ongoing technological revolution by improving and refining existing systems and by discovering and developing new systems to meet the needs of our ever-changing society.

This text introduces you to electrical engineering using the analysis and design of linear circuits. We begin by presenting an overview of electrical engineering, some ideas about an engineering point of view as it relates to circuit analysis, and a review of the International System of Units. We then describe generally what circuit analysis entails. Next, we introduce the concepts of voltage and current. We continue by discussing the ideal basic element and the need for a polarity reference system. We conclude the chapter by describing how current and voltage relate to power and energy.

Practical Perspective

Balancing Power

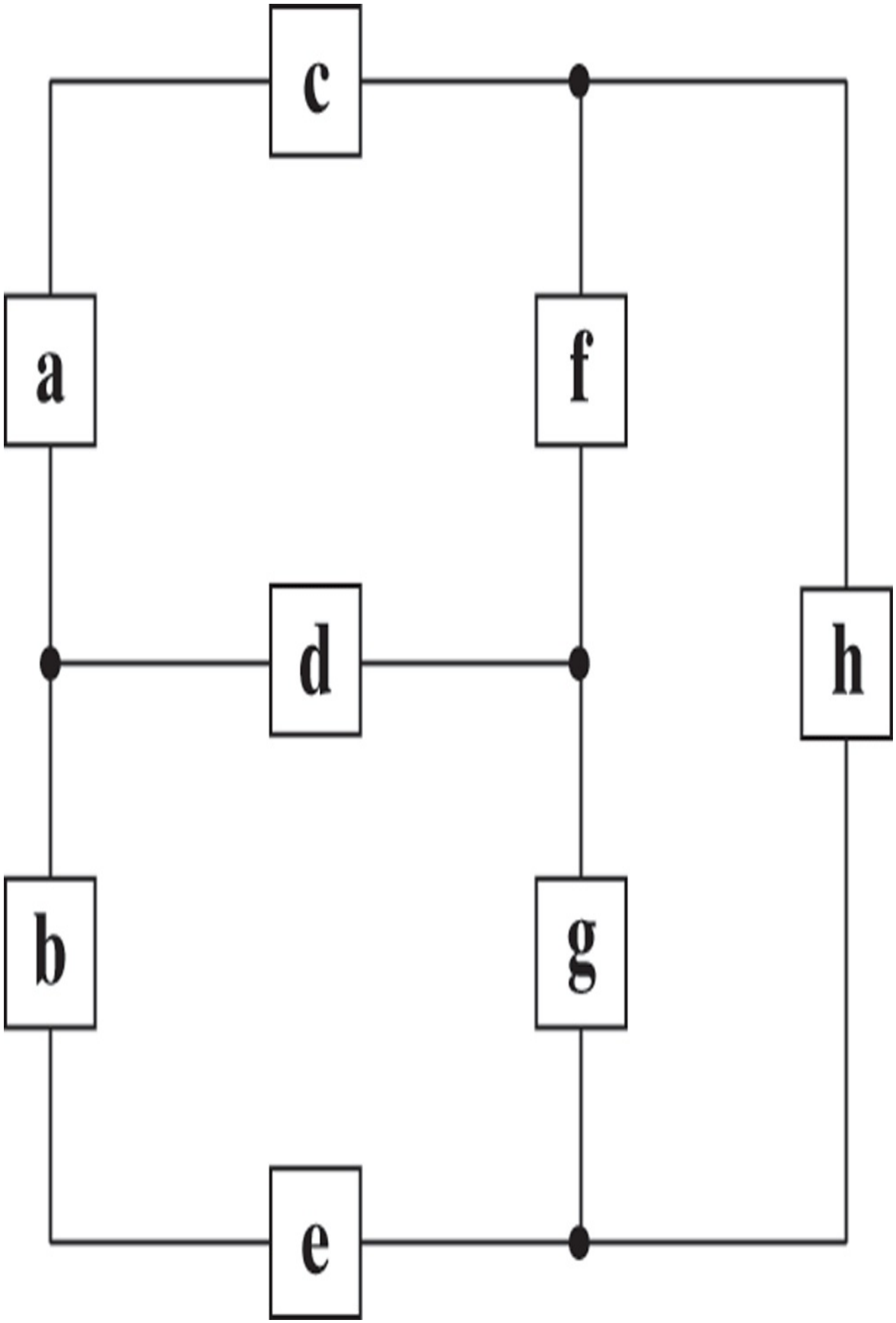
One of the most important skills you will develop is the ability to check your answers for the circuits you design and analyze using the tools developed in this text. A common method used to check for valid answers is to calculate the power in the circuit. The linear circuits we study have no net power, so the sum of the power associated with all circuit components must be zero. If the total power for the circuit is zero, we say that the power balances, but if the total power is not zero, we need to find the errors in our calculation.

As an example, we will consider a simple model for distributing electricity to a typical home. (Note that a more realistic model will be investigated in the Practical Perspective for Chapter 9.) The components labeled a and b represent the source of electrical power for the home. The components labeled c, d, and e represent the wires that carry the electrical current from the source to the devices in the home requiring electrical power. The components labeled f, g, and h represent lamps, televisions, hair dryers, refrigerators, and other devices that require power.

romakoma/Shutterstock

PhotoSerg/Shutterstock

Once we have introduced the concepts of voltage, current, power, and energy, we will examine this circuit model in detail, and use a power balance to determine whether the results of analyzing this circuit are correct.


c
a
f
d
h
b
g
e

1.1-3 Full Alternative Text

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1.1 Electrical Engineering: An Overview

The electrical engineering profession focuses on systems that produce, transmit, and measure electric signals. Electrical engineering combines the physicist's models of natural phenomena with the mathematician's tools for manipulating those models to produce systems that meet practical needs. Electrical systems pervade our lives; they are found in homes, schools, workplaces, and transportation vehicles everywhere. We begin by presenting a few examples from each of the five major classifications of electrical systems:

- communication systems
- computer systems
- control systems
- power systems
- signal-processing systems

Then we describe how electrical engineers analyze and design such systems.

Communication systems are electrical systems that generate, transmit, and distribute information. Well-known examples include television equipment, such as cameras, transmitters, receivers, and monitors; radio telescopes, used to explore the universe; satellite systems, which return images of other planets and our own; radar systems, used to coordinate plane flights; and telephone systems.

Figure 1.1 depicts the major components of a modern telephone system that supports mobile phones, landlines, and international calling. Inside a telephone, a microphone turns sound waves into electric signals. These signals are carried to local or mobile exchanges, where they are combined

with the signals from tens, hundreds, or thousands of other telephones. The form of the signals can be radio waves traveling through air, electrical signals traveling in underground coaxial cable, light pulses traveling in fiber-optic cable, or microwave signals that travel through space. The combined signals are broadcast from a transmission antenna to a receiving antenna. There the combined signals are separated at an exchange, and each is routed to the appropriate telephone, where an earphone acts as a speaker to convert the received electric signals back into sound waves. At each stage of the process, electric circuits operate on the signals. Imagine the challenge involved in designing, building, and operating each circuit in a way that guarantees that all of the hundreds of thousands of simultaneous calls have high-quality connections.

Figure 1.1 A telephone system.

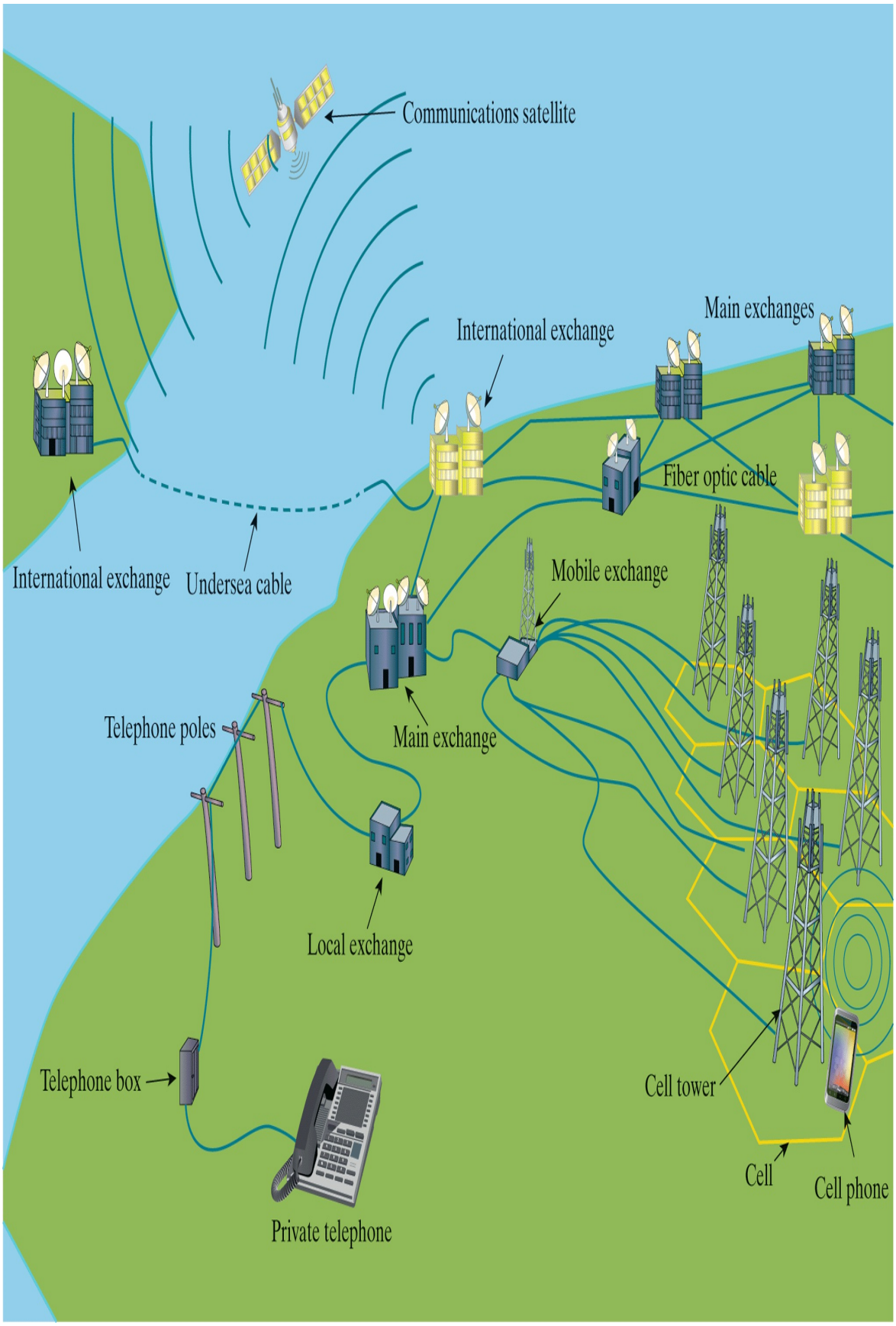

Communications satellite
International exchange
Main exchanges
Fiber optic cable
International exchange
Undersea cable
Mobile exchange
Main exchange
Telephone poles
Local exchange
Telephone box
Private telephone
Cell tower
Cell
Cell phone

Figure 1.1 Full Alternative Text

Computer systems use electric signals to process information ranging from word processing to mathematical computations. Systems range in size and power from simple calculators to personal computers to supercomputers that perform such complex tasks as processing weather data and modeling chemical interactions of complex organic molecules. These systems include networks of integrated circuits—miniature assemblies of hundreds, thousands, or millions of electrical components that often operate at speeds and power levels close to fundamental physical limits, including the speed of light and the thermodynamic laws.

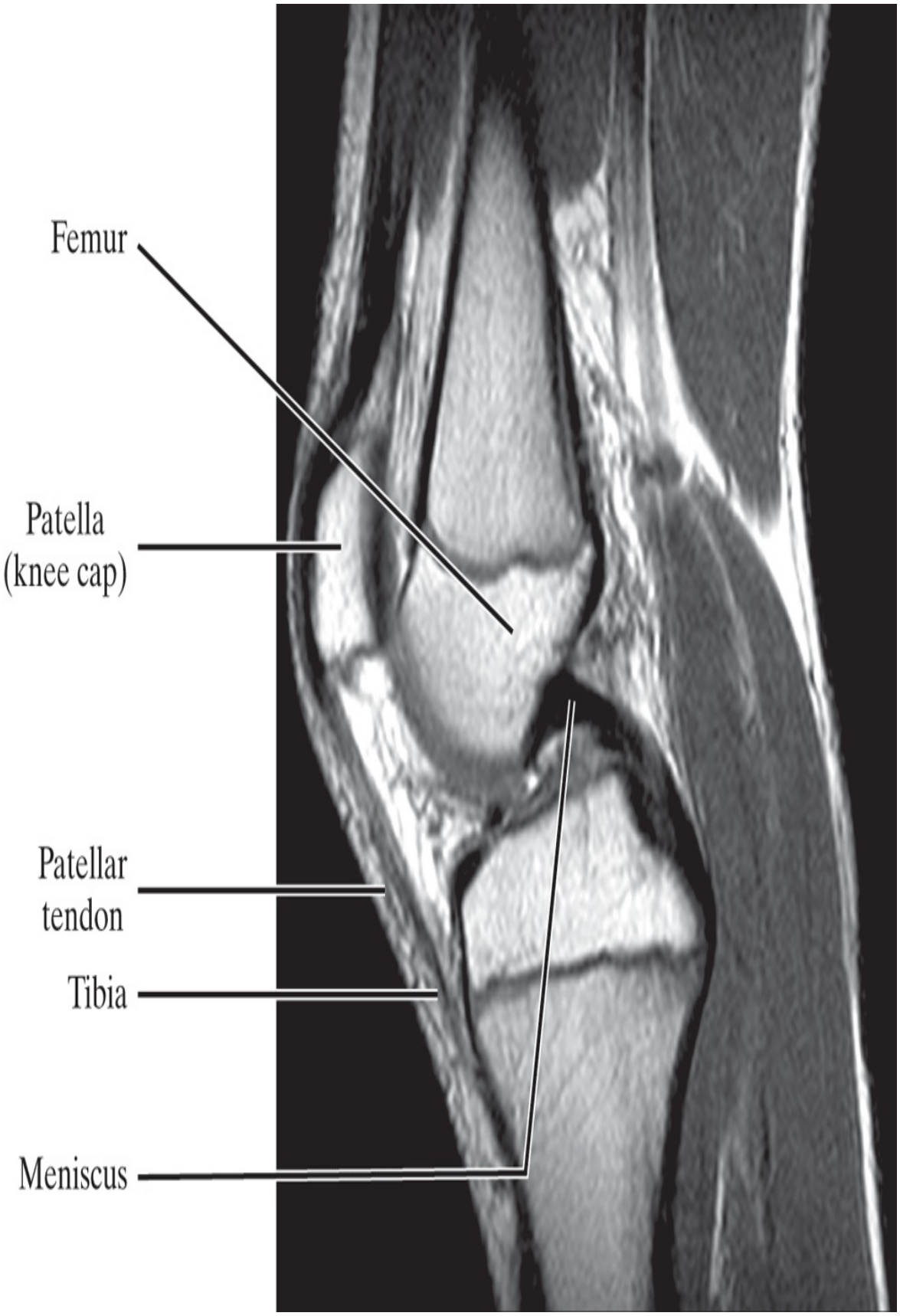
Control systems use electric signals to regulate processes. Examples include the control of temperatures, pressures, and flow rates in an oil refinery; the fuel–air mixture in a fuel-injected automobile engine; mechanisms such as the motors, doors, and lights in elevators; and the locks in the Panama Canal. The autopilot and autolanding systems that help to fly and land airplanes are also familiar control systems.

Power systems generate and distribute electric power. Electric power, which is the foundation of our technology-based society, usually is generated in large quantities by nuclear, hydroelectric, solar, and thermal (coal-, oil-, or gas-fired) generators. Power is distributed by a grid of conductors that crisscross the country. A major challenge in designing and operating such a system is to provide sufficient redundancy and control so that failure of any piece of equipment does not leave a city, state, or region completely without power.

Signal-processing systems act on electric signals that represent information. They transform the signals and the information contained in them into a more suitable form. There are many different ways to process the signals and their information. For example, image-processing systems gather massive quantities of data from orbiting weather satellites, reduce the amount of data to a manageable level, and transform the remaining data into a video image for the evening news broadcast. A magnetic resonance imaging (MRI) scan is another example of an image-processing system. It takes signals generated by powerful magnetic fields and radio waves and transforms them into a

detailed, three-dimensional image such as the one shown in Fig. 1.2, which can be used to diagnose disease and injury.

Figure 1.2 An MRI scan of an adult knee joint.


Femur
Patella
(knee cap)
Patellar
tendon
Tibia
Meniscus

Neil Borden/Science Source/Getty Images

Figure 1.2 Full Alternative Text

Considerable interaction takes place among the engineering disciplines involved in designing and operating these five classes of systems. Thus, communications engineers use digital computers to control the flow of information. Computers contain control systems, and control systems contain computers. Power systems require extensive communications systems to coordinate safely and reliably the operation of components, which may be spread across a continent. A signal-processing system may involve a communications link, a computer, and a control system.

A good example of the interaction among systems is a commercial airplane, such as the one shown in Fig. 1.3. A sophisticated communications system enables the pilot and the air traffic controller to monitor the plane's location, permitting the air traffic controller to design a safe flight path for all of the nearby aircraft and enabling the pilot to keep the plane on its designated path. An onboard computer system manages engine functions, implements the navigation and flight control systems, and generates video information screens in the cockpit. A complex control system uses cockpit commands to adjust the position and speed of the airplane, producing the appropriate signals to the engines and the control surfaces (such as the wing flaps, ailerons, and rudder) to ensure the plane remains safely airborne and on the desired flight path. The plane must have its own power system to stay aloft and to provide and distribute the electric power needed to keep the cabin lights on, make the coffee, and activate the entertainment system. Signal-processing systems reduce the noise in air traffic communications and transform information about the plane's location into the more meaningful form of a video display in the cockpit. Engineering challenges abound in the design of each of these systems and their integration into a coherent whole. For example, these systems must operate in widely varying and unpredictable environmental conditions. Perhaps the most important engineering challenge is to guarantee that sufficient redundancy is incorporated in the designs, ensuring that passengers arrive safely and on time at their desired destinations.

Figure 1.3 Interacting systems on a commercial aircraft.

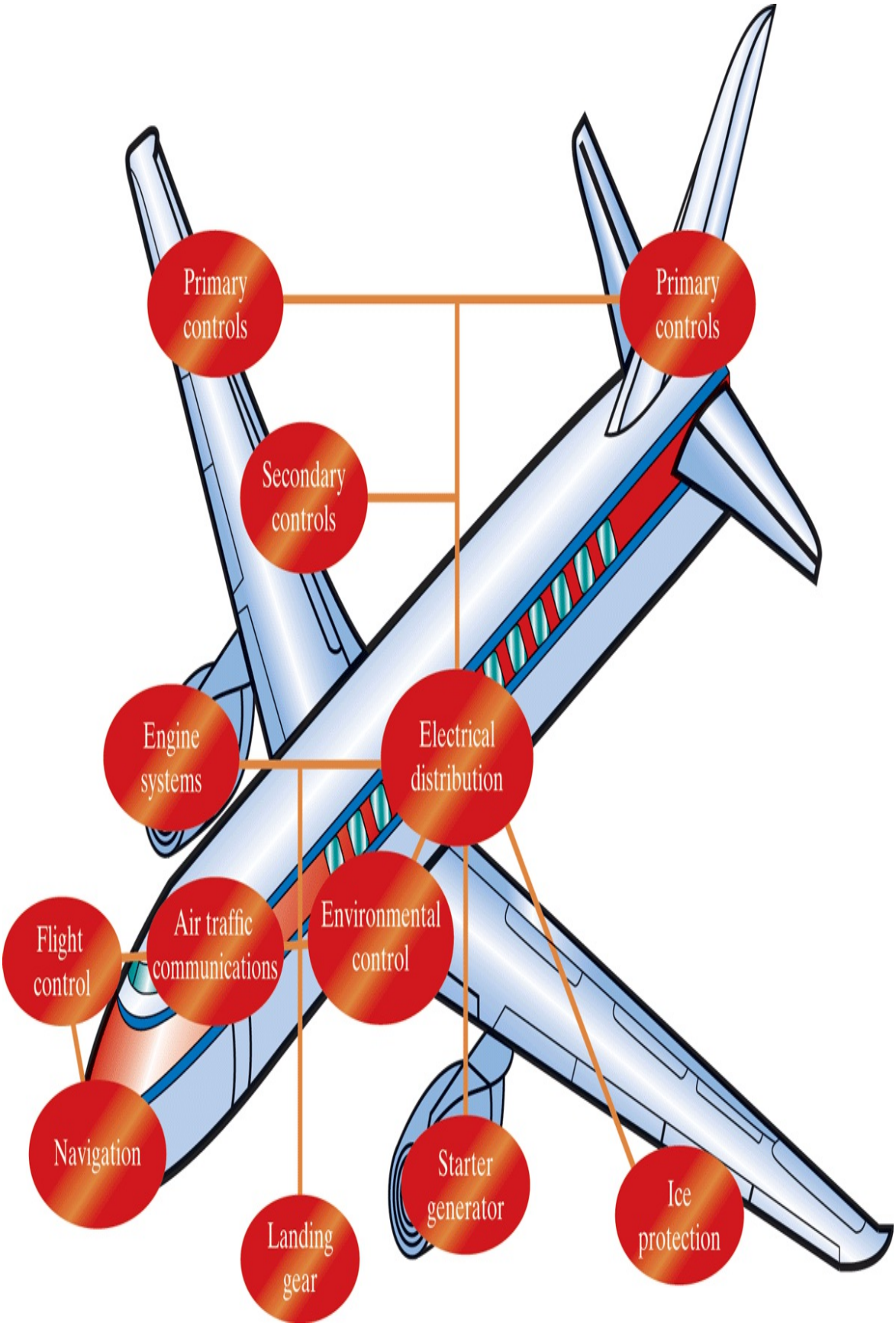

Primary controls
Primary controls
Secondary controls
Engine systems
Electrical distribution
Flight control
Air traffic communications
Environmental control
Navigation
Landing gear
Starter generator
Ice protection

Figure 1.3 Full Alternative Text

Although electrical engineers may be interested primarily in one area, they must also be knowledgeable in other areas that interact with this area of interest. This interaction is part of what makes electrical engineering a challenging and exciting profession. The emphasis in engineering is on making things work, so an engineer is free to acquire and use any technique from any field that helps to get the job done.

Circuit Theory

An **electric circuit** is a mathematical model that approximates the behavior of an actual electrical system. Since electric circuits are found in every branch of electrical engineering, they provide an important foundation for learning how to design and operate systems such as those just described. The models, the mathematical techniques, and the language of circuit theory will form the intellectual framework for your future engineering endeavors.

Note that the term *electric circuit* is commonly used to refer to an actual electrical system as well as to the model that represents it. In this text, when we talk about an electric circuit, we always mean a model, unless otherwise stated. It is the modeling aspect of circuit theory that has broad applications across engineering disciplines.

Circuit theory is a special case of electromagnetic field theory: the study of static and moving electric charges. But applying generalized field theory to the study of electric signals is cumbersome and requires advanced mathematics. Consequently, a course in electromagnetic field theory is not a prerequisite to understanding the material in this book. We do, however, assume that you have had an introductory physics course in which electrical and magnetic phenomena were discussed.

Three basic assumptions permit us to use circuit theory, rather than electromagnetic field theory, to study a physical system represented by an electric circuit.

1. Electrical effects happen instantaneously throughout a system. We can make this assumption because we know that electric signals travel at or near the speed of light. Thus, if the system is physically small, electric signals move through it so quickly that we can consider them to affect every point in the system simultaneously. A system that is small enough so that we can make this assumption is called a **lumped-parameter system**.

2. The net charge on every component in the system is always zero. Thus, no component can collect a net excess of charge, although some components, as you will learn later, can hold equal but opposite separated charges.

3. There is no magnetic coupling between the components in a system. As we demonstrate later, magnetic coupling can occur *within* a component.

That's it; there are no other assumptions. Using circuit theory provides simple solutions (of sufficient accuracy) to problems that would become hopelessly complicated if we were to use electromagnetic field theory. These benefits are so great that engineers sometimes specifically design electrical systems to ensure that these assumptions are met. The importance of assumptions 2 and 3 becomes apparent after we introduce the basic circuit elements and the rules for analyzing interconnected elements.

Let's take a closer look at assumption 1. The question is, "How small does a physical system have to be to qualify as a lumped-parameter system?" To get a quantitative answer to this question, remember that electric signals propagate as waves. If the wavelength of the signal is large compared to the physical dimensions of the system, we have a lumped-parameter system. The wavelength λ is the velocity divided by the repetition rate, or **frequency**, of the signal; that is, $\lambda=c/f$. The frequency f is measured in hertz (Hz). For example, power systems in the United States operate at 60 Hz. If we use the speed of light ($c=3\times108$ m/s) as the velocity of propagation, the wavelength is 5×106 m. If the power system of interest is physically smaller than this wavelength, we can represent it as a lumped-parameter system and use circuit theory to analyze its behavior. How do we define *smaller*? A good rule is the *rule of 1/10th*: If the dimension of the system is less than 1/10th the dimension of the wavelength, you have a lumped-parameter system. Thus, as

long as the physical dimension of the power system is less than 5×105 m (which is about 310 miles), we can treat it as a lumped-parameter system.

Now consider a communication system that sends and receives radio signals. The propagation frequency of radio signals is on the order of 109 Hz, so the wavelength is 0.3 m. Using the rule of 1N10th, a communication system qualifies as a lumped-parameter system if its dimension is less than 3 cm. Whenever any of the pertinent physical dimensions of a system under study approaches the wavelength of its signals, we must use electromagnetic field theory to analyze that system. Throughout this book we study circuits derived from lumped-parameter systems.

Problem Solving

As a practicing engineer, you will not be asked to solve problems that have already been solved. Whether you are improving the performance of an existing system or designing a new system, you will be working on unsolved problems. As a student, however, you will read and discuss problems with known solutions. Then, by solving related homework and exam problems on your own, you will begin to develop the skills needed to attack the unsolved problems you'll face as a practicing engineer.

Let's review several general problem-solving strategies. Many of these pertain to thinking about and organizing your solution strategy *before* proceeding with calculations.

1. Identify what's given and what's to be found. In problem solving, you need to know your destination before you can select a route for getting there. What is the problem asking you to solve or find? Sometimes the goal of the problem is obvious; other times you may need to paraphrase or make lists or tables of known and unknown information to see your objective.

   On one hand, the problem statement may contain extraneous information that you need to weed out before proceeding. On the other hand, it may offer incomplete information or more complexities than can

be handled by the solution methods you know. In that case, you'll need to make assumptions to fill in the missing information or simplify the problem context. Be prepared to circle back and reconsider supposedly extraneous information and/or your assumptions if your calculations get bogged down or produce an answer that doesn't seem to make sense.

2. Sketch a circuit diagram or other visual model. Translating a verbal problem description into a visual model is often a useful step in the solution process. If a circuit diagram is already provided, you may need to add information to it, such as labels, values, or reference directions. You may also want to redraw the circuit in a simpler, but equivalent, form. Later in this text you will learn the methods for developing such simplified equivalent circuits.

3. Think of several solution methods and decide on a way of choosing among them. This course will help you build a collection of analytical tools, several of which may work on a given problem. But one method may produce fewer equations to be solved than another, or it may require only algebra instead of calculus to reach a solution. Such efficiencies, if you can anticipate them, can streamline your calculations considerably. Having an alternative method in mind also gives you a path to pursue if your first solution attempt bogs down.

4. Calculate a solution. Your planning up to this point should have helped you identify a good analytical method and the correct equations for the problem. Now comes the solution of those equations. Paper-and-pencil, calculator, and computer methods are all available for performing the actual calculations of circuit analysis. Efficiency and your instructor's preferences will dictate which tools you should use.

5. Use your creativity. If you suspect that your answer is off base or if the calculations seem to go on and on without moving you toward a solution, you should pause and consider alternatives. You may need to revisit your assumptions or select a different solution method. Or you may need to take a less conventional problem-solving approach, such as working backward from a solution. This text provides answers to all of the Assessment Problems and many of the Chapter Problems so that you may work backward when you get stuck. In the real world, you won't be

given answers in advance, but you may have a desired problem outcome in mind from which you can work backward. Other creative approaches include allowing yourself to see parallels with other types of problems you've successfully solved, following your intuition or hunches about how to proceed, and simply setting the problem aside temporarily and coming back to it later.

6. Test your solution. Ask yourself whether the solution you've obtained makes sense. Does the magnitude of the answer seem reasonable? Is the solution physically realizable? Are the units correct? You may want to rework the problem using an alternative method to validate your original answer and help you develop your intuition about the most efficient solution methods for various kinds of problems. In the real world, safety-critical designs are always checked by several independent means. Getting into the habit of checking your answers will benefit you both as a student and as a practicing engineer.

These problem-solving steps cannot be used as a recipe to solve every problem in this or any other course. You may need to skip, change the order of, or elaborate on certain steps to solve a particular problem. Use these steps as a guideline to develop a problem-solving style that works for you.

1.2 The International System of Units

Engineers use quantitative measures to compare theoretical results to experimental results and compare competing engineering designs. Modern engineering is a multidisciplinary profession in which teams of engineers work together on projects, and they can communicate their results in a meaningful way only if they all use the same units of measure. The International System of Units (abbreviated SI) is used by all the major engineering societies and most engineers throughout the world; hence we use it in this book.

The SI units are based on seven *defined* quantities:

- length
- mass
- time
- electric current
- thermodynamic temperature
- amount of substance
- luminous intensity

These quantities, along with the basic unit and symbol for each, are listed in Table 1.1. Although not strictly SI units, the familiar time units of minute (60 s), hour (3600 s), and so on are often used in engineering calculations. In addition, defined quantities are combined to form **derived** units. Some quantities, such as force, energy, power, and electric charge, you already know through previous physics courses. Table 1.2 lists the derived units used in this book.

Table 1.1 The International System of Units (SI)

Quantity	Basic Unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	degree kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd

National Institute of Standards and Technology Special Publication 330, 2008 Edition, Natl. Inst. Stand. Technol. Spec. Pub. 330, 2008 Ed., 96 pages (March 2008)

Table 1.1 Full Alternative Text

Table 1.2 Derived Units in SI

Quantity	Unit Name (Symbol)	Formula
Frequency	hertz (Hz)	s^{-1}
Force	newton (N)	$kg \cdot m/s^2$
Energy or work	joule (J)	$N \cdot m$
Power	watt (W)	J/s
Electric charge	coulomb (C)	$A \cdot s$
Electric potential	volt (V)	J/C
Electric resistance	ohm (Ω)	V/A
Electric conductance	siemens (S)	A/V
Electric capacitance	farad (F)	C/V
Magnetic flux	weber (Wb)	$V \cdot s$
Inductance	henry (H)	Wb/A

National Institute of Standards and Technology Special Publication 330, 2008 Edition, Natl. Inst. Stand. Technol. Spec. Pub. 330, 2008 Ed., 96 pages (March 2008)

Table 1.2 Full Alternative Text

In many cases, the SI unit is either too small or too large to use conveniently. Standard prefixes corresponding to powers of 10, as listed in Table 1.3, are then applied to the basic unit. Engineers often use only the prefixes for powers divisible by 3; thus centi, deci, deka, and hecto are used rarely. Also, engineers often select the prefix that places the base number in the range between 1 and 1000. Suppose that a time calculation yields a result of 10^{-5} s, that is, 0.00001 s. Most engineers would describe this quantity as 10 μs, that is, $10^{-5} = 10 \times 10^{-6}$ s, rather than as 0.01 ms or 10,000 ns.

Table 1.3 Standardized Prefixes to Signify Powers of 10

Prefix	Symbol	Power
atto	a	10^{-18}
femto	f	10^{-15}
pico	p	10^{-12}
nano	n	10^{-9}
micro	μ	10^{-6}
milli	m	10^{-3}
centi	c	10^{-2}
deci	d	10^{-1}
deka	da	10
hecto	h	10^{2}
kilo	k	10^{3}
mega	M	10^{6}
giga	G	10^{9}
tera	T	10^{12}

National Institute of Standards and Technology Special Publication 330, 2008 Edition, Natl. Inst. Stand. Technol. Spec. Pub. 330, 2008 Ed., 96 pages (March 2008)

Table 1.3 Full Alternative Text

Example 1.1 illustrates a method for converting from one set of units to another and also uses power-of-10 prefixes.

Example 1.1 Using SI Units and Prefixes for Powers of 10

If a signal can travel in a cable at 80% of the speed of light, what length of cable, in inches, represents 1 ns?

Solution

First, note that 1 ns=10−9 s. Also, recall that the speed of light c=3×108 m/s. Then, 80% of the speed of light is 0.8c=(0.8)(3×108)=2.4×108 m/s. Using a product of ratios, we can convert 80% of the speed of light from meters per second to inches per nanosecond. The result is the distance in inches traveled in 1 nanosecond:

$$2.4\times 10^{8}\ \text{meters}\ 1\ \text{second}\cdot 1\ \text{second}\ 10^{9}\ \text{nanoseconds}\cdot 100\ \text{centimeters}\ 1\ \text{meter}\cdot 1\ \text{inch}\ 2.54\ \text{centimeters}=9.45\ \text{inches/nanosecond}.$$

Therefore, a signal traveling at 80% of the speed of light will cover 9.45 inches of cable in 1 nanosecond.

Assessment Problems

Objective 1—Understand and be able to use SI units and the standard

prefixes for powers of 10

1. 1.1 Assume a telephone signal travels through a cable at two-thirds the speed of light. How long does it take the signal to get from New York City to Miami if the distance is approximately 1100 miles?

   Answer: 8.85 ms.

2. 1.2 How many dollars per millisecond would the federal government have to collect to retire a deficit of $100 billion in one year?

   Answer: $3.17/ms.

SELF-CHECK: Also try Chapter Problems 1.2, 1.3, and 1.6.

1.3 Circuit Analysis: An Overview

We look broadly at engineering design, specifically the design of electric circuits, before becoming involved in the details of circuit analysis. This overview provides you with a perspective on where circuit analysis fits within the whole of circuit design. Even though this book focuses on circuit analysis, we try to provide opportunities for circuit design where appropriate.

All engineering designs begin with a need 1, as shown in Fig. 1.4. This need may come from the desire to improve on an existing design, or it may be something brand new. A careful assessment of the need results in design specifications, which are measurable characteristics of a proposed design. Once a design is proposed, the design specifications 2 allow us to assess whether or not the design actually meets the need.

Figure 1.4 A conceptual model for electrical engineering design.

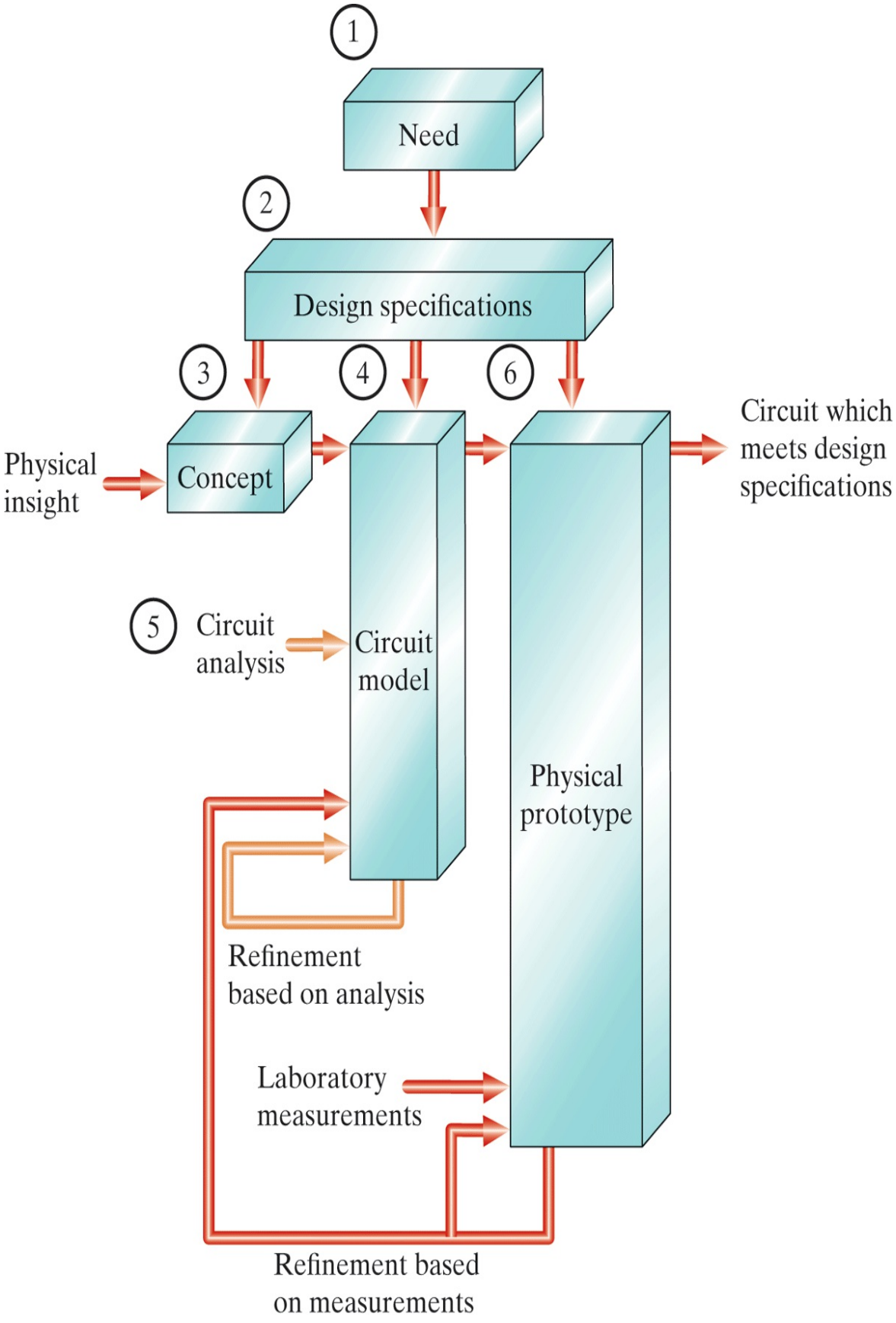

1
Need
2
Design specifications
3
4
6
Physical insight
Concept
Circuit which meets design specifications
5
Circuit analysis
Circuit model
Physical prototype
Refinement based on analysis
Laboratory measurements
Refinement based on measurements

Figure 1.4 Full Alternative Text

A concept 3 for the design comes next. The concept derives from a complete understanding of the design specifications coupled with an insight into the need, which comes from education and experience. The concept may be realized as a sketch, as a written description, or as some other form. Often the next step is to translate the concept into a mathematical model. A commonly used mathematical model for electrical systems is a circuit model 4.

The elements that make up the circuit model are called ideal circuit components. An **ideal circuit component** is a mathematical model of an actual electrical component, like a battery or a light bulb. The ideal circuit components used in a circuit model should represent the behavior of the actual electrical components to an acceptable degree of accuracy. The tools of circuit analysis 5, the focus of this book, are then applied to the circuit. **Circuit analysis** uses mathematical techniques to predict the behavior of the circuit model and its ideal circuit components. A comparison between the desired behavior, from the design specifications, and the predicted behavior, from circuit analysis, may lead to refinements in the circuit model and its ideal circuit elements. Once the desired and predicted behaviors are in agreement, a physical prototype 6 can be constructed.

The **physical prototype** is an actual electrical system, constructed from actual electrical components. Measurements determine the quantitative behavior of the physical system. This actual behavior is compared with the desired behavior from the design specifications and the predicted behavior from circuit analysis. The comparisons may result in refinements to the physical prototype, the circuit model, or both. This iterative process, in which models, components, and systems are continually refined, usually produces a design that accurately satisfies the design specifications and thus meets the need.

Circuit analysis clearly plays a very important role in the design process. Because circuit analysis is applied to circuit models, practicing engineers try to use mature circuit models so that the resulting designs will meet the design specifications in the first iteration. In this book, we use models that have been tested for at least 40 years; you can assume that they are mature. The ability

to model actual electrical systems with ideal circuit elements makes circuit theory extremely useful to engineers.

Saying that the interconnection of ideal circuit elements can be used to quantitatively predict the behavior of a system implies that we can describe the interconnection with mathematical equations. For the mathematical equations to be useful, we must write them in terms of measurable quantities. In the case of circuits, these quantities are voltage and current, which we discuss in Section 1.4. The study of circuit analysis involves understanding the behavior of each ideal circuit element in terms of its voltage and current and understanding the constraints imposed on the voltage and current as a result of interconnecting the ideal elements.

1.4 Voltage and Current

The concept of electric charge is the basis for describing all electrical phenomena. Let’s review some important characteristics of electric charge.

- Electric charge is bipolar, meaning that electrical effects are described in terms of positive and negative charges.
- Electric charge exists in discrete quantities, which are integer multiples of the electronic charge, 1.6022×10−19 C.
- Electrical effects are attributed to both the separation of charge and charges in motion.

In circuit theory, the separation of charge creates an electric force (voltage), and the motion of charge creates an electric fluid (current).

The concepts of voltage and current are useful from an engineering point of view because they can be expressed quantitatively. Whenever positive and negative charges are separated, energy is expended. **Voltage** is the energy per unit charge created by the separation. We express this ratio in differential form as

Definition of Voltage

v=dwdq, (1.1)

where

v=the voltage in volts,w=the energy in joules,q=the charge in coulombs.

The electrical effects caused by charges in motion depend on the rate of charge flow. The rate of charge flow is known as the **electric current**, which is expressed as

Definition of Current

i=dqdt, (1.2)

where

i=the current in amperes,q=the charge in coulombs,t=the time in seconds.

Equations 1.1 and 1.2 define the magnitude of voltage and current, respectively. The bipolar nature of electric charge requires that we assign polarity references to these variables. We will do so in Section 1.5.

Although current is made up of discrete moving electrons, we do not need to consider them individually because of the enormous number of them. Rather, we can think of electrons and their corresponding charge as one smoothly flowing entity. Thus, i is treated as a continuous variable.

One advantage of using *circuit models* is that we can model a component strictly in terms of the voltage and current at its terminals. Thus, two physically different components could have the same relationship between the terminal voltage and terminal current. If they do, for purposes of circuit analysis, they are identical. Once we know how a component behaves at its terminals, we can analyze its behavior in a circuit. However, when developing *component models*, we are interested in a component's internal behavior. We might want to know, for example, whether charge conduction is taking place because of free electrons moving through the crystal lattice structure of a metal or whether it is because of electrons moving within the covalent bonds of a semiconductor material. These concerns are beyond the realm of circuit theory, so in this book we use component models that have already been developed.

1.5 The Ideal Basic Circuit Element

An **ideal basic circuit element** has three attributes.

1. It has only two terminals, which are points of connection to other circuit components.

2. It is described mathematically in terms of current and/or voltage.

3. It cannot be subdivided into other elements.

Using the word *ideal* implies that a basic circuit element does not exist as a realizable physical component. Ideal elements can be connected in order to model actual devices and systems, as we discussed in Section 1.3. Using the word *basic* implies that the circuit element cannot be further reduced or subdivided into other elements. Thus, the basic circuit elements form the building blocks for constructing circuit models, but they themselves cannot be modeled with any other type of element.

Figure 1.5 represents an ideal basic circuit element. The box is blank because we are making no commitment at this time as to the type of circuit element it is. In Fig. 1.5, the voltage across the terminals of the box is denoted by v, and the current in the circuit element is denoted by i. The plus and minus signs indicate the polarity reference for the voltage, and the arrow placed alongside the current indicates its reference direction. Table 1.4 interprets the voltage polarity and current direction, given positive or negative numerical values of v and i. Note that algebraically the notion of positive charge flowing in one direction is equivalent to the notion of negative charge flowing in the opposite direction.

Figure 1.5 An ideal basic circuit element.

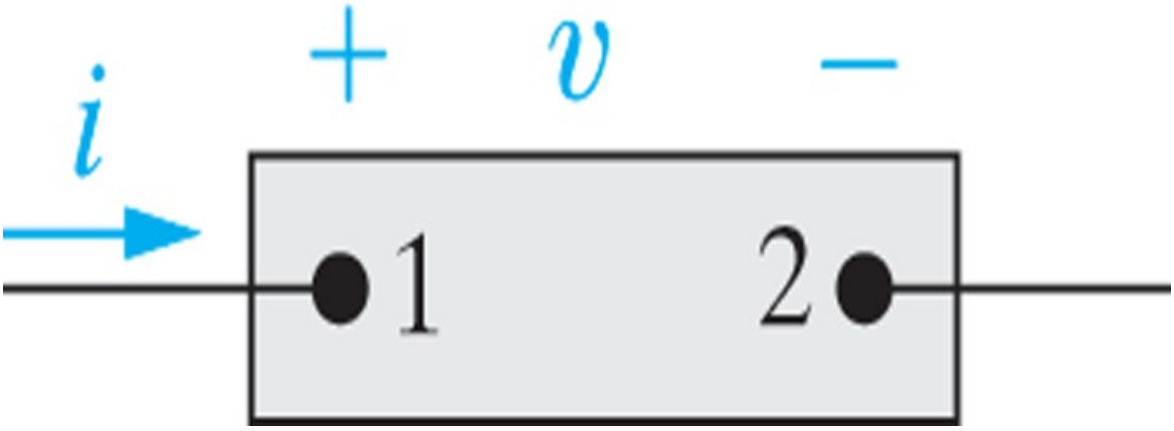


Figure 1.5 Full Alternative Text

Table 1.4 Interpretation of Reference Directions in Fig. 1.5

	Positive Value	Negative Value
v	voltage drop from terminal 1 to terminal 2	voltage rise from terminal 1 to terminal 2
	or	*or*
	voltage rise from terminal 2 to terminal 1	voltage drop from terminal 2 to terminal 1
i	positive charge flowing from terminal 1 to terminal 2	positive charge flowing from terminal 2 to terminal 1
	or	*or*
	negative charge flowing from terminal 2 to terminal 1	negative charge flowing from terminal 1 to terminal 2

Table 1.4 Full Alternative Text

Assigning the reference polarity for voltage and the reference direction for current is entirely arbitrary. However, once you have assigned the references, you must write all subsequent equations to agree with the chosen references. The most widely used sign convention applied to these references is called the **passive sign convention**, which we use throughout this book.

Passive Sign Convention

Whenever the reference direction for the current in an element is in the direction of the reference voltage drop across the element (as in Fig. 1.5), use a positive sign in any expression that relates the voltage to the current. Otherwise, use a negative sign.

We apply this sign convention in all the analyses that follow. Our purpose for introducing it even before we have introduced the different types of basic circuit elements is to emphasize that selecting polarity references is *not* a function either of the basic elements or the type of interconnections made with the basic elements. We apply and interpret the passive sign convention for power calculations in Section 1.6.

Example 1.2 illustrates one use of the equation defining current.

Example 1.2 Relating Current and Charge

No charge exists at the upper terminal of the element in Fig. 1.5 for $t<0$. At $t=0$, a 5 A current begins to flow into the upper terminal.

1. Derive the expression for the charge accumulating at the upper terminal of the element for $t>0$.

2. If the current is stopped after 10 seconds, how much charge has accumulated at the upper terminal?

Solution

1. From the definition of current given in Eq. 1.2, the expression for charge accumulation due to current flow is

$q(t)=\int_0^t i(x)dx$.

Therefore,

$q(t)=\int_0^t 5dx=5x|_0^t=5t-5(0)=5t$ C for $t>0$.

2. The total charge that accumulates at the upper terminal in 10 seconds due to a 5 A current is $q(10)=5(10)=50$ C.

Assessment Problems

Objective 2—Know and be able to use the definitions of *voltage* and *current*

1. 1.3 The current at the terminals of the element in Fig. 1.5 is

   $i=0, t<0; i=20e^{-5000t}$, A, $t\geq 0$;

   Calculate the total charge (in microcoulombs) entering the element at its upper terminal.

   Answer: 4000 μC.

2. 1.4 The expression for the charge entering the upper terminal of Fig. 1.5 is

   $q=\frac{1}{\alpha^2}-(t\alpha+\frac{1}{\alpha^2})e^{-\alpha t}$ C.

   Find the maximum value of the current entering the terminal if $\alpha=0.03679\ s^{-1}$.

   Answer: 10 A.

SELF-CHECK: Also try Chapter Problem 1.7.

1.6 Power and Energy

Power and energy calculations are important in circuit analysis. Although voltage and current are useful variables in the analysis and design of electrically based systems, the useful output of the system often is nonelectrical (e.g., sound emitted from a speaker or light from a light bulb), and this output is conveniently expressed in terms of power or energy. Also, all practical devices have limitations on the amount of power that they can handle. In the design process, therefore, voltage and current calculations by themselves are not sufficient to determine whether or not a design meets its specifications.

We now relate power and energy to voltage and current and at the same time use the power calculation to illustrate the passive sign convention. Recall from basic physics that power is the time rate of expending or absorbing energy. (A water pump rated 75 kW can deliver more liters per second than one rated 7.5 kW.) Mathematically, energy per unit time is expressed in the form of a derivative, or

Definition of Power

p=dwdt, (1.3)

where

p=the power in watts,w=the energy in joules,t=the time in seconds.

Thus, 1 W is equivalent to 1 J/s.

The power associated with the flow of charge follows directly from the definition of voltage and current in Eqs. 1.1 and 1.2, or

p=dwdt=(dwdq)(dqdt),

so

Power Equation

p=vi,(1.4)

where

p=the power in watts,v=the voltage in volts,i=the current in amperes.

Equation 1.4 shows that the **power** associated with a basic circuit element is the product of the current in the element and the voltage across the element. Therefore, power is a quantity associated with a circuit element, and we have to determine from our calculation whether power is being delivered to the circuit element or extracted from it. This information comes from correctly applying and interpreting the passive sign convention (Section 1.5).

If we use the passive sign convention, Eq. 1.4 is correct if the reference direction for the current is in the direction of the reference voltage drop across the terminals. Otherwise, Eq. 1.4 must be written with a minus sign. In other words, if the current reference is in the direction of a reference voltage rise across the terminals, the expression for the power is

p=−vi.

The algebraic sign of power is based on charge movement through voltage drops and rises. As positive charges move through a drop in voltage, they lose energy, and as they move through a rise in voltage, they gain energy. Figure 1.6 summarizes the relationship between the polarity references for voltage and current and the expression for power.

Figure 1.6 Polarity references and the

expression for power.

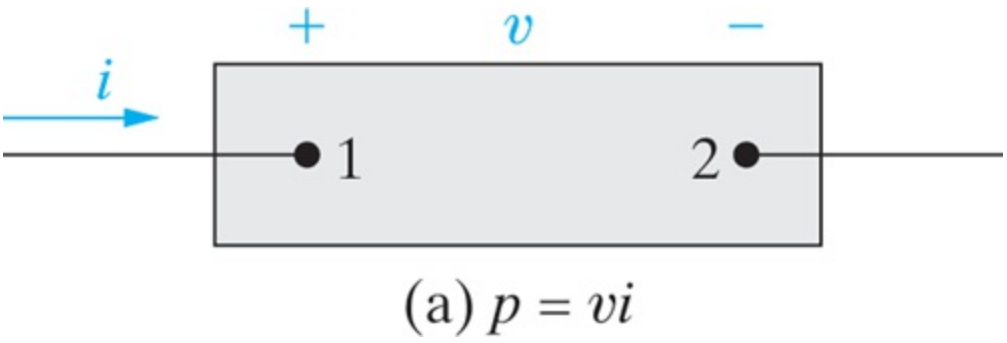
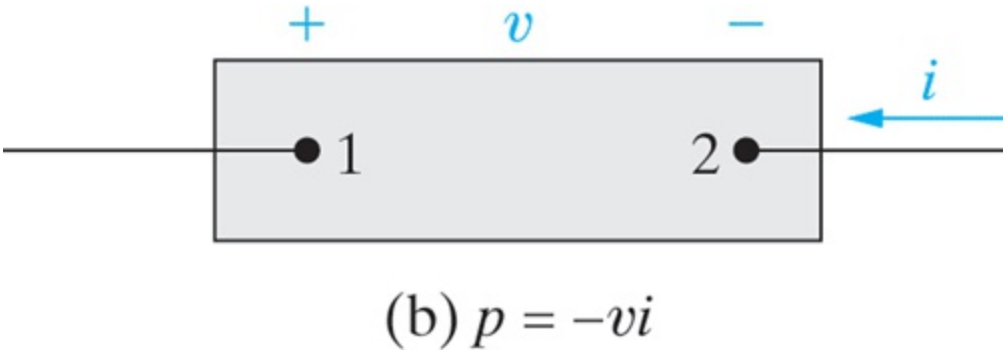
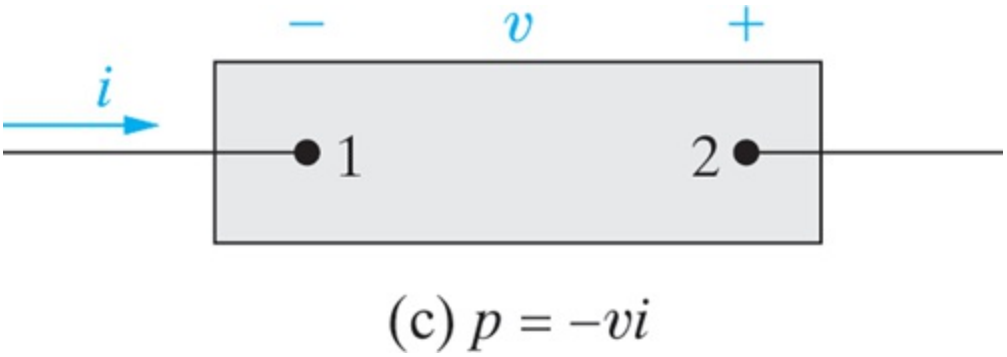


Figure 1.6 Full Alternative Text



1.6-9 Full Alternative Text



1.6-9 Full Alternative Text

(d) $p = vi$

1.6-9 Full Alternative Text

We can now state the rule for interpreting the algebraic sign of power:

Interpreting Algebraic Sign of Power

- If the power is positive (that is, if p>0), power is being delivered to the circuit element represented by the box.
- If the power is negative (that is, if p<0), power is being extracted from the circuit element.

Example 1.3 shows that the passive sign convention generates the correct sign for power regardless of the voltage polarity and current direction you choose.

Example 1.3 Using the Passive Sign Convention

1. Suppose you have selected the polarity references shown in Fig. 1.6(b). Your calculations for the current and voltage yield the following numerical results:

$i=4\ \text{A}$ and $v=-10\ \text{V}$.

Calculate the power associated with the circuit element and determine whether it is absorbing or supplying power.

2. Your classmate is solving the same problem but has chosen the reference polarities shown in Fig. 1.6(c). Her calculations for the current and voltage show that

   $i=-4\ \text{A}$ and $v=10\ \text{V}$.

   What power does she calculate?

Solution

1. The power associated with the circuit element in Fig. 1.6(b) is

   $p=-(-10)(4)=40\ \text{W}$.

   Thus, the circuit element is absorbing 40 W.

2. Your classmate calculates that the power associated with the circuit element in Fig. 1.6(c) is

   $p=-(10)(-4)=40\ \text{W}$.

   Using the reference system in Fig. 1.6(c) gives the same conclusion as using the reference system in Fig. 1.6(b)—namely, that the circuit element is absorbing 40 W. In fact, any of the reference systems in Fig. 1.6 yields this same result.

Example 1.4 illustrates the relationship between voltage, current, power, and energy for an ideal basic circuit element and the use of the passive sign convention.

Example 1.4 Relating Voltage,

Current, Power, and Energy

Assume that the voltage at the terminals of the element in Fig. 1.5, whose current was defined in Assessment Problem 1.3, is

$v=0 t<0; v=10e-5000t$ kV, $t\geq 0$.

1. Calculate the power supplied to the element at 1 ms.
2. Calculate the total energy (in joules) delivered to the circuit element.

Solution

1. Since the current is entering the + terminal of the voltage drop defined for the element in Fig. 1.5, we use a "+" sign in the power equation.

   $p = vi = (10{,}000e-5000t)(20e-5000t) = 200{,}000e-10{,}000t$ W. $p(0.001)= 200{,}000e - 10{,}000(0.0001) = 200{,}000e-10= 200{,}000(45.4\times 10-6) = 9.08$ W.

2. From the definition of power given in Eq. 1.3, the expression for energy is

   $w(t)=\int 0tp(x)dx$.

   To find the total energy delivered, integrate the expresssion for power from zero to infinity. Therefore,

   $wtotal=\int 0\infty 200{,}000e-10{,}000x\ dx=200{,}000e-10{,}000x-10{,}000|0\infty =-20e-\infty-(-20e-0\ )=0+20=20$ J.

   Thus, the total energy supplied to the circuit element is 20 J.

Assessment Problems

Objective 3—Know and use the definitions of *power* and *energy*;
Objective 4—Be able to use the passive sign convention

1. 1.5 Assume that a 20 V voltage drop occurs across an element from terminal 2 to terminal 1 and that a current of 4 A enters terminal 2.

    1. Specify the values of v and i for the polarity references shown in Fig. 1.6(a)–(d).

    2. Calculate the power associated with the circuit element.

    3. Is the circuit element absorbing or delivering power?

    Answer:

    1. Circuit 1.6(a): $v = -20$ V,$i=-4$ A; circuit 1.6(b): $v = -20$ V,$v= -20$ V,$i=4$ A; circuit 1.6(c): $v = 20$ V,$i=-4$ A; circuit 1.6(d): $v = 20$ V,$i=4$ A;

    2. 80 W;

    3. absorbing.

2. 1.6 The voltage and current at the terminals of the circuit element in Fig. 1.5 are zero for $t<0$. For $t\geq 0$, they are

    $v = 80{,}000t\, e^{-500t}$ V, $t\geq 0$; $i = 15t\, e^{-500t}$ A, $t\geq 0$.

    1. Find the time when the power delivered to the circuit element is maximum.

    2. Find the maximum value of power.

    3. Find the total energy delivered to the circuit element.

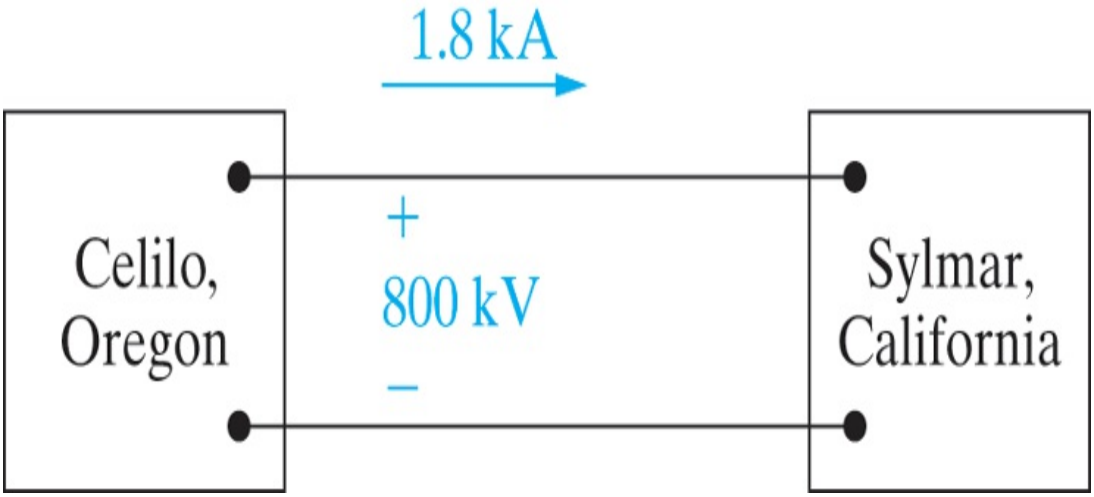
    Answer:

    1. 2 ms;

2. 649.6 mW;

3. 2.4 mJ.

3. 1.7 A high-voltage direct-current (dc) transmission line between Celilo, Oregon, and Sylmar, California, is operating at 800 kV and carrying 1800 A, as shown. Calculate the power (in megawatts) at the Oregon end of the line and state the direction of power flow.



Answer: 1440 MW, Celilo to Sylmar

SELF-CHECK: Also try Chapter Problems 1.15, 1.18, and 1.25.

Practical Perspective

Balancing Power

A circuit model for distributing power to a typical home is shown in Fig. 1.7, with voltage polarities and current directions defined for all of the circuit components. Circuit analysis gives values for all of these voltages and currents, as summarized in Table 1.5. To determine whether or not the values given are correct, calculate the power associated with each component. Use the passive sign convention in the power calculations, as shown in the

following.

Figure 1.7 Circuit model for power distribution in a home, with voltages and currents defined.

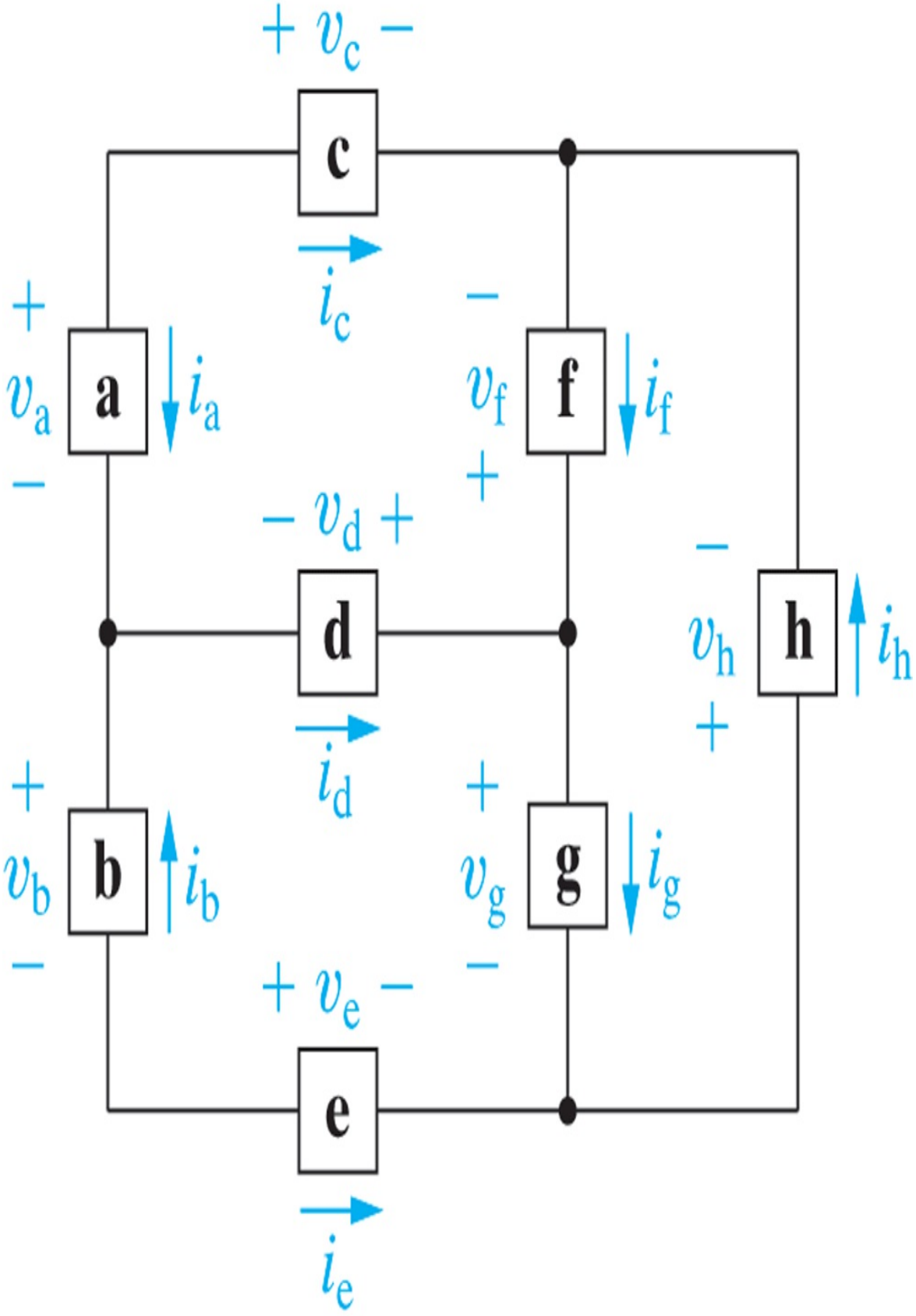

+ v_c −
c
i_c
+
v_a
−
a
i_a
−
v_f
+
f
i_f
− v_d +
d
i_d
−
v_h
+
h
i_h
+
v_b
−
b
i_b
+
v_g
−
g
i_g
+ v_e −
e
i_e

Figure 1.7 Full Alternative Text

Table 1.5 Voltage and Current Values for the Circuit in Fig. 1.7

Component	v(V)	i(A)
a	120	−10
b	120	9
c	10	10
d	10	1
e	−10	−9
f	−100	5
g	120	4
h	−220	−5

Table 1.5 Full Alternative Text

p a = v a i a =(120)(−10)=−1200 W p b = − v b i b =−(120)(9)=−1080 W p c = v c i c =(10)(10)=100 W p d = − v d i d =−(10)(1)=−10 W p e = v e i e =(−10)(−9)=90 W p f = − v f i f =−(−100)(5)=500 W p g = v g i g =(

120)(4)=480 W p h = v h i h =(−220)(−5)=1100 W

The power calculations show that components a, b, and d are supplying power, since the power values are negative, while components c, e, f, g, and h are absorbing power. Now check to see if the power balances by finding the total power supplied and the total power absorbed.

psupplied=pa+pb+pd=−1200−1080−10=
−2290 Wpabsorbed=pc+pe+pf+pg+ph=100+90+500+480+1100=2270 Wpsup
−2290+2270=−20 W

Something is wrong—if the values for voltage and current in this circuit are correct, the total power should be zero! There is an error in the data, and we can find it from the calculated powers if the error exists in the sign of a single component. Note that if we divide the total power by 2, we get −10 W, which is the power calculated for component d. If the power for component d is +10 W, the total power would be 0. Circuit analysis techniques from upcoming chapters can be used to show that the current through component d should be −1 A, not +1 A as given in Table 1.5.

SELF-CHECK: Assess your understanding of the Practical Perspective by trying Chapter Problems 1.34 and 1.35.

Summary

- The International System of Units (SI) enables engineers to communicate in a meaningful way about quantitative results. Table 1.1 summarizes the SI units; Table 1.2 presents some useful derived SI units. (See page 10.)

- A circuit model is a mathematical representation of an electrical system. Circuit analysis, used to predict the behavior of a circuit model, is based on the variables of voltage and current. (See page 12.)

    - **Voltage** is the energy per unit charge created by charge separation and has the SI unit of volt. (See page 13.)

      $v=dw/dq$

    - **Current** is the rate of charge flow and has the SI unit of ampere. (See page 13.)

      $i=dq/dt$

- The **ideal basic circuit element** is a two-terminal component that cannot be subdivided; it can be described mathematically in terms of its terminal voltage and current. (See page 14.)

- The **passive sign convention** uses a positive sign in the expression that relates the voltage and current at the terminals of an element when the reference direction for the current through the element is in the direction of the reference voltage drop across the element. (See page 14.)

- **Power** is energy per unit of time and is equal to the product of the terminal voltage and current; it has the SI unit of watt. (See page 16.)

  $p=dw/dt=vi$

  The algebraic sign of power is interpreted as follows:

- If $p>0$, power is being delivered to the circuit or circuit component.
- If $p<0$, power is being extracted from the circuit or circuit component. (See page 16.)

Problems

Section 1.2

1. 1.1 The line described in Assessment Problem 1.7 is 845 mi in length. The line contains four conductors, each weighing 2526 lb per 1000 ft. How many kilograms of conductor are in the line?

2. 1.2 A 32-inch monitor contains 3840×2160 picture elements, or pixels. Each pixel is represented in 24 bits of memory. A byte of memory is 8 bits.

    1. a) How many megabytes (MB) of memory are required to store the information displayed on the monitor?

    2. b) To display a video on the monitor, the image must be refreshed 30 times per second. How many terabytes (TB) of memory are required to store a 2 hr video?

    3. c) For the video described in part (b), how fast must the image data in memory be moved to the monitor? Express your answer in gigabits per second (Gb/s).

3. 1.3 Some species of bamboo can grow (250 mm/day). Assume individual cells in the plant are 10 μm long.

    1. a) How long, on average, does it take a bamboo stalk to grow 1 cell length?

    2. b) How many cell lengths are added in one week, on average?

4. 1.4 A hand-held video player displays 480×320 picture elements (pixels) in each frame of the video. Each pixel requires 2 bytes of memory. Videos are displayed at a rate of 30 frames per second. How many hours of video will fit in a 32 gigabyte memory?

5. 1.5 The 16 gigabyte (GB=230 bytes) flash memory chip for an MP3 player is 11 mm by 15 mm by 1 mm. This memory chip holds 20,000

photos.

1. a) How many photos fit into a cube whose sides are 1 mm?

2. b) How many bytes of memory are stored in a cube whose sides are 200 μm?

6. 1.6 There are approximately 260 million passenger vehicles registered in the United States. Assume that the battery in the average vehicle stores 540 watt-hours (Wh) of energy. Estimate (in gigawatt-hours) the total energy stored in US passenger vehicles.

Section 1.4

1. 1.7 The current entering the upper terminal of Fig. 1.5 is

   $i=24\ \cos\ 4000t$ A

   Assume the charge at the upper terminal is zero at the instant the current is passing through its maximum value. Find the expression for q(t).

2. 1.8 How much energy is imparted to an electron as it flows through a 6 V battery from the positive to the negative terminal? Express your answer in attojoules.

3. 1.9 In electronic circuits it is not unusual to encounter currents in the microampere range. Assume a 35 μA current, due to the flow of electrons. What is the average number of electrons per second that flow past a fixed reference cross section that is perpendicular to the direction of flow?

4. 1.10 There is no charge at the upper terminal of the element in Fig. 1.5 for $t<0$. At $t=0$ a current of $125e^{-2500t}$ mA enters the upper terminal.

   1. a) Derive the expression for the charge that accumulates at the upper terminal for $t>0$.

   2. b) Find the total charge that accumulates at the upper terminal.

   3. c) If the current is stopped at $t=0.5$ ms, how much charge has accumulated at the upper terminal?

5. 1.11 The current at the terminals of the element in Fig. 1.5 is

   $i = 0, t<0; i = 40te^{-500t}A, t\geq 0.$

   1. a) Find the expression for the charge accumulating at the upper terminal.

2. b) Find the charge that has accumulated at t=1 ms.

Sections 1.5–1.6

1. 1.12 When a car has a dead battery, it can often be started by connecting the battery from another car across its terminals. The positive terminals are connected together as are the negative terminals. The connection is illustrated in Fig. P1.12. Assume the current i in Fig. P1.12 is measured and found to be 40 A.

    1. a) Which car has the dead battery?

    2. b) If this connection is maintained for 1.5 min, how much energy is transferred to the dead battery?

Figure P1.12

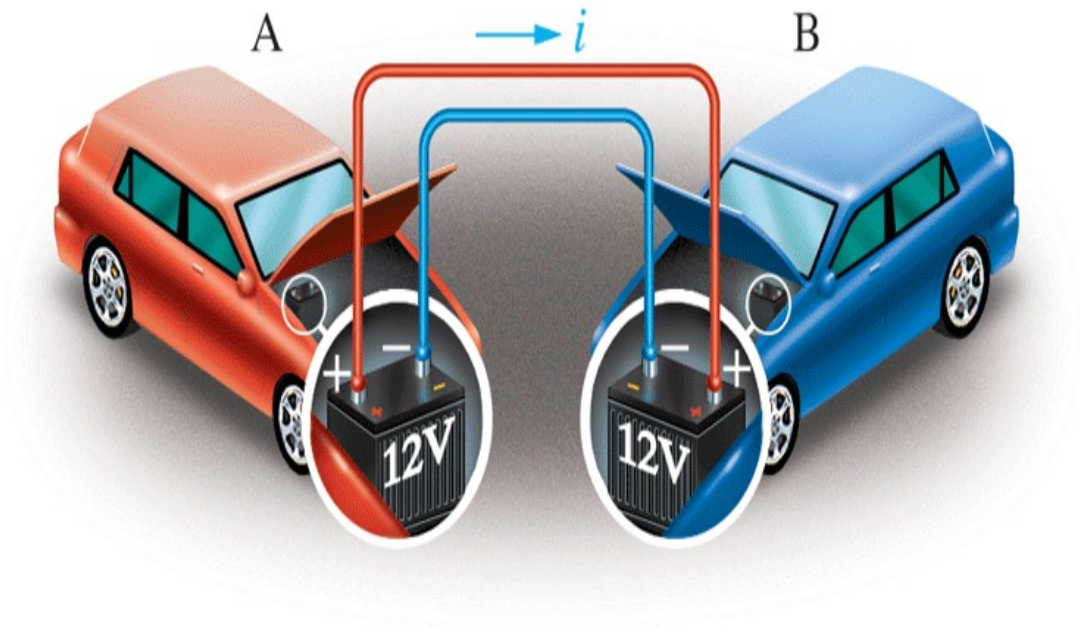


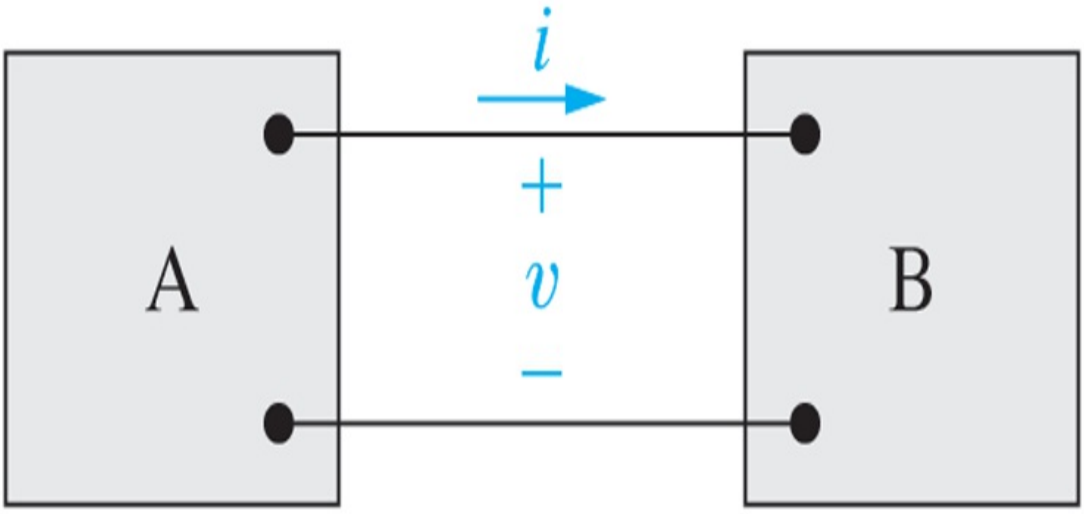
Figure P1.12 Full Alternative Text

2. 1.13 Two electric circuits, represented by boxes A and B, are connected

as shown in Fig. P1.13. The reference direction for the current i in the interconnection and the reference polarity for the voltage v across the interconnection are as shown in the figure. For each of the following sets of numerical values, calculate the power in the interconnection and state whether the power is flowing from A to B or vice versa.

1. a) i=6 A,v=30 V
2. b) i=−8 A,v=−20 V
3. c) i=4 A,v=−60 V
4. d) i=−9 A,v=40 V

Figure P1.13



3. 1.14 One 12 V battery supplies 100 mA to a boom box. How much energy does the battery supply in 4 h?

4. 1.15 The references for the voltage and current at the terminals of a circuit element are as shown in Fig. 1.6(d). The numerical values for v and i are −20 V and 5 A.

    1. Calculate the power at the terminals and state whether the power is

being absorbed or delivered by the element in the box.

2. Given that the current is due to electron flow, state whether the electrons are entering or leaving terminal 2.

3. Do the electrons gain or lose energy as they pass through the element in the box?

5. 1.16 Repeat Problem 1.15 with a current of −5 A.

6. 1.17 The manufacturer of a 6 V dry-cell flashlight battery says that the battery will deliver 15 mA for 60 continuous hours. During that time the voltage will drop from 6 V to 4 V. Assume the drop in voltage is linear with time. How much energy does the battery deliver in this 60 h interval?

7. 1.18 PSPICEMULTISIM The voltage and current at the terminals of the circuit element in Fig. 1.5 are zero for $t<0$. For $t\geq 0$ they are

   $v=75-75e^{-1000t}$ V, $i=50e^{-1000t}$ mA.

   1. a) Find the maximum value of the power delivered to the circuit.

   2. b) Find the total energy delivered to the element.

8. 1.19 The voltage and current at the terminals of the circuit element in Fig. 1.5 are zero for $t<0$. For $t\geq 0$ they are

   $v=15e^{-250t}$ V, $i=40e^{-250t}$ mA.

   1. a) Calculate the power supplied to the element at 10 ms.

   2. b) Calculate the total energy delivered to the circuit element.

9. 1.20 PSPICEMULTISIM The voltage and current at the terminals of the circuit element in Fig. 1.5 are zero for $t<0$. For $t\geq 0$ they are

   $v=(1500t+1)e^{-750t}$ V, $t\geq 0$; $i=40e^{-750t}$ mA, $t\geq 0$.

1. a) Find the time when the power delivered to the circuit element is maximum.

2. b) Find the maximum value of p in milliwatts.

3. c) Find the total energy delivered to the circuit element in microjoules.

10. 1.21 The voltage and current at the terminals of the circuit element in Fig. 1.5 are zero for $t<0$. For $t\geq 0$ they are

    $v=50e-1600t-50e-400t$ V, $i=5e-1600t-5e-400t$ mA.

    1. a) Find the power at $t=625$ μs.

    2. b) How much energy is delivered to the circuit element between 0 and 625 μs?

    3. c) Find the total energy delivered to the element.

11. 1.22 PSPICEMULTISIM The voltage and current at the terminals of the circuit element in Fig. 1.5 are zero for $t<0$. For $t\geq 0$ they are

    $v = (10{,}000t + 5) e-400t$ V, $i = (40t + 0.05) e-400t$ A.

    1. a) At what instant of time is maximum power delivered to the element?

    2. b) Find the maximum power in watts.

    3. c) Find the total energy delivered to the element in microjoules.

12. 1.23 PSPICEMULTISIM The voltage and current at the terminals of the circuit element in Fig. 1.5 are zero for $t<0$ and $t>40$ s. In the interval between 0 and 40 s the expressions are

    $v=t(1-0.025t)$ V, $0<t<40$ s; $i=4-0.2t$ A, $0<t<40$ s.

    1. a) At what instant of time is the power being delivered to the circuit

element maximum?

2. b) What is the power at the time found in part (a)?

3. c) At what instant of time is the power being extracted from the circuit element maximum?

4. d) What is the power at the time found in part (c)?

5. e) Calculate the net energy delivered to the circuit at 0, 10, 20, 30 and 40 s.

13. 1.24 PSPICEMULTISIM The voltage and current at the terminals of the element in Fig. 1.5 are

    $v=250\cos 800\pi t$ V, $i=8\sin 800\pi t$ A.

    1. a) Find the maximum value of the power being delivered to the element.

    2. b) Find the maximum value of the power being extracted from the element.

    3. c) Find the average value of p in the interval $0\leq 1\leq 2.5$ ms.

    4. d) Find the average value of p in the interval $0\leq t\leq 15.625$ ms.

14. 1.25 PSPICEMULTISIM The voltage and current at the terminals of the circuit element in Fig. 1.5 are zero for $t<0$. For $t\geq 0$ they are

    $v=100e^{-50t}\sin 150t$ V, $i=20e^{-50t}\sin 150t$ A.

    1. a) Find the power absorbed by the element at t=20 ms.

    2. b) Find the total energy absorbed by the element.

15. 1.26 An industrial battery is charged over a period of several hours at a constant voltage of 120 V. Initially, the current is 10 mA and increases linearly to 15 mA in 10 ks. From 10 ks to 20 ks, the current is constant

at 15 mA. From 20 ks to 30 ks the current decreases linearly to 10 mA. At 30 ks the power is disconnected from the battery.

1. a) Sketch the current from t=0 to t=30 ks.

2. b) Sketch the power delivered to the battery from t=0 to t=30 ks.

3. c) Using the sketch of the power, find the total energy delivered to the battery.

16. 1.27 PSPICEMULTISIM The voltage and current at the terminals of an automobile battery during a charge cycle are shown in Fig. P1.27.

Figure P1.27

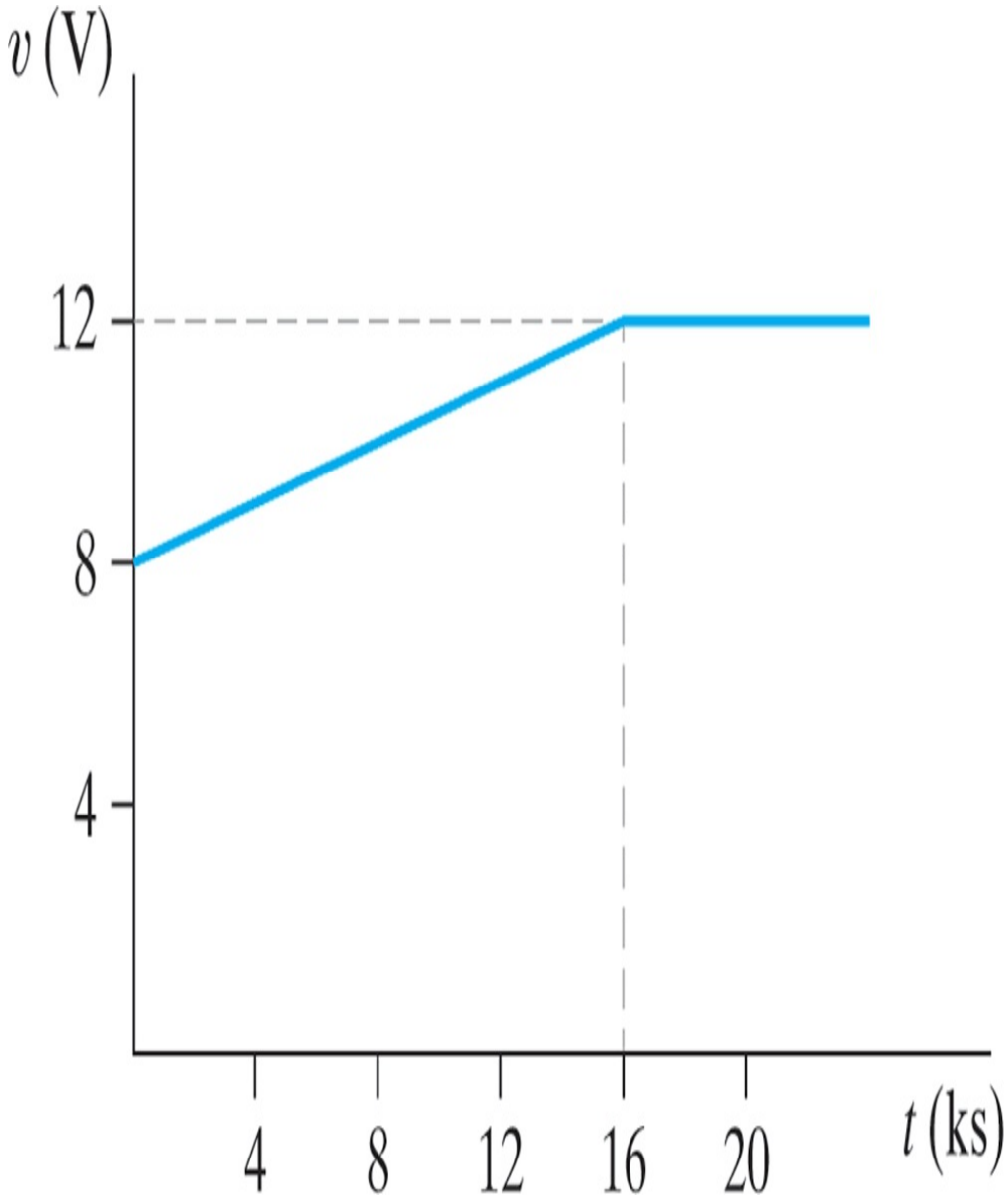
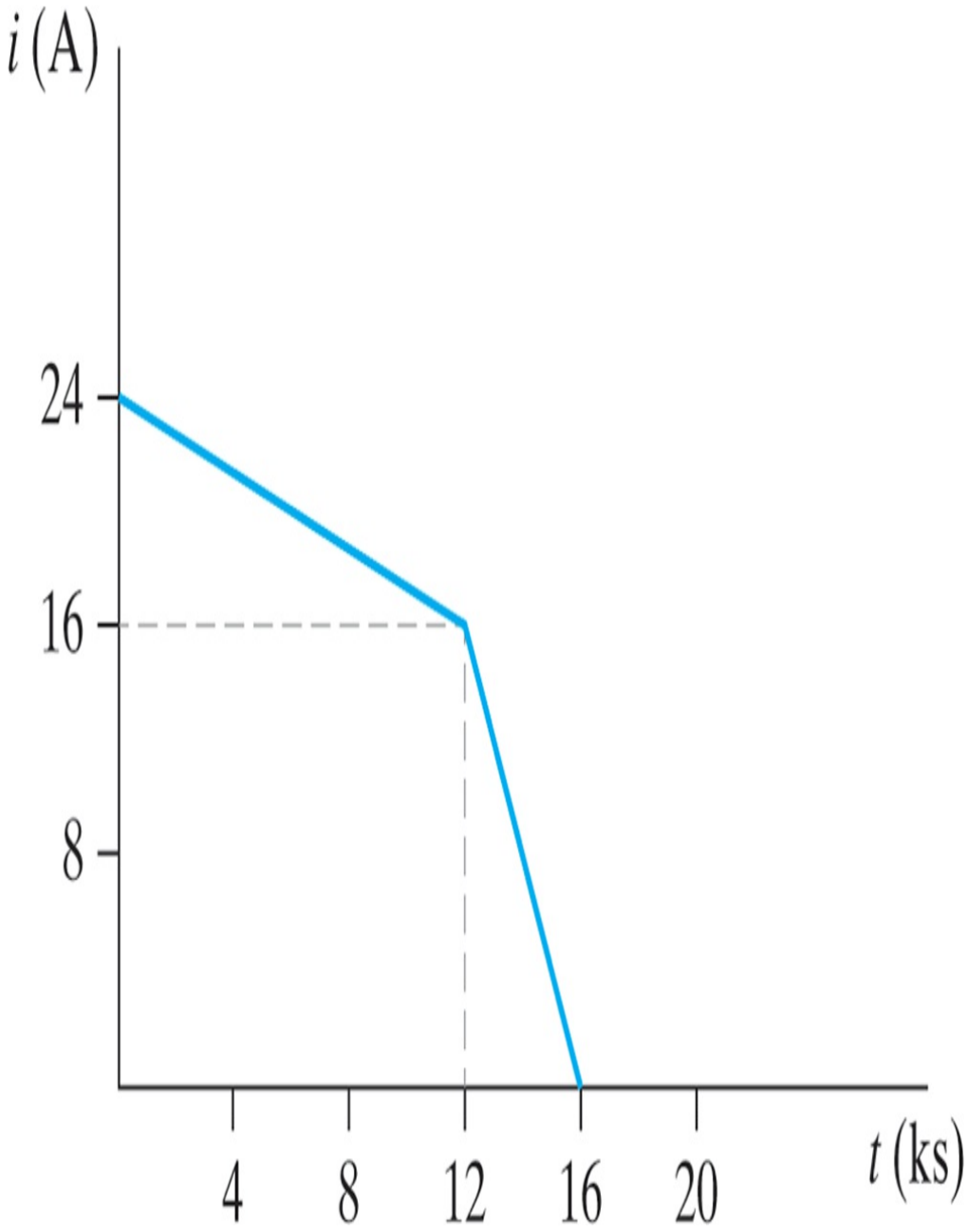


Figure P1.27 Full Alternative Text



1.2-11 Full Alternative Text

1. a) Calculate the total charge transferred to the battery.
2. b) Calculate the total energy transferred to the battery.
3.

17. 1.28 The voltage and current at the terminals of the circuit element in

Fig. 1.5 are shown in Fig. P1.28.

Figure P1.28

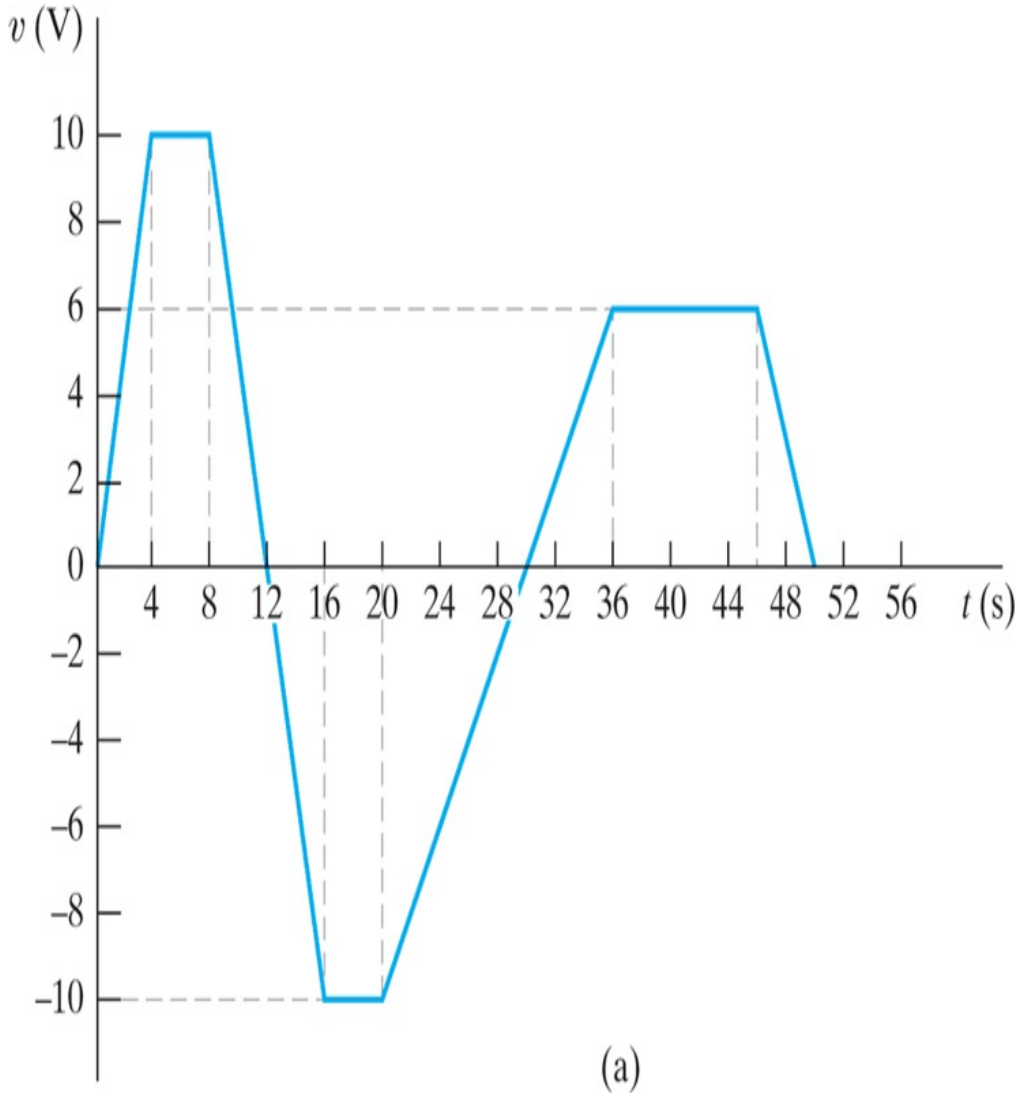
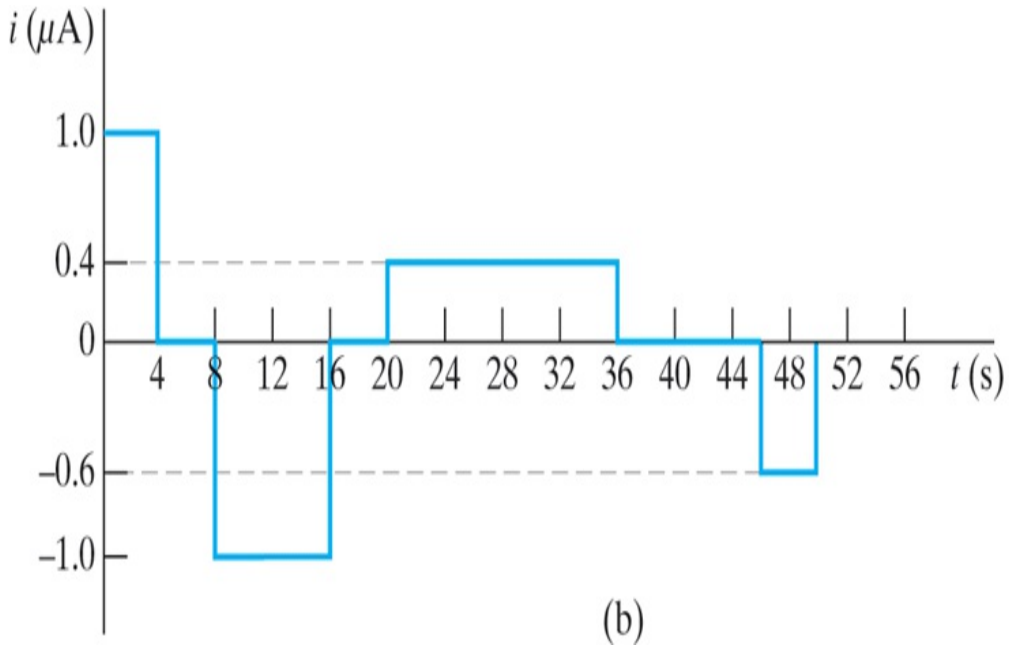


Figure P1.28 Full Alternative Text



1.2-11 Full Alternative Text

1. a) Sketch the power versus t plot for 0≤t≤50 s.

2. b) Calculate the energy delivered to the circuit element at t=4, 12, 36, and 50 s.

18. 1.29 The numerical values for the currents and voltages in the circuit in Fig. P1.29 are given in Table P1.29. Find the total power developed in the circuit.

Figure P1.29

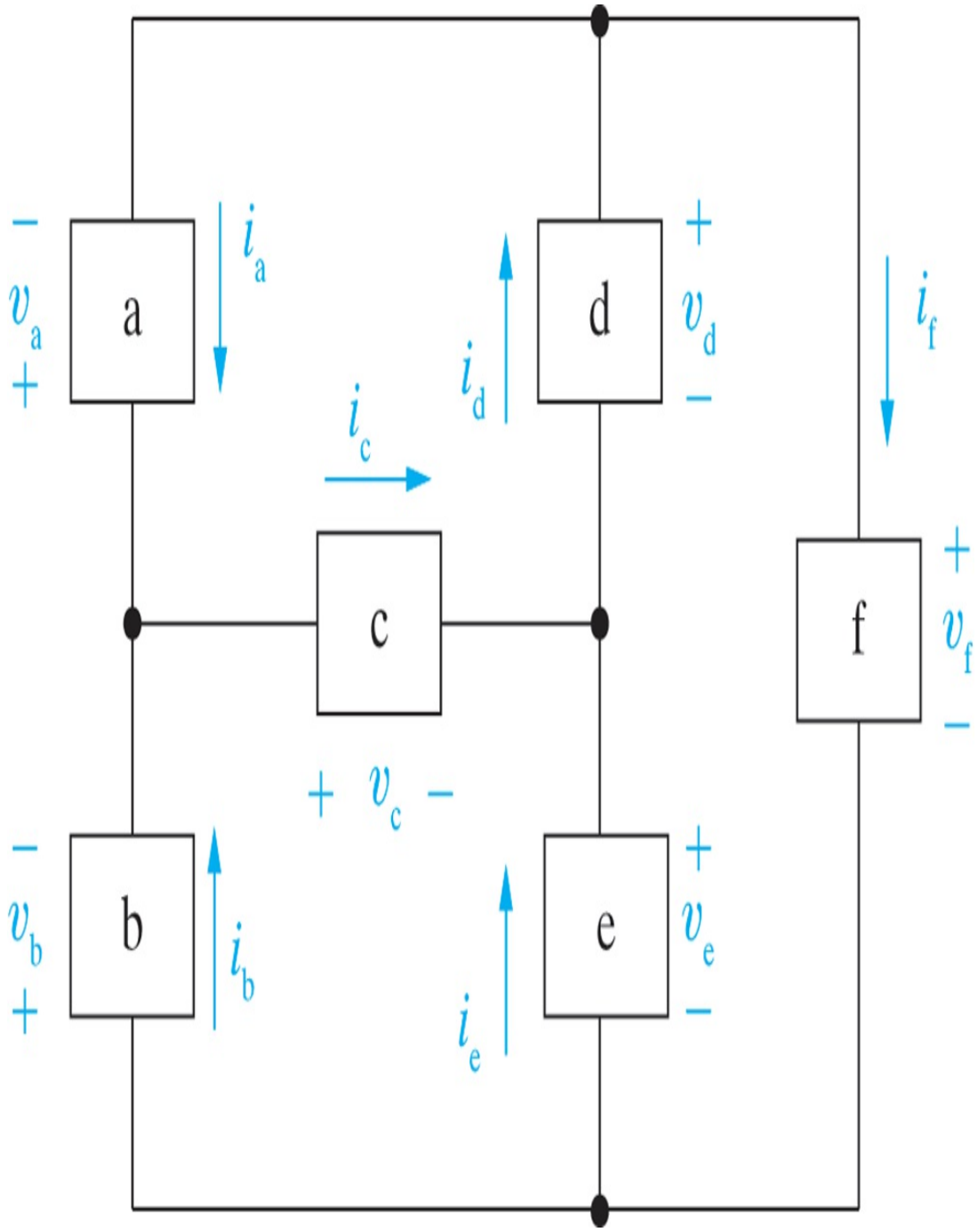


Figure P1.29 Full Alternative Text

Table P1.29

Element	Voltage (V)	Current (mA)
a	−18	−51
b	−18	45
c	2	−6
d	20	−20
e	16	−14
f	36	31

Table P1.29 Full Alternative Text

19. 1.30 The voltage and power values for each of the elements shown in Fig. P1.30 are given in Table P1.30.

Figure P1.30

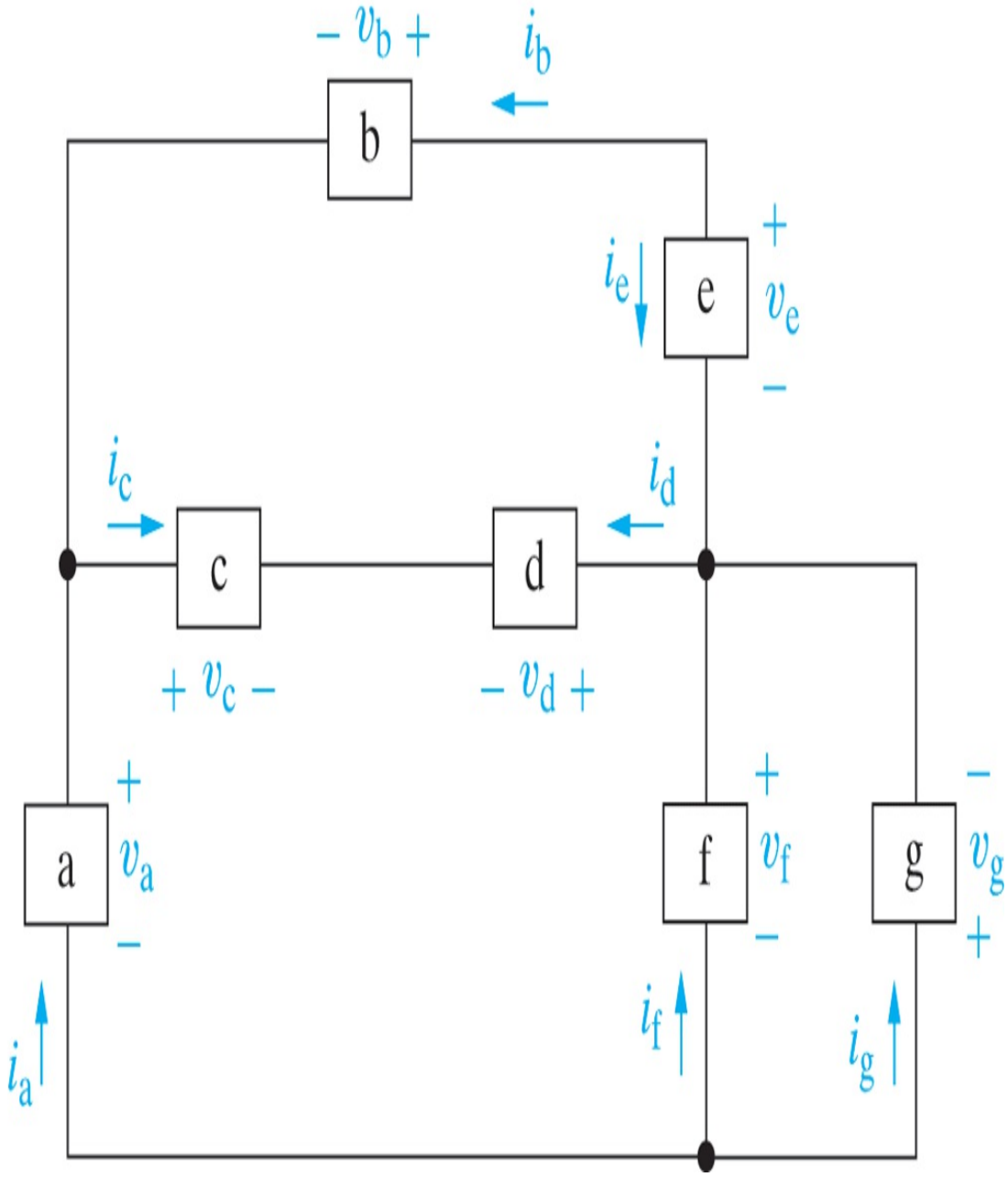


Figure P1.30 Full Alternative Text

1. a) Show that the interconnection of the elements satisfies the power check.

2. b) Find the value of the current through each of the elements using the values of power and voltage and the current directions shown in the figure.

Table P1.30

Element	Power (kW)	Voltage (V)
a	0.6 supplied	400
b	0.05 supplied	−100
c	0.4 absorbed	200
d	0.6 supplied	300
e	0.1 absorbed	−200
f	2.0 absorbed	500
g	1.25 supplied	−500

Table P1.30 Full Alternative Text

20. 1.31 The numerical values of the voltages and currents in the interconnection seen in Fig. P1.31 are given in Table P1.31. Does the interconnection satisfy the power check?

Figure P1.31

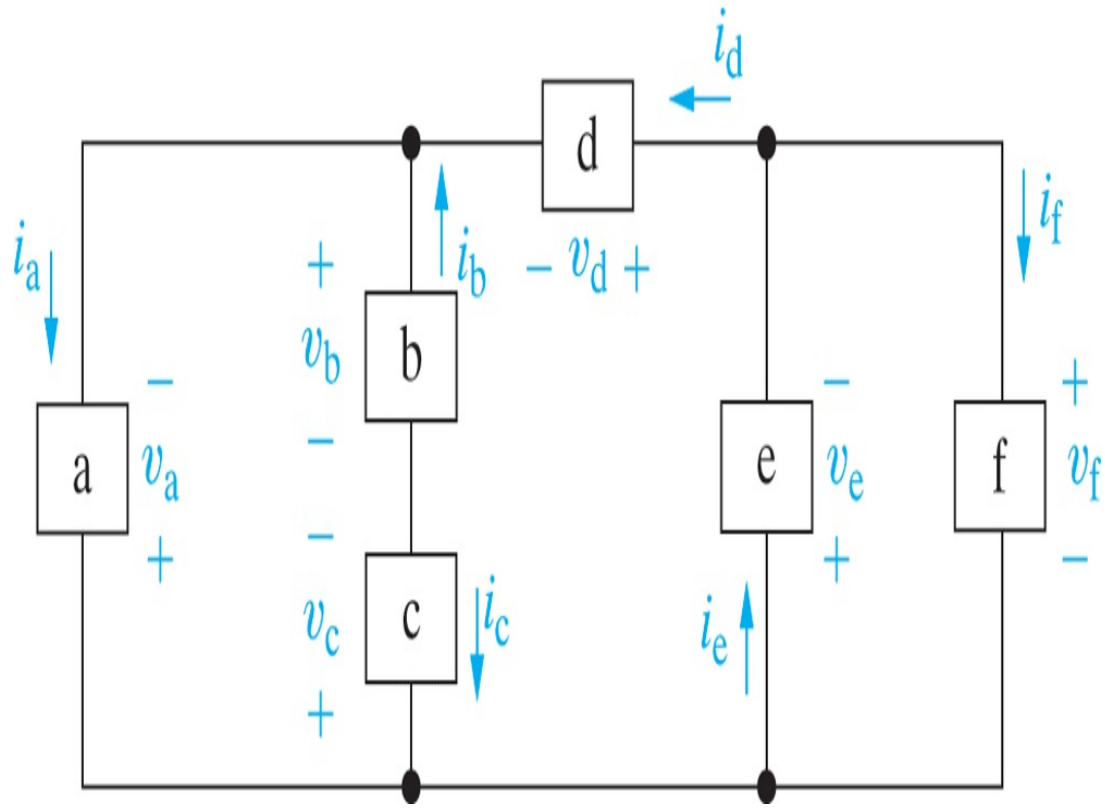


Figure P1.31 Full Alternative Text

Table P1.31

Element	Voltage (kV)	Current (mA)
a	−3	−250
b	4	−400
c	1	400
d	1	150
e	−4	200
f	4	50

Table P1.31 Full Alternative Text

21. 1.32 The current and power for each of the interconnected elements in Fig. P1.32 is measured. The values are listed in Table P1.32.

Figure P1.32

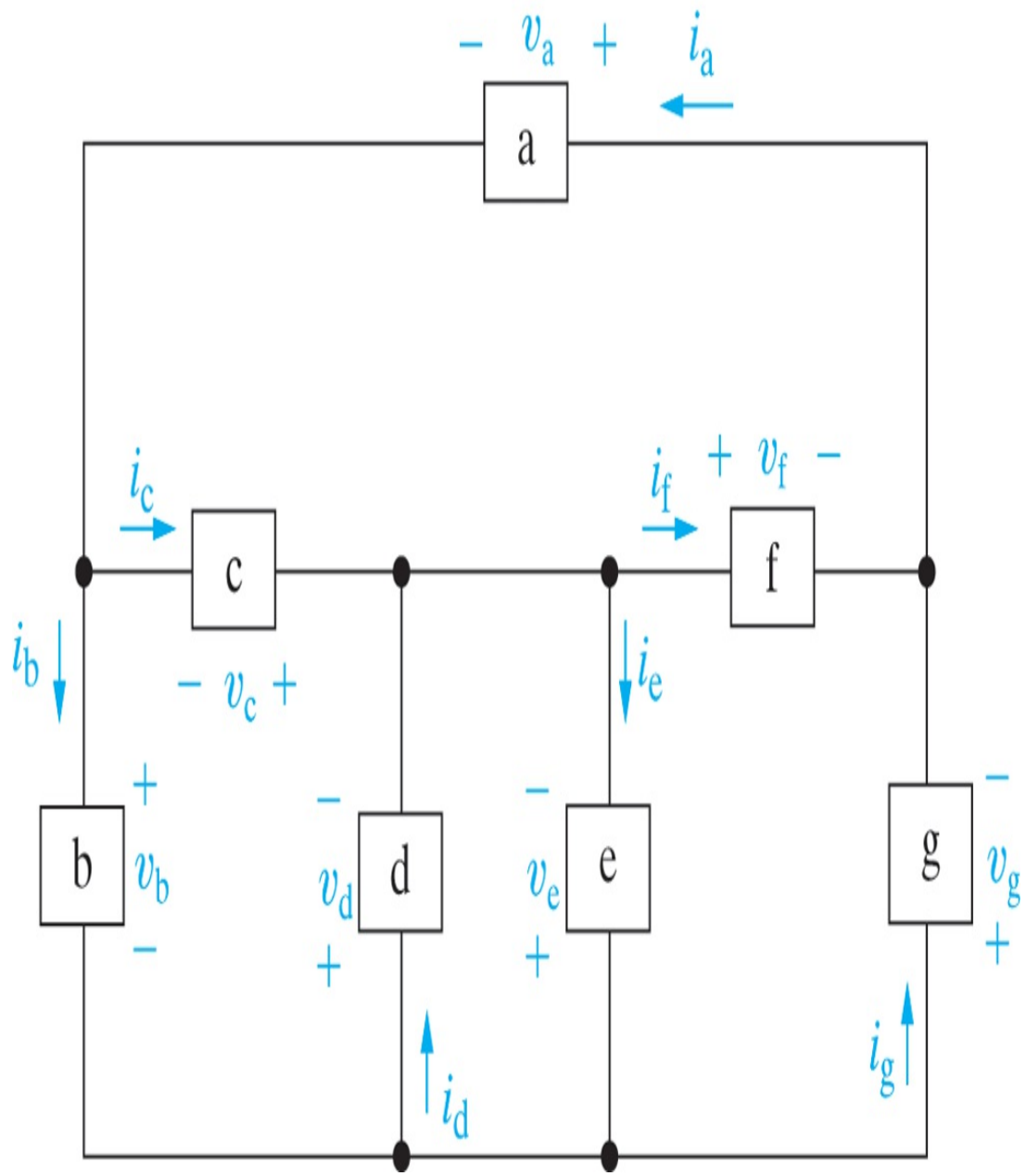


Figure P1.32 Full Alternative Text

1. a) Show that the interconnection satisfies the power check.
2. b) Identify the elements that absorb power.
3. c) Find the voltage for each of the elements in the interconnection, using the values of power and current and the voltage polarities shown in the figure.

Table P1.32

Element	Power (mW)	Current (mA)
a	175	25
b	375	75
c	150	−50
d	−320	40
e	160	20
f	120	−30
g	−660	55

Table P1.32 Full Alternative Text

22. 1.33 Assume you are an engineer in charge of a project and one of your subordinate engineers reports that the interconnection in Fig. P1.33 does not pass the power check. The data for the interconnection are given in Table P1.33.

Figure P1.33

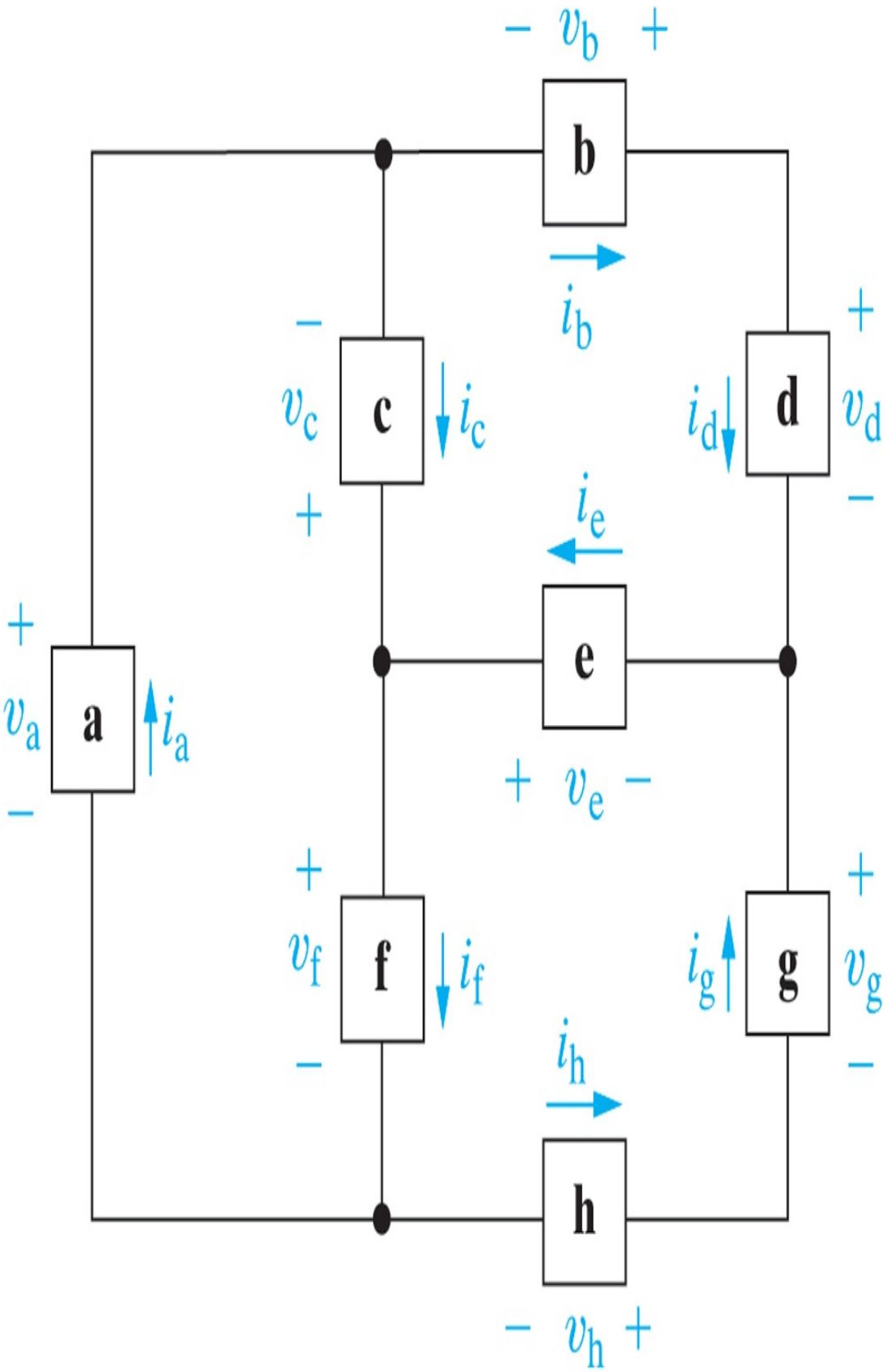

− v_b +
b
i_b
−
v_c
c
i_c
+
i_d
d
+
v_d
−
i_e
+
v_a
−
a
i_a
e
+ v_e −
+
v_f
−
f
i_f
i_g
g
+
v_g
−
i_h
h
− v_h +

Figure P1.33 Full Alternative Text

1. a) Is the subordinate correct? Explain your answer.
2. b) If the subordinate is correct, can you find the error in the data?

Table P1.33

Element	Voltage (V)	Current (A)
a	900	−22.5
b	105	−52.5
c	−600	−30.0
d	585	−52.5
e	−120	30.0
f	300	60.0
g	585	82.5
h	−165	82.5

Table P1.33 Full Alternative Text

23. 1.34 PRACTICALPERSPECTIVE Show that the power balances for the circuit shown in Fig. 1.7, using the voltage and current values given in Table 1.5, with the value of the current for component d changed to −1 A.

24. 1.35 PRACTICALPERSPECTIVE Suppose there is no power lost in the

wires used to distribute power in a typical home.

1. a) Create a new model for the power distribution circuit by modifying the circuit shown in Fig 1.7. Use the same names, voltage polarities, and current directions for the components that remain in this modified model.

2. b) The following voltages and currents are calculated for the components:

va=120 V ia=−10 A
vb=120 V ib=10 A
vf=−120 V if=3 A
vg=120 V
vh=−240 V ih=−7 A

If the power in this modified model balances, what is the value of the current in component g?

Chapter 2 Circuit Elements

Chapter Contents

1. 2.1 Voltage and Current Sources
2. 2.2 Electrical Resistance (Ohm’s Law)
3. 2.3 Constructing a Circuit Model
4. 2.4 Kirchhoff’s Laws
5. 2.5 Analyzing a Circuit Containing Dependent Sources

Chapter Objectives

1. Understand the symbols for and the behavior of the following ideal basic circuit elements: independent voltage and current sources, dependent voltage and current sources, and resistors.

2. Be able to state Ohm's law, Kirchhoff's current law, and Kirchhoff's voltage law, and be able to use these laws to analyze simple circuits.

3. Know how to calculate the power for each element in a simple circuit and be able to determine whether or not the power balances for the whole circuit.

There are five ideal basic circuit elements:

- voltage sources
- current sources
- resistors
- inductors
- capacitors

In this chapter, we discuss the characteristics of the first three circuit elements—voltage sources, current sources, and resistors. Although this may seem like a small number of elements, many practical systems can be modeled with just sources and resistors. They are also a useful starting point because of their relative simplicity; the mathematical relationships between voltage and current in sources and resistors are algebraic. Thus, you will be able to begin learning the basic techniques of circuit analysis with only algebraic manipulations.

We will postpone introducing inductors and capacitors until Chapter 6, because their use requires that you solve integral and differential equations.

However, the basic analytical techniques for solving circuits with inductors and capacitors are the same as those introduced in this chapter. So, by the time you need to begin manipulating more difficult equations, you should be very familiar with the methods of writing them.

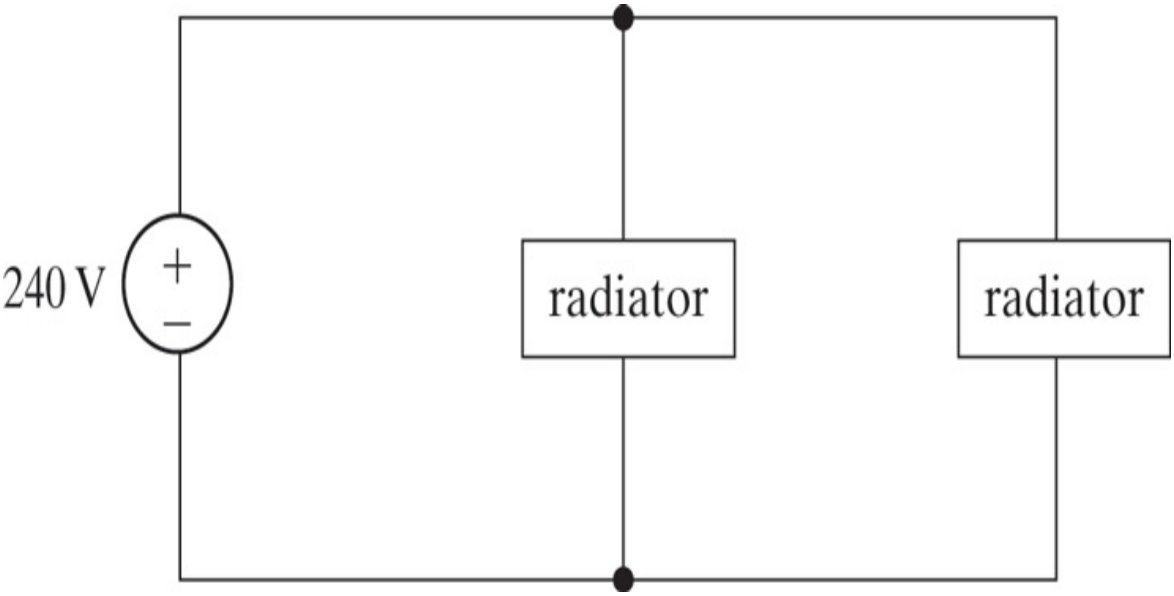
Practical Perspective

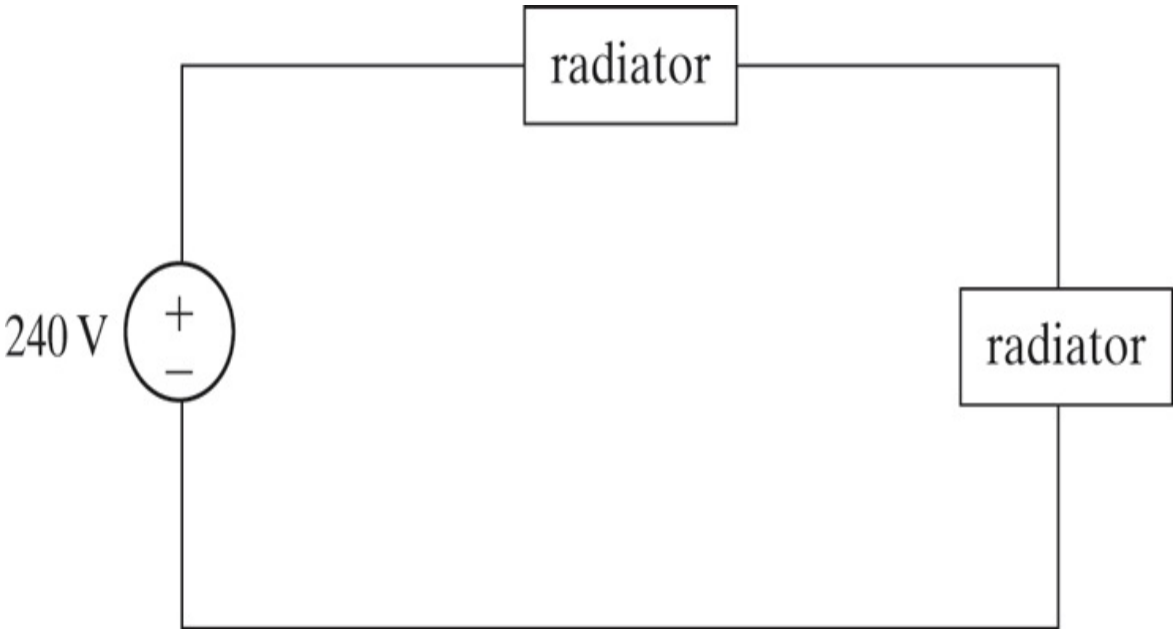
Heating with Electric Radiators

You want to heat your small garage using a couple of electric radiators. The power and voltage requirements for each radiator are 1200 W, 240 V. But you are not sure how to wire the radiators to the power supplied to the garage. Should you use the wiring diagram on the left or the one on the right? Does it make any difference?

Once you have studied the material in this chapter, you will be able to answer these questions and determine how to heat the garage. The Practical Perspective at the end of this chapter guides you through the analysis of two circuits based on the two wiring diagrams shown below.

Style-Photography/Fotolia




radiator
240 V
+
−
radiator

2.1 Voltage and Current Sources

An **electrical source** is a device capable of converting nonelectric energy to electric energy and vice versa. For example, a discharging battery converts chemical energy to electric energy, whereas a charging battery converts electric energy to chemical energy. A *dynamo* is a machine that converts mechanical energy to electric energy and vice versa. For operations in the mechanical-to-electric mode, it is called a *generator*. For transformations from electric to mechanical energy, it is called a *motor*. Electric sources either deliver or absorb electric power while maintaining either voltage or current. This behavior led to the creation of the ideal voltage source and the ideal current source as basic circuit elements.

- An **ideal voltage source** is a circuit element that maintains a prescribed voltage across its terminals regardless of the current flowing in those terminals.

- An **ideal current source** is a circuit element that maintains a prescribed current through its terminals regardless of the voltage across those terminals.

These circuit elements do not exist as practical devices—they are idealized models of actual voltage and current sources.

Using an ideal model for current and voltage sources constrains the mathematical descriptions of these components. For example, because an ideal voltage source provides a steady voltage even if the current in the element changes, it is impossible to specify the current in an ideal voltage source as a function of its voltage. Likewise, if the only information you have about an ideal current source is the value of current supplied, it is impossible to determine the voltage across that current source. We have sacrificed our ability to relate voltage and current in a practical source for the simplicity of using ideal sources in circuit analysis.

Ideal voltage and current sources can be further described as either

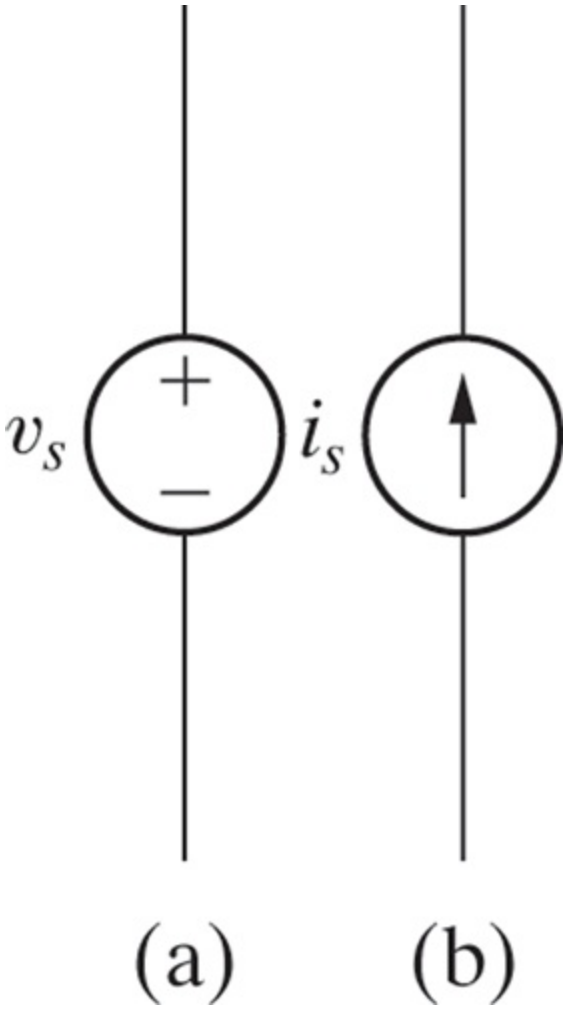
independent sources or *dependent sources*.

- An **independent source** establishes a voltage or current in a circuit without relying on voltages or currents elsewhere in the circuit. The value of the voltage or current supplied is specified by the value of the independent source alone.

- A **dependent source**, in contrast, establishes a voltage or current whose value depends on the value of a voltage or current elsewhere in the circuit. You cannot specify the value of a dependent source unless you know the value of the voltage or current on which it depends.

The circuit symbols for the ideal independent sources are shown in Fig. 2.1. Note that a circle is used to represent an independent source. To completely specify an ideal independent voltage source in a circuit, you must include the value of the supplied voltage and the reference polarity, as shown in Fig. 2.1(a). Similarly, to completely specify an ideal independent current source, you must include the value of the supplied current and its reference direction, as shown in Fig. 2.1(b).

Figure 2.1 The circuit symbols for (a) an ideal independent voltage source and (b) an ideal independent current source.



The circuit symbol for an ideal dependent source is a diamond, as shown in Fig. 2.2. There are four possible variations because both dependent current sources and dependent voltage sources can be controlled by either a voltage or a current elsewhere in the circuit. Dependent sources are sometimes called controlled sources.

Figure 2.2 (a) (b) Circuit symbols for ideal dependent voltage sources and (c) (d) ideal dependent current sources.

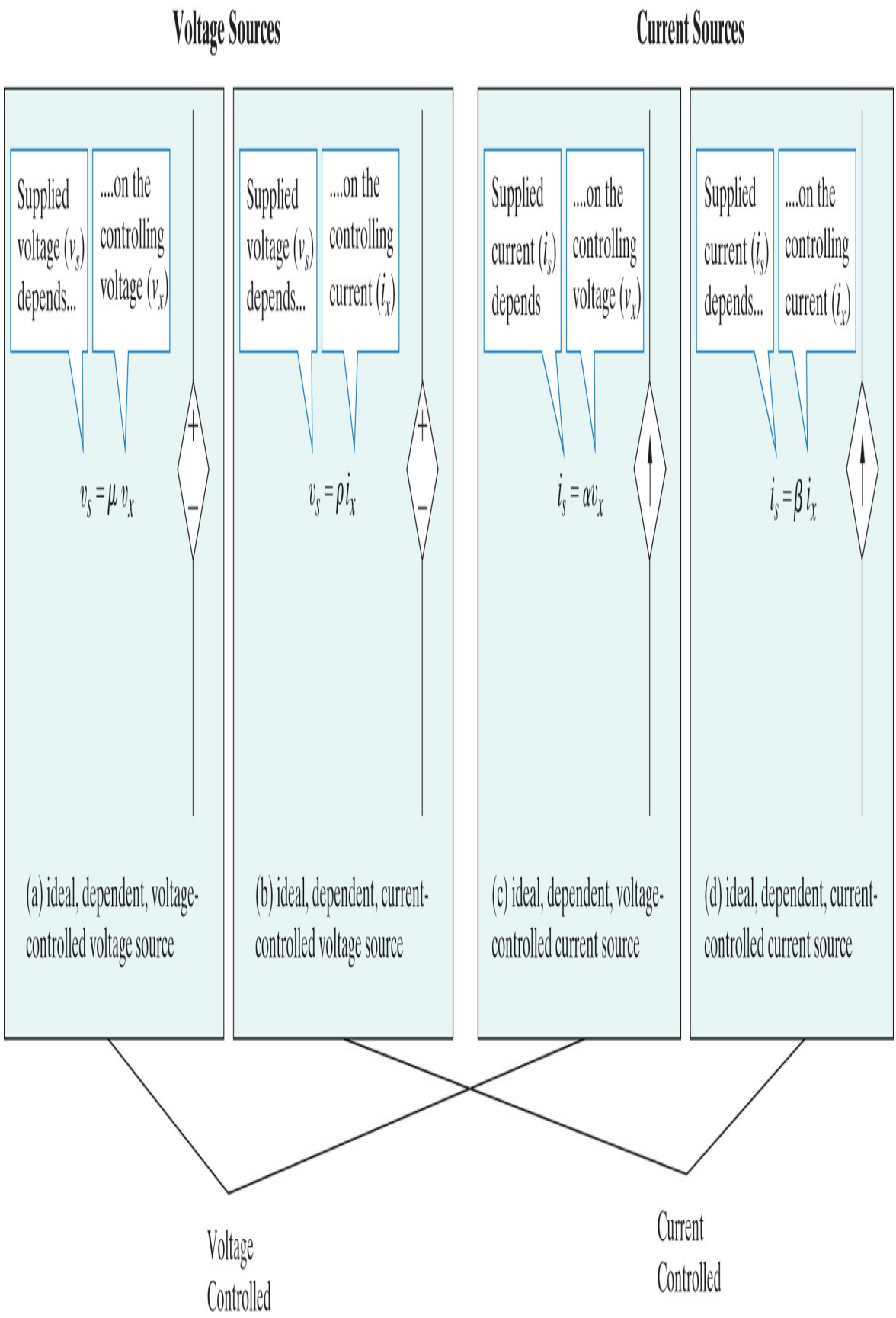

Voltage Sources
Current Sources
Supplied voltage (v_s) depends...
....on the controlling voltage (v_x)
$v_s = \mu v_x$
Supplied voltage (v_s) depends...
....on the controlling current (i_x)
$v_s = \rho i_x$
Supplied current (i_s) depends
....on the controlling voltage (v_x)
$i_s = \alpha v_x$
Supplied current (i_s) depends...
....on the controlling current (i_x)
$i_s = \beta i_x$
(a) ideal, dependent, voltage-controlled voltage source
(b) ideal, dependent, current-controlled voltage source
(c) ideal, dependent, voltage-controlled current source
(d) ideal, dependent, current-controlled current source
Voltage Controlled
Current Controlled

Figure 2.2 Full Alternative Text

To completely specify an ideal dependent voltage-controlled voltage source, you must identify the controlling voltage, the equation that permits you to compute the supplied voltage from the controlling voltage, and the reference polarity for the supplied voltage. For example, in Fig. 2.2(a), the controlling voltage is vx, the equation that determines the supplied voltage vs is

$$v_s = \mu v_x,$$

and the reference polarity for vs is as indicated. Note that μ is a multiplying constant that is dimensionless.

Similar requirements exist for completely specifying the other ideal dependent sources. In Fig. 2.2(b), the controlling current is ix, the equation for the supplied voltage vs is

$$v_s = \rho i_x,$$

the reference polarity is as shown, and the multiplying constant ρ has the dimension volts per ampere. In Fig. 2.2(c), the controlling voltage is vx, the equation for the supplied current is is

$$i_s = \alpha v_x,$$

the reference direction is as shown, and the multiplying constant α has the dimension amperes per volt. In Fig. 2.2(d), the controlling current is ix, the equation for the supplied current is is

$$i_s = \beta i_x,$$

the reference direction is as shown, and the multiplying constant β is dimensionless.

Note that the ideal independent and dependent voltage and current sources generate either constant voltages or currents, that is, voltages and currents that are invariant with time. Constant sources are often called **dc sources**. The *dc* stands for *direct current*, a description that has a historical basis but

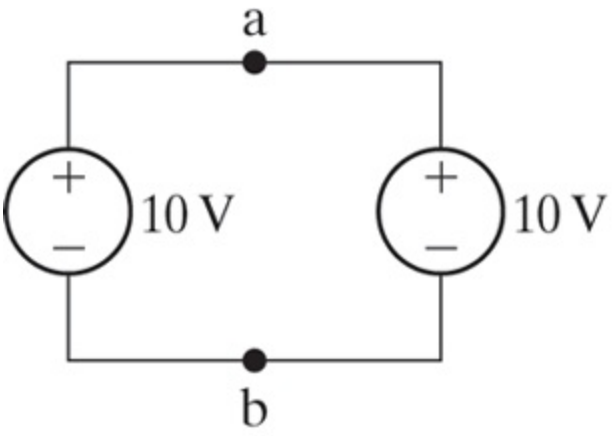
can seem misleading now. Historically, a direct current was defined as a current produced by a constant voltage. Therefore, a constant voltage became known as a direct current, or dc, voltage. The use of *dc* for *constant* stuck, and the terms *dc current* and *dc voltage* are now universally accepted in science and engineering to mean constant current and constant voltage.

Finally, we note that ideal sources are examples of active circuit elements. An **active element** is one that models a device capable of generating electric energy. **Passive elements** model physical devices that cannot generate electric energy. Resistors, inductors, and capacitors are examples of passive circuit elements. Examples 2.1 and 2.2 illustrate how the characteristics of ideal independent and dependent sources limit the types of permissible interconnections of the sources.

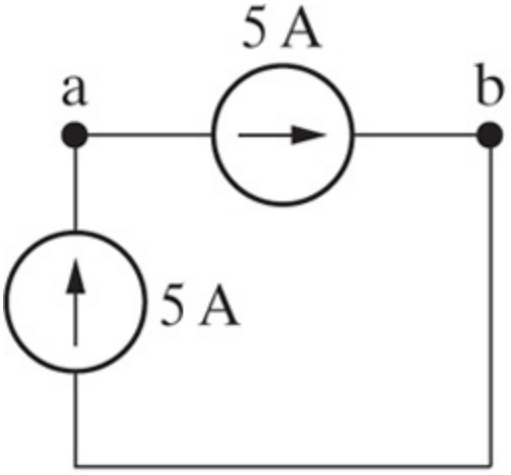
Example 2.1 Testing Interconnections of Ideal Sources

Use the definitions of the ideal independent voltage and current sources to determine which interconnections in Fig. 2.3 are permitted and which violate the constraints imposed by the ideal sources.

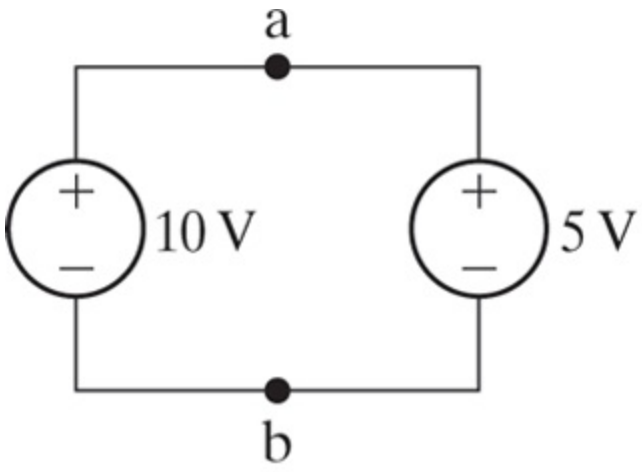
Figure 2.3 The circuits for Example 2.1.



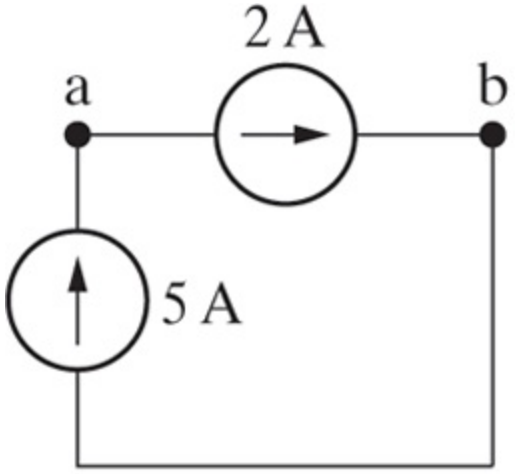
(a)



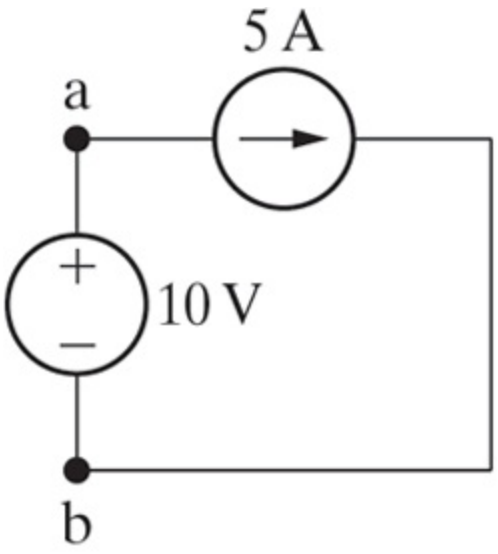
(b)



(c)



(d)



(e)

Solution

Connection (a) is permitted. Each source supplies voltage across the same pair of terminals, marked a and b. This requires that each source supply the same voltage with the same polarity, which they do.

Connection (b) is permitted. Each source supplies current through the same pair of terminals, marked a and b. This requires that each source supply the same current in the same direction, which they do.

Connection (c) is not permitted. Each source supplies voltage across the same pair of terminals, marked a and b. This requires that each source supply the same voltage with the same polarity, which they do not.

Connection (d) is not permitted. Each source supplies current through the same pair of terminals, marked a and b. This requires that each source supply the same current in the same direction, which they do not.

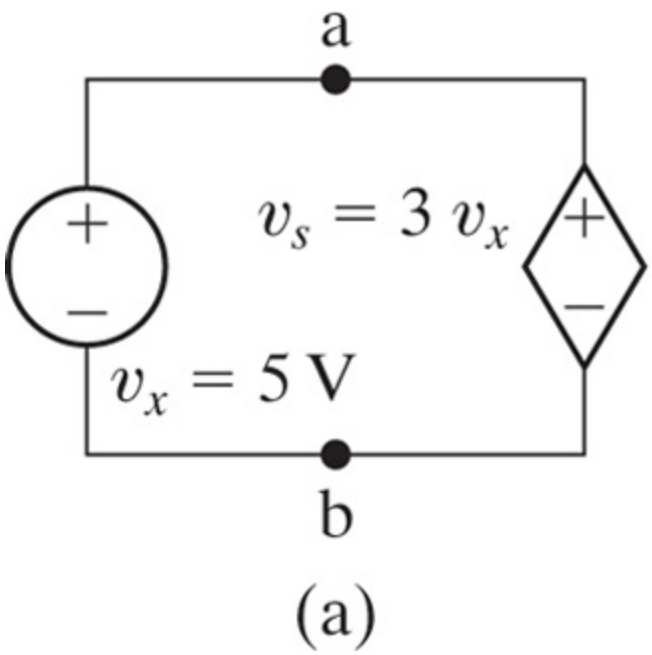
Connection (e) is permitted. The voltage source supplies voltage across the pair of terminals marked a and b. The current source supplies current through the same pair of terminals. Because an ideal voltage source supplies the same

voltage regardless of the current, and an ideal current source supplies the same current regardless of the voltage, this connection is permitted.

Example 2.2 Testing Interconnections of Ideal Independent and Dependent Sources

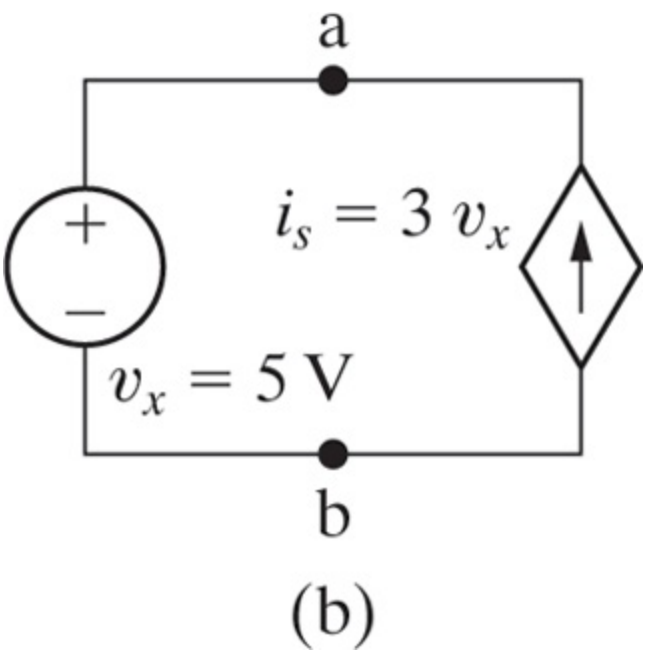
State which interconnections in Fig. 2.4 are permitted and which violate the constraints imposed by the ideal sources, using the definitions of the ideal independent and dependent sources.

Figure 2.4 The circuits for Example 2.2.



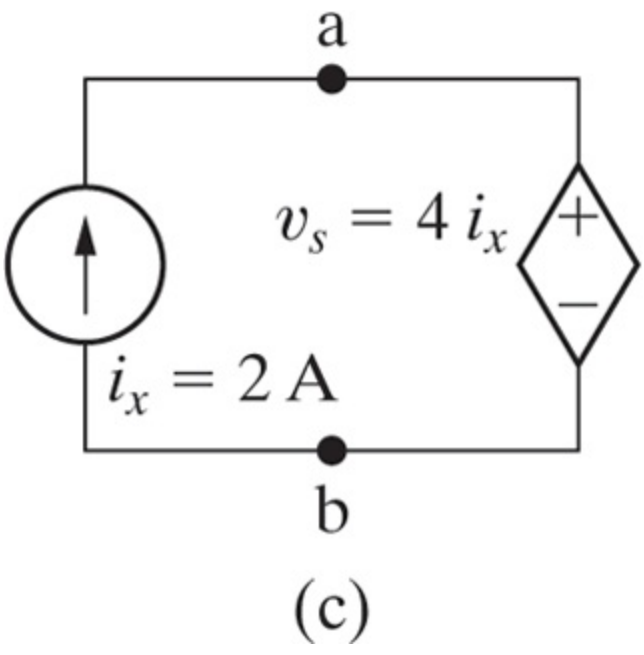
(a)

Figure 2.4 Full Alternative Text



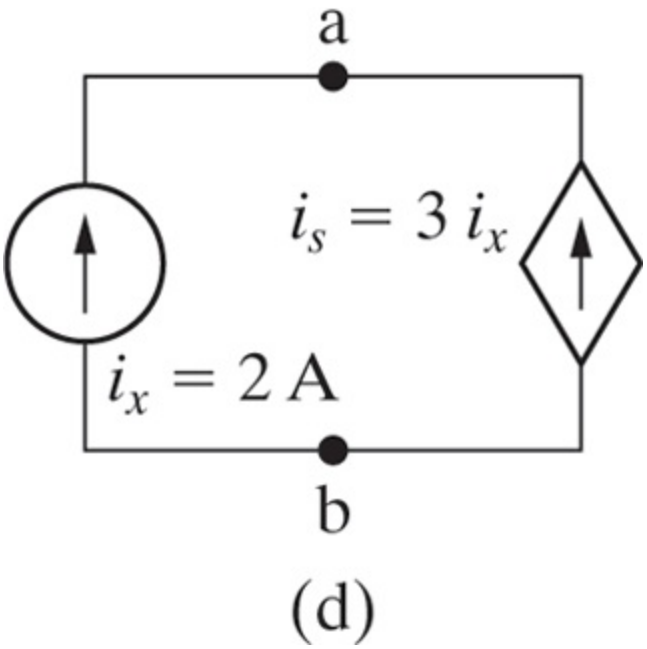
(b)

2.1-4 Full Alternative Text



(c)

2.1-4 Full Alternative Text



(d)

2.1-4 Full Alternative Text

Solution

Connection (a) is not permitted. Both the independent source and the dependent source supply voltage across the same pair of terminals, labeled a and b. This requires that each source supply the same voltage with the same polarity. The independent source supplies 5 V, but the dependent source supplies 15 V.

Connection (b) is permitted. The independent voltage source supplies voltage across the pair of terminals marked a and b. The dependent current source supplies current through the same pair of terminals. Because an ideal voltage source supplies the same voltage regardless of current, and an ideal current source supplies the same current regardless of voltage, this is a valid connection.

Connection (c) is permitted. The independent current source supplies current through the pair of terminals marked a and b. The dependent voltage source supplies voltage across the same pair of terminals. Because an ideal current source supplies the same current regardless of voltage, and an ideal voltage

source supplies the same voltage regardless of current, this is a valid connection.

Connection (d) is not permitted. Both the independent source and the dependent source supply current through the same pair of terminals, labeled a and b. This requires that each source supply the same current in the same direction. The independent source supplies 2 A, but the dependent source supplies 6 A in the opposite direction.

Assessment Problems

Objective 1—Understand ideal basic circuit elements

1. 2.1 For the circuit shown,

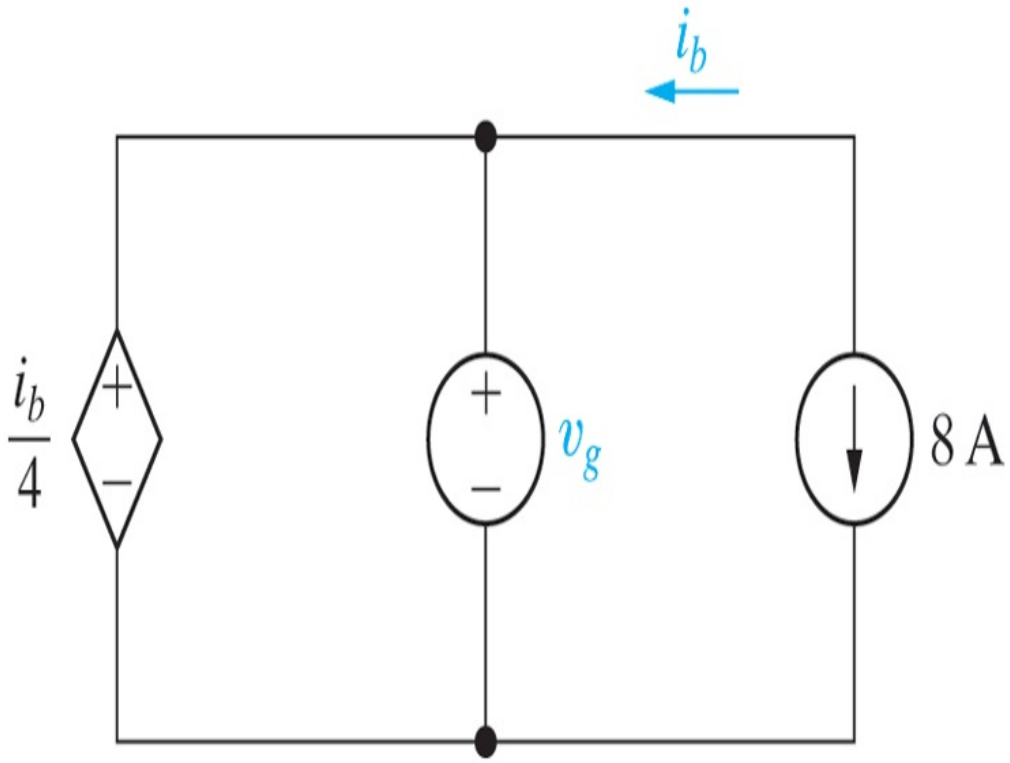
    1. What value of vg is required in order for the interconnection to be valid?

    2. For this value of vg, find the power associated with the 8 A source.

    Answer

    1. −2 V;

    2. −16 W (16 W delivered).



2.1-4 Full Alternative Text

2. 2.2 For the circuit shown,

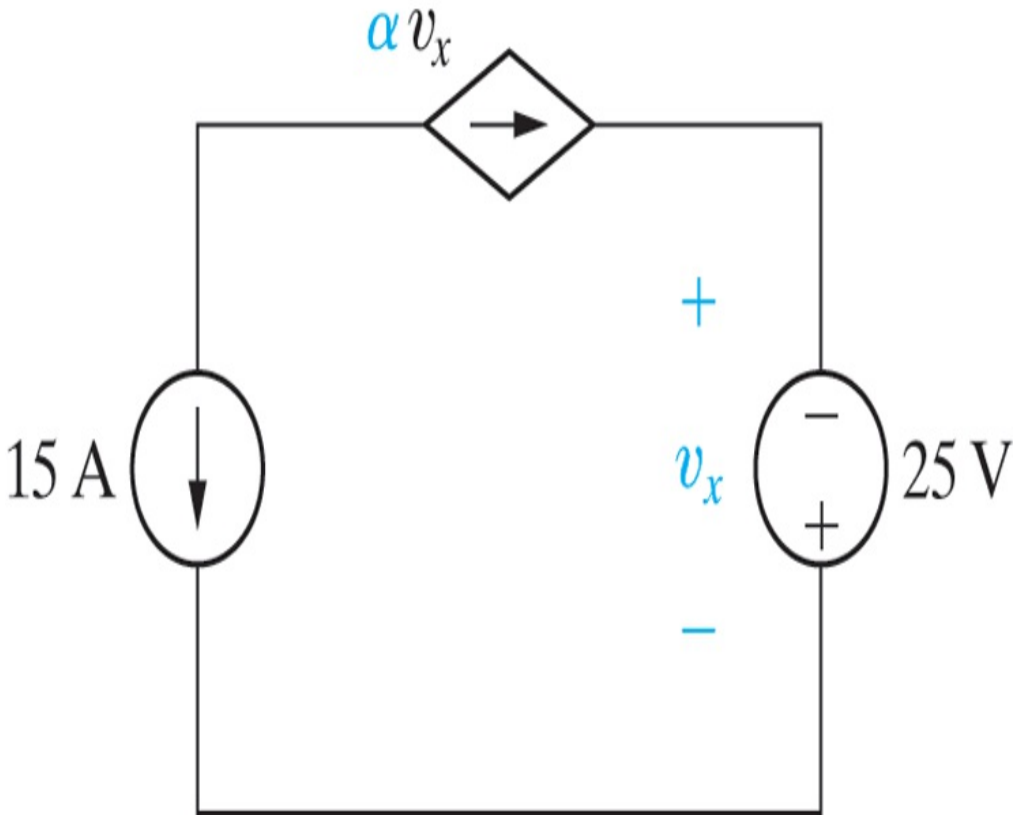
    1. What value of α is required in order for the interconnection to be valid?

    2. For the value of α calculated in part (a), find the power associated with the 25 V source.

    Answer

    1. 0.6 A/V;

    2. 375 W (375 W absorbed).



2.1-5 Full Alternative Text

SELF-CHECK: *Also try Chapter Problems 2.6 and 2.7.*

2.2 Electrical Resistance (Ohm's Law)

Resistance is the capacity of materials to impede the flow of current or, more specifically, the flow of electric charge. The circuit element modeling this behavior is the **resistor**. Figure 2.5 shows the resistor's circuit symbol, with R denoting the resistance value of the resistor.

Figure 2.5 The circuit symbol for a resistor having a resistance R.

To understand resistance, think about the electrons that make up electric current moving through, interacting with, and being resisted by the atomic structure of some material. The interactions convert some electric energy to thermal energy, dissipated as heat. Many useful electrical devices take advantage of resistance heating, including stoves, toasters, irons, and space heaters.

Most materials resist electric current; the amount of resistance depends on the material. Metals like copper and aluminum have small values of resistance, so they are often used as wires conducting electric current. When represented

in a circuit diagram, copper or aluminum wiring isn't usually modeled as a resistor; the wire's resistance is so small compared to the resistance of other circuit elements that we can neglect the wiring resistance to simplify the diagram.

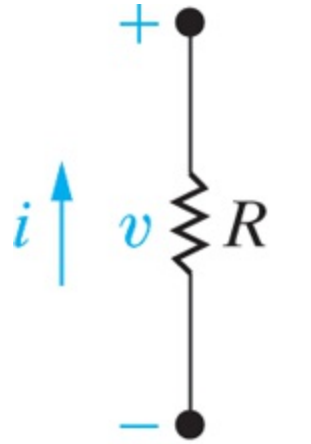
A resistor is an ideal basic circuit element, which is described mathematically using its voltage and current. The relationship between voltage and current for a resistor is known as **Ohm's law**, after Georg Simon Ohm, a German physicist who established its validity early in the nineteenth century. Consider the resistor shown in Fig. 2.6(a), where the current in the resistor is in the direction of the voltage drop across the resistor. For this resistor, Ohm's law is

Figure 2.6 Two possible reference choices for the current and voltage at the terminals of a resistor and the resulting equations.

$v = iR$

(a)

Figure 2.6 Full Alternative Text



$v = -iR$

(b)

2.2-6 Full Alternative Text

Ohm’s Law

v=iR, (2.1)

where

v=the voltage in volts,i=the current in amperes,R=the resistance in ohms.

For the resistor in Fig. 2.6(b), Ohm’s law is

v=−iR, (2.2)

where v, i, and R are again measured in volts, amperes, and ohms, respectively. We use the passive sign convention (Section 1.5) in determining the algebraic signs in Eqs. 2.1 and 2.2.

Resistance is measured in the SI unit ohms. The Greek letter Omega (Ω) is the standard symbol for an ohm. The circuit diagram symbol for an 8 Ω resistor is shown in Fig. 2.7.

Figure 2.7 The circuit symbol for an 8 Ω resistor.

Ohm’s law expresses the voltage as a function of the current. However, expressing the current as a function of the voltage also is convenient. Thus, from Eq. 2.1,

$$i = \frac{v}{R}, \quad (2.3)$$

or, from Eq. 2.2,

$$i = -\frac{v}{R}. \quad (2.4)$$

The reciprocal of the resistance is referred to as **conductance**, is symbolized by the letter G, and is measured in siemens (S). Thus,

$$G = \frac{1}{R}. \quad (2.5)$$

An 8 Ω resistor has a conductance value of 0.125 S.

Ideal resistors model the behavior of physical devices. The word *ideal* reminds us that the resistor model makes several simplifying assumptions about the behavior of actual resistive devices. Assuming the resistance of the ideal resistor is constant, so that its value does not vary over time, is the most important of these simplifications. Most actual resistive devices have a time-varying resistance, often because the temperature of the device changes over time. The ideal resistor model represents a physical device whose resistance doesn't vary much from some constant value over the time period of interest in the circuit analysis. In this book, we assume that the simplifying assumptions about resistance devices are valid, and we thus use ideal resistors in circuit analysis.

We can calculate the power at the terminals of a resistor in several ways. The first approach is to use the defining equation (Section 1.6) to calculate the product of the terminal voltage and current. For the resistor shown in Fig. 2.6(a), we write

$$p = vi, \quad (2.6)$$

and for the resistor shown in Fig. 2.6(b), we write

$$p = -vi. \quad (2.7)$$

A second method expresses resistor power in terms of the current and the resistance. Substituting Eq. 2.1 into Eq. 2.6, we obtain

$$p = vi = (i\,R)i$$

So

Power in a Resistor in Terms of Current

$$p = i^2 R. \quad (2.8)$$

Likewise, substituting Eq. 2.2 into Eq. 2.7, we have

$$p = -vi = -(-i\,R)i = i\,2R. \quad (2.9)$$

Equations 2.8 and 2.9 are identical, demonstrating that regardless of voltage polarity and current direction, the power at the terminals of a resistor is positive. Therefore, resistors absorb power from the circuit.

A third method expresses resistor power in terms of the voltage and resistance. The expression is independent of the polarity references, so

Power in a Resistor in Terms of Voltage

$$p = v^2 R. \quad (2.10)$$

Sometimes a resistor's value will be expressed as a conductance rather than as a resistance. Using the relationship between resistance and conductance given in Eq. 2.5, we can also write Eqs. 2.9 and 2.10 in terms of the conductance, or

$$p = i^2 G, \quad (2.11)$$
$$p = v^2 G. \quad (2.12)$$

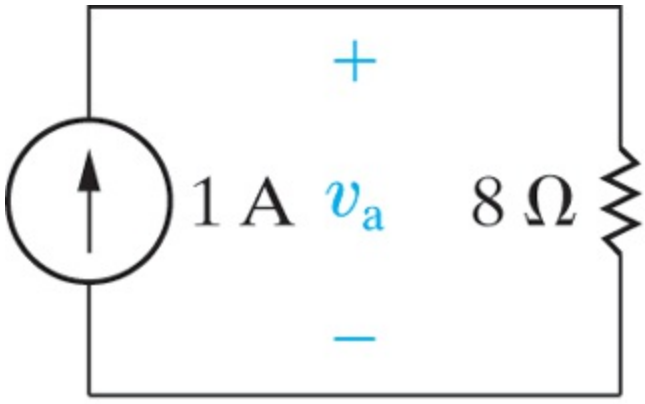
Equations 2.6–2.12 provide a variety of methods for calculating the power absorbed by a resistor. Each yields the same answer. In analyzing a circuit, look at the information provided and choose the power equation that uses that information directly.

Example 2.3 illustrates Ohm's law for a circuit with an ideal source and a resistor. Power calculations at the terminals of a resistor also are illustrated.

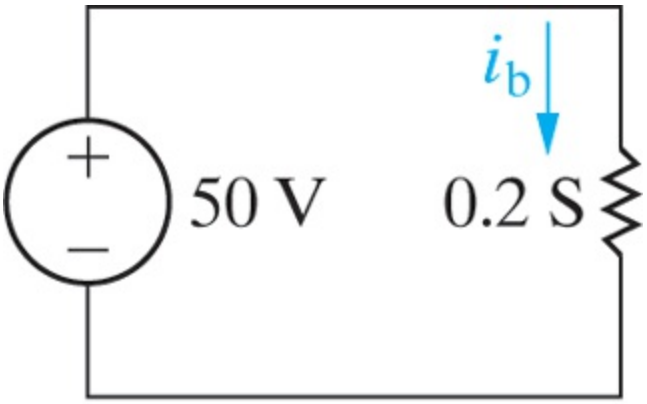
Example 2.3 Calculating Voltage, Current, and Power for a Simple Resistive Circuit

In each circuit in Fig. 2.8, either the value of v or i is not known.

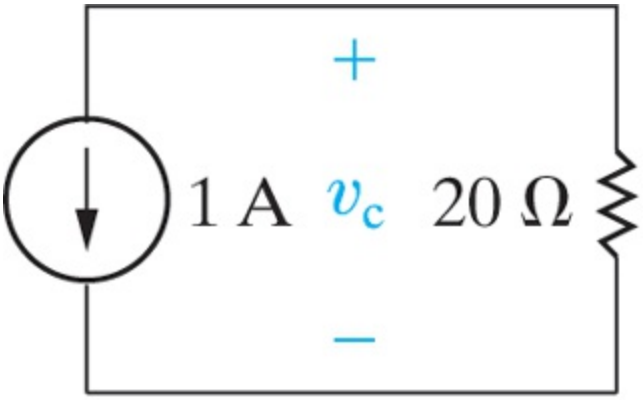
Figure 2.8 The circuits for Example 2.3.



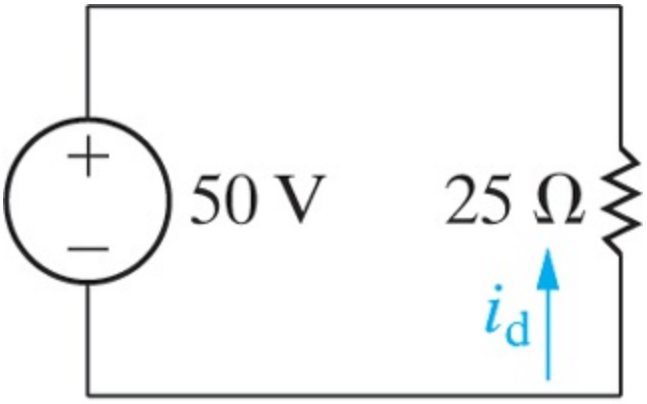
(a)



(b)



(c)



(d)

1. Calculate the values of v and i.

2. Determine the power dissipated in each resistor.

Solution

1. The voltage va in Fig. 2.8(a) is a drop in the direction of the resistor current. The resistor voltage is the product of its current and its resistance, so,

   va=(1)(8)=8 V.

   The current ib in the resistor with a conductance of 0.2 S in Fig. 2.8(b) is in the direction of the voltage drop across the resistor. The resistor current is the product of its voltage and its conductance, so

   ib=(50)(0.2)=10 A.

   The voltage vc in Fig. 2.8(c) is a rise in the direction of the resistor current. The resistor voltage is the product of its current and its resistance, so

   vc=−(1)(20)=−20 V.

   The current id in the 25 Ω resistor in Fig. 2.8(d) is in the direction of the voltage rise across the resistor. The resistor current is its voltage divided by its resistance, so

   id=−5025=−2 A.

2. The power dissipated in each of the four resistors is

   - p8Ω=(8)28=(1)2(8)=8 W(using Eq. 2.10 and Eq. 2.9);
   - p0.2S=(50)2(0.2)=500 W(using Eq. 2.12);
   - p20Ω=(−20)220=(1)2(20)=20 W(using Eq. 2.10 and Eq. 2.9);
   - p25Ω=(50)225=(−2)2(25)=100 W(using Eq. 2.10 and Eq. 2.9).

Assessment Problems

Objective 2—Be able to state and use Ohm's law

1. 2.3 For the circuit shown,

    1. If vg=1 kV and ig=5 mA, find the value of R and the power absorbed by the resistor.

    2. If ig=75 mA and the power delivered by the voltage source is 3 W, find vg, R, and the power absorbed by the resistor.

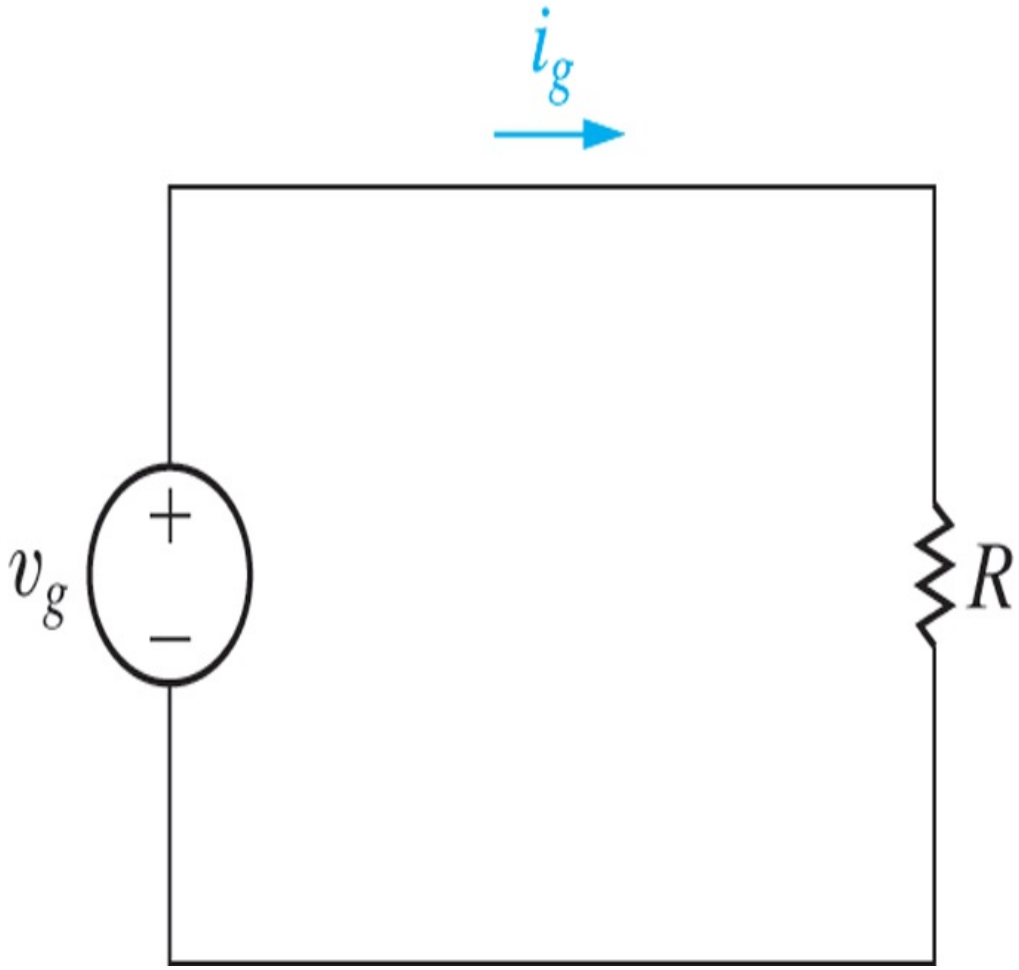
    3. If R=300 Ω and the power absorbed by R is 480 mW, find ig and vg.

    Answer

    1. 200 kΩ, 5 W;

    2. 40 V, 533.33 Ω, 3 W;

    3. 40 mA, 12 V.



2. 2.4 For the circuit shown,

    1. If ig=0.5 A and G=50 mS, find vg and the power delivered by the current source.

    2. If vg=15 V and the power delivered to the conductor is 9 W, find the conductance G and the source current ig.

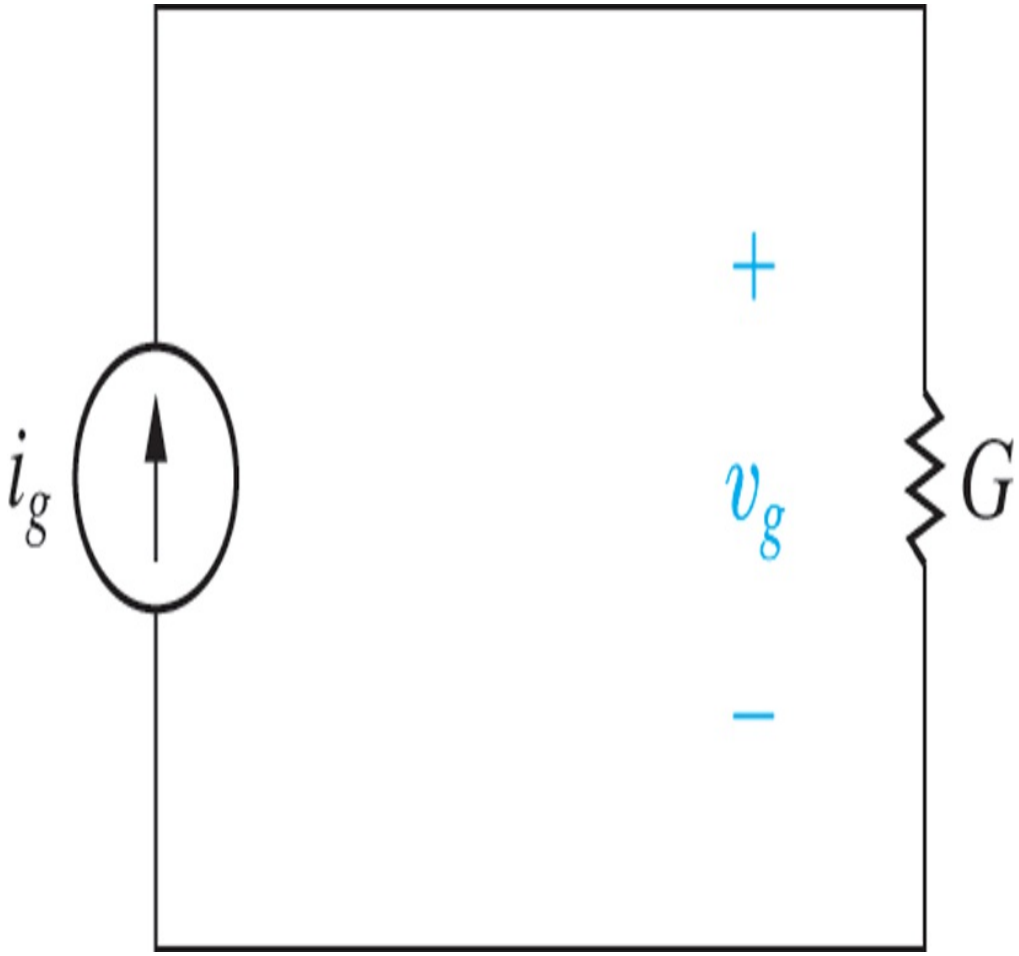
    3. If G=200 μS and the power delivered to the conductance is 8 W, find ig and vg.

    Answer

    1. 10 V, 5 W;

    2. 40 mS, 0.6 A;

3. 40 mA, 200 V.



SELF-CHECK: *Also try Chapter Problems 2.11 and 2.12.*

2.3 Constructing a Circuit Model

Let’s now move on to using ideal sources and resistors to construct circuit models of real-world systems. Developing a circuit model of a device or system is an important skill. Although this text emphasizes circuit-solving skills, as an electrical engineer you’ll need other skills as well, one of the most important of which is modeling.

We develop circuit models in the next two examples. In Example 2.4, we construct a circuit model based on knowing how the system’s components behave and how the components are interconnected. In Example 2.5, we create a circuit model by measuring the terminal behavior of a device.

Example 2.4 Constructing a Circuit Model of a Flashlight

Construct a circuit model of a flashlight. Figure 2.9 shows a photograph of a widely available flashlight.

Figure 2.9 A flashlight can be viewed as an electrical system.

Thom Lang/Corbis/Getty Imagess

Solution

When a flashlight is regarded as an electrical system, the components of primary interest are the batteries, the lamp, the connector, the case, and the switch. Figure 2.10 shows these components. We now consider the circuit model for each component.

Figure 2.10 The arrangement of flashlight components.

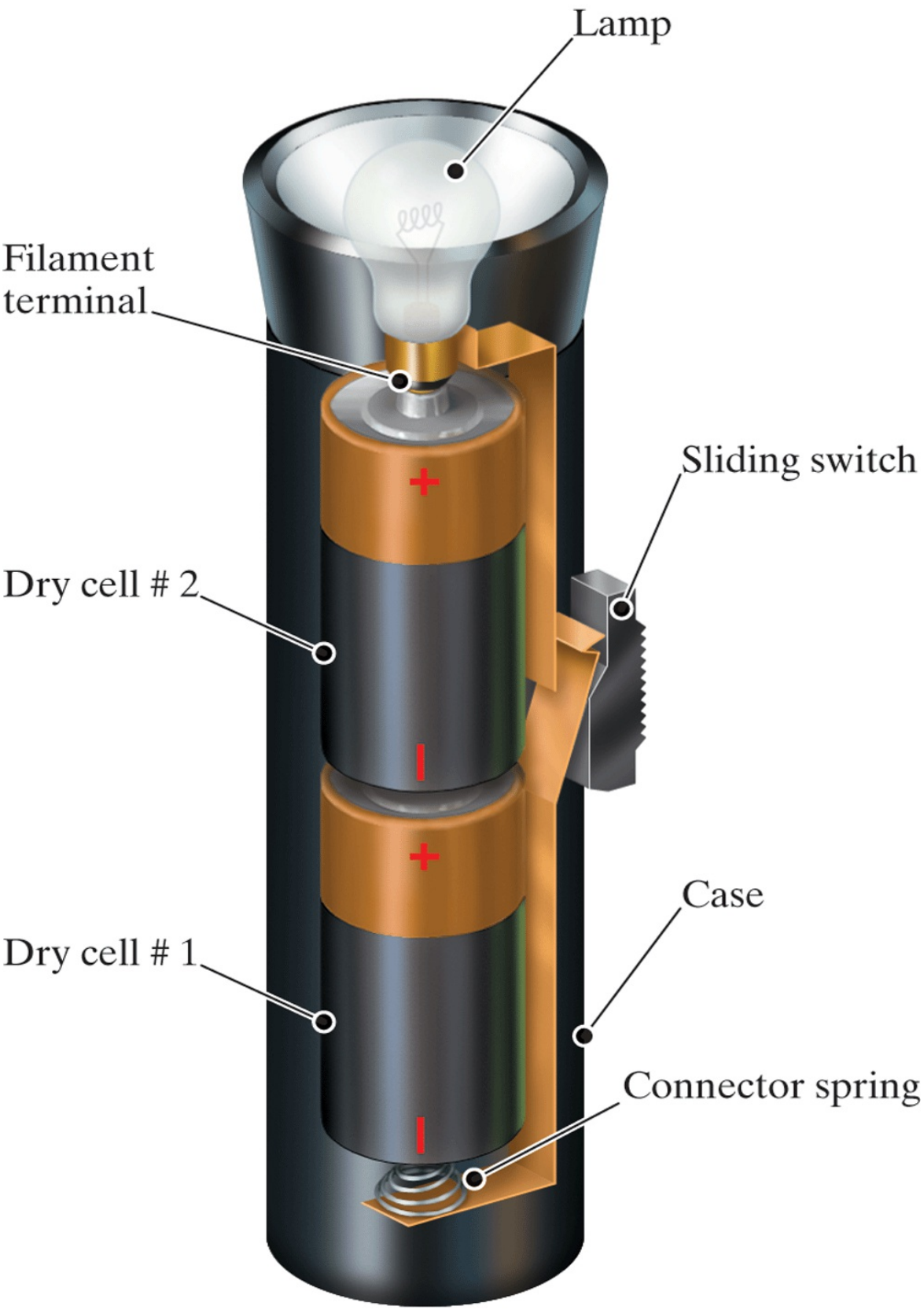

Lamp
Filament terminal
Sliding switch
Dry cell # 2
Case
Dry cell # 1
Connector spring

Figure 2.10 Full Alternative Text

- A dry-cell battery maintains a reasonably constant terminal voltage if the current demand is not excessive. Thus, if the dry-cell battery is operating within its intended limits, we can model it with an ideal voltage source. The prescribed voltage is constant and equal to the sum of two dry-cell values.

- The ultimate output of the lamp is light energy, the result of heating the lamp's filament to a temperature high enough to cause radiation in the visible range. We can model the lamp with an ideal resistor. The resistor accounts for the amount of electric energy converted to thermal energy, but it does not predict how much of the thermal energy is converted to light energy. The resistor representing the lamp also predicts the steady current drain on the batteries, a characteristic of the system that also is of interest. In this model, Rl symbolizes the lamp resistance.

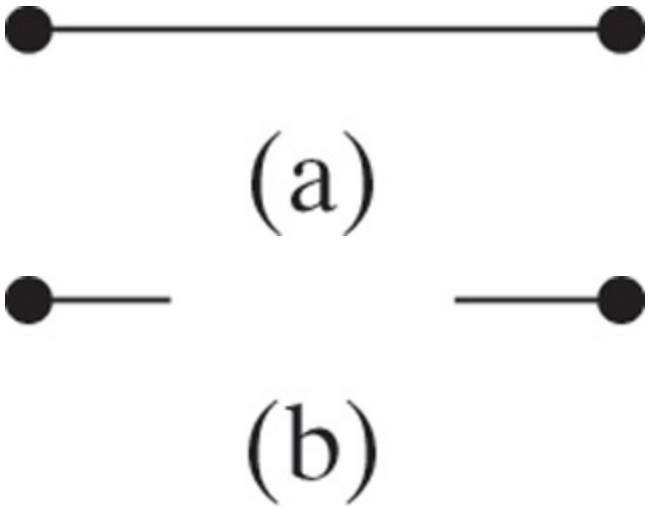
- The connector used in the flashlight serves a dual role. First, it provides an electrical conductive path between the dry cells and the case. Second, it is formed into a springy coil that applies mechanical pressure to the contact between the batteries and the lamp, maintaining contact between the two dry cells and between the dry cells and the lamp. Hence, in choosing the wire for the connector, we may find that its mechanical properties are more important than its electrical properties for the flashlight design. Electrically, we can model the connector with an ideal resistor with resistance R1.

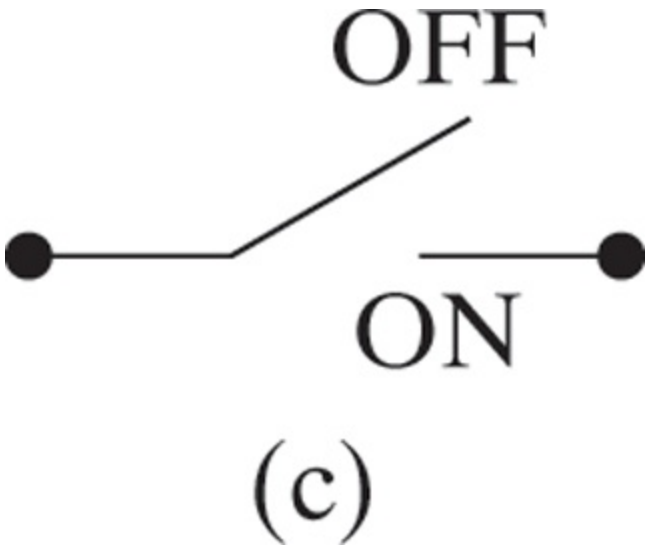
- The case also serves both a mechanical and an electrical purpose. Mechanically, it contains all the other components and provides a grip for the person using the flashlight. Electrically, it provides a connection between other elements in the flashlight. If the case is metal, it conducts current between the batteries and the lamp. If it is plastic, a metal strip inside the case connects the coiled connector to the switch. An ideal resistor with resistance Rc models the electrical connection provided by the case.

- The switch has two electrical states: on or off. An ideal switch in the on

state offers no resistance to the current, but it offers infinite resistance to current in the off state. These two states represent the limiting values of a resistor; that is, the on state corresponds to a zero resistance, called a **short circuit** (R=0), and the off state corresponds to an infinite resistance called an **open circuit** (R=∞). Figures 2.11(a) and (b) depict a short circuit and an open circuit, respectively. The symbol shown in Fig. 2.11(c) represents a switch that can be either a short circuit or an open circuit, depending on the position of its contacts.

Figure 2.11 Circuit symbols. (a) Short circuit. (b) Open circuit. (c) Switch.





(c)

We now construct the circuit model of the flashlight shown in Fig. 2.10. Starting with the dry-cell batteries, the positive terminal of the first cell is connected to the negative terminal of the second cell. The positive terminal of the second cell is connected to one terminal of the lamp. The other terminal of the lamp makes contact with one side of the switch, and the other side of the switch is connected to the metal case. The metal spring connects the metal case to the negative terminal of the first dry cell. Note that the connected elements in Fig. 2.10 form a closed path or circuit. Figure 2.12 shows a circuit model for the flashlight.

Figure 2.12 A circuit model for a flashlight.

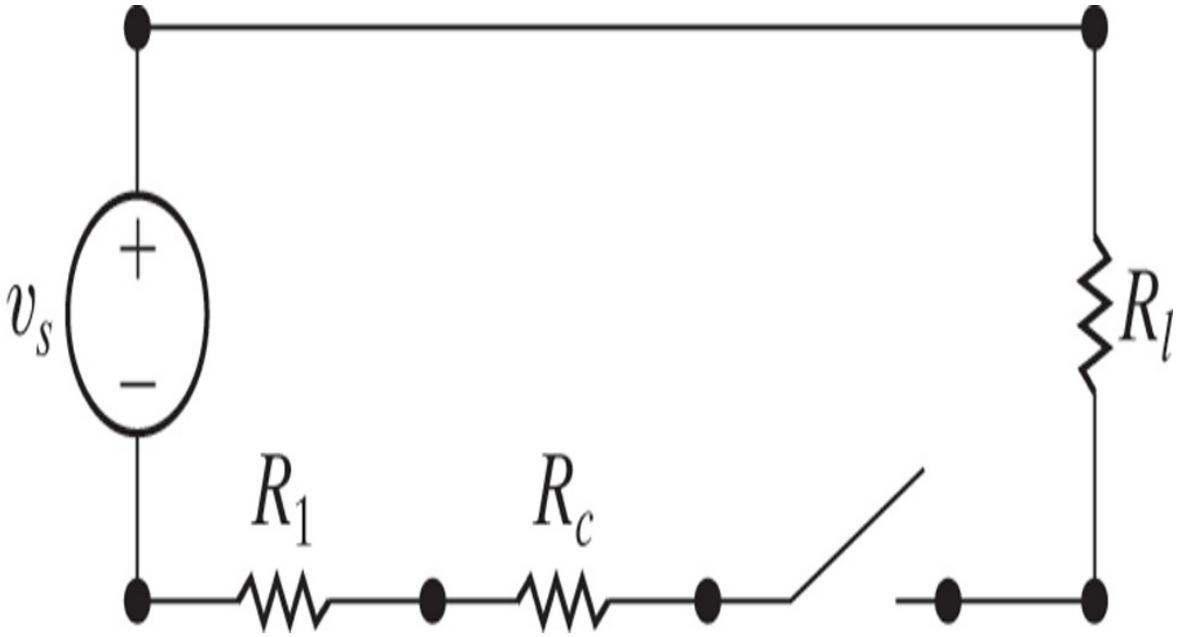


Figure 2.12 Full Alternative Text

Our flashlight example provides some general modeling guidelines.

1. The *electrical* behavior of each physical component is of primary interest in a circuit model. In the flashlight model, three very different physical components—a lamp, a coiled wire, and a metal case—are all represented by resistors because each circuit component resists the current flowing through the circuit.

2. Circuit models may need to account for undesired as well as desired electrical effects. For example, the heat resulting from the lamp resistance produces the light, a desired effect. However, the heat resulting from the case and coil resistance represents an unwanted or parasitic resistance. It drains the dry cells and produces no useful output. Such parasitic effects must be considered, or the resulting model may not adequately represent the system.

3. Modeling requires approximation. We made several simplifying assumptions in developing the flashlight's circuit model. For example, we assumed an ideal switch, but in practical switches, contact resistance may be large enough to interfere with proper operation of the system. Our model does not predict this behavior. We also assumed that the

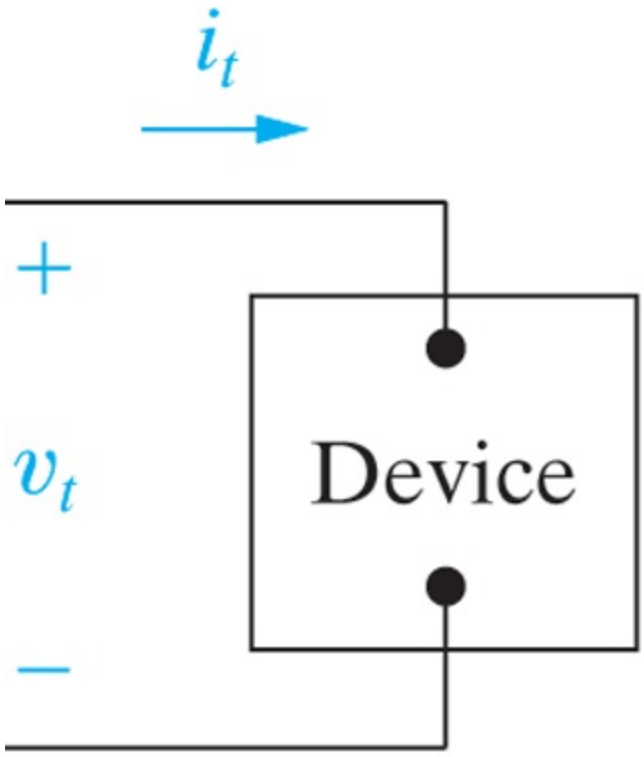
coiled connector exerts enough pressure to eliminate any contact resistance between the dry cells. Our model does not predict the effect of inadequate pressure. Our use of an ideal voltage source ignores any internal dissipation of energy in the dry cells, which might be due to the parasitic heating just mentioned. We could account for this by adding an ideal resistor between the source and the lamp resistor. Our model assumes the internal loss to be negligible.

We used a basic understanding of the internal components of the flashlight to construct its circuit model. However, sometimes we know only the terminal behavior of a device and must use this information to construct the model. Example 2.5 presents such a modeling problem.

Example 2.5 Constructing a Circuit Model Based on Terminal Measurements

The voltage and current are measured at the terminals of the device illustrated in Fig. 2.13(a), and the values of vt and it are tabulated in Fig. 2.13(b). Construct a circuit model of the device inside the box.

Figure 2.13 The (a) device and (b) data for Example 2.5.



(a)

v_t (V)	i_t (A)
−40	−10
−20	−5
0	0
20	5
40	10

(b)

2.3-8 Full Alternative Text

Solution

Plotting the voltage as a function of the current yields the graph shown in Fig. 2.14(a). The equation of the line in this figure is vt=4it, so the terminal voltage is directly proportional to the terminal current. Using Ohm's law, the device inside the box behaves like a 4 Ω resistor. Therefore, the circuit model for the device inside the box is a 4 Ω resistor, as seen in Fig. 2.14(b).

Figure 2.14 (a) The values

of υi versus it for the device in Fig. 2.13. (b) The circuit model for the device in Fig. 2.13.

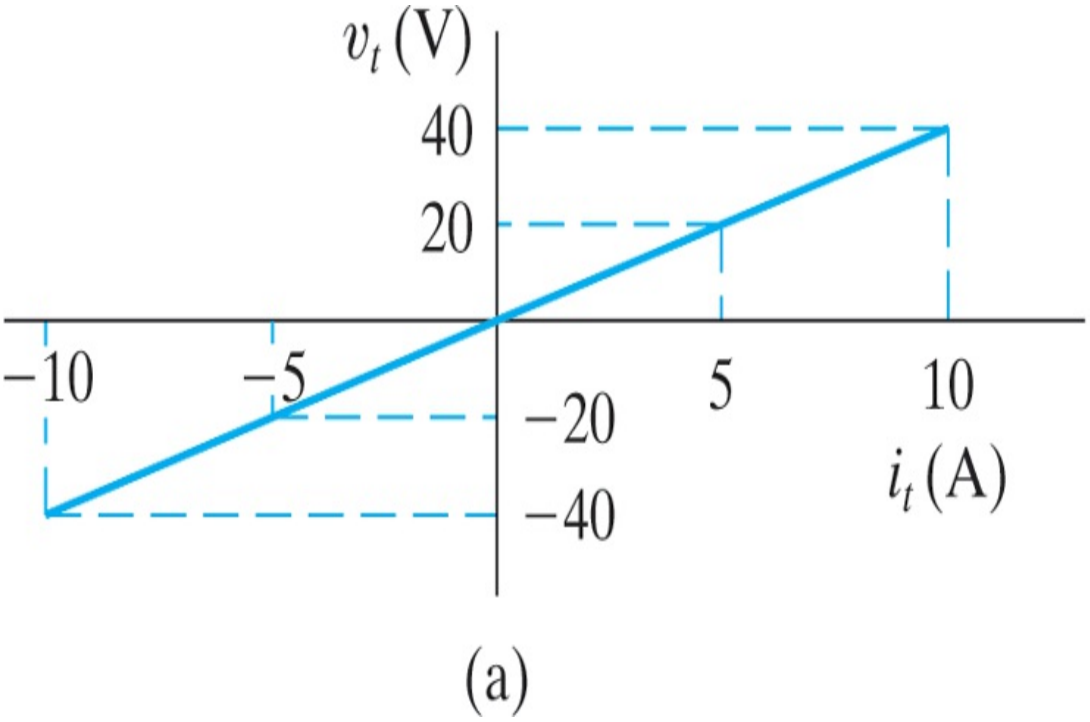
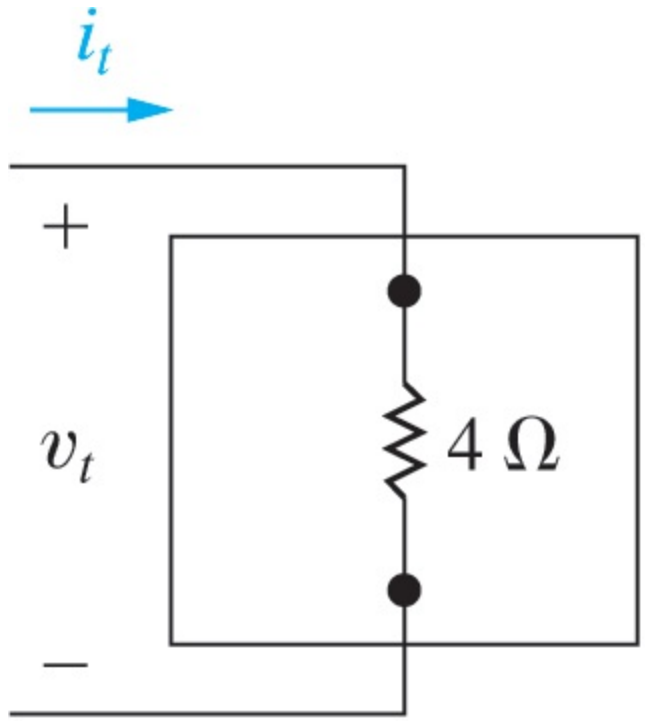


Figure 2.14 Full Alternative Text



(b)

We come back to this technique of using terminal characteristics to construct a circuit model after introducing Kirchhoff's laws and circuit analysis.

SELF-CHECK: *Assess your understanding of this example by trying Chapter Problems 2.14 and 2.16.*

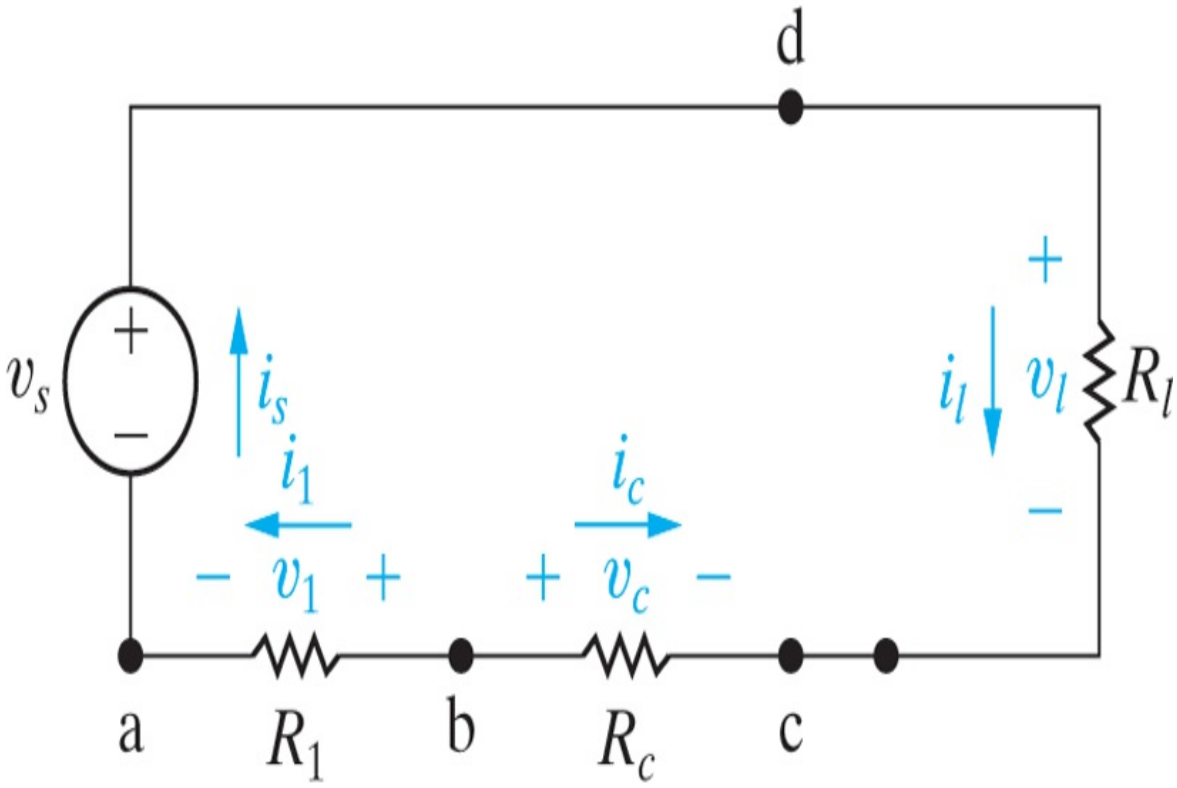
2.4 Kirchhoff’s Laws

A circuit is **solved** when we determine the voltage across and the current in every element. While Ohm’s law is an important tool for solving a circuit, it may not be enough to provide a complete solution. Generally, we need two additional algebraic relationships, known as Kirchhoff’s laws, to solve most circuits.

Kirchhoff’s Current Law

Let’s try to solve the flashlight circuit from Example 2.4. We begin by redrawing the circuit as shown in Fig. 2.15, with the switch in the on state. We have labeled the current and voltage variables associated with each resistor and the current associated with the voltage source, including reference polarities. For convenience, we use the same subscript for the voltage, current, and resistor labels. In Fig. 2.15, we also removed some of the terminal dots of Fig. 2.12 and have inserted nodes. Terminal dots are the start and end points of an individual circuit element. A **node** is a point where two or more circuit elements meet. In Fig. 2.15, the nodes are labeled a, b, c, and d. Node d connects the battery and the lamp and stretches all the way across the top of the diagram, though we label a single point for convenience. The dots on either side of the switch indicate its terminals, but only one is needed to represent a node, labeled node c.

Figure 2.15



Circuit model of the flashlight with assigned voltage and current variables.

Figure 2.15 Full Alternative Text

The circuit in Fig. 2.15 has seven unknowns: is, i1, ic, il, v1, vc, and vl. Recall that vs=3 V, as it represents the sum of the terminal voltages of the two dry cells. To solve the flashlight circuit, we must find values for the seven unknown variables. From algebra, you know that to find n unknown quantities you must solve n simultaneous independent equations. Applying Ohm's law (Section 2.2) to each of the three resistors gives us three of the seven equations we need:

v1=i1R1, (2.13)
vc=icRc, (2.14)
vl=ilRl. (2.15)

What about the other four equations?

Connecting the circuit elements constrains the relationships among the terminal voltages and currents. These constraints are called Kirchhoff's laws, after Gustav Kirchhoff, who first stated them in a paper published in 1848. The two laws that state the constraints in mathematical form are known as Kirchhoff's current law and Kirchhoff's voltage law.

We can now state **Kirchhoff's current law (KCL)**:

Kirchhoff's Current Law (KCL)

The algebraic sum of all the currents at any node in a circuit equals zero.

To use Kirchhoff's current law at a node, assign an algebraic sign corresponding to the current's reference direction for every current at the node. Assigning a positive sign to a current leaving the node requires assigning a negative sign to a current entering a node. Conversely, giving a negative sign to a current leaving a node requires giving a positive sign to a current entering a node.

We apply Kirchhoff's current law to the four nodes in the circuit shown in Fig. 2.15, using the convention that currents leaving a node are positive. The four equations are:

node a $is - i1 = 0$, (2.16)
node b $i1 + ic = 0$, (2.17)
node c $-ic - il = 0$, (2.18)
node d $il - is = 0$. (2.19)

But Eqs. 2.16–2.19 are not an independent set because any one of the four can be derived from the other three. In any circuit with n nodes, n−1 independent equations can be derived from Kirchhoff's current law.1 Let's disregard Eq. 2.19 so that we have six independent equations, namely, Eqs. 2.13–2.18. We need one more, which we can derive from Kirchhoff's voltage law.

1 We say more about this observation in Chapter 4.

Kirchhoff’s Voltage Law

Before we can state Kirchhoff’s voltage law, we must define a **closed path or loop**. Starting at an arbitrarily selected node, we trace a closed path in a circuit through selected basic circuit elements and return to the original node without passing through any intermediate node more than once. The circuit shown in Fig. 2.15 has only one closed path or loop. For example, choosing node a as the starting point and tracing the circuit clockwise, we form the closed path by moving through nodes d, c, b, and back to node a. We can now state **Kirchhoff’s voltage law (KVL)**:

Kirchhoff’s Voltage Law (KVL)

The algebraic sum of all the voltages around any closed path in a circuit equals zero.

To use Kirchhoff’s voltage law, assign an algebraic sign (reference direction) to each voltage in the loop. As we trace a closed path, a voltage will appear either as a rise or a drop in the tracing direction. Assigning a positive sign to a voltage rise requires assigning a negative sign to a voltage drop. Conversely, giving a negative sign to a voltage rise requires giving a positive sign to a voltage drop.

We now apply Kirchhoff’s voltage law to the circuit shown in Fig. 2.15, tracing the closed path clockwise and assigning a positive algebraic sign to voltage drops. Starting at node d leads to the expression

$$vl - vc + v1 - vs = 0. \quad (2.20)$$

Now we have the seven independent equations needed to find the seven unknown circuit variables in Fig. 2.15.

Solving seven simultaneous equations for the simple flashlight lamp seems excessive. In the coming chapters, we present analytical techniques that solve a simple one-loop circuit like the one shown in Fig. 2.15 using a single

equation. Before leaving the flashlight circuit, we observe two analysis details that are important for the techniques presented in subsequent chapters.

1. If you know the current in a resistor, you also know the voltage across the resistor because current and voltage are directly related through Ohm's law. Thus, you can associate one unknown variable with each resistor, either the current or the voltage. For example, choose the current as the unknown variable. Once you solve for the unknown current in the resistor, you can find the voltage across the resistor. In general, if you know the current in a passive element, you can find the voltage across it, greatly reducing the number of simultaneous equations to solve. In the flashlight circuit, choosing the current as the unknown variable eliminates the voltages vc, vl, and v1 as unknowns, and reduces the analytical task to solving four simultaneous equations rather than seven.

2. When only two elements connect to a node, if you know the current in one of the elements, you also know it in the second element by applying Kirchhoff's current law at the node. When just two elements connect at a single node, the elements are said to be **in series**, and you need to define only one unknown current for the two elements. Note that each node in the circuit shown in Fig. 2.15 connects only two elements, so you need to define only one unknown current. Equations 2.16–2.18 lead directly to

   $$is = i1 = -ic = il,$$

   which states that if you know any one of the element currents, you know them all. For example, choosing is as the unknown eliminates i1, ic, and il. The problem is reduced to determining one unknown, namely, is.

Examples 2.6 and 2.7 illustrate how to write circuit equations based on Kirchhoff's laws. Example 2.8 illustrates how to use Kirchhoff's laws and Ohm's law to find an unknown current. Example 2.9 expands on the technique presented in Example 2.5 for constructing a circuit model for a device whose terminal characteristics are known.

Example 2.6 Using Kirchhoff's Current Law

Sum the currents at each node in the circuit shown in Fig. 2.16. Note that there is no connection dot (•) in the center of the diagram, where the 4 Ω branch crosses the branch containing the ideal current source ia.

Figure 2.16 The circuit for Example 2.6.

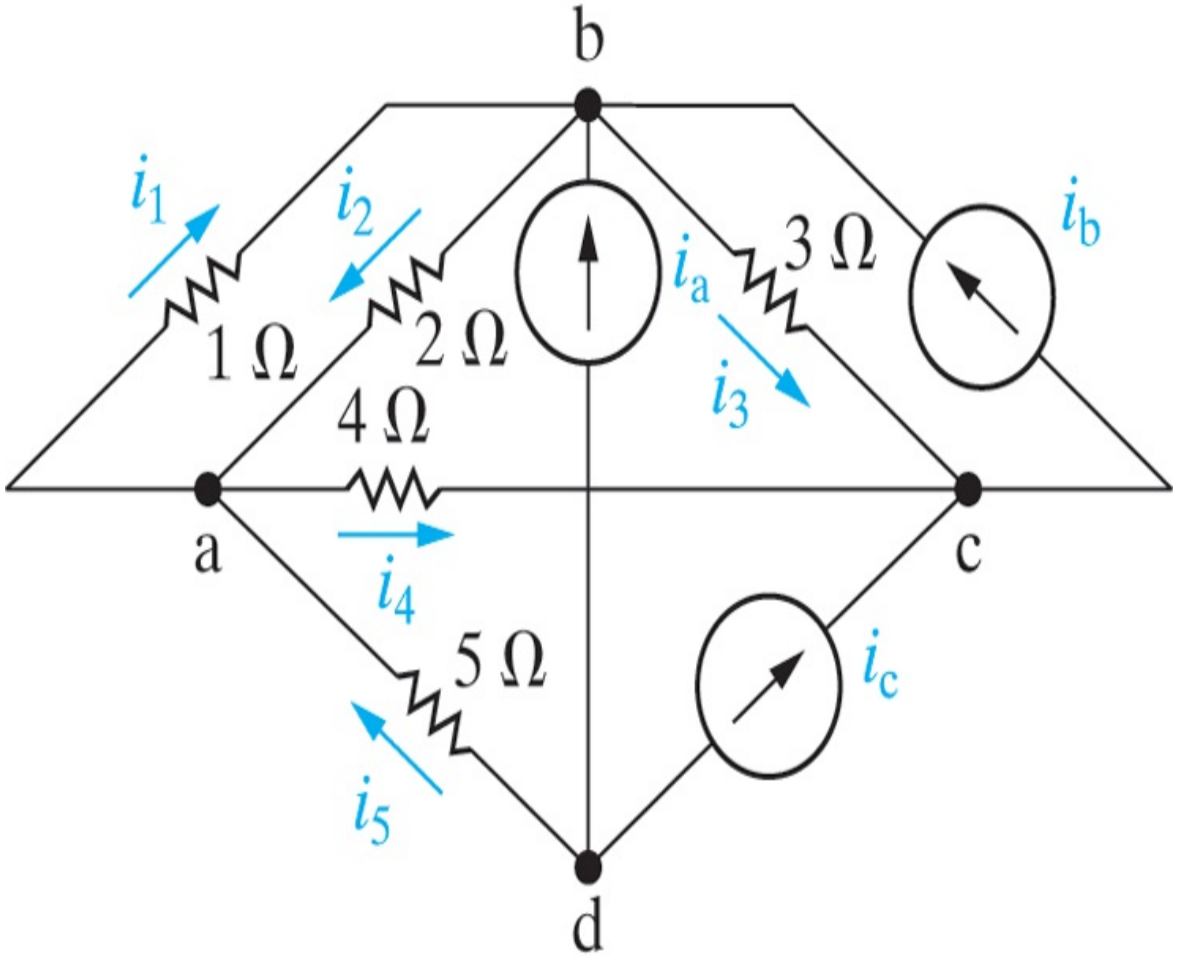


Figure 2.16 Full Alternative Text

Solution

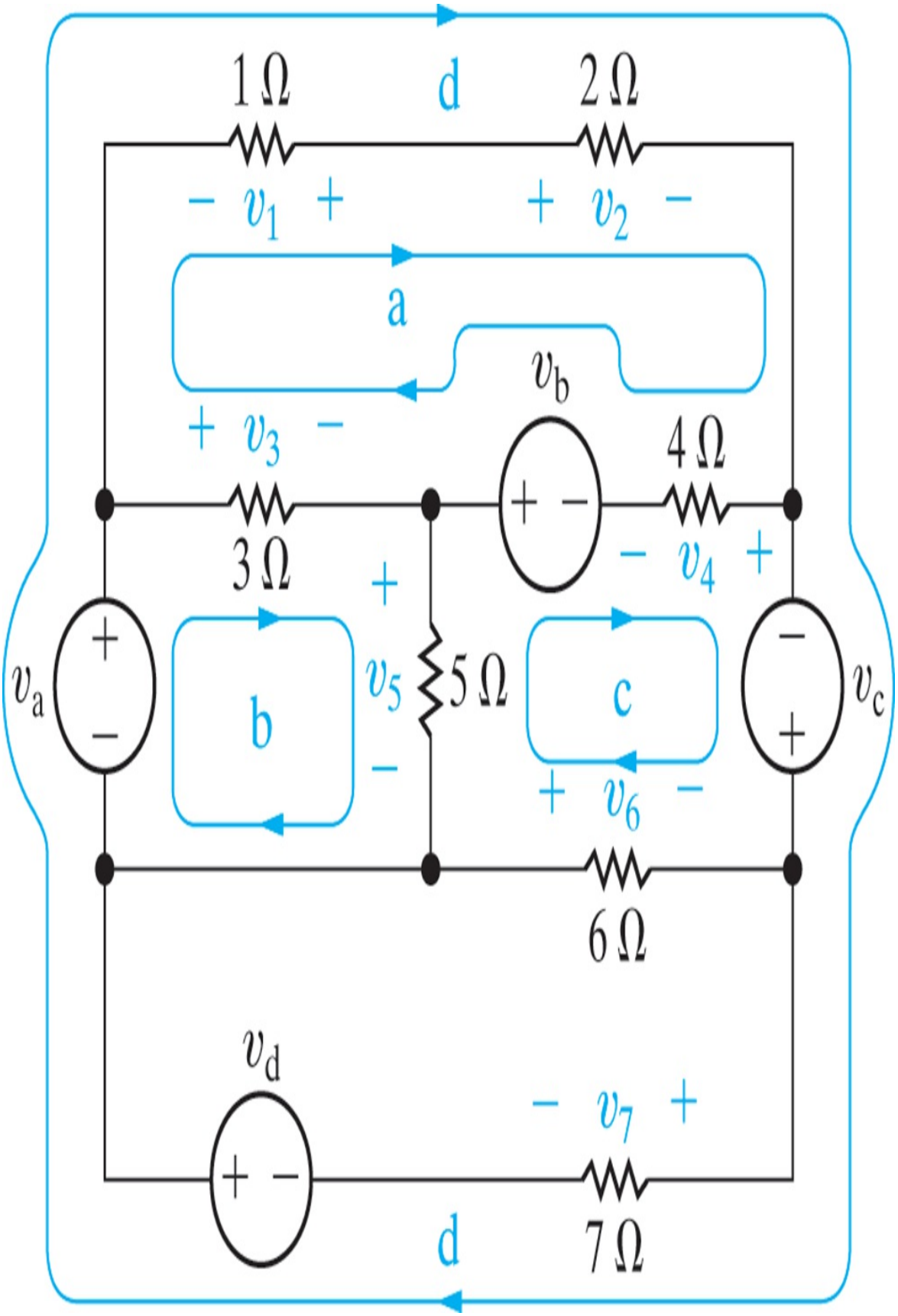
In writing the equations, we use a positive sign for a current leaving a node. The four equations are

node ai1+ i4−i2−i5=0,node bi2+i3−i1−ib−ia=0,node cib −i3−i4−ic=0,node di5+ia+ic=0.

Example 2.7 Using Kirchhoff's Voltage Law

Sum the voltages around each designated path in the circuit shown in Fig. 2.17.

Figure 2.17


1 Ω
d
2 Ω
− v1 +
+ v2 −
a
vb
+ v3 −
4 Ω
3 Ω
+ −
− v4 +
+
va
+
−
b
v5
5 Ω
c
−
−
+
vc
+ v6 −
6 Ω
vd
− v7 +
+ −
d
7 Ω

The circuit for Example 2.7.

Figure 2.17 Full Alternative Text

Solution

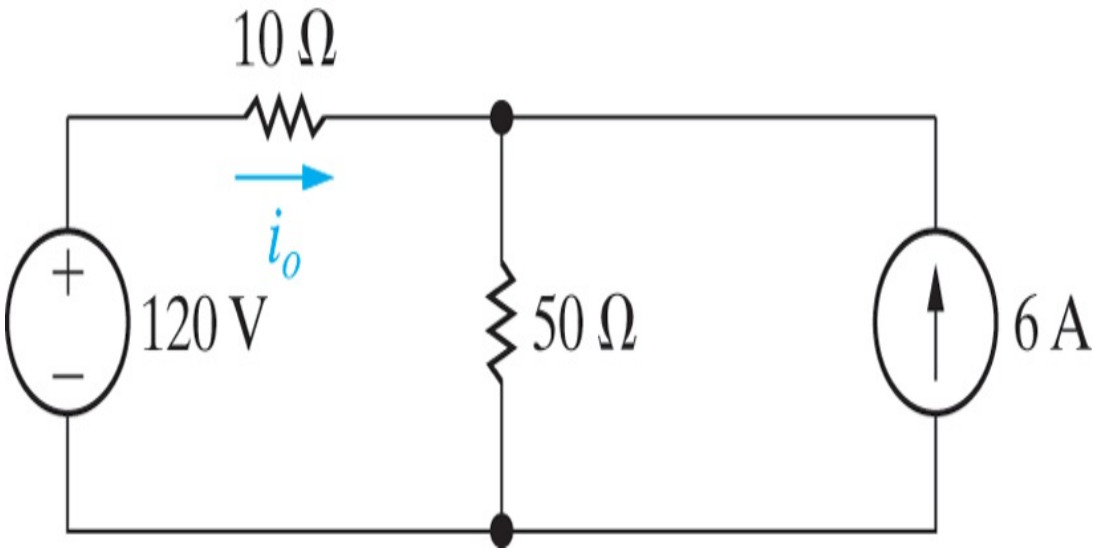
In writing the equations, we use a positive sign for a voltage drop. The four equations are

path a−v1+ v2+v4−vb−v3=0,path b−va+v3+v5=0,path cvb−v4−vc −v6−v5=0,path d−va−v1+v2−vc+v7−vd=0.

Example 2.8 Applying Ohm’s Law and Kirchhoff’s Laws to Find an Unknown Current

1. Use Kirchhoff’s laws and Ohm’s law to find io in the circuit shown in Fig. 2.18.

Figure 2.18



The circuit for Example 2.8.

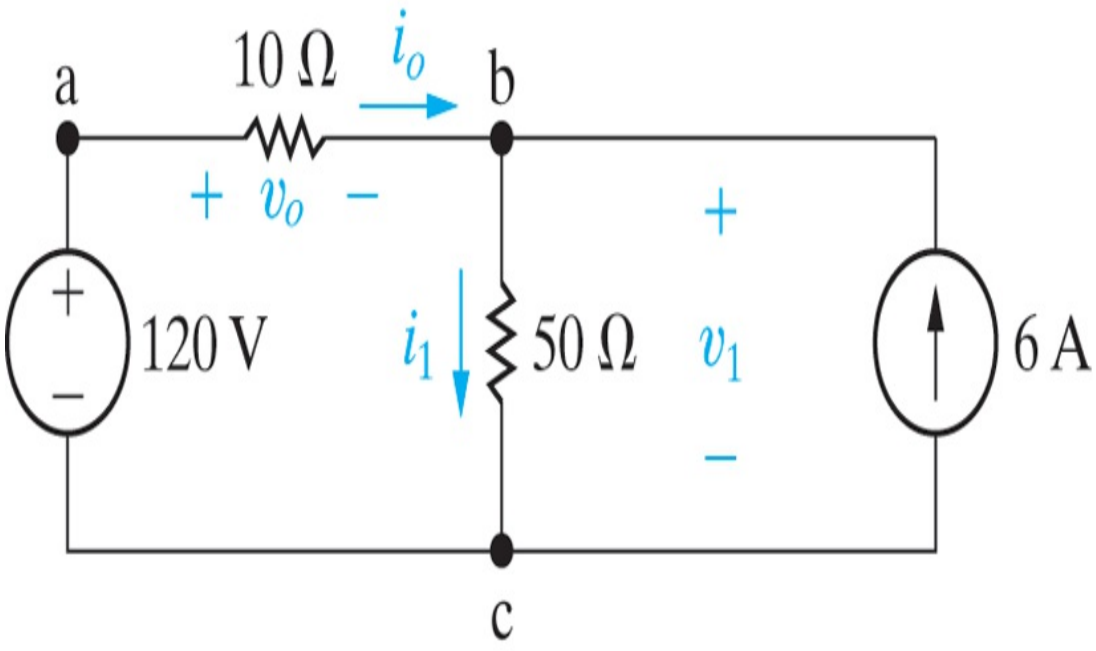
Figure 2.18 Full Alternative Text

2. Test the solution for io by verifying that the total power generated equals the total power dissipated.

Solution

1. We begin by redrawing the circuit and assigning an unknown current to the 50 Ω resistor and unknown voltages across the 10 Ω and 50 Ω resistors. Figure 2.19 shows the circuit. The nodes are labeled a, b, and c to aid the discussion.

Figure 2.19



The circuit shown in Fig. 2.18, with the unknowns i1, υ1, and υ1 defined.

Figure 2.19 Full Alternative Text

Because io also is the current in the 120 V source, we have two unknown currents and therefore must derive two simultaneous equations involving io and i1. One of the equations results from applying Kirchhoff's current law to either node b or c. Summing the currents at node b and assigning a positive sign to the currents leaving the node gives

i1−io− 6=0.

We obtain the second equation from Kirchhoff's voltage law in combination with Ohm's law. Noting from Ohm's law that vo=10io and v1=50i1, we sum the voltages clockwise around the closed path c-a-b-c to obtain

−120+10io+50i1=0.

In writing this equation, we assigned a positive sign to voltage drops in the clockwise direction. Solving these two equations (see Appendix A)

for io and i1 yields

io=−3 A and i1=3 A.

2. The power for the 50 Ω resistor is

   p50Ω=(i1)2(50)=(3)2(50)=450 W.

   The power for the 10 Ω resistor is

   p10Ω=(io)2(10)=(−3)2(10)=90 W.

   The power for the 120 V source is

   p120V=−120io=−120(−3)=360 W.

   The power for the 6 A source is

   p6A=
   −v1(6), and v1=50i1=50(3)=150 V; therefore p6A=−150(6)=
   −900 W.

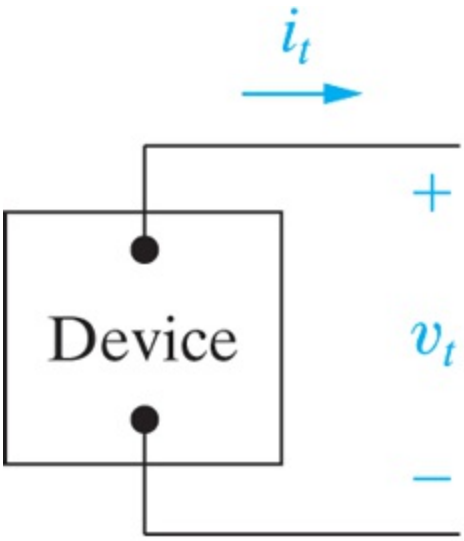
   The 6 A source is delivering 900 W, and the 120 V source and the two resistors are absorbing power. The total power absorbed is p6A+p50Ω
   +p10Ω=360+450+90=900 W.

   Therefore, the solution verifies that the power delivered equals the power absorbed.

Example 2.9 Constructing a Circuit Model Based on Terminal Measurements

We measured the terminal voltage and terminal current on the device shown in Fig. 2.20(a) and tabulated the values of vt and it in Fig. 2.20(b).

Figure 2.20 (a) Device and (b) data for Example 2.9.



v_t (V)	i_t (A)
30	0
15	3
0	6

(a) (b)

Figure 2.20 Full Alternative Text

1. Construct a circuit model of the device inside the box.

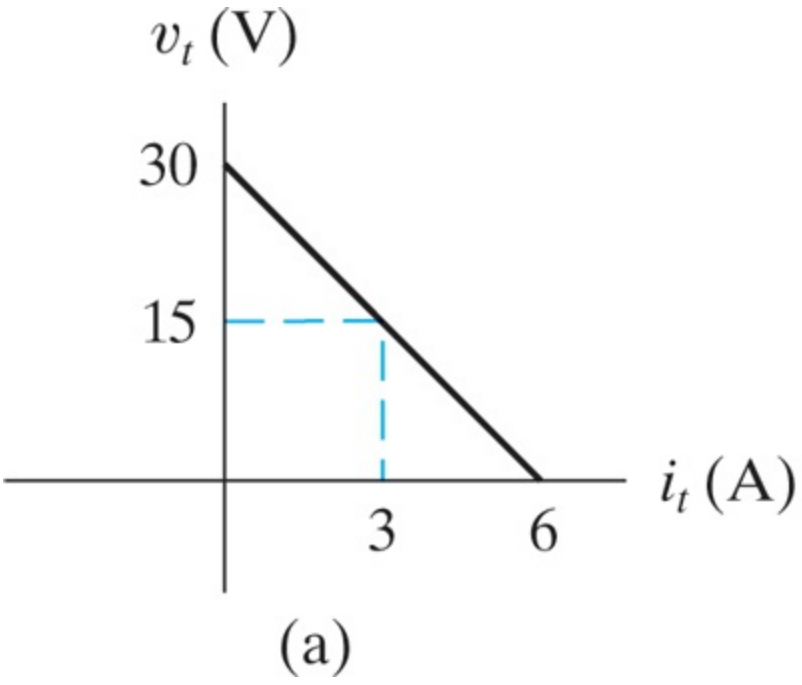
2. Using this circuit model, predict the power this device will deliver to a 10 Ω resistor.

Solution

1. Plotting the voltage as a function of the current yields the graph shown in Fig. 2.21(a). The equation of the line plotted is

   vt=30−5it.

Figure 2.21 (a) The graph of υi versus it for the device in Fig. 2.20(a). (b) The resulting circuit model for the device in Fig. 2.20(a), connected to a 10 Ω resistor.



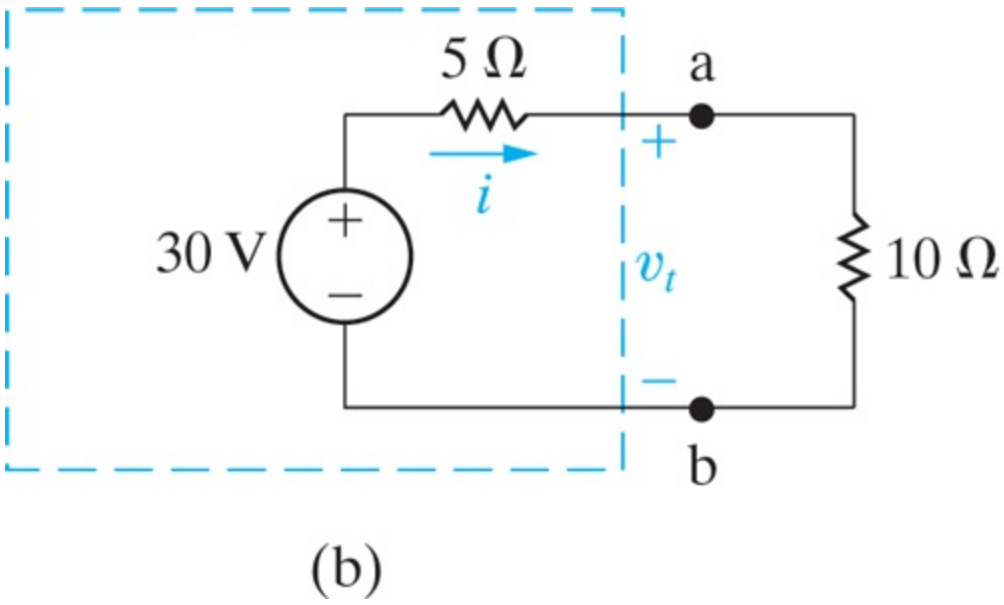


Figure 2.21 Full Alternative Text

What circuit model components produce this relationship between voltage and current? Kirchhoff's voltage law tells us that the voltage drops across two components in series add. From the equation, one of those components produces a 30 V drop regardless of the current, so this component's model is an ideal independent voltage source. The other component produces a positive voltage drop in the direction of the current it. Because the voltage drop is proportional to the current, Ohm's law tells us that this component's model is an ideal resistor with a value of 5 Ω. The resulting circuit model is depicted in the dashed box in Fig. 2.21(b).

2. Now we attach a 10 Ω resistor to the device in Fig. 2.21(b) to complete the circuit. Kirchhoff's current law tells us that the current in the 10 Ω resistor equals the current in the 5 Ω resistor. Using Kirchhoff's voltage law and Ohm's law, we can write the equation for the voltage drops around the circuit, starting at the voltage source and proceeding clockwise:

   $-30+5i+10i=0.$

   Solving for i, we get

i=2 A.

This is the value of current flowing in the 10 Ω resistor, so compute the resistor’s power using the equation p=i2R:

p10Ω=(2)2(10)=40 W.

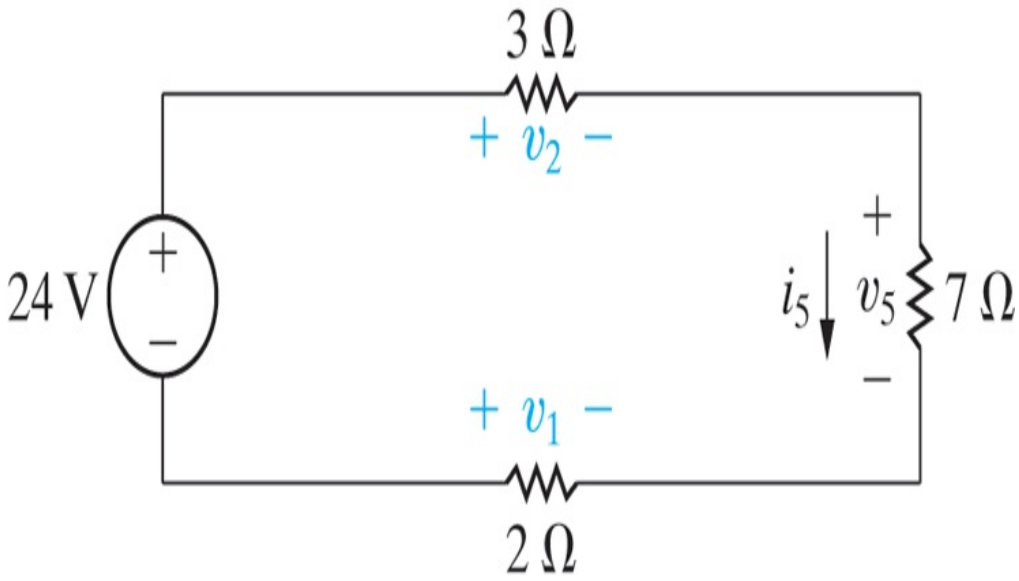
Assessment Problems

Objective 3—Be able to state and use Ohm’s law and Kirchhoff’s current and voltage laws

1. 2.5 For the circuit shown, calculate (a) i5; (b) v1; (c) v2; (d) v5; and (e) the power delivered by the 24 V source.

   Answer

   1. 2 A;
   2. –4 V;
   3. 6 V;
   4. 14 V;
   5. 48 W.

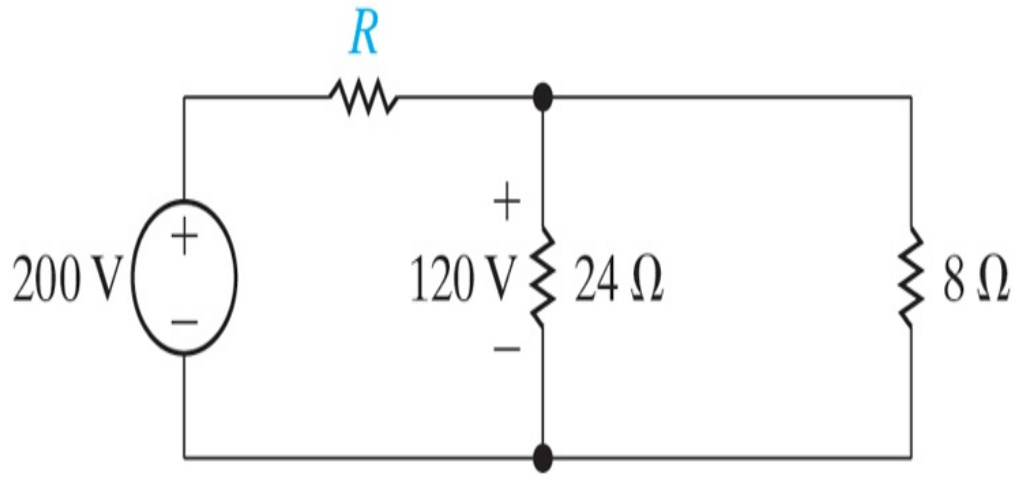


2.4-8 Full Alternative Text

2. 2.6 Use Ohm's law and Kirchhoff's laws to find the value of R in the circuit shown.

   Answer

   1. R=4 Ω.



2.4-9 Full Alternative Text

3. 2.7

a)

1. The terminal voltage and terminal current were measured on the device shown. The values of vt and it are provided in the table. Using these values, create the straight-line plot of vt versus it. Compute the equation of the line and use the equation to construct a circuit model for the device using an ideal voltage source and a resistor.

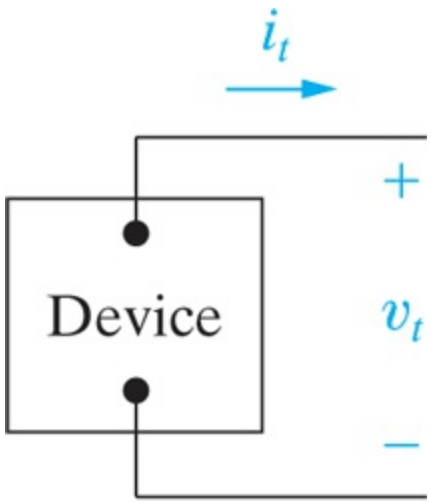
2. Use the model constructed in (a) to predict the power that the device will deliver to a 25 Ω resistor.

Answer

(a)

1. A 25 V source in series with a 100 Ω resistor;

2. 1 W.



(a)

v_t (V)	i_t (A)
25	0
15	0.1
5	0.2
0	0.25

(b)

2.4-11 Full Alternative Text

4. 2.8 Repeat Assessment Problem 2.7, but use the equation of the graphed line to construct a circuit model containing an ideal current source and a resistor.

   Answer

   1. A 0.25 A current source connected between the terminals of a 100 Ω resistor;

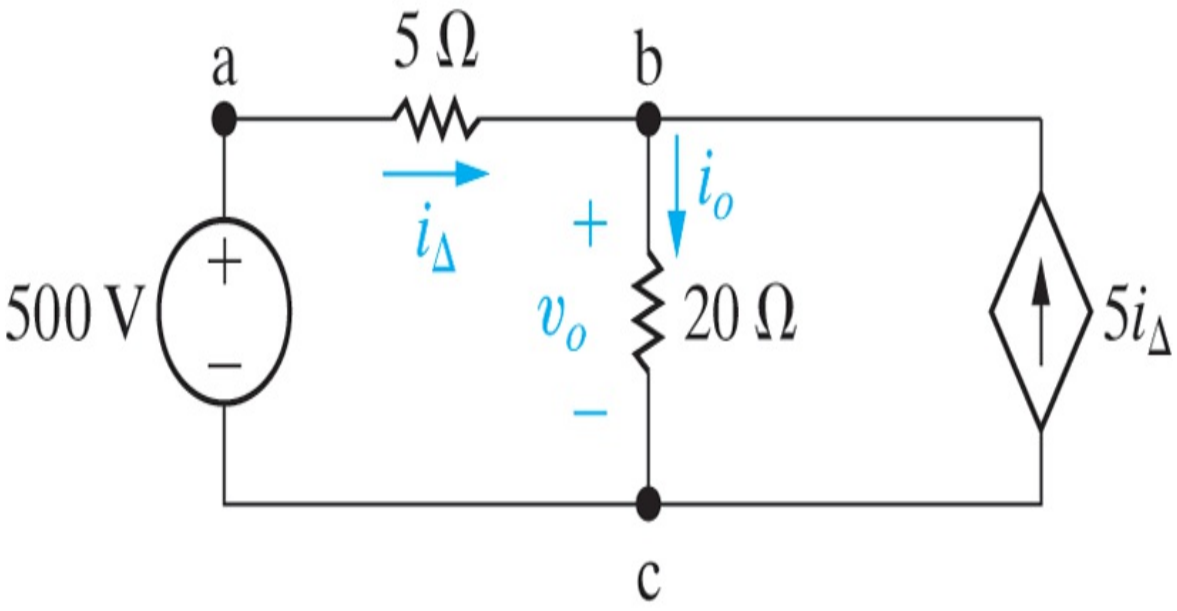
   2. 1 W.

SELF-CHECK: *Also try Chapter Problems 2.17, 2.18, 2.29, and 2.30.*

2.5 Analyzing a Circuit Containing Dependent Sources

We conclude this introduction to elementary circuit analysis by considering circuits with dependent sources. One such circuit is shown in Fig. 2.22.

Figure 2.22



A circuit with a dependent source.

Figure 2.22 Full Alternative Text

We want to use Kirchhoff's laws and Ohm's law to find vo in this circuit. Before writing equations, it is good practice to examine the circuit diagram closely. This will help us identify the information that is known and the information we must calculate. It may also help us devise a strategy for solving the circuit using only a few calculations.

A look at the circuit in Fig. 2.22 reveals that:

- Once we know io, we can calculate vo using Ohm's law.
- Once we know iΔ, we also know the current supplied by the dependent source 5iΔ.
- The current in the 500 V source is iΔ, using Kirchhoff's current law at node a.

There are thus two unknown currents, iΔ and io. We need to construct and solve two independent equations involving these two currents to produce a value for vo. This is the approach used in Example 2.10.

Example 2.10 Analyzing a Circuit with a Dependent Source

Find the voltage vo for the circuit in Fig. 2.22.

Solution

The closed path consisting of the voltage source, the 5 Ω resistor, and the 20 Ω resistor contains the two unknown currents. Apply Kirchhoff's voltage law around this closed path, using Ohm's law to express the voltage across the resistors in terms of the currents in those resistors. Starting at node c and traversing the path clockwise gives:

$$-500+5i_\Delta+20i_o=0.$$

Now we need a second equation containing these two currents. We can't apply Kirchhoff's voltage law to the closed path formed by the 20 Ω resistor and the dependent current source because we don't know the value of the voltage across the dependent current source. For this same reason, we cannot apply Kirchhoff's voltage law to the closed path containing the voltage

source, the 5 Ω resistor, and the dependent source.

We turn to Kirchhoff's current law to generate the second equation. Either node b or node c can be used to construct the second equation from Kirchhoff's current law, since we have already used node a to determine that the current in the voltage source and the 5 Ω resistor is the same. We select node b and produce the following equation, summing the currents leaving the node:

$-i\Delta+io-5i\Delta=0.$

Solve the KCL equation for io in terms of iΔ(io=6iΔ), and then substitute this expression for io into the KVL equation to give

$500=5i\Delta+20(6i\Delta)=125i\Delta.$

Therefore,

$i\Delta=500/125=4$ A and $io=6(4)=24$ A.

Using io and Ohm's law for the 20 Ω resistor, we can solve for the voltage vo:

$vo=20io=480$ V.

Think about a circuit analysis strategy before beginning to write equations because not every closed path yields a useful Kirchhoff's voltage law equation and not every node yields a useful Kirchhoff's current law equation. Think about the problem and select a fruitful approach and useful analysis tools to reduce the number and complexity of equations to be solved. Example 2.11 applies Ohm's law and Kirchhoff's laws to another circuit with a dependent source. Example 2.12 involves a much more complicated circuit, but with a careful choice of analysis tools, the analysis is relatively uncomplicated.

Example 2.11 Applying Ohm's Law

and Kirchhoff's Laws to Find an Unknown Voltage

1. Use Kirchhoff's laws and Ohm's law to find the voltage vo as shown in Fig. 2.23.

Figure 2.23 The circuit for Example 2.11.

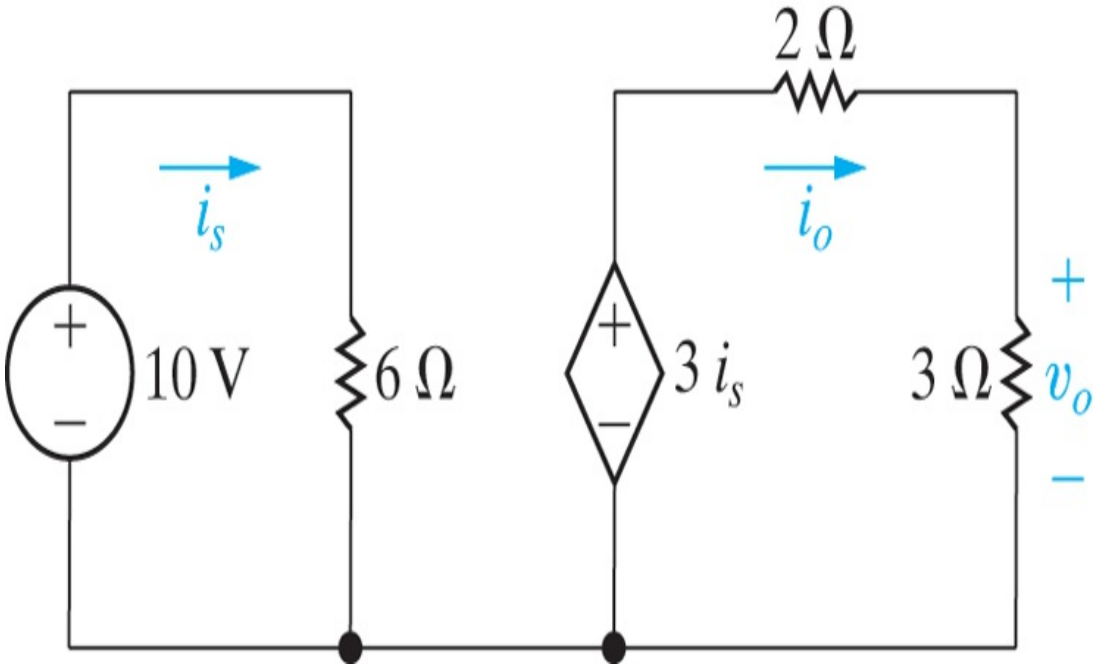


Figure 2.23 Full Alternative Text

2. Show that your solution is consistent with the requirement that the total power developed in the circuit equals the total power dissipated.

Solution

1. A close look at the circuit in Fig. 2.23 reveals that:

    - There are two closed paths, the one on the left with the current is and the one on the right with the current io.
    - Once io is known, we can compute vo using Ohm's law.

    We need two equations for the two currents. Because there are two closed paths and both have voltage sources, we can apply Kirchhoff's voltage law to each, using Ohm's law to express the voltage across the resistors in terms of the current in those resistors. The resulting equations are:

    $-10+6i_s=0$ and $-3i_s+2i_o+3i_o=0$.

    Solving for the currents yields

    $i_s=1.67$ A and $i_o=1$ A.

    Applying Ohm's law to the 3 Ω resistor gives the desired voltage:

    $v_o=3i_o=3$ V.

2. To compute the power delivered to the voltage sources, we use the power equation, $p=vi$, together with the passive sign convention. The power for the independent voltage source is

    $p=-10i_s=-10(1.67)=-16.7$ W.

    The power for the dependent voltage source is

    $p=-(3i_s)i_o=-(5)(1)=-5$ W.

    Both sources are supplying power, and the total power supplied is 21.7 W.

    To compute the power for the resistors, we use the power equation, $p=i^2R$. The power for the 6 Ω resistor is

p=(1.67)2(6)=16.7 W.

The power for the 2 Ω resistor is

p=(1)2(2)=2 W.

The power for the 3 Ω resistor is

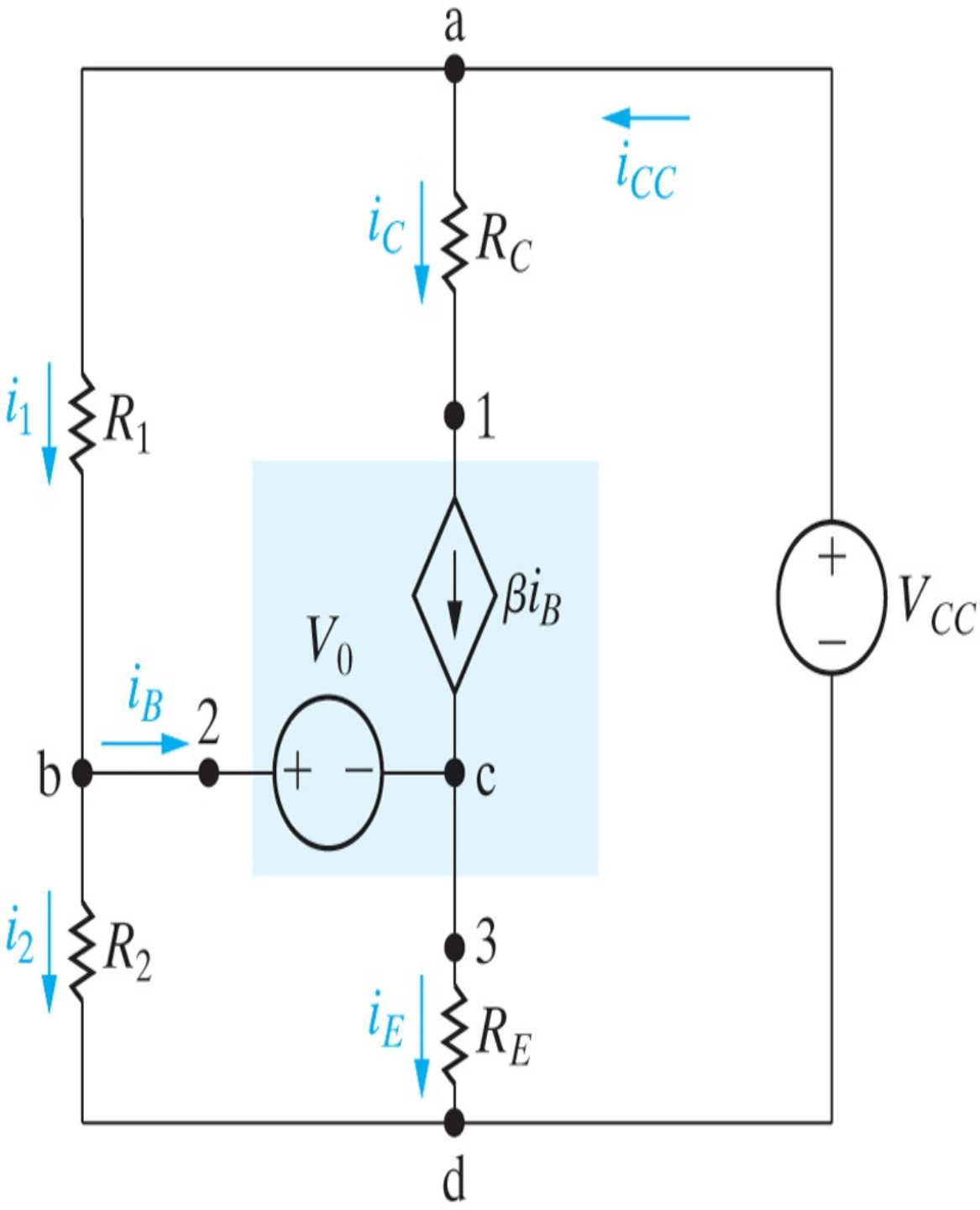
p=(1)2(3)=3 W.

The resistors all absorb power, and the total power absorbed is 21.7 W, equal to the total power supplied by the sources.

Example 2.12 Applying Ohm's Law and Kirchhoff's Law in an Amplifier Circuit

The circuit in Fig. 2.24 represents a common configuration encountered in the analysis and design of transistor amplifiers. Assume that the values of all the circuit elements—R1, R2, RC, RE, VCC, and V0—are known.

Figure 2.24



The circuit for Example 2.12.

Figure 2.24 Full Alternative Text

1. Develop the equations needed to determine the current in each element of this circuit.

2. From these equations, devise a formula for computing iB in terms of the circuit element values.

Solution

Carefully examine the circuit to identify six unknown currents, designated i1, i2, iB, iC, iE, and iCC. In defining these six unknown currents, we observed that the resistor RC is in series with the dependent current source βiB, so these two components have the same current. We now must derive six independent equations involving these six unknowns.

1. We can derive three equations by applying Kirchhoff's current law to any three of the nodes a, b, c, and d. Let's use nodes a, b, and c and label the currents away from the nodes as positive:

    1. $i1+ iC-iCC=0$,
    2. $iB+ i2-i1=0$,
    3. $iE-iB-iC=0$.

    A fourth equation results from imposing the constraint presented by the series connection of RC and the dependent source:

    4. $iC=\beta iB$.

    We use Kirchhoff's voltage law to derive the remaining two equations. We must select two closed paths, one for each Kirchhoff's voltage law equation. The voltage across the dependent current source is unknown and cannot be determined from the source current βiB, so select two closed paths that do not contain this dependent current source.

    We choose the paths b-c-d-b and b-a-d-b, then use Ohm's law to express resistor voltage in terms of resistor current. Traverse the paths in the clockwise direction and specify voltage drops as

positive to yield

5. V0+iERE−i2R2=0,
6. −i1R1+ VCC−i2R2=0.

2. To get a single equation for iB in terms of the known circuit variables, you can follow these steps:
    - Solve Eq. (6) for i1, and substitute this solution for i1 into Eq. (2).
    - Solve the transformed Eq. (2) for i2, and substitute this solution for i2 into Eq. (5).
    - Solve the transformed Eq. (5) for iE, and substitute this solution for iE into Eq. (3). Use Eq. (4) to eliminate iC in Eq. (3).
    - Solve the transformed Eq. (3) for iB, and rearrange the terms to yield

iB=(VCCR2)/(R1+R2)−V0(R1R2)/(R1+R2)+(1+β)RE. (2.21)

Problem 2.38 asks you to verify these steps. Note that once we know iB, we can easily obtain the remaining currents.

Assessment Problems

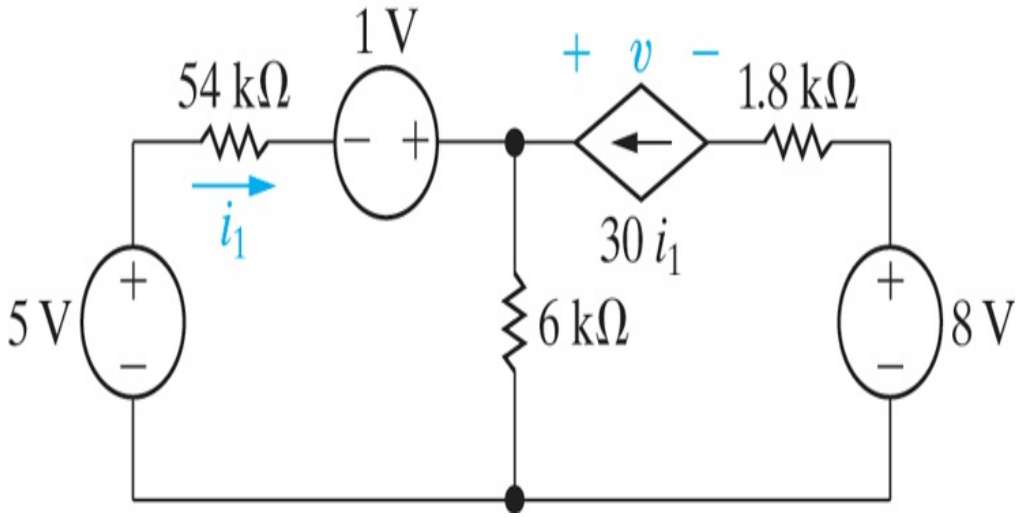
Objective 4—Know how to calculate power for each element in a simple circuit

1. 2.9 For the circuit shown, find (a) the current i1 in microamperes, (b) the voltage v in volts, (c) the total power generated, and (d) the total power absorbed.

    Answer

    1. 25 μA;

2. –2 V;
3. 6150 μW;
4. 6150 μW.

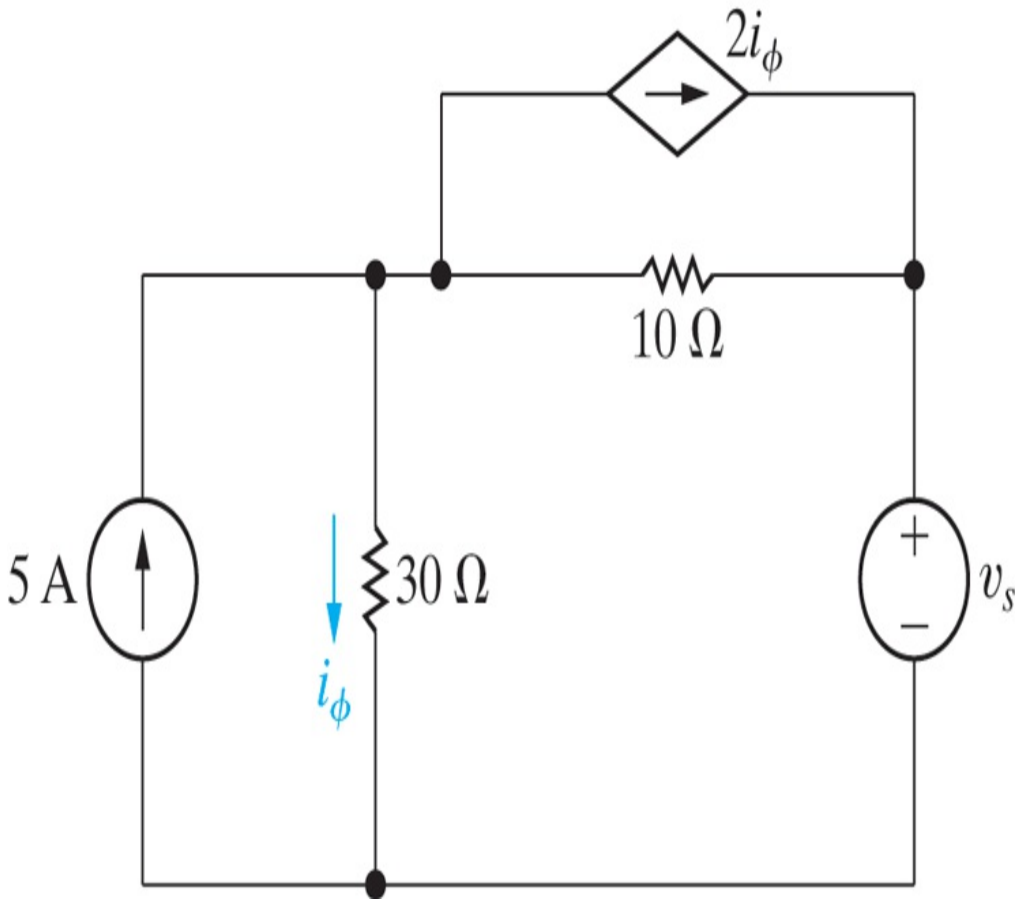


2.5-12 Full Alternative Text

2. 2.10 The current iϕ in the circuit shown is 2 A. Calculate
    1. vs,
    2. the power absorbed by the independent voltage source,
    3. the power delivered by the independent current source,
    4. the power delivered by the controlled current source,
    5. the total power dissipated in the two resistors.

    Answer
    1. 70 V;
    2. 210 W;
    3. 300 W;

4. 40 W;
5. 130 W.



2.5-13 Full Alternative Text

SELF-CHECK: *Also try Chapter Problems 2.33 and 2.34.*

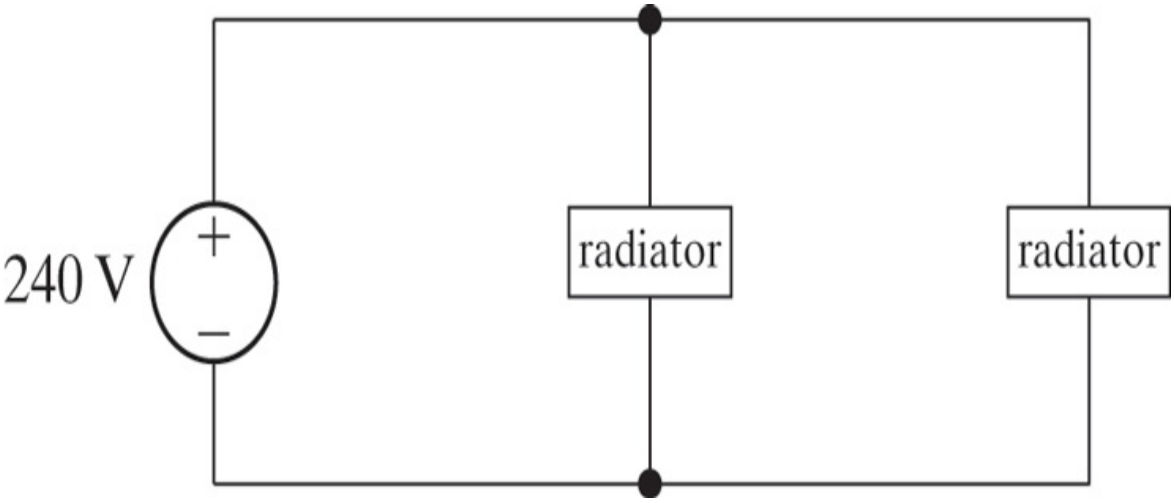
Practical Perspective

Heating with Electric Radiators

Let’s determine which of the two wiring diagrams introduced at the beginning of this chapter should be used to wire the electric radiators to the

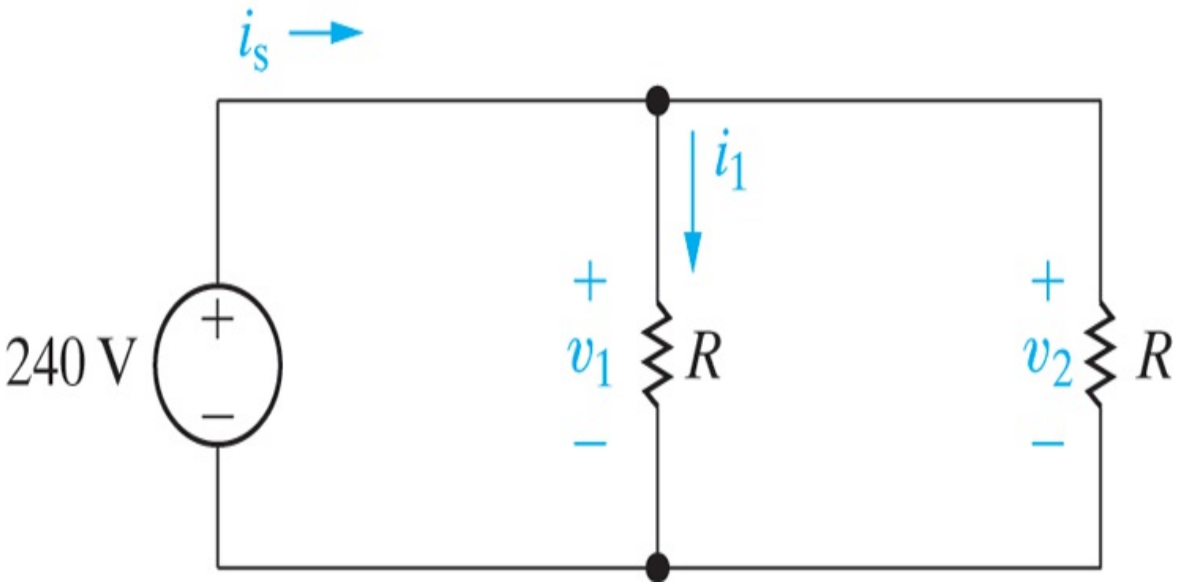
power supplied to the garage. We begin with the diagram shown in Fig. 2.25. We can turn this into a circuit by modeling the radiators as resistors. The resulting circuit is shown in Fig. 2.26. Note that each radiator has the same resistance, R, and is labeled with a voltage and current value.

Figure 2.25



A wiring diagram for two radiators.

Figure 2.26



A circuit based on Fig. 2.25.

Figure 2.26 Full Alternative Text

To find the unknown voltages and currents for the circuit in Fig. 2.26, begin by using Kirchhoff's voltage law to sum the voltage drops around the path on the circuit's left side:

$-240+v1=0 \Rightarrow v1=240$ V.

Now use Kirchhoff's voltage law to sum the voltage drops around the path on the circuit's right side:

$-v1+v2=0 \Rightarrow v2=v1=240$ V.

Remember that the power and voltage specifications for each radiator are 1200 W, 240 V. Therefore, the configuration shown in Fig. 2.25 satisfies the voltage specification, since each radiator would have a supplied voltage of 240 V.

Next, calculate the value of resistance R that correctly models each radiator. We want the power associated with each radiator to be 1200 W. Use the equation for resistor power that involves the resistance and the voltage:

$P1=v12R=v22R=P2 \Rightarrow R=v12P1=24021200=48\ \Omega$.

Each radiator can be modeled as a 48 Ω resistor, with a voltage drop of 240 V and power of 1200 W. The total power for two radiators is thus 2400 W.

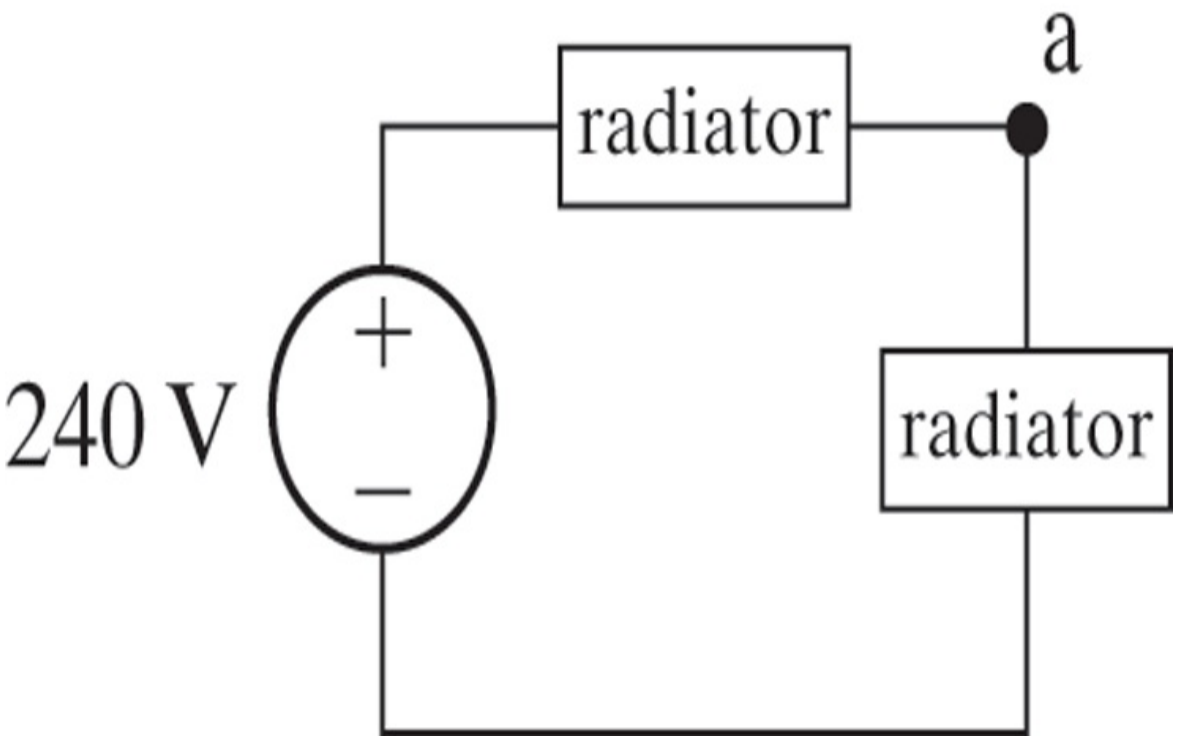
Finally, calculate the power for the 240 V source. To do this, calculate the current in the voltage source, is, using Kirchhoff's current law to sum the currents leaving the top node in Fig. 2.26. Then use is to find the power for the voltage source.

$-is+i1+i2=0 \quad \Rightarrow \quad is=i1+i2=v1R+v2R=24048 + 24048=10$ A.
$ps=-(240)(is)=-(240)(10)=-2400$ W.

Thus, the total power in the circuit is $-2400+2400=0$, and the power balances.

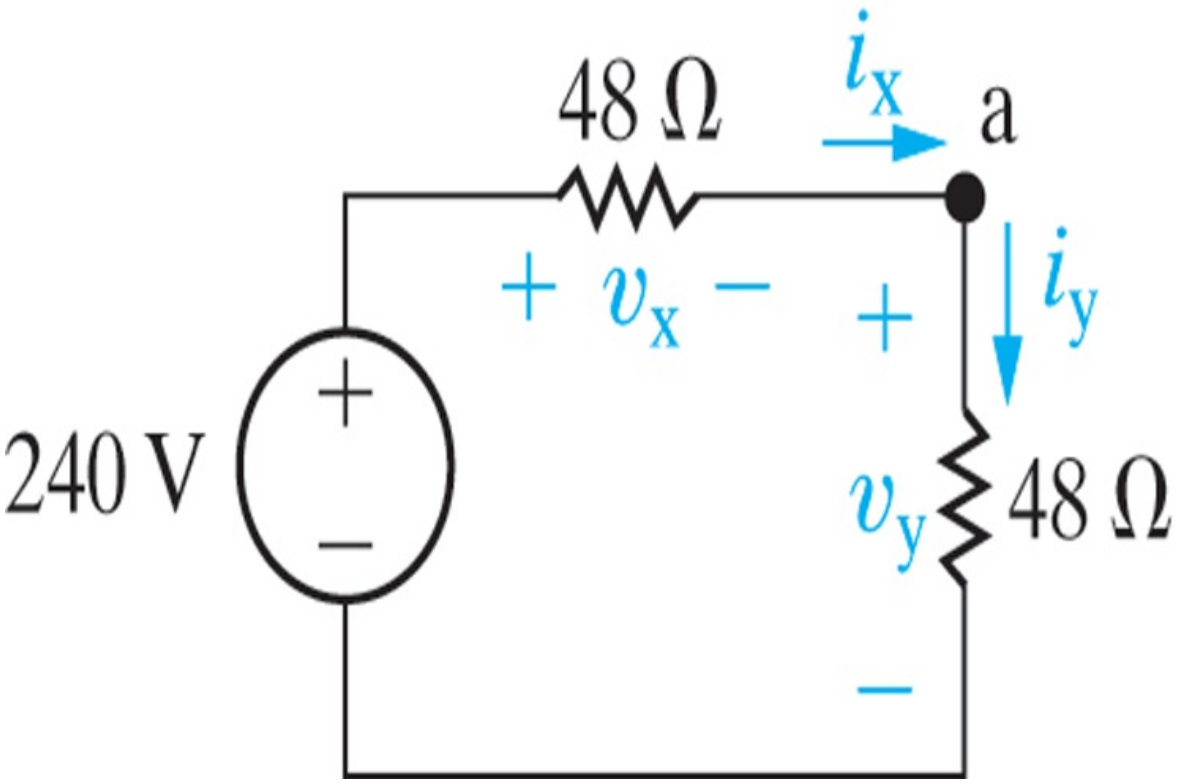
Now look at the other wiring diagram for the radiators, shown in Fig. 2.27. We know that the radiators can be modeled using 48 Ω resistors, which are used to turn the wiring diagram into the circuit in Fig. 2.28.

Figure 2.27



Another way to wire two radiators.

Figure 2.28



A circuit based on Fig. 2.27.

Figure 2.28 Full Alternative Text

Start analyzing the circuit in Fig. 2.28 by using Kirchhoff's voltage law to sum the voltage drops around the closed path:

$-240+v_x+v_y=0 \quad \Rightarrow \quad v_x+v_y=240.$

Next, use Kirchhoff's current law to sum to currents leaving the node labeled a:

$-i_x+i_y=0 \quad \Rightarrow \quad i_x=i_y=i.$

The current in the two resistors is the same, and we can use that current in Ohm's law equations to replace the two unknown voltages in the Kirchhoff's voltage law equation:

$48i + 48i = 240 = 96i \quad \Rightarrow \quad i = \frac{240}{96} = 2.5 \text{ A}.$

Use the current in the two resistors to calculate the power for the two radiators.

$P_x = P_y = Ri^2 = (48)(2.5)^2 = 300 \text{ W}.$

Thus, if the radiators are wired as shown in Fig. 2.27, their total power is 600 W. This is insufficient to heat the garage.

Therefore, the way the radiators are wired has a big impact on the amount of heat that will be supplied. When they are wired using the diagram in Fig. 2.25, 2400 W of power will be available, but when they are wired using the diagram in Fig. 2.27, only 600 W of power will be available.

SELF-CHECK: Assess your understanding of the Practical Perspective by solving Chapter Problems 2.41–2.43.

Summary

- The circuit elements introduced in this chapter are voltage sources, current sources, and resistors:
    - An **ideal voltage source** maintains a prescribed voltage regardless of the current in the source. An **ideal current source** maintains a prescribed current regardless of the voltage across the source. Voltage and current sources are either **independent**, that is, not influenced by any other current or voltage in the circuit, or **dependent**, that is, determined by some other current or voltage in the circuit. (See .)
    - A **resistor** constrains its voltage and current to be proportional to each other. The value of the proportional constant relating voltage and current in a resistor is called its **resistance** and is measured in ohms. (See .)
- **Ohm's law** establishes the proportionality of voltage and current in a resistor. Specifically,

  $v=iR$

  if the current flow in the resistor is in the direction of the voltage drop across it, or

  $v=-iR$

  if the current flow in the resistor is in the direction of the voltage rise across it. (See .)
- By combining the equation for power, $p=vi$, with Ohm's law, we can determine the power absorbed by a resistor:

  $p=i2R=v2/R.$

(See pages 33–34.)

- Circuits have nodes and closed paths. A **node** is a point where two or more circuit elements join. When just two elements connect to form a node, they are said to be **in series**. A **closed path** is a loop traced through connecting elements, starting and ending at the same node and encountering intermediate nodes only once each. (See pages 39–40.)

- The voltages and currents of interconnected circuit elements obey Kirchhoff's laws:

    - **Kirchhoff's current law** states that the algebraic sum of all the currents at any node in a circuit equals zero. (See page 39.)

    - **Kirchhoff's voltage law** states that the algebraic sum of all the voltages around any closed path in a circuit equals zero. (See page 40.)

- A circuit is solved when the voltage across and the current in every element have been determined. By combining an understanding of independent and dependent sources, Ohm's law, and Kirchhoff's laws, we can solve many simple circuits.

Problems

Section 2.1

1. 2.1 If the interconnection in Fig. P2.1 is valid, find the power developed by the current sources. If the interconnection is not valid, explain why.

Figure P2.1

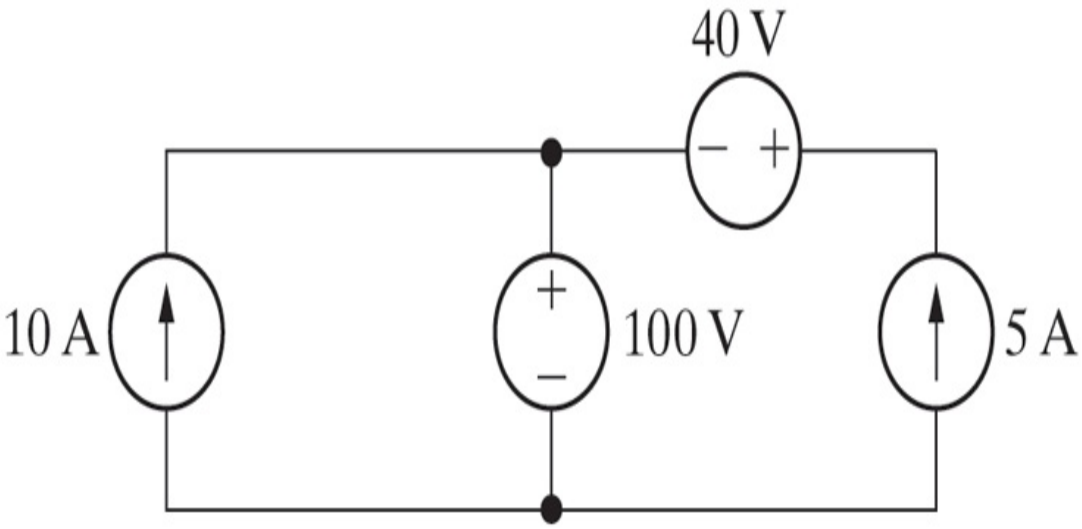


Figure P2.1 Full Alternative Text

2. 2.2

    1. Is the interconnection of ideal sources in the circuit in Fig. P2.2 valid? Explain.

    2. Identify which sources are developing power and which sources are absorbing power.

    3. Verify that the total power developed in the circuit equals the total power absorbed.

    4. Repeat (a)–(c), reversing the polarity of the 18 V source.

Figure P2.2

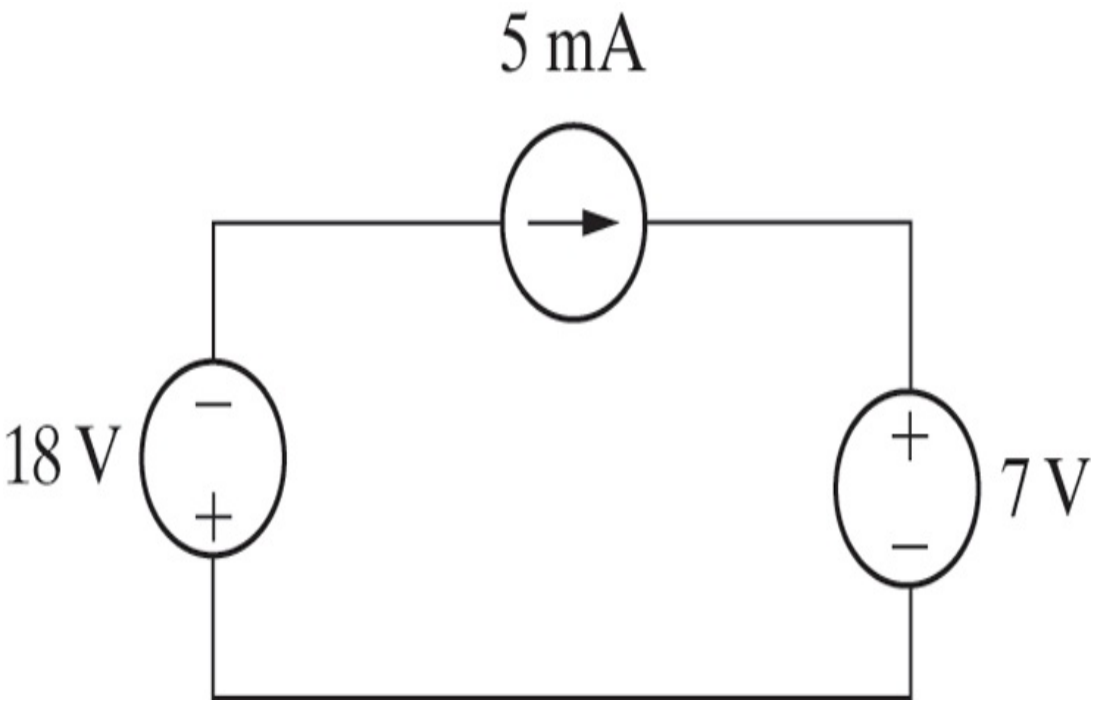


Figure P2.2 Full Alternative Text

3. 2.3 If the interconnection in Fig. P2.3 is valid, find the total power developed by the voltage sources. If the interconnection is not valid, explain why.

Figure P2.3

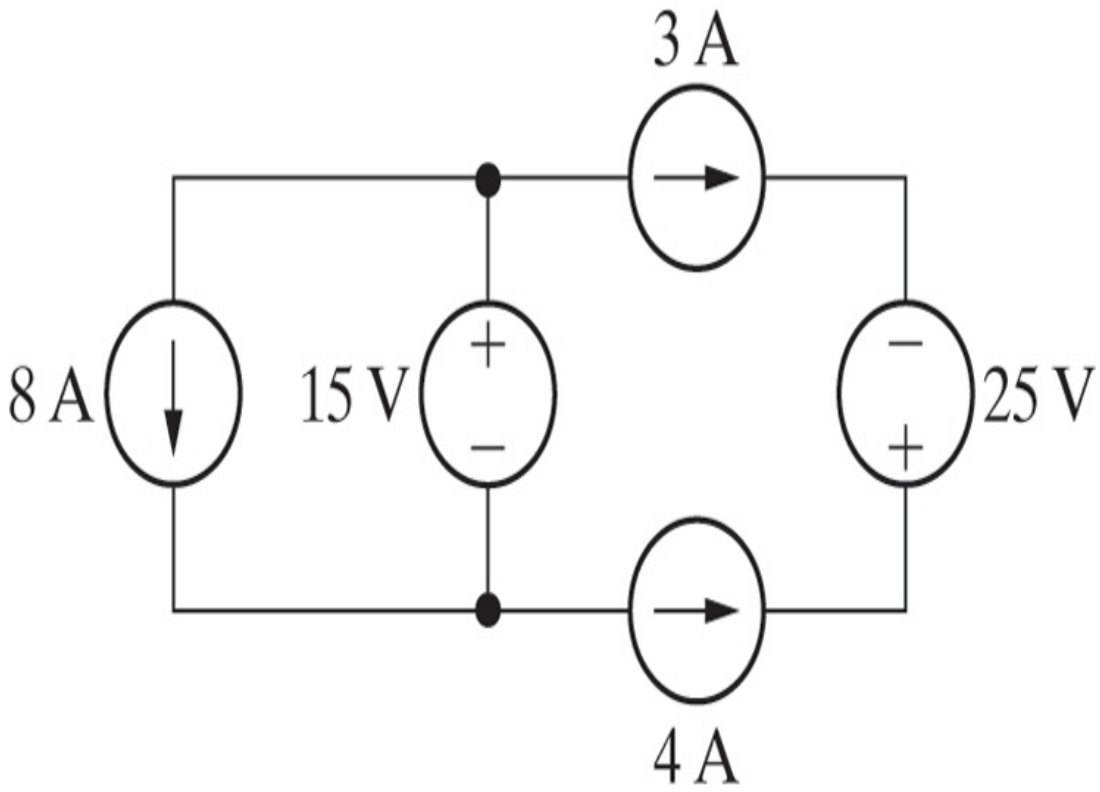
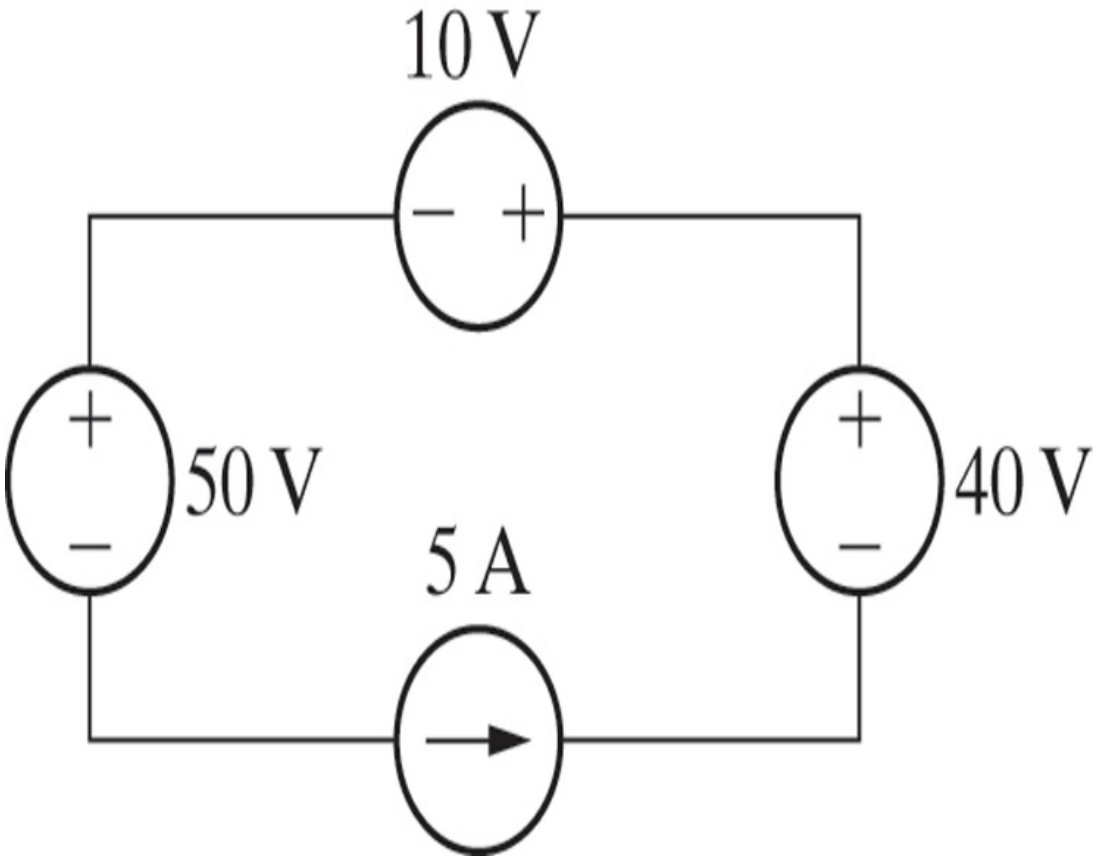


Figure P2.3 Full Alternative Text

4. 2.4 If the interconnection in Fig. P2.4 is valid, find the total power developed in the circuit. If the interconnection is not valid, explain why.

Figure P2.4



5. 2.5 The interconnection of ideal sources can lead to an indeterminate solution. With this thought in mind, explain why the solutions for v1 and v2 in the circuit in Fig. P2.5 are not unique.

Figure P2.5

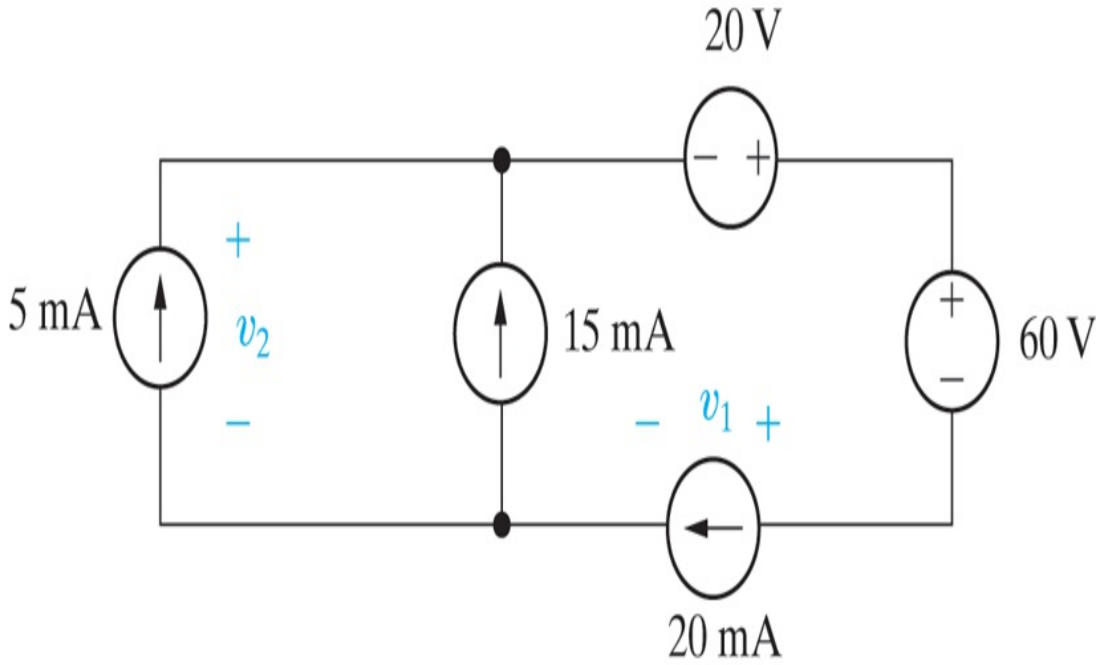


Figure P2.5 Full Alternative Text

6. 2.6 Consider the interconnection shown in Fig. P2.6.

Figure P2.6

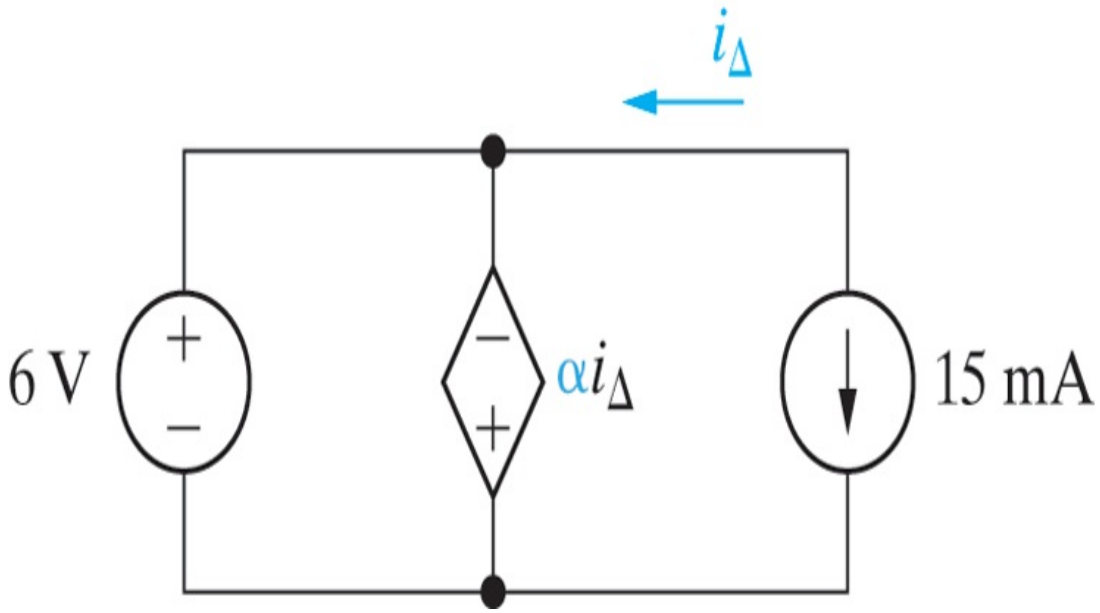


Figure P2.6 Full Alternative Text

1. What value of α is required to make this a valid interconnection?
2. For this value of α, find the power associated with the current source.
3. Is the current source supplying or absorbing power?

7. 2.7 Consider the interconnection shown in Fig. P2.7.

Figure P2.7

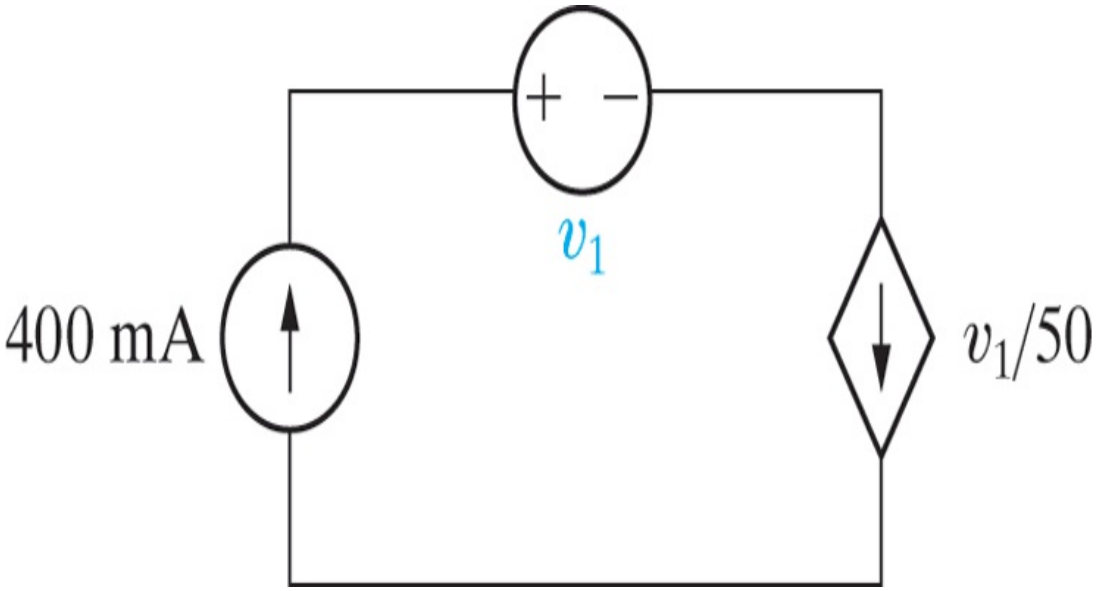


Figure P2.7 Full Alternative Text

1. What value of v1 is required to make this a valid interconnection?
2. For this value of v1, find the power associated with the voltage source.

8. 2.8 If the interconnection in Fig. P2.8 is valid, find the total power developed in the circuit. If the interconnection is not valid, explain why.

Figure P2.8

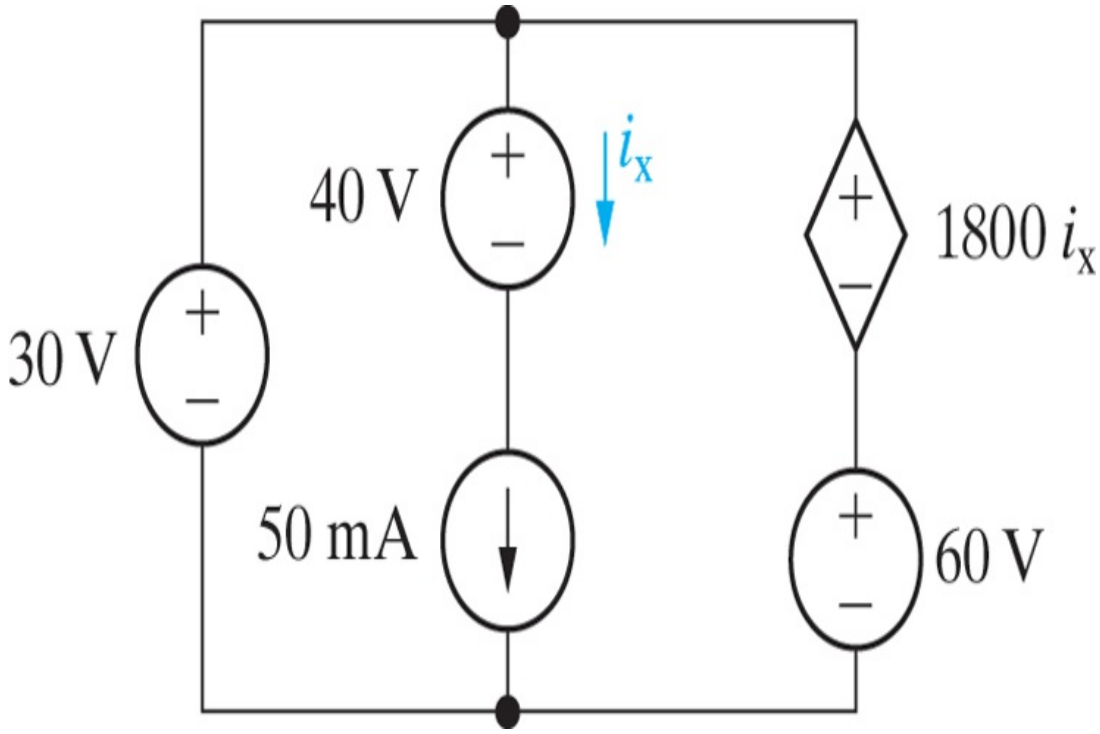


Figure P2.8 Full Alternative Text

9. 2.9 Find the total power developed in the circuit in Fig. P2.9 if vo=5 V.

Figure P2.9

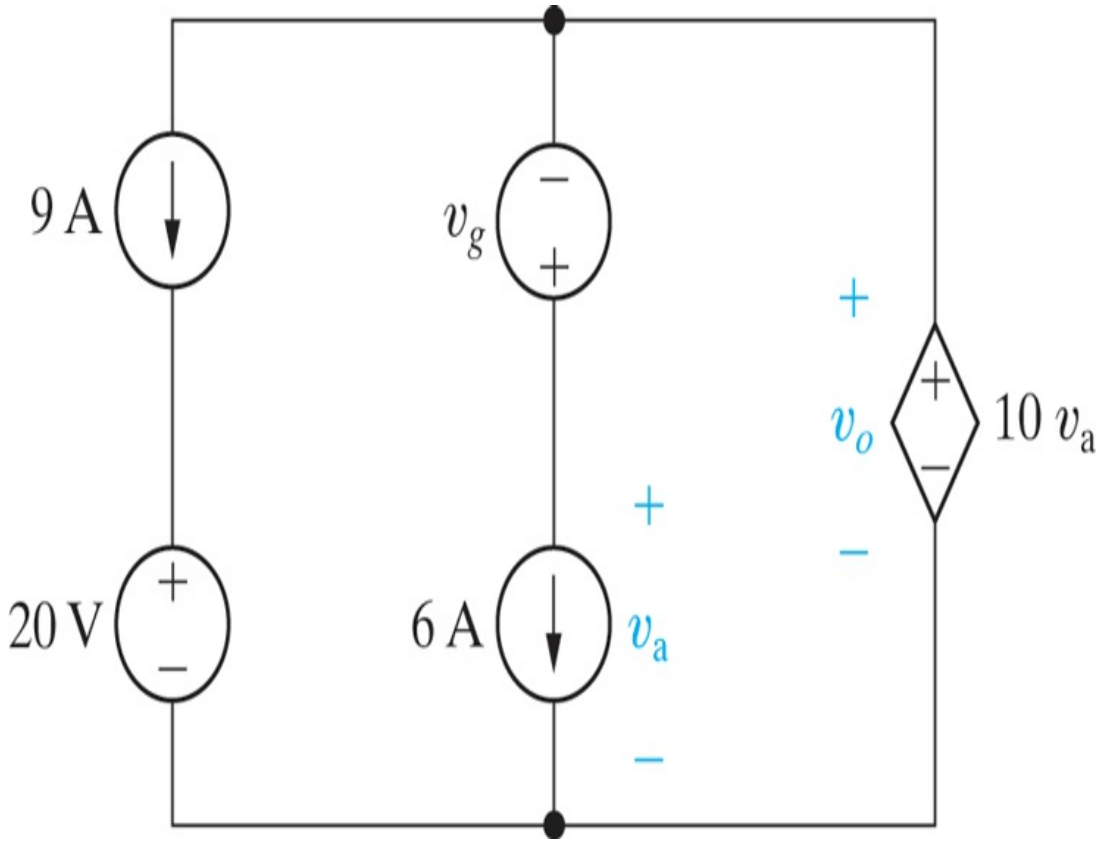


Figure P2.9 Full Alternative Text

10. 2.10

a)

1. Is the interconnection in Fig. P2.10 valid? Explain.

Figure P2.10

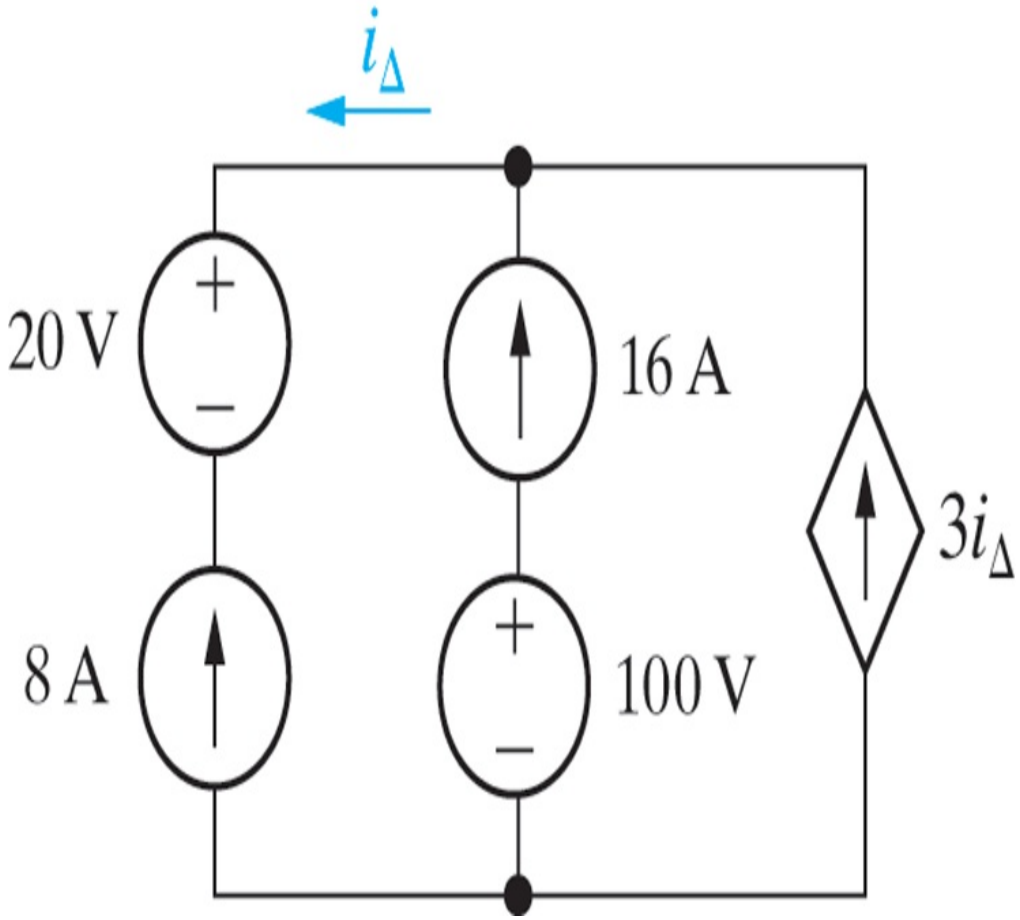


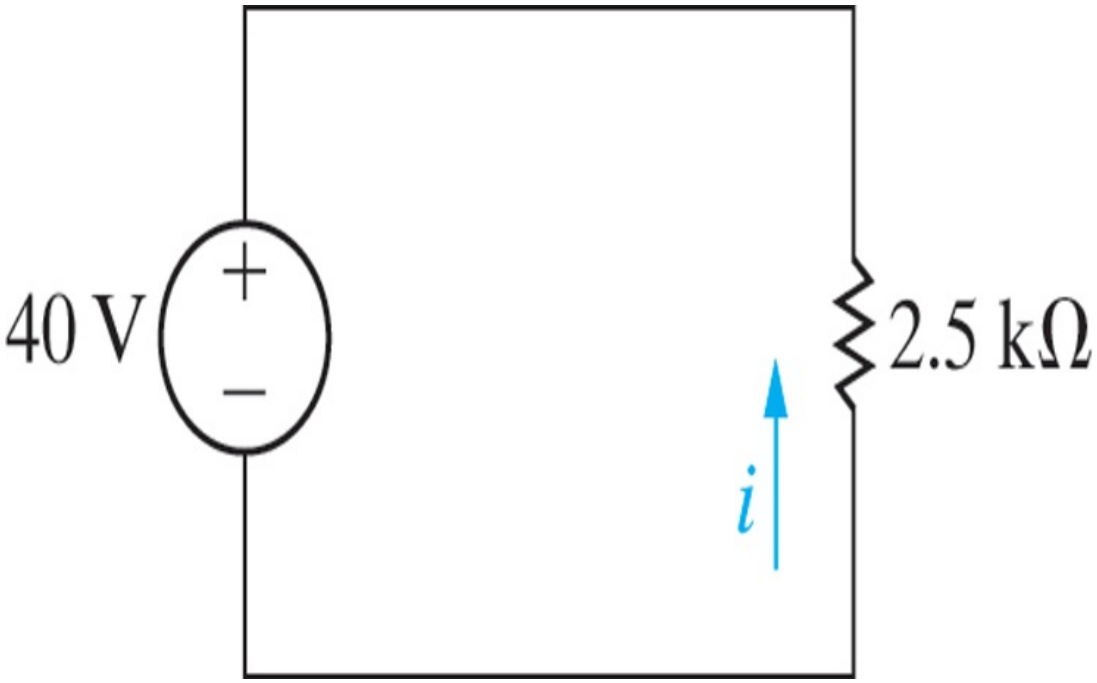
Figure P2.10 Full Alternative Text

2. Can you find the total energy developed in the circuit? Explain.

Sections 2.2–2.3

1. 2.11 For the circuit shown in Fig. P2.11

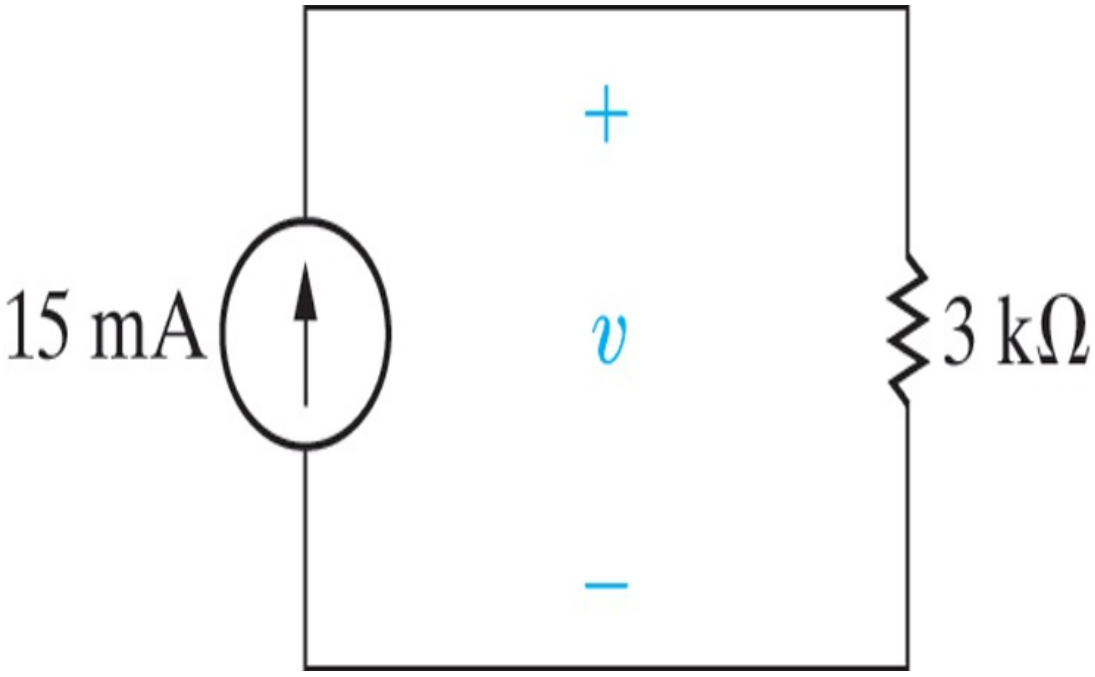
Figure P2.11



1. Find i.
2. Find the power supplied by the voltage source.
3. Reverse the polarity of the voltage source and repeat parts (a) and (b).

2. 2.12 For the circuit shown in Fig. P2.12

Figure P2.12



1. Find υ.
2. Find the power absorbed by the resistor.
3. Reverse the direction of the current source and repeat parts (a) and (b).

3. 2.13 A pair of automotive headlamps is connected to a 12 V battery via the arrangement shown in Fig. P2.13. In the figure, the triangular symbol is used to indicate that the terminal is connected directly to the metal frame of the car.

Figure P2.13

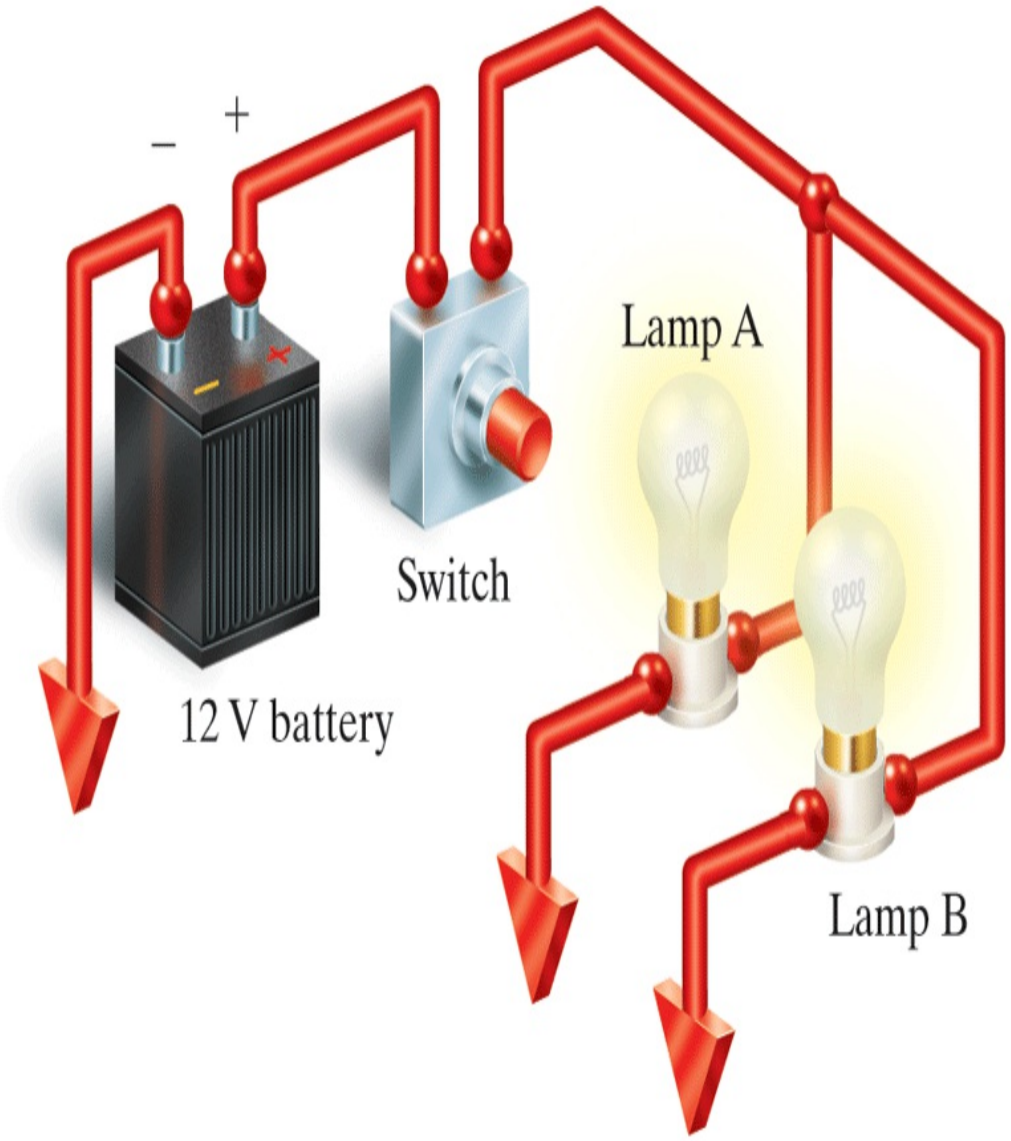


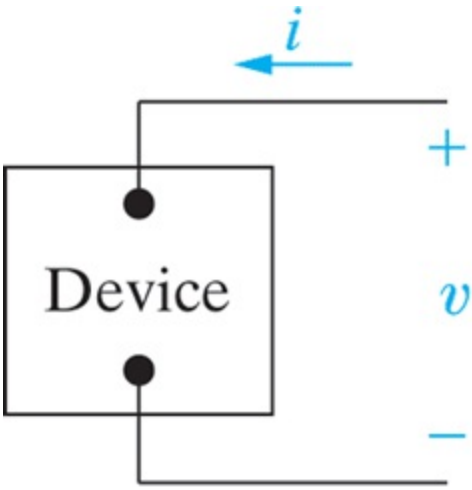
Figure P2.13 Full Alternative Text

1. Construct a circuit model using resistors and an independent voltage source.

2. Identify the correspondence between the ideal circuit element and the symbol component that it represents.

4. 2.14 The terminal voltage and terminal current were measured on the device shown in Fig. P2.14(a). The values of v and i are given in the table of Fig. P2.14(b). Use the values in the table to construct a circuit model for the device consisting of a single resistor from Appendix H.

Figure P2.14



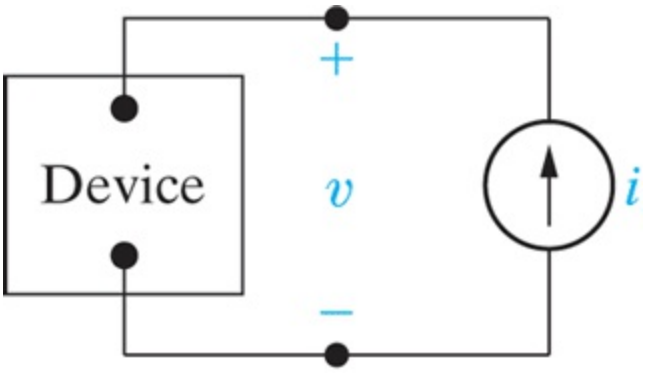
i (mA)	v (V)
−10	−120
−5	−60
5	60
10	120
15	180

(a) (b)

Figure P2.14 Full Alternative Text

5. 2.15 A variety of current source values were applied to the device shown in Fig. P2.15(a). The power absorbed by the device for each value of current is recorded in the table given in Fig. P2.15(b). Use the values in the table to construct a circuit model for the device consisting of a single resistor from Appendix H.

Figure P2.15



(a)

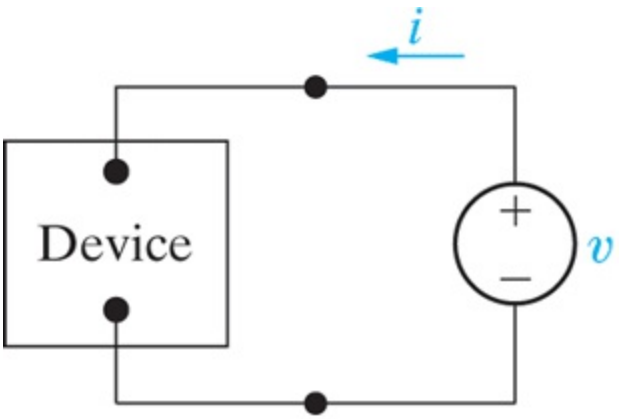
i (mA)	p (mW)
0.5	8.25
1.0	33.00
1.5	74.25
2.0	132.00
2.5	206.25
3.0	297.00

(b)

Figure P2.15 Full Alternative Text

6. 2.16 A variety of voltage source values were applied to the device shown in Fig. P2.16(a). The power absorbed by the device for each value of voltage is recorded in the table given in Fig. P2.16(b). Use the values in the table to construct a circuit model for the device consisting of a single resistor from Appendix H.

Figure P2.16



(a)

v (V)	p (mW)
−8	640
−4	160
4	160
8	640
12	1440
16	2560

(b)

Figure P2.16 Full Alternative Text

Section 2.4

1. 2.17 PSPICEMULTISIM

   a)

   1. Find the currents i1 and i2 in the circuit in Fig. P2.17.

Figure P2.17

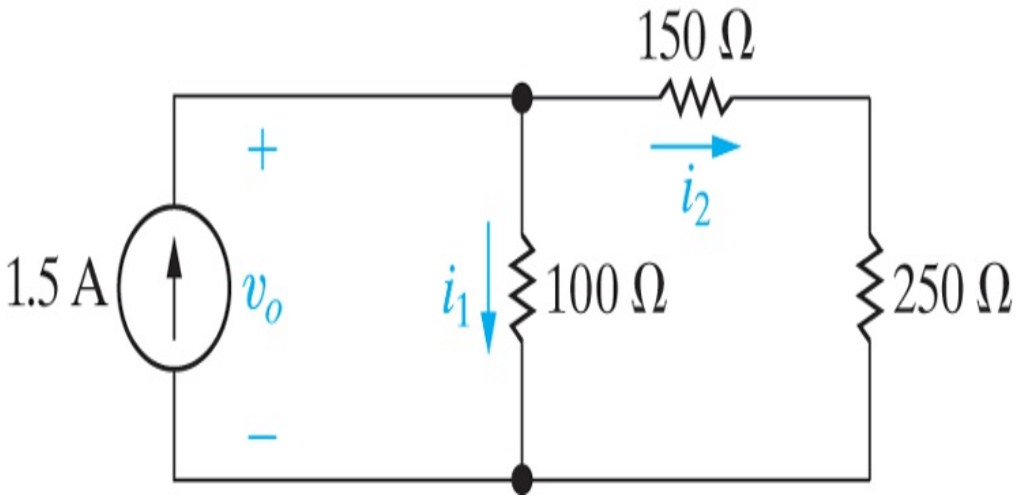


Figure P2.17 Full Alternative Text

   2. Find the voltage vo.

   3. Verify that the total power developed equals the total power dissipated.

2. 2.18 PSPICEMULTISIM Given the circuit shown in Fig. P2.18, find

Figure P2.18

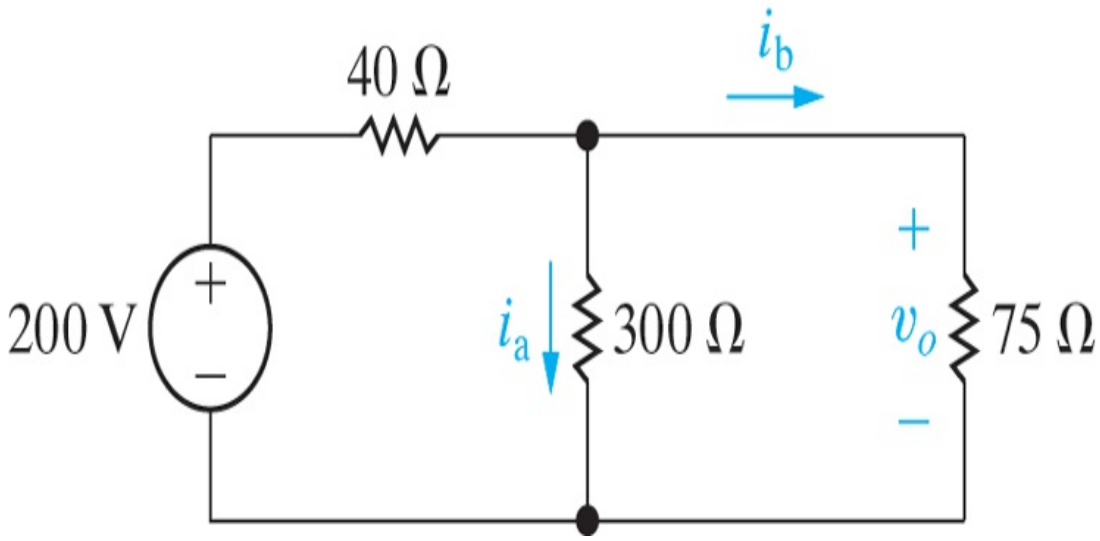


Figure P2.18 Full Alternative Text

1. the value of ia,
2. the value of ib,
3. the value of vo,
4. the power dissipated in each resistor,
5. the power delivered by the 200 V source.

3. 2.19 PSPICEMULTISIM The current ia in the circuit shown in Fig. P 2.19 is 20 A. Find (a) io; (b) ig; and (c) the power delivered by the independent current source.

Figure P2.19

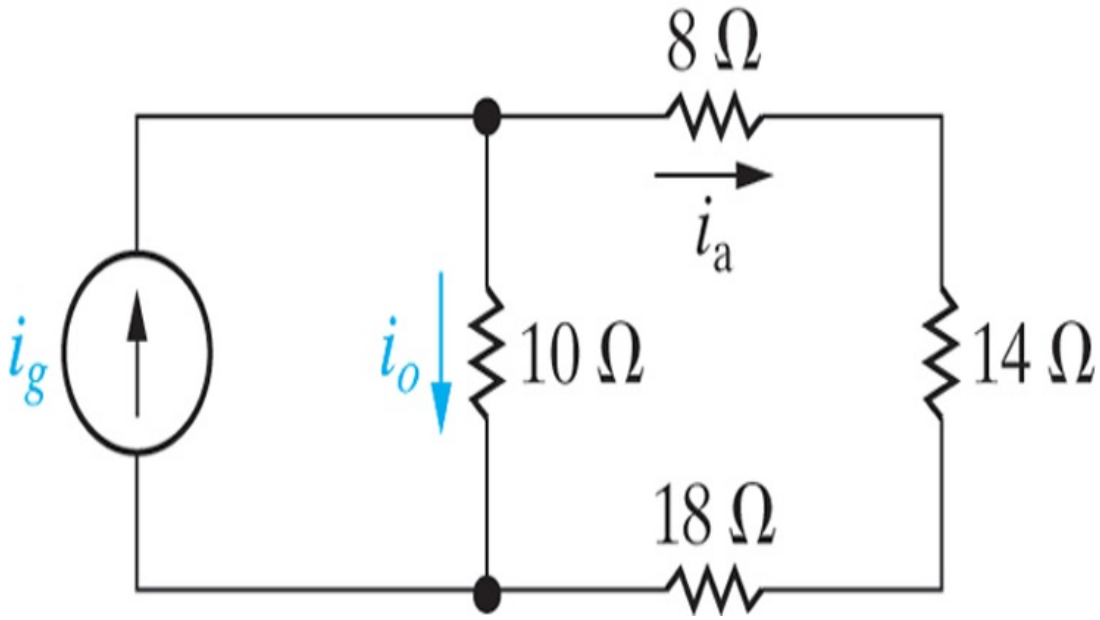


Figure P2.19 Full Alternative Text

4. 2.20 Consider the circuit shown in Fig. P2.20.

Figure P2.20

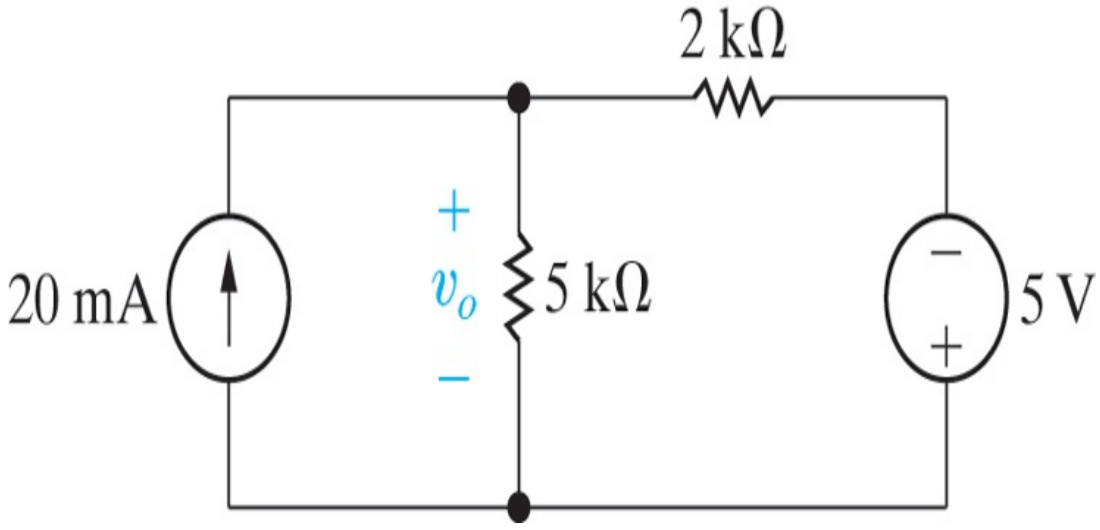


Figure P2.20 Full Alternative Text

1. Find vo using Kirchoff's laws and Ohm's law.

2. Test the solution for vo by verifying that the total power supplied equals the total power absorbed.

5. 2.21 The current ix in the circuit shown in Fig. P2.21 is 50 mA, and the voltage vx is 3.5 V. Find (a) i1; (b) v1; (c) vg; and (d) the power supplied by the voltage source.

Figure P2.21

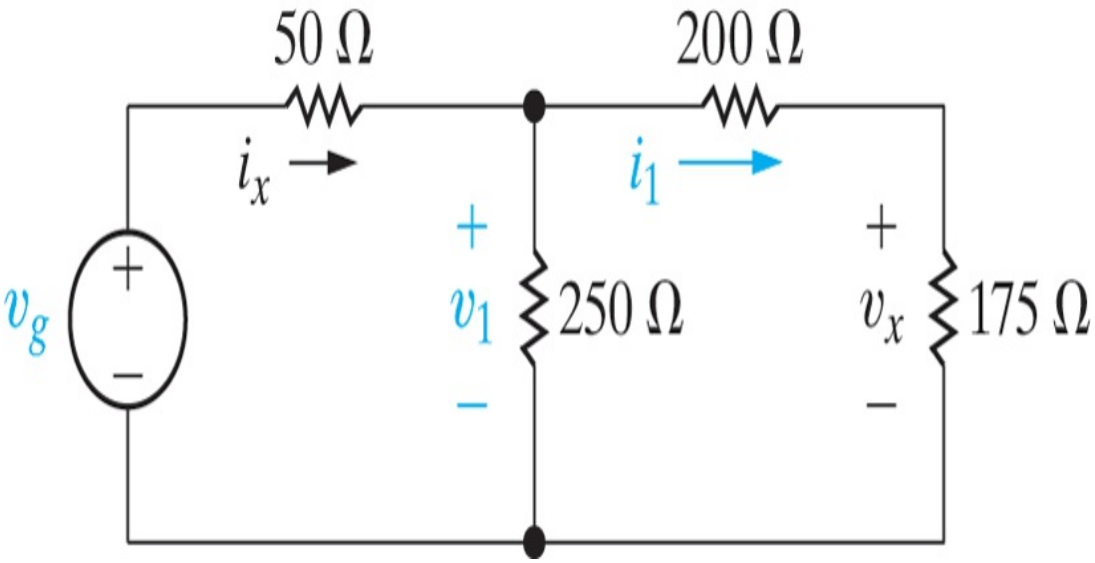


Figure P2.21 Full Alternative Text

6. 2.22 PSPICEMULTISIM The current io in the circuit in Fig. P2.22 is 2 A.

Figure P2.22

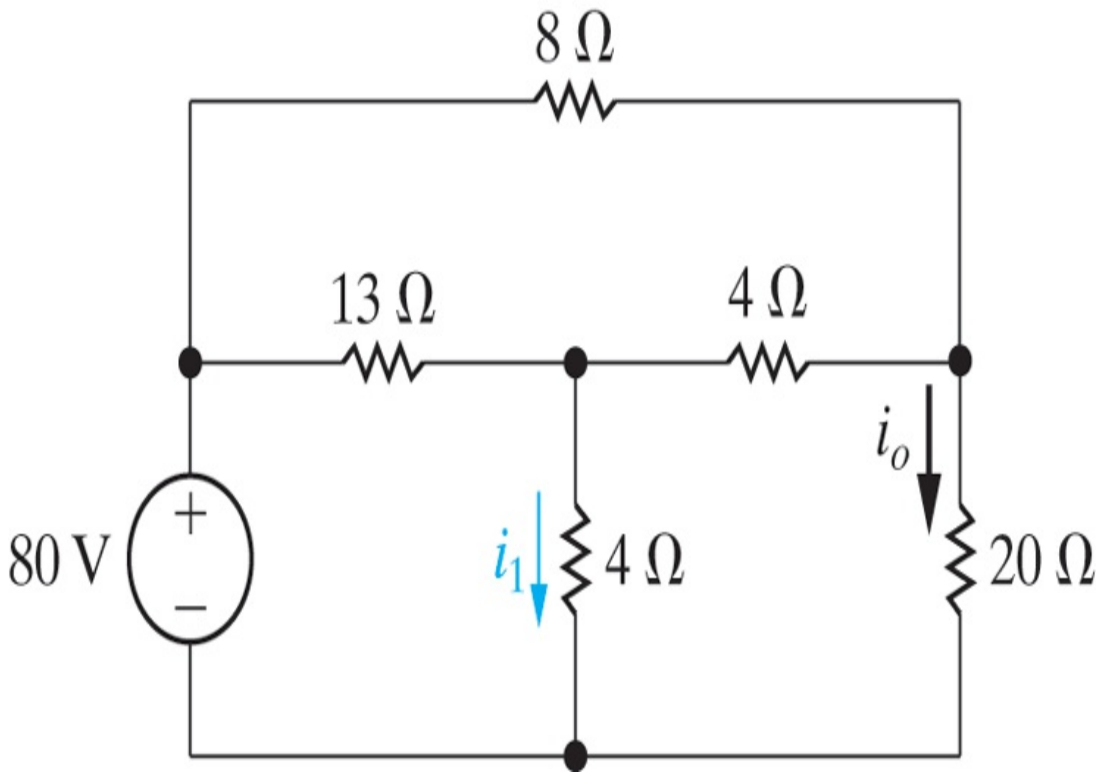


Figure P2.22 Full Alternative Text

1. Find i1.
2. Find the power dissipated in each resistor.
3. Verify that the total power dissipated in the circuit equals the power developed by the 80 V source.

7. 2.23 PSPICEMULTISIM The voltage across the 22.5 Ω resistor in the circuit in Fig. P2.23 is 90 V, positive at the upper terminal.

Figure P2.23

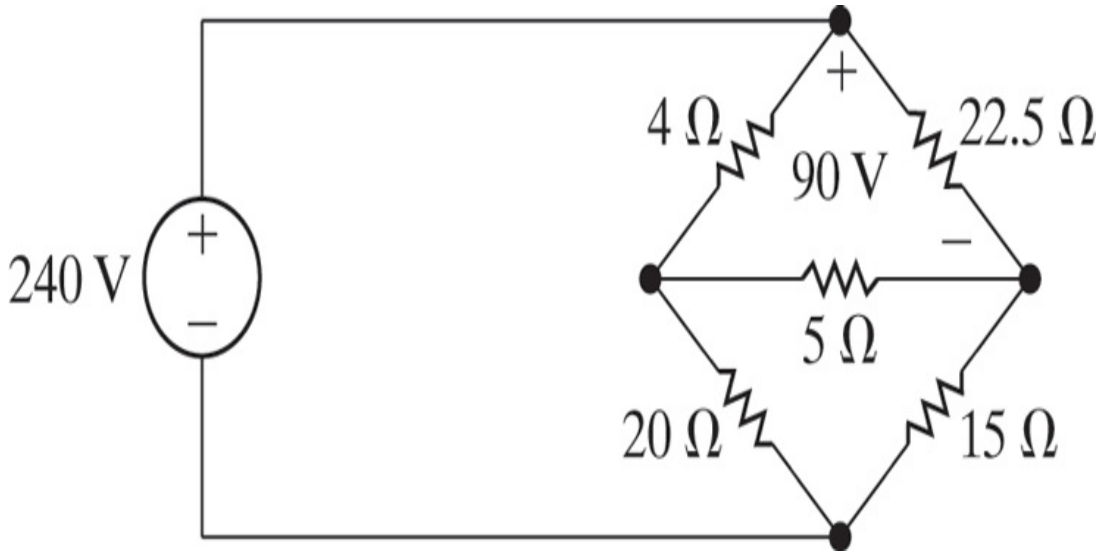


Figure P2.23 Full Alternative Text

1. Find the power dissipated in each resistor.
2. Find the power supplied by the 240 V ideal voltage source.
3. Verify that the power supplied equals the total power dissipated.

8. 2.24 The currents i1 and i2 in the circuit in Fig. P2.24 are 20 A and 15 A, respectively.

Figure P2.24

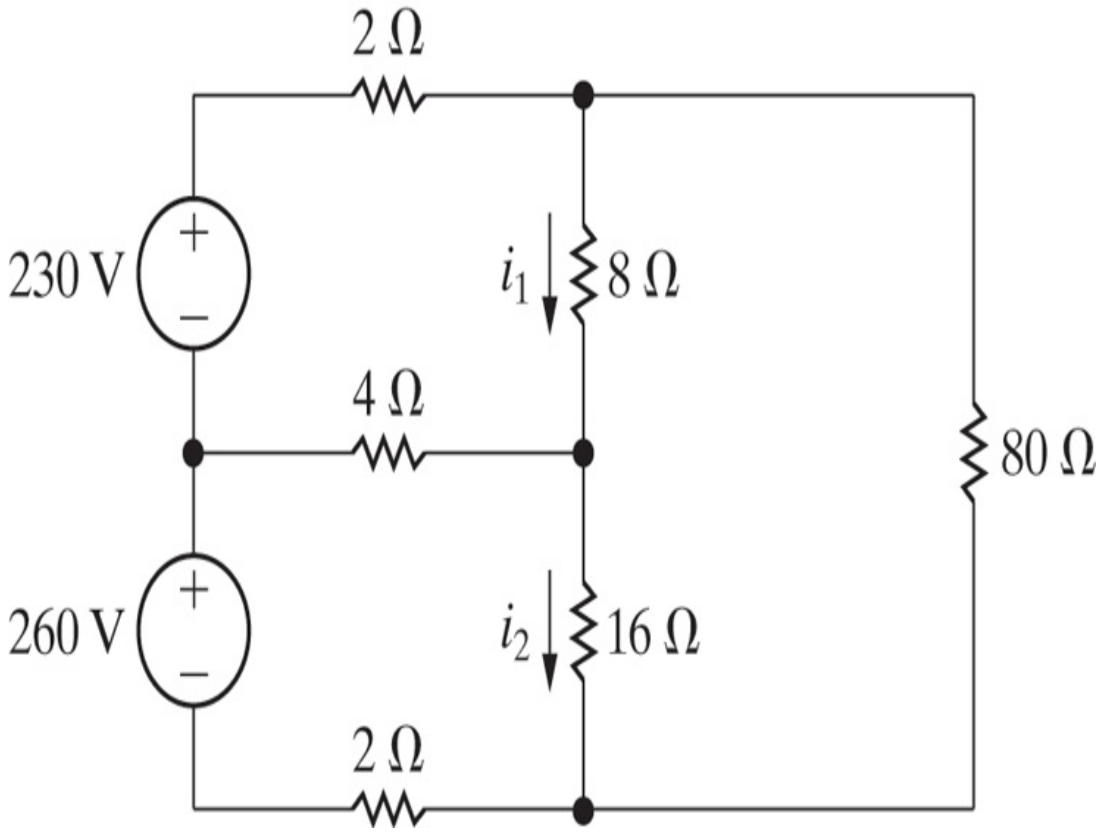


Figure P2.24 Full Alternative Text

1. Find the power supplied by each voltage source.
2. Show that the total power supplied equals the total power dissipated in the resistors.

9. 2.25 PSPICEMULTISIM The currents ia and ib in the circuit in Fig. P 2.25 are 4 A and −2 A, respectively.

Figure P2.25

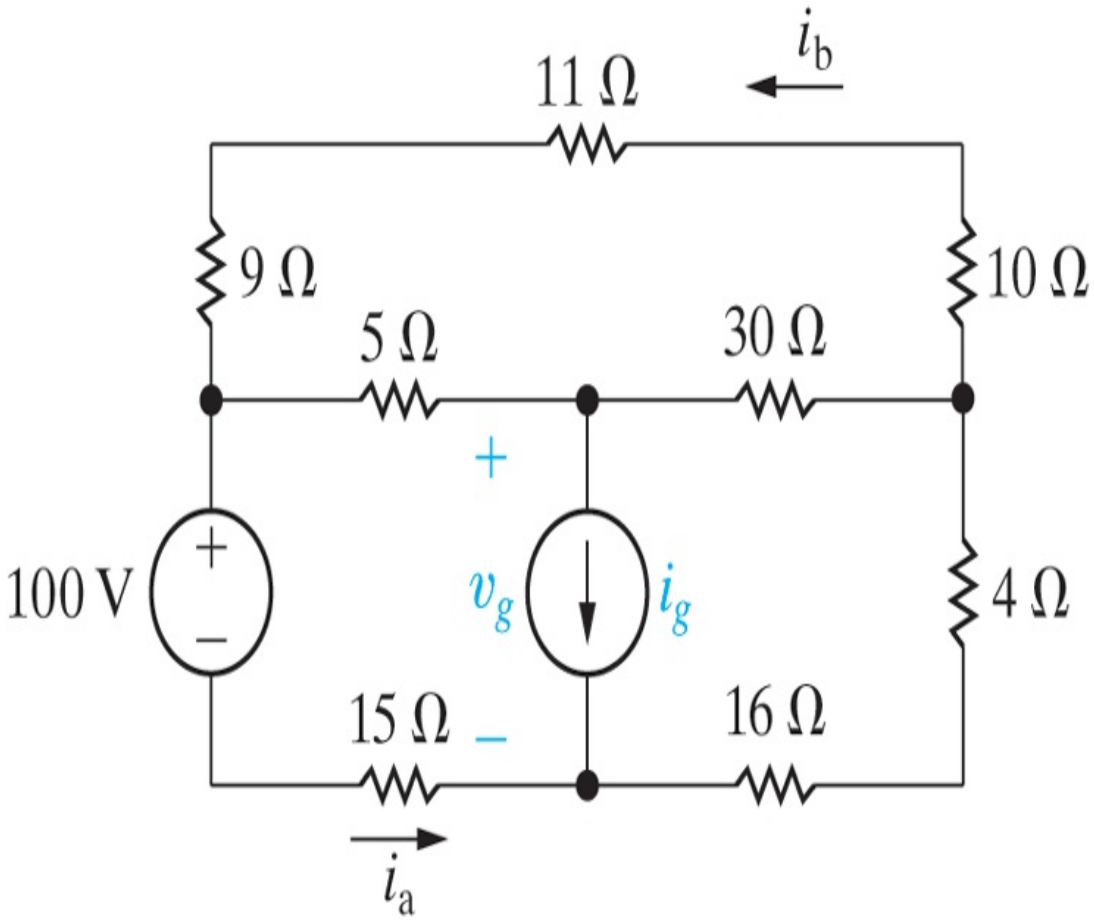


Figure P2.25 Full Alternative Text

1. Find ig.
2. Find the power dissipated in each resistor.
3. Find vg.
4. Show that the power delivered by the current source is equal to the power absorbed by all the other elements.

10. 2.26 PSPICEMULTISIM For the circuit shown in Fig. P2.26, find (a) R and (b) the power supplied by the 240 V source.

Figure P2.26

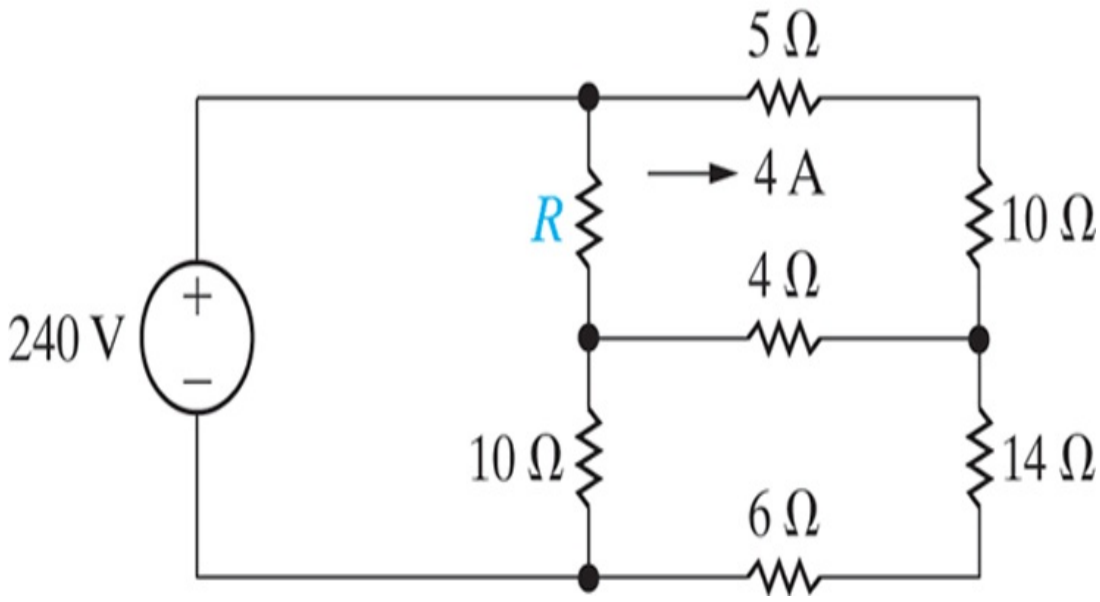


Figure P2.26 Full Alternative Text

11. 2.27 PSPICEMULTISIM The variable resistor R in the circuit in Fig. P 2.27 is adjusted until io equals 10 mA. Find the value of R.

Figure P2.27

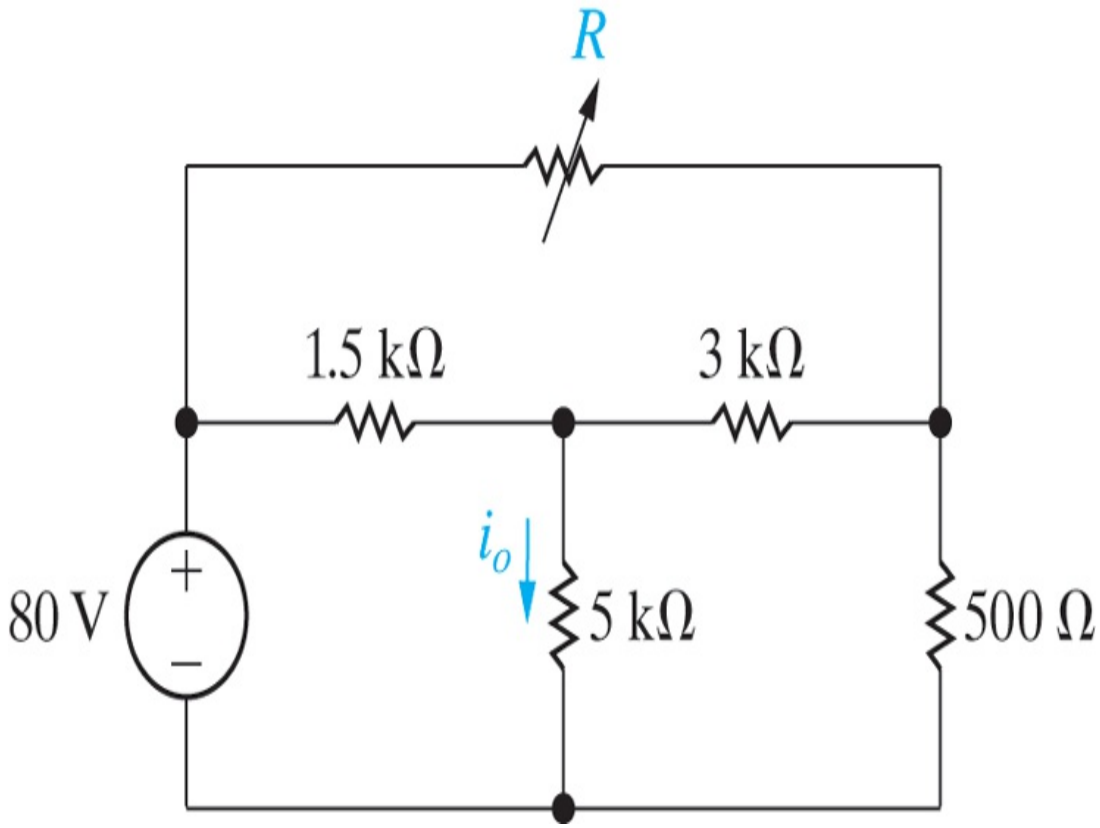
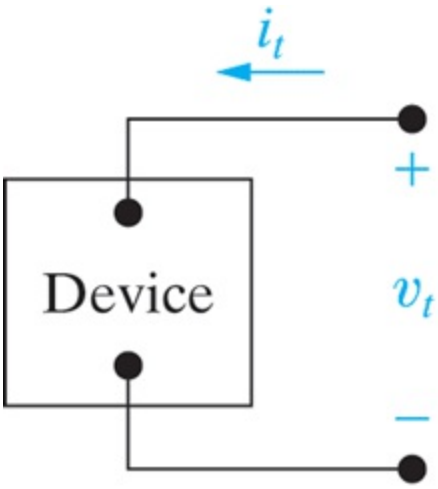


Figure P2.27 Full Alternative Text

12. 2.28 The voltage and current were measured at the terminals of the device shown in Fig. P2.28(a). The results are tabulated in Fig. P 2.28(b).

Figure P2.28



v_t (V)	i_t (A)
–30	0
10	2
50	4
90	6
130	8

(a) (b)

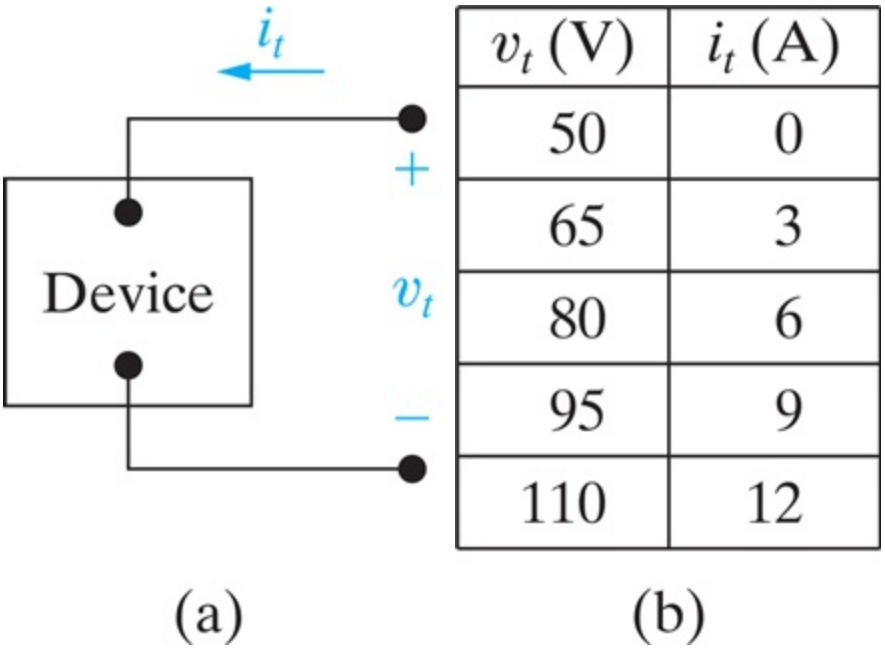
Figure P2.28 Full Alternative Text

1. Construct a circuit model for this device using an ideal voltage source in series with a resistor.

2. Use the model to predict the it the amount of power the device will deliver to a 40 Ω resistor.

13. 2.29 The voltage and current were measured at the terminals of the device shown in Fig. P2.29(a). The results are tabulated in Fig. P 2.29(b).

Figure P2.29



v_t (V)	i_t (A)
50	0
65	3
80	6
95	9
110	12

(b)

Figure P2.29 Full Alternative Text

1. Construct a circuit model for this device using an ideal current source in parallel with a resistor.

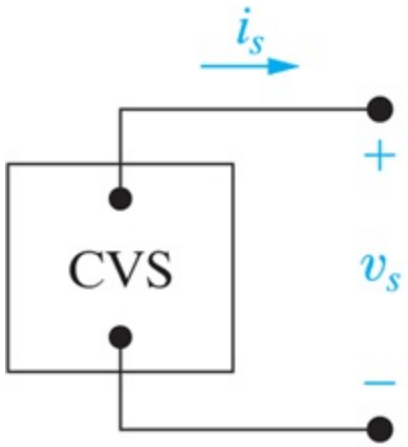
2. Use the model to predict the amount of power the device will deliver to a 20 Ω resistor.

14. 2.30 The table in Fig. P2.30(a) gives the relationship between the terminal voltage and current of the practical constant voltage source shown in Fig. P2.30(b).

Figure P2.30

v_s (V)	i_s (mA)
24	0
22	8
20	16
18	24
15	32
10	40
0	48

(a)



(b)

Figure P2.30 Full Alternative Text

1. Plot vs versus is.

2. Construct a circuit model of the practical source that is valid for 0≤is≤24 mA, based on the equation of the line plotted in (a). (Use an ideal voltage source in series with an ideal resistor.)

3. Use your circuit model to predict the current delivered to a 1 kΩ resistor connected to the terminals of the practical source.

4. Use your circuit model to predict the current delivered to a short circuit connected to the terminals of the practical source.

5. What is the actual short-circuit current?

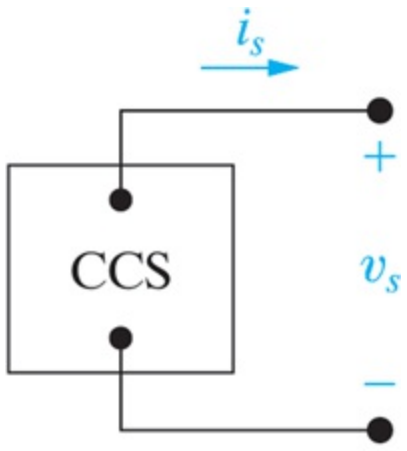
6. Explain why the answers to (d) and (e) are not the same.

15. 2.31 The table in Fig. P2.31(a) gives the relationship between the terminal current and voltage of the practical constant current source shown in Fig. P2.31(b).

Figure P2.31

i_s (mA)	v_s (V)
40	0
35	10
30	20
25	30
18	40
8	50
0	55

(a)



(b)

Figure P2.31 Full Alternative Text

1. Plot is versus vs.
2. Construct a circuit model of this current source that is valid for 0≤vs≤30 V, based on the equation of the line plotted in (a).
3. Use your circuit model to predict the current delivered to a 3 kΩ resistor.
4. Use your circuit model to predict the open- circuit voltage of the current source.
5. What is the actual open-circuit voltage?
6. Explain why the answers to (d) and (e) are not the same.

16. 2.32 Consider the circuit shown in Fig. P2.32.

    1. Find io.

2. Verify the value of io by showing that the power generated in the circuit equals the power absorbed in the circuit.

Figure P2.32

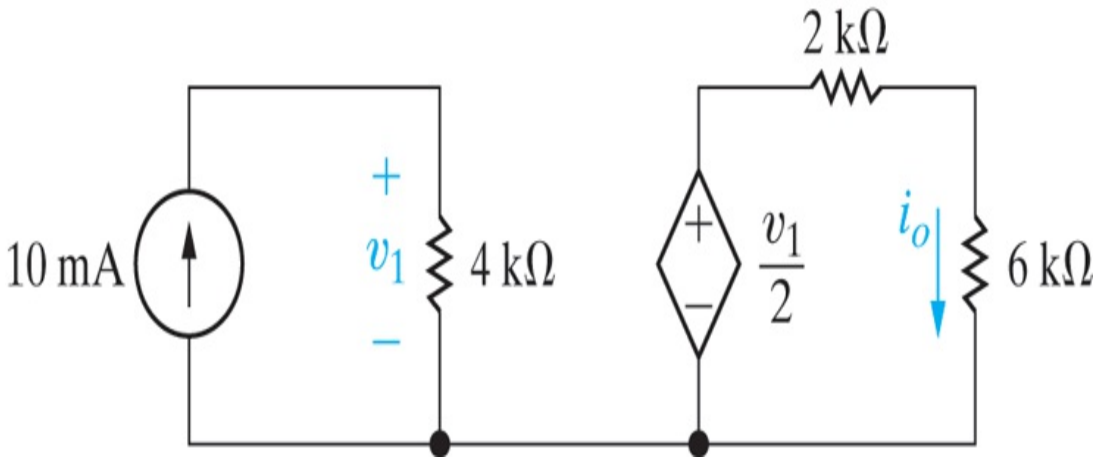


Figure P2.32 Full Alternative Text

17. 2.33 For the circuit shown in Fig. P2.33, find vo and the total power absorbed in the circuit.

Figure P2.33

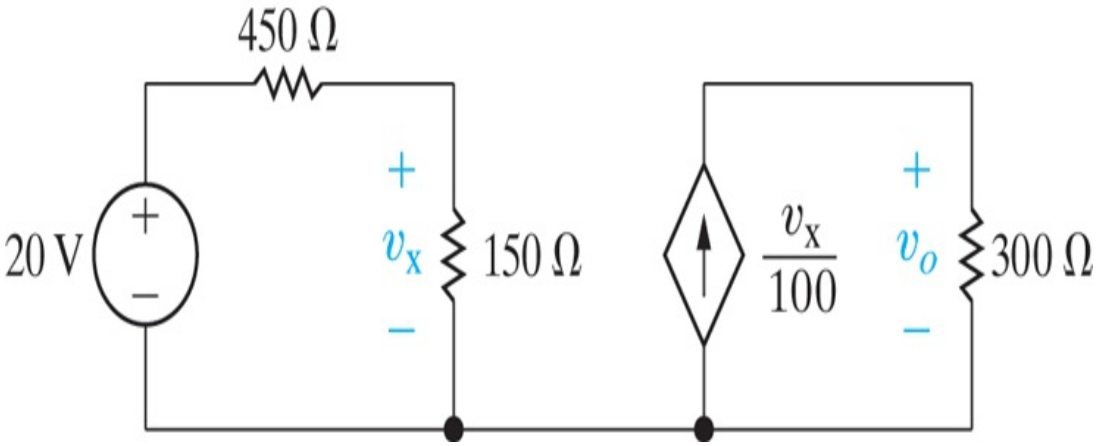


Figure P2.33 Full Alternative Text

18. 2.34 For the circuit shown in Fig. P2.34, find vo and the total power supplied in the circuit.

Figure P2.34

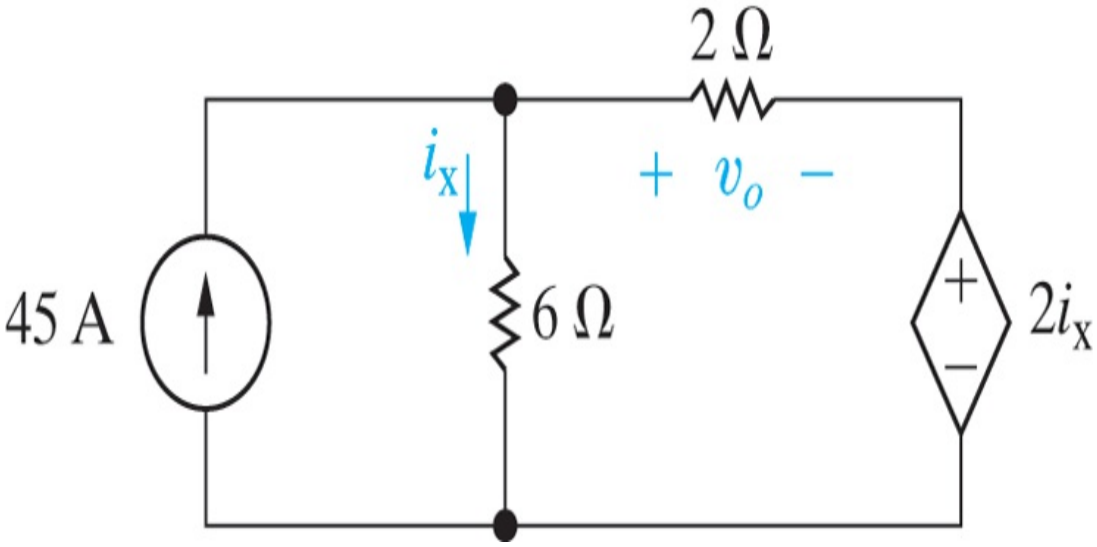


Figure P2.34 Full Alternative Text

19. 2.35 PSPICEMULTISIM Find (a) io, (b) i1, and (c) i2 in the circuit in Fig. P2.35.

Figure P2.35

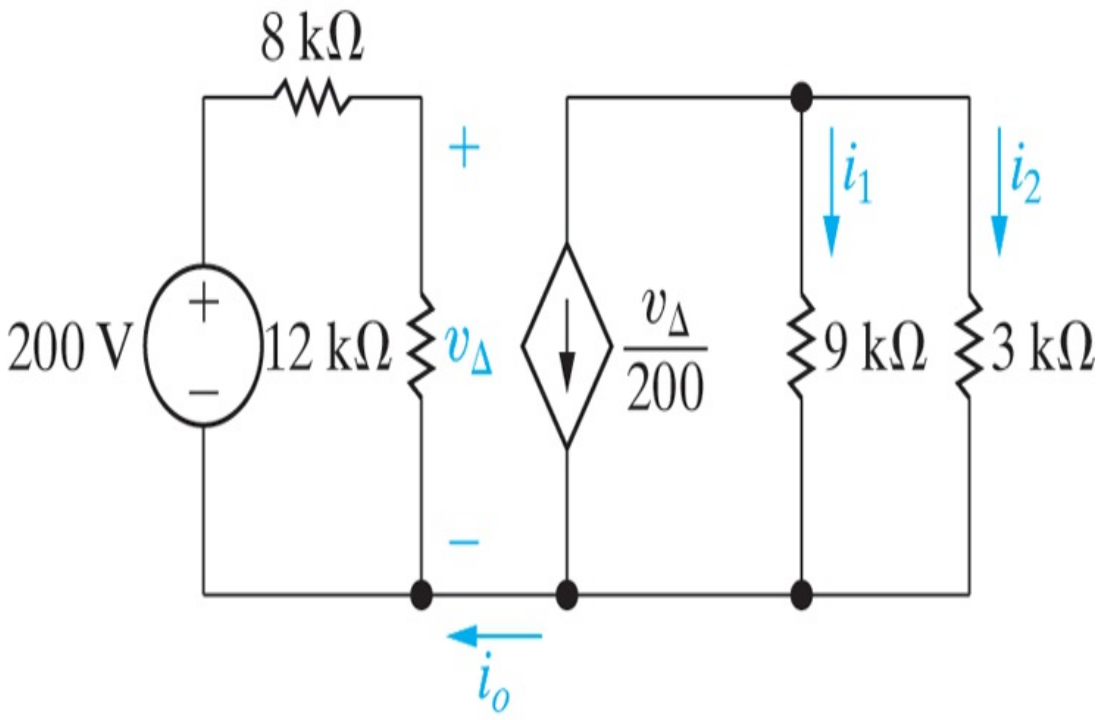


Figure P2.35 Full Alternative Text

20. 2.36 PSPICEMULTISIM For the circuit shown in Fig. P2.36, calculate (a) iΔ and vo, and (b) show that the power developed equals the power absorbed.

Figure P2.36

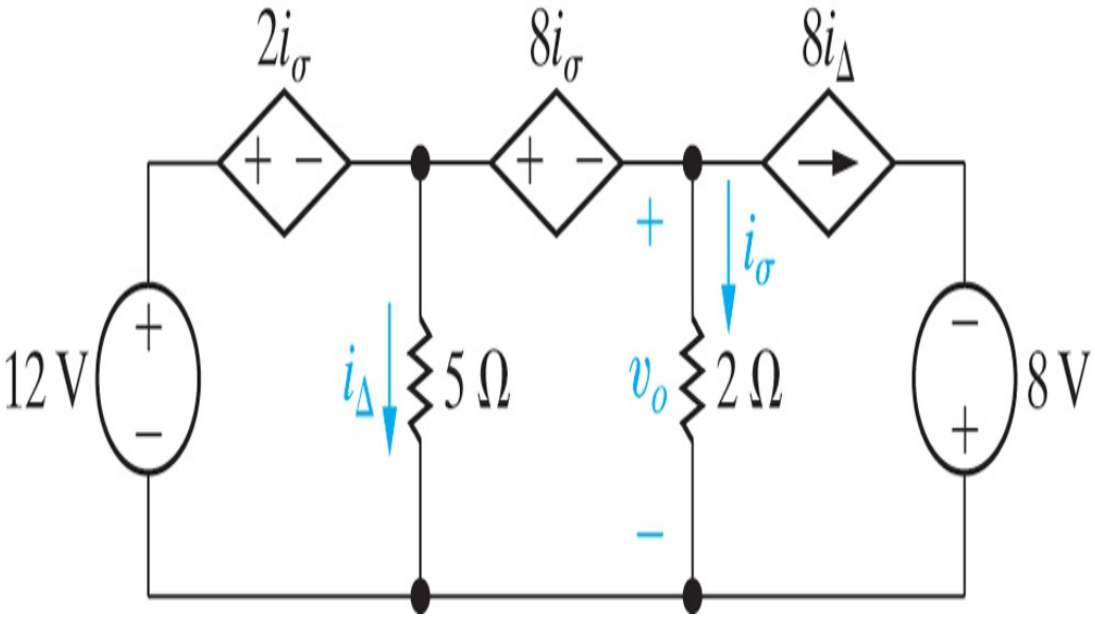


Figure P2.36 Full Alternative Text

21. 2.37 Find v1 and vg in the circuit shown in Fig. P2.37 when vo equals 5 V. (*Hint:* Start at the right end of the circuit and work back toward vg.)

Figure P2.37

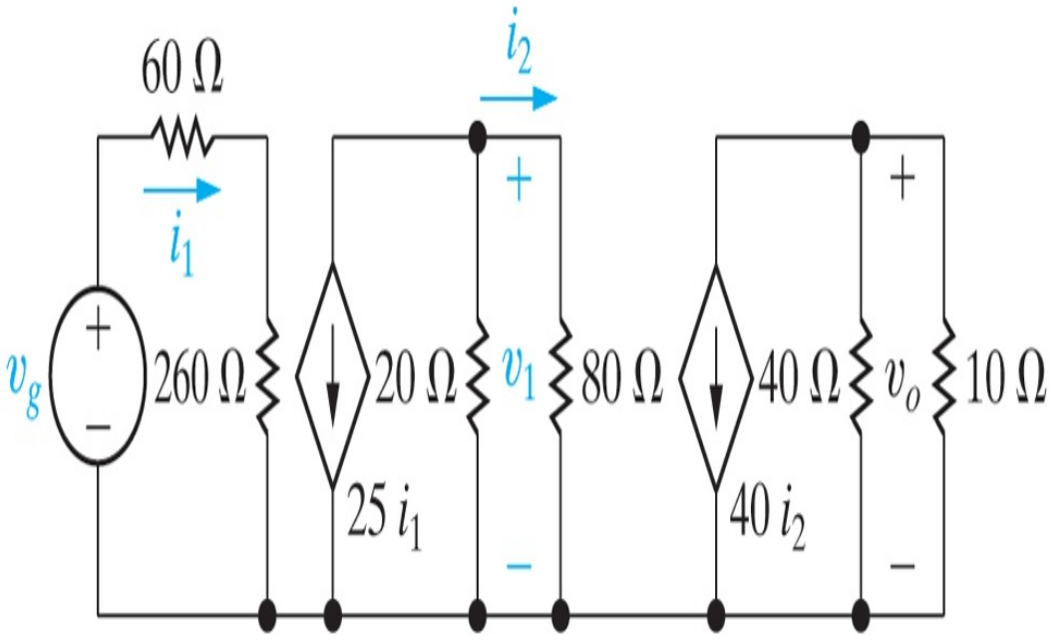
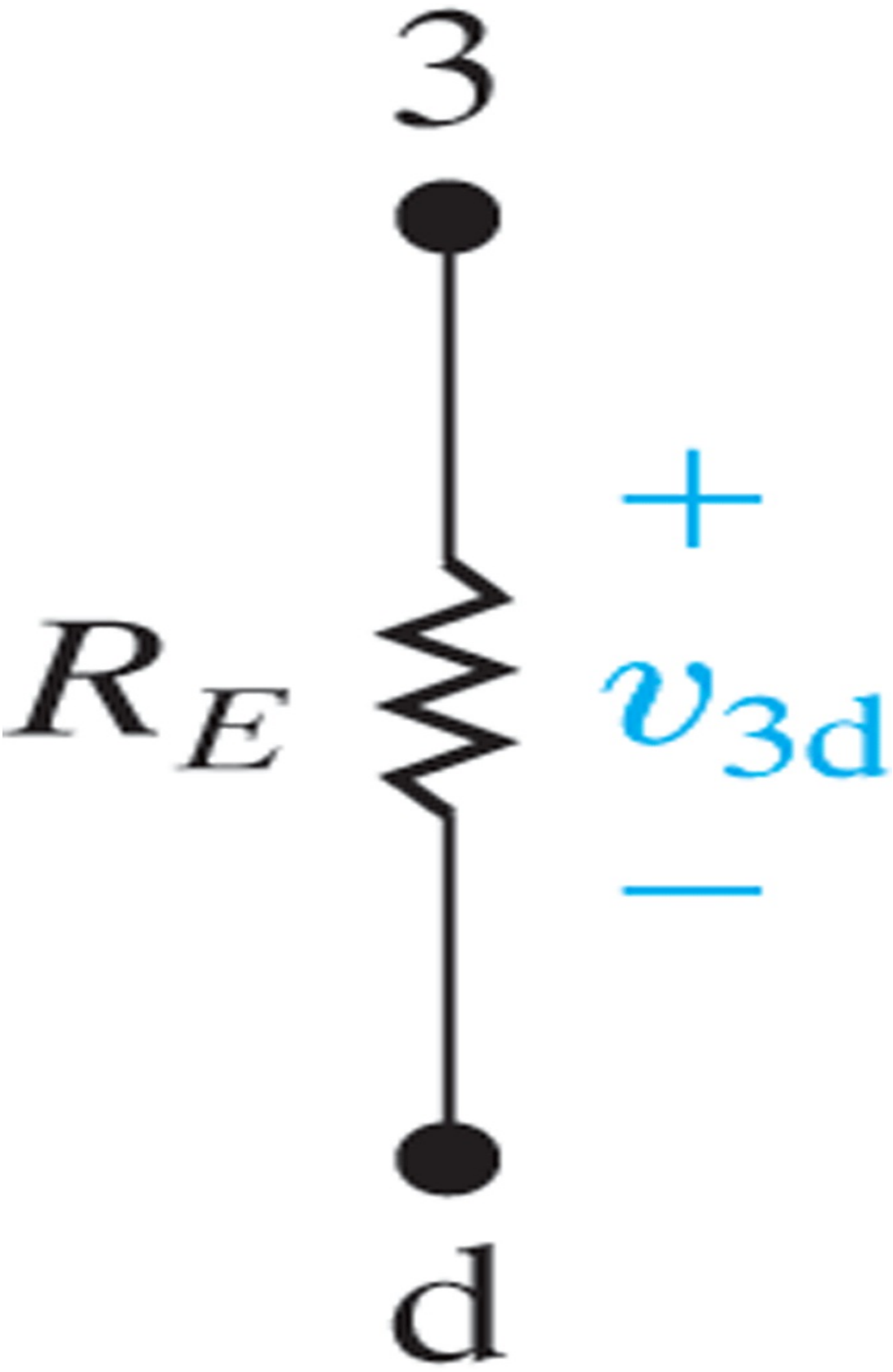


Figure P2.37 Full Alternative Text

22. 2.38 Derive Eq. 2.21. *Hint:* Use Eqs. (3) and (4) from Example 2.12 to express iE as a function of iB. Solve Eq. (2) for i2 and substitute the result into both Eqs. (5) and (6). Solve the "new" Eq. (6) for i1 and substitute this result into the "new" Eq. (5). Replace iE in the "new" Eq. (5) and solve for iB. Note that because iCC appears only in Eq. (1), the solution for iB involves the manipulation of only five equations.

23. 2.39 PSPICEMULTISIM For the circuit shown in Fig. 2.24, R1=40 kΩ R2=60 kΩ, RC=750 Ω, RE=120 Ω, VCC=10 V, V0=600 mV, and β=49. Calculate iB, iC, iE, v3d, vbd, i2, i1, vab, iCC, and v13 . (*Note:* In the double subscript notation on voltage variables, the first subscript is positive with respect to the second subscript. See Fig. P2.39.)

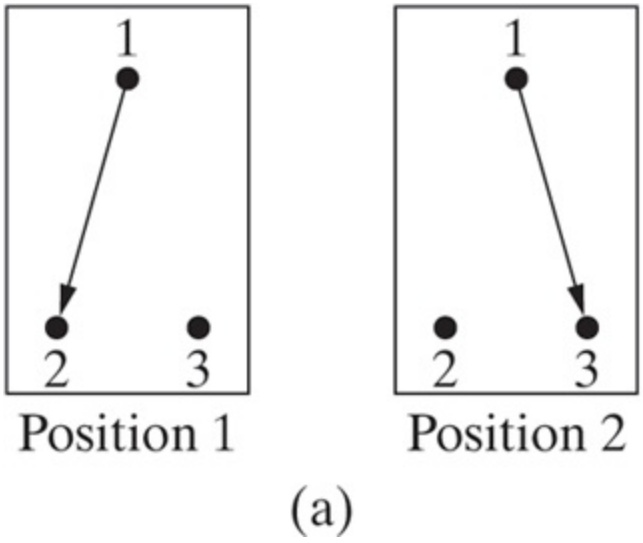
Figure P2.39


3
R_E
+
v_{3d}
−
d

Sections 2.1–2.5

1. 2.40 PSPICEMULTISIM It is often desirable in designing an electric wiring system to be able to control a single appliance from two or more locations, for example, to control a lighting fixture from both the top and bottom of a stairwell. In home wiring systems, this type of control is implemented with three-way and four-way switches. A three-way switch is a three-terminal, two-position switch, and a four-way switch is a four-terminal, two-position switch. The switches are shown schematically in Fig. P2.40(a), which illustrates a three-way switch, and P2.40(b), which illustrates a four-way switch.

Figure P2.40



(a)

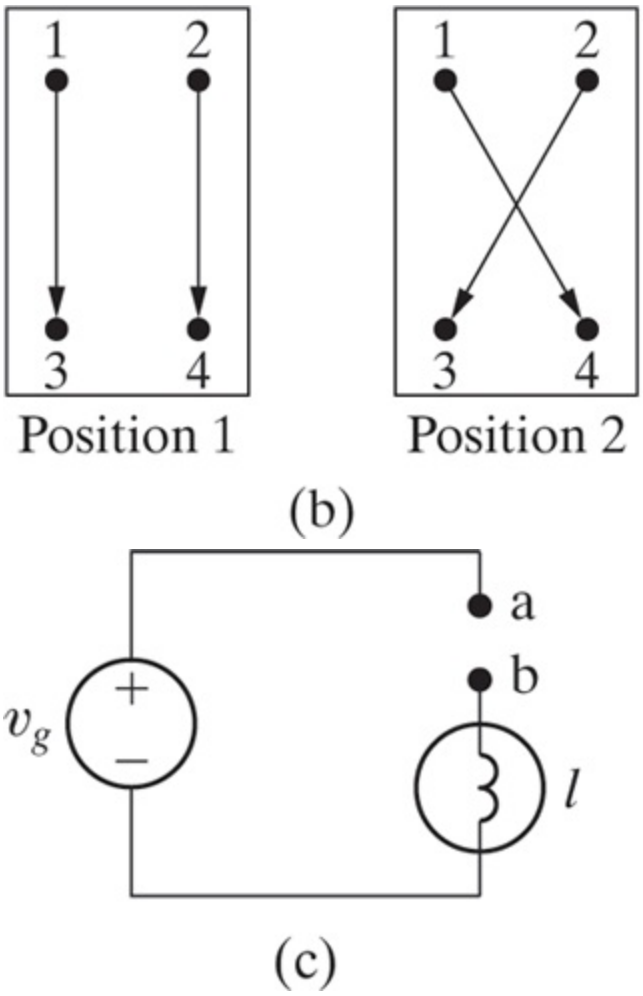


Figure P2.40 Full Alternative Text

1. Show how two three-way switches can be connected between a and b in the circuit in Fig. P2.40(c) so that the lamp l can be turned on or off from two locations.

2. If the lamp (appliance) is to be controlled from more than two locations, four-way switches are used in conjunction with two three-way switches. One four-way switch is required for each location in excess of two. Show how one four-way switch plus two three-way switches can be connected between a and b in Fig. P 2.40(c) to control the lamp from three locations. (*Hint:* The four-way switch is placed between the three-way switches.)

2. 2.41 PSPICEMULTISIM Suppose you want to add a third radiator to

your garage that is identical to the two radiators you have already installed. All three radiators can be modeled by 48 Ω resistors. Using the wiring diagram shown in Fig. P2.41, calculate the total power for the three radiators.

Figure P2.41

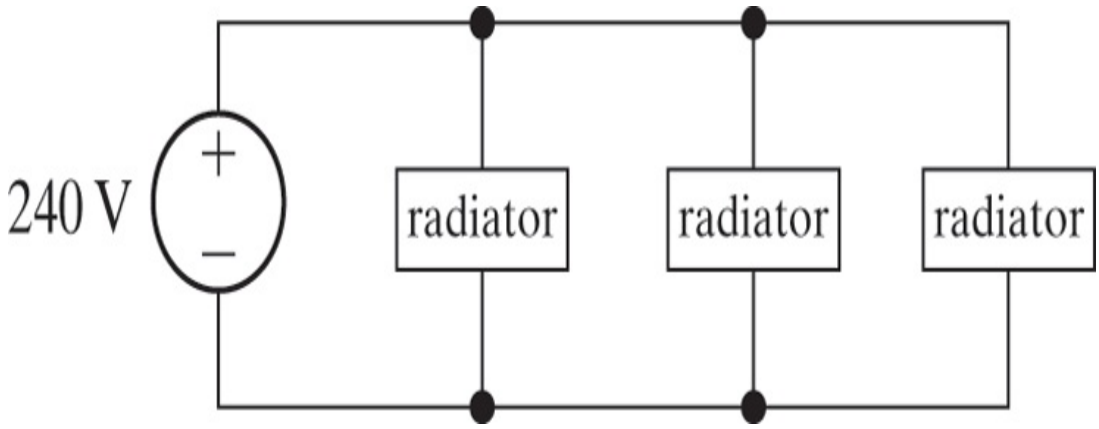


Figure P2.41 Full Alternative Text

3. 2.42 PSPICEMULTISIM Repeat Problem 2.41 using the wiring diagram shown in Fig. P2.42. Compare the total radiator power in this configuration with the total radiator power in the configuration shown in Fig. P2.41.

Figure P2.42

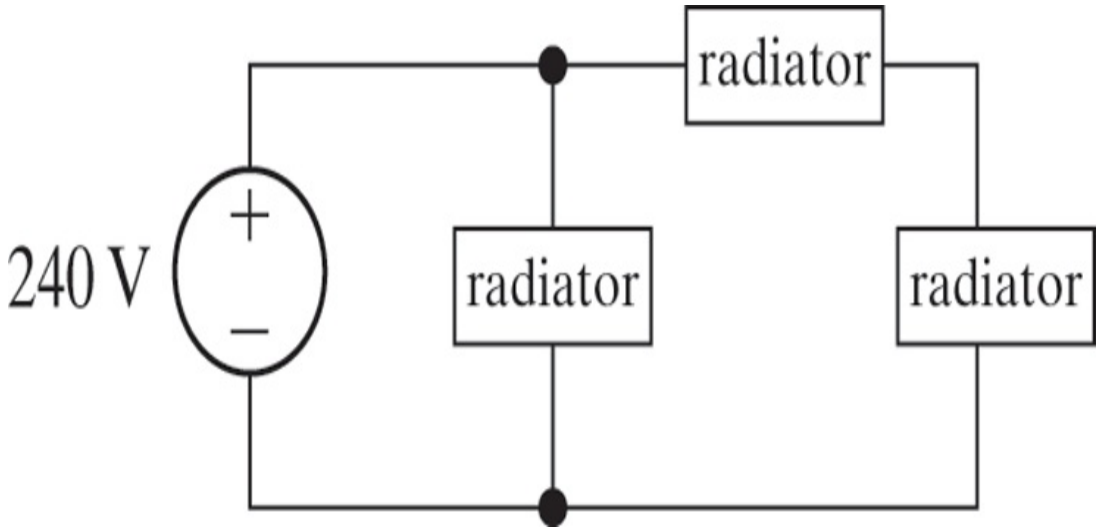


Figure P2.42 Full Alternative Text

4. 2.43 PSPICEMULTISIM Repeat Problem 2.41 using the wiring diagram shown in Fig. P2.43. Compare the total radiator power in this configuration with the total radiator power in the configuration shown in Fig. P2.41.

Figure P2.43

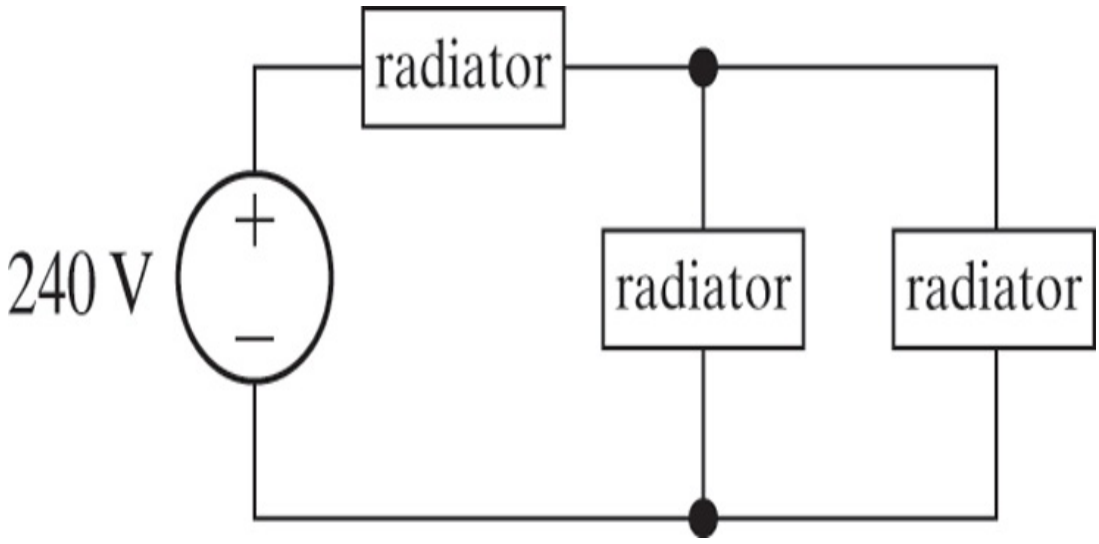
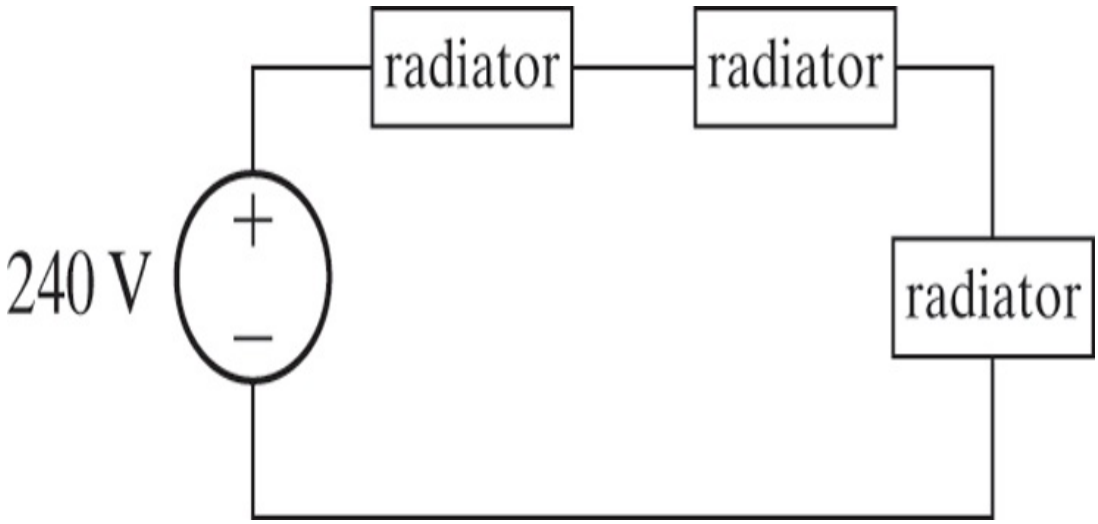


Figure P2.43 Full Alternative Text

5. 2.44 PSPICEMULTISIM Repeat Problem 2.41 using the wiring diagram shown in Fig. P2.44. Compare the total radiator power in this configuration with the total radiator power in the configuration shown in Fig. P2.41.

Figure P2.44



Chapter 3 Simple Resistive Circuits

Chapter Contents

1. 3.1 Resistors in Series
2. 3.2 Resistors in Parallel
3. 3.3 The Voltage-Divider and Current-Divider Circuits
4. 3.4 Voltage Division and Current Division
5. 3.5 Measuring Voltage and Current
6. 3.6 Measuring Resistance—The Wheatstone Bridge
7. 3.7 Delta-to-Wye (Pi-to-Tee) Equivalent Circuits

Chapter Objectives

1. Be able to recognize resistors connected in series and in parallel and use the rules for combining series-connected resistors and parallel-connected resistors to yield equivalent resistance.

2. Know how to design simple voltage-divider and current-divider circuits.

3. Be able to use voltage division and current division appropriately to solve simple circuits.

4. Be able to determine the reading of an ammeter when added to a circuit to measure current; be able to determine the reading of a voltmeter when added to a circuit to measure voltage.

5. Understand how a Wheatstone bridge is used to measure resistance.

6. Know when and how to use delta-to-wye equivalent circuits to solve simple circuits.

This chapter focuses on two important circuit element interconnections: series connections and parallel connections.

- When resistors are connected in series and in parallel, we can combine these resistors into equivalent resistors, reducing the number of circuit elements and the circuit's complexity, and simplifying the circuit analysis.

- When a voltage source is connected to two or more *resistors in series*, the supplied *voltage* divides across the resistors. This common configuration is used as a voltage divider that outputs a specific (desired) voltage, smaller than the supplied voltage. We introduce voltage division, a new tool that simplifies the analysis of these circuits.

- When a current source is connected to two or more *resistors in parallel*, the supplied *current* divides among the resistors. This common

configuration is used as a current divider that outputs a specific (desired) current, smaller than the supplied current. We introduce current division, a new tool that simplifies the analysis of these circuits.

Finally, we look at three important measurement instruments.

- The ammeter, which is a practical application of the current divider, is used to measure current.
- The voltmeter, which is a practical application of the voltage divider, is used to measure voltage.
- The Wheatstone bridge, which introduces two new interconnections known as delta and wye connections, is used to measure resistance.

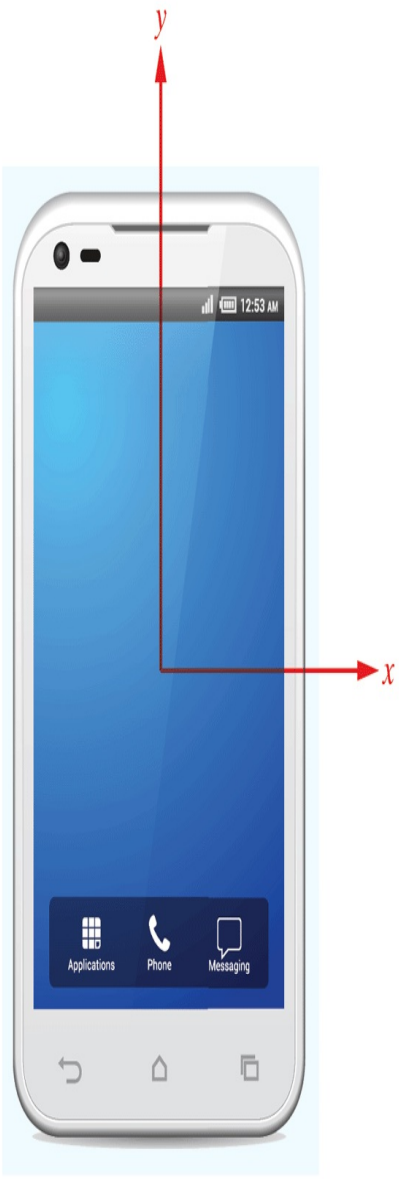
Practical Perspective

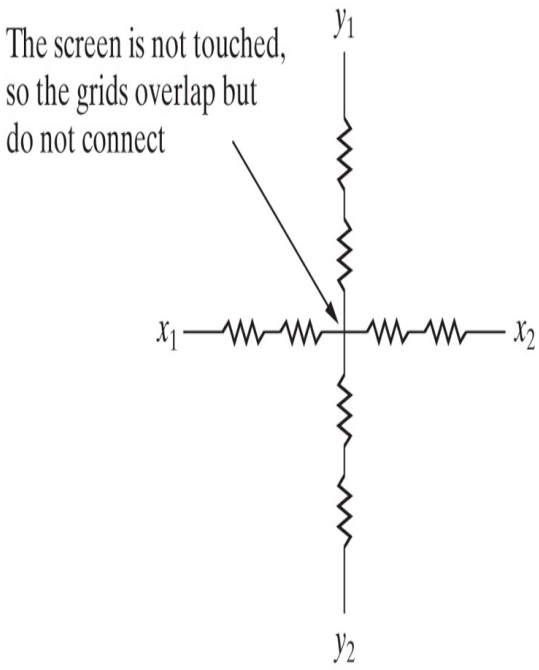
Resistive Touch Screens

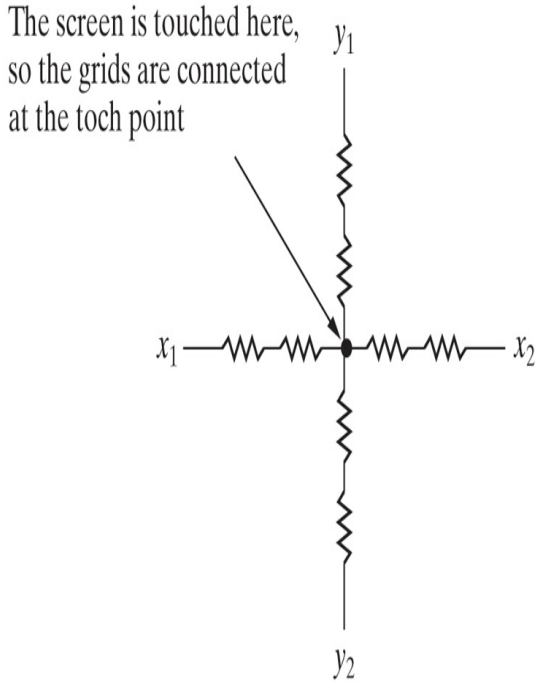
Many mobile phones and tablet computers use touch screens created by applying a transparent resistive material to the glass or acrylic screens. Two screens are typically used, separated by a transparent insulating layer. We can model a touch screen as a grid of resistors in the x-direction and a grid of resistors in the y-direction. The figure on the right depicts one row of the grid in the x-direction, with terminals x1 and x2, and one column of the grid in the y-direction, with terminals y1 and y2.

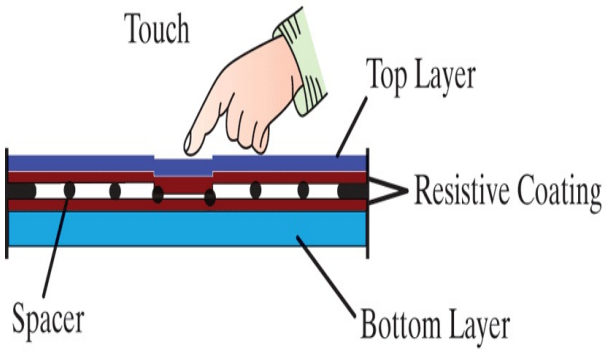
A separate electronic circuit repeatedly applies a voltage drop across the grid in the x-direction (between the points x1 and x2), then removes that voltage and applies a voltage drop across the grid in the y-direction (between points y1 and y2). When the screen is touched, the two resistive layers are pressed together, creating a voltage that is sensed in the x-grid and another voltage that is sensed in the y-grid. These two voltages precisely locate the point where the screen was touched.

How is the voltage created by touching the screen related to the position where the screen was touched? How are the properties of the grids used to calculate the touch position? We will answer these questions in the Practical Perspective at the end of this chapter. The circuit analysis required to answer these questions uses some of the new tools developed in this chapter.


y
x
12:53 AM
Applications
Phone
Messaging


The screen is not touched, so the grids overlap but do not connect
y_1
x_1
x_2
y_2


The screen is touched here, so the grids are connected at the toch point
y_1
x_1
x_2
y_2


Touch
Top Layer
Resistive Coating
Spacer
Bottom Layer

3.1-1 Full Alternative Text

3.1 Resistors in Series

In Chapter 2, we learned that two elements connected at a single node are said to be *in series*. For example, the seven resistors in Fig. 3.1 are connected in series. **Series-connected circuit elements** carry the same current. Applying Kirchhoff's current law to each node in the circuit, we can show that these resistors carry the same current. The series interconnection in Fig. 3.1 requires that

Figure 3.1 Resistors connected in series.

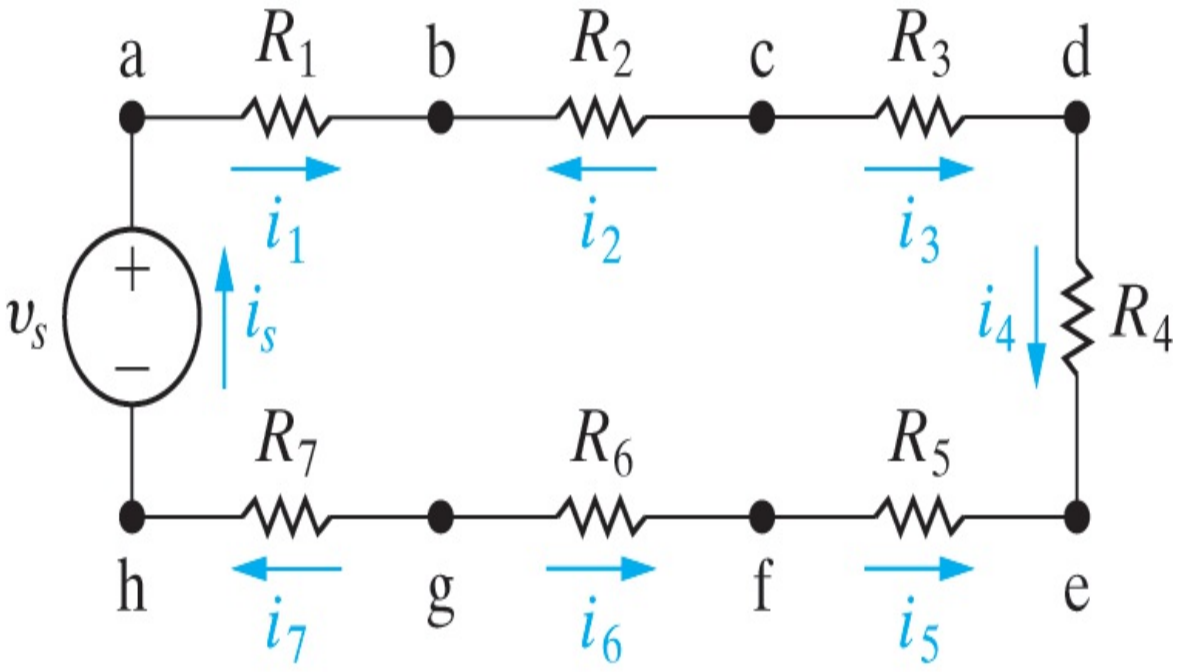


Figure 3.1 Full Alternative Text
is=i1= −i2=i3=i4=−i5=−i6=i7,

so if we know any one of the seven currents, we know them all. Thus, we can redraw Fig. 3.1 as shown in Fig. 3.2, using the single current is.

Figure 3.2 Series resistors with a single unknown current is.

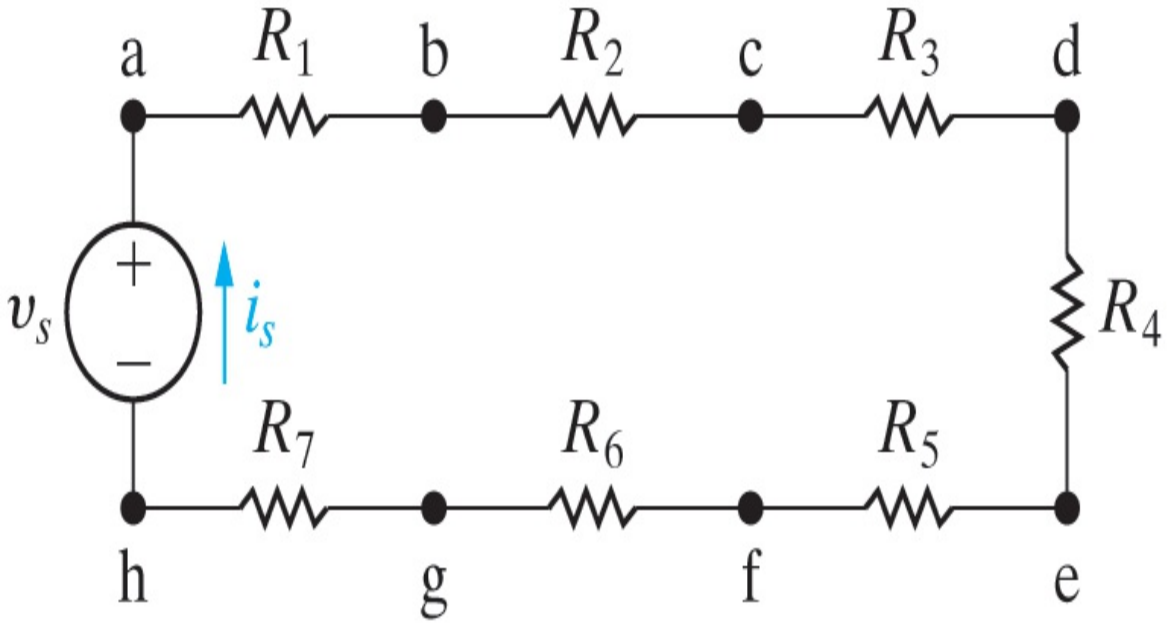


Figure 3.2 Full Alternative Text

To find is, we apply Kirchhoff's voltage law around the single closed loop in the clockwise direction. Defining the voltage across each resistor as a drop in the direction of is (Ohm's law) gives

−vs+isR1+isR2+isR3+isR4+isR5+isR6+isR7=0,

or

vs=is(R1+R2+R3+R4+R5+R6+R7).

This equation tells us we can simplify the circuit in Fig. 3.2 by replacing the seven resistors with a single equivalent resistor, Req, whose numerical value is the sum of the individual resistors; that is,

Req=R1+R2+R3+R4+R5+R6+R7

and

vs=isReq.

Thus, we can redraw Fig. 3.2 as shown in Fig. 3.3, which is a much simpler circuit.

Figure 3.3 A simplified version of the circuit shown in Fig. 3.2.

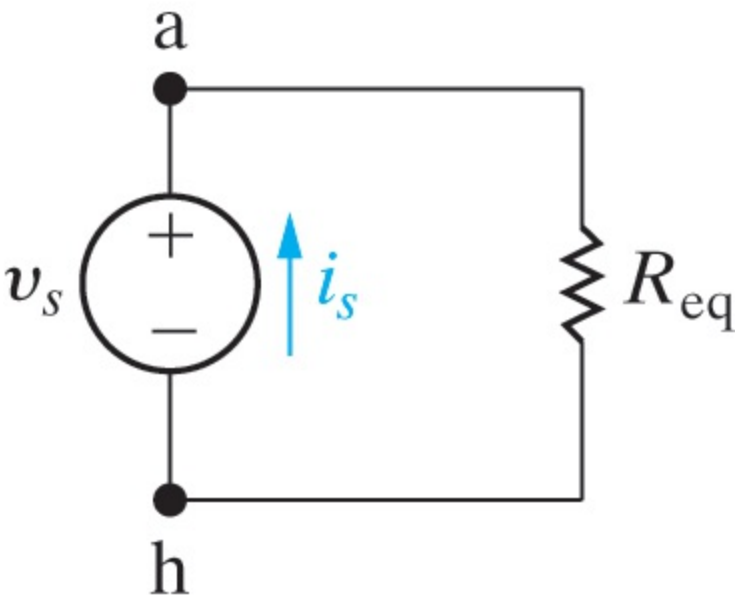


Figure 3.3 Full Alternative Text

In general, if k resistors are connected in series, the equivalent single resistor has a resistance equal to the sum of the k resistances, or

Combining Resistors in Series

Req=∑i=1kRi=R1+R2+ ⋯ + Rk.(3.1)

Note that the resistance of the equivalent resistor is always larger than the largest resistor in the series connection.

Think about equivalent resistance by visualizing the series-connected resistors inside a black box, depicted on the left of Fig. 3.4. (An electrical engineer uses the term **black box** to imply an opaque container; that is, the contents are hidden from view.) The single equivalent resistor is in a second black box, on the right of Fig. 3.4. We can derive the equation for the equivalent resistor by writing the equations relating voltage and current for each black box and finding the condition that makes these two equations equivalent, given in Eq. 3.1 when k=7.

Figure 3.4 The black box equivalent of the circuit shown in Fig. 3.2.

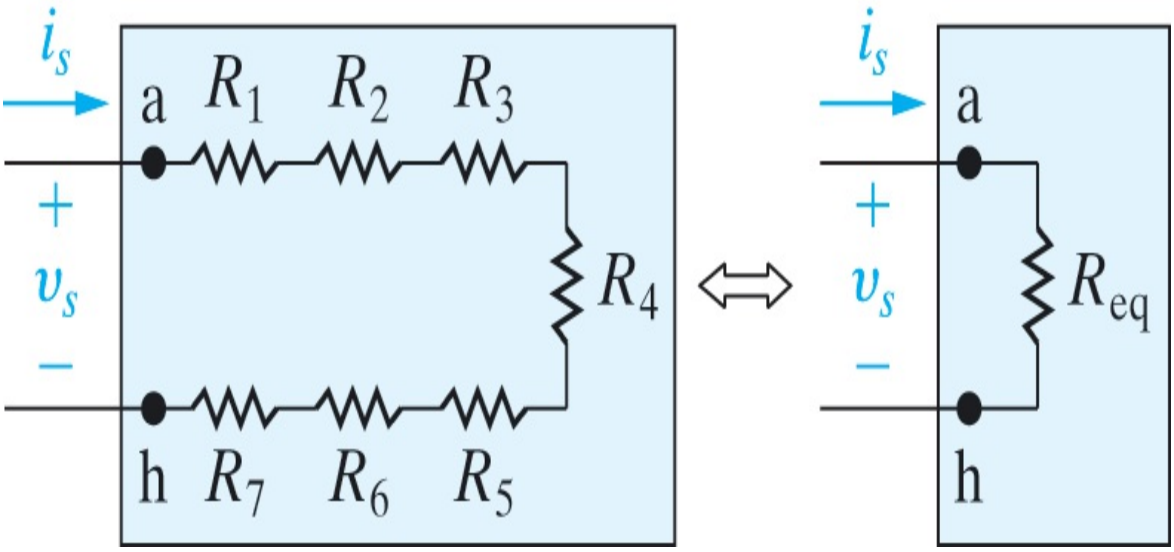


Figure 3.4 Full Alternative Text

3.2 Resistors in Parallel

Two elements connected at both of their nodes are said to be *in parallel*. For example, the four resistors in the circuit in Fig. 3.5 are in parallel. **Parallel-connected circuit elements** have the same voltage across their terminals. Don't assume that two elements are parallel connected merely because they are lined up in parallel in a circuit diagram. The defining characteristic of parallel-connected elements is that they have the same voltage across their terminals. In Fig. 3.6, you can see that R1 and R3 are not parallel connected because, between their respective terminals, another resistor dissipates some of the voltage.

Figure 3.5 Resistors in parallel.

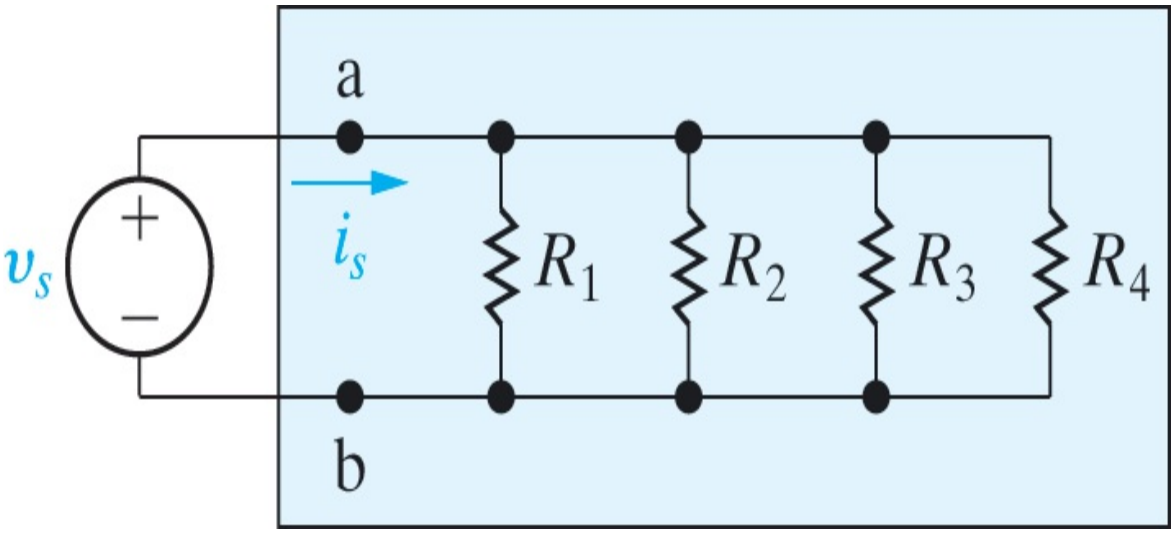


Figure 3.5 Full Alternative Text

Figure 3.6 Nonparallel resistors.

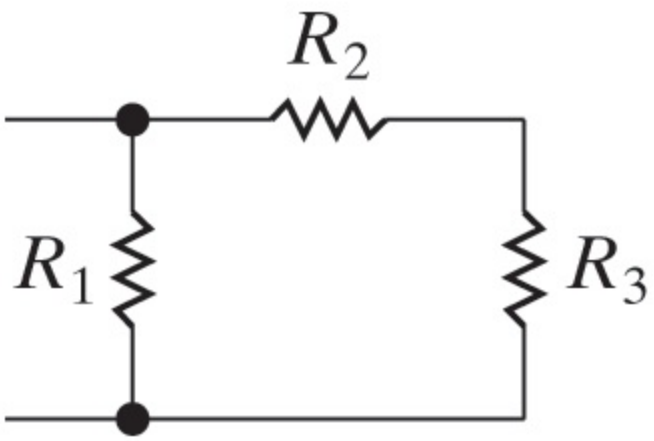


Figure 3.6 Full Alternative Text

We can reduce resistors in parallel to a single equivalent resistor using Kirchhoff's current law and Ohm's law. In Fig. 3.5, we let the currents i1, i2, i3, and i4 be the currents in the resistors R1 through R4, respectively. Note that the positive reference direction for each resistor current is through the resistor from node a to node b. From Kirchhoff's current law,

is=i1+i2+i3+i4.

The parallel connection of the resistors means that the voltage across each resistor must be the same. Hence, from Ohm's law,

i1R1=i2R2=i3R3=i4R4=vs.

Therefore,

i1=vsR1,i2=vsR2,i3=vsR3,andi4=vsR4.

Substituting the expressions for the four branch currents into the KCL equation and simplifying yields

is=vs (1R1+1R2+1R3+1R4),

from which

isvs=1Req=1R1+1R2+1R3+1R4.

This equation shows that the four resistors in Fig. 3.5 can be replaced by a single equivalent resistor, thereby simplifying the circuit. The circuit in Fig. 3.7 illustrates the substitution. The equivalent resistance of k resistors connected in parallel is

Figure 3.7 Replacing the four parallel resistors shown in Fig. 3.5 with a single equivalent resistor.

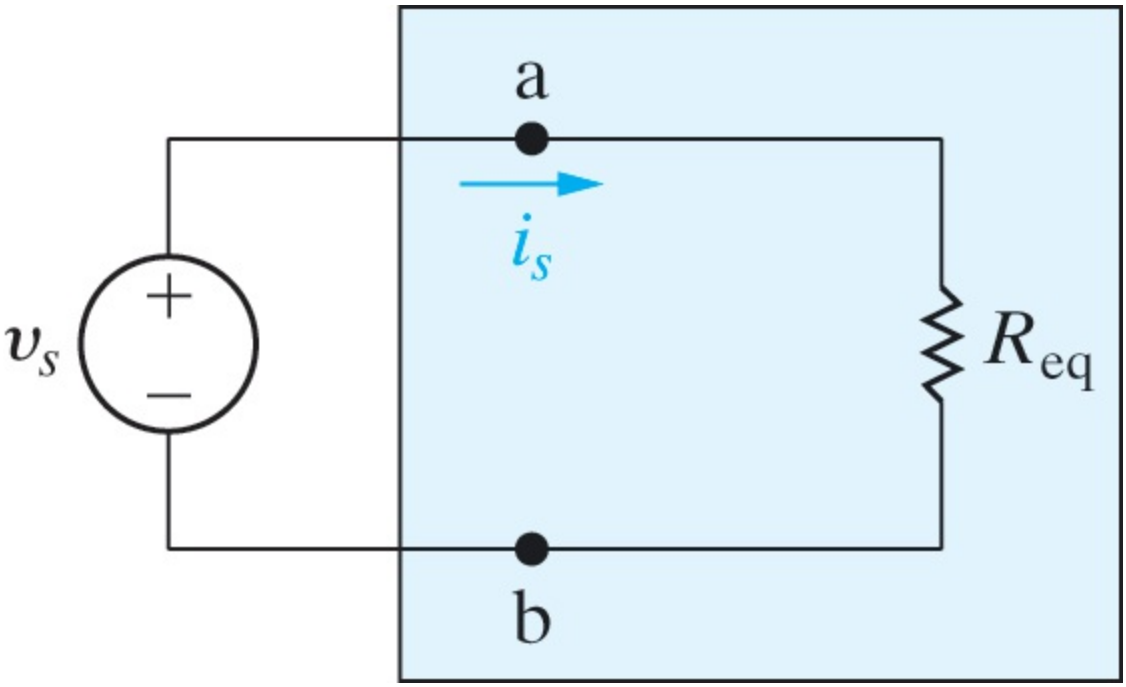


Figure 3.7 Full Alternative Text

Combining Resistors in Parallel

1Req=∑i=1k 1Ri=1R1+1R2+ ··· + 1Rk.(3.2)

Note that the resistance of the equivalent resistor is always smaller than the resistance of the smallest resistor in the parallel connection. Using conductance when dealing with resistors connected in parallel is sometimes more convenient. In that case, Eq. 3.2 becomes

$$G_{eq} = \sum_{i=1}^{k} G_i = G_1 + G_2 + \cdots + G_k.$$

Many times only two resistors are connected in parallel. Figure 3.8 illustrates this special case. We calculate the equivalent resistance from Eq. 3.2:

Figure 3.8 Two resistors connected in parallel.

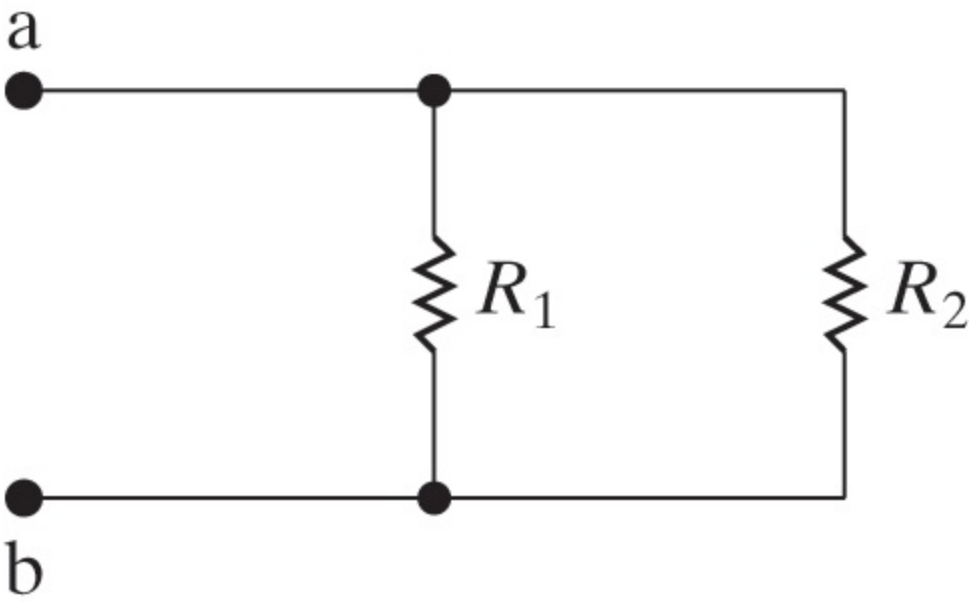


Figure 3.8 Full Alternative Text

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1 R_2},$$

or

Combining Two Resistors in

Parallel

Req=R1R2R1+R2.(3.3)

Thus, for just two resistors in parallel, the equivalent resistance equals the product of the resistances divided by the sum of the resistances. Remember that you can only use this result in the special case of just two resistors in parallel.

Work through Examples 3.1 and 3.2 to practice using series and parallel simplifications in circuit analysis.

Example 3.1 Applying Series-Parallel Simplification

1. Find the equivalent resistance seen by the current source in Fig. 3.9, using series and parallel simplifications.

Figure 3.9 The circuit for Example 3.1.

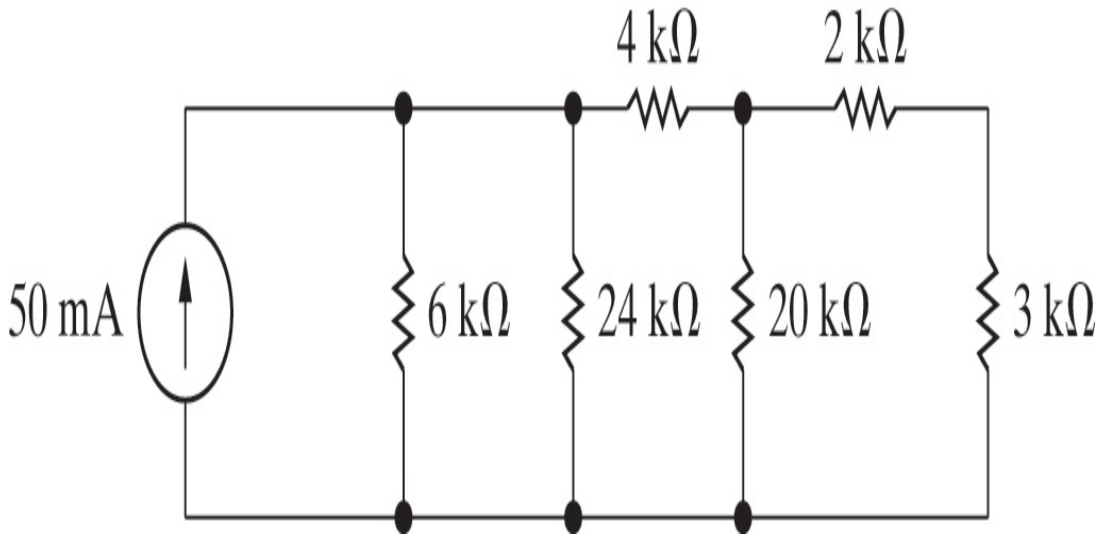


Figure 3.9 Full Alternative Text

2. Use your results in part (a) to find the power delivered by the current source.

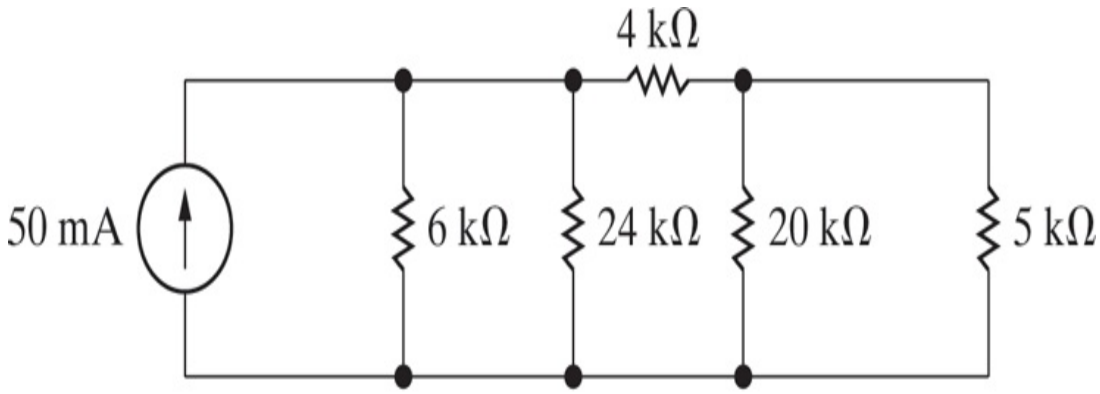
Solution

1. Our goal is a circuit with the 50 mA current source and a single resistor. We start simplifying the circuit's right-hand side, moving left toward the current source. The 2 kΩ and 3 kΩ resistors are in series and can be replaced by a single resistor whose value is

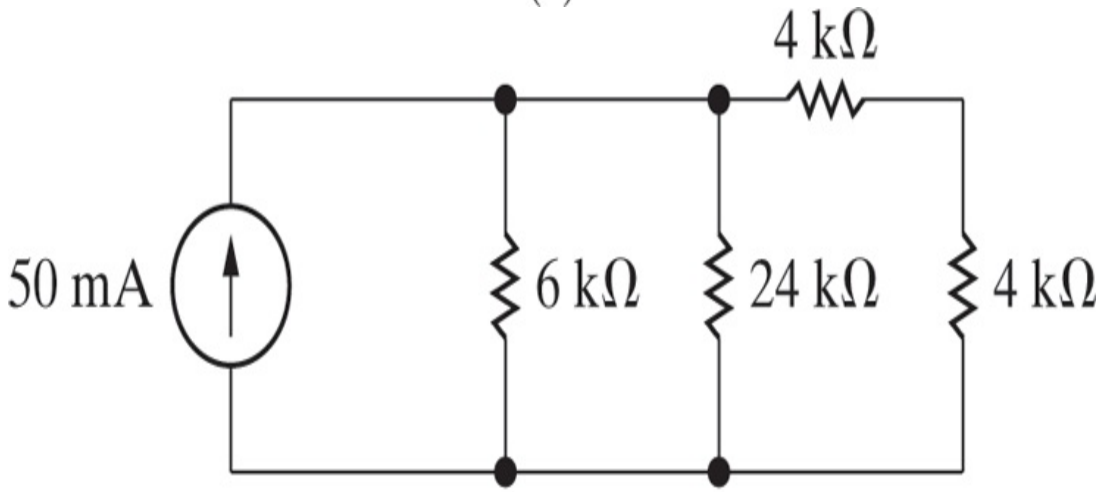
   2000+3000=5000=5 kΩ.

   Figure 3.10(a) shows this simplified circuit where the 20 kΩ and the 5 kΩ resistors are now in parallel. We replace these parallel-connected resistors with a single equivalent resistor, calculating its value using the "product over the sum" equation (Eq. 3.3):

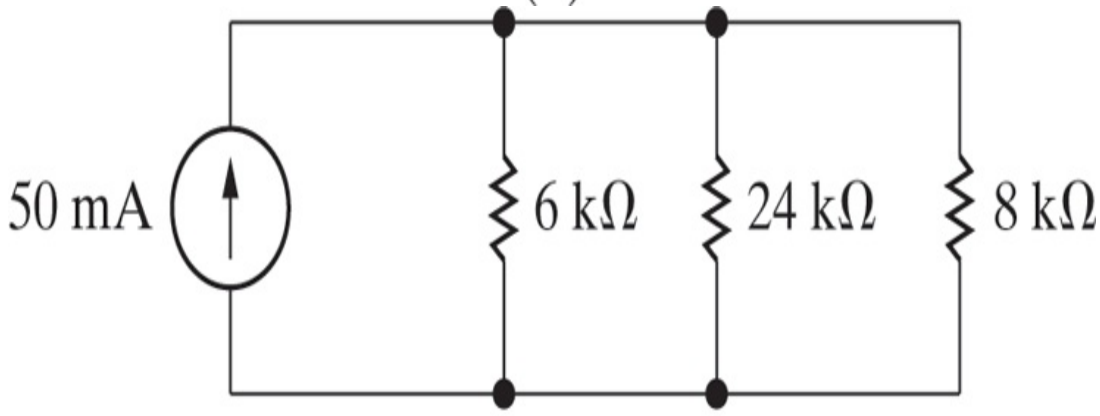
Figure 3.10 Simplifying the circuit in Fig. 3.9.


4 kΩ
50 mA
6 kΩ
24 kΩ
20 kΩ
5 kΩ

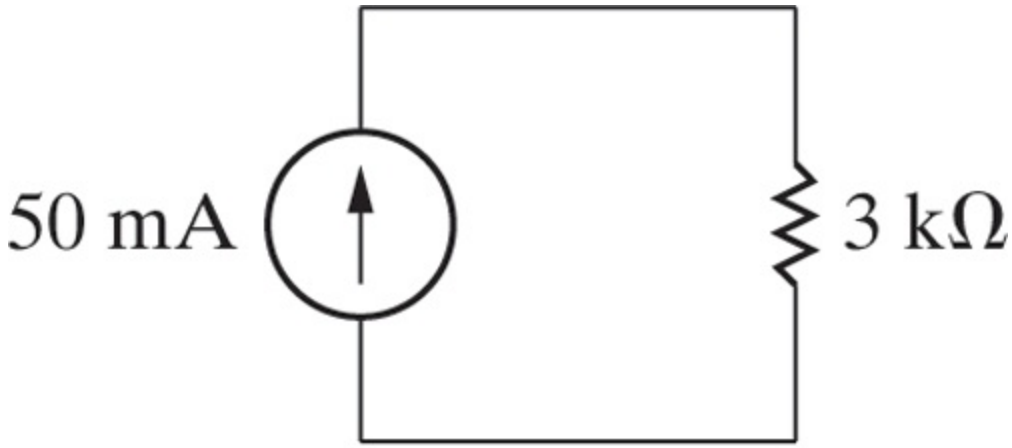
(a)


4 kΩ
50 mA
6 kΩ
24 kΩ
4 kΩ

(b)


50 mA
6 kΩ
24 kΩ
8 kΩ

(c)



(d)

Figure 3.10 Full Alternative Text

(20,000)(5000)20,000+5000=4000=4 kΩ.

Figure 3.10(b) shows this result, and now we see the two 4 kΩ resistors are in series. They can be replaced with a single resistor whose value is

4000+4000=8000=8 kΩ.

Figure 3.10(c) shows this simplification. Now the 24 kΩ, 6 kΩ, and 8 kΩ resistors are in parallel. To find their equivalent, we add their inverses and invert the result (Eq. 3.2):

(124,000+16000+18000)−1=(124,000+424,000+324,000)−1
=(824,000)−1=24,0008=3000=3 kΩ.

The equivalent resistance seen by the current source is 3 kΩ, as shown in Fig. 3.10(d).

2. The power of the source and the power of the equivalent 3 kΩ resistor must sum to zero. Using Fig. 3.10(d), we can easily calculate the resistor's power using its current and resistance to give

   p=(0.05)2(3000)=7.5 W.

   The equivalent resistor is absorbing 7.5 W, so the current source must be

delivering 7.5 W.

Example 3.2 Solving a Circuit Using Series-Parallel Simplification

Find is, i1, and i2 in the circuit shown in Fig. 3.11.

Figure 3.11 The circuit for Example 3.2.

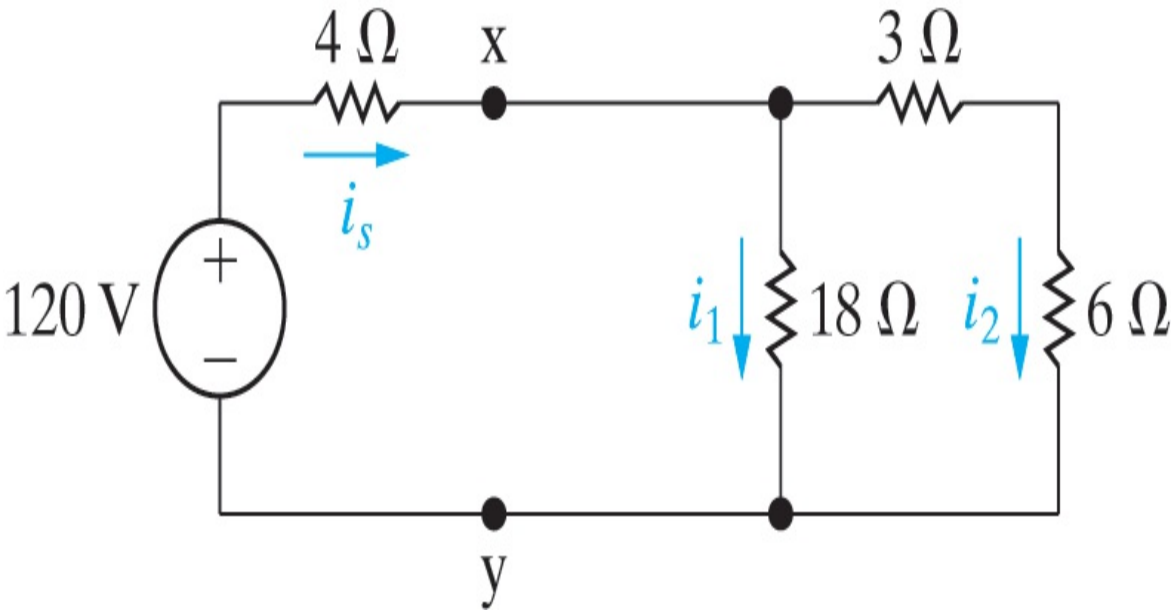


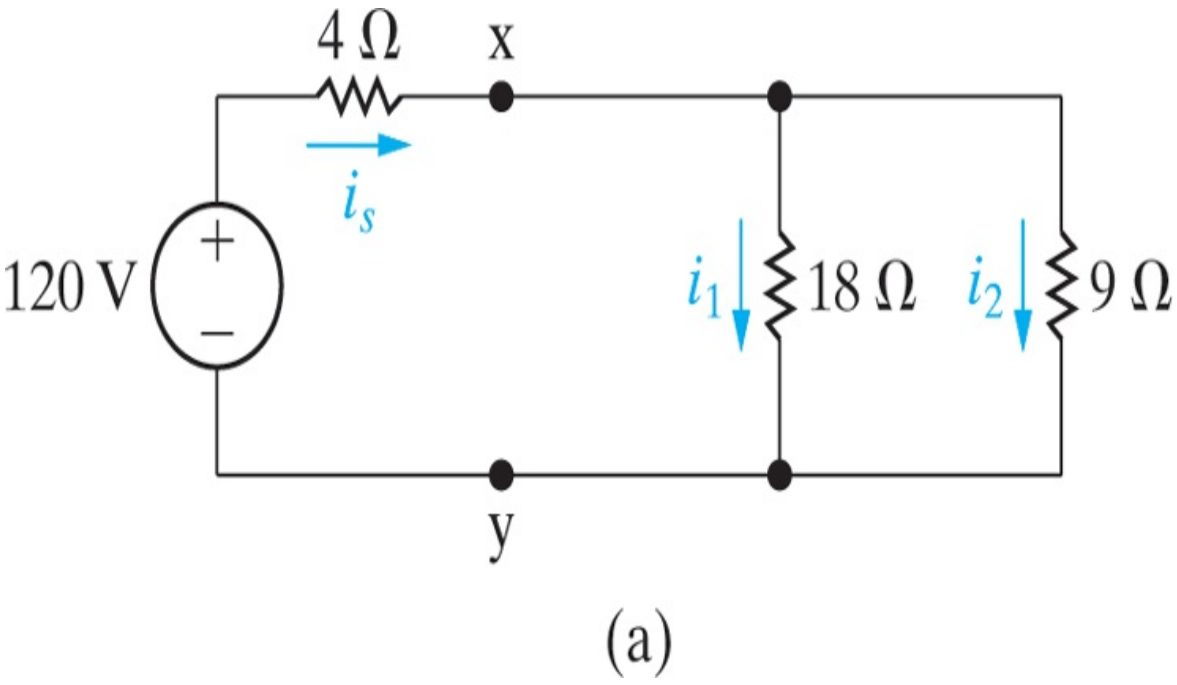
Figure 3.11 Full Alternative Text

Solution

Using series-parallel simplifications, we reduce the resistors to the right of the x-y terminals to a single equivalent resistor. On the circuit's right-hand

side, the 3 Ω and 6 Ω resistors are in series. We replace this series combination with a 9 Ω resistor, reducing the circuit to the one shown in Fig. 3.12(a). Then we replace the parallel combination of the 9 Ω and 18 Ω resistors with a single equivalent resistance of (18×9)/(18 +9), or 6 Ω. Figure 3.12(b) shows the resulting circuit. The nodes x and y marked on all diagrams should help you trace through the circuit simplification.

Figure 3.12 A simplification of the circuit shown in Fig. 3.11.



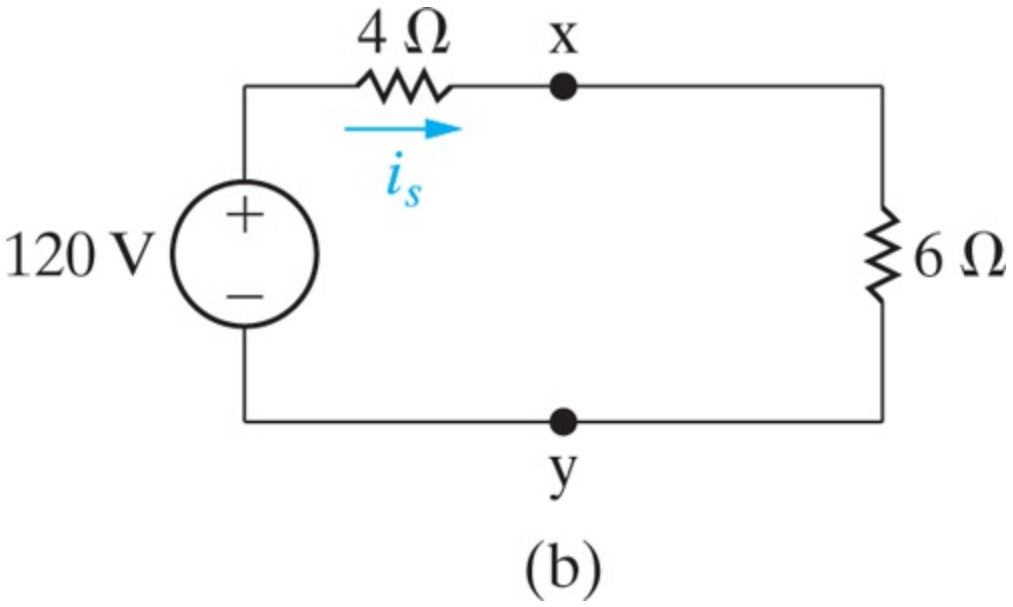


Figure 3.12 Full Alternative Text

From Fig. 3.12(b) you can use Ohm's law to verify that

is=120(6+4)=12 A.

Figure 3.13 shows this result and includes the voltage v1 to help clarify the subsequent discussion. Using Ohm's law, we compute the value of v1:

Figure 3.13 The circuit of Fig. 3.12(b) showing the numerical value of is.

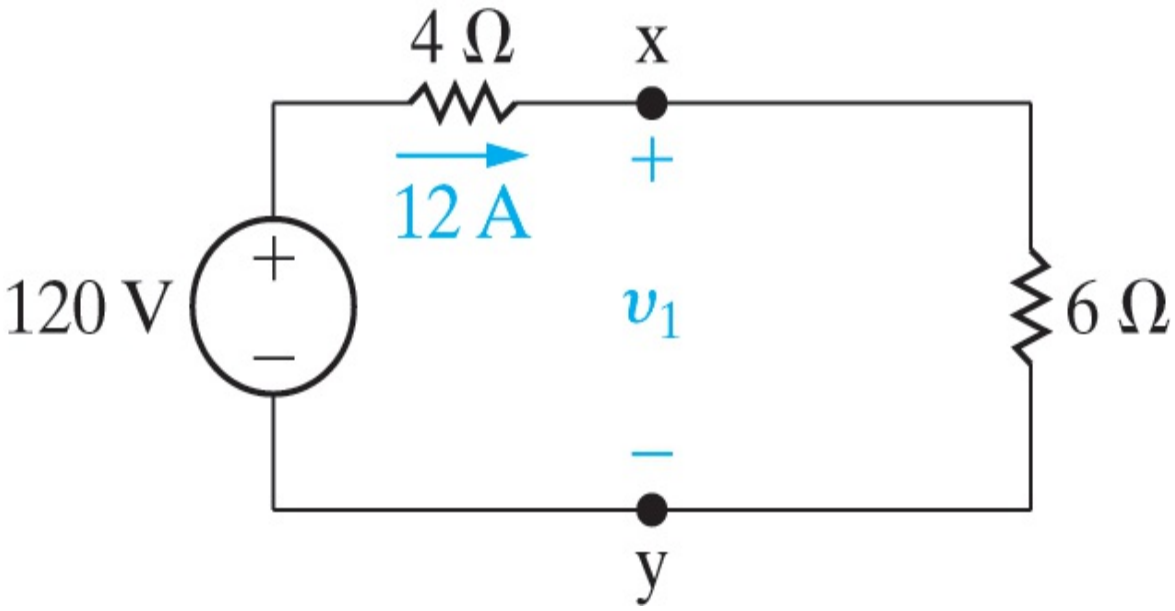


Figure 3.13 Full Alternative Text
v1=(12)(6)=72 V.

Since v1 is the voltage drop from node x to node y, we can return to the circuit shown in Fig. 3.12(a) and again use Ohm's law to calculate i1 and i2. Thus,

i1=v118=7218=4 A,
i2=v19=729=8 A.

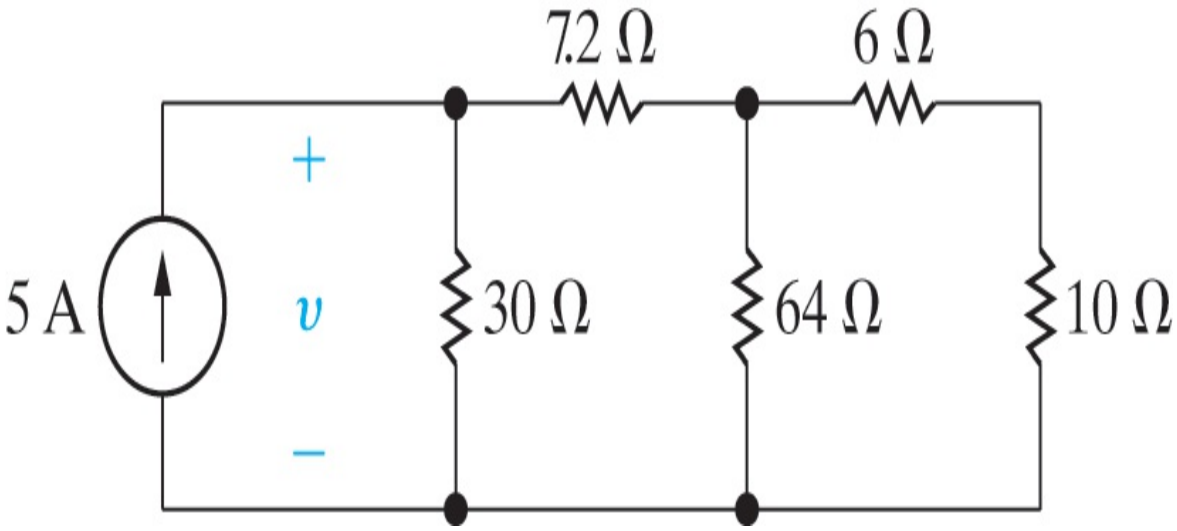
We have found the three specified currents by using series-parallel reductions in combination with Ohm's law.

Before leaving Example 3.2, you should verify that the solution satisfies Kirchhoff's current law at every node and Kirchhoff's voltage law around every closed path. (There are three closed paths that can be tested.) You can also show that the power delivered by the voltage source equals the total power dissipated in the resistors, and thus the power in the circuit balances. (See Problems 3.8 and 3.9.)

Assessment Problem

Objective 1—Be able to recognize resistors connected in series and in parallel

3.1 For the circuit shown, find (a) the voltage v, (b) the power delivered to the circuit by the current source, and (c) the power dissipated in the 10 Ω resistor.



3.2-2 Full Alternative Text

Answer:

1. a) 60 V;
2. b) 300 W;
3. c) 57.6 W.

SELF-CHECK: *Also try Chapter Problems 3.1–3.4.*

3.3 The Voltage-Divider and Current-Divider Circuits

The Voltage-Divider Circuit

A **voltage-divider circuit** produces two or more smaller voltages from a single voltage supply. This is especially useful in electronic circuits, where a single circuit may require voltages of + 15 V, −15 V, and +5 V. An example of a voltage-divider circuit that creates two voltages is shown in Fig 3.14. We introduce the current i, as shown in Fig. 3.14(b), and recognize (from Kirchhoff's current law) that R1 and R2 carry this current. Using Ohm's law to determine the resistor voltages from the current i and applying Kirchhoff's voltage law around the closed loop yields

Figure 3.14 (a) A voltage-divider circuit and (b) the voltage-divider circuit with current i indicated.

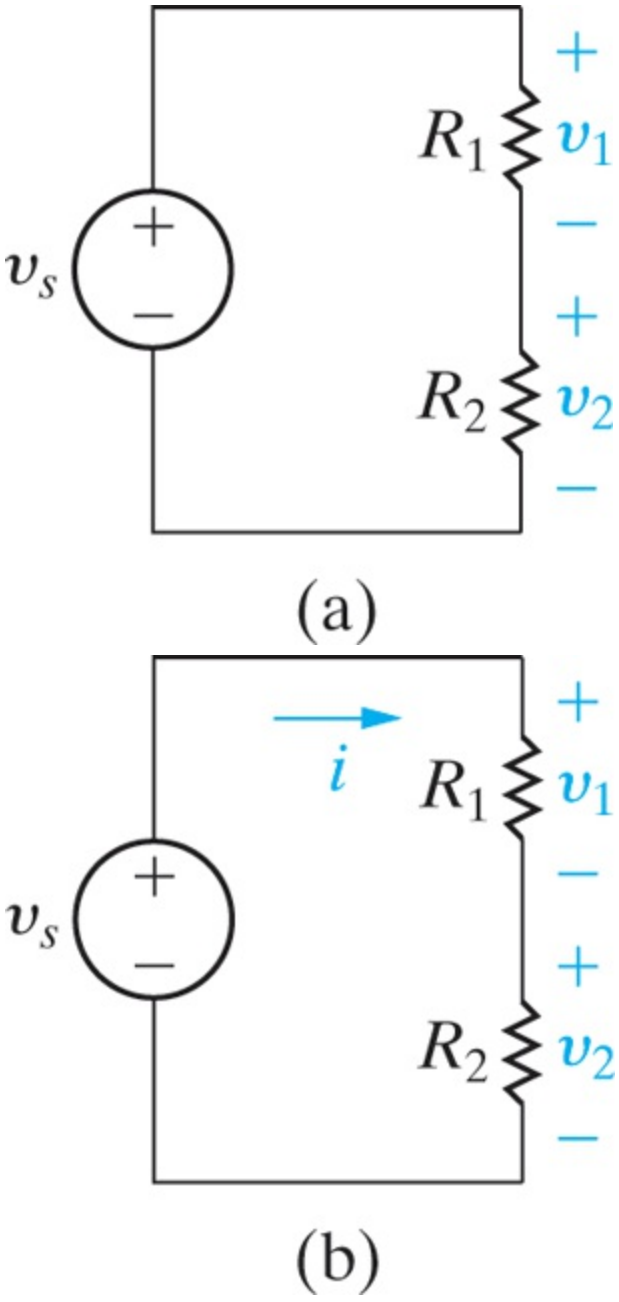


Figure 3.14 Full Alternative Text
vs=iR1+ iR2,

or

i=vsR1+ R2.

Using Ohm’s law and the expression for i, we calculate v1 and v2:

v1=iR1=vsR1R1+R2,(3.4)
v2=iR2=vsR2R1+R2.(3.5)

Equations 3.4 and 3.5 show that v1 and v2 are fractions of vs. Expressed in words, each fraction is

the resistance across which the divided voltage is definedthe sum of the two re

Because this ratio is always less than 1.0, the divided voltages v1 and v2 are always less than the source voltage vs.

Work through Example 3.3 to design a simple voltage divider.

Example 3.3 Designing a Simple Voltage Divider

The voltage divider in Fig. 3.14 has a source voltage of 20 V. Determine the values of the resistors R1 and R2 to give v1=15 V and v2=5 V.

Solution

From Eqs. 3.4 and 3.5,

15=R1R1+R2(20)and5=R2R1+R2(20).

Unfortunately, these two equations are not independent. If you solve each equation for R1, you get R1=3R2. An infinite number of combinations of R1 and R2 yield the correct values for y1 and y2. For example, if you choose R2=10 kΩ, then R1=30 kΩ gives the correct voltages, but if you choose R2=400 Ω, then R1=1200 Ω gives the correct voltages.

When selecting values for R1 and R2, you should consider the power the resistors must dissipate and the effects of connecting the voltage-divider circuit to other circuit components. Example 3.4 uses the voltage divider

designed in Example 3.3 to supply 5 V to a 10 kΩ resistor.

Example 3.4 Adding a Resistive Load to a Voltage Divider

1. For the voltage divider designed in Example 3.3, suppose resistors R2=10 kΩ and R1=30 kΩ. Connect a resistor RL=10 kΩ in parallel with R2 and determine the voltage across RL.

2. Repeat part (a) using resistors R2=400 Ω and R1=1200 Ω, but the same value of RL.

Solution

1. The voltage divider with the resistor RL is shown in Fig. 3.15. The resistor RL acts as a load on the voltage-divider circuit. A **load** on any circuit consists of one or more circuit elements that draw power from the circuit. The parallel combination of the two 10 kΩ resistors, one from the voltage divider and the other the load resistor RL, gives an equivalent resistance of 5 kΩ. Therefore, from Eq. 3.5,

Figure 3.15 The voltage divider from Example 3.3 with a resistive load.

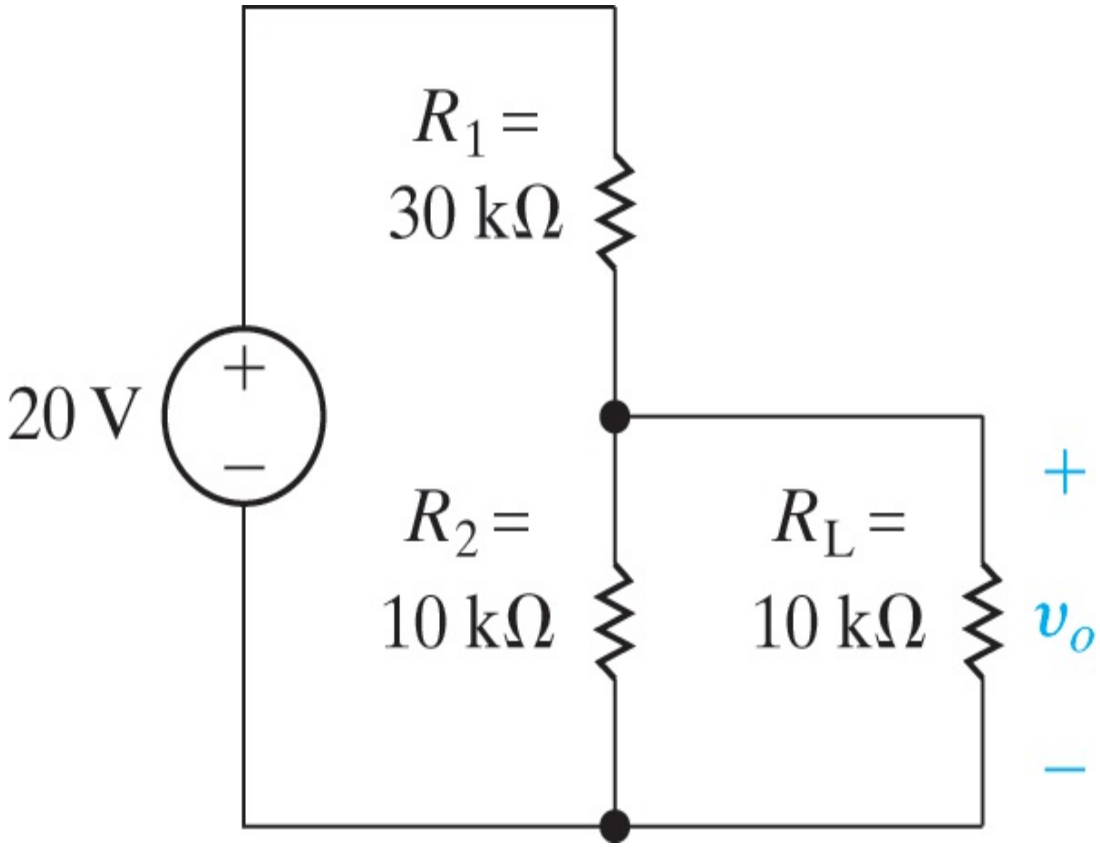


Figure 3.15 Full Alternative Text
vo=500030,000+5000(20)=2.86 V.

This is certainly not the 5 V we were expecting the voltage divider to deliver to the load, because adding the load resistor changed the voltage-divider circuit.

2. The voltage divider with a different set of resistors and the same load resistor is shown in Fig. 3.16. Again, we expect the load resistor to change the voltage-divider circuit. The parallel combination of the 400 Ω and 10 kΩ resistors gives an equivalent resistance of 384.615 Ω. Therefore, from Eq. 3.5,

Figure 3.16 The voltage divider from Example 3.3

with a different choice of R1 and R2 resistors and a resistive load.

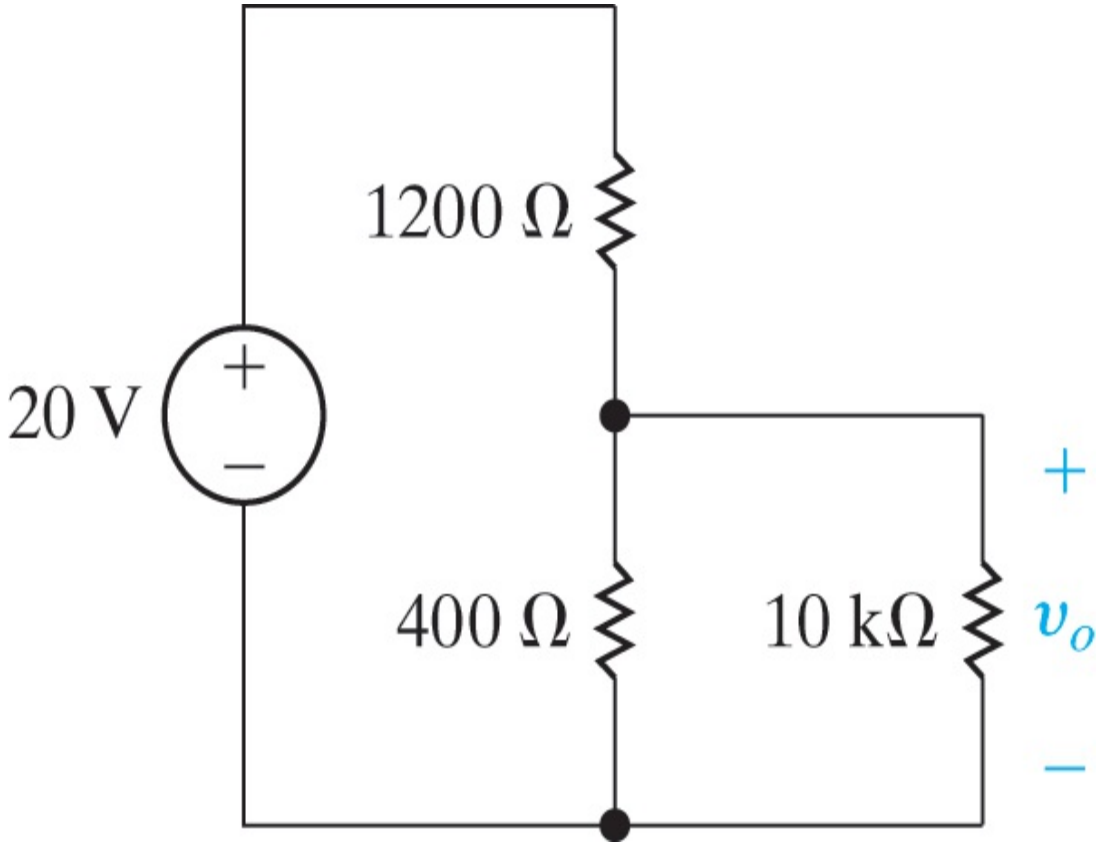


Figure 3.16 Full Alternative Text
vo=384.6151200+384.615(20)=4.85 V.

This is much closer to the 5 V we expected the voltage divider to deliver to the load. The effect of the load resistor is minimal because the load resistor value is much larger than the value of R2 in the voltage divider.

Figure 3.17 shows a general voltage divider with a load RL connected. The expression for the output voltage is

Figure 3.17 A voltage divider connected to a load RL.

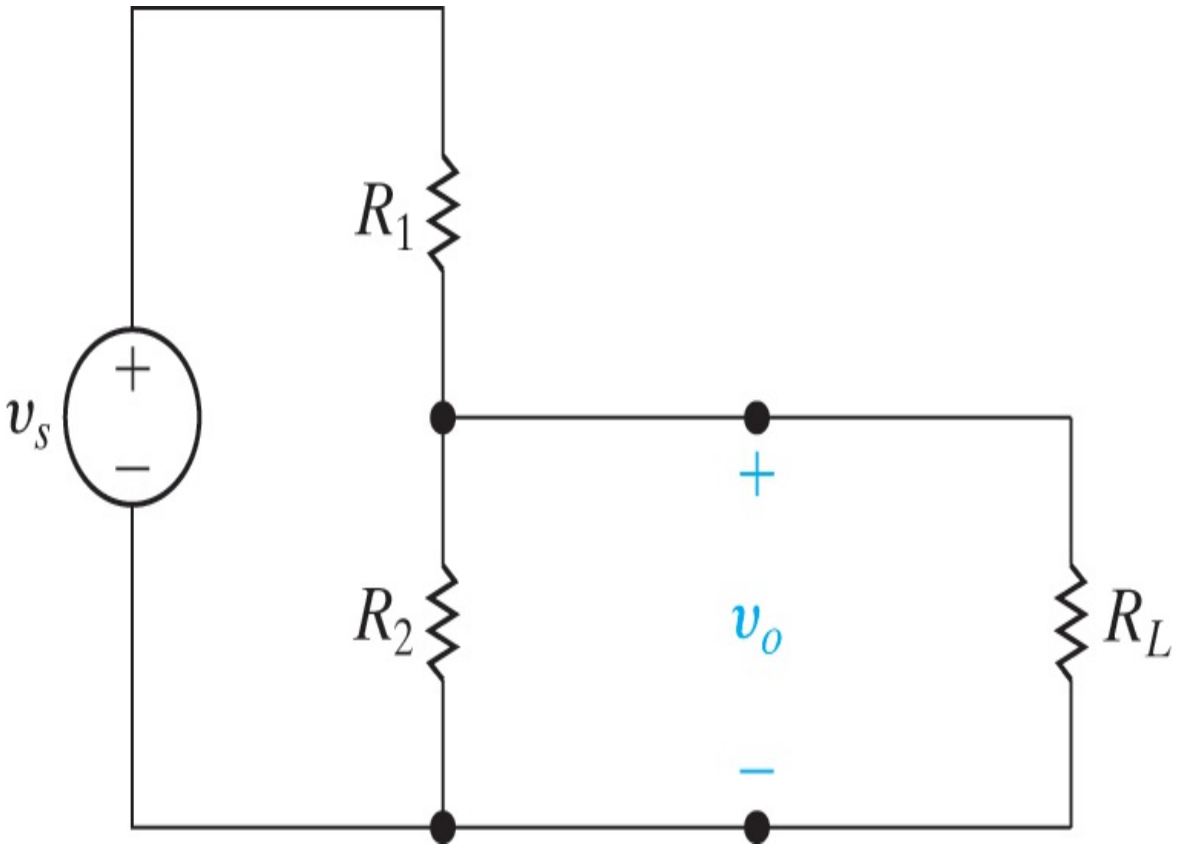


Figure 3.17 Full Alternative Text
vo=ReqR1+Reqvs,

where

Req=R2RLR2+RL.

Substituting the expression for Req into the equation for vo and simplifying yields

vo=R2R1[1+(R2/RL)]+R2vs.(3.6)

Note that Eq. 3.6 reduces to Eq. 3.5 as RL → ∞, as it should. Equation 3.6

shows that, as long as RL≫R2, the voltage ratio vo/vs is essentially undisturbed by adding a load to the divider, as we saw in Example 3.4.

Another characteristic of the voltage-divider circuit is its sensitivity to the tolerances of the resistors. By *tolerance* we mean a range of possible values. The resistances of commercially available resistors always vary within some percentage of their stated value. Example 3.5 illustrates the effect of resistor tolerances in a voltage-divider circuit.

Example 3.5 The Effect of Resistor Tolerance on the Voltage-Divider Circuit

The resistors used in the voltage-divider circuit shown in Fig. 3.18 have a tolerance of ±10%. Find the maximum and minimum value of vo.

Figure 3.18 The circuit for Example 3.5.

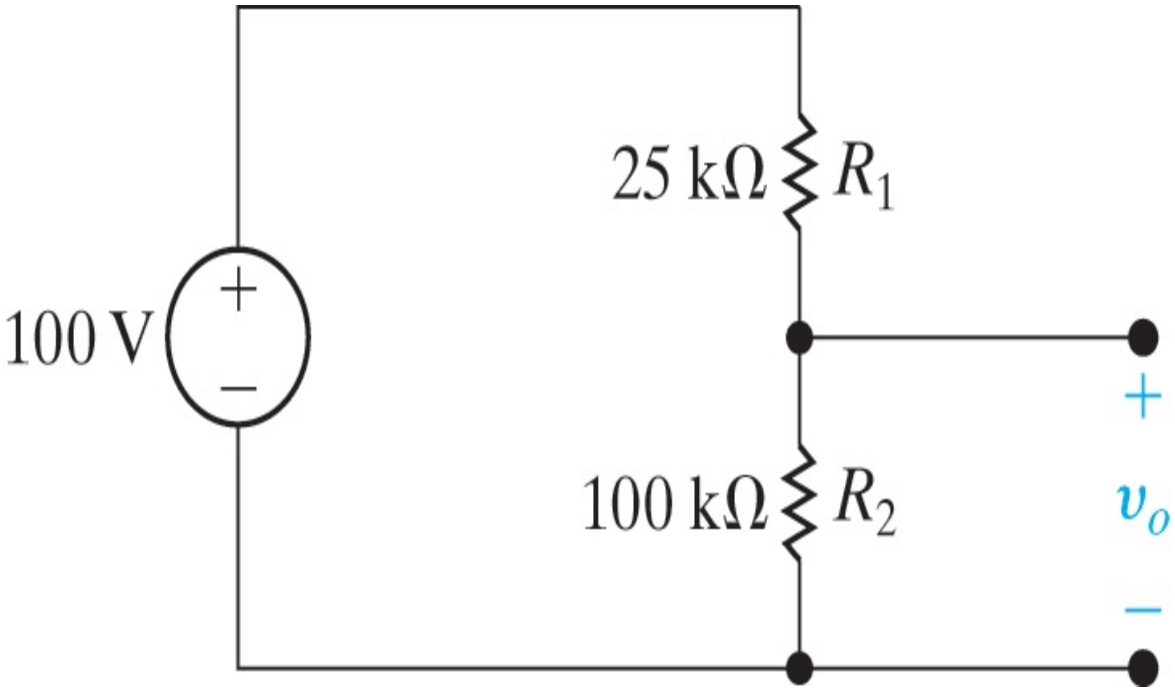


Figure 3.18 Full Alternative Text

Solution

From Eq. 3.5, the maximum value of vo occurs when R2=110 kΩ (10% high) and R1=22.5 kΩ (10% low), and the minimum value of vo occurs when R2=90 kΩ (10% low) and R1=27.5 kΩ (10% high). Therefore

vo(max)=110 k110 k+22.5 k(100)=83.02 V

and

vo(min)=90 k90 k+27.5 k(100)=76.60 V.

If we choose 10% resistors for this voltage divider, the no-load output voltage will lie between 76.60 and 83.02 V.

The Current-Divider Circuit

The **current-divider circuit** shown in Fig. 3.19 consists of two resistors connected in parallel across a current source. It divides the current is between R1 and R2. What is the relationship between the current is and the current in each resistor (i1 and i2)? The voltage across the parallel resistors can be expressed in three ways: as the product of the R1 resistor and its current i1, as the product of the R2 resistor and its current i2, and as the product of the equivalent resistance seen by the source and the source current. These three expressions for the voltage are given as

Figure 3.19 The current-divider circuit.

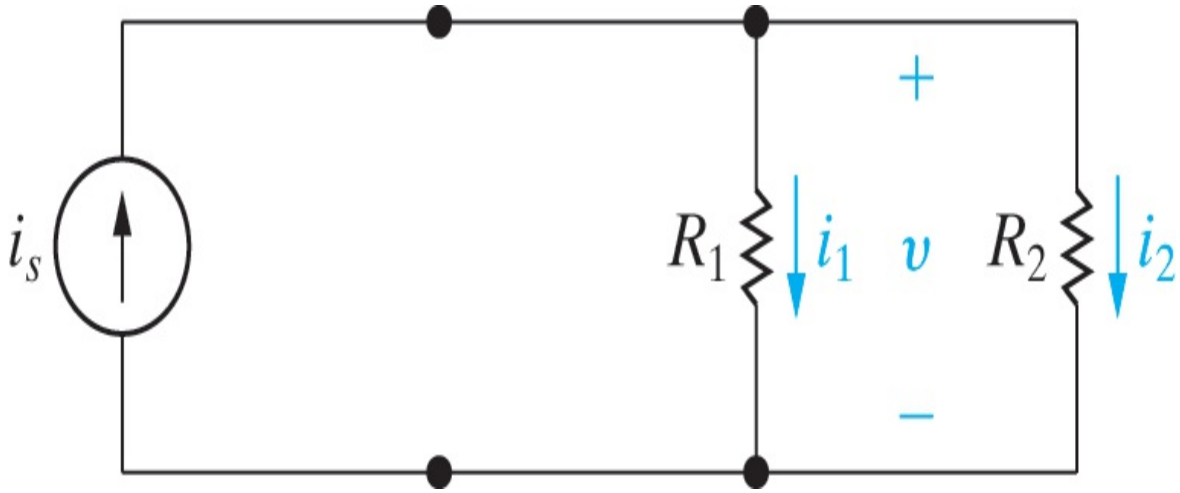


Figure 3.19 Full Alternative Text

$$v = i_1 R_1 = i_2 R_2 = \frac{R_1 R_2}{R_1 + R_2} i_s.$$

Therefore,

$$i_1 = \frac{R_2}{R_1 + R_2} i_s, \quad (3.7)$$

$$i_2 = \frac{R_1}{R_1 + R_2} i_s. \quad (3.8)$$

Equations 3.7 and 3.8 show that when the current divides between two resistors in parallel, the current in one resistor equals the current entering the parallel pair multiplied by the other resistance and divided by the sum of the resistors. See how to design a current divider by working through Example

3.6.

Example 3.6 Designing a Current-Divider Circuit

Suppose the current source for the current divider shown in Fig. 3.19 is 100 mA. Assuming you have 0.25 W resistors available, what is the largest R2 resistor you can use to get i2=50 mA?

Solution

While you can use Eq. 3.8 to find the ratio of resistors, it should be clear that if the current in one resistor is 50 mA, the current in the other resistor must also be 50 mA, so both resistors must have the same value. Therefore, there are an infinite number of different resistor values which, when used for R1 and R2 will give i2=50 mA. If the resistors have the same value and the same current, they absorb the same amount of power, which cannot exceed 0.25 W. From the power equation for resistors,

$$p=i2R=(0.05)2R=0.25$$

so

$$R=0.25(0.05)2=100\ \Omega.$$

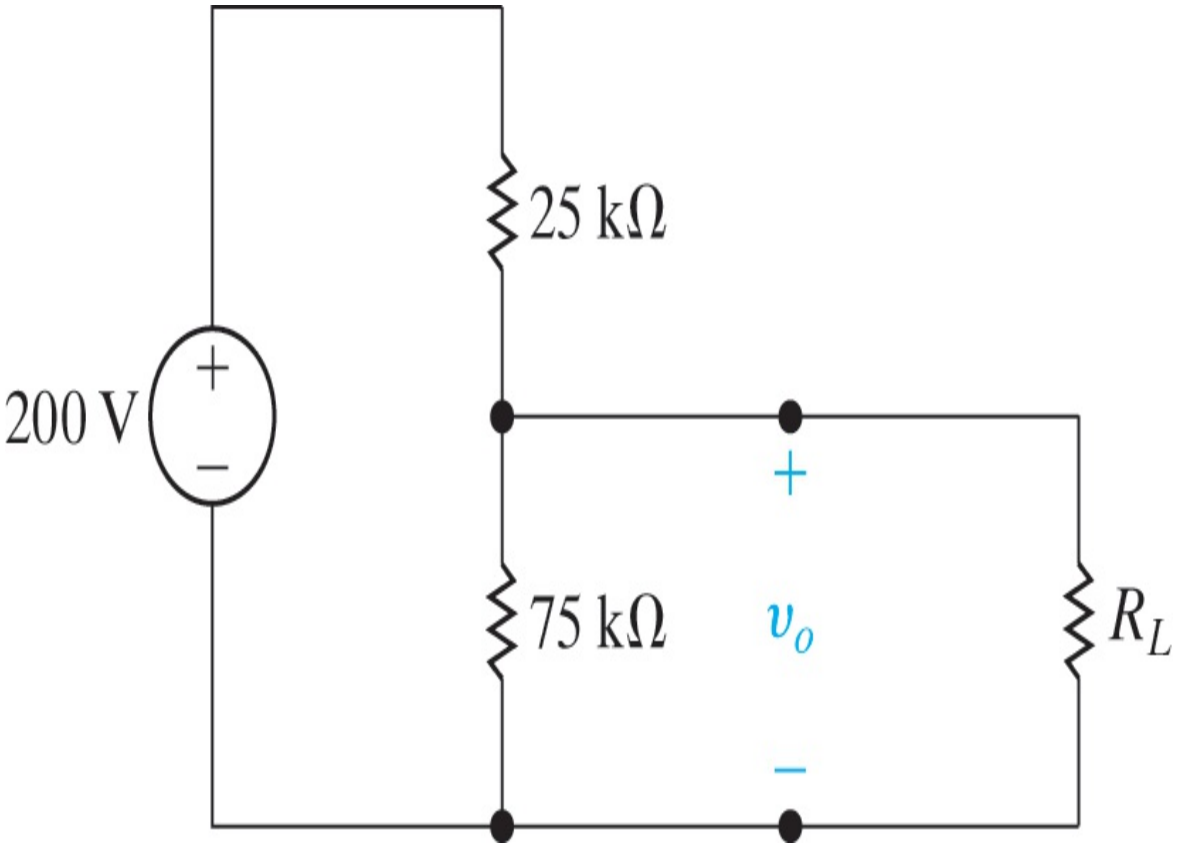
The largest 0.25 W resistors that can be used to create a current i2=50 mA are 100 Ω resistors.

Assessment Problems

Objective 2—Know how to design simple voltage-divider and current-divider circuits

3.2

1. Find the no-load value of vo in the circuit shown.
2. Find vo when RL is 150 kΩ.
3. How much power is dissipated in the 25 kΩ resistor if the load terminals are accidentally short-circuited?
4. What is the maximum power dissipated in the 75 kΩ resistor?



[3.3-3 Full Alternative Text]

Answer:

1. 150 V;
2. 1.6 W;

3. 133.33 V;

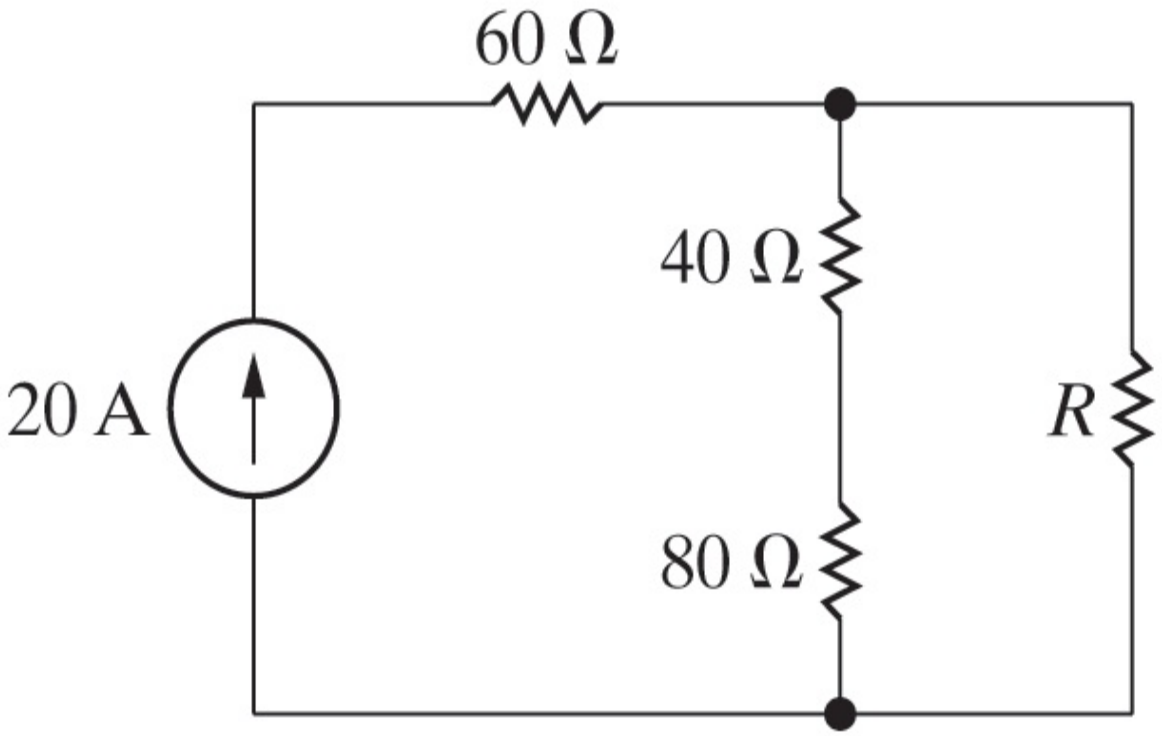
4. 0.3 W.

3.3

1. Find the value of R that will cause 4 A of current to flow through the 80 Ω resistor in the circuit shown.

2. How much power will the resistor R from part (a) need to dissipate?

3. How much power will the current source generate for the value of R from part (a)?



3.3-4 Full Alternative Text

Answer:

1. 30 Ω;

2. 7680 W;

3. 33,600 W.

SELF-CHECK: *Also try Chapter Problems 3.13, 3.14, and 3.20.*

3.4 Voltage Division and Current Division

We now introduce two additional and very useful circuit analysis techniques known as **voltage division** and **current division**. These techniques generalize the results from analyzing the voltage-divider circuit in Fig. 3.14 and the current-divider circuit in Fig. 3.19. We begin with voltage division.

Voltage Division

Consider the circuit shown in Fig. 3.20, where the box on the left contains a single voltage source or any other combination of basic circuit elements that results in the voltage v shown in the figure. To the right of the box are n resistors connected in series. We are interested in finding the voltage drop vj across an arbitrary resistor Rj in terms of the voltage v. We start by using Ohm's law to calculate i, the current through all of the resistors in series, in terms of the current v and the n resistors:

Figure 3.20 Circuit used to illustrate voltage division.

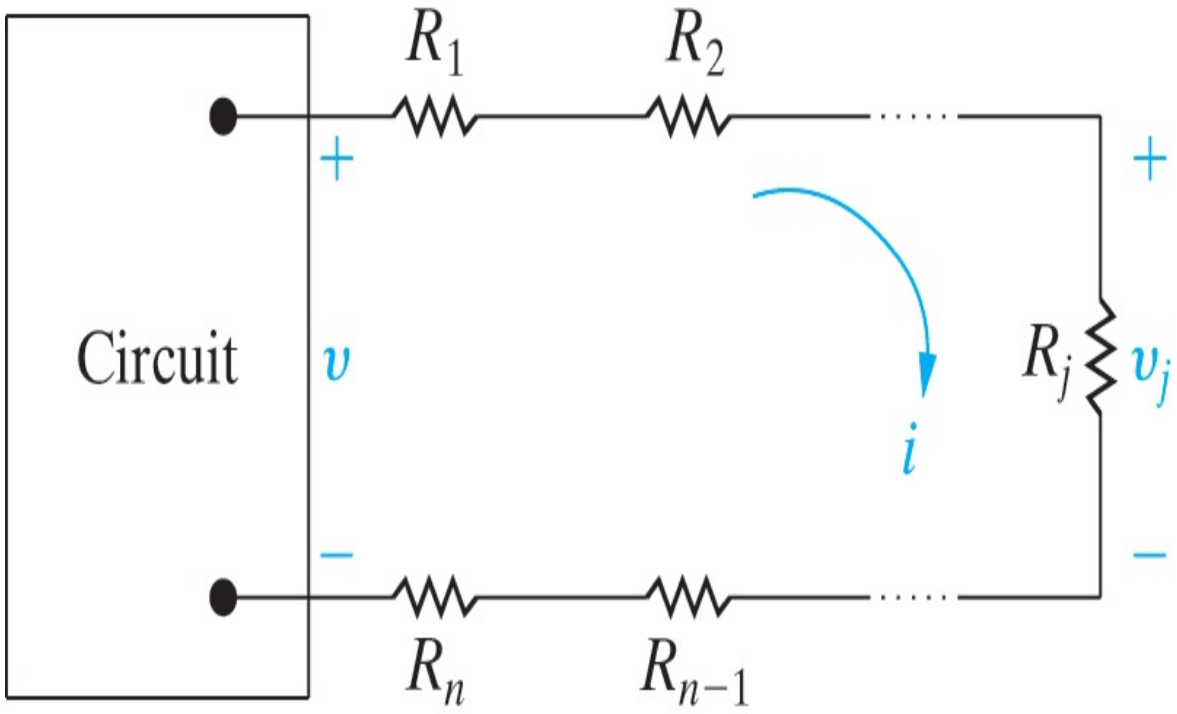


Figure 3.20 Full Alternative Text
i=vR1+R2+···+Rn=vReq.

The equivalent resistance, Req, is the sum of the n resistor values because the resistors are in series, as shown in Eq. 3.1. We apply Ohm's law a second time to calculate the voltage drop vj across the resistor Rj, substituting v/Req for i:

Voltage Division Equation

vj=iRj=RjReqv.(3.9)

Equation 3.9 is the voltage division equation. It says that the voltage drop vj across a single resistor Rj from a collection of series-connected resistors is proportional to the total voltage drop v across the set of series-connected resistors. The constant of proportionality is the ratio of the single resistance to the equivalent resistance of the series-connected set of resistors, or Rj/Req.

Current Division

Now consider the circuit shown in Fig. 3.21, where the box on the left contains a single current source or any other combination of basic circuit elements that results in the current i shown in the figure. To the right of the box are n resistors connected in parallel. We are interested in finding the current ij through an arbitrary resistor Rj in terms of the current i. We start by using Ohm's law to calculate v, the voltage drop across each of the resistors in parallel, in terms of the current i and the n resistors:

Figure 3.21 Circuit used to illustrate current division.

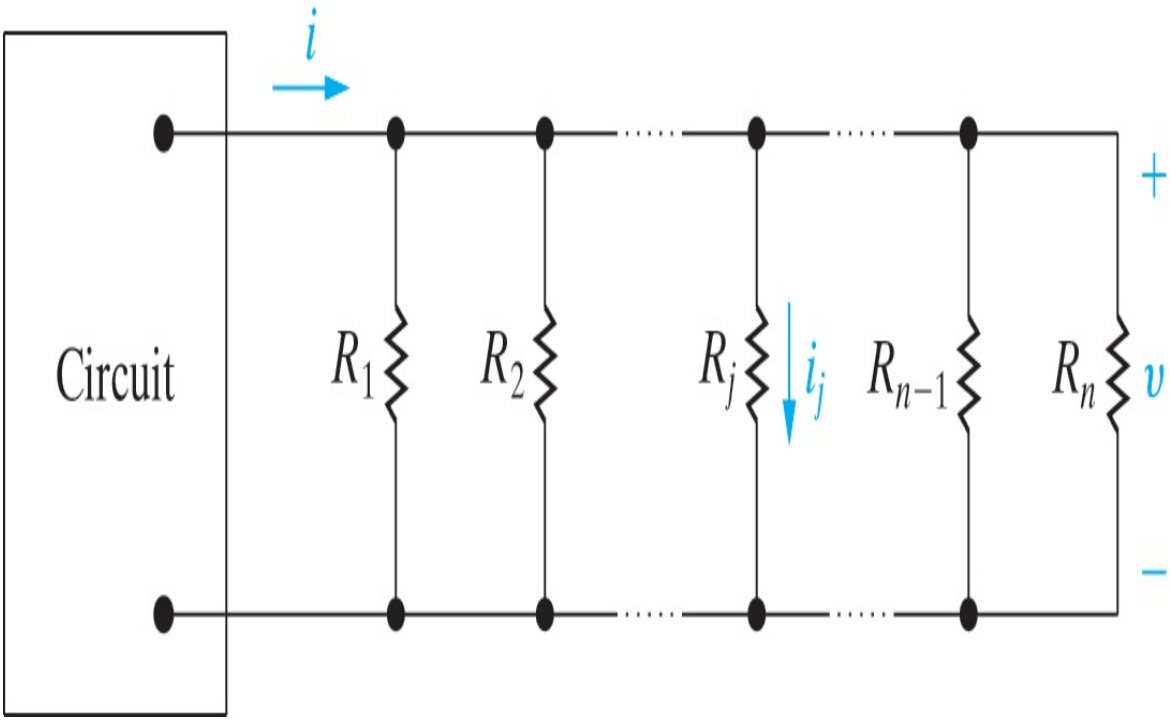


Figure 3.21 Full Alternative Text
v=i(R1 || R2||···||Rn)=iReq.

The equivalent resistance of n resistors in parallel, Req, can be calculated

using Eq. 3.2. We apply Ohm's law a second time to calculate the current ij through the resistor Rj, replacing v with iReq:

Current Division Equation

$$i_j = v_{Rj} = R_{eq} R_j i. \quad (3.10)$$

Equation 3.10 is the current division equation. It says that the current ij through a single resistor Rj from a collection of parallel-connected resistors is proportional to the total current i supplied to the set of parallel-connected resistors. The constant of proportionality is the ratio of the equivalent resistance of the parallel-connected set of resistors to the single resistance, or Req/Rj. Note that the constant of proportionality in the current division equation is the inverse of the constant of proportionality in the voltage division equation!

Example 3.7 uses voltage division and current division to solve for voltages and currents in a circuit.

Example 3.7 Using Voltage Division and Current Division to Solve a Circuit

Use current division to find the current io and use voltage division to find the voltage vo for the circuit in Fig. 3.22.

Figure 3.22 The circuit for Example 3.7.

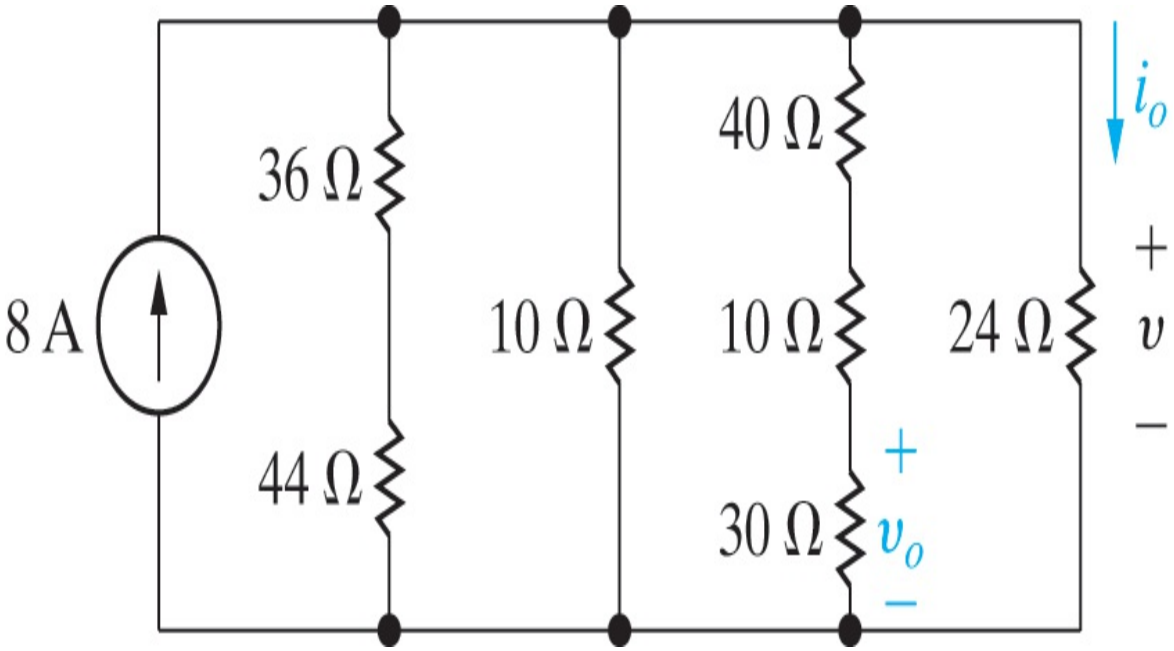


Figure 3.22 Full Alternative Text

Solution

We can use Eq. 3.10 if we can find the equivalent resistance of the four parallel branches containing resistors. Using "+" to represent series-connected resistors and "||" to represent parallel-connected resistors, the equivalent resistance is

Req=(36+44)|| 10 ||(40+10+30) || 24
=80 || 10 || 80 || 24=1180+110+180+124=6 Ω.

Applying Eq. 3.10,

io=624(8 A)=2 A.

We can use Ohm's law to find the voltage drop across the 24 Ω resistor:

v=(24)(2)=48 V.

This is also the voltage drop across the branch containing the 40 Ω, the 10 Ω, and the 30 Ω resistors in series. Use voltage division to determine the voltage

drop vo across the 30 Ω resistor from the voltage drop across the series-connected resistors, using Eq. 3.9. The equivalent resistance of the series-connected resistors is 40+10+30=80 Ω, so

vo=3080(48 V)=18 V.

Assessment Problem

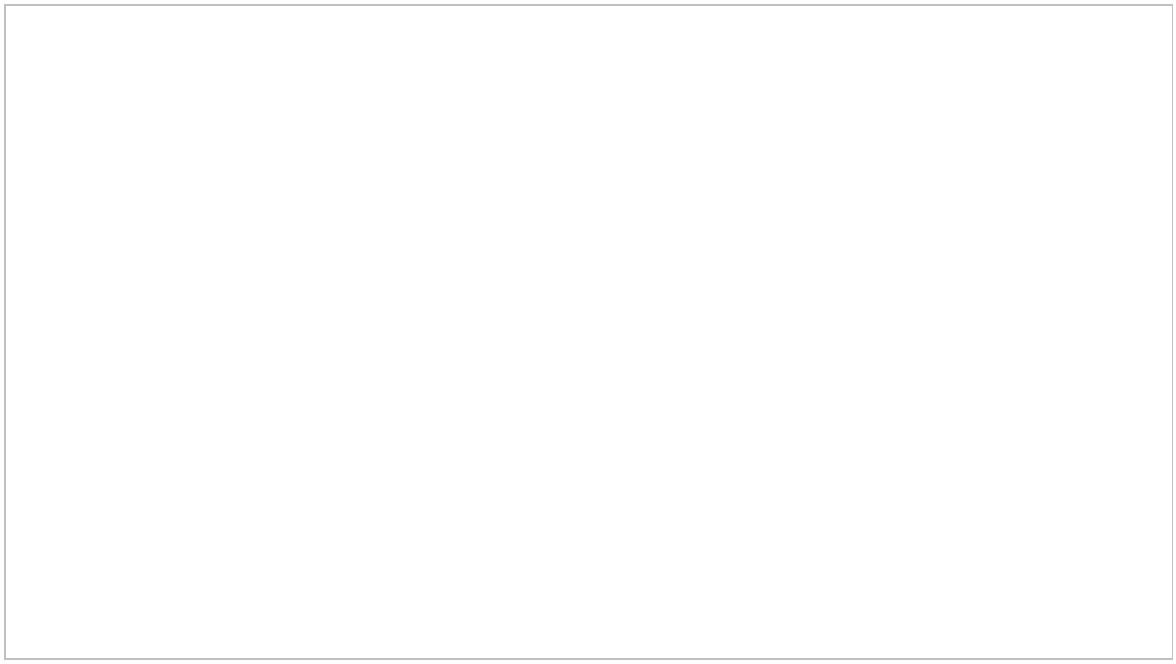
Objective 3—Be able to use voltage and current division to solve simple circuits

3.4

1. Use voltage division to determine the voltage vo across the 40 Ω resistor in the circuit shown.

2. Use vo from part (a) to determine the current through the 40 Ω resistor, and use this current and current division to calculate the current in the 30 Ω resistor.

3. How much power is absorbed by the 50 Ω resistor?



3.4-5 Full Alternative Text

Answer:

1. 20 V;
2. 166.67 mA;
3. 347.22 mW.

SELF-CHECK: *Also try Chapter Problems 3.26 and 3.27.*

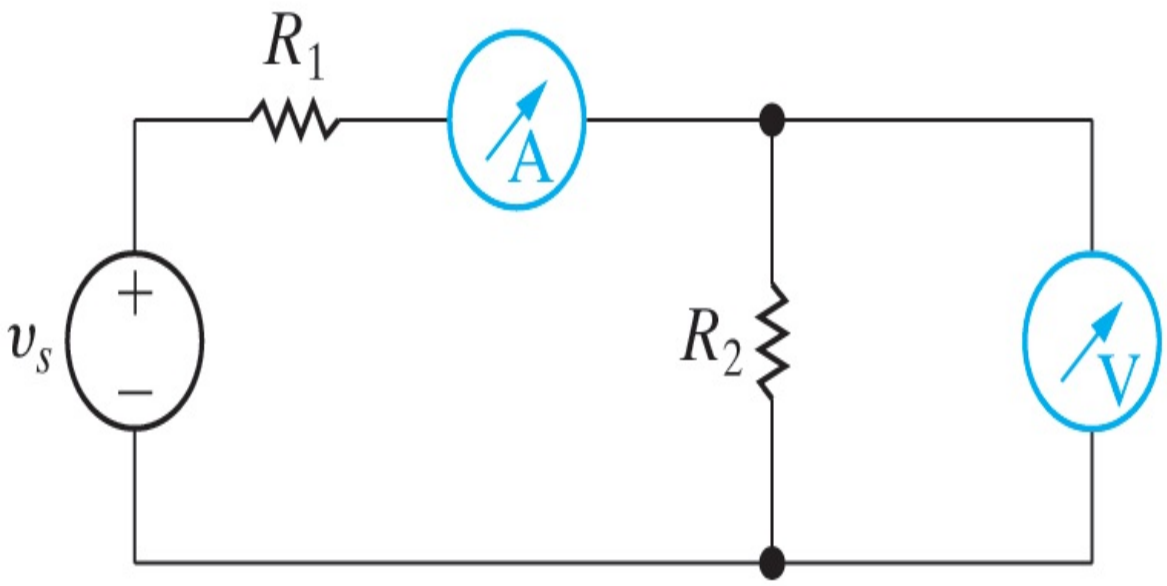
3.5 Measuring Voltage and Current

Working with actual circuits often requires making voltage and current measurements. The next two sections explore several common devices used to make these measurements. The devices are relatively simple to analyze and offer practical examples of the current- and voltage-divider configurations. We begin by looking at ammeters and voltmeters.

- An **ammeter** is an instrument designed to measure current; it is placed in series with the circuit element whose current is being measured.
- A **voltmeter** is an instrument designed to measure voltage; it is placed in parallel with the element whose voltage is being measured.

Ideal ammeters and voltmeters have no effect on the circuit variable they are designed to measure. That is, an ideal ammeter has an equivalent resistance of 0 Ω and functions as a short circuit in series with the element whose current is being measured. An ideal voltmeter has an infinite equivalent resistance and functions as an open circuit in parallel with the element whose voltage is being measured. Figure 3.23 measures the current in R1 using an ammeter and measures the voltage across R2 using a voltmeter. The ideal models for these meters in the same circuit are shown in Fig. 3.24.

Figure 3.23 An ammeter connected to measure the current in R1, and a voltmeter connected to measure the voltage across R2.



Denis Semenchenko/Shutterstock

Figure 3.23 Full Alternative Text

Figure 3.24 A short-circuit model for the ideal ammeter, and an open-circuit model for the ideal voltmeter.

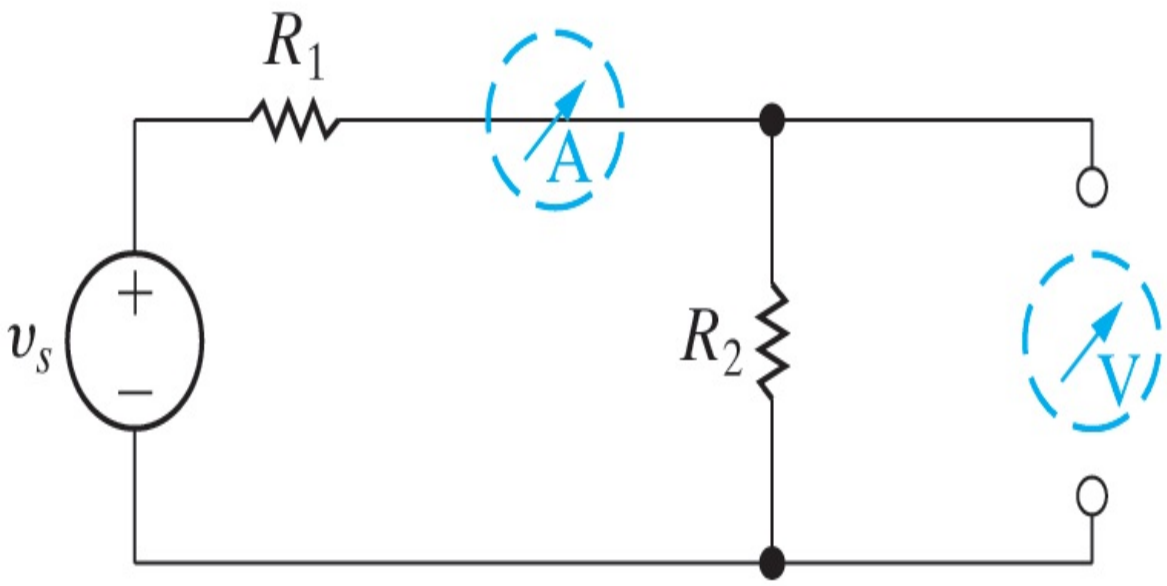


Figure 3.24 Full Alternative Text

There are two broad categories of meters used to measure continuous voltages and currents: analog meters and digital meters.

Analog Meters

Analog meters are based on the d'Arsonval meter movement, which includes a dial readout pointer as shown in Fig. 3.25. The d'Arsonval meter movement consists of a movable coil placed in the field of a permanent magnet. When current flows in the coil, it creates a torque on the coil, causing it to rotate and move a pointer across a calibrated scale. By design, the deflection of the pointer is directly proportional to the current in the movable coil. The coil is characterized by both a voltage rating and a current rating. For example, one commercially available meter movement is rated at 50 mV and 1 mA. This means that when the coil is carrying 1 mA, the voltage drop across the coil is 50 mV and the pointer is deflected to its full-scale position.

Figure 3.25 A schematic

diagram of a d’Arsonval meter movement.

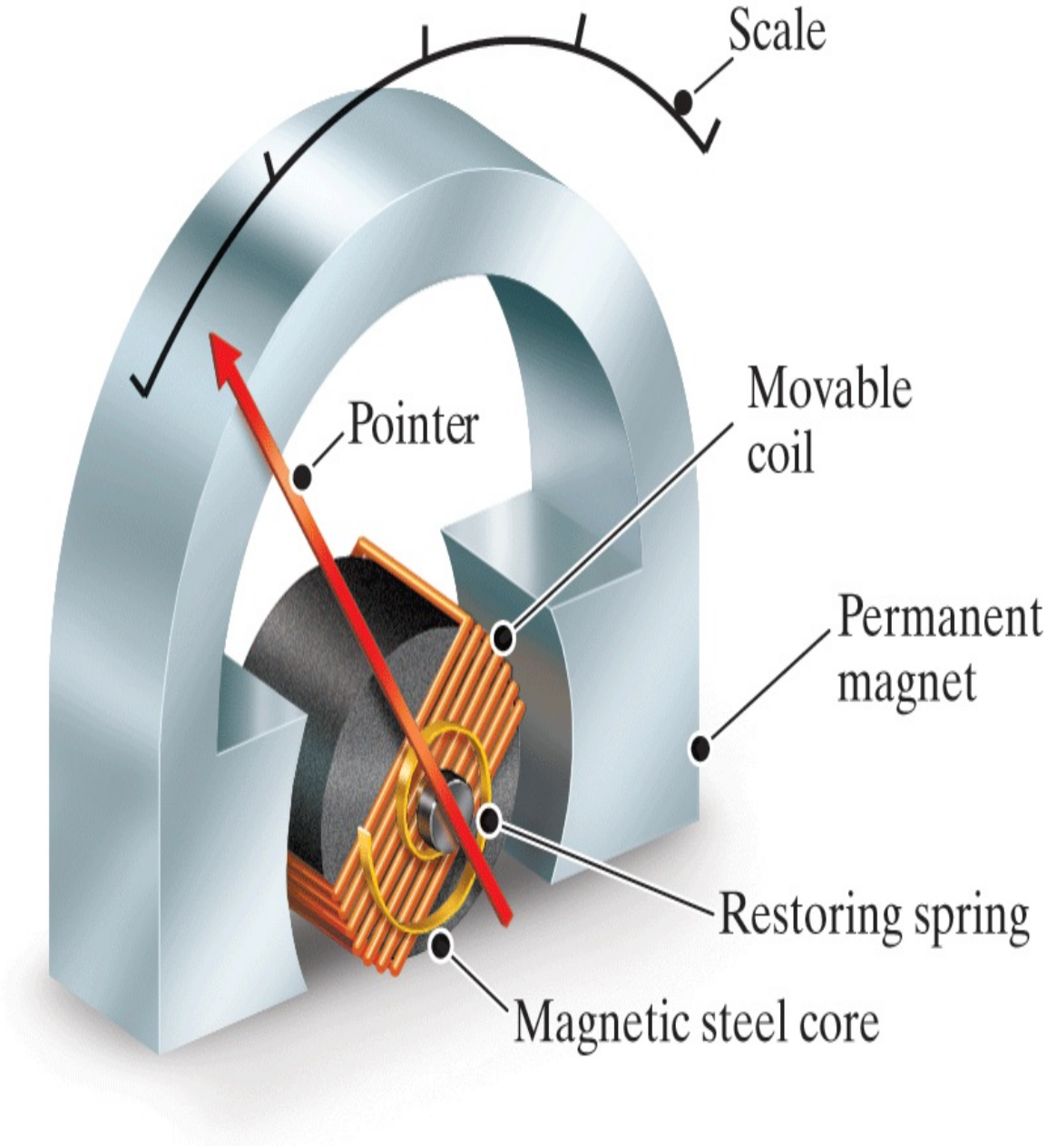


Figure 3.25 Full Alternative Text

An analog ammeter consists of a d’Arsonval movement in parallel with a resistor, as shown in Fig. 3.26. The parallel resistor limits the amount of

current in the movement's coil by shunting some of it through RA. In contrast, an analog voltmeter consists of a d'Arsonval movement in series with a resistor, as shown in Fig. 3.27. Here, the resistor limits the voltage drop across the meter's coil. In both meters, the added resistor determines the full-scale reading of the meter movement.

Figure 3.26 An analog ammeter circuit.

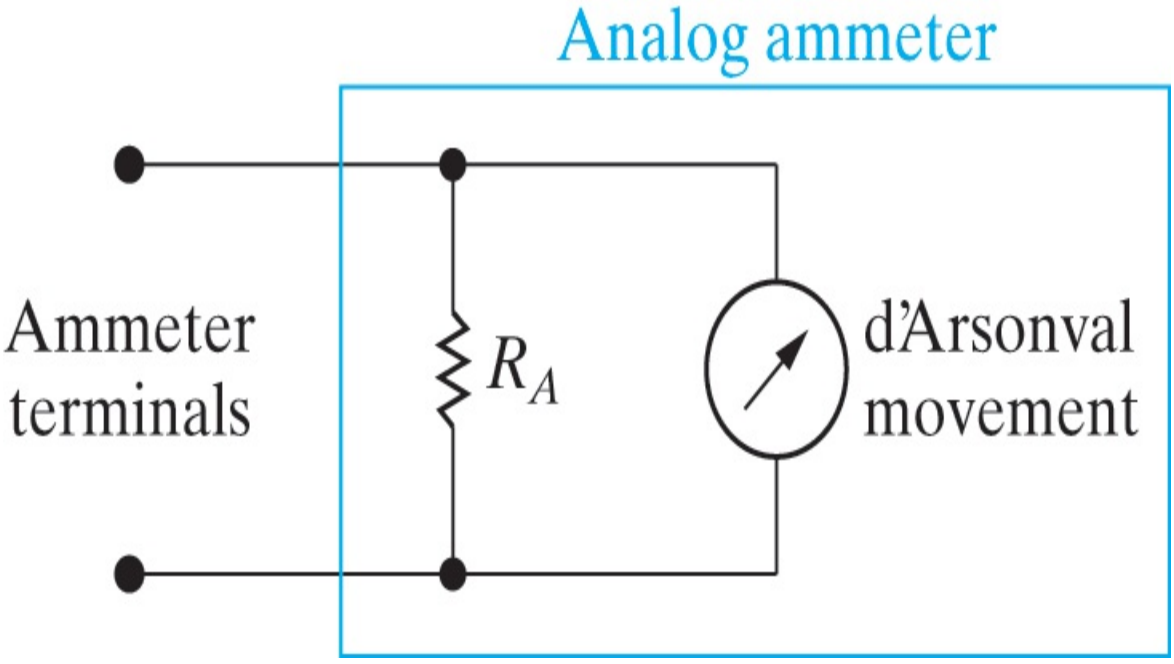


Figure 3.26 Full Alternative Text

Figure 3.27 An analog voltmeter circuit.

A diagram shows an analog voltmeter circuit.

Figure 3.27 Full Alternative Text

From these descriptions we see that an analog meter is nonideal; both the added resistor and the meter movement introduce resistance in the circuit where the meter is attached. In fact, any instrument used to make physical measurements extracts energy from the system while making measurements. The more energy extracted by the instruments, the more severely the measurement is disturbed. The equivalent resistance of a real ammeter is not zero, so it adds resistance to the circuit in series with the element whose current is being read. The equivalent resistance of a real voltmeter is not infinite, so it adds resistance to the circuit in parallel with the element whose voltage is being read.

How much these meters disturb the circuit being measured depends on the effective resistance of the meters compared with the resistance in the circuit. For example, using the rule of 1/10th, the effective resistance of an ammeter should be no more than 1/10th of the value of the smallest resistance in the circuit to be sure that the current being measured is nearly the same with or without the ammeter. But in an analog meter, the value of resistance is determined by the desired full-scale reading we wish to make, and it cannot be arbitrarily selected. Examples 3.8 and 3.9 illustrate how to calculate the resistance needed in an analog ammeter or voltmeter. The examples also determine the effective resistance of the meter when it is inserted in a circuit.

Example 3.8 Using a d'Arsonval Ammeter

1. A 50 mV, 1 mA d'Arsonval movement is to be used in an ammeter with a full-scale reading of 10 mA. Determine RA.

2. Repeat (a) for a full-scale reading of 1 A.

3. How much resistance is added to the circuit when the 10 mA ammeter is inserted to measure current?

4. Repeat (c) for the 1 A ammeter.

Solution

1. Look at the analog ammeter circuit in Fig. 3.26. The current in the ammeter must divide between the branch with the resistor RA and the branch with the meter movement. From the problem statement we know that when the current in the ammeter is 10 mA, 1 mA is flowing through the meter coil, which means that 9 mA must be diverted through RA. We also know that when the movement carries 1 mA, the voltage across its terminals is 50 mV, which is also the voltage across RA. Using Ohm's law,

   $9\times10-3RA=50\times10-3,$

   or

   $RA=50/9=5.555\ \Omega.$

2. When the full-scale deflection of the ammeter is 1 A, RA must carry 999 mA when the movement carries 1 mA. In this case,

   $999\times10-3RA=50\times10-3,$

or

$R_A = 50/999 \approx 50.05\ m\Omega$.

3. Let Rm represent the equivalent resistance of the ammeter. When the ammeter current is 10 mA, its voltage drop is 50 mV, so from Ohm's law,

   $R_m = \frac{0.05}{0.01} = 5\ \Omega$.

   Alternatively, the resistance of the ammeter is the equivalent resistance of the meter movement in parallel with RA. The resistance of the meter movement is the ratio of its voltage to its current, or $0.05/0.001 = 50\ \Omega$. Therefore,

   $R_m = 50 \| (50/9) = \frac{(50)(50/9)}{50 + (50/9)} = 5\ \Omega$.

4. For the 1 A ammeter

   $R_m = \frac{0.05}{1} = 0.05\ \Omega$,

   or, alternatively,

   $R_m = 50 \| (50/999) = \frac{(50)(50/999)}{50 + (50/999)} = 0.05\ \Omega$.

Example 3.9 Using a d'Arsonval Voltmeter

1. A 50 mV, 1 mA d'Arsonval movement is to be used in a voltmeter in which the full-scale reading is 150 V. Determine Rv.
2. Repeat (a) for a full-scale reading of 5 V.
3. How much resistance does the 150 V meter insert into the circuit?
4. Repeat (c) for the 5 V meter.

Solution

1. Look at the analog voltmeter circuit in Fig. 3.27. The voltage across the voltmeter must divide between the resistor Rv and the meter movement. From the problem statement we know that when the voltage across the voltmeter is 150 V, the voltage across the meter coil must be 50 mV. The remaining 149.95 V must be the voltage across Rv. We also know that when the movement's voltage drop is 50 mV, its current is 1 mA, which is also the current in Rv. Using Ohm's law,

   $$Rv = \frac{149.95}{0.001} = 149{,}950\ \Omega.$$

2. For a full-scale reading of 5 V, the voltage across Rv is 4.95 V and the current in Rv is still 1 mA, so from Ohm's law,

   $$Rv = \frac{4.95}{0.001} = 4950\ \Omega.$$

3. Let Rm represent the equivalent resistance of the voltmeter. When the voltage across the voltmeter is 150 V, its current is 1 mA, so from Ohm's law,

   $$Rm = \frac{150}{10^{-3}} = 150{,}000\ \Omega.$$

   Alternatively, the resistance of the voltmeter is the equivalent resistance of Rv in series with the meter movement. The resistance of the meter movement is the ratio of its voltage to its current, or
   50 m V/1 mA = 50 Ω. Therefore,

   $$Rm = 149{,}950 + 50 = 150{,}000\ \Omega.$$

4. For the 5 V voltmeter,

   $$Rm = \frac{5}{10^{-3}} = 5000\ \Omega.$$

   or, alternatively,

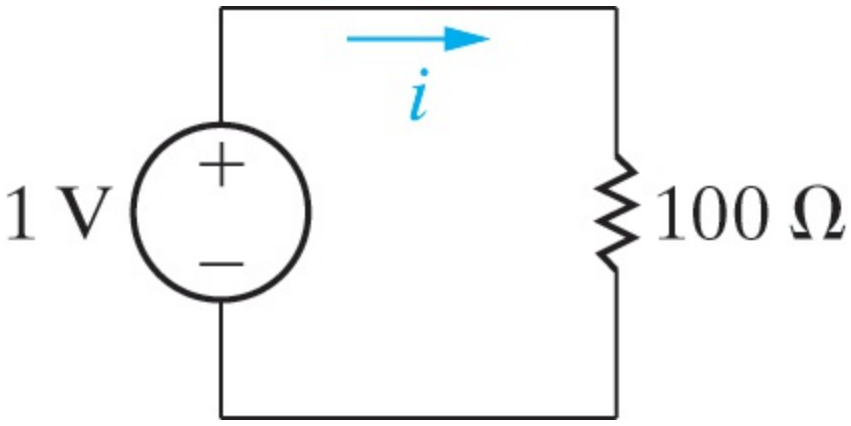
   $$Rm = 4950 + 50 = 5000\ \Omega.$$

Assessment Problems

Objective 4—Be able to determine the reading of ammeters and voltmeters

3.5

1. Find the current in the circuit shown.
2. If the ammeter in Example 3.8(a) is used to measure the current, what will it read?

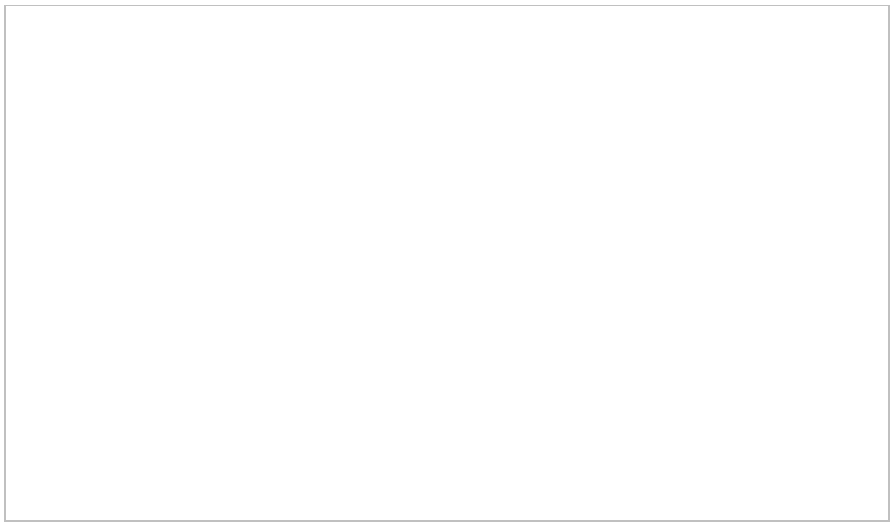


Answer:

1. 10 mA;
2. 9.524 mA.

3.6

1. Find the voltage v across the 75 kΩ resistor in the circuit shown.
2. If the 150 V voltmeter of Example 3.9(a) is used to measure the voltage, what will be the reading?



Answer:

1. 50 V;
2. 46.15 V.

SELF-CHECK: *Also try Chapter Problems 3.33 and 3.37.*

Digital Meters

Digital meters measure the continuous voltage or current signal at discrete points in time, called the sampling times. The signal is thus converted from an analog signal, which is continuous in time, to a digital signal, which exists only at discrete instants in time. A more detailed explanation of the workings of digital meters is beyond the scope of this text and course. However, you are likely to see and use digital meters in lab settings because they offer several advantages over analog meters. They introduce less resistance into the circuit to which they are connected (though they are still nonideal), are easier to connect, and take more precise measurements owing to the nature of their readout mechanism.

3.6 Measuring Resistance—The Wheatstone Bridge

While many different circuit configurations are used to measure resistance, here we will focus on just one, the Wheatstone bridge. The Wheatstone bridge circuit is used to precisely measure resistances of medium values, that is, in the range of 1 Ω to 1 MΩ. In commercial models of the Wheatstone bridge, accuracies on the order of ±0.1% are possible. The bridge circuit, shown in Fig. 3.28, consists of four resistors, a dc voltage source, and a detector. The resistance of one of the four resistors can be varied, which is indicated by the arrow through R3. The dc voltage source is usually a battery, which is indicated by the battery symbol for the voltage source v in Fig. 3.28. The detector is generally a d'Arsonval movement in the microamp range and is called a *galvanometer*. In Fig. 3.28, R1, R2, and R3 are known resistors and Rx is the unknown resistor.

Figure 3.28 The Wheatstone bridge circuit.

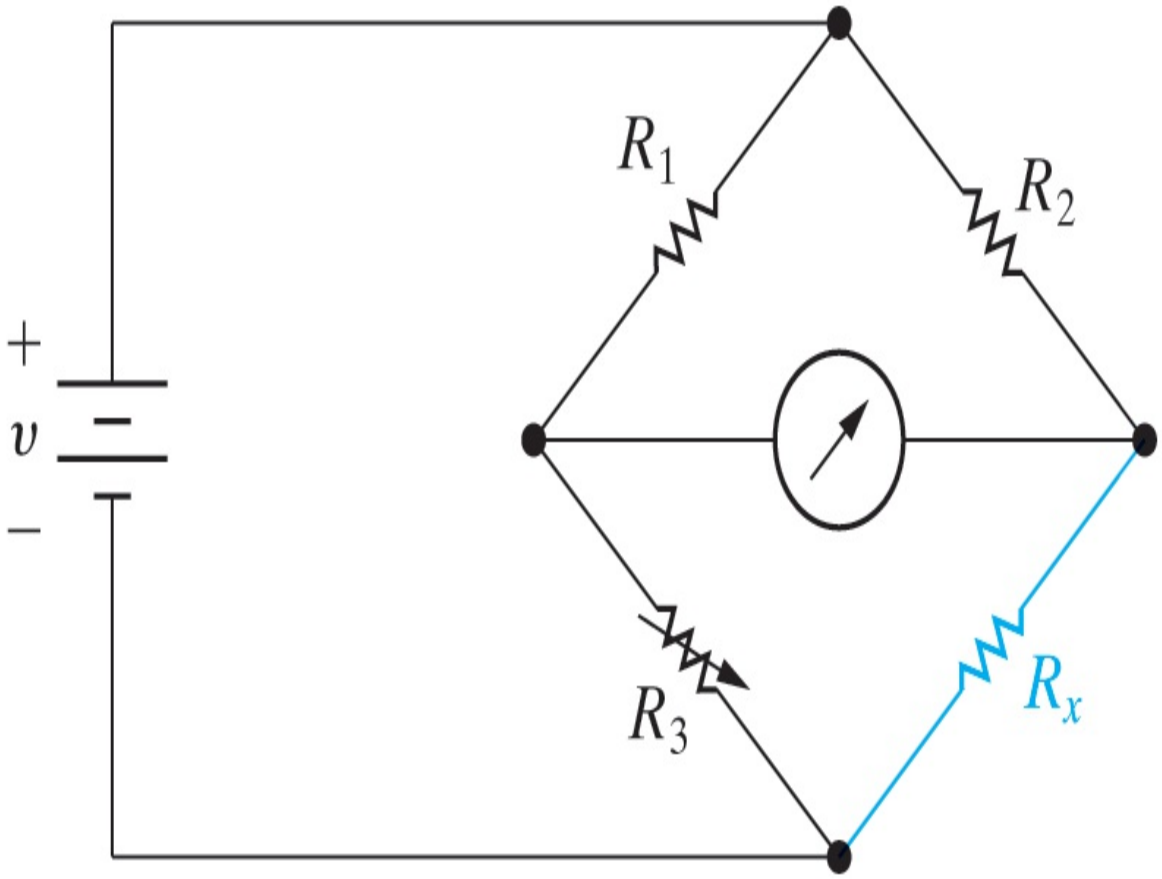


Figure 3.28 Full Alternative Text

To find the value of Rx, we adjust the variable resistor R3 until there is no current in the galvanometer. We then calculate the unknown resistor from the simple expression

Rx=R2R1R3.(3.11)

We derive Eq. 3.11 by applying Kirchhoff's laws to the bridge circuit. We redraw the bridge circuit as Fig. 3.29 to show the branch currents in the bridge. When ig is zero, we say the bridge is *balanced*. At node a, Kirchhoff's current law requires that

Figure 3.29 A balanced Wheatstone bridge (ig=0).

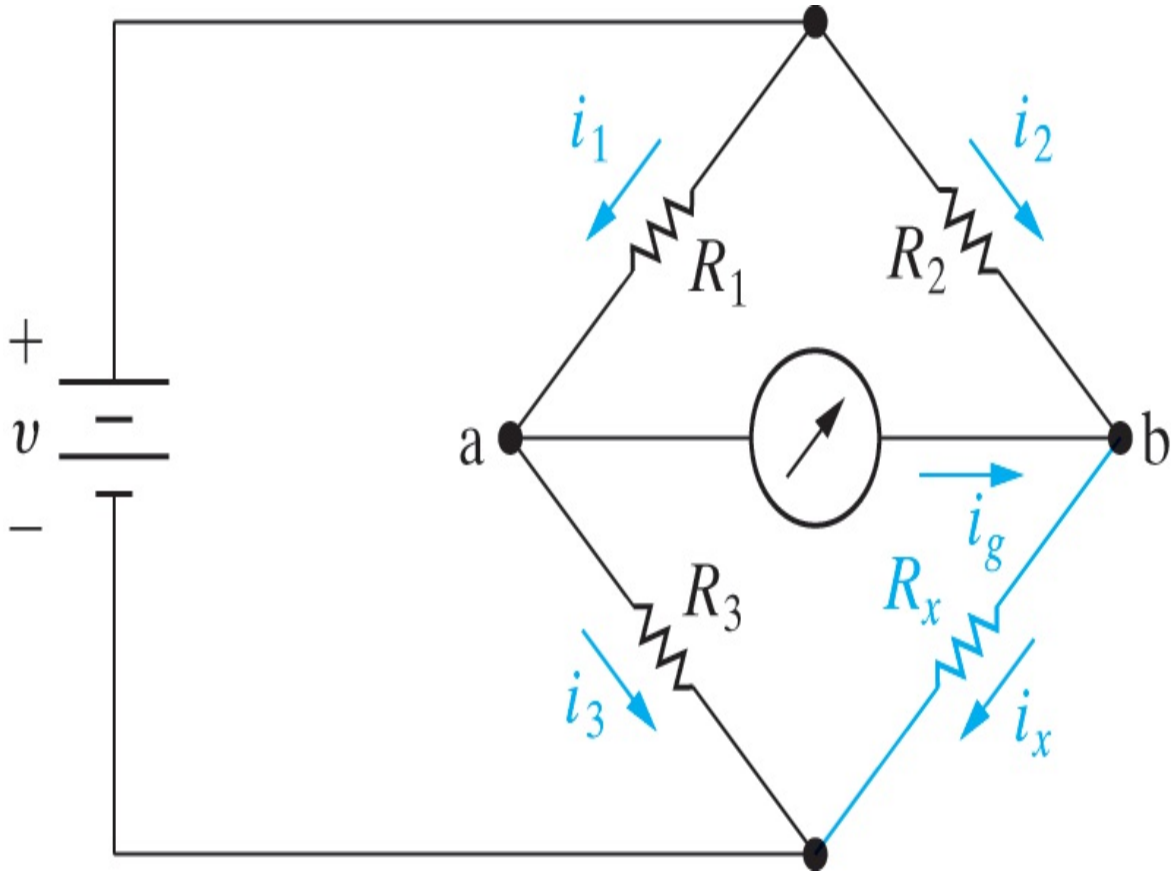


Figure 3.29 Full Alternative Text

i1=i3,

while at node b, Kirchhoff’s current law requires that

i2=ix.

Now, because ig is zero, the voltage drop across the detector is also zero, so nodes a and b are at the same potential. Thus, when the bridge is balanced, Kirchhoff’s voltage law for the clockwise path containing the galvanometer and resistors R3 and Rx gives

0+Rxix−R3i3=0

so

i3R3=ixRx.

Using Kirchhoff’s voltage law for the path containing the galvanometer and resistors R1 and R2 gives

i1R1=i2R2.

Divide the first KVL equation by the second KVL equation to give

i3R3i1R1=ixRxi2R2,

Eliminate the currents (because i1 = i3 and i2 = ix) and solve for Rx to get Eq. 3.11.

Now that we have verified the validity of Eq. 3.11, some comments about the result are in order. First, note that if R2/R1=1, the unknown resistor Rx equals R3, so R3 must vary over a range that includes the value Rx. For example, if the unknown resistance were 1000 Ω and R3 could be varied from 0 to 100 Ω, the bridge could never be balanced. Thus, to cover a wide range of unknown resistors, we must be able to vary the ratio R2/R1. In a commercial Wheatstone bridge, R1 and R2 consist of decimal values of resistances that can be switched into the bridge circuit. Normally, the decimal values are 1, 10, 100, and 1000 Ω, so that the ratio R2/R1 can be varied from 0.001 to 1000 in decimal steps. The variable resistor R3 is usually adjustable in integral values of resistance from 1 to 11,000 Ω.

Second, although Eq. 3.11 implies that Rx can vary from zero to infinity, the practical range of Rx is approximately 1 Ω to 1 MΩ. Resistances smaller than 1 Ω are difficult to measure on a standard Wheatstone bridge because of thermoelectric voltages generated at the junctions of dissimilar metals and because of thermal heating effects—that is, i2R effects. Resistances larger than 1 MΩ are difficult to measure accurately because of leakage currents. In other words, if Rx is large, the current leakage in the electrical insulation may be comparable to the current in the branches of the bridge circuit.

Example 3.10 uses a Wheatstone bridge to measure a range of unknown resistors.

Example 3.10 Using a Wheatstone

Bridge to Measure Resistance

For the Wheatstone bridge in Fig. 3.30, R3 can be varied from 10 Ω to 2 kΩ. What range of resistor values can this bridge measure?

Figure 3.30 The circuit for Example 3.10.

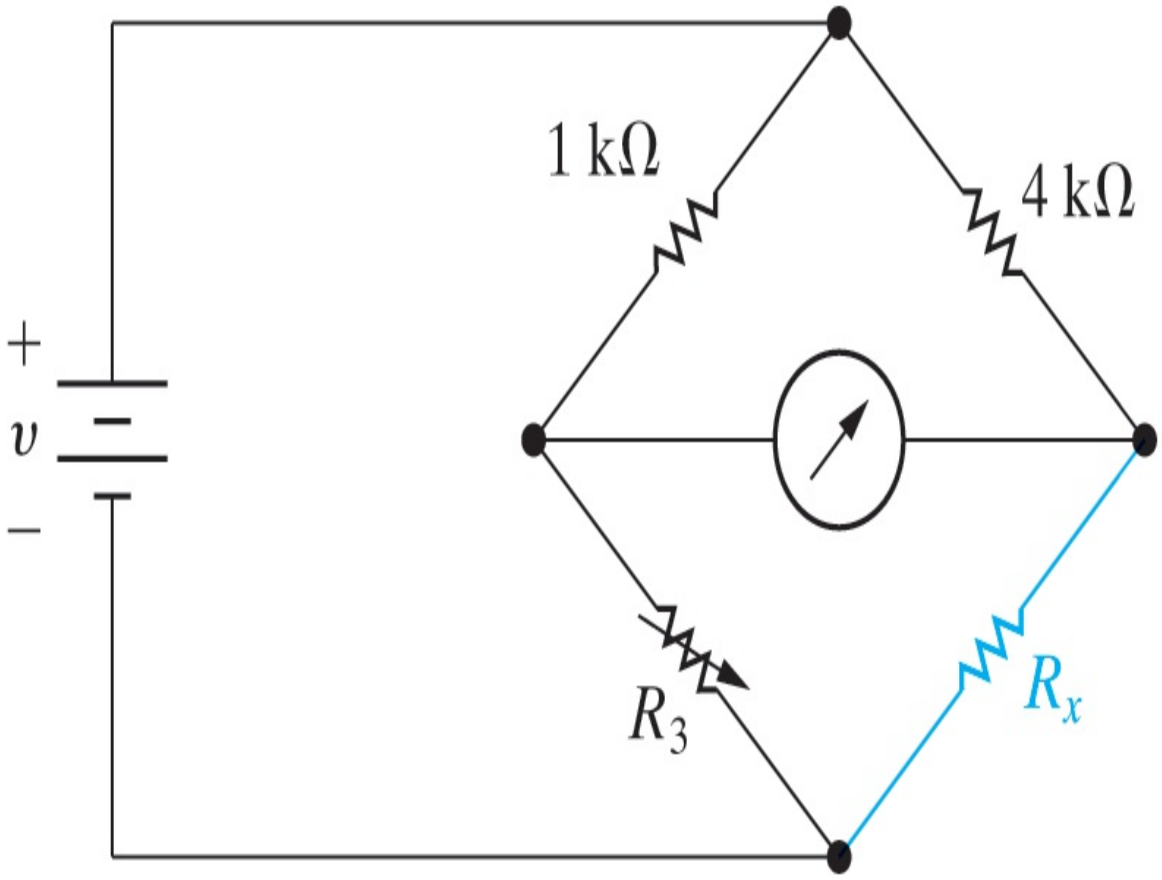


Figure 3.30 Full Alternative Text

Solution

When R3=10 Ω, the bridge is balanced when

Rx=40001000(10)=40 Ω.

When R3=2 kΩ, the bridge is balanced when

Rx=40001000(2000)=8 kΩ.

Therefore, the range of resistor values the bridge can measure is 40 Ω to 8 kΩ.

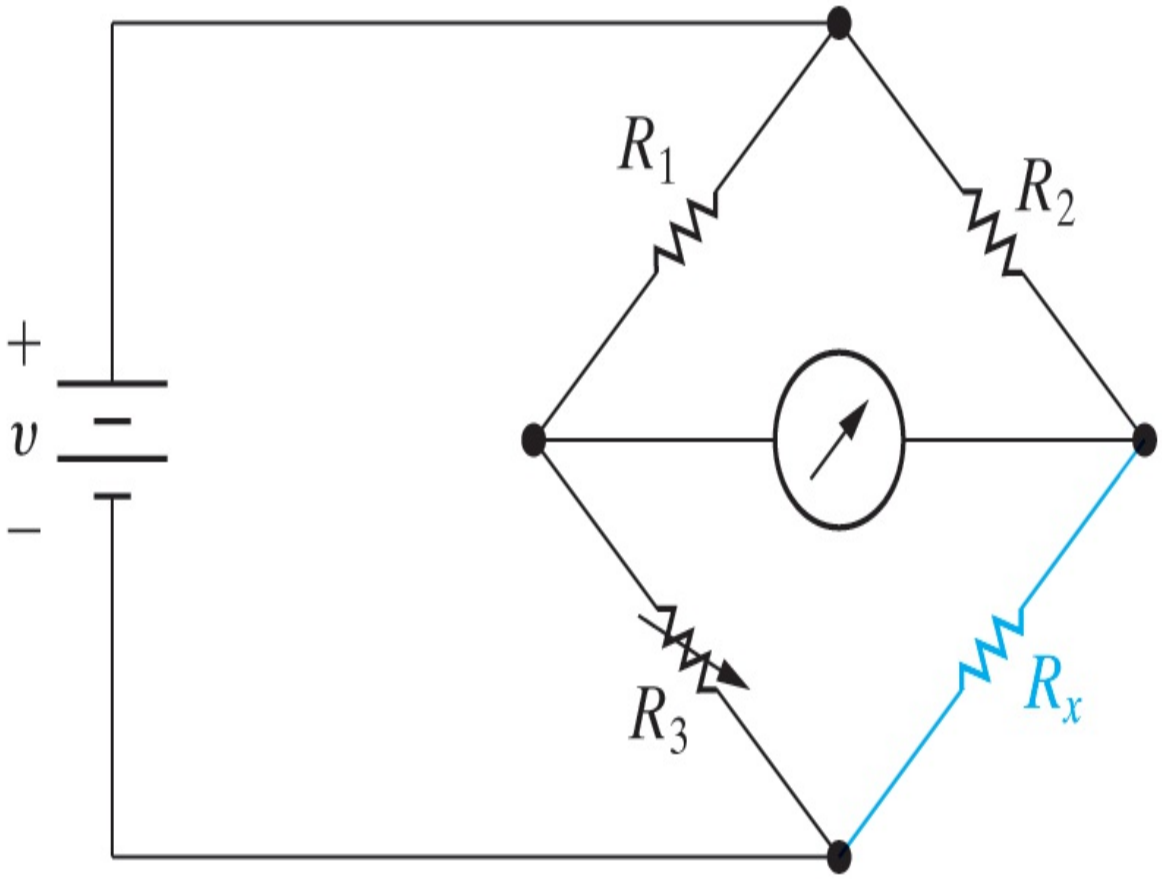
Assessment Problem

Objective 5—Understand how a Wheatstone bridge is used to measure resistance

3.7 The bridge circuit shown is balanced when R1=100 Ω, R2=1000 Ω, and R3=150 Ω. The bridge is energized from a 5 V dc source.

1. What is the value of Rx?

2. Suppose each bridge resistor is capable of dissipating 250 mW. Can the bridge be left in the balanced state without exceeding the power-dissipating capacity of the resistors, thereby damaging the bridge?



3.6-8 Full Alternative Text

Answer:

1. a) 1500 Ω;
2. b) yes.

SELF-CHECK: *Also try Chapter Problem 3.51.*

3.7 Delta-to-Wye (Pi-to-Tee) Equivalent Circuits

Look again at the Wheatstone bridge in Fig. 3.28; if we replace the galvanometer with its equivalent resistance Rm, we get the circuit shown in Fig. 3.31 (p. 76). Because this circuit does not have any series-connected or parallel-connected resistors, we cannot simplify it using the simple series or parallel equivalent circuits introduced earlier in this chapter. But the five interconnected resistors can be reduced to a single equivalent resistor using a delta-to-wye (Δ-to-Y) or pi-to-tee (π-to-T) equivalent circuit.[1]

1 Δ and Y structures are present in a variety of useful circuits, not just resistive networks. Hence, the Δ-to-Y transformation is a helpful tool in circuit analysis.

Figure 3.31 A resistive network generated by a Wheatstone bridge circuit.

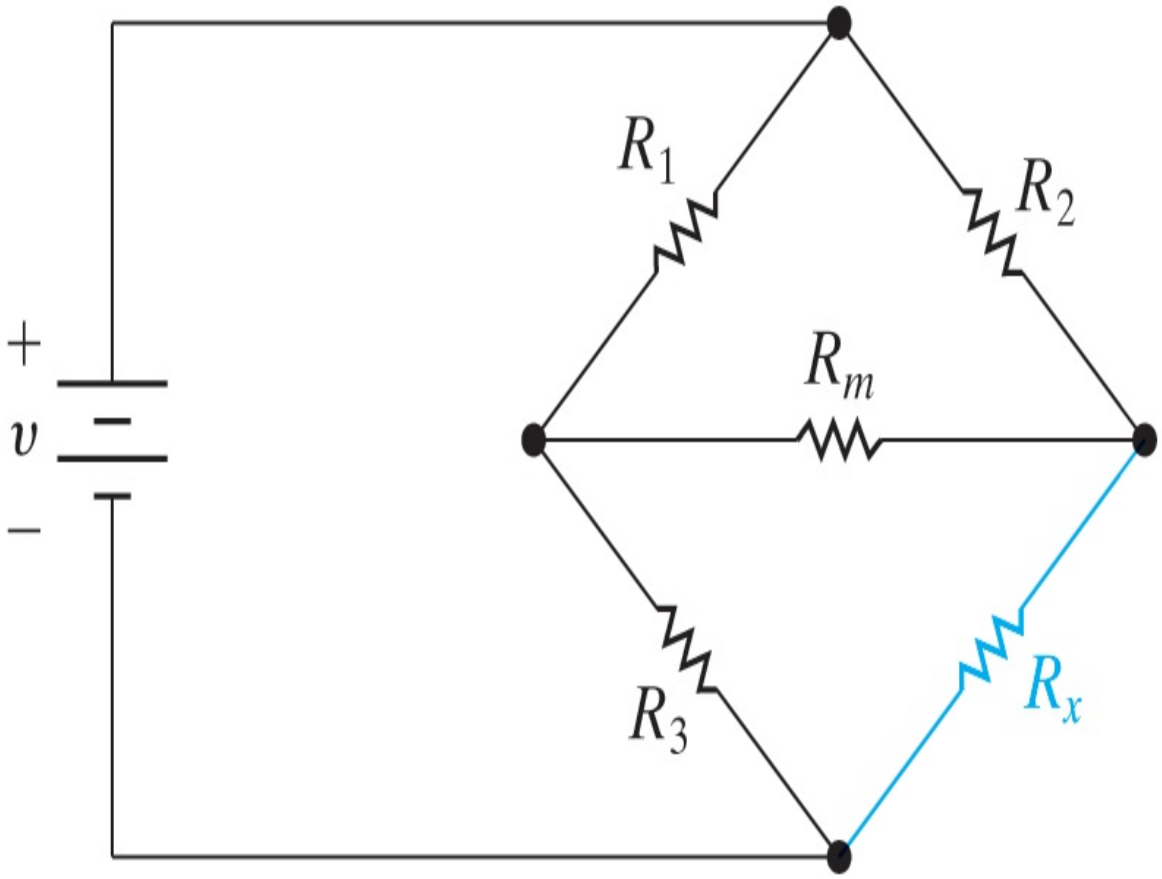


Figure 3.31 Full Alternative Text

The resistors R1, R2, and Rm (or R3, Rm and Rx) in the circuit shown in Fig. 3.31 form a **delta (Δ) interconnection** because the interconnection looks like the Greek letter Δ. It is also called a **pi interconnection** because the Δ can be reshaped like the Greek letter π without disturbing the electrical equivalence of the two configurations, as shown in Fig. 3.32.

Figure 3.32 A Δ configuration viewed as a π configuration.

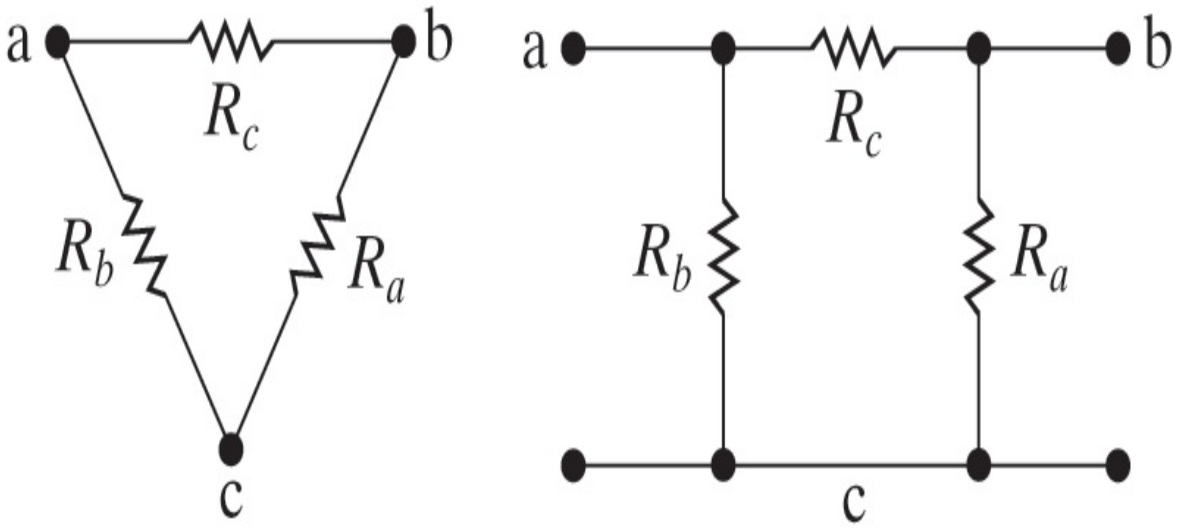


Figure 3.32 Full Alternative Text

The resistors R1, Rm, and R3 (or R2, Rm and Rx) in the circuit shown in Fig. 3.31 form a **wye (Y) interconnection** because the interconnection can be shaped to look like the letter Y. It is easier to see the Y shape of the interconnection in Fig. 3.33. The Y configuration is also called a **tee (T) interconnection** because the Y structure can be reshaped into a T structure without disturbing the electrical equivalence of the two structures, as shown in Fig. 3.33.

Figure 3.33 A Y structure viewed as a T structure.

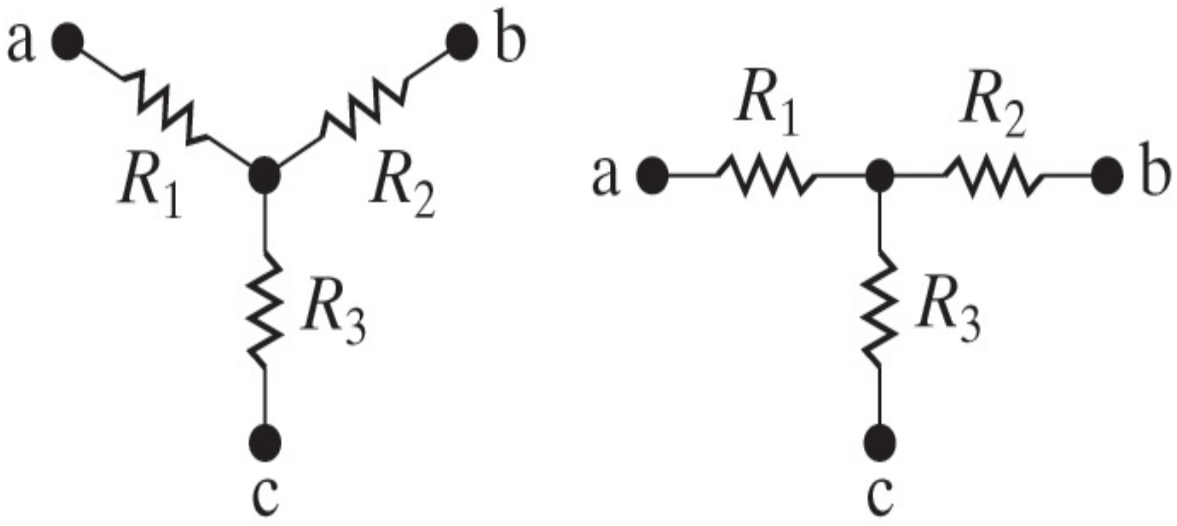


Figure 3.33 Full Alternative Text

Figure 3.34 illustrates the Δ-to-Y (or π-to-T) equivalent circuit transformation. But we cannot transform the Δ interconnection into the Y interconnection simply by changing the shape of the interconnections. Saying the Δ-connected circuit is equivalent to the Y-connected circuit means that the Δ configuration can be replaced with a Y configuration without changing the terminal behavior. Thus, if each circuit is placed in a black box, we can't tell whether the box contains a set of Δ-connected resistors or a set of Y-connected resistors by making external measurements. This condition is true only if the resistance between corresponding terminal pairs is the same for each box. For example, the resistance between terminals a and b must be the same whether we use the Δ-connected set or the Y-connected set. For each pair of terminals in the Δ-connected circuit, compute the equivalent resistance using series and parallel simplifications. For each pair of terminals in the Y-connected circuit, compute the equivalent resistance using only series simplification. The three equivalent resistance equations are

Figure 3.34 The -to-Y transformation.

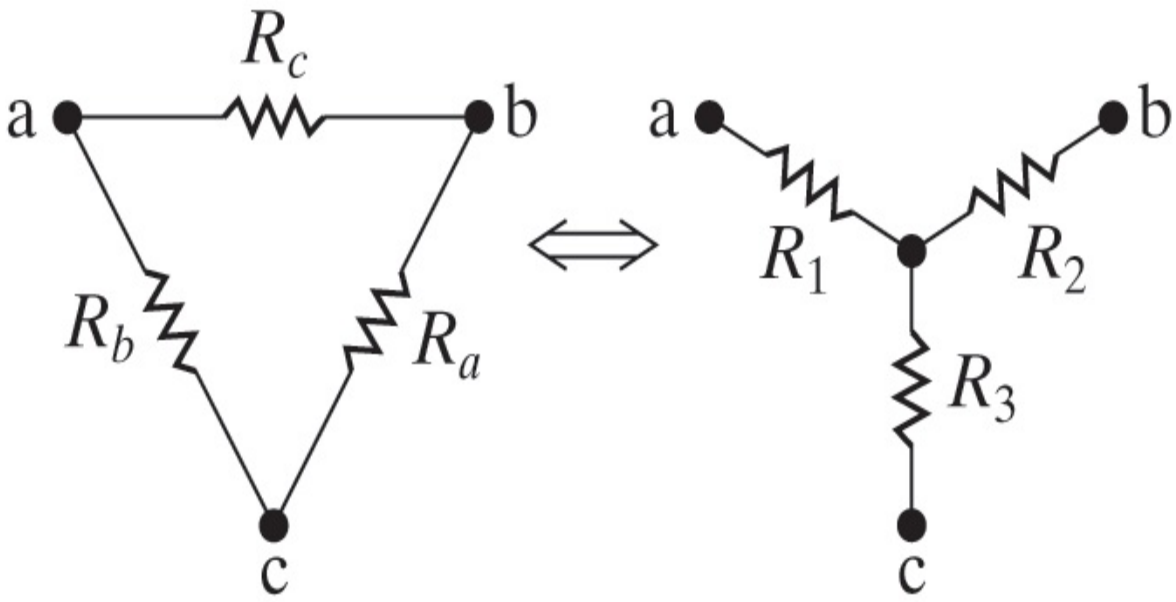


Figure 3.34 Full Alternative Text

$$R_{ab} = R_c(R_a + R_b)R_a + R_b + R_c = R_1 + R_2, \quad (3.12)$$

$$R_{bc} = R_a(R_b + R_c)R_a + R_b + R_c = R_2 + R_3, \quad (3.13)$$

$$R_{ca} = R_b(R_c + R_a)R_a + R_b + R_c = R_1 + R_3. \quad (3.14)$$

Straightforward algebraic manipulation of Eqs. 3.12–3.14 gives values for the Y-connected resistors in terms of the Δ-connected resistors. Use these equations when transforming three Δ-connected resistors into an equivalent Y connection:

$$R_1 = R_b\, R_c R_a + R_b + R_c, \quad (3.15)$$

$$R_2 = R_c R_a R_a + R_b + R_c, \quad (3.16)$$

$$R_3 = R_a R_b R_a + R_b + R_c. \quad (3.17)$$

Reversing the Δ-to-Y transformation also is possible, so we can start with a Y structure and replace it with an equivalent Δ structure. The expressions for the three Δ-connected resistors as functions of the three Y-connected resistors are

$$R_a = R_1 R_2 + R_2 R_3 + R_3 R_1 R_1, \quad (3.18)$$

$$R_b = R_1 R_2 + R_2 R_3 + R_3 R_1 R_2, \quad (3.19)$$

$$R_c = R_1 R_2 + R_2 R_3 + R_3 R_1 R_3. \quad (3.20)$$

Example 3.11 uses a Δ-to-Y transformation to simplify a circuit and its

analysis.

Example 3.11 Applying a Delta-to-Wye Transform

Find the current and power supplied by the 40 V source in the circuit shown in Fig. 3.35.

Figure 3.35 The circuit for Example 3.11.

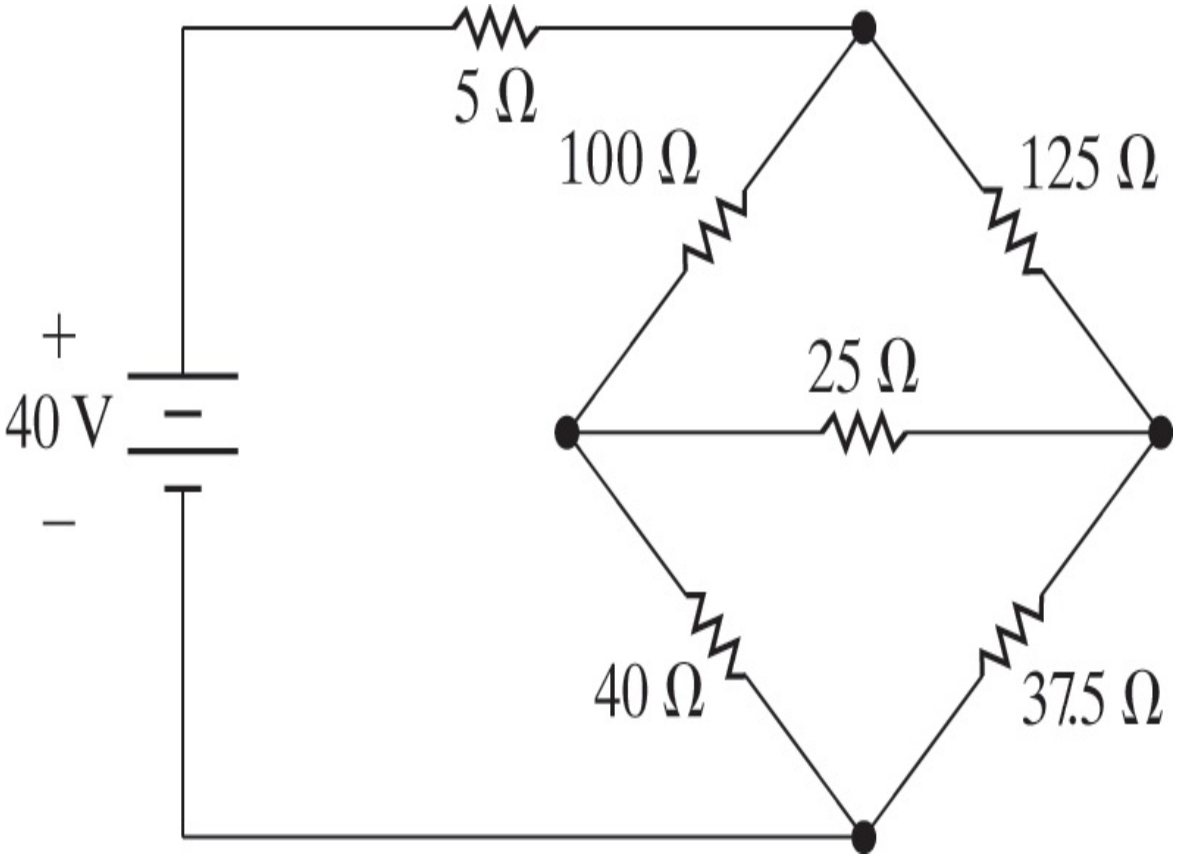


Figure 3.35 Full Alternative Text

Solution

This problem is easy to solve if we can find the equivalent resistance seen by the source. Begin this simplification by replacing either the upper Δ (100, 125, 25 Ω) or the lower Δ (40, 25, 37.5 Ω) with its equivalent Y. We choose to replace the upper Δ by computing the three Y resistances, defined in Fig. 3.36, using Eqs. 3.15 to 3.17. Thus,

Figure 3.36 The equivalent Y resistors.

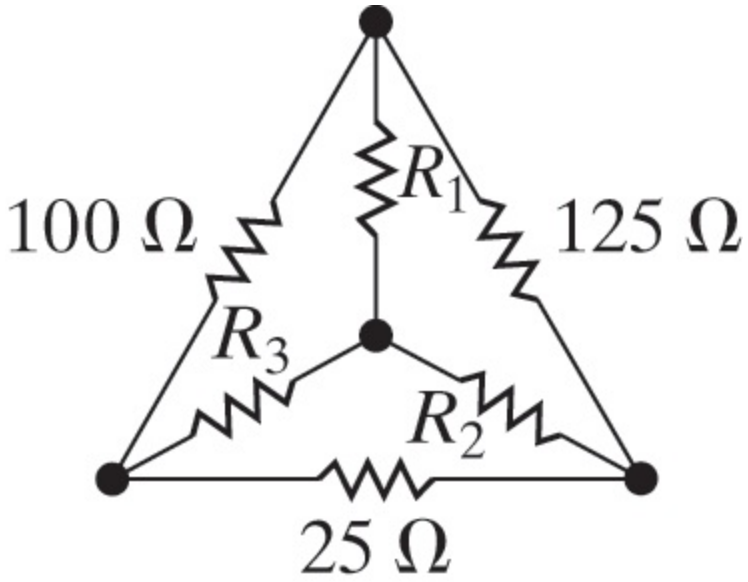


Figure 3.36 Full Alternative Text

$R1=100\times125250=50\ \Omega,$
$R2=125\times25250=12.5\ \Omega,$
$R3=100\times25250=10\ \Omega.$

Substituting the Y resistors into the circuit shown in Fig. 3.35 produces the circuit shown in Fig. 3.37. From Fig. 3.37 we can easily calculate the resistance seen by the 40 V source using series-parallel simplifications:

Figure 3.37 A transformed version of the circuit shown in Fig. 3.35.

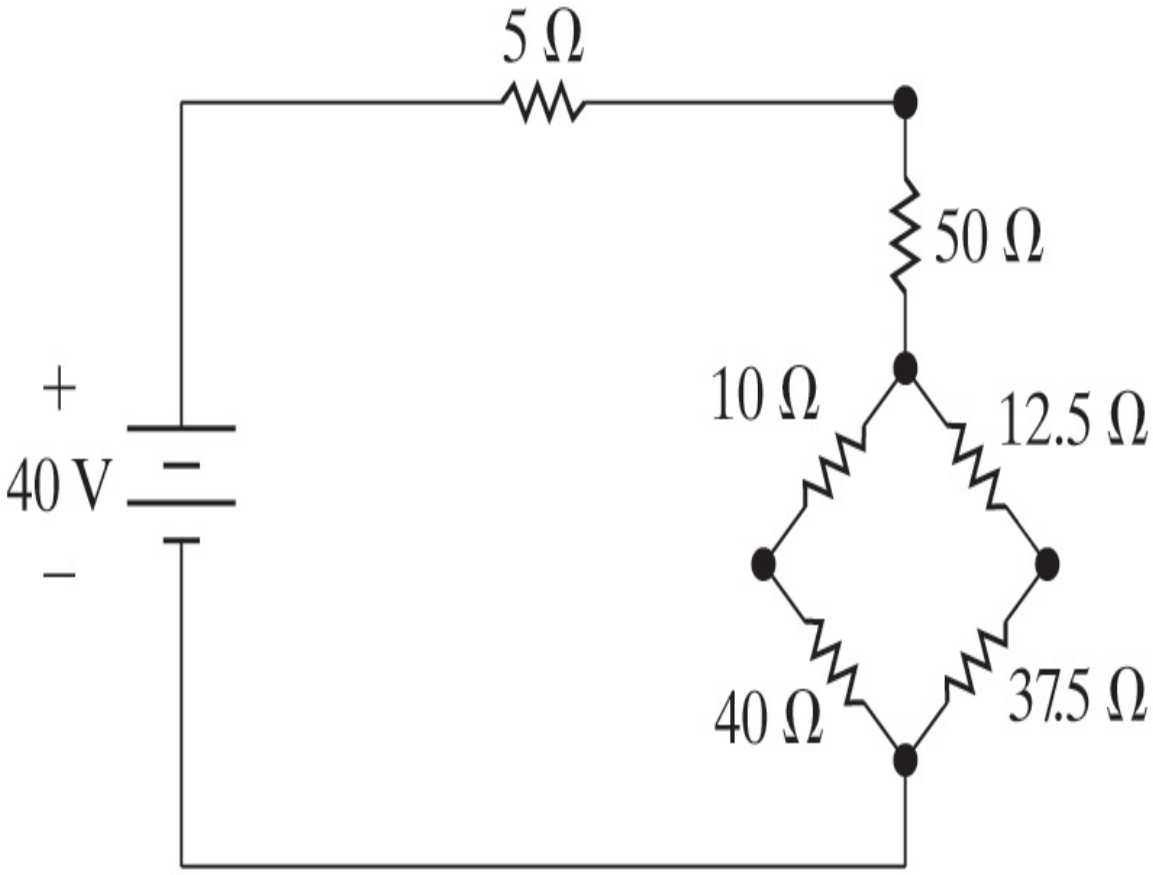


Figure 3.37 Full Alternative Text

Req=5+50+(10+40)||(12.5+37.5)=55+(50)(50)50+50=80 Ω.

The circuit simplifies to an 80 Ω resistor across a 40 V source, as shown in Fig. 3.38, so the 40 V source delivers current i=40/80=0.5 A and power p=40(0.5)=20 W is delivered to the circuit.

Figure 3.38 The final step in the

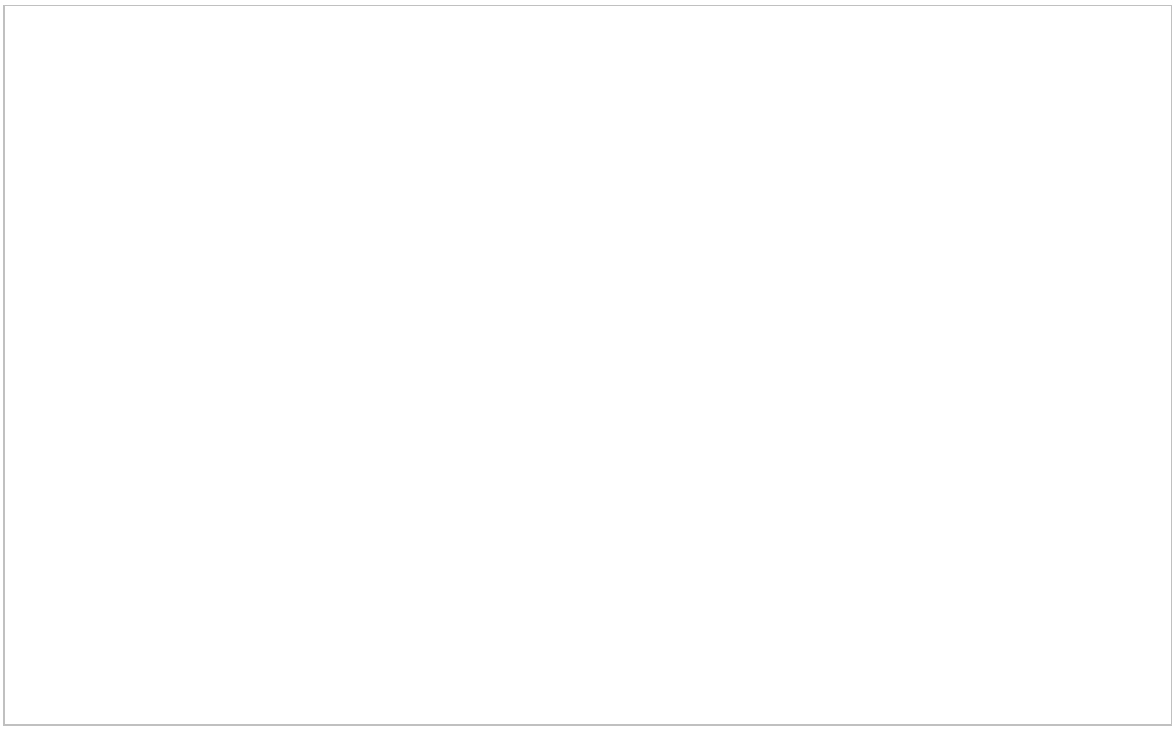
simplification of the circuit shown in Fig. 3.35.

Assessment Problem

Objective 6—Know when and how to use delta-to-wye equivalent circuits

3.8 Use a Y-to-Δ transformation to find the voltage v in the circuit shown.

Answer: 35 V.



3.7-9 Full Alternative Text

SELF-CHECK: *Also try Chapter Problems 3.59, 3.60, and 3.63.*

Practical Perspective

Resistive Touch Screens

Let’s analyze the resistive grid in the x-direction. We model the resistance of the grid in the x-direction with the resistance Rx, as shown in Fig. 3.39. The x-location where the screen is touched is indicated by the arrow. Touching the screen effectively divides the total resistance, Rx, into two separate resistances α Rx and (1−α)Rx. The resulting voltage drop across the resistance α Rx is Vx.

Figure 3.39 The resistive touch

screen grid in the x-direction.

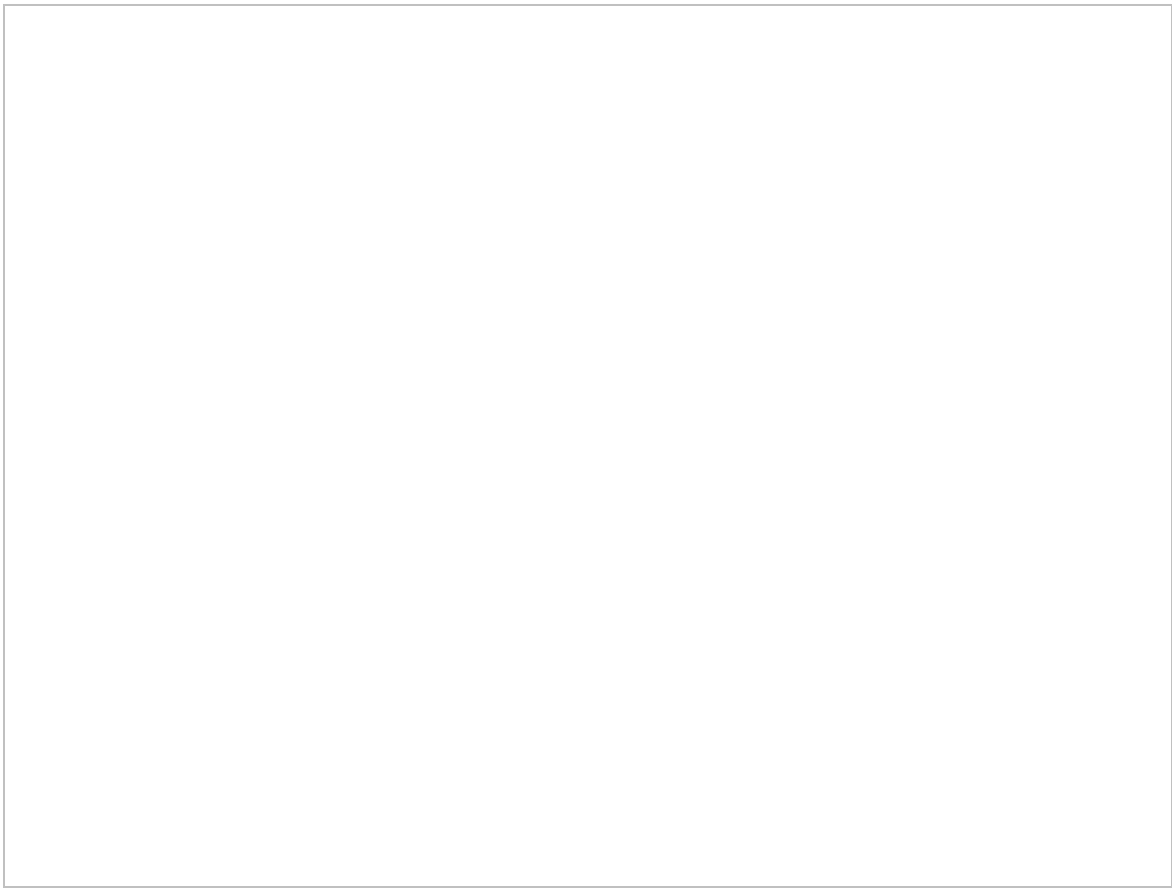


Figure 3.39 Full Alternative Text

From the figure you can see that when the touch is on the far right side of the screen, $\alpha=0$ and $Vx=0$. Similarly, when the touch is on the far left side of the screen, $\alpha=1$ and $Vx=Vs$. If the touch is inbetween the two edges of the screen, the value of α is between 0 and 1 and the two parts of the resistance Rx form a voltage divider. We can calculate the voltage Vx using the equation for voltage division:

$$Vx=\alpha Rx\alpha Rx+(1-\alpha)RxVs=\alpha RxRxVs=\alpha Vs.$$

We can find the value of α, which represents the location of the touch point with respect to the far right side of the screen, by dividing the voltage across the grid resistance starting at the touch point, Vx, by the voltage applied

across the entire resistive grid in the x-direction, Vs:

$\alpha = V_x V_s$.

Now we want to use the value of α to determine the x-coordinate of the touch location on the screen. Typically, the screen coordinates are specified in terms of pixels (short for “picture elements”). For example, the screen of a mobile phone is a grid of pixels, with px pixels in the x-direction and py pixels in the y-direction. Each pixel is identified by its x-location (a number between 0 and $p_x - 1$) and its y-location (a number between 0 and $p_y - 1$). The pixel with the location (0, 0) is in the upper-left-hand corner of the screen, as shown in Fig. 3.40.

Figure 3.40 The pixel coordinates of a screen with px pixels in the x-direction and py pixels in the y-direction.

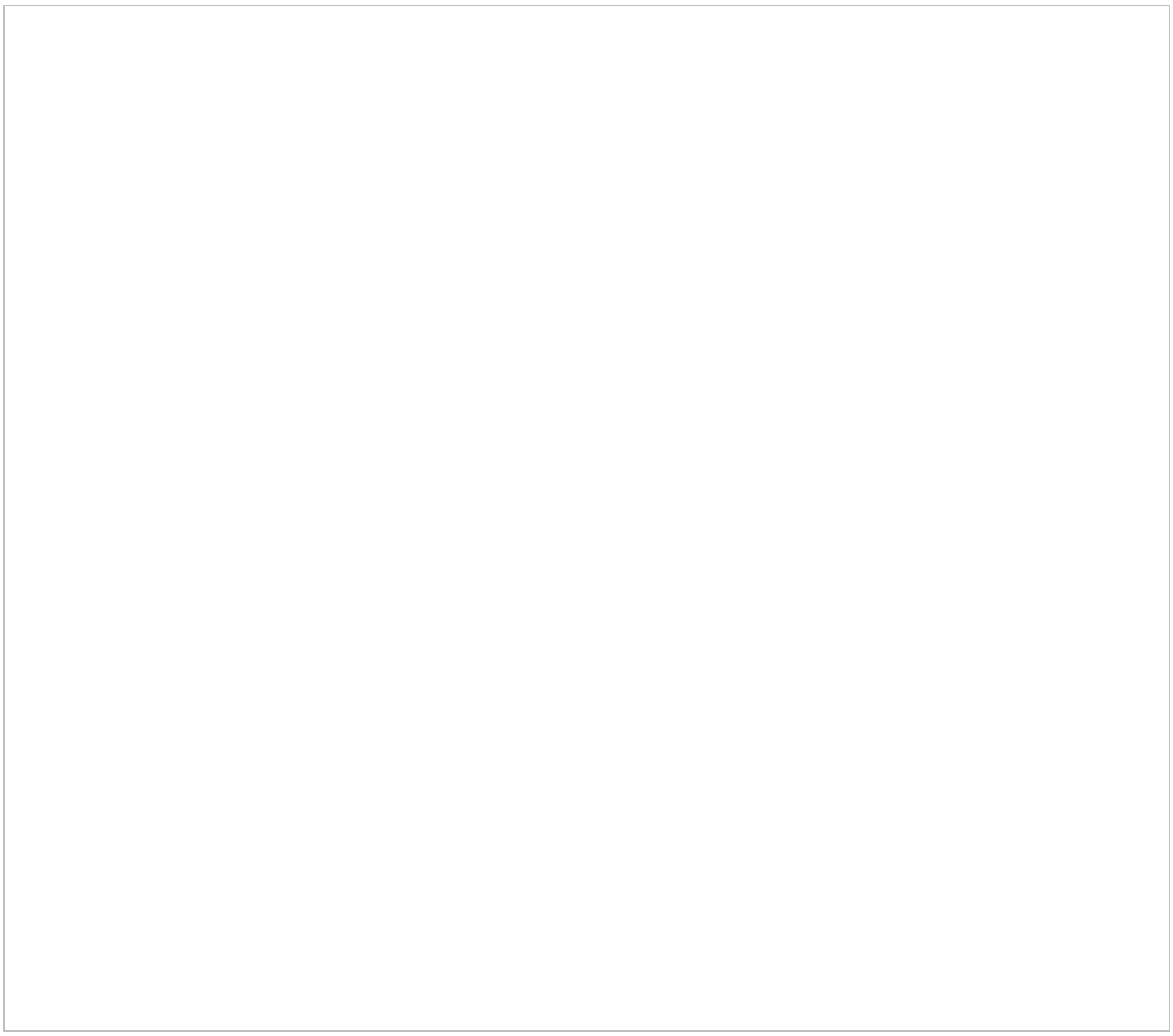


Figure 3.40 Full Alternative Text

Since α represents the location of the touch point with respect to the right side of the screen, (1−α) represents the location of the touch point with respect to the left side of the screen. Therefore, the x-coordinate of the pixel corresponding to the touch point is

$x=(1-\alpha)p_x$.

Note that the value of x is capped at (p_x-1).

Using the model of the resistive screen grid in the y-direction shown in Fig. 3.41, it is easy to show that the voltage created by a touch at the arrow is given by

Figure 3.41 The resistive touch screen grid in the y-direction.

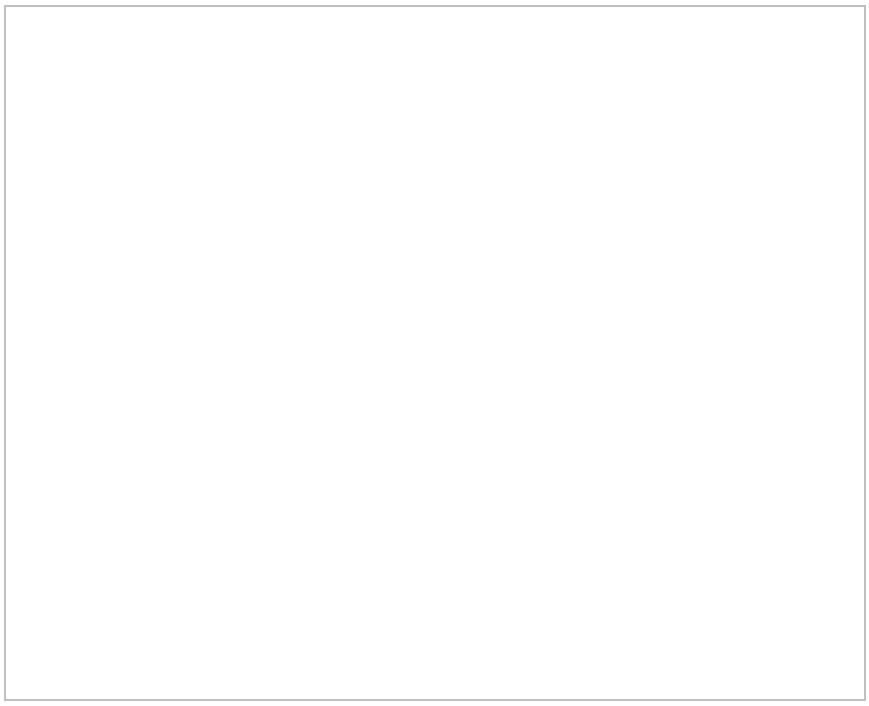


Figure 3.41 Full Alternative Text
Vy=βVs.

Therefore, the y-coordinate of the pixel corresponding to the touch point is

y=(1−β)py,

where the value of y is capped at (py−1). (See Problem 3.72.)

SELF-CHECK: *Assess your understanding of the Practical Perspective by solving Chapter Problems 3.72–3.75.*

Summary

- **Series resistors** can be combined to obtain a single equivalent resistance according to the equation

  $$R_{eq} = \sum_{i=1}^{k} R_i = R_1 + R_2 + \cdots + R_k.$$

  (See page 60.)

- **Parallel resistors** can be combined to obtain a single equivalent resistance according to the equation

  $$\frac{1}{R_{eq}} = \sum_{i=1}^{k} \frac{1}{R_i} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_k}.$$

  When just two resistors are in parallel, the equation for equivalent resistance can be simplified to give

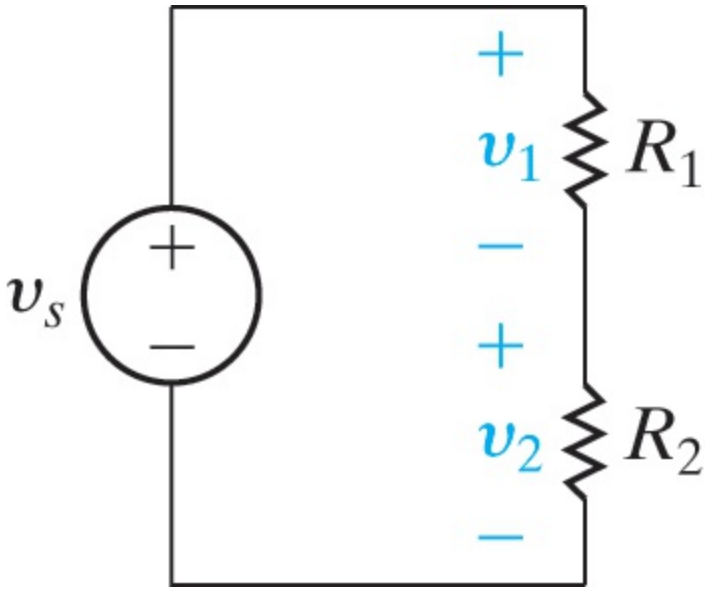
  $$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}.$$

  (See pages 61–62.)

- When voltage is divided between series-connected resistors, as shown in the figure, the voltage across each resistor can be found according to the equations

  $$v_1 = \frac{R_1}{R_1 + R_2} v_s,$$
  $$v_2 = \frac{R_2}{R_1 + R_2} v_s.$$

  (See page 64.)

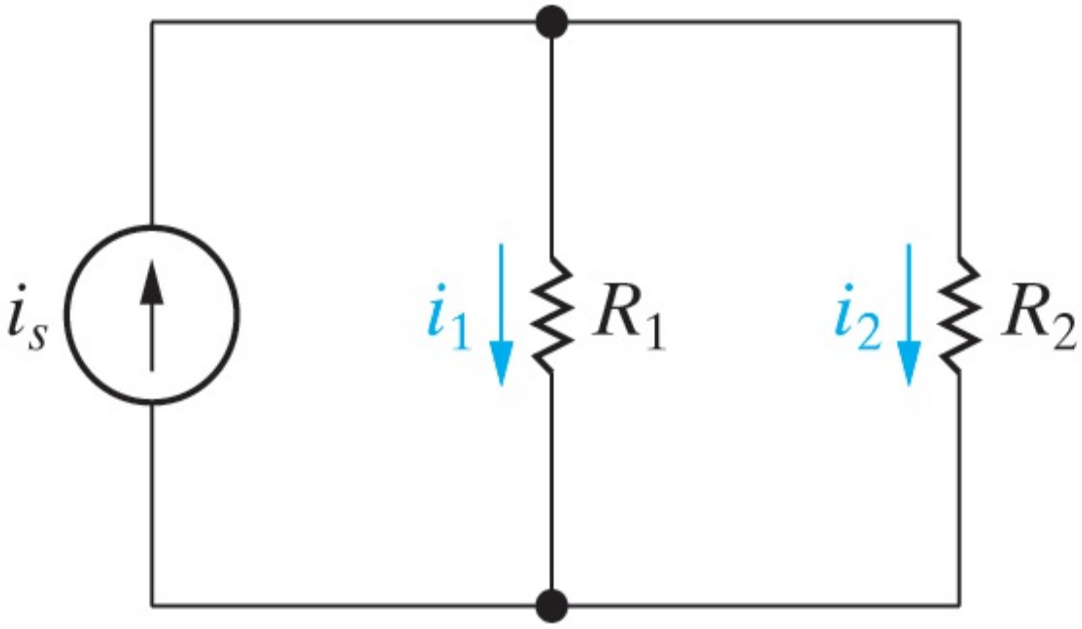


3.1-10 Full Alternative Text

- When current is divided between parallel-connected resistors, as shown in the figure, the current in each resistor can be found according to the equations

  i1=R2R1+R2is.
  i2=R1R1+R2is.

  (See page 67.)



3.1-11 Full Alternative Text

- **Voltage division** is a circuit analysis tool used to find the voltage drop across a single resistance from a collection of series-connected resistances when the voltage drop across the collection is known:

  vj=RjReqv,

  where vj is the voltage drop across the resistance Rj and v is the voltage drop across the series-connected resistances whose equivalent resistance is Req. (See page 68.)

- **Current division** is a circuit analysis tool used to find the current through a single resistance from a collection of parallel-connected resistances when the current into the collection is known:

  ij=ReqRji,

  where ij is the current through the resistance Rj and i is the current into the parallel-connected resistances whose equivalent resistance is Req. (See page 69.)

- A **voltmeter** measures voltage and must be placed in parallel with the

voltage being measured. An ideal voltmeter has infinite internal resistance and thus does not alter the voltage being measured. (See page 70.)

- An **ammeter** measures current and must be placed in series with the current being measured. An ideal ammeter has zero internal resistance and thus does not alter the current being measured. (See page 70.)

- **Digital meters** and **analog meters** have internal resistance, which influences the value of the circuit variable being measured. Meters based on the d'Arsonval meter movement deliberately include internal resistance as a way to limit the current in the movement's coil. (See pages 71–73.)

- The **Wheatstone bridge** circuit is used to make precise measurements of a resistor's value using four resistors, a dc voltage source, and a galvanometer. A Wheatstone bridge is balanced when the resistors obey Eq. 3.11, resulting in a galvanometer reading of 0 A. (See pages 73–74.)

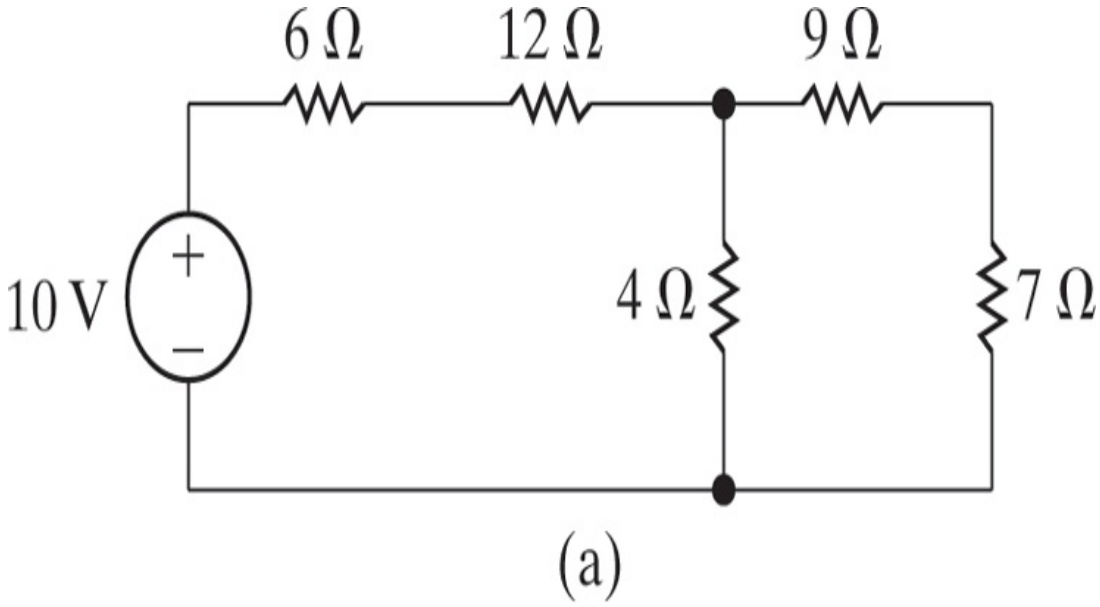
- A circuit with three resistors connected in a Δ configuration (or a π configuration) can be transformed into an equivalent circuit in which the three resistors are Y connected (or T connected). The Δ-to-Y transformation is given by Eqs. 3.15–3.17; the Y-to-Δ transformation is given by Eqs. 3.18–3.20. (See pages 75–77.)

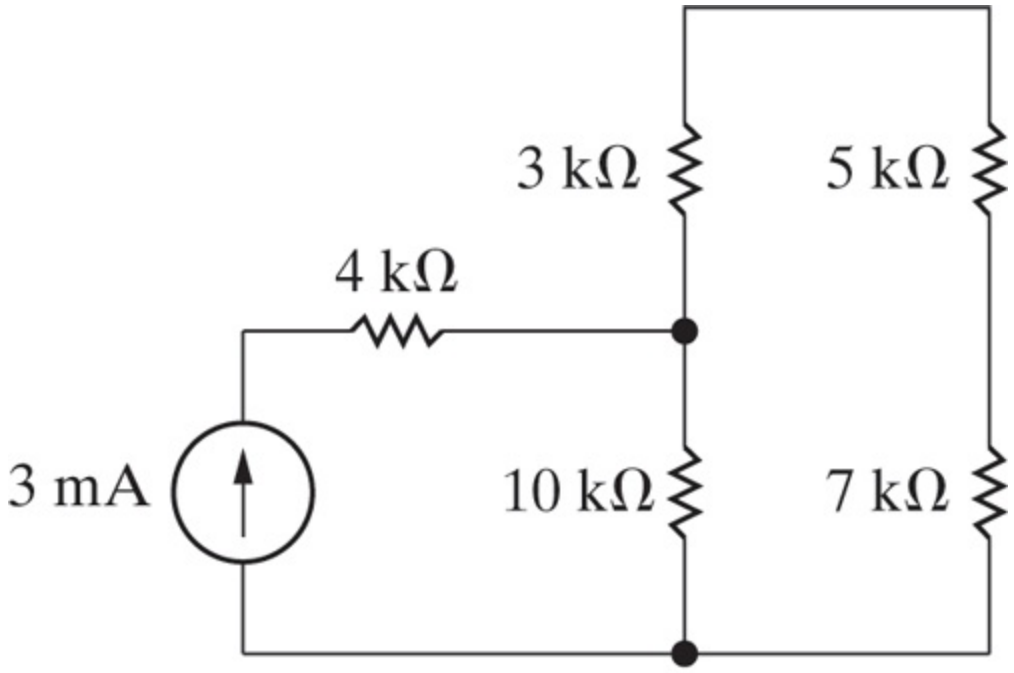
Problems

Sections 3.1–3.2

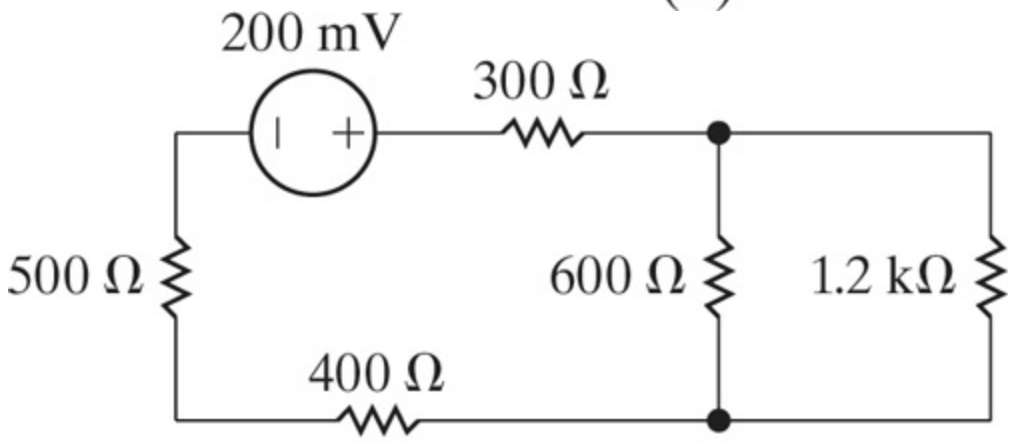
1. 3.1 For each of the circuits shown in Fig. P 3.1,

Figure P3.1

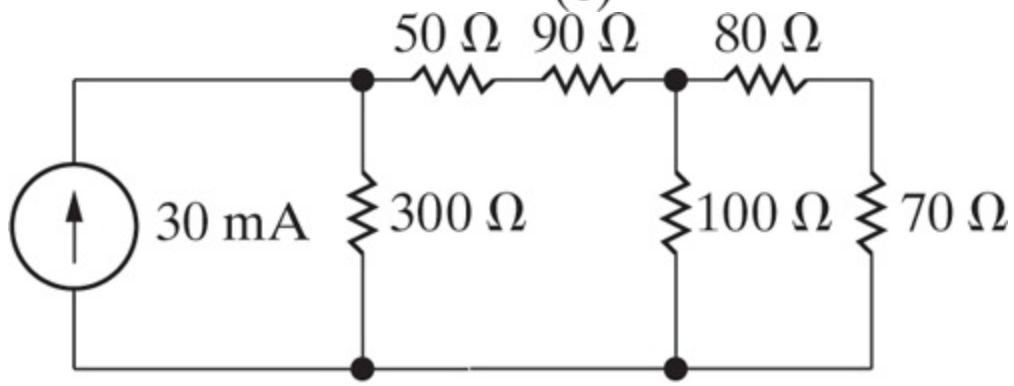



3 kΩ
5 kΩ
4 kΩ
3 mA
10 kΩ
7 kΩ

(b)


200 mV
300 Ω
500 Ω
600 Ω
1.2 kΩ
400 Ω

(c)


50 Ω
90 Ω
80 Ω
30 mA
300 Ω
100 Ω
70 Ω

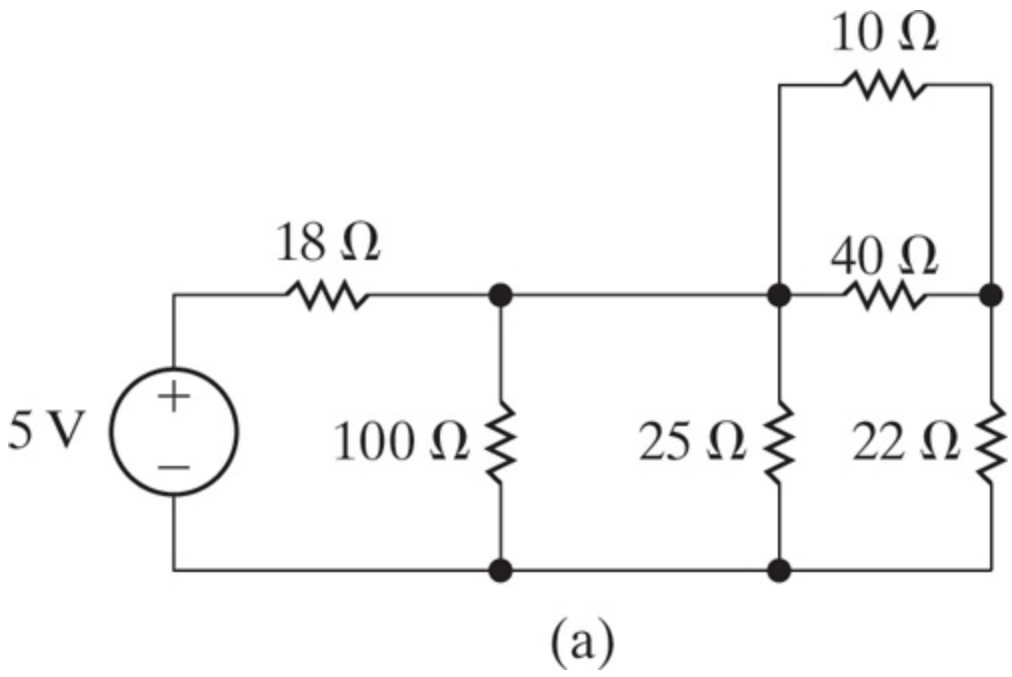
(d)

Figure P3.1 Full Alternative Text

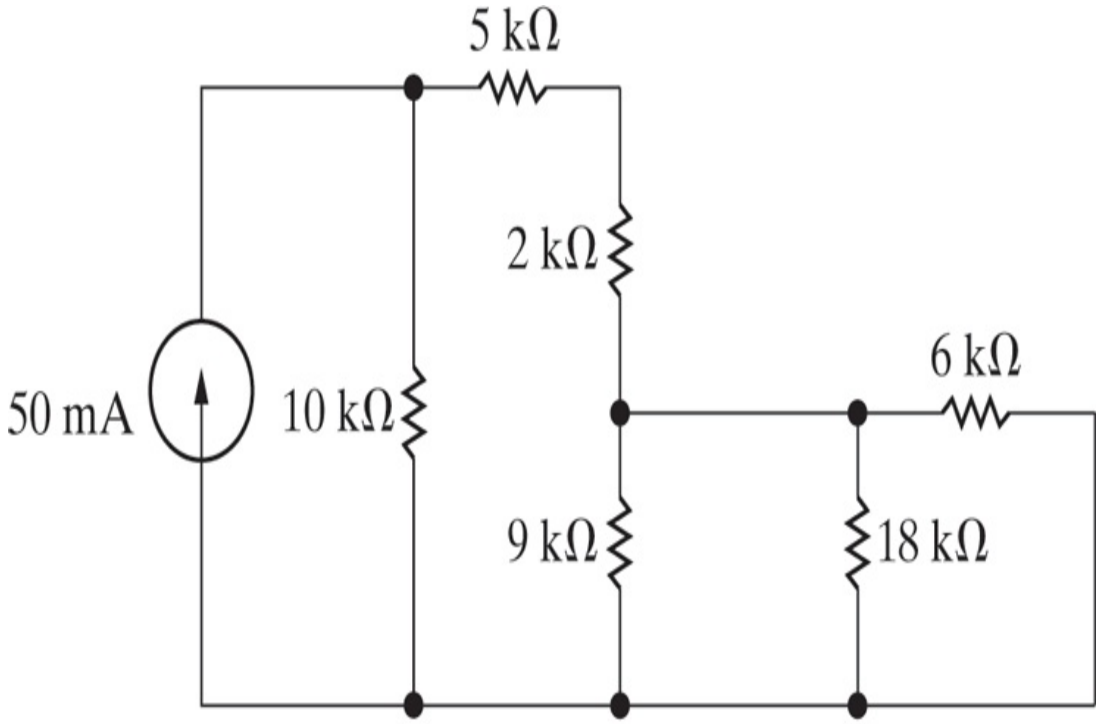
1. a) identify the resistors connected in series,
2. b) simplify the circuit by replacing the series-connected resistors with equivalent resistors.

2. 3.2 For each of the circuits shown in Fig. P 3.1,
    1. a) find the equivalent resistance seen by the source,
    2. b) find the power developed by the source.
3. 3.3 For each of the circuits shown in Fig. P 3.3,

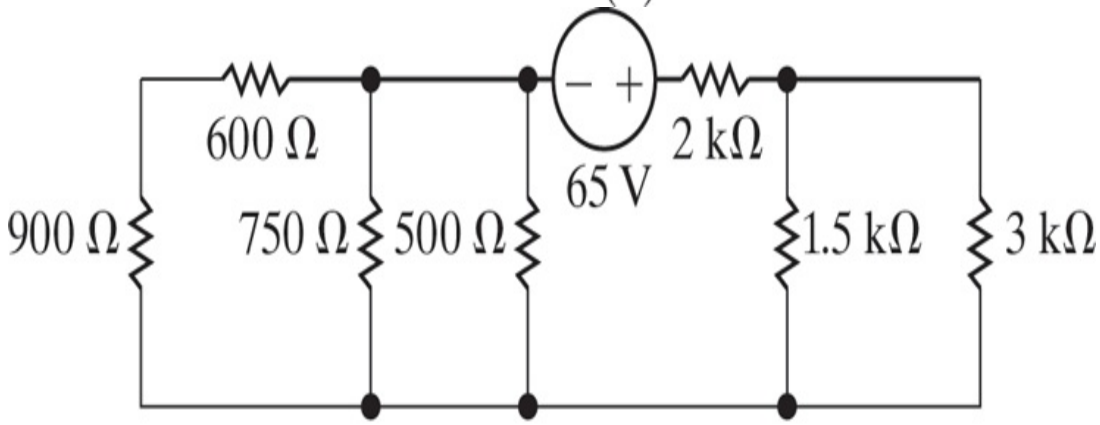
Figure P3.3



(a)


5 kΩ
2 kΩ
6 kΩ
50 mA
10 kΩ
9 kΩ
18 kΩ

(b)


600 Ω
65 V
2 kΩ
900 Ω
750 Ω
500 Ω
1.5 kΩ
3 kΩ

(c)

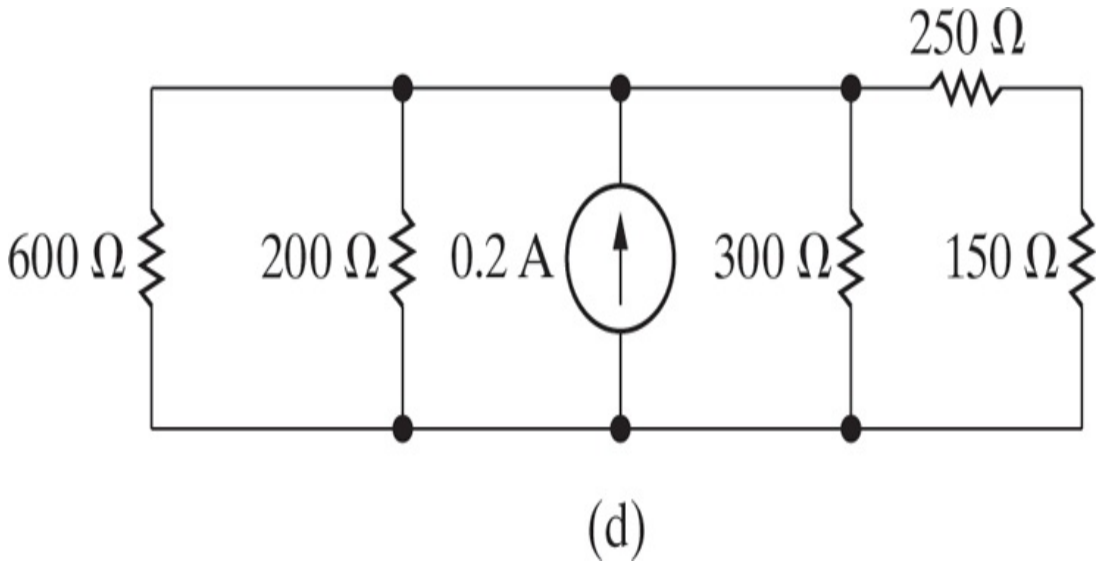
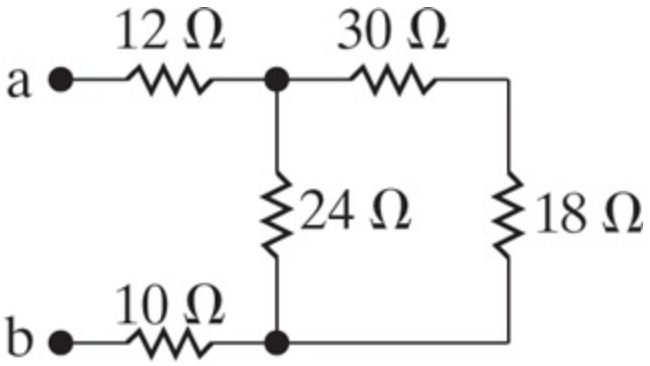


Figure P3.3 Full Alternative Text

1. a) identify the resistors connected in parallel,
2. b) simplify the circuit by replacing the parallel-connected resistors with equivalent resistors.

4. 3.4 For each of the circuits shown in Fig. P 3.3,
    1. a) find the equivalent resistance seen by the source,
    2. b) find the power developed by the source.
5. 3.5 PSPICEMULTISIM Find the equivalent resistance Rab for each of the circuits in Fig. P 3.5.

Figure P3.5



(a)

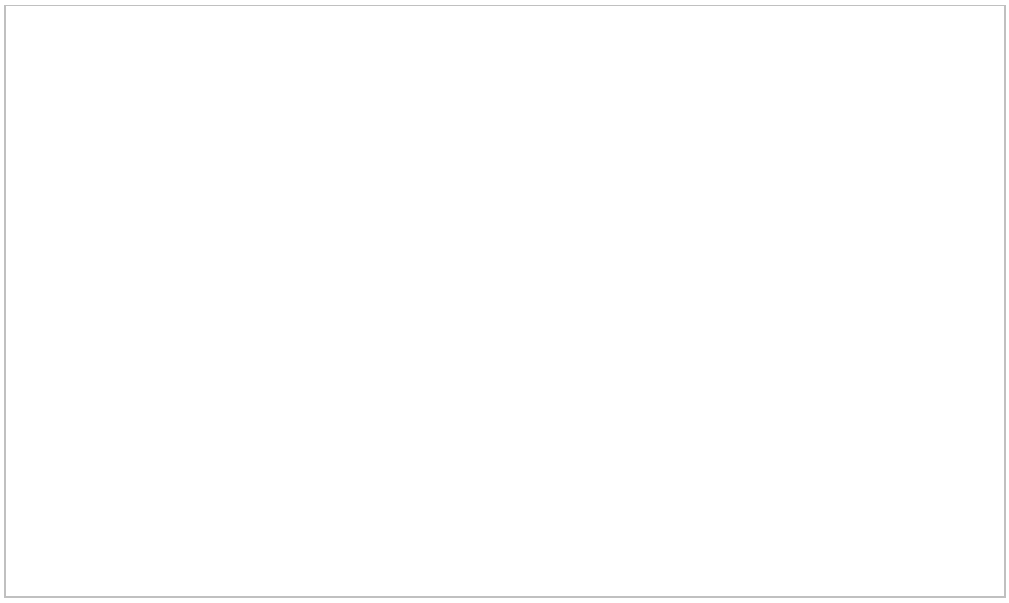
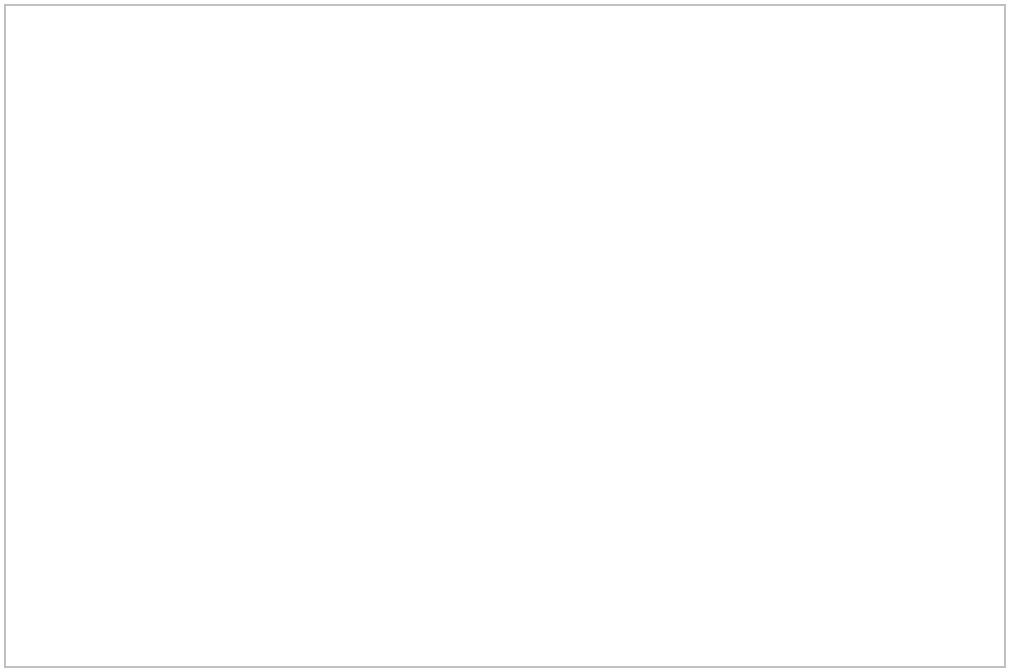
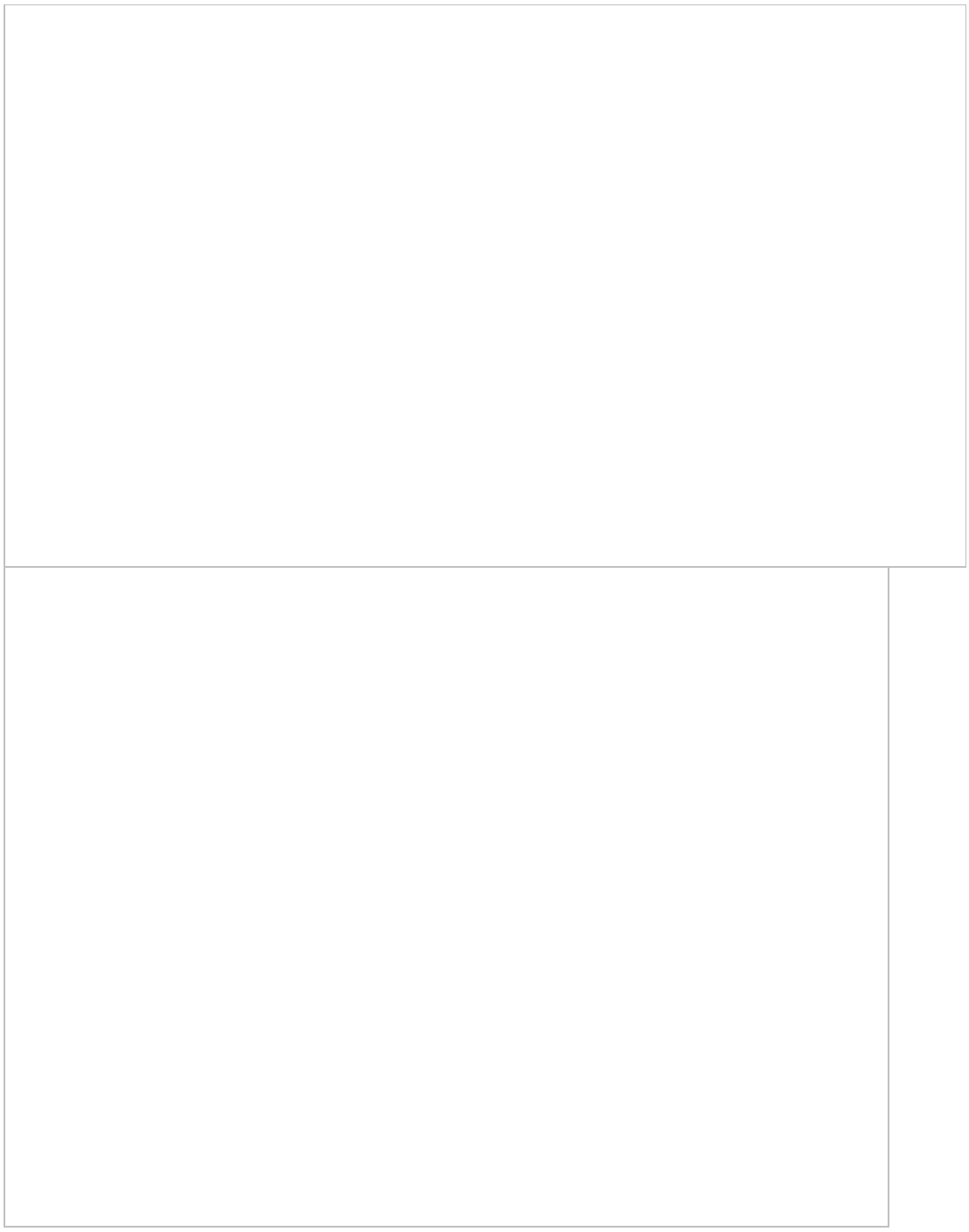


Figure P3.5 Full Alternative Text

6. 3.6 PSPICEMULTISIM Find the equivalent resistance Rab for each of the circuits in Fig. P 3.6.

Figure P3.6





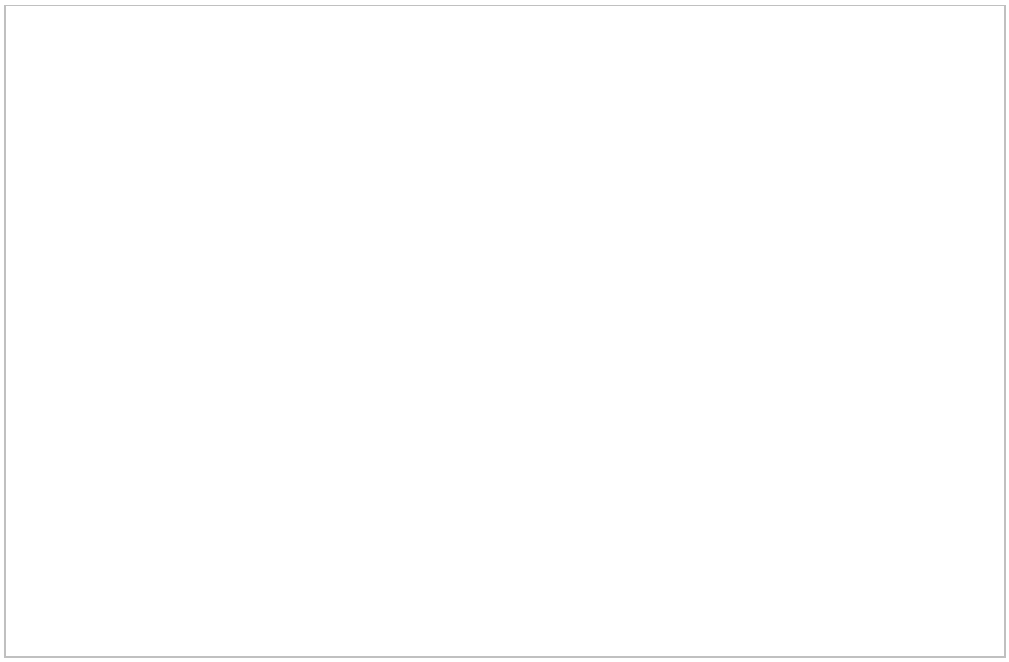
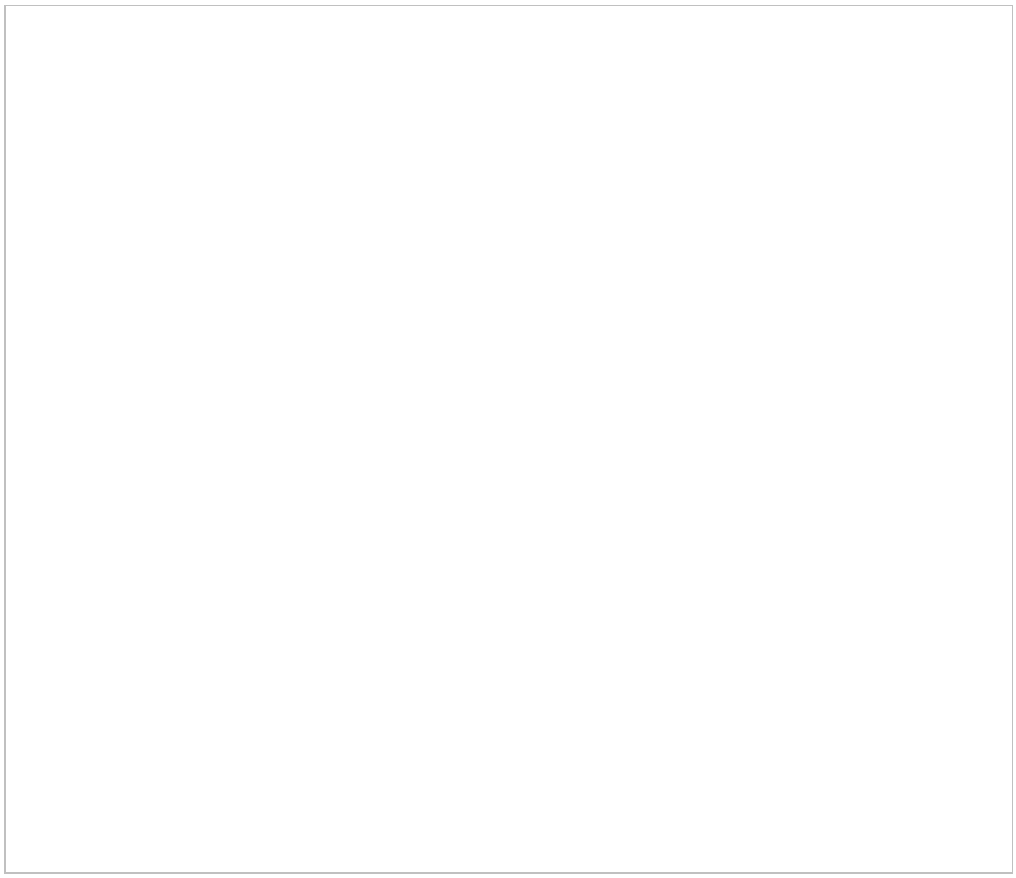


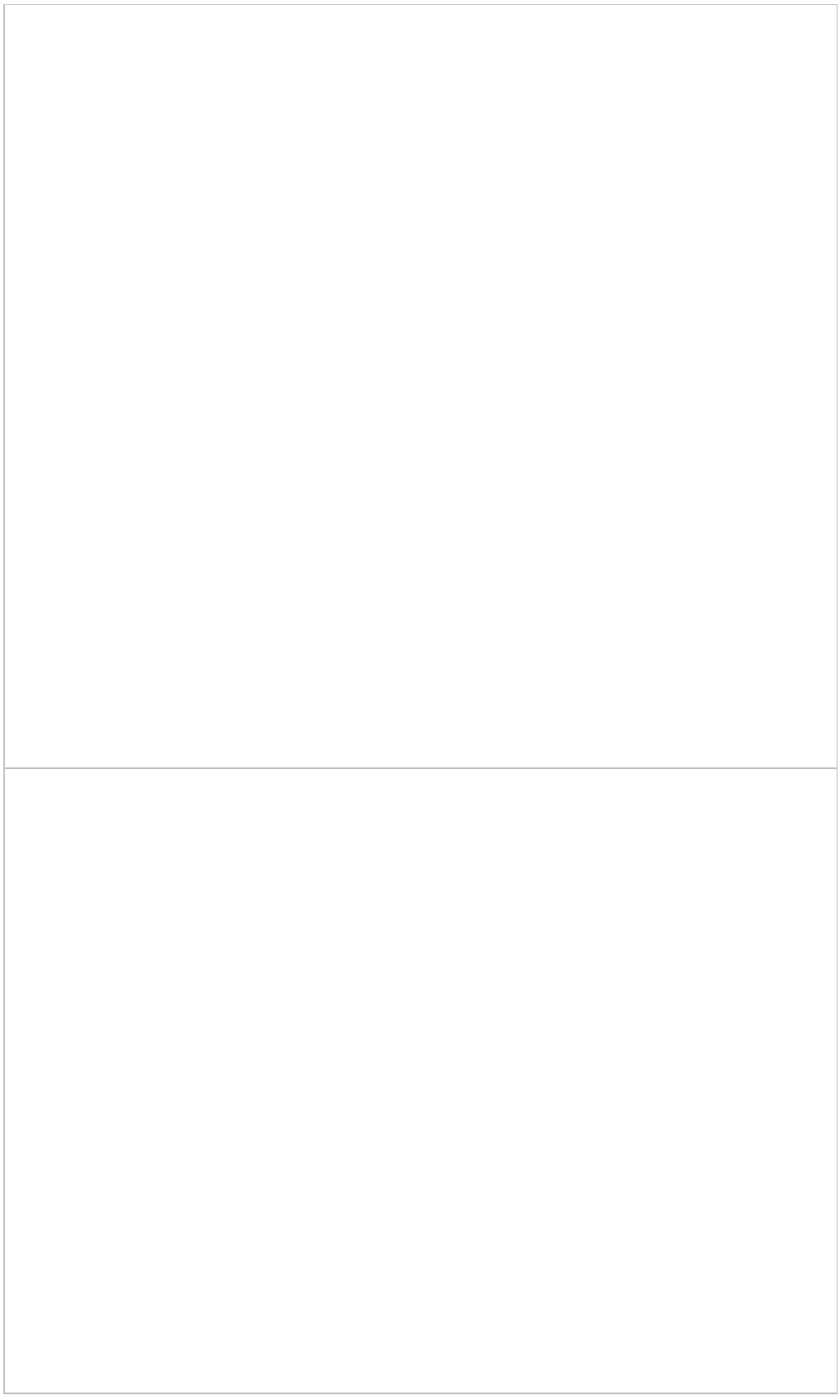
Figure P3.6 Full Alternative Text

7. 3.7 PSPICEMULTISIM

    1. a) In the circuits in Fig. P 3.7(a)–(d), find the equivalent resistance seen by the source.

Figure P3.7





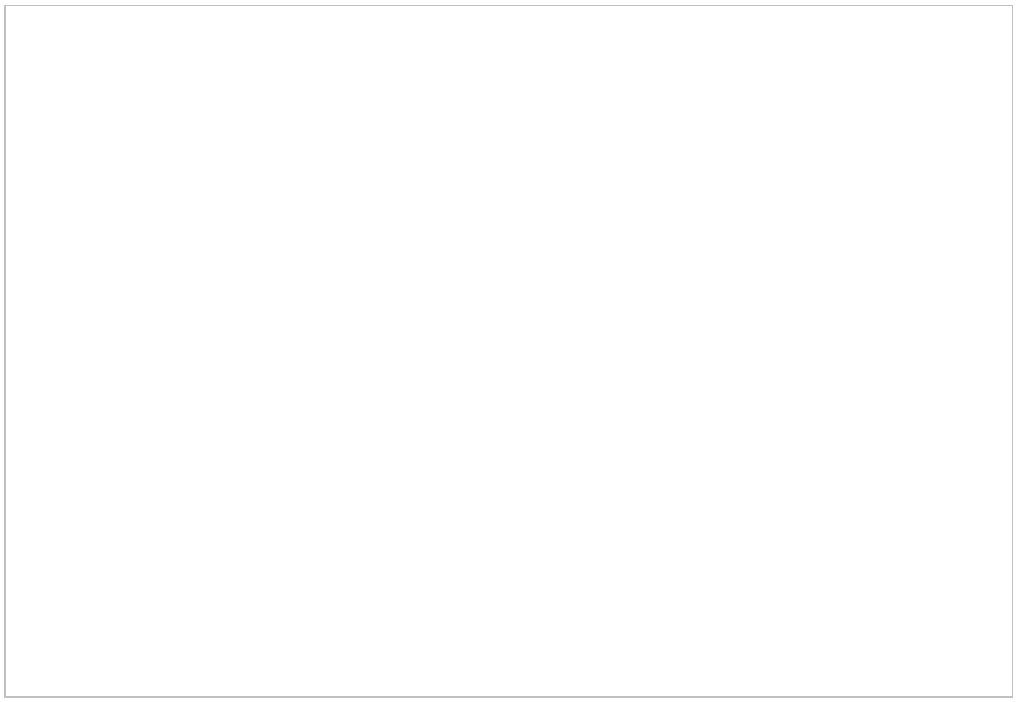


Figure P3.7 Full Alternative Text

2. b) For each circuit find the power delivered by the source.

8. 3.8 PSPICEMULTISIM

    1. a) Show that the solution of the circuit in Fig. 3.11 (see Example 3.1) satisfies Kirchhoff's current law at junctions x and y.

    2. b) Show that the solution of the circuit in Fig. 3.11 satisfies Kirchhoff's voltage law around every closed loop.

9. 3.9 PSPICEMULTISIM

    1. a) Find the power dissipated in each resistor in the circuit shown in Fig. 3.11.

    2. b) Find the power delivered by the 120 V source.

    3. c) Show that the power delivered equals the power dissipated.

10. 3.10

1. a) Find an expression for the equivalent resistance of two resistors of value R in series.

2. b) Find an expression for the equivalent resistance of n resistors of value R in series.

3. c) Using the results of (a), design a resistive network with an equivalent resistance of 3 kΩ using two resistors with the same value from Appendix H.

4. d) Using the results of (b), design a resistive network with an equivalent resistance of 4 kΩ using a minimum number of identical resistors from Appendix H.

11. 3.11

    1. a) Find an expression for the equivalent resistance of two resistors of value R in parallel.

    2. b) Find an expression for the equivalent resistance of n resistors of value R in parallel.

    3. c) Using the results of (a), design a resistive network with an equivalent resistance of 5 kΩ using two resistors with the same value from Appendix H.

    4. d) Using the results of (b), design a resistive network with an equivalent resistance of 4 kΩ using a minimum number of identical resistors from Appendix H.

Section 3.3

1. 3.12 PSPICEMULTISIM In the voltage-divider circuit shown in Fig. P 3.12, the no-load value of v0 is 4 V. When the load resistance RL is attached across the terminals a and b, vo drops to 3 V. Find RL.

Figure P3.12

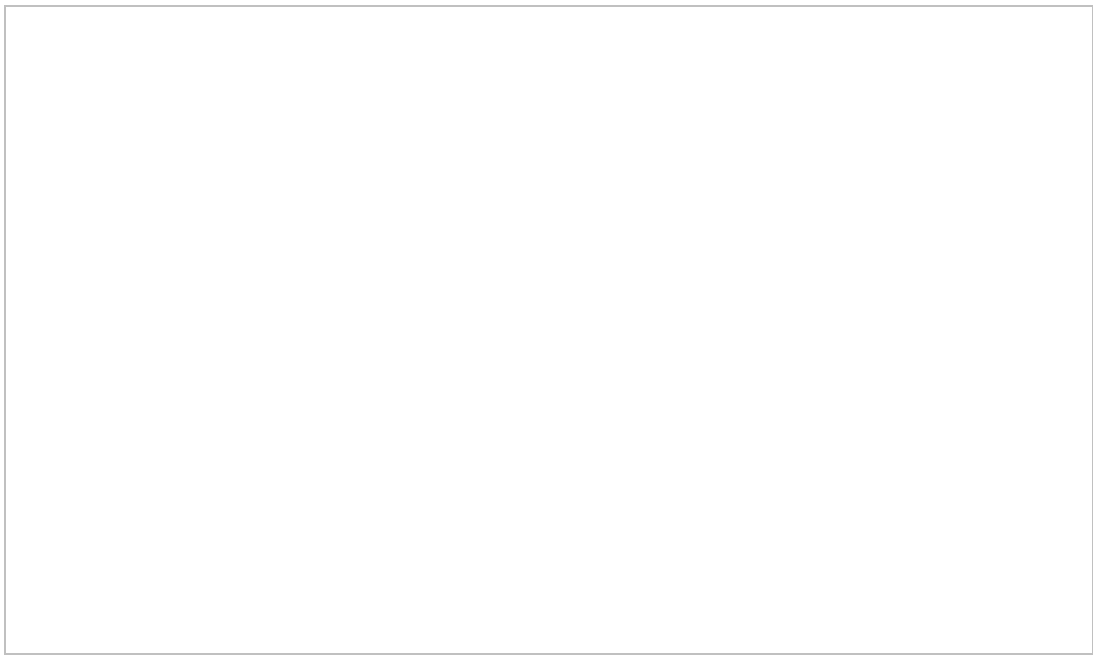


Figure P3.12 Full Alternative Text

2. 3.13 DESIGNPROBLEMPSPICEMULTISIM

    1. a) Calculate the no-load voltage v0 for the voltage-divider circuit shown in Fig. P 3.13.

Figure P3.13

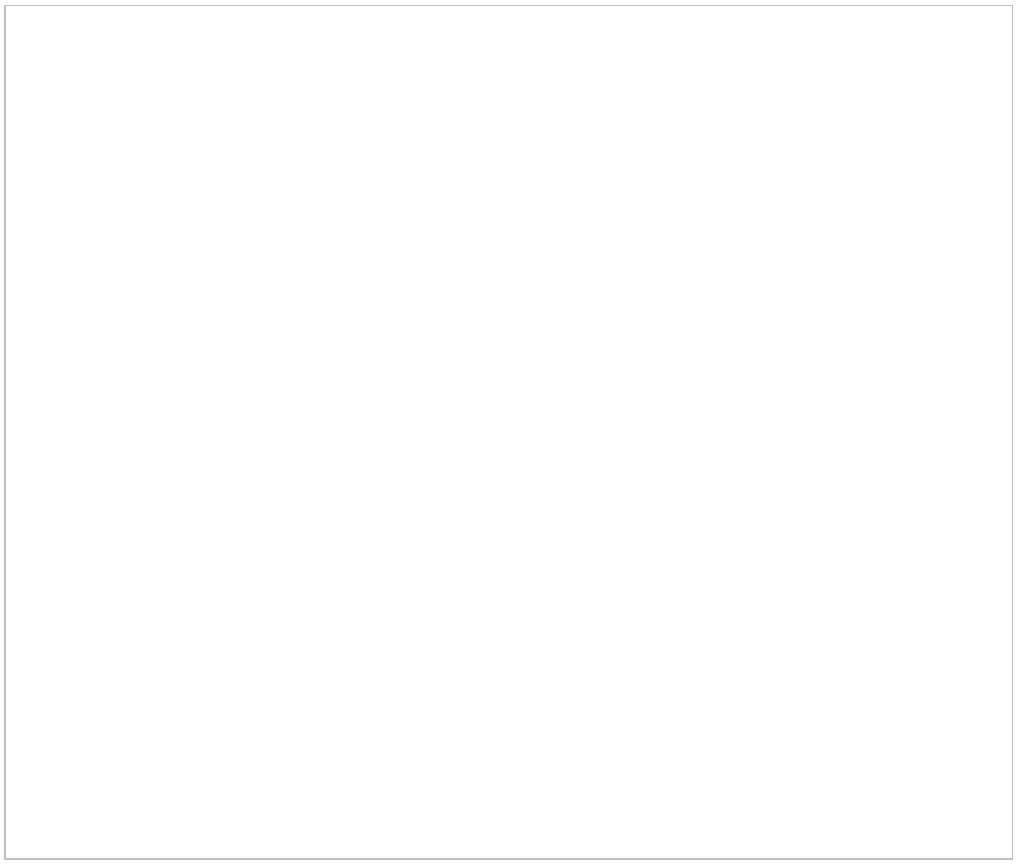


Figure P3.13 Full Alternative Text

2. b) Calculate the power dissipated in R1 and R2.

3. c) Assume that only 1 W resistors are available. The no-load voltage is to be the same as in (a). Specify the smallest ohmic values of R1 and R2.

3. 3.14 DESIGNPROBLEMPSPICEMULTISIM The no-load voltage in the voltage-divider circuit shown in Fig. P3.14 is 8 V. The smallest load resistor that is ever connected to the divider is 3.6 kΩ. When the divider is loaded, vo is not to drop below 7.5 V.

Figure P3.14

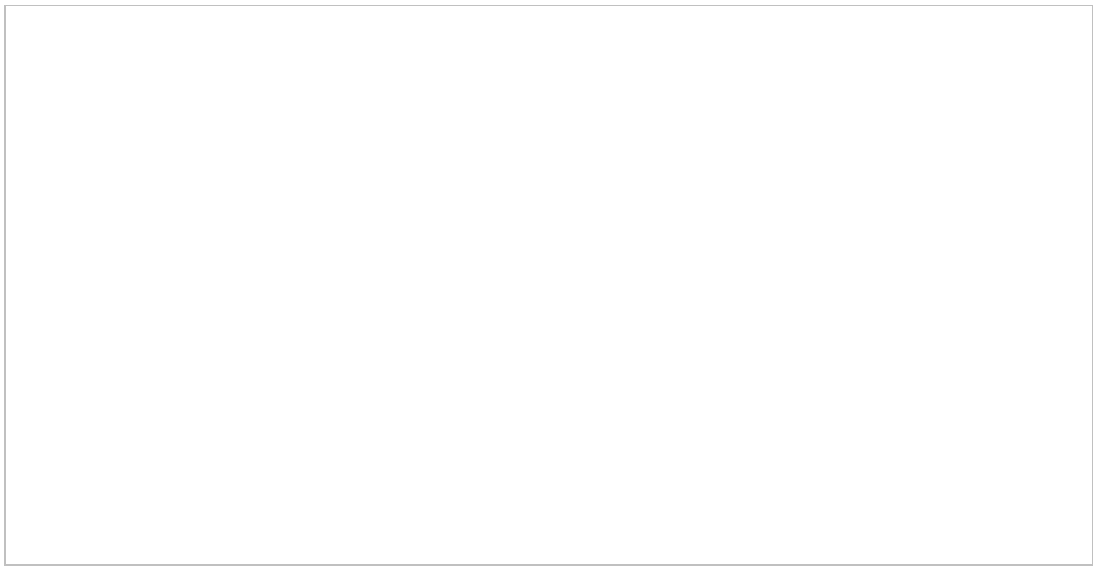


Figure P3.14 Full Alternative Text

1. a) Design the divider circuit to meet the specifications just mentioned. Specify the numerical values of R1 and R2.

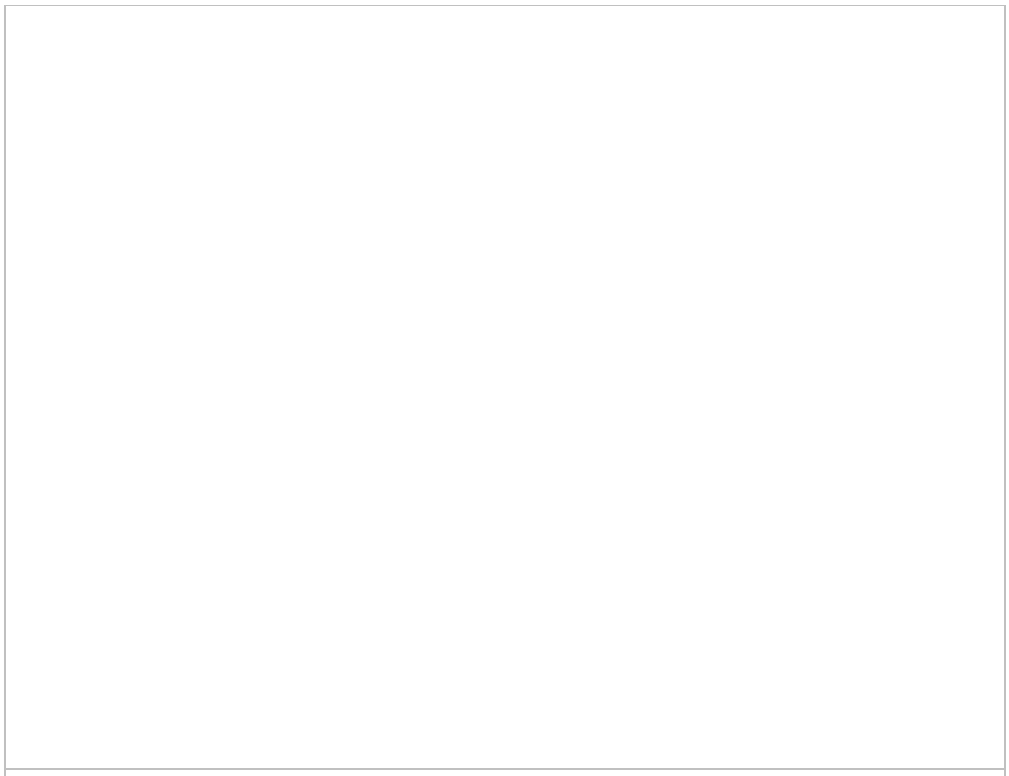
2. b) Assume the power ratings of commercially available resistors are 1/16, 1/8, 1/4, 1, and 2 W. What power rating would you specify?

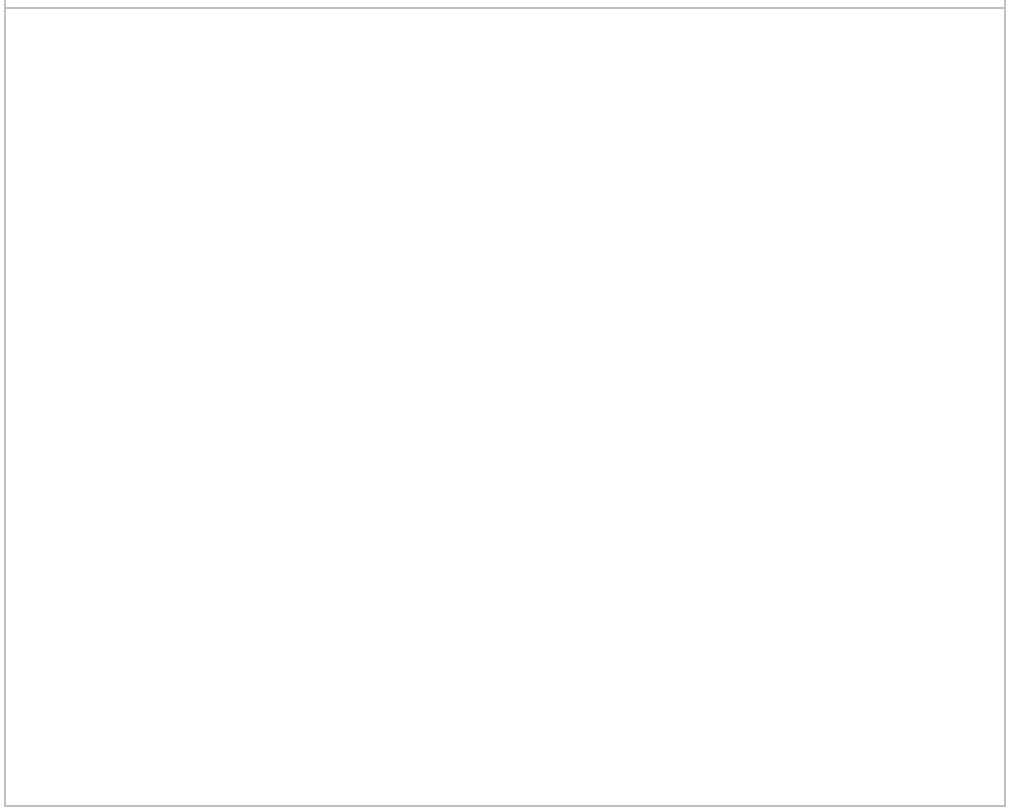
4. 3.15 Assume the voltage divider in Fig. P3.14 has been constructed from 1 W resistors. What is the smallest resistor from Appendix H that can be used as RL before one of the resistors in the divider is operating at its dissipation limit?

5. 3.16 PSPICEMULTISIM

    1. a) The voltage divider in Fig. P3.16(a) is loaded with the voltage divider shown in Fig. P3.16(b); that is, a is connected to a′, and b is connected to b′. Find vo.

Figure P3.16





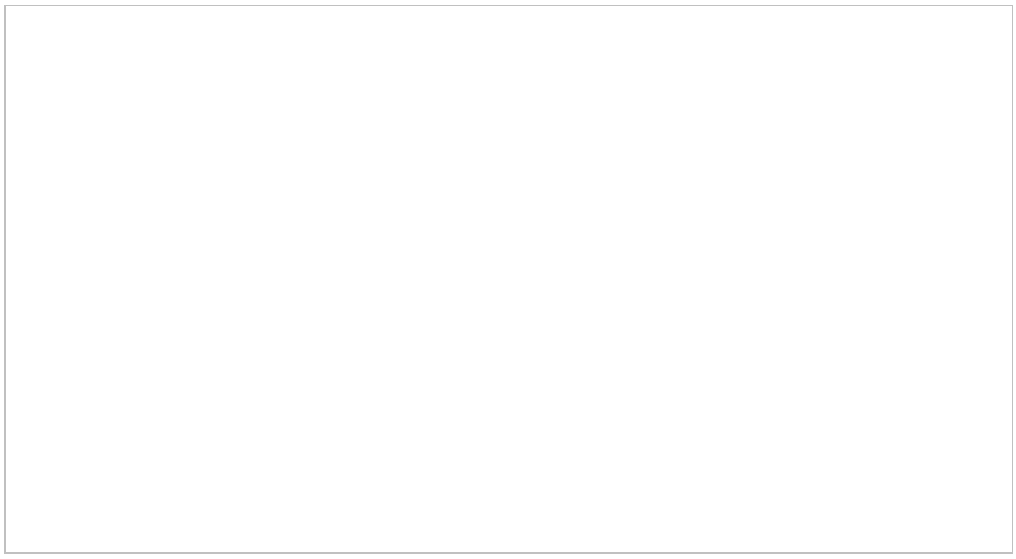


Figure P3.16 Full Alternative Text

2. b) Now assume the voltage divider in Fig. P3.16(b) is connected to the voltage divider in Fig. P3.16(a) by means of a current-controlled voltage source as shown in Fig. P3.16(c). Find vo.

3. c) What effect does adding the dependent-voltage source have on the operation of the voltage divider that is connected to the 240 V source?

6. 3.17 DESIGNPROBLEM_ There is often a need to produce more than one voltage using a voltage divider. For example, the memory components of many personal computers require voltages of −12 V, 5 V, and +12 V, all with respect to a common reference terminal. Select the values of R1, R2, and R3 in the circuit in Fig. P3.17 to meet the following design requirements:

Figure P3.17

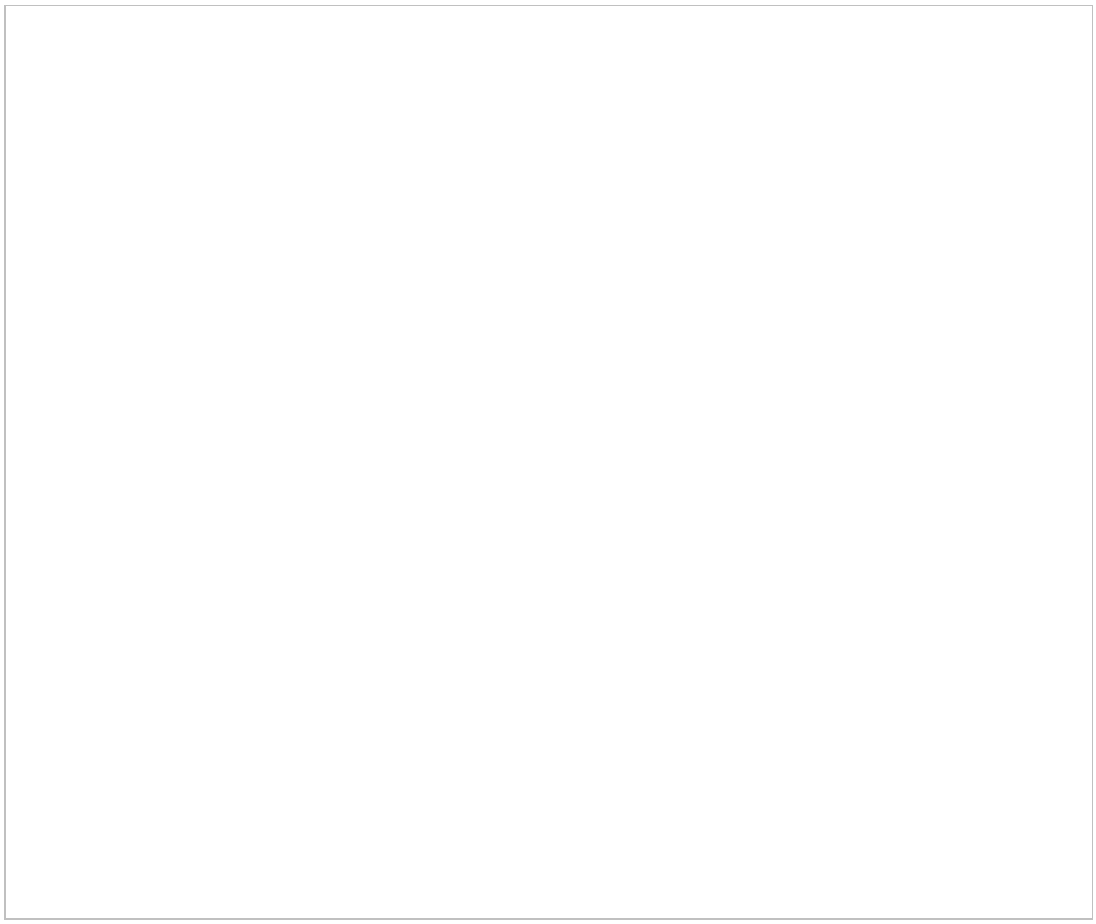


Figure P3.17 Full Alternative Text

1. a) The total power supplied to the divider circuit by the 24 V source is 80 W when the divider is unloaded.

2. b) The three voltages, all measured with respect to the common reference terminal, are v1=12 V, v2=5 V, and v3=−12 V.

7. 3.18 DESIGNPROBLEM_ A voltage divider like that in Fig. 3.17 is to be designed so that vo=kvs at no load (RL=∞) and vo=αvs at full load (RL=Ro). Note that by definition α<k<1.

    1. a) Show that

        R1=k−ααkRo

        and

$R2=k-\alpha\alpha(1-k)Ro$.

2. b) Specify the numerical values of R1 and R2 if k = 0.85, α=0.80, and Ro=34 kΩ.

3. c) If vs=60 V, specify the maximum power that will be dissipated in R1 and R2.

4. d) Assume the load resistor is accidentally short circuited. How much power is dissipated in R1 and R2?

8. 3.19 PSPICEMULTISIM For the current-divider circuit in Fig. P3.19 calculate

Figure P3.19

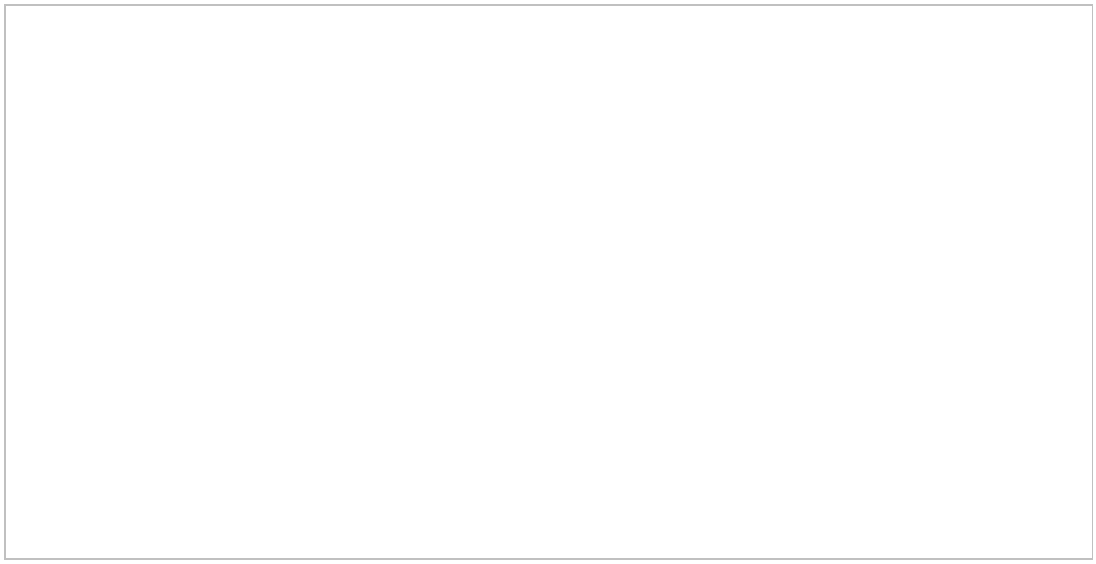


Figure P3.19 Full Alternative Text

1. a) io and vo.

2. b) the power dissipated in the 6 Ω resistor.

3. c) the power developed by the current source.

9. 3.20 PSPICEMULTISIM Find the power dissipated in the 30 Ω resistor in the current-divider circuit in Fig. P3.20.

Figure P3.20

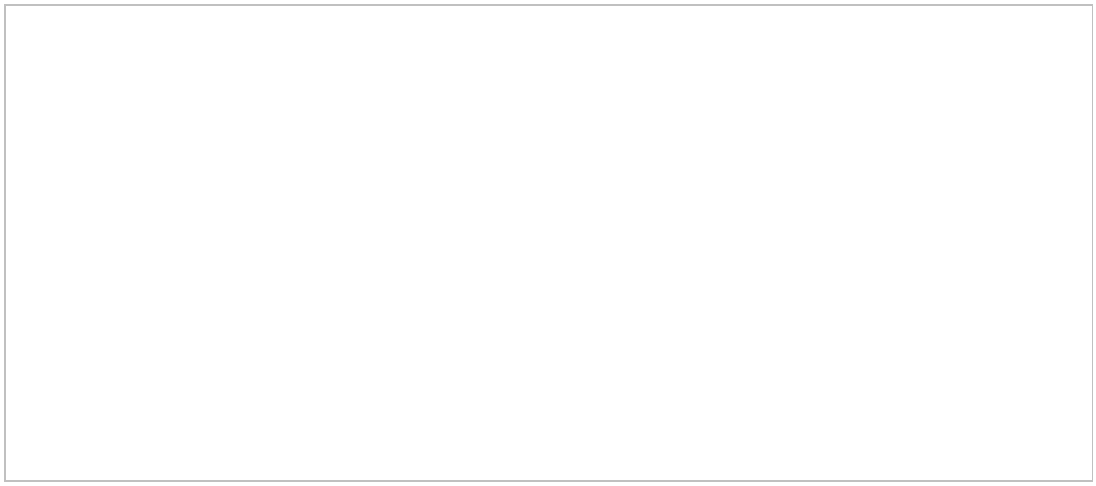


Figure P3.20 Full Alternative Text

10. 3.21 DESIGNPROBLEM_ Specify the resistors in the current-divider circuit in Fig. P3.21 to meet the following design criteria:

Figure P3.21

Figure P3.21 Full Alternative Text

ig=50 mA; vg=25 V; i1=0.6i2;
i3=2i2; and i4=4i1.

11. 3.22 PSPICEMULTISIM

    1. a) Show that the current in the kth branch of the circuit in Fig. P3.22(a) is equal to the source current ig times the conductance of the kth branch divided by the sum of the conductances, that is,

Figure P3.22

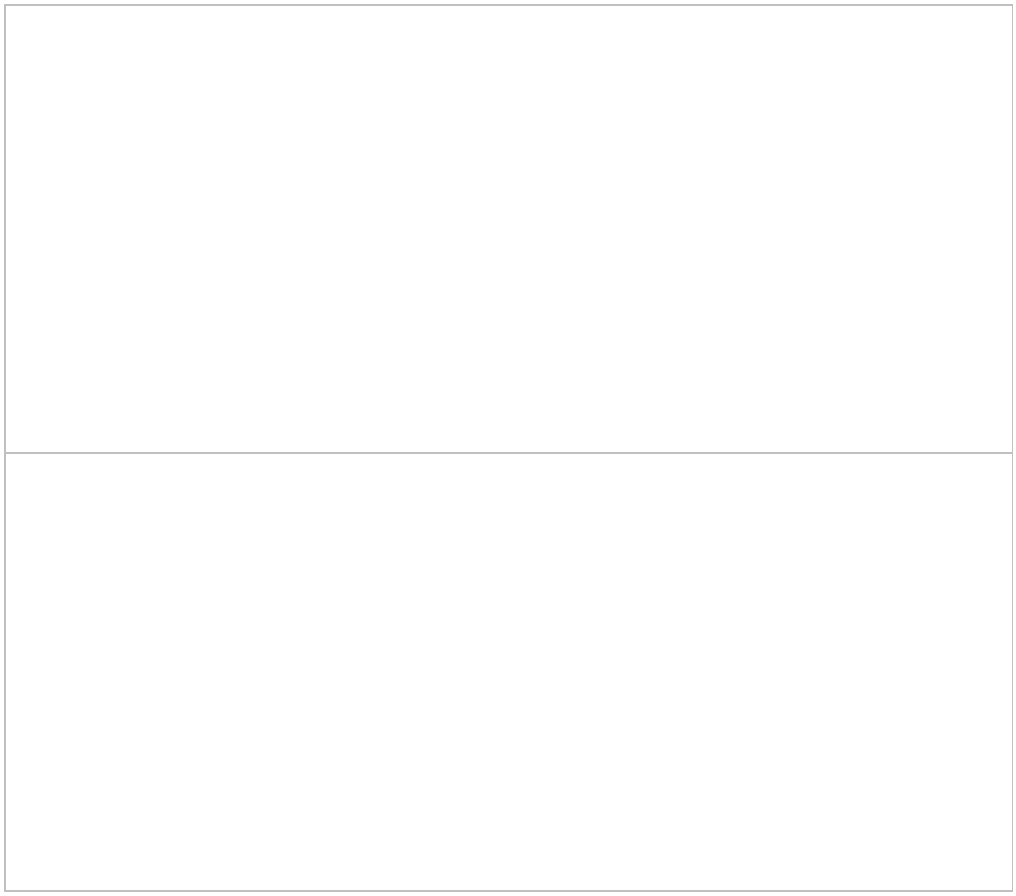


Figure P3.22 Full Alternative Text

ik=igGkG1+G2+G3+···+Gk+···+GN.

    2. b) Use the result derived in (a) to calculate the current in the 5 Ω resistor in the circuit in Fig. P3.22(b).

Section 3.4

1. 3.23 Look at the circuit in Fig. P3.1(a).
    1. a) Use voltage division to find the voltage across the 4 Ω resistor, positive at the top.
    2. b) Use the result from part (a) and voltage division to find the voltage across the 9 Ω resistor, positive on the left.
2. 3.24 Look at the circuit in Fig. P3.1(d).
    1. a) Use current division to find the current in the 50 Ω resistor from left to right.
    2. b) Use the result from part (a) and current division to find the current in the 70 Ω resistor from top to bottom.
3. 3.25 Attach a 6 V voltage source between the terminals a–b in Fig. P3.6(b), with the positive terminal at the top.
    1. a) Use voltage division to find the voltage across the 4 Ω resistor, positive at the top.
    2. b) Use the result from part (a) to find the current in the 4 Ω resistor from left to right.
    3. c) Use the result from part (b) and current division to find the current in the 16 Ω resistor from left to right.
    4. d) Use the result from part (c) and current division to find the current in the 10 Ω resistor from top to bottom.
    5. e) Use the result from part (d) to find the voltage across the 10 Ω resistor, positive at the top.

6. f) Use the result from part (e) and voltage division to find the voltage across the 18 Ω resistor, positive at the top.

4. 3.26 Look at the circuit in Fig. P3.7(a).

    1. a) Use current division to find the current in the 360 Ω resistor from top to bottom.

    2. b) Using your result from (a), find the voltage drop across the 360 Ω resistor, positive at the top.

    3. c) Starting with your result from (b), use voltage division to find the voltage across the 90 Ω resistor, positive on the left.

    4. d) Using your result from (c), find the current in the 90 Ω resistor from left to right.

    5. e) Starting with your result from (d), use current division to find the current in the 160 Ω resistor from top to bottom.

5. 3.27 Attach a 450 mA current source between the terminals a–b in Fig. P3.6(a), with the current arrow pointing up.

    1. a) Use current division to find the current in the 36 Ω resistor from top to bottom.

    2. b) Use the result from part (a) to find the voltage across the 36 Ω resistor, positive at the top.

    3. c) Use the result from part (b) and voltage division to find the voltage across the 18 Ω resistor, positive at the top.

    4. d) Use the result from part (c) and voltage division to find the voltage across the 10 Ω resistor, positive at the top.

6. 3.28 For the circuit in Fig. P3.28, find ig and then use current division to find io.

Figure P3.28

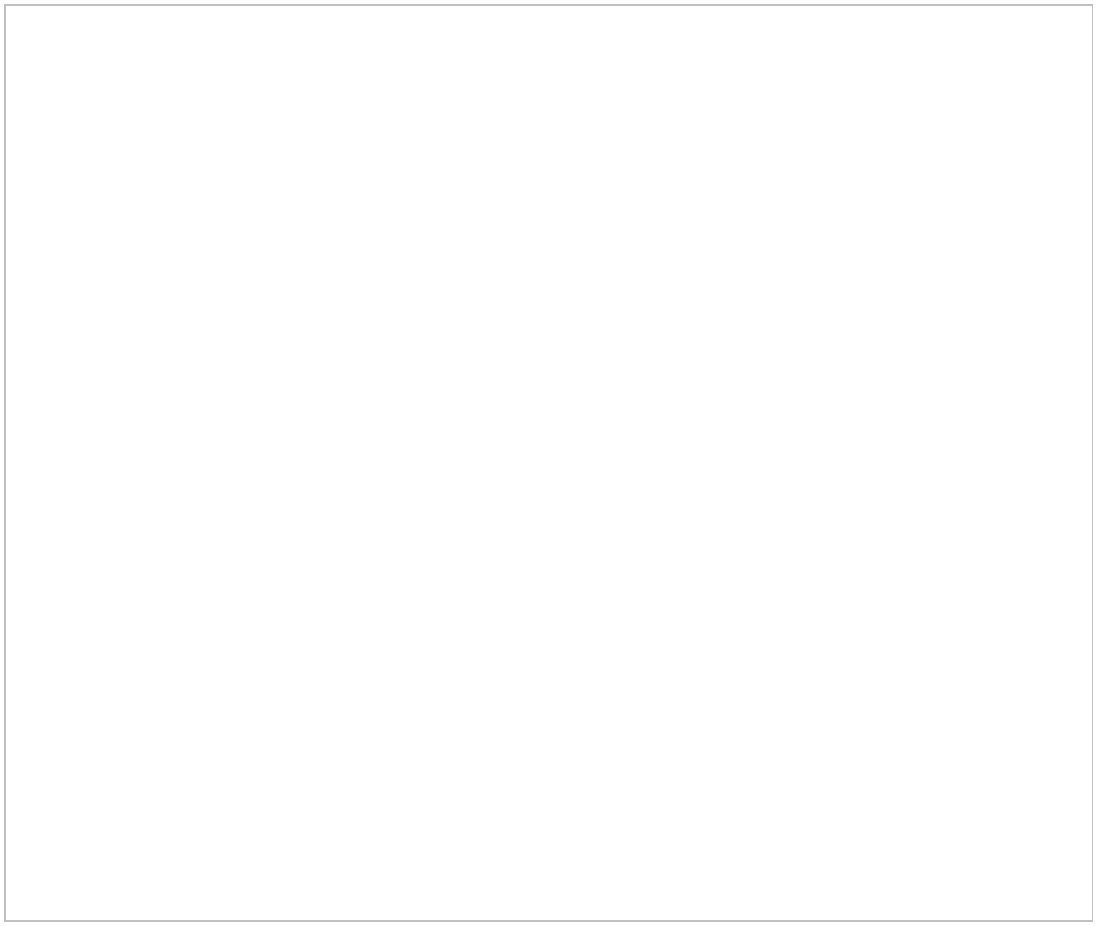


Figure P3.28 Full Alternative Text

7. 3.29 PSPICEMULTISIM For the circuit in Fig. P3.29, calculate i1 and i2 using current division.

Figure P3.29

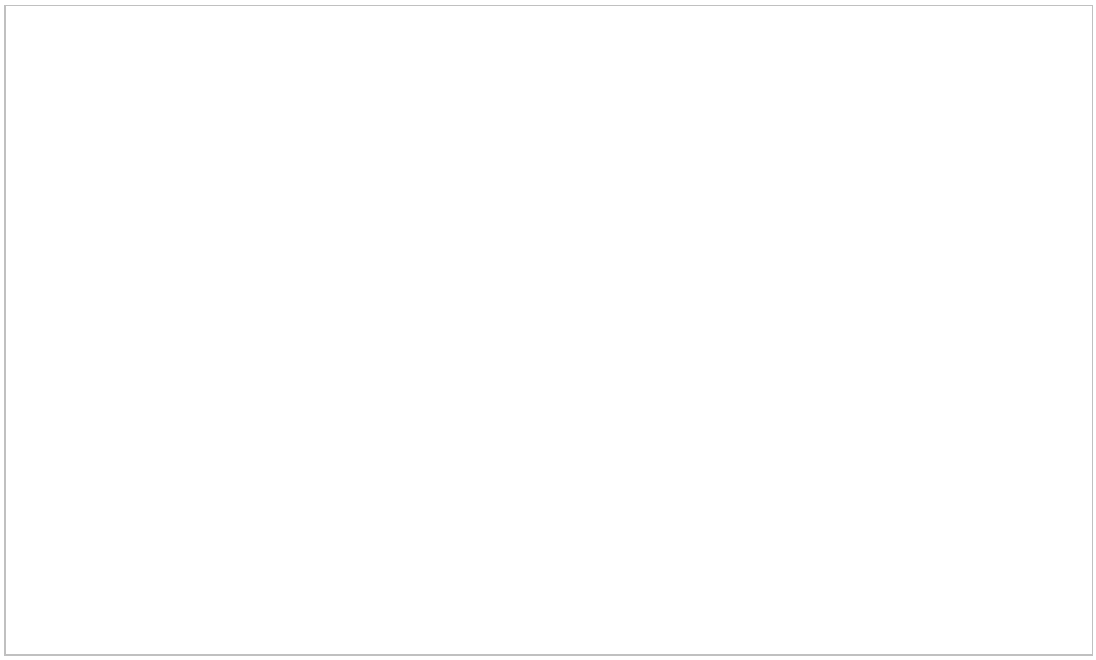


Figure P3.29 Full Alternative Text

8. 3.30 PSPICEMULTISIM Find v1 and v2 in the circuit in Fig. P3.30 using voltage and/or current division.

Figure P3.30

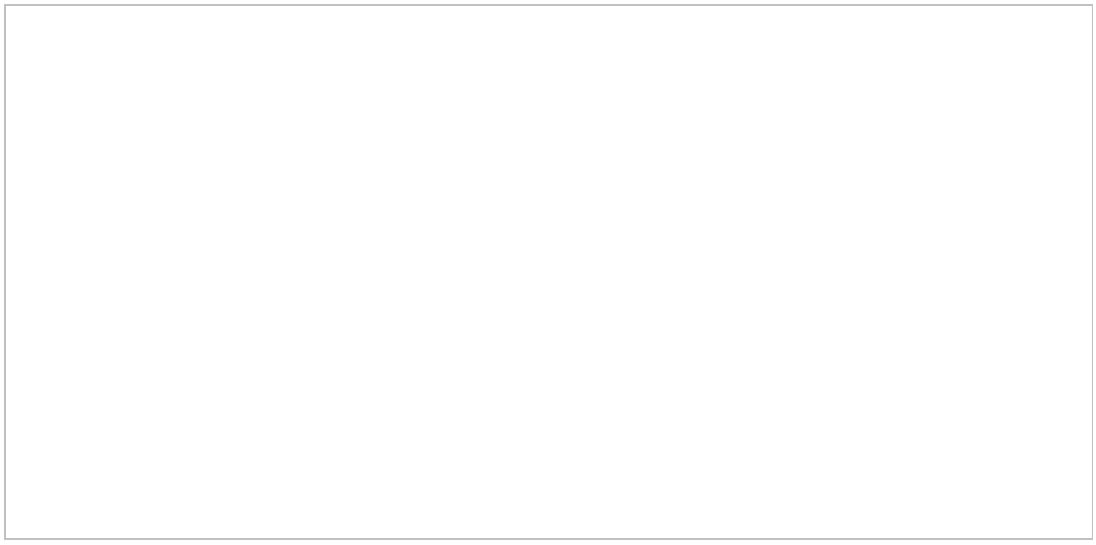


Figure P3.30 Full Alternative Text

9. 3.31 PSPICEMULTISIM Find vo in the circuit in Fig. P3.31 using voltage and/or current division.

Figure P3.31

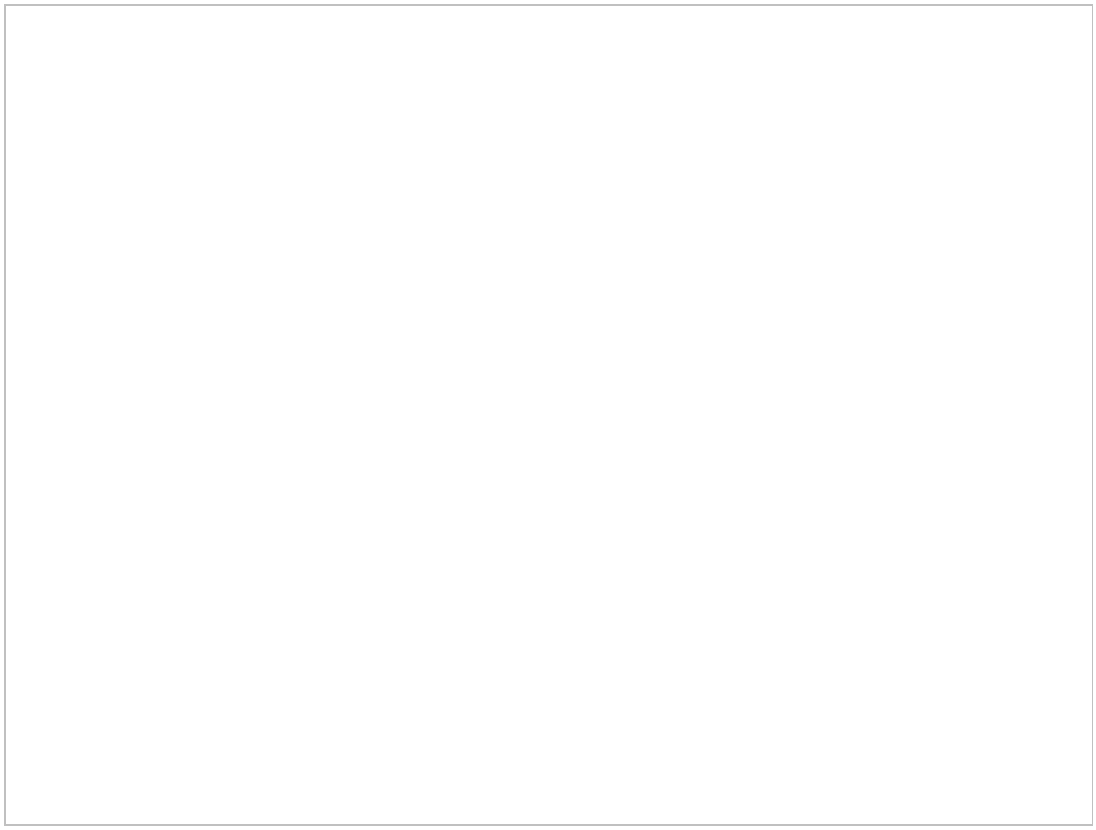


Figure P3.31 Full Alternative Text

10. 3.32 PSPICEMULTISIM

    1. a) Find the voltage vx in the circuit in Fig. P3.32 using voltage and/or current division.

Figure P3.32

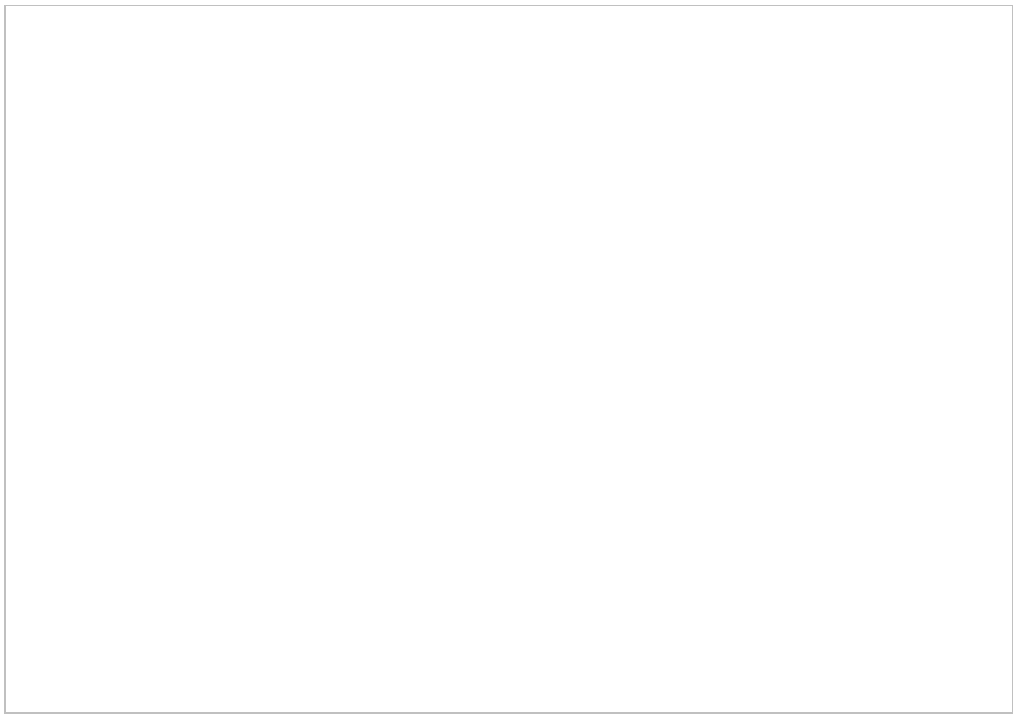


Figure P3.32 Full Alternative Text

2. b) Replace the 45 V source with a general voltage source equal to Vs. Assume Vs is positive at the upper terminal. Find vx as a function of Vs.

Section 3.5

1. 3.33 A shunt resistor and a 50 mV, 1 mA d'Arsonval movement are used to build a 5 A ammeter. A resistance of 20 mΩ is placed across the terminals of the ammeter. What is the new full-scale range of the ammeter?

2. 3.34

    1. a) Show for the ammeter circuit in Fig. P3.34 that the current in the d'Arsonval movement is always 1/100th of the current being measured.

Figure P3.34

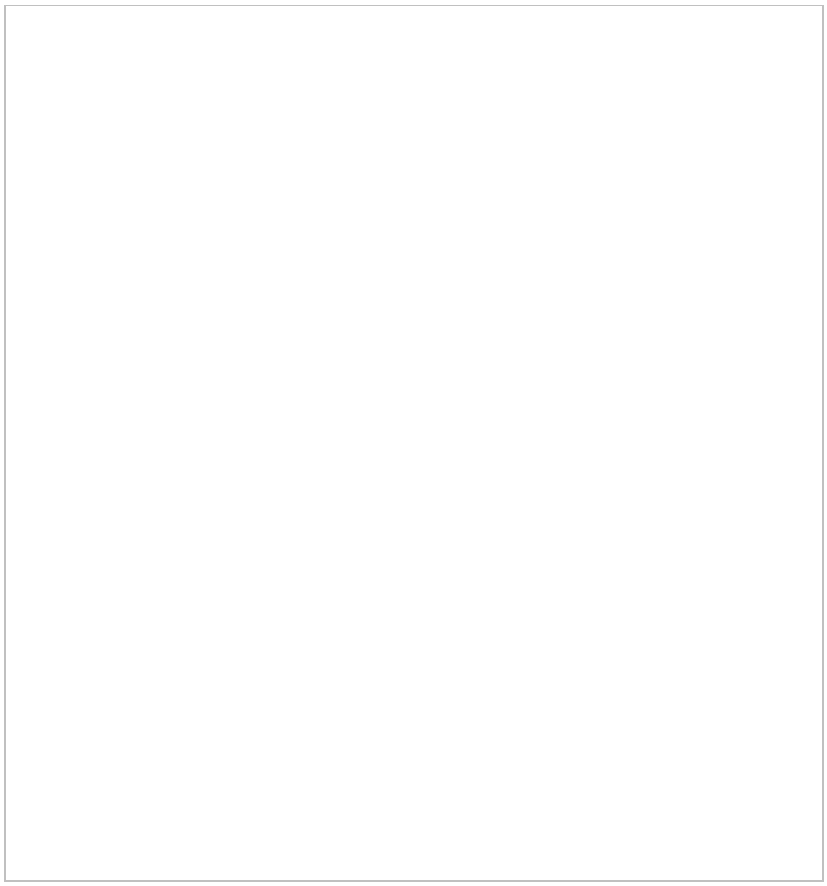


Figure P3.34 Full Alternative Text

2. b) What would the fraction be if the 100 μ V, 2 μA movement were used in a 1 A ammeter?

3. c) Would you expect a uniform scale on a dc d'Arsonval ammeter?

3. 3.35 A d'Arsonval ammeter is shown in Fig. P3.35.

Figure P3.35

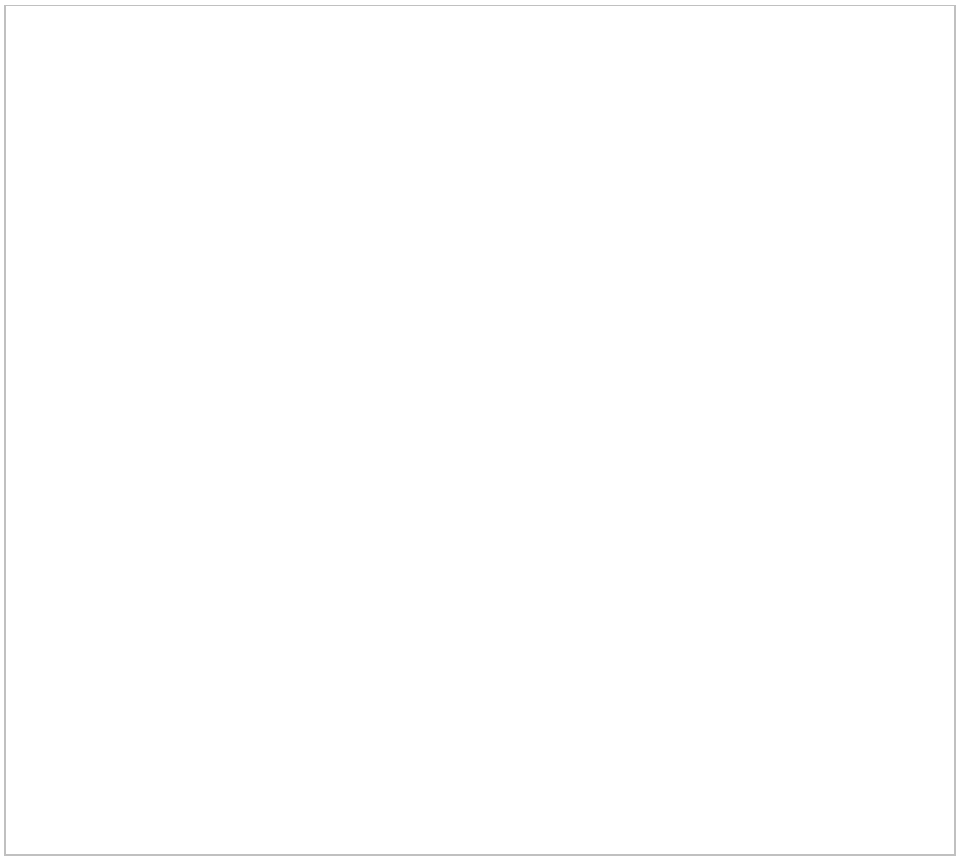


Figure P3.35 Full Alternative Text

1. a) Calculate the value of the shunt resistor, RA, to give a full-scale current reading of 5 A.

2. b) How much resistance is added to a circuit when the 5 A ammeter in part (a) is inserted to measure current?

3. c) Calculate the value of the shunt resistor, RA, to give a full-scale current reading of 100 mA.

4. d) How much resistance is added to a circuit when the 100 mA ammeter in part (c) is inserted to measure current?

4. 3.36 DESIGNPROBLEM_ A d'Arsonval movement is rated at 2 mA and 100 mV. Assume 0.25 W precision resistors are available to use as shunts. What is the largest full-scale-reading ammeter that can be designed using a single resistor? Explain.

5. 3.37 A d'Arsonval voltmeter is shown in Fig. P3.37. Find the value of Rv for each of the following full-scale readings: (a) 50 V, (b) 5 V, (c) 250 mV, and (d) 25 mV.

Figure P3.37

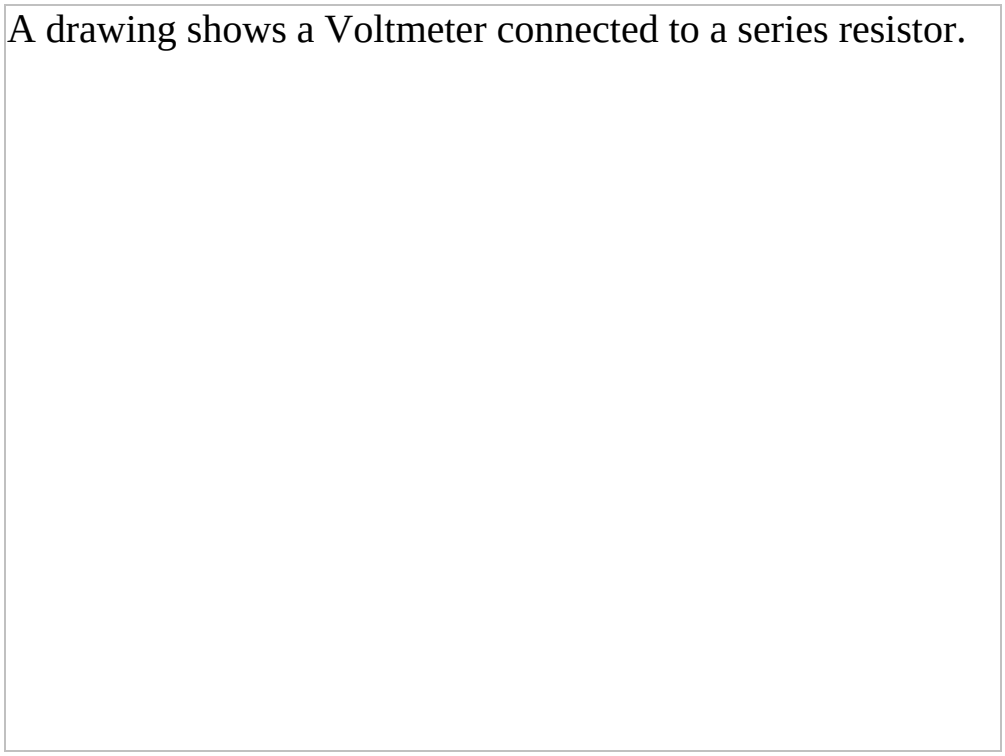


Figure P3.37 Full Alternative Text

6. 3.38 Suppose the d'Arsonval voltmeter described in Problem 3.37(b) is used to measure the voltage across the 45 Ω resistor in Fig. P3.38.

Figure P3.38

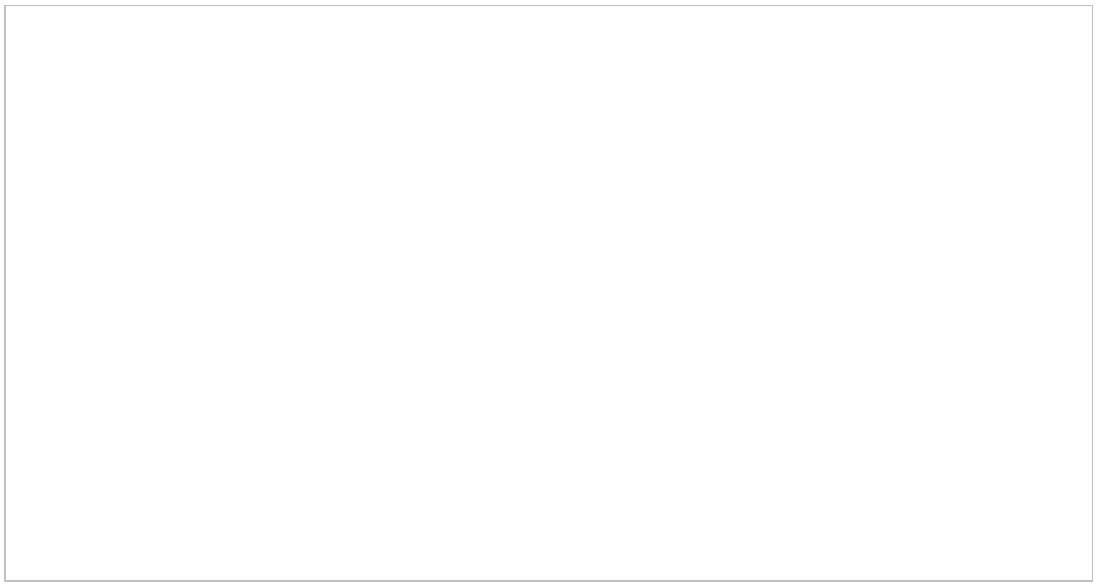


Figure P3.38 Full Alternative Text

1. a) What will the voltmeter read?

2. b) Find the percentage of error in the voltmeter reading if

% error=(measured valuetrue value−1)×100.

7. 3.39 The ammeter in the circuit in Fig. P3.39 has a resistance of 0.1 Ω. Using the definition of the percentage error in a meter reading found in Problem 3.38, what is the percentage of error in the reading of this ammeter?

Figure P3.39

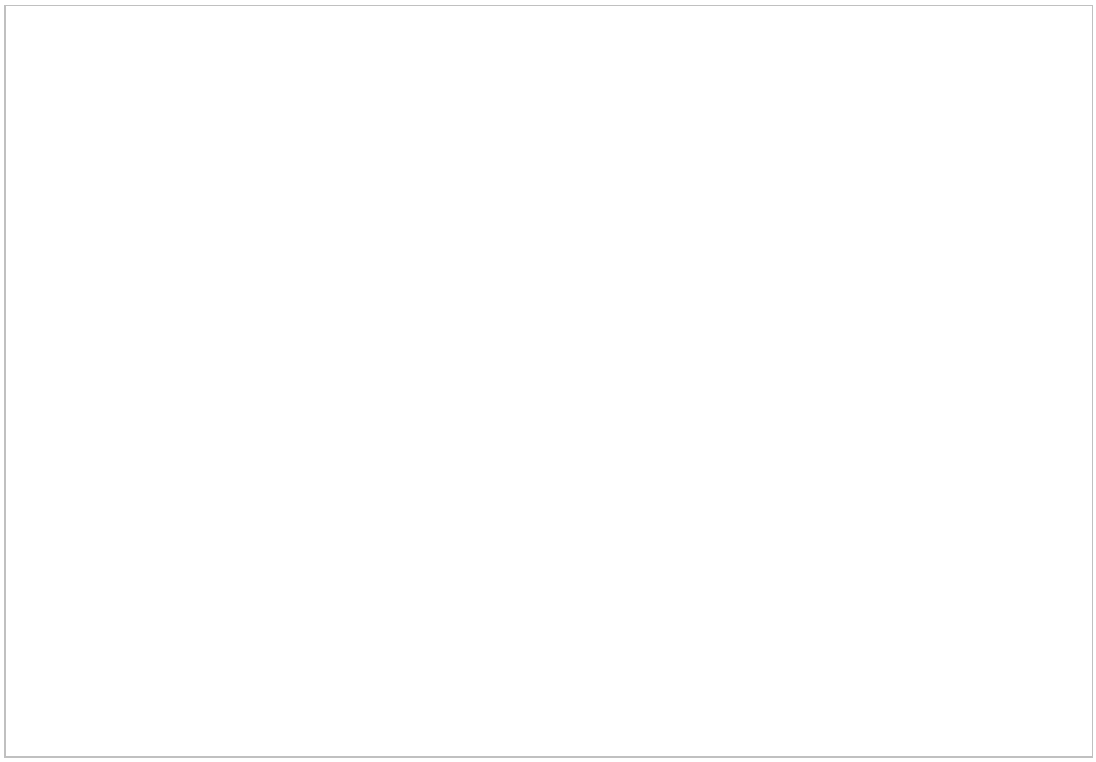


Figure P3.39 Full Alternative Text

8. 3.40 The ammeter described in Problem 3.39 is used to measure the current io in the circuit in Fig. P3.38. What is the percentage of error in the measured value?

9. 3.41 PSPICEMULTISIM The elements in the circuit in Fig2.24. have the following values: R1=20 kΩ, R2=80 kΩ, RC=0.82 kΩ, RE=0.2 kΩ, VCC=7.5 V, V0=0.6 V, and β=39.

    1. a) Calculate the value of iB in microamperes.

    2. b) Assume that a digital multimeter, when used as a dc ammeter, has a resistance of 1 kΩ. If the meter is inserted between terminals b and 2 to measure the current iB, what will the meter read?

    3. c) Using the calculated value of iB in (a) as the correct value, what is the percentage of error in the measurement?

10. 3.42 The voltmeter shown in Fig. P3.42(a) has a full-scale reading of 500 V. The meter movement is rated 100 mV and 0.5 mA. What is the

percentage of error in the meter reading if it is used to measure the voltage v in the circuit of Fig. P3.42(b)?

Figure P3.42

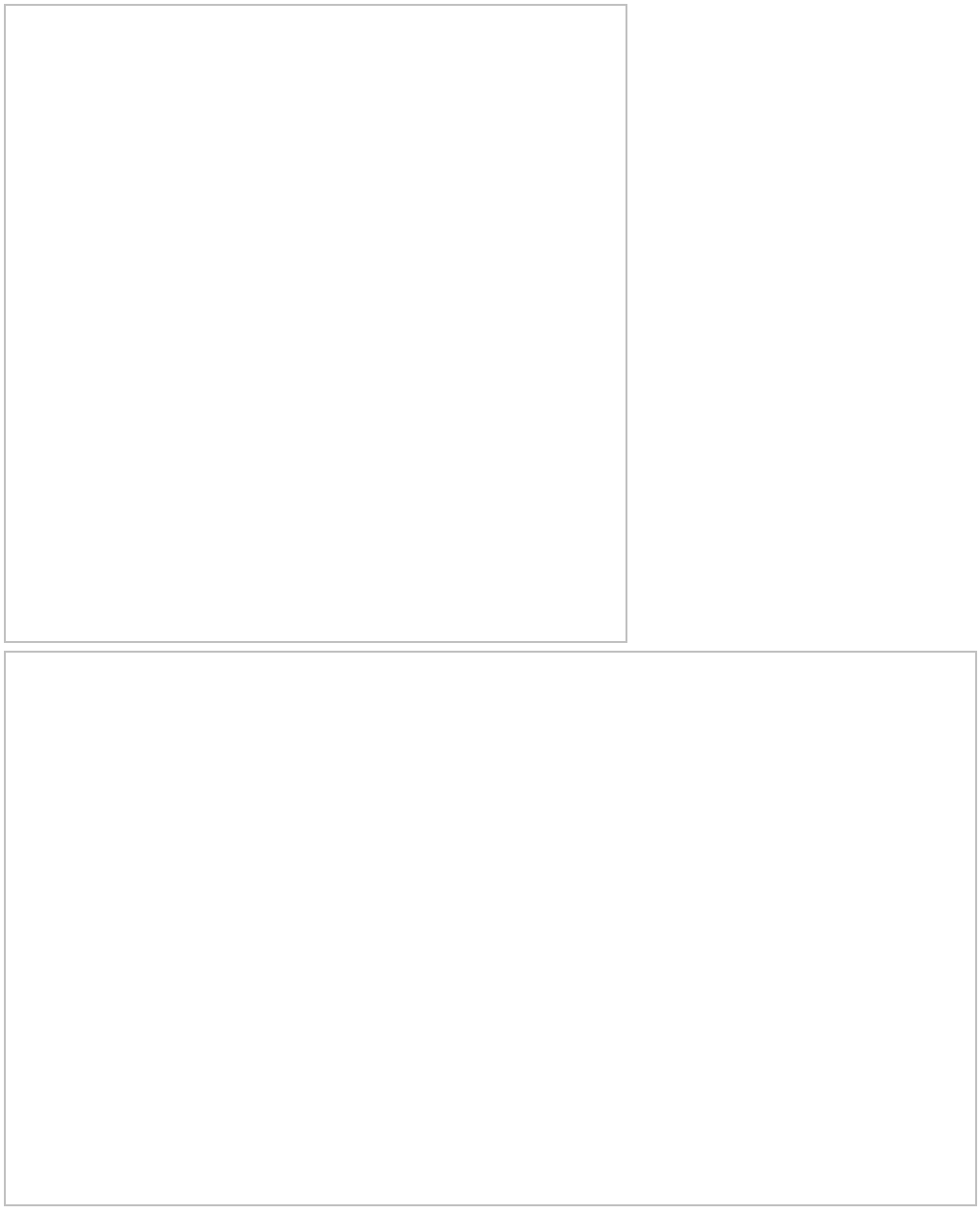


Figure P3.42 Full Alternative Text

11. 3.43 DESIGNPROBLEM_ Assume in designing the multirange

voltmeter shown in Fig. P3.43 that you ignore the resistance of the meter movement.

Figure P3.43

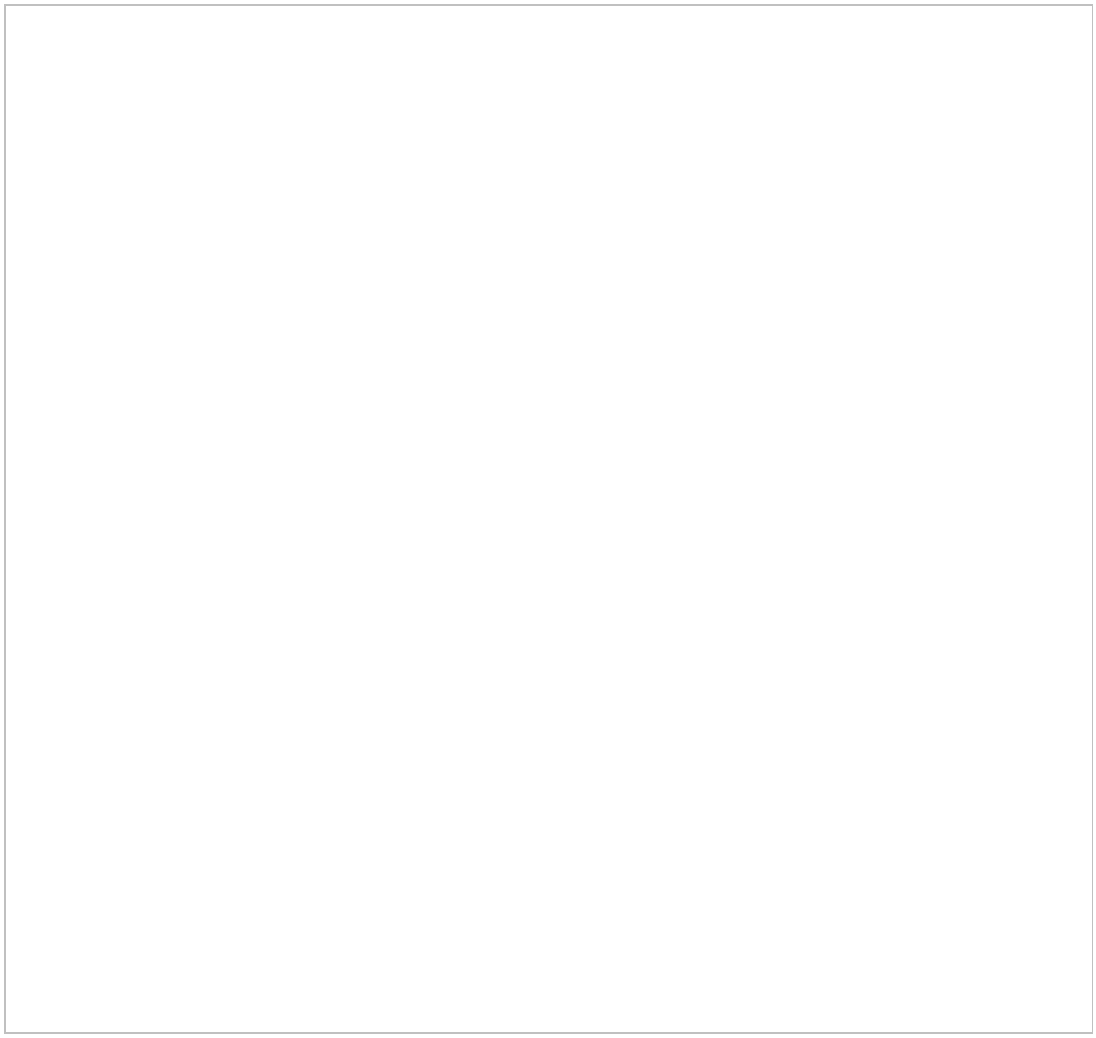


Figure P3.43 Full Alternative Text

1. a) Specify the values of R1, R2, and R3.

2. b) For each of the three ranges, calculate the percentage of error that this design strategy produces.

12. 3.44 The voltage-divider circuit shown in Fig. P3.44 is designed so that

the no-load output voltage is 7/9ths of the input voltage. A d'Arsonval voltmeter having a sensitivity of 100 Ω/V and a full-scale rating of 200 V is used to check the operation of the circuit.

Figure P3.44

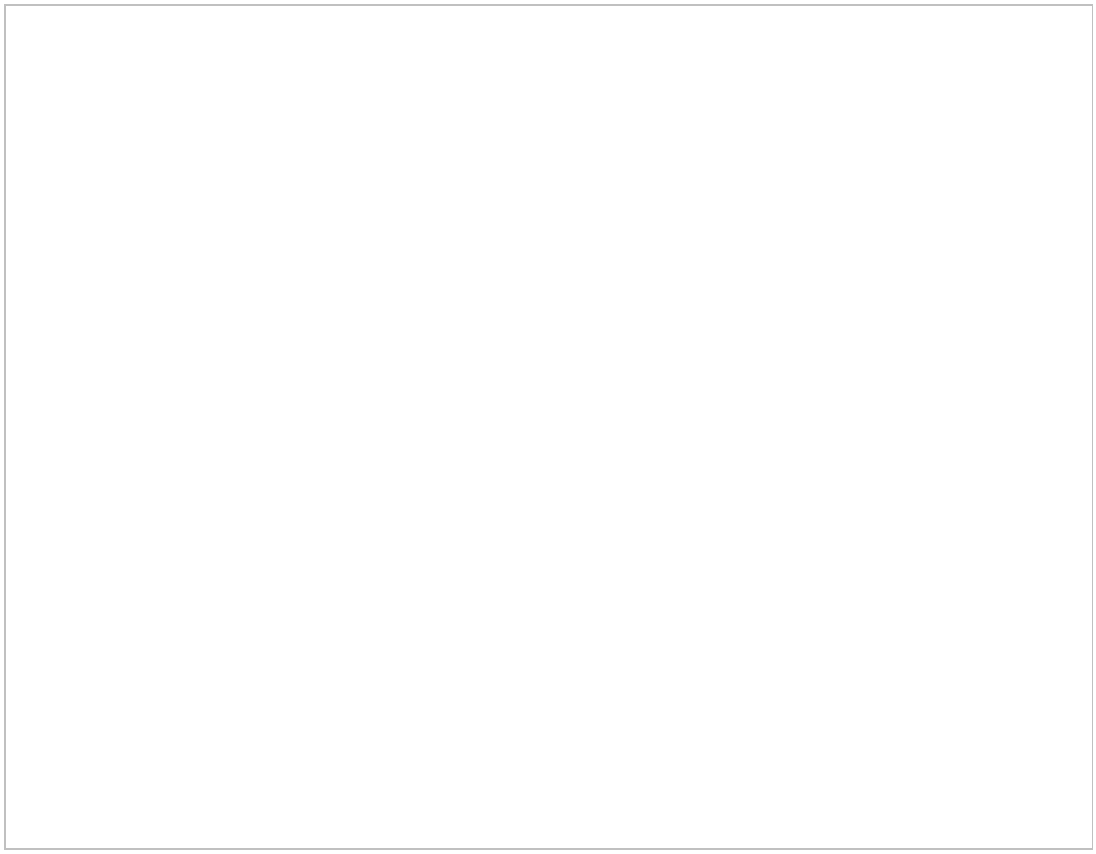


Figure P3.44 Full Alternative Text

1. a) What will the voltmeter read if it is placed across the 180 V source?

2. b) What will the voltmeter read if it is placed across the 70 kΩ resistor?

3. c) What will the voltmeter read if it is placed across the 20 kΩ resistor?

4. d) Will the voltmeter readings obtained in parts (b) and (c) add to the reading recorded in part (a)? Explain why or why not.

13. 3.45 The circuit model of a dc voltage source is shown in Fig. P3.45. The following voltage measurements are made at the terminals of the source: (1) With the terminals of the source open, the voltage is measured at 50 mV, and (2) with a 15 MΩ resistor connected to the terminals, the voltage is measured at 48.75 mV. All measurements are made with a digital voltmeter that has a meter resistance of 10 MΩ.

Figure P3.45

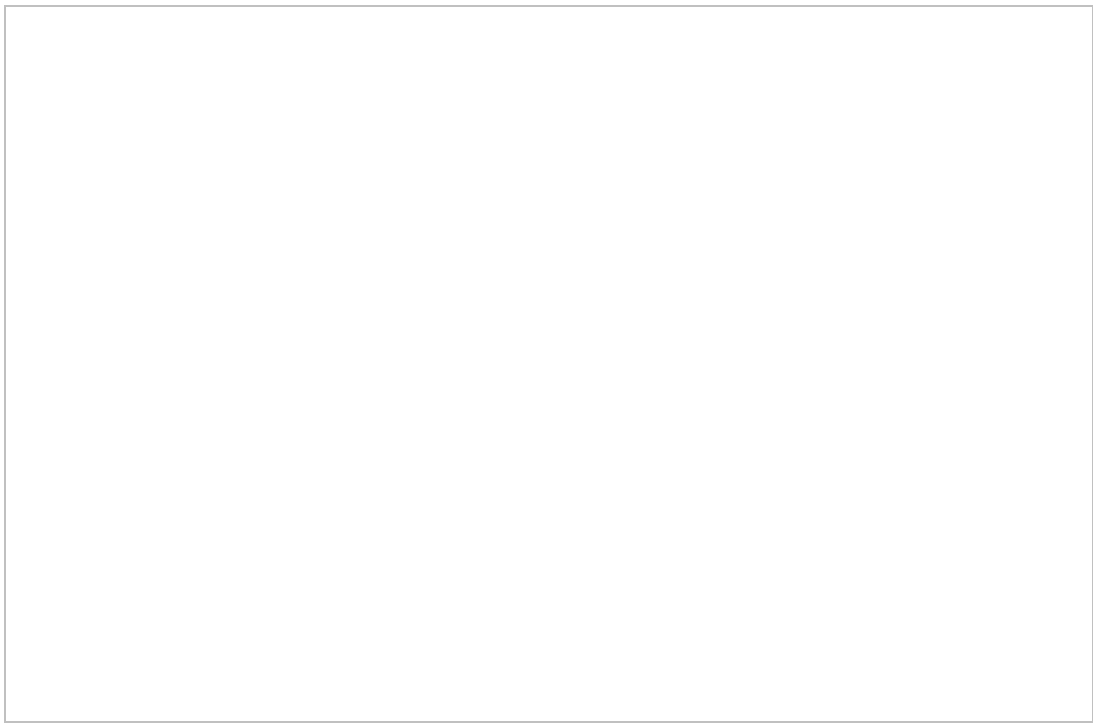


Figure P3.45 Full Alternative Text

1. a) What is the internal voltage of the source (vs) in millivolts?

2. b) What is the internal resistance of the source (Rs) in kiloohms?

14. 3.46 You have been told that the dc voltage of a power supply is about 400 V. When you go to the instrument room to get a dc voltmeter to

measure the power supply voltage, you find that there are only two dc voltmeters available. One voltmeter is rated 300 V full scale and has a sensitivity of 1000 Ω/V. The other voltmeter is rated 150 V full scale and has a sensitivity of 800 Ω/V. (*Hint:* you can find the effective resistance of a voltmeter by multiplying its rated full-scale voltage and its sensitivity.)

1. a) How can you use the two voltmeters to check the power supply voltage?

2. b) What is the maximum voltage that can be measured?

3. c) If the power supply voltage is 399 V, what will each voltmeter read?

15. 3.47 Assume that in addition to the two voltmeters described in Problem 3.46, an 80 kΩ precision resistor is also available. The 50 kΩ resistor is connected in series with the series-connected voltmeters. This circuit is then connected across the terminals of the power supply. The reading on the 300 V meter is 288 V and the reading on the 150 V meter is 115.2 V. What is the voltage of the power supply?

16. 3.48 DESIGNPROBLEM_ Design a d'Arsonval voltmeter that will have the three voltage ranges shown in Fig. P3.48.

Figure P3.48

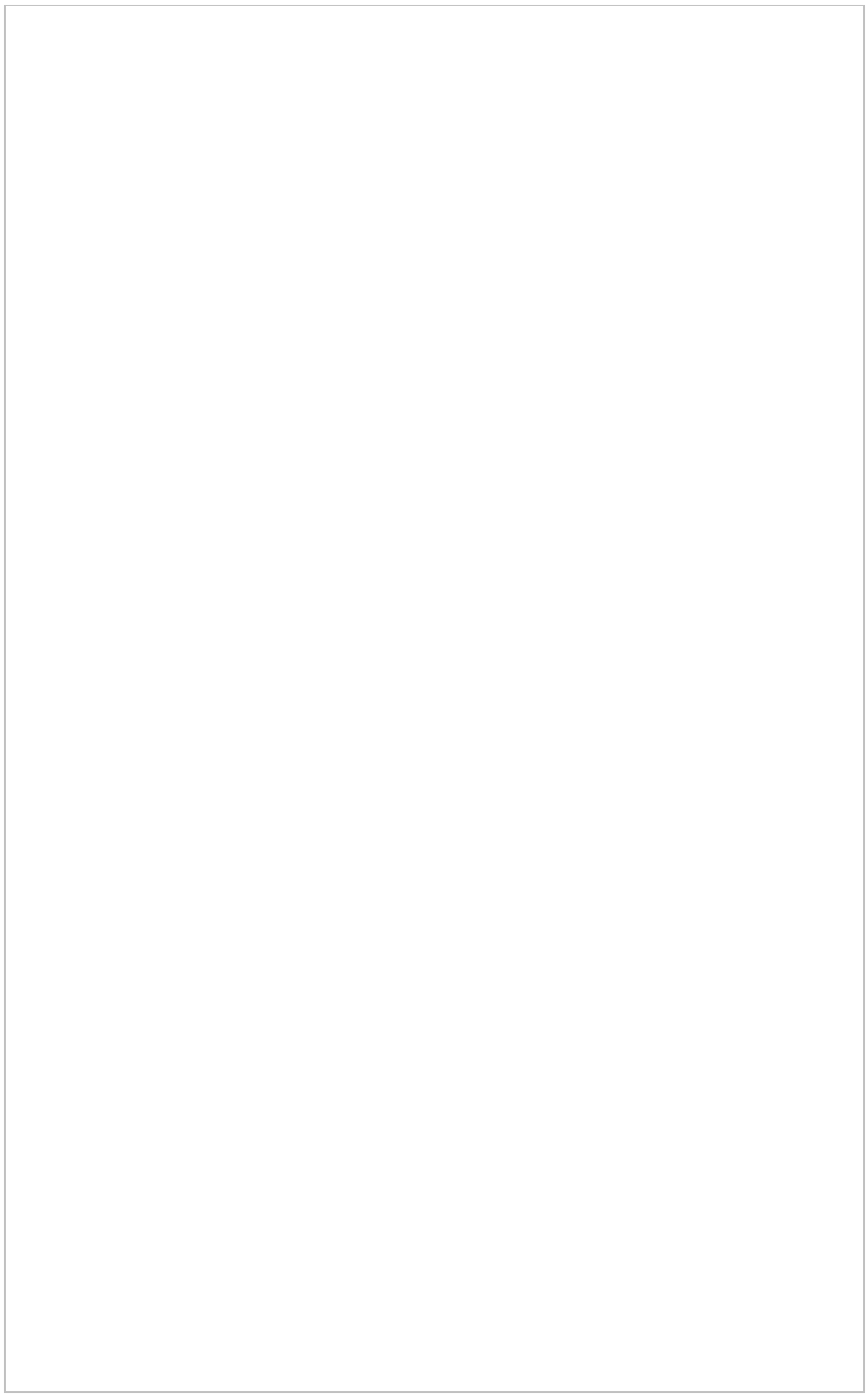


Figure P3.48 Full Alternative Text

1. a) Specify the values of R1, R2, and R3.

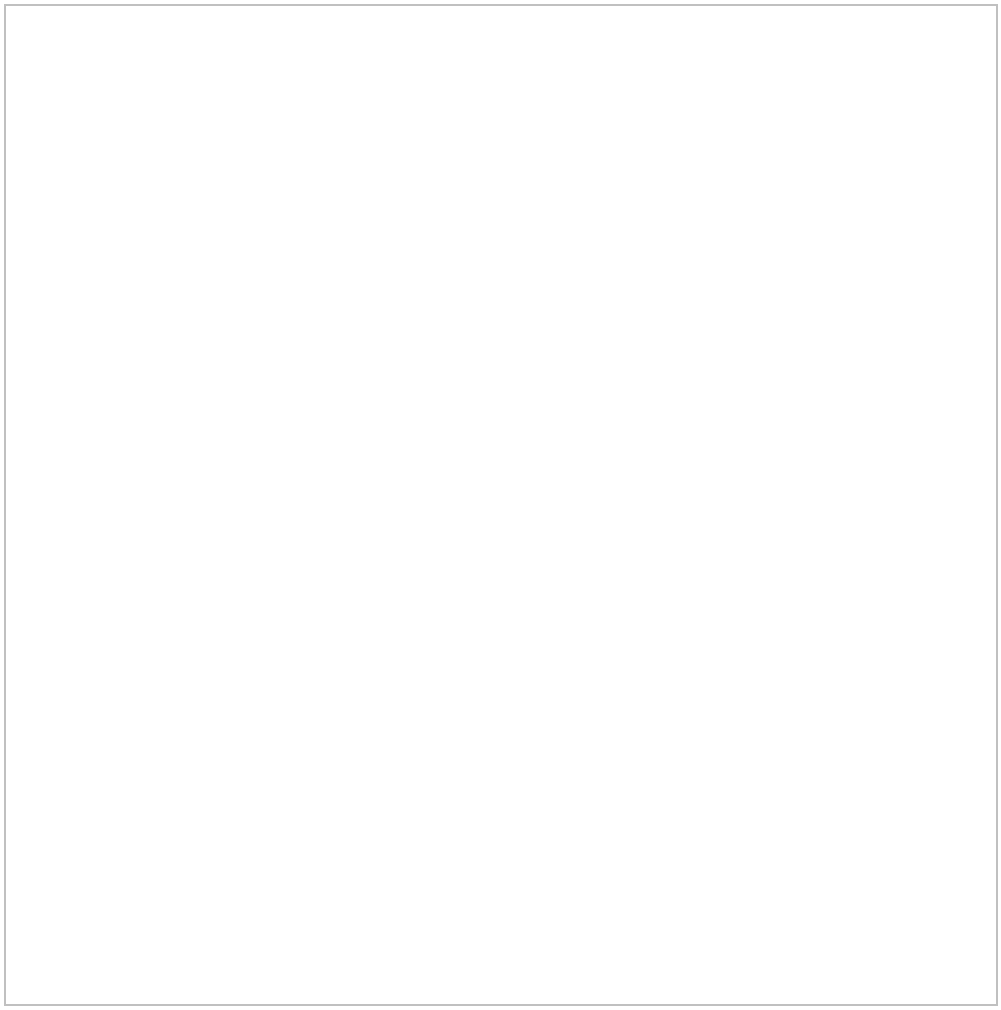
2. b) Assume that a 750 kΩ resistor is connected between the 150 V terminal and the common terminal. The voltmeter is then connected

to an unknown voltage using the common terminal and the 300 V terminal. The voltmeter reads 288 V. What is the unknown voltage?

3. c) What is the maximum voltage the voltmeter in (b) can measure?

17. 3.49 A 200 kΩ resistor is connected from the 100 V terminal to the common terminal of a dual-scale voltmeter, as shown in Fig. P3.49(a). This modified voltmeter is then used to measure the voltage across the 600 kΩ resistor in the circuit in Fig. P3.49(b).

Figure P3.49



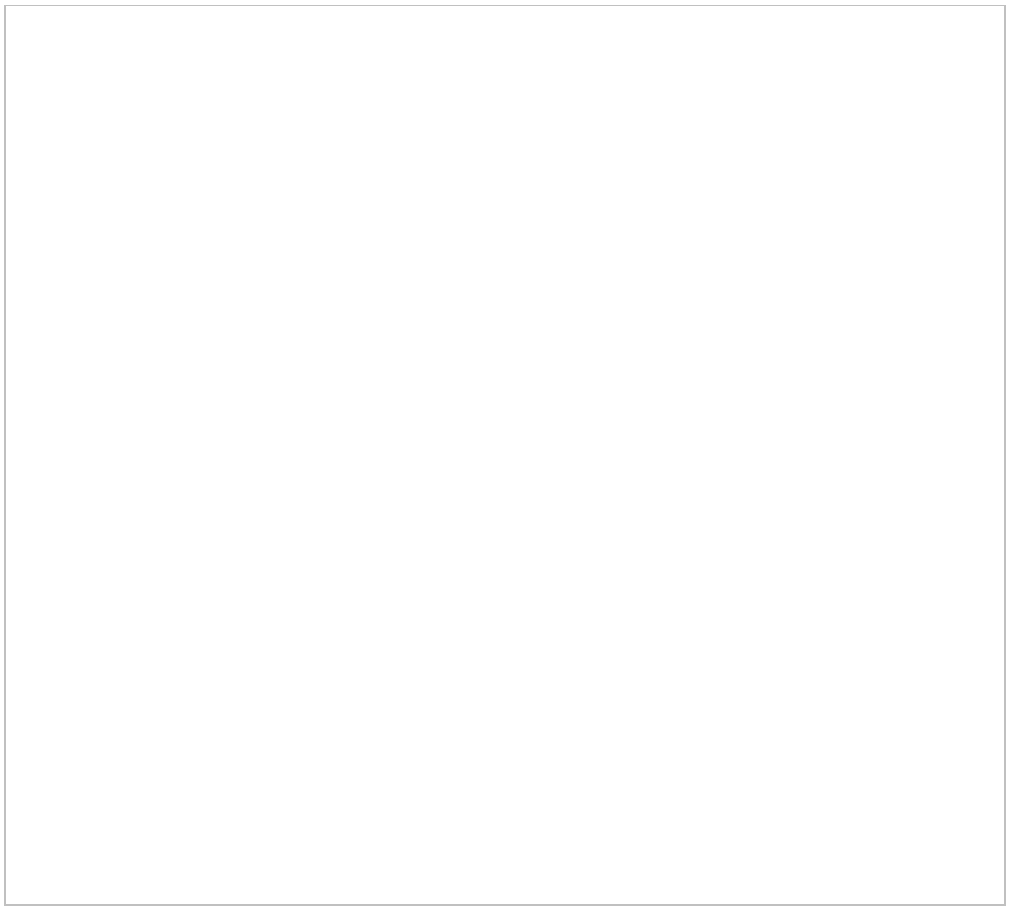


Figure P3.49 Full Alternative Text

1. a) What is the reading on the 820 V scale of the meter?
2. b) What is the percentage of error in the measured voltage?

Section 3.6

1. 3.50 PSPICEMULTISIM Assume the ideal voltage source in Fig. 3.28 is replaced by an ideal current source. Show that Eq. 3.11 is still valid.

2. 3.51 PSPICEMULTISIM The bridge circuit shown in Fig. 3.28 is energized from a 21 V dc source. The bridge is balanced when R1=800, R2=1200 Ω, and R3=600 Ω.

    1. a) What is the value of Rx?

    2. b) How much current (in milliamperes) does the dc source supply?

    3. c) Which resistor in the circuit absorbs the most power? How much power does it absorb?

    4. d) Which resistor absorbs the least power? How much power does it absorb?

3. 3.52 Find the detector current id in the unbalanced bridge in Fig. P3.52 if the voltage drop across the detector is negligible.

Figure P3.52

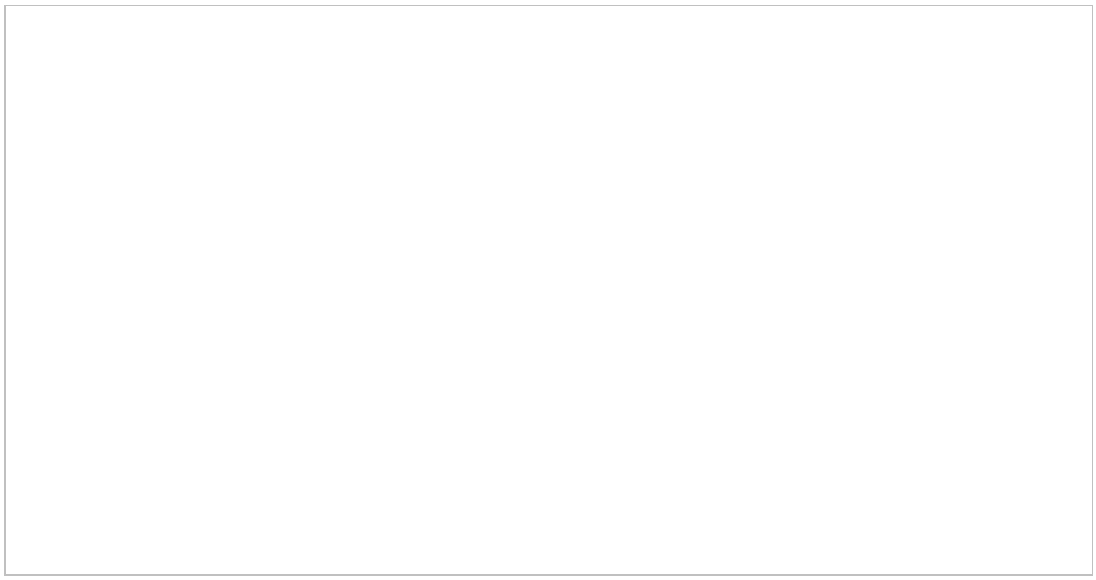


Figure P3.52 Full Alternative Text

4. 3.53 PSPICEMULTISIM Find the power dissipated in the 18 Ω resistor in the circuit in Fig. P3.53.

Figure P3.53

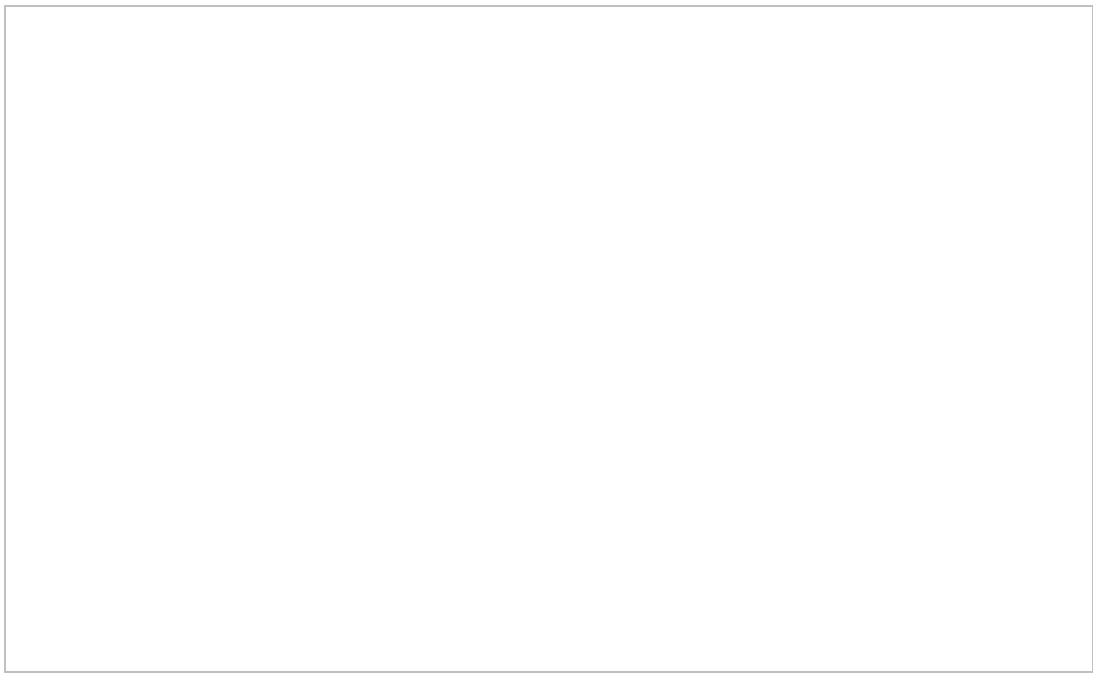


Figure P3.53 Full Alternative Text

5. 3.54 PSPICEMULTISIM In the Wheatstone bridge circuit shown in Fig. 3.28, the ratio R2/R1 can be set to the following values: 0.001, 0.01, 0.1, 1, 10, 100, and 1000. The resistor R3 can be varied from 1 to 11,110 Ω, in increments of 1 Ω. An unknown resistor is known to lie between 4 and5 Ω. What should be the setting of the R2/R1 ratio so that the unknown resistor can be measured to four significant figures?

Section 3.7

1. 3.55 Find the current and power supplied by the 40 V source in the circuit for Example 3.11 (Fig. 3.35) by replacing the lower Δ (25, 37.5, and 40 Ω) with its equivalent Y.

2. 3.56 Find the current and power supplied by the 40 V source in the circuit for Example 3.11 (Fig. 3.35) by replacing the Y on the left (25, 40, and 100 Ω) with its equivalent Δ.

3. 3.57 Find the current and power supplied by the 40 V source in the circuit for Example 3.11 (Fig. 3.35) by replacing the Y on the right (25, 37.5, and 125 Ω) with its equivalent Δ.

4. 3.58 PSPICEMULTISIM Use a Δ-to-Y transformation to find the voltages s1 and v2 in the circuit in Fig. P3.58.

Figure P3.58

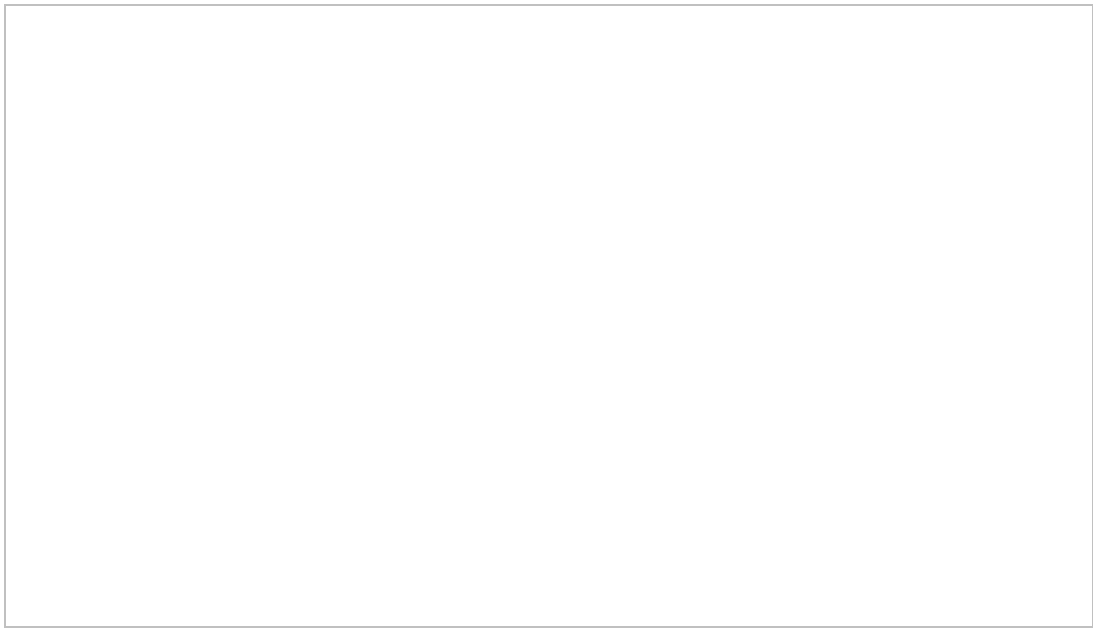


Figure P3.58 Full Alternative Text

5. 3.59 PSPICEMULTISIM For the circuit shown in Fig. P3.59, find (a) s1, (b) v, (c) i2, and (d) the power supplied by the current source.

Figure P3.59

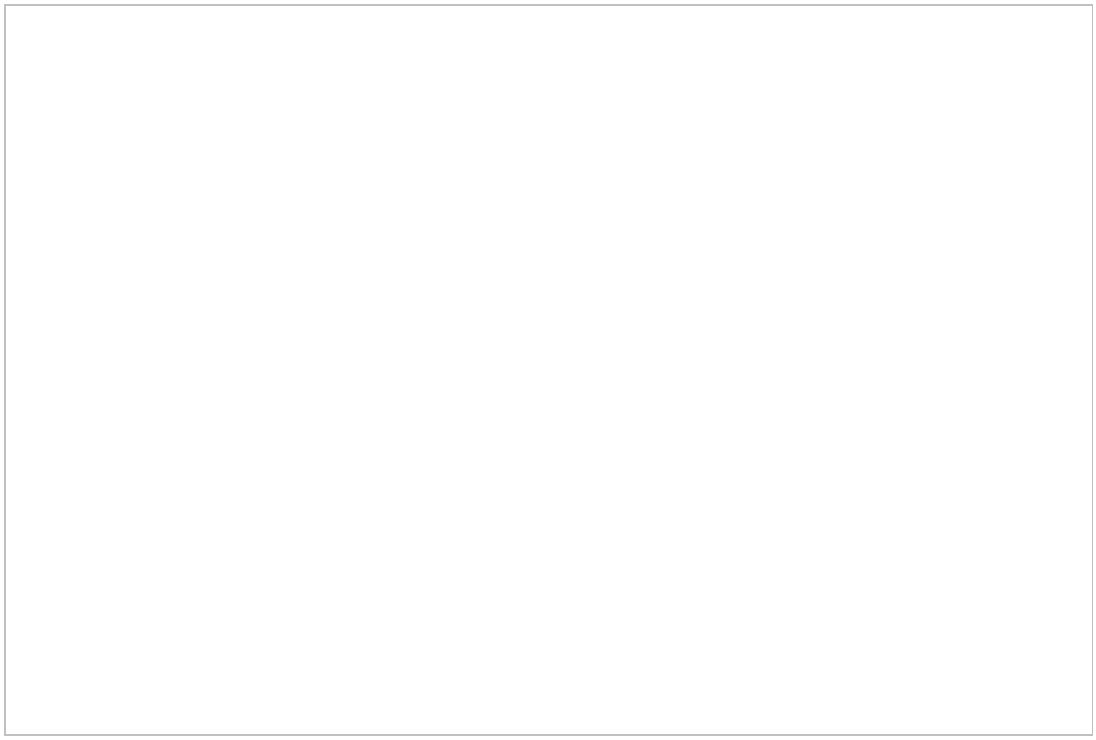


Figure P3.59 Full Alternative Text

6. 3.60 PSPICEMULTISIM Find io and the power dissipated in the 140 Ω resistor in the circuit in Fig. P3.60.

Figure P3.60

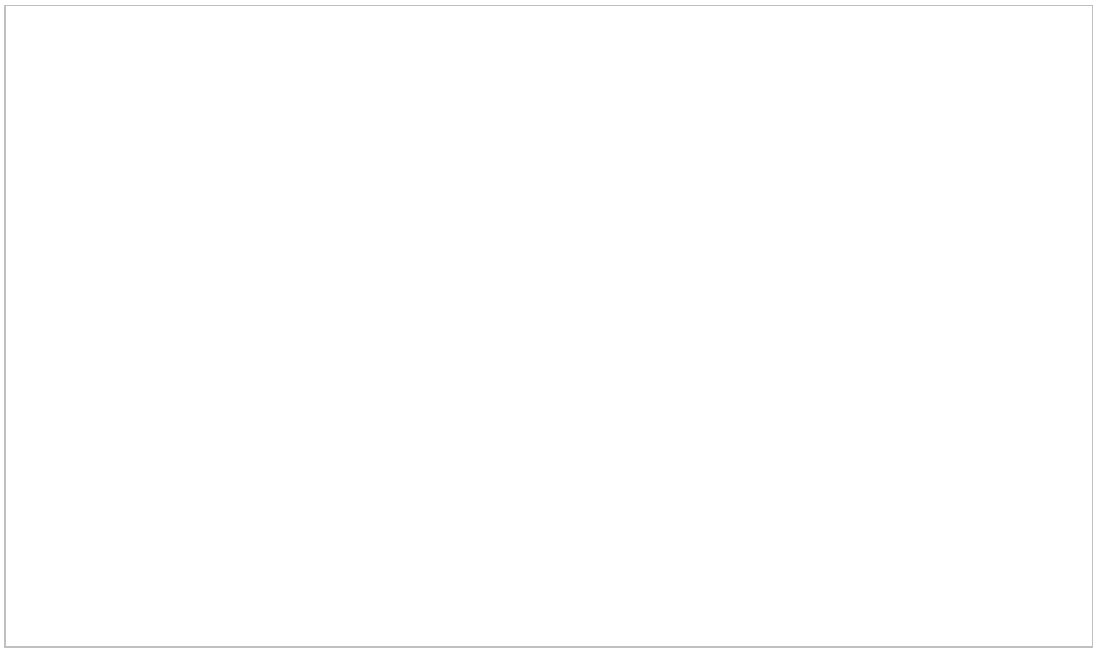


Figure P3.60 Full Alternative Text

7. 3.61 PSPICEMULTISIM

    1. a) Find the equivalent resistance Rab in the circuit in Fig. P3.61 by using a Y-to-Δ transformation involving resistors R2, R3, and R5.

Figure P3.61

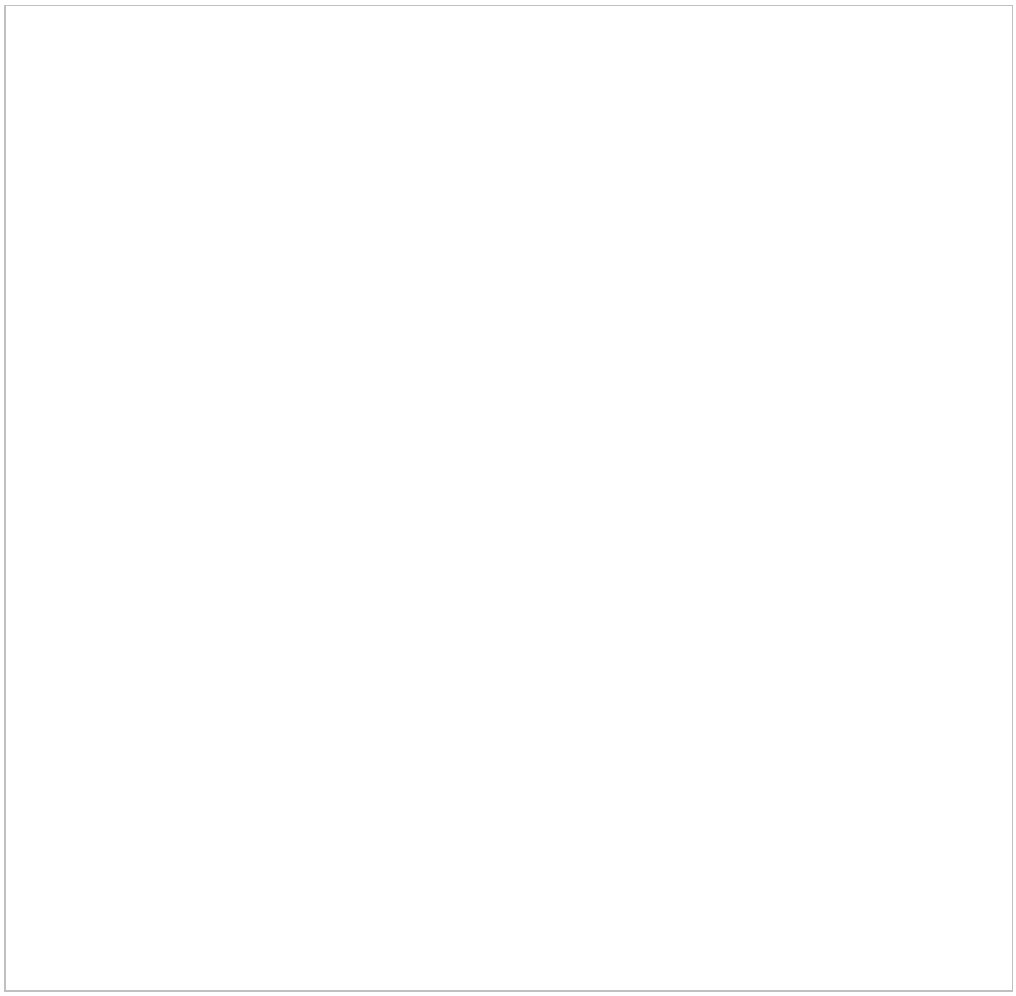


Figure P3.61 Full Alternative Text

2. b) Repeat (a) using a Δ-to-Y transformation involving resistors R3, R4, and R5.

3. c) Give two additional Δ-to-Y or Y-to-Δ transformations that could be used to find Rab.

8. 3.62 PSPICEMULTISIM Use a Y-to-Δ transformation to find (a) io; (b) s1; (c) i2; and (d) the power delivered by the ideal voltage source in the circuit in Fig. P3.62.

Figure P3.62

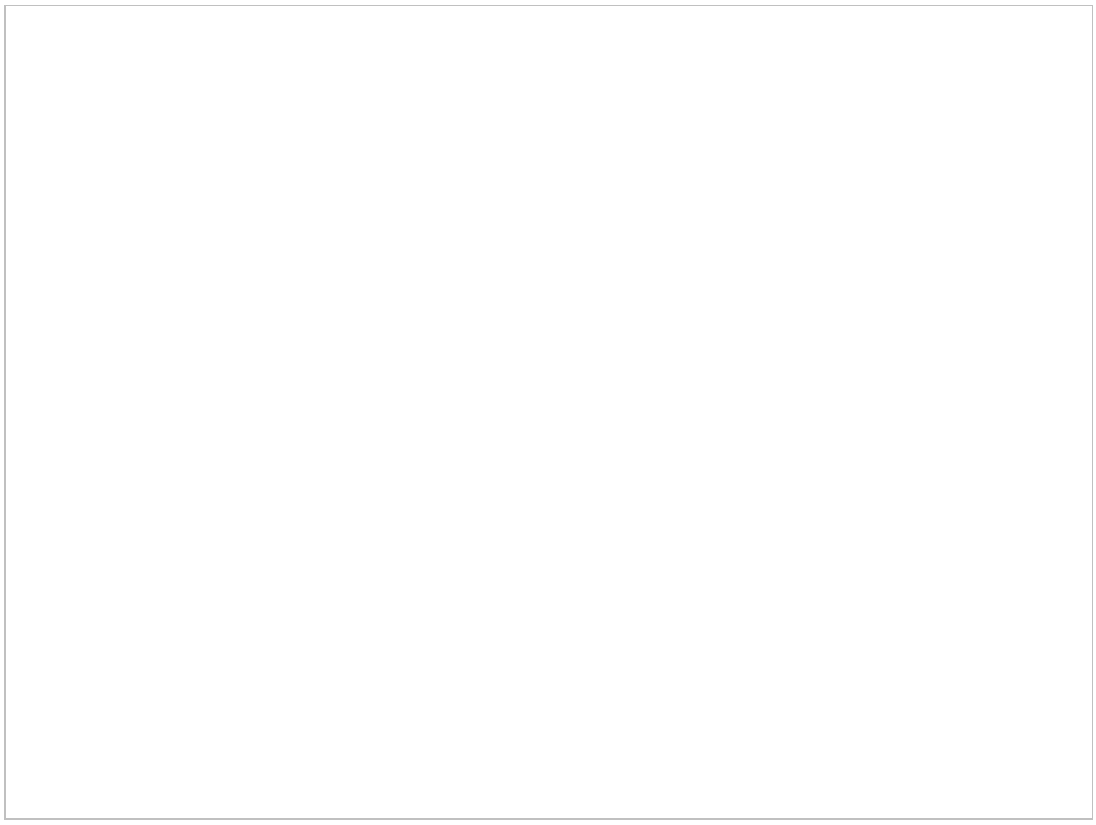


Figure P3.62 Full Alternative Text

9. 3.63 PSPICEMULTISIM

    1. a) Find the resistance seen by the ideal voltage source in the circuit in Fig. P3.63.

Figure P3.63

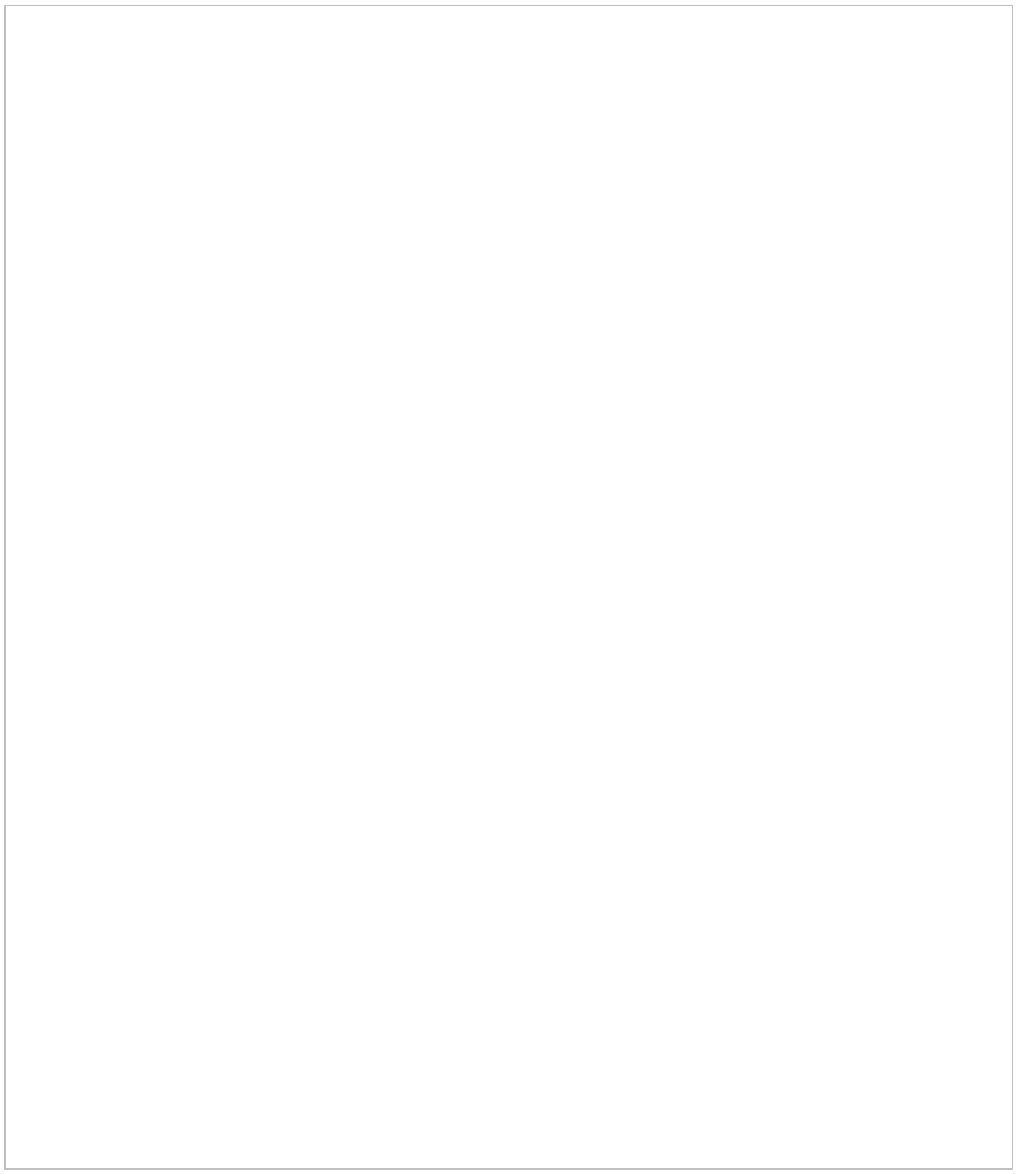


Figure P3.63 Full Alternative Text

2. b) If vab equals 400 V, how much power is dissipated in the 31 Ω resistor?

10. 3.64 Show that the expressions for Δ conductances as functions of the three Y conductances are

Ga=G2G3G1+G2+G3,Gb=G1G3G1+G2+G3,Gc=G1G2G1+G2+G3,

where

Ga=1Ra,G1=1R1,etc.

11. 3.65 Derive Eqs. 3.15–3.20 from Eqs. 3.12–3.14. The following two hints should help you get started in the right direction:

    1. To find R1 as a function of Ra, Rb, and Rc, first subtract Eq. 3.13 from Eq. 3.14 and then add this result to Eq. 3.12. Use similar manipulations to find R2 and R3 as functions of Ra, Rb, and Rc.

    2. To find Rb as a function of R1, R2, and R3, take advantage of the derivations obtained by hint (1), namely, Eqs. 3.15–3.17. Note that these equations can be divided to obtain

       R2R3=RcRb, or Rc=R2R3Rb,

       and

       R1R2=RbRa, or Ra=R2R1Rb.

       Now use these ratios in Eq. 3.14 to eliminate Ra and Rc. Use similar manipulations to find Ra and Rc as functions of R1, R2, and R3.

Sections 3.1–3.7

1. 3.66 DESIGNPROBLEM_ Resistor networks are sometimes used as volume- control circuits. In this application, they are referred to as *resistance attenuators* or *pads*. A typical fixed-attenuator pad is shown in Fig. P3.66. In designing an attenuation pad, the circuit designer will select the values of R1 and R2 so that the ratio of vo/vi and the resistance seen by the input voltage source Rab both have a specified value.

Figure P3.66

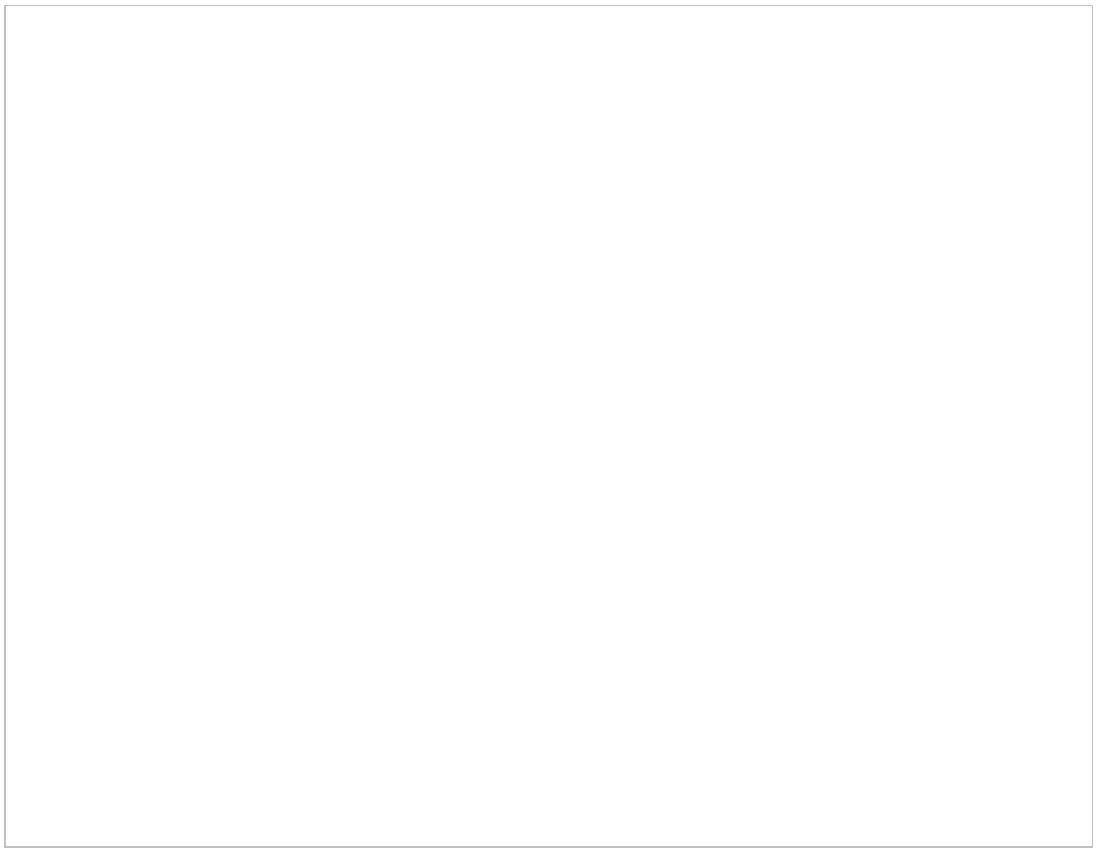


Figure P3.66 Full Alternative Text

1. a) Show that if Rab=RL, then

   RL2=4R1(R1+R2).
   vovi=R22R1+R2+RL.

2. b) Select the values of R1 and R2 so that Rab=RL=300 Ω and vo/vi=0.5.

3. c) Choose values from Appendix H that are closest to R1 and R2 from part (b). Calculate the percent error in the resulting values for Rab and v0/v1 if these new resistor values are used.

2. 3.67 DESIGNPROBLEM_

   1. a) The fixed-attenuator pad shown in Fig. P3.67 is called a *bridged tee*. Use a Y-to-Δ transformation to show that Rab=RL if R=RL.

Figure P3.67

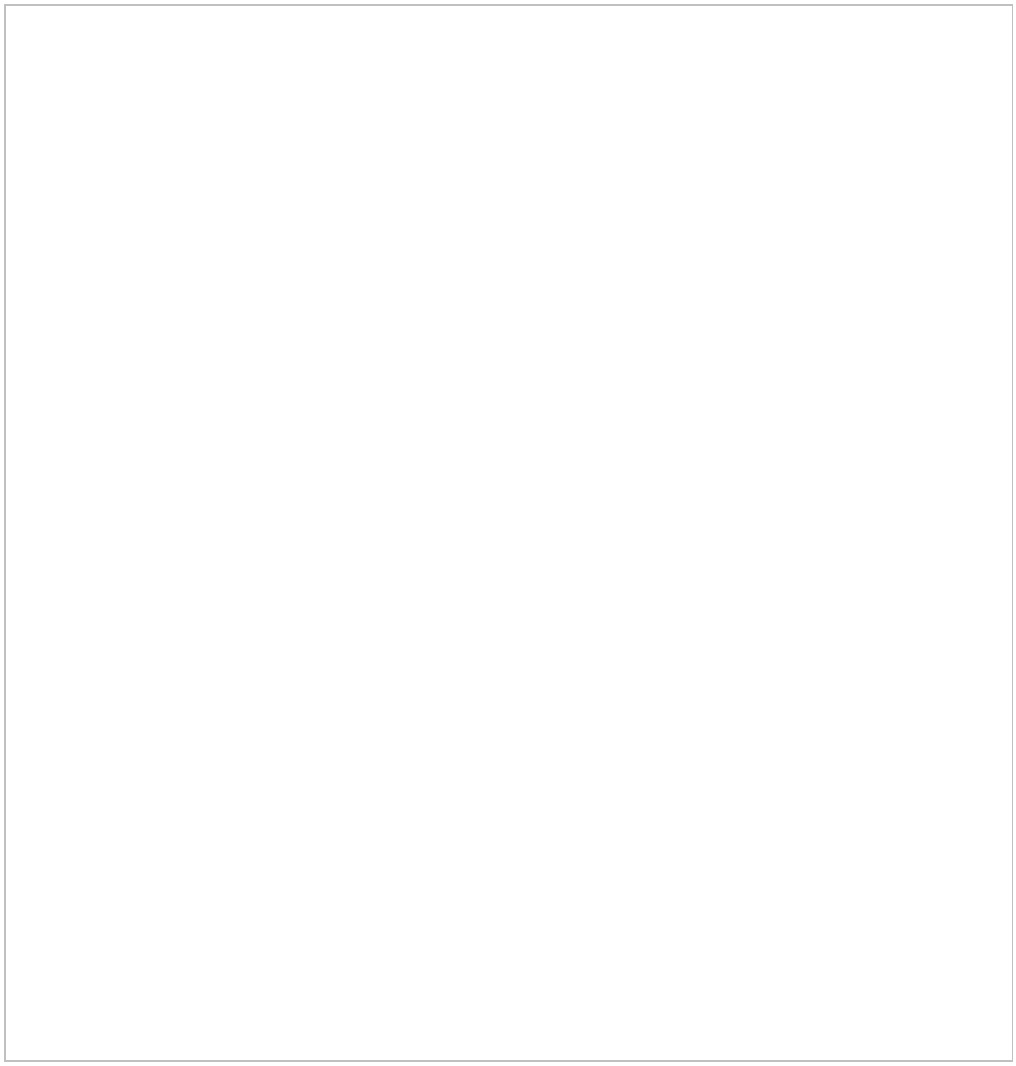


Figure P3.67 Full Alternative Text

2. b) Show that when R=RL, the voltage ratio vo/vi equals 0.50.

3. 3.68 PSPICEMULTISIM The design equations for the bridged-tee attenuator circuit in Fig. P3.68 are

Figure P3.68

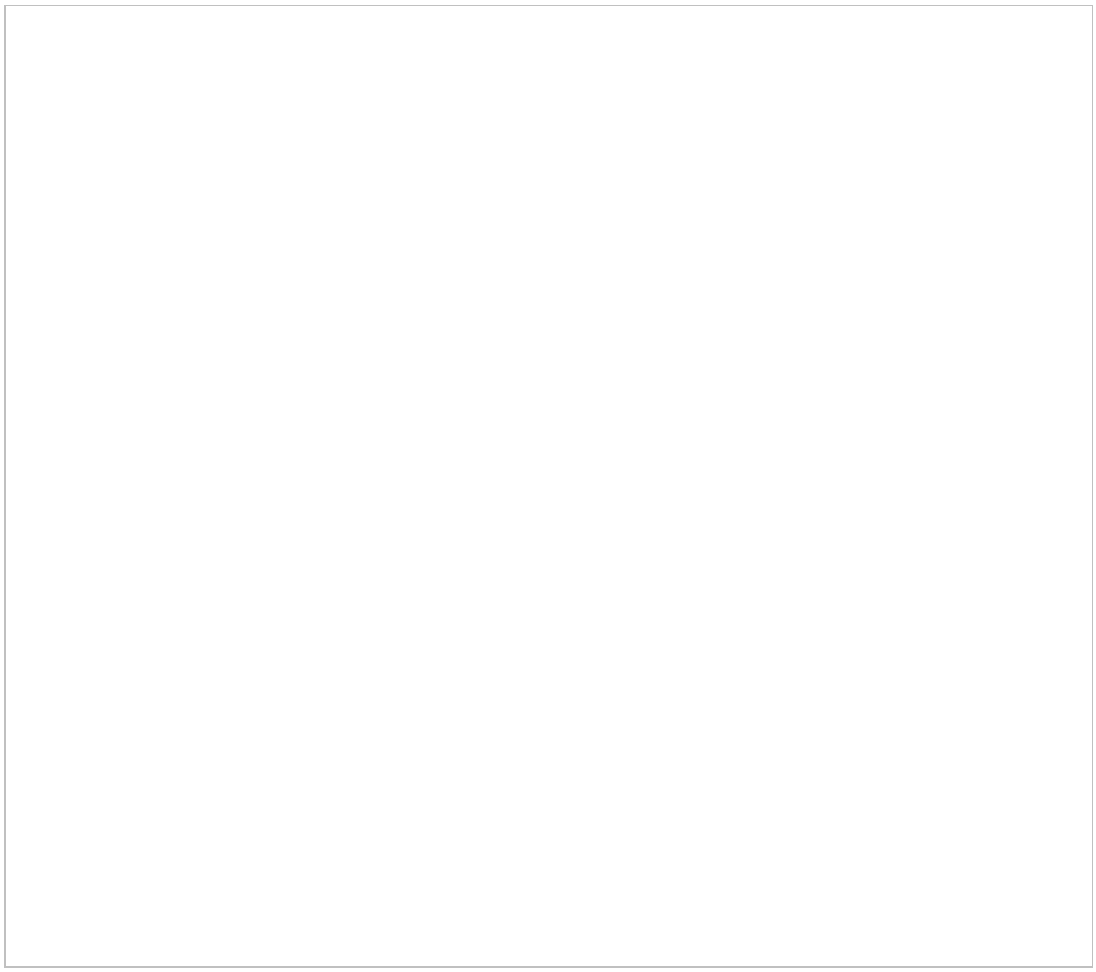


Figure P3.68 Full Alternative Text

$R_2 = \frac{2RR_L}{3R_2 - R_L}$,

$\frac{v_o}{v_i} = \frac{3R - R_L}{3R + R_L}$,

when R_2 has the value just given.

1. a) Design a fixed attenuator so that $v_i = 3.5v_o$ when $R_L = 300\ \Omega$.
2. b) Assume the voltage applied to the input of the pad designed in (a) is 42 V. Which resistor in the pad dissipates the most power?
3. c) How much power is dissipated in the resistor in part (b)?
4. d) Which resistor in the pad dissipates the least power?
5. e) How much power is dissipated in the resistor in part (d)?

4. 3.69 PSPICEMULTISIM

    1. a) For the circuit shown in Fig. P3.69 the bridge is balanced when $\Delta R=0$. Show that if $\Delta R \ll Ro$ the bridge output voltage is approximately

Figure P3.69

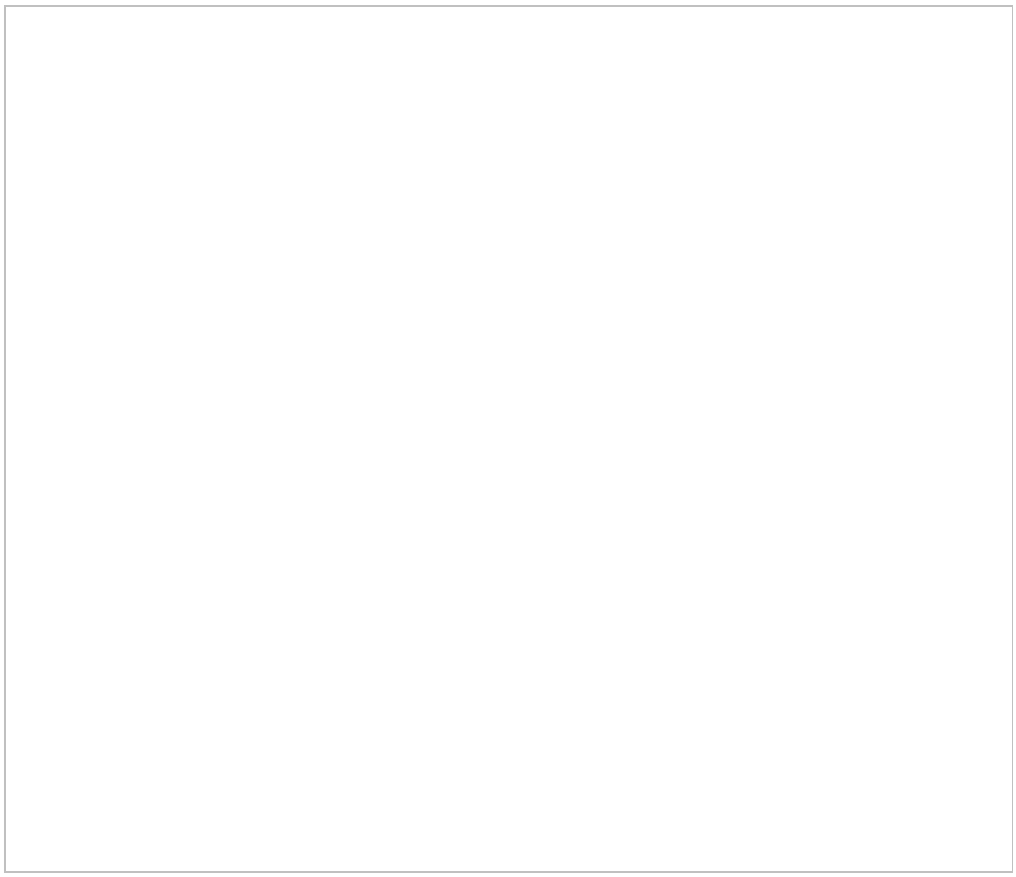


Figure P3.69 Full Alternative Text

$$v_o \approx -\frac{\Delta R}{R4}\frac{(Ro+R4)}{2}v_{in}$$

    2. b) Given R2=1 kΩ, R3=500 Ω, R4=5 kΩ, and vin=6 V, what is the approximate bridge output voltage if Δ R is 3% of Ro?

    3. c) Find the actual value of vo in part (b).

5. 3.70

    1. a) If percent error is defined as

       % error=[approximate valuetrue value−1]×100,

       show that the percent error in the approximation of vo in Problem 3.69 is

       % error=−(ΔR)R3(R2+R3)R4×100.

    2. b) Calculate the percent error in vo, using the values in Problem 3.69(b).

6. 3.71 DESIGNPROBLEM_ Assume the error in vo in the bridge circuit in Fig. P3.69 is not to exceed 0.5%. What is the largest percent change in Ro that can be tolerated?

7. 3.72 PRACTICALPERSPECTIVE_

    1. a) Using Fig. 3.41 derive the expression for the voltage Vy.

    2. b) Assuming that there are py pixels in the y-direction, derive the expression for the y-coordinate of the touch point, using the result from part (a).

8. 3.73 PRACTICALPERSPECTIVEPSPICEMULTISIM A resistive touch screen has 5 V applied to the grid in the x-direction and in the y-direction. The screen has 480 pixels in the x-direction and 800 pixels in the y-direction. When the screen is touched, the voltage in the x-grid is 1 V and the voltage in the y-grid is 3.75 V.

    1. a) Calculate the values of α and β.

    2. b) Calculate the x- and y-coordinates of the pixel at the point where the screen was touched.

9. 3.74 PRACTICALPERSPECTIVEDESIGNPROBLEMPSPICEMULTISIM

A resistive touch screen has 640 pixels in the x-direction and 1024 pixels in the y-direction. The resistive grid has 8 V applied in both the x- and y-directions. The pixel coordinates at the touch point are (480, 192). Calculate the voltages Vx and Vy.

10. 3.75 PRACTICALPERSPECTIVE_ Suppose the resistive touch screen described in Problem 3.74 is simultaneously touched at two points, one with coordinates (480, 192) and the other with coordinates (240, 384).

    1. a) Calculate the voltage measured in the x- and y-grids.

    2. b) Which touch point has your calculation in (a) identified?

Chapter 4 Techniques of Circuit Analysis

Chapter Contents

1. 4.1 Terminology
2. 4.2 Introduction to the Node-Voltage Method
3. 4.3 The Node-Voltage Method and Dependent Sources
4. 4.4 The Node-Voltage Method: Some Special Cases
5. 4.5 Introduction to the Mesh-Current Method
6. 4.6 The Mesh-Current Method and Dependent Sources
7. 4.7 The Mesh-Current Method: Some Special Cases
8. 4.8 The Node-Voltage Method Versus the Mesh-Current Method
9. 4.9 Source Transformations
10. 4.10 Thévenin and Norton Equivalents
11. 4.11 More on Deriving the Thévenin Equivalent
12. 4.12 Maximum Power Transfer
13. 4.13 Superposition

Chapter Objectives

1. Understand and be able to use the node-voltage method to solve a circuit.

2. Understand and be able to use the mesh-current method to solve a circuit.

3. Be able to decide whether the node-voltage method or the mesh-current method is the preferred approach to solving a particular circuit.

4. Understand source transformation and be able to use it to solve a circuit.

5. Understand the concept of the Thévenin and Norton equivalent circuits and be able to construct a Thévenin or Norton equivalent for a circuit.

6. Know the condition for maximum power transfer to a resistive load and be able to calculate the value of the load resistor that satisfies this condition.

This chapter introduces two powerful circuit analysis techniques: the *node-voltage method* and the *mesh-current method*. The power of these methods comes from their ability to describe complex circuits using a minimum number of simultaneous equations. The alternative—applying Kirchhoff's laws and Ohm's law—becomes cumbersome for complex circuits.

We also introduce new techniques for simplifying circuits: *source transformations* and *Thévenin* and *Norton equivalent circuits*. Adding these skills to your existing knowledge of series-parallel reductions and Δ-to-Y transformations greatly expands your ability to simplify and solve complex circuits.

Finally, we explore the concepts of *maximum power transfer* and *superposition*. You will learn to use Thévenin equivalent circuits to establish the conditions needed to ensure maximum power is delivered to a resistive load. Superposition helps us analyze circuits that have more than one

independent source.

Practical Perspective

Circuits with Realistic Resistors

In the last chapter, we examined the effect of imprecise resistor values on the performance of a voltage divider. Resistors are manufactured for only a small number of discrete values, and any given resistor from a batch of resistors will vary from its stated value within some tolerance. Resistors with a tolerance of 1% are more expensive than resistors with a tolerance of 10%. If we know that a particular resistor must be very close to its stated value for the circuit to function correctly, we can then decide to spend the extra money necessary to achieve a tighter tolerance on that resistor's value.

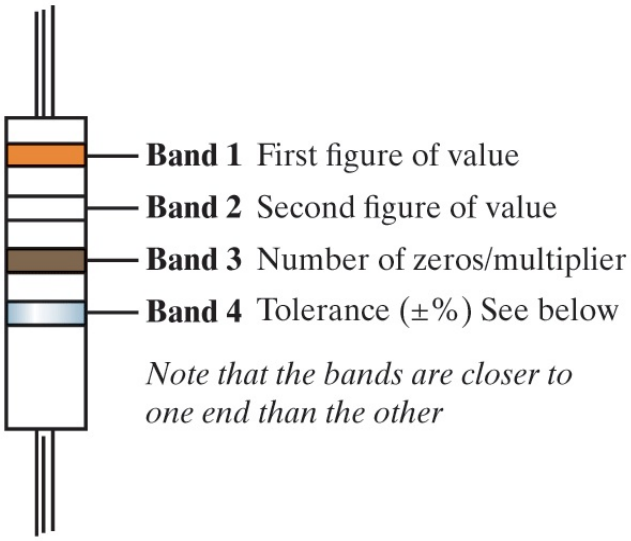
Therefore, we want to predict the effect of varying the value of each resistor in a circuit on the output of that circuit, a technique known as **sensitivity analysis**. Once we have presented additional circuit analysis methods, the topic of sensitivity analysis will be examined.

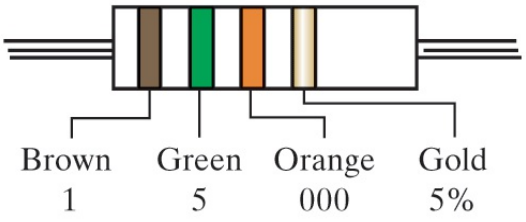
Resistor color code

Band color	Value
Black	0
Brown	1
Red	2
Orange	3
Yellow	4
Green	5
Blue	6
Violet	7
Grey	8
White	9
Gold	0.1
Silver	0.01

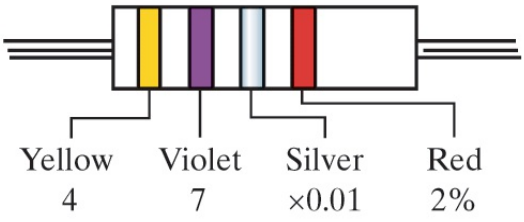
Tolerance color code

Band color	±%
Brown	1
Red	2
Gold	5
Silver	10
None	20

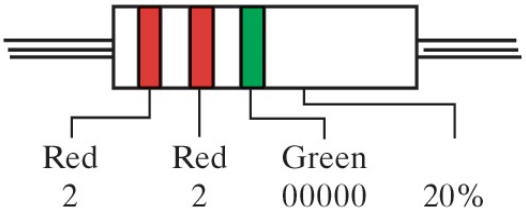




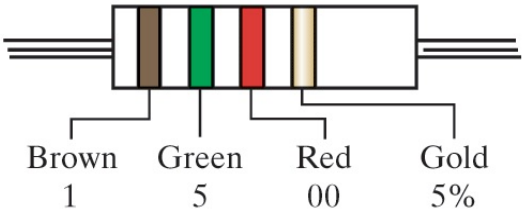
Resistor is 15000 Ω or 15 kΩ ± 5%



Resistor is 47 × 0.01 Ω or 0.47 Ω ± 2%



Resistor is 2200000 Ω or 2.2 MΩ ± 20%

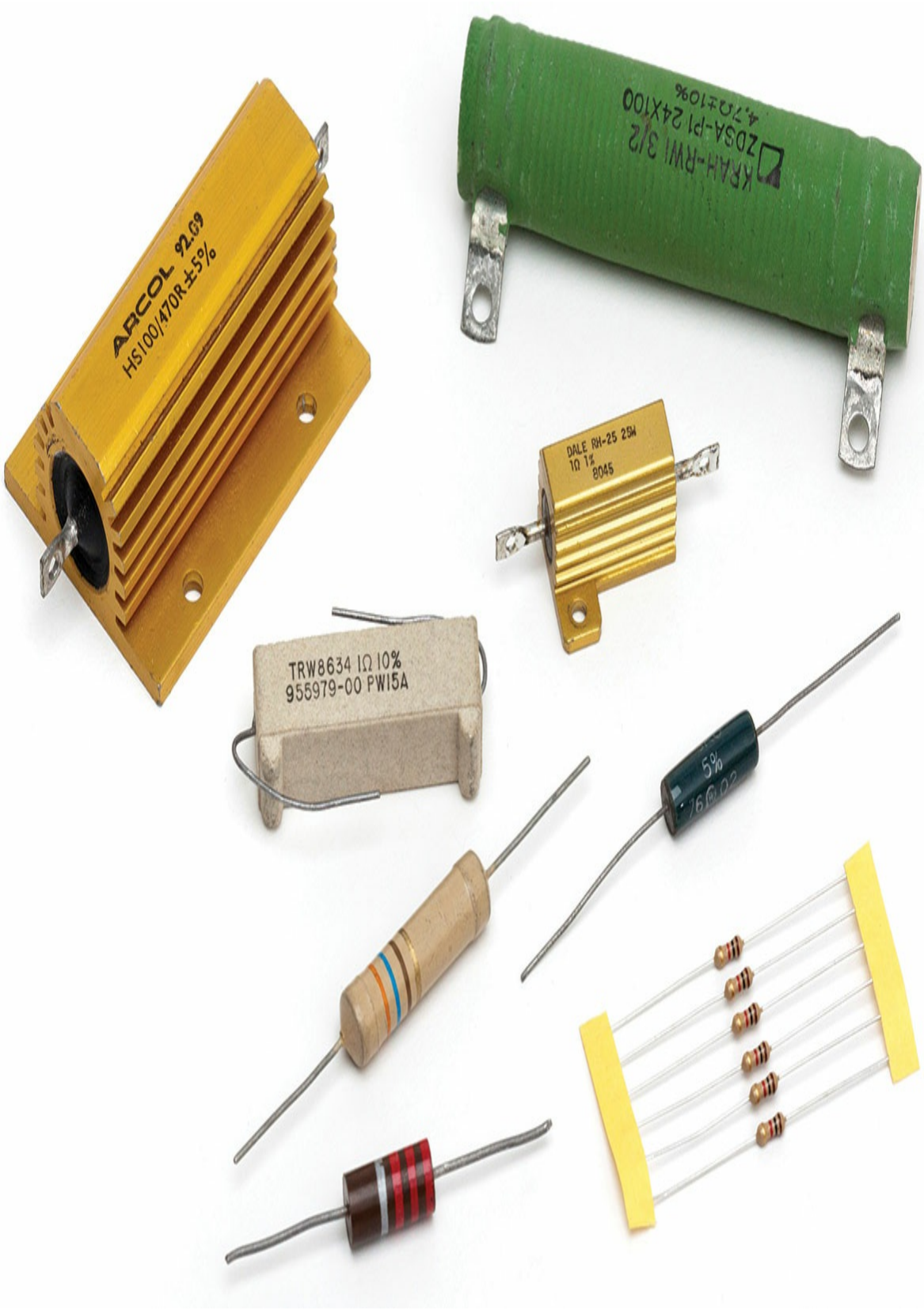


Resistor is 1500 Ω or 1.5 kΩ ± 5%

HL Studios/Pearson Education Ltd

4.1-1 Full Alternative Text

Roomanald/Shutterstock


ARCOL 92.69
HS100/470R±5%
KRAH-RW1 3/2
ZDSA-P1 24X100
4.7Ω±10%
DALE RH-25 25W
1Ω 1%
8045
TRW8634 1Ω 10%
955979-00 PW15A
5%

David J. Green - electrical/Alamy Stock Photo

4.1 Terminology

Before discussing the node-voltage and mesh-current methods of circuit analysis, we must define a few basic terms. So far, all the circuits presented have been **planar circuits**—that is, those circuits that can be drawn on a plane with no crossing branches. A circuit that is drawn with crossing branches still is considered planar if it can be redrawn with no crossing branches. For example, the circuit shown in Fig. 4.1(a) can be redrawn as Fig. 4.1(b); the circuits are equivalent because all the node connections have been maintained. Therefore, Fig. 4.1(a) is a planar circuit because it can be redrawn as one. Figure 4.2 shows a nonplanar circuit—it cannot be redrawn in such a way that all the node connections are maintained and no branches overlap. Identifying a circuit as planar or nonplanar is important, because

Figure 4.1 (a) A planar circuit. (b) The same circuit redrawn to verify that it is planar.

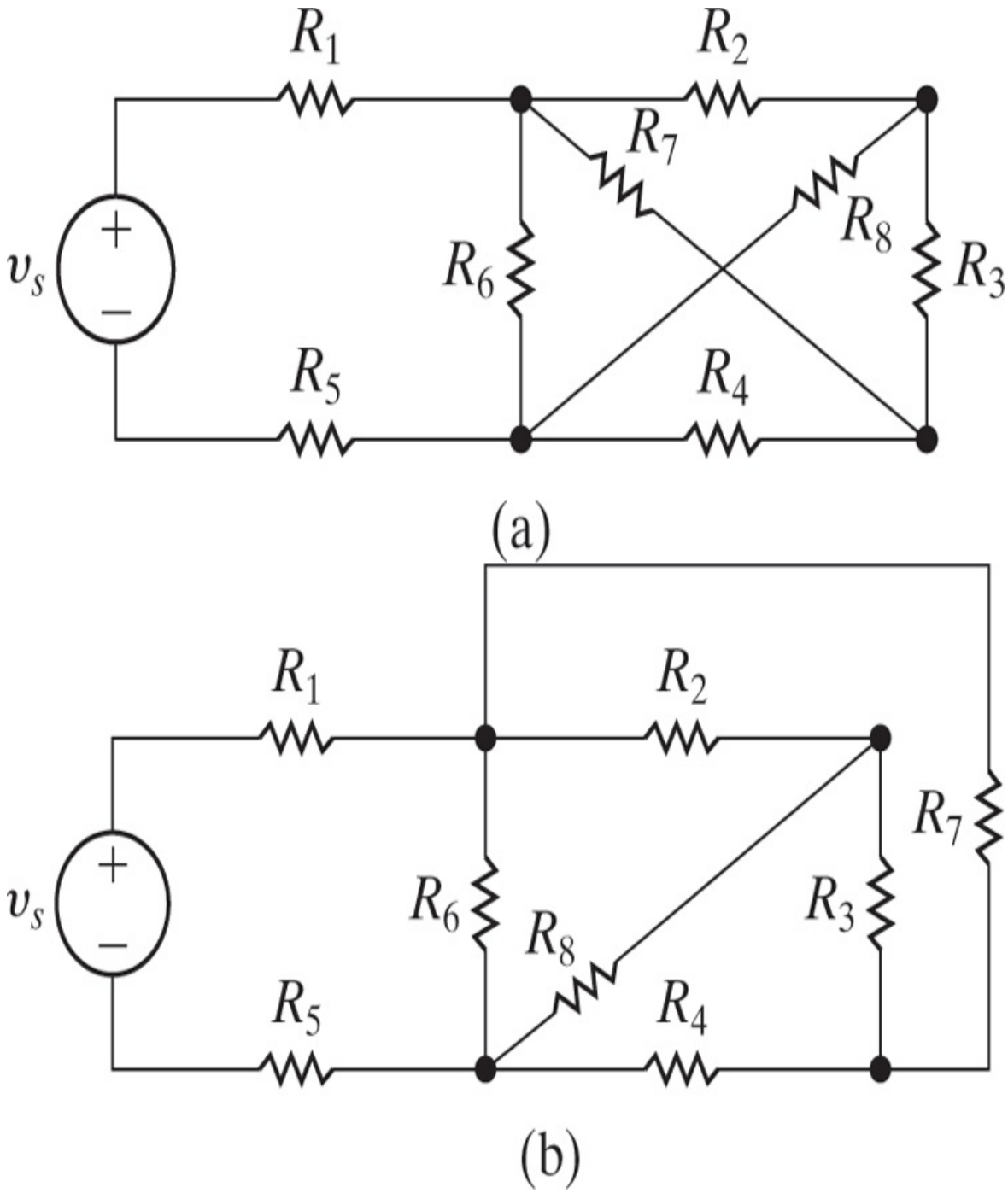


Figure 4.1 Full Alternative Text

Figure 4.2 A nonplanar circuit.

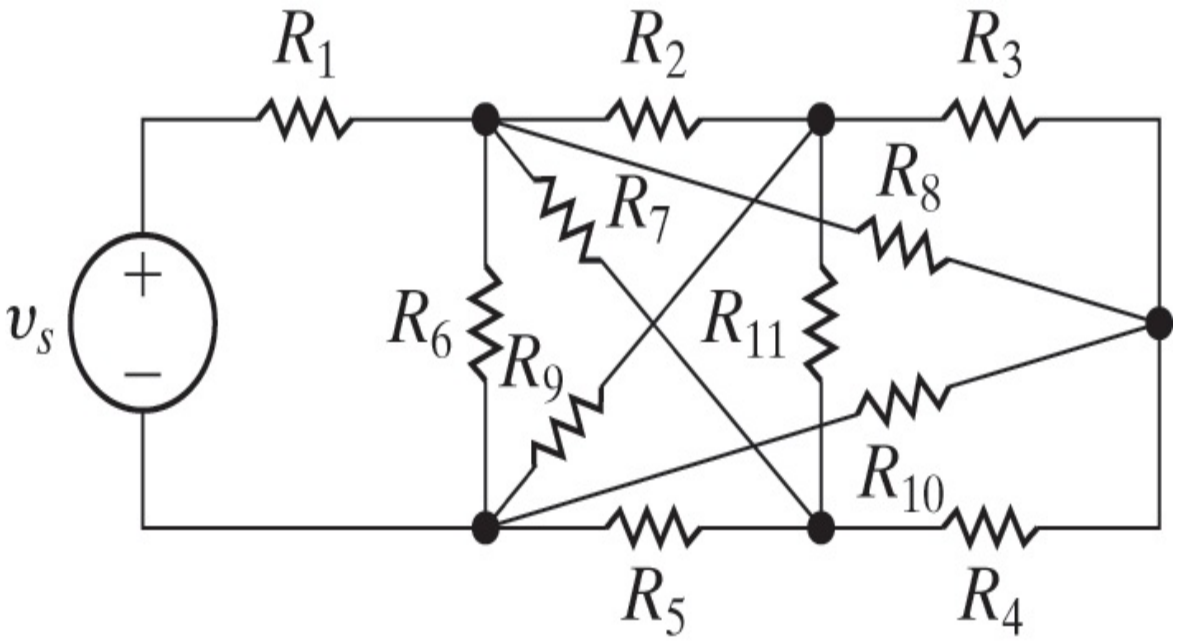


Figure 4.2 Full Alternative Text

- the node-voltage method is applicable to both planar and nonplanar circuits;
- the mesh-current method is limited to planar circuits.

Describing a Circuit—The Vocabulary

When ideal basic circuit elements (Section 1.5) are interconnected to form a circuit, the resulting interconnection is described in terms of nodes, paths, loops, branches, and meshes. Two of these terms, nodes and loops, were defined in Section 2.4. Now we'll define the rest of these terms. For your convenience, all of these definitions are presented in Table 4.1. The table also includes examples of each definition taken from the circuit in Fig. 4.3, which are developed in Example 4.1.

Table 4.1 Terms for Describing

Circuits

Name	Definition	Example from Fig. 4.3
node	A point where two or more circuit elements join	a
essential node	A node where three or more circuit elements join	b
path	A trace of adjoining basic elements with no elements included more than once	$v_1 - R_1 - R_5 - R_6$
branch	A path that connects two nodes	R_1
essential branch	A path that connects two essential nodes without passing through an essential node	$v_1 - R_1$
loop	A path whose last node is the same as the starting node	$v_1 - R_1 - R_5 - R_6 - R_4 - v_2$
mesh	A loop that does not enclose any other loops	$v_1 - R_1 - R_5 - R_3 - R_2$
planar circuit	A circuit that can be drawn on a plane with no crossing branches	Fig. 4.3 is a planar circuit. Fig. 4.2 is a nonplanar circuit.

Table 4.1 Full Alternative Text

Figure 4.3 A circuit illustrating nodes, branches, meshes, paths, and loops.

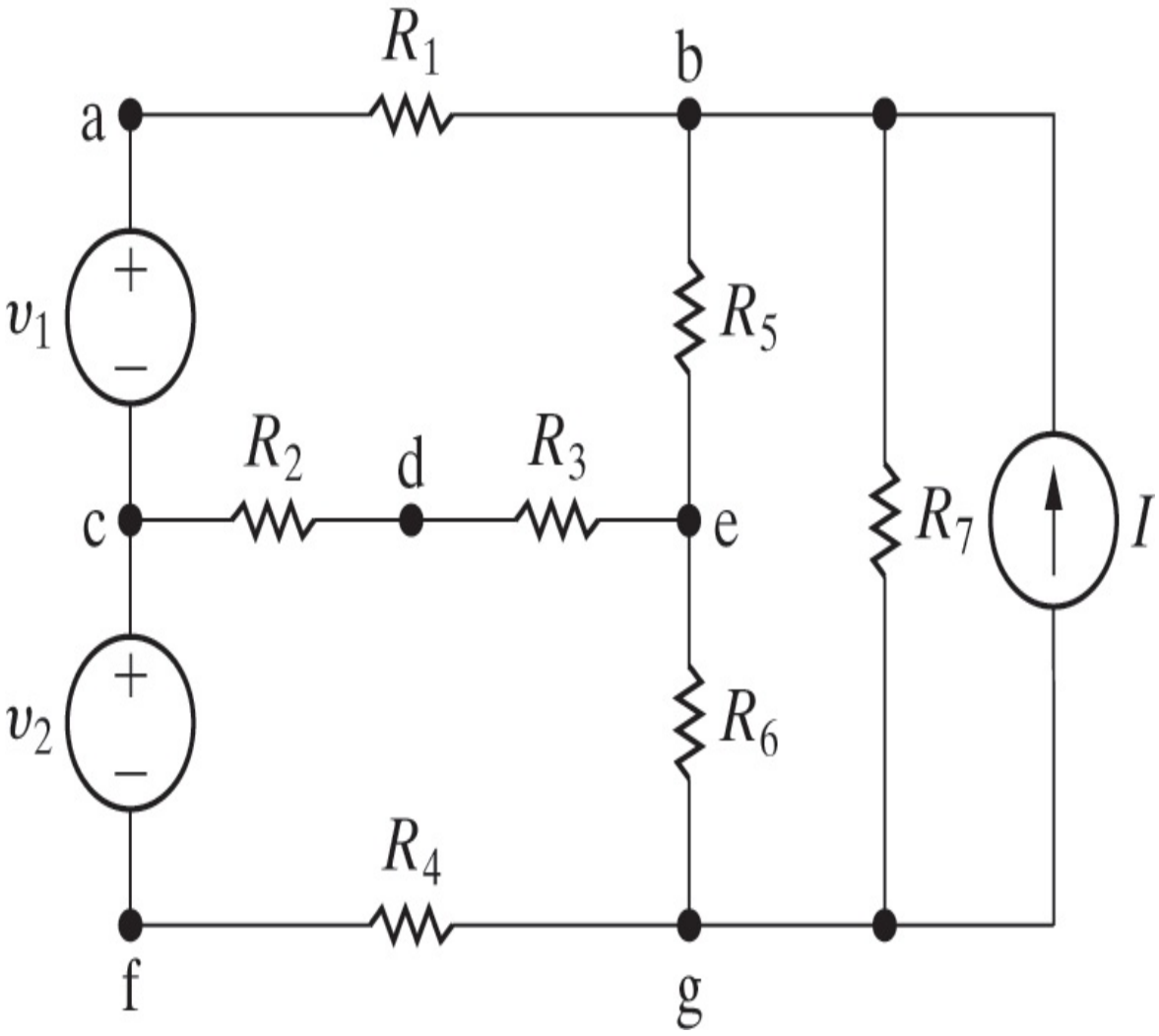


Figure 4.3 Full Alternative Text

Example 4.1 Identifying Node, Branch, Mesh, and Loop in a Circuit

For the circuit in Fig. 4.3, identify

1. all nodes.

2. all essential nodes.
3. all branches.
4. all essential branches.
5. all meshes.
6. two paths that are not loops or essential branches.
7. two loops that are not meshes.

Solution

1. The nodes are a, b, c, d, e, f, and g.
2. The essential nodes are b, c, e, and g.
3. The branches are v1, v2, R1, R2, R3, R4, R5, R6, R7, and I.
4. The essential branches are v1−R1, R2−R3, v2−R4, R5, R6, R7, and I.
5. The meshes are v1−R1−R5−R3−R2, v2−R2−R3−R6−R4, R5−R7−R6, and R7−I.
6. R1−R5−R6 is a path, but it is not a loop (because it does not have the same starting and ending nodes), nor is it an essential branch (because it does not connect two essential nodes). v2−R2 is also a path but is neither a loop nor an essential branch, for the same reasons.
7. v1−R1−R5−R6−R4−v2 is a loop but is not a mesh because there are two loops within it. I−R5−R6 is also a loop but not a mesh.

SELF-CHECK: Assess your understanding of this material by trying Chapter Problem 4.1.

Simultaneous Equations—How Many?

Recall that we need b independent equations to solve a circuit with b unknown currents. In Fig. 4.3, for example, the circuit has nine branches with unknown currents, so b=9; we need nine independent equations to solve for the unknown currents. Some of the equations can be written by applying Kirchhoff's current law (KCL) to a set of the circuit's nodes. In fact, if the circuit has n nodes, we can derive n−1 independent equations by applying Kirchhoff's current law to *any* set of n−1 nodes.[1] To obtain the rest of the needed b−(n−1) equations, we apply Kirchhoff's voltage law (KVL) to circuit loops or meshes.

1 Applying KCL to the last unused node (the nth node) does *not* generate an independent equation. See Problem 4.4.

To reduce the number of independent equations needed, we can use essential nodes and essential branches instead of nodes and branches. This is because the number of essential nodes in a circuit is less than or equal to the number of nodes, and the number of essential branches is less than or equal to the number of branches. Thus, our systematic method for writing the necessary equations to solve for the circuit's unknown currents is as follows:

- Count the number of essential nodes, ne.
- Count the number of essential branches, be, where the current is unknown.
- Write ne−1 equations by applying KCL to any set of ne−1 nodes.
- Write be−(ne−1) equations by applying KVL around a set of be−(ne−1) loops or meshes.

Remember that the voltage for each component in every loop or mesh must be known or must be described in terms of the component's current using Ohm's law.

Let’s illustrate our systematic approach by applying it to the circuit from Example 4.1, as seen in Example 4.2.

Example 4.2 Using Essential Nodes and Essential Branches to Write Simultaneous Equations

The circuit in Fig. 4.4 has six essential branches, denoted i1 − i6, where the current is unknown. Use the systematic approach to write the six equations needed to solve for the six unknown currents.

Figure 4.4 The circuit shown in Fig. 4.3 with six unknown branch currents defined.

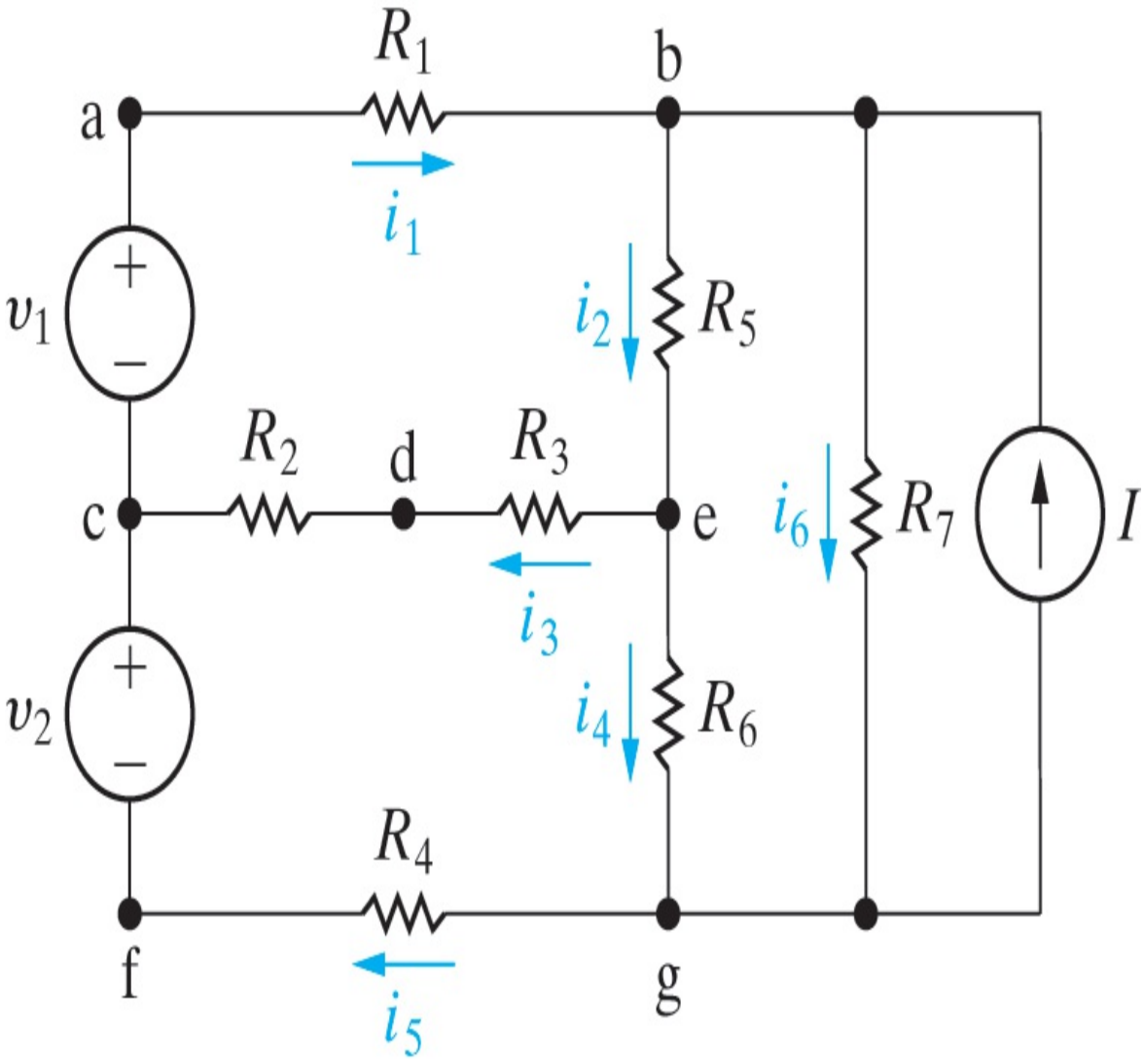


Figure 4.4 Full Alternative Text

Solution

The essential nodes in the circuit are labeled b, c, e, and g, so ne=4. From the problem statement we know that the number of essential branches where the current is unknown is be=6. Note that there are seven essential branches in the circuit, but the current in the essential branch containing the current source is known. We need to write six independent equations because there are six unknown currents.

We derive three of the six independent equations by applying Kirchhoff's current law to any three of the four essential nodes. We use the nodes b, c, and e to get

$$-i1+i2+i6-I=0, i1-i3-i5=0, i3+i4-i2=0.$$

We derive the remaining three equations by applying Kirchhoff's voltage law around three meshes. Remember that the voltage across every component in each mesh must be known or must be expressed as the product of the component's resistance and its current using Ohm's law. Because the circuit has four meshes, we need to dismiss one mesh. We eliminate the R7−I mesh because we don't know the voltage across I.[2] Using the other three meshes gives

2 We say more about this decision in Section 4.7.

$$R1i1+R5i2+i3(R2+R3)-v1=0, -i3(R2+R3)+i4R6+i5R4-v2=0, -i2R5+i6R7-i4$$

Rearranging the six equations to facilitate their solution yields the set

$$-i1+i2+0i3+0i4+0i5+i6=I, i1+0i2-i3+0i4-i5+0i6=0, 0i1-i2+i3+i4+0i5+0i6=0,$$
$$(R2+R3)i3+0i4+0i5+0i6=v1, 0i1+0i2-(R2+R3)i3+R6i4+R4i5+0i6=v2, 0i1-R5$$

SELF-CHECK: Assess your understanding of this material by trying Chapter Problems 4.2, 4.3, and 4.5.

4.2 Introduction to the Node-Voltage Method

The **node-voltage method** generates independent simultaneous equations by applying Kirchoff's current law at the essential nodes of the circuit. Solving the simultaneous equations yields the voltage drops between all but one of the essential nodes and a reference essential node. You can use these voltages to calculate voltages and currents for every component in the circuit, thereby solving the circuit. We illustrate the step-by-step procedure using the circuit in Fig. 4.5.

Figure 4.5 A circuit used to illustrate the node-voltage method of circuit analysis.

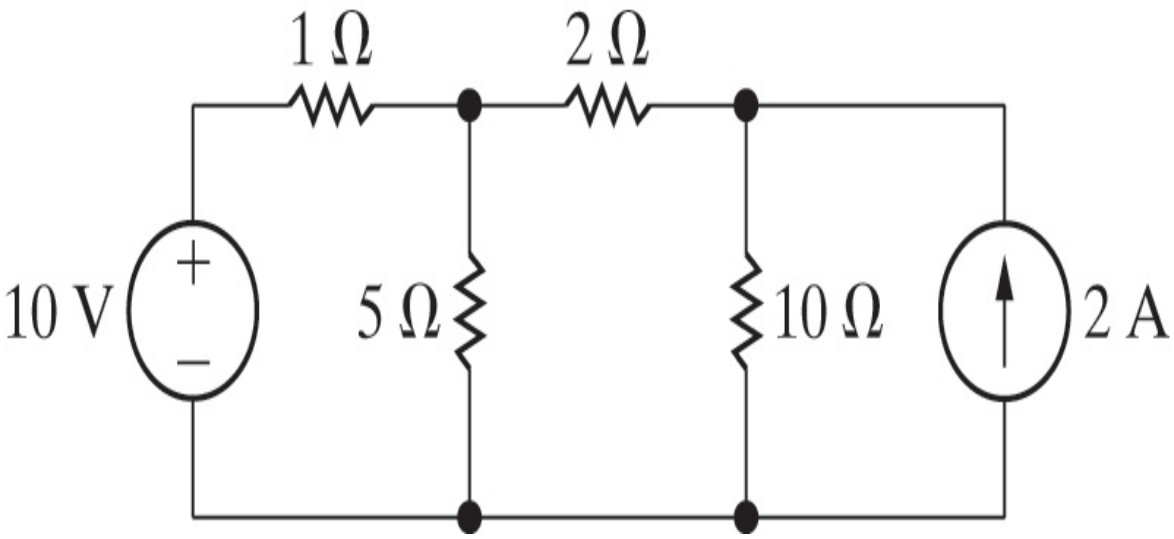


Figure 4.5 Full Alternative Text

Step 1 is to make a neat layout of the circuit so that no branches cross and to

mark the essential nodes on the circuit diagram, as in Fig. 4.6. This circuit has three essential nodes (ne=3); therefore, we need two (ne−1) KCL equations to describe the circuit.

Figure 4.6 The circuit shown in Fig. 4.5 with a reference node and the node voltages.

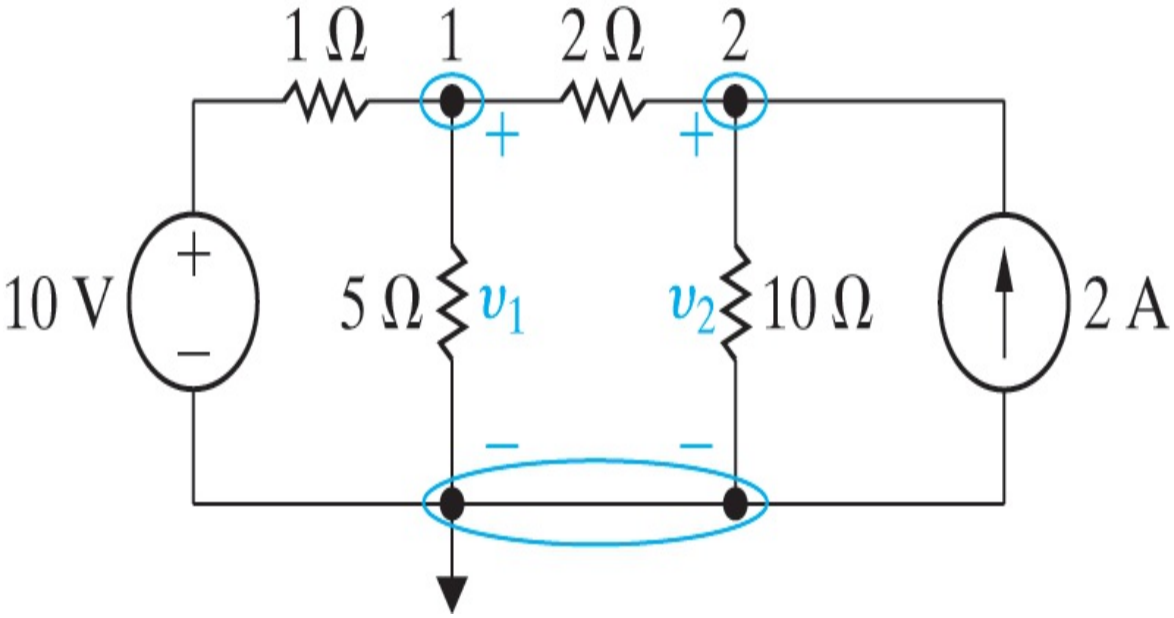


Figure 4.6 Full Alternative Text

Step 2 is to select one of the three essential nodes as a reference node. Although in theory the choice is arbitrary, there is often an obvious and practical choice. Choosing the reference node becomes easier with practice. For example, the node with the most branches is usually a good choice, so we select the lower node in Fig. 4.5 as the reference node. The reference node is identified by the symbol ▼, as in Fig. 4.6. Complete this step by labeling the remaining essential node voltages on the circuit diagram. A **node voltage** is defined as the voltage rise from the reference node to a nonreference essential node. For this circuit, we must define two node voltages, which are denoted

v1 and v2 in Fig. 4.6.

In **Step 3** we generate the KCL equations. To do this, write the current leaving each branch connected to a nonreference node as a function of the node voltages and sum the currents to zero in accordance with Kirchhoff's current law. Let's look at node 1. Ohm's law tells us that the current leaving node 1 through the 1 Ω resistor equals the voltage across the resistor (v1 – 10) divided by its resistance (1 Ω). That is, the current equals (v1−10)/1 . Figure 4.7 depicts these observations. Repeating this reasoning, the current leaving node 1 through the 5 Ω resistor is v1/5, and the current leaving node 1 through the 2 Ω resistor is (v1−v2)/2. Because the sum of the three currents leaving node 1 must equal zero, we can write the KCL equation at node 1 as

Figure 4.7 Computation of the branch current i.

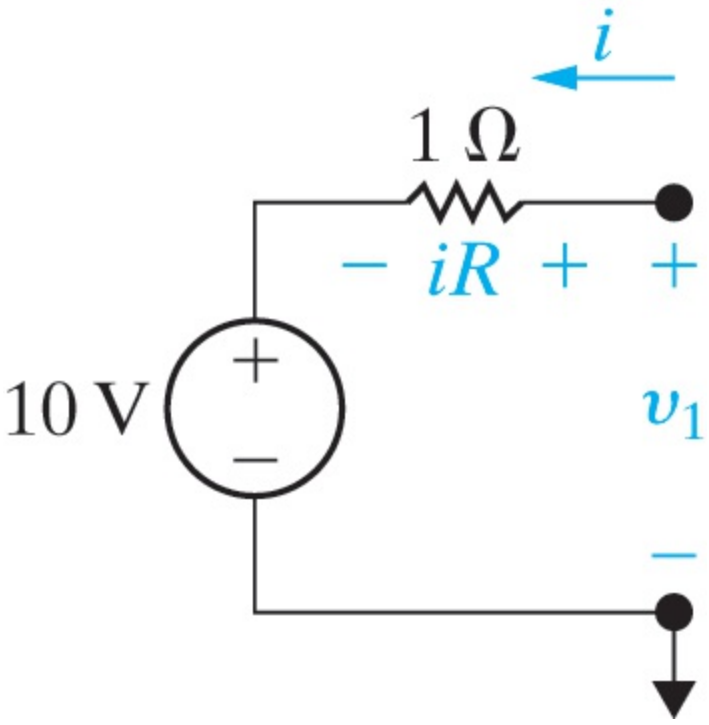


Figure 4.7 Full Alternative Text

v1−101+v15+v1−v22=0. (4.1)

Repeating the process for node 2 gives

$$v2-v12+v210-2=0. \quad (4.2)$$

Note that the first term in Eq. 4.2 is the current leaving node 2 through the 2 Ω resistor, the second term is the current leaving node 2 through the 10 Ω resistor, and the third term is the current leaving node 2 through the current source.

In **Step 4**, solve the simultaneous equations (see Appendix A). Equations 4.1 and 4.2 are the two simultaneous equations that describe the circuit in terms of the node voltages v1 and v2. Solving for v1 and v2 yields

$$v1=10011=9.09 \text{ V}, v2=12011=10.91 \text{ V}.$$

Step 5 uses the node voltages to solve for the remaining unknowns in the circuit. Once the node voltages are known, all branch currents can be calculated. Once these are known, the component voltages and powers can be calculated.

A condensed version of the node-voltage method is shown in Analysis Method 4.1. To practice the node-voltage method, work through Example 4.3.

Node-Voltage Method

1. **Identify each essential node.**
2. **Pick and label a reference node;** then label the node voltages at the remaining essential nodes.
3. **Write a KCL equation** for every nonreference essential node.
4. **Solve the equations** to find the node-voltage values.
5. **Solve the circuit** using node voltages from Step 4 to find component currents, voltages, and power values.

Analysis Method 4.1 The basic version of the node-voltage method.

Example 4.3 Using the Node-Voltage Method

1. Use the node-voltage method of circuit analysis to find the branch currents ia, ib, and ic in the circuit shown in Fig. 4.8.

Figure 4.8 The circuit for Example 4.3.

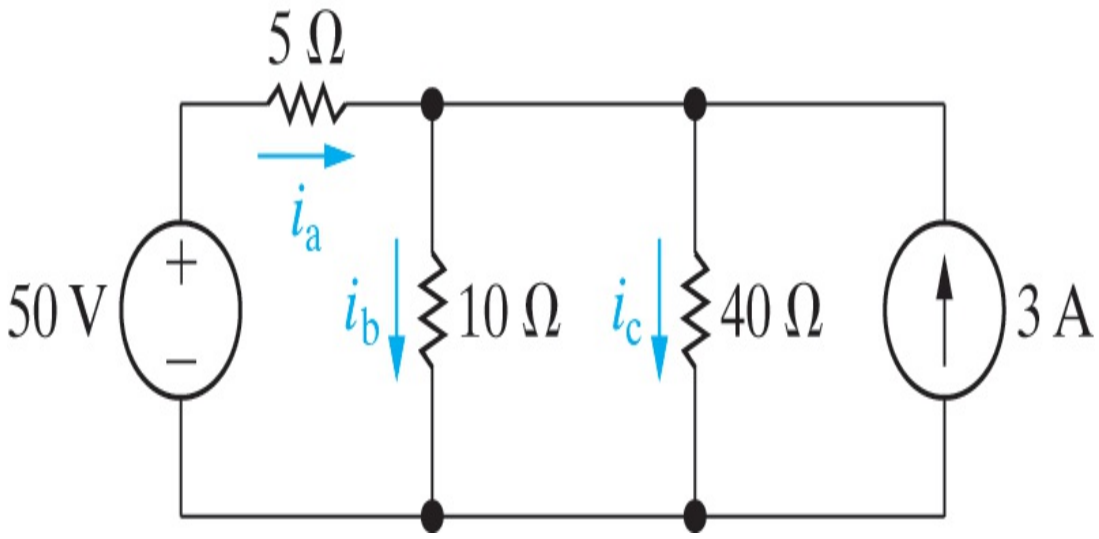


Figure 4.8 Full Alternative Text

2. Find the power associated with each source, and state whether the source is delivering or absorbing power.

Solution

1. We begin by noting that the circuit has two essential nodes; thus, we need to write a single KCL equation. **Step 1**: Identify the two essential nodes. **Step 2**: Select the lower node as the reference node and define the unknown node voltage as v1. Figure 4.9 illustrates these decisions. **Step 3**: Write a KCL equation at the nonreference essential node by summing the currents leaving node 1:

Figure 4.9 The circuit shown in Fig. 4.8 with a reference node and the unknown node voltage v1.

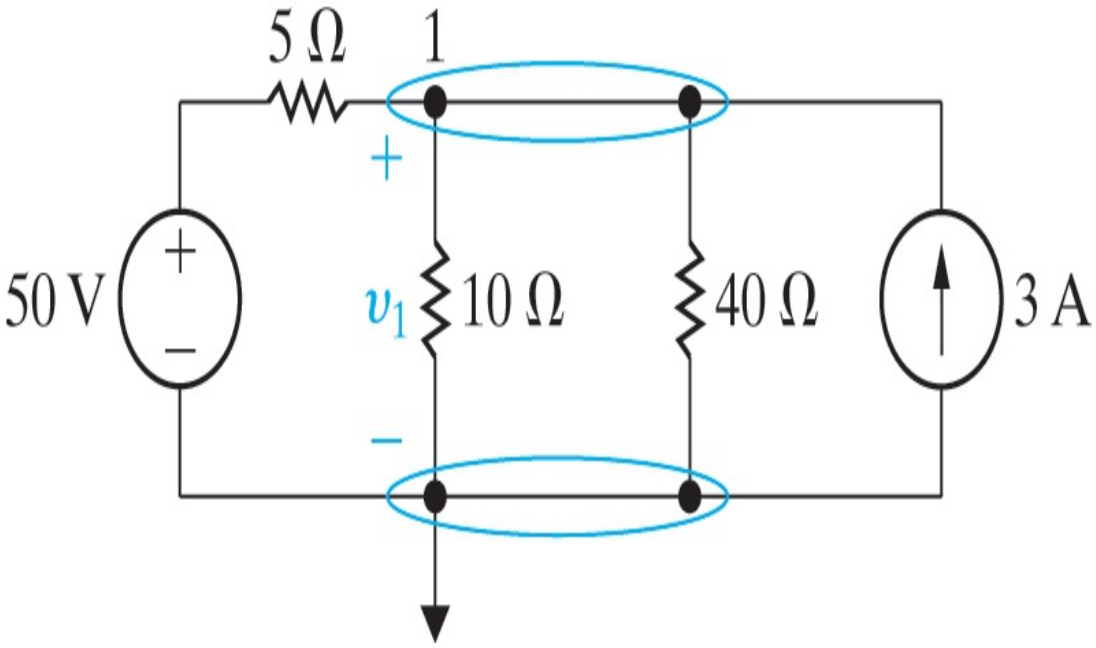


Figure 4.9 Full Alternative Text

v1−505+v110+v140−3=0.

Step 4: Solve the equation for v1, giving

v1=40 V.

Step 5: Use the node voltage v1 and Ohm's law to find the requested branch currents:

$ia=50-v15=50-405=2$ A,$ib=v110=4010=4$ A,$ic=v140=4040=1$ A.

2. The power associated with the 50 V source is

   $p50V=-50ia=-100$ W (delivering).

   The power associated with the 3 A source is

   $p3A=-3v1=-3(40)=-120$ W (delivering).

   We check these calculations by noting that the total delivered power is 220 W. The total power absorbed by the three resistors is 4(5)+16(10)+1(40) or 220 W, which equals the total delivered power.

Assessment Problems

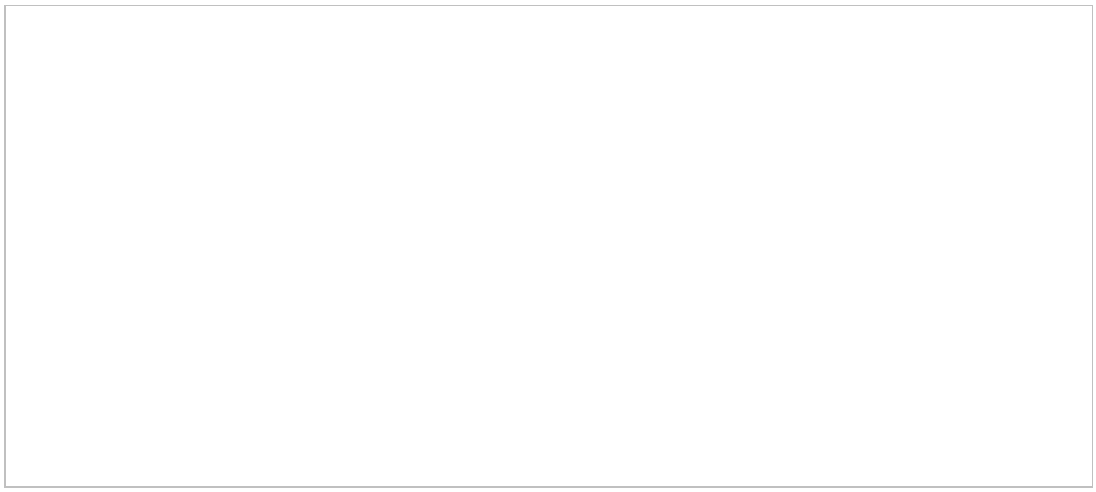
Objective 1—Understand and be able to use the node-voltage method

1. 4.1

   1. a) For the circuit shown, use the node-voltage method to find v1, v2, and i1.

   2. b) How much power is delivered to the circuit by the 15 A source?

   3. c) Repeat (b) for the 5 A source.

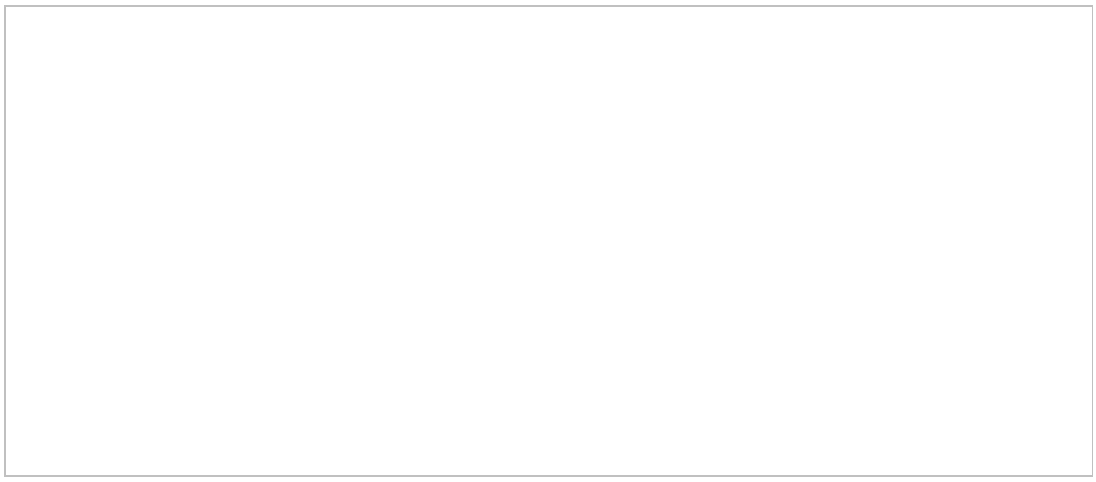


4.2-5 Full Alternative Text

Answer:

1. (a) 60 V, 10 V, 10 A;
2. (b) 900 W;
3. (c) −50 W.

2. 4.2 Use the node-voltage method to find v in the circuit shown.



4.2-6 Full Alternative Text

Answer:

15 V.

SELF-CHECK: Also try Chapter Problems 4.6, 4.10, and 4.12.

4.3 The Node-Voltage Method and Dependent Sources

If the circuit contains dependent sources, the KCL equations must be supplemented with the constraint equations imposed by the dependent sources. We will modify Step 3 in the node-voltage method to accommodate dependent sources. Example 4.4 illustrates the application of the node-voltage method to a circuit containing a dependent source.

Example 4.4 Using the Node-Voltage Method with Dependent Sources

Use the node-voltage method to find the power dissipated in the 5 Ω resistor in the circuit shown in Fig. 4.10.

Figure 4.10 The circuit for Example 4.4.

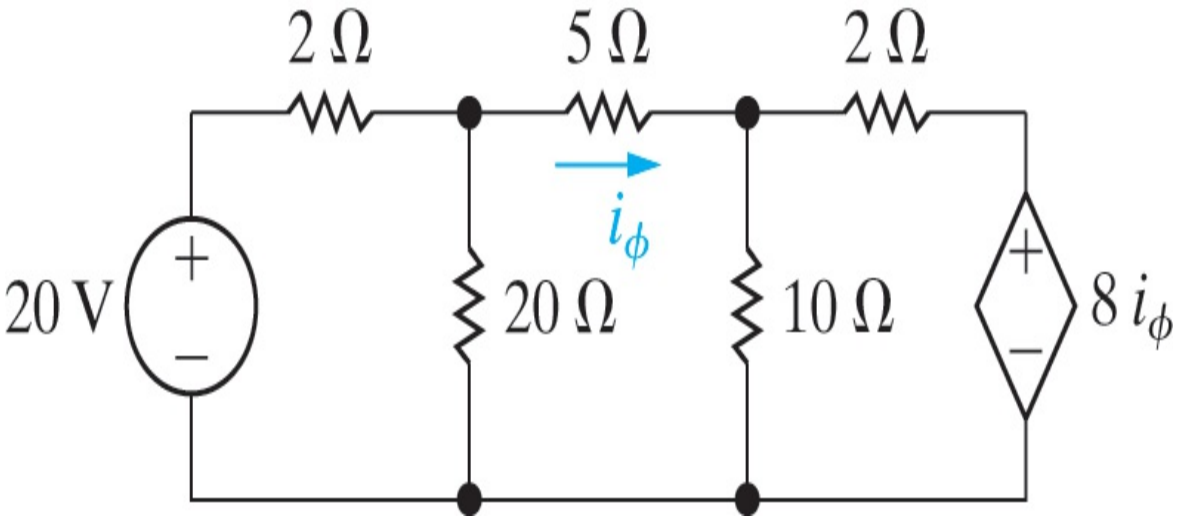


Figure 4.10 Full Alternative Text

Solution

1. Step 1: Identify the circuit's three essential nodes. We will need two KCL equations to describe the circuit.

2. Step 2: Since four branches terminate on the lower node, we select it as the reference node and label the node voltages at the remaining essential nodes. The results of the first two steps are shown in Fig. 4.11.

Figure 4.11 The circuit shown in Fig. 4.10, with a reference node and the node voltages.

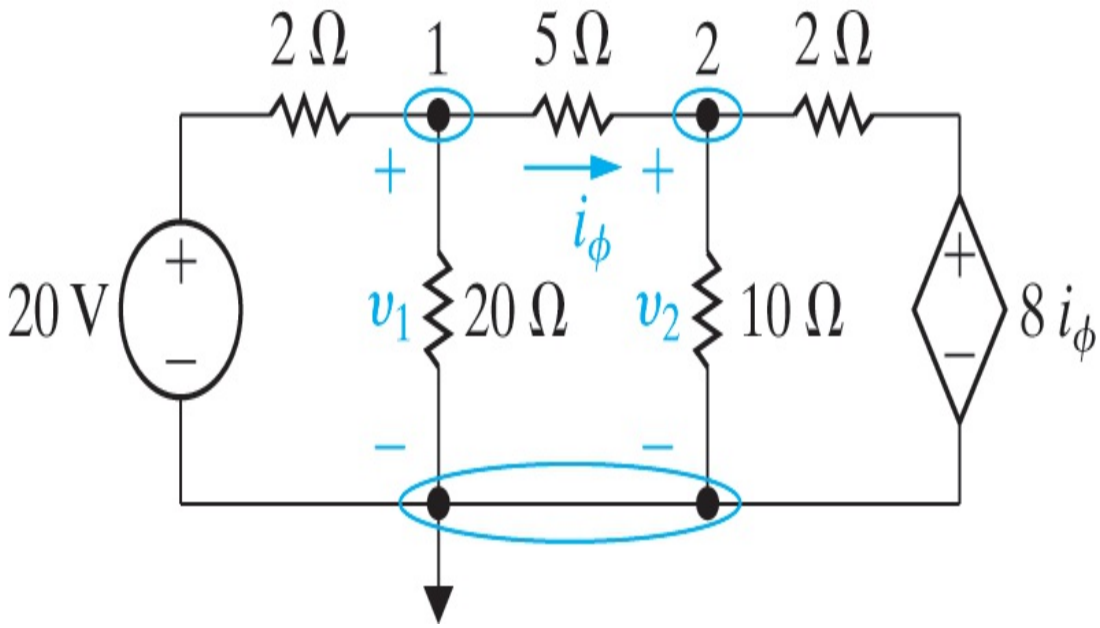


Figure 4.11 Full Alternative Text

3. Step 3: Generate the simultaneous equations by applying KCL at the nonreference essential nodes. Summing the currents leaving node 1 gives the equation

   v1−202+v120+v1−v25=0.

   Summing the currents leaving node 2 yields

   v2−v15+v210+v2−8iϕ2=0.

   As written, these two node-voltage equations contain three unknowns, namely, v1, v2, and iϕ. We need a third equation, which comes from the constraint imposed by the dependent source. This equation expresses the controlling current of the dependent source, iϕ, in terms of the node voltages, or

   iϕ=v1−v25.

   As you can see, we need to modify Step 3 in the node-voltage procedure to remind us to write a constraint equation whenever a dependent source is present.

4. Step 3: Write a KCL equation at each nonreference essential node. If the circuit contains dependent sources, write a dependent source constraint equation that defines the controlling voltage or current of the dependent source in terms of the node voltages.

   The condensed form for Step 3 is shown in Analysis Method 4.2.

Node-Voltage Method

3. **Write a KCL equation** for every nonreference essential node.
   - If there are dependent sources, write a constraint equation for each one.

Analysis Method 4.2 Modified Step 3 for the node-voltage method.

5. Step 4: Solve for v1, v2, and iϕ, giving

   v1=16 V, v2=10 V, and iϕ=1.2 A.

6. Step 5: Use the node voltage values to find the current in the 5 Ω resistor and the power dissipated in that resistor:

   iφ=v1−v25=16−105=1.2 A,
   p5Ω=5iφ2=5(1.2)2=7.2 W.

   A good exercise to build your problem-solving intuition is to reconsider this example, using node 2 as the reference node. Does it make the analysis easier or harder?

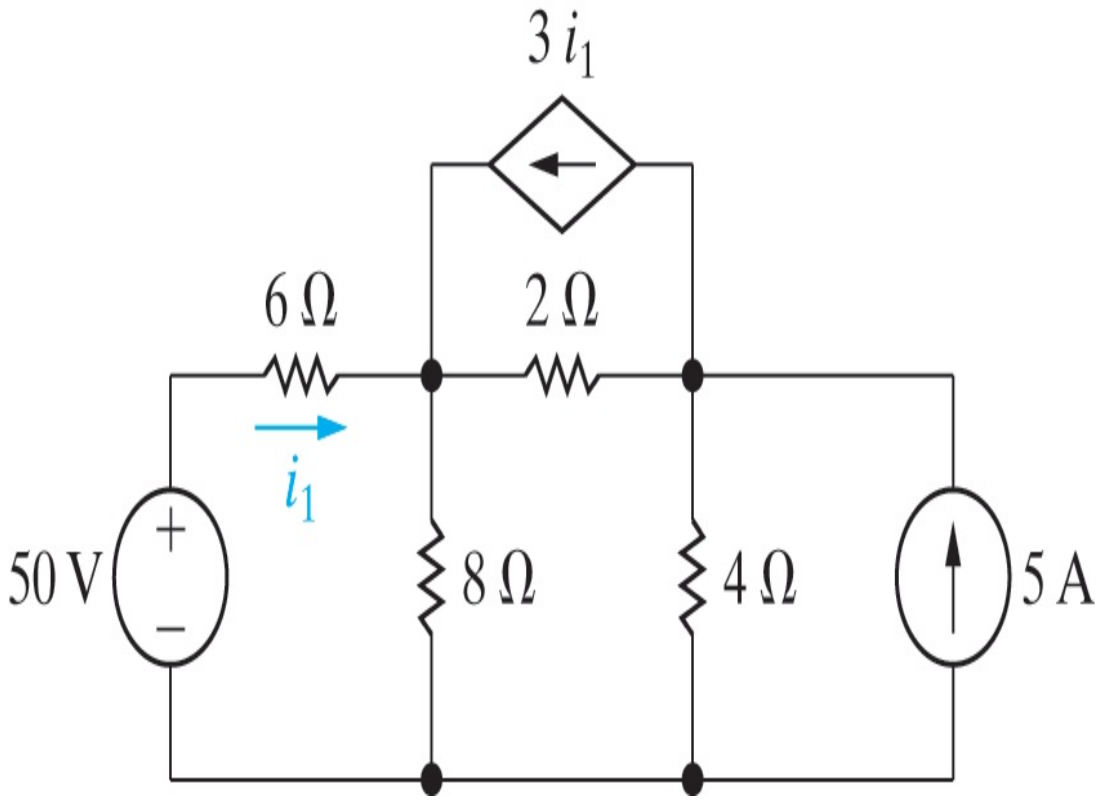
Assessment Problem

Objective 1—Understand and be able to use the node-voltage method

1. 4.3

1. a) Use the node-voltage method to find the power associated with each source in the circuit shown.

2. b) State whether the source is delivering power to the circuit or extracting power from the circuit.



4.3-7 Full Alternative Text

Answer:

1. (a) p50V=−150 W, p3i1=−144 W, p5A=−80 W;

2. (b) all sources are delivering power to the circuit.

SELF-CHECK: Also try Chapter Problems 4.17 and 4.19.

4.4 The Node-Voltage Method: Some Special Cases

Let’s explore the special case in which a voltage source is the only element between two essential nodes. The five-step node-voltage method still applies as long as we modify Step 2.

As an example, let’s look at the circuit in Fig. 4.12. There are three essential nodes in this circuit, which means two simultaneous equations are needed. Apply Steps 1 and 2 to identify the essential nodes, choose a reference node, and label the remaining nodes. Notice that the 100 V source constrains the voltage between node 1 and the reference node to 100 V. We now modify Step 2 in order to take advantage of this simplification.

Figure 4.12 A circuit with a known node voltage.

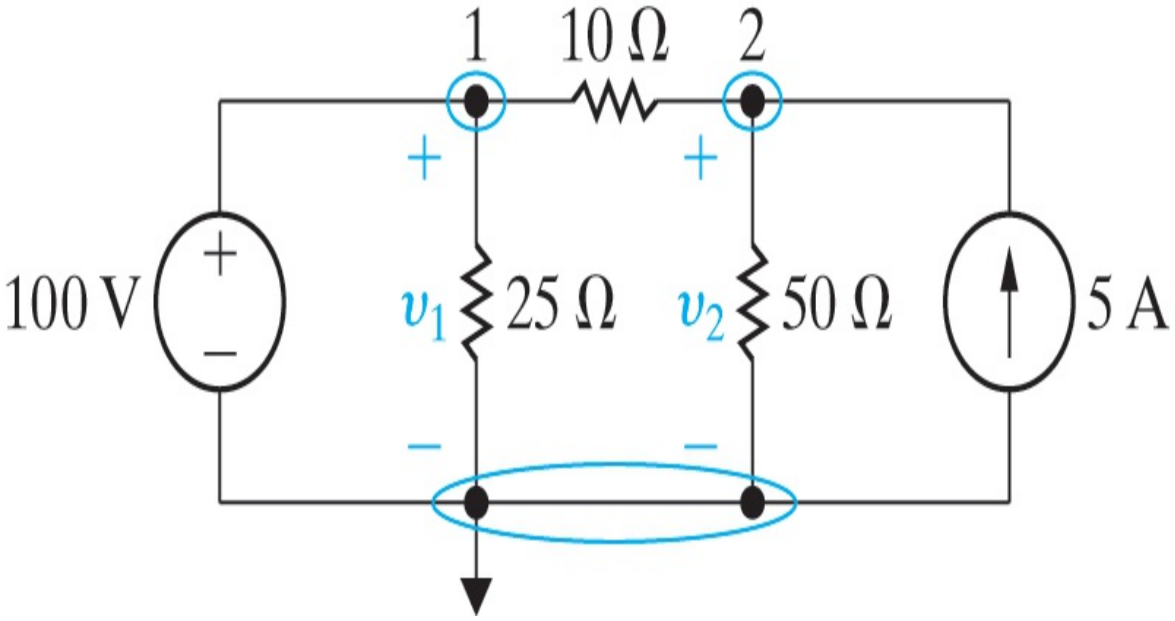


Figure 4.12 Full Alternative Text

Step 2: Pick and label a reference node, then label the node voltages at the remaining essential nodes. If a voltage source is the only element between an essential node and the reference node, replace the node voltage label with the value of the voltage source.

Thus, we can replace the voltage v1 in the circuit with its value, 100 V, as shown in Fig. 4.13.

Figure 4.13 The circuit in Fig. 4.12 with the node voltage v1 replaced with its value, 100 V.

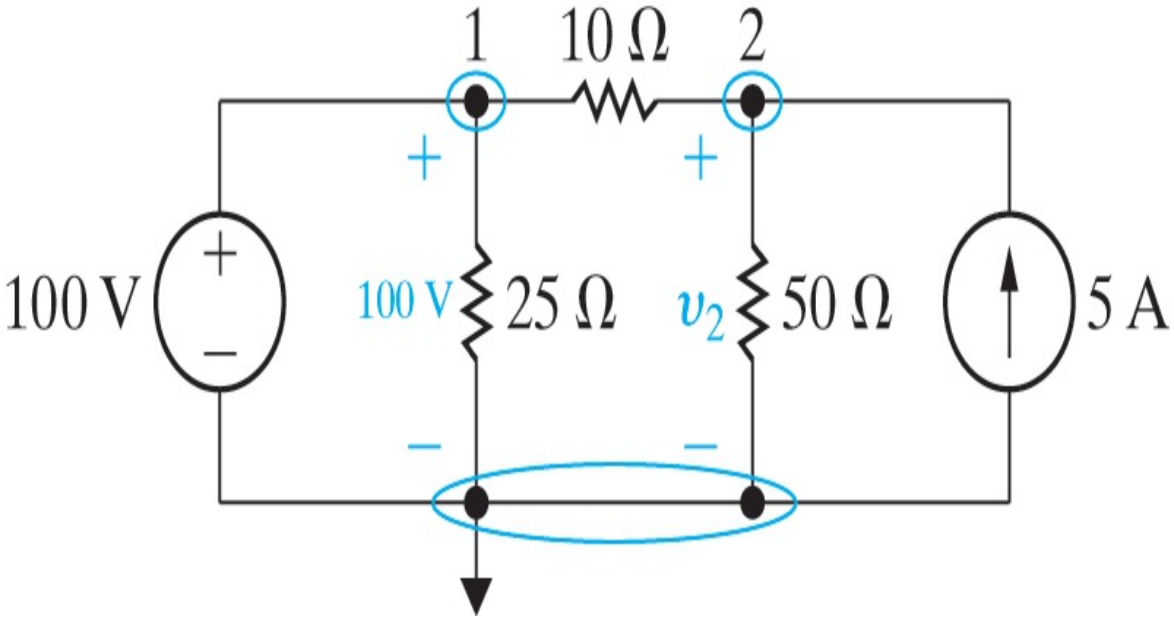


Figure 4.13 Full Alternative Text

Now Step 3 requires only a single KCL equation at node 2:

v2−10010+v250−5=0. (4.3)

In Step 4, solve Eq. 4.3 for v2:

v2=125 V.

Knowing v2, we can calculate the current in every branch. Use Step 5 to verify that the current into node 1 in the branch containing the independent voltage source is 1.5 A.

As another example, consider the circuit shown in Fig. 4.14, which has four essential nodes and requires three simultaneous equations. However, two essential nodes are connected by an independent voltage source, and two other essential nodes are connected by a current-controlled dependent voltage source. Making simple modifications to the node-voltage method allows us to take advantage of these observations.

Figure 4.14 A circuit with a dependent voltage source connected between nodes.

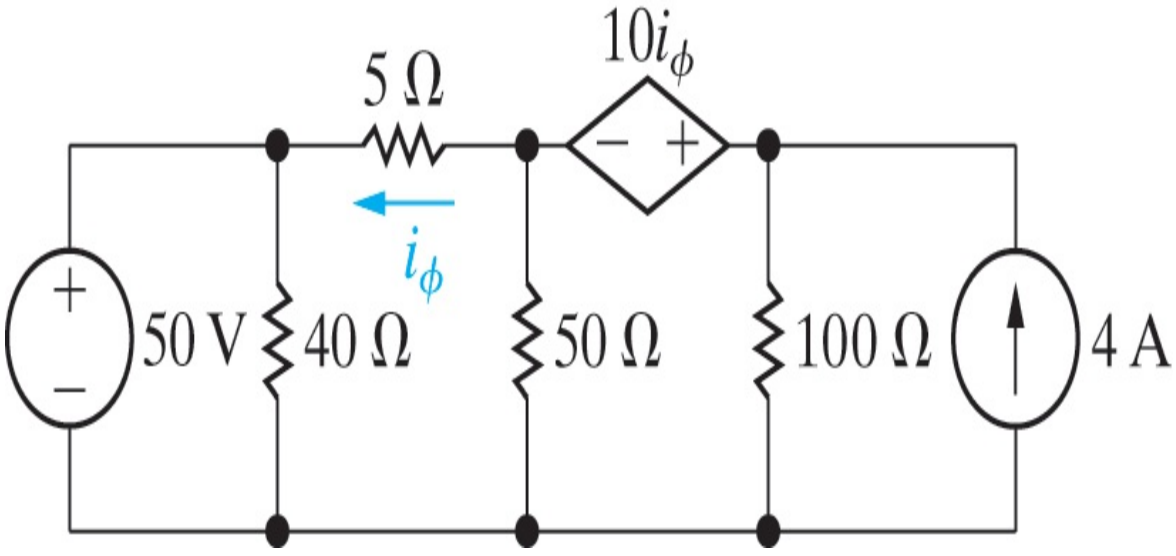


Figure 4.14 Full Alternative Text

There are several possibilities for the reference node. The nodes on each side of the dependent voltage source look attractive because, if chosen, one of the node voltages would be either +10iϕ (left node is the reference) or −10iϕ (right node is the reference). The lower node looks even better because one node voltage is immediately known (50 V) and five branches terminate there. We therefore opt for the lower node as the reference.

Figure 4.15 shows the redrawn circuit, after Steps 1 and 2. Notice that we introduced the current i because we cannot express the current in the dependent voltage source branch as a function of the node voltages v2 and v3. We write a KCL equation at node 2 to give

Figure 4.15. The circuit shown in Fig. 4.14 with the selected node voltages defined.

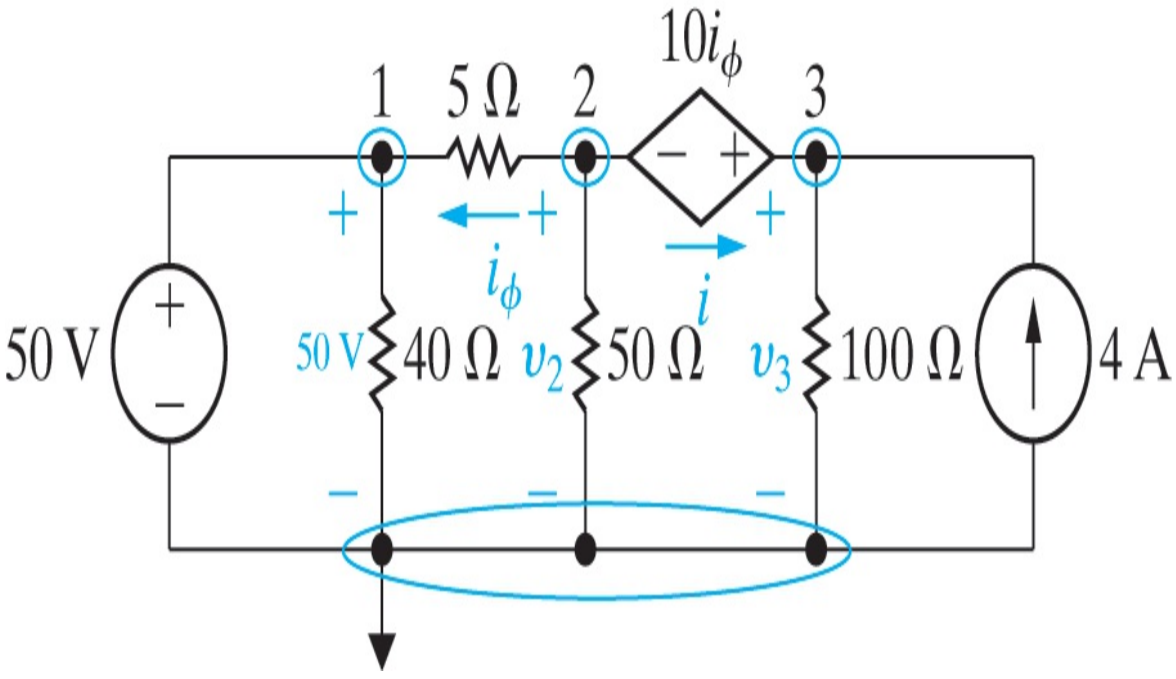


Figure 4.15. Full Alternative Text

v2−505+v250+i=0, (4.4)

and at node 3 to give

v3100−i−4=0. (4.5)

We eliminate i simply by adding Eqs. 4.4 and 4.5 to get

v2−505+v250+v3100−4=0. (4.6)

We will continue the steps of the node-voltage method after we introduce a new concept, the supernode.

The Concept of a Supernode

When a voltage source is between two essential nodes, we can combine those nodes and the source to form a **supernode**. Let’s apply the supernode concept to our circuit from Fig. 4.15. Figure 4.16 shows the circuit redrawn with a supernode created by combining nodes 2 and 3. We can remember to look for supernodes by modifying Step 2 one final time:

Figure 4.16 Combining nodes 2 and 3 to form a supernode.

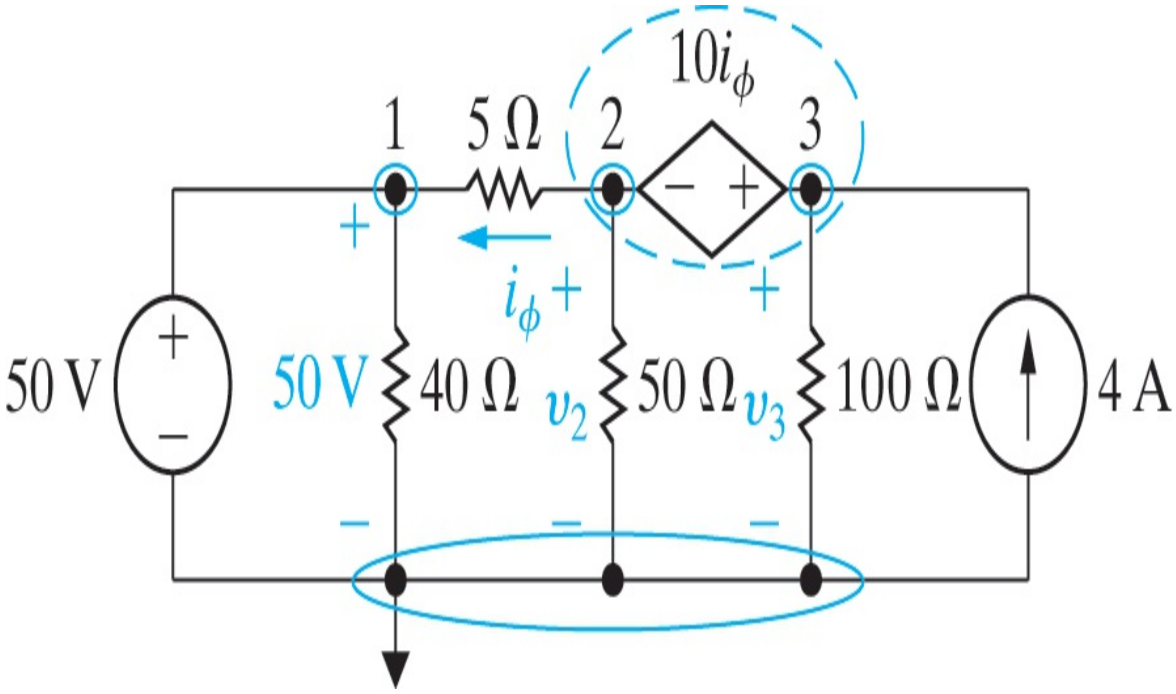


Figure 4.16 Full Alternative Text

Step 2: Pick and label a reference node, then label the node voltages at the remaining essential nodes. If a voltage source is the only element between an essential node and the reference node, replace the node voltage label with the value of the voltage source. If a voltage source is the only element between two nonreference essential nodes, combine the two essential nodes and the voltage source into a single supernode.

Step 3 also needs to be modified. Obviously, Kirchhoff's current law must hold for supernodes, so we can write a single KCL equation for the supernode. The supernode also constrains the difference between the node voltages used to create the supernode to the value of the voltage source within the supernode. We therefore need to write a supernode constraint equation. Thus, we arrive at the final version of Step 3.

Step 3: Write a KCL equation at each supernode, then write a KCL equation at each remaining nonreference essential node where the voltage is unknown. If there are any dependent sources, write a dependent source constraint equation for each one, and if there are any supernodes, write a supernode constraint equation for each one.

The final version of the node-voltage method can be found in the end-of-chapter Summary; a condensed version is given in Analysis Method 4.3.

Node-Voltage Method

1. **Identify each essential node.**
2. **Pick and label a reference node;** then label the node voltage at the remaining essential nodes.
    - If a voltage source is the only element between an essential node and the reference node, replace the node voltage label with the value of the voltage source.
    - If a voltage source is the only element between two nonreference essential nodes, combine the nodes and the source into a single supernode.
3. **Write a KCL equation** for each supernode and every remaining nonreference essential node where the voltage is unknown.
    - If there are dependent sources, write a constraint equation for each one.
    - If there are supernodes, write a constraint equation for each one.
4. **Solve the equations** to find the node voltage values and any other unknowns.
5. **Solve the circuit** using the values from Step 4 to find component currents, voltages, and power values.

Analysis Method 4.3 Complete form of the node-voltage method.

We can now use Step 3 to create the equations for the circuit in Fig. 4.16. First, write the KCL equation for the supernode. Starting with the 5 Ω branch and moving counterclockwise around the supernode, we generate the

equation

$$v2-505+v250+v3100-4=0, \quad (4.7)$$

which is identical to Eq. 4.6. Creating a supernode at nodes 2 and 3 has made writing this equation much easier. Next, write the supernode constraint equation by setting the value of the voltage source in the supernode to the difference between the two node voltages in the supernode:

$$v3-v2=10i\varphi. \quad (4.8)$$

Finally, express the current controlling the dependent voltage source as a function of the node voltages:

$$i\phi=v2-505. \quad (4.9)$$

Use Step 5 to solve Eqs. 4.7, 4.8, and 4.9 for the three unknowns v2, v3, and $i\phi$ to give

$$v2=60 \text{ V}, \qquad v3=80 \text{ V}, \qquad \text{and} \qquad i\varphi=2 \text{ A}.$$

Let’s use the node-voltage method to analyze the amplifier circuit first introduced in Section 2.5 and shown again in Fig. 4.17. When we analyzed this circuit using Ohm’s law, KVL, and KCL in Section 2.5, we faced the task of writing and solving six simultaneous equations. Work through Example 4.5 to see how the node-voltage method can simplify the circuit analysis.

Figure 4.17 The transistor amplifier circuit shown in Fig 2.24.

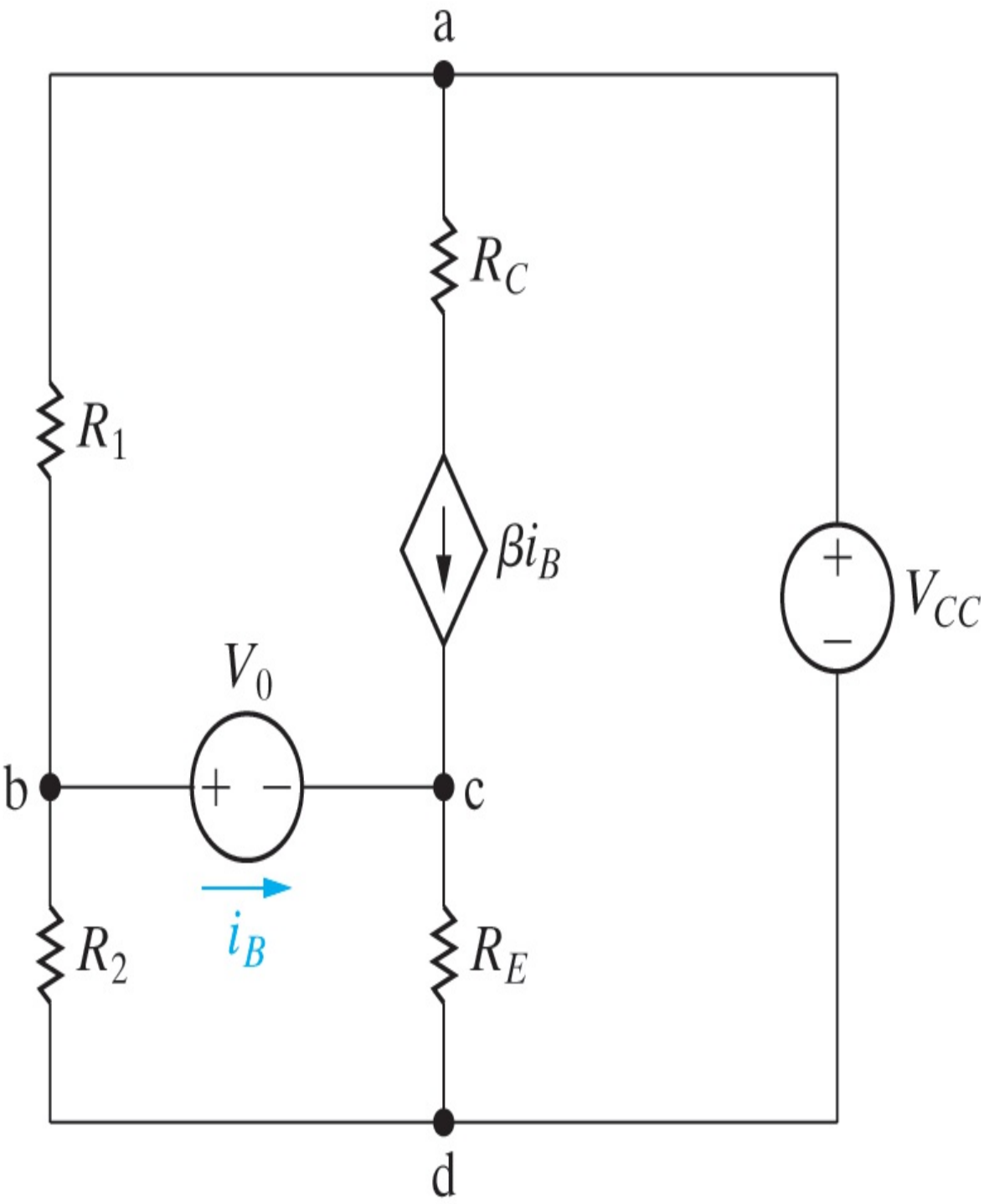


Figure 4.17 Full Alternative Text

Example 4.5 Node-Voltage Analysis

of the Amplifier Circuit

Use the node-voltage method to find iB in the amplifier circuit shown in Fig. 4.17.

Solution

Step 1: We identify the four essential nodes, which are labeled a, b, c, and d. **Step 2**: Choose node d as the reference node. Then label the voltages at the remaining three essential nodes. **Step 3**: Before writing equations, we notice two special cases. The voltage source VCC in the branch connecting node a and the reference node constrains the voltage between those nodes, so va=VCC, and the voltage source V0 in the branch between nodes b and c constrains the voltage between those nodes and creates a supernode. The results of Steps 1 and 2 and the modifications prompted by the special cases are depicted in Fig. 4.18.

Figure 4.18 The circuit shown in Fig. 4.17, with voltages and the supernode identified.

Figure 4.18 Full Alternative Text

Continuing Step 3, write the supernode KCL equation to give

$$\frac{v_b}{R_2}+\frac{v_b-V_{CC}}{R_1}+\frac{v_c}{R_E}-\beta i_B=0. \quad (4.10)$$

Now write the dependent source constraint equation, which defines the controlling current iB in terms of the node voltages. Since iB is the current in a voltage source, we cannot use Ohm's law, so instead, write a KCL equation at node c:

$$iB=vcRE-\beta iB. \quad (4.11)$$

The last part of Step 3 is the supernode constraint equation

$$vb-vc=V0. \quad (4.12)$$

Step 3 gave us three equations with three unknowns. To solve these equations, we use back-substitution to eliminate the variables vb and vc. Begin by rearranging Eq. 4.11 to give

$$iB=vcRE(1+\beta). \quad (4.13)$$

Next, solve Eq. 4.12 for vc to give

$$vc=vb-V0. \quad (4.14)$$

Substituting Eqs. 4.13 and 4.14 into Eq. 4.10 and rearranging yields

$$vb[1R1+1R2+1(1+\beta)RE]=VCCR1+V0(1+\beta)RE. \quad (4.15)$$

Solving Eq. 4.15 for vb yields

$$vb=VCCR2(1+\beta)RE+V0R1R2R1R2+(1+\beta)RE(R1+R2). \quad (4.16)$$

You should verify that, when Eq. 4.16 is combined with Eqs. 4.13 and 4.14, the solution for iB is

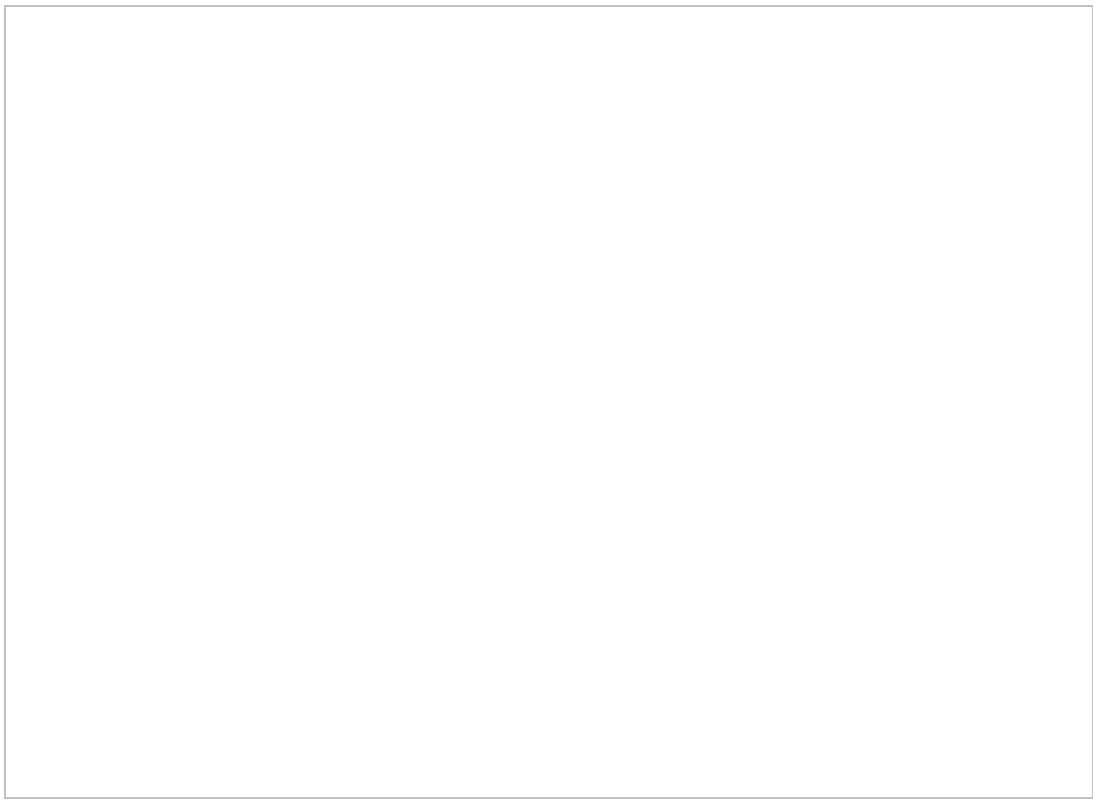
$$iB=(VCCR2)/(R1+R2)-V0(R1R2)/(R1+R2)+(1+\beta)RE, \quad (4.17)$$

which is identical to Eq. 2.21. (See Problem 4.31.) Using the node-voltage method to analyze this circuit reduces the problem from manipulating six simultaneous equations (see Problem 2.38) to manipulating three simultaneous equations.

Assessment Problems

Objective 1—Understand and be able to use the node-voltage method

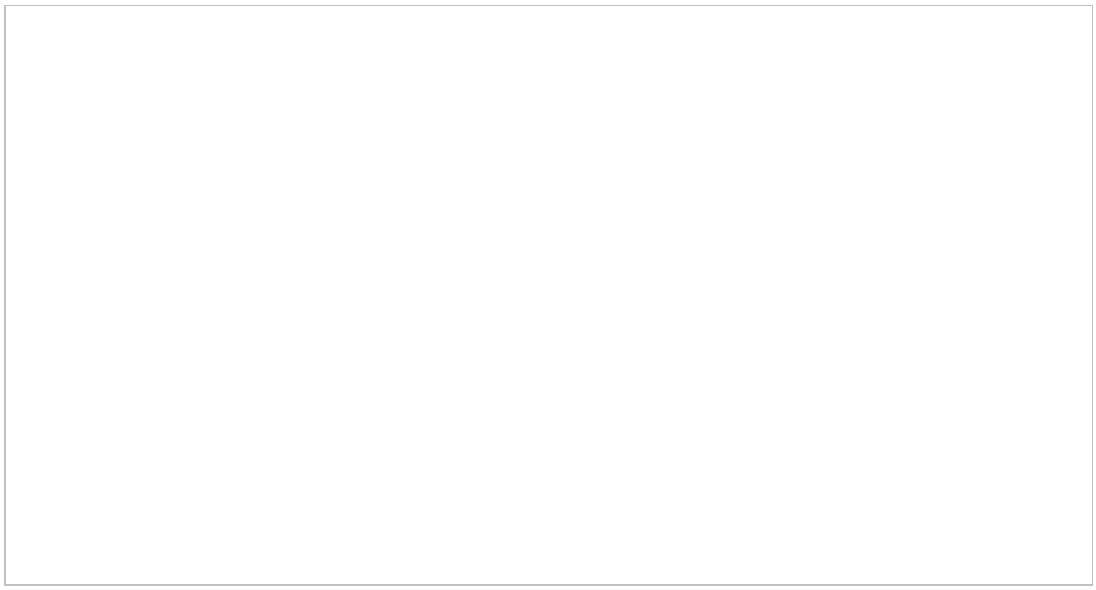
1. 4.4 Use the node-voltage method to find vo in the circuit shown.



4.4-8 Full Alternative Text

Answer: 24 V.

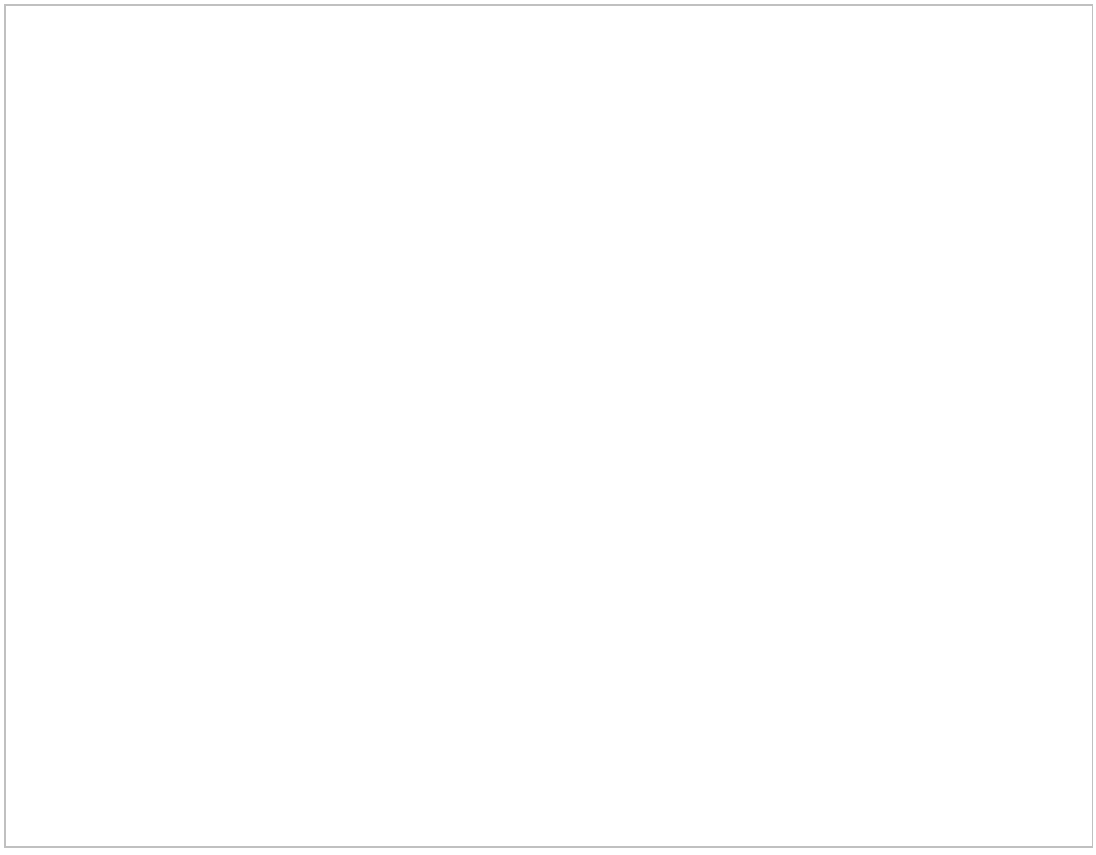
2. 4.5 Use the node-voltage method to find v in the circuit shown.



4.4-9 Full Alternative Text

Answer: 8 V.

3. 4.6 Use the node-voltage method to find v1 in the circuit shown.



4.4-10 Full Alternative Text

Answer: 48 V.

SELF-CHECK: Also try Chapter Problems 4.22, 4.27, and 4.28.

4.5 Introduction to the Mesh-Current Method

Before we learn about the mesh-current method for solving circuits, let's summarize the node-voltage method. The node-voltage method presented in Sections 4.2–4.4 is used to solve a circuit by writing simultaneous KCL equations at essential nodes. If a dependent source is present, a dependent source constraint equation is required. Special cases exist when a voltage source is the only component in a branch connecting two essential nodes. If one of the essential nodes is the reference node, the node voltage at the other node is the value of the voltage source and no KCL equation is required at that node. If neither of the essential nodes is the reference node, the two nodes and the voltage source are combined into a supernode. A KCL equation is written for the supernode as well as a supernode constraint equation. The simultaneous equations are solved to find the node voltages, and the node voltages can be used to find the voltage, current, and power for every circuit component.

We now turn to the mesh-current method, presented in Sections 4.5–4.7. The mesh-current method is used to solve a circuit by writing simultaneous KVL equations for the circuit's meshes. If a dependent source is present, a dependent source constraint equation is required. Special cases exist when a current source is a component of a mesh. If a current source is a component in only one mesh, the mesh current must have the same value as the current source and no KVL equation is needed for that mesh. If a current source is shared by two adjacent meshes, the two meshes are combined to form a supermesh. A KVL equation is written for the supermesh as well as a supermesh constraint equation. The simultaneous equations are solved to find the mesh currents, and the mesh currents can be used to find the voltage, current, and power for every circuit component.

Did you notice the symmetries in the descriptions of the node-voltage method and the mesh-current method? In engineering, this symmetry is called **duality**, and we will encounter it throughout this text. In comparing the two

circuit analysis techniques, we see that essential nodes and meshes are duals, KVL and KCL are duals, voltages and currents are duals, supernodes and supermeshes are duals, and so on. Recognizing the existence of duality can help you master the techniques for circuit analysis presented in this book.

Applying the Mesh-Current Method

Recall from the terms defined in Table 4.1 that a mesh is a loop that does not contain any other loops. You should review the definitions of loop and path in Table 4.1, too. A **mesh current** is the current that exists on the perimeter of a mesh. We represent a mesh current on a circuit diagram using a curved arrow that follows the mesh perimeter, where the arrowhead indicates the current's direction.

Just like the node-voltage method, the mesh-current method is a step-by-step procedure. Let's use the mesh-current method for the circuit shown in Fig. 4.19 to solve for the currents i1, i2, and i3. Because the circuit has two meshes, we expect to write two simultaneous equations.

Figure 4.19 A circuit used to illustrate development of the mesh-current method of circuit analysis.

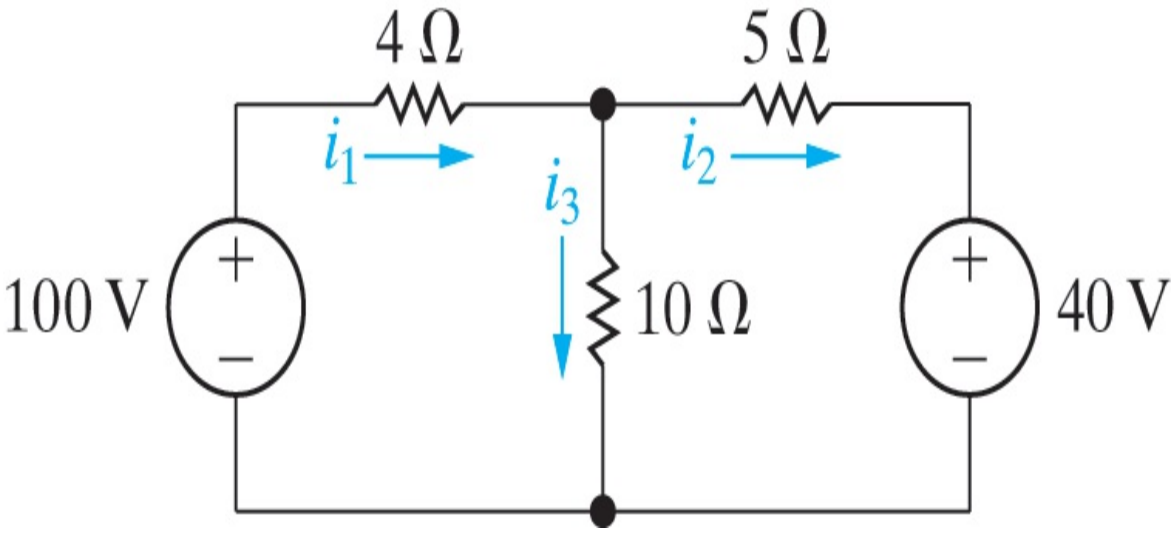


Figure 4.19 Full Alternative Text

In **Step 1**, we identify the meshes using a directed curved arrow that follows the perimeter of the mesh.

Step 2 labels the mesh currents using the labels ia and ib. The results of Steps 1 and 2 are shown in Fig. 4.20. We can see from this figure that the branch current i1 equals the mesh current ia and that the branch current i2 equals the mesh current ib. Note that to avoid confusion we use different names for the branch currents and the mesh currents in this circuit.

Figure 4.20 Mesh currents ia and ib.

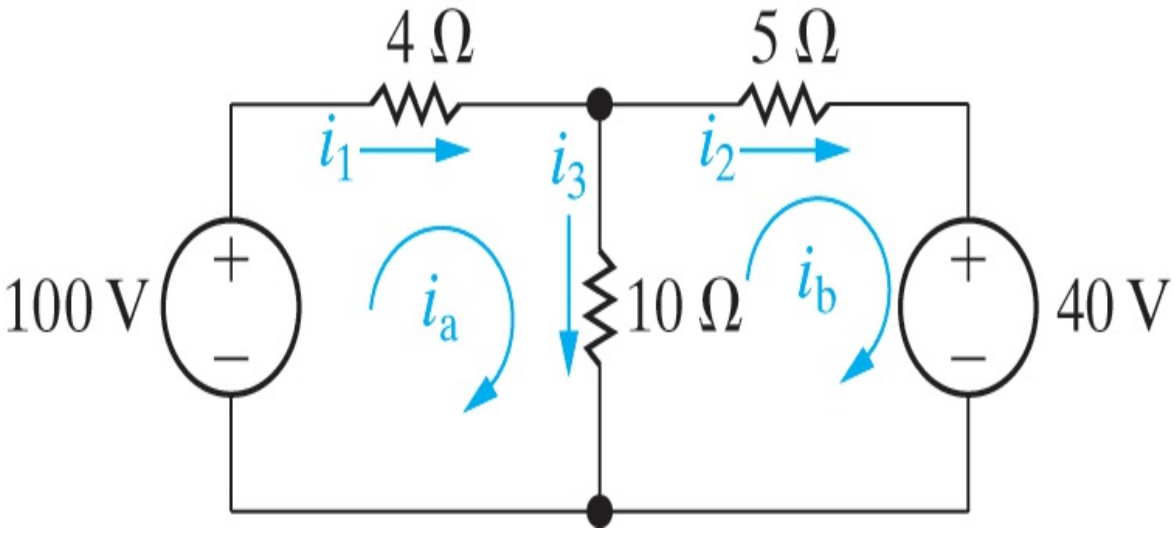


Figure 4.20 Full Alternative Text

In **Step 3,** we write the KVL equation for each mesh. Let's start with the mesh whose mesh current is ia. Pick a starting point anywhere on the perimeter of the mesh and sum the voltage drops for each component in the mesh in the direction of the mesh current until you return to the starting point. Start below the 100 V source and travel in the clockwise direction of the mesh current. The first voltage is due to the 100 V source and has the value – 100. The next voltage is across the 4 Ω resistor, whose current is ia, so from Ohm's law the voltage is 4ia. The next voltage is across the 10 Ω resistor, whose current is (ia – ib), so from Ohm's law the voltage is 10(ia – ib). We are back to the starting point, so KVL tells us that the sum of these three voltages is zero:

$-100+4ia+10(ia-ib)=0.$

Repeat this process to get the KVL equation for the ib mesh. Remember that you can start anywhere on the perimeter of this mesh, so let's start at the left of the 5 Ω resistor. Again we determine the voltage drops for each component in the direction of the ib mesh. The voltage drop for the 5 Ω resistor is 5ib, the voltage drop for the 40 V source is 40, and the voltage drop for the 10 Ω resistor is 10(ib – ia). We have returned to the starting point, so KVL tells us that the sum of these three voltages is zero:

$5ib+40+10(ib-ia)=0.$

In **Step 4,** we solve these simultaneous mesh current equations (see Appendix A) to find that the mesh current values are

$i_a = 10$ A

and

$i_b = 4$ A.

Step 5 uses the mesh currents to solve for the currents, voltages, and power for all components in the circuit. Let's calculate the three branch currents to illustrate:

$i_1 = i_a = 10$ A,
$i_2 = i_b = 4$ A,
$i_3 = i_a - i_b = 6$ A.

The ability to calculate the branch currents using the mesh currents is crucial to the mesh-current method of circuit analysis. Once you know the mesh currents, you also know the branch currents. And once you know the branch currents, you can compute any voltages or powers of interest. A condensed version of the mesh-current method is given in Analysis Method 4.4.

Mesh-Current Method

1. **Identify the meshes** with curved directed arrows that follow the perimeter of each mesh.
2. **Label the mesh currents** for each mesh.
3. **Write the KVL equations** for each mesh.
4. **Solve the KVL equations** to find the mesh current values.
5. **Solve the circuit** using mesh currents from Step 4 to find component currents, voltages, and power values.

Analysis Method 4.4 The basic version of the mesh-current method.

Example 4.6 illustrates how the mesh-current method is used to find source powers and a branch voltage.

Example 4.6 Using the Mesh-Current Method

1. Use the mesh-current method to determine the power associated with each voltage source in the circuit shown in Fig. 4.21.

Figure 4.21 The circuit for Example 4.6.

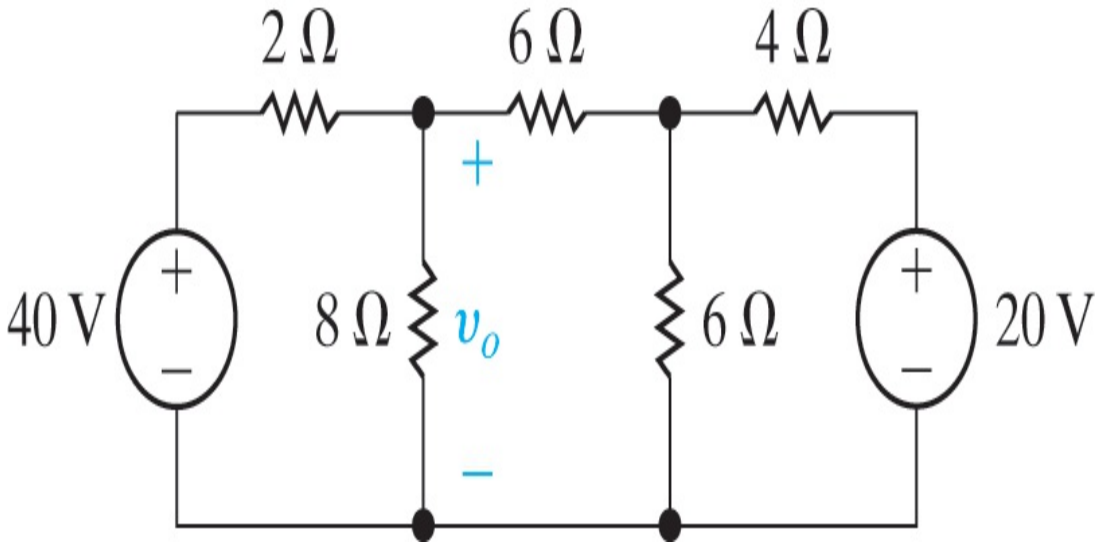


Figure 4.21 Full Alternative Text

2. Calculate the voltage vo across the 8 Ω resistor.

Solution

1. Step 1: We identify the three meshes in the circuit and draw the mesh currents as directed curved arrows following the perimeter of each mesh. It is best to define all mesh currents in the same direction.

   Step 2: Label the mesh currents; the results of the first two steps are depicted in Fig. 4.22.

Figure 4.22 The three mesh currents used to analyze the circuit shown in Fig. 4.21.

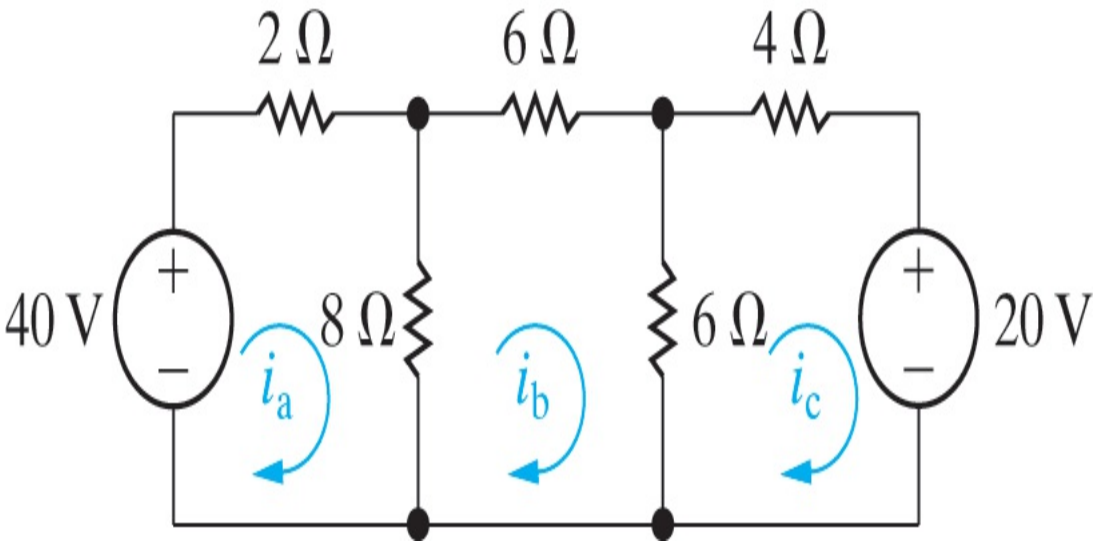


Figure 4.22 Full Alternative Text

Step 3: We use KVL to generate an equation for each mesh by summing the voltages in the direction of the mesh current. In the ia mesh, start just below the 40 V source and sum the voltages in the clockwise direction to give

$-40+2ia+8(ia-ib)=0.$

In the ib mesh, start below the 8 Ω resistor and sum the voltages in the clockwise direction to give

$8(ib-ia)+6ib+6(ib-ic)=0.$

In the ic mesh, start below the 6 Ω resistor and sum the voltages in the clockwise direction to give

$6(ic-ib)+4ic+20=0.$

Step 4: Solve the three simultaneous mesh current equations from Step 3 to give

$ia=5.6\ A, ib=2.0\ A, ic=-0.80\ A.$

Step 5: Use the mesh currents to find the power for each source. The mesh current ia equals the branch current in the 40 V source, so the power associated with this source is

$p_{40V}=-40ia=-224\ W.$

The minus sign means that this source is delivering power to the network. The current in the 20 V source equals the mesh current ic; therefore

$p_{20\ V}=20ic=-16\ W.$

The 20 V source also is delivering power to the network.

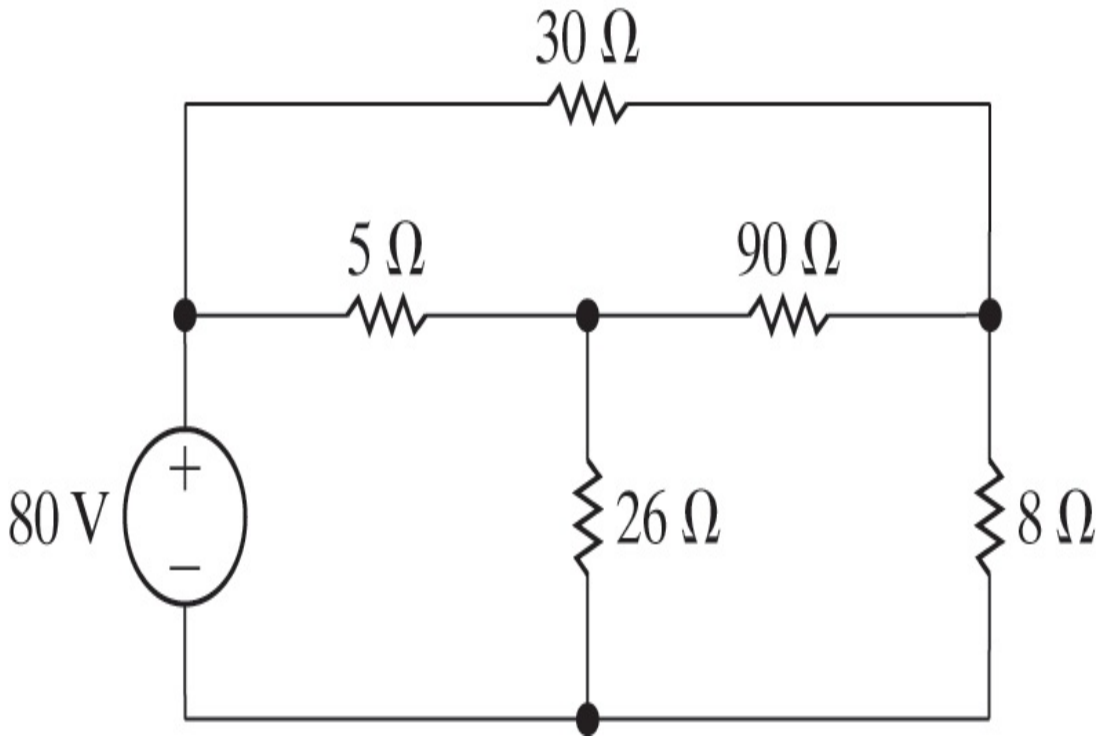
2. The branch current in the 8 Ω resistor in the direction of the voltage drop vo is ia−ib. Therefore

   $vo=8(ia-ib)=8(3.6)=28.8\ V.$

Assessment Problem

Objective 2—Understand and be able to use the mesh-current method

1. 4.7 Use the mesh-current method to find (a) the power delivered by the 80 V source to the circuit shown and (b) the power dissipated in the 8 Ω resistor.



4.5-11 Full Alternative Text

Answer:

1. (a) 400 W;
2. (b) 50 W.

SELF-CHECK: Also try Chapter Problems 4.36 and 4.37.

4.6 The Mesh-Current Method and Dependent Sources

If the circuit contains dependent sources, we must modify the equation-writing step in the mesh-current analysis method, just as we did in the node-voltage analysis method.

Step 3: Write the KVL equation for each mesh; if the circuit contains a dependent source, write a dependent source constraint equation that defines the controlling variable for the dependent source in terms of the mesh currents.

The condensed form for Step 3 is shown in Analysis Method 4.5. Example 4.7 applies the mesh-current method to a circuit with a dependent source.

Mesh-Current Method

3. **Write the KVL equations** for each mesh.
    - If there are dependent sources, write a dependent source constraint equation for each.

Analysis Method 4.5 Modified Step 3 for the mesh-current method.

Example 4.7 Using the Mesh-Current Method with Dependent Sources

Use the mesh-current method to find the power dissipated in the 4 Ω resistor

in the circuit shown in Fig. 4.23.

Figure 4.23 The circuit for Example 4.7.

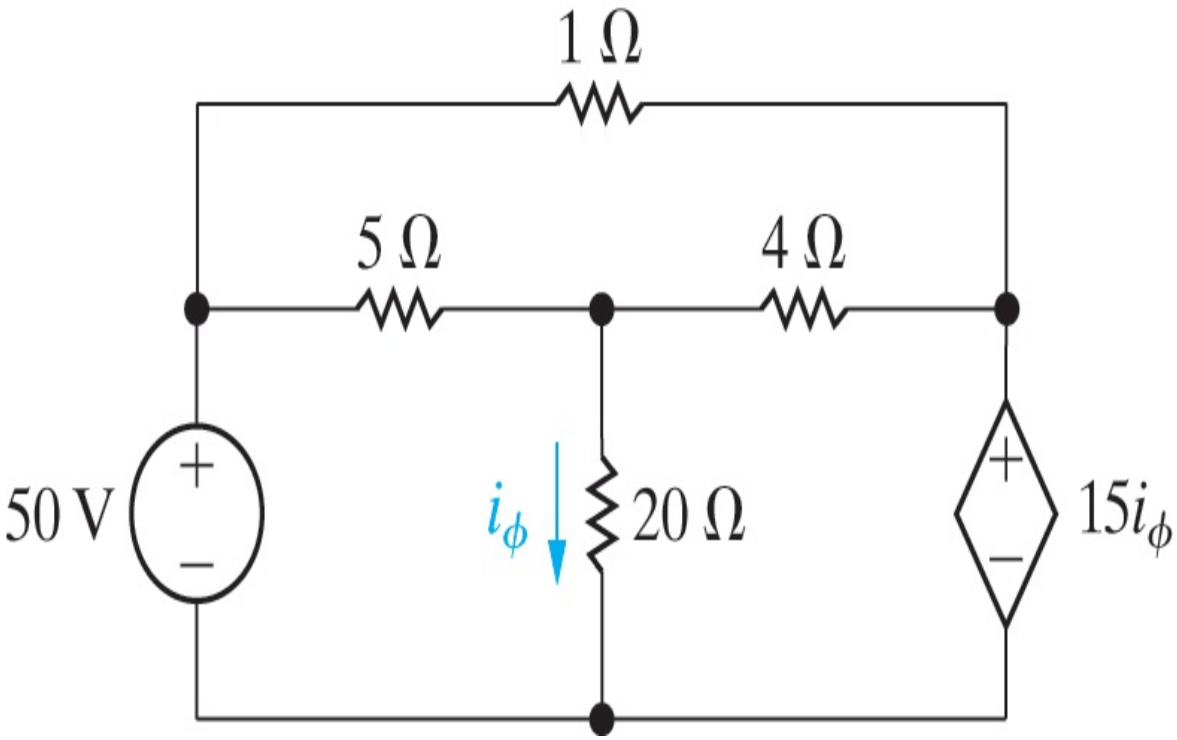


Figure 4.23 Full Alternative Text

Solution

1. Step 1: Begin by drawing the mesh currents in each of the three meshes.
2. Step 2: Label each mesh current. The resulting circuit is shown in Fig. 4.24.

Figure 4.24 The circuit

shown in Fig. 4.23 with the three mesh currents.

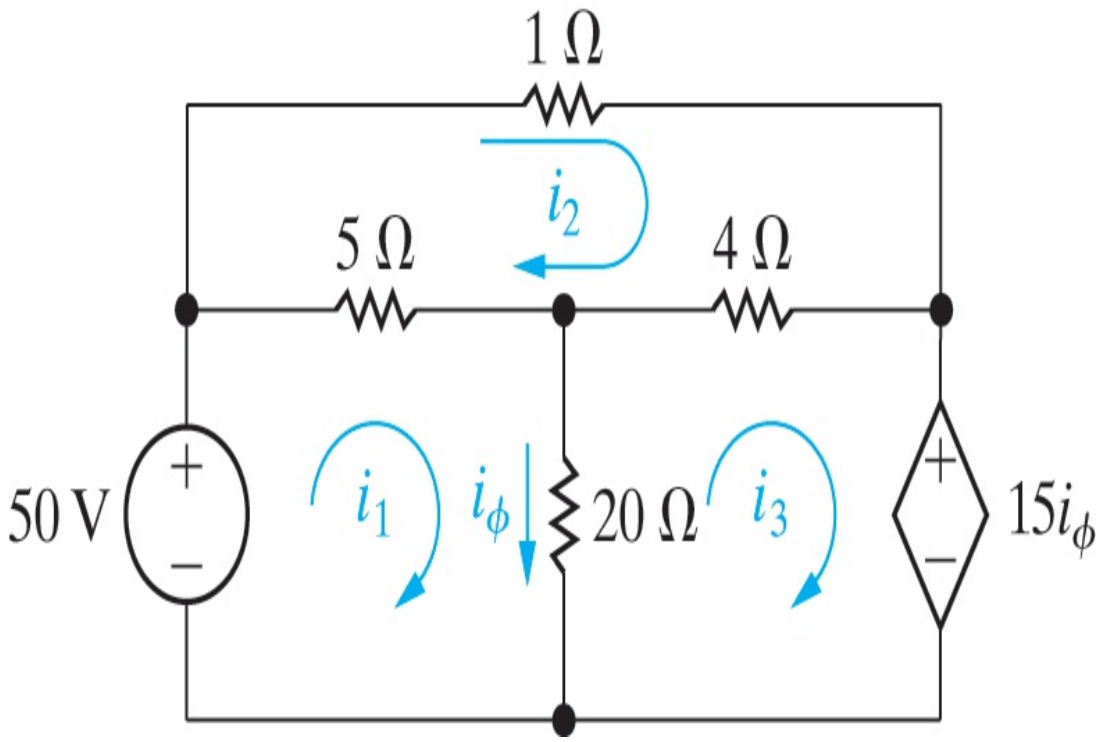


Figure 4.24 Full Alternative Text

3. Step 3: Write a KVL equation for each mesh by picking a starting point anywhere in the mesh and summing the voltages around the mesh in the direction of the mesh current. When you return to the starting point, set the sum equal to zero. The three mesh-current equations are

   5(i1−i2)+20(i1−i3)−50=0,
   5(i2−i1)+1i2+4(i2−i3)=0,
   20(i3−i1)+4(i3−i2)+15iφ=0.

   To complete Step 3, express the branch current controlling the dependent voltage source in terms of the mesh currents as

   iϕ=i1−i3.

4. Step 4: Solve the four equations generated in Step 3 to find the four unknown currents:

   $i1=29.6\ \text{A}, i2=26\ \text{A}, i3=28\ \text{A}, i\phi=1.6\ \text{A}.$

5. Step 5: Use the mesh currents to find the power for the 4 Ω resistor. The current in the 4 Ω resistor oriented from left to right is $i3-i2$, or 2 A. Therefore, the power dissipated is

   $p4\Omega=(i3-i2)2(4)=(2)2(4)=16\ \text{W}.$

   What if you had not been told to use the mesh-current method? Would you have chosen the node-voltage method? It reduces the problem to finding one unknown node voltage because of the presence of two voltage sources between essential nodes. We say more about making such choices in Section 4.8.

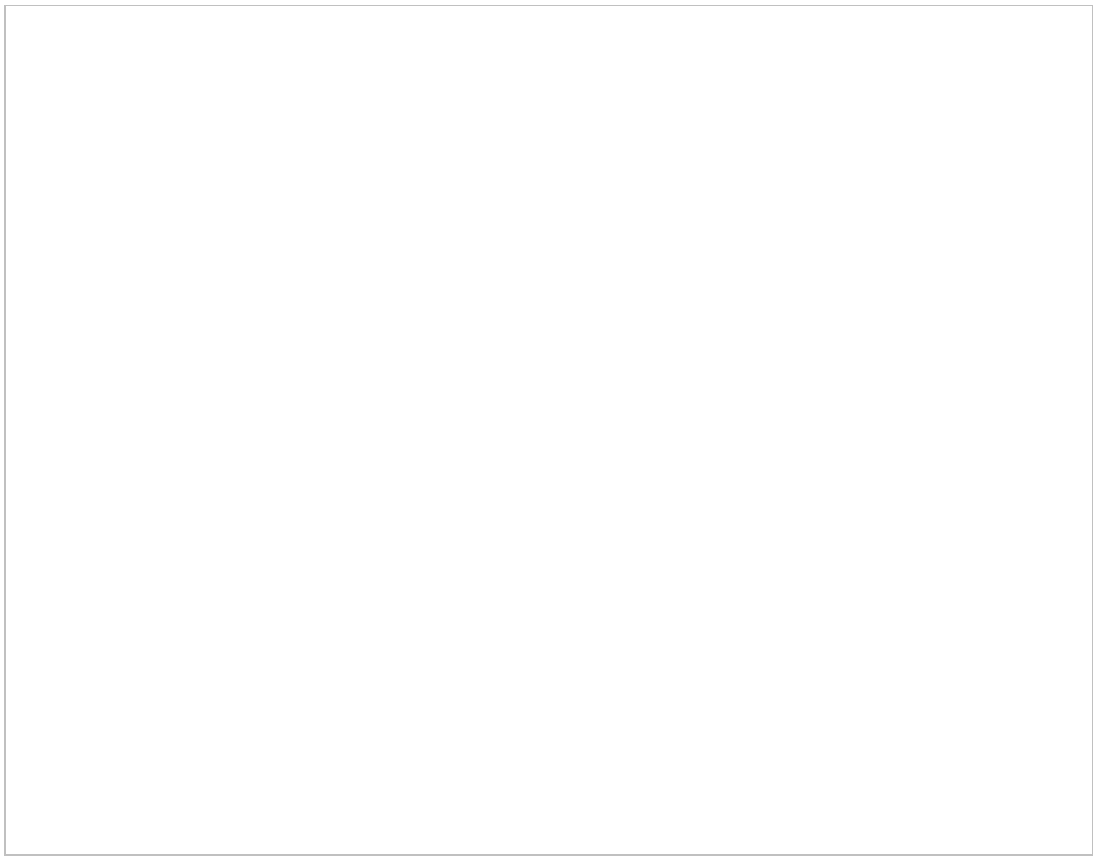
Assessment Problems

Objective 2—Understand and be able to use the mesh-current method

1. 4.8

   1. a) Determine the number of mesh-current equations needed to solve the circuit shown.

   2. b) Use the mesh-current method to find how much power is being delivered by the dependent voltage source.

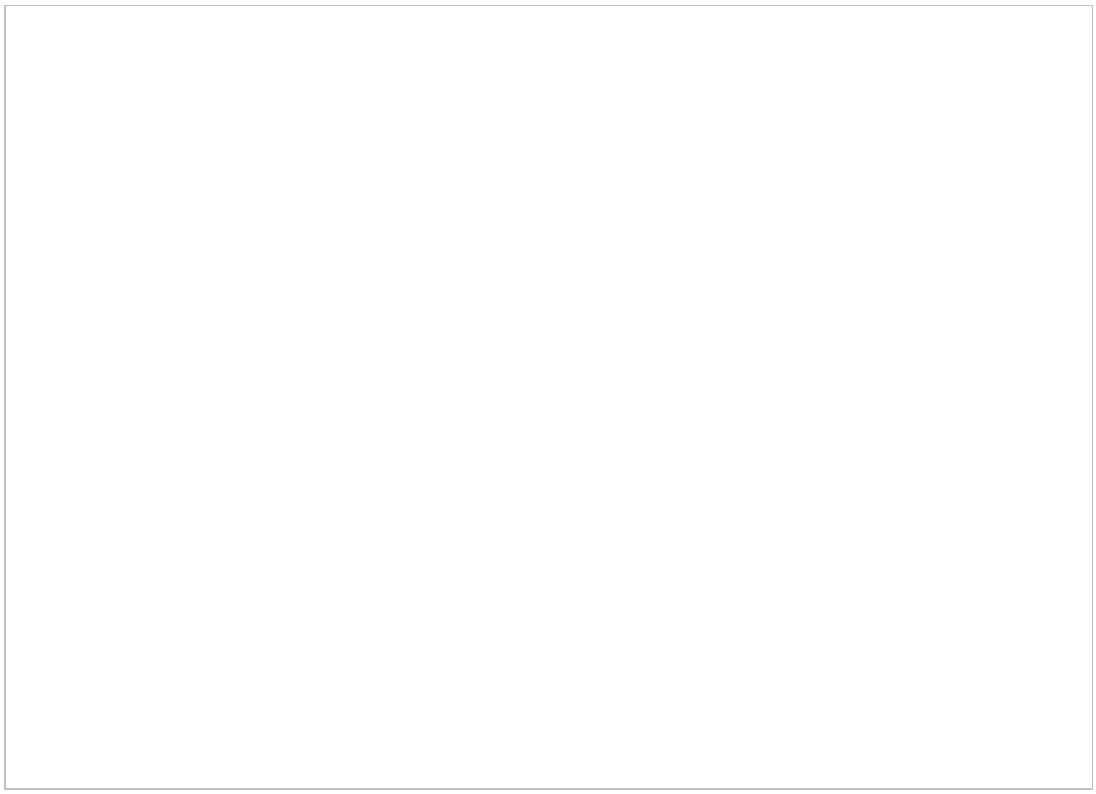


4.6-12 Full Alternative Text

Answer:

1. (a) 3;
2. (b) −36 W.

2. 4.9 Use the mesh-current method to find vo in the circuit shown.



4.6-13 Full Alternative Text

Answer: 16V.

SELF-CHECK: Also try Chapter Problems 4.39 and 4.41.

4.7 The Mesh-Current Method: Some Special Cases

Recall that *voltage* sources present special cases when using the node-*voltage* method (Section 4.4). So, it is no surprise that *current* sources present special cases when using the mesh-*current* method. There are two special cases, one that occurs when a current source is in a single mesh, and the other that occurs when a current source is shared by two adjacent meshes.

When a current source is in a single mesh, the value of the mesh current is known, since it must equal the current of the source. Therefore, we label the mesh current with its value, and we do not need to write a KVL equation for that mesh. This leads to the following modification in Step 2 of the mesh-current method.

Step 2: Label the mesh current for each mesh; if there is a current source in a single mesh, label the mesh current with the value of the current source.

This special case is illustrated in Example 4.8.

Example 4.8 A Special Case in the Mesh-Current Method

Use the mesh-current method to find branch currents ia, ib, and ic in the circuit for Example 4.3, repeated here as Fig. 4.25.

Figure 4.25 The circuit for Example 4.8.

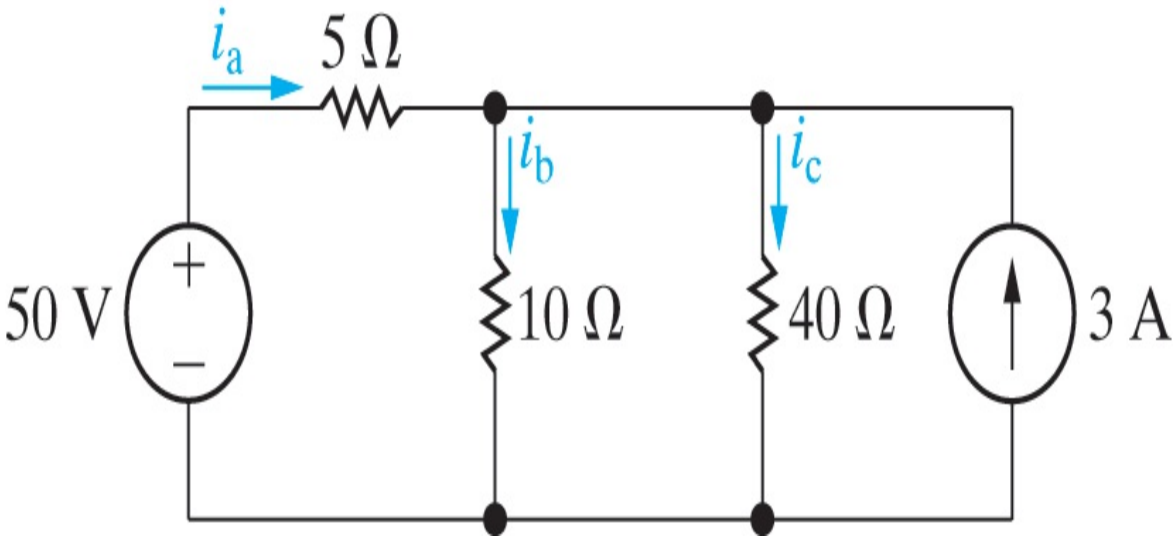


Figure 4.25 Full Alternative Text

Solution

1. Step 1: Use directed arrows that traverse the mesh perimeters to identify the three mesh currents.

2. Step 2: Label the mesh currents as i1, i2, and i3. The modification in Step 2 reminds us to look for current sources, and the i3 mesh has a current source that is not shared by any other mesh. Therefore, the i3 mesh current equals the current supplied by the source. Note that i3 and the current source are in opposite directions, so the current in this mesh should be labeled −3 A. The results of Steps 1 and 2 are shown in Fig. 4.26.

Figure 4.26 The circuit shown in Fig. 4.25 with the mesh currents identified and labeled.

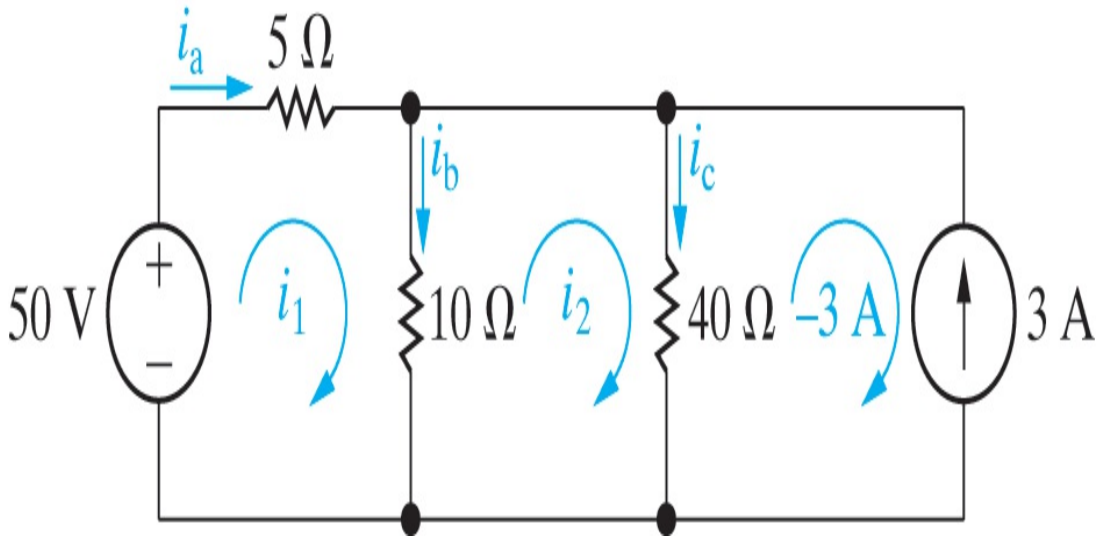


Figure 4.26 Full Alternative Text

3. Step 3: Write the KVL equations for the meshes whose mesh currents are unknown, which in this example are the i1 and i2 meshes. Remember to pick a starting point anywhere along the mesh, sum the voltages in the direction of the mesh current, and set the sum equal to zero when you return to the starting point. The resulting simultaneous mesh current equations are

   $-50+5i1+10(i1-i2)=0 \text{and} 10(i2-i1)+40(i2-(-3))=0.$

4. Step 4: Solving the simultaneous mesh current equations gives

   $i1=2 \text{ A} \text{and} i2=-2 \text{ A}.$

5. Step 5: Finally, we use the mesh currents to calculate the branch currents in the circuit, ia, ib, and ic.

   $ia=i1=2 \text{ A}, ib=i1-i2=4 \text{ A}, ic=i2+3=1 \text{ A}.$

   These are the same branch current values as those calculated in Example 4.3. Which of the two circuit analysis methods is better when calculating the branch currents? Which method is better when calculating the power associated with the sources?

Now we turn our attention to the other special case, created by a current

source that is shared between two adjacent meshes. The circuit shown in Fig. 4.27 depicts this situation. Applying Steps 1 and 2, we define the mesh currents ia, ib, and ic, as well as the voltage across the 5 A current source. In Step 3, we write the KVL equations for each mesh; let's start with mesh a and pick the starting point just below the 100 V source. The first two voltages we encounter when we traverse the mesh in the direction of the mesh current are −100 V and 3(ia – ib). But when we get to the current source, we must label the voltage drop across it as v and use this variable in the equation. Thus, for mesh a:

Figure 4.27 A circuit illustrating mesh analysis when a branch contains an independent current source.

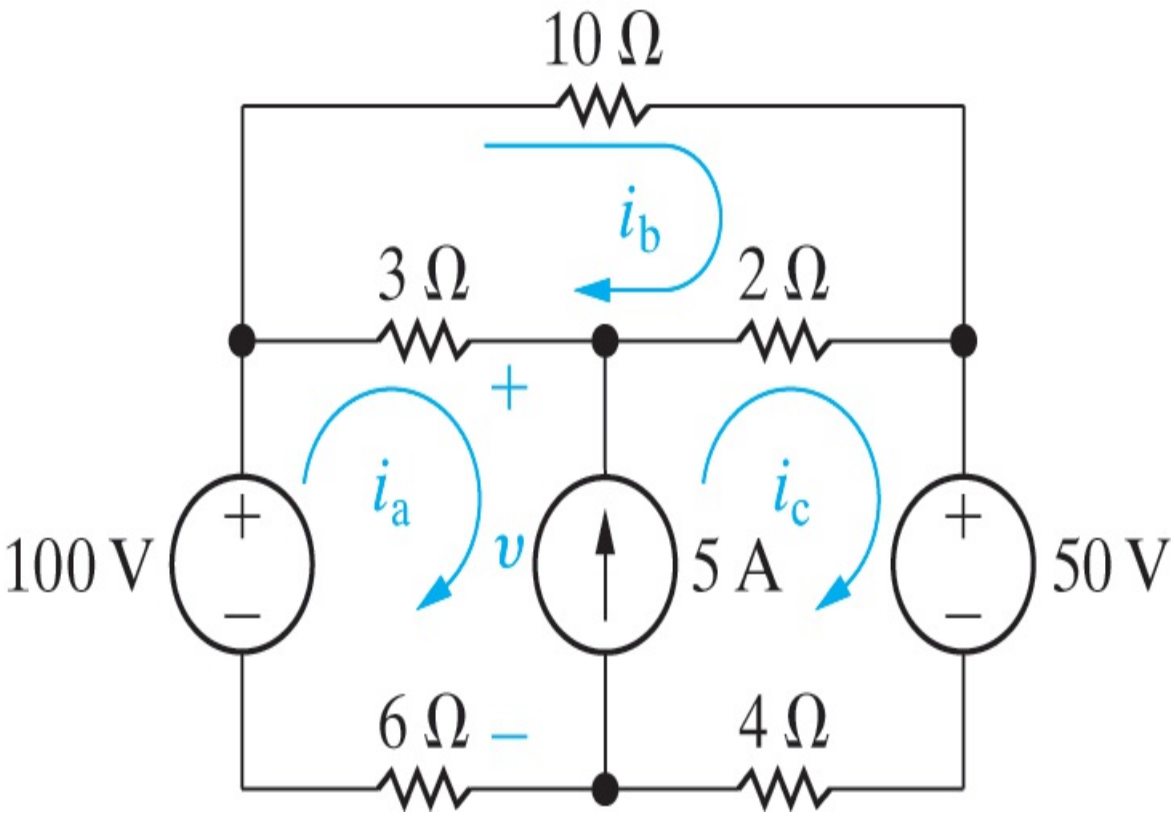


Figure 4.27 Full Alternative Text

$$-100+3(ia-ib)+v+6ia=0. \quad (4.18)$$

The same situation arises in mesh c, to give

$$50+4ic-v+2(ic-ib)=0. \quad (4.19)$$

We now add Eqs. 4.18 and 4.19 to eliminate v; when simplified, the result is

$$-50+9ia-5ib+6ic=0. \quad (4.20)$$

We will complete Steps 4 and 5 in the mesh-current method after introducing a new concept, the supermesh.

The Concept of a Supermesh

When a current source is shared between two meshes, we can combine these meshes to form a **supermesh**, which traverses the perimeters of the two meshes and avoids the branch containing the shared current source. Figure 4.28 illustrates the supermesh concept. Using a modified Step 3, we write a KVL equation around the supermesh (denoted by the dashed line), using the original mesh currents to give

Figure 4.28 The circuit shown in Fig. 4.27, illustrating the concept of a supermesh.

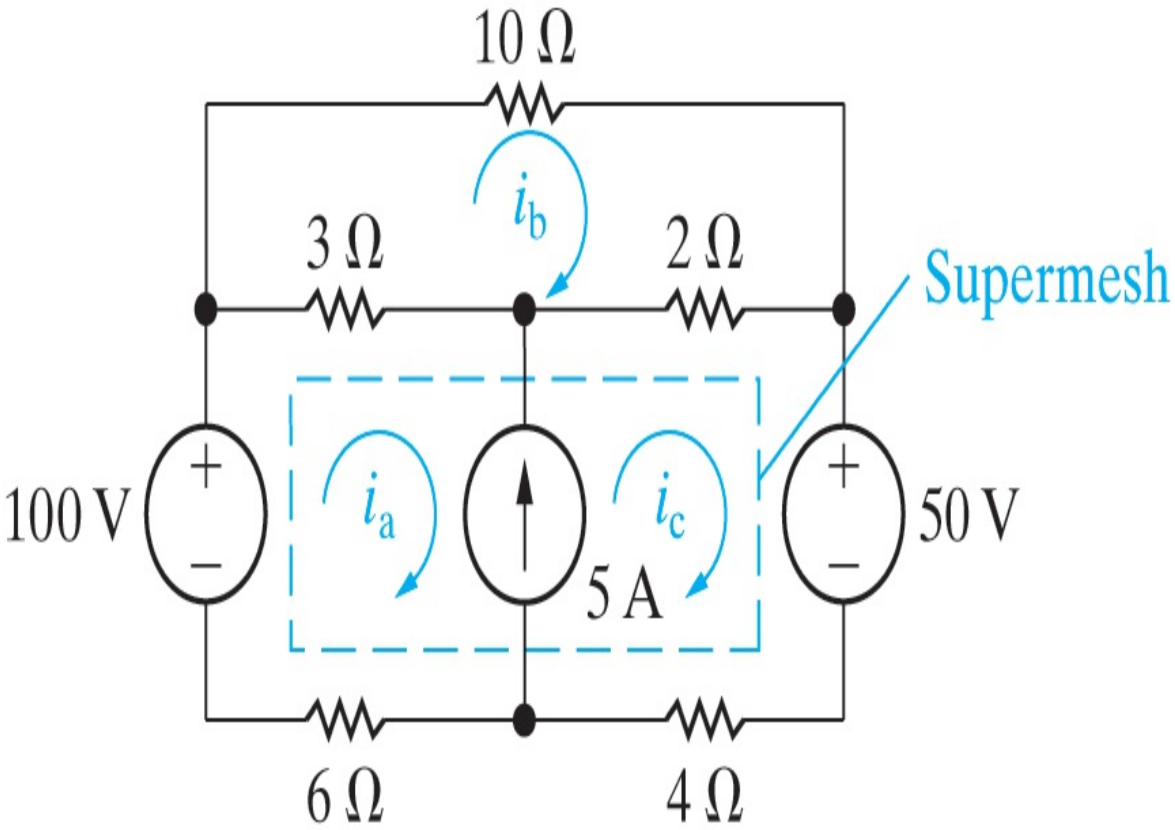


Figure 4.28 Full Alternative Text

−100+3(ia−ib)+2(ic−ib)+50+4ic+6ia=0,

which simplifies to

−50+9ia−5ib+6ic=0. (4.21)

Note that Eqs. 4.20 and 4.21 are identical. Thus, the supermesh has eliminated the need for introducing the unknown voltage across the current source.

The KVL equation for the b mesh is

10ib+2(ib−ic)+3(ib−ia)=0. (4.22)

We have two simultaneous equations, Eqs. 4.21 and 4.22, but three unknowns. Remember that the presence of a supernode in the node-voltage method requires a KCL equation at the supernode and a supernode constraint equation that defines the difference between the node voltages in the

supernode as the value of the voltage source in the supernode. In a like manner, the presence of a supermesh in the mesh-current method requires a KVL equation around the supermesh and a supermesh constraint equation that defines the difference between the mesh currents in the supermesh as the value of the shared current source. From Fig. 4.28, the supermesh constraint equation is

$$i_c - i_a = 5. \quad (4.23)$$

The final version of Steps 2 and 3 in the mesh-current method reminds us how to handle current sources in our circuits:

- Step 2: Label the mesh current for each mesh; if there is a current source in a single mesh, label the mesh current with the value of the source. If there is a current source shared between two meshes, create a supermesh by combining the two meshes and mentally erasing the current source.

- Step 3: Write a KVL equation around each supermesh and each single mesh where the mesh current is unknown. If there is a dependent source, write a constraint equation defining the controlling quantity for the dependent source in terms of the mesh currents. If there is a supermesh, write a supermesh constraint equation that defines the difference between the two mesh currents in the supermesh as the value of the shared current source.

The final version of the mesh-current method can be found in the end-of-chapter Summary; a condensed version is given in Analysis Method 4.6.

Mesh-Current Method

1. **Identify the meshes** with curved directed arrows that follow the perimeter of each mesh.

2. **Label the mesh currents** for each mesh.

    - If a current source is in a single mesh, label the mesh current with the value of the current source.

- If a current source is shared between two meshes, combine the meshes to create a supermesh and mentally erase the current source.

3. **Write the KVL equations** for each supermesh and each single mesh where the current is unknown.
    - If there are dependent sources, write a dependent source constraint equation for each.
    - If there are supermeshes, write a supermesh constraint equation for each.
4. **Solve the KVL equations** and any constraint equations to find the mesh-current values and other unknowns.
5. **Solve the circuit** using mesh currents from Step 4 to find component currents, voltages, and power values.

Analysis Method 4.6 Complete form of the mesh-current method.

Use Step 4 to solve Eqs. 4.21–4.23 and confirm that the solutions for the three mesh currents are

$i_a = 1.75$ A, $i_b = 1.25$ A, and $i_c = 6.75$ A.

Work through Example 4.9 to see how the mesh-current method can be used to solve the amplifier circuit from Example 4.5.

Example 4.9 Mesh-Current Analysis of the Amplifier Circuit

Use the mesh-current method to find iB for the amplifier circuit in Fig. 4.29.

Figure 4.29 The circuit shown in Fig 2.24.

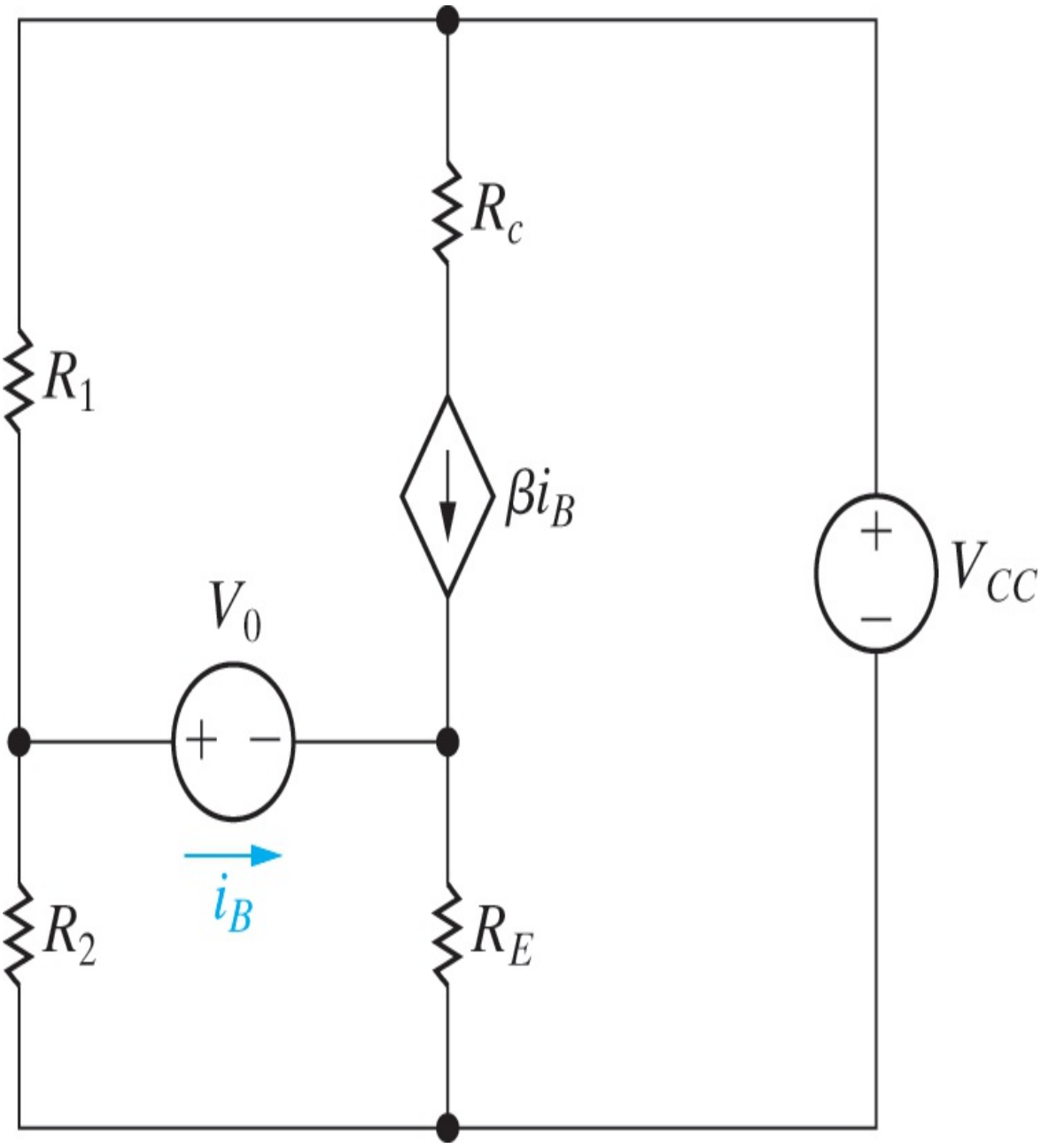


Figure 4.29 Full Alternative Text

Solution

1. Step 1: Use directed arrows that traverse the mesh perimeters to identify the three mesh currents.

2. Step 2: Label the mesh currents as ia, ib, and ic. Then recognize the current source that is shared between the ia and ic meshes. Combine these meshes, bypassing the branch with the shared current source, to create a supermesh. The result of the first two steps is the circuit shown in Fig. 4.30.

Figure 4.30 The circuit shown in Fig. 4.29, depicting the supermesh created by the presence of the dependent current source.

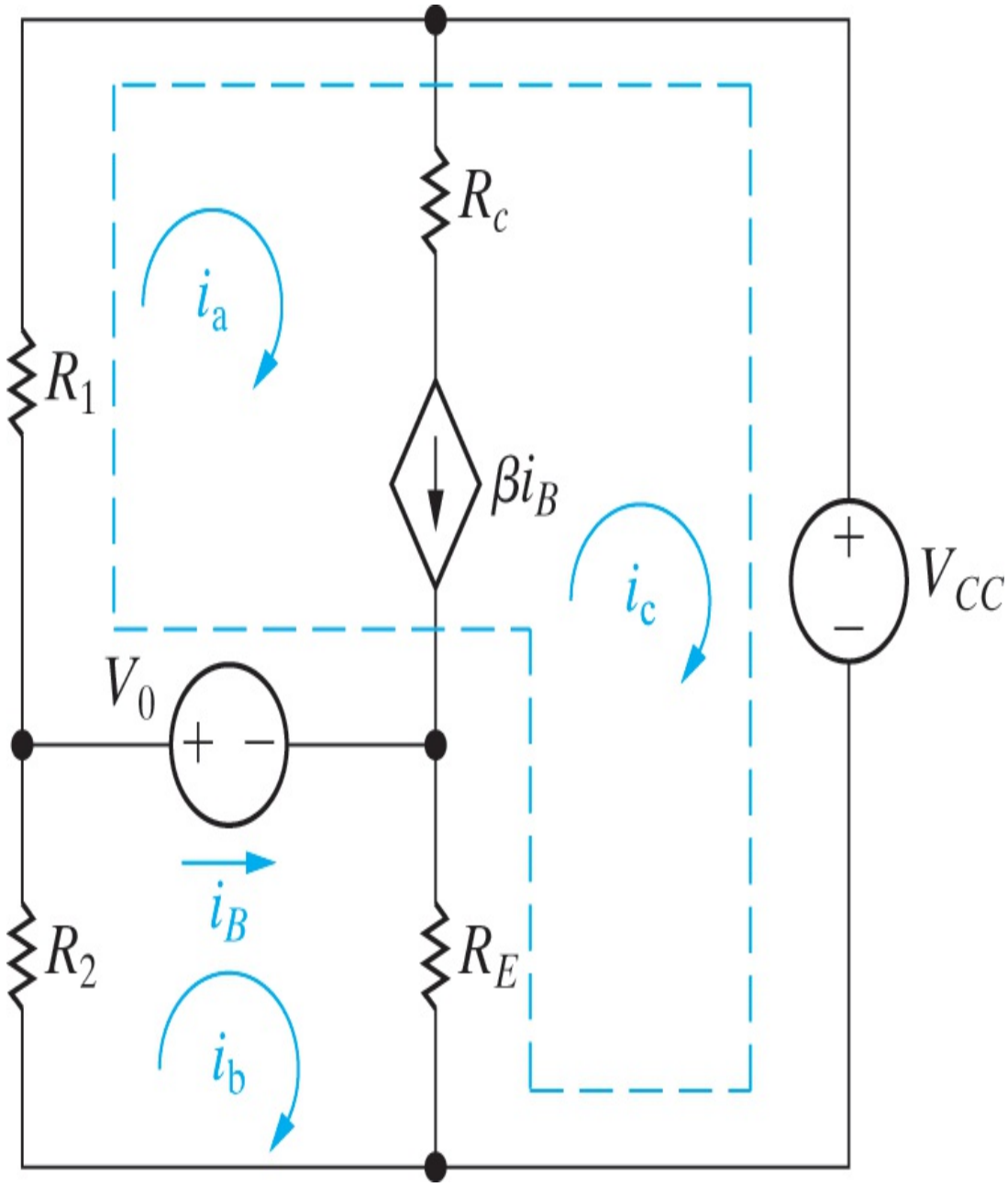


Figure 4.30 Full Alternative Text

3. Step 3: Using KVL, sum the voltages around the supermesh in terms of the mesh currents ia, ib, and ic to obtain

   R1ia+vCC+RE(ic−ib)−V0=0. (4.24)

   The KVL equation for mesh b is

$$R2ib+V0+RE(ib-ic)=0. \quad (4.25)$$

The constraint imposed by the dependent current source is

$$iB=ib-ia. \quad (4.26)$$

The supermesh constraint equation is

$$\beta iB=ib-ic. \quad (4.27)$$

4. Step 4: Use back-substitution to solve Eqs. 4.24–4.27. Start by combining Eqs. 4.26 and 4.27 to eliminate iB and solve for ic to give

   $$ic=(1+\beta)ia-\beta ib. \quad (4.28)$$

   We now use Eq. 4.28 to eliminate ic from Eqs. 4.24 and 4.25:

   $$[R1+(1+\beta)RE]ia-(1+\beta)REib=V0-VCC, \quad (4.29)$$
   $$-(1+\beta)REia+[R2+(1+\beta)RE]ib=-V0. \quad (4.30)$$

   You should verify that the solution of Eqs. 4.29 and 4.30 for ia and ib gives

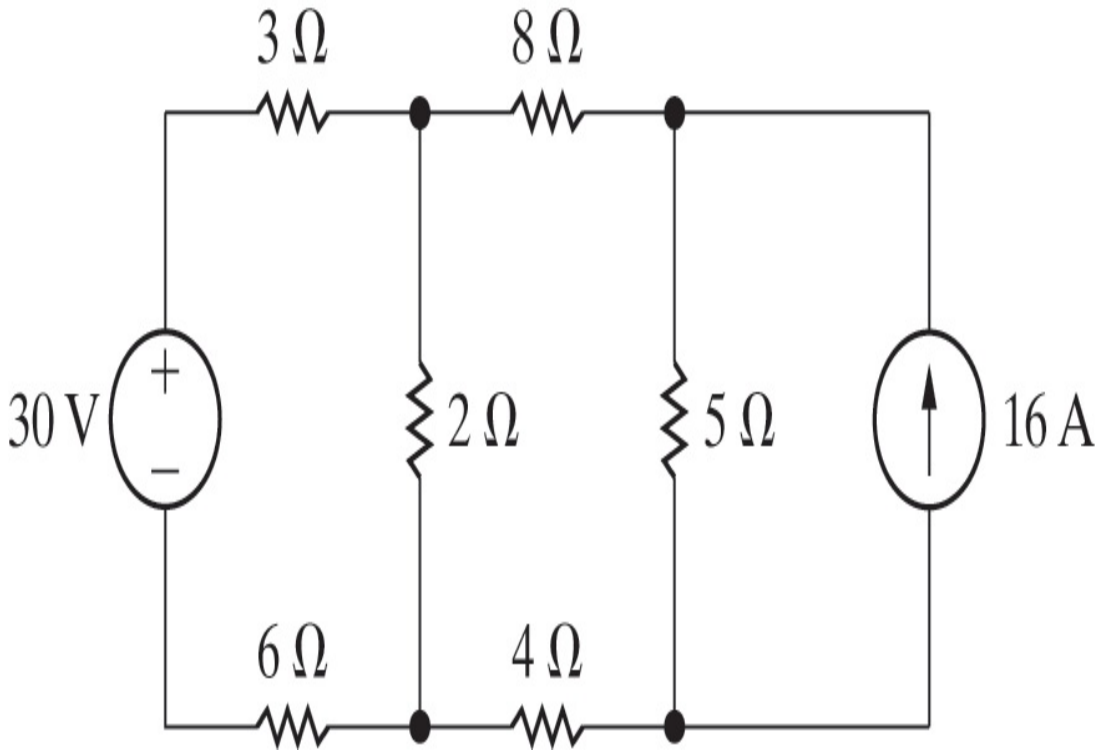
   $$ia=V0R2-VCCR2-VCC(1+\beta)RER1R2+(1+\beta)RE(R1+R2), \quad (4.31)$$
   $$ib=-V0R1-(1+\beta)REVCCR1R2+(1+\beta)RE(R1+R2). \quad (4.32)$$

5. Step 5: Use the two mesh currents from Eqs. 4.31 and 4.32, together with the definition for iB in Eq. 4.26, to find iB. You should verify that the result is the same as that given by Eq. 2.21.

Assessment Problems

Objective 2—Understand and be able to use the mesh-current method

1. 4.10 Use the mesh-current method to find the power dissipated in the 2 Ω resistor in the circuit shown.

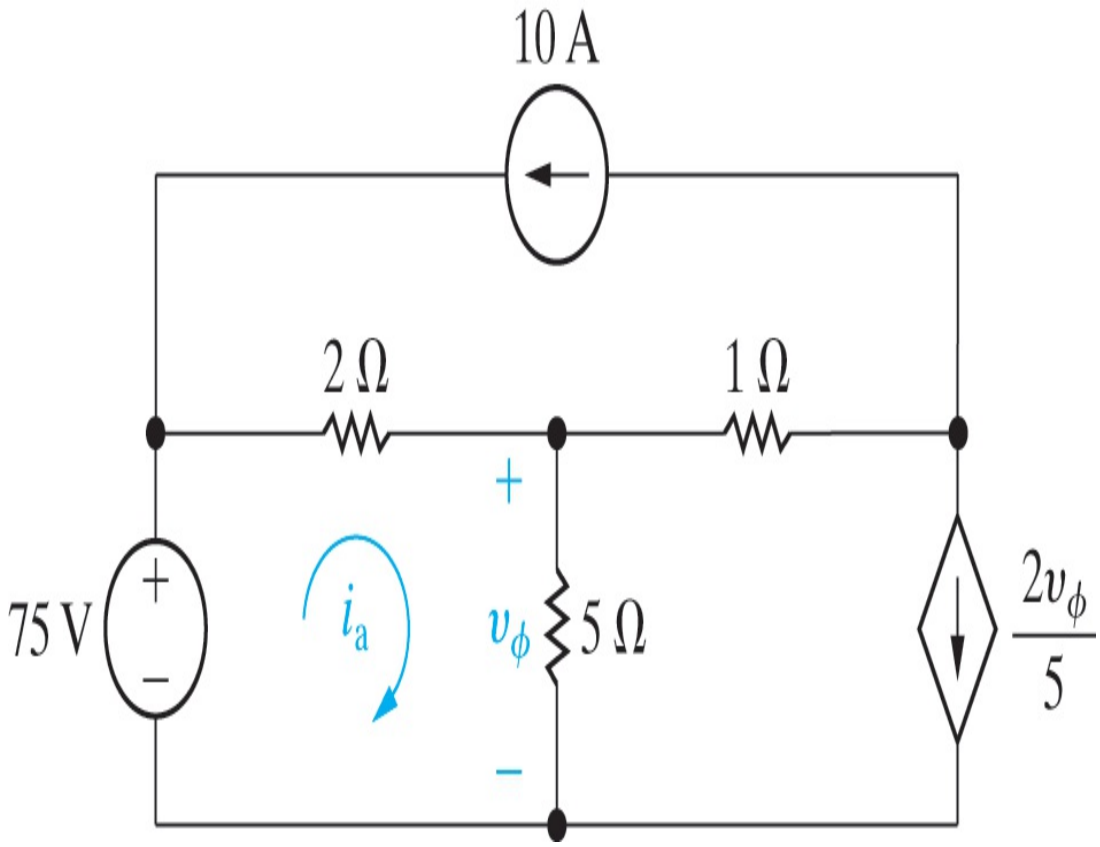


4.7-14 Full Alternative Text

Answer:

72 W.

2. 4.11 Use the mesh-current method to find the mesh current ia in the circuit shown.

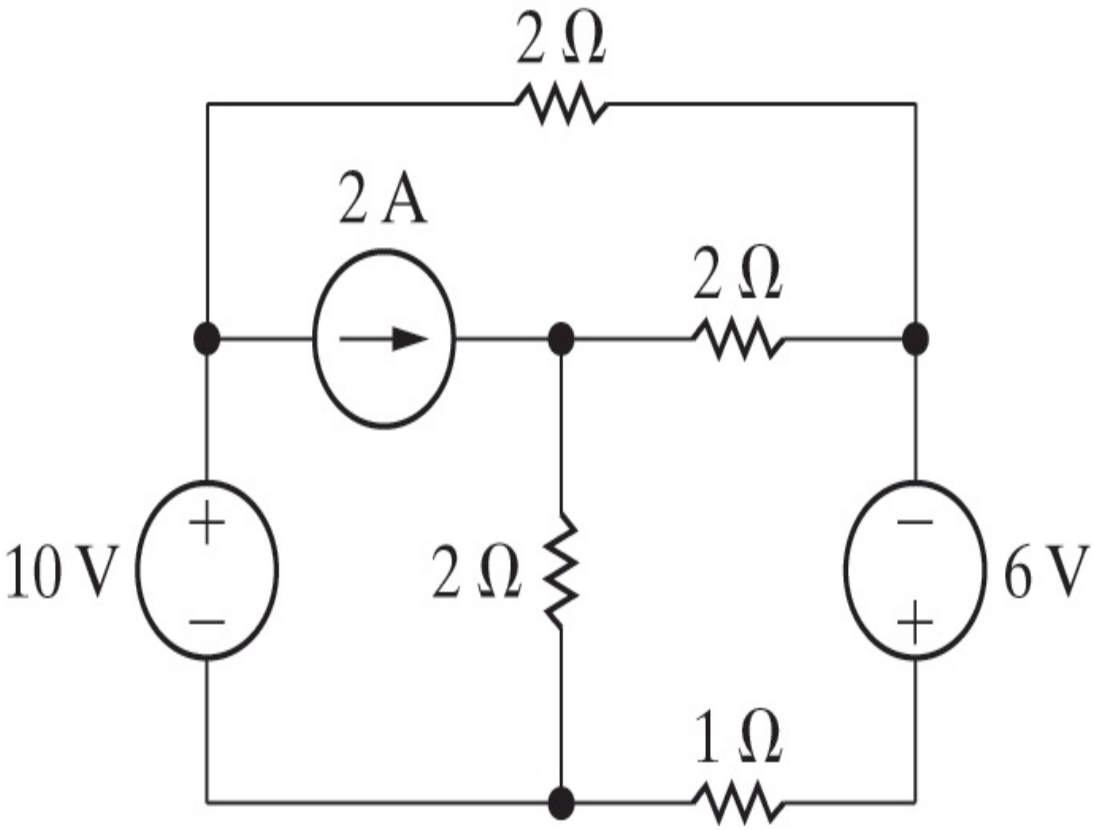


[4.7-15 Full Alternative Text]

Answer:

15 A.

3. 4.12 Use the mesh-current method to find the power dissipated in the 1 Ω resistor in the circuit shown.



4.7-16 Full Alternative Text

Answer:

36 W.

SELF-CHECK: Also try Chapter Problems 4.45, 4.46, 4.50, and 4.52.

4.8 The Node-Voltage Method Versus the Mesh-Current Method

It is natural to ask, “When is the node-voltage method preferred to the mesh-current method and vice versa?” As you might suspect, there is no clear-cut answer. Asking a number of questions, however, may help you identify the more efficient method before plunging into the solution process:

- Does one of the methods result in fewer simultaneous equations to solve?
- Is there a branch between two essential nodes that contains only a voltage source? If so, making one of the essential nodes the reference node and using the node-voltage method reduces the number of equations to be solved.
- Is there a mesh containing a current source that is not shared with an adjacent mesh? If so, using the mesh-current method allows you to reduce the number of equations to be solved.
- Will solving some portion of the circuit give the requested solution? If so, which method is most efficient for solving just the pertinent portion of the circuit?

Perhaps the most important observation is that, for any situation, some time spent thinking about the problem in relation to the various analytical approaches available is time well spent. Examples 4.10 and 4.11 illustrate the process of deciding between the node-voltage and mesh-current methods.

Example 4.10 Understanding the Node-Voltage Method Versus Mesh-

Current Method

Find the power dissipated in the 300 Ω resistor in the circuit shown in Fig. 4.31.

Figure 4.31 The circuit for Example 4.10.

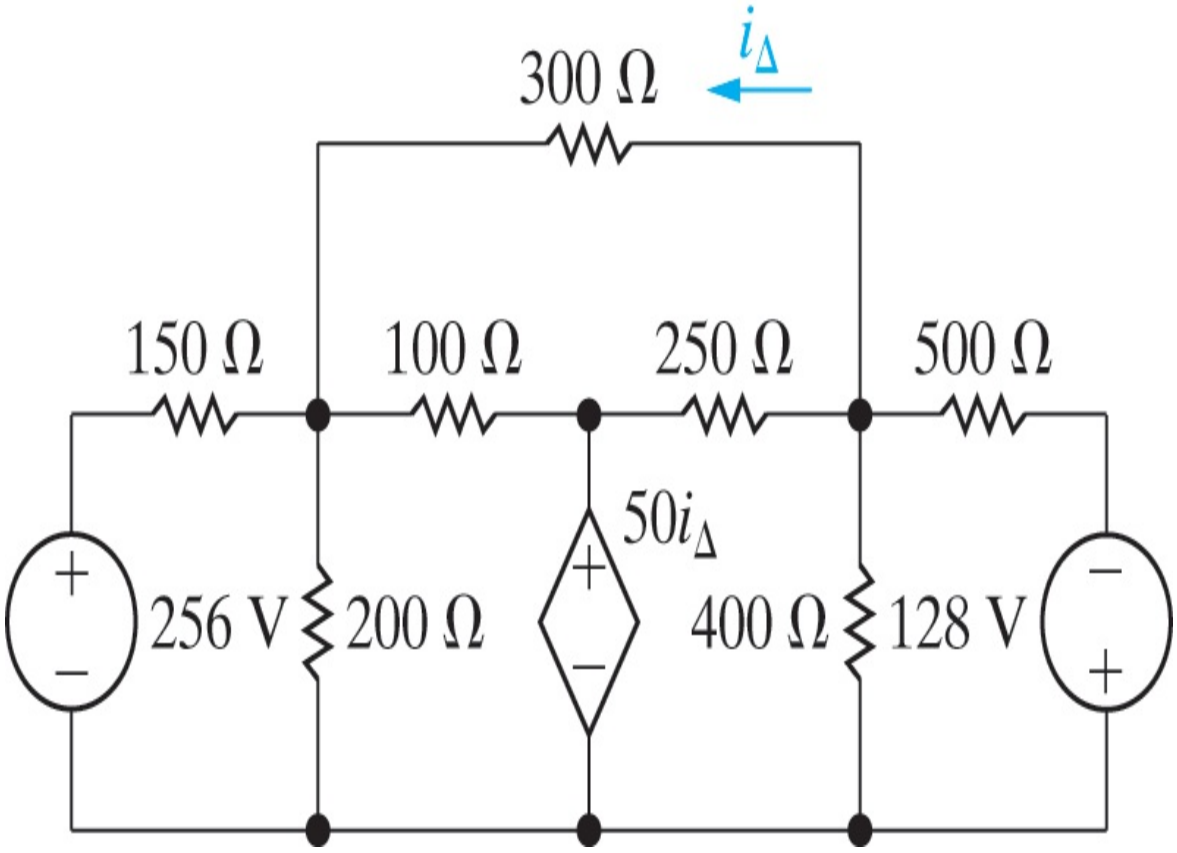


Figure 4.31 Full Alternative Text

Solution

To find the power dissipated in the 300 Ω resistor, we need to find either the

current in the resistor or the voltage across it. The mesh-current method yields the current in the resistor; this approach requires solving five mesh equations, as depicted in Fig. 4.32, and a dependent source constraint equation, for a total of six simultaneous equations.

Figure 4.32 The circuit shown in Fig. 4.31, with the five mesh currents.

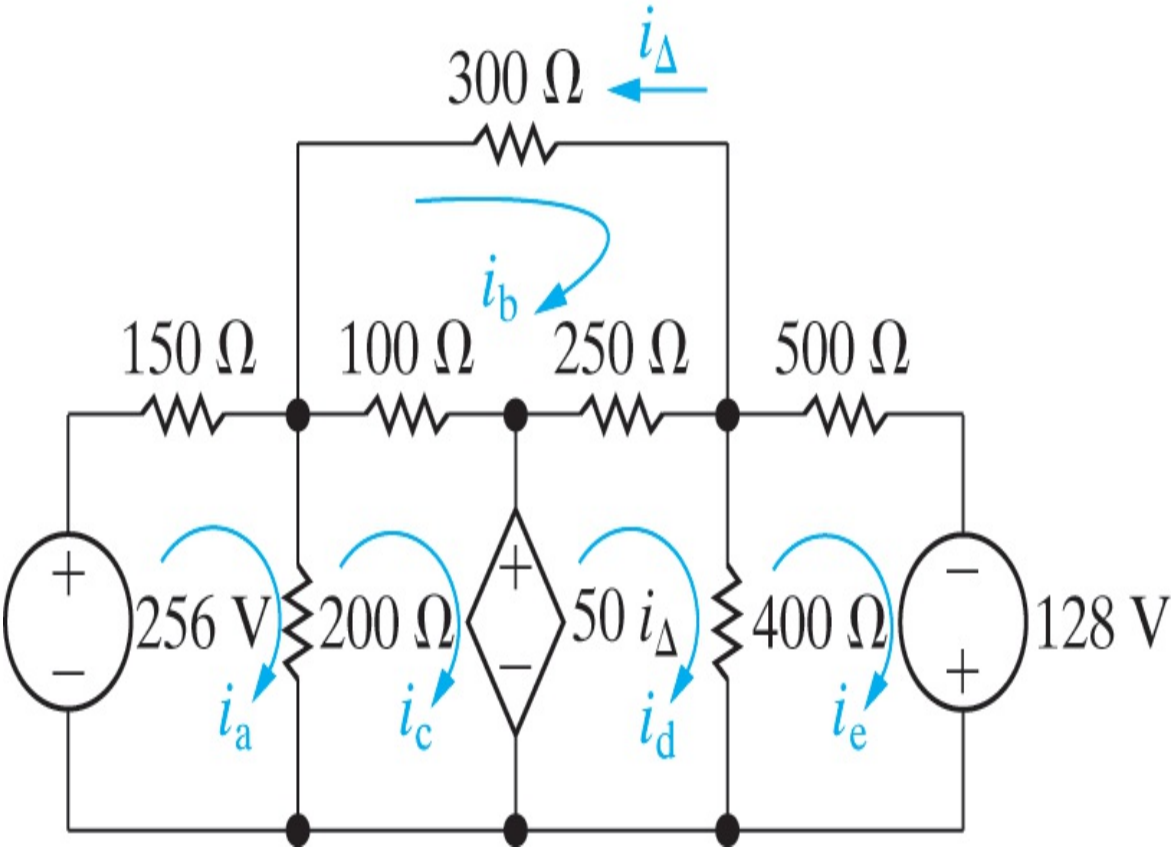


Figure 4.32 Full Alternative Text

Let’s now consider using the node-voltage method. The circuit has four essential nodes, and therefore only three node-voltage equations are required to describe the circuit. The dependent voltage source between two essential

nodes forms a supernode, requiring a KCL equation and a supernode constraint equation. We have to sum the currents at the remaining essential node, and we need to write the dependent source constraint equation, for a total of four simultaneous equations. Thus, the node-voltage method is the more attractive approach.

1. Step 1: We begin by identifying the four essential nodes in the circuit of Fig. 4.31. The three black dots at the bottom of the circuit identify a single essential node, and the three black dots in the middle of the circuit are the three remaining essential nodes.

2. Step 2: Select a reference node. Two essential nodes in the circuit in Fig. 4.31 merit consideration. The first is the reference node in Fig. 4.33, where we also defined the three node voltages v1, v2, and v3, and indicated that nodes 1 and 3 form a supernode because they are connected by a dependent voltage source. If the reference node in Fig. 4.33 is selected, one of the unknown node voltages is the voltage across the 300 Ω resistor, namely, v2 in Fig. 4.33. Once we know this voltage, we calculate the power in the 300 Ω resistor by using the expression

   p300Ω=v22/300.

Figure 4.33 The circuit shown in Fig. 4.31, with a reference node.

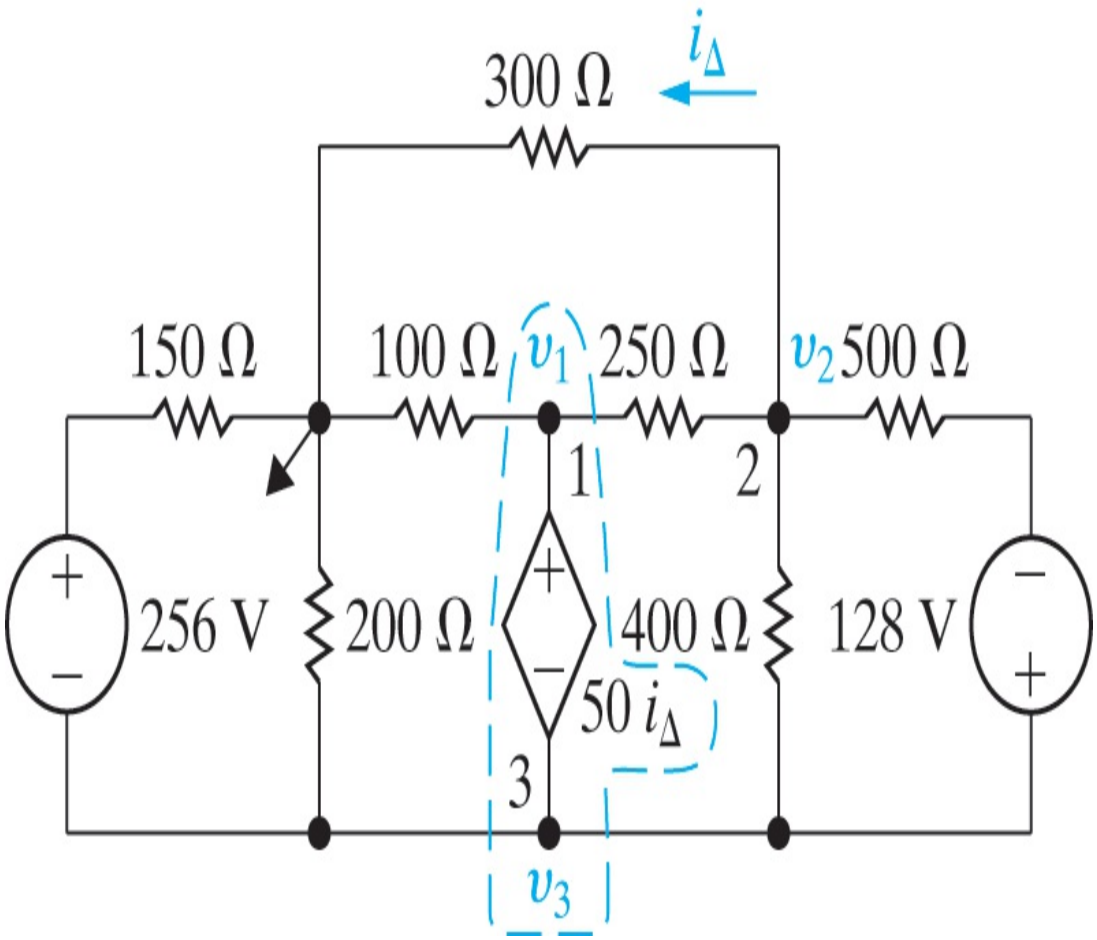


Figure 4.33 Full Alternative Text

The second node worth considering as the reference node is the bottom node in the circuit, as shown in Fig. 4.34. If this reference node is chosen, one of the unknown node voltages is eliminated because vb=50iΔ. We would need to write two KCL equations and a dependent source constraint equation, and solve these three simultaneous equations. However, to find either the current in the 300 Ω resistor or the voltage across it requires an additional calculation once we know the node voltages va and vc.

Figure 4.34 The circuit shown in Fig. 4.31 with an

alternative reference node.

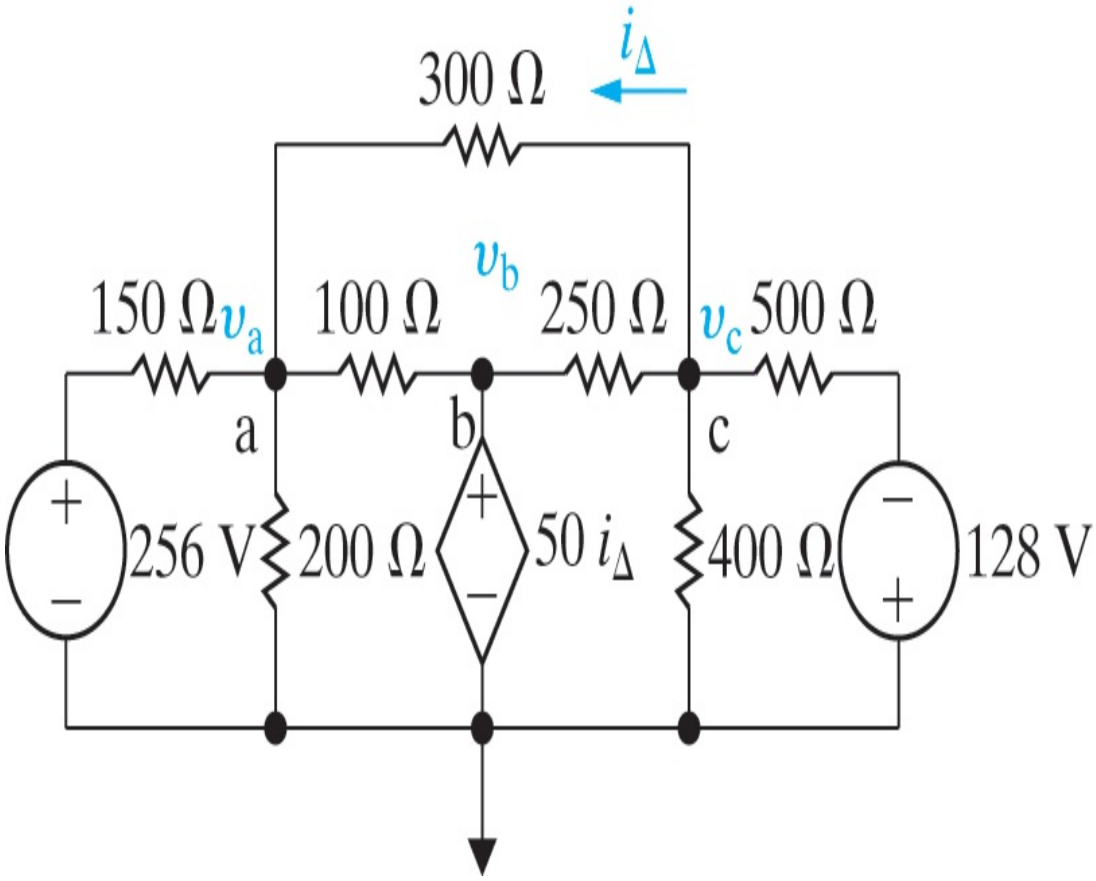


Figure 4.34 Full Alternative Text

3. Step 3: We compare these two possible reference nodes by generating two sets of KCL equations and constraint equations. The first set pertains to the circuit shown in Fig. 4.33, and the second set is based on the circuit shown in Fig. 4.34.

    - Set 1 (Fig. 4.33)

      At the supernode,

      v1100+v1−v2250+v3200+v3−v2400+v3−(v2+128)500
      +v3+256150=0.

At v2,

$$\frac{v_2}{300}+\frac{v_2-v_1}{250}+\frac{v_2-v_3}{400}+\frac{v_2+128-v_3}{500}=0.$$

The dependent source constraint equation is

$$i_\Delta=\frac{v_2}{300}.$$

The supernode constraint equation is

$$v_1-v_3=50i_\Delta.$$

- Set 2 (Fig. 4.34); remember that $v_b=50i_\Delta$.

  At va,

  $$\frac{v_a}{200}+\frac{v_a-256}{150}+\frac{v_a-50i_\Delta}{100}+\frac{v_a-v_c}{300}=0.$$

  At vc,

  $$\frac{v_c}{400}+\frac{v_c+128}{500}+\frac{v_c-50i_\Delta}{250}+\frac{v_c-v_a}{300}=0.$$

  The dependent source constraint equation is

  $$i_\Delta=\frac{v_c-v_a}{300}.$$

4. Step 4: Solve each set of equations.
5. Step 5: Verify that both solutions lead to a power calculation of 16.57 W dissipated in the 300 Ω resistor.

Example 4.11 Comparing the Node-Voltage and Mesh-Current Methods

Find the voltage vo in the circuit shown in Fig. 4.35.

Figure 4.35 The circuit for Example 4.11.

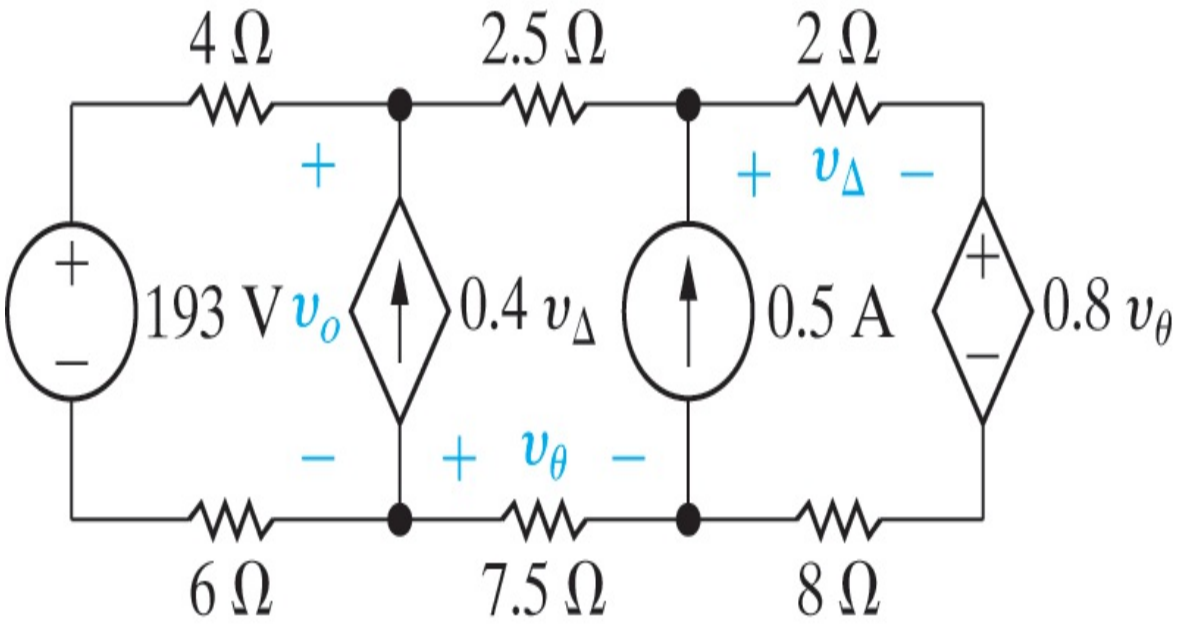


Figure 4.35 Full Alternative Text

Solution

We first consider using the mesh-current method.

1. Step 1: Identify the three mesh currents in the circuit using directed arrows that follow the mesh perimeters.

2. Step 2: Label the three mesh currents. Because there are two currents sources, each shared by two meshes, we can combine all three meshes into a single supermesh that traverses the perimeter of the entire circuit and avoids the two branches with current sources. The result of these two steps is shown in Fig. 4.36.

Figure 4.36 The circuit shown in Fig. 4.35 with the three mesh currents.

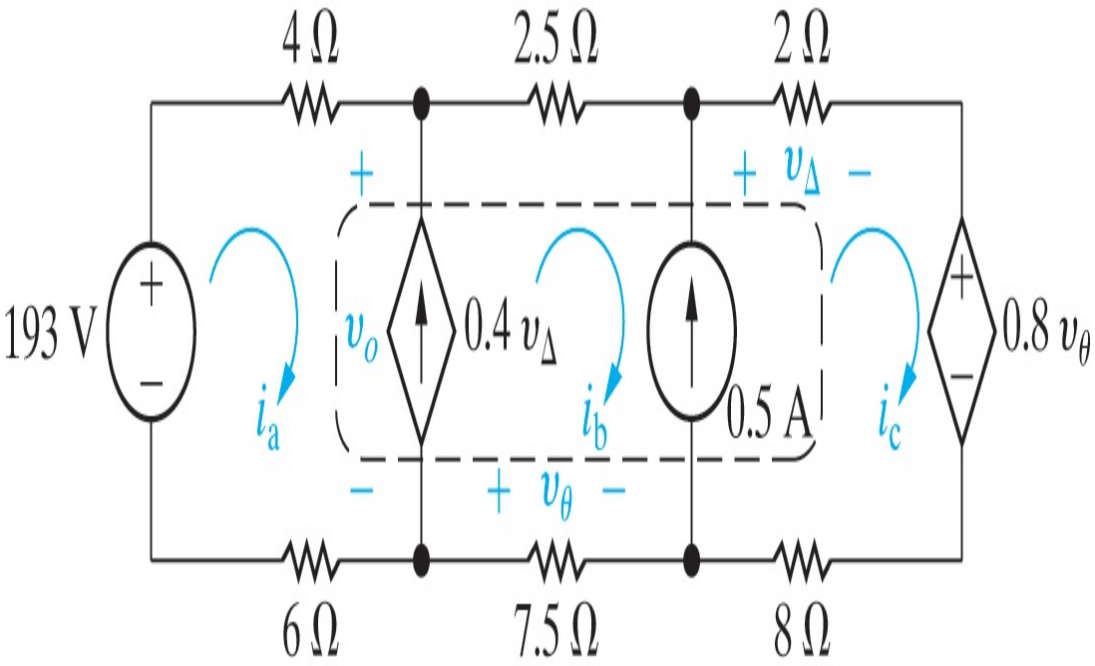


Figure 4.36 Full Alternative Text

3. Step 3: Write the KCL equation for the supermesh:

   −193+4ia+2.5ib+2ic+0.8vθ+8ic+7.5ib+6ia=0.

   The supermesh constraint equations are

   ib−ia=0.4vΔandic−ib=0.5,

   and the two dependent source constraint equations are

   vΔ=2icandvθ=−7.5ib.

4. Step 4: We must solve the five simultaneous equations generated in Step

3.

5. Step 5: We need one additional equation to find vo from the mesh current ia:

   vo=193−10ia.

   Now let’s consider using the node-voltage method.

6. Step 1: There are four essential nodes in the circuit of Fig. 4.35, identified by the four black dots in the figure.

7. Step 2: We can make the unknown voltage vo one of the three node voltages by choosing the bottom left node as the reference node. After labeling the remaining two node voltages, we have the circuit in Fig. 4.37.

Figure 4.37 The circuit shown in Fig. 4.35 with node voltages.

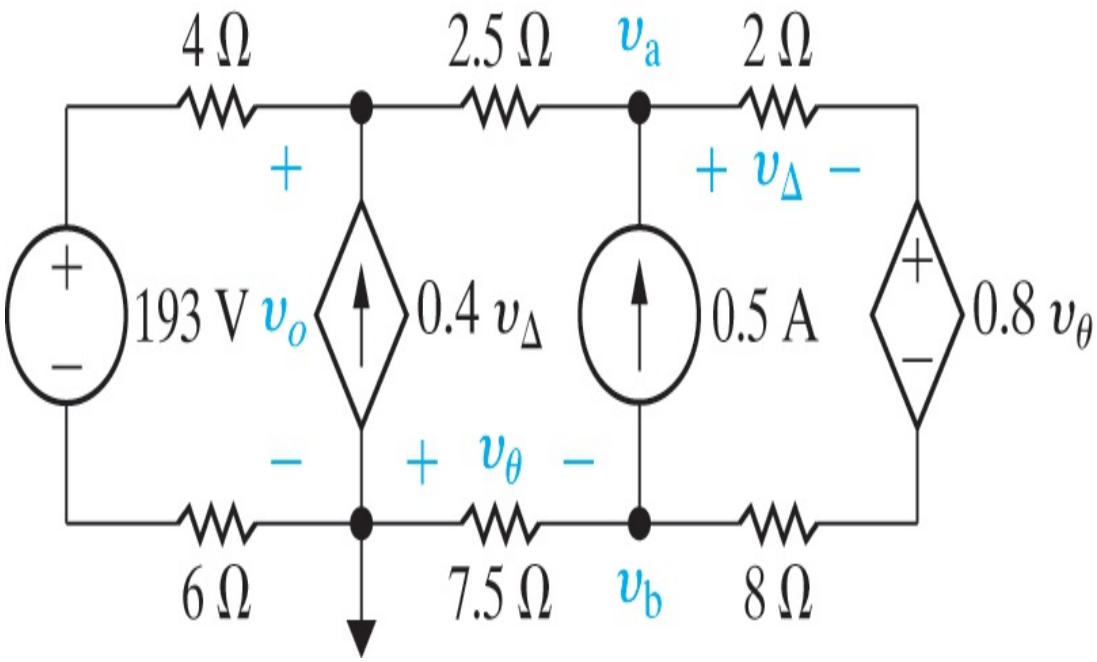


Figure 4.37 Full Alternative Text

8. Step 3: The KCL equations are

   $\frac{v_o - 193}{10} - 0.4v_\Delta + \frac{v_o - v_a}{2.5} = 0,$
   $\frac{v_a - v_o}{2.5} - 0.5 + \frac{v_a - (v_b + 0.8v_\theta)}{10} = 0,$
   $\frac{v_b}{7.5} + 0.5 + \frac{v_b + 0.8v_\theta - v_a}{10} = 0.$

   The dependent source constraint equations are

   $v_\theta = -v_b \text{ and } v_\Delta = 2[v_a - (v_b + 0.8v_\theta)10].$

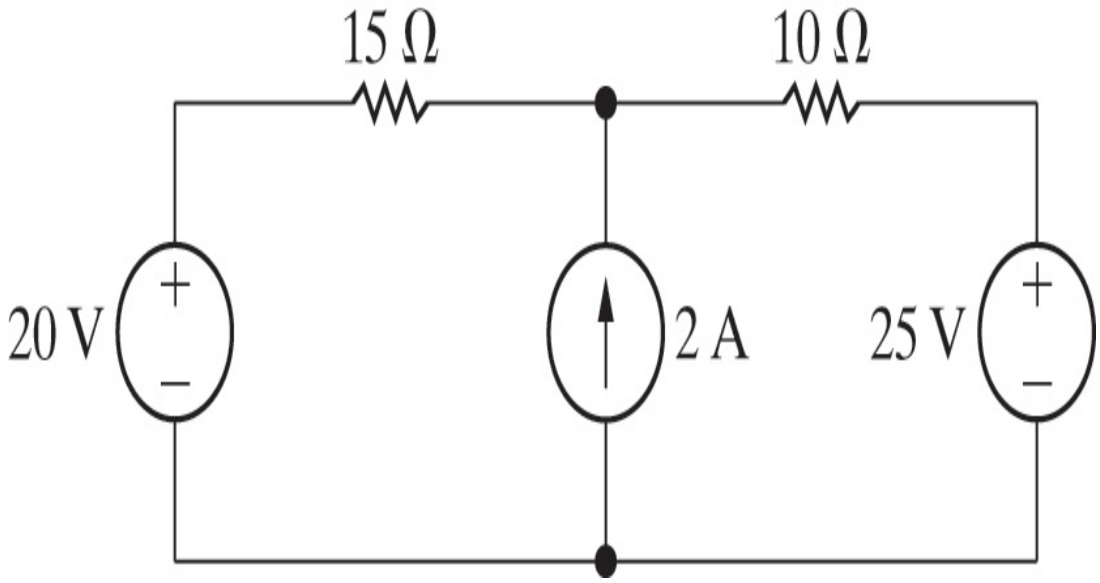
9. Step 4: Once we solve these five simultaneous equations, we have the value of vo without writing an additional equation, so Step 5 is not needed.

   Based on the comparison of the two methods, the node-voltage method involves a bit less work. You should verify that both approaches give $v_o = 173$ V.

Assessment Problems

Objective 3—Deciding between the node-voltage and mesh-current methods

1. 4.13 Find the power delivered by the 2 A current source in the circuit shown.

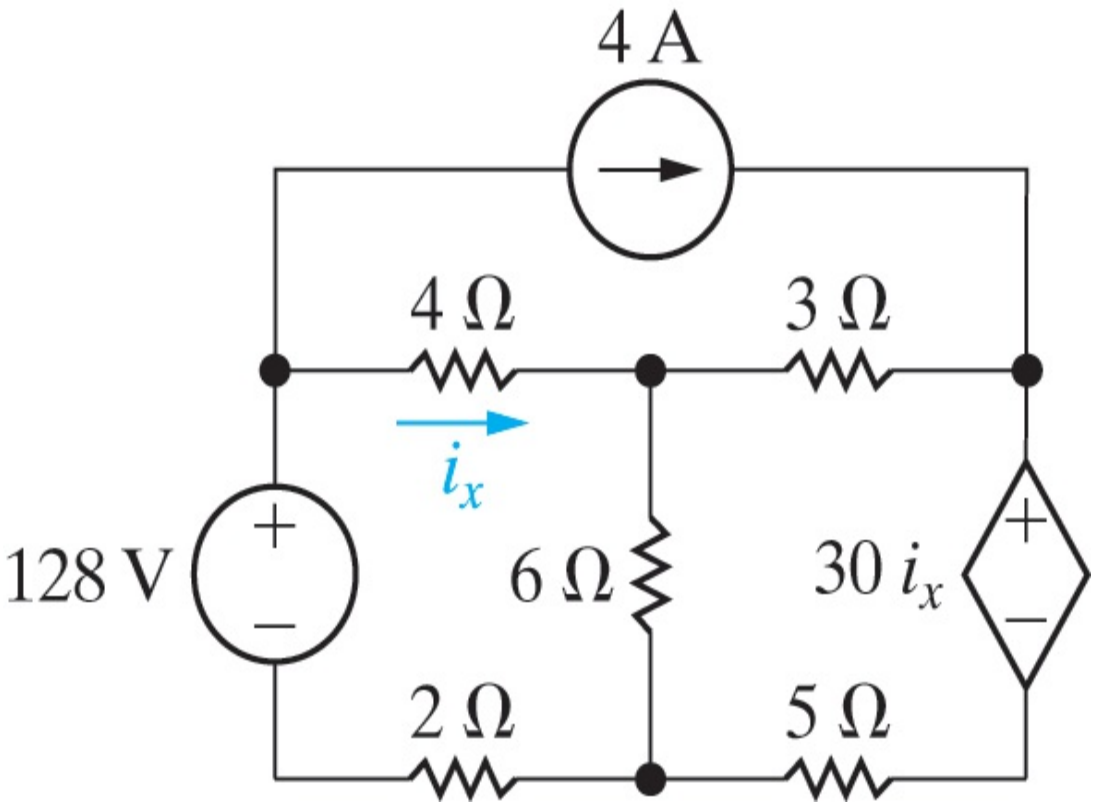


[4.8-17 Full Alternative Text]

Answer:

70 W.

2. 4.14 Find the power delivered by the 4 A current source in the circuit shown.



4.8-18 Full Alternative Text

Answer:

40 W.

SELF-CHECK: Also try ChapterProblems 4.54 and 4.56.

4.9 Source Transformations

We are always interested in methods that simplify circuits. Series-parallel reductions and Δ-to-Y transformations are already on our list of simplifying techniques. We now expand the list with source transformations. A **source transformation** allows a voltage source in series with a resistor to be replaced by a current source in parallel with the same resistor or vice versa. Figure 4.38 shows a source transformation. The double-headed arrow emphasizes that a source transformation is bilateral; that is, we can start with either configuration and derive the other.

Figure 4.38 Source transformations.

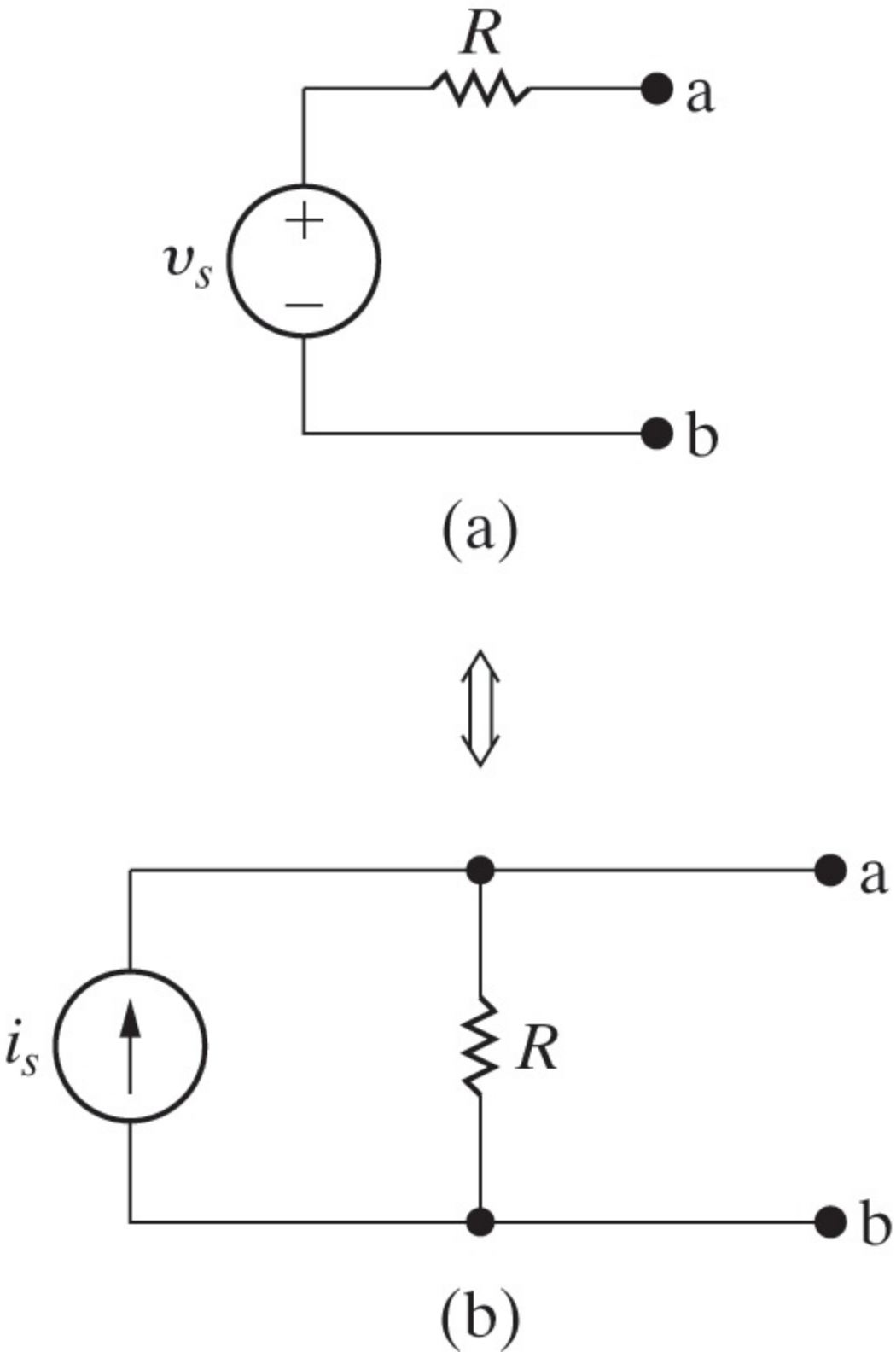


Figure 4.38 Full Alternative Text

We need to find the relationship between vs and is that guarantees the two configurations in Fig. 4.38 are equivalent with respect to nodes a and b. Equivalence is achieved if any resistor RL has the same current and thus the same voltage drop, whether connected between nodes a and b in Fig. 4.38(a) or Fig. 4.38(b).

Suppose RL is connected between nodes a and b in Fig. 4.38(a). Using Ohm's law, we find that the current in RL is

$$i_L = \frac{v_s}{R + R_L}. \quad (4.33)$$

Now suppose the same resistor RL is connected between nodes a and b in Fig. 4.38(b). Using current division, we see that the current in RL is

$$i_L = \frac{R}{R + R_L} i_s. \quad (4.34)$$

If the two circuits in Fig. 4.38 are equivalent, these resistor currents must be the same. Equating the right-hand sides of Eqs. 4.33 and 4.34 and simplifying gives the **condition of equivalence**:

$$i_s = \frac{v_s}{R}. \quad (4.35)$$

When Eq. 4.35 is satisfied for the circuits in Fig. 4.38, the current in RL connected between nodes a and b is the same for both circuits for all values of RL. If the current in RL is the same for both circuits, then the voltage drop across RL is the same for both circuits, and the circuits are equivalent at nodes a and b. If the polarity of vs is reversed, the orientation of is must be reversed to maintain equivalence.

Example 4.12 uses source transformations to simplify a circuit-analysis problem.

Example 4.12 Using Source Transformations to Solve a Circuit

Find the power associated with the 6 V source for the circuit shown in Fig.

4.39 and state whether the 6 V source is absorbing or delivering the power.

Figure 4.39 The circuit for Example 4.12.

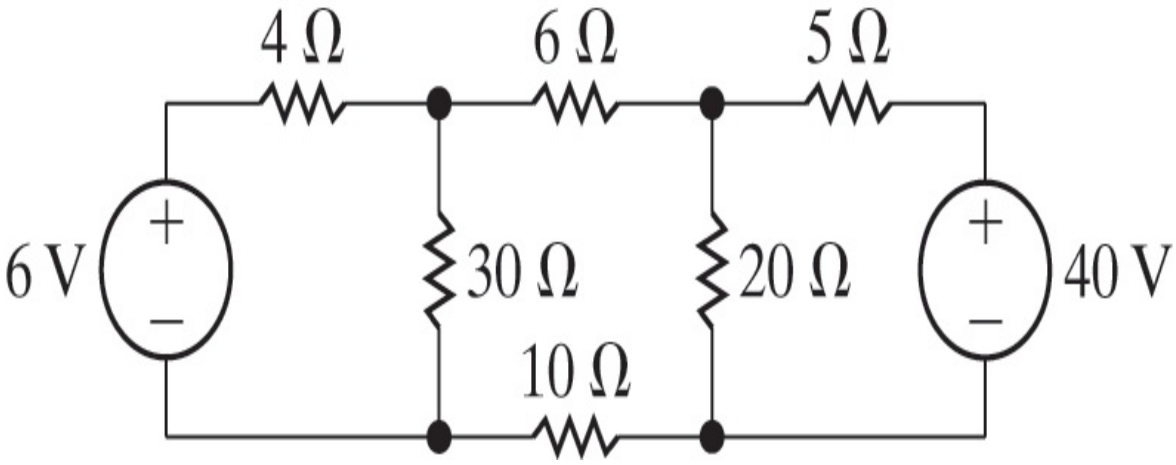
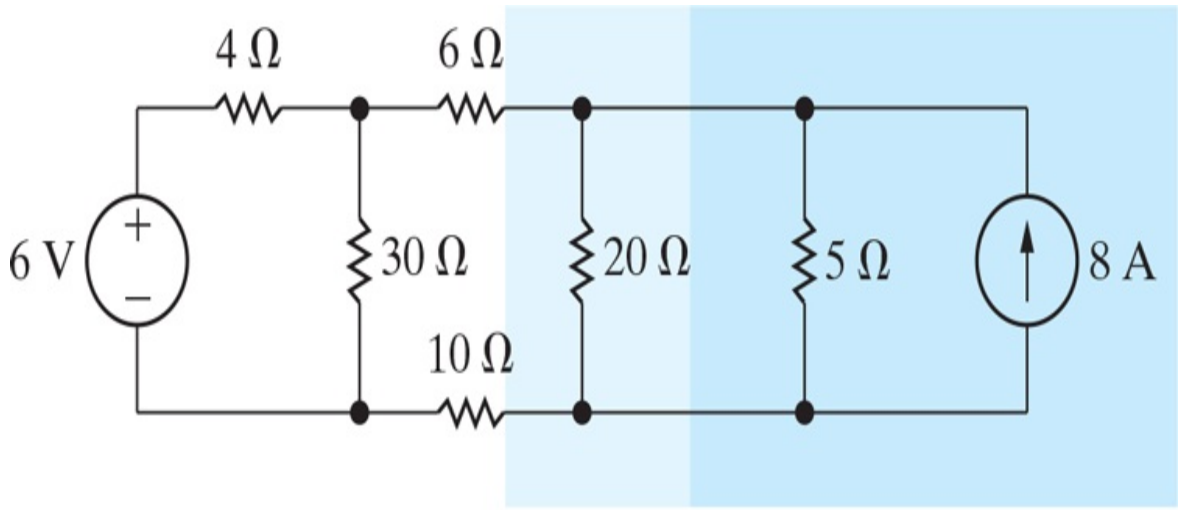


Figure 4.39 Full Alternative Text

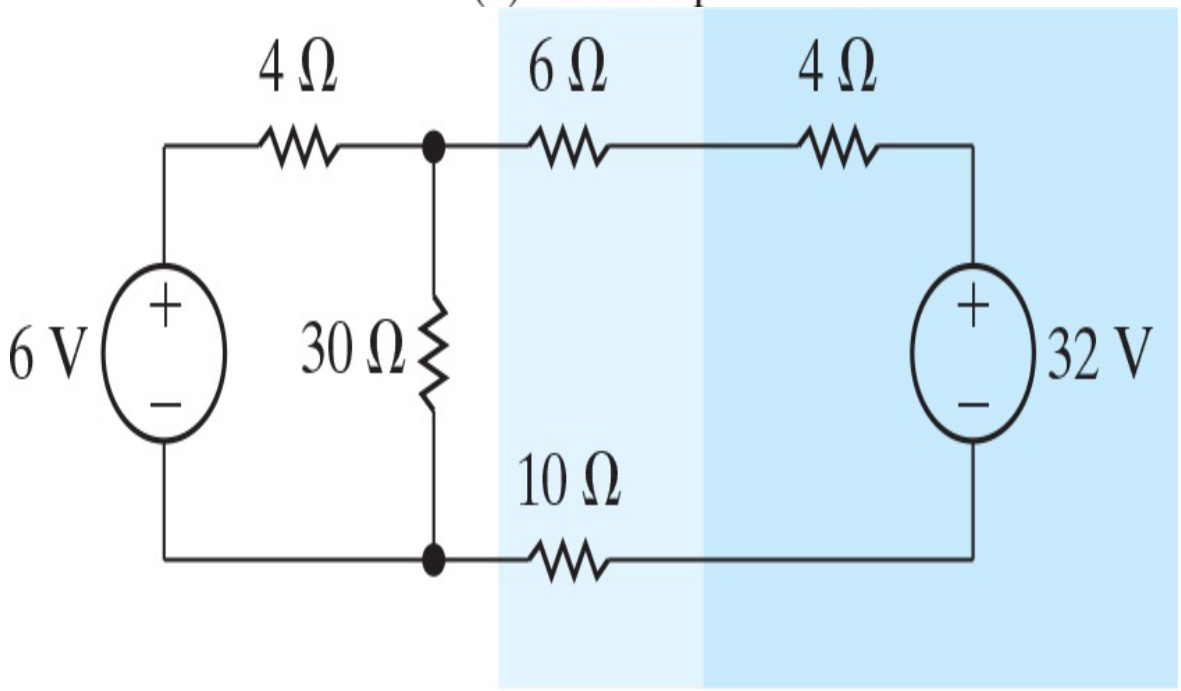
Solution

If we study the circuit shown in Fig. 4.39, we see ways to simplify the circuit by using source transformations. But we must simplify the circuit in a way that preserves the branch containing the 6 V source. Therefore, begin on the right side of the circuit with the branch containing the 40 V source. We can transform the 40 V source in series with the 5 Ω resistor into an 8 A current source in parallel with a 5 Ω resistor, as shown in Fig. 4.40(a).

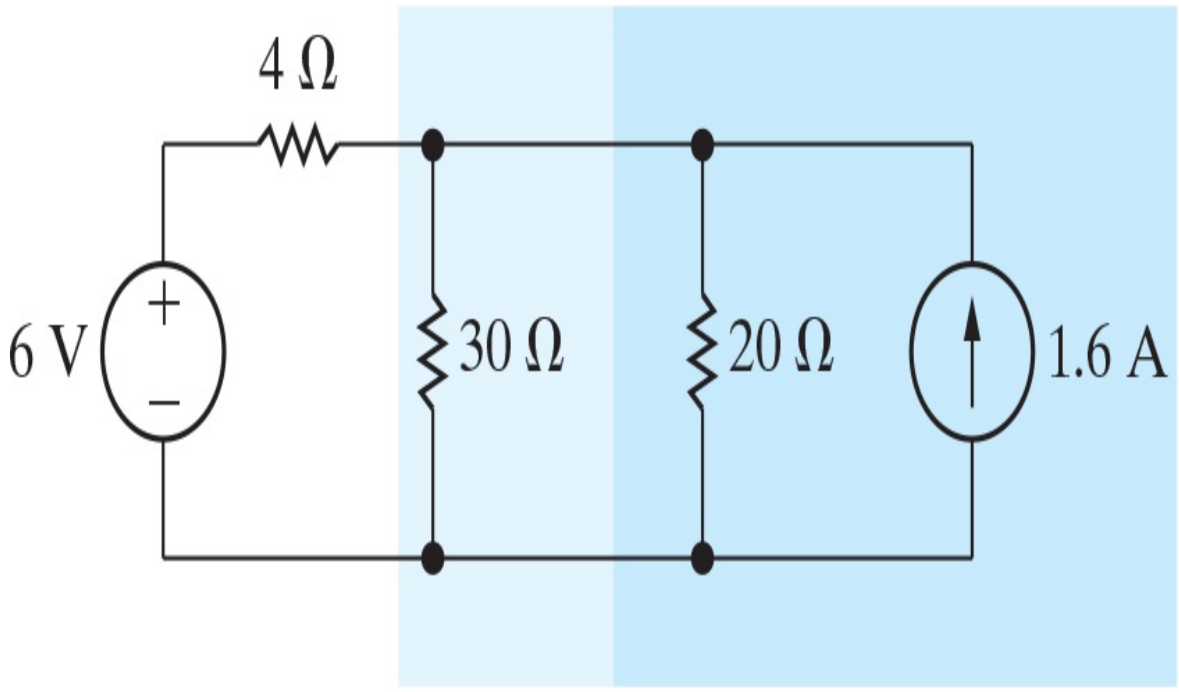
Figure 4.40 Step-by-step simplification of the circuit shown in Fig. 4.39.



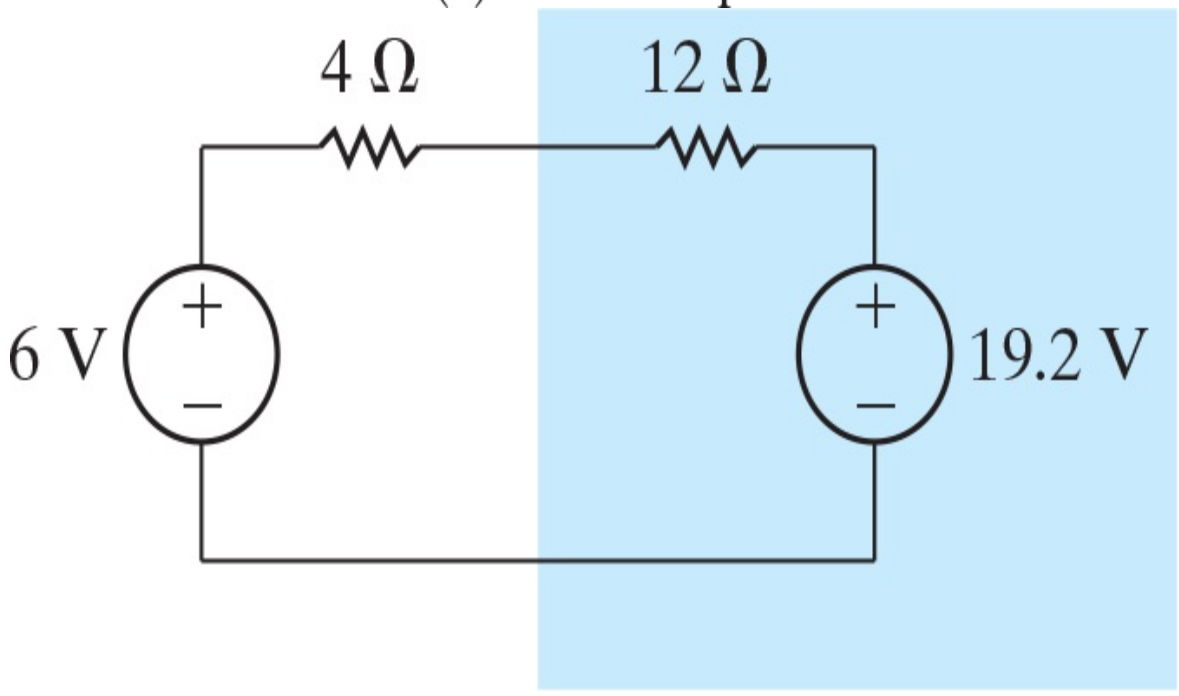
(a) First step



(b) Second step



(c) Third step



(d) Fourth step

Figure 4.40 Full Alternative Text

Next, replace the parallel combination of the20 Ω and 5 Ω resistors with a 4 Ω resistor. This 4 Ω resistor is in parallel with the 8 A source and therefore can be replaced with a 32 V source in series with a 4 Ω resistor, as shown in Fig. 4.40(b). The 32 V source is in series with 20 Ω of resistance and, hence, can be replaced by a current source of 1.6 A in parallel with 20 Ω, as shown in Fig. 4.40(c). The 20 Ω and 30 Ω parallel resistors can be reduced to a single 12 Ω resistor. The parallel combination of the 1.6 A current source and the 12 Ω resistor transforms into a voltage source of 19.2 V in series with 12 Ω. Figure 4.40(d) shows the result of this last transformation. The current in the direction of the voltage drop across the 6 V source is (19.2−6)/16, or 0.825 A. Therefore, the power associated with the 6 V source is

$$p_{6V}=(6)\,(0.825)=4.95\ \text{W}$$

and the voltage source is absorbing power.

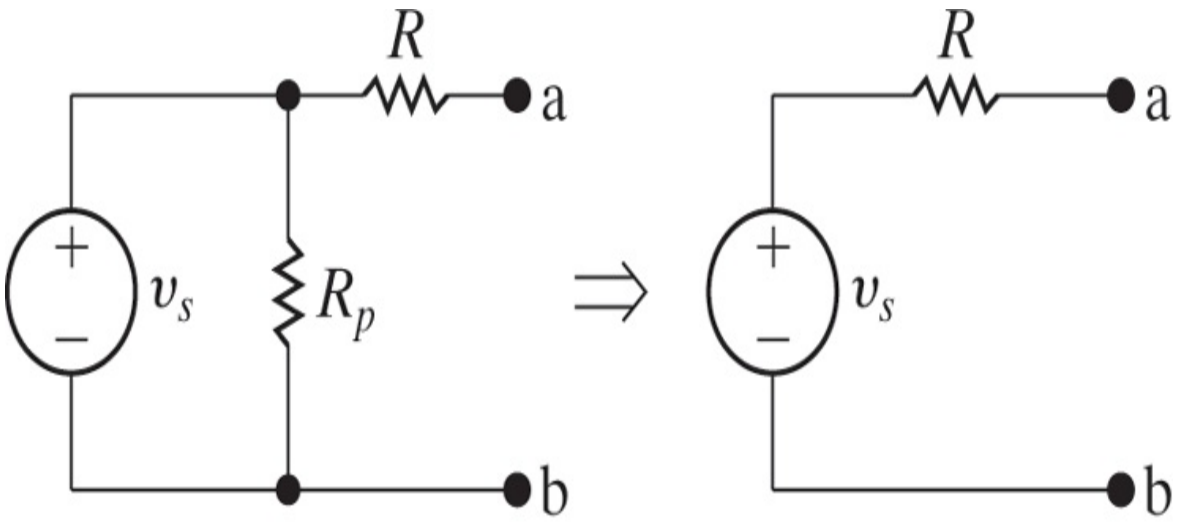
Practice your circuit analysis skills by using either the node-voltage method or the mesh-current method to solve this circuit and verify that you get the same answer.

A couple of questions arise from the source transformation depicted in Fig. 4.40.

- What happens if there is a resistance Rp in parallel with the voltage source?
- What happens if there is a resistance Rs in series with the current source?

In both cases, the resistance can be removed to create a simpler equivalent circuit with respect to terminals a and b. Figure 4.41 summarizes this observation. The two circuits depicted in Fig. 4.41(a) are equivalent with respect to terminals a and b because they produce the same voltage and current in any resistor RL inserted between nodes a and b. The same can be said for the circuits in Fig. 4.41(b). Example 4.13 illustrates an application of the equivalent circuits depicted in Fig. 4.41.

Figure 4.41 (a) Generating a simplified equivalent circuit from a circuit with a resistor in parallel with a voltage source; (b) generating a simplified circuit from a circuit with a resistor in series with a current source.



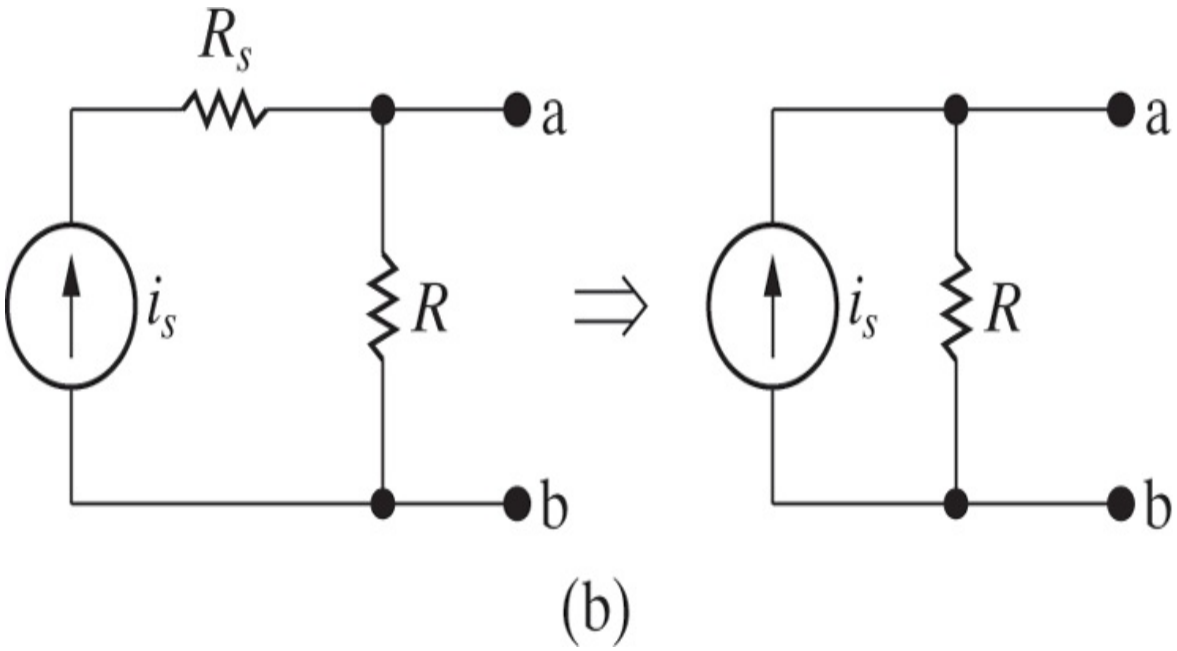


Figure 4.41 Full Alternative Text

Example 4.13 Using Special Source Transformation Techniques

1. Use source transformations to find the voltage vo in the circuit shown in Fig. 4.42.

Figure 4.42 The circuit for Example 4.13.

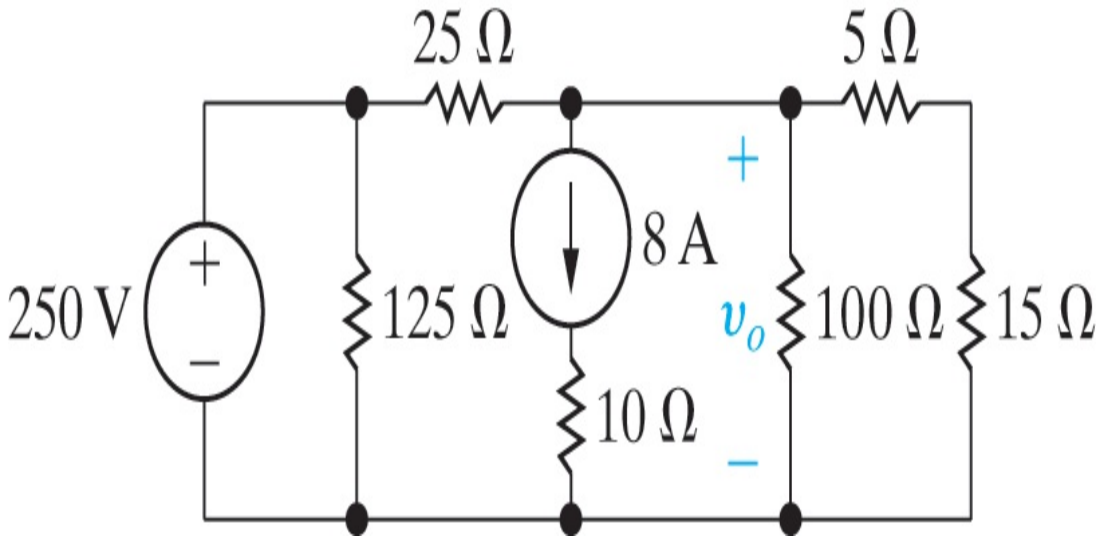


Figure 4.42 Full Alternative Text

2. Find the power developed by the 250 V voltage source.
3. Find the power developed by the 8 A current source.

Solution

1. We begin by removing the 125 Ω and 10 Ω resistors because the 125 Ω resistor is connected in parallel with the 250 V voltage source and the 10 Ω resistor is connected in series with the 8 A current source. We also combine the series-connected resistors in the rightmost branch into a single resistance of 20 Ω. Figure 4.43 shows the simplified circuit.

Figure 4.43 A simplified version of the circuit shown in Fig. 4.42.

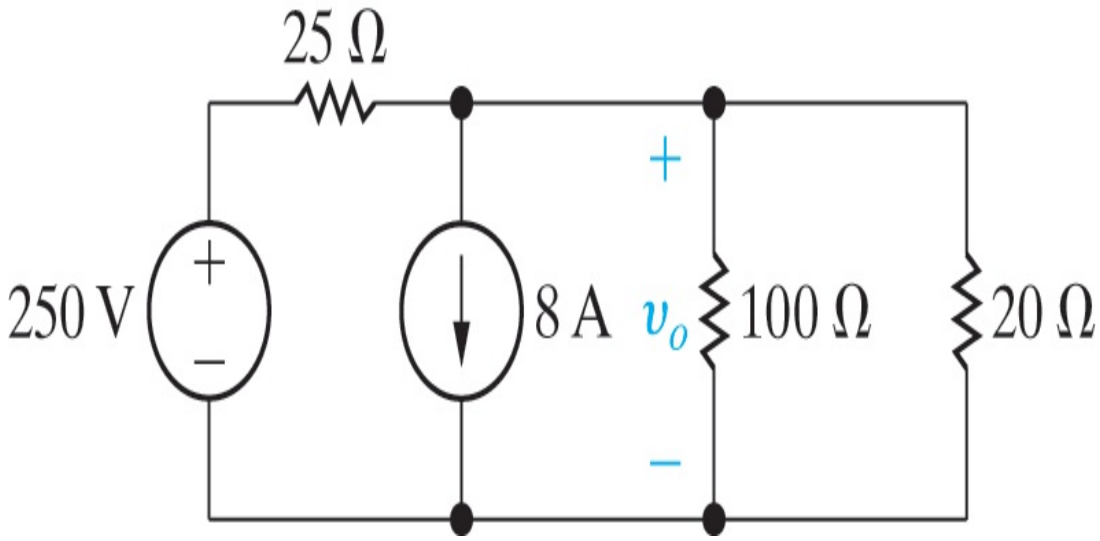


Figure 4.43 Full Alternative Text

Now use a source transformation to replace the series-connected 250 V source and 25 Ω resistor with a 10 A source in parallel with the 25 Ω resistor, as shown in Fig. 4.44. We can then use Kirchhoff's current law to combine the parallel current sources into a single source. The parallel resistors combine into a single resistor. Figure 4.45 shows the result. Hence vo=20 V.

Figure 4.44 The circuit shown in Fig. 4.43 after a source transformation.

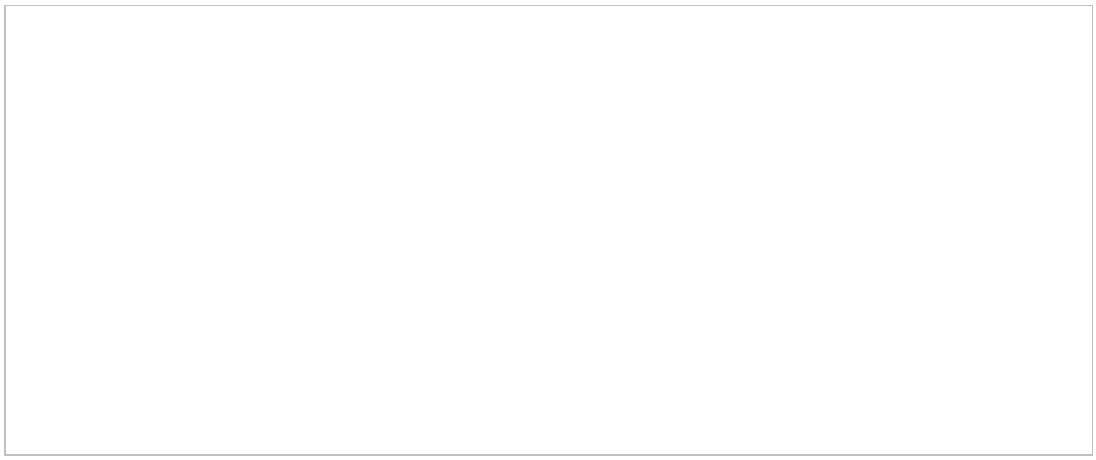


Figure 4.44 Full Alternative Text

Figure 4.45 The circuit shown in Fig. 4.44 after combining sources and resistors.

Figure 4.45 Full Alternative Text

2. We need to return to the original circuit in Fig. 4.42 to calculate the power associated with the sources. While a resistor connected in parallel with a voltage source or a resistor connected in series with a current

source can be removed without affecting the terminal behavior of the circuit, these resistors play an important role in how the power is dissipated throughout the circuit. The current supplied by the 250 V source, represented as is, equals the current in the 125 Ω resistor plus the current in the 25 Ω resistor. Thus,

is=250125+ 250−2025=11.2 A.

Therefore, the power developed by the voltage source is

p250V(developed)=(250) (11.2)=2800 W.

3. To find the power developed by the 8 A current source, we first find the voltage across the source. If we let vs represent the voltage across the source, positive at the upper terminal of the source, we obtain

   vs+8(10)=vo=20, or vs=−60 V,

   and the power developed by the 8 A source is 480 W. Note that the 125 Ω and 10 Ω resistors do not affect the value of vo but do affect the power calculations. Check your power calculations by determining the power absorbed by all of the resistors in the circuit.

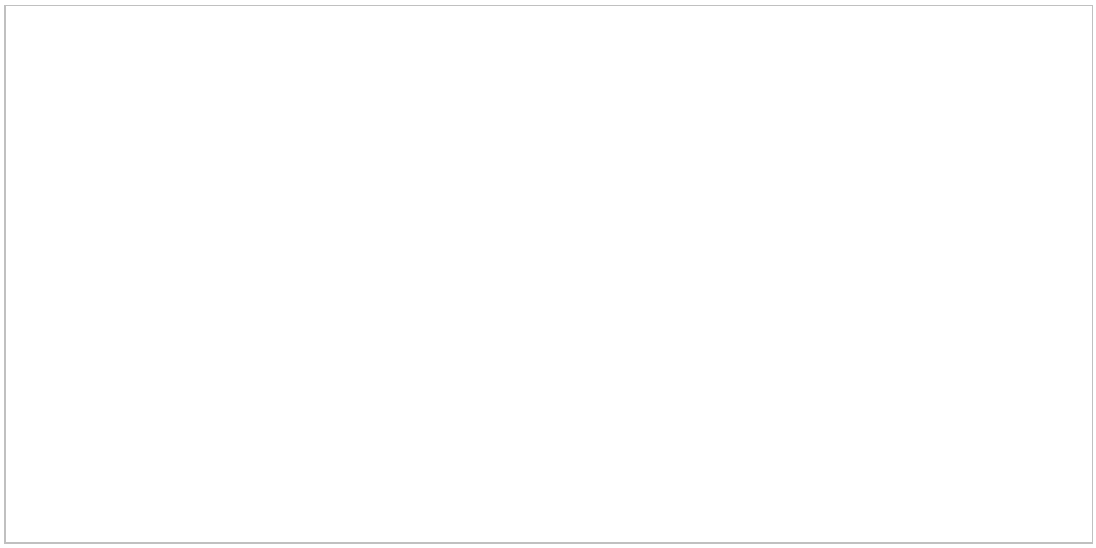
Assessment Problem

Objective 4—Understand source transformation

1. 4.15

   1. a) Use a series of source transformations to find the voltage v in the circuit shown.

   2. b) How much power does the 120 V source deliver to the circuit?



4.9-19 Full Alternative Text

Answer:

1. (a) 48 V;
2. (b) 374.4 W.

SELF-CHECK: Also try Chapter Problems 4.59 and 4.62.

4.10 Thévenin and Norton Equivalents

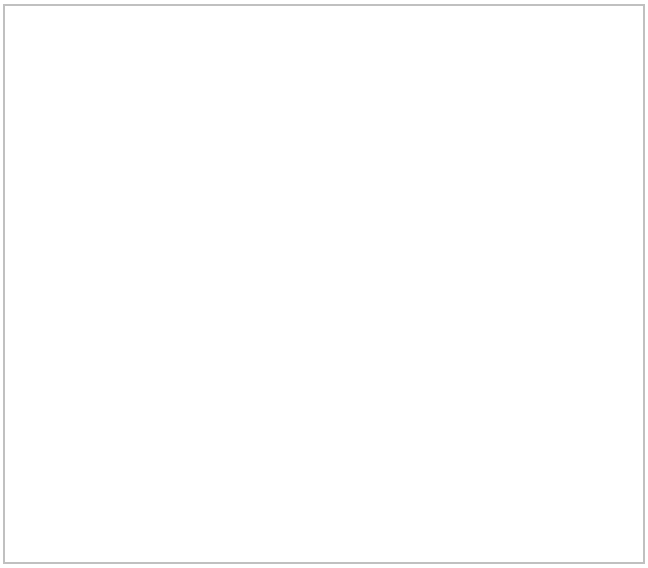
At times in circuit analysis, we want to concentrate on what happens at a specific pair of terminals. For example, when we plug a toaster into an outlet, we are interested primarily in the voltage and current at the terminals of the toaster. We have little or no interest in the effect that connecting the toaster has on voltages or currents elsewhere in the circuit supplying the outlet. We can expand this interest in terminal behavior to a set of appliances, each requiring a different amount of power. We then are interested in how the voltage and current delivered at the outlet change as we change appliances. In other words, we want to focus on the behavior of the circuit supplying the outlet, but only at the outlet terminals.

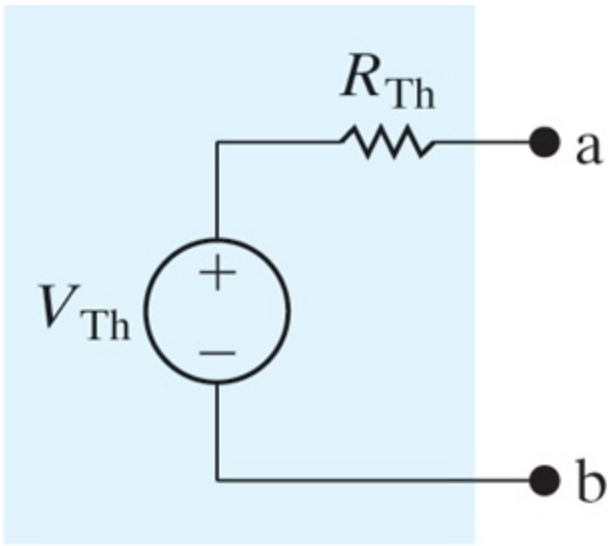
Thévenin and Norton equivalents are circuit simplification techniques. These equivalent circuits retain no information about the internal behavior of the original circuit and focus only on terminal behavior. They are extremely valuable when analyzing complex circuits where one portion of the circuit is fixed, so it can be replaced by a simple Thévenin or Norton equivalent, and another portion of the circuit is changing. Although here we discuss them as they pertain to resistive circuits, Thévenin and Norton equivalent circuits may be used to represent any circuit made up of linear elements.

The Thévenin equivalent circuit is the simplest equivalent for a given circuit and consists of a single voltage source in series with a single resistor. The Norton equivalent circuit is the source transform of the Thévenin equivalent circuit. To better grasp the concept of a Thévenin equivalent circuit, imagine a circuit with a complex interconnection of resistors, independent sources, and dependent sources, as shown in Fig. 4.46(a). We are interested in simplifying this complex circuit with respect to the terminals a and b. The simplified equivalent circuit, shown in Fig. 4.46(b), is the series combination of a voltage source VTh and a resistor RTh called the **Thévenin equivalent circuit**. It is equivalent to the original circuit in the sense that, if we connect the same load across the terminals a and b of each circuit, we get the same

voltage and current at the terminals of the load. This equivalence holds for all possible values of load resistance.

Figure 4.46 (a) A general circuit. (b) The Thévenin equivalent circuit.





(b)

The Thévenin Equivalent

To represent the original circuit by its Thévenin equivalent, we must calculate the Thévenin voltage VTh and the Thévenin resistance RTh. First, we note that if the load resistance is infinitely large, we have an open-circuit condition. The open-circuit voltage at the terminals a and b in the circuit shown in Fig. 4.46(b) is VTh. By hypothesis, this must be the same as the open-circuit voltage at the terminals a and b in the original circuit. Therefore, to find the Thévenin voltage VTh, calculate the open-circuit voltage in the original circuit.

Reducing the load resistance to zero gives us a short-circuit condition. If we place a short circuit across the terminals a and b of the Thévenin equivalent circuit, the short-circuit current directed from a to b is

isc=VThRTh. (4.36)

By hypothesis, this short-circuit current must be identical to the short-circuit current that exists in a short circuit placed across the terminals a and b of the original network. From Eq. 4.36,

RTh=VThisc. (4.37)

Thus, the Thévenin resistance is the ratio of the open-circuit voltage to the short-circuit current. Work through Example 4.14 to see how to find the Thévenin equivalent of a circuit.

Example 4.14 Finding a Thévenin Equivalent

Find the Thévenin equivalent of the circuit in Fig. 4.47.

Figure 4.47 A circuit used to

illustrate a Thévenin equivalent.

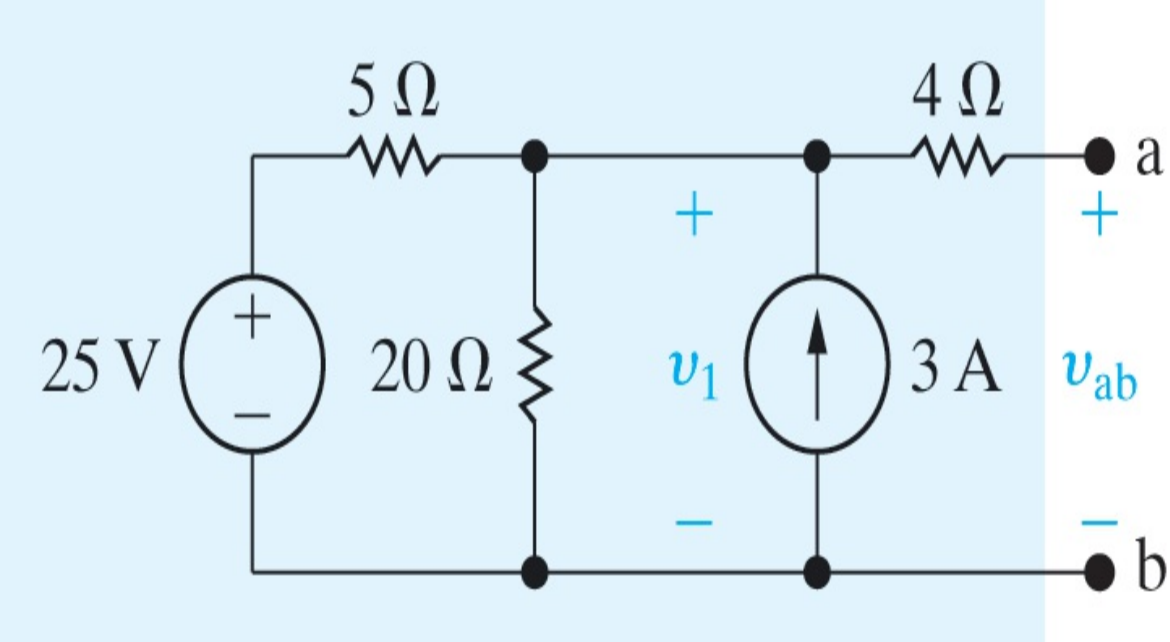


Figure 4.47 Full Alternative Text

Solution

To find the Thévenin equivalent of the circuit shown in Fig. 4.47, we first calculate the open-circuit voltage vab. Note that when the terminals a, b are open, there is no current in the 4 Ω resistor. Therefore the open-circuit voltage vab is identical to the voltage across the 3 A current source, labeled v1. We find the voltage by solving a single KCL equation. Choosing the lower node as the reference node, we get

v1−255+v120−3=0.

Solving for v1 yields

v1=32 V=VTh.

Hence, the Thévenin voltage for the circuit is 32 V.

The next step is to place a short circuit across the terminals a and b and calculate the resulting short-circuit current. Figure 4.48 shows the circuit with the short in place. Note that the short-circuit current is in the direction of the open-circuit voltage drop across the terminals a and b. If the short-circuit current is in the direction of the open-circuit voltage rise across the terminals, a minus sign must be inserted in. Eq. 4.37.

Figure 4.48 The circuit shown in Fig. 4.47 with terminals a and b short-circuited.

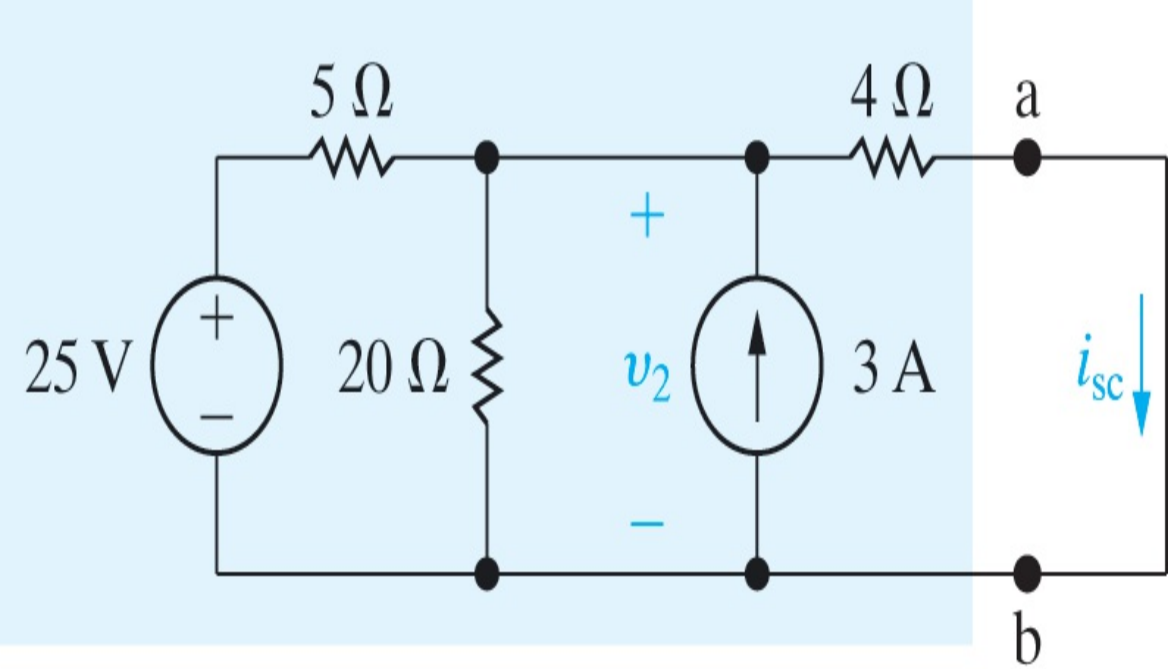


Figure 4.48 Full Alternative Text

The short-circuit current (isc) is found easily once v2 is known. Therefore, the problem reduces to finding v2 with the short in place. Again, if we use the lower node as the reference node, the KCL equation at the node labeled v2 is

v2−255+ v220−3+ v24=0.

Solving for v2 gives

v2=16 V.

Hence, the short-circuit current is

isc=164=4 A.

We now find the Thévenin resistance by substituting the numerical values for the Thévenin voltage, VTh, and the short-circuit current, isc, into Eq. 4.37:

RTh=VThisc=324=8 Ω.

Figure 4.49 shows the Thévenin equivalent for the circuit shown in Fig. 4.45.

Figure 4.49 The Thévenin equivalent of the circuit shown in Fig. 4.47.

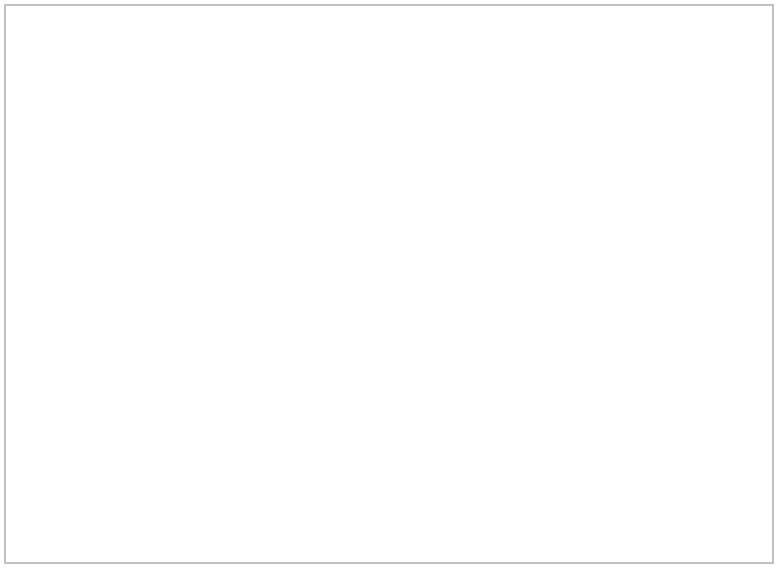


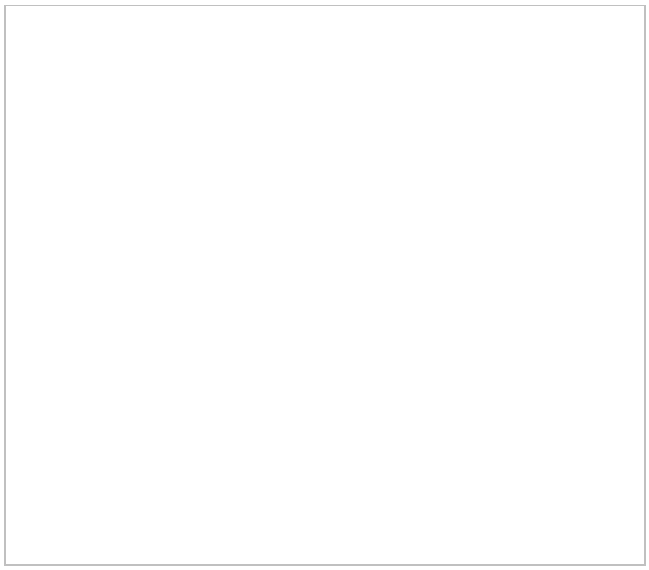
Figure 4.49 Full Alternative Text

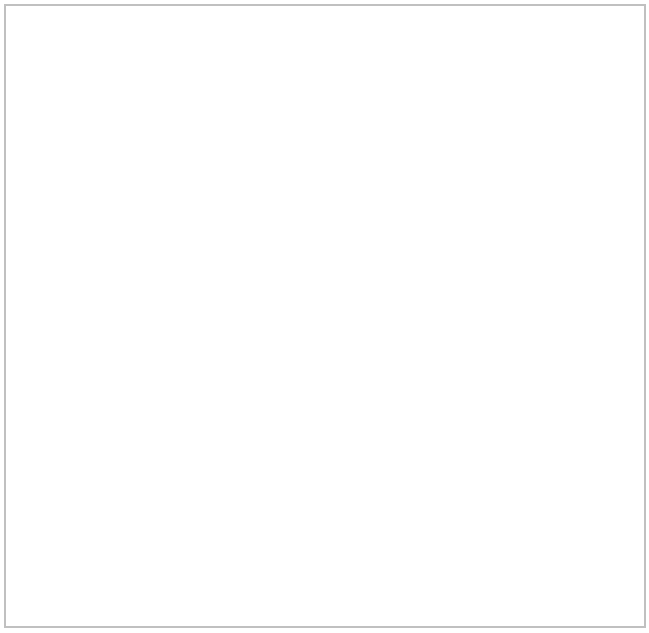
You should verify that, if a 24 Ω resistor is connected across the terminals a and b in Fig. 4.47, the voltage across the resistor will be 24 V and the current in the resistor will be 1 A, as would be the case with the Thévenin circuit in Fig. 4.49. This same equivalence between the circuits in Figs. 4.47 and 4.49 holds for any resistor value connected between nodes a and b.

The Norton Equivalent

A **Norton equivalent circuit** consists of an independent current source in parallel with the Norton equivalent resistance, as shown in Fig. 4.50. We can derive it directly from the original circuit by calculating the open-circuit voltage and the short-circuit current, just as we did when calculating the Thévenin equivalent. The Norton current equals the short-circuit current at the terminals of interest, and the Norton resistance is the ratio of the open-circuit voltage to the short-circuit current, so it is identical to the Thévenin resistance:

Figure 4.50 (a) A general circuit. (b) The Norton equivalent circuit.





IN=isc;
RN=vocisc=RTh. (4.38)

If we already have a Thévenin equivalent circuit, we can derive the Norton equivalent circuit from it simply by making a source transformation.

Using Source Transformations

Sometimes we can use source transformations to derive a Thévenin or Norton

equivalent circuit. This technique works best when the network contains only independent sources. Dependent sources require us to retain the identity of the controlling voltages and/or currents, and this constraint usually prohibits simplification of the circuit by source transformations. Work through Example 4.15 to see how a series of source transformations leads to a Norton equivalent circuit.

Example 4.15 Finding a Norton Equivalent

Find the Norton equivalent of the circuit in Fig. 4.47 by making a series of source transformations.

Solution

We start on the left side of the circuit and transform the series-connected 25 V source and 5 Ω resistor to a parallel-connected 5 A source and 5 Ω resistor, as shown in Step 1 of Fig. 4.51. Use KCL to combine the parallel-connected 5 A and 3 A sources into a single 8 A source, and combine the parallel-connected 5 Ω and 25 Ω resistors into a single 4 Ω resistor, as shown in Step 2 of Fig. 4.51. Now transform the parallel-connected 8 A source and 4 Ω resistor to a series-connected 32 V source and 4 Ω resistor, and combine the two series-connected 4 Ω resistors into a single 8 Ω resistor, as shown in Step 3 of Fig. 4.51. Note that the result of Step 3 is the Thévenin equivalent circuit we derived in Example 4.12. Finally, transform the series-connected 32 V source and 8 Ω resistor into a parallel-connected 4 A source and 8 Ω resistor, which is the Norton equivalent circuit, as shown in Step 4 of Fig. 4.51.

Figure 4.51 Step-by-step derivation of the Thévenin and

Norton equivalents of the circuit shown in Fig. 4.47.

Figure 4.51 Full Alternative Text

Gain additional practice with Thévenin equivalent circuits and see how to cope with the presence of a dependent source in the original circuit by working through Example 4.16.

Example 4.16 Finding the Thévenin Equivalent of a Circuit with a Dependent Source

Find the Thévenin equivalent for the circuit containing dependent sources shown in Fig. 4.52.

Figure 4.52 A circuit used to illustrate a Thévenin equivalent when the circuit contains dependent sources.

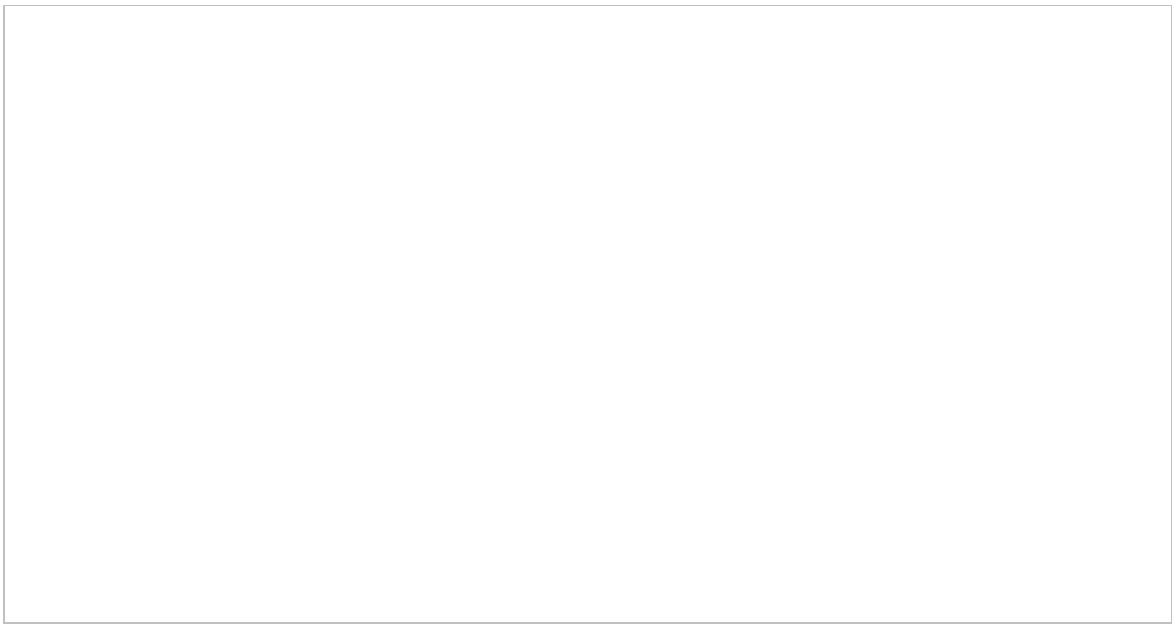


Figure 4.52 Full Alternative Text

Solution

The first step in analyzing the circuit in Fig. 4.52 is to recognize that the current labeled ix must be zero. (Note the absence of a return path for ix to enter the left-hand portion of the circuit.) The open- circuit, or Thévenin, voltage is the voltage across the 25 Ω resistor. Since ix=0,

VTh=vab=(−20i)(25)=−500i.

The current i is

i=5−3v2000=5−3VTh2000.

In writing the equation for i, we recognize that the Thévenin voltage is identical to v. When we combine these two equations, we obtain

VTh=−5 V.

To calculate the short-circuit current, we place a short circuit across a and b. When the terminals a and b are shorted together, the control voltage v is

reduced to zero. Therefore, with the short in place, the circuit shown in Fig. 4.52 becomes the one shown in Fig. 4.53. With the short circuit shunting the 25 Ω resistor, all of the current from the dependent current source appears in the short, so

Figure 4.53 The circuit shown in Fig. 4.52 with terminals a and b short-circuited.

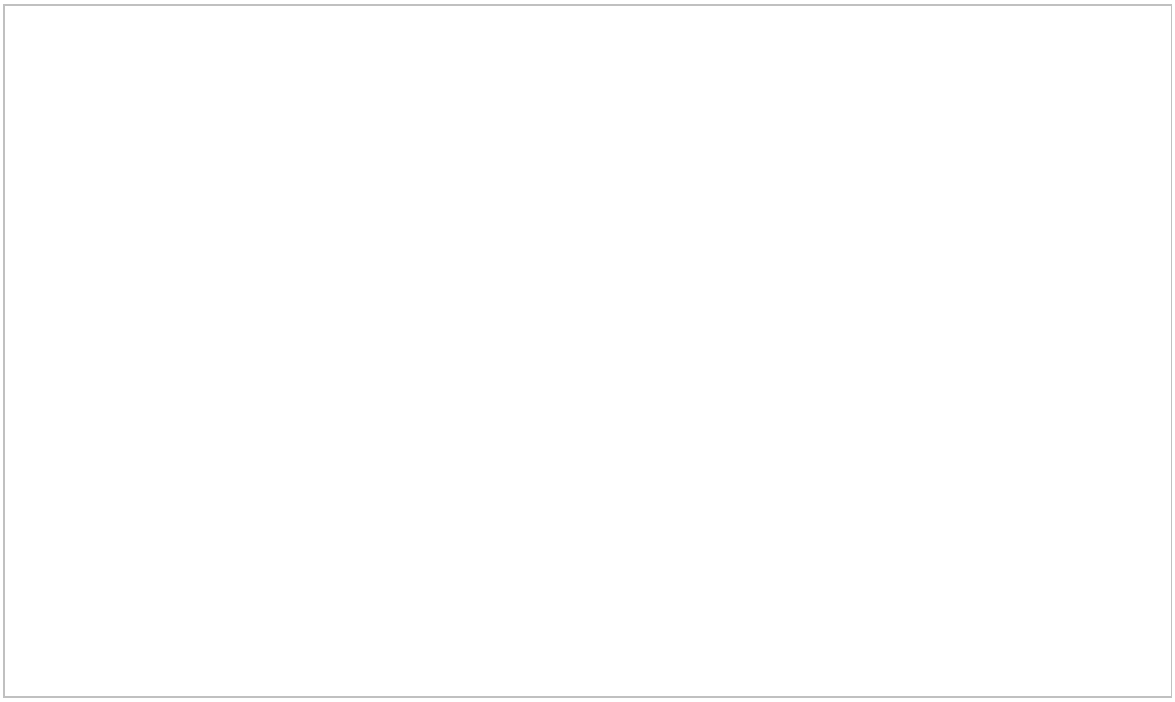


Figure 4.53 Full Alternative Text

isc=−20i.

As the voltage controlling the dependent voltage source has been reduced to zero, the current controlling the dependent current source is

i=52000=2.5 mA.

Combining these two equations yields a short-circuit current of

isc=−20(2.5)=−50 mA.

From isc and VTh we get

RTh=vThisc=−5−0.05=100 Ω.

Figure 4.54 illustrates the Thévenin equivalent for the circuit shown in Fig. 4.52. Note that the reference polarity marks on the Thévenin voltage source in Fig. 4.54 agree with the preceding equation for VTh.

Figure 4.54 The Thévenin equivalent for the circuit shown in Fig. 4.52.

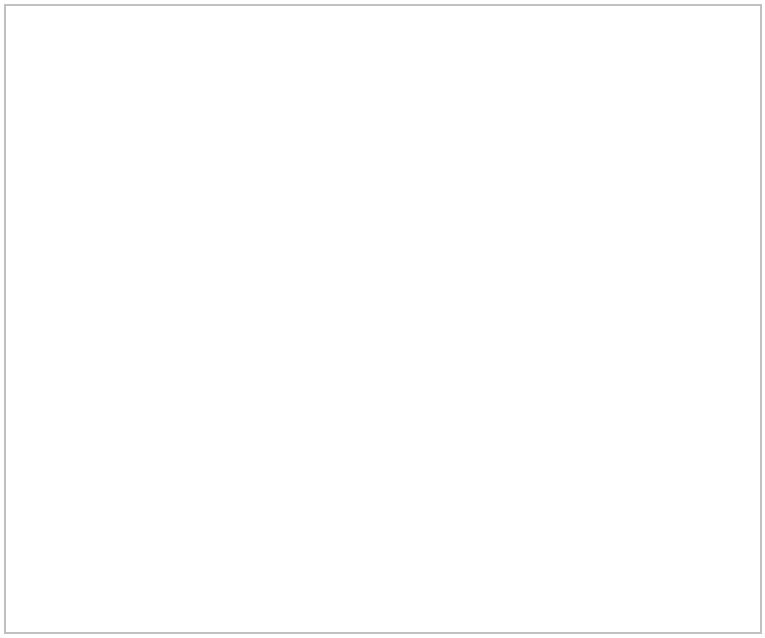
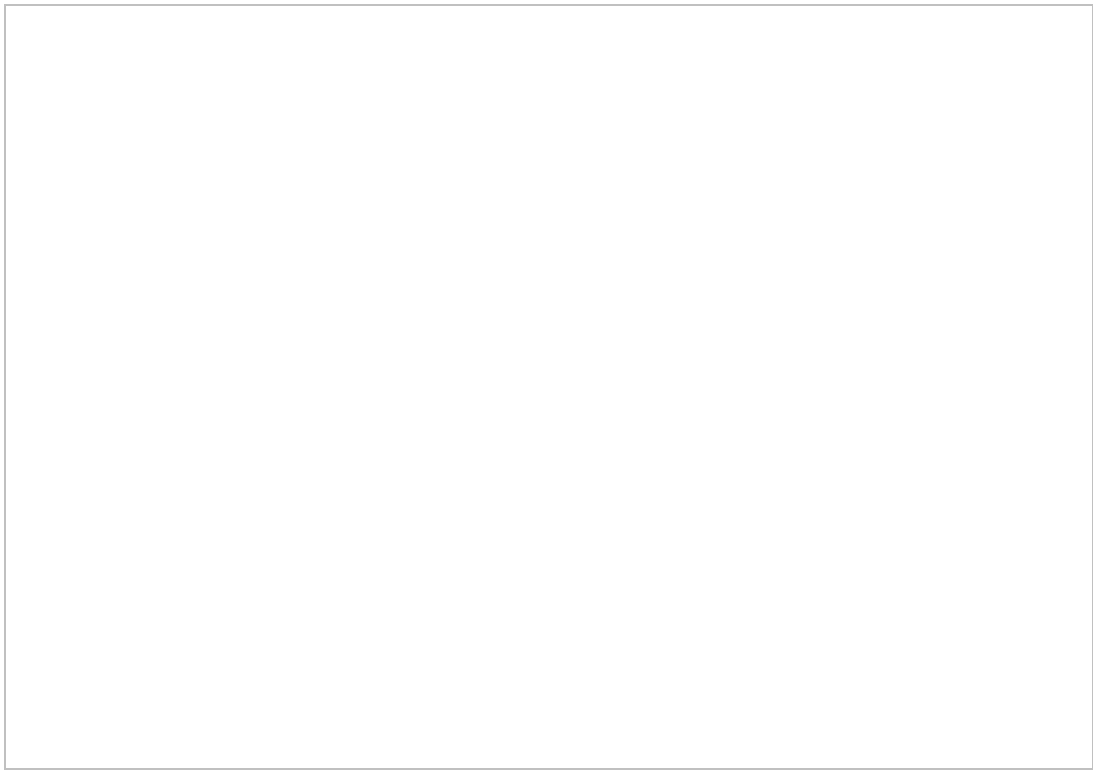


Figure 4.54 Full Alternative Text

Assessment Problems

Objective 5—Understand Thévenin and Norton equivalents

1. 4.16 Find the Thévenin equivalent circuit with respect to the terminals a, b for the circuit shown.

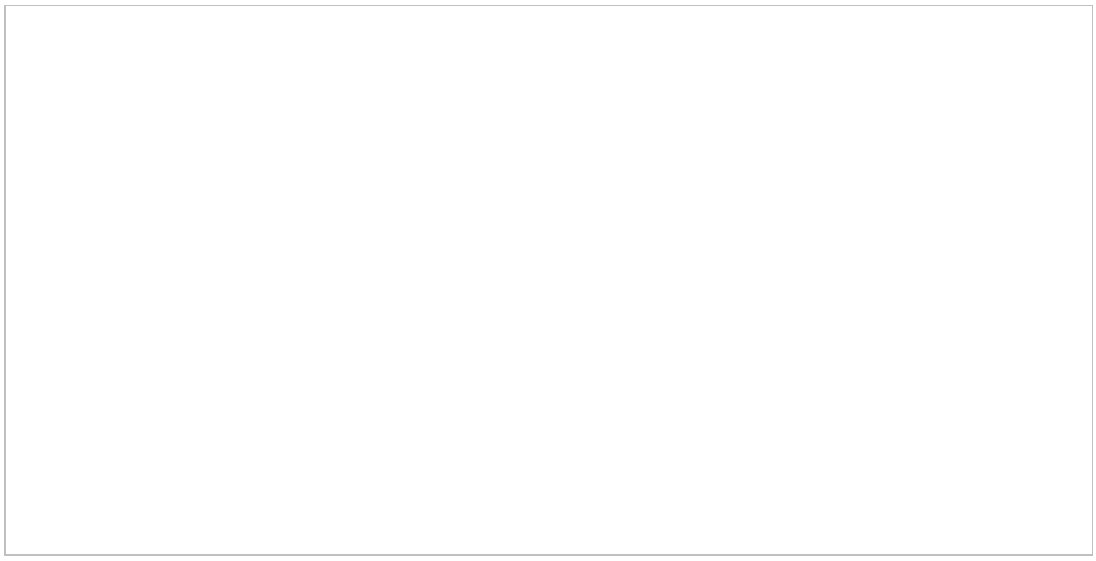


4.10-20 Full Alternative Text

Answer:

Vab=VTh=64.8 V, RTh=6 Ω.

2. 4.17 Find the Norton equivalent circuit with respect to the terminals a, b for the circuit shown.

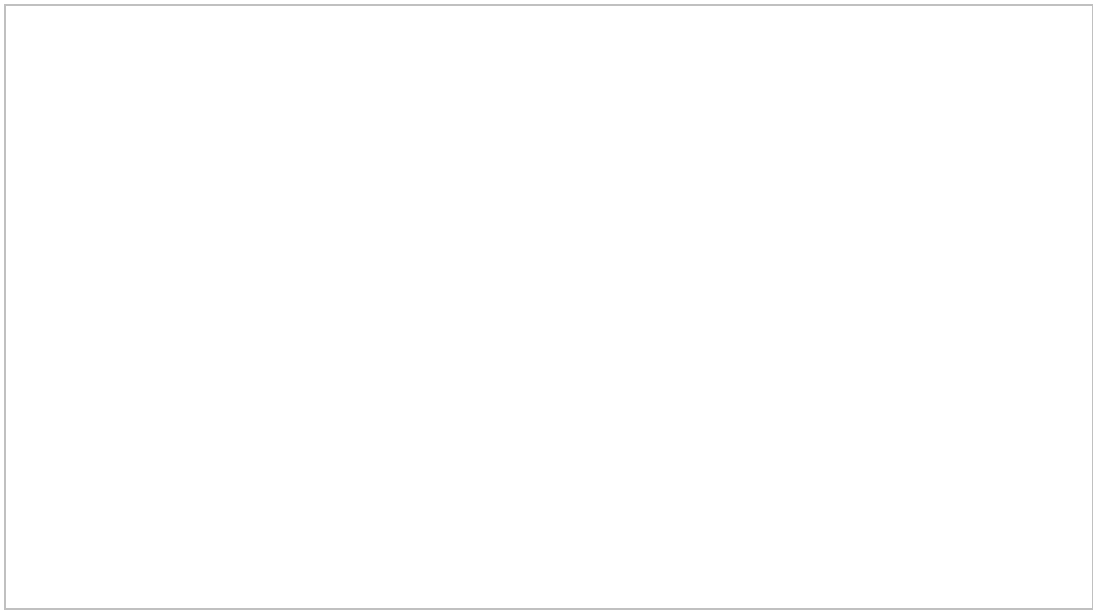


4.10-21 Full Alternative Text

Answer:

IN=6 A (directed toward a), RN=7.5 Ω.

3. 4.18 A voltmeter with an internal resistance of 100 kΩ is used to measure the voltage vAB in the circuit shown. What is the voltmeter reading?



4.10-22 Full Alternative Text

Answer:

120 V.

SELF-CHECK: Also try Chapter Problems 4.64, 4.68, and 4.72.

4.11 More on Deriving the Thévenin Equivalent

We can calculate the Thévenin resistance, RTh, directly from the circuit rather than calculating it as the ratio of the open-circuit voltage to the short-circuit current (Eq. 4.37). If the circuit contains only independent sources and resistors, we can determine RTh by deactivating all independent sources and then calculating the resistance seen looking into the network at the designated terminal pair. A voltage source is deactivated by replacing it with a short circuit, while a current source is deactivated by replacing it with an open circuit. Example 4.17 illustrates this direct method for determining RTh.

Example 4.17 Finding the Thévenin Equivalent Resistance Directly from the Circuit

Find RTh the circuit shown in Fig. 4.55.

Figure 4.55 A circuit used to illustrate a Thévenin equivalent.

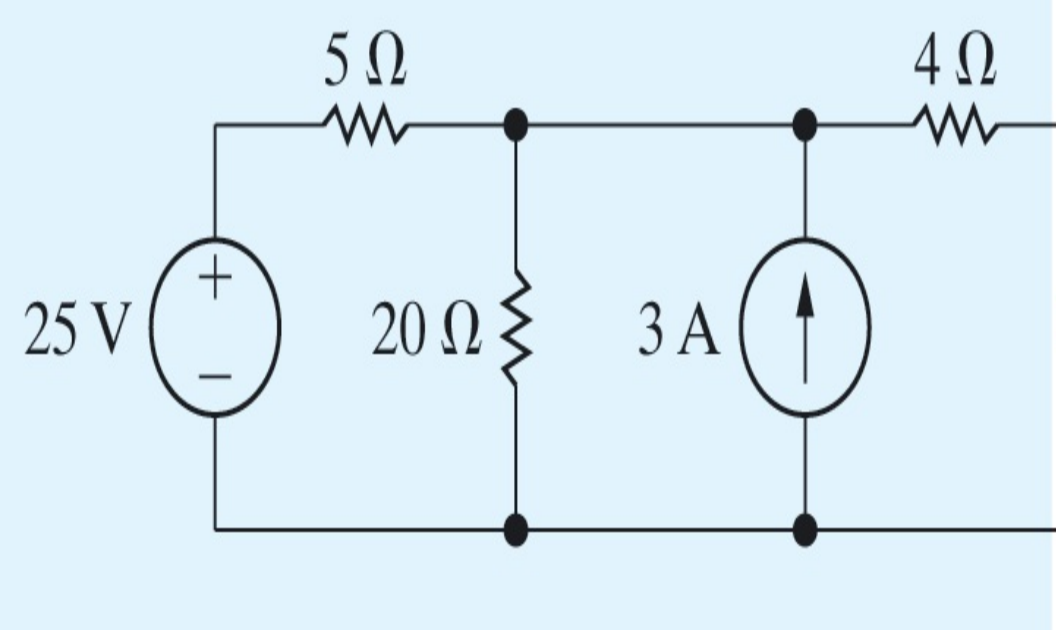


Figure 4.55 Full Alternative Text

Solution

Deactivating the independent sources simplifies the circuit to the one shown in Fig. 4.56. The resistance seen looking into the terminals a and b is denoted Rab, which consists of the 4 Ω resistor in series with the parallel combination of the 5 and 20 Ω resistors. Thus,

Figure 4.56 The circuit shown in Fig. 4.55 after deactivation of the independent sources.

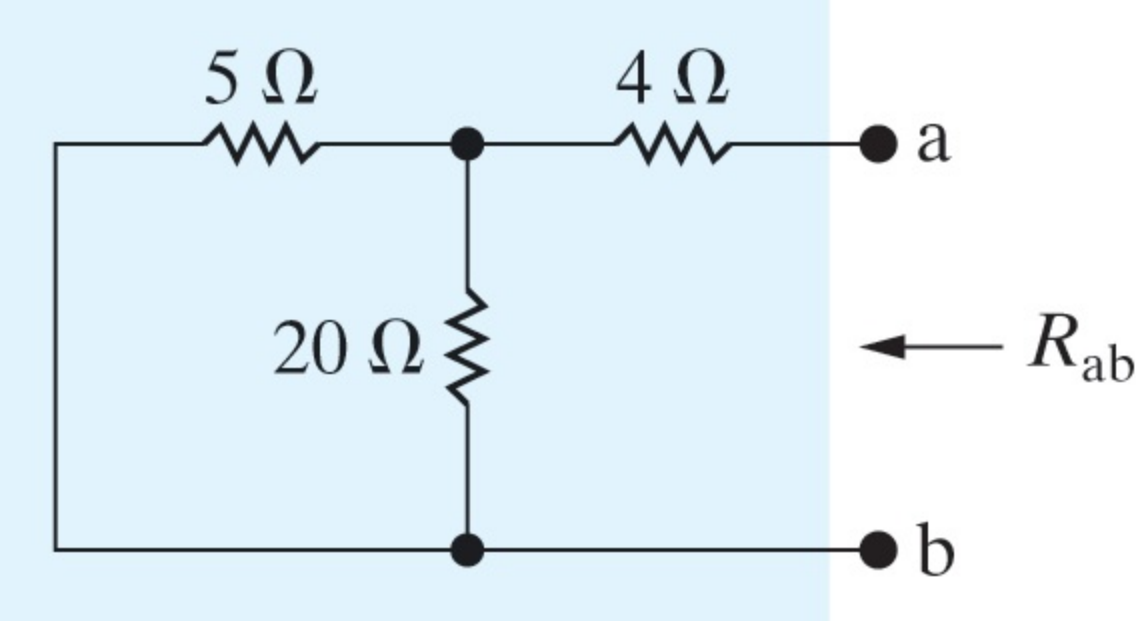


Figure 4.56 Full Alternative Text
Rab=RTh=4+5× 2025=8 Ω.

Note that deriving RTh directly from the circuit is much simpler than finding RTh from Eq. 4.37, as we did in Example 4.14.

If the circuit or network contains dependent sources, a direct method for finding the Thévenin resistance RTh is as follows. We first deactivate all independent sources, and then we apply either a test voltage source or a test current source to the Thévenin terminals a and b. The Thévenin resistance equals the ratio of the voltage across the test source to the current delivered by the test source. Example 4.18 illustrates this alternative procedure for finding RTh, using the same circuit as Example 4.16.

Example 4.18 Finding the Thévenin Equivalent Resistance Using a Test Source

Find the Thévenin resistance RTh for the circuit in Fig. 4.52, using the test

source method.

Solution

Begin by deactivating the independent voltage source and exciting the circuit from the terminals a and b with either a test voltage source or a test current source. If we apply a test voltage source, we will know the voltage of the dependent voltage source and hence the controlling current i. Therefore, we opt for the test voltage source. Figure 4.57 shows the circuit for computing the Thévenin resistance.

Figure 4.57 An alternative method for computing the Thévenin resistance.

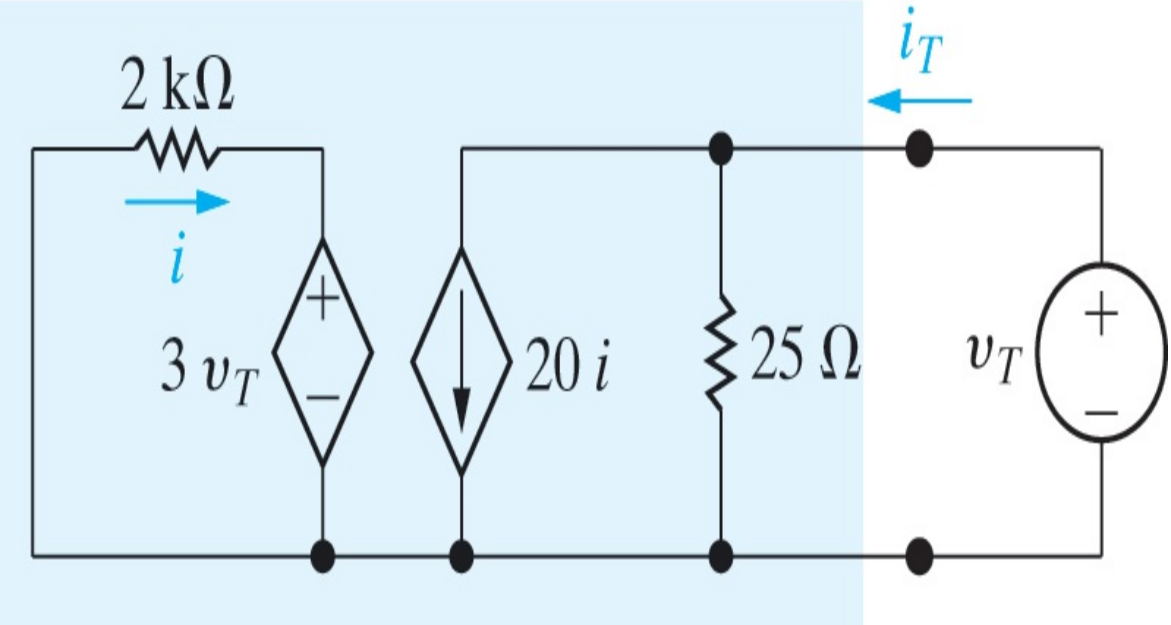


Figure 4.57 Full Alternative Text

The test voltage source is denoted vT, and the current that it delivers to the

circuit is labeled iT. To find the Thévenin resistance, we solve the circuit for the ratio of the voltage to the current at the test source; that is, RTh=vT/iT. From Fig. 4.57,

iT=vT25+20i,i=−3vT2000.

We then substitute the expression for i into the equation for iT and solve the resulting equation for the ratio iT/vT:

iT=vT25−60vT2000,
iTvT=125−6200=505000=1100.

The Thévenin resistance is the inverse of the ratio iT/vT, so

RTh=vTiT=100 Ω.

In a network containing only resistors and dependent sources, the Thévenin equivalent voltage VTh=0 and the Norton equivalent current IN=0. It should be clear that if the circuit you start with has no independent sources, its equivalent circuit cannot have any independent sources either. Therefore, the Thévenin and Norton equivalents for a circuit with only dependent sources and resistors is a single equivalent resistance, whose value must be determined using the test source method. The procedure is illustrated in Example 4.19.

Example 4.19 Finding the Thévenin Equivalent of a Circuit with Dependent Sources and Resistors

Find the Norton equivalent for the circuit in Fig. 4.58.

Figure 4.58 A circuit used to

determine a Thévenin equivalent when the circuit contains only dependent sources and resistors.

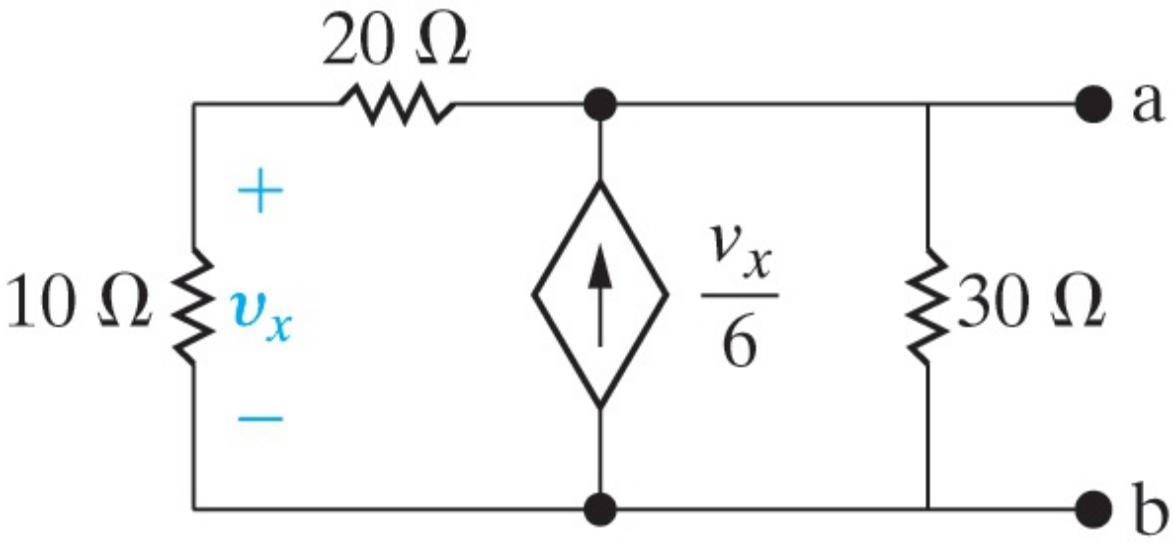


Figure 4.58 Full Alternative Text

Solution

The circuit in Fig. 4.58 has no independent sources. Therefore, the Norton equivalent current is zero, and the Norton equivalent circuit consists only of the Norton resistance, RN. Applying a test source to the terminals a and b is the only way to determine RN. We have applied a test current source, whose value is iT, as shown in Fig. 4.59. Analyze this circuit to calculate the voltage across the test source, vT, and then calculate the Norton equivalent resistance using the ratio of vT to iT.

Figure 4.59 The circuit in Fig.4.58 with a test current source.

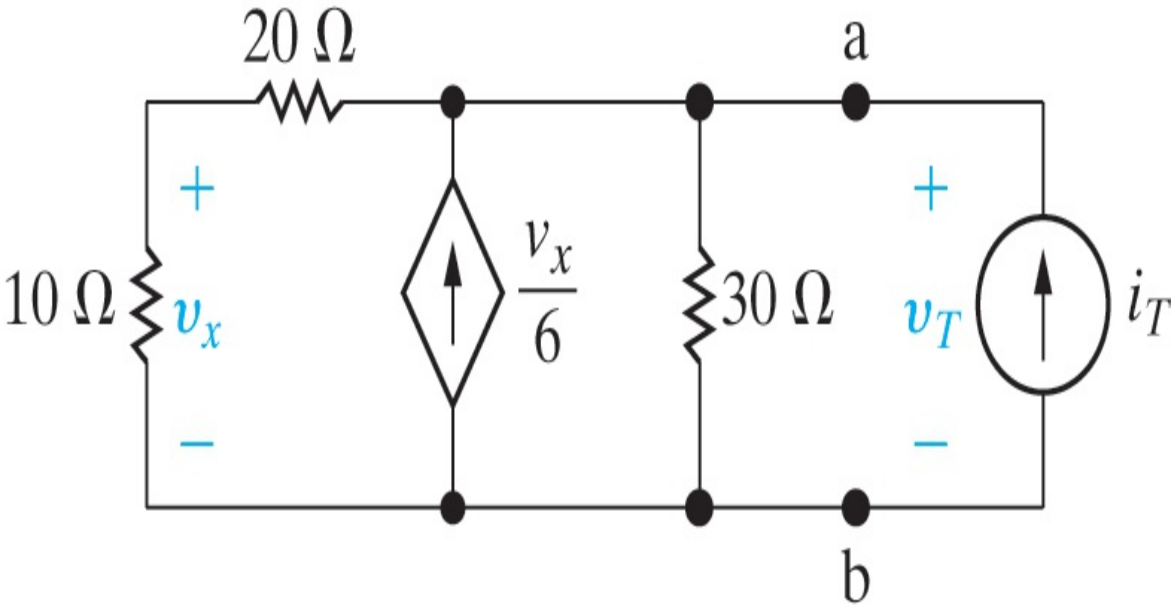


Figure 4.59 Full Alternative Text

Write a KCL equation at the top essential node to give

iT=vT20+10−vx6+vT30.

Use voltage division to find the voltage across the 10 Ω resistor:

vx=1020+10vT=vT3.

Then,

iT=vT30−vT18+vT30=vT90.

Therefore, the Norton equivalent of the circuit in Fig. 4.58 is a single resistor whose resistance RN=vT/iT=90 Ω.

We conclude this discussion of Thévenin and Norton equivalents with one final example of their application in circuit analysis. Sometimes we can use a Thévenin equivalent to simplify one portion of a circuit, thereby greatly simplifying analysis of the larger network. Let's return to the amplifier circuit first introduced in Section 2.5 and subsequently analyzed in Sections 4.4 and 4.7. Study Example 4.20 to see how a Thévenin equivalent of one portion of this circuit helps us in the analysis of the whole circuit.

Example 4.20 Using a Thévenin Equivalent to Analyze the Amplifier Circuit

Use a Thévenin equivalent of the left side of the amplifier circuit, shown in Fig. 4.60, to find the current iB.

Figure 4.60 The application of a Thévenin equivalent in circuit analysis.

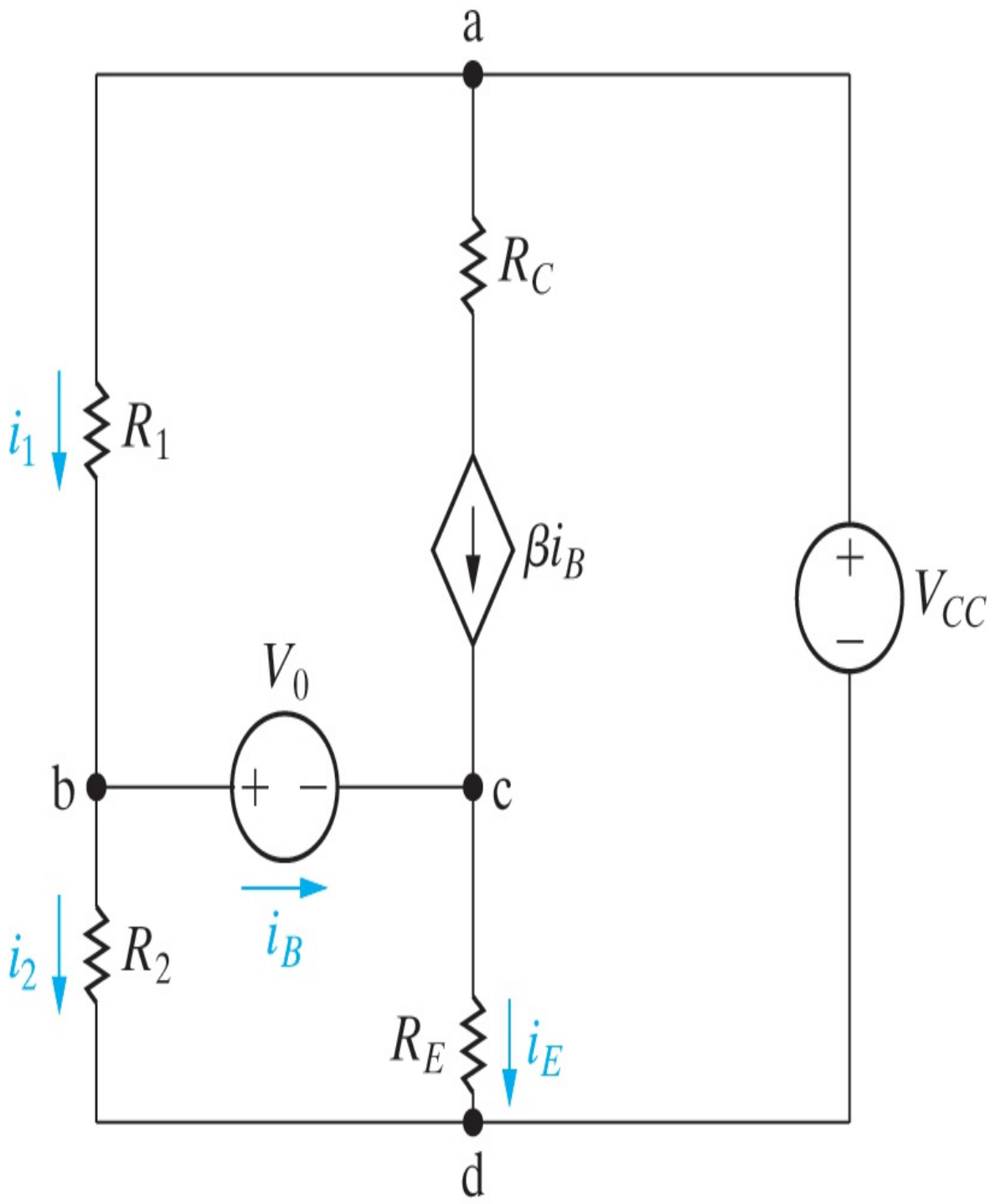


Figure 4.60 Full Alternative Text

Solution

We redraw the circuit as shown in Fig. 4.61 to prepare to replace the subcircuit to the left of V0 with its Thévenin equivalent. You should be able to determine that this modification has no effect on the branch currents i1, i2, iB, and iE. Then replace the circuit made up of VCC, R1, and R2 with a Thévenin equivalent, with respect to the terminals b and d. The Thévenin voltage and resistance are

Figure 4.61 A modified version of the circuit shown in Fig. 4.60.

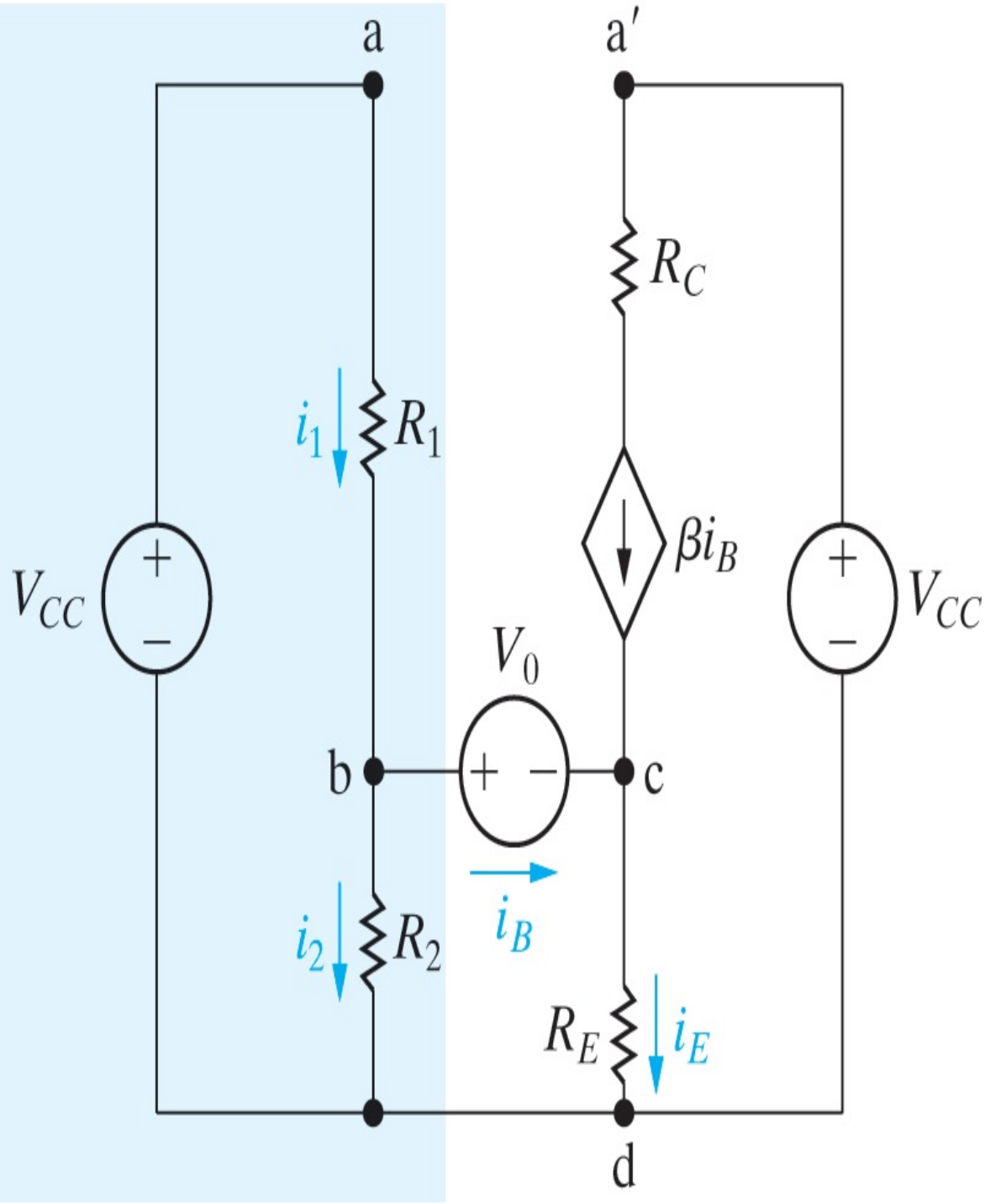


Figure 4.61 Full Alternative Text
VTh=VCCR2R1+R2, (4.39)
RTh=R1R2R1+R2. (4.40)

With the Thévenin equivalent, the circuit in Fig. 4.61 becomes the one shown

in Fig. 4.62.

Figure 4.62 The circuit shown in Fig. 4.61 modified by a Thévenin equivalent.

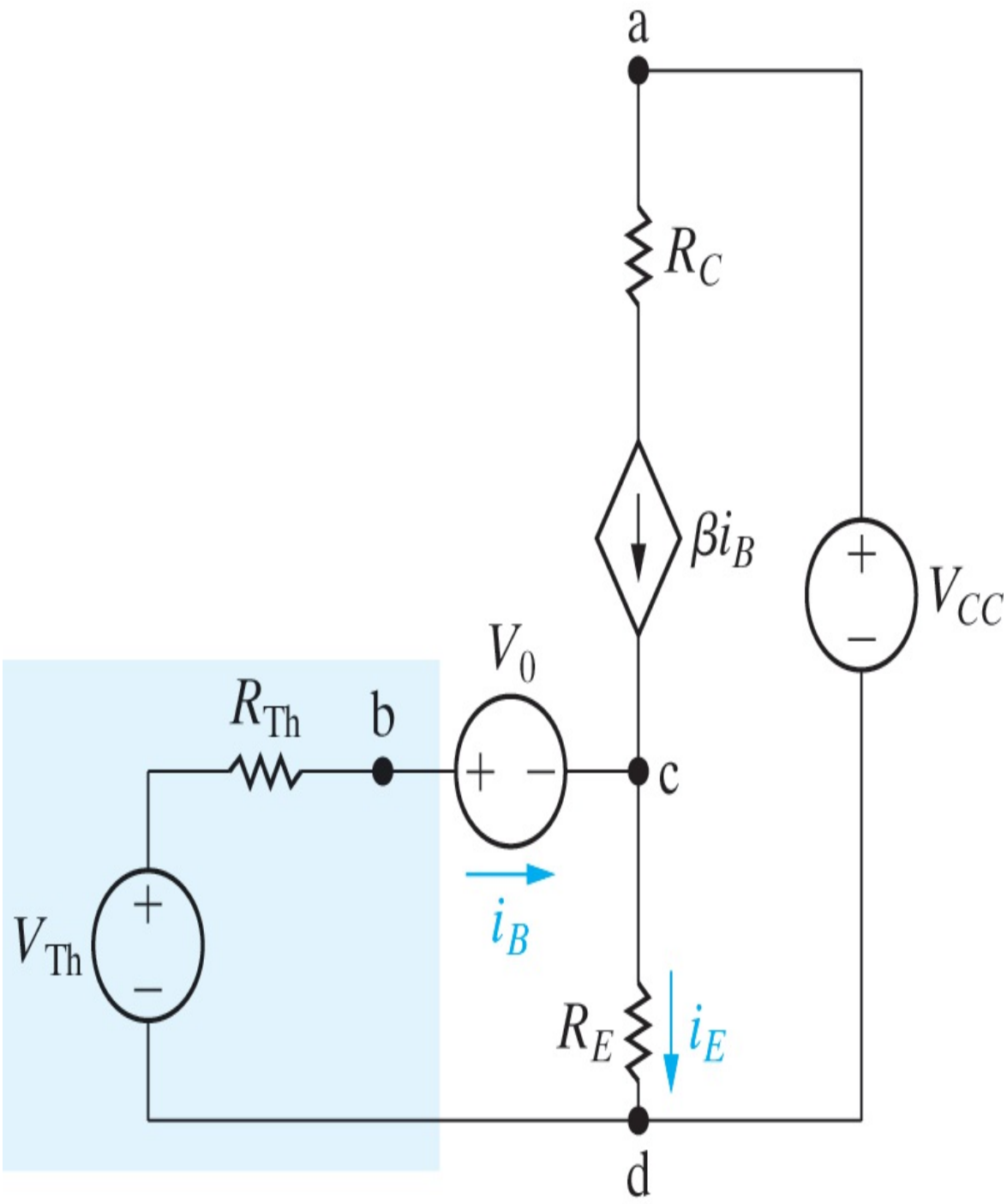


Figure 4.62 Full Alternative Text

We now derive an equation for iB by summing the voltages around the left mesh. In writing this mesh equation, we recognize that iE=(1+β)iB. Thus,

$V_{Th} = R_{Th} i_B + V_0 + R_E(1+\beta) i_B,$

from which

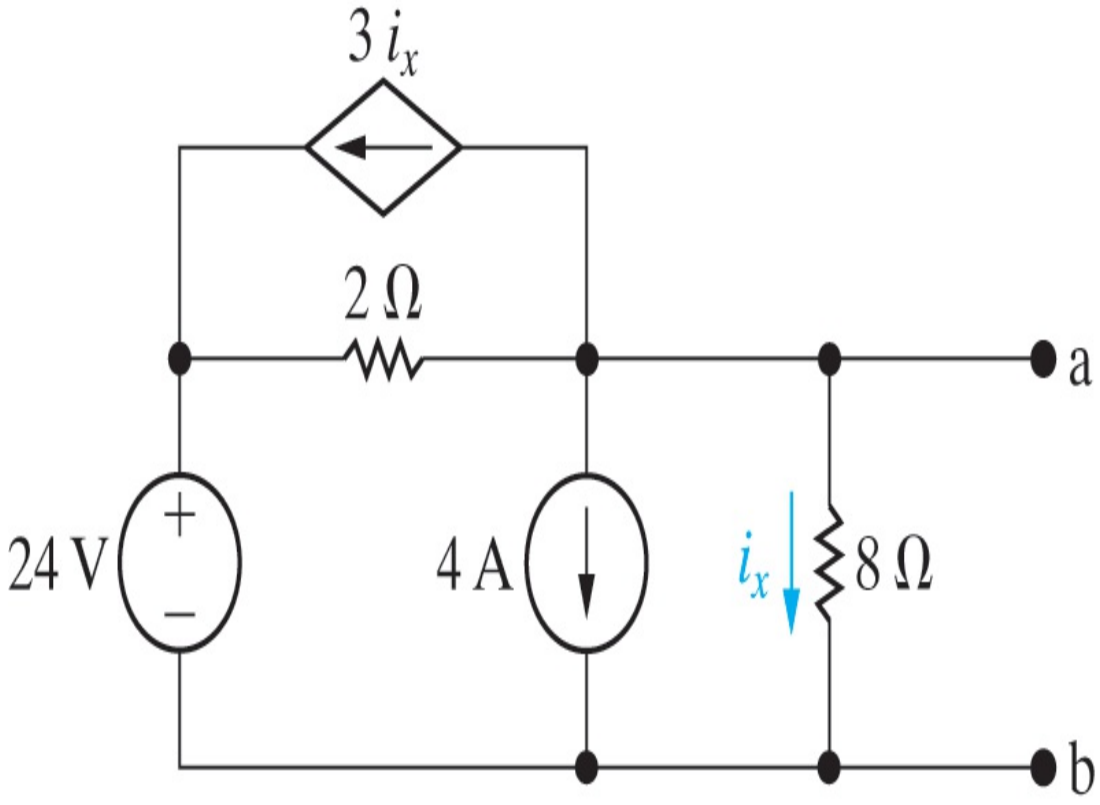
$$i_B = \frac{V_{Th} - V_0}{R_{Th} + (1+\beta)R_E}. \quad (4.41)$$

When we substitute Eqs. 4.39 and 4.40 into Eq. 4.41, we get the same expression obtained in Eq. 2.25. Note that once we have incorporated the Thévenin equivalent into the original circuit, we can obtain the solution for i_B by writing a single equation.

Assessment Problems

Objective 5—Understand Thévenin and Norton equivalents

1. 4.19 Find the Thévenin equivalent circuit with respect to the terminals a, b for the circuit shown.

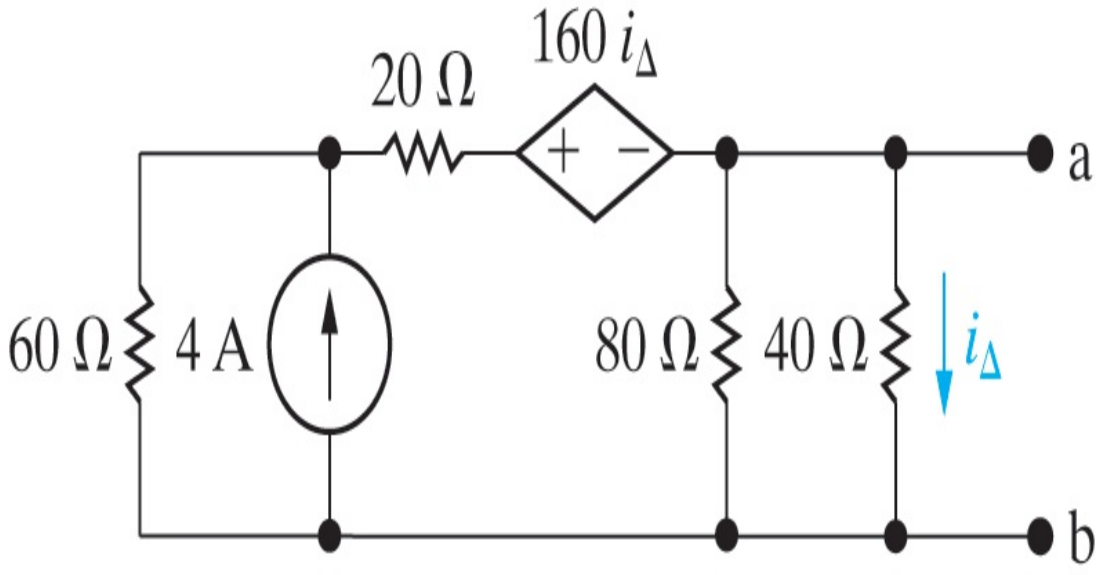


4.11-23 Full Alternative Text

Answer:

VTh=vab=8 V, RTh=1 Ω.

2. 4.20 Find the Thévenin equivalent circuit with respect to the terminals a, b for the circuit shown. (*Hint:* Define the voltage at the left-most node as v, and write two nodal equations with VTh as the right node voltage.)



4.11-24 Full Alternative Text

Answer:

VTh=vab=30 V, RTh=10 Ω.

SELF-CHECK: Also try Chapter Problems 4.74 and 4.79.

4.12 Maximum Power Transfer

Circuit analysis plays an important role in the analysis of systems designed to transfer power from a source to a load. This text discusses power transfer in two basic types of systems.

- **Systems Optimized for Maximum Efficiency** Power utility systems are a good example of this type because they generate, transmit, and distribute large quantities of electric power. If a power utility system is inefficient, a large percentage of the power generated is lost in the transmission and distribution processes, and thus wasted. We will look at these types of systems in Sections 9.10 and 9.11.

- **Systems Optimized for Maximum Power** Communication and instrumentation systems are good examples because when information (data) is transmitted via electric signals, the power available at the transmitter or detector is limited. Thus, transmitting as much of this power as possible to the receiver (load) is desirable. In such applications, the amount of power being transferred is small, so the transfer efficiency is not a primary concern. Here we consider maximum power transfer in systems that can be modeled by a purely resistive circuit.

Maximum power transfer can best be described with the aid of the circuit shown in Fig. 4.63. We assume a resistive network containing independent and dependent sources and a designated pair of terminals, a and b, to which a load, RL, is connected. The problem is to determine the value of RL that permits maximum power delivery to RL. The first step in this process is to recognize that a resistive network can always be replaced by its Thévenin equivalent. Therefore, we redraw the circuit shown in Fig. 4.63 as the one shown in Fig. 4.64. Replacing the original network by its Thévenin equivalent greatly simplifies the task of finding RL. To derive RL, begin by expressing the power dissipated in RL as a function of the three circuit parameters VTh, RTh, and RL. Thus

Figure 4.63 A circuit describing maximum power transfer.

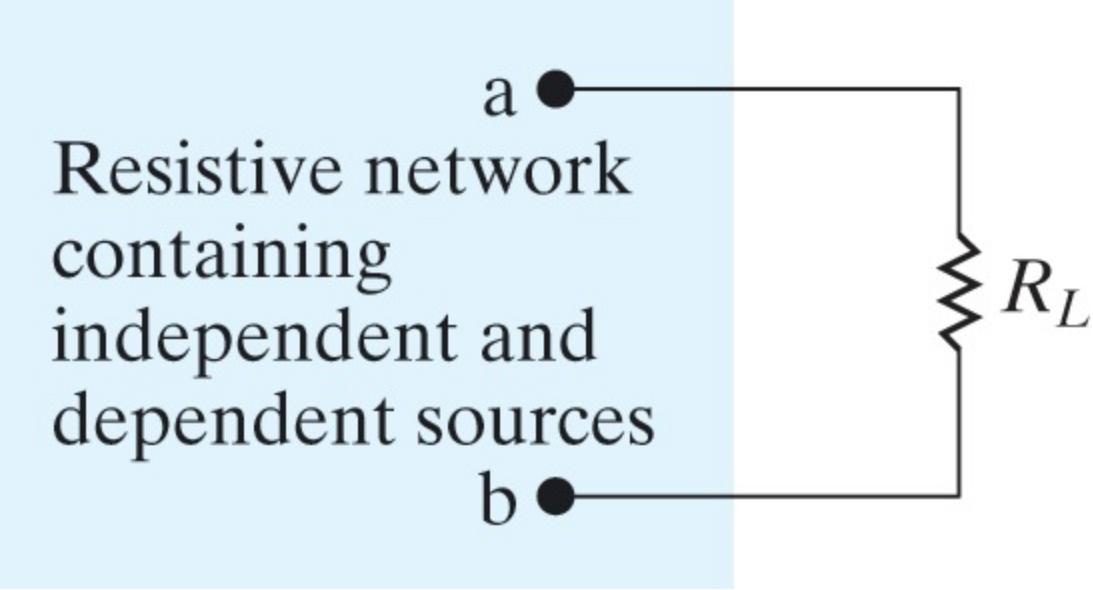


Figure 4.63 Full Alternative Text

Figure 4.64 A circuit used to determine the value of RL for maximum power transfer.

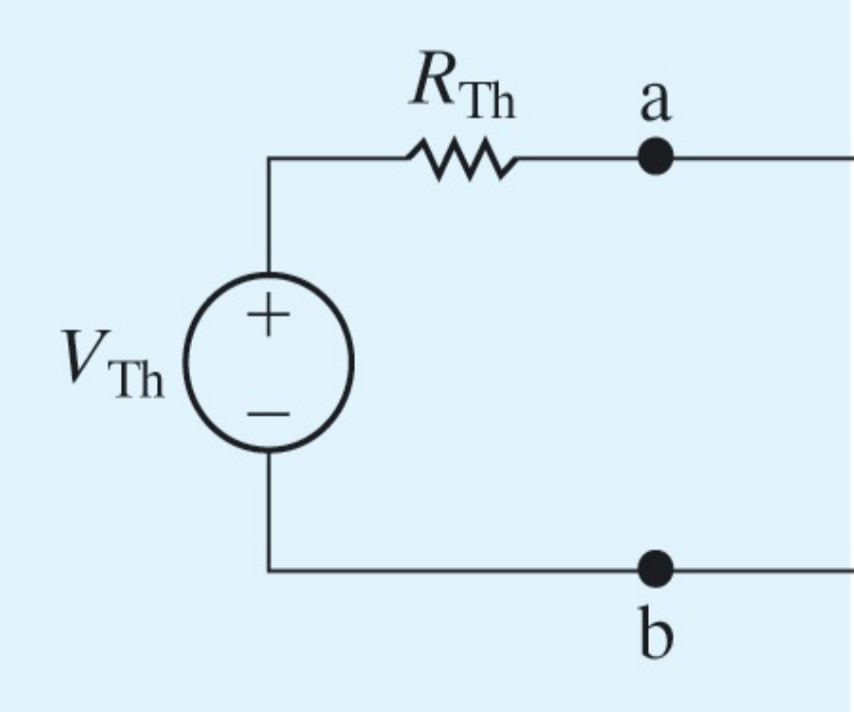


Figure 4.64 Full Alternative Text
p=i2 RL=(VThRTh+RL)2RL. (4.42)

Next, we recognize that for a given circuit, VTh and RTh will be fixed. Therefore, the power dissipated is a function of the single variable RL. To find the value of RL that maximizes the power, we use elementary calculus to write an equation for the derivative of p with respect to RL:

dpdRL=VTh2[(RTh+RL)2−RL · 2(RTh+RL)(RTh+RL)4].

The derivative is zero and p is maximized when

(RTh+RL)2=2RL(RTh + RL).

Solving for the load resistance RL yields

Condition for Maximum Power Transferred to a Resistive Load

RL=RTh. (4.43)

Thus, **maximum power transfer** occurs when the load resistance RL equals the Thévenin resistance RTh. To find the maximum power delivered to RL, substitute Eq. 4.43 into Eq. 4.42:

Maximum Power Transferred to a Resistive Load

pmax=VTh2RL(2RL)2=VTh24RL. (4.44)

Analyzing a circuit to calculate the load resistor required for maximum power transfer is illustrated in Example 4.21.

Example 4.21 Calculating the Condition for Maximum Power Transfer

1. For the circuit shown in Fig. 4.65, find the value of RL that results in maximum power being transferred to RL.

Figure 4.65 The circuit for Example 4.21.

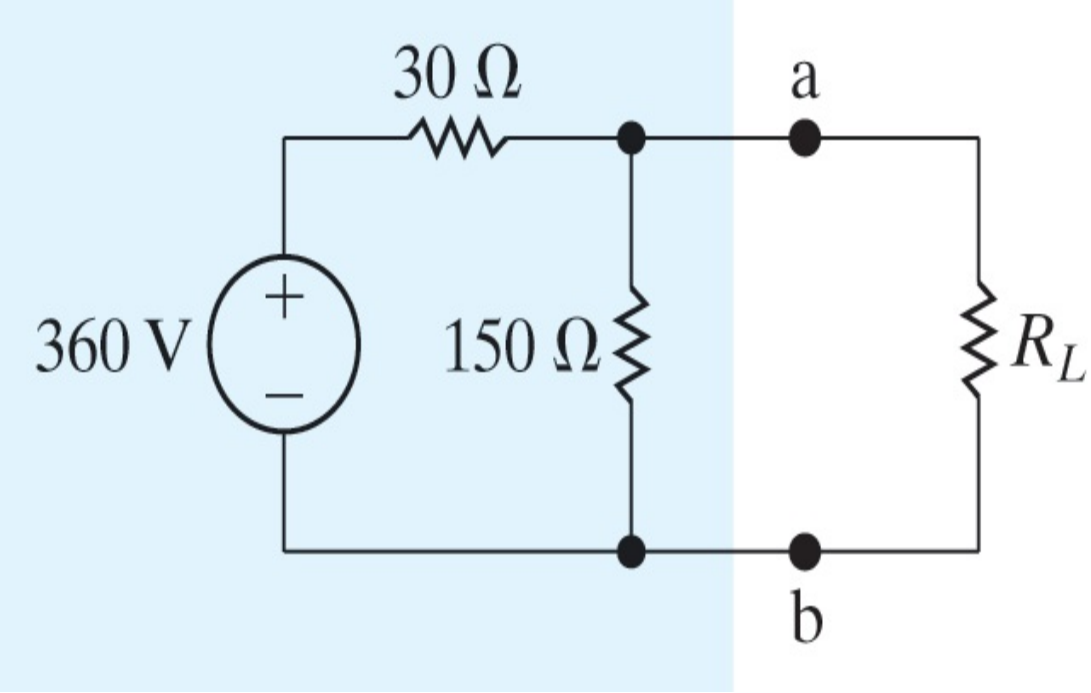


Figure 4.65 Full Alternative Text

2. Calculate the maximum power that can be delivered to RL.

3. When RL is adjusted for maximum power transfer, what percentage of the power delivered by the 360 V source reaches RL?

Solution

1. The Thévenin voltage for the circuit to the left of the terminals a and b is

   VTh=150180(360)=300 V.

   The Thévenin resistance is

   RTh=(150)(30)180=25 Ω

   Replacing the circuit to the left of the terminals a and b with its Thévenin equivalent gives us the circuit shown in Fig. 4.66, so RL must equal 25 Ω for maximum power transfer.

Figure 4.66 Reduction of the circuit shown in Fig. 4.66 by means of a Thévenin equivalent.

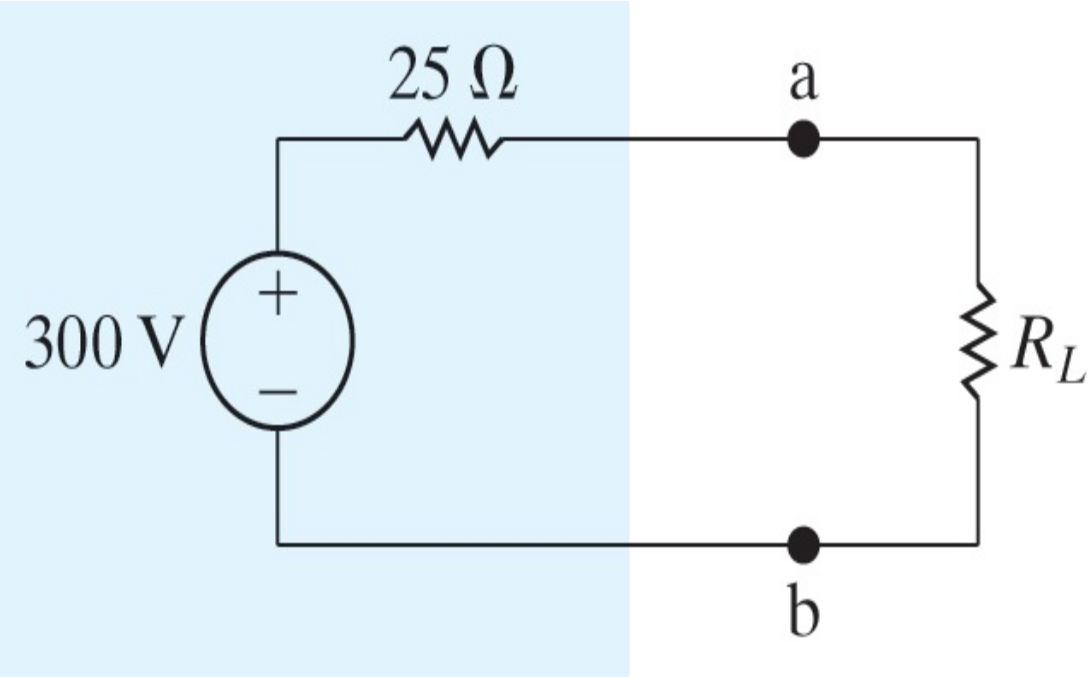


Figure 4.66 Full Alternative Text

2. The maximum power that can be delivered to RL is

   pmax=(30050)2(25)=900 W.

3. When RL equals 25 Ω, the voltage vab is

   vab=(30050)(25)=150 V.

   From Fig. 4.65, when vab equals 150 V, the current in the voltage source in the direction of the voltage rise across the source is

$i_s = 36 - \frac{150}{30} = \frac{210}{30} = 7$ A.

Therefore, the source is delivering 2520 W to the circuit, or

$p_s = -i_s(36) = -2520$ W.

The percentage of the source power delivered to the load is

$\frac{2520}{9002} \times 100 = 35.71$ %.

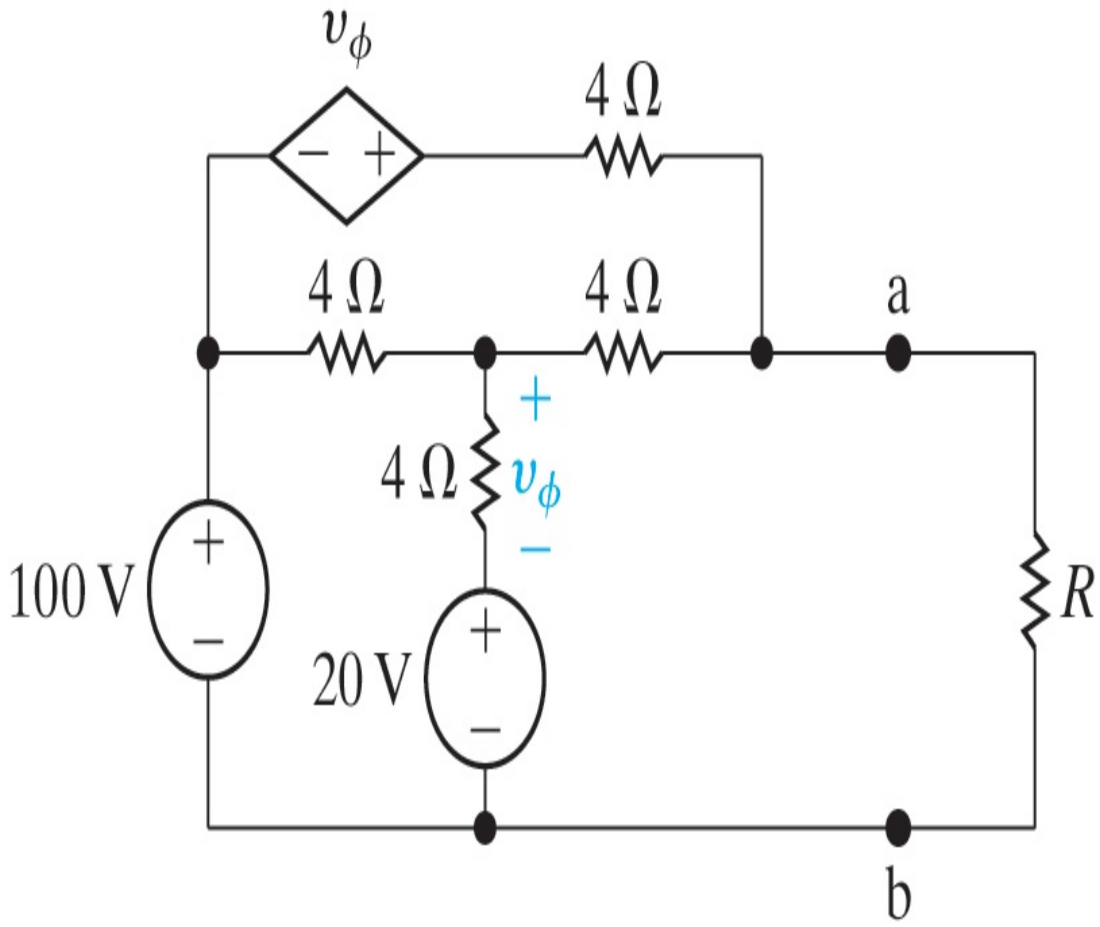
Assessment Problems

Objective 6—Know the condition for and calculate maximum power transfer to resistive load

1. 4.21

    1. a) Find the value of R that enables the circuit shown to deliver maximum power to the terminals a, b.

    2. b) Find the maximum power delivered to R.



4.12-25 Full Alternative Text

Answer:

1. (a) 3 Ω;
2. (b) 1.2 kW.

2. 4.22 Assume that the circuit in Assessment Problem 4.21 is delivering maximum power to the load resistor R.
    1. a) How much power is the 100 V source delivering to the network?
    2. b) Repeat (a) for the dependent voltage source.
    3. c) What percentage of the total power generated by these two

sources is delivered to the load resistor R?

Answer:

1. (a) 3000 W;
2. (b) 800 W;
3. (c) 31.58%.

SELF-CHECK: Also try Chapter Problems 4.87 and 4.89.

4.13 Superposition

A linear system obeys the principle of **superposition**, which states that whenever a linear system is excited, or driven, by more than one independent source of energy, the total response is the sum of the individual responses. An individual response is the result of an independent source acting alone. Because we are dealing with circuits made up of interconnected linear-circuit elements, we can apply the principle of superposition directly to the analysis of such circuits when they are driven by more than one independent energy source. At present, we restrict the discussion to simple resistive networks; however, the principle is applicable to any linear system.

Superposition is applied in both the analysis and design of circuits. In analyzing a complex circuit with multiple independent voltage and current sources, there are often fewer, simpler equations to solve when the effects of the independent sources are considered one at a time. Applying superposition can thus simplify circuit analysis. Be aware, though, that sometimes applying superposition actually complicates the analysis, producing more equations to solve than with an alternative method. Superposition is required only if the independent sources in a circuit are fundamentally different. In these early chapters, all independent sources are dc sources, so superposition is not required. We introduce superposition here in anticipation of later chapters in which circuits will require it.

Superposition is applied in design to synthesize a desired circuit response that could not be achieved in a circuit with a single source. If the desired circuit response can be written as a sum of two or more terms, the response can be realized by including one independent source for each term of the response. This approach to the design of circuits with complex responses allows a designer to consider several simple designs instead of one complex design.

We demonstrate the superposition principle in Example 4.22.

Example 4.22 Using Superposition

to Solve a Circuit

Use the superposition principle to find the branch currents in the circuit shown in Fig. 4.67.

Figure 4.67 A circuit used to illustrate superposition.

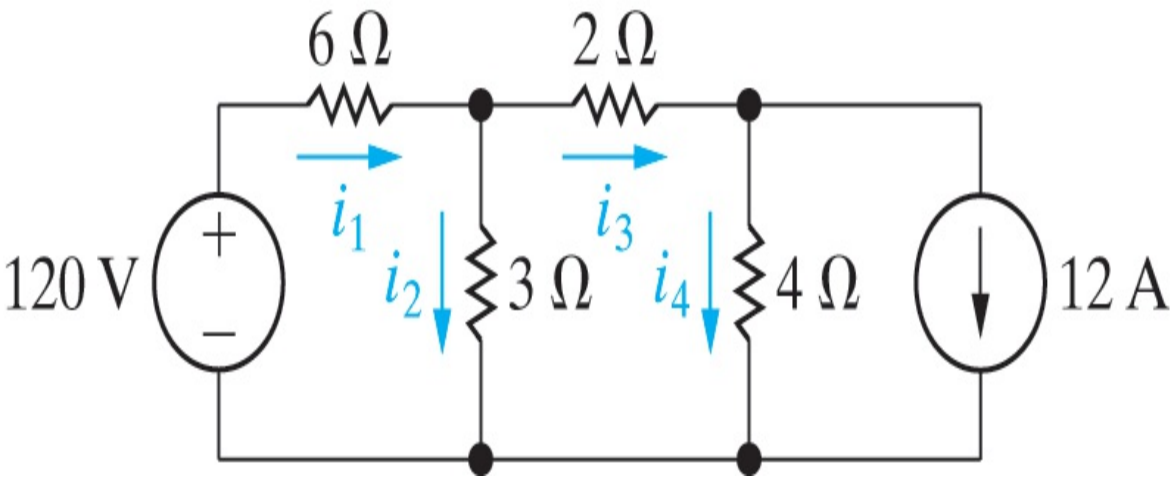


Figure 4.67 Full Alternative Text

Solution

We begin by finding the branch currents resulting from the 120 V voltage source. We denote those currents with a prime. Replacing the ideal current source with an open circuit deactivates it; Fig. 4.68 shows this. The branch currents in this circuit are the result of only the voltage source.

Figure 4.68 The circuit shown

in Fig. 4.67 with the current source deactivated.

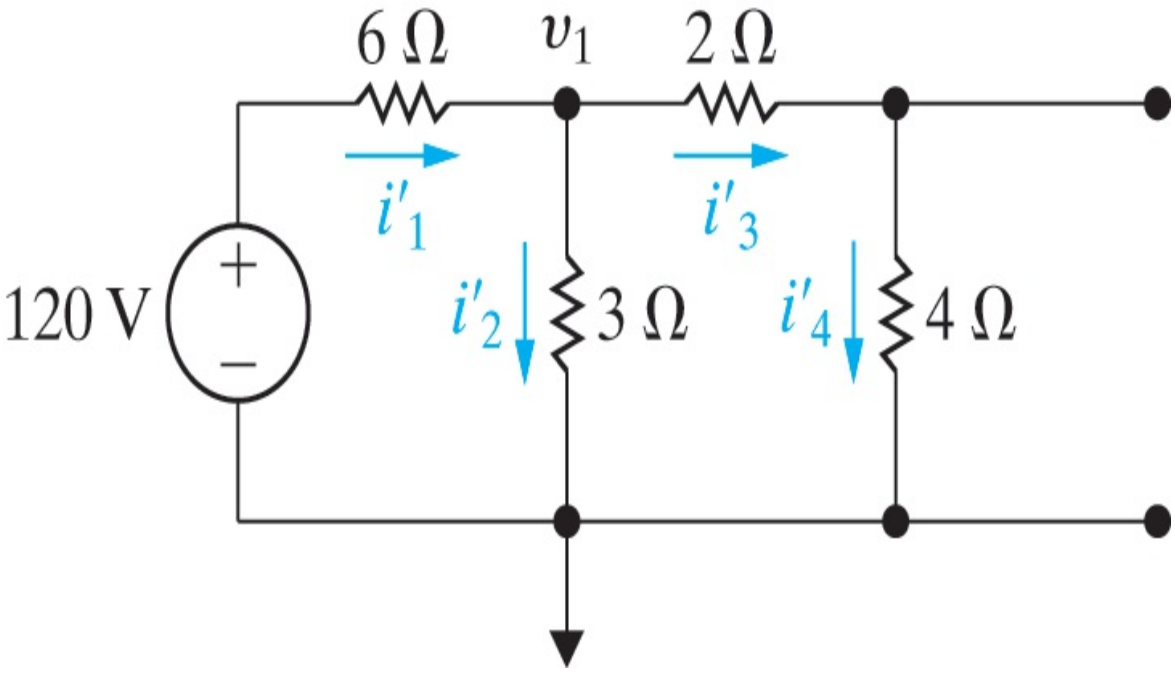


Figure 4.68 Full Alternative Text

We can easily find the branch currents in the circuit in Fig. 4.68 once we know the node voltage across the 3 Ω resistor. Denoting this voltage v1, we write

v1−1206+ v13+ v12+4=0,

from which

v1=30 V.

Now we can write the expressions for the branch currents i′1 − i′4 directly:

i′1=120−306=15 A,
i′2=303=10 A,
i′3=i′4=306=5 A.

To find the component of the branch currents resulting from the current source, we deactivate the ideal voltage source and solve the circuit shown in Fig. 4.69. The double-prime notation for the currents indicates they are the components of the total current resulting from the ideal current source.

Figure 4.69 The circuit shown in Fig. 4.67 with the voltage source deactivated.

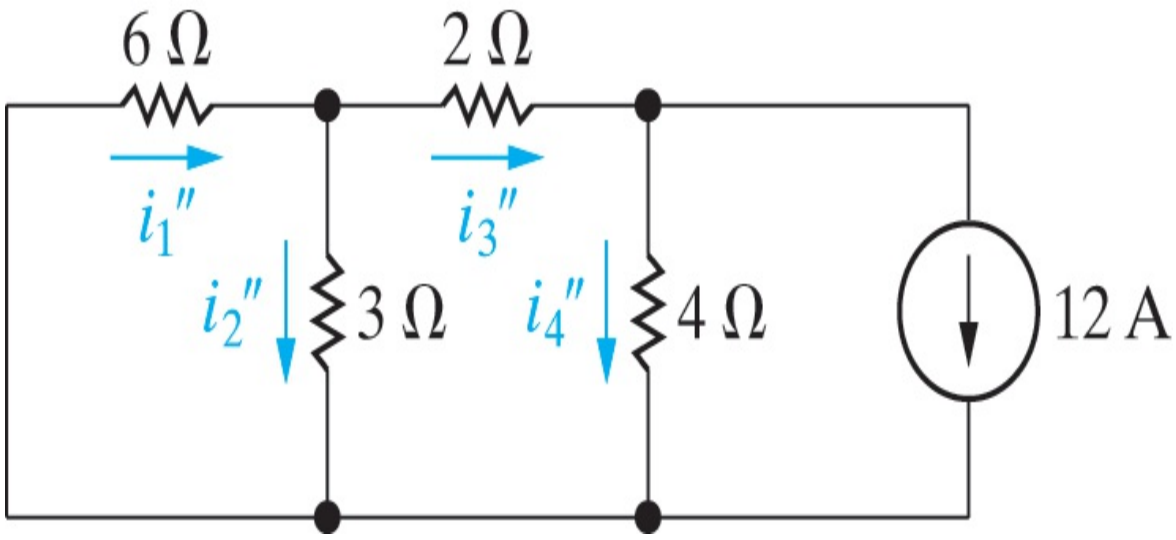


Figure 4.69 Full Alternative Text

We determine the branch currents in the circuit shown in Fig. 4.69 by first solving for the node voltages across the 3 and 4 Ω resistors, respectively. Figure 4.70 shows the two node voltages. The two KCL equations that describe the circuit are

Figure 4.70 The circuit shown in Fig. 4.69 showing the node

voltages v3 and v4.

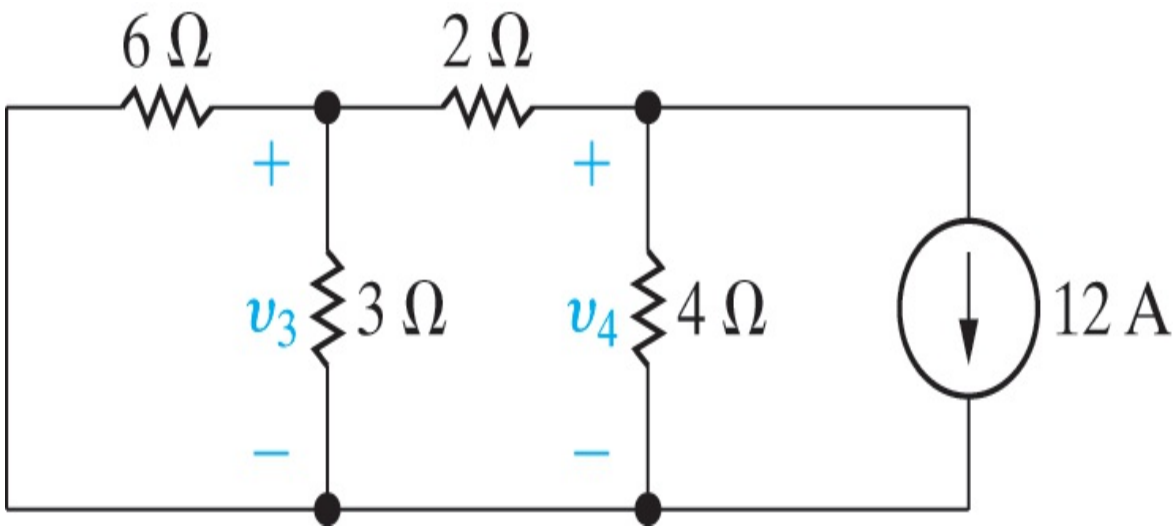


Figure 4.70 Full Alternative Text

v33+v36+v3−v42=0,v4−v32+ v44 +12=0.

Solving the simultaneous KCL equations for v3 and v4, we get

v3=−12 V,v4=−24 V.

Now we can write the branch currents i″1 through i″4 directly in terms of the node voltages v3 and v4:

i″1=−v36=126=2 A,
i″2=v33=−123=−4 A,
i″3=v3−v42=−12+242=6 A,
i″4=v44=−244=−6 A.

To find the branch currents in the original circuit, that is, the currents i1, i2, i3, and i4 in Fig. 4.67, we simply add the single-primed currents to the double-primed currents:

i1=i′1+ i″1=15+2=17 A,i2=i′2+ i″2=10−4=6 A,i3=i′3+ i″3=5+6=11 A,i4=i ′4+ i″4=5−6=−1 A.

You should verify that the currents i1, i2, i3, and i4 have the correct values for the branch currents in the circuit shown in Fig. 4.67.

When applying superposition to linear circuits containing both independent and dependent sources, you must recognize that the dependent sources are never deactivated. Example 4.23 applies superposition when a circuit contains both dependent and independent sources.

Example 4.23 Using Superposition to Solve a Circuit with Dependent Sources

Use the principle of superposition to find vo in the circuit shown in Fig. 4.71.

Figure 4.71 The circuit for Example 4.23.

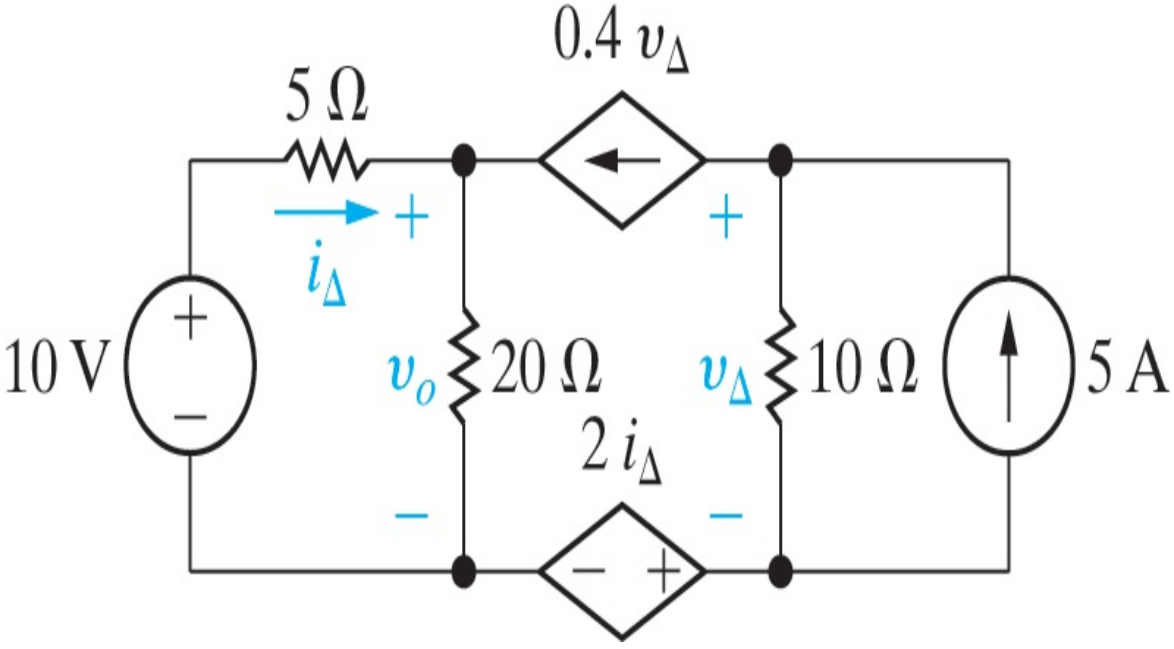


Figure 4.71 Full Alternative Text

Solution

We begin by finding the component of vo resulting from the 10 V source. Figure 4.72 shows the circuit. With the 5 A source deactivated, v′Δ must equal (−0.4v′Δ) (10). Hence, v′Δ must be zero, the branch containing the two dependent sources is open, and

Figure 4.72 The circuit shown in Fig. 4.71 with the 5 A source deactivated.

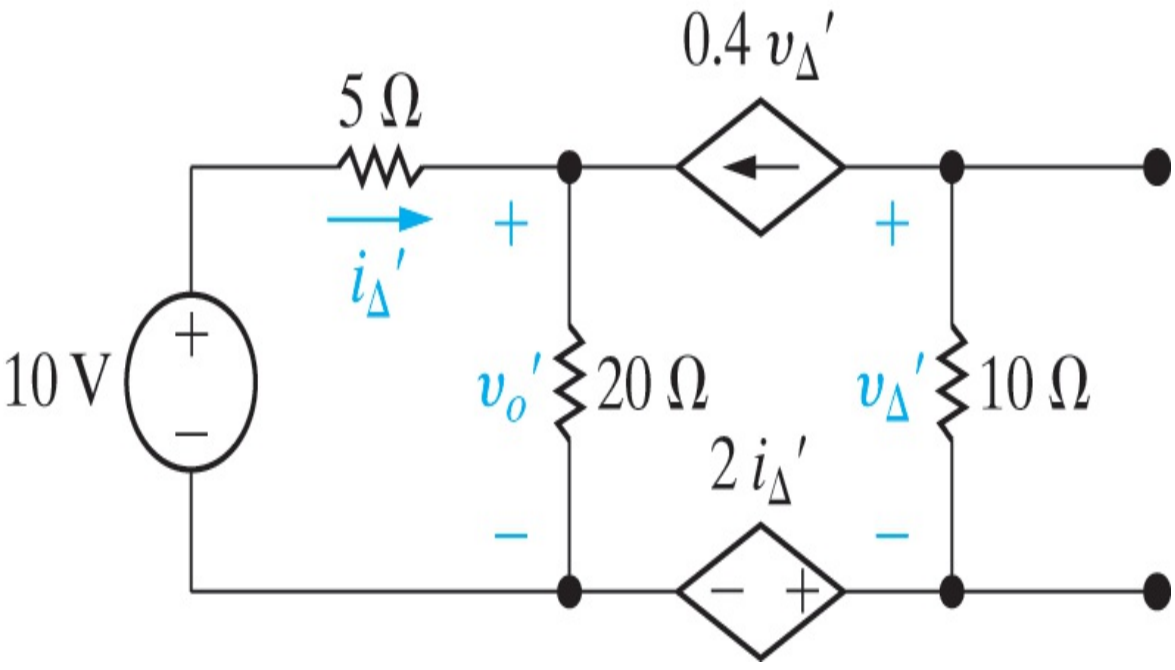


Figure 4.72 Full Alternative Text

v′o=2025(10)=8 V.

When the 10 V source is deactivated, the circuit reduces to the one shown in

Fig. 4.73. We have added a reference node and the node designations a, b, and c to aid the discussion. Summing the currents away from node a yields

Figure 4.73 The circuit shown in Fig. 4.71 with the 10 V source deactivated.

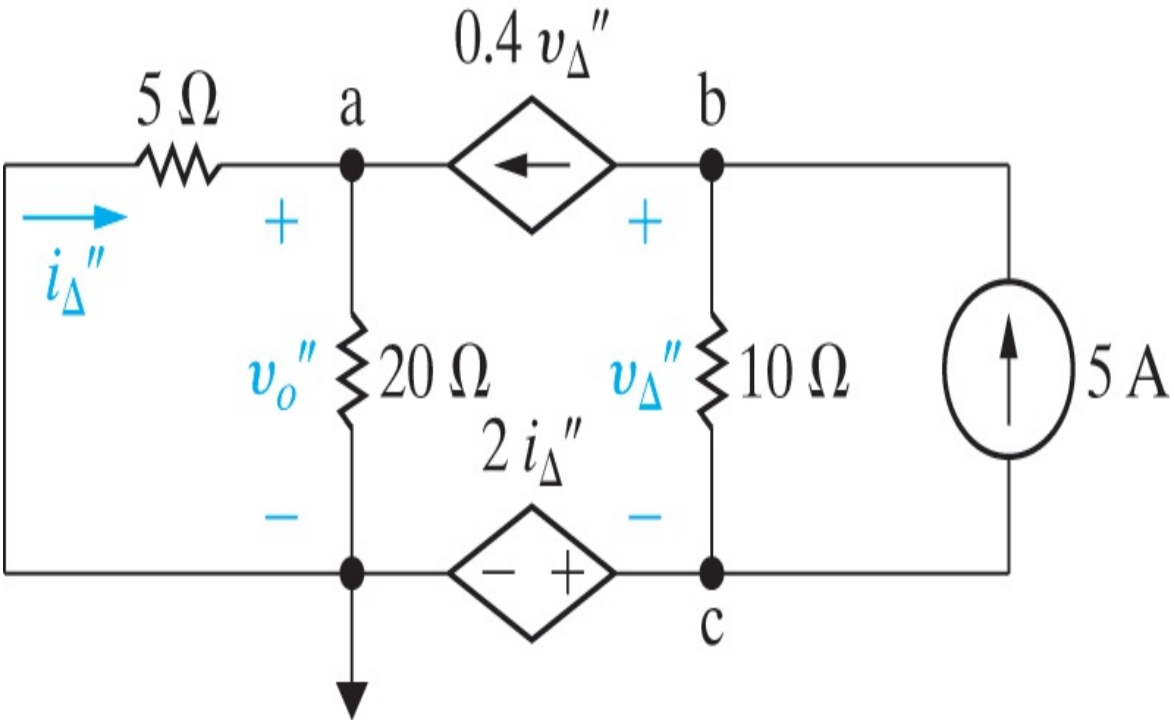


Figure 4.73 Full Alternative Text

v″o20+ v″o5−0.4v″Δ=0, or 5v″o− 8v″Δ=0.

Summing the currents away from node b gives

0.4v″Δ+vb−2i″Δ10−5 =0, or
4v″Δ+ vb−2i″Δ=50.

We now use

$v_b = 2i'' + v''$

to find the value for v''. Thus,

$5v'' = 50$, or $v'' = 10$ V.

From the node a equation,

$5v''_0 = 80$, or $v''_0 = 16$ V.

The value of v_o is the sum of v'_o and v''_o, or 24 V.

SELF-CHECK: Assess your understanding of this material by trying Chapter Problems 4.92 and 4.97.

Practical Perspective

Circuits with Realistic Resistors

It is not possible to fabricate identical electrical components. For example, resistors produced from the same manufacturing process can vary in value by as much as 20%. Therefore, in creating an electrical system, the designer must consider the impact that component variation will have on the performance of the system. Sensitivity analysis permits the designer to calculate the impact of variations in the component values on the output of the system. We will see how this information enables a designer to specify an acceptable component value tolerance for each of the system's components.

Consider the circuit shown in Fig. 4.74. We will use sensitivity analysis to determine the sensitivity of the node voltages v1 and v2 to changes in the resistor R1. Using the node-voltage method, we derive the expressions for v1 and v2 as functions of the circuit resistors and source currents. The results are given in Eqs. 4.45 and 4.46:

Figure 4.74 Circuit used to introduce sensitivity analysis.

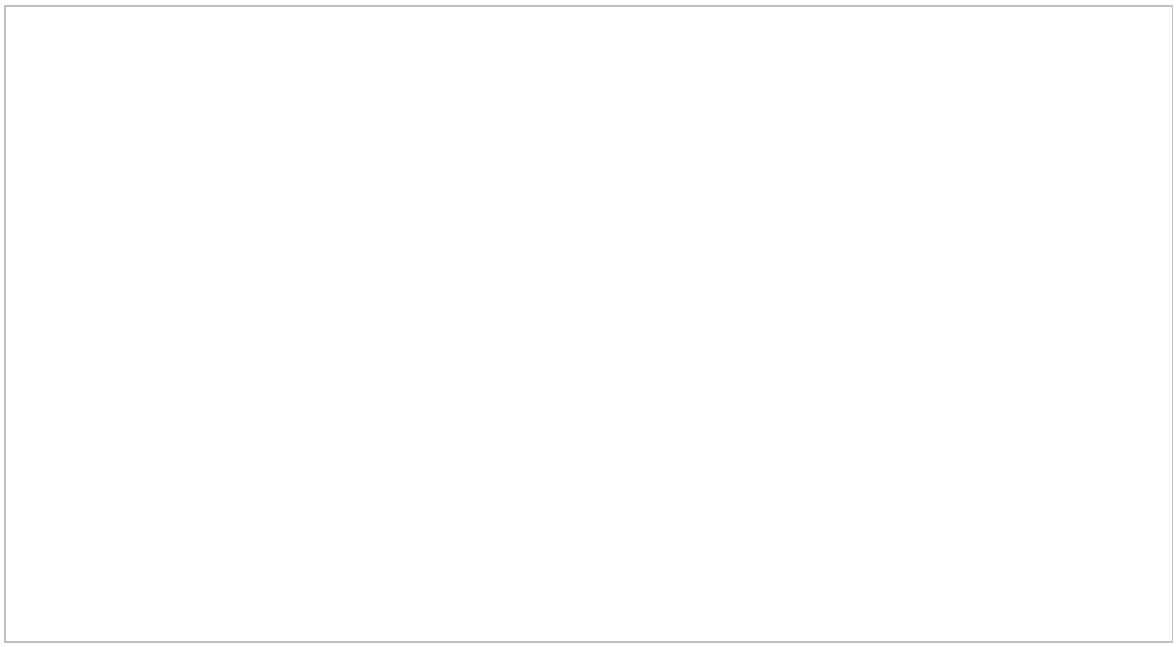


Figure 4.74 Full Alternative Text

$$v1=R1\{R3R4Ig2-[R2(R3+R4)+R3R4]Ig1\}(R1+R2)(R3+R4)+R3R4, \quad (4.45)$$

$$v2=R3R4[(R1+R2)Ig2-R1Ig1](R1+R2)\,(R3+R4)+\,R3R4. \quad (4.46)$$

The sensitivity of v1 with respect to R1 is found by differentiating Eq. 4.45 with respect to R1, and similarly the sensitivity of v2 with respect to R1 is found by differentiating Eq. 4.46 with respect to R1. We get

$$dv1d\,R1=[R3R4+R2(R3+R4)]\,\{R3R4Ig2-[R3R4+R2(R3+R4)]Ig1\}\,[(R1+R2)(R3+R4)+R3R4]2, \quad (4.47)$$

$$dv2d\,R1=R3R4\{R3R4Ig2-[R2(R3+R4)+\,R3R4]Ig1\}[(R1+R2)\,(R3+R4)+R3R4]2. \quad (4.48)$$

We now consider an example with actual component values to illustrate the use of Eqs. 4.47 and 4.48.

Example

Assume the nominal values of the components in the circuit in Fig. 4.74 are: R1=25 Ω; R2=5 Ω; R3=50 Ω; R4=75 Ω; Ig1=12 A; and Ig2=16 A. Use sensitivity analysis to predict the values of v1 and v2 if the value of R1 is different by 10% from its nominal value.

Solution

From Eqs. 4.45 and 4.46 we find the nominal values of v1 and v2. Thus

$$v1=25\{3750(16)-[5(125)+3750]12\}30(125)+3750=25 \text{ V}, \quad (4.49)$$

and

$$v2=3750[30(16)-25(12)]30(125)+3750=90 \text{ V}. \quad (4.50)$$

Now from Eqs. 4.47 and 4.48 we can find the sensitivity of v1 and v2 to changes in R1. Hence,

$$dv1dR1=[3750+5(125)]-\{ 3750(16)-[3750+5(125)]12 \}[(30)(125)+3750]2=712 \text{ V}/\Omega, \quad (4.51)$$

and

$$dv2dR1=3750\{ 3750(16)-[5(125)+3750]12 \}](7500)2=0.5 \text{ V}/\Omega. \quad (4.52)$$

How do we use the results given by Eqs. 4.51 and 4.52? Assume that R1 is 10% less than its nominal value, that is, R1=22.5 Ω. Then $\Delta R1=-2.5$ Ω and Eq. 4.51 predicts that $\Delta v1$ will be

$$\Delta v1=(712)(-2.5)=-1.4583 \text{ V}.$$

Therefore, if R1 is 10% less than its nominal value, our analysis predicts that v1 will be

$$v1=25-1.4583=23.5417 \text{ V}. \quad (4.53)$$

Similarly, for Eq. 4.52 we have

$$\Delta v2=0.5(-2.5)=-1.25 \text{ V},$$
$$v2=90-1.25=88.75 \text{ V}. \quad (4.54)$$

We attempt to confirm the results in Eqs. 4.53 and 4.54 by substituting the value R1=22.5 Ω into Eqs. 4.45 and 4.46. When we do, the results are

$$v1=23.4780 \text{ V},$$
$$v2=88.6960 \text{ V}.$$

Why is there a difference between the values predicted from the sensitivity analysis and the exact values computed by substituting for R1 in the equations for v1 and v2? We can see from Eqs. 4.47 and 4.48 that the sensitivity of v1 and v2 with respect to R1 is a function of R1 because R1 appears in the denominator of both Eqs. 4.47 and 4.48. This means that as R1 changes, the sensitivities change; hence, we cannot expect Eqs. 4.47 and 4.48 to give exact results for large changes in R1. Note that for a 10% change in R1, the percent error between the predicted and exact values of v1 and v2 is small. Specifically, the percent error in v1=0.2713% and the percent error in v2=0.0676%.

From this example, we can see that a tremendous amount of work is involved if we are to determine the sensitivity of v1 and v2 to changes in the remaining component values, namely, R2, R3, R4, Ig1, and Ig2. Fortunately, PSpice[2] has a sensitivity function that will perform sensitivity analysis for us. The sensitivity function in PSpice calculates two types of sensitivity. The first is known as the one-unit sensitivity, and the second is known as the 1% sensitivity. In the example circuit, a one-unit change in a resistor would change its value by 1 Ω and a one-unit change in a current source would change its value by 1 A. In contrast, 1% sensitivity analysis determines the effect of changing resistors or sources by 1% of their nominal values.

2 See the PSpice supplement that accompanies this text.

The result of PSpice sensitivity analysis of the circuit in Fig. 4.69 is shown in

Table 4.2. Because we are analyzing a linear circuit, we can use superposition to predict values of v1 and v2 if more than one component's value changes. For example, let us assume R1 decreases to 24 Ω and R2 decreases to 4 Ω. From Table 4.2 we can combine the unit sensitivity of v1 to changes in R1 and R2 to get

Table 4.2 PSpice Sensitivity Analysis Results

Table 4.2 Full Alternative Text

$\Delta v1\Delta R1 + \Delta v1\Delta R2 = 0.5833 - 5.417 = -4.8337$ V/Ω.

Similarly,

$\Delta v2\Delta R1+\Delta v2\Delta R2=0.5+6.5=7.0$ V/Ω.

Thus, if both R1 and R2 decreased by 1 Ω, we would predict

$v1=25+ 4.8227=29.8337$ V,
$v2=90-7=83$ V.

If we substitute R1=24 Ω and R2=4 Ω into Eqs. 4.45 and 4.46, we get

$v1=29.793$ V,
$v2=82.759$ V.

In both cases, our predictions are within a fraction of a volt of the actual node voltage values.

Circuit designers use the results of sensitivity analysis to determine which component value variation has the greatest impact on the output of the circuit. As we can see from the PSpice sensitivity analysis in Table 4.2, the node voltages v1 and v2 are much more sensitive to changes in R2 than to changes in R1. Specifically, v1 is (5.417/0.5833) or approximately 9 times more sensitive to changes in R2 than to changes in R1, and v2 is (6.5/0.5) or 13 times more sensitive to changes in R2 than to changes in R1. Hence, in the example circuit, the tolerance on R2 must be more stringent than the tolerance on R1 if it is important to keep v1 and v2 close to their nominal values.

SELF-CHECK: Assess your understanding of this Practical Perspective by trying Chapter Problems 4.105–4.107.

Summary

- For the topics in this chapter, mastery of some basic terms, and the concepts they represent, is necessary. Those terms are **node**, **essential node**, **path**, **branch**, **essential branch**, **mesh**, and **planar circuit**. Table 4.1 provides definitions and examples of these terms. (See page 95.)

- Two new circuit analysis techniques were introduced in this chapter:

    - The **node-voltage method** works with both planar and nonplanar circuits. The steps in the node-voltage method are in Table 4.3.(See page 135.)

Table 4.3 Steps in the Node-Voltage Method and the Mesh-Current Method

	Node-Voltage Method	Mesh-Current Method
Step 1 Identify nodes/meshes	Identify the essential nodes by circling them on the circuit diagram	Identify the meshes by drawing directed arrows inside each mesh
Step 2 Label node voltages/mesh currents Recognize special cases	Pick and label a reference node; then label the remaining essential node voltages • If a voltage source is the only component in a branch connecting the reference node and another essential node, label the essential node with the value of the voltage source • If a voltage source is the only component in a branch connecting two nonreference essential nodes, create a supernode that includes the voltage source and the two nodes on either side	Label each mesh current • If a current source is in a single mesh, label the mesh current with the value of the current source • If a current source is shared by two adjacent meshes, create a supermesh by combining the two adjacent meshes and temporarily eliminating the branch that contains the current source
Step 3 Write the equations	Write the following equations: • A KCL equation for any supernodes • A KCL equation for any remaining essential nodes where the voltage is unknown • A constraint equation for each dependent source that defines the controlling variable for the dependent source in terms of the node voltages • A constraint equation for each supernode that equates the difference between the two node voltages in the supernode to the voltage source in the supernode	Write the following equations: • A KVL equation for any supermeshes • A KVL equation for any remaining meshes where the current is unknown • A constraint equation for each dependent source that defines the controlling variable for the dependent source in terms of the mesh currents • A constraint equation for each supermesh that equates the difference between the two mesh currents in the supermesh to the current source eliminated to form the supermesh
Step 4 Solve the equations	Solve the equations to find the node voltages	Solve the equations to find the mesh currents
Step 5 Solve for other unknowns	Use the node voltage values to find any unknown voltages, currents, or powers	Use the mesh current values to find any unknown voltages, currents, or powers

Table 4.3 Full Alternative Text

- The **mesh-current method** works only with planar circuits. The

steps in the mesh-current method are in Table 4.3. (See page 135.)

- Several new circuit simplification techniques were introduced in this chapter:

  - **Source transformations** allow us to exchange a voltage source (vs) and a series resistor (R) for a current source (is) and a parallel resistor (R) and vice versa. The combinations must be equivalent in terms of their terminal voltage and current. Terminal equivalence holds, provided that

    is=vsR.

    (See page 115–116.)

- **Thévenin equivalents** and **Norton equivalents** allow us to simplify a circuit composed of sources and resistors into an equivalent circuit consisting of a voltage source and a series resistor (Thévenin) or a current source and a parallel resistor (Norton). The simplified circuit and the original circuit must be equivalent in terms of their terminal voltage and current. Note that

  1. The Thévenin voltage (VTh) is the open-circuit voltage across the terminals of the original circuit;

  2. The Thévenin resistance (RTh) is the ratio of the Thévenin voltage to the current in a short circuit connecting the terminals of the original circuit;

  3. The Norton equivalent is obtained by performing a source transformation on a Thévenin equivalent.

  (See page 118–121.)

- **Maximum power transfer** is a technique for calculating the maximum value of p that can be delivered to a load, RL. Maximum power transfer occurs when RL=RTh, the Thévenin resistance as seen from the resistor RL. The equation for the maximum power transferred is

p=VTh24RL.

(See page 126–127.)

- In a circuit with multiple independent sources, **superposition** allows us to activate one source at a time and calculate voltages and currents due to each source. To determine the voltages and currents that exist when all independent sources are active, sum the voltages and currents that resulted from each of the sources. Dependent sources are never deactivated when applying superposition. (See page 129.)

Problems

Section 4.1

1. 4.1 For the circuit shown in Fig. P4.1, state the numerical value of the number of (a) branches, (b) branches where the current is unknown, (c) essential branches, (d) essential branches where the current is unknown, (e) nodes, (f) essential nodes, and (g) meshes.

Figure P4.1

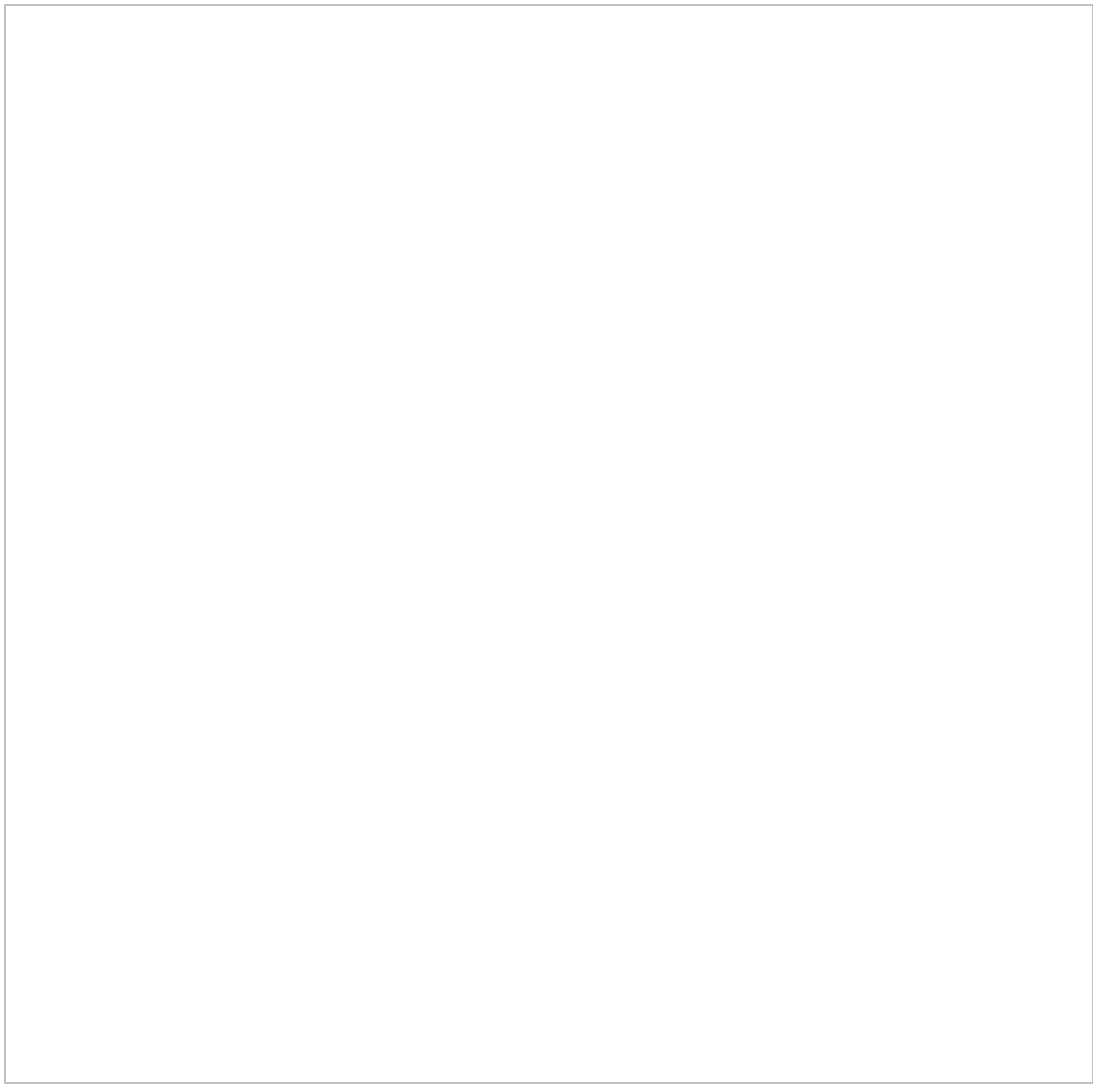


Figure P4.1 Full Alternative Text

2. 4.2

    1. a) If only the essential nodes and branches are identified in the circuit in Fig. P4.1, how many simultaneous equations are needed to describe the circuit?

    2. b) How many of these equations can be derived using Kirchhoff's current law?

    3. c) How many must be derived using Kirchhoff's voltage law?

    4. d) What two meshes should be avoided in applying the voltage law?

3. 4.3 Assume the voltage vs in the circuit in Fig. P4.3 is known. The resistors R1−R7 are also known.

    1. a) How many unknown currents are there?

    2. b) How many independent equations can be written using Kirchhoff's current law (KCL)?

    3. c) Write an independent set of KCL equations.

    4. d) How many independent equations can be derived from Kirchhoff's voltage law (KVL)?

    5. e) Write a set of independent KVL equations.

Figure P4.3

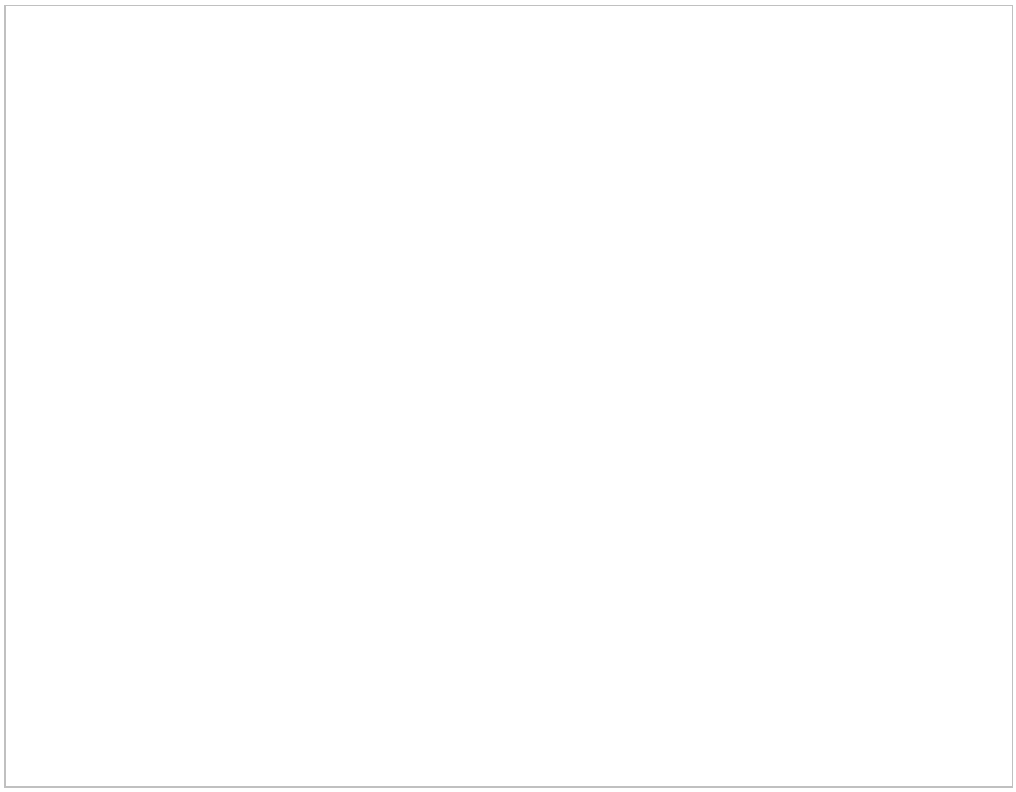


Figure P4.3 Full Alternative Text

4. 4.4 A current leaving a node is defined as positive.
    1. a) Sum the currents at each essential node in the circuit shown in Fig. P4.3.
    2. b) Show that any one of the equations in (a) can be derived from the remaining three equations.
5. 4.5 Look at the circuit in Fig. 4.4.
    1. a) Write the KCL equation at the essential node labeled g.
    2. b) Show that the KCL equation in part (a) can be derived from the KCL equations at nodes b, c, and e (see Example 4.2).

Section 4.2

1. 4.6 PSPICEMULTISIM Use the node-voltage method to find vo in the circuit in Fig. P4.6.

Figure P4.6

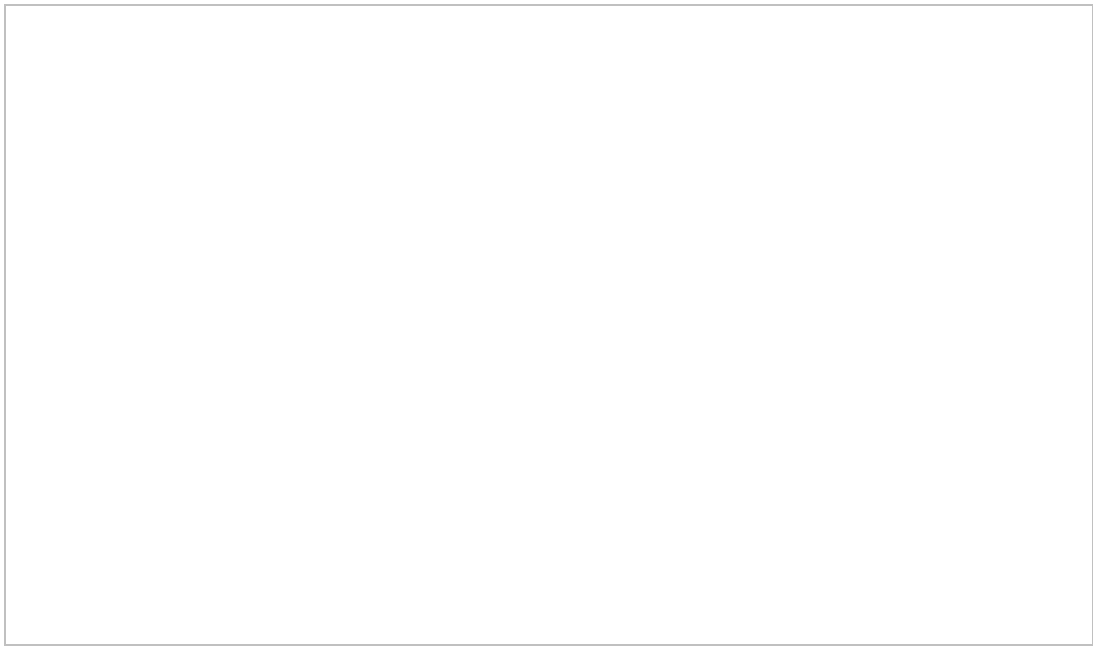


Figure P4.6 Full Alternative Text

2. 4.7 PSPICEMULTISIM

    1. a) Find the power developed by the 40 mA current source in the circuit in Fig. P4.6.

    2. b) Find the power developed by the 25 V voltage source in the circuit in Fig. P4.6.

    3. c) Verify that the total power developed equals the total power dissipated.

3. 4.8 PSPICEMULTISIM A 100 Ω resistor is connected in series with the 40 mA current source in the circuit in Fig. P4.6.

    1. a) Find vo.

    2. b) Find the power developed by the 40 mA current source.

    3. c) Find the power developed by the 25 V voltage source.

    4. d) Verify that the total power developed equals the total power dissipated.

    5. e) What effect will any finite resistance connected in series with the 40 mA current source have on the value of vo?

4. 4.9 PSPICEMULTISIM Use the node-voltage method to find how much power the 2 A source extracts from the circuit in Fig. P4.9.

Figure P4.9

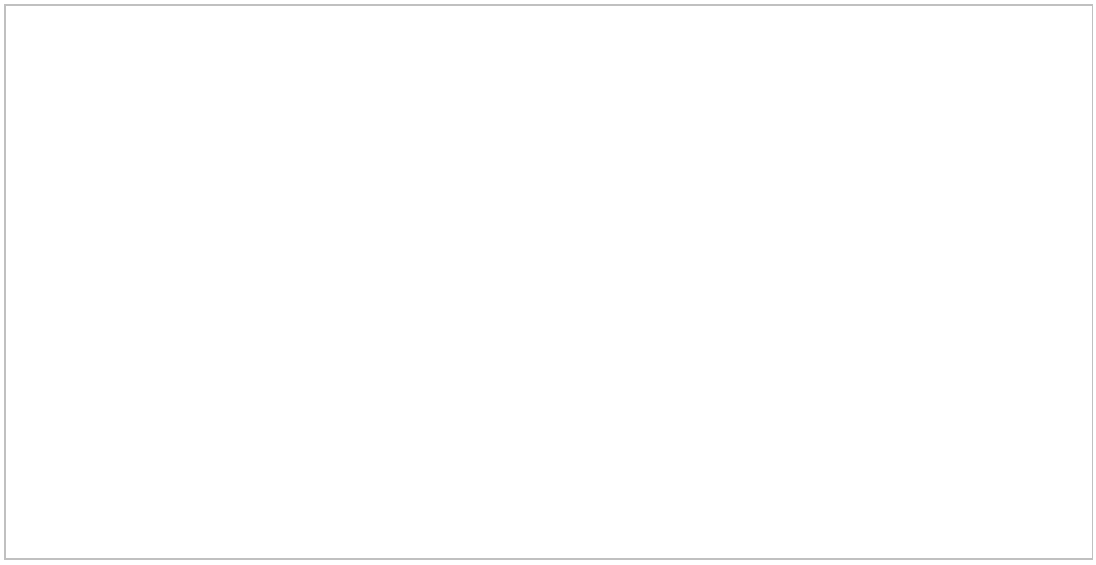


Figure P4.9 Full Alternative Text

5. 4.10 PSPICEMULTISIM Use the node-voltage method to find v1 and v2 in the circuit shown in Fig. P4.10.

Figure P4.10

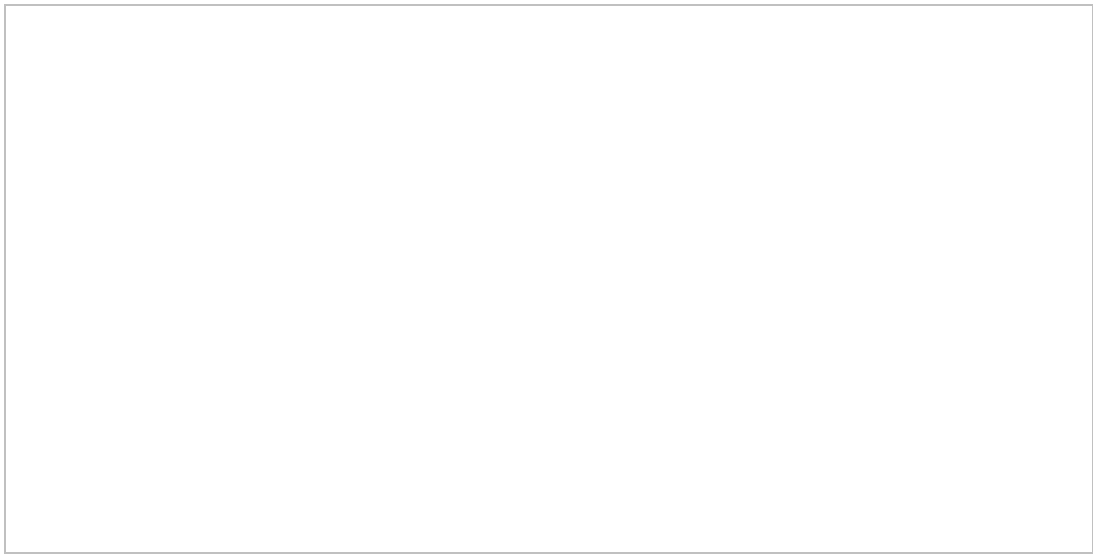


Figure P4.10 Full Alternative Text

6. 4.11 PSPICEMULTISIM Use the node-voltage method to find v1 and v2 in the circuit in Fig. P4.11.

Figure P4.11

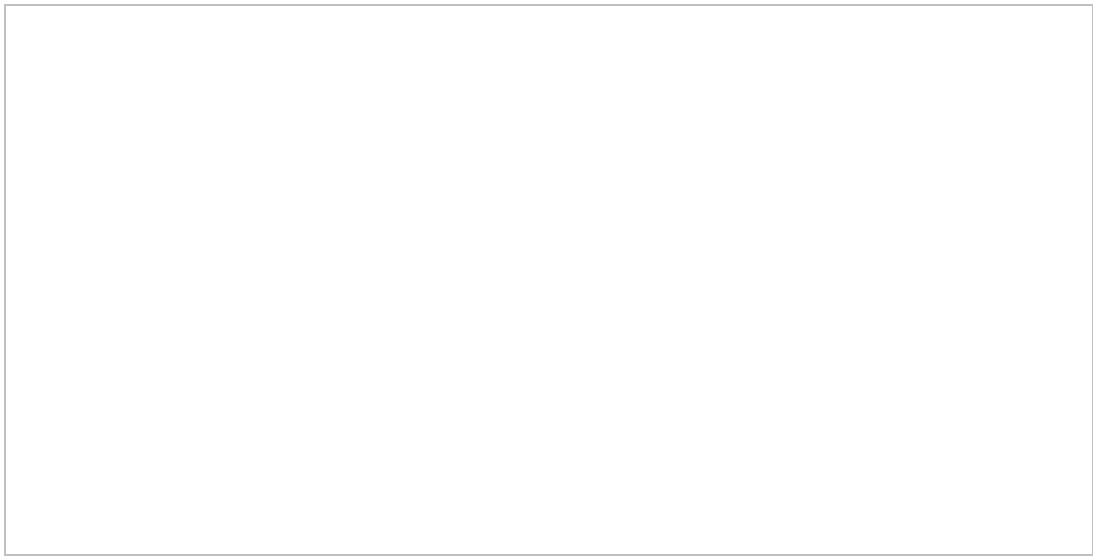


Figure P4.11 Full Alternative Text

7. 4.12 PSPICEMULTISIM

    1. a) Use the node-voltage method to find the branch currents ia−ie in the circuit shown in Fig. P4.12.

    2. b) Find the total power developed in the circuit.

Figure P4.12

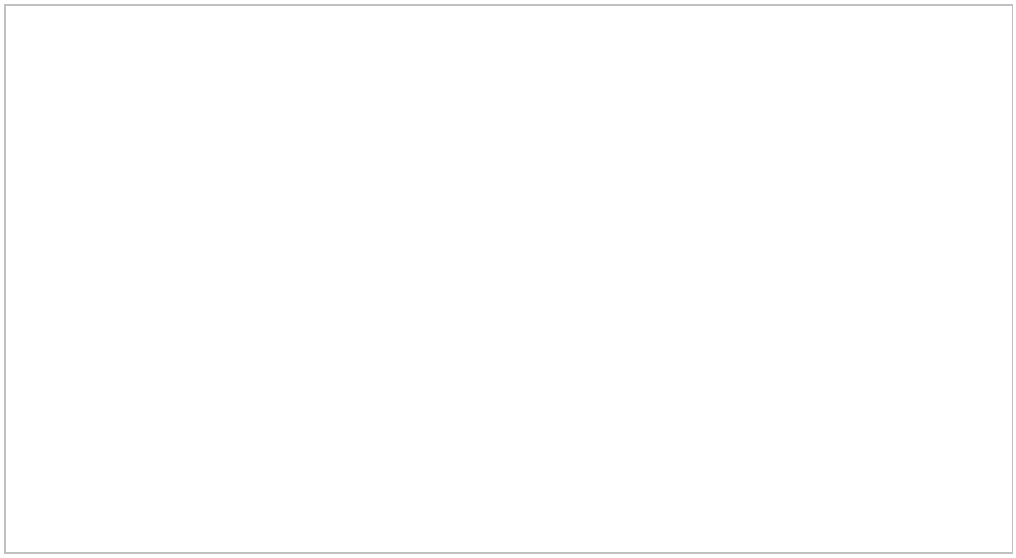


Figure P4.12 Full Alternative Text

8. 4.13 PSPICEMULTISIM

    1. a) Use the node-voltage method to find v1, v2, and v3 in the circuit in Fig. P4.13.

    2. b) How much power does the 40 V voltage source deliver to the circuit?

Figure P4.13

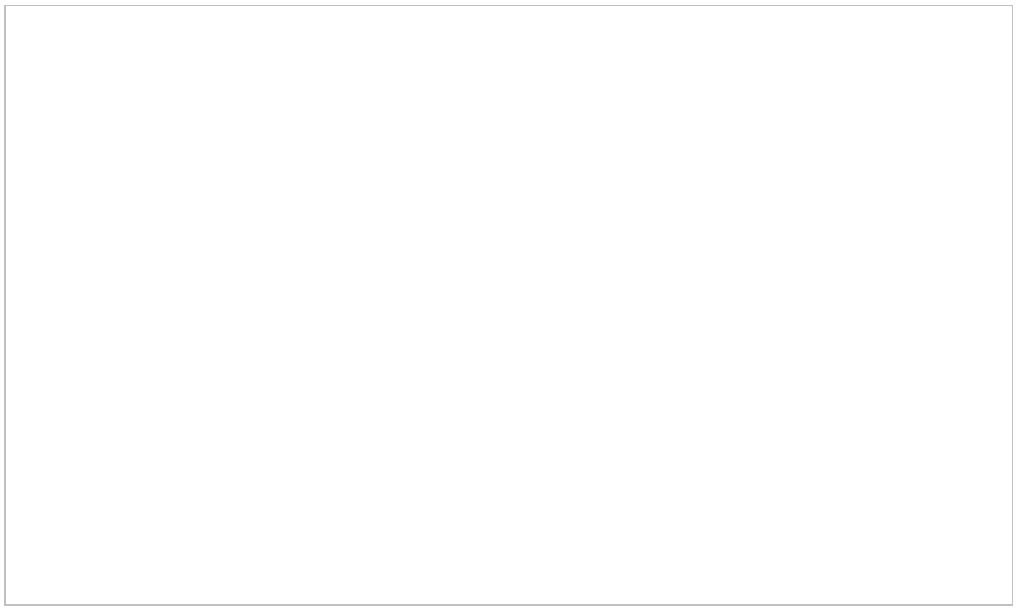


Figure P4.13 Full Alternative Text

9. 4.14 PSPICEMULTISIM The circuit shown in Fig. P4.14 is a dc model of a residential power distribution circuit.

    1. a) Use the node-voltage method to find the branch currents i1−i6.

    2. b) Test your solution for the branch currents by showing that the total power dissipated equals the total power developed.

Figure P4.14

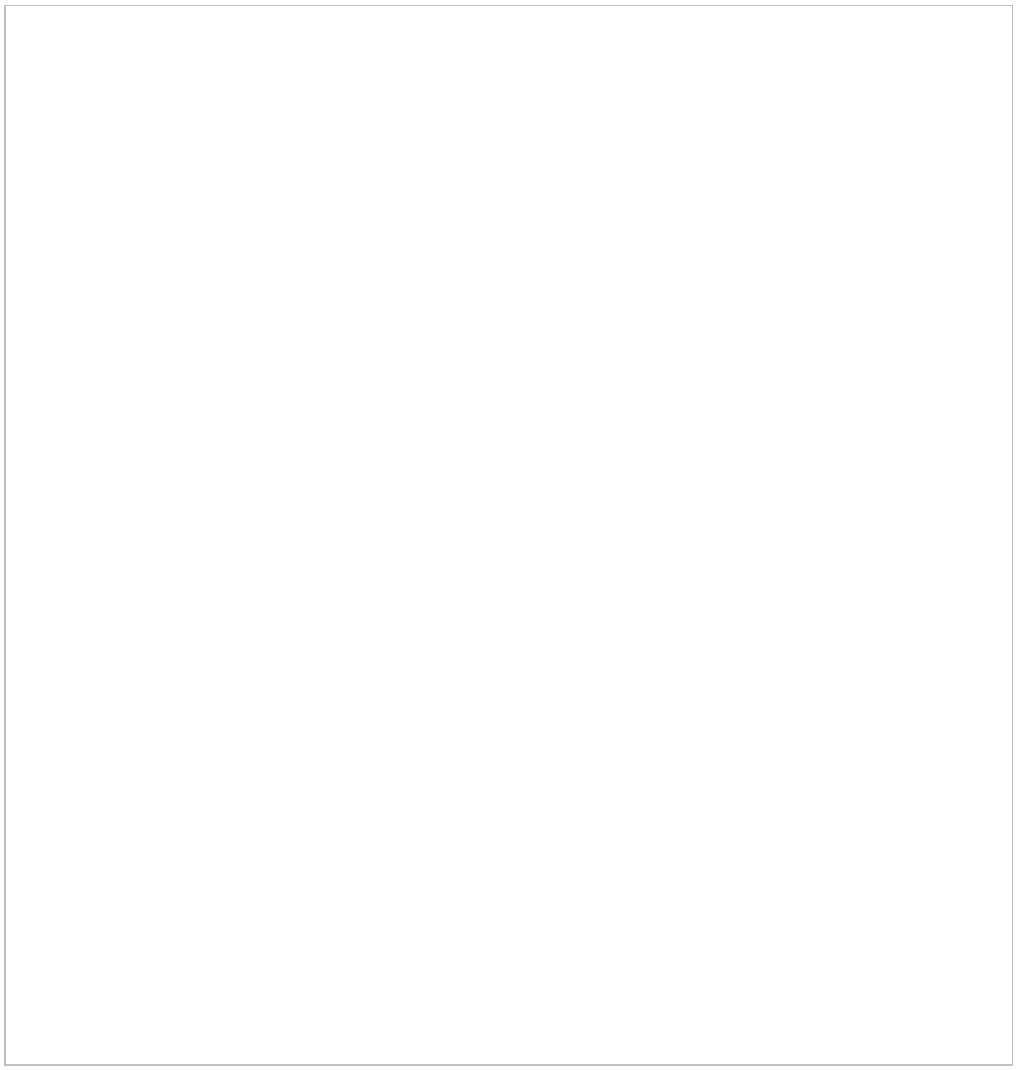


Figure P4.14 Full Alternative Text

10. 4.15 PSPICEMULTISIM Use the node-voltage method to find the total power dissipated in the circuit in Fig. P4.15.

Figure P4.15

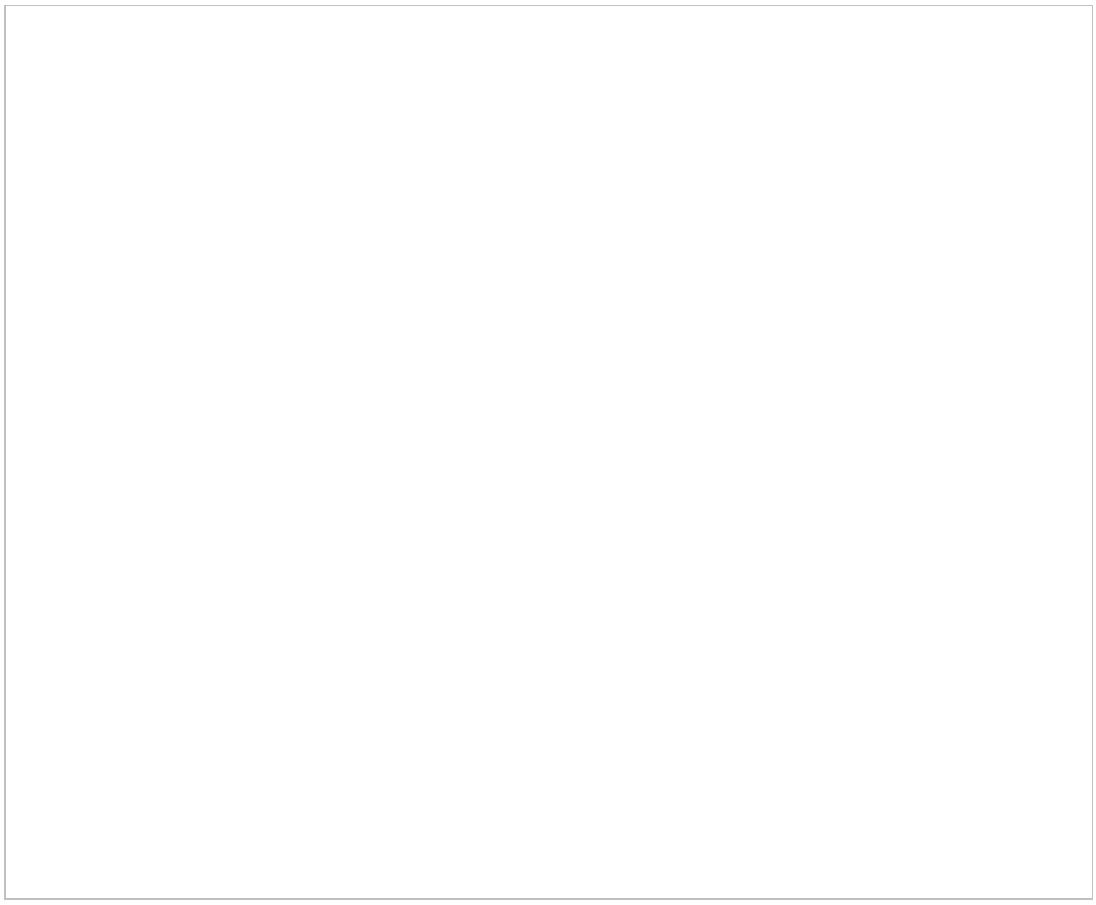


Figure P4.15 Full Alternative Text

11. 4.16 PSPICEMULTISIM

    1. a) Use the node-voltage method to show that the output voltage vo in the circuit in Fig. P4.16 is equal to the average value of the source voltages.

    2. b) Find vo if v1=120 V, v2=60 V, and v3=−30 V.

Figure P4.16

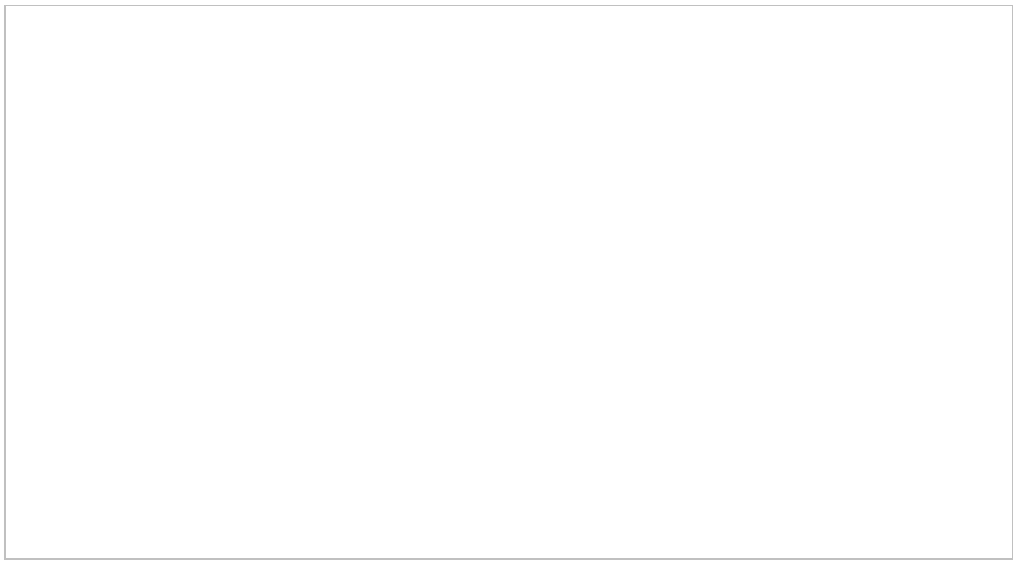


Figure P4.16 Full Alternative Text

Section 4.3

1. 4.17 PSPICEMULTISIM Use the node-voltage method to calculate the power delivered by the dependent voltage source in the circuit in Fig. P4.17.

Figure P4.17

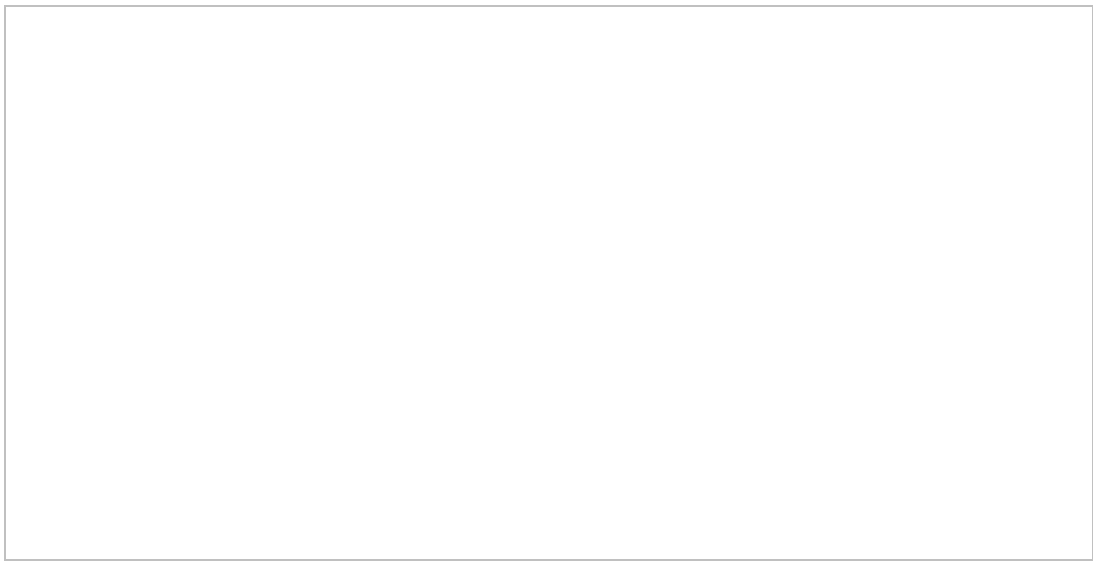


Figure P4.17 Full Alternative Text

2. 4.18

    1. a) Use the node voltage method to find vo for the circuit in Fig. P4.18.

    2. b) Find the total power supplied in the circuit.

Figure P4.18

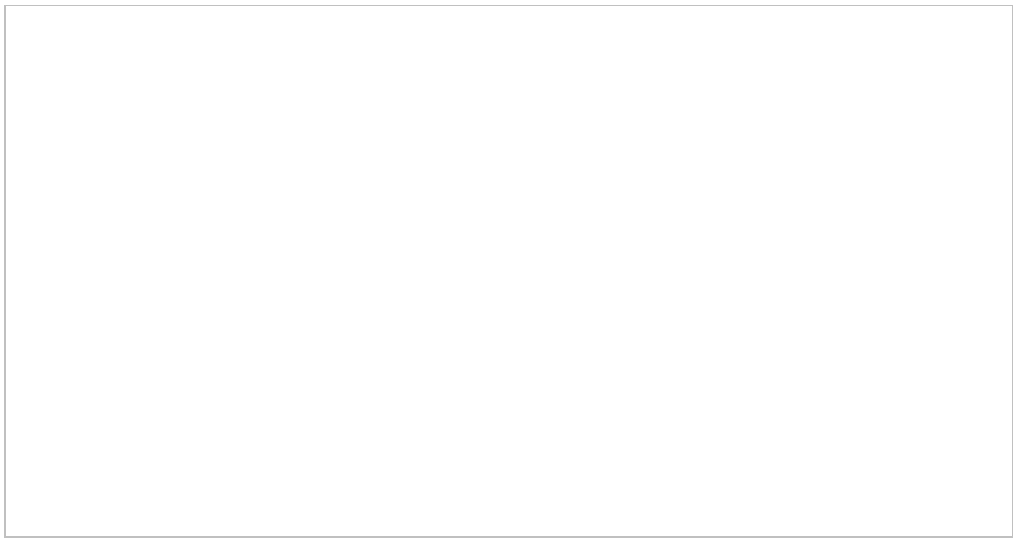


Figure P4.18 Full Alternative Text

3. 4.19 PSPICEMULTISIM

    1. a) Use the node-voltage method to find the total power developed in the circuit in Fig. P4.19.

    2. b) Check your answer by finding the total power absorbed in the circuit.

Figure P4.19

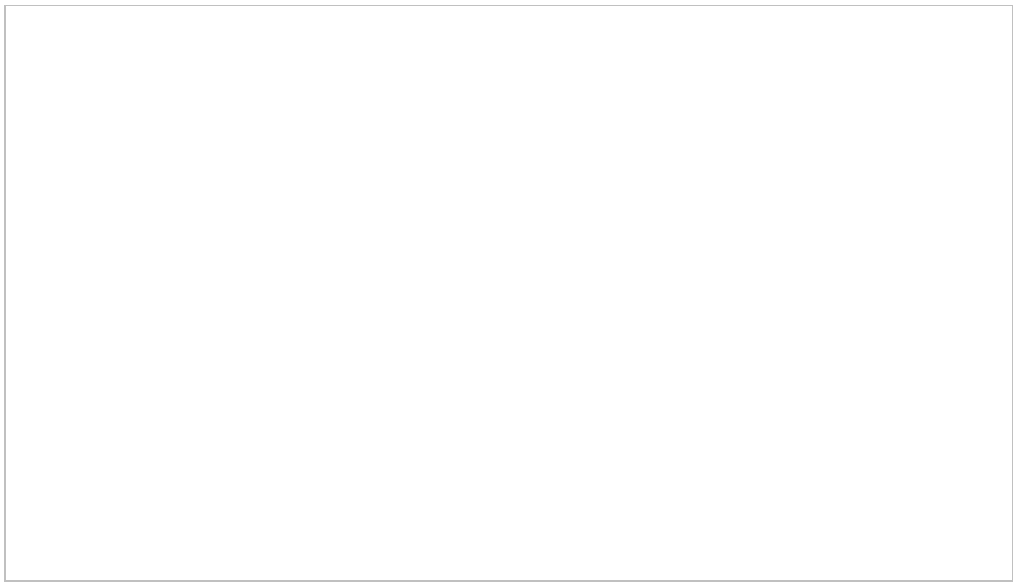


Figure P4.19 Full Alternative Text

4. 4.20 PSPICEMULTISIM

    1. a) Use the node-voltage method to find vo in the circuit in Fig. P4.20.

    2. b) Find the power absorbed by the dependent source.

    3. c) Find the total power developed by the independent sources.

Figure P4.20

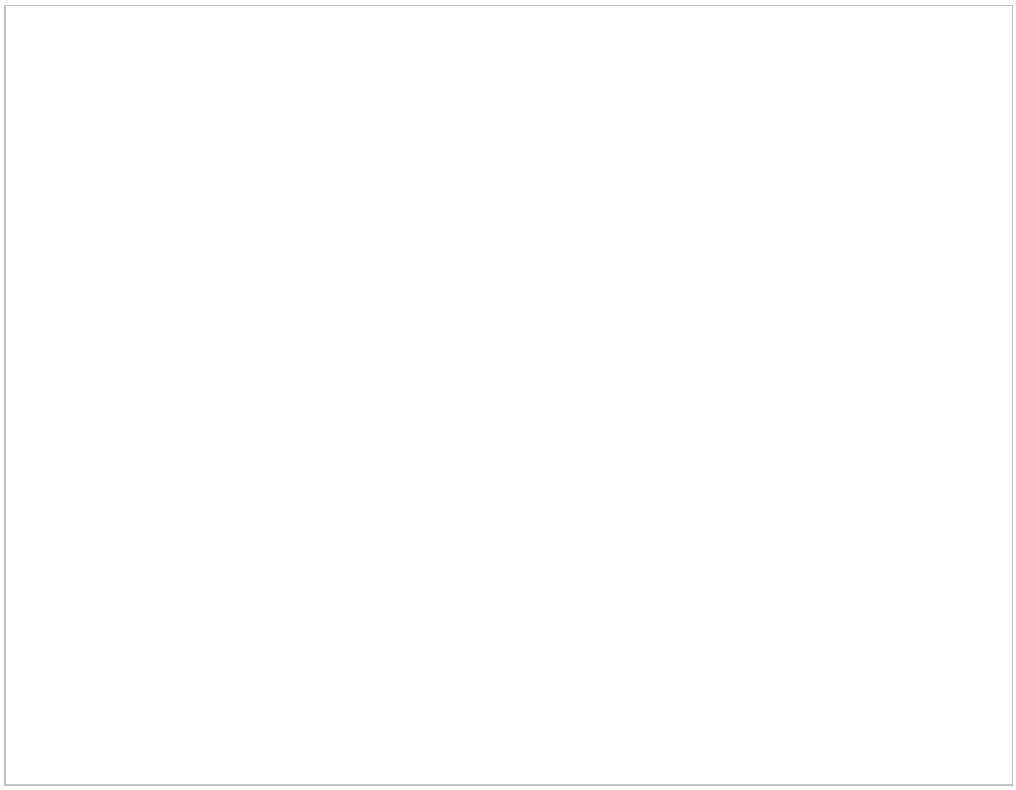


Figure P4.20 Full Alternative Text

5. 4.21 PSPICEMULTISIM

   a) Find the node voltages v1, v2, and v3 in the circuit in Fig. P4.21.

   1.

   2. b) Find the total power dissipated in the circuit.

Figure P4.21

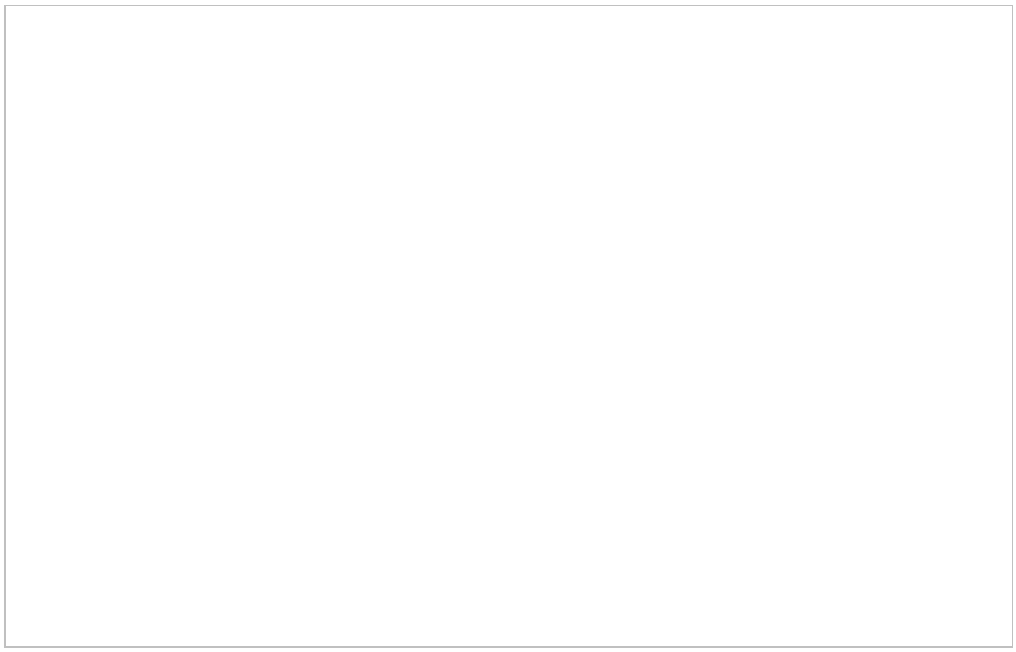


Figure P4.21 Full Alternative Text

Section 4.4

1. 4.22 PSPICEMULTISIM Use the node-voltage method to find the value of vo in the circuit in Fig. P4.22.

Figure P4.22

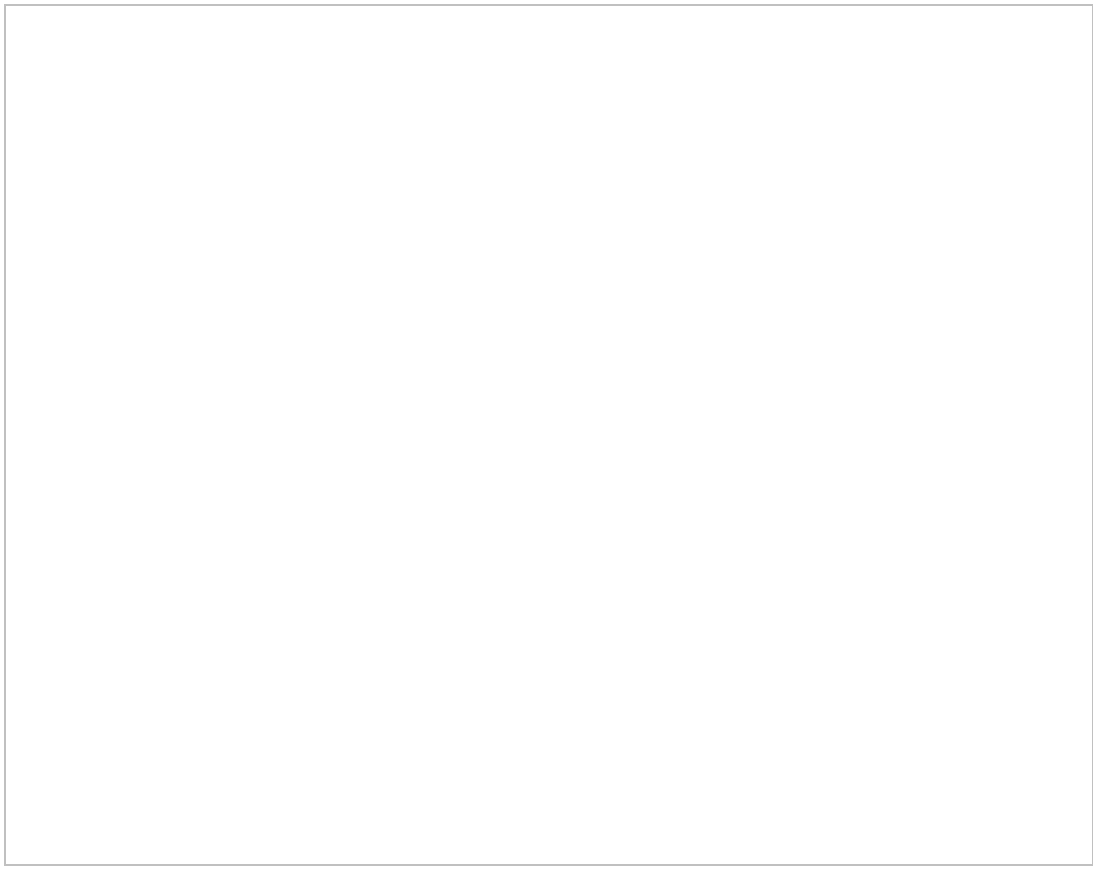


Figure P4.22 Full Alternative Text

2. 4.23 PSPICEMULTISIM

    1. a) Use the node-voltage method to find the branch currents i1, i2, and i3 in the circuit in Fig. P4.23.

2. b) Check your solution for i1, i2, and i3 by showing that the power dissipated in the circuit equals the power developed.

Figure P4.23

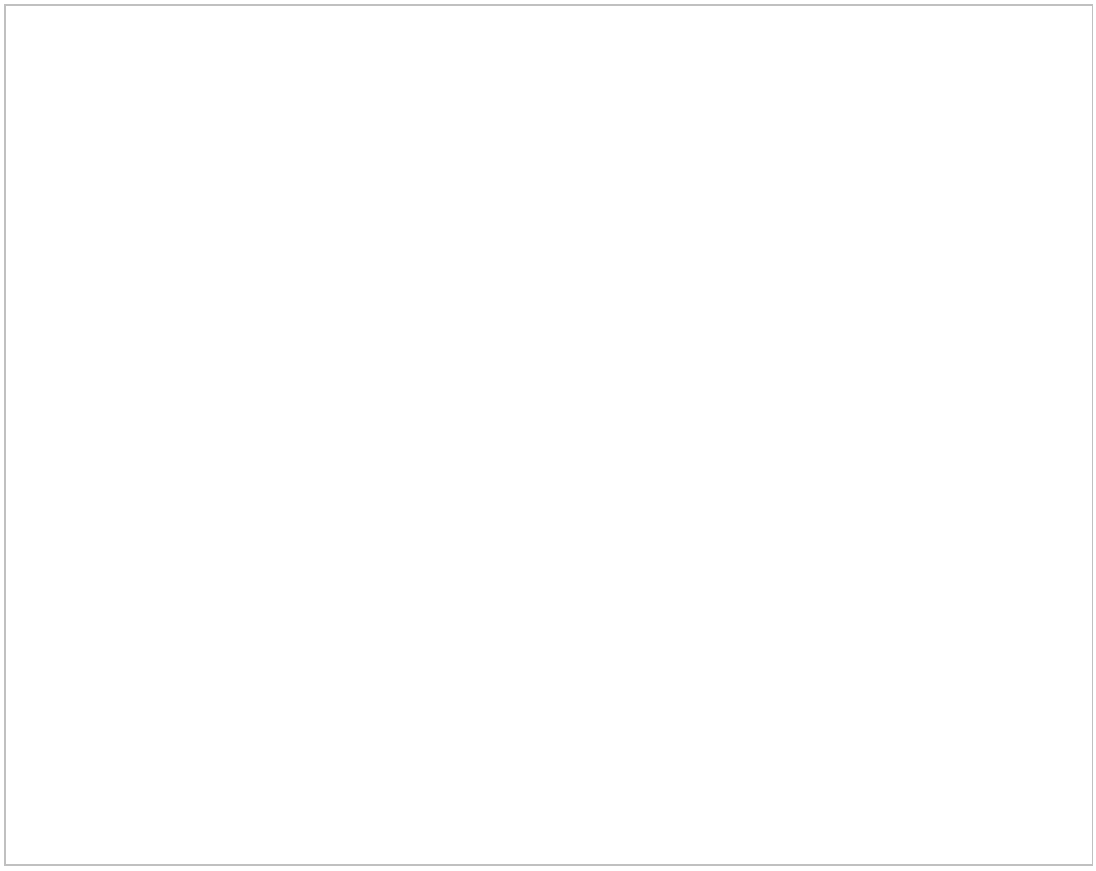


Figure P4.23 Full Alternative Text

3. 4.24 PSPICEMULTISIM Use the node-voltage method to find the value of vo in the circuit in Fig. P4.24.

Figure P4.24

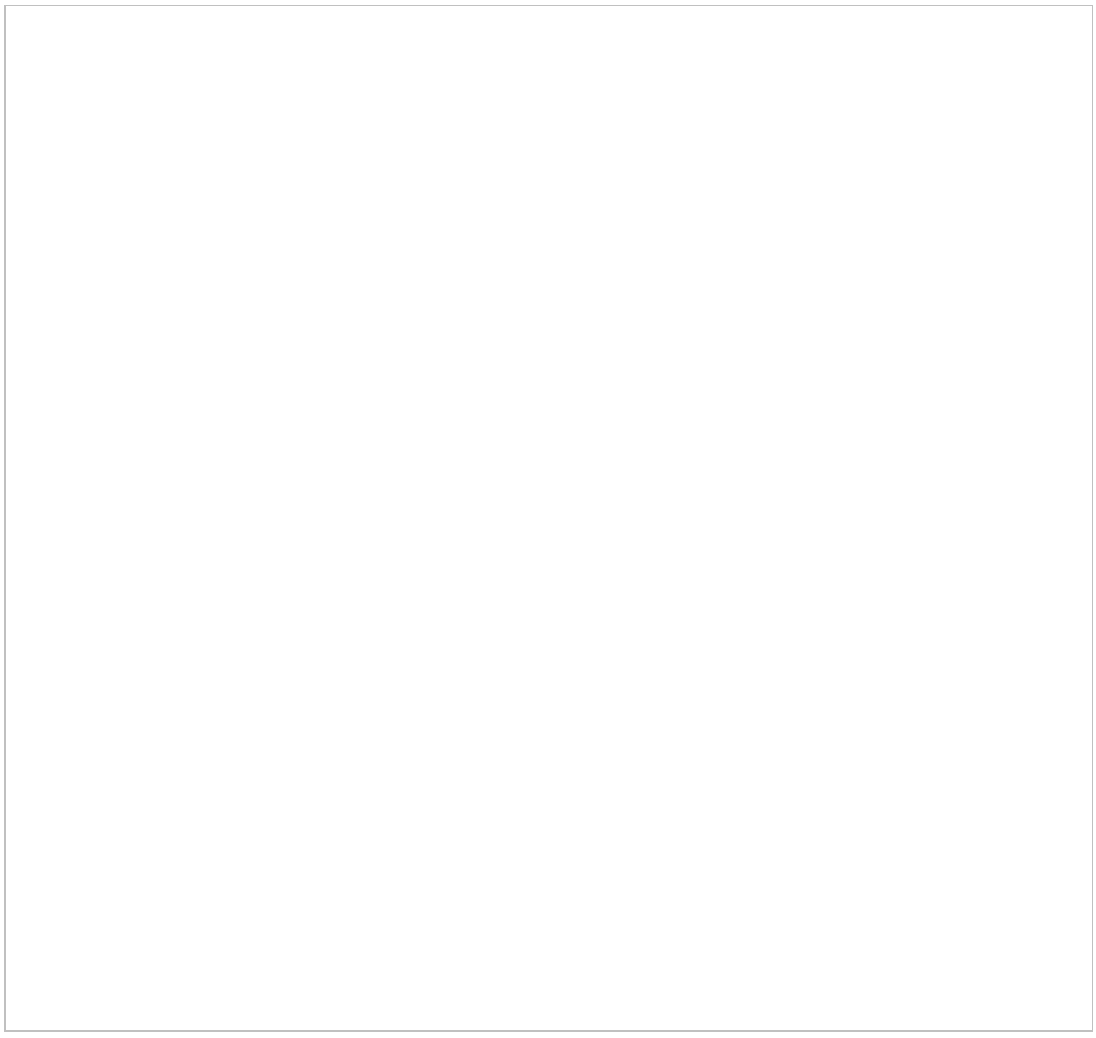


Figure P4.24 Full Alternative Text

4. 4.25

    1. a) Use the node-voltage method to find the power dissipated in the 2 Ω resistor in the circuit in Fig. P4.25.

    2. b) Find the power supplied by the 230 V source.

Figure P4.25

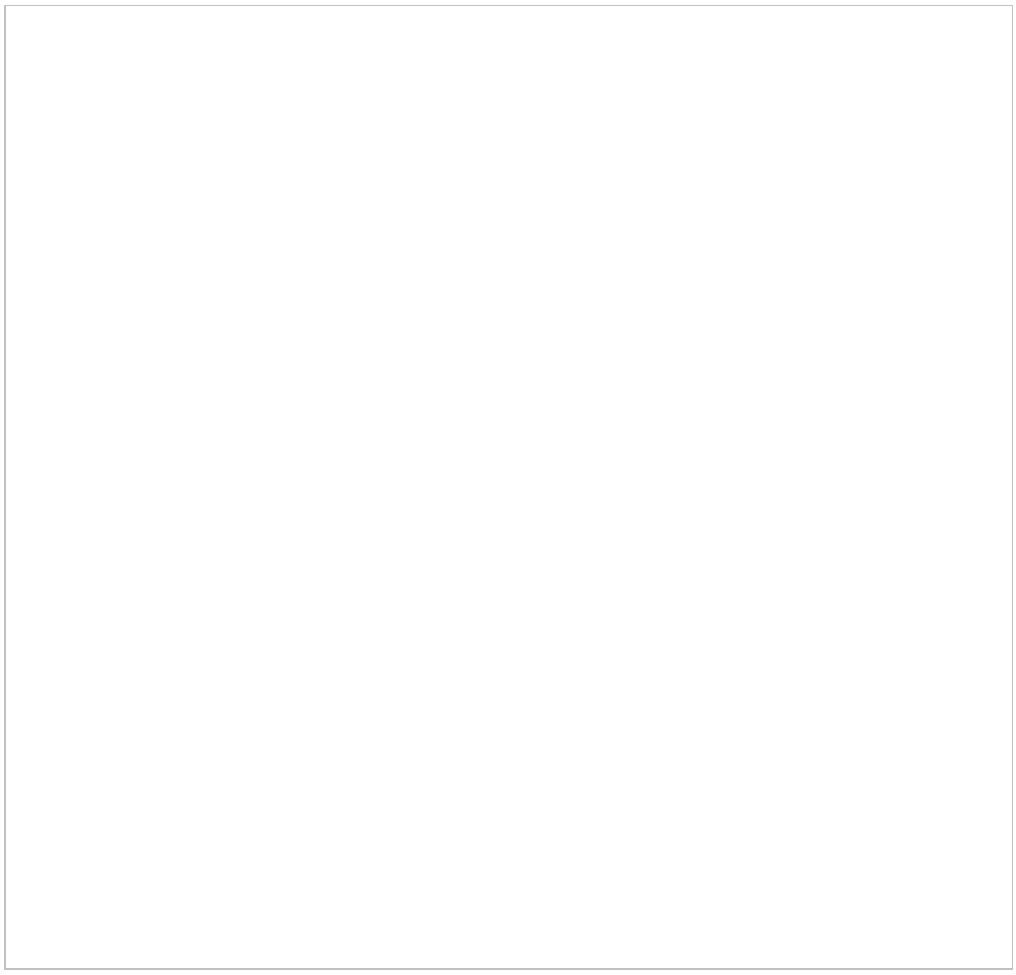


Figure P4.25 Full Alternative Text

5. 4.26 PSPICEMULTISIM Use the node-voltage method to find io in the circuit in Fig. P4.26.

Figure P4.26

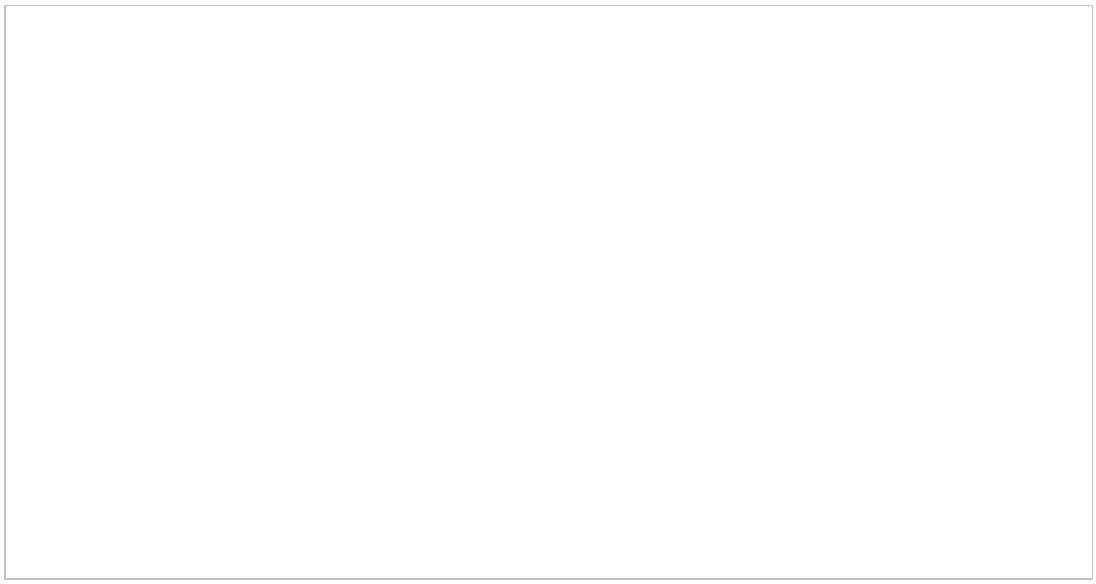


Figure P4.26 Full Alternative Text

6. 4.27 PSPICEMULTISIM

    1. a) Use the node-voltage method to find vo and the power delivered by the 2 A current source in the circuit in Fig. P4.27. Use node a as the reference node.

    2. b) Repate part (a), but use node b as the reference node.

    3. c) Compare the choice of reference node in (a) and (b) Which is better, and why?

Figure P4.27

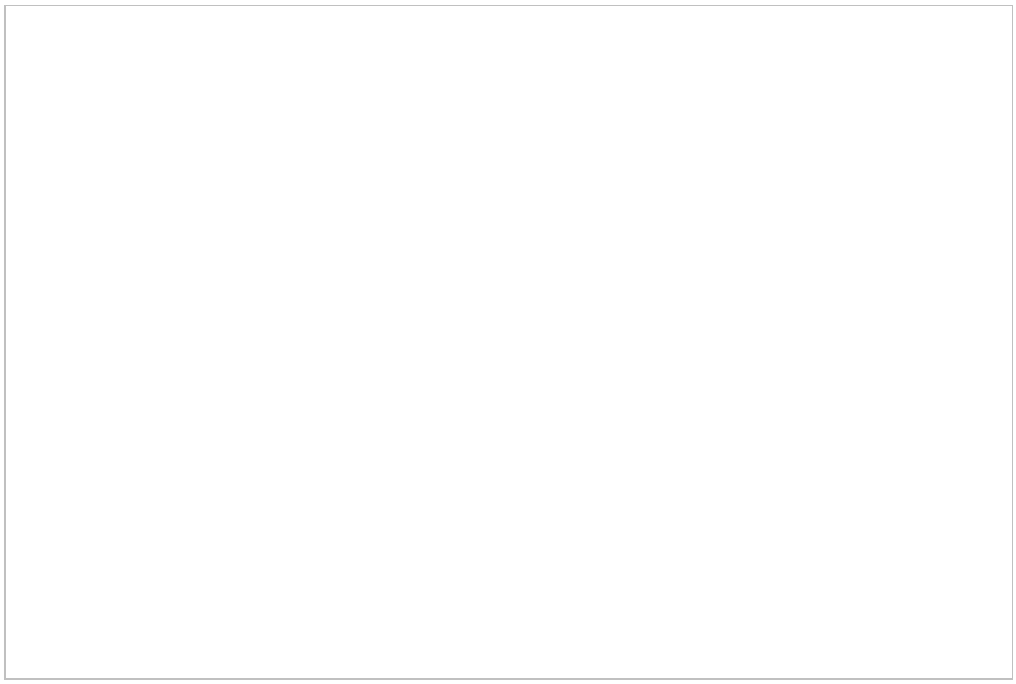


Figure P4.27 Full Alternative Text

7. 4.28 PSPICEMULTISIM Use the node-voltage method to find vo in the circuit in Fig. P4.28.

Figure P4.28

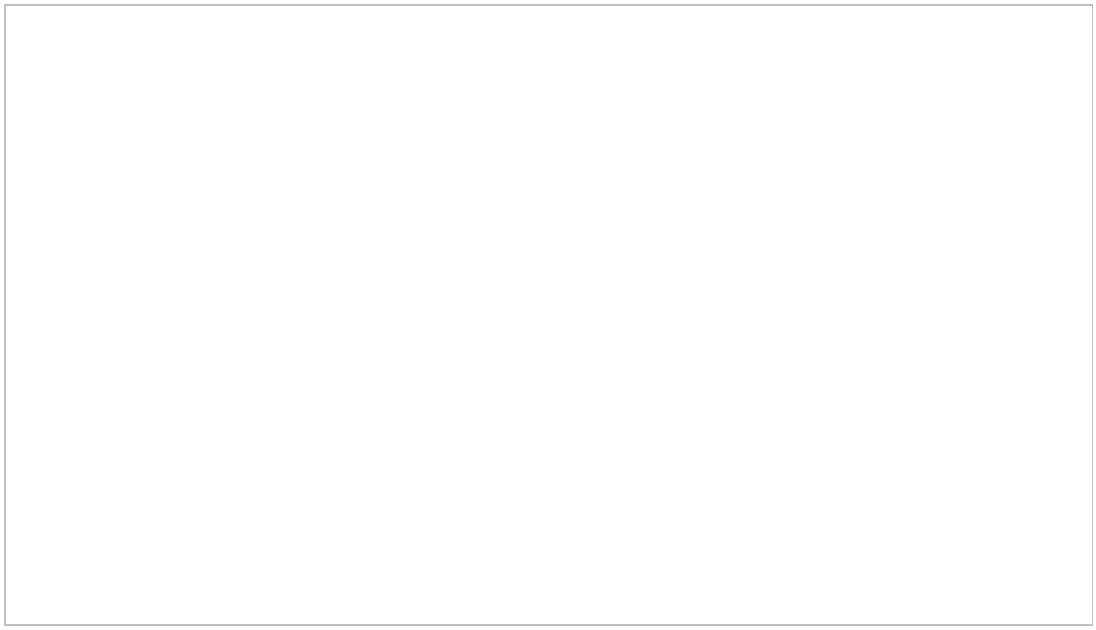


Figure P4.28 Full Alternative Text

8. 4.29 PSPICEMULTISIM Use the node-voltage method to find the power developed by the 20 V source in the circuit in Fig. P4.29.

Figure P4.29

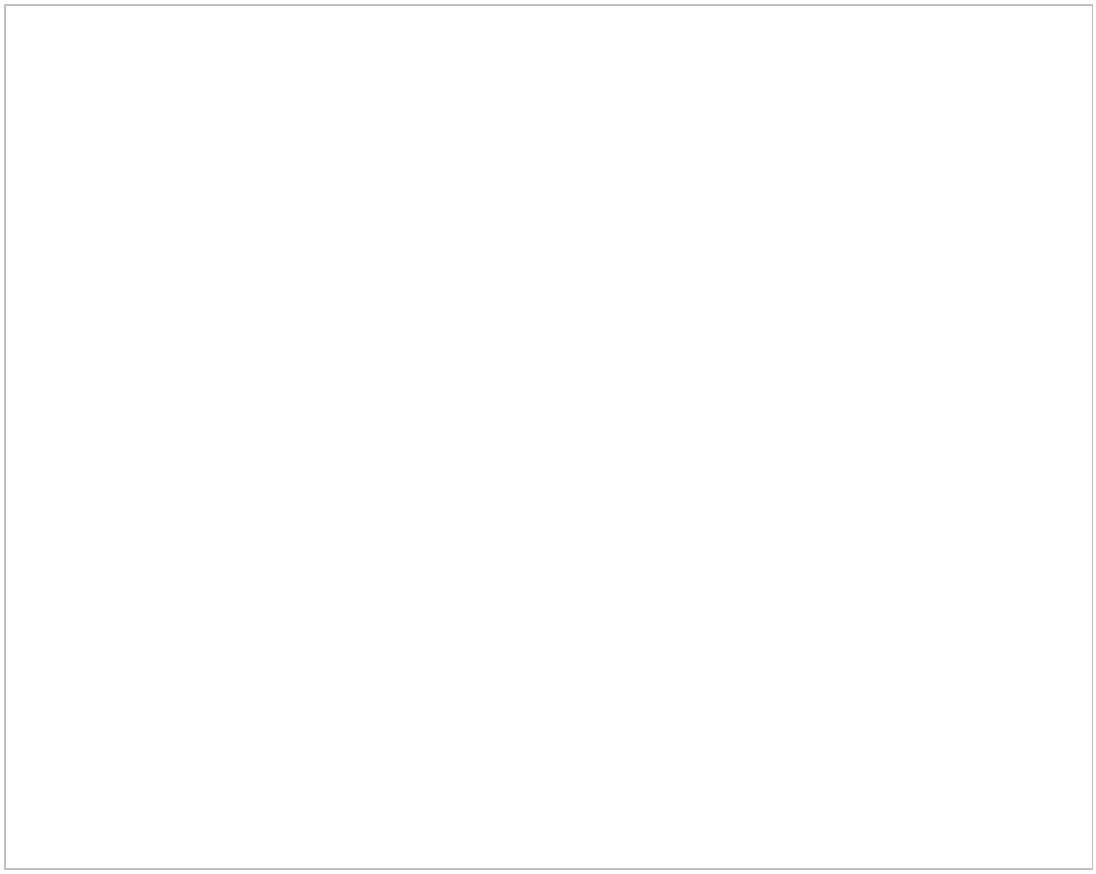


Figure P4.29 Full Alternative Text

9. 4.30 Assume you are a project engineer and one of your staff is assigned to analyze the circuit shown in Fig. P4.30. The reference node and node numbers given on the figure were assigned by the analyst. Her solution gives the values of v1 and v2 as 105 V and 85 V, respectively.

    1. a) What values did the analyst use for the left-most and right-most node voltages when writing KCL equations at nodes 1 and 2?

2. b) Use the values supplied by the analyst to calculate the total power developed in the circuit and the total power dissipated in the circuit.

3. c) Do you agree with the solution submitted by the analyst?

Figure P4.30

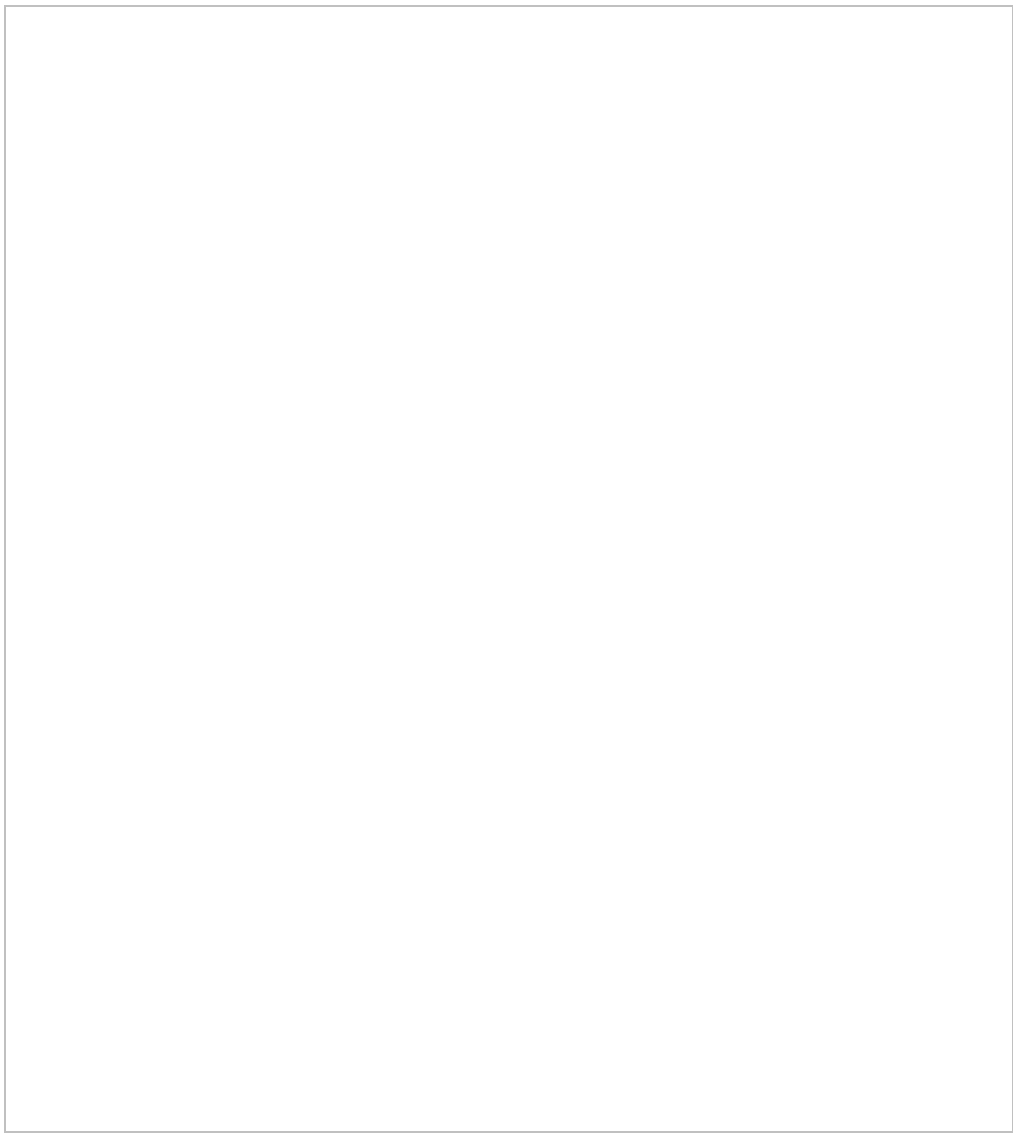


Figure P4.30 Full Alternative Text

10. 4.31 Show that when Eqs. 4.13, 4.14, and 4.16 are solved for iB, the

result is identical to Eq. 2.21.

Section 4.5

1. 4.32 Solve Problem 4.12 using the mesh-current method.
2. 4.33 Solve Problem 4.14 using the mesh-current method.
3. 4.34 Solve Problem 4.25 using the mesh-current method.
4. 4.35 Solve Problem 4.26 using the mesh-current method.
5. 4.36 PSPICEMULTISIM
    1. a) Use the mesh-current method to find the branch currents ia, ib, and ic in the circuit in Fig. P4.36.
    2. b) Repeat (a) if the polarity of the 140 V source is reversed.

Figure P4.36

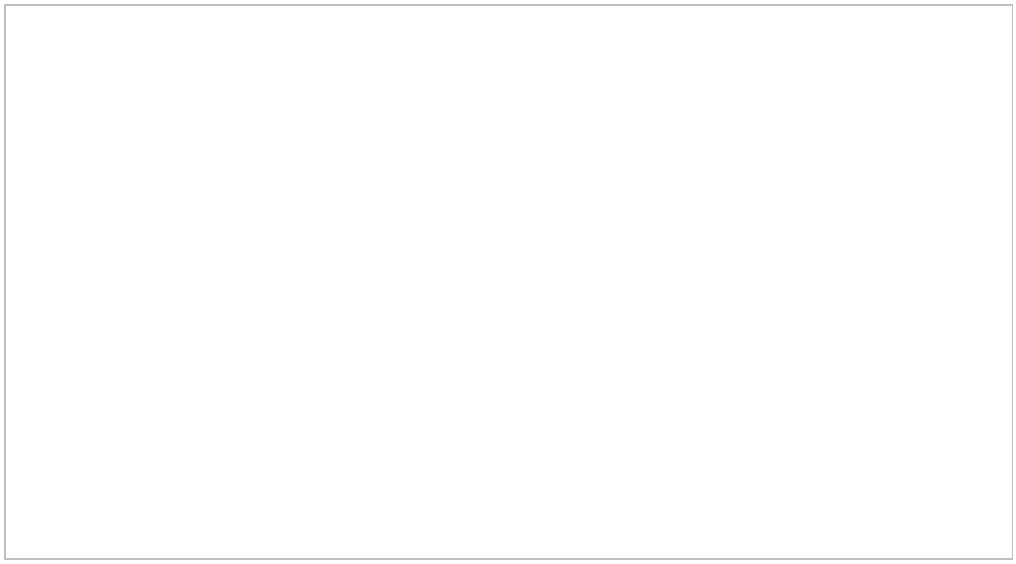


Figure P4.36 Full Alternative Text

6. 4.37 PSPICEMULTISIM

    1. a) Use the mesh-current method to find the total power developed in the circuit in Fig. P4.37.

    2. b) Check your answer by showing that the total power developed equals the total power dissipated.

Figure P4.37

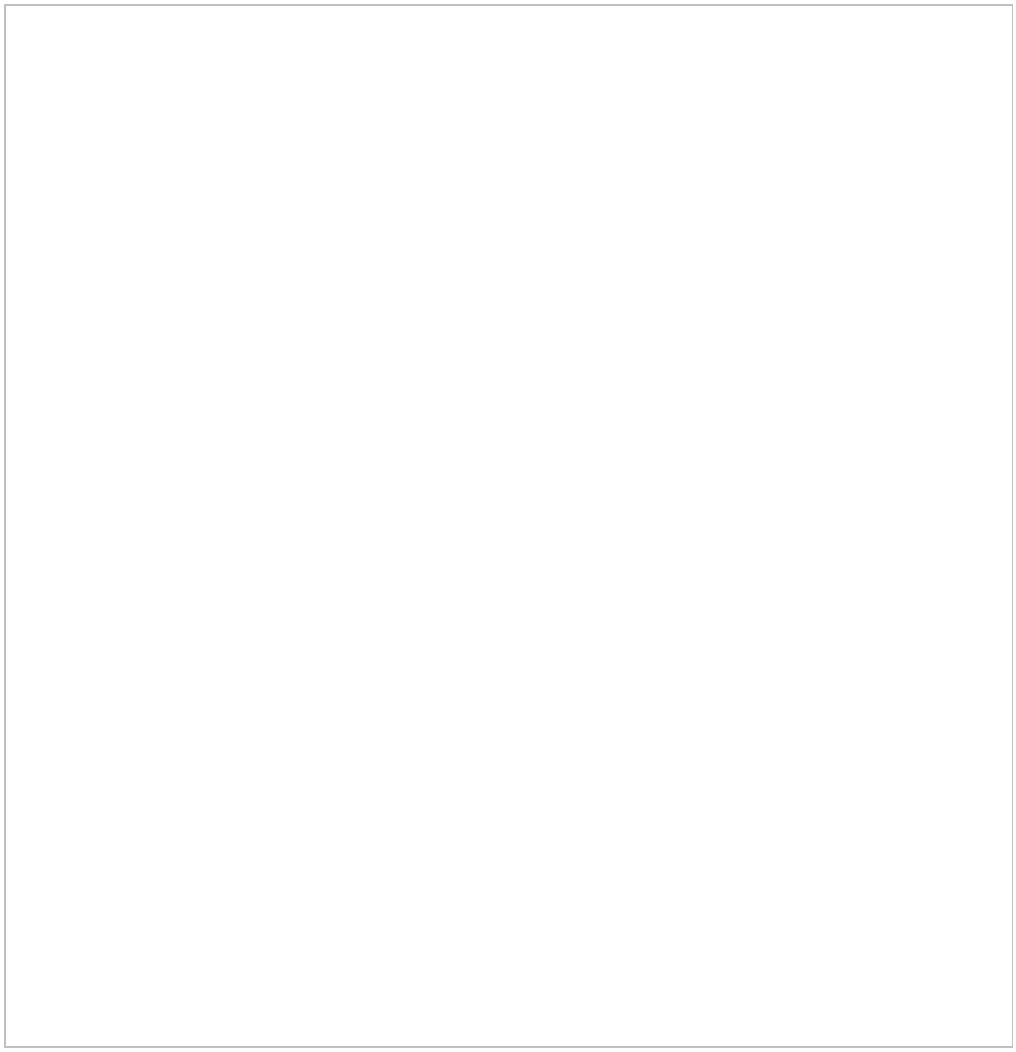


Figure P4.37 Full Alternative Text

Section 4.6

1. 4.38 Solve Problem 4.17 using the mesh-current method.

2. 4.39 PSPICEMULTISIM Use the mesh-current method to find the power dissipated in the 15 Ω resistor in the circuit in Fig. P4.39.

Figure P4.39

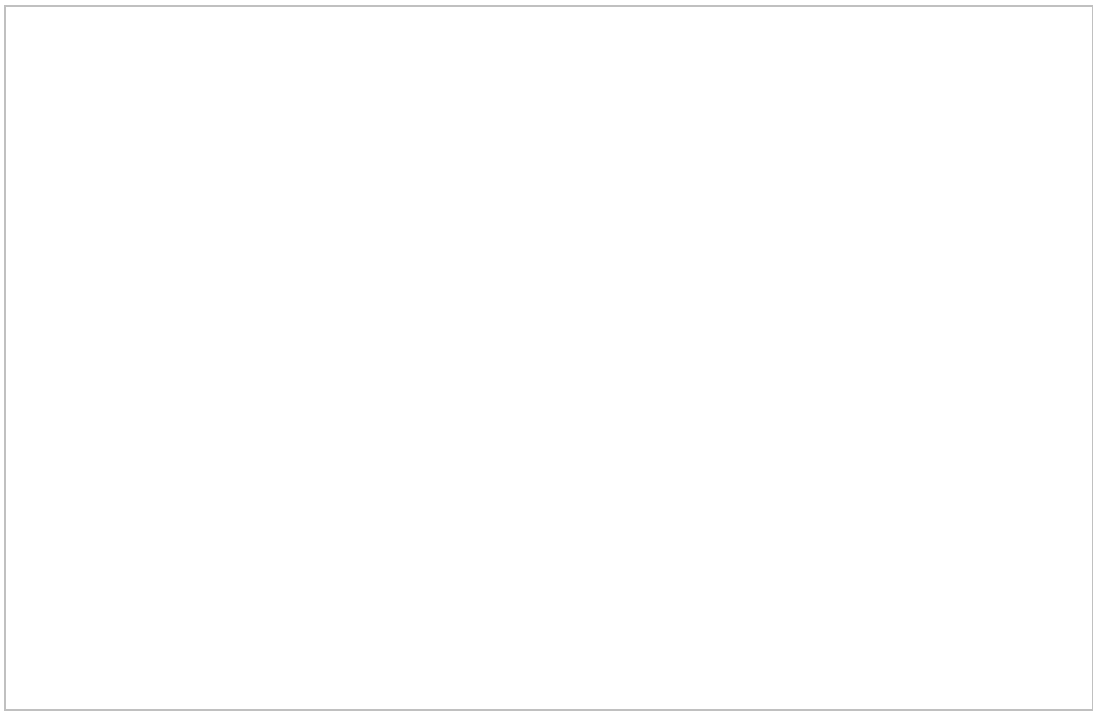


Figure P4.39 Full Alternative Text

3. 4.40 PSPICEMULTISIM Use the mesh-current method to find the power developed in the dependent voltage source in the circuit in Fig. P4.40.

Figure P4.40

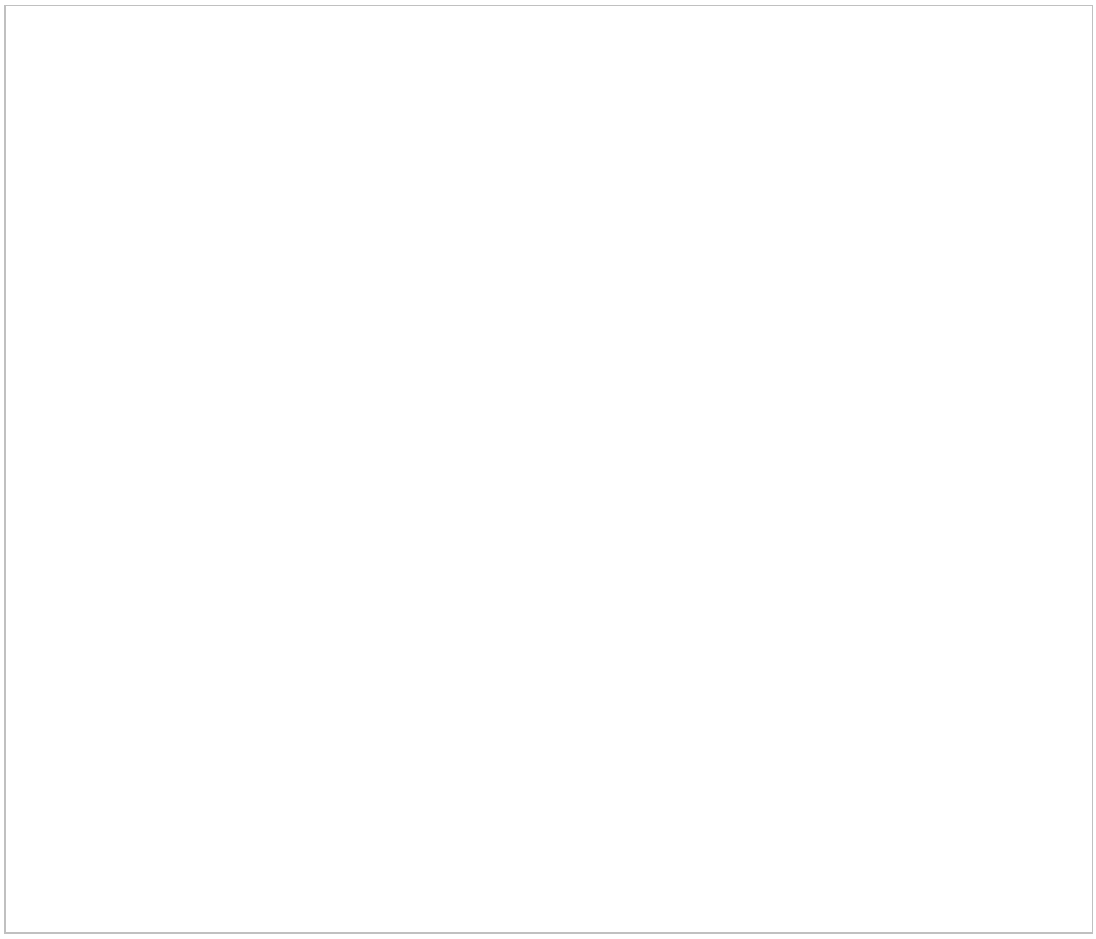


Figure P4.40 Full Alternative Text

4. 4.41 PSPICEMULTISIM Use the mesh-current method to find the power delivered by the dependent voltage source in the circuit seen in Fig. P4.41.

Figure P4.41

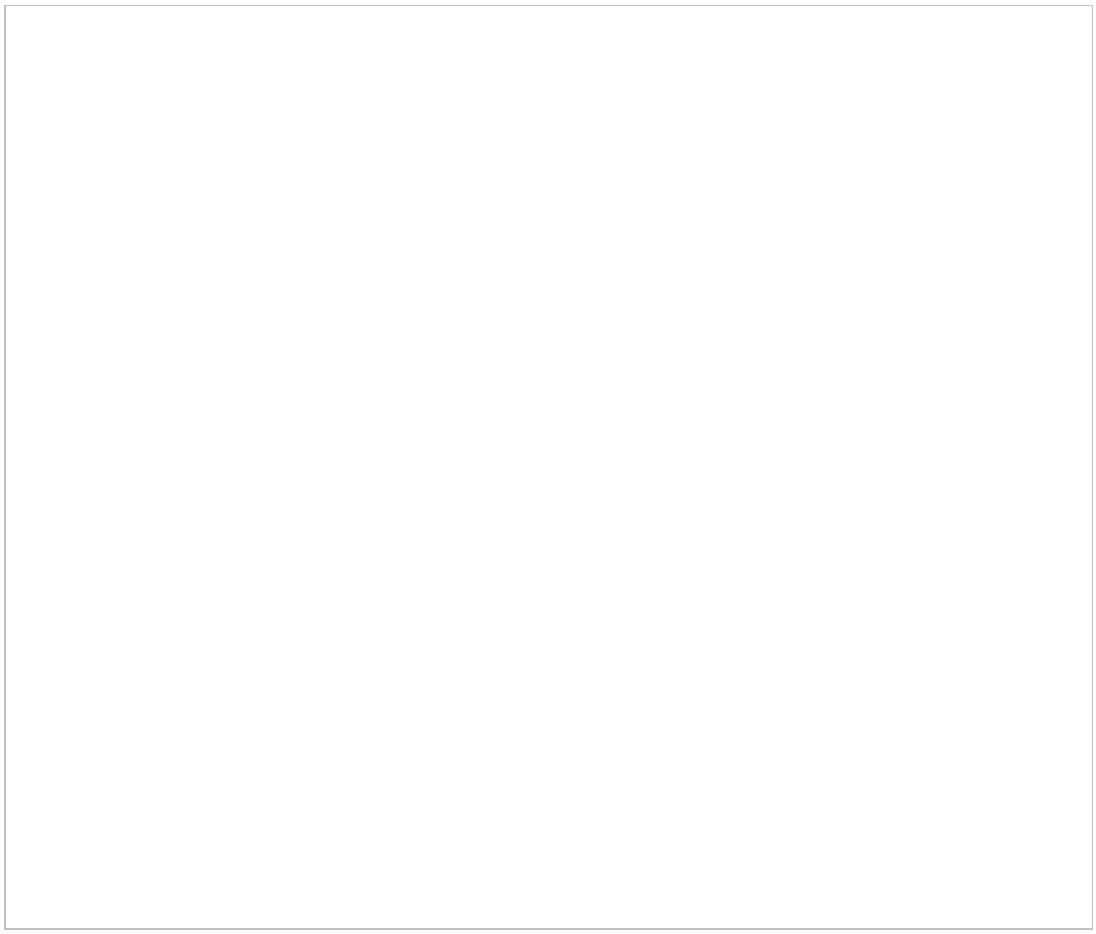


Figure P4.41 Full Alternative Text

5. 4.42 PSPICEMULTISIM

    1. a) Use the mesh-current method to find vo in the circuit in Fig. P4.42.

    2. b) Find the power delivered by the dependent source.

Figure P4.42

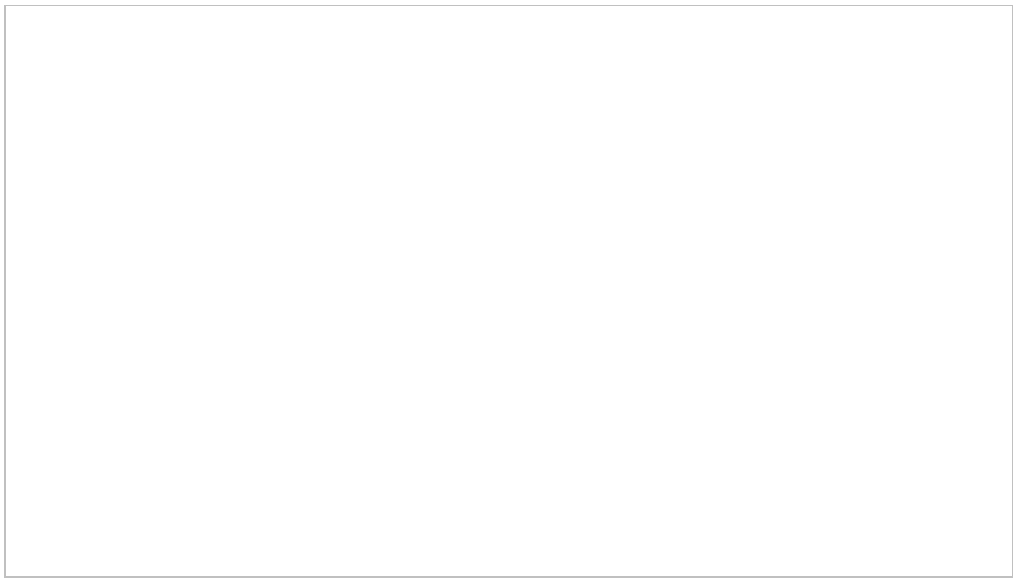


Figure P4.42 Full Alternative Text

Section 4.7

1. 4.43 Solve Problem 4.10 using the mesh-current method.
2. 4.44 Solve Problem 4.21 using the mesh-current method.
3. 4.45
    1. a) Use the mesh-current method to find how much power the 5 A current source delivers to the circuit in Fig. P4.45.
    2. b) Find the total power delivered to the circuit.
    3. c) Check your calculations by showing that the total power developed in the circuit equals the total power dissipated

Figure P4.45

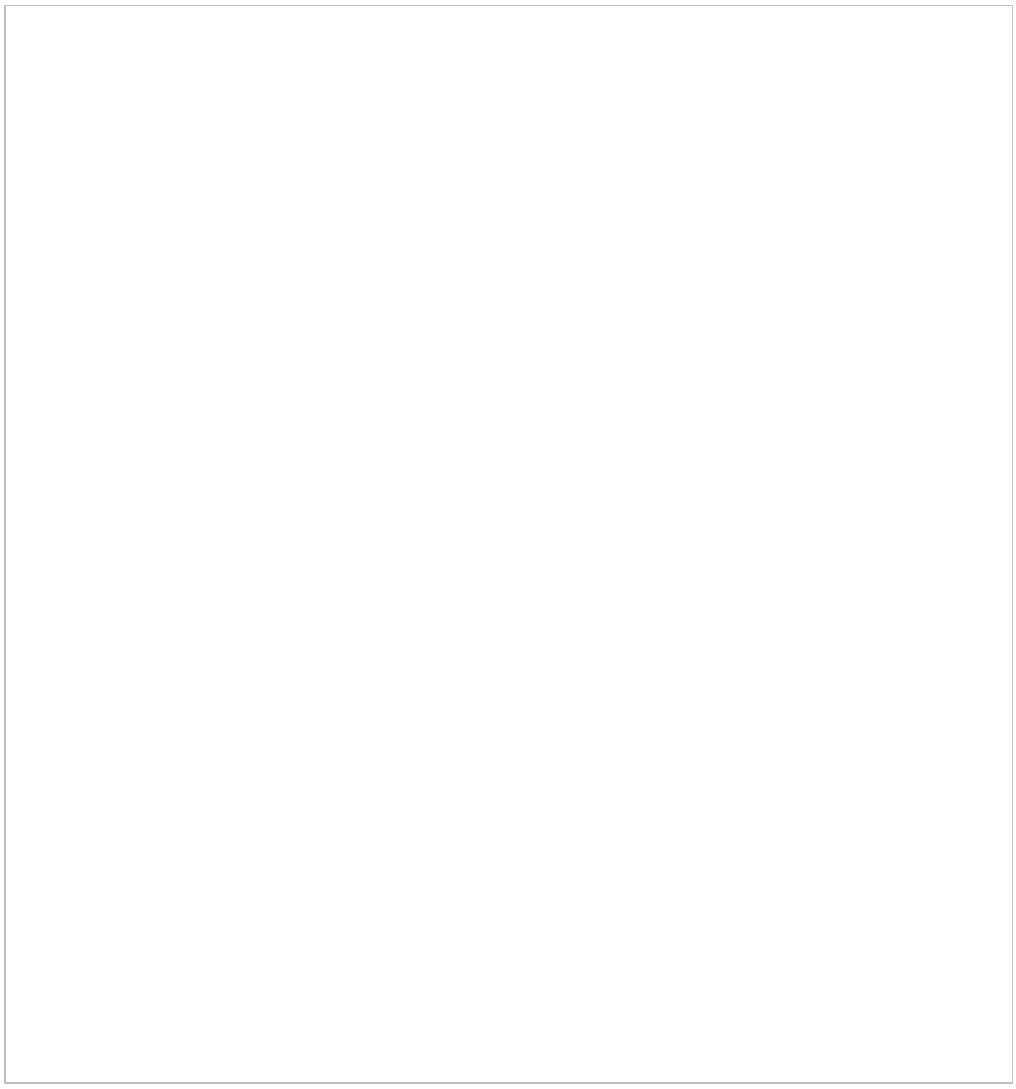


Figure P4.45 Full Alternative Text

4. 4.46 PSPICEMULTISIM

    1. a) Use the mesh-current method to solve for iΔ in the circuit in Fig. P4.46.

    2. b) Find the power delivered by the independent current source.

    3. c) Find the power delivered by the dependent voltage source.

Figure P4.46

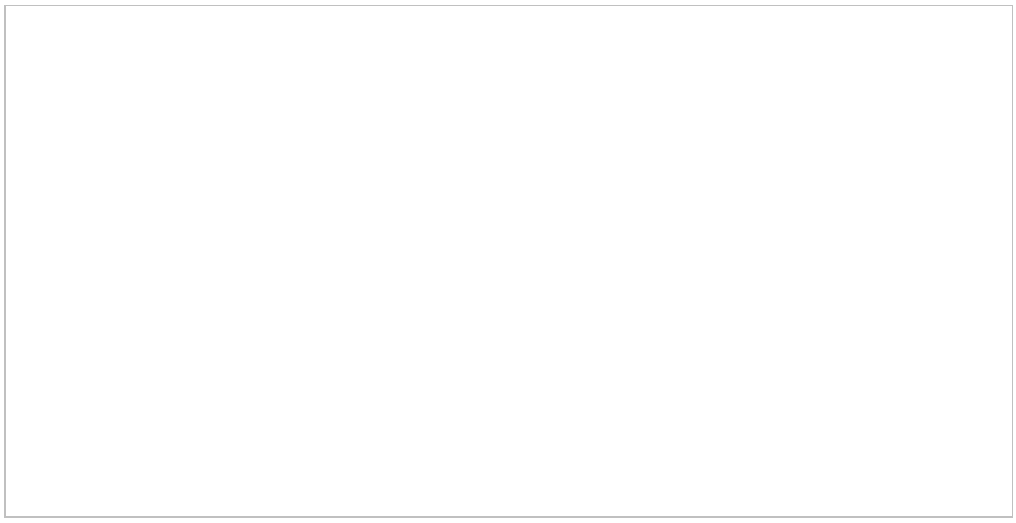


Figure P4.46 Full Alternative Text

5. 4.47 PSPICEMULTISIM

    1. a) Use the mesh-current method to determine which sources in the circuit in Fig. P4.47 are generating power.

    2. b) Find the total power dissipated in the circuit.

Figure P4.47

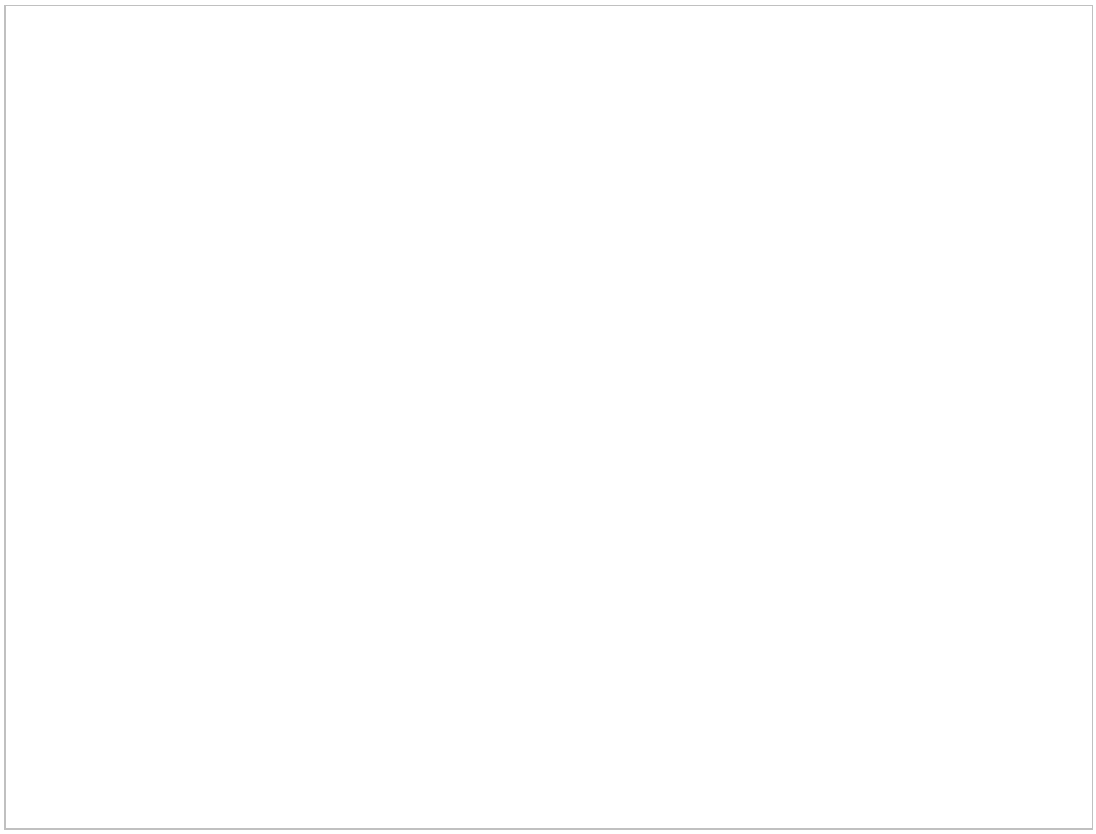


Figure P4.47 Full Alternative Text

6. 4.48 PSPICEMULTISIM Use the mesh-current method to find the total power developed in the circuit in Fig. P4.48.

Figure P4.48

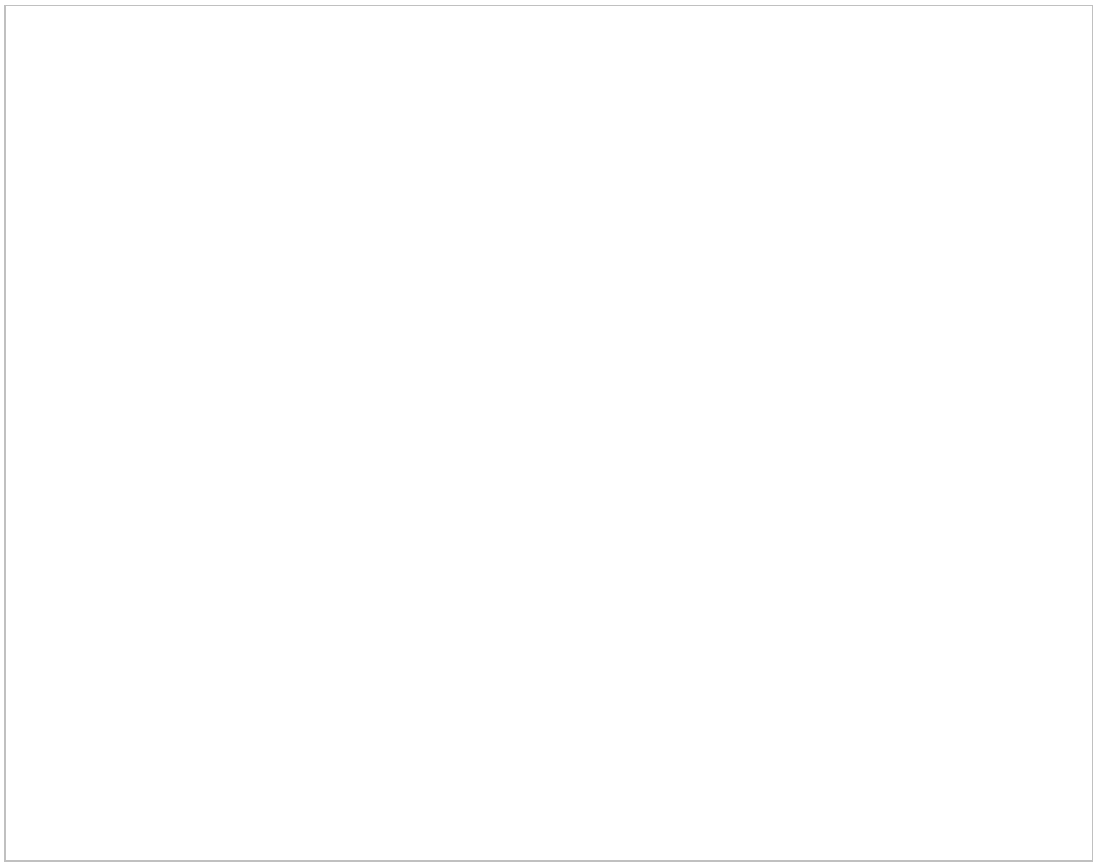


Figure P4.48 Full Alternative Text

7. 4.49 Solve Problem 4.23 using the mesh-current method.

8. 4.50 PSPICEMULTISIM Use the mesh-current method to find the total power dissipated in the circuit in Fig. P4.50.

Figure P4.50

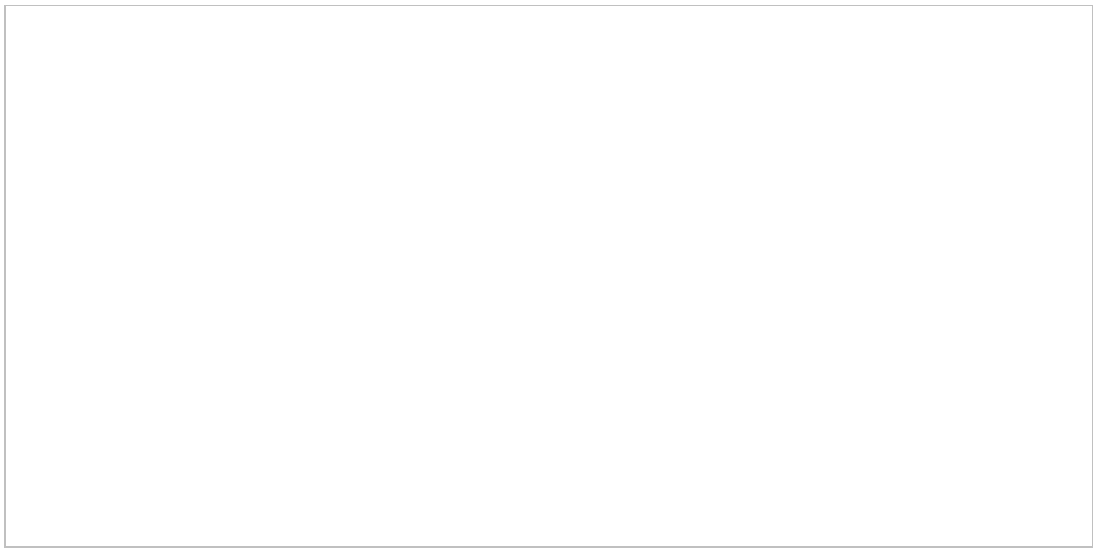


Figure P4.50 Full Alternative Text

9. 4.51

    1. a) Assume the 20 V source in the circuit in Fig. P4.50 is changed to 60 V. Find the total power dissipated in the circuit.

    2. b) Repeat (a) with the 6 A current source replaced by a short circuit.

    3. c) Explain why the answers to (a) and (b) are the same.

    4. d) Now assume you wish to change the value of the 90 V source, instead of the 20 V source, in the circuit in Fig. P4.50 to get the same power dissipated by the current source that you found in (a) and (b). Use the results in part (c) to calculate the new value of this voltage source.

10. 4.52 PSPICEMULTISIM

    1. a) Use the mesh-current method to find the branch currents in ia−ie in the circuit in Fig. P4.52.

    2. b) Check your solution by showing that the total power developed in the circuit equals the total power dissipated.

Figure P4.52

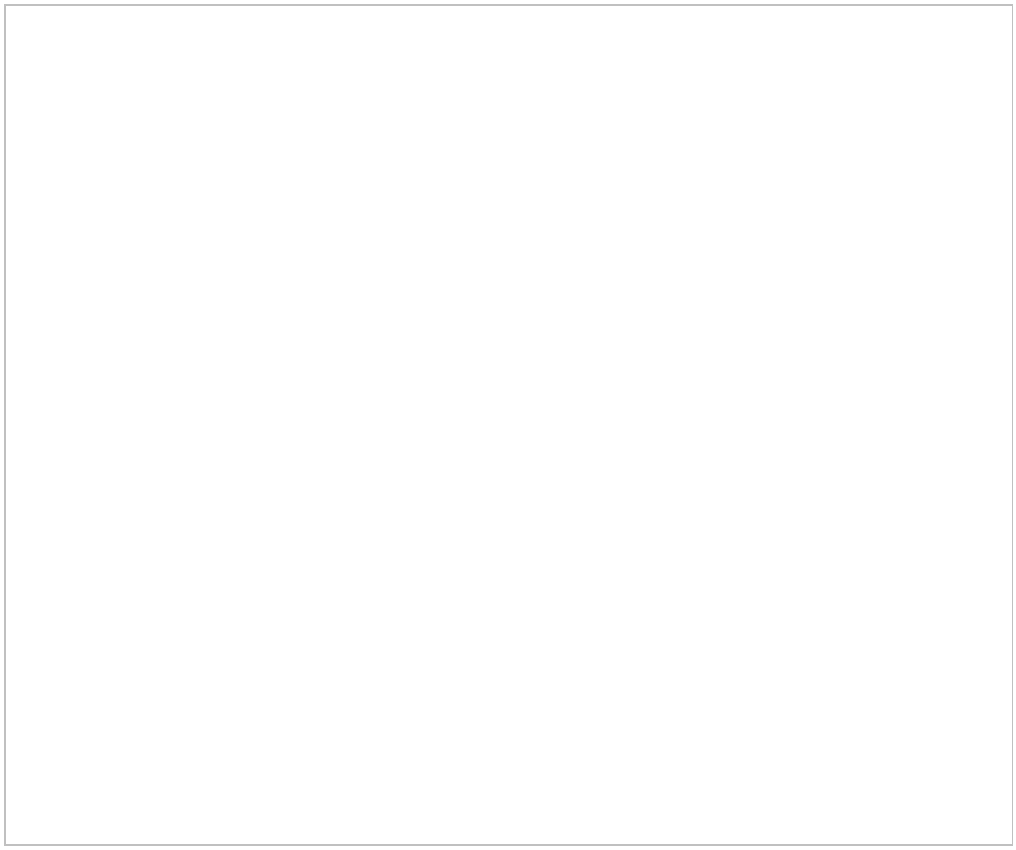


Figure P4.52 Full Alternative Text

11. 4.53 PSPICEMULTISIM

    1. a) Find the branch currents ia−ie for the circuit shown in Fig. P4.53.

    2. b) Check your answers by showing that the total power generated equals the total power dissipated.

Figure P4.53

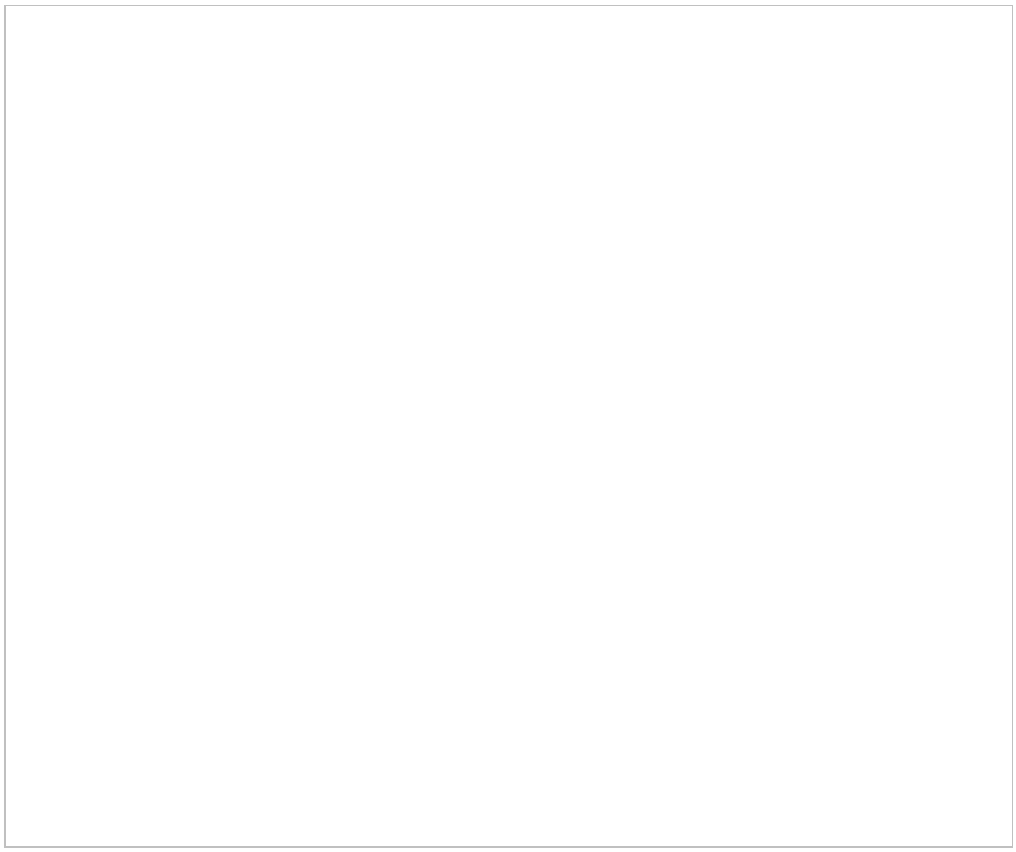


Figure P4.53 Full Alternative Text

Section 4.8

1. 4.54 PSPICEMULTISIM The variable dc voltage source in the circuit in Fig. P4.54 is adjusted so that io is zero.

    1. a) Would you use the node-voltage or mesh- current method to find vdc? Explain your choice.

    2. b) Find the value of vdc, using the method selected in (a).

    3. c) Check your solution by showing the power developed equals the power dissipated.

Figure P4.54

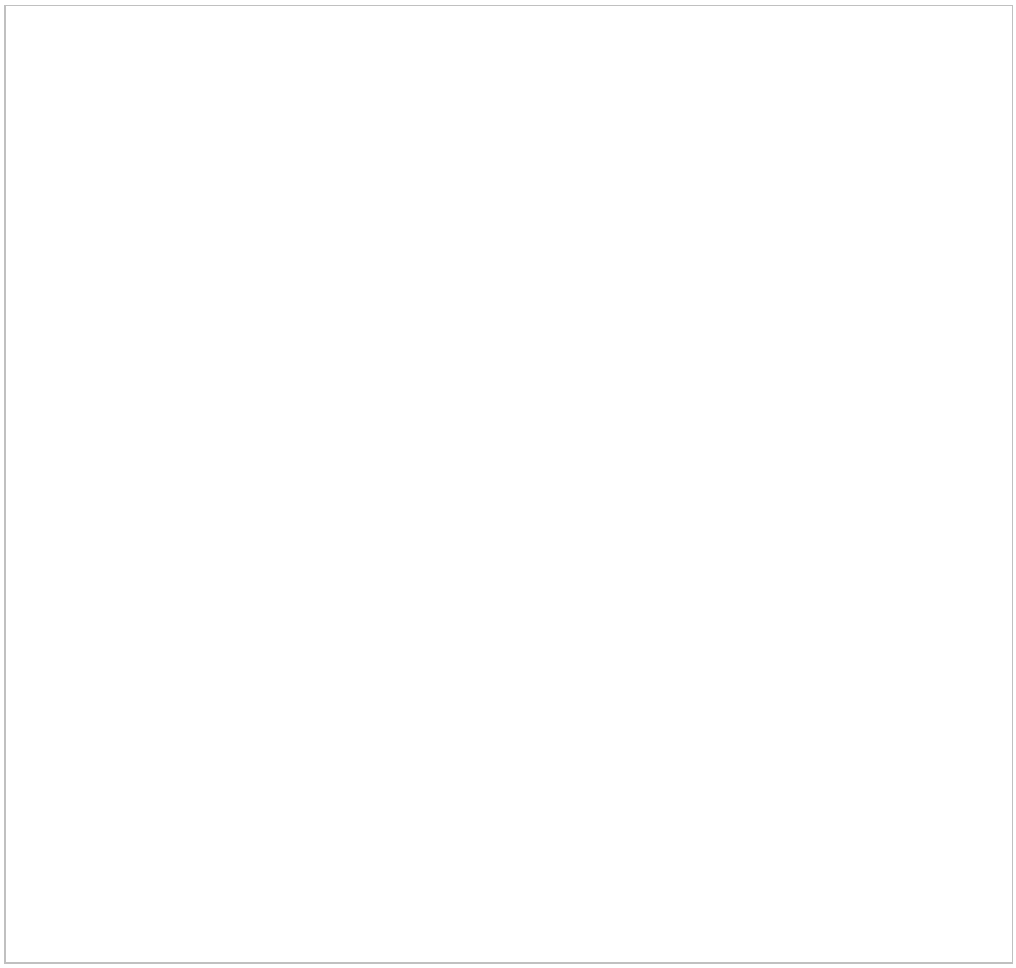


Figure P4.54 Full Alternative Text

2. 4.55 PSPICEMULTISIM The variable dc current source in the circuit in Fig. P4.55 is adjusted so that the power developed by the 4 A current source is zero. You want to find the value of idc.

    1. a) Would you use the node-voltage or mesh-current method to find idc? Explain your choice.

    2. b) Use the method selected in (a) to find idc.

Figure P4.55

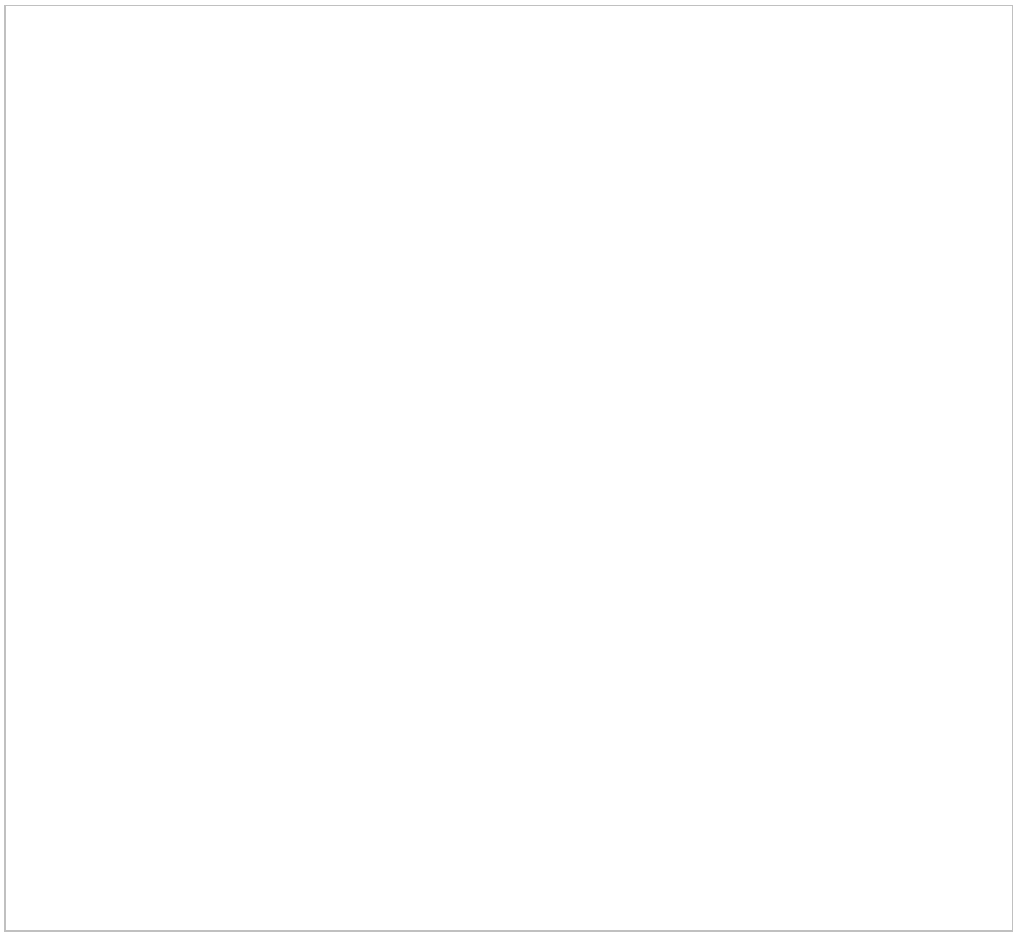


Figure P4.55 Full Alternative Text

3. 4.56 PSPICEMULTISIM Assume you have been asked to find the power dissipated in the horizontal 1 kΩ resistor in the circuit in Fig. P4.56.

    1. a) Which method of circuit analysis would you recommend? Explain why.

    2. b) Use your recommended method of analysis to find the power dissipated in the horizontal 1 kΩ resistor.

    3. c) Would you change your recommendation if the problem had been to find the power developed by the 10 mA current source? Explain.

    4. d) Find the power delivered by the 10 mA current source.

Figure P4.56

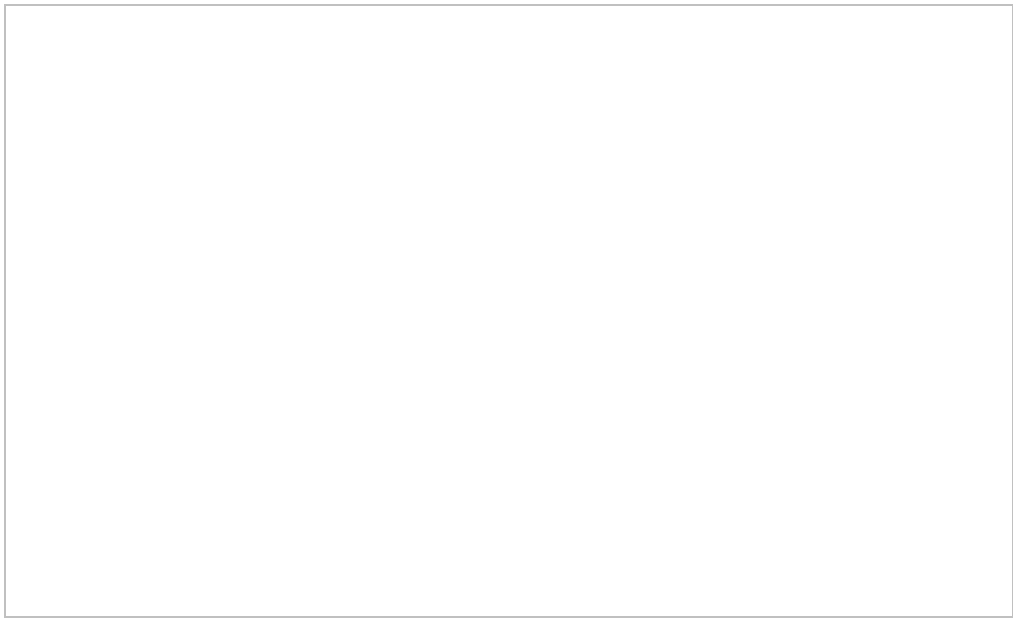


Figure P4.56 Full Alternative Text

4. 4.57 PSPICEMULTISIM A 4 kΩ resistor is placed in parallel with the 10 mA current source in the circuit in Fig. P4.56. Assume you have been asked to calculate the power developed by the current source.

    1. a) Which method of circuit analysis would you recommend? Explain why.

    2. b) Find the power developed by the current source.

5. 4.58 PSPICEMULTISIM

    1. a) Would you use the node-voltage or mesh- current method to find the power absorbed by the 20 V source in the circuit in Fig. P4.58? Explain your choice.

    2. b) Use the method you selected in (a) to find the power.

Figure P4.58

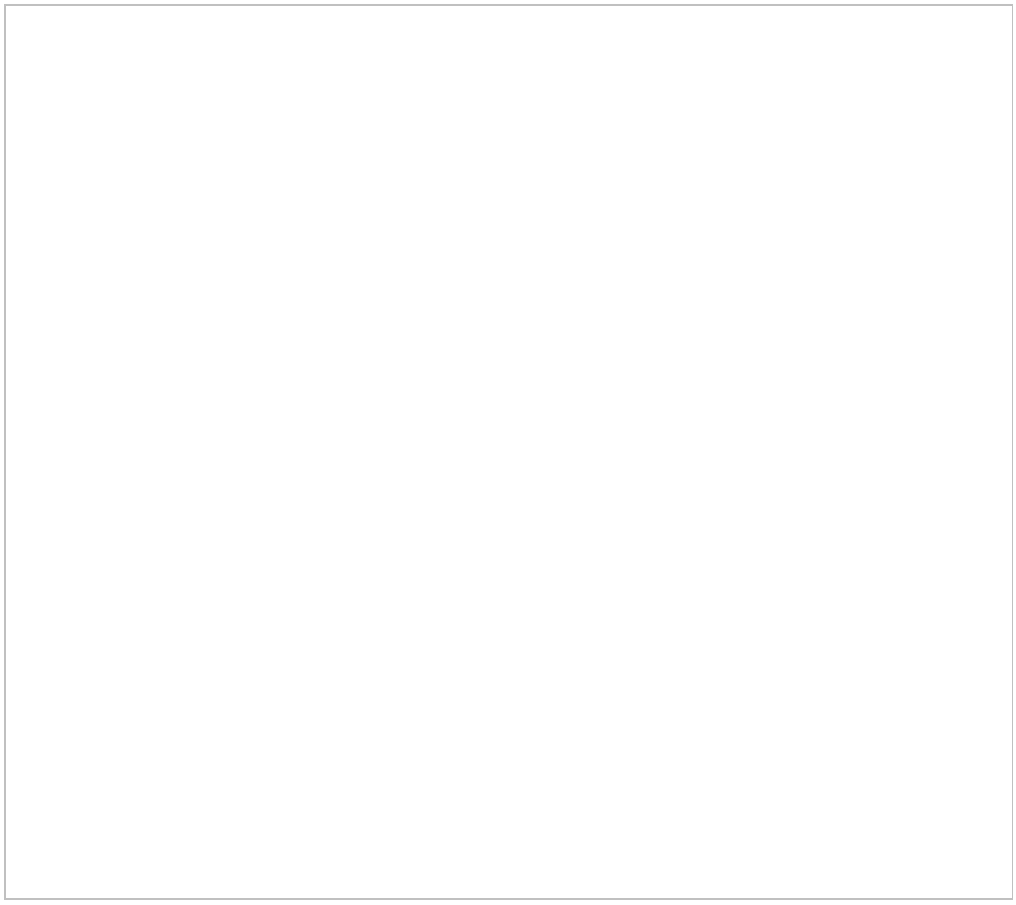


Figure P4.58 Full Alternative Text

Section 4.9

1. 4.59 PSPICEMULTISIM

    1. a) Use source transformations to find the current io in the circuit in Fig. P4.59.

    2. b) Verify your solution by using the node-voltage method to find io.

Figure P4.59

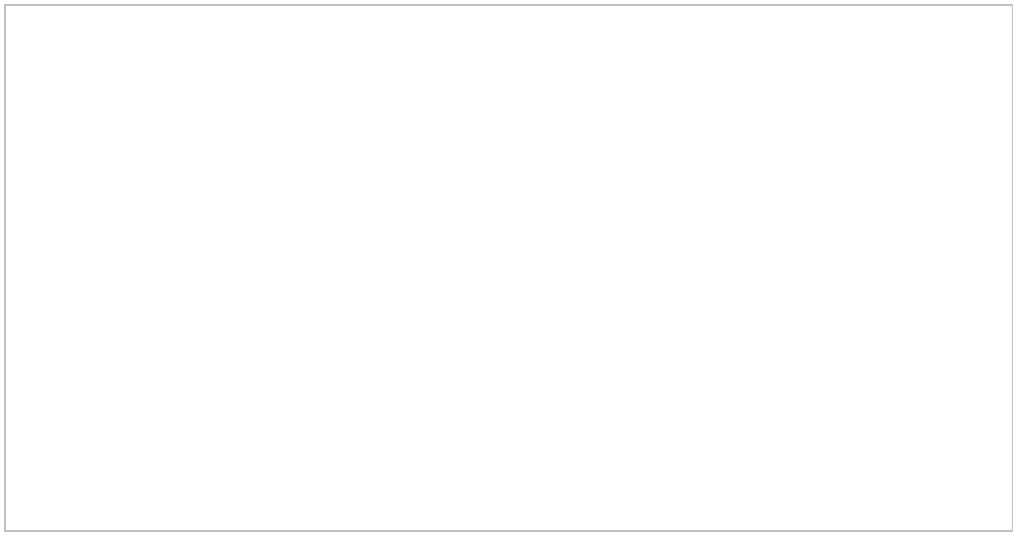


Figure P4.59 Full Alternative Text

2. 4.60 PSPICEMULTISIM

    1. a) Find the current io in the circuit in Fig. P4.60 by making a succession of appropriate source transformations.

    2. b) Using the result obtained in (a), work back through the circuit to find the power developed by the 120 V source.

Figure P4.60

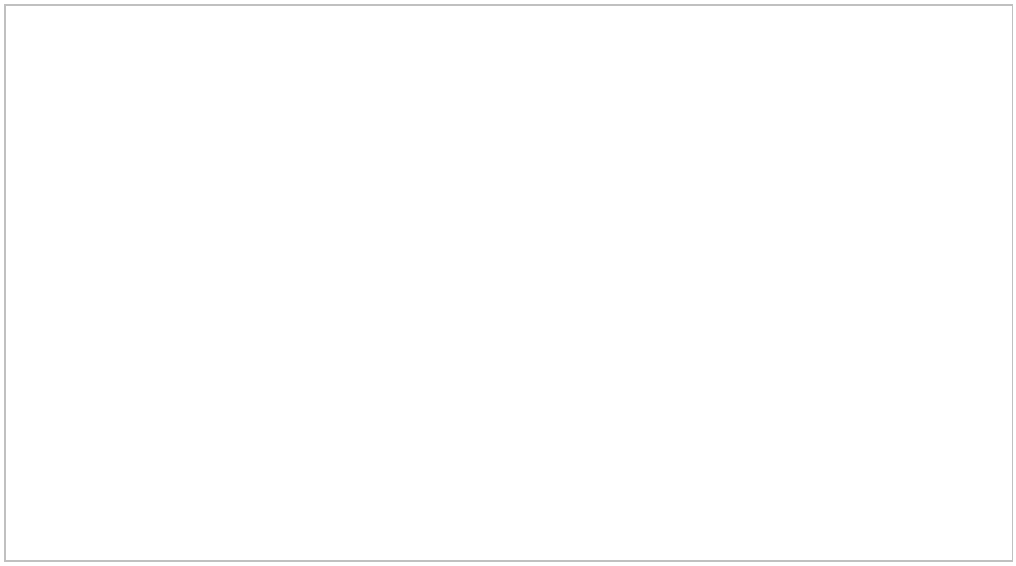


Figure P4.60 Full Alternative Text

3. 4.61

    1. a) Make a series of source transformations to find the voltage v0 in the circuit in Fig. P4.61.

    2. b) Verify your solution using the mesh-current method.

Figure P4.61

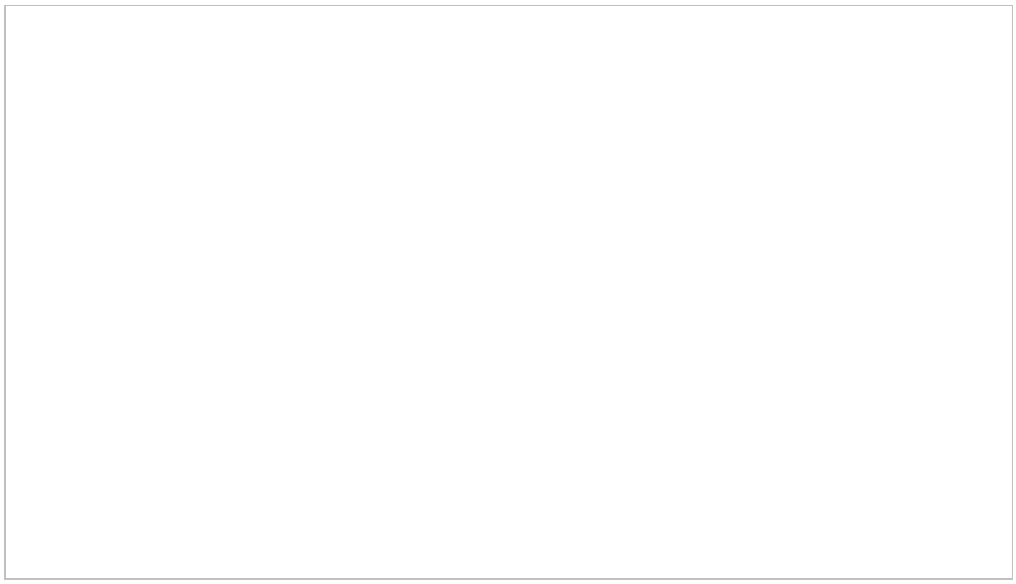


Figure P4.61 Full Alternative Text

4. 4.62 PSPICEMULTISIM

    1. a) Use a series of source transformations to find io in the circuit in Fig. P4.62.

    2. b) Verify your solution by using the mesh-current method to find io.

Figure P4.62

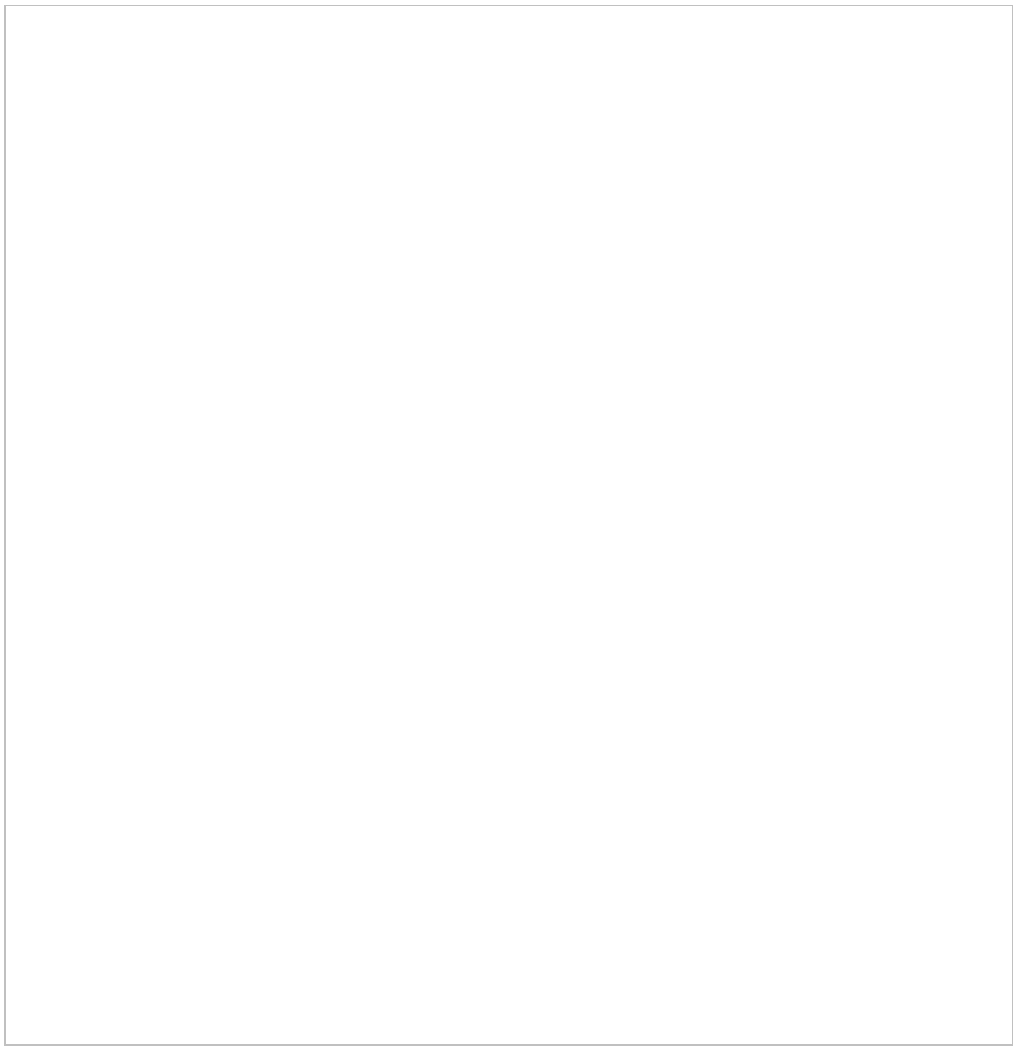


Figure P4.62 Full Alternative Text

5. 4.63 PSPICEMULTISIM

    1. a) Use source transformations to find vo in the circuit in Fig. P4.63.
    2. b) Find the power developed by the 520 V source.
    3. c) Find the power developed by the 1 A current source.
    4. d) Verify that the total power developed equals the total power dissipated.

Figure P4.63

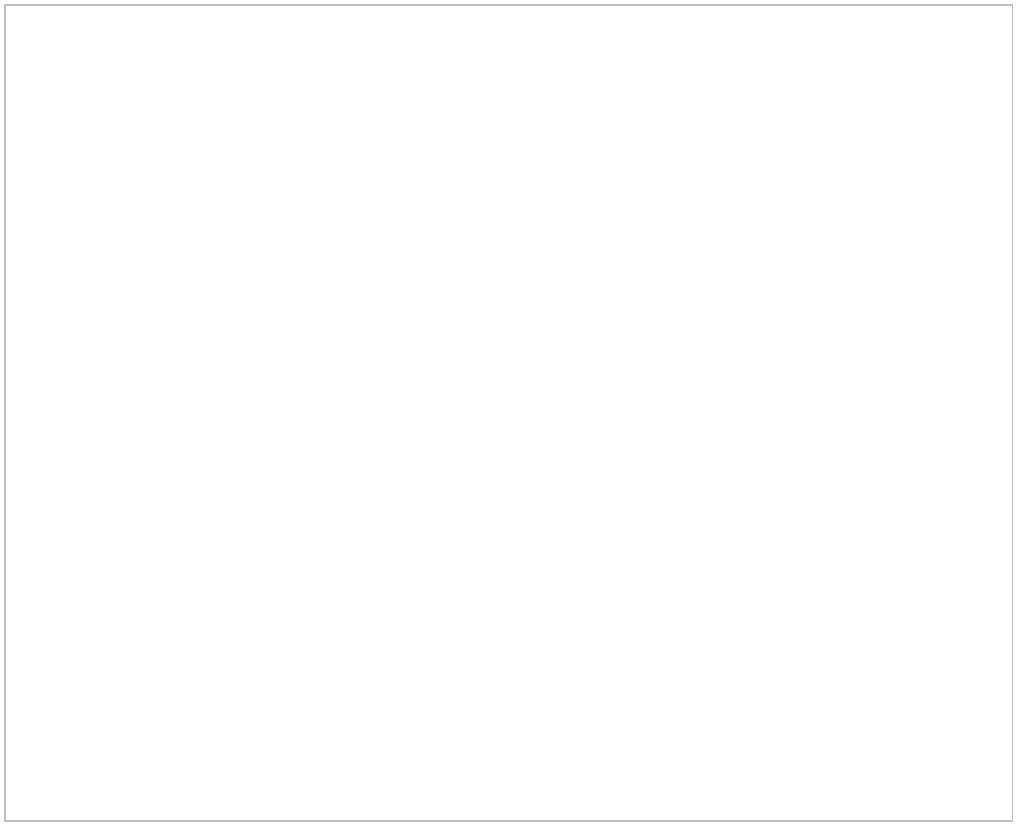


Figure P4.63 Full Alternative Text

Section 4.10

1. 4.64 PSPICEMULTISIM Find the Thévenin equivalent with respect to the terminals a, b for the circuit in Fig. P4.64.

Figure P4.64

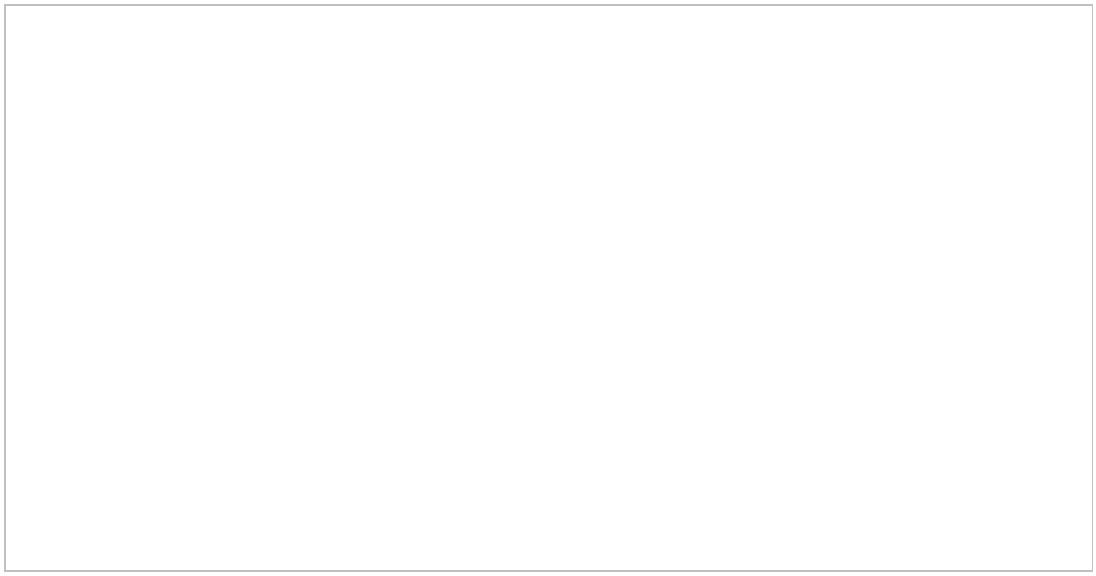


Figure P4.64 Full Alternative Text

2. 4.65 Find the Norton equivalent with respect to the terminals a, b for the circuit in Fig. P4.65.

Figure P4.65

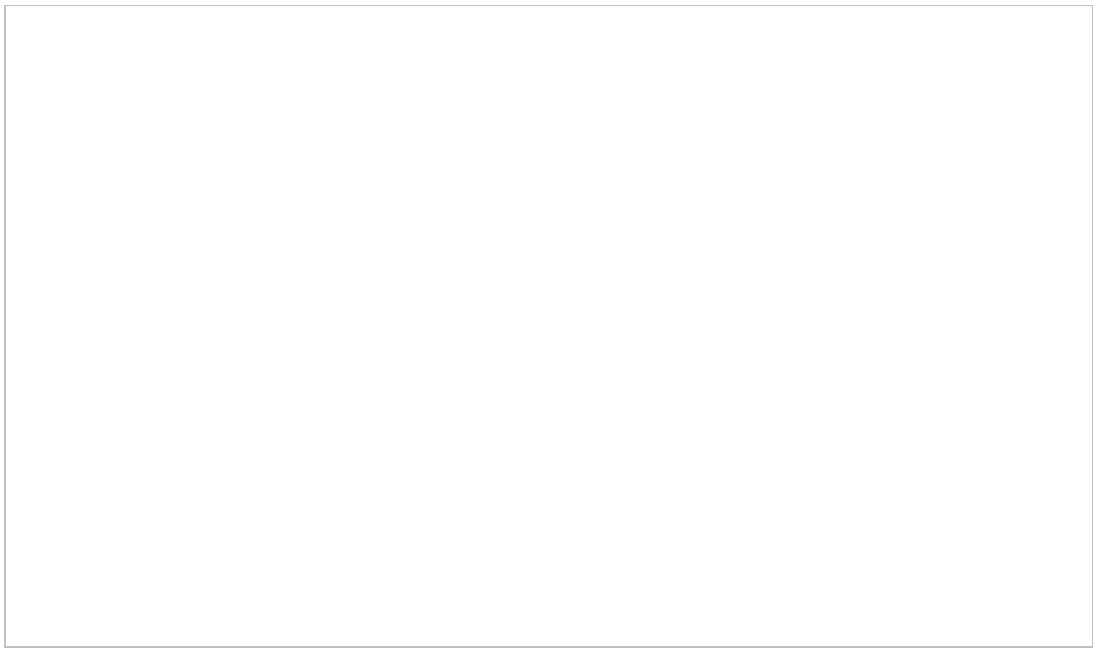


Figure P4.65 Full Alternative Text

3. 4.66 PSPICEMULTISIM Find the Norton equivalent with respect to the terminals a, b for the circuit in Fig. P4.66.

Figure P4.66

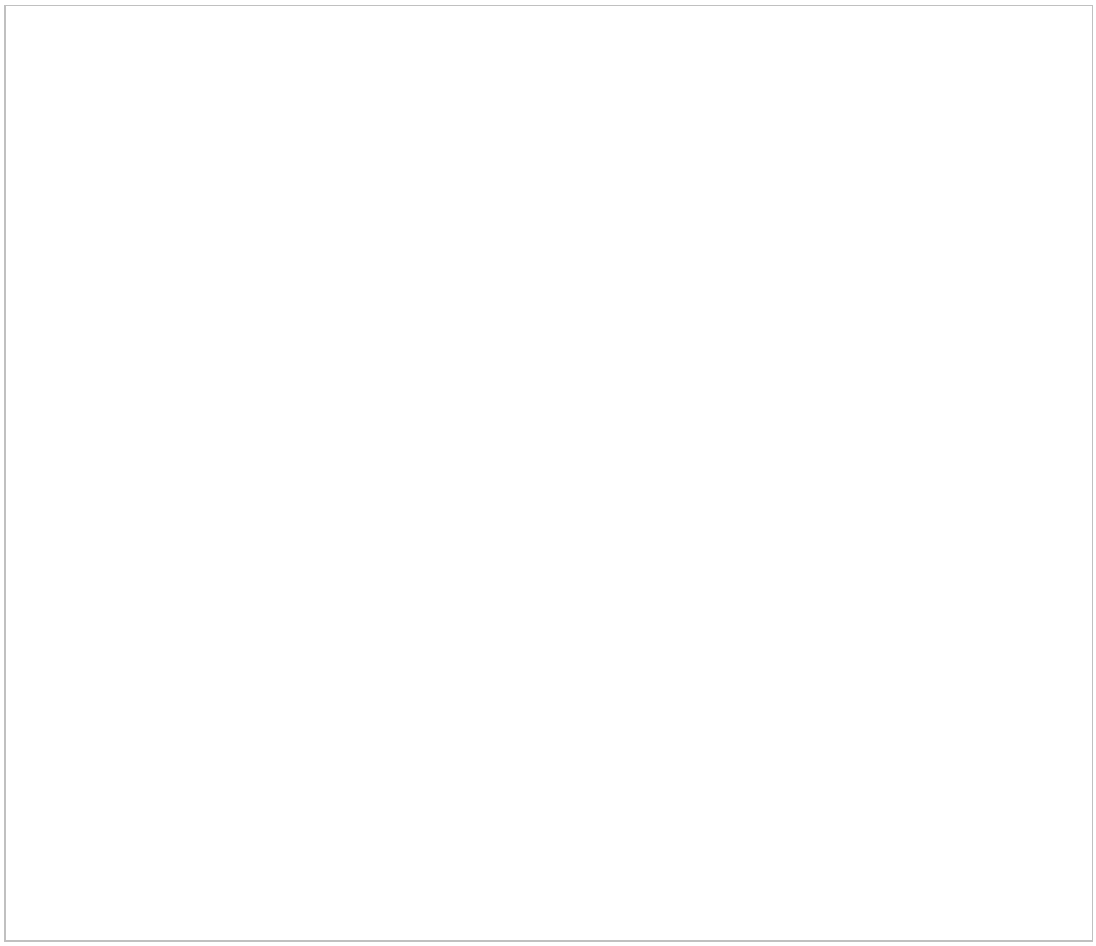


Figure P4.66 Full Alternative Text

4. 4.67 PSPICEMULTISIM Find the Thévenin equivalent with respect to the terminals a, b for the circuit in Fig. P4.67.

Figure P4.67

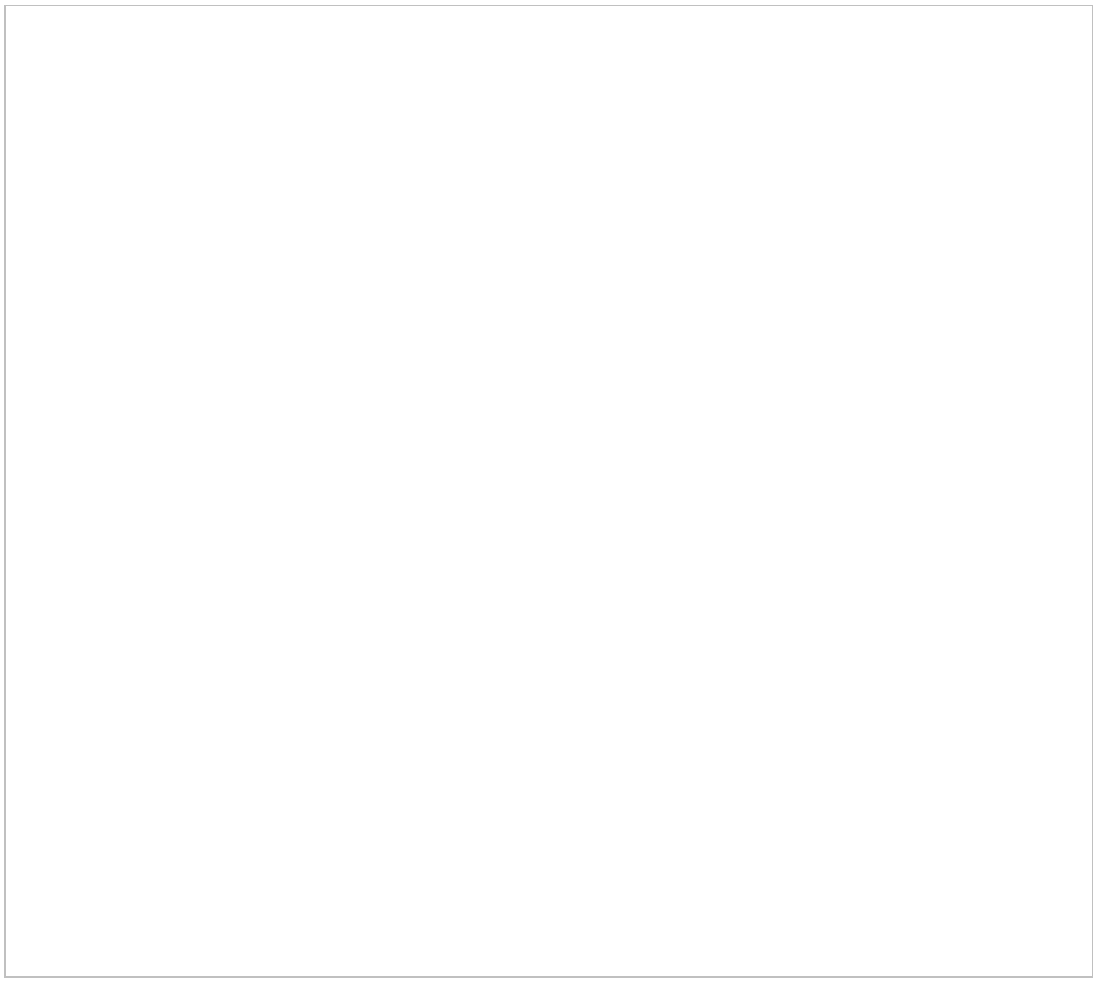


Figure P4.67 Full Alternative Text

5. 4.68 PSPICEMULTISIM Find the Norton equivalent with respect to the terminals a, b in the circuit in Fig. P4.68.

Figure P4.68

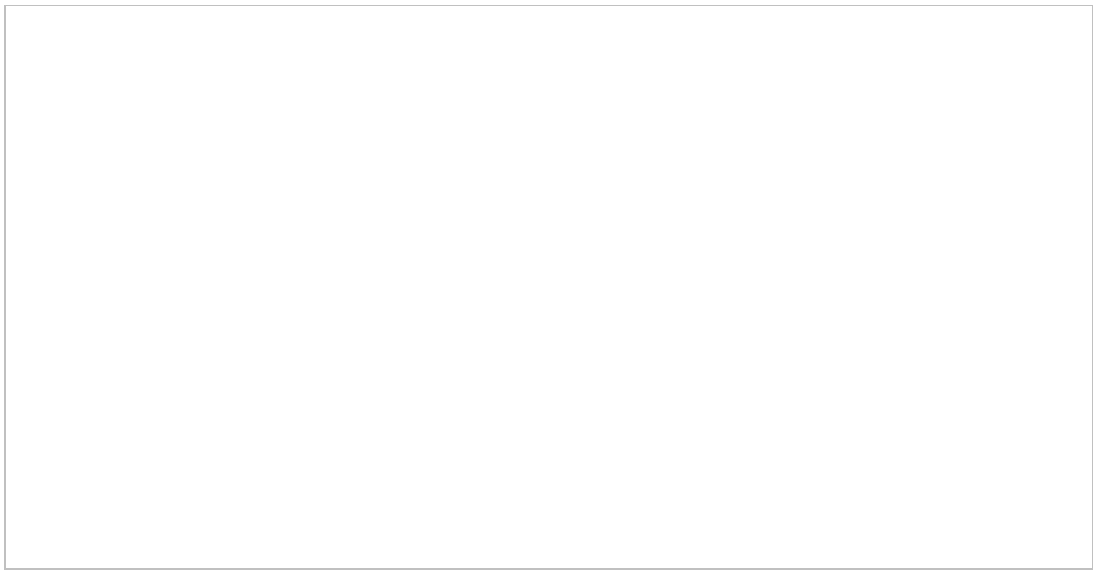


Figure P4.68 Full Alternative Text

6. 4.69 PSPICEMULTISIM Determine io and vo in the circuit shown in Fig. P4.69 when Ro is a resistor from Appendix H such that 100 Ω≤Ro<200 Ω.

Figure P4.69

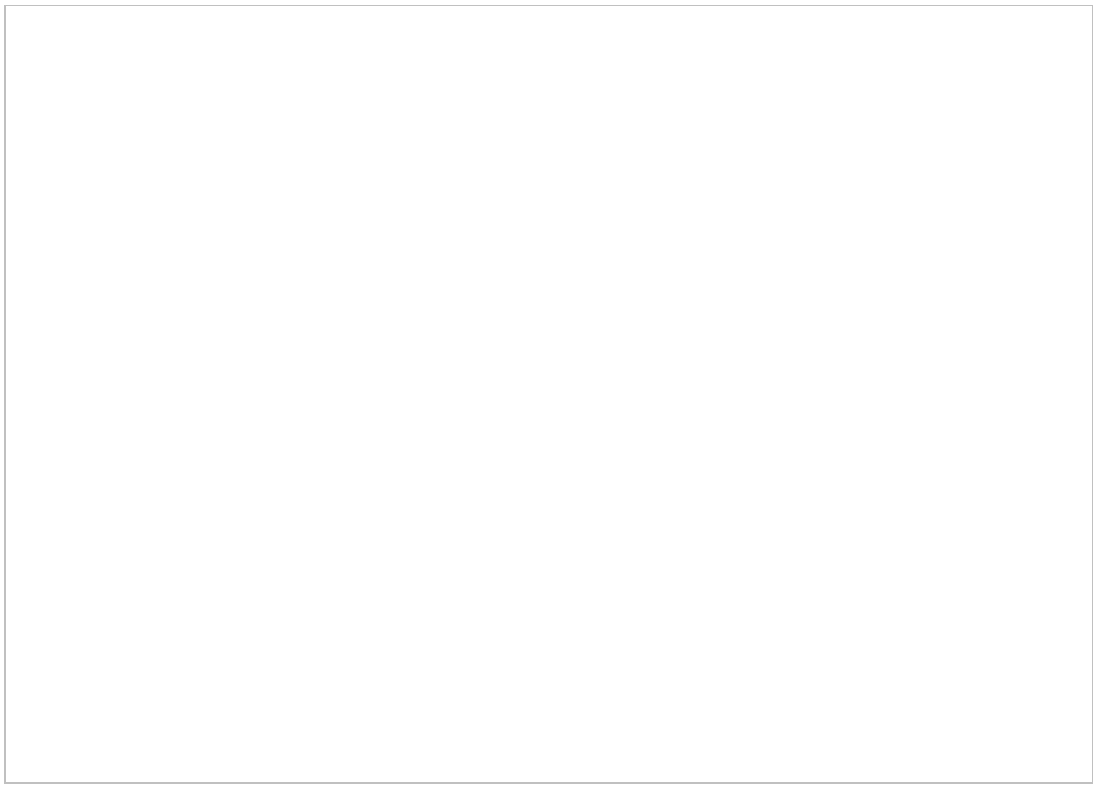


Figure P4.69 Full Alternative Text

7. 4.70 An automobile battery, when connected to a car radio, provides 12.72 V to the radio. When connected to a set of headlights, it provides 12 V to the headlights. Assume the radio can be modeled as a 6.36 Ω resistor and the headlights can be modeled as a 0.6 Ω resistor. What are the Thévenin and Norton equivalents for the battery?

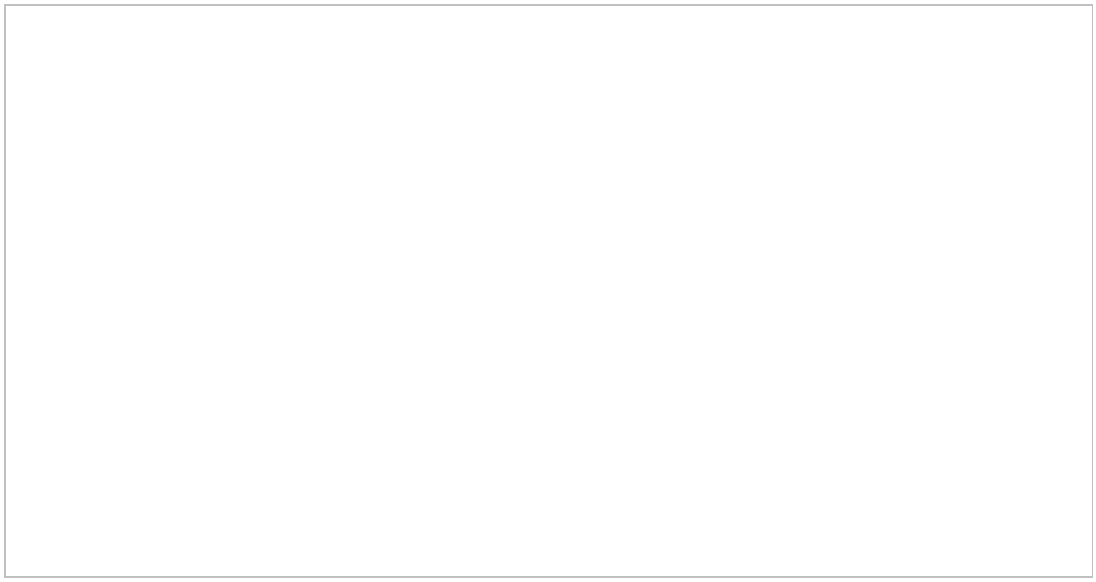
8. 4.71 A Thévenin equivalent can also be determined from measurements made at the pair of terminals of interest. Assume the following measurements were made at the terminals a,b in the circuit in Fig. P4.71.

   When a 20 Ω resistor is connected to the terminals a, b, the voltage vab is measured and found to be 100 V.

   When a 50 Ω resistor is connected to the terminals a, b, the voltage is measured and found to be 200 V.

   Find the Thévenin equivalent of the network with respect to the terminals a, b.

Figure P4.71



9. 4.72 PSPICEMULTISIM A voltmeter with a resistance of 85.5 kΩ is used to measure the voltage vab in the circuit in Fig. P4.72.

    1. a) What is the voltmeter reading?

    2. b) What is the percentage of error in the voltmeter reading if the percentage of error is defined as [(measured−actual)/actual]×100?

Figure P4.72

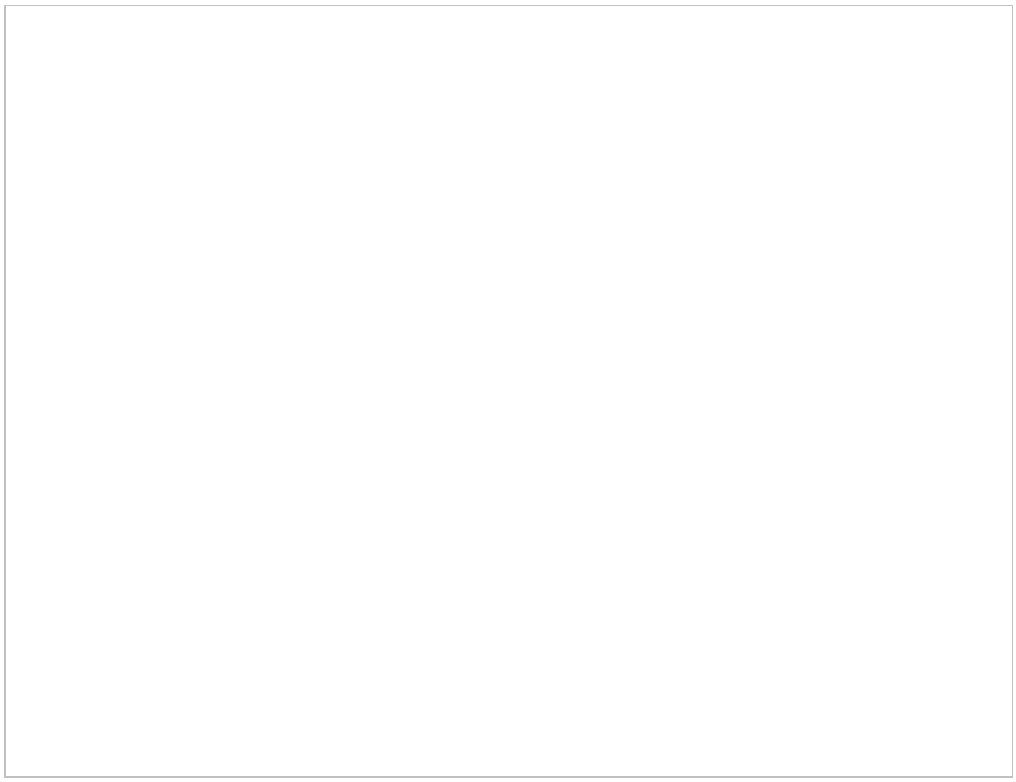


Figure P4.72 Full Alternative Text

10. 4.73 PSPICEMULTISIM The Wheatstone bridge in the circuit shown in Fig. P4.73 is balanced when R3 equals 500 Ω. If the galvanometer has a resistance of 50 Ω, how much current will the galvanometer detect, when the bridge is unbalanced by setting R3 to 501 Ω? (*Hint:* Find the Thévenin equivalent with respect to the galvanometer terminals when R3=501 Ω. Note that once we have found this Thévenin equivalent, it is easy to find the amount of unbalanced current in the galvanometer branch for different galvanometer movements.)

Figure P4.73

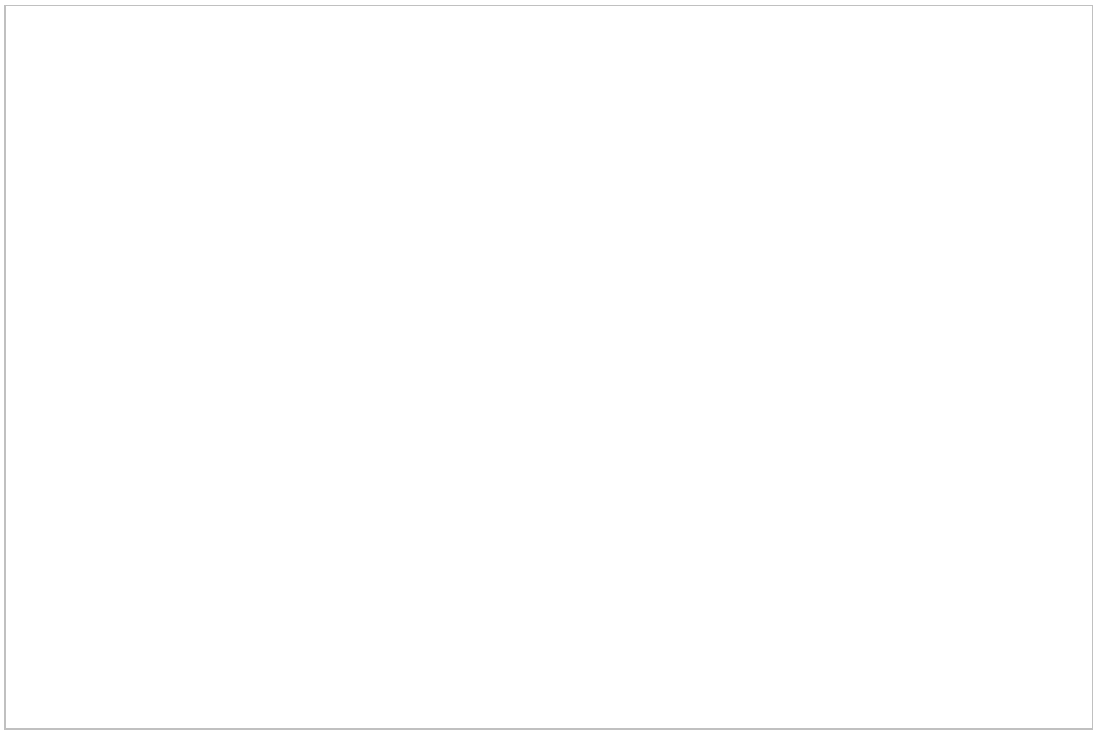


Figure P4.73 Full Alternative Text

11. 4.74 PSPICEMULTISIM Determine the Thévenin equivalent with respect to the terminals a, b for the circuit shown in Fig. P4.74.

Figure P4.74

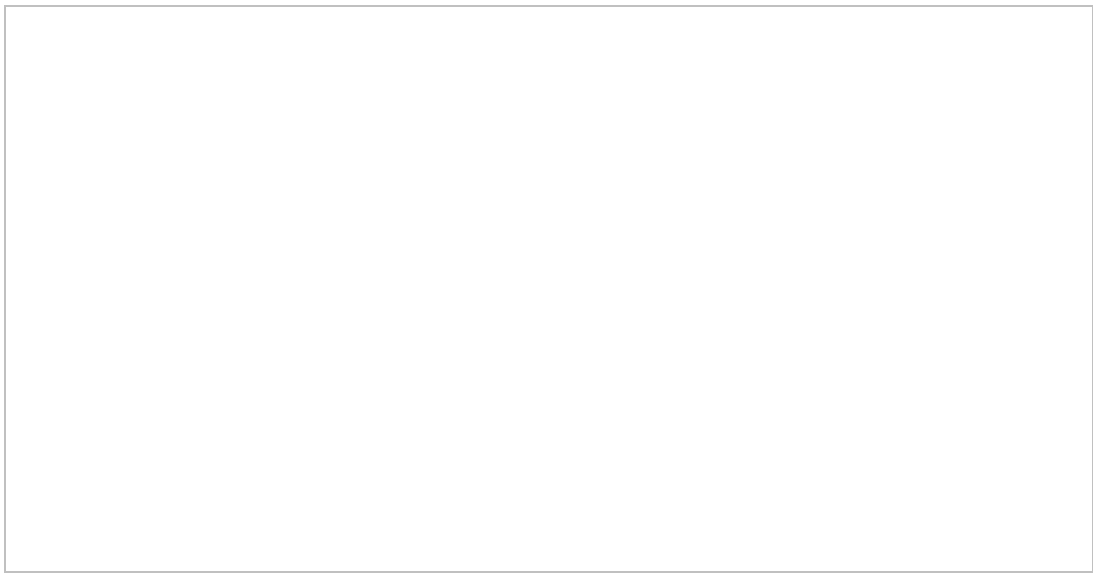


Figure P4.74 Full Alternative Text

12. 4.75 PSPICEMULTISIM Find the Norton equivalent with respect to the terminals a, b for the circuit seen in Fig. P4.75.

Figure P4.75

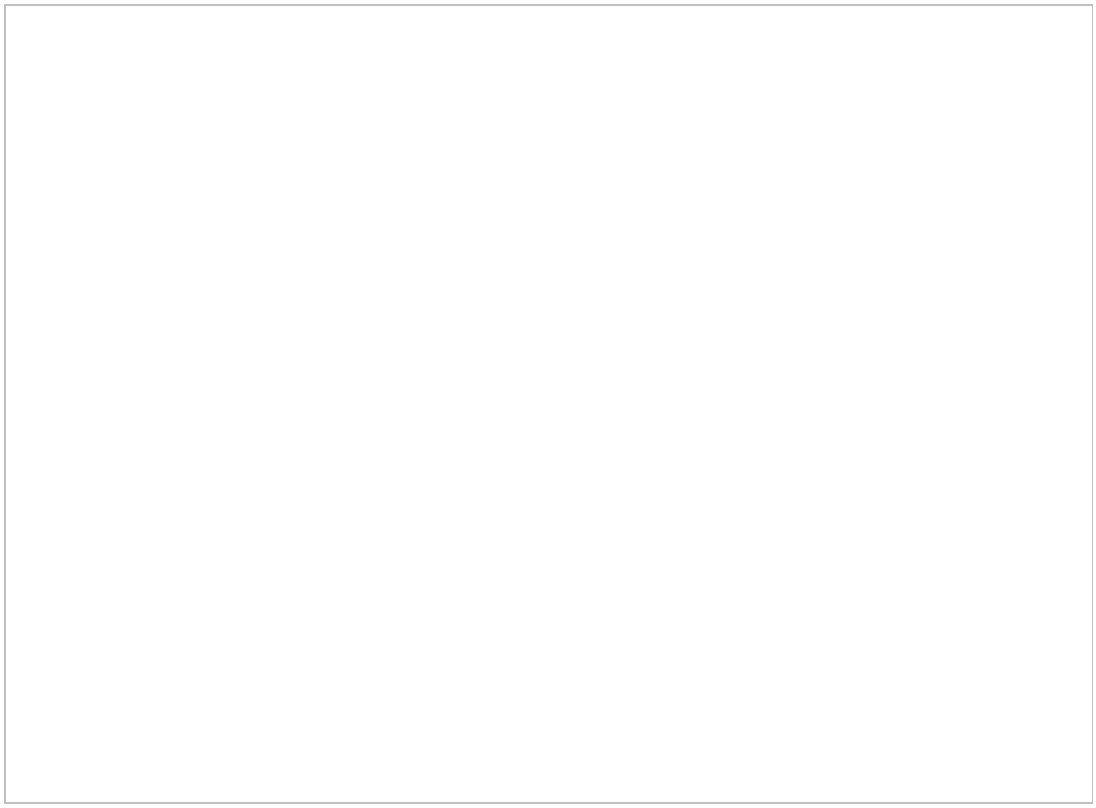


Figure P4.75 Full Alternative Text

13. 4.76 PSPICEMULTISIM When an ammeter is used to measure the current iϕ in the circuit shown in Fig. P4.76, it reads 10 A.

    1. a) What is the resistance of the ammeter?

    2. b) What is the percentage of error in the current measurement?

Figure P4.76

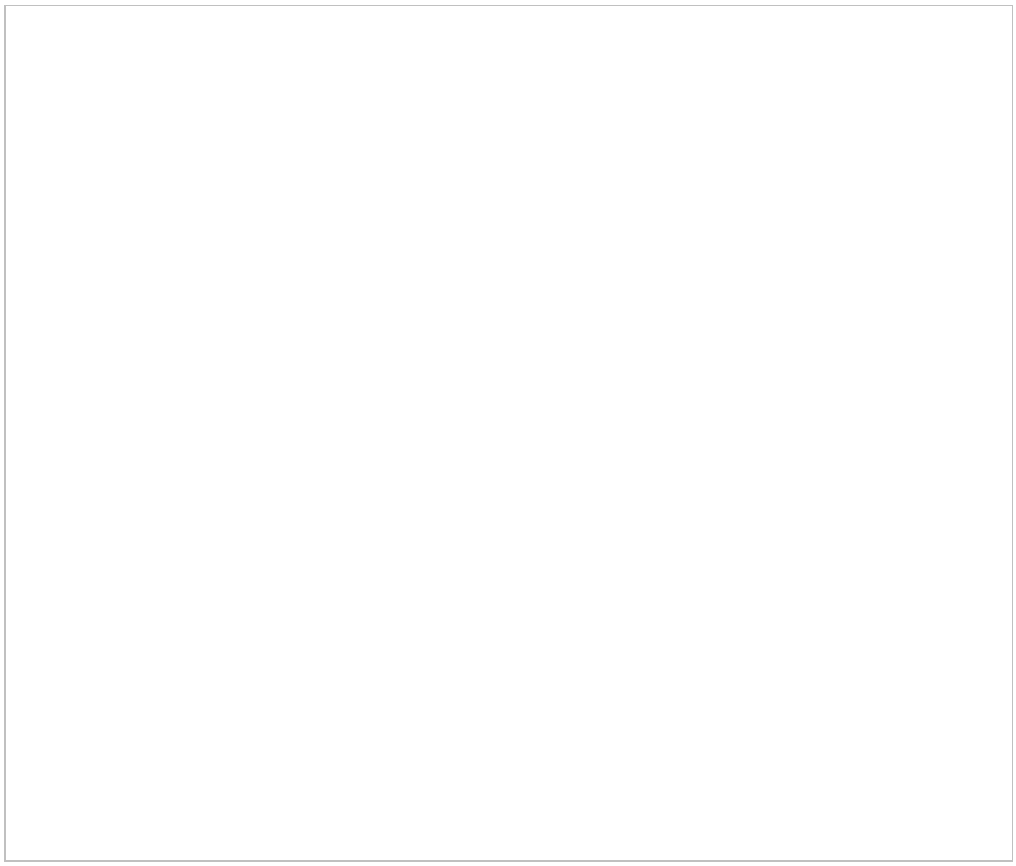


Figure P4.76 Full Alternative Text

Section 4.11

1. 4.77

    1. a) Find the Thévenin equivalent resistance with respect to the terminals a, b in the circuit in Fig. P4.64 without finding either the open circuit voltage or the short circuit current.

    2. b) Find the Norton equivalent resistance with respect to the terminals a, b in the circuit in Fig. P4.66 without finding either the open circuit voltage or the short circuit current.

2. 4.78 PSPICEMULTISIM

    1. a) Find the Thévenin equivalent with respect to the terminals a, b for the circuit in Fig. P4.78 by finding the open-circuit voltage and the short-circuit current.

    2. b) Solve for the Thévenin resistance by removing the independent sources. Compare your result to the Thévenin resistance found in (a).

Figure P4.78

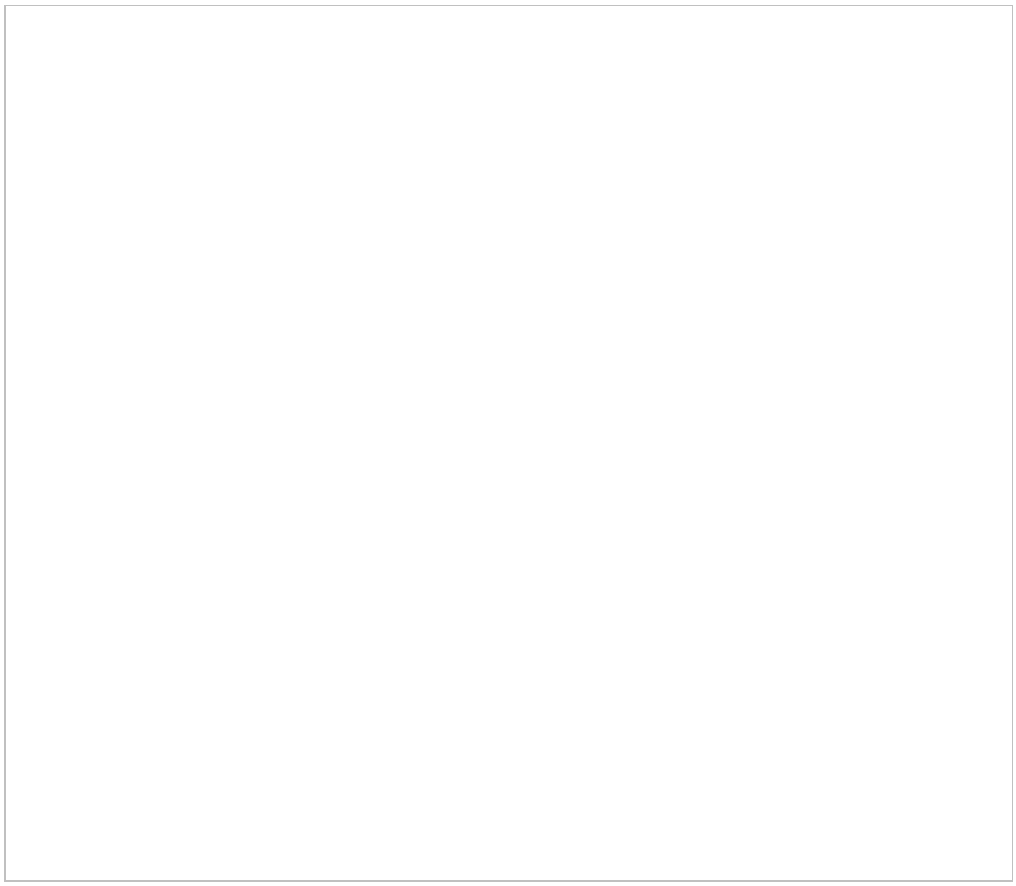


Figure P4.78 Full Alternative Text

3. 4.79 Find the Thévenin equivalent with respect to the terminals a,b in the circuit in Fig. P4.79.

Figure P4.79

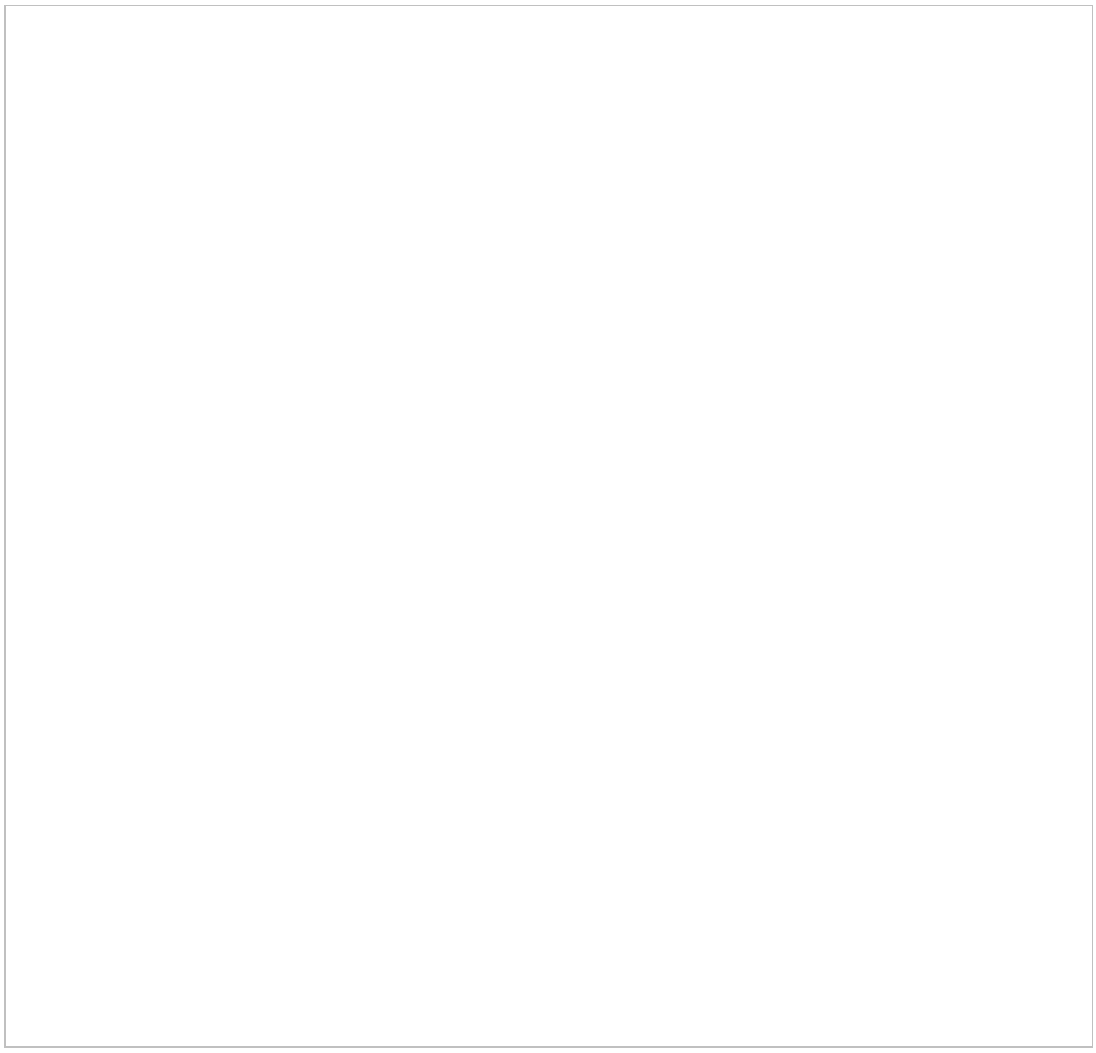


Figure P4.79 Full Alternative Text

4. 4.80 Find the Thévenin equivalent with respect to the terminals a, b in the circuit in Fig. P4.80.

Figure P4.80

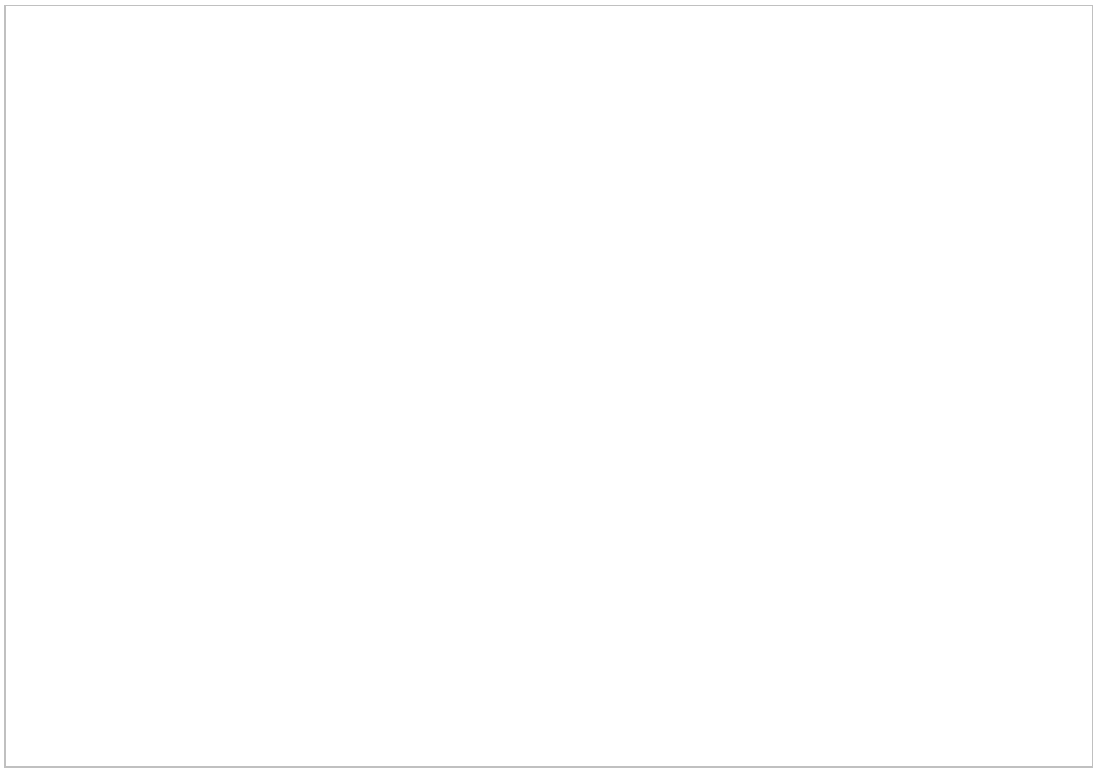


Figure P4.80 Full Alternative Text

5. 4.81 Find the Norton equivalent with respect to the terminals a, b for the circuit seen in Fig. P4.81.

Figure P4.81

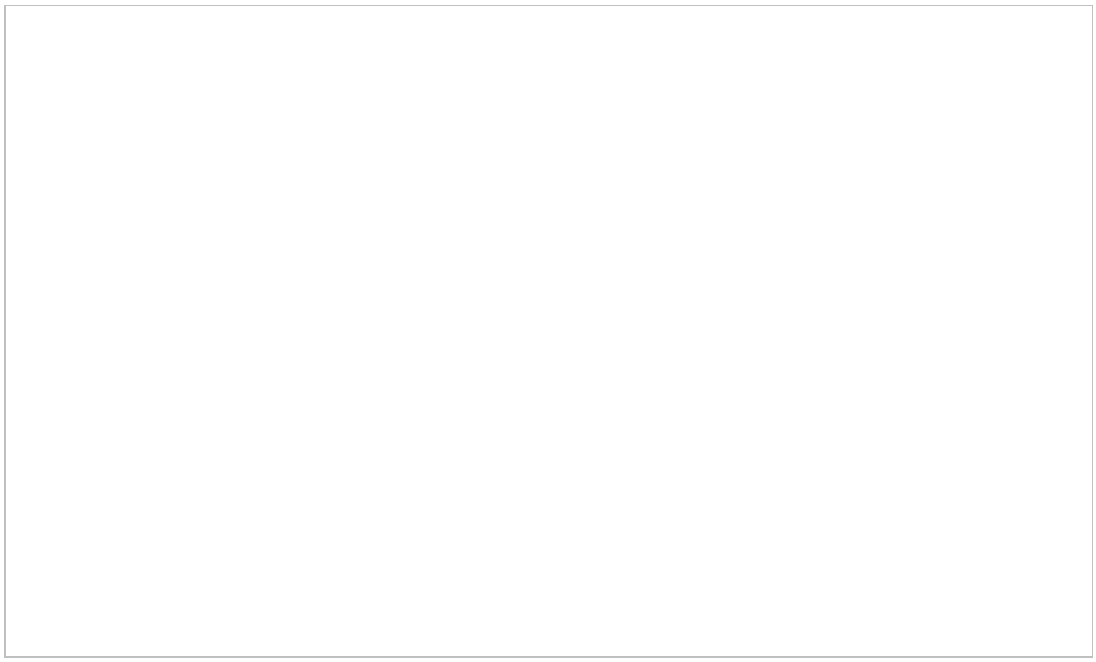


Figure P4.81 Full Alternative Text

Section 4.12

1. 4.82 PSPICEMULTISIM The variable resistor in the circuit in Fig. P4.82 is adjusted for maximum power transfer to Ro.

    1. a) Find the value of Ro.

    2. b) Find the maximum power that can be delivered to Ro.

    3. c) Find a resistor in Appendix H closest to the value in part (a). How much power is delivered to this resistor?

Figure P4.82

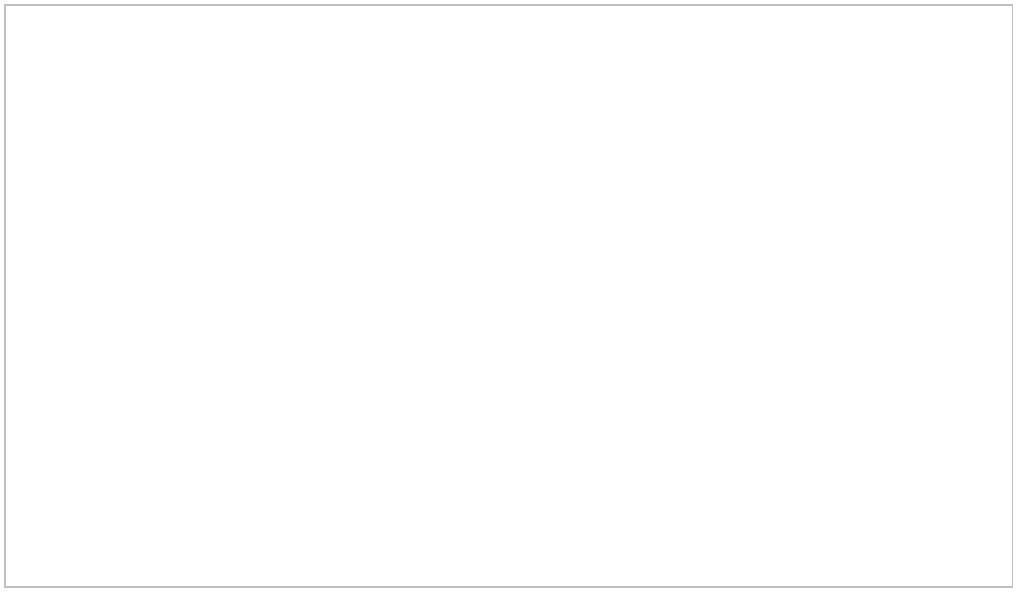


Figure P4.82 Full Alternative Text

2. 4.83 PSPICEMULTISIM What percentage of the total power developed in the circuit in Fig. P4.82 is delivered to Ro when Ro is set for maximum power transfer?

3. 4.84

    1. a) Calculate the power delivered for each value of Ro used in Problem 4.69.

    2. b) Plot the power delivered to Ro versus the resistance Ro.

    3. c) At what value of Ro is the power delivered to Ro a maximum?

4. 4.85

    1. a) Find the value of the variable resistor Ro in the circuit in Fig. P4.85 that will result in maximum power dissipation in the 6 Ω resistor. (*Hint:* Hasty conclusions could be hazardous to your career.)

    2. b) What is the maximum power that can be delivered to the 6 Ω resistor?

Figure P4.85

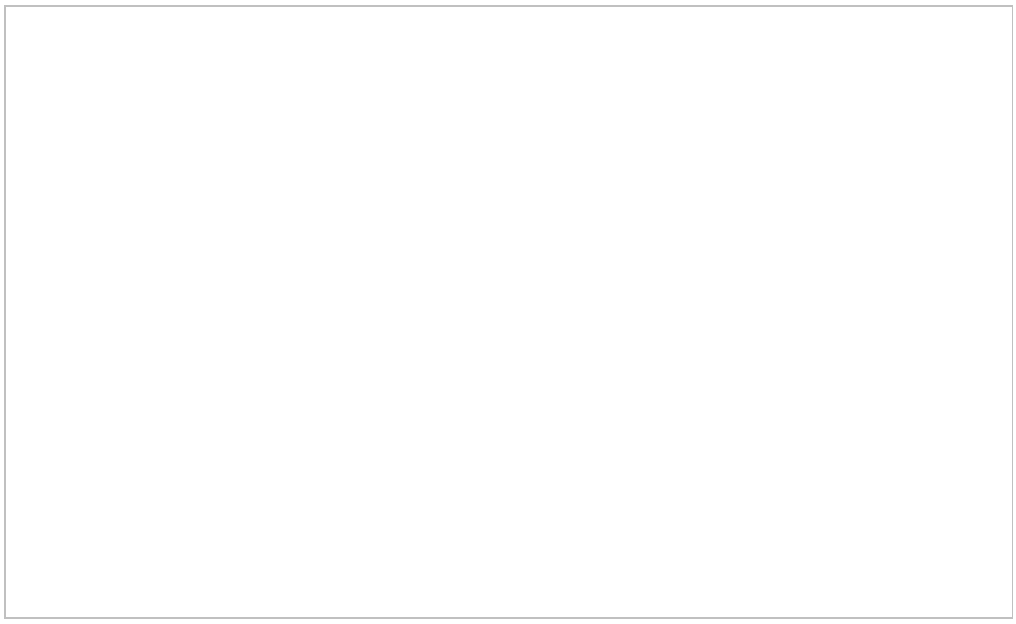


Figure P4.85 Full Alternative Text

5. 4.86 PSPICEMULTISIM A variable resistor Ro is connected across the terminals a, b in the circuit in Fig. P4.75. The variable resistor is adjusted until maximum power is transferred to Ro.

    1. a) Find the value of Ro.

    2. b) Find the maximum power delivered to Ro.

    3. c) Find the percentage of the total power developed in the circuit that is delivered to Ro.

    4. d) Find the resistor from Appendix H closest in value to the Ro. from part (a).

    5. e) Find the percentage of the total power developed in the circuit that is delivered to the resistor in part (d).

6. 4.87 PSPICEMULTISIM The variable resistor (Ro) in the circuit in Fig. P4.87 is adjusted until the power dissipated in the resistor is 250 W. Find the values of Ro that satisfy this condition.

Figure P4.87

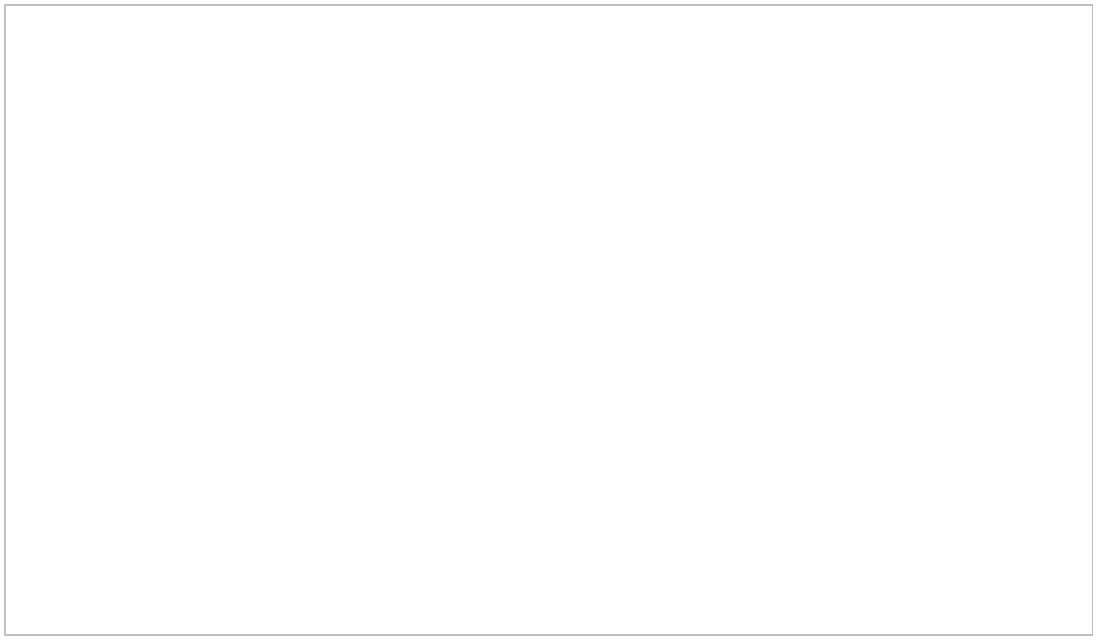


Figure P4.87 Full Alternative Text

7. 4.88 PSPICEMULTISIM The variable resistor (Ro) in the circuit in Fig. P4.88 is adjusted until it absorbs maximum power from the circuit.

    1. a) Find the value of Ro.

    2. b) Find the maximum power.

    3. c) Find the percentage of the total power developed in the circuit that is delivered to Ro.

Figure P4.88

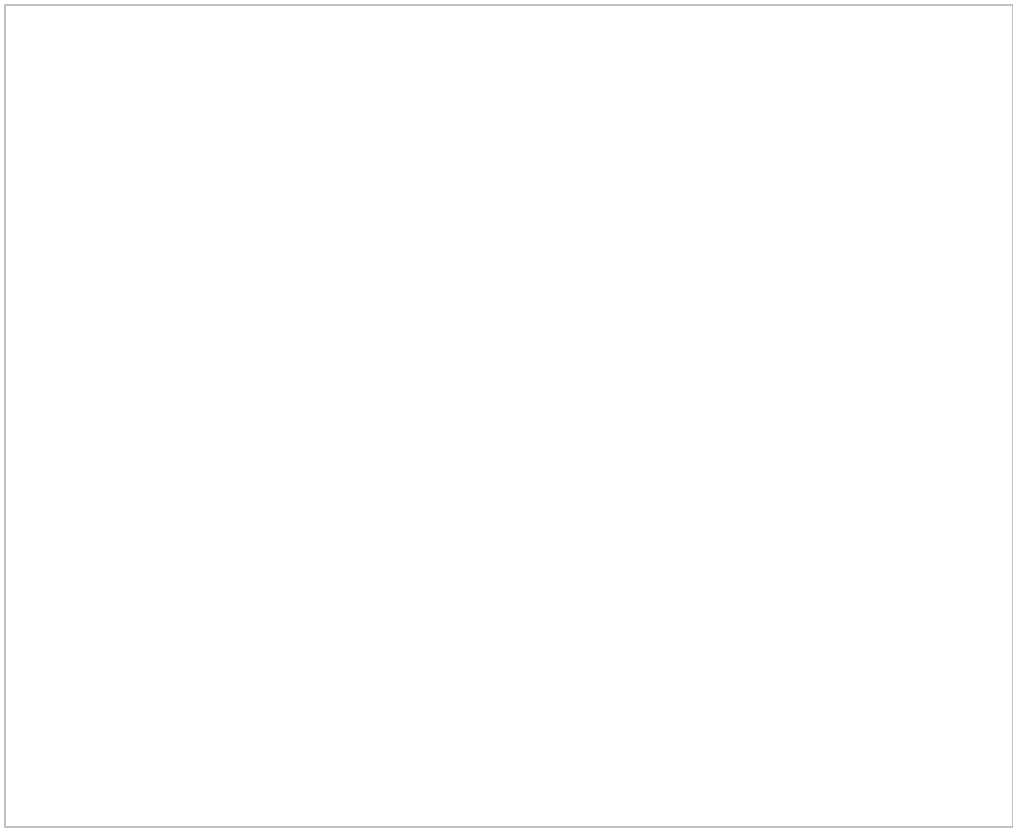


Figure P4.88 Full Alternative Text

8. 4.89 PSPICEMULTISIM The variable resistor (RL) in the circuit in Fig. P4.89 is adjusted for maximum power transfer to RL.

1. a) Find the numerical value of RL.
2. b) Find the maximum power transferred to RL.

Figure P4.89

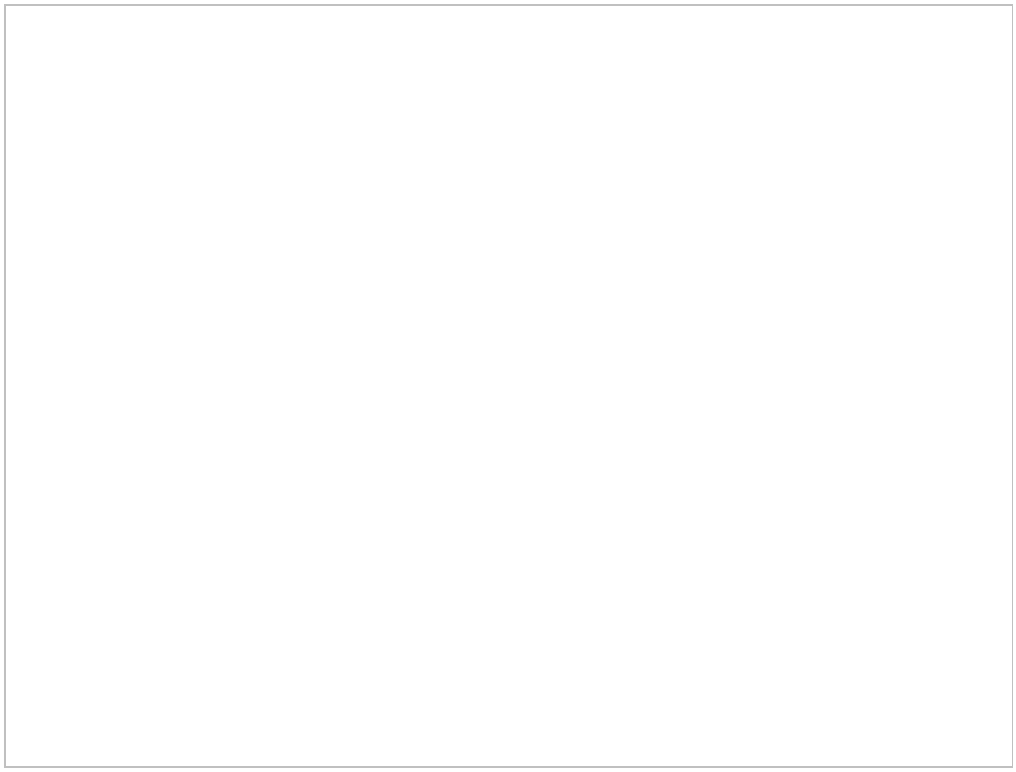


Figure P4.89 Full Alternative Text

9. 4.90 PSPICEMULTISIM The variable resistor (Ro) in the circuit in Fig. P4.90 is adjusted for maximum power transfer to Ro.
    1. a) Find the value of Ro.
    2. b) Find the maximum power that can be delivered to Ro.
    3. c) What percentage of the total power developed in the circuit is delivered to Ro found in part(a)?
    4. d) If Ro is selected from Appendix H, which resistor value will

result in the greatest amount of power delivered to Ro?

Figure P4.90

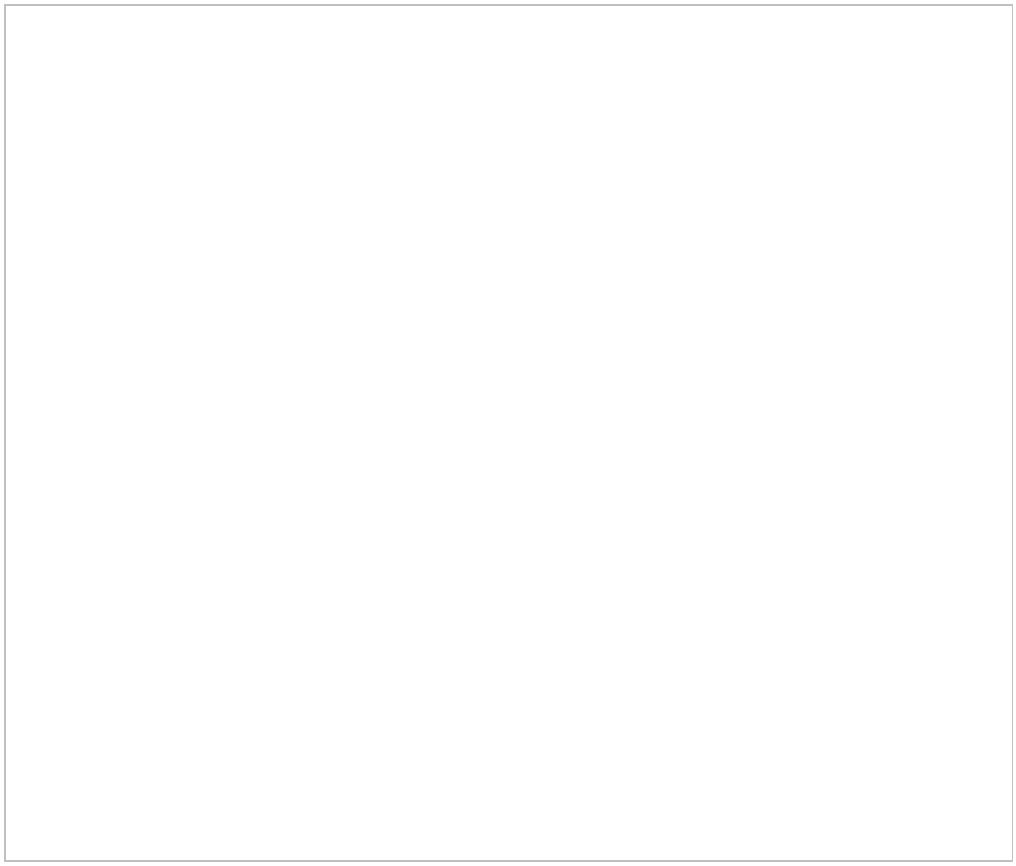


Figure P4.90 Full Alternative Text

10. 4.91 PSPICEMULTISIM The variable resistor in the circuit in Fig. P4.91 is adjusted for maximum power transfer to Ro.

    1. a) Find the numerical value of Ro.

    2. b) Find the maximum power delivered to Ro.

    3. c) How much power does the 180 V source deliver to the circuit when Ro is adjusted to the value found in (a)?

Figure P4.91

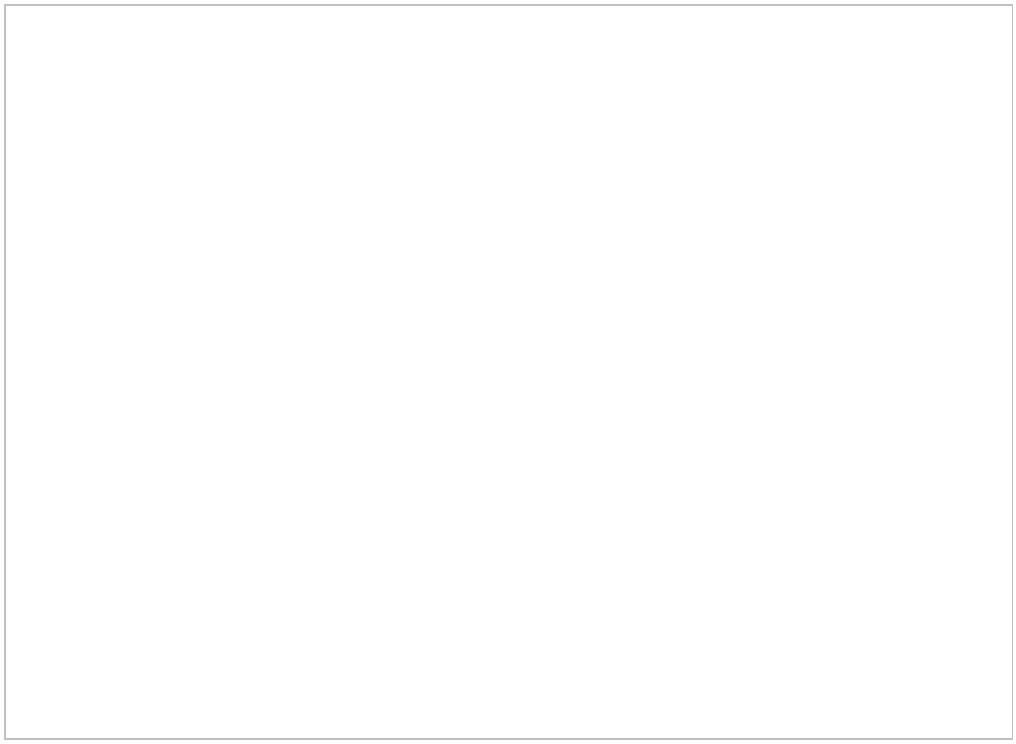


Figure P4.91 Full Alternative Text

11. 4.92 PSPICEMULTISIM

    1. Use the principle of superposition to find the voltage v in the circuit of Fig. P4.92.

    2. Find the power dissipated in the 10 Ω resistor.

Figure P4.92

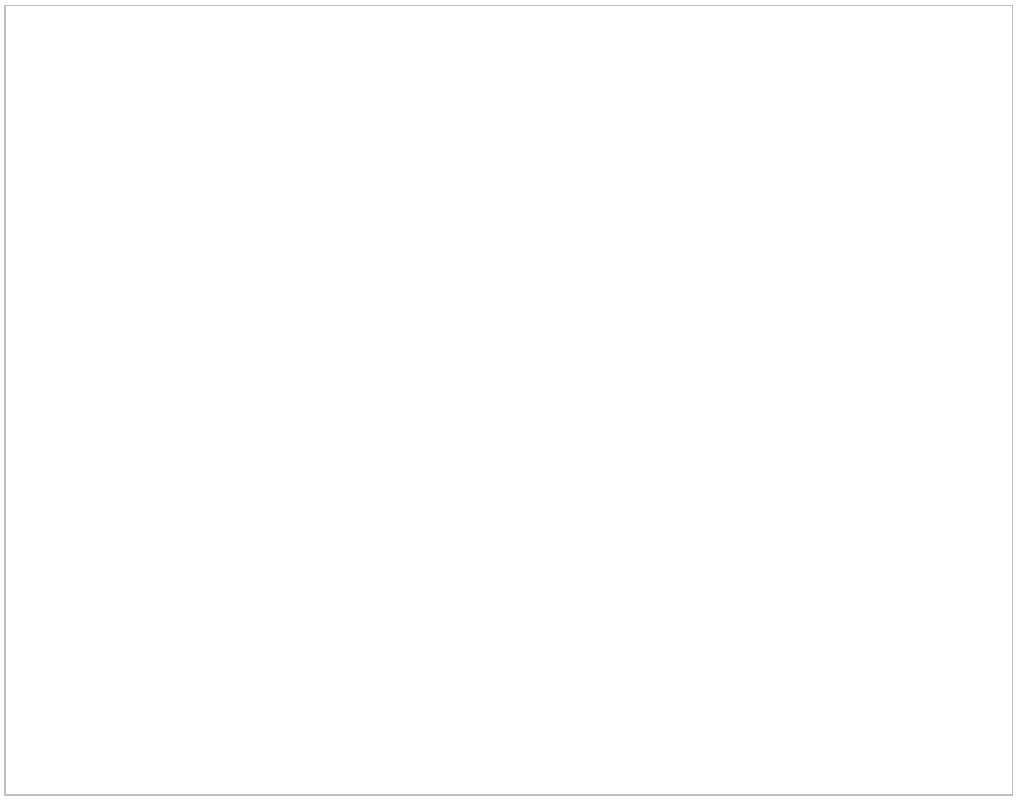


Figure P4.92 Full Alternative Text

12. 4.93 Use superposition to solve for io and vo in the circuit in Fig. P4.93.

Figure P4.93

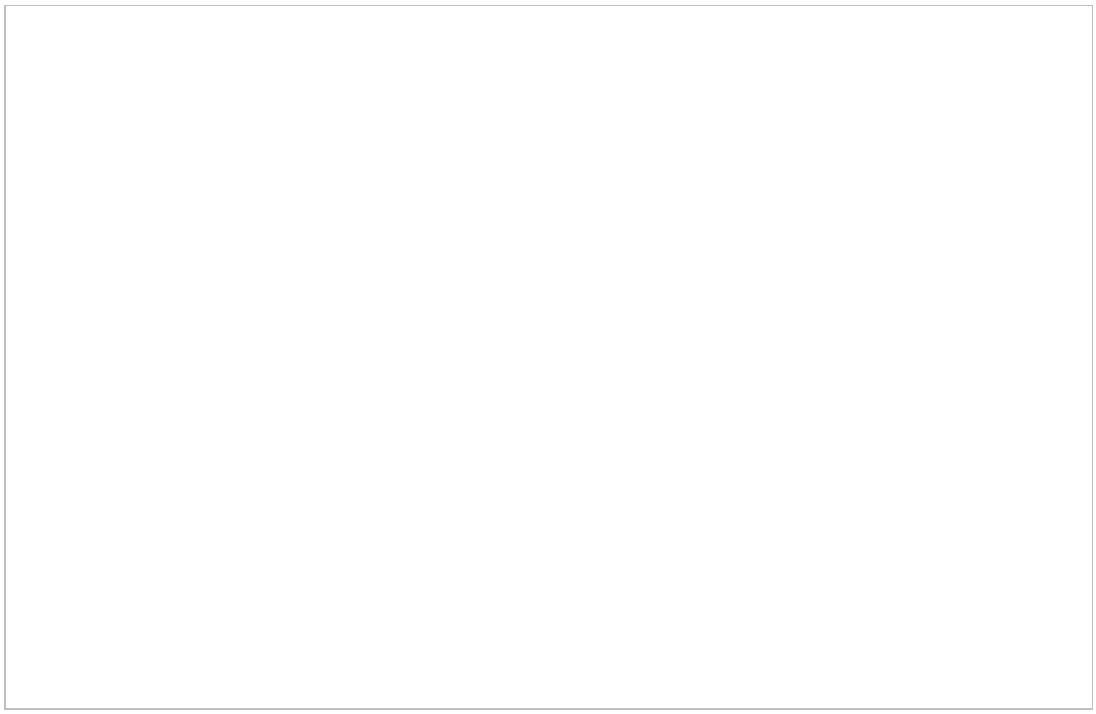


Figure P4.93 Full Alternative Text

13. 4.94 PSPICEMULTISIM Use the principle of superposition to find the voltage vo in the circuit in Fig. P4.94.

Figure P4.94

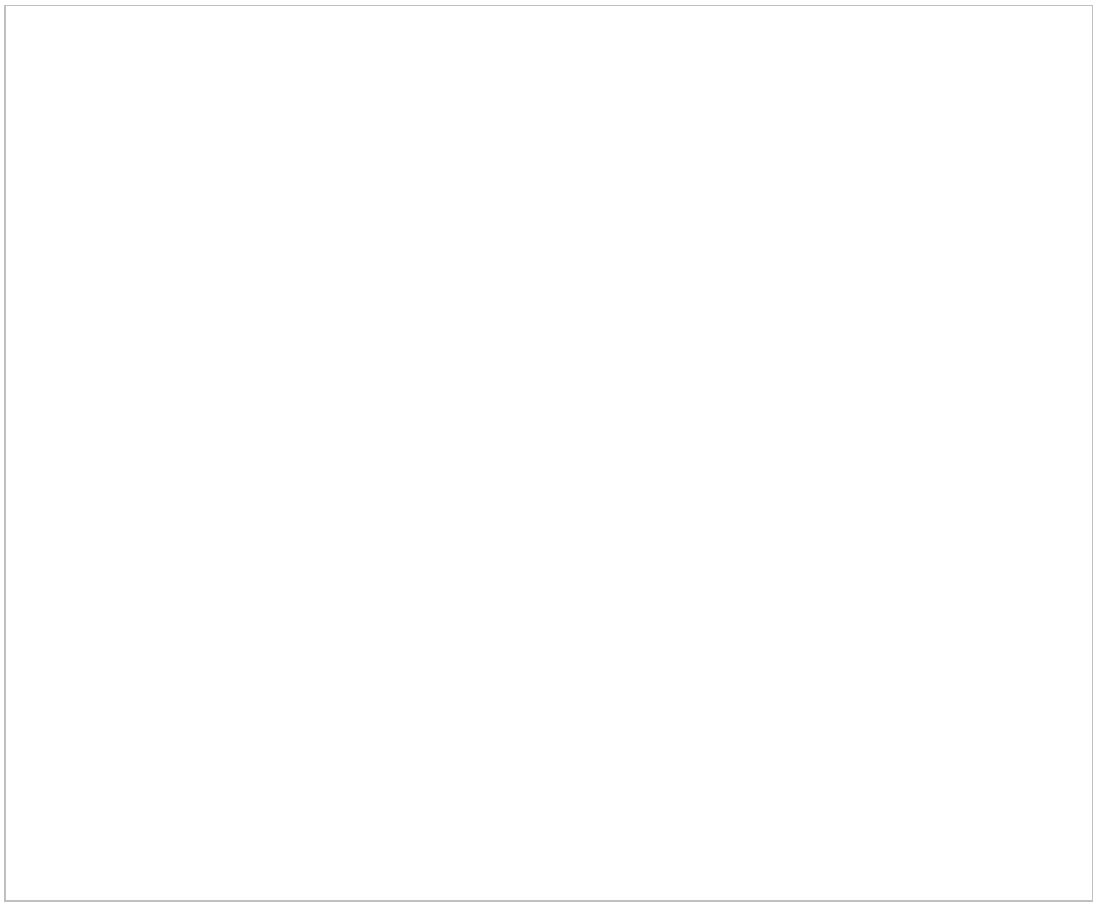


Figure P4.94 Full Alternative Text

Section 4.13

1. 4.95 PSPICEMULTISIM

    1. a) In the circuit in Fig. P4.95, before the 10 mA current source is attached to the terminals a, b, the current io is calculated and found to be 1.5 mA. Use superposition to find the value of io after the current source is attached.

    2. b) Verify your solution by finding io when all three sources are acting simultaneously.

Figure P4.95

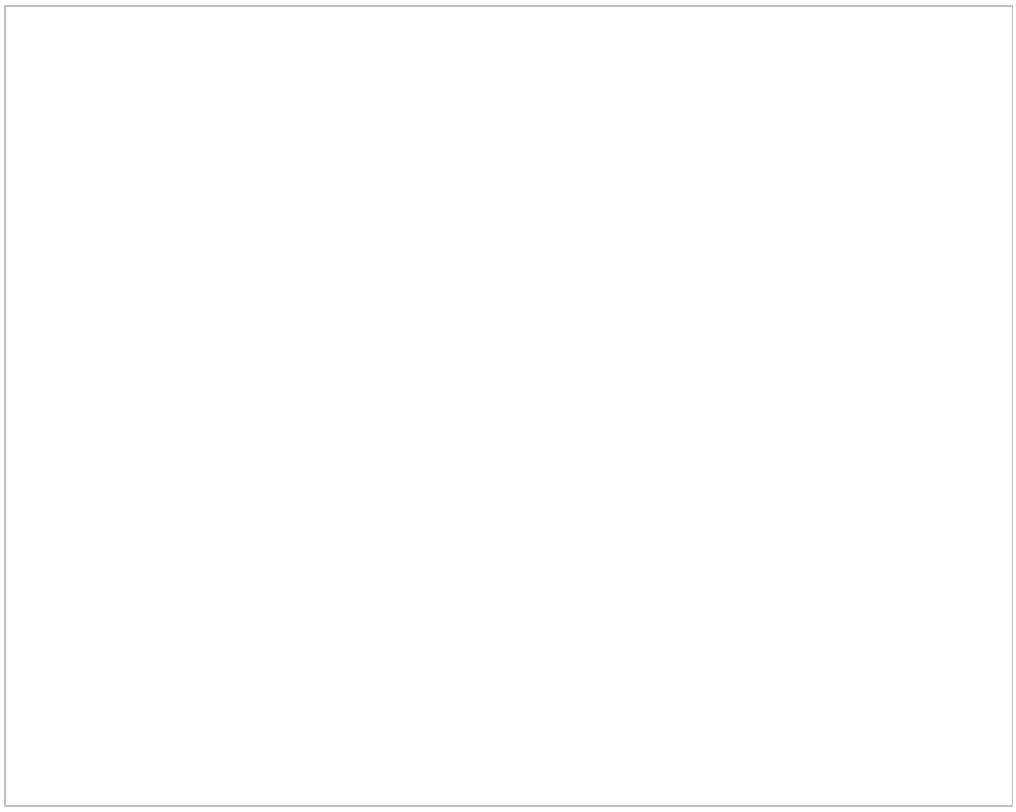


Figure P4.95 Full Alternative Text

2. 4.96 PSPICEMULTISIM Use the principle of superposition to find the current io in the circuit shown in Fig. P4.96.

Figure P4.96

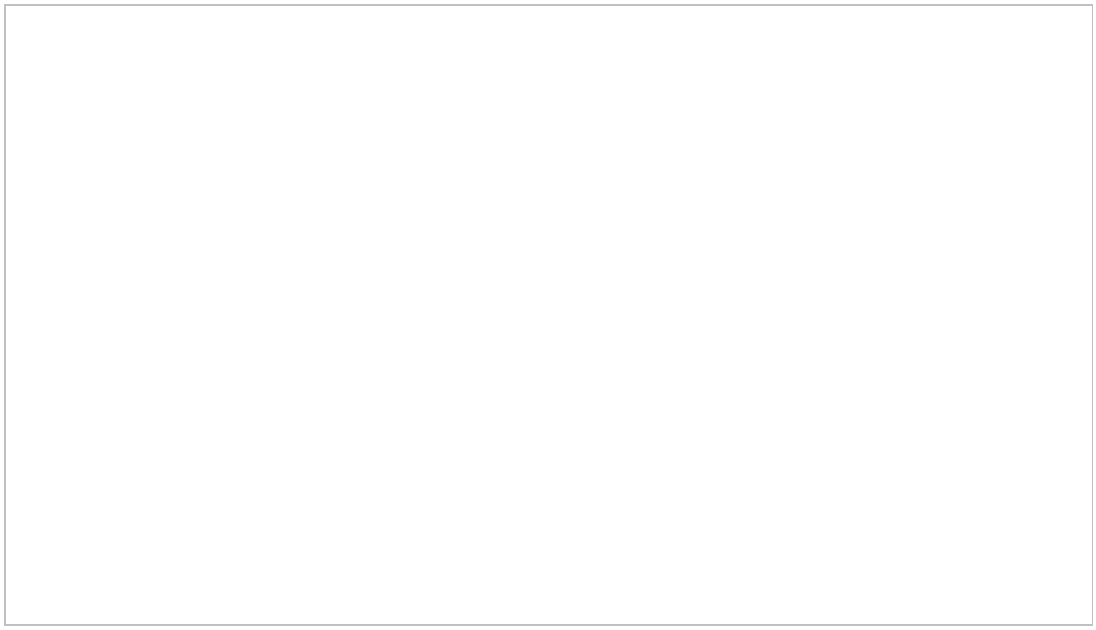


Figure P4.96 Full Alternative Text

3. 4.97 PSPICEMULTISIM Use the principle of superposition to find the current i in the circuit of Fig. P4.97.

Figure P4.97

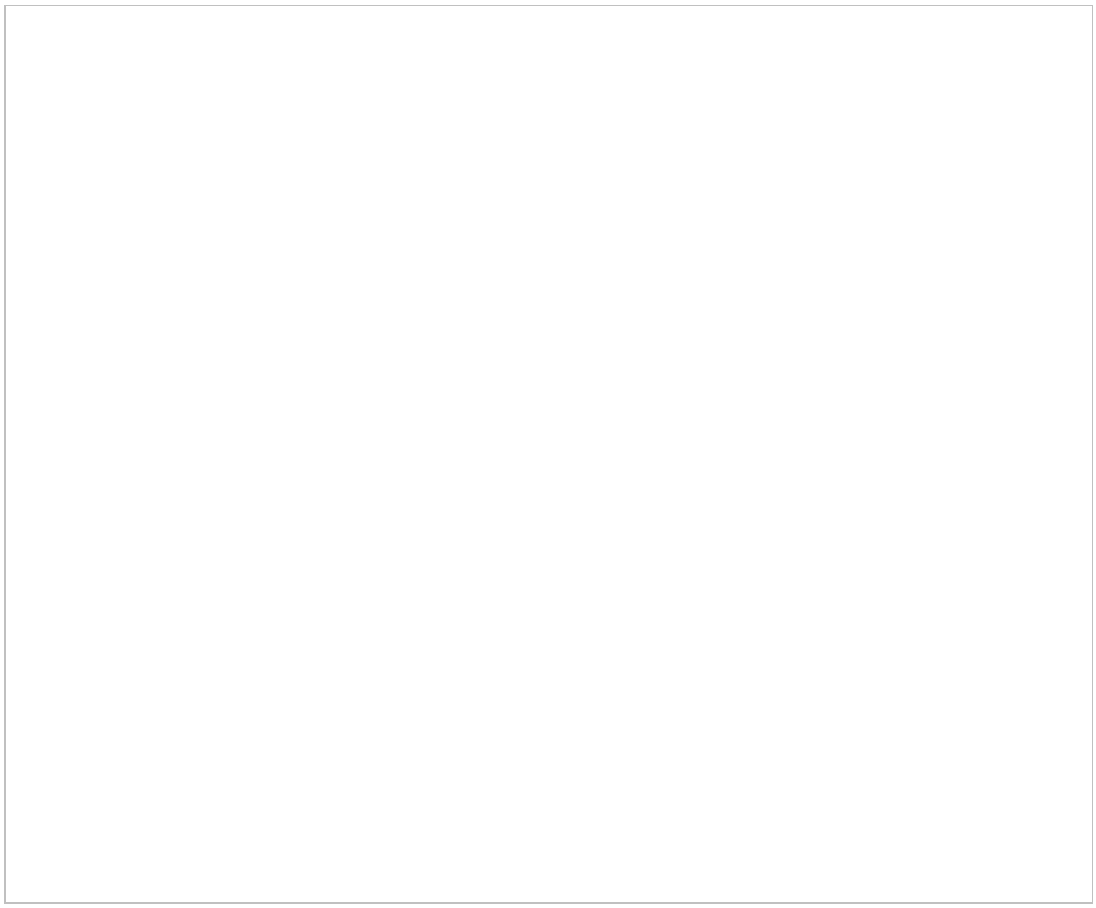


Figure P4.97 Full Alternative Text

4. 4.98 PSPICEMULTISIM Use the principle of superposition to find vo in the circuit in Fig. P4.98.

Figure P4.98

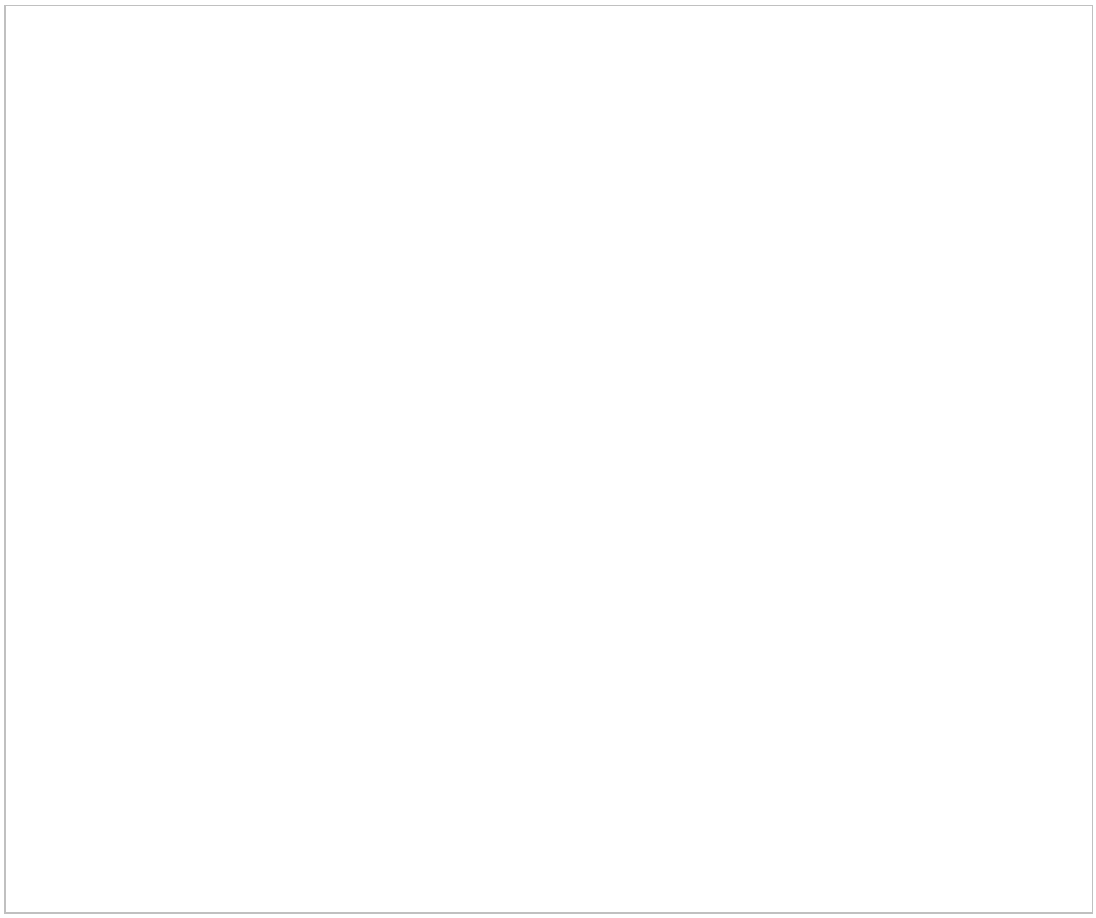


Figure P4.98 Full Alternative Text

5. 4.99 PSPICEMULTISIM Find v1, v2, and v3 in the circuit in Fig. P4.99.

Figure P4.99

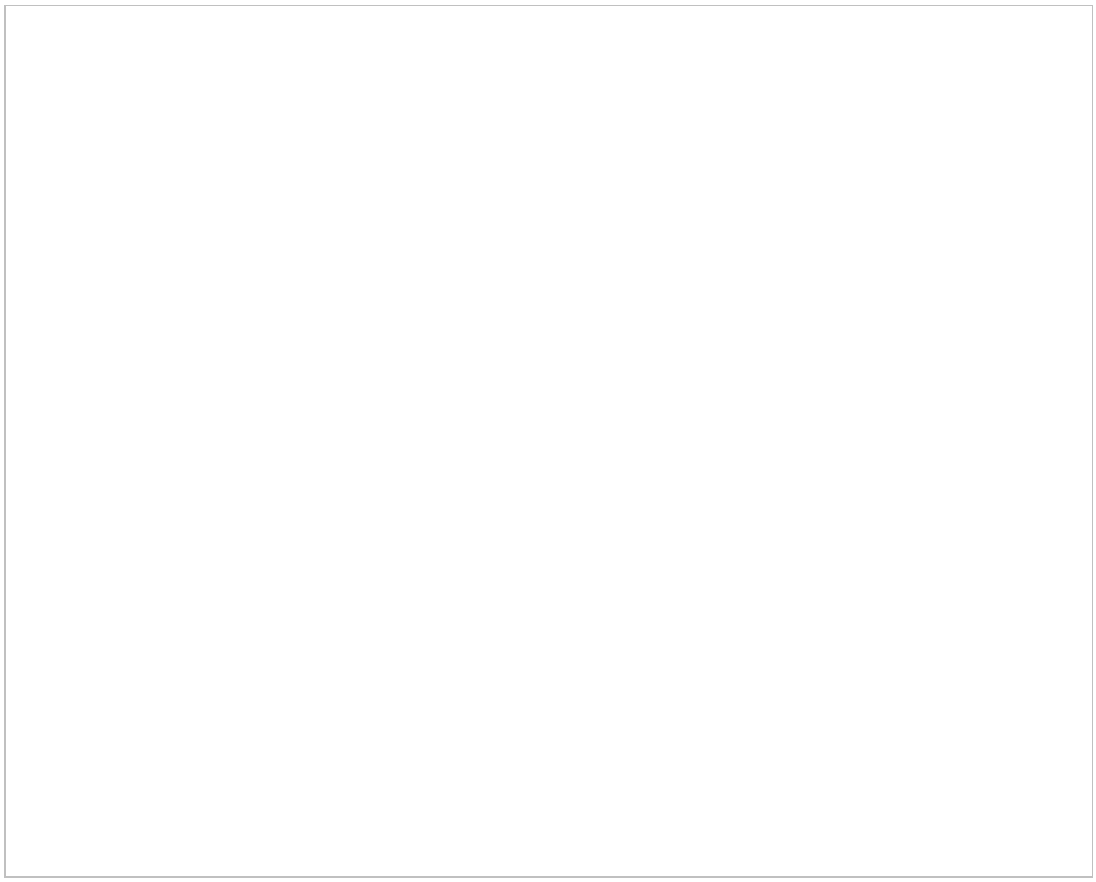


Figure P4.99 Full Alternative Text

6. 4.100 PSPICEMULTISIM Find i1 and i2 in the circuit in Fig. P4.100.

Figure P4.100

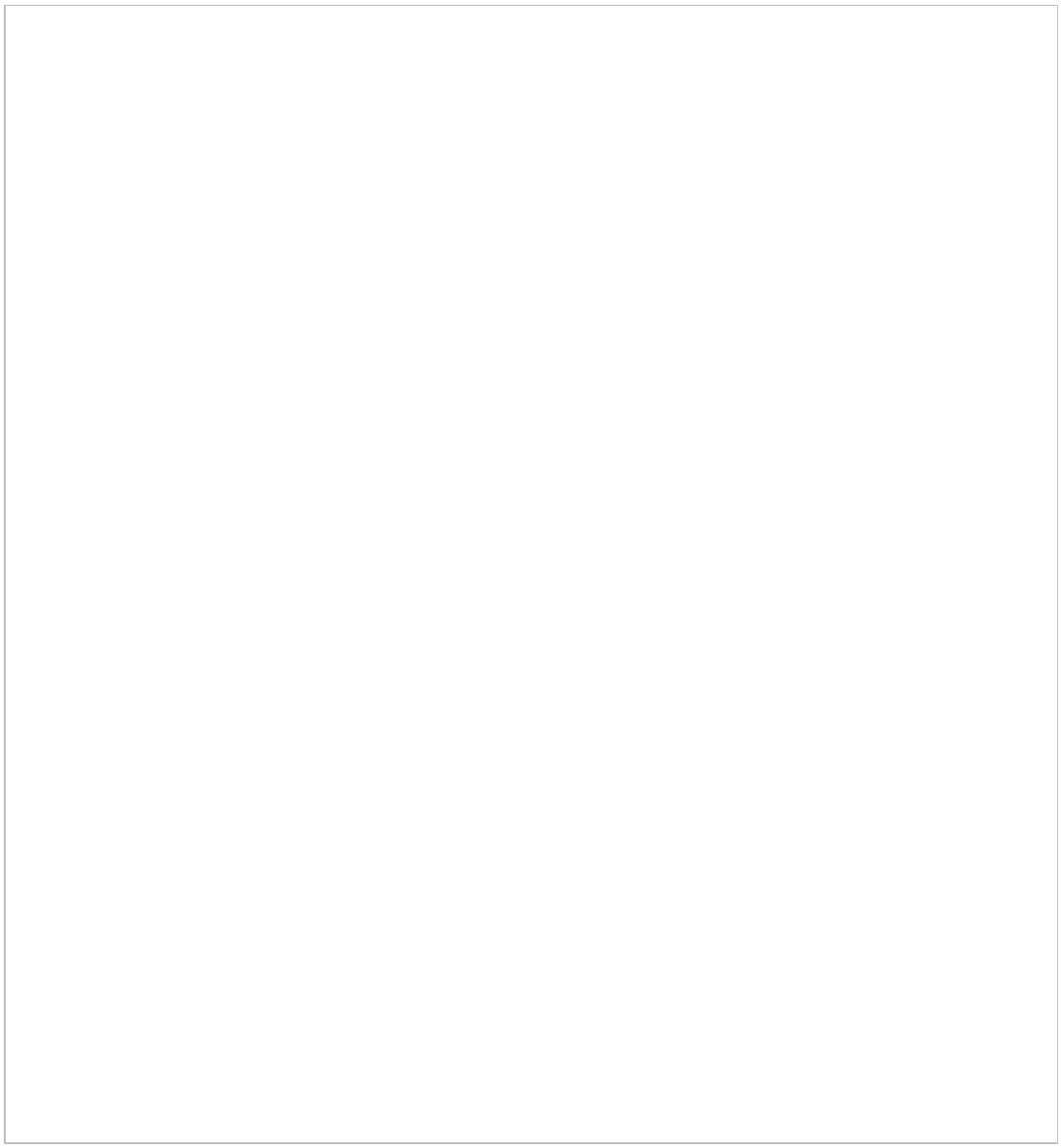


Figure P4.100 Full Alternative Text

Sections 4.1–4.13

1. 4.101 Assume your supervisor has asked you to determine the power developed by the 50 V source in the circuit in Fig. P4.101. Before calculating the power developed by the 50 V source, the supervisor asks you to submit a proposal describing how you plan to attack the problem. Furthermore, he asks you to explain why you have chosen your

proposed method of solution.

1. a) Describe your plan of attack, explaining your reasoning.
2. b) Use the method you have outlined in (a) to find the power developed by the 50 V source.

Figure P4.101

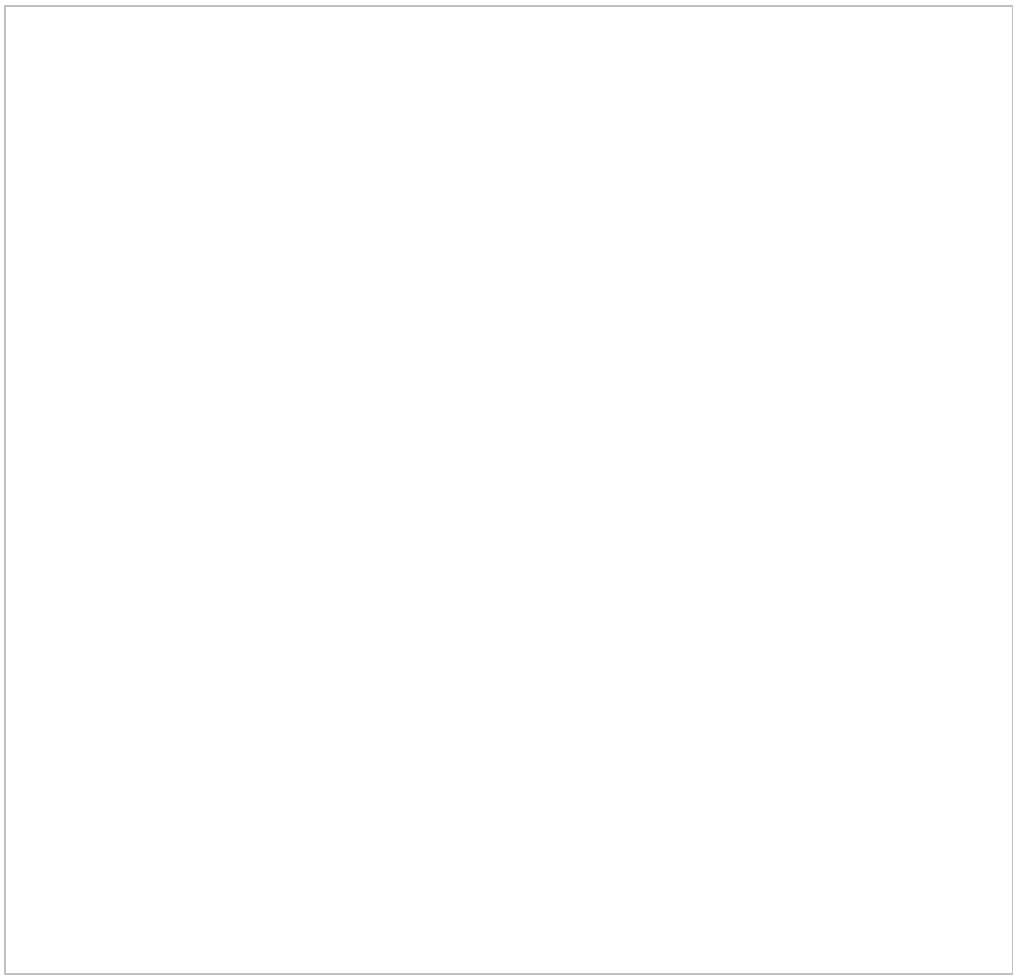


Figure P4.101 Full Alternative Text

2. 4.102 Two ideal dc voltage sources are connected by electrical conductors that have a resistance of r Ω/m, as shown in Fig. P4.102. A load having a resistance of R Ω moves between the two voltage sources.

Let x equal the distance between the load and the source v1, and let L equal the distance between the sources.

1. a) Show that

   $$v=v1RL+R(v2-v1)xRL+2rLx-2rx2.$$

2. b) Show that the voltage v will be minimum when

   $$x=Lv2-v1[-v1\pm v1v2-R2rL(v1-v2)2].$$

3. c) Find x when L=16 km, v1=1000 V, v2=1200 V, R=3.9 Ω, and $r=5\times10-5$ Ω/m.

4. d) What is the minimum value of v for the circuit of part (c)?

Figure P4.102

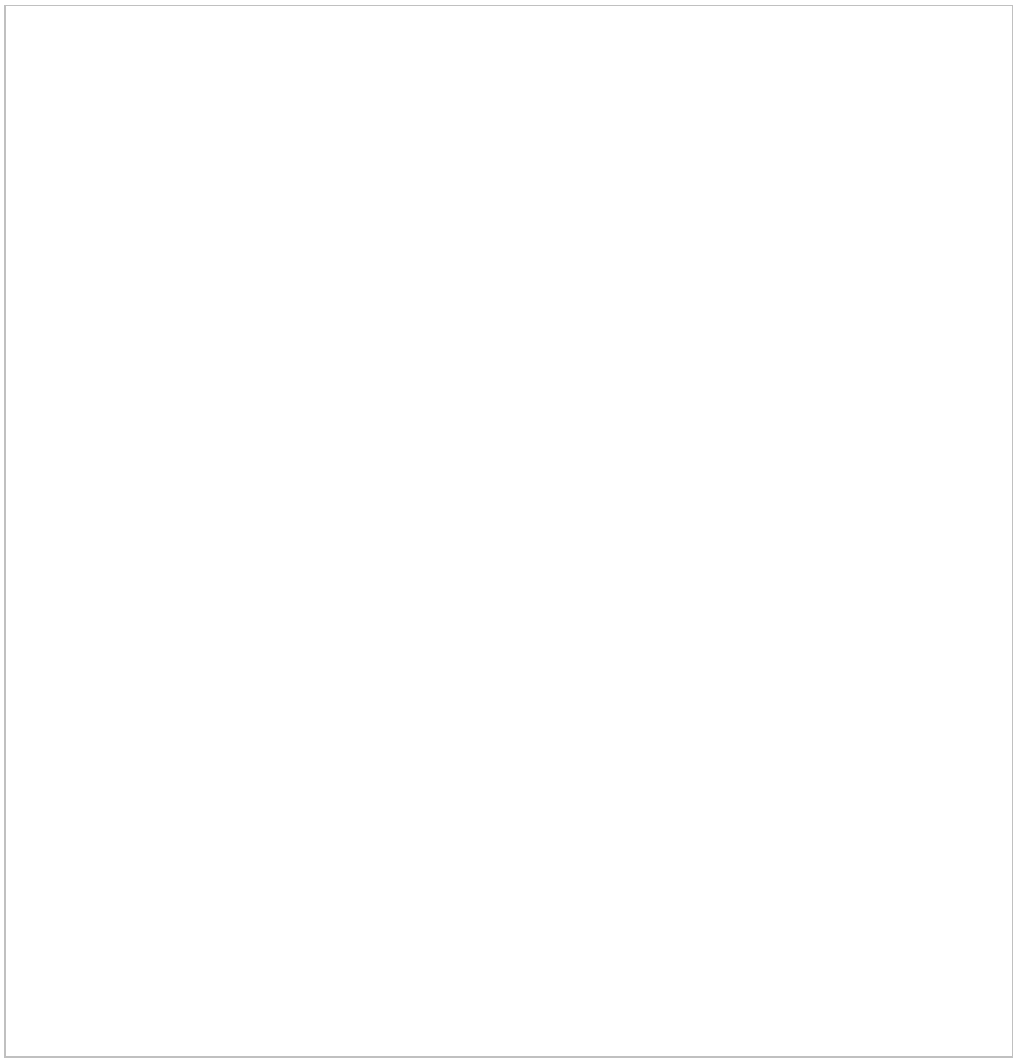


Figure P4.102 Full Alternative Text

3. 4.103 PSPICEMULTISIM Laboratory measurements on a dc voltage source yield a terminal voltage of 75 V with no load connected to the source and 60 V when loaded with a 20 Ω resistor.

    1. a) What is the Thévenin equivalent with respect to the terminals of the dc voltage source?

    2. b) Show that the Thévenin resistance of the source is given by the expression

       $R_{Th}=(v_{Thvo}-1)R_L,$

where

vTh=Thévenin voltage,vo=the terminal voltage corresponding to the

4. 4.104 PRACTICALPERSPECTIVE_ For the circuit in Fig. 4.74 derive the expressions for the sensitivity of v1 and v2 to changes in the source currents Ig11 and Ig2.

5. 4.105 PRACTICALPERSPECTIVEPSPICEMULTISIM Assume the nominal values for the components in the circuit in Fig. 4.74 are: R1=25 Ω; R2=5 Ω; R3=50 Ω; R4=75 Ω; Ig1=12 A; and Ig2=16 A. Predict the values of v1 and v2 if Ig1 decreases to 11 A and all other components stay at their nominal values. Check your predictions using a tool like PSpice or MATLAB.

6. 4.106 PRACTICALPERSPECTIVE_ Repeat Problem 4.105 if Ig2 increases to 17 A, and all other components stay at their nominal values. Check your predictions using a tool like PSpice or MATLAB.

7. 4.107 PRACTICALPERSPECTIVEPSPICEMULTISIM Repeat Problem 4.105 if Ig1 decreases to 11 A and Ig2 increases to 17 A. Check your predictions using a tool like PSpice or MATLAB.

8. 4.108 PRACTICALPERSPECTIVE_ Use the results given in Table 4.2 to predict the values of v1 and v2 if R1 and R3 increase to 10% above their nominal values and R2 and R4 decrease to 10% below their nominal values. Ig1 and Ig2 remain at their nominal values. Compare your predicted values of v1 and v2 with their actual values.

Chapter 5 The Operational Amplifier

Chapter Contents

1. 5.1 Operational Amplifier Terminals
2. 5.2 Terminal Voltages and Currents
3. 5.3 The Inverting-Amplifier Circuit
4. 5.4 The Summing-Amplifier Circuit
5. 5.5 The Noninverting-Amplifier Circuit
6. 5.6 The Difference-Amplifier Circuit
7. 5.7 A More Realistic Model for the Operational Amplifier

Chapter Objectives

1. Be able to name the five op amp terminals and describe and use the voltage and current constraints and the resulting simplifications they lead to in an ideal op amp.

2. Be able to analyze simple circuits containing ideal op amps and recognize the following op amp circuits: inverting amplifier, summing amplifier, noninverting amplifier, and difference amplifier.

3. Understand the more realistic model for an op amp and be able to use this model to analyze simple circuits containing op amps.

This chapter analyzes circuits containing sources, resistors, and a new component, the operational amplifier (op amp). Unlike sources and resistors, the op amp is not an ideal basic circuit element. Instead, it is a complicated integrated circuit consisting of many electronic components such as transistors and diodes that are beyond the scope of this text. We can use op amps in introductory circuits, however, by taking a black box approach that focuses solely on the terminal behavior of the op amp (not on its internal structure or the currents and voltages that exist in this structure).

Operational amplifier circuits were first used as basic building blocks in analog computers. The term *operational* refers to op amp circuits that implement mathematical operations such as integration, differentiation, addition, sign changing, and scaling. While the range of applications has broadened beyond implementing mathematical operations, the original name for the circuit persists.

We do not introduce new circuit analysis techniques in this chapter. Instead, we apply tools we have already introduced to analyze and design interesting and useful op amp circuits that perform scaling, addition, and subtraction. Once we introduce inductors and capacitors in Chapter 6, we will present op amp circuits that integrate and differentiate electric signals.

Initially, we employ an ideal model of the op amp's terminal behavior. At the conclusion of this chapter, we consider a more realistic op amp model that employs a dependent source. This provides additional opportunities to practice analyzing circuits with these sources.

Practical Perspective

Strain Gages

How could you measure the amount of bending in metal cables, such as the ones shown in the figure, without physically contacting the cables? One method uses a strain gage, which is a type of **transducer**. A transducer is a device that measures a quantity by converting it into a more convenient form. The quantity we wish to measure in the metal bar is the bending angle, but measuring the angle directly is quite difficult and could even be dangerous. Instead, we attach a strain gage (shown in the line drawing here) to the metal bar. A strain gage is a grid of thin wires whose resistance changes when the wires are lengthened or shortened, according to the equation

$$\Delta R = 2R\Delta LL,$$

where R is the resistance of the gage at rest, $\Delta L / L$ is the fractional lengthening of the gage (which is the definition of "strain"), 2 is a constant typical of the manufacturer's gage factor, and ΔR is the change in resistance due to the bending of the bar. Typically, pairs of strain gages are attached to opposite sides of a bar. When the bar is bent, the wires in one pair of gages get longer and thinner, increasing the resistance, while the wires in the other pair of gages get shorter and thicker, decreasing the resistance.

Horiyan/Shutterstock

How can the change in resistance be measured? One way would be to use an ohmmeter. However, the change in resistance experienced by the strain gage is typically much smaller than an ohmmeter can measure accurately. Usually, the pairs of strain gages are connected to form a Wheatstone bridge, and the voltage difference between two legs of the bridge is measured. We use an op amp circuit to amplify, or increase, the voltage difference to make an accurate measurement. After we introduce the operational amplifier and some of the important circuits that employ them, we conclude with an analysis of a strain gage circuit that measures the bending in the metal bar.

5.1 Operational Amplifier Terminals

We begin by looking at the terminal behavior of a commercially available op amp, the μA 741. Fairchild Semiconductor introduced this widely used device in 1968. This op amp is available in several different packages. For our discussion, we assume an eight-lead DIP.[1] Figure 5.1 shows a top view of the package, with the terminals, their names, and numbers. We focus on the following terminals:

[1] DIP is an abbreviation for *dual in-line package*. This means that the terminals on each side of the package are in line and that the terminals on opposite sides of the package also line up.

- inverting input,
- noninverting input,
- output,
- positive power supply (V+),
- negative power supply (V−).

Figure 5.1 The eight-lead DIP package (top view).

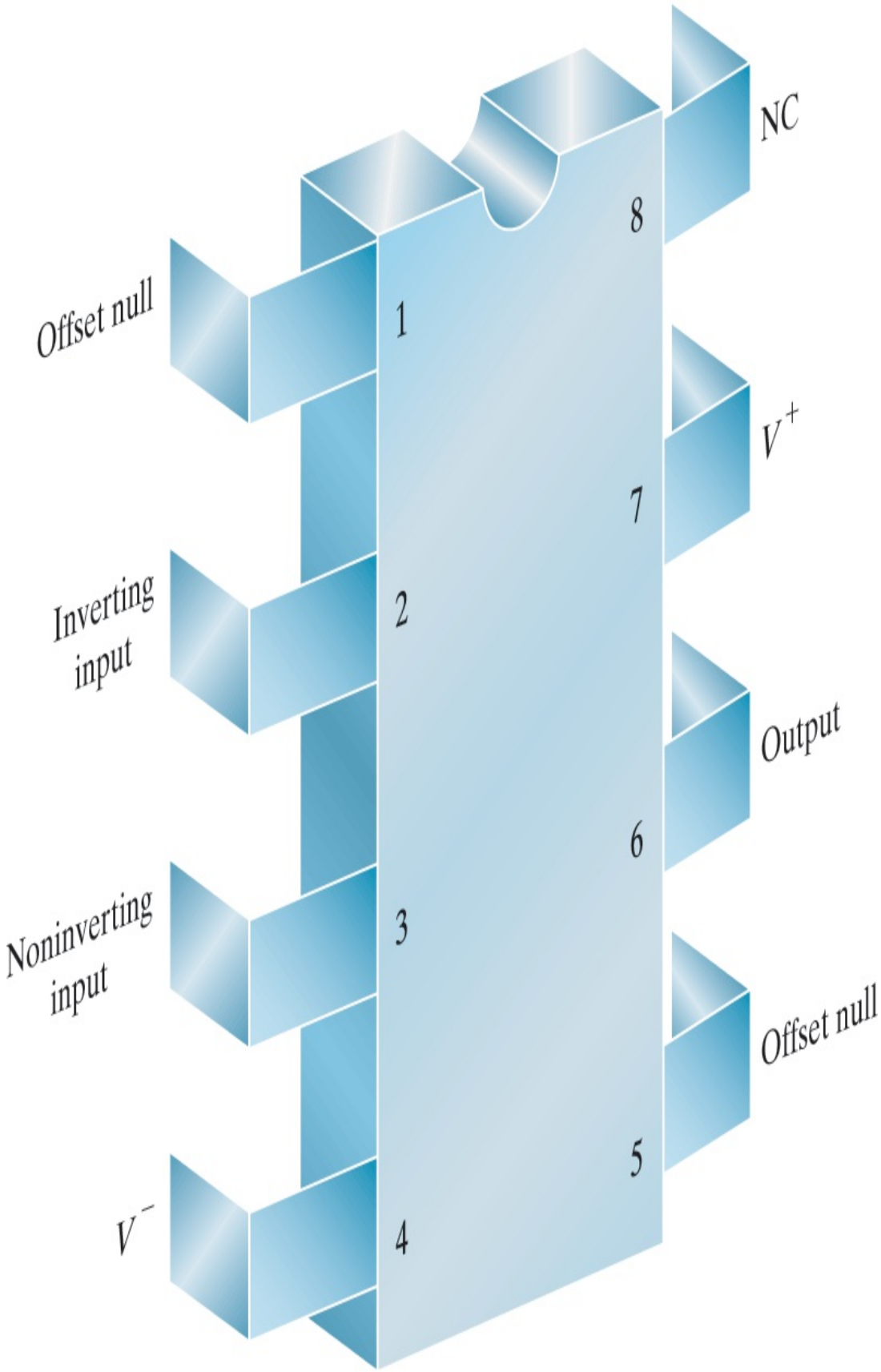

NC
8
Offset null
1
V^+
7
Inverting input
2
Output
6
Noninverting input
3
Offset null
5
V^-
4

Figure 5.1 Full Alternative Text

The remaining three terminals are of little or no concern. The two offset null terminals may be used in an auxiliary circuit that compensates for performance degradation owing to aging and imperfections. These terminals are seldom used because degradation is usually negligible. Terminal 8 is an unused terminal; NC stands for no connection, which means that the terminal is not connected to the op amp circuit.

Figure 5.2 shows a common circuit symbol for an op amp that contains the five terminals of primary interest. Because word labels are inconvenient in circuit diagrams, we simplify the terminal designations in the following way. The noninverting input terminal is labeled plus (+), and the inverting input terminal is labeled minus (−). The power supply terminals, which are always drawn outside the triangle, are marked V+ and V−. The terminal at the apex of the triangle is always understood to be the output terminal. Figure 5.3 summarizes these simplifications.

Figure 5.2 The circuit symbol for an operational amplifier (op amp).

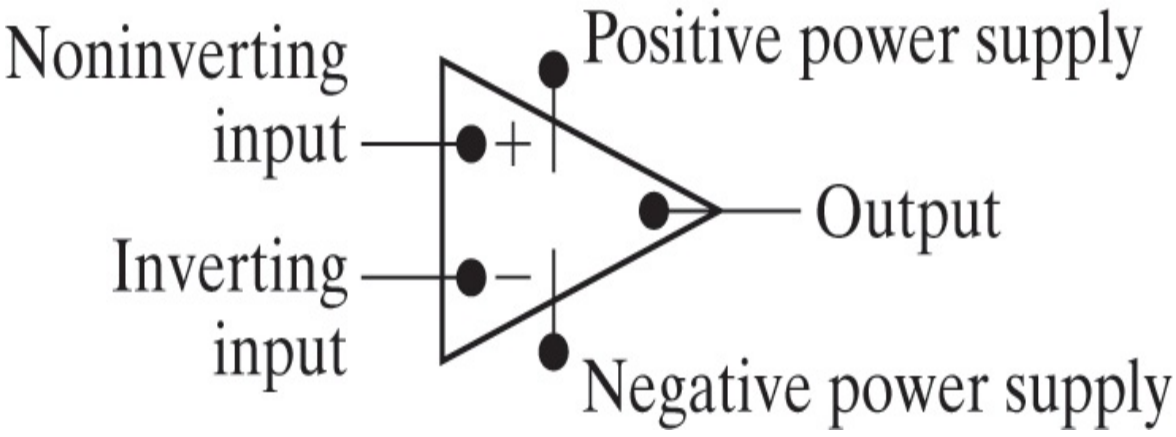


Figure 5.2 Full Alternative Text

Figure 5.3 A simplified circuit symbol for an op amp.

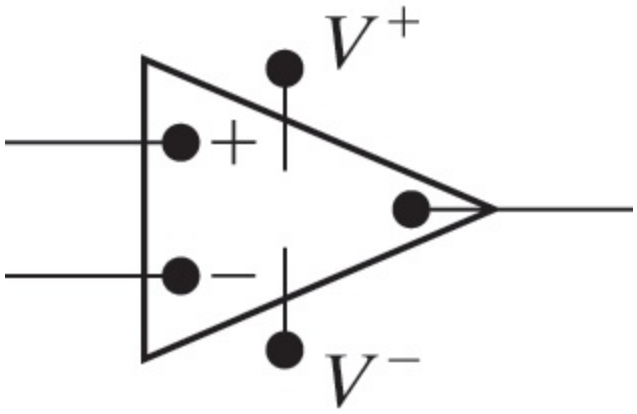


Figure 5.3 Full Alternative Text

5.2 Terminal Voltages and Currents

We now describe the behavior of the op amp using the terminal voltages and currents. The voltage variables are measured from a common reference node.[2] Figure 5.4 shows the five voltage variables with their reference polarities.

[2] The common node is external to the op amp. It is the reference terminal of the circuit in which the op amp is embedded.

Figure 5.4 Terminal voltage variables.

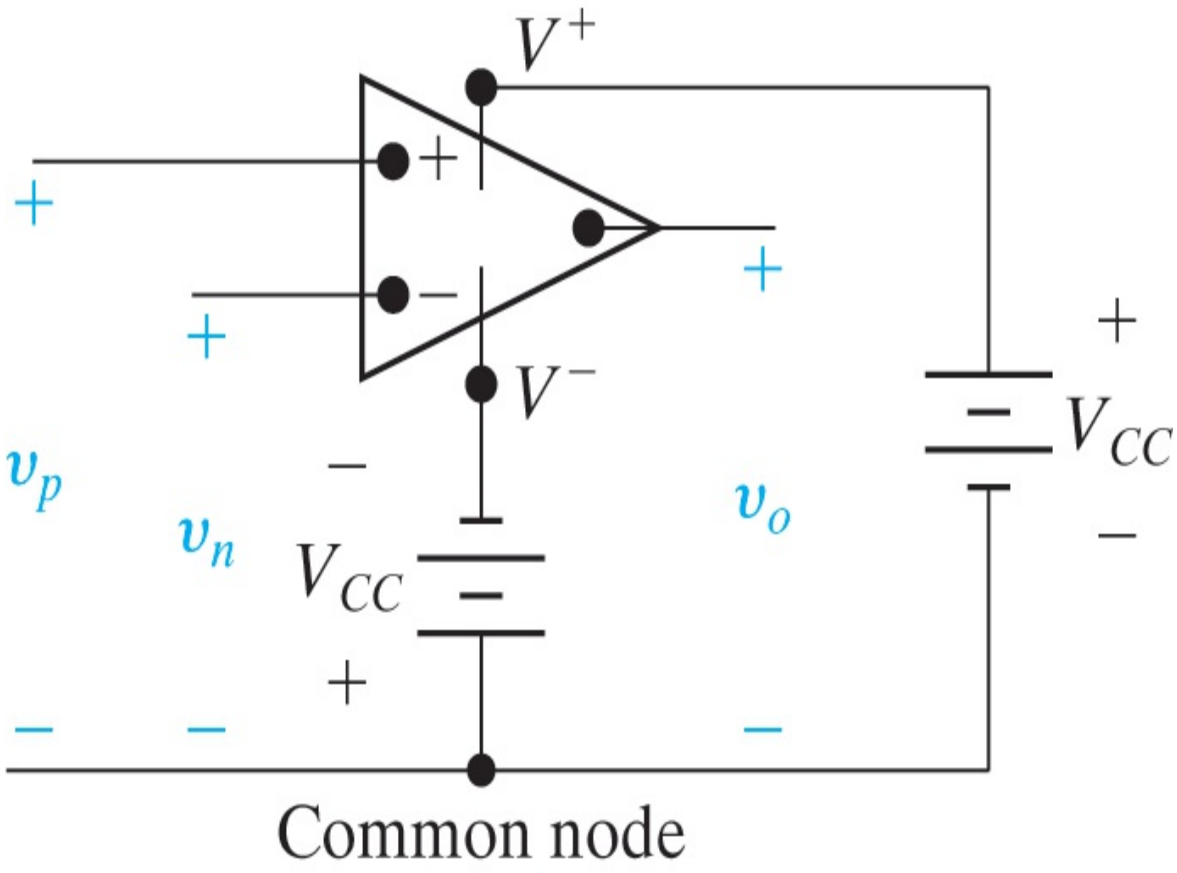


Figure 5.4 Full Alternative Text

All voltages are considered as voltage rises from the common node, a convention we also used in the node-voltage analysis method. A positive supply voltage (VCC) is connected between V+ and the common node. A negative supply voltage (−VCC) is connected between V− and the common node. The voltage between the inverting input terminal and the common node is denoted vn. The voltage between the noninverting input terminal and the common node is designated as vp. The voltage between the output terminal and the common node is denoted vo.

Figure 5.5 shows the current variables with their reference directions, all of which are into the terminals of the operational amplifier: in is the current into the inverting input terminal; ip is the current into the noninverting input terminal; io is the current into the output terminal; ic+ is the current into the positive power supply terminal; and ic− is the current into the negative power supply terminal.

Figure 5.5 Terminal current variables.

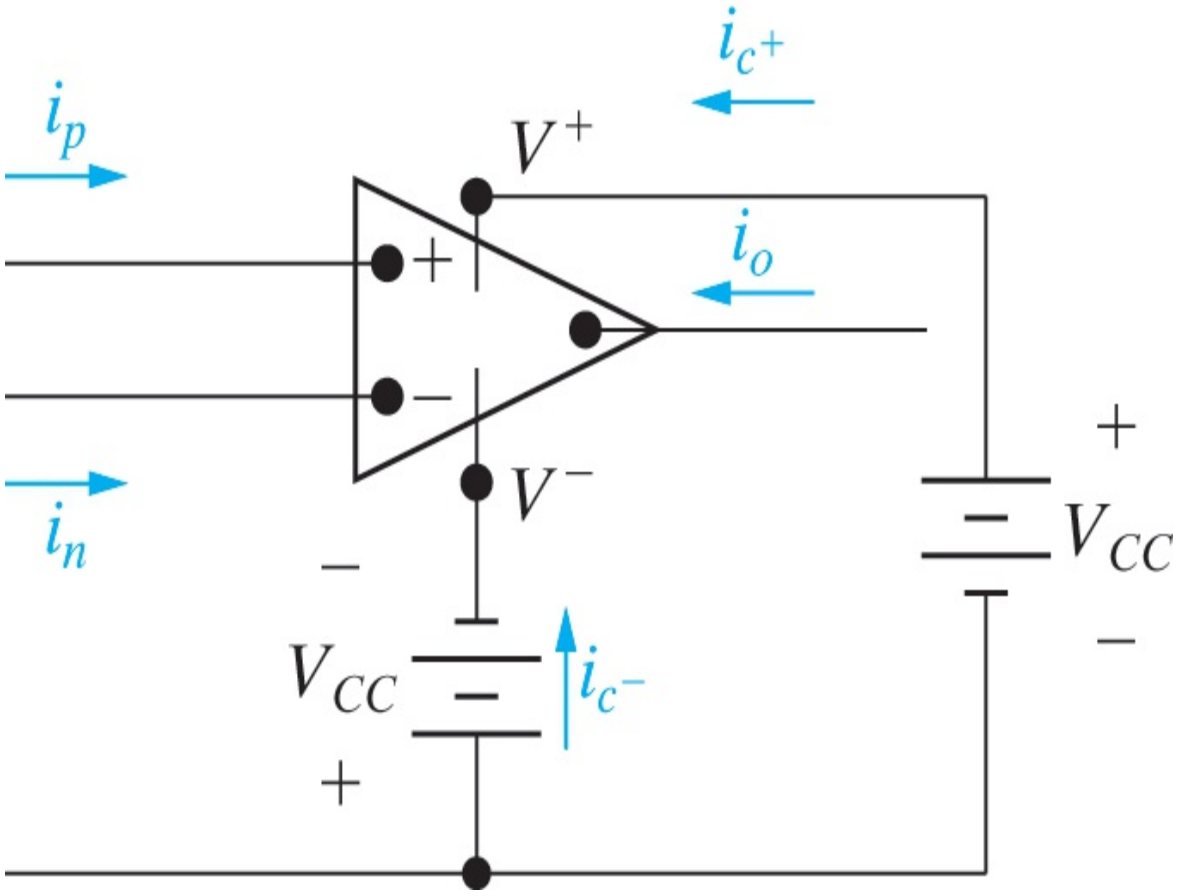


Figure 5.5 Full Alternative Text

The terminal behavior of the op amp as a linear circuit element is characterized by constraints on the input voltages and the input currents. The voltage constraint is derived from the voltage transfer characteristic of the op amp integrated circuit, pictured in Fig. 5.6.

Figure 5.6 The voltage transfer characteristic of an op amp.

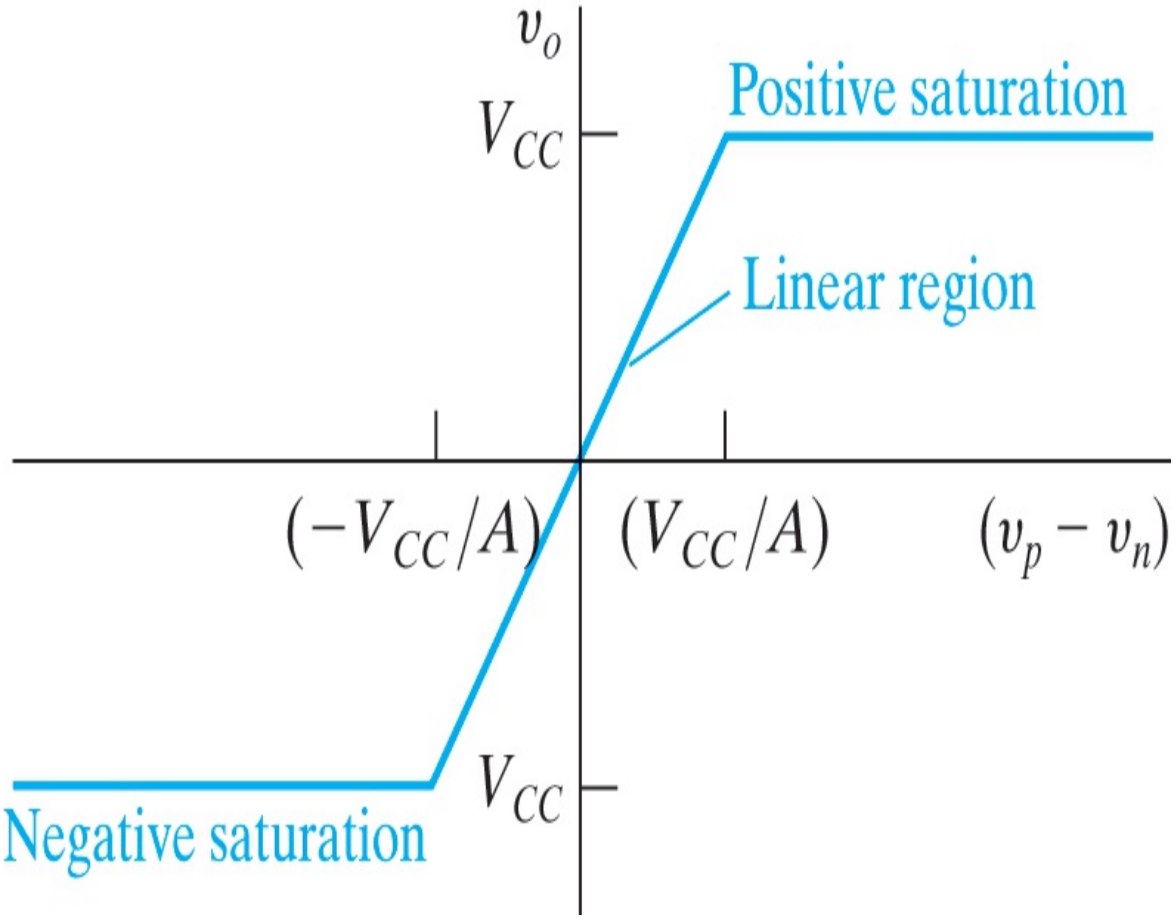


Figure 5.6 Full Alternative Text

The voltage transfer characteristic describes how the output voltage varies as a function of the input voltages—that is, how voltage is transferred from the input to the output. Note that the op amp's output voltage is a function of the difference between its input voltages, vp−vn. The equation for the voltage transfer characteristic is

vo={−VCCA(vp−vn)<−VCC,A(vp−vn)−VCC≤A(vp
−vn)≤+VCC,+ VCCA(vp−vn)>+VCC. (5.1)

We see from Fig. 5.6 and Eq. 5.1 that the op amp has three distinct regions of operation: negative saturation, linear region, and positive saturation. When the magnitude of the input voltage difference (|vp−vn|) is small, the op amp behaves as a linear device, so the output voltage is a linear function of the input voltages. More specifically, the output voltage is equal to the difference

between the input voltages times the multiplying constant, or **gain**, A. Outside this linear region are two saturation regions. When the output of the op amp saturates, the op amp behaves as a nonlinear device; its output voltage is no longer a linear function of the input voltages.

OP AMP Input Voltage Constraints

When we confine the op amp to its linear operating region, a constraint is imposed on the input voltages, vp and vn. The constraint is based on typical numerical values for VCC and A in Eq. 5.1. For most op amps, the recommended dc power supply voltages seldom exceed 20 V, and the gain, A, is rarely less than 10,000, or 104. We see from both Fig. 5.6 and Eq. 5.1 that in the linear region, the magnitude of the input voltage difference (|vp − vn|) must be less than 20/104, or 2 mV.

Node voltages in the circuits we study are typically much larger than 2 mV, so a voltage difference of less than 2 mV means the two voltages are essentially equal. Thus, when an op amp is constrained to its linear operating region and the node voltages are much larger than 2 mV, the constraint on the input voltages of the op amp is

Input Voltage Constraint for an OP AMP

vp=vn. (5.2)

Note that Eq. 5.2 characterizes the relationship between the input voltages for an ideal op amp—that is, an op amp whose value of A is infinite.

We can use Eq. 5.2 only if the op amp is confined to its linear operating region. The op amp stays in its linear region if the op amp circuit includes a signal path from the op amp's output terminal to its inverting input terminal. This configuration is known as **negative feedback** because the signal is fed back from the output and is subtracted from the input signal. The negative

feedback causes the input voltage difference to decrease. Because the output voltage is proportional to the input voltage difference, the output voltage is also decreased, and the op amp operates in its linear region.

If a circuit containing an op amp does not provide a negative feedback path from the op amp output to the inverting input, then the op amp will normally saturate. But even if the circuit provides a negative feedback path for the op amp, linear operation is not guaranteed. So how do we know whether the op amp is operating in its linear region?

The answer is, we don't! We deal with this dilemma by assuming linear operation, performing the circuit analysis, and then checking our results for contradictions. For example, suppose we assume that an op amp in a circuit is operating in its linear region, and we compute the output voltage of the op amp to be 10 V. On examining the circuit, we discover that VCC is 6 V, resulting in a contradiction, because the op amp's output voltage can be no larger than VCC. Thus, our assumption of linear operation was invalid, and the op amp output must be saturated at 6 V.

We have identified a constraint on the input voltages that is based on the voltage transfer characteristic of the op amp, the assumption that the op amp is restricted to its linear operating region, and typical values for VCC[3] and A. Equation 5.2 represents the voltage constraint for an ideal op amp, that is, with a value of A that is infinite.

[3] The positive and negative power supply voltages do not have to be equal in magnitude. In the linear operating region, vo must lie between the two supply voltages. For example, if V+=15 V and V-=-10 V, then −10 V≤vo≤15 V.

OP AMP Input Current Constraints

We now turn our attention to the constraint on the input currents. Analysis of the op amp integrated circuit reveals that the equivalent resistance seen by the input terminals of the op amp is very large, typically 1 MΩ or more. Ideally, the equivalent input resistance is infinite, resulting in the current constraint

Input Current Constraint for an Ideal OP AMP

ip=in=0.(5.3)

Note that the current constraint is not based on assuming that the op amp is confined to its linear operating region, as was the voltage constraint. Together, Eqs. 5.2 and 5.3 form the constraints on terminal behavior that define our ideal op amp model.

From Kirchhoff's current law we know that the sum of the currents entering the operational amplifier is zero, or

ip+in+io+ic++ic−=0.

Substituting the constraint given by Eq. 5.3 into this KCL equation gives

io=−(ic++ ic−).

The equation for io tells us that, even though the current at the input terminals is negligible, there may still be appreciable current at the output terminal.

When we use Eqs. 5.2 and 5.3 in analyzing a circuit with an op amp, we are effectively using an ideal model of that op amp. Let's summarize the circuit analysis steps:

Analyzing a Circuit With an Ideal OP AMP

1. Check for a negative feedback path. If it exists, assume the op amp operates in its linear region.

2. Write a KCL equation at the inverting input terminal.

3. Solve the KCL equation and use the solution to find the op amp's output voltage.

4. Compare the op amp's output voltage to the power supply voltages to determine if the op amp is operating in its linear region or if it is saturated.

Analysis Method 5.1 Analyzing an ideal op amp circuit with a negative feedback path.

1. Step 1: Check for the presence of a negative feedback path; if it exists, we can assume the op amp is operating in its linear region.

2. Step 2: Write a KCL equation at the inverting input terminal, using the input current constraint (Eq. 5.3), the value of vn (Eq. 5.2), and Ohm's law to find the currents. This equation will usually contain the unknown voltage at the op amp's output terminal.

3. Step 3: Solve the KCL equation and calculate the voltage at the op amp's output terminal.

4. Step 4: Compare the voltage at the op amp's output terminal to the power supply voltages to determine whether the op amp is actually in its linear region or whether it has saturated.

These steps are summarized in Analysis Method 5.1. Example 5.1 analyzes an op amp circuit using this analysis method.

Example 5.1 Analyzing an OP AMP Circuit

The op amp in the circuit shown in Fig. 5.7 is ideal.

Figure 5.7 The circuit for

Example 5.1.

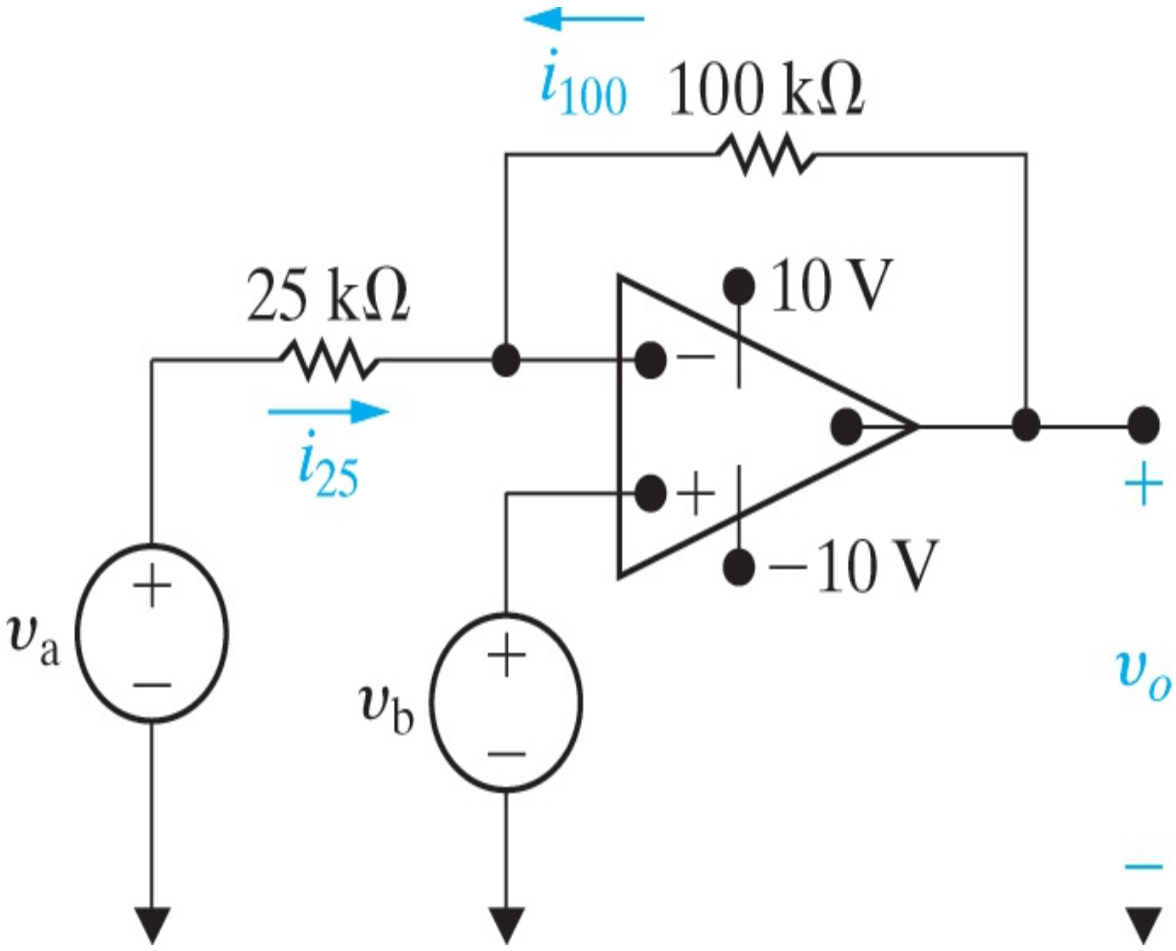


Figure 5.7 Full Alternative Text

1. Calculate vo if va=1 V and vb=0 V.
2. Repeat (a) for va=1 V and vb=2 V.
3. If va=1.5 V, specify the range of vb that avoids amplifier saturation.

Solution

1. Step 1: A negative feedback path exists from the op amp's output to its inverting input through the 100 kΩ resistor, so we assume the op amp is

confined to its linear operating region.

Step 2: The voltage at the inverting input terminal is 0 because vp=vb=0 from the connected voltage source, and vn=vp from the voltage constraint in Eq. 5.2.

Step 3: Use KCL to sum the currents entering the node labeled vn to get

i25+i100-in=0.

Remember that in is the current entering the inverting op amp terminal. From Ohm's law,

i25=va−vn25,000=1−025,000=125,000;i100=vo−vn100,000=vo −0100,000=vo100,000.

The current constraint requires in=0. Substituting the values for the three currents into the KCL equation, we obtain

125+vo100=0.

Hence, vo is -4V.

Step 4: Because vo lies between ±10 V, our assumption that the op amp is in its linear region of operation is confirmed.

2. Using the same steps as in (a), we get

   vp=vb=vn=2V,i25=−i100.i25=va−vn25,000=1−225,000= −125,000 ;i100=vo−vn100,000=vo−2100,000 .

   Therefore, vo=6V. Again, vo lies within ±10 V.

3. As before, vn=vp=vb, and i25=-i100. Because va=1.5 V,

   1.5-vb25,000=vo-vb100,000 .

   Solving for vb as a function of vo gives

vb=15(6+vo).

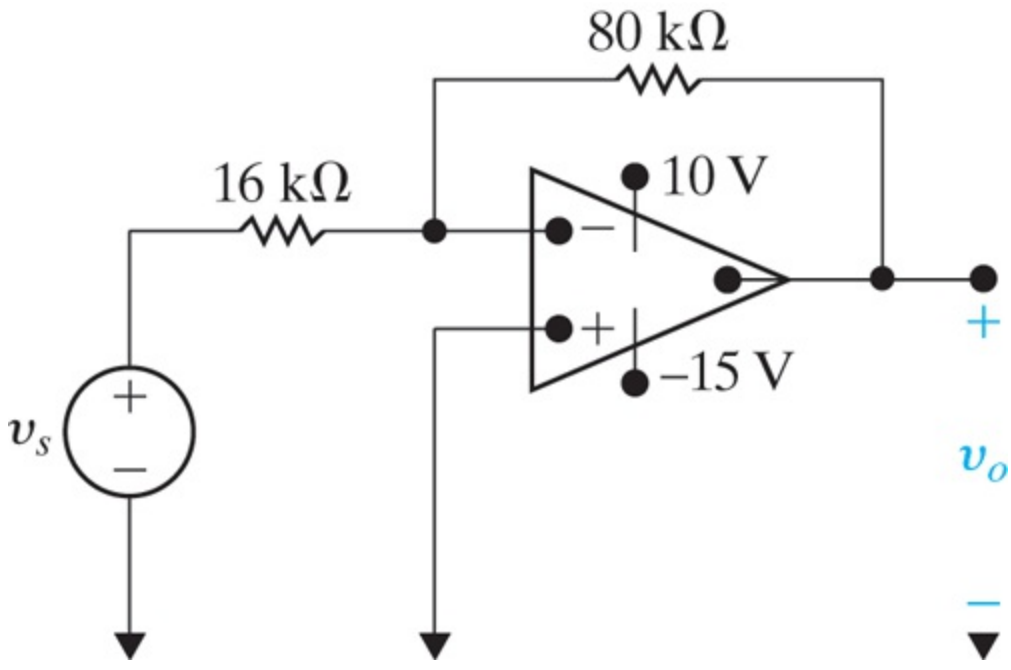
Now, if the amplifier operates within its linear region, −10V≤vo≤10V. Substituting these limits on vo into the expression for vb, we find the range for vb is

−0.8V≤vb≤3.2V.

Assessment Problem

Objective 1—Use voltage and current constraints in an ideal op amp

1. 5.1 Assume that the op amp in the circuit shown is ideal.
    1. Calculate vo for the following values of vs: 0.4, 2.0, 3.5, −0.6, −1.6, and −2.4 V.
    2. Specify the range of vs required to avoid amplifier saturation.



5.2-3 Full Alternative Text

Answer:

1. −2, −10, −15, 3, 8, and 10 V;
2. $-2\ \text{V} \le v_s \le 3\ \text{V}$.

SELF-CHECK: Also try Chapter Problems 5.1, 5.3, and 5.4.

5.3 The Inverting-Amplifier Circuit

This section and the three that follow present some important op amp circuits. We begin with the inverting-amplifier circuit, shown in Fig. 5.8. This circuit contains an ideal op amp, two resistors (Rf and Rs), a voltage source (vs), and a short circuit connecting the noninverting input terminal and the common node.

Figure 5.8 An inverting-amplifier circuit.

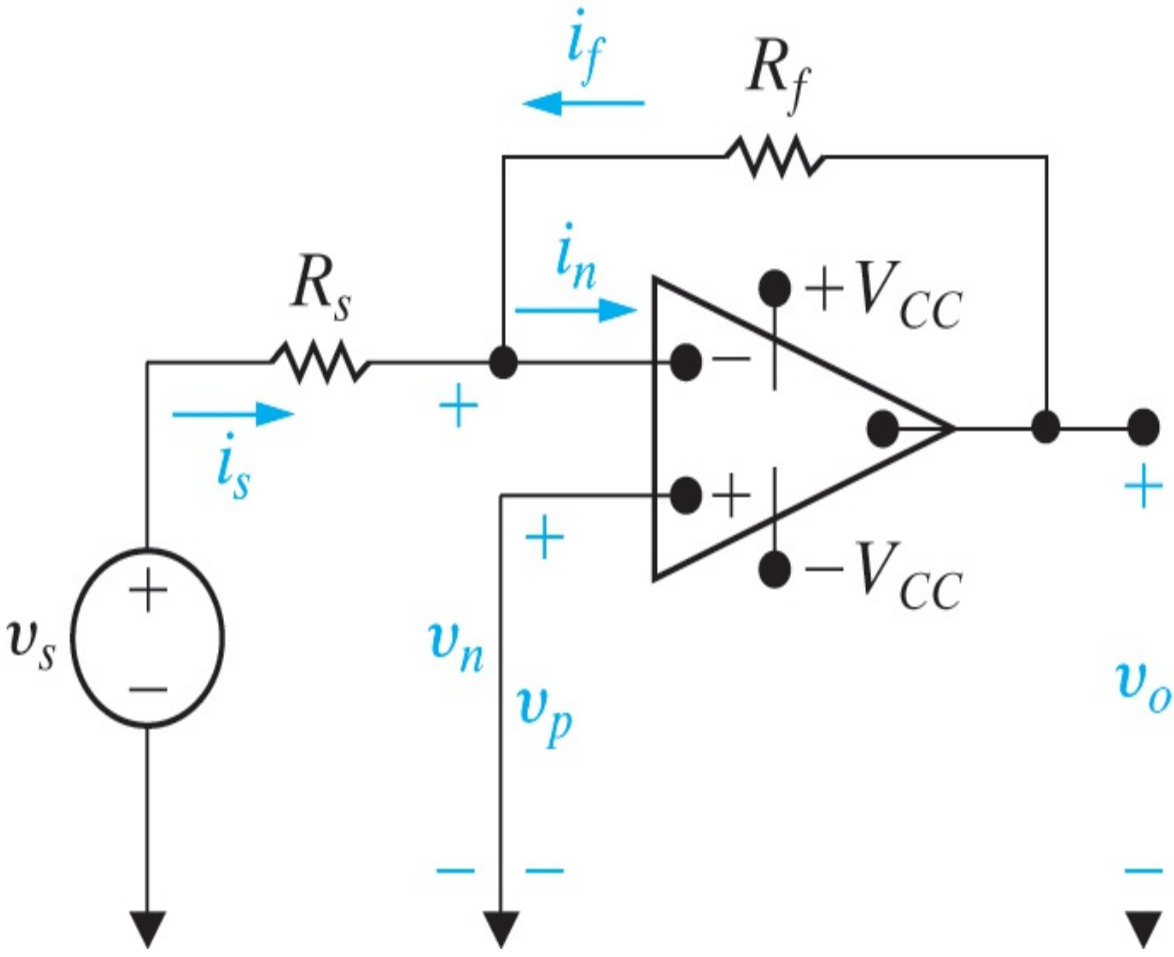


Figure 5.8 Full Alternative Text

We can analyze this circuit to obtain an expression for the output voltage, vo, as a function of the source voltage, vs. Starting with Step 1, we note the circuit's negative feedback path, so assume the op amp is in its linear operating region. In Step 2, the voltage constraint of Eq. 5.2 sets the voltage at vn=0, because the voltage at vp=0. Step 3 generates a single KCL equation at the inverting terminal of the op amp, given as

is+if=in.

From Ohm's law,

is=vsRs,if=voRf.

Now we invoke the constraint stated in Eq. 5.3, namely,

in=0.

Substituting the expressions for is, if, and in into the KCL equation and solving for vo yields

Inverting Amplifier Equation

vo=−RfRsvs.(5.4)

Note that the output voltage is an inverted, scaled replica of the input. The sign reverses, or inverts, from input to output. The scaling factor, or gain, is the ratio Rf/Rs,which is usually greater than 1, so |vo|>|vs|. Hence, we call this circuit an inverting amplifier.

Using Step 4, we see that the result given by Eq. 5.4 is valid only if the op amp shown in the circuit in Fig. 5.8 is operating in its linear region. Even if the op amp is not ideal, Eq. 5.4 is a good approximation. (We demonstrate this in Section 5.7.) Equation 5.4 specifies the gain of the inverting amplifier with the external resistors Rf and Rs. In Step 4, the upper limit on the gain, Rf/Rs, is determined by the power supply voltages and the value of the signal

voltage vs. If we assume equal power supply voltages, that is, V+=−V −=VCC, we get

|vo | ≤VCC, |RfRsvs| ≤ VCC, RfRs≤|VCCvs|.

For example, if VCC=15 V and vs=10 mV, the ratio Rf/Rs must be less than 1500.

In the inverting amplifier circuit shown in Fig. 5.8, the resistor Rf provides the negative feedback connection. That is, it connects the output terminal to the inverting input terminal. If Rf is removed, the feedback path is opened and the amplifier is said to be operating *open loop*. Figure 5.9 shows the open-loop operation.

Figure 5.9 An inverting amplifier operating open loop.

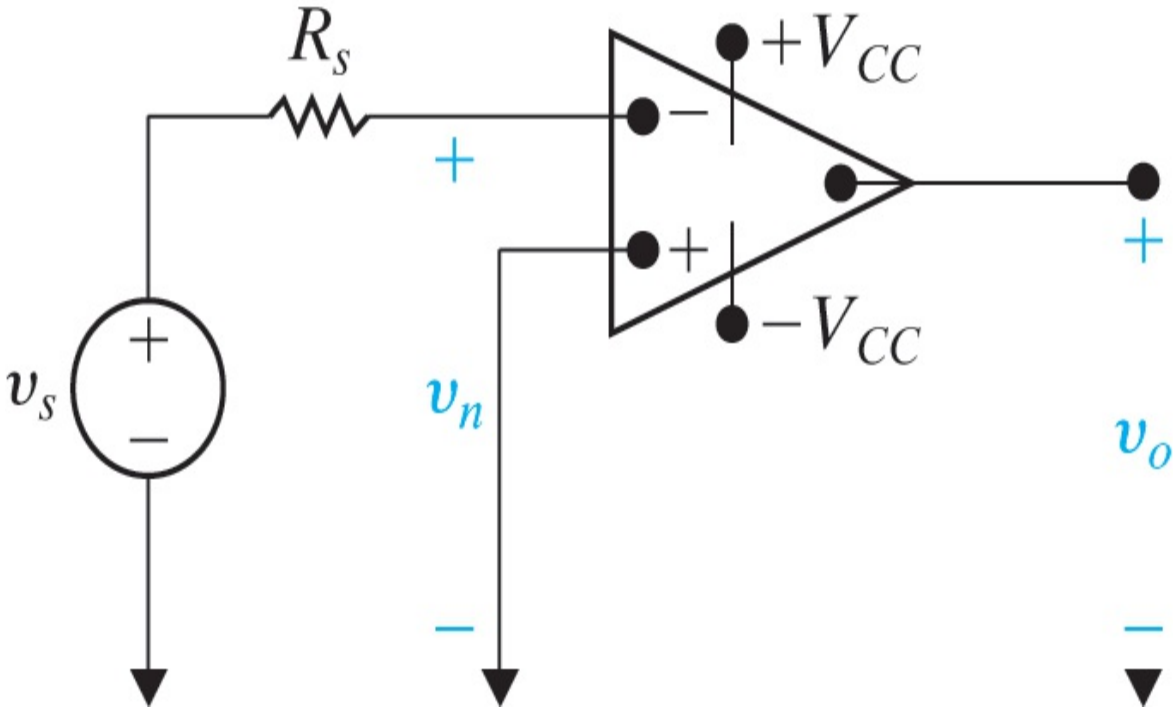


Figure 5.9 Full Alternative Text

Opening the feedback path drastically changes the behavior of the circuit. To understand the open-loop circuit, we do not replace the op amp with its ideal model; although A and the input resistance are both large, they are not infinite. Now the output voltage is

$$v_o = -Av_n, \quad (5.5)$$

assuming as before that $V+ = -V- = V_{CC}$; then $|v_n| < V_{CC}/A$ for linear operation. Because the inverting input current is almost zero, the voltage drop across Rs is almost zero, and the inverting input voltage nearly equals the signal voltage, vs ; that is, $v_n \approx v_s$. Hence, the op amp can operate open loop in the linear mode only if $|v_s| < V_{CC}/A$. If $|v_s| > V_{CC}/A$, the op amp simply saturates. In particular, if $v_s < -V_{CC}/A$, the op amp saturates at +VCC, and if $v_s > V_{CC}/A$, the op amp saturates at −VCC. Because the relationship shown in Eq. 5.5 occurs when there is no feedback path, the value of A is often called the **open-loop gain** of the op amp.

Example 5.2 uses the inverting-amplifier equation to design an inverting amplifier using realistic resistor values.

Example 5.2 Designing an Inverting Amplifier

1. Design an inverting amplifier (see Fig. 5.8) with a gain of 12. Use ±15 V power supplies and an ideal op amp.

2. What range of input voltages, vs, allows the op amp in this design to remain in its linear operating region?

Solution

1. We need to find two resistors whose ratio is 12 from the realistic resistor values listed in Appendix H. There are lots of different possibilities, but let’s choose Rs=1 kΩ and Rf=12 kΩ. Use the inverting-amplifier

equation (Eq. 5.4) to verify the design:

vo=−RfRsvs=−12,000 1000 vs=−12vs.

Thus, we have an inverting amplifier with a gain of 12, as shown in Fig. 5.10.

Figure 5.10 Inverting amplifier for Example 5.2.

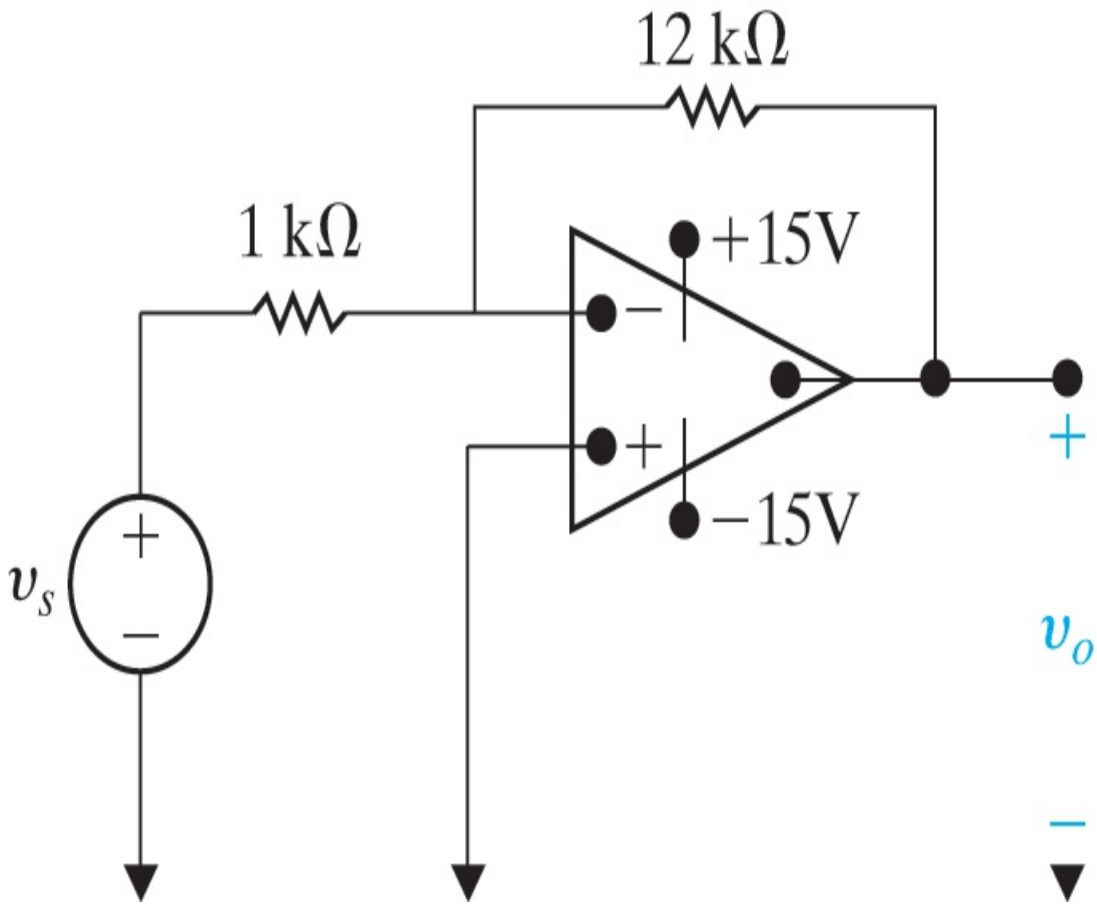


Figure 5.10 Full Alternative Text

2. Solve two different versions of the inverting-amplifier equation for vs, first using vo=+15 V and then using vo=−15 V:

$15 = -12v_s$ so $v_s = -1.25$ V; $-15 = -12v_s$ so $v_s = 1.25$ V.

Thus, if the input voltage is greater than or equal to −1.25 V and less than or equal to +1.25 V, the op amp in the inverting amplifier will remain in its linear operating region.

Assessment Problem

Objective 2—Be able to analyze simple circuits containing ideal op amps

1. 5.2 The source voltage v_s in the circuit in Assessment Problem 5.1 is −640 mV. The 80 kΩ feedback resistor is replaced by a variable resistor Rx. What range of Rx allows the inverting amplifier to operate in its linear region?

   Answer: $0 \le R_x \le 250$ kΩ.

SELF-CHECK: Also try Chapter Problems 5.8 and 5.10.

5.4 The Summing-Amplifier Circuit

Figure 5.11 shows a summing amplifier with three input voltages. The output voltage of a summing amplifier is an inverted, scaled sum of the voltages applied to the input of the amplifier. We can find the relationship between the output voltage vo and the three input voltages, va, vb, and vc,using Analysis Method 5.1.

Figure 5.11 A summing amplifier.

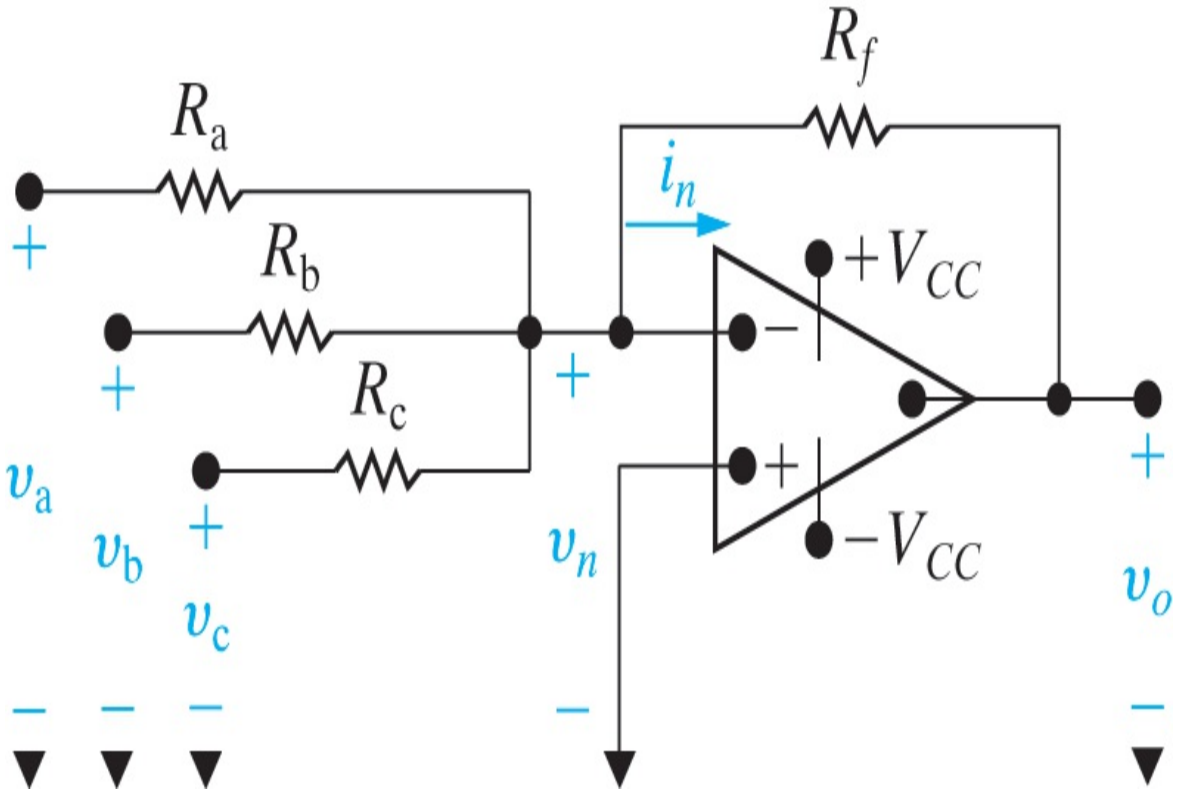


Figure 5.11 Full Alternative Text

The summing amplifier has a negative feedback path that includes the resistor

Rf, so in Step 1 we assume the op amp is in its linear region. We then use the ideal op amp voltage constraint in Step 2 together with the ground imposed by the circuit at vp to determine that vn=vp=0. In Step 3, we write a KCL equation at the inverting input terminal, using Ohm's law to specify the current in each resistor in terms of the voltage across that resistor, to get

vn−vaRa+vn−vbRb+vn−vcRc+vn−voRf+in=0.

Apply the voltage constraint from Step 2 and the current constraint in=0 to the KCL equation, then solve for vo to get

Inverting Summing-Amplifier Equation

vo=−(RfRava+RfRbvb+RfRcvc).(5.6)

Equation 5.6 shows that the output voltage is an inverted, scaled sum of the three input voltages. According to Step 4, this equation is valid only if the value of vo is between the two power supply voltages.

If Ra=Rb=Rc=Rs, then Eq. 5.6 reduces to

vo=−RfRs(va+vb+vc).

Finally, if we make Rf=Rs, the output voltage is just the inverted sum of the input voltages. That is,

vo=−(va+vb+vc).

Although we analyzed the summing amplifier with three input signals, the number of input voltages can be increased or decreased as needed. For example, you might wish to sum 16 individually recorded audio signals to form a single audio signal. The summing-amplifier configuration in Fig. 5.11 could include 16 different input resistors whose values specify different amplification factors for each of the input audio tracks. The summing amplifier thus plays the role of an audio mixer. As with inverting-amplifier

circuits, the scaling factors in summing-amplifier circuits are determined by the external resistors Rf, Ra, Rb, Rc, …, Rn.

Example 5.3 uses the summing-amplifier equation to design an inverting summing amplifier.

Example 5.3 Designing a Summing Amplifier

1. Design a summing amplifier (see Fig. 5.11) whose output voltage is

   vo=−4va−vb−5vc.

   Use an ideal op amp with ±12 V power supplies and a 20 kΩ feedback resistor.

2. Suppose va=2 V and vc=−1 V. What range of input voltages for vb allows the op amp in this design to remain in its linear operating region?

3. Suppose va=2 V, vb=3 V, and vc=−1 V. Using the input resistor values found in part (a), how large can the feedback resistor be before the op amp saturates?

Solution

1. Use the summing-amplifier equation (Eq. 5.6) and the feedback resistor value to find the three input resistor values:

   −RfRa=−4soRa=20 k4=5 kΩ;−RfRb=−1soRb=20 k1=20 kΩ;−RfRc=−5soRc=20 k5=4 kΩ.

   The resulting circuit is shown in Fig. 5.12.

Figure 5.12 The summing amplifier for Example 3.3(a).

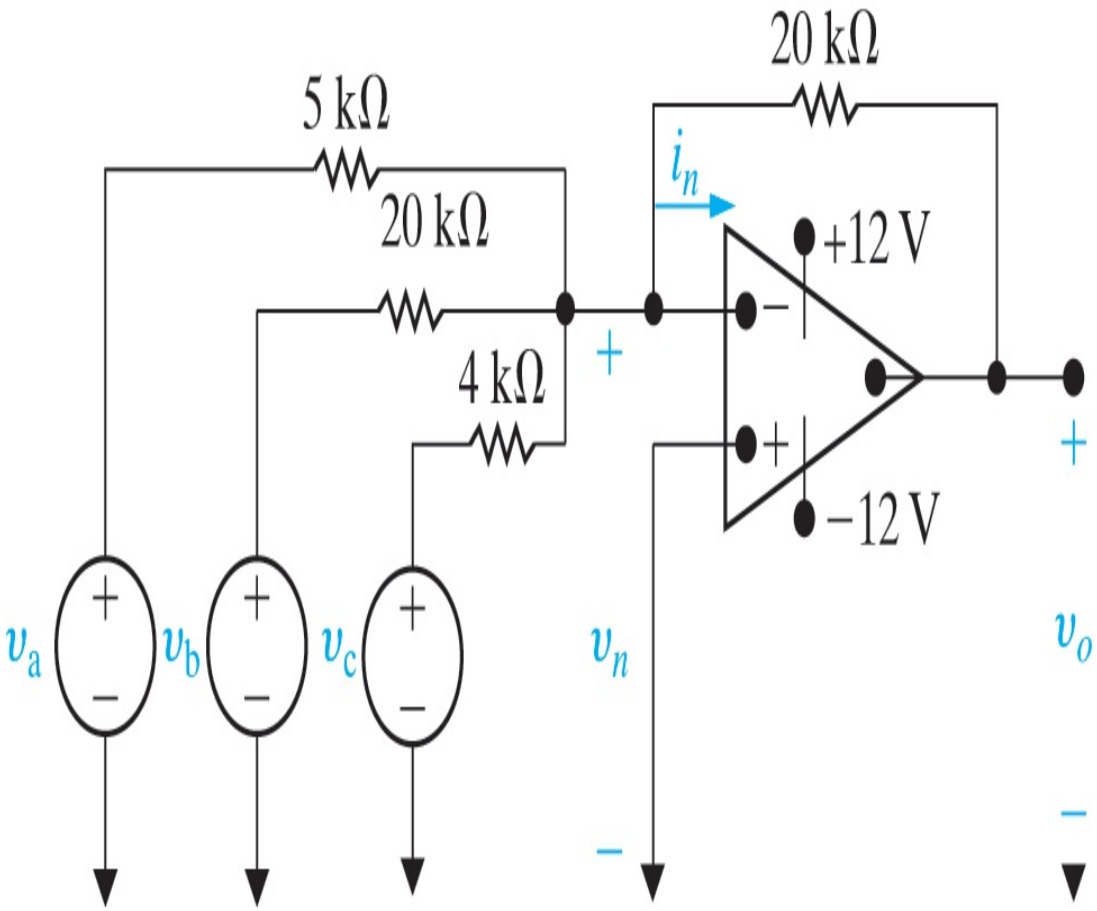


Figure 5.12 Full Alternative Text

2. Substitute the values for va and vc into the equation for vo given in the problem statement to get

   vo=−4(2)−vb−5(−1)=−3−vb.

   Solving this equation for vb in terms of vo gives

   vb=−3−vo.

   Now substitute the two power supply voltages for the output voltage to

find the range of vb values that keeps the op amp in its linear region:

$-15\ \text{V} \le v_b \le 9\ \text{V}$.

3. Starting with the summing-amplifier equation, Eq. 5.6, substitute the input resistor values found in part (a) and the specified input voltage values. Remember that the feedback resistor is an unknown in this equation:

   $v_o = -\frac{R_f}{5000}(2) - \frac{R_f}{20{,}000}(3) - \frac{R_f}{4000}(-1) = -\frac{6R_f}{20{,}000}$.

   From this equation, it should be clear that if the op amp saturates, it will do so at its negative power supply value, −12 V. Using this voltage for v_o in the above equation and solving for the feedback resistance gives

   $R_f = 40\ \text{k}\Omega$.

   Given the specified input voltages, this is the largest value of feedback resistance that keeps the op amp in its linear region.

Assessment Problem

Objective 2—Be able to analyze simple circuits containing ideal op amps

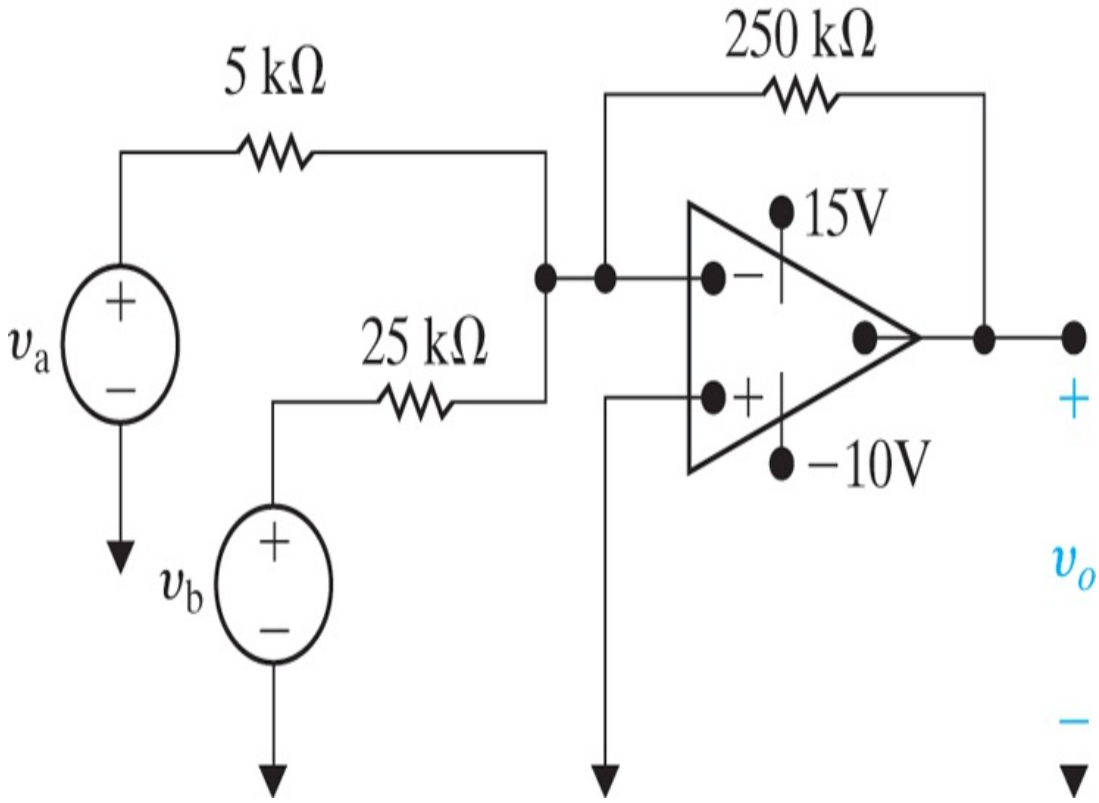
1. 5.3

   1. Find v_o in the circuit shown if $v_a = 0.1$ V and $v_b = 0.25$ V.

   2. If $v_b = 0.25$ V, how large can v_a be before the op amp saturates?

   3. If $v_a = 0.10$ V, how large can v_b be before the op amp saturates?

   4. Repeat (a), (b), and (c) with the polarity of v_b reversed.



5.4-4 Full Alternative Text

Answer:

1. −7.5 V;
2. 0.15 V;
3. 0.5 V;
4. −2.5, 0.25, and 2 V.

SELF-CHECK: Also try Chapter Problems 5.12, 5.14, and 5.16.

5.5 The Noninverting-Amplifier Circuit

Figure 5.13 depicts a noninverting-amplifier circuit. We use Analysis Method 5.1 to find the expression for the output voltage as a function of the source voltage vg. Starting with Step 1, we note that the noninverting-amplifier circuit has a negative feedback path with the resistor Rf, so we assume that the op amp is in its linear region. In Step 2, the voltage constraint equation (Eq. 5.2) tells us that vn=vp. However, the voltage at the noninverting terminal is not 0 because the terminal is not connected to the common node. The current constraint equation (Eq. 5.3) tells us the noninverting input current is zero. Since this current equals the current in the resistor Rs, there is no voltage drop across Rs and vp=vg. Therefore, vn=vg as well.

Figure 5.13 A noninverting amplifier.

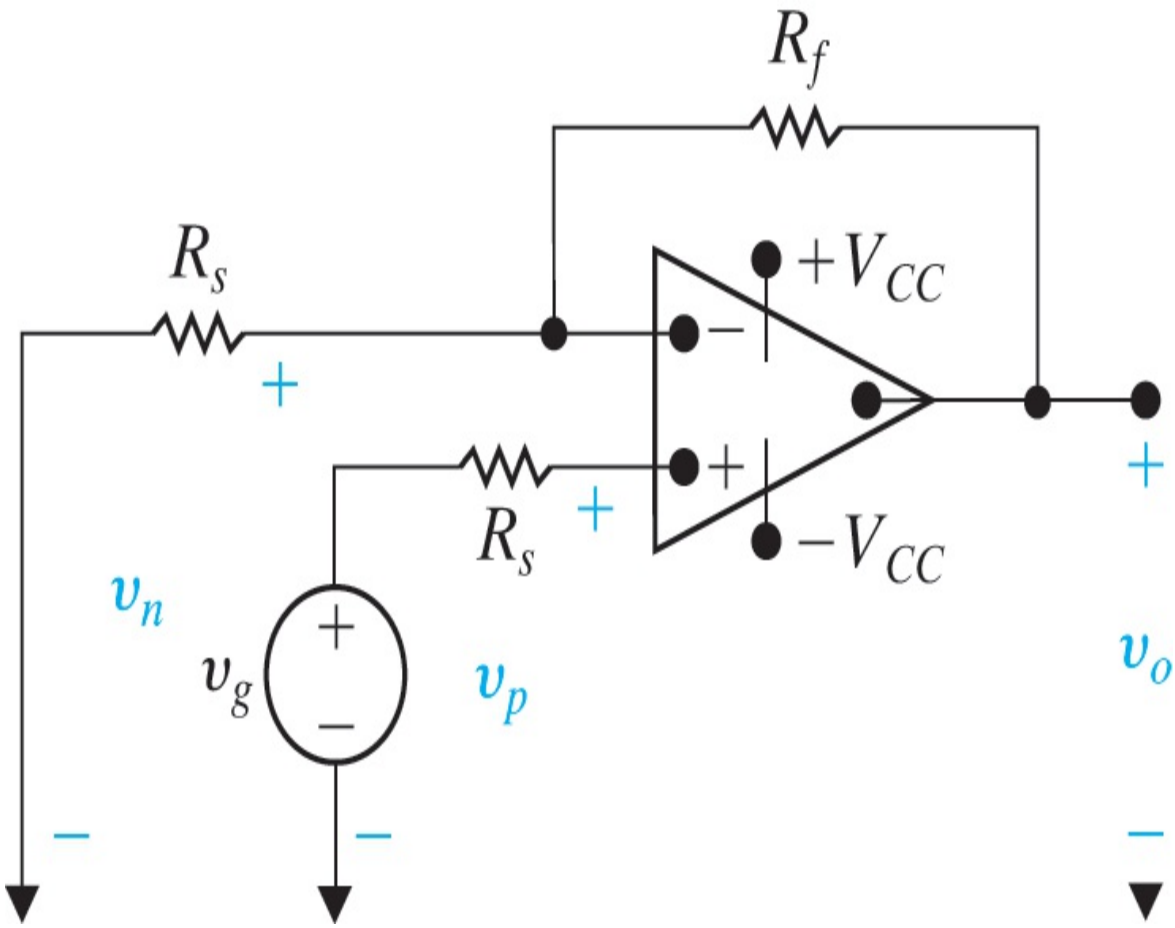


Figure 5.13 Full Alternative Text

In Step 3, we write a KCL equation at the noninverting terminal, using the result of Step 2 and the current constraint equation to give

vgRs+vg−voRf=0.

Solve the KCL equation for vo to get

Noninverting-Amplifier Equation

vo=Rs+RfRsvg.(5.7)

From Step 4 we know that keeping the op amp in its linear region requires that

Rs+RfRs<|VCCvg|.

Once again, the ideal op amp assumption allows us to express the output voltage as a function of the input voltage and the external resistors Rs and Rf.

Example 5.4 illustrates the design of a noninverting amplifier using realistic resistor values.

Example 5.4 Designing a Noninverting Amplifier

1. Design a noninverting amplifier (see Fig. 5.13) with a gain of 6. Assume the op amp is ideal.

2. Suppose we wish to amplify a voltage vg, where $-1.5\ \text{V} \le \text{vg} \le +1.5\ \text{V}$. What are the smallest power supply voltages that could be used with the resistors selected in part (a) to ensure that the op amp remains in its linear region?

Solution

1. Using the noninverting-amplifier equation (Eq. 5.7),

   vo=Rs+RfRsvg=6vg so Rs+Rf Rs =6.

   Therefore,

   Rs+Rf=6Rs, so Rf=5Rs.

   Look at the realistic resistor values listed in Appendix H. Let's choose Rf=10 kΩ, so Rs=2 kΩ. But there is not a 2 kΩ resistor in Appendix H. We can create an equivalent 2 kΩ resistor by combining two 1 kΩ resistors in series. We can use a third 1 kΩ resistor for Rg. The resulting circuit is shown in Fig. 5.14.

Figure 5.14 The noninverting amplifier design of Example 5.3.

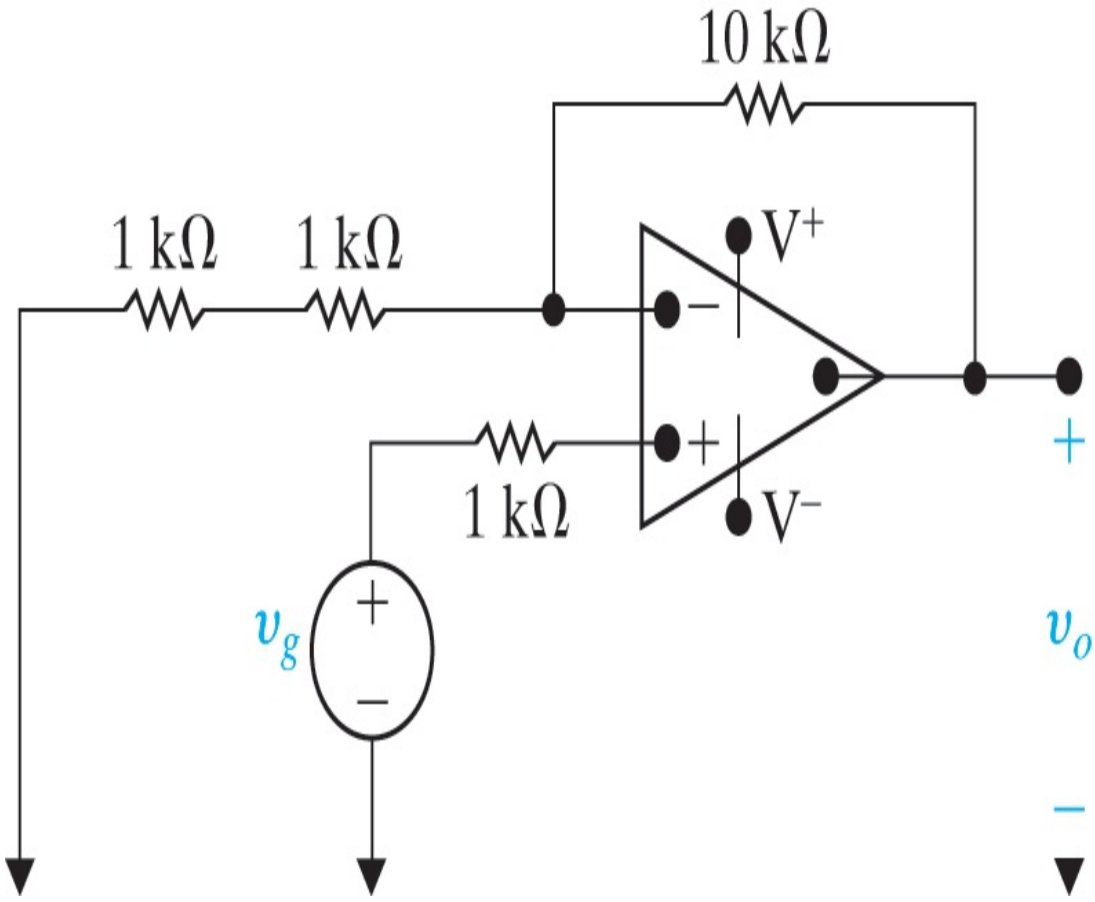


Figure 5.14 Full Alternative Text

2. Solve two different versions of the noninverting-amplifier equation for vo, first using vg=+1.5 V and then using vg=−1.5 V:

   vo=6(1.5)=9 V;vo=6(−1.5)=−9 V.

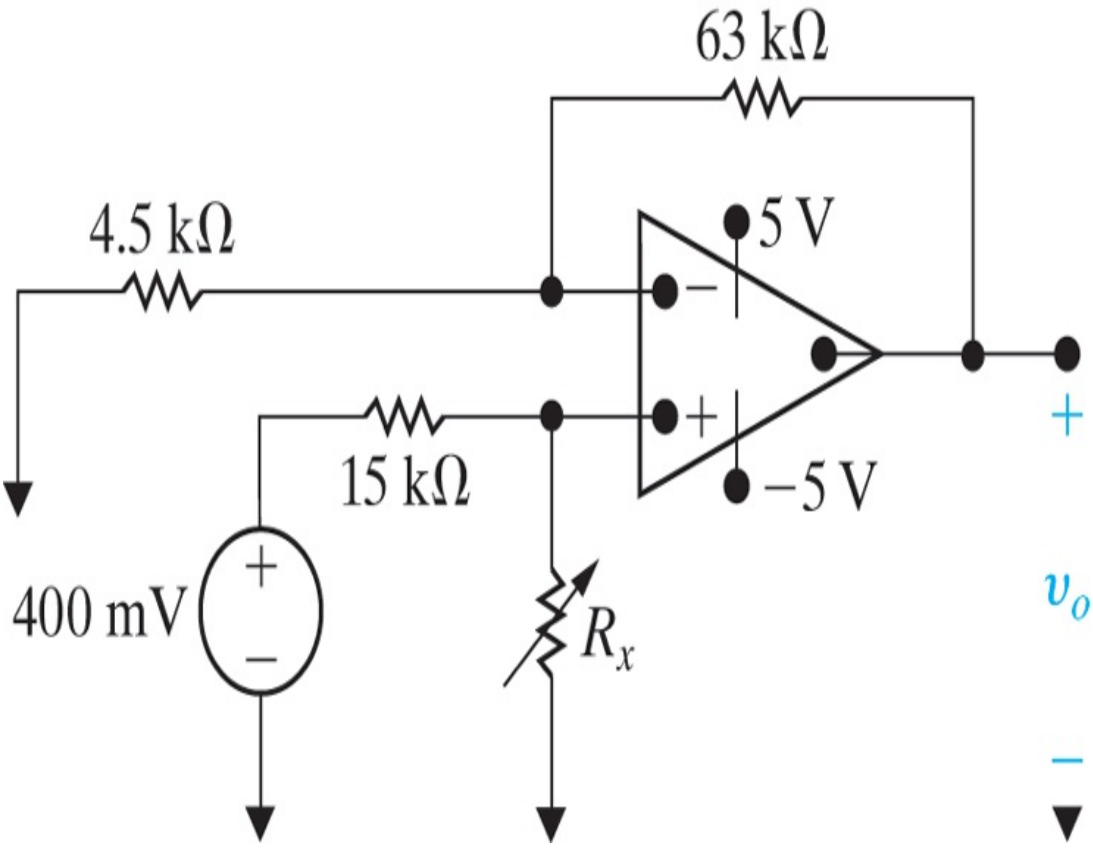
   Thus, if we use ±9 V power supplies for the noninverting amplifier designed in part (a) and −1.5 V≤vg≤ +1.5 V, the op amp will remain in its linear operating region. The circuit resulting from the analysis in

parts (a) and (b) is shown in Fig. 5.14, with V+=9 V and V−=−9 V.

Assessment Problem

Objective 2—Be able to analyze simple circuits containing ideal op amps

1. 5.4 Assume that the op amp in the circuit shown is ideal.
    1. Find the output voltage when the variable resistor is set to 60 kΩ.
    2. How large can Rx be before the amplifier saturates?



5.5-5 Full Alternative Text

Answer:

1. 4.8 V;

2. 75 kΩ.

SELF-CHECK: Also try Chapter Problems 5.18 and 5.20.

5.6 The Difference-Amplifier Circuit

The output voltage of a difference amplifier is proportional to the difference between the two input voltages. To demonstrate, we analyze the difference-amplifier circuit shown in Fig. 5.15, using Analysis Method 5.1. In Step 1, we note the negative feedback path that includes the resistor Rb and thereby assume the op amp is in its linear region. In Step 2, we use the voltage constraint equation (Eq. 5.2) to recognize that vn=vp. Now we need an expression for vp. Let's focus on the subcircuit at the noninverting input; employing the current constraint equation (Eq. 5.3), we note that there is no current into the noninverting input. Therefore, the current in the vb source remains in the loop containing that source and the resistors Rc and Rd. The voltage at the noninverting terminal is the voltage across Rd, which we can find using voltage division:

vp=RdRc+Rdvb=vn.

Figure 5.15 A difference amplifier.

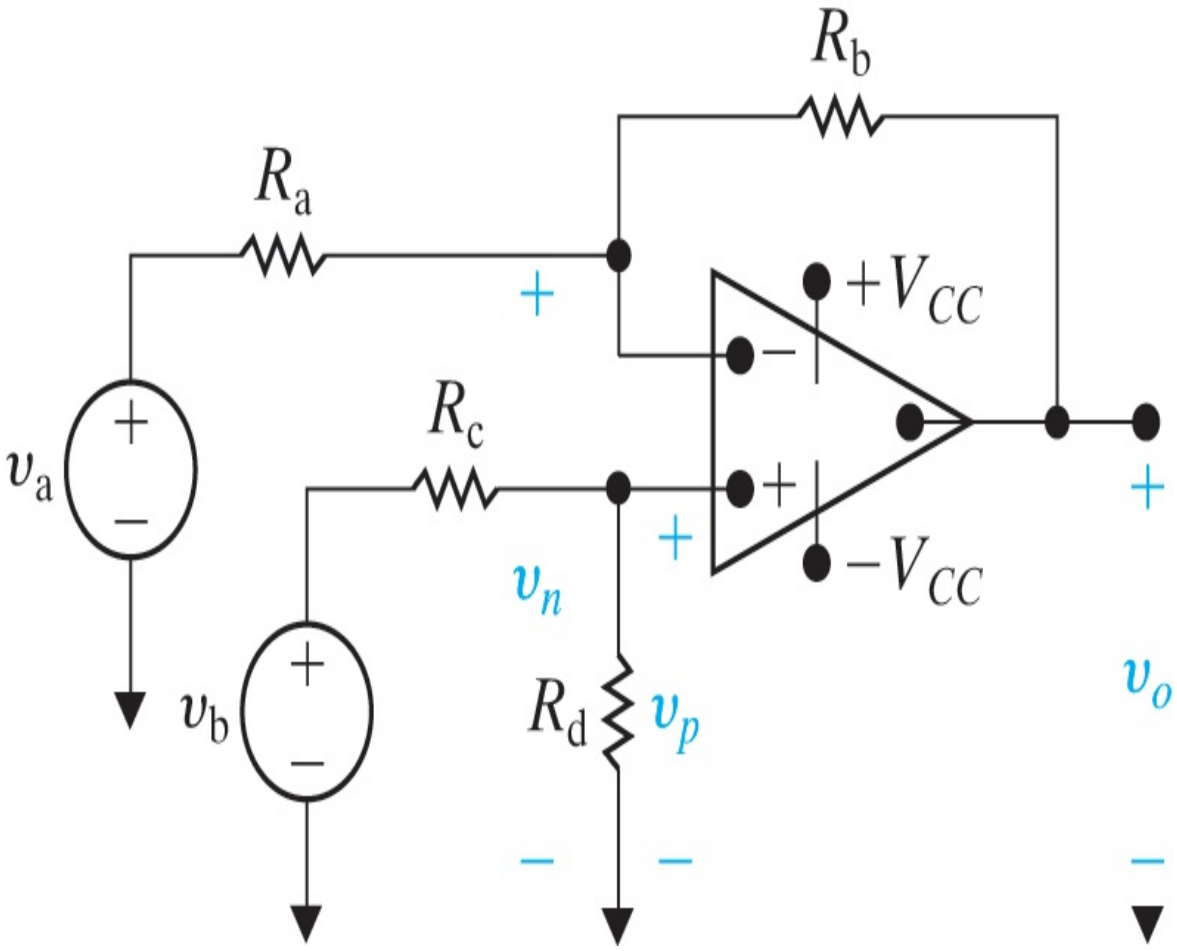


Figure 5.15 Full Alternative Text

At Step 3, write a KCL equation at the inverting terminal, employing the current constraint equation again to see that the current into the inverting terminal is zero:

vn−vaRa+vn−voRb=0.

Solving the KCL equation for vo as a function of both va and vn, we get

vo=(Ra+RbRa)vn−(RbRa)va.

Substituting the equation for vn into the equation for vo gives the desired relationship:

Difference-Amplifier Equation

$$v_o=\frac{R_d(R_a+R_b)}{R_a(R_c+R_d)}v_b-\frac{R_b}{R_a}v_a. \quad (5.8)$$

Equation 5.8 shows that the output voltage is proportional to the difference between a scaled replica of vb and a scaled replica of va. In general, the scaling factor applied to vb is not the same as that applied to va. However, the scaling factor applied to each input voltage can be made equal by setting

$$\frac{R_a}{R_b}=\frac{R_c}{R_d}. \quad (5.9)$$

When Eq. 5.9 is satisfied, the expression for the output voltage reduces to

Simplified Difference-Amplifier Equation

$$v_o=\frac{R_b}{R_a}(v_b-v_a). \quad (5.10)$$

If Eq. 5.9 is satisfied, the output voltage is a scaled replica of the difference between the input voltages vb and va. As in the previous ideal amplifier circuits, the scaling is controlled by the external resistors. Furthermore, the relationship between the output voltage and the input voltages is not affected by connecting a nonzero load resistance across the output of the amplifier.

Follow Example 5.5 to design a difference amplifier using realistic resistor values.

Example 5.5 Designing a Difference Amplifier

1. Design a difference amplifier (see Fig. 5.15) that amplifies the

difference between two input voltages by a gain of 8, using an ideal op amp and ±8 V power supplies.

2. Suppose va=1 V in the difference amplifier designed in part (a). What range of input voltages for vb will allow the op amp to remain in its linear operating region?

Solution

1. Using the simplified difference-amplifier equation (Eq. 5.10),

   $v_o = \frac{R_b}{R_a}(v_b - v_a) = 8(v_b - v_a)$ so $\frac{R_b}{R_a} = 8$.

   We want two resistors whose ratio is 8. Look at the realistic resistor values listed in Appendix H. Let's choose $R_b = 12\ k\Omega$, so $R_a = 1.5\ k\Omega$, although there are many other possibilities. Note that the simplified difference-amplifier equation requires that

   $\frac{R_a}{R_b} = \frac{R_c}{R_d}$.

   A simple choice for Rc and Rd is $R_c = R_a = 1.5\ k\Omega$ and $R_d = R_b = 12\ k\Omega$. The resulting circuit is shown in Fig. 5.16.

Figure 5.16 The difference amplifier designed in Example 5.4.

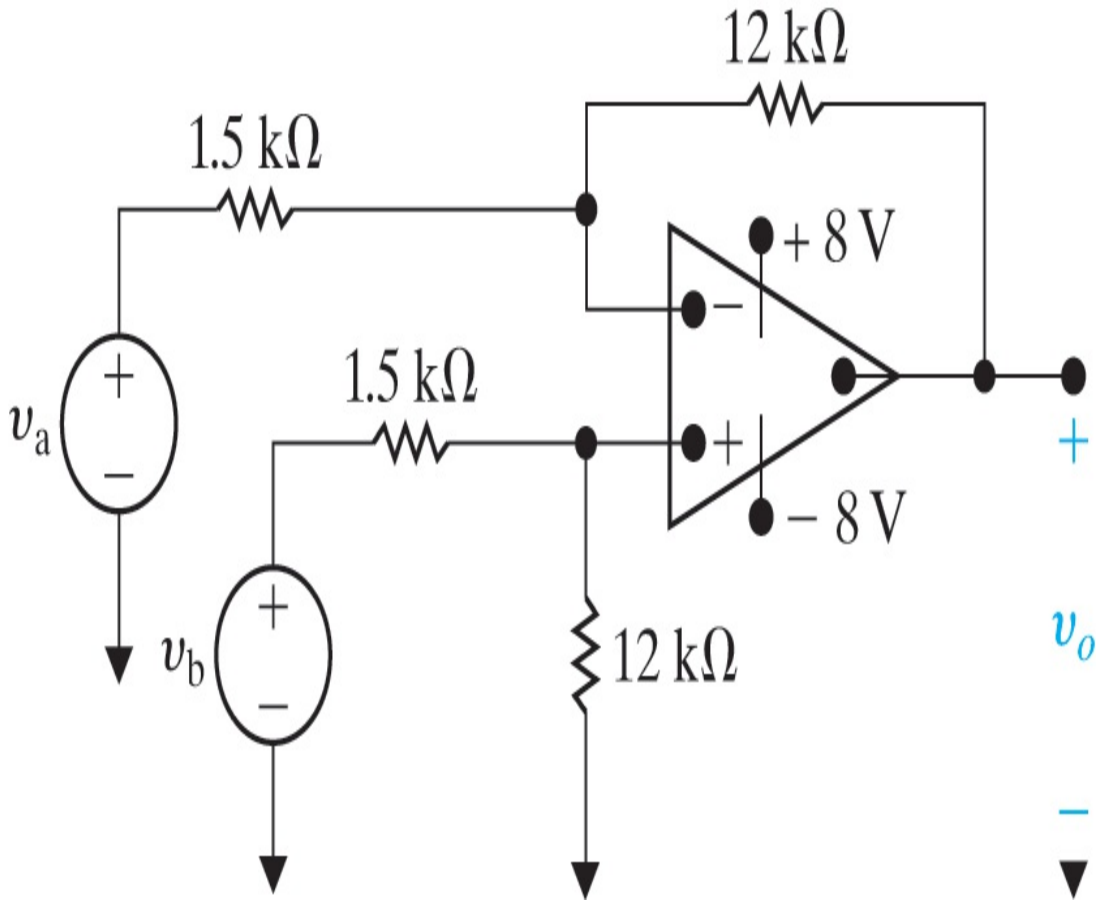


Figure 5.16 Full Alternative Text

2. Using va=1, solve two different versions of the simplified difference-amplifier equation (Eq. 5.10) for vb in terms of vo. Then substitute the two limiting values for the output voltage, vo=+8 V and vo=−8 V:

   vb=vo8+1=88+1=2 V;vb=vo8+1=−88+1=0 V.

   Thus, if va=1 V in the difference amplifier from part (a), the op amp will remain in its linear region if 0 V≤vb≤+2 V.

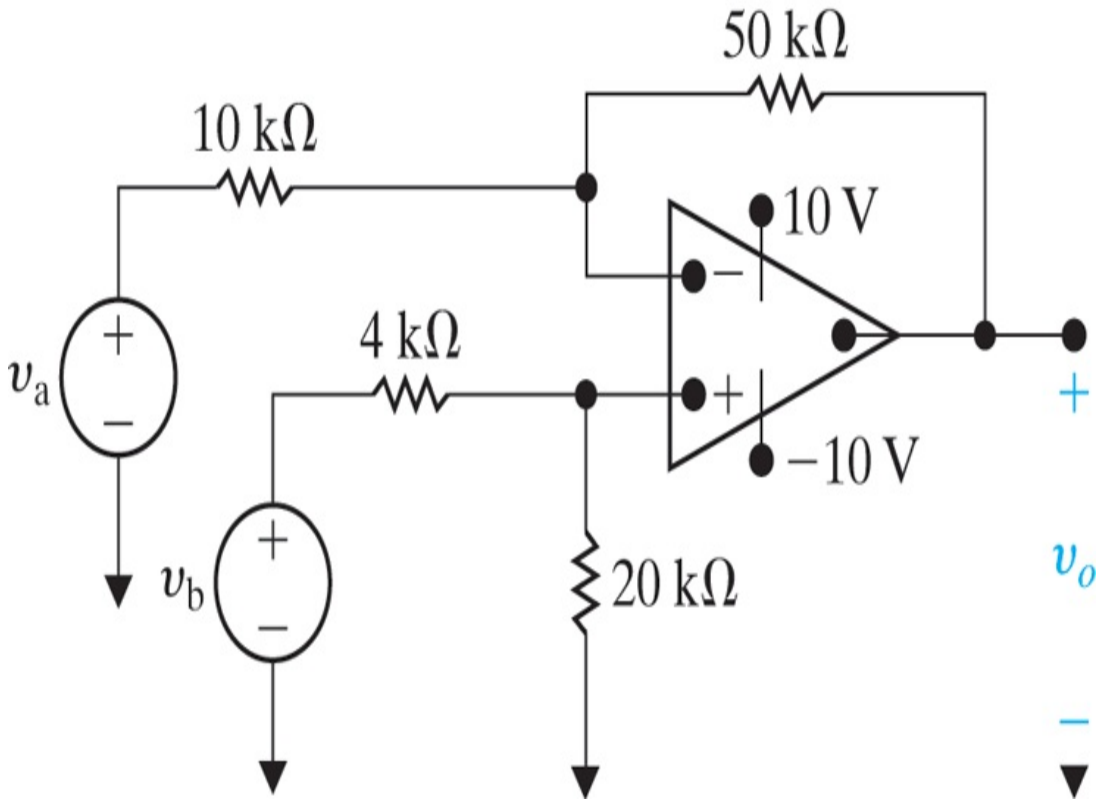
Assessment Problem

Objective 2—Be able to analyze simple circuits containing ideal op amps

1. 5.5

1. In the difference amplifier shown, vb=4.0 V. What range of values for va will result in linear operation?

2. Repeat (a) with the 20 kΩ resistor decreased to 8 kΩ.



5.6-6 Full Alternative Text

Answer:

1. 2 V≤va≤6 V;

2. 1.2 V≤va≤5.2 V.

SELF-CHECK: Also try Chapter Problems 5.25, 5.27, and 5.31.

The Difference Amplifier—Another Perspective

Let’s examine the difference-amplifier behavior more closely by redefining its inputs in terms of two other voltages. The first is the **differential mode** input, which is the difference between the two input voltages in Fig. 5.15:

$$v_{dm}=v_b-v_a. \quad (5.11)$$

The second is the **common mode** input, which is the average of the two input voltages in Fig. 5.15:

$$v_{cm}=(v_a+v_b)/2. \quad (5.12)$$

Using Eqs. 5.11 and 5.12, we can now represent the original input voltages, va and vb, in terms of the differential mode and common mode voltages, vdm and vcm:

$$v_a=v_{cm}-12v_{dm}, \quad (5.13)$$
$$v_b=v_{cm}+12v_{dm}. \quad (5.14)$$

Substituting Eqs. 5.13 and 5.14 into Eq. 5.8 gives the output of the difference amplifier in terms of the differential mode and common mode voltages:

$$v_o=\left[\ \frac{R_aR_d-R_bR_c}{R_a(R_c+R_d)}\ \right]v_{cm}+\left[\ \frac{R_d(R_a+R_b)+R_b(R_c+R_d)}{2R_a(R_c+R_d)}\ \right]v_{dm}=A_{cm}v_{cm}+A_{dm}v_{dm}, \quad (5.15)$$

where Acm is the common mode gain and Adm is the differential mode gain. Now, substitute Rc=Ra and Rd=Rb, which are possible values for Rc and Rd that satisfy Eq. 5.9, into Eq. 5.15:

$$v_o=(0)v_{cm}+\left(\frac{R_b}{R_a}\right)v_{dm}. \quad (5.16)$$

Thus, an ideal difference amplifier has Acm=0, amplifies only the differential mode portion of the input voltage, and eliminates the common mode portion of the input voltage. Figure 5.17 shows a difference-amplifier circuit with differential mode and common mode input voltages in place of va and vb.

Figure 5.17 A difference

amplifier with common mode and differential mode input voltages.

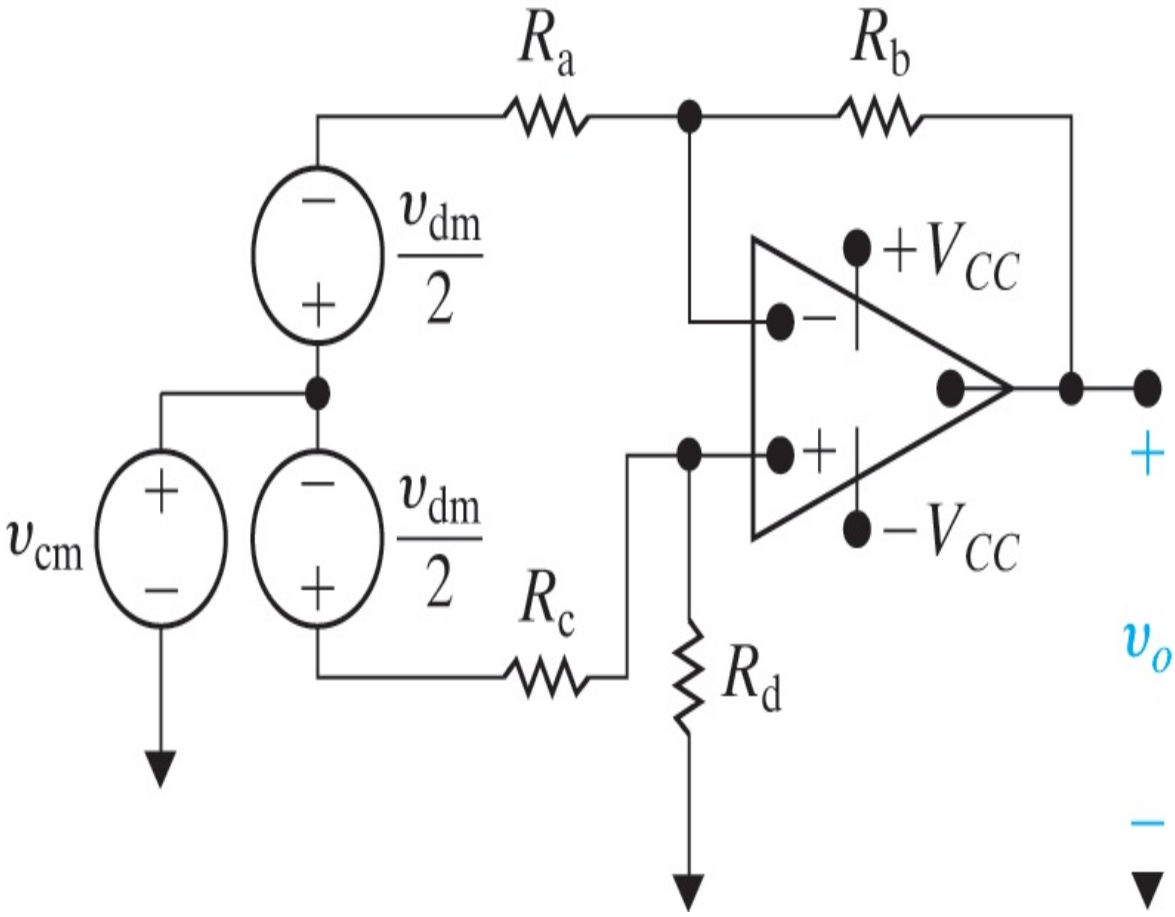


Figure 5.17 Full Alternative Text

Equation 5.16 provides an important perspective on the function of the difference amplifier. In many applications it is the differential mode signal that contains the information of interest, while the common mode signal represents the noise found in all electric signals. For example, an electrocardiograph electrode measures the voltages produced by your body that regulate your heartbeat. These voltages have very small magnitudes compared with the electrical noise that the electrode picks up from lights and

other electrical equipment in the room. The noise appears as the common mode portion of the measured voltage, whereas the heart rate voltages comprise the differential mode portion. Thus, an ideal difference amplifier (one whose resistors satisfy Eq. 5.9) amplifies only the voltage of interest and suppresses the noise.

Measuring Difference-Amplifier Performance—The Common Mode Rejection Ratio

An ideal difference amplifier has zero common mode gain and nonzero (and usually large) differential mode gain. Two factors influence the ideal common mode gain—resistance mismatches (that is, Eq. 5.9 is not satisfied) or a nonideal op amp (that is, Eqs. 5.2 and 5.3 are not satisfied). We focus first on how resistance mismatches affect the performance of a difference amplifier.

Suppose that resistor values are chosen that do not precisely satisfy Eq. 5.9. Instead, the relationship among the resistors Ra, Rb, Rc, and Rd is

$$RaRb=(1-\in)RcRd,$$

so

$$Ra=(1-\in)Rc \quad \text{and} \quad Rb=Rd,$$

or

$$Rd=(1-\in)Rb \quad \text{and} \quad Ra=Rc, \tag{5.17}$$

where $\in$ is a very small number. We can see the effect of this resistance mismatch on the common mode gain of the difference amplifier by substituting Eq. 5.17 into Eq. 5.15 and simplifying the expression for Acm:

$$Acm=Ra(1-\in)Rb-RaRbRa[Ra+(1-\in)Rb]=-\in RbRa+(1-\in)Rb\approx$$

$-\in RbRa+Rb.(5.18)$

The approximation in Eq. 5.18 is valid because $\in$ is very small, and therefore $(1-\in)$ is approximately 1. Note that when the resistors in the difference amplifier satisfy Eq. 5.9, $\in=0$ and Eq. 5.18 gives Acm=0.

Now calculate the effect of the resistance mismatch on the differential mode gain by substituting Eq. 5.17 into Eq. 5.15 and simplifying the expression for Adm:

$Adm=(1-\in)Rb(Ra+Rb)+Rb[Ra+(1-\in)Rb]2Ra[Ra+(1-\in)Rb]=RbRa[1-(\in/2)RaRa+(1-\in)Rb]\approx RbRa[1-(\in/2)RaRa+Rb].(5.19)$

We use the same rationale for the approximation in Eq. 5.19 as in the computation of Acm. When the resistors in the difference amplifier satisfy Eq. 5.9, $\in=0$ and Eq. 5.19 gives Adm=Rb/Ra.

The **common mode rejection ratio (CMRR)** measures a difference amplifier's performance. It is defined as the ratio of the differential mode gain to the common mode gain:

$CMRR=|AdmAcm|.(5.20)$

The larger the CMRR, the closer the difference amplifier's behavior is to ideal. We can see the effect of resistance mismatch on the CMRR by substituting Eqs. 5.18 and 5.19 into Eq. 5.20:

$CMRR\approx| RbRa[1-(Ra\in/2)/(Ra+Rb)]-\in Rb/(Ra+Rb) |\approx| Ra(1-\in/2)+Rb -\in Ra |\approx| 1+Rb/Ra-\in |.(5.21)$

From Eq. 5.21, if the resistors in the difference amplifier are matched, $\in=0$ and CMRR= ∞. Even if the resistors are mismatched, we can minimize the impact of the mismatch by making the differential mode gain (Rb/Ra) very large, thereby making the CMRR large.

The second reason for nonzero common mode gain is a nonideal op amp. Remember that the op amp is itself a difference amplifier because in the linear operating region, its output is proportional to the difference of its

inputs; that is, $v_o = A(v_p - v_n)$. The output of a nonideal op amp is not strictly proportional to the difference between the inputs (the differential mode input) but also includes a common mode signal. Internal mismatches in the integrated circuit components make the behavior of the op amp nonideal, in the same way that the resistor mismatches in the difference-amplifier circuit make its behavior nonideal. Even though a discussion of nonideal op amps is beyond the scope of this text, note that the CMRR is used to rate op amps in practice by assessing how nearly ideal an op amp's behavior is.

Example 5.6 examines how resistor mismatches affect the CMRR of a difference amplifier.

Example 5.6 Calculating the CMRR

1. Suppose the Rc resistor in the difference amplifier designed in Example 5.5, shown in Fig. 5.16, is 10% larger than its nominal value. All other resistor values are unchanged. Calculate the common mode gain, the difference mode gain, and the CMRR for the difference amplifier.

2. Repeat part (a) assuming the Rd resistor value is 10% larger than its nominal value and all other resistor values are unchanged.

Solution

1. Use the common mode gain equation in Eq. 5.15 with $R_c = 1500(1.1) = 1650\ \Omega$ to get

   $$A_{cm} = \frac{(1500)(12{,}000) - (12{,}000)(1650)}{1500(1650 + 12{,}000)} = -0.0879.$$

   Then use the difference mode gain equation in Eq. 5.15 with $R_c = 1500(1.1) = 1650\ \Omega$ to get

   $$A_{dm} = 12{,}000(1500 + 12{,}000) + 12{,}000(1650 + 12{,}000)2(1500)(1650 + 12{,}000) = 7.956.$$

The CMRR (Eq. 5.20) is thus

CMRR=|7.956−0.0879|=90.5.

2. Use the common mode gain equation in Eq. 5.15 with Rd=12,000(1.1)=13,200 Ω to get

   Acm=(1500)(13,200)−(12,000)(1500)1500(1500+13,200)=0.08163.

   Then use the difference mode gain equation in Eq. 5.15 with Rd=12,000(1.1)=13,200 Ω to get

   Adm=13,200(1500+12,000)+12,000(1500+13,200)2(1500)(1500+13,200)=8.0408.

   The CMRR (Eq. 5.20) is thus

   CMRR=|8.04080.08163|=98.5.

Assessment Problem

Objective 2—Be able to analyze simple circuits containing ideal op amps

1. 5.6 Suppose the 12 kΩ resistor Rd in the difference amplifier in Fig. 5.16 is replaced by a variable resistor.

   What range of Rd values will ensure the difference amplifier has a CMRR≥ |100|?

   Answer: 11 kΩ≤Rd≤13.18 kΩ.

SELF-CHECK: Assess your understanding of this material by trying Chapter Problems 5.32 and 5.33.

5.7 A More Realistic Model for the Operational Amplifier

We now consider a more realistic model, shown in Fig. 5.18, that predicts the performance of an op amp in its linear region of operation. This model includes three modifications to the ideal op amp: (1) a finite input resistance, Ri; (2) a finite open-loop gain, A; and (3) a nonzero output resistance, Ro.

Figure 5.18 An equivalent circuit for an operational amplifier.

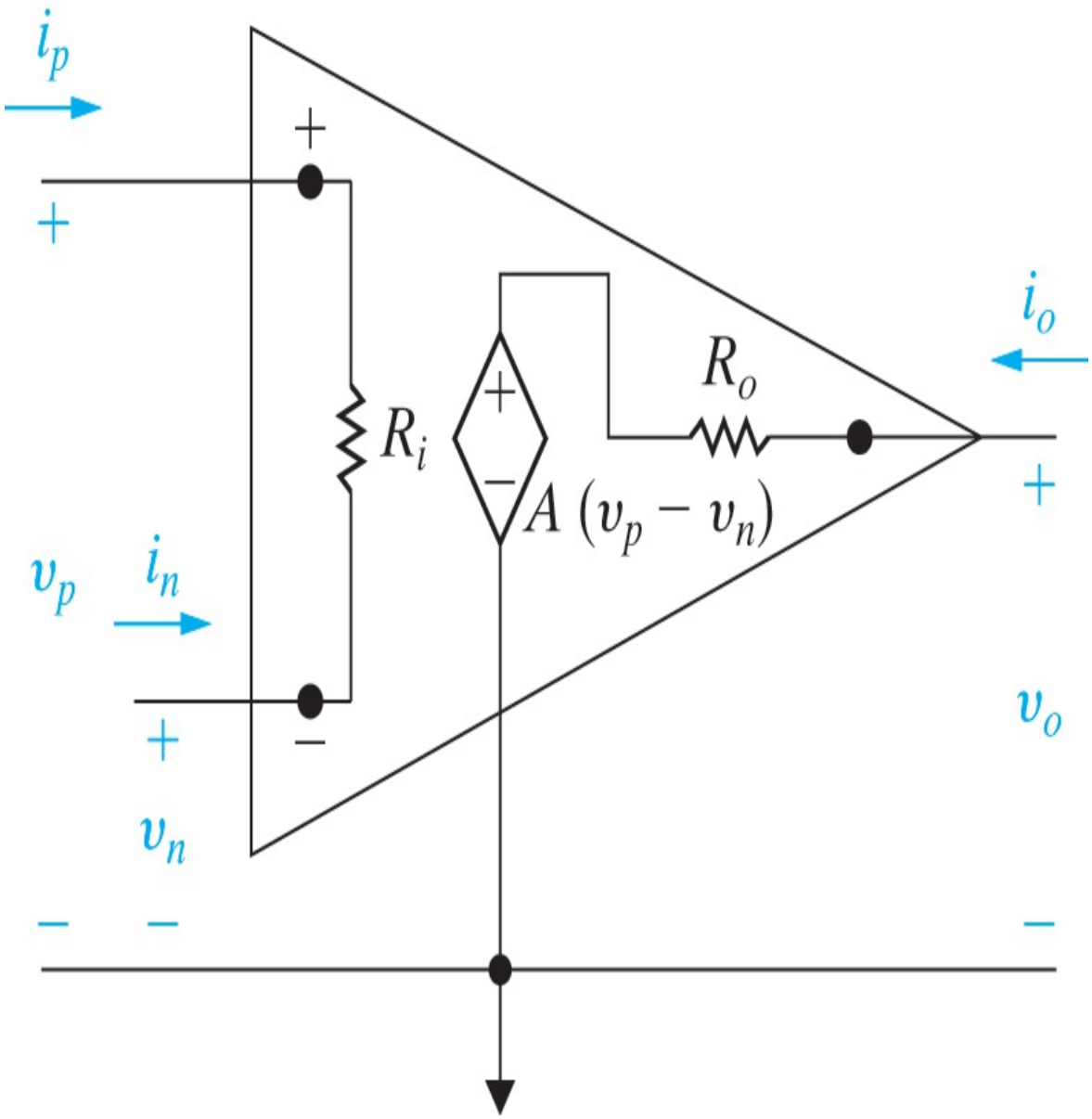


Figure 5.18 Full Alternative Text

When using the equivalent circuit shown in Fig. 5.18, the assumptions that vn=vp (Eq. 5.2) and in=ip=0 (Eq. 5.3) are invalid. Equation 5.1 is also invalid because the output resistance, Ro, is not zero.

Although the presence of A, Ri and Ro makes op amp circuit analysis more cumbersome, it remains straightforward. To illustrate, we use the equivalent circuit shown in Fig. 5.18 when analyzing both an inverting and a noninverting amplifier. We begin with the inverting amplifier.

Analyzing an Inverting-Amplifier Circuit Using a More Realistic Op Amp Model

The circuit for the inverting amplifier, using the op amp circuit shown in Fig. 5.18, is depicted in Fig. 5.19. We can find the output voltage, vo, as a function of the source voltage, vs, by writing two KCL equations at the nodes labeled a and b in Fig. 5.19. Note that vp=0 due to the external short-circuit connection at the noninverting input terminal. The equations are:

node a:vn−vsRs+vnRi+vn−voRf=0,node b:vo−vnRf+vo−A(−vn)Ro=0.

Figure 5.19 An inverting-amplifier circuit.

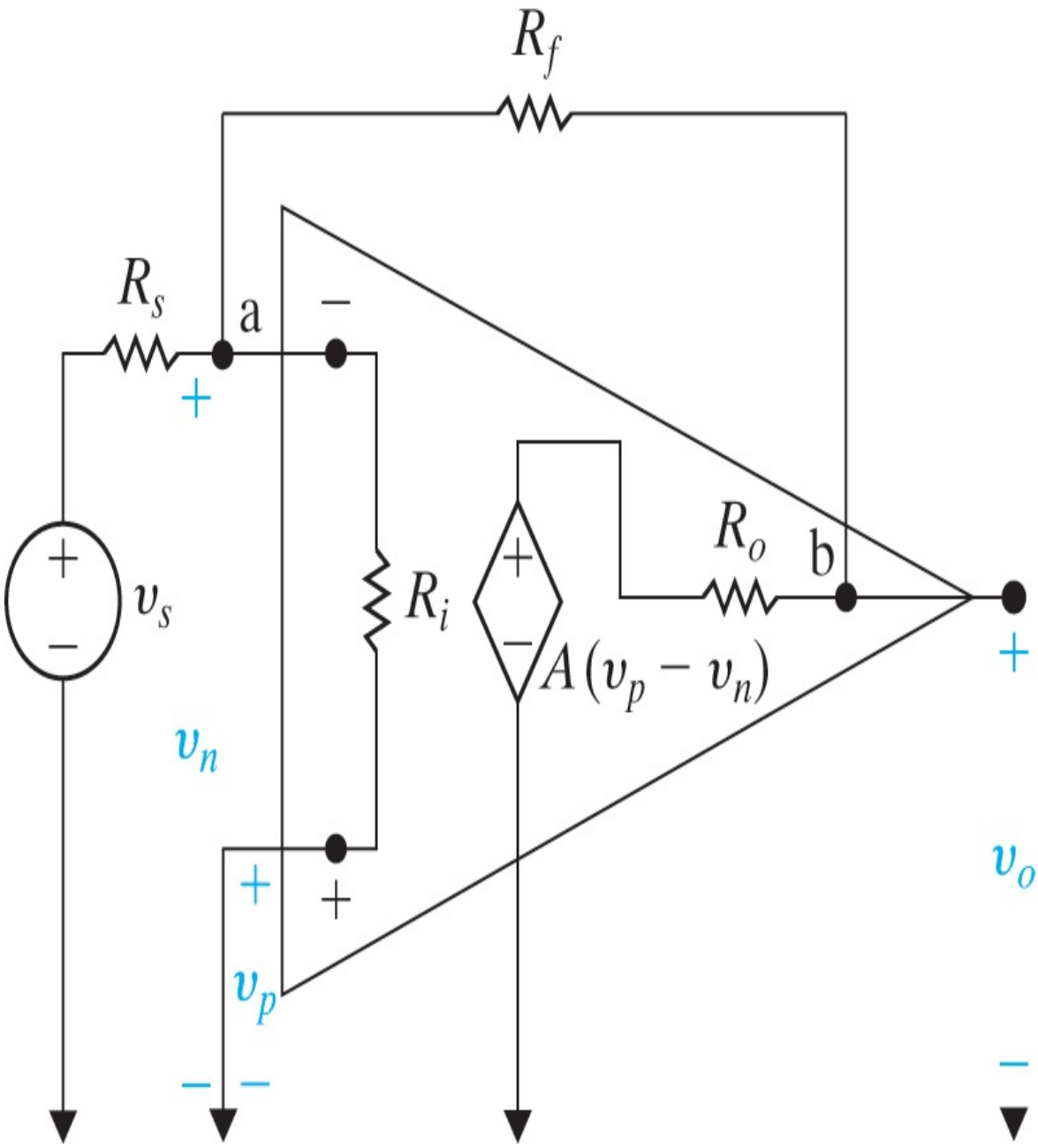


Figure 5.19 Full Alternative Text

We rearrange the KCL equations, preparing to use either back-substitution or Cramer's method to solve for vo:

(1Rs+1Ri+1Rf)vn−1Rfvo=1Rsvs,(ARo−1Rf)vn+(1Rf+1Ro)vo=0.

Solving for vo yields

$$v_o = \frac{-A + (R_o/R_f)}{R_s R_f (1 + A + R_o R_i) + (R_s R_i + 1) + R_o R_f} v_s. \quad (5.22)$$

Note that Eq. 5.22 reduces to Eq. 5.4 as $R_o \to 0$, $R_i \to \infty$, and $A \to \infty$.

If the inverting amplifier shown in Fig. 5.19 has a load resistance, RL, at its output terminal, the relationship between vo and vs is

$$v_o = \frac{-A + (R_o/R_f)}{R_s R_f (1 + A + R_o R_i + R_o R_L) + (1 + R_o R_L)(1 + R_s R_i) + R_o R_f} v_s.$$

Analyzing a Noninverting-Amplifier Circuit Using a More Realistic Op Amp Model

Using the equivalent circuit shown in Fig. 5.18 to analyze a noninverting amplifier, we obtain the circuit depicted in Fig. 5.20. Here, the voltage source vg in series with the resistance Rg represents the signal source. The resistor RL denotes the load on the amplifier. We derive an expression for vo as a function of vg by writing KCL equations at nodes a and b:

$$\text{node a:} \frac{v_n}{R_s} + \frac{v_n - v_g}{R_g} + \frac{v_n}{R_i} + \frac{v_n - v_o}{R_f} = 0, \quad (5.23)$$

$$\text{node b:} \frac{v_o - v_n}{R_f} + \frac{v_o}{R_L} + \frac{v_o - A(v_p - v_n)}{R_o} = 0. \quad (5.24)$$

Figure 5.20 A noninverting-amplifier circuit.

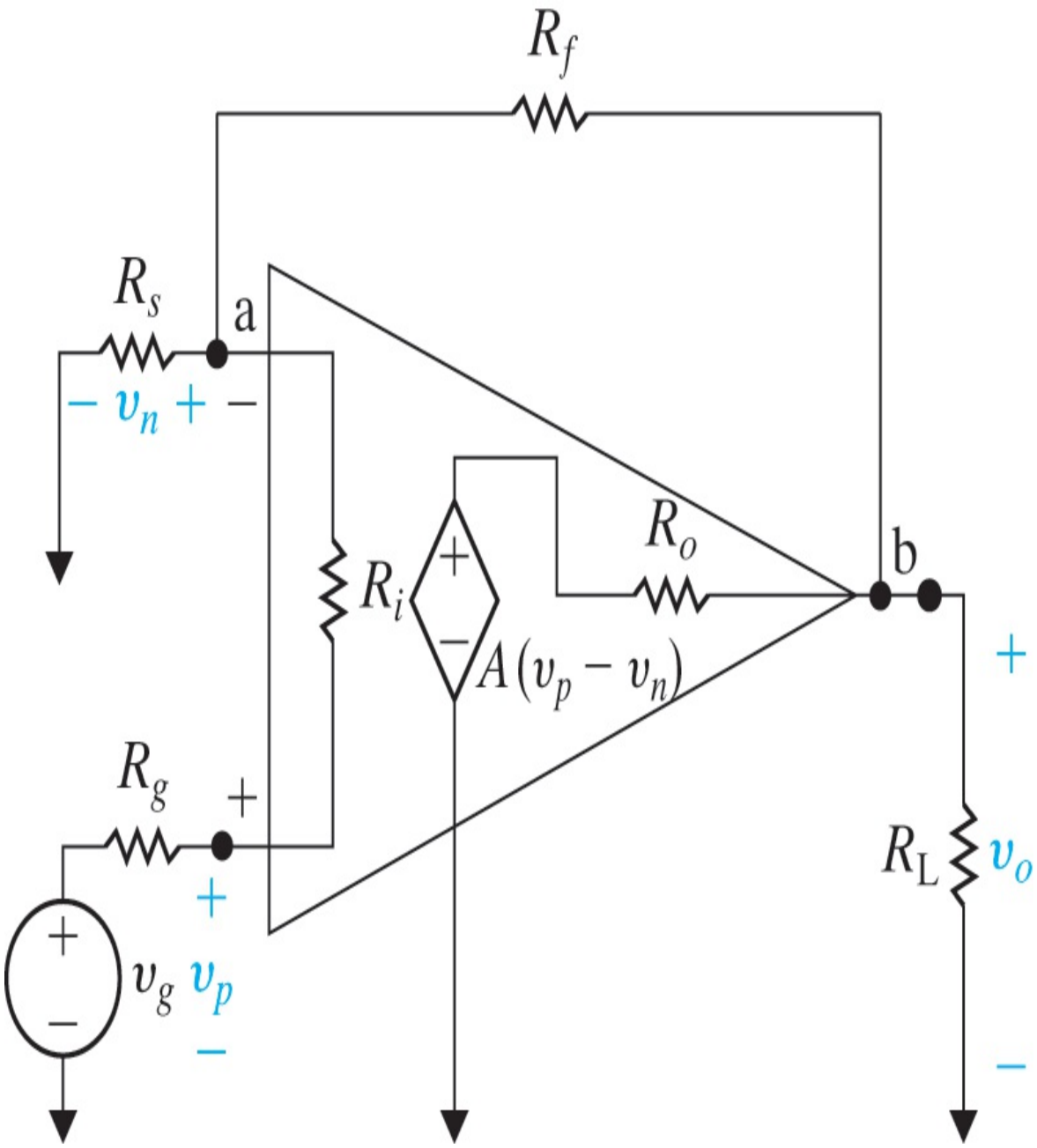


Figure 5.20 Full Alternative Text

The current in Rg is the same as in Ri, so

vp−vgRg=vn−vgRi+Rg. (5.25)

Use Eq. 5.25 to eliminate vp from Eq. 5.24, giving

vn(1Rs+1Rg+Ri+1Rf)−vo(1Rf)=vg(1Rg+Ri),vn[ARiRo(Ri+Rg)−1Rf

]+vo(1Rf+1Ro+1RL)=vg[ARiRo(Ri+Rg)].

Solving for vo yields

vo=[(Rf+Rs)+(RsRo/ARi)]vgRs+RoA(1+Kr)+RfRs+(Rf+Rs)(Ri+Rg)ARi, (5.26)

where

Kr=Rs+RgRi+Rf+RsRL+RfRs+RfRg+RgRsRiRL.

Note that Eq. 5.26 reduces to Eq. 5.7 when Ro → 0, A → ∞, and Ri → ∞. For the unloaded (RL=∞) noninverting amplifier, Kr reduces to (Rs+Rg)/Ri and the expression for vo becomes

vo=[(Rf+Rs)+RsRo/ARi]vgRs+RoA(1+Rs+RgRi)+1ARi[RfRs+(Rf+Rs)(Ri+Rg)].(5.27)

Example 5.7 analyzes a noninverting-amplifier circuit that employs the more realistic op amp model.

Example 5.7 Analyzing a Noninverting-Amplifier Circuit using a Realistic Op Amp Model

Here we analyze the noninverting amplifier designed in Example 5.5 using the realistic op amp model in Fig. 5.18. Assume that the open-loop gain A=50,000, the input resistance Ri=100 kΩ, and the output resistance Ro=7.5 kΩ. The circuit is shown in Fig. 5.21; note that there is no load resistance at the output.

Figure 5.21 The difference

amplifier from Example 5.5, using a realistic op amp model with A=50,000, Ri=100 kΩ, and Ro=7.5 kΩ.

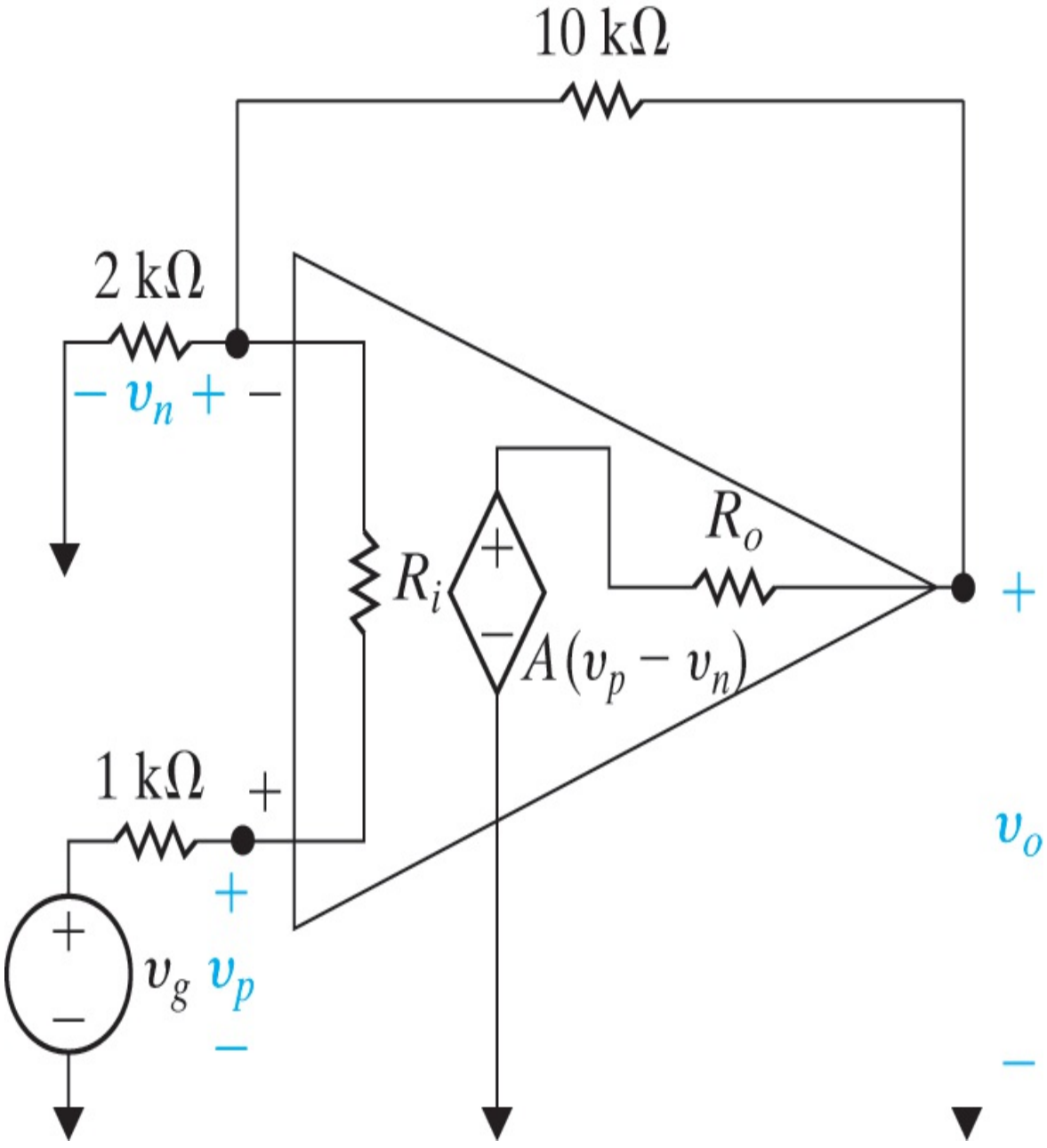


Figure 5.21 Full Alternative Text

1. Calculate the ratio of the output voltage to the source voltage, vo/vg.

2. Find the voltages at the op amp input terminals op amp, vn and vp, with respect to the common node, when vg=1 V.

3. Find the voltage difference at the op amp input terminals, (vp–vn), when vg=1 V.

4. Find the current in the signal source, ig, when the voltage of the source vg=1 V.

Solution

1. Using Eq. 5.27,

   vovg=10 k+2 k+(2 k)(7.5 k)(100 k)
   (50,000)2 k+7.5 k50,000(1+2 k+1 k100 k)+150,000(100 k)[(10 k)
   (2 k)+(10 k+2 k)(100 k+1 k)]=5.9988.

   Note how close this value is to the gain of 6 specified and achieved in Example 5.5 using the ideal op amp model.

2. From part (a), when vg=1 V, vo=5.9988 V. Now use Eq. 5.23 to solve for vn in terms of vo and vg:

   vn(12 k+11 k+100 k+110 k)
   =1100 k+1 k+5.998810 k;vn=0.999803 V.

   Use Eq. 5.25 to solve for vp:

   vpRg(vn−vg)Ri+Rg+vg=1 k(0.999803−1)100 k+1 k+1=0.999996 V.

3. Using the results from part (b), we find that the voltage difference at the op amp input terminals is

$v_p - v_n = 192.895\ \mu V$.

While this voltage difference is very small, it is not zero, as we assume when using the ideal op amp model.

4. The current in the signal source is the current in the resistor Rg. Using Ohm's law,

   $i_g = \frac{v_g - v_p}{R_g} = \frac{1 - 0.9999610}{1000} = 3.86\ nA$.

   This is also the current into the noninverting op amp terminal. It is very small but is not zero, as we assume when using the ideal op amp model.

Assessment Problem

Objective 3—Understand the more realistic model for an op amp

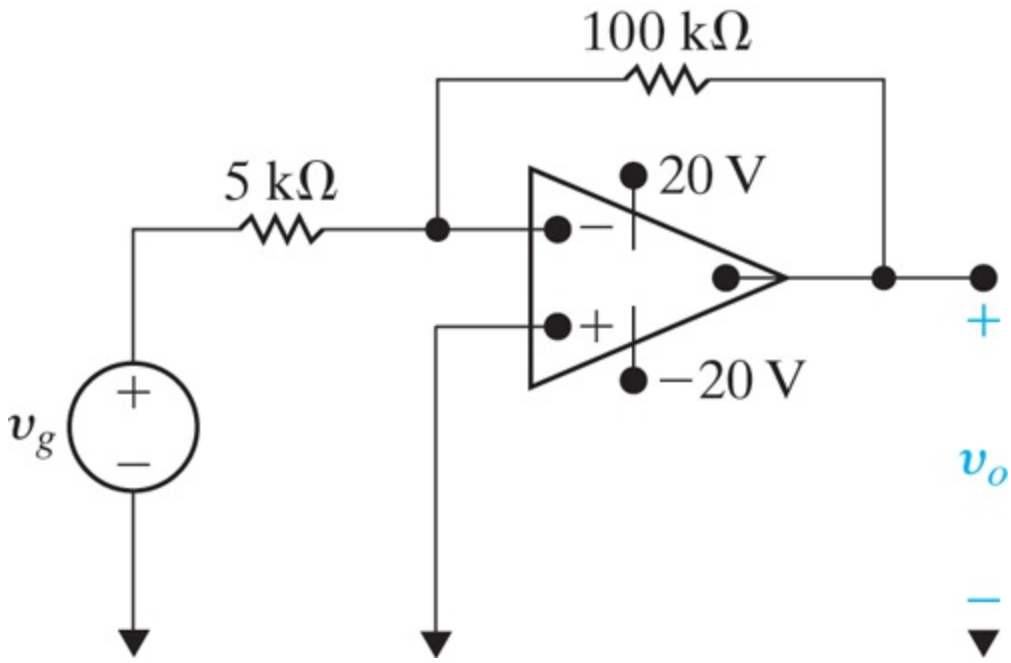
1. 5.7 The inverting amplifier in the circuit shown has an input resistance of 500 kΩ, an output resistance of 5 kΩ, and an open-loop gain of 300,000. Assume that the amplifier is operating in its linear region.

   1. Calculate the voltage gain (vo/vg) of the amplifier.

   2. Calculate the value of vn in microvolts when vg=1 V.

   3. Calculate the resistance seen by the signal source (vg).

   4. Repeat (a)–(c) using the ideal model for the op amp.



5.7-7 Full Alternative Text

Answer:

1. −19.9985;
2. 69.995 μV;
3. 5000.35 Ω;
4. −20, 0 μV, 5 kΩ.

SELF-CHECK: Also try Chapter Problems 5.44 and 5.48.

Practical Perspective

Strain Gages

Changes in the shape of elastic solids are of great importance to engineers who design structures such as aircraft frames that twist, stretch, or bend when

subjected to external forces. To use strain gages, you need information about the physical structure of the gage, methods of bonding the gage to the surface of the structure, and the orientation of the gage relative to the forces exerted on the structure. Strain gage measurements are important in many engineering applications, and knowledge of electric circuits is germane to their proper use.

The circuit shown in Fig. 5.22 provides one way to measure the change in resistance experienced by strain gages in applications like the one in the chapter opener. As we will see, this circuit is the familiar difference amplifier, with the strain gage bridge providing the two voltages whose difference is amplified. The pair of strain gages that are lengthened once the bar is bent have the values $R+\Delta R$ in the bridge feeding the difference amplifier, whereas the pair of strain gages that are shortened have the values $R-\Delta R$. We will analyze this circuit to discover the relationship between the output voltage, vo and the change in resistance, ΔR experienced by the strain gages.

Figure 5.22 An op amp circuit used for measuring the change in strain gage resistance.

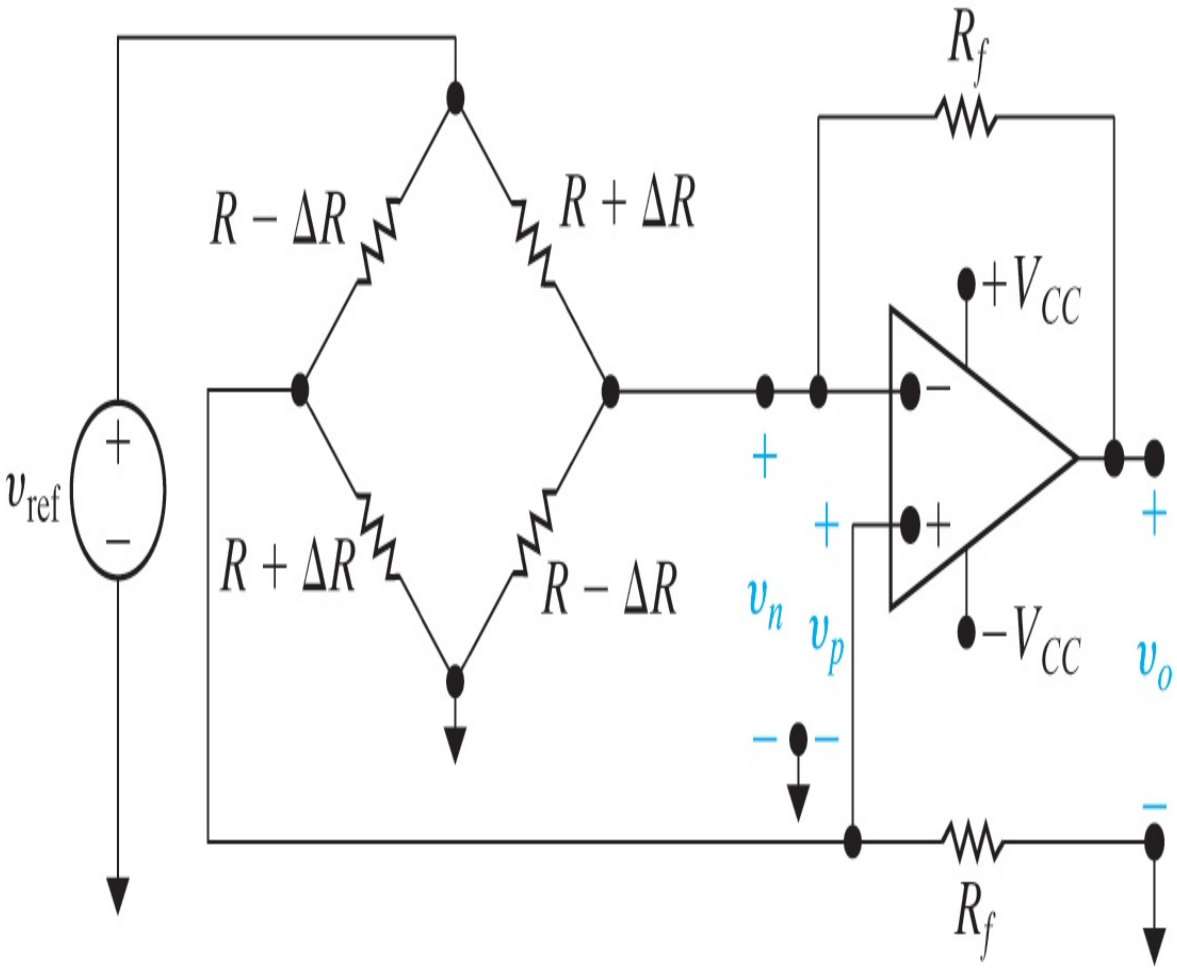


Figure 5.22 Full Alternative Text

To begin, assume that the op amp is ideal. Writing the KCL equations at the inverting and noninverting input terminals of the op amp, we see

vref−vnR + ΔR=vnR − ΔR+vn−voRf, (5.28)
vref− vpR − ΔR=vpR + ΔR+vpRf. (5.29)

Now rearrange Eq. 5.29 to get an expression for the voltage at the noninverting terminal of the op amp:

vp=vref(R−ΔR)(1R+ΔR+1R−ΔR+1Rf). (5.30)

As usual, we will assume that the op amp is operating in its linear region, so vp=vn, and the expression for vp in Eq. 5.30 must also be the expression for vn. We can thus substitute the right-hand side of Eq. 5.30 for vn in Eq. 5.28

and solve for vo. After some algebraic manipulation,

$$v_o = R_f \frac{(2\Delta R)R_2 - (\Delta R)^2}{} v_{ref}. \quad (5.31)$$

Because the change in resistance experienced by strain gages is very small, $(\Delta R) << R_2$ so $R_2 - (\Delta R)^2 \approx R_2$ and Eq. 5.31 becomes

$$v_o \approx R_f \frac{R_2}{} \Delta v_{ref},$$

where $\Delta = \Delta R/R$. By adjusting the value of Rf, we can amplify the small change in resistance to get a large, measurable output voltage, as long as the output voltage value is between the two power supply values.

SELF-CHECK: Assess your understanding of this Practical Perspective by trying Chapter Problem 5.49.

Summary

- The operational amplifier (op amp) is a complex electronic circuit with two input terminals, two power supply terminals and one output terminal. The voltage at the inverting input terminal is vn, the voltage at the noninverting input terminal is vp, and the voltage at the output terminal is vo, all with respect to a common node. The current into the inverting input terminal is in, while the current into the noninverting input terminal is ip. (See page 152.)

- The equation that defines the voltage transfer characteristic of an ideal op amp is

  $$vo=\{-VCC, A(vp-vn)<-VCC, A(vp-vn), -VCC\leq A(vp-vn)\leq +VCC, +VCC, A(vp-vn)>+VCC,$$

  where A is a proportionality constant known as the open-loop gain, and VCC represents the power supply voltages. (See page 153.)

- A feedback path between an op amp's output and its inverting input can constrain the op amp to its linear operating region where $vo=A(vp-vn)$. (See page 153.)

- A voltage constraint exists when the op amp is confined to its linear operating region due to typical values of VCC and A. If the ideal modeling assumptions are made—meaning A is assumed to be infinite—the ideal op amp model is characterized by the voltage constraint

  $$vp=vn.$$

  (See page 153.)

- A current constraint further characterizes the ideal op amp model, assuming the ideal input resistance of the op amp integrated circuit is infinite. This current constraint is given by

ip=in=0.

(See page 154.)

- To analyze an ideal op amp circuit, follow these steps:
    - Check for the presence of a negative feedback path; if it exists, we can assume the op amp is operating in its linear region.
    - Write a KCL equation at the inverting input terminal, using the input current constraint (Eq. 5.3), the value of vn, and Ohm's law to find the currents. This equation will usually contain the unknown voltage at the op amp's output terminal.
    - Solve the KCL equation and determine the voltage at the op amp's output terminal.
    - Compare the voltage at the op amp's output terminal to the power supply voltages to determine whether the op amp is actually in its linear region or whether it has saturated.
- An inverting amplifier is an op amp circuit producing an output voltage that is an inverted, scaled replica of the input. (See page 156.)
- A summing amplifier is an op amp circuit producing an output voltage that is a scaled sum of the input voltages. (See page 158.)
- A noninverting amplifier is an op amp circuit producing an output voltage that is a scaled replica of the input voltage. (See page 160.)
- A difference amplifier is an op amp circuit producing an output voltage that is a scaled replica of the input voltage difference. (See page 162.)
- The two voltage inputs to a difference amplifier can be used to calculate the common mode and difference mode voltage inputs, vcm and vdm. The output from the difference amplifier can be written in the form

  vo=Acmvcm+Admvdm,

where Acm is the common mode gain and Adm is the differential mode gain. (See page 164.)

- In an ideal difference amplifier, Acm=0. To measure how nearly ideal a difference amplifier is, we use the common mode rejection ratio:

  CMRR=|AdmAcm|.

  An ideal difference amplifier has an infinite CMRR. (See page 166.)

- We considered both a simple, ideal op amp model and a more realistic model in this chapter. The differences between the two models are as follows:

Simplified Model	More Realistic Model
Infinite input resistance	Finite input resistance
Infinite open-loop gain	Finite open-loop gain
Zero output resistance	Nonzero output resistance

(See page 167.)

Problems

Sections 5.1–5.2

1. 5.1 PSPICEMULTISIM The op amp in the circuit in Fig. P5.1 is ideal.

    1. Label the five op amp terminals with their names.

    2. What ideal op amp constraint determines the value of in? What is this value?

    3. What ideal op amp constraint determines the value of (vp−vn)? What is this value?

    4. Calculate vo.

Figure P5.1

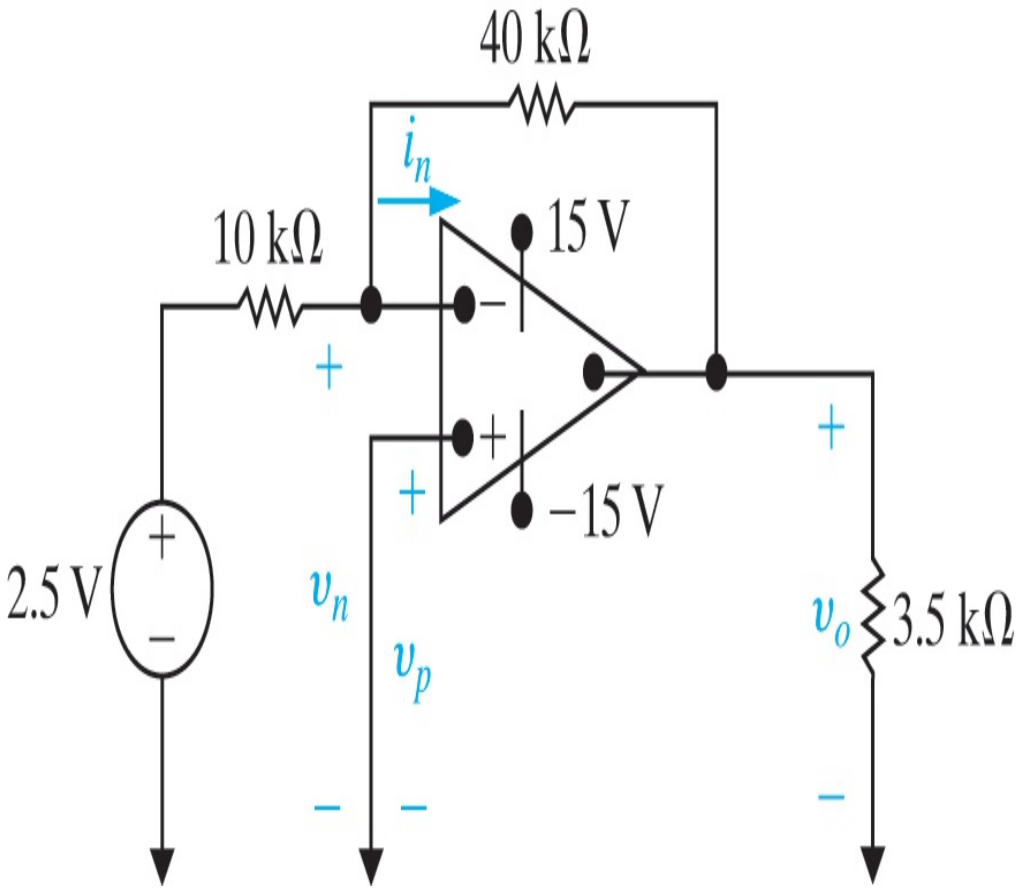


Figure P5.1 Full Alternative Text

2. 5.2

    1. Replace the 2.5 V source in the circuit in Fig. P5.1 and calculate vo for each of the following source values: -6 V, -3.5 V, -1.25 V, 1 V, 2.4 V, 5.4 V.

    2. Specify the range of voltage source values that will not cause the op amp to saturate.

3. 5.3 PSPICEMULTISIM Find io in the circuit in Fig. P5.3 if the op amp is ideal.

Figure P5.3

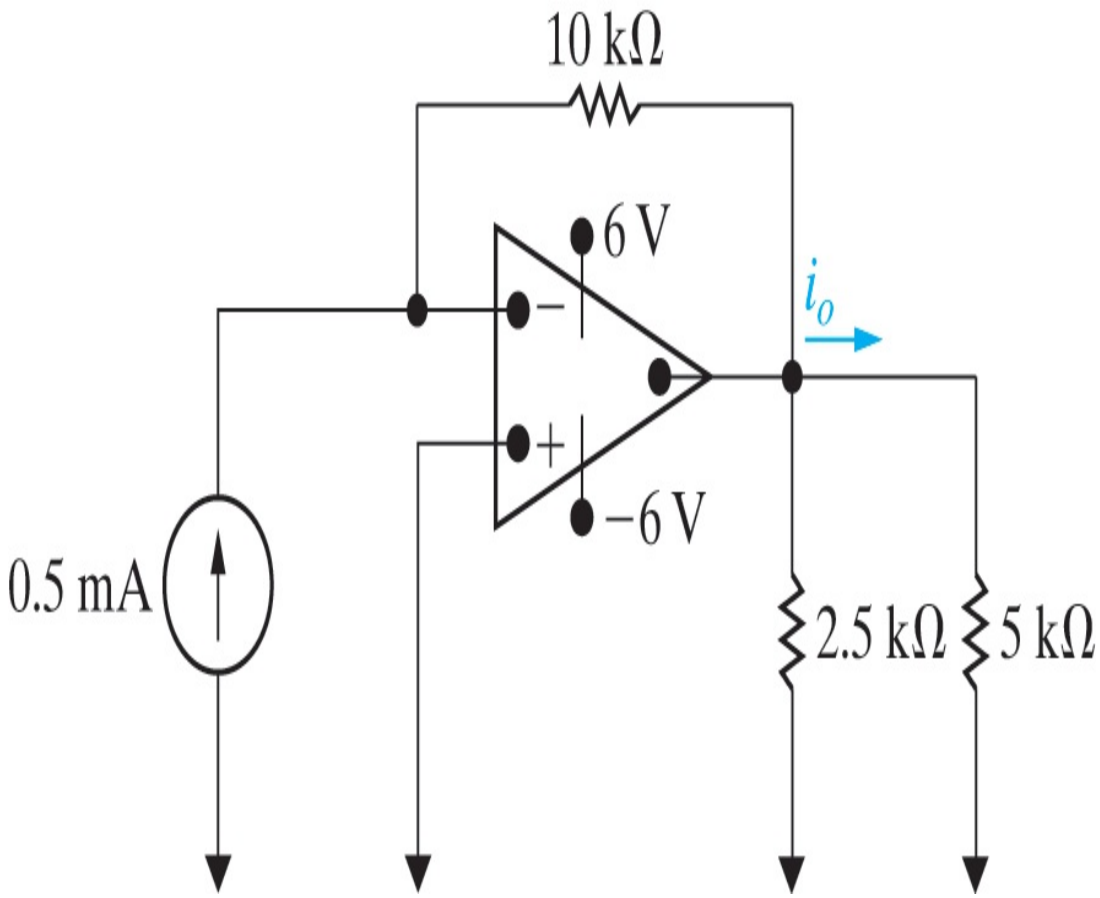


Figure P5.3 Full Alternative Text

4. 5.4 PSPICEMULTISIM The op amp in the circuit in Fig. P5.4 is ideal.
    1. Calculate vo if va=4 V and vb=0 V.
    2. Calculate vo if va=2 V and vb=0 V.
    3. Calculate vo if va=2 V and vb=1 V.
    4. Calculate vo if va=1 V and vb=2 V.
    5. If vb=1.6 V, specify the range of va such that the amplifier does not saturate.

Figure P5.4

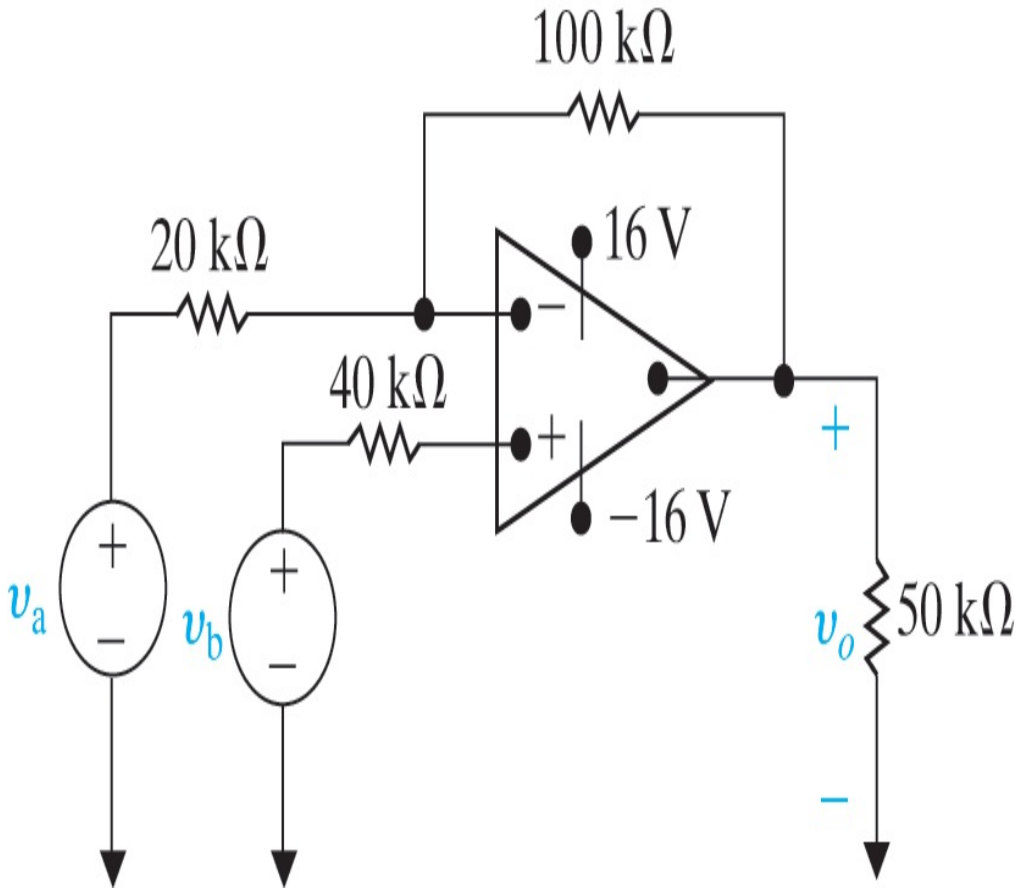


Figure P5.4 Full Alternative Text

5. 5.5 PSPICEMULTISIM The op amp in the circuit in Fig. P5.5 is ideal. Calculate the following:

    1. ia;
    2. va;
    3. vo;
    4. io.

Figure P5.5

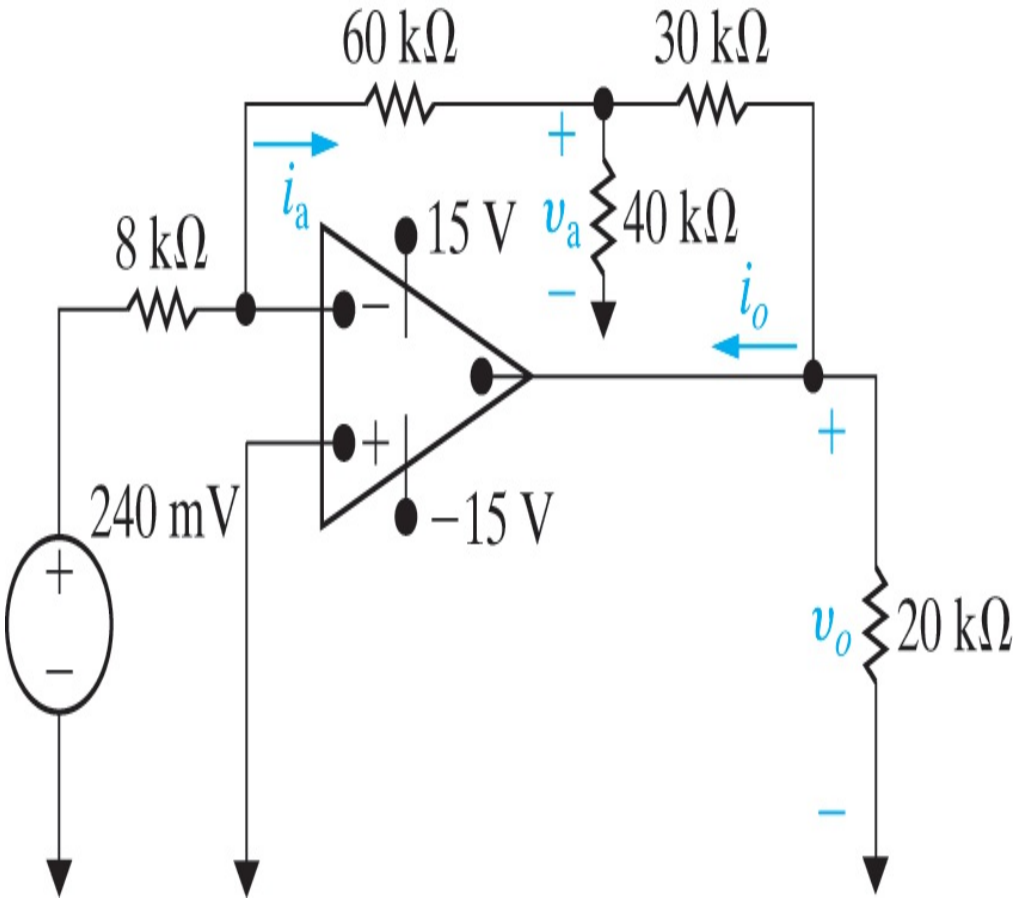


Figure P5.5 Full Alternative Text

6. 5.6 PSPICEMULTISIM Find iL (in milliamperes) in the circuit in Fig. P5.6.

Figure P5.6

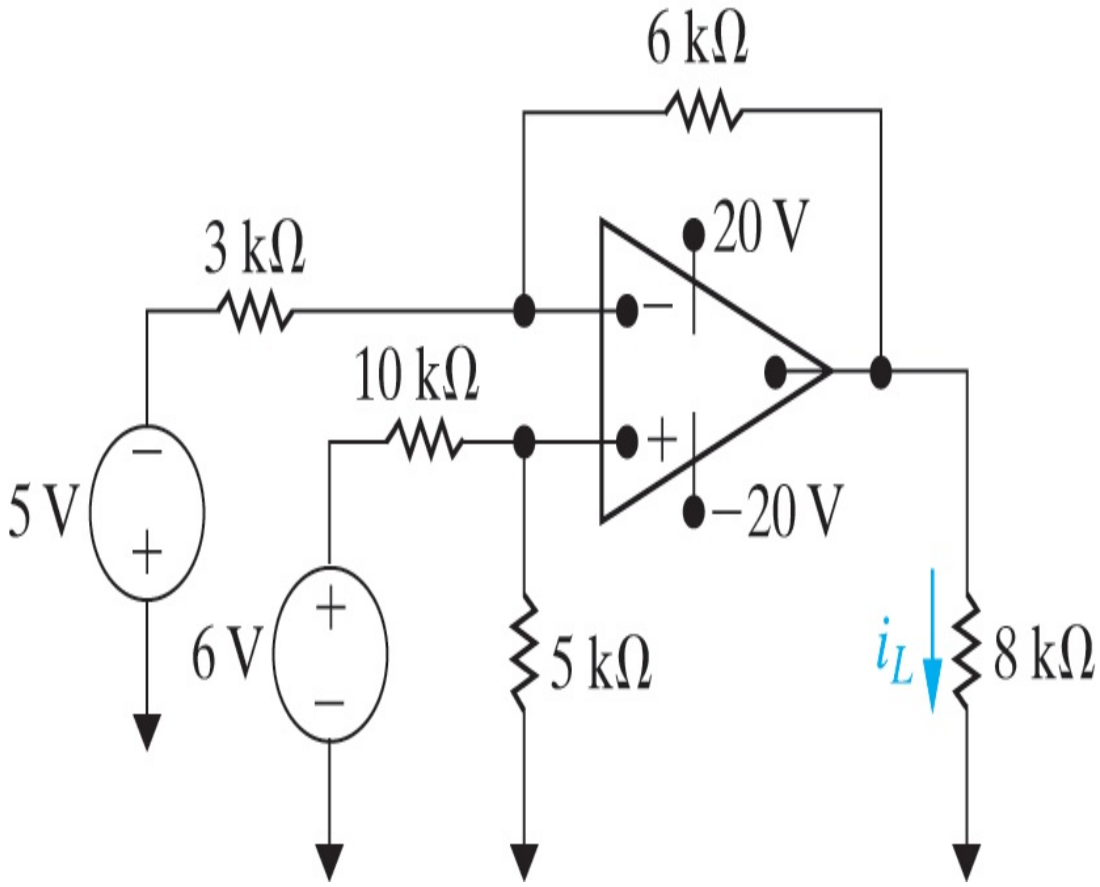


Figure P5.6 Full Alternative Text

7. 5.7 PSPICEMULTISIM A voltmeter with a full-scale reading of 10 V is used to measure the output voltage in the circuit in Fig. P5.7. What is the reading of the voltmeter? Assume the op amp is ideal.

Figure P5.7

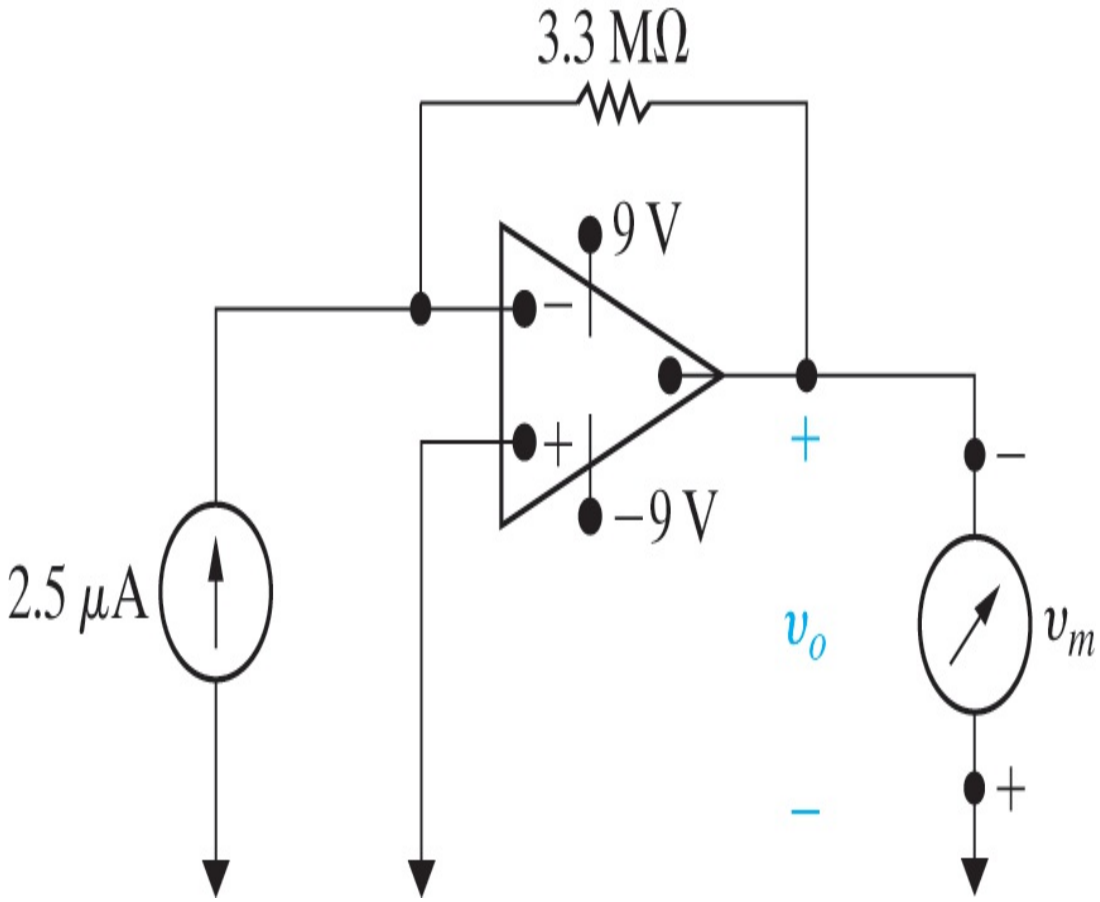


Figure P5.7 Full Alternative Text

Section 5.3

1. 5.8 DESIGNPROBLEM_

    1. Design an inverting amplifier with a gain of 2.5, using an ideal op amp. Use a set of identical resistors from Appendix H.

    2. If you wish to amplify signals between -2 V and +3 V using the circuit you designed in part (a), what are the smallest power supply voltages you can use?

2. 5.9

    1. Design an inverting amplifier with a gain of 4. Use an ideal op amp, a 30 kΩ resistor in the feedback path, and ±12 V power supplies.

    2. Using your design from part (a), determine the range of input voltages that will keep the op amp in its linear operating region.

    3. Suppose you wish to amplify a 2 V signal, using your design from part (a) with a variable feedback resistor. What is the largest value of feedback resistance that keeps the op amp in its linear operation region? Using this resistor value, what is the new gain of the inverting amplifier?

3. 5.10 PSPICEMULTISIM The op amp in the circuit in Fig. P5.10 is ideal.

    1. Find the range of values for σ in which the op amp does not saturate.

    2. Find io (in microamperes) when σ=0.272.

Figure P5.10

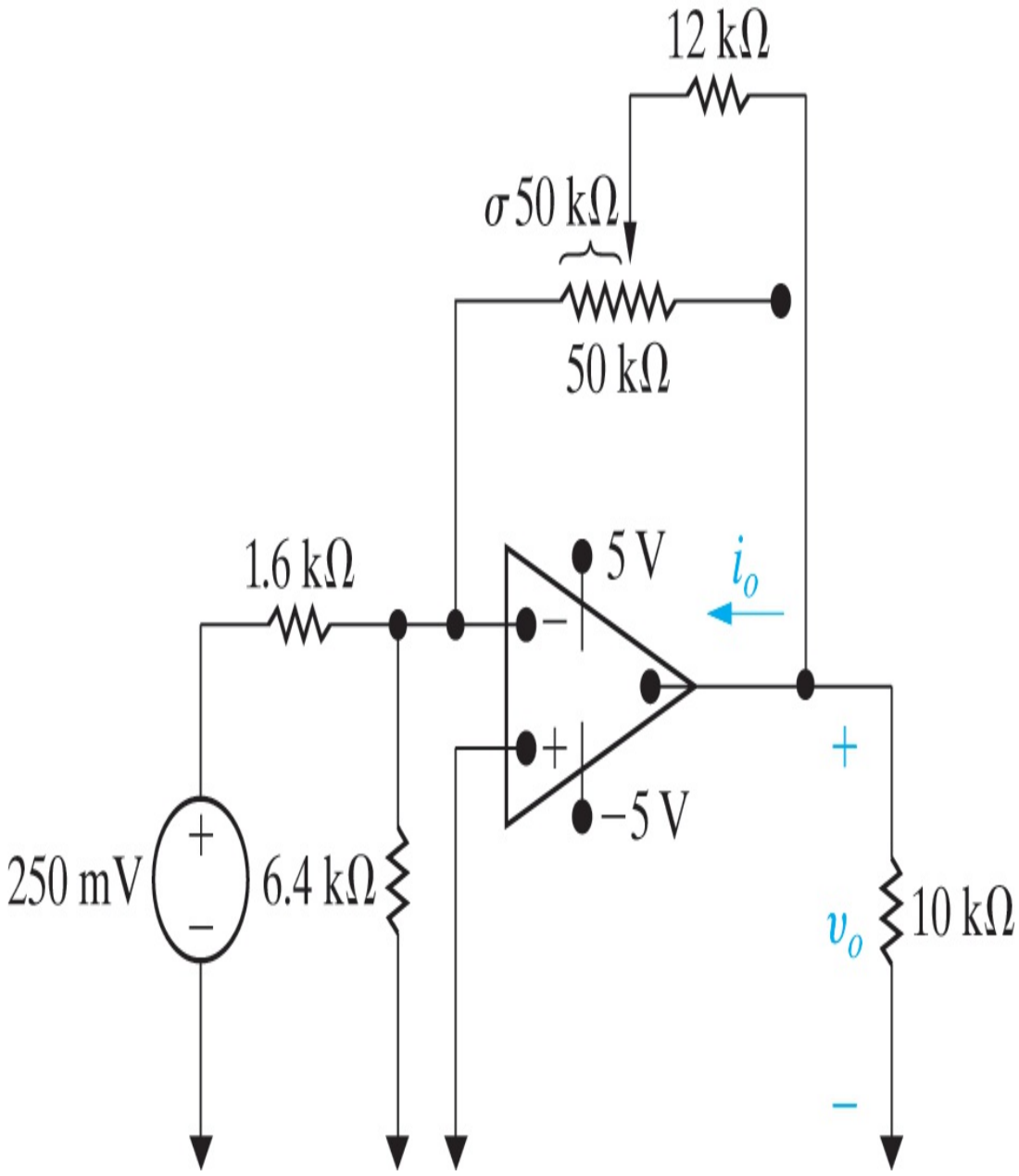


Figure P5.10 Full Alternative Text

4. 5.11 PSPICEMULTISIM

    1. The op amp in the circuit shown in Fig. P5.11 is ideal. The adjustable resistor RΔ has a maximum value of 100 kΩ, and a is restricted to the range of 0.2 =a=1. Calculate the range of vo if vg=40 mV.

2. If a is not restricted, at what value of a will the op amp saturate?

Figure P5.11

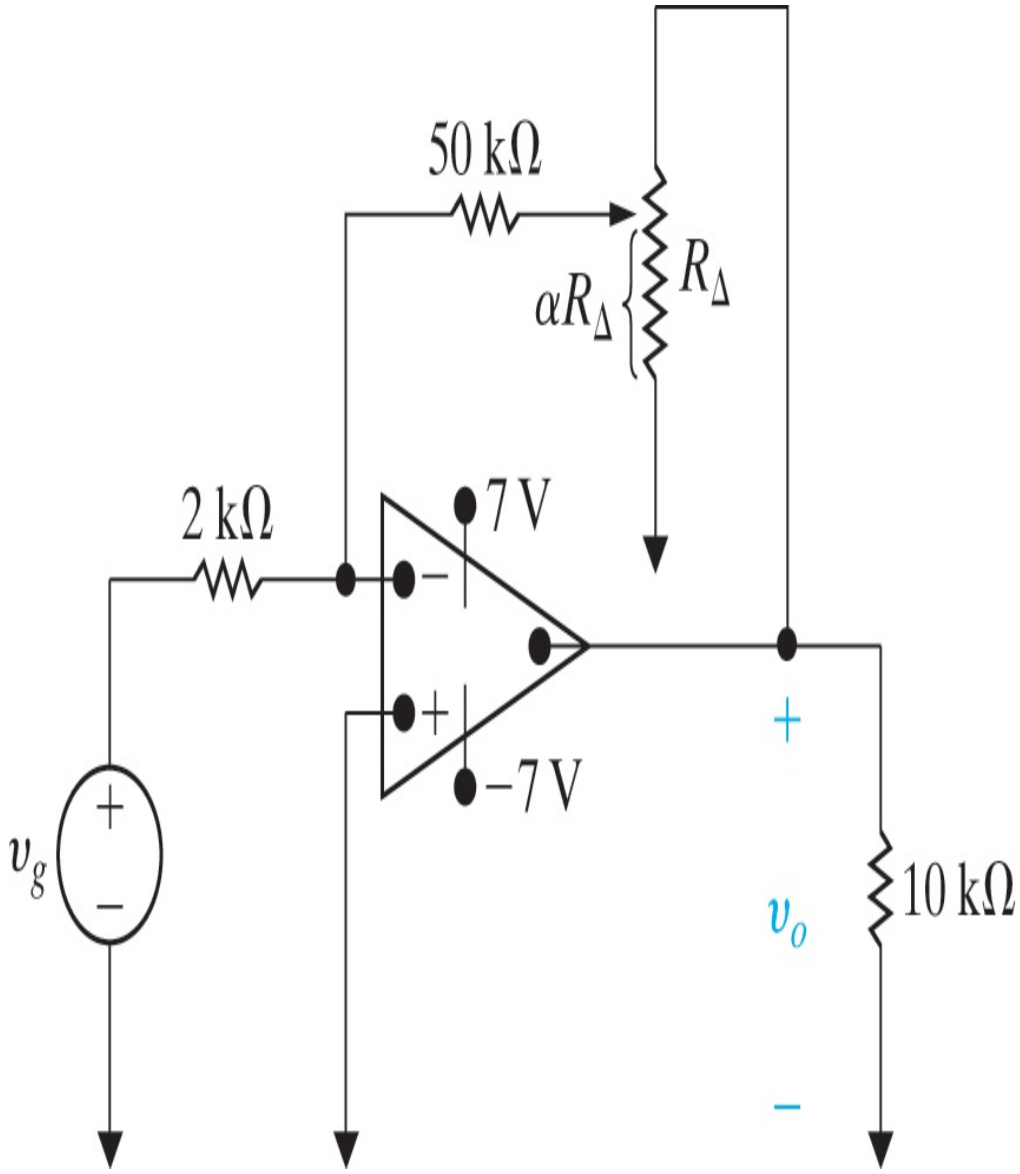


Figure P5.11 Full Alternative Text

Section 5.4

1. 5.12 PSPICEMULTISIM The op amp in Fig. P5.12 is ideal.
    1. What circuit configuration is shown in this figure?
    2. Find vo if va=1.2 V, vb=-1.5 V, and vc=4 V.
    3. The voltages va and vc remain at 1.2 V and 4 V, respectively. What are the limits on vb if the op amp operates within its linear region?

Figure P5.12

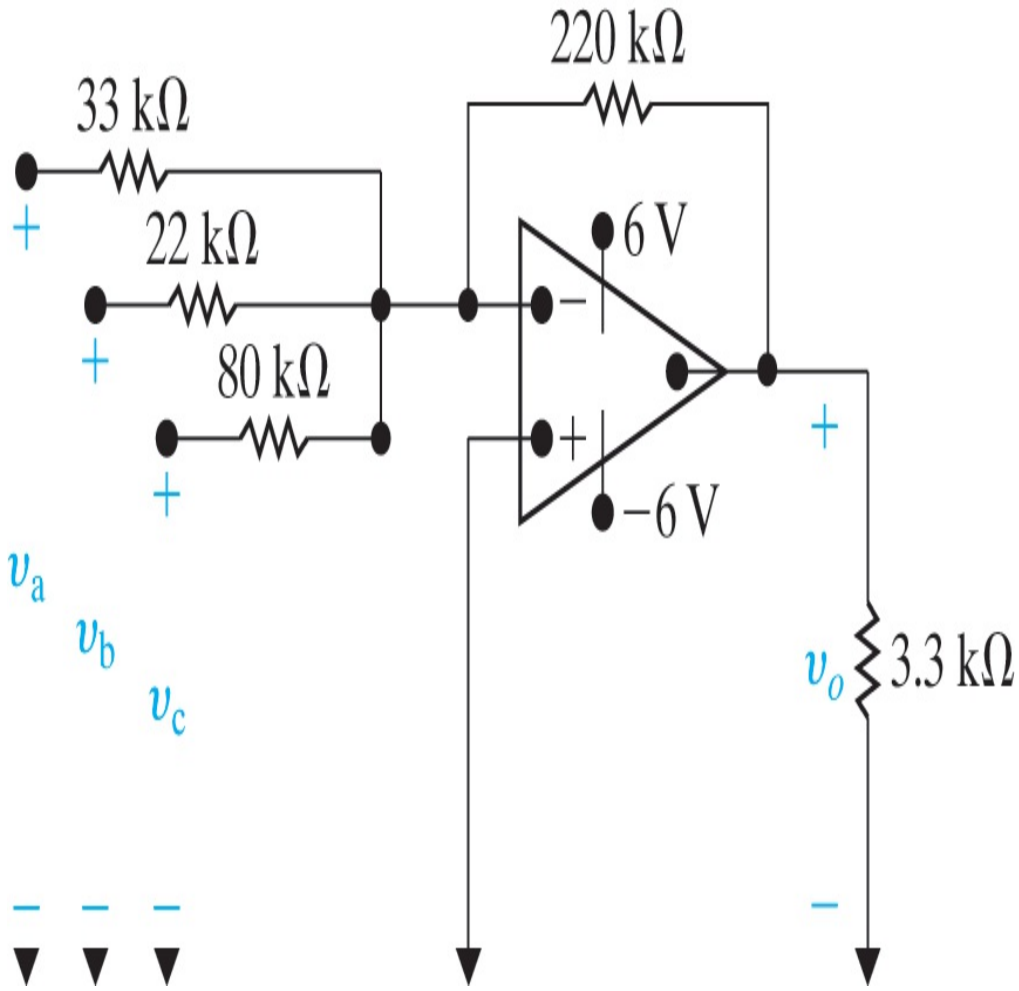


Figure P5.12 Full Alternative Text

2. 5.13

    1. Design an inverting-summing amplifier using a 120 kΩ resistor in the feedback path so that

        vo=-(8va+5vb+12vc).

        Use ±15 V power supplies.

    2. Suppose va=2 V and vc=-1 V. What range of values for vb will keep the op amp in its linear operating region?

3. 5.14 PSPICEMULTISIM Refer to the circuit in Fig. 5.11, where the op amp is assumed to be ideal. Given that Ra=3 kΩ, Rb=5 kΩ, Rc=25 kΩ,

va=150 mV, vb=100 mV, vc=250 mV, and VCC=±6 V, specify the range of Rf for which the op amp operates within its linear region.

4. 5.15 DESIGNPROBLEM_ PSPICEMULTISIM Design an inverting-summing amplifier so that

   vo=-(8va+4vb+10vc+6vd).

   Start by choosing a feedback resistor (Rf) from Appendix H. Then choose single resistors or construct resistor networks using resistor values in Appendix H to satisfy the design values for Ra, Rb, Rc, and Rd. Draw your final circuit diagram.

5. 5.16 PSPICEMULTISIM

   1. The op amp in Fig. P5.16 is ideal. Find vo if va=4 V, vb=9 V, vc=13 V, and vd=8 V.

   2. Assume vb, vc, and vd retain their values as given in (a). Specify the range of va such that the op amp operates within its linear region.

Figure P5.16

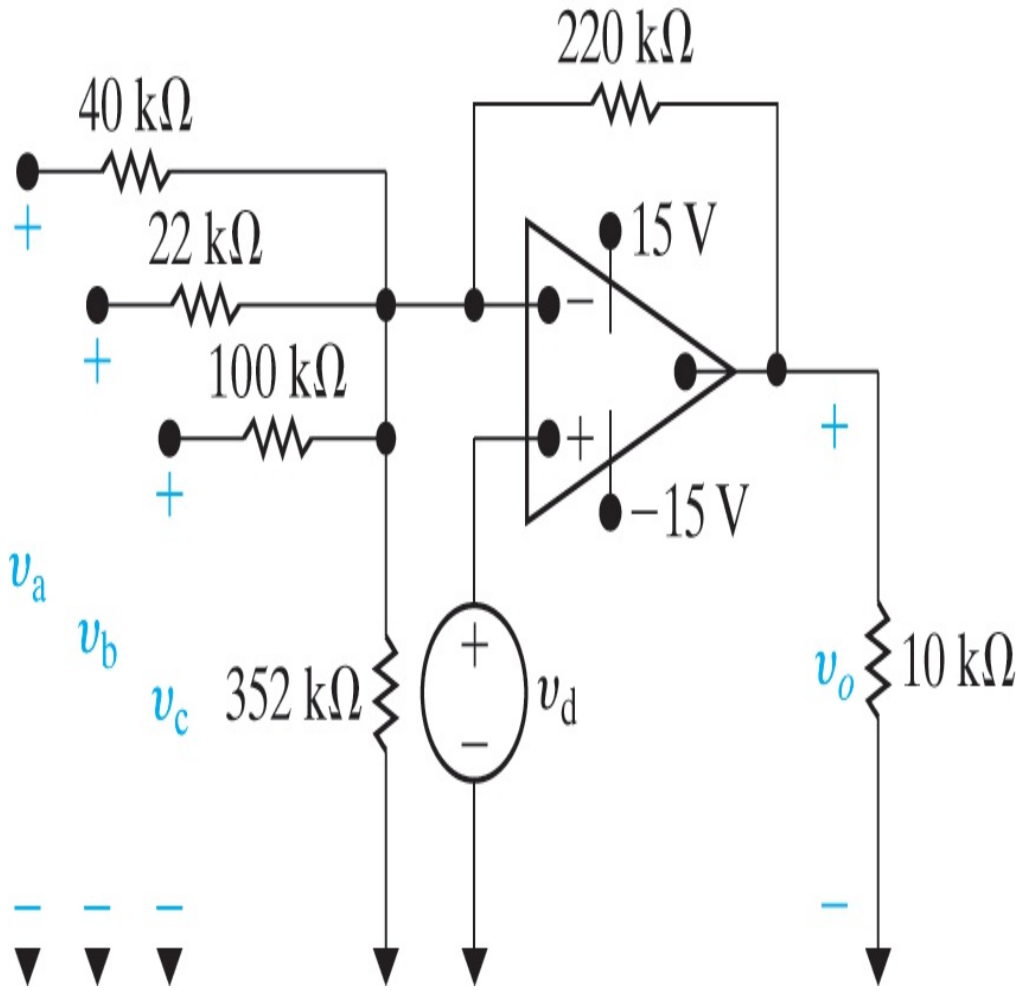


Figure P5.16 Full Alternative Text

6. 5.17 PSPICEMULTISIM The 220 kO feedback resistor in the circuit in Fig. P5.16 is replaced by a variable resistor Rf. The voltages va-vd have the same values as given in Problem 5.16(a).

    1. What value of Rf will cause the op amp to saturate? Note that $0 \leq Rf \leq \infty$.

    2. When Rf has the value found in (a), what is the current (in microamperes) into the output terminal of the op amp?

Section 5.5

1. 5.18 The op amp in the circuit of Fig. P5.18 is ideal.
    1. What op amp circuit configuration is this?
    2. Find vo in terms of vs.
    3. Find the range of values for vs such that vo does not saturate and the op amp remains in its linear region of operation.

Figure P5.18

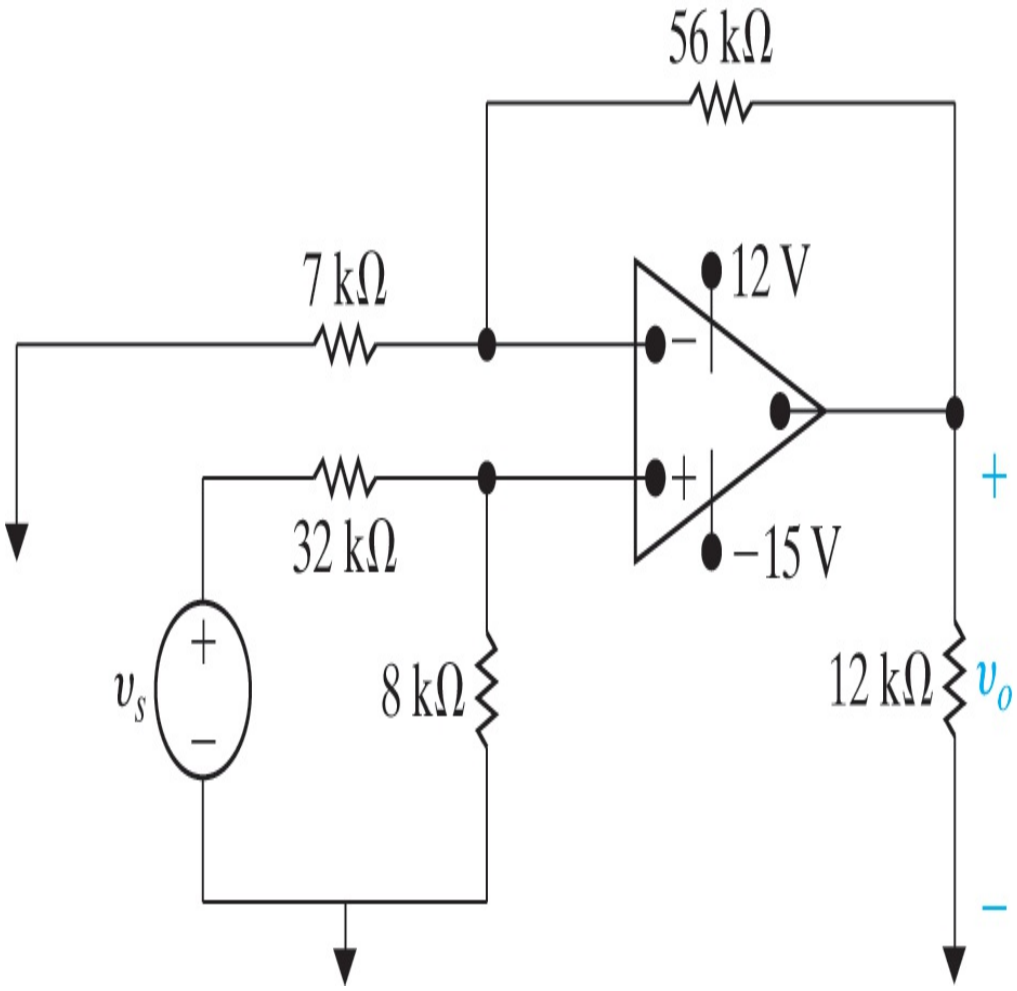


Figure P5.18 Full Alternative Text

2. 5.19 PSPICEMULTISIM The op amp in the circuit of Fig. P5.19 is ideal.

    1. What op amp circuit configuration is this?

    2. Calculate vo.

Figure P5.19

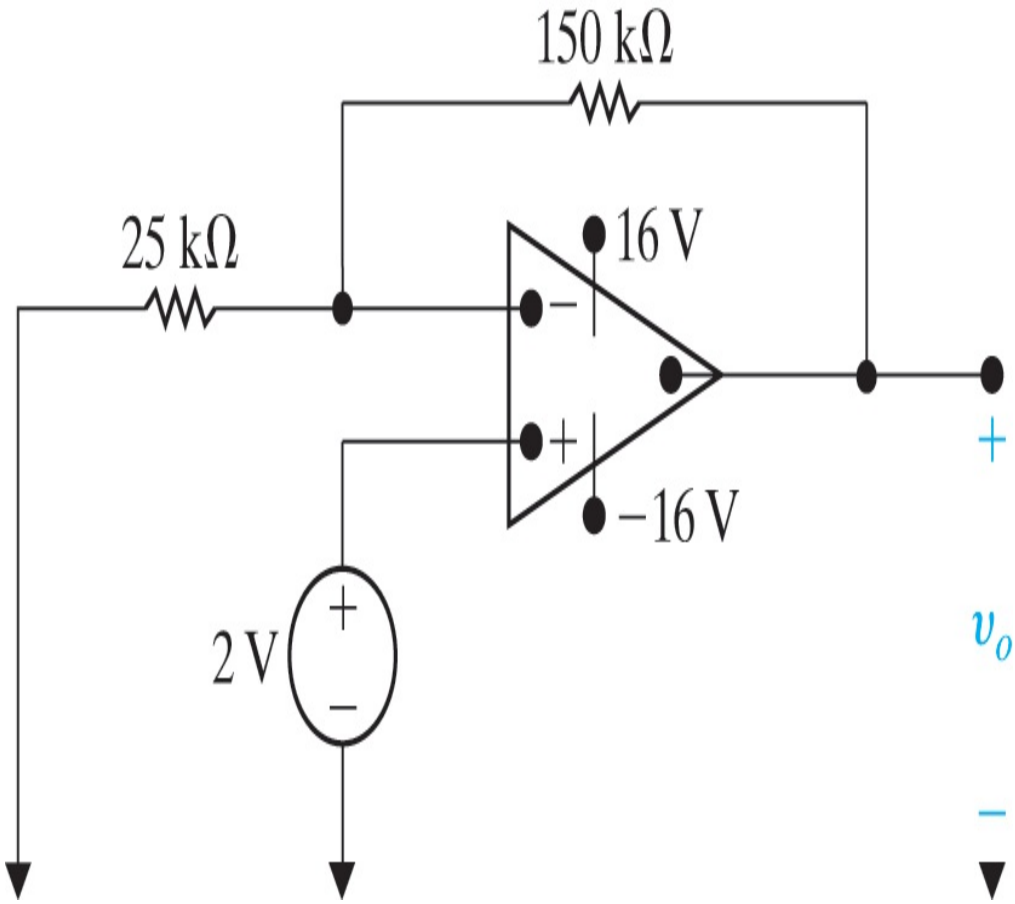


Figure P5.19 Full Alternative Text

3. 5.20 PSPICEMULTISIM The op amp in the circuit shown in Fig. P5.20 is ideal,

    1. Calculate vo when vg equals 3 V.

    2. Specify the range of values of vg so that the op amp operates in a linear mode.

    3. Assume that vg equals 5 V and that the 48 kΩ resistor is replaced with a variable resistor. What value of the variable resistor will cause the op amp to saturate?

Figure P5.20

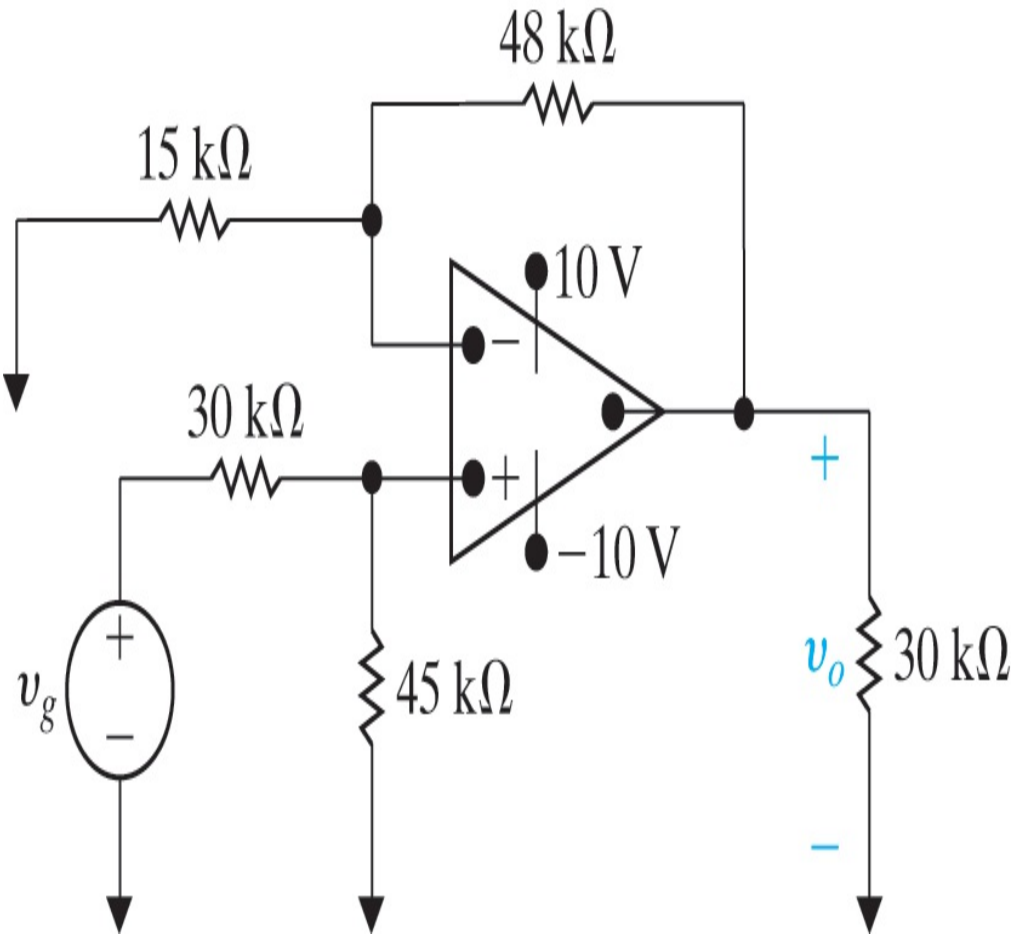


Figure P5.20 Full Alternative Text

4. 5.21

    1. Design a noninverting amplifier (see Fig. 5.13) with a gain of 2.5. Use resistors from Appendix H. You might need to combine resistors in series and in parallel to get the desired resistance. Draw your final circuit.

    2. If you use ±16 V power supplies for the op amp, what range of input values will allow the op amp to stay in its linear operating region?

5. 5.22

    1. Design a noninverting amplifier (see Fig. 5.13) with a gain of 6, using a 75 kΩ resistor in the feedback path. Draw your final circuit

diagram.

2. Suppose you wish to amplify input signals in the range $-2.5\ \text{V} \leq v_g \leq 1.5\ \text{V}$. What are the minimum values of the power supplies that will keep the op amp in its linear operating region?

6. 5.23 PSPICEMULTISIM The op amp in the circuit of Fig. P5.23 is ideal.

    1. What op amp circuit configuration is this?

    2. Find v_o in terms of v_s.

    3. Find the range of values for v_s such that v_o does not saturate and the op amp remains in its linear region of operation.

Figure P5.23

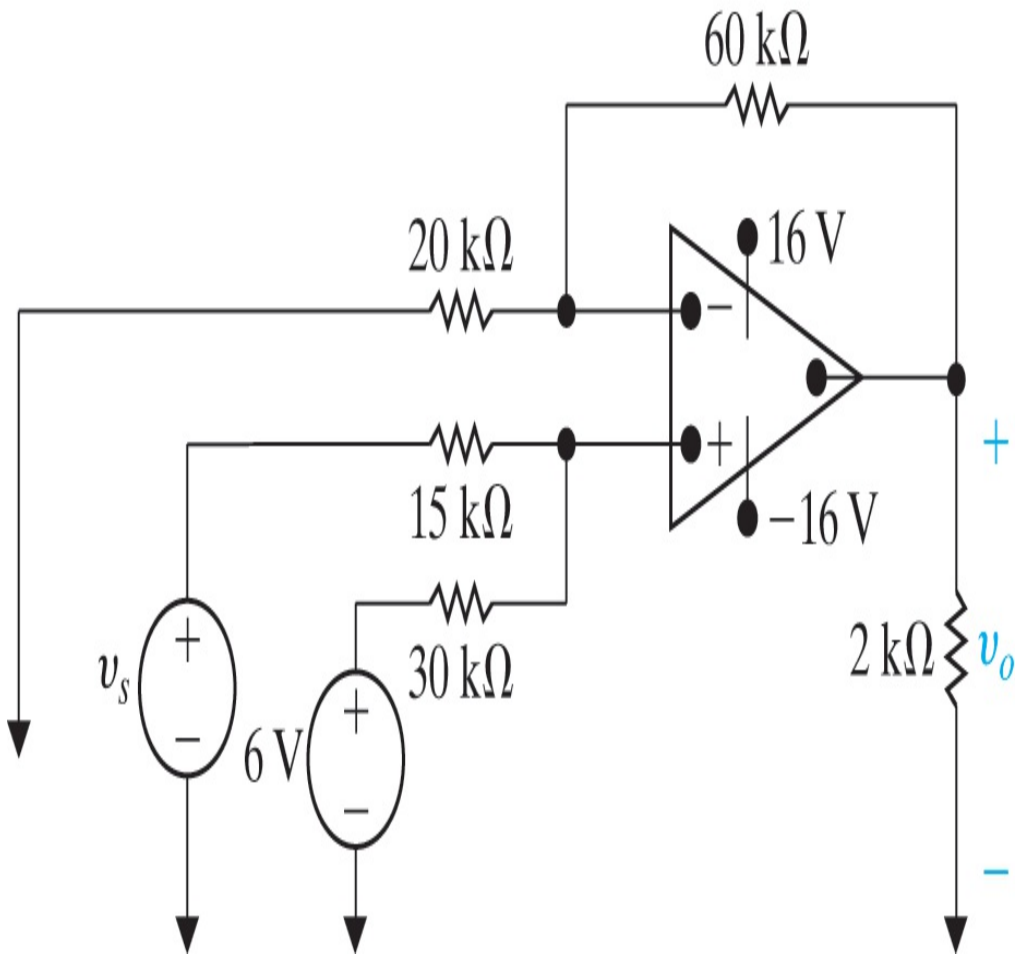


Figure P5.23 Full Alternative Text

7. 5.24 DESIGNPROBLEM_ PSPICEMULTISIM The circuit in Fig. P5.24 is a noninverting summing amplifier. Assume the op amp is ideal. Design the circuit so that

   vo=va+2vb+3vc.

   1. Specify the numerical values of Ra and Rc.

   2. Calculate ia, ib, and ic (in microamperes) when va=0.7 V, vb=0.4 V, and vc=1.1 V.

Figure P5.24

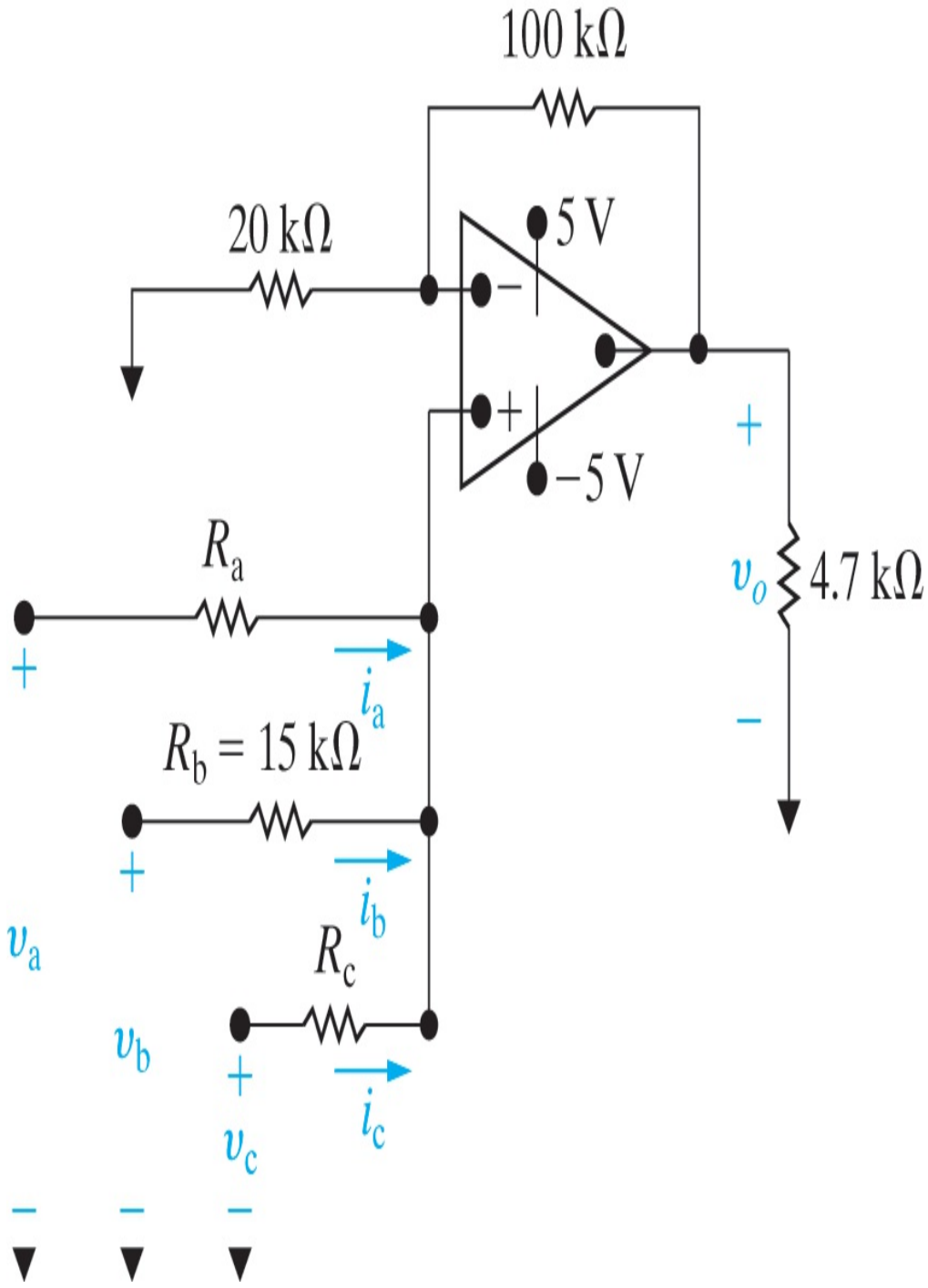


Figure P5.24 Full Alternative Text

Section 5.6

1. 5.25 The op amp in the circuit of Fig. P5.25 is ideal.
    1. What op amp circuit configuration is this?
    2. Find an expression for the output voltage vo in terms of the input voltage va.
    3. Suppose va=2 V. What value of Rf will cause the op amp to saturate?

Figure P5.25

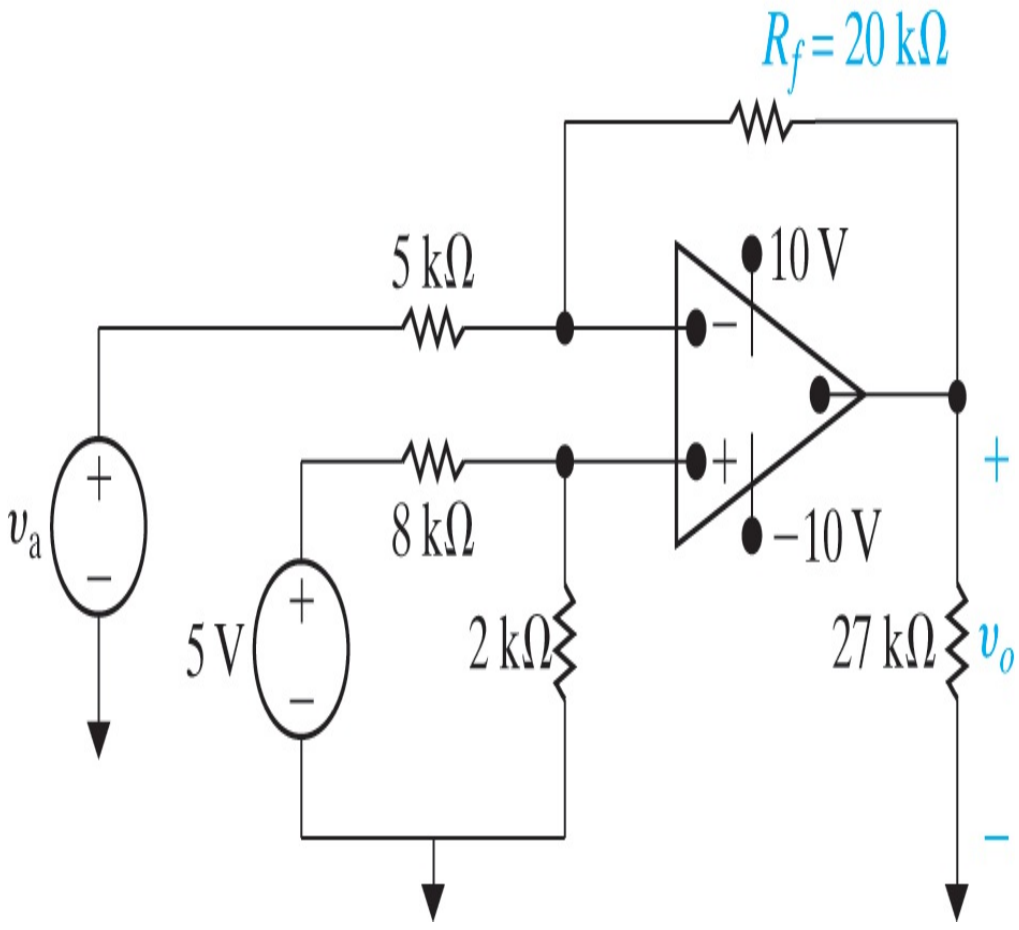


Figure P5.25 Full Alternative Text

2. 5.26 The resistor Rf in the circuit in Fig. P5.26 is adjusted until the ideal op amp saturates. Specify Rf in kilohms.

Figure P5.26

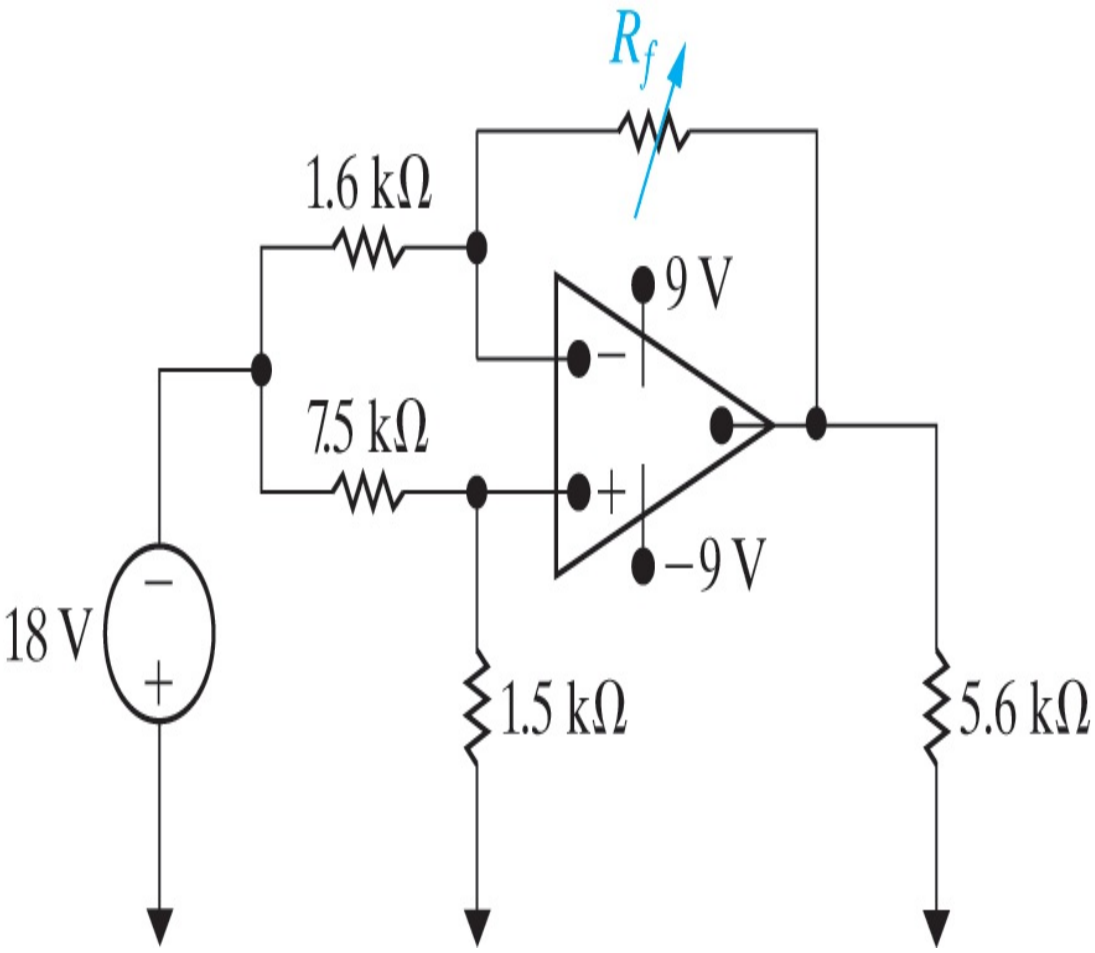


Figure P5.26 Full Alternative Text

3. 5.27 PSPICEMULTISIM The resistors in the difference amplifier shown in Fig. 5.15 are Ra=10 kΩ, Rb=100 kΩ, Rc=33 kΩ and Rd=47 kΩ. The signal voltages va and vb are 0.67 and 0.8 V, respectively, and VCC= ±5 V.

    1. Find vo.

2. What is the resistance seen by the signal source va?

3. What is the resistance seen by the signal source vb?

4. 5.28 DESIGNPROBLEM_ PSPICEMULTISIM Design a difference amplifier (Fig. 5.15) to meet the following criteria: vo=3vb-4va. The resistance seen by the signal source vb is 470 kΩ, and the resistance seen by the signal source va is 22 kΩ when the output voltage vo is zero. Specify the values of Ra, Rb, Rc, and Rd using single resistors or combinations of resistors from Appendix H.

5. 5.29

    1. Use the principle of superposition to derive Eq. 5.8.

    2. Derive Eqs. 5.9 and 5.10.

6. 5.30 DESIGNPROBLEM_ PSPICEMULTISIM Select the values of Rb and Rf in the circuit in Fig. P5.30 so that

    vo=8000(ib-ia).

    Use single resistors or combinations of resistors from Appendix H. The op amp is ideal.

Figure P5.30

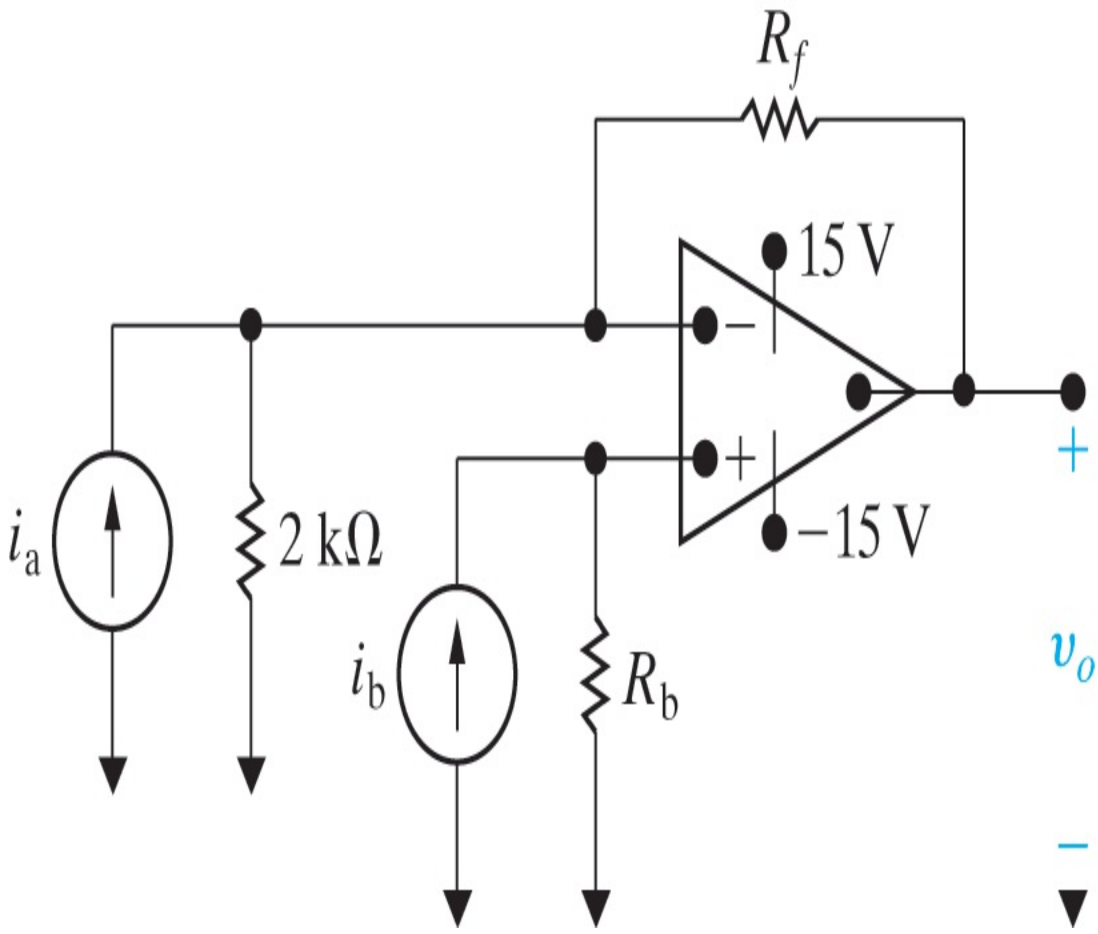


Figure P5.30 Full Alternative Text

7. 5.31 PSPICEMULTISIM The op amp in the adder-subtracter circuit shown in Fig. P5.31 is ideal.

    1. Find vo when va=1 V, vb=2 V, vc=3 V, and vd=4 V.

    2. If va, vb, and vd are held constant, what values of vc will not saturate the op amp?

Figure P5.31

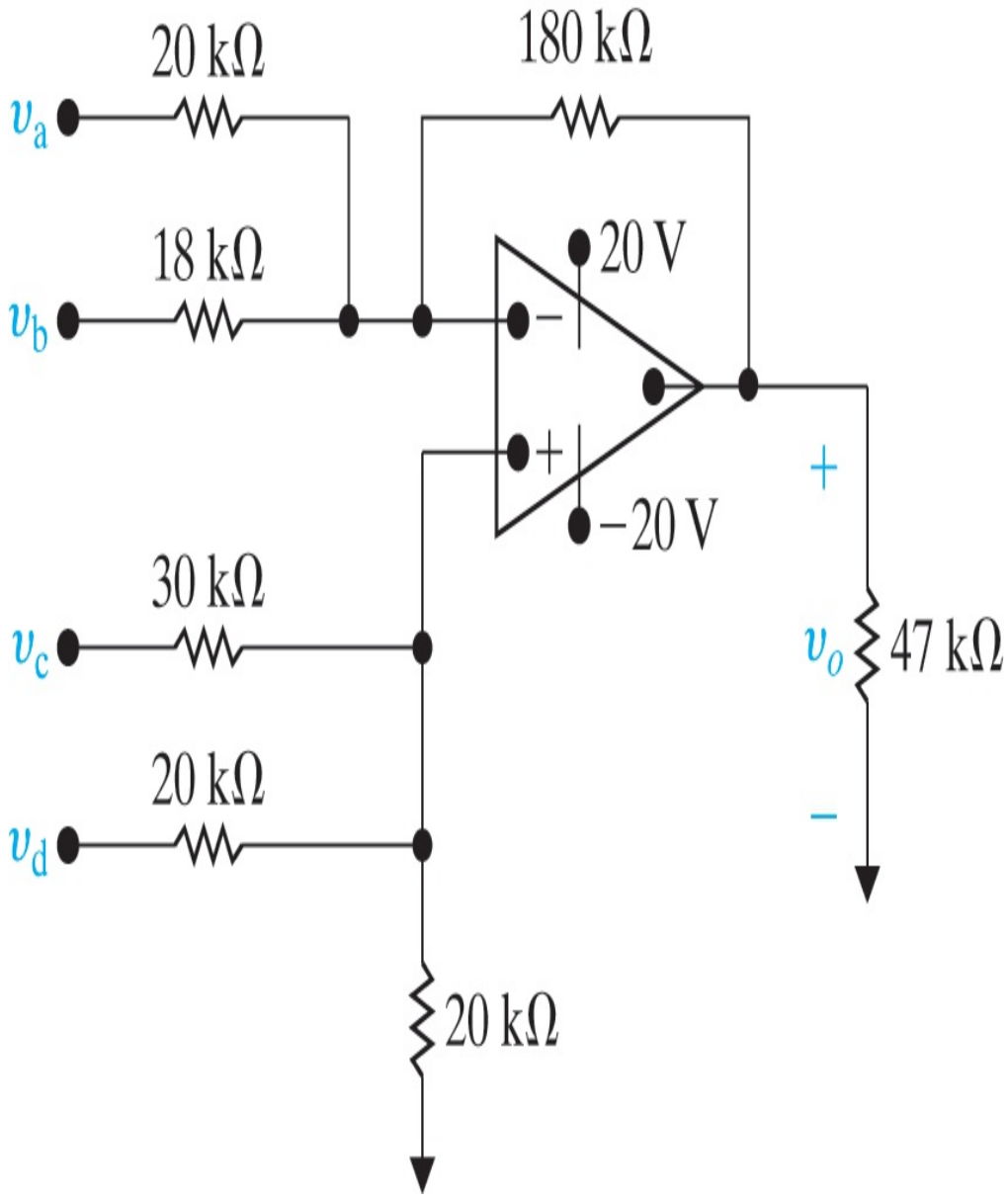


Figure P5.31 Full Alternative Text

8. 5.32 In the difference amplifier shown in Fig. P5.32, compute (a) the differential mode gain, (b) the common mode gain, and (c) the CMRR.

Figure P5.32

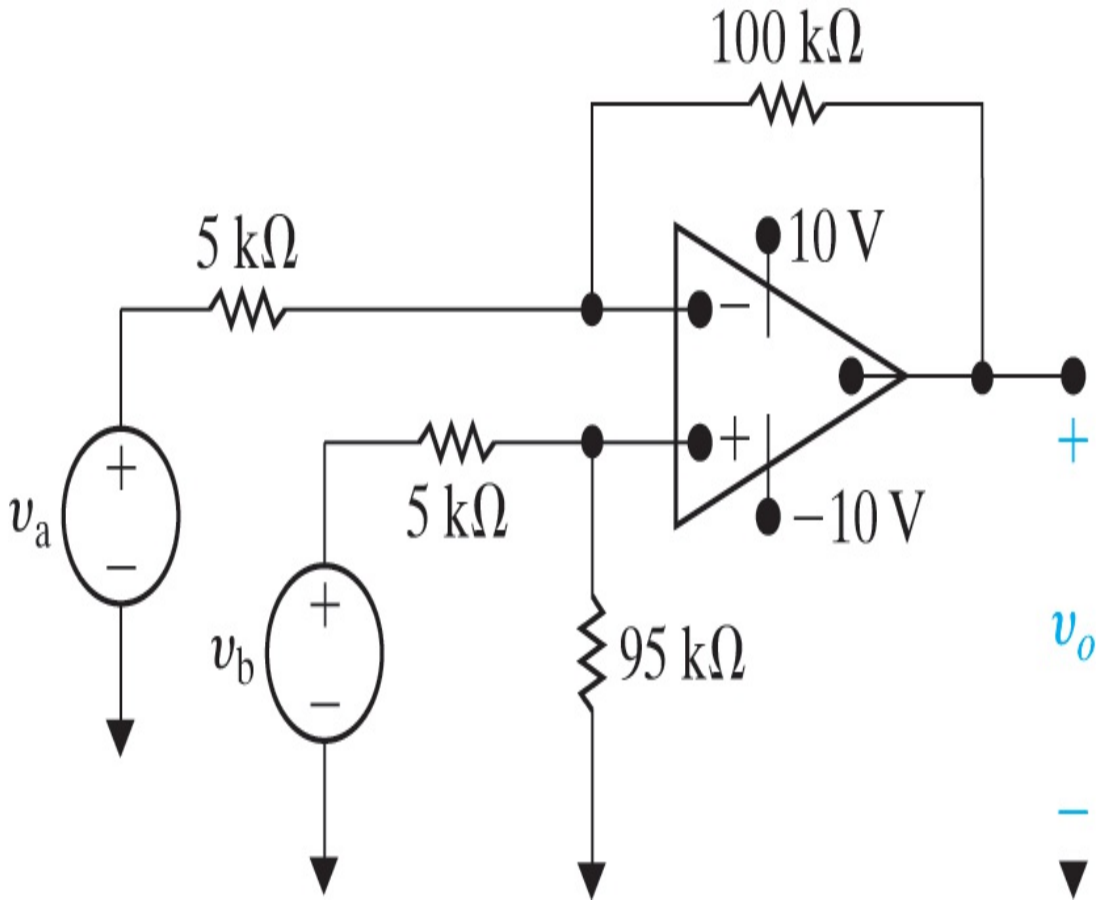


Figure P5.32 Full Alternative Text

9. 5.33 In the difference amplifier shown in Fig. P5.33, what range of values of Rx yields a CMRR≥1000?

Figure P5.33

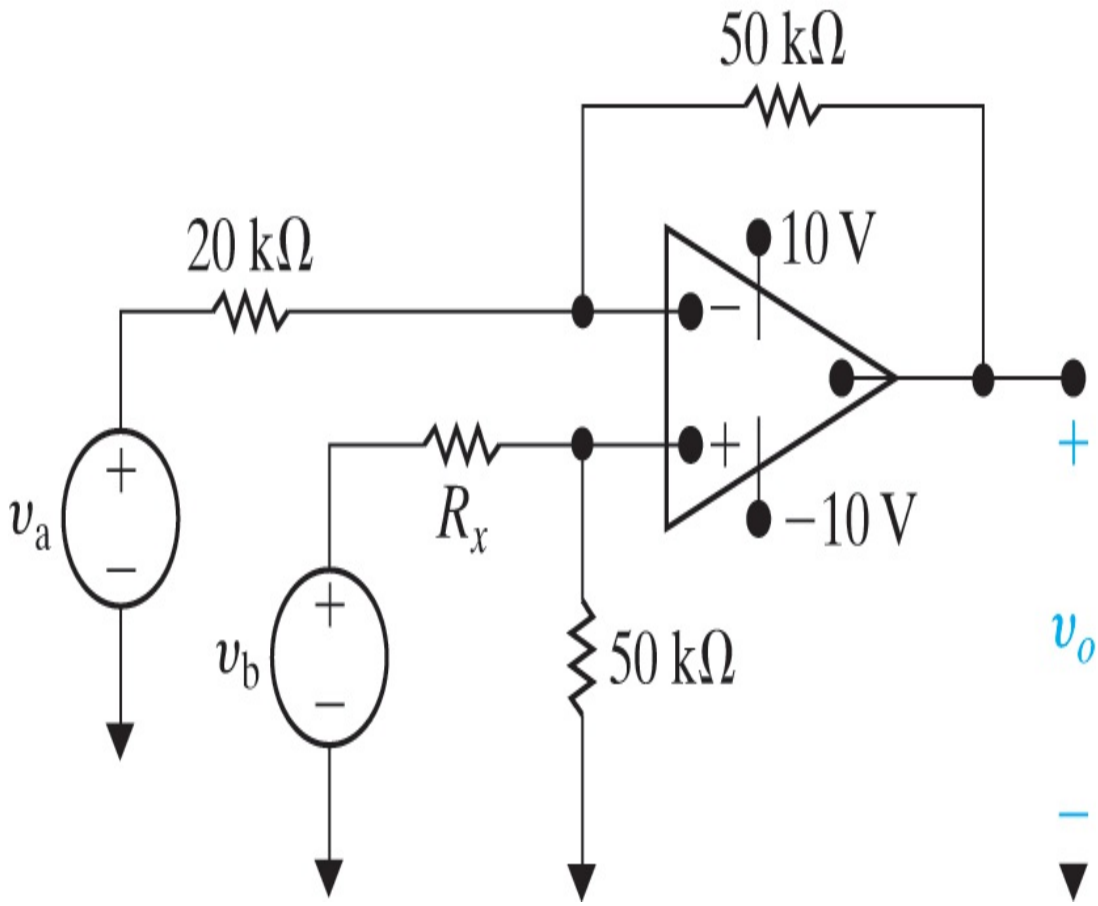


Figure P5.33 Full Alternative Text

10. 5.34 The op amp in the circuit of Fig. P5.34 is ideal.

    1. Plot vo versus a when Rf=4R1 and vg=2 V. Use increments of 0.1 and note by hypothesis that 0≤a≤1.0.

    2. Write an equation for the straight line you plotted in (a). How are the slope and intercept of the line related to vg and the ratio Rf/R1?

    3. Using the results from (b), choose values for vg and the ratio Rf/R1 such that vo=-6a+4.

Figure P5.34

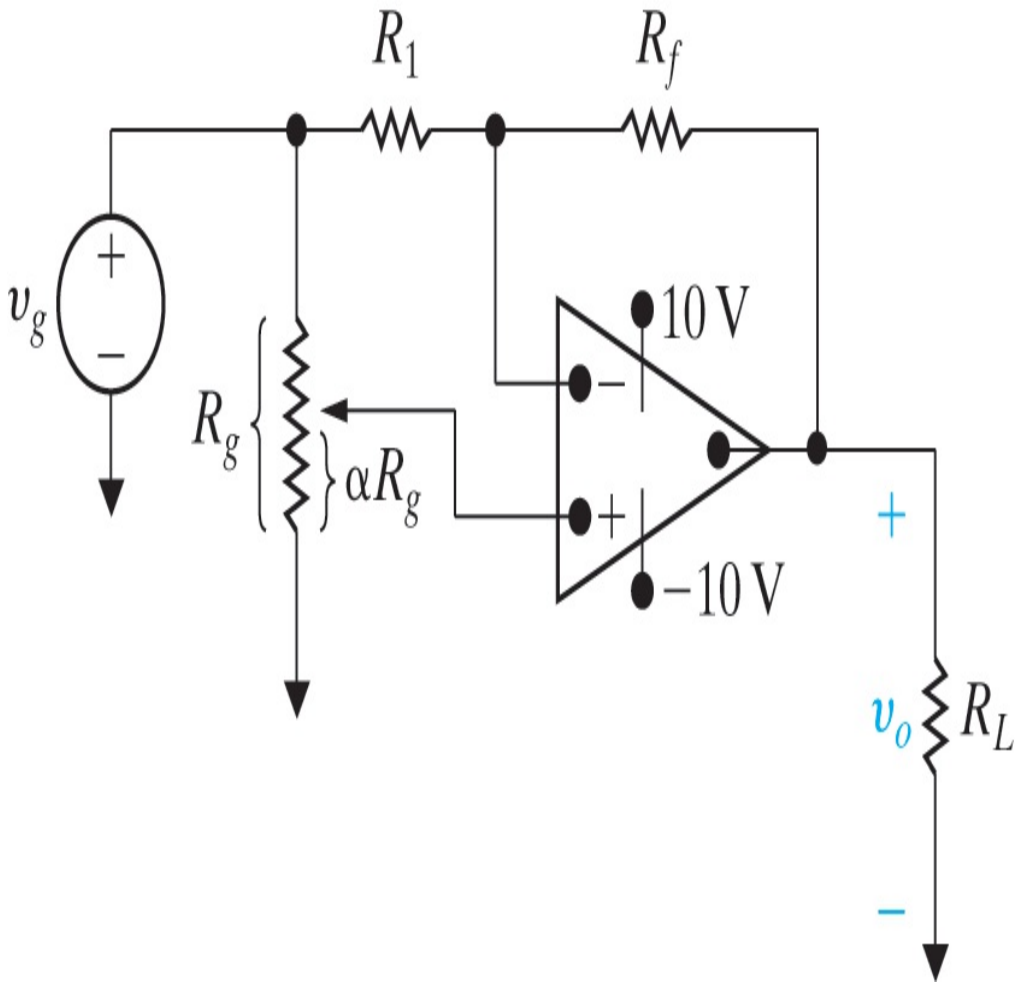


Figure P5.34 Full Alternative Text

Sections 5.1–5.6

1. 5.35 Assume that the ideal op amp in the circuit seen in Fig. P5.35 is operating in its linear region.

    1. Show that vo=[(R1+R2)/R1]vs.

    2. What happens if R1 → ∞ and R2 → 0?

    3. Explain why this circuit is referred to as a voltage follower when R1=∞ and R2=0.

Figure P5.35

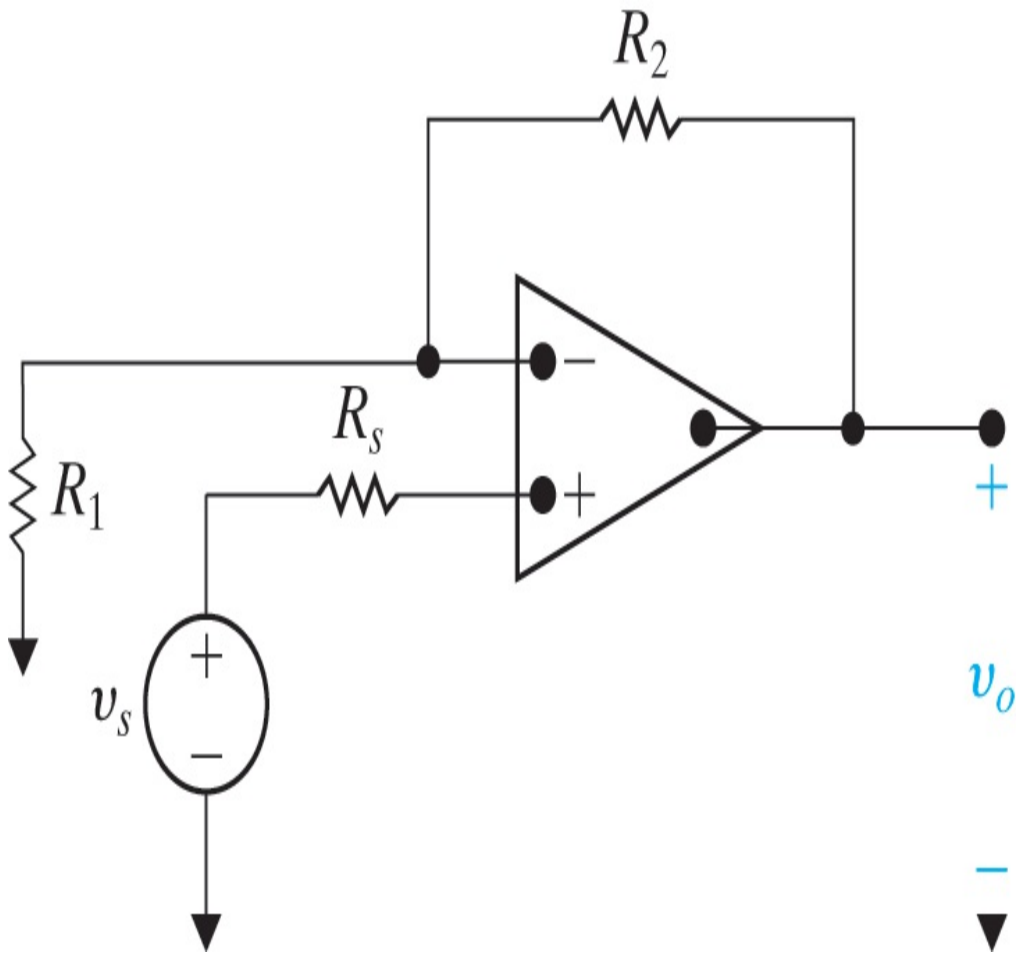
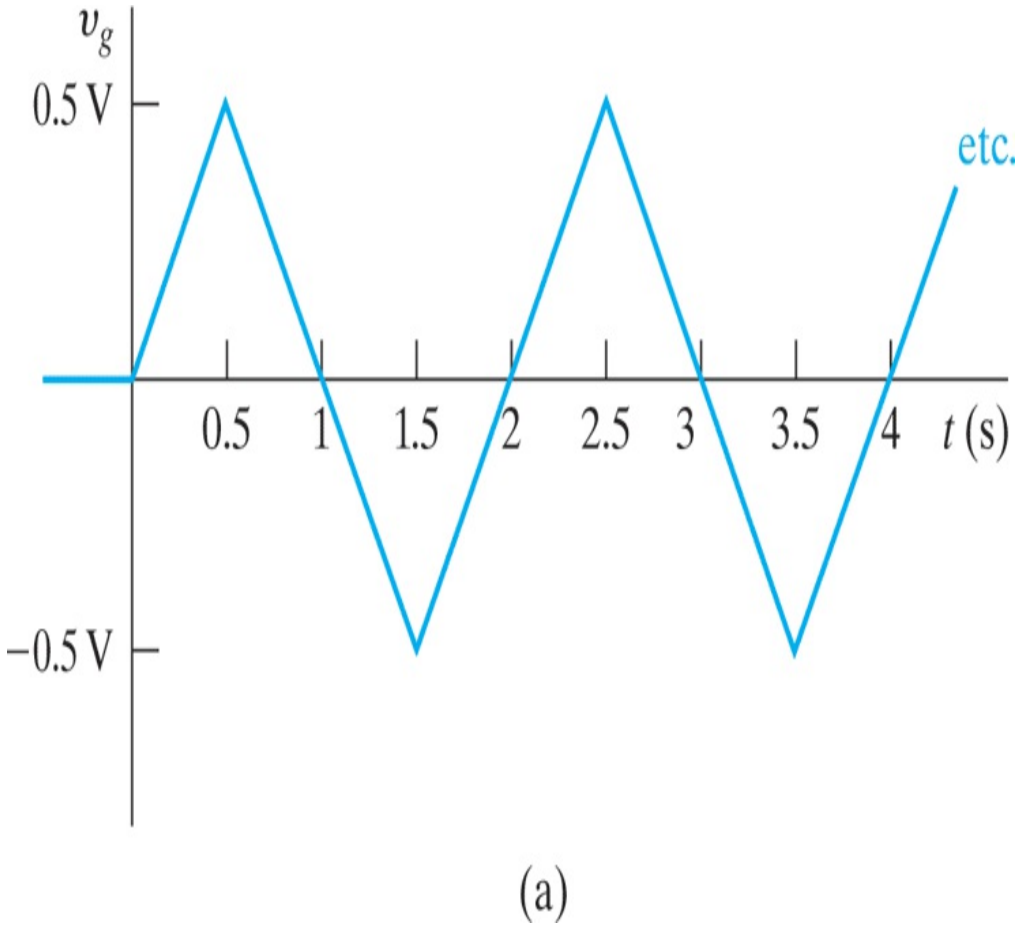


Figure P5.35 Full Alternative Text

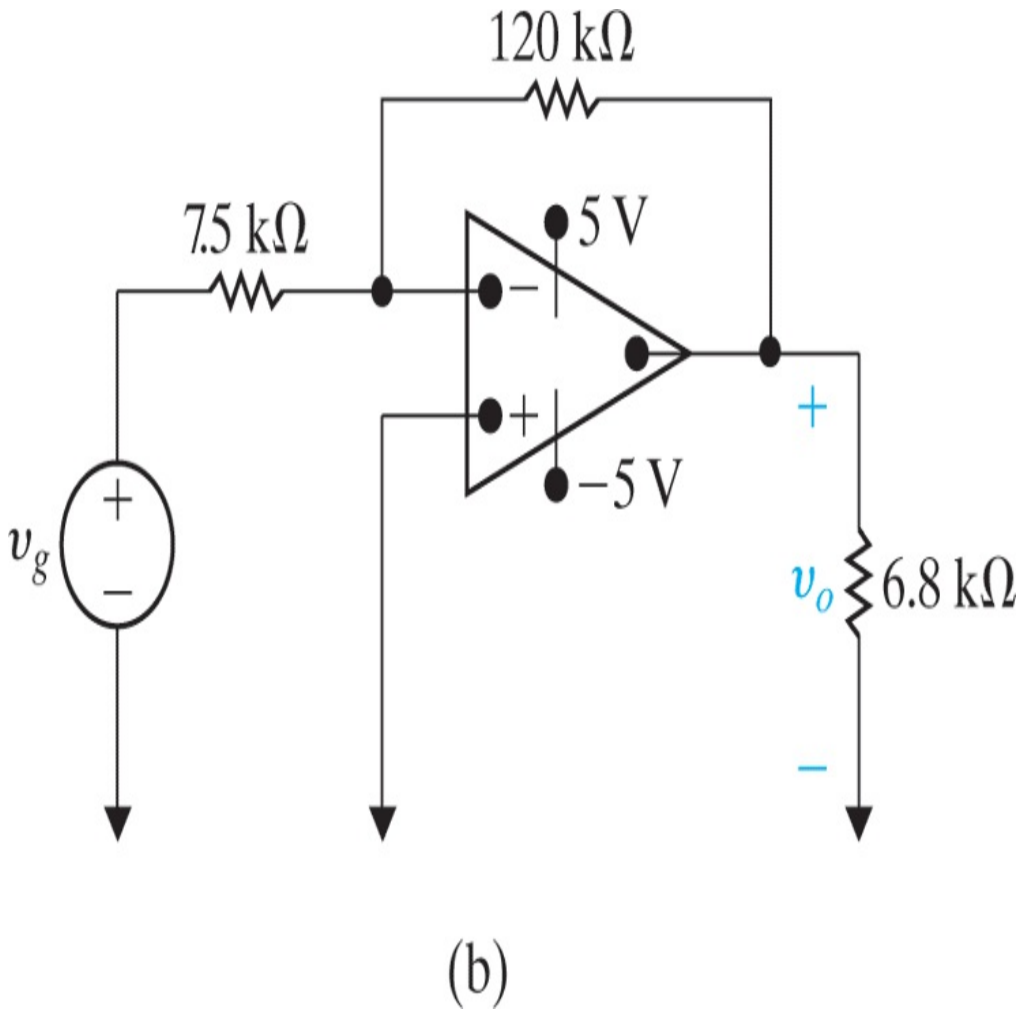
2. 5.36 PSPICEMULTISIM The voltage vg shown in Fig. P5.36(a) is applied to the inverting amplifier shown in Fig. P5.36(b). Sketch vo versus t, assuming the op amp is ideal.

Figure P5.36



(a)

5.2-9 Full Alternative Text



5.2-9 Full Alternative Text

3. 5.37 PSPICEMULTISIM The signal voltage vg in the circuit shown in Fig. P5.37 is described by the following equations:

   vg=0, t≤0,vg=4cos(π4)t V,0≤t≤∞.

   Sketch vo versus t, assuming the op amp is ideal.

Figure P5.37

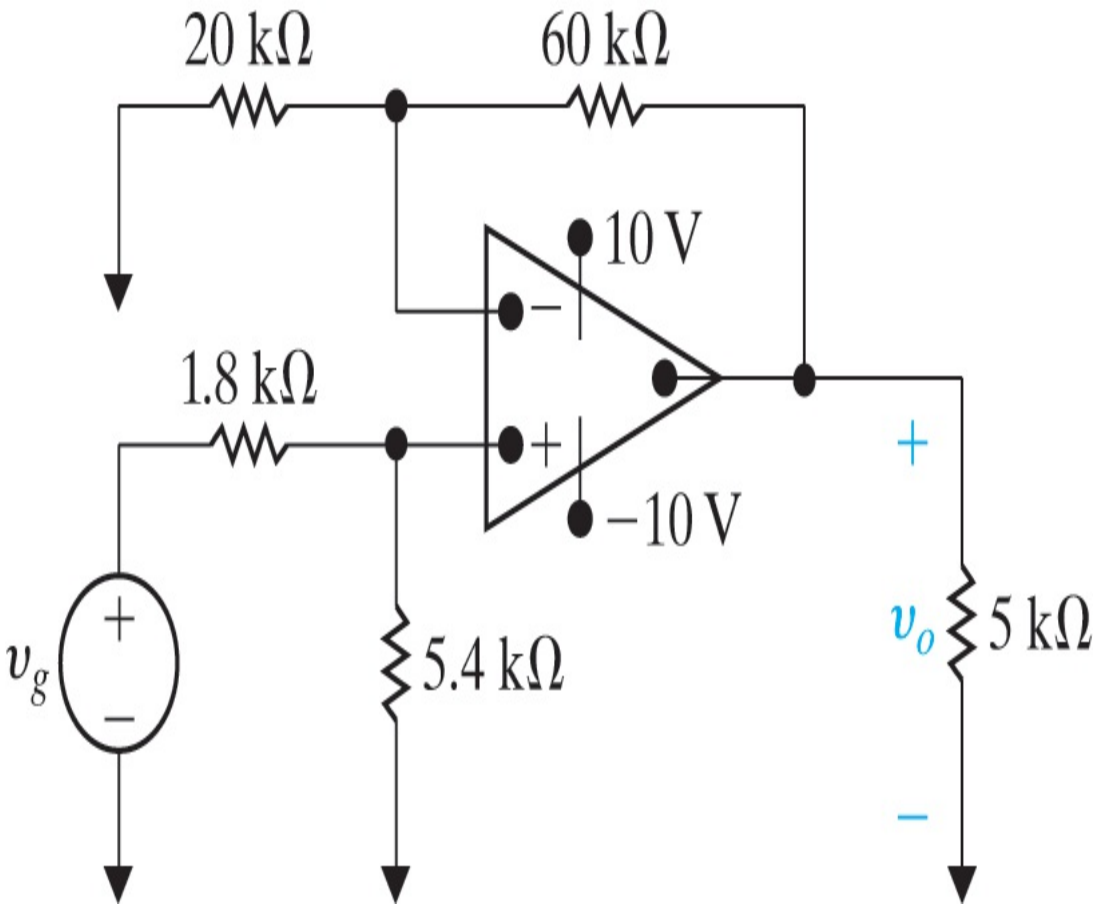


Figure P5.37 Full Alternative Text

4. 5.38

    1. Show that when the ideal op amp in Fig. P5.38 is operating in its linear region,

       ia=3vgR.

    2. Show that the ideal op amp will saturate when

       Ra=R(±VCC-2vg)3vg.

Figure P5.38

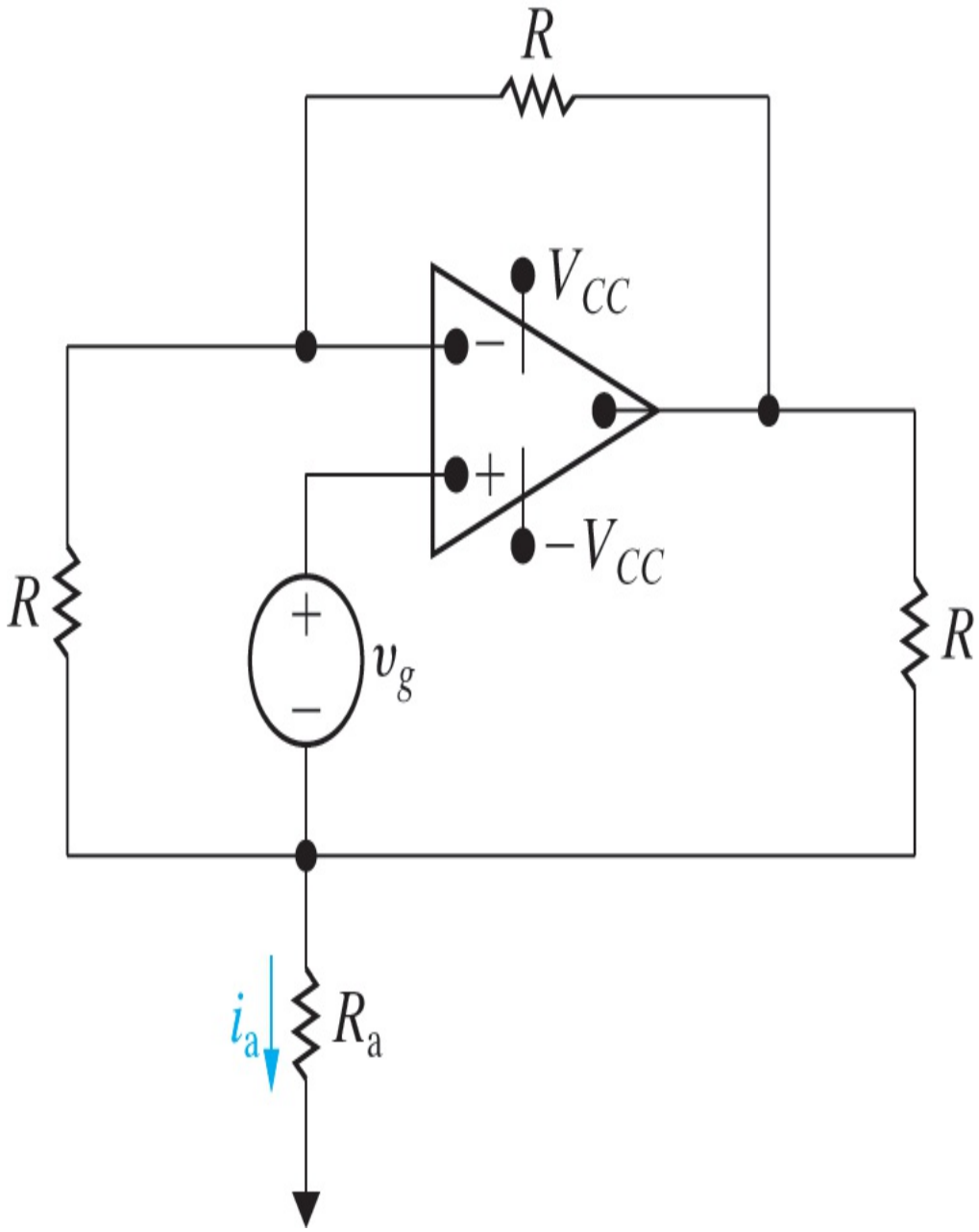


Figure P5.38 Full Alternative Text

5. 5.39 PSPICEMULTISIM The op amps in the circuit in Fig. P5.39 are ideal.

    1. Find ia.

    2. Find the value of the left source voltage for which ia=0.

Figure P5.39

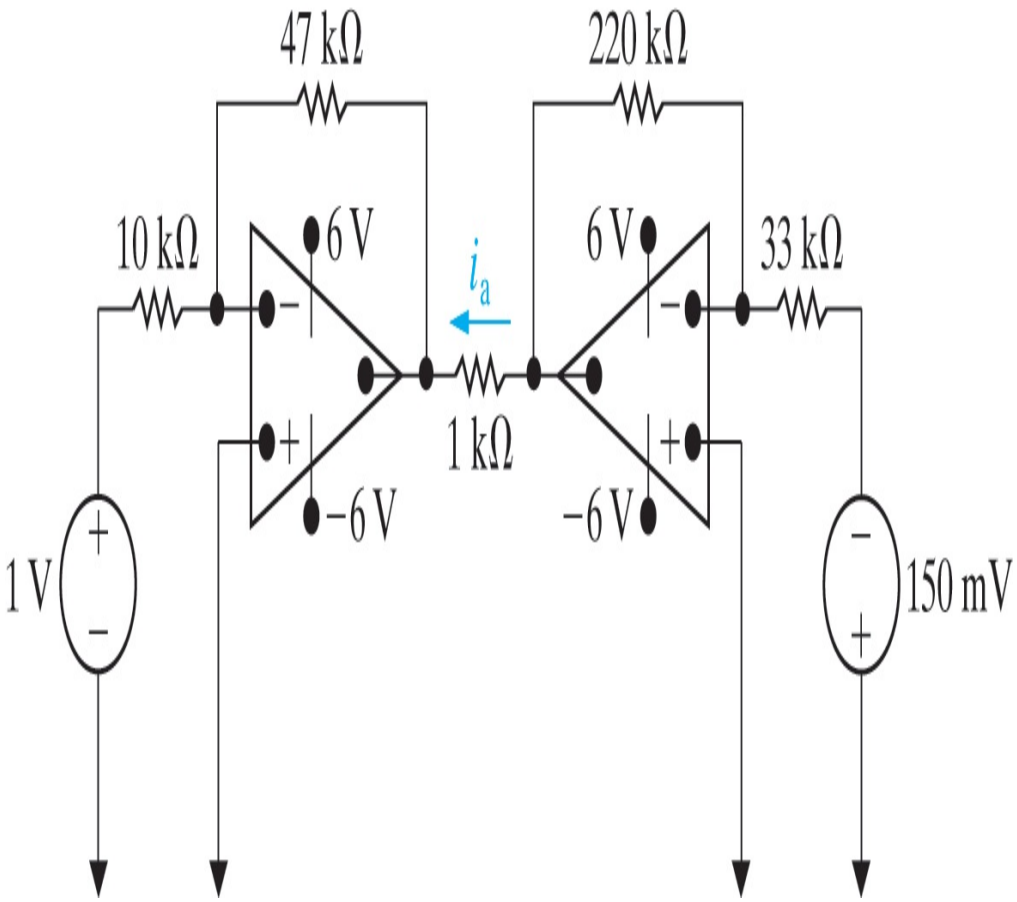


Figure P5.39 Full Alternative Text

6. 5.40 PSPICEMULTISIM The two op amps in the circuit in Fig. P5.40 are ideal. Calculate vo1 and vo2.

Figure P5.40

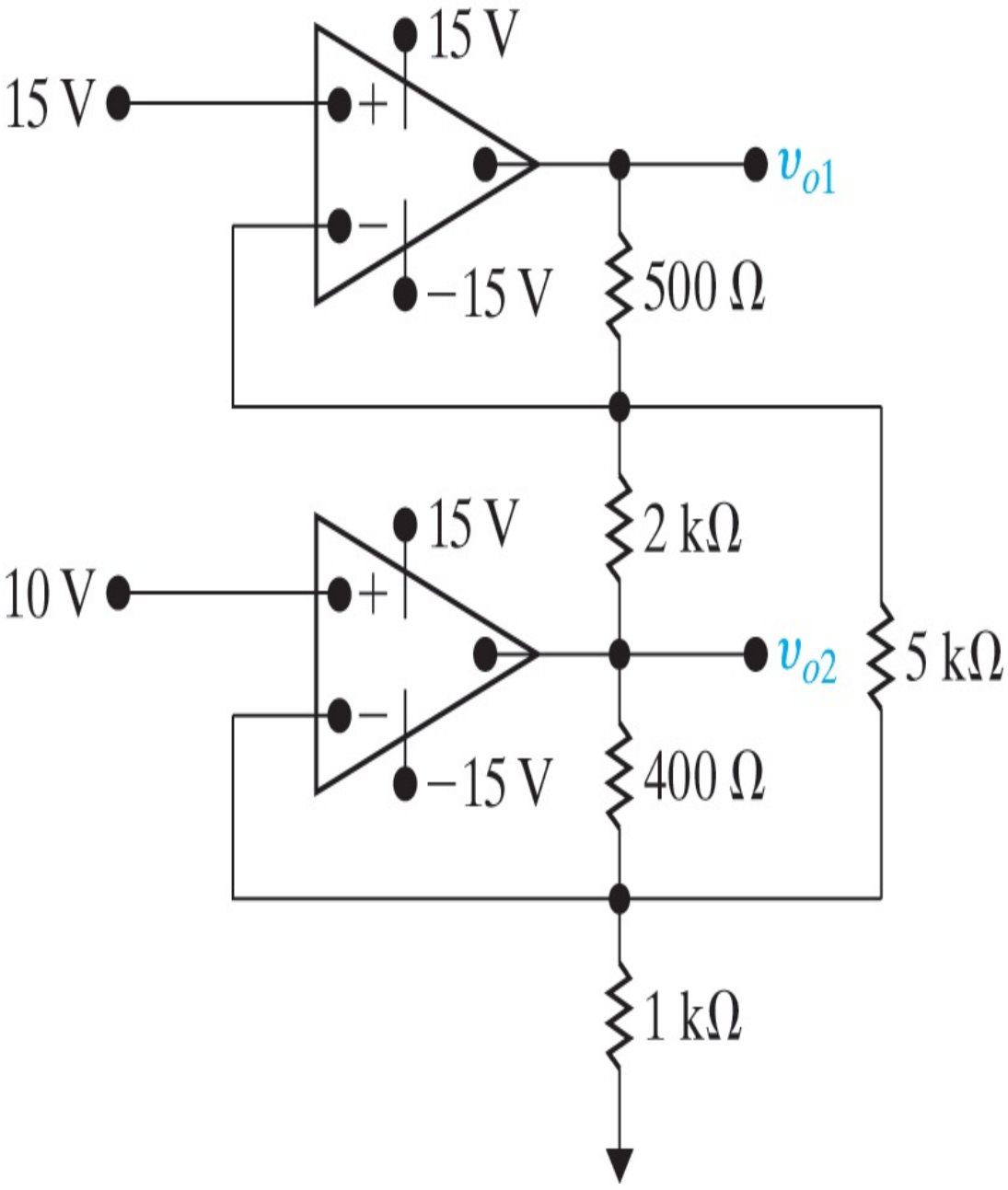


Figure P5.40 Full Alternative Text

7. 5.41 PSPICEMULTISIM The circuit inside the shaded area in Fig. P5.41 is a constant current source for a limited range of values of RL.

    1. Find the value of iL for RL=4 kΩ.

    2. Find the maximum value for RL for which iL will have the value in (a).

3. Assume that RL=16 kΩ. Explain the operation of the circuit. You can assume that in=ip≈0 under all operating conditions.

4. Sketch iL versus RL for 0≤RL≤16 kΩ.

Figure P5.41

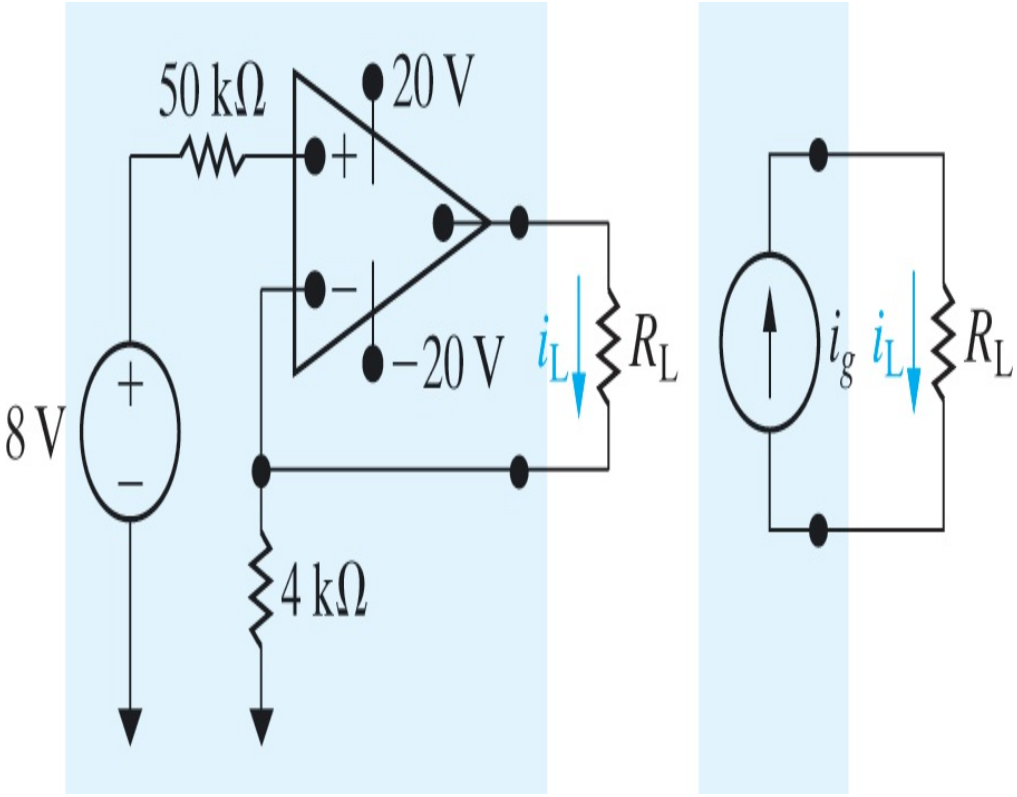


Figure P5.41 Full Alternative Text

8. 5.42 PSPICEMULTISIM Assume that the ideal op amp in the circuit in Fig. P5.42 is operating in its linear region.

    1. Calculate the power delivered to the 16 kΩ resistor.

    2. Repeat (a) with the op amp removed from the circuit, that is, with the 16 kΩ resistor connected in the series with the voltage source and the 48 kΩ resistor.

3. Find the ratio of the power found in (a) to that found in (b).

4. Does the insertion of the op amp between the source and the load serve a useful purpose? Explain.

Figure P5.42

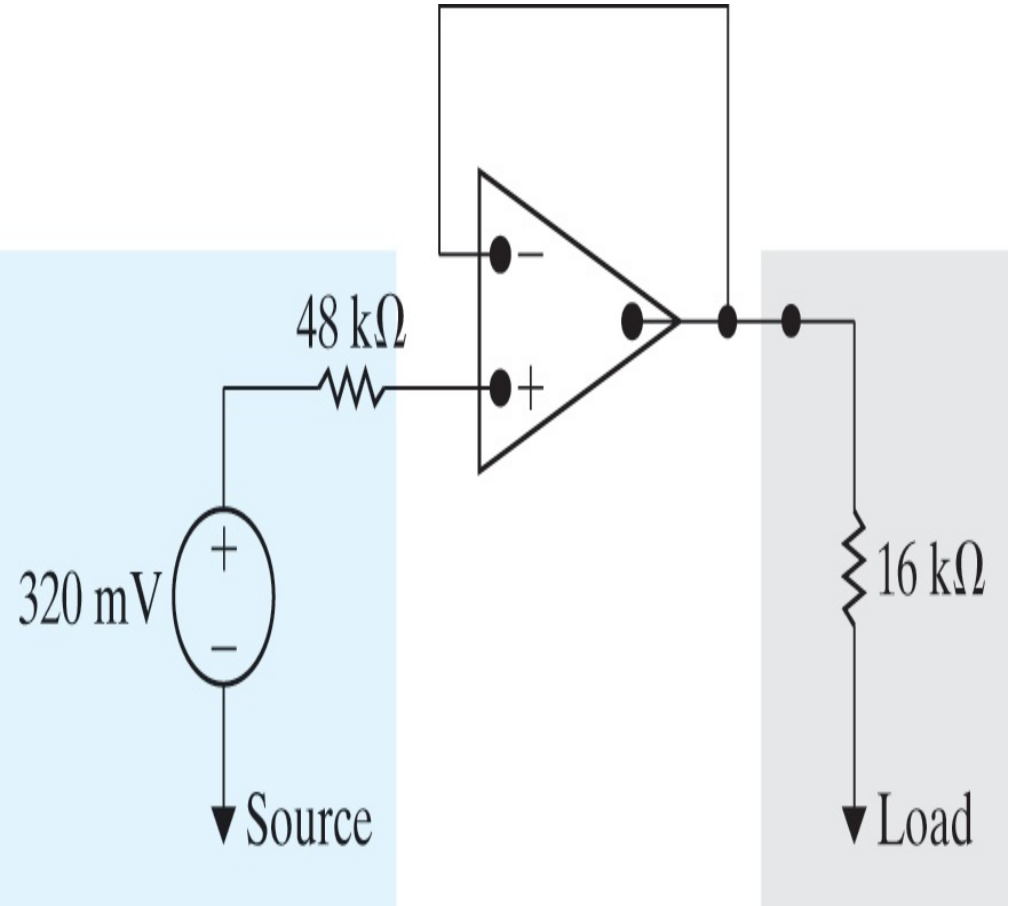


Figure P5.42 Full Alternative Text

Section 5.7

1. 5.43 Derive Eq. 5.31.

2. 5.44 PSPICEMULTISIM Repeat Assessment Problem 5.7, given that the inverting amplifier is loaded with a 500 Ω resistor.

3. 5.45 PSPICEMULTISIM

    1. Find the Thévenin equivalent circuit with respect to the output terminals a, b for the inverting amplifier of Fig. P5.45. The dc signal source has a value of 880 mV. The op amp has an input resistance of 500 kΩ, an output resistance of 2 kΩ, and an open-loop gain of 100,000.

    2. What is the output resistance of the inverting amplifier?

    3. What is the resistance (in ohms) seen by the signal source vs when the load at the terminals a, b is 330 Ω?

Figure P5.45

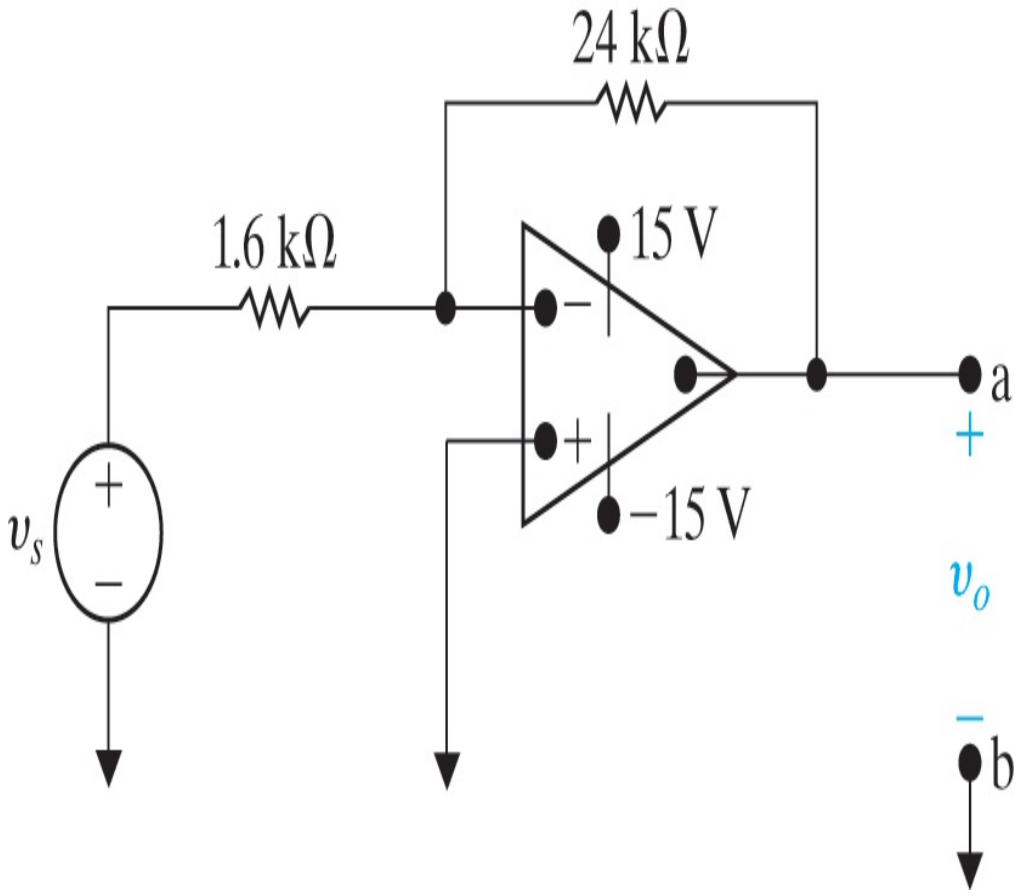


Figure P5.45 Full Alternative Text

4. 5.46 PSPICEMULTISIM Repeat Problem 5.45 assuming an ideal op amp.

5. 5.47 PSPICEMULTISIM Assume the input resistance of the op amp in Fig. P5.47 is infinite and its output resistance is zero.

    1. Find vo as a function of vg and the open-loop gain A.

    2. What is the value of vo if vg=0.4 V and A=90?

    3. What is the value of vo if vg=0.4 V and A=8?

    4. How large does A have to be so that vo is 95% of its value in (c)?

Figure P5.47

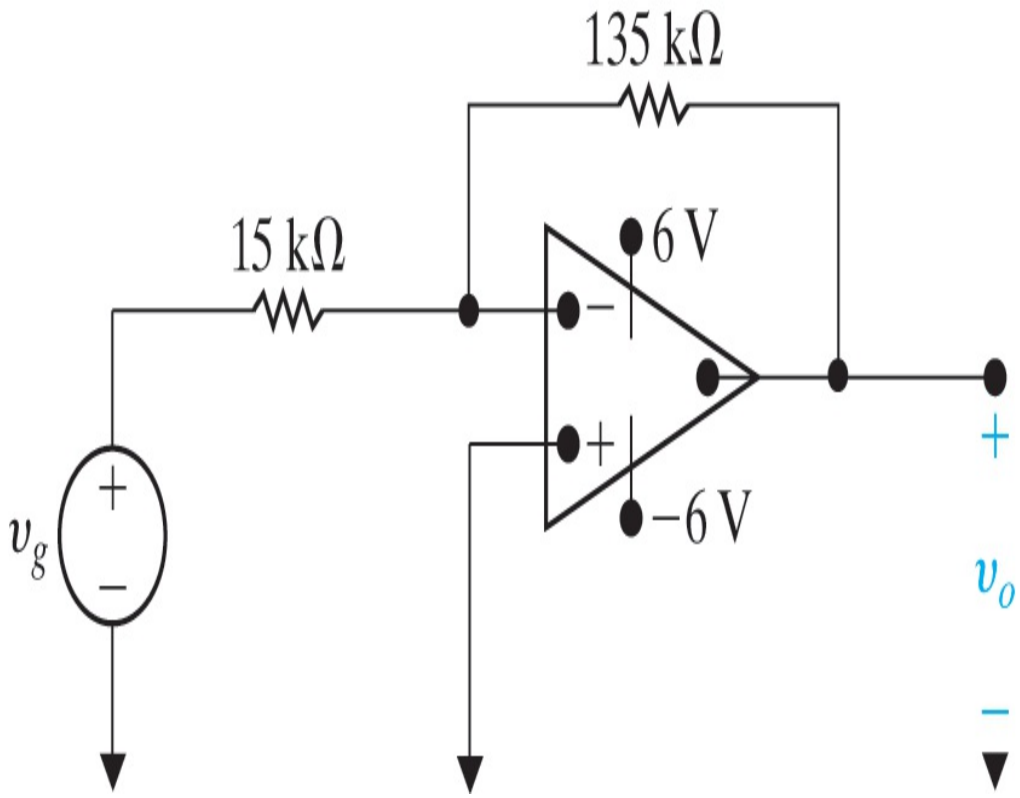


Figure P5.47 Full Alternative Text

6. 5.48 PSPICEMULTISIM The op amp in the noninverting amplifier circuit of Fig. P5.48 has an input resistance of 400 kΩ, an output resistance of 5 kΩ, and an open-loop gain of 20,000. Assume that the op amp is operating in its linear region.

    1. Calculate the voltage gain (vo/vg).

    2. Find the inverting and noninverting input voltages vn and vp (in millivolts) if vg=1 V.

    3. Calculate the difference (vp-vn) in microvolts when vg=1 V.

    4. Find the current drain in picoamperes on the signal source vg when vg=1 V.

5. Repeat (a)–(d) assuming an ideal op amp.

Figure P5.48

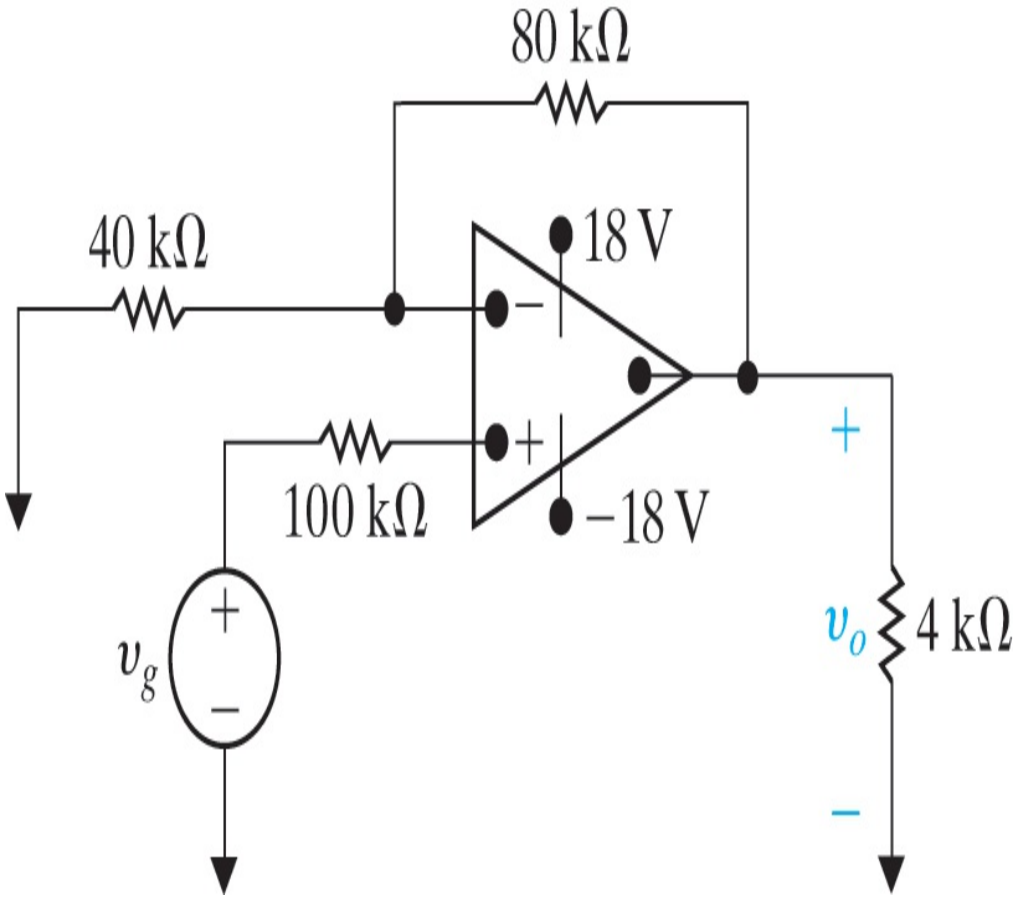


Figure P5.48 Full Alternative Text

Sections 5.1–5.7

1. 5.49 PRACTICALPERSPECTIVE_ Suppose the strain gages in the bridge in Fig. 5.22 have the value 120 O±1%. The power supplies to the op amp are ±15 V, and the reference voltage, vref, is taken from the positive power supply.

    1. Calculate the value of Rf so that when the strain gage that is

lengthening reaches its maximum length, the output voltage is 5 V.

2. Suppose that we can accurately measure 50 mV changes in the output voltage. What change in strain gage resistance can be detected in milliohms?

2. 5.50 PRACTICALPERSPECTIVE_ PSPICEMULTISIM

    1. For the circuit shown in Fig. P5.50, show that if $\Delta R << R$, the output voltage of the op amp is approximately

        $$v_o \approx \frac{R_f(R+R_f)}{R^2(R+2R_f)}(-\Delta R)v_{in}.$$

    2. Find v_o if $R_f = 470\ k\Omega$, $R = 10\ k\Omega$, $\Delta R = 95\ \Omega$, and $v_{in} = 15$ V.

    3. Find the actual value of v_o in (b).

Figure P5.50

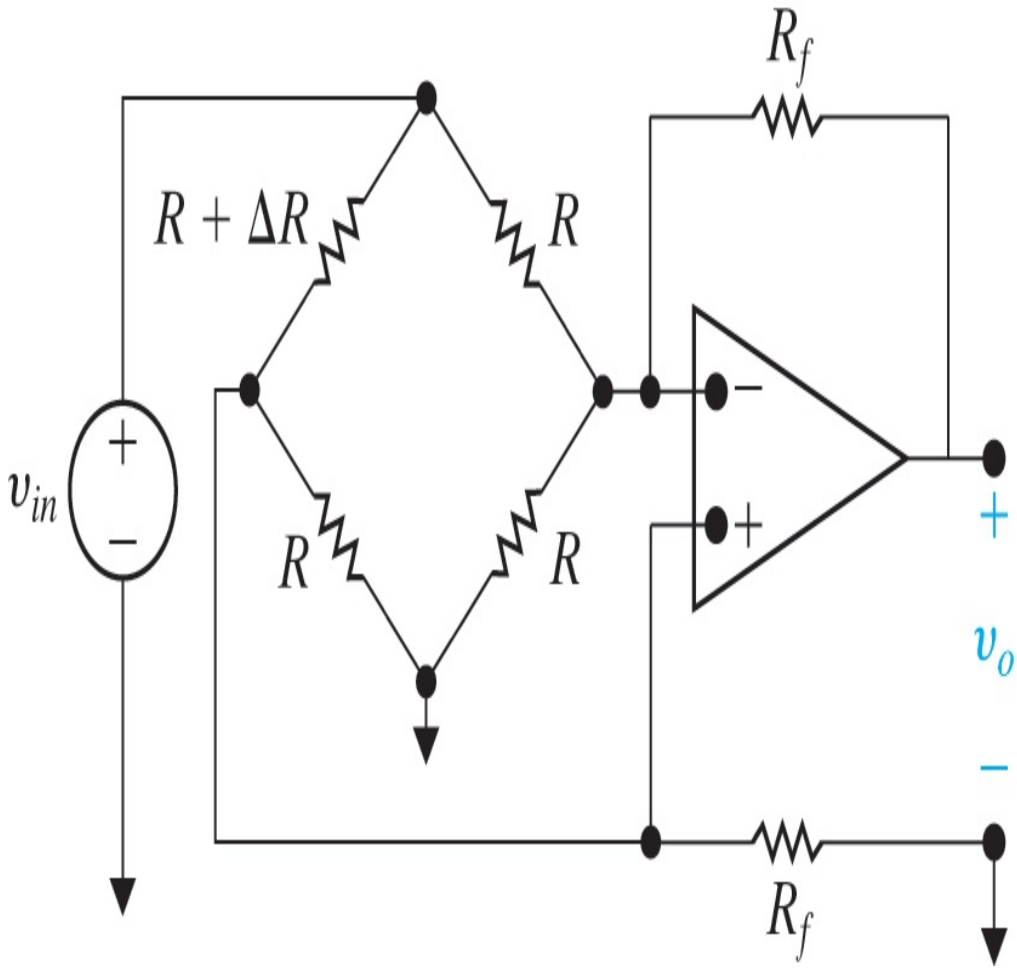


Figure P5.50 Full Alternative Text

3. 5.51 PRACTICALPERSPECTIVE_ PSPICEMULTISIM

    1. If percent error is defined as

        % error=[approximate valuetrue value-1]×100,

        show that the percent error in the approximation of vo in Problem 5.50 is

        % error=ΔRR(R+Rf)(R+2Rf)×100.

    2. Calculate the percent error in vo for Problem 5.50(b).

4. 5.52 PRACTICALPERSPECTIVE_ PSPICEMULTISIM Assume the

percent error in the approximation of vo in the circuit in Fig. P5.50 is not to exceed 1%. What is the largest percent change in R that can be tolerated?

5. 5.53 PRACTICALPERSPECTIVE_ PSPICEMULTISIM Assume the resistor in the variable branch of the bridge circuit in Fig. P5.50 is R-ΔR instead of R+ΔR.

    1. What is the expression for vo if $\Delta R \ll R$?

    2. What is the expression for the percent error in vo as a function of R, Rf, and ΔR?

    3. Assume the resistance in the variable arm of the bridge circuit in Fig. P5.50 is 9810 Ω and the values of R, Rf, and vin are the same as in Problem 5.50(b). What is the approximate value of vo?

    4. What is the percent error in the approximation of vo when the variable arm resistance is 9810 Ω?

Chapter 6 Inductance, Capacitance, and Mutual Inductance

Chapter Contents

1. 6.1 The Inductor
2. 6.2 The Capacitor
3. 6.3Series-Parallel Combinations of Inductance and Capacitance
4. 6.4 Mutual Inductance
5. 6.5 A Closer Look at Mutual Inductance

Chapter Objectives

1. Know and be able to use the equations for voltage, current, power, and energy in an inductor; understand how an inductor behaves in the presence of constant current; and understand the requirement that the current be continuous in an inductor.

2. Know and be able to use the equations for voltage, current, power, and energy in a capacitor; understand how a capacitor behaves in the presence of constant voltage; and understand the requirement that the voltage be continuous in a capacitor.

3. Be able to combine inductors with initial conditions in series and in parallel to form a single equivalent inductor with an initial condition; be able to combine capacitors with initial conditions in series and in parallel to form a single equivalent capacitor with an initial condition.

4. Understand the basic concept of mutual inductance and be able to write mesh-current equations for a circuit containing magnetically coupled coils using the dot convention correctly.

Here we introduce inductors and capacitors, the last two ideal circuit elements mentioned in Chapter 2. Fortunately, the circuit analysis techniques you learned in Chapters 3 and 4 apply to circuits containing inductors and capacitors as well. Once you understand the terminal behavior of these elements in terms of current and voltage, you can use Kirchhoff's laws to describe any interconnections with the other basic elements.

Like other components, inductors and capacitors are easier to describe in terms of circuit variables rather than electromagnetic field variables. Hence, Sections 6.1 and 6.2 briefly review the field concepts underlying inductors and capacitors before focusing on the circuit descriptions. These sections also examine energy in inductors and capacitors. Energy can be stored in both magnetic and electric fields, so inductors and capacitors can store energy. For example, energy can be stored in an inductor and then released to fire a spark

plug, or stored in a capacitor and then released to fire a strobe light. In ideal inductors and capacitors, you can extract only as much energy as you have stored. Because inductors and capacitors cannot generate energy, they are classified as **passive elements**.

Section 6.3 describes circuit simplification using series and parallel combinations of capacitors or inductors.

In Sections 6.4 and 6.5, we consider two circuits linked by a magnetic field and thus magnetically coupled. The voltage induced in one circuit is related to the time-varying current in the other circuit by a parameter known as **mutual inductance**. The practical significance of magnetic coupling unfolds as we study the relationships between current, voltage, power, and several new parameters specific to mutual inductance. We introduce these relationships here and then describe their utility in a device called a transformer in Chapters 9 and 10.

Practical Perspective

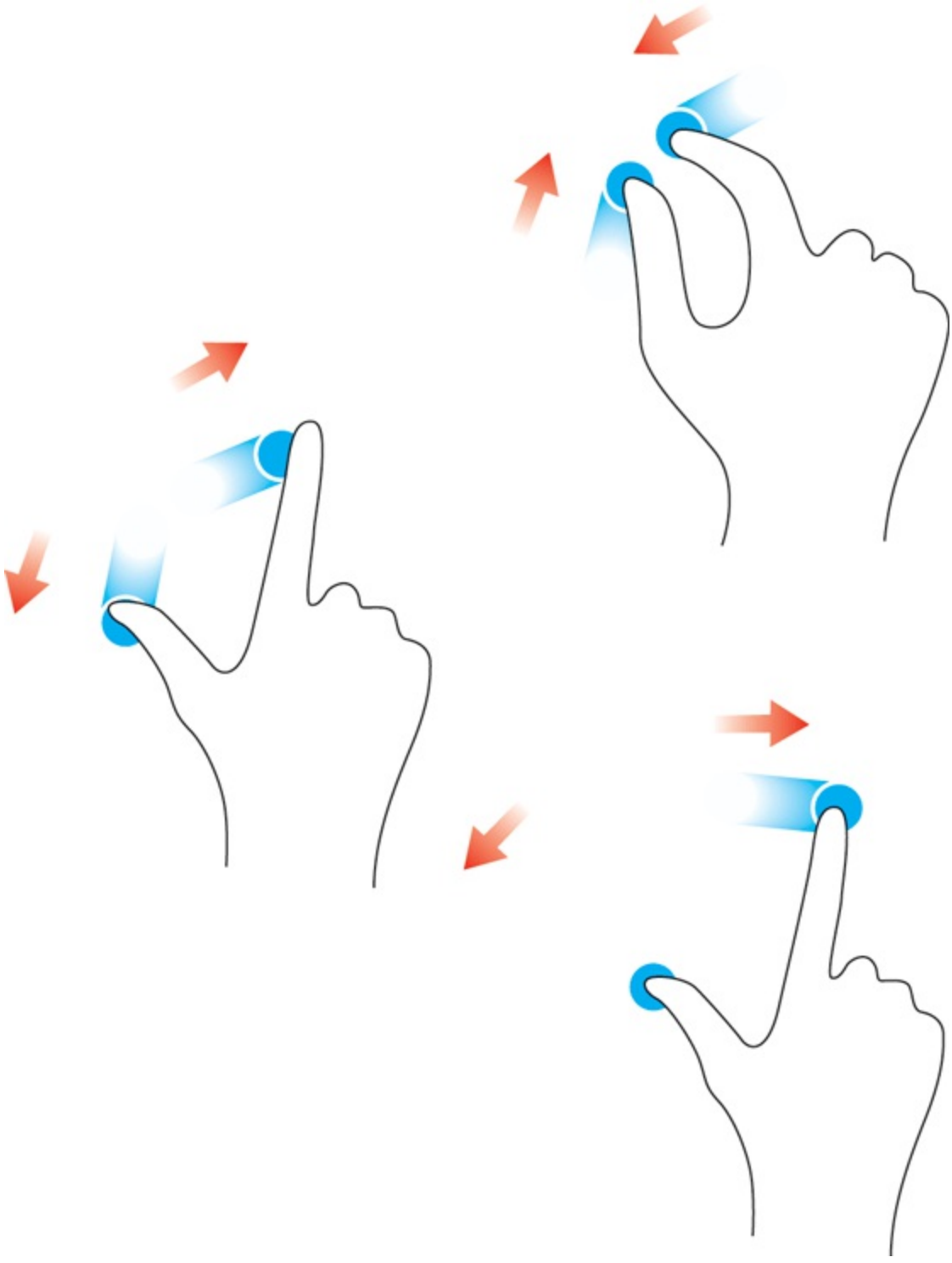
Capacitive Touch Screens

The Practical Perspective in Chapter 3 used a grid of resistors to create a touch screen for a phone or computer monitor. But resistive touch screens have some limitations, the most important of which is that the screen can only process a single touch at any instant in time (see Problem 3.75). This means a resistive touch screen cannot process the “pinch” gesture used by many devices to enlarge or shrink the image on the screen.

Multi-touch screens use a different component within a grid below the screen —capacitors. When you touch a capacitive touch screen, the capacitor’s value changes, causing a voltage change. Once you have learned the basic behavior of capacitors and know how they combine in series and in parallel, we will present two possible designs for a multi-touch screen using a grid of capacitors.

4:44 pm
SMS
23°C

cobalt88 /Shutterstock

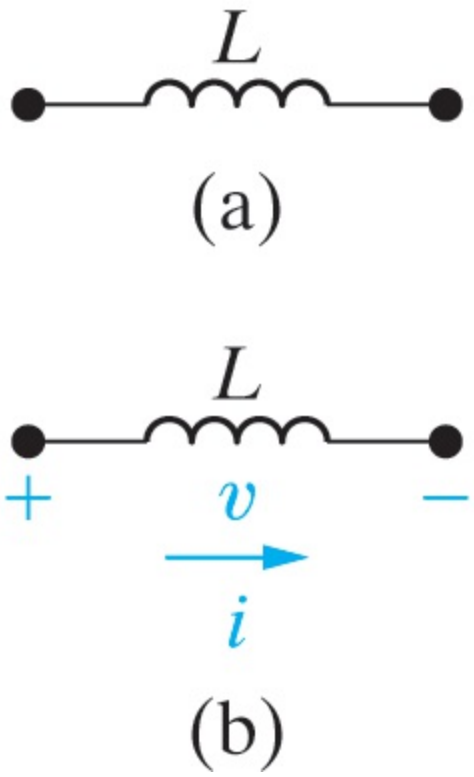


6.1 The Inductor

An inductor is an electrical component that opposes any change in electrical current. It is composed of a coil of wire wound around a supporting core whose material can be magnetic or nonmagnetic. The behavior of inductors is based on phenomena associated with magnetic fields. The source of the magnetic field is charge in motion, or current. If the current is varying with time, the magnetic field is varying with time. A time-varying magnetic field induces a voltage in any conductor linked by the field. The circuit parameter of **inductance** relates the induced voltage to the current.

Figure 6.1(a) shows an inductor, represented graphically as a coiled wire. Its inductance is symbolized by the letter L and is measured in henrys (H). Assigning the reference direction of the current in the direction of the voltage drop across the terminals of the inductor, as shown in Fig. 6.1(b), and using the passive sign convention yields

Figure 6.1 (a) The graphic symbol for an inductor with an inductance of L henrys. (b) Assigning reference voltage and current to the inductor, following the passive sign convention.



The Inductor v–i Equation

v=Ldidt,(6.1)

where v is measured in volts, L in henrys, i in amperes, and t in seconds. If the current reference is in the direction of the voltage rise, Eq. 6.1 is written with a minus sign.

Note from Eq. 6.1 that the voltage across the terminals of an inductor is proportional to the time rate of change of the current in the inductor. We can make two important observations here. First, if the current is constant, the voltage across the ideal inductor is zero. Thus, the inductor behaves as a short circuit in the presence of a constant, or dc, current. Second, current cannot change instantaneously in an inductor; that is, the current cannot change by a finite amount in zero time. Equation 6.1 tells us that this change would require an infinite voltage, and infinite voltages are not possible. For example, when someone opens the switch on an inductive circuit in an actual system, the current initially continues to flow in the air across the switch, a

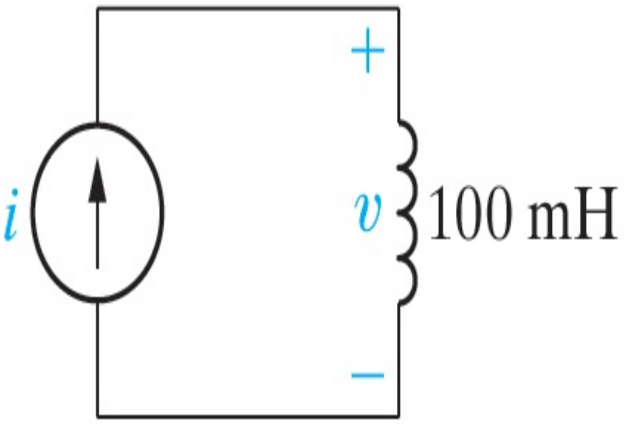
phenomenon called *arcing*. The arc across the switch prevents the current from dropping to zero instantaneously.

Example 6.1 illustrates the application of Eq. 6.1 to a simple circuit.

Example 6.1 Determining the Voltage, Given the Current, at the Terminals of an Inductor

The independent current source in the circuit shown in Fig. 6.2 generates zero current for t<0 and the pulse 10te−5tA for t>0.

Figure 6.2 The circuit for Example 6.1.



$$i = 0, \qquad t < 0$$

$$i = 10te^{-5t}\text{A}, \qquad t > 0$$

1. Sketch the current waveform.
2. At what instant of time is the current maximum?
3. Express the voltage across the terminals of the 100 mH inductor as a function of time.

4. Sketch the voltage waveform.
5. Are the voltage and the current at a maximum at the same time?
6. At what instant of time does the voltage change polarity?
7. Is there ever an instantaneous change in voltage across the inductor? If so, at what time?

Solution

1. Figure 6.3 shows the current waveform.

Figure 6.3 The current waveform for Example 6.1.

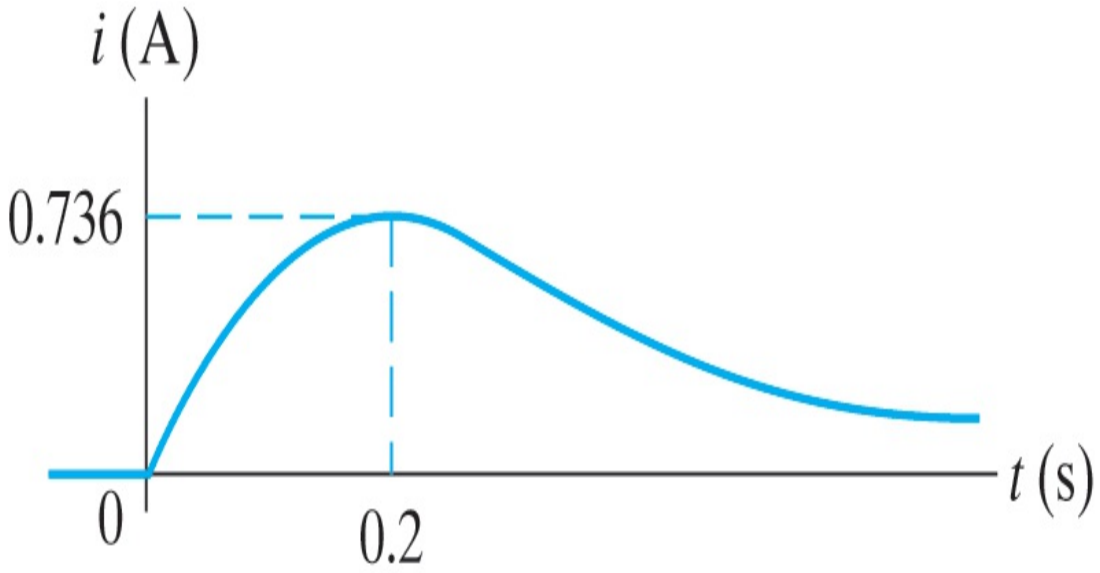


Figure 6.3 Full Alternative Text

2. di/dt=10(−5te−5t+e−5t)=10e−5t(1−5t) A/s; di/dt=0 when t=0.2s. (See Fig. 6.3.)

3. $v=Ldi/dt=(0.1)10e^{-5t}(1-5t)=e^{-5t}(1-5t)$ V, $t>0$. $v=0$; $t<0$.

4. Figure 6.4 shows the voltage waveform.

Figure 6.4 The voltage waveform for Example 6.1.

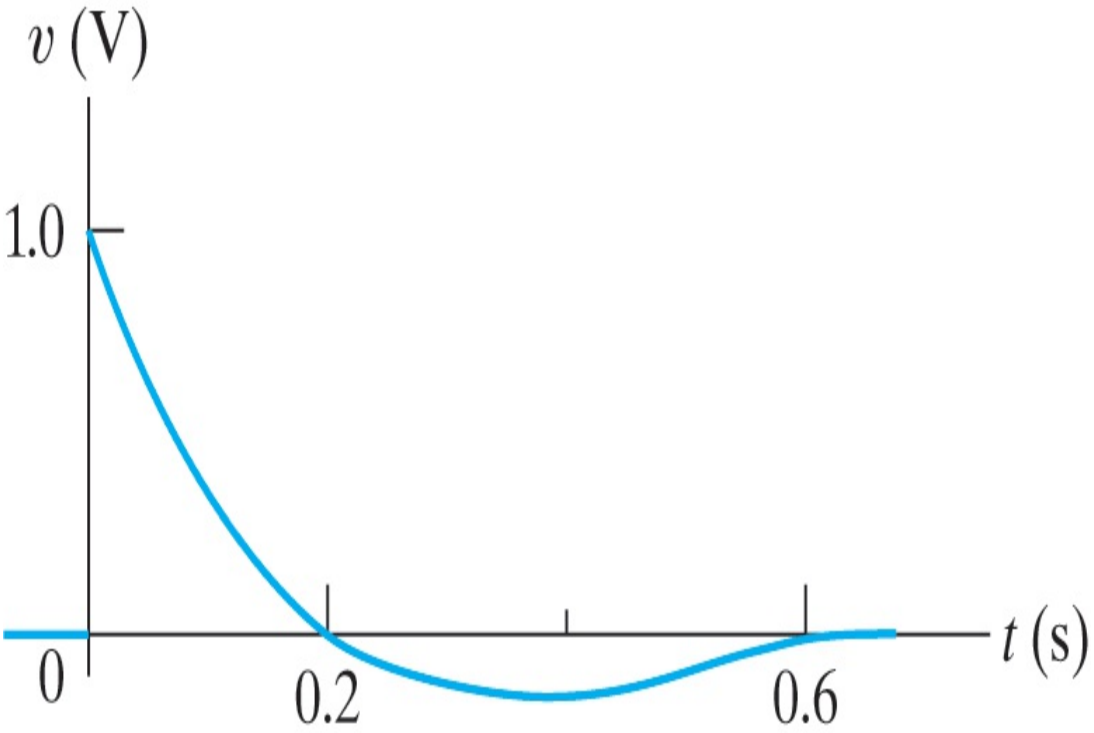


Figure 6.4 Full Alternative Text

5. No; the voltage is proportional to di/dt, not i.

6. At 0.2 s, which corresponds to the moment when di/dt is passing through zero and changing sign.

7. Yes, at t=0. Note that the voltage can change instantaneously across the terminals of an inductor, even though the current in the inductor cannot change instantaneously.

Current in an Inductor in Terms of the Voltage Across the Inductor

Equation 6.1 expresses the voltage across the terminals of an inductor as a function of the current in the inductor. Now we express the current as a function of the voltage. To find i as a function of v, start by multiplying both sides of Eq. 6.1 by a differential time dt:

v dt=L(didt) dt.

Multiplying the rate at which i varies with t by a differential change in time generates a differential change in i, so the expression simplifies to

v dt=L di.

We next integrate both sides of the simplified expression. For convenience, we interchange the two sides of the equation and write

L ∫i(t0)i(t)dx=∫t0tv dτ.

Note that we use x and τ as the variables of integration, so i and t become limits on the integrals. Then, divide both sides of the integral equation by L and solve for the inductor current to get

The Inductor i–v Equation

i(t)=1L ∫t0tv dτ+i(t0),(6.2)

where i(t0) is the value of the inductor current at the time when we initiate the integration, namely, t0. In many practical applications, t0 is zero and Eq. 6.2 becomes

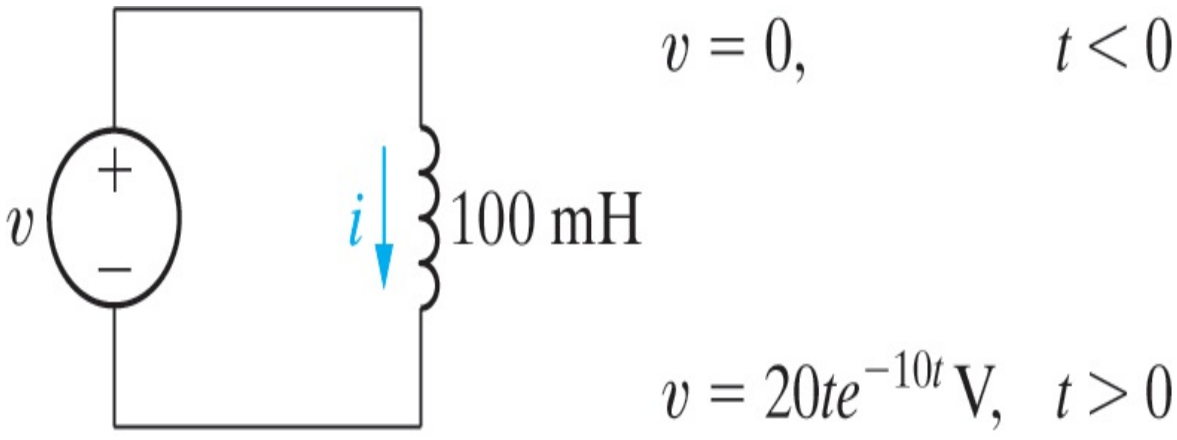
i(t)=1L ∫0tv dτ+i(0).

Equations 6.1 and 6.2 both give the relationship between the voltage and current at the terminals of an inductor. In both equations, the reference direction for the current is in the direction of the voltage drop across the terminals. Pay attention to the algebraic sign of i(t0). If the initial current direction and the reference direction for i are the same, the initial current is positive. If the initial current is in the opposite direction, it is negative. Example 6.2 illustrates the application of Eq. 6.2.

Example 6.2 Determining the Current, Given the Voltage, at the Terminals of an Inductor

The voltage pulse applied to the 100 mH inductor shown in Fig. 6.5 is 0 for t<0 and is given by the expression

Figure 6.5 The circuit for Example 6.2.



v(t)=20te−10t V

for t>0. Also assume i=0 for t≤0.

1. Sketch the voltage as a function of time.
2. Find the inductor current as a function of time.
3. Sketch the current as a function of time.

Solution

1. The voltage as a function of time is shown in Fig. 6.6.

Figure 6.6 The voltage waveform for Example 6.2.

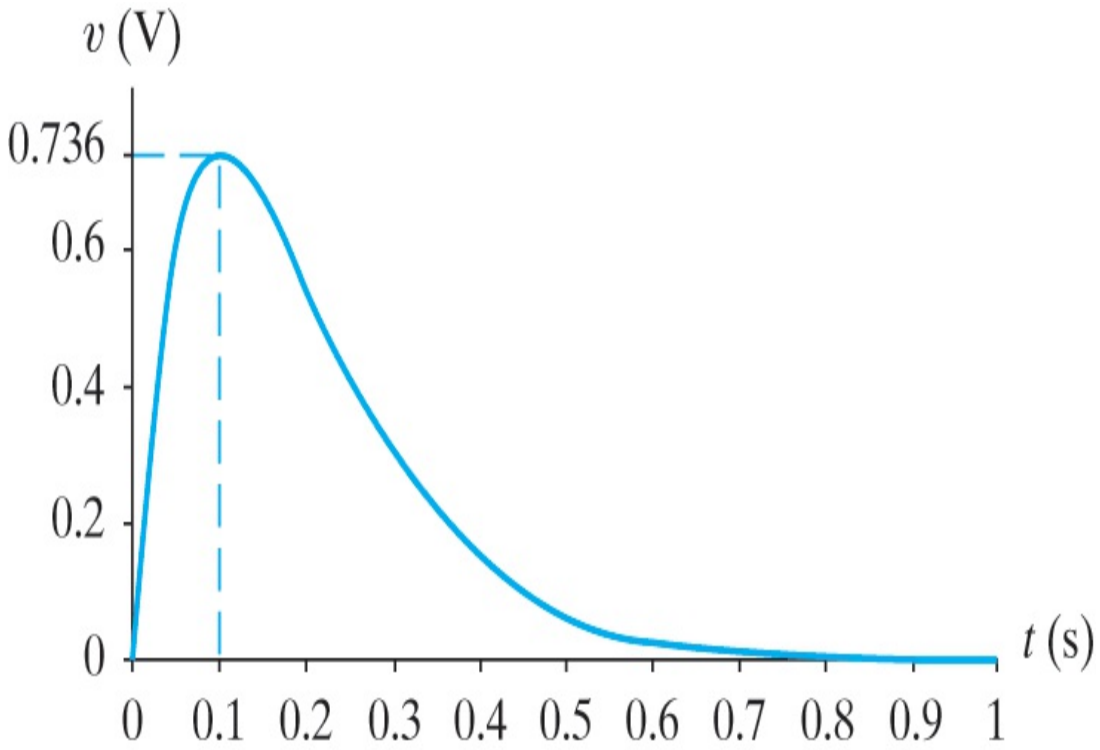


Figure 6.6 Full Alternative Text

2. The current in the inductor is 0 at t=0. Therefore, the current for t>0 is

i = 200[− e −10τ 100 (10τ+1)] | 0 t , = 2(1−10t e −10t − e −10t) A,t>0. = 1 0.1 ∫ 0 t 20τ e −10τ dτ+0

3. Figure 6.7 shows the current as a function of time.

Figure 6.7 The current waveform for Example 6.2.

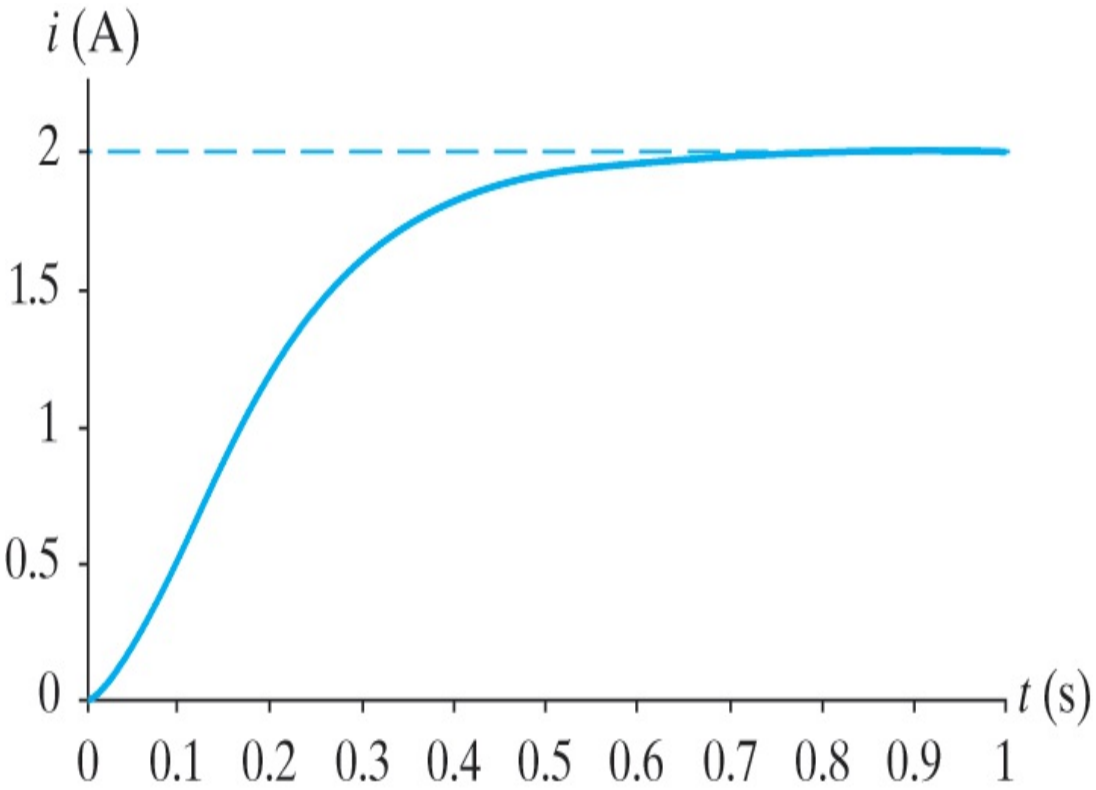


Figure 6.7 Full Alternative Text

In Example 6.2, i approaches a constant value of 2 A as t increases. We say more about this result after discussing the energy stored in an inductor.

Power and Energy in the Inductor

The power and energy relationships for an inductor can be derived directly from the current and voltage relationships. If the current reference is in the direction of the voltage drop across the terminals of the inductor, the power is

p=vi.(6.3)

Remember that power is in watts, voltage is in volts, and current is in amperes. If we express the inductor voltage as a function of the inductor current, the expression for inductor power becomes

Power in an Inductor

p=Lididt.(6.4)

We can also express the current in terms of the voltage:

p=v[1L ∫t0tv dτ+i(t0)].

We can use Eq. 6.4 to find the energy stored in the inductor. Power is the time rate of expending energy, so

p=dwdt=Lididt.

Multiplying both sides by a differential time gives the differential relationship

dw=Li di.

Integrate both sides of the differential relationship, recognizing that the reference for zero energy corresponds to zero current in the inductor. Thus

∫0w dx=L ∫0iy dy,

so

Energy in an Inductor

w=12Li2.(6.5)

As before, we use different symbols of integration to avoid confusion with the limits placed on the integrals. In Eq. 6.5, the energy is in joules, inductance is in henrys, and current is in amperes. Example 6.3 applies Eqs. 6.3 and 6.5 to the circuits in Examples 6.1 and 6.2 to examine power and energy in these circuits.

Example 6.3 Determining the Current, Voltage, Power, and Energy for an Inductor

1. For Example 6.1, plot i, v, p, and w versus time. Line up the plots vertically to allow easy assessment of each variable's behavior.
2. In what time interval is energy being stored in the inductor?
3. In what time interval is energy being extracted from the inductor?
4. What is the maximum energy stored in the inductor?
5. Evaluate the integrals

   ∫00.2p dt and∫0.2∞p dt,

   and comment on their significance.
6. Repeat (a)–(c) for Example 6.2.
7. In Example 6.2, why is there a sustained current in the inductor as the voltage approaches zero?

Solution

1. The plots of i and v follow directly from the expressions for i and v obtained in Example 6.1. Applying Eq. 6.3,

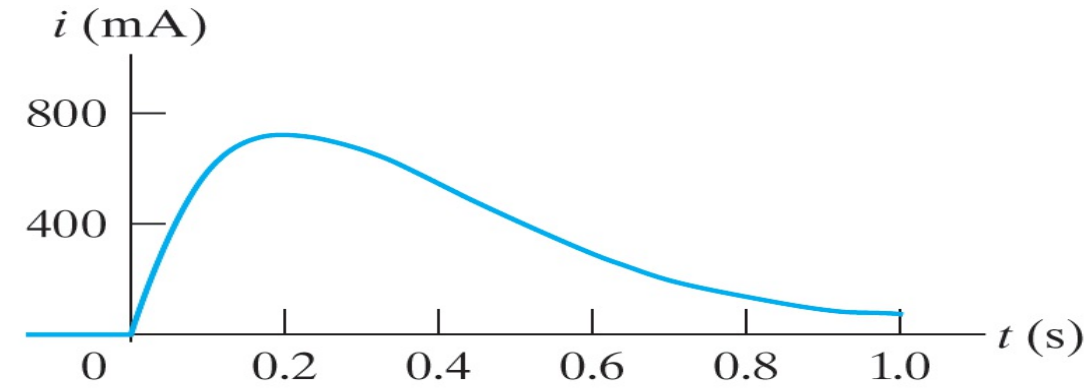
   $p = vi = [\, e^{-5t} (1-5t) \,](10t\, e^{-5t}) = 10t\, e^{-10t} (1-5t)$ W.

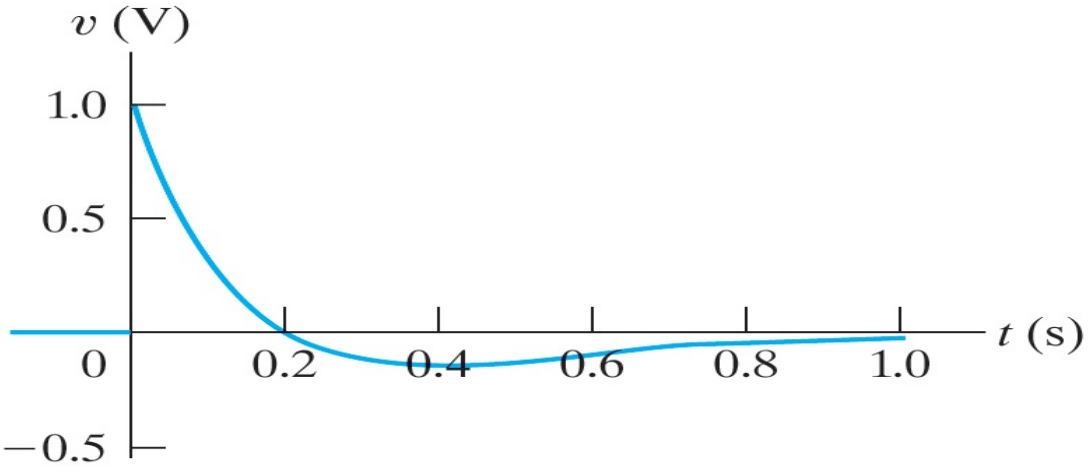
   Applying Eq. 6.5,

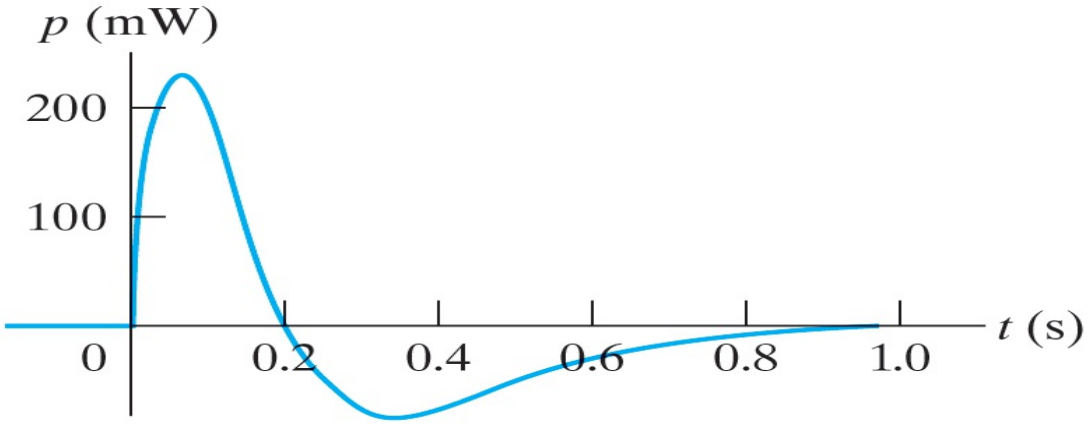
   $w = \frac{1}{2}Li = \frac{1}{2}(0.1)(10te^{-5t})^2 = 5t^2e^{-10t}$ J.

   The plots of i, v, p, and w are shown in Fig. 6.8.

Figure 6.8 The variables i, v, p, and w versus t for Example 6.1.


i (mA)
800
400
0
0.2
0.4
0.6
0.8
1.0
t (s)


v (V)
1.0
0.5
0
−0.5
0.2
0.4
0.6
0.8
1.0
t (s)


p (mW)
200
100
0
0.2
0.4
0.6
0.8
1.0
t (s)

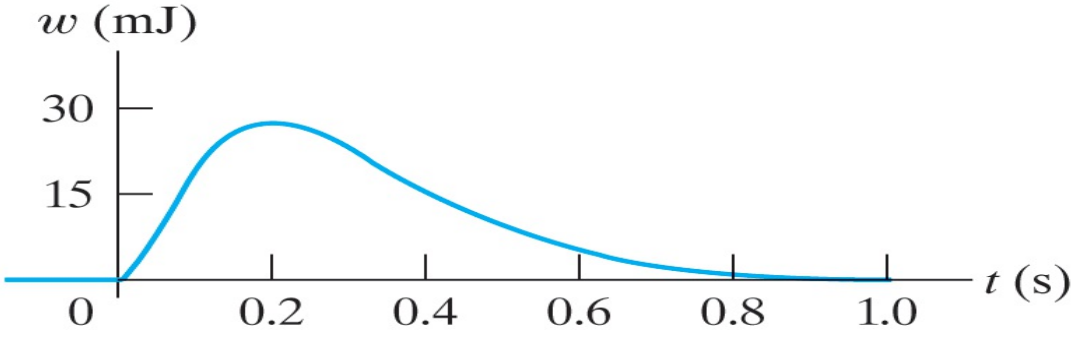

w (mJ)
30
15
0
0.2
0.4
0.6
0.8
1.0
t (s)

Figure 6.8 Full Alternative Text

2. When the energy curve increases, energy is being stored. Thus, from Fig. 6.8, energy is being stored in the time interval 0 to 0.2 s. This corresponds to the interval when $p>0$.

3. When the energy curve decreases, energy is being extracted. Thus, from Fig. 6.8, energy is being extracted in the time interval 0.2 s to ∞. This corresponds to the interval when $p<0$.

4. Equation 6.5 tells us that energy is at a maximum when current is at a maximum; the graphs in Fig. 6.8 confirm this. From Example 6.1, $i_{max}=0.736$ A. Therefore, $w_{max}=27.07$ mJ.

5. From part (a),

   $p=10te^{-10t}(1-5t)=10te^{-10t}=50t2e^{-10t}$.

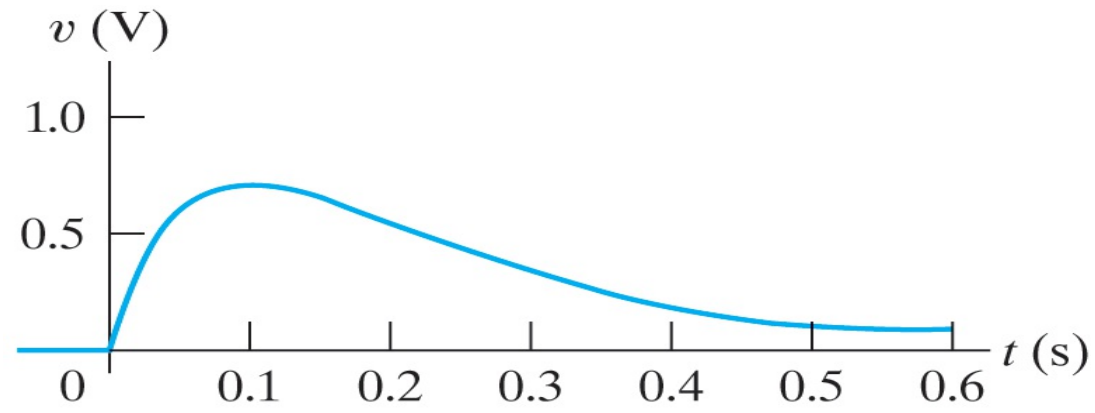
   Thus

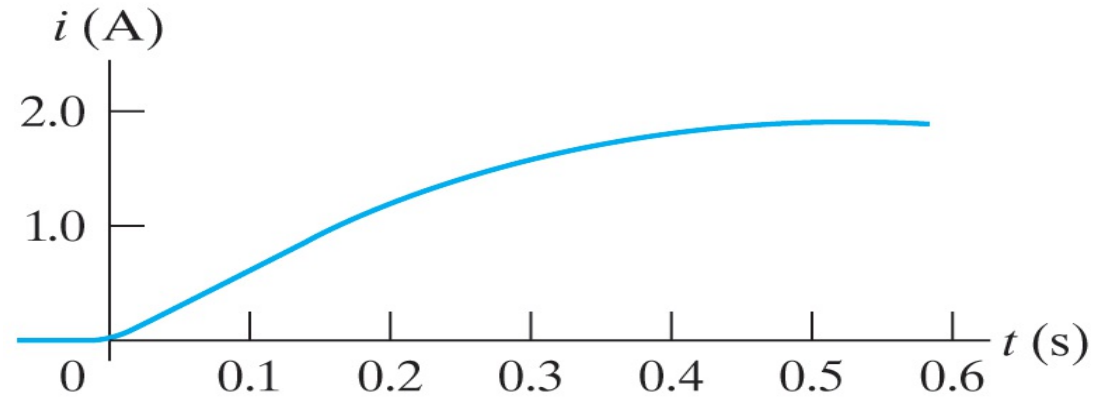
   $$\int_0^{0.2} p\,dt = 10\left[e^{-10t}100(-10t-1)\right]_0^{0.2} - 50\{t^2e^{-10t} - 10 + 2\,10\left[e^{-10t}100(-10t-1)\right]\}_0^{0.2} = 0.2e^{-2} = 27.07 \text{ mJ},$$
   $$\int_{0.2}^{\infty} p\,dt = 10\left[e^{-10t}100(-10t-1)\right]_{0.2}^{\infty} - 50\{t^2e^{-10t} - 10 + 2\,10\left[e^{-10t}100(-10t-1)\right]\}_{0.2}^{\infty} = -0.2e^{-2} = -27.07 \text{ mJ}.$$

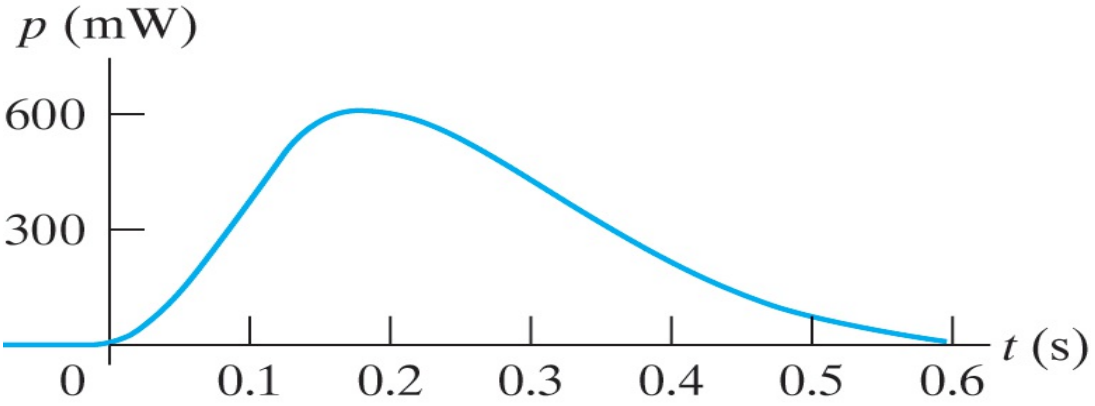
   Based on the definition of p, the area under the plot of p versus t represents the energy expended over the interval of integration. Hence, integrating the power between 0 and 0.2 s represents the energy stored in the inductor during this time interval. Integrating p between 0.2 s and infinity gives the energy extracted. Note that in this time interval, all the energy originally stored is removed, so after the current peak, no energy is stored in the inductor.

6. The plots of i, v, p, and w follow directly from the expressions for v and i given in Example 6.2 and are shown in Fig. 6.9. Note that in this case the power is always positive, and hence energy is always being stored during the voltage pulse.

Figure 6.9 The variables i, v, p, and w versus t for Example 6.2.


v (V)
1.0
0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
t (s)


i (A)
2.0
1.0
0
0.1
0.2
0.3
0.4
0.5
0.6
t (s)


p (mW)
600
300
0
0.1
0.2
0.3
0.4
0.5
0.6
t (s)

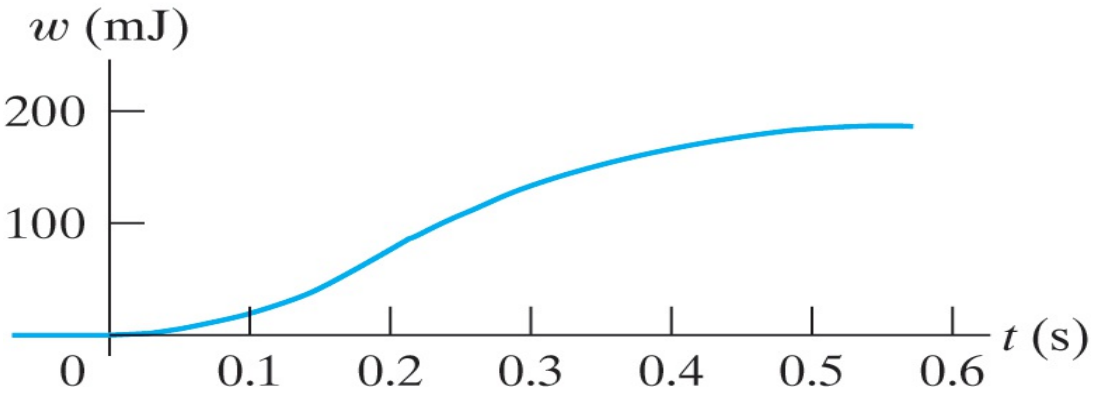

w (mJ)
200
100
0
0.1
0.2
0.3
0.4
0.5
0.6
t (s)

Figure 6.9 Full Alternative Text

7. The inductor stores energy when the voltage pulse is applied. Because the inductor is ideal, the energy cannot dissipate after the voltage subsides to zero, and thus current continues to circulate in the circuit. Practical inductors require a resistor in the circuit model, which we will examine later in this text.

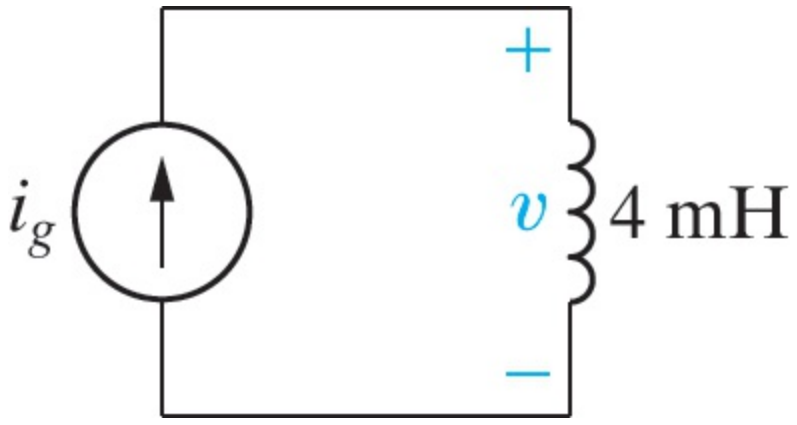
Assessment Problem

Objective 1—Know and be able to use the equations for voltage, current, power, and energy in an inductor

1. 6.1 The current source in the circuit shown generates the current pulse

   $i_g(t) = 0,\ t<0,\ i_g(t) = 8e^{-300t} - 8e^{-1200t}\ \text{A},\ t\geq 0.$

   Find (a) v(0); (b) the instant of time, greater than zero, when the voltage v passes through zero; (c) the expression for the power delivered to the inductor; (d) the instant when the power delivered to the inductor is maximum; (e) the maximum power; (f) the instant of time when the stored energy is maximum; and (g) the maximum energy stored in the inductor.



   Answer:

1. (a) 28.8 V;
2. (b) 1.54 ms;
3. (c) $-76.8e^{-600t} + 384e^{-1500t} - 307.2e^{-2400t}$ W, $t \geq 0$;
4. (d) 411.05 μs;
5. (e) 32.72 W;
6. (f) 1.54 ms;
7. (g) 28.57 mJ.

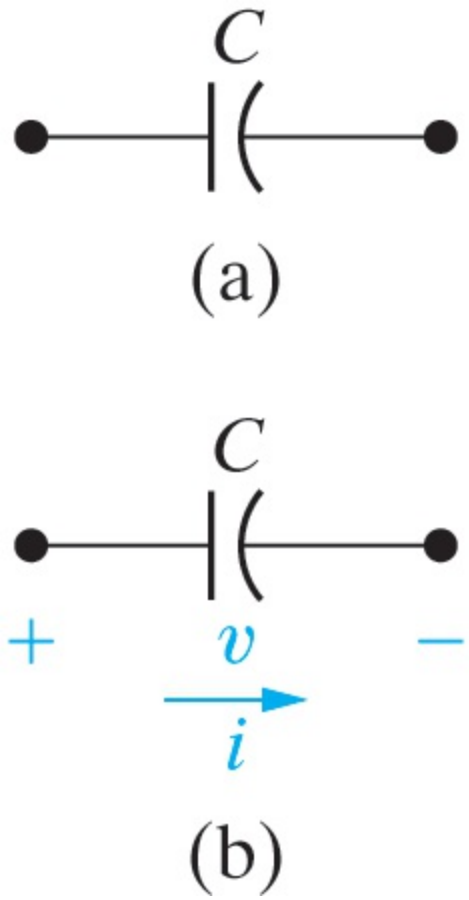
SELF-CHECK: *Also try Chapter Problems 6.3 and 6.11*.

6.2 The Capacitor

A capacitor is an electrical component consisting of two conductors separated by an insulator or dielectric material. The capacitor is the only device other than a battery that can store electrical charge. The behavior of capacitors is based on phenomena associated with electric fields. The source of the electric field is separation of charge, or voltage. If the voltage is varying with time, the electric field is varying with time. A time-varying electric field produces a displacement current in the space occupied by the field. The circuit parameter of **capacitance** relates the displacement current to the voltage, where the displacement current is equal to the conduction current at the terminals of the capacitor.

The circuit symbol for a capacitor is two short parallel conductive plates, as shown in Fig. 6.10(a). The capacitance is represented by the letter C and is measured in farads (F). Because the farad is an extremely large quantity of capacitance, practical capacitor values usually lie in the picofarad (pF) to microfarad (μF) range.

Figure 6.10 (a) The circuit symbol for a capacitor. (b) Assigning reference voltage and current to the capacitor, following the passive sign convention.



The capacitor’s symbol reminds us that capacitance occurs whenever electrical conductors are separated by a dielectric, or insulating, material. This condition implies that electric charge is not transported through the capacitor. Although applying a voltage to the terminals of the capacitor cannot move a charge through the dielectric, it can displace a charge within the dielectric. As the voltage varies with time, the displacement of charge also varies with time, causing what is known as the **displacement current**.

The displacement current is indistinguishable from the conduction current at the capacitor’s terminals. The current is proportional to the rate at which the voltage across the capacitor varies with time, so

Capacitor i–v Equation

i=Cdvdt,(6.6)

where i is measured in amperes, C in farads, v in volts, and t in seconds. In Fig. 6.10(b), the current reference is in the direction of the voltage drop across the capacitor. Using the passive sign convention, we write Eq. 6.6 with a positive sign. If the current reference is in the direction of the voltage rise, Eq. 6.6 is written with a minus sign.

Two important observations follow from Eq. 6.6. First, if the voltage across the terminals is constant, the capacitor current is zero because a conduction current cannot be established in the dielectric material of the capacitor. Only a time-varying voltage can produce a displacement current. Thus, a capacitor behaves as an open circuit in the presence of a constant voltage. Second, voltage cannot change instantaneously across the terminals of a capacitor. Equation 6.6 indicates that such a change would produce infinite current, a physical impossibility.

Equation 6.6 gives the capacitor current as a function of the capacitor voltage. To express the voltage as a function of the current, we multiply both sides of Eq. 6.6 by a differential time dt and then integrate the resulting differentials:

$i\,dt = C\,dv$ or $\int_{v(t_0)}^{v(t)} dx = \frac{1}{C}\int_{t_0}^{t} i\,d\tau$.

Carrying out the integration of the left-hand side of the second equation and rearranging gives

Capacitor v–i Equation

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i\,d\tau + v(t_0). \quad (6.7)$$

In many practical applications of Eq. 6.7, the initial time is zero; that is, $t_0 = 0$. Thus, Eq. 6.7 becomes

$$v(t) = \frac{1}{C}\int_{0}^{t} i\,d\tau + v(0).$$

We can easily derive the power and energy relationships for the capacitor. From the definition of power,

Capacitor Power Equation

$$p = vi = Cv\frac{dv}{dt}, \quad (6.8)$$

or

$$p = i\left[\frac{1}{C}\int_{t0}^{t} i\, d\tau + v(t0)\right].$$

Combining the definition of energy with Eq. 6.8 yields

$$dw = Cv\, dv,$$

from which

$$\int_{0}^{w} dx = C\int_{0}^{v} y\, dy,$$

or

Capacitor Energy Equation

$$w = \frac{1}{2}Cv^2. \quad (6.9)$$

In deriving Eq. 6.9, the reference for zero energy corresponds to zero voltage.

Examples 6.4 and 6.5 illustrate the current, voltage, power, and energy relationships for a capacitor.

Example 6.4 Determining Current, Voltage, Power, and Energy for a Capacitor

The voltage pulse across the terminals of a 0.5 μF capacitor is:

$v(t)=\{0, t\leq 0\ s; 4t\ V, 0\ s\leq t\leq 1\ s; 4e^{-(t-1)}\ V, t\geq 1\ s.$

1. Derive the expressions for the capacitor current, power, and energy.
2. Sketch the voltage, current, power, and energy as functions of time. Line up the plots vertically.
3. Specify the time interval when energy is being stored in the capacitor.
4. Specify the time interval when energy is being delivered by the capacitor.
5. Evaluate the integrals

   $\int_0^1 p\ dt$ and $\int_1^\infty p\ dt$

   and comment on their significance.

Solution

1. From Eq. 6.6,

   $i=\{(0.5\ \mu)(0)=0, t<0 s; (0.5\ \mu)(4)=2\ \mu A, 0\ s<t<1\ s; (0.5\ \mu)(-4e^{-(t-1)})=-2e^{-(t-1)}\mu A, t>1\ s.$

   The expression for the power is derived from Eq. 6.8:

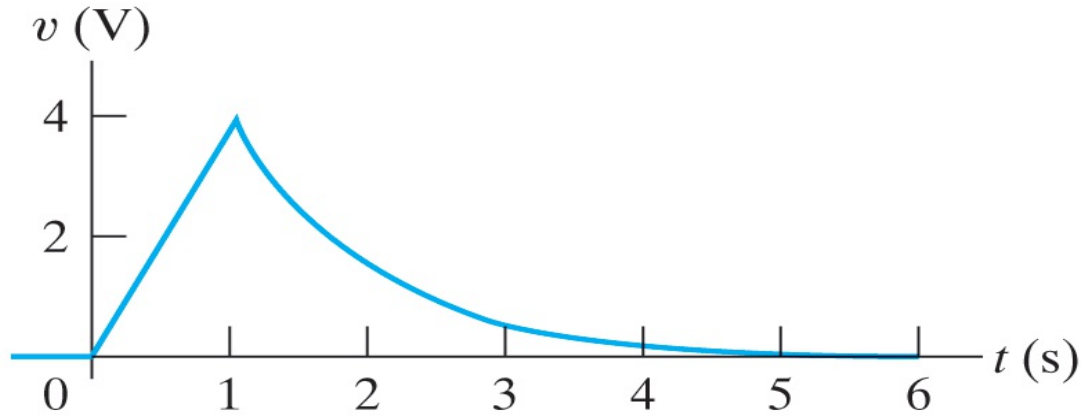
   $p=\{0, t\leq 0\ s; (4t)(2)=8t\ \mu\ W, 0\ s\leq t<1\ s; (4e^{-(t-1)})(-2\mu e^{-(t-1)})=-8e^{-2(t-1)}\mu W, t>1\ s.$

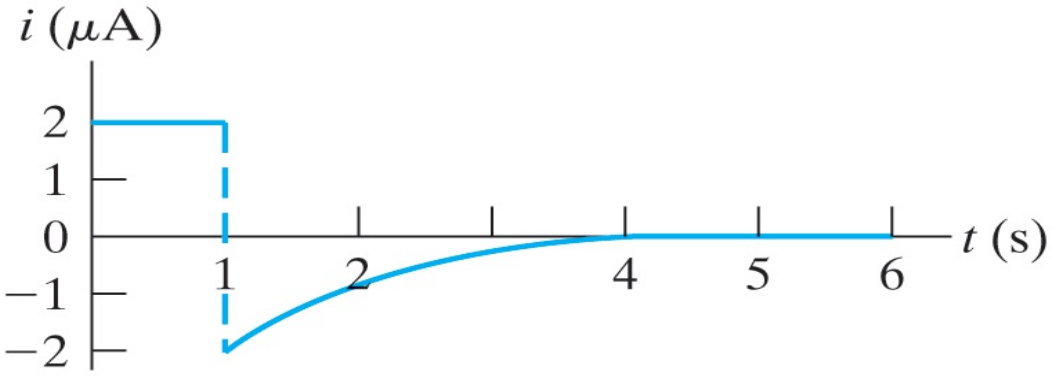
   The energy expression follows directly from Eq. 6.9:

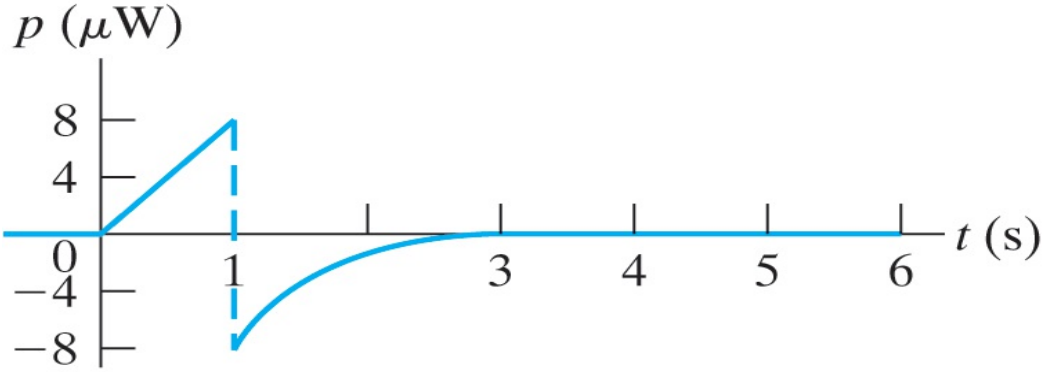
   $w=\{0, t\leq 0\ s; 12(0.5\ \mu)16t^2=4t^2\mu J, 0\ s\leq t<1\ s; 12(0.5\ \mu)16e^{-2(t-1)}=4e^{-2(t-1)}\mu J, t\geq 1\ s.$

2. Figure 6.11 shows the voltage, current, power, and energy as functions of time.

Figure 6.11 The variables i, v, p, and w versus t for Example 6.4.


v (V)
4
2
0
1
2
3
4
5
6
t (s)


i (μA)
2
1
0
−1
−2
1
2
4
5
6
t (s)


p (μW)
8
4
0
−4
−8
1
3
4
5
6
t (s)

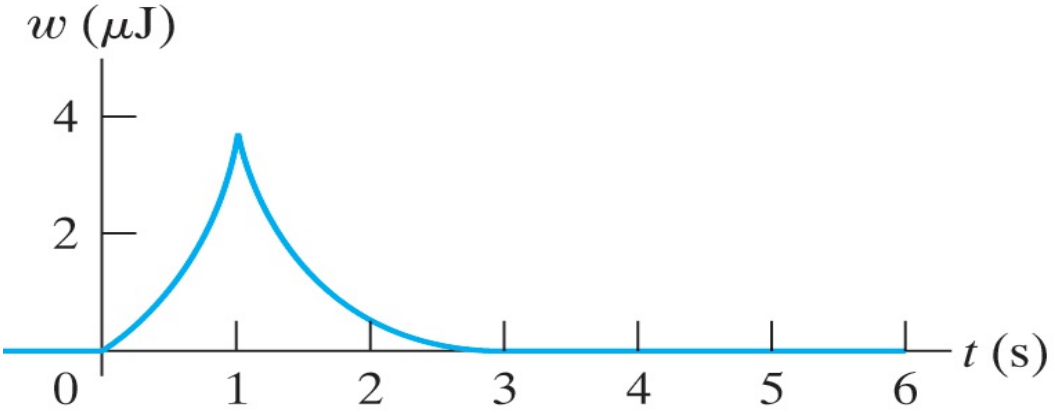

w (μJ)
4
2
0
1
2
3
4
5
6
t (s)

Figure 6.11 Full Alternative Text

3. Energy is being stored in the capacitor whenever the power is positive. Hence, energy is being stored in the interval from 0 to 1 s.

4. Energy is being delivered by the capacitor whenever the power is negative. Thus, energy is being delivered for all t greater than 1 s.

5. The integral of p dt is the energy associated with the time interval corresponding to the integral's limits. Thus, the first integral represents the energy stored in the capacitor between 0 and 1 s, whereas the second integral represents the energy returned, or delivered, by the capacitor in the interval 1 s to ∞:

   $$\int_0^1 p\,dt=\int_0^1 8t\,dt=4t^2\big|_0^1=4\ \mu J,$$
   $$\int_1^\infty p\,dt=\int_1^\infty(-8e^{-2(t-1)})dt=(-8)e^{-2(t-1)}-2\big|_1^\infty=-4\ \mu J.$$

   The voltage applied to the capacitor returns to zero as time increases, so the energy returned by this ideal capacitor must equal the energy stored.

Example 6.5 Finding v, p, and w Induced by a Triangular Current Pulse for a Capacitor

An uncharged 0.2 μF capacitor is driven by a triangular current pulse. The current pulse is described by

$$i(t)=\{0,t\le 0;5000t\ A,0\le t\le 20\ \mu s;0.2-5000t\ A,20\le t\le 40\ \mu s;0,t\ge 40\ \mu s.$$

1. Derive the expressions for the capacitor voltage, power, and energy for each of the four time intervals needed to describe the current.

2. Plot i, v, p, and w versus t. Align the plots as specified in the previous examples.

3. Why does a voltage remain on the capacitor after the current returns to zero?

Solution

1. For $t \leq 0$, v, p, and w all are zero.

   For $0 \leq t \leq 20\ \mu s$,

   $v = 10.2 \times 10^{-6} \int_0^t (5000\tau) d\tau + 0 = 12.5 \times 10^9 t^2 V, p = vi, = 62.5 \times 10^{12} t^3\ W, w = 12Cv^2 = 15.625 \times 10^{12} t^4\ J.$

   For $20\ \mu s \leq t \leq 40\ \mu s$,

   $v(20 \times 10^{-6}) = 12.5 \times 10^9 (20 \times 10^{-6})^2 = 5\ V.$

   Then,

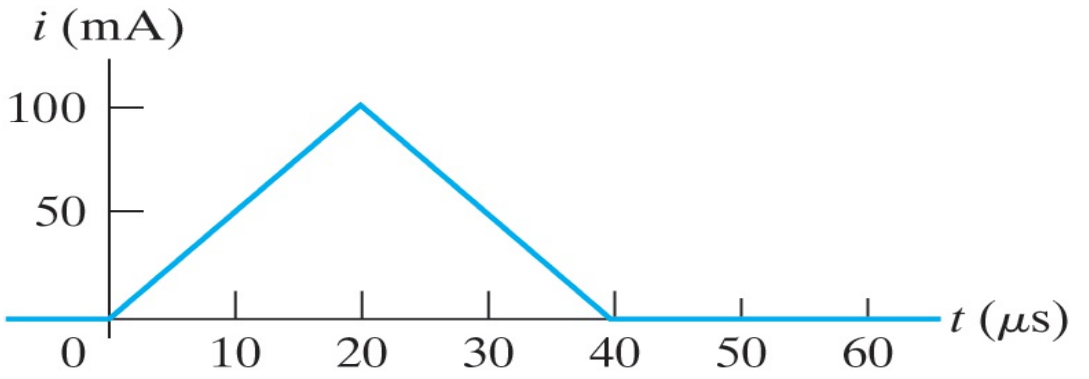
   $v = 1\ 0.2 \times\ 10^{-6} \int_{20\ \mu s}^{t} (\ 0.2 - 5000\tau\ ) d\tau + 5 = (\ 10^6 t - 12.5 \times\ 10^9 t^2\ -10\ )\ V,$ $P = vi, = (\ 62.5 \times\ 10^{12} t^3 - 7.5 \times\ 10^9 t^2 + 2.5 \times\ 10^5 t - 2\ )\ W,$ $w = 1\ 2\ C\ v^2\ , = (15.625 \times\ 10^{12} t^4 - 2.5 \times\ 10^9 t^3 + 0.125 \times\ 10^6 t^2 - 2t +\ \ 10^{-5}\ )\ J.$

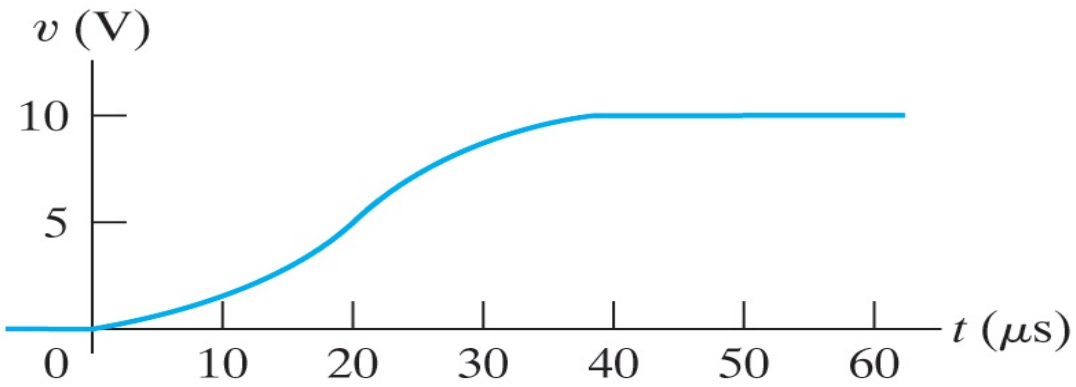
   For $t \geq 40\ \mu s$,

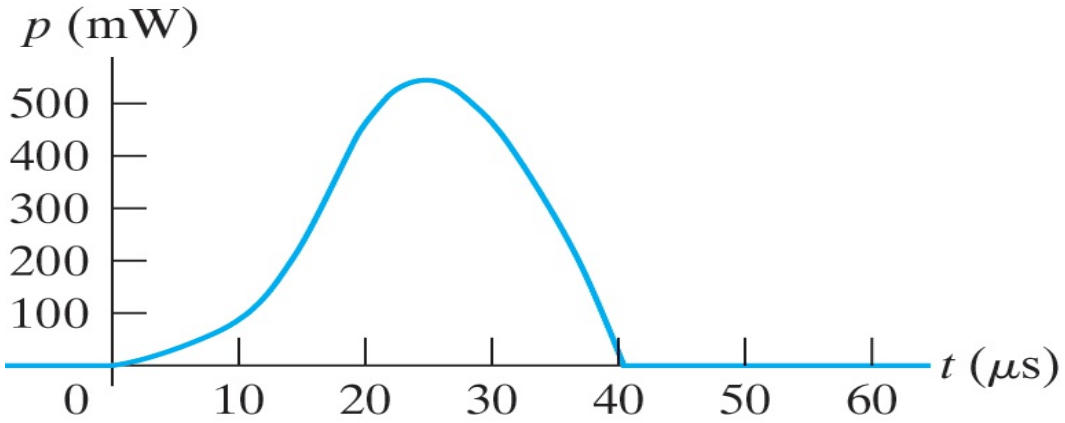
   $v = 10\ V, p = vi = 0, w = 12Cv^2 = 10\ \mu J.$

2. The excitation current and the resulting voltage, power, and energy are plotted in Fig. 6.12.

Figure 6.12 The variables i, v, p, and w versus t for Example 6.5.


i (mA)
100
50
0
10
20
30
40
50
60
t (μs)


v (V)
10
5
0
10
20
30
40
50
60
t (μs)


p (mW)
500
400
300
200
100
0
10
20
30
40
50
60
t (μs)

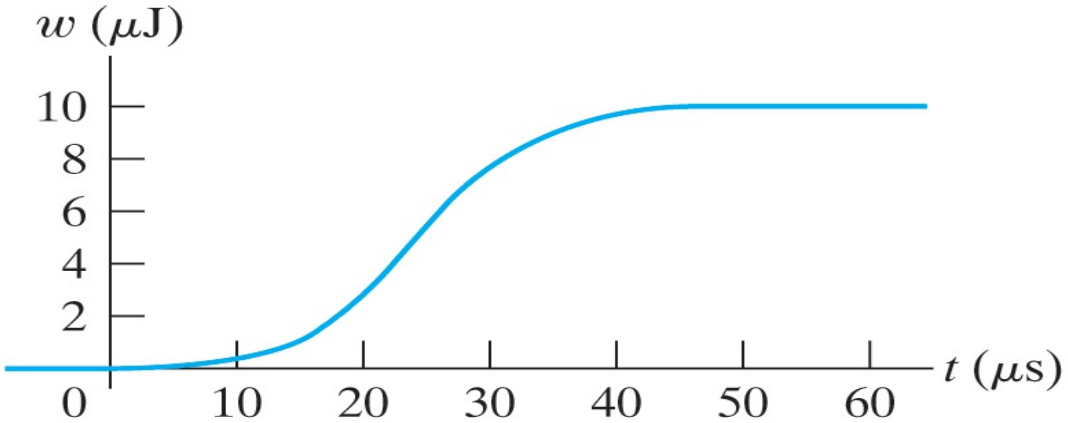

w (μJ)
10
8
6
4
2
0
10
20
30
40
50
60
t (μs)

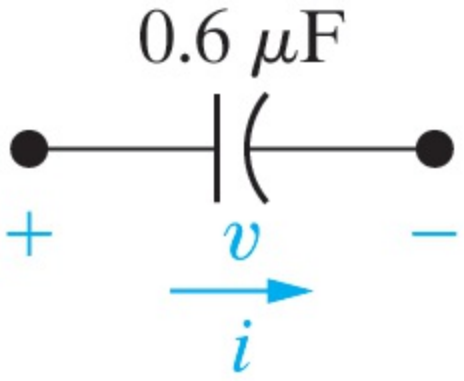
Figure 6.12 Full Alternative Text

3. Note that the power is always positive for the duration of the current pulse, which means that energy is continuously being stored in the capacitor. When the current returns to zero, the stored energy is trapped because the ideal capacitor cannot dissipate energy. Thus, a voltage remains on the capacitor after its current returns to zero.

Assessment Problems

Objective 2—Know and be able to use the equations for voltage, current, power, and energy in a capacitor

1. 6.2 The voltage at the terminals of the 0.6 μF capacitor shown in the figure is 0 for t<0 and 40e−15,000tsin30,000t V for t≥0. Find (a) i(0); (b) the power delivered to the capacitor at t=π/80 ms; and (c) the energy stored in the capacitor at t=π/80 ms.



Answer:

1. (a) 0.72 A;
2. (b) −649.2 mW;
3. (c) 126.13 μJ.

2. 6.3 The current in the capacitor of Assessment Problem 6.2 is 0 for $t<0$ and 3 cos 50,000t A for $t\geq 0$. Find (a) v(t); (b) the maximum power delivered to the capacitor at any one instant of time; and (c) the maximum energy stored in the capacitor at any one instant of time.

   Answer:

   1. (a) 100 sin 50,000t V, $t\geq 0$;

   2. (b) 150 W; (c) 3 mJ.

SELF-CHECK: *Also try Chapter Problems 6.17 and 6.21*.

6.3 Series-Parallel Combinations of Inductance and Capacitance

Just as series-parallel combinations of resistors can be reduced to a single equivalent resistor, series-parallel combinations of inductors or capacitors can be reduced to a single inductor or capacitor.

Inductors in Series and Parallel

Figure 6.13 shows inductors in series. The series connection means the inductors all have the same current, so we define only one current for the series combination. The voltage drops across the individual inductors are

Figure 6.13 Inductors in series.

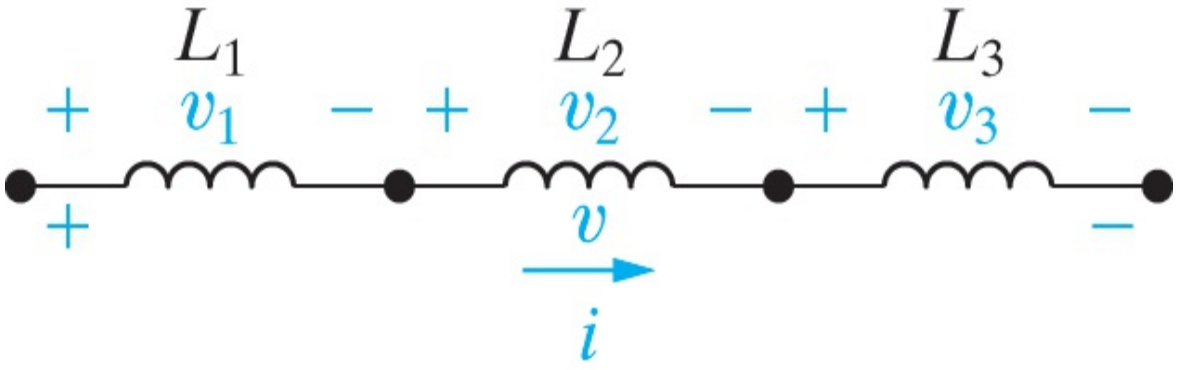


Figure 6.13 Full Alternative Text
v1=L1didt, v2=L2didt, and v3=L3didt.

The voltage across the series connection is

v=v1+v2+v3=(L1+L2+L3)didt.

Thus, the equivalent inductance of series-connected inductors is the sum of the individual inductances. For n inductors in series,

Combining Inductors in Series

$$L_{eq} = \sum_{i=1}^{n} L_i. \quad (6.10)$$

If the original inductors carry an initial current of i(t0), the equivalent inductor carries the same initial current. Figure 6.14 shows the equivalent circuit for series inductors carrying an initial current.

Figure 6.14 An equivalent circuit for inductors in series carrying an initial current i(t0).

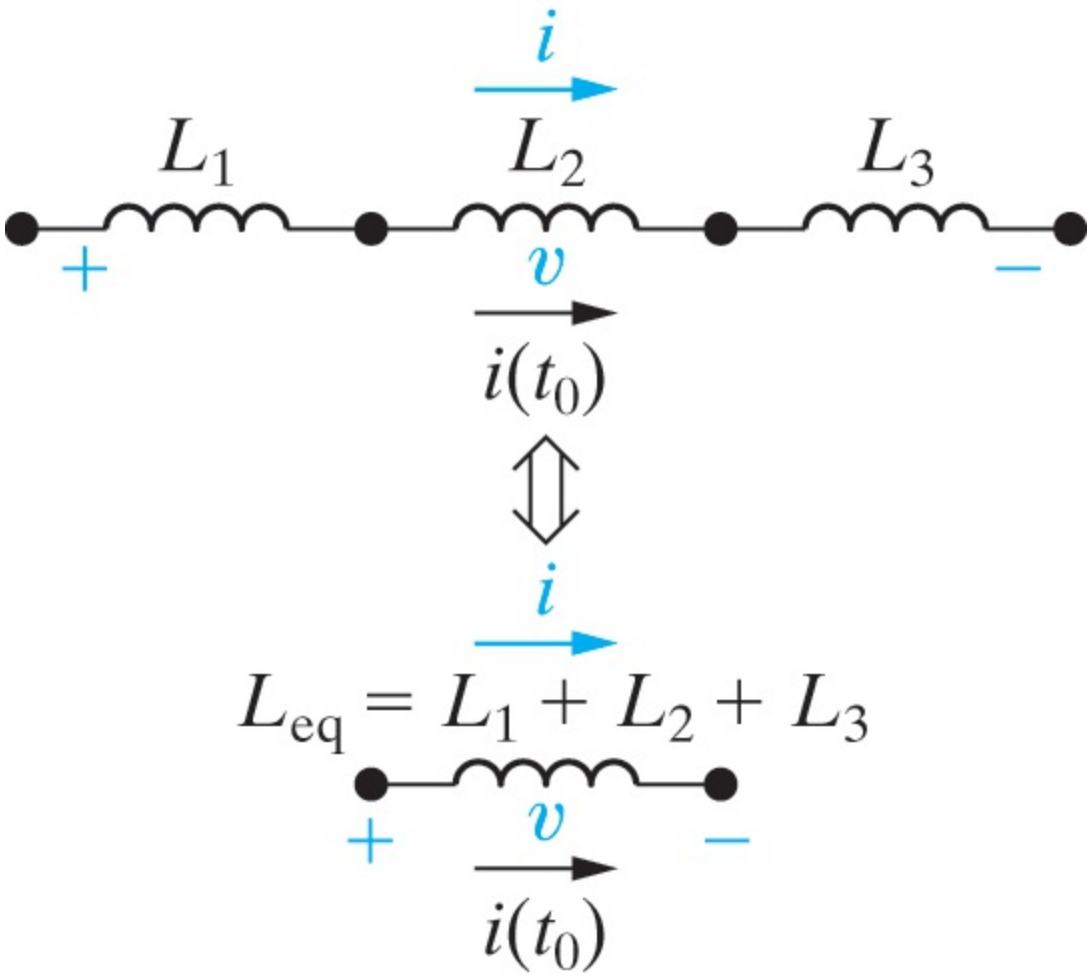


Figure 6.14 Full Alternative Text

Inductors in parallel have the same terminal voltage, so the current in each inductor is a function of the terminal voltage and the initial current in that inductor. For the three inductors in parallel shown in Fig. 6.15, the currents for the individual inductors are

Figure 6.15 Three inductors in parallel.

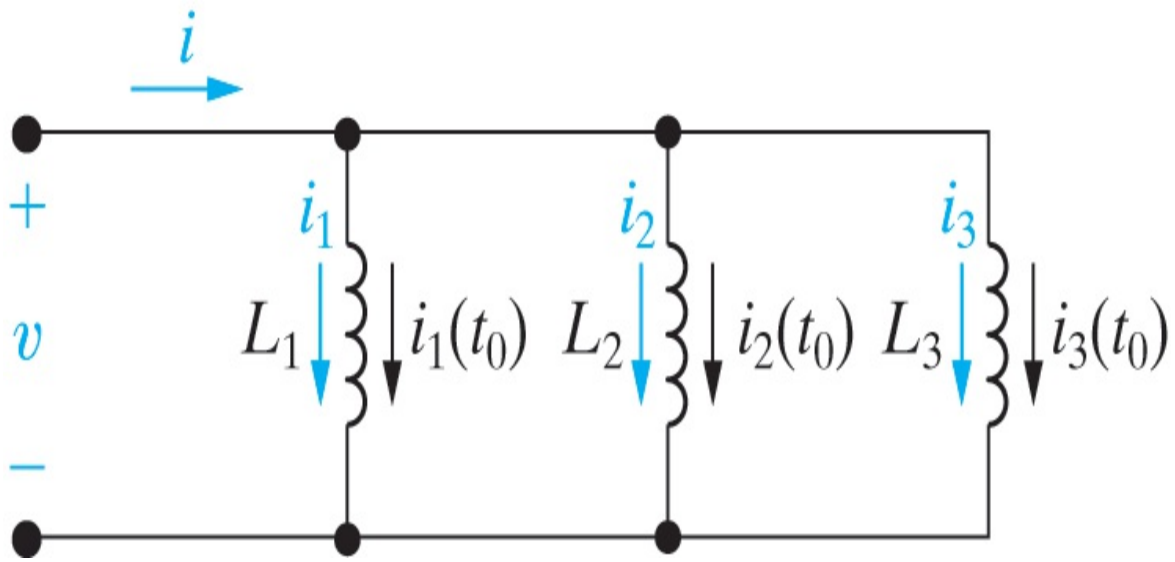


Figure 6.15 Full Alternative Text
i1=1L1∫t0tv dτ+i1(t0),
i2=1L2 ∫t0tv dτ+i2(t0),
i3=1L3∫t0tv dτ+ i3(t0).

The current entering the top node shared by the three parallel inductors is the sum of the inductor currents:

i=i1+i2+ i3.

Substituting the expressions for i1, i2, and i3 into the sum yields

i=(1L1+1L2+1L3)∫t0tv dτ+i1(t0)+i2(t0)+i3(t0).(6.11)

The expression for the current as a function of the voltage for a single equivalent inductor is

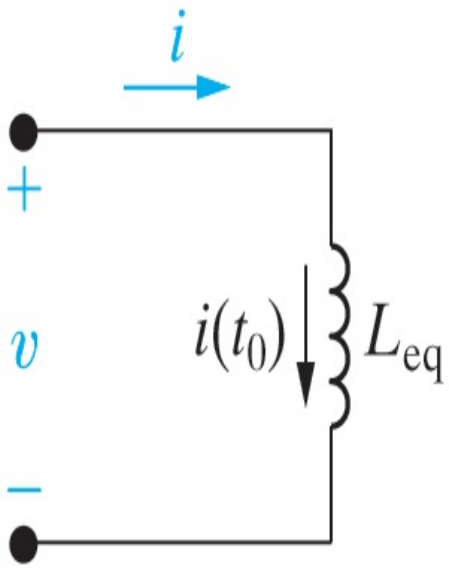
i=1Leq∫t0tv dτ + i(t0).(6.12)

Comparing Eq. 6.12 with Eq. 6.11 yields

1Leq=1L1+1L2+1L3,i(t0)=i1(t0)+i2(t0)+i3(t0).

Figure 6.16 shows the equivalent circuit for the three parallel inductors in Fig. 6.15.

Figure 6.16 An equivalent circuit for three inductors in parallel.



$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

$$i(t_0) = i_1(t_0) + i_2(t_0) + i_3(t_0)$$

Figure 6.16 Full Alternative Text

The results for three inductors in parallel can be extended to n inductors in parallel:

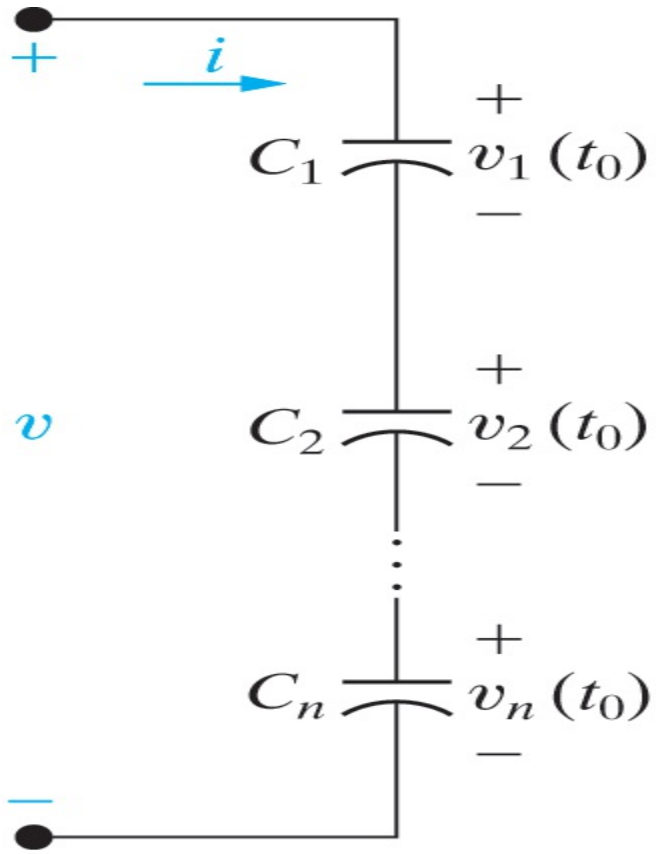
Combining Inductors and Their Initial Currents in Parallel

1 L eq = ∑ i=1 n 1 L i , (6.13)
i(t 0) = ∑ j=1 n i j (t 0), (6.14)

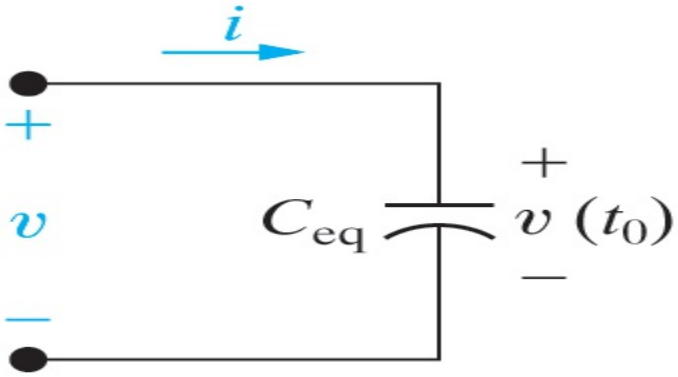
Capacitors in Series and Parallel

Capacitors connected in series can be reduced to a single equivalent capacitor. The reciprocal of the equivalent capacitance is equal to the sum of the reciprocals of the individual capacitances. The initial voltage on the equivalent capacitor is the algebraic sum of the initial voltages on the individual capacitors. Figure 6.17 and the following equations summarize these observations for n series-connected capacitors:

Figure 6.17 An equivalent circuit for capacitors connected in series. (a) The series capacitors. (b) The equivalent circuit.



(a)



$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$$

$$v(t_0) = v_1(t_0) + v_2(t_0) + \cdots + v_n(t_0)$$

(b)

Figure 6.17 Full Alternative Text

Combining Capacitors and Their Initial Voltages in Series

$$\frac{1}{C_{eq}} = \sum_{i=1}^{n} \frac{1}{C_i}, \quad (6.15)$$
$$v(t_0) = \sum_{j=1}^{n} v_j(t_0). \quad (6.16)$$

We leave the derivation of the equivalent circuit for series-connected capacitors as an exercise. (See Problem 6.27.)

The equivalent capacitance of capacitors connected in parallel is the sum of the individual capacitances, as Fig. 6.18 page 196 and the following equation show:

Figure 6.18 An equivalent circuit for capacitors connected in parallel. (a) Capacitors in parallel. (b) The equivalent circuit.

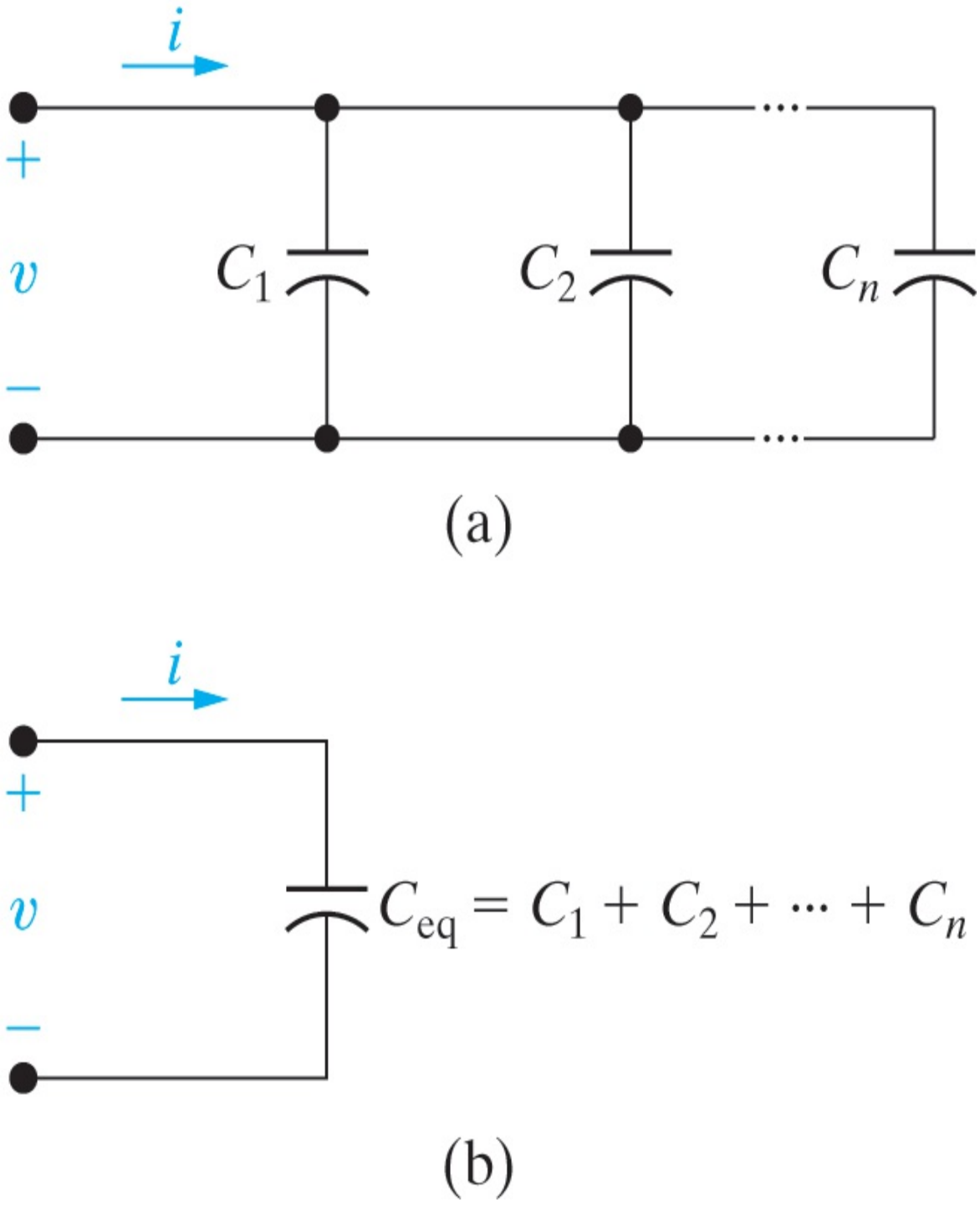


Figure 6.18 Full Alternative Text

COMBINING CAPACITORS IN

PARALLEL

$$C_{eq} = \sum_{i=1}^{n} C_i \quad (6.17)$$

Capacitors connected in parallel must carry the same voltage. Therefore, the initial voltage across the original parallel capacitors equals the initial voltage across the equivalent capacitance Ceq. The derivation of the equivalent circuit for parallel capacitors is left as an exercise. (See Problem 6.28.)

Examples 6.6 and 6.7 use series and parallel combinations to simplify a circuit with multiple inductors and a circuit with multiple capacitors.

Example 6.6 Finding the Equivalent Inductance

Figure 6.19 shows four interconnected inductors. The initial currents for two of the inductors are also shown in Fig. 6.19. A single equivalent inductor, together with its initial current, is shown in Fig. 6.20.

Figure 6.19 Interconnected inductors for Example 6.6.

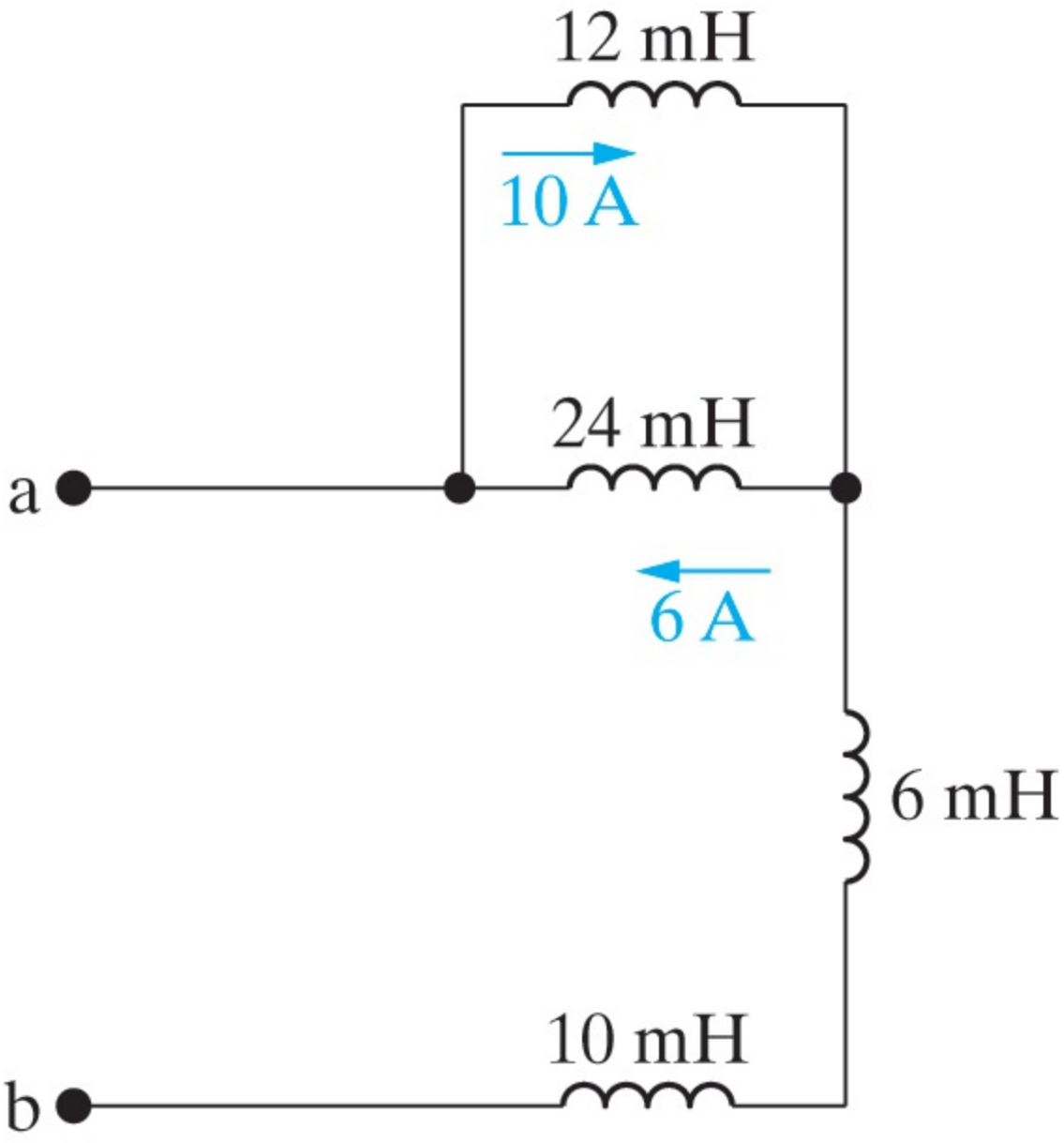


Figure 6.19 Full Alternative Text

Figure 6.20 The equivalent inductor for the inductors in Fig. 6.19

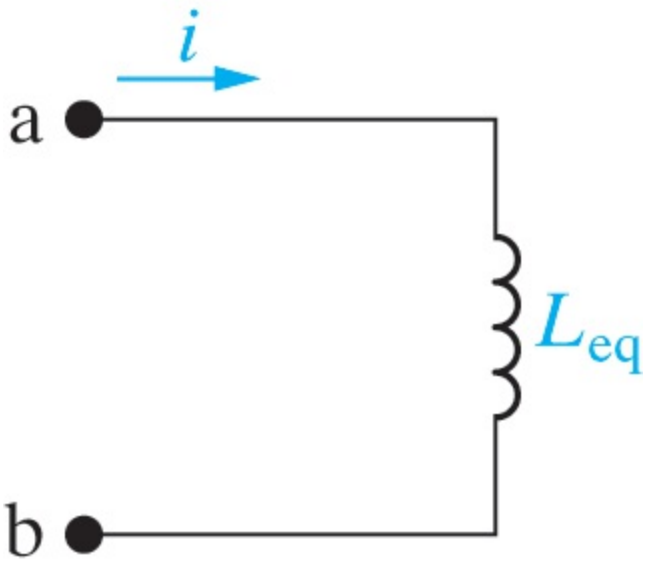


Figure 6.20 Full Alternative Text

1. Find the equivalent inductance, Leq.
2. Find the initial current in the equivalent inductor.

Solution

1. Begin by replacing the parallel-connected 12 mH and 24 mH inductors with a single equivalent inductor whose inductance is

   $(10.012+10.024)-1=0.008=8$ mH.

   Now the 8 mH, 6 mH, and 10 mH inductors are in series. Combining them gives

   $Leq=0.008+0.006+0.010=0.024=24$ mH.

2. The initial current in the equivalent inductor, i, is the same as the current entering the node to the left of the 24 mH inductor. The KCL equation at that node, summing the currents entering the node, is

   $i-10+6=0$.

Therefore, the initial current in the equivalent inductor is i = 4 A.

Example 6.7 Finding the Equivalent Capacitance

Figure 6.21 shows four interconnected capacitors. The initial voltages for three of the capacitors are also shown in Fig. 6.21. A single equivalent capacitor, together with its initial voltage, is shown in Fig. 6.22.

Figure 6.21 Interconnected capacitors for Example 6.7.

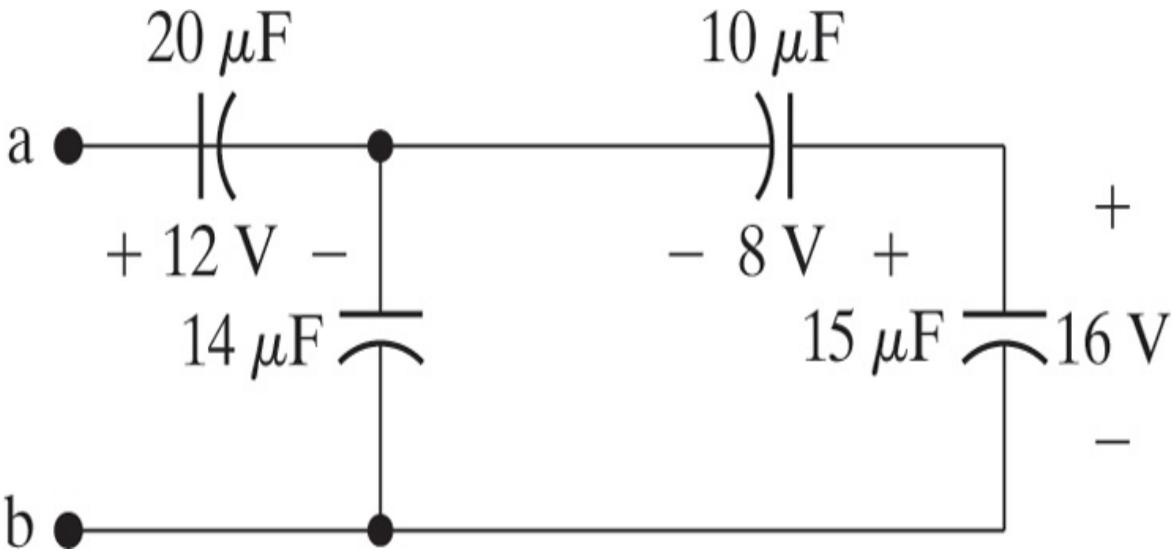


Figure 6.21 Full Alternative Text

Figure 6.22 The equivalent capacitor for the capacitors in

Fig. 6.21.

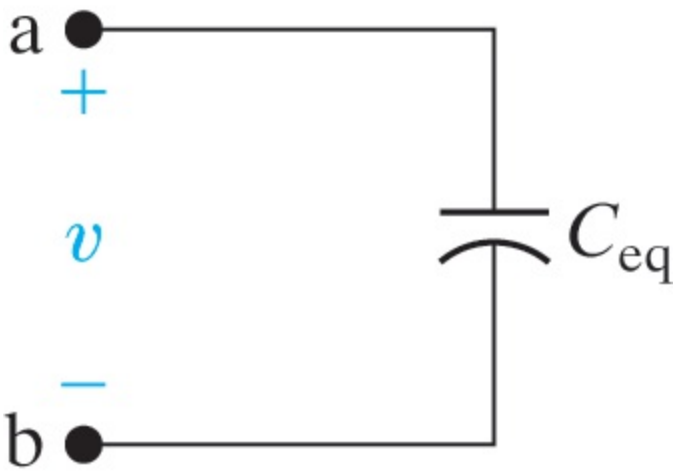


Figure 6.22 Full Alternative Text

1. Find the equivalent capacitance, Ceq.
2. Find the initial voltage across the equivalent capacitor.

Solution

1. Begin by replacing the 10 μF and 15 μF capacitors with a single equivalent capacitor whose capacitance is

   $(110\times10-6+115\times10-6)-1=6\times10-6=6$ μF.

   Next, combine the 6 μF capacitor from the first simplification with the 14 μF capacitor to give

   $6\times10-6+14\times10-6=20\times10-6=20$ μF.

   Finally, combine the 20 μF from the previous simplification with the 20 μF on the left side of the circuit to give

   $Ceq=(120\times10-6+120\times10-6)-1=10\times10-6=10$ μF.

2. To find the initial voltage from a to b, use KVL to sum the initial voltages for the capacitors on the perimeter of the circuit. This gives

   $12-8+16=20$ V.

   Therefore, the initial voltage across the equivalent capacitor is 20 V.

Assessment Problems

Objective 3—Be able to combine inductors or capacitors in series and in parallel to form a single equivalent inductor

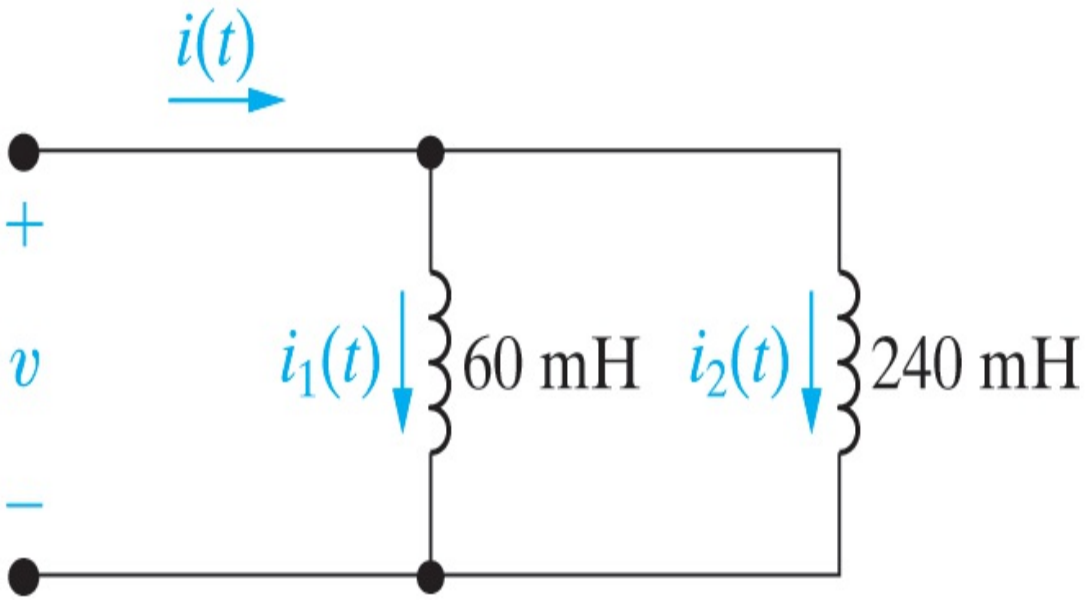
1. 6.4 The initial values of i1 and i2 in the circuit shown are + 3 A and −5 A, respectively. The voltage at the terminals of the parallel inductors for $t \geq 0$ is $-30e^{-5t}$ mV.

   1. a) If the parallel inductors are replaced by a single inductor, what is its inductance?

   2. b) Find the initial current and its reference direction in the equivalent inductor.

   3. c) Use the equivalent inductor to find i(t).

   4. d) Find i1(t) and i2(t). Verify that the solutions for i1(t), i2(t), and i(t) satisfy Kirchhoff's current law.

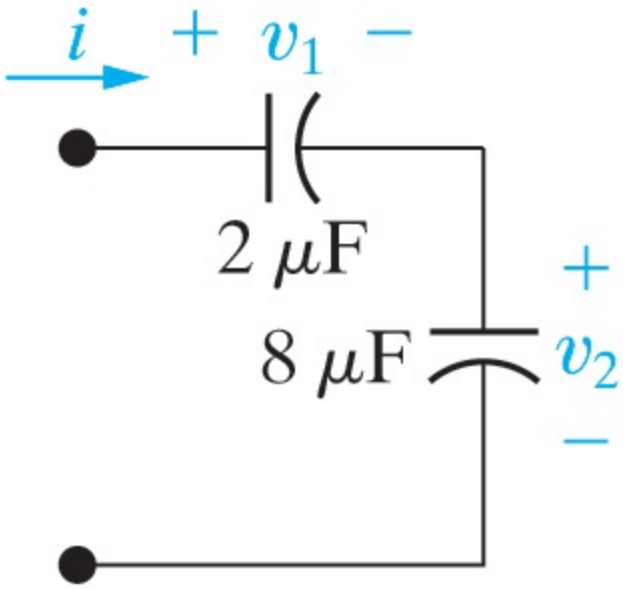


6.3-5 Full Alternative Text

Answer:

1. (a) 48 mH;
2. (b) 2 A, up;
3. (c) 0.125e−5t−2.125 A, t≥0;
4. (d) i1(t)=0.1e−5t+ 2.9 A, t≥0, i2(t)=0.025e−5t− 5.025 A, t≥0.

2. 6.5 The current at the terminals of the two capacitors shown is 240e −10tμA for t≥0. The initial values of v1 and v2 are −10 V and −5 V, respectively. Calculate the total energy trapped in the capacitors as t→∞. (*Hint:* Don't combine the capacitors in series—find the energy trapped in each, and then add.)



6.3-6 Full Alternative Text

Answer:

20 μJ.

SELF-CHECK: *Also try Chapter Problems 6.22, 6.24, 6.27, and 6.31*.

Inductor and Capacitor Symmetry

We introduced the concept of symmetry, or duality, in Chapter 4. In that chapter, we recognized several examples of duality, including

- essential nodes and meshes;
- KCL and KVL;
- the node voltage method and the mesh current method.

There are also examples of duality in Sections 6.1 through 6.3; these are summarized in Table 6.1. From this table we note many dual relationships, including

Table 6.1 Inductor and Capacitor Duality

	Inductors	Capacitors
Primary *v-i* equation	$v(t) = L\dfrac{di(t)}{dt}$	$i(t) = C\dfrac{dv(t)}{dt}$
Alternate *v-i* equation	$i(t) = \dfrac{1}{L}\displaystyle\int_{t_0}^{t} v(\tau)d\tau + i(t_0)$	$v(t) = \dfrac{1}{C}\displaystyle\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$
Initial condition	$i(t_0)$	$v(t_0)$
Behavior with a constant source	If $i(t) = I, v(t) = 0$ and the inductor behaves like a short circuit	If $v(t) = V, i(t) = 0$ and the capacitor behaves like an open circuit
Continuity requirement	$i(t)$ is continuous for all time so $v(t)$ is finite	$v(t)$ is continuous for all time so $i(t)$ is finite
Power equation	$p(t) = v(t)i(t) = Li(t)\dfrac{di(t)}{dt}$	$p(t) = v(t)i(t) = Cv(t)\dfrac{dv(t)}{dt}$
Energy equation	$w(t) = \dfrac{1}{2}Li(t)^2$	$w(t) = \dfrac{1}{2}Cv(t)^2$
Series-connected equivalent	$L_{eq} = \displaystyle\sum_{j=1}^{n} L_j$ $i_{eq}(t_0) = i_j(t_0)$ for all j	$\dfrac{1}{C_{eq}} = \displaystyle\sum_{j=1}^{n} \dfrac{1}{C_j}$ $v_{eq}(t_0) = \displaystyle\sum_{j=1}^{n} v_j(t_0)$
Parallel-connected equivalent	$\dfrac{1}{L_{eq}} = \displaystyle\sum_{j=1}^{n} \dfrac{1}{L_j}$ $i_{eq}(t_0) = \displaystyle\sum_{j=1}^{n} i_j(t_0)$	$C_{eq} = \displaystyle\sum_{j=1}^{n} C_j$ $v_{eq}(t_0) = v_j(t_0)$ for all j

Table 6.1 Full Alternative Text

- voltage and current;
- open circuits and short circuits;
- inductance and capacitance;
- series connections and parallel connections.

Recognizing the symmetry in the characteristics of inductors and capacitors makes it easier to understand and remember these characteristics.

6.4 Mutual Inductance

A time-varying current in an inductor coil creates a time-varying magnetic field that surrounds that coil, as we saw in Section 6.1. Inductance relates the voltage drop across an inductor to the time-varying current in that inductor. As you will see, it makes sense to rename this parameter self-inductance.

Now we consider circuits containing two inductors with time-varying currents, each creating a time-varying magnetic field. In these circuits, the magnetic field generated by one coil's current envelops the other coil and vice versa, thereby magnetically coupling the two inductors and the circuits that contain them. Figure 6.23 shows an example of such a circuit, where L1 and L2 are the self-inductances of the individual inductors and M is the mutual inductance associated with the magnetic coupling. Note the double-headed arrow adjacent to M; the arrows indicate the pair of coils with this value of mutual inductance. We need this notation to accommodate circuits with more than one pair of magnetically coupled coils.

Figure 6.23 Two magnetically coupled coils.

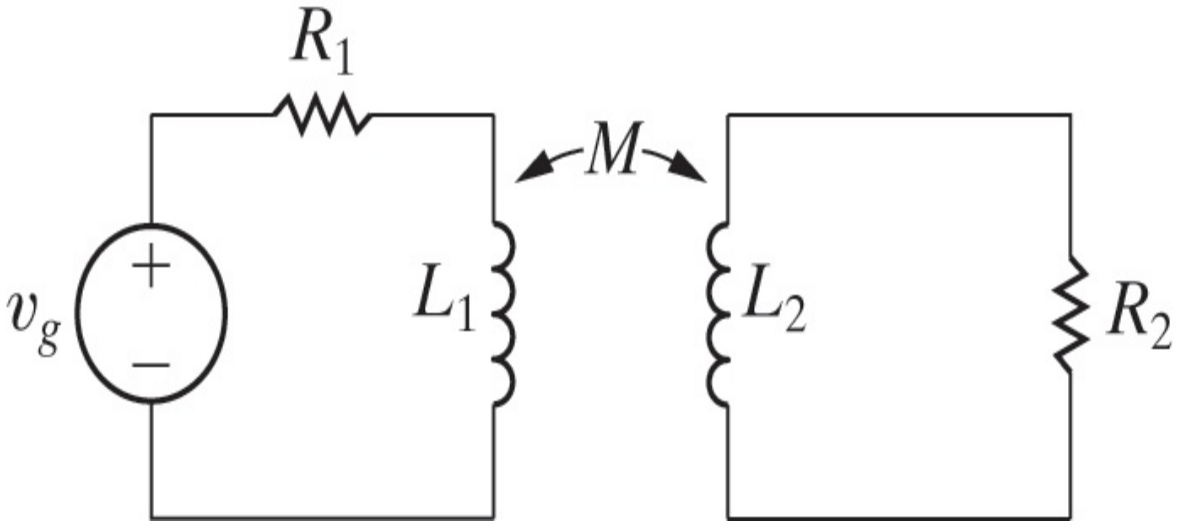
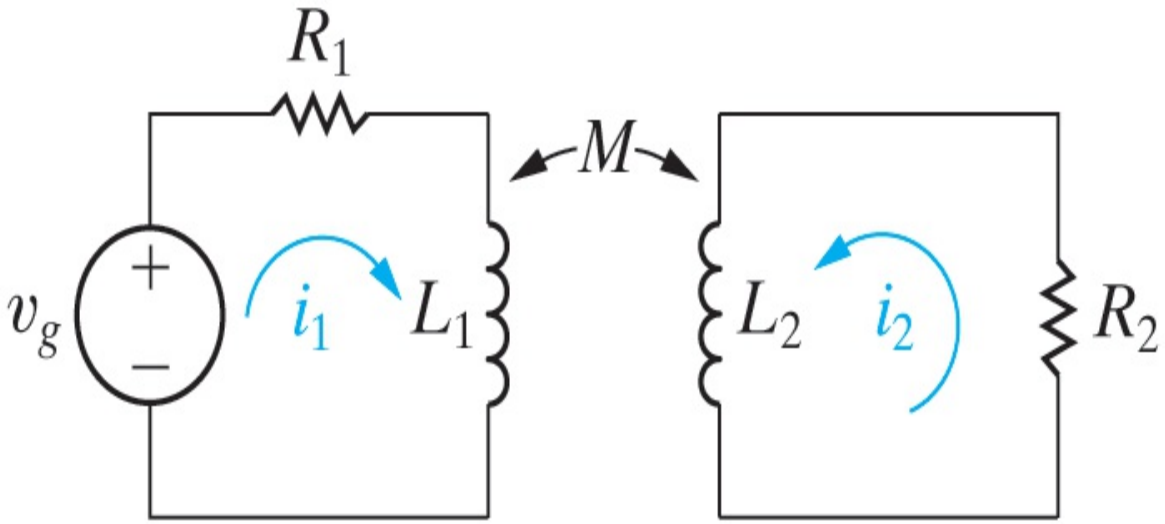


Figure 6.23 Full Alternative Text

The mesh-current method is the easiest way to analyze circuits with magnetically coupled inductors. Following Analysis Method 4.6 (page 110), in Step 1 we identify the meshes, and in Step 2 we label the mesh currents. The result is shown in Fig. 6.24. In Step 3, we write a KVL equation around each mesh, summing the voltages across each mesh component. There will be two voltages across each inductor coil because the coils are magnetically coupled. One voltage is the self-induced voltage, which is the product of the self-inductance of the coil and the first derivative of that coil's current. The other voltage is the mutually induced voltage, which is the product of the mutual inductance of the coils and the first derivative of the current in the other coil. Consider the inductor on the left in Fig. 6.24 with self-inductance L1. The self-induced voltage across this coil is L1(di1/dt), and the mutually induced voltage across this coil is M(di2/dt). But what about the polarities of these two voltages?

Figure 6.24 Coil currents i1 and i2 used to describe the circuit shown in Fig. 6.23.



The circuit on the left shows an inductor L1 on the right wire, a resistor R1 on the top wire, and an independent voltage source, v sub g, on the left wire. Mesh current in this circuit is indicated by a clockwise pointing curved arrow, i1.

The circuit on the right shows an inductor L2 on the left wire and a resistor R2 on the right wire. The mesh current in this circuit is indicated by an anticlockwise pointing curved arrow, i2.

Mutual inductance of two coils is marked as M, with a double sided arrow pointing to both inductors.

Using the passive sign convention, we find that the self-induced voltage is a voltage drop in the direction of the current producing the voltage. But the polarity of the mutually induced voltage depends on the way the inductor coils are wound in relation to the reference direction of coil currents. Showing the details of mutually coupled windings is very cumbersome. Instead, we use the **dot convention** to determine the mutually induced voltage polarities. A dot is placed on one terminal of each winding, as shown in Fig. 6.25. These dots carry the sign information, so we can draw the inductor coils schematically rather than showing how they wrap around a core structure.

Figure 6.25 The circuit of Fig. 6.24 with dots added to the coils indicating the polarity of the mutually induced voltages.

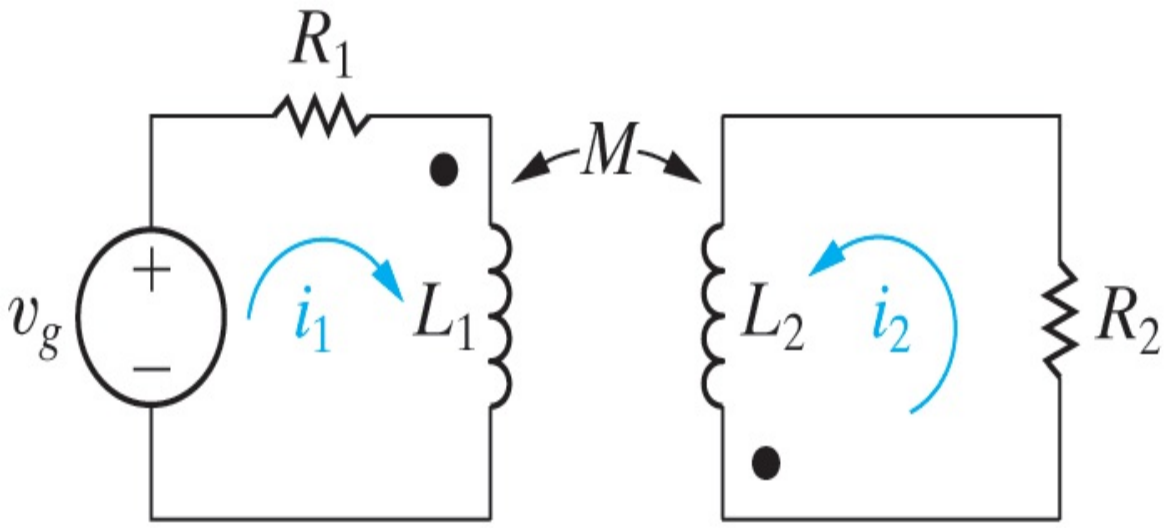


Figure 6.25 Full Alternative Text

The dot convention can be summarized as follows:

Dot Convention for Magnetically Coupled Inductor Coils

When the reference direction for a current enters the dotted terminal of a coil, the reference polarity of the voltage that it induces in the other coil is positive at its dotted terminal.

Or, stated alternatively,

Alternate Dot Convention

When the reference direction for a current leaves the dotted terminal of a coil, the reference polarity of the voltage that it induces in the other coil is negative at its dotted terminal.

Usually, dot markings will be provided for you in the circuit diagrams in this text, as they are in Fig. 6.25. Let's use the dot convention to complete Step 3 in the mesh current method. Start with the mesh on the left in Fig. 6.25 and sum the voltages around the mesh in the direction of the i1 mesh current to get

$-vg + i1R1 + L1di1dt - Mdi2dt = 0,$

The first three terms in the sum, including their signs, should be familiar to you. Look at the fourth term: note that the i2 current enters the undotted terminal of the L2 inductor and creates a mutually induced voltage across the L1 inductor that is positive at its undotted terminal. This is a voltage rise with respect to the direction of i1, so the sign of Mdi2/dt in the sum is negative.

Now consider the mesh on the right in Fig. 6.25, and sum the voltages around the mesh in the direction of the i2 mesh current to get

$$i2R2+L2di2dt−Mdi1dt=0.$$

Look at the third term in the sum: the i1 current enters the dotted terminal of the L1 inductor and creates a mutually induced voltage across the L2 inductor that is positive at its dotted terminal. This is a voltage rise with respect to the direction of i2, so the sign of Mdi1/dt in the sum is negative. Figure 6.26 shows the self- and mutually induced voltages across both inductor coils, together with their polarity marks.

Figure 6.26 The self- and mutually induced voltages appearing across the coils shown in Fig. 6.25.

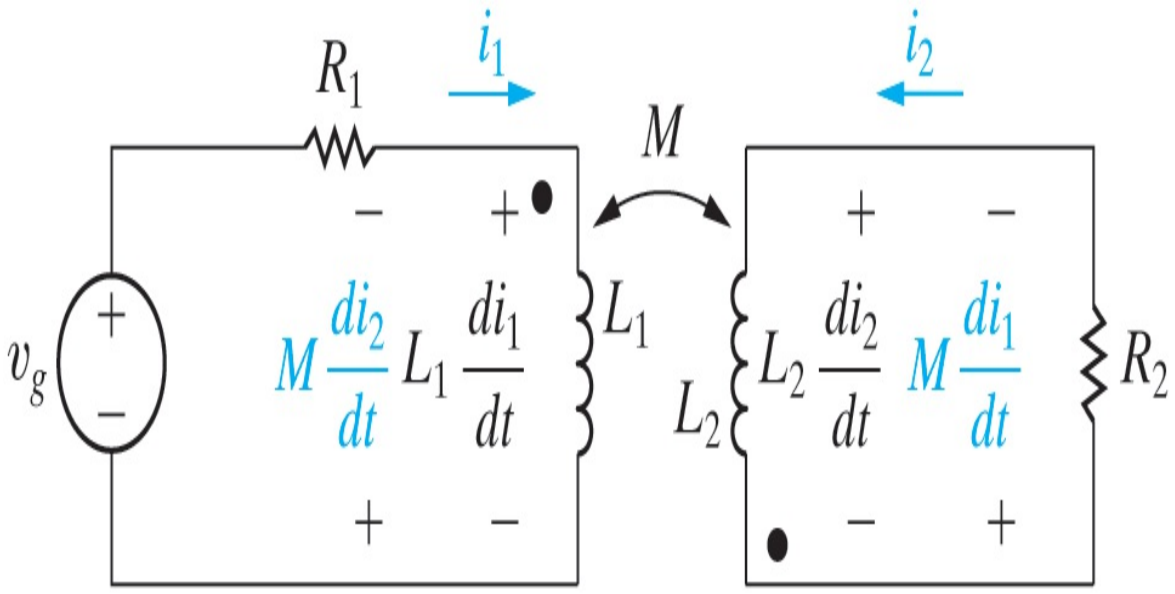


Figure 6.26 Full Alternative Text

The Procedure for Determining Dot Markings

If the polarity dots are not given, you can often determine their locations by examining the physical configuration of the actual circuit or by testing it in the laboratory. We present both of these procedures.

The first procedure assumes that we know the physical arrangement of the two coils and the mode of each winding in a magnetically coupled circuit. Use the following six steps, applied here to Fig. 6.27, to create the dot markings:

Figure 6.27 A set of coils showing a method for determining a set of dot

markings.

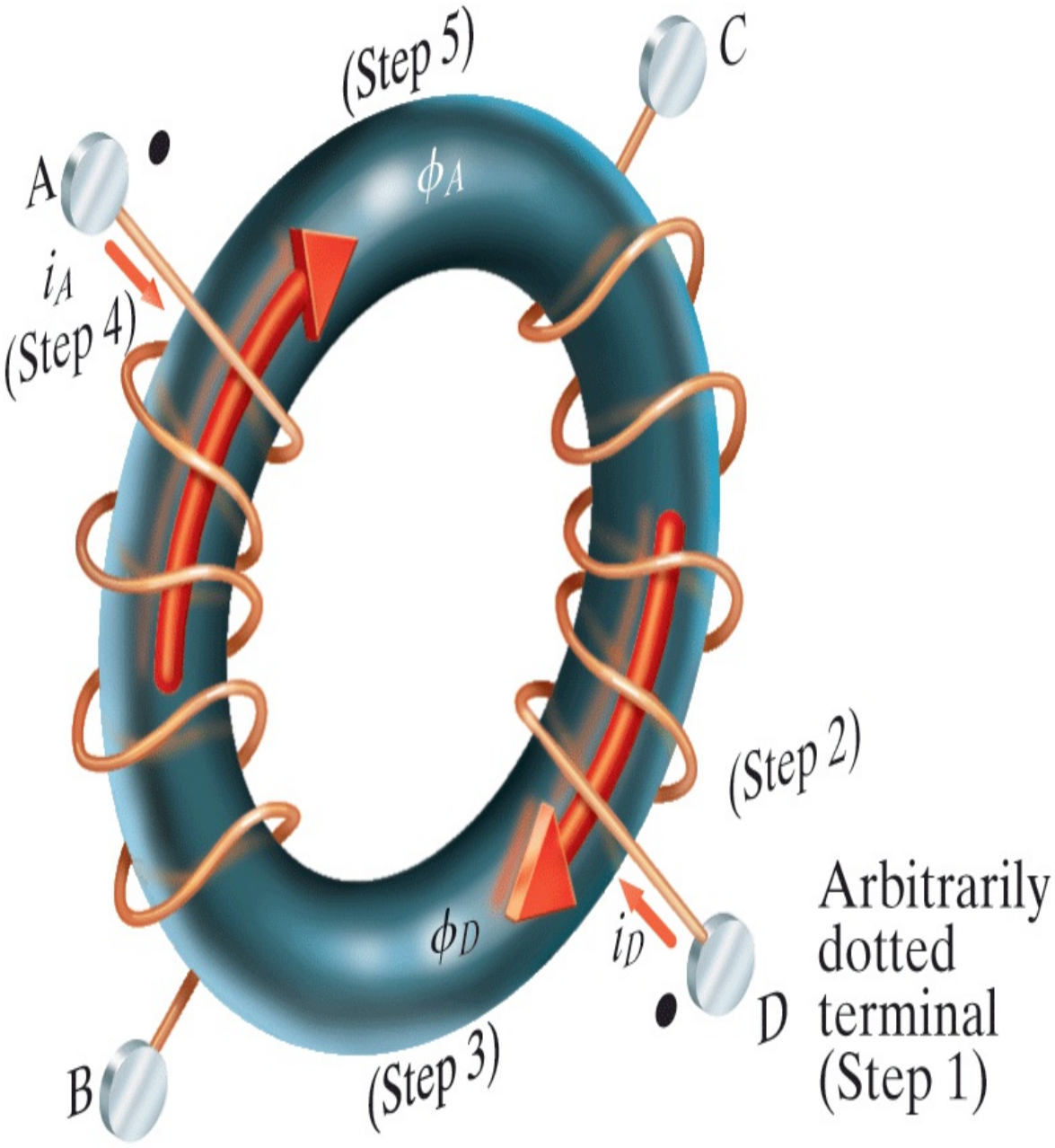


Figure 6.27 Full Alternative Text

1. Arbitrarily select one terminal—say, the D terminal—of one coil and mark it with a dot.

2. Assign a current into the dotted terminal and label it iD.

3. Use the right-hand rule[1] to determine the direction of the magnetic field established by iD *inside* the coupled coils and label this field ϕD.

   1 See discussion of Faraday's law on page 203.

4. Arbitrarily pick one terminal of the second coil—say, terminal A—and assign a current into this terminal, showing the current as iA.

5. Use the right-hand rule to determine the direction of the flux established by iA *inside* the coupled coils and label this flux ϕA.

6. Compare the directions of the two fluxes ϕD and ϕA. If the fluxes have the same reference direction, place a dot on the terminal of the second coil where the test current (iA) enters. If the fluxes have different reference directions, place a dot on the terminal of the second coil where the test current leaves. In Fig. 6.27, the fluxes ϕD and ϕA have the same reference direction, and therefore a dot goes on terminal A.

The second procedure determines the relative polarities of magnetically coupled coils experimentally. This method is used when it is impossible to determine how the coils are wound on the core. One experimental method is to connect a dc voltage source, a resistor, a switch, and a dc voltmeter to the pair of coils, as shown in Fig. 6.28. The shaded box covering the coils implies that physical inspection of the coils is not possible. The resistor R limits the magnitude of the current supplied by the dc voltage source.

Figure 6.28 An experimental setup for determining polarity marks.

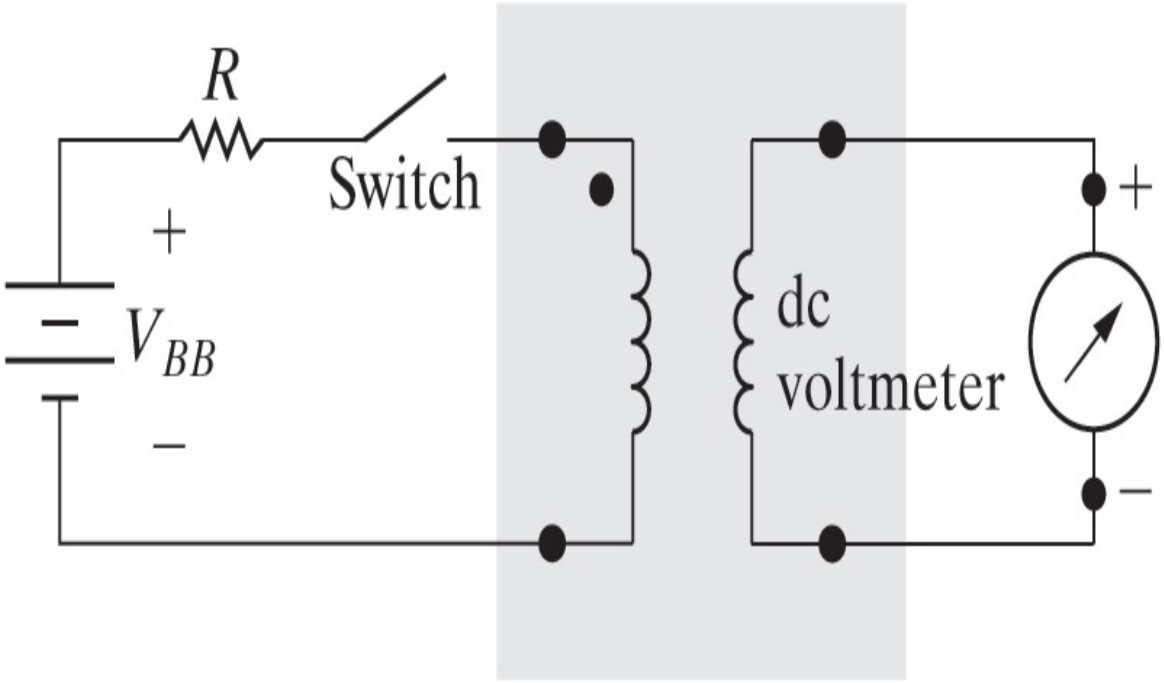


Figure 6.28 Full Alternative Text

Begin by marking the coil terminal connected to the positive terminal of the dc source, via the switch and limiting resistor, with a dot, as shown in Fig. 6.28. When the switch is closed, observe the voltmeter deflection. If the momentary deflection is upscale, place a dot on the coil terminal connected to the voltmeter's positive terminal. If the deflection is downscale, place a dot on the coil terminal connected to the voltmeter's negative terminal.

Example 6.8 constructs the equations for a circuit with magnetically coupled coils, using the dot convention.

Example 6.8 Finding Mesh-Current Equations for a Circuit with Magnetically Coupled Coils

1. Use the mesh-current method to write equations for the circuit in Fig. 6.29 in terms of the currents i1 and i2.

Figure 6.29 The circuit for Example 6.8.

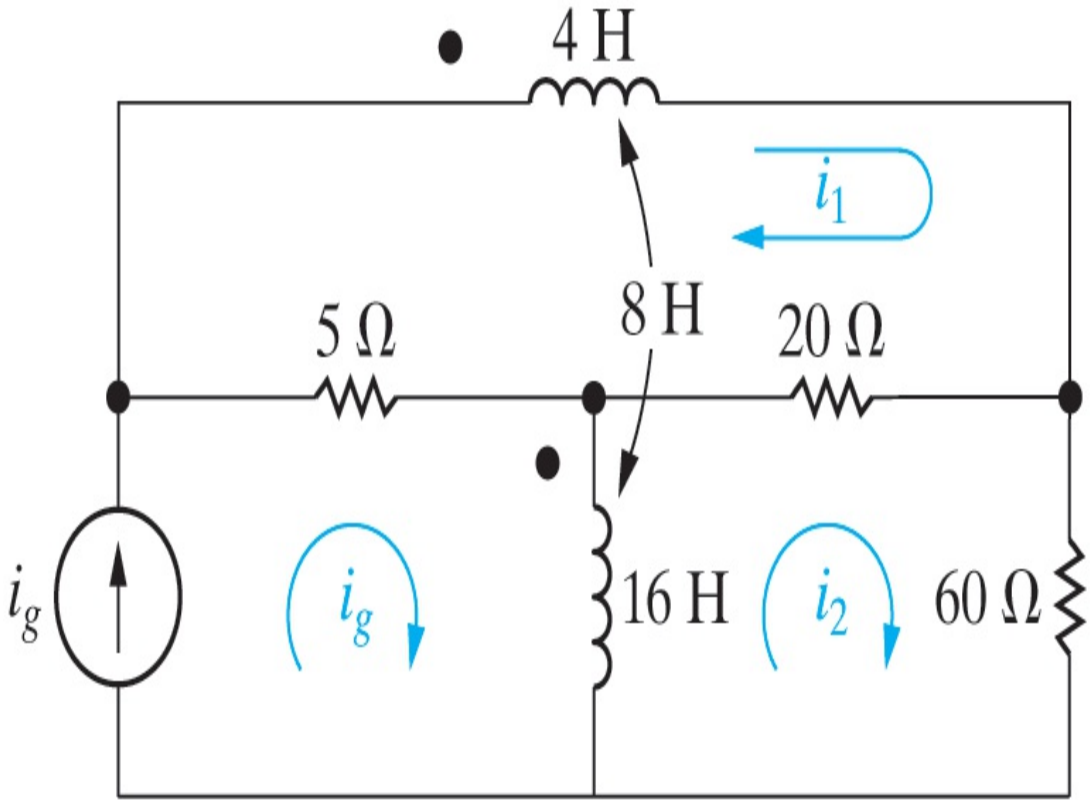


Figure 6.29 Full Alternative Text

2. Verify that if there is no energy stored in the circuit at t=0 and if ig=16−16e−5t A, the solutions for i1 and i2 are

   i1=4+ 64e−5t−68e−4t A,
   i2=1−52e−5t+ 51e−4t A.

Solution

1. Follow the steps in Analysis Method 4.6. Steps 1 and 2 identify the meshes and label the mesh currents, as shown in Fig. 6.29. In Step 3, we

write a KVL equation for each mesh where the current is unknown. Summing the voltages around the i1 mesh yields

$$4\frac{di_1}{dt}+8\frac{d}{dt}(i_g - i_2)+20(i_1-i_2)+5(i_1-i_g)=0.$$

Look carefully at the second term in this equation and make certain you understand how the dot convention was used. Note that the voltage across the 4 H coil due to the current (i_g-i_2), that is, $8d(i_g- i_2)/dt$, is a voltage drop in the direction of i_1, so this term has a positive sign.

The KVL equation for the i2 mesh is

$$20(i_2-i_1)+ 60i_2+16\frac{d}{dt}(i_2-i_g)-8\frac{di_1}{dt}=0.$$

Look carefully at the fourth term in this equation and make certain you understand how the dot convention was used. The voltage induced in the 16 H coil by the current i1, that is, $8di_1/dt$, is a voltage rise in the direction of i2, so this term has a negative sign.

2. To check the validity of i1 and i2, we begin by testing the initial and final values of i1 and i2. We know by hypothesis that $i_1(0)=i_2(0)=0$. From the given solutions we have

   $$i_1(0)=4+64-68=0,$$
   $$i_2(0)=1-52+51=0.$$

   Now we observe that as t approaches infinity, the source current (ig) approaches a constant value of 16 A, and therefore the magnetically coupled coils behave as short circuits. Hence, at $t=\infty$ the circuit reduces to that shown in Fig. 6.30. From Fig. 6.30 we see that at $t=\infty$ the three resistors are in parallel across the 16 A source. The equivalent resistance is 3.75 Ω, and thus the voltage across the 16 A current source is 60 V. Write a KCL equation at node a, using Ohm's law to find the currents in the 20 Ω and 60 Ω resistors to give

Figure 6.30 The circuit of

Example 6.8 when t = ∞

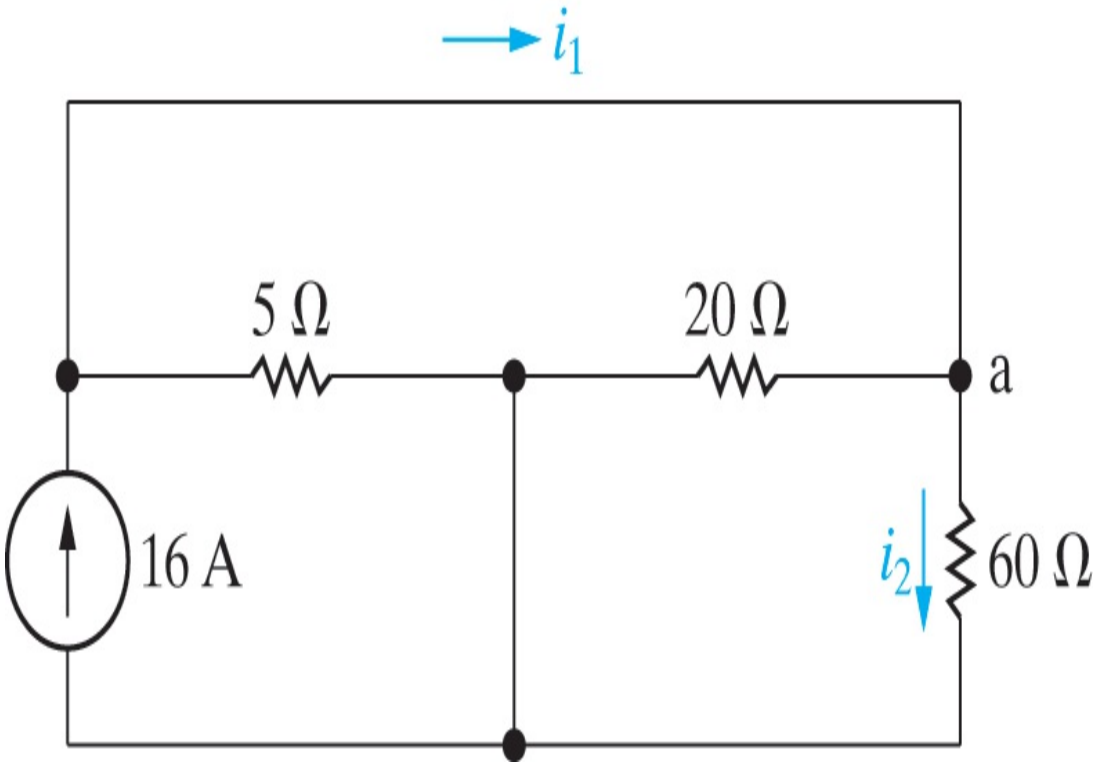


Figure 6.30 Full Alternative Text
i1(∞)=6020+6060=4 A.

Using Ohm's law,

i2(∞)=6060=1 A.

These values agree with the final values predicted by the solutions for i1 and i2:

i1(∞)=4+64(0)−68(0)=4 A,
i2(∞)=1−52(0)+51(0)=1 A.

Finally, we check the solutions to see if they satisfy the differential equations derived in (a). We will leave this final check to the reader via Problem 6.36.

Assessment Problem

Objective 4—Use the dot convention to write mesh-current equations for mutually coupled coils

1. 6.6

    1. a) Write a set of mesh-current equations for the circuit in Example 6.8 if the dot on the 4 H inductor is at the right-hand terminal, the reference direction of ig is reversed, and the 60 Ω resistor is increased to 780 Ω.

    2. b) Verify that if there is no energy stored in the circuit at t=0, and if ig=1.96−1.96e−4t A, the solutions to the differential equations derived in (a) of this Assessment Problem are

    $$i1=-0.4-11.6e^{-4t}+12e^{-5t}\ A,$$
    $$i2=-0.01-0.99e^{-4t}+e^{-5t}\ A.$$

    Answer:

    1. (a) $4(di1/dt)+25i1+8(di2/dt)-20i2=-5ig-8(dig/dt)$

        and

        $8(di1/dt)-20i1+16(di2/dt)+800i2=-16(dig/dt);$

    2. (b) verification.

SELF-CHECK: *Also try Chapter Problem 6.41*.

6.5 A Closer Look at Mutual Inductance

Here we take a closer look at self-inductance, and then we turn to a deeper inspection of mutual inductance, examining the limitations and assumptions made in Section 6.4.

A Review of Self-Inductance

Michael Faraday, who studied inductance in the early 1800s, envisioned a magnetic field consisting of lines of force surrounding the current-carrying conductor. Picture these lines of force as energy-storing elastic bands that close on themselves. As the current increases and decreases, the elastic bands (that is, the lines of force) spread and collapse about the conductor. The voltage induced in the conductor is proportional to the number of lines that collapse into, or cut, the conductor. This image of induced voltage is expressed by Faraday's law:

$$v = d\lambda dt, \quad (6.18)$$

where λ is the flux linkage, measured in weber-turns.

So, how is Faraday's law related to inductance as defined in Section 6.1? Let's look at the coil depicted in Fig. 6.31. The lines threading the N turns, labeled ϕ, represent the magnetic lines of force that make up the magnetic field, which has a spatial orientation and a strength. Use the right-hand rule to determine the spatial orientation: When the fingers of the right hand are wrapped around the coil and point in the direction of the current, the thumb points in the direction of that portion of the magnetic field inside the coil.

Figure 6.31 Representation of a

magnetic field linking an N-turn coil.

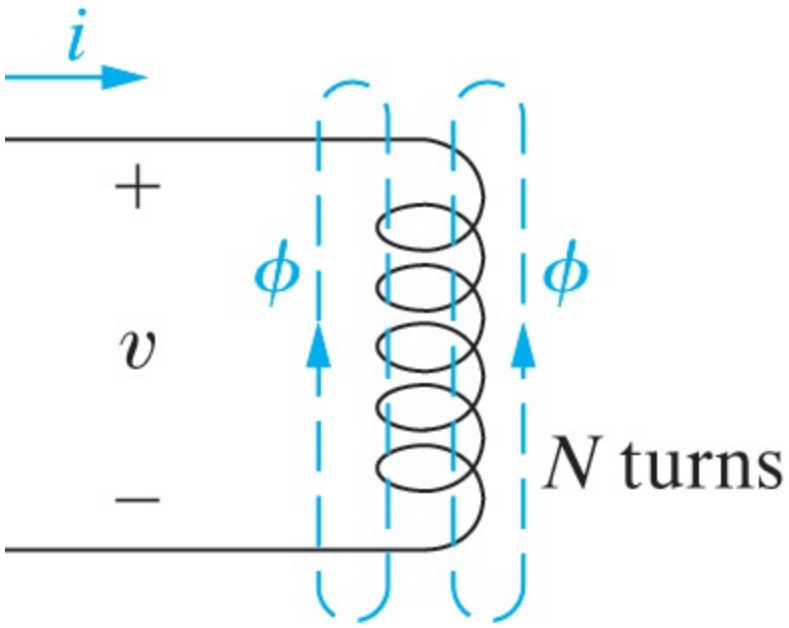


Figure 6.31 Full Alternative Text

To determine the magnetic field strength, begin by defining flux linkage, introduced in Faraday's law (Eq. 6.18). The flux linkage is the product of the magnetic field, ϕ, measured in webers (Wb), and the number of turns linked by the field, N:

$\lambda = N\phi$.

The magnitude of the flux, ϕ, is related to the magnitude of the coil current by the relationship

$\varphi = PNi$

where N is the number of turns on the coil, and P is the permeance of the space occupied by the flux. When the space containing the flux is made up of magnetic materials (such as iron, nickel, and cobalt), the permeance varies with the flux, giving a nonlinear relationship between ϕ and i. But when the

space containing the flux is composed of nonmagnetic materials, the permeance is constant, giving a linear relationship between ϕ and i. Note that the flux is also proportional to the number of turns on the coil.

Here, we assume that the core material—the space containing the flux—is nonmagnetic. Then, substituting the expressions for flux (ϕ) and flux linkage (λ) into Eq. 6.18 takes us from Faraday's law to the inductor equation:

$$v = d\lambda\, dt = d(N\phi)\, dt = N\, d\phi\, dt = N\, d\, dt\, (PNi) = N\, 2\, P\, di\, dt = L\, di\, dt\ . \quad (6.19)$$

The polarity of the induced voltage in the circuit in Fig. 6.31 depends on the current creating the magnetic field. For example, when i is increasing, di/dt is positive and v is positive, so energy is required to establish the magnetic field. The product vi gives the rate at which energy is stored in the field. When i is decreasing, di/dt is negative, v is negative, and the field collapses about the coil, returning energy to the circuit.

Given this deeper look at self-inductance, we now reexamine mutual inductance.

The Concept of Mutual Inductance

Figure 6.32 shows two magnetically coupled coils. Use the procedure presented in Section 6.4 to verify that the dot markings on the two coils agree with the direction of the coil windings and currents shown. The number of turns on each coil are N1 and N2, respectively. Coil 1 is energized by a time-varying current source that establishes the current i1 in the N1 turns. Coil 2 is not energized and is open. The coils are wound on a nonmagnetic core. The flux produced by the current i1 can be divided into two components, labeled ϕ11 and ϕ21. The flux component ϕ11 is the flux produced by i1 that links only the N1 turns. The component ϕ21 is the flux produced by i1 that links both the N2 and N1 turns. The first digit in the flux subscript gives the coil number, and the second digit refers to the coil current. Thus, ϕ11 is a flux linking coil 1 and produced by a current in coil 1, whereas ϕ21 is a flux linking coil 2 and produced by a current in coil 1.

Figure 6.32 Two magnetically coupled coils.

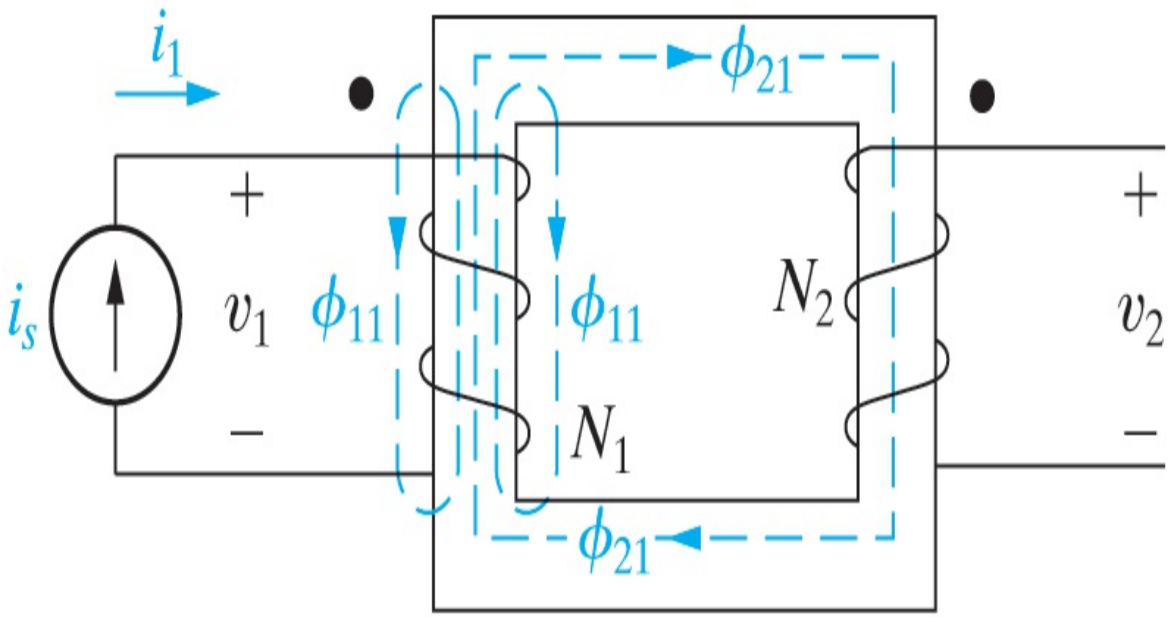


Figure 6.32 Full Alternative Text

The total flux linking coil 1 is ϕ1, the sum of ϕ11 and ϕ21:

ϕ1=ϕ11+ϕ21.

The flux ϕ1 and its components ϕ11 and ϕ21 are related to the coil current i1 as follows:

ϕ1=P1N1i1,
ϕ11=P11N1i1,
ϕ21=P21N1i1,

where P1 is the permeance of the space occupied by the flux ϕ1, P11 is the permeance of the space occupied by the flux ϕ11, and P21 is the permeance of the space occupied by the flux ϕ21. Combining these four equations and simplifying yields the relationship between the permeance of the space occupied by the total flux ϕ1 and the permeances of the spaces occupied by

its components ϕ11 and ϕ21:

P1=P11+P21.(6.20)

We use Faraday's law to derive expressions for v1 and v2:

v 1 = d λ 1 dt = d(N 1 ϕ 1) dt = N 1 d dt (ϕ 11 + ϕ 21) = N 1 2 (P 11 + P 21) d i 1 dt = N 1 2 P 1 d i 1 dt = L 1 d i 1 dt , (6.21)

and

v 2 = d λ 2 dt = d(N 2 ϕ 21) dt = N 2 d dt (P 21 N 1 i 1) = N 2 N 1 P 21 d i 1 dt = M 21 d i 1 dt .

The coefficient of di1/dt in the equation for v1 is the self-inductance of coil 1. The coefficient of di1/dt in the equation for v2 is the mutual inductance between coils 1 and 2. Thus

M21=N2N1P21.(6.22)

The subscript on M specifies an inductance that relates the voltage induced in coil 2 to the current in coil 1.

Figure 6.33 again shows two magnetically coupled coils, but now coil 2 is energized by a time-varying current source (i2) and coil 1 is open. The total flux linking coil 2 is

Figure 6.33 The magnetically coupled coils of Fig. 6.32, with coil 2 excited and coil 1 open.

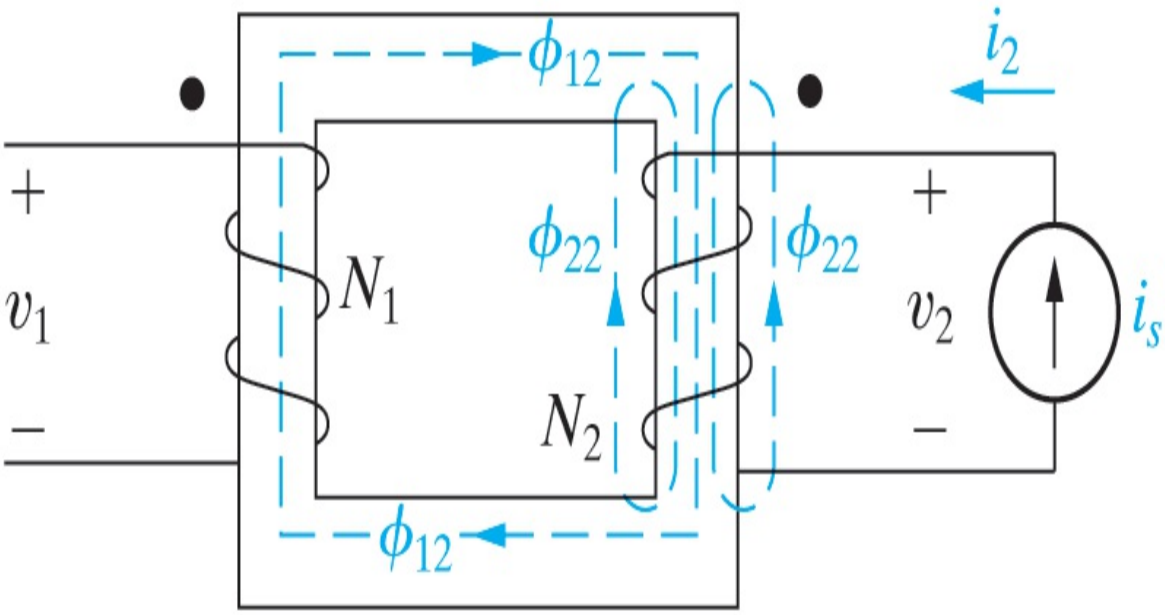


Figure 6.33 Full Alternative Text
ϕ2=ϕ22+ ϕ12.

The flux ϕ2 and its components ϕ22 and ϕ12 are related to the coil current i2 as follows:

ϕ2=P2N2i2,
ϕ22=P22N2i2,
ϕ12=P12N2i2.

The voltages v2 and v1 are

v2=dλ2dt=N22P2di2dt=L2di2dt,v1=dλ1dt=ddt(N1ϕ12)=N1N2P12di2dt=M12 (6.23)

The coefficient of mutual inductance that relates the voltage induced in coil 1 to the time-varying current in coil 2 is

M12=N1N2P12.(6.24)

For nonmagnetic materials, the permeances P12 and P21 are equal, so from Eqs. 6.22 and 6.24,

M12=M21=M.

Hence, for linear circuits with just two magnetically coupled coils, attaching subscripts to the coefficient of mutual inductance is not necessary.

Mutual Inductance in Terms of Self-Inductance

Here we derive the relationship between mutual inductance and self-inductance. From Eqs. 6.21 and 6.23,

$$L_1 = N_{12}P_1,$$
$$L_2 = N_{22}P_2,$$

so

$$L_1L_2 = N_{12}N_{22}P_1P_2.$$

Use Eq. 6.20 and the corresponding expression for P2 to write

$$L_1L_2 = N_{12}N_{22}(P_{11}+P_{21})(P_{22}+P_{12}).$$

But for a linear system, P21=P12, so the expression for L1L2 becomes

$$L_1L_2 = (N_1N_2P_{12})^2(1+P_{11}P_{12})(1+P_{22}P_{12})$$
$$= M^2(1+P_{11}P_{12})\,(1+P_{22}P_{12}). \quad (6.25)$$

Now replace the two terms involving permeances by a single constant, defined as

$$1 - k^2 = (1+P_{11}P_{12})\,(1+P_{22}P_{12}). \quad (6.26)$$

Substituting Eq. 6.26 into Eq. 6.25 and rearranging yields

$$M^2 = k^2L_1L_2$$

or

Relating Self-Inductances and Mutual Inductance

M=kL1L2,(6.27)

where the constant k is called the coefficient of coupling. From Eq. 6.26,

1k2≥1 so k≤1.

In fact, the coefficient of coupling must lie between 0 and 1, or

0≤k≤1.(6.28)

The coefficient of coupling is 0 when the two coils have no common flux; that is, when ϕ12=ϕ21=0. This condition implies that P12=0, and Eq. 6.26 indicates that 1/k2= ∞, or k=0. If there is no flux linkage between the coils, obviously M is 0.

The coefficient of coupling is equal to 1 when ϕ11 and ϕ22 are 0. This condition implies that all the flux that links coil 1 also links coil 2, so P11=P22=0, which represents an ideal state. In reality, winding two coils so that they share precisely the same flux is physically impossible. Magnetic materials (such as alloys of iron, cobalt, and nickel) create a space with high permeance and are used to establish coefficients of coupling that approach unity. (We say more about this important quality of magnetic materials in Chapter 9.)

Energy Calculations

We conclude by calculating the total energy stored in magnetically coupled coils. Along the way, we confirm two observations made earlier: For linear magnetic coupling, (1) M12=M21=M, and (2) M=kL1L2, where 0≤k≤1.

Look at the circuit shown in Fig. 6.34. Initially, assume that the currents i1

and i2 are zero and that this zero-current state corresponds to zero energy stored in the coils. Then let i1 increase from zero to some arbitrary value I1 and compute the energy stored when i1=I1. Because i2=0, the total power input into the pair of coils is v1i1, and the energy stored is

Figure 6.34 The circuit used to derive the basic energy relationships.

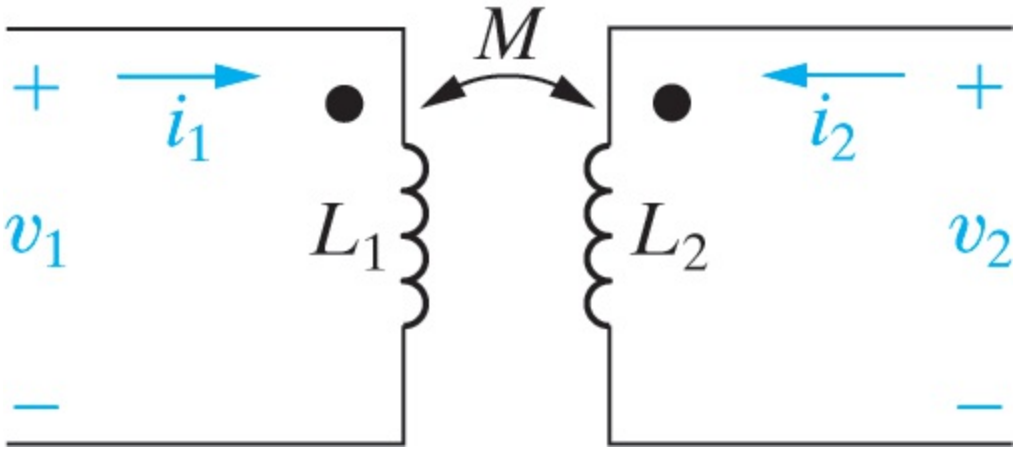


Figure 6.34 Full Alternative Text

∫0W1dw=L1∫0I1i1di1,

W1=12L1I12.

Now we hold i1 constant at I1 and increase i2 from zero to some arbitrary value I2. During this time interval, the voltage induced in coil 2 by i1 is zero because I1 is constant. The voltage induced in coil 1 by i2 is M12di2/dt. Therefore, the power input to the pair of coils is

p=I1M12di2dt+i2v2.

The total energy stored in the pair of coils when i2=I2 is

∫W1Wdw=∫0I2I1M12di2+∫0I2L2i2di2,

or

$$W=W1+I1I2M12+12L2I22,$$
$$=12L1I12+12L2I22+I1I2M12. \quad (6.29)$$

If we reverse the procedure—that is, if we first increase i2 from zero to I2 and then increase i1 from zero to I1—the total energy stored is

$$W=12L1I12+12L2I22+I1I2M21. \quad (6.30)$$

Equations 6.29 and 6.30 express the total energy stored in a pair of linearly coupled coils as a function of the coil currents, the self-inductances, and the mutual inductance. Note that the only difference between these equations is the coefficient of the current product I1I2. We use Eq. 6.29 if i1 is established first and Eq. 6.30 if i2 is established first.

When the coupling medium is linear, the total energy stored is the same regardless of the order used to establish I1 and I2 because the resultant magnetic flux depends only on the final values of i1 and i2, not on how the currents reached their final values. If the resultant flux is the same, the stored energy is the same. Therefore, for linear coupling, M12=M21. Also, because I1 and I2 are arbitrary values of i1 and i2, respectively, we represent the coil currents by their instantaneous values i1 and i2. Thus, at any instant of time, the total energy stored in the coupled coils is

$$w(t)=12L1i12+12L2i22+Mi1i2. \quad (6.31)$$

We derived Eq. 6.31 by assuming that both coil currents entered dotted terminals. We leave it to you to verify that, if one current enters a dotted terminal while the other leaves such a terminal, the algebraic sign of the term Mi1i2 reverses. Thus, in general,

Energy Stored in Magnetically Coupled Coils

$$w(t)=12L1i12+12L2i22\pm Mi1i2. \quad (6.32)$$

We can use Eq. 6.32 to show that M cannot exceed L1L2. The magnetically coupled coils are passive elements, so the total energy stored can never be negative. If w(t) can never be negative, Eq. 6.32 indicates that

$$12L1i12+12L2i22-Mi1i2$$

when i1 and i2 are either both positive or both negative. The limiting value of M occurs when

$$12L1i12+12L2i22-Mi1i2=0. \quad (6.33)$$

To find the limiting value of M, we add and subtract the term i1i2L1L2 to the left-hand side of Eq. 6.33. Doing so generates a term that is a perfect square:

$$(L12i1-L22i2)2+i1i2(L1L2-M)=0. \quad (6.34)$$

The squared term in Eq. 6.34 can never be negative, but it can be zero. Therefore, $w(t)\geq 0$ only if

$$L1L2\geq M, \quad (6.35)$$

which is another way of saying that

$$M=kL1L2 \qquad (0\leq k\leq 1).$$

We derived Eq. 6.35 by assuming that i1 and i2 are either both positive or both negative. However, we get the same result if i1 and i2 have opposite signs because in this case we obtain the limiting value of M by selecting the plus sign in Eq. 6.32.

Work through Example 6.9 to practice calculating the coupling coefficient and the stored energy for magnetically coupled coils.

Example 6.9 Calculating the

Coupling Coefficient and Stored Energy for Magnetically Coupled Coils

The mutual inductance and self-inductances of the coils in Fig. 6.34 are M=40 mH, L1=25 mH, and L2=100 mH.

1. Calculate the coupling coefficient.
2. Calculate the energy stored in the coupled coils when i1=10 A and i2=15 A.
3. If the coupling coefficient is increased to 1 and i1=10 A, what value of i2 results in zero stored energy?

Solution

1. k=ML1L2=0.04(0.025)(0.1)=0.8.
2. w=12(0.025)(10)2+12(0.1)(15)2+(0.04)(10)(15)=18.5 J.
3. When k=1, M=(0.025)(0.1)=0.05=50mH. The energy in the coils is now

   12(0.025)(10)2+12(0.1)(i2)2+(0.05)(10)(i2)=0

   so i2 must satisfy the quadratic equation

   0.05i22 + 0.5i2 + 1.25 = 0.

   Use the quadratic formula to find i2:

   i2=−0.5±0.52−4(0.05)(1.25)2(0.05)=−5 A.

   You should verify that the energy is zero for this value of i2, when k=1.

Assessment Problem

Objective 4—Understand the concept of mutual inductance

1. 6.7 Consider the magnetically coupled coils described in Example 6.9. Assume that the physical arrangement of the coils results in P1=P2. If coil 1 has 500 turns, how many turns does coil 2 have?

   Answer:

   1000 turns

SELF-CHECK: *Assess your understanding of this material by trying Chapter Problems 6.45 and 6.48.*

Practical Perspective

Capacitive Touch Screens

Capacitive touch screens are often used in applications where two or more simultaneous touch points must be detected. We will discuss two designs for a multi-touch screen. The first design employs a grid of electrodes, as shown in Fig. 6.35. When energized, a small parasitic capacitance, Cp, exists between each electrode strip and ground, as shown in Fig. 6.36(a). When the screen is touched, say at position x, y on the screen, a second capacitance exists due to the transfer of a small amount of charge from the screen to the human body, which acts like a conductor. This introduces a second capacitance at the point of touch with respect to ground, as shown in Fig. 6.36(b).

Figure 6.35 Multi-touch screen

with grid of electrodes.

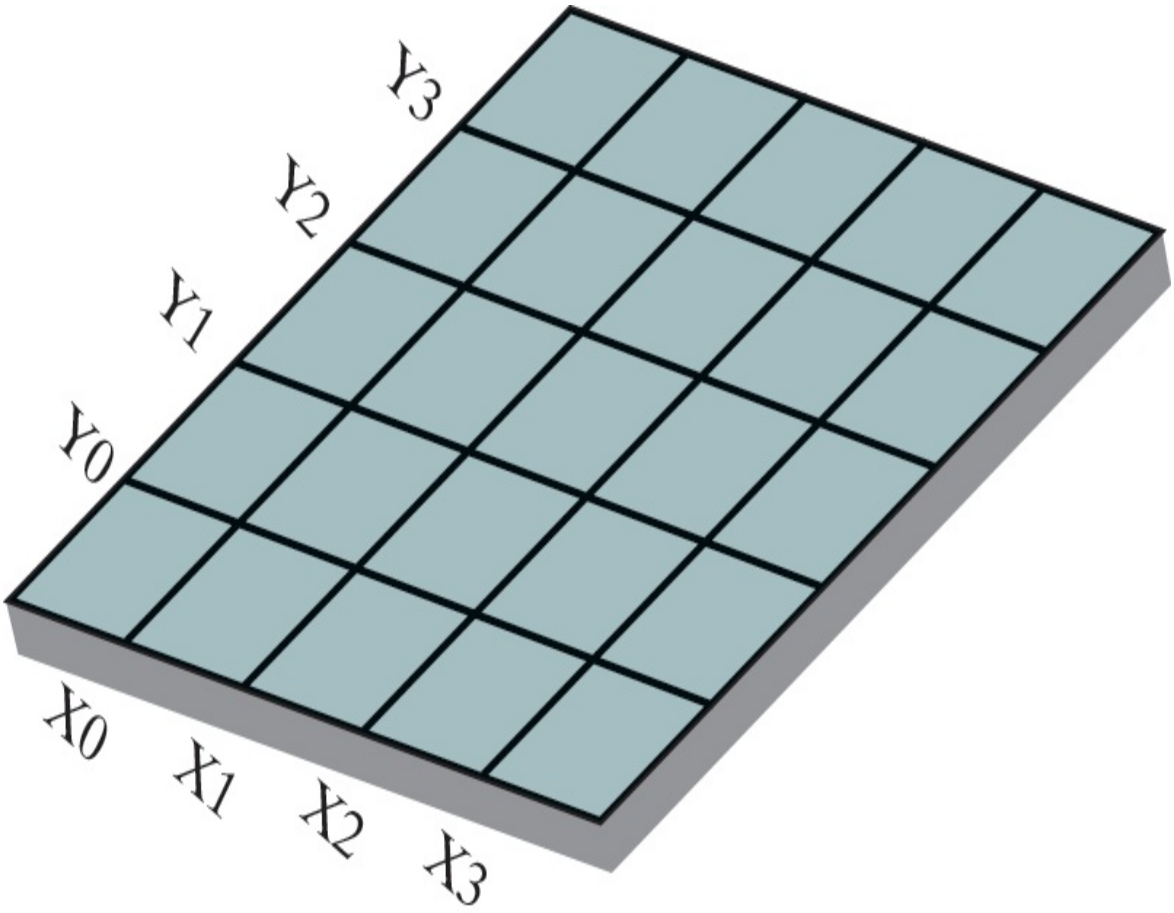


Figure 6.35 Full Alternative Text

Figure 6.36 (a) Parasitic capacitance between electrode and ground with no touch; (b) Additional capacitance introduced by a touch.

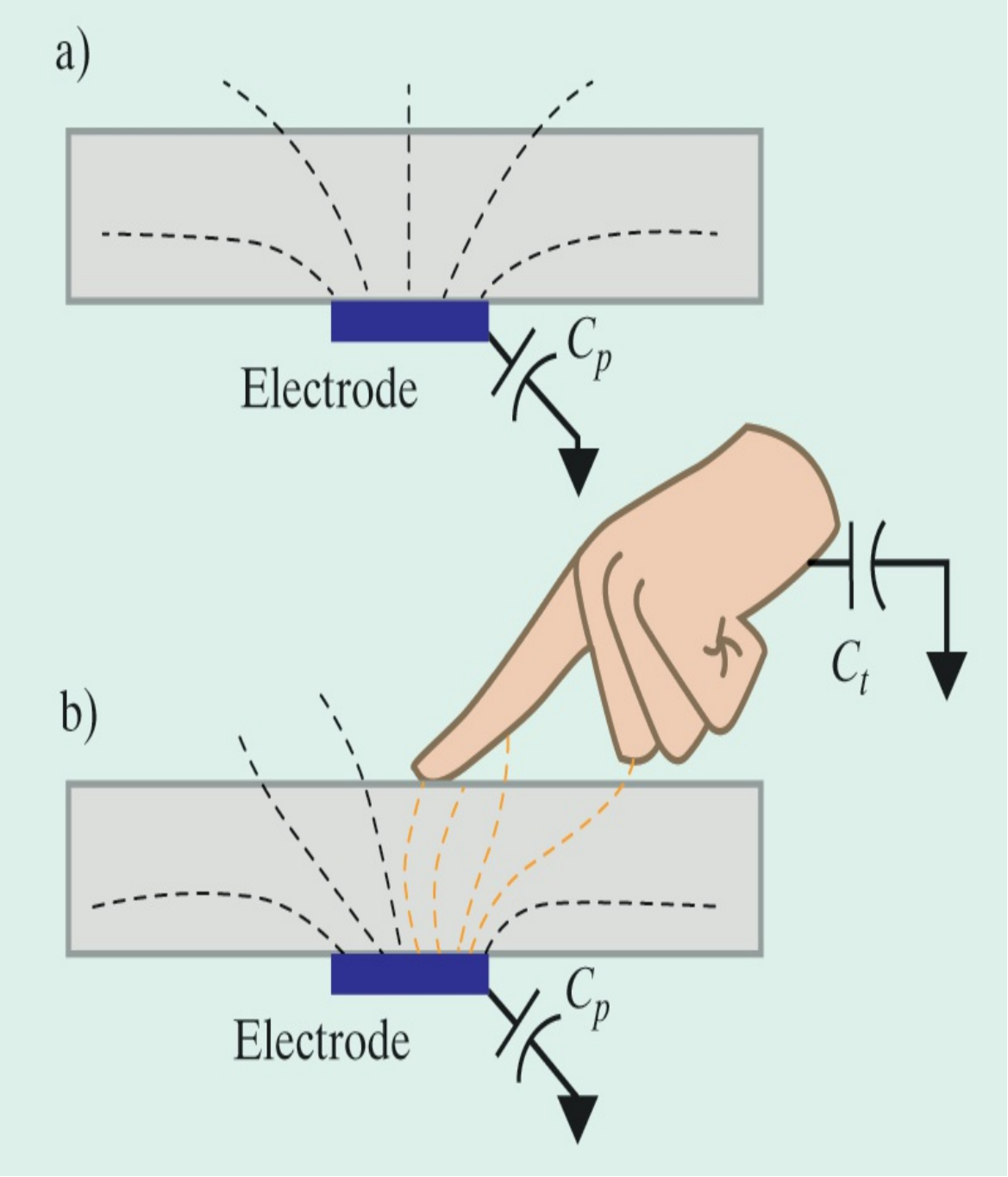


Figure 6.36 Full Alternative Text

A touch-screen controller is continually monitoring the capacitance between the electrodes in the grid and ground. If the screen is untouched, the capacitance between every electrode in the x-grid and ground is Cp; the same is true for the capacitance between every electrode in the y-grid and ground.

When the screen is touched at a single point, Ct and Cp combine in parallel. The equivalent capacitance between the x-grid electrode closest to the touch point and ground is now

Ctx=Ct+Cp.

Likewise, the equivalent capacitance between the y-grid electrode closest to the touch point and ground is now

Cty=Ct+Cp.

Thus, a screen touch increases the capacitance between the electrodes and ground for the x- and y-grid electrodes closest to the touch point.

What happens when there are two simultaneous points where the screen is touched? Assume that the first touch point has coordinates x1, y1 and the second touch point has coordinates x2, y2. Now there are four screen locations that correspond to an increase in capacitance: x1, y1; x1, y2; x2, y1; and x2, y2. Two of those screen locations match the two touch points, and the other two points are called “ghost” points because the screen was not touched at those points. Therefore, this method for implementing a capacitive touch screen cannot accurately identify more than a single touch point.

Most modern capacitive touch screens do not use the “self-capacitance” design. Instead of measuring the capacitance between each x-grid electrode and ground, and each y-grid electrode and ground, the capacitance between each x-grid electrode and each y-grid electrode is measured. This capacitance is known as “mutual” capacitance and is shown in Fig. 6.37(a).

Figure 6.37 (a) Mutual capacitance between an x-grid and a y-grid electrode; (b) Additional capacitance

introduced by a touch.

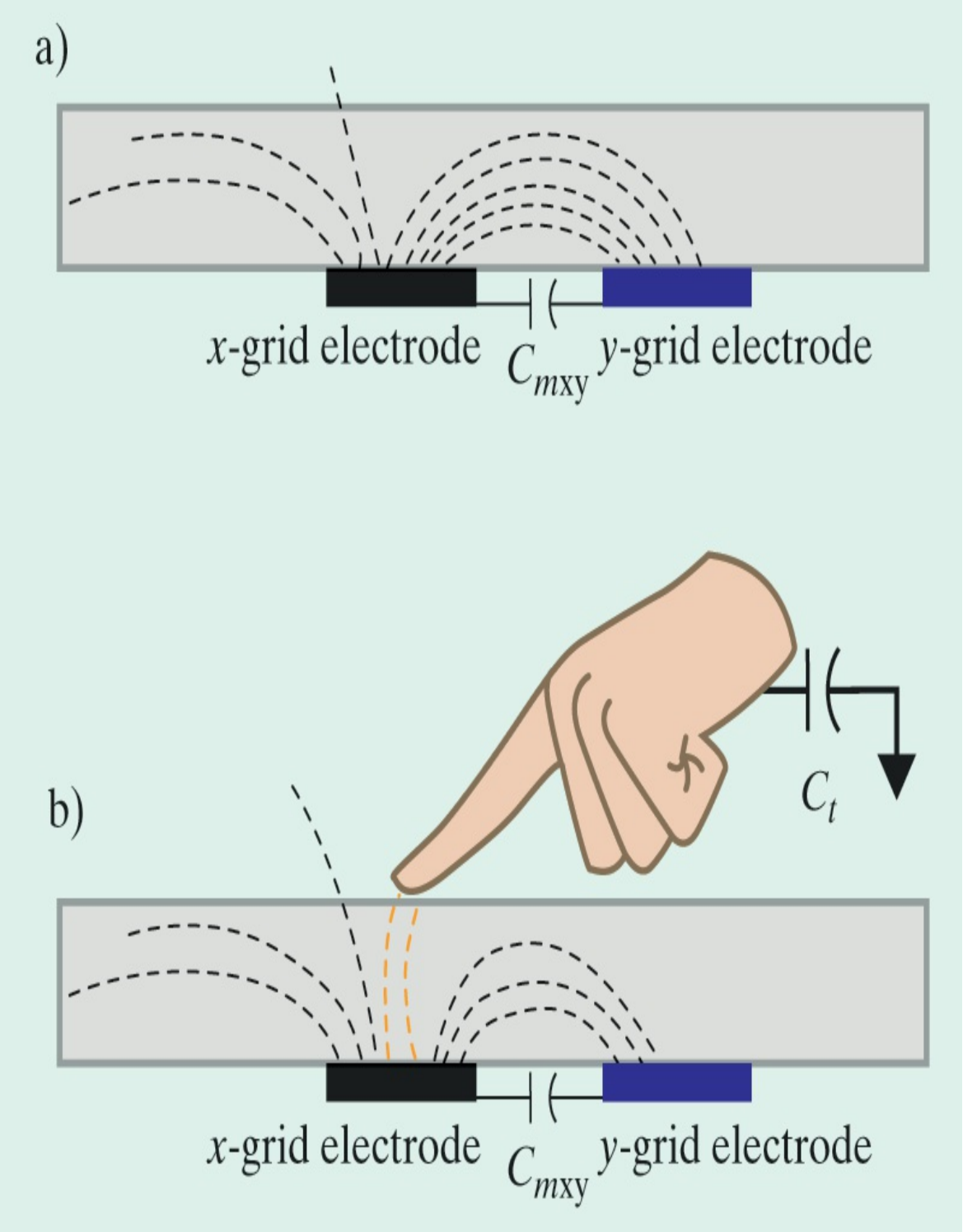


Figure 6.37 Full Alternative Text

When the screen is touched, say at position x, y on the screen, a second capacitance again exists due to the transfer of a small amount of charge from the screen to the human body. The second capacitance exists at the point of touch with respect to ground, as shown in Fig. 6.37(b). Therefore, whenever there is a change in the mutual capacitance, Cmxy, the screen touch point can be uniquely identified as x, y. If the screen is touched at the points x1, y1 and x2, y2 then precisely two mutual capacitances change: Cmx1y1 and Cmx2y2. There are no “ghost” points identified, as there were in the self-capacitance design. The mutual capacitance design produces a multi-touch screen capable of identifying two or more touch points uniquely and accurately.

SELF-CHECK: *Assess your understanding of the Practical Perspective by solving Chapter Problems 6.51–6.53.*

Summary

- **Inductance** is a linear circuit parameter that relates the voltage induced by a time-varying magnetic field to the current producing the field. (See page 184.)
- **Capacitance** is a linear circuit parameter that relates the current induced by a time-varying electric field to the voltage producing the field. (See page 189.)
- Inductors and capacitors are passive elements; they can store and release energy, but they cannot generate or dissipate energy. (See page 182.)
- The instantaneous power at the terminals of an inductor or capacitor can be positive or negative, depending on whether energy is being delivered to or extracted from the element.
- An inductor:
    - does not permit an instantaneous change in its terminal current,
    - does permit an instantaneous change in its teminal voltage, and
    - behaves as a short circuit in the presence of a constant terminal current. (See page 184.)
- A capacitor:
    - does not permit an instantaneous change in its terminal voltage,
    - does permit an instantaneous change in its terminal current, and
    - behaves as an open circuit in the presence of a constant terminal voltage. (See page 190.)
- Equations for voltage, current, power, and energy in ideal inductors and

capacitors are given in Table 6.1. (See page 198.)

- Inductors in series or in parallel can be replaced by an equivalent inductor. Capacitors in series or in parallel can be replaced by an equivalent capacitor. The equations are summarized in Table 6.1. The table includes the initial conditions for series and parallel equivalent circuits involving inductors and capacitors.

- **Mutual inductance**, M, is the circuit parameter relating the voltage induced in one circuit to a time-varying current in another circuit. Specifically,

  $v1=L1di1dt+ M12di2dt$
  $v2=M21di1dt+L2di2dt,$

  where v1 and i1 are the voltage and current in circuit 1, and v2 and i2 are the voltage and current in circuit 2. For coils wound on nonmagnetic cores, $M12=M21=M$. (See page 199.)

- The **dot convention** establishes the polarity of mutually induced voltages:

  When the reference direction for a current enters the dotted terminal of a coil, the reference polarity of the voltage that the current induces in the other coil is positive at its dotted terminal.

  Or, alternatively,

  When the reference direction for a current leaves the dotted terminal of a coil, the reference polarity of the voltage that the current induces in the other coil is negative at its dotted terminal.

  (See page 199.)

- The relationship between the self-inductance of each winding and the mutual inductance between windings is

  $M=kL1L2.$

The **coefficient of coupling**, k, is a measure of the degree of magnetic coupling. By definition, $0 \leq k \leq 1$. (See page 206.)

- The energy stored in magnetically coupled coils in a linear medium is related to the coil currents and inductances by the relationship

  $w = 12L1i12 + 12L2i22 \pm Mi1i2.$

  (See page 208.)

Problems

Section 6.1

1. 6.1 PSPICEMULTISIM The current in a 50μH inductor is known to be

   $i_L = 18te^{-10t}$ A for $t \geq 0$.

   1. Find the voltage across the inductor for $t > 0$. (Assume the passive sign convention.)

   2. Find the power (in microwatts) at the terminals of the inductor when $t = 200$ ms.

   3. Is the inductor absorbing or delivering power at 200 ms?

   4. Find the energy (in microjoules) stored in the inductor at 200 ms.

   5. Find the maximum energy (in microjoules) stored in the inductor and the time (in milliseconds) when it occurs.

2. 6.2 PSPICEMULTISIM The voltage at the terminals of the 200 μH inductor in Fig. P6.2(a) is shown in Fig. P6.2(b). The inductor current i is known to be zero for $t \leq 0$.

   1. Derive the expressions for i for $t \geq 0$.

   2. Sketch i versus t for $0 \leq t \leq \infty$.

Figure P6.2

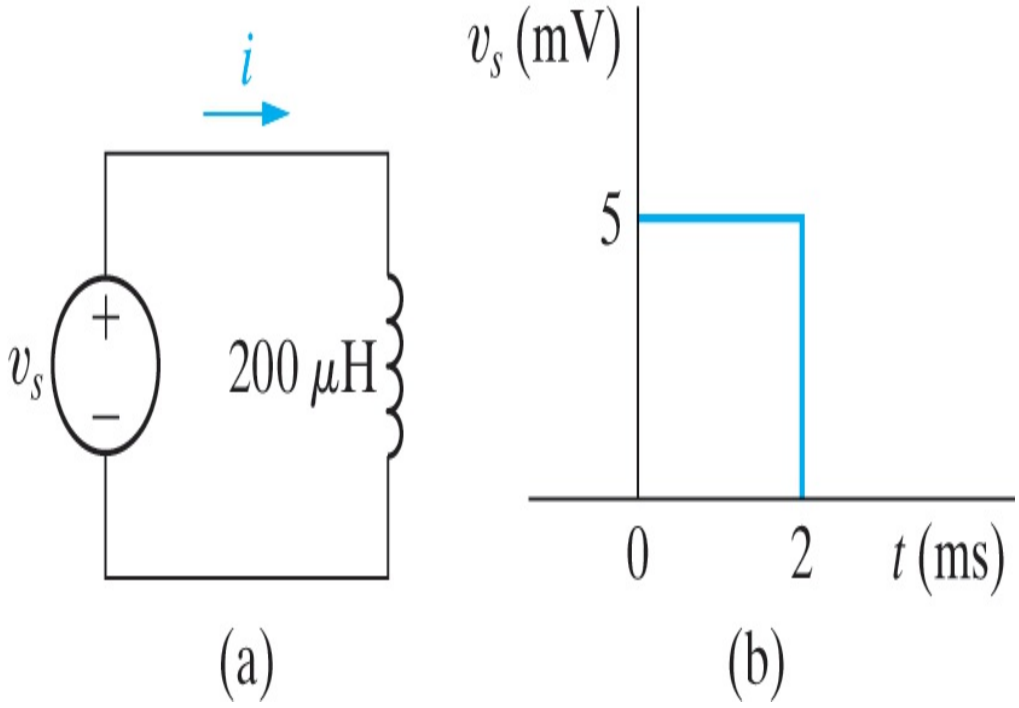


Figure P6.2 Full Alternative Text

3. 6.3 PSPICEMULTISIM The triangular current pulse shown in Fig. P6.3 is applied to a 20 mH inductor.

    1. Write the expressions that describe i(t) in the four intervals t<0, 0≤t≤5 ms, 5 ms≤t≤10 ms, and t>10 ms.

    2. Derive the expressions for the inductor voltage, power, and energy. Use the passive sign convention.

Figure P6.3

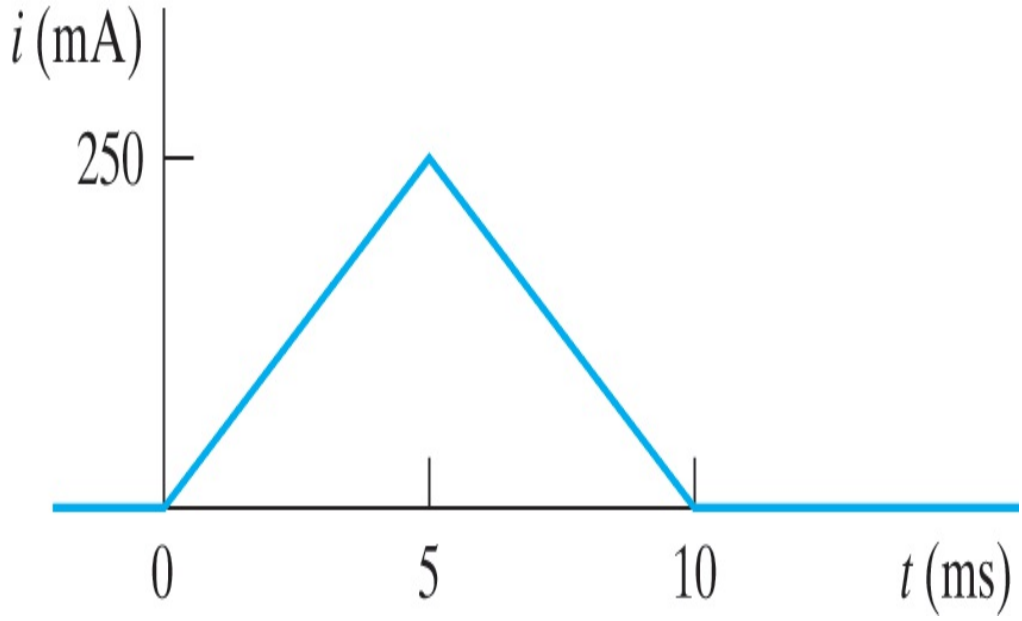


Figure P6.3 Full Alternative Text

4. 6.4 The current in a 200 mH inductor is

   i = 75 mA, t≤0; i = (B 1 cos 200t+ B 2 sin 200t) e −50t A, t≥0.

   The voltage across the inductor (passive sign convention) is 4.25 V at t=0. Calculate the power at the terminals of the inductor at t=25 ms. State whether the inductor is absorbing or delivering power.

5. 6.5 The current in a 20 mH inductor is known to be

   i=40 mA,t≤0;
   i=A1e−10,000t+A2e−40.000tA,t≥0.

   The voltage across the inductor (passive sign convention) is 28 V at t=0.

   1. Find the expression for the voltage across the inductor for t>0.

   2. Find the time, greater than zero, when the power at the terminals of the inductor is zero.

6. 6.6 Assume in Problem 6.5 that the value of the voltage across the inductor at t=0 is −68 V instead of 28 V.

1. Find the numerical expressions for i and v for t≥0.

2. Specify the time intervals when the inductor is storing energy and the time intervals when the inductor is delivering energy.

3. Show that the total energy extracted from the inductor is equal to the total energy stored.

7. 6.7 Evaluate the integral

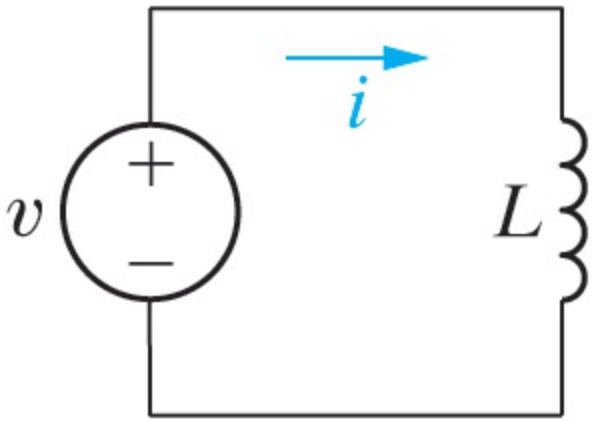
   $\int_0^\infty p\,dt$

   for Example 6.2. Comment on the significance of the result.

8. 6.8 PSPICEMULTISIM

   1. Find the inductor current in the circuit in Fig. P6.8 if $v=30\cos 500t$ V, L=15 mH, and $i(0)=-4$ A.

Figure P6.8



   2. Sketch i, v, p, and w versus t. In making these sketches, use the format used in Fig. 6.8. Plot over one complete cycle of the voltage waveform.

   3. Describe the subintervals in the time interval between 0 and 4π ms when power is being absorbed by the inductor. Repeat for the

subintervals when power is being delivered by the inductor.

9. 6.9 PSPICEMULTISIM The current in a 25 mH inductor is known to be −10 A For $t \leq 0$ and $(-10 \cos 400t - 5 \sin 400t)\, e^{-200t}$ A for $t \geq 0$. Assume the passive sign convention.

    1. At what instant of time is the voltage across the inductor maximum?

    2. What is the maximum voltage?

10. 6.10 The current in and the voltage across a 5 H inductor are known to be zero for $t \leq 0$. The voltage across the inductor is given by the graph in Fig. P6.10 for $t \geq 0$.

    1. Derive the expression for the current as a function of time in the intervals $0 \leq t \leq 1$ s, $1 \text{ s} \leq t \leq 3$ s, $3 \text{ s} \leq t \leq 5$ s, $5 \text{ s} \leq t \leq 6$ s, and $6 \text{ s} \leq t < \infty$.

Figure P6.10

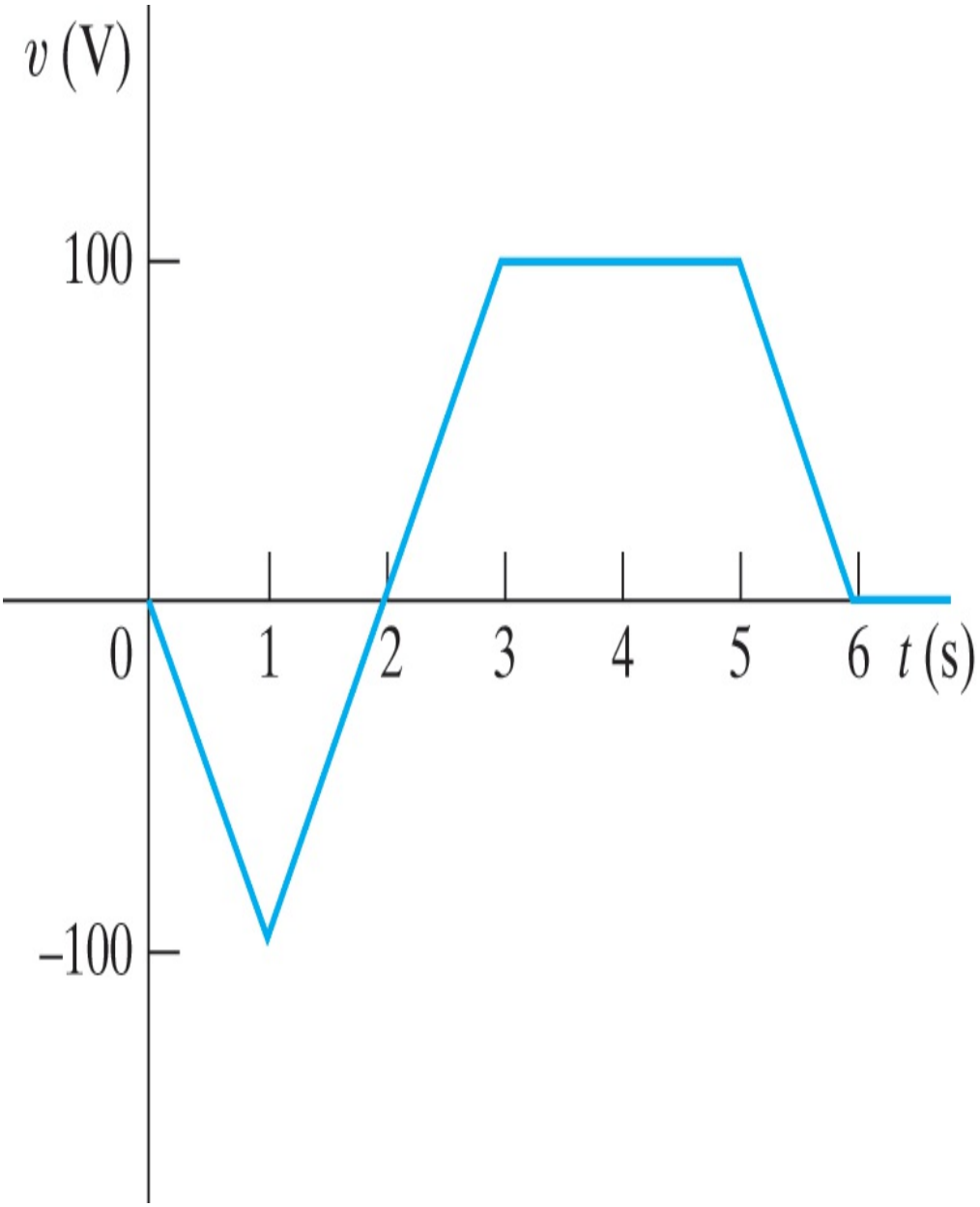


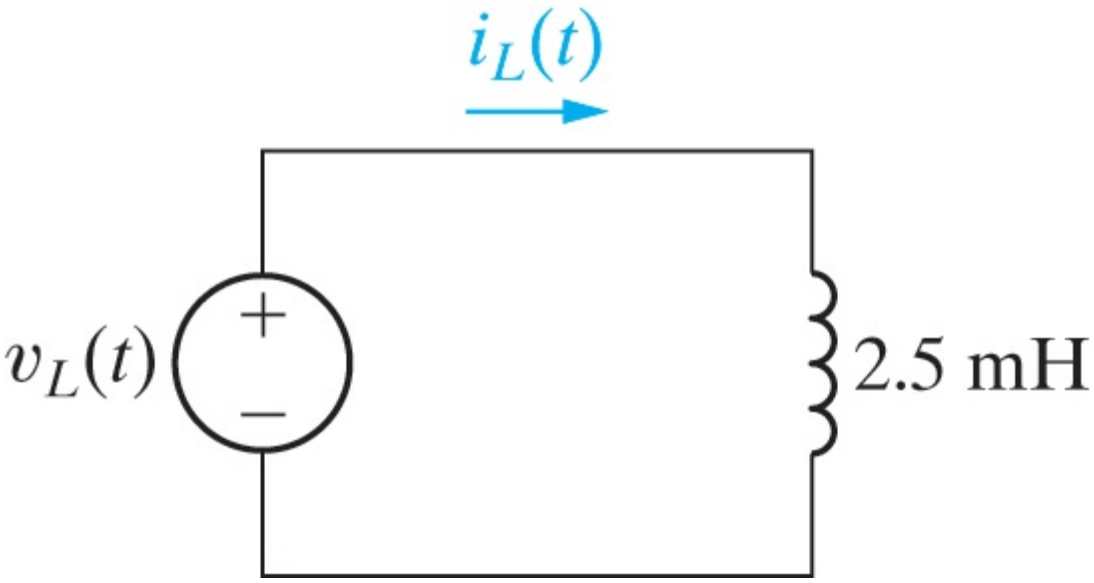
Figure P6.10 Full Alternative Text

2. For t>0, what is the current in the inductor when the voltage is zero?

3. Sketch i versus t for $0 \le t < \infty$.

11. 6.11 The current in the 2.5 mH inductor in Fig. P6.11 is known to be 1 A for t<0. The inductor voltage for $t \ge 0$ is given by the expression

Figure P6.11



vL(t)=3e−4t mV, 0+≤t≤2 svL(t)=−3e−4(t−2) mV, 2 s≤t<∞

Sketch vL(t) and iL(t) for 0≤t<∞.

12. 6.12 Initially there was no energy stored in the 5 H inductor in the circuit in Fig. P6.12 when it was placed across the terminals of the voltmeter. At t=0 the inductor was switched instantaneously to position b where it remained for 1.6 s before returning instantaneously to position a. The d'Arsonval voltmeter has a full-scale reading of 20 V and a sensitivity of 1000 Ω/V. What will the reading of the voltmeter be at the instant the switch returns to position a if the inertia of the d'Arsonval movement is negligible?

Figure P6.12

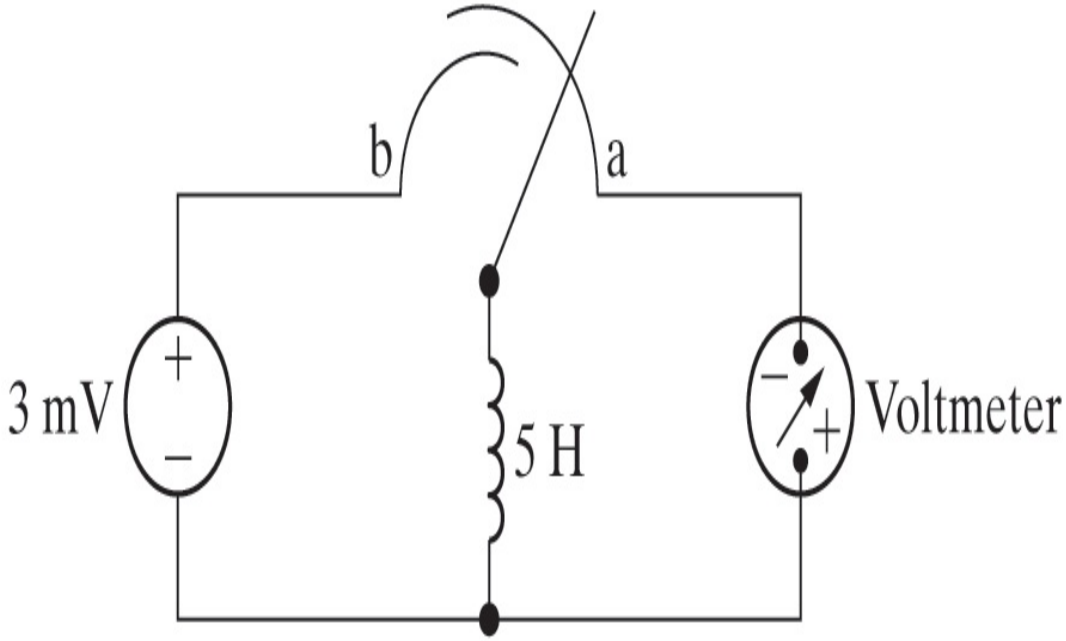


Figure P6.12 Full Alternative Text

Section 6.2

1. 6.13 The voltage across a 5 μF capacitor is known to be

   $v_c = 500te^{-2500t}$ V for $t \geq 0$.

   1. Find the current through the capacitor for $t > 0$. Assume the passive sign convention.
   2. Find the power at the terminals of the capacitor when $t = 100$ μs.
   3. Is the capacitor absorbing or delivering power at $t = 100$ μs ?
   4. Find the energy stored in the capacitor at $t = 100$ μs.
   5. Find the maximum energy stored in the capacitors and the time when the maximum occurs.

2. 6.14 PSPICEMULTISIM The voltage at the terminals of the capacitor in Fig. 6.10 is known to be

   $v = \{-10 \text{ V}, t \leq 0; 40 - 1e^{-1000t}(50 \cos 500t + 20 \sin 500t) \text{ V}, t \geq 0$.

   Assume $C = 0.8$μF.

   1. Find the current in the capacitor for $t < 0$.
   2. Find the current in the capacitor for $t > 0$.
   3. Is there an instantaneous change in the voltage across the capacitor at $t = 0$?
   4. Is there an instantaneous change in the current in the capacitor at $t = 0$?
   5. How much energy (in millijoules) is stored in the capacitor at $t = \infty$?

3. 6.15 The triangular voltage pulse shown in Fig. P6.15 is applied to a 200 μF capacitor.

Figure P6.15

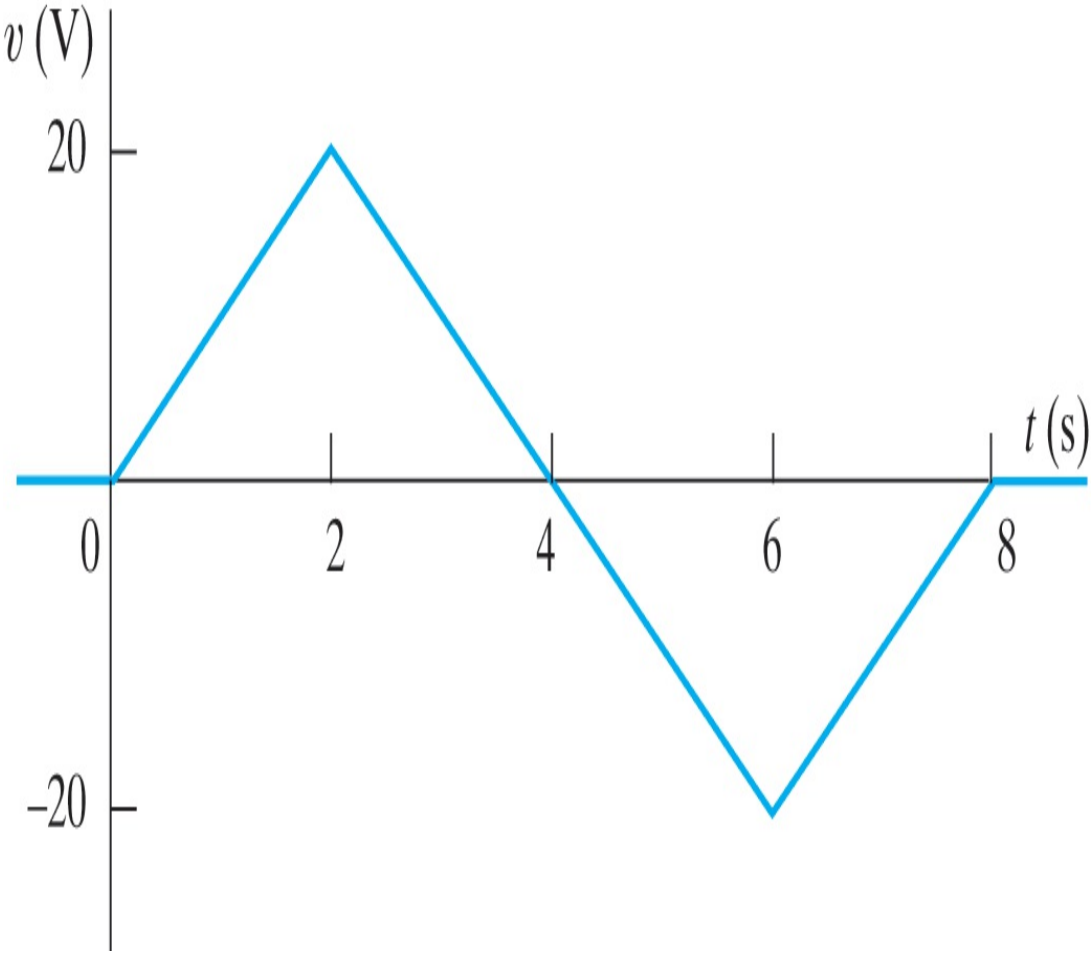


Figure P6.15 Full Alternative Text

1. Write the expressions that describe v(t) in the five time intervals $t<0$, $0 \le t \le 2$ s, $2 \text{ s} \le t \le 6$ s, $6 \text{ s} \le t \le 8$ s, and $t>8$ s.

2. Derive the expressions for the capacitor current, power, and energy for the time intervals in part (a). Use the passive sign convention.

3. Identify the time intervals between 0 and 8 s when power is being

delivered by the capacitor. Repeat for the time intervals when power is being absorbed by the capacitor.

4. 6.16 The expressions for voltage, power, and energy derived in Example 6.5 involved both integration and manipulation of algebraic expressions. As an engineer, you cannot accept such results on faith alone. That is, you should develop the habit of asking yourself, “Do these results make sense in terms of the known behavior of the circuit they purport to describe?” With these thoughts in mind, test the expressions of Example 6.5 by performing the following checks:

    1. Check the expressions to see whether the voltage is continuous in passing from one time interval to the next.

    2. Check the power expression in each interval by selecting a time within the interval and seeing whether it gives the same result as the corresponding product of v and i. For example, test at 10 and 30 μs.

    3. Check the energy expression within each interval by selecting a time within the interval and seeing whether the energy equation gives the same result as 12Cv2. Use 10 and 30 μs as test points.

5. 6.17 A 20μF capacitor is subjected to a voltage pulse having a duration of 1 s. The pulse is described by the following equations:

    $$v_c(t)=\{30t^2\text{ V},0\le t\le 0.5\text{ s};30(t-1)^2\text{ V},0.5\text{ s}\le t\le 1\text{ s};0\text{elsewhere.}$$

    Sketch the current pulse that exists in the capacitor during the 1 s interval.

6. 6.18 PSPICEMULTISIM The voltage across the terminals of a 250 nF capacitor is

    $$v=\{50\text{ V},t\le 0;(A1e^{-4000t}+A2te^{-4000t})\text{ V},t\ge 0.$$

    The initial current in the capacitor is 400 mA. Assume the passive sign convention.

1. What is the initial energy stored in the capacitor?
2. Evaluate the coefficients A1 and A2.
3. What is the expression for the capacitor current?

7. 6.19 PSPICEMULTISIM The initial voltage on the 0.5 μF capacitor shown in Fig. P6.19(a) is −20 V. The capacitor current has the waveform shown in Fig. P6.19(b).
    1. How much energy, in microjoules, is stored in the capacitor at t=500 μs?

Figure P6.19

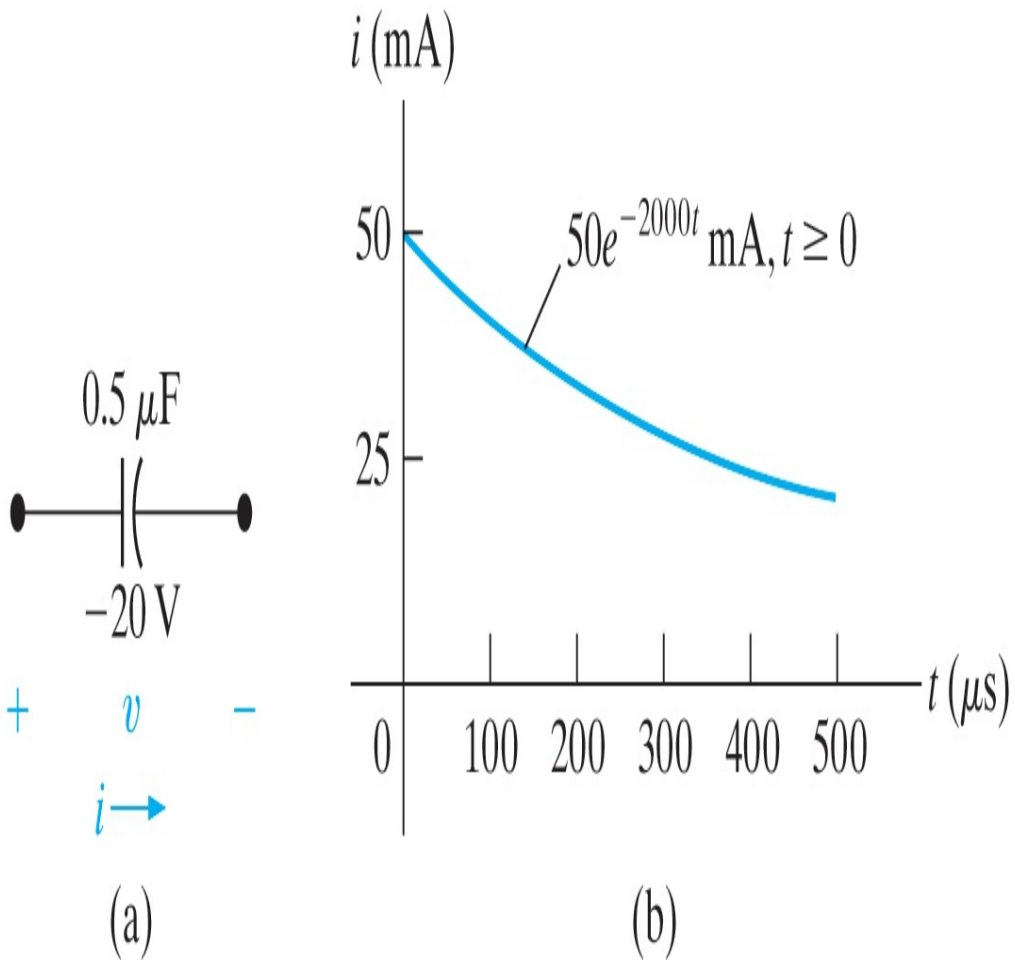


Figure P6.19 Full Alternative Text

2. Repeat (a) for t=∞.

8. 6.20 PSPICEMULTISIM The current shown in Fig. P6.20 is applied to a 250 nF capacitor. The initial voltage on the capacitor is zero.

    1. a) Find the charge on the capacitor at t=30 μs.

Figure P6.20

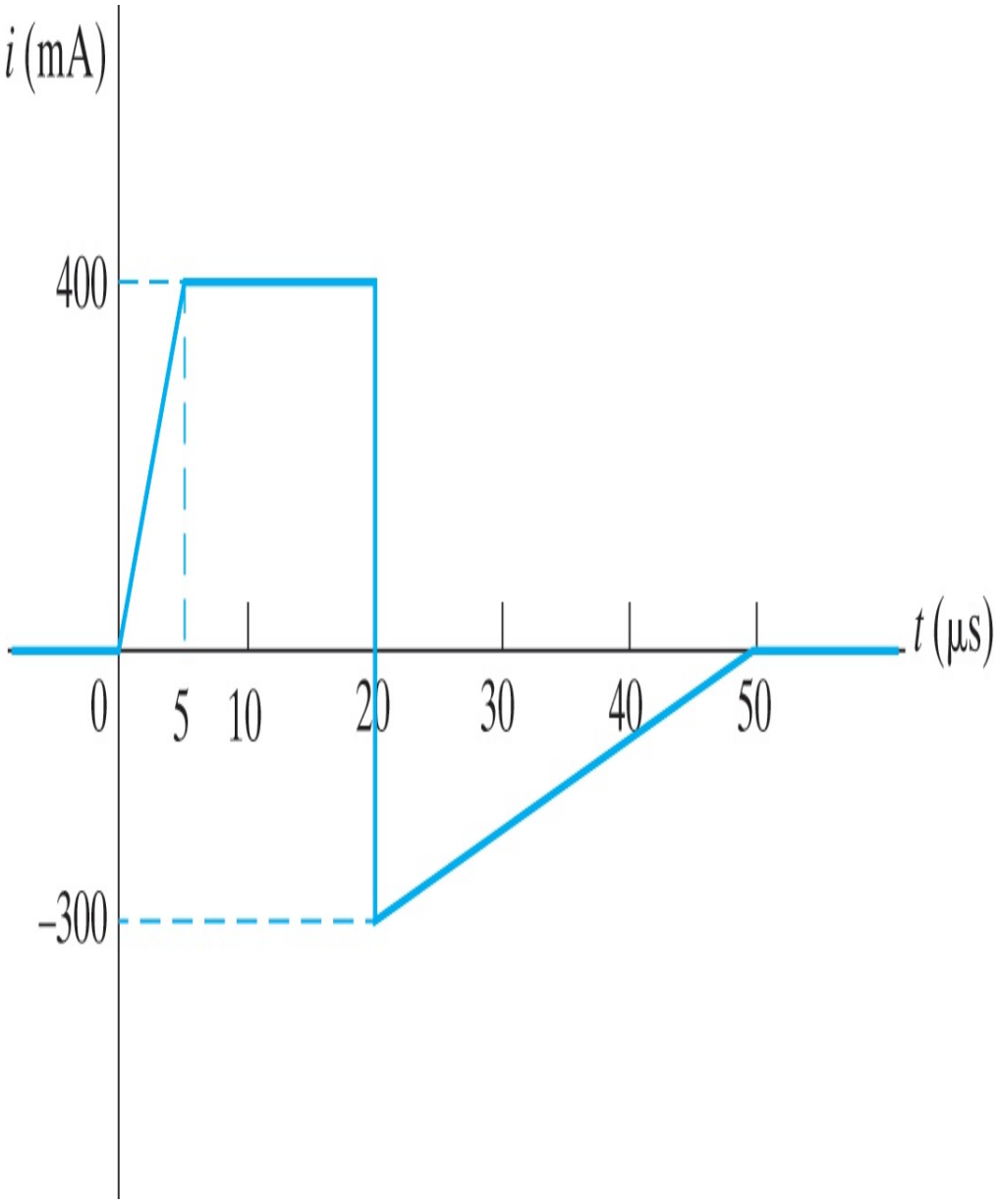


Figure P6.20 Full Alternative Text

2. b) Find the voltage on the capacitor at t=50 μs.

3. c) How much energy is stored in the capacitor by this current?

9. 6.21 PSPICEMULTISIM The rectangular-shaped current pulse shown in Fig. P6.21 is applied to a 5 μF capacitor. The initial voltage on the capacitor is a 12 V drop in the reference direction of the current. Derive the expression for the capacitor voltage for the time intervals in (a)–(e).

Figure P6.21

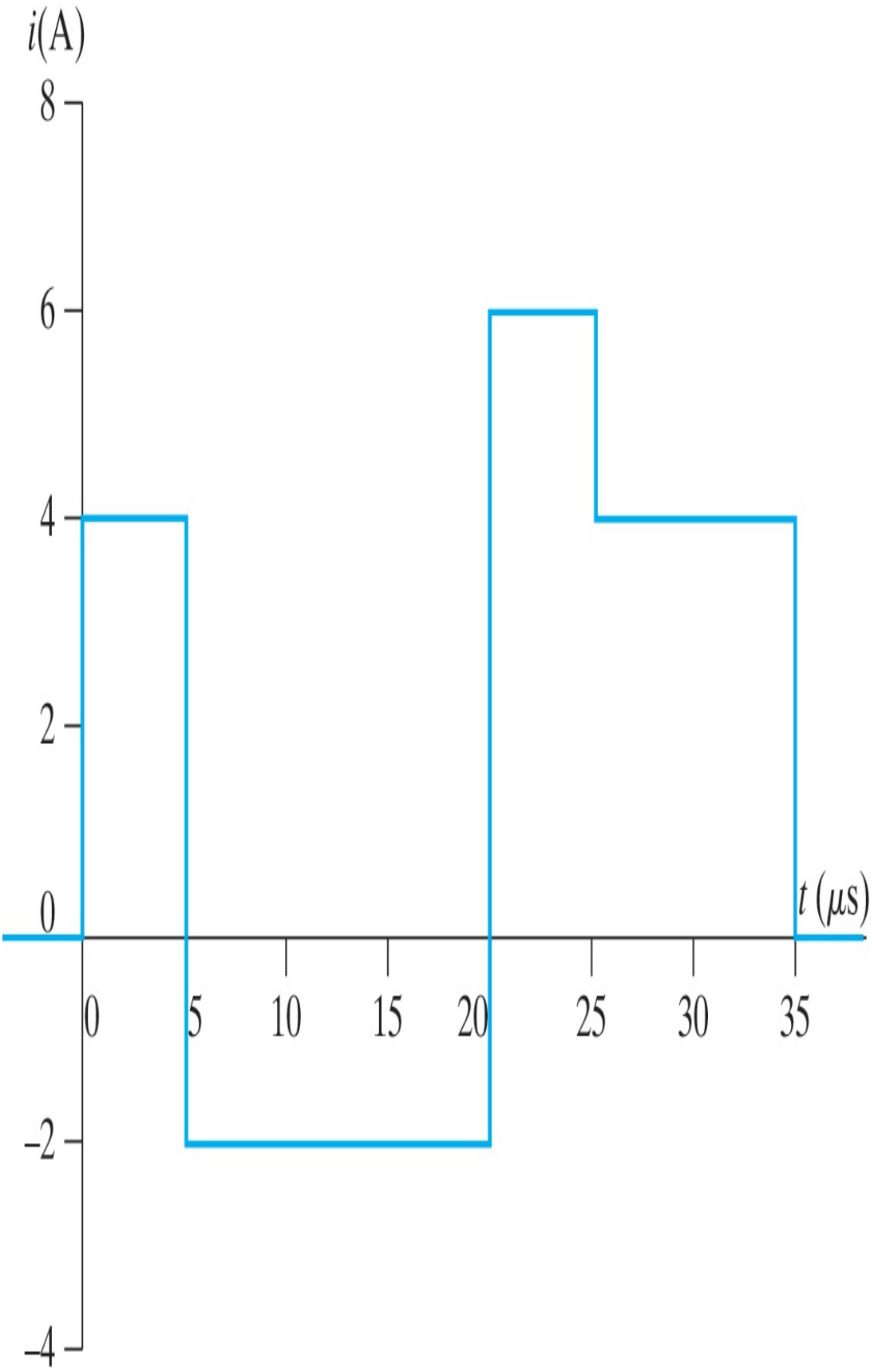

i(A)
8
6
4
2
0
–2
–4
t (μs)
0
5
10
15
20
25
30
35

Figure P6.21 Full Alternative Text

1. $0 \le t \le 5\ \mu s$;
2. $5\ \mu s \le t \le 20\ \mu s$;
3. $20\ \mu s \le t \le 25\ \mu s$
4. $25\ \mu s \le t \le 35\ \mu s$
5. $35\ \mu s \le t < \infty$
6. Sketch v(t) over the interval $-20\ \mu s \le t \le 40\ \mu s$

Section 6.3

1. 6.22 Use realistic inductor values from Appendix H to construct series and parallel combinations of inductors to yield the equivalent inductances specified below. Try to minimize the number of inductors used. Assume that no initial energy is stored in any of the inductors.

    1. 8 mH;

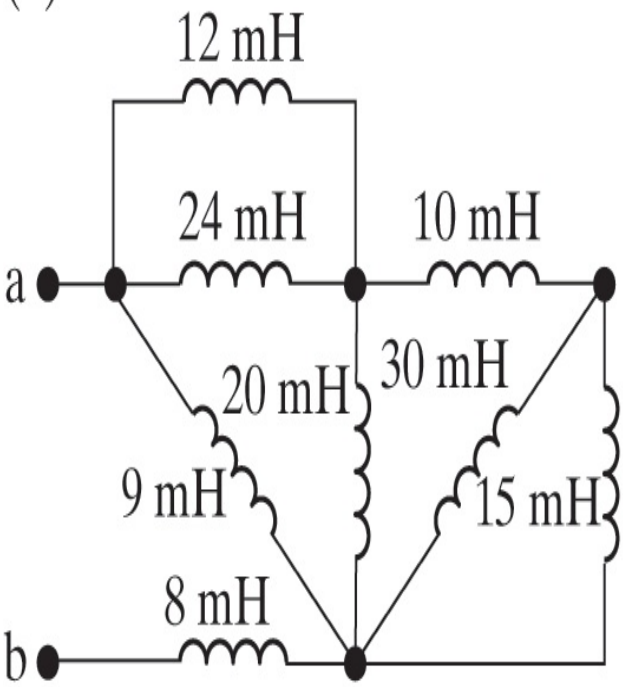
    2. 45 μH;

    3. 180 μH.

2. 6.23 PSPICEMULTISIM Assume that the initial energy stored in the inductors of Figs. P6.23(a) and (b) is zero. Find the equivalent inductance with respect to the terminals a, b.

Figure P6.23

(a)


12 mH
24 mH
10 mH
a
20 mH
30 mH
9 mH
15 mH
8 mH
b

(b)

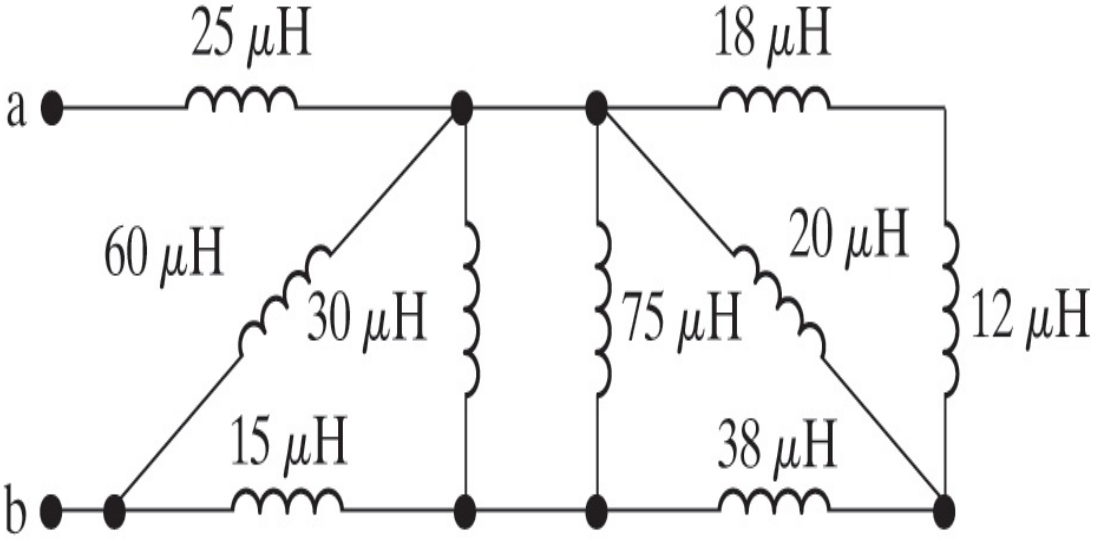

25 μH
18 μH
a
60 μH
20 μH
30 μH
75 μH
12 μH
15 μH
38 μH
b

Figure P6.23 Full Alternative Text

3. 6.24 PSPICEMULTISIM The three inductors in the circuit in Fig. P6.24 are connected across the terminals of a black box at t=0. The resulting voltage for t>0 is known to be

Figure P6.24

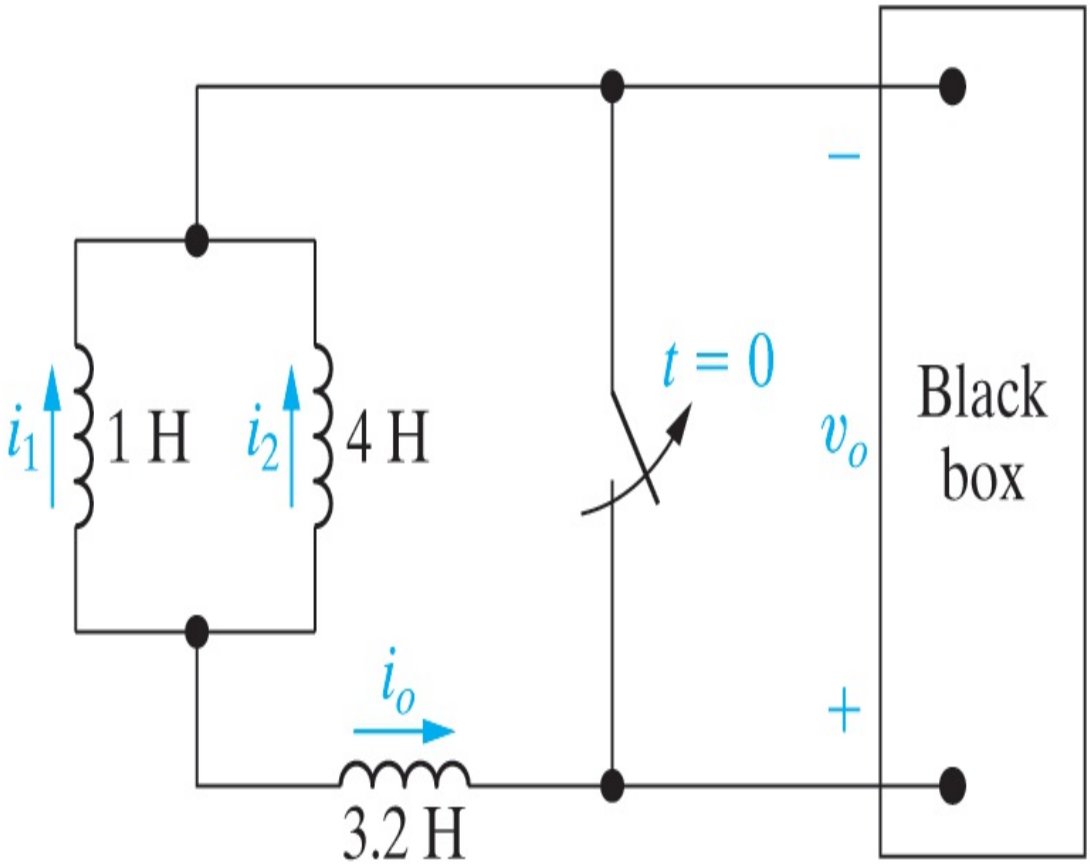


Figure P6.24 Full Alternative Text
vo=2000e−100t V.

If i1(0)=−6 A and i2(0)=1 A, find

1. io(0);

2. io(t), t≥0;
3. t1(t), t≥0;
4. i2(t), t≥0;
5. the initial energy stored in the three inductors;
6. the total energy delivered to the black box; and
7. the energy trapped in the ideal inductors.

4. 6.25 For the circuit shown in Fig. P6.24, how many milliseconds after the switch is opened is the energy delivered to the black box 80% of the total energy delivered?

5. 6.26 The two parallel inductors in Fig. P6.26 are connected across the terminals of a black box at t=0. The resulting voltage v for t>0 is known to be 12e−t V. It is also known that i1(0)=2 A and i2(0)=4 A.

    1. Replace the original inductors with an equivalent inductor and find i(t) for t≥0.

Figure P6.26

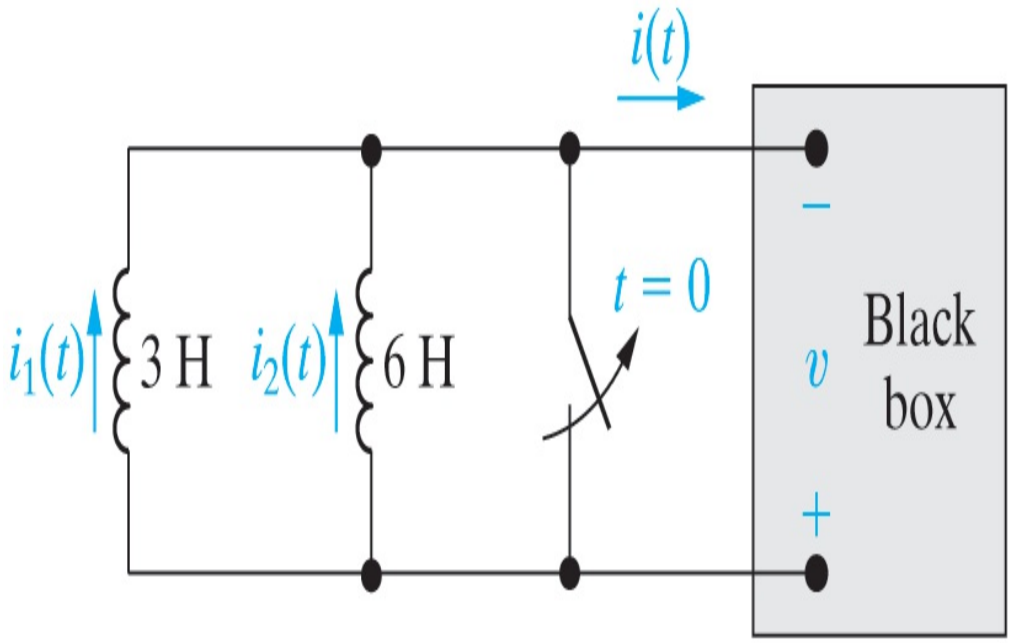


Figure P6.26 Full Alternative Text

2. Find i1(t) for t≥0.
3. Find i2(t) for t≥0.
4. How much energy is delivered to the black box in the time interval 0≤t<∞?
5. How much energy was initially stored in the parallel inductors?
6. How much energy is trapped in the ideal inductors?
7. Show that your solutions for i1 and i2 agree with the answer obtained in (f).

6. 6.27 Derive the equivalent circuit for a series connection of ideal capacitors. Assume that each capacitor has its own initial voltage. Denote these initial voltages as v1(t0), v2(t0), and so on. (*Hint*: Sum the voltages across the string of capacitors, recognizing that the series connection forces the current in each capacitor to be the same.)
7. 6.28 Derive the equivalent circuit for a parallel connection of ideal

capacitors. Assume that the initial voltage across the paralleled capacitors is v(t0). (*Hint*: Sum the currents into the string of capacitors, recognizing that the parallel connection forces the voltage across each capacitor to be the same.)

8. 6.29 Use realistic capacitor values from Appendix H to construct series and parallel combinations of capacitors to yield the equivalent capacitances specified below. Try to minimize the number of capacitors used. Assume that no initial energy is stored in any of the capacitors.

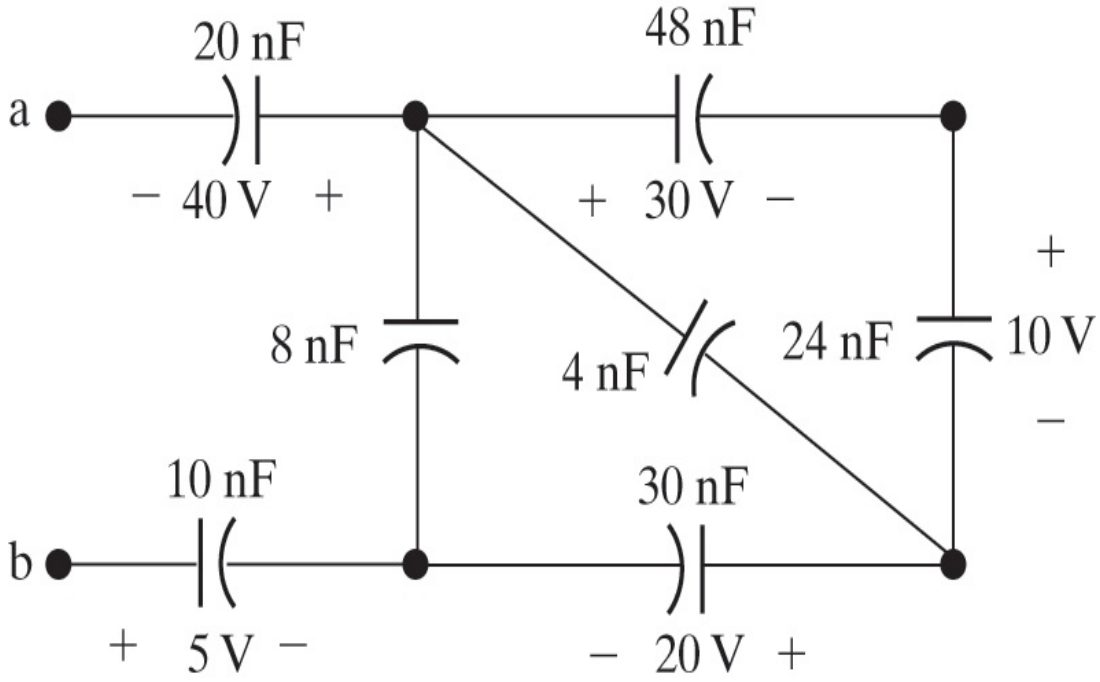
    1. 480 pF;

    2. 600 nF;

    3. 120 μF.

9. 6.30 Find the equivalent capacitance with respect to the terminals a, b for the circuits shown in Fig. P6.30.

Figure P6.30


20 nF
48 nF
a
− 40 V +
+ 30 V −
+
8 nF
4 nF
24 nF
10 V
−
10 nF
30 nF
b
+ 5 V −
− 20 V +

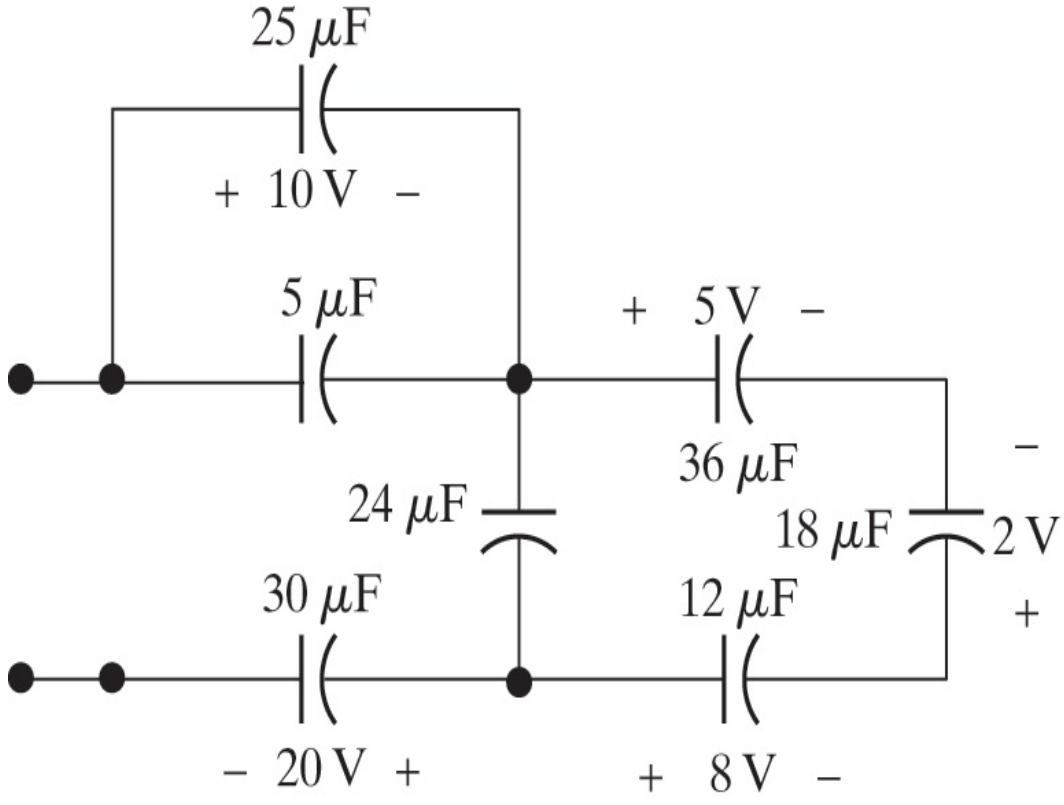

25 μF
+ 10 V −
5 μF
+ 5 V −
36 μF
−
24 μF
18 μF
2 V
+
30 μF
12 μF
− 20 V +
+ 8 V −

Figure P6.30 Full Alternative Text

10. 6.31 The two series-connected capacitors in Fig. P6.31 are connected to the terminals of a black box at t=0. The resulting current i(t) for t>0 is known to be 20e−t μA.

Figure P6.31

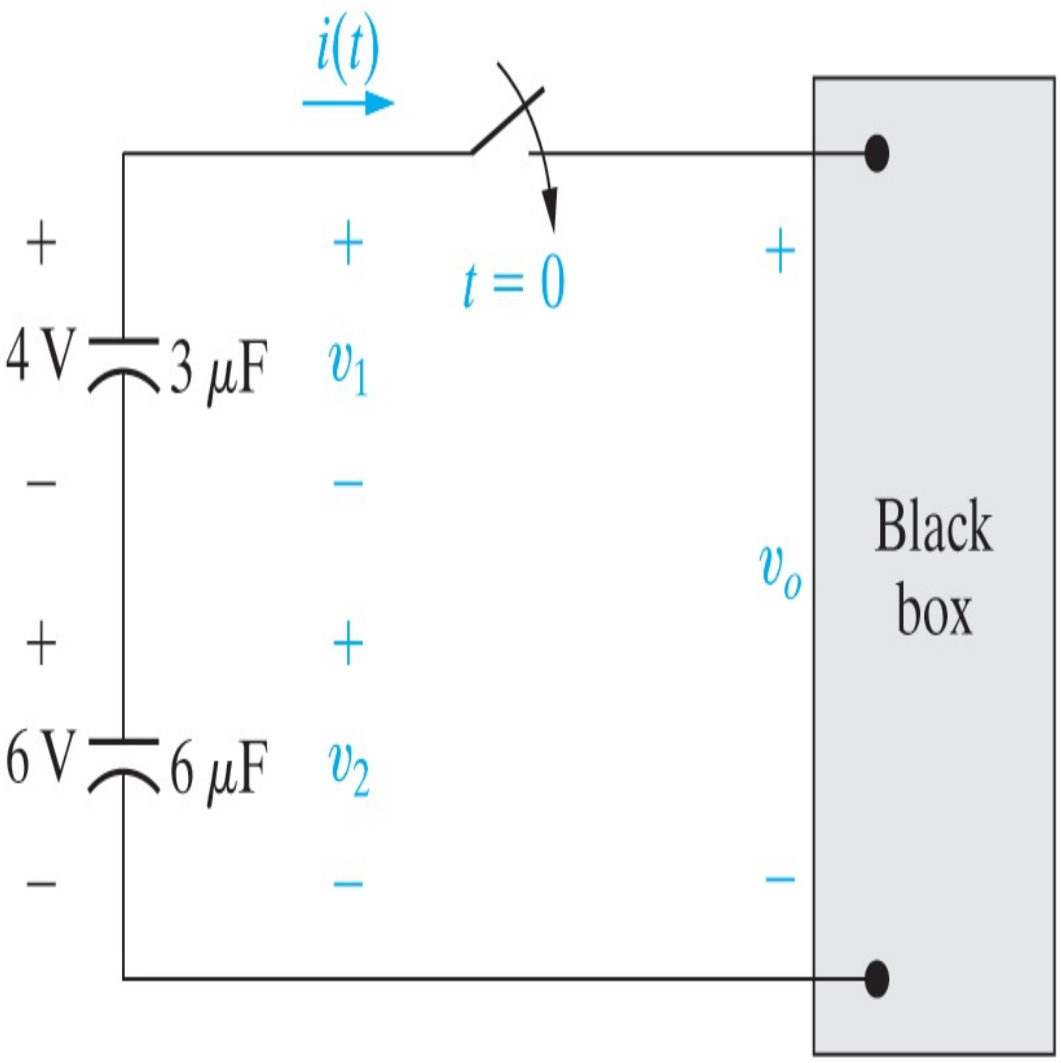


Figure P6.31 Full Alternative Text

1. Replace the original capacitors with an equivalent capacitor and

find vo(t) for t≥0.

2. Find v1(t) for t≥0.
3. Find v2(t) for t≥0.
4. How much energy is delivered to the black box in the time interval $0 \le t < \infty$?
5. How much energy was initially stored in the series capacitors?
6. How much energy is trapped in the ideal capacitors?
7. Show that the solutions for v1 and v2 agree with the answer obtained in (f).

11. 6.32 The four capacitors in the circuit in Fig. P6.32 are connected across the terminals of a black box at t=0. The resulting current ib for t>0 is known to be

Figure P6.32

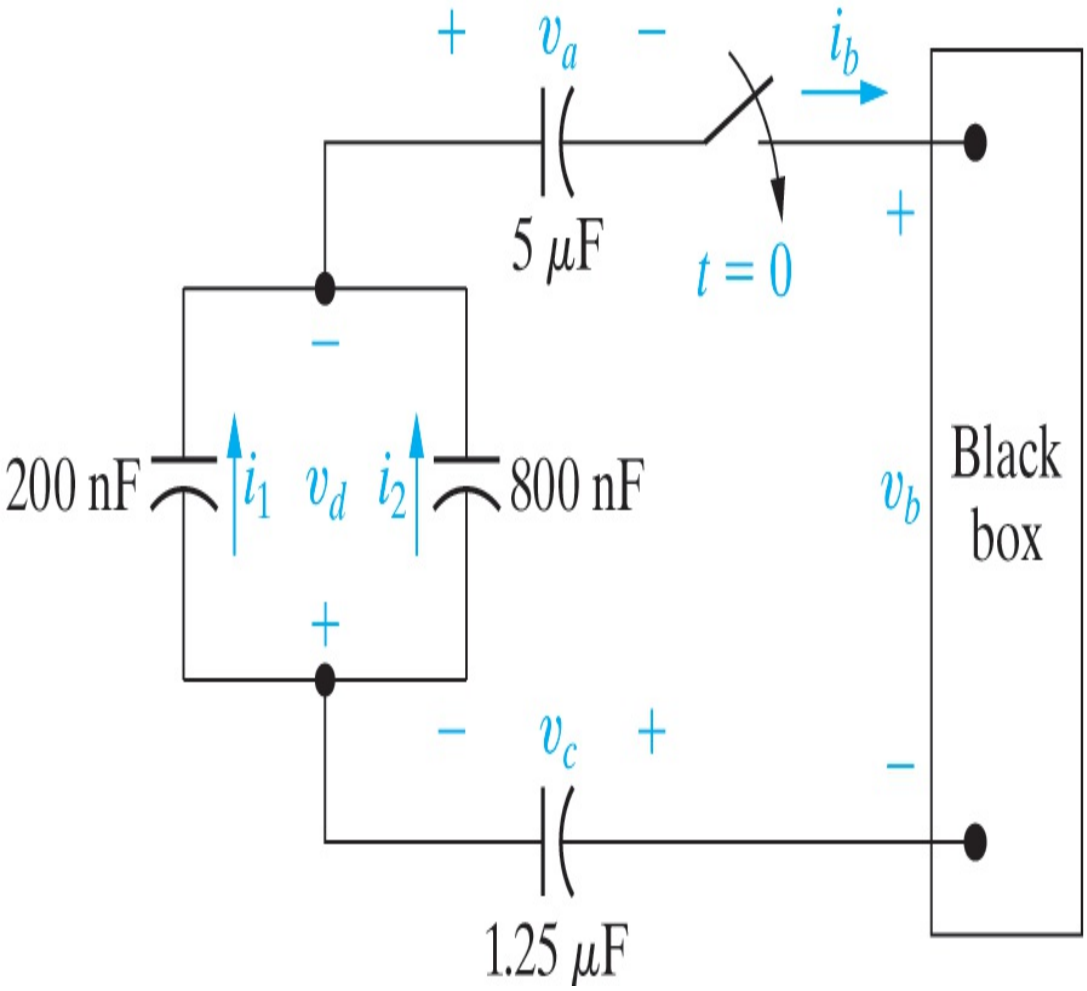


Figure P6.32 Full Alternative Text
ib=−5e−50t mA.

If va(0)=−20 V, vc(0)=−30 V, and vd(0)=250 V, find the following for t≥0: (a) vb(t), (b) va(t), (c) vc(t), (d) vd(t), (e) i1(t), and (f) i2(t).

12. 6.33 For the circuit in Fig. P6.32, calculate

    1. the initial energy stored in the capacitors;
    2. the final energy stored in the capacitors;
    3. the total energy delivered to the black box;
    4. the percentage of the initial energy stored that is delivered to the

black box; and

5. the time, in milliseconds, it takes to deliver 7.5 mJ to the black box.

13. 6.34 At t=0, a series-connected capacitor and inductor are placed across the terminals of a black box, as shown in Fig. P6.34. For t>0, it is known that

Figure P6.34

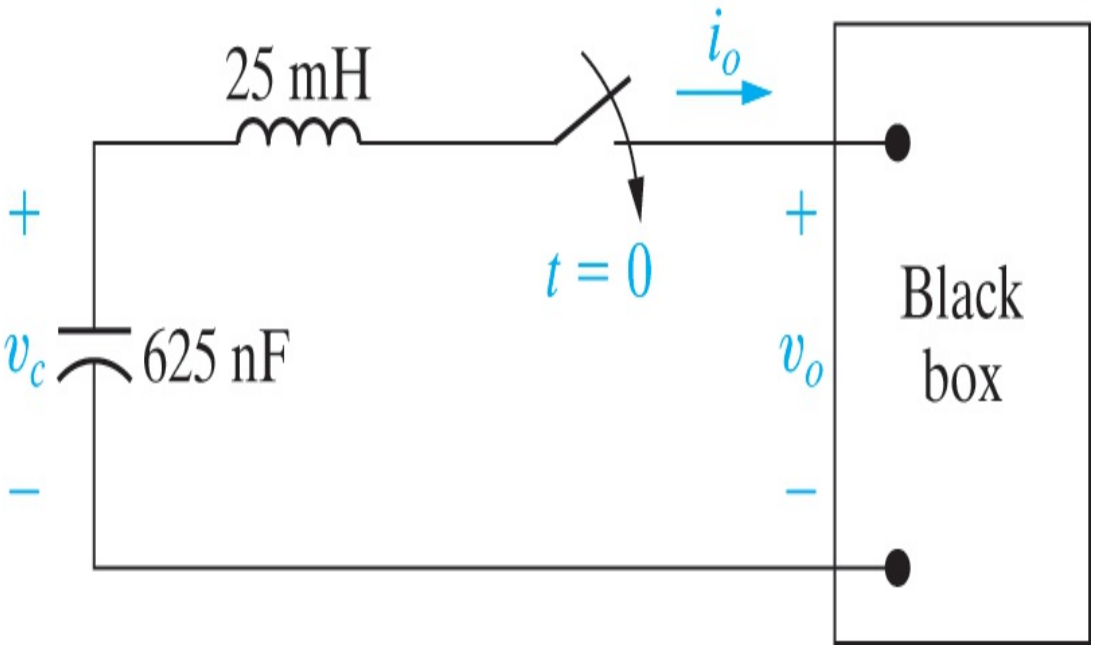


Figure P6.34 Full Alternative Text
io=1.5e−16,000t−0.5e−16,000t A
.

If vc(0)=−50 V find vo for t≥0.

14. 6.35 The current in the circuit in Fig. P6.35 is known to be

Figure P6.35

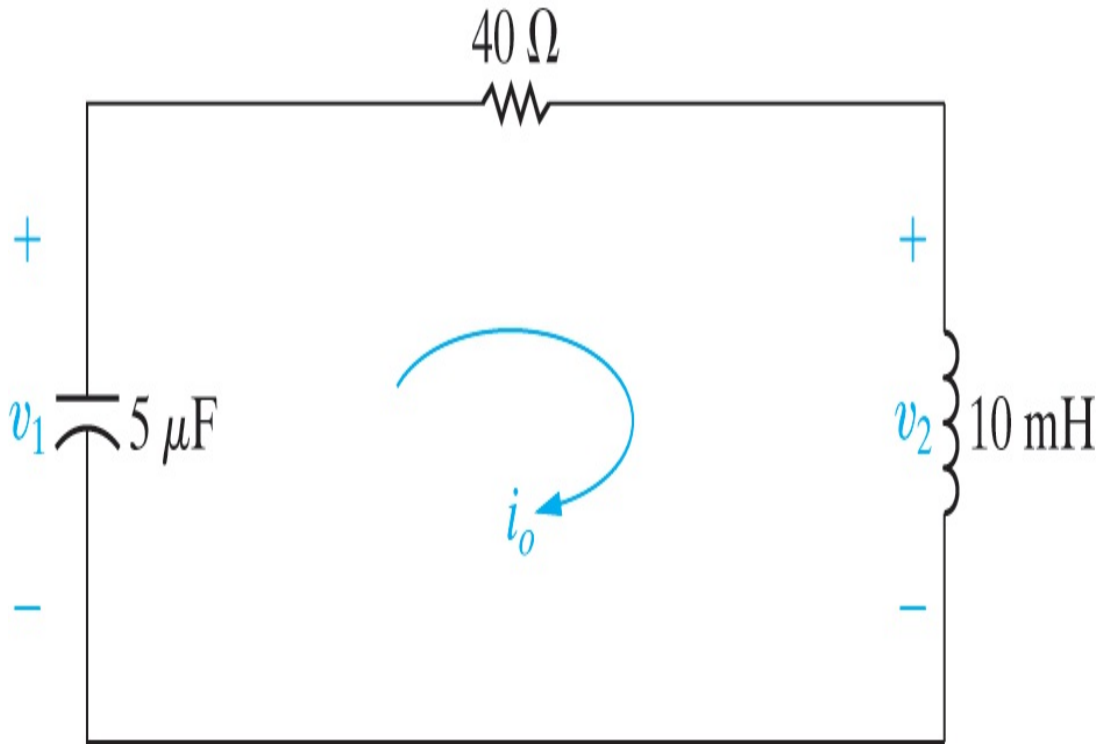


Figure P6.35 Full Alternative Text

io=5e−2000t(2cos 4000t+sin 4000t)

for t≥0+. Find v1(0+) and v2(0+).

Section 6.4

1. 6.36

    1. Show that the differential equations derived in (a) of Example 6.8 can be rearranged as follows:

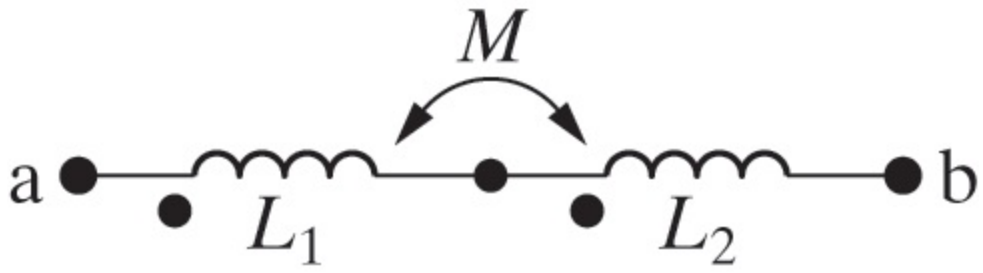
        $4\frac{di_1}{dt}+25i-8\frac{di_2}{dt}-20i_2=5i_g-8\frac{di_g}{dt}$;
        $-8\frac{di_1}{dt}-20i_1+16\frac{di_2}{dt}+80i_2=16\frac{di_g}{dt}$.

    2. Show that the solutions for i1, and i2 given in (b) of Example 6.8 satisfy the differential equations given in part (a) of this problem.

2. 6.37

    1. Show that the two coupled coils in Fig. P6.37 can be replaced by a single coil having an inductance of $L_{ab}=L_1+L_2+2M$. (*Hint:* Express vab as a function of iab.)

Figure P6.37



    2. Show that if the connections to the terminals of the coil labeled L2 are reversed, $L_{ab}=L_1+L_2-2M$.

3. 6.38

    1. Show that the two magnetically coupled coils in Fig. P6.38 can be

replaced by a single coil having an inductance of

Figure P6.38

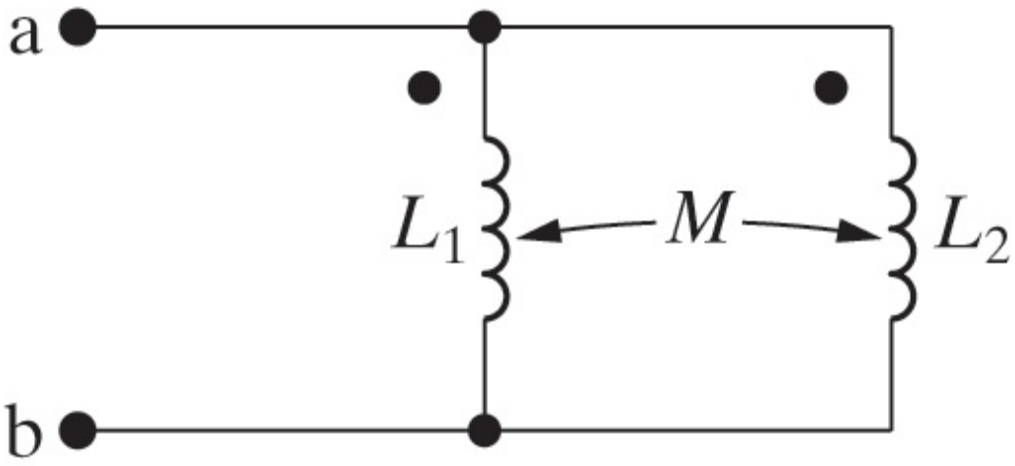


Figure P6.38 Full Alternative Text
Lab=L1L2−M2L1+L2−2M.

(*Hint:* Let i1 and i2 be clockwise mesh currents in the left and right "windows" of Fig. P6.38, respectively. Sum the voltages around the two meshes. In mesh 1 let vab be the unspecified applied voltage. Solve for di1/dt as a function of vab.)

2. Show that if the magnetic polarity of coil 2 is reversed, then

   Lab=L1L2−M2L1+L2+2M.

4. 6.39 Let vg represent the voltage across the current source in the circuit in Fig. 6.29. The reference for vg is positive at the upper terminal of the current source.

   1. Find vg as a function of time when ig=16−16e−51 A.

   2. What is the initial value of vg?

   3. Find the expression for the power developed by the current source.

4. How much power is the current source developing when t is infinite?

5. Calculate the power dissipated in each resistor when t is infinite.

5. 6.40 Let vo represent the voltage across the 16 H inductor in the circuit in Fig. 6.29. Assume vo is positive at the dot. As in Example 6.8, ig=16−16e−5t A.

    1. Can you find vo without having to differentiate the expressions for the currents? Explain.

    2. Derive the expression for vo.

    3. Check your answer in (b) using the appropriate current derivatives and inductances.

6. 6.41 There is no energy stored in the circuit in Fig. P6.41 at the time the switch is opened.

    1. Derive the differential equation that governs the behavior of i2 if L1=5 H, L2=0.2 H, M=0.5 H, and Ro=10 Ω.

Figure P6.41

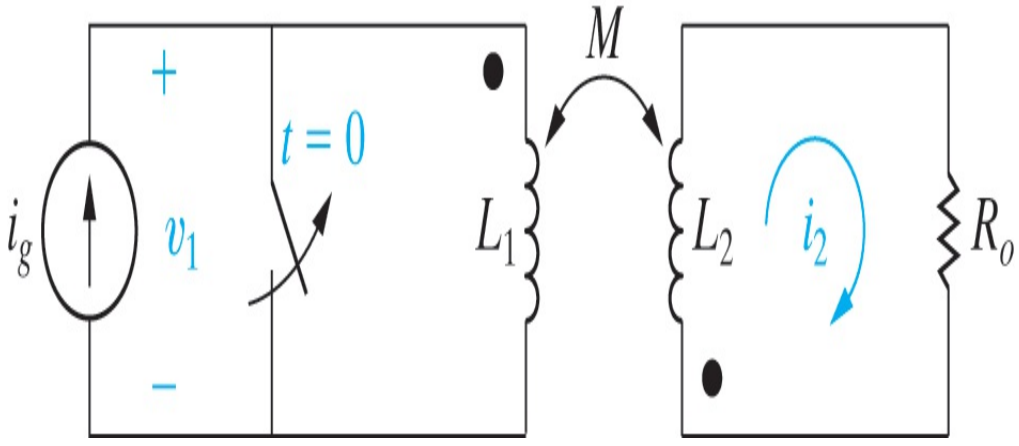


Figure P6.41 Full Alternative Text

2. Show that when ig=e−10t−10 A, t≥0, the differential equation derived in (a) is satisfied when i2=625e−10t−250e−50t mA, t≥0.

3. Find the expression for the voltage v1 across the current source.

4. What is the initial value of v1? Does this make sense in terms of known circuit behavior?

7. 6.42 The polarity markings on two coils are to be determined experimentally. The experimental setup is shown in Fig. P6.42. Assume that the terminal connected to the positive terminal of the battery has been given a polarity mark as shown. When the switch is *opened*, the dc voltmeter kicks downscale. Where should the polarity mark be placed on the coil connected to the voltmeter?

Figure P6.42

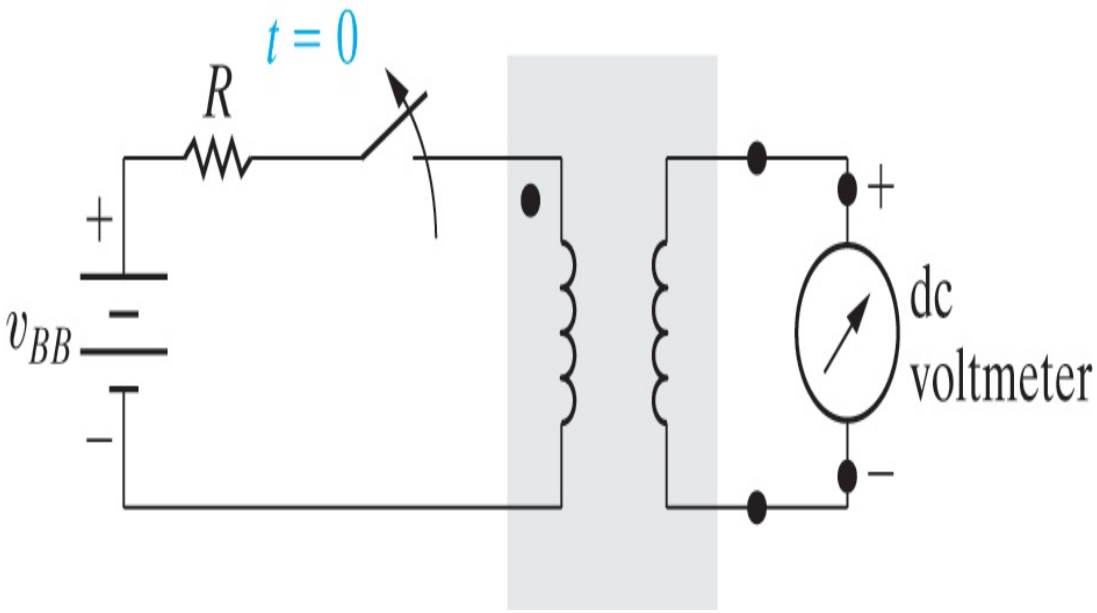


Figure P6.42 Full Alternative Text

8. 6.43 The physical construction of two pairs of magnetically coupled coils is shown in Fig. P6.43. Assume that the magnetic flux is confined to the core material in each structure. Show two possible locations for

the dot markings on each pair of coils.

Figure P6.43

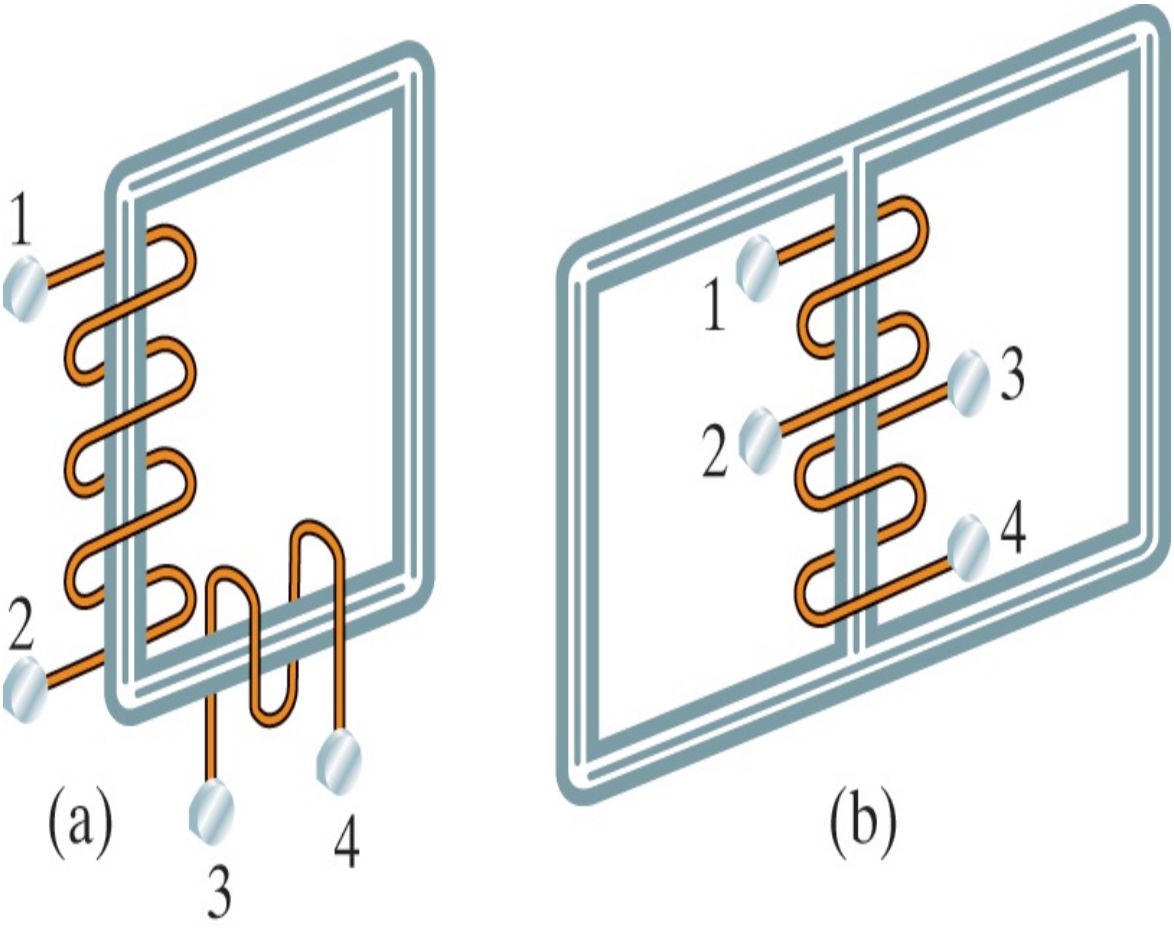


Figure P6.43 Full Alternative Text

Section 6.5

1. 6.44

    1. Starting with Eq. 6.26, show that the coefficient of coupling can also be expressed as

       $k=(\phi 21\phi 1)(\phi 12\phi 2)$.

    2. On the basis of the fractions $\phi 21/\phi 1$ and $\phi 12/\phi 2$, explain why k is less than 1.0.

2. 6.45 The self-inductances of the coils in Fig. 6.34 are L1=18 mH and L2=32 mH. If the coefficient of coupling is 0.85, calculate the energy stored in the system in millijoules when (a) i1=6 A, i2=9 A; (b) i1= −6 A, i2=−9 A; (c) i1=−6 A, i2=9 A; and (d) i1=6 A, i2=−9A.

3. 6.46 The coefficient of coupling in Problem 6.45 is increased to 1.0.

    1. If i1 equals 6 A, what value of i2 results in zero stored energy?

    2. Is there any physically realizable value of i2 that can make the stored energy negative?

4. 6.47 Two magnetically coupled coils are wound on a nonmagnetic core. The self-inductance of coil 1 is 288 mH, the mutual inductance is 90 mH, the coefficient of coupling is 0.75, and the physical structure of the coils is such that P11=P22.

    1. Find L2 and the turns ratio N1/N2.

    2. If N1=1200, what is the value of P1 and P2?

5. 6.48 Two magnetically coupled coils have self-inductances of 60 mH and 9.6 mH, respectively. The mutual inductance between the coils is 22.8 mH.

1. What is the coefficient of coupling?

2. For these two coils, what is the largest value that M can have?

3. Assume that the physical structure of these coupled coils is such that P1=P2. What is the turns ratio N1/N2 if N1 is the number of turns on the 60 mH coil?

6. 6.49 The self-inductances of two magnetically coupled coils are L1=180 μH and L2=500 μH. The coupling medium is nonmagnetic. If coil 1 has 300 turns and coil 2 has 500 turns, find P11 and P21 (in nanowebers per ampere) when the coefficient of coupling is 0.6.

7. 6.50 The self-inductances of two magnetically coupled coils are 72 mH and 40.5 mH, respectively. The 72 mH coil has 250 turns, and the coefficient of coupling between the coils is 23.. The coupling medium is nonmagnetic. When coil 1 is excited with coil 2 open, the flux linking only coil 1 is 0.2 as large as the flux linking coil 2.

    1. How many turns does coil 2 have?

    2. What is the value of P2 in nanowebers per ampere?

    3. What is the value of P11 in nanowebers per ampere?

    4. What is the ratio (ϕ22/ϕ12)?

Sections 6.1–6.5

1. 6.51 PRACTICAL PERSPECTIVE _ Suppose a capacitive touch screen that uses the mutual-capacitance design, as shown in Fig. 6.37, is touched at the point x, y. Determine the mutual capacitance at that point, C′mxy, in terms of the mutual capacitance at the point without a touch, Cmxy, and the capacitance introduced by the touch, Ct.

2. 6.52

PRACTICAL PERSPECTIVE _

1. Assume the parasitic capacitance in the self-capacitance design, Cp=30 pF, and the capacitance introduced by a touch is 15 pF (see Fig. 6.36[b]). What is the capacitance at the touch point with respect to ground for the x-grid and y-grid electrodes closest to the touch point?

2. Assume the mutual capacitance in the mutual- capacitance design, Cmxy=30 pF, and the capacitance introduced by a touch is 15 pF (see Fig. 6.37[b]). What is the mutual capacitance between the x- and y-grid electrodes closest to the touch point?

3. Compare your results in parts (a) and (b)—does touching the screen increase or decrease the capacitance in these two different capacitive touch screen designs?

3. 6.53

PRACTICAL PERSPECTIVE _

1. As shown in the Practical Perspective, the self-capacitance design does not permit a true multi-touch screen—if the screen is touched at two difference points, a total of four touch points are identified, the two actual touch points and two ghost points. If a self-capacitance touch screen is touched at the x, y coordinates (2.1, 4.3) and (3.2, 2.5), what are the four touch locations that will be identified? (Assume the touch coordinates are measured in inches from the upper left corner of the screen.)

2. A self-capacitance touch screen can still function as a multi-touch screen for several common gestures. For example, suppose at time t1 the two touch points are those identified in part (a), and at time t2 four touch points associated with the x, y coordinates (1.8, 4.9) and (3.9, 1.8) are identified. Comparing the four points at t1 with the four points at t2, software can recognize a pinch gesture—should the screen be zoomed in or zoomed out?

3. Repeat part (b), assuming that at time t2 four touch points associated with the x, y coordinates (2.8, 3.9) and (3.0, 2.8) are identified.

Chapter 7 Response of First-Order RL and RC Circuits

Chapter Contents

1. 7.1 The Natural Response of an RL Circuit
2. 7.2 The Natural Response of an RC Circuit
3. 7.3 The Step Response of RL and RC Circuits
4. 7.4 A General Solution for Step and Natural Responses
5. 7.5 Sequential Switching
6. 7.6 Unbounded Response
7. 7.7 The Integrating Amplifier

Chapter Objectives

1. Be able to determine the natural response of both RL and RC circuits.
2. Be able to determine the step response of both RL and RC circuits.
3. Know how to analyze circuits with sequential switching.
4. Be able to analyze op amp circuits containing resistors and a single capacitor.

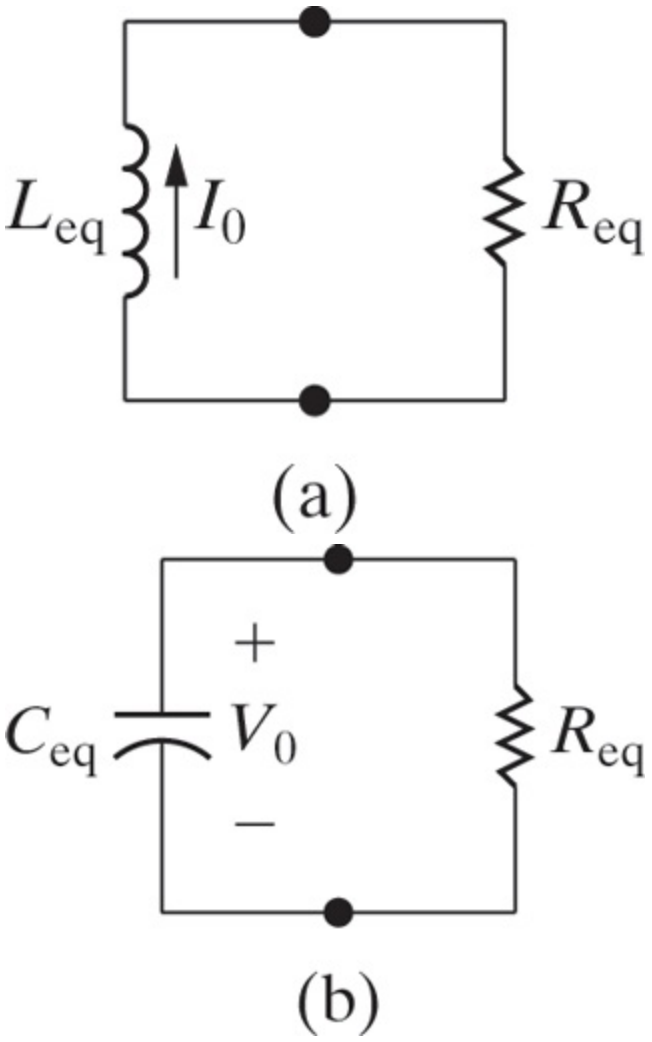
In this chapter, we focus on circuits that consist only of sources, resistors, and either (but not both) inductors or capacitors. We call these circuits *RL* (resistor-inductor) and *RC* (resistor-capacitor) **circuits**. In Chapter 6, we saw that inductors and capacitors can store energy. We analyze *RL* and *RC* circuits to determine the currents and voltages that arise when energy is either released or acquired by an inductor or capacitor in response to an abrupt change in a dc voltage or current source.

We divide our analysis of *RL* and *RC* circuits into three phases.

- First Phase: Find the currents and voltages that arise when stored energy in an inductor or capacitor is suddenly released to a resistive network. This happens when the inductor or capacitor is abruptly disconnected from its dc source. Thus, we can reduce the circuit to one of the two equivalent forms shown in Fig. 7.1 on page 222. These currents and voltages characterize the **natural response** of the circuit because the nature of the circuit itself, not external sources of excitation, determines its behavior.

Figure 7.1 The two forms of the circuits for natural

response. (a) *RL* circuit. (b) *RC* circuit.

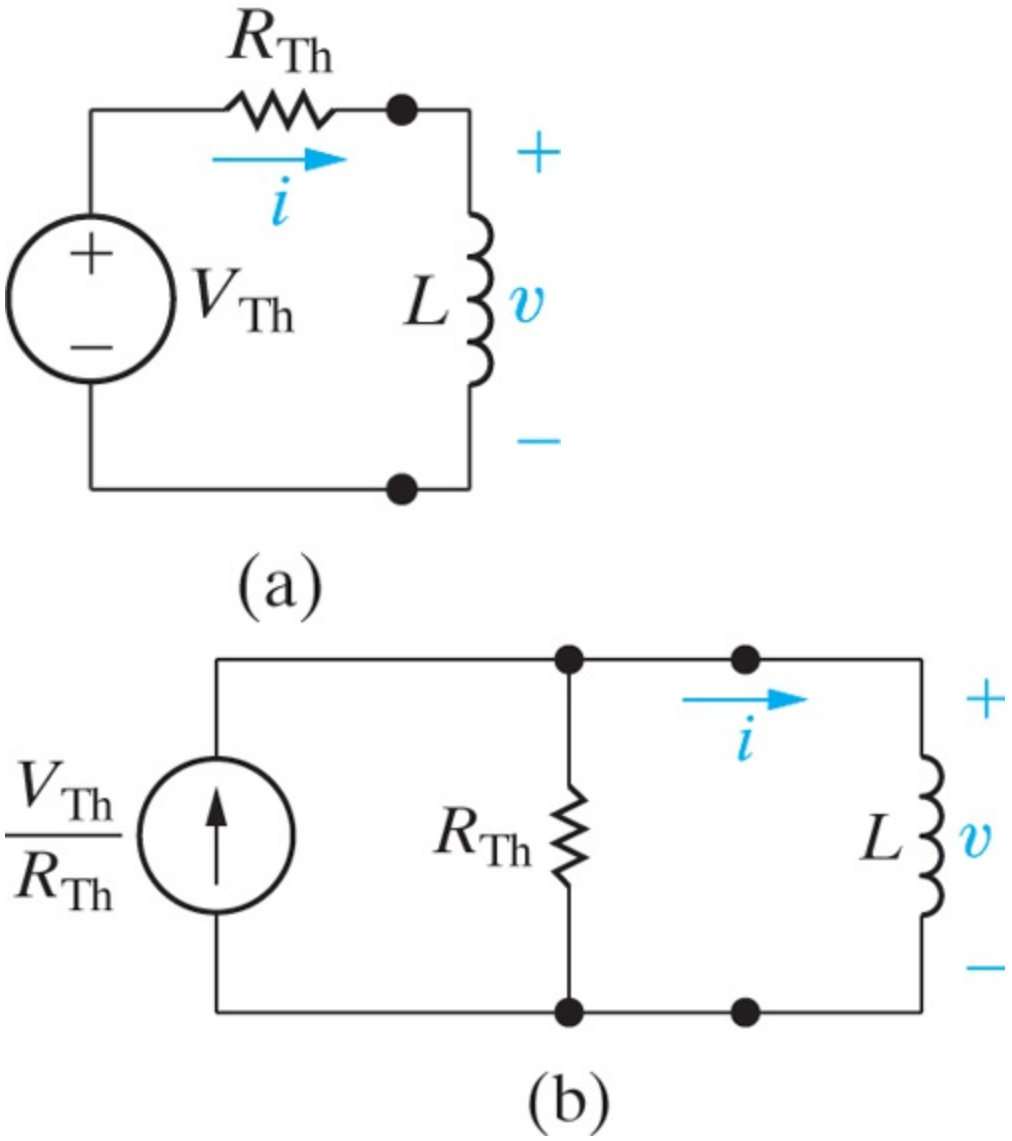


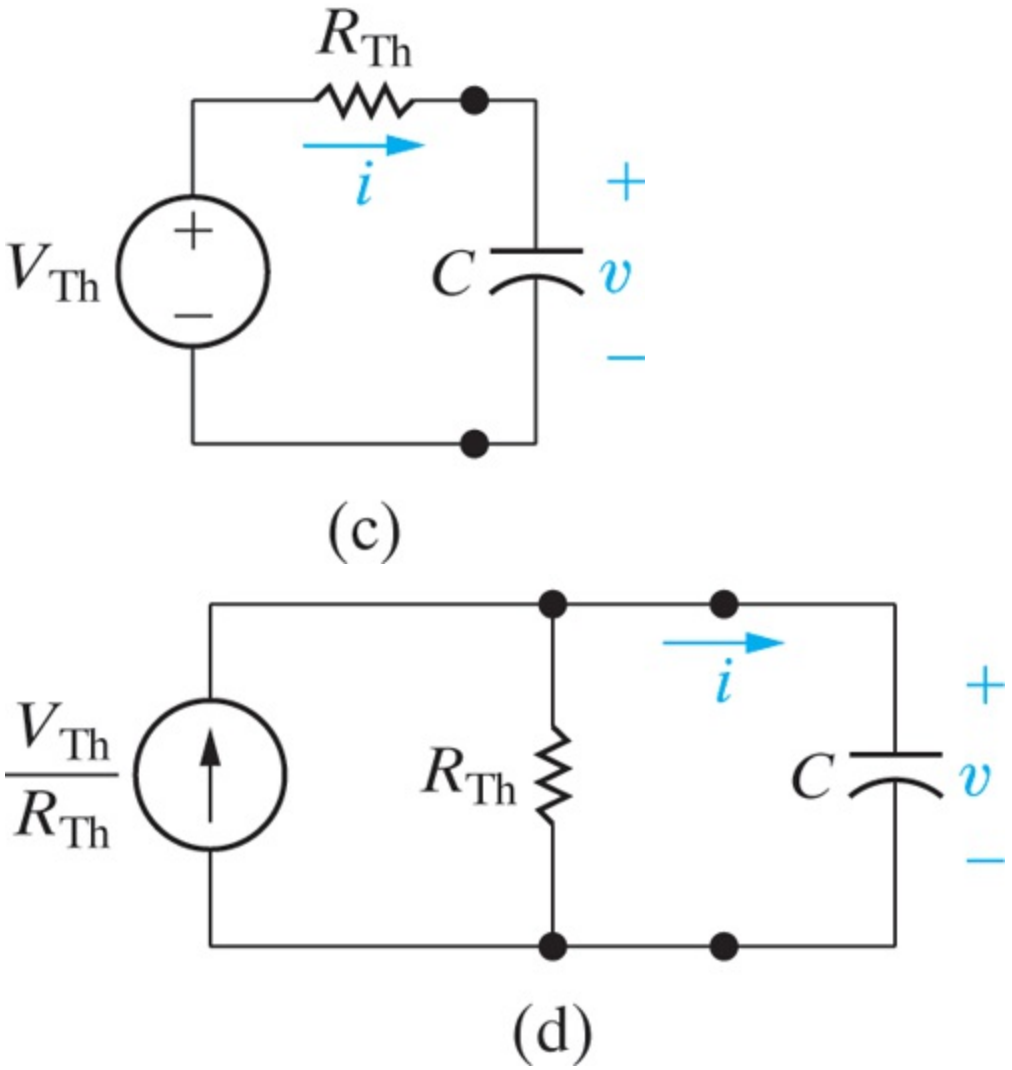
- Second Phase: Find the currents and voltages that arise when energy is being acquired by an inductor or capacitor when a dc voltage or current source is suddenly applied. This response is called the **step response**.

- Third Phase: Develop a general method for finding the response of *RL* and *RC* circuits to any abrupt change in a dc voltage or current source. A general method exists because the process for finding both the natural and step responses is the same.

Figure 7.2 (page 222) shows the four general configurations of *RL* and *RC* circuits. Note that when there are no independent sources in the circuit, the Thévenin voltage or Norton current is zero, and the circuit reduces to one of those shown in Fig. 7.1; that is, we have a natural-response problem.

Figure 7.2 Four possible first-order circuits.





(a) An inductor connected to a Thévenin equivalent.

(b) An inductor connected to a Norton equivalent.

(c) A capacitor connected to a Thévenin equivalent.

(d) A capacitor connected to a Norton equivalent.

Figure 7.2 Full Alternative Text

RL and *RC* circuits are also known as **first-order circuits** because their voltages and currents are described by first-order differential equations. No matter how complex a circuit may appear, if it can be reduced to a Thévenin

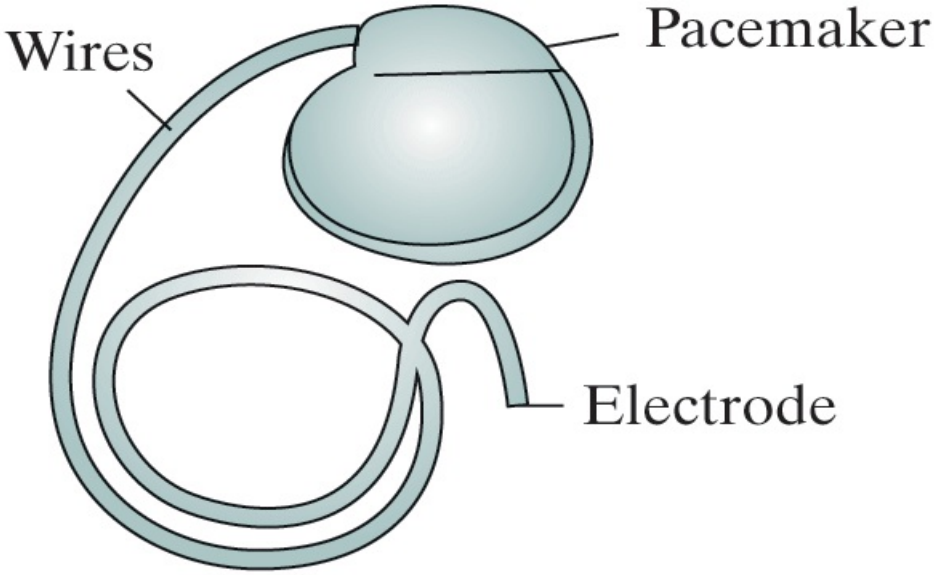
or Norton equivalent connected to the terminals of an equivalent inductor or capacitor, it is a first-order circuit. If the original circuit has two or more inductors or capacitors, they must be interconnected so that they can be replaced by a single equivalent element.

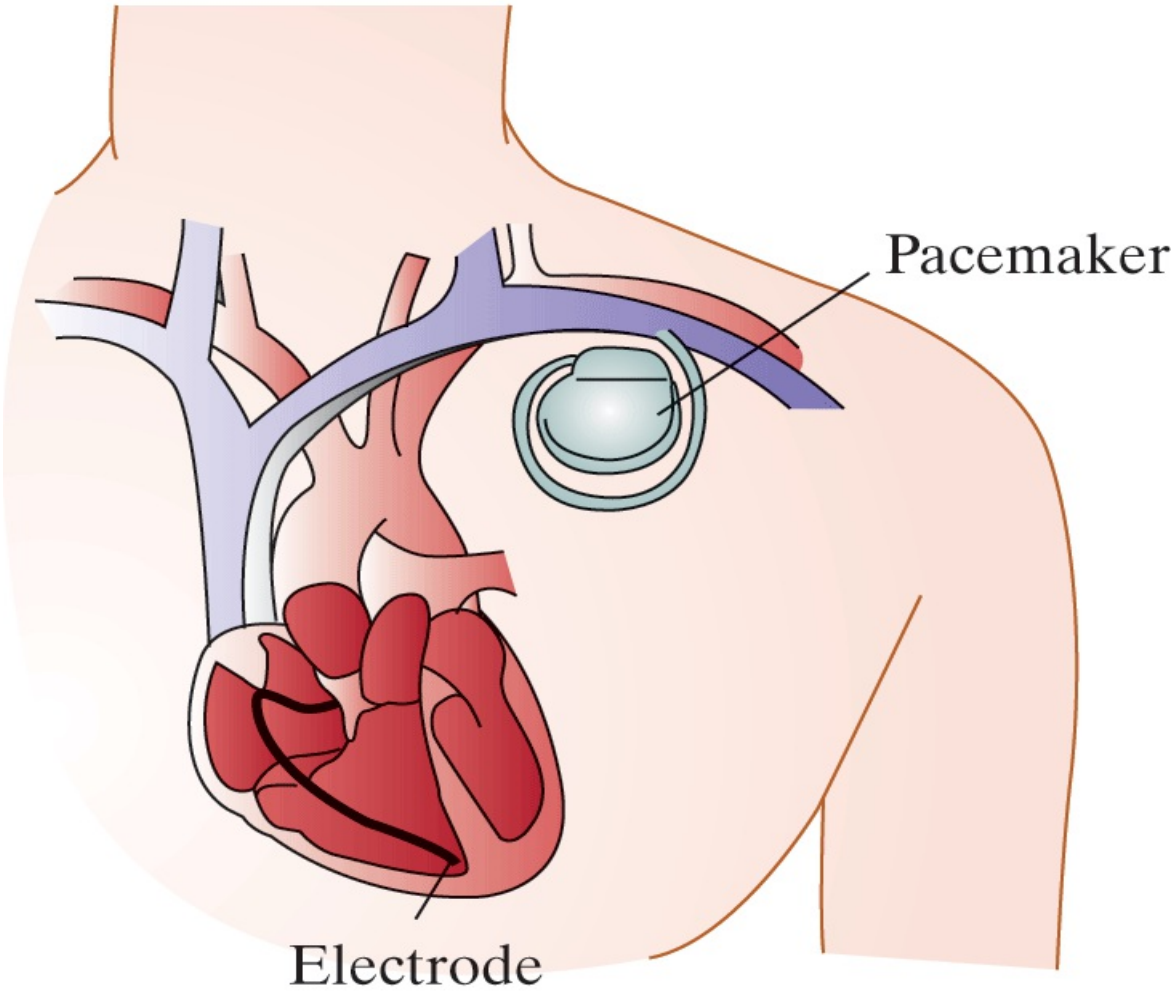
Practical Perspective

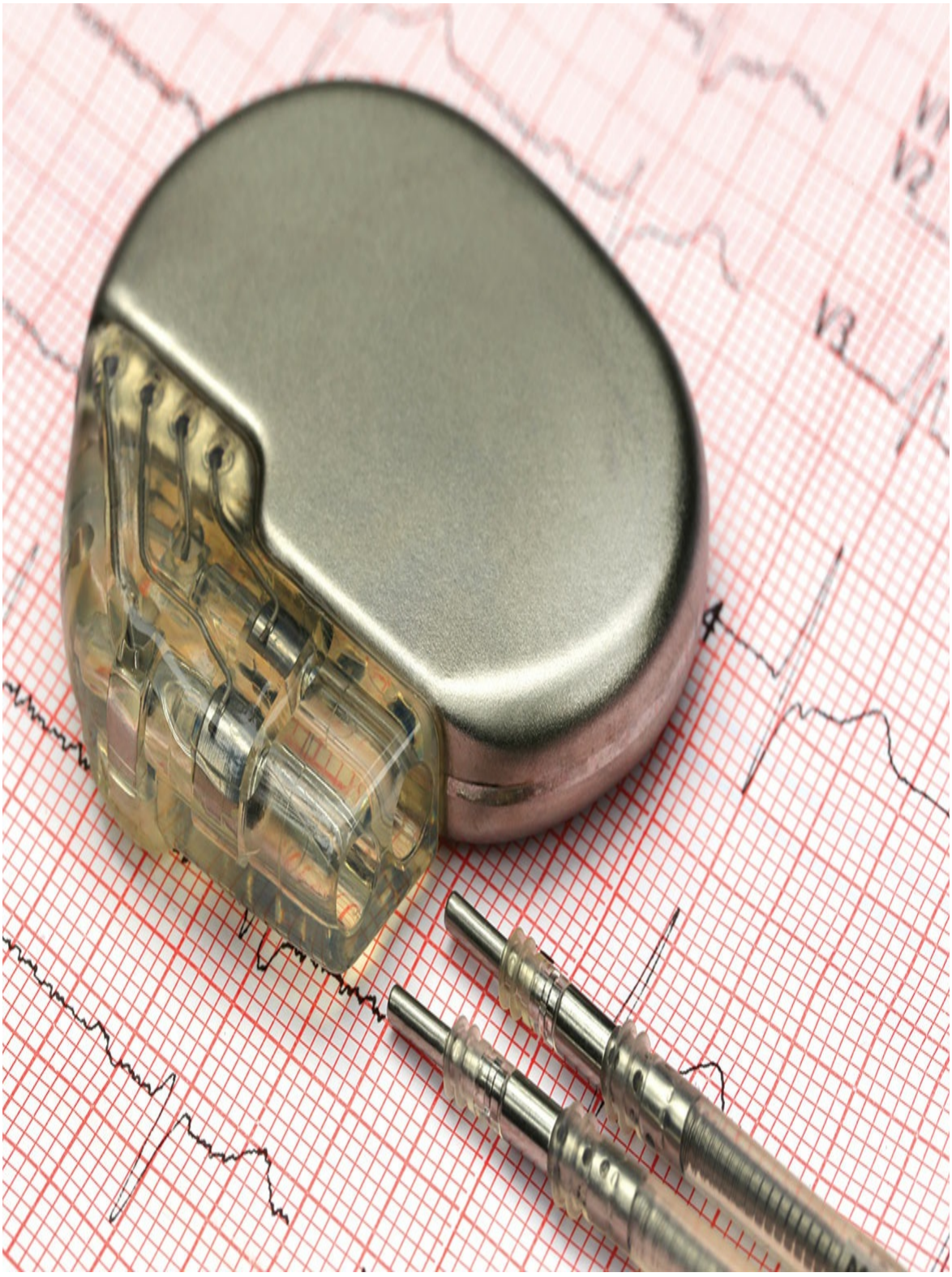
Artificial Pacemaker

The muscle that makes up the heart contracts due to rhythmical electrical impulses. Pacemaker cells control the impulse frequency. In adults, the pacemaker cells establish a resting heart rate of about 72 beats per minutes. Sometimes, however, damaged pacemaker cells produce a very low resting heart rate (a condition known as bradycardia) or a very high resting heart rate (a condition known as tachycardia). When either happens, a normal heart rhythm can be restored by implanting an artificial pacemaker that mimics the pacemaker cells by delivering electrical impulses to the heart. Examples of internal and external artificial pacemakers are shown in the figures below.

Artificial pacemakers are very small and lightweight. They have a programmable microprocessor that adjusts the heart rate based on several parameters, an efficient battery with a life of up to 15 years, and a circuit that generates the pulse. The simplest circuit consists of a resistor and a capacitor. After we introduce and study the RC circuit, we will look at an RC circuit design for an artificial pacemaker.


Wires
Pacemaker
Electrode


Pacemaker
Electrode



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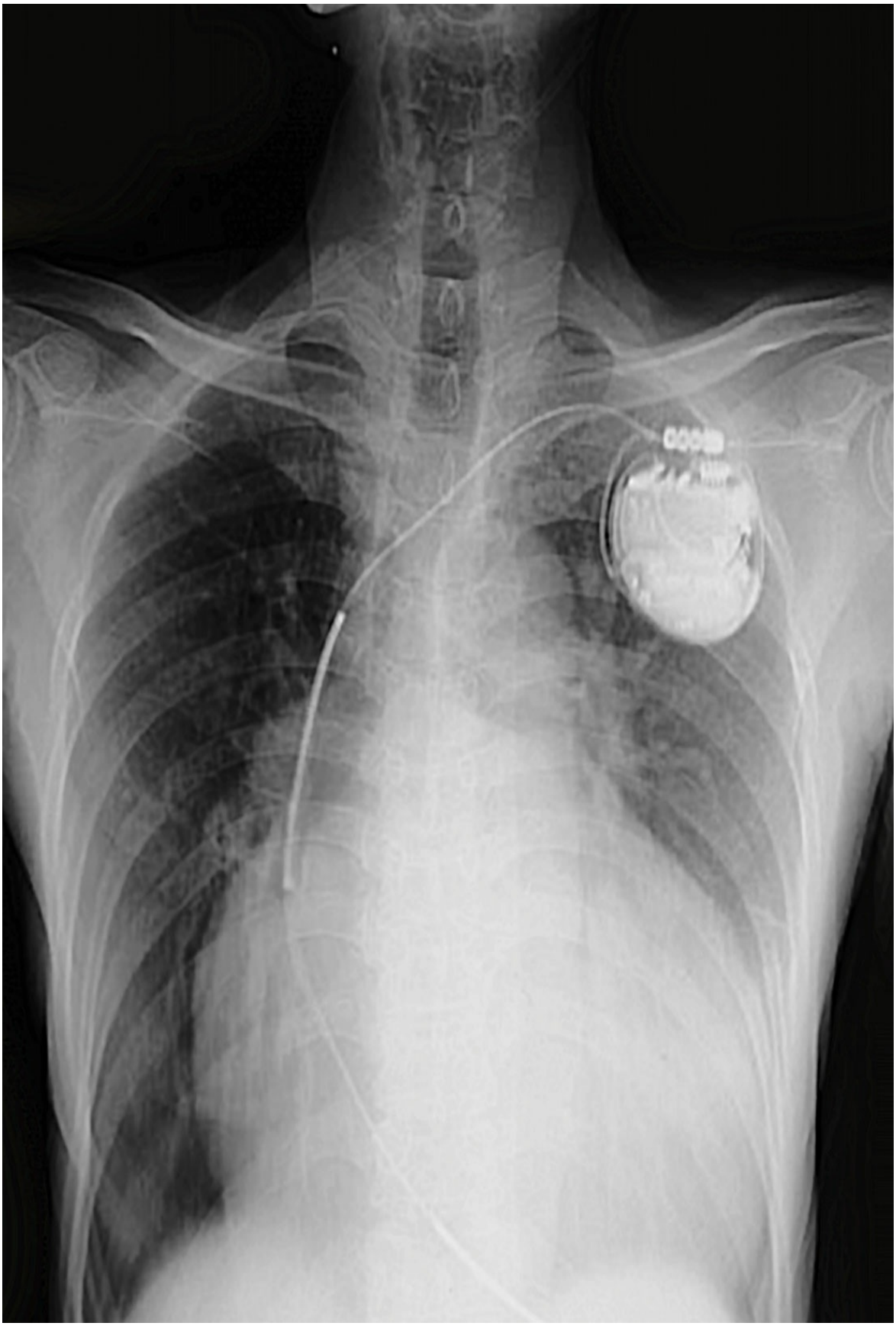

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After introducing the techniques for analyzing the natural and step responses of first-order circuits, we will discuss three special cases:

- Sequential switching (circuits in which switching occurs at two or more instants in time);
- Circuits with an unbounded response;
- The integrating amplifier circuit (containing an ideal op amp).

7.1 The Natural Response of an *RL* Circuit

We can describe the natural response of an *RL* circuit using the circuit shown in Fig. 7.3. We assume that the independent current source generates a constant current Is and that the switch has been in a closed position for a long time. We define the phrase *a long time* more accurately later in this section. For now it means that all currents and voltages have reached a constant value. Thus only constant, or dc, currents exist in the circuit just before the switch opens, and the voltage across the inductor is zero (Ldi/dt=0). Therefore, before the stored energy is released,

Figure 7.3 An RL circuit.

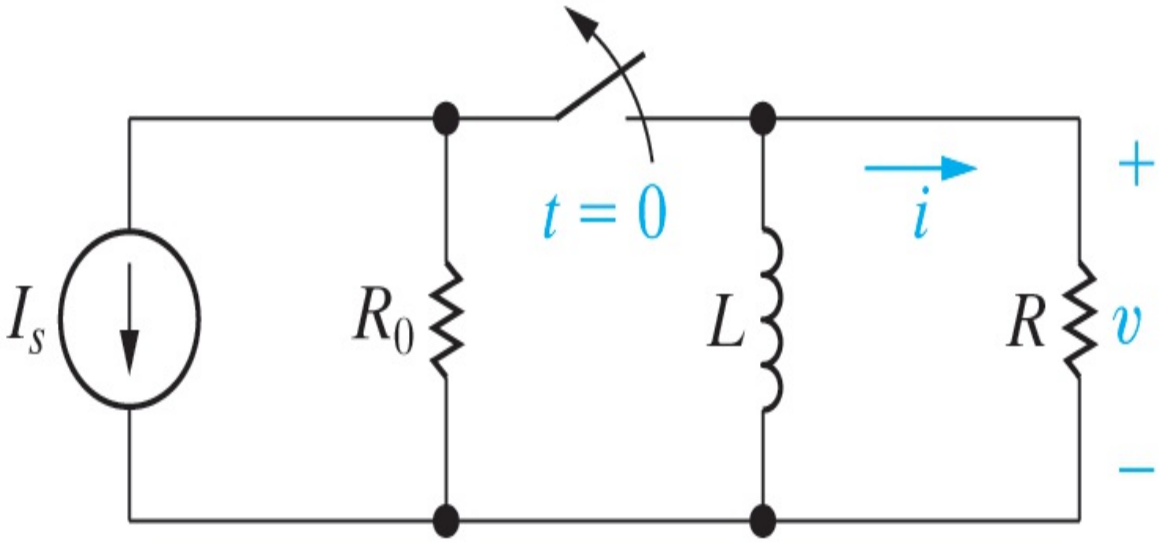


Figure 7.3 Full Alternative Text

- The inductor behaves like a short circuit;
- The entire source current Is appears in the inductive branch; and

- There is no current in either R0 or R.

To find the natural response, we find the voltage and current at the terminals of the resistor R after the switch has been opened—that is, after the source and its parallel resistor R0 have been disconnected and the inductor begins releasing energy. If we let t=0 denote the instant when the switch is opened, we find v(t) and i(t) for t≥0. For t≥0, the circuit shown in Fig. 7.3 reduces to the one shown in Fig. 7.4.

Figure 7.4 The circuit shown in Fig. 7.3, for t≥0.

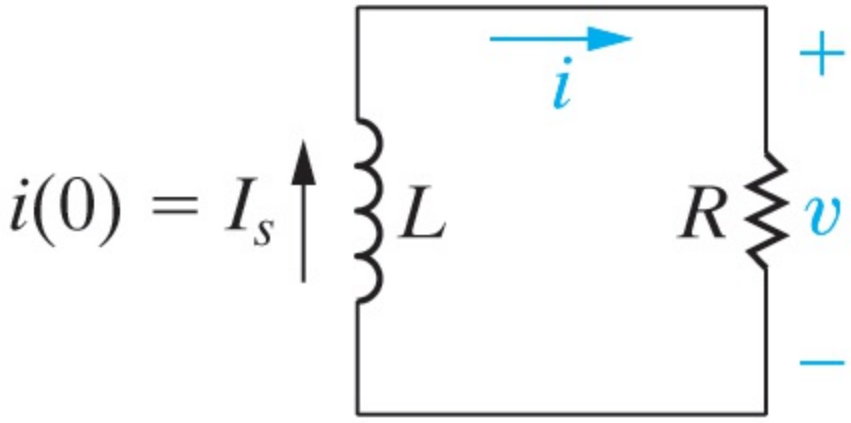


Figure 7.4 Full Alternative Text

Deriving the Expression for the Current

To find i(t), we write an expression involving i, R, and L for the circuit in Fig. 7.4 using Kirchhoff's voltage law. Summing the voltages around the closed loop gives

Ldidt+Ri=0,(7.1)

where we used the passive sign convention. Equation 7.1 is a **first-order** ordinary differential equation because it involves the ordinary derivative of the unknown, di/dt, and the highest order derivative appearing in the equation is 1.

We can go one step further in describing this equation. The coefficients in the equation, R and L, are constants; that is, they are not functions of either the dependent variable i or the independent variable t. Thus, the equation can also be described as an ordinary differential equation with constant coefficients.

To solve Eq. 7.1, we divide by L, move the term involving i to the right-hand side, and then multiply both sides by a differential time dt. The result is

didtdt=−RLi dt.

Next, we recognize the left-hand side of this equation simplifies to a differential change in the current i, that is, di. We now divide through by i, getting

dii=−RLdt.

We obtain an explicit expression for i as a function of t by integrating both sides. Using x and y as variables of integration yields

∫i(t0)i(t)dxx=−RL∫t0tdy,

where i(t0) is the current at time t0 and i(t) is the current at time t. Here, t0=0. Therefore, carrying out the indicated integration gives

lni(t)i(0)=−RLt.

Based on the definition of the natural logarithm, we can solve for the current to get

i(t)=i(0)e−(R/L)t.

Recall from Chapter 6 that the inductor current cannot change instantaneously. Therefore, in the first instant after the switch has been

opened, the current in the inductor remains unchanged. If 0− is the time just prior to switching and 0+ is the time immediately following switching, then

Initial Inductor Current

$$i(0-)=i(0+)=I0. \quad (7.2)$$

I0 is the inductor's initial current, as in Fig. 7.1(a), and has the same direction as the reference direction of i. Hence, the equation for the current becomes

$$i(t)=I0e-(R/L)t, \quad t\geq 0.$$

Figure 7.5 shows this response, where the current has an initial value I0 and decreases exponentially toward zero as t increases.

Figure 7.5 The current response for the circuit shown in Fig. 7.4.

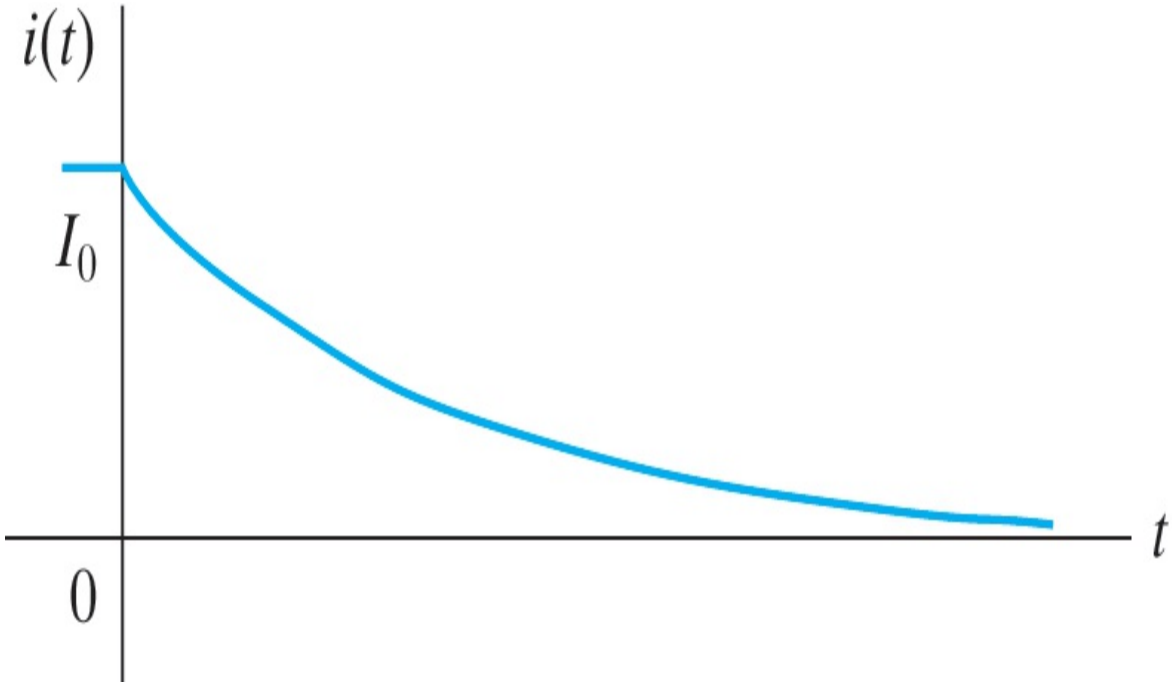


Figure 7.5 Full Alternative Text

Note that the expression for i(t) includes the term e−(R/L)t. The coefficient of t—namely, R/L—determines the rate at which the current approaches zero. The reciprocal of this coefficient is the **time constant** of the circuit, denoted

Time Constant for RL Circuit

τ=LR.(7.3)

Using the time constant, we write the expression for current as

Natural Response of an RL Circuit

i(t)=I0e−t/τ, t≥0,(7.4)

Now we have a step-by-step method for finding the natural response of an RL circuit.

1. Step 1: Determine the initial current, I0, in the inductor. This usually involves analyzing the circuit for $t<0$.

2. Step 2: Calculate the time constant, τ. To do this, you need to find the equivalent resistance attached to the inductor for $t\geq 0$.

3. Step 3: Write the equation for the inductor current for $t\geq 0$ by substituting the values for the initial current and the time constant into Eq. 7.4.

4. Step 4: Calculate any other quantities of interest, such as resistor current and voltage, using resistive circuit analysis techniques.

This method is summarized in Analysis Method 7.1 and is illustrated in Example 7.1.

RL Natural-Response Method

1. **Determine the initial inductor current,** I0, by analyzing the circuit for $t<0$

2. **Calculate the time constant,** $\tau=L/R$, where R is the equivalent resistance connected to the inductor for $t\geq 0$.

3. **Write the equation for the inductor current**, $i(t)=I0e^{-t/\tau}$, for $t\geq 0$

4. **Calculate other quantities of interest** using the inductor current.

Analysis Method 7.1 Finding the RL natural response.

Example 7.1 Determining the Natural Response of an RL Circuit

The switch in the circuit shown in Fig. 7.6 has been closed for a long time before it is opened at t=0. Find

Figure 7.6 The circuit for Example 7.1.

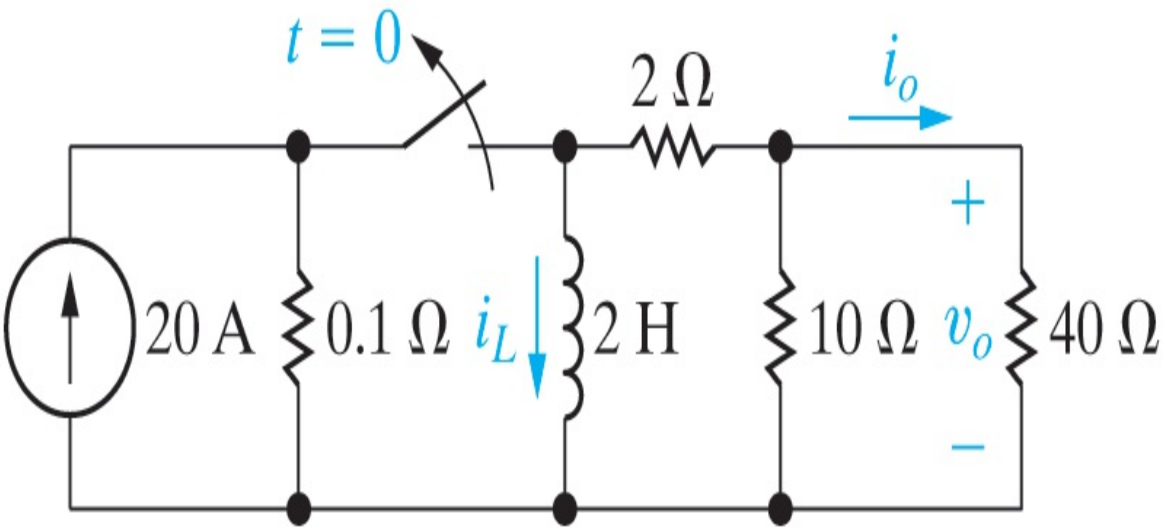


Figure 7.6 Full Alternative Text

1. iL(t) for t≥0,
2. io(t) for t≥0+,
3. vo(t) for t≥0+,
4. the percentage of the total energy stored in the 2 H inductor that is dissipated in the 10 Ω resistor.

Solution

Use Analysis Method 7.1.

1.
    1. Step 1: To determine the initial current in the inductor, draw the

circuit in Fig. 7.6 for t<0. The switch has been closed for a long time prior to t=0, so we know the inductor voltage is zero at t=0– and the inductor can be replaced by a short circuit. The result is shown in Fig. 7.7. The short circuit shunts all of the resistors, so it has all of the current from the source. Therefore, the current in the inductor at t=0– is 20 A and

Figure 7.7 The circuit for Example 7.1 when t<0.

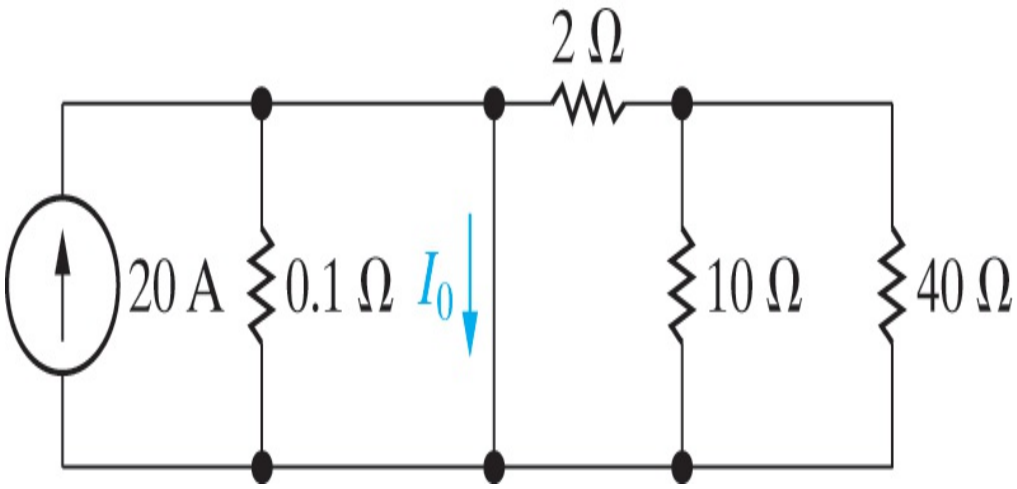


Figure 7.7 Full Alternative Text
I0=iL(0−)=iL(0+)=20A.

2. Step 2: To calculate the time constant, τ, we need to find the equivalent resistance attached to the inductor when t≥0. To do this, draw the circuit in Fig. 7.6 for t≥0. Since the switch is now open, the current source and its parallel resistor are removed from the circuit, as shown in Fig. 7.8. From this circuit you can see that the equivalent resistance attached to the inductor is

Figure 7.8 The circuit for

Example 7.1 when t≥0.

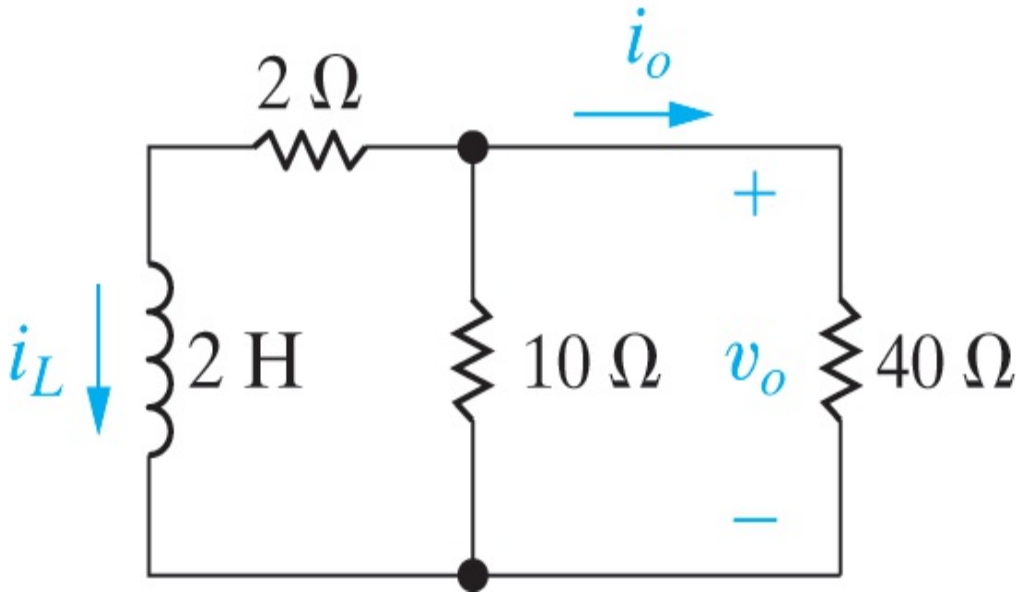


Figure 7.8 Full Alternative Text
Req=2+(40 || 10)=10 Ω

and the time constant of the circuit is

τ=LReq=210=0.2s.

3. Step 3: Write the equation for the inductor current by substituting the values for the initial current and the time constant into Eq. 7.4 to give

   iL(t)=I0e−t/τ=20e−t/0.2=20e−5tA, t≥0.

4. Step 4: We use resistive circuit analysis in the remaining parts of this problem to find additional currents and voltages.

2. We find the current in the 40 Ω resistor in Fig. 7.8 using current division; that is,

   io=−iL1010+ 40.

Note that this expression is valid for $t \geq 0+$ because $i_o = 0$ at $t = 0–$, so the resistor current i_o changes instantaneously. Thus,

$$i_o(t) = -0.25 i_L(t) = -4e^{-5t} A, \quad t \geq 0+.$$

3. We find the voltage v_o in Fig. 7.8 by applying Ohm's law:

$$v_o(t) = 40 i_o = -160e^{-5t} \text{ V}, \quad t \geq 0+.$$

4. The power dissipated in the 10 Ω resistor in Fig. 7.8 is

$$p_{10\Omega}(t) = v_o^2 10 = 2560e^{-10t} \text{ W}, \quad t \geq 0+.$$

The total energy dissipated in the 10 Ω resistor is

$$w_{10\Omega}(t) = \int_0^\infty 2560e^{-10t}\, dt = 256 \text{ J}.$$

The initial energy stored in the 2 H inductor is

$$w(0) = \frac{1}{2} L i_L^2(0) = \frac{1}{2}(2)(20)^2 = 400 \text{ J}.$$

Therefore, the percentage of energy dissipated in the 10 Ω resistor is

$$\frac{256}{400}(100) = 64\%.$$

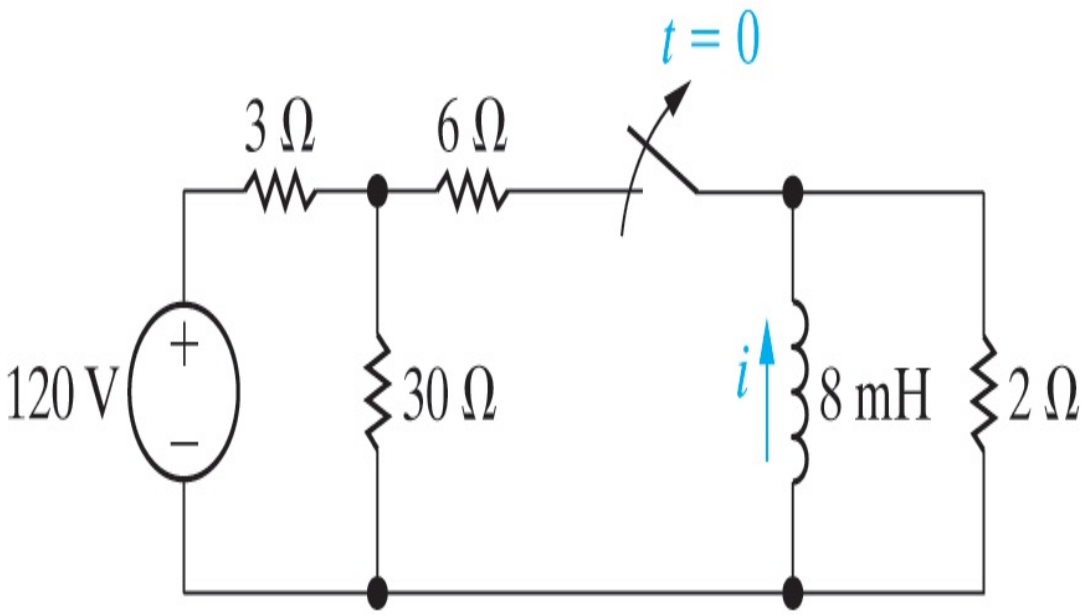
Assessment Problems

Objective 1—Be able to determine the natural response of both RL and RC circuits

1. 7.1 The switch in the circuit shown has been closed for a long time and is opened at t=0.
    1. a) Calculate the initial value of i.
    2. b) Calculate the initial energy stored in the inductor.
    3. c) What is the time constant of the circuit for t>0?

4. d) What is the numerical expression for i(t) for t≥0?

5. e) What percentage of the initial energy stored has been dissipated in the 2 Ω resistor 5 ms after the switch has been opened?



7.1-4 Full Alternative Text

Answer:

1. a) −12.5A;

2. b) 625 mJ;

3. c) 4 ms;

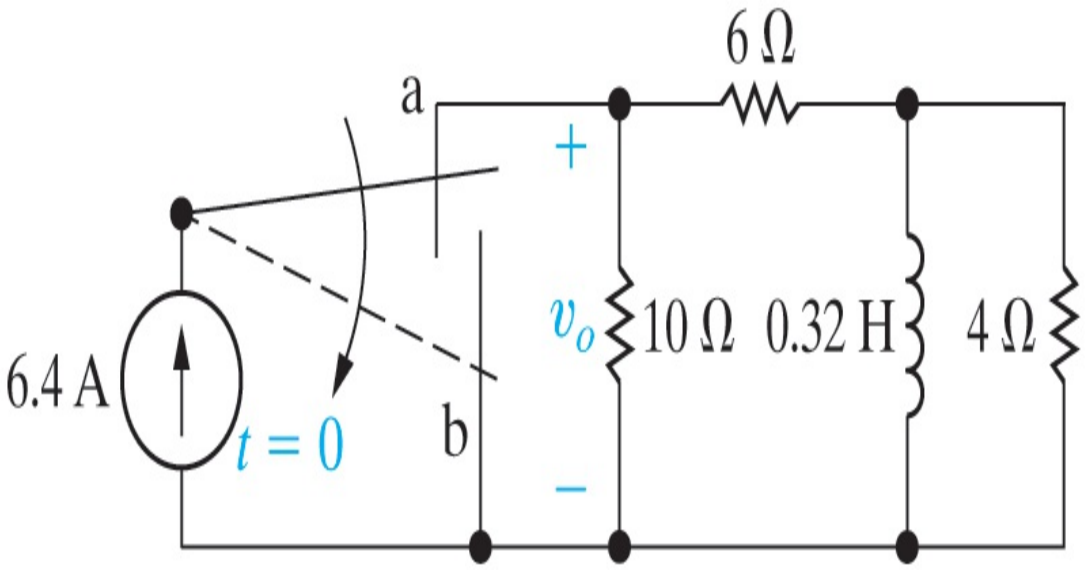
4. d) −12.5e−250tA, t≥0;

5. e) 91.8%.

2. 7.2 At t=0, the switch in the circuit shown moves instantaneously from position a to position b.

    1. a) Calculate vo for t≥0+.

2. b) What percentage of the initial energy stored in the inductor is eventually dissipated in the 4 Ω resistor?



7.1-5 Full Alternative Text

Answer:

1. a) −8e−10t V, t≥0;
2. b) 80%.

SELF-CHECK: Also try Chapter Problems 7.1, 7.2, and 7.4.

Deriving the Expressions for Voltage, Power, and Energy

We derive the voltage across the resistor in Fig. 7.4 using Ohm's law:

$v = iR = I_0 R\, e^{-t/\tau}, \quad t \geq 0+ \quad (7.5)$

Note that while the current is defined for t≥0 (Eq. 7.4), the voltage is defined only for t≥0+, not at t=0. At t=0 a step change occurs in the voltage. For t<0,

the derivative of the current is zero, so the voltage is also zero (v=Ldi/dt=0). Thus

$$v(0-)=0, v(0+)=I_0R,$$

where v(0+) is obtained from Eq. 7.5 with t=0+1.[1]. The value of the voltage at t=0 is undefined owing to the step change at t=0. Thus, we use $t \geq 0+$ when defining the region of validity for the voltage in Eq. 7.5.

[1]We can define the expressions 0− and 0+ more formally. The expression x(0−) refers to the limit of the variable x as $t \to 0$ from the left, or from negative time. The expression x(0+) refers to the limit of the variable x as $t \to 0$ from the right, or from positive time.

We derive the power dissipated in the resistor from any of the following expressions:

$$p=vi, \quad p=i^2R, \quad \text{or} \quad p=v^2R.$$

Whichever form is used, the resulting expression can be reduced to

$$p=I_0^2R\, e^{-2t/\tau}, \quad t \geq 0+.$$

The energy delivered to the resistor during any interval of time after the switch has been opened is

$$w=\int_0^t p\,dx=\int_0^t I_0^2Re^{-2x/\tau}dx=\tau 2I_0^2R(1-e^{-2t/\tau})=\frac{L}{2R}I_0^2R(1-e^{-2t/\tau})=\frac{1}{2}LI_0^2(1-e^{-2t/\tau}), t \geq 0+.$$

Note from the energy equation that as t becomes infinite, the energy dissipated in the resistor approaches the initial energy stored in the inductor.

The Significance of the Time Constant

The time constant is an important parameter for first-order circuits. You can

express the time elapsed after switching as an integer multiple of τ. For example, one time constant after the inductor begins releasing its stored energy to the resistor, the current has been reduced to e−1, or approximately 0.37 of its initial value.

Table 7.1 gives the value of e−t/τ for integer multiples of τ from 1 to 10. Note that the current is less than 1% of its initial value when the elapsed time exceeds five time constants. So, five time constants after switching has occurred, the currents and voltages have essentially reached their final values. After switching, the changes to the currents and voltages are momentary events and represent the **transient response** of the circuit.

Table 7.1 Value of e−t/τ For t Equal to Integral Multiples of τ

t	$e^{-t/\tau}$	t	$e^{-t/\tau}$
τ	3.6788×10^{-1}	6τ	2.4788×10^{-3}
2τ	1.3534×10^{-1}	7τ	9.1188×10^{-4}
3τ	4.9787×10^{-2}	8τ	3.3546×10^{-4}
4τ	1.8316×10^{-2}	9τ	1.2341×10^{-4}
5τ	6.7379×10^{-3}	10τ	4.5400×10^{-5}

Table 7.1 Full Alternative Text

By contrast, the phrase *a long time* implies that five or more time constants have elapsed, for first-order circuits. The response that exists a long time after switching is called the **steady-state response**. The phrase *a long time* then also means the time it takes the circuit to reach its steady-state value.

The time constant also represents the time required for the current to reach its final value if the current continues to change at its initial rate. To illustrate, we evaluate di/dt at 0+ and assume that the current continues to change at this rate:

didt(0+)=−I0τe−0+/τ=−I0τ.

Now, if i starts at I0 and decreases at a constant rate of I0/τ amperes per second, the expression for i becomes

$i = I_0 - I_0 \frac{\tau}{t}$.

This expression indicates that i would reach its final value of zero in τ seconds. Figure 7.9 shows how this graphic interpretation can be used to estimate a circuit's time constant from a plot of its natural response. Such a plot could be generated on an oscilloscope measuring output current. Drawing the tangent to the natural-response plot at t=0 and reading the value at which the tangent intersects the time axis gives the value of τ. This allows you to determine the time constant of a circuit even if you don't know its component values.

Figure 7.9 A graphic interpretation of the time constant of the RL circuit shown in Fig. 7.4.

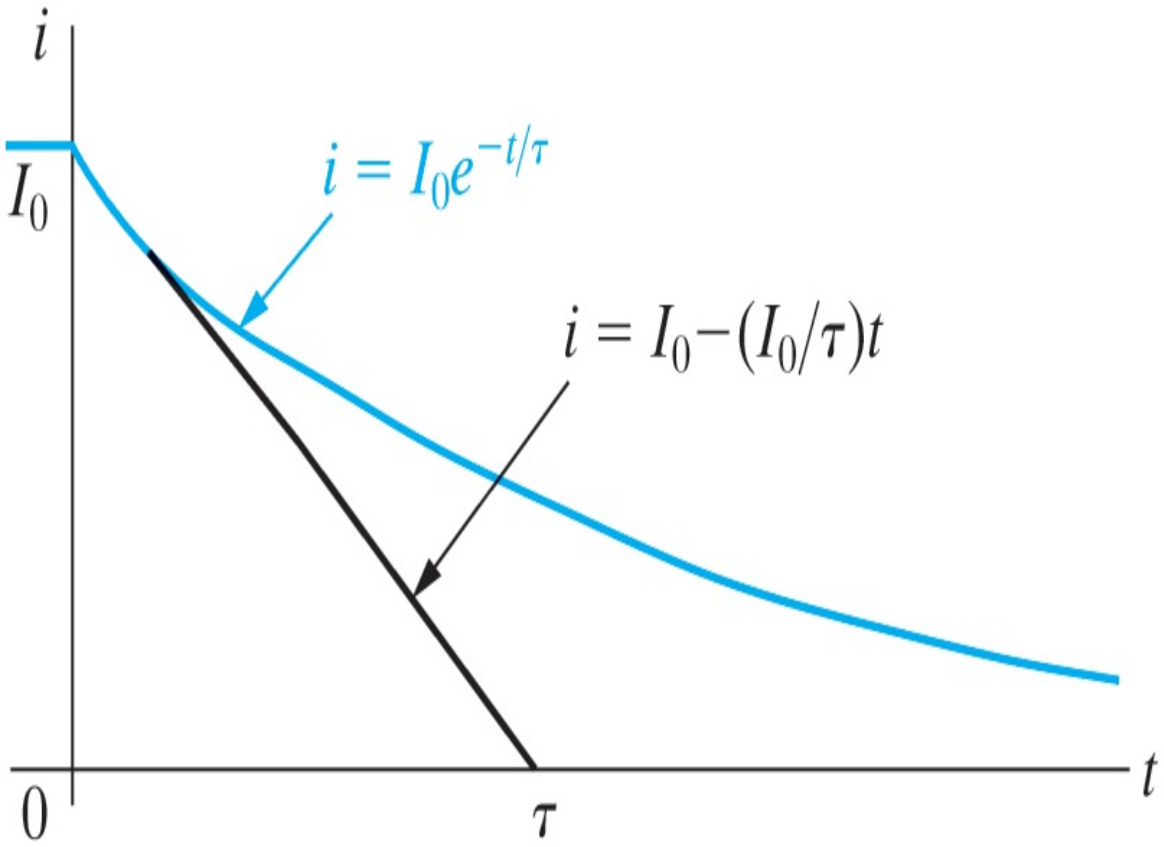


Figure 7.9 Full Alternative Text

Up to this point, we have dealt with circuits having a single inductor. But the techniques presented apply to circuits with multiple inductors if the inductors can be combined into a single equivalent inductor. Example 7.2 finds the natural response of an RL circuit that contains two inductors.

Example 7.2 Determining the Natural Response of an RL Circuit with Parallel Inductors

In the circuit shown in Fig. 7.10, the initial currents in inductors L1 and L2 have been established by sources not shown. The switch is opened at t=0.

Figure 7.10 The circuit for Example 7.2.

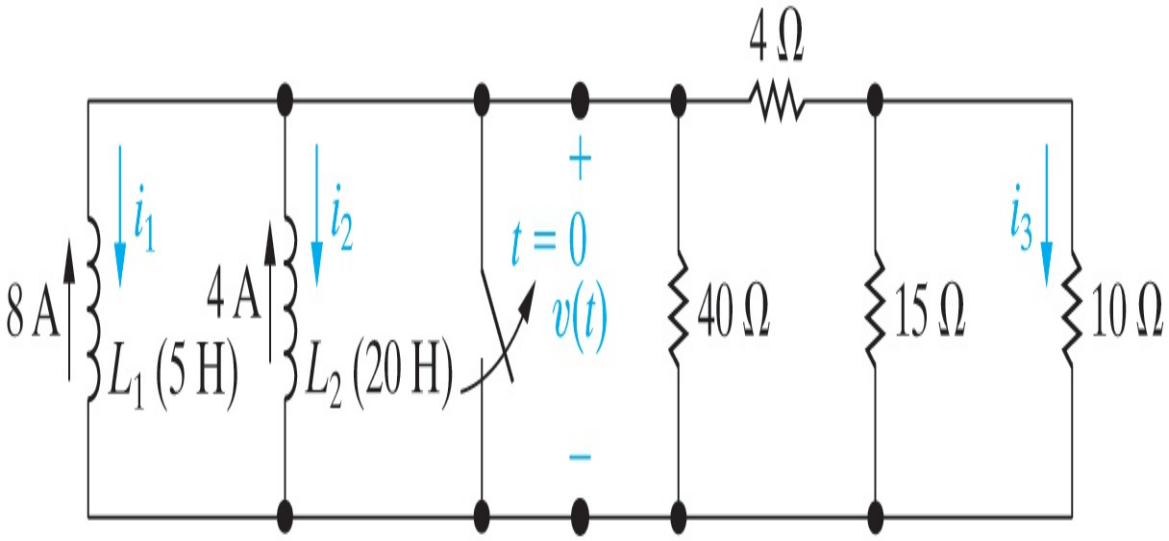


Figure 7.10 Full Alternative Text

1. Find i1, i2, and i3 for t≥0.
2. Calculate the initial energy stored in the parallel inductors.
3. Determine how much energy is stored in the inductors as t→ ∞.
4. Show that the total energy delivered to the resistive network equals the difference between the results obtained in (b) and (c).

Solution

1. The key to finding currents i1, i2, and i3 lies in knowing the voltage v(t). We can easily find v(t) if we simplify the circuit shown in Fig. 7.10 to the equivalent form shown in Fig. 7.11. The parallel inductors combine to give an equivalent inductance of 4 H, carrying an initial current of 12 A. The resistive network reduces to a single resistance of 40 || [4+

(15 || 10)]=8 Ω. We can now use Analysis Method 7.1.

Figure 7.11 A simplification of the circuit shown in Fig. 7.10.

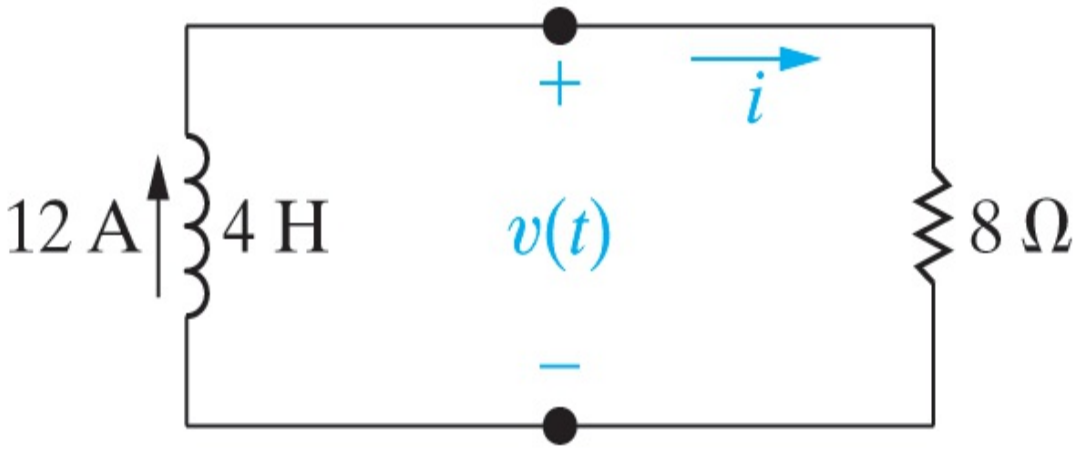


Figure 7.11 Full Alternative Text

1. Step 1: The initial current in the inductor in Fig. 7.11 is I0=12 A.

2. Step 2: The equivalent resistance attached to the inductor in Fig. 7.11 is 8 Ω. Therefore, the time constant is

   $\tau = \frac{L}{R} = \frac{4}{8} = 0.5$ s.

3. Step 3: The inductor current in Fig. 7.11 is

   $i(t) = I_0 e^{-t/\tau} = 12e^{-t/0.5} = 12e^{-2t}$ A, $t \geq 0$.

4. Step 4: We will use additional circuit analysis techniques to find the currents i1, i2, and i3. To begin, note that in Fig. 7.11, v(t)=8i(t), so

   $v(t) = 96e^{-2t}$ V, $t \geq 0+$.

From the circuit in Fig. 7.10, we see that that v(t)=0 at t=0−, so the expression for v(t) is valid for t≥0+. After obtaining v(t), we can calculate i1 and i2 using the relationship between current and voltage in inductors:

$$i1=15\int_0^t 96e^{-2x}\,dx-8=1.6-9.6e^{-2t}\ A, t\geq 0, i2=120\int_0^t 96e^{-2x}\,dx-4=-1.6-2.4e^{-2t}\ A, t\geq 0.$$

We will use two steps to find i3; in the first step, calculate the voltage across the parallel 15 Ω and 10 Ω resistors using voltage division. Calling that voltage $v_{15||10}$, positive at the top of the circuit, we get

$$v_{15||10}=15||10\,4+15||10\,v=610(96e^{-2t})=57.6e^{-2t}V, \quad t\geq 0+.$$

Now use Ohm's law to calculate i3, giving

$$i3=v_{15||10}10=5.76e^{-2t}A, \quad t\geq 0.+$$

Note that the expressions for the inductor currents i1 and i2 are valid for t≥0, whereas the expression for the resistor current i3 is valid for t≥0+.

2. The initial energy stored in the inductors is

   $$w=12(5)(8)2+12(20)(4)2=320J.$$

3. As t → ∞, i1 → 1.6A and i2 → −1.6 A. Therefore, a long time after the switch opens, the energy stored in the two inductors is

   $$w=12(5)(1.6)2+\ 12(20)(-1.6)2=32\ J.$$

4. We obtain the total energy delivered to the resistive network by integrating the expression for the instantaneous power from zero to infinity:

   $$w=\int_0^\infty p\,dt=\int_0^\infty (96e^{-2t})(12e^{-2t})dt=1152e^{-4t}-4|_0^\infty=288\ \ J.$$

   This result is the difference between the initially stored energy (320 J)

and the energy trapped in the parallel inductors (32 J). Also, note that the equivalent inductor for the parallel inductors (which predicts the terminal behavior of the parallel combination) has an initial energy of 12(4)(12)2=288 J; that is, the energy stored in the equivalent inductor represents the amount of energy that will be delivered to the resistive network at the terminals of the original inductors.

SELF-CHECK: Also try Chapter Problem 7.18.

7.2 The Natural Response of an RC Circuit

The form of an RC circuit's natural response is analogous to that of an RL circuit. Consequently, we don't treat the RC circuit in as much detail as we did the RL circuit.

We develop the natural response of an RC circuit using the circuit shown in Fig. 7.12. Begin by assuming that the switch has been in position a for a long time, allowing the loop containing the dc voltage source Vg, the resistor R1, and the capacitor C to reach a steady-state condition. Recall from Chapter 6 that a capacitor behaves as an open circuit in the presence of a constant voltage, so the source voltage appears across the capacitor terminals. In Section 7.3, we will discuss how the capacitor voltage builds to the steady-state value of the dc voltage source. Here it is important to remember that when the switch is moved from position a to position b (at t=0), the voltage on the capacitor is Vg. Because there can be no instantaneous change in the voltage at the terminals of a capacitor, the problem reduces to solving the circuit shown in Fig. 7.13.

Figure 7.12 An RC circuit.

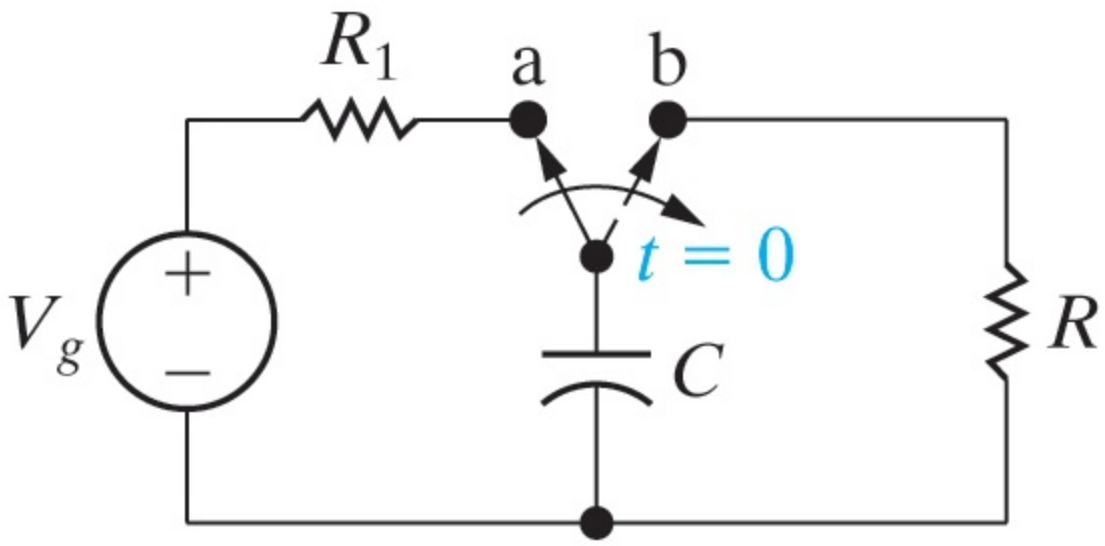


Figure 7.12 Full Alternative Text

Figure 7.13 The circuit shown in Fig. 7.12, after switching.

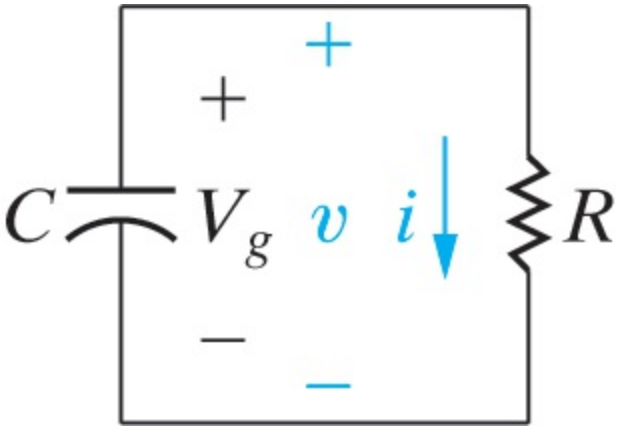


Figure 7.13 Full Alternative Text

Deriving the Expression for the Voltage

We can easily find the voltage v(t) by writing a KCL equation. Using the lower node between R and C as the reference node and summing the currents away from the upper node between R and C gives

$$C\frac{dv}{dt} + \frac{v}{R} = 0. \quad (7.6)$$

Comparing Eq. 7.6 with Eq. 7.1, you should see that the mathematical techniques used to find i(t) in the RL circuit can be used to find v(t) in the RC circuit. We leave it to you to show that

$$v(t) = v(0)e^{-t/RC}, \quad t \geq 0.$$

As we have already noted, the initial voltage on the capacitor equals the voltage source voltage Vg, or

Capacitor Voltage

$$v(0^-) = v(0) = v(0^+) = Vg = V0, \quad (7.7)$$

where V0 denotes the initial voltage on the capacitor. The time constant for the RC circuit equals the product of the resistance and capacitance, namely,

Time Constant for RC Circuit

$$\tau = RC. \quad (7.8)$$

Therefore, the general expression for the voltage becomes

Natural Response of an RC Circuit

$$v(t) = V0e^{-t/\tau}, \quad t \geq 0, \quad (7.9)$$

which indicates that the natural response of an RC circuit is an exponential decay of the initial voltage. The time constant RC governs the rate of decay.

Figure 7.14 shows the plot of Eq. 7.9 and the graphic interpretation of the time constant.

Figure 7.14 The natural response of an RC circuit.

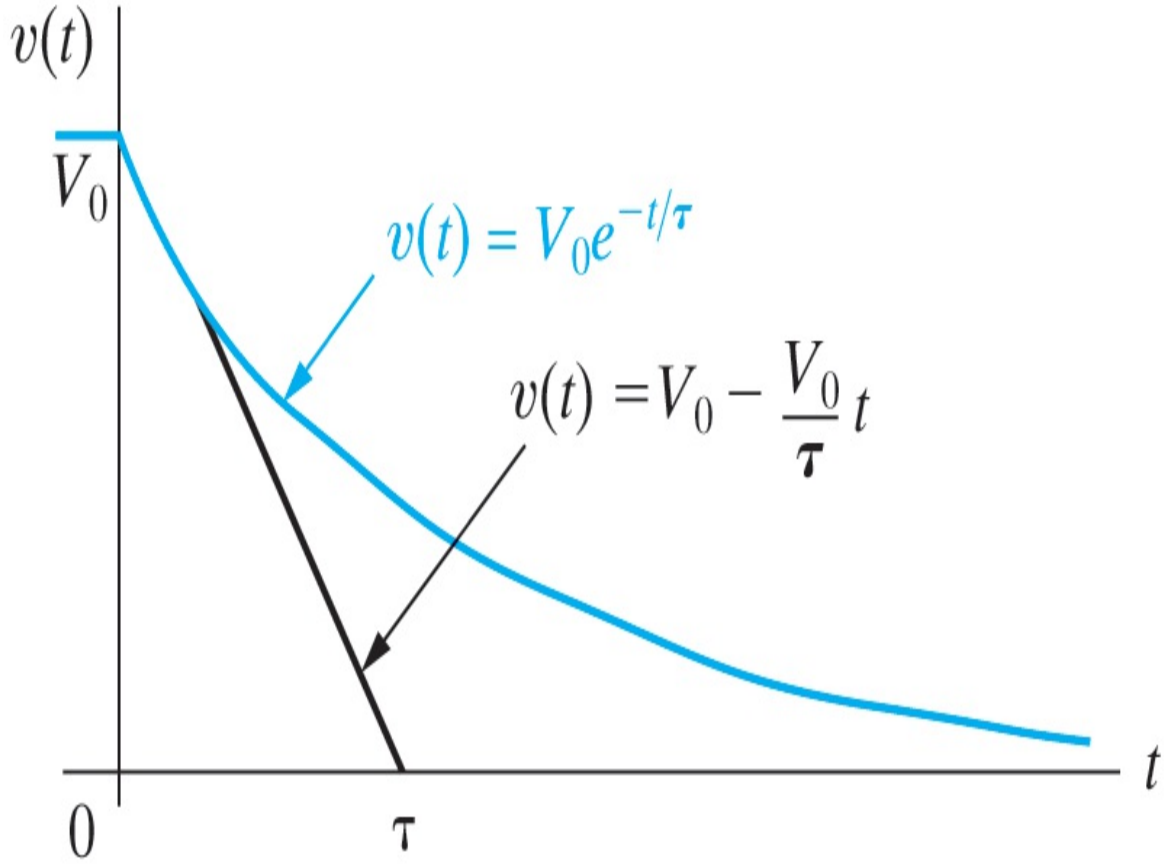


Figure 7.14 Full Alternative Text

Now we have a step-by-step method for finding the natural response of an RC circuit.

1. Step 1: Determine the initial voltage, V0, across the capacitor. This usually involves analyzing the circuit for t<0.

2. Step 2: Calculate the time constant, τ. To do this, you need to find the

equivalent resistance attached to the capacitor for $t \geq 0$.

3. Step 3: Write the equation for the capacitor voltage for $t \geq 0$ by substituting the values for the initial voltage and the time constant into Eq. 7.9.

4. Step 4: Calculate any other quantities of interest, such as resistor current and voltage, using resistive circuit analysis techniques.

This method is summarized in Analysis Method 7.2.

RC Natural-Response Method

1. **Determine the initial capacitor voltage,** V_0, by analyzing the circuit for $t<0$.

2. **Calculate the time constant,** $\tau = RC$, where R is the equivalent resistance connected to the capacitor for $t \geq 0$

3. **Write the equation for capacitor voltage,** $v(t) = V_0 e^{-t/\tau}$, for $t \geq 0$

4. **Calculate other quantities of interest** using the capacitor voltage.

Analysis Method 7.2 Finding the RC natural response.

After determining v(t), we can easily derive the expressions for i, p, and w:

$$i(t) = \frac{v(t)}{R} = \frac{V_0}{R} e^{-t/\tau}, t \geq 0+, \quad p = vi = \frac{V_0^2}{R} e^{-2t/\tau}, t \geq 0+, \quad w = \int_0^t p \, dx = \int_0^t \frac{V_0^2}{R} e^{-2x/\tau} \, dx = \frac{1}{2} C V_0^2 (1 - e^{-2t/\tau}), t \geq 0$$

Example 7.3 uses Analysis Method 7.2 to determine the natural response of an RC circuit. Analysis Method 7.2 applies to circuits with a single capacitor, but it can also be used to analyze circuits with multiple capacitors if they can all be combined into a single equivalent capacitor. Example 7.4 considers a circuit with two capacitors.

Example 7.3 Determining the Natural Response of an RC Circuit

The switch in the circuit shown in Fig. 7.15 has been in position x for a long time. At t=0, the switch moves instantaneously to position y. Find

Figure 7.15 The circuit for Example 7.3.

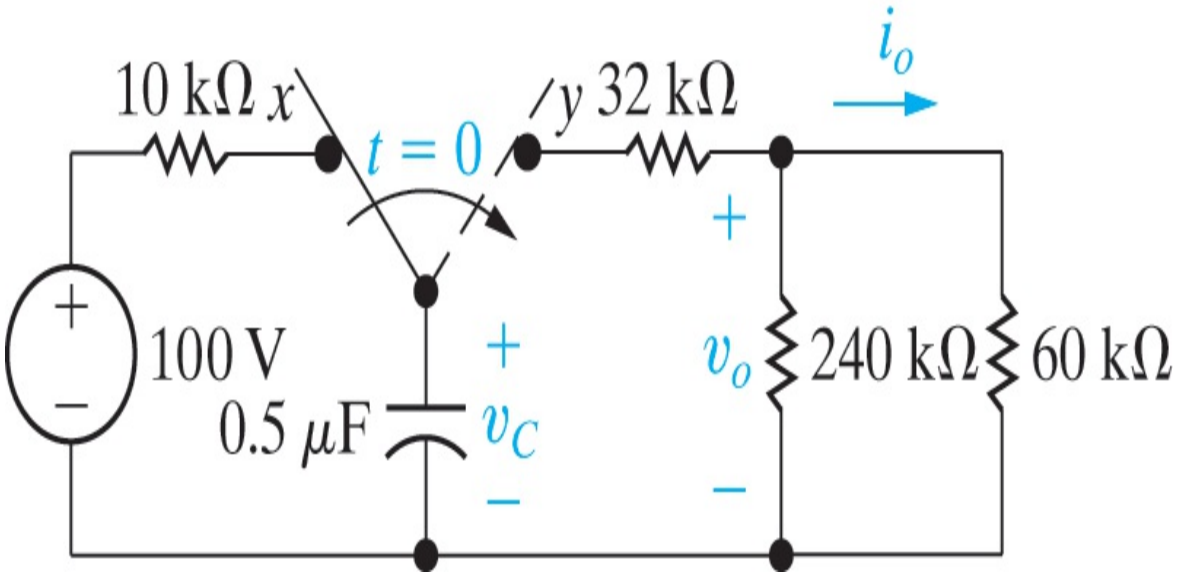


Figure 7.15 Full Alternative Text

1. vC(t) for t≥0,
2. vo(t) for t≥0+,
3. io(t) for t≥0+, and
4. the total energy dissipated in the 60 kΩ resistor.

Solution

Use Analysis Method 7.2.

1. 1. Step 1: Determine the initial capacitor voltage V0 by drawing the circuit in Fig. 7.15 for t < 0. The result is shown in Fig. 7.16, and since the capacitor behaves like an open circuit, its initial voltage equals the source voltage:

Figure 7.16 The circuit in Fig. 7.15 for t<0

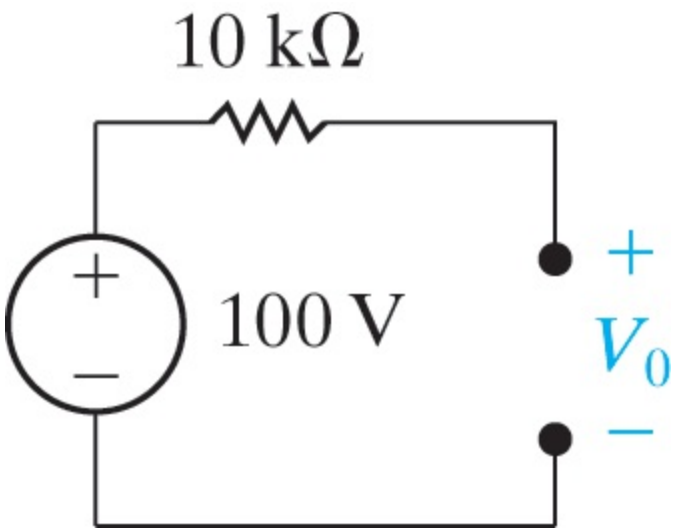


Figure 7.16 Full Alternative Text
V0=100 V.

2. Step 2: Calculate the time constant. To do this, draw the circuit in Fig. 7.15 for t≥0, as shown in Fig. 7.17, and find the equivalent resistance attached to the capacitor:

Figure 7.17 The circuit in

Fig. 7.15 for t≥0

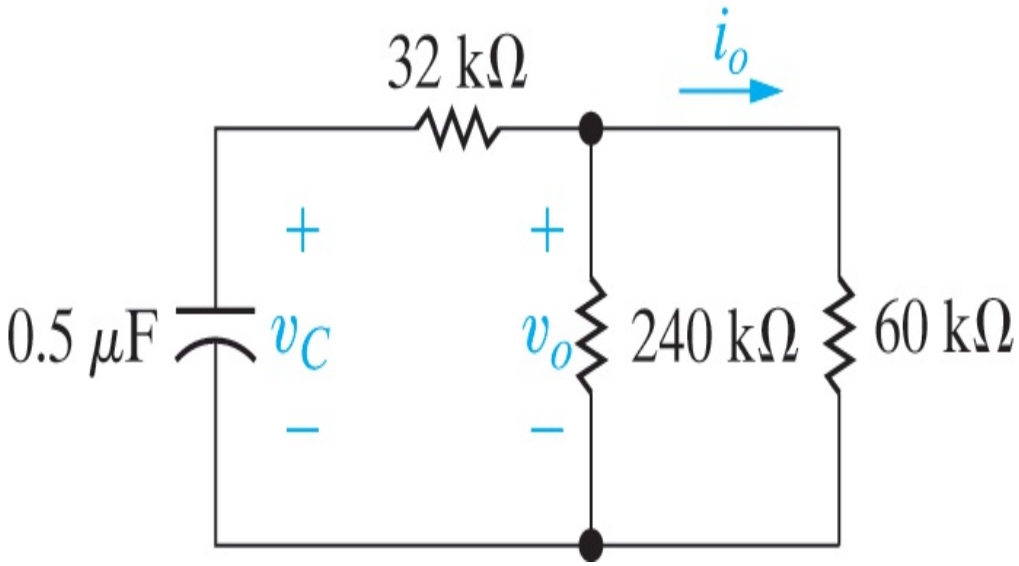


Figure 7.17 Full Alternative Text

Req=32×103+(240×103||60×103)=80 kΩ,τ=ReqC=(80×103)(0.5×10−6)=40 ms.

3. Step 3: Write the equation for the capacitor voltage by substituting the values for V0 and τ into Eq. 7.9:

   vC(t)=100e−t/0.04=100e−25tV, t≥0.

4. Step 4: Determine the remaining quantities using resistive circuit analysis techniques for the circuit in Fig. 7.17.

2. To find vo(t) in Fig. 7.17, note that the resistive circuit forms a voltage divider across the terminals of the capacitor. Thus

   vo(t)=240×103||60×10332×103+(240×103||60×103)vC(t)=0.6(100e−25t)=60e−25tV, t≥0+.

   This expression for vo(t) is valid for t≥0+ because vo(0−) is zero. Thus, we have an instantaneous change in the voltage across the 240 kΩ resistor.

3. We find the current io(t) from Ohm’s law:

   $$i_o(t)=\frac{v_o(t)}{60\times 103}=e^{-25t}\ \text{mA},\quad t\geq 0+.$$

4. The power dissipated in the 60 kΩ resistor is

   $$p_{60k\Omega}(t)=i_o^2(t)(60\times 103)=60e^{-50t}\ \text{mW},\quad t\geq 0+.$$

   The total energy dissipated is

   $$w_{60\ k\Omega}=\int_0^\infty i_o^2(t)(60\times 103)dt=1.2\ \text{mJ}.$$

Example 7.4 Determining the Natural Response of an RC Circuit with Series Capacitors

The initial voltages on capacitors C1 and C2 in the circuit shown in Fig. 7.18 have been established by sources not shown. The switch is closed at t=0.

Figure 7.18 The circuit for Example 7.4.

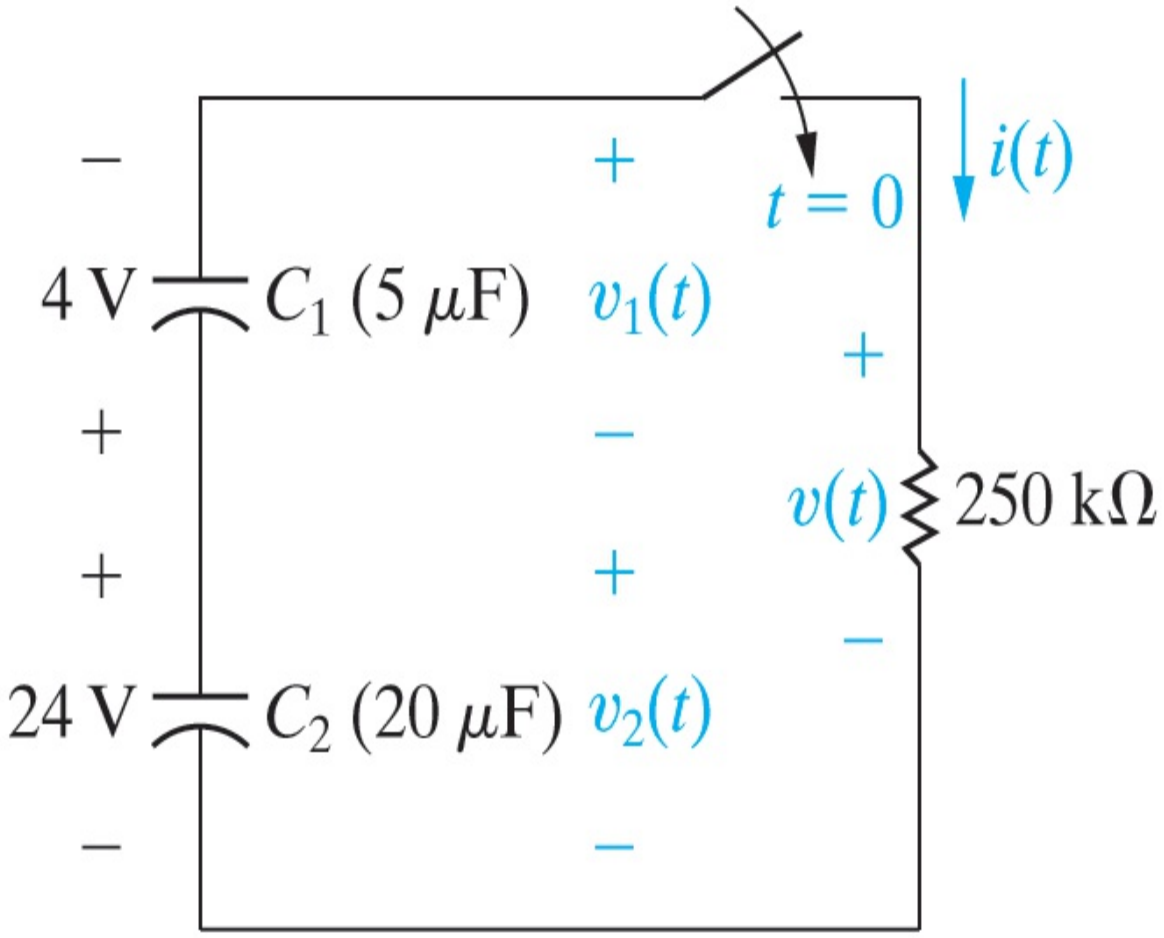


Figure 7.18 Full Alternative Text

1. Find v1(t), v2(t), and v(t) for t≥0 and i(t) for t≥0+.
2. Calculate the initial energy stored in the capacitors C1 and C2.
3. Determine how much energy is stored in the capacitors as t→ ∞.
4. Show that the total energy delivered to the 250 kΩ resistor is the difference between the results obtained in (b) and (c).

Solution

1. Once we know v(t), we can obtain the current i(t) from Ohm's law. After determining i(t), we can calculate v1(t) and v2(t) because the

voltage across a capacitor is a function of the capacitor current. To find v(t), we replace the series-connected capacitors with an equivalent capacitor. It has a capacitance of 4 μF and is charged to a voltage of 20 V. Therefore, the circuit shown in Fig. 7.18 reduces to the one shown in Fig. 7.19. We can now use Analysis Method 7.2 to determine v(t).

Figure 7.19 A simplification of the circuit shown in Fig. 7.18.

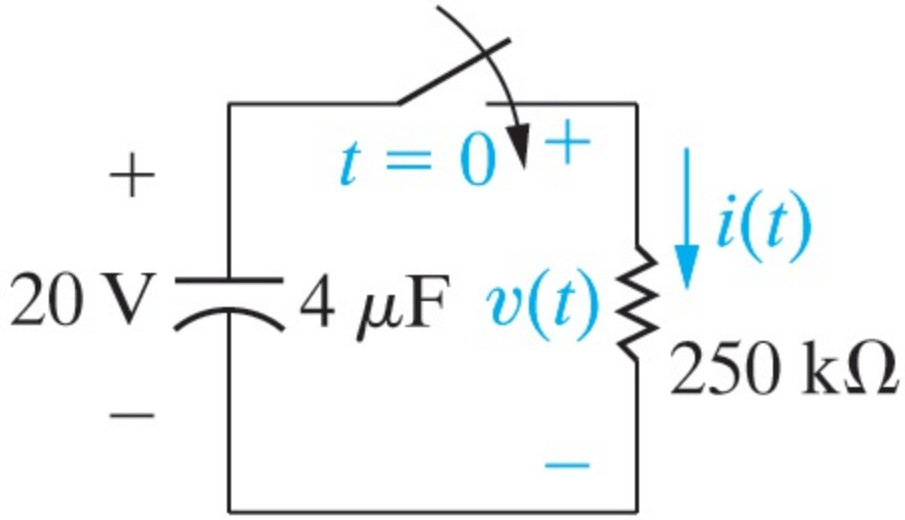


Figure 7.19 Full Alternative Text

1. Step 1: The initial voltage across the capacitor in Fig. 7.19 is V0=20 V.

2. Step 2: The resistance attached to the capacitor in Fig. 7.19 is 250 kΩ. Therefore, the time constant is τ=(250×103)(4×10−6)=1 s.

3. Step 3: Write the equation for the capacitor voltage by substituting the values for V0 and τ in Eq. 7.9 to give

   v(t)=20e−t V, t≥0.

4. Step 4: Determine the currents and voltages requested using the techniques described at the start of the Solution.

For the circuit in Fig. 7.19, use Ohm's law to find the current i(t):

$$i(t)=v(t)250{,}000=80e-t\ \mu A,\quad t\geq 0+.$$

Knowing i(t), we calculate the expressions for v1(t) and v2(t) for the circuit in Fig. 7.18:

$$v1(t)=-15\times 10-6\int 0t80\times 10-6e-xdx-4=(16e-t-20)\ V,\quad t\geq 0, v2(t)=-120\times 10-6\int 0t80\times 10-6e-xdx+24=(4e-t+20)\ V, t\geq 0.$$

2. The initial energy stored in C1 is

$$w1=12(5\times 10-6)(4)2=40\ \mu J.$$

The initial energy stored in C2 is

$$w2=12(20\times 10-6)(24)2=5760\ \mu J.$$

The total energy stored in the two capacitors is

$$wo=40+5760=5800\ \mu J.$$

3. As $t\rightarrow\ \infty$,

$v1\rightarrow -20V$ and $v2\rightarrow +20V$.

Therefore, the energy stored in the two capacitors is

$$w\infty=12(5\times 10-6)(-20)2+12(20\times 10-6)(20)2=5000\ \mu J.$$

4. The total energy delivered to the 250 kΩ resistor is

$$w=\int 0\infty pdt=\int 0\infty(20e-t)(80\times 10-6e-t)dt=800\ \mu J.$$

Comparing the results obtained in (b) and (c) shows that

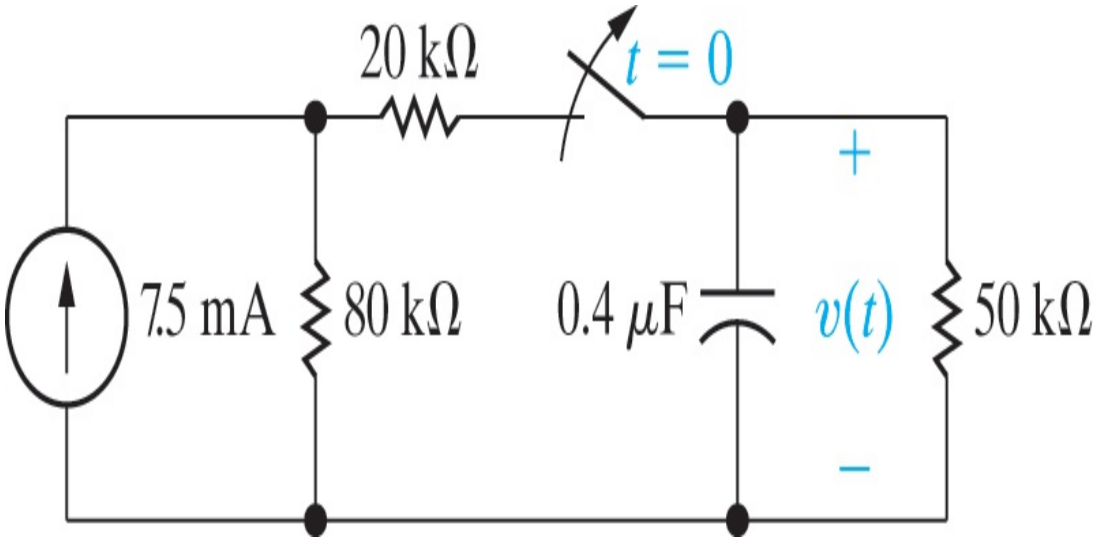
800 μJ=(5800−5000) μJ.

The energy stored in the equivalent capacitor in Fig. 7.19 is 12(4×10–6)(20)2=800 μJ. Because this capacitor predicts the terminal behavior of the original series-connected capacitors, the energy stored in the equivalent capacitor is the energy delivered to the 250 kΩ resistor.

Assessment Problems

Objective 1—Be able to determine the natural response of both RL and RC circuits

1. 7.3 The switch in the circuit shown has been closed for a long time and is opened at t=0. Find
    1. a) the initial value of v(t),
    2. b) the time constant for t>0,
    3. c) the numerical expression for v(t) after the switch has been opened,
    4. d) the initial energy stored in the capacitor, and
    5. e) the length of time required to dissipate 75% of the initially stored energy.

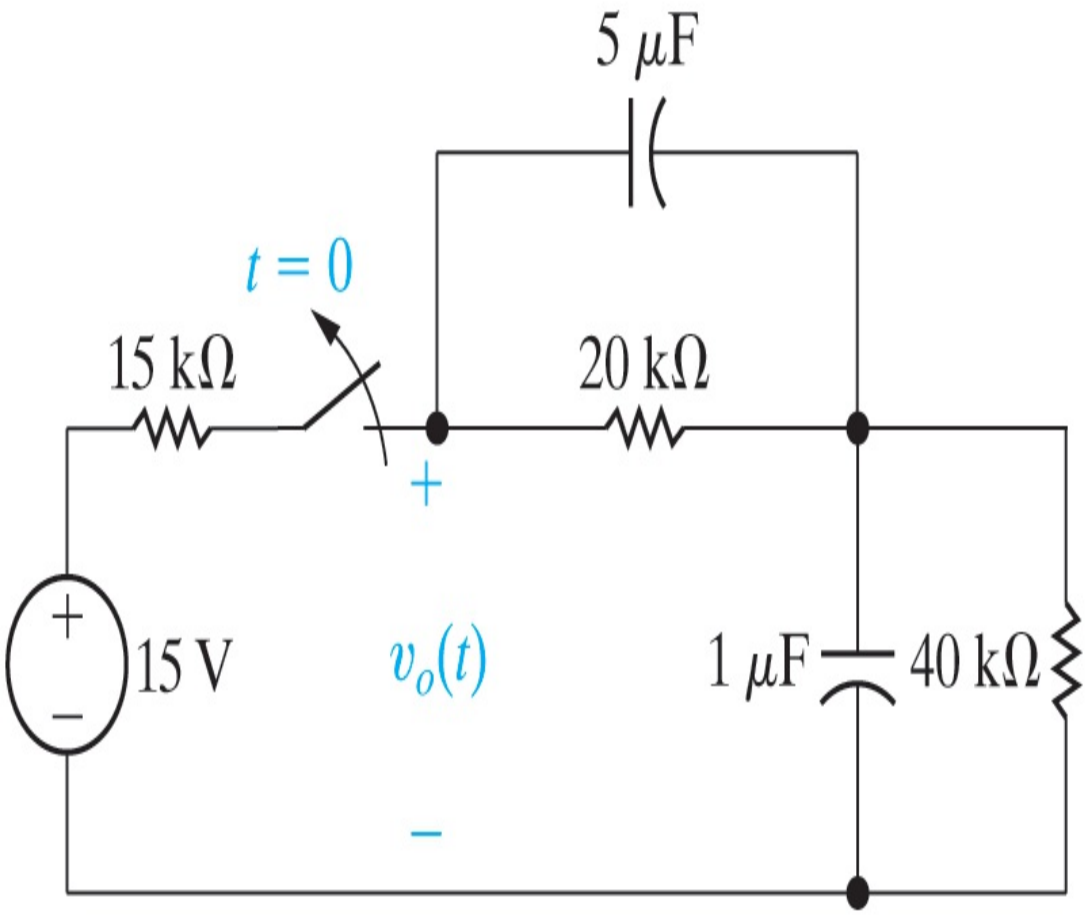


7.2-7 Full Alternative Text

Answer:

1. a) 200 V;
2. b) 20 ms;
3. c) 200e−50t V, t≥0;
4. d) 8 mJ;
5. e) 13.86 ms.

2. 7.4 The switch in the circuit shown has been closed for a long time before being opened at t=0.
    1. a) Find vo(t) for t≥0.
    2. b) What percentage of the initial energy stored in the circuit has been dissipated after the switch has been open for 60 ms?



7.2-8 Full Alternative Text

Answer:

1. a) $8e^{-25t} + 4e^{-10t}$ V, $t \geq 0$;
2. b) 81.05%.

SELF-CHECK: Also try Chapter Problems 7.22 and 7.33.

7.3 The Step Response of RL and RC Circuits

The response of a circuit to the sudden application of a constant voltage or current source is called the **step response** of the circuit. In this section, we find the step response of first-order RL and RC circuits by describing the currents and voltages generated when either dc voltage or current sources are suddenly applied. We also show how the circuit responds when energy is being stored in the inductor or capacitor. We begin with the step response of an RL circuit.

The Step Response of an RL Circuit

We modify the first-order circuit shown in Fig. 7.2(a) by adding a switch and develop the step response of an RL circuit using the resulting circuit, shown in Fig. 7.20. We assume a nonzero initial current i(0), so the inductor has stored energy at the time the switch is closed. We want to find the expressions for the current in the circuit and for the voltage across the inductor after the switch has been closed. We use circuit analysis to derive the differential equation that describes the circuit in terms of the variable of interest, and then we use elementary calculus to solve the equation, just as we did in Section 7.1.

Figure 7.20 A circuit used to illustrate the step response of a first-order RL circuit.

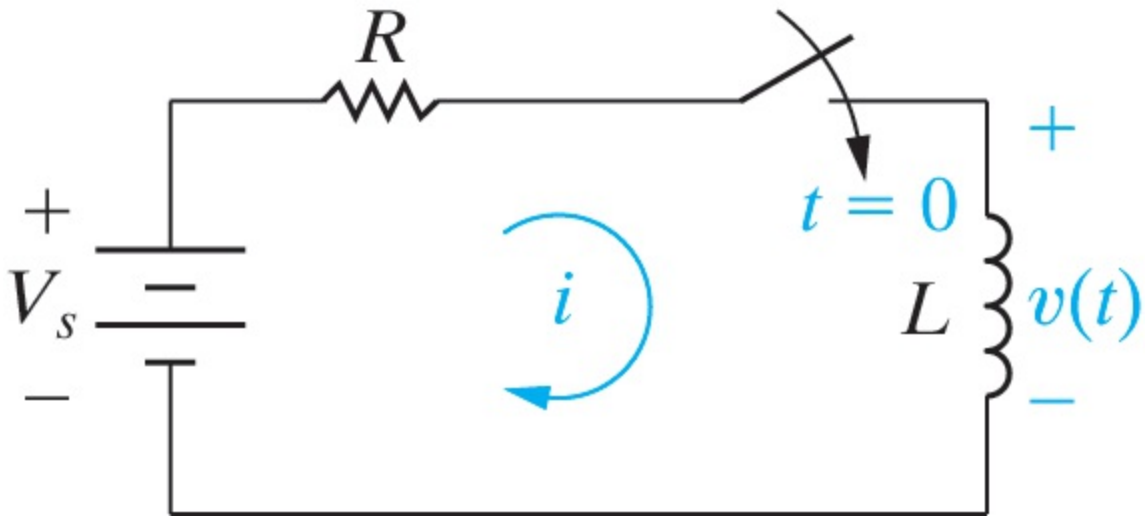


Figure 7.20 Full Alternative Text

After the switch in Fig. 7.20 has been closed, KVL requires that

Vs=Ri+Ldidt,(7.10)

which can be solved for the current by separating the variables i and t and then integrating. We begin by solving Eq. 7.10 for the derivative di /dt :

didt=−Ri+ VsL=−RL(i− VsR).

Next, we multiply both sides of the equation for di/dt by a differential time dt. This step reduces the left-hand side of the equation to a differential change in the current. Thus

di=−RL(i−VsR)dt.

We now separate the variables in the equation for di to get

dii−(Vs/R)=−RLdt,

and then integrate both sides. Using x and y as variables for the integration, we obtain

∫I0i(t)dxx−(Vs/R)=−RL∫0tdy,

where I0 is the current at t=0 and i(t) is the current at any t>0. Evaluating the integrals gives

lni(t) − (Vs/R)I0−(Vs/R)=−RLt,

from which

i(t)−(Vs/R)I0−(Vs/R)=e−(R/L)t,

or

i(t)=VsR+(I0−VsR)e−(R/L)t.(7.11)

Equation 7.11 indicates that after the switch is closed, the current changes exponentially from its initial value I0 to a final value If=Vs/R. The time constant of the circuit, τ=L/R, determines the rate of change. We can determine the final value of the current by analyzing the circuit in Fig. 7.20 as $t \to \infty$, and we can calculate the time constant by finding the equivalent resistance attached to the inductor in the circuit shown in Fig. 7.20 for t≥0. Now express the equation for inductor current (Eq. 7.11) in terms of the initial value of the current, the final value of the current, and the time constant of the circuit to give

Step Response of an RL Circuit

i(t)=If+(I0−If)e−t/τ.(7.12)

Using Eq. 7.12, we can construct a step-by-step procedure to calculate the step response of an RL circuit.

1. Step 1: Determine the initial current, I0, in the inductor. This usually involves analyzing the circuit for t<0.

2. Step 2: Calculate the time constant, τ. To do this, you need to find the equivalent resistance attached to the inductor for t≥0.

3. Step 3: Calculate the final value of the inductor current, If, by analyzing

the circuit as t approaches infinity.

4. Step 4: Write the equation for the inductor current when $t \geq 0$ by substituting the values for the initial current, the time constant, and the final current into Eq. 7.12.

5. Step 5: Calculate any other quantities of interest, such as resistor current and voltage, using resistive circuit analysis techniques.

This method is summarized in Analysis Method 7.3 and is applied to a specific circuit in Example 7.5.

RL Step-Response Method

1. **Determine the initial inductor current,** $I0$, by analyzing the circuit for $t<0$.

2. **Calculate the time constant,** $\tau=L/R$, where R is the equivalent resistance connected to the inductor for $t \geq 0$.

3. **Find the final value for the inductor current**, If, by analyzing the circuit as $t \rightarrow \infty$

4. **Write the equation for inductor current,** $i(t)=If+(I0-If)\, e^{-t/\tau}$, for $t \geq 0$

5. **Calculate other quantities of interest** using the inductor current.

Analysis Method 7.3 Finding the RL step response.

Example 7.5 Determining the Step Response of an RL Circuit

The switch in the circuit shown in Fig. 7.21 has been in position a for a long time. At t=0, the switch moves from position a to position b. The switch is a

make-before-break type; that is, the connection at position b is established before the connection at position a is broken, so the inductor current is continuous.

Figure 7.21 The circuit for Example 7.5.

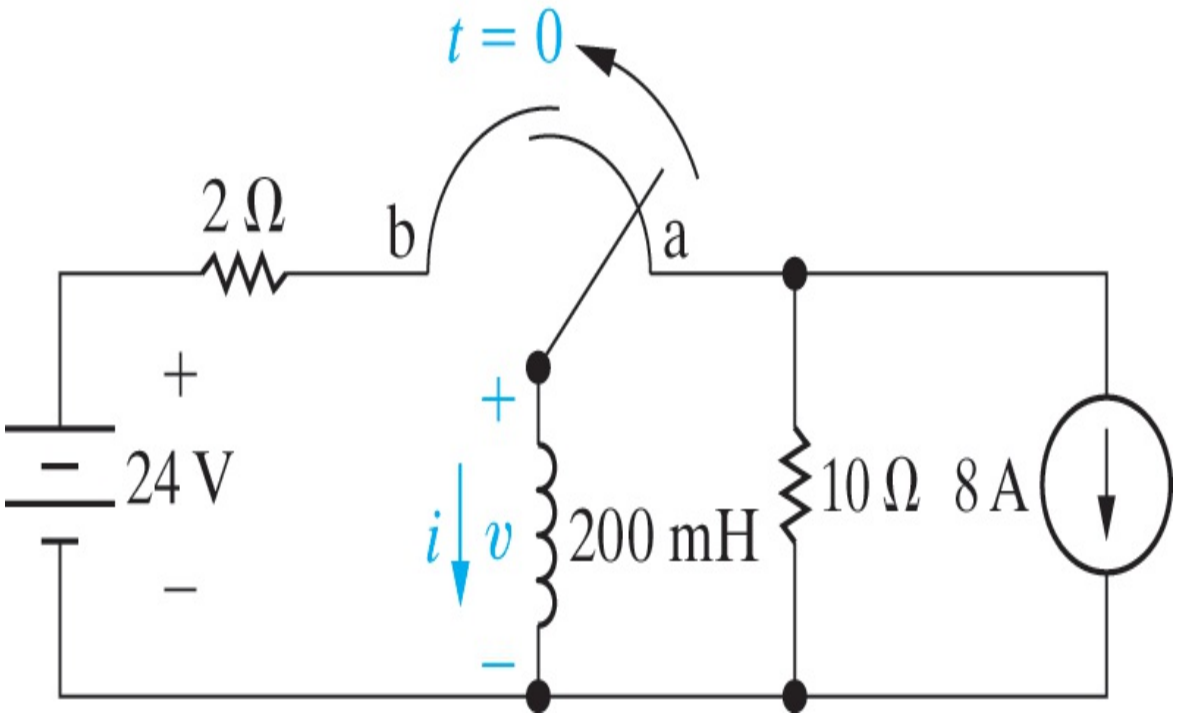


Figure 7.21 Full Alternative Text

1. Find the expression for i(t) for t≥0.
2. What is the initial voltage across the inductor just after the switch has been moved to position b?
3. Does this initial voltage make sense in terms of circuit behavior?
4. How many milliseconds after the switch has been moved does the inductor voltage equal 24 V?

5. Plot both i(t) and v(t) versus t.

Solution

1. Use Analysis Method 7.3 to find the inductor current.

    1. Step 1:Determine the initial current in the inductor. To do this, draw the circuit in Fig. 7.21 when t<0 and the switch is in position a, as shown in Fig. 7.22. Note that since the switch has been in position a for a long time, the inductor behaves like a short circuit that carries all of the current from the 8 A current source. Therefore, I0=−8 A because the inductor current and the source current are in opposite directions.

Figure 7.22 The circuit in Fig. 7.21 for t<0.

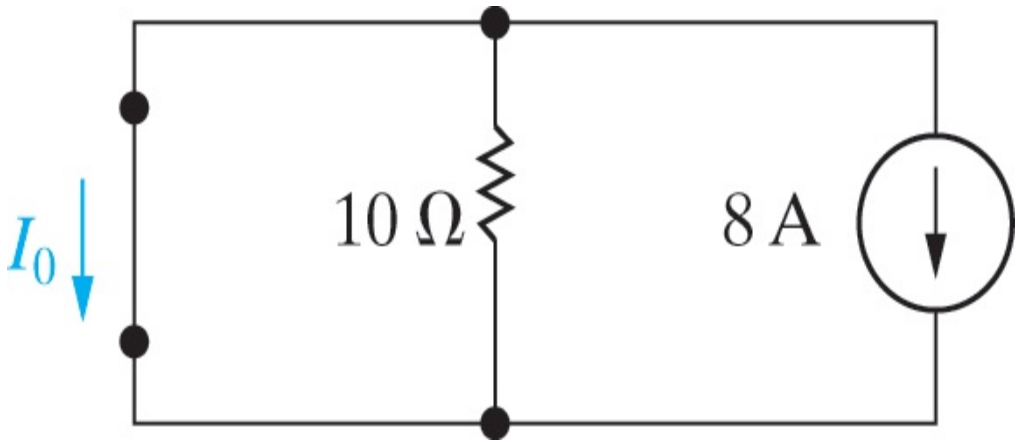


Figure 7.22 Full Alternative Text

    2. Step 2:Calculate the time constant for the circuit. Start by drawing the circuit in Fig. 7.21 when t≥0 and the switch is in position b, as shown in Fig. 7.23. Then determine the Thévenin equivalent resistance for the circuit attached to the inductor. Since the circuit

attached to the inductor is already a Thévenin equivalent circuit, the Thévenin equivalent resistance is 2 Ω and τ=0.2/2=0.1 s.

Figure 7.23 The circuit in Fig. 7.21 for

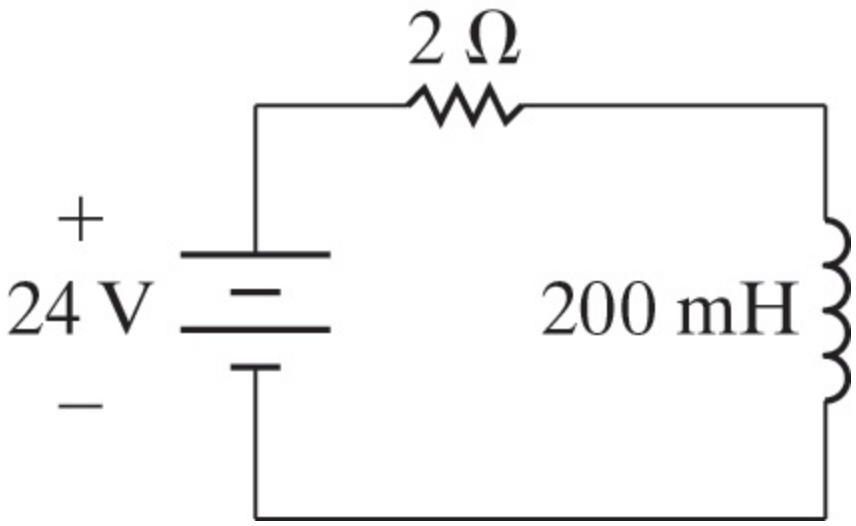


Figure 7.23 Full Alternative Text

3. Step 3:Calculate the final value for the inductor current. To do this, draw the circuit in Fig. 7.21 as $t \to \infty$, when the switch is in position b, as shown in Fig. 7.24. Since the switch has been in position b for a long time, the inductor behaves like a short circuit, as seen in Fig. 7.24, and the current can be found from Ohm's law. Therefore, If=24/2=12 A.

Figure 7.24 The circuit in Fig. 7.21 as t<0.

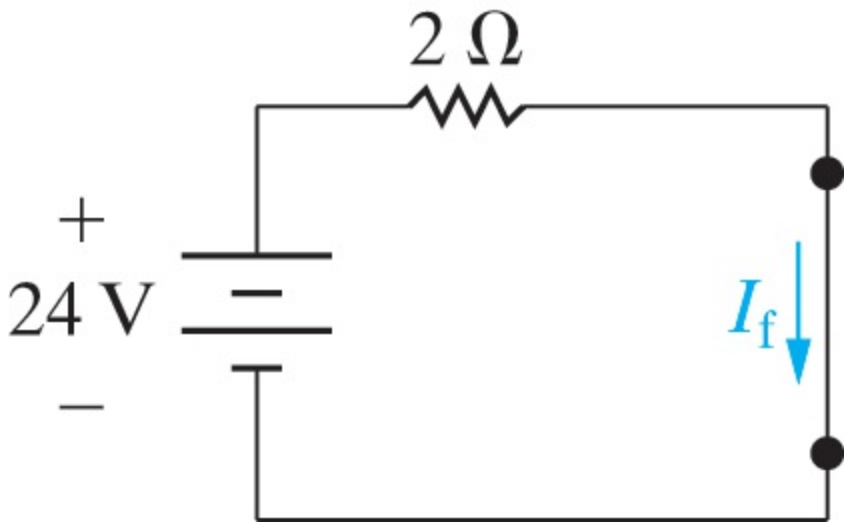


Figure 7.24 Full Alternative Text

4. Step 4:Write the equation for the inductor current when t≥0 by substituting the values for the initial current, the time constant, and the final current into Eq. 7.12 to give

   $i=I_f+(I_0-I_f)e^{-t/\tau}=12+(-8-12)e^{-t/0.1}=12-20e^{-10t}A, \quad t\geq 0.$

5. Step 5:Calculate any other quantities of interest, which we do in the remainder of this example.

2. The voltage across the inductor is

   $v=L\frac{di}{dt}=0.2(200e^{-10t})=40e^{-10t}\ V, \quad t\geq 0+.$

   The initial inductor voltage is

   $v(0+)=40V.$

3. Yes. In the instant after the switch has been moved to position b, the inductor current is 8 A counterclockwise around the newly formed closed path. This current causes a 16 V drop across the 2 Ω resistor. This voltage drop adds to the 24 V drop across the source, producing a 40 V drop across the inductor.

4. We find the time at which the inductor voltage equals 24 V by solving

the expression

$24=40e-10t$

for t:

$t=110 \ln 4024=51.08$ ms.

5. Figure 7.25 shows the graphs of i(t) and v(t) versus t. Note that at the instant of time when the current equals zero, the inductor voltage equals the source voltage of 24 V, as predicted by Kirchhoff's voltage law.

Figure 7.25 The current and voltage waveforms for Example 7.5.

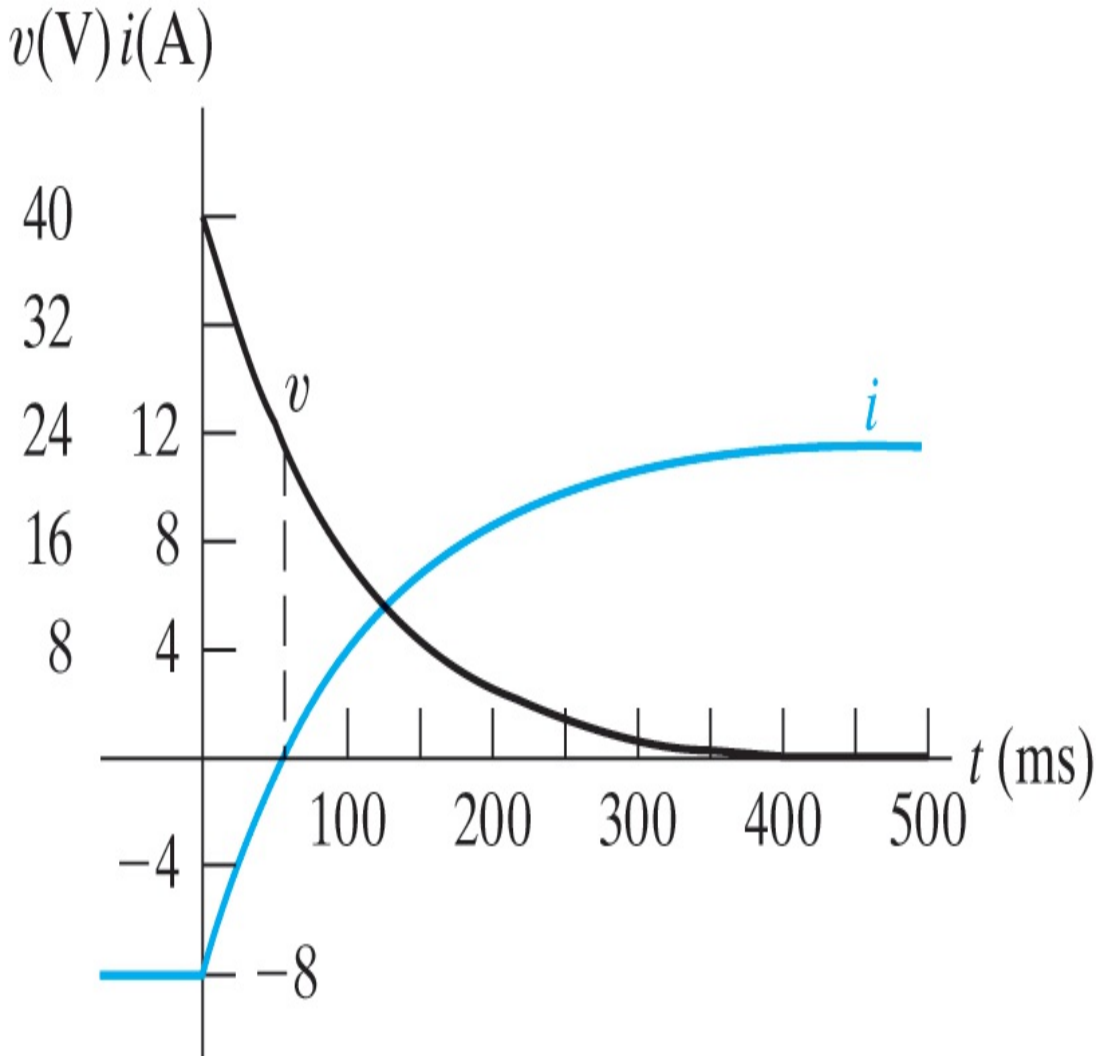


Figure 7.25 Full Alternative Text

Assessment Problem

Objective 2—Be able to determine the step response of both RL and RC circuits

1. 7.5 Assume that the switch in the circuit shown in Fig. 7.21 has been in position b for a long time and that at t=0 it moves to position a. Find (a) i(0+); (b) v(0+); (c) τ, t>0; (d) i(t), t≥0; and (e) v(t), t≥0+.

   Answer:

1. a) 12 A;
2. b) −200V;
3. c) 20 ms;
4. d) −8+20e−50t A, t≥0;
5. e) −200e−50t V, t≥0+.

SELF-CHECK: Also try Chapter Problems 7.35, 7.36, and 7.38.

Observations on the Step Response of an RL Circuit

Let’s take a closer look at the RL step response of the circuit shown in Fig. 7.20. If the initial energy in the inductor is zero, I0 is zero and Eq. 7.11 reduces to

$$i(t)=V_{sR}-V_{sR}e^{-(R/L)t}. \quad (7.13)$$

One time constant after the switch has been closed, the current will have reached approximately 63% of its final value, or

$$i(\tau)=V_{sR}-V_{sR}e^{-1}\approx 0.6321V_{sR}.$$

If the current were to continue to increase at its initial rate, it would reach its final value at $t=\tau$; that is, because

$$\frac{di}{dt}=-V_{sR}(-1\tau)e^{-t/\tau}=V_{sL}e^{-t/\tau},$$

the initial rate at which i(t) increases is

$$\frac{di}{dt}(0)=V_{sL}.$$

If the current were to continue to increase at this rate, the expression for i

would be

$$i = \frac{V_s}{L} t, \quad (7.14)$$

so at $t = \tau$,

$$i = \frac{V_s}{L} \frac{L}{R} = \frac{V_s}{R}.$$

Equations 7.13 and 7.14 are plotted in Fig. 7.26. The values for $i(\tau)$ and I_f are also shown in this figure.

Figure 7.26 The step response of the RL circuit shown in Fig. 7.20 when I0=0

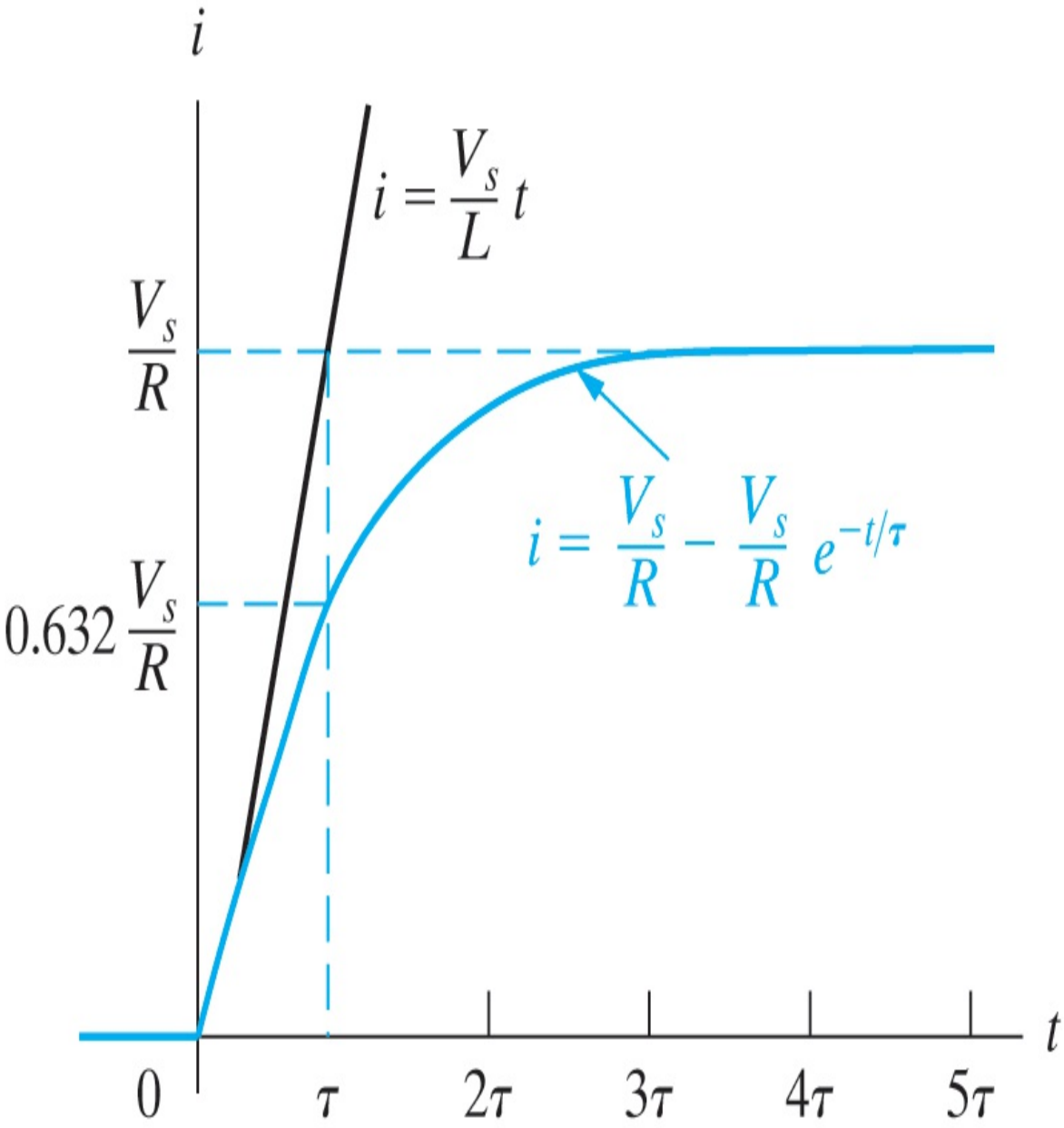


Figure 7.26 Full Alternative Text

The voltage across an inductor is Ldi/dt, so from Eq. 7.11, for t≥0+,

v=L(−RL)(I0−VsR)e−(R/L)t=(Vs−I0R)e−(R/L)t.(7.15)

The voltage across the inductor is zero before the switch is closed because the inductor is behaving like a short circuit. The voltage equation indicates that the inductor voltage jumps to Vs−I0R at the instant the switch is closed and

then decays exponentially to zero.

Does the value of v at t=0+ make sense? Because the initial current is I0 and the inductor prevents an instantaneous change in current, the current is I0 in the instant after the switch has been closed. The voltage drop across the resistor is I0R, so the voltage across the inductor is the source voltage minus the resistor voltage, that is, Vs−I0R.

When the initial inductor current is zero, Eq. 7.15 simplifies to

$$v = Vse^{-(R/L)t}.$$

If the initial current is zero, the voltage across the inductor jumps to Vs when the switch closes. We also expect the inductor voltage to approach zero as t increases because the current in the circuit is approaching the constant value of Vs/R. Figure 7.27 shows the plot of the simplified voltage equation and the relationship between the time constant and the initial rate at which the inductor voltage is decreasing.

Figure 7.27 Inductor voltage versus time.

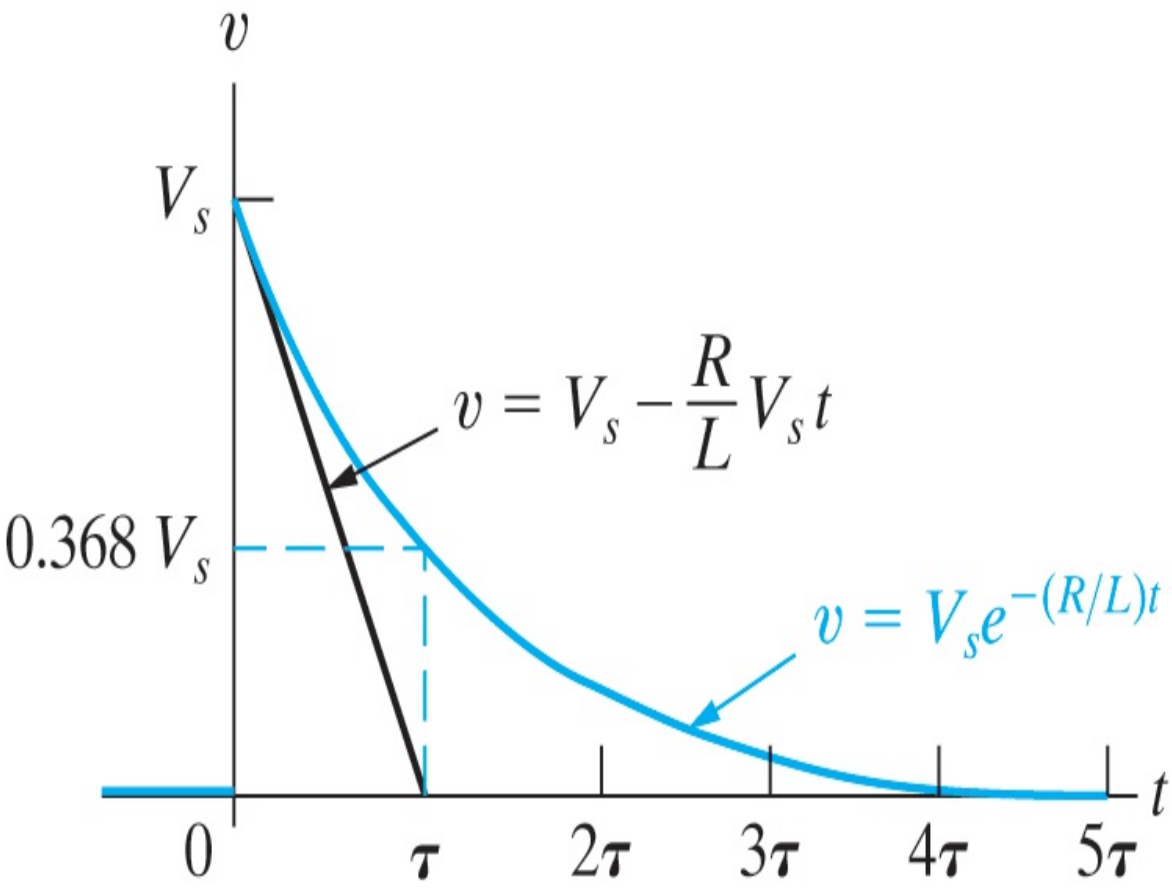


Figure 7.27 Full Alternative Text

We can also describe the voltage v(t) across the inductor in Fig. 7.20 directly, without first calculating the circuit current. We begin by noting that the voltage across the resistor is the difference between the source voltage and the inductor voltage. Using Ohm's law, we write

i(t)=VsR− v(t)R,

where Vs is a constant. Differentiating both sides with respect to time yields

didt=−1Rdvdt.

Then, if we multiply each side of this equation by the inductance L, we get an expression for the voltage across the inductor on the left-hand side, or

v=− LRdvdt.

Putting this differential equation into standard form yields

dvdt+RLv=0.(7.16)

You should verify (in Problem 7.39) that the solution to Eq. 7.16 is identical to that given in Eq. 7.15.

At this point, a general observation about the step response of an RL circuit is pertinent. (This observation will prove helpful later.) When we derived the differential equation for the inductor current, we obtained Eq. 7.10, which we can rewrite as

didt+RLi=VsL.(7.17)

Observe that Eqs. 7.16 and 7.17 have the same form. Specifically, each equates the sum of the first derivative of the variable and a constant times the variable to a constant value. In Eq. 7.16, the constant on the right-hand side happens to be zero; hence, this equation takes on the same form as the natural-response equations in Section 7.1. In both Eq. 7.16 and Eq. 7.17, the constant multiplying the dependent variable is the reciprocal of the time constant; that is, R/L=1/τ. We will encounter a similar situation in the derivations for the step response of an RC circuit. In Section 7.4, we will use these observations to develop a general approach to finding the natural and step responses of RL and RC circuits.

The Step Response of an RC Circuit

We can find the step response of a first-order RC circuit by analyzing the circuit shown in Fig. 7.28. For mathematical convenience, we choose the Norton equivalent of the network connected to the equivalent capacitor. Summing the currents away from the top node in Fig. 7.28 generates the differential equation

Figure 7.28 A circuit used to

illustrate the step response of a first-order RC circuit.

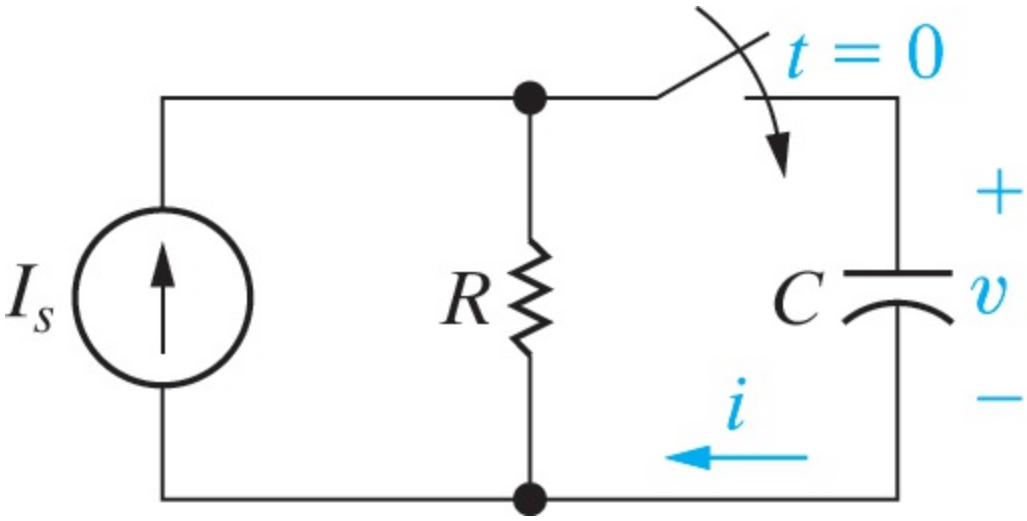


Figure 7.28 Full Alternative Text
CdvCdt+ vCR=Is.

Dividing both sides of the differential equation by C gives

dvCdt+vCRC=IsC.(7.18)

Comparing Eq. 7.18 with Eq. 7.17 reveals that the form of the solution for vC is the same as that for the current in the inductive circuit, namely, Eq. 7.11. Therefore, by substituting the appropriate variables and coefficients, we can write the solution for vC directly. The translation requires that Is replace Vs, C replace L, 1/R replace R, and V0 replace I0. We get

vC=IsR+(V0−IsR)e−t/RC, t≥0.(7.19)

Equation 7.19 indicates that after the switch has been closed, the voltage changes exponentially from its initial value V0 to a final value Vf=IsR. The time constant of the circuit, τ=RC, determines the rate of change. We can determine the final value of the voltage by analyzing the circuit in Fig. 7.28 as t→0, and we can calculate the time constant by finding the equivalent

resistance attached to the capacitor in the circuit shown in Fig. 7.28 for t≥0. Now express the equation for capacitor voltage (Eq. 7.19) in terms of the initial value of the voltage, the final value of the voltage, and the time constant of the circuit to give

Step Response of an RC Circuit

$$v(t)=Vf+(V0-Vf)e^{-t/\tau}. \quad (7.20)$$

Using Eq. 7.20, we can construct a step-by-step procedure to calculate the step response of an RC circuit.

1. Step 1: Determine the initial voltage, V0, across the capacitor. This usually involves analyzing the circuit for t<0.

2. Step 2: Calculate the time constant, τ. To do this, you need to find the equivalent resistance attached to the capacitor for t≥0.

3. Step 3: Calculate the final value of the capacitor voltage, Vf, by analyzing the circuit as t approaches infinity.

4. Step 4: Write the equation for the capacitor voltage when t≥0 by substituting the values for the initial voltage, the time constant, and the final voltage into Eq. 7.20.

5. Step 5: Calculate any other quantities of interest, such as resistor current and voltage, using resistive circuit analysis techniques.

This method is summarized in Analysis Method 7.4.

RC Step-Response Method

1. **Determine the initial capacitor voltage,** V0, by analyzing the circuit for t<0.

2. **Calculate the time constant,** $\tau=RC$, where R is the equivalent resistance connected to the inductor for $t \geq 0$

3. **Calculate the final value for the capacitor voltage**, Vf, by analyzing the circuit as $t \rightarrow \infty$

4. **Write the equation for the capacitor voltage,** $v(t)=Vf+(V0-Vf)\ e^{-t/\tau}$, for $t \geq 0$

5. **Calculate other quantities of interest** using the capacitor voltage.

Analysis Method 7.4 Finding the RC step response.

A similar derivation for the current in the capacitor yields the differential equation

$$didt+1RCi=0.(7.21)$$

Equation 7.21 has the same form as Eq. 7.16, so the solution for i is obtained by using the same translations used for the solution of Eq. 7.18. Thus

$$i=(Is-V0R)e^{-t/RC}, \quad t \geq 0+,(7.22)$$

where V0 is the initial voltage across the capacitor.

We obtained Eqs. 7.19 and 7.22 by applying a mathematical analogy to the solution for the step response of the RL circuit. Let's see whether these solutions for the RC circuit make sense in terms of known circuit behavior. From Eq. 7.19, we have already observed that the initial capacitor voltage is V0, the final capacitor voltage is IsR, and the time constant of the circuit is RC. Also note that the solution for vC is valid for $t \geq 0$. These observations are consistent with the behavior of a capacitor in parallel with a resistor when driven by a constant current source.

Equation 7.22 predicts that the current in the capacitor at $t=0+$ is $Is-V0/R$. This prediction makes sense because the capacitor voltage cannot change instantaneously, and therefore the initial current in the resistor is V0/R. The capacitor branch current changes instantaneously from zero at $t=0-$ to Is $-V0/R$ at $t=0+$. The capacitor current is zero at $t= \infty$.

Example 7.6 illustrates how to use Analysis Method 7.4 to find the step response of a first-order RC circuit.

Example 7.6 Determining the Step Response of an RC Circuit

The switch in the circuit shown in Fig. 7.29 has been in position 1 for a long time. At t=0, the switch moves to position 2. Find

Figure 7.29 The circuit for Example 7.6.

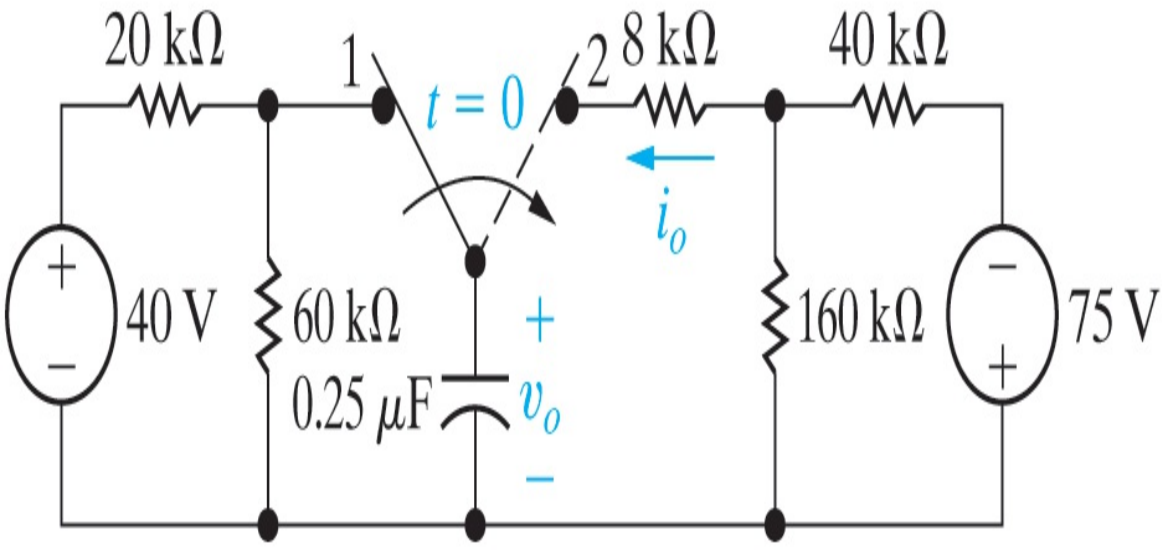


Figure 7.29 Full Alternative Text

1. vo(t) for t≥0 and
2. io(t) for t≥0+.

Solution

Use Analysis Method 7.4.

1. 1. Step 1:Determine the initial voltage across the capacitor by analyzing the circuit in Fig. 7.29 for t<0. Do this by redrawing the circuit with the switch in position 1, as shown in Fig. 7.30. Note that the capacitor behaves like an open circuit because the switch has been in position 1 for a long time. The capacitor's initial voltage is the same as the voltage across the 60 kΩ resistor, which we can find using voltage division:

Figure 7.30 The circuit in Fig. 7.29 when t<0.

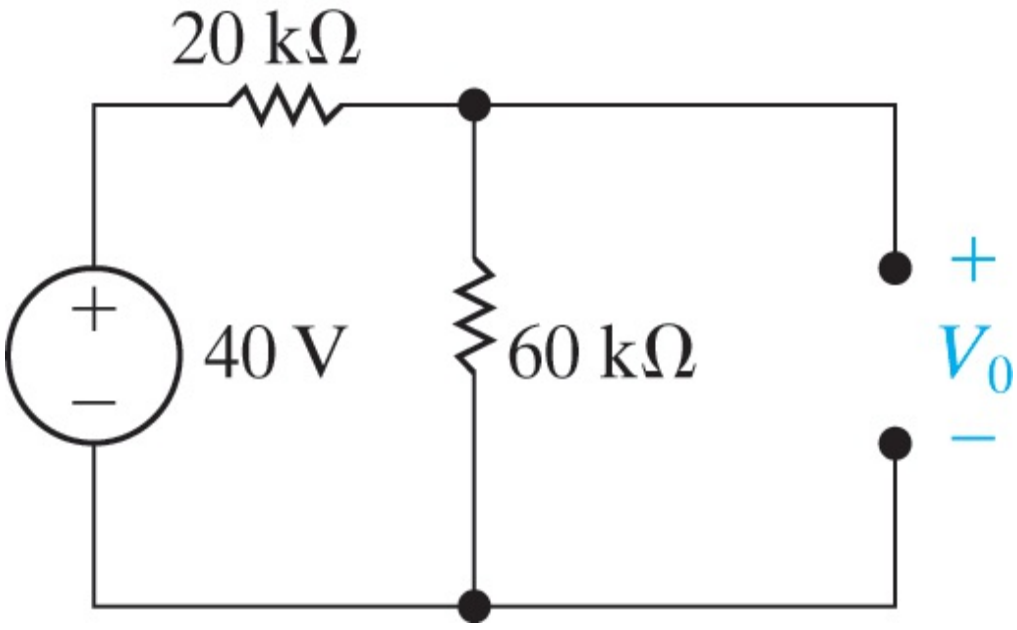


Figure 7.30 Full Alternative Text
V0=60,00060,000+20,000(40)=30V.

2. Step 2:Calculate the time constant by finding the equivalent

resistance attached to the capacitor for $t \geq 0$ in the circuit of Fig. 7.29. Begin by drawing the circuit in Fig. 7.29 with the switch in position 2, as shown in Fig. 7.31(a). Then find the Norton equivalent with respect to the terminals of the capacitor. Begin by computing the open-circuit voltage, which is given by the -75V source divided across the 40 kΩ and 160 kΩ resistors:

1. 1. **Figure 7.31 (a) The circuit in Fig. 7.29 when t≤0; (b) replacing the circuit to the right of the capacitor in part (a) with its Norton equivalent.**

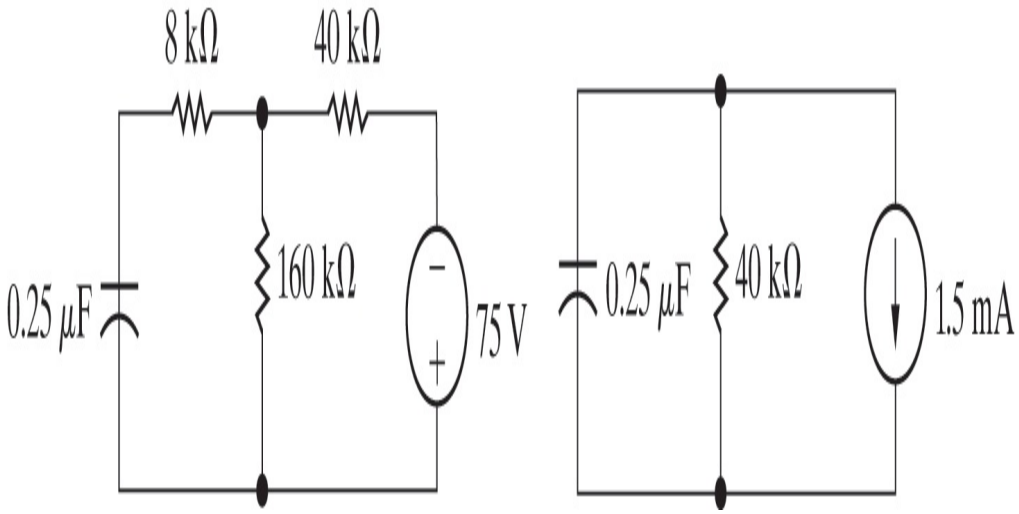


Figure 7.31 Full Alternative Text

Voc=160×103(40+160)×103(−75)=−60 V.

Next, calculate the Thévenin resistance, as seen to the right of the capacitor, by shorting the −75V source and making series and parallel combinations of the resistors:

RTh=8000+40,000 || 160,000=40 kΩ

The value of the Norton current source is the ratio of the open-circuit voltage to the Thévenin resistance, or −60/(40×103)=

−1.5mA. The resulting Norton equivalent circuit is shown in Fig. 7.31(b). From Fig. 7.31(b) we see that the equivalent resistance attached to the capacitor is 40 kΩ, so the time constant is

$\tau = RC = (40 \times 103)(0.25 \times 10 - 6) = 10\text{ms}$.

2. Step 3:Calculate the final value of the capacitor voltage by analyzing the circuit in Fig. 7.29 as $t \rightarrow \infty$. The circuit is shown in Fig. 7.32, and since the switch has been in position 2 for a long time, the capacitor behaves like an open circuit. The final capacitor voltage equals the voltage across the 40 kΩ resistor, so

Figure 7.32 The circuit in Fig. 7.29 as

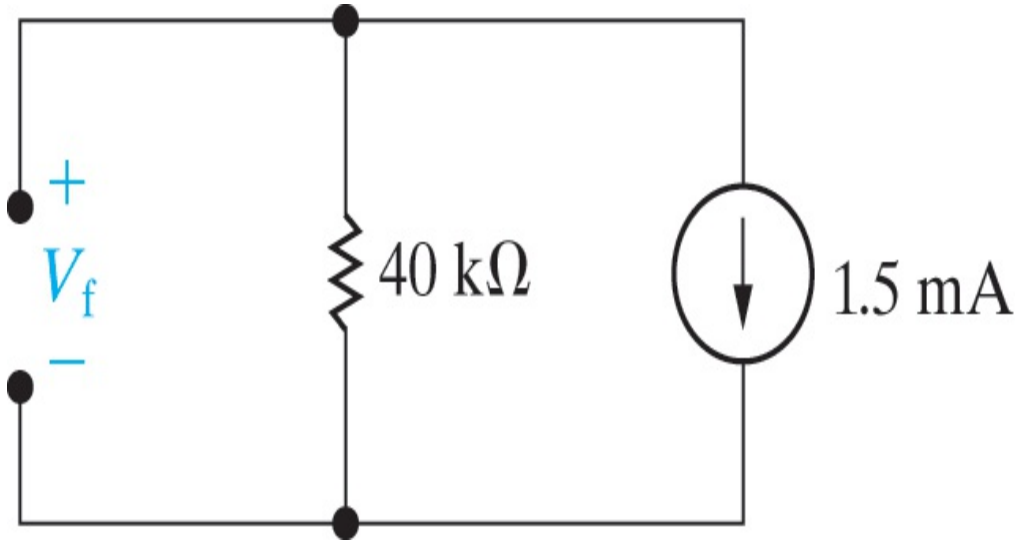


Figure 7.32 Full Alternative Text

$V_f = -(40 \times 103)(1.5 \times 10 - 3) = -60\text{V}$.

3. Step 4:Write the equation for capacitor voltage by substituting the values for initial capacitor voltage, time constant, and final capacitor voltage into Eq. 7.20 to give

$v_o = V_f + (V_0 - V_f)e^{-t/\tau} = -60 + [30 - (-60)]e^{-t/0.01} = -60 + 90e^{-100t}\text{V}, \quad t = 0.$

4. Step 5:We calculate the other quantity of interest, io, in part (b).

2. Write the solution for io using the relationship between current and voltage in a capacitor to give

   io=Cdvodt=(0.25×10-6)(-9000e-100t)=-2.25e-100t mA.

   Because dvo(0−)/dt=0, the expression for io clearly is valid only for t≥0+.

Assessment Problem

Objective 2—Be able to determine the step response of both RL and RC circuits

1. 7.6

   1. a) Find the expression for the voltage across the 160 kΩ resistor in the circuit shown in Fig. 7.29. Let this voltage be denoted vA, and assume that the reference polarity for the voltage is positive at the upper terminal of the 160 kΩ resistor.

   2. b) Specify the interval of time for which the expression obtained in (a) is valid.

   Answer:

   1. a) −60+72e−100t V;

   2. b) t≥0+.

SELF-CHECK: Also try Chapter Problems 7.52 and 7.55.

7.4 A General Solution for Step and Natural Responses

We can construct a general approach to finding either the natural response or the step response of the first-order RL and RC circuits because their differential equations all have the same form. To generalize the solution of these four possible circuits, we let x(t) represent the unknown quantity, where x(t) represents the circuit variable that is required to be continuous for all time. Thus, x(t) is the inductor current for RL natural- and step-response circuits and is the capacitor voltage for RC natural- and step-response circuits. From Eqs. 7.16, 7.17, 7.18, and 7.21, we know that the differential equation describing both the natural and step responses of the RL and RC circuits takes the form

$$\frac{dx}{dt}+\frac{x}{\tau}=K,$$

where the value of the constant K is zero for the natural response and nonzero for the step response.

How are the natural and step responses of RL circuits different, and how are they the same? Compare the circuits we analyzed to determine the natural and step responses of RL circuits using Figs. 7.3 and 7.20. The step-response circuit contains an independent source for $t \geq 0$, but the natural-response circuit does not. Now compare the analysis of the RL natural response and the RL step response (Analysis Methods 7.1 and 7.3). There is one extra calculation in the step-response analysis that computes the final value of the inductor current. We can make Analysis Methods 7.1 and 7.3 exactly the same by recognizing that the final value of the inductor current in the natural-response circuit is 0. These same observations hold when comparing both the natural and step responses of RC circuits (Figs. 7.12 and 7.28) and the natural-response analysis and step-response analysis of RC circuits (Analysis Methods 7.2 and 7.4). So we can make Analysis Methods 7.2 and 7.4 exactly the same by recognizing that the final value of the capacitor voltage in the natural-response circuit is 0.

Next, let's look at how the step response of an RL circuit is similar to and different from the step response of an RC circuit. Comparing Analysis Methods 7.2 and 7.4, we identify four important differences:

- In the RL circuit, we find the inductor current for $t \geq 0$; in the RC circuit, we find the capacitor voltage for $t \geq 0$.
- We analyze the RL circuit when $t < 0$ to find the initial inductor current; we analyze the RC circuit when $t < 0$ to find the initial capacitor voltage.
- We analyze the RL circuit when $t \geq 0$ to find the equivalent resistance attached to the inductor and use it to calculate the circuit's time constant, $\tau = L/R$; we analyze the RC circuit when $t \geq 0$ to find the equivalent resistance attached to the capacitor and use it to calculate the circuit's time constant, $\tau = RC$.
- We analyze the RL circuit as $t \to \infty$ to find the final inductor current; we analyze the RC circuit as $t \to \infty$ to find the final capacitor voltage. If the circuit exhibits a natural response instead of a step response, we know that the final values are zero without performing circuit analysis.

Based on these comparisons, we can create a general step-by-step method to calculate the natural and step responses of both RL and RC circuits.

1. Step 1:Identify the variable x(t), which is the quantity that is required to be continuous for all time. This is the inductor current in RL circuits and the capacitor voltage in RC circuits.
2. Step 2:Calculate the initial value X0, by analyzing the circuit to find x(t) for $t < 0$.
3. Step 3:Calculate the time constant, τ, for the circuit by analyzing the circuit for $t \geq 0$ to find the equivalent resistance attached to the inductor or capacitor. For RL circuits, $\tau = L/R$, and for RC circuits, $\tau = RC$.
4. Step 4:Calculate the final value Xf, by analyzing the circuit to find x(t) as $t \to \infty$. If the circuit exhibits a natural response, Xf=0, so no calculation is needed.

5. Step 5:Write the equation for x(t) by substituting the initial value X0, the time constant τ, and the final value Xf, into the expression

General Solution for Natural and Step Responses of RL and RC Circuits

$$x(t)= Xf+(X0−Xf)e−t/\tau,\ t\geq 0. (7.23)$$

6. Step 6:Use x(t) to find any other quantities of interest in the circuit.

These general steps are summarized in Analysis Method 7.5. Examples 7.7–7.9 illustrate how to use Analysis Method 7.5 to find the natural or step responses of RC or RL circuits.

General Method for Natural and Step Response of RL and RC Circuits

1. **Identify the variable x(t),** which is the inductor current for RL circuits and capacitor voltage for RC circuits.

2. **Calculate the initial value X0**, by analyzing the circuit to find x(t) for $t<0$

3. **Calculate the time constant** τ; for RL circuits $\tau=L/R$ and for RC circuits $\tau=RC$, where R is the equivalent resistance connected to the inductor or capacitor for $t\geq 0$

4. **Calculate the final value Xf**, by analyzing the circuit to find x(t) as $t\rightarrow\infty$; for the natural response, $Xf=0$

5. **Write the equation for x(t),** $x(t)=Xf+(X0-Xf)\ e^{-t/\tau}$, for $t \geq 0$.
6. **Calculate other quantities of interest** using x(t).

Analysis Method 7.5 Finding the RL and RC natural and step response.

Example 7.7 Using the General Solution Method to Find an RL Circuit's Natural Response

The switch in the circuit shown in Fig. 7.33 has been closed for a long time. At t=0 the switch opens and remains open.

Figure 7.33 The circuit for Example 7.7.

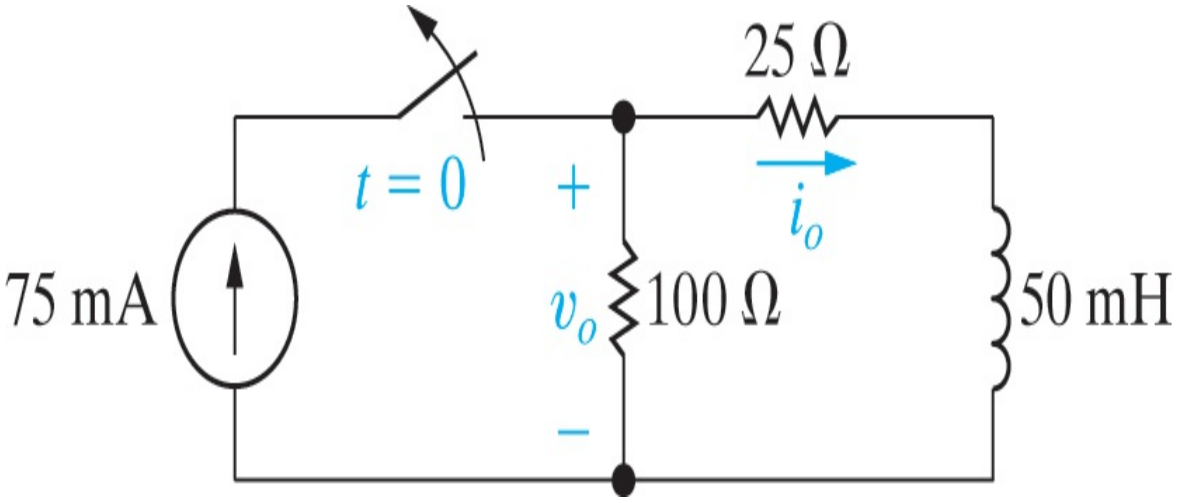


Figure 7.33 Full Alternative Text

1. What is the initial value of io?

2. What is the time constant of the circuit when the switch is open?
3. What is the final value of io?
4. What is the expression for io(t) when t≥0?
5. What is the expression for vo(t) when t≥0?
6. Find vo(0−) and vo(0+).

Solution

Use Analysis Method 7.5.

1. Step1: Identify the inductor current, io, as the variable of interest, because this is an RL circuit.

   Step2: Calculate the initial value of io. The switch has been closed for a long time, so the inductor behaves like a short circuit. Therefore, the current through the inductor is the current in the 25 Ω resistor. Using current division, the current in the 25 Ω resistor is [(100||25)/25] (0.075)=60 mA, so I0=io(0)=60 mA.

2. Step3: Calculate the time constant τ=L/R. When t≥0, the equivalent resistance attached to the inductor is the series combination of the 100 Ω and 25 Ω resistors, or 125 Ω and. Therefore,

   τ=0.05125=0.4ms.

3. Step4: Calculate the final value for the inductor current, If. This is a natural-response problem because for t≥0 there is no source in the circuit. Eventually, all of the energy stored in the inductor before the switch opens is dissipated by the resistors and the inductor current is zero, so If=0.

4. Step5: Write the equation for the inductor current by substituting the values for Io, τ, and If into Eq. 7.23 to give

$$i_o(t) = I_f + (I_0 - I_f)e^{-t/\tau} = 0 + (0.06 - 0)e^{-t/0.4\times10^{-3}} = 60e^{-2500t}\,\text{mA}, \quad t \geq 0.$$

5. Step6: Use the inductor current to find the voltage across the 100 Ω, using Ohm's law. The result is

   $$v_o(t) = -100i_o = -6e^{-2500t}\ \text{V}, \quad t \geq 0+.$$

6. From part (a), when t<0 the switch is closed, and the current divides between the 100 Ω and 25 Ω resistors. We know that the current in the 25 Ω is 60 mA, so the current in the 100 Ω must be 75–60=15 mA. Using Ohm's law,

   $$v_o(0-) = 100(0.015) = 1.5\text{V}.$$

   From part (e)

   $$v_o(0+) = -6e^{-2500(0+)} = -6\text{V}.$$

   There is a discontinuity in the voltage across the 100 Ω resistor at t=0.

Example 7.8 Using the General Solution Method to Find an RC Circuit's Step Response

The switch in the circuit shown in Fig. 7.34 has been in position a for a long time. At t=0 the switch is moved to position b.

Figure 7.34 The circuit for Example 7.8.

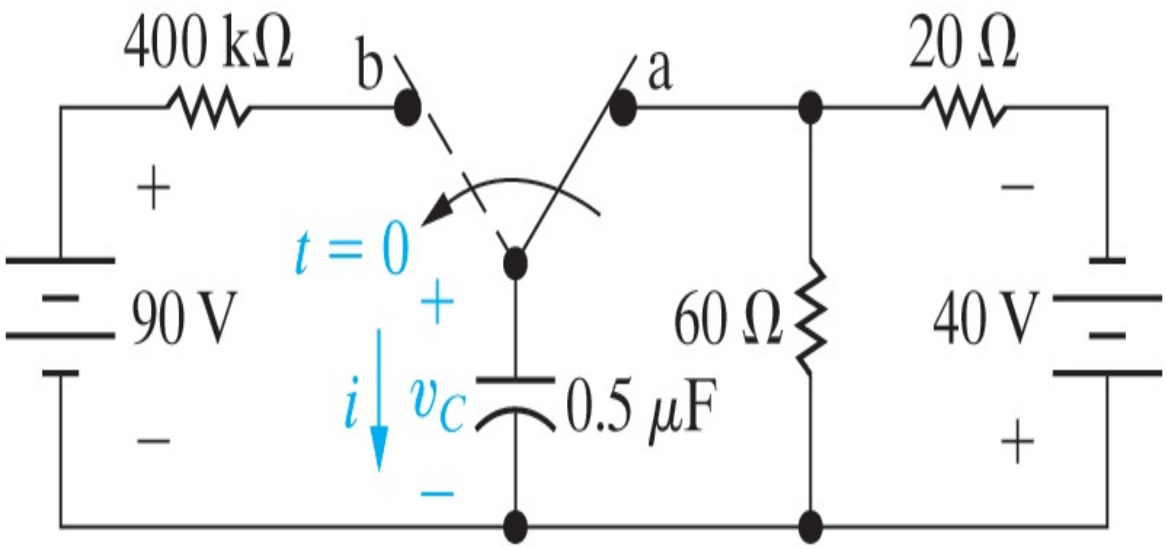


Figure 7.34 Full Alternative Text

1. What is the expression for vC(t) when t≥0?
2. What is the expression for i(t) when t≥0+?
3. How long after the switch is in position b does the capacitor voltage equal zero?
4. Plot vC(t) and i(t) versus t.

Solution

Use Analysis Method 7.5.

1. 1. Step 1: Identify the capacitor voltage, vC, as the variable of interest, because this is an RC circuit.

   2. Step 2: Calculate the initial value of vC. The switch has been in position a for a long time, so the capacitor looks like an open circuit. Therefore, the voltage across the capacitor is the voltage across the 60 Ω resistor. Using voltage division, the voltage across the 60 Ω resistor is [60/(60+20)](40)=30 V, positive at the lower

terminal of the resistor. But vC is positive at the upper terminal of the capacitor, so V0=vC(0)=−30 V.

3. Step 3: Calculate the time constant τ=RC. When t≥0, the equivalent resistance attached to the capacitor has the value 400 kΩ. Therefore,

   τ=(400×103)(0.5×10−6)=0.2s.

4. Step 4: Calculate the final value for the capacitor voltage, Vf. As t→∞, the switch has been in position b for a long time, and the capacitor behaves like an open circuit in the presence of the 90 V source. Because of the open circuit, there is no current in the 400 kΩ resistor, so Vf=90 V.

5. Step 5: Write the equation for capacitor voltage by substituting the values for Vo, τ, and Vf into Eq. 7.23 to give

   vC(t)=Vf+(V0−Vf)e−t/τ=90+(−30−90)e−t/0.2=90−120e−5t V, t≥0.

2. Step6: Use the relationship between voltage and current for capacitors to find the capacitor voltage. The result is

   i(t)=CdvCdt=(0.5×10−6)[−5(−120e−5t)]=300e−5tμA, t≥0+.

3. To find how long the switch must be in position b before the capacitor voltage becomes zero, we solve the equation derived in (a) for the time when vC(t)=0:

   120e−5t=90 or e5t=12090,

   so

   t=15 ln (43)=57.54 ms.

   Note that when vC=0, the voltage drop across the 400 kΩ resistor is 90 V so i=225 μA.

4. Figure 7.35 shows the graphs of vC(t) and i(t) versus t.

Figure 7.35 The current and voltage waveforms for Example 7.8.

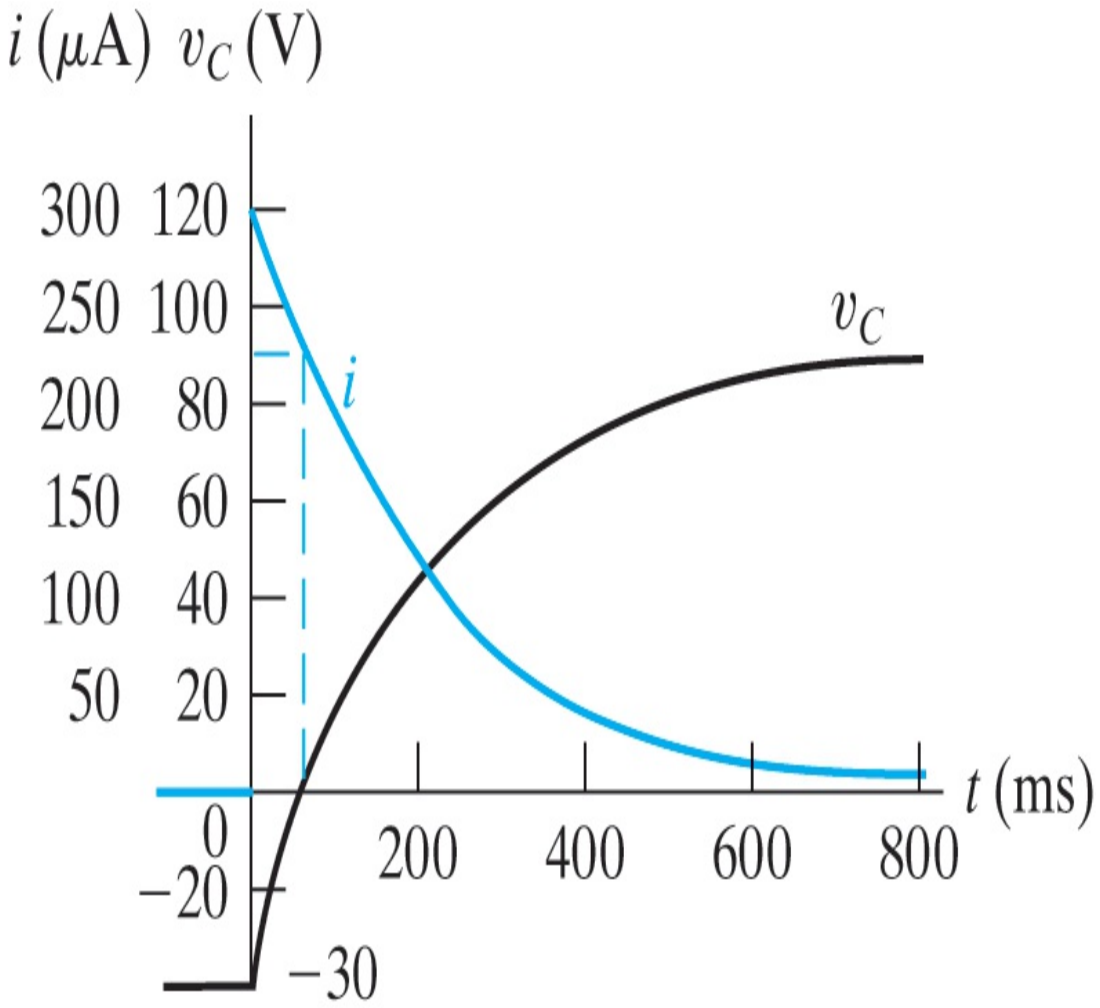


Figure 7.35 Full Alternative Text

Example 7.9 Using the General

Solution Method to Find an RL Circuit's Step Response

The switch in the circuit shown in Fig. 7.36 has been open for a long time. At t=0 the switch is closed. Find the expression for

Figure 7.36 The circuit for Example 7.9.

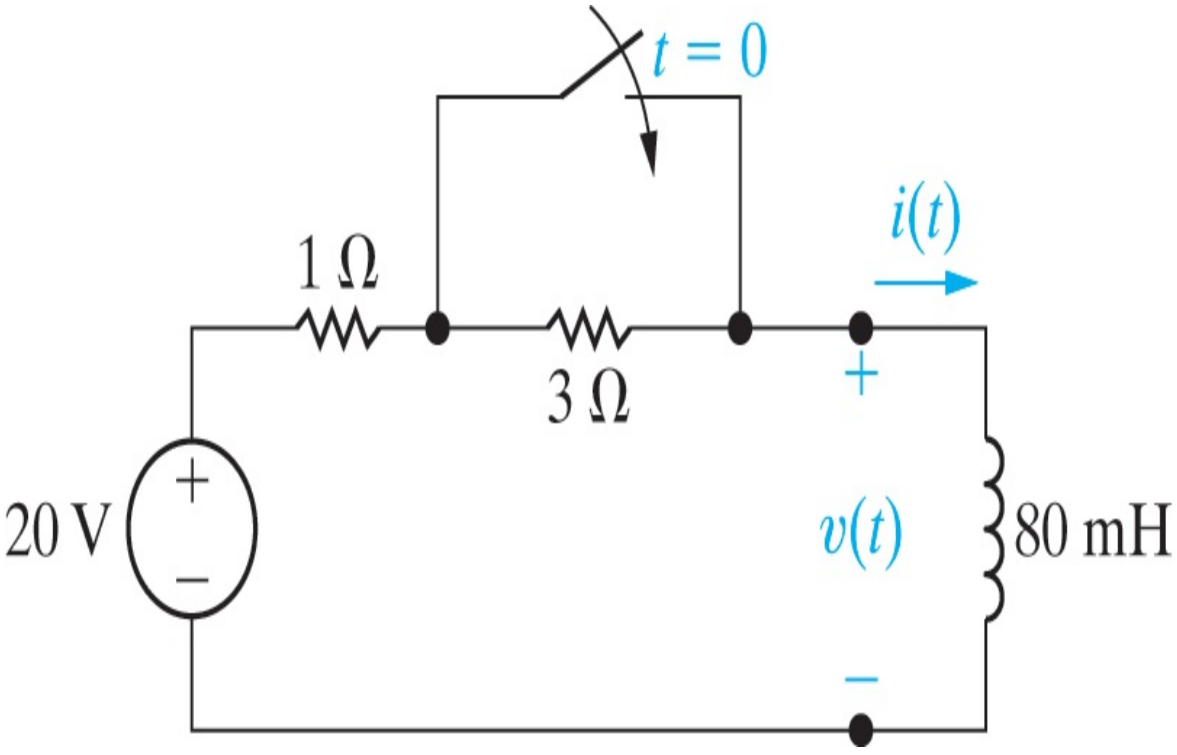


Figure 7.36 Full Alternative Text

1. i(t) when t≥0 and
2. v(t) when t≥0+.

Solution

Use Analysis Method 7.5.

1. 1. Step 1: Identify the inductor current, i, as the variable of interest, because this is an RL circuit.

   2. Step 2: Calculate the initial value of i. The switch has been open for a long time, so from Ohm's law, the initial current in the inductor is 20/(1+3)=5 A. Thus, I0=i(0)=5 A.

   3. Step 3: Calculate the time constant $\tau=L/R$. When $t\geq 0$, the switch is closed, shunting the 3 Ω resistor. The remaining resistance attached to the inductor has the value 1 Ω. Therefore,

      $\tau=80\times10-31=80$ ms.

   4. Step 4: Calculate the final value for the inductor current. As $t\rightarrow\infty$, the switch has been closed for a long time, and the inductor behaves like a short circuit in the presence of the 20 V source. Using Ohm's law, the current in the inductor is 20/1=20 A, so If=20 A.

   5. Step 5: Write the equation for inductor current by substituting the values for Io, τ, and If into Eq. 7.23 to give

      $i(t)=If+(I0-If)e-t/\tau=20+(5-20)e-t/0.08=20-15e-12.5tA, \quad t\geq 0.$

2. Step6: Use the relationship between voltage and current for inductors to find the inductor voltage. The result is

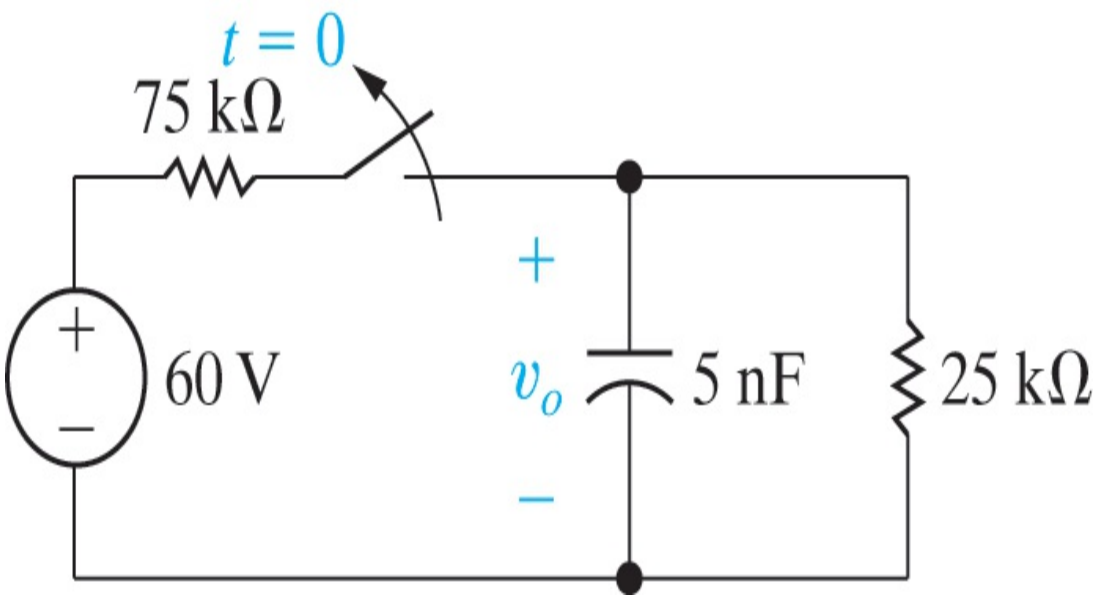
$v(t)=Ldidt=(80\times10-3)[\ -12.5(-15e-12.5t)\ ]=15e-12.5tV, \quad t\geq 0+.$

Assessment Problems

Objectives 1 and 2—Be able to determine the natural and step response

of both RL and RC circuits

1. 7.7 The switch in the circuit has been closed for a long time. At t=0 the switch opens and stays open. Find the expression for vo(t) for t≥0.

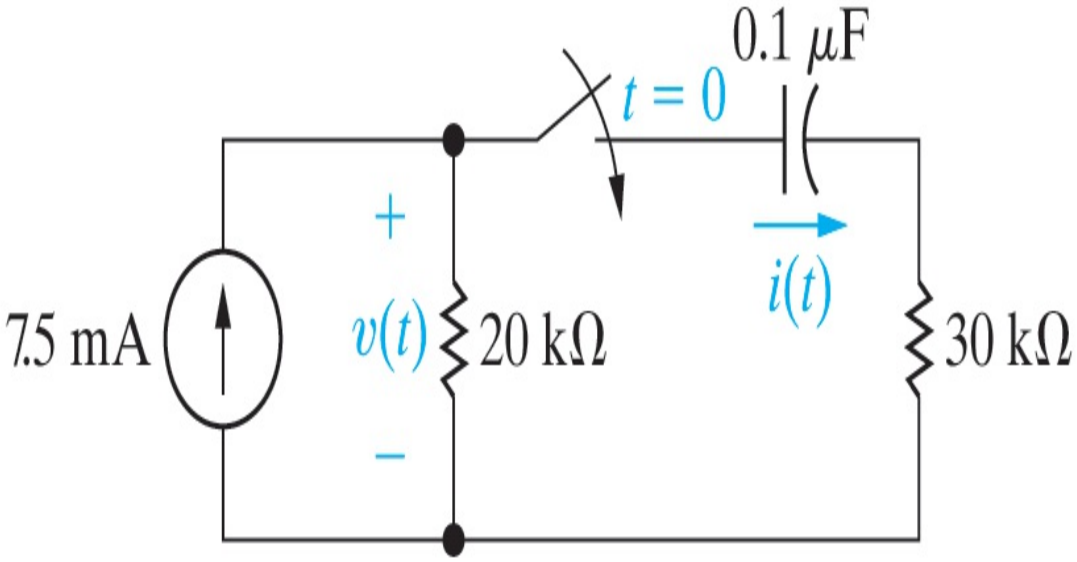


7.4-9 Full Alternative Text

Answer:

15e−8000t V, t≥0

2. 7.8 The switch in the circuit has been open for a long time. The initial charge on the capacitor is zero. At t=0 the switch is closed. Find the expression for



7.4-10 Full Alternative Text

1. a)

   i(t) for t≥0+ and

2. b)

   v(t) for t≥0+.

Answer:

1. a) 3e−200t mA, t≥0+;
2. b) (150–60e−200t V, t≥0+

SELF-CHECK: Also try Chapter Problems 7.56 and 7.59.

Example 7.10 shows that Eq. 7.23 can even be used to find the step response of some circuits containing magnetically coupled coils.

Example 7.10 Determining the Step Response of a Circuit with

Magnetically Coupled Coils

There is no energy stored in the circuit in Fig. 7.37 at the time the switch is closed.

Figure 7.37 The circuit for Example 7.10.

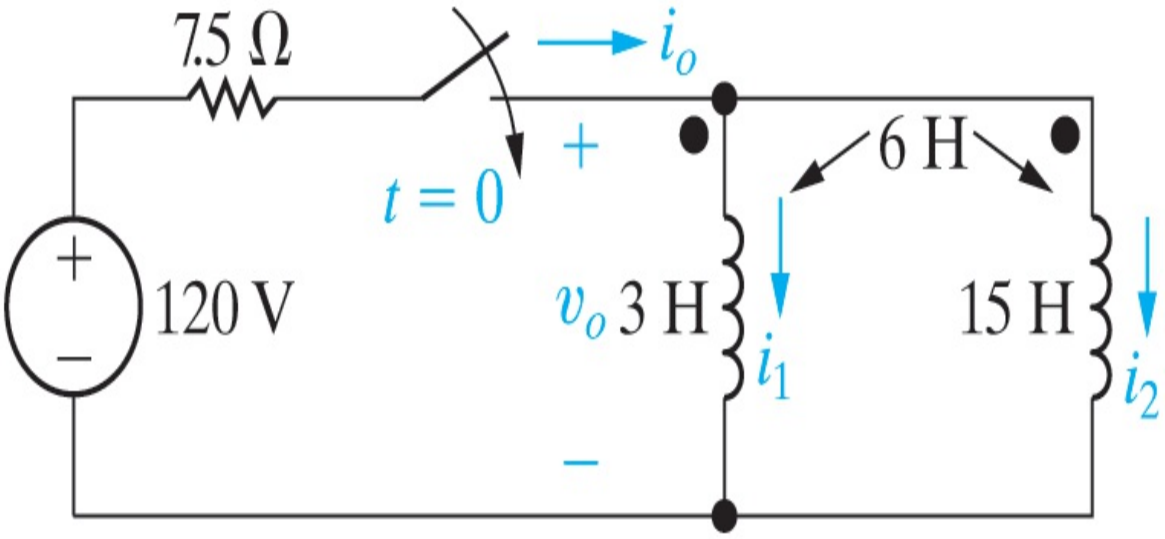


Figure 7.37 Full Alternative Text

1. Find the solutions for io, vo, i1, and i2.

2. Show that the solutions obtained in (a) make sense in terms of known circuit behavior.

Solution

1. For the circuit in Fig. 7.37, the magnetically coupled coils can be replaced by a single inductor having an inductance of

Leq=L1L2−M2L1+L2−2M=45−3618−12=1.5 H.

(See Problem 6.38.) It follows that the circuit in Fig. 7.37 can be simplified, as shown in Fig. 7.38. We can apply Analysis Method 7.5 to the circuit in Fig. 7.38.

Figure 7.38 The circuit in Fig. 7.37 with the magnetically coupled coils replaced by an equivalent coil.

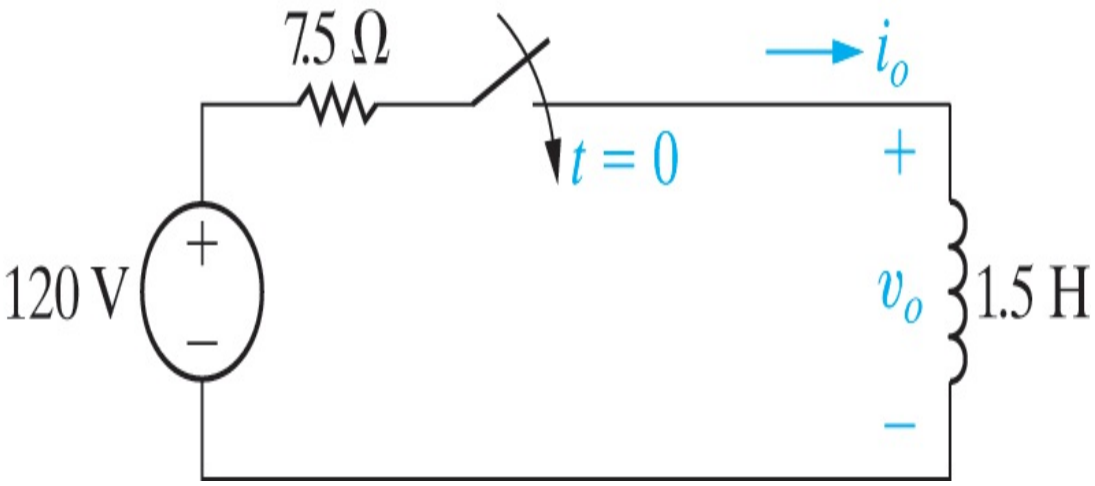


Figure 7.38 Full Alternative Text

1. Step 1: Identify the inductor current, io, as the variable of interest, because this is an RL circuit.

2. Step 2: Calculate the initial value of i. By hypothesis, there is no initial energy stored in the coils, so there is no initial current in the equivalent 1.5 H inductor. Thus, I0=i(0)=0.

3. Step 3: Calculate the time constant $\tau = L/R$. When $t \geq 0$, the switch is closed and the resistance attached to the inductor has the value 7.5 Ω. Therefore,

   $\tau = 1.5 7.5 = 0.2 s.$

4. Step 4: Calculate the final value for the inductor current. As $t \rightarrow \infty$, the switch has been closed for a long time, and the inductor behaves like a short circuit in the presence of the 120 V source. Using Ohm's law, the current in the inductor is 120/7.5=16 A, so If=16 A.

5. Step 5: Write the equation for inductor current by substituting the values for Io, τ, and If into Eq. 7.23 to give

   $i_o(t) = I_f + (I_0 - I_f)e^{-t/\tau} = 16 + (0 - 16)e^{-t/0.2} = 16 - 16e^{-5t} A, \quad t \geq 0.$

6. Step 6: Use the inductor current and the relationship between voltage and current for inductors to find the inductor voltage. The result is

   $v(t) = L\frac{di}{dt} = (1.5)[\ -5(-16e^{-5t})\ ] = 120e^{-5t} V, \quad t \geq 0+.$

   To find i1 and i2 we use KVL for the mesh containing the coupled coils in Fig. 7.37 to see that

   $3\frac{di_1}{dt} + 6\frac{di_2}{dt} = 6\frac{di_1}{dt} + 15\frac{di_2}{dt}$

   or

   $\frac{di_1}{dt} = -3\frac{di_2}{dt}.$

   It also follows from Fig. 7.37 and KCL that $i_o = i_1 + i_2$, so

   $\frac{di_o}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} = -3\frac{di_2}{dt} + \frac{di_2}{dt} = -2\frac{di_2}{dt}.$

   Therefore

   $80e^{-5t} = -2\frac{di_2}{dt}.$

Because i2(0) is zero, we have

$$i2=\int 0t-40e-5x\,dx=-8+8e-5t\ A,\quad t\geq 0.$$

Using io=i1+ i2, we get

$$i1=(16-16e-5t)-(-8+8e-5t)=24-24e-5tA,\quad t\geq 0.$$

2. First, we observe that io(0), i1(0), and i2(0) are all zero, which is consistent with the statement that no energy is stored in the circuit at the instant the switch is closed. Then we observe that vo(0+)=120 V, which is consistent with the fact that io(0)=0. Now we see that the solutions for i1 and i2 are consistent with the solution for vo by observing

   $$vo=3di1dt+6di2dt=360e-5t-240e-5t=120e-5t\ V,\quad t\geq 0+,$$

   or

   $$vo=6di1dt+15di2dt=720e-5t-600e-5t=120e-5t\ V,\quad t\geq 0+.$$

   The final values of i1 and i2 can be checked using flux linkages. The flux linking the 3 H coil ($\lambda 1$) must be equal to the flux linking the 15 H coil ($\lambda 2$) because

   $$vo=d\lambda 1dt=d\lambda 2dt.$$

   Now

   $$\lambda 1=3i1+6i2\ \text{Wb-turns}$$

   and

   $$\lambda 2=6i1+15i2\ \text{Wb-turns}.$$

   Regardless of which expression we use, we obtain

   $$\lambda 1=\lambda 2=24-24e-5t\ \text{Wb-turns}.$$

   Note the solution for $\lambda 1$ or $\lambda 2$ is consistent with the solution for vo.

The final value of the flux linking either coil 1 or coil 2 is 24 Wb-turns; that is,

$\lambda 1(\infty)=\lambda 2(\infty)=24$ Wb-turns.

The final value of i1 is

$i1(\infty)=24$ A

and the final value of i2 is

$i2(\infty)=-8$ A.

The consistency between these final values for i1 and i2 and the final value of the flux linkage can be seen from the expressions:

$\lambda 1(\infty)=3i1(\infty)+6i2(\infty)=3(24)+6(-8)=24$ Wb-turns,$\lambda 2(\infty)=6i1(\infty)+15i2(\infty)=6(24)+15(-8)=24$ Wb-turns.

The final values of i1 and i2 can only be checked via flux linkage because at $t=\infty$ the two coils are ideal short circuits. We cannot use current division when the two branches have no resistance

SELF-CHECK: Assess your understanding of this material by using the general solution method to solve Chapter Problems 7.68 and 7.69.

7.5 Sequential Switching

Whenever switching occurs at two or more distinct times in a circuit, we have **sequential switching**. For example, a single, two-position switch may be in position 1 at t1 and in position 2 at t2, or multiple switches may be opened or closed in sequence. We determine the voltages and currents generated by a switching sequence using the techniques described previously in this chapter, primarily Analysis Method 7.5. We derive the expressions for v(t) and i(t) for a given position of the switch or switches and then use these solutions to determine the initial conditions for the next position of the switch or switches.

With sequential switching problems, a premium is placed on obtaining the initial value x(t0). Recall that anything but inductive currents and capacitive voltages can change instantaneously at the time of switching. Thus, solving first for inductive currents and capacitive voltages is even more pertinent in sequential switching problems. Drawing the circuit that pertains to each time interval in such a problem is often helpful in the solution process.

Examples 7.11 and 7.12 illustrate the analysis techniques for circuits with sequential switching. The first example is a natural-response problem with two switching times, and the second is a step-response problem.

Example 7.11 Analyzing an RL Circuit that has Sequential Switching

The two switches in the circuit shown in Fig. 7.39 have been closed for a long time. At t=0, switch 1 is opened. Then, 35 ms later, switch 2 is opened.

Figure 7.39 The circuit for Example 7.11.

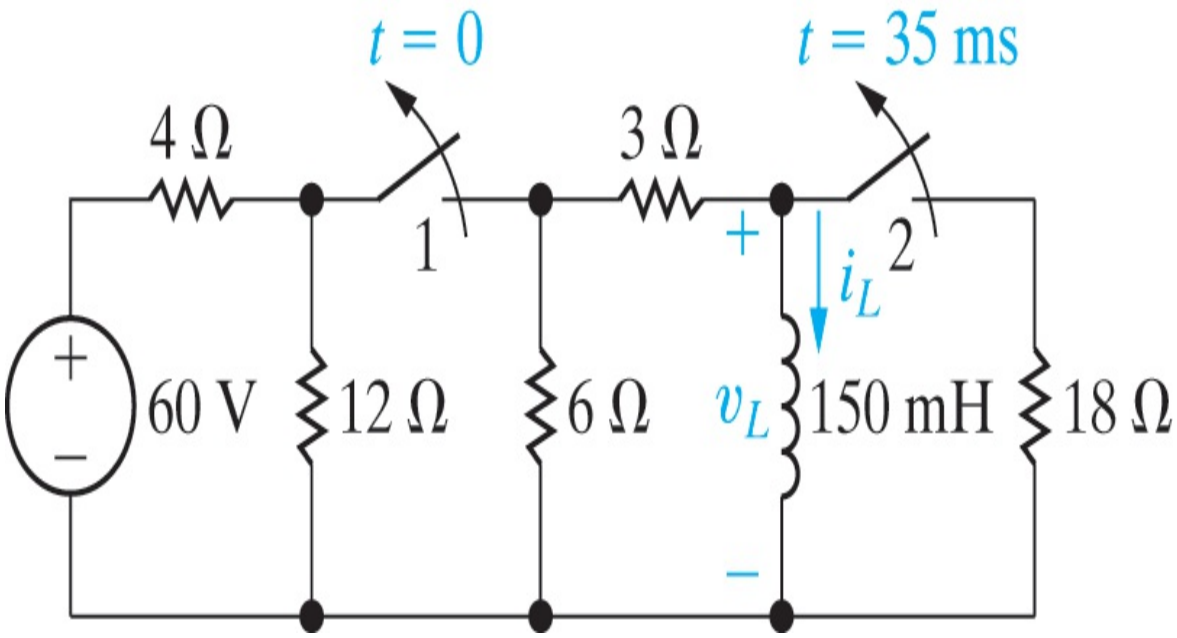


Figure 7.39 Full Alternative Text

1. Find iL(t) for 0≤t≤35 ms.
2. Find iL for t≥35 ms.
3. What percentage of the initial energy stored in the 150 mH inductor is dissipated in the 18 Ω resistor?
4. Repeat (c) for the 3 Ω resistor.
5. Repeat (c) for the 6 Ω resistor.

Solution

1. We use Analysis Method 7.5 to solve this problem.

1. Step 1: Identify the inductor current, iL, as the variable of interest, because this is an RL circuit.

2. Step 2: Calculate the initial value of i. For t<0 both switches are closed, causing the 150 mH inductor to short-circuit the 18 Ω resistor. The equivalent circuit is shown in Fig. 7.40. We determine the initial current in the inductor by solving for iL(0−) in the circuit shown in Fig. 7.40. After making several source transformations, we find iL(0−) to be 6 A, so I0=6 A.

Figure 7.40 The circuit shown in Fig. 7.39, for t<0.

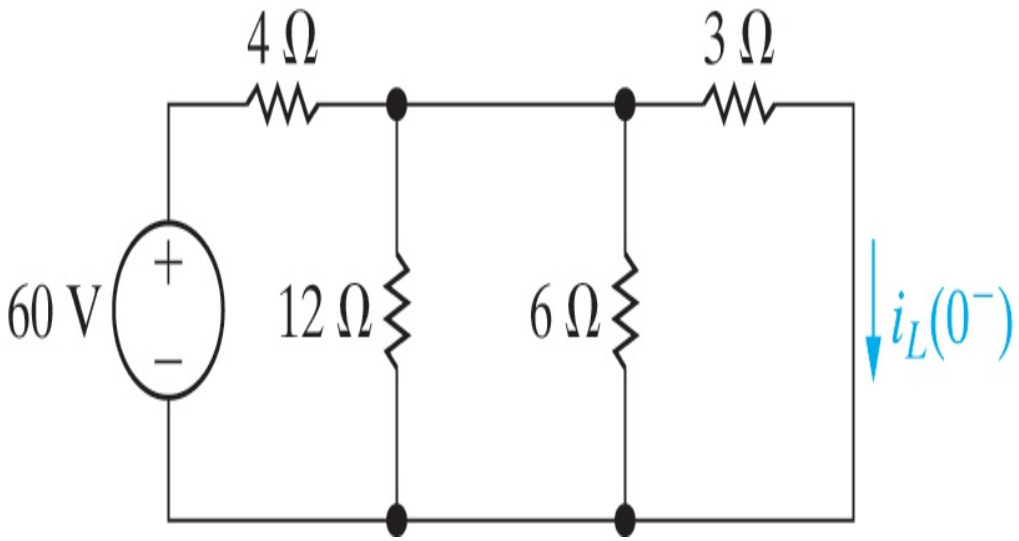


Figure 7.40 Full Alternative Text

3. Step 3: Calculate the time constant τ=L/R. For 0≤t≤35 ms, switch 1 is open (switch 2 is closed), which disconnects the 60 V voltage source and the 4 Ω and 12 Ω resistors from the circuit. The inductor is no longer behaving as a short circuit (because the dc source is no longer in the circuit), so the 18 Ω resistor is no longer short-circuited. The equivalent circuit is shown in Fig. 7.41. Note that the equivalent resistance across the terminals of the inductor is the

parallel combination of 9 Ω and 18 Ω, or 6 Ω. Therefore,

Figure 7.41 The circuit shown in Fig. 7.39, for 0≤t≤35 ms.

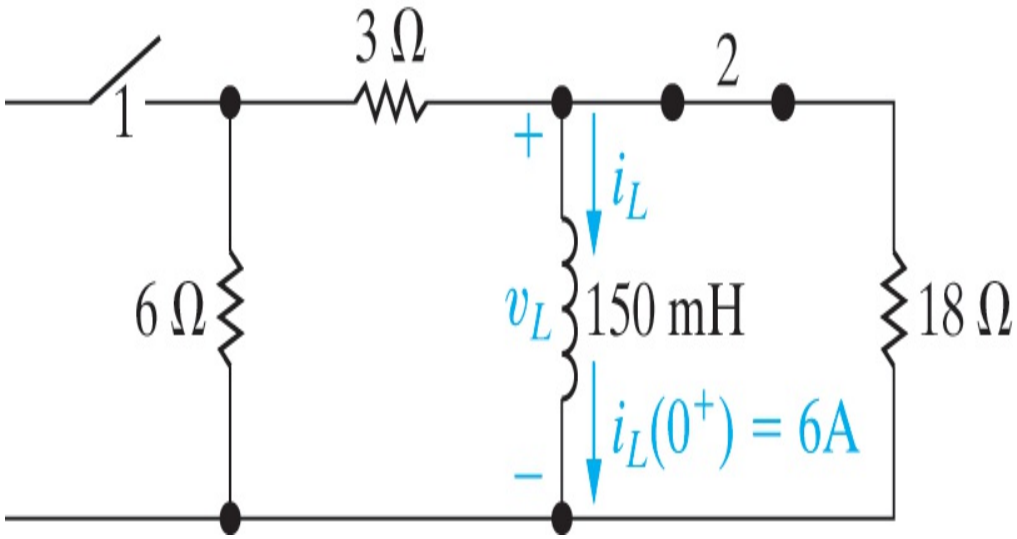


Figure 7.41 Full Alternative Text
τ=0.156=25ms.

4. Step 4:Calculate the final value for the inductor current. For 0≤t≤35 ms there is no source in the circuit, so during this time period we have a natural-response problem and the final value of the inductor current is zero. Thus, If=0.

5. Step 5:Write the equation for the inductor current for 0≤t≤35 ms by substituting the values for Io, τ, and If into Eq. 7.23 to give

   iL(t)=If+(I0−If)e−t/τ=0+(6−0)e−t/0.025=6e−40tA,0≤t≤35 ms.

   Now we repeat Steps 2–5 for t≥35 ms.

6. Step 2:Calculate the initial value of the inductor current for this

time segment. When t=35ms, the value of the inductor current is determined from the inductor current equation for the previous time segment because the inductor current must be continuous for all time. So,

iL(35×10−3)=6e−40(35×10−3)=6e−1.4=1.48A.

Thus, for t≥35 ms, I0=1.48 A.

7. Step 3:Calculate the time constant τ=L/R. For t≥35 ms, both switches are open, and the circuit reduces to the one shown in Fig. 7.42. Note that the equivalent resistance across the terminals of the inductor is the series combination of 3 Ω and 6 Ω or 9 Ω. Therefore,

Figure 7.42 The circuit shown in Fig. 7.39, for t≥35 ms.

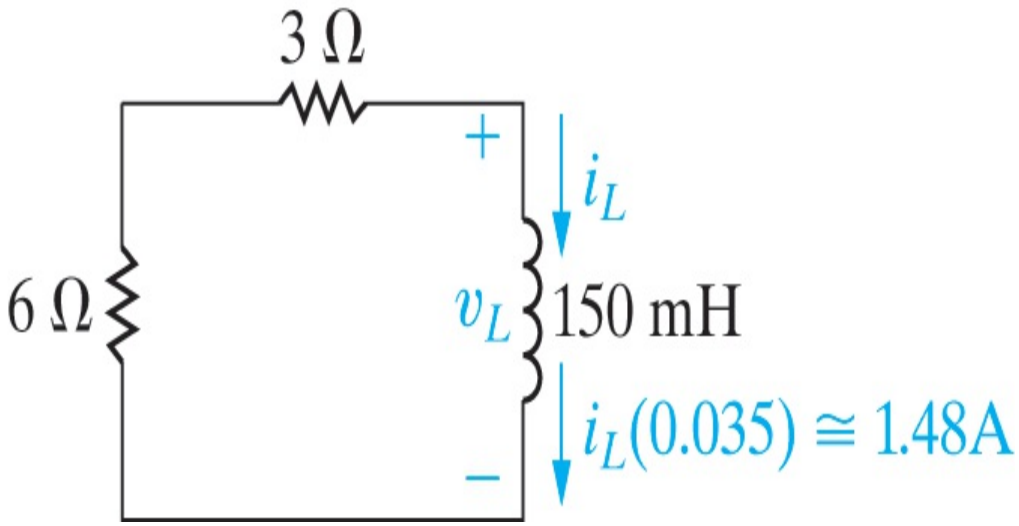


Figure 7.42 Full Alternative Text
τ=0.159=16.67ms.

8. Step 4:Calculate the final value for the inductor current. For $t \geq 35$ ms there is no source in the circuit, so during this time period we have a natural-response problem, and the final value of the inductor current is zero. Thus, $I_f = 0$.

9. Step 5:Write the equation for inductor current when $t \geq 35$ ms by substituting the values for I_o, τ, and I_f into Eq. 7.23 to give

   $i_L(t) = I_f + (I_0 - I_f)e^{-(t-t_0)/\tau} = 0 + (1.48 - 0)e^{-(t-0.035)/0.01667} = 1.48e^{-60(t-0.035)}$ A, $t \geq 35$ ms.

   Note that when switch 2 is opened, the time constant changes and the exponential function is shifted in time by 35 ms.

10. Step 6:Use the inductor current to solve the remaining parts of this problem.

3. The 18 Ω resistor is in the circuit only during the first 35 ms of the switching sequence. During this interval, the voltage across the resistor is

   $v_L = 0.15\frac{d}{dt}(6e^{-40t}) = -36e^{-40t}$ V, $0 < t < 35$ ms.

   The power dissipated in the 18 Ω resistor is

   $p = \frac{v_L^2}{18} = 72e^{-80t}$ W, $0 < t < 35$ ms.

   Hence, the energy dissipated is

   $w = \int_0^{0.035} 72e^{-80t}\, dt = 72 - 80e^{-80t}\Big|_0^{0.035} = 0.9(1 - e^{-2.8}) = 845.27$ mJ.

   The initial energy stored in the 150 mH inductor is

   $w_0 = \frac{1}{2}(0.15)(6)^2 = 2.7\text{J} = 2700$ mJ.

   Therefore, $(845.27/2700) \times 100$, or 31.31% of the initial energy stored in the 150 mH inductor is dissipated in the 18 Ω resistor.

4. For $0 < t < 35$ ms, the voltage across the 3 Ω resistor is

$v_{3\Omega} = (v_{L9})(3) = \frac{1}{3}v_L = -12e^{-40t}$ V.

Therefore, the energy dissipated in the 3 Ω resistor in the first 35 ms is

$$w_{3\Omega} = \int_0^{0.035} (-12e^{-40t})^2 3\,dt = 0.6(1 - e^{-2.8}) = 563.51 \text{ mJ}.$$

For $t > 35$ms, the current in the 3 Ω resistor is

$$i_{3\Omega} = i_L = (6e^{-1.4})e^{-60(t-0.035)} \text{ A}.$$

Hence, the energy dissipated in the 3 Ω resistor for $t > 35$ ms is

$$w_{3\Omega} = \int_{0.035}^{\infty} 3i_{3\Omega}^2\,dt = \int_{0.035}^{\infty} 3(6e^{-1.4})^2(e^{-60(t-0.035)})^2\,dt = 108e^{-2.8} \times e^{-120(t-0.035)}{-120}\Big|_{0.035}^{\infty} = \frac{108}{120}e^{-2.8} = 54.73 \text{ mJ}.$$

The total energy dissipated in the 3 Ω resistor is

$$w_{3\Omega(\text{total})} = 563.51 + 54.73 = 618.24 \text{ mJ}.$$

The percentage of the initial energy stored is

$$\frac{618.24}{2700} \times 100 = 22.90\%.$$

5. Because the 6 Ω resistor is in series with the 3 Ω resistor, the energy dissipated and the percentage of the initial energy stored will be twice that of the 3 Ω resistor:

   $w_{6\Omega(\text{total})} = 1236.48$ mJ,

   and the percentage of the initial energy stored is 45.80%. We check these calculations by observing that

   $$1236.48 + 618.24 + 845.27 = 2699.99 \text{ mJ}$$

   and

   $$31.31 + 22.90 + 45.80 = 100.01\%.$$

   The small discrepancies in the summations are the result of roundoff

errors.

Example 7.12 Analyzing an RC Circuit that Has Sequential Switching

The uncharged capacitor in the circuit shown in Fig. 7.43 is initially switched to terminal a of the three-position switch. At t=0, the switch is moved to position b, where it remains for 15 ms. After the 15 ms delay, the switch is moved to position c, where it remains indefinitely.

Figure 7.43 The circuit for Example 7.12.

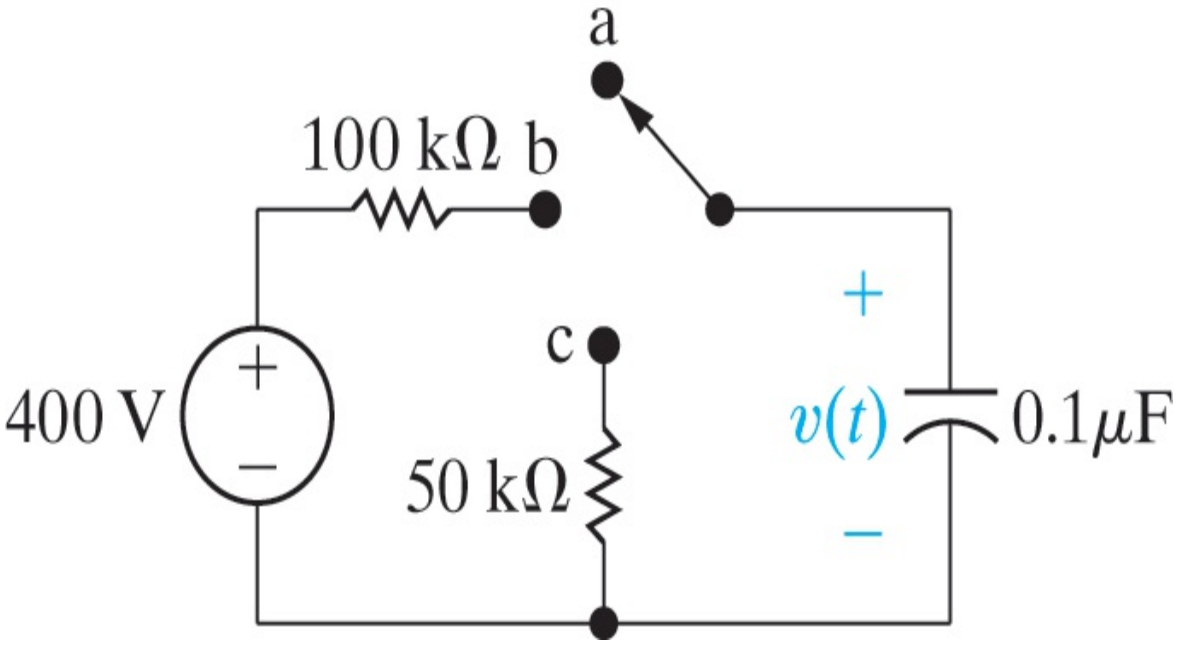


Figure 7.43 Full Alternative Text

1. Derive the numerical expression for the voltage across the capacitor.
2. Plot the capacitor voltage versus time.
3. When will the voltage on the capacitor equal 200 V?

Solution

1. We use Analysis Method 7.5 to solve this problem.
    1. Step 1: Identify the capacitor voltage, v, as the variable of interest, because this is an RC circuit.
    2. Step 2:Calculate the initial value of v. For $t<0$, the capacitor is initially uncharged, so $V0=v(0)=0$ V.
    3. Step 3:Calculate the time constant $t=RC$. When $0\leq t\leq 15$ ms, the equivalent resistance attached to the capacitor has the value 100 kΩ. Therefore,

        $\tau=(100\times 103)(0.1\times 10-6)=10$ ms.
    4. Step 4:Calculate the final value for the capacitor voltage, Vf. If the switch were to remain in position b for a long time, the capacitor would eventually behave like an open circuit in the presence of the 400 V source. Because of the open circuit, there would be no current in the 100 kΩ resistor, so $Vf=400$ V.
    5. Step 5:Write the equation for capacitor voltage by substituting the values for Vo, τ, and Vf into Eq. 7.23 to give

        $v(t)=Vf+(V0-Vf)e-t/\tau=400+(0-400)e-t/0.01=400-400e$ $-100t$ V,$0\leq t\leq 15$ ms.

        Now we repeat Steps 2–5 for the next time interval, $t\geq 15$ ms.
    6. Step 2:Calculate the initial value of v. At $t=15$ ms, the capacitor

voltage is determined by the equation we derived for the previous time interval. So,

$v(0.015)=400-400e-100(0.015)=400-400e-1.5=310.75$ V.

Thus, $V0=v(0.015)=310.75$ V.

7. Step 3:Calculate the time constant $t=RC$. When $t\geq 15$ ms, the equivalent resistance attached to the capacitor has the value 50 kΩ. Therefore,

   $\tau=(50\times 103)(0.1\times 10-6)=5$ ms.

8. Step 4:Calculate the final value for the capacitor voltage, Vf. For $t\geq 15$ ms, the switch remains in position c for a long time, and there is no source in the circuit. During this time interval, the circuit exhibits a natural response, so $Vf=0$.

9. Step 5:Write the equation for capacitor voltage by substituting the values for Vo, τ, and Vf into Eq. 7.23 to give

   $v(t)=Vf+(V0-Vf)e-(t-t0)/\tau=0+(310.75-0)e-(t-0.015)/0.005=310.75e-200(t-0.015)$ V, $t\geq 15$ ms.

10. Step 6:Use the capacitor voltage to solve the remaining parts of this problem.

2. Figure 7.44 shows the plot of v versus t.

Figure 7.44 The capacitor voltage for Example 7.12.

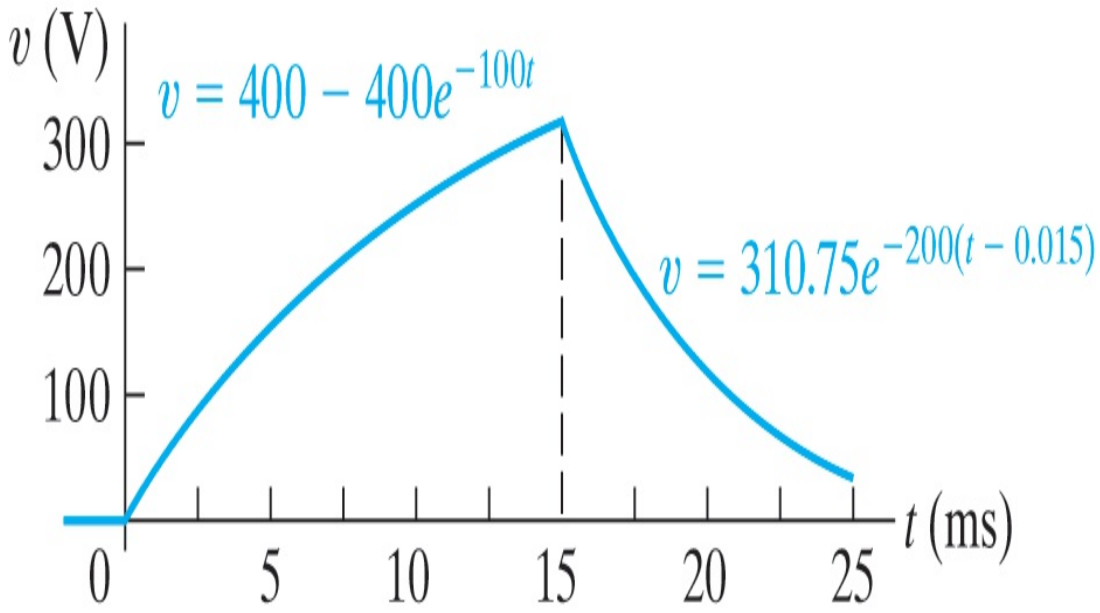


Figure 7.44 Full Alternative Text

3. The plot in Fig. 7.44 reveals that the capacitor voltage will equal 200 V at two different times: once in the interval between 0 and 15 ms and once after 15 ms. We find the first time by solving the expression

   200=400−400e−100t1,

   which yields t1=6.93 ms. We find the second time by solving the expression

   200=310.75e−200(t2−0.015).

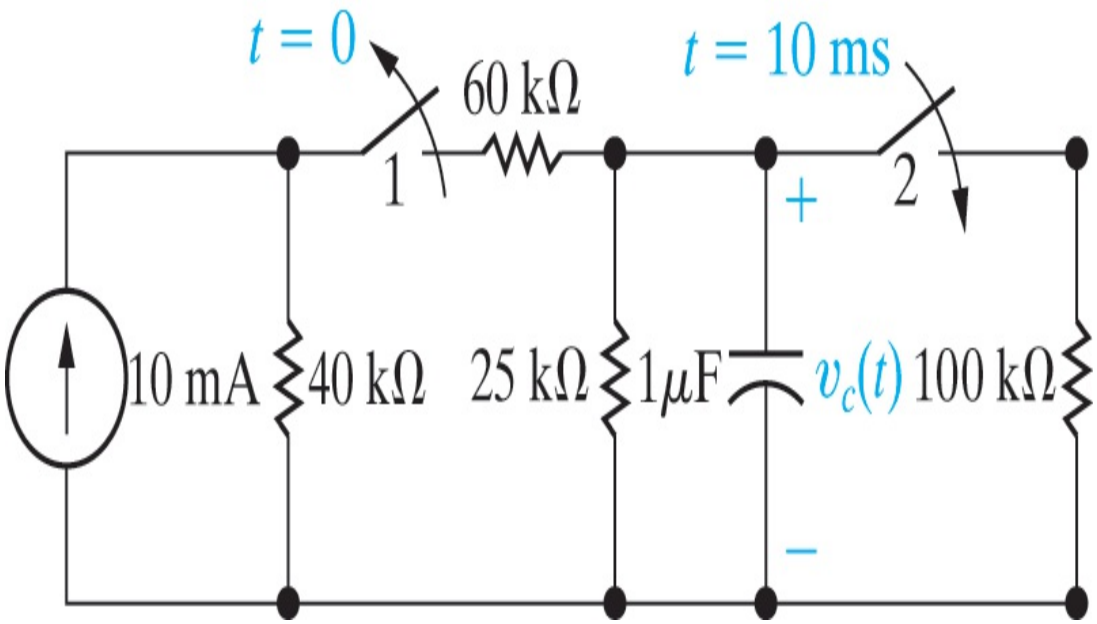
   In this case, t2=17.20 ms.

Assessment Problems

Objective 3—Know how to analyze circuits with sequential switching

1. 7.9 In the circuit shown, switch 1 has been closed, and switch 2 has been open for a long time. At t=0, switch 1 is opened. Then 10 ms later, switch 2 is closed. Find

1. a) vc(t) for 0≤t≤0.01 s,
2. b) vc(t) for t≥0.01 s,
3. c) the total energy dissipated in the 25 kΩ resistor, and
4. d) the total energy dissipated in the 100 kΩ resistor.



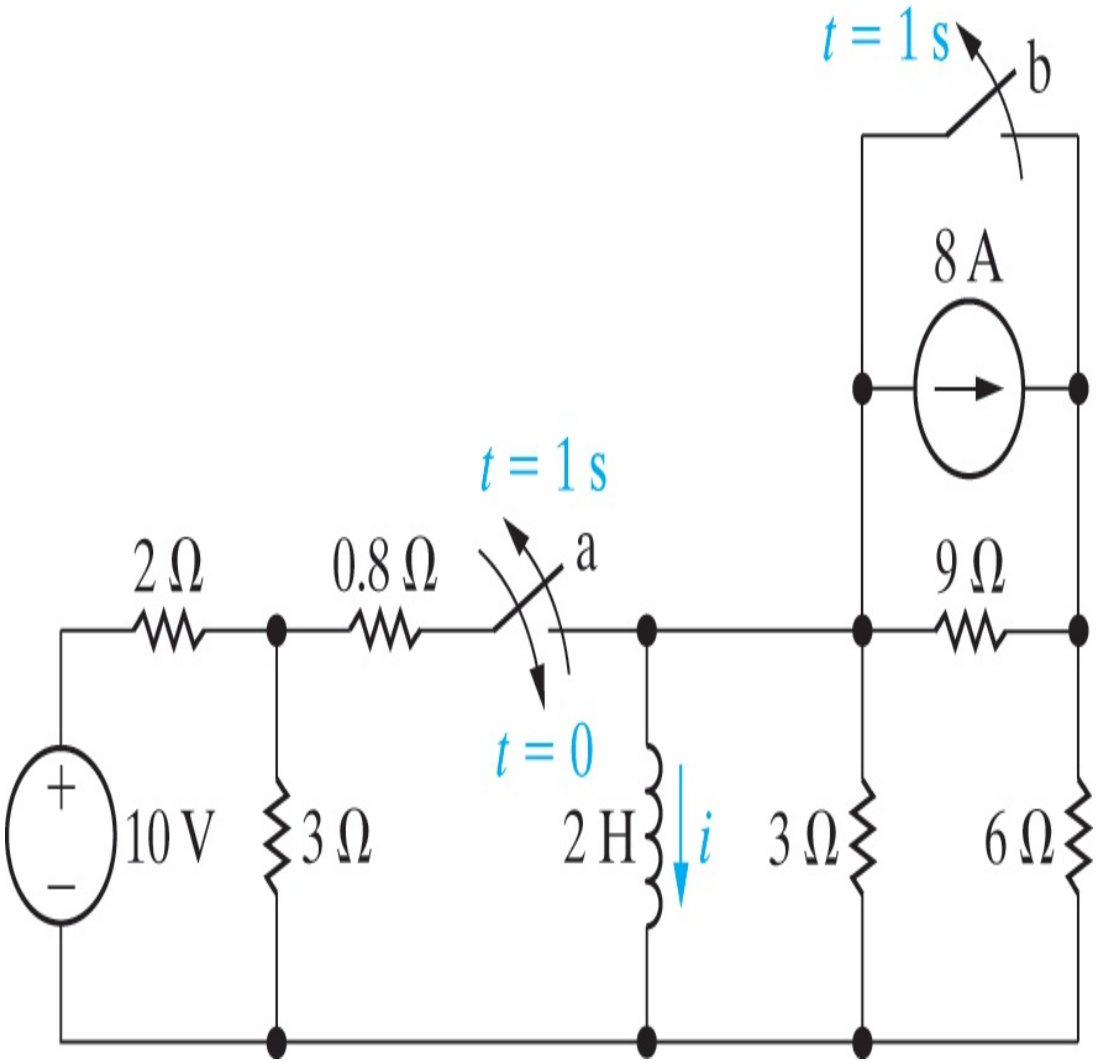
7.5-11 Full Alternative Text

Answer:

1. a) 80e−40t V;
2. b) 53.63e−50(t−0.01) V;
3. c) 2.91 mJ;
4. d) 0.29 mJ.

2. 7.10 Switch a in the circuit shown has been open for a long time, and switch b has been closed for a long time. Switch a is closed at t=0 and, after remaining closed for 1 s, is opened again. Switch b is opened simultaneously, and both switches remain open indefinitely. Determine

the expression for the inductor current i that is valid when (a) 0≤t≤1 s and (b) t≥1 s.



7.5-12 Full Alternative Text

Answer:

1. a) (3−3e−0.5t) A;
2. b) (−4.8+5.98e−1.25(t−1)) A.

SELF-CHECK: Also try Chapter Problems 7.73 and 7.78.

7.6 Unbounded Response

A circuit response may grow, rather than decay, exponentially with time. This type of response, called an **unbounded response**, is possible if the circuit contains dependent sources. In circuits with an unbounded response, the Thévenin equivalent resistance with respect to the terminals of either an inductor or a capacitor is negative. This negative resistance generates a negative time constant, and the resulting currents and voltages increase without limit. In an actual circuit, a component eventually breaks down or saturates, halting the unbounded response.

We cannot use Analysis Method 7.5 to analyze a circuit with an unbounded response because calculating a final value of voltage or current is not possible. Instead, we must derive the differential equation describing the circuit's response and then solve it using the separation of variables technique. Example 7.13 analyzes a circuit with an unbounded response to illustrate this technique.

Example 7.13 Finding the Unbounded Response in an RC Circuit

1. When the switch is closed in the circuit shown in Fig. 7.45, the voltage on the capacitor is 10 V. Find the expression for vo for t≥0.

Figure 7.45 The circuit for Example 7.13.

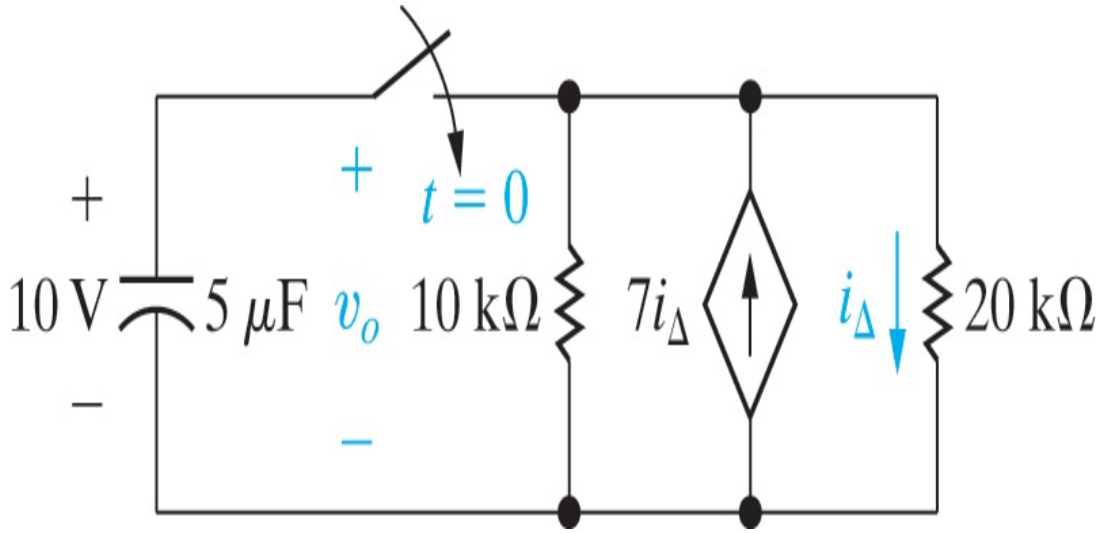


Figure 7.45 Full Alternative Text

2. Assume that the capacitor short-circuits when its terminal voltage reaches 150 V. How many milliseconds elapse before the capacitor short-circuits?

Solution

1. We need to write the differential equation that describes the capacitor voltage, vo. To make this task easier, let's simplify the circuit attached to the capacitor by replacing it with its Thévenin equivalent. This subcircuit is shown in Fig. 7.46, and as you can see, it does not contain an independent source. Thus, the Thévenin equivalent consists of a single resistor. To find the Thévenin equivalent resistance for the circuit in Fig. 7.46, we use the test-source method (see Example 4.18, p. 124), where vT is the test voltage and iT is the test current. Writing a KCL equation at the top node, we get

Figure 7.46 The test-source method used to find RTh.

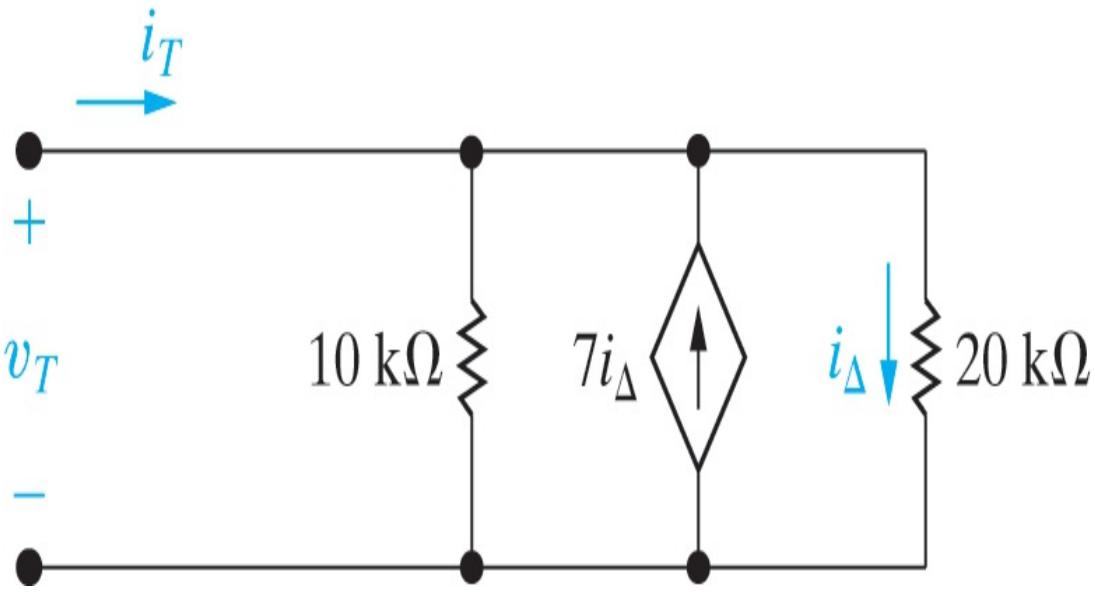


Figure 7.46 Full Alternative Text

iT=vT10×103−7(vT20×103)+vT20×103

Solving for the ratio vT/iT yields the Thévenin resistance:

RTh=vTiT=−5 kΩ.

We replace the two resistors and the dependent source in Fig. 7.45 with RTh to get the circuit shown in Fig. 7.47. For t≥0, write a KCL equation at the top node to construct the differential equation describing this circuit:

Figure 7.47 A simplification of the circuit shown in Fig. 7.45.

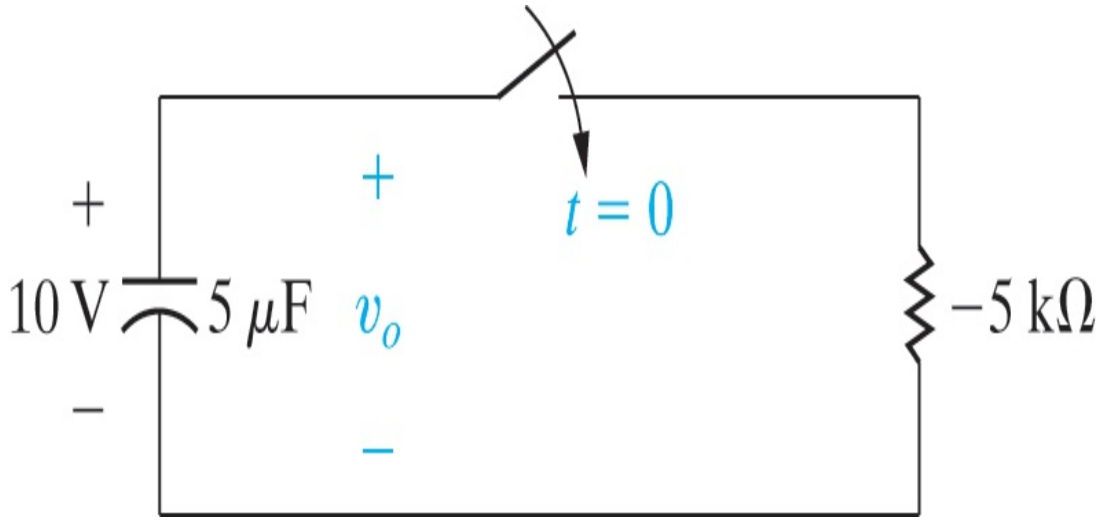


Figure 7.47 Full Alternative Text

$(5\times10-6)\frac{dvo}{dt}+vo-5000=0$

Dividing by the coefficient of the first derivative yields

$\frac{dvo}{dt}-40vo=0.$

This equation has the same form as Eq. 7.1, so we can find vo(t) using the same separation of variables technique applied to Eq. 7.1 (see p. 223). Thus, the capacitor voltage is

$vo(t)=10e40t$ V, $t\geq0$.

2. vo=150 V when $e40t=15$. Therefore, $40t=\ln 15$, and t=67.70 ms.

SELF-CHECK: Assess your understanding of this material by trying Chapter Problems 7.87 and 7.88.

As an engineer, you should be aware that interconnected circuit elements may create unbounded responses. If such interconnections are unintended, the resulting circuit may experience unexpected, and potentially dangerous, component failures.

7.7 The Integrating Amplifier

We are now ready to analyze an integrating-amplifier circuit, shown in Fig. 7.48, which is based on the op amp presented in Chapter 5. This circuit generates an output voltage proportional to the integral of the input voltage. In Fig. 7.48, we added the branch currents if and is, along with the node voltages vn and vp, to aid our analysis. We assume that the op amp is ideal. Write a KCL equation at the inverting input node, and remember that the current into the ideal op amp at its input terminals is zero, to get

Figure 7.48 An integrating amplifier.

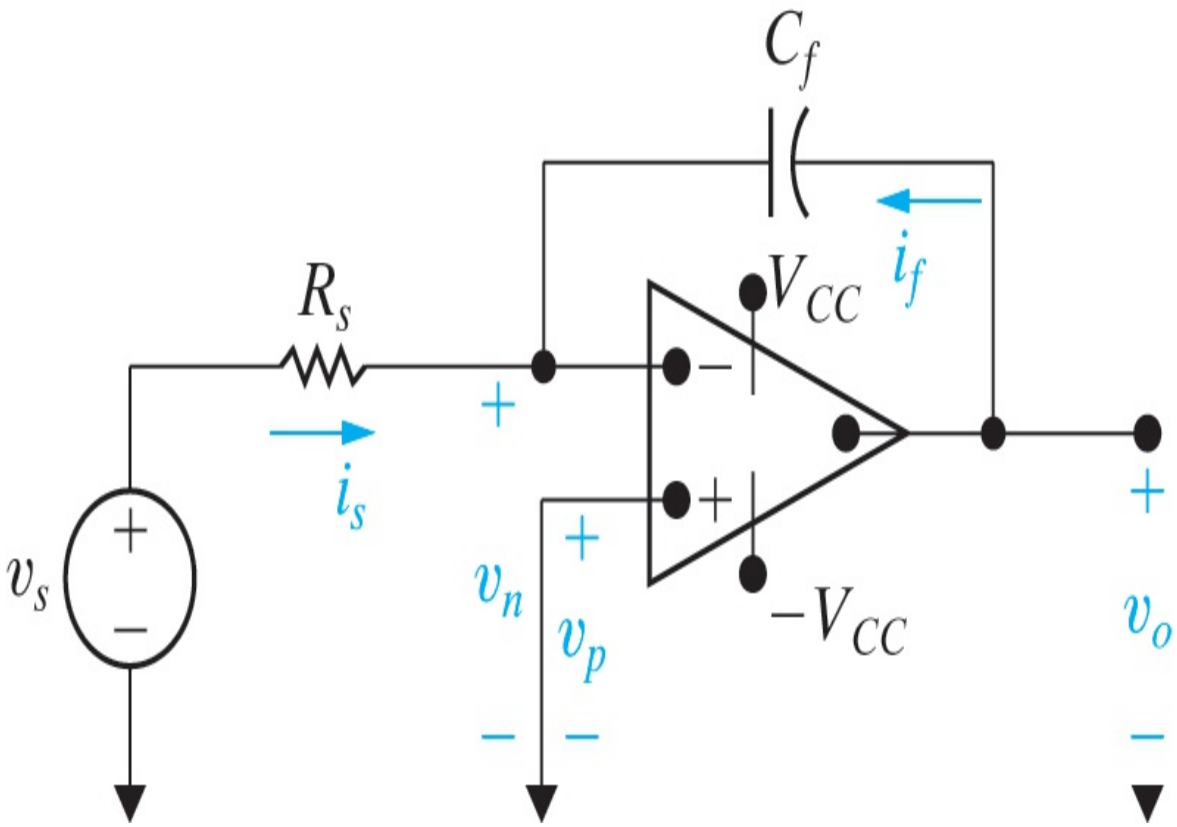


Figure 7.48 Full Alternative Text
if+is=0.

Also, the ideal op amp constrains the voltages at its two input terminals to give

vn=vp.

In the integrating amplifier circuit, vp=0, so using Ohm's law

is=vsRs,

and using the relationship between voltage and current for a capacitor,

if=Cfdvodt.

Substituting the expressions for is and if into the KCL equation and solving for dvo/dt, we get

dvodt=−1RsCfvs.

Multiplying both sides of this equation by a differential time dt and then integrating from t0 to t generates the equation

vo(t)=− 1RsCf∫t0tvs dy+vo(t0).(7.24)

In Eq. 7.24, t0 represents the instant in time when we begin the integration. Thus, vo(t0) is the value of the output voltage at that time. Also, because vn=vp=0, vo(t0) is identical to the initial voltage on the feedback capacitor Cf.

Equation 7.24 states that the output voltage of an integrating amplifier equals the initial value of the voltage on the capacitor plus an inverted (minus sign), scaled (1/RsCf) replica of the integral of the input voltage. If no energy is stored in the capacitor when integration commences, Eq. 7.24 reduces to

vo(t)=−1RsCf∫t0tvs dy.

For example, assume that the input voltage is the rectangular voltage pulse

shown in Fig. 7.49. Assume also that the initial value of vo(t) is zero at the instant vs steps from 0 to Vm. Using Eq. 7.24, we see that

Figure 7.49 An input voltage signal.

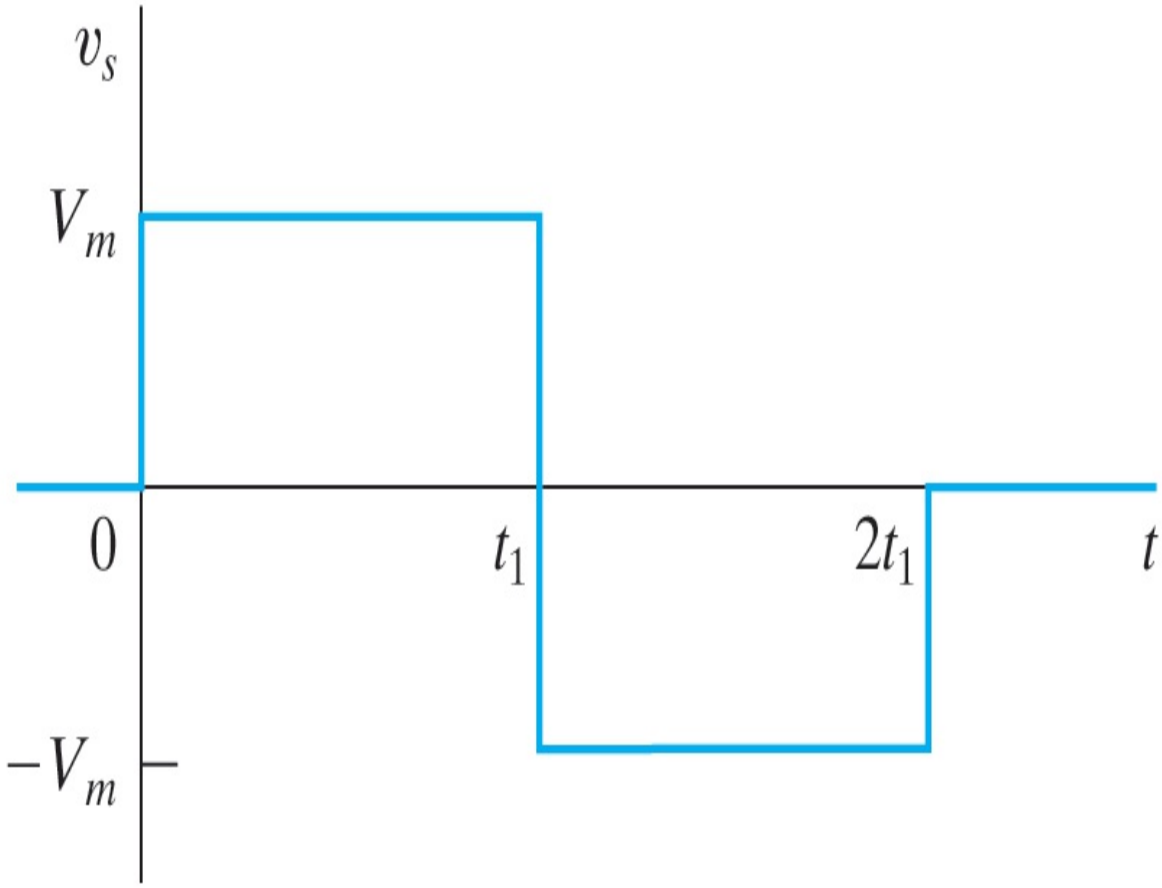


Figure 7.49 Full Alternative Text
vo=−1RsCfVmt+ 0, 0≤t≤t1.

When t lies between t1 and 2s1,

vo=−1RsCf∫t1t (−Vm) dy−1RsCfVmt1=VmRsCft− 2VmRsCft1,t1≤t≤2t1.

Figure 7.50 shows a plot of vo(t) versus t. Clearly, the output voltage is an inverted, scaled replica of the integral of the input voltage.

Figure 7.50 The output voltage of an integrating amplifier.

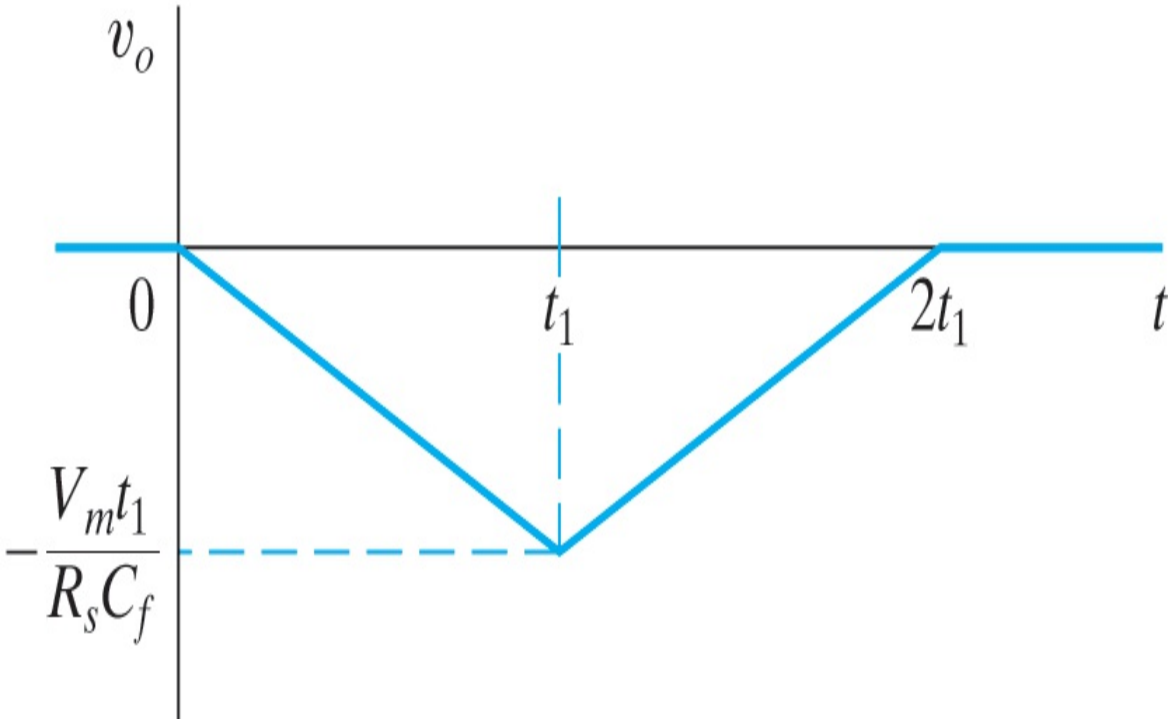


Figure 7.50 Full Alternative Text

The output voltage is proportional to the integral of the input voltage only if the op amp operates within its linear range—that is, if it doesn't saturate. Examples 7.14 and 7.15 further illustrate the analysis of the integrating amplifier.

Example 7.14 Analyzing an Integrating Amplifier

Assume that the numerical values for the voltage shown in Fig. 7.49 are Vm=50 mV and t1=1 s. We apply this voltage to the integrating-amplifier circuit shown in Fig. 7.48. The circuit parameters of the amplifier are

$R_s = 100\ k\Omega$, $C_f = 0.1\ \mu F$, and $V_{CC} = 6$ V. The capacitor's initial voltage is zero.

1. Calculate $v_o(t)$.
2. Plot $v_o(t)$ versus t.

Solution

1. For $0 \le t \le 1$ s,

   $$v_o = -1(100 \times 103)(0.1 \times 10 - 6)50 \times 10 - 3t + 0 = -5t \text{ V}, 0 \le t \le 1 \text{ s}.$$

   For $1 \le t \le 2$ s,

   $$v_o = (5t - 10) \text{ V}.$$

2. Figure 7.51 shows a plot of $v_o(t)$ versus t.

Figure 7.51 The output voltage for Example 7.14.

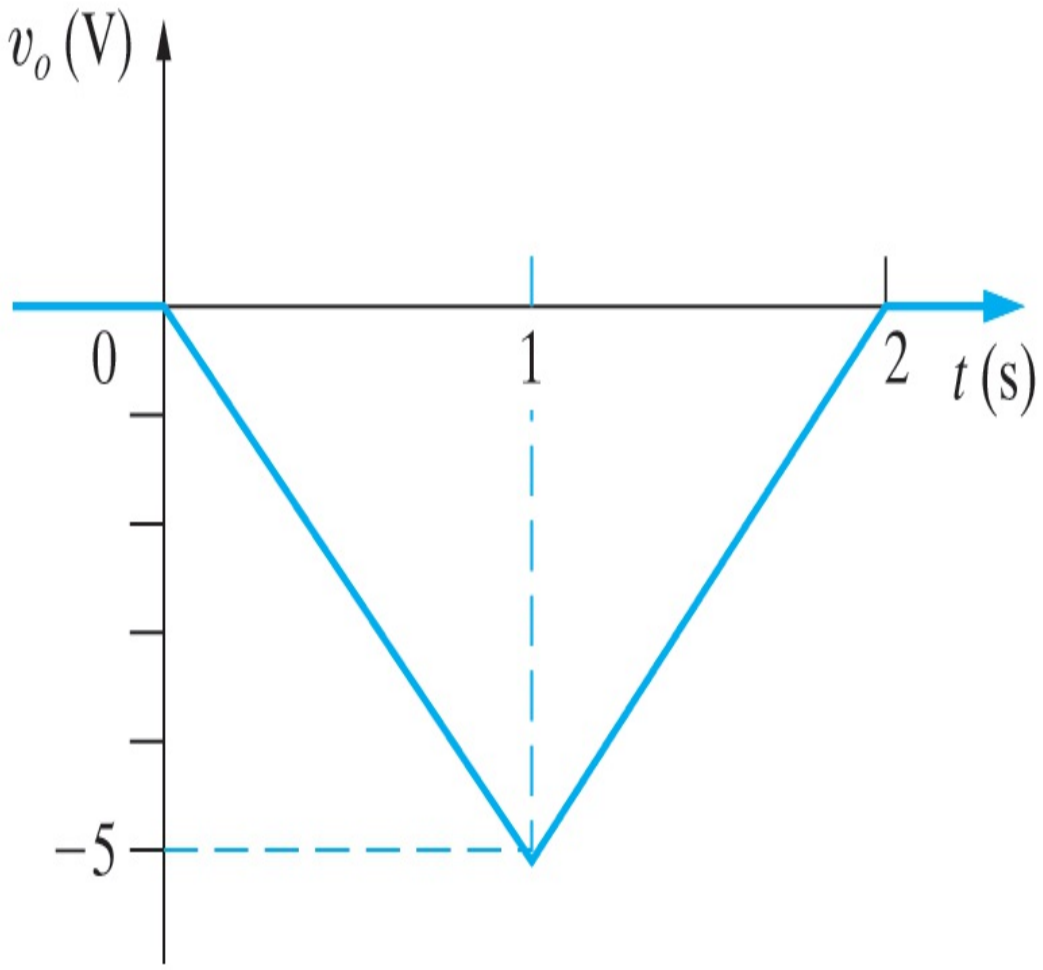


Figure 7.51 Full Alternative Text

Example 7.15 Analyzing an Integrating Amplifier that Has Sequential Switching

At the instant the switch makes contact with terminal a in the circuit shown in Fig. 7.52, the voltage on the 0.1 μF capacitor is 5 V. The switch remains at terminal a for 9 ms and then moves instantaneously to terminal b. How many milliseconds after making contact with terminal b does the op amp saturate?

Figure 7.52 The circuit for Example 7.15.

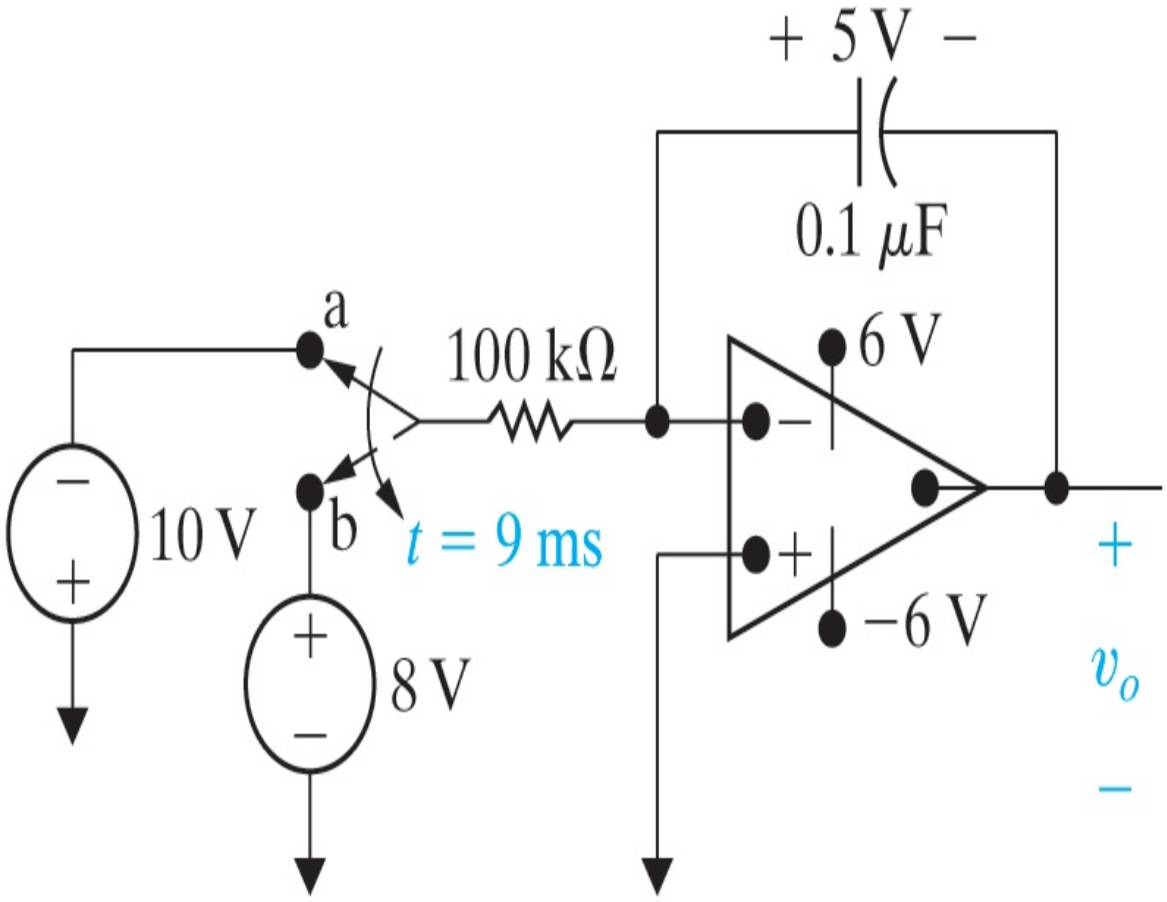


Figure 7.52 Full Alternative Text

Solution

The expression for the output voltage during the time the switch is at terminal a is

vo=−1(100×103)(0.1×10−6)∫0t(−10)dy+(−5)=(−5+1000t) V.

Thus, 9 ms after the switch makes contact with terminal a, the output voltage

is 1000(9×10–3)–5=4 V. Note that the op amp does not saturate during its first 9 ms of operation.

The expression for the output voltage after the switch moves to terminal b is

$$v_o = -1(100 \times 103)(0.1 \times 10-6)\int 9 \times 10-3t8dy+4 = -800(t-9 \times 10-3)+4 = (11.2-800t)\ \text{V}.$$

When the switch is at terminal b, the voltage is decreasing, and the op amp eventually saturates at −6 V. Therefore, we set the expression for vo equal to −6 V to obtain the saturation time ts:

$$11.2-800t_s = -6,$$

or

$$t_s = 21.5\ \text{ms}.$$

Thus, the integrating amplifier saturates 21.5 ms after making contact with terminal b.

From the examples, we see that the integrating amplifier can perform the integration function very well but only within specified limits that avoid saturating the op amp. The op amp saturates because charge accumulates on the feedback capacitor. We can prevent saturation by placing a resistor in parallel with the feedback capacitor. We examine such a circuit in Chapter 8.

Note that we can convert the integrating amplifier to a differentiating amplifier by interchanging the input resistance Rs and the feedback capacitor Cf. Then

$$v_o = -R_s C_f \frac{dv_s}{dt}.$$

We leave the derivation of this expression as an exercise for you. The differentiating amplifier is seldom used because in practice it is a source of unwanted or noisy signals.

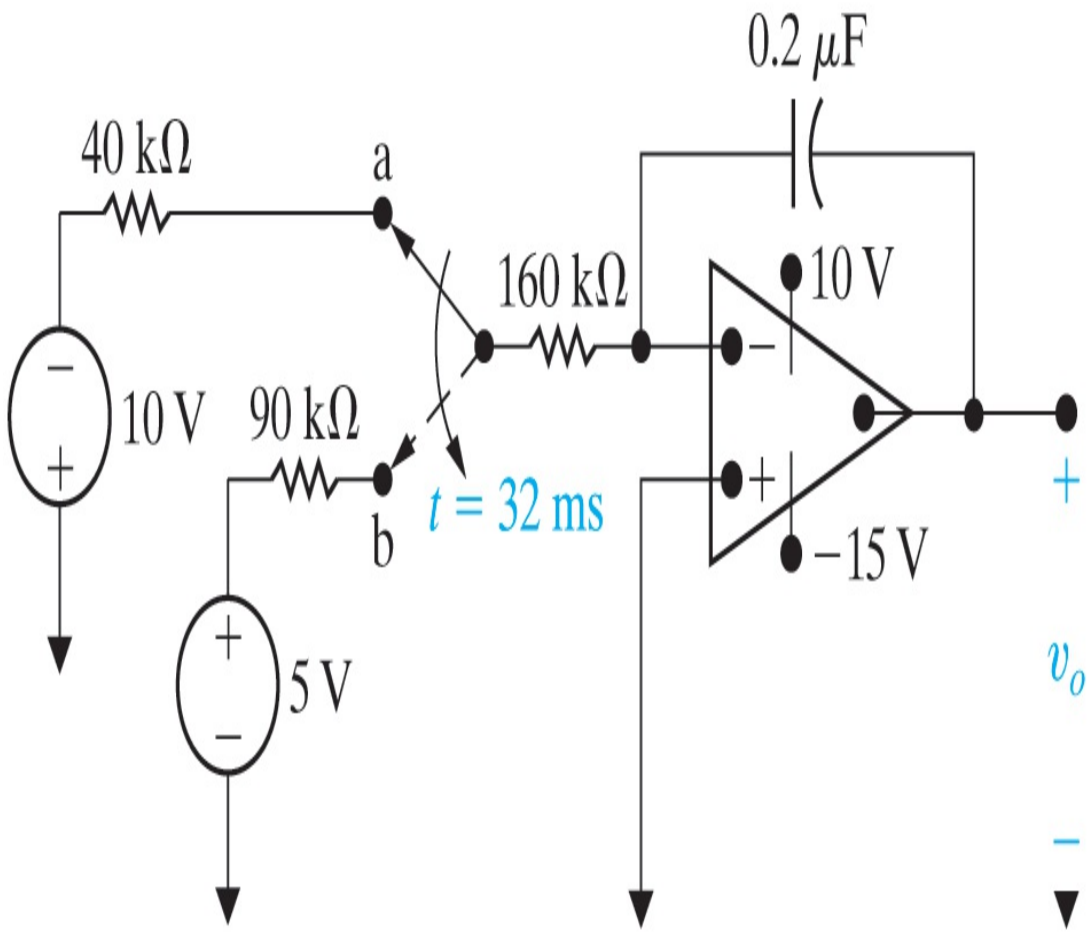
Finally, we can use an inductor instead of a capacitor to create both integrating- and differentiating-amplifier circuits. Since it is easier to

fabricate capacitors for integrated-circuit devices, inductors are rarely used in integrating amplifiers.

Assessment Problems

Objective 4—Be able to analyze op amp circuits containing resistors and a single capacitor

1. 7.11 There is no energy stored in the capacitor at the time the switch in the circuit makes contact with terminal a. The switch remains at position a for 32 ms and then moves instantaneously to position b. How many milliseconds after making contact with terminal a does the op amp saturate?



7.7-13 Full Alternative Text

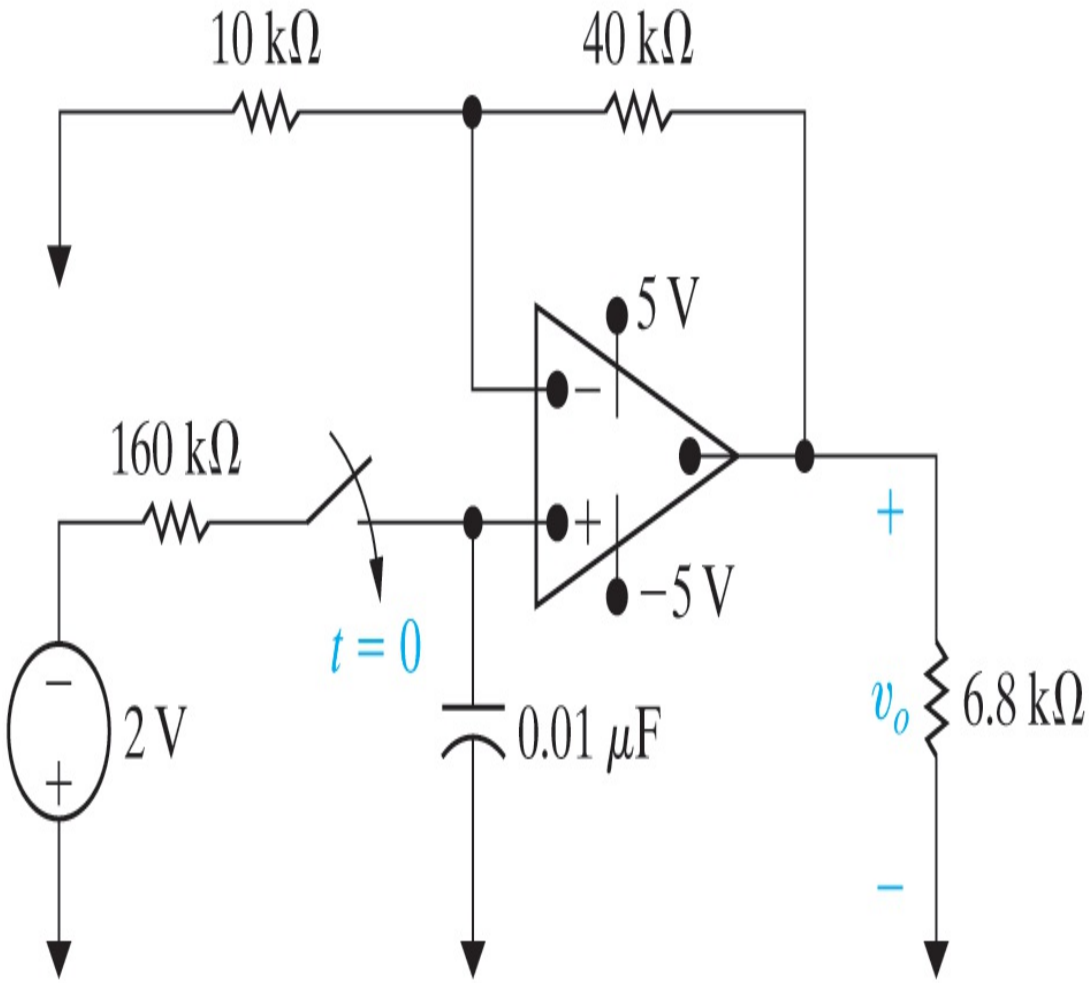
Answer:

262 ms.

2. 7.12

    1. a) When the switch closes in the circuit shown, there is no energy stored in the capacitor. How long does it take to saturate the op amp?

    2. b) Repeat (a) with an initial voltage on the capacitor of 1 V, positive at the upper terminal.



7.7-14 Full Alternative Text

Answer:

1. a) 1.11 ms;
2. b) 1.76 ms.

SELF-CHECK: Also try Chapter Problems 7.94 and 7.95.

Practical Perspective

Artificial Pacemaker

The RC circuit shown in Fig. 7.53 can be used in an artificial pacemaker to establish a normal heart rhythm by generating periodic electrical impulses. The box labeled “controller” behaves as an open circuit until the voltage drop across the capacitor reaches a preset limit. Once that limit is reached, the capacitor discharges its stored energy in the form of an electrical impulse to the heart, starts to recharge, and then the process repeats.

Figure 7.53 An artificial pacemaker circuit.

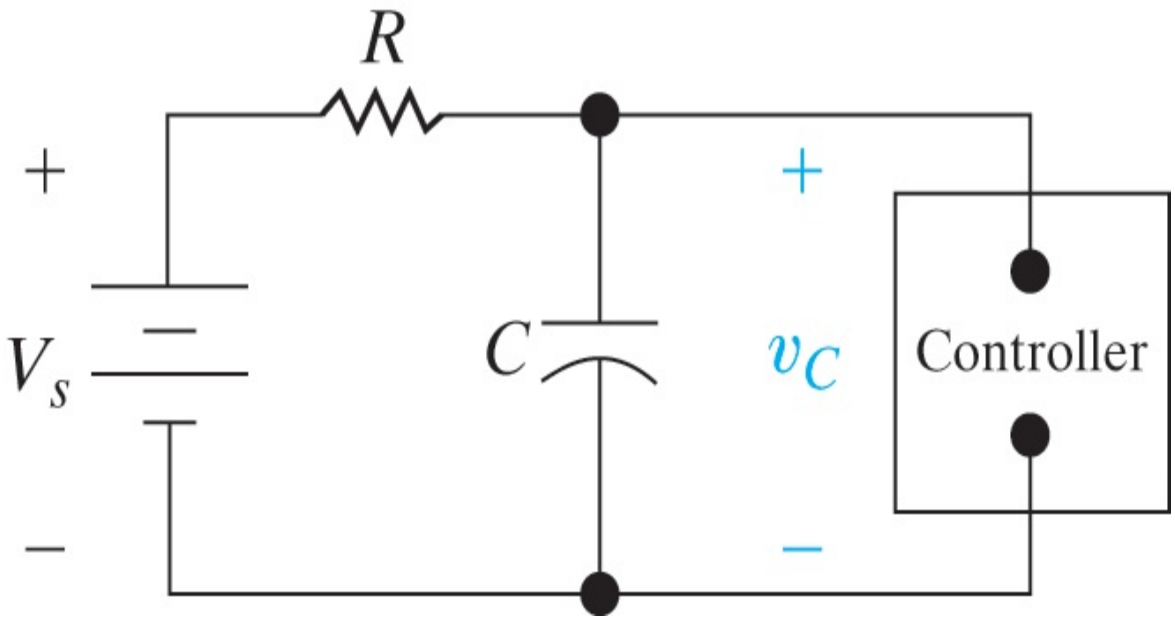


Figure 7.53 Full Alternative Text

Before we develop the analytical expressions that describe the circuit's behavior, let's get a feel for how the circuit works. First, when the controller behaves like an open circuit, the dc voltage source will charge the capacitor via the resistor R, toward a value of Vs volts. But once the capacitor voltage reaches Vmax, the controller behaves like a short circuit, enabling the capacitor to discharge. Once the capacitor discharge is complete, the controller again acts like an open circuit and the capacitor starts to recharge. This cycle of charging and discharging the capacitor establishes the desired heart rhythm, as shown in Fig. 7.54.

Figure 7.54 Capacitor voltage versus time for the circuit in Fig. 7.55.

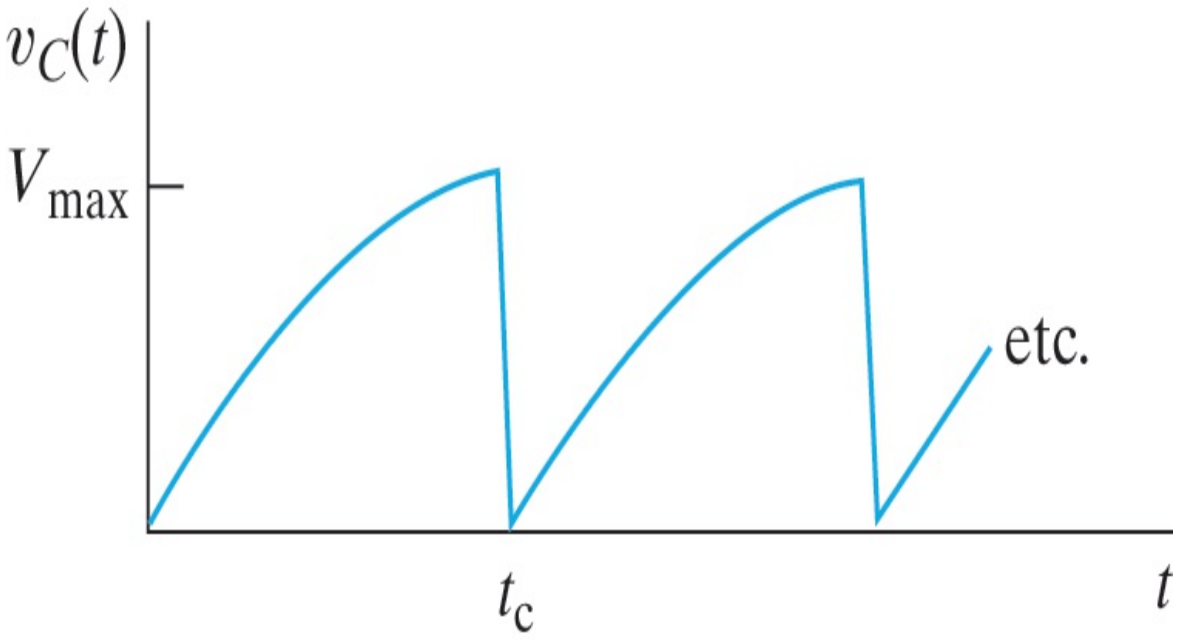


Figure 7.54 Full Alternative Text

In drawing Fig. 7.54, we have chosen t=0 at the instant the capacitor starts to charge. This plot also assumes that the circuit has reached the repetitive stage of its operation and that the time to discharge the capacitor is negligible when compared to the recharge time. We need an equation for vC(t) as a function of Vs, R, and C to design the artificial pacemaker circuit.

To begin the analysis, we assume that the circuit has been in operation for a long time. Let t=0 at the instant when the capacitor has completely discharged and the controller is acting as an open circuit. From the circuit we find the initial and final values of the capacitor voltage and the circuit's time constant:

V0=vC(0)=0;
Vf=vC(∞)=Vs; and τ=RC.

To find the capacitor voltage while the capacitor is charging, substitute the initial and final values of the capacitor voltage and the circuit's time constant into Eq. 7.23 and simplify to get

vC(t)=Vs (1−e−t/RC).

Suppose the controller is programmed to generate an electrical pulse that stimulates the heart when vC=0.75 Vs=Vmax. Given values of R and C, we can determine the resulting heart rate, H, in beats per minute as follows:

H=60−RC ln 0.25 [beats per minute]

A more realistic design problem requires you to calculate the value of resistance, R, given Vmax as a percentage of Vs, C, and the desired heart rate in beats per minute. Developing an equation for resistance, R, is the focus of Problem 7.106.

Figure 7.55 The artificial pacemaker circuit at t=0, when the capacitor is charging.

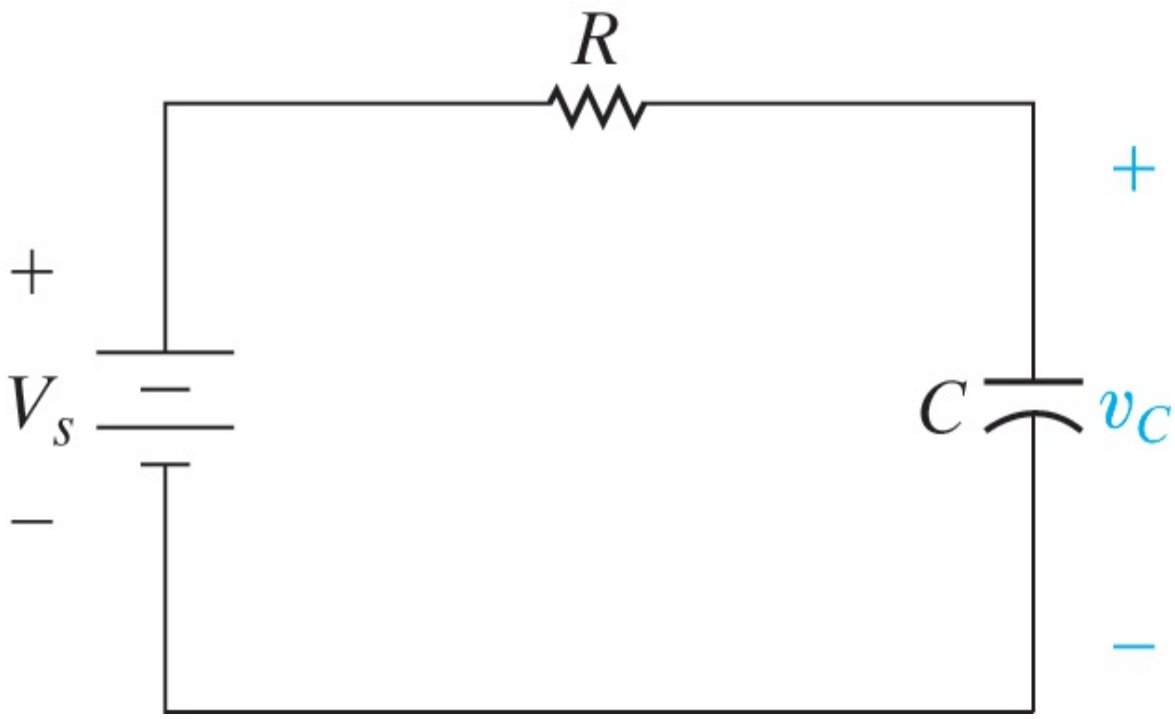


Figure 7.55 Full Alternative Text

SELF-CHECK: Assess your understanding of the Practical Perspective by

solving Chapter Problems 7.104–7.107.

Summary

- A first-order circuit may be reduced to a Thévenin (or Norton) equivalent connected to either a single equivalent inductor or capacitor. (See page 220.)

- The **natural response** is the currents and voltages that exist when stored energy is released to a circuit that contains no independent sources. (See page 220.)

- The **time constant** of an RL circuit equals the equivalent inductance divided by the Thévenin resistance as viewed from the terminals of the equivalent inductor. (See page 223.)

- The **time constant** of an RC circuit equals the equivalent capacitance times the Thévenin resistance as viewed from the terminals of the equivalent capacitor. (See page 229.)

- The **step response** is the currents and voltages that result from abrupt changes in dc sources connected to a circuit. Stored energy may or may not be present at the time the abrupt changes take place. (See page 233.)

- Analysis Method 7.5 can be used to find the solution for the natural and step responses of both RL and RC circuits:

    1. Step 1: Identify the variable x(t), which is the quantity that is required to be continuous for all time. This is the inductor current in RL circuits and the capacitor voltage in RC circuits.

    2. Step 2: Calculate the initial value X0, by analyzing the circuit to find x(t) for $t<0$.

    3. Step 3: Calculate the time constant, τ, for the circuit by analyzing the circuit for $t\geq 0$ to find the equivalent resistance attached to the inductor or capacitor. For RL circuits, $\tau=L/R$, and for RC circuits, $\tau=RC$.

4. Step 4: Calculate the final value Xf, by analyzing the circuit to find $x(t)$ as $t \to \infty$. If the circuit exhibits a natural response, $Xf=0$, so no calculation is needed.

5. Step 5: Write the equation for $x(t)$ by substituting the initial value $X0$, the time constant τ, and the final value Xf, into the expression $x(t)= Xf+(X0-Xf)e^{-t/\tau}$, $t \geq 0$.

6. Step 6: Use $x(t)$ to find any other quantities of interest in the circuit. (See page 242.)

- **Sequential switching** in first-order circuits is analyzed by dividing the analysis into time intervals corresponding to specific switch positions. Initial values for a particular interval are determined from the solution corresponding to the immediately preceding interval. (See page 246.)

- An **unbounded response** occurs when the Thévenin resistance is negative, which is possible when the first-order circuit contains dependent sources. (See page 250.)

- An integrating amplifier consists of an ideal op amp, a capacitor in the negative feedback branch, and a resistor in series with the signal source. It outputs the integral of the signal source, within specified limits that avoid saturating the op amp. (See page 252.)

Problems

Section 7.1

1. 7.1 PSPICEMULTISIM The switch in the circuit in Fig. P 7.1 has been closed for a long time before opening at t=0.

Figure P7.1

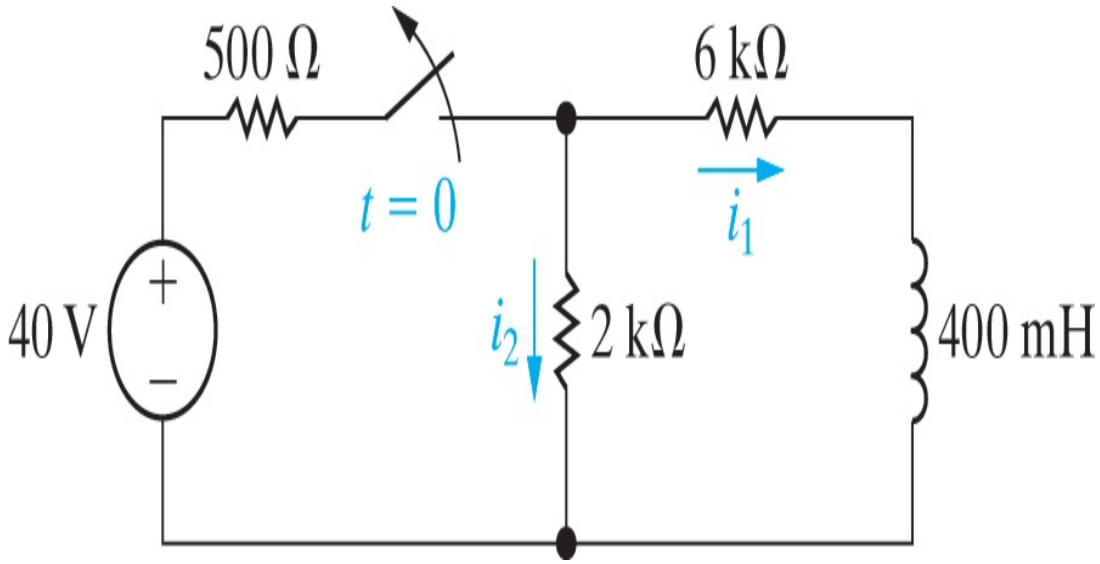


Figure P7.1 Full Alternative Text

   1. a) Find i1(0−) and i2(0−).
   2. b) Find i1(0+) and i2(0+).
   3. c) Find i1(t) for t≥0.
   4. d) Find i2(t) for t≥0+.
   5. e) Explain why i2(0−)≠i2(0+).

2. 7.2 In the circuit shown in Fig. P 7.2, the switch makes contact with position b just before breaking contact with position a. This is known as

a make-before-break switch and it ensures that the inductor current is continuous. The interval of time between “making” and “breaking” is assumed to be negligible. The switch has been in the a position for a long time. At t=0 the switch is thrown from position a to position b.

Figure P7.2

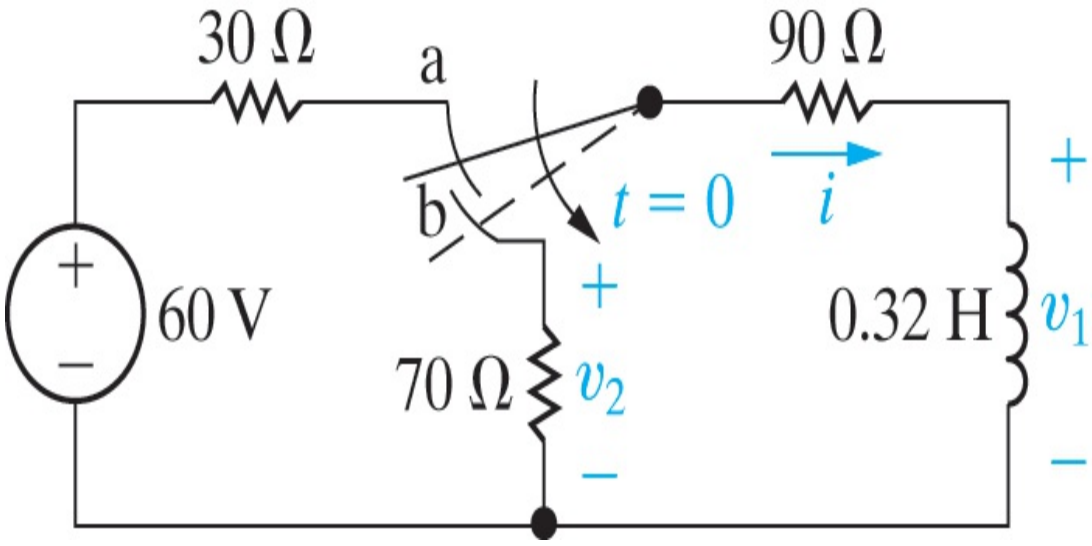


Figure P7.2 Full Alternative Text

1. a) Determine the initial current in the inductor.
2. b) Determine the time constant of the circuit for t>0.
3. c) Find i, v1, and v2 for t≥0.
4. d) What percentage of the initial energy stored in the inductor is dissipated in the 90 Ω resistor 1 ms after the switch is thrown from position a to position b?

3. 7.3 PSPICEMULTISIM The switch in the circuit in Fig. P 7.3 has been open for a long time. At t=0 the switch is closed.

Figure P7.3

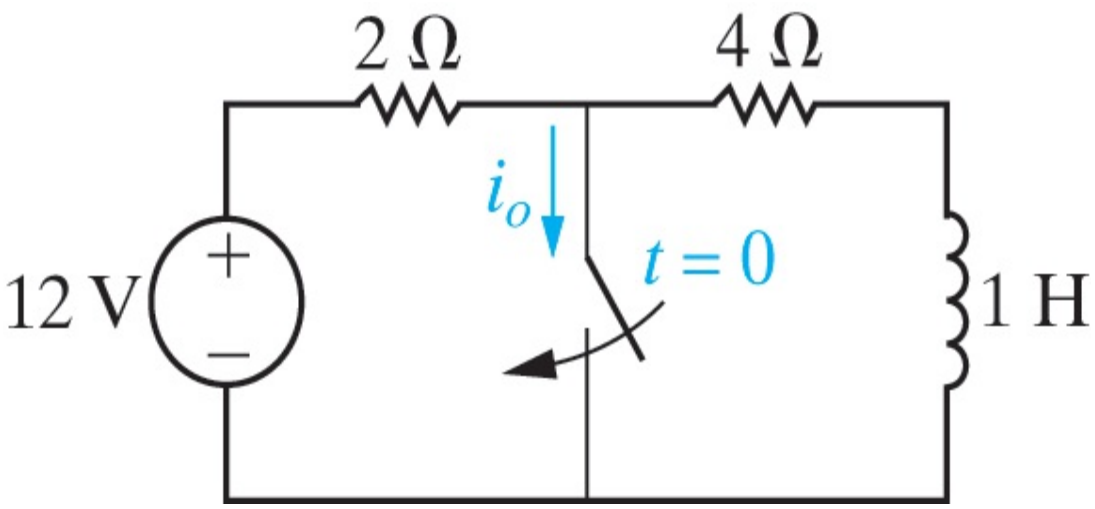


Figure P7.3 Full Alternative Text

1. a) Determine io(0) and io(∞).
2. b) Determine io(t) for t≥0.
3. c) How many milliseconds after the switch has been closed will io equal 5 A?

4. 7.4 The switch shown in Fig. P 7.4 has been open for a long time before closing at t=0.

Figure P7.4

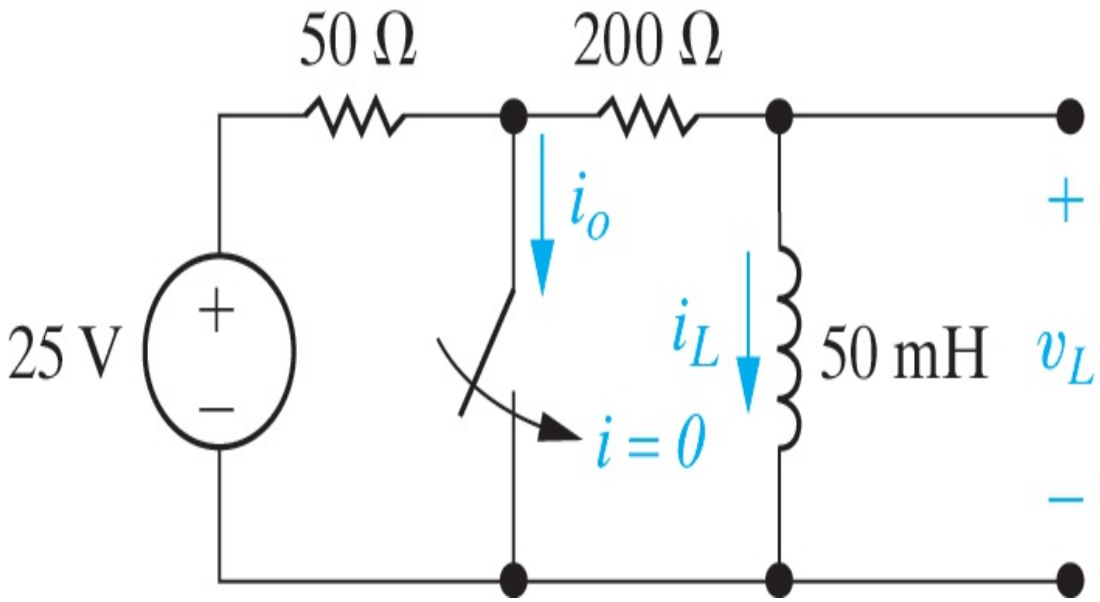


Figure P7.4 Full Alternative Text

1. a) Find io(0−), iL(0−), and vL(0−).
2. b) Find io(0+), iL(0+), and vL(0+).
3. c) Find io(∞), iL(∞), and vL(∞).
4. d) Write the expression for iL(t) for t≥0.
5. e) Write the expression for io(t) for t≥0+.
6. f) Write the expression for vL(t) for t≥0+.

5. 7.5 PSPICEMULTISIM The switch in the circuit in Fig. P 7.5 has been in position 1 for a long time. At t=0, the switch moves instantaneously to position 2. Find vo(t) for t≥0+.

Figure P7.5

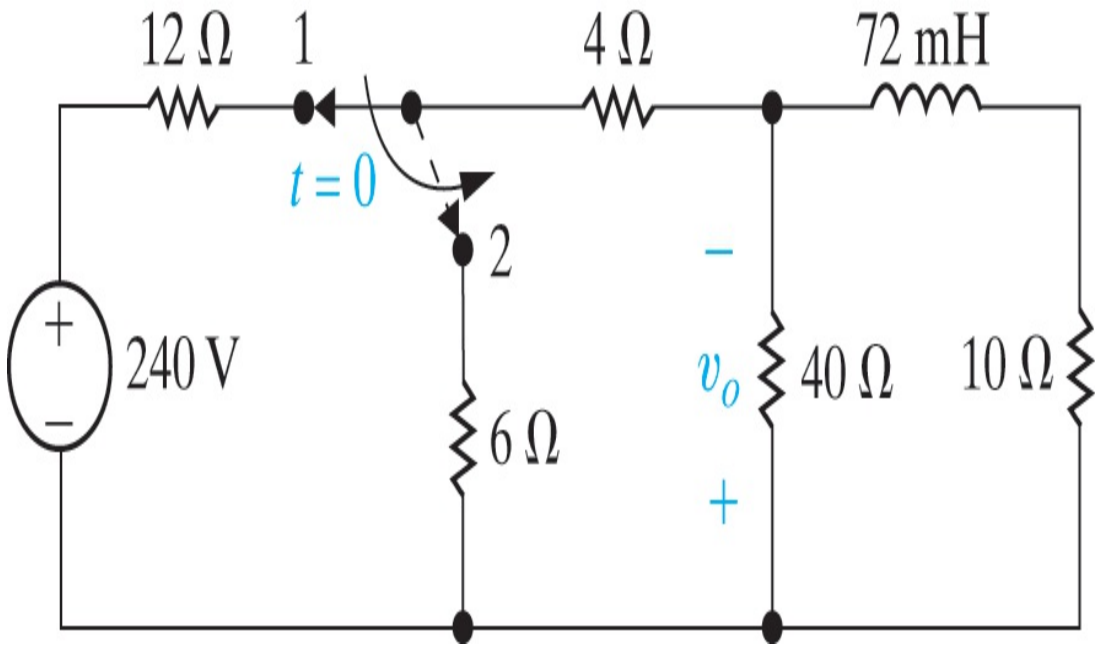


Figure P7.5 Full Alternative Text

6. 7.6 For the circuit of Fig. P 7.5, what percentage of the initial energy stored in the inductor is eventually dissipated in the 40 Ω resistor?

7. 7.7 PSPICEMULTISIM The switch in the circuit in Fig. P 7.7 has been closed for a long time. At t=0 it is opened.

Figure P7.7

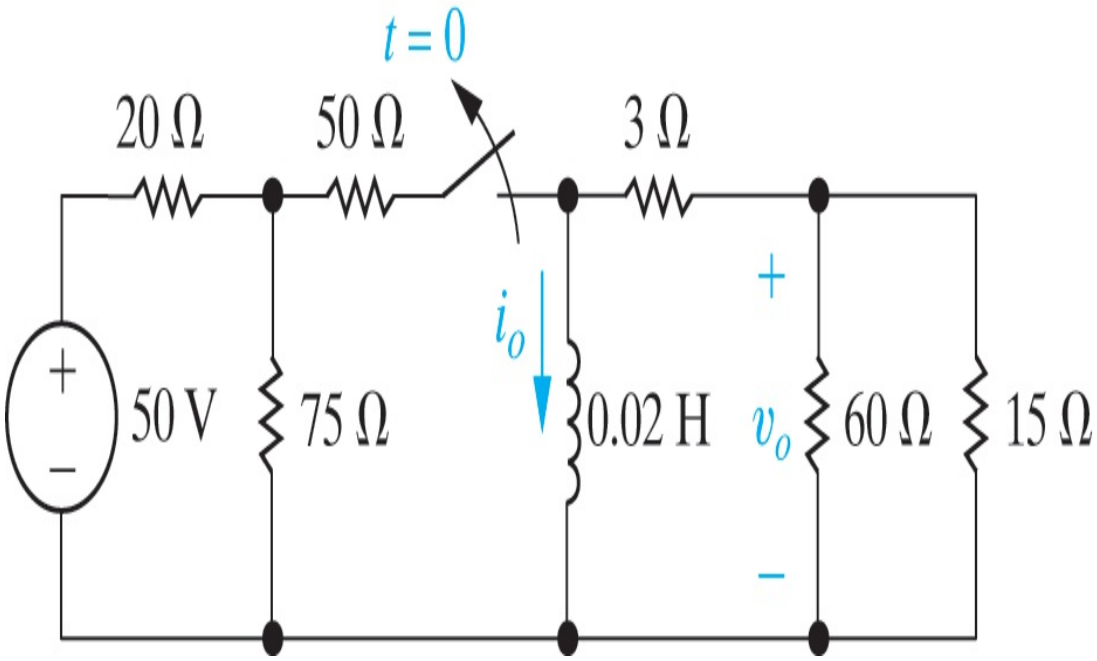
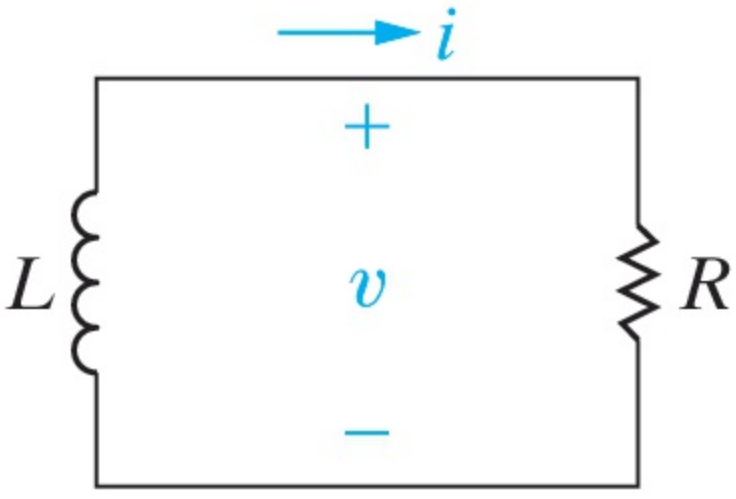


Figure P7.7 Full Alternative Text

1. a) Write the expression for io(t) for t≥0.
2. b) Write the expression for vo(t) for t≥0+.

8. 7.8 In the circuit in Fig. P 7.8, the voltage and current expressions are

Figure P7.8



$v=400e^{-5t}V, \quad t\geq 0+; i=10e^{-5t}A, \quad t\geq 0.$

Find

1. a) R.
2. b) τ (in milliseconds).
3. c) L.
4. d) the initial energy stored in the inductor.
5. e) the time (in milliseconds) it takes to dissipate 80% of the initial stored energy.

9. 7.9

    1. a) Use component values from Appendix H to create a first-order RL circuit (see Fig. 7.4) with a time constant of 1 ms. Use a single inductor and a network of resistors, if necessary. Draw your circuit.
    2. b) Suppose the inductor you chose in part (a) has an initial current of 10 mA. Write an expression for the current through the inductor for $t\geq 0$.
    3. c) Using your result from part (b), calculate the time at which half of the initial energy stored in the inductor has been dissipated by the resistor.

10. 7.10 In the circuit in Fig. P 7.10, the switch has been closed for a long time before opening at t=0.

Figure P7.10

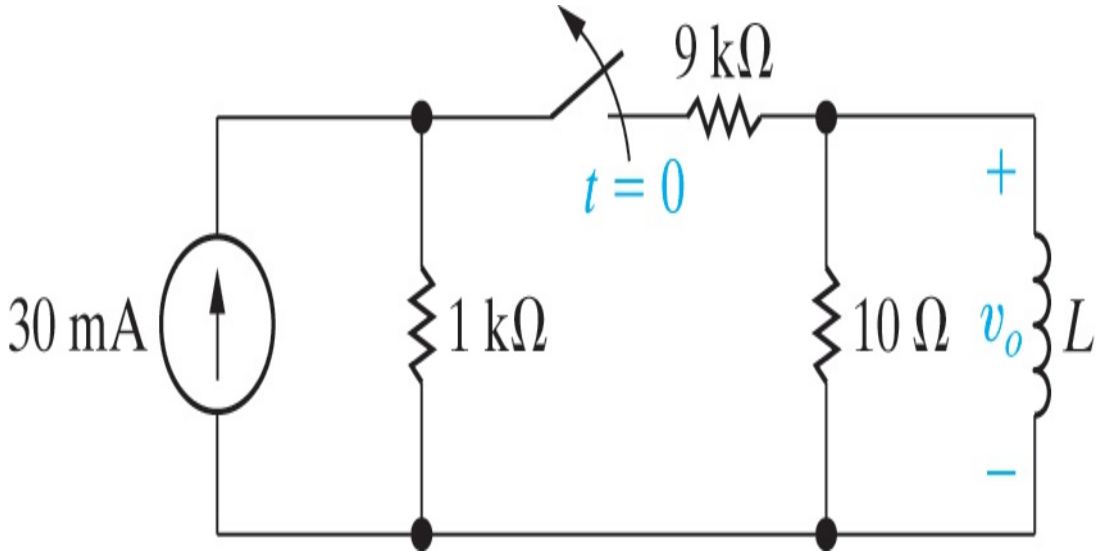


Figure P7.10 Full Alternative Text

1. a) Find the value of L so that vo(t) equals 0.5 vo(0+) when t=1 ms.

2. b) Find the percentage of the stored energy that has been dissipated in the 10 Ω resistor when t=1 ms.

11. 7.11 The switch in the circuit seen in Fig. P 7.11 has been in position 1 for a long time. At t=0, the switch moves instantaneously to position 2. Find the value of R so that 20% of the initial energy stored in the 30 mH inductor is dissipated in R in 15 μs.

Figure P7.11

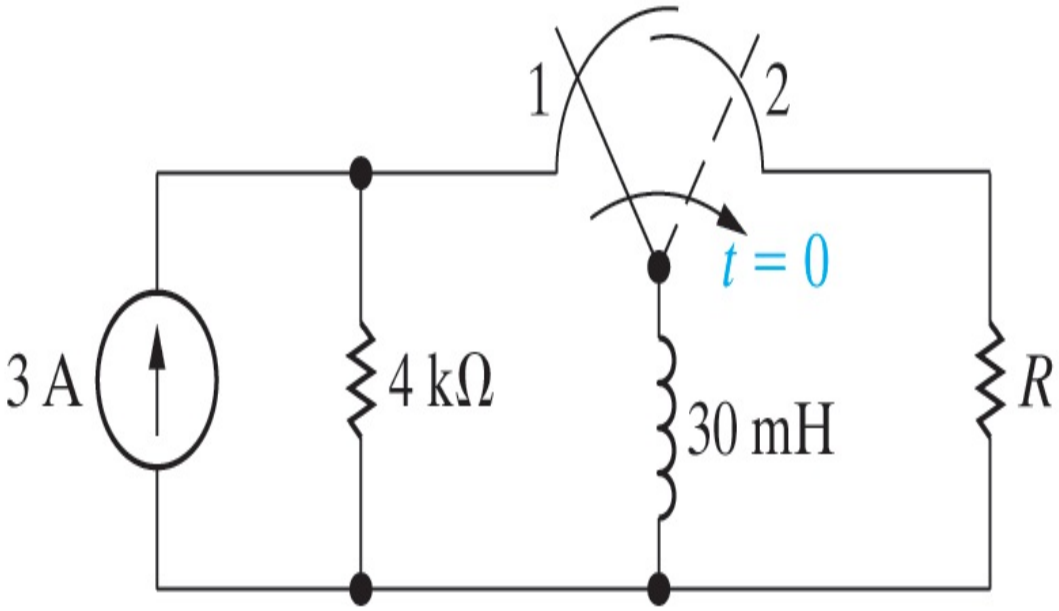


Figure P7.11 Full Alternative Text

12. 7.12 In the circuit in Fig. P 7.11, let Ig represent the dc current source, σ represent the fraction of initial energy stored in the inductor that is dissipated in to seconds, and L represent the inductance.

    1. a) Show that

       R=Lln[1/(1−σ)]2to.

    2. b) Test the expression derived in (a) by using it to find the value of R in Problem 7.11.

13. 7.13 The two switches in the circuit seen in Fig. P 7.13 are synchronized. The switches have been closed for a long time before opening at t=0.

Figure P7.13

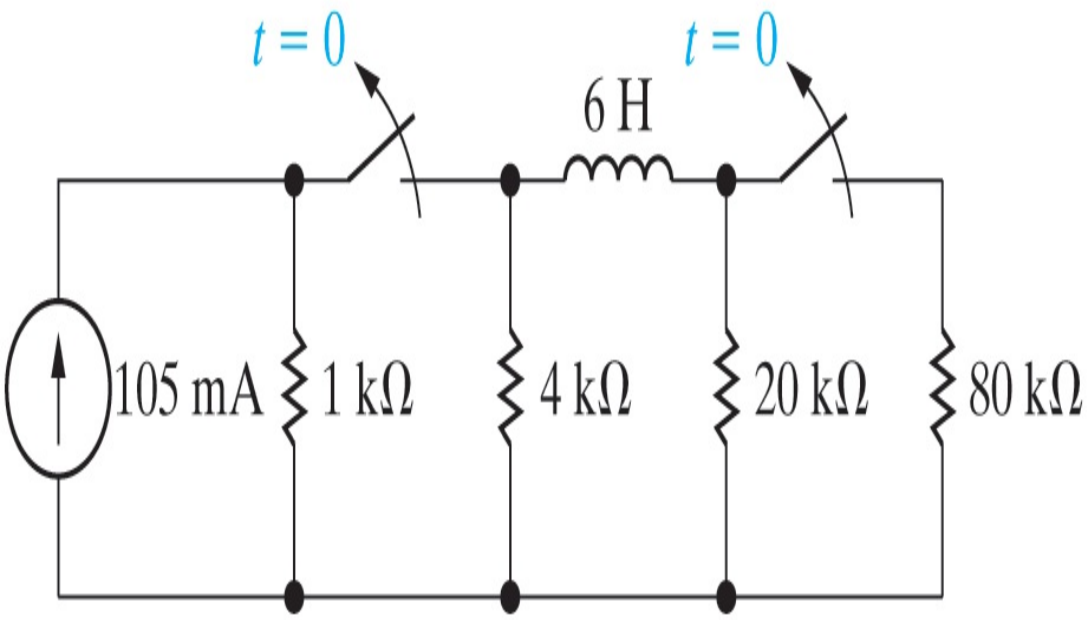


Figure P7.13 Full Alternative Text

1. a) How many microseconds after the switches are open is the energy dissipated in the 4 kΩ resistor 10% of the initial energy stored in the 6 H inductor?

2. b) At the time calculated in (a), what percentage of the total energy stored in the inductor has been dissipated?

14. 7.14 PSPICEMULTISIM The switch in the circuit in Fig. P 7.14 has been closed for a long time before opening at t=0. Find vo(t) for t≥0+.

Figure P7.14

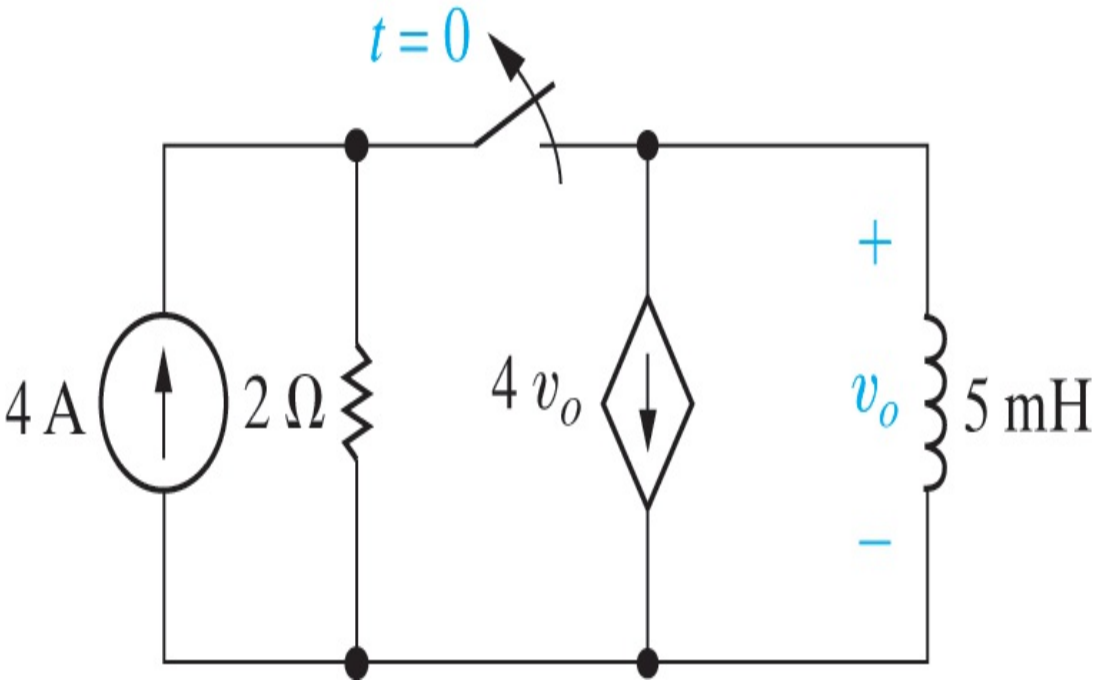


Figure P7.14 Full Alternative Text

15. 7.15 The switch in Fig. P 7.15 has been closed for a long time before opening at t=0. Find

Figure P7.15

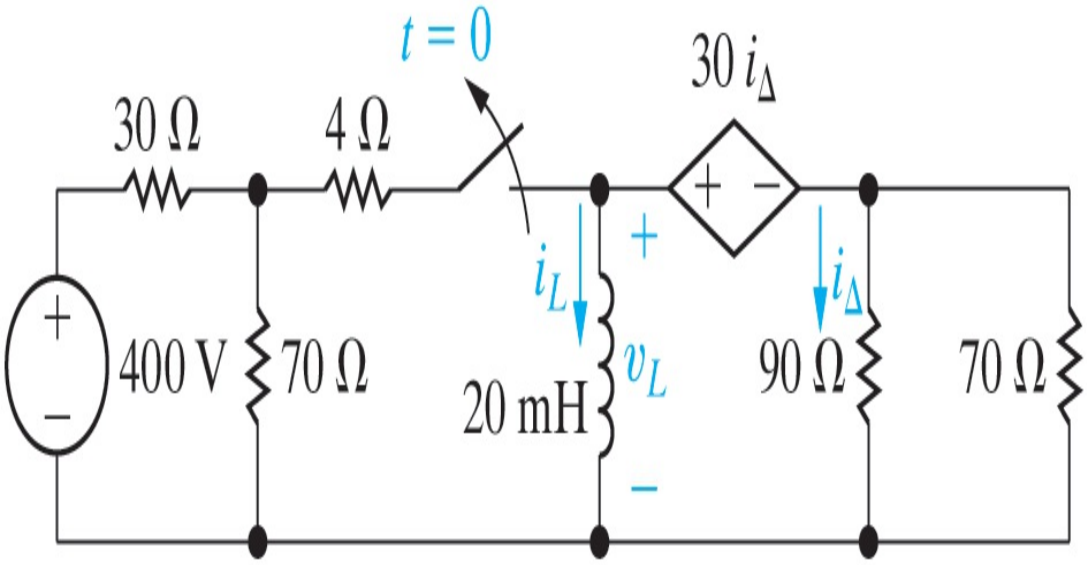


Figure P7.15 Full Alternative Text

1. a) $i_L(t)$, $t \geq 0$.
2. b) $v_L(t)$, $t \geq 0+$.
3. c) $i_\Delta(t)$, $t \geq 0+$.

16. 7.16 What percentage of the initial energy stored in the inductor in the circuit in Fig. P 7.15 is dissipated by the current-controlled voltage source?

17. 7.17 PSPICEMULTISIM The 165 V, 5 Ω source in the circuit in Fig. P 7.17 is inadvertently short-circuited at its terminals a, b. At the time the fault occurs, the circuit has been in operation for a long time.

Figure P7.17

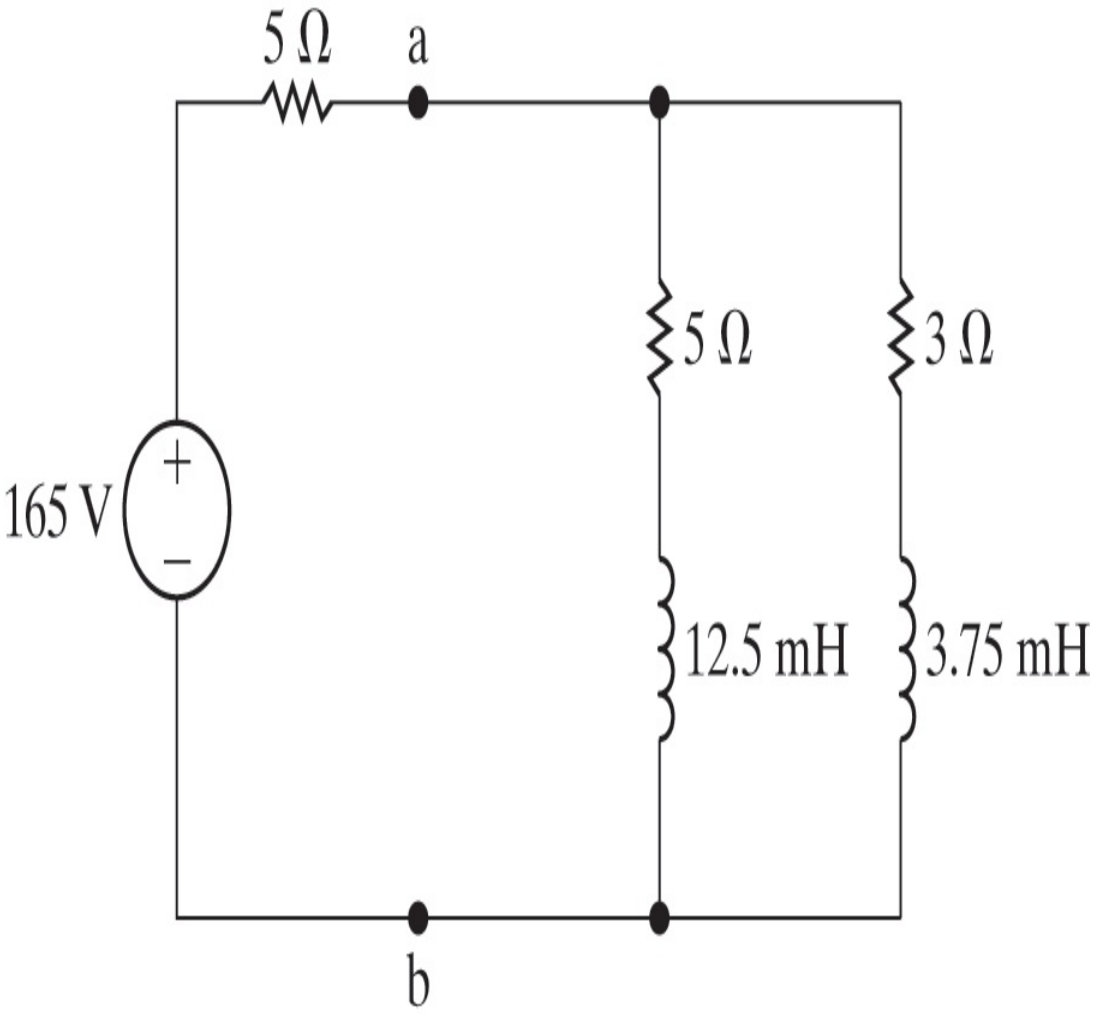


Figure P7.17 Full Alternative Text

1. a) What is the initial value of the current iab in the short-circuit connection between terminals a, b?

2. b) What is the final value of the current iab?

3. c) How many microseconds after the short circuit has occurred is the current in the short equal to 19 A?

18. 7.18 PSPICEMULTISIM In the circuit shown in Fig. P 7.18, the switch has been in position a for a long time. At t=0, it moves instantaneously from a to b.

Figure P7.18

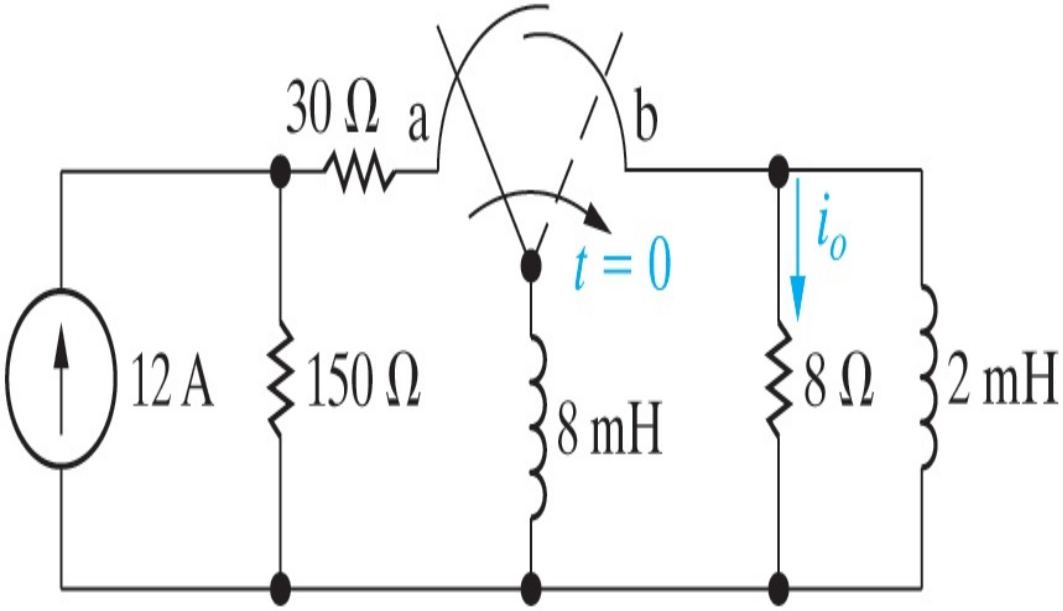


Figure P7.18 Full Alternative Text

1. a) Find io(t) for t≥0.
2. b) What is the total energy delivered to the 8 Ω resistor?
3. c) How many time constants does it take to deliver 95% of the energy found in (b)?

19. 7.19 PSPICEMULTISIM The two switches shown in the circuit in Fig. P 7.19 operate simultaneously. Prior to t=0 each switch has been in its indicated position for a long time. At t=0 the two switches move instantaneously to their new positions. Find

Figure P7.19

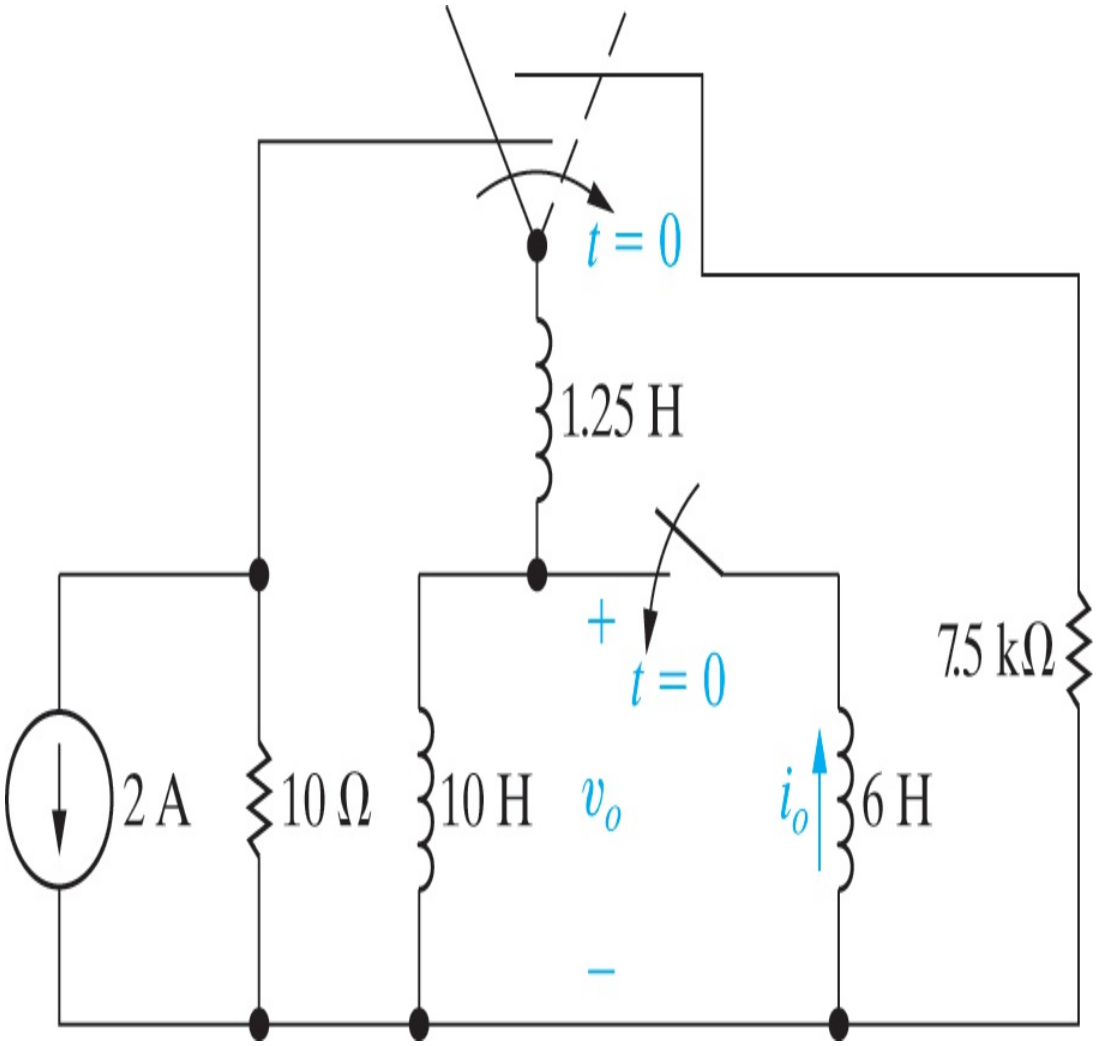


Figure P7.19 Full Alternative Text

1. a) vo(t), t≥0+.
2. b) io(t), t≥0.

20. 7.20 For the circuit seen in Fig. P 7.19, find
    1. a) the total energy dissipated in the 7.5 kΩ resistor.
    2. b) the energy trapped in the ideal inductors.

Section 7.2

1. 7.21 The switch in the circuit in Fig. P 7.21 has been in the left position for a long time. At t=0 it moves to the right position and stays there.

Figure P7.21

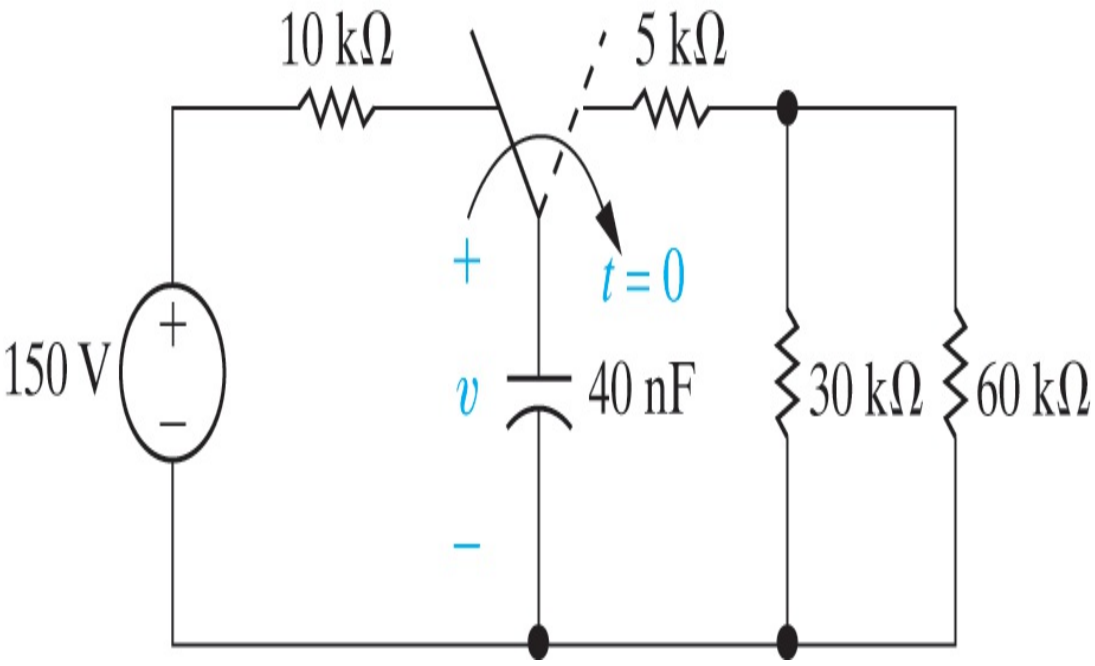


Figure P7.21 Full Alternative Text

   1. a) Find the initial voltage drop across the capacitor.
   2. b) Find the initial energy stored by the capacitor.
   3. c) Find the time constant of this circuit for t>0.
   4. d) Write the expression for the capacitor voltage v(t) for t≥0.

2. 7.22 The switch in the circuit in Fig. P 7.22 has been in the left position for a long time. At t=0 it moves to the right position and stays there.

Figure P7.22

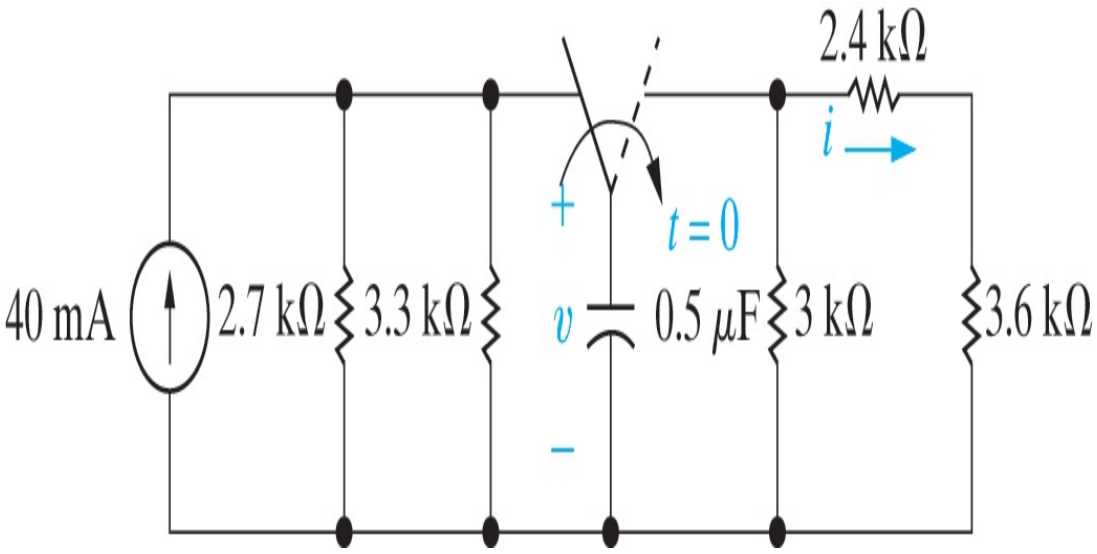


Figure P7.22 Full Alternative Text

1. a) Write the expression for the capacitor voltage, v(t), for t≥0.

2. b) Write the expression for the current through the 40 kΩ resistor, i(t), for t≥0+.

3. 7.23 What percentage of the initial energy stored in the capacitor in Fig. P 7.22 is dissipated by the 3 kΩ resistor 500 μs after the switch is thrown?

4. 7.24 The switch shown in Fig. P 7.24 has been open for a long time before closing at t=0. Write the expression for the capacitor voltage, v(t), for t≥0.

Figure P7.24

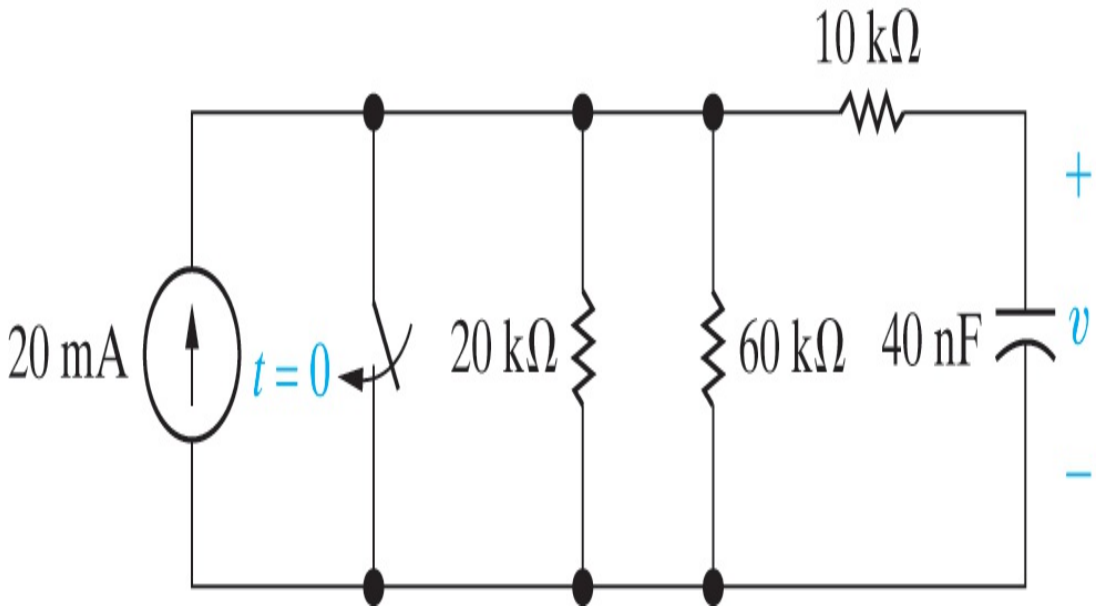


Figure P7.24 Full Alternative Text

5. 7.25 PSPICEMULTISIM The switch in the circuit in Fig. P 7.25 is closed at t=0 after being open for a long time.

Figure P7.25

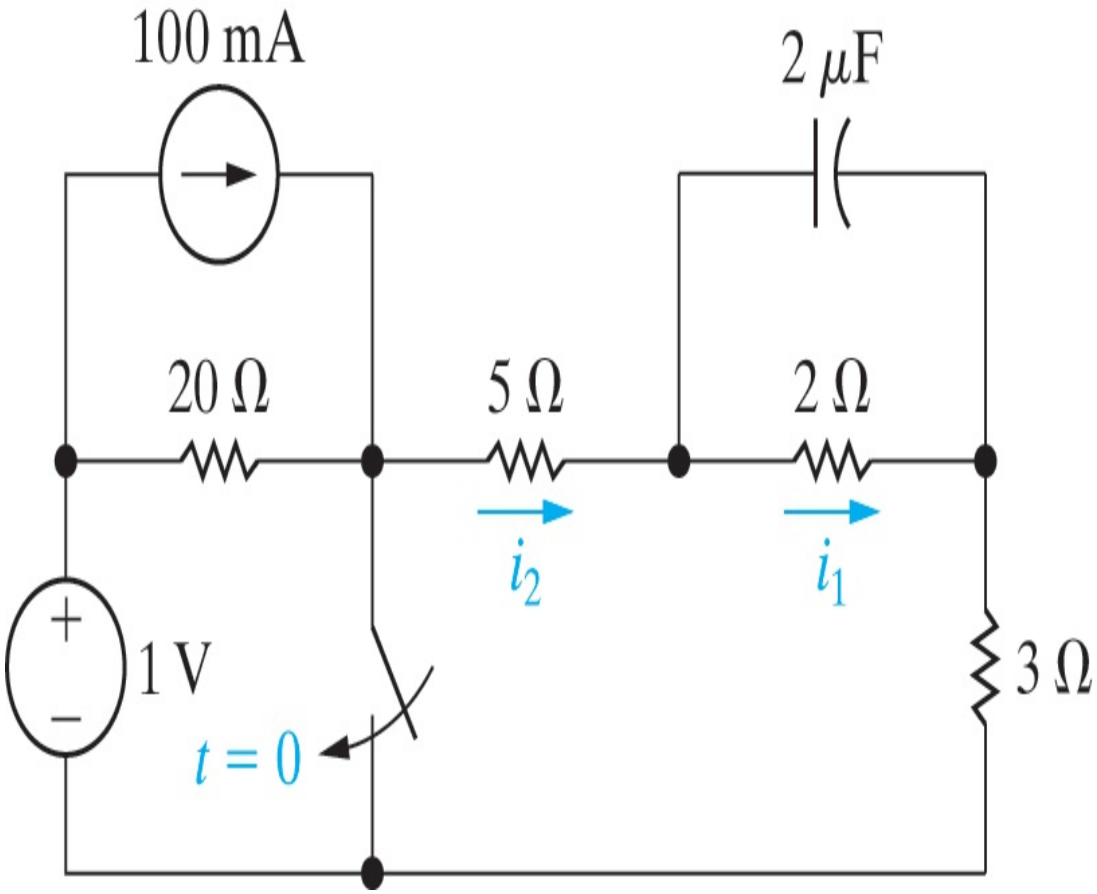


Figure P7.25 Full Alternative Text

1. a) Find i1(0−) and i2(0−).
2. b) Find i1(0+) and i2(0+).
3. c) Explain why i1(0−)=i1(0+).
4. d) Explain why i2(0−)≠i2(0+).
5. e) Find i1(t) for t≥0.
6. f) Find i2(t) for t≥0+.

6. 7.26 In the circuit shown in Fig. P 7.26, both switches operate together; that is, they either open or close at the same time. The switches are closed a long time before opening at t=0.

Figure P7.26

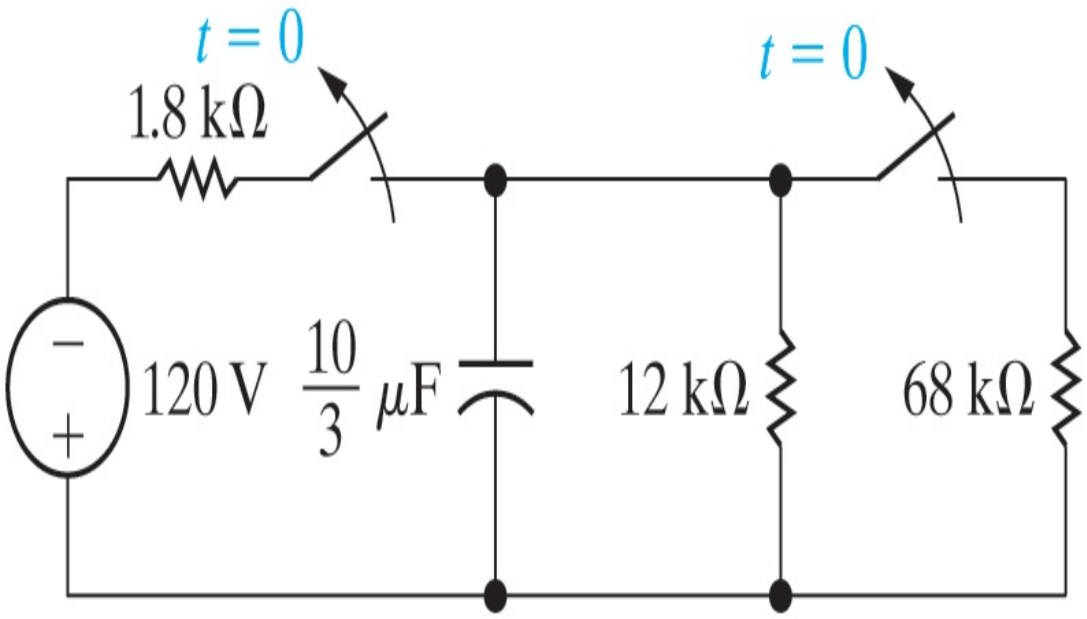


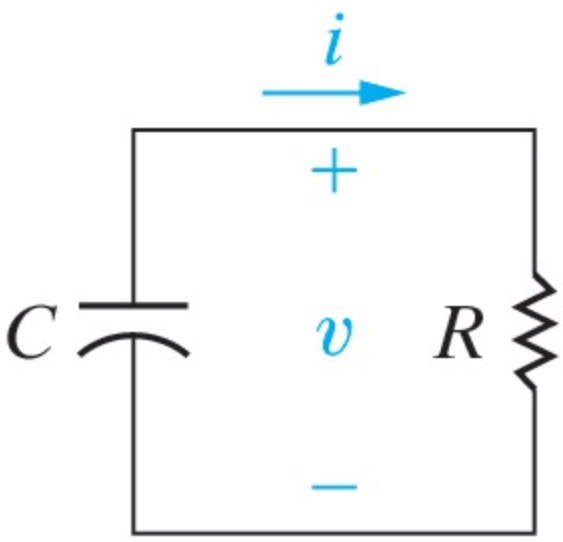
Figure P7.26 Full Alternative Text

1. a) How many microjoules of energy have been dissipated in the 12 kΩ resistor 12 ms after the switches open?

2. b) How long does it take to dissipate 75% of the initially stored energy?

7. 7.27 In the circuit in Fig. P 7.27 the voltage and current expressions are

Figure P7.27



v=48e−25t V, t≥0;
i=12e−25t mA, t≥0+.

Find

1. a) R.
2. b) C.
3. c) τ (in milliseconds).
4. d) the initial energy stored in the capacitor.
5. e) the amount of energy that has been dissipated by the resistor 60 ms after the voltage begins to decay.

8. 7.28

    1. a) Use component values from Appendix H to create a first-order RC circuit (see Fig. 7.13) with a time constant of 50 ms. Use a single capacitor and a network of resistors, if necessary. Draw your circuit.
    2. b) Suppose the capacitor you chose in part (a) has an initial voltage drop of 50 V. Write an expression for the voltage drop across the

capacitor for t≥0.

3. c) Using you result from part (b), calculate the time at which the voltage drop across the capacitor has reached 10 V.

9. 7.29 PSPICEMULTISIM The switch in the circuit in Fig. P 7.29 has been in position 1 for a long time before moving to position 2 at t=0. Find io(t) for t≥0+.

Figure P7.29

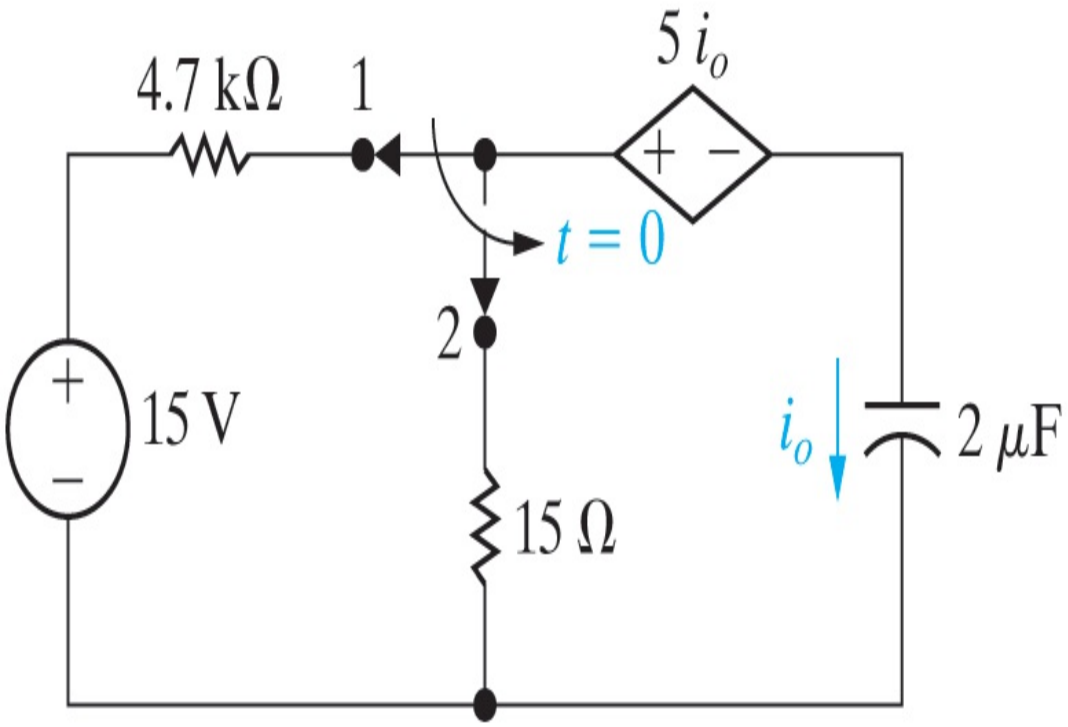


Figure P7.29 Full Alternative Text

10. 7.30 The switch in the circuit seen in Fig. P 7.30 has been in position x for a long time. At t=0, the switch moves instantaneously to position y.

Figure P7.30

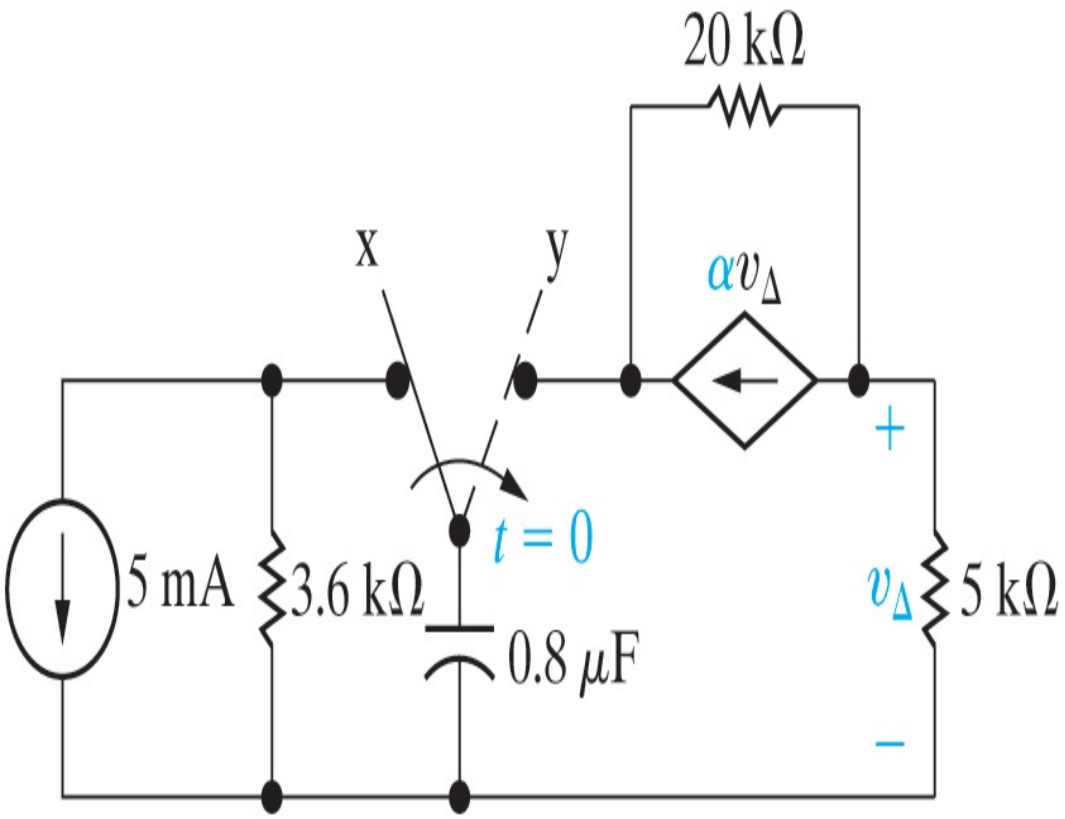


Figure P7.30 Full Alternative Text

1. a) Find α so that the time constant for t>0 is 40 ms.

2. b) For the α found in (a), find vΔ.

11. 7.31

    1. a) In Problem 7.30, how many microjoules of energy are generated by the dependent current source during the time the capacitor discharges to 0 V?

    2. b) Show that for t≥0 the total energy stored and generated in the capacitive circuit equals the total energy dissipated.

12. 7.32 PSPICEMULTISIM At the time the switch is closed in the circuit in Fig. P 7.32, the voltage across the parallel capacitors is 50 V and the voltage on the 250 nF capacitor is 40 V.

Figure P7.32

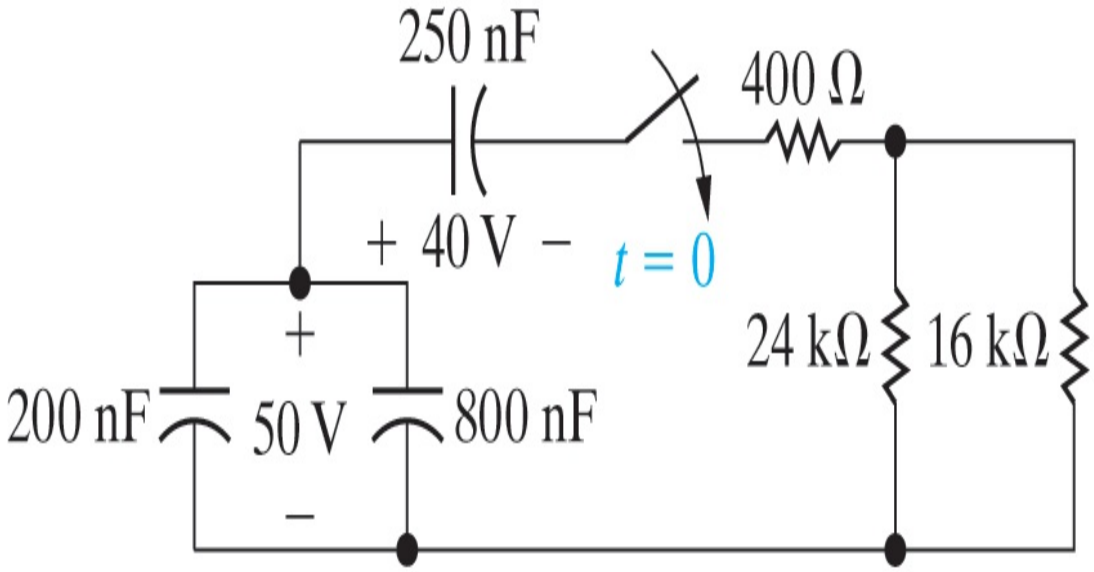


Figure P7.32 Full Alternative Text

1. a) What percentage of the initial energy stored in the three capacitors is dissipated in the 24 kΩ resistor?

2. b) Repeat (a) for the 400 Ω and 16 kΩ resistors.

3. c) What percentage of the initial energy is trapped in the capacitors?

13. 7.33 The switch in the circuit in Fig. P 7.33 has been in position a for a long time and v2=0 V. At t=0, the switch is thrown to position b. Calculate

Figure P7.33

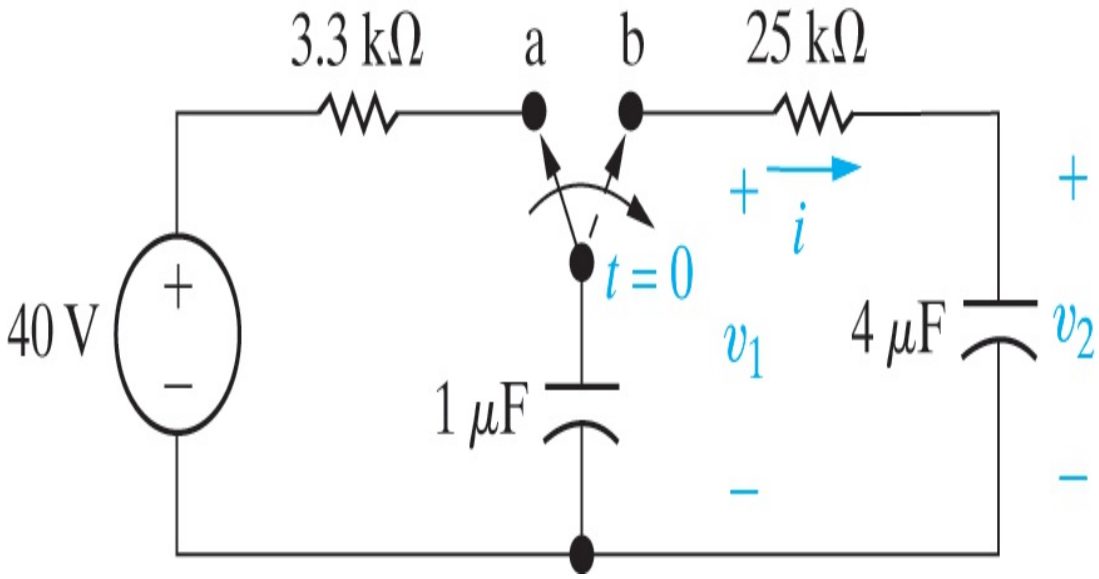


Figure P7.33 Full Alternative Text

1. a) i, v1, and v2 for t≥0+,

2. b) the energy stored in the 1 μF capacitor at t=0, and

3. c) the energy trapped in the circuit and the total energy dissipated in the 25 kΩ resistor if the switch remains in position b indefinitely.

14. 7.34 PSPICEMULTISIM After the circuit in Fig. P 7.34 has been in operation for a long time, a screwdriver is inadvertently connected across the terminals a, b. Assume the resistance of the screwdriver is negligible.

Figure P7.34

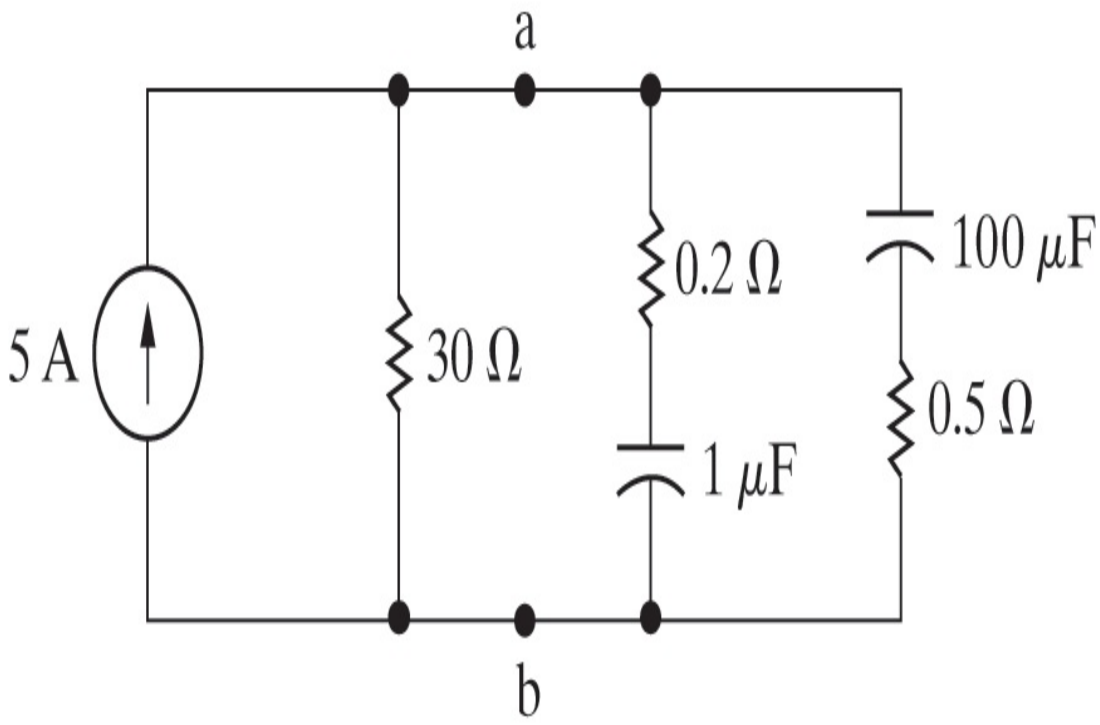


Figure P7.34 Full Alternative Text

1. a) Find the current in the screwdriver at t=0+ and t=∞.
2. b) Derive the expression for the current in the screwdriver for t≥0+.

Section 7.3

1. 7.35 After the switch in the circuit of Fig. P 7.35 has been open for a long time, it is closed at t=0. Calculate (a) the initial value of i; (b) the final value of i; (c) the time constant for t≥0; and (d) the numerical expression for i(t) when t≥0.

Figure P7.35

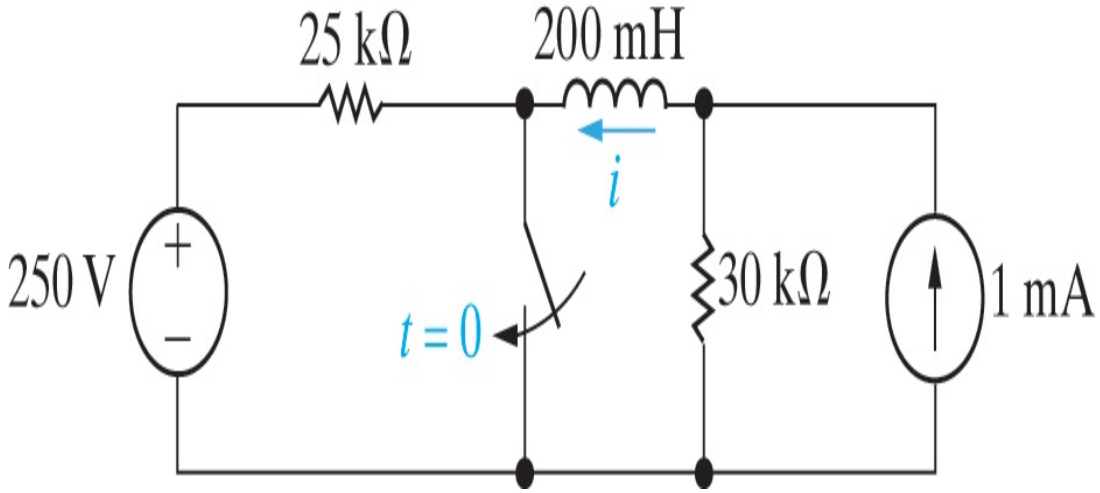


Figure P7.35 Full Alternative Text

2. 7.36 PSPICEMULTISIM The switch in the circuit shown in Fig. P 7.36 has been in position a for a long time. At t=0, the switch moves instantaneously to position b.

Figure P7.36

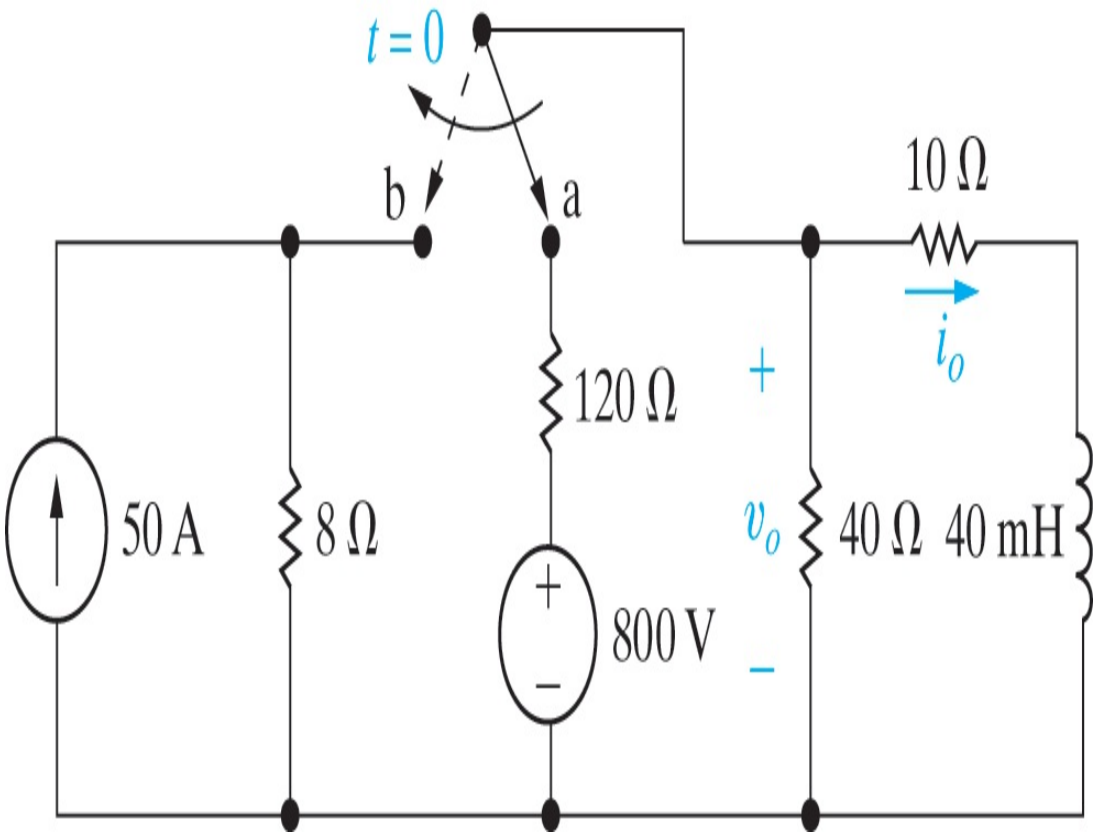


Figure P7.36 Full Alternative Text

1. a) Find the numerical expression for io(t) when t≥0.

2. b) Find the numerical expression for vo(t) for t≥0+.

3. 7.37 Repeat Problem 7.36 assuming that the switch in the circuit in Fig. P 7.36 has been in position b for a long time and then moves to position a at t=0 and stays there.

4. 7.38 PSPICEMULTISIM The switch in the circuit shown in Fig. P 7.38 has been in position a for a long time before moving to position b at t=0.

Figure P7.38

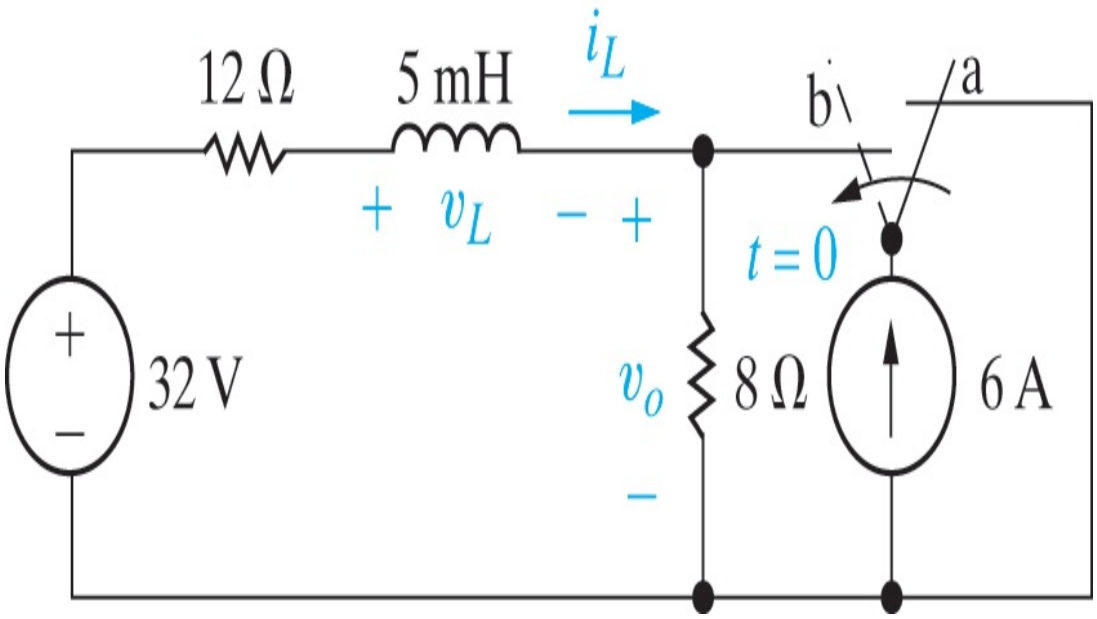


Figure P7.38 Full Alternative Text

1. a) Find the numerical expressions for iL(t) and vo(t) for t≥0.
2. b) Find the numerical values of vL(0+) and vo(0+).

5. 7.39
    1. a) Derive Eq. 7.16 by first converting the Thévenin equivalent in Fig. 7.20 to a Norton equivalent and then summing the currents away from the upper node, using the inductor voltage v as the variable of interest.
    2. b) Use the separation of variables technique to find the solution to Eq. 7.16. Verify that your solution agrees with the solution given in Eq. 7.15.

6. 7.40 The current and voltage at the terminals of the inductor in the circuit in Fig. 7.20 are

    $i(t)=(4+4e^{-40t})$ A, $t\geq 0$; $v(t)=-80e^{-40t}$ V, $t\geq 0+$.

    1. a) Specify the numerical values of Vs, R, Io, and L.

2. b) How many milliseconds after the switch has been closed does the energy stored in the inductor reach 9 J?

7. 7.41 The switch in the circuit shown in Fig. P 7.41 has been closed for a long time. The switch opens at t=0. For t≥0+:

Figure P7.41

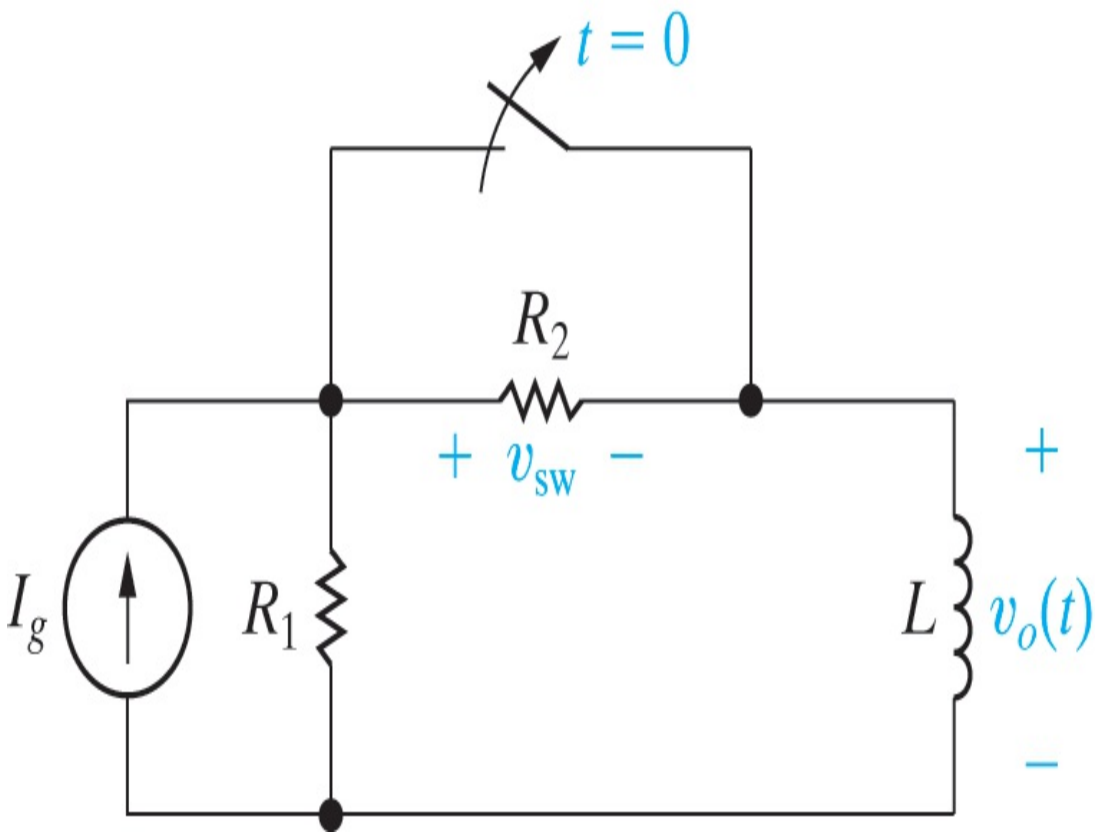


Figure P7.41 Full Alternative Text

1. a) Find vo(t) as a function of Ig, R1, R2, and L.
2. b) Explain what happens to vo(t) as R2 gets larger and larger.
3. c) Find vSW as a function of Ig, R1, R2, and L.
4. d) Explain what happens to vSW as R2 gets larger and larger.

8. 7.42 The switch in the circuit in Fig. P 7.42 has been closed for a long time. A student abruptly opens the switch and reports to her instructor that when the switch opened, an electric arc with noticeable persistence was established across the switch, and at the same time the voltmeter placed across the coil was damaged. On the basis of your analysis of the circuit in Problem 7.41, can you explain to the student why this happened?

Figure P7.42

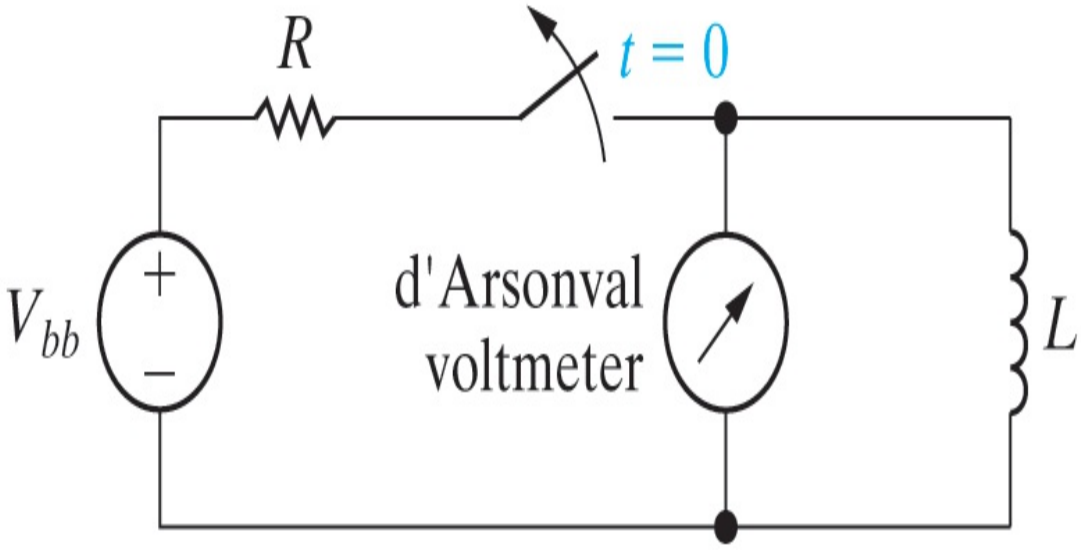


Figure P7.42 Full Alternative Text

9. 7.43

    1. a) Use component values from Appendix H to create a first-order RL circuit (see Fig. 7.20) with a time constant of 8 μs. Use a single inductor and a network of resistors, if necessary. Draw your circuit.

    2. b) Suppose the inductor you chose in part (a) has no initial stored energy. At t=0, a switch connects a voltage source with a value of 25 V in series with the inductor and equivalent resistance. Write an expression for the current through the inductor for t≥0.

3. c) Using your result from part (b), calculate the time at which the current through the inductor reaches 75% of its final value.

10. 7.44 PSPICEMULTISIM The switch in the circuit in Fig. P 7.44 has been open a long time before closing at t=0. Find vo(t) for t≥0+.

Figure P7.44

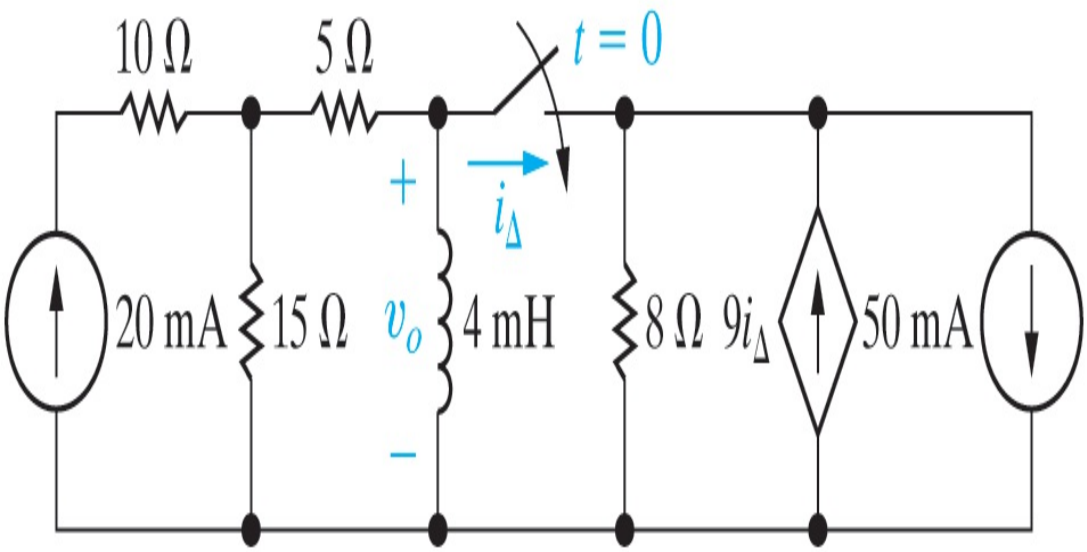


Figure P7.44 Full Alternative Text

11. 7.45 PSPICEMULTISIM The switch in the circuit in Fig. P 7.45 has been open a long time before closing at t=0. Find io(t) for t≥0.

Figure P7.45

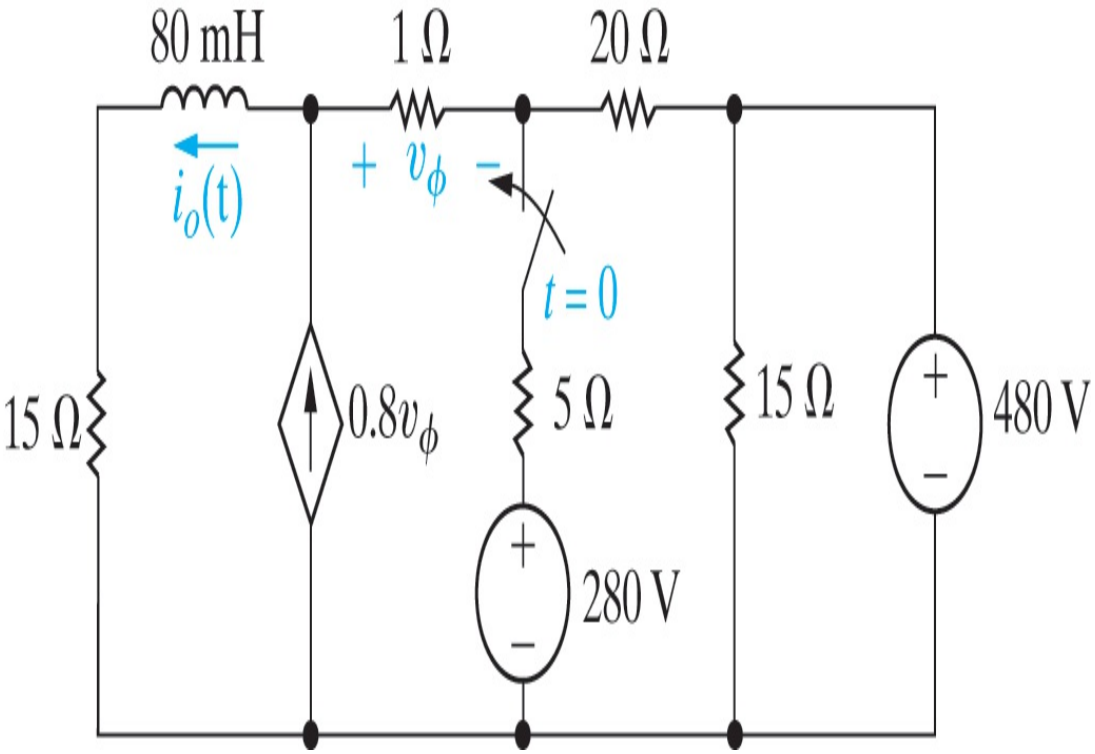


Figure P7.45 Full Alternative Text

12. 7.46 PSPICEMULTISIM The switch in the circuit in Fig. P 7.46 has been in position 1 for a long time. At t=0 it moves instantaneously to position 2. How many milliseconds after the switch moves does vo equal 100 V?

Figure P7.46

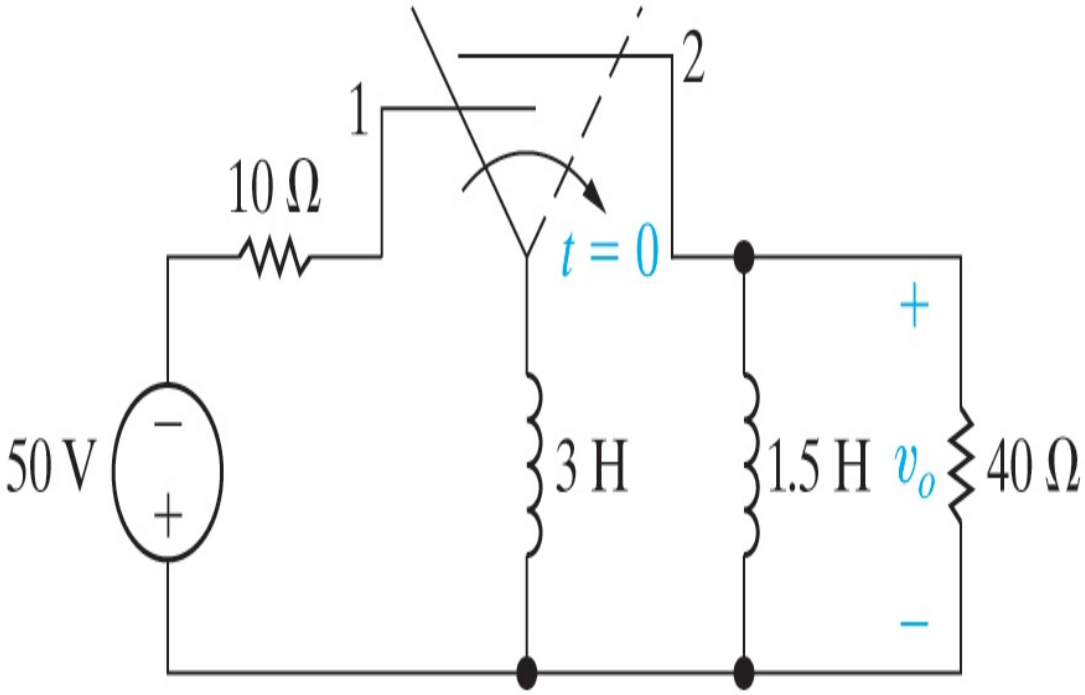


Figure P7.46 Full Alternative Text

13. 7.47 For the circuit in Fig. P 7.46, find (in joules):
    1. a) the total energy dissipated in the 40 Ω resistor,
    2. b) the energy trapped in the inductors, and
    3. c) the initial energy stored in the inductors.
14. 7.48 PSPICEMULTISIM The switch in the circuit in Fig. P 7.48 has been open a long time before closing at t=0. Find vo(t) for t≥0+.

Figure P7.48

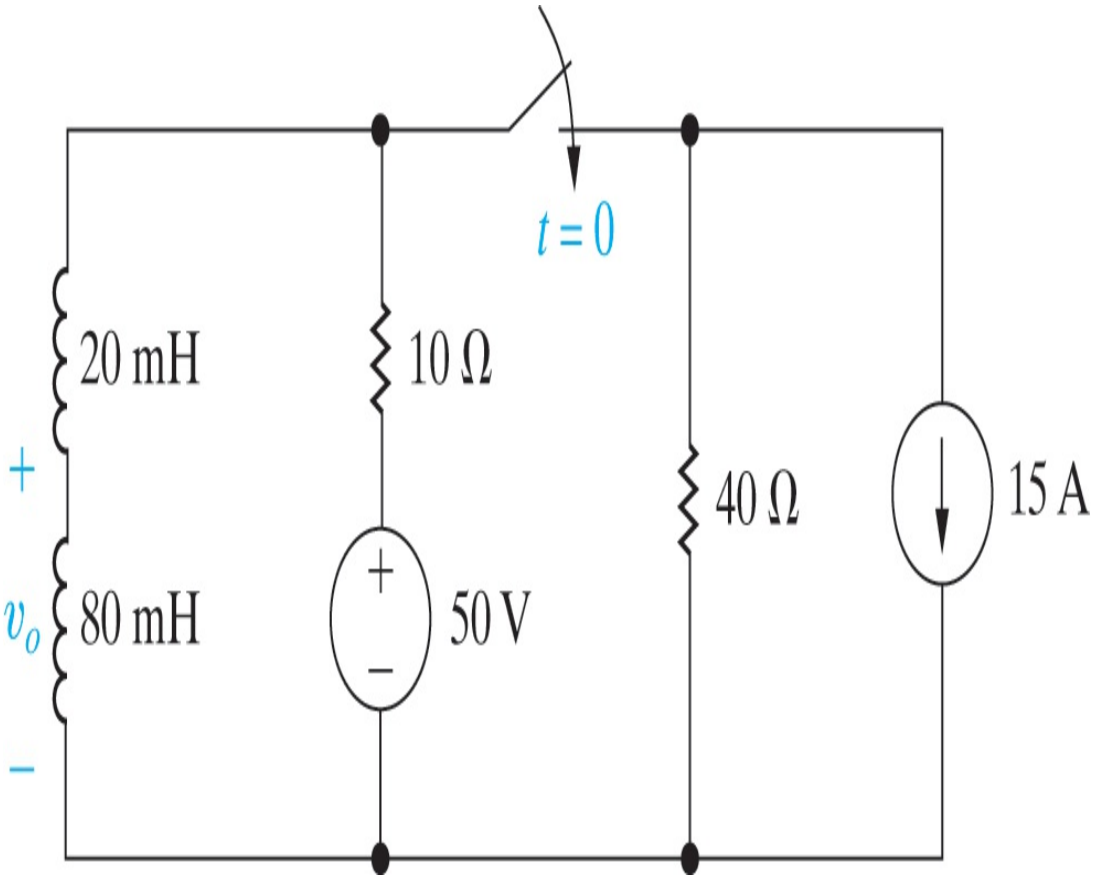


Figure P7.48 Full Alternative Text

15. 7.49 PSPICEMULTISIM The make-before-break switch in the circuit of Fig. P 7.49 has been in position a for a long time. At t=0, the switch moves instantaneously to position b. Find

Figure P7.49

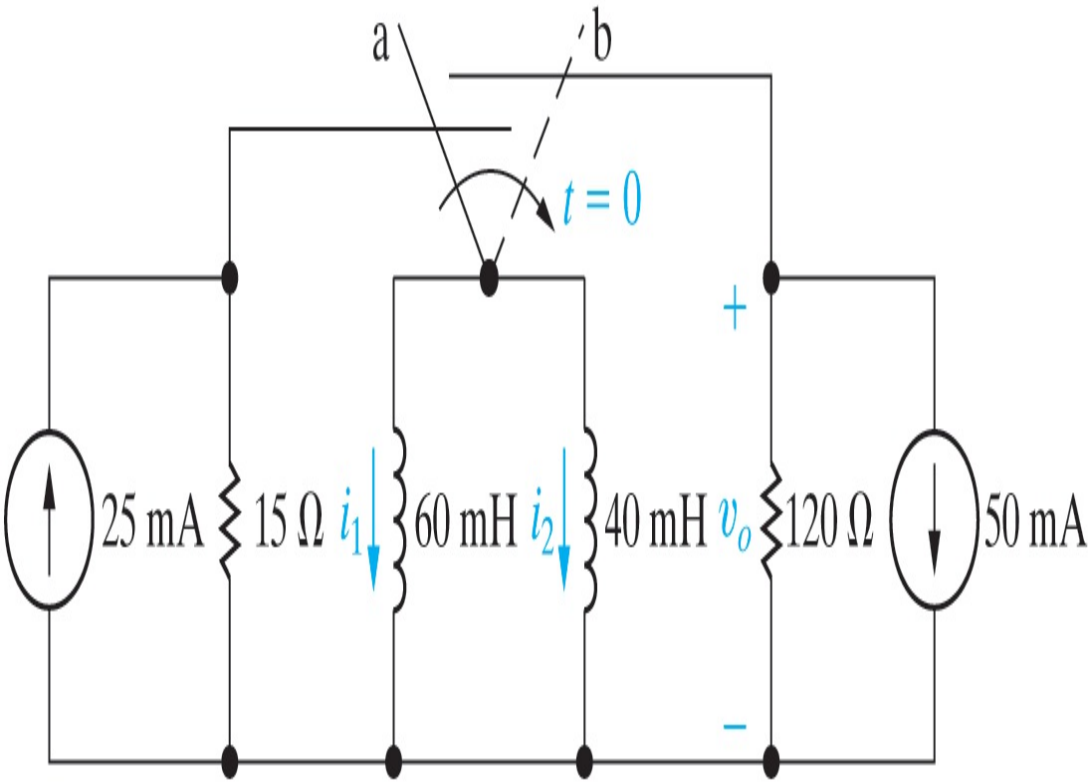


Figure P7.49 Full Alternative Text

1. a) vo(t), t≥0+.
2. b) i1(t), t≥0.
3. c) i2(t), t≥0.

16. 7.50 There is no energy stored in the inductors L1 and L2 at the time the switch is opened in the circuit shown in Fig. P 7.50.

Figure P7.50

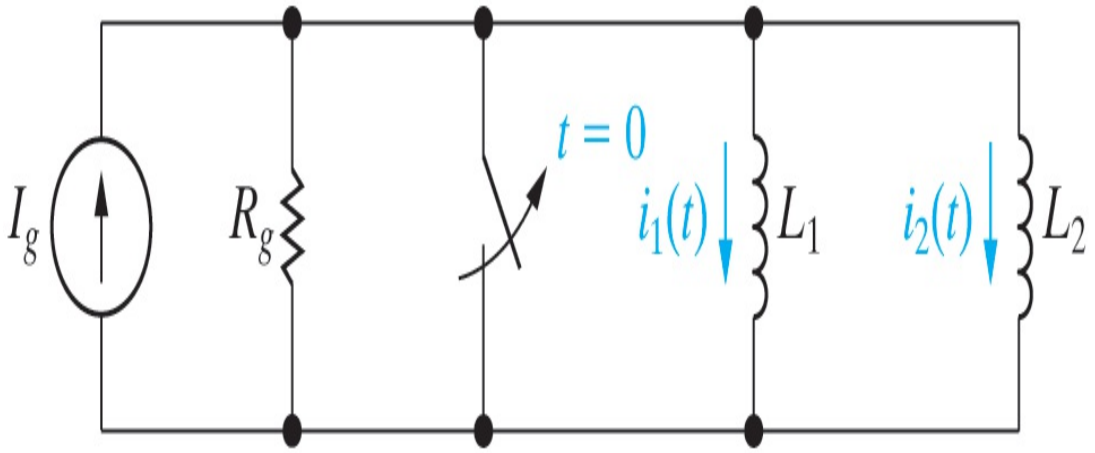


Figure P7.50 Full Alternative Text

1. a) Derive the expressions for the currents i1(t) and i2(t) for t≥0.

2. b) Use the expressions derived in (a) to find i1(∞) and i2(∞).

17. 7.51 Assume that the switch in the circuit of Fig. P 7.51 has been in position a for a long time and that at t=0 it is moved to position b. Find (a) vC(0+); (b) τ for t>0; (c) vC(∞); (d) i(0+); (e) vC, t≥0; and (f) i, t≥0+.

Figure P7.51

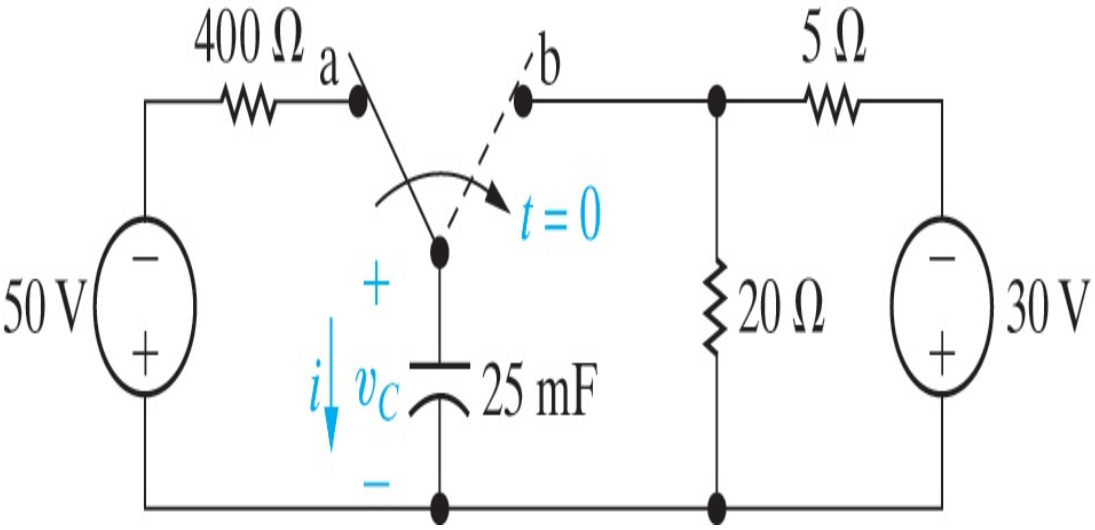


Figure P7.51 Full Alternative Text

18. 7.52 PSPICEMULTISIM The switch in the circuit seen in Fig. P 7.52 has been in position a for a long time. At t=0, the switch moves instantaneously to position b. For t≥0+, find

Figure P7.52

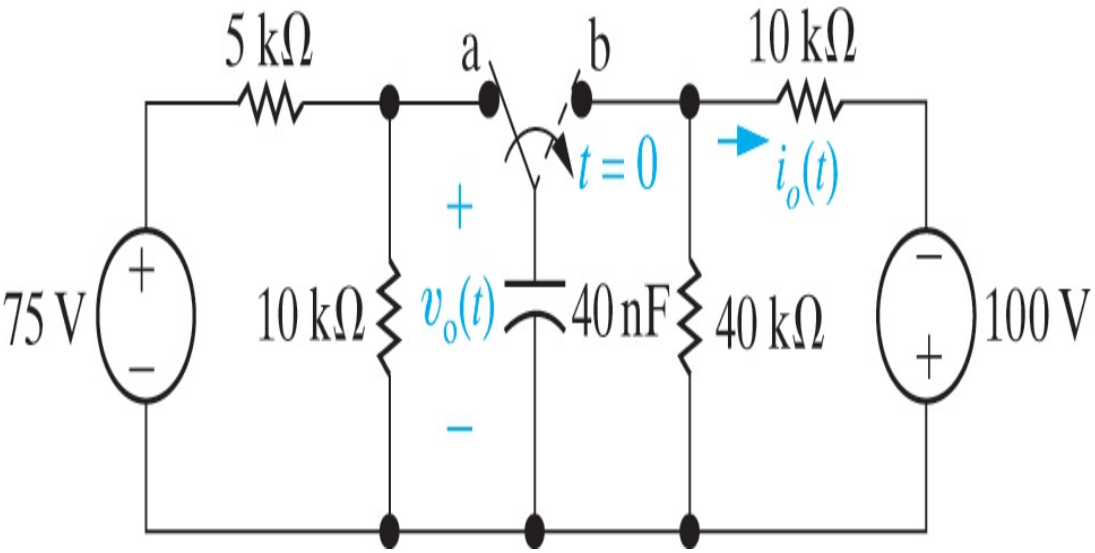


Figure P7.52 Full Alternative Text

1. a) vo(t).
2. b) io(t).

19. 7.53

Figure P7.53

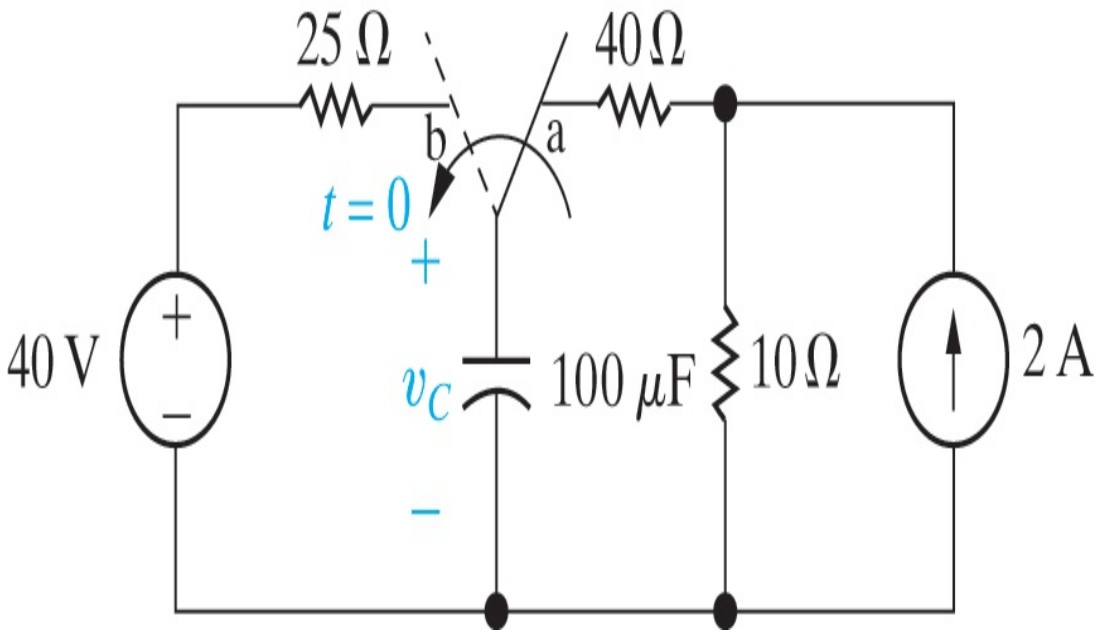


Figure P7.53 Full Alternative Text

1. a) The switch in the circuit in Fig. P 7.53 has been in position a for a long time. At t=0, the switch moves instantaneously to position b and stays there. Find the initial and final values of the capacitor voltage, the time constant for t≥0, and the expression for the capacitor voltage for t≥0.

2. b) Now suppose the switch in the circuit in Fig. P 7.53 has been in position b for a long time. At t=0, the switch moves instantaneously to position a and stays there. Find the initial and final values of the capacitor voltage, the time constant for t≥0, and the expression for the capacitor voltage for t≥0.

20. 7.54 PSPICEMULTISIM The circuit in Fig. P 7.54 has been in operation for a long time. At t=0, the voltage source reverses polarity and the current source drops from 3 mA to 2 mA. Find vo(t) for t≥0.

Figure P7.54

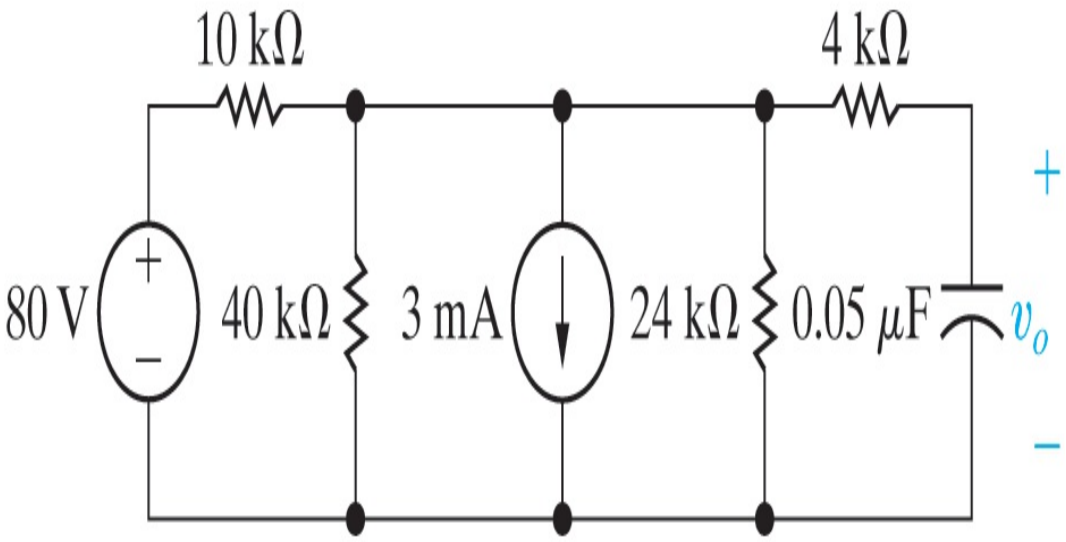


Figure P7.54 Full Alternative Text

21. 7.55 The switch in the circuit of Fig. P 7.55 has been in position a for a long time. At t=0 the switch is moved to position b. Calculate (a) the initial voltage on the capacitor; (b) the final voltage on the capacitor; (c) the time constant (in microseconds) for t>0; and (d) the length of time (in microseconds) required for the capacitor voltage to reach zero after the switch is moved to position b.

Figure P7.55

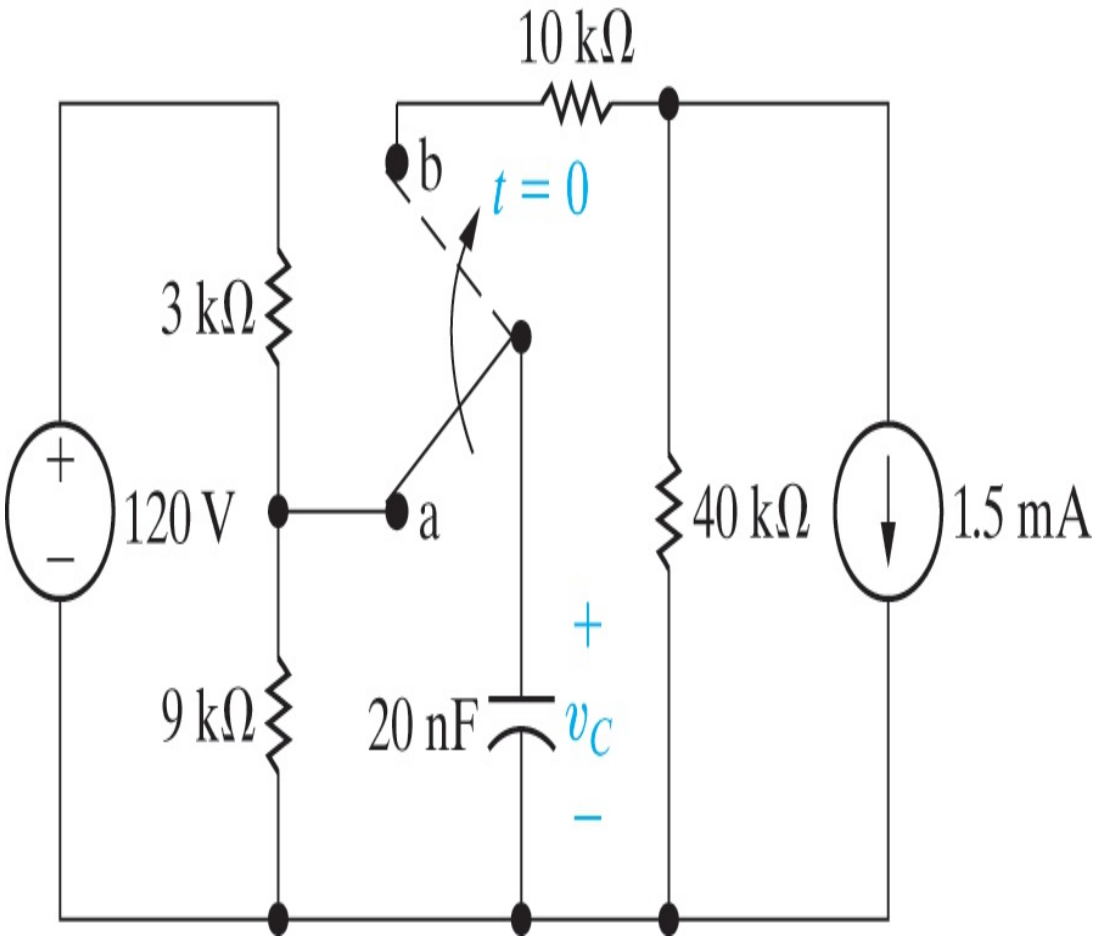


Figure P7.55 Full Alternative Text

22. 7.56 PSPICEMULTISIM The switch in the circuit seen in Fig. P 7.56 has been in position a for a long time. At t=0, the switch moves instantaneously to position b. Find vo(t) and io(t) for t≥0+.

Figure P7.56

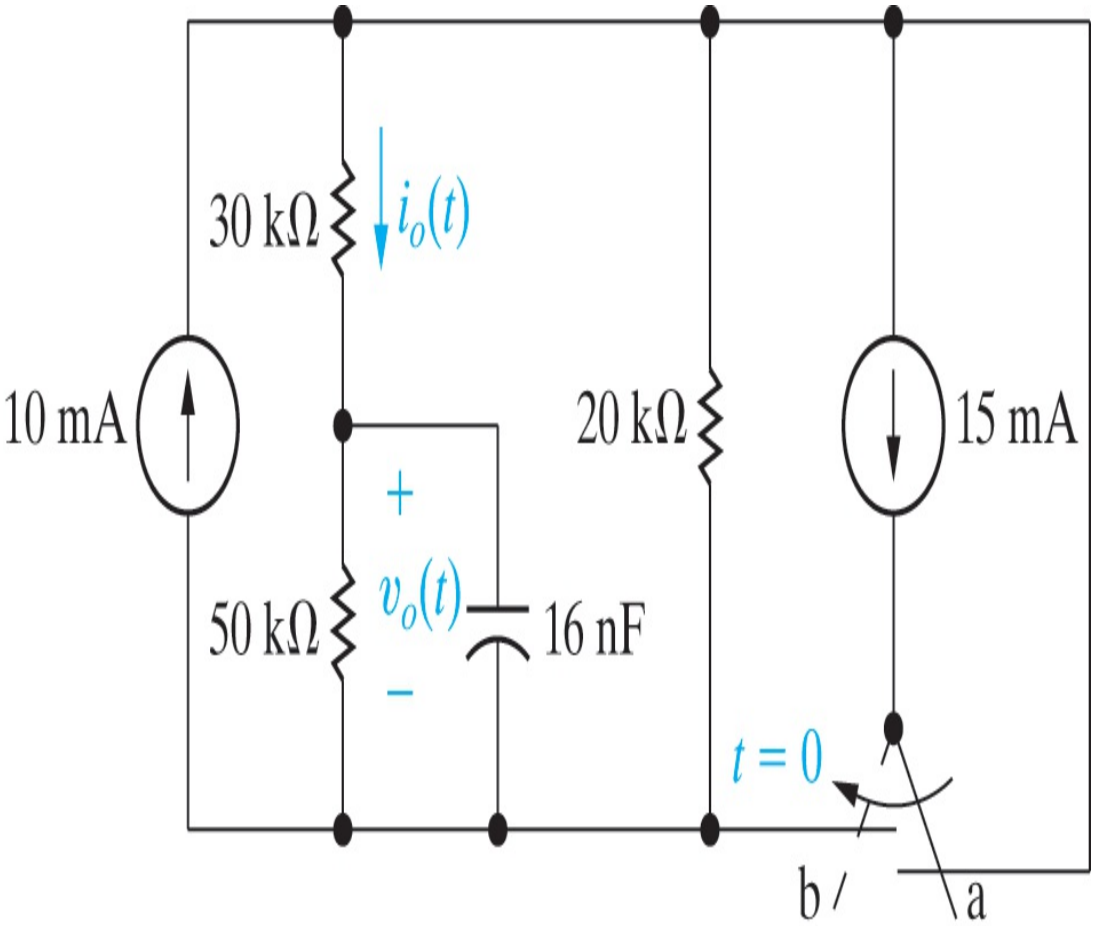


Figure P7.56 Full Alternative Text

23. 7.57 The current and voltage at the terminals of the capacitor in the circuit in Fig. 7.28 are

    $i(t)=3e^{-2500t}$ mA, $t\geq 0+$; $v(t)=(40-24e^{-2500t})$ V, $t\geq 0$.

    1. a) Specify the numerical values of Is, Vo, R, C, and τ.

    2. b) How many microseconds after the switch has been closed does the energy stored in the capacitor reach 81% of its final value?

24. 7.58

    1. a) Use component values from Appendix H to create a first-order RC circuit (see Fig. 7.28) with a time constant of 250 ms. Use a single capacitor and a network of resistors, if necessary. Draw your

circuit.

2. b) Suppose the capacitor you chose in part (a) has an initial voltage drop of 100 V. At t=0, a switch connects a current source with a value of 1 mA in parallel with the capacitor and equivalent resistance. Write an expression for the voltage drop across the capacitor for t≥0.

3. c) Using your result from part (b), calculate the time at which the voltage drop across the capicitor reaches 50 V.

25. 7.59 PSPICEMULTISIM The switch in the circuit in Fig. P 7.59 has been in position a for a long time. At t=0, the switch moves instantaneously to position b. At the instant the switch makes contact with terminal b, switch 2 opens. Find vo(t) for t≥0.

Figure P7.59

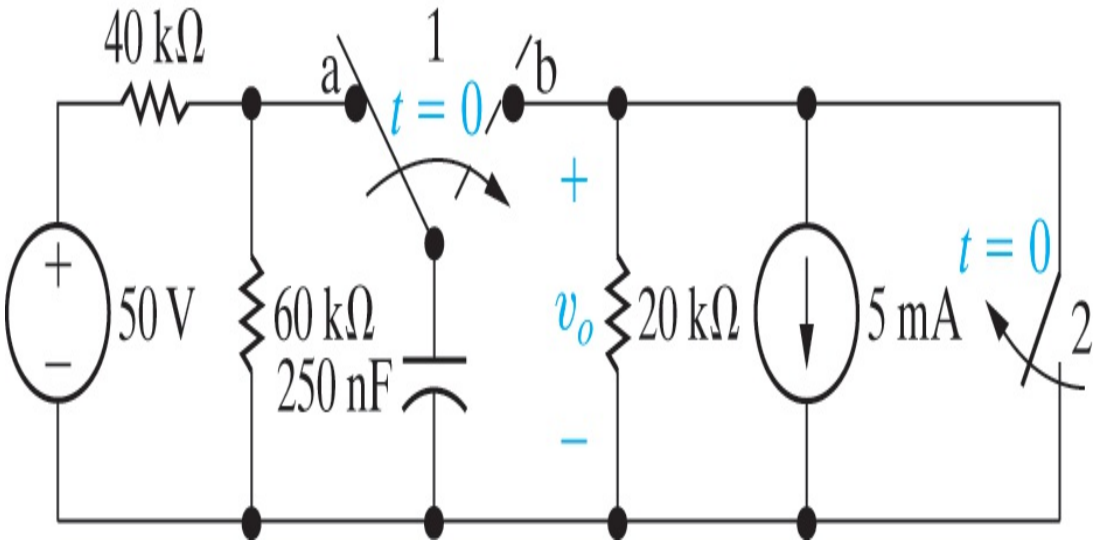


Figure P7.59 Full Alternative Text

26. 7.60

1. a) Derive Eq. 7.17 by first converting the Norton equivalent circuit shown in Fig. 7.28 to a Thévenin equivalent and then summing the

voltages around the closed loop, using the capacitor current i as the relevant variable.

2. b) Use the separation of variables technique to find the solution to Eq. 7.17. Verify that your solution agrees with that of Eq. 7.22.

27. 7.61 PSPICEMULTISIM The switch in the circuit shown in Fig. P 7.61 has been in the off position for a long time. At t=0, the switch moves instantaneously to the on position. Find vo(t) for t≥0.

Figure P7.61

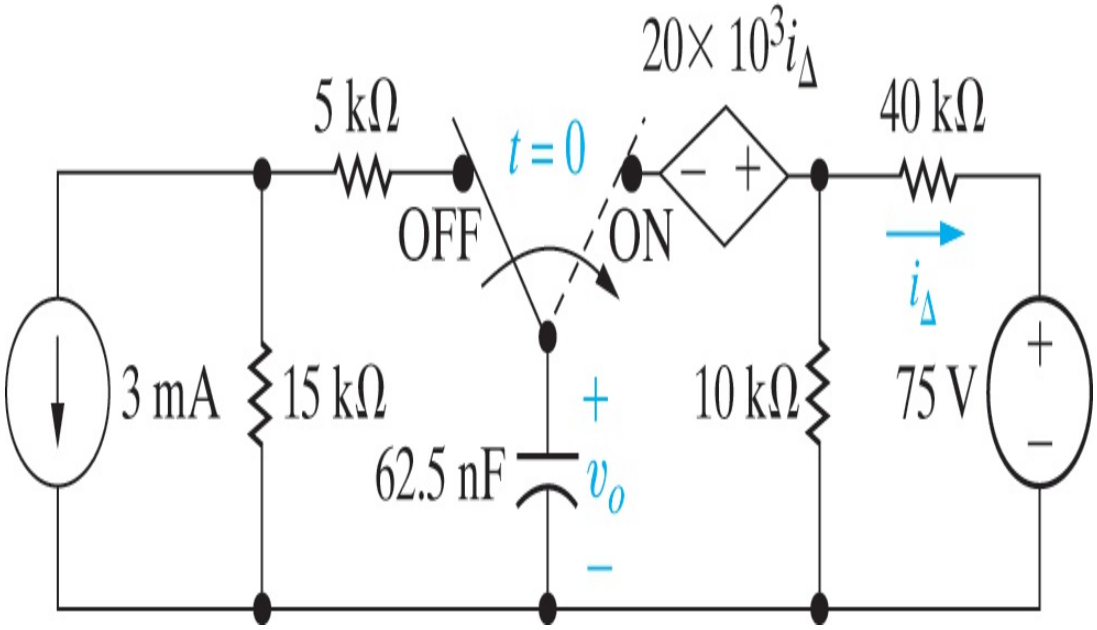


Figure P7.61 Full Alternative Text

28. 7.62 PSPICEMULTISIM Assume that the switch in the circuit of Fig. P 7.61 has been in the on position for a long time before switching instantaneously to the off position at t=0. Find vo(t) for t≥0.

29. 7.63 PSPICEMULTISIM The switch in the circuit shown in Fig. P 7.63 opens at t=0 after being closed for a long time. How many milliseconds after the switch opens is the energy stored in the capacitor 36% of its

final value?

Figure P7.63

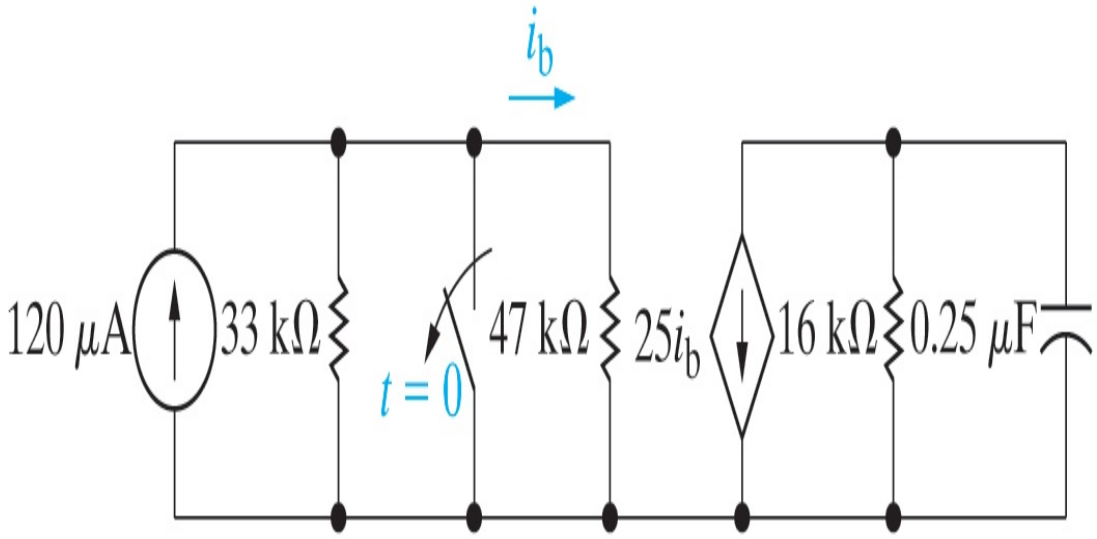


Figure P7.63 Full Alternative Text

30. 7.64 PSPICEMULTISIM The switch in the circuit of Fig. P 7.64 has been in position a for a long time. At t=0, it moves instantaneously to position b. For t≥0+, find

Figure P7.64

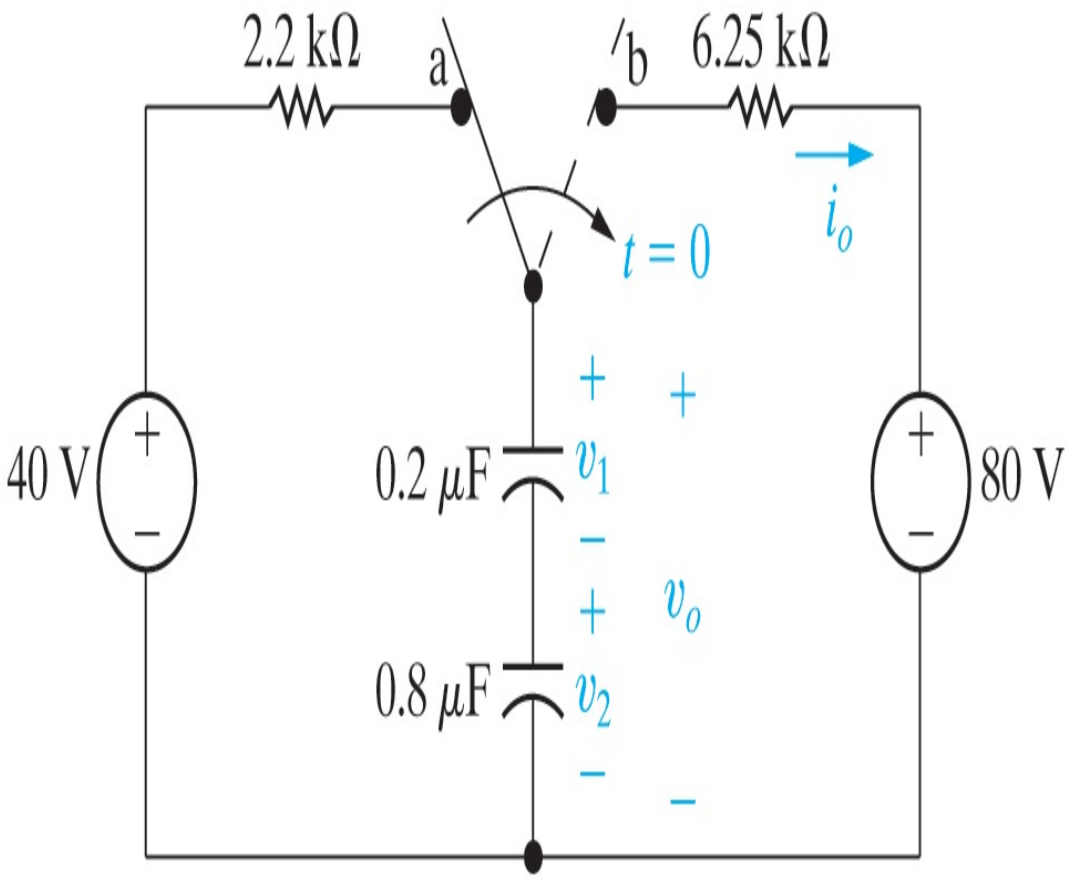


Figure P7.64 Full Alternative Text

1. a) vo(t).
2. b) io(t).
3. c) v1(t).
4. d) v2(t).
5. e) the energy trapped in the capacitors as $t \rightarrow \infty$.

31. 7.65 The switch in the circuit in Fig. P 7.65 has been in position x for a long time. The initial charge on the 60 nF capacitor is zero. At t=0, the switch moves instantaneously to position y.

Figure P7.65

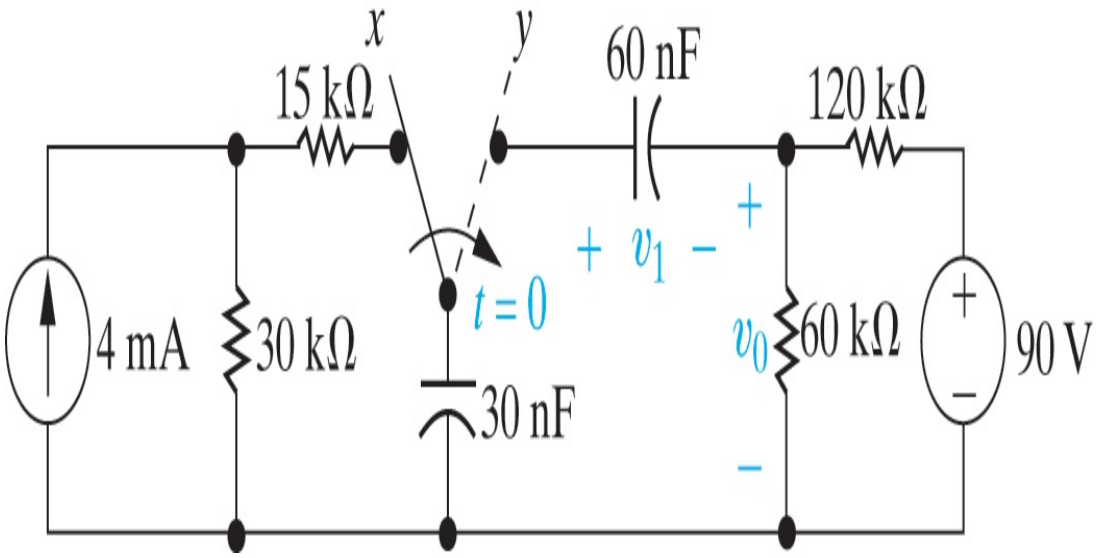


Figure P7.65 Full Alternative Text

1. a) Find vo(t) for t≥0+.
2. b) Find v1(t) for t≥0.

32. 7.66 There is no energy stored in the capacitors C1 and C2 at the time the switch is closed in the circuit seen in Fig. P 7.66.

Figure P7.66

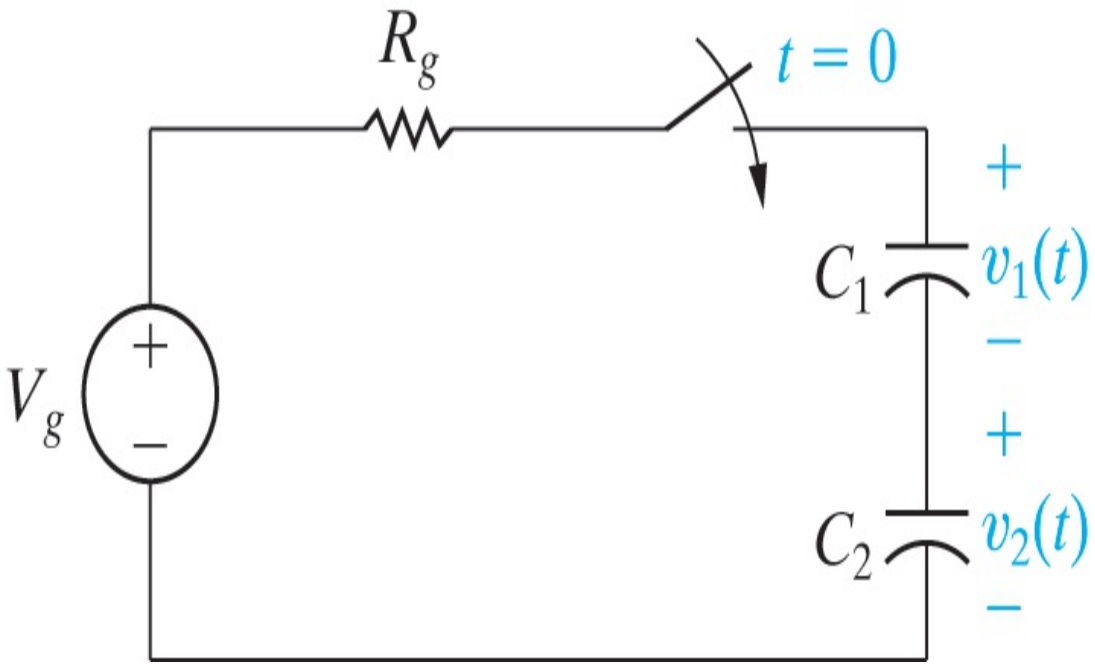


Figure P7.66 Full Alternative Text

1. a) Derive the expressions for v1(t) and v2(t) for t≥0.
2. b) Use the expressions derived in (a) to find v1(∞) and v2(∞).

Section 7.4

1. 7.67 Repeat (a) and (b) in Example 7.10 if the mutual inductance is reduced to zero.

2. 7.68 There is no energy stored in the circuit of Fig. P 7.68 at the time the switch is closed.

Figure P7.68

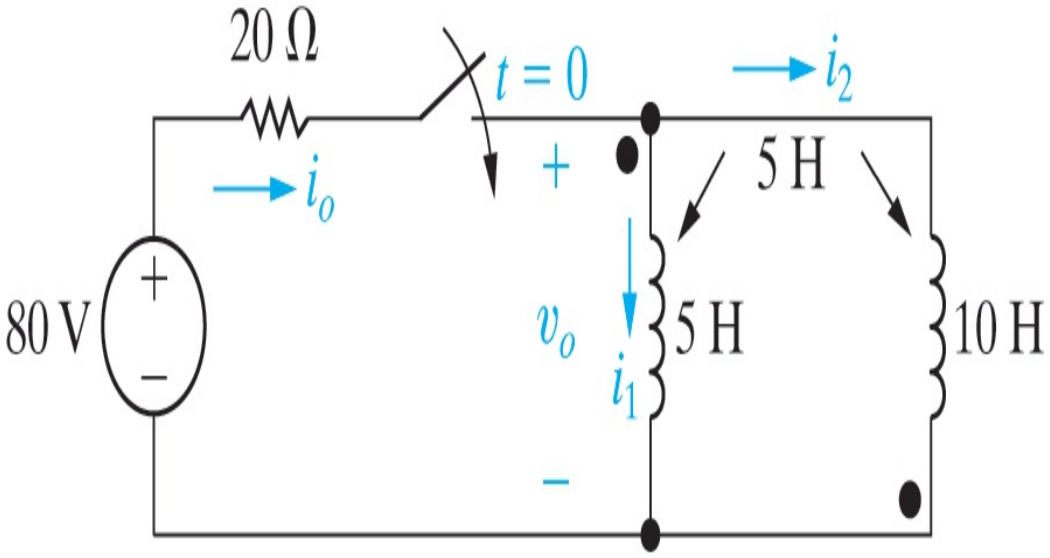


Figure P7.68 Full Alternative Text

1. a) Find io(t) for t≥0.

2. b) Find vo(t) for t≥0+.

3. c) Find i1(t) for t≥0.

4. d) Find i2(t) for t≥0.

5. e) Do your answers make sense in terms of known circuit behavior?

3. 7.69 PSPICEMULTISIM There is no energy stored in the circuit in Fig. P 7.69 at the time the switch is closed.

Figure P7.69

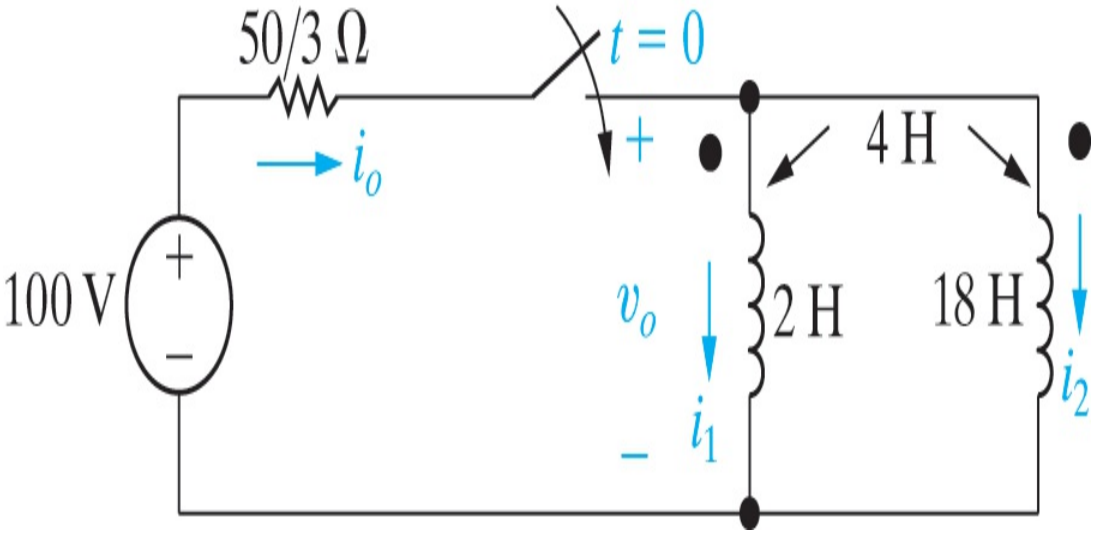


Figure P7.69 Full Alternative Text

1. a) Find io(t) for t≥0.
2. b) Find vo(t) for t≥0+.
3. c) Find i1(t) for t≥0.
4. d) Find i2(t) for t≥0.
5. e) Do your answers make sense in terms of known circuit behavior?

4. 7.70 PSPICEMULTISIM There is no energy stored in the circuit in Fig. P 7.70 at the time the switch is closed.

Figure P7.70

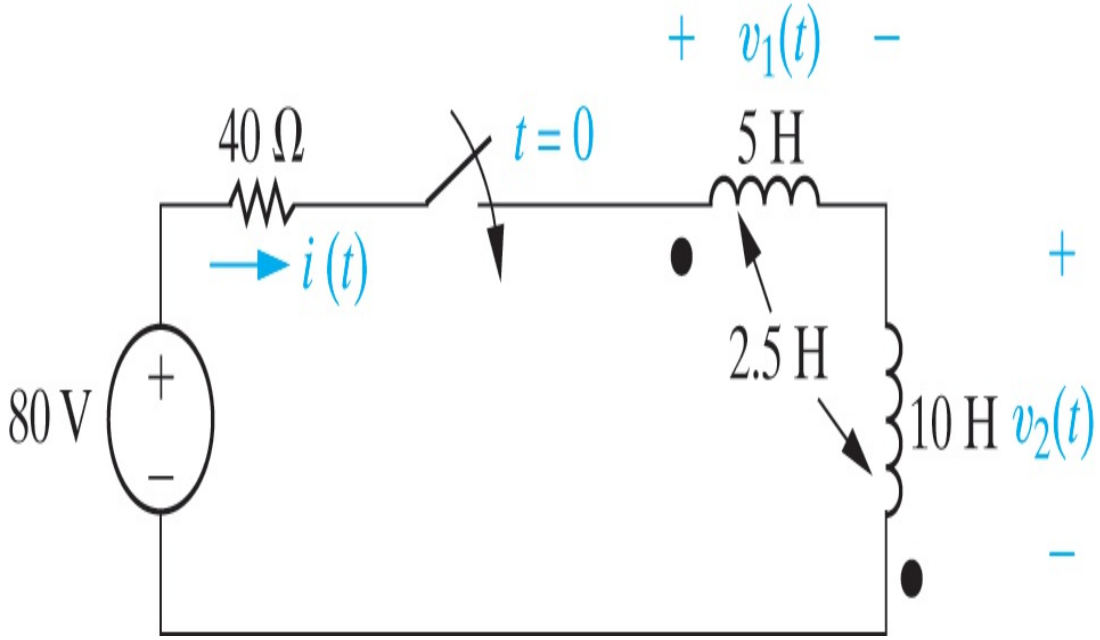


Figure P7.70 Full Alternative Text

1. a) Find i(t) for t≥0.
2. b) Find v1(t) for t≥0+.
3. c) Find v2(t) for t≥0.
4. d) Do your answers make sense in terms of known circuit behavior?

5. 7.71 PSPICEMULTISIM Repeat Problem 70 if the dot on the 10 H coil is at the top of the coil.

Section 7.5

1. 7.72 PSPICEMULTISIM In the circuit in Fig. P 7.72, switch A has been open and switch B has been closed for a long time. At t=0, switch A closes. Five seconds after switch A closes, switch B opens. Find iL(t) for t≥0.

Figure P7.72

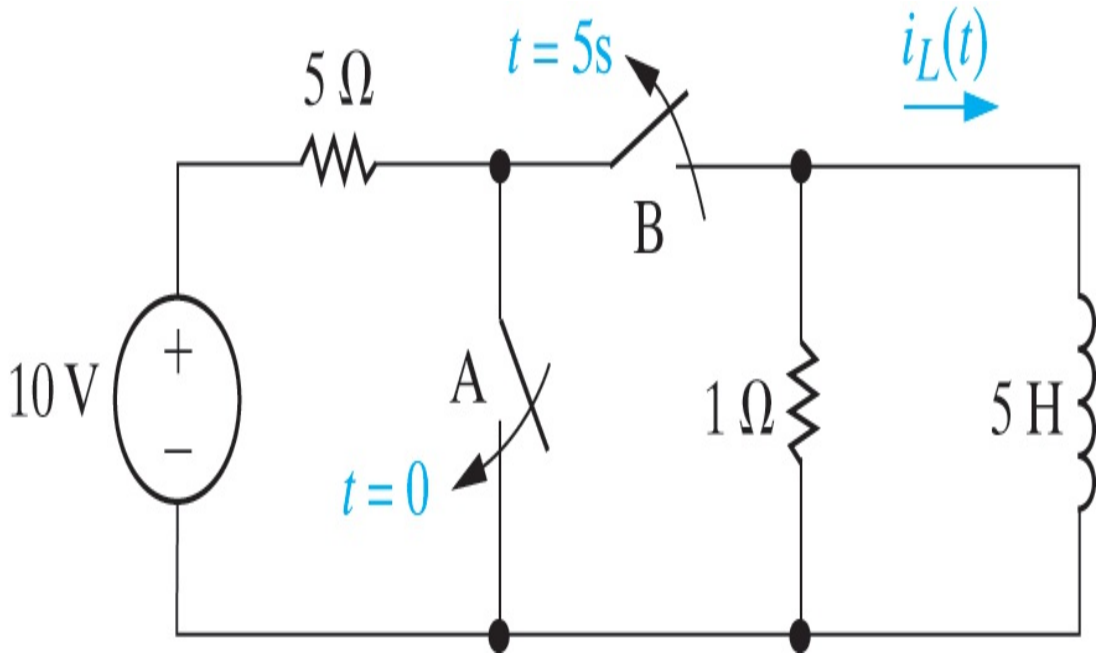


Figure P7.72 Full Alternative Text

2. 7.73 PSPICEMULTISIM The action of the two switches in the circuit seen in Fig. P 7.73 is as follows. For t<0, switch 1 is in position a and switch 2 is open. This state has existed for a long time. At t=0, switch 1 moves instantaneously from position a to position b, while switch 2 remains open. Ten milliseconds after switch 1 operates, switch 2 closes, remains closed for 10 ms and then opens. Find vo(t) 25 ms after switch 1 moves to position b.

Figure P7.73

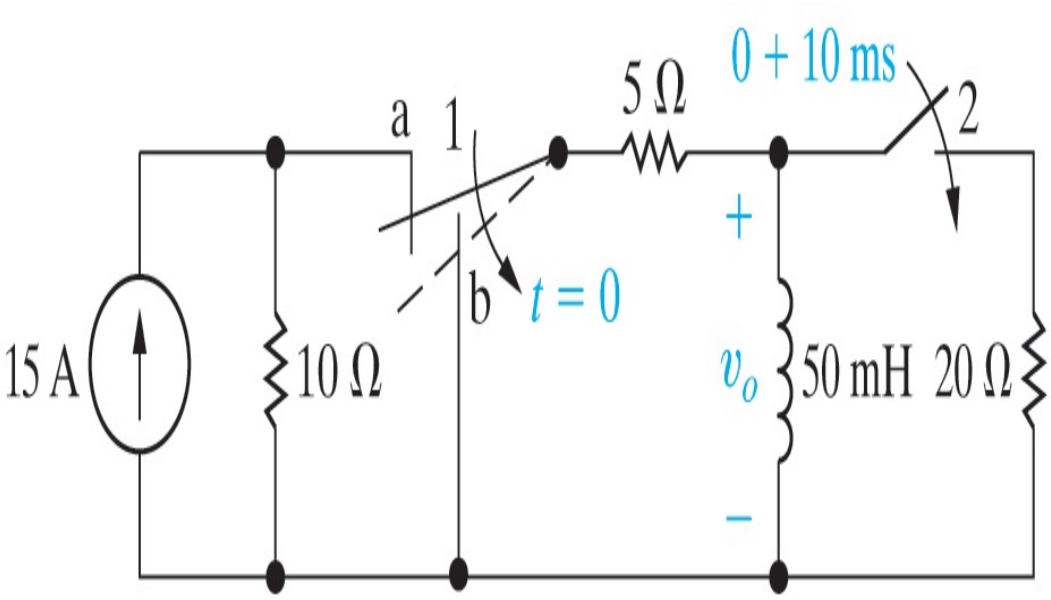


Figure P7.73 Full Alternative Text

3. 7.74 For the circuit in Fig. P 7.73, how many milliseconds after switch 1 moves to position b is the energy stored in the inductor 4% of its initial value?

4. 7.75 PSPICEMULTISIM The switch in the circuit shown in Fig. P 7.75 has been in position a for a long time. At t=0, the switch is moved to position b, where it remains for 1 ms. The switch is then moved to position c, where it remains indefinitely. Find

Figure P7.75

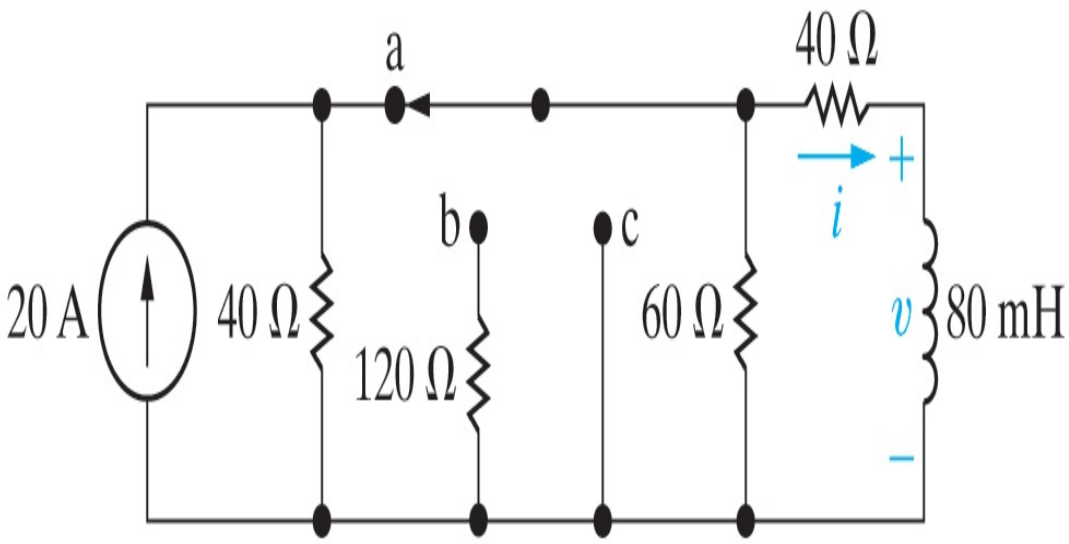


Figure P7.75 Full Alternative Text

1. a) i(0+).
2. b) i(200 μs).
3. c) i(6 ms).
4. d) v(1−1 ms).
5. e) v(1+ ms).

5. 7.76 PSPICEMULTISIM There is no energy stored in the capacitor in the circuit in Fig. P 7.76 when switch 1 closes at t=0. Switch 2 closes 10 microseconds later. Find vo(t) for t≥0.

Figure P7.76

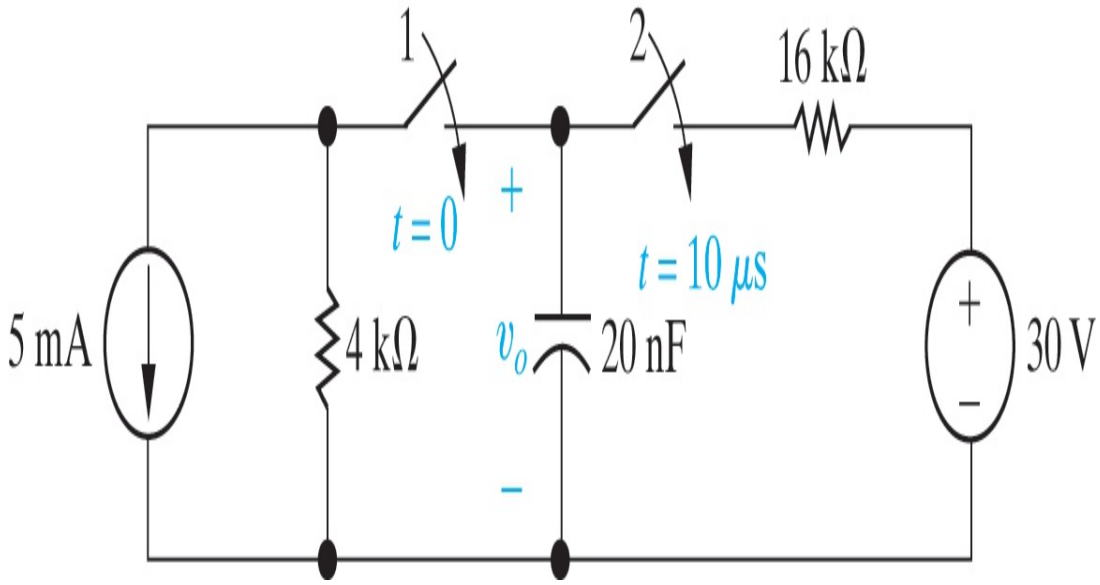


Figure P7.76 Full Alternative Text

6. 7.77 PSPICEMULTISIM The capacitor in the circuit seen in Fig. P 7.77 has been charged to 300 V. At t=0, switch 1 closes, causing the capacitor to discharge into the resistive network. Switch 2 closes 200 μs after switch 1 closes. Find the magnitude and direction of the current in the second switch 300 μs after switch 1 closes.

Figure P7.77

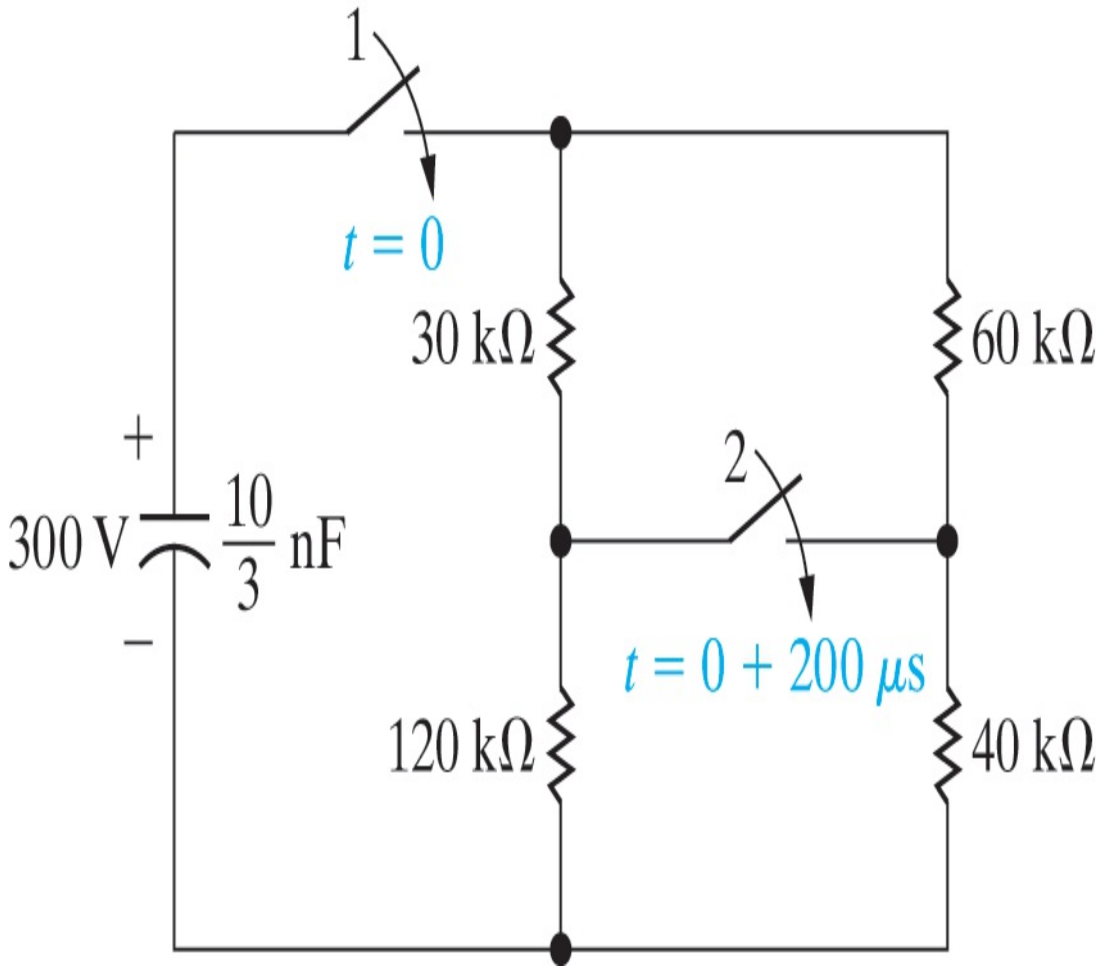


Figure P7.77 Full Alternative Text

7. 7.78 PSPICEMULTISIM The switch in the circuit in Fig. P 7.78 has been in position a for a long time. At t=0, it moves instantaneously to position b, where it remains for 50 ms before moving instantaneously to position c. Find vo for t≥0.

Figure P7.78

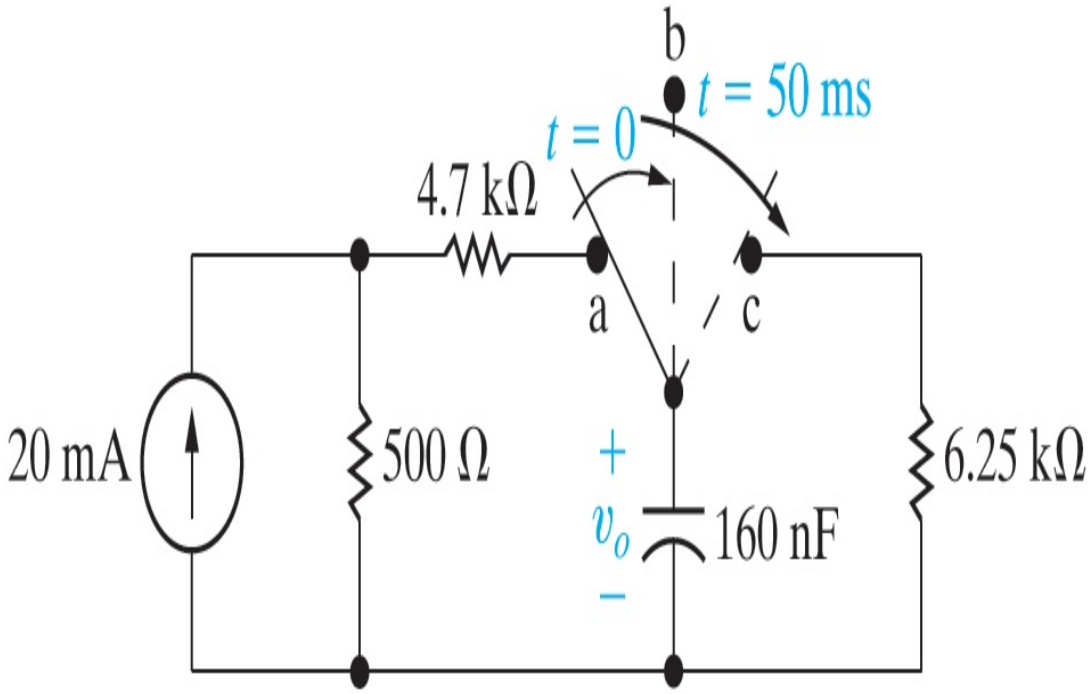


Figure P7.78 Full Alternative Text

8. 7.79 In the circuit in Fig. P 7.79, switch 1 has been in position a and switch 2 has been closed for a long time. At t=0, switch 1 moves instantaneously to position b. Eight hundred microseconds later, switch 2 opens, remains open for 300 μs, and then recloses. Find vo 1.5 ms after switch 1 makes contact with terminal b.

Figure P7.79

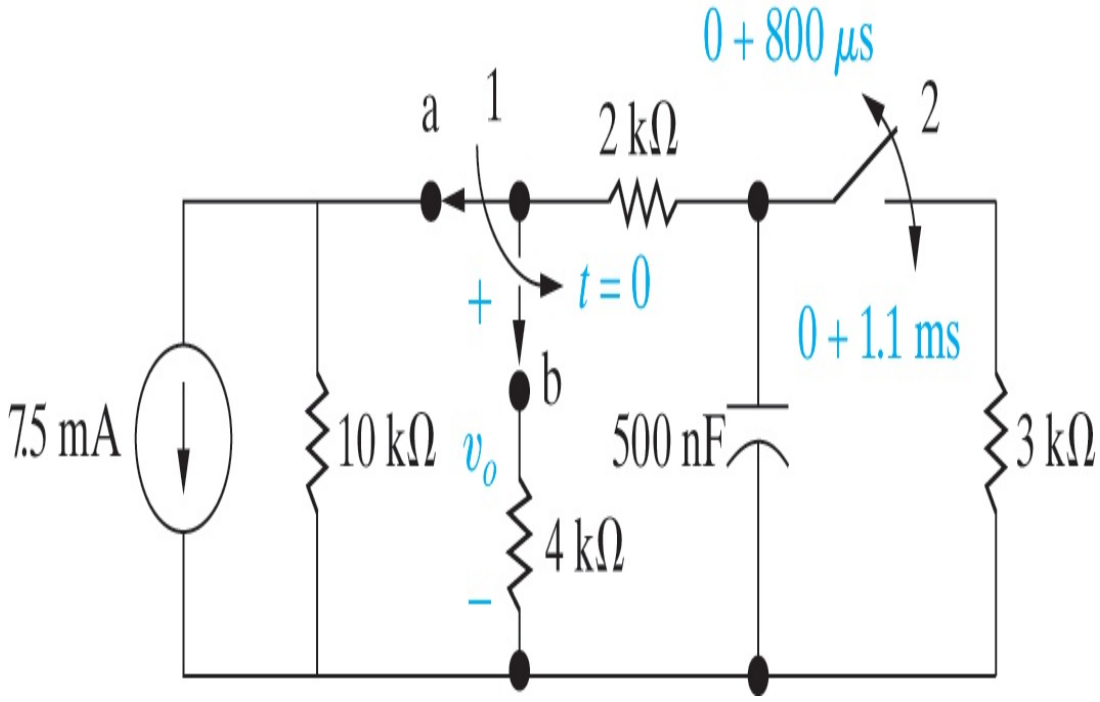


Figure P7.79 Full Alternative Text

9. 7.80 PSPICEMULTISIM For the circuit in Fig. P 7.79, what percentage of the initial energy stored in the 500 nF capacitor is dissipated in the 3 Ωk resistor?

10. 7.81 PSPICEMULTISIM The current source in the circuit in Fig. P 7.81(a) generates the current pulse shown in Fig. P 7.81(b). There is no energy stored at t=0.

Figure P7.81

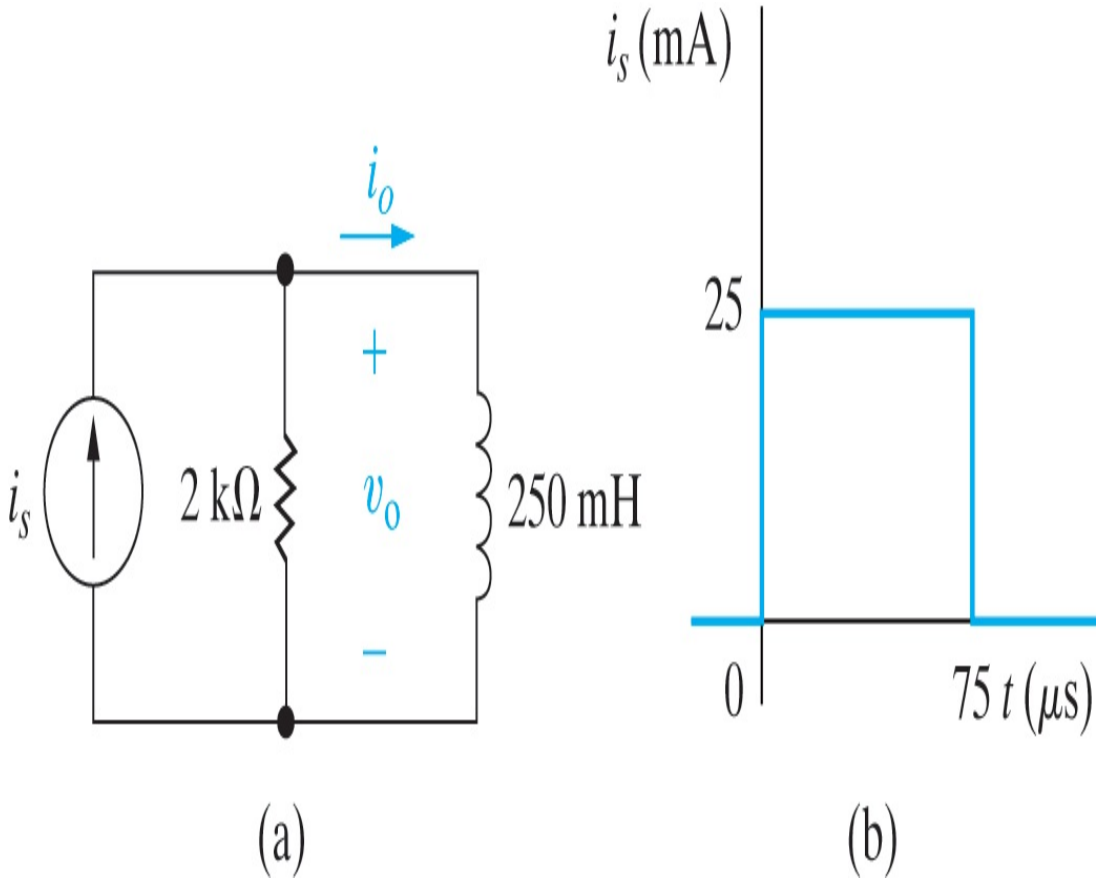


Figure P7.81 Full Alternative Text

1. a) Derive the numerical expressions for vo(t) for the time intervals t<0, 0≤t≤75 μs, and 75 μs≤t<∞.

2. b) Calculate vo (75− μs) and vo(75+ μs).

3. c) Calculate io(75− μs) and io(75+ μs).

11. 7.82 PSPICEMULTISIM The voltage waveform shown in Fig. P 7.82(a) is applied to the circuit of Fig. P 7.82(b). The initial current in the inductor is zero.

Figure P7.82

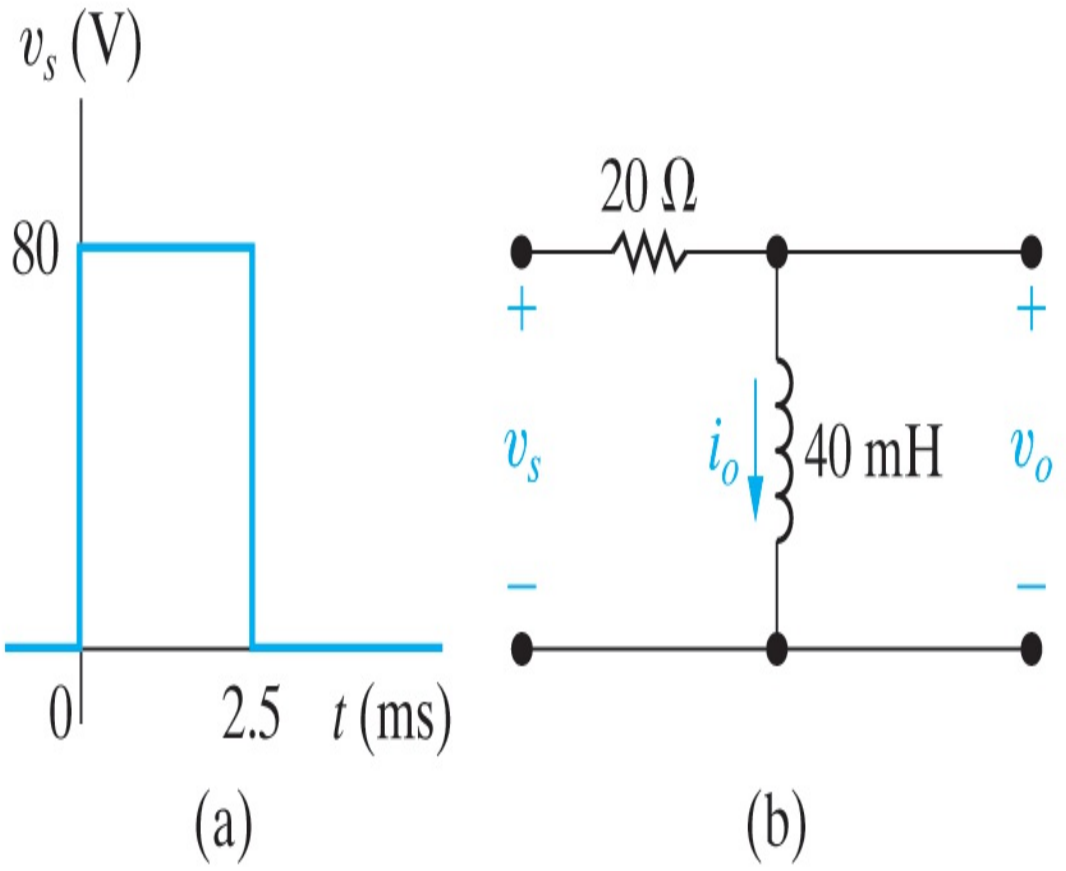


Figure P7.82 Full Alternative Text

1. a) Calculate vo(t).
2. b) Make a sketch of vo(t) versus t.
3. c) Find io at t=5 ms.

12. 7.83 PSPICEMULTISIM The voltage signal source in the circuit in Fig. P 7.83(a) is generating the signal shown in Fig. P 7.83(b). There is no stored energy at t=0.

Figure P7.83

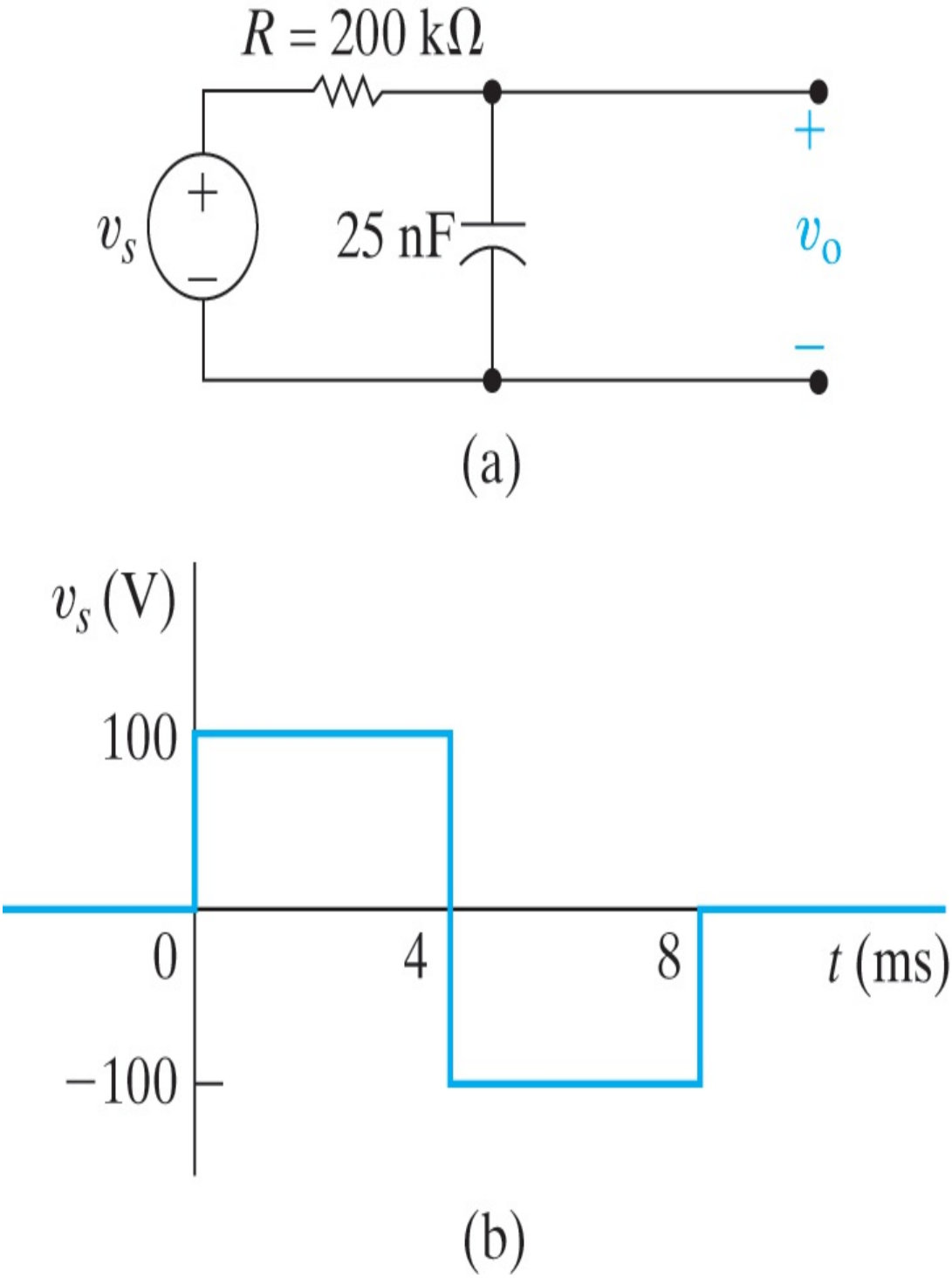


Figure P7.83 Full Alternative Text

1. a) Derive the expressions for vo(t) that apply in the intervals $t<0$; $0\le t\le 4$ ms; 4 ms$\le t\le$8 ms; and 8 ms$\le t<\infty$.

2. b) Sketch vo and vs on the same coordinate axes.

3. c) Repeat (a) and (b) with R reduced to 50 kΩ.

13. 7.84 PSPICEMULTISIM The voltage waveform shown in Fig. P 7.84(a) is applied to the circuit of Fig. P 7.84(b). The initial voltage on the capacitor is zero.

Figure P7.84

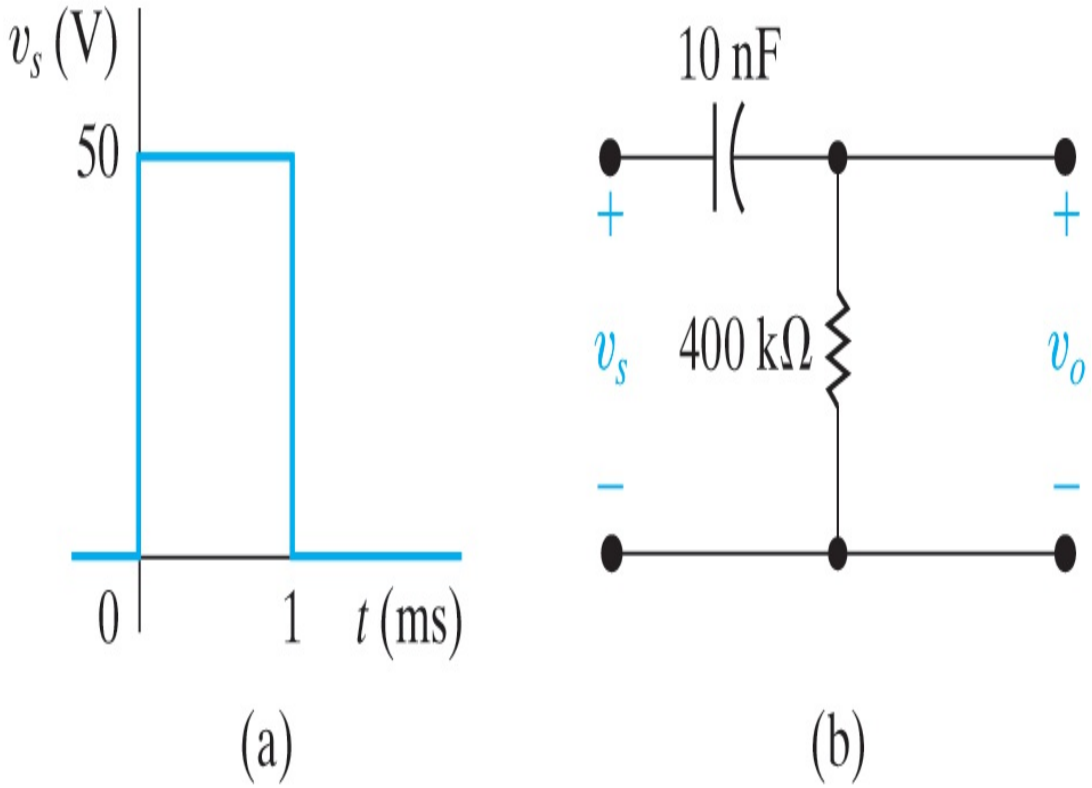


Figure P7.84 Full Alternative Text

1. a) Calculate vo(t).

2. b) Make a sketch of vo(t) versus t.

Section 7.6

1. 7.85 PSPICEMULTISIM The inductor current in the circuit in Fig. P 7.85 is 25 mA at the instant the switch is opened. The inductor will malfunction whenever the magnitude of the inductor current equals or exceeds 5 A. How long after the switch is opened does the inductor malfunction?

Figure P7.85

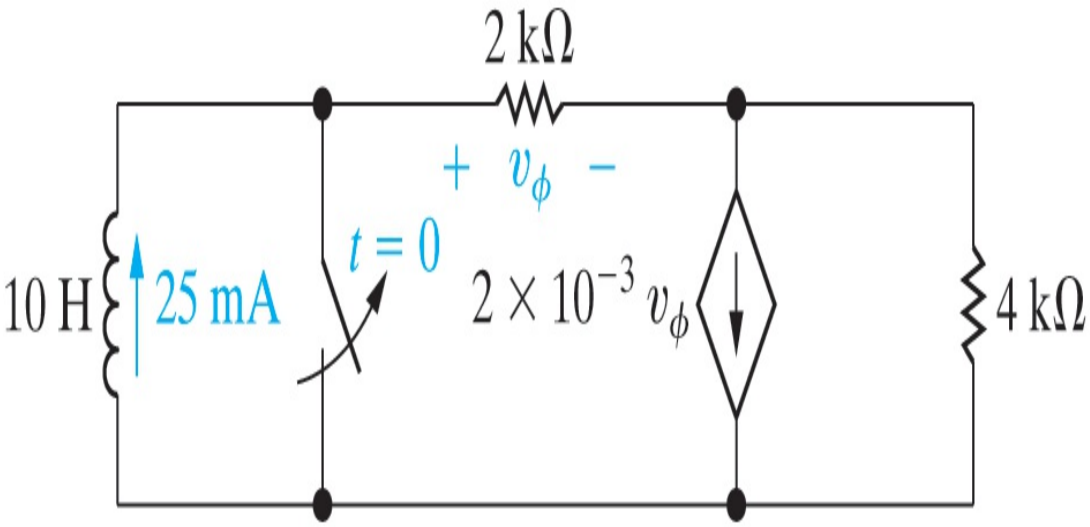


Figure P7.85 Full Alternative Text

2. 7.86 PSPICEMULTISIM The capacitor in the circuit shown in Fig. P 7.86 is charged to 20 V at the time the switch is closed. If the capacitor ruptures when its terminal voltage equals or exceeds 20 kV, how long does it take to rupture the capacitor?

Figure P7.86

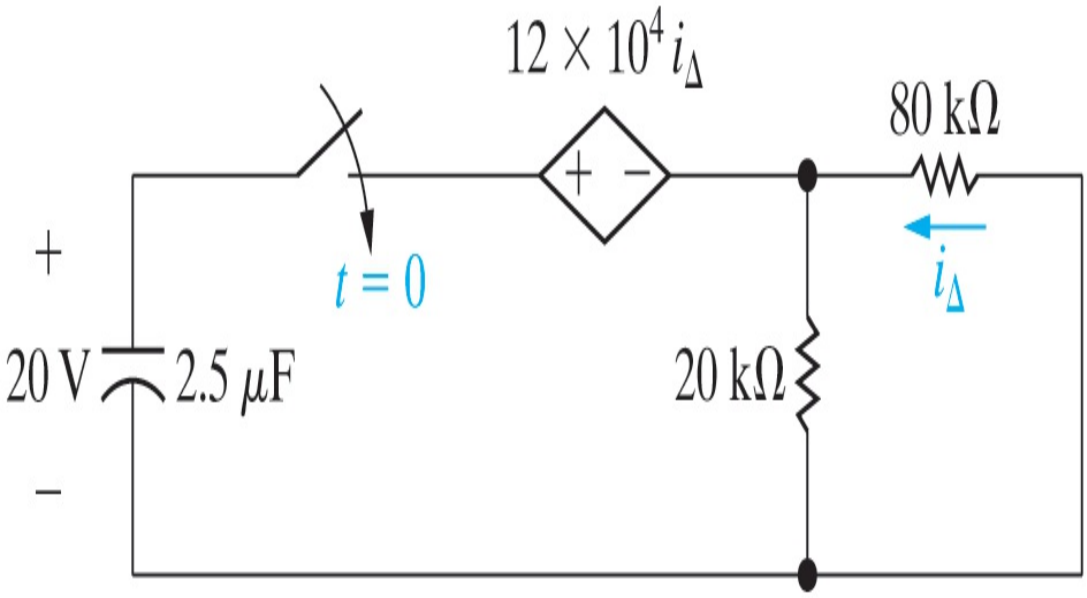


Figure P7.86 Full Alternative Text

3. 7.87 PSPICEMULTISIM The switch in the circuit in Fig. P 7.87 has been closed for a long time. The maximum voltage rating of the 1.6 μF capacitor is 14.4 kV. How long after the switch is opened does the voltage across the capacitor reach the maximum voltage rating?

Figure P7.87

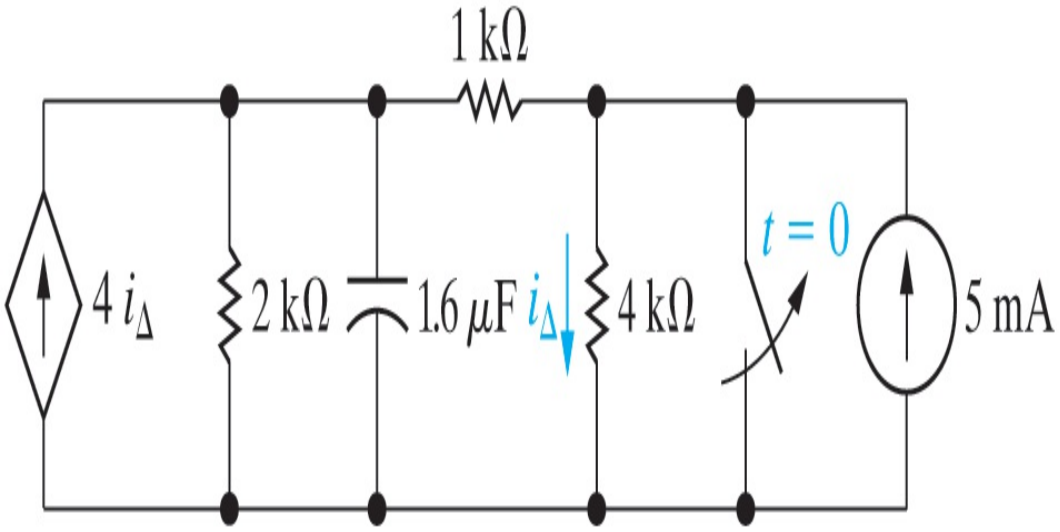


Figure P7.87 Full Alternative Text

4. 7.88 PSPICEMULTISIM The gap in the circuit seen in Fig. P 7.88 will arc over whenever the voltage across the gap reaches 45 kV. The initial current in the inductor is zero. The value of β is adjusted so the Thévenin resistance with respect to the terminals of the inductor is −5 kΩ.

Figure P7.88

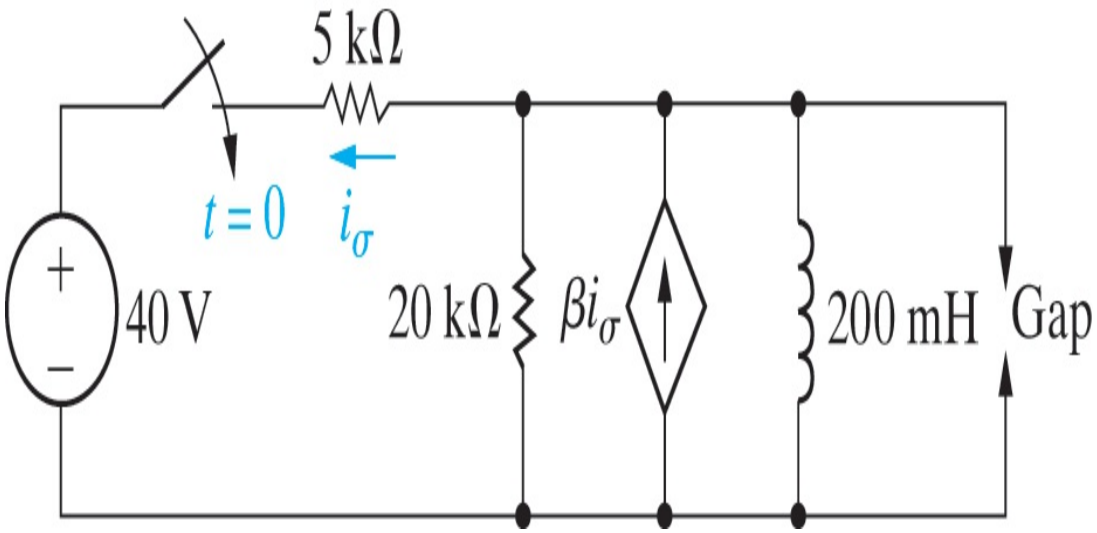


Figure P7.88 Full Alternative Text

1. a) What is the value of β?

2. b) How many microseconds after the switch has been closed will the gap arc over?

5. 7.89 The circuit shown in Fig. P 7.89 is used to close the switch between a and b for a predetermined length of time. The electric relay holds its contact arms down as long as the voltage across the relay coil exceeds 5 V. When the coil voltage equals 5 V, the relay contacts return to their initial position by a mechanical spring action. The switch between a and b is initially closed by momentarily pressing the push button. Assume that the capacitor is fully charged when the push button is first pushed

down. The resistance of the relay coil is 25 kΩ, and the inductance of the coil is negligible.

Figure P7.89

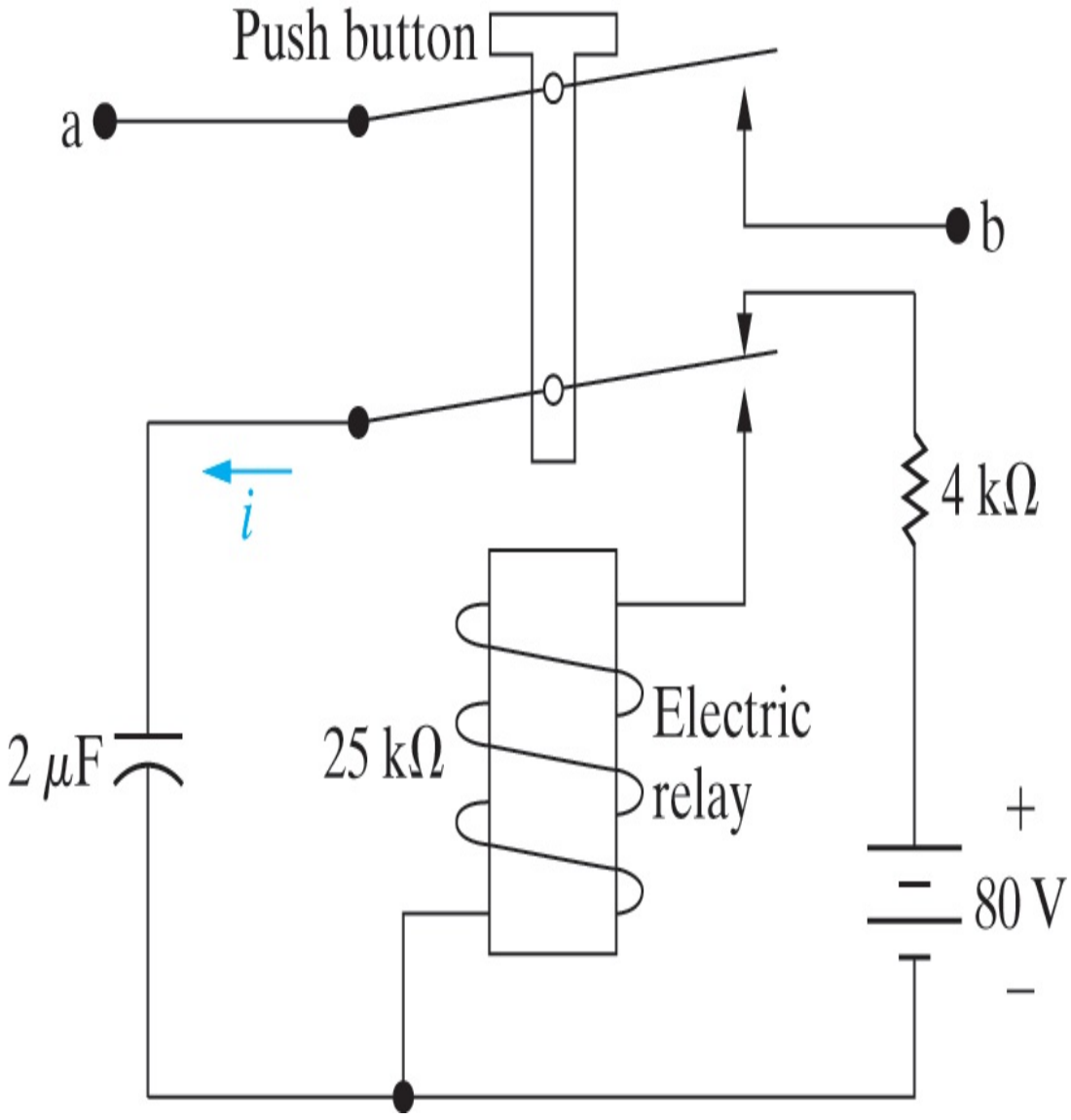


Figure P7.89 Full Alternative Text

1. a) How long will the switch between a and b remain closed?
2. b) Write the numerical expression for i from the time the relay

contacts first open to the time the capacitor is completely charged.

3. c) How many milliseconds (after the circuit between a and b is interrupted) does it take the capacitor to reach 85% of its final value?

Section 7.7

1. 7.90 PSPICEMULTISIM The energy stored in the capacitor in the circuit shown in Fig. P 7.90 is zero at the instant the switch is closed. The ideal operational amplifier reaches saturation in 15 ms. What is the numerical value of R?

Figure P7.90

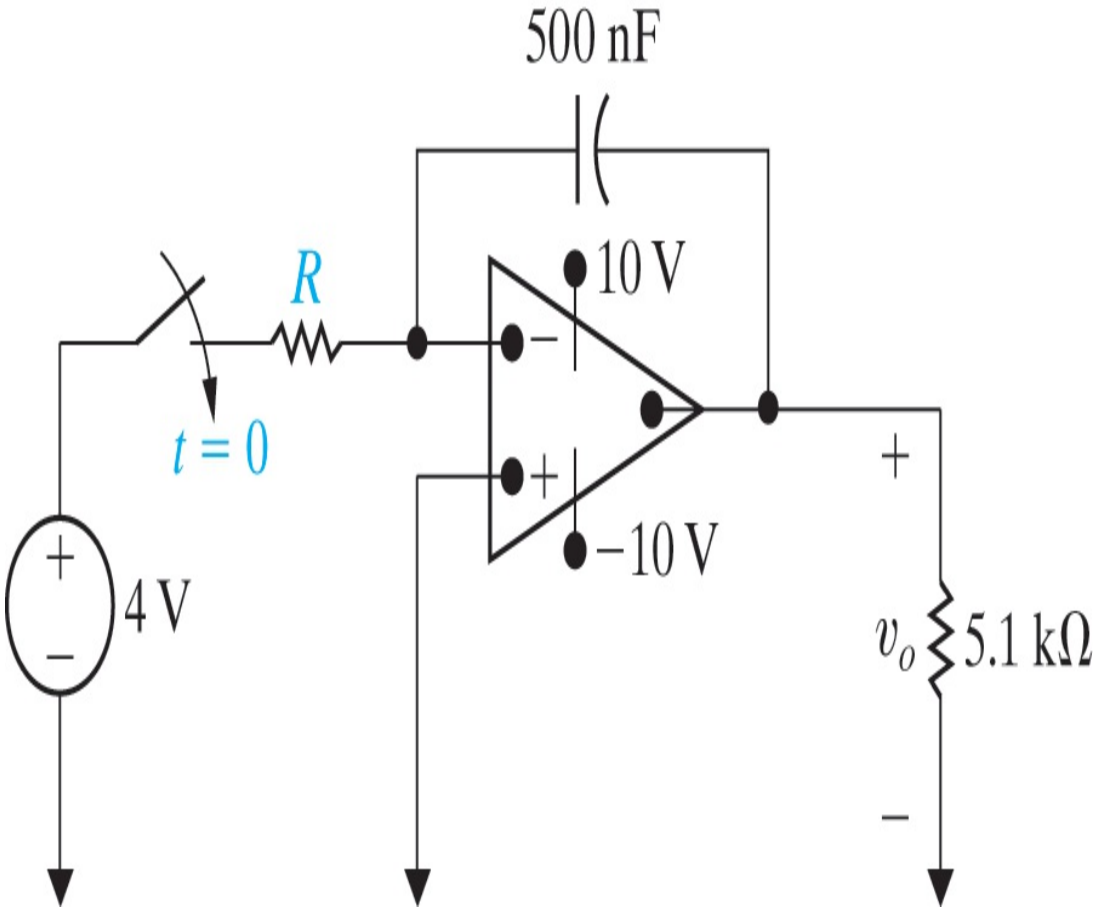


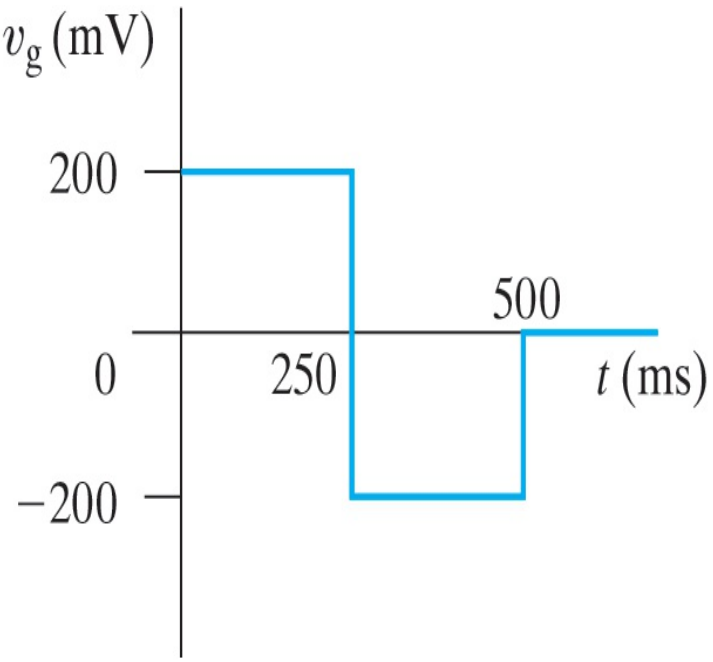
Figure P7.90 Full Alternative Text

2. 7.91 PSPICEMULTISIM At the instant the switch is closed in the circuit of Fig. P 7.90, the capacitor is charged to 6 V, positive at the

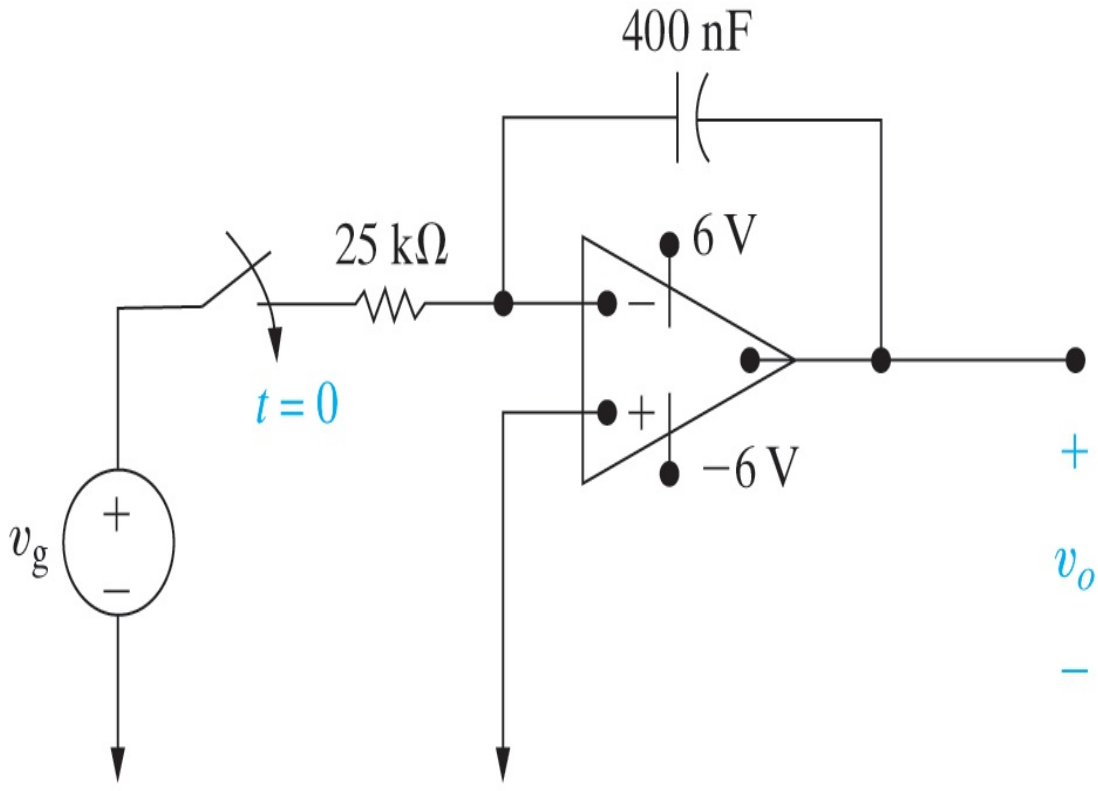
right-hand terminal. If the ideal operational amplifier saturates in 40 ms, what is the value of R?

3. 7.92 PSPICEMULTISIM The voltage pulse shown in Fig. P 7.92(a) is applied to the ideal integrating amplifier shown in Fig. P 7.92(b). Derive the numerical expression for vo(t) when vo=(0)=0 for the time intervals

Figure P7.92



(a)



(b)

Figure P7.92 Full Alternative Text

1. a) $t<0$.
2. b) $0 \leq t \leq 250$ ms.
3. c) 250 ms$\leq t \leq$500 ms.
4. d) 500 ms$\leq t$.

4. 7.93 PSPICEMULTISIM Repeat Problem 7.92 with a 5 MΩ resistor placed across the 400 nF feedback capacitor.

5. 7.94 PSPICEMULTISIM There is no energy stored in the capacitors in the circuit shown in Fig. P 7.94 at the instant the two switches close. Assume the op amp is ideal.

Figure P7.94

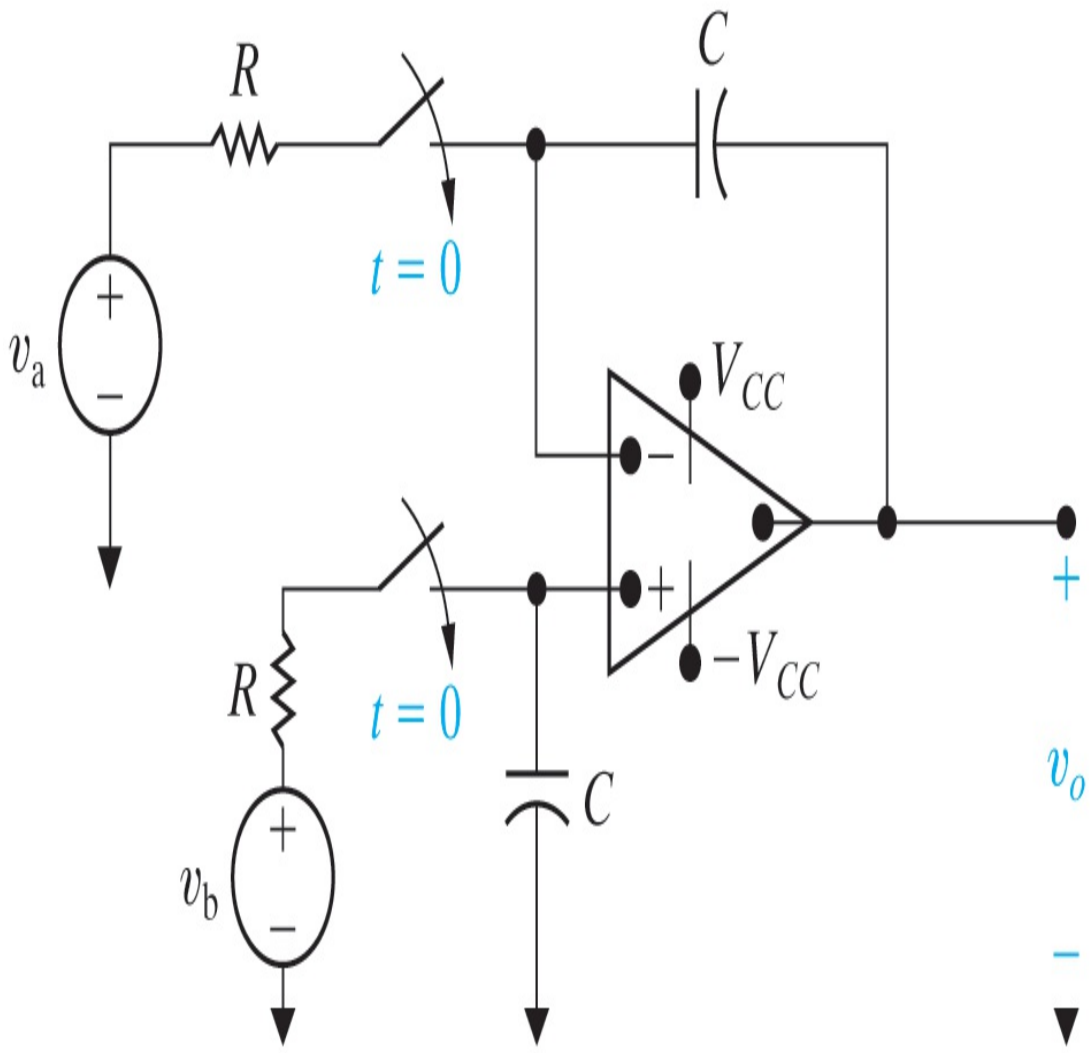


Figure P7.94 Full Alternative Text

1. a) Find vo as a function of va, vb, R, and C.

2. b) On the basis of the result obtained in (a), describe the operation of the circuit.

3. c) How long will it take to saturate the amplifier if va=40 mV; vb=15 mV; R=50 kΩ; C=10 nF; and VCC=6 V?

6. 7.95 PSPICEMULTISIM At the instant the switch of Fig. P 7.95 is closed, the voltage on the capacitor is 56 V. Assume an ideal op amp. How many milliseconds after the switch is closed will the output voltage vo equal zero?

Figure P7.95

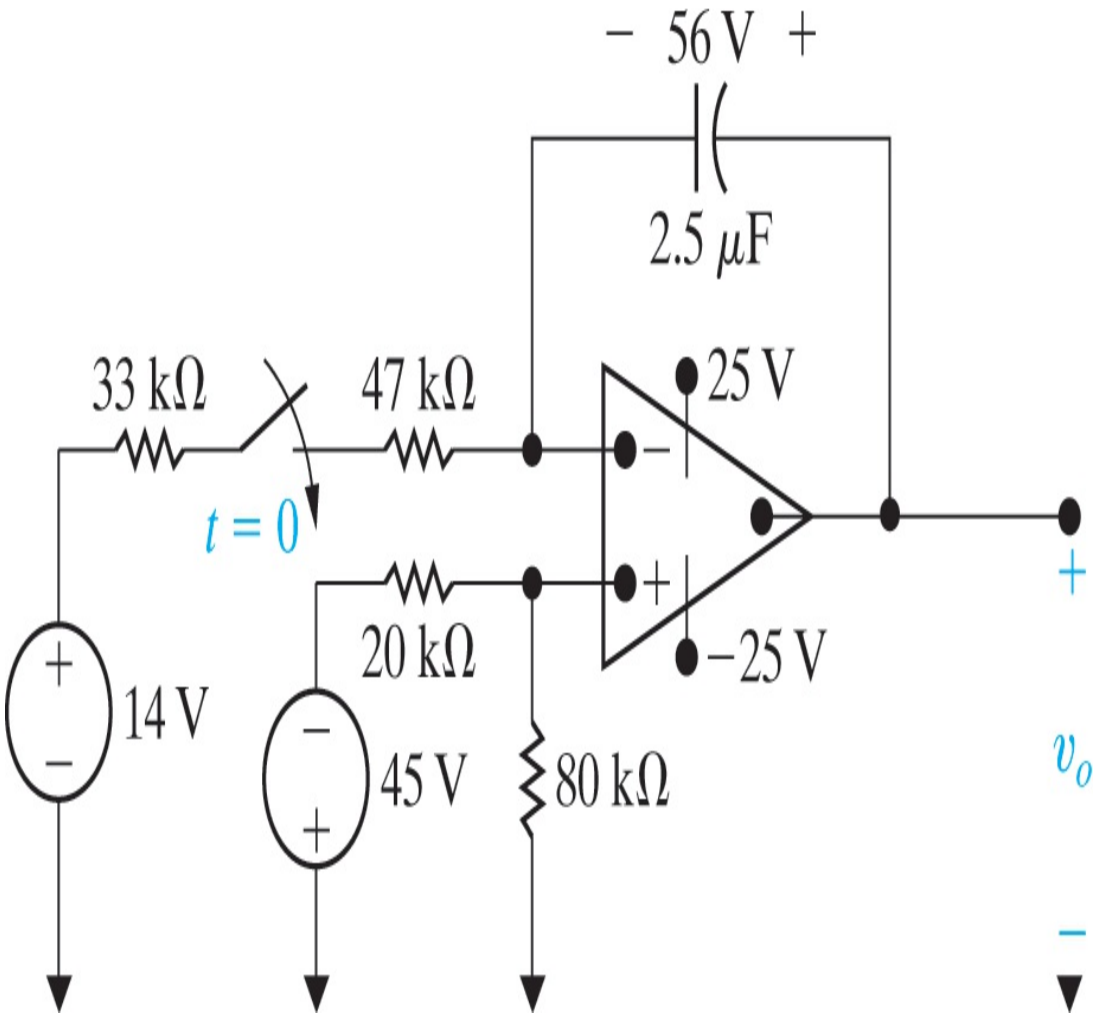
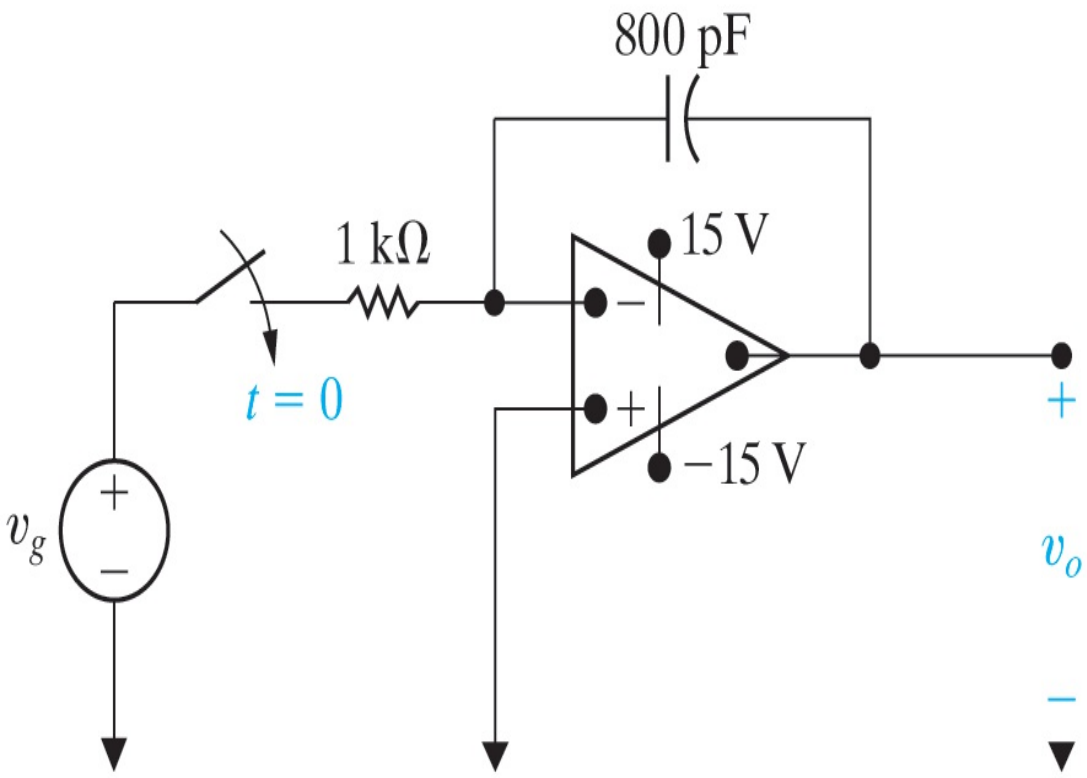


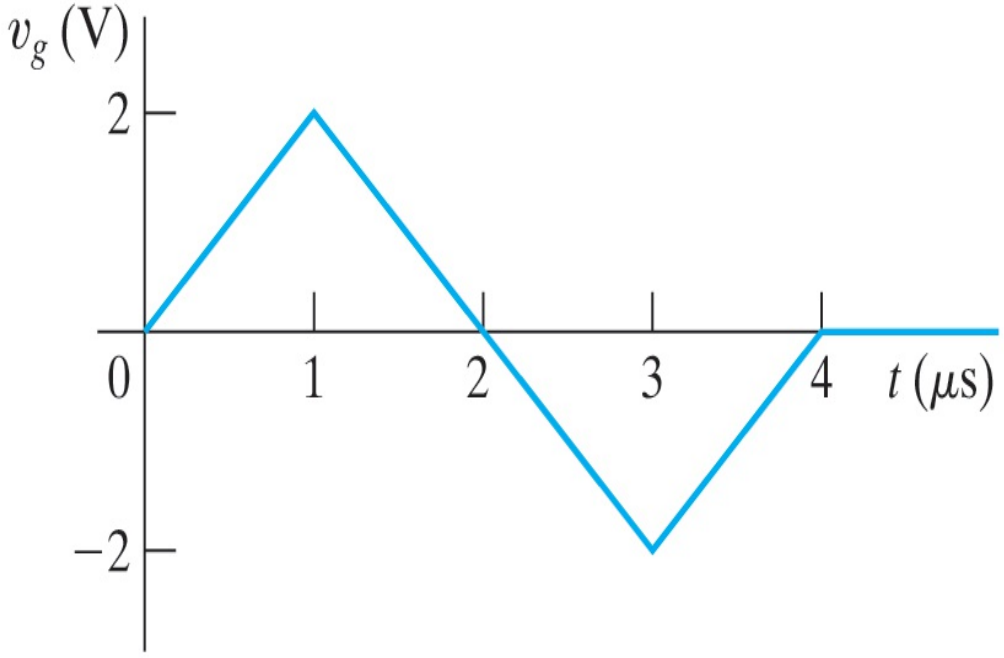
Figure P7.95 Full Alternative Text

7. 7.96 PSPICEMULTISIM The voltage source in the circuit in Fig. P 7.96(a) is generating the triangular waveform shown in Fig. P 7.96(b). Assume the energy stored in the capacitor is zero at t=0 and the op amp is ideal.

Figure P7.96



(a)



(b)

Figure P7.96 Full Alternative Text

1. a) Derive the numerical expressions for vo(t) for the following time intervals: $0 \le t \le 1$ μs; 1 μs$\le t \le$3 μs; and 3 μs$\le t \le$4 μs.

2. b) Sketch the output waveform between 0 and 4 μs.

3. c) If the triangular input voltage continues to repeat itself for $t > 4$ μs, what would you expect the output voltage to be? Explain.

Sections 7.1–7.7

1. 7.97 PSPICEMULTISIM The circuit shown in Fig. P 7.97 is known as a *monostable multivibrator*. The adjective *monostable* is used to describe the fact that the circuit has one stable state. That is, if left alone, the electronic switch T2 will be on, and T1 will be off. (The operation of the ideal transistor switch is described in detail in Problem 7.99.) T2 can be turned off by momentarily closing the switch S. After S returns to its open position, T2 will return to its on state.

Figure P7.97

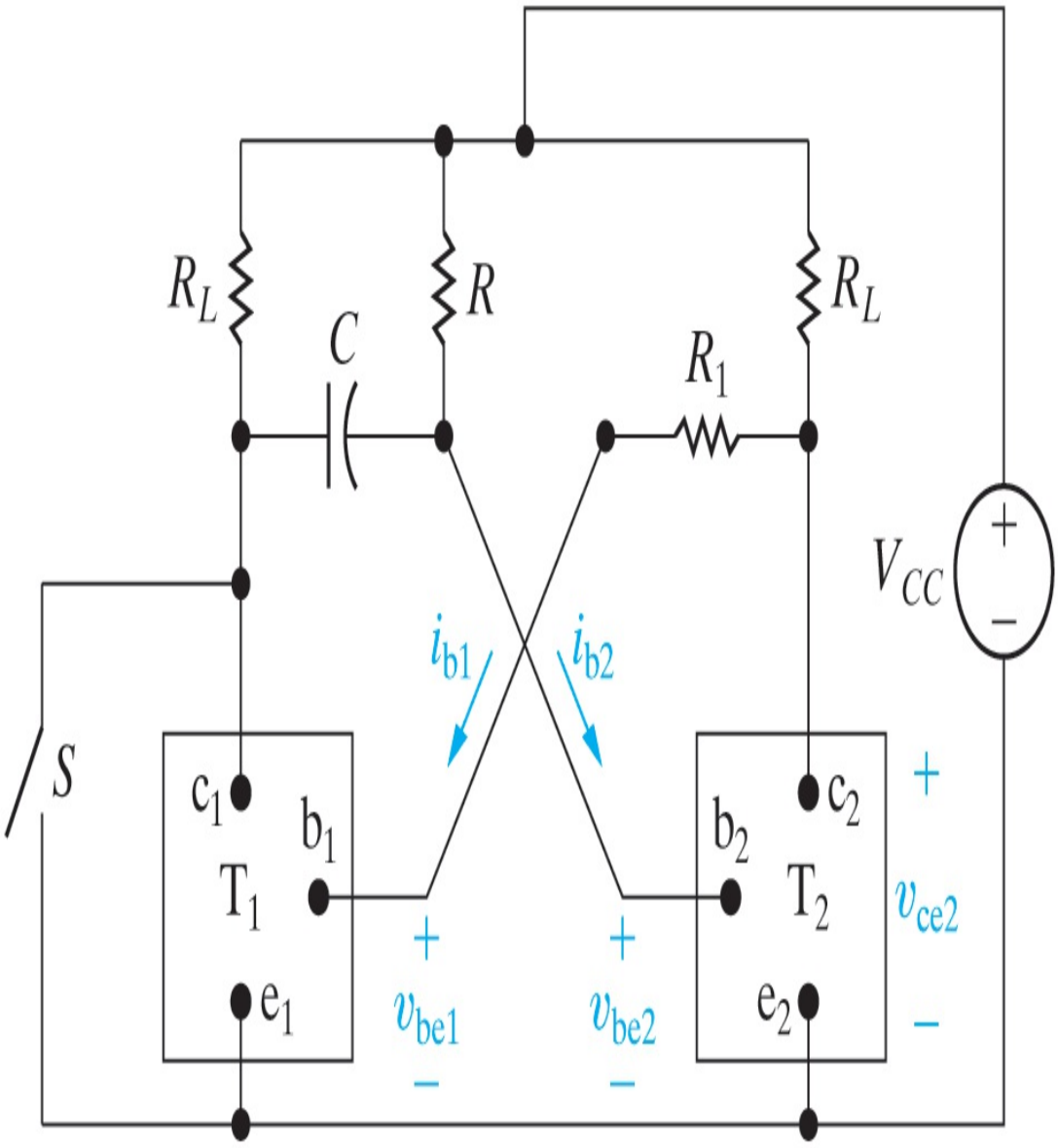


Figure P7.97 Full Alternative Text

1. a) Show that if T2 is on, T1 is off and will stay off.
2. b) Explain why T2 is turned off when S is momentarily closed.
3. c) Show that T2 will stay off for RC ln 2 s.

2. 7.98 The parameter values in the circuit in Fig. P 7.97 are VCC=6 V; R1=5.0 kΩ; RL=20 kΩ; C=250 pF; and R=23,083 Ω.

1. a) Sketch vce2 versus t, assuming that after S is momentarily closed, it remains open until the circuit has reached its stable state. Assume S is closed at t=0. Make your sketch for the interval $-5 \leq t \leq 10$ μs.

2. b) Repeat (a) for ib2 versus t.

3. 7.99 PSPICEMULTISIM The circuit shown in Fig. P 7.99 is known as an *astable multivibrator* and finds wide application in pulse circuits. The purpose of this problem is to relate the charging and discharging of the capacitors to the operation of the circuit. The key to analyzing the circuit is to understand the behavior of the ideal transistor switches T1 and T2. The circuit is designed so that the switches automatically alternate between on and off. When T1 is off, T2 is on and vice versa. Thus in the analysis of this circuit, we assume a switch is either on or off. We also assume that the ideal transistor switch can change its state instantaneously. In other words, it can snap from off to on and vice versa. When a transistor switch is on, (1) the base current ib is greater than zero, (2) the terminal voltage vbe is zero, and (3) the terminal voltage vce is zero. Thus, when a transistor switch is on, it presents a short circuit between the terminals b,e and c,e. When a transistor switch is off, (1) the terminal voltage vbe is negative, (2) the base current is zero, and (3) there is an open circuit between the terminals c,e. Thus when a transistor switch is off, it presents an open circuit between the terminals b,e and c,e. Assume that T2 has been on and has just snapped off, while T1 has been off and has just snapped on. You may assume that at this instance, C2 is charged to the supply voltage VCC, and the charge on C1 is zero. Also assume C1=C2 and R1=R2=10RL.

Figure P7.99

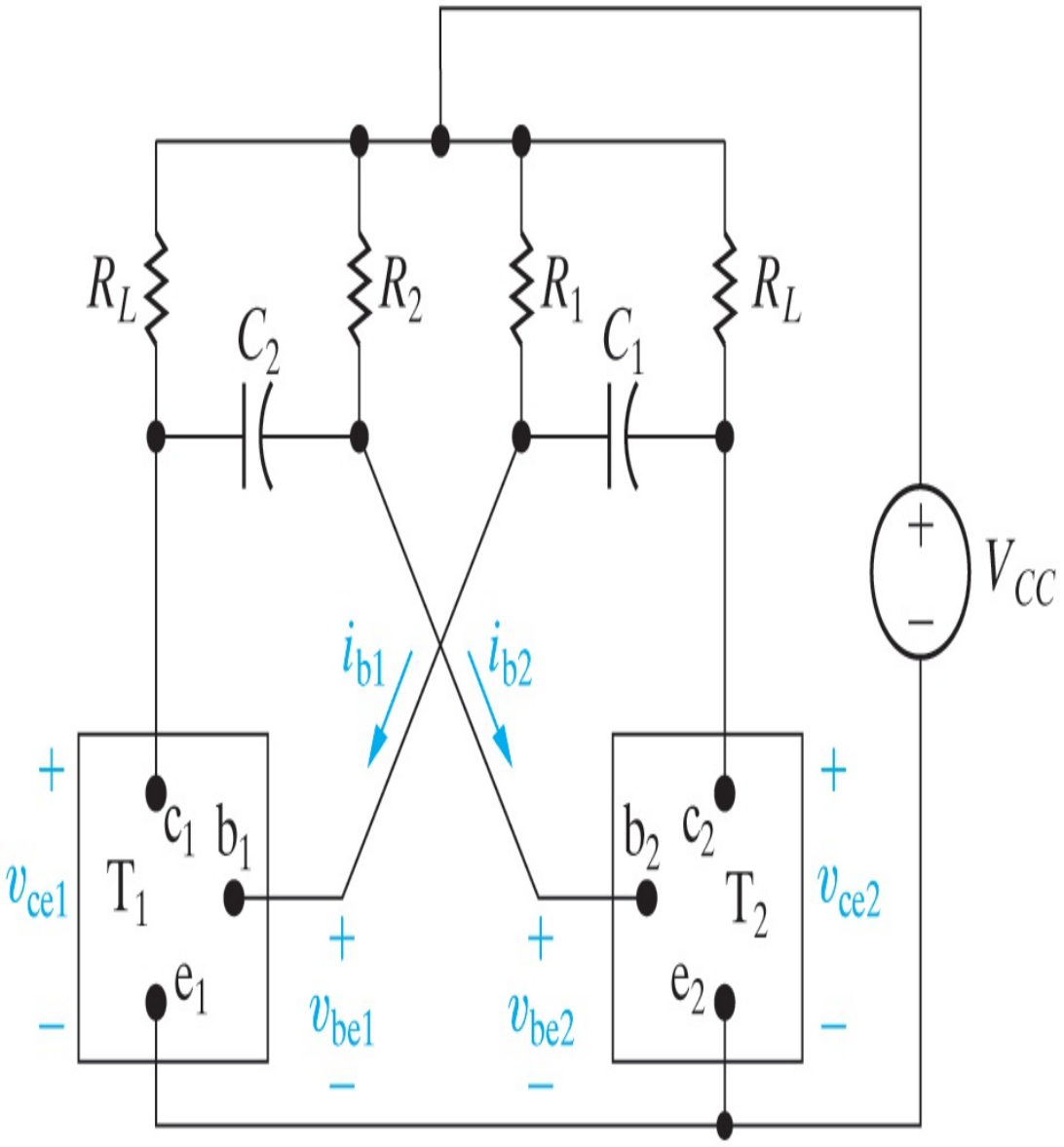


Figure P7.99 Full Alternative Text

1. a) Derive the expression for vbe2 during the interval that T2 is off.
2. b) Derive the expression for vce2 during the interval that T2 is off.
3. c) Find the length of time T2 is off.
4. d) Find the value of vce2 at the end of the interval that T2 is off.
5. e) Derive the expression for ib1 during the interval that T2 is off.

6. f) Find the value of ib1 at the end of the interval that T2 is off.
7. g) Sketch vce2 versus t during the interval that T2 is off.
8. h) Sketch ib1 versus t during the interval that T2 is off.

4. 7.100 The component values in the circuit of Fig. P 7.99 are VCC=10 V; RL=1 kΩ; C1=C2=1 nF; and R1=R2=14.43 kΩ.
    1. a) How long is T2 in the off state during one cycle of operation?
    2. b) How long is T2 in the on state during one cycle of operation?
    3. c) Repeat (a) for T1.
    4. d) Repeat (b) for T1.
    5. e) At the first instant after T1 turns on, what is the value of ib1?
    6. f) At the instant just before T1 turns off, what is the value of ib1?
    7. g) What is the value of vce2 at the instant just before T2 turns on?
5. 7.101 Repeat Problem 7.100 with C2=0.8 nF. All other component values are unchanged.
6. 7.102 The astable multivibrator circuit in Fig. P 7.99 is to satisfy the following criteria: (1) One transistor switch is to be on for 48 μs and off for 36 μs for each cycle; (2) RL=2 kΩ; (3) VCC=5 V; (4) R1=R2; and (5) 6RL≤R1≤50 RL. What are the limiting values for the capacitors C1 and C2?
7. 7.103 The relay shown in Fig. P 7.103 connects the 30 V dc generator to the dc bus as long as the relay current is greater than 0.4 A. If the relay current drops to 0.4 A or less, the spring-loaded relay immediately connects the dc bus to the 30 V standby battery. The resistance of the relay winding is 60 Ω. The inductance of the relay winding is to be determined.

Figure P7.103

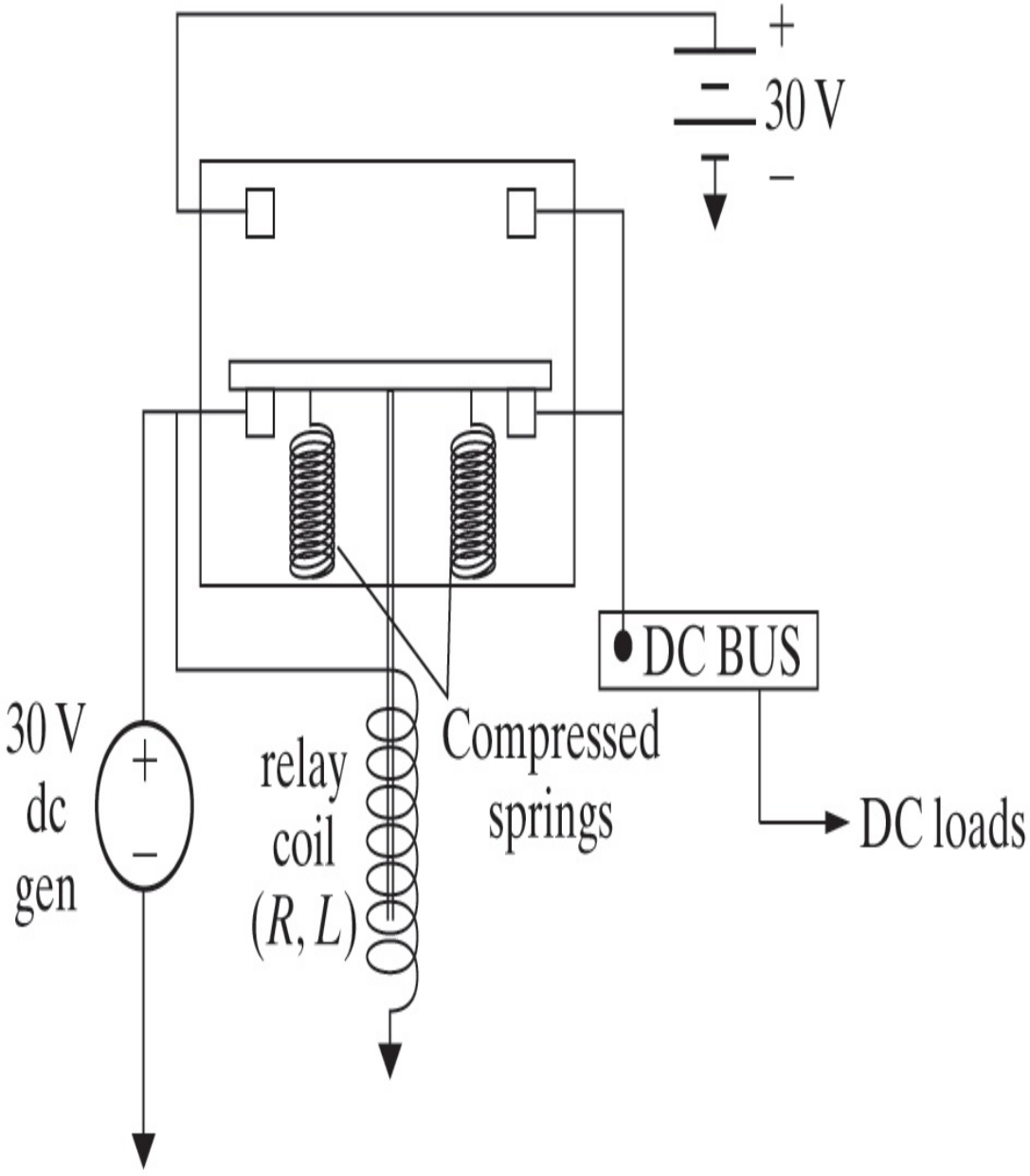


Figure P7.103 Full Alternative Text

1. a) Assume the prime motor driving the 30 V dc generator abruptly slows down, causing the generated voltage to drop suddenly to 21 V. What value of L will assure that the standby battery will be connected to the dc bus in 0.5 seconds?

2. b) Using the value of L determined in (a), state how long it will take the relay to operate if the generated voltage suddenly drops to zero.

8. 7.104PRACTICALPERSPECTIVE_ Derive the expression for heart rate in beats per minute given the values of R and C and assuming that the capacitor discharges when its voltage reaches 75% of the source voltage Vs. The expression, given in the Practical Perspective, is repeated here for convenience:

   $H=60-RC \ln 0.25$ [beats per minute].

9. 7.105 PRACTICALPERSPECTIVE_ Use an expression similar to the one derived in Problem 7.104 to calculate the heart rate in beats per minute for $R=150$ kΩ, $C=6$ μF, if the capacitor discharges when its voltage reaches 60% of the source voltage Vs.

10. 7.106 PRACTICALPERSPECTIVE_ Show that the resistance required to achieve a heart rate H, in beats per minute, is given by the equation

    $R=-60HC \ln(1-VmaxVs)$,

    where C is the capacitance, Vs. is the source voltage, and Vmax is the capacitor voltage at which discharge occurs.

11. 7.107 PRACTICALPERSPECTIVE_ Use the expression derived in Problem 7.106 to calculate the resistance required to achieve a heart rate of 70 beats per minute using a capacitance of 2.5 μF and assuming that the capacitor discharges when its voltage reaches 68% of the source voltage.

Chapter 8 Natural and Step Responses of RLC Circuits

Chapter Contents

1. 8.1 Introduction to the Natural Response of a Parallel RLC Circuit
2. 8.2 The Forms of the Natural Response of a Parallel RLC Circuit
3. 8.3 The Step Response of a Parallel RLC Circuit
4. 8.4 The Natural and Step Response of a Series RLC Circuit
5. 8.5 A Circuit with Two Integrating Amplifiers

Chapter Objectives

1. Be able to determine the natural response and the step response of parallel RLC circuits.

2. Be able to determine the natural response and the step response of series RLC circuits.

In this chapter, we discuss the natural response and step response of circuits containing a resistor, an inductor, and a capacitor, known as RLC circuits. We limit our analysis to two simple structures: the parallel RLC circuit and the series RLC circuit.

We begin with the natural response of a parallel RLC circuit and cover this material in two sections: one section discusses the solution of the second-order differential equation that describes the circuit, and the other presents the three distinct forms that the solution can take. After introducing these three forms, we show that the same forms apply to the step response of a parallel RLC circuit as well as to the natural and step responses of series RLC circuits. The chapter concludes with an introduction to an op-amp-based circuit whose output is also characterized by a second-order differential equation.

Parallel RLC Circuits

We characterize the natural response of a parallel RLC circuit by finding the voltage across the parallel branches created by the release of energy stored in the inductor or capacitor, or both. The circuit is shown in Fig. 8.1 on page 274. The initial voltage on the capacitor, V0, represents the initial energy stored in the capacitor. The initial current in the inductor, I0, represents the initial energy stored in the inductor. You can find the individual branch currents after determining the voltage.

Figure 8.1 A circuit used to illustrate the natural response of a parallel RLC circuit.

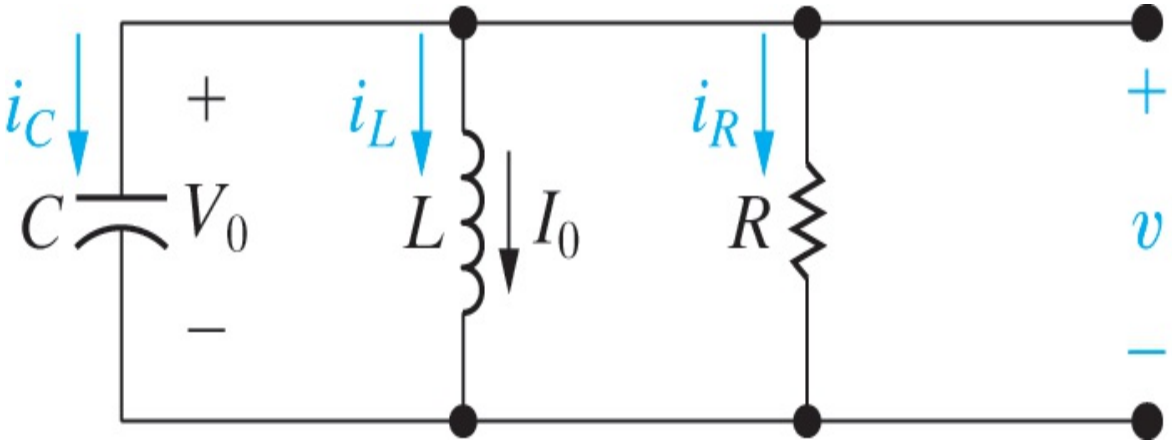


Figure 8.1 Full Alternative Text

We derive the step response of a parallel RLC circuit by using Fig. 8.2 on page 274. We determine the circuit's response when a dc current source is applied suddenly. Energy may or may not be stored in the circuit when the current source is applied.

Figure 8.2 A circuit used to illustrate the step response of a parallel RLC circuit.

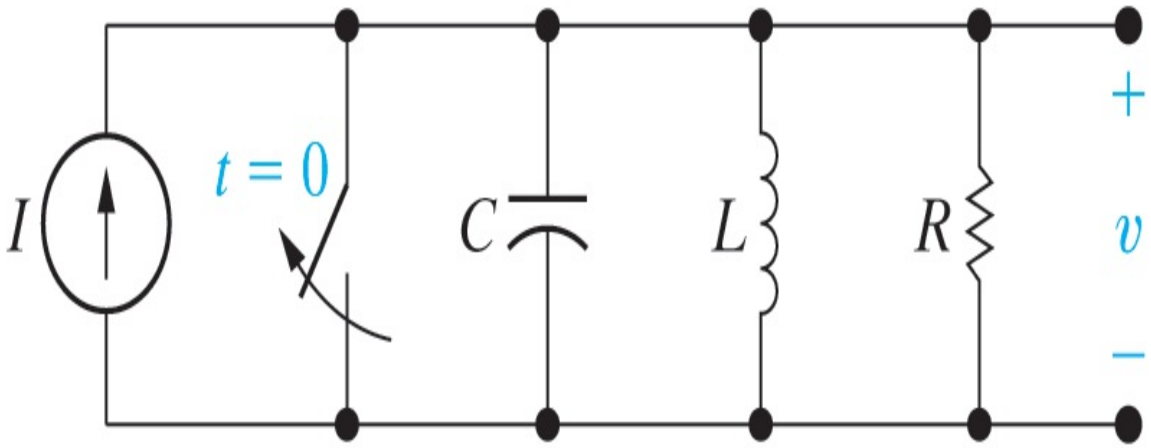


Figure 8.2 Full Alternative Text

Series RLC Circuits

We characterize the natural response of a series RLC circuit by finding the current generated in the series-connected elements by the release of initially stored energy in the inductor, capacitor, or both. The circuit is shown in Fig. 8.3 on page 274. As before, the initial inductor current, I0, and the initial capacitor voltage, V0, represent the initially stored energy. You can find the individual element voltages after determining the current.

Figure 8.3 A circuit used to illustrate the natural response of a series RLC circuit.

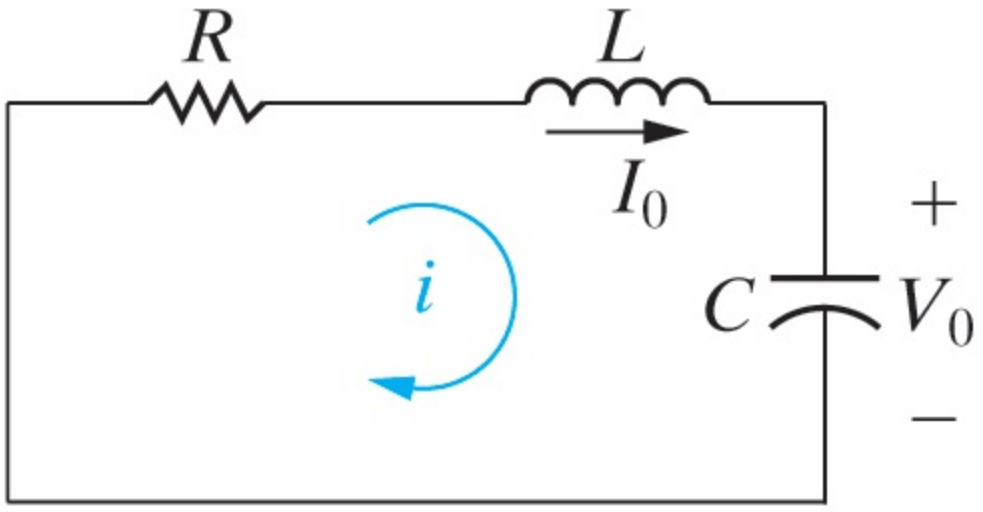


Figure 8.3 Full Alternative Text

We describe the step response of a series RLC circuit using the circuit shown in Fig. 8.4. We determine the circuit's response to the sudden application of the dc voltage source. Energy may or may not be stored in the circuit when the switch is closed.

Figure 8.4 A circuit used to illustrate the step response of a series RLC circuit.

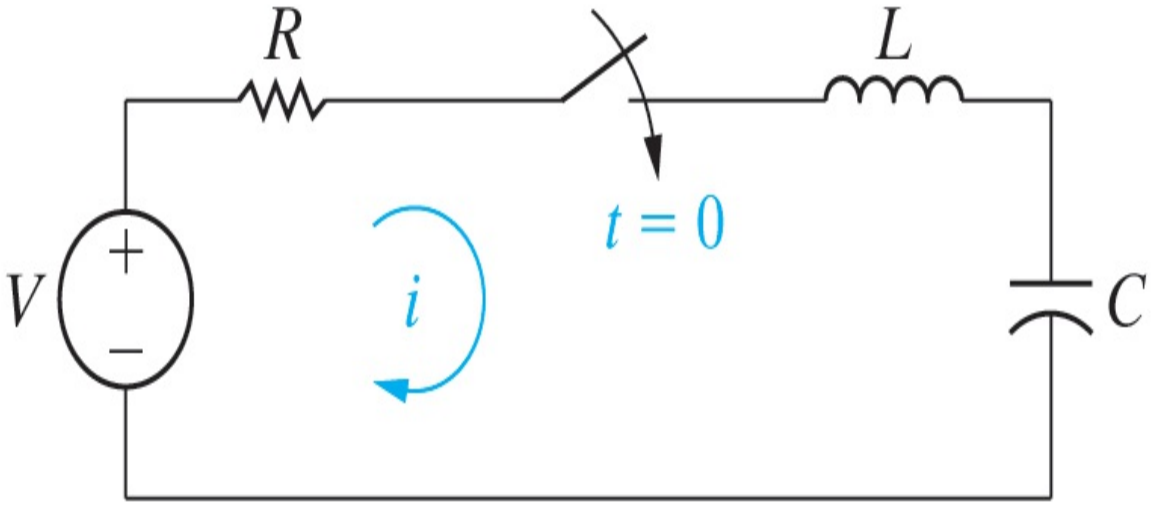


Figure 8.4 Full Alternative Text

Practical Perspective

Clock for Computer Timing

The digital circuits found in most computers require a timing signal that synchronizes the operation of the circuits. Consider a laptop computer whose processor speed is 2 GHz. This means that the central processing unit for this computer can perform about 2×109 simple operations every second.

The timing signal, produced by a clock generator chip, is typically a square wave with the required clock frequency. The square wave is obtained from a sinusoidal wave with the required clock frequency. Typically, the sinusoidal wave is generated by a precisely cut quartz crystal with an applied voltage. The crystal produces a stable frequency suitable for synchronizing digital circuits.

We can also generate a sinusoidal wave using a circuit with an inductor and a capacitor. By choosing the values of inductance and capacitance, we can create a sinusoid with a specific frequency. We will examine such a design once we have presented the fundamental concepts of second-order circuits.

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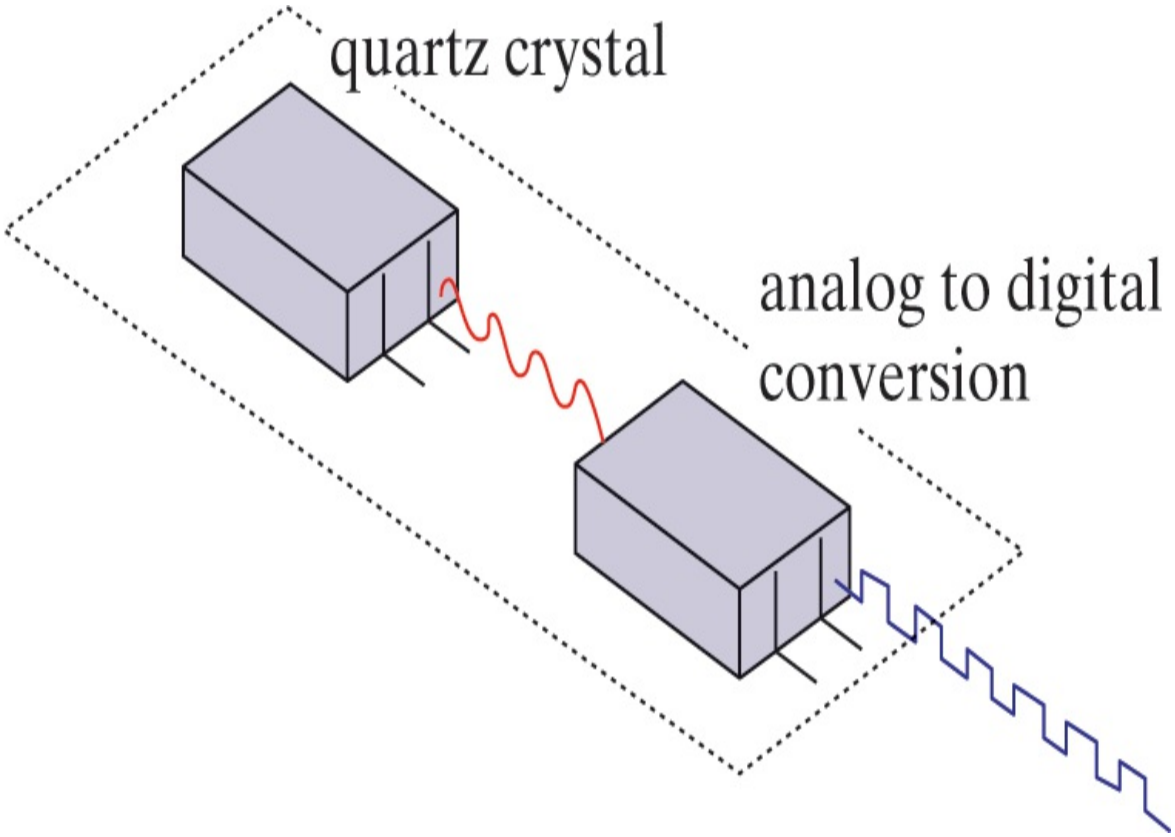
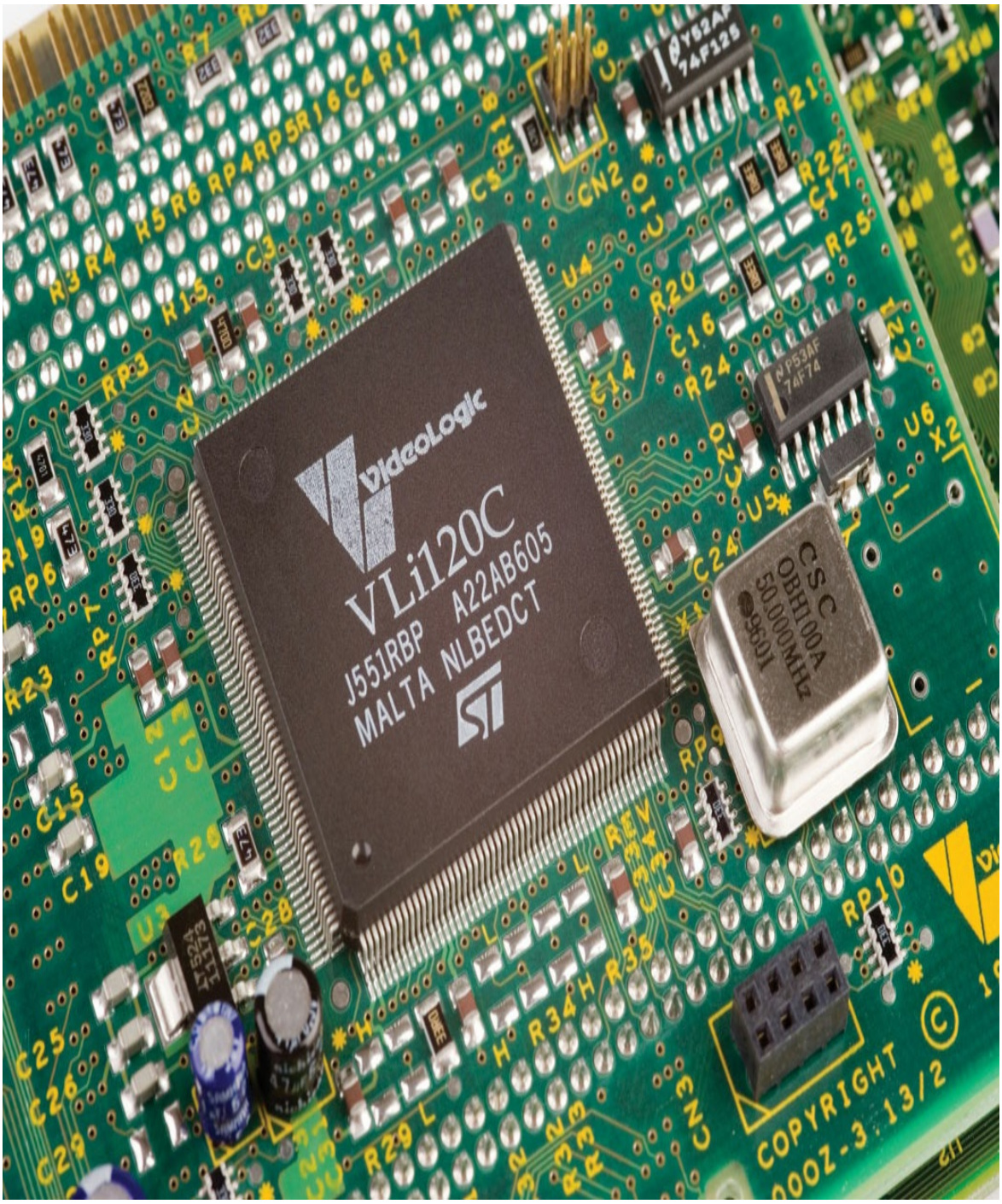

quartz crystal
analog to digital
conversion



David J. Green/Alamy Stock Photo

8.1 Introduction to the Natural Response of a Parallel RLC Circuit

To find the natural response of the circuit shown in Fig. 8.1, we begin by deriving the differential equation that the voltage v satisfies. We choose to find the voltage because it is the same for each component. Once we know the voltage, we can find every branch current by using the current–voltage relationship for the branch component. We write the differential equation for the voltage using KCL to sum the currents leaving the top node, where each current is expressed as a function of the unknown voltage v:

$$\frac{v}{R}+\frac{1}{L}\int_0^t v\,d\tau+I_0+C\frac{dv}{dt}=0.$$

We eliminate the integral in the KCL equation by differentiating once with respect to t, and because I0 is a constant, we get

$$\frac{1}{R}\frac{dv}{dt}+\frac{v}{L}+C\frac{d^2v}{dt^2}=0.$$

We now divide the equation by the capacitance C and arrange the derivatives in descending order:

$$\frac{d^2v}{dt^2}+\frac{1}{RC}\frac{dv}{dt}+\frac{v}{LC}=0. \quad (8.1)$$

Equation 8.1 is an ordinary, second-order differential equation with constant coefficients because it describes a circuit with both an inductor and a capacitor. Therefore, we also call RLC circuits **second-order circuits**.

The General Solution of the Second-Order Differential Equation

We can’t solve Eq. 8.1 by separating the variables and integrating, as we were able to do with the first-order equations in Chapter 7. Instead, we solve

Eq. 8.1 by assuming that the voltage is of the form

$$v=Aest, \quad (8.2)$$

where A and s are unknown constants.

Why did we choose an exponential form for v, given in Eq. 8.2? The reason is that Eq. 8.1 requires the sum of the second derivative of v, and the first derivative of v times a constant, and v times a constant equal zero for all values of t. This can occur only if higher-order derivatives of v have the same form as v. The exponential function satisfies this criterion. Furthermore, note that the solutions of the first-order equations we derived in Chapter 7 were all exponential; thus, it seems reasonable to assume that the solution of the second-order equation is also exponential.

If Eq. 8.2 is a solution of Eq. 8.1, it must satisfy Eq. 8.1 for all values of t. Substituting Eq. 8.2 into Eq. 8.1 generates the expression

$$As2est+ AsRCest+ AestLC=0,$$

or

$$Aest(s2+sRC+1LC)=0,$$

which can be satisfied for all values of t only if A is zero or the parenthetical term is zero because est≠0 for any finite values of st. We cannot use A=0 as a general solution because to do so implies that the voltage is zero for all time —a physical impossibility if energy is stored in either the inductor or capacitor. Therefore, in order for Eq. 8.2 to be a solution of Eq. 8.1, the parenthetical term must be zero, or

Characteristic Equation, Parallel RLC Circuit

$$s2+sRC+1LC=0. \quad (8.3)$$

Equation 8.3 is called the **characteristic equation** of the differential equation because the roots of this quadratic equation determine the mathematical character of v(t).

The two roots of Eq. 8.3 are

$$s1=-12RC+(12RC)2-1LC,$$
$$s2=-12RC-(12RC)2-1LC\ .$$

If either root is substituted into Eq. 8.2, v satisfies the differential equation in Eq. 8.1, regardless of the value of A. Therefore, both

$$v=A1es1t \text{ and}$$
$$v=A2es2t$$

satisfy Eq. 8.1. Denoting these two solutions v1 and v2, respectively, we can show that their sum also is a solution. Specifically, if we let

$$v=v1+v2=A1es1t+A2es2t,$$

then

$$dvdt=A1s1es1t+A2s2es2t,$$
$$d2vdt2=A1s12es1t+A2s22es2t.$$

Substituting these expressions for v and its first and second derivatives into Eq. 8.1 gives

$$A1es1t(s12+1RCs1+1LC)+A2es2t(s22+1RCs2+1LC)=0.$$

But each parenthetical term is zero because by definition s1 and s2 are roots of the characteristic equation. Hence, the natural response of the parallel RLC circuit shown in Fig. 8.1 is

$$v=A1es1t+A2es2t. \quad (8.4)$$

In Eq. 8.4, the constants s1 and s2, which are the roots of the characteristic equation, are determined by the circuit parameters R, L, and C. The constants A1 and A2 are determined by the initial conditions for the inductor and the

capacitor.

To find the natural response (Eq. 8.4), we begin by finding the roots of the characteristic equation, s1 and s2, which we first wrote in terms of the circuit parameters. We now rewrite them as follows:

$$s1=-\alpha+\sqrt{\alpha^2-\omega_0^2}, \quad (8.5)$$
$$s2=-\alpha-\sqrt{\alpha^2-\omega_0^2}, \quad (8.6)$$

where

Neper Frequency, Parallel RLC Circuit

$$\alpha=\frac{1}{2RC}, \quad (8.7)$$

Resonant Radian Frequency, Parallel RLC Circuit

$$\omega_0=\frac{1}{\sqrt{LC}}. \quad (8.8)$$

These results are summarized in Table 8.1.

Table 8.1 Natural-Response Parameters of the Parallel RLC Circuit

Parameter	Terminology	Value in Natural Response
s_1, s_2	Characteristic roots	$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$
		$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$
α	Neper frequency	$\alpha = \frac{1}{2RC}$
ω_0	Resonant radian frequency	$\omega_0 = \frac{1}{\sqrt{LC}}$

Table 8.1 Full Alternative Text

The exponent of e must be dimensionless, so both s1 and s2 (and hence α and ω0) must have the dimension of the inverse of time, or frequency. To distinguish among the frequencies s1, s2, α, and ω0, we use the following terminology: s1 and s2 are the *complex frequencies*, α is the *neper frequency*, and ω0 is the *resonant radian frequency*. The full significance of this terminology unfolds as we move through the remaining chapters of this book. These frequencies all have the dimension of angular frequency per time. For the complex frequencies, the neper frequency, and the resonant radian frequency, we specify values using the unit *radians per second* (rad/s).

The form of the roots s1 and s2 depends on the values of α and ω0. There are three possibilities.

- If ω02<α2, both roots will be real and distinct. For reasons to be discussed later, we call the voltage response **overdamped**.

- If $\omega_0^2 > \alpha^2$, both s1 and s2 will be complex and, in addition, will be conjugates of each other. In this situation, we call the voltage response **underdamped**.

- If $\omega_0^2 = \alpha^2$, s1 and s2 will be real and equal. Here, we call the voltage response **critically damped**.

As we shall see, damping affects the way the voltage response reaches its final (or steady-state) value. We discuss each case separately in Section 8.2.

Example 8.1 illustrates how the values of R, L, and C determine the numerical values of s1 and s2.

Example 8.1 Finding the Roots of the Characteristic Equation of a Parallel RLC Circuit

1. Find the roots of the characteristic equation that governs the transient behavior of the voltage shown in Fig. 8.5 if $R = 200\,\Omega$, $L = 50$ mH, and $C = 0.2\ \mu$F.

Figure 8.5 A circuit used to illustrate the natural response of a parallel RLC circuit.

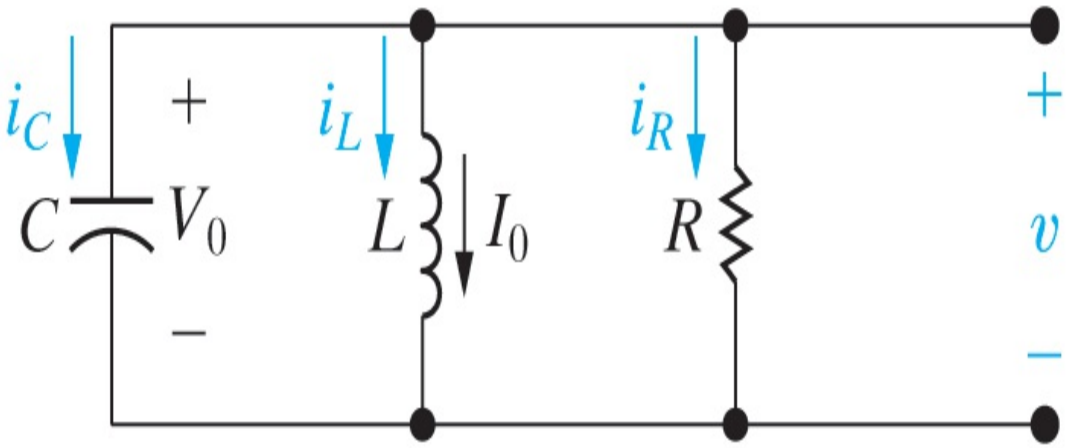


Figure 8.5 Full Alternative Text

2. Will the response be overdamped, underdamped, or critically damped?
3. Repeat (a) and (b) for R=312.5 Ω.
4. What value of R causes the response to be critically damped?

Solution

1. For the given values of R, L, and C,

   $\alpha = \frac{1}{2RC} = \frac{1}{2(200)(0.2\times10^{-6})} = 12{,}500 \text{ rad/s},$
   $\omega_0^2 = \frac{1}{LC} = \frac{1}{(50\times10^{-3})(0.2\times10^{-6})} = 10^8 \text{ rad}^2/\text{s}^2.$

   From Eqs. 8.5 and 8.6,

   $s_1 = -12{,}500 + \sqrt{(12{,}500)^2 - 10^8} = -12{,}500 + 7500 = -5000 \text{ rad/s},$
   $s_1 = -12{,}500 - \sqrt{(12{,}500)^2 - 10^8} = -12{,}500 - 7500 = -20{,}000 \text{ rad/s}.$

2. The voltage response is overdamped because $\omega_0^2 < \alpha^2$.
3. For R=312.5 Ω,

   $\alpha = \frac{1}{2RC} = \frac{1}{2(312.5)(0.2\times10^{-6})} = 8000 \text{ rad/s},$

Since ω02 remains at 108 rad2/s2,

$s\ 1 = -8000 + (\ 8000\ )\ 2 - 10\ 8 = -8000 + j6000 \text{rad/s},$
$s\ 2 = -8000 - (\ 8000\ )\ 2 - 10\ 8 = -8000 - j6000 \text{rad/s}.$

(In electrical engineering, the imaginary number −1 is represented by the letter j because the letter i represents current.)

In this case, the voltage response is underdamped since $\omega_0^2 > \alpha^2$.

4. For critical damping, $\alpha^2 = \omega_0^2$, so

   $(12RC)2 = 1LC = 108,$

   or

   $12RC = 104,$

   and

   $R = 12(104)(0.2 \times 10^{-6}) = 250\Omega.$

Assessment Problem

Objective 1—Be able to determine the natural response and the step response of parallel RLC circuits

8.1

1. The resistance and inductance of the circuit in Fig. 8.5 are 100Ω and 20 mH, respectively.

   1. Find the value of C that makes the voltage response critically damped.

   2. If C is adjusted to give a neper frequency of 5krad/s, find the value of C and the roots of the characteristic equation.

3. If C is adjusted to give a resonant frequency of 20krad/s, find the value of C and the roots of the characteristic equation.

Answer:

1. 500nF;
2. C=1 μF, s1=−5000+j5000 rad/s, s2=−5000−j5000 rad/s;
3. C=125nF, s1=−5359 rad/s, s1=−5359 rad/s,

SELF-CHECK: Also try Chapter Problem 8.3.

8.2 The Forms of the Natural Response of a Parallel RLC Circuit

In this section, we find the natural response for each of the three types of damping: overdamped, underdamped, and critically damped. As we have already seen, the values of s1 and s2 determine the type of damping. We need to find values for the two coefficients A1 and A2 so that we can completely characterize the natural response given in Eq. 8.4. This requires two equations based on the following observations:

- The initial value of the voltage in Eq. 8.4 must be the same as the initial value of the voltage in the circuit.
- The initial value of the first derivative of the voltage in Eq. 8.4 must be the same as the initial value of the first derivative of the voltage in the circuit.

As we will see, the natural-response equations, as well as the equations for evaluating the unknown coefficients, are slightly different for each of the three types of damping. This is why the first task that presents itself when finding the natural response is to determine whether the response is overdamped, underdamped, or critically damped.

The Overdamped Response

When the roots of the characteristic equation are real and distinct, the response of a parallel RLC circuit is overdamped. The solution for the voltage is

Parallel RLC Natural Response:

Overdamped

v=A1es1t+ A2es2t,(8.9)

where s1 and s2 are the roots of the characteristic equation. The constants A1 and A2 are determined by the initial conditions, specifically from the values of v(0+) and dv(0+)/dt, which in turn are determined from the initial voltage on the capacitor, V0, and the initial current in the inductor, I0.

To determine the values of A1 and A2, we need two independent equations. The first equation sets the initial value v from Eq. 8.9 equal to the initial value of v in the circuit, which is the initial voltage for the capacitor, V0. The resulting equation is

v(0+)=A1+A2=V0. (8.10)

The second equation sets the initial value of dv/dt from Eq. 8.9 equal to the initial value of dv/dt in the circuit. The initial value of dv/dt from Eq. 8.9 is

dv(0+)dt=s1A1+s2A2.

But how do we find the initial value of dv/dt from the circuit? Remember that dv/dt appears in the equation relating voltage and current for a capacitor,

iC=Cdvdt.

We can solve the capacitor equation for dv/dt and find its initial value in terms of the initial current in the capacitor:

dv(0+)dt=iC(0+)C.

Now we use KCL to find the initial current in the capacitor. We know that the sum of the three branch currents at t=0+ must be zero. The initial current in the resistive branch is the initial voltage V0 divided by the resistance, and the initial current in the inductive branch is I0. Using the reference system depicted in Fig. 8.5, we obtain

$$i_C(0+) = -\frac{V_0}{R} - I_0.$$

Now we have the second equation needed to find the values of A1 and A2 in Eq. 8.9:

$$\frac{dv(0+)}{dt} = A_1 s_1 + A_2 s_2 = \frac{1}{C}\left(-\frac{V_0}{R} - I_0\right). \quad (8.11)$$

The method for finding the voltage for the parallel RLC circuit in Fig. 8.5 is as follows:

1. Step 1: Determine the initial values of capacitor voltage, V0, and inductor current, I0, by analyzing the parallel RLC circuit for $t<0$.

2. Step 2: Determine the values of the neper frequency, α, and the resonant radian frequency, ω_0, using the values of R, L, and C and the equations in Table 8.1.

3. Step 3: Compare α^2 and ω_0^2. If $\alpha^2 > \omega_0^2$, the response is overdamped:

   $$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}, t \geq 0.$$

4. Step 4: If the response is overdamped, calculate the values of s1 and s2 from α and ω_0, using the equations in Table 8.1.

5. Step 5: If the response is overdamped, calculate the values of A1 and A2 by solving Eqs. 8.10 and 8.11 simultaneously:

   $$A_1 + A_2 = V_0;$$
   $$A_1 s_1 + A_2 s_2 = \frac{1}{C}\left(-\frac{V_0}{R} - I_0\right).$$

6. Step 6: Write the equation for v(t) from Step 3 using the results from Steps 4 and 5. Find any desired branch current using the relationship between voltage and current for the component in the branch.

A condensed version of this method is given in Analysis Method 8.1. Examples 8.2 and 8.3 use this method to find the overdamped response of a parallel RLC circuit.

Natural Response of a Parallel RLC Circuit

1. **Determine the initial capacitor voltage (V0) and inductor current (I0)** from the circuit.
2. **Determine the values of** α **and** $\omega 0$ using the equations in Table 8.1.
3. **If** $\alpha 2 > \omega 02$, **the response is overdamped** and $v(t)=A1es1t+A2es2t, t \geq 0$
4. **If the response is overdamped, calculate s1 and s2** using the equations in Table 8.1.
5. **If the response is overdamped, calculate A1 and A2** by simultaneously solving Eqs. 8.10 and 8.11.
6. **Write the equation for v(t) from Step 3** using the results from Steps 4 and 5; find any desired branch currents.

Analysis Method 8.1 The natural response of an overdamped parallel RLC circuit.

Example 8.2 Finding the Overdamped Natural Response of a Parallel RLC Circuit

For the circuit in Fig. 8.6, $v(0+)=12$ V, and $iL(0+)=30$ mA.

Figure 8.6 The circuit for Example 8.2.

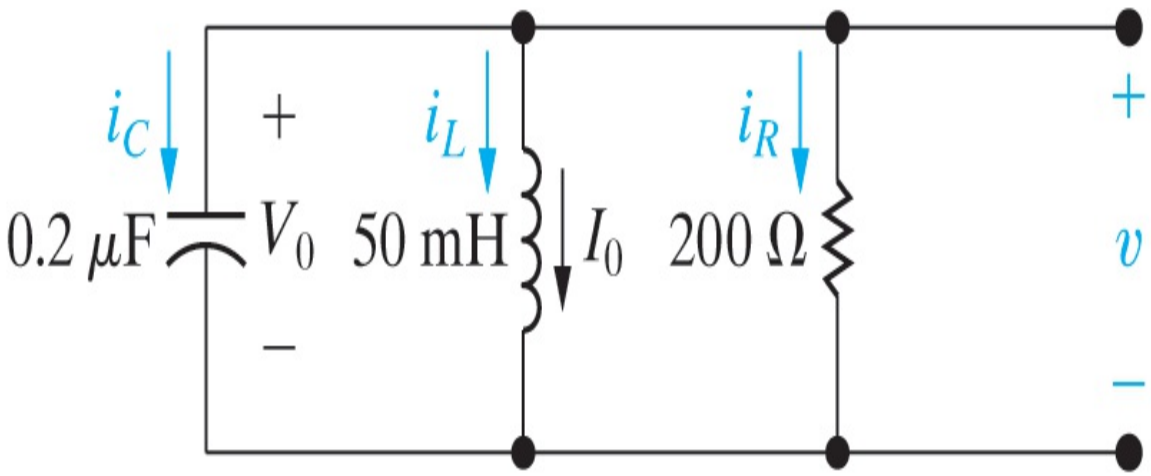


Figure 8.6 Full Alternative Text

1. Find the expression for v(t).
2. Sketch v(t) in the interval 0≤t≤250 μs.

Solution

1. We use Analysis Method 8.1 to find the voltage.
    1. Step 1: Determine the initial values of capacitor voltage, V0, and inductor current, I0; since these values are given in the problem statement, no circuit analysis is required.
    2. Step 2: Determine the values of α and ω0 using the equations in Table 8.1:

        $\alpha = 12RC = 12(200)(0.2\times10^{-6}) = 12{,}500$ rad/s,
        $\omega_0^2 = 1LC = 1(50\times10^{-3})(0.2\times10^{-6}) = 108$ rad2/s2.
    3. Step 3: Compare α2 and ω02; since α2>ω02, the response is overdamped and

        $v(t) = A_1e^{s_1t} + A_2e^{s_2t}, \quad t \geq 0.$

4. Step 4: Since the response is overdamped, calculate the values of s1 and s2:

   $s1=-a+a2-\omega02=-12,500+(12,500)2-108=-12,500+7500=-5000$ rad/s;$s2=-a-a2-\omega02=-12,500-(12,500)2-108=-12,500-7500=-20,000$ rad/s.

5. Step 5: Since the response is overdamped, calculate the values of A1 and A2 by simultaneously solving

   $A1+A2=V0=12;$
   $A1s1+A2s2=1C(-V0R-I0)$ so
   $-5000A1-20,000A2=10.2\times10-6(-12200-0.03)=-450,000.$

   Solving,

   $A1=-14$ V and $A2=26$ V.

6. Step 6: Write the equation for v(t) using the results from Steps 4 and 5:

   $v(t)=(-14e-5000t+26e-20,000t)$ V, $t\geq0$.

2. Figure 8.7 shows a plot of v(t) versus t over the interval $0\leq t\leq250$ μs.

Figure 8.7 The voltage response for Example 8.2.

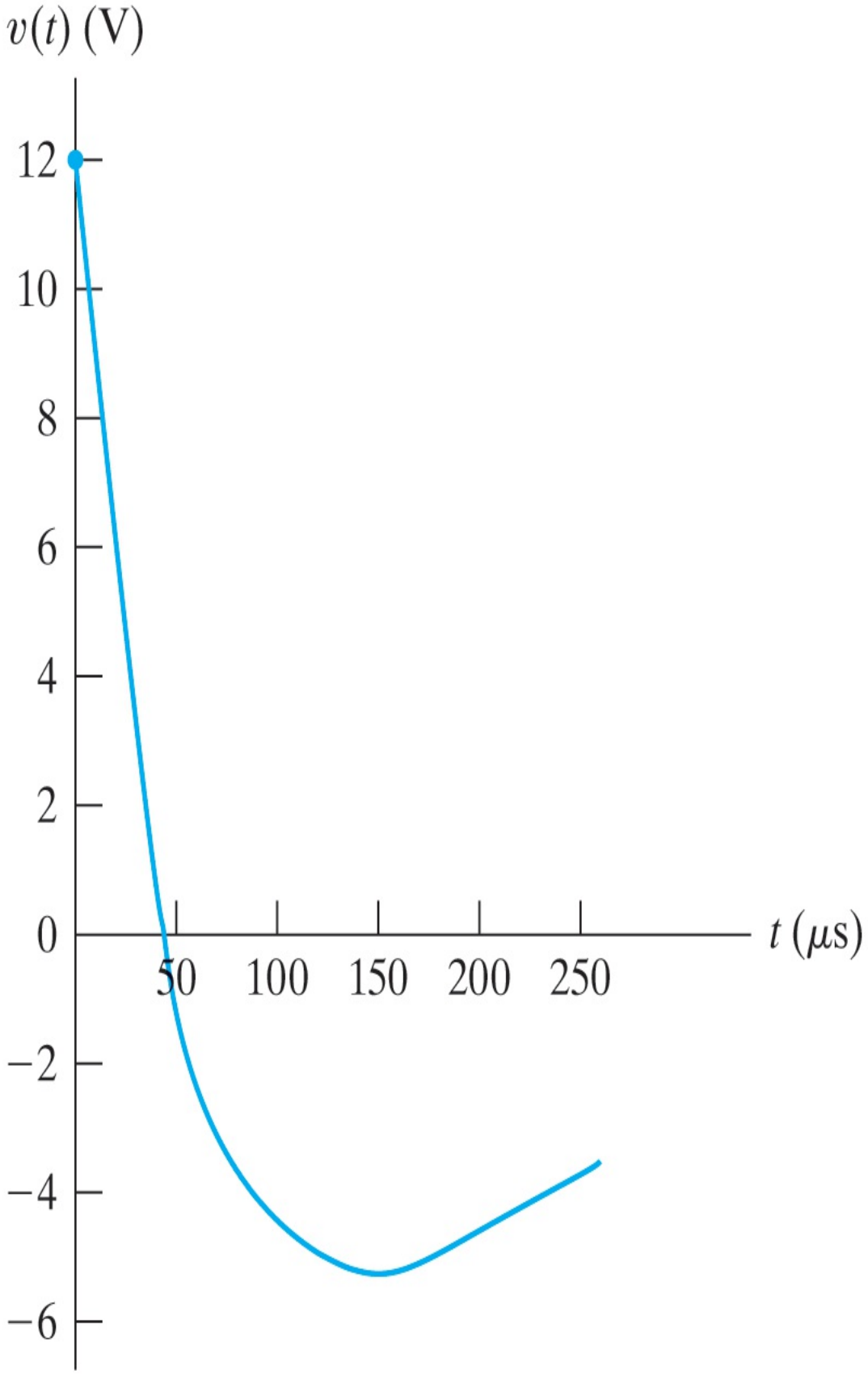

v(t) (V)
12
10
8
6
4
2
0
−2
−4
−6
50
100
150
200
250
t (μs)

Figure 8.7 Full Alternative Text

Example 8.3 Calculating Branch Currents in the Natural Response of a Parallel RLC Circuit

Derive the expressions for the three branch currents iR, iL, and iC in Example 8.2 (Fig. 8.6) during the time the stored energy is being released.

Solution

We know the voltage across the three branches from the solution in Example 8.2 is

$$v(t)=(-14e^{-5000t}+26e^{-20,000t})\ \text{V}, \quad t\geq 0.$$

The current in the resistive branch is then

$$i_R(t)=\frac{v(t)}{200}=(-70e^{-5000t}+130e^{-20,000t})\ \text{mA}, \quad t\geq 0.$$

There are two ways to find the current in the inductive branch. One way is to use the integral relationship that exists between the current and the voltage at the terminals of an inductor:

$$i_L(t)=\frac{1}{L}\int_0^t v_L(x)\,dx+I_0.$$

A second approach is to find the current in the capacitive branch first and then use the fact that $i_R+i_L+i_C=0$. Let's use this approach. The current in the capacitive branch is

$$i_C(t)=C\frac{dv}{dt}=0.2\times 10^{-6}(70,000e^{-5000t}-520,000e^{-20,000t})=(14e^{-5000t}-104e^{-20,000t})\ \text{mA}, \quad t\geq 0^+.$$

Now find the inductive branch current from the relationship

$$i_L(t) = -i_R(t) - i_C(t) = (56e^{-5000t} - 26e^{-20{,}000t})\ \text{mA}, \quad t \geq 0.$$

We leave it to you, in Assessment Problem 8.2, to show that the integral relationship between voltage and current in an inductor leads to the same result. Note that the expression for iL agrees with the initial inductor current, as it must.

Assessment Problems

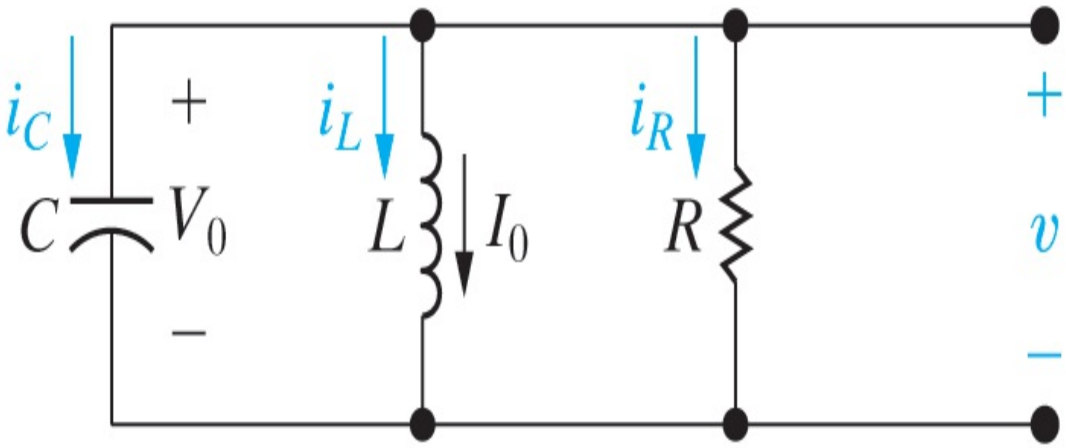
Objective 1—Be able to determine the natural response and the step response of parallel RLC circuits

1. 8.2 Use the integral relationship between iL and v to find the expression for iL in Fig. 8.6.

   Answer:

   $i_L(t) = (56e^{-5000t} - 26e^{-20{,}000t})$ mA, $t \geq 0$.

2. 8.3 The element values in the circuit shown are R=2 kΩ, L=250 mH, and C=10 nF. The initial current I0 in the inductor is −4 A, and the initial voltage on the capacitor is 0 V. The output signal is the voltage v. Find (a) iR(0+); (b) iC(0+); (c) dv(0+)/dt; (d) A1; (e) A2; and (f) v(t) when t≥0.



8.2-5 Full Alternative Text

Answer:

1. (a) 0;
2. 4 A;
3. 4×108 V/s;
4. 13,333 V;
5. −13,333 V;
6. 13,333(e−10,000t−e−40,000t) V.

SELF-CHECK: Also try Chapter Problems 8.6 and 8.16.

The Underdamped Voltage Response

When ω02>α2, the roots of the characteristic equation are complex numbers, and the response is underdamped. For convenience, we express the roots s1 and s2 as

$s_1 = -a + -(\omega_0^2 - a^2) = -a + j\omega_0^2 - a^2 = -a + j\omega_d$, (8.12)
$s_2 = -a - j\omega_d$, (8.13)

where

Damped Radian Frequency

$\omega_d = \omega_0^2 - \alpha^2$. (8.14)

The term ω_d is called the **damped radian frequency**. Later we explain the reason for this terminology.

The underdamped voltage response of a parallel RLC circuit is

Parallel RLC Natural Response

$v(t) = B_1 e^{-\alpha t} \cos \omega_d t + B_2 e^{-\alpha t} \sin \omega_d t$, (8.15)

which follows from Eq. 8.9. In making the transition from Eq. 8.9 to Eq. 8.15, we use the Euler identity:

$e^{\pm j\theta} = \cos\theta \pm j \sin\theta$.

Thus,

$v(t) = A_1 e^{(-\alpha + j\omega_d)t} + A_2 e^{-(\alpha + j\omega_d)t} = A_1 e^{-\alpha t} e^{j\omega_d t} + A_2 e^{-\alpha t} e^{-j\omega_d t} = e^{-\alpha t}(A_1 \cos \omega_d t + j A_1 \sin \omega_d t + A_2 \cos \omega_d t - j A_2 \sin \omega_d t) = e^{-\alpha t}[(A_1 + A_2)\cos \omega_d t + j(A_1 - A_2)\sin \omega_d t]$.

At this point in the transition from Eq. 8.9 to Eq. 8.15, replace the arbitrary constants $A_1 + A_2$ and $j(A_1 - A_2)$ with new arbitrary constants denoted B_1 and B_2 to get

$v = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t) = B_1 e^{-\alpha t} \cos \omega_d t + B_2 e^{-\alpha t} \sin \omega_d t$.

The constants B1 and B2 are real, not complex, because the voltage is a real function. Don't be misled by the fact that B2=j(A1−A2). In this underdamped case, A1 and A2 are complex conjugates, and thus B1 and B2 are real. (See Problems 8.22 and 8.23.) Defining the underdamped response in terms of the coefficients B1 and B2 yields a simple expression for the voltage, v. We determine B1 and B2 in the same way that we found A1 and A2 for the overdamped response—by solving two simultaneous equations. The first equation sets the initial value v from Eq. 8.15 equal to the initial value of v in the circuit. The second equation sets the initial value of dv/dt from Eq. 8.15 equal to the initial value of dv/dt in the circuit. Note that the initial values of v and dv/dt in the circuit are the same in both the underdamped and overdamped cases. For the underdamped response, the two simultaneous equations that determine B1 and B2 are

υ(0 +)= B 1 = V 0 , (8.216)
dυ(0 +) dt =−α B 1 + ω d B 2 = 1 C (− V 0 R − I 0). (8.17)

The overall process for finding the underdamped response is the same as that for the overdamped response, although the response equations and the simultaneous equations used to find the constants are slightly different. We can modify Steps 3, 4, and 5 in the method for finding the voltage for the parallel RLC circuit to accommodate the differences.

1. Step 3: Compare α2 and ω02. If α2>ω02, the response is overdamped:

   v(t)=A1es1t+A2es2t, t≥0.

   If α2<ω02, the response is overdamped:

   v(t)=B1eαt cos ωdt+B2eαt sin ωdt, t≥0.

2. Step 4: If the response is overdamped, calculate the values of s1 and s2 from α and ω0, using the equations in Table 8.1. If the response is underdamped, calculate the value of ωd from

   ωd=ω02−α2.

3. Step 5: If the response is overdamped, calculate the values of A1 and A2

by solving Eqs. 8.10 and 8.11 simultaneously:

$$A_1 + A_2 = V_0 \; ; \; A_1 s_1 + A_2 s_2 = \frac{1}{C}\left(-V_0 R - I_0\right).$$

If the response is underdamped, calculate the values of B1 and B2 by solving Eqs. 8.16 and 8.17 simultaneously:

$$B_1 = V_0;$$
$$-\alpha B_1 + \omega_d B_2 = \frac{1}{C}\left(-\frac{V_0}{R} - I_0\right).$$

These modified steps are condensed in Analysis Method 8.2. We examine the characteristics of the underdamped response following Example 8.4, which analyzes a circuit whose response is underdamped.

Natural Response of a Parallel RLC Circuit

1. **Determine the initial capacitor voltage (V0) and inductor current (I0)** from the circuit.

2. **Determine the values of** α **and** ω_0 using the equations in Table 8.1.

3. **If** $\alpha^2 > \omega_0^2$, **the response is overdamped** and
   $v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}, \quad t \geq 0;$

   If $\alpha^2 < \omega_0^2$, **the response is underdamped** and
   $v(t) = B_1 e^{\alpha t} \cos \omega_d t + B_2 e^{\alpha t} \sin \omega_d t, \quad t \geq 0$

4. **If the response is overdamped, calculate s1 and s2** using the equations in Table 8.1;

   If the response is underdamped, calculate ω_d using $\omega_d = \omega_0^2 - \alpha^2$

5. **If the response is overdamped, calculate A1 and A2** by simultaneously solving Eqs. 8.10 and 8.11;

If the response is underdamped, calculate B1 and B2 by simultaneously solving Eqs. 8.16 and 8.17.

6. **Write the equation for v(t) from Step 3** using the results from Steps 4 and 5; find any desired branch currents.

Analysis Method 8.2 The natural response of an overdamped or underdamped parallel RLC circuit.

Example 8.4 Finding the Underdamped Natural Response of a Parallel RLC Circuit

In the circuit shown in Fig. 8.8, V0=0, and I0=−12.25 mA.

Figure 8.8 The circuit for Example 8.4.

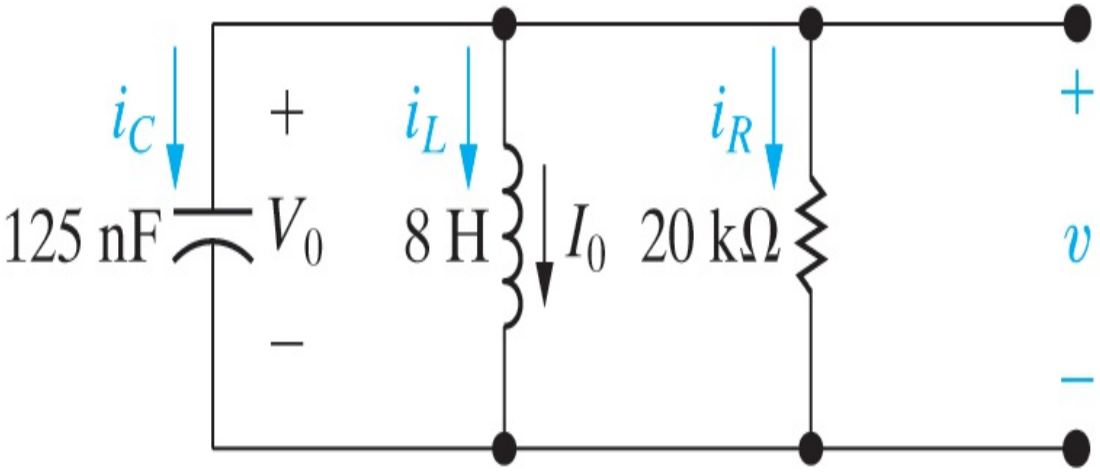


Figure 8.8 Full Alternative Text

1. Calculate the voltage response for $t \geq 0$.
2. Plot $v(t)$ versus t for the time interval $0 \leq t \leq 11$ ms.

Solution

Use Analysis Method 8.2.

1. 1. Step 1: Determine the initial values of capacitor voltage, V_0, and inductor current, I_0; since these values are given in the problem statement, no circuit analysis is required.

   2. Step 2: Determine the values of α and ω_0 using the equations in Table 8.1:

      $\alpha = \frac{1}{2RC} = \frac{1}{2(20{,}000)(125 \times 10^{-9})} = 200$ rad/s,
      $\omega_0^2 = \frac{1}{LC} = \frac{1}{(8)(125 \times 10^{-6})} = 10^6$ rad^2/s^2.

   3. Step 3: Compare α^2 and ω_0^2; since $\alpha^2 < \omega_0^2$, the response is underdamped and

      $v(t) = B_1 e^{\alpha t} \cos \omega_d t + B_2 e^{\alpha t} \sin \omega_d t, \quad t \geq 0.$

   4. Step 4: Since the response is underdamped, calculate the value of ω_d:

      $\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{10^8 - (200)^2} = 979.80$ rad/s.

   5. Step 5: Since the response is underdamped, calculate the values of B_1 and B_2 by simultaneously solving

      $B_1 = V_0 = 0; \quad -\alpha B_1 + \omega_d B_2 = \frac{1}{C}\left(-\frac{V_0}{R} - I_0\right) \quad \text{so} \quad -200 B_1 - 979.80 B_2 = \frac{1}{125 \times 10^{-9}}\left(-\frac{0}{20{,}000} - \left(-12.25 \times 10^{-3}\right)\right) = 98{,}000.$

      Solving,

$B1=0\ V \quad \text{and} \quad B2=100\ V.$

6. Step 6: Write the equation for v(t) using the results from Steps 4 and 5:

   $v(t)=100e^{-200t}\sin 979.80t\ V, \quad t\geq 0.$

2. Figure 8.9 shows the plot of v(t) versus t for the first 11 ms after the stored energy is released. It clearly indicates that the underdamped response is a damped oscillation. The voltage v(t) approaches its final value, alternating between values that are greater than and less than the final value. Furthermore, these swings about the final value decrease exponentially with time.

Figure 8.9 The voltage response for Example 8.4.

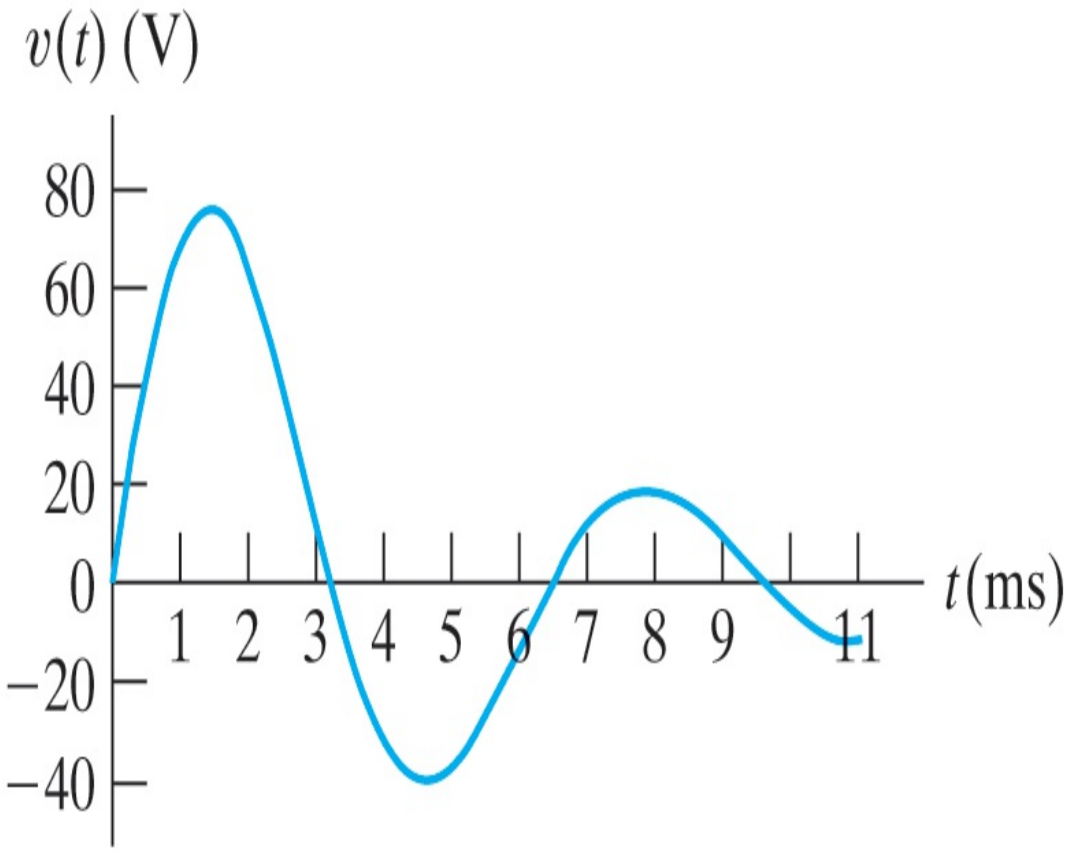


Figure 8.9 Full Alternative Text

Assessment Problem

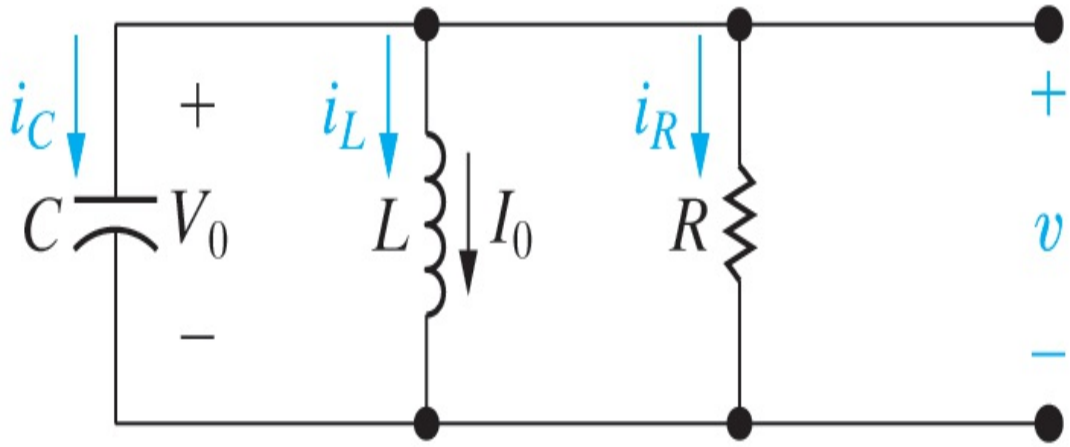
Objective 1—Be able to determine the natural and the step response of parallel RLC circuits

1. 8.4 A 10 mH inductor, a 1 μF capacitor, and a variable resistor are connected in parallel in the circuit shown. The resistor is adjusted so that the roots of the characteristic equation are −8000±j6000 rad/s. The initial voltage on the capacitor is 10 V, and the initial current in the inductor is 80 mA. Find

    1. R;

    2. dv(0+)/dt;

3. B1 and B2 in the solution for v; and
4. iL(t).



8.2-6 Full Alternative Text

Answer:

1. 62.5 Ω;
2. −240,000 V/s;
3. B1=10 V, B2=−80/3 V;
4. 10e−8000t[8 cos 6000t+(82/3) sin 6000t] mA when t≥0.

SELF-CHECK: Also try Chapter Problems 8.8 and 8.14.

Characteristics of the Underdamped Response

Let’s look at the general nature of the underdamped response. From Eq. 8.15 and the plot in Fig. 8.9 we know that the voltage alternates, or oscillates between positive and negative values. The voltage oscillates because there are two types of energy-storage elements in the circuit: the inductor and the

capacitor. (A mechanical analogy of this electric circuit is that of a mass suspended on a spring, where oscillation is possible because energy can be stored in both the spring and the moving mass.) The oscillation rate is fixed by vd and the oscillation amplitude decreases exponentially at a rate determined by α, so α is also called the **damping factor** or **damping coefficient**. That explains why ωd is called the damped radian frequency.

If there is no damping, α=0 and the frequency of oscillation is ω0. Whenever there is a dissipative element, R, in the circuit, α is not zero and the frequency of oscillation, ωd, is less than ω0. Thus, when α is not zero, the frequency of oscillation is said to be damped. As the dissipative losses in the circuit decrease, the persistence of the oscillations increases, and the frequency of the oscillations approaches ω0. In other words, as $R \to \infty$, the energy dissipation in the circuit in Fig. 8.8 approaches zero because $p=v^2/R \to 0$. As $R \to \infty$, $\alpha \to 0$ and $\omega d \to \omega 0$; thus, the voltage oscillates and its amplitude does not decay.

In Example 8.4, if R is increased to infinity, the solution for v(t) becomes

$$v(t)=98 \sin 1000t \text{ V}, \quad t \geq 0.$$

In this case, the oscillation is sustained at a frequency of 1000 rad/s and the maximum amplitude of the voltage is 98 V.

Let's examine the qualitative differences between an underdamped and an overdamped response. In an underdamped system, the response oscillates, or "bounces," about its final value. This oscillation is also called *ringing*. In an overdamped system, the response approaches its final value without ringing or in what is sometimes described as a "sluggish" manner. When specifying the desired response of a second-order system, you have two options:

- Reach the final value in the shortest time possible, without concern for the small oscillations about that final value that will eventually cease. Therefore, design the system components to achieve an underdamped response.

- Do not allow the response to exceed its final value, perhaps to ensure that components are not damaged. Therefore, design the system

components to achieve an overdamped response, and accept a relatively slow rise to the final value.

The Critically Damped Voltage Response

The response of a parallel RLC circuit is critically damped when $\omega_0^2=\alpha^2$, or $\omega_0=\alpha$. When a circuit is critically damped, the response is on the verge of oscillating, and the roots of the characteristic equation are real and equal:

$$s_1=s_2=-\alpha=-\frac{1}{2RC}. \quad (8.18)$$

If we substitute $s_1=s_2=-\alpha$, into the voltage equation (Eq. 8.9), the equation becomes

$$v=(A_1+A_2)e^{-\alpha t}=A_0e^{-\alpha t},$$

where A_0 is an arbitrary constant. But this expression for v cannot satisfy two independent initial conditions (V_0, I_0) with only one constant, A_0.

Thus, when the roots of the characteristic equation are equal, the solution for the differential equation (Eq. 8.1) must take a different form, namely,

Parallel RLC Natural Response– Critically Damped

$$v(t)=D_1te^{-\alpha t}+D_2e^{-\alpha t}. \quad (8.19)$$

This solution involves a simple exponential term plus the product of a linear term and an exponential term. The justification of Eq. 8.19 is left for an introductory course in differential equations.

There are only two unknowns in Eq. 8.19, D_1 and D_2. We find their values in

the same way we found A1 and A2 for the overdamped response and B1 and B2for the underdamped response—by solving two simultaneous equations.

One equation sets the initial value v from Eq. 8.19 equal to the initial value of v in the circuit. The second equation sets the initial value of dv/dt from Eq. 8.19 equal to the initial value of dv/dt in the circuit. Note that the initial values of v and dv/dt in the circuit are the same in the underdamped, overdamped, and critically damped cases. For the critically damped response, the two simultaneous equations that determine D1 and D2 are

$$\upsilon(0+) = D2 = V0, \quad (8.20)$$
$$d\upsilon(0+)\,dt = D1 - \alpha D2 = 1C(-V0R - I0). \quad (8.21)$$

The overall process for finding the critically damped response is the same as that for the overdamped and underdamped responses, but again, the response equation and the simultaneous equations used to find the constants are slightly different. We can modify Steps 3, 4, and 5 in the method for finding the voltage for the parallel RLC circuit to accommodate the differences.

1. Step 3: Compare $\alpha 2$ and $\omega 02$. If $\alpha 2 > \omega 02$, the response is overdamped:

   $v(t) = A1es1t + A2es2t, \quad t \geq 0.$

   If $\alpha 2 < \omega 02$, the response is overdamped:

   $v(t) = B1e\alpha t \cos \omega dt + B2e\alpha t \sin \omega dt, \quad t \geq 0.$

   If $\alpha 2 = \omega 02$, the response is critically damped:

   $v(t) = D1te-\alpha t + D2e-\alpha t, \quad t \geq 0.$

2. Step 4: If the response is overdamped, calculate the values of s1 and s2 from α and $\omega 0$, using the equations in Table 8.1. If the response is underdamped, calculate the value of ωd from

   $\omega d = \omega 02 - \alpha 2.$

   If the response is critically damped, you can skip this step.

3. Step 5: If the response is overdamped, calculate the values of A1 and A2 by solving Eqs. 8.10 and 8.11 simultaneously:

   $A1+A2=V0;$
   $A1s1+A2s2=1C(-V0R-I0).$

   If the response is underdamped, calculate the values of B1 and B2 by solving Eqs. 8.16 and 8.17 simultaneously:

   $B1=V0;$

   If the response is critically damped, calculate the values of D1 and D2 by solving Eqs. 8.20 and 8.21 simultaneously:

   $D 2 = V 0 ; D 1 - \alpha D 2 = 1 C (- V 0 R - I 0).$

   These modified steps are condensed in Analysis Method 8.3, while Table 8.2 collects all of the equations needed to find the natural response of any parallel RLC circuit.

Table 8.2 Equations for analyzing the natural response of parallel RLC circuits

Characteristic equation	$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$
Neper, resonant, and damped frequencies	$\alpha = \frac{1}{2RC} \quad \omega_0 = \sqrt{\frac{1}{LC}} \quad \omega_d = \sqrt{\omega_0^2 - \alpha^2}$
Roots of the characteristic equation	$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}, \quad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$
$\alpha^2 > \omega_0^2$: overdamped	$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}, t \geq 0$
	$v(0^+) = A_1 + A_2 = V_0$
	$\frac{dv(0^+)}{dt} = s_1 A_1 + s_2 A_2 = \frac{1}{C}\left(\frac{-V_0}{R} - I_0\right)$
$\alpha^2 < \omega_0^2$: underdamped	$v(t) = B_1 e^{-\alpha t} \cos \omega_d t + B_2 e^{-\alpha t} \sin \omega_d t, t \geq 0$
	$v(0^+) = B_1 = V_0$
	$\frac{dv(0^+)}{dt} = -\alpha B_1 + \omega_d B_2 = \frac{1}{C}\left(\frac{-V_0}{R} - I_0\right)$
$\alpha^2 = \omega_0^2$: critically damped	$v(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}, t \geq 0$
	$v(0^+) = D_2 = V_0$
	$\frac{dv(0^+)}{dt} = D_1 - \alpha D_2 = \frac{1}{C}\left(\frac{-V_0}{R} - I_0\right)$

(Note that the equations in the last three rows assume that the reference direction for the current in every component is in the direction of the reference voltage drop across that component.)

Table 8.2 Full Alternative Text

(Note that the equations in the last three rows assume that the reference direction for the current in every component is in the direction of the reference voltage drop across that component.)

Natural Response of a Parallel RLC Circuit

1. **Determine the initial capacitor voltage (V0) and inductor current (I0)** from the circuit.

2. **Determine the values of** α **and** ω_0 using the equations in Table 8.2.

3. **If** $\alpha^2 > \omega_0^2$, **the response is overdamped** and $v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}, \quad t \geq 0;$

   If $\alpha^2 < \omega_0^2$, **the response is underdamped** and $v(t) = B_1 e^{\alpha t} \cos \omega_d t + B_2 e^{\alpha t} \sin \omega_d t, \quad t \geq 0;$

   If $\alpha^2 = \omega_0^2$ **the response is critically damped** and $v(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}, \quad t \geq 0$

4. **If the response is overdamped, calculate s1 and s2** using the equations in Table 8.2;

   If the response is underdamped, calculate ω_d using the equation in Table 8.2.

5. **If the response is overdamped, calculate A1 and A2** by simultaneously solving the equations in Table 8.2;

   If the response is underdamped, calculate B1 and B2 by simultaneously solving the equations in Table 8.2;

   If the response is critically damped, calculate D1 and D2 by simultaneously solving the equations in Table 8.2.

6. **Write the equation for v(t) from Step 3** using the results from Steps 4 and 5; find any desired branch currents.

Analysis Method 8.3 The natural response of parallel RLC circuits.

You will rarely encounter critically damped systems in practice, largely because ω_0 must equal α exactly. Both of these quantities depend on circuit parameters, and in a real circuit it is very difficult to choose component values that satisfy an exact equality relationship. Even so, Example 8.5 illustrates the approach for finding the critically damped response of a parallel RLC circuit.

(Note that the equations in the last three rows assume that the reference direction for the current in every component is in the direction of the reference voltage drop across that component.)

Example 8.5 Finding the Critically Damped Natural Response of a Parallel RLC Circuit

1. For the circuit in Example 8.4 (Fig. 8.8), find the value of R that results in a critically damped voltage response.

2. Calculate $v(t)$ for $t \geq 0$.

3. Plot $v(t)$ versus t for $0 \leq t \leq 7$ ms.

Solution

1. From Example 8.4, we know that $\omega_0^2 = 106$. Therefore, for critical damping,

   $\alpha = 103 = 12RC$,

or

$R=106(2000)(0.125)=4000\ \Omega$.

2. Follow the steps in Analysis Method 8.3 to find the voltage v.

    1. Step 1: Determine the initial values of capacitor voltage, V0, and inductor current, I0; since these values are given in Example 8.4, no circuit analysis is required.

    2. Step 2: From part (a), we know that $\alpha=\omega 0=1000$ rad/s.

    3. Step 3: Compare $\alpha 2$ and $\omega 02$; since $\alpha 2=\omega 02$, the response is critically damped and

        $v(t)=D1te^{-\alpha t}+D2e^{-\alpha t}, \quad t\geq 0.$

    4. Step 4: Since the response is critically damped, this step is not needed.

    5. Step 5: Since the response is critically damped, calculate the values of D1 and D2:

        $D 2 = V 0 = 0; D 1 - \alpha D 2 = 1 C (- V 0 R - I 0)$ so $D 1 - 1000 D 2 = 1 125 \times 10 - 9 (- 0 4000 - (- 12.25 \times 10 - 3)) = 98,000.$

        Therefore,

        $D1=98,000$ V/s and $D2=0$ V

    6. Step 6: Write the equation for v(t) using the results from Steps 4 and 5:

        $v(t)=98,000te^{-1000t}$ V, $\quad t\geq 0.$

3. Figure 8.10 shows a plot of v(t) versus t in the interval $0\leq t\leq 7$ ms.

Figure 8.10 The voltage

response for Example 8.10

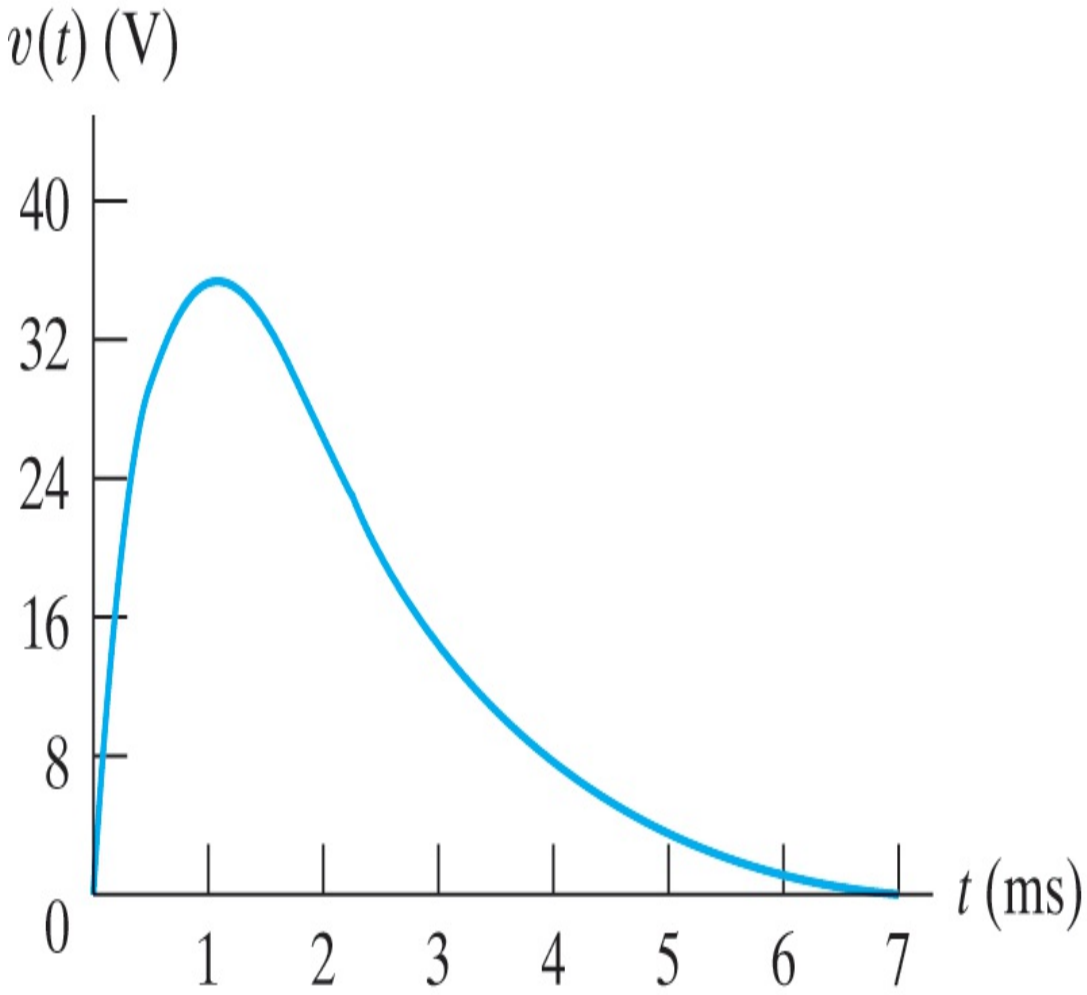


Figure 8.10 Full Alternative Text

Assessment Problem

Objective 1—Be able to determine the natural and the step response of parallel RLC circuits

1. 8.5 The resistor in the circuit in Assessment Problem 8.4 is adjusted for critical damping. The inductance and capacitance values are 0.4 H and

10 μF, respectively. The initial energy stored in the circuit is 25 mJ and is distributed equally between the inductor and capacitor. Find (a) R; (b) V0; (c) I0; (d) D1 and D2 in the solution for v; and (e) iR, $t \geq 0+$.

Answer:

1. 100 Ω;
2. 50 V;
3. 250 mA;
4. −50,000 V/s, 50 V;
5. $iR(t)=(-500te^{-500t}+0.50e^{-500t})$ A, $t \geq 0+$.

SELF-CHECK: Also try Chapter Problems 8.9 and 8.15.

8.3 The Step Response of a Parallel RLC Circuit

Now we find the step response of a parallel RLC circuit, represented by the circuit shown in Fig. 8.11. The step response results from the sudden application of a dc current source. Energy may or may not be stored in the circuit when the current source is applied. We develop a general approach to the step response by finding the current in the inductive branch (iL).

Figure 8.11 A circuit used to describe the step response of a parallel RLC circuit.

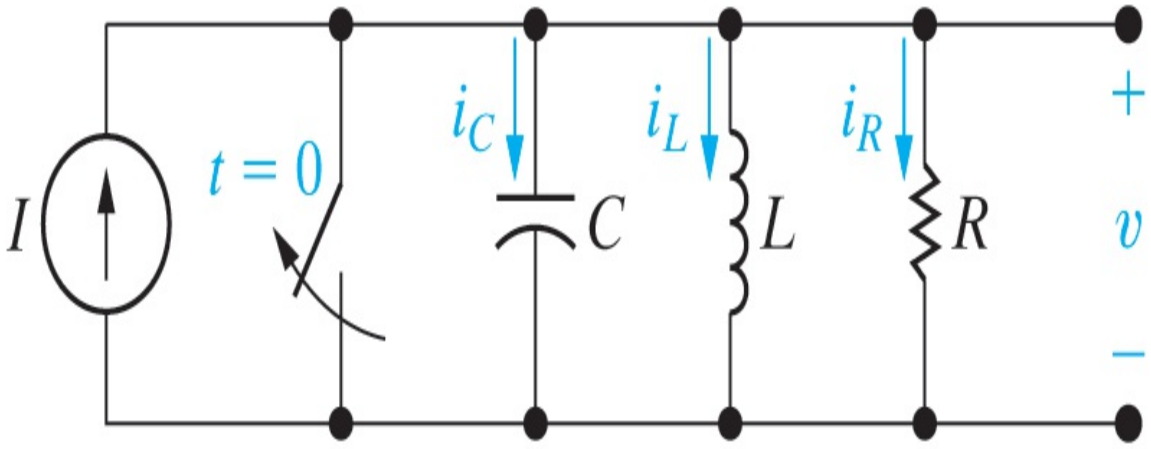


Figure 8.11 Full Alternative Text

Why do we find the inductor current? Remember that for the natural response, we found the voltage because it was the same for all of the parallel-connected components, and we could use the voltage to find the current in any branch. But to find the step response, we need to satisfy three constraints.

Two of the constraints are established by the initial values of the capacitor voltage and the inductor current, just as in the natural-response problem. For the step-response problem, a third constraint arises from a nonzero final value that exists because there is a source in the circuit for t≥0.

We draw the circuit in Fig. 8.11 as $t \to \infty$. In the presence of the dc current source, the capacitor behaves like an open circuit, and the inductor behaves like a short circuit, which shunts the resistor. The resulting circuit is shown in Fig. 8.12, where we see that the only nonzero final value is the inductor current. This explains why we find the inductor current and not the voltage in the parallel RLC step response.

Figure 8.12 The circuit in Fig. 8.11 as $t \to \infty$

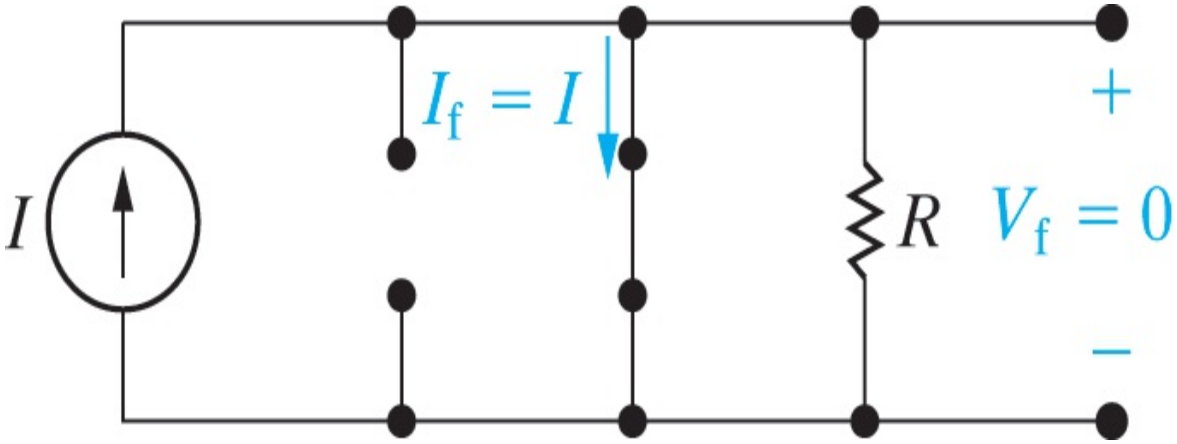


Figure 8.12 Full Alternative Text

To find the inductor current iL in the circuit in Fig. 8.11, we begin with a KCL equation for the circuit's top node:

iL+iR+ iC=I,

or

iL+vR+ Cdvdt=I. (8.22)

Because

$$v = L\frac{di_L}{dt},$$

we get

$$\frac{dv}{dt} = L\frac{d^2i_L}{dt^2}.$$

Now we can write Eq. 8.22 using only the inductor current and its first and second derivatives, to give

$$i_L + LR\frac{di_L}{dt} + LC\frac{d^2i_L}{dt^2} = I.$$

For convenience, we divide through by LC and rearrange terms:

$$\frac{d^2i_L}{dt^2} + \frac{1}{RC}\frac{di_L}{dt} + \frac{i_L}{LC} = \frac{I}{LC}. \quad (8.23)$$

Compare Eq. 8.23 with Eq. 8.1—they have the same form, but note the nonzero constant on the right-hand side of Eq. 8.23. Before showing how to solve Eq. 8.23 directly, we find its solution indirectly. When we know the solution of Eq. 8.23, explaining the direct approach will be easier.

The Indirect Approach

We can solve for i_L indirectly by first finding the voltage v. We use the techniques introduced in Section 8.2 because the differential equation that v must satisfy is identical to Eq. 8.1. To see this, we simply return to Eq. 8.22 and express i_L as a function of v; thus

$$\frac{1}{L}\int_0^t v\,d\tau + I_0 + \frac{v}{R} + C\frac{dv}{dt} = I,$$

where I_0 is the initial current in the inductor. Differentiating once with respect to t reduces the right-hand side to zero because I is a constant and eliminates I_0 from the left-hand side for the same reason. Thus

$$\frac{v}{L} + \frac{1}{R}\frac{dv}{dt} + C\frac{d^2v}{dt^2} = 0,$$

or

$$\frac{d^2v}{dt^2}+\frac{1}{RC}\frac{dv}{dt}+\frac{v}{LC}=0.$$

As discussed in Section 8.2, the solution for v depends on the roots of the characteristic equation. Thus, the three possible solutions are

$$v=A_1e^{s_1t}+A_2e^{s_2t},\quad v=B_1e^{-\alpha t}\cos\omega_d t+B_2e^{-\alpha t}\sin\omega_d t,\quad v=D_1te^{-\alpha t}+D_2e^{-\alpha t}.$$

A word of caution: Because there is a source in the circuit for t>0, you must take into account the value of the source current at t=0+ when you evaluate the coefficients in the three expressions for v.

To find the three possible solutions for iL, we substitute the three expressions for v into Eq. 8.22. When this has been done, you should be able to verify that the three solutions for iL will be

Parallel RLC Step-Response Forms

$$i_L=I+A_1'e^{s_1t}+A_2'e^{s_2t}\quad(\text{overdamped})\quad(8.24)$$
$$i_L=I+B_1'e^{-\alpha t}\cos\omega_d t+B_2'e^{-\alpha t}\sin\omega_d t\quad(\text{underdamped})\quad(8.25)$$
$$i_L=I+D_1'te^{-\alpha t}\quad D_2'e^{-\alpha t}\quad(\text{critically damped})\quad(8.26)$$

where A′1, A′2, B′1, B′2, D′1, and D′2, are arbitrary constants. In each case, the primed constants can be found indirectly in terms of the arbitrary constants associated with the voltage solution. However, this approach is cumbersome.

The Direct Approach

As we have just seen, the solution for a second-order differential equation with a constant forcing function equals the forced response, plus a response function identical in form to the natural response. Thus, we can always write the solution for the step response in the form

i=If+{function of the same formas the natural response},

or

v=Vf+{function of the same formas the natural response},

where If and Vf represent the final value of the response function. The final value may be zero, as we saw for the voltage v in the circuit in Fig. 8.12.

As we have already noted, the only quantity with a nonzero final value in the circuit of Fig. 8.11 is the inductor current. Let's look at how to alter the parallel RLC natural-response method to construct a method for finding the parallel RLC step response for the inductor current.

1. Step 1: Determine the initial values of capacitor voltage, V0, and inductor current, I0, by analyzing the parallel RLC circuit for t<0, In this step, we also need to find the final value of the inductor current, If, by analyzing the circuit as $t \rightarrow \infty$.

2. Step 2: Determine the values of the neper frequency, α, and the resonant radian frequency, ω0, using the values of R, L, and C. No modifications are needed for this step.

3. Step 3: Compare α2 and ω02. Here we replace the natural response for the voltage with the step response for the inductor current, given in Eqs. 8.24–8.26.

4. Step 4: If the response is overdamped, calculate the values of s1 and s2. If the response is underdamped, calculate the value of ωd. If the response is critically damped, you can skip this step. No changes are needed in this step.

5. Step 5: Calculate the values of the A′, B′, and D′ coefficients by simultaneously solving two equations. To construct the first equation, we evaluate the expression for iL(t) from Step 3 at t=0+ and set it equal to the initial inductor current, I0. For example, in the overdamped case the first equation is

$If+A'1+A'2=I0;$

in the underdamped case, the first equation is

$If+B'1=I0;$

and in the critically damped case, the first equation is

$If+D'2=I0.$

To construct the second equation, we find the diL/dt from the inductor current in Step 3, evaluate it at t=0+, and set it equal to the initial value of diL/dt from the circuit. How do we find diL/dt from the circuit? We use the relationship between voltage and current in an inductor to get

$diL(0+)dt=vL(0+)L=V0L.$

In the overdamped case, the second equation is

$A'1s1+A'2s2=V0L;$

in the underdamped case, the second equation is

$-\alpha B'1+\omega dB'2=V0L;$

and in the critically damped case, the second equation is

$D'1-\alpha D'2=V0L.$

6. Step 6: Write the equation for iL(t) from Step 3 using the results from Steps 4 and 5. Find the voltage v and the remaining branch currents using the relationship between voltage and current for the component in each branch.

The steps for finding the step response of a parallel RLC circuit are condensed into Analysis Method 8.4. All of the equations you will need are collected in Table 8.3. Examples 8.6–8.10 illustrate how to use Table 8.3 and Analysis Method 8.4 when finding the step response of a parallel *RLC circuit*.

Table 8.3 Equations for analyzing the step response of parallel RLC circuits

Characteristic equation	$s^2 + \frac{1}{RC}s + \frac{1}{LC} = \frac{I}{LC}$
Neper, resonant, and damped frequencies	$\alpha = \frac{1}{2RC} \quad \omega_0 = \sqrt{\frac{1}{LC}} \quad \omega_d = \sqrt{\omega_0^2 - \alpha^2}$
Roots of the characteristic equation	$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}, \quad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$
$\alpha^2 > \omega_0^2$: overdamped	$i_L(t) = I_f + A_1' e^{s_1 t} + A_2' e^{s_2 t}, \quad t \geq 0$
	$i_L(0^+) = I_f + A_1' + A_2' = I_0$
	$\frac{di_L(0^+)}{dt} = s_1 A_1' + s_2 A_2' = \frac{V_0}{L}$
$\alpha^2 < \omega_0^2$: underdamped	$i_L(t) = I_f + B_1' e^{-\alpha t} \cos \omega_d t + B_2' e^{-\alpha t} \sin \omega_d t, \quad t \geq 0$
	$i_L(0^+) = I_f + B_1' = I_0$
	$\frac{di_L(0^+)}{dt} = -\alpha B_1' + \omega_d B_2' = \frac{V_0}{L}$
$\alpha^2 = \omega_0^2$: critically damped	$i_L(t) = I_f + D_1' t e^{-\alpha t} + D_2' e^{-\alpha t}, \quad t \geq 0$
	$i_L(0^+) = I_f + D_2' = I_0$
	$\frac{di_L(0^+)}{dt} = D_1' - \alpha D_2' = \frac{V_0}{L}$

(Note that the equations in the last three rows assume that the

reference direction for the current in every component is in the direction of the reference voltage drop across that component.)

Table 8.3 Full Alternative Text

Step Response of a Parallel RLC Circuit

1. **Determine the initial capacitor voltage (V0), the initial inductor current (I0), and the final inductor current (If)** from the circuit.

2. **Determine the values of** α **and** $\omega 0$ using the equations in Table 8.3.

3. **If** $\alpha 2 > \omega 02$, **the response is overdamped** and $iL(t) = If + A'1es1t + A'2es2t$, $t \geq 0+$;

   If $\alpha 2 > \omega 02$ **the response is underdamped** and $iL(t) = If + B'1e^{-\alpha t}\cos\omega dt + B'2e^{-\alpha t}\sin\omega dt$, $t \geq 0+$;

   If $\alpha 2 = \omega 02$, **the response is critically damped** and $iL(t) = If + D'1te^{-\alpha t} + D'2e^{-\alpha t}$, $t \geq 0+$

4. **If the response is overdamped, calculate s1 and s2** using the equations in Table 8.3;

   If the response is underdamped, calculate ωd using the equation in Table 8.3.

5. **If the response is overdamped, calculate** $A1'$ **and** $A2'$ by simultaneously solving the equations in Table 8.3;

   If the response is underdamped, calculate $B1'$ **and** $B2'$ by simultaneously solving the equations in Table 8.3;

   If the response is critically damped, calculate $D1'$ **and** $D2'$ by simultaneously solving the equations in Table 8.3.

6. **Write the equation for iL(t) from Step 3** using the results from Steps 4 and 5; find the inductor voltage and any desired branch currents.

Analysis Method 8.4 The step response of parallel RLC circuits.

Example 8.6 Finding the Overdamped Step Response of a Parallel RLC Circuit

The initial energy stored in the circuit in Fig. 8.13 is zero. At t=0, a dc current source of 24 mA is applied to the circuit. The value of the resistor is 400Ω.. Find iL(t) for t≥0.

Figure 8.13 The circuit for Example 8.6.

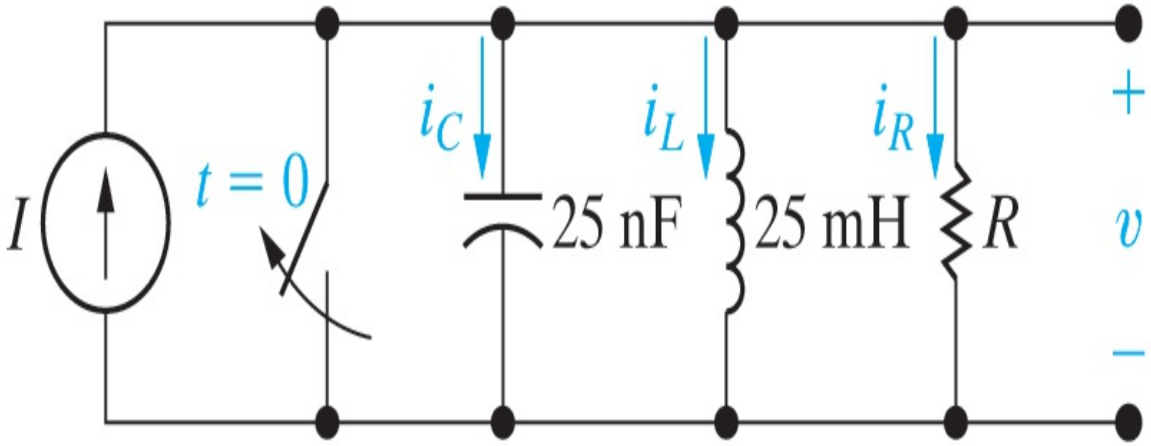


Figure 8.13 Full Alternative Text

Solution

Follow the steps in Analysis Method 8.4.

1. Step 1: The initial values of capacitor voltage, V0, and inductor current, I0 are both zero because the initial stored energy is zero. As $t \to \infty$, the capacitor behaves like an open circuit and the inductor behaves like a short circuit that shunts the resistor, so all of the current from the source is in the inductor. Thus, If=24 mA.

2. Step 2: Using the equations in the second row of Table 8.3,

   $\alpha = 12RC = 12(400)(25\times10-9) = 50{,}000 \text{rad/s};$
   $\omega 0 = 1LC = 1(0.025)(25\times10-9) = 40{,}000 \text{rad/s}.$

3. Step 3: Compare α2 and ω02; since α2>ω02, the response is overdamped and from Table 8.3,

   $iL(t) = If + A'1es1t + A'2es2t,\ t \geq 0.$

4. Step 4: Calculate s1 and s2 using the equations in the third row of Table 8.3:

   $s\,1 = -\alpha + \alpha\,2 - \omega\,0\,2 = -50{,}000 + 50,\,000\,2 - 40,\,000\,2 = -50{,}000 + 30{,}000 = -20{,}000 \text{rad/s};$ $s\,2 = -\alpha - \alpha\,2 - \omega\,0\,2 = -50{,}000 - 50,\,000\,2 - 40,\,000\,2 = -50{,}000 - 30{,}000 = -80{,}000 \text{rad/s}.$

5. Step 5: Calculate the values of A1′ and A2′ by simultaneously solving the equations from row 4 in Table 8.3:

   $If + A'1 + A'2 = Io \quad \text{so} \quad 0.024 + A'1 + A'2 = 0;$
   $s1A'1 + s2A'2 = VoL \quad \text{so} \quad -20{,}000A'1 - 80{,}000A'2 = 0.$

   Solving,

   $A'1 = -32\text{mA} \quad \text{and} \quad A'2 = 8\text{mA}.$

6. Step 6: Write the equation from Step 3 using the results from Steps 4 and 5 to give

   $iL(t) = (-32e-20{,}000t + 8e-80{,}000t)\text{mA}, t \geq 0.$

Example 8.7 Finding the Underdamped Step Response of a Parallel RLC Circuit

The resistor in the circuit in Example 8.6 (Fig. 8.13) is increased to 625Ω. Find iL(t) for t≥0.

Solution

Follow the steps in Analysis Method 8.4.

1. Step 1: From Step 1 of Example 8.6, V0=0, I0=0, If=24 mA.

2. Step 2: Since only R has changed, ω0=40,000 rad/s, from Example 8.6 and

   $$\alpha = \frac{1}{2RC} = \frac{1}{2(625)(25\times10-9)} = 32{,}000 \text{rad/s}.$$

3. Step 3: Compare α2 and ω02; since α2<ω02, the response is underdamped and from Table 8.3,

   $$i_L(t) = I_f + B'_1 e^{-\alpha t}\cos\omega_d t + B'_2 e^{-\alpha t}\sin\omega_d t,\ t \geq 0.$$

4. Step 4: Calculate ωd using the equation in the second row of Table 8.3:

   $$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{40{,}000^2 - 32{,}000^2} = 24{,}000 \text{rad/s}.$$

5. Step 5: Calculate the values of B1′ and B2′ by simultaneously solving the equations from row 5 in Table 8.3:

   $$I_f + B'_1 = I_o \quad \text{so} \quad 0.024 + B'_1 = 0;$$
   $$-\alpha B'_1 + \omega_d B'_2 = \frac{V_o}{L} \quad \text{so} \quad -32{,}000B'_1 + 24{,}000B'_2 = 0.$$

   Solving,

$B'1 = -24\text{mA}$ and $B'2 = -32\text{mA}$.

6. Step 6: Write the equation from Step 3 using the results from Steps 4 and 5 to give

   $i_L(t) = (24 - 24e^{-32{,}000t}\cos 24{,}000t - 32e^{-32{,}000t}\sin 24{,}000t)\text{mA}, \; t \geq 0.$

Example 8.8 Finding the Critically Damped Step Response of a Parallel RLC Circuit

The resistor in the circuit in Example 8.6 (Fig. 8.13) is set at 500Ω. Find i_L for $t \geq 0$.

Solution

Follow the steps in Analysis Method 8.4.

1. Step 1: From Step 1 of Example 8.6,

   $V_0 = 0, \; I_0 = 0, \; I_f = 24 \text{ mA}.$

2. Step 2: Since only R has changed,

   $\omega_0 = 40{,}000 \text{ rad/s},$

   and from the second row of Table 8.3

   $\alpha = \frac{1}{2RC} = \frac{1}{2(500)(25 \times 10^{-9})} = 40{,}000 \text{ rad/s}.$

3. Step 3: Compare α^2 and ω_0^2; since $\alpha^2 = \omega_0^2$, the response is critically damped and from Table 8.3,

$$i_L(t) = I_f + D'_1 te^{-\alpha t} + D'_2 e^{-\alpha t},\ t \geq 0.$$

4. Step 4: The response is critically damped, so this step is not needed.

5. Step 5: Calculate the values of $D1'$ and $D2'$ by simultaneously solving the equations from row 6 in Table 8.3:

   $$I_f + D'_2 = I_o \quad \text{so} \quad 0.024 + D'_2 = 0.029;$$
   $$D'_1 - \alpha D'_2 = V_{oL} \quad \text{so} \quad D'_1 - 40{,}000 D'_2 = 2000.$$

   Solving,

   $$D'_1 = 2.2 \times 106 \text{mA/s} \quad \text{and} \quad D'_2 = 5\text{mA}.$$

6. Step 6: Write the equation from Step 3 using the results from Steps 4 and 5 to give

   $$i_L(t) = (24 - 2.2 \times 106te^{-40{,}000t} + 5e^{-40{,}000t})\text{mA},\ t \geq 0.$$

Example 8.9 Comparing the Three-Step Response Forms

1. Plot on a single graph, over a range from 0 to 220μs, the overdamped, underdamped, and critically damped responses derived in Examples 8.6–8.8.

2. Use the plots of (a) to find the time required for iL to reach 90% of its final value.

3. On the basis of the results obtained in (b), which response would you specify in a design that puts a premium on reaching 90% of the final value of the output in the shortest time?

4. Which response would you specify in a design that must ensure that the final value of the current is never exceeded?

Solution

1. See Fig. 8.14.

Figure 8.14 The current plots for Example 8.9.

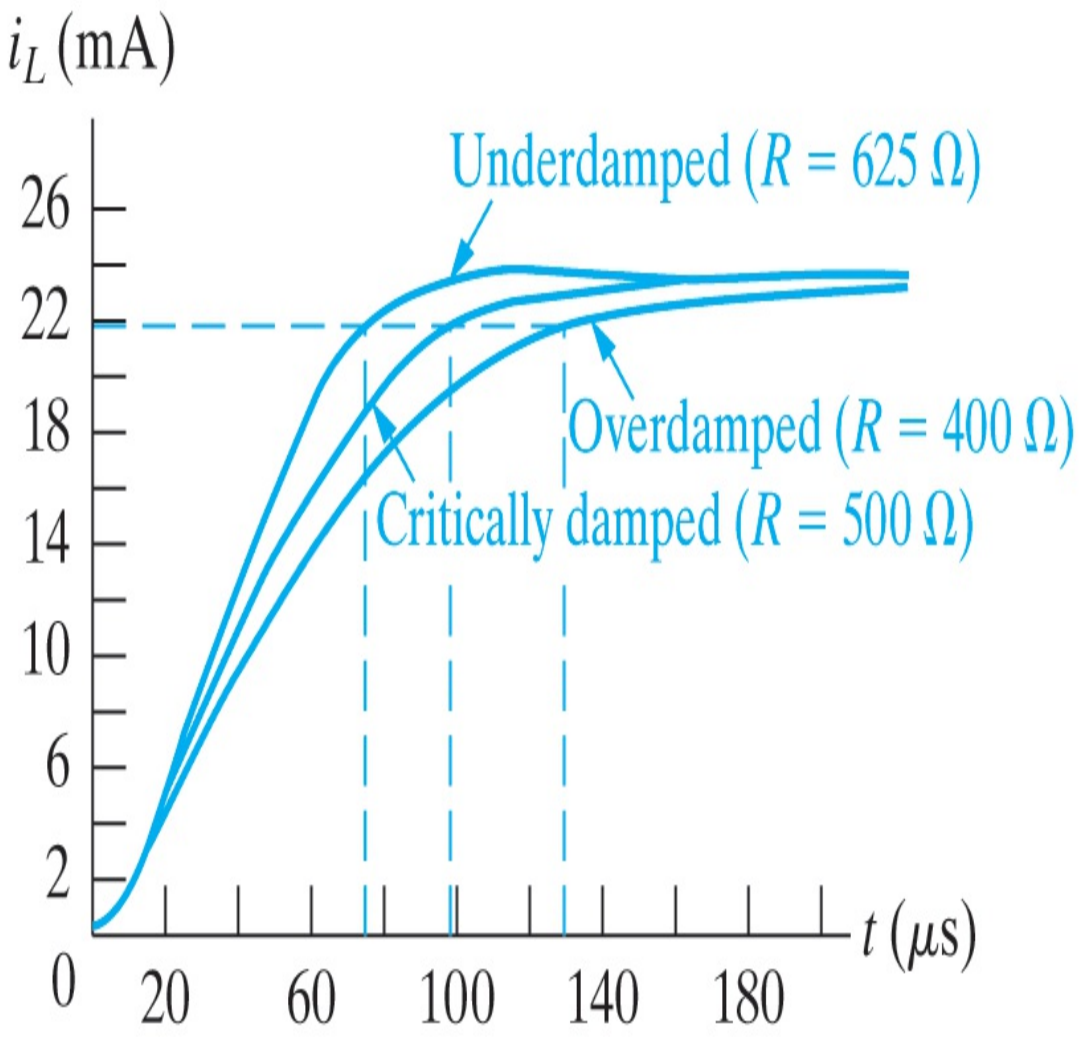


Figure 8.14 Full Alternative Text

2. The final value of iL is 24 mA, so we can read the times off the plots corresponding to iL=21.6mA. Thus, tod=130 μs, tcd=97 μs, and tud=74 μs.

3. The underdamped response reaches 90% of the final value in the fastest time, so it is the desired response type when speed is the most important design specification.

4. From the plot, you can see that the underdamped response overshoots the final value of current, whereas neither the critically damped nor the overdamped response produces currents in excess of 24 mA. Although specifying either of the latter two responses would meet the design specification, it is best to use the overdamped response. It would be impractical to require a design to achieve the exact component values that ensure a critically damped response.

Example 8.10 Finding Step Response of a Parallel RLC Circuit with Initial Stored Energy

Energy is stored in the circuit in Example 8.8 (Fig. 8.13, with R=500Ω) at the instant the dc current source is applied. The initial current in the inductor is 29 mA, and the initial voltage across the capacitor is 50 V. Find iL(t) for t≥0 and v(t) for t≥0.

Solution

Follow Analysis Method 8.4.

1. Step 1: From the problem statement, V0=50 V and I0=29 mA. The final value of the inductor current is unchanged from the problem in Example 8.8, so If=24 mA.

2. Step 2: From Example 8.8, $\alpha=40{,}000$ rad/s and $\omega_0=40{,}000$ rad/s.

3. Step 3: Compare α^2 and ω_0^2; since $\alpha^2=\omega_0^2$, the response is critically damped and from Table 8.3,

   $$i_L(t)=I_f+D'_1te^{-\alpha t}+D'_2e^{-\alpha t},\ t\geq 0.$$

4. Step 4: The response is critically damped, so this step is not needed.

5. Step 5: Calculate the values of D_1' and D_2' by simultaneously solving the equations from row 6 in Table 8.3:

   $$I_f+D'_2=I_o \quad \text{so} \quad 0.024+D'_2=0.029;$$
   $$D'_1-\alpha D'_2=V_oL \quad \text{so} \quad D'_1-40{,}000D'_2=500.025=2000.$$

   Solving,

   $$D'_1=2.2\times 106\text{mA/s} \quad \text{and} \quad D'_2=5\text{mA}.$$

6. Step 6: Write the equation from Step 3 using the results from Steps 4 and 5 to give

   $$i_L(t)=(24-2.2\times 106te^{-40{,}000t}+5e^{-40{,}000t})\text{mA},\ t\geq 0.$$

We can get the expression for $v(t), t\geq 0$ by using the relationship between the voltage and current in an inductor:

$$v(t)=L\frac{di_L}{dt}=(25\times 10^{-3})[(2.2\times 10^6)(-40{,}000)te^{-40{,}000t}+2.2\times 10^6 e^{-40{,}000t}+(5)(-40{,}000)e^{-40{,}000t}]\times 10^{-3}=(-2.2\times 10^6 te^{-40{,}000t}+50e^{-40{,}000t})\text{V},\ t\geq 0.$$

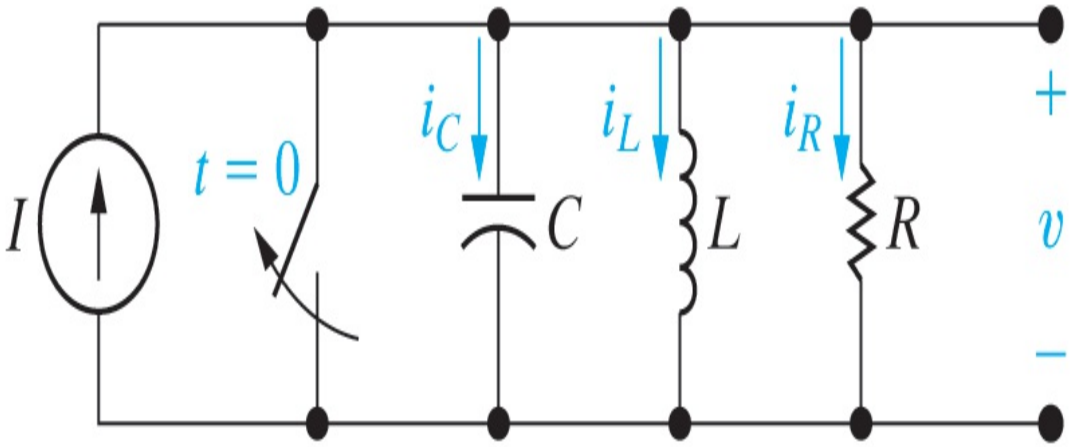
To check this result, let's verify that the initial voltage across the inductor is 50 V:

$$v(0)=-2.2\times 106(0)(1)+50(1)=50\text{ V}.$$

Assessment Problem

Objective 1—Be able to determine the natural response and the step response of parallel RLC circuits

1. 8.6 In the circuit shown, R=500Ω, L=0.64H, C=1 μF, and I=−1 A. The initial voltage drop across the capacitor is 40 V and the initial inductor current is 0.5 A. Find (a) iR(0+); (b) iC(0+); (c) diL(0+)/dt; (d) s1, s2; (e) iL(t) for t≥0; and (f) v(t) for t≥0+.



8.3-9 Full Alternative Text

Answer:

1. 80 mA;
2. −1.58A;
3. 62.5A/s;
4. (−1000 + j 750) rad/s, (−1000 − j 750) rad/s;
5. [−1+e−1000t[1.5 cos 750t+ 2.0833 sin 750t] A, for t≥0;
6. e−1000t(40 cos 750t−2053.33sin 750t) V, for t≥0+.

SELF-CHECK: Also try Chapter Problems 8.32–8.34.

8.4 The Natural and Step Response of a Series RLC Circuit

The procedures for finding the natural or step responses of a series RLC circuit are the same as those used to find the natural or step responses of a parallel RLC circuit because both circuits are described by differential equations that have the same form. For the natural-response problem, we solve for the current because it is the same for all circuit components. We begin by summing the voltages, expressed in terms of the current, around the closed path in the circuit shown in Fig. 8.15. Thus

Figure 8.15 A circuit used to illustrate the natural response of a series RLC circuit.

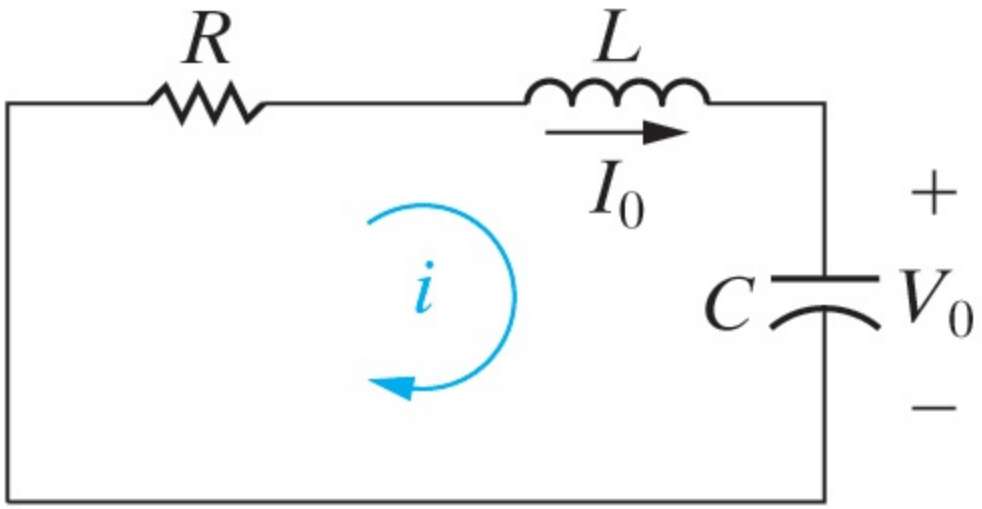


Figure 8.15 Full Alternative Text

$$Ri+L\frac{di}{dt}+\frac{1}{C}\int_0^t i\,d\tau + V_0=0.$$

We now differentiate once with respect to t to get

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0,$$

which we can rearrange as

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0. \quad (8.27)$$

Comparing Eq. 8.27 with Eq. 8.1 reveals that they have the same form. Therefore, to find the solution of Eq. 8.27, we follow the same process that led us to the solution of Eq. 8.1.

From Eq. 8.27, the characteristic equation for the series RLC circuit is

Characteristic Equation, Series RLC Circuit

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0. \quad (8.28)$$

The roots of the characteristic equation are

$$s_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}},$$

or

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}.$$

The neper frequency (α) for the series RLC circuit is

Neper Frequency, Series RLC Circuit

$$\alpha = \frac{R}{2L} \text{ rad/s}, \quad (8.29)$$

and the expression for the resonant radian frequency is

Resonant Radian Frequency, Series RLC Circuit

ω0=1LC rad/s.(8.30)

Note that the equation for the neper frequency of the series RLC circuit differs from that of the parallel RLC circuit, but the equations for the resonant radian frequencies are the same.

The current response will be overdamped, underdamped, or critically damped according to whether ω02< α2, ω02> α2, or ω02=α2, respectively. Thus, the three possible solutions for the current are as follows:

Series RLC Natural-Response Forms

i(t)= A 1 e s 1 t + A 2 e s 2 t (overdamped), (8.31)
i(t)= B 1 e −αt cos ω d t + B 2 e −αt sin ω d t (underdamped), (8.32)
i(t)= D 1 t e −αt D 2 e −αt (critically damped). (8.33)

Once you know the current you can find the voltage across any circuit element.

Let’s look at how to alter the parallel RLC natural-response method to construct a method for finding the series RLC natural response for the circuit’s current.

1. Step 1: Determine the initial values of capacitor voltage, V0, and inductor current, I0, by analyzing the RLC circuit. This step is unchanged.

2. Step 2: Determine the values of the neper frequency, α, and the resonant radian frequency, ω_0, using the values of R, L, and C. While the equation for ω_0 is the same for both circuits, the equation for α is different: for the parallel RLC, $\alpha=1/(2RC)$, and for the series RLC, $\alpha=R/(2L)$.

3. Step 3: Compare α^2 and ω_0^2 to determine the response form. Use the appropriate equation for the circuit current from Eqs. 8.31–8.33.

4. Step 4: If the response is overdamped, calculate the values of s_1 and s_2. If the response is underdamped, calculate the value of ω_d. If the response is critically damped, you can skip this step. No changes are needed in this step.

5. Step 5: Calculate the values of the A, B, and D coefficients by simultaneously solving two equations. To construct the first equation, we evaluate the expression for i(t) from Step 3 at $t=0+$ and set it equal to the initial inductor current, I_0. For example, in the overdamped case the first equation is

   $$A'_1+A'_2=I_0;$$

   in the underdamped case, the first equation is

   $$B'_1=I_0;$$

   and in the critically damped case, the first equation is

   $$D'_2=I_0.$$

   To construct the second equation, we find di/dt from the circuit current in Step 3, evaluate it at $t=0+$ and set it equal to the initial value of di/dt from the circuit. How do we find the initial value of di/dt from the circuit? We use the relationship between voltage and current in an inductor to get

   $$\frac{di(0+)}{dt}=\frac{v_L(0+)}{L}.$$

   But we don’t know the initial voltage across the inductor, so we use

KVL to find it. We know that the sum of the three component voltages at t=0+ must be zero. The voltage across the resistor at t=0+ is the product of the initial current (I0) and the resistance, and the voltage across the capacitor at t=0+ is V0. Using the reference system in Fig. 8.15, we obtain

$$v_L(0+) = -RI0 - V0.$$

So the initial value of di/dt from the circuit is

$$di(0+)dt = 1L(-RI0 - V0).$$

Thus, in the overdamped case, the second equation is

$$A'_1 s_1 + A'_2 s_2 = 1 L (-R I_0 - V_0)$$

in the underdamped case, the second equation is

$$-\alpha B'_1 + \omega_d B'_2 = (-R I_0 - V_0);$$
;

and in the critically damped case, the second equation is

$$D'_1 - \alpha D' = 1 L (-R I_0 - V_0).$$

6. Step 6: Write the equation for i(t) from Step 3 using the results from Steps 4 and 5. Find the voltage for any component using its relationship between voltage and current.

The steps for finding the natural response for a parallel RLC circuit are condensed into Analysis Method 8.5. All of the equations you will need are collected in Table 8.4.

Table 8.4 Equations for analyzing the natural response

of series RLC circuits

Characteristic equation	$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$
Neper, resonant, and damped frequencies	$\alpha = \frac{R}{2L} \quad \omega_0 = \sqrt{\frac{1}{LC}} \quad \omega_d = \sqrt{\omega_0^2 - \alpha^2}$
Roots of the characteristic equation	$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}, \quad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$
$\alpha^2 > \omega_0^2$: overdamped	$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}, t \geq 0$
	$i(0^+) = A_1 + A_2 = I_0$
	$\frac{di(0^+)}{dt} = s_1 A_1 + s_2 A_2 = \frac{1}{L}(-RI_0 - V_0)$
$\alpha^2 < \omega_0^2$: underdamped	$i(t) = B_1 e^{-\alpha t} \cos \omega_d t + B_2 e^{-\alpha t} \sin \omega_d t, \quad t \geq 0$
	$i(0^+) = B_1 = I_0$
	$\frac{di(0^+)}{dt} = -\alpha B_1 + \omega_d B_2 = \frac{1}{L}(-RI_0 - V_0)$
$\alpha^2 = \omega_0^2$: critically damped	$i(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}, \quad t \geq 0$
	$i(0^+) = D_2 = I_0$
	$\frac{di(0^+)}{dt} = D_1 - \alpha D_2 = \frac{1}{L}(-RI_0 - V_0)$

Table 8.4 Full Alternative Text

Natural Response of a Series RLC circuits

1. **Determine the initial capacitor voltage (V0) and inductor current (I0)** from the circuit.

2. **Determine the values of** α **and** $\omega 0$ using the equations in Table 8.4.

3. **If** $\alpha 2 > \omega 02$, **the response is overdamped** and $i(t)=A1es1t+A2es2t, t\geq 0$;

   If $\alpha 2 < \omega 02$ **the response is underdamped** and $i(t)=B1e\alpha t\cos\omega dt+B2e\alpha t\sin\omega dt, t\geq 0$;

   If $\alpha 2=\omega 02$, **the response is critically damped** and $i(t)=D1te-\alpha t+D2e-\alpha t, t\geq 0$

4. **If the response is overdamped, calculate s1 and s2** using the equations in Table 8.4;

   If the response is underdamped, calculate ωd using the equation in Table 8.4.

5. **If the response is overdamped, calculate A1 and A2** by simultaneously solving the equations in Table 8.4;

   If the response is underdamped, calculate B1 and B2 by simultaneously solving the equations in Table 8.4;

   If the response is critically damped, calculate D1 and D2 by simultaneously solving the equations in Table 8.4.

6. **Write the equation for i(t) from Step 3** using the results from Steps 4 and 5; find any desired component voltages.

Analysis Method 8.5 The natural response of series RLC circuits.

To verify that the procedure for finding the step response of a series RLC circuit is similar to that for a parallel RLC circuit, we show that the differential equation that describes the capacitor voltage in Fig. 8.16 has the same form as the differential equation that describes the inductor current in Fig. 8.11. Applying KVL to the circuit shown in Fig. 8.16 gives

Figure 8.16 A circuit used to illustrate the step response of a series RLC circuit.

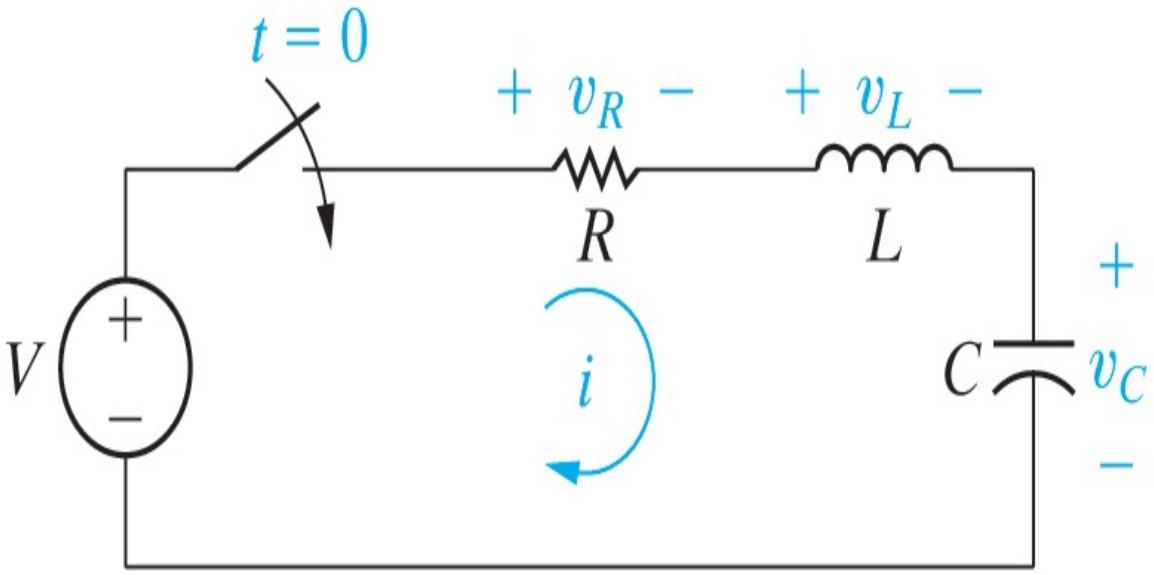


Figure 8.16 Full Alternative Text
V=Ri+Ldidt+ vC.

The current (i) is related to the capacitor voltage (vC) by the expression

i=CdvCdt,

from which

didt=Cd2vCdt2.

Substitute the expressions for the current and its first derivative into the KVL equation to get

d2vCdt2+RLdvCdt+vCLC=VLC. (8.34)

Equation 8.34 has the same form as Eq. 8.23; therefore, the procedure for finding vC parallels that for finding iL. The three possible solutions for vC are as follows:

PARALLEL RLC STEP-RESPONSE FORMS

$$\upsilon_c = V_f + A_1' e^{s_1 t} + A_2' e^{s_2 t} \text{ (overdamped)}, \quad (8.35)$$
$$\upsilon_c = V_f + B_1' e^{-\alpha t} \cos \omega_d t + B_2' e^{-\alpha t} \sin \omega_d t \text{ (underdamped)}, \quad (8.36)$$
$$\upsilon_c = V_f D_1' t e^{-\alpha t} \; D_2' e^{-\alpha t} \text{ (critically damped)}. \quad (8.37)$$

where Vf is the final value of vC. Hence, from the circuit shown in Fig. 8.15, the final value of vC is the dc source voltage V.

Let’s look at how to alter the series RLC natural-response method (Analysis Method 8.5) to find the series RLC step response for the capacitor voltage.

1. Step 1: Determine the initial values of capacitor voltage, V0, and inductor current, I0, by analyzing the RLC circuit. In this step, we also need to find the final value of the capacitor voltage, Vf, by analyzing the circuit as $t \rightarrow \infty$.

2. Step 2: Determine the values of the neper frequency, α, and the resonant radian frequency, ω0, using the values of R, L, and C. No modifications are needed for this step.

3. Step 3: Compare α2 and ω02. Here we replace the natural response for the current with the step response for the capacitor voltage, given in Eqs. 8.35–8.37.

4. Step 4: If the response is overdamped, calculate the values of s1 and s2. If the response is underdamped, calculate the value of ωd. If the response is critically damped, you can skip this step. No changes are needed in this step.

5. Step 5: Here we calculate the values of the A′, B′, and D′ coefficients by simultaneously solving two equations. To construct the first equation, we evaluate the expression for vC(t) from Step 3 at t=0+ and set it equal to the initial capacitor voltage, V0. For example, in the overdamped case the first equation is

   $$Vf+A'1+A'2=V0;$$

   in the underdamped case, the first equation is

   $$Vf+B'1=V0;$$

   and in the critically damped case, the first equation is

   $$Vf+D'2=V0.$$

   To construct the second equation, we find dvC/dt from the capacitor voltage in Step 3, evaluate it at t=0+ and set it equal to the initial value of dvC/dt from the circuit. How do we find dvC/dt at t=0+ from the circuit? We use the relationship between voltage and current in a capacitor to get

   $$dvC(0+)dt=iC(0+)C=I0C.$$

   In the overdamped case, the second equation is

   $$A'1s1+A'2s2=I0C;$$

   in the underdamped case, the second equation is

   $$-\alpha B'1+\omega dB'2=I0C;$$

   and in the critically damped case, the second equation is

$D'1 - \alpha D'2 = I0C$.

6. Step 6: Here we must write the equation for vC(t) from Step 3 using the results from Steps 4 and 5. Find the current i and the remaining component voltages using the relationships between voltage and current.

The steps for finding the step response for a series RLC circuit are condensed into Analysis Method 8.6. All of the equations you will need are collected in Table 8.5.

Table 8.5 Equations for analyzing the step response of series RLC circuits

Characteristic equation	$s^2 + \frac{R}{L}s + \frac{1}{LC} = \frac{V}{LC}$
Neper, resonant, and damped frequencies	$\alpha = \frac{R}{2L} \quad \omega_0 = \sqrt{\frac{1}{LC}} \quad \omega_d = \sqrt{\omega_0^2 - \alpha^2}$
Roots of the characteristic equation	$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}, \quad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$
$\alpha^2 > \omega_0^2$: overdamped	$v_C(t) = V_f + A_1' e^{s_1 t} + A_2' e^{s_2 t}, t \geq 0$
	$v_C(0^+) = V_f + A_1' + A_2' = V_0$
	$\frac{dv_C(0^+)}{dt} = s_1 A_1' + s_2 A_2' = \frac{I_0}{C}$
$\alpha^2 < \omega_0^2$: underdamped	$v_C(t) = V_f + B_1' e^{-\alpha t} \cos \omega_d t + B_2' e^{-\alpha t} \sin \omega_d t, t \geq 0$
	$v_C(0^+) = V_f + B_1' = V_0$
	$\frac{dv_C(0^+)}{dt} = -\alpha B_1' + \omega_d B_2' = \frac{I_0}{C}$
$\alpha^2 = \omega_0^2$: critically damped	$v_C(t) = V_f + D_1' t e^{-\alpha t} + D_2' e^{-\alpha t}, t \geq 0$
	$v_C(0^+) = V_f + D_2' = V_0$
	$\frac{dv_C(0^+)}{dt} = D_1' - \alpha D_2' = \frac{I_0}{C}$

Table 8.5 Full Alternative Text

(Note that the equations in the last three rows assume that the reference

direction for the current in every component is in the direction of the reference voltage drop across that component.)

Step Response of a Series RLC circuits

1. **Determine the initial capacitor voltage (V0), the initial inductor current (I0), and the final capacitor voltage (Vf)** from the circuit.

2. **Determine the values of** α **and** $\omega 0$ using the equations in Table 8.5.

3. **If** $\alpha 2 > \omega 02$, **the response is overdamped** and $vC(t) = Vf + A'1es1t + A'2es2t$, $t \geq 0+$;

   If $\alpha 2 < \omega 02$, **the response is underdamped** and $vC(t) = Vf + B'1e^{-\alpha t}\cos\omega dt + B'2e^{-\alpha t}\sin\omega dt$, $t \geq 0+$;

   If $\alpha 2 = \omega 02$, **the response is critically damped** and $vC(t) = Vf + D'1te^{-\alpha t} + D'2e^{-\alpha t}$, $t \geq 0+$

4. **If the response is overdamped, calculate s1 and s2** using the equations in Table 8.5;

   If the response is underdamped, calculate ωd using the equation in Table 8.5.

5. **If the response is overdamped, calculate** $A1'$ **and** $A2'$ by simultaneously solving the equations in Table 8.5;

   If the response is underdamped, calculate $B1'$ **and** $B2'$ by simultaneously solving the equations in Table 8.5;

   If the response is critically damped, calculate $D1'$ **and** $D2'$ by simultaneously solving the equations in Table 8.5.

6. **Write the equation for vC(t) from Step 3** using the results from Steps

4 and 5; find the inductor voltage and any desired branch currents.

Analysis Method 8.6 The step response of series RLC circuits.

Examples 8.11 and 8.12 find the natural and step responses of a series RLC circuit.

Example 8.11 Finding the Natural Response of a Series RLC Circuit

The 0.1μF capacitor in the circuit shown in Fig. 8.17 is charged to 100 V. At t=0 the capacitor is discharged through a series combination of a 100 mH inductor and a 560Ω resistor.

Figure 8.17 The circuit for Example 8.11.

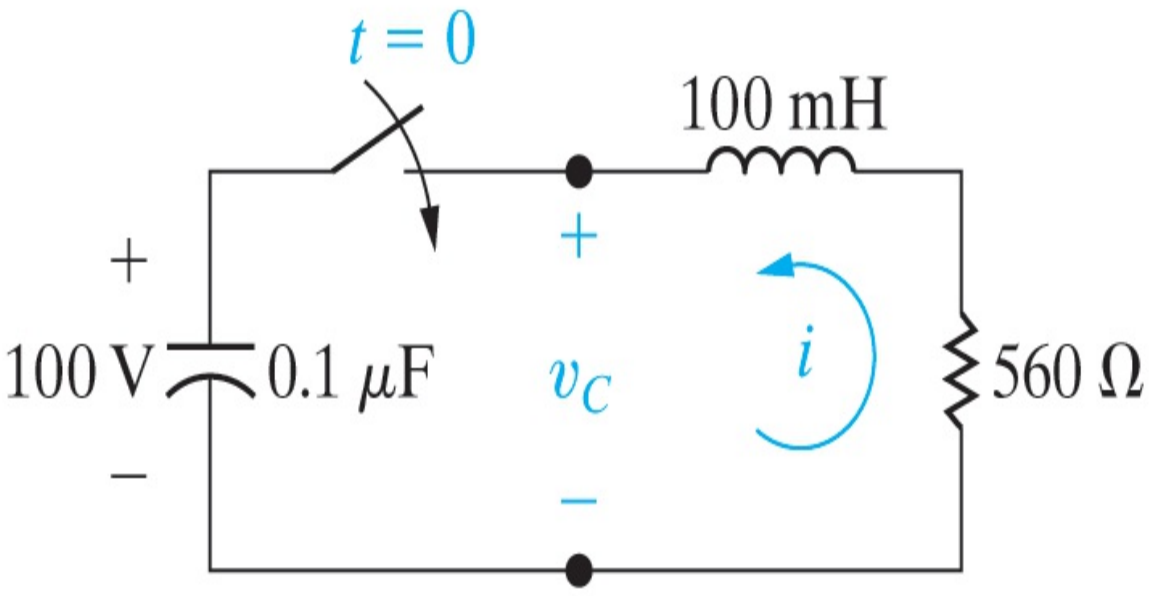


Figure 8.17 Full Alternative Text

1. Find i(t) for t≥0.

2. Find vC(t) for t≥0.

Solution

1. This is a natural-response problem because there is no source in the circuit for t≥0; follow Analysis Method 8.5, which uses Table 8.4.

    1. Step 1: Determine the initial values of capacitor voltage, V0, and inductor current, I0. From the problem statement and the circuit configuration, V0=100 V and I0=0.

    2. Step 2: Determine the values of α and ω0 using the equations in Table 8.4:

        α = R 2L = 560 2(0.1) =2800rad/s, ω 0 2 = 1 LC = 1 (0.1)(0.1× 10 −6) = 10 8 rad 2 /s 2 .

    3. Step 3: Compare α2 and ω02; since α2<ω02, the response is underdamped and from row 5 in Table 8.4,

        i(t)=B1e−αtcosωdt+B2e−αtsinωdt, t≥0.

    4. Step 4: Since the response is underdamped, calculate the value of ωd from row 2 in Table 8.4:

        ωd=ω02−α2=108−28002=9600rad/s.

    5. Step 5: Since the response is underdamped, calculate the values of B1 and B2 by simultaneously solving the equations from row 5 in Table 8.4:

        B1=Io=0;
        −αB1+ωdB2=1L(−RIo−Vo) so
        −2800B1+9600B2=10.1(−(560)(0)−100)=−1000

Solving,

$B1=0$ and $B2=-0.1042A$.

6. Step 6: Write the equation for i(t) using the results from Steps 4 and 5:

   $i(t)=-0.1042e^{-2800t}\sin 9600t$ A, $t\geq 0$.

2. To find vC(t), we can use either of the following relationships:

   $vC=1C\int_0^t i d\tau+100$ or $vC=-(Ri+L\frac{di}{dt})$.

   Whichever expression is used (the second is recommended), the result is

   $vC(t)=(100\cos 9600t+29.17\sin 9600t)e^{-2800t}V$, $t\geq 0$.

Example 8.12 Finding the Step Response of a Series RLC Circuit

No energy is stored in the 100 mH inductor or the 0.4μF capacitor when the switch in the circuit shown in Fig. 8.18 is closed. Find vC(t) for t≥0.

Figure 8.18 The circuit for Example 8.12.

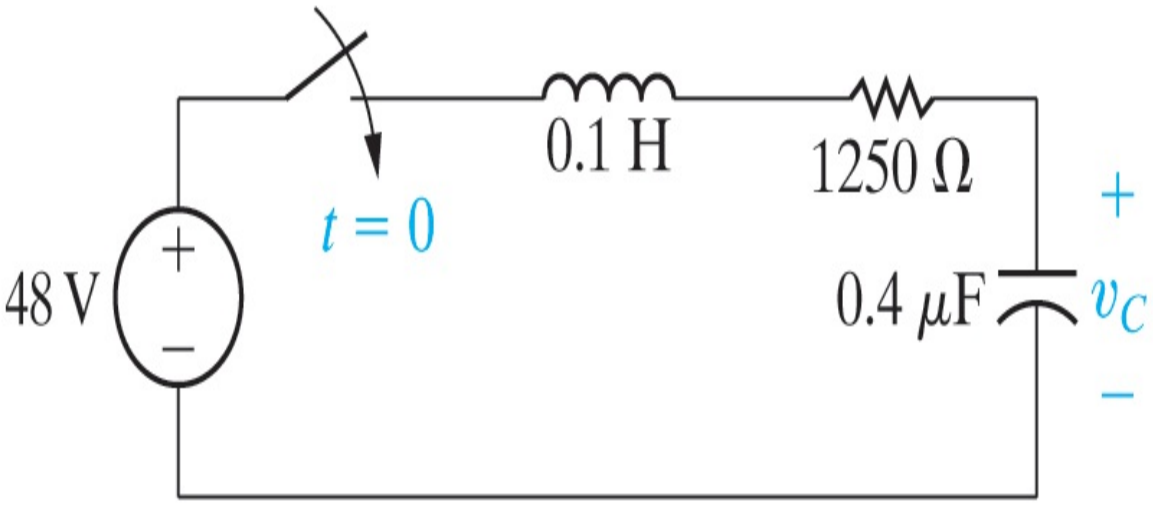


Figure 8.18 Full Alternative Text

Solution

This is a step-response problem because there is a source in the circuit for t≥0; follow Analysis Method 8.6, which uses Table 8.5.

1. Step 1: Determine the initial values of capacitor voltage, V0, and inductor current, I0, and the final value of the capacitor voltage, Vf. From the problem statement and the circuit configuration, V0=0, I0=0, and Vf=48 V.

2. Step 2: Determine the values of α and ω0 using the equations in Table 8.5:

   α = R 2L = 1250 2(0.1) =6250rad/s; ω 0 2 = 1 LC = 1 (0.1)(0.4× 10 −6) =25× 10 6 rad 2 /s 2 .

3. Step 3: Compare α2 and ω02; since α2>ω02, the response is overdamped and from row 4 in Table 8.5:

   vC(t)=Vf+A′1es1t+A′2es2t, t≥0.

4. Step 4: Since the response is overdamped, calculate the values of s1 and

s2 from row 2 in Table 8.5:

$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -6250 + \sqrt{6250^2 - 5000^2} = -6250 + 3750 = -2500 \text{rad/s}$; $s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -6250 - \sqrt{6250^2 - 5000^2} = -6250 - 3750 = -10{,}000 \text{rad/s}$.

5. Step 5: Since the response is overdamped, calculate the values of A1′ and A2′ by simultaneously solving the equations from row 4 in Table 8.5:

   $V_f + A'_1 + A'_2 = V_0$ so $48 + A'_1 + A'_2 = 0$; $s_1 A'_1 + s_2 A'_2 = I_0/C$ so $-2500 A'_1 - 10{,}000 A'_2 = 0$.

   Solving,

   $A'_1 = -64\text{V}$ and $A'_2 = 16\text{V}$.

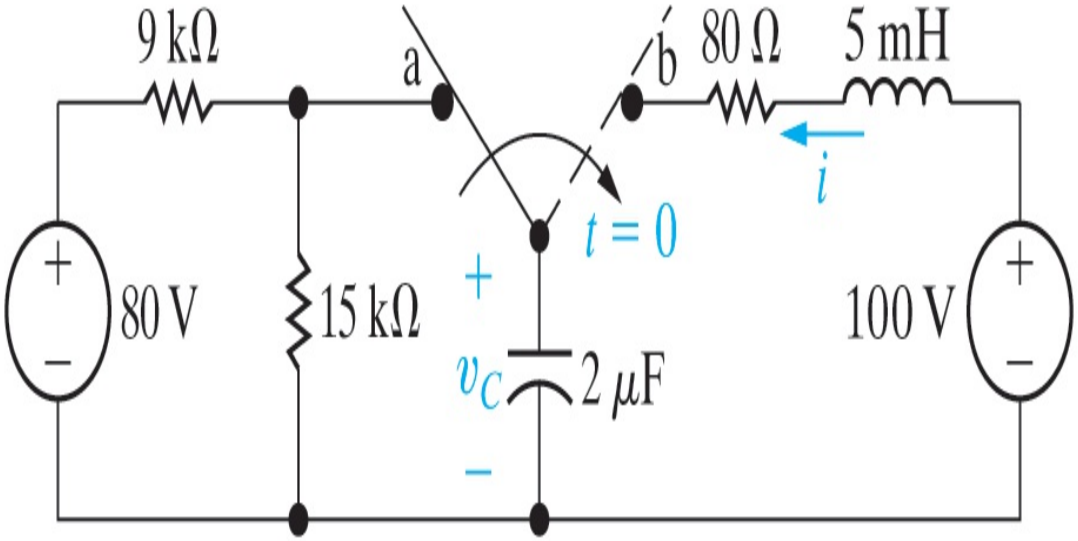
6. Step 6: Write the equation for vC(t) in Step 3 using the results from Steps 4 and 5:

   $v_C(t) = (48 - 64e^{-2500t} + 16e^{-10{,}000t})\text{V}, \quad t \geq 0$.

Assessment Problems

Objective 2—Be able to determine the natural response and the step response of series RLC circuits

1. 8.7 The switch in the circuit shown has been in position a for a long time. At t=0, it moves to position b. Find vC(t) for $t \geq 0$.



8.4-12 Full Alternative Text

Answer:

[100−e−8000t (50 cos 6000t+66.67 sin 6000t)] V for t≥0.

2. 8.8 Find i(t) for t≥0 for the circuit in Assessment Problem 8.7.

   Answer:

   (1.67e−8000t sin 6000t) A for t≥0.

3. 8.9 Repeat Assessment Problems 8.7 and 8.8 if the 80 Ω resistor is replaced by a 100 Ω resistor.

   Answer:

   v C (t) = (100−500,000t e −10,000t −50 e −10,000t)V,t≥0; i(t) = −10,000t e −10,000t A,t≥0.

SELF-CHECK: Also try Chapter Problems 8.50–8.52.

8.5 A Circuit with Two Integrating Amplifiers

A circuit containing two integrating amplifiers connected in cascade[1] is also a second-order circuit; that is, the output voltage of the second integrator is related to the input voltage of the first by a second-order differential equation. We begin our analysis of a circuit containing two cascaded amplifiers with the circuit shown in Fig. 8.19.

1 In a cascade connection, the output signal of the first amplifier (vo1 in Fig. 8.19) is the input signal for the second amplifier.

Figure 8.19 Two integrating amplifiers connected in cascade.

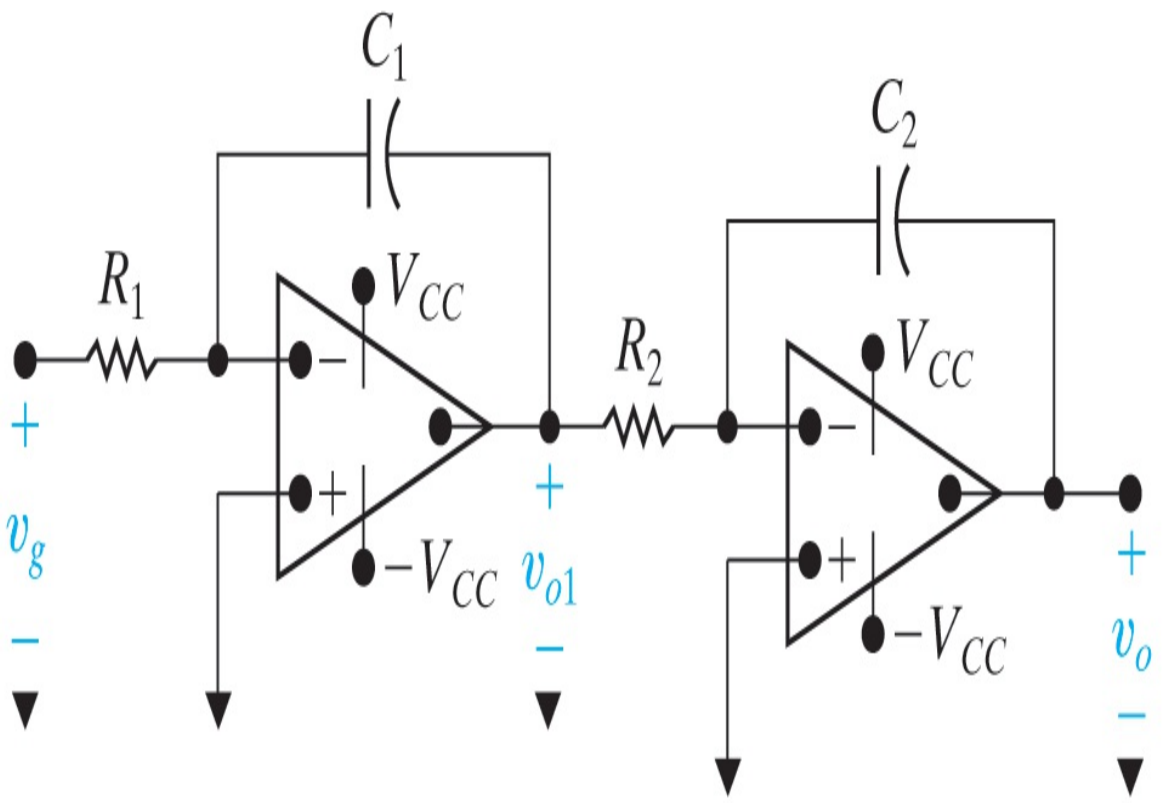


Figure 8.19 Full Alternative Text

We derive the differential equation that defines the relationship between vo and vg., assuming that the op amps are ideal. Begin the derivation by summing the currents at the inverting input terminal of the first integrator:

0−vgR1+C1ddt(0−vo1)=0.

Simplifying and rearranging, we get

dvo1dt=−1R1C1vg. (8.38)

Now, sum the currents away from the inverting input terminal of the second integrating amplifier:

0−vo1R2+C2ddt(0−vo)=0,

or

dvodt=−1R2C2vo1.

Differentiating both sides of this equation gives

d2vodt2=−1R2C2dvo1dt.

We find the differential equation that governs the relationship between vo and vg by substituting for dvo1/dt, using Eq. 8.38:

d2vodt2=1R1C11R2C2vg. (8.39)

Example 8.13 illustrates the step response of a circuit containing two cascaded integrating amplifiers.

Example 8.13 Analyzing Two Cascaded Integrating Amplifiers

No energy is stored in the circuit shown in Fig. 8.20 when the input voltage vg jumps instantaneously from 0 to 25 mV.

Figure 8.20 The circuit for Example 8.13.

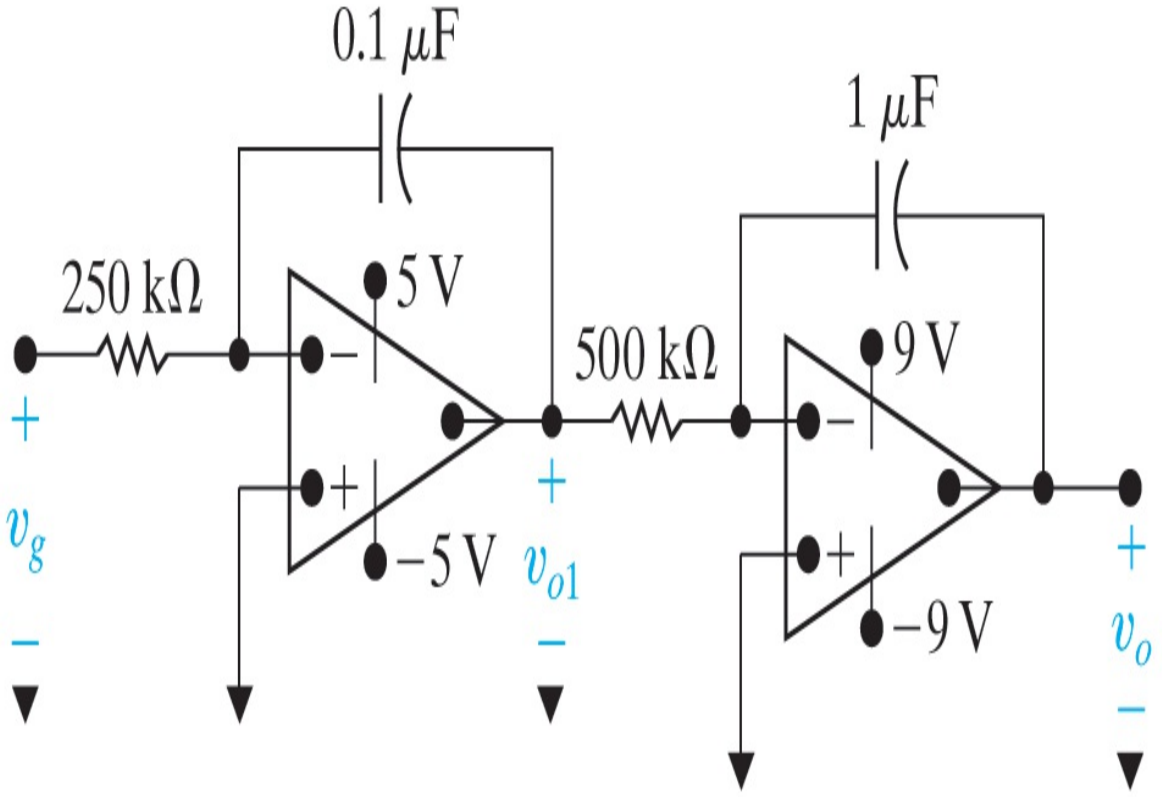


Figure 8.20 Full Alternative Text

1. Derive the expression for vo(t) for 0≤t≤tsat.
2. Find the time tsat when the circuit saturates.

Solution

1. Figure 8.20 indicates that the amplifier scaling factors are

   1 R 1 C 1 = 1000 (250)(0.1) =40, 1 R 2 C 2 = 1000 (500)(1) =2.

   Now, because vg=25mV for t>0, Eq. 8.39 becomes

   d2vodt2=(40)(2)(25×10−3)=2.

   To solve for vo, we let

$g(t)=\frac{dv_o}{dt}$,

then,

$\frac{dg(t)}{dt}=2, \text{and} dg(t)=2dt.$

Hence

$\int g(0)g(t)dy=2\int 0tdx,$

from which

$g(t)-g(0)=2t.$

However,

$g(0)=\frac{dv_o(0)}{dt}=0,$

because the energy stored in the circuit initially is zero, and the op amps are ideal. (See Problem 8.59.) Then,

$\frac{dv_o}{dt}=2t, \text{and} v_o=t2+ v_o(0).$

But $v_o(0)=0$, so the expression for v_o becomes

$v_o=t2, 0\leq t\leq t_{sat}.$

2. The second integrating amplifier saturates when v_o reaches 9 V or t=3s. But it is possible that the first integrating amplifier saturates before t=3s. To explore this possibility, use Eq. 8.38 to find dv_{o1}/dt:

$\frac{dv_{o1}}{dt}=-40(25)\times 10-3=-1.$

Solving for v_{o1} yields

$v_{o1}=-t.$

Thus, at t=3s, $v_{o1}=-3V$, and, because the power supply voltage on the first integrating amplifier is ±5V, the circuit reaches saturation when the

second amplifier saturates. When one of the op amps saturates, we no longer can use the linear model to predict the behavior of the circuit.

SELF-CHECK: Assess your understanding of this material by trying Chapter Problem 8.63.

Two Integrating Amplifiers with Feedback Resistors

Figure 8.21 depicts a variation of the circuit shown in Fig. 8.19. Recall from Section 7.7 that the op amp in the integrating amplifier saturates due to the feedback capacitor's accumulation of charge. Here, a resistor is placed in parallel with each feedback capacitor (C1 and C2) to overcome this problem. We derive the equation for the output voltage, vo, and determine the impact of these feedback resistors on the integrating amplifiers from Example 8.13.

Figure 8.21 Cascaded integrating amplifiers with feedback resistors.

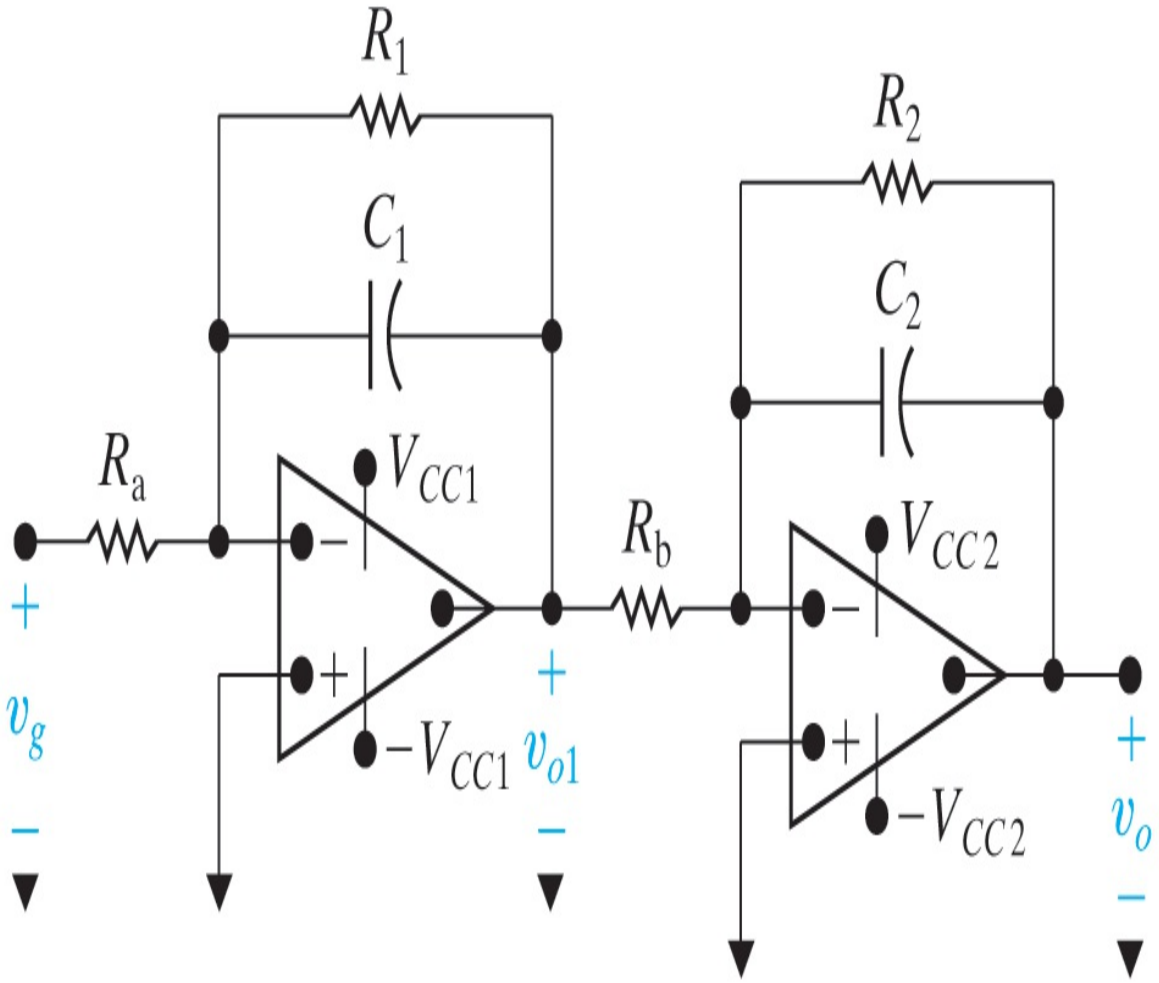


Figure 8.21 Full Alternative Text

We begin the derivation of the second-order differential equation that relates vo1 to vg by summing the currents at the inverting input node of the first integrator:

0−vgRa+0−vo1R1+C1ddt(0−vo1)=0.

Simplifying and rearranging, we get

dvo1dt+1R1C1vo1=−vgRaC1.

For convenience, we let τ1=R1C1 and write Eq. 8.41

dvo1dt+vo1τ1=−vgRaC1. (8.40)

Next, sum the currents at the inverting input terminal of the second integrator:

$$0-v_{o1}R_b+0-v_oR_2+C_2\frac{d}{dt}(0-v_o)=0.$$

Simplifying and rearranging, we get

$$\frac{dv_o}{dt}+v_o\tau_2=-v_{o1}R_bC_2, \quad (8.41)$$

where $\tau_2=R_2C_2$. Differentiating both sides of Eq. 8.40 yields

$$\frac{d^2v_o}{dt^2}+\frac{1}{\tau_2}\frac{dv_o}{dt}=-\frac{1}{R_bC_2}\frac{dv_{o1}}{dt}.$$

From Eq. 8.40,

$$\frac{dv_{o1}}{dt}=-\frac{v_{o1}}{\tau_1}-\frac{v_g}{R_aC_1},$$

and from Eq. 8.41,

$$v_{o1}=-R_bC_2\frac{dv_o}{dt}-R_bC_2\tau_2v_o.$$

Finally, eliminate v_{o1} and dv_{o1}/dt from the second-order differential equation and obtain the desired relationship:

$$\frac{d^2v_o}{dt^2}+\left(\frac{1}{\tau_1}+\frac{1}{\tau_2}\right)\frac{dv_o}{dt}+\left(\frac{1}{\tau_1\tau_2}\right)v_o=\frac{v_g}{R_aC_1R_bC_2}. \quad (8.42)$$

From Eq. 8.42, the characteristic equation is

$$s^2+\left(\frac{1}{\tau_1}+\frac{1}{\tau_2}\right)s+\frac{1}{\tau_1\tau_2}=0.$$

The roots of the characteristic equation are real, namely,

$$s_1=-\frac{1}{\tau_1}, \quad s_2=-\frac{1}{\tau_2}.$$

Example 8.14 determines the step response of two cascaded integrating amplifiers when the feedback capacitors are shunted with feedback resistors.

Example 8.14 Analyzing Two

Cascaded Integrating Amplifiers with Feedback Resistors

The parameters for the circuit shown in Fig. 8.21 are Ra=100 kΩ, R1=500 kΩ, C1=0.1 μF, Rb=25kΩ, R2=100 kΩ, and C2=1 μF. The power supply voltage for each op amp is ±6V. The signal voltage (vg) for the cascaded integrating amplifiers jumps from 0 to 250 mV at t=0. No energy is stored in the feedback capacitors at the instant the signal is applied.

1. Find differential equation that governs vo.
2. Find vo(t) for t≥0.
3. Find the differential equation that governs vo1.
4. Find vo1(t) for t≥0.

Solution

1. From the numerical values of the circuit parameters, we have τ1=R1C1=0.05 s; τ2=R2C2=0.10 s, and vg/RaC1RbC2=1000 V/s2. Substituting these values into Eq. 8.42 gives

   d2vodt2+30dvodt+200vo=1000.

2. The roots of the characteristic equation are s1=−20rad/s and s2= −10rad/s. The final value of vo is the product of the input voltage and the gain of each stage because the capacitors behave as open circuits as t→ ∞. Thus,

   vo(∞)=(250×10−3)(−500)100(−100)25=5 V.

   The solution for vo thus takes the form:

$v_o = 5 + A'_1 e^{-10t} + A'_2 e^{-20t}$.

With $v_o(0) = 0$ and $dv_o(0)/dt = 0$, the numerical values of A'_1 and A'_2 are $A'_1 = -10$ V and $A'_2 = 5$ V. Therefore, the solution for v_o is

$v_o(t) = (5 - 10e^{-10t} + 5e^{-20t})$ V, $t \geq 0$.

The solution assumes that neither op amp saturates. We have already noted that the final value of v_o is 5 V, which is less than 6 V; hence, the second op amp does not saturate. The final value of v_{o1} is $(250 \times 10^{-3})(-500/100)$, or -1.25V. Therefore, the first op amp does not saturate, and our assumption and solution are correct.

3. Substituting the numerical values of the parameters into Eq. 8.40 generates the desired differential equation:

   $dv_{o1}/dt + 20v_{o1} = -25$.

4. We have already noted the initial and final values of v_{o1}, along with the time constant τ_1. Thus, we write the solution in accordance with Analysis Method 7.5, developed in Section 7.4:

   $v_{o1} = -1.25 + [\,0 - (-1.25)\,]\, e^{-20t} = -1.25 + 1.25\, e^{-20t}$ V, $t \geq 0$.

SELF-CHECK: Assess your understanding of this material by trying Chapter Problem 8.64.

Practical Perspective

Clock for Computer Timing

Consider the circuit in Fig. 8.22, where the output is the voltage drop across the capacitor. For $t \geq 0$ this circuit looks like the series RLC natural-response circuit of Fig. 8.3 without its resistor. When we analyze this LC circuit, we will discover that its output is an undamped sinusoid, which a computer's

clock generator could use instead of the typical quartz crystal oscillator. We will be able to specify the frequency of the clock by selecting appropriate values for the inductor and capacitor.

Figure 8.22 An LC natural response circuit.

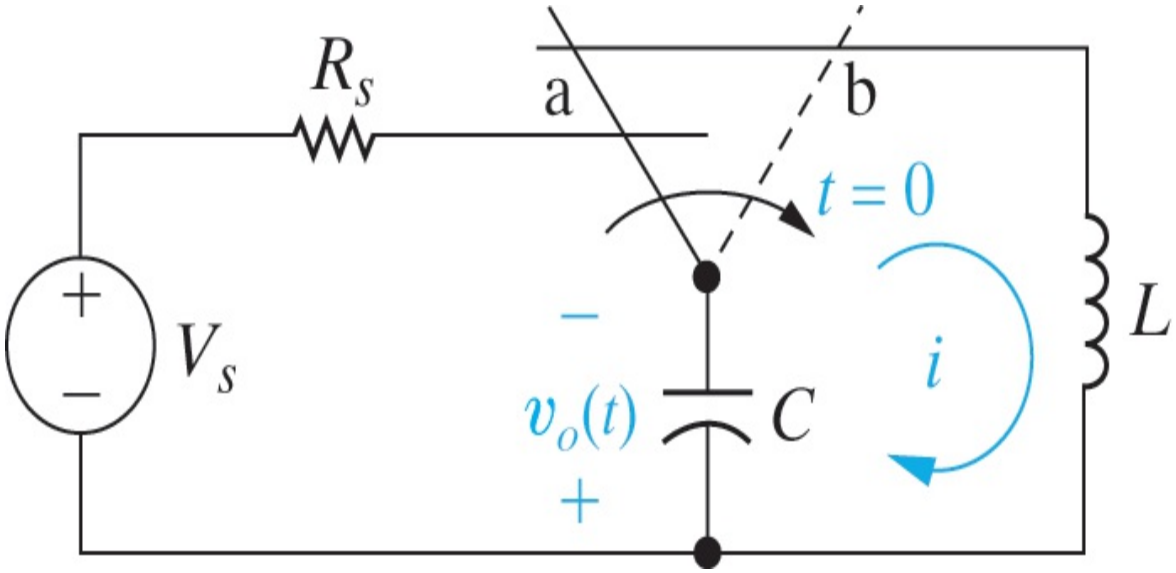


Figure 8.22 Full Alternative Text

Since this is a series RLC natural-response problem, we follow Analysis Method 8.5, which uses Table 8.4. Remember that R=0 for t≥0.

1. Step 1: Determine the initial values of capacitor voltage,V0, and inductor current, I0. How does the circuit behave when t<0? When the switch is in the a position, all of the components are connected. The capacitor acts like an open circuit, whose voltage is V0, and the inductor acts like a short circuit, whose current is I0. The capacitor and inductor have the same voltage, and since the inductor's voltage is 0, V0=0. The current in the inductor is the current in the loop containing the voltage source, the resistor, and the inductor, which is V0=VS/RS.

2. Step 2: Determine the values of α and ω0 using the equations in Table

8.4:

$\alpha = R\ 2L = 0\ 2L = 0,\ \omega\ 0\ 2 = 1\ LC$.

3. Step 3: Compare $\alpha 2$ and $\omega 02$; since $\alpha 2 < \omega 02$, the response is underdamped and from row 5 in Table 8.4:

   $i(t) = B1e^{-\alpha t}\cos\omega dt + B2e^{-\alpha t}\sin\omega dt,\ t \geq 0.$

4. Step 4: Since the response is underdamped, calculate the value of ωd from row 2 in Table 8.4:

   $\omega d = \omega 02 - \alpha 2 = \omega 02 - 02 = \omega 0.$

5. Step 5: Since the response is underdamped, calculate the values of B1 and B2 by simultaneously solving the equations from row 5 in Table 8.4:

   $B1 = Io = VSRS$
   $-\alpha B1 + \omega dB2 = 1L(-RSIo - Vo)$ so
   $-(0)B1 + \omega 0B2 = 1L(-(0)(VSRS) - 0) = 0$

   Solving,

   $B1 = VSRS$ and $B2 = 0.$

6. Step 6: Write the equation for i(t) using the results from Steps 4 and 5:

   $i(t) = VSRSe^{-(0)t}\cos\omega 0t, t \geq 0.$

   We can now use the expression for the current in the circuit to find the voltage output by the capacitor:

   $vo(t) = 1C\int 0ti(x)dx = 1C\int 0tVSRS\cos\omega 0xdx = VS\omega 0RSC\sin\omega 0t, t \geq 0.$

By choosing values for L and C, we can use the circuit in Fig. 8.22 to generate an undamped sinusoid when $t \geq 0$ for a computer's clock generator.

So why is a quartz crystal used to generate the sinusoid for the clock

generator instead of the LC circuit of Fig. 8.22? Remember that our analysis of the LC circuit assumed that the inductor and capacitor are ideal. But ideal inductors and capacitors do not exist—real inductors and capacitors have a small amount of resistance. We leave it to you to examine the effect of this small amount of resistance on the performance of an LC oscillator in the Chapter Problems.

SELF-CHECK: Assess your understanding of the Practical Perspective by solving Chapter Problems 8.66–8.68.

Summary

- The **characteristic equation** for both the parallel and series RLC circuits has the form

  $s2+2\alpha s+\omega02=0,$

  where $\alpha=1/2RC$ for the parallel circuit, $\alpha=R/2L$ for the series circuit, and $\omega02=1/LC$ for both the parallel and series circuits. (See pages 275 and 297.)

- The roots of the characteristic equation are

  $s1,2=-\alpha\pm\alpha2-\omega02.$

  (See page 276.)

- The form of the natural and step responses of series and parallel RLC circuits depends on the values of $\alpha2$ and $\omega02$; such responses can be **overdamped**, **underdamped**, or **critically damped**. These terms describe the impact of the dissipative element (R) on the response. The **neper frequency**, α, reflects the effect of R. (See pages 276 and 277.)

- To determine the natural response of a parallel RLC circuit, follow the steps in Analysis Method 8.3, using the equations in Table 8.2. (See page 287.)

- To determine the step response of a parallel RLC circuit, follow the steps in Analysis Method 8.4, using the equations in Table 8.3. (See page 292.)

- To determine the natural response of a series RLC circuit, follow the steps in Analysis Method 8.5, using the equations in Table 8.4. (See page 299.)

- To determine the step response of a series RLC circuit, follow the steps

in Analysis Method 8.6, using the equations in Table 8.5. (See page 301.)

- When two integrating amplifiers with ideal op amps are connected in cascade, the output voltage of the second integrator is related to the input voltage of the first by an ordinary, second-order differential equation. Therefore, the techniques developed in this chapter may be used to analyze the behavior of a cascaded integrator. (See pages 303 and 304.)

- We can overcome the limitation of a simple integrating amplifier—the saturation of the op amp due to charge accumulating in the feedback capacitor—by placing a resistor in parallel with the capacitor in the feedback path. (See page 305.)

Problems

Sections 8.1–8.2

1. 8.1 The resistance, inductance, and capacitance in a parallel RLC circuit are 1 kΩ, 12.5 H, and 2 μF, respectively.

    1. Calculate the roots of the characteristic equation that describe the voltage response of the circuit.

    2. Will the response be over-, under-, or critically damped?

    3. What value of R will yield a damped frequency of 120 rad/s?

    4. What are the roots of the characteristic equation for the value of R found in (c)?

    5. What value of R will result in a critically damped response?

2. 8.2 PSPICEMULTISIM The circuit elements in the circuit in Fig. 8.1 are R=200 Ω, L=50 mH, and C=0.2 μF. The initial inductor current is −45 mA and the initial capacitor voltage is 15 V.

    1. Calculate the initial current in each branch of the circuit.

    2. Find v(t) for t≥0.

    3. Find iL(t) for t≥0.

3. 8.3 PSPICEMULTISIM The resistance in Problem 8.2 is increased to 250 Ω. Find the expression for v(t) for t≥0.

4. 8.4 PSPICEMULTISIM The resistance in Problem 8.2 is increased to 312.5 Ω. Find the expression for v(t) for t≥0.

5. 8.5 PSPICEMULTISIM The resistor in the circuit in Example 8.4 is changed to 3200 Ω.

    1. Find the numerical expression for v(t) when t≥0.

    2. Plot v(t) versus t for the time interval 0≤t≤7 ms. Compare this response with the one in Example 8.4 (R=20 kΩ) and Example 8.5 (R=4 kΩ). In particular, compare peak values of v(t) and the times when these peak values occur.

6. 8.6 Suppose the inductor in the circuit shown in Fig. 8.1 has a value of 10 mH. The voltage response for t≥0 is

    $v(t)=40e^{-1000t}-90e^{-4000t}$ V.

    1. Determine the numerical values of ω0, α, C, and R.

    2. Calculate iR(t), iL(t), and iC(t) for t≥0+.

7. 8.7 The natural response for the circuit shown in Fig. 8.1 is known to be

    $v(t)=3e^{-100t}+3e^{-900t}$ V, t≥0.

    If C=2.5 μF and L=(40/9) H, find iL(0+) in milliamperes.

8. 8.8 The natural voltage response of the circuit in Fig. 8.1 is

    $v(t)=120e^{-400t}\cos 300t+80e^{-400t}\sin 300t$ V,

    when the capacitor is 250 μF. Find (a) L; (b) R; (c) V0; (d) I0; and (e) iL(t).

9. 8.9 The voltage response for the circuit in Fig. 8.1 is known to be

    $v(t)=D1te^{-500t}+D2e^{-500t}$, t≥0.

    The initial current in the inductor (I0) is −10 mA, and the initial voltage on the capacitor (V0) is 8 V. The inductance is 4 H.

1. Find the values of R, C, D1, and D2.

2. Find iC(t) for t≥0+.

10. 8.10 PSPICEMULTISIM In the circuit shown in Fig. 8.1, a 20 mH inductor is shunted by a 500 nF capacitor, the resistor R is adjusted for critical damping, V0 = 40 V, and I0=120 mA.

    1. Calculate the numerical value of R.

    2. Calculate v(t) for t≥0.

    3. Find v(t) when iC(t)=0.

    4. What percentage of the initially stored energy remains stored in the circuit at the instant iC (t) is 0?

11. 8.11

    1. Design a parallel RLC circuit (see Fig. 8.1) using component values from Appendix H, with a resonant radian frequency of 5000 rad/s. Choose a resistor or create a resistor network so that the response is critically damped. Draw your circuit.

    2. Calculate the roots of the characteristic equation for the resistance in part (a).

12. 8.12

    1. Change the resistance for the circuit you designed in Problem 8.11(a) so that the response is underdamped. Continue to use components from Appendix H. Calculate the roots of the characteristic equation for this new resistance.

    2. Change the resistance for the circuit you designed in Problem 8.11(a) so that the response is overdamped. Continue to use components from Appendix H. Calculate the roots of the characteristic equation for this new resistance.

13. 8.13 The initial value of the voltage v in the circuit in Fig. 8.1 is zero, and the initial value of the capacitor current, ic(0+), is 45 mA. The expression for the capacitor current is known to be

    $ic(t)=A1e^{-200t}+A2e^{-800t}, \quad t\geq 0+,$

    when R is 250 Ω. Find

    1. the values of α, ω0, L, C, A1, and A2

    (Hint:$diC(0+)dt=-diL(0+)dt-diR(0+)dt=-v(0)L-1RiC(0+)C$)

    1. the expression for v(t), t≥0,
    2. the expression for iR(t)≥0,
    3. the expression for iL(t)≥0.

14. 8.14 PSPICEMULTISIM The two switches in the circuit seen in Fig. P8.14 operate synchronously. When switch 1 is in position a, switch 2 is in position d. When switch 1 moves to position b, switch 2 moves to position c. Switch 1 has been in position a for a long time. At t=0, the switches move to their alternate positions. Find vo(t) for t≥0.

Figure P8.14

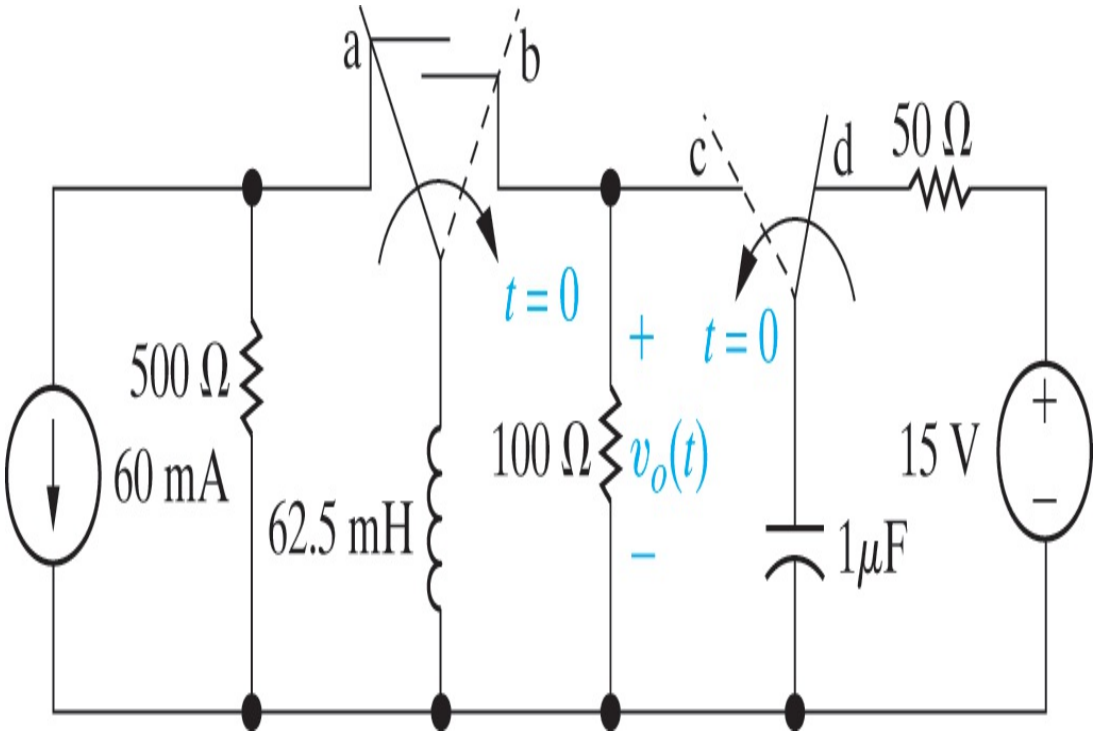


Figure P8.14 Full Alternative Text

15. 8.15 PSPICEMULTISIM The resistor in the circuit of Fig. P8.14 is increased from 100 Ω to 125 Ω. Find vo(t) for t≥0.

16. 8.16 PSPICEMULTISIM The resistor in the circuit of Fig. P8.14 is increased from 100 Ω to 200 Ω. Find vo(t) for t≥0.

17. 8.17 PSPICEMULTISIM The switch in the circuit of Fig. P8.17 has been in position a for a long time. At t=0 the switch moves instantaneously to position b. Find vo(t) for t≥0.

Figure P8.17

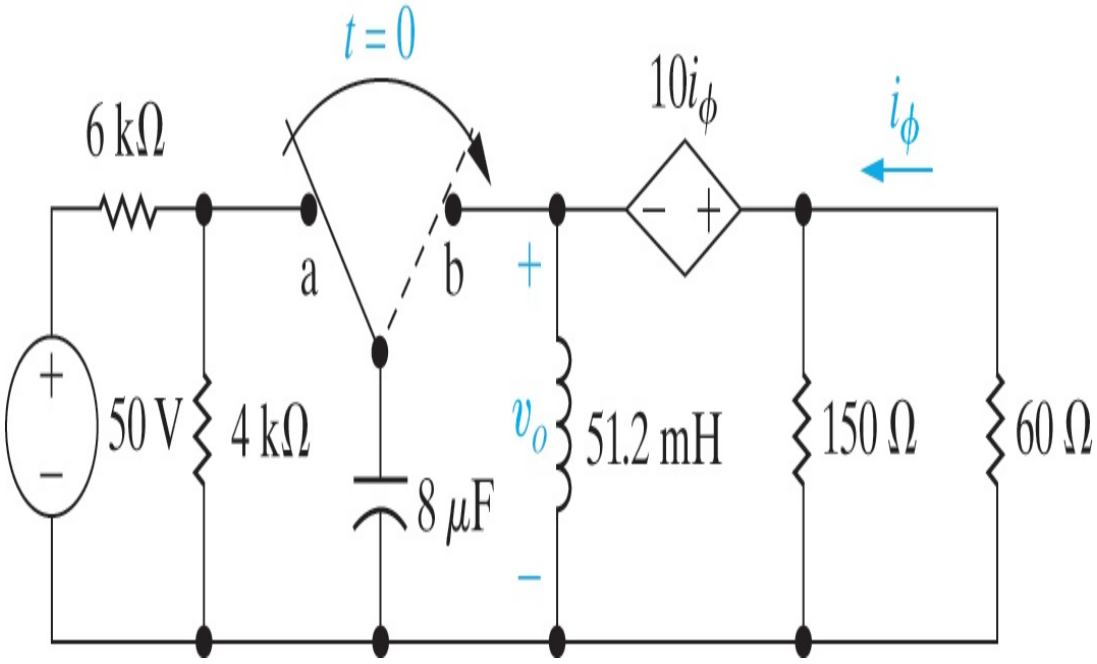


Figure P8.17 Full Alternative Text

18. 8.18 The inductor in the circuit of Fig. P8.17 is increased to 80 mH. Find vo(t) for t≥0.

19. 8.19 The inductor in the circuit of Fig. P8.17 is increased to 125 mH. Find vo(t) for t≥0.

20. 8.20 PSPICEMULTISIMIn the circuit in Fig. 8.1, R=12.5 Ω, C=80 mF L=(50/101) H, V0=0 V, and I0=−4 A.

    1. Find v(t) for t≥0.

    2. Find the first three values of t for which dv/dt is zero. Let these values of t be denoted t1, t2, and t3.

    3. Show that t3−t1=Td.

    4. Show that t2−t1=Td/2.

    5. Calculate v(t1), v(t2), and v(t3).

    6. Sketch v(t) versus t for 0≤t≤t2.

21. 8.21 PSPICEMULTISIM

    1. Find v(t) for t≥0 in the circuit in Problem 8.20 if the 12.5 Ω resistor is removed from the circuit.

    2. Calculate the frequency of v(t) in hertz.

    3. Calculate the maximum amplitude of v(t) in volts.

22. 8.22 Assume the underdamped voltage response of the circuit in Fig. 8.1 is written as

    $$v(t)=(A1+A2)e^{-\alpha t}\cos\omega dt+j(A1-A2)e^{-\alpha t}\sin\omega dt$$

    The initial value of the inductor current is I0, and the initial value of the capacitor voltage is V0. Show that A2 is the conjugate of A1. (Hint: Use the same process as outlined in the chapter to find A1 and A2.)

23. 8.23 Show that the results obtained from Problem 8.22—that is, the expressions for A1 and A2—are consistent with Eqs. 8.16 and 8.17 in the text.

Section 8.3

1. 8.24 PSPICEMULTISIM For the circuit in Example 8.6, find, for t≥0, (a) v(t); (b) iR(t); and (c) iC(t).

2. 8.25 For the circuit in Example 8.7, find, for t≥0, (a) v(t) and (b) iC(t).

3. 8.26 PSPICEMULTISIM For the circuit in Example 8.8, find v(t) for t≥0.

4. 8.27 PSPICEMULTISIM The switch in the circuit in Fig. P8.27 has been open for a long time before closing at t=0. Find io(t) for t≥0.

Figure P8.27

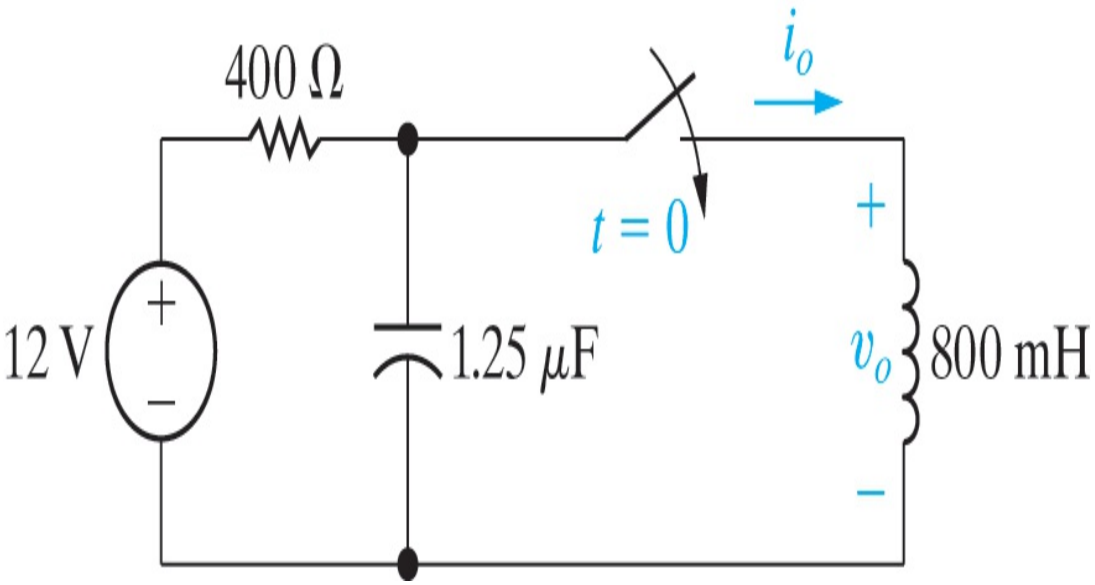


Figure P8.27 Full Alternative Text

5. 8.28 PSPICEMULTISIM

    1. For the circuit in Fig. P8.27, find vo for t≥0.

2. Show that your solution for vo is consistent with the solution for io in Problem 8.27.

6. 8.29 PSPICEMULTISIM The switch in the circuit in Fig. P8.29 has been open a long time before closing at t=0. At the time the switch closes, the capacitor has no stored energy. Find vo for t≥0.

Figure P8.29

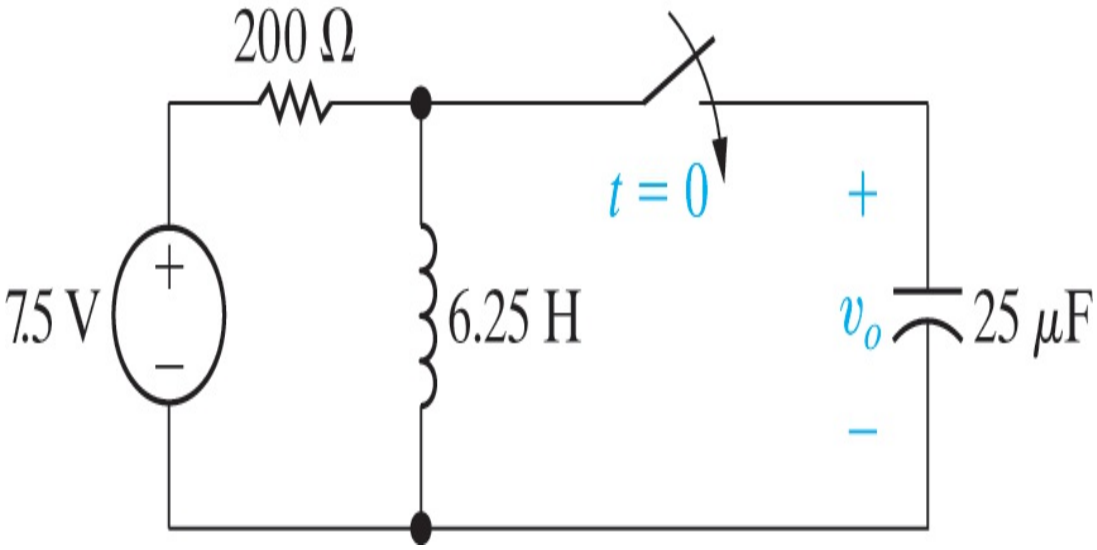


Figure P8.29 Full Alternative Text

7. 8.30 PSPICEMULTISIM There is no energy stored in the circuit in Fig. P8.30 when the switch is closed at t=0. Find io(t) for t≥0.

Figure P8.30

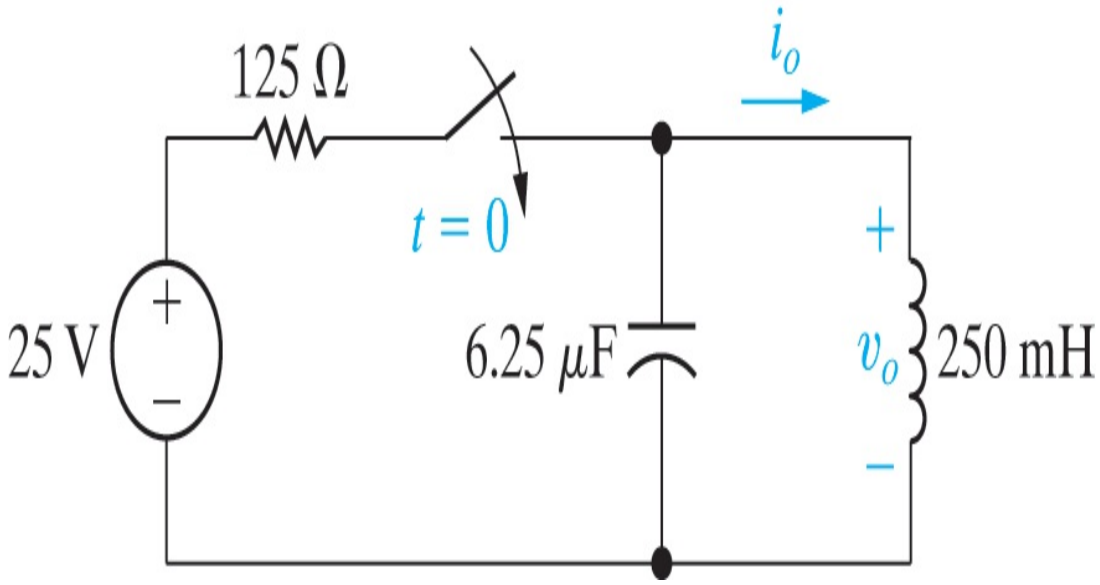


Figure P8.30 Full Alternative Text

8. 8.31 PSPICEMULTISIM

    1. For the circuit in Fig. P8.30, find vo for t≥0.

    2. Show that your solution for vo is consistent with the solution for io in Problem 8.30.

9. 8.32 PSPICEMULTISIM Assume that at the instant the 2 A current source is applied to the circuit in Fig. P8.32, the initial current in the 25 mH inductor is 1 A, and the initial voltage on the capacitor is 50 V (positive at the upper terminal). Find the expression for iL(t) for t≥0 if R equals 12.5 Ω.

Figure P8.32

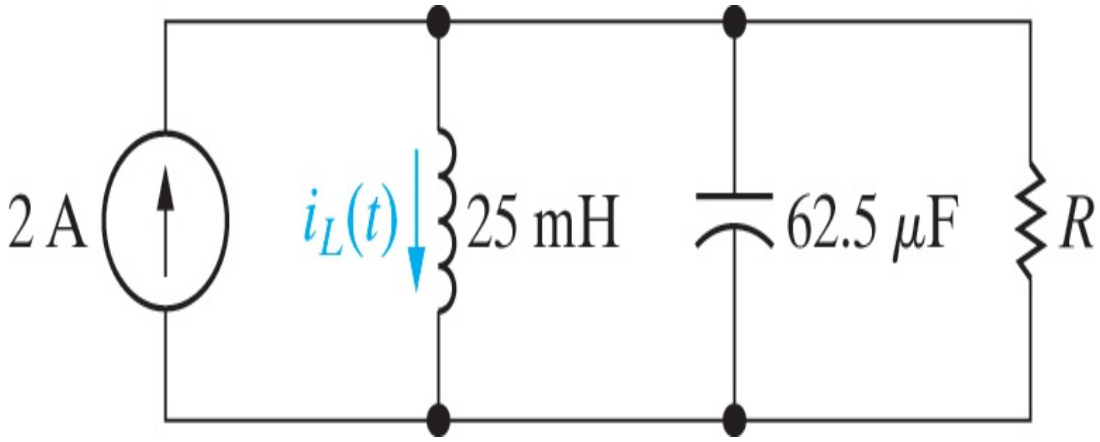


Figure P8.32 Full Alternative Text

10. 8.33 PSPICEMULTISIM The resistance in the circuit in Fig. P8.32 is changed to 8 Ω. Find iL(t) for t≥0.

11. 8.34 PSPICEMULTISIM The resistance in the circuit in Fig. P8.32 is changed to 10 Ω. Find iL(t) for t≥0.

12. 8.35 PSPICEMULTISIM Switches 1 and 2 in the circuit in Fig. P8.35 are synchronized. When switch 1 is opened, switch 2 closes and vice versa. Switch 1 has been open a long time before closing at t=0. Find iL(t) for t≥0.

Figure P8.35

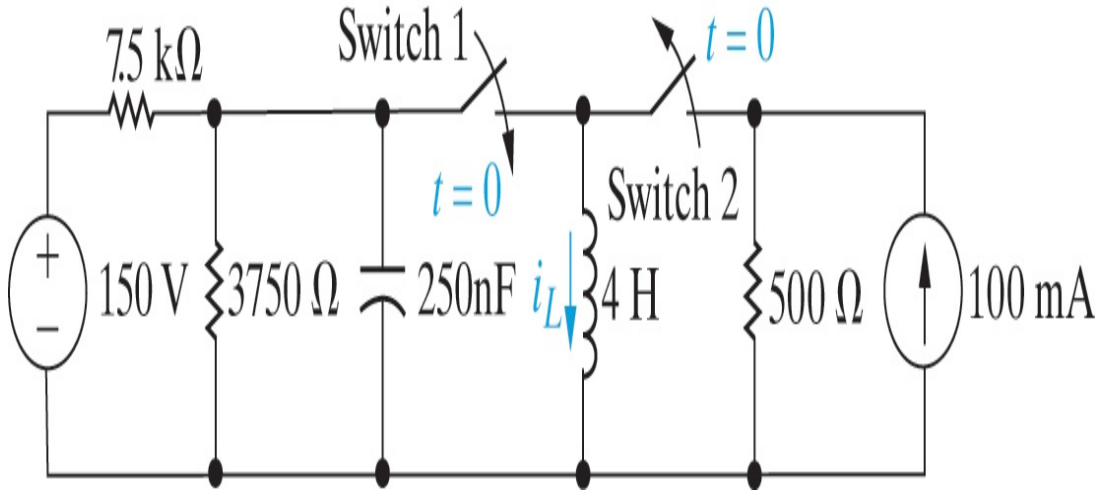


Figure P8.35 Full Alternative Text

13. 8.36 PSPICEMULTISIM The switch in the circuit in Fig. P8.36 has been in the left position for a long time before moving to the right position at t=0. Find

    1. iL(t) for t≥0,
    2. vC(t) for t≥0.

Figure P8.36

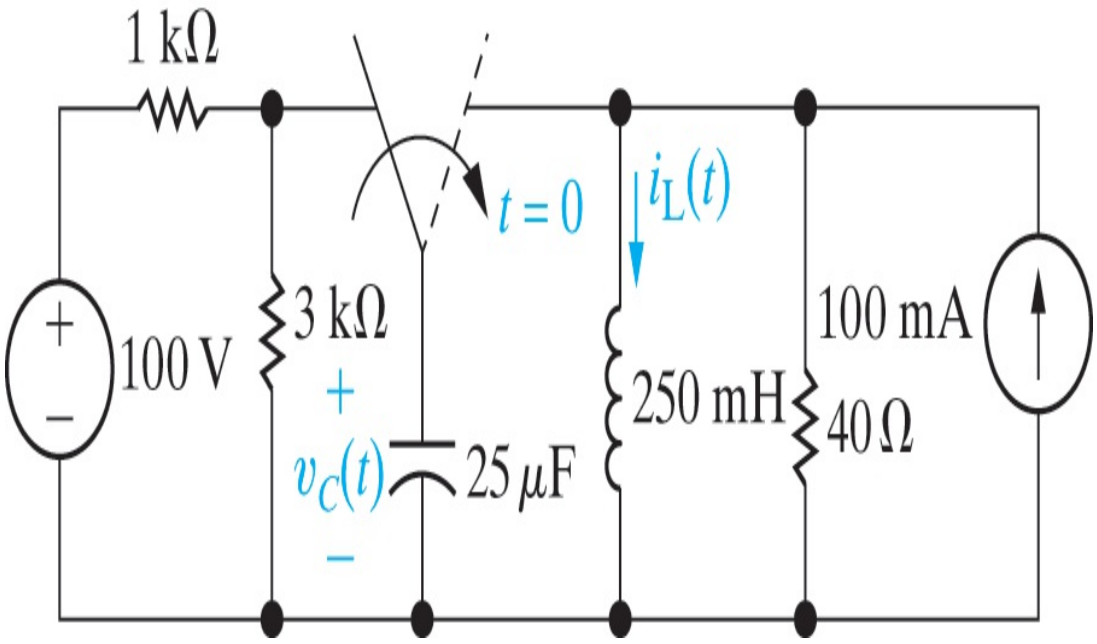


Figure P8.36 Full Alternative Text

14. 8.37 PSPICEMULTISIM Consider the circuit in Fig. P8.36.

    1. Find the total energy delivered to the inductor.
    2. Find the total energy delivered to the 40 Ω resistor.
    3. Find the total energy delivered to the capacitor.

4. Find the total energy delivered by the current source.

5. Check the results of parts (a) through (d) against the conservation of energy principle.

15. 8.38 PSPICEMULTISIM The switch in the circuit in Fig. 38 has been open a long time before closing at t=0. Find iL(t) for t≥0.

Figure P8.38

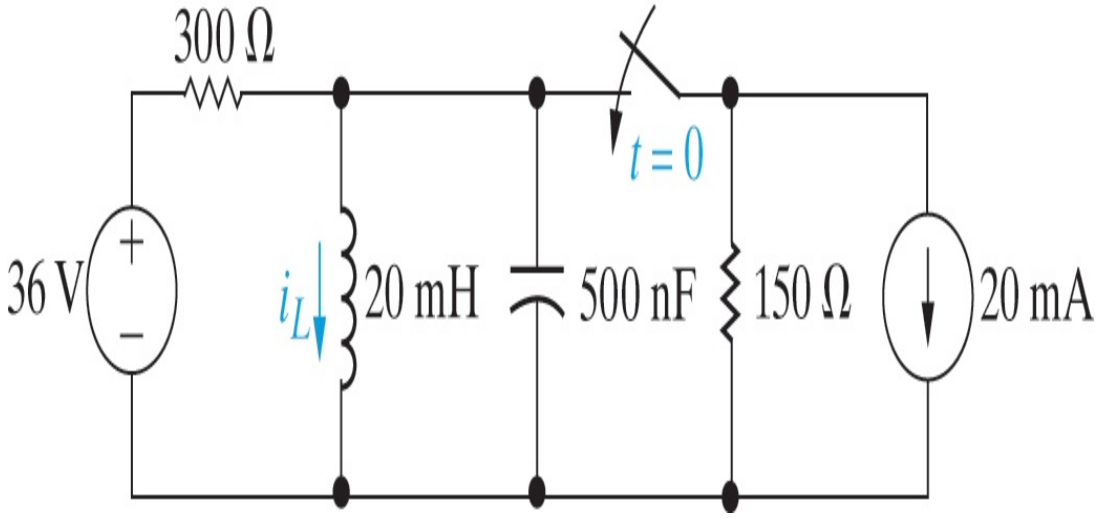


Figure P8.38 Full Alternative Text

Section 8.4

1. 8.39 PSPICEMULTISIM In the circuit in Fig. P8.39, the resistor is adjusted for critical damping. The initial capacitor voltage is 20 V, and the initial inductor current is 30 mA.

    1. Find the numerical value of R.

    2. Find the numerical values of i and di/dt immediately after the switch is closed.

    3. Find vC(t) for t≥0.

Figure P8.39

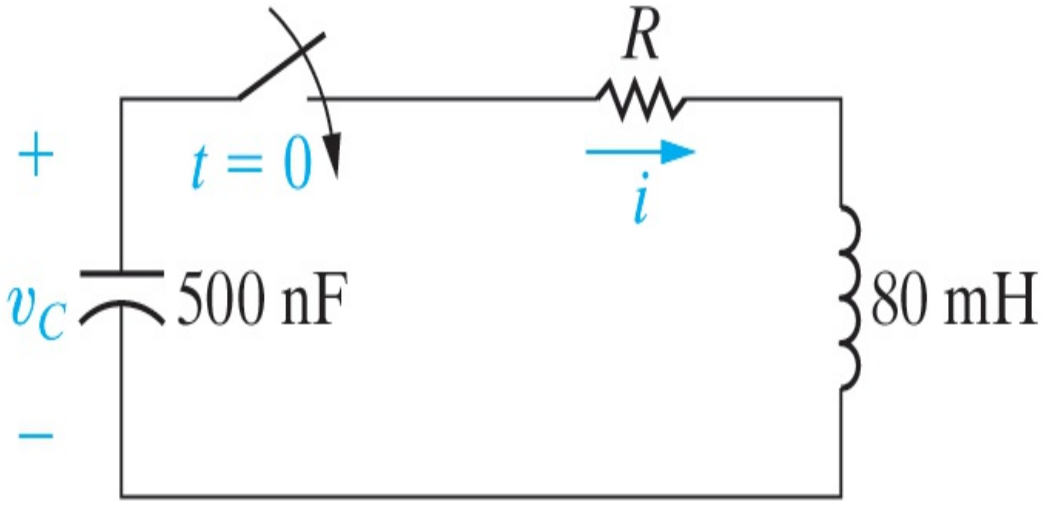


Figure P8.39 Full Alternative Text

2. 8.40 The initial energy stored in the 50 nF capacitor in the circuit in Fig. P8.40 is 90 μJ. The initial energy stored in the inductor is zero. The roots of the characteristic equation that describes the natural behavior of the current i are −1000 s−1 and −4,000 s−1

1. Find the numerical values of R and L.
2. Find the numerical values of i(0) and di(0)/dt immediately after the switch has been closed.
3. Find i(t) for t≥0.
4. How many microseconds after the switch closes does the current reach its maximum value?
5. What is the maximum value of i in milliamperes?
6. Find vL(t) for t≥0.

Figure P8.40

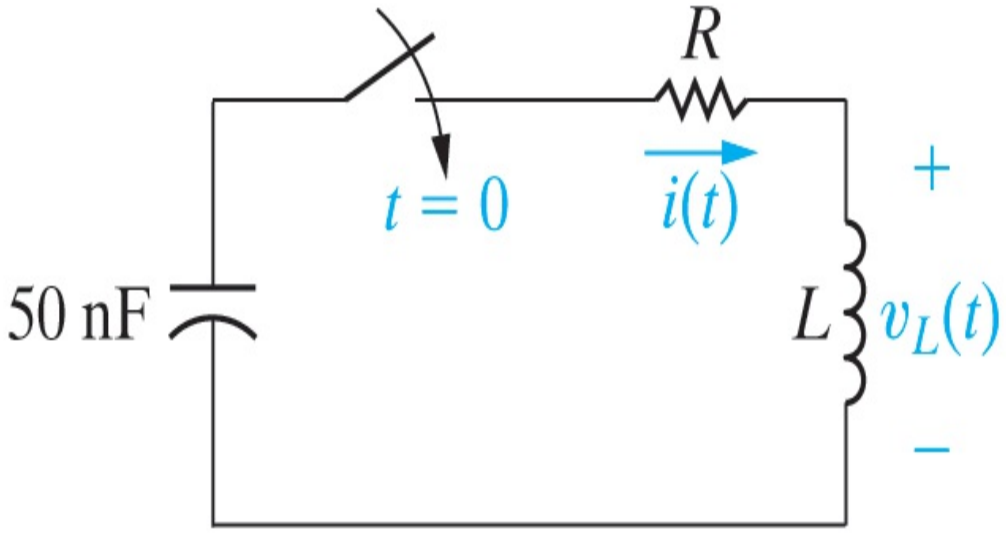


Figure P8.40 Full Alternative Text

3. 8.41 The current in the circuit in Fig. 8.3 is known to be

   $i=B_1e^{-2000t}\cos 1500t+B_2e^{-2000t}\sin 1500t, \quad t\geq 0.$

   The capacitor has a value of 80 nF; the initial value of the current is 7.5

mA; and the initial voltage on the capacitor is –30 V. Find the values of R, L, B1, and B2.

4. 8.42 Find the voltage across the 80 nF capacitor for the circuit described in Problem 8.41. Assume the reference polarity for the capacitor voltage is positive at the upper terminal.

5. 8.43

    1. Design a series RLC circuit (see Fig. 8.3) using component values from Appendix H, with a resonant radian frequency of 20 krad/s. Choose a resistor or create a resistor network so that the response is critically damped. Draw your circuit.

    2. Calculate the roots of the characteristic equation for the resistance in part (a).

6. 8.44

    1. Change the resistance for the circuit you designed in Problem 8.43(a) so that the response is underdamped. Continue to use components from Appendix H. Calculate the roots of the characteristic equation for this new resistance.

    2. Change the resistance for the circuit you designed in Problem 8.43(a) so that the response is overdamped. Continue to use components from Appendix H. Calculate the roots of the characteristic equation for this new resistance.

7. 8.45 The switch in the circuit shown in Fig. 45 has been in position a for a long time. At t=0, the switch is moved instantaneously to position b. Find i(t) for $t \geq 0$.

Figure P8.45

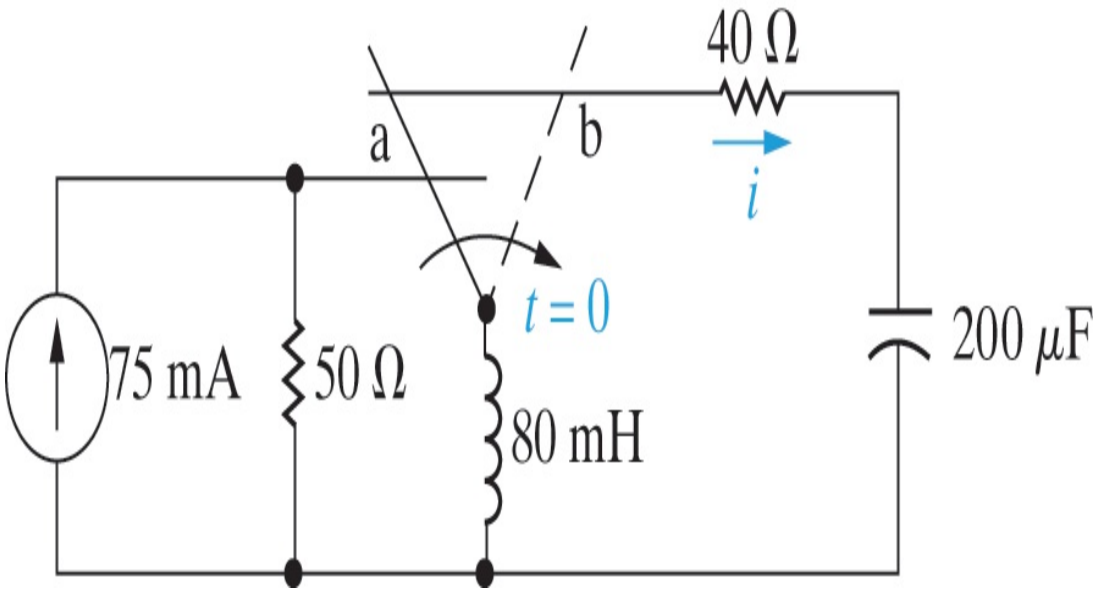


Figure P8.45 Full Alternative Text

8. 8.46 PSPICEMULTISIM The switch in the circuit in Fig. P8.46 has been in position a for a long time. At t=0, the switch moves instantaneously to position b.

    1. What is the initial value of va?
    2. What is the initial value of dva/dt?
    3. What is the numerical expression for va(t) for t≥0?

Figure P8.46

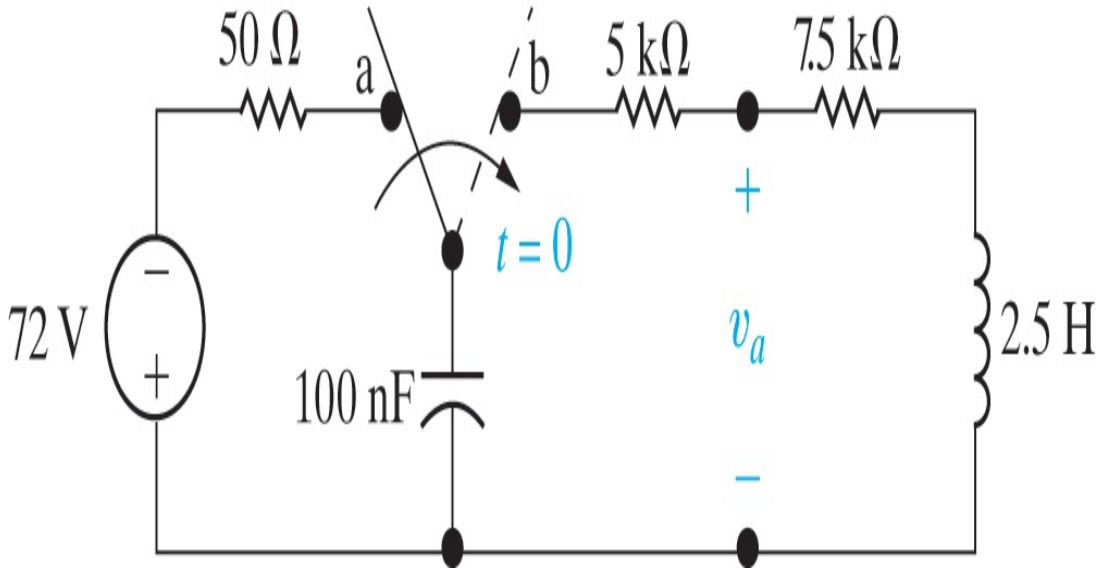


Figure P8.46 Full Alternative Text

9. 8.47 PSPICEMULTISIM The circuit shown in Fig. P8.47 has been in operation for a long time. At t=0, the two switches move to the new positions shown in the figure. Find

    1. io(t) for t≥0,

    2. vo(t) for t≥0.

Figure P8.47

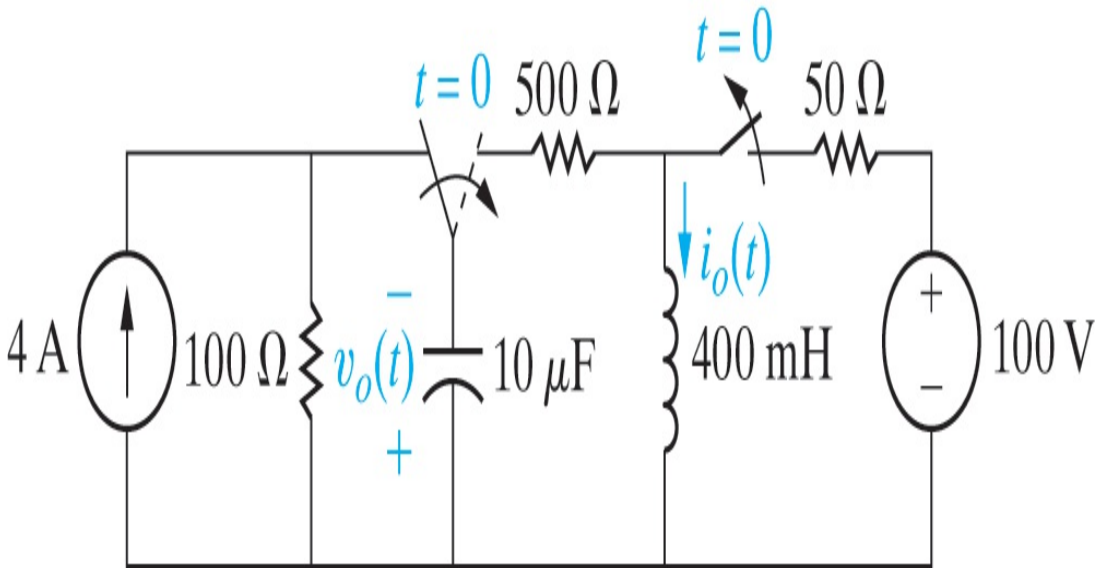


Figure P8.47 Full Alternative Text

10. 8.48 The switch in the circuit shown in Fig. P8.48 has been closed for a long time. The switch opens at t=0. Find vo(t) for t≥0+.

Figure P8.48

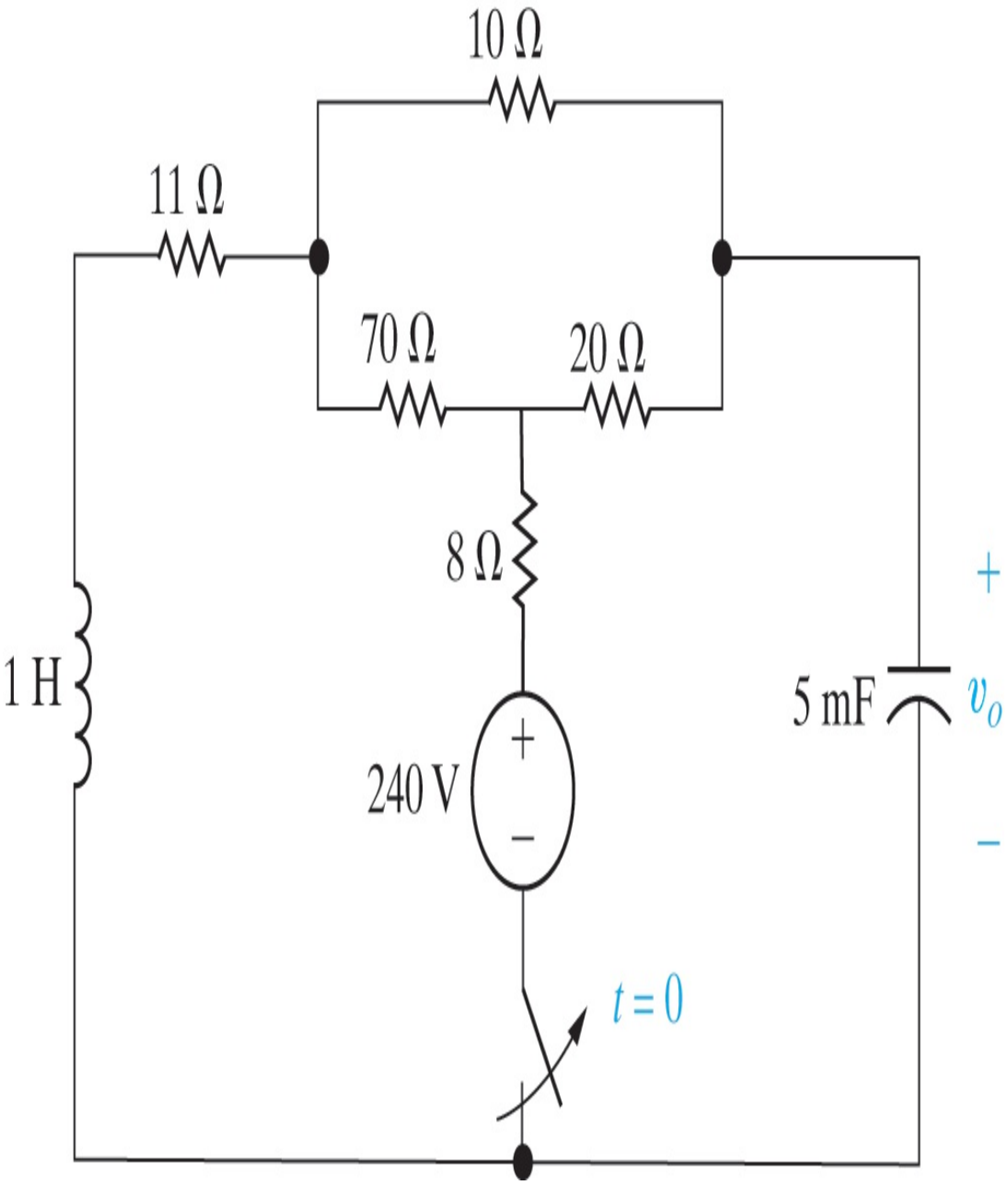


Figure P8.48 Full Alternative Text

11. 8.49 PSPICEMULTISIM The circuit shown in Fig. P8.49 has been in operation for a long time. At t=0, the source voltage suddenly increases to 250 V. Find vo(t) for t≥0.

Figure P8.49

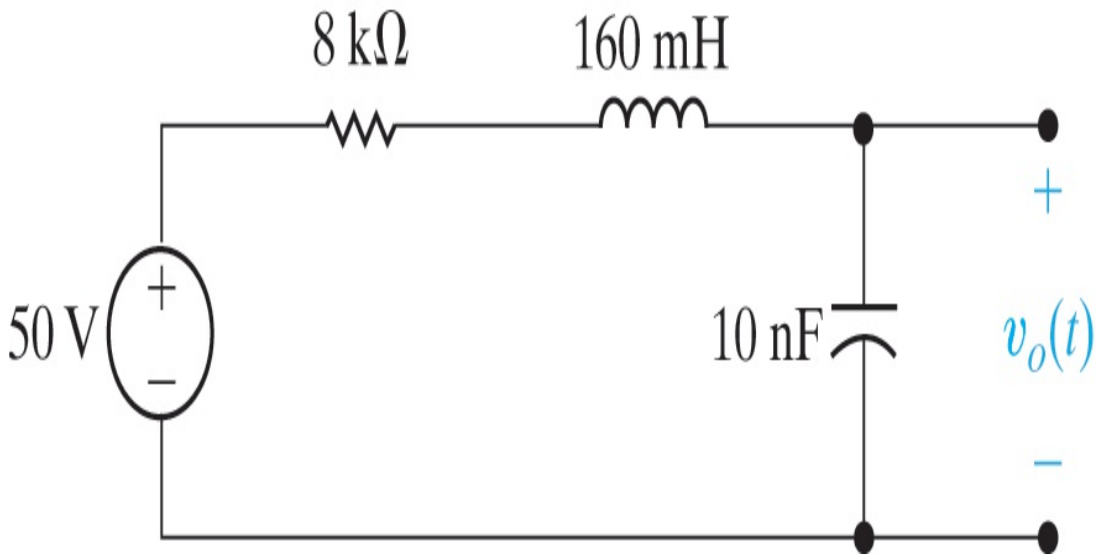


Figure P8.49 Full Alternative Text

12. 8.50 PSPICEMULTISIM The initial energy stored in the circuit in Fig. P8.50 is zero. Find vo(t) for t≥0.

Figure P8.50

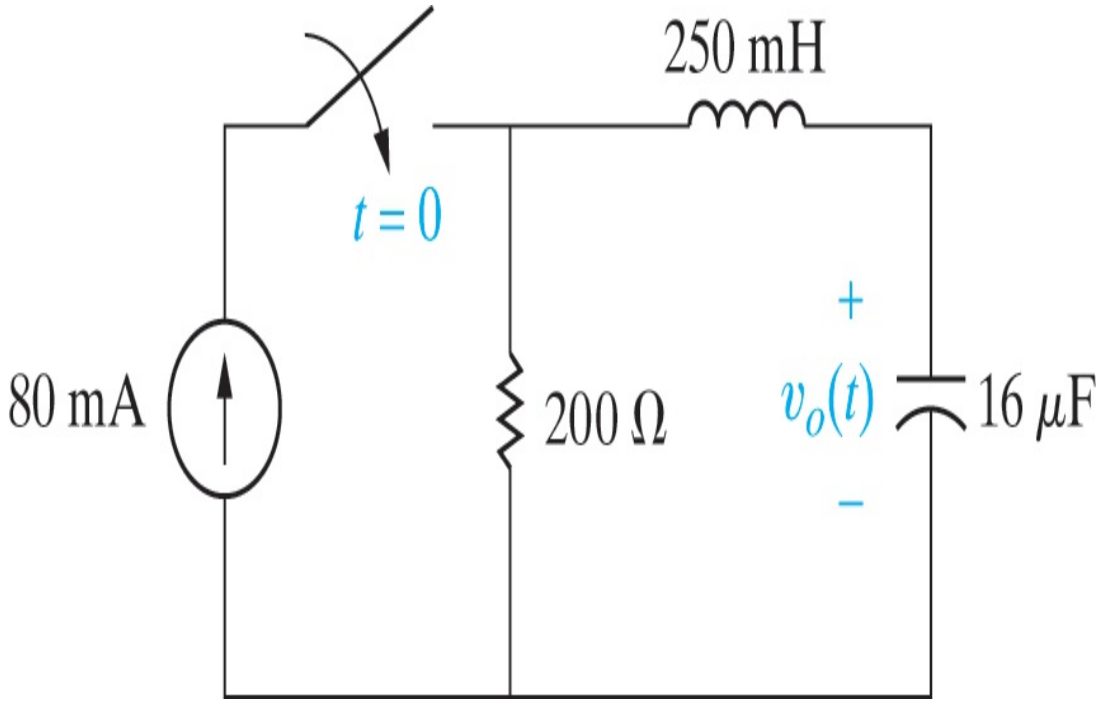


Figure P8.50 Full Alternative Text

13. 8.51 PSPICEMULTISIM The resistor in the circuit shown in Fig. P8.50 is changed to 250 Ω. The initial energy stored is still zero. Find vo(t) for t≥0.

14. 8.52 The resistor in the circuit shown in Fig. P8.50 is changed to 312.5 Ω. The initial energy stored is still zero. Find vo(t) for t≥0.

15. 8.53 PSPICEMULTISIM The two switches in the circuit seen in Fig. P8.53 operate synchronously. When switch 1 is in position a, switch 2 is closed. When switch 1 is in position b, switch 2 is open. Switch 1 has been in position a for a long time. At t=0, it moves instantaneously to position b. Find vC(t) for t≥0.

Figure P8.53

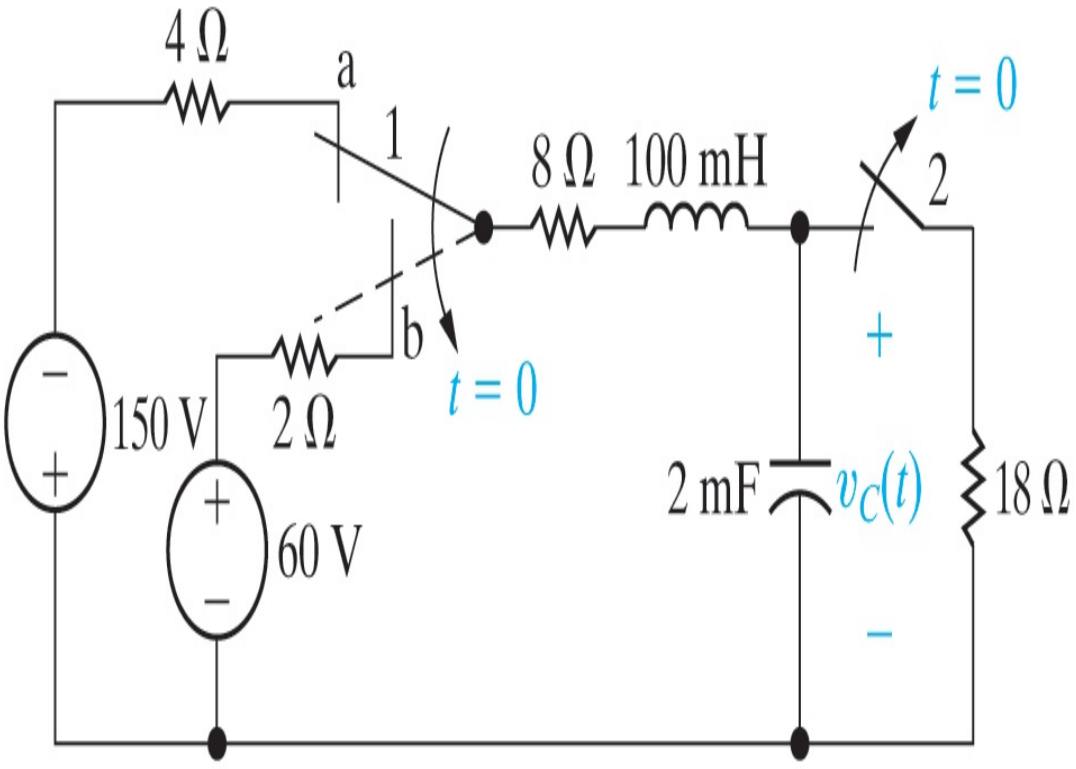


Figure P8.53 Full Alternative Text

16. 8.54 PSPICEMULTISIM The switch in the circuit of Fig. P8.54 has been in position a for a long time. At t=0 the switch moves instantaneously to position b. Find vo(t) for t≥0.

Figure P8.54

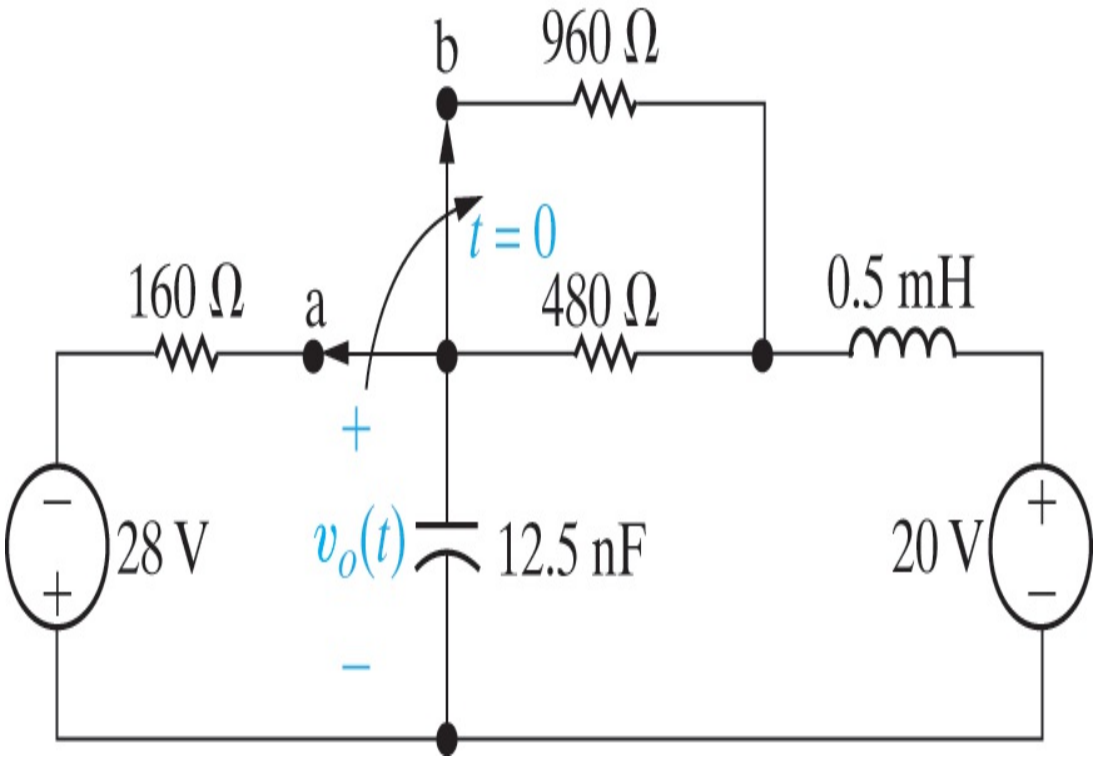


Figure P8.54 Full Alternative Text

17. 8.55 The switch in the circuit shown in Fig. P8.55 has been closed for a long time before it is opened at t=0. Assume that the circuit parameters are such that the response is underdamped.

    1. Derive the expression for vo(t) as a function of Vg, α, ωd, C, and R for t≥0.

    2. Derive the expression for the value of t when the magnitude of vo is maximum.

Figure P8.55

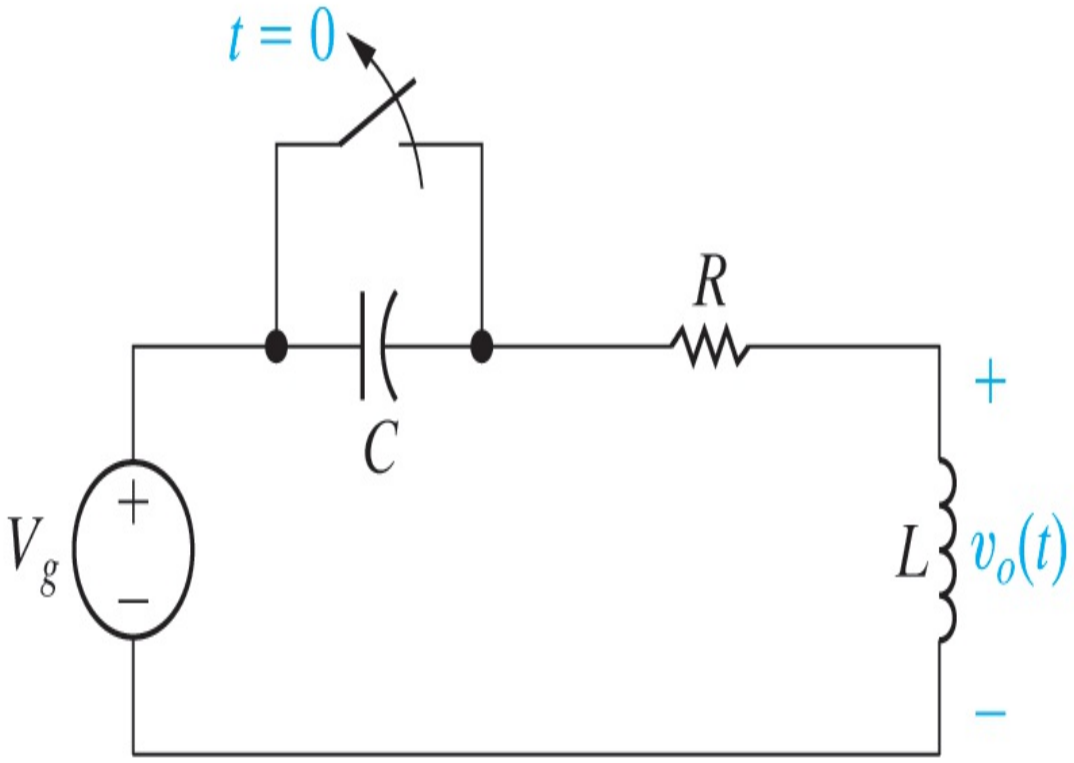


Figure P8.55 Full Alternative Text

18. 8.56 PSPICEMULTISIM The circuit parameters in the circuit of Fig. P8.55 are R=4800 Ω, L=64 mH, C=4 nF, and vg=−72 V.

    1. Express vo(t) numerically for t≥0.

    2. How many microseconds after the switch opens is the inductor voltage maximum?

    3. What is the maximum value of the inductor voltage?

    4. Repeat (a)–(c) with R reduced to 480 Ω.

19. 8.57 Assume that the capacitor voltage in the circuit of Fig. 8.16 is underdamped. Also assume that no energy is stored in the circuit

elements when the switch is closed.

1. Show that $dv_C/dt=(\omega_0^2/\omega_d)Ve^{-\alpha t}\sin\omega_d t$.
2. Show that $dv_C/dt=0$ when $t=n\pi/\omega_d$, where $n=0, 1, 2, \ldots$.
3. Let $t_n=n\pi/\omega_d$, and show that $v_C(t_n)=V-V(-1)^n e^{-\alpha n\pi/\omega_d}$.
4. Show that

   $$\alpha=\frac{1}{T_d}\ln\frac{v_C(t_1)-V}{v_C(t_3)-V},$$

   where $T_d=t_3-t_1$.

20. 8.58 The voltage across a 100 nF capacitor in the circuit of Fig. 8.16 is described as follows: After the switch has been closed for several seconds, the voltage is constant at 100 V. The first time the voltage exceeds 100 V, it reaches a peak of 163.84 V. This occurs $\pi/7$ ms after the switch has been closed. The second time the voltage exceeds 100 V, it reaches a peak of 126.02 V. This second peak occurs $3\pi/7$ after the switch has been closed. At the time when the switch is closed, there is no energy stored in either the capacitor or the inductor. Find the numerical values of R and L. (Hint: Work Problem 8.57 first.)

Section 8.5

1. 8.59 Show that, if no energy is stored in the circuit shown in Fig. 8.20 at the instant vg jumps in value, then dvo/dt equals zero at t=0.
2. 8.60 PSPICEMULTISIM
    1. Find the equation for vo(t) for $0 \leq t \leq t_{sat}$ in the circuit shown in Fig. 8.20 if vo1(0)=5 V and vo(0)=8 V.
    2. How long does the circuit take to reach saturation?
3. 8.61
    1. Rework Example 8.14 with feedback resistors R1 and R2 removed.
    2. Rework Example 8.14 with vo1(0)=−2 V and vo(0)=4 V.
4. 8.62
    1. Derive the differential equation that relates the output voltage to the input voltage for the circuit shown in Fig. P8.62.
    2. Compare the result with Eq. 8.39 when R1C1=R2C2=RC in Fig. 8.19.
    3. What is the advantage of the circuit shown in Fig. P8.62?

Figure P8.62

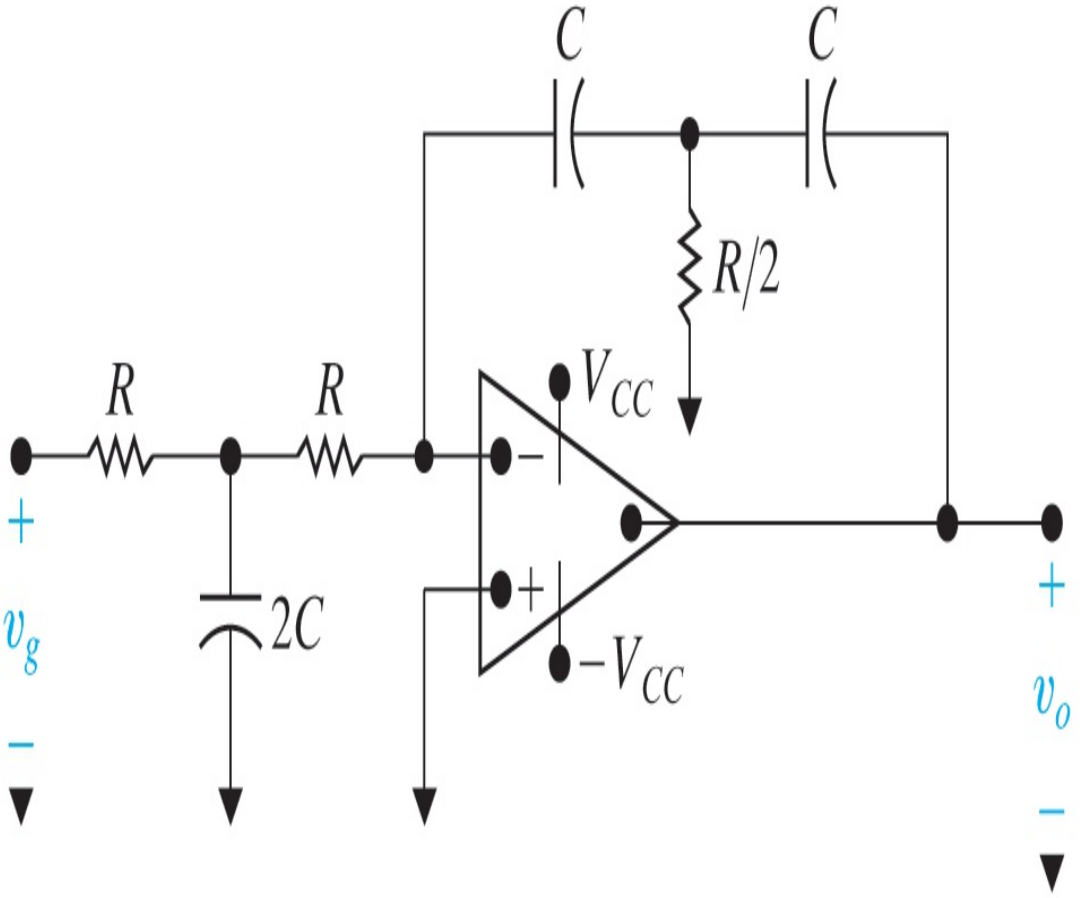


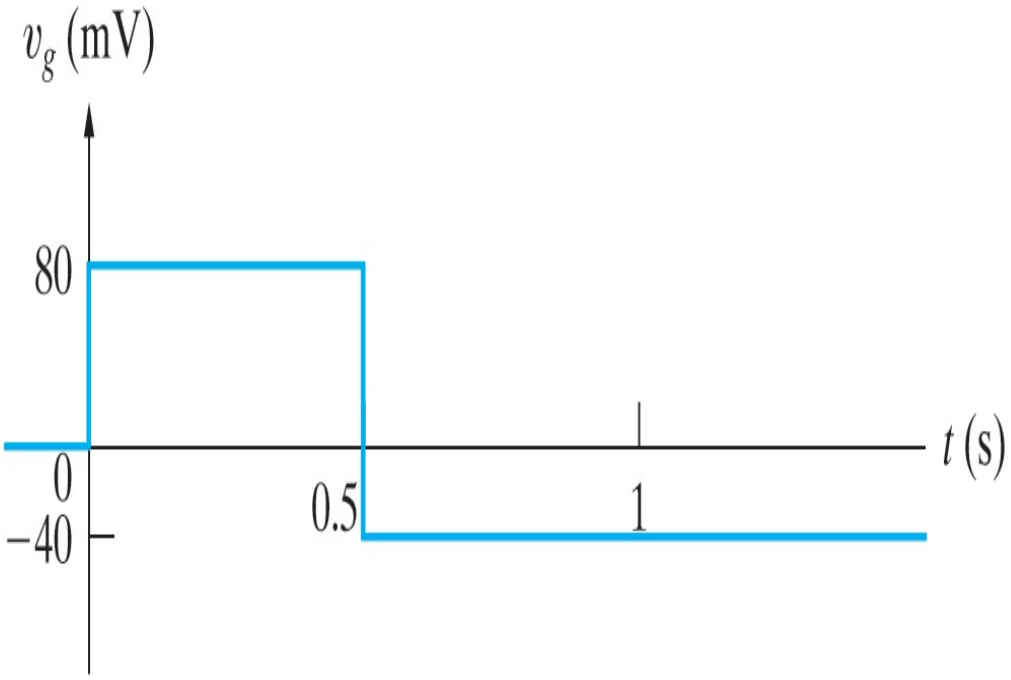
Figure P8.62 Full Alternative Text

5. 8.63 PSPICEMULTISIM The voltage signal of Fig. P8.63(a) is applied to the cascaded integrating amplifiers shown in Fig. P8.63(b). There is no energy stored in the capacitors at the instant the signal is applied.

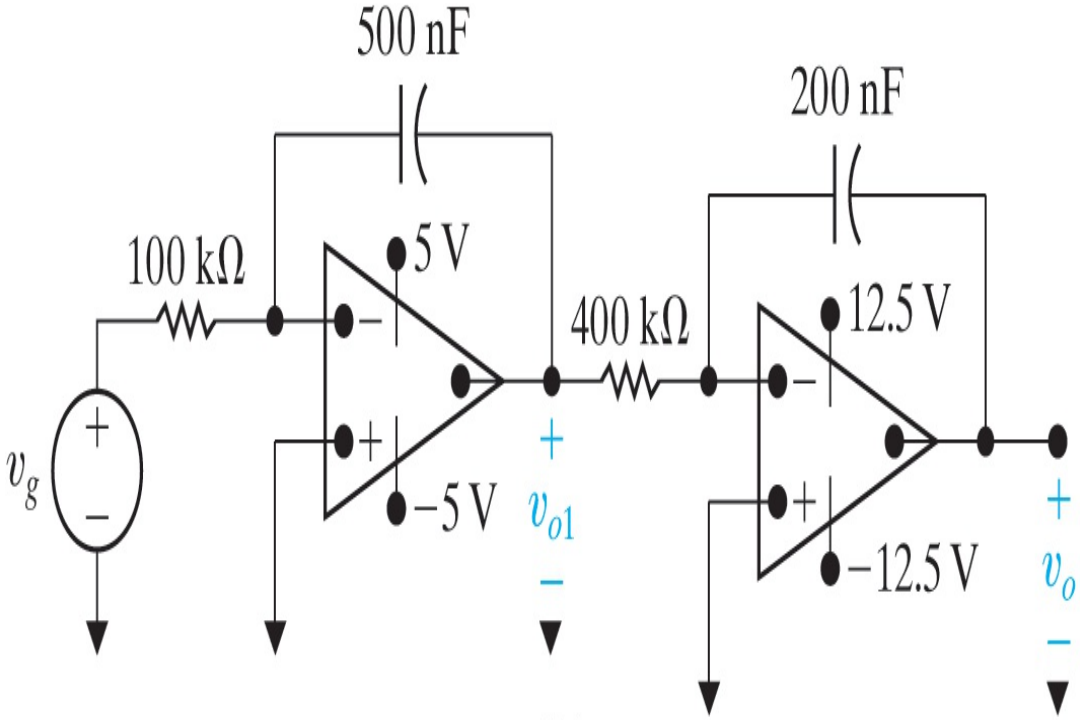
    1. Derive the numerical expressions for vo(t) and vo1(t) for the time intervals 0≤t≤0.5 s and 0.5 s≤t≤tsat.

    2. Compute the value of tsat.

Figure P8.63


v_g (mV)
80
0
−40
0.5
1
t (s)

(a)


500 nF
200 nF
100 kΩ
5 V
400 kΩ
12.5 V
v_g
+
−
−5 V
v_{o1}
−12.5 V
v_o

(b)

Figure P8.63 Full Alternative Text

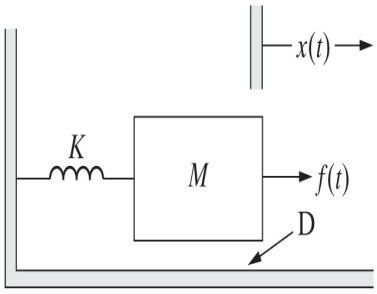
6. 8.64 PSPICEMULTISIM The circuit in Fig. P8.63(b) is modified by adding a 1 MΩ resistor in parallel with the 500 nF capacitor and a 5 MΩ resistor in parallel with the 200 nF capacitor. As in Problem 8.63, there is no energy stored in the capacitors at the time the signal is applied. Derive the numerical expressions for vo(t) and vo1(t) for the time intervals $0 \le t \le 0.5$ s and $t \ge 0.5$ s.

7. 8.65 We now wish to illustrate how several op amp circuits can be interconnected to solve a differential equation.

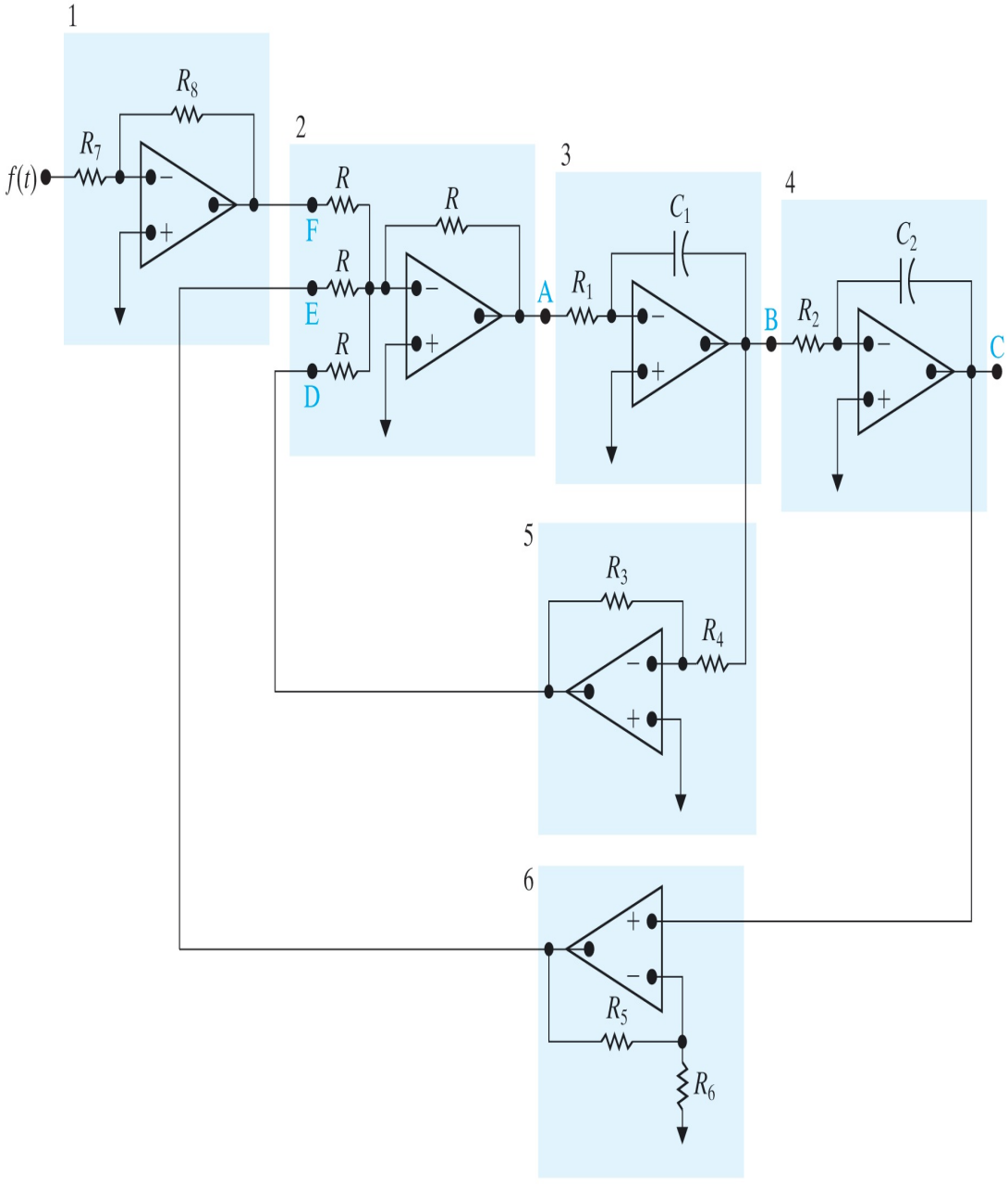
    1. Derive the differential equation for the spring-mass system shown in Fig. P8.65(a). Assume that the force exerted by the spring is directly proportional to the spring displacement, that the mass is constant, and that the frictional force is directly proportional to the velocity of the moving mass.

    2. Rewrite the differential equation derived in (a) so that the highest order derivative is expressed as a function of all the other terms in the equation. Now assume that a voltage equal to d2x/dt2 is available and by successive integrations generates dx/dt and x. We can synthesize the coefficients in the equations by scaling amplifiers, and we use a summing amplifier to combine the terms required to generate d2x/dt2. With these ideas in mind, analyze the interconnection shown in Fig. P8.65(b). In particular, describe the purpose of each shaded area in the circuit and describe the signal at the points labeled B, C, D, E, and F, assuming the signal at A represents d2x/dt2. Also discuss the parameters R; R1, C1; R2, C2; R3, R4; R5, R6; and R7, R8 in terms of the coefficients in the differential equation.

Figure P8.65



(a)



(b)

Figure P8.65 Full Alternative Text

Sections 8.18.5

1. 8.66 PRACTICALPERSPECTIVE_

    1. Suppose the circuit in Fig. 8.22 has a 5 nH inductor and a 2 pF capacitor. Calculate the frequency, in GHz, of the sinusoidal output for $t \geq 0$.

    2. The dc voltage source and series-connected resistor in Fig. 8.22 are used to establish the initial energy in the inductor. If V=10 V and R=25 Ω, calculate the initial energy stored in the inductor.

    3. What is the total energy stored in the LC circuit for any time $t \geq 0$?

2. 8.67 PRACTICALPERSPECTIVE_ Consider the LC oscillator circuit in Fig. 8.22. Assume that V=4 V, R=10 Ω, and L=1 nH.

    1. Calculate the value of capacitance, C, that will produce a sinusoidal output with a frequency of 2 GHz for $t \geq 0$.

    2. Write the expression for the output voltage, $v_o(t)$, for $t \geq 0$.

3. 8.68 PRACTICALPERSPECTIVE_ Suppose the inductor and capacitor in the LC oscillator circuit in Fig. 8.22 are not ideal, but instead have some small resistance that can be lumped together. Assume that V=10 V, R=25 Ω, L=5 nH, and C=2 pF, just as in Problem 8.66. Suppose the resistance associated with the inductor and capacitor is 10 mΩ.

    1. Calculate the values of the neper frequency, α, and the resonant radian frequency, ω_0.

    2. Is the response of this circuit overdamped, underdamped, or critically damped?

3. What is the actual frequency of oscillation, in GHz?
4. Approximately how long will the circuit oscillate?

Chapter 9 Sinusoidal Steady-State Analysis

Chapter Contents

1. 9.1 The Sinusoidal Source
2. 9.2 The Sinusoidal Response
3. 9.3 The Phasor
4. 9.4 The Passive Circuit Elements in the Frequency Domain
5. 9.5 Kirchhoff’s Laws in the Frequency Domain
6. 9.6 Series, Parallel, and Delta-to-Wye Simplifications
7. 9.7 Source Transformations and Thévenin–Norton Equivalent Circuits
8. 9.8 The Node-Voltage Method
9. 9.9 The Mesh-Current Method
10. 9.10 The Transformer
11. 9.11 The Ideal Transformer
12. 9.12 Phasor Diagrams

Chapter Objectives

1. Understand phasor concepts and be able to perform a phasor transform and an inverse phasor transform.

2. Be able to transform a circuit with a sinusoidal source into the frequency domain using phasor concepts.

3. Know how to use the following circuit analysis techniques to solve a circuit in the frequency domain:

    - Kirchhoff's laws;

    - Series, parallel, and delta-to-wye simplifications;

    - Voltage and current division;

    - Thévenin and Norton equivalents;

    - Node-voltage method; and

    - Mesh-current method.

4. Be able to analyze circuits containing linear transformers using phasor methods.

5. Understand the ideal transformer constraints and be able to analyze circuits containing ideal transformers using phasor methods.

Thus far, we have focused on circuits with constant sources; in this chapter we are now ready to consider circuits energized by sinusoidal voltage or current sources. For these circuits, we will calculate the values of the specified output voltages and currents in the steady state. This means we will not know the complete response of the circuits, which in general is the sum of the transient (or natural) response and the steady-state response. Our analysis will only characterize a circuit's response once the transient component has

decayed to zero.

Sinusoidal sources and their effect on circuit behavior form an important area of study for several reasons.

- Generating, transmitting, distributing, and consuming electric energy occurs under essentially sinusoidal steady-state conditions.
- Understanding sinusoidal behavior makes it possible to predict the behavior of circuits with nonsinusoidal sources.
- Specifying the behavior of an electrical system in terms of its steady-state sinusoidal response simplifies the design. If the system satisfies the specifications, the designer knows that the circuit will respond satisfactorily to nonsinusoidal inputs.

The remaining chapters of this book are largely based on the techniques used when analyzing circuits with sinusoidal sources. Fortunately, the circuit analysis and simplification techniques from Chapters 1–4 work for circuits with sinusoidal as well as dc sources, so some of the material in this chapter will be very familiar to you. The challenges of sinusoidal analysis include developing the appropriate component models, writing the equations that describe the resulting circuit, and working with complex numbers.

Practical Perspective

A Household Distribution Circuit

Power systems that generate, transmit, and distribute electrical power are designed to operate in the sinusoidal steady state. The standard household distribution circuit used in the United States supplies both 120 V and 240 V.

Consider the following situation. At the end of a day of fieldwork, a farmer returns to his farmstead, checks his hog confinement building, and finds to his dismay that the hogs are dead. The problem is traced to a blown fuse that

caused a 240 V fan motor to stop. The loss of ventilation led to the suffocation of the livestock. The interrupted fuse is located in the main switch that connects the farmstead to the electrical service.

Before the insurance company settles the claim, it wants to know if the electric circuit supplying the farmstead functioned properly. The lawyers for the insurance company are puzzled because the farmer's wife, who was in the house on the day of the accident convalescing from minor surgery, was able to watch TV during the afternoon. Furthermore, when she went to the kitchen to start preparing the evening meal, the electric clock indicated the correct time. The lawyers have hired you to explain why the electric clock in the kitchen and the television set in the living room continued to operate after the fuse in the main switch blew.

We will explore this situation and answer the question after learning how to calculate the steady-state response of circuits with sinusoidal sources.

Yulia Grigoryeva/Shutterstock

Björn Erlandsson /123RF

H5 (sec 1)
208Y/120V 3PH 4W
WARNING
DANGER
CLASS CTL PANELBOARD
CIRCUIT

Tetra Images/Alamy Stock Photo

9.1 The Sinusoidal Source

A **sinusoidal voltage source** (independent or dependent) produces a voltage that varies sinusoidally with time. **A sinusoidal current source** (independent or dependent) produces a current that varies sinusoidally with time. We begin by reviewing the sinusoidal function, using a voltage source as an example, but our observations also apply to current sources.

We can express a sinusoidally varying function with either the sine function or the cosine function. Although they work equally well, we cannot use both functional forms simultaneously. We will use the cosine function throughout our discussion. Hence, we write a sinusoidally varying voltage as

$v=V_m \cos(\omega t + \phi)$. (9.1)

To aid discussion of the parameters in Eq. 9.1, we show the voltage versus time plot in Fig. 9.1. The coefficient Vm gives the maximum **amplitude** of the sinusoidal voltage. Because ±1 bounds the cosine function, ±Vm bounds the amplitude, as seen in Fig. 9.1. You can also see that the sinusoidal function repeats at regular intervals; therefore, it is a periodic function. A periodic function is characterized by the time required for the function to pass through all its possible values. This time is the **period** of the function, T. and is measured in seconds. The reciprocal of T gives the number of cycles per second, or the frequency, of the periodic function, and is denoted f, so

f=1T. (9.2)

Figure 9.1 A sinusoidal voltage.

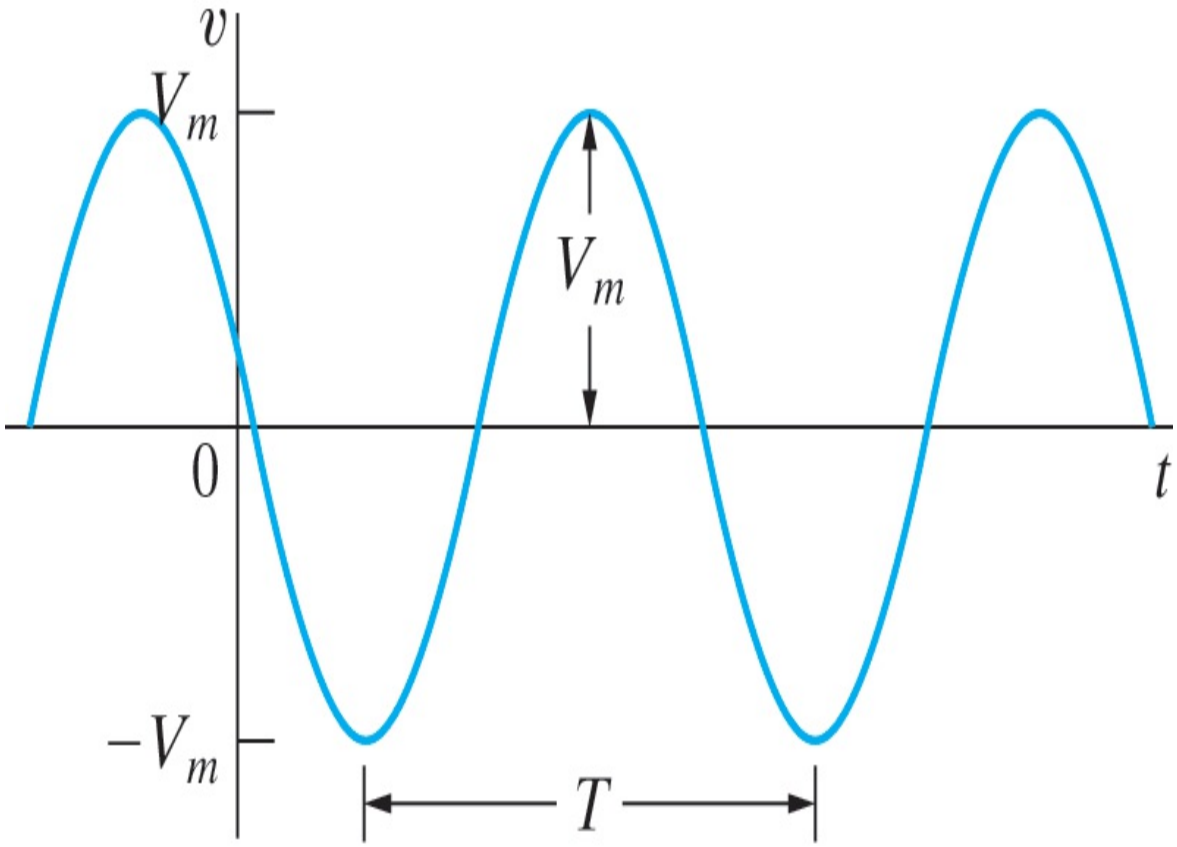


Figure 9.1 Full Alternative Text

A cycle per second is called a hertz, abbreviated Hz. (The term *cycles per second* rarely is used in contemporary technical literature.)

Now look at the coefficient of t in Eq. 9.1. Omega (ω) represents the **angular frequency** of the sinusoidal function and is related to both T and f:

$\omega = 2\pi f = 2\pi/T$ (radians/second). (9.3)

Equation 9.3 tells us that the cosine (or sine) function passes through a complete set of values each time its argument, ωt, passes through 2π rad (360°). From Eq. 9.3, we see that whenever t is an integral multiple of T, the argument ωt increases by an integral multiple of 2π rad.

The angle ϕ in Eq. 9.1 is the **phase angle** of the sinusoidal voltage. It determines the value of the sinusoidal function at t=0; therefore, it fixes the point on the periodic wave where we start measuring time. Changing the

phase angle ϕ shifts the sinusoidal function along the time axis but has no effect on either the amplitude (Vm) or the angular frequency (ω). Note, for example, that reducing ϕ to zero shifts the sinusoidal function shown in Fig. 9.1 ϕ/ω time units to the right, as shown in Fig. 9.2. When compared with a sinusoidal function with ϕ=0, a sinusoidal function with a positive ϕ is shifted to the left, while a sinusoidal function with a negative ϕ is shifted to the right. (See Problem 9.4.)

Figure 9.2 The sinusoidal voltage from Fig. 9.1 shifted to the right when ϕ=0.

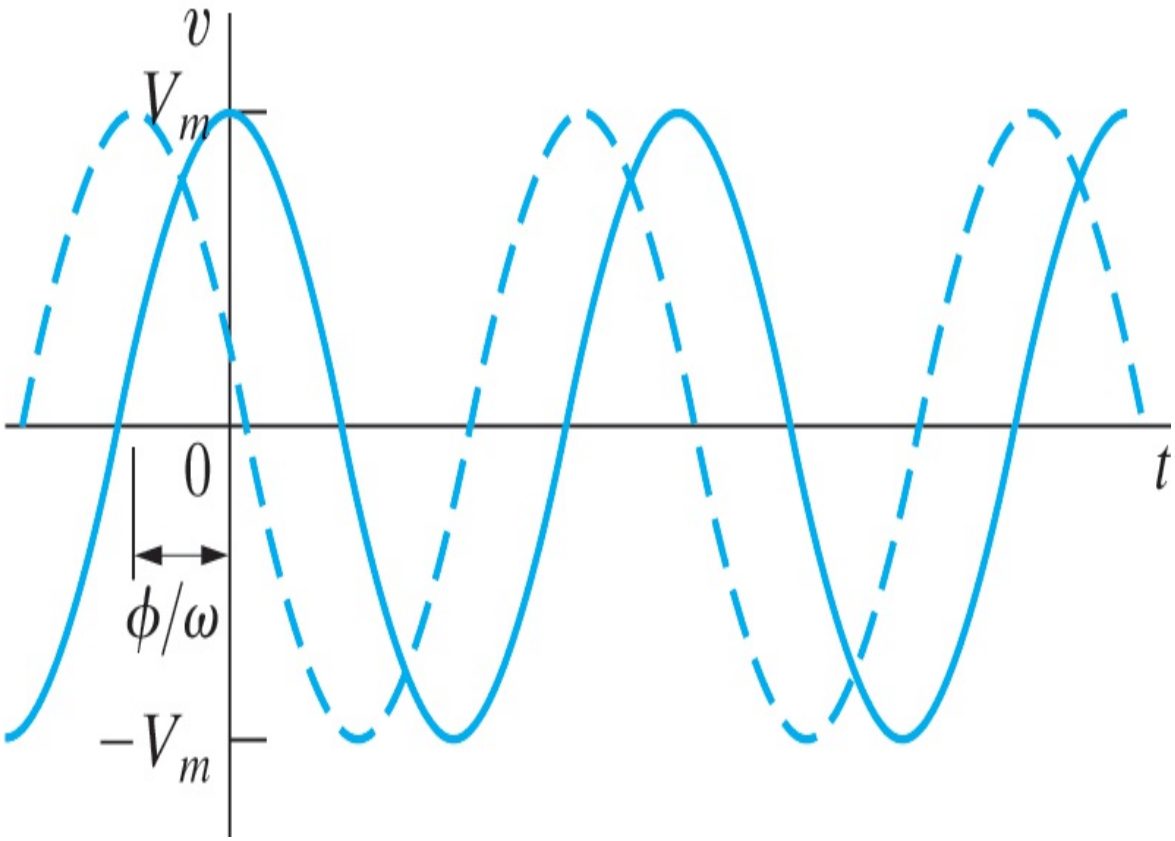


Figure 9.2 Full Alternative Text

Remember that ωt and ϕ must carry the same units because the argument of the sinusoidal function is (ωt+ϕ). With ωt expressed in radians, you would expect ϕ to also be in radians. However, ϕ normally is given in degrees, and ωt is converted from radians to degrees before the two quantities are added. The conversion from radians to degrees is given by

(number of degrees)=180° π (number of radians).

Another important characteristic of the sinusoidal voltage (or current) is its **rms value**. The rms value of a periodic function is defined as the square **r**oot of the **m**ean value of the **s**quared function. Hence, if v=Vmcos(ωt+ϕ), the rms value of v is

Vrms=1T∫t0t0+TVm2 cos2(ωt + ϕ) dt. (9.4)

Note from Eq. 9.4 that we obtain the mean value of the squared voltage by integrating v2 over one period (that is, from t0 to t0 + T) and then dividing by the range of integration, T. Note further that the starting point for the integration t0 is arbitrary.

The quantity under the square root sign in Eq. 9.4 reduces to Vm2/2. (See Problem 9.7.) Hence, the rms value of v is

RMS Value of a Sinusoidal Voltage Source

Vrms=Vm2. (9.5)

The rms value of the sinusoidal voltage depends only on the maximum amplitude of v, namely, Vm. The rms value is not a function of either the frequency or the phase angle. In Chapter 10, we explain the importance of the rms value and use it extensively to calculate power in circuits with sinusoidal sources.

We can completely describe a specific sinusoidal signal if we know its

frequency, phase angle, and amplitude. Examples 9.1, 9.2, and 9.3 illustrate these basic properties of the sinusoidal function. In Example 9.4, we calculate the rms value of a periodic function, and in so doing we clarify the meaning of *root mean square*.

Example 9.1 Finding the Characteristics of a Sinusoidal Current

A sinusoidal current has a maximum amplitude of 20 A. The current passes through one complete cycle in 1 ms. The magnitude of the current at t=0 is 10 A.

1. What is the frequency of the current in hertz?
2. What is the frequency in radians per second?
3. Write the expression for i(t) using the cosine function. Express ϕ in degrees.
4. What is the rms value of the current?

Solution

1. From the statement of the problem, T=1ms; hence, f=1/T=1/0.001=1000Hz.
2. $\omega=2\pi f=2\pi(1000)=2000\pi$ rad/sec.
3. We have

   $i(t) = I_m \cos(\omega t + \phi) = 20 \cos(2000\pi t + \phi)$A,

but i(0)=10A. Therefore, 10=20 cos ϕ, so ϕ=60°. Thus, the expression for i(t) becomes

i(t)=20cos(2000πt+60°)A.

4. From Eq. 9.5, the rms value of a sinusoidal current is Im/2. Therefore, the rms value is 20/2, or 14.14 A.

Example 9.2 Finding the Characteristics of a Sinusoidal Voltage

A sinusoidal voltage is given by the expression v=300 cos(120πt+30°) V.

1. What is the period of the voltage in milliseconds?
2. What is the frequency in hertz?
3. What is the magnitude of v at t=2.778 ms?
4. What is the rms value of v?

Solution

1. From the expression for v, ω=120π rad/s. Because ω=2π/T, T=2π/ω=160 s, or 16.667 ms.
2. The frequency is 1/T, or 60 Hz.
3. From (a), ω=2π/16.667; thus, at t=2.778 ms, ωt is nearly 1.047 rad, or 60°. Therefore, v(2.778 ms)=300 cos (60° + 30°)=0 V.
4. Vrms=300/2=212.13 V.

Example 9.3 Translating a Sine Expression to a Cosine Expression

We can translate the sine function to the cosine function by subtracting 90° (π/2 rad) from the argument of the sine function.

1. Verify this translation by showing that

   $\sin(\omega t + \theta) = \cos(\omega t + \theta - 90°)$.

2. Use the result in (a) to express $\sin(\omega t + 30°)$ as a cosine function.

Solution

1. Verification involves direct application of the trigonometric identity

   $\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$.

   We let $\alpha = \omega t + \theta$ and $\beta = 90°$. From the trigonometric identity,

   $\cos(\omega t + \theta - 90°) = \cos(\omega t + \theta)\cos(90°) + \sin(\omega t + \theta)\sin(90°)$.

   Since $\cos 90° = 0$ and $\sin 90° = 1$, we have

   $\cos(\omega t + \theta - 90°) = \sin(\omega t + \theta)$.

2. From (a) we have

   $\sin(\omega t + 30°) = \cos(\omega t + 30° - 90°) = \cos(\omega t - 60°)$.

Example 9.4 Calculating the rms Value of a Triangular Waveform

Calculate the rms value of the periodic triangular current shown in Fig. 9.3. Express your answer in terms of the peak current Ip.

Figure 9.3 Periodic triangular current.

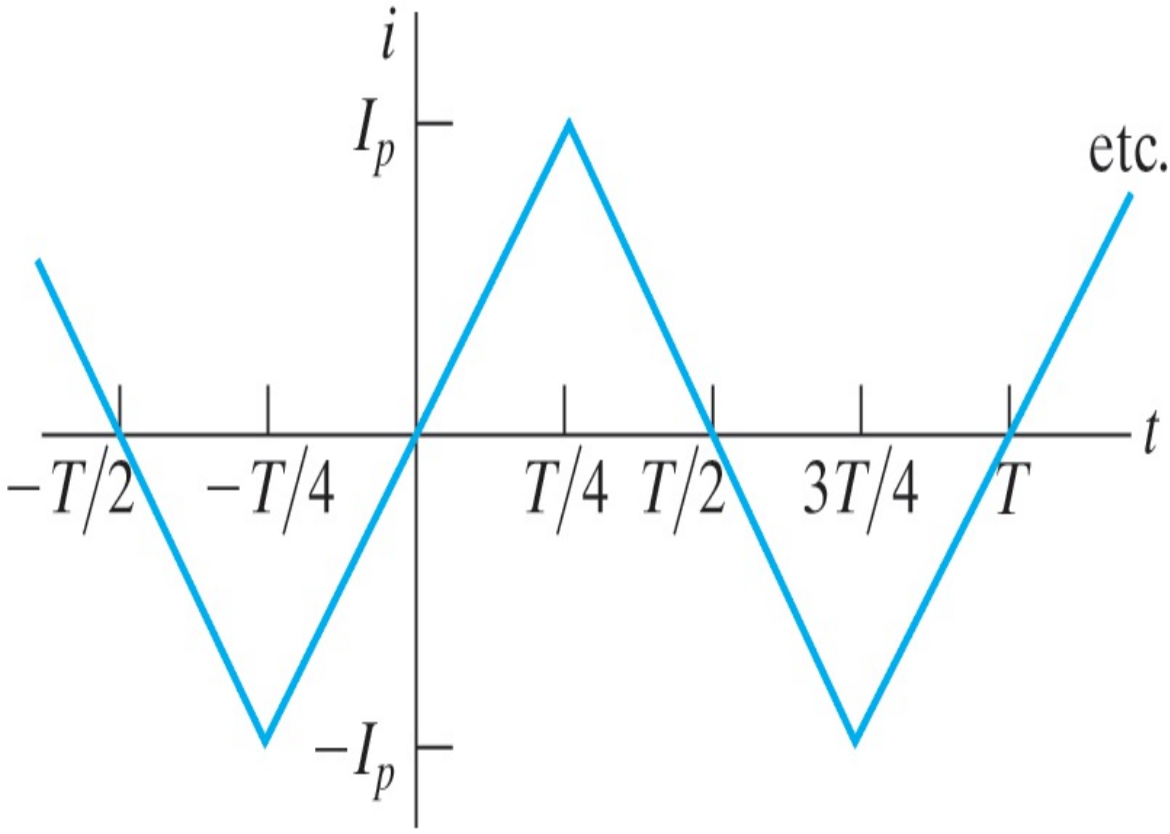


Figure 9.3 Full Alternative Text

Solution

From the definition of rms, the rms value of i is

Irms=1T∫t0t0+Ti2dt.

Interpreting the integral under the square root sign as the area under the squared function for an interval of one period helps us find the rms value. The squared function, with the area between 0 and T shaded, is shown in Fig. 9.4. Notice that the area under the squared current for an interval of one period is equal to four times the area under the squared current for the interval 0 to T/4 seconds; that is,

Figure 9.4 i2 versus t.

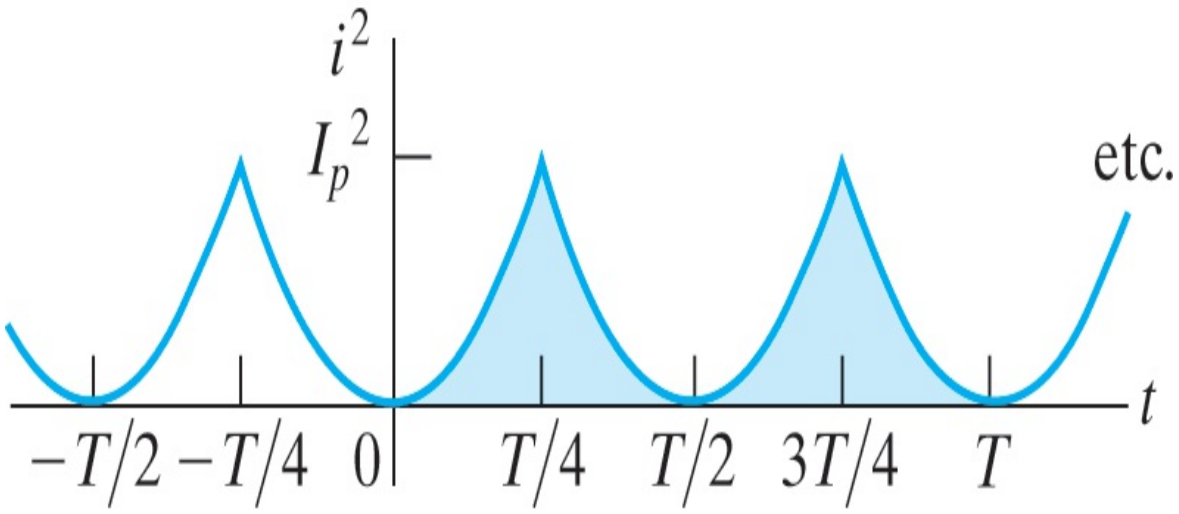


Figure 9.4 Full Alternative Text
∫t0t0+Ti2dt=4∫0T/4i2dt.

The analytical expression for i in the interval 0 to T/4 is

i=4IpTt, 0<t<T/4.

The area under the squared function for one period is

∫t0t0+Ti2dt=4∫0T/416Ip2T2t2dt=Ip2T3.

The mean, or average, value of the function is simply the area for one period

divided by the period. Thus

imean=1T Ip2T3=13Ip2.

The rms value of the current is the square root of this mean value. Hence

Irms=Ip3.

SELF-CHECK: Assess your understanding of this material by trying *Chapter Problems 9.1, 9.2, and 9.8.*

9.2 The Sinusoidal Response

As stated in the Introduction, this chapter focuses on the steady-state response to sinusoidal sources. But we begin by characterizing the total response, which will help you keep the steady-state solution in perspective.

The circuit shown in Fig. 9.5 describes the general problem, where vs is a sinusoidal voltage described by

$v_s = V_m \cos(\omega t + \phi)$.

Figure 9.5 An RL circuit excited by a sinusoidal voltage source.

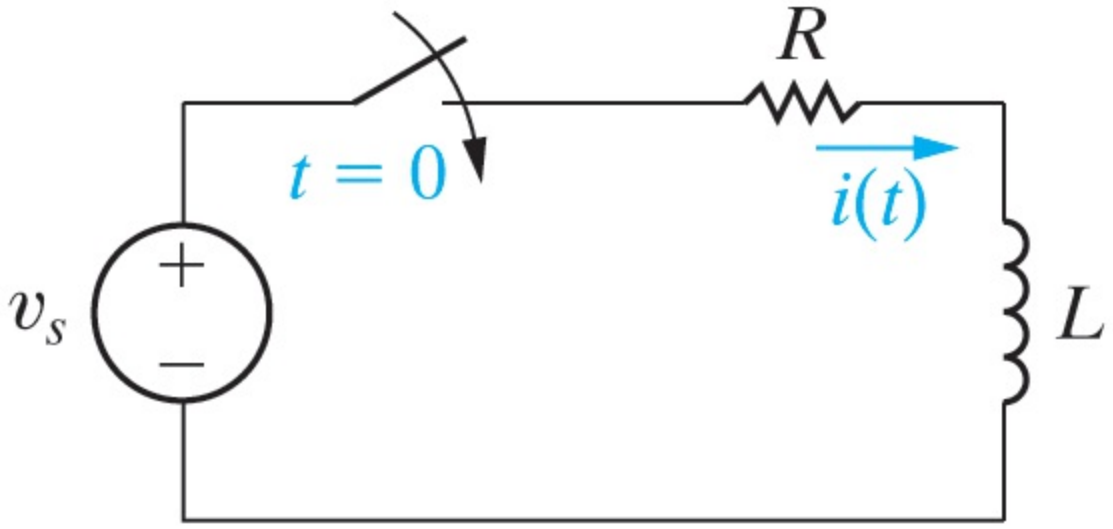


Figure 9.5 Full Alternative Text

For convenience, we assume the circuit's initial current is zero, and we measure time from the moment the switch is closed. We want to find i(t) for

$t \geq 0$, using a method similar to the one used when finding the step response of an RL circuit (Chapter 7). But here, the voltage source is time-varying sinusoidal voltage rather than a constant voltage. Applying KVL to the circuit in Fig. 9.5 gives us the ordinary differential equation

$$L\frac{di}{dt} + Ri = V_m \cos(\omega t + \phi). \quad (9.6)$$

The solution for Eq. 9.6 is discussed in an introductory course in differential equations. We ask those of you who have not yet studied differential equations to accept that the solution for i is

$$i = -\frac{V_m}{R^2 + \omega^2 L^2}\cos(\phi - \theta)e^{-(R/L)t} + \frac{V_m}{R^2 + \omega^2 L^2}\cos(\omega t + \phi - \theta), \quad (9.7)$$

where

$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right).$$

Thus, we can easily determine θ for a circuit driven by a sinusoidal source of known frequency. We can check that Eq. 9.7 is valid by showing that it satisfies Eq. 9.6 for all values of $t \geq 0$; this exercise is left for your exploration in Problem 9.9.

Look carefully at the two terms on the right-hand side of Eq. 9.7. The first term is a decaying exponential function whose time constant is $\tau = L/R$. This term is the **transient component** of the current because it decays to zero as $t \to 0$. Remember from Chapter 7 that this transient component has less than 1% of its initial value when $t = 5\tau$.

The second term is a cosine whose frequency is ω, the same as the frequency of the voltage source. This is the **steady-state component** of the current because it persists as long as the switch remains closed and the source continues to supply the sinusoidal voltage. In this chapter, we find only the steady-state response of circuits with sinusoidal sources; that is, we find the response once its transient component has decayed to zero. We develop a technique for calculating the steady-state response directly, thus avoiding the problem of solving the differential equation. However, when we use this technique, we cannot find either the transient component or the total response.

Using the steady-state component of Eq. 9.7, we identify four important characteristics of the steady-state solution:

1. The steady-state solution is a cosine function, just like the circuit's source.

2. The frequency of the solution is identical to the frequency of the source. This condition is always true in a linear circuit when the circuit parameters, R, L, and C, are constant. (If frequencies in the solution are not present in the source, there is a nonlinear element in the circuit.)

3. The maximum amplitude of the steady-state response, in general, differs from the maximum amplitude of the source. For the circuit in Fig. 9.5, the maximum amplitude of the current is $V_m/R^2+\omega^2L^2$, while the maximum amplitude of the source is V_m.

4. The phase angle of the steady-state response, in general, differs from the phase angle of the source. For the circuit being discussed, the phase angle of the current is $\phi - \theta$, and that of the voltage source is ϕ.

These characteristics motivate the phasor method, which we introduce in Section 9.3. Note that finding only the steady-state response means finding only its maximum amplitude and phase angle. The waveform and frequency of the steady-state response are already known because they are the same as the circuit's source.

SELF-CHECK: Assess your understanding of this material by trying *Chapter Problem 9.10*.

9.3 The Phasor

A **phasor** is a complex number that carries the amplitude and phase angle information of a sinusoidal function.1 The phasor concept is rooted in Euler's identity, which relates the exponential function to the trigonometric function:

1 You can review complex numbers by reading Appendix B.

1You can review complex numbers by reading Appendix B.

$$e^{\pm j\theta} = \cos\theta \pm j \sin\theta.$$

Euler's identity gives us another way of representing the cosine and sine functions. We can think of the cosine function as the real part of the exponential function and the sine function as the imaginary part of the exponential function; that is,

$$\cos\theta = R\{e^{j\theta}\}$$

and

$$\sin\theta = I\{e^{j\theta}\},$$

where R means "the real part of" and I means "the imaginary part of."

Because we chose to use the cosine function to represent sinusoidal signals (see Section 9.1), we can apply Euler's identity directly. In particular, we write the sinusoidal voltage function given in Eq. 9.1 by replacing the cosine function with the real part of the complex exponential:

$$\begin{aligned} v &= V_m \cos(\omega t + \phi) \\ &= V_m R\{e^{j(\omega t + \phi)}\} \\ &= V_m R\{e^{j\omega t} e^{j\phi}\}. \end{aligned}$$

We can move the constant Vm inside the argument of the R function without altering the equation. We can also reverse the order of the two exponential

functions inside the argument and write the voltage as

$v=R\{V_m e^{j\phi} e^{j\omega t}\}$.

In this expression for the voltage, note that the quantity $V_m e^{j\phi}$ is a complex number that carries the amplitude and phase angle of the cosine function we started with (Eq. 9.1). We define this complex number as the **phasor representation**, or **phasor transform**, of the given sinusoidal function. Thus

Phasor Transform

$$V=V_m e^{j\phi}=P\{V_m \cos(\omega t+\phi)\}, \quad (9.8)$$

where the notation $P\{V_m \cos(\omega t+\phi)\}$ is read as "the phasor transform of $V_m \cos(\omega t+\phi)$." Thus, the phasor transform transfers the sinusoidal function from the time domain to the complex-number domain, which is also called the **frequency domain**, since the response depends, in general, on ω. As in Eq. 9.8, throughout this book we represent a phasor quantity by using a boldface capital letter.

Equation 9.8 is the polar form of a phasor, but we also can express a phasor in rectangular form. Thus, we rewrite Eq. 9.8 as

$$V=V_m \cos\phi + jV_m \sin\phi.$$

Both polar and rectangular forms are useful in circuit applications of the phasor concept.

We see from Eq. 9.8 that phasors always have the form $Ae^{j\phi}$, where A is the amplitude of the underlying voltage or current. It is common to abbreviate phasors using the angle notation $A\angle\phi^\circ$, where

$$A\angle\phi^\circ \equiv A e^{j\phi}.$$

We use this angle notation extensively in the material that follows.

Inverse Phasor Transform

Using Eq. 9.8, we can transform a sinusoidal function to a phasor. We can also reverse the process; that is, we can transform a phasor back to the original sinusoidal function. If $V=100 - 26^\circ$ V, the expression for v is 100 cos (ωt−26°) V because we have decided to use the cosine function for all sinusoids. Notice that the phasor cannot give us the value of ω because it carries only amplitude and phase information. When we transform a phasor to the corresponding time-domain expression, we use the *inverse phasor transform* function, as shown in the equation

Inverse Phasor Transform

$$P^{-1}\{V_m e^{j\theta}\}=R\{V_m e^{j\theta} e^{j\omega t}\}=V_m \cos(\omega t+\theta^\circ) \quad (9.9)$$

where the notation $P^{-1}\{V_m e^{j\phi}\}$ is read as "the inverse phasor transform of $V_m e^{j\phi}$." Using Eq. 9.9, we find the inverse phasor transform by multiplying the phasor by $e^{j\omega t}$ and extracting the real part of the product.

Before applying the phasor transform to circuit analysis, we use it to solve a problem with which you are already familiar: adding sinusoidal functions. Example 9.5 shows how the phasor transform greatly simplifies this type of problem.

Example 9.5 Adding Cosines Using Phasors

If $y_1=20 \cos(\omega t - 30^\circ)$ and $y_2=40 \cos(\omega t + 60^\circ)$, express $y=y_1 + y_2$ as a single sinusoidal function.

1. Solve by using trigonometric identities.

2. Solve by using the phasor concept.

Solution

1. First, we expand both y1 and y2, using the cosine of the sum of two angles, to get

   $y1=20 \cos \omega t \cos 30° + 20 \sin \omega t \sin 30°;$
   $y2=40 \cos \omega t \cos 60° - 40 \sin \omega t \sin 60°.$

   Adding y1 and y2, we obtain

   $y = (20 \cos 30° + 40 \cos 60°) \cos \omega t + (20 \sin 30° - 40 \sin 60°) \sin \omega t = 37.32 \cos \omega t - 24.64 \sin \omega t.$

   To combine these two terms, we treat the coefficients of the cosine and sine as sides of a right triangle (Fig. 9.6) and then multiply and divide the right-hand side by the hypotenuse. Our expression for y becomes

   $y = 44.72(37.32\ 44.72\ \cos \omega t - 24.64\ 44.72\ \sin \omega t) = 44.72(\cos 33.43° \cos \omega t - \sin 33.43° \sin \omega t).$

Figure 9.6 A right triangle used in the solution for y.

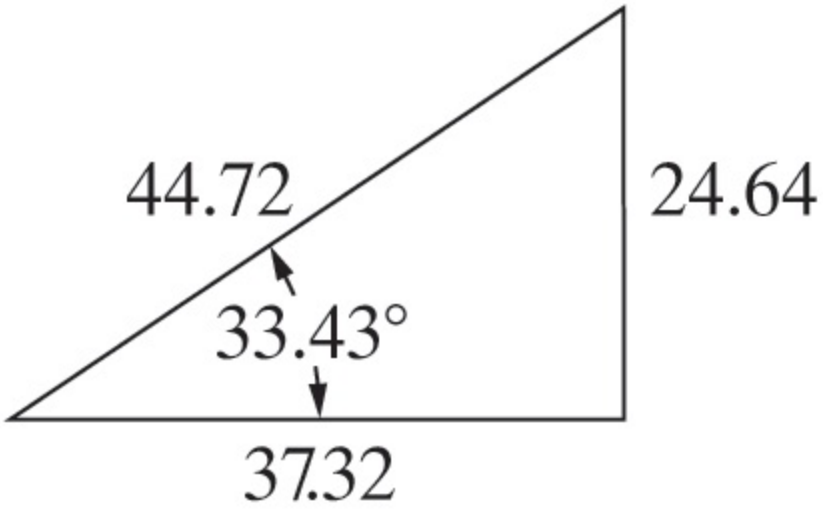


Figure 9.6 Full Alternative Text

Again, we invoke the identity involving the cosine of the sum of two angles and write

$y=44.72\cos(\omega t+33.43°)$.

2. The sum of the two cosines is

   $y=20\cos(\omega t-30°)+40\cos(\omega t+60°)$.

   Use Euler's identity to rewrite the righthand side of this equation as

   $y=R\{20e^{-j30°}e^{j\omega}\}+R\{40e^{j60°}e^{j\omega}\}$
   $=R\{20e^{-j30°}e^{j\omega}+40e^{j60°}e^{j\omega}\}$.

   Factoring out the term $e^{j\omega}$ from each term gives

   $y=R\{(20e^{-j30°}+40e^{j60°})e^{j\omega}\}$.

   We can calculate the sum of the two phasors using the angle notation:

   $20\angle -30° + 40\angle 60° = (17.32-j10)+(20+j34.64) = 37.32+j24.64 = 44.72\angle 33.43°$.

   Therefore,

$y = R\{ 44.72\, e^{j33.43°}\, e^{j\omega} \} = 44.72\cos(\omega t + 33.43°)$.

Adding sinusoidal functions using phasors is clearly easier than using trigonometric identities. Note that it requires the ability to move back and forth between the polar and rectangular forms of complex numbers.

Assessment Problems

Objective 1—Understand phasor concepts and be able to perform a phasor transform and an inverse phasor transform

1. 9.1 Find the phasor transform of each trigonometric function:
    1. $v=170\cos(377t - 40°)$ V.
    2. $i=10\sin(1000t + 20°)$ A.
    3. $i=[5\cos(\omega t + 36.87°) + 10\cos(\omega t - 53.13°)]$ A.
    4. $v=[300\cos(20{,}000\pi t + 45°) - 100\sin(20{,}000\pi t + 30°)]$ mV.

    Answer:

    1. $170\ \angle -40^\circ$ V;
    2. $10\ \angle -70°$ A;
    3. $11.18\ \angle -26.57°$ A;
    4. $339.90\ \angle 61.51°$ mV.
2. 9.2 Find the time-domain expression corresponding to each phasor:
    1. $V=18.6\ \angle -54°$ V.
    2. $I=(20\ \angle 45° - 50\ \angle -30°)$mA.
    3. $V=(20+j80-30\ \angle 15°)$V.

Answer:

1. $18.6 \cos(\omega t - 54°)$ V;
2. $48.81 \cos(\omega t + 126.68°)$ mA;
3. $72.79 \cos(\omega t + 97.08°)$ V.

SELF-CHECK: Also try *Chapter Problem 9.11*.

9.4 The Passive Circuit Elements in the Frequency Domain

Applying the phasor transform in circuit analysis is a two-step process.

1. Establish the relationship between the phasor current and the phasor voltage at the terminals of the passive circuit elements. We complete this step in this section, analyzing the resistor, inductor, and capacitor in the phasor domain.

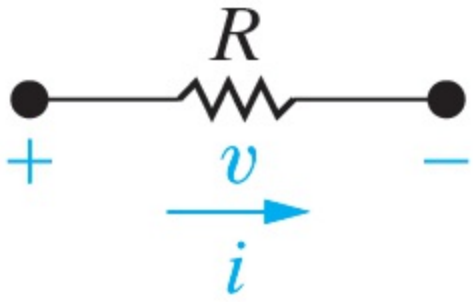
2. Develop the phasor-domain version of Kirchhoff's laws, which we discuss in Section 9.5.

The V-I Relationship for a Resistor

From Ohm's law, if the current in a resistor is $i=I_m\cos(\omega t + \theta_i)$, the voltage at the terminals of the resistor, as shown in Fig. 9.7, is

$$v = R[\, I_m \cos(\omega t + \theta_i)\,] = R\, I_m [\, \cos(\omega t + \theta_i)\,],$$

Figure 9.7 A resistive element carrying a sinusoidal current.



where Im is the maximum amplitude of the current in amperes and θi is the phase angle of the current.

The phasor transform of this voltage is

$$V = R\, I_m e^{j\theta_i} = R\, I_m \angle \theta_i .$$

But $I_m \angle \theta_i$ is the phasor representation of the sinusoidal current, so we can write the voltage phasor as

Relationship Between Phasor Voltage and Phasor Current for a Resistor

$$V = RI, \quad (9.10)$$

which states that the phasor voltage at the terminals of a resistor is the resistance times the phasor current—the phasor version of Ohm's law. Figure 9.8 shows the circuit diagram for a resistor in the frequency domain.

Figure 9.8 The frequency-domain equivalent circuit of a resistor.

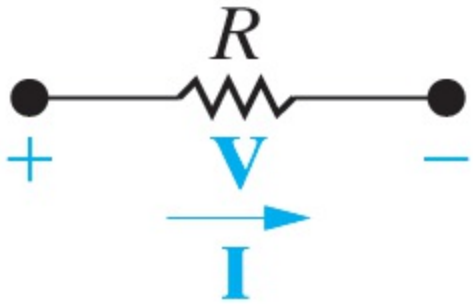


Figure 9.8 Full Alternative Text

Equation 9.10 contains an important piece of information—namely, that at the terminals of a resistor, there is no phase shift between the current and voltage. Figure 9.9 depicts this phase relationship, where the phase angle of both the voltage and the current waveforms is 60°. The signals are said to be **in phase** because they both reach corresponding values on their respective curves at the same time (for example, they are at their positive maxima at the same instant).

Figure 9.9 A plot showing that the voltage and current at the terminals of a resistor are in phase.

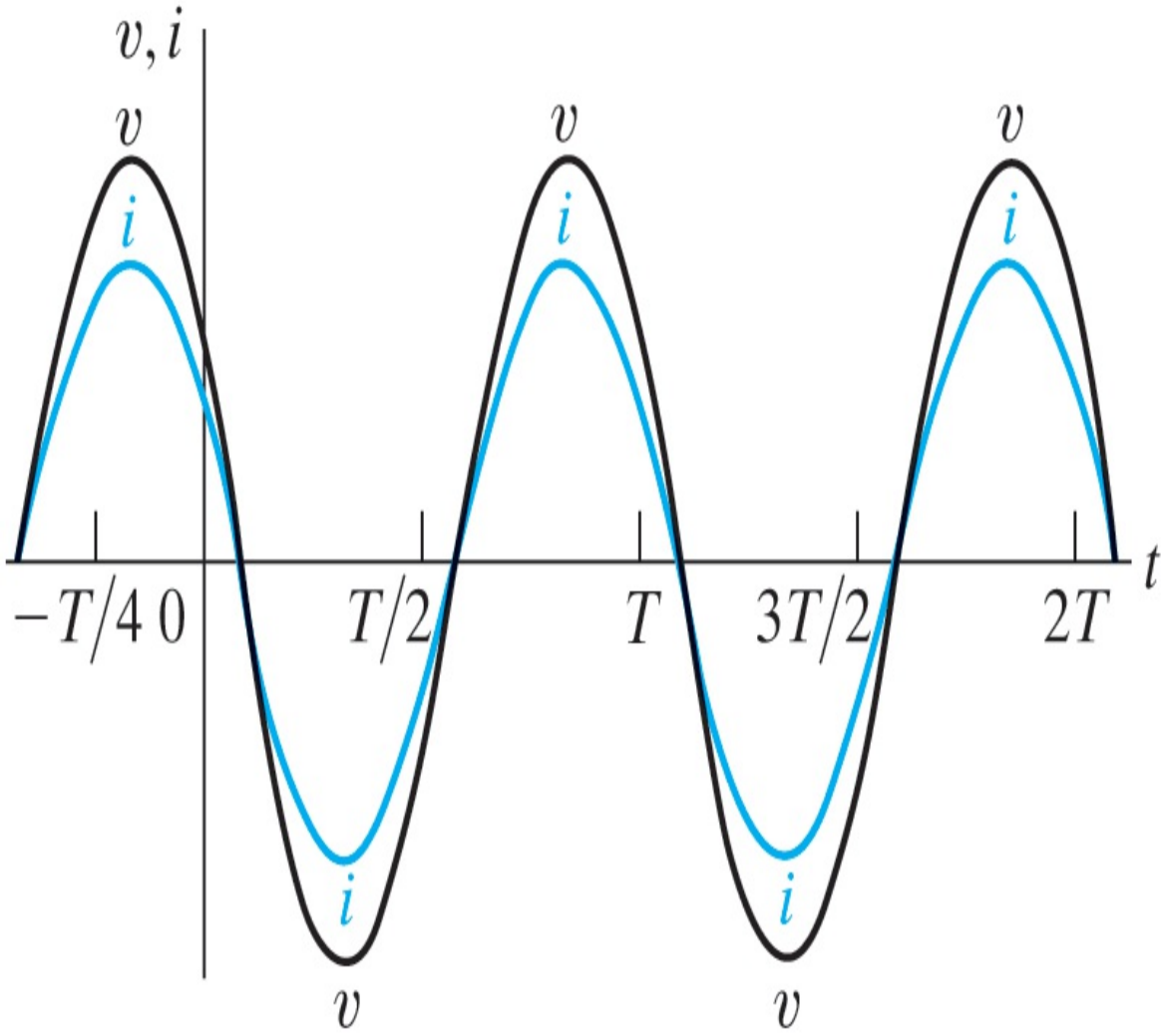


Figure 9.9 Full Alternative Text

The V-I Relationship for an Inductor

We derive the relationship between the phasor current and phasor voltage at the terminals of an inductor by assuming a sinusoidal current and using Ldi/dt to establish the corresponding voltage. Thus, for i=Im cos (ωt + θi), the expression for the voltage is

$v = L\frac{di}{dt} = -\omega L I_m \sin(\omega t + \theta_i)$.

We now replace the sine function with the cosine function:

$v = -\omega L I_m \cos(\omega t + \theta_i - 90°)$.

The phasor representation of the voltage is then

$V = -\omega L I_m e^{j(\theta_i - 90°)} = -\omega L I_m e^{j\theta_i} e^{-j90°} = j\omega L I_m e^{j\theta_i} = j\omega L I_m \angle \theta_i$.

Note that, in deriving the expression for the phasor voltage, we used the identity

$e^{-j90°} = \cos 90° - j \sin 90° = -j$.

Also, $I_m \angle \theta_i$ is the phasor representation of the sinusoidal current, so we can express the phasor voltage in terms of the phasor current:

Relationship Between Phasor Voltage and Phasor Current for an Inductor

$V = j\omega L I$. (9.11)

Equation 9.11 states that the phasor voltage at the terminals of an inductor equals jωL times the phasor current. Figure 9.10 shows the frequency-domain equivalent circuit for the inductor. Note that the relationship between phasor voltage and phasor current for an inductor also applies for the mutual inductance in one coil due to current flowing in another mutually coupled coil. That is, the phasor voltage at the terminals of one coil in a mutually coupled pair of coils equals jωM times the phasor current in the other coil.

Figure 9.10 The frequency-domain equivalent circuit for an inductor.

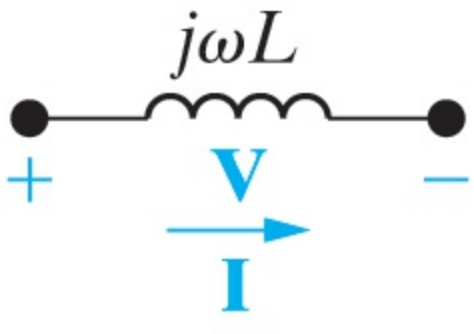


Figure 9.10 Full Alternative Text

We can rewrite Eq. 9.11 as

$$V=(\omega L\ 90°)\ I\ m\ \theta\ i =\omega L\ I\ m\ (\ \theta\ i\ +90)°\ ,$$

which indicates that the voltage and current are out of phase by exactly90°, Specifically, the voltage leads the current by 90°, or, equivalently, the current lags the voltage by 90°. Figure 9.11 illustrates the concept of *voltage leading current* or *current lagging voltage*. For example, the voltage reaches its negative peak exactly 90° before the current reaches its negative peak. The same observation can be made with respect to the zero-going-positive crossing or the positive peak.

Figure 9.11 A plot showing the phase relationship between the current and voltage at the

terminals of an inductor ($\theta_i = 60^\circ$)

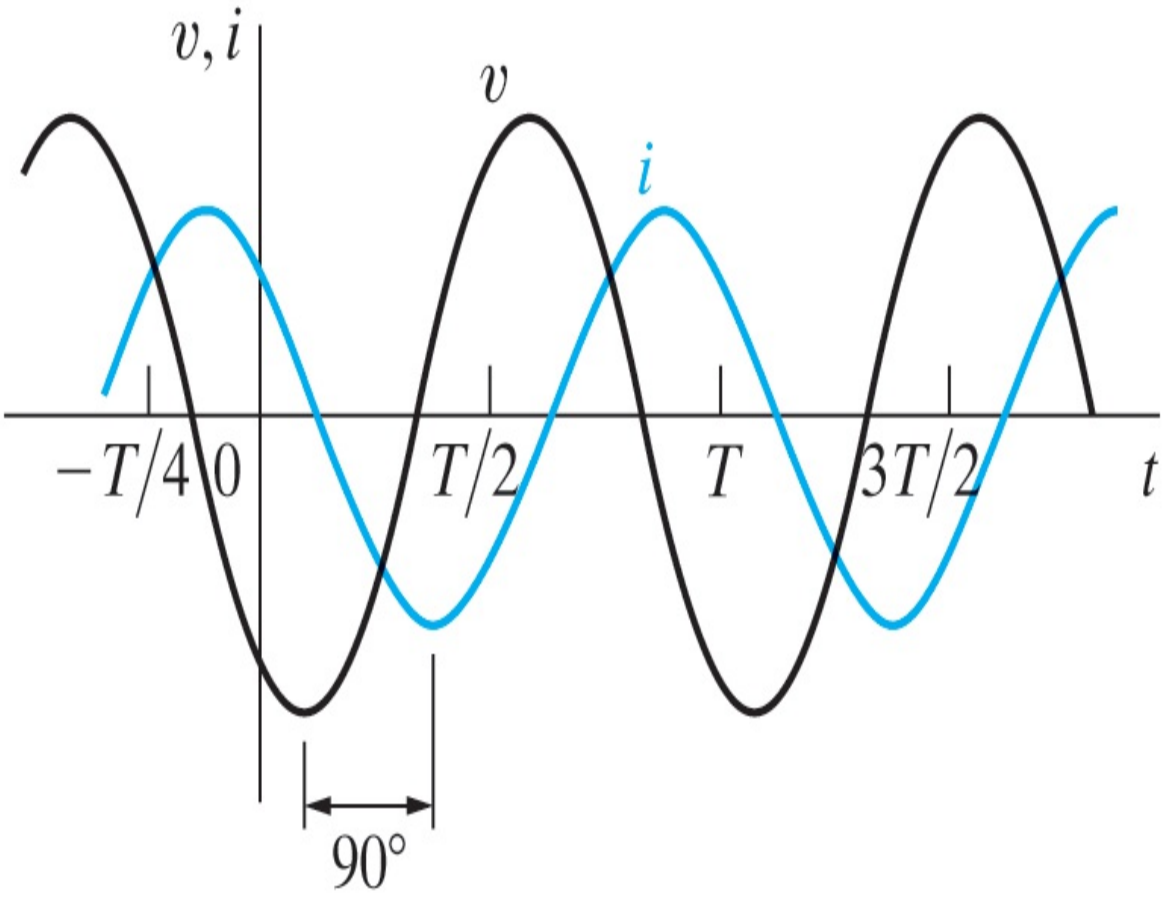


Figure 9.11 Full Alternative Text

We can also express the phase shift in seconds. A phase shift of 90° corresponds to one-fourth of a period; hence, the voltage leads the current by T/4, or 14f second.

The V-I Relationship for a Capacitor

To determine the relationship between the phasor current and phasor voltage at the terminals of a capacitor, we start with the relationship between current and voltage for a capacitor in the time domain,

$i=C\frac{dv}{dt}$,

and assume that

$v=V_m \cos(\omega t + \theta_v)$.

Therefore,

$i=C\frac{dv}{dt}=-\omega C V_m \sin(\omega t+\theta_v)$.

We now rewrite the expression for the current using the cosine function:

$i=-\omega C V_m \cos(\omega t+\theta_v-90°)$.

The phasor representation of the current is

$I=-\omega C V_m e^{j(\theta_v-90°)}=-\omega C V_m e^{j\theta_v} e^{-j90°}=j\omega C V_m e^{j\theta_v}=j\omega C V_m \theta_v$.

Since $V_m \theta_v$ is the phasor representation of the sinusoidal voltage, we can express the current phasor in terms of the voltage phasor as

$I=j\omega CV$.

Now express the voltage phasor in terms of the current phasor, to conform to the phasor equations for resistors and inductors:

Relationship Between Phasor Voltage and Phasor Current for a Capacitor

V=1jωCI (9.12)

Equation 9.12 demonstrates that the equivalent circuit for the capacitor in the phasor domain is as shown in Fig. 9.12.

Figure 9.12 The frequency domain equivalent circuit of a capacitor.

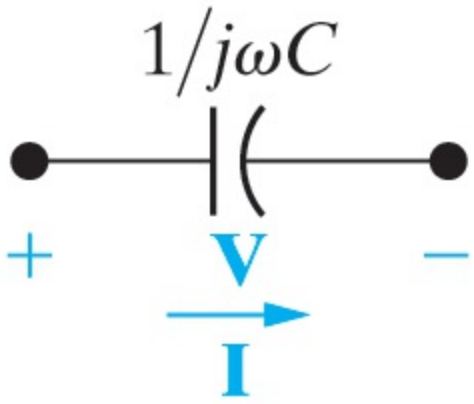


Figure 9.12 Full Alternative Text

The voltage across the terminals of a capacitor lags behind the current by 90°. We can show this by rewriting Eq. 9.12 as

V = (1 ωC −90°) I m θ i ° = I m ωC (θ i − 90)° .

Thus, we can also say that the current leads the voltage by 90°. Figure 9.13 shows the phase relationship between the current and voltage at the terminals of a capacitor.

Figure 9.13 A plot showing the phase relationship between the

current and voltage at the terminals of a capacitor (θ i = 60 °).

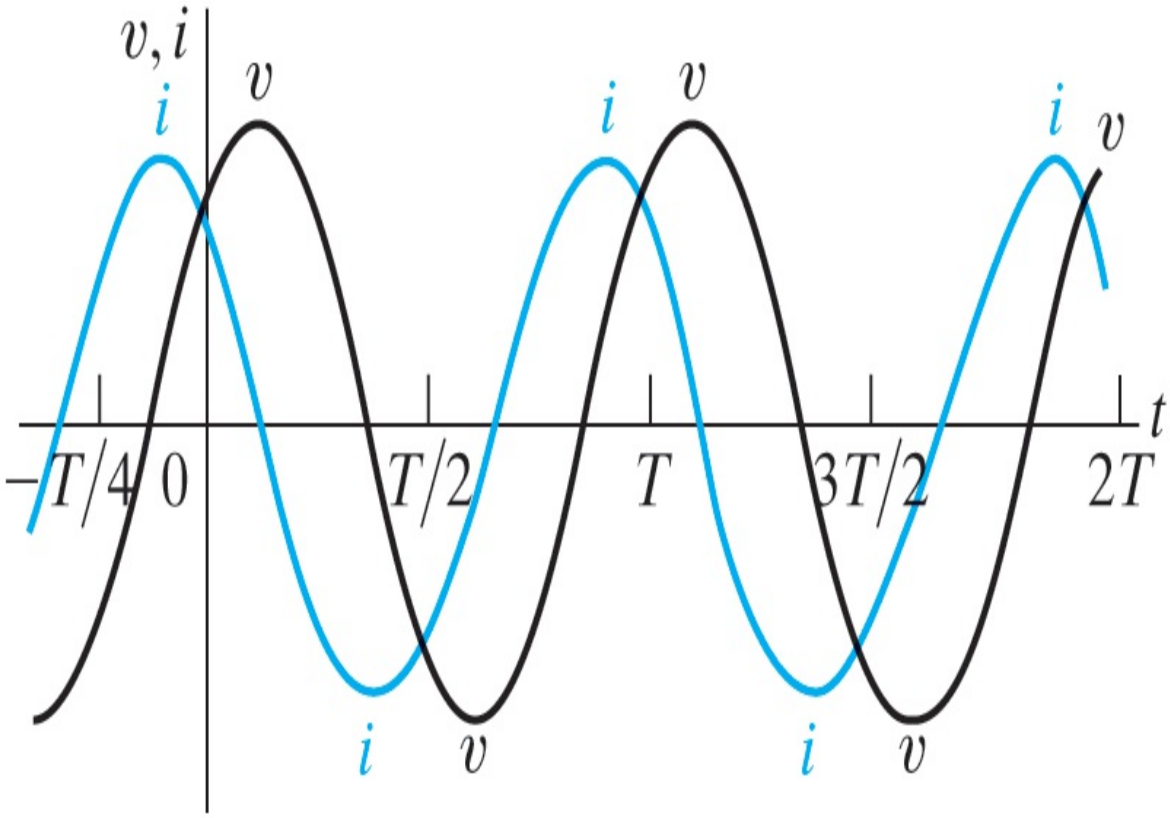


Figure 9.13 Full Alternative Text

Impedance and Reactance

We conclude this discussion of passive circuit elements in the frequency domain with an important observation. When we compare Eqs. 9.10, 9.11, and 9.12, we note that they are all of the form

Definition of Impedance

$$V=ZI, \quad (9.13)$$

where Z represents the impedance of the circuit element. Solving for Z in Eq. 9.13, you can see that impedance is the ratio of a circuit element's voltage phasor to its current phasor. Thus, the impedance of a resistor is R, the impedance of an inductor is $j\omega L$, the impedance of mutual inductance is $j\omega M$, and the impedance of a capacitor is $1/j\omega C$. In all cases, impedance is measured in ohms. Note that, although impedance is a complex number, it is not a phasor. Remember, a phasor is a complex number that results from the phasor transform of a cosine waveform. Thus, although all phasors are complex numbers, not all complex numbers are phasors.

Impedance in the frequency domain is the quantity analogous to resistance, inductance, and capacitance in the time domain. The imaginary part of the impedance is called reactance. The values of impedance and reactance for each of the component values are summarized in Table 9.1.

Table 9.1 Impedance and Reactance Values

Circuit Element	Impedance	Reactance
Resistor	R	—
Inductor	$j\omega L$	ωL
Capacitor	$j(-1/\omega C)$	$-1/\omega C$

Table 9.1 Full Alternative Text

And, finally, a reminder. The passive sign convention holds in the frequency domain. If the reference direction for the current phasor in a circuit element is in the direction of the voltage phasor rise across the element, you must insert a minus sign into the equation that relates the voltage phasor to the current phasor.

Work through Example 9.6 to practice transforming circuit components from the time domain to the phasor domain.

Example 9.6 Calculating Component Voltages Using Phasor Techniques

Figure 9.14 shows a resistor and an inductor connected in series. The current in these components is

Figure 9.14 The components for Example 9.6.

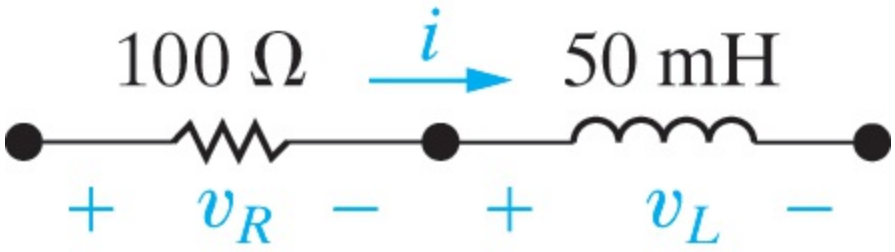


Figure 9.14 Full Alternative Text
i=50cos(1000t+45°)mA.

The phasor transform of these components is shown in Fig. 9.15. Find

1. ZR;
2. ZL;
3. **I**;
4. VR;
5. VL.

Figure 9.15 The phasor transform of the components in Fig. 9.14.

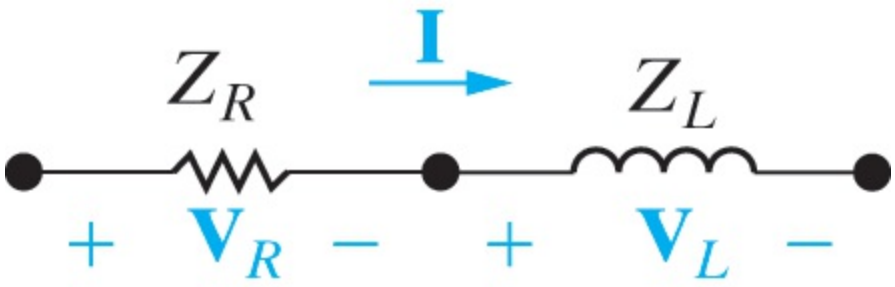


Figure 9.15 Full Alternative Text

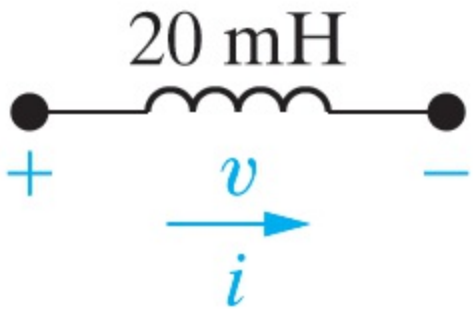
Solution

1. ZR=R=100 Ω.
2. ZL=jωL=j(1000)(0.05)=j50 Ω.
3. I=P{ 0.05cos(1000t+ 45 ° }=0.05 45° =50 45 ° mA.
4. V R = Z R I=(100)(0.05 45 °)=5 45 ° V.
5. V L .= Z L I=(j50)(0.05 45 °)=2.5 135 ° V.

Assessment Problems

Objective 2—Be able to transform a circuit with a sinusoidal source into the frequency domain using phasor concepts

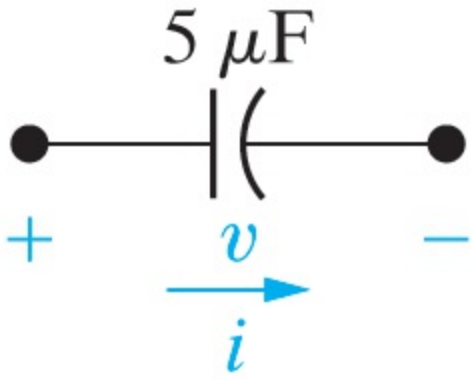
1. 9.3 The current in the 20 mH inductor is 10 cos(10,000t + 30°)mA. Calculate (a) the inductive reactance; (b) the impedance of the inductor; (c) the phasor voltage **V**; and (d) the steady-state expression for v(t).



Answer:

1. 200 Ω;
2. j200 Ω;
3. 2 120° V;
4. 2 cos(10,000t + 120°) V.

2. 9.4 The voltage across the terminals of the 5μF capacitor is 30 cos(4000t + 25°) V. Calculate (a) the capacitive reactance; (b) the impedance of the capacitor; (c) the phasor current **I**; and (d) the steady-state expression for i(t).



Answer:

1. −50 Ω;
2. −j50 Ω;

3. 0.6 115° A;

4. 0.6 cos(4000t + 115°) A.

SELF-CHECK: Also try *Chapter Problems 9.13 and 9.14*.

9.5 Kirchhoff’s Laws in the Frequency Domain

Kirchhoff’s Voltage Law in the Frequency Domain

We begin by assuming that $v_1, v_2, \ldots, v_n$ represent voltages around a closed path in a circuit. We also assume that the circuit is operating in a sinusoidal steady state. Thus, Kirchhoff’s voltage law requires that

$$v1 + v2 + \cdots + vn=0,$$

which in the sinusoidal steady state becomes

$$Vm1 \cos(\omega t + \theta 1) + Vm2 \cos(\omega t + \theta 2) + \cdots + Vmn \cos(\omega t + \theta n)=0.$$

We now use Euler’s identity to write the KVL equation as

$$R \{Vm1ej\theta 1ej\omega t\} + R \{Vm2ej\theta 2ej\omega t\} +\ldots+ R \{Vmnej\theta nej\omega t\}=0.$$

which we simplify as

$$R \{Vm1ej\theta 1ej\omega t + Vm2ej\theta 2ej\omega t + \cdots + Vmnej\theta nej\omega t\}=0.$$

Factoring the term $ej\omega t$ from each term yields

$$R \{(Vm1ej\theta 1 + Vm2ej\theta 2 + \cdots + Vmnej\theta n)ej\omega t\}=0,$$

or

$$R\{(V1 + V2 + \cdots + Vn)ej\omega t\}=0.$$

But $ej\omega t \neq 0$, so

KVL in the Frequency Domain

$V1 + V2 + \cdots + Vn = 0$, (9.14)

which is the statement of Kirchhoff's voltage law as it applies to phasor voltages.

Kirchhoff's Current Law in the Frequency Domain

A similar derivation applies to a set of sinusoidal currents. Thus, if

$i1 + i2 + \cdots + in = 0$,

then

KCL in the Frequency Domain

$I1 + I2 + \cdots + In = 0$, (9.15)

where I1, I2, …, In are the phasor representations of the individual currents i1, i2, …, in. Thus, Eq. 9.15 states Kirchhoff's current law as it applies to phasor currents.

Equations 9.13, 9.14, and 9.15 form the basis for circuit analysis in the frequency domain. Note that Eq. 9.13 has the same algebraic form as Ohm's law and that Eqs. 9.14 and 9.15 state Kirchhoff's laws for phasor quantities. Therefore, you can use all the techniques developed for analyzing resistive circuits to find phasor currents and voltages. No new analytical techniques are needed; the basic circuit analysis and simplification tools covered in Chapters 2–4 can all be used to analyze circuits in the frequency domain. Phasor-circuit analysis consists of two fundamental tasks: (1) You must be able to construct the frequency-domain model of a circuit; and (2) you must

be able to manipulate complex numbers and/or quantities algebraically.

Example 9.7 illustrates the use of KVL in the frequency domain.

Example 9.7 Using KVL in the Frequency Domain

1. Use the results from Example 9.6 to calculate the phasor voltage drop, from left to right, across the series combination of the resistive and inductive impedances in Fig. 9.15.

2. Use the phasor voltage found in (a) to calculate the steady-state voltage drop, from left to right, across the series combination of resistor and inductor in Fig. 9.14.

Solution

1. Using KVL, the phasor voltage drop from left to right in Fig. 9.15 is

   $$V = V_1 + V_2 = 5\,45° + 2.5\,135° = 5.59\,71.565° \text{ V}.$$

2. To find the steady-state voltage drop across the resistor and inductor in Fig. 9.14, we need to apply the inverse phasor transform to the phasor **V** from part (a). We need the frequency of the current defined in Example 9.6, which is $\omega = 1000$ rad/s:

   $$v_{ss}(t) = P^{-1}\{V\} = P^{-1}\{5.59\,71.565°\} = 5.59\cos(1000t + 71.565°)\text{V}.$$

Assessment Problem

Objective 3—Know how to use circuit analysis techniques to solve a circuit

in the frequency domain

1. 9.5 Four branches terminate at a common node. The reference direction of each branch current (i1, i2, i3, and i4) is toward the node. If $i_1 = 100\cos(\omega t + 25°)$ A, $i_2 = 100\cos(\omega t + 145°)$ A, and $i_3 = 100\cos(\omega t - 95°)$ A, find i_4.

   Answer: i4=0.

SELF-CHECK: Also try *Chapter Problem 9.15*.

9.6 Series, Parallel, and Delta-to-Wye Simplifications

The rules for combining impedances in series or parallel and for making delta-to-wye transformations are the same as those for resistors. The only difference is that combining impedances involves the algebraic manipulation of complex numbers.

Combining Impedances in Series

Impedances in series can be combined into a single equivalent impedance whose value is the sum of the individual impedances. The circuit shown in Fig. 9.16 defines the problem in general terms. The impedances Z1, Z2, …, Zn are connected in series between terminals a,b. When impedances are in series, they carry the same phasor current **I**. From Eq. 9.13, the voltage drop across each impedance is Z1I, Z2I, …, ZnI, and from Kirchhoff's voltage law,

$$V_{ab} = Z_1 I + Z_2 I + ... + Z_n I = (Z_1 + Z_2 + ... + Z_n)I.$$

Figure 9.16 Impedances in series.

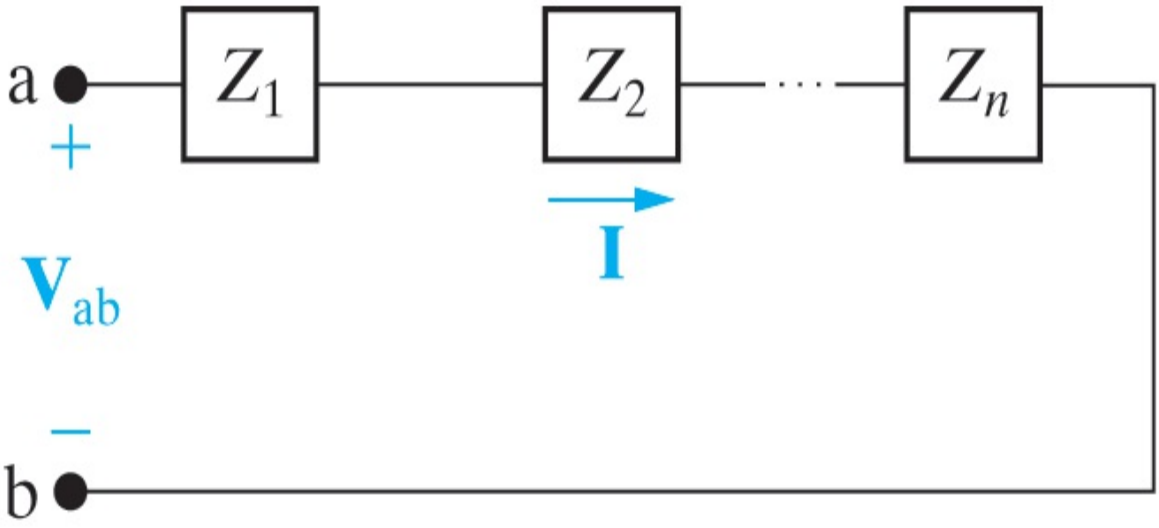


Figure 9.16 Full Alternative Text

The equivalent impedance between terminals a,b is

Combining Impedances in Series

Zab = V ab I =Z1 +Z 2+ . . .+ Zn. (9.16)

Remember from Chapter 3 that we can use voltage division to find the voltage across a single component from a collection of series-connected components whose total voltage is known (Eq. 3.9). We derived the voltage division equation using the equation for the equivalent resistance of series-connected resistors. Using the same process, we can derive the voltage division equation for frequency-domain circuits, where Vs is the voltage applied to a collection of series-connected impedances, Vj is the voltage across the impedance Zj, and Zeq is the equivalent impedance of the series-connected impedances:

Voltage Division in the Frequency Domain

$$V_i = \frac{Z_i}{Z_{eq}} V_s \quad (9.17)$$

Example 9.8 illustrates the following frequency-domain circuit analysis techniques: combining impedances in series, Ohm's law for phasors, and voltage division.

Example 9.8 Combining Impedances in Series

A 90 Ω resistor, a 32 mH inductor, and a 5μF capacitor are connected in series across the terminals of a sinusoidal voltage source, as shown in Fig. 9.17. The steady-state expression for the source voltage vs is 750 cos (5000t + 30°) V.

1. Construct the frequency-domain equivalent circuit.
2. Calculate the phasor voltage **V** using voltage division for the circuit from part (a).
3. Find the steady-state voltage v using the inverse phasor transform.

Figure 9.17 The circuit for Example 9.8.

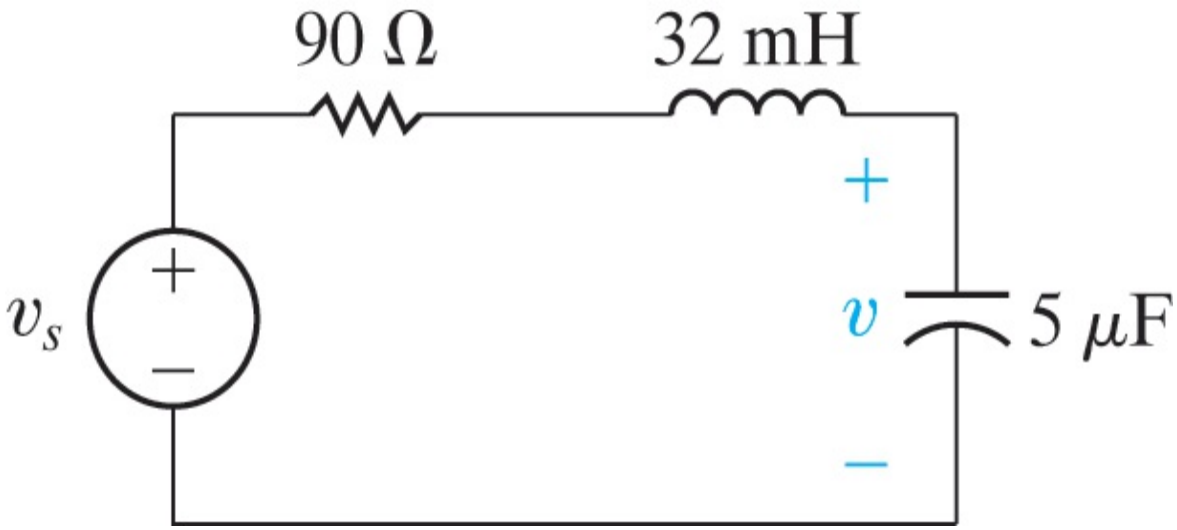


Figure 9.17 Full Alternative Text

Solution

1. From the expression for vs, we have ω=5000 rad/s. Therefore, the impedance of the inductor is

   ZL=jωL=j(5000)(32 × 10−3)=j160 Ω,

   and the impedance of the capacitor is

   ZC=j−1ωC=−j1(5000)(5×10−6)=−j40 Ω

   The phasor transform of vs is

   V s =750 30° V.

   Figure 9.18 illustrates the frequency-domain equivalent circuit of the circuit shown in Fig. 9.17.

2. Using voltage division, we see that the phasor voltage **V** is proportional to the source voltage; from Eq. 9.17,

   V= −j40 90+j160−j40 (750 30°)=200 −113.13° V.

Note that we used Eq. 9.16 to find the equivalent impedance of the series-connected impedances in the circuit.

3. Find the steady-state voltage v using the inverse phasor transform of **V** from part (b). Remember that the source frequency is 5000 rad/s:

   $v(t)=200\cos(5000t-113.13°)V$.

Figure 9.18 The frequency-domain equivalent circuit of the circuit shown in Fig. 9.17.

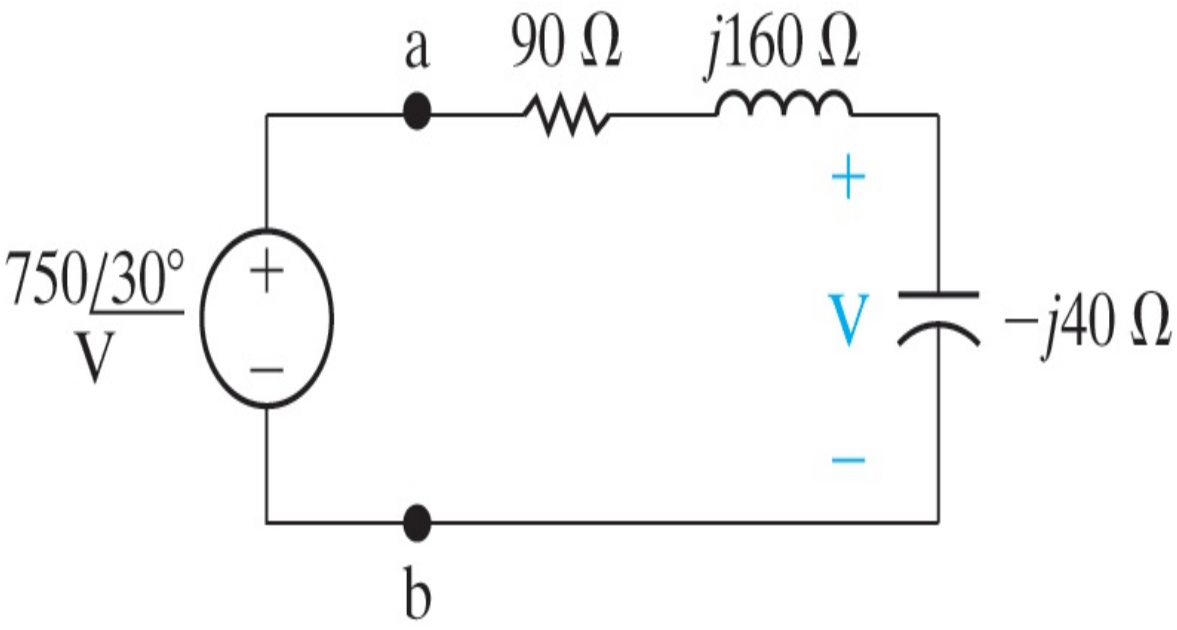


Figure 9.18 Full Alternative Text

This voltage is the steady-state component of the complete response, which is what remains once the transient component has decayed to zero.

Assessment Problem

Objective 3—Know how to use circuit analysis techniques to solve a circuit in the frequency domain

1. 9.6 Using the values of resistance and inductance in the circuit of Fig. 9.15 , let V s =125 −60° V and ω=5000 rad/s. Find

    1. the value of capacitance that yields a steady-state output current i with a phase angle of −105°.

    2. the magnitude of the steady-state output current i.

    Answer:

    1. 2.86 μF;

    2. 0.982 A.

SELF-CHECK: Also try *Chapter Problem 9.17*.

Combining Impedances in Parallel

Impedances connected in parallel can be reduced to an equivalent impedance using the reciprocal relationship

Combining Impedances in Parallel

1Zab=1Z1+1Z2+···+1Zn. (9.18)

Figure 9.19 depicts the parallel connection of impedances. Note that when impedances are in parallel, they have the same voltage across their terminals. We derive Eq. 9.18 directly from Fig. 9.19 by combining Kirchhoff's current law with the phasor-domain version of Ohm's law, that is, Eq. 9.13. From Fig. 9.19,

I=I1+I2+···+In,

or

VZab=VZ1+VZ2+···+VZn.

Figure 9.19 Impedances in parallel.

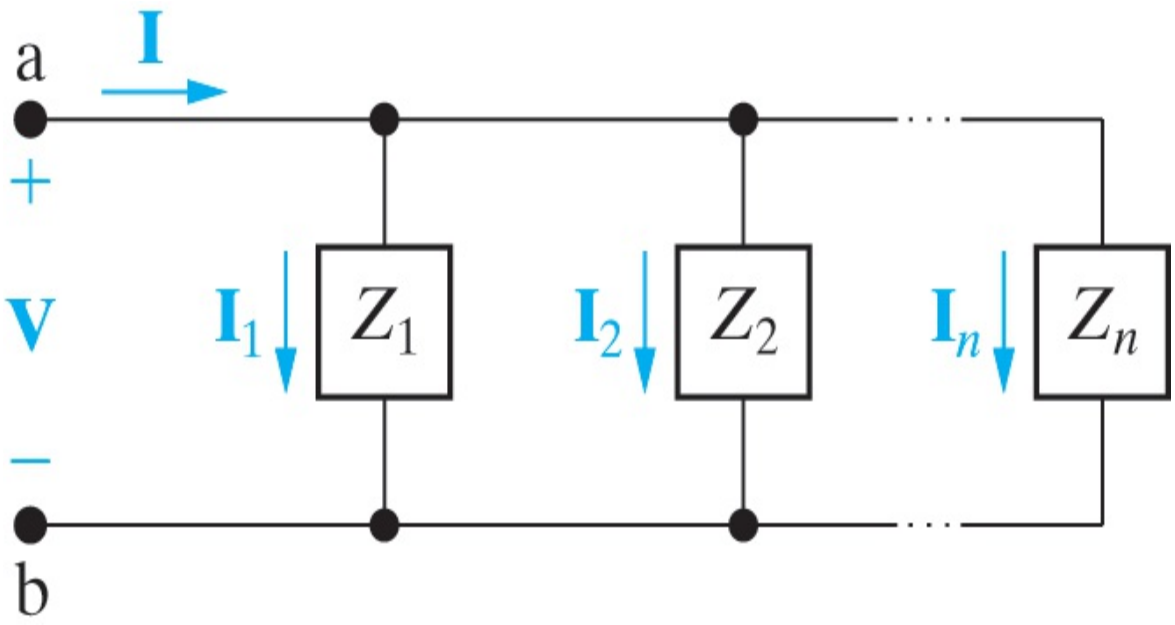


Figure 9.19 Full Alternative Text

Canceling the common voltage term from both sides gives us Eq. 9.18.

From Eq. 9.18, for the special case of just two impedances in parallel,

Zab=Z1Z2Z1+Z2. (9.19)

We can also express Eq. 9.18 in terms of **admittance**, defined as the reciprocal of impedance and denoted Y. Thus

Y=1Z=G+jB (siemens).

Admittance is a complex number whose real part, G, is called **conductance**

and whose imaginary part, B, is called **susceptance**. Like admittance, conductance and susceptance are measured in siemens (S). Replacing impedances with admittances in Eq. 9.18, we get

$$Y_{ab}=Y_1+Y_2+\cdots+Y_n.$$

The admittance of each of the ideal passive circuit elements also is worth noting and is summarized in Table 9.2.

Table 9.2 Admittance and Susceptance Values

Circuit Element	Admittance (Y)	Susceptance
Resistor	G (conductance)	—
Inductor	$j(-1/\omega L)$	$-1/\omega L$
Capacitor	$j\omega C$	ωC

Table 9.2 Full Alternative Text

Finally, remember from Chapter 3 that we can use current division to find the current in a single branch from a collection of parallel-connected branches whose total current is known (Eq. 3.10). We derived the current division equation using the equation for the equivalent resistance of parallel-connected resistors. Using the same process, we can derive the current division equation for frequency-domain circuits, where Is is the current supplied to a collection of parallel-connected impedances, Ij is the current in the branch containing impedance Zj, and Zeq is the equivalent impedance of the parallel-connected impedances:

Current Division in the Frequency

Domain

Ii=ZeqZiIs. (9.20)

We use Eq. 9.18 to calculate the equivalent impedance in Eq. 9.20.

Example 9.9 analyzes a circuit in the frequency domain using series and parallel combinations of impedances and current division.

Example 9.9 Combining Impedances in Series and in Parallel

The sinusoidal current source in the circuit shown in Fig. 9.20 produces the current is=8 cos 200,000t A.

1. Construct the frequency-domain equivalent circuit.
2. Find the equivalent admittance to the right of the current source.
3. Use the equivalent admittance from part (b) to find the phasor voltage **V**.
4. Find the phasor current **I**, using current division.
5. Find the steady-state expressions for v and i.

Figure 9.20 The circuit for Example 9.9.

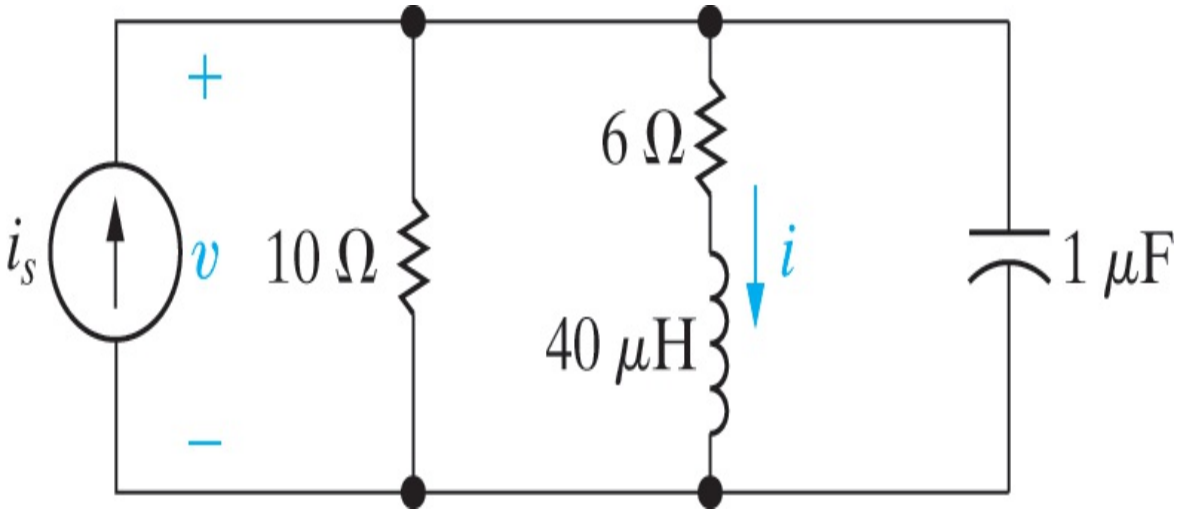


Figure 9.20 Full Alternative Text

Solution

1. The phasor transform of the current source is 8 0° ; the resistors transform directly to the frequency domain as 10 and 6 Ω; the 40 μH inductor has an impedance of j8 Ω at the given frequency of 200,000 rad/s; and at this frequency the 1 μF capacitor has an impedance of −j5 Ω. Figure 9.21 shows the frequency-domain equivalent circuit and symbols representing the phasor transforms of the unknowns.

2. We first find the equivalent admittance to the right of the current source by adding the admittances of each branch. The admittance of the first branch is

   $Y1=110=0.1$ S,

   the admittance of the second branch is

   $Y2=16+j8=0.06-j0.08$S.

   and the admittance of the third branch is

   $Y3=1-j5=j0.2$ S.

The admittance of the three branches is

$Y_{eq} = Y_1 + Y_2 + Y_3 = 0.16+j0.12\ S = 0.2\ 36.87°\ S.$

3. Use the equivalent admittance from part (b) to find the phasor voltage **V**.

   The impedance seen by the current source is

   $Z_{eq} = 1\ Y_{eq} = 5\ -36.87°\ \Omega.$

   The phasor voltage **V** is

   $V = Z_{eq}\ I = 40\ -36.87°\ V.$

4. Using Eq. 9.20, together with the equivalent impedance found in part (c), we get

   $I = 5\ -36.87°\ 6+j8\ (\ 8\ 0°\ ) = 4\ -90°\ A.$

   You can verify this answer using the phasor voltage across the branch, **V**, and the impedance of the branch, (6+j8) Ω.

5. From the phasors found in parts (c) and (d), the steady-state time-domain expressions are

   $v(t) = 40\cos(200{,}000t - 36.87°)\ V, i(t) = 4\cos(200{,}000t - 90°)\ A.$

Figure 9.21 The frequency-domain equivalent circuit.

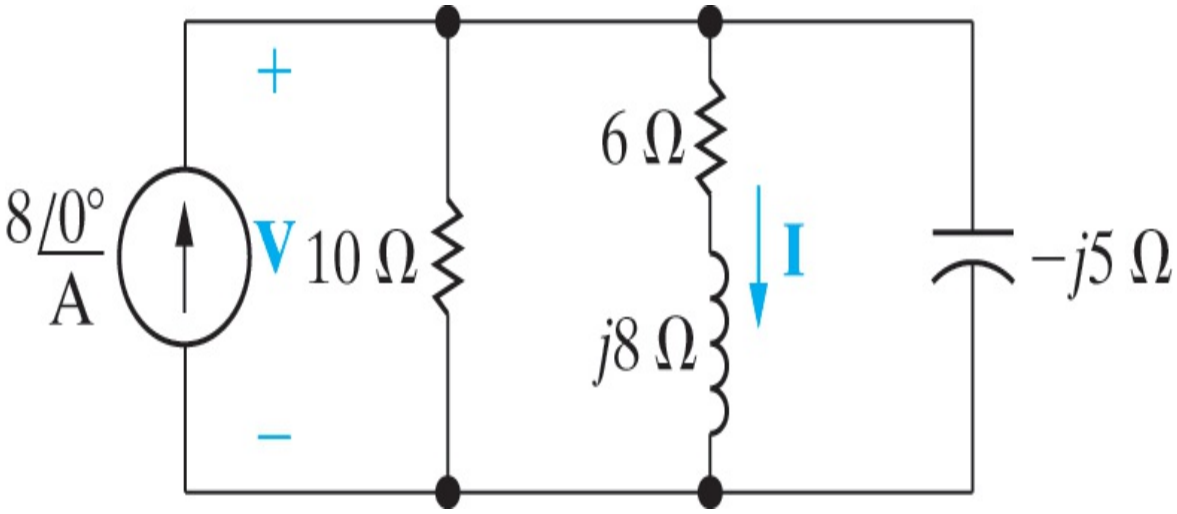


Figure 9.21 Full Alternative Text

Assessment Problems

Objective 3—Know how to use circuit analysis techniques to solve a circuit in the frequency domain

1. 9.7 A 20 Ω resistor is connected in parallel with a 5 mH inductor. This parallel combination is connected in series with a 5 Ω resistor and a 25 μF capacitor.

    1. Calculate the impedance of this interconnection if the frequency is 2 krad/s.

    2. Repeat (a) for a frequency of 8 krad/s.

    3. At what finite frequency does the impedance of the interconnection become purely resistive?

    4. What is the impedance at the frequency found in (c)?

    Answer:

    1. 9−j12 Ω;

2. 21+j3 Ω;
3. 4 krad/s;
4. 15 Ω.

2. 9.8 The interconnection described in Assessment Problem 9.7 is connected across the terminals of a voltage source that is generating v=150 cos 4000t V. What is the maximum amplitude of the current in the 5 mH inductor?

   Answer: 7.07 A.

SELF-CHECK: Also try *Chapter Problems 9.28, 9.31, and 9.36*.

Delta-to-Wye Transformations

The Δ-to-Y transformation for resistive circuits, discussed in Section 3.7, also applies to impedances. Figure 9.22 defines the Δ-connected impedances along with the Y-equivalent circuit. The Y impedances as functions of the Δ impedances are

$$Z_1 = \frac{Z_b Z_c}{Z_a + Z_b + Z_c}, \quad (9.21)$$
$$Z_2 = \frac{Z_c Z_a}{Z_a + Z_b + Z_c}, \quad (9.22)$$
$$Z_3 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c}. \quad (9.23)$$

Figure 9.22 The delta-to-wye transformation.

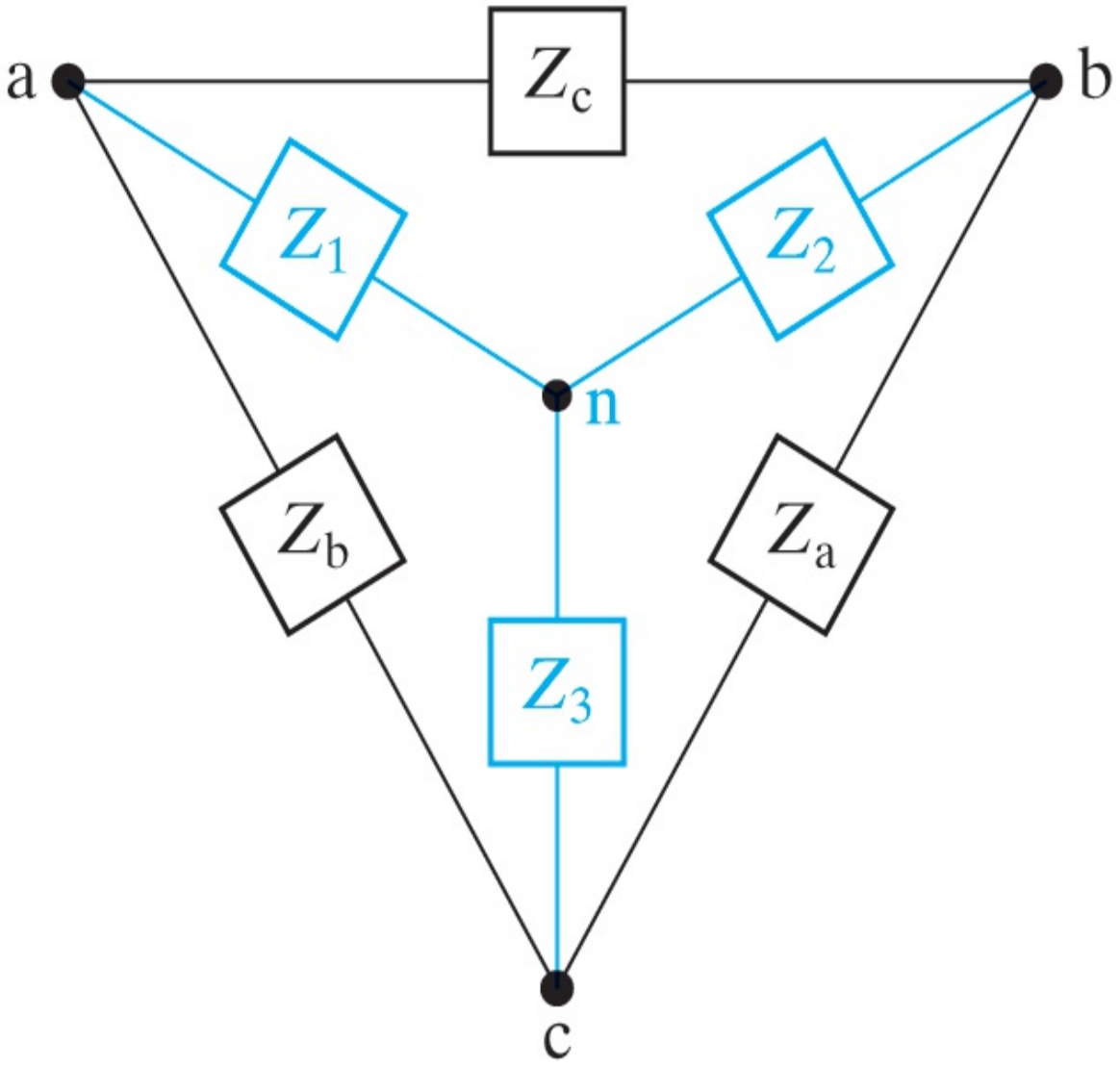


Figure 9.22 Full Alternative Text

The Δ-to-Y transformation also may be reversed; that is, we can start with the Y structure and replace it with an equivalent Δ structure. The Δ impedances as functions of the Y impedances are

Za=Z1Z2+Z2Z3+Z3Z1Z1, (9.24)
Zb=Z1Z2+Z2Z3+Z3Z1Z2, (9.25)
Zc=Z1Z2+Z2Z3+Z3Z1Z3. (9.26)

The process used to derive Eqs. 9.21–9.23 or Eqs. 9.24–9.26 is the same as that used to derive the corresponding equations for resistive circuits. In fact,

comparing Eqs. 3.15–3.17 with Eqs. 9.21–9.23 and Eqs. 3.18–3.20 with Eqs. 9.24–9.26 reveals that the symbol Z has replaced the symbol R. You may want to review Problem 3.65 concerning the derivation of the Δ-to-Y transformation.

Example 9.10 uses the Δ-to-Y transformation in phasor-circuit analysis.

Example 9.10 Using a Delta-to-Wye Transform in the Frequency Domain

Use a Δ-to-Y impedance transformation to find I0, I1, I2, I3, I4, I5, V1, and V2 in the circuit in Fig. 9.23.

Figure 9.23 The circuit for Example 9.10.

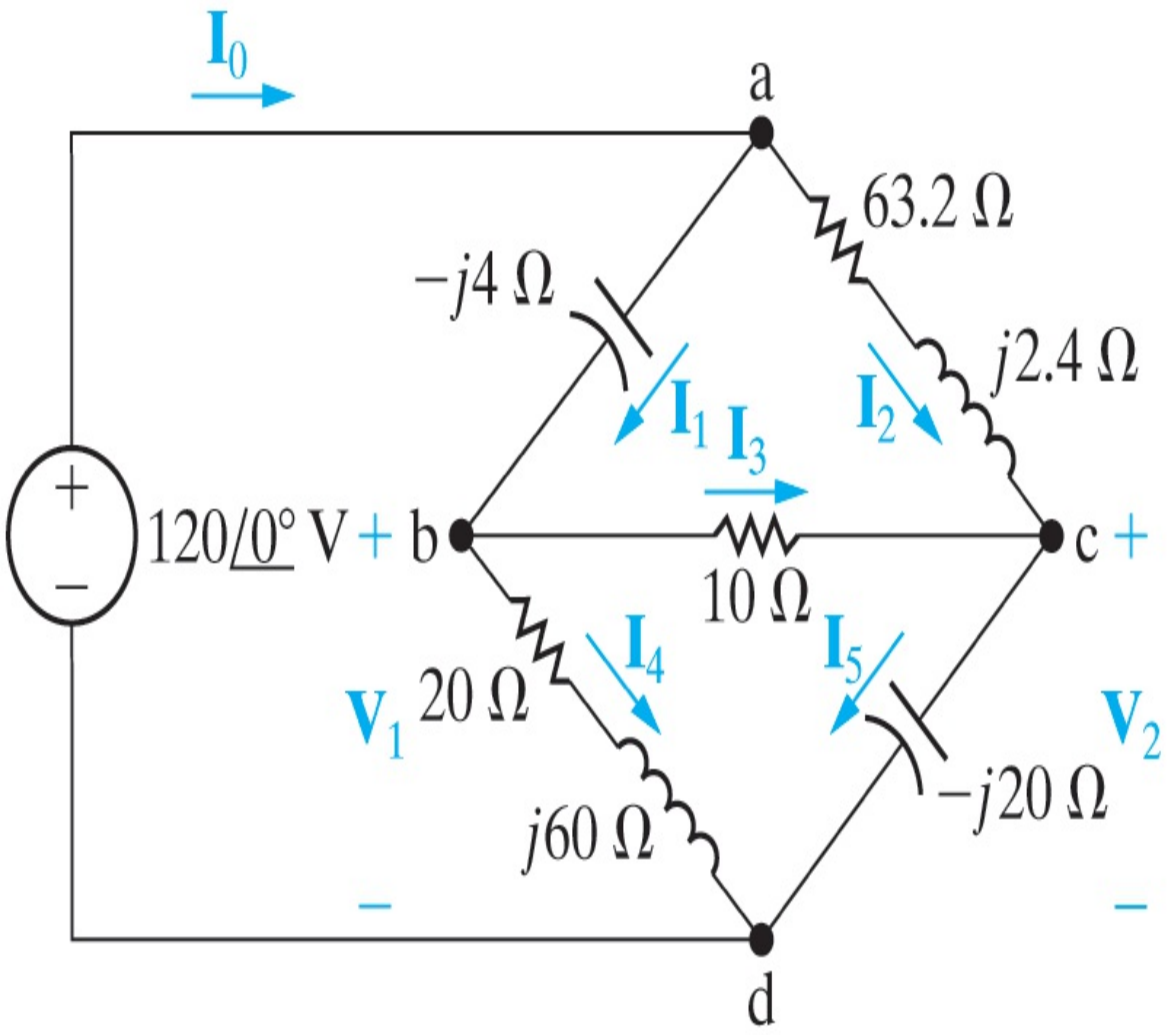


Figure 9.23 Full Alternative Text

Solution

It is not possible to simplify the circuit in Fig. 9.23 using series and parallel combinations of impedances. But if we replace either the upper delta (abc) or the lower delta (bcd) with its Y equivalent, we can simplify the resulting circuit by series-parallel combinations. To decide which delta to replace, find the sum of the impedances around each delta. This quantity forms the denominator for the equivalent Y impedances. The sum around the lower delta is 30+j40, so we choose to transform the lower delta to its equivalent Y. The Y impedance connecting to terminal b is

$Z1=(20+j60)(10)30+j40=12+j4\Omega$,

the Y impedance connecting to terminal c is

$Z2=10(-j20)30+j40=-3.2-j2.4\ \Omega$,

and the Y impedance connecting to terminal d is

$Z3=(20+j60)(-j20)30+j40=8-j24\ \Omega$.

Inserting the Y-equivalent impedances into the circuit results in the circuit shown in Fig. 9.24.

Figure 9.24 The circuit shown in Fig. 9.23, with the lower delta replaced by its equivalent wye.

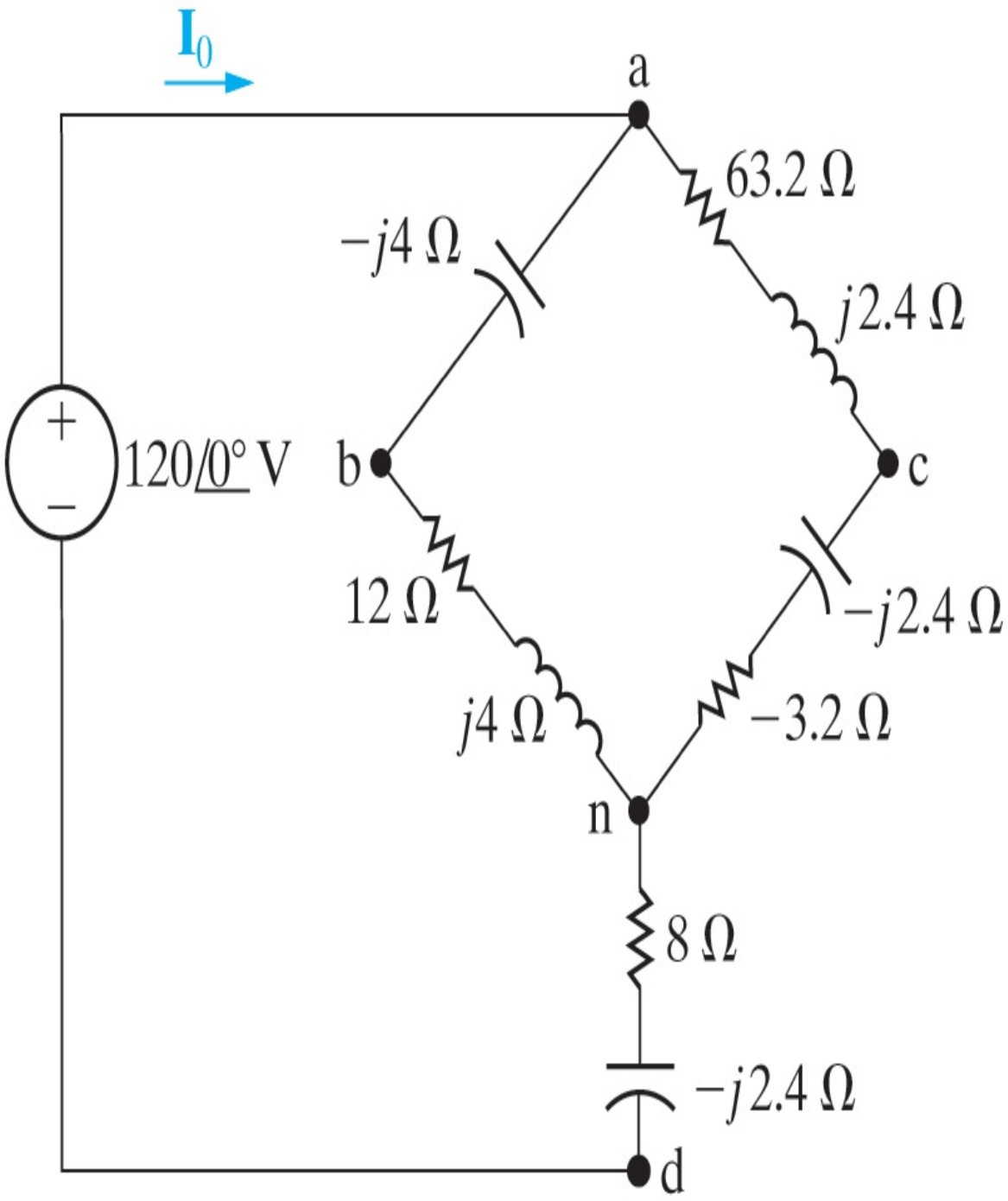


Figure 9.24 Full Alternative Text

We can simplify the circuit in Fig. 9.24 by making series-parallel combinations. The impedence of the abn branch is

$Z_{abn}=12+j4-j4=12\ \Omega$,

and the impedance of the acn branch is

Zacn=63.2+j2.4−j2.4−3.2=60 Ω.

Note that the abn branch is in parallel with the acn branch. Therefore, we may replace these two branches with a single branch having an impedance of

Zan=(60)(12)72=10 Ω.

Combining this 10 Ω resistor with the impedance between n and d reduces the circuit to the one shown in Fig. 9.25. From that circuit,

I 0 = 120 0° 18−j 24 =4 53.13° =2.4+j3.2 A.

Figure 9.25 A simplified version of the circuit shown in Fig. 9.24.

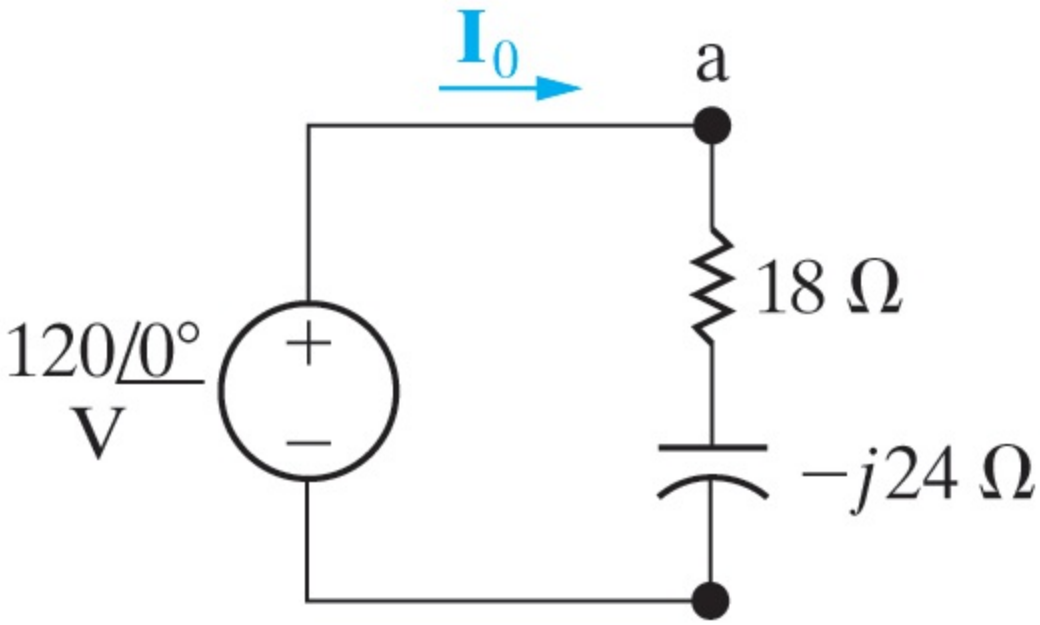


Figure 9.25 Full Alternative Text

Once we know I0, we can work back through the equivalent circuits to find the branch currents in the original circuit. We begin by noting that I0 is the current in the branch nd of Fig. 9.24. Therefore,

$$V_{nd}=(8-j24)I_0=96-j32 \text{ V}.$$

We can now calculate the voltage Van because

$$V=V_{an}+V_{nd}$$

where **V** is the phasor voltage of the source and Vnd is known. Thus

$$V_{an}=120-(96-j32)=24+j32 \text{ V}.$$

We now compute the branch currents Iabn and Iacn using Van and the equivalent impedance of each branch:

$$I_{abn}=\frac{24+j32}{12}=2+j83\text{A}, I_{acn}=\frac{24+j32}{60}=410+j815\text{A}.$$

In terms of the branch currents defined in Fig. 9.23,

$$I_1=I_{abn}=2+j83\text{A}, I_2=I_{acn}=410+j815\text{A}.$$

We check the calculations of I1 and I2 by noting that

$$I_1+I_2=2.4+j3.2=I_0.$$

To find the branch currents I3, I4, and I5, we must first calculate the voltages V1 and V2. Referring to Fig. 9.23, we note that

$$V_1=120\angle 0^\circ-(-j4)I_1=328\sqrt{3}+j8 \text{ V}, V_2=120\angle 0^\circ-(63.2+j2.4)I_2=96-j104\sqrt{3} \text{ V}.$$

We now calculate the branch currents I3, I4, and I5:

$$I_3=\frac{V_1-V_2}{10}=43+j12.83\text{A}, I_4=\frac{V_1}{20+j60}=23-j1.6\text{A}, I_5=\frac{V_2}{-j20}=2615+j4.8\text{A}.$$

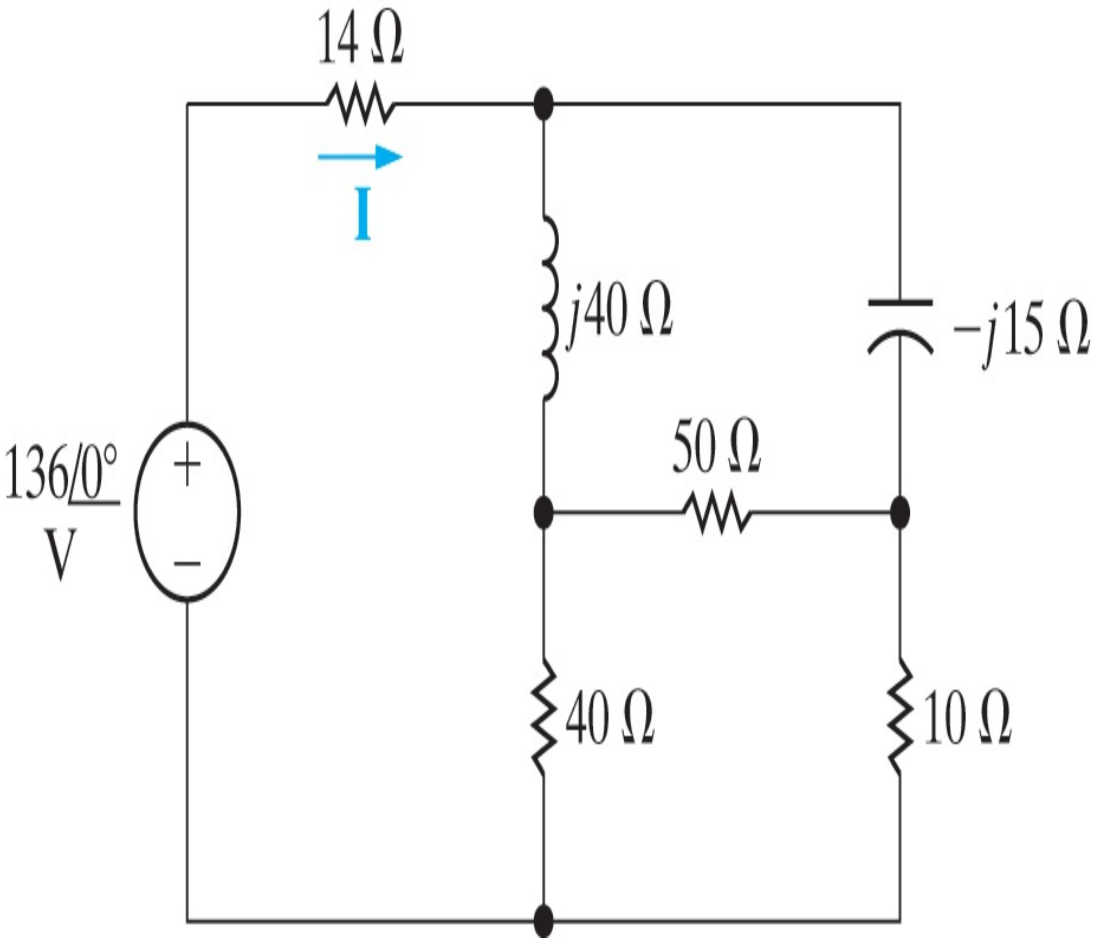
We check the calculations by noting that

I4+I5=23+2615-j1.6+j4.8=2.4+j3.2=I0,I3+I4=43+23+j12.83-j1.6=2+j83=I1,I3+I2=43+410+j12.83+j815=2615+j4.8=I5.

Assessment Problem

Objective 3—Know how to use circuit analysis techniques to solve a circuit in the frequency domain

1. 9.9 Use a Y-to-Δ transformation to find the current **I** in the circuit shown.



9.6-8 Full Alternative Text

Answer: I=4 28.07° A.

SELF-CHECK: Also try *Chapter Problem 9.42*.

9.7 Source Transformations and Thévenin–Norton Equivalent Circuits

The source transformations introduced in Section 4.9 and the Thévenin–Norton equivalent circuits discussed in Section 4.10 are analytical techniques that also can be applied to frequency-domain circuits. We prove these techniques are valid by following the same process used in Sections 4.9 and 4.10, except that we substitute impedance (Z) for resistance (R). Figure 9.26 shows a source-transformation equivalent circuit in the frequency domain.

Figure 9.26 A source transformation in the frequency domain.

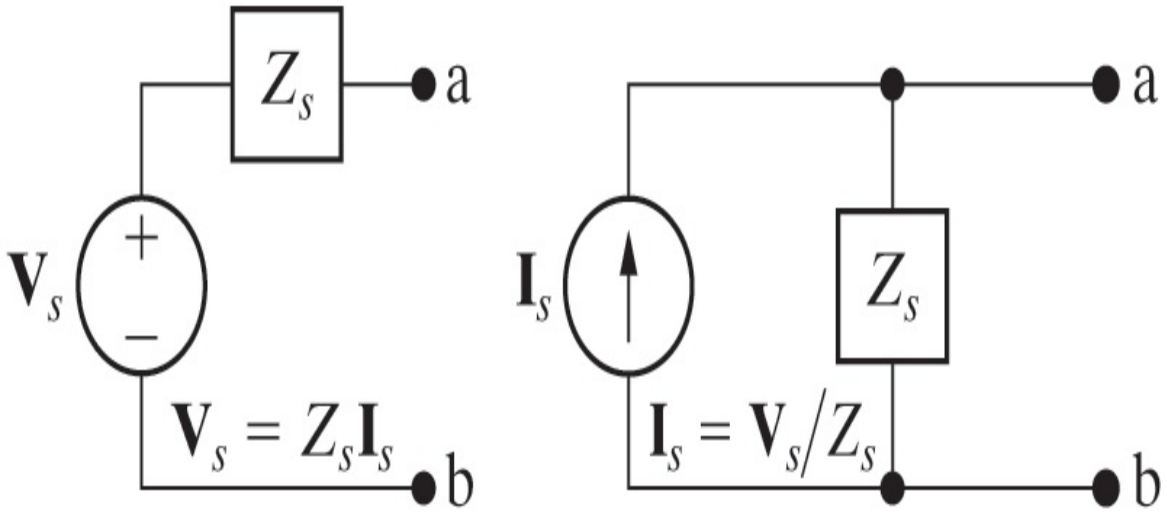


Figure 9.26 Full Alternative Text

Figure 9.27 illustrates the frequency-domain version of a Thévenin equivalent circuit, and Figure 9.28 shows the frequency-domain equivalent of a Norton equivalent circuit. The techniques for finding the Thévenin equivalent voltage and impedance are identical to those used for resistive circuits, except that the frequency-domain equivalent circuit involves phasors and complex numbers. The same holds for finding the Norton equivalent current and impedance.

Figure 9.27 The frequency-domain version of a Thévenin equivalent circuit.

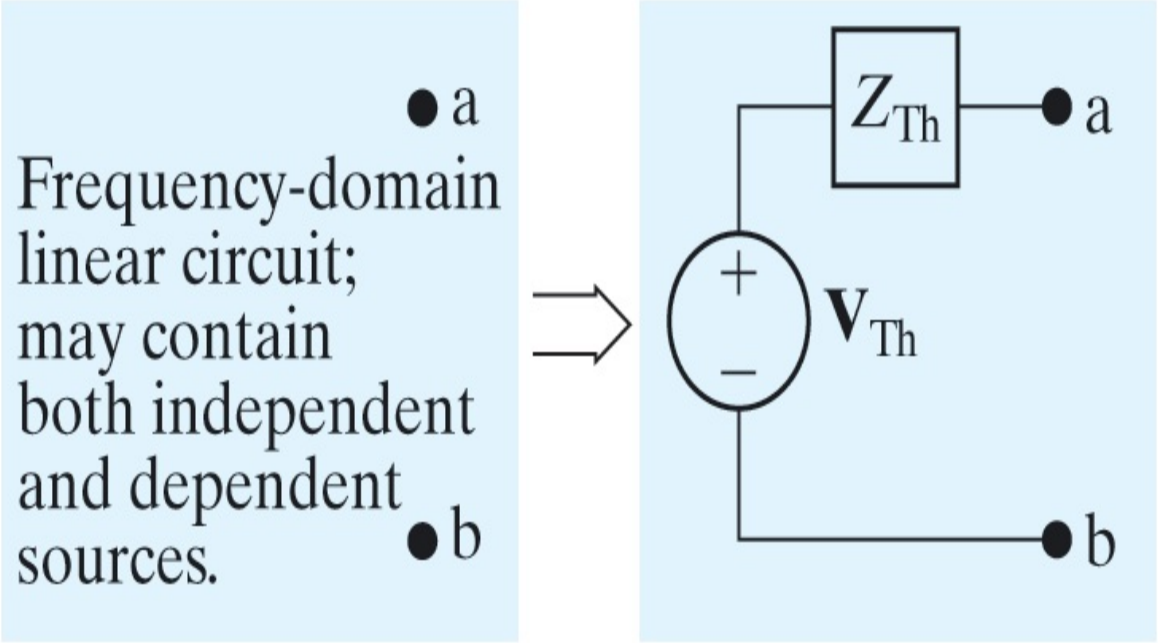


Figure 9.27 Full Alternative Text

Figure 9.28 The frequency-domain version of a Norton

equivalent circuit.

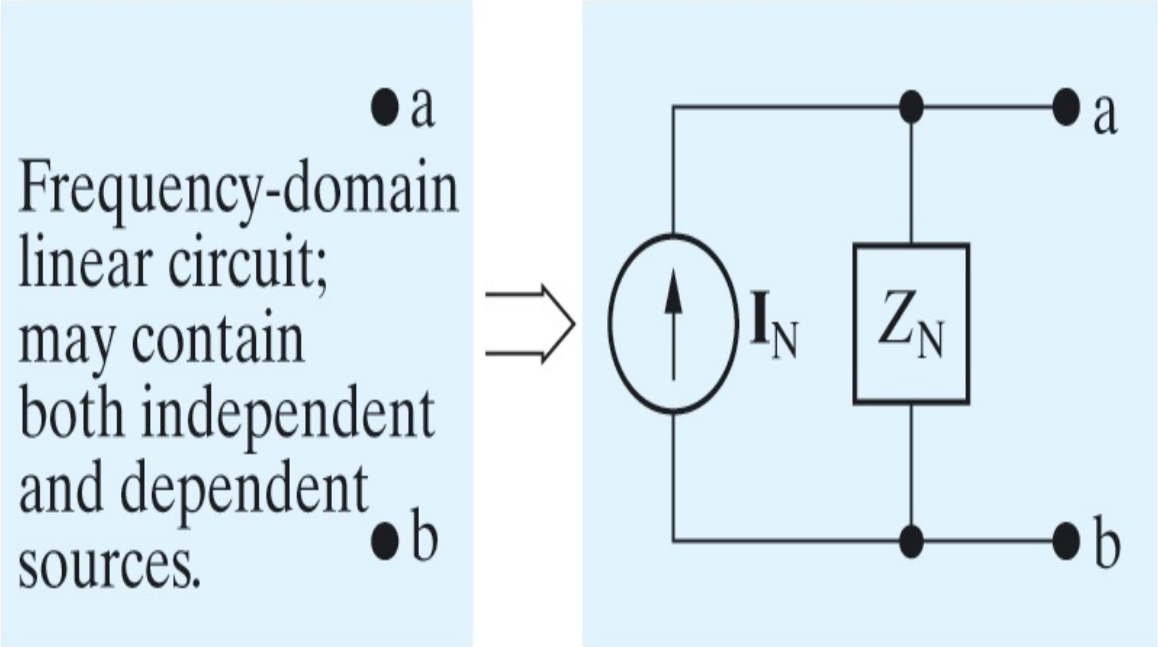


Figure 9.28 Full Alternative Text

Example 9.11 demonstrates the application of the source-transformation equivalent circuit to frequency-domain analysis. Example 9.12 illustrates the details of finding a Thévenin equivalent circuit in the frequency domain.

Example 9.11 Performing Source Transformations in the Frequency Domain

Use a series of source transformations to find the phasor voltage V0 in the circuit shown in Fig. 9.29.

Figure 9.29 The circuit for

Example 9.11.

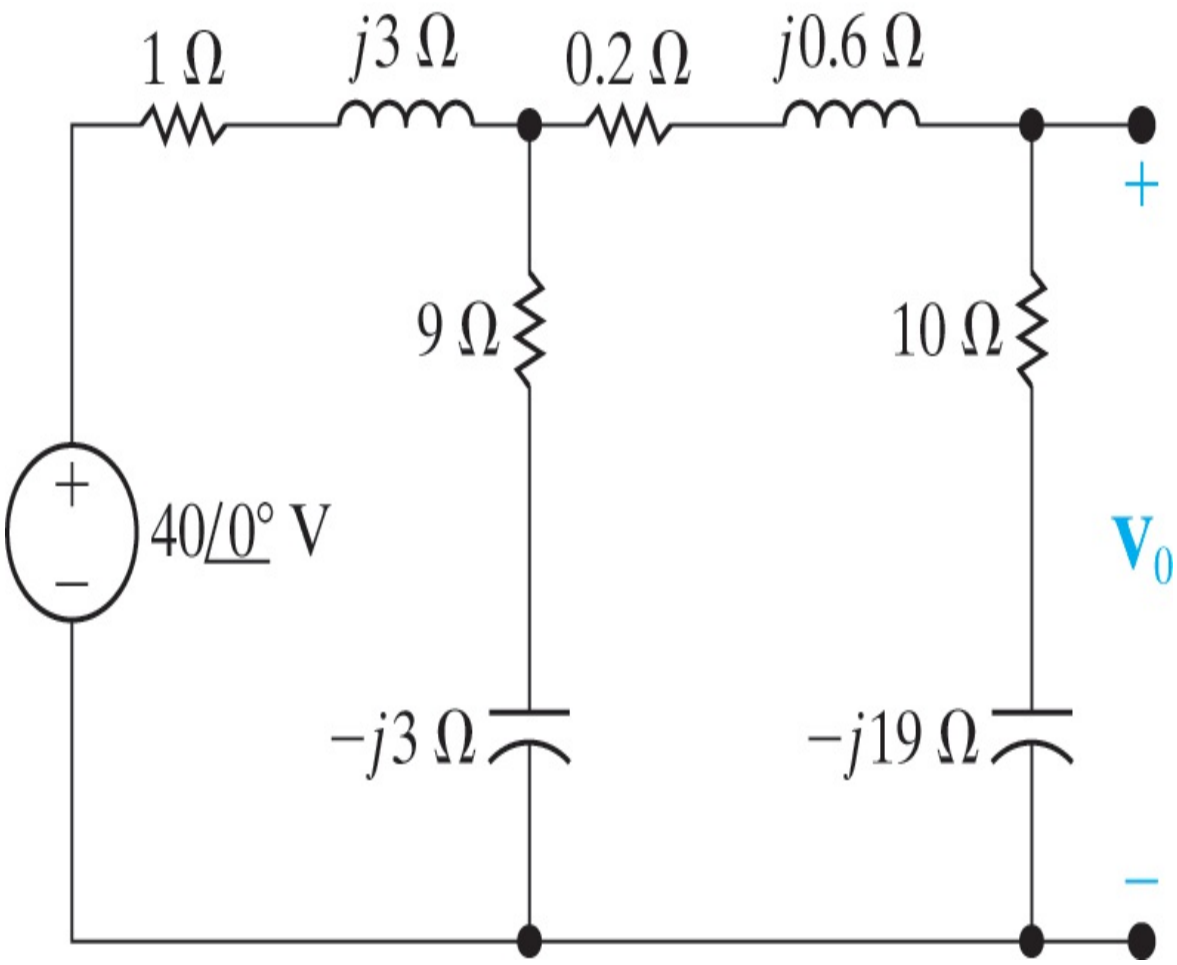


Figure 9.29 Full Alternative Text

Solution

Begin by replacing the series combination of the voltage source (40 0°) and the impedance of 1+j3 Ω with the parallel combination of a current source and the 1+j 3 Ω impedance. The current source is

I= 40 1+j 3 =4−j12 A.

The resulting circuit is shown in Fig. 9.30. We used the polarity of the 40 V

source to determine the direction for **I**.

Figure 9.30 The first step in reducing the circuit shown in Fig. 9.29.

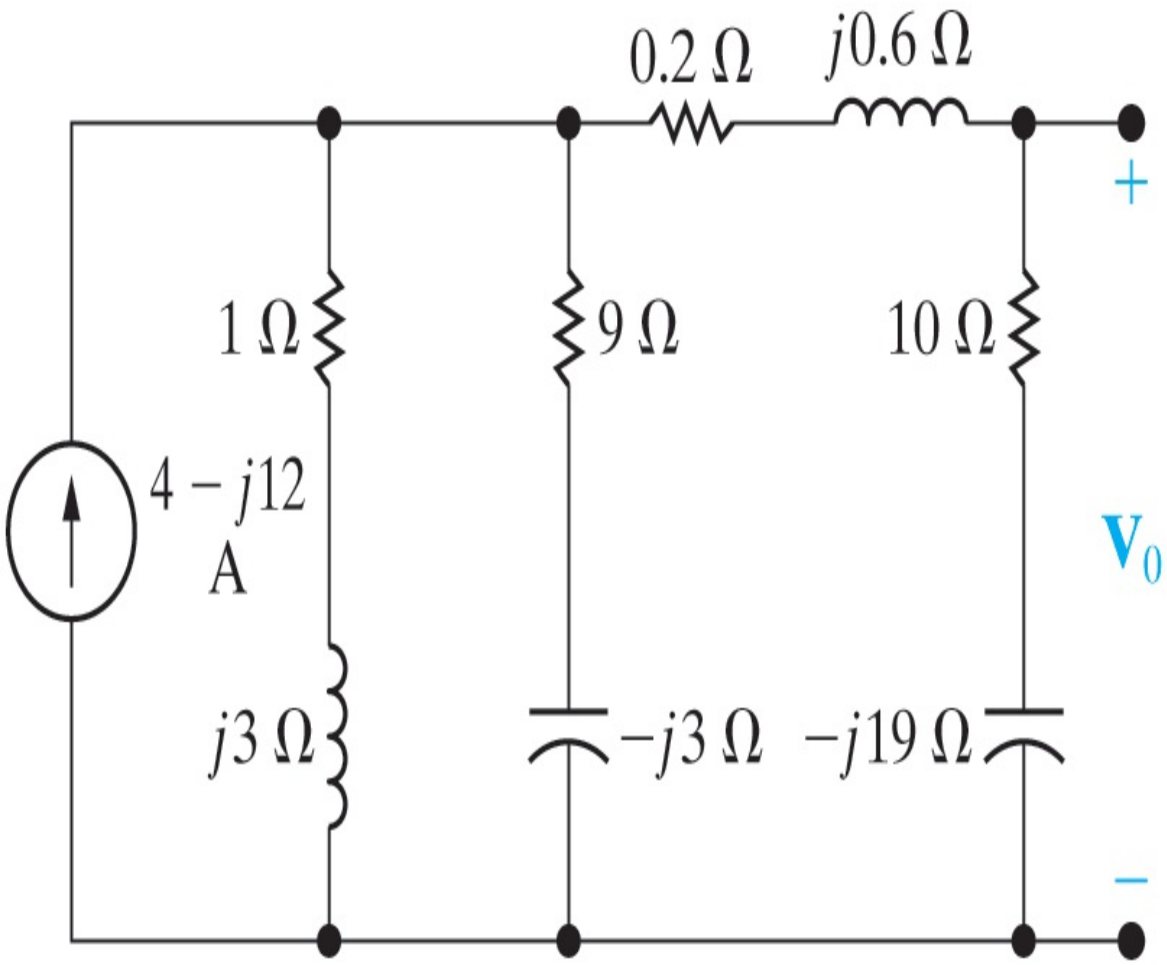


Figure 9.30 Full Alternative Text

Next, we combine the two parallel branches into a single impedance,

$Z=(1+j\ 3)(9-j3)10=1.8+j2.4\ \Omega$,

which is in parallel with the current source of 4−j12 A. Another source transformation converts this parallel combination to a series combination of a voltage source and the impedance of 1.8+j2.4 Ω. The voltage of the voltage source is

$$V=(4-j12)(1.8+j2.4)=36-j12 \text{ V}.$$

The resulting circuit is shown in Fig. 9.31. We added the current I0 to this circuit to assist us in finding V0.

Figure 9.31 The second step in reducing the circuit shown in Fig. 9.29.

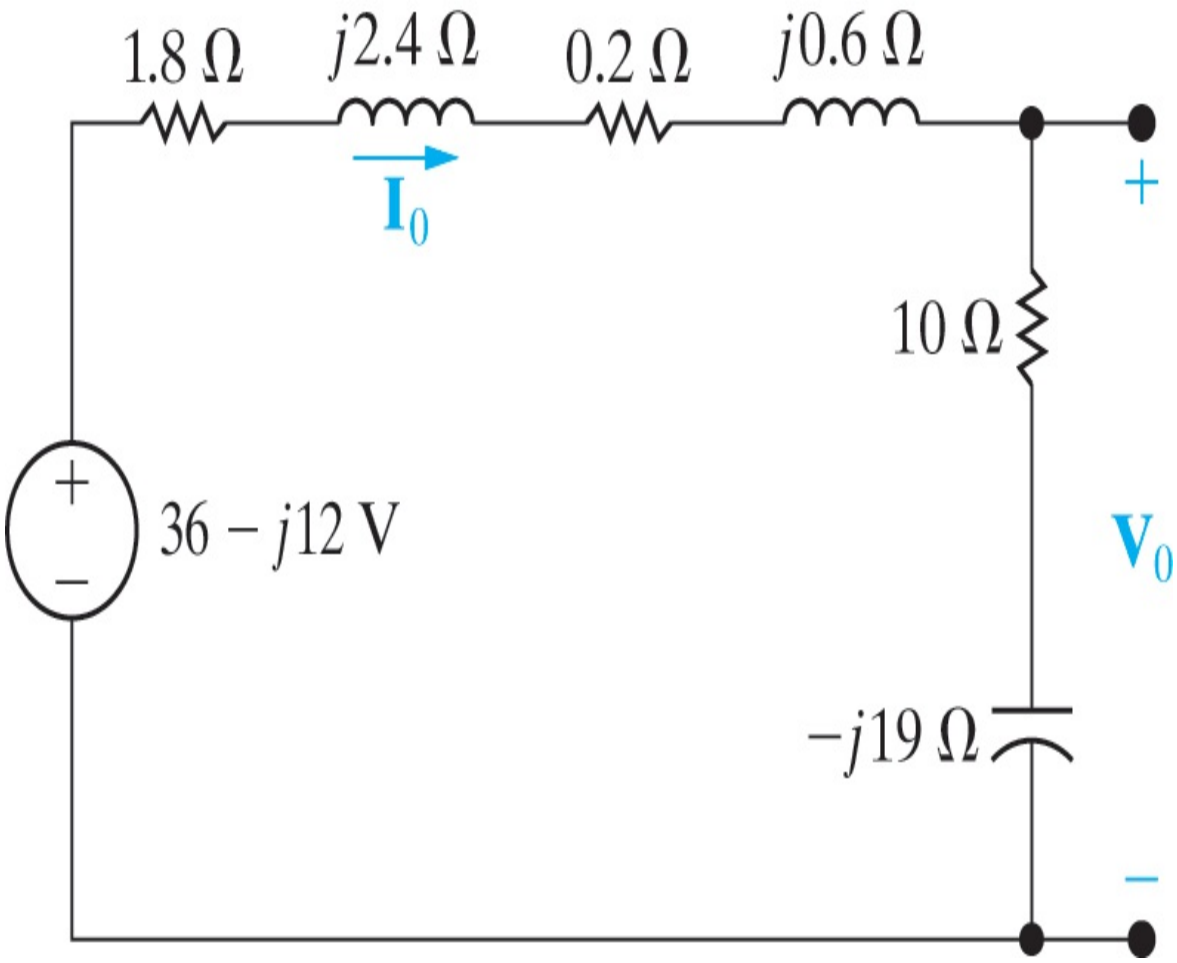


Figure 9.31 Full Alternative Text

We have now reduced the circuit to a simple series connection. We calculate the current I0 by dividing the voltage of the source by the total series impedance:

I 0 = 36−j12 12−j16 = 1.56+j1.08 A.

Now multiply I0 by the impedance 10−j19 to get V0

V0=(1.56+j1.08)(10−j19)=36.12−j18.84 V.

You can verify this result by using voltage division to calculate V0.

Example 9.12 Finding a Thévenin

Equivalent in the Frequency Domain

Find the Thévenin equivalent circuit with respect to terminals a,b for the circuit shown in Fig. 9.32.

Figure 9.32 The circuit for Example 9.12.

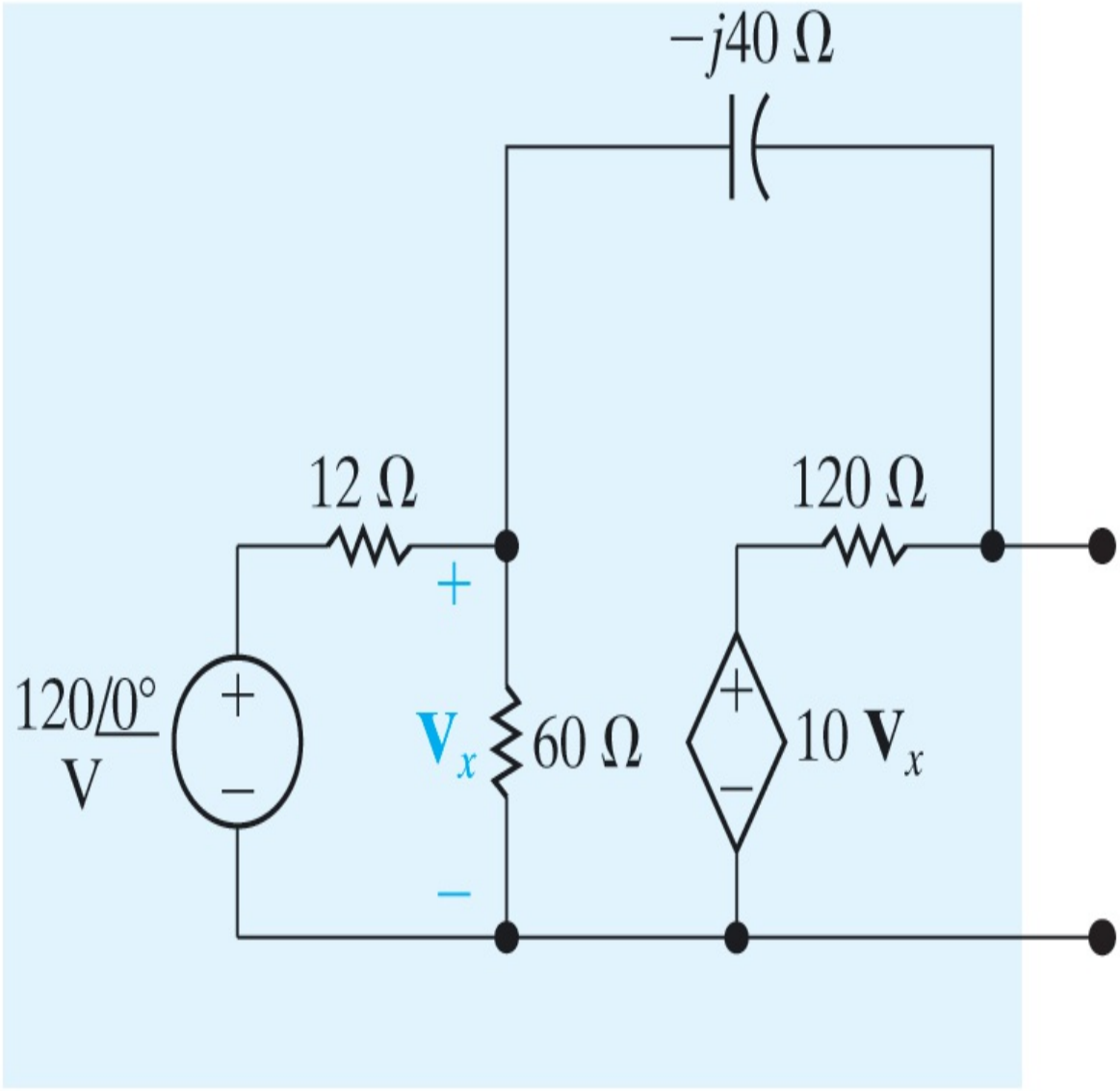


Figure 9.32 Full Alternative Text

Solution

We first determine the Thévenin equivalent voltage. This voltage is the open-circuit voltage appearing at terminals a,b. We choose the reference for the Thévenin voltage as positive at terminal a. We can make two source transformations using the 120 V, 12 Ω, and 60 Ω circuit elements to simplify the left-hand side of the circuit. These transformations must preserve the identity of the controlling voltage Vx because of the dependent voltage

source.

The first source transformation replaces the series combination of the 120 V source and 12 Ω resistor with a 10 A current source in parallel with 12 Ω. Next, we replace the parallel combination of the 12 and 60 Ω resistors with a single 10 Ω resistor. Finally, we replace the parallel-connected 10 A source and 10 Ω resistor with a 100 V source in series with 10 Ω. Figure 9.33 shows the resulting circuit.

Figure 9.33 A simplified version of the circuit shown in Fig. 9.32.

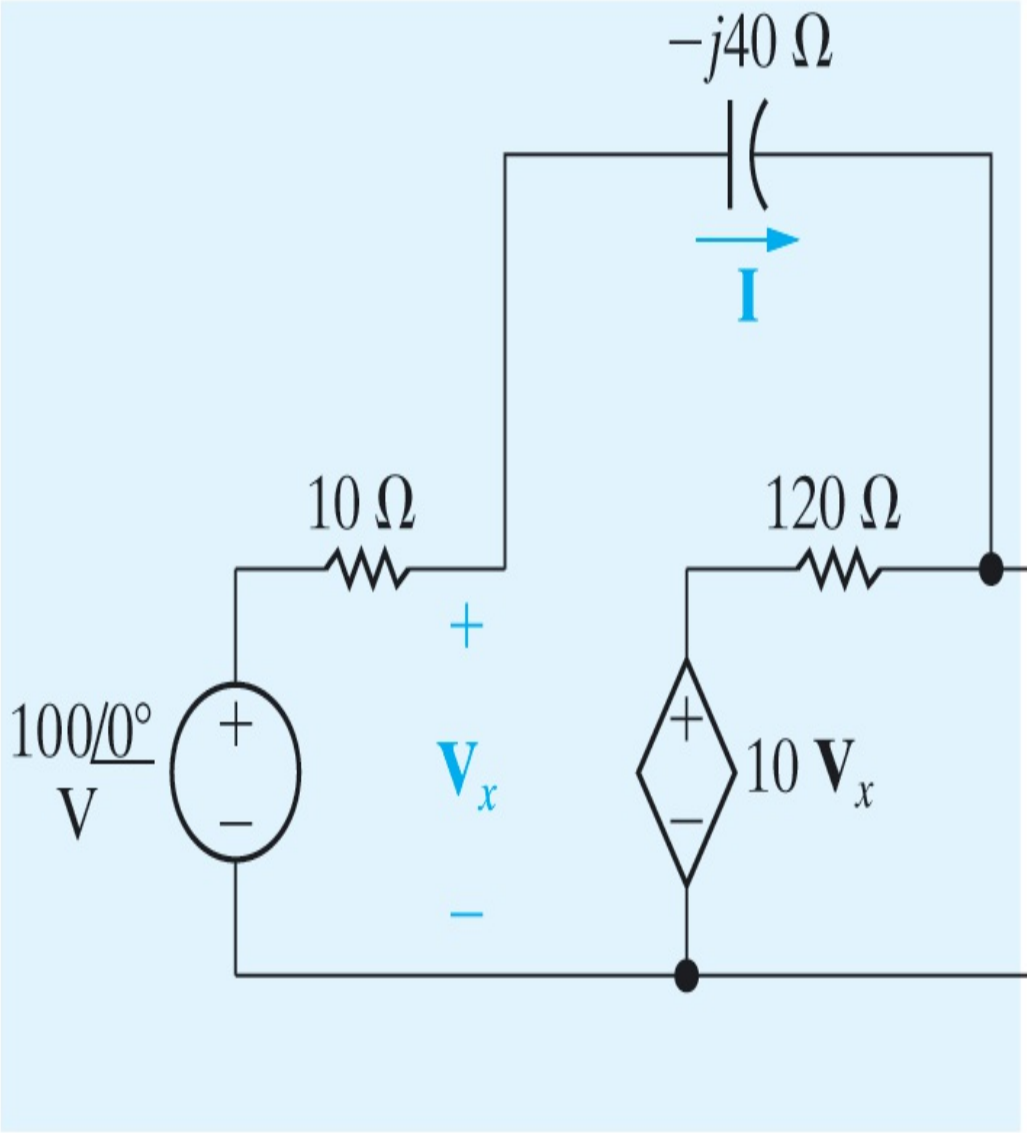


Figure 9.33 Full Alternative Text

We added the current **I** to Fig. 9.33; note that once we know its value, we can compute the Thévenin voltage. Use KVL to find **I** by summing the voltages around the closed path in the circuit. Hence

$100=10I-j40I+120I+10Vx=(130-j40)I+10Vx.$

We relate the controlling voltage Vx to the current **I** by noting from Fig. 9.33 that

$Vx=100-10I.$

Then,

$$I = \frac{-90}{30 - j40} = -10.8 - j14.4 \text{ A.}$$

Finally, we note from Fig. 9.33 that

$$V_{Th} = 10V_x + 120I = 10(100 - 10I) + 120I = 1000 + 20(-10.8 - j14.4) = 784 - j288 \text{ V.}$$

We can find the Thévenin impedance using any of the techniques in Sections 4.10–4.11 for finding Thévenin resistance. We use the test-source method in this example. We begin by deactivating all independent sources in the circuit, and then we apply either a test-voltage source or a test-current source to the terminals of interest. The ratio of the voltage to the current at the test source is the Thévenin impedance. Figure 9.34 presents the result of applying this technique to the circuit shown in Fig. 9.32 while preserving the identity of Vx.

Figure 9.34 A circuit for calculating the Thévenin equivalent impedance.

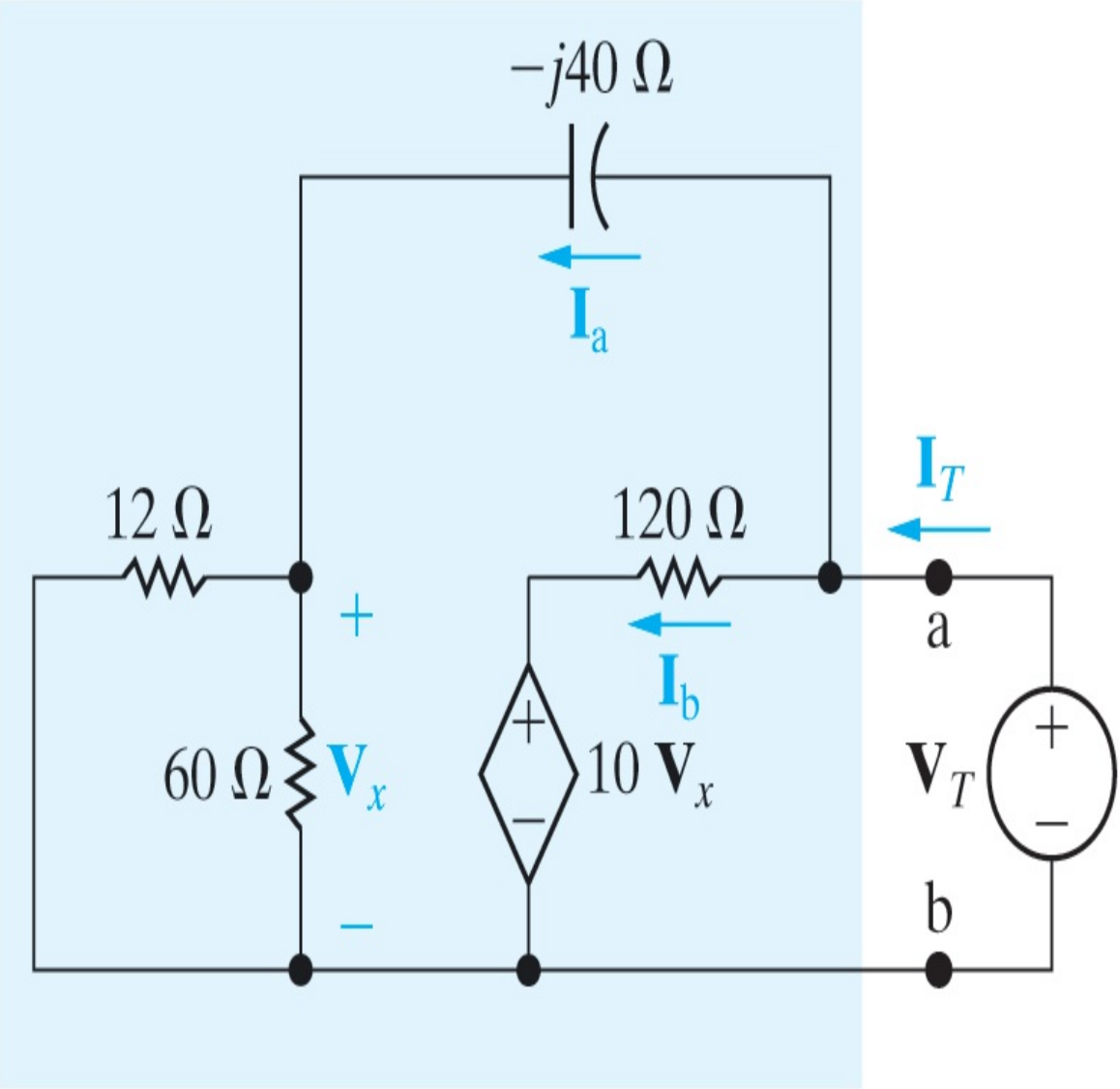


Figure 9.34 Full Alternative Text

We added branch currents Ia and Ib to simplify the calculation of IT. You should verify the following relationships by applying Ohm’s law, KVL, and KCL for phasors:

I a = V T 10-j40 , V x =10 I a , I b = V T -10 V x 120 = - V T (9+j4) 120(1-j4) , I T = I a + I b = V T 10-j40 ?(1- 9+j4 12) = V T (3-j4) 12(10-j40) , Z Th = V T I T =91.2-j38.4?O.

Figure 9.35 depicts the Thévenin equivalent circuit.

Figure 9.35 The Thévenin equivalent for the circuit shown in Fig. 9.32.

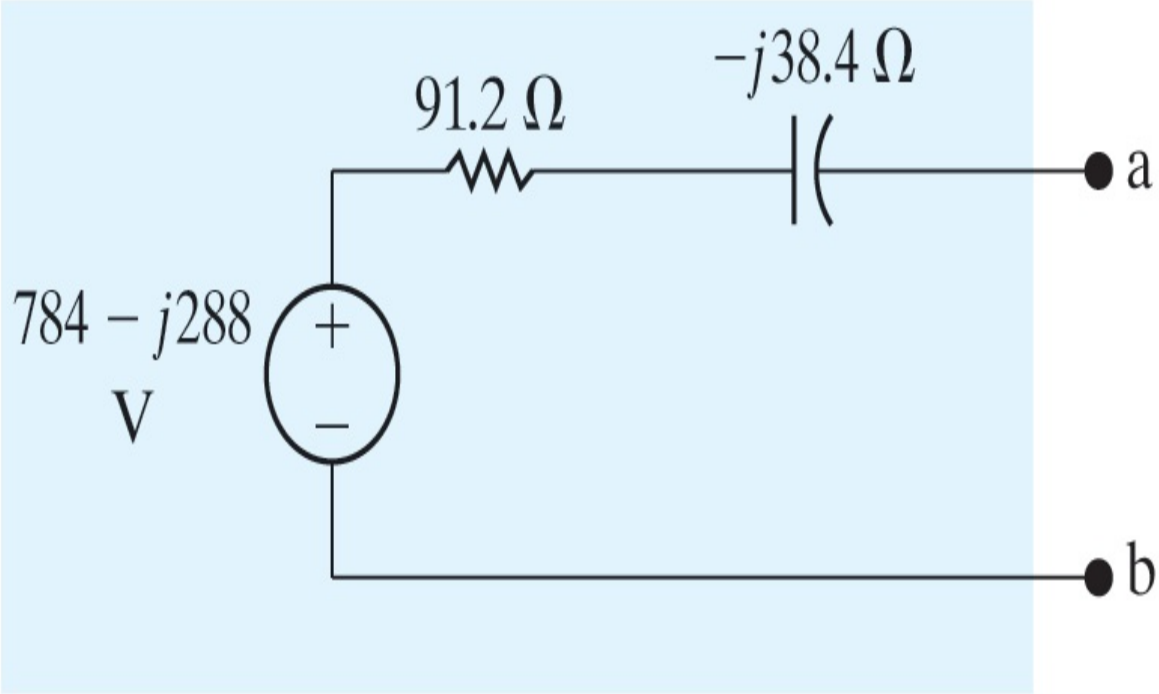


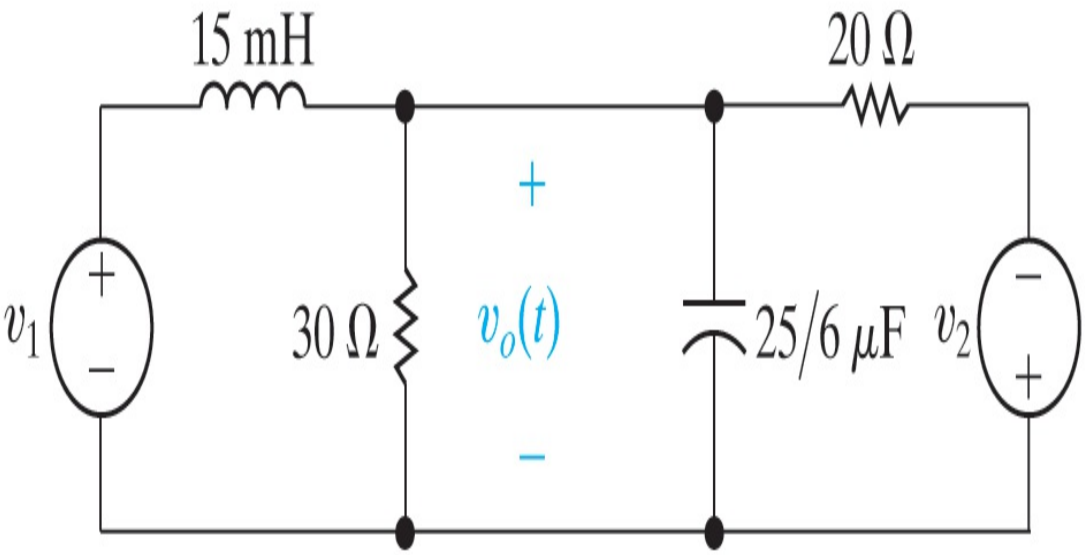
Figure 9.35 Full Alternative Text

Assessment Problems

Objective 3—Know how to use circuit analysis techniques to solve a circuit in the frequency domain

1. 9.10 Find the steady-state expression for vo(t) in the circuit shown by using source transformations. The sinusoidal voltage sources are

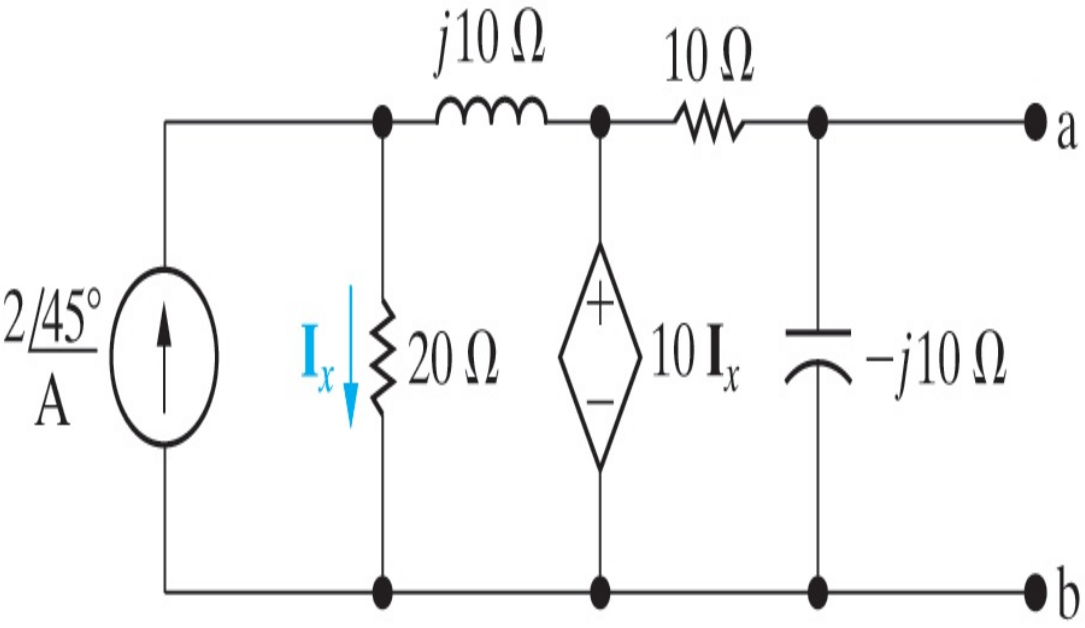
   v1=240 cos(4000t+53.13°)V,v2=96 sin4000tV.



9.7-9 Full Alternative Text

Answer: 48 cos(4000t + 36.87°) V.

2. 9.11 Find the Thévenin equivalent with respect to terminals a,b in the circuit shown.



9.7-10 Full Alternative Text

Answer:

$V_{Th} = V_{ab} = 10 \angle 45°$ V ; $Z_{Th} = 5 - j5\ \Omega$.

SELF-CHECK: Also try *Chapter Problems 9.43, 9.44, and 9.48.*

9.8 The Node-Voltage Method

In Sections 4.2–4.4, we introduced the node-voltage method of circuit analysis, culminating in Analysis Method 4.3 (p. 102). We can use this analysis method to find the steady-state response for circuits with sinusoidal sources. We need to make a few modifications:

- If the circuit is in the time domain, it must be transformed to the appropriate frequency domain. To do this, transform known voltages and currents to phasors, replace unknown voltages and currents with phasor symbols, and replace the component values for resistors, inductors, mutually coupled coils, and capacitors with their impedance values.

- Follow the steps in Analysis Method 4.3 to find the values of the unknown voltage and current phasors of interest.

- Apply the inverse phasor transform to the voltage and current phasors to find the steady-state values of the corresponding voltages and currents in the time domain.

Example 9.13 illustrates these steps. Assessment Problem 9.12 and many of the chapter problems give you an opportunity to use the node-voltage method to solve for steady-state sinusoidal responses.

Example 9.13 Using the Node-Voltage Method in the Frequency Domain

Use the node-voltage method to find the branch currents ia, ib, and ic in the steady-state, for the circuit shown in Fig. 9.36. The value of the current source in this circuit is is=10.6 cos(500t) A.

Figure 9.36 The circuit for Example 9.13.

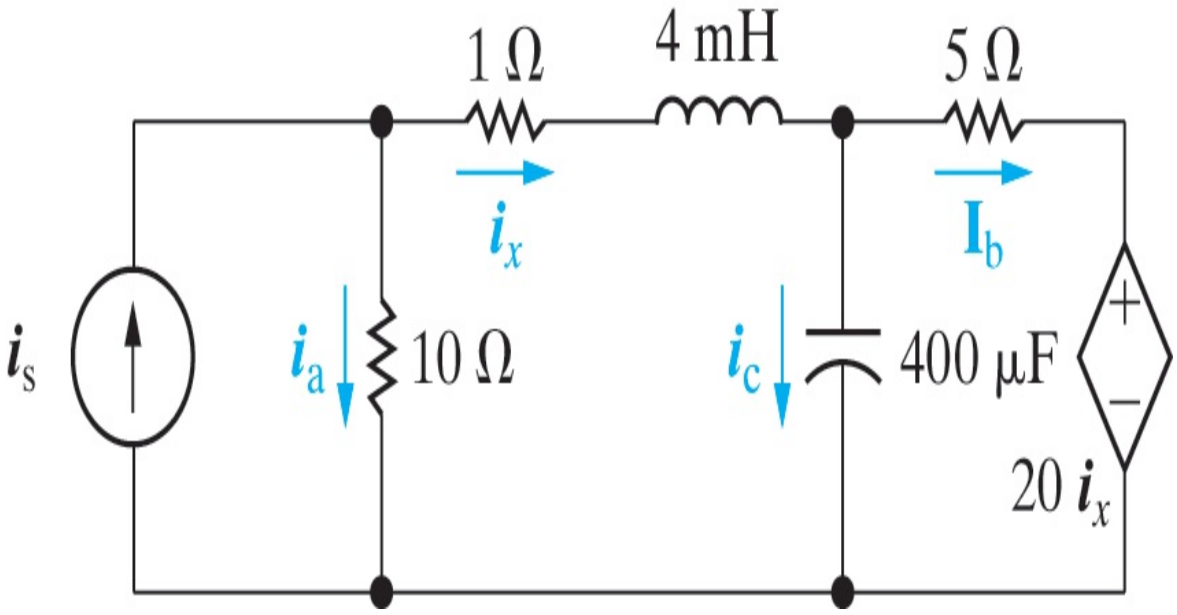


Figure 9.36 Full Alternative Text

Solution

We begin by transforming the circuit into the frequency domain. To do this, we replace the value of the current source with its phasor transform, 10.6 0° A. We also replace the currents ia, ib, ic, and ix with corresponding phasor symbols Ia, Ib, Ic, and Ix. Then we replace the inductor and capacitor values with their impedances, using the frequency of the source:

ZL=j(500)(4×10-3)=j2Ω;ZC=-j(500)(400×10-6)=-j5Ω.

The resulting frequency-domain circuit is shown in Fig. 9.37.

Figure 9.37 The circuit in Fig.

9.36, transformed into the frequency domain.

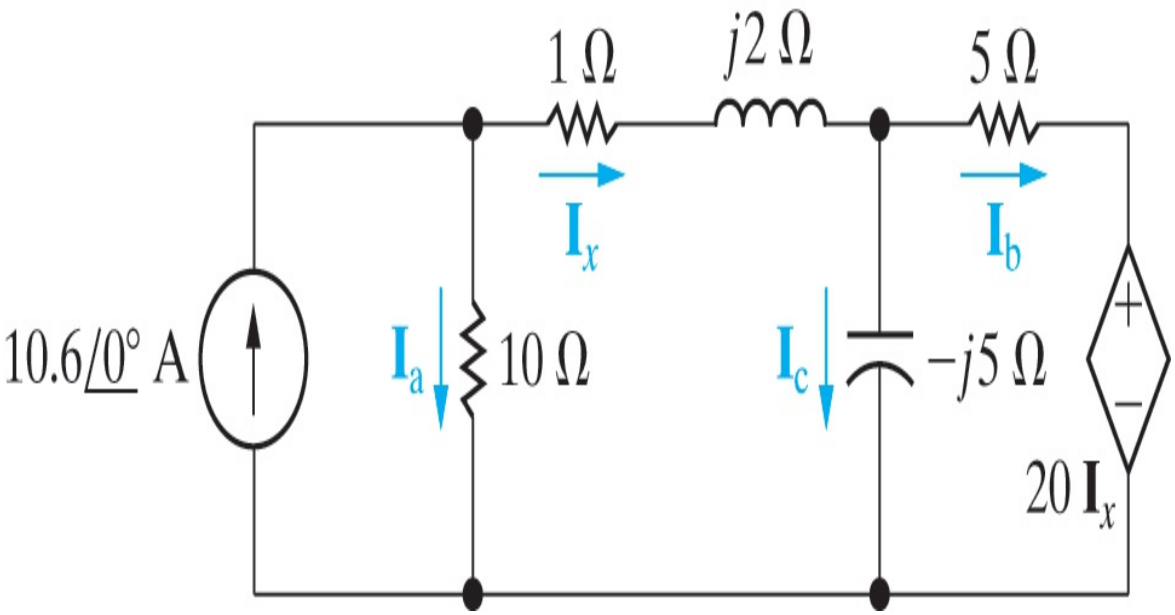


Figure 9.37 Full Alternative Text

Now we can employ Analysis Method 4.3

1. Step 1: The circuit has three essential nodes, two at the top and one on the bottom. We will need two KCL equations to describe the circuit.

2. Step 2: Four branches terminate on the bottom node, so we select it as the reference node and label the node voltages at the remaining essential nodes. The results of the first two steps are shown in Fig. 9.38 .

Figure 9.38 The circuit shown in Fig. 9.37, with the node voltages defined.

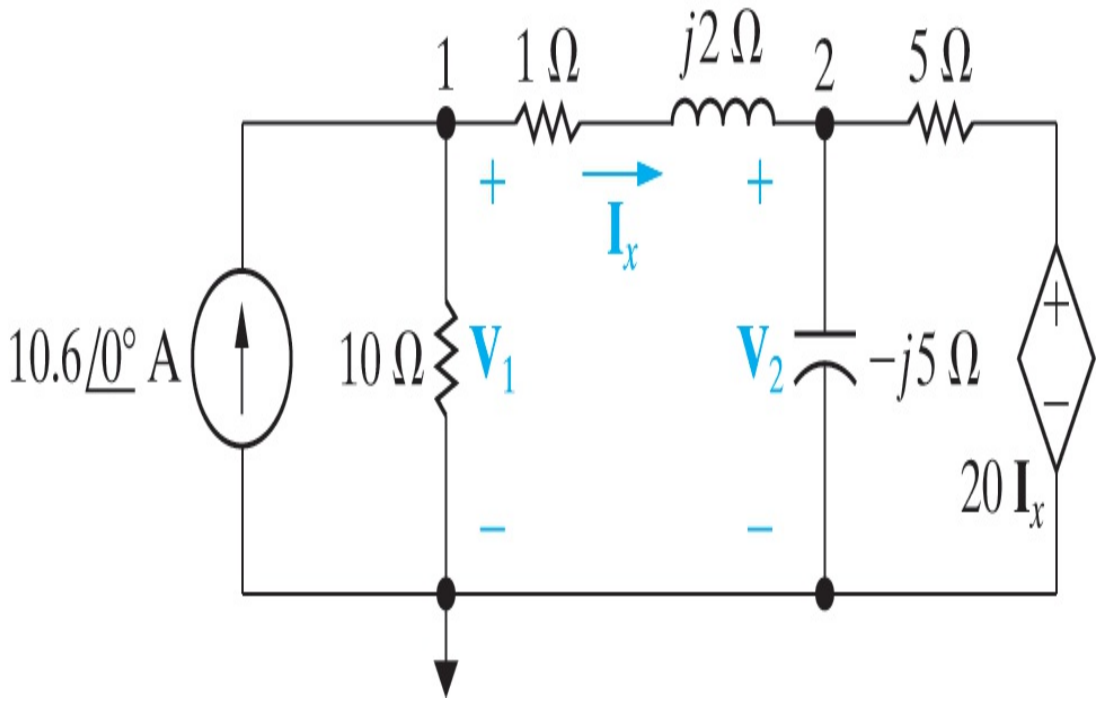


Figure 9.38 Full Alternative Text

3. Step 3: Apply KCL at the nonreference essential nodes to give

   $-10.6+\frac{V1}{10}+\frac{V1-V2}{1+j2}=0$

   and

   $\frac{V2-V1}{1+j2}+\frac{V2}{-j5}+\frac{V2-20Ix}{5}=0.$

   The circuit has a dependent source, so we need a dependent source constraint equation that defines Ix in terms of the node voltages:

   $Ix=\frac{V1-V2}{1+j2}.$

4. Step 4: Solve the three equations from Step 3 for V1, V2, and Ix:

   $V1=68.4-j16.8\ V, V2=68-j26\ V, Ix=3.76+j1.68\ A.$

5. Step 5: Use the phasor values from Step 4 to find the three branch currents from Fig. 9.37 :

I a = V 1 10 =6.84−j1.68 A=7.04 −13.8° A, I b = V 2 −20 I x 5 = −1.44−j11.92 A=12 −96.89° A, I c = V 2 −j5 =5.2+j13.6 A=14.56 69.08° A.

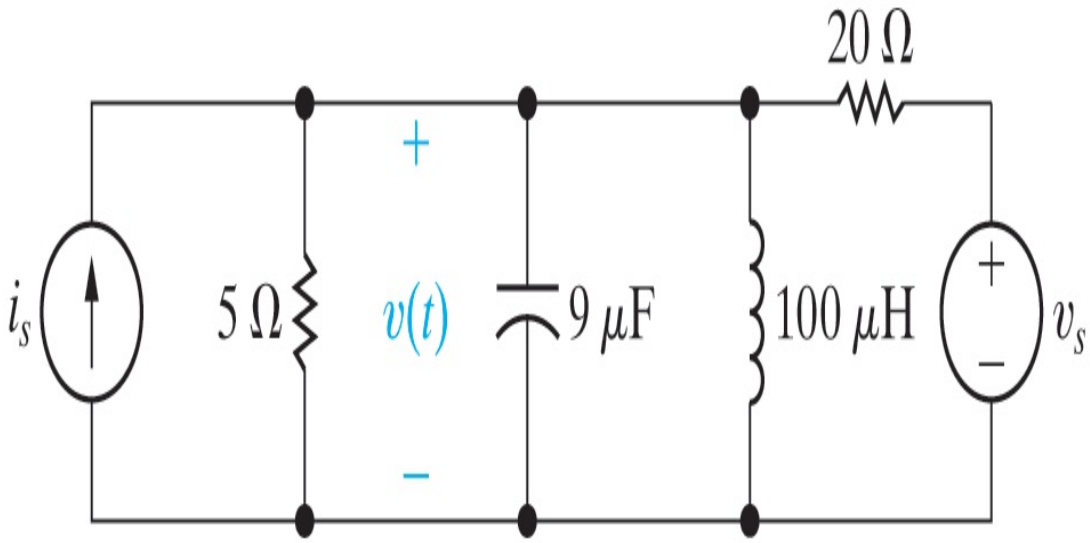
We find the steady-state values of the branch currents in the time-domain circuit of Fig. 9.37 by applying the inverse phasor transform to the results of Step 5. Remember that the frequency of the current source in the circuit is 500 rad/s. The results are

ia=7.04cos(500t-13.8°) A,ib=12cos(500t-96.89°) A,ic=14.56cos(500t+69.08°) A.

Assessment Problem

Objective 3—Know how to use circuit analysis techniques to solve a circuit in the frequency domain

1. 9.12 Use the node-voltage method to find the steady-state expression for v(t) in the circuit shown. The sinusoidal sources are is=10 cos ωt A and vs=100 sin ωt V, where ω=50 krad/s.



9.8-11 Full Alternative Text

Answer: $v(t)=31.62\cos(50{,}000t-71.57°)$ V.

SELF-CHECK: Also try *Chapter 9.55 and 9.59*.

9.9 The Mesh-Current Method

We can also use the mesh-current method to analyze frequency-domain circuits. If a problem begins with a circuit in the time domain, it needs to be transformed into the frequency domain. Then, Analysis Method 4.6 (p. 110) can be used to find the mesh-current phasors, just as it was used to find mesh currents in resistive circuits. Finally, apply the inverse phasor transform to the phasor voltages and currents to find the steady-state voltages and currents in the time domain. We use the mesh current method to analyze a frequency-domain circuit in Example 9.14.

Example 9.14 Using the Mesh-Current Method in the Frequency Domain

Use the mesh-current method to find the voltages V1, V2, and V3 in the circuit shown in Fig. 9.39.

Figure 9.39 The circuit for Example 9.14.

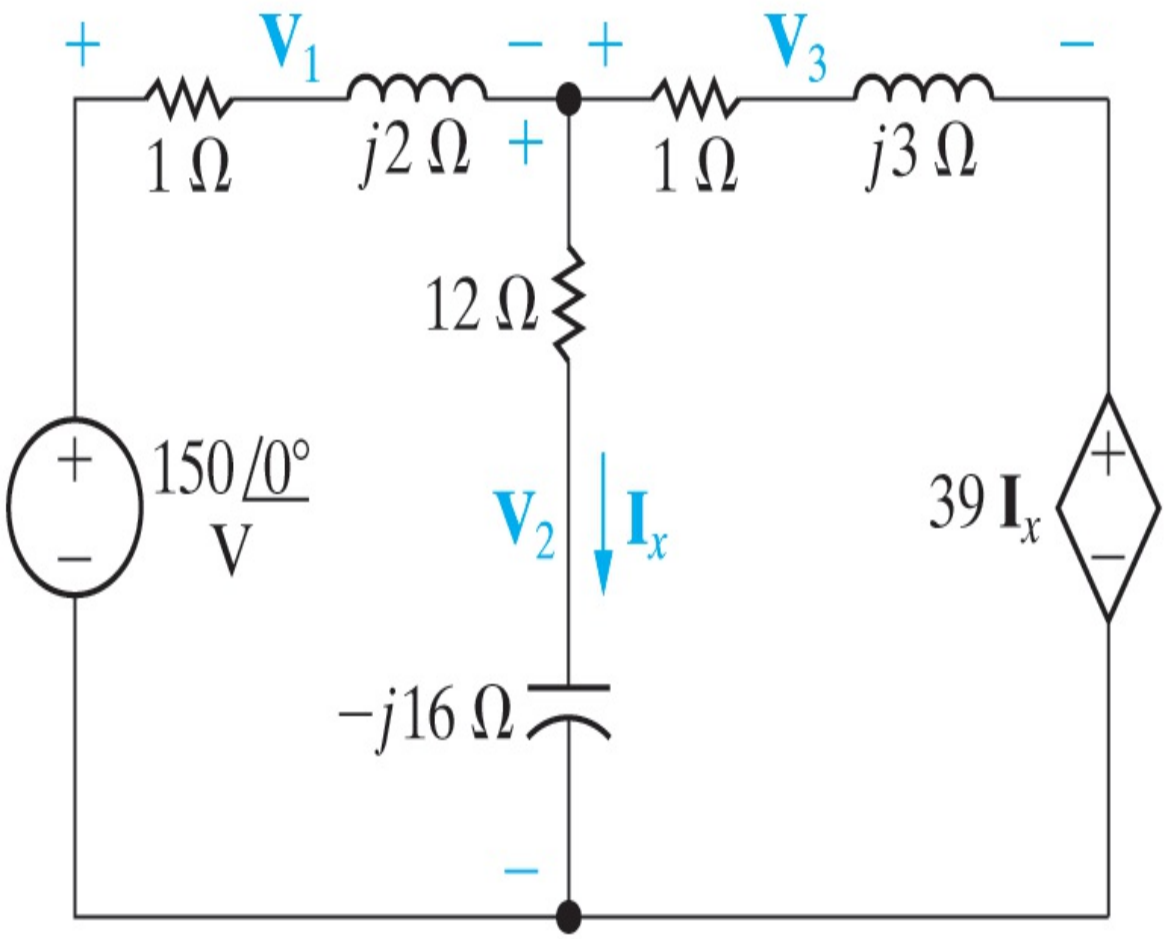


Figure 9.39 Full Alternative Text

Solution

The circuit is already in the frequency domain, so we apply Analysis Method 4.6

1. Step 1: Use directed arrows that traverse the mesh perimeters to identify the two mesh current phasors.

2. Step 2: Label the mesh current phasors as I1 and I2, as shown in Fig. 9.40 .

3. Step 3: Write the KVL equations for the meshes:

150=(1+j2)I1+(12−j16)(I1−I2),
0=(12−j16)(I2−I1)+(1+j3)I2+39Ix.

The circuit in Fig. 9.40 has a dependent source, so we need a constraint equation that defines Ix in terms of the mesh currents. The resulting equation is

Ix=I1−I2.

4. Step 4: Solving the simultaneous equations in Step 3 gives

   I1=-26-j52 A,I2=-24-j58 A,Ix=-2+j6 A.

5. Step 5: Finally, we use the mesh-current phasors from Step 4 to find the phasor voltages identified in the circuit of Fig. 9.39 :

   V1=(1+j2)I1=78-j104 V,V2=(12-j16)Ix=72+j104 V,V3=(1+j3)I2=150-j130 V.

   Also

   39Ix=−78+j234 V.

   We check these calculations by summing the voltages around closed paths:

   −150+ V 1 + V 2 = −150+78−j104+72 +j104 = 0, − V 2 + V 3 +39 I x = −72−j104+150−j130 −78+j234 = 0 −150+ V 1 + V 3 +39 I x = −150+78−j104+150 −j130−78+j234 = 0.

Figure 9.40 Mesh currents used to solve the circuit shown in Fig. 9.39.

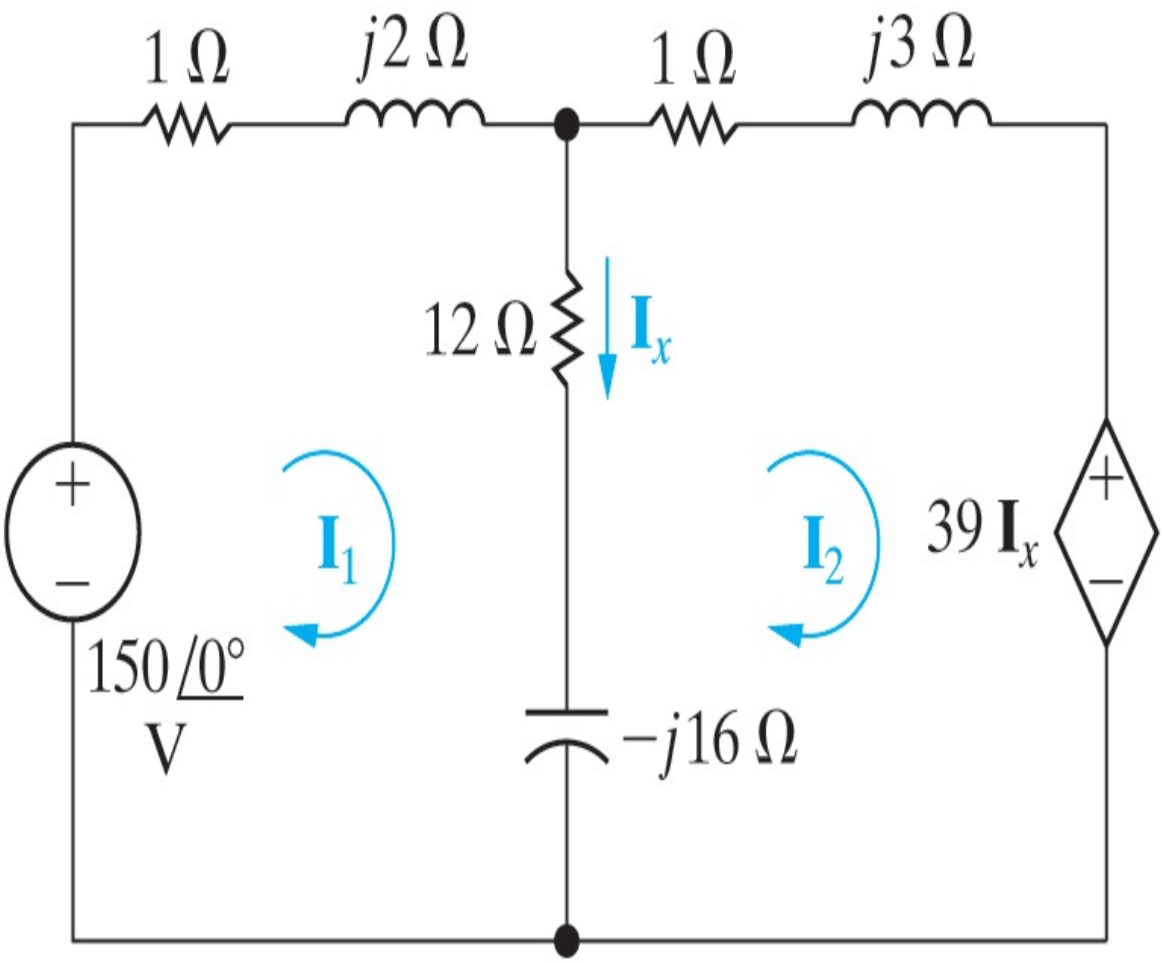
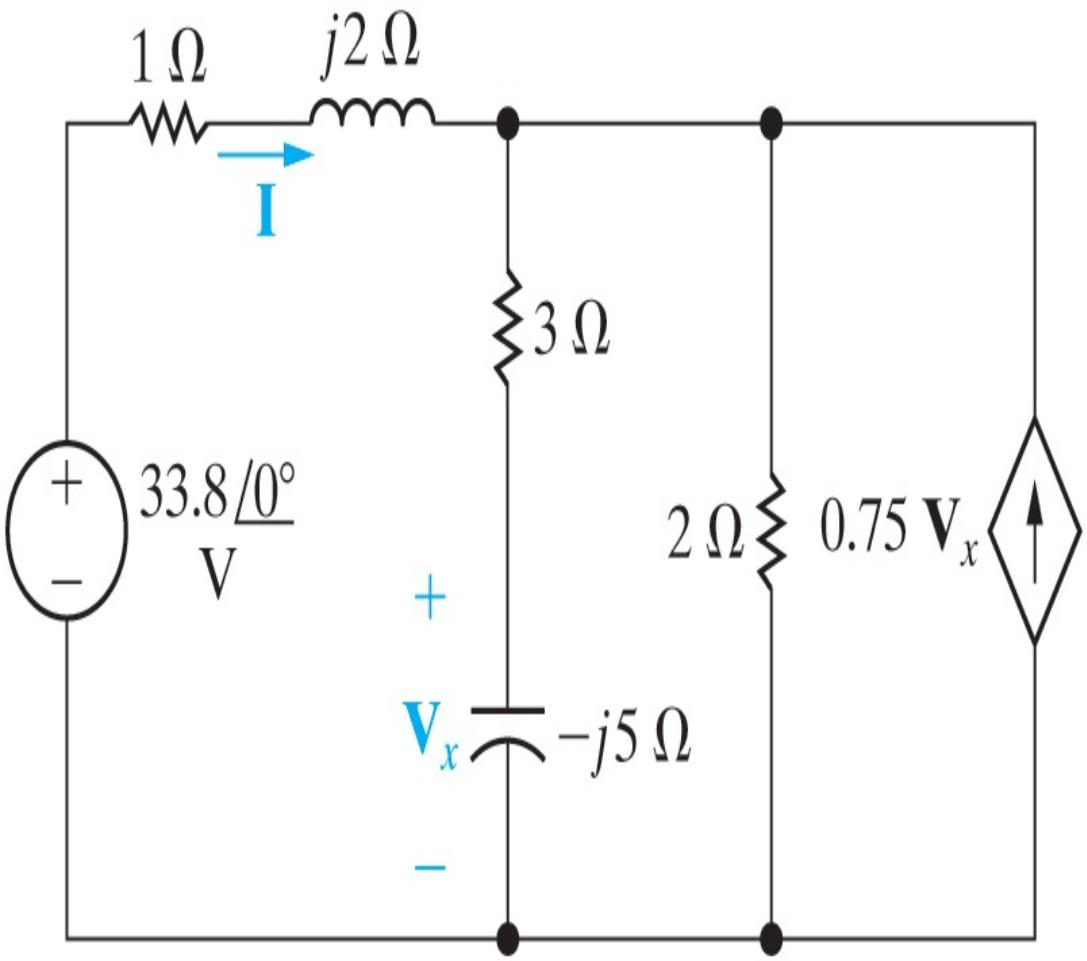


Figure 9.40 Full Alternative Text

Assessment Problem

Objective 3—Know how to use circuit analysis techniques to solve a circuit in the frequency domain

1. 9.13 Use the mesh-current method to find the phasor current **I** in the circuit shown.



9.9-12 Full Alternative Text

Answer:

29+j2=29.07 3.95° A.

SELF-CHECK: Also try *Chapter Problems 9.61 and 9.64*.

9.10 The Transformer

A transformer is a device based on the magnetic coupling that characterizes mutually coupled inductor coils. Transformers are used in both communication and power circuits. In communication circuits, the transformer is used to match impedances and eliminate dc signals from portions of the system. In power circuits, transformers are used to establish ac voltage levels that facilitate the transmission, distribution, and consumption of electrical power. We need to know how a transformer behaves in the sinusoidal steady state when analyzing both communication and power systems. In this section, we will discuss the sinusoidal steady-state behavior of the **linear transformer**, which is found primarily in communication circuits. In Section 9.11, we will present the **ideal transformer**, which is used to model the ferromagnetic transformers found in power systems.

When analyzing circuits containing mutually coupled inductor coils, we use the mesh-current method. The node-voltage method is hard to use when mutual inductance is present because the currents in the coupled coils cannot be written by inspection as functions of the node voltages.

The Analysis of a Linear Transformer Circuit

A simple **transformer** is formed when two coils are wound on a single core to ensure magnetic coupling. Figure 9.41 shows the frequency-domain circuit model of a system that uses a transformer to connect a load to a source. The transformer winding connected to the source is called the **primary winding**, and the winding connected to the load is called the **secondary winding**. The transformer circuit parameters are

- R1=the resistance of the primary winding,
- R2=the resistance of the secondary winding,

- L1=the self-inductance of the primary winding,
- L2=the self-inductance of the secondary winding,
- M=the mutual inductance.

Figure 9.41 The frequency domain circuit model for a transformer used to connect a load to a source.

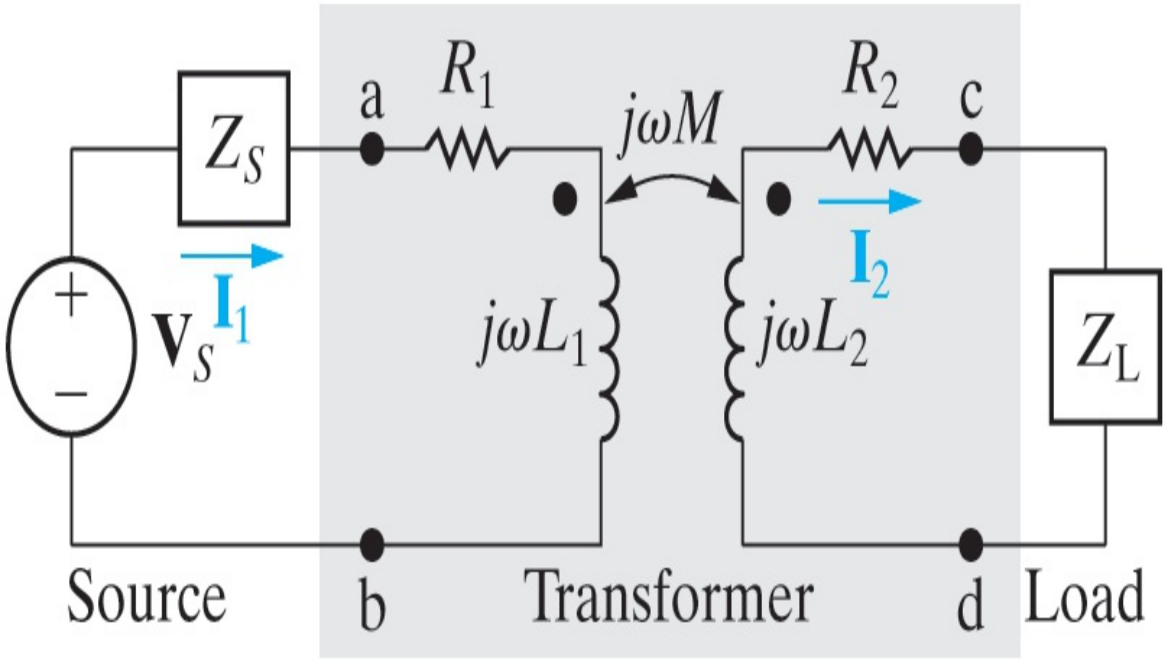


Figure 9.41 Full Alternative Text

The internal voltage of the sinusoidal source is Vs, and the internal impedance of the source is Zs. The impedance ZL represents the load connected to the secondary winding of the transformer. The phasor currents I1 and I2 are the primary and secondary currents of the transformer,

respectively.

We analyze the circuit in Fig. 9.41 to find I1 and I2 as functions of the circuit parameters Vs, Zs, R1, L1, L2, R2, M, ZL, and ω. Let's write the two KVL equations that describe the circuit:

$$Vs=(Zs+R1+j\omega L1)I1-j\omega MI2, 0=-j\omega MI1+(R2+j\omega L2+ZL)I2.$$

We define

$$Z_{11} = Z_s + R_1 + j\omega L_1 , \quad (9.27)$$
$$Z_{22} = R_2 + j\omega L_2 + Z_L , \quad (9.28)$$

where Z11 is the total self-impedance of the mesh containing the primary winding of the transformer and Z22 is the total self-impedance of the mesh containing the secondary winding. Using the impedances defined in Eqs. 9.27 and 9.28, we solve the mesh-current equations for I1 and I2to give

$$I1=Z22Z11Z22+\omega 2M2Vs, \quad (9.29)$$
$$I2=j\omega MZ11Z22+\omega 2M2Vs=j\omega MZ22I1. \quad (9.30)$$

We are also interested in finding the impedance seen when we look into the transformer from the terminals a and b. The internal source voltage VS is attached to an equivalent impedance whose value is the ratio of the source-voltage phasor to the primary current phasor, or

$$VsI1=Zint=Z11Z22+\omega 2M2Z22=Z11+\omega 2M2Z22.$$

The impedance at the terminals of the source is Zint − Zs, so

$$Zab=Z11+\omega 2M2Z22-Zs=R1+j\omega L1+\omega 2M2(R2+j\omega L2+ZL). \quad (9.31)$$

Note that the impedance Zab is independent of the magnetic polarity of the transformer because the mutual inductance M appears in Eq. 9.31 as a squared quantity. The impedance Zab is interesting because it describes how the transformer affects the impedance of the load as seen from the source. Without the transformer, the load would be connected directly to the source, and the source would see a load impedance of ZL; with the transformer, the load is connected to the source through the transformer, and the source sees a

load impedance that is a modified version of ZL, as seen in the third term of Eq. 9.31.

Reflected Impedance

The third term in Eq. 9.31 is called the **reflected impedance** (Zr) because it is the equivalent impedance of the secondary coil and load impedance transmitted, or reflected, to the primary side of the transformer. Note that the reflected impedance exists because of the mutual inductance. If the two coils are not coupled, M is zero, Zr is zero, and Zab is the self-impedance of the primary coil.

To consider reflected impedance in more detail, we first express the load impedance in rectangular form:

$$Z_L = R_L + jX_L,$$

where the load reactance XL carries its own algebraic sign. That is, XL is a positive number if the load is inductive and a negative number if the load is capacitive. We can now write the reflected impedance in rectangular form:

$$Z_r = \omega^2 M^2 R_2 + R_L + j(\omega L_2 + X_L) = \omega^2 M^2 [(R_2 + R_L) - j(\omega L_2 + X_L)] (R_2 + R_L)^2 + (\omega L_2 + X_L)^2 = \omega^2 M^2 |Z_{22}|^2 [(R_2 + R_L) - j(\omega L_2 + X_L)]. \quad 9.32$$

The derivation of Eq. 9.32 uses the fact that, when ZL is written in rectangular form, the self-impedance of the mesh containing the secondary winding is

$$Z_{22} = R_2 + R_L + j(\omega L_2 + X_L).$$

In Eq. 9.32 we see that the self-impedance of the secondary circuit is reflected into the primary circuit by a scaling factor of $(\omega M/|Z_{22}|)^2$, and that the sign of the reactive component $(\omega L_2 + X_L)$ is reversed. Thus, the linear transformer reflects the complex conjugate of the self-impedance of the secondary circuit (Z_{22}^*) into the primary winding with a scalar multiplier.

Example 9.15 analyzes a circuit with a linear transformer.

Example 9.15 Analyzing a Linear Transformer in the Frequency Domain

The parameters of a linear transformer are R1=200 Ω, R2=100 Ω, L1=9 H, L2=4 H, and k=0.5. The transformer couples a load impedance with an 800 Ω resistor in series with a 1 μF capacitor to a sinusoidal voltage source. The 300 V (rms) source has an internal impedance of 500+j100 Ω and a frequency of 400 rad/s.

1. Construct a frequency-domain equivalent circuit of the system.
2. Calculate the self-impedance of the primary circuit.
3. Calculate the self-impedance of the secondary circuit.
4. Calculate the impedance reflected into the primary winding.
5. Calculate the scaling factor for the reflected impedance.
6. Calculate the impedance seen looking into the primary terminals of the transformer.
7. Calculate the Thévenin equivalent with respect to the terminals of the load impedance.

Solution

1. Figure 9.42 shows the frequency-domain equivalent circuit. Note that the internal voltage of the source serves as the reference phasor because it is assigned a phase angle of 0°, and that V1 and V2 represent the

terminal voltages of the transformer. In constructing the circuit in Fig. 9.42, we made the following calculations:

jω L 1 = j(400)(9)=j3600 Ω, jω L 2 = j(400)(4)=j1600 Ω, M = 0.5 (9)(4) =3 H, jωM = j(400)(3)=j1200 Ω, −j ωC = −j (400)(1× 10 −6) =−j2500 Ω.

Figure 9.42 The frequency-domain equivalent circuit for Example 9.15.

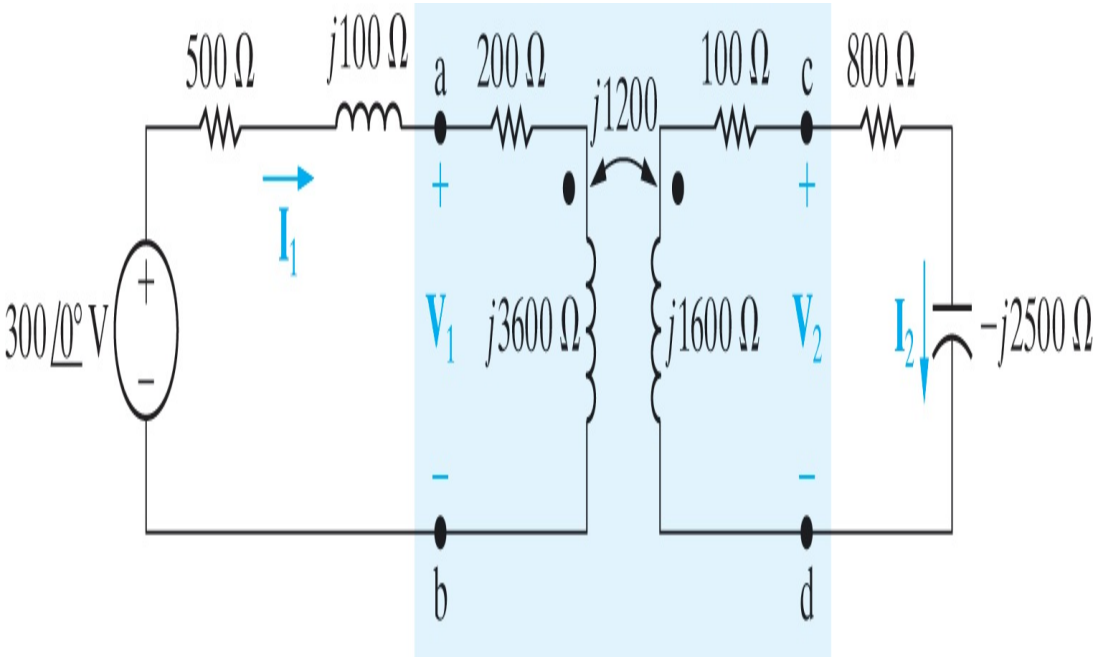


Figure 9.42 Full Alternative Text

2. From Eq. 9.27, the self-impedance of the primary circuit is

   Z11=500+j100+200+j3600=700+j3700 Ω.

3. From Eq. 9.28, the self-impedance of the secondary circuit is

$Z_{22}=100+j1600+800-j2500=900-j900\ \Omega$.

4. From Eq. 9.32, the impedance reflected into the primary winding is

   $Z_r=(1200|900-j900|)2(900+j900)=89\ (900+j900)=800+j800O$.

5. The scaling factor by which Z22* is reflected is 8/9.

6. The impedance seen looking into the primary terminals of the transformer is the impedance of the primary winding, Z11, plus the reflected impedance, Zr; thus

   $Z_{ab}=200+j3600+800+j800=1000+j4400\ \Omega$.

7. The Thévenin voltage is the open-circuit value of Vcd, which equals j1200 times the open-circuit value of I1. The open-circuit value of I1 is

   $I_1 = 300\ \angle 0° \ 700+j3700 = 79.67\ \angle -79.29°$ mA.

   Therefore

   $V_{Th} = j1200(79.67\ \angle -79.29°)\times 10^{-3} = 95.60\ \angle 10.71°$ V.

   The Thévenin impedance equals the impedance of the secondary winding, plus the impedance reflected from the primary when the voltage source is replaced by a short circuit. Thus

   $Z_{Th}=100+j1600+(1200|700+j3700|)2(700-j3700)=171.09+j1224.26\ \Omega$.

   The Thévenin equivalent is shown in Fig. 9.43.

Figure 9.43 The Thévenin equivalent circuit for Example 9.15.

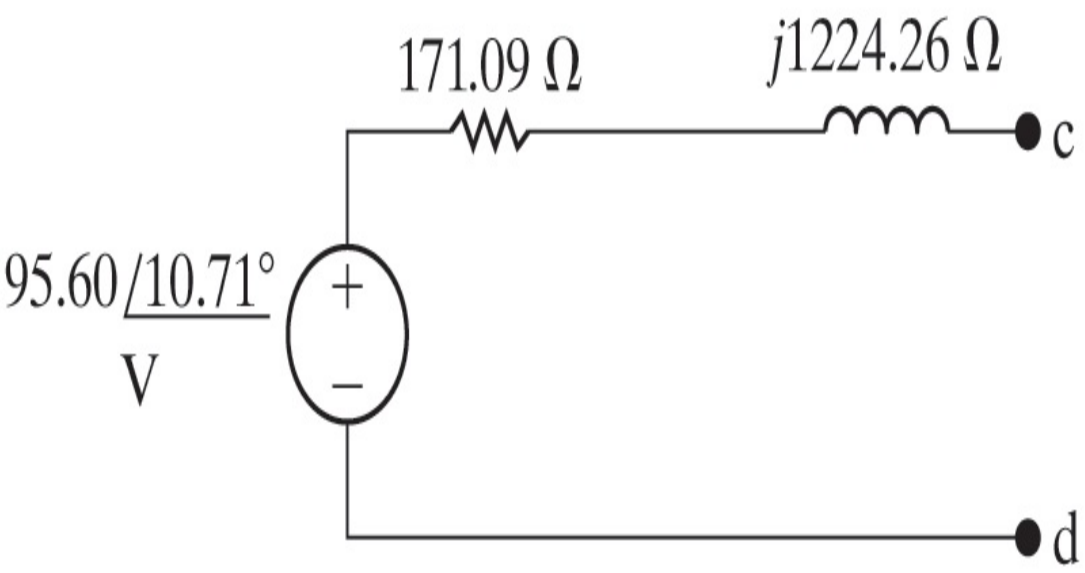
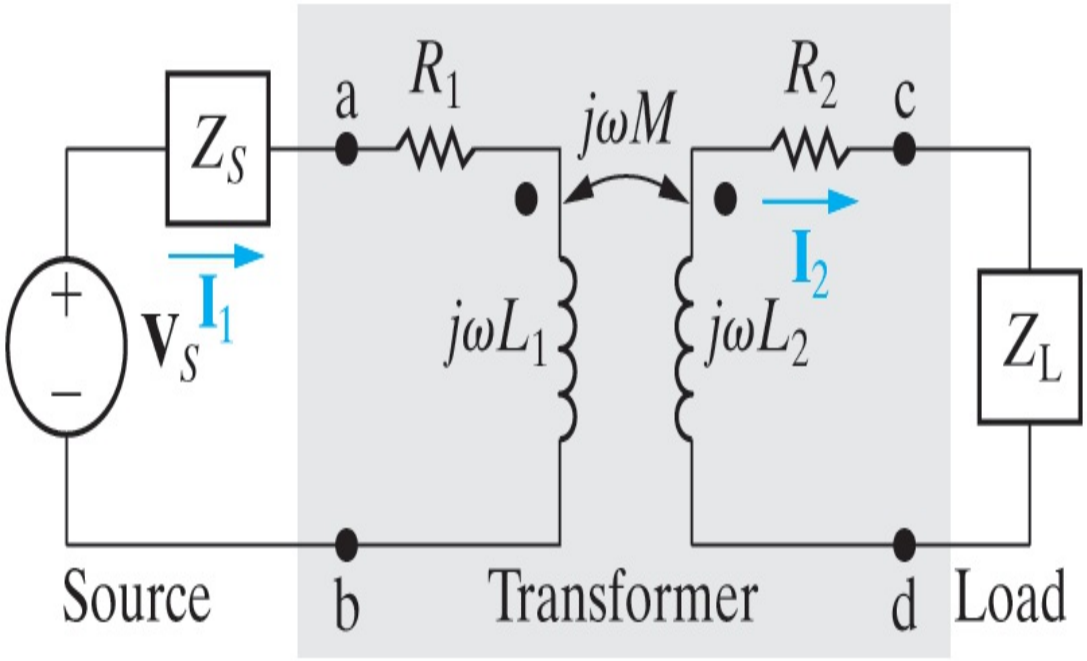


Figure 9.43 Full Alternative Text

Assessment Problem

Objective 4—Be able to analyze circuits containing linear transformers using phasor methods

1. 9.14 A linear transformer couples a load consisting of a 360 Ω resistor in series with a 0.25 H inductor to a sinusoidal voltage source, as shown. The voltage source has an internal impedance of 184+j0 Ω and a maximum voltage of 245.20 V, and it is operating at 800 rad/s. The transformer parameters are R1=100 Ω, L1=0.5 H, R2=40 Ω, L2=0.125 H, and k=0.4. Calculate (a) the reflected impedance; (b) the primary current; and (c) the secondary current.



9.10-13 Full Alternative Text

Answer:

1. 10.24−j7.68 Ω;
2. 0.5 cos(800t−53.13°) A;
3. 80 cos 800t mA.

SELF-CHECK: Also try *Chapter Problems 9.74 and 9.77*.

9.11 The Ideal Transformer

An **ideal transformer** consists of two magnetically coupled coils having N1 and N2 turns, respectively, and exhibiting these three properties:

1. The coefficient of coupling is unity (k=1).
2. The self-inductance of each coil is infinite (L1=L2=∞).
3. The coil losses, due to parasitic resistance, are negligible.

Understanding the behavior of ideal transformers begins with Eq. 9.31, which describes the impedance at the terminals of a source connected to a linear transformer. We repeat this equation in the following discussion and examine it further.

Exploring Limiting Values

Equation 9.31, repeated here as Eq. 9.33, defines the relationship between the input impedance (Zab) and load impedance (ZL) for a linear transformer:

$$Z_{ab}=Z_{11}+\omega^2M^2Z_{22}-Z_s$$
$$=R1+j\omega L1+\omega^2M2(R2+j\omega L2+ZL). \quad (9.33)$$

Let's consider what happens to Eq. 9.33 as L1 and L2 each become infinitely large and, at the same time, the coefficient of coupling approaches unity. Transformers wound on ferromagnetic cores can approach these conditions. Even though such transformers are nonlinear, we can obtain some useful information using an ideal model that ignores the nonlinearities.

To show how Zab changes when k=1 and L1 and L2 approach infinity, we first introduce the notation

$$Z_{22}=R2+RL+j(\omega L2+XL)=R22+jX22$$

and then rearrange Eq.9.33:

$$Z_{ab} = R_1 + \omega^2 M^2 R_{22} R_{22}^2 + X_{22}^2 + j(\omega L_1 - \omega^2 M^2 X_{22} R_{22}^2 + X_{22}^2) = R_{ab} + j X_{ab} . \quad (9.34)$$

At this point, we must be careful with the imaginary part of Zab because, as L1 and L2 approach infinity, Xab is the difference between two large quantities. Thus, before letting L1 and L2 increase, we write the imaginary part of Zab as

$$X_{ab}=\omega L_1-(\omega L_1)(\omega L_1)X_{22}R_{22}^2+X_{22}^2=\omega L_1\,(1-\omega L_2 X_{22}R_{22}^2+X_{22}^2),$$

where we recognize that, when k=1, M2=L1L2. Putting the term multiplying ωL1 over a common denominator gives

$$X_{ab}=\omega L_1(R_{22}^2+\omega L_2 X_L+X_L^2 R_{22}^2+X_{22}^2).$$

Factoring ωL2 out of the numerator and (ωL2)2 out of the denominator, then simplifying, yields

$$X_{ab}=L_1L_2X_L+(R_{22}^2+X_L^2)/\omega L_2(R_{22}/\omega L_2)^2+[1+(X_L/\omega L_2)]^2.$$

As k approaches 1.0, the ratio L1/L2 approaches the constant value of (N1/N2)2. This follows from the relationship between L1 and N1 (Eq. 6.21), the relationship between L2 and N2 (Eq. 6.23), and the fact that, as the coupling becomes extremely tight, the two permeances P1 and P2 become equal. The expression for Xab simplifies to

$$X_{ab}=(N_1N_2)^2\,X_L,$$

as $L_1 \to \infty$, $L_2 \to \infty$, and $k \to 1.0$.

The same reasoning leads to simplification of the reflected resistance in Eq. 9.34:

$$\omega^2 M^2 R_{22} R_{22}^2 + X_{22}^2 = L_1 L_2 R_{22} = (N_1 N_2)^2 R_{22}.$$

Substituting the simplified forms for Xab and the reflected resistance in Eq. 9.34 yields

Zab=R1+(N1N2)2R2+(N1N2)2(RL+jXL).

Compare this expression for Zab with the one given in Eq. 9.33. We see that when the coefficient of coupling approaches unity and the self-inductances of the coupled coils approach infinity, the transformer reflects the secondary winding resistance and the load impedance to the primary side by a scaling factor equal to the turns ratio (N1/N2) squared. Hence, we may describe the terminal behavior of the ideal transformer in terms of two characteristics. First, the magnitude of the volts per turn is the same for each coil, or

|V1N1|=|V2N2|. (9.35)

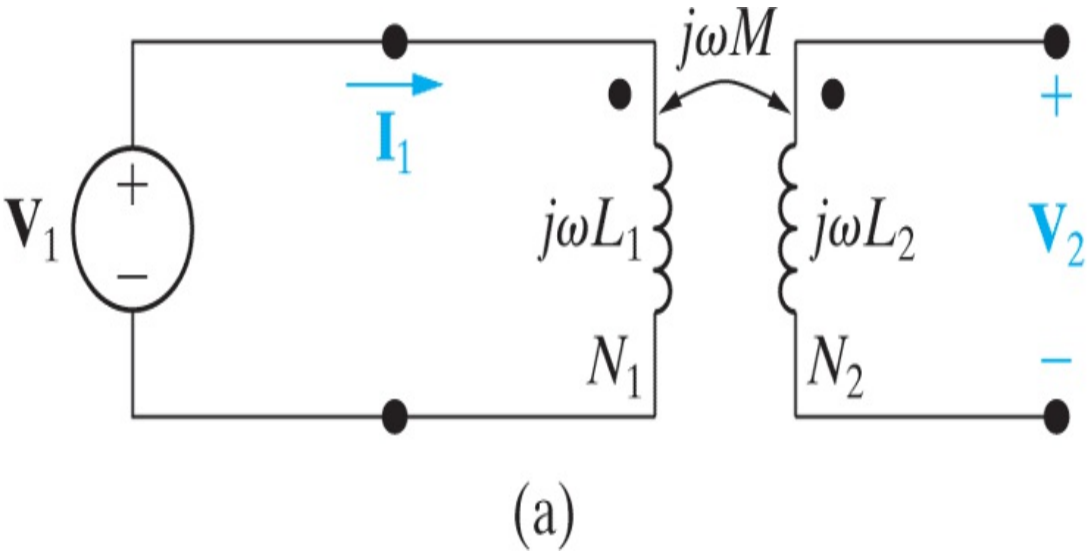
Second, the magnitude of the ampere-turns is the same for each coil, or

|I1N1|=|I2N2|. (9.36)

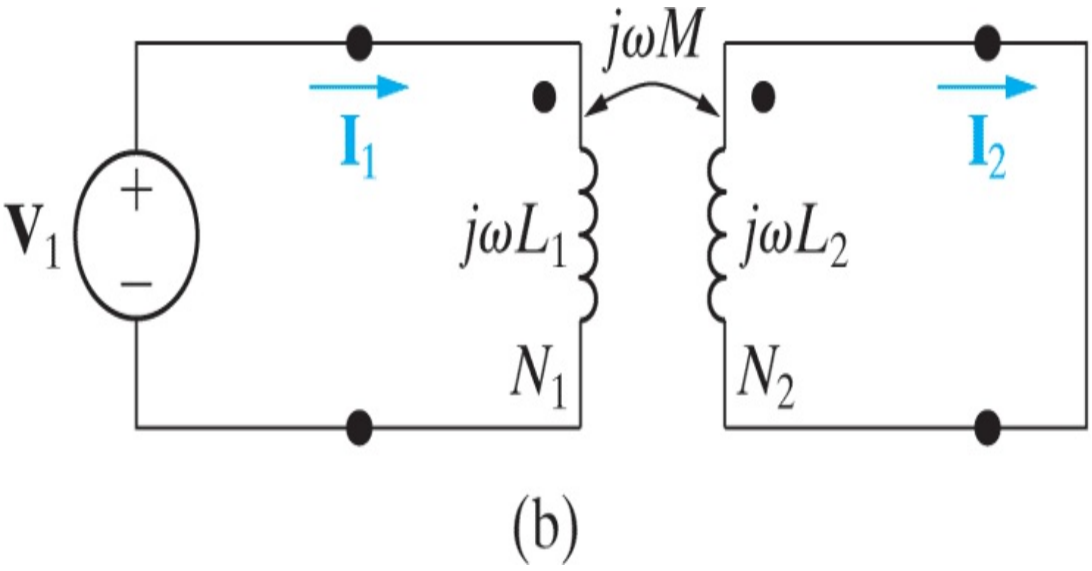
We use magnitude signs in Eqs. 9.35 and 9.36 because we have not yet established reference polarities for the currents and voltages; we discuss the removal of the magnitude signs shortly.

Figure 9.44 shows two lossless (R1=R2=0) magnetically coupled coils. We use Fig. 9.44 to validate Eqs. 9.35 and 9.36. In Fig. 9.44(a), coil 2 is open; in Fig. 9.44(b), coil 2 is shorted. Although we carry out the following analysis in terms of sinusoidal steady-state operation, the results also apply to instantaneous values of v and i.

Figure 9.44 The circuits used to verify Eqs. 9.35 and 9.36 for an ideal transformer.



9.11-14 Full Alternative Text



9.11-14 Full Alternative Text

Determining the Voltage and Current Ratios

Note in Fig. 9.44(a) that the voltage at the terminals of the open-circuit coil is entirely the result of the current in coil 1; therefore,

V2=jωMI1.

The current in coil 1 is

I1=V1jωL1.

Thus,

V2=ML1V1.

For unity coupling (k=1), the mutual inductance equals L1L2, so the expression for V2 becomes

V2=L2L1 V1.

For unity coupling, the flux linking coil 1 is the same as the flux linking coil 2, so we need only one permeance to describe the self-inductance of each coil. Thus,

V2=N22PN12PV1=N2N1V1

or

Voltage Relationship for an Ideal Transformer

V1N1=V2N2. (9.37)

Summing the voltages around the shorted coil of Fig. 9.44(b) yields

0=−jωMI1+jωL2I2,

which, when k=1, gives

I1I2=L2M=L2L1L2=L2L1=N2N1.

Therefore,

Current Relationship for an Ideal Transformer

I1N1=I2N2. (9.38)

Figure 9.45 shows the graphic symbol for an ideal transformer. The vertical lines in the symbol represent the layers of magnetic material from which ferromagnetic cores are often made. Coils wound on a ferromagnetic core behave very much like an ideal transformer, for several reasons. The ferromagnetic material creates a space with high permeance. Thus, most of the magnetic flux is trapped inside the core material, establishing tight magnetic coupling between coils that share the same core. High permeance also means high self-inductance because L=N2P. Finally, ferromagnetically coupled coils efficiently transfer power from one coil to the other. Efficiencies in excess of 95% are common, so neglecting losses is a valid approximation for many applications.

Figure 9.45 The graphic symbol for an ideal transformer.

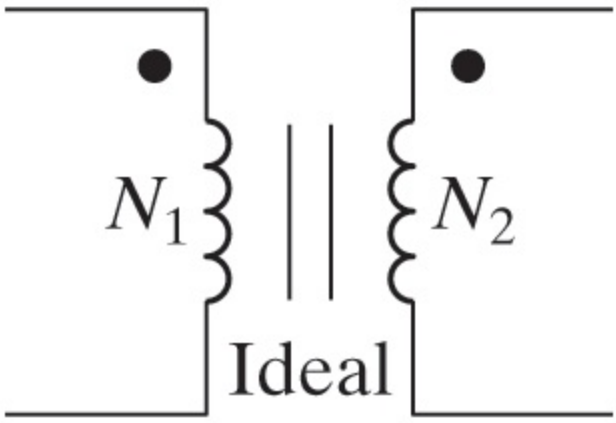


Figure 9.45 Full Alternative Text

Determining the Polarity of the Voltage and Current Ratios

We now turn to the removal of the magnitude signs from Eqs. 9.35 and 9.36. Note that magnitude signs do not appear in Eqs. 9.37 and 9.38 because we established reference polarities for voltages and reference directions for currents in Fig. 9.44. In addition, we specified the magnetic polarity dots of the two coupled coils.

The rules for assigning the proper algebraic sign to Eqs. 9.35 and 9.36 are as follows:

Dot Convention for Ideal Transformers

If the coil voltages V1 and V2 are both positive or negative at the dot-marked terminals, use a plus sign in Eq. 9.35. Otherwise, use a negative sign.

If the coil currents I1 and I2 are both directed into or out of the dot-marked terminals, use a minus sign in Eq. 9.36. Otherwise, use a plus sign.

The four circuits shown in Fig. 9.46 illustrate these rules.

Figure 9.46 Circuits that show the proper algebraic signs for relating the terminal voltages

and currents of an ideal transformer.

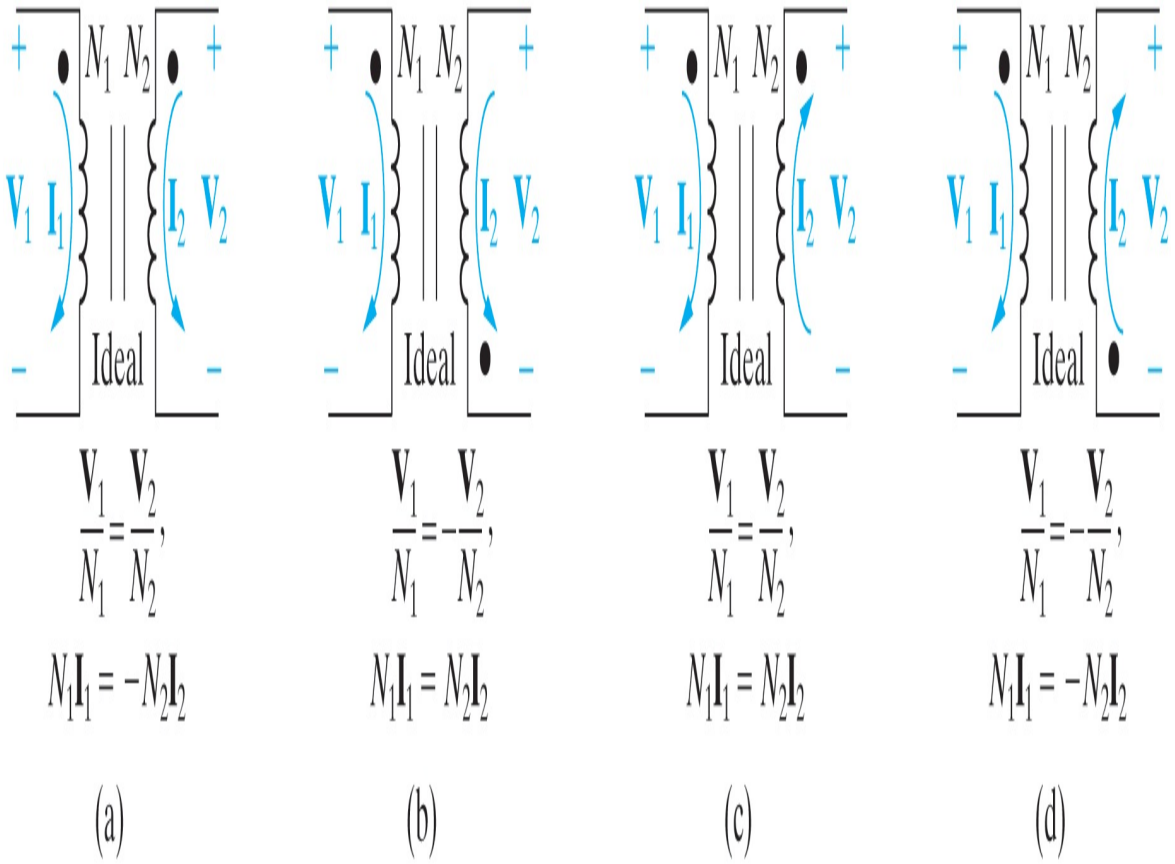


Figure 9.46 Full Alternative Text

The turns ratio for the two windings is an important parameter of the ideal transformer. In this text, we use a to denote the ratio N2/N1, so

a=N2N1. (9.39)

Figure 9.47 shows three ways to represent the turns ratio of an ideal transformer. Figure 9.47(a) shows the number of turns in each coil explicitly. Figure 9.47(b) shows that the ratio N2/N1 is 5 to 1, and Fig. 9.47(c) shows that the ratio N2/N1 is 1 to 15.

Figure 9.47 Three ways to show that the turns ratio of an ideal transformer is 5.

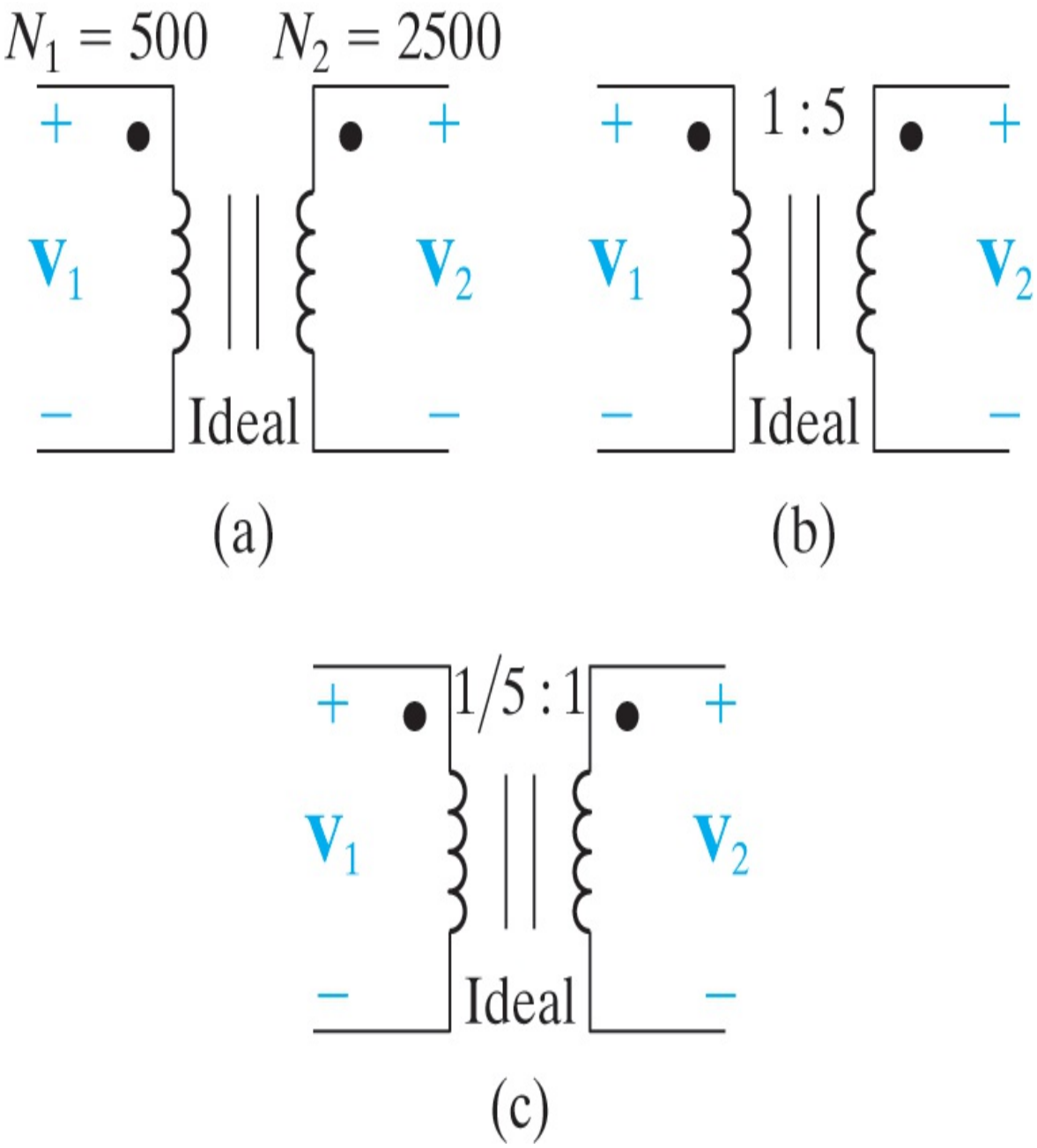


Figure 9.47 Full Alternative Text

Example 9.16 analyzes a circuit containing an ideal transformer.

Example 9.16 Analyzing an Ideal Transformer Circuit in the Frequency Domain

The load impedance connected to the secondary winding of the ideal transformer in Fig. 9.48 is a 237.5m Ω resistor in series with a 125μH inductor.

Figure 9.48 The circuit for Example 9.16.

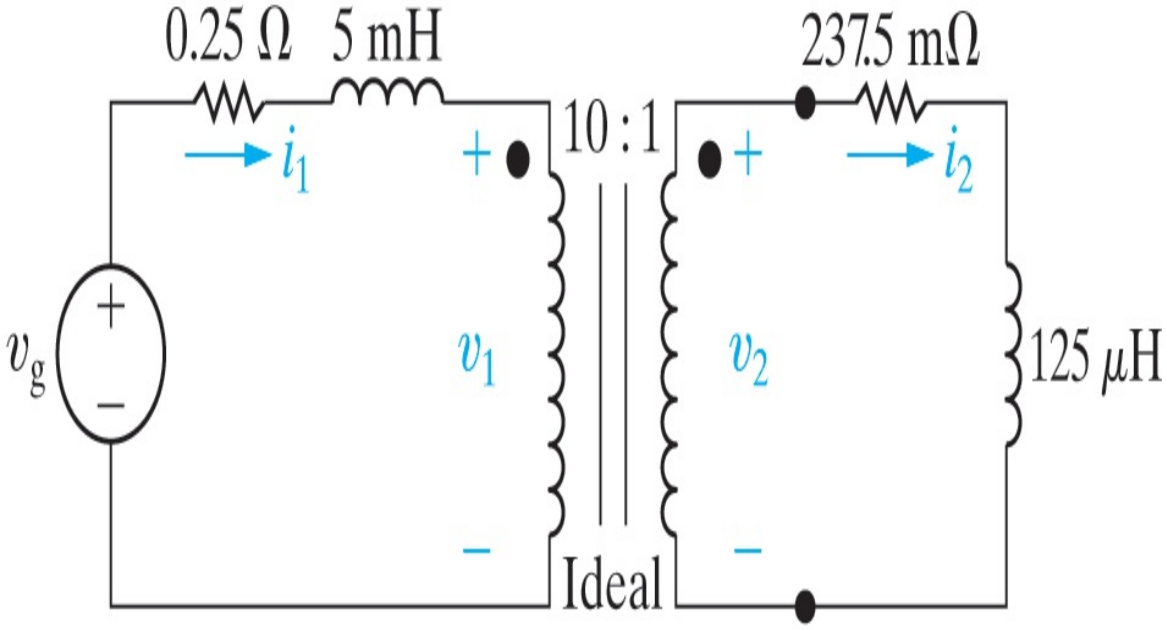


Figure 9.48 Full Alternative Text

If the sinusoidal voltage source (vg) is generating the voltage

2500 cos 400t V, find the steady-state expressions for: (a) i1; (b) v1; (c) i2; and (d) v2.

Solution

1. We begin by transforming the circuit to the frequency domain. The voltage source has the phasor value 2500 $\angle 0°$ V; the 5 mH inductor has an impedance of j2 Ω; and the 125 μH inductor has an impedance of j0.05 Ω. The resulting frequency domain circuit is shown in Fig. 9.49.

Figure 9.49 Phasor domain circuit for Example 9.16.

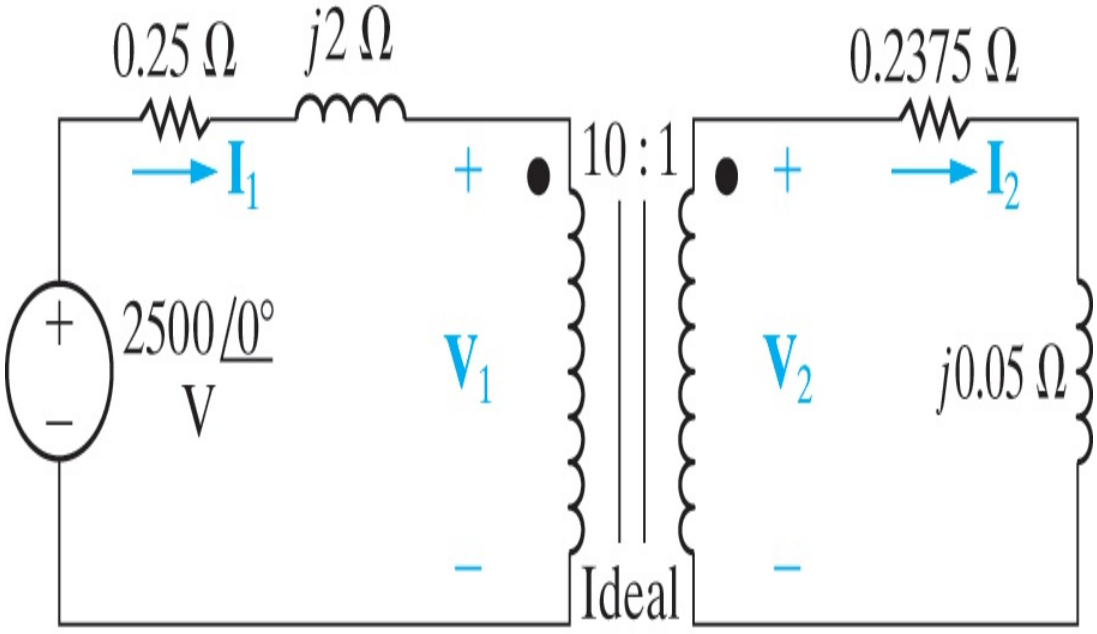


Figure 9.49 Full Alternative Text

Writing a KCL equation for the left-hand mesh in Fig. 9.49 gives

$$2500 \angle 0° = (0.25 + j2)\, I_1 + V_1 .$$

Using the relationship between V1 and V2 for the ideal transformer and using Ohm's law to express V2 in terms of I2, we have

$V1=10V2=10[(0.2375+j0.05)I2]$.

Because

$I2=10I1$

we have

$V1=10(0.2375+j0.05)(10I1)=(23.75+j5)I1$.

Therefore

$2500\ 0° =(24+j7) I\ 1$,

or

$I\ 1 =100\ -16.26°$ A.

Thus, the steady-state expression for i1 is

$i1=100\cos(400t-16.26°)$A.

2. $V\ 1 = 2500\ 0° - (100\ -16.26°)(0.25 + j2) = 2420-j185=2427.06\ -4.37°$ V.

   Hence, in the steady state

   $v1=2427.06\cos(400t-4.37°)$ V.

3. $I\ 2 =10\ I\ 1 =1000\ -16.26°$ A.

   Therefore, in the steady state

   $i2=1000\cos(400t-16.26°)$ A.

4. $V\ 2 =0.1\ V\ 1 =242.71\ -4.37°$ V,

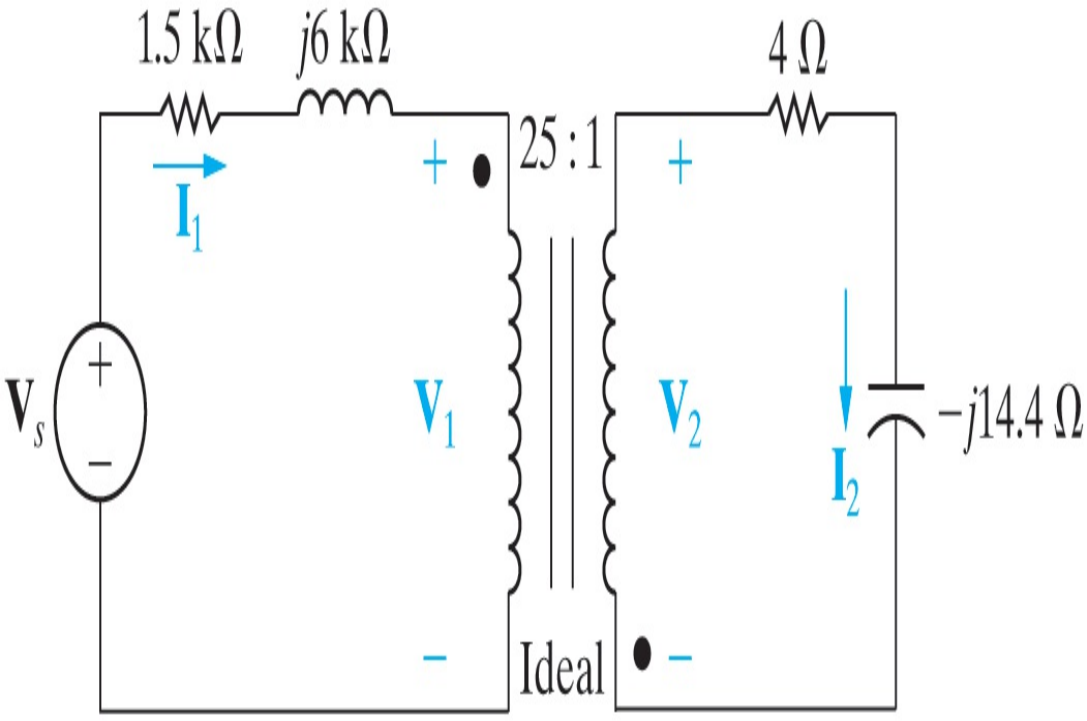
so in the steady state,

v2=242.71cos(400t−4.37°) V.

Assessment Problem

Objective 5—Be able to analyze circuits with ideal transformers

1. 9.15 The source voltage in the phasor domain circuit in the accompanying figure is 25 0° kV. Find the amplitude and phase angle of V2 and I2.



9.11-14 Full Alternative Text

Answer:

V 2 =1868.15 142.39° V;

I 2 =125 216.87° A.

SELF-CHECK: Also try *Chapter Problem 9.79*.

Using an Ideal Transformer for Impedance Matching

Ideal transformers can be used to increase or decrease the impedance level of a load, as illustrated by the circuit shown in Fig. 9.50. The impedance seen by the practical voltage source (Vs in series with Zs) is V1/I1. The voltage and current at the terminals of the load impedance (V2 and I2) are related to V1 and I1 by the transformer turns ratio; thus

V1=V2a,

Figure 9.50 Using an ideal transformer to couple a load to a source.

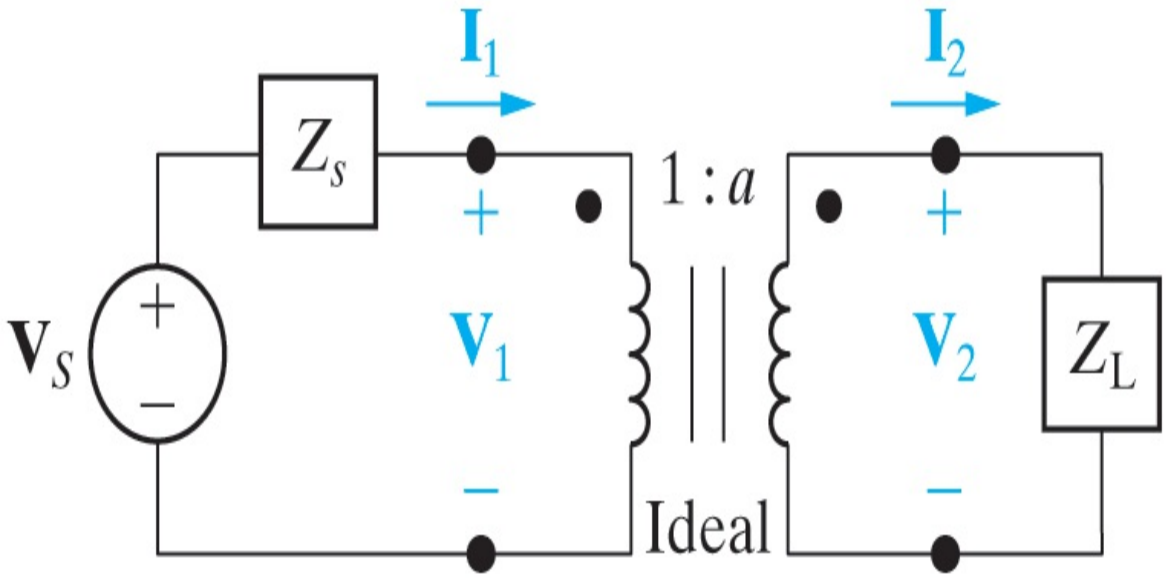


Figure 9.50 Full Alternative Text

and

$I_1 = aI_2$.

Therefore, the impedance seen by the practical source is

$Z_{IN} = \frac{V_1}{I_1} = \frac{1}{a^2}\frac{V_2}{I_2}$,

but the ratio V_2/I_2 is the load impedance Z_L, so the expression for Z_{IN} becomes

$$Z_{IN} = \frac{1}{a^2} Z_L. \quad (9.40)$$

Thus, the ideal transformer's secondary coil reflects the load impedance back to the primary coil, with the scaling factor $1/a^2$.

Note that the ideal transformer changes the magnitude of Z_L but does not affect its phase angle. Whether Z_{IN} is greater or less than Z_L depends on the turns ratio a.The ideal transformer—or its practical counterpart, the ferromagnetic core transformer—can be used to match the magnitude of Z_L to the magnitude of Z_s. We will discuss why this may be desirable in Chapter 10.

Ideal transformers are also used to increase or decrease voltages from a source to a load, as we will see in Chapter 10. Thus, ideal transformers are used widely in the electric utility industry, where it is desirable to decrease, or step down, the voltage level at the power line to safer residential voltage levels.

9.12 Phasor Diagrams

When we analyze the sinusoidal steady-state operation of a circuit using phasors, a diagram of the phasor currents and voltages can give us additional insight into the circuit's behavior. A phasor diagram shows the magnitude and phase angle of each phasor quantity in the complex-number plane. Phase angles are measured counterclockwise from the positive real axis, and magnitudes are measured from the origin of the axes. For example, Fig. 9.51 shows the phasor quantities $10\ \angle 30°$, $12\ \angle 150°$, $5\ \angle -45°$, and $8\ \angle -170°$.

Figure 9.51 A graphic representation of phasors.

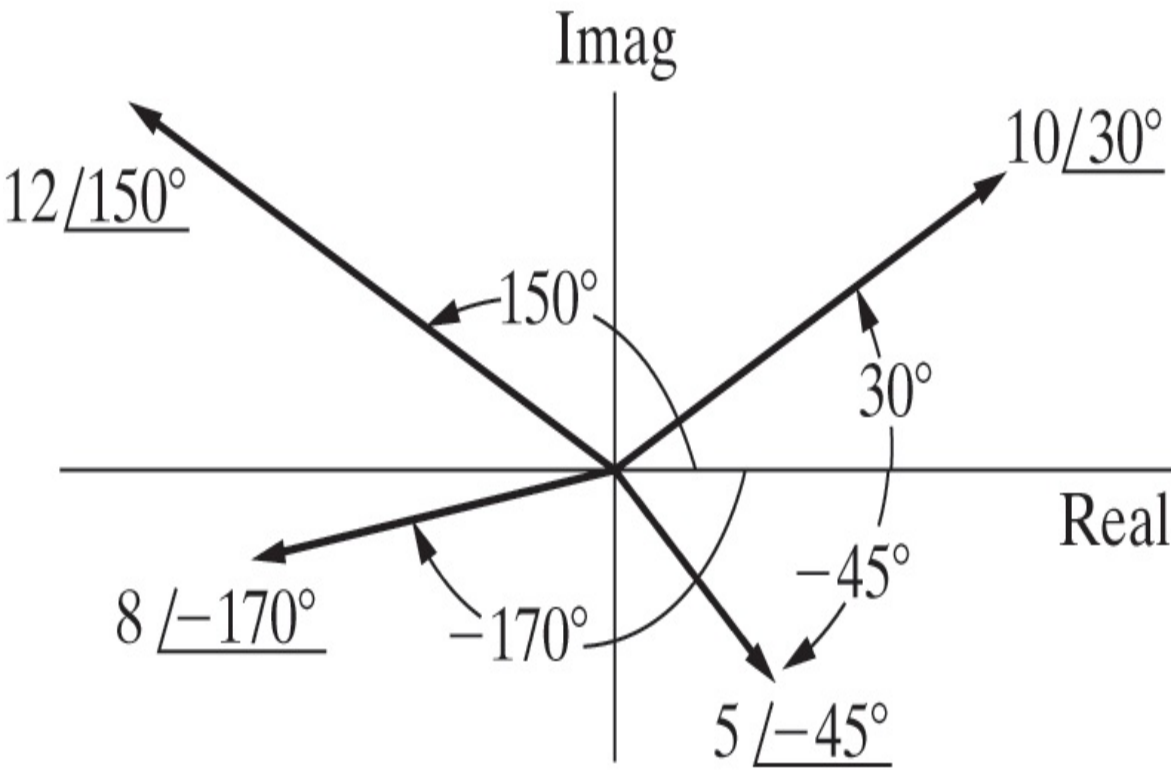


Figure 9.51 Full Alternative Text

Because constructing phasor diagrams for circuits usually involves both currents and voltages, we use two different magnitude scales, one for currents and one for voltages. Visualizing a phasor on the complex-number plane is a good way to check your calculations. For example, suppose you convert the phasor −7–j3 to polar form. Before making your calculation, you should anticipate a magnitude greater than 7 and an angle in the third quadrant that is more negative than −135° or less positive than 225°, as illustrated in Fig. 9.52.

Figure 9.52 The complex number −7 −j3 = 7.62 − 156.80°

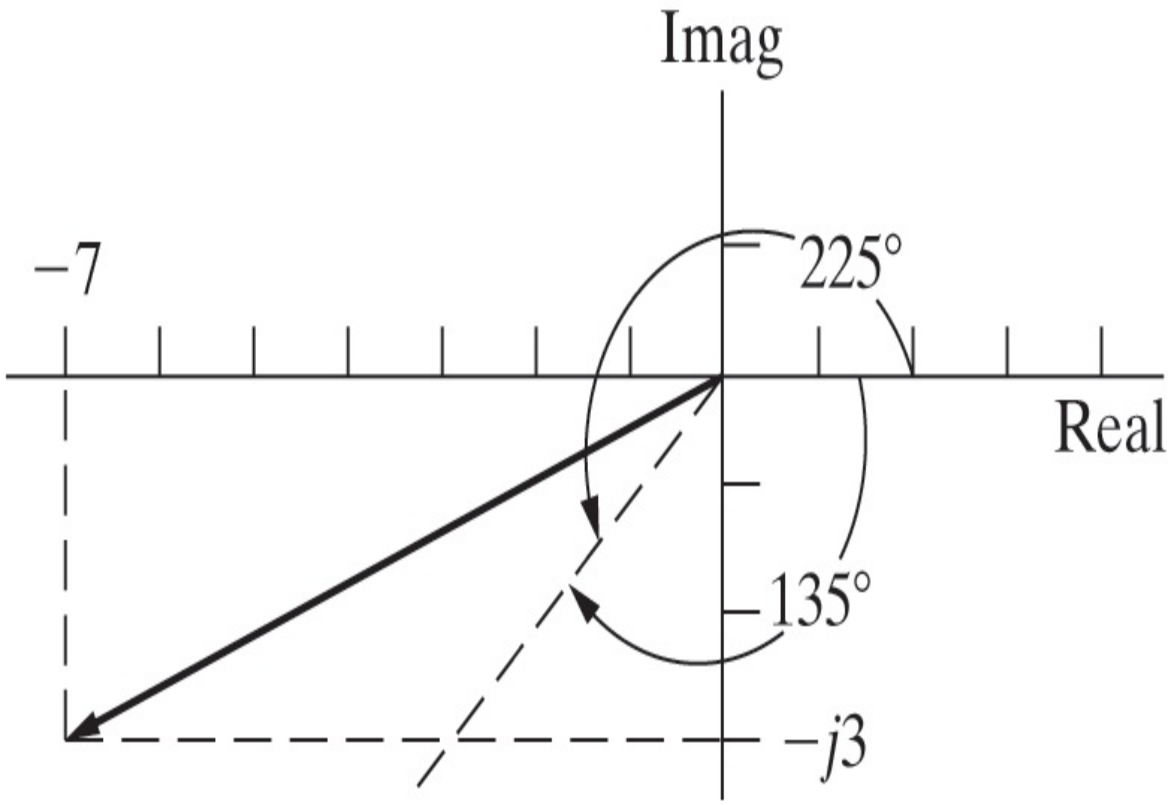


Figure 9.52 Full Alternative Text

Examples 9.17 and 9.18 construct and use phasor diagrams. You can use phasor diagrams to get additional insight into the steady-state sinusoidal operation of a circuit. For example, Problem 9.85 uses a phasor diagram to explain the operation of a phase-shifting circuit.

Example 9.17 Using Phasor Diagrams to Analyze a Circuit

Use a phasor diagram for the circuit in Fig. 9.53 to find the value of R that causes the current through that resistor, iR, to lag the source current, is, by 45° when ω=5 krad/s.

Figure 9.53 The circuit for Example 9.17.

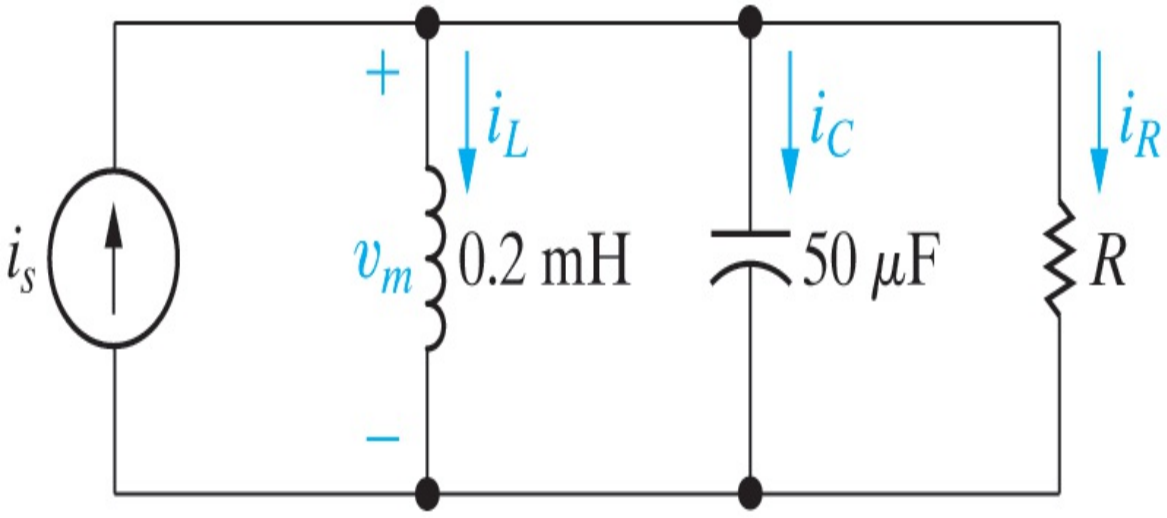


Figure 9.53 Full Alternative Text

Solution

According to KCL, the sum of the currents IR, IL, and IC must equal the source current Is. If we assume that the phase angle of the voltage Vm is zero, we can draw the current phasors for each of the components. The current phasor for the inductor is

$$I_L = V_m \angle 0° \, j(5000)(0.2 \times 10^{-3}) = V_m \angle -90°,$$

whereas the current phasor for the capacitor is

$$I_C = V_m \angle 0° \, -j(5000)(50 \times 10^{-6}) = 4 V_m \angle 90°,$$

and the current phasor for the resistor is

$$I_R = V_m \angle 0° \, R = V_m R \angle 0°.$$

These phasors are shown in Fig. 9.54. The phasor diagram also shows the source current phasor, sketched as a dashed line, which must be the sum of the current phasors of the three circuit components and must be at an angle that is 45° more positive than the current phasor for the resistor. As you can see, summing the phasors makes an isosceles triangle, so the length of the current phasor for the resistor must equal 3Vm. Therefore, the value of the resistor is 13 Ω.

Figure 9.54 The phasor diagram for the currents in Fig. 9.53.

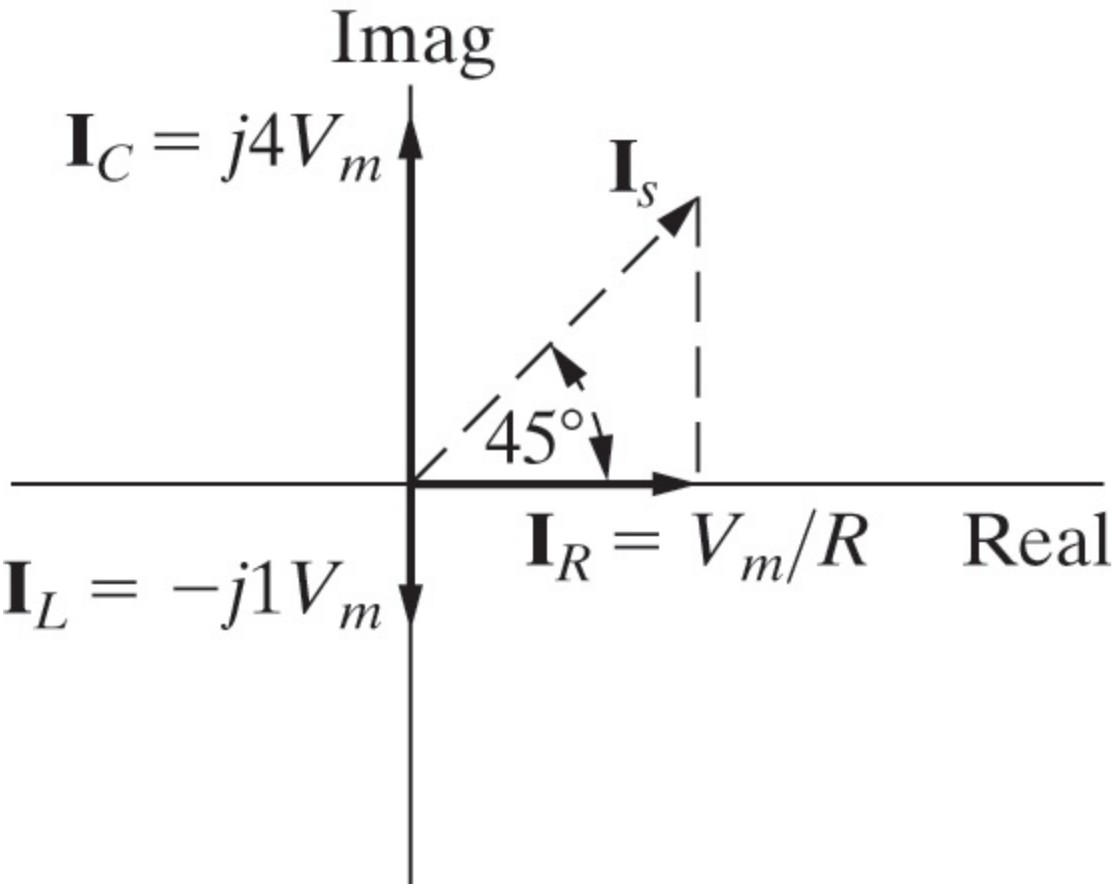


Figure 9.54 Full Alternative Text

Example 9.18 Using Phasor Diagrams to Analyze Capacitive Loading Effects

The circuit in Fig. 9.55 has a load consisting of the parallel combination of the resistor and inductor. Use phasor diagrams to explore the effect of adding a capacitor across the terminals of the load on the amplitude of Vs if we adjust Vs so that the amplitude of VL remains constant. Utility companies use this technique to control the voltage drop on their lines.

Figure 9.55 The circuit for Example 9.18.

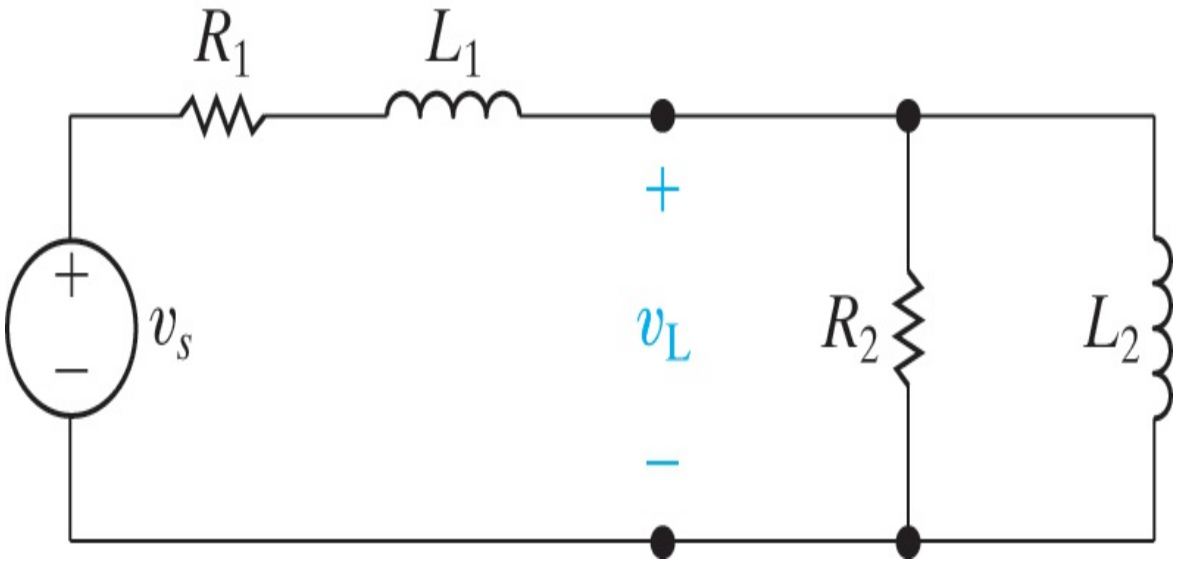


Figure 9.55 Full Alternative Text

Solution

We begin by assuming zero capacitance across the load. After constructing the phasor diagram for the zero-capacitance case, we can add the capacitor and study its effect on the amplitude of Vs, holding the amplitude of VL constant. Figure 9.56 shows the frequency-domain equivalent of the circuit in Fig. 9.55. We added the phasor branch currents **I**, Ia, and Ib to Fig. 9.56 to assist our analysis.

Figure 9.56 The frequency-domain equivalent of the circuit in Fig. 9.55.

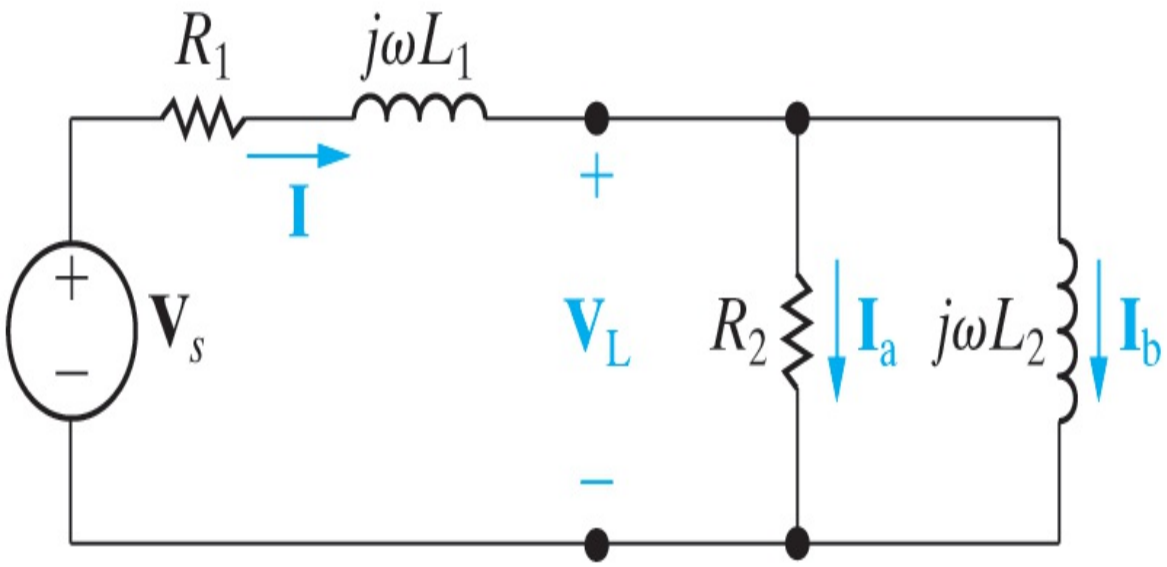


Figure 9.56 Full Alternative Text

Figure 9.57 shows the stepwise evolution of the phasor diagram. Keep in mind that in this example we are not interested in specific phasor values and positions, but rather in the general effect of adding a capacitor across the terminals of the load. Thus, we want to develop the relative positions of the phasors before and after the capacitor is added.

Figure 9.57 The step-by-step evolution of the phasor diagram for the circuit in Fig. 9.56.

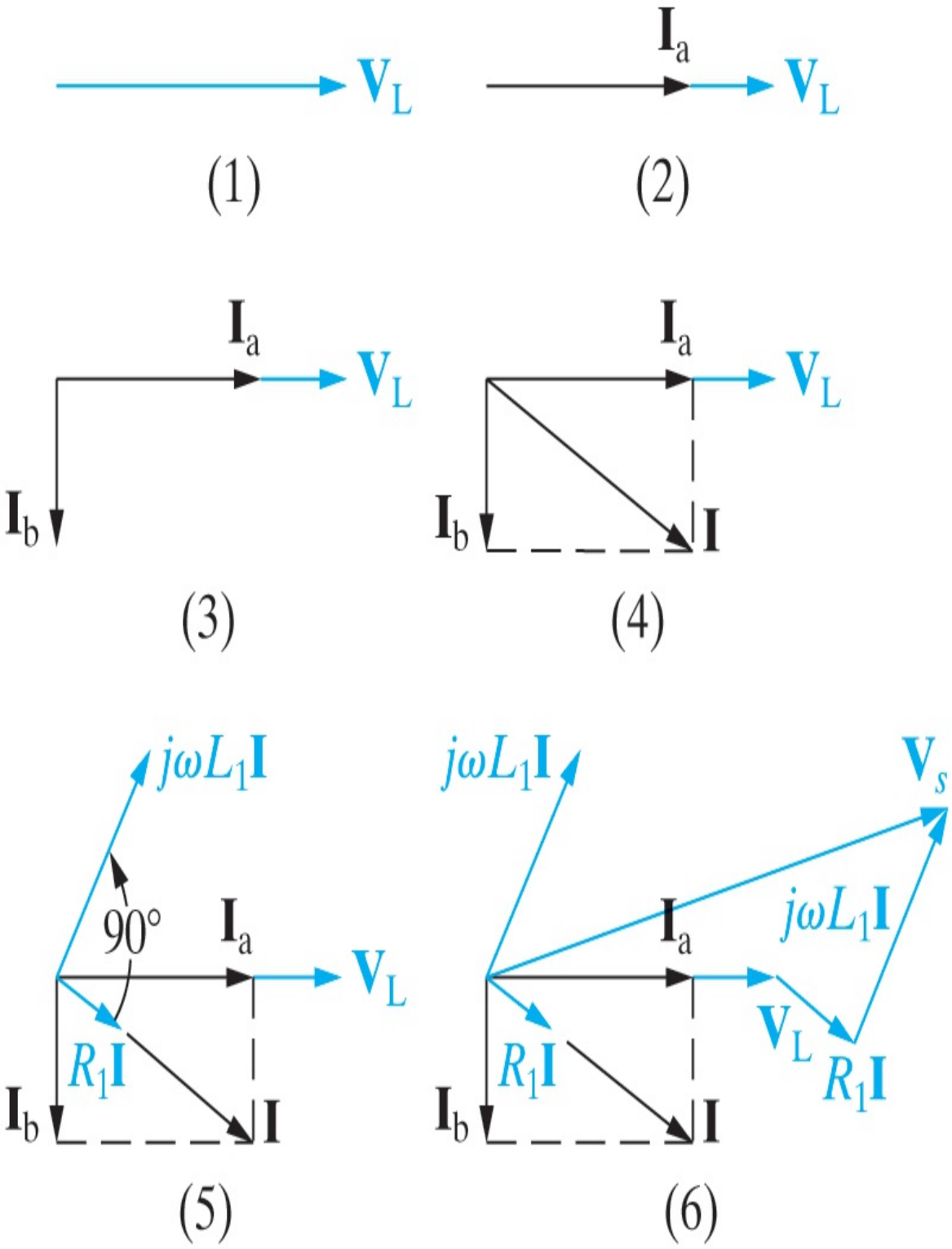


Figure 9.57 Full Alternative Text

Relating the phasor diagram to the circuit shown in Fig. 9.56, we make the

following observations:

- We choose VL as our reference because we are holding its amplitude constant. For convenience, we place this phasor on the positive real axis.
- We know that Ia is in phase with VL and that its magnitude is |VL|/R2. (On the phasor diagram, the magnitude scale for the current phasors is independent of the magnitude scale for the voltage phasors.)
- We know that Ib lags behind VL by 90° and that its magnitude is |VL|/ ωL2.
- The current **I** is equal to the sum of Ia and Ib.
- The voltage drop across R1 is in phase with the current **I**, and the voltage drop across jωL1 leads **I** by 90°.
- The source voltage is given by Vs=VL + (R1 + jωL1)I.

The completed phasor diagram shown in Step 6 of Fig. 9.57 shows the amplitude and phase angle relationships among all the currents and voltages in Fig. 9.56.

Now add the capacitor branch shown in Fig. 9.58. We are holding VL constant, so we construct the phasor diagram for the circuit in Fig. 9.58 following the same steps as those in Fig. 9.57, except that, in Step 4, we add the capacitor current Ic to the diagram, where Ic leads VL by 90°, with |IC|=|VLωC|. Figure 9.59 shows the effect of Ic on the current **I**: Both the magnitude and phase angle of **I** change as the magnitude of Ic changes. As **I** changes, so do the magnitude and phase angle of the voltage drop across the impedance (R1+jωL1), resulting in changes to the magnitude and phase angle of Vs. The phasor diagram shown in Fig. 9.60 depicts these observations. The dashed phasors represent the pertinent currents and voltages before adding the capacitor.

Figure 9.58 The addition of a

capacitor to the circuit shown in Fig. 9.56.

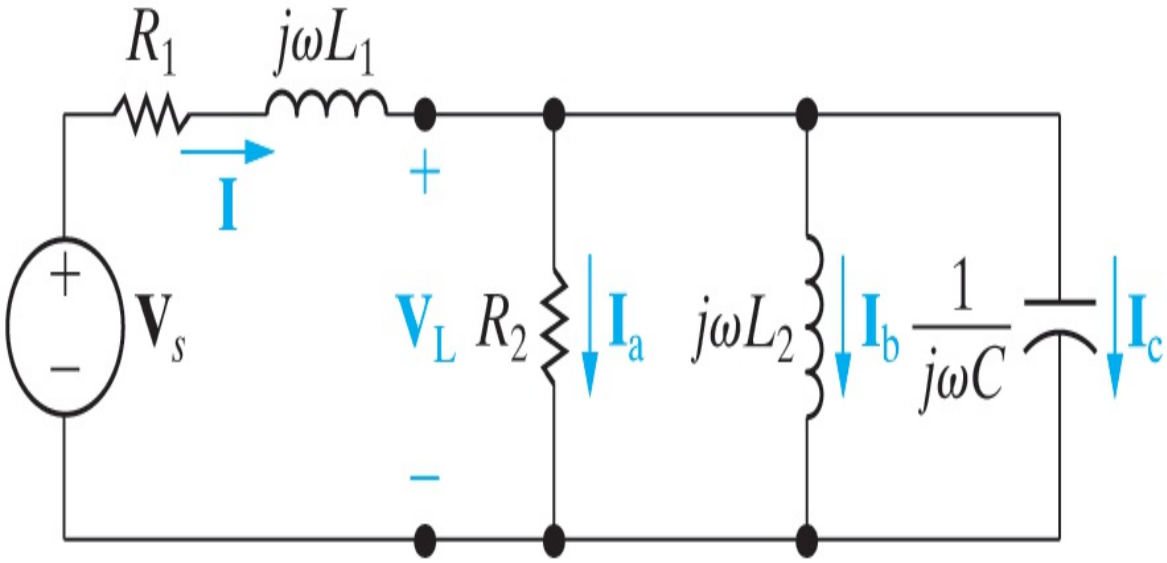


Figure 9.58 Full Alternative Text

Figure 9.59 The effect of the capacitor current Ic on the line current I.

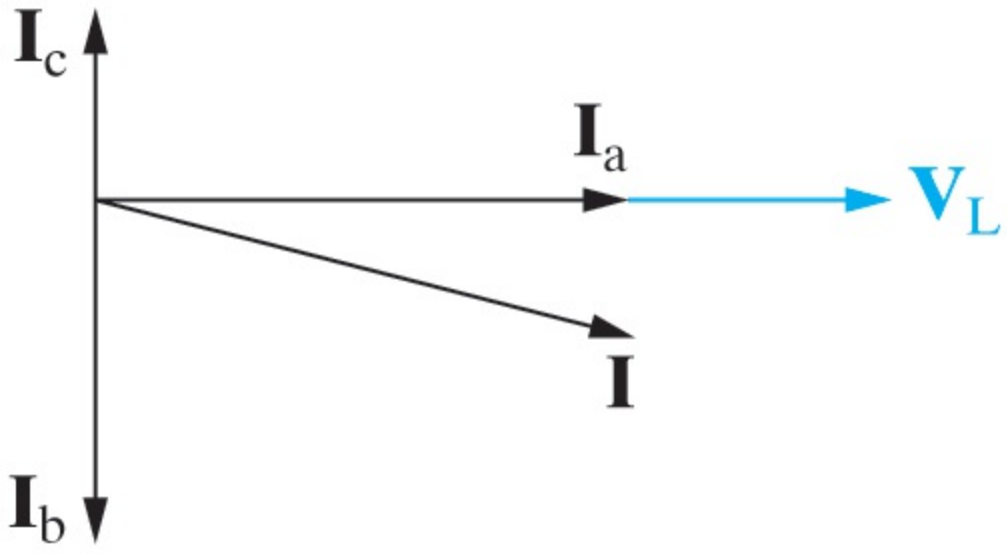


Figure 9.59 Full Alternative Text

Figure 9.60 The effect of adding a load-shunting capacitor to the circuit shown in Fig. 9.53 if VL is held constant.

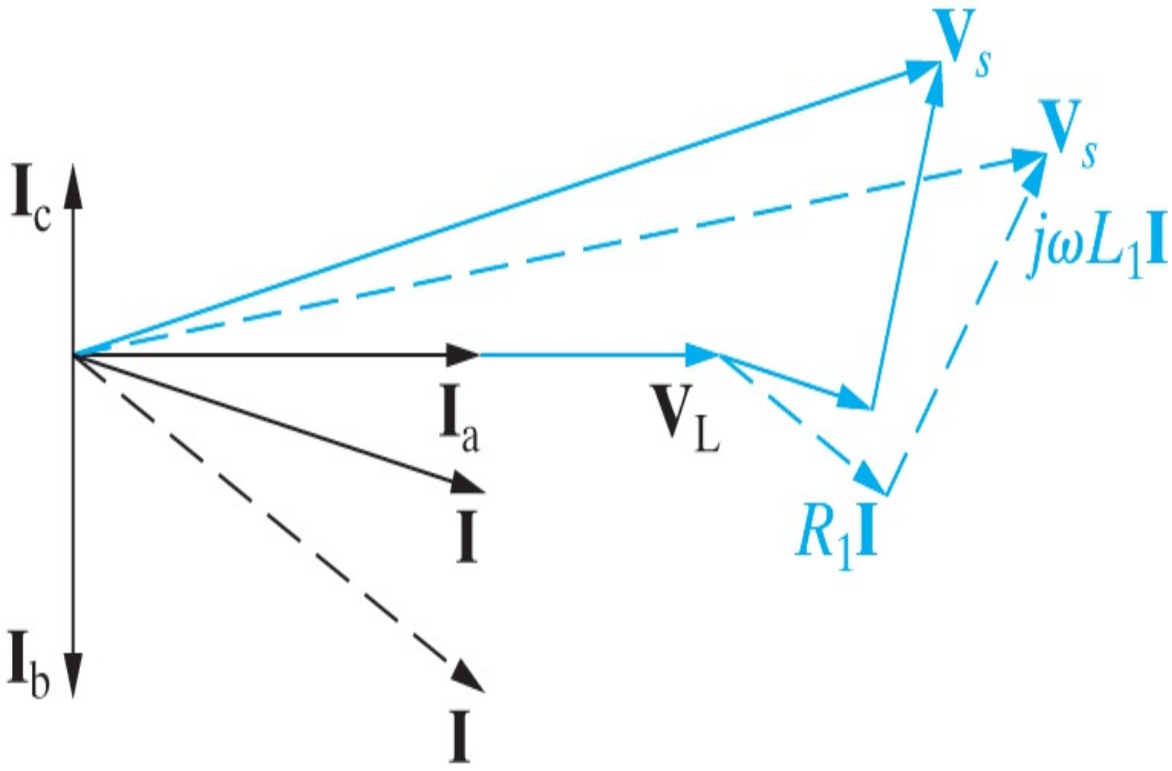


Figure 9.60 Full Alternative Text

Thus, comparing the dashed phasors of **I**, R1I, jωL1I, and Vs with their solid counterparts clearly shows the effect of adding C to the circuit. In particular, adding the capacitor reduces the source voltage amplitude while maintaining the load voltage amplitude. Practically, this result means that, as the load increases (i.e., as Ia and Ib increase), we can add capacitors to the system (i.e., increase Ic), so that under heavy load conditions we can maintain VL without increasing the amplitude of the source voltage.

SELF-CHECK: Assess your understanding of this material by trying *Chapter Problems 9.83 and 9.84.*

Practical Perspective

A Household Distribution Circuit

After determining the loads on the three-wire distribution circuit prior to the interruption of fuse A, you are able to construct the frequency-domain circuit model shown in Fig. 9.61. The impedances of the wires connecting the source to the loads are assumed negligible.

Figure 9.61 The three-wire household distribution circuit model.

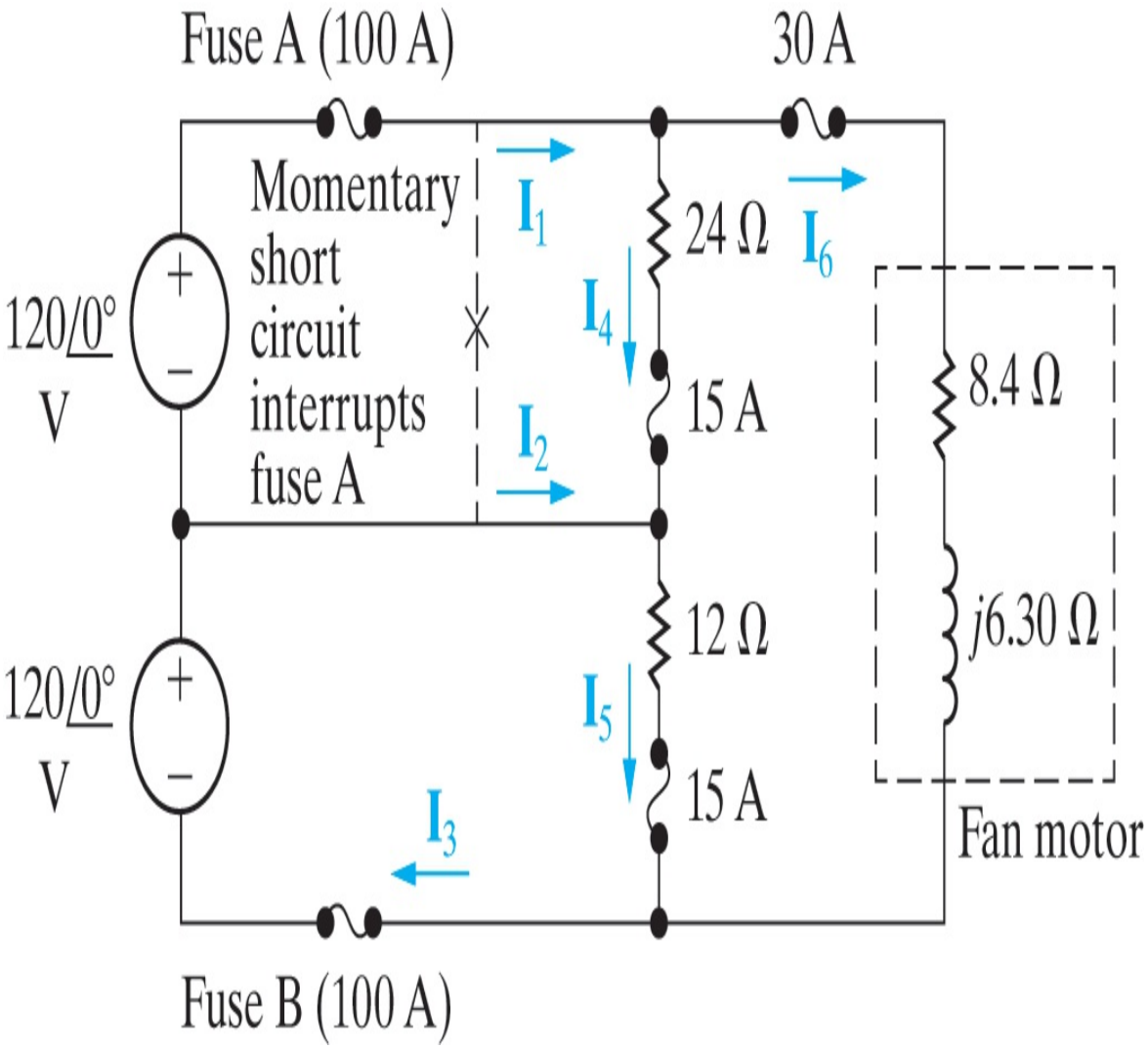


Figure 9.61 Full Alternative Text

Let’s begin by calculating all of the branch current phasors in the figure, prior to the interruption of Fuse A. We calculate I4, I5, and I6 using Ohm’s law:

$$I_4 = \frac{120}{24} = 5\ 0°\ A;\quad I_5 = \frac{120}{12} = 10\ 0°\ A;\quad I_6 = \frac{240}{8.4+j6.3} = 18.29 - j13.71 A = 22.86\ -36.87°\ A.$$

We calculate I1, I2, and I3 using KCL and the other branch currents:

$$I_1 = I_4 + I_6 = 23.29 - j13.71 A = 27.02\ -30.5°\ A;\quad I_2 = I_5 - I_4 = 5\ 0°\ A;\quad I_3 = I_5 + I_6 = 28.29 - j13.71 A = 31.44\ -25.87°\ A.$$

Now calculate those same branch current phasors after Fuse A is interrupted. We assume that the fan motor behaves like a short circuit when it stalls, and the interrupted fuse behaves like an open circuit. The circuit model now looks like Fig. 9.62. To analyze this circuit, we write two mesh current equations using the mesh current phasors shown in Fig. 9.62:

$$12(Ia - Ib) = 120;\quad 24Ib + 12(Ib - Ia) = 0.$$

Figure 9.62 The circuit in Fig. 9.61 after Fuse A is interrupted and the fan motor stalls.

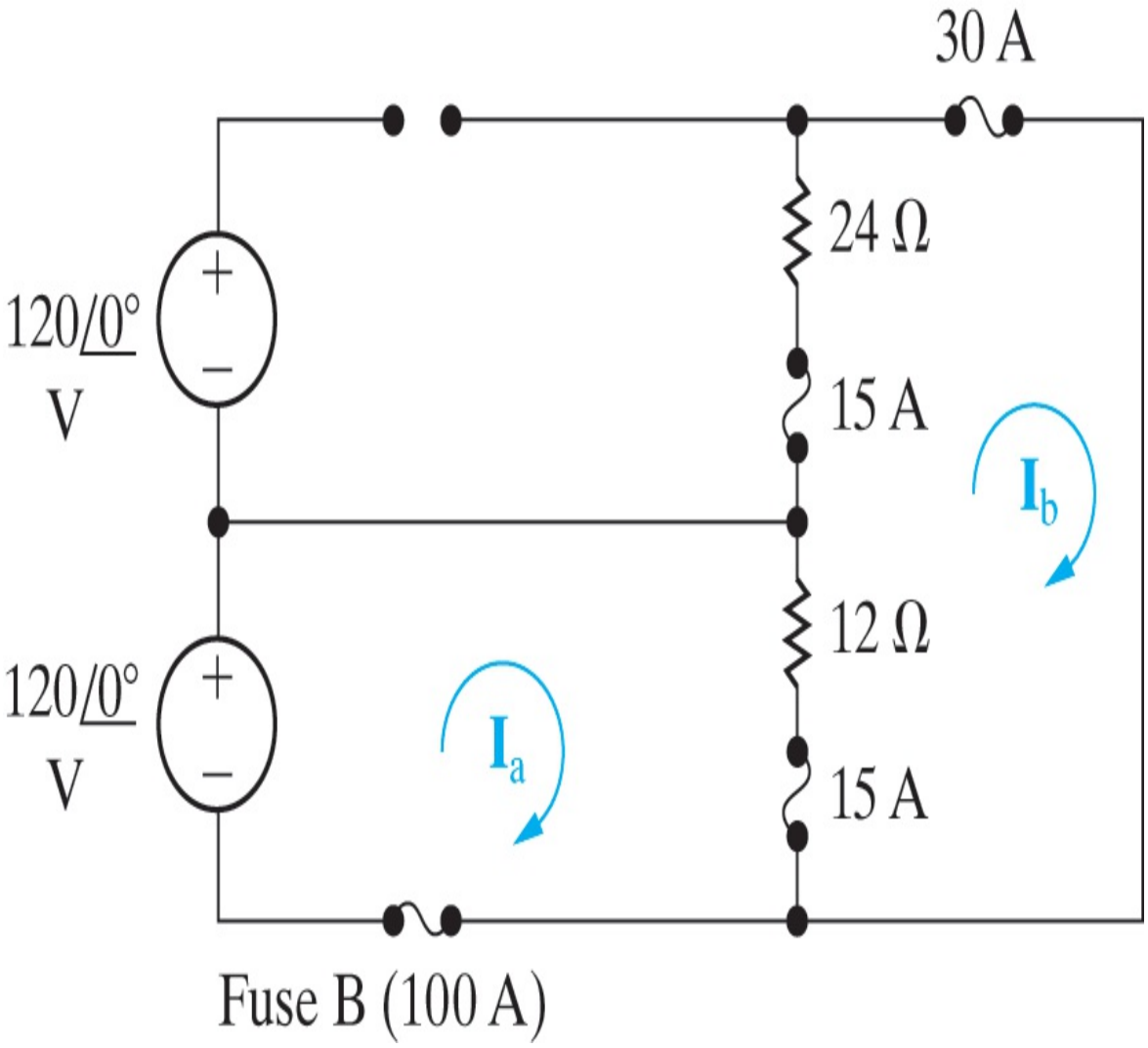


Figure 9.62 Full Alternative Text

Solving the mesh current equations, we get

Ia=15 A;Ib=5A.

Using these mesh current phasors to calculate the new branch current phasors from Fig. 9.61, we get

I 1 = 0A; I 2 = I 3 = I a =15A; I 6 = I b =5A; I 4 = I 1 − I 6 =−5A; I 5 = I 2 + I 4 =10A.

We can see that even though Fuse A is interrupted and the I1 branch current is zero, all of the other branch currents are nonzero. The television and clock

continued to operate because they are represented by the 12 Ω load that still has an ample supply of current.

SELF-CHECK: Assess your understanding of this Practical Perspective by trying *Chapter Problems 9.86 and 9.87*.

Summary

- The general equation for a **sinusoidal source** is

  $v=V_m \cos(\omega t + \phi)$ (voltage source),

  or

  $i=I_m \cos(\omega t + \phi)$ (current source),

  where V_m (or I_m) is the maximum amplitude, ω is the frequency, and ϕ is the phase angle. (See page 320.)

- The frequency, ω, of a sinusoidal response is the same as the frequency of the sinusoidal source driving the circuit. The amplitude and phase angle of the response are usually different from those of the source. (See page 323.)

- The best way to find the steady-state voltages and currents in a circuit driven by sinusoidal sources is to perform the analysis in the frequency domain. The following mathematical transforms allow us to move between the time and frequency domains.

  - The phasor transform (from the time domain to the frequency domain):

    $V=V_m e^{j\phi}=P\{V_m \cos(\omega t+\phi)\}$.

  - The inverse phasor transform (from the frequency domain to the time domain):

    $P^{-1}\{V_m e^{j\phi}=R\{V_m e^{j\phi} e^{j\omega t}\}$.

  (See pages 324–325.)

- In a circuit with a sinusoidal source, the voltage leads the current by 90°

at the terminals of an inductor, and the current leads the voltage by 90° at the terminals of a capacitor. (See pages 327–331.)

- **Impedance** (Z) relates the phasor current and phasor voltage for resistors, inductors, and capacitors in an equation that has the same form as Ohm's law,

  V=ZI,

  where the reference direction for **I** obeys the passive sign convention. The reciprocal of impedance is **admittance** (Y), so another way to express the current-voltage relationship for resistors, inductors, and capacitors in the frequency domain is

  V=I/Y.

  (See pages 330 and 336.)

- The equations for impedance and admittance for resistors, inductors, and capacitors are summarized in Table 9.3.

Table 9.3 Impedance and Related Values

Element	Impedance (Z)	Reactance	Admittance (Y)	Susceptance
Resistor	R (resistance)	–	G (conductance)	–
Capacitor	$j(-1/\omega C)$	$-1/\omega C$	$j\omega C$	ωC
Inductor	$j\omega L$	ωL	$j(-1/\omega L)$	$-1/\omega L$

Table 9.3 Full Alternative Text

- All of the circuit analysis techniques developed in Chapters 2–4 for resistive circuits also apply to sinusoidal steady-state circuits in the frequency domain. These techniques include KVL, KCL, series, and parallel combinations of impedances, voltage and current division, node-voltage and mesh-current methods, Thévenin and Norton equivalents, and source transformation.

- The two-winding **linear transformer** is a coupling device made up of two coils wound on the same nonmagnetic core. **Reflected impedance** is the impedance of the secondary circuit as seen from the terminals of the primary circuit, or vice versa. The reflected impedance of a linear transformer seen from the primary side is the complex conjugate of the self-impedance of the secondary circuit scaled by the factor $(\omega M/|Z22|)2$. (See pages 347–349.)

- The two-winding **ideal transformer** is a linear transformer with the following special properties: perfect coupling ($k=1$), infinite self-inductance in each coil ($L1=L2=\infty$), and lossless coils ($R1=R2=0$). The circuit behavior is governed by the turns ratio $a=N2/N1$. In particular, the volts per turn is the same for each winding, or

  $V1N1=\pm V2N2,$

  and the ampere turns are the same for each winding, or

  $N1I1=\pm N2I2.$

  (See page 356.)

Problems

Section 9.1

1. 9.1 Consider the sinusoidal voltage

   $v(t)=40\cos(100\pi t+60^\circ)$ V.

   1. What is the maximum amplitude of the voltage?
   2. What is the frequency in hertz?
   3. What is the frequency in radians per second?
   4. What is the phase angle in radians?
   5. What is the phase angle in degrees?
   6. What is the period in milliseconds?
   7. What is the first time after t=0 that v=−40 V?
   8. The sinusoidal function is shifted 10/3 ms to the right along the time axis. What is the expression for v(t)?
   9. What is the minimum number of milliseconds that the function must be shifted to the left if the expression for v(t) is $40\sin 100\pi t$ V?

2. 9.2 A sinusoidal voltage is given by the expression

   $v=10\cos(3769.91t-53.13^\circ)$ V.

   Find (a) f in hertz; (b) T in milliseconds; (c) Vm; (d) v(0); (e) ϕ in degrees and radians; (f) the smallest positive value of t at which v=0; and (g) the smallest positive value of t at which dv/dt=0.

3. 9.3 At t=−2 ms, a sinusoidal voltage is known to be zero and going

positive. The voltage is next zero at t=8 ms. It is also known that the voltage is 80.9 V at t=0.

1. What is the frequency of v in hertz?

2. What is the expression for v?

4. 9.4 In a single graph, sketch $v=100\cos(\omega t+\phi)$ versus ωt for $\phi = -60^\circ, -30^\circ, 0^\circ, 30^\circ$, and 60°.

    1. State whether the voltage function is shifting to the right or left as ϕ becomes more positive.

    2. What is the direction of shift if ϕ changes from 0 to -30°?

5. 9.5 A sinusoidal voltage is zero at $t=(-2\pi/3)$ ms and increasing at a rate of 80,000 V/s. The maximum amplitude of the voltage is 80 V.

    1. What is the frequency of v in radians per second?

    2. What is the expression for v?

6. 9.6 The rms value of the sinusoidal voltage supplied to the convenience outlet of a home in the United States is 120 V. What is the maximum value of the voltage at the outlet?

7. 9.7 Show that

    $\int_{t0}^{t0+T} V_m^2 \cos^2(\omega t+\phi)\,dt = V_m^2 T^2.$

8. 9.8 Find the rms value of the half-wave rectified sinusoidal voltage shown in Fig. P9.8 .

Figure P9.8

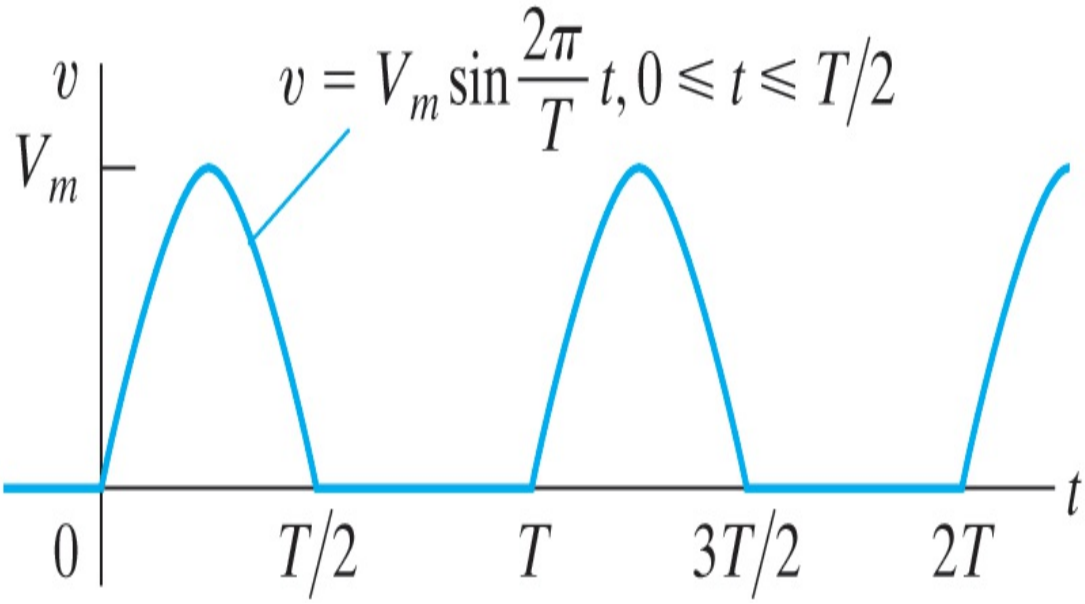


Figure P9.8 Full Alternative Text

Section 9.2

1. 9.9

    1. Verify that Eq. 9.7 is the solution of Eq. 9.6 . This can be done by substituting Eq. 9.7 into the left-hand side of Eq. 9.6 and then noting that it equals the right-hand side for all values of t>0. At t=0, Eq. 9.7 should reduce to the initial value of the current.

    2. Because the transient component vanishes as time elapses and because our solution must satisfy the differential equation for all values of t, the steady-state component, by itself, must also satisfy the differential equation. Verify this observation by showing that the steady-state component of Eq. 9.7 satisfies Eq. 9.6 .

2. 9.10 The voltage applied to the circuit shown in Fig. 9.5 at t=0 is $20\cos(800t+25^\circ)$ V. The circuit resistance is 80 Ω and the initial current in the 75 mH inductor is zero.

    1. Find i(t) for $t \geq 0$.

    2. Write the expressions for the transient and steady-state components of i(t).

    3. Find the numerical value of i after the switch has been closed for 1.875 ms.

    4. What are the maximum amplitude, frequency (in radians per second), and phase angle of the steady-state current?

    5. By how many degrees are the voltage and the steady-state current out of phase?

Sections 9.3-9.4

1. 9.11 Use the concept of the phasor to combine the following sinusoidal functions into a single trigonometric expression:
    1. $y=50\cos(500t+60^\circ)+100\cos(500t-30^\circ)$,
    2. $y=200\cos(377t+50^\circ)-100\sin(377t+150^\circ)$,
    3. $y=80\cos(100t+30^\circ)-100\sin(100t-135^\circ)+50\cos(100t-90^\circ)$,
    4. $y=250\cos\omega t+250\cos(\omega t+120^\circ)+250\cos(\omega t-120^\circ)$.
2. 9.12 The expressions for the steady-state voltage and current at the terminals of the circuit seen in Fig. P9.12 are

    $v_g=300\cos(5000\pi t+78°)V, i_g=6\sin(5000\pi t+123°)A$

    1. What is the impedance seen by the source?
    2. By how many microseconds is the current out of phase with the voltage?

Figure P9.12

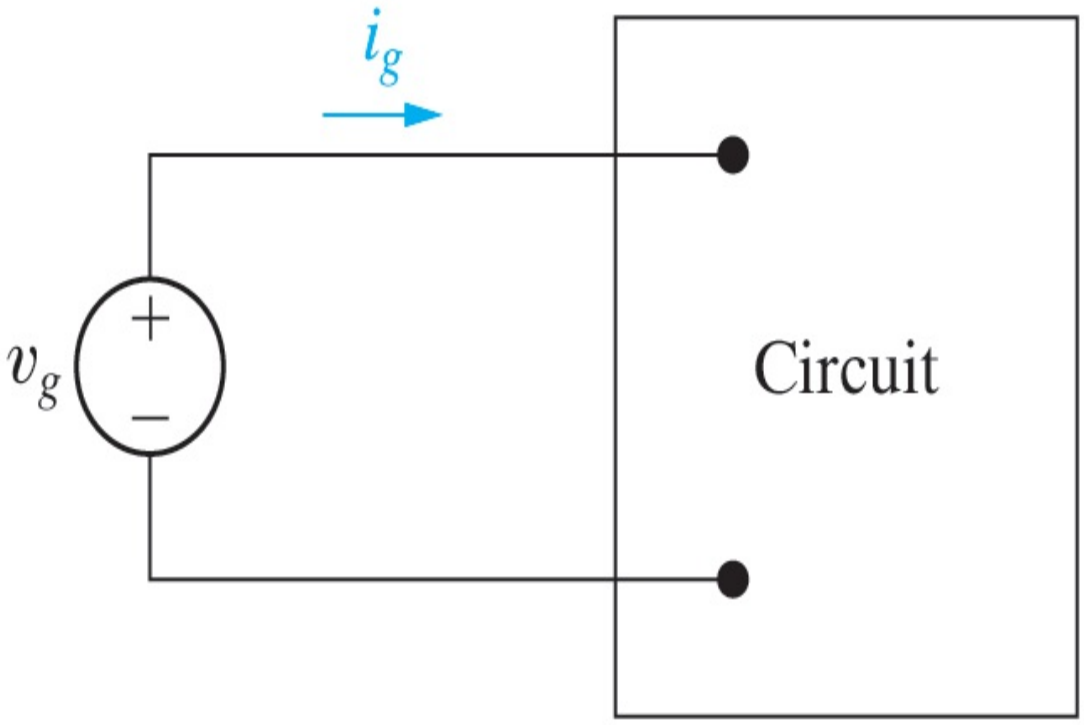


Figure P9.12 Full Alternative Text

3. 9.13 A 400 Hz sinusoidal voltage with a maximum amplitude of 100 V at t=0 is applied across the terminals of an inductor. The maximum amplitude of the steady-state current in the inductor is 20 A.

    1. What is the frequency of the inductor current?

    2. If the phase angle of the voltage is zero, what is the phase angle of the current?

    3. What is the inductive reactance of the inductor?

    4. What is the inductance of the inductor in millihenrys?

    5. What is the impedance of the inductor?

4. 9.14 A 50 kHz sinusoidal voltage has zero phase angle and a maximum amplitude of 10 mV. When this voltage is applied across the terminals of a capacitor, the resulting steady-state current has a maximum amplitude of 628.32 μA.

1. What is the frequency of the current in radians per second?
2. What is the phase angle of the current?
3. What is the capacitive reactance of the capacitor?
4. What is the capacitance of the capacitor in microfarads?
5. What is the impedance of the capacitor?

Sections 9.5 and 9.6

1. 9.15 PSPICEMULTISIM A 40 Ω resistor, a 5 mH inductor, and a 1.25 μF capacitor are connected in series. The series-connected elements are energized by a sinusoidal voltage source whose voltage is $600\cos(8000t+20^\circ)$ V.
    1. Draw the frequency-domain equivalent circuit.
    2. Reference the current in the direction of the voltage rise across the source, and find the phasor current.
    3. Find the steady-state expression for i(t).
2. 9.16 PSPICEMULTISIM A 10 Ω resistor and a 5 μF capacitor are connected in parallel. This parallel combination is also in parallel with the series combination of a 8 Ωresistor and a 300 μH inductor. These three parallel branches are driven by a sinusoidal current source whose current is $922\cos(20{,}000t+30^\circ)$ A.
    1. Draw the frequency-domain equivalent circuit.
    2. Reference the voltage across the current source as a rise in the direction of the source current, and find the phasor voltage.
    3. Find the steady-state expression for v(t).

3. 9.17 Three branches having impedances of 3+j4 Ω, 16−j12 Ω, and −j4 Ω, respectively, are connected in parallel. What are the equivalent (a) admittance, (b) conductance, and (c) susceptance of the parallel connection in millisiemens? (d) If the parallel branches are excited from a sinusoidal current source where $i=8\cos\omega t$ A, what is the maximum amplitude of the current in the purely capacitive branch?

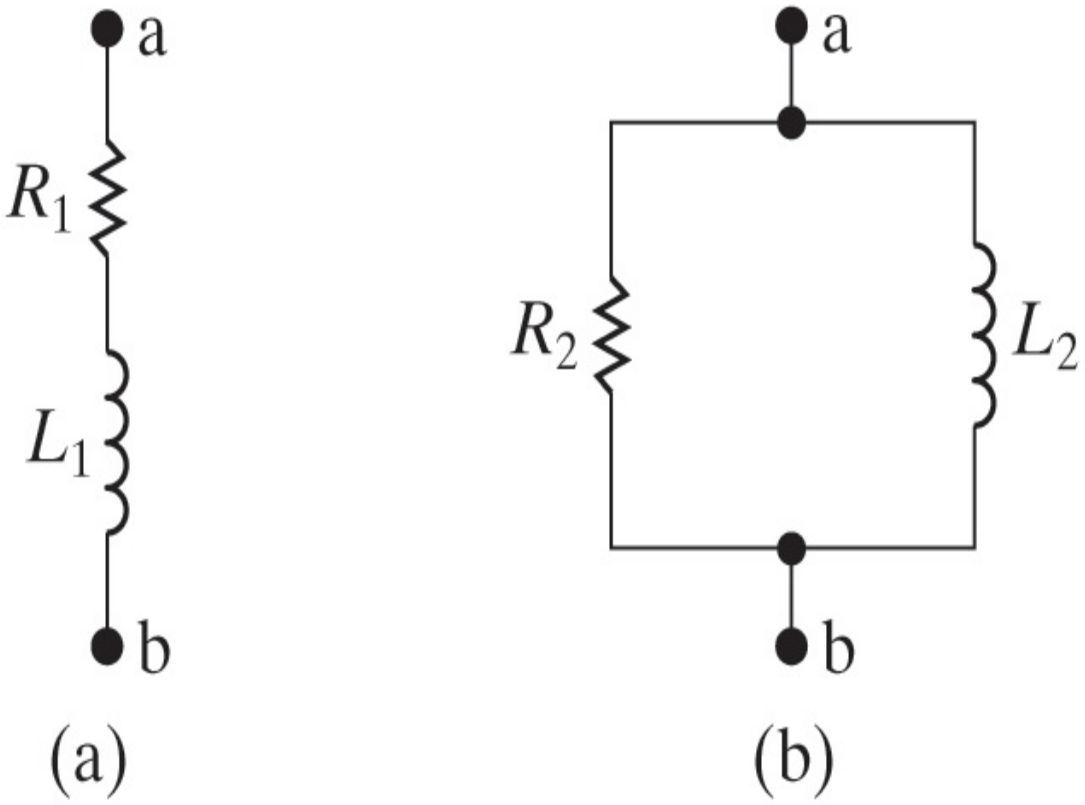
4. 9.18

    1. Show that, at a given frequency ω, the circuits in Fig. P9.18 (a) and (b) will have the same impedance between the terminals a,b if

       $R_1=\omega^2 L_2^2 R_2/(R_2^2+\omega^2 L_2^2), \quad L_1=R_2^2 L_2/(R_2^2+\omega^2 L_2^2).$

    2. Find the values of resistance and inductance that when connected in series will have the same impedance at 4 krad/s as that of a 5 kΩ resistor connected in parallel with a 1.25 H inductor.

Figure P9.18



5. 9.19

    1. Show that at a given frequency ω, the circuits in Fig. P9.18(a) and (b) will have the same impedance between the terminals a,b if

        R2=R12+ω2L12R1, L2=R12+ω2L12ω2L1.

        (Hint: The two circuits will have the same impedance if they have the same admittance.)

    2. Find the values of resistance and inductance that when connected in parallel will have the same impedance at 1 krad/s an 8 kΩresistor connected in series with a 4 H inductor.

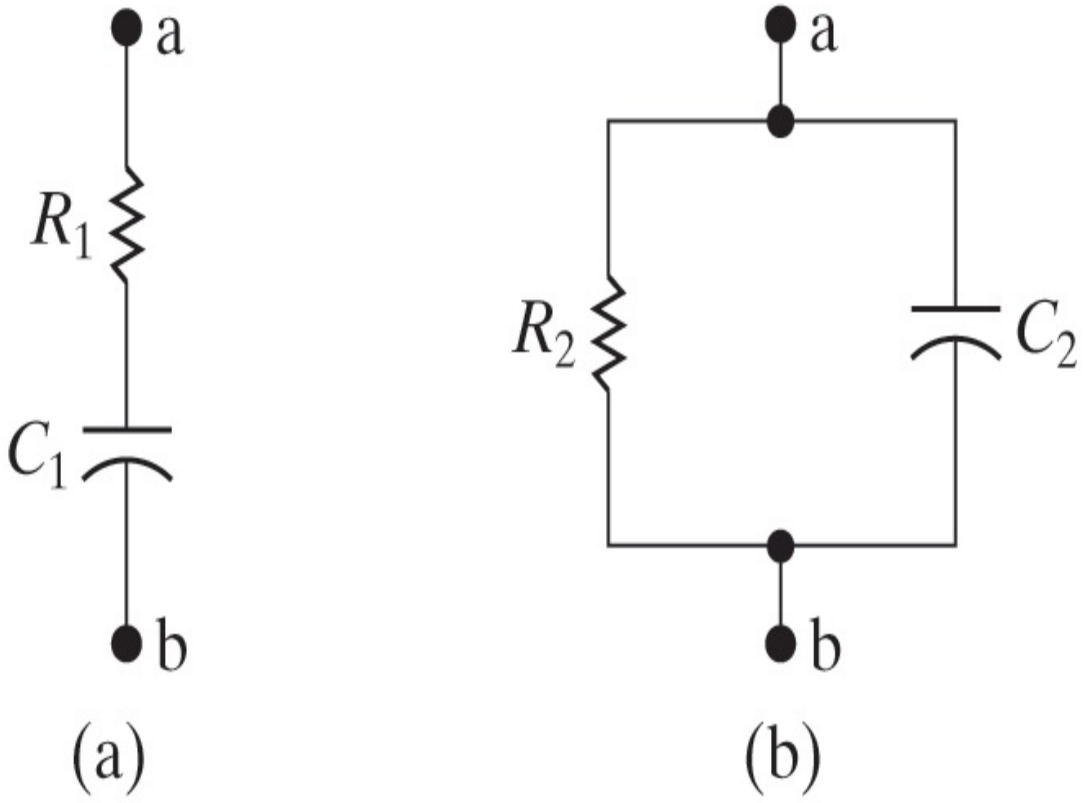
6. 9.20

    1. Show that at a given frequency ω, the circuits in Fig. P9.20 (a) and (b) will have the same impedance between the terminals a,b if

$R1=R21+\omega 2R22C22, C1=1+\omega 2R22C22\omega 2R22C2$.

2. Find the values of resistance and capacitance that when connected in series will have the same impedance at 40 krad/s as that of a 1000 Ω resistor connected in parallel with a 50 nF capacitor.

Figure P9.20



7. 9.21

1. Show that at a given frequency ω, the circuits in Fig 9.20 (a) and (b) will have the same impedance between the terminals a,b if

   $R2=1+\omega 2R12C12\omega 2R1C12, C2=C11+\omega 2R12C12$.

   (Hint: The two circuits will have the same impedance if they have the same admittance.)

2. Find the values of resistance and capacitance that when connected in parallel will give the same impedance at 50 krad/s as that of a 1 kΩ resistor connected in series with a capacitance of 40 nF.

8. 9.22

    1. Using component values from Appendix H , combine at least one resistor, inductor, and capacitor in series to create an impedance of 300−j400 Ω at a frequency of 10,000 rad/s.

    2. At what frequency does the circuit from part (a) have an impedance that is purely resistive?

9. 9.23

    1. Using component values from Appendix H , combine at least one resistor and one inductor in parallel to create an impedance of 40+j20 Ω at a frequency of 5000 rad/s. (Hint: Use the results of Problem 9.19 .)

    2. Using component values from Appendix H , combine at least one resistor and one capacitor in parallel to create an impedance of 40−j20 Ω at a frequency of 5000 rad/s. (Hint: Use the result of Problem 9.21 .)

10. 9.24

    1. Using component values from Appendix H , find a single capacitor or a network of capacitors that, when combined in parallel with the RL circuit from Problem 9.23 (a), gives an equivalent impedance that is purely resistive at a frequency of 5000 rad/s.

    2. Using component values from Appendix H , find a single inductor or a network of inductors that, when combined in parallel with the RC circuit from Problem 9.23 (b), gives an equivalent impedance that is purely resistive at a frequency of 5000 rad/s.

11. 9.25 Find the admittance Yab in the circuit seen in Fig. P9.25 . Express

Yab in both polar and rectangular form. Give the value of Yab in millisiemens.

Figure P9.25

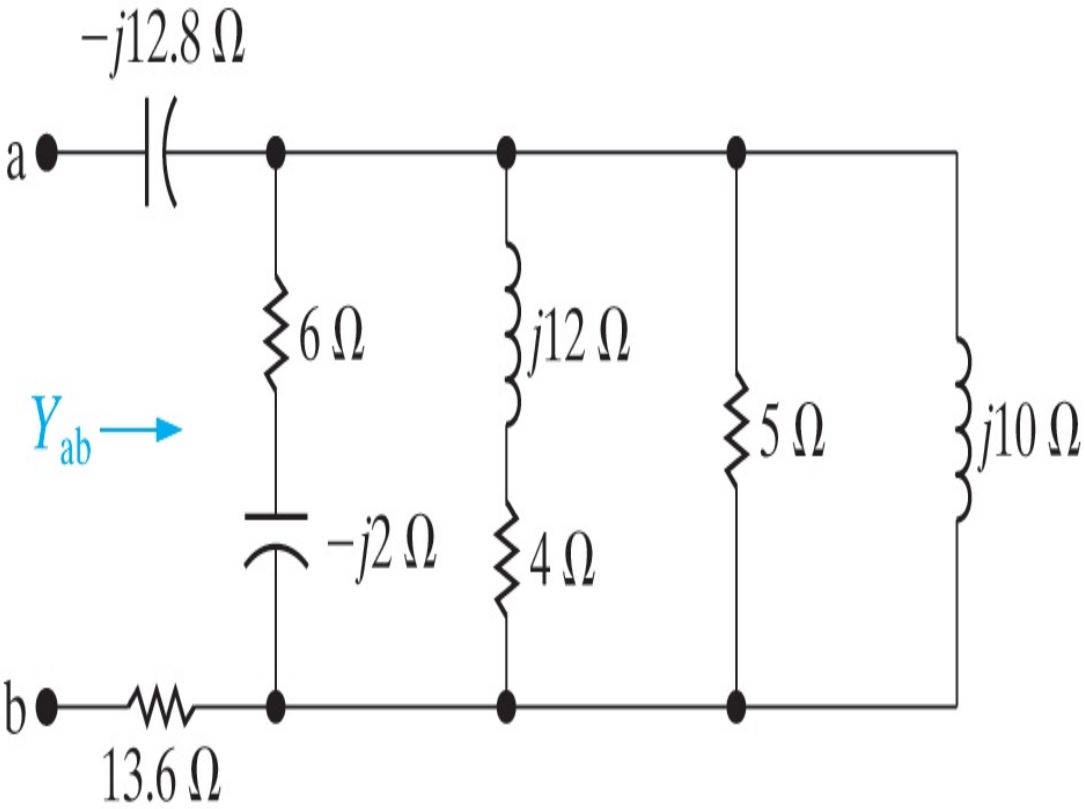


Figure P9.25 Full Alternative Text

12. 9.26 Find the impedance Zab in the circuit seen in Fig. P9.26 . Express Zab in both polar and rectangular form.

Figure P9.26

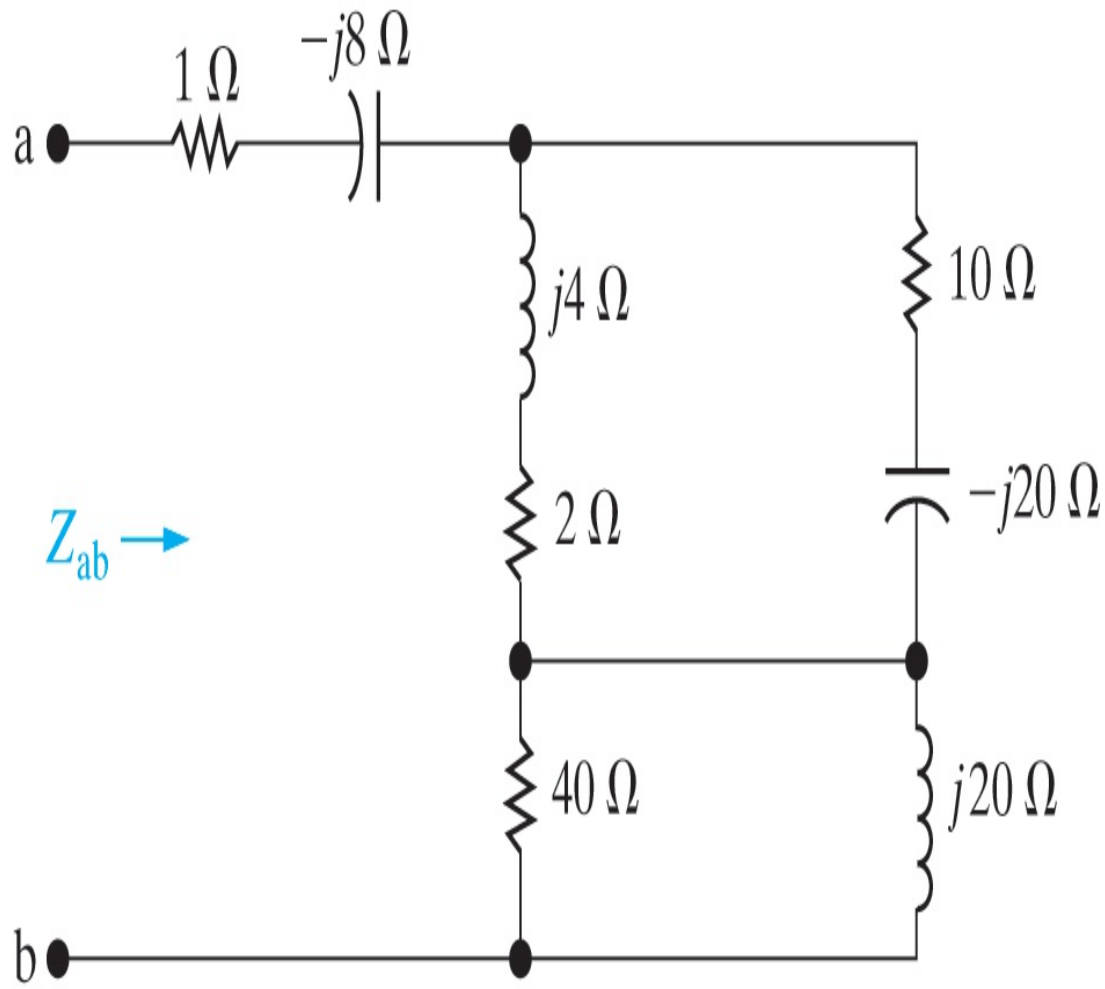


Figure P9.26 Full Alternative Text

13. 9.27 PSPICEMULTISIM

    1. For the circuit shown in Fig. P9.27 , find the frequency (in radians per second) at which the impedance Zab is purely resistive.

    2. Find the value of Zab at the frequency of (a).

Figure P9.27

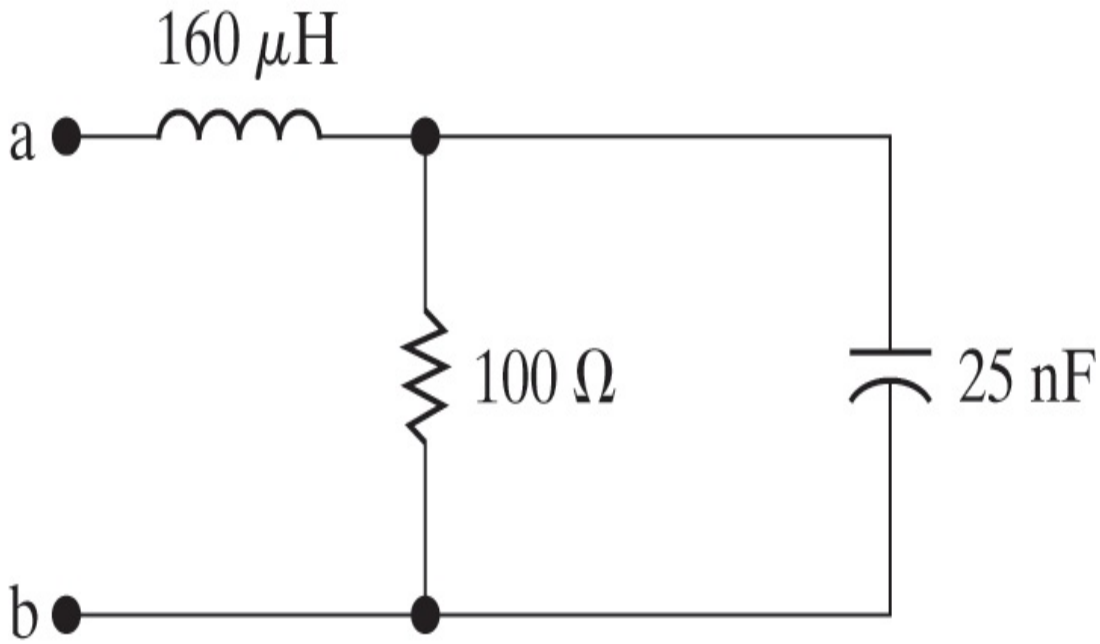


Figure P9.27 Full Alternative Text

14. 9.28 PSPICEMULTISIM The circuit in Fig. P9.28 is operating in the sinusoidal steady state. Find the steady-state expression for vo(t) if vg=40 sin 50,000t V.

Figure P9.28

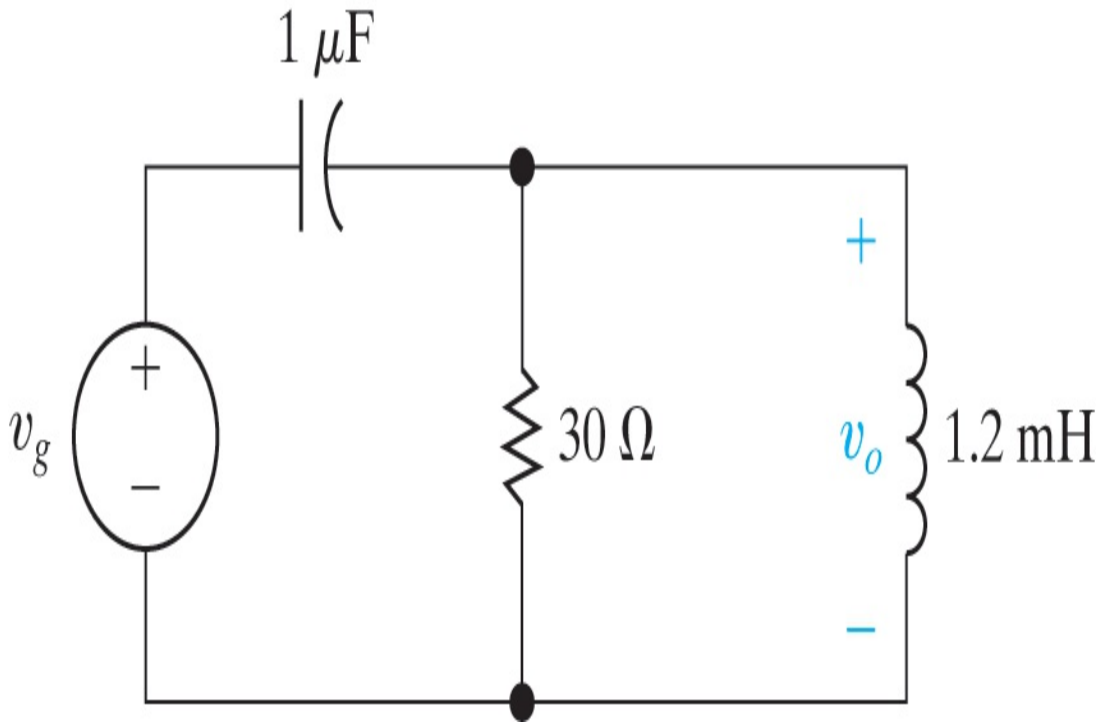


Figure P9.28 Full Alternative Text

15. 9.29 PSPICEMULTISIM Find the steady-state expression for io(t) in the circuit in Fig. P9.29 if vs=100 sin 50t mV.

Figure P9.29

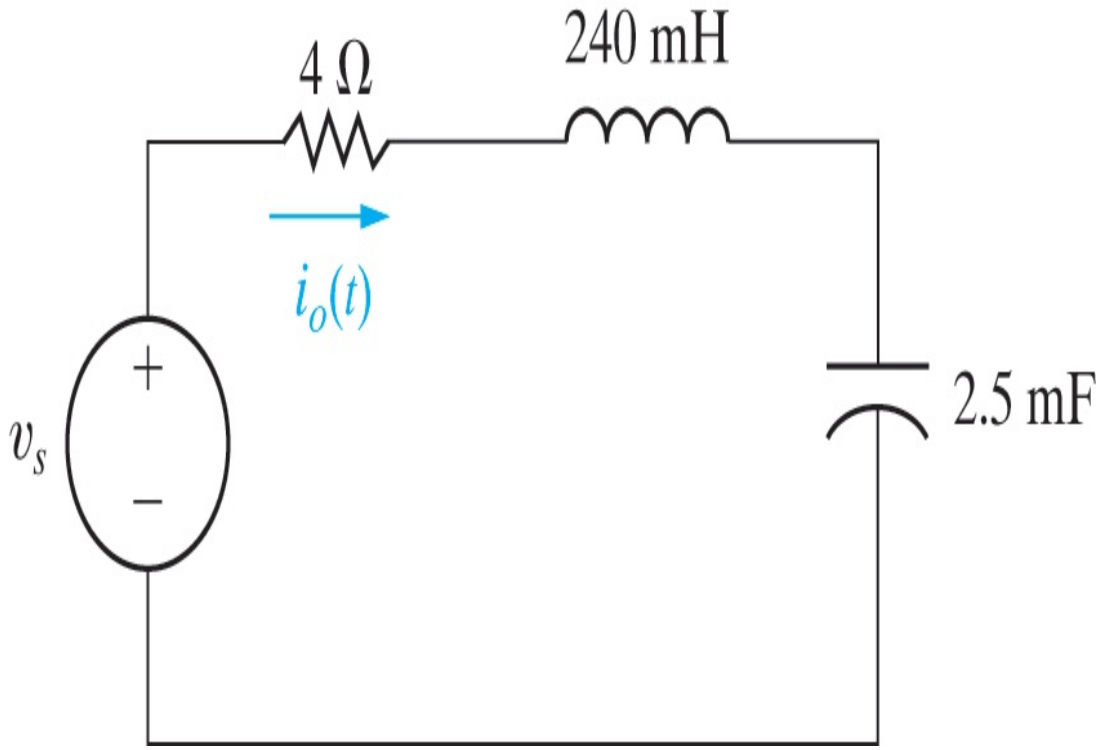


Figure P9.29 Full Alternative Text

16. 9.30 PSPICEMULTISIM The circuit in Fig. P9.30 is operating in the sinusoidal steady state. Find io(t) if vs(t)=25 sin 4000t V.

Figure P9.30

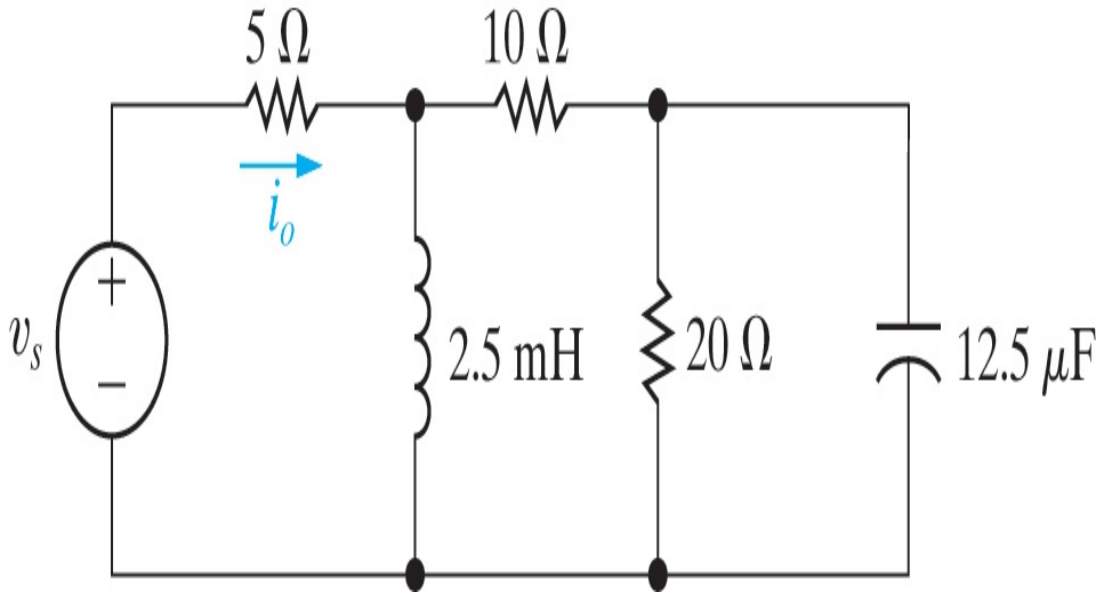


Figure P9.30 Full Alternative Text

17. 9.31 Find the steady-state expression for vo in the circuit of Fig. P9.31 if ig=0.5 cos 2000t A.

Figure P9.31

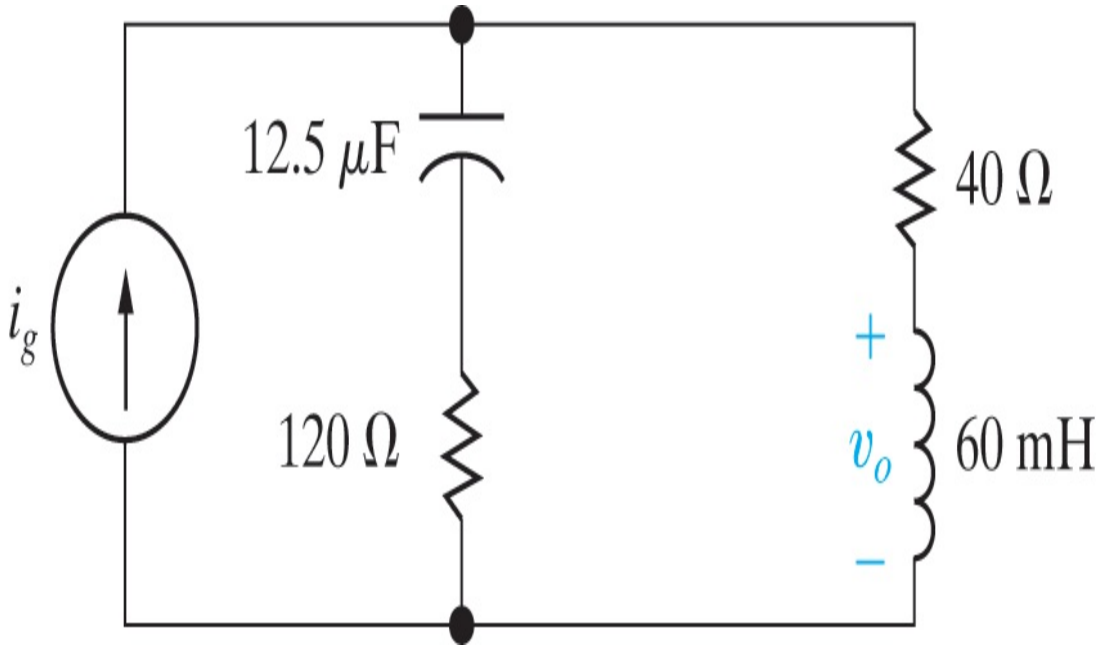


Figure P9.31 Full Alternative Text

18. 9.32 PSPICEMULTISIM The phasor current Ib in the circuit shown in Fig. P9.32 is 25 0 . mA.

    1. Find Ia, Ic, and Ig.

    2. If ω=1500 rad/s, write expressions for ia(t), ic(t), and ig(t).

Figure P9.32

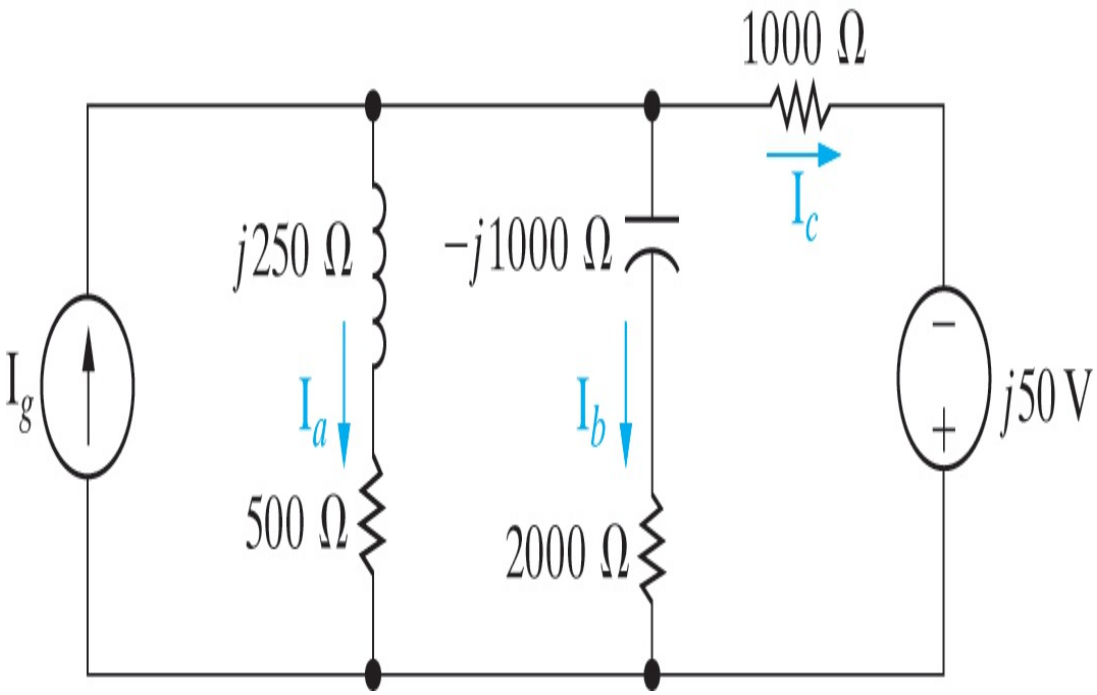


Figure P9.32 Full Alternative Text

19. 9.33 PSPICEMULTISIM

    1. For the circuit shown in Fig. P9.33 , find the steady-state expression for vo if ig=25 cos 50,000t mA.

    2. By how many microseconds does vo lead ig?

Figure P9.33

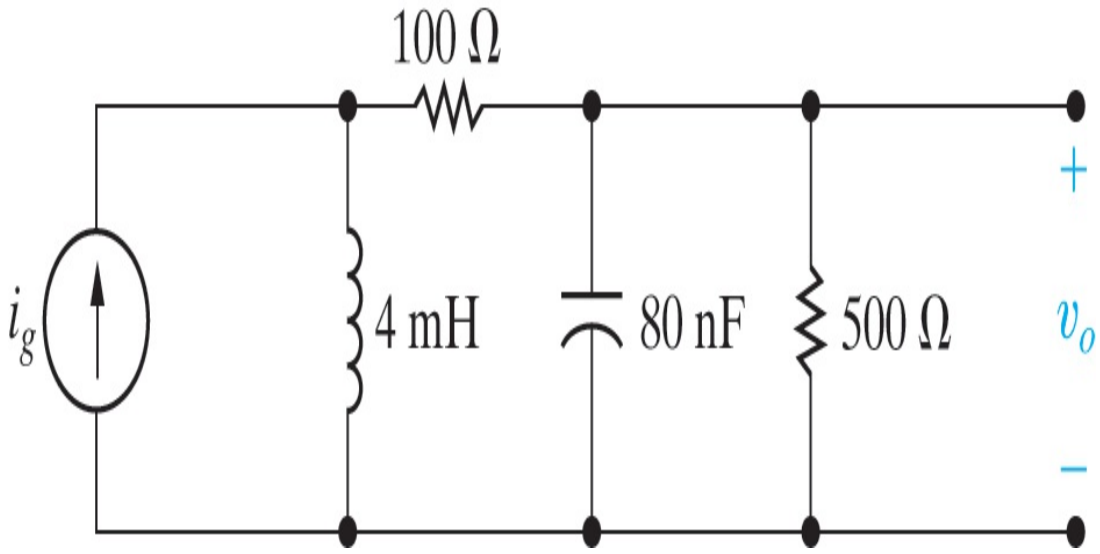


Figure P9.33 Full Alternative Text

20. 9.34 Find the value of Z in the circuit seen in Fig. P9.34 if Vg=100−j50 V, Ig=30+j20 A, and V1=140+j30 V.

Figure P9.34

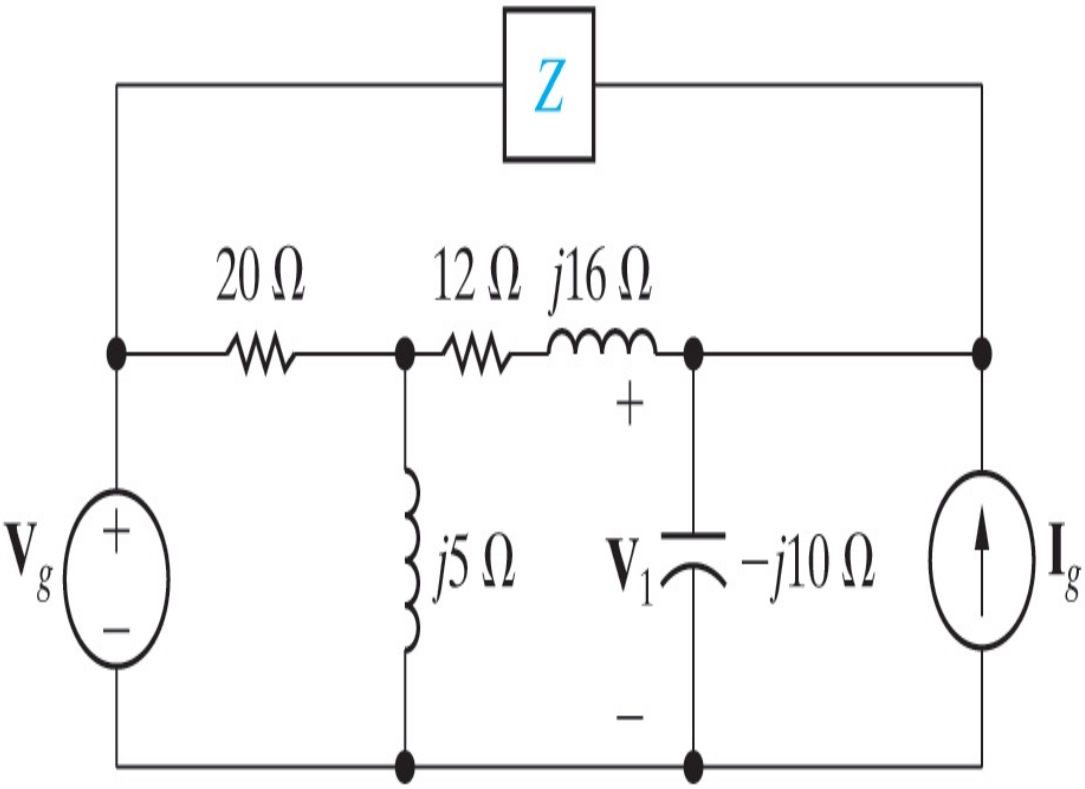


Figure P9.34 Full Alternative Text

21. 9.35 Find Ib and Z in the circuit shown in Fig. P9.35 if V g =25 0 ° V and I a =5 90 ° A.

Figure P9.35

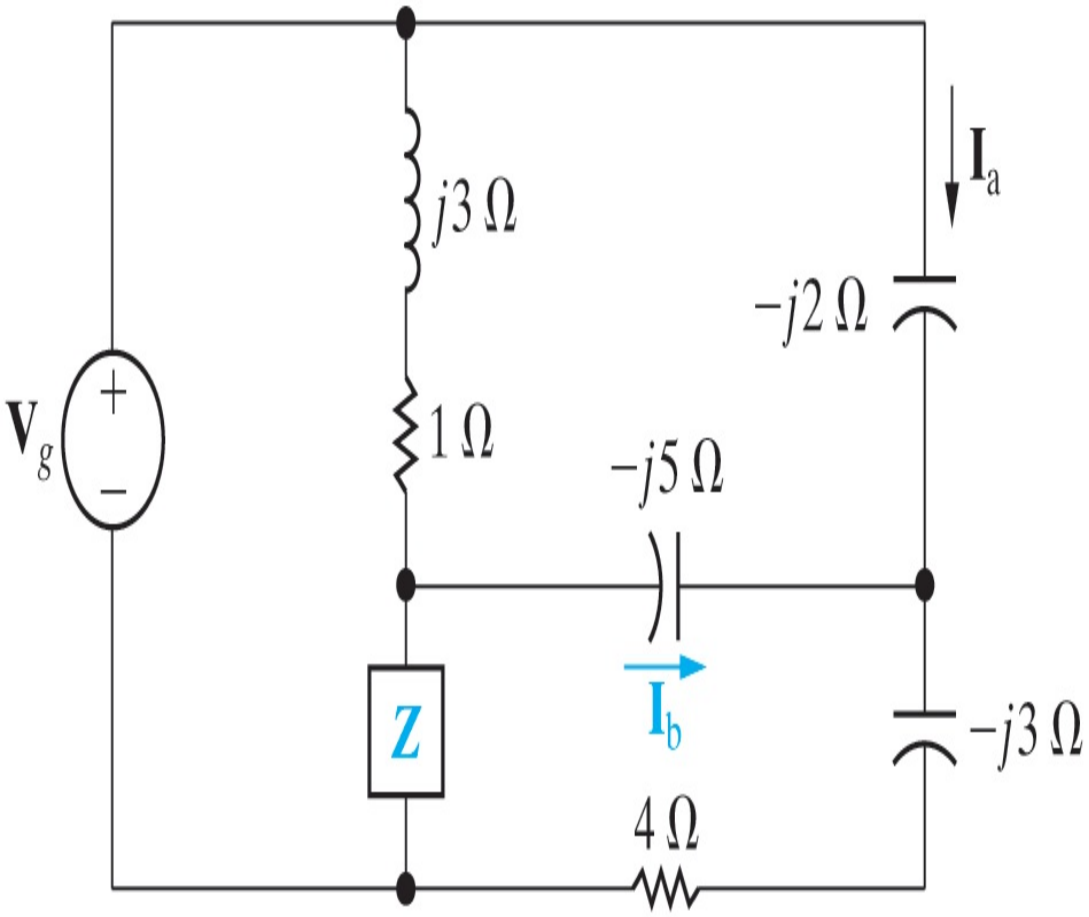


Figure P9.35 Full Alternative Text

22. 9.36 The circuit shown in Fig. P9.36 is operating in the sinusoidal steady state. Find the value of ω if

    io=100 sin(ωt+173.13°) mA,vg=500 cos(ωt+30°) V.

Figure P9.36

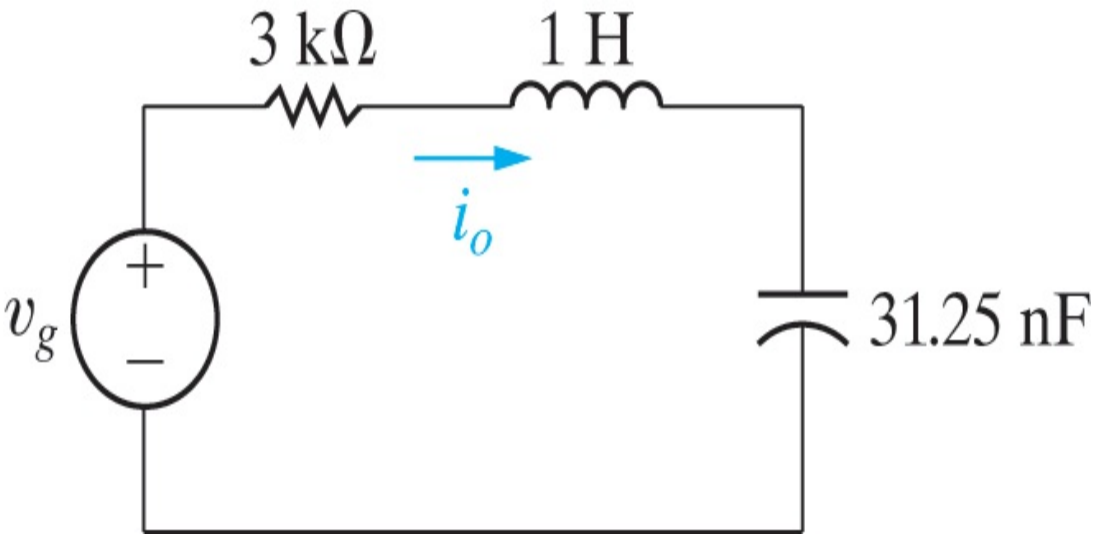


Figure P9.36 Full Alternative Text

23. 9.37 PSPICEMULTISIM The frequency of the sinusoidal voltage source in the circuit in Fig. P9.37 is adjusted until ig is in phase with vg.

    1. What is the value of ω in radians per second?

    2. If vg=15 cosωt V (where ω is the frequency found in [a]), what is the steady-state expression for ig?

Figure P9.37

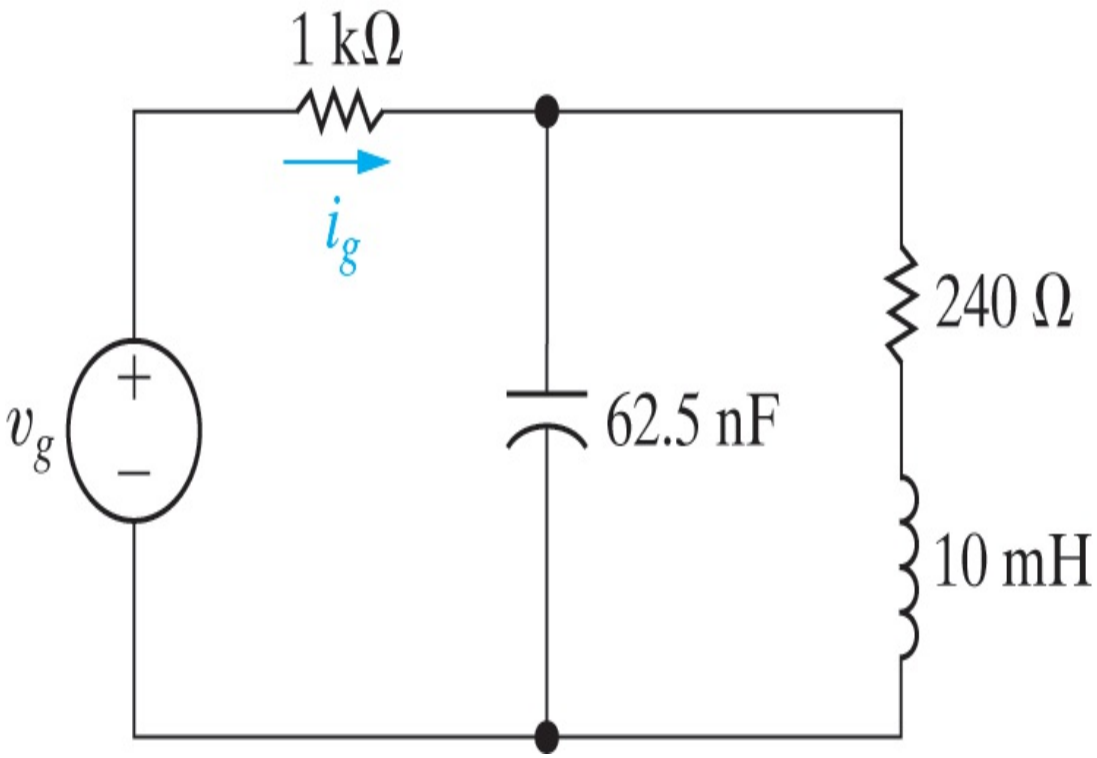


Figure P9.37 Full Alternative Text

24. 9.38 PSPICEMULTISIM The frequency of the sinusoidal voltage source in the circuit in Fig. P9.38 is adjusted until the current io is in phase with vg.

    1. Find the frequency in hertz.

    2. Find the steady-state expression for ig (at the frequency found in [a]) if vg=90 cosωt V.

Figure P9.38

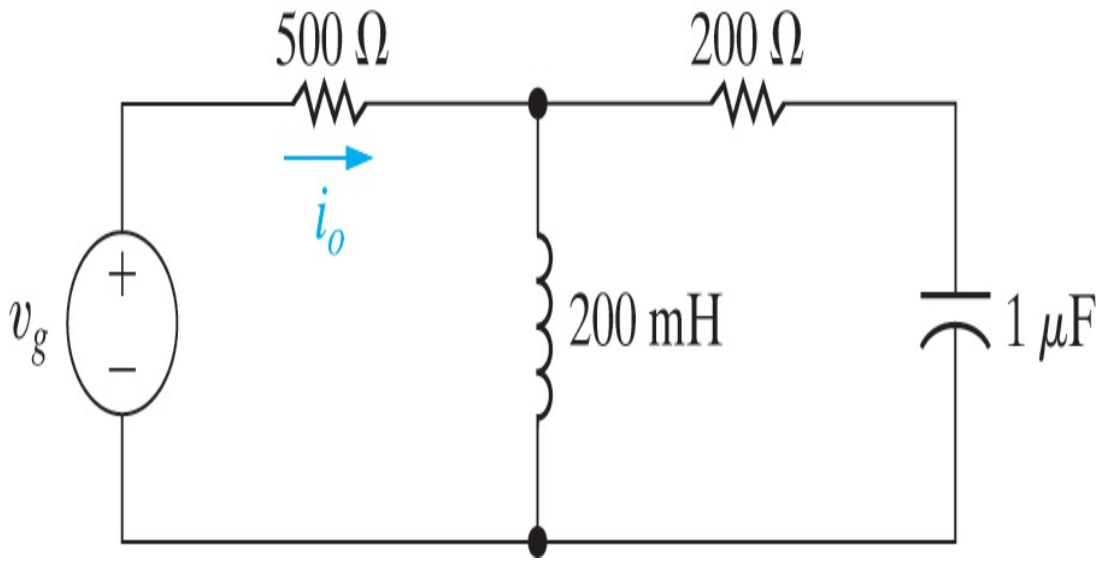


Figure P9.38 Full Alternative Text

25. 9.39 PSPICEMULTISIM

    1. The frequency of the source voltage in the circuit in Fig. P9.39 is adjusted until vg is in phase with ig. What is the value of ω in radians per second?

    2. If ig=60 cosωt mA (where ω is the frequency found in [a]), what is the steady-state expression for vg?

Figure P9.39

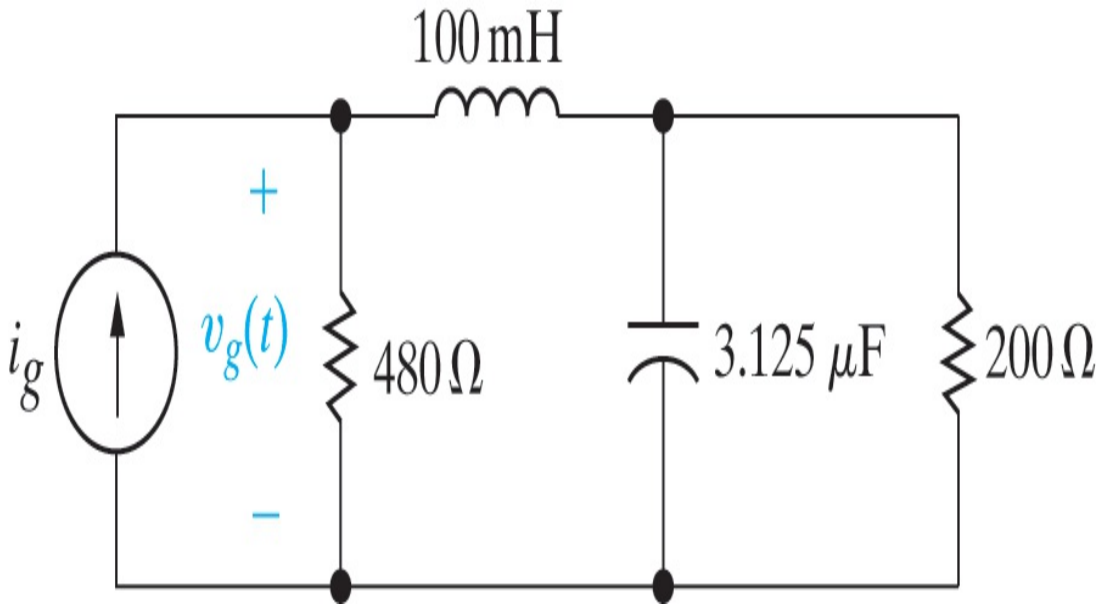


Figure P9.39 Full Alternative Text

26. 9.40 PSPICEMULTISIM The circuit shown in Fig. P9.40 is operating in the sinusoidal steady state. The capacitor is adjusted until the current ig is in phase with the sinusoidal voltage vg.

    1. Specify the capacitance in microfarads if vg=80 cos5000t V.

    2. Give the steady-state expression for ig when C has the value found in (a).

Figure P9.40

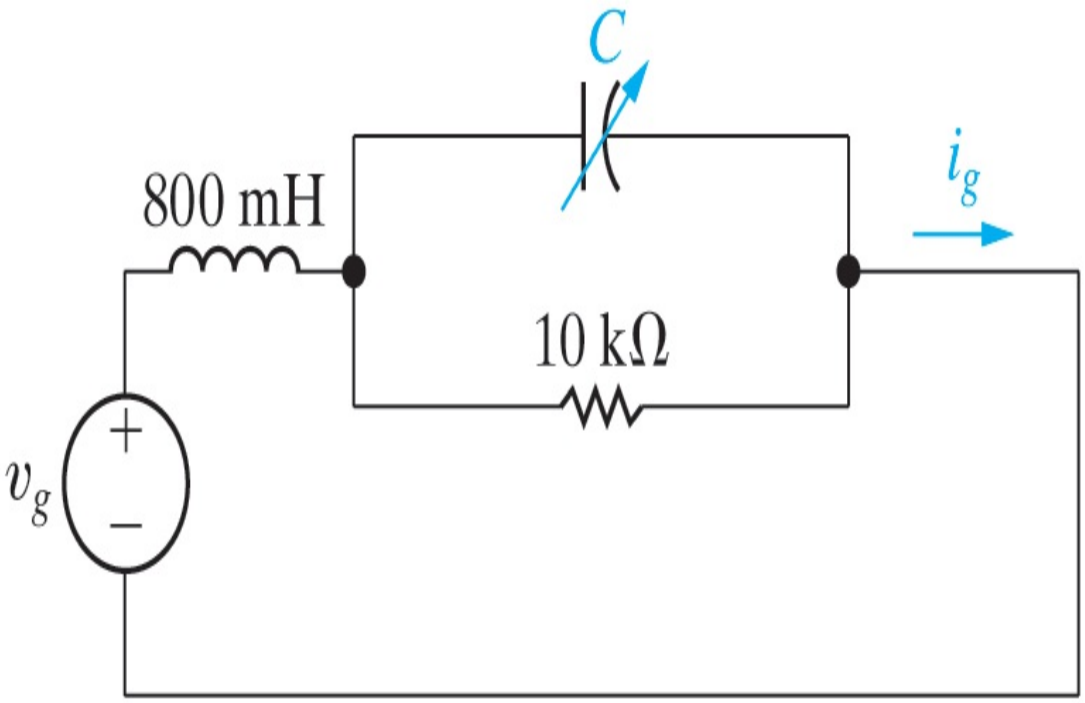


Figure P9.40 Full Alternative Text

27. 9.41 PSPICEMULTISIM

    1. The source voltage in the circuit in Fig. P9.41 is vg=200 cos500t V. Find the values of L such that ig is in phase with vg when the circuit is operating in the steady state.

    2. For the values of L found in (a), find the steady-state expressions for ig.

Figure P9.41

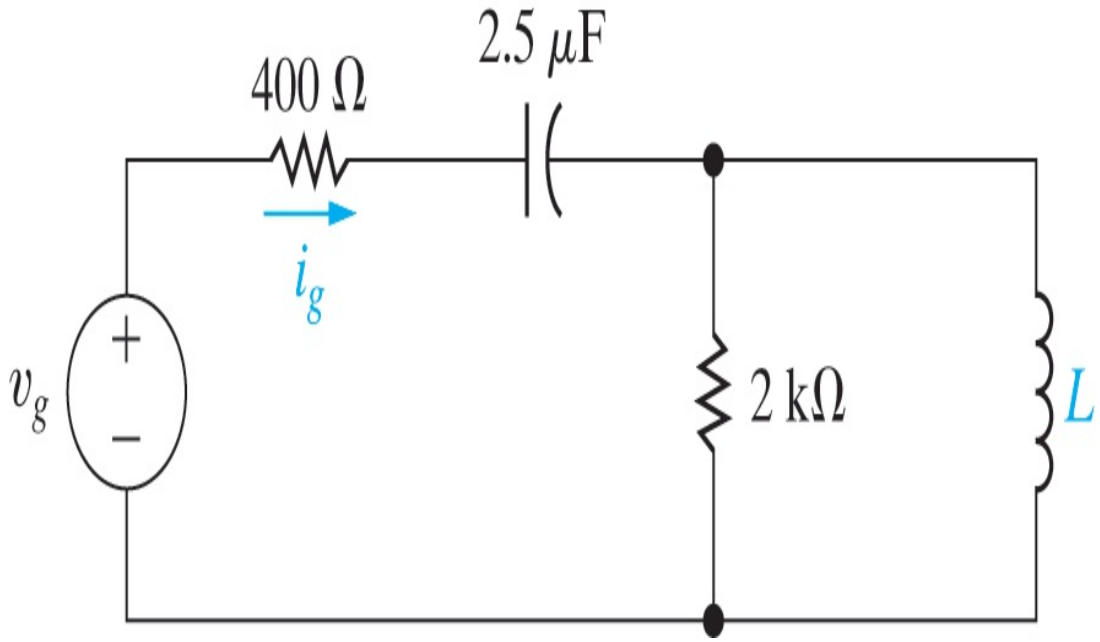


Figure P9.41 Full Alternative Text

28. 9.42 Find Zab for the circuit shown in Fig P9.42 .

Figure P9.42

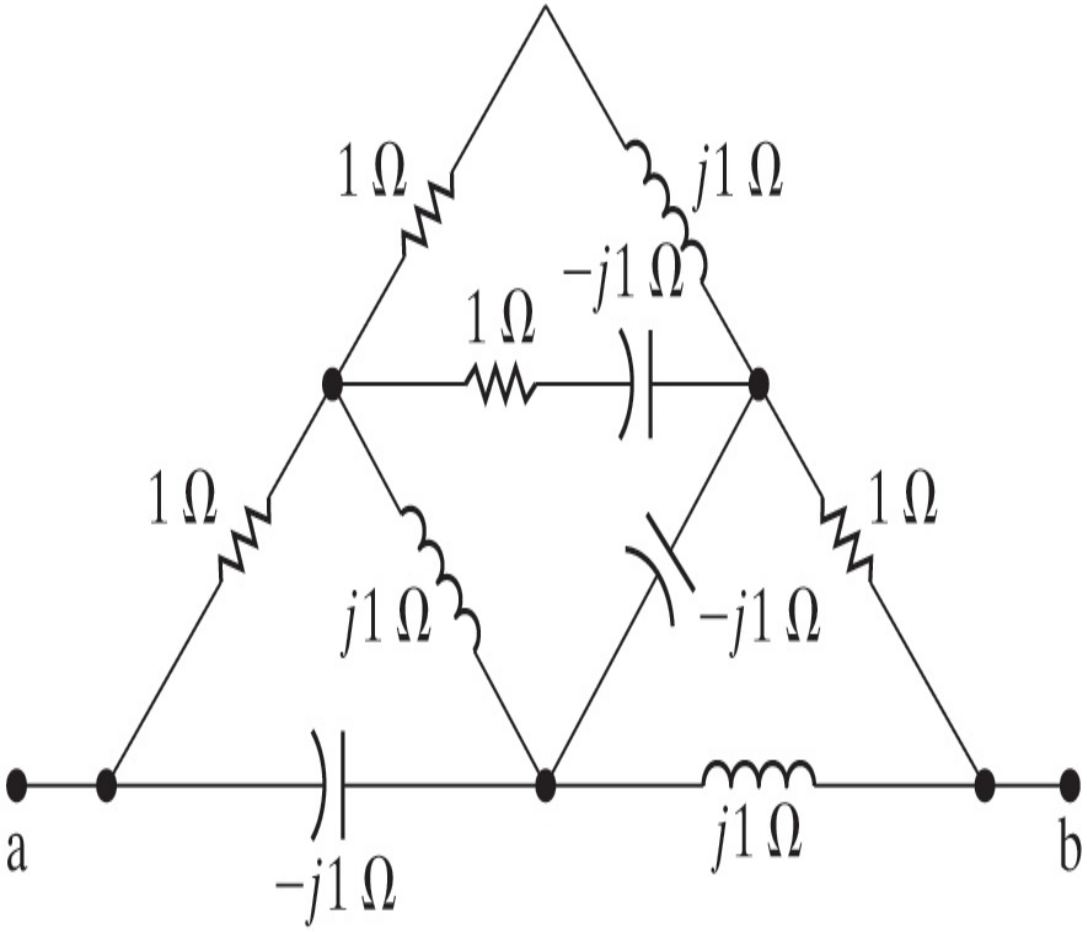


Figure P9.42 Full Alternative Text

Section 9.7

1. 9.43 Use source transformations to find the Thévenin equivalent circuit with respect to the terminals a,b for the circuit shown in Fig. P9.43 .

Figure P9.43

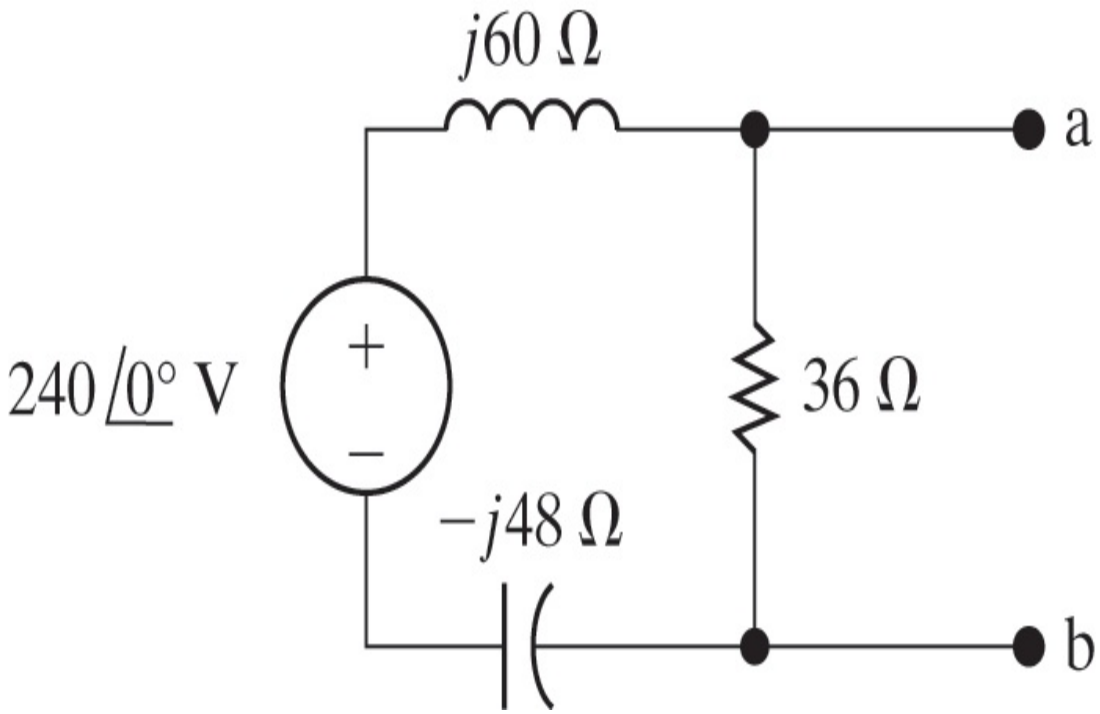


Figure P9.43 Full Alternative Text

2. 9.44 Use source transformations to find the Norton equivalent circuit with respect to the terminals a,b for the circuit shown in Fig. P9.44 .

Figure P9.44

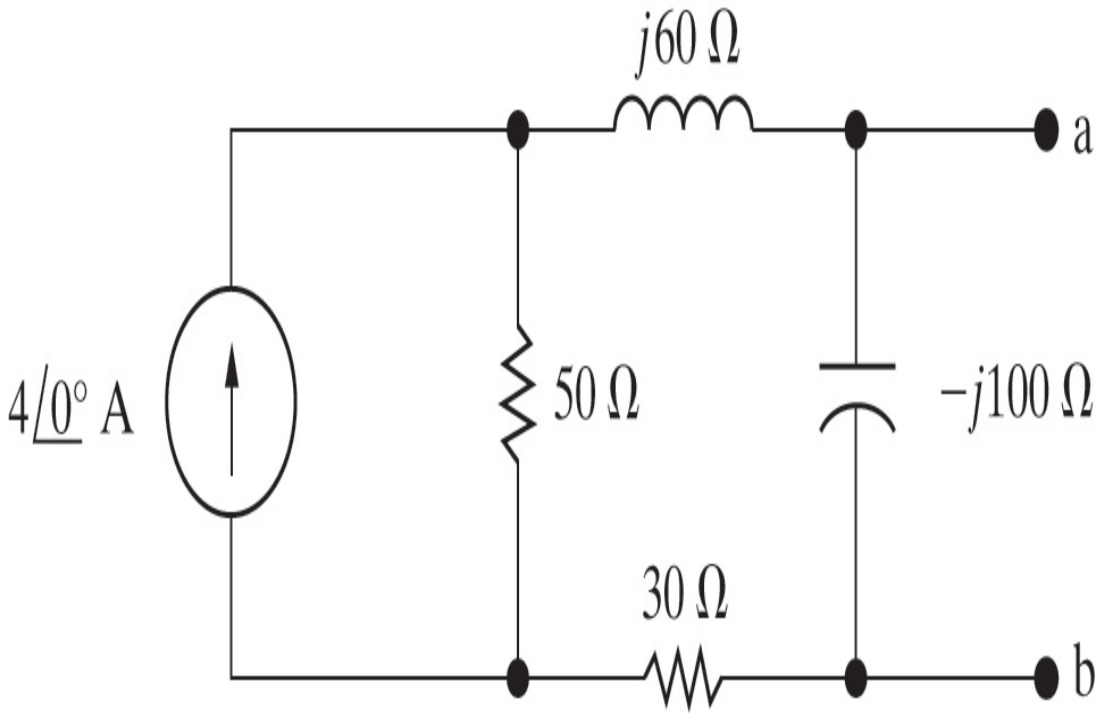


Figure P9.44 Full Alternative Text

3. 9.45 The sinusoidal voltage source in the circuit in Fig. P9.45 is developing a voltage equal to 50 sin 400t V.

    1. Find the Thévenin voltage with respect to the terminals a,b.
    2. Find the Thévenin impedance with respect to the terminals a,b.
    3. Draw the Thévenin equivalent.

Figure P9.45

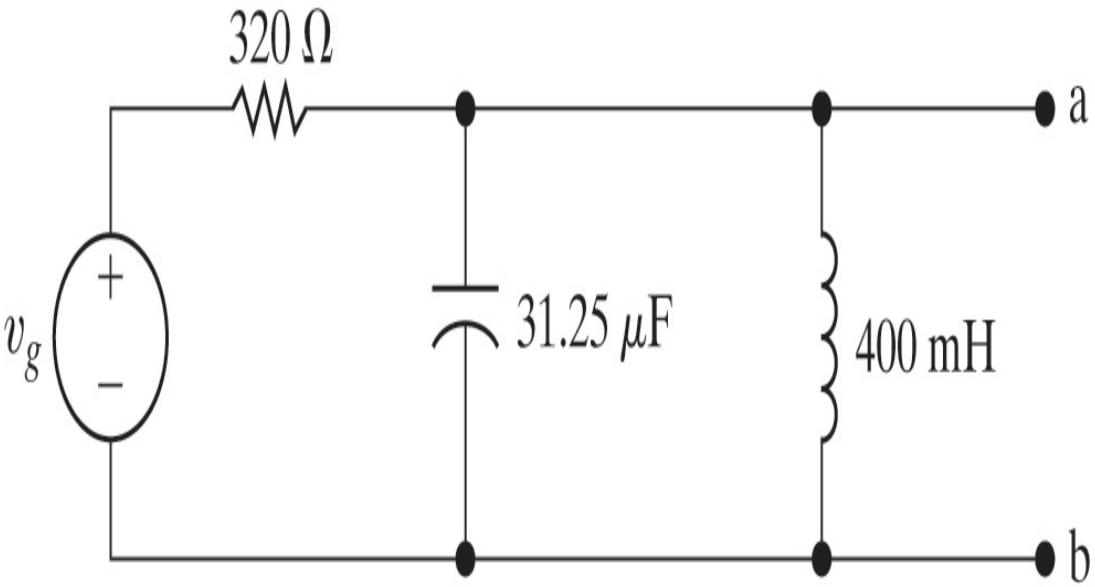


Figure P9.45 Full Alternative Text

4. 9.46 Find the Norton equivalent circuit with respect to the terminals a,b for the circuit shown in Fig. P9.46 .

Figure P9.46

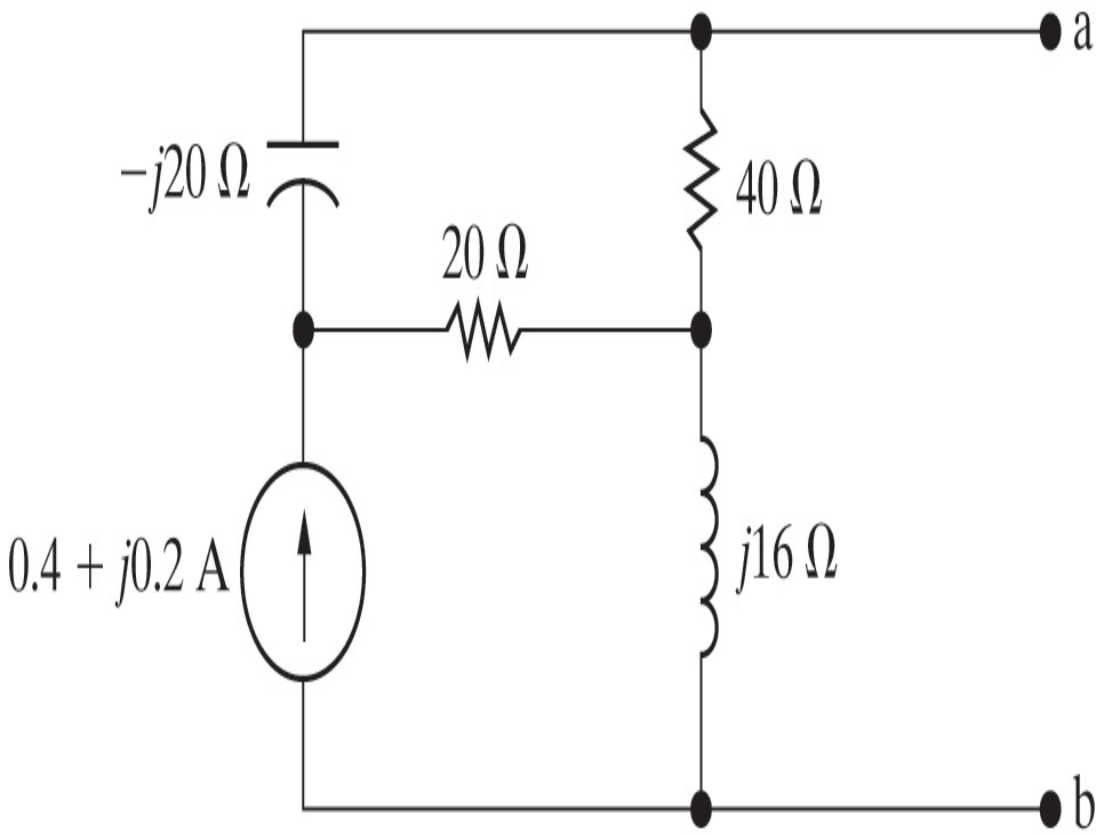
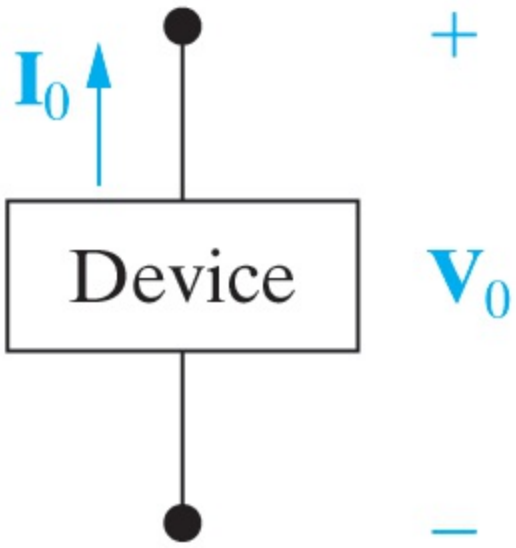


Figure P9.46 Full Alternative Text

5. 9.47 The device in Fig. P9.47 is represented in the frequency domain by a Norton equivalent. When a resistor having an impedance of 5 kΩ is connected across the device, the value of V0 is (5–j15) V. When a capacitor having an impedance of −j3 kΩ is connected across the device, the value of I0 is (4.5–j6) mA. Find the Norton current IN and the Norton impedance ZN.

Figure P9.47



6. 9.48 Find the Norton equivalent with respect to terminals a,b in the circuit of Fig. P9.48 .

Figure P9.48

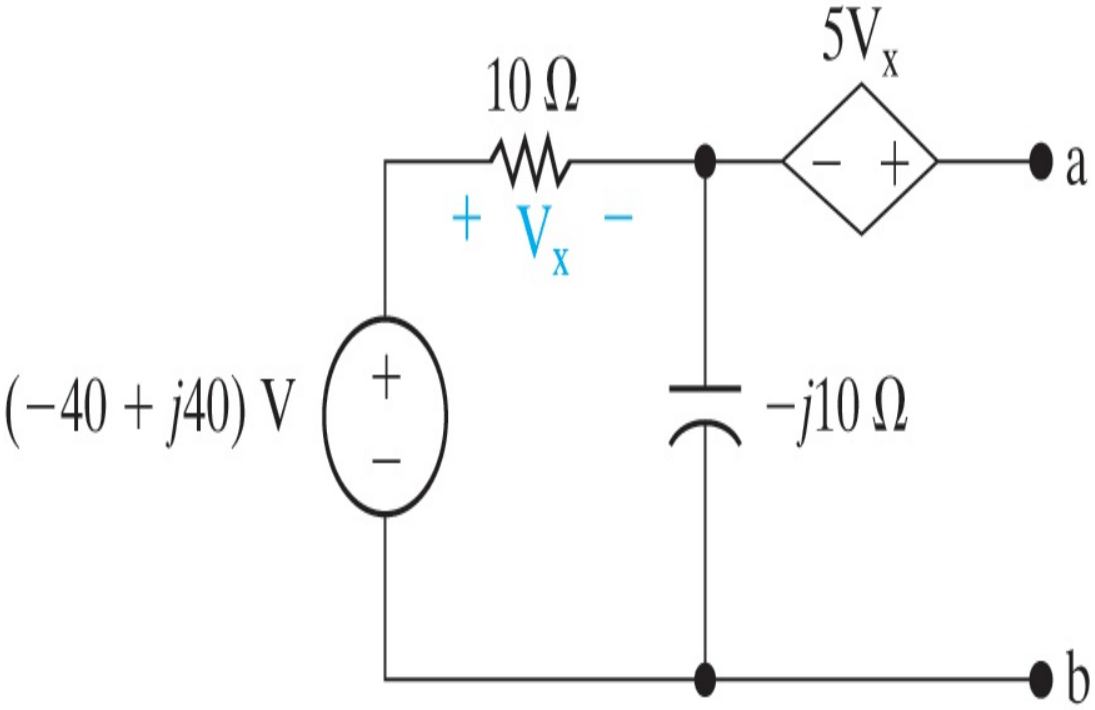


Figure P9.48 Full Alternative Text

7. 9.49 Find the Norton equivalent circuit with respect to the terminals a,b for the circuit shown in Fig. P9.49 when V s =5 0 ° V.

Figure P9.49

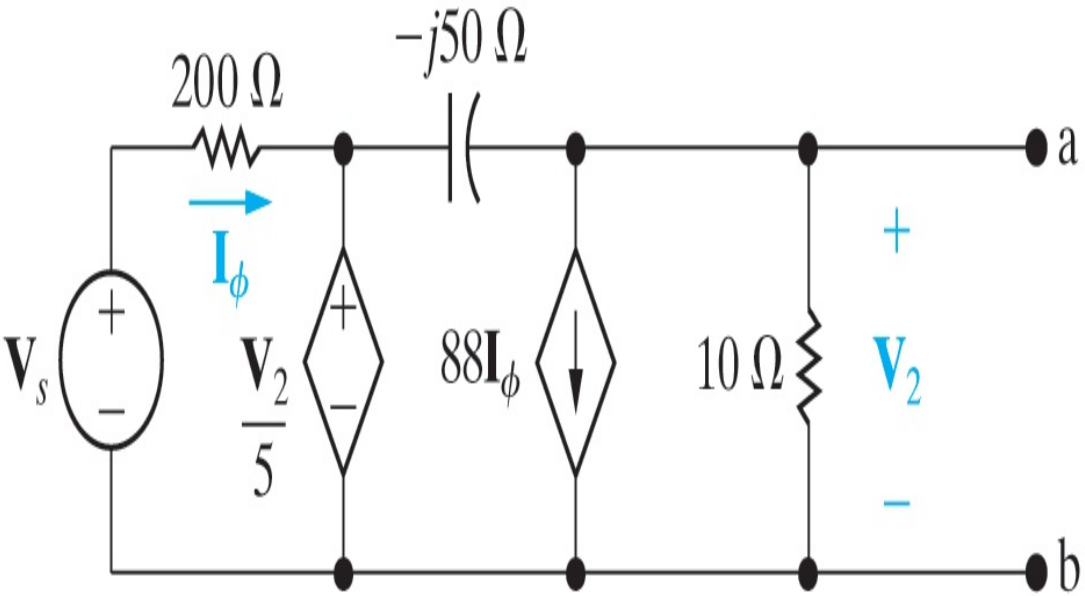


Figure P9.49 Full Alternative Text

8. 9.50 Find the Thévenin equivalent circuit with respect to the terminals a,b of the circuit shown in Fig. P9.50 .

Figure P9.50

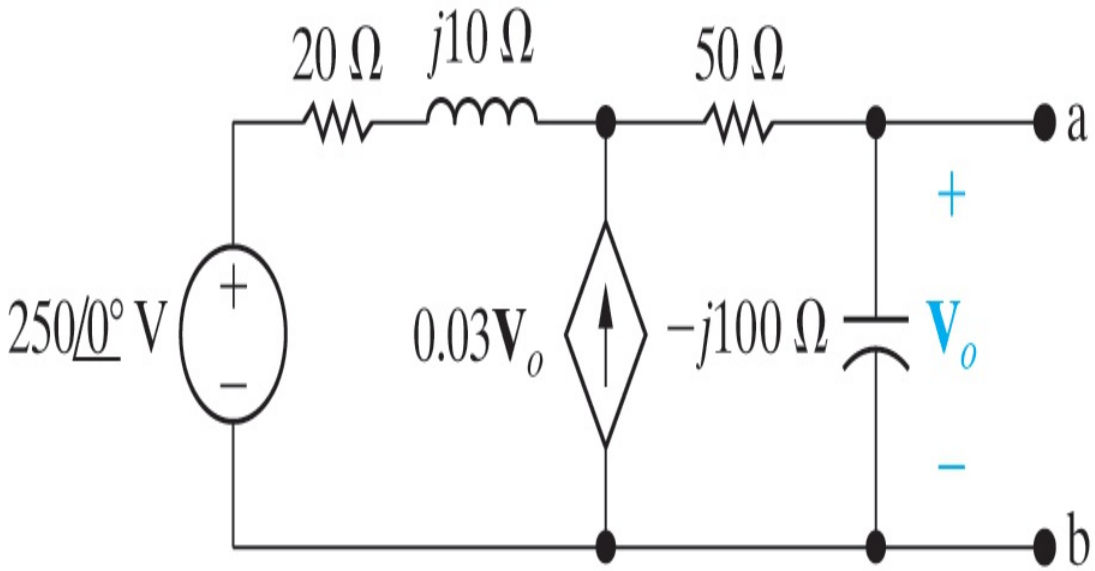


Figure P9.50 Full Alternative Text

9. 9.51 Find the Thévenin impedance seen looking into the terminals a,b of the circuit in Fig. P9.51 if the frequency of operation is (200/π) kHz.

Figure P9.51

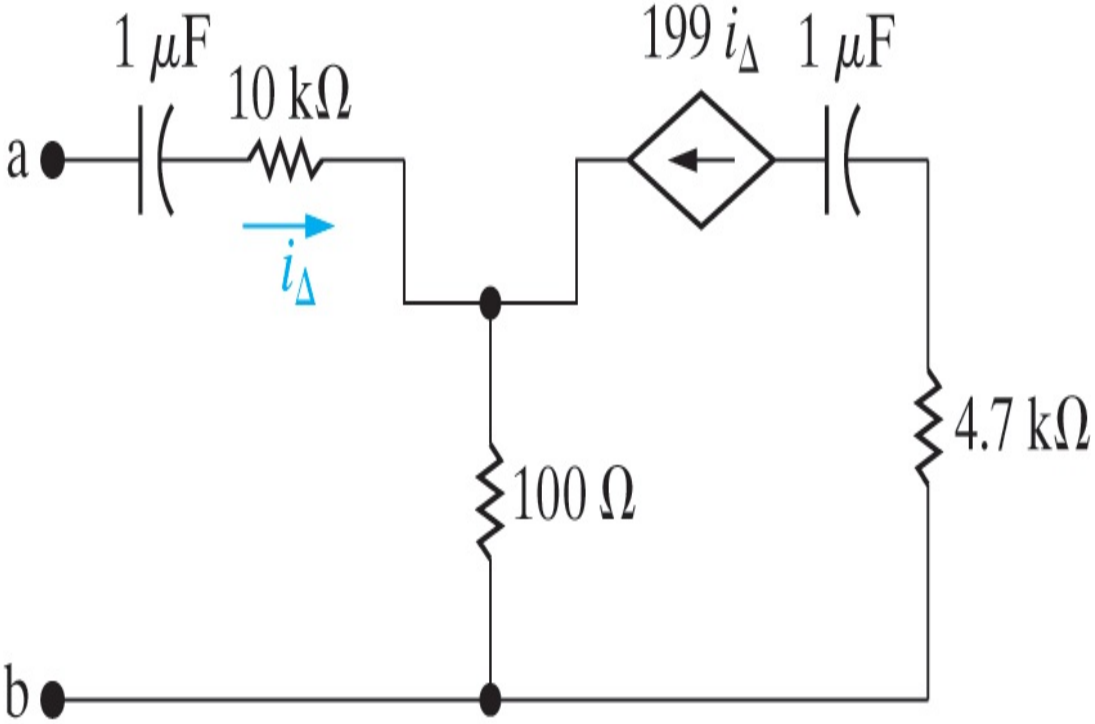


Figure P9.51 Full Alternative Text

10. 9.52 Find Zab in the circuit shown in Fig. P9.52 when the circuit is operating at a frequency of 100 krad/s.

Figure P9.52

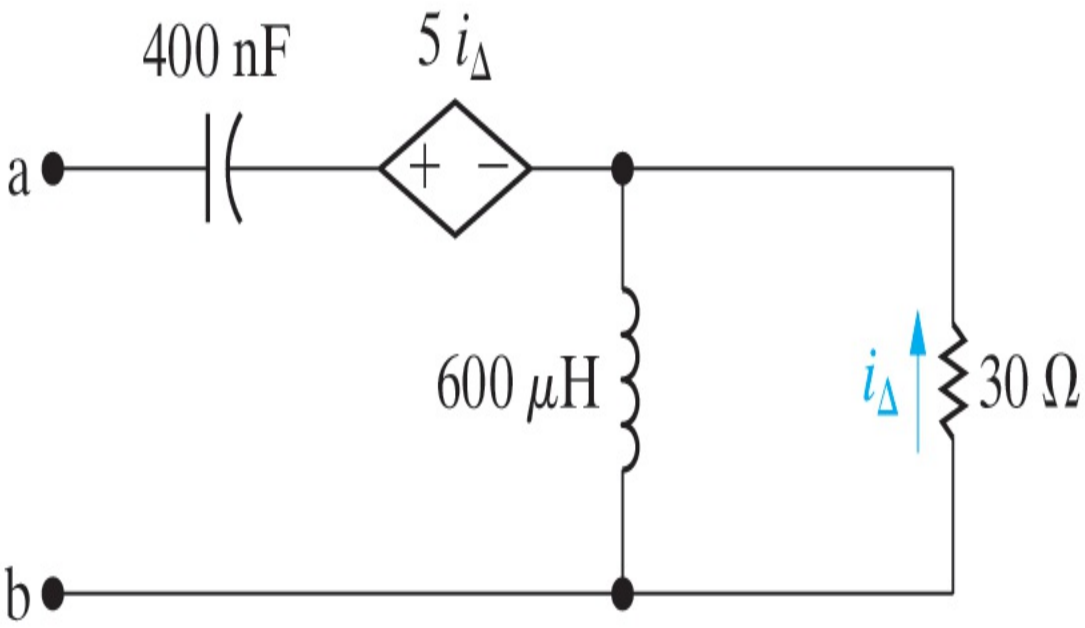


Figure P9.52 Full Alternative Text

11. 9.53 The circuit shown in Fig. P9.53 is operating at a frequency of 10 rad/s. Assume α is real and lies between −10 and +10, that is, −10≤α≤10.

    1. Find the value of α so that the Thévenin impedance looking into the terminals a,b is purely resistive.

    2. What is the value of the Thévenin impedance for the α found in (a)?

    3. Can α be adjusted so that the Thévenin impedance equals 500−j500 Ω? If so, what is the value of α?

    4. For what values of α will the Thévenin impedance be inductive?

Figure P9.53

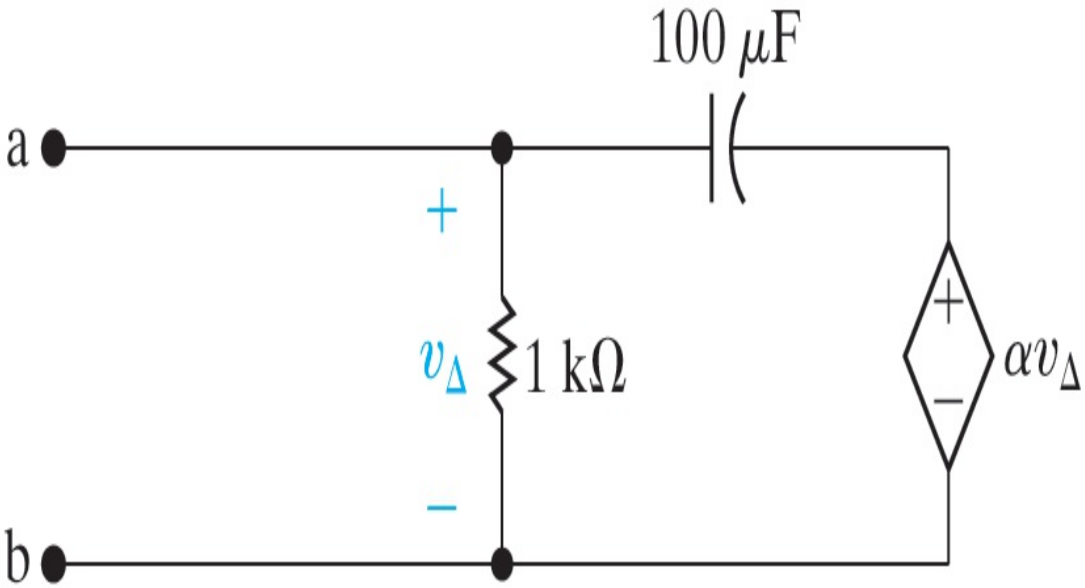


Figure P9.53 Full Alternative Text

Section 9.8

1. 9.54 PSPICEMULTISIM Use the node-voltage method to find the steady-state expression for vo(t) in the circuit in Fig. P9.54 if

   vg1=25 sin(400t+143.13°) V,vg2=18.03 cos(400t+33.69°) V.

Figure P9.54

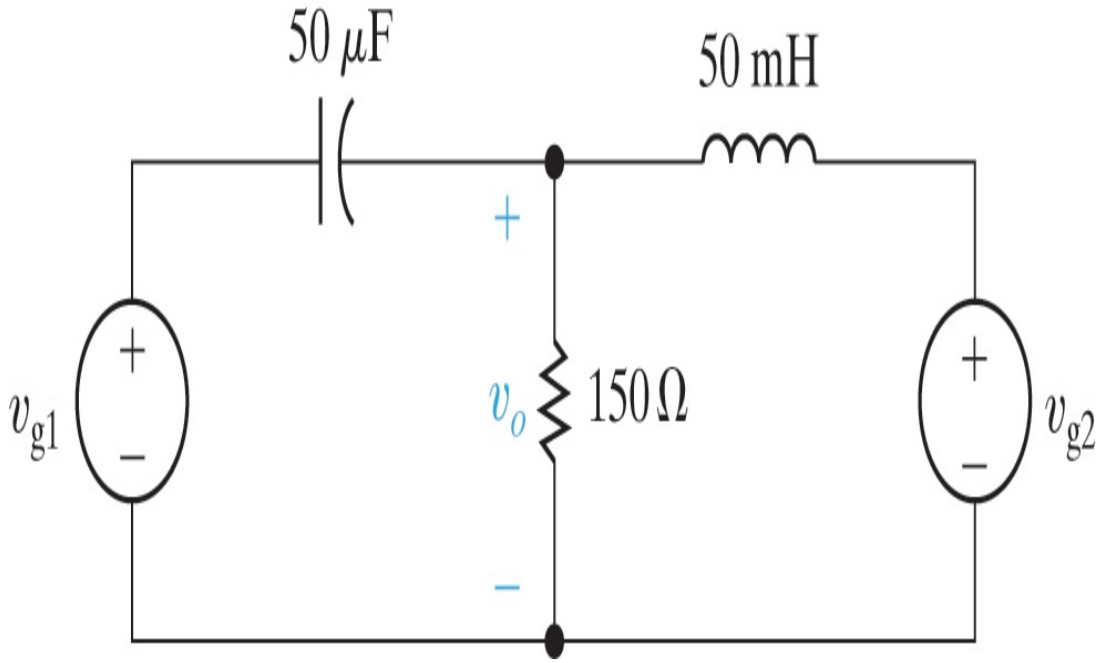


Figure P9.54 Full Alternative Text

2. 9.55 Use the node-voltage method to find Vo in the circuit in Fig. P9.55 .

Figure P9.55

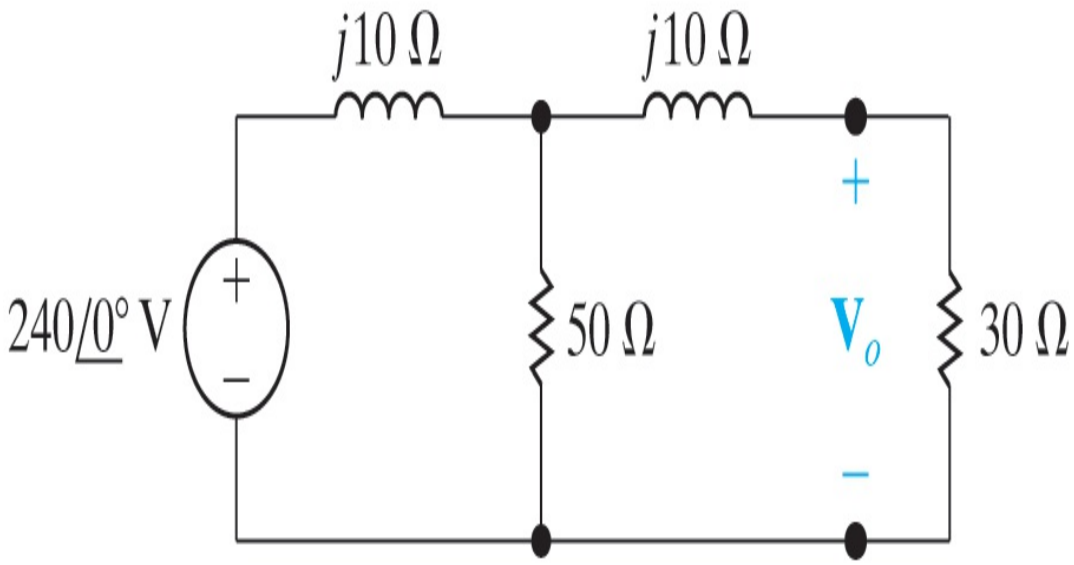


Figure P9.55 Full Alternative Text

3. 9.56 PSPICEMULTISIM Use the node voltage method to find the steady-state expression for io in the circuit seen in Fig. P9.56 if ig=5 cos 2500t A and vg=20 cos(2500t+90°) V.

Figure P9.56

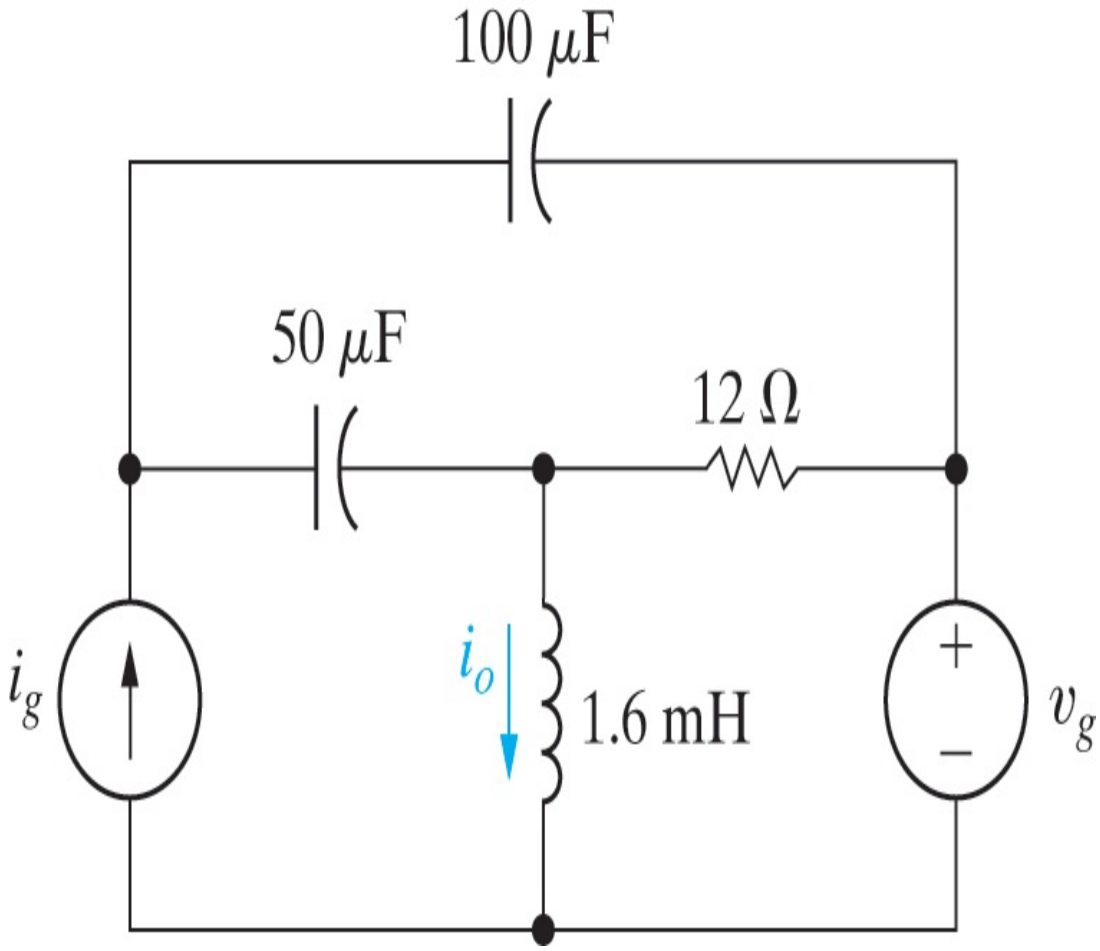


Figure P9.56 Full Alternative Text

4. 9.57 Use the node-voltage method to find Vo and Io in the circuit seen in Fig. P9.57 .

Figure P9.57

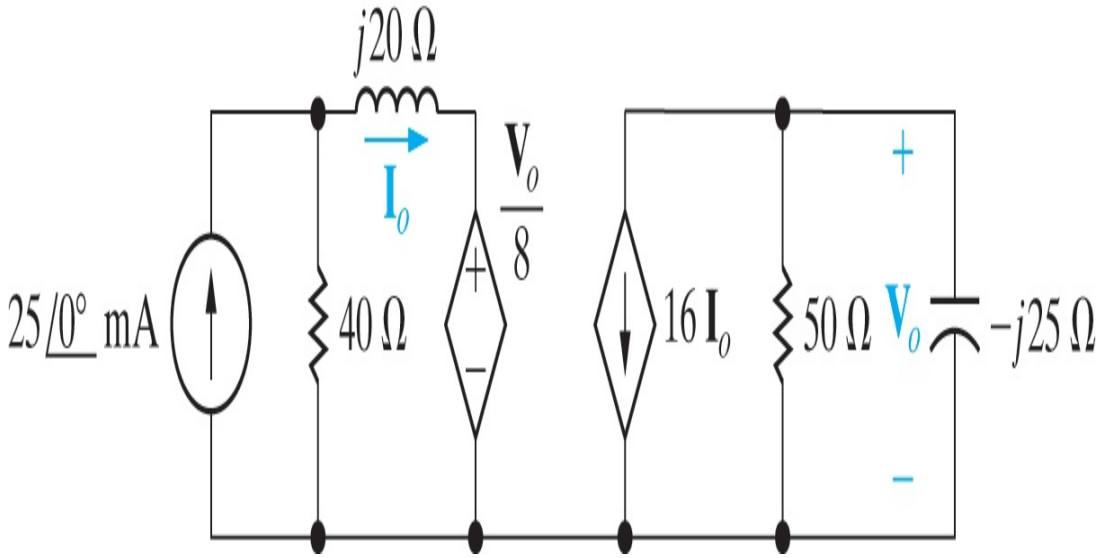


Figure P9.57 Full Alternative Text

5. 9.58 Use the node-voltage method to find the phasor voltage V0 in the circuit shown in Fig. P9.58 . Express the voltage in both polar and rectangular form.

Figure P9.58

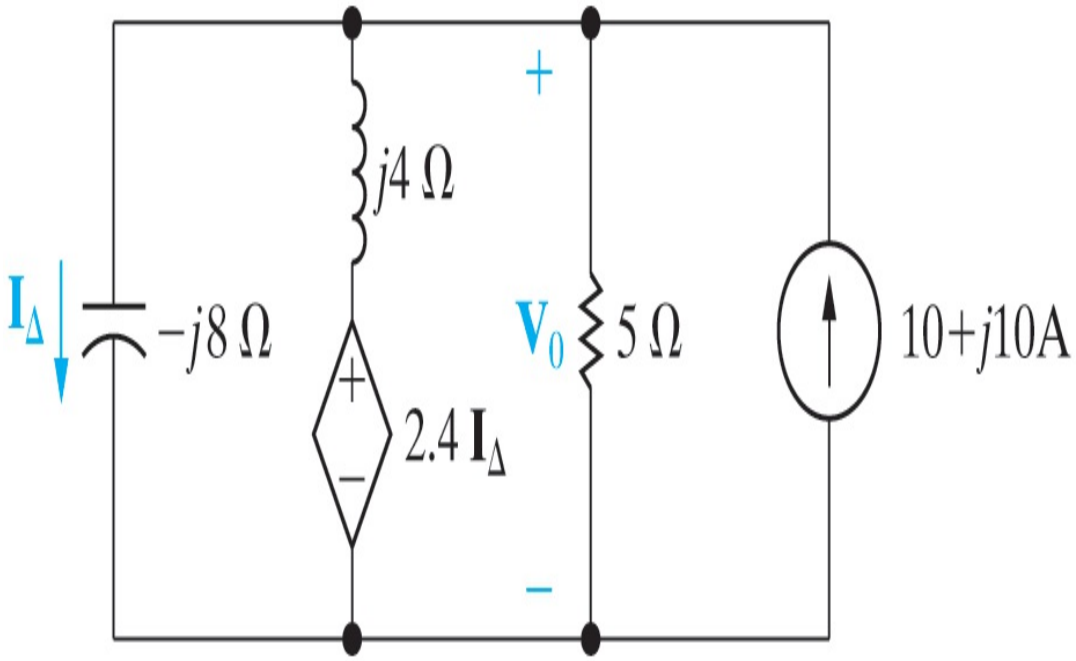


Figure P9.58 Full Alternative Text

Section 9.9

1. 9.59 PSPICEMULTISIM Use the mesh-current method to find the steady-state expression for the branch currents ia and ib in the circuit seen in Fig.P9.59 if v a = 50 sin 10 6 t V and v b = 25 cos(10 6 t + 90 °)V.

Figure P9.59

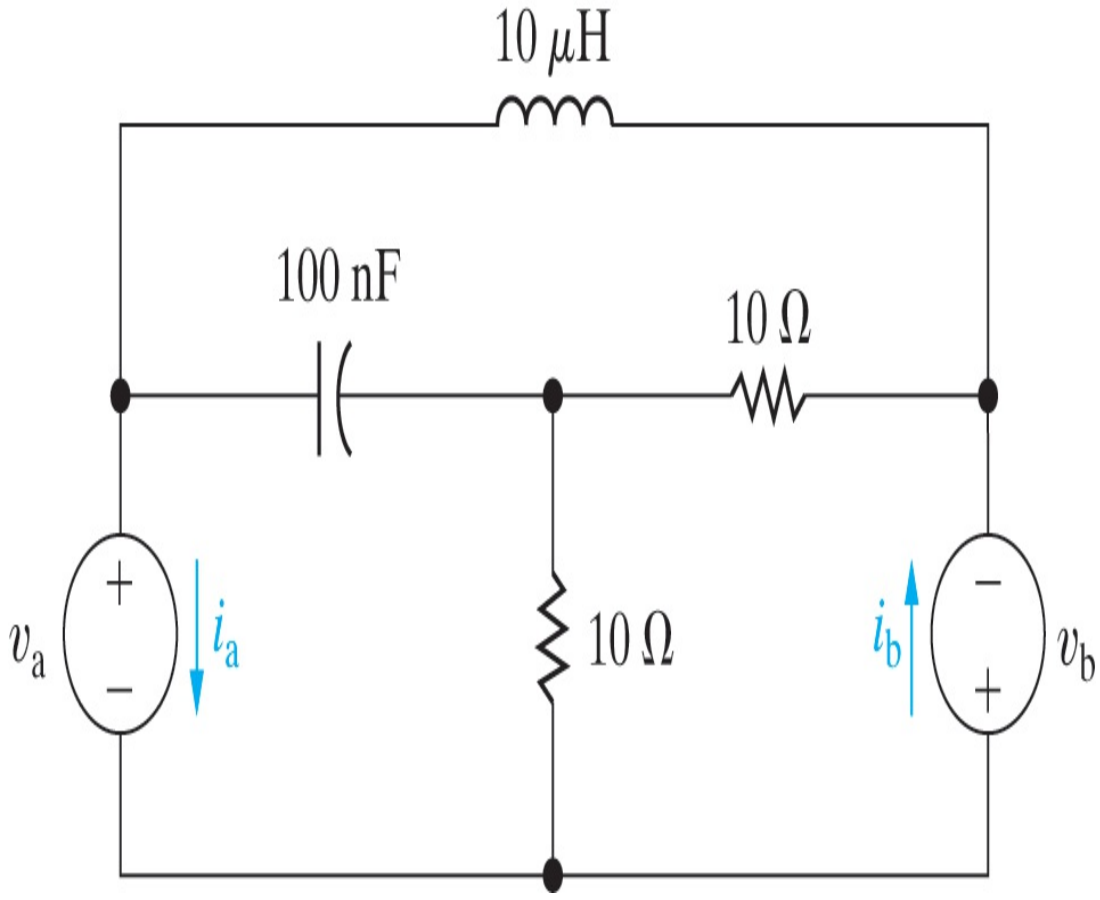


Figure P9.59 Full Alternative Text

2. 9.60 Use the mesh-current method to find the steady-state expression for io(t) in the circuit in Fig. P9.60 if

va=100 cos 50,000t V,vb=100 sin(50,000t+180°) V.

Figure P9.60

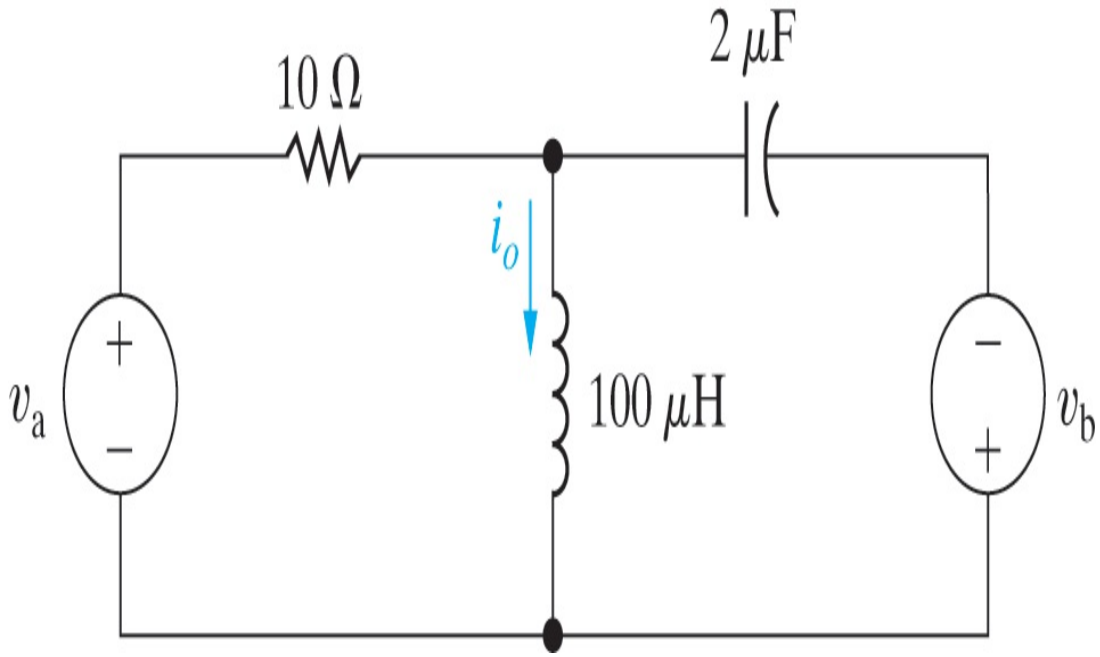


Figure P9.60 Full Alternative Text

3. 9.61 Use the mesh-current method to find the steady-state expression for vo(t) in the circuit in Fig. P9.54 .

4. 9.62 Use the mesh-current method to find the steady-state expression for io(t) in the circuit in Fig. P9.56 .

5. 9.63 Use the mesh-current method to find the branch currents Ia, Ib, Ic, and Id in the circuit shown in Fig. P9.63 .

Figure P9.63

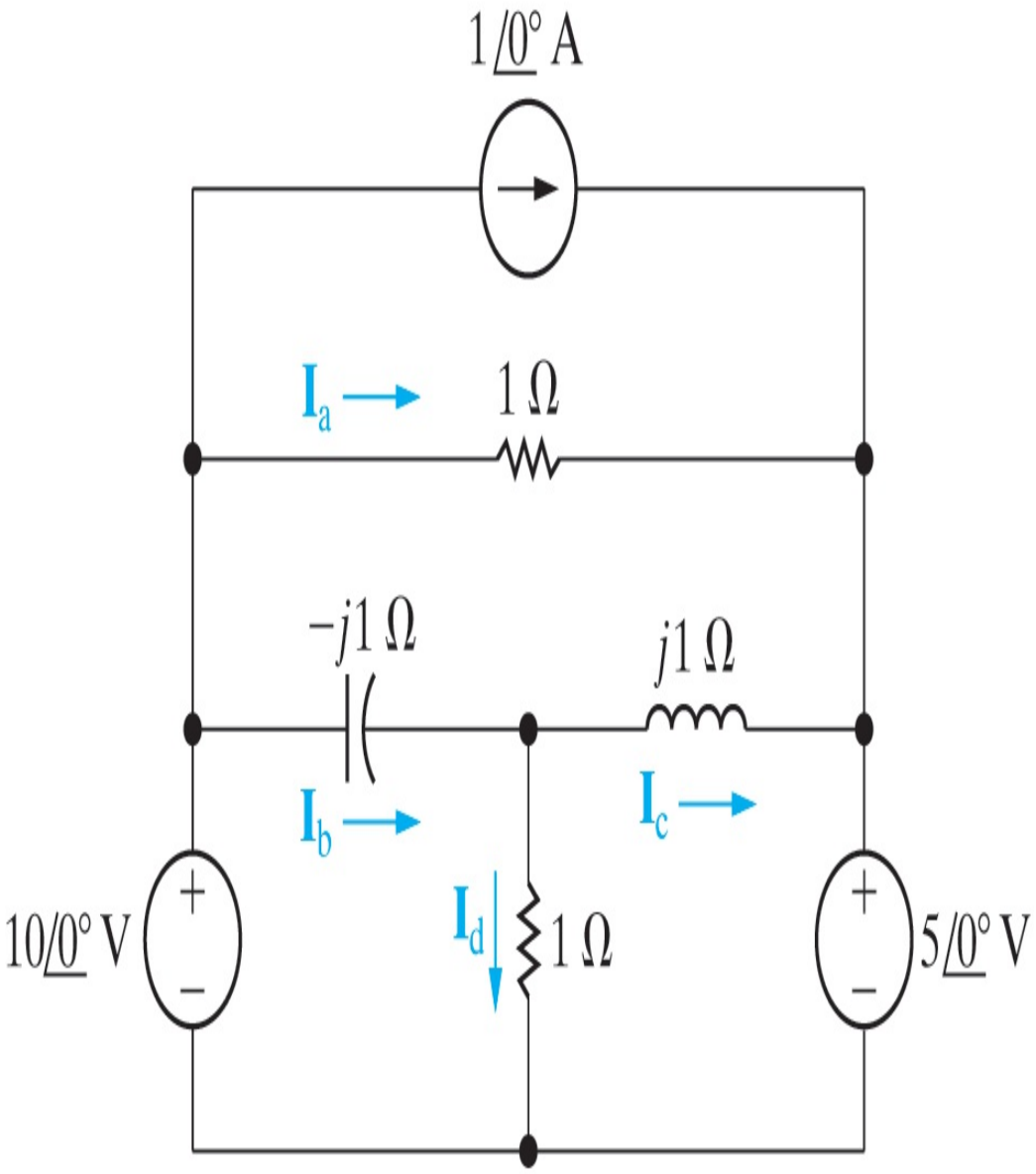


Figure P9.63 Full Alternative Text

6. 9.64 PSPICEMULTISIM Use the mesh-current method to find the steady-state expression for vo in the circuit seen in Fig. P9.64 if vg equals 400 cos 5000t V.

Figure P9.64

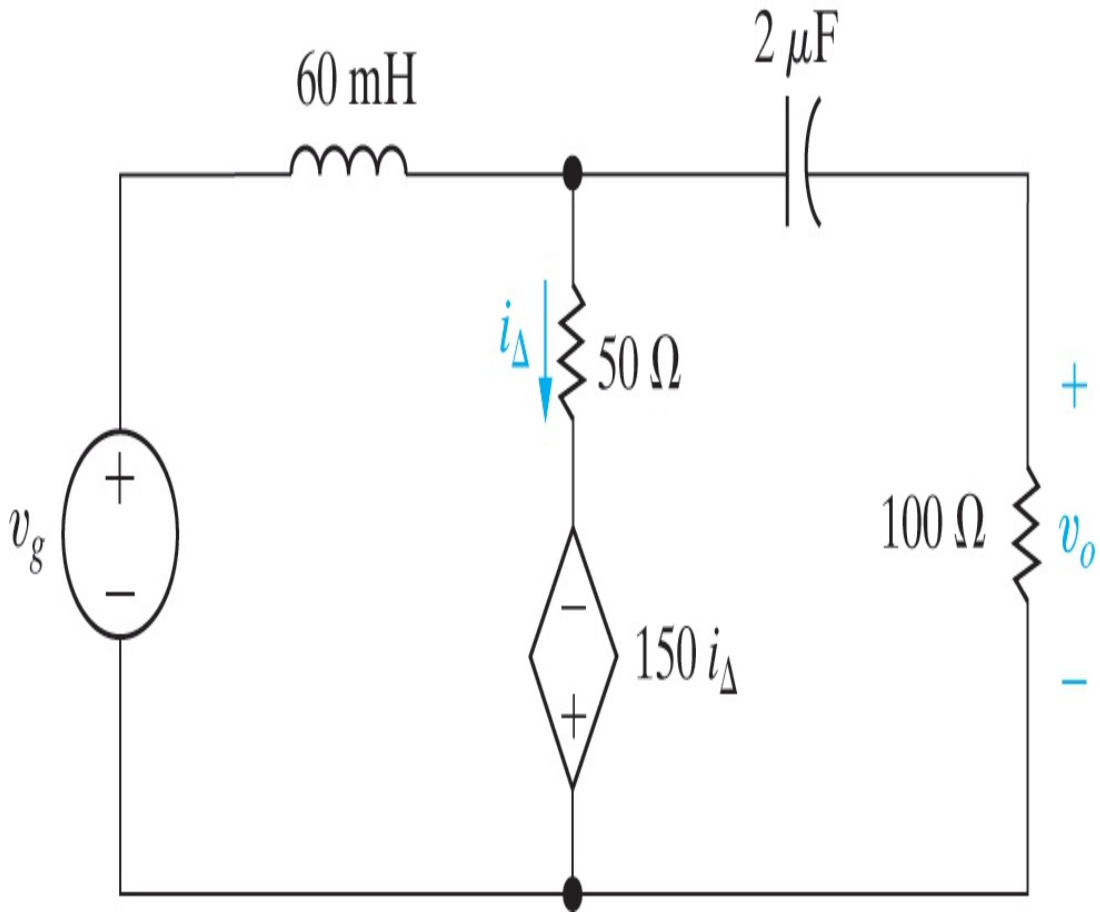


Figure P9.64 Full Alternative Text

Sections 9.5-9.9

1. 9.65 PSPICEMULTISIM Use voltage division to find the steady-state expression for vo(t) in the circuit in Fig. P9.65 if vg=100 cos 8000tV.

Figure P9.65

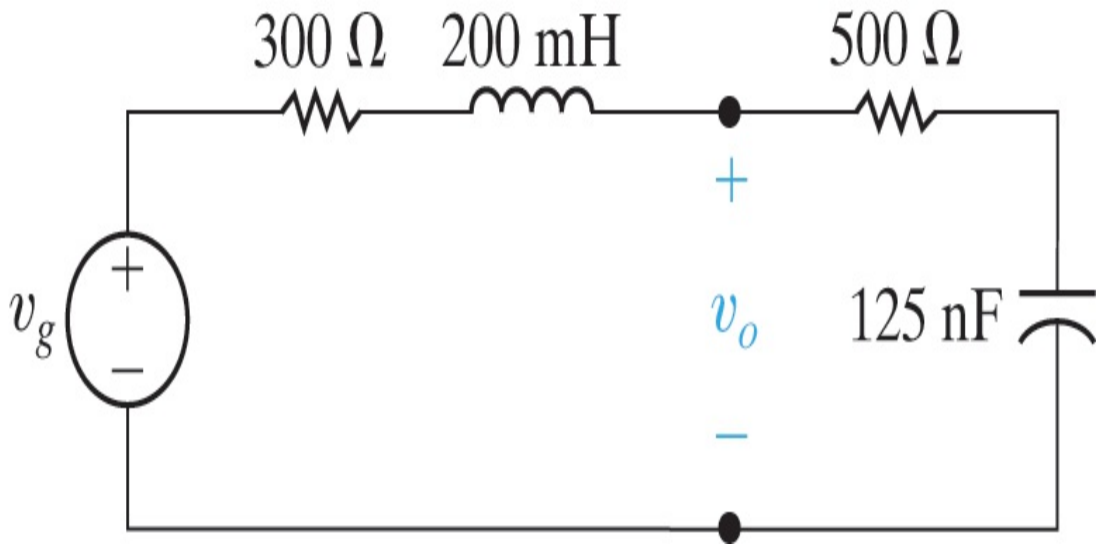


Figure P9.65 Full Alternative Text

2. 9.66 PSPICEMULTISIM Use current division to find the steady-state expression for io in the circuit in Fig. P9.66 if ig=400 cos 20,000t mA.

Figure P9.66

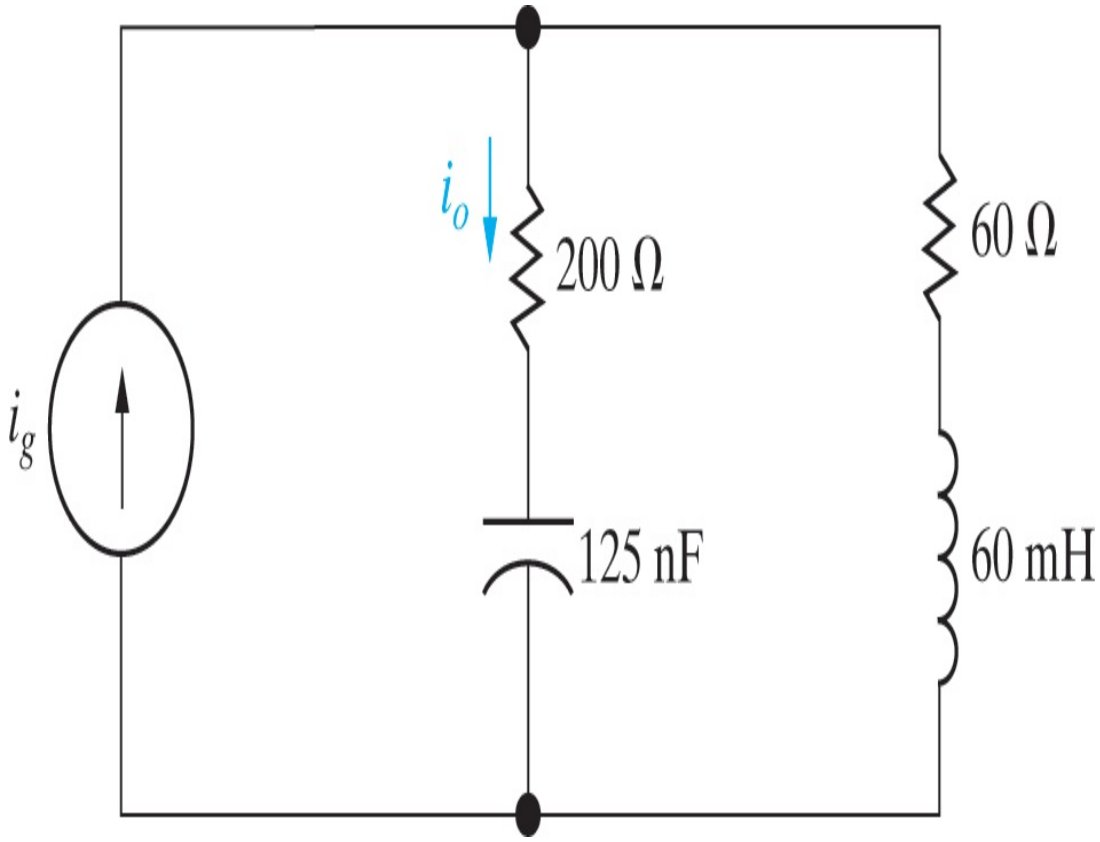


Figure P9.66 Full Alternative Text

3. 9.67 For the circuit in Fig. P9.67 , suppose

   v1=20 cos(2000t−36.87°) V
   v2=10 cos(5000t+16.26°) V

   1. What circuit analysis technique must be used to find the steady-state expression for vo(t)?

   2. Find the steady-state expression for vo(t).

Figure P9.67

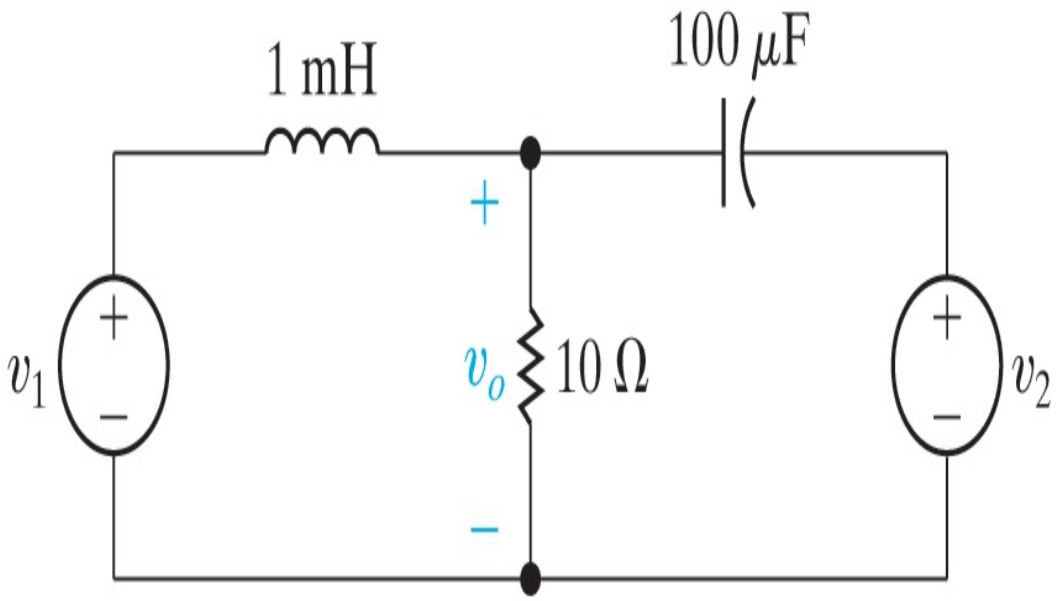


Figure P9.67 Full Alternative Text

4. 9.68 For the circuit in Fig. P9.68 , suppose

   va=10 cos 16,000t V
   vb=20 cos 4000t V

   1. What circuit analysis technique must be used to find the steady-state expression for io(t)?

   2. Find the steady-state expression for io(t).

Figure P9.68

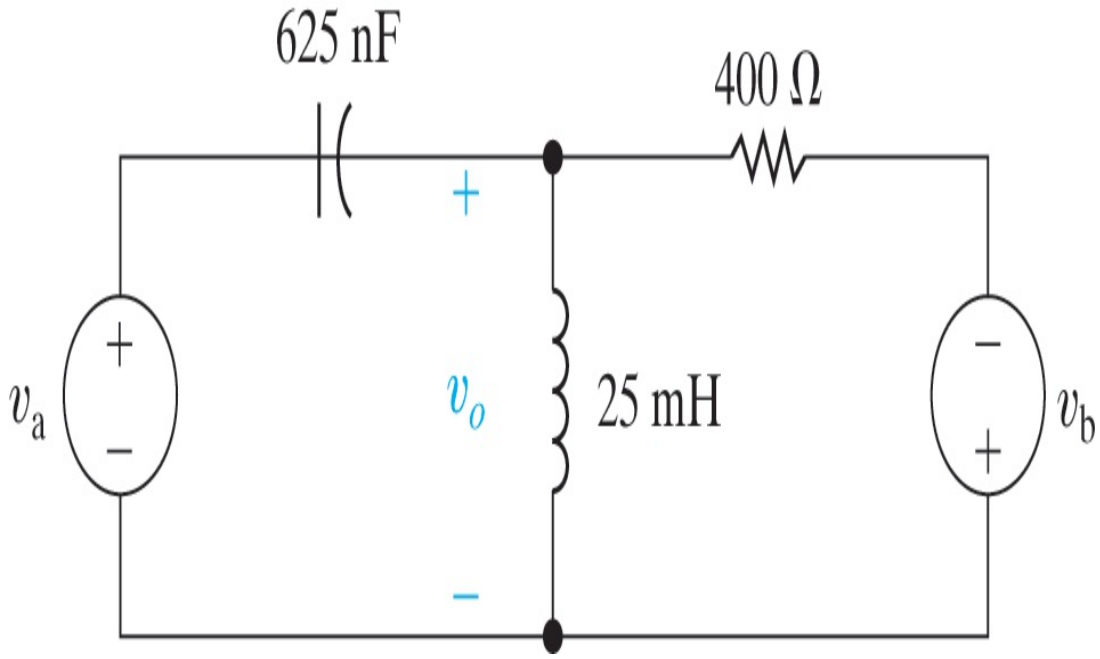


Figure P9.68 Full Alternative Text

5. 9.69 PSPICEMULTISIM The op amp in the circuit in Fig. P9.69 is ideal.

    1. Find the steady-state expression for vo(t).
    2. How large can the amplitude of vg be before the amplifier saturates?

Figure P9.69

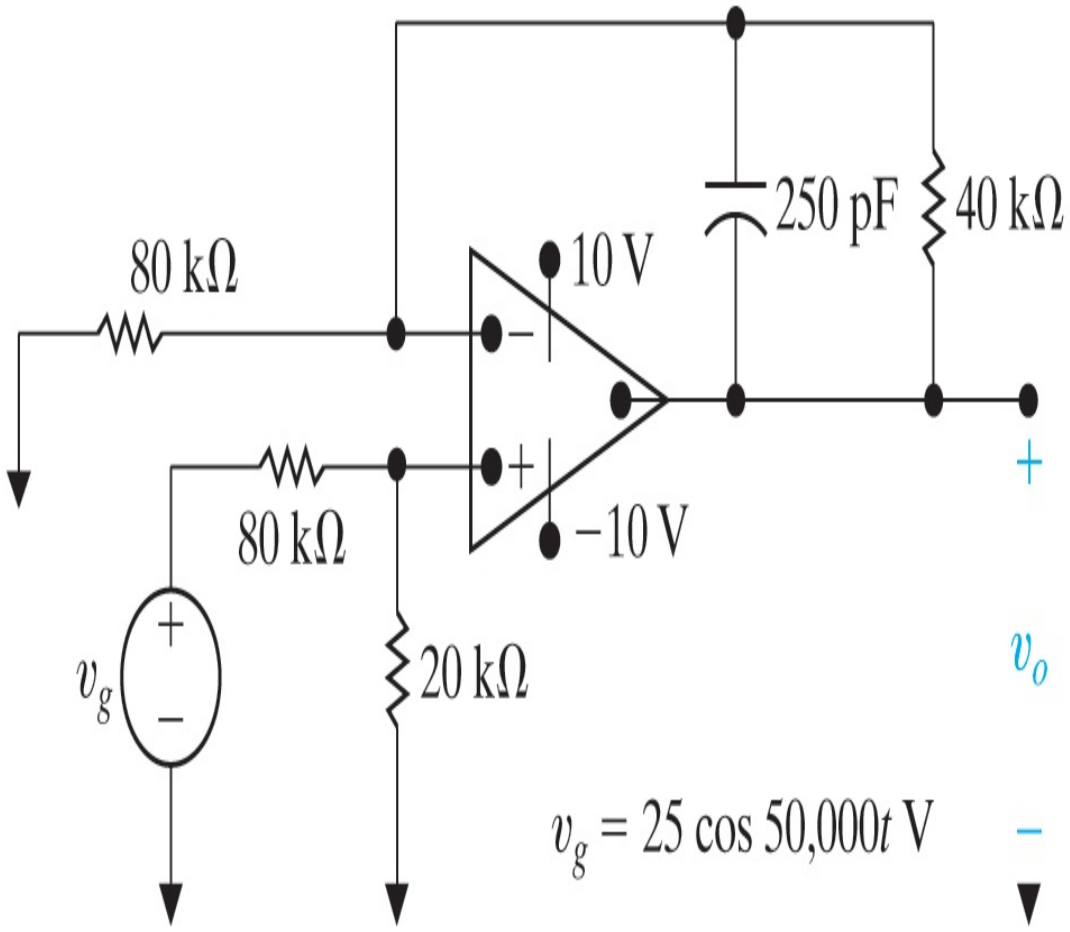


Figure P9.69 Full Alternative Text

6. 9.70 PSPICEMULTISIM The op amp in the circuit seen in Fig. P9.70 is ideal. Find the steady-state expression for vo(t) when vg=2 cos 106t V.

Figure P9.70

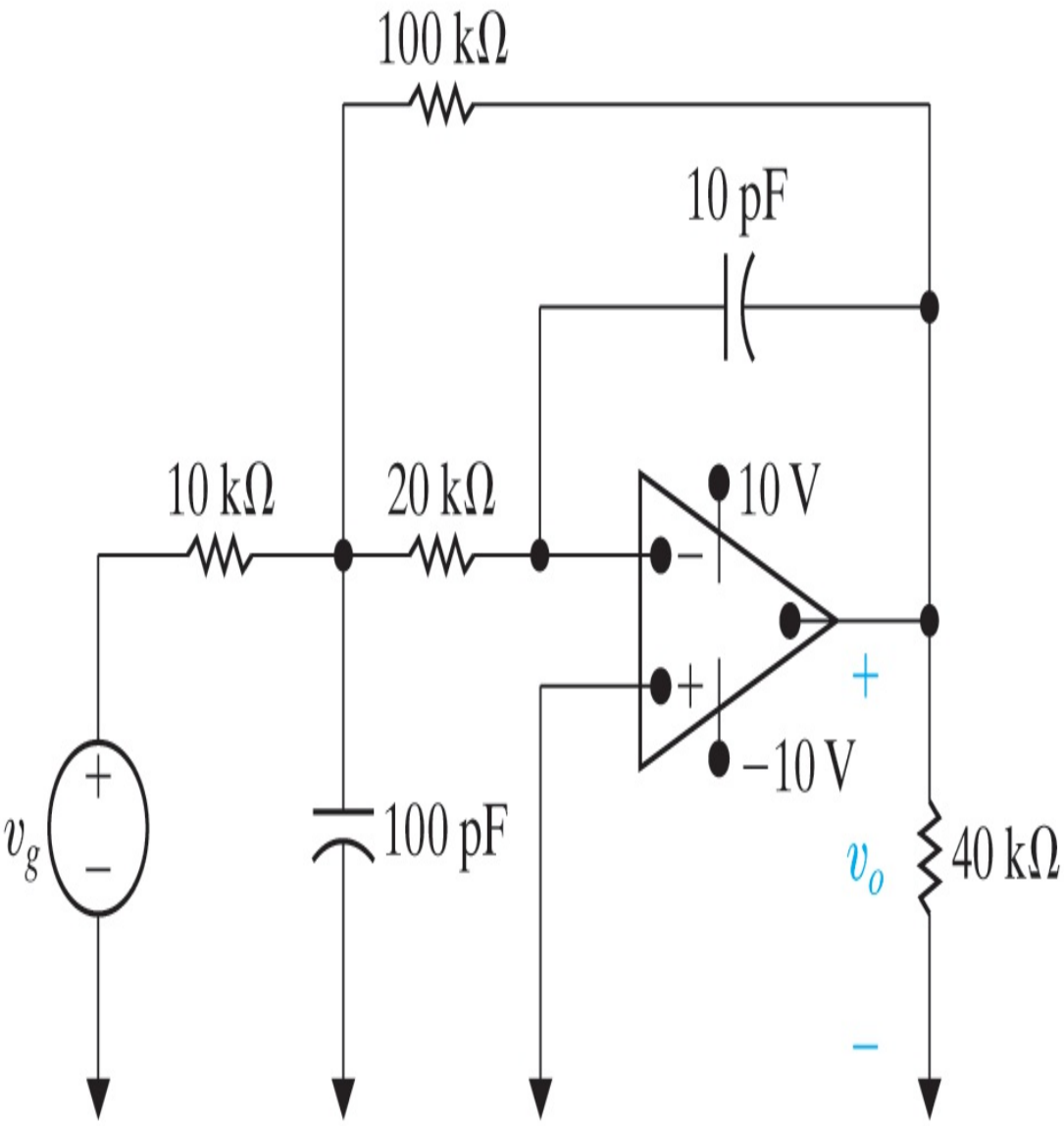


Figure P9.70 Full Alternative Text

7. 9.71 PSPICEMULTISIM The sinusoidal voltage source in the circuit shown in Fig. P9.71 is generating the voltage vg=4cos200t V. If the op amp is ideal, what is the steady-state expression for vo(t)?

Figure P9.71

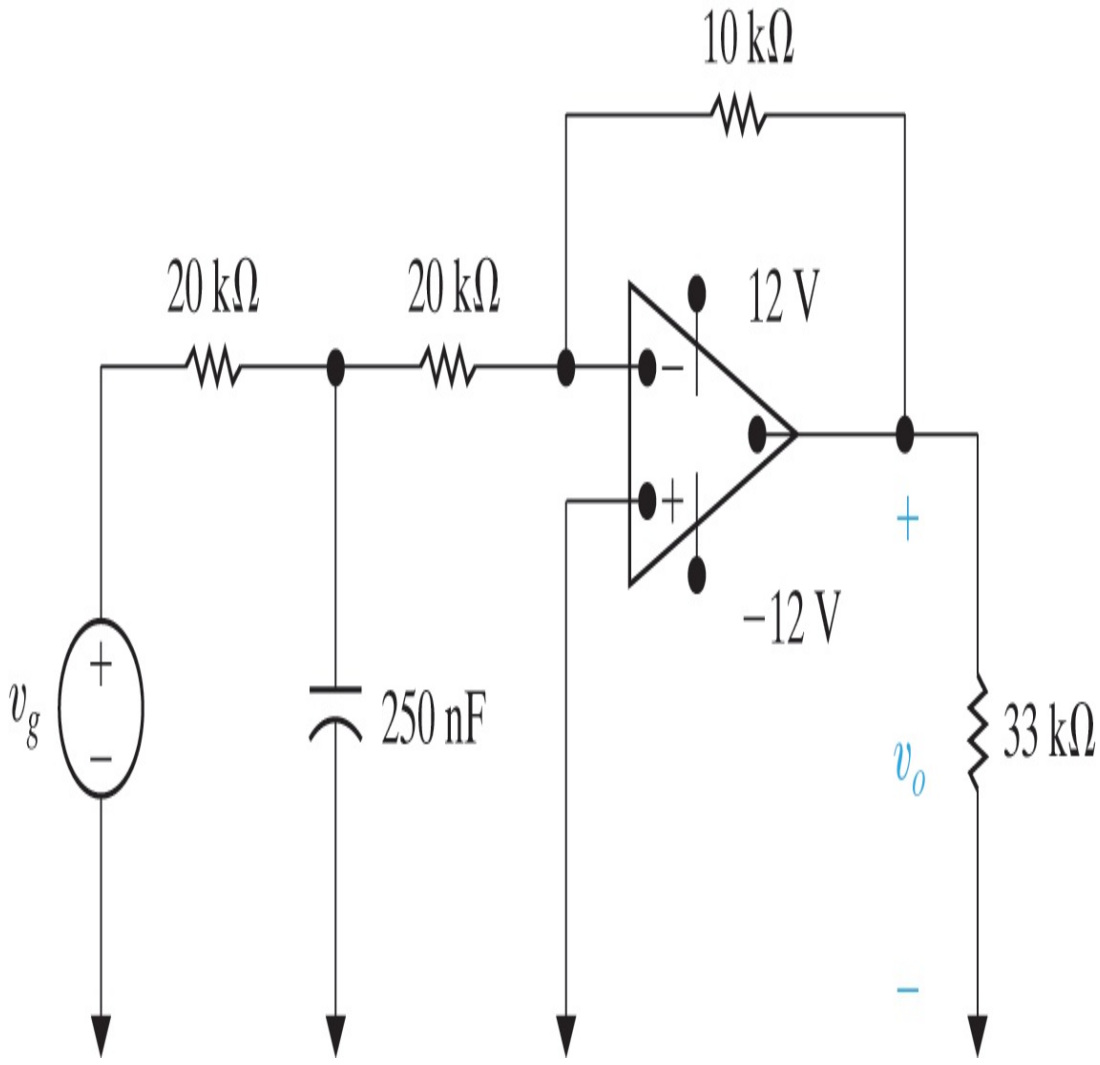


Figure P9.71 Full Alternative Text

8. 9.72 PSPICEMULTISIM The 250 nF capacitor in the circuit seen in Fig. P9.71 is replaced with a variable capacitor. The capacitor is adjusted until the output voltage leads the input voltage by 135°.

    1. Find the value of C in microfarads.

    2. Write the steady-state expression for vo(t) when C has the value found in (a).

9. 9.73 PSPICEMULTISIM The op amp in the circuit shown in Fig. P9.73 is ideal. The voltage of the ideal sinusoidal source is vg=6 cos 105t V.

1. How small can Co be before the steady-state output voltage no longer has a pure sinusoidal waveform?

2. For the value of Co found in (a), write the steady-state expression for vo.

Figure P9.73

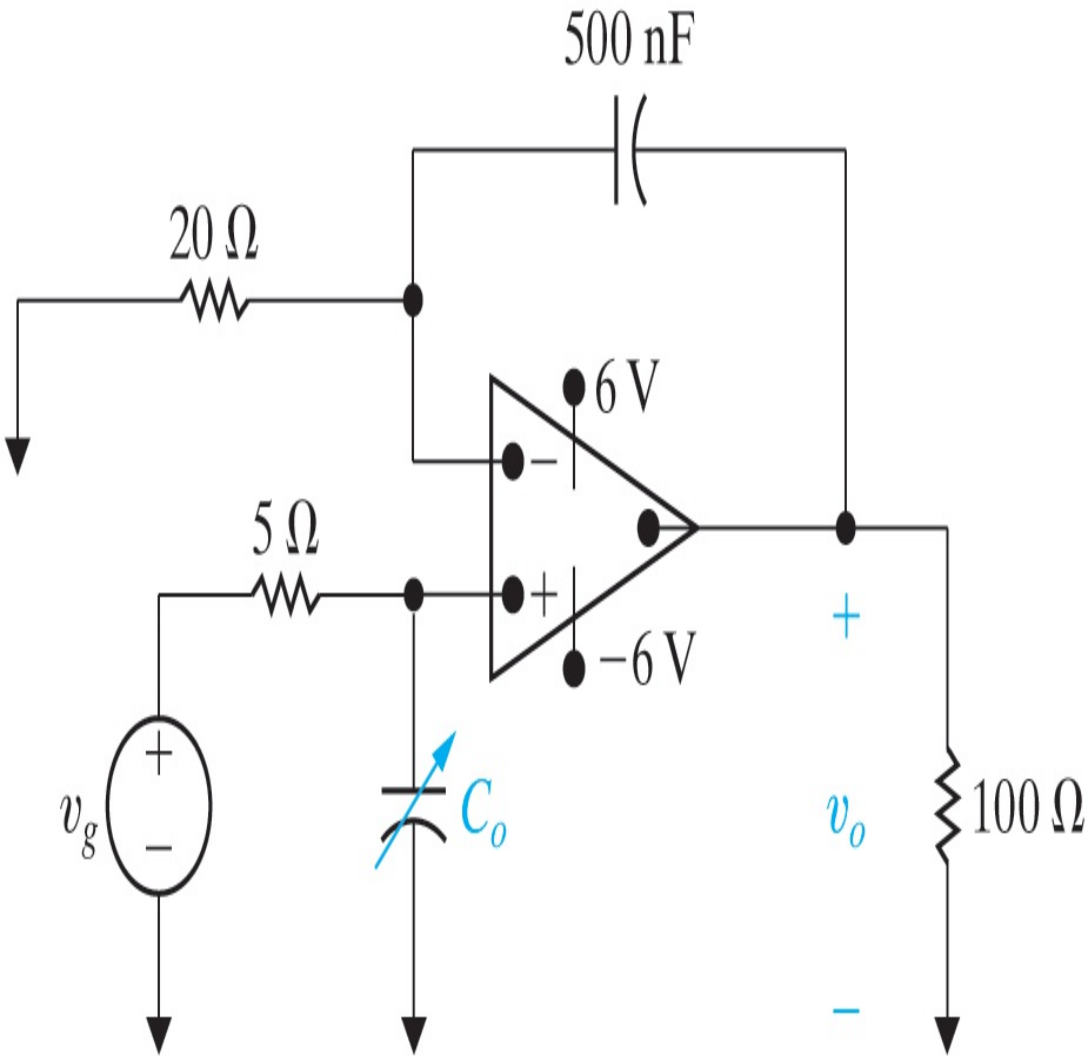


Figure P9.73 Full Alternative Text

Section 9.10

1. 9.74 PSPICEMULTISIM

    1. Find the steady-state expressions for the currents ig and iL in the circuit in Fig. P9.74 when vg=70 cos 5000t V.

    2. Find the coefficient of coupling.

    3. Find the energy stored in the magnetically coupled coils at t=100π μs and t=200π μs.

Figure P9.74

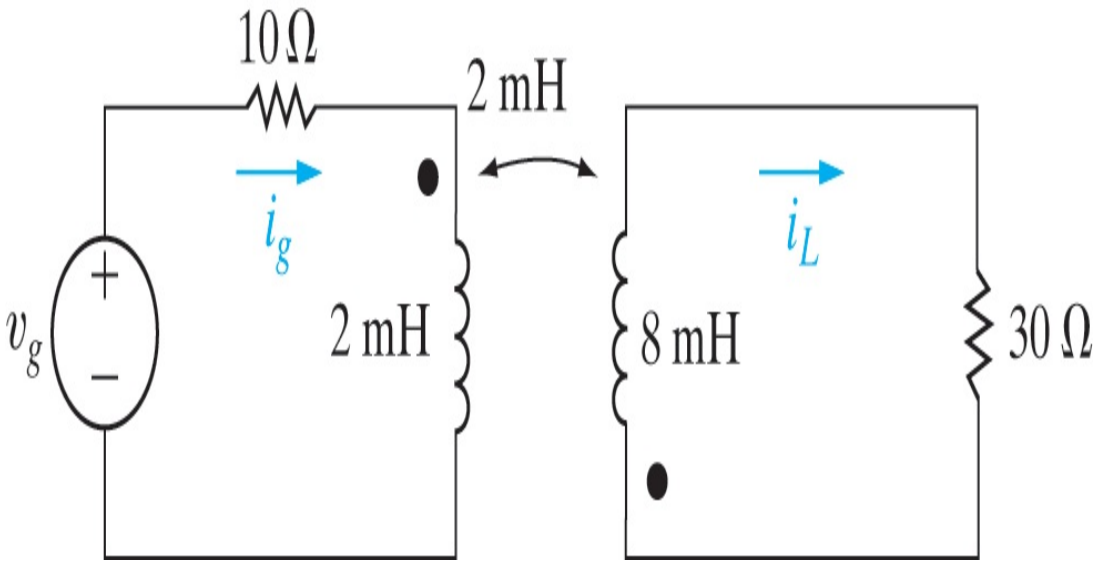


Figure P9.74 Full Alternative Text

2. 9.75 For the circuit in Fig. P9.75 , find the Thévenin equivalent with respect to the terminals c,d.

Figure P9.75

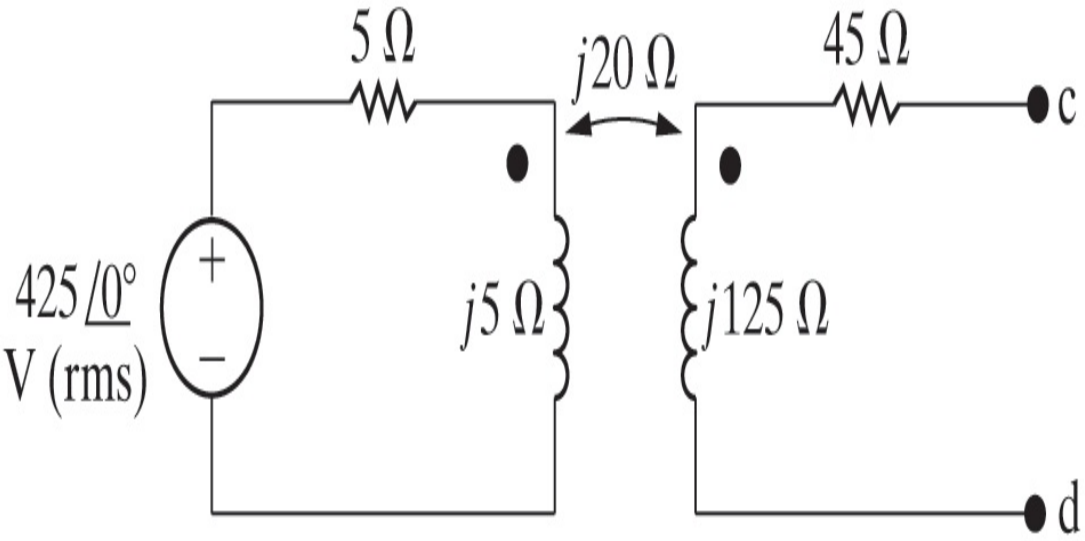


Figure P9.75 Full Alternative Text

3. 9.76 The value of k in the circuit in Fig. P9.76 is adjusted so that Zab is purely resistive when ω=4 krad/s. Find Zab.

Figure P9.76

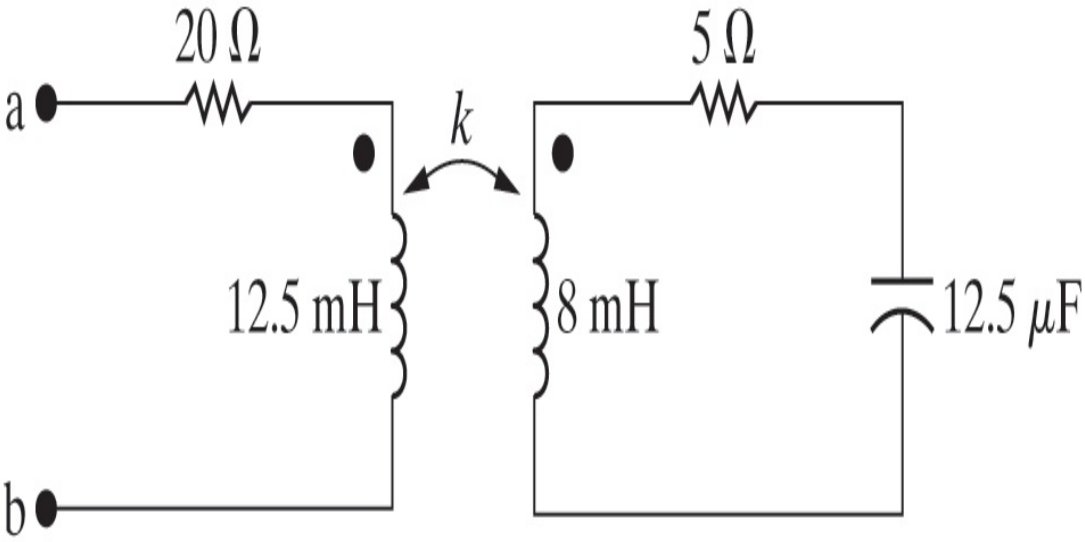


Figure P9.76 Full Alternative Text

4. 9.77 PSPICEMULTISIM The sinusoidal voltage source in the circuit seen in Fig. P9.77 is operating at a frequency of 200 krad/s. The coefficient of coupling is adjusted until the peak amplitude of i1 is maximum.

    1. What is the value of k?

    2. What is the peak amplitude of i1 if vg=560 cos(2×105t) V?

Figure P9.77

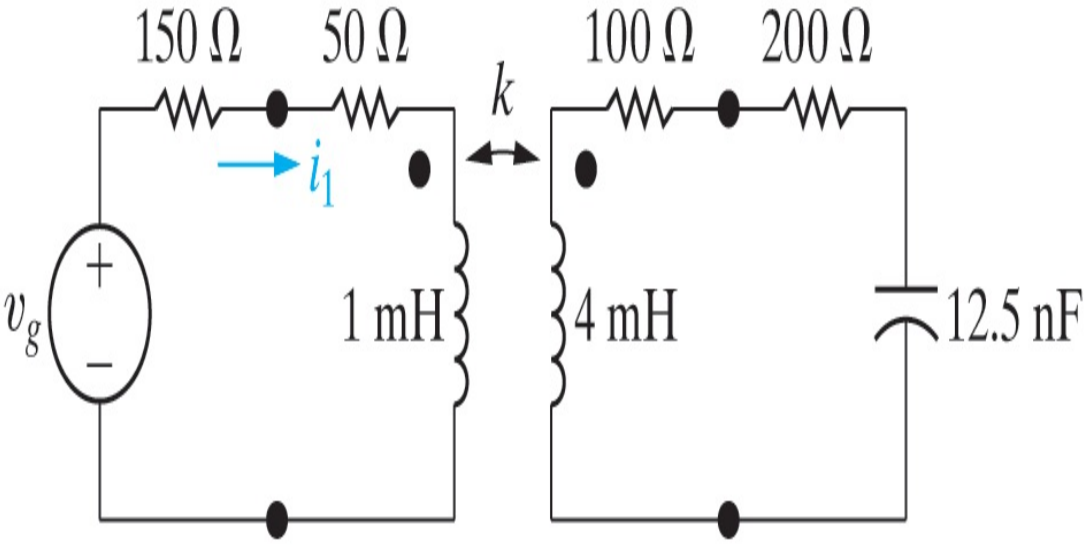


Figure P9.77 Full Alternative Text

5. 9.78 A series combination of a 300Ω resistor and a 100 mH inductor is connected to a sinusoidal voltage source by a linear transformer. The source is operating at a frequency of 1000 rad/s. At this frequency, the internal impedance of the source is (100+j13.74)Ω. The rms voltage at the terminals of the source is 50 V when it is not loaded. The parameters of the linear transformer are R1=41.68 Ω, L1=180 mH, R2=500 Ω, L2=500 mH, and M=270 mH.

1. What is the value of the impedance reflected into the primary?

2. What is the value of the impedance seen from the terminals of the practical source?

Section 9.11

1. 9.79 Find the impedance Zab in the circuit in Fig. P9.79 if Z L =200 −45° Ω .

Figure P9.79

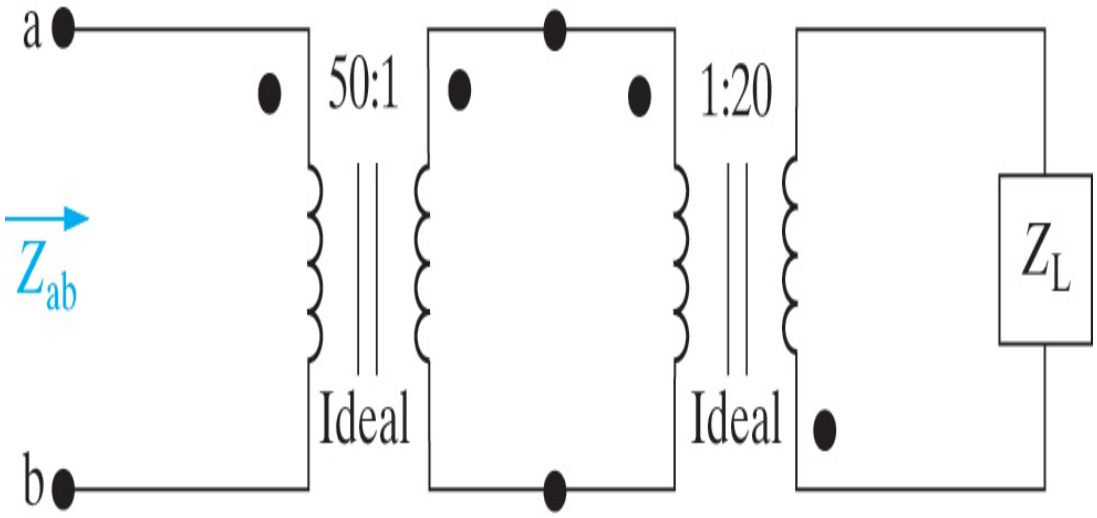


Figure P9.79 Full Alternative Text

2. 9.80 At first glance, it may appear from Eq.9. 34 that an inductive load could make the reactance seen looking into the primary terminals (i.e., Xab) look capacitive. Intuitively, we know this is impossible. Show that Xab can never be negative if XL is an inductive reactance.

3. 9.81

    1. Show that the impedance seen looking into the terminals a,b in the circuit in Fig. P9.81 is given by the expression

        Zab=(1+N1N2)2ZL.

2. Show that if the polarity terminals of either one of the coils is reversed,

   Zab=(1−N1N2)2ZL.

Figure P9.81

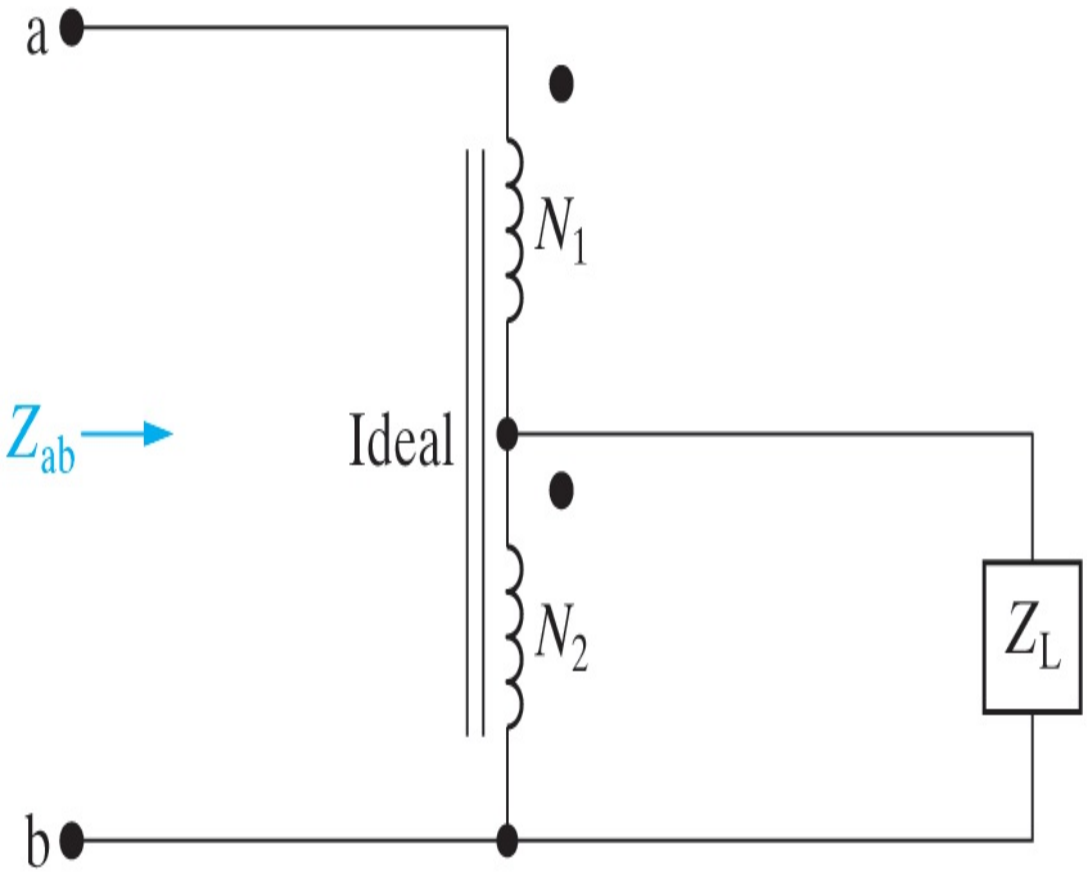


Figure P9.81 Full Alternative Text

4. 9.82

   1. Show that the impedance seen looking into the terminals a,b in the circuit in Fig. P9.82 is given by the expression

      Zab=ZL(1+N1N2)2.

2. Show that if the polarity terminal of either one of the coils is reversed that

   Zab=ZL(1−N1N2)2.

Figure P9.82

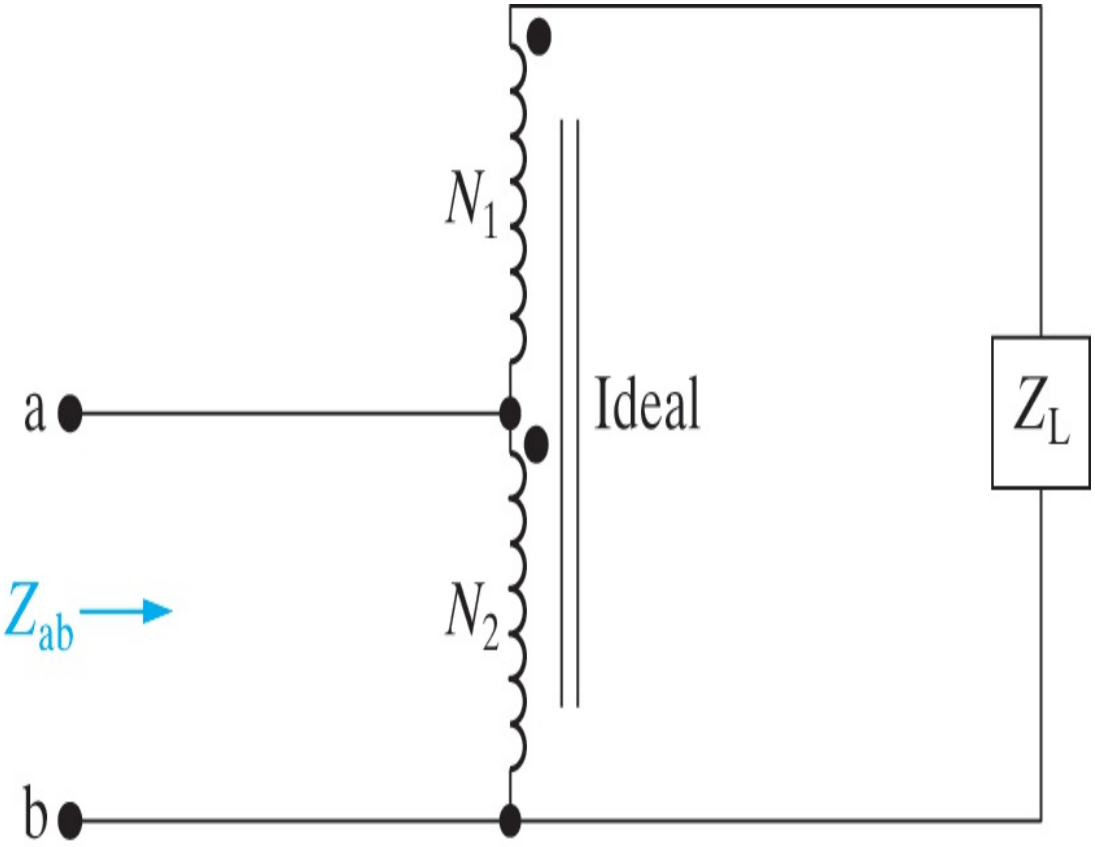


Figure P9.82 Full Alternative Text

Section 9.12

1. 9.83 The parameters in the circuit shown in Fig. 9.56 are R1=0.1 Ω, ωL1=0.8 Ω, R2=24 Ω, ωL2=32 Ω, and VL=240+j0 V.

    1. Calculate the phasor voltage Vs.

    2. Connect a capacitor in parallel with the inductor, hold VL constant, and adjust the capacitor until the magnitude of **I** is a minimum. What is the capacitive reactance? What is the value of Vs?

    3. Find the value of the capacitive reactance that keeps the magnitude of **I** as small as possible and that at the same time makes

        |Vs|=|VL|=240 V

2. 9.84

    1. For the circuit shown in Fig. P9.84 , compute Vs and Vl.

    2. Construct a phasor diagram showing the relationship between Vs, Vl, and the load voltage of 120 0° V .

    3. Repeat parts (a) and (b), given that the load resistance changes from 7.5 Ω to 2.5 Ω and the load reactance changes from 12 Ω to 4 Ω. Assume that the load voltage remains constant at 120∠0° V. How much must the amplitude of Vs be increased in order to maintain the load voltage at 120 V?

    4. Repeat part (c), given that at the same time the load resistance and reactance change, a capacitive reactance of −2 Ω is connected across the load terminals.

Figure P9.84

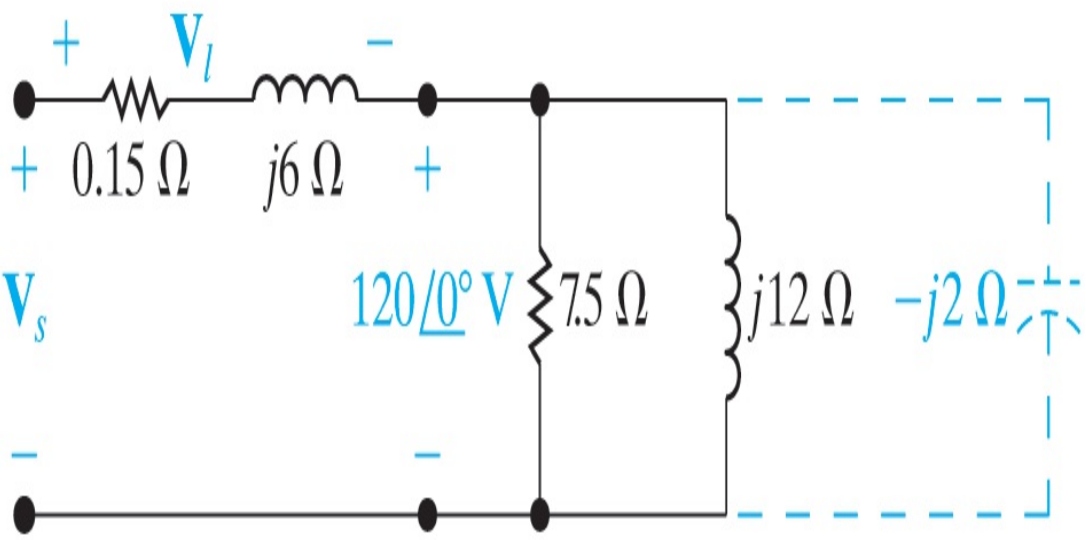


Figure P9.84 Full Alternative Text

3. 9.85 PSPICEMULTISIM Show by using a phasor diagram what happens to the magnitude and phase angle of the voltage vo in the circuit in Fig. P9.85 as Rx is varied from zero to infinity. The amplitude and phase angle of the source voltage are held constant as Rx varies.

Figure P9.85

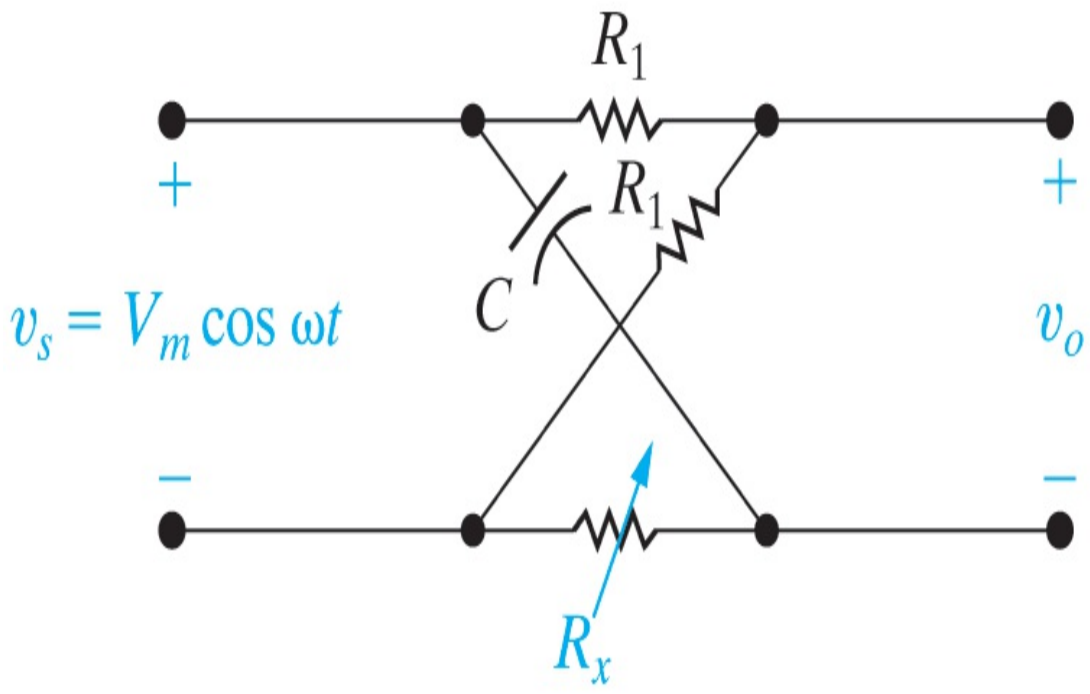


Figure P9.85 Full Alternative Text

Section 9.1-9.12

1. 9.86 PRACTICALPERSPECTIVE_
    1. Calculate the branch currents I1−I6 in the circuit in Fig. P9.86 .
    2. Find the primary current Ip.

Figure P9.86

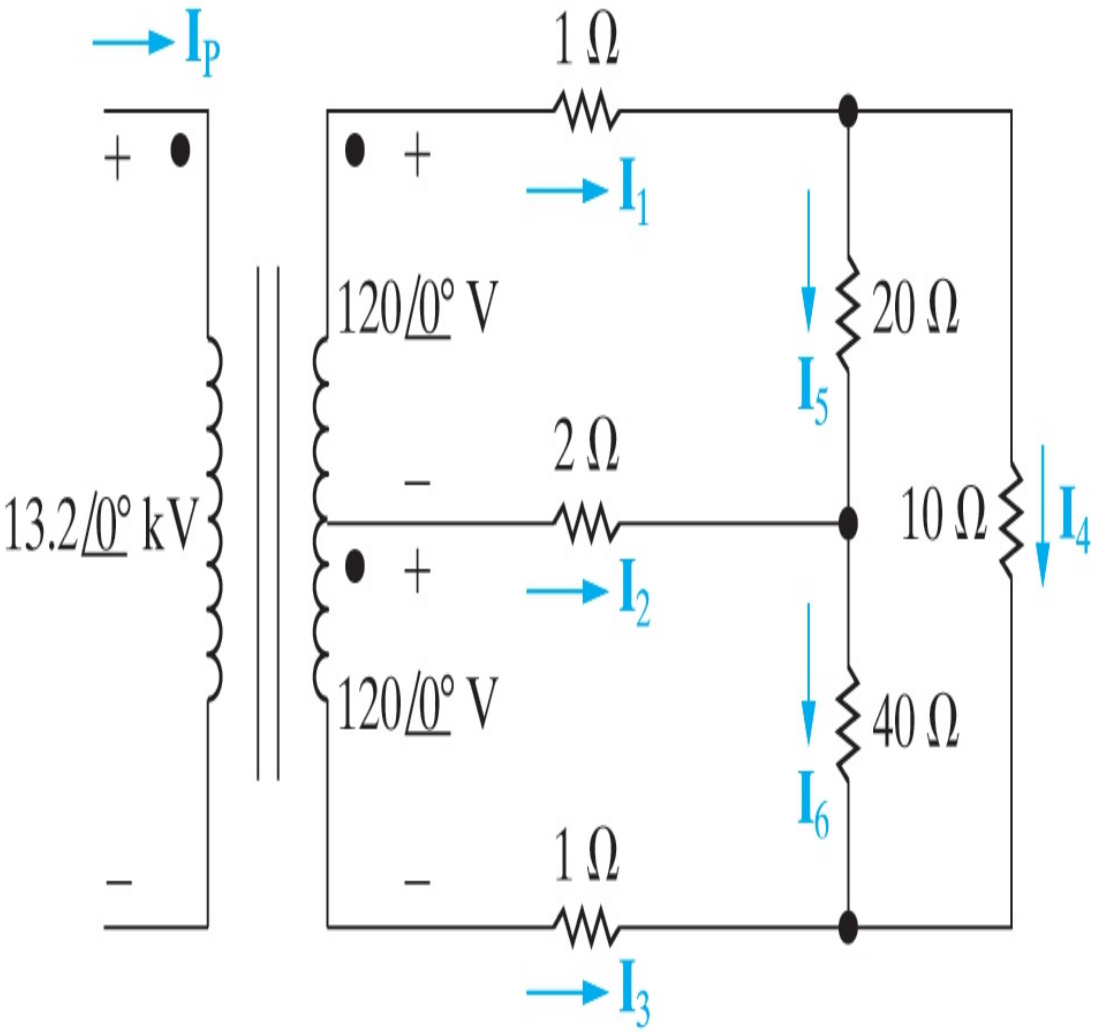


Figure P9.86 Full Alternative Text

2. 9.87 PRACTICALPERSPECTIVE_ Suppose the 40 Ω resistance in the distribution circuit in Fig. P9.86 is replaced by a 20 Ω resistance.

    1. Recalculate the branch current in the 2 Ω resistor, I2.

    2. Recalculate the primary current, Ip.

    3. On the basis of your answers, is it desirable to have the resistance of the two 120 V loads be equal?

3. 9.88 PRACTICALPERSPECTIVE_ A residential wiring circuit is shown in Fig. P9.88 . In this model, the resistor R3 is used to model a 250 V appliance (such as an electric range), and the resistors R1 and R2 are used to model 125 V appliances (such as a lamp, toaster, and iron). The branches carrying I1 and I2 are modeling what electricians refer to as the hot conductors in the circuit, and the branch carrying In is modeling the neutral conductor. Our purpose in analyzing the circuit is to show the importance of the neutral conductor in the satisfactory operation of the circuit. You are to choose the method for analyzing the circuit.

    1. Show that In is zero if R1=R2.

    2. Show that V1=V2 if R1=R2.

    3. Open the neutral branch and calculate V1 and V2 if R1=40 Ω, R2=400 Ω, and R3=8 Ω.

    4. Close the neutral branch and repeat (c).

    5. On the basis of your calculations, explain why the neutral conductor is never fused in such a manner that it could open while the hot conductors are energized.

Figure P9.88

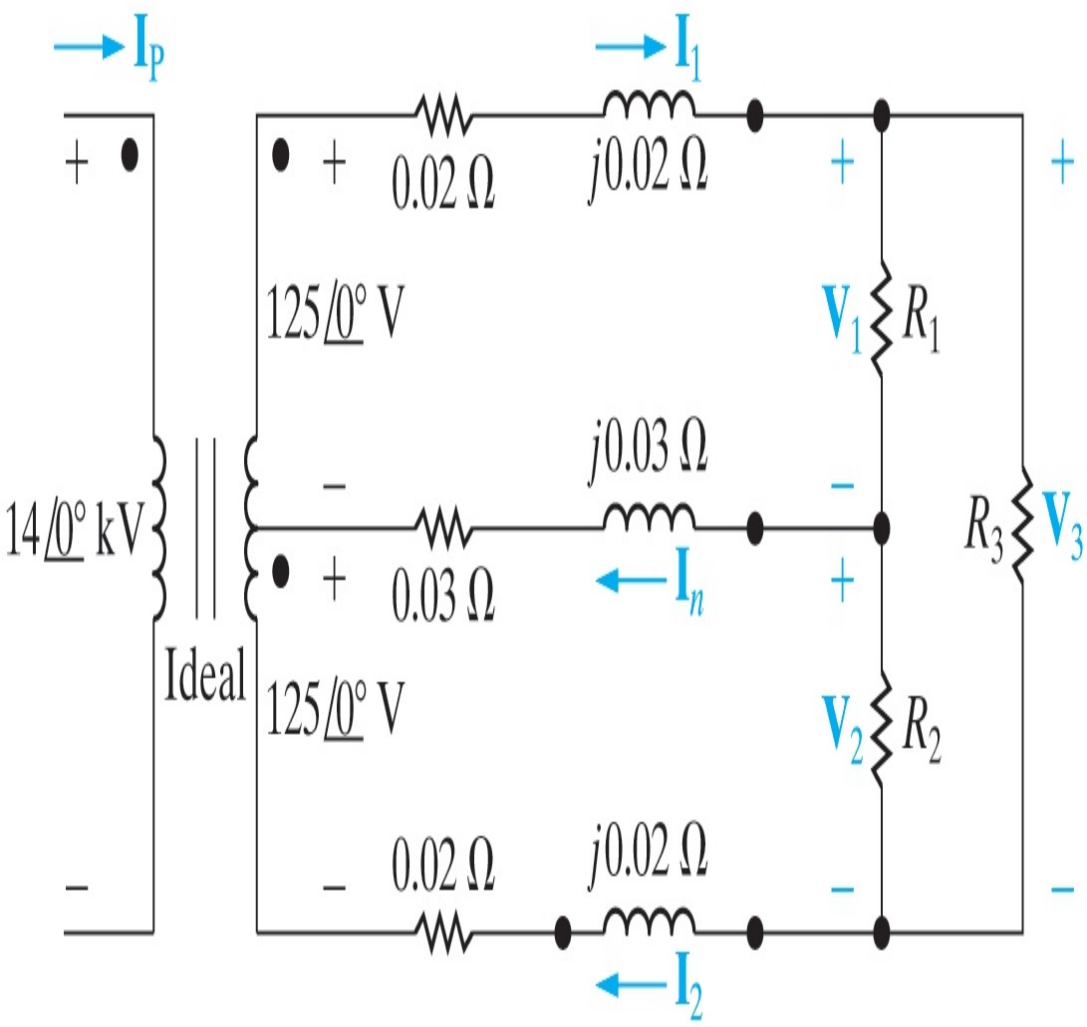


Figure P9.88 Full Alternative Text

4. 9.89 PRACTICALPERSPECTIVE_

    1. Find the primary current IP for (c) and (d) in Problem 9.88.

    2. Do your answers make sense in terms of known circuit behavior?

5. 9.90 PRACTICALPERSPECTIVE_ Assume the fan motor in Fig. 9.61 is equipped with a thermal cutout designed to interrupt the motor circuit if the motor current becomes excessive. Would you expect the thermal cutout to operate? Explain.

6. 9.91 PRACTICALPERSPECTIVE_ Explain why fuse B in Fig. 9.61 is

not interrupted when the fan motor stalls.

Chapter 10 Sinusoidal Steady-State Power Calculations

Chapter Contents

1. 10.1 Instantaneous Power
2. 10.2 Average and Reactive Power
3. 10.3 The rms Value and Power Calculations
4. 10.4 Complex Power
5. 10.5 Power Calculations
6. 10.6 Maximum Power Transfer

Chapter Objectives

1. Understand the following ac power concepts, their relationships to one another, and how to calculate them in a circuit:
    - Instantaneous power;
    - Average (real) power;
    - Reactive power;
    - Complex power; and
    - Power factor.
2. Understand the condition for maximum real power delivered to a load in an ac circuit and be able to calculate the load impedance required to deliver maximum real power to the load.
3. Be able to calculate all forms of ac power in ac circuits with linear transformers and in ac circuits with ideal transformers.

Nearly all electric energy is supplied by sinusoidal voltages and currents. Thus, after our Chapter 9 discussion of sinusoidal circuits, we now consider sinusoidal steady-state power calculations. We are primarily interested in the average power delivered to or supplied by a pair of terminals in the sinusoidal steady state. We also present other power quantities, including reactive power, complex power, and apparent power.

We begin and end this chapter with two concepts that should be very familiar to you from previous chapters: the basic equation for power (Section 10.1) and maximum power transfer (Section 10.6). In between, we discuss the general techniques for calculating power, which will be familiar from your studies in Chapters 1 and 4, although some additional mathematical techniques are required here to deal with sinusoidal, rather than dc, signals. We also revisit the rms value of a sinusoid, briefly introduced in Chapter 9,

because it is used extensively in power calculations.

A wide variety of problems deal with the delivery of energy to do work, ranging from determining the power rating for safely and efficiently operating an appliance to designing the vast array of generators, transformers, and wires that provide electric energy to household and industrial consumers. Thus, power engineering is an important and exciting subdiscipline in electrical engineering.

Practical Perspective

Vampire Power

Even when we are not using many of the common electrical devices found in our homes, schools, and businesses, they can still be consuming power. This “standby power” can run an internal clock, charge batteries, display time or other quantities, monitor temperature or other environmental measures, or search for signals to receive. Devices such as microwave ovens, DVRs, televisions, remote controls, and computers all consume power when not in use.

The ac adapters used to charge many portable devices are a common source of standby power. Even when the device is unplugged from the adapter, the adapter can continue to consume power if it is plugged into the wall outlet. Because the plug on the adapter looks somewhat like vampire fangs, standby power has become known as “vampire power.” It is power that is used even while we sleep.

How much vampire power do electrical devices in our home use over the course of a year? Is there a way to reduce or eliminate vampire power? These questions are explored in the Practical Perspective example at the end of the chaptertext and in the chapter problems.

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10.1 Instantaneous Power

Consider the familiar circuit in Fig. 10.1. Here, v and i are steady-state sinusoidal signals, given by

Figure 10.1 The black box representation of a circuit used for calculating power.

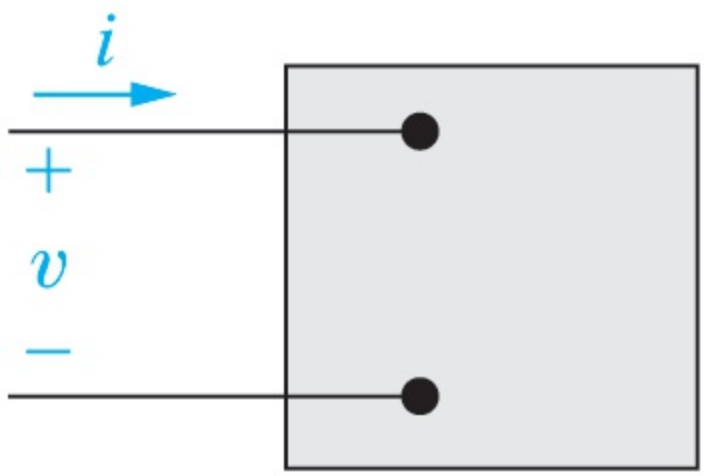


Figure 10.1 Full Alternative Text

$v=V_m\cos(\omega t+\theta_v), i=I_m\cos(\omega t+\theta_i),$

where θ_v is the voltage phase angle and θ_i is the current phase angle. Using the passive sign convention, we find that the power at any instant of time is

$p=vi.$

This is **instantaneous power**. Instantaneous power is measured in watts when the voltage is in volts and the current is in amperes.

Because the circuit operates in the sinusoidal steady state, we can choose any convenient reference for zero time. It is convenient to define zero time at the instant the current passes through a positive maximum. This reference system requires a shift of both the voltage and current by θ_i. Thus, the equations for voltage and current become

$$v = V_m \cos(\omega t + \theta_v - \theta_i), \quad (10.1)$$
$$i = I_m \cos \omega t. \quad (10.2)$$

When we substitute Eqs. 10.1 and 10.2 into the power equation, the expression for the instantaneous power becomes

$$p = V_m I_m \cos(\omega t + \theta_v - \theta_i) \cos \omega t.$$

We could use this equation directly to find the average power. Instead, we use a couple of trigonometric identities to construct a much more informative expression. We begin with the trigonometric identity[1]

[1] See Appendix F.

$$\cos \alpha \cos \beta = \frac{1}{2}\cos(\alpha - \beta) + \frac{1}{2}\cos(\alpha + \beta)$$

and let $\alpha = \omega t + \theta_v - \theta_i$ and $\beta = \omega t$ to give

$$p = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(2\omega t + \theta_v - \theta_i).$$

Now use the trigonometric identity

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

to expand the second term on the right-hand side of the expression for p, which gives

Instantaneous Power

$$p = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)\cos 2\omega t - \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)\sin 2\omega t. \quad (10.3)$$

Examine the three terms on the right-hand side of Eq. 10.3. The first term is a constant; it is not a function of time. The other two terms are sinusoids, each with a frequency that is double the frequency of the voltage and current in Eqs. 10.1 and 10.2. You can make these same observations in the plot of Fig. 10.2, which depicts v, i, and p, assuming $\theta v=60°$ and $\theta i=0°$. You can see that the frequency of the instantaneous power is twice the frequency of the voltage or current. Therefore, the instantaneous power goes through two complete cycles for every cycle of either the voltage or the current.

Figure 10.2 Instantaneous power, voltage, and current versus ωt for steady-state sinusoidal operation.

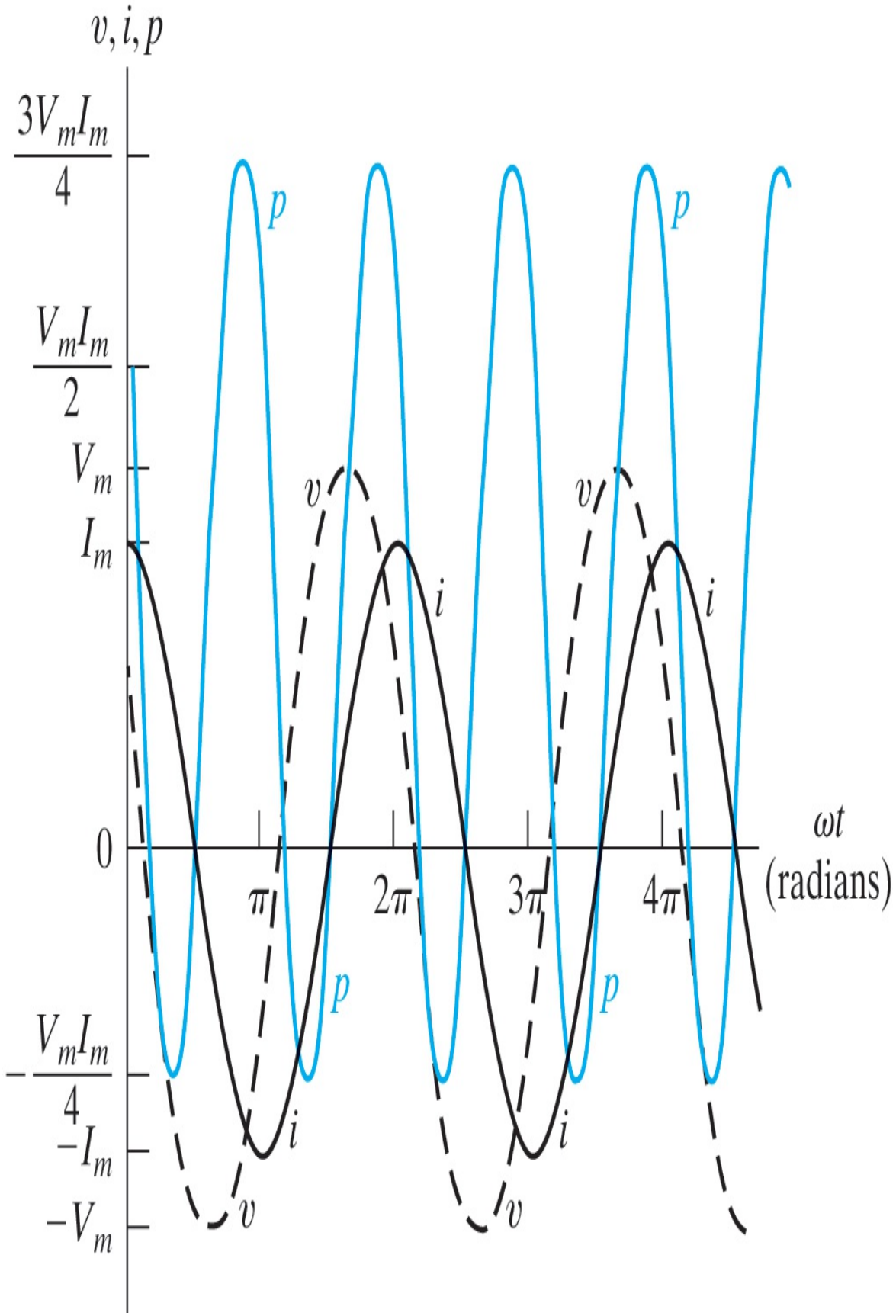

v, i, p
3V_mI_m / 4
V_mI_m / 2
V_m
I_m
0
−V_mI_m / 4
−I_m
−V_m
p
v
i
ωt
(radians)
π
2π
3π
4π

Figure 10.2 Full Alternative Text

Also note that the instantaneous power may be negative for a portion of each cycle. When the power is negative, the energy stored in the inductors or capacitors is being extracted. The instantaneous power varies with time when a circuit operates in the sinusoidal steady state. As a result, some motor-driven appliances (such as refrigerators) experience vibration and require resilient motor mountings to prevent excessive vibration.

In the next section, we use Eq. 10.3 to find the average power at the terminals of the circuit in Fig. 10.1 and also introduce the concept of reactive power.

10.2 Average and Reactive Power

As we have already noted, Eq. 10.3 has three terms, which we can rewrite as follows:

$$p = P + P\cos 2\omega t - Q\sin 2\omega t, \quad (10.4)$$

where

Average (Real) Power

$$P = V_m I_m 2\cos(\theta_v - \theta_i), \quad (10.5)$$

Reactive Power

$$Q = V_m I_m 2\sin(\theta_v - \theta_i). \quad (10.6)$$

P is the **average power**, and Q is the **reactive power**. Average power is sometimes called **real power** because it describes the power in a circuit that is transformed from electric to nonelectric energy. Although the two terms are interchangeable, we primarily use the term *average power* in this text.

It is easy to see why P is called the average power: it is the average of the instantaneous power over one period. In equation form,

$$P = \frac{1}{T}\int_{t_0}^{t_0+T} p\, dt, \quad (10.7)$$

where T is the period of the sinusoidal function. The limits on the integral imply that we can initiate the integration process at any convenient time t_0 but that we must terminate the integration exactly one period later. (We could integrate over nT periods, where n is an integer, provided we multiply the integral by 1/nT.)

We could find the average power by substituting Eq. 10.3 directly into Eq. 10.7 and integrating. But the average value of p is given by the first term on the right-hand side of Eq. 10.3 because the integral of both cos 2ωt and sin 2ωt over one period is zero. Thus, the average power is given in Eq. 10.5.

We can develop a better understanding of all the terms in Eq. 10.4 and the relationships among them by examining the power in circuits that are purely resistive, purely inductive, or purely capacitive.

Power for Purely Resistive Circuits

If the circuit between the terminals in Fig. 10.1 is purely resistive, the voltage and current are in phase, which means that $\theta v = \theta i$. Equation 10.4 then reduces to

$$p = P + P \cos 2\omega t.$$

The instantaneous power for a resistor is called the **instantaneous real power**. Figure 10.3 shows a graph of p for a purely resistive circuit with $\omega = 377$ rad/s. By definition, the average power, P, is the average of p over one period. Looking at the graph, we see that $P = 1$ for this circuit. Note that the instantaneous real power can never be negative, which is seen in its equation and is also shown in Fig. 10.3. In other words, power cannot be extracted from a purely resistive network. Resistors dissipate electric energy in the form of thermal energy.

Figure 10.3 Instantaneous power and average power for a purely resistive circuit.

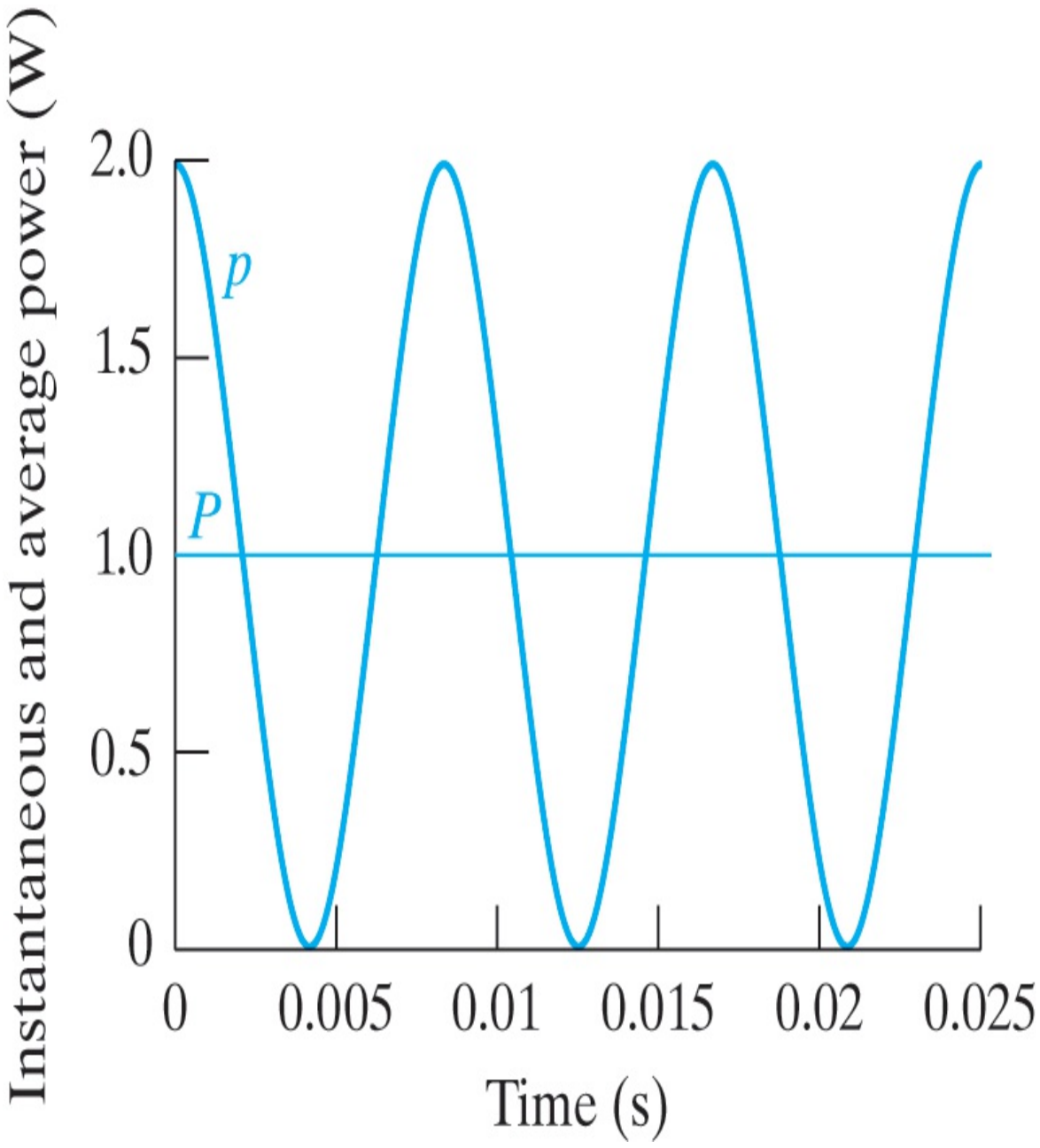


Figure 10.3 Full Alternative Text

Power for Purely Inductive Circuits

If the circuit between the terminals in Fig. 10.1 is purely inductive, the current lags the voltage by 90° (that is, $\theta_i = \theta_v - 90°$); therefore, $\theta_v - \theta_i = +90°$. The expression for the instantaneous power then reduces to

$p = -Q\sin 2\omega t$.

In a purely inductive circuit, the average power is zero, and energy is not transformed from electric to nonelectric form. Instead, the instantaneous power in a purely inductive circuit is continually exchanged between the circuit and the source driving the circuit, at a frequency of 2ω. When p is positive, energy is stored in the magnetic fields associated with the inductive elements, and when p is negative, energy is extracted from the magnetic fields.

We measure the power of purely inductive circuits using the reactive power Q. The name *reactive power* recognizes an inductor as a reactive element; its impedance is purely reactive. Note that average power P and reactive power Q carry the same dimension. To distinguish between average and reactive power, we use the units watt (W) for average power and **var** (*volt-amp reactive*, or VAR) for reactive power. Figure 10.4 plots the instantaneous power for a purely inductive circuit, assuming $\omega = 377$ rad/s and Q=1 VAR.

Figure 10.4 Instantaneous power, average power, and reactive power for a purely inductive circuit.

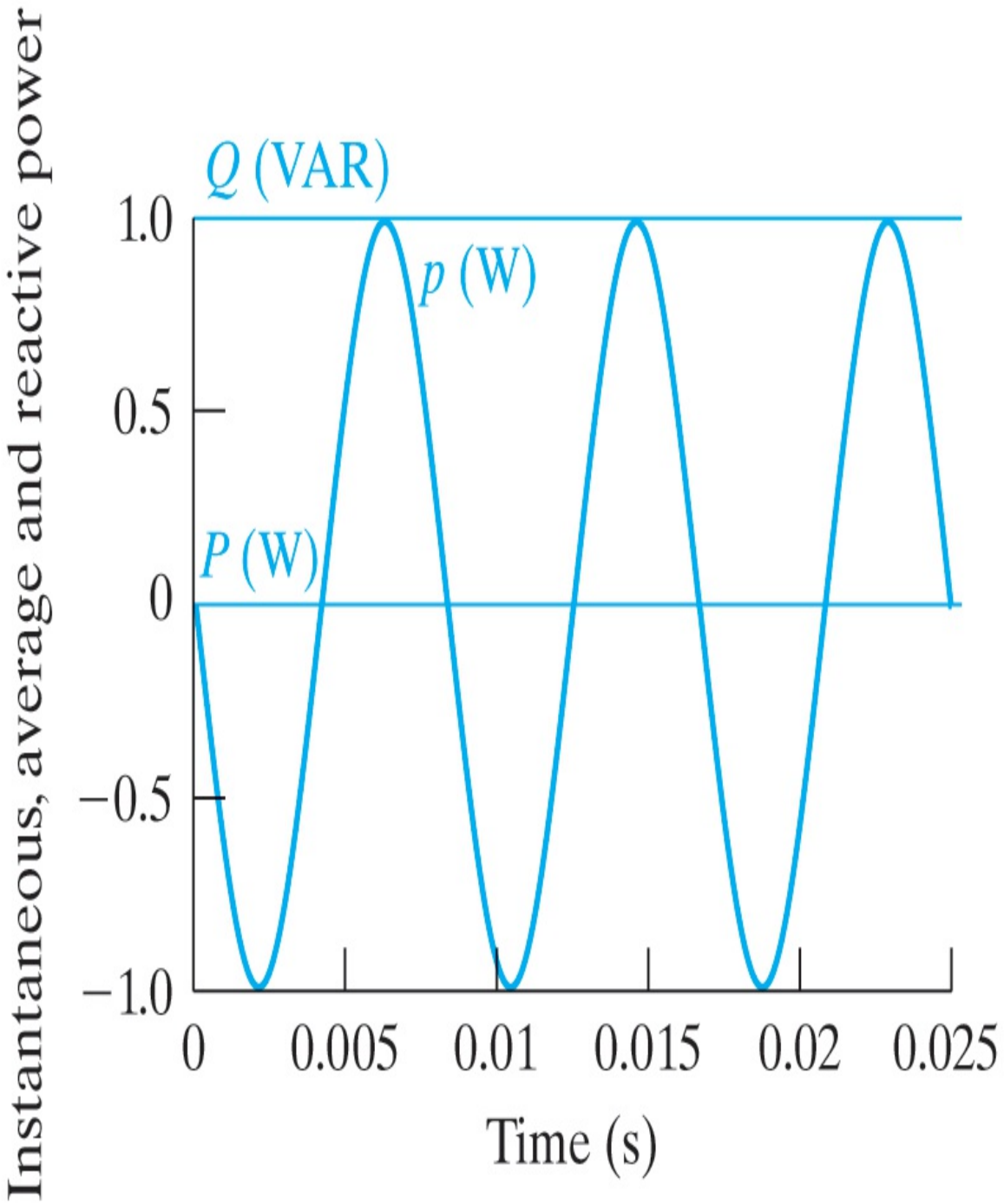


Figure 10.4 Full Alternative Text

Power for Purely Capacitive

Circuits

If the circuit between the terminals in Fig. 10.1 is purely capacitive, the current leads the voltage by 90° (that is, $\theta i = \theta v + 90°$); thus, $\theta v - \theta i = -90°$. The expression for the instantaneous power then becomes

$$p = -Q \sin 2\omega t.$$

Again, the average power is zero, and energy is not transformed from electric to nonelectric form. Instead, the power is continually exchanged between the source driving the circuit and the electric field associated with the capacitive elements. Figure 10.5 plots the instantaneous power for a purely capacitive circuit, assuming $\omega = 377$ rad/s and $Q = -1$ VAR.

Figure 10.5 Instantaneous power, average power, and reactive power for a purely capacitive circuit.

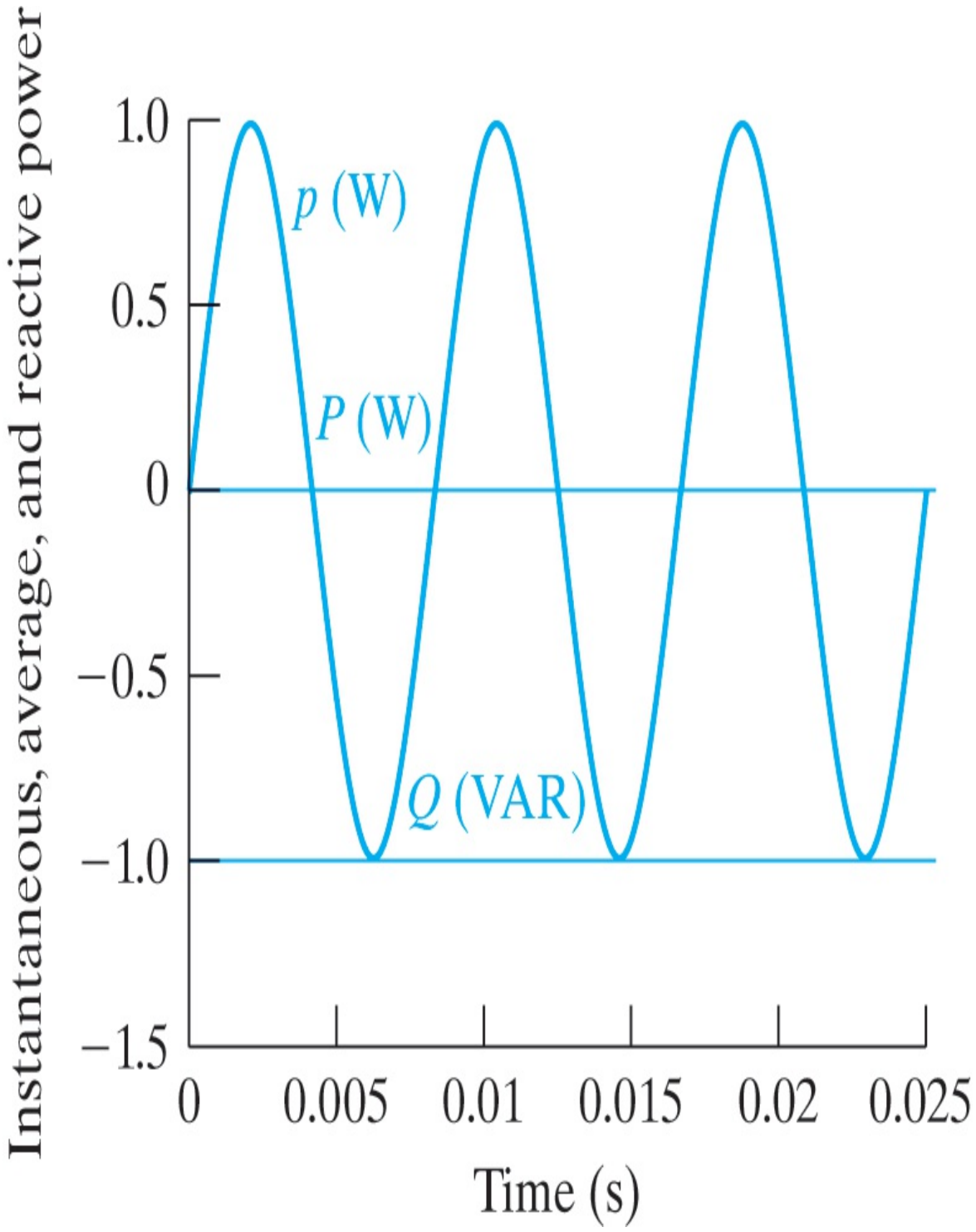


Figure 10.5 Full Alternative Text

Note that the decision to use the current as the reference (see Eq. 10.2) means that Q is positive for inductors (because $\theta v - \theta i = 90°$) and negative for

capacitors (because $\theta v - \theta i = -90°$). Power engineers recognize this difference in the algebraic sign of Q by saying that inductors demand (or absorb) magnetizing vars and capacitors furnish (or deliver) magnetizing vars. We say more about this convention later.

The Power Factor

The angle $\theta v - \theta i$ is used when computing both average and reactive power and is referred to as the **power factor angle**. The cosine of this angle is called the **power factor**, abbreviated pf, and the sine of this angle is called the **reactive factor**, abbreviated rf. Thus

Power Factor

$$pf = \cos(\theta v - \theta i), \quad (10.8)$$
$$rf = \sin(\theta v - \theta i).$$

Even if you know the value of the power factor, you cannot determine the power factor angle because $\cos(\theta v - \theta i) = \cos(\theta i - \theta v)$. To completely describe this angle, we use the phrases **lagging power factor and leading power factor**. Lagging power factor means that current lags voltage—hence, an inductive load. Leading power factor means that current leads voltage—hence, a capacitive load. Both the power factor and the reactive factor are convenient quantities to use in describing electrical loads.

Example 10.1 illustrates the interpretation of P and Q using numerical calculations.

Example 10.1 Calculating Average and Reactive Power

1. Calculate the average power and the reactive power at the terminals of

the network shown in Fig. 10.6 if

Figure 10.6 A pair of terminals used for calculating power.

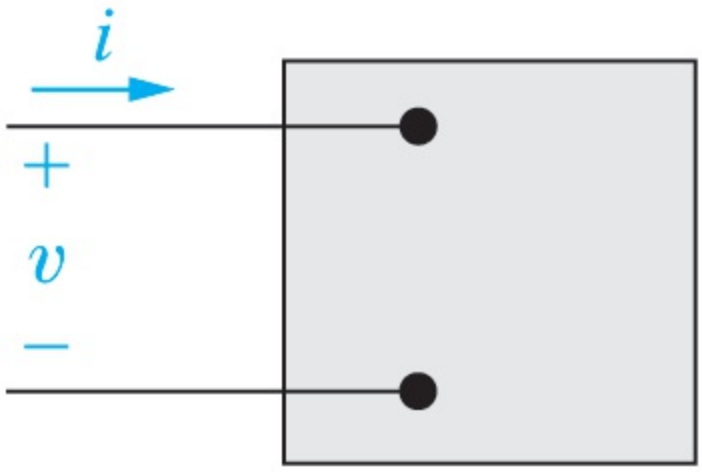


Figure 10.6 Full Alternative Text
v=100 cos (ωt+15°) V,i=4 sin (ωt−15°) A.

2. State whether the network inside the box is absorbing or delivering average power.

3. State whether the network inside the box is absorbing or supplying magnetizing vars.

Solution

1. Because i is expressed in terms of the sine function, the first step in calculating P and Q is to rewrite i as a cosine function:

   i=4 cos (ωt−105°) A.

We now calculate P and Q directly from Eqs. 10.5 and 10.6, using the passive sign convention. Thus

$P=12(100)(4)\cos[\ 15^\circ -(-105^\circ)\ ]=-100\ \text{W}, Q=12100(4)\sin[\ 15^\circ -(-105^\circ)\ ]=173.21\ \text{VAR}.$

2. The value of P is negative, so the network inside the box is delivering average power to the terminals.

3. The value of Q is positive, so the network inside the box is absorbing magnetizing vars at its terminals.

Assessment Problems

Objective 1—Understand ac power concepts, their relationships to one another, and how to calcuate them in a circuit

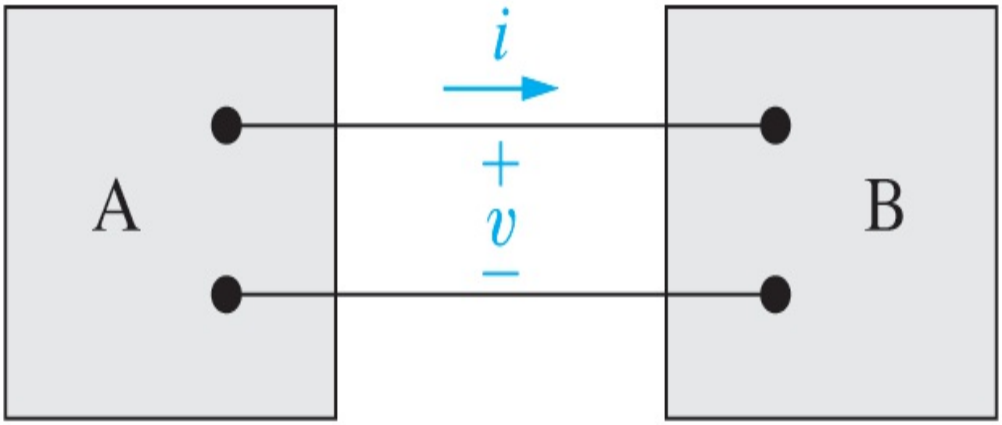
1. 10.1 For each of the following sets of voltage and current, calculate the real and reactive power in the line between networks A and B in the circuit shown. In each case, state whether the power flow is from A to B or vice versa. Also state whether magnetizing vars are being transferred from A to B or vice versa.

    1. a) $v=100\cos(\omega t-45°)\ \text{V}; i=20\cos(\omega t+15°)\text{A}.$

    2. b) $v=100\cos(\omega t-45°)\ \text{V}; i=20\cos(\omega t+165°)\ \text{A}.$

    3. c) $v=100\cos(\omega t-45°)\ \text{V}; i=20\cos(\omega t-105°)\ \text{A}.$

    4. d) $v=100\cos\omega t\ \text{V}; i=20\cos(\omega t+120°)\ \text{A}.$



Answer:

1. (a) P=500 W(A to B);Q=−866.03 VAR(B to A).
2. (b) P=−866.03 W(B to A);Q=500 VAR(A to B).
3. (c) P=500 W(A to B);Q=866.03 VAR(A to B).
4. (d) P=−500 W(B to A);Q=−866.03 VAR(B to A).

2. 10.2 Compute the power factor and the reactive factor for the network inside the box in Fig. 10.6, whose voltage and current are described in Example 10.1.

   Hint: Use −i to calculate the power factor and reactive factor.

   Answer: pf=0.5 leading; rf=−0.866.

SELF-CHECK: Also try Chapter Problem 10.1.

Appliance Ratings

Average power is used to quantify the power needs of household appliances. Your monthly electric bill is based on the number of kilowatt-hours used by the household. Table 10.1 presents data for some common appliances, including the average hours per month and the number of months the

appliance is used, the annual kilowatt-hour (kwh) consumption, and the annual cost of operation. For example, a coffee maker has a monthly use of 30 hours, or one hour per day, is used every month of the year, and consumes 60 kwh per year, at a cost of about $10. Therefore, the coffee maker consumes 60 kwh/360 hours=0.167 kW=167 W of power during the hour it operates each day.

Table 10.1 Annual Energy Requirements of Electric Household Appliances

Appliance	Hours in Use per Month	Months Used	Annual kWH	Annual Cost
A/C–central	120	3	1080	$173
Clothes dryer–electric	24	12	901	$144
Clothes washer (does not include cost of hot water)	28	12	108	$17
Coffee maker (residential)	30	12	60	$10
Computer–desktop with monitor	*	*	127	$20
Computer–laptop	*	*	23	$4
Dishwasher–heat dry (does not include hot water)	30	12	293	$47
DVD player	60	12	18	$3
Fan–ceiling (does not include lights)	150	6	72	$12
Fan–table/box/floor	60	3	28	$4
Game console (includes standby/phantom load)	*	*	65	$10
Heating system–electric heat–baseboard, 10 ft	240	5	1500	$240
Humidifier	240	12	360	$58
Lighting–incandescent, 75 Watt	60	12	54	$9
Lighting–CFL, 20 Watt (75W incandescent equivalent)	90	12	22	$3
Lighting–LED, 10 Watt (75W incandescent equivalent)	90	12	12	$2
Microwave	9	12	101	$16
Oven–electric	9	12	284	$45
Refrigerator–19–21.4 cu ft –2001–2008	720	12	533	$85
Refrigerator–19–21.4 cu ft (new ENERGY STAR)	720	12	336	$54
Set-top box, cable/satellite receiver	720	12	249	$40
Television–50"+ non-ENERGY STAR TV	*	*	215	$34
Water heater–electric (newer base model .95 energy factor)	n/a	12	4559	$729

Table 10.1 Full Alternative Text

*Draws power in standby mode

Notes:

- Hours in Use per Month is based on a typical four-person household in a northern US state.
- Annual kWh may vary considerably depending on model, age, and use.
- Annual Cost is based on 16 cents per kilowatt hour (kWH).
- Data used with permission from https://www.efficiencyvermont.com/tips-tools/tools/electric-usage-chart-tool.

Example 10.2 uses Table 10.1 to determine whether four common appliances can all be in operation without exceeding the current-carrying capacity of the household.

Example 10.2 Making Power Calculations Involving Household Appliances

The branch circuit supplying the outlets in a typical home kitchen is wired with #12 conductor and is protected by either a 20 A fuse or a 20 A circuit breaker. Assume that the following 120 V appliances are in operation at the same time: a coffee maker, microwave, dishwasher, and older refrigerator. Will the circuit be interrupted by the protective device?

Solution

We have already estimated that the average power used by the coffee maker is 167 W. Using Table 10.1, we find that the average power used by the other three appliances is

$P_{microwave} = 101(9)(12) = 0.935 \text{ kW} = 935 \text{ W}$, $P_{dishwasher} = 293(30)(12) = 0.814 \text{ kW} = 814 \text{ W}$, $P_{refrigerator} = 533(720)(12) = 0.062 \text{ kW} = 62 \text{ W}$.

The total average power used by the four appliances is

$$P_{total} = 167 + 935 + 814 + 62 = 1978 \text{ W}.$$

The total current in the protective device is

$$I = \frac{P_{total}}{V} = \frac{1978}{120} = 16.5 \text{ A}.$$

Since the current is less than 20 A, the protective device will not interrupt the circuit.

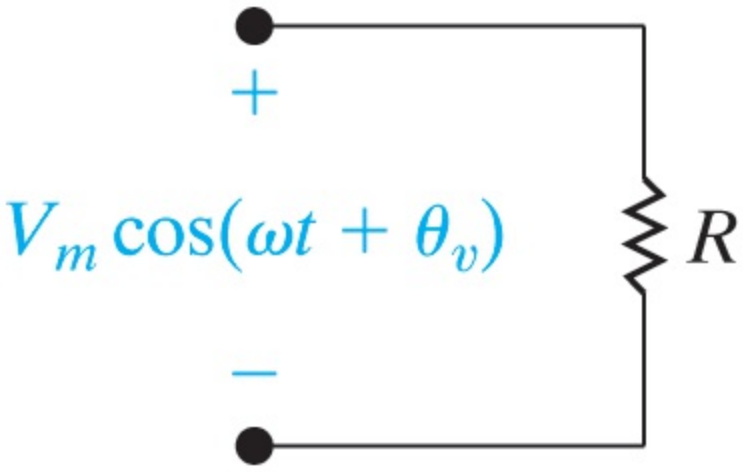
SELF-CHECK: Assess your understanding of this material by trying Chapter Problem 10.2.

10.3 The rms Value and Power Calculations

When we introduced the rms value of a sinusoidal voltage (or current) in Section 9.1, we mentioned that it would play an important role in power calculations. We now discuss this role.

Assume a sinusoidal voltage is applied to the terminals of a resistor, as shown in Fig. 10.7, and that we want to determine the average power delivered to the resistor. From Eq. 10.7,

Figure 10.7 A sinusoidal voltage applied to the terminals of a resistor.



P=1Tʃt0t0+TVm2 cos2(ωt+ ϕv)R dt=1R[1Tʃt0t0+TVm2 cos2(ωt+ ϕv) dt].

From Eq. 9.4, we see that the bracketed term is the rms value of the voltage squared. Therefore, the average power delivered to R is

P=Vrms2R.

If the resistor has a sinusoidal current, say, Im cos (ωt+ ϕi), the average power delivered to the resistor is

P=Irms2R.

The rms value is also referred to as the **effective value** of the sinusoidal voltage (or current). The rms value has an interesting property: Given an equivalent resistive load, R, and an equivalent time period, T, the rms value of a sinusoidal source delivers the same energy to R as does a dc source of the same value. For example, a dc source of 100 V delivers the same energy in T seconds that a sinusoidal source of 100 V(rms) delivers, assuming equivalent load resistances (see Problem 10.16). Figure 10.8 demonstrates this equivalence. The effect of the two sources is identical with respect to energy delivery. So we use the terms *effective value* and *rms value* interchangeably.

Figure 10.8 The effective value of vs [100 V(rms)] delivers the same power to R as the dc voltage vs [100 V(dc)].

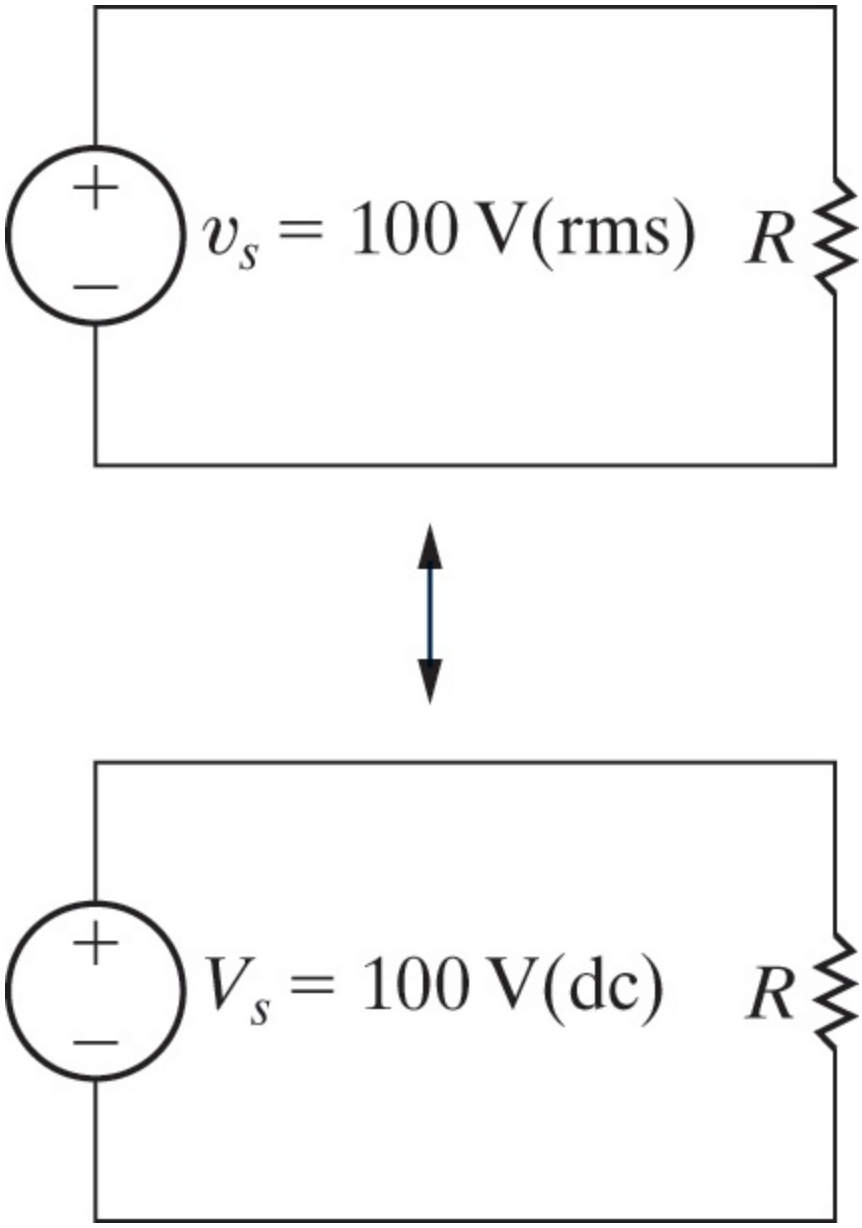


Figure 10.8 Full Alternative Text

The average power given by Eq. 10.5 and the reactive power given by Eq. 10.6 can be written in terms of effective values:

$$P=VmIm2\cos(\theta v-\theta i)=Vm2Im2\cos(\theta v-\theta i)=VrmsIrms\cos(\theta v-\theta i) \quad (10.9)$$

and, by similar manipulation,

$$Q=VrmsIrms\sin(\theta v-\theta i). \quad (10.10)$$

Using the effective values of sinusoidal signals in power calculations is so widespread that we specify the voltage and current ratings of circuits and equipment using rms values. For example, the voltage rating of residential electric wiring is often 240 V/120 V service. These voltages are the rms values of the sinusoidal voltages supplied by the utility company, which provides power at two voltage levels, accommodating low-voltage appliances (such as televisions) and higher-voltage appliances (such as electric ranges). Appliances such as electric lamps, irons, and toasters all carry rms ratings on their nameplates. For example, a 120 V, 100 W lamp has a resistance of 1202/100, or 144 Ω, and draws an rms current of 120/144, or 0.833 A. The peak value of the lamp current is 0.8332, or 1.18 A.

The phasor transform of a sinusoidal function may also be expressed as an rms value. The magnitude of the rms phasor is equal to the rms value of the sinusoidal function. We indicate that a phasor is based on an rms value using either an explicit statement, a parenthetical “rms” adjacent to the phasor’s units, or the subscript “rms.”

In Example 10.3, we use rms values to calculate power.

Example 10.3 Determining Average Power Delivered to a Resistor by a Sinusoidal Voltage

1. A sinusoidal voltage having a maximum amplitude of 625 V is applied to the terminals of a 50 Ω resistor. Find the average power delivered to the resistor.

2. Repeat (a) by first finding the current in the resistor.

Solution

1. The rms value of the sinusoidal voltage is 625/2, or approximately 441.94 V. The average power delivered to the 50 Ω resistor is

   $P = V_{rms}^2 R = (441.94)250 = 3906.25$ W.

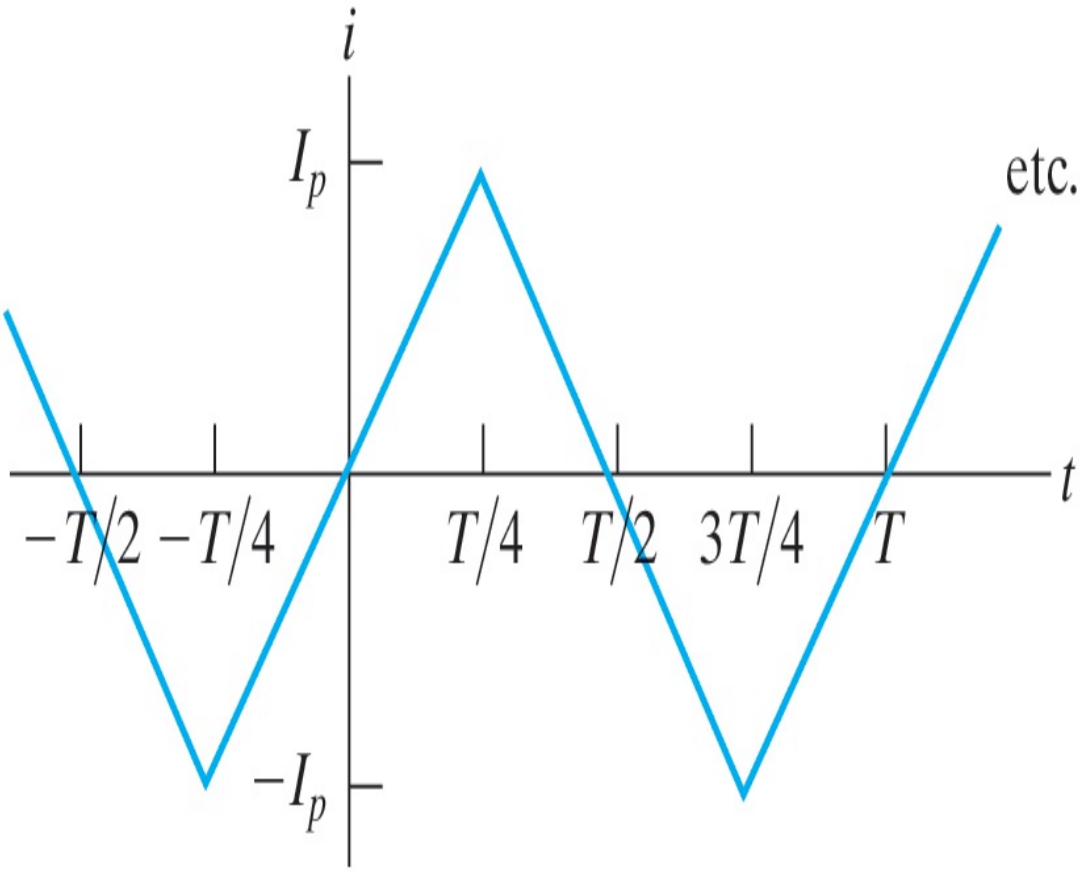
2. The maximum amplitude of the current in the resistor is 625/50, or 12.5 A. The rms value of the current is 12.5/2, or approximately 8.84 A. Hence, the average power delivered to the resistor is

   $P = (8.84)250 = 3906.25$ W.

Assessment Problem

Objective 1—Understand ac power concepts, their relationships to one another, and how to calculate them in a circuit

1. 10.3 The periodic triangular current in Example 9.4, repeated here, has a peak value of 180 mA. Find the average power that this current delivers to a 5 kΩ resistor.



10.3-6 Full Alternative Text

Answer: 54 W.

SELF-CHECK: Also try Chapter Problem 10.14.

10.4 Complex Power

Before discussing the methods for calculating real and reactive power in circuits operating in the sinusoidal steady state, we introduce and define complex power. **Complex power** is the complex sum of real power and reactive power, or

Complex Power

$$S=P+jQ. \quad (10.11)$$

As you will see, we can compute the complex power using the voltage and current phasors for a circuit. Equation 10.11 can then be used to determine the average and reactive power, because P=R{S} and Q=I {S}.

Complex power has the same units as average or reactive power. However, to distinguish complex power from both average and reactive power, we use the units **volt-amps** (VA). Thus, we use volt-amps for complex power, watts for average power, and vars for reactive power, as summarized in Table 10.2.

Table 10.2 Three Power Quantities and Their Units

Quantity	Units
Complex power	volt-amps
Average power	watts
Reactive power	var

Complex power provides a geometric relationship among several different power quantities. In Eq. 10.11, envision P, Q, and |S| as the sides of a right triangle, as shown in Fig. 10.9. We can show that the angle θ in the power triangle is the power factor angle θv− θi. For the right triangle shown in Fig. 10.9,

Figure 10.9 A power triangle.

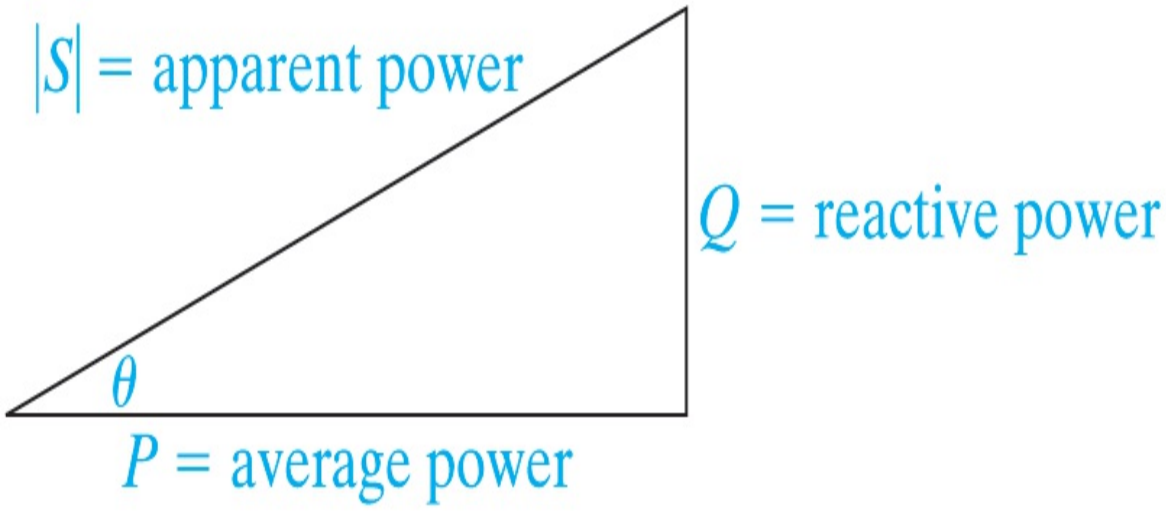


Figure 10.9 Full Alternative Text
tanθ=QP.

But from the definitions of P and Q (Eqs. 10.5 and 10.6, respectively),

QP=(VmIm/2) sin (θv− θi)(VmIm/2) cos (θv− θi)=tan(θv−θi).

Therefore, θ=θv− θi. The geometric relationships for a right triangle mean that the four power triangle dimensions (the three sides and the power factor angle) can be determined if any two of the four are known.

The magnitude of complex power is referred to as **apparent power**. Specifically,

Apparent Power

$$|S|=P2+Q2. \quad (10.12)$$

Apparent power, like complex power, is measured in volt-amps. The apparent power, or volt-amp, requirement of a device designed to convert electric energy to a nonelectric form is more useful than the average power requirement. The apparent power represents the volt-amp capacity required to supply the average power used by the device. As you can see from the power triangle in Fig. 10.9, unless the power factor angle is 0° (that is, the device is purely resistive, pf=1, and Q=0), the volt-amp capacity required by the device is larger than the average power used by the device.

Many appliances (including refrigerators, fans, air conditioners, fluorescent lighting fixtures, and washing machines) and most industrial loads operate at a lagging power factor. The power factor of these loads can be *corrected* either by adding a capacitor to the device itself or by connecting capacitors across the line feeding the load; the latter method is often used for large industrial loads. Many of the problems at the end of the chapter explore methods for correcting a lagging power factor load and improving the operation of a circuit.

Example 10.4 uses a power triangle to calculate several quantities associated with power in an electrical load.

Example 10.4 Calculating Complex Power

An electrical load operates at 240 V(rms). The load absorbs an average power of 8 kW at a lagging power factor of 0.8.

1. Calculate the complex power of the load.

2. Calculate the impedance of the load, Z.

Solution

1. Because the power factor is lagging, we know that the load is inductive and that the algebraic sign of the reactive power is positive. From the power triangle shown in Fig. 10.10,

Figure 10.10 A power triangle.

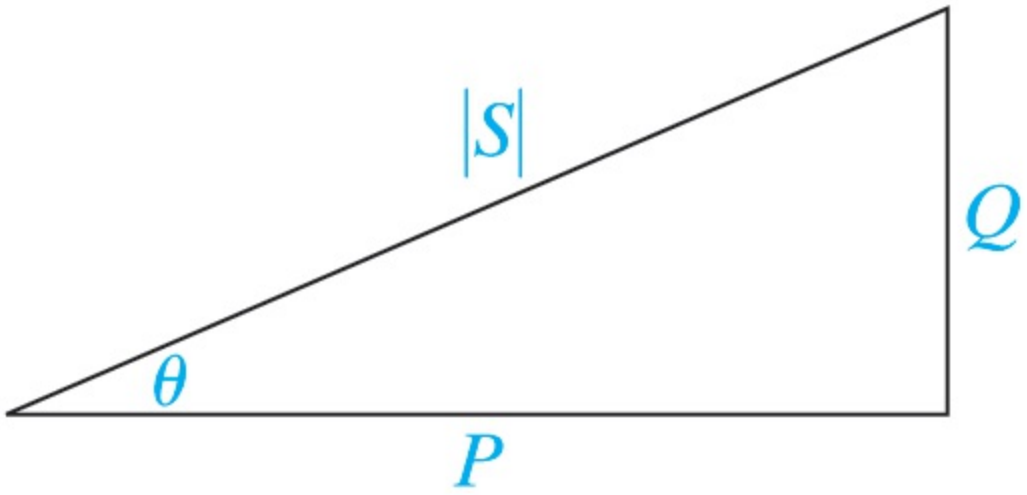


Figure 10.10 Full Alternative Text

$P=|S|\cos\theta, Q=|S|\sin\theta.$

Since $\cos\theta=0.8$, we know that $\sin\theta=0.6$. Therefore

$$|S|=P\cos\theta=80000.8=10{,}000=10\text{ kVA}, Q=|S|\sin\theta=(10{,}000)(0.6)=6\text{ kvar},$$

and

$$S=P+jQ=8+j6\text{ kVA}.$$

2. From the problem statement, we know that P = 8 kW for the load. Using Eq. 10.9,

$$P=V_{rms}I_{rms}\cos(\theta_v-\theta_i)=(240)I_{rms}(0.8)=8000\text{ W}.$$

Solving for Irms,

Irms=41.67 A(rms).

We already know the angle of the load impedance because it is the power factor angle:

$\theta = \cos^{-1}(0.8) = 36.87°$.

We also know that θ is positive because the power factor is lagging, indicating an inductive load. Compute the load impedance magnitude using its definition as the ratio of the magnitude of the load voltage to the magnitude of the load current:

$|Z| = |V_{rms}| / |I_{rms}| = 240/41.67 = 5.76$.

Hence,

$Z = 5.76 \angle 36.87° \ \Omega = 4.608 + j3.456 \ \Omega$.

Assessment Problem

Objective 1—Understand ac power concepts, their relationships to one another, and how to calculate them in a circuit

1. 10.4 A load consisting of a 1.35 kΩ resistor in parallel with a 405 mH inductor is connected across the terminals of a sinusoidal voltage source vg, where vg = 90 cos 2500t V. Find
    1. a) the average power delivered to the load,
    2. b) the reactive power for the load,
    3. c) the apparent power for the load, and
    4. d) the power factor of the load.

Answer:

1. (a) 3 W;
2. (b) 4 VAR;
3. (c) 5 VA;
4. (d) 0.6 lagging.

SELF-CHECK: Also try Chapter Problem 10.17.

10.5 Power Calculations

We now develop additional equations for calculating real, reactive, and complex power. We begin by combining Eqs. 10.5, 10.6, and 10.11 to get

$$S = V_m I_m 2 \cos(\theta_\upsilon - \theta_i) + j V_m I_m 2 \sin(\theta_\upsilon - \theta_i) = V_m I_m 2 [\cos(\theta_\upsilon - \theta_i) + j\sin(\theta_\upsilon - \theta_i)] = V_m I_m 2 e^{j(\theta_\upsilon - \theta_i)} = 1\,2\, V_m I_m (\theta_\upsilon - \theta_i) .$$

If we use the rms values of the sinusoidal voltage and current, the expression for the complex power becomes

$$S = V_{rms} I_{rms} (\theta_v - \theta_i).$$

Therefore, if the phasor current and voltage are known at a pair of terminals, the complex power associated with that pair of terminals is either one half the product of the phasor voltage and the conjugate of the phasor current, or the product of the rms phasor voltage and the conjugate of the rms phasor current. We can show this for the rms phasor voltage and current in Fig. 10.11 as follows:

Figure 10.11 The phasor voltage and current associated with a pair of terminals.

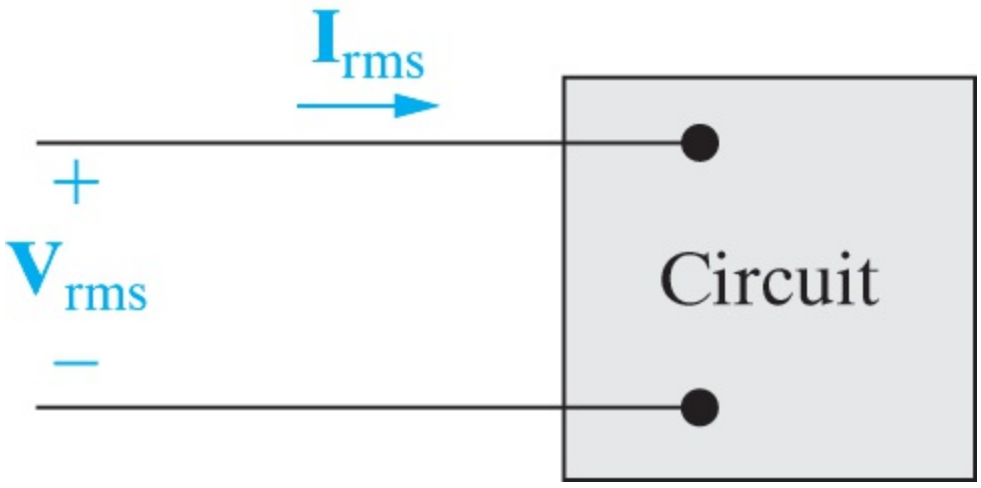


Figure 10.11 Full Alternative Text

$S = V_{rms} I_{rms} \angle(\theta_v - \theta_i) = V_{rms} I_{rms} e^{j(\theta_v - \theta_i)} = V_{rms}\, e^{j\theta_v} I_{rms} e^{-j\theta_i}$

so

Complex Power, Alternate Form

$$S = V_{rms} I_{rms}^*. \quad (10.13)$$

Note that $I_{rms}^* = I_{rms} e^{-j\theta_i}$ follows from Euler's identity and the trigonometric identities $\cos(-\theta) = \cos(\theta)$ and $\sin(-\theta) = -\sin(\theta)$:

$$I_{rms} e^{-j\theta_i} = I_{rms}\cos(-\theta_i) + jI_{rms}\sin(-\theta_i) = I_{rms}\cos(\theta_i) - jI_{rms}\sin(\theta_i) = I_{rms}^*.$$

If the voltage and current phasors are not specified as rms values, the derivation technique used for Eq. 10.13 yields

$$S = \frac{1}{2} V I^*. \quad (10.14)$$

Both Eqs. 10.13 and 10.14 use the passive sign convention. If the current reference is in the direction of the voltage rise across the terminals, we insert a minus sign on the right-hand side of each equation.

Example 10.5 uses Eq. 10.14 in a power calculation, with the phasor representation of the voltage and current from Example 10.1

Example 10.5 Calculating Power Using Phasor Voltage and Current

1. Calculate the average power and the reactive power at the terminals of the network shown in Fig. 10.12 if

Figure 10.12 The circuit for Example 10.5.

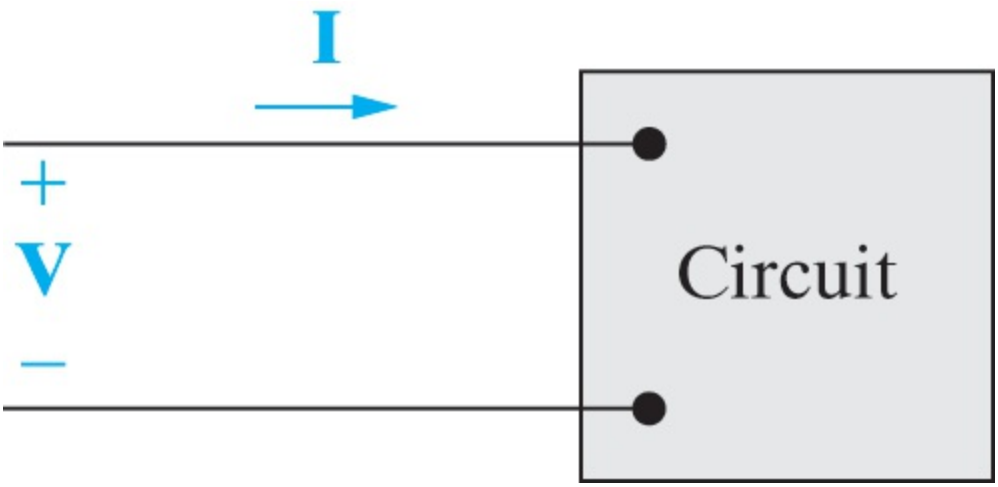


Figure 10.12 Full Alternative Text
$V=100\ \angle 15°$ V, $I=4\ \angle -105°$ A.

2. State whether the network inside the box is absorbing or delivering average power.

3. State whether the network inside the box is absorbing or supplying magnetizing vars.

Solution

1. From Eq. 10.14,

   $S=12(100\ \ 15°)(4\ \ +105°)=200\ \ 120°=-100+j173.21$ VA.

   Once we calculate the complex power, we can read off both the real and reactive powers, because S=P+jQ. Thus

   $P=-100$ W,$Q=173.21$ var.

2. The value of P is negative, so the network inside the box is delivering average power to the terminals.

3. The value of Q is positive, so the network inside the box is absorbing magnetizing vars at its terminals.

Alternate Forms for Complex Power

Equations 10.13 and 10.14 have several useful variations. We use the rms form of the equations throughout because voltages and currents are most often given as rms values in power computations.

The first variation of Eq. 10.13 replaces the voltage with the product of the current times the impedance. We can always represent the circuit inside the box of Fig. 10.11 by an equivalent impedance, as shown in Fig. 10.13. Then,

Figure 10.13 The general circuit of Fig. 10.11 replaced with an equivalent impedance.

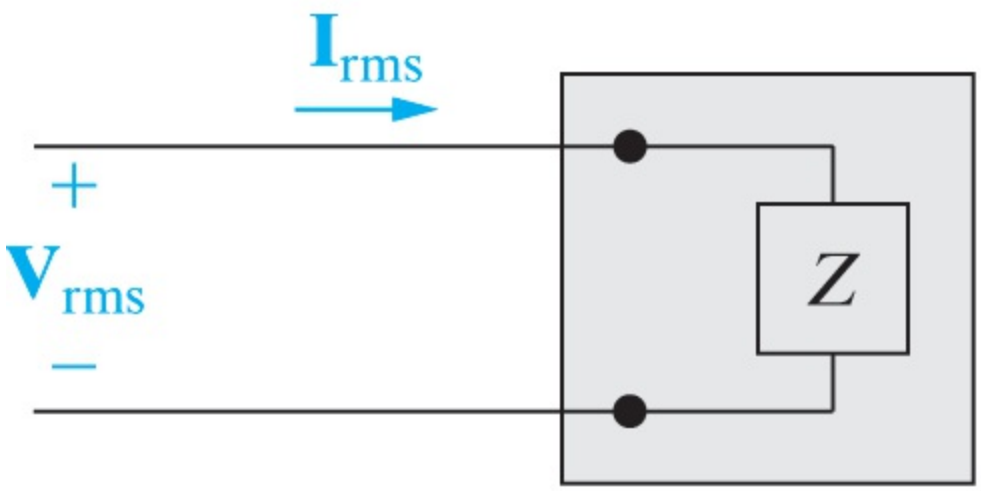


Figure 10.13 Full Alternative Text
Vrms=ZIrms.

Replacing the rms voltage phasor in Eq. 10.13 yields

S=Z IrmsIrms*= | Irms |2 Z= | Irms |2(R + jX)=| Irms|2R+j |Irms|2X=P+jQ, (10.15)

from which

P=|Irms|2R=12Im2R,(10.16)
Q=|Irms|2X=12Im2X.(10.17)

In Eqs. 10.15 and 10.17, X is the reactance of either the equivalent inductance or the equivalent capacitance of the circuit. Remember that reactance is positive for inductive circuits and negative for capacitive circuits.

A second variation of Eq. 10.13 replaces the current with the voltage divided by the impedance:

S=Vrms(VrmsZ)*=|Vrms|2Z*=P+jQ.(10.18)

Note that if Z is a pure resistive element,

P=|Vrms|2R,(10.19)

and if Z is a pure reactive element,

Q=|Vrms|2X.(10.20)

In Eq. 10.20, X is positive for an inductor and negative for a capacitor.

Examples 10.6–10.8 demonstrate various power calculations in circuits operating in the sinusoidal steady state.

Example 10.6 Calculating Average and Reactive Power

In the circuit shown in Fig. 10.14, a load having an impedance of 39+j26 Ω is fed from a voltage source through a line having an impedance of 1+j4 Ω. The source voltage is 250 V(rms).

Figure 10.14 The circuit for Example 10.6.

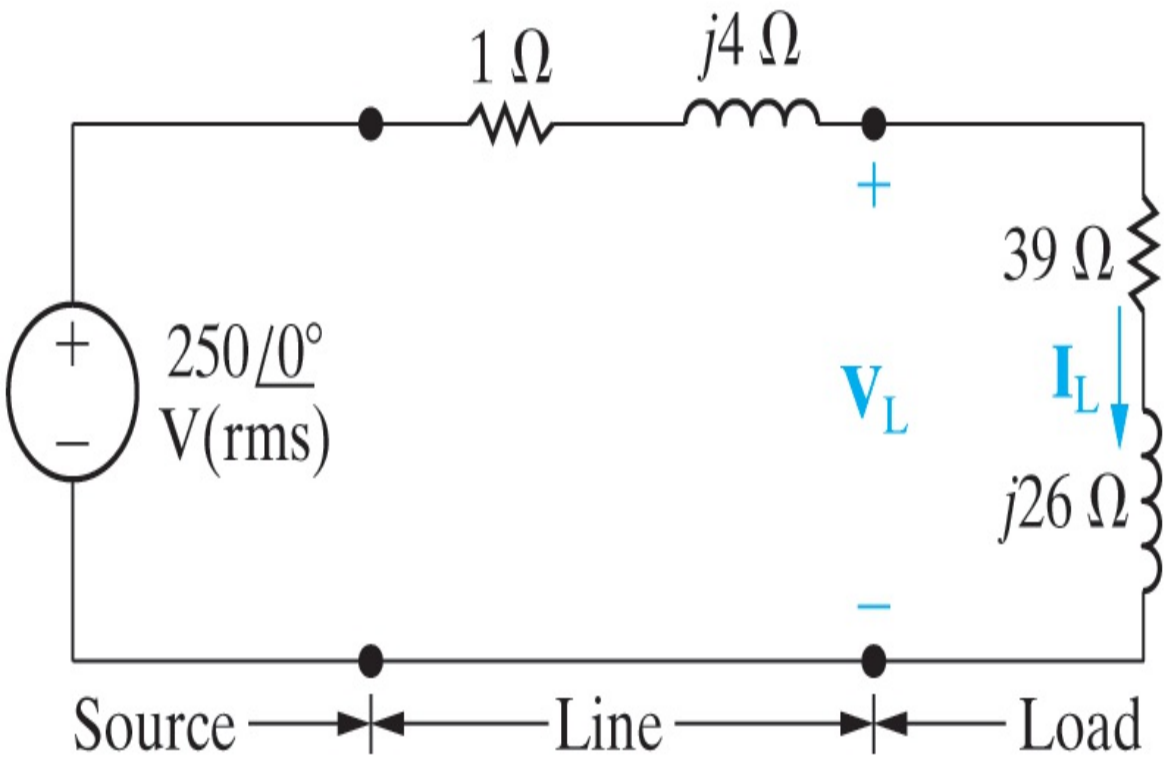


Figure 10.14 Full Alternative Text

1. Calculate the load current phasor IL and voltage phasor VL.
2. Calculate the average and reactive power delivered to the load.
3. Calculate the average and reactive power delivered to the line.
4. Calculate the average and reactive power supplied by the source.

Solution

1. The line and load impedances are in series across the voltage source, so the load current equals the voltage divided by the total impedance, or

   IL=250 0°40+j30=4−j3=5 −36.87° A (rms).

   Because the voltage is given as an rms value, the current value is also

rms. The load voltage is the product of the load current and load impedance:

$V_L=(39 + j26)I_L=234-j13=234.36\ -3.18°$ V (rms).

2. Use Eq. 10.13 to find the average and reactive power delivered to the load. Therefore

   $S=V_L I_L^*=(234-j13)(4+j3)=975 + j650$ VA.

   Thus, the load is absorbing an average power of 975 W and a reactive power of 650 var.

3. Because the line current is known, the average and reactive power delivered to the line are most easily calculated using Eqs. 10.16 and 10.17. Thus

   $P=(5)2(1)=25$ W, $Q=(5)2(4)=100$ var.

   Note that the reactive power associated with the line is positive because the line reactance is inductive.

4. We can calculate the average and reactive power delivered by the source by adding the complex power delivered to the line to that delivered to the load, or

   $S=25+j100+975+j650=1000+j750$ VA.

   The complex power at the source can also be calculated from Eq. 10.13:

   $S_s=-250I_L^*$.

   The minus sign is inserted in Eq. 10.13 whenever the current reference is in the direction of a voltage rise. Thus

   $S_s=-250(4+j3)=-(1000+j750)$ VA.

   The minus sign implies that both average power and magnetizing reactive power are being delivered by the source. This result agrees with

the previous calculation of S, as it must, because the source supplies all the average and reactive power absorbed by the line and load.

Example 10.7 Calculating Power in Parallel Loads

The two loads in the circuit shown in Fig. 10.15 can be described as follows: Load 1 absorbs 8 kW at a leading power factor of 0.8. Load 2 absorbs 20 kVA at a lagging power factor of 0.6.

Figure 10.15 The circuit for Example 10.7.

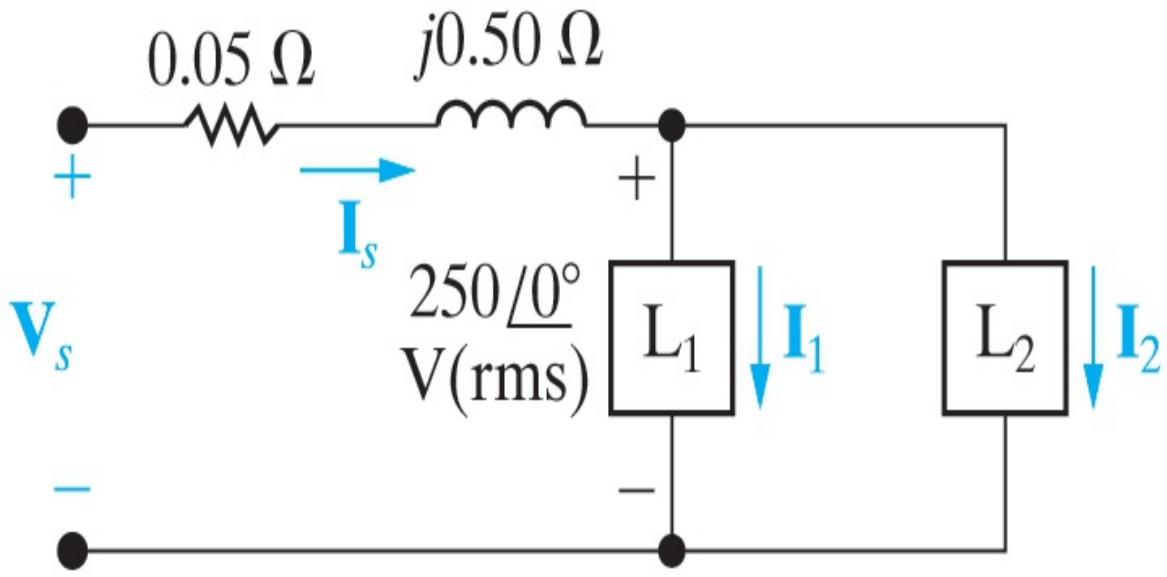


Figure 10.15 Full Alternative Text

1. Determine the power factor of the two loads in parallel.

2. Determine the apparent power required to supply the loads, the magnitude of the current, Is, and the average power loss in the

transmission line.

3. Given that the frequency of the source is 60 Hz, compute the value of the capacitor that would correct the power factor to 1 if placed in parallel with the two loads. Recalculate the values in (b) for the load with the corrected power factor.

Solution

1. All voltage and current phasors in this problem are rms values. Note from the circuit diagram in Fig. 10.15 that Is=I1 + I2. The total complex power absorbed by the two loads is

   S=(250)Is*=(250)(I1+I2)*=(250)I1*+(250)I2*=S1+S2.

   We can sum the complex powers geometrically, using the power triangles for each load, as shown in Fig. 10.16. By hypothesis,

Figure 10.16 (a) The power triangle for load 1; (b) The power triangle for load 2; (c) The sum of the power triangles.

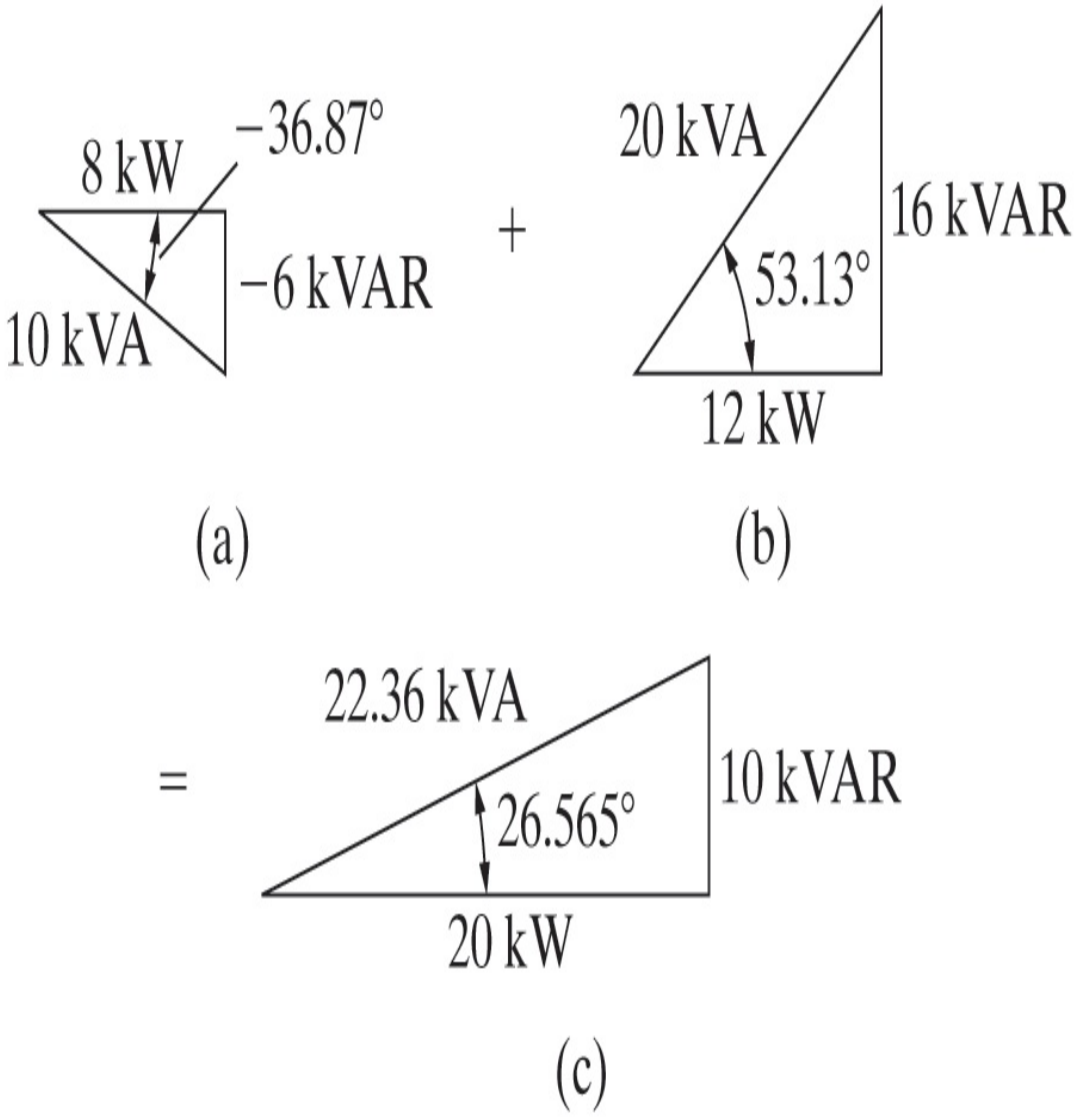


Figure 10.16 Full Alternative Text

S1=8000−j8000(.6)
(.8)=8000−j6000 VA,S2=20,000(.6)+j 20,000(.8)=12,000+j16,000 VA.

It follows that

S=20,000+j10,000 VA,

and

Is*=20,000+j10,000250=80+j40 A(rms).

Therefore

$Is=80-j40=89.44\ -26.57°$ A(rms).

Thus, the power factor of the combined load is

pf=cos(0+26.57°)=0.8944 lagging.

The power factor of the two loads in parallel is lagging because the net reactive power is positive.

2. The apparent power supplied to the two loads is

   $|S|=|20{,}000+j10{,}000|=22.36$ kVA.

   The magnitude of the current supplying this apparent power is

   $|Is|=|80-j40|=89.44$ A(rms).

   The average power lost in the line results from the current in the line resistance:

   $Pline=|Is|2R=(89.44)2(0.05)=400$ W.

   Note that the power supplied totals 20,000+400=20,400 W, even though the loads require a total of only 20,000 W.

3. As we can see from the power triangle in Fig. 10.16(c), we can correct the power factor to 1 if we place a capacitor in parallel with the existing loads that supplies 10 kVAR of magnetizing reactive power. The value of the capacitor is calculated as follows. First, find the capacitive reactance from Eq. 10.20:

   $X=|\ Vrms\ |2Q=(250)2-10{,}000=-6.25\ \Omega$.

   Recall that the reactive impedance of a capacitor is $-1/\omega C$, and $\omega=2\pi(60)=376.99$ rad/s, because the source frequency is 60 Hz. Thus,

   $C=-1\omega X=-1(376.99)(-6.25)=424.4\ \mu F$.

Adding the capacitor as the third load is represented in geometric form as the sum of the two power triangles shown in Fig. 10.17. When the power factor is 1, the apparent power and the average power are the same, as seen from the power triangle in Fig. 10.17(c). Therefore, once the power factor has been corrected, the apparent power is

Figure 10.17 (a) The sum of the power triangles for loads 1 and 2; (b) The power triangle for a 424.4 μF capacitor at 60 Hz; (c) The sum of the power triangles in (a) and (b).

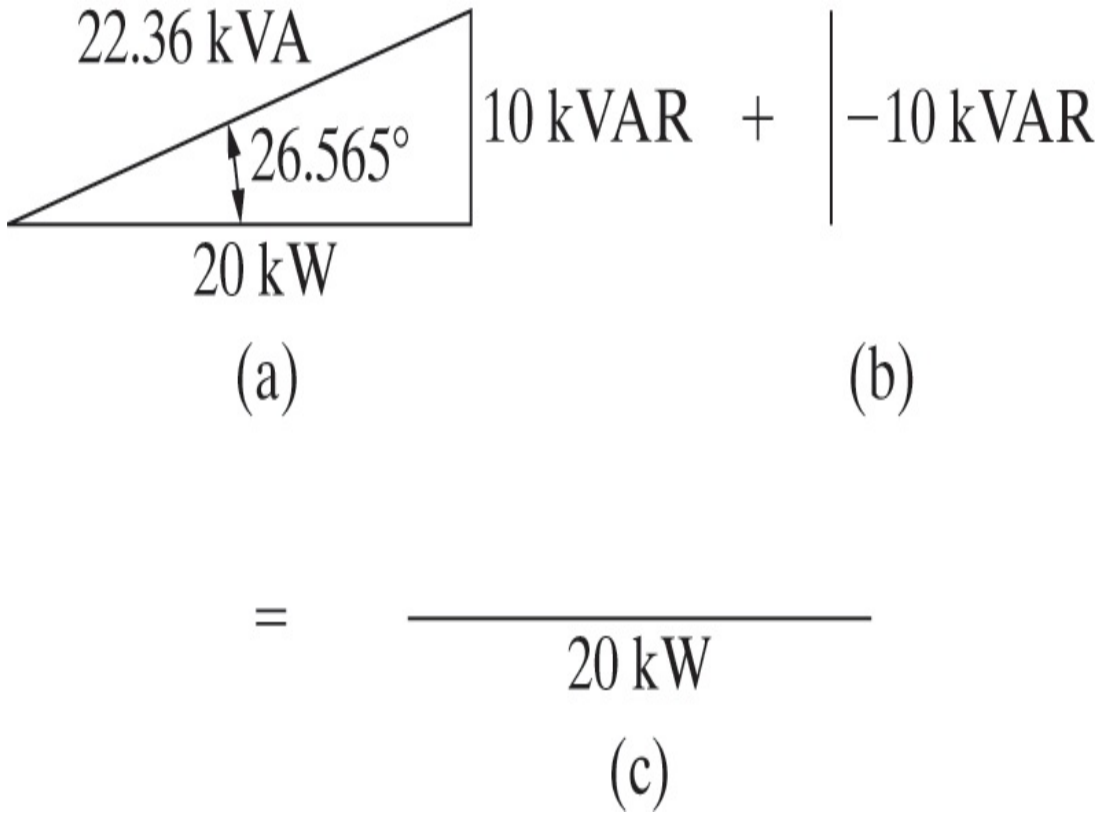


Figure 10.17 Full Alternative Text

|S|=P=20 kVA.

The magnitude of the current that supplies this apparent power is

|Is|=20,000250=80 A(rms).

The average power lost in the line is thus reduced to

P line=|Is|2R=(80)2(0.05)=320 W.

Now, the power supplied totals

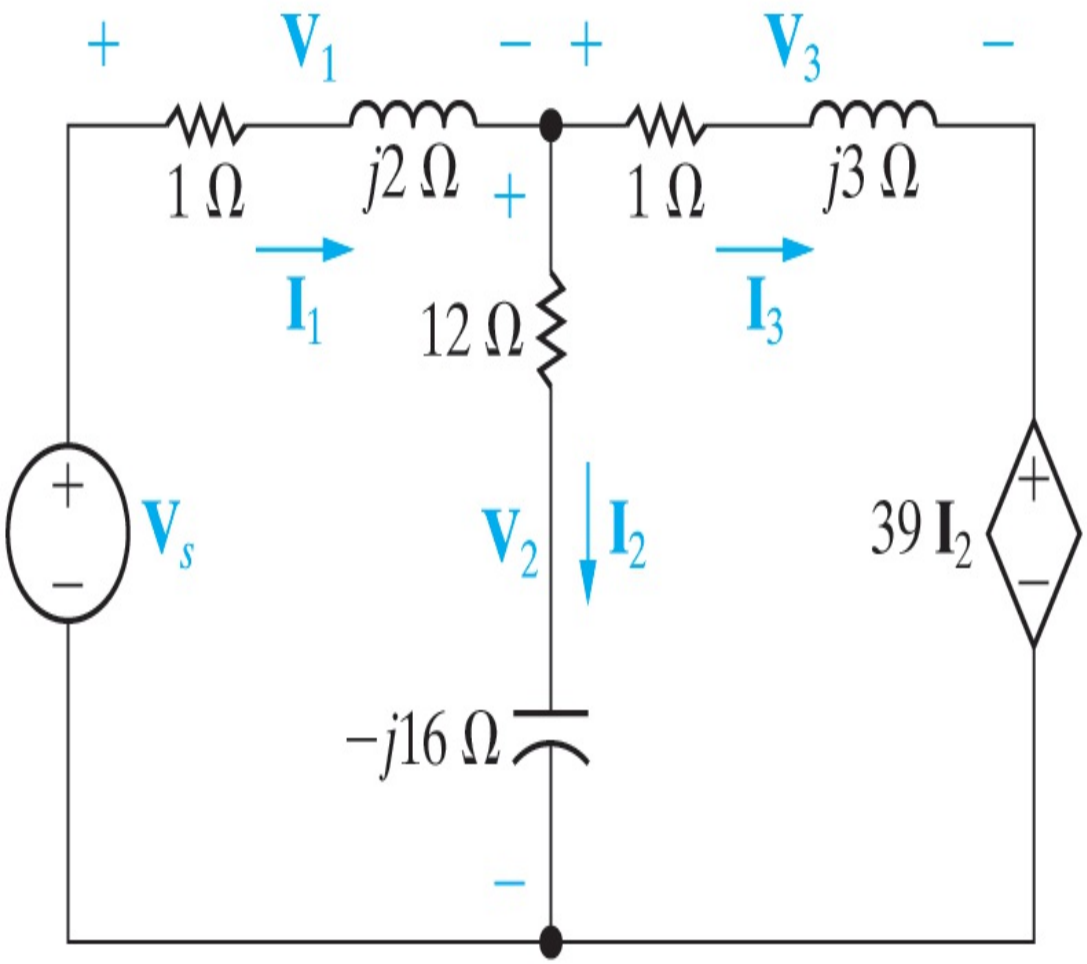
20,000+320=20,320 W.

Note that the addition of the capacitor has reduced the line loss by 20%, from 400 W to 320 W.

Example 10.8 Balancing Power Delivered with Power Absorbed in an AC Circuit

1. Calculate the total average and reactive power delivered to each impedance in the circuit shown in Fig. 10.18.

Figure 10.18 The circuit, with solution, for Example 10.8.



$$\mathbf{V}_s = 150\angle 0^\circ \text{ V}$$

$$\mathbf{V}_1 = (78 - j104)\text{ V} \qquad \mathbf{I}_1 = (-26 - j52)\text{ A}$$

$$\mathbf{V}_2 = (72 + j104)\text{ V} \qquad \mathbf{I}_2 = (-2 + j6)\text{ A}$$

$$\mathbf{V}_3 = (150 - j130)\text{ V} \qquad \mathbf{I}_3 = (-24 - j58)\text{ A}$$

Figure 10.18 Full Alternative Text

2. Calculate the average and reactive powers associated with each source in the circuit.

3. Verify that the average power delivered equals the average power absorbed and that the magnetizing reactive power delivered equals the magnetizing reactive power absorbed.

Solution

1. The complex power delivered to the (1+j2) Ω impedance is

   $S1=12V1I1^*=P1+jQ1=12(78-j104)(-26+j52)=12(3380+j6760)=1690+j3380$ VA.

   Thus, this impedance is absorbing an average power of 1690 W and a reactive power of 3380 VAR. The complex power delivered to the (12−j16) Ω impedance is

   $S2=12V2I2^*=P2+jQ2=12(72+j104)(-2-j6)=240-j320$ VA.

   Therefore, the impedance in the vertical branch is absorbing 240 W and delivering 320 VAR. The complex power delivered to the (1+j3) Ω impedance is

   $S3=12V3I3^*=P3+jQ3=12(150-j130)(-24+j58)=1970+j5910$ VA.

   This impedance is absorbing 1970 W and 5910 VAR.

2. The complex power associated with the independent voltage source is

   $Ss=-12VsI1^*=Ps+jQs=-12(150)(-26+j52)=1950-j3900$ VA.

   Note that the independent voltage source is absorbing an average power of 1950 W and delivering 3900 VAR. The complex power associated with the current-controlled voltage source is

   $Sx=12(39Ix)(I2^*)=Px+jQx=12(-78+j234)(-24+j58)=-5850-j5070$ VA.

   Both average power and magnetizing reactive power are being delivered by the dependent source.

3. The total power absorbed by the passive impedances and the independent voltage source is

   Pabsorbed=P1+P2+P3+Ps=5850 W.

   The dependent voltage source is the only circuit element delivering average power. Thus

   Pdelivered=5850 W.

   Magnetizing reactive power is being absorbed by the two horizontal branches. Thus

   Qabsorbed=Q1+Q3=9290 VAR.

   Magnetizing reactive power is being delivered by the independent voltage source, the capacitor in the vertical impedance branch, and the dependent voltage source. Therefore

   Qdelivered=9290 VAR.

Assessment Problems

Objective 1—Understand ac power concepts, their relationships to one another, and how to calculate them in a circuit

1. 10.5 The load impedance in the circuit shown is shunted by a capacitor having a capacitive reactance of −52 Ω. Calculate:

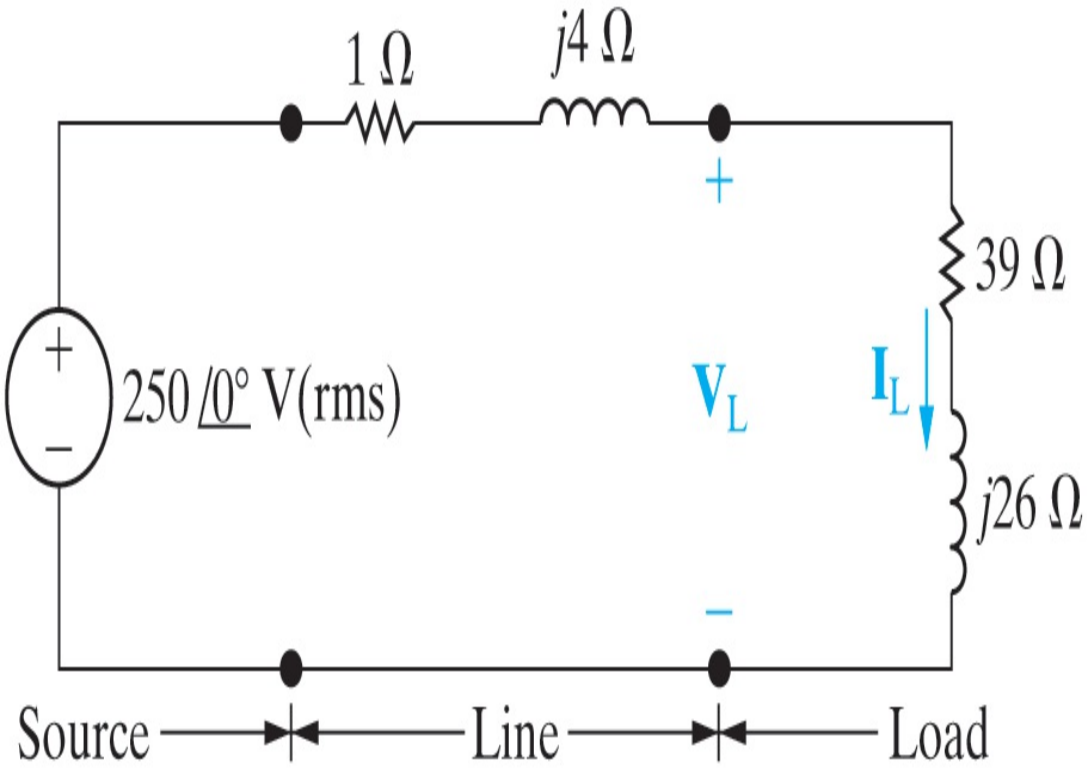
   1. a) the rms phasors VL and IL,

   2. b) the average power and magnetizing reactive power absorbed by the (39+j26) Ω load impedance,

   3. c) the average power and magnetizing reactive power absorbed by the (1+j4) Ω line impedance,

4. d) the average power and magnetizing reactive power delivered by the source, and

5. e) the magnetizing reactive power delivered by the shunting capacitor.



[10.5-8 Full Alternative Text]

Answer:

1. (a) 252.20 −4.54° V (rms), 5.38 −38.23° A (rms);

2. (b) 1129.09 W, 752.73 VAR;

3. (c) 23.52 W, 94.09 VAR;

4. (d) 1152.62 W, −376.36 VAR;

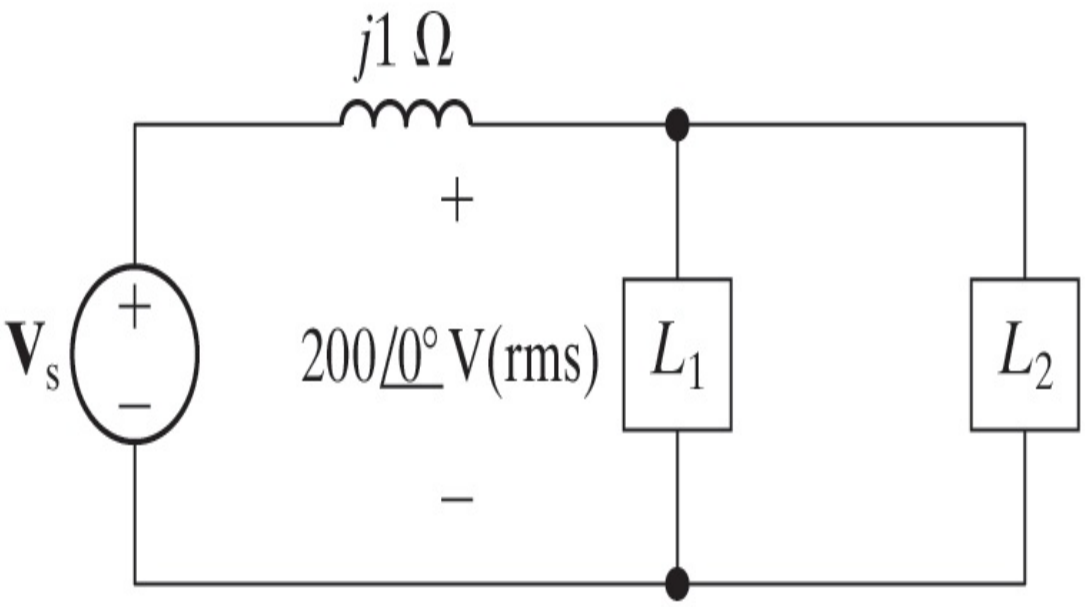
5. (e) 1223.18 VAR.

2. 10.6 The voltage at the terminals of a load is 250 V(rms). The load is absorbing an average power of 40 kW and delivering a magnetizing reactive power of 30 kVAR. Derive two equivalent impedance models of the load.

   Answer:

   1 Ω in series with 0.75 Ω of capacitive reactance; 1.5625 Ω in parallel with 2.083 Ω of capacitive reactance.

3. 10.7 Find the phasor voltage Vs in the circuit shown if loads L1 and L2 are absorbing 15 kVA at 0.6 pf lagging and 6 kVA at 0.8 pf leading, respectively. Express Vs in polar form.



   10.5-9 Full Alternative Text

   Answer:

   251.64 ∠ 15.91°_ V(rms).

SELF-CHECK: Also try Chapter Problems 10.18, 10.24, and 10.27.

10.6 Maximum Power Transfer

Recall from Chapter 4 that certain systems—for example, those that transmit information via electric signals—need to transfer a maximum amount of power from the source to the load. We now determine the condition for maximum power transfer in sinusoidal steady-state networks, beginning with Fig. 10.19. We must determine the load impedance ZL that maximizes the average power delivered to terminals a and b. Any linear network can be replaced by a Thévenin equivalent circuit, so we will use the circuit in Fig. 10.20 to find the value of ZL that results in maximum average power delivered to ZL.

Figure 10.19 A circuit describing maximum power transfer.

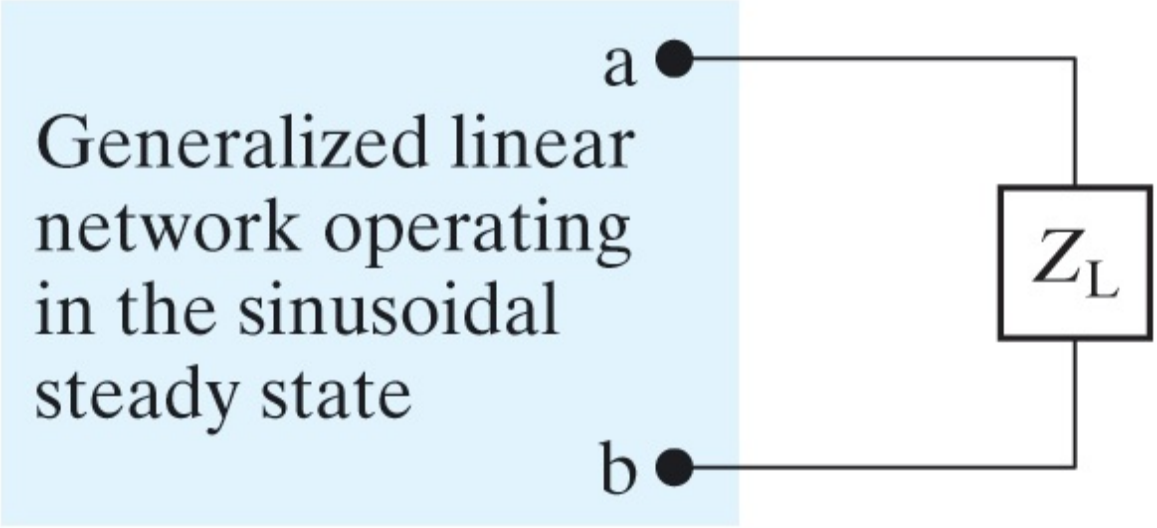


Figure 10.19 Full Alternative Text

Figure 10.20 The circuit shown in Fig. 10.19, with the network replaced by its Thévenin equivalent.

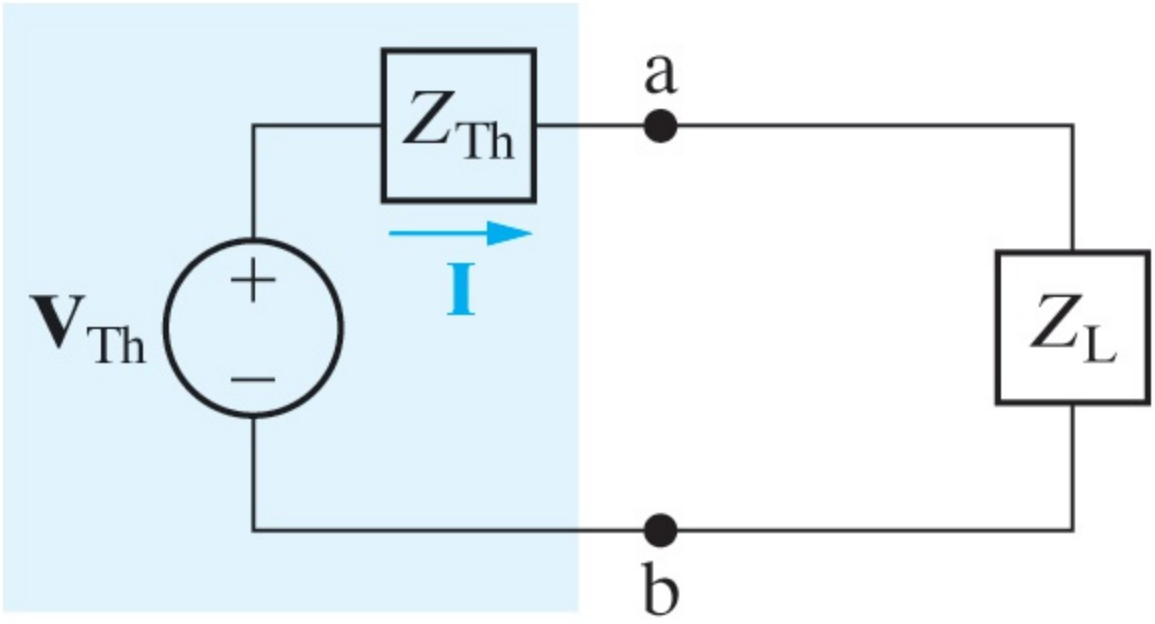


Figure 10.20 Full Alternative Text

For maximum average power transfer, ZL must equal the conjugate of the Thévenin impedance; that is,

Condition for Maximum Average Power Transfer

ZL=ZTh*.(10.21)

We derive Eq. 10.21 by a straightforward application of elementary calculus. We begin by expressing ZTh and ZL in rectangular form:

$$Z_{Th} = R_{Th} + jX_{Th}, \quad Z_L = R_L + jX_L.$$

In these impedance equations, the reactance term carries its own algebraic sign—positive for inductance and negative for capacitance. We assume that the Thévenin voltage amplitude is an rms value. We also use the Thévenin voltage as the reference phasor, that is, the phasor whose phase angle is 0°. Then, from Fig. 10.20, the rms value of the load current **I** is

$$I = \frac{V_{Th}}{(R_{Th} + R_L) + j(X_{Th} + X_L)}.$$

The average power delivered to the load is

$$P = |I|^2 R_L.$$

Therefore,

$$P = \frac{|V_{Th}|^2 R_L}{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2}. \quad (10.22)$$

In Eq. 10.22, remember that VTh, RTh, and XTh are fixed quantities, whereas RL and XL are independent variables. Therefore, to maximize P, we must find the values of RL and XL that make both $\partial P/\partial R_L$ and $\partial P/\partial X_L$ zero.

From Eq. 10.22,

$$\frac{\partial P}{\partial X_L} = -\frac{|V_{Th}|^2 2R_L(X_L + X_{Th})}{[(R_L + R_{Th})^2 + (X_L + X_{Th})^2]^2}, \quad \frac{\partial P}{\partial R_L} = \frac{|V_{Th}|^2[(R_L + R_{Th})^2 + (X_L + X_{Th})^2 - 2R_L(R_L + R_{Th})]}{[(R_L+R_{Th})^2 + (X_L+X_{Th})^2]^2}.$$

From its equation, $\partial P/\partial X_L$ is zero when

$$X_L = -X_{Th}.$$

From its equation, $\partial P/\partial R_L$ is zero when

$$R_L = R_{Th}^2 + (X_L+X_{Th})^2.$$

Note that when we combine the expressions for XL and RL, both partial derivatives are zero when ZL=ZTh*.

The Maximum Average Power Absorbed

When ZL=ZTh*, we can use the circuit in Fig. 10.20 to calculate the maximum average power that is delivered to ZL. The rms load current is VTh/2RL because ZL=ZTh*, and the maximum average power delivered to the load is

Pmax=|VTh|2RL4RL2=14 |VTh|2RL.(10.23)

If the Thévenin voltage phasor is expressed using its maximum amplitude rather than its rms amplitude, Eq. 10.23 becomes

Pmax=18 Vm2RL.(10.24)

Maximum Power Transfer When ZL is Restricted

Maximum average power can be delivered to ZL only if ZL can be set equal to the conjugate of ZTh. In some situations, this is not possible. First, RL and XL may be restricted to a limited range of values. To maximize power in this situation, set XL as close to −XTh as possible and then adjust RL as close to RTh2+(XL+XTh)2 as possible (see Example 10.10).

A second type of restriction occurs when the magnitude of ZL can be varied, but its phase angle cannot. Under this restriction, maximum power is delivered to the load when the magnitude of ZL is set equal to the magnitude of ZTh; that is, when

|ZL|=|ZTh|.

The proof of this is left to you as Problem 10.45.

For purely resistive networks, maximum power transfer occurs when the load resistance equals the Thévenin resistance. Note that we first derived this result in the introduction to maximum power transfer in Chapter 4.

Examples 10.9–10.11 calculate the load impedance ZL that produces maximum average power transfer to the load, for several different situations. Example 10.12 finds the condition for maximum power transfer to a load for a circuit with an ideal transformer.

Example 10.9 Determining Maximum Power Transfer without Load Restrictions

1. For the circuit shown in Fig. 10.21, determine the impedance ZL that results in maximum average power transferred to ZL.

Figure 10.21 The circuit for Example 10.9.

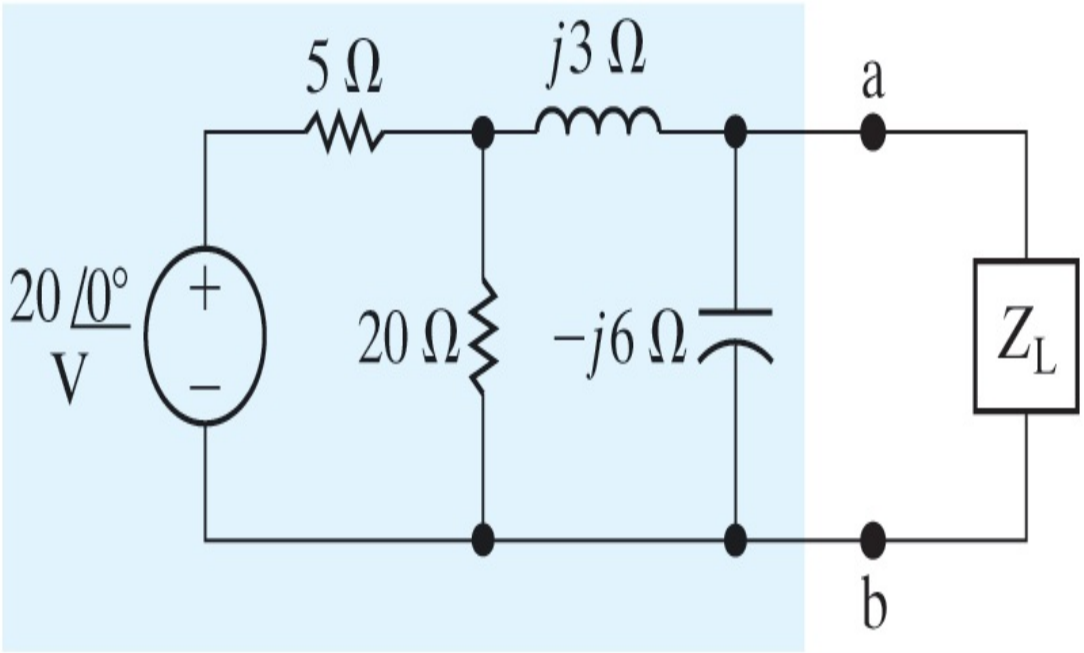


Figure 10.21 Full Alternative Text

2. What is the maximum average power transferred to the load impedance determined in (a)?

Solution

1. To begin, determine the Thévenin equivalent with respect to the load terminals a, b. After two source transformations involving the 20 V source, the 5 Ω resistor, and the 20 Ω resistor, we simplify the circuit shown in Fig. 10.21 to the one shown in Fig. 10.22. Use voltage division in the simplified circuit to get

Figure 10.22 A simplification of Fig. 10.21 by source transformations.

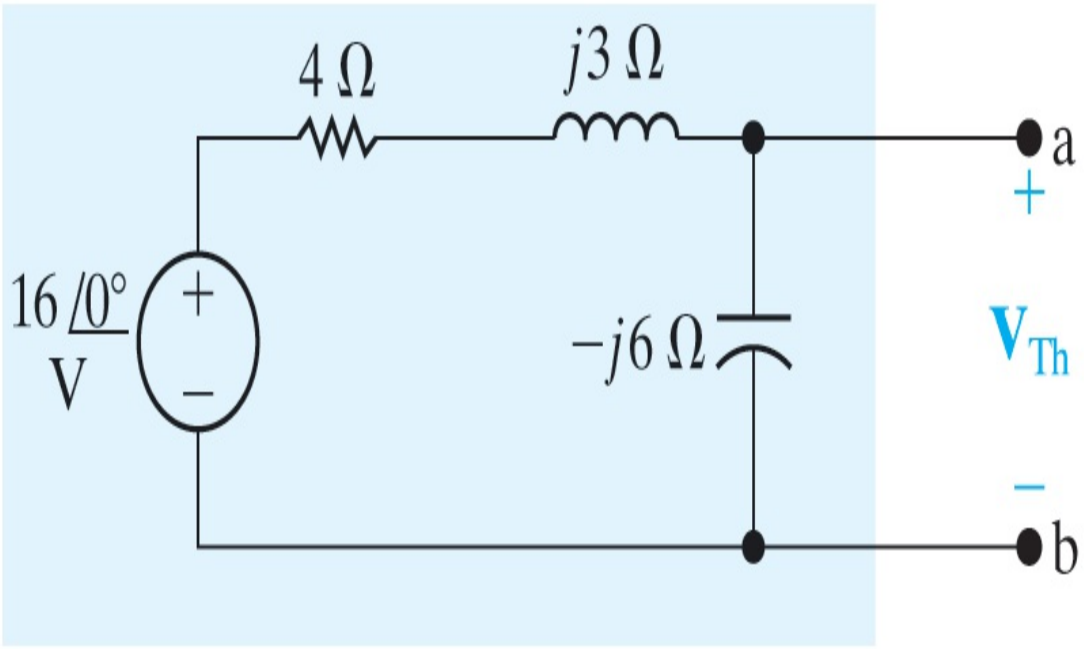


Figure 10.22 Full Alternative Text

$V_{Th} = -j6\ 4+j3-j6(160°) = 19.2\ \angle -53.13° = 11.52-j15.36$ V.

To find the Thévenin impedance, deactivate the source in Fig. 10.22 and calculate the impedance seen looking into the terminals a and b. Thus,

$Z_{Th} = -j6 \| (4+j3) = \frac{(-j6)(4+j3)}{4+j3-j6} = 5.76-j1.68\ \Omega.$

For maximum average power transfer, the load impedance must be the conjugate of ZTh, so

$Z_L = 5.76+j1.68\ \Omega.$

2. We calculate the maximum average power delivered to ZL using the circuit in Fig. 10.23, which has the Thévenin equivalent of the original network attached to the load impedance calculated in part (a). From Fig. 10.23, the rms magnitude of the load current **I** is

Figure 10.23 The circuit shown in Fig. 10.21, with the

original network replaced by its Thévenin equivalent.

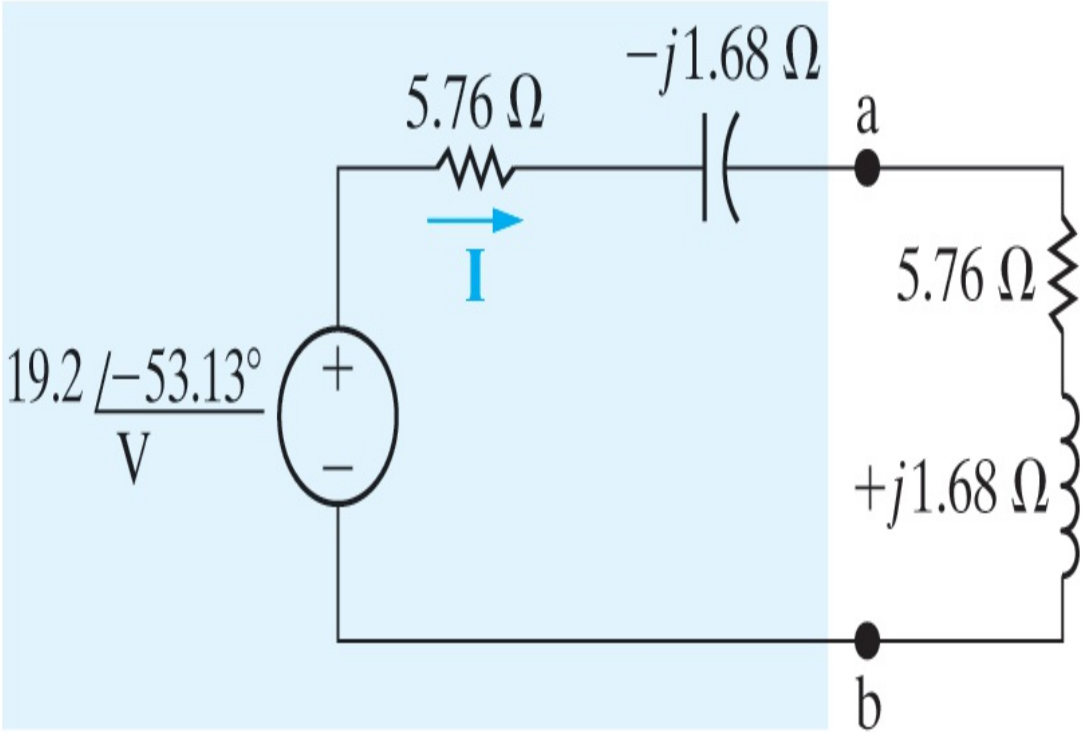


Figure 10.23 Full Alternative Text
Irms=19.2/22(5.76)=1.1785 A(rms).

The average power delivered to the load is

P=Irms2(5.76)=8 W.

Example 10.10 Determining Maximum Power Transfer with Load Impedance Restriction

1. For the circuit shown in Fig. 10.24, what value of ZL results in maximum average power transfer to ZL? What is the maximum power in milliwatts?

Figure 10.24 The circuit for Examples 10.10 and 10.11.

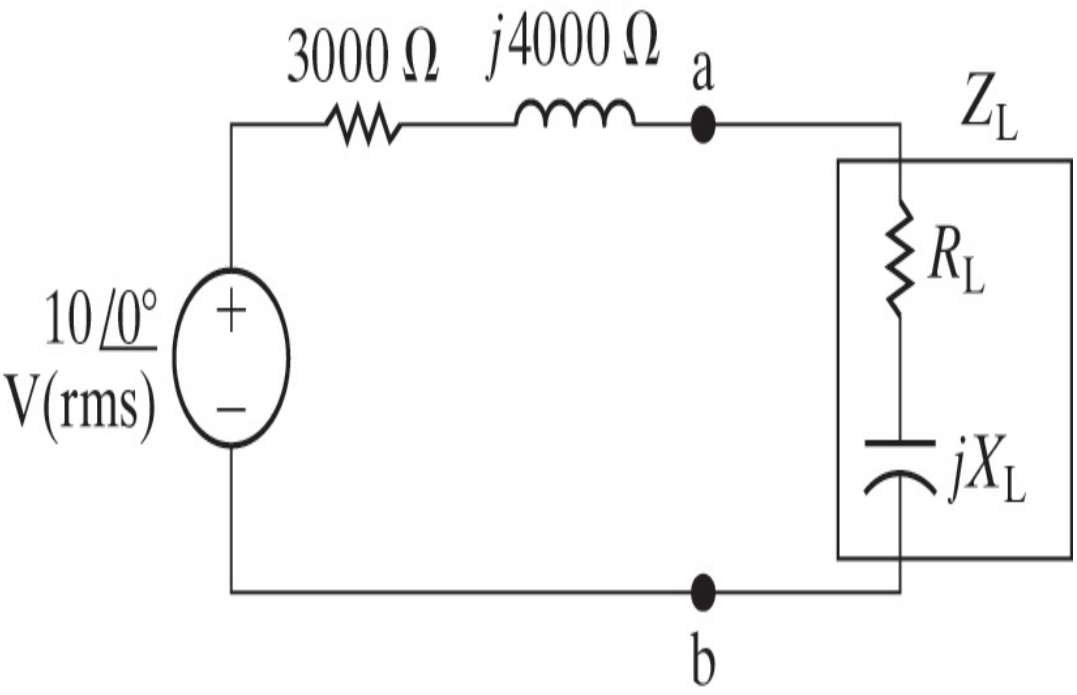


Figure 10.24 Full Alternative Text

2. Assume that the load resistance can be varied between 0 and 4000 Ω and that the capacitive reactance of the load can be varied between 0 and −2000 Ω. What settings of RL and XL transfer the most average power to the load? What is the maximum average power that can be transferred under these restrictions?

Solution

1. If there are no restrictions on RL and XL, maximum average power is delivered to the load if the load impedance equals the conjugate of the Thévenin impedance. Therefore we set

   $RL=3000\ \Omega$ and $XL=-4000\ \Omega$,

   or

   $ZL=3000-j4000\ \Omega$.

   Because the source voltage is an rms value, the average power delivered to ZL is

   $P=14\ 1023000=253\ \text{mW}=8.33\ \text{mW}$.

2. Now RL and XL are restricted, so first we set XL as close to $-4000\ \Omega$ as possible; thus, $XL=-2000\ \Omega$. Next, we set RL as close to $RTh2+(XL+XTh)2$ as possible. Thus

   $RL=30002+(-2000+4000)2=3605.55\ \Omega$.

   Since RL can be varied from 0 to $4000\ \Omega$, we can set RL to $3605.55\ \Omega$. Therefore, the load impedance value is

   $ZL=3605.55-j2000\ \Omega$.

   For this value of ZL, the value of the load current is

   $Irms=10\ \ 0°6605.55+j2000=1.4489\ \ -16.85°\ \text{mA(rms)}$.

   The average power delivered to the load is

   $P=(1.4489\times10-3)2(3605.55)=7.57\ \text{mW}$.

   This is the maximum power delivered to a load with the specified restrictions on RL and XL. Note that this is less than the 8.33 mW that can be delivered if there are no restrictions, as we found in part (a).

Example 10.11 Finding Maximum Power Transfer with Impedance Angle Restrictions

A load impedance having a constant phase angle of −36.87° is connected across the terminals a and b in the circuit shown in Fig. 10.24. The magnitude of ZL is varied until the average power delivered is maximized under the given restriction.

1. Specify ZL in rectangular form.
2. Calculate the average power delivered to ZL.

Solution

1. When only the magnitude of ZL can be varied, maximum power is delivered to the load when the magnitude of ZL equals the magnitude of ZTh. So,

   $|ZL|=|ZTh|=|3000+j4000|=5000\ \Omega.$

   Therefore,

   $ZL=5000\ \angle -36.87°=4000-j3000\ \Omega.$

2. When ZL equals 4000−j3000 Ω, the load current is

   $Irms=107000+j1000=1.4142\ \angle -8.13°\ mA(rms),$

   and the average power delivered to the load is

   $P=(1.4142\times10-3)2(4000)=8\ mW.$

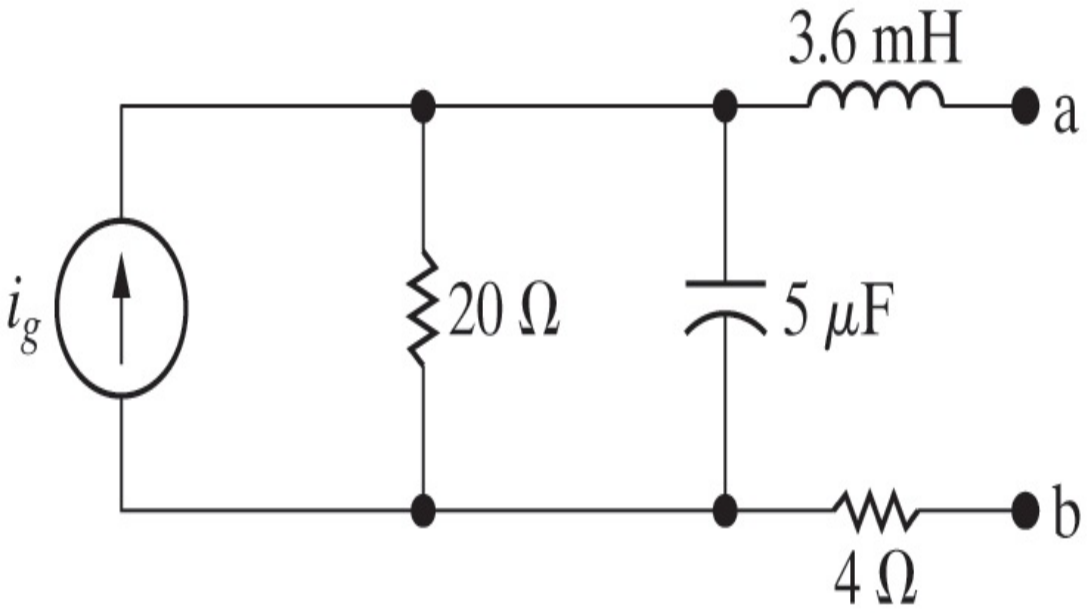
   This quantity is the maximum power that can be delivered by this circuit

to a load impedance whose angle is constant at −36.87°. Again, this quantity is less than the maximum power that can be delivered if there are no restrictions on ZL.

Assessment Problem

Objective 2—Understand the condition for maximum real power delivered to a load in an ac circuit

1. 10.8 The source current in the circuit shown is 3 cos 5000t A.
    1. a) What impedance should be connected across terminals a and b for maximum average power transfer?
    2. b) What is the average power transferred to the impedance in (a)?
    3. c) Assume that the load is restricted to pure resistance. What size resistor connected across a and b will result in the maximum average power transferred?
    4. d) What is the average power transferred to the resistor in (c)?



10.6-10 Full Alternative Text

Answer:

1. (a) 20−j10 Ω;
2. (b) 18 W;
3. (c) 22.36 Ω;
4. (d) 17 W.

SELF-CHECK: Also try Chapter Problems 10.43, 10.49, and 10.50.

Example 10.12 Finding Maximum Power Transfer in a Circuit with an Ideal Transformer

The variable resistor in the circuit in Fig. 10.25 is adjusted until maximum average power is delivered to RL.

Figure 10.25 The circuit for Example 10.12.

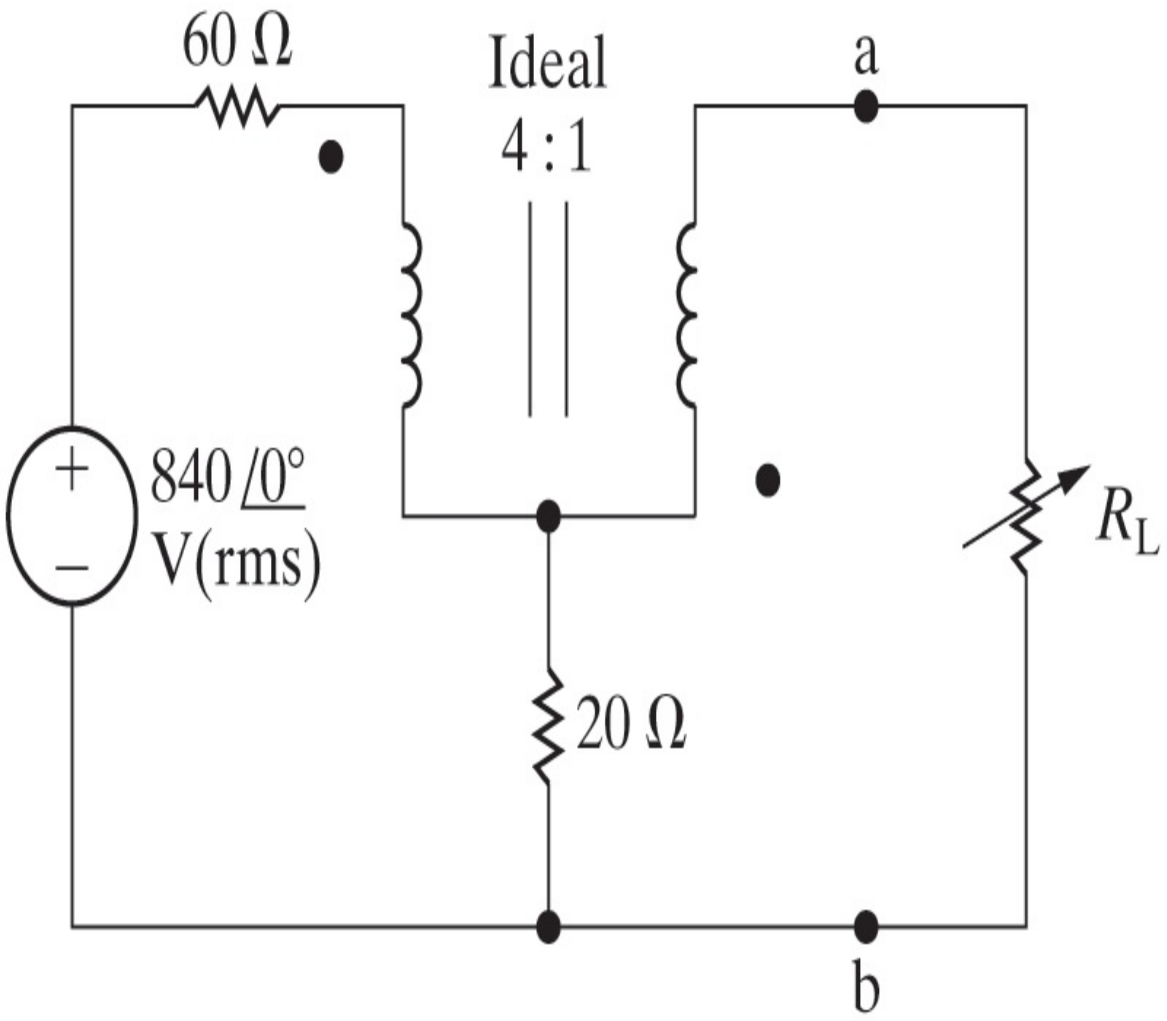


Figure 10.25 Full Alternative Text

1. What is the value of RL in ohms?
2. What is the maximum average power (in watts) delivered to RL?

Solution

1. We first find the Thévenin equivalent with respect to the terminals of RL. We determine the open-circuit voltage using the circuit in Fig. 10.26. The variables V1, V2, I1, and I2 have been added to aid the discussion.

Figure 10.26 The circuit used to find the Thévenin voltage.

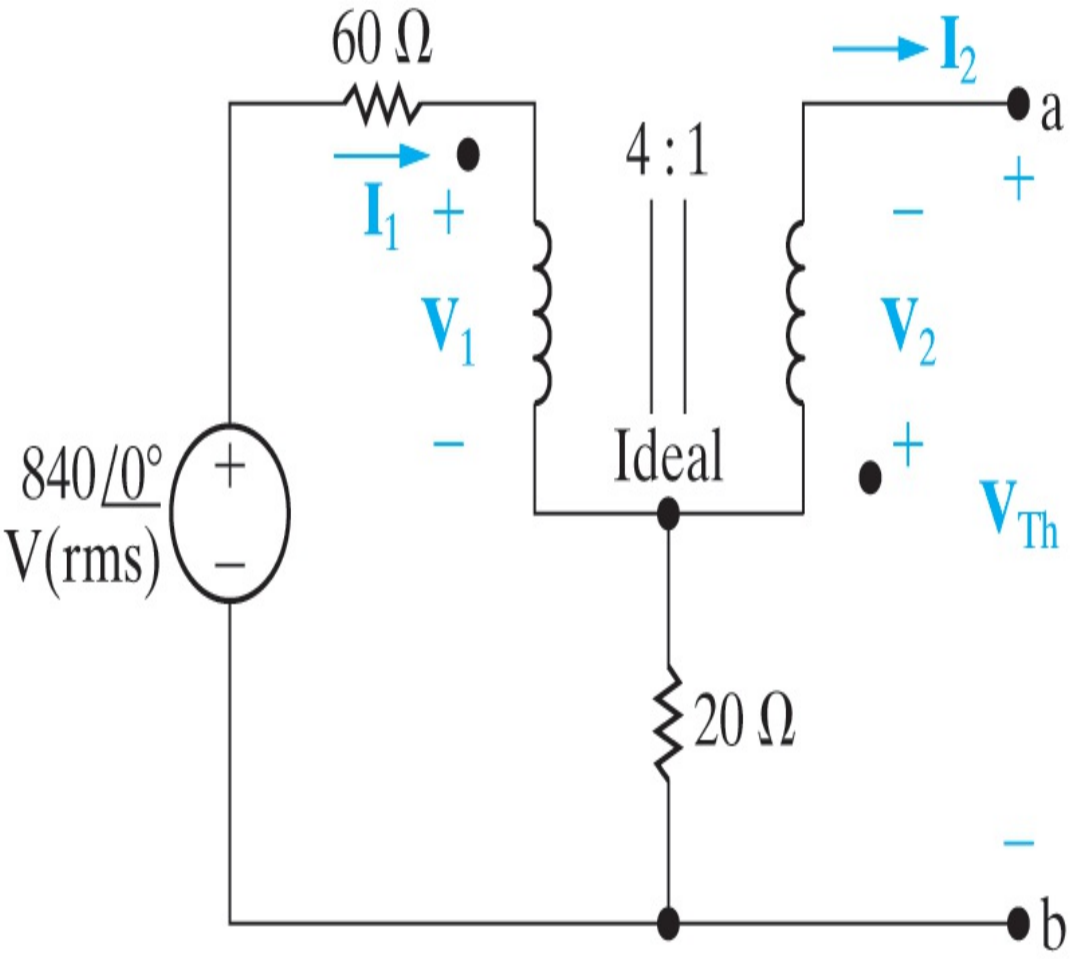


Figure 10.26 Full Alternative Text

The ideal transformer imposes the following constraints on the variables V1, V2, I1, and I2:

V2=14V1, I1=−14I2.

The open-circuit value of I2 is zero; hence, I1 is zero. It follows that

V1=840 0° V(rms), V2=210 0° V(rms).

From Fig. 10.26 we note that VTh is the negative of V2, because there is no current in the 20 Ω resistor. Hence

$V_{Th} = -210\ 0°$ V(rms).

We determine the short-circuit current using the circuit in Fig. 10.27. Since I1 and I2 are mesh currents, write a KVL equation for each mesh:

Figure 10.27 The circuit used to calculate the short circuit current.

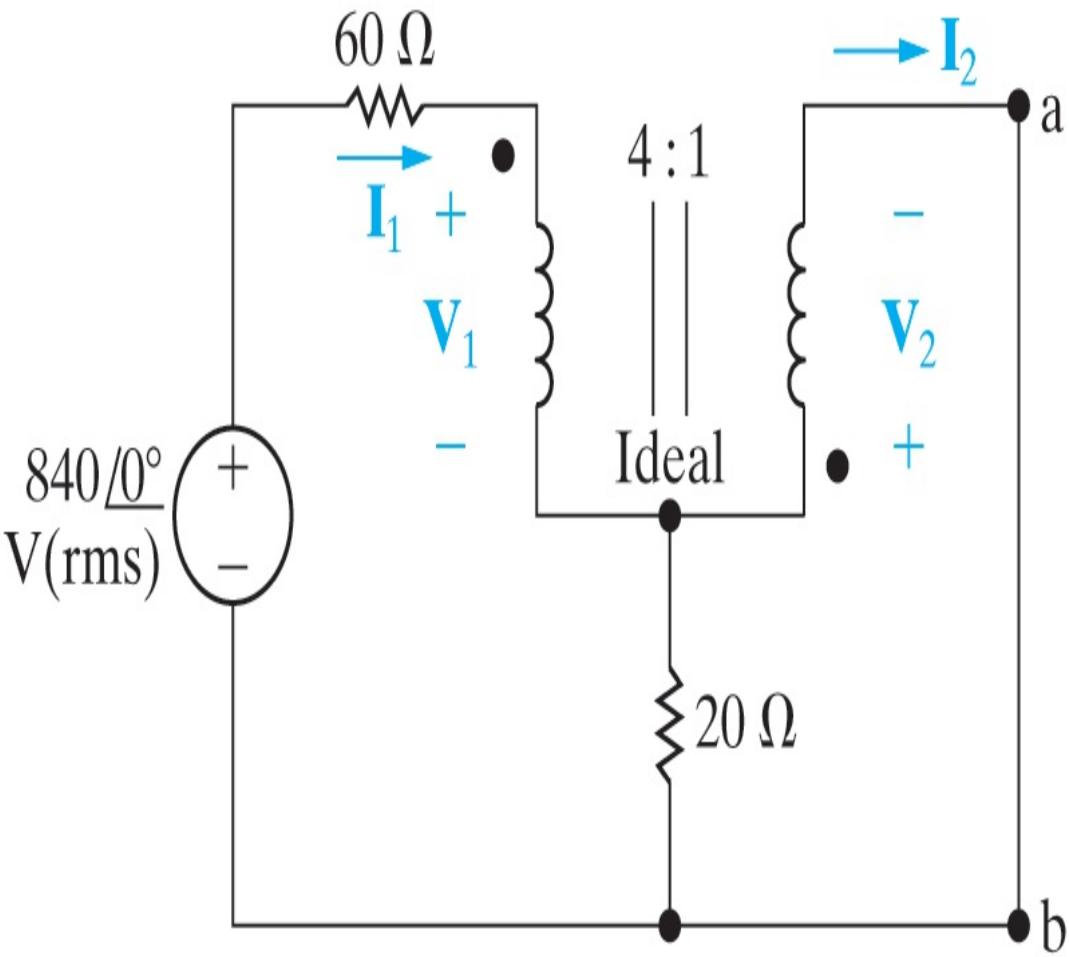


Figure 10.27 Full Alternative Text
$840\ 0° = 80I1 - 20I2 + V1, 0 = 20I2 - 20I1 + V2.$

Combine these two KVL equations with the constraint equations to get

$840\ 0° = -40I2 + V1, 0 = 25I2 + V14.$

Solving for the short circuit value of I2 yields

$I2 = -6$ A.

Therefore, the Thévenin resistance is

$RTh = -210 - 6 = 35\ \Omega.$

Maximum power will be delivered to RL when RL equals 35 Ω.

2. We determine the maximum power delivered to RL using the Thévenin equivalent circuit in Fig. 10.28. From this circuit, the rms current in the load resistor is (−210/70) A(rms). Therefore,

Figure 10.28 The Thévenin equivalent loaded for maximum power transfer.

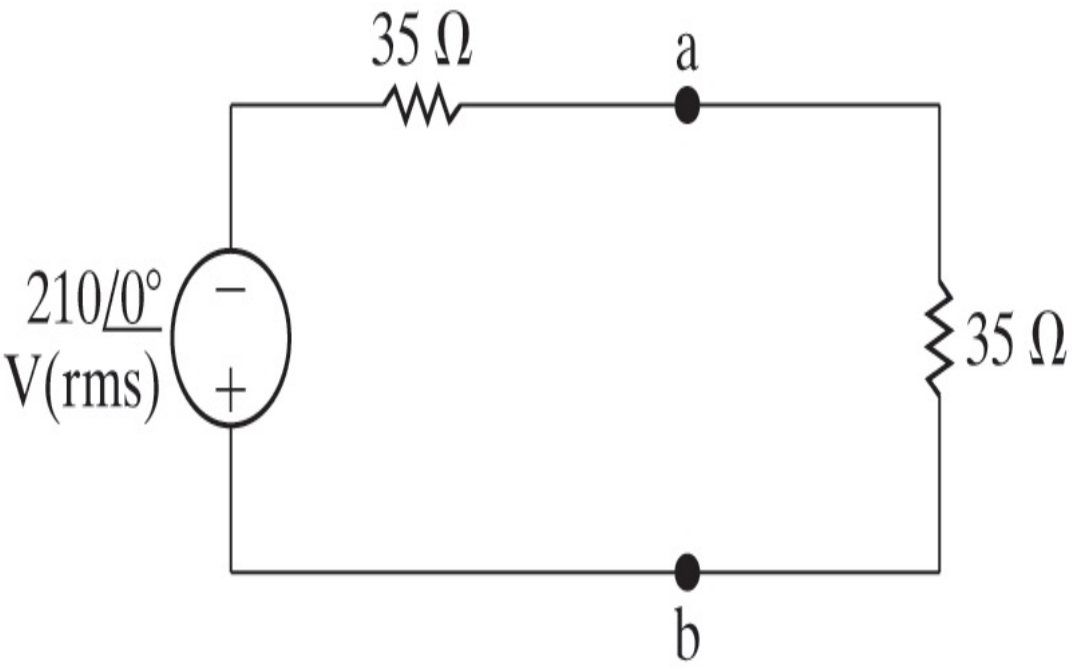
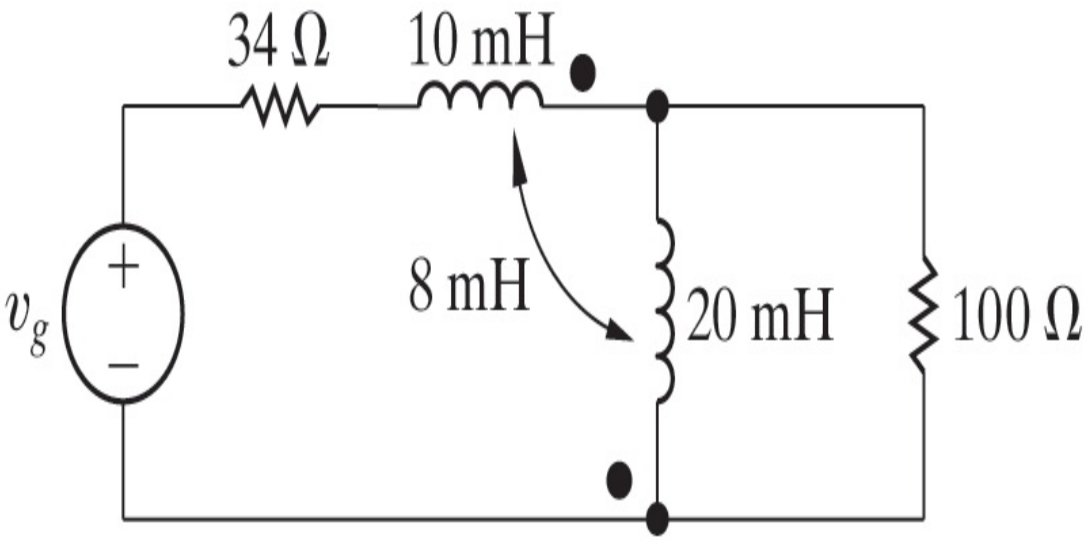


Figure 10.28 Full Alternative Text
Pmax=(−21070)2(35)=315 W.

Assessment Problems

Objective 3—Be able to calculate all forms of ac power in ac circuits with linear transformers and ideal transformers

1. 10.9 Find the average power delivered to the 100 Ω resistor in the circuit shown if vg=660 cos 5000t V.



10.6-11 Full Alternative Text

Answer:

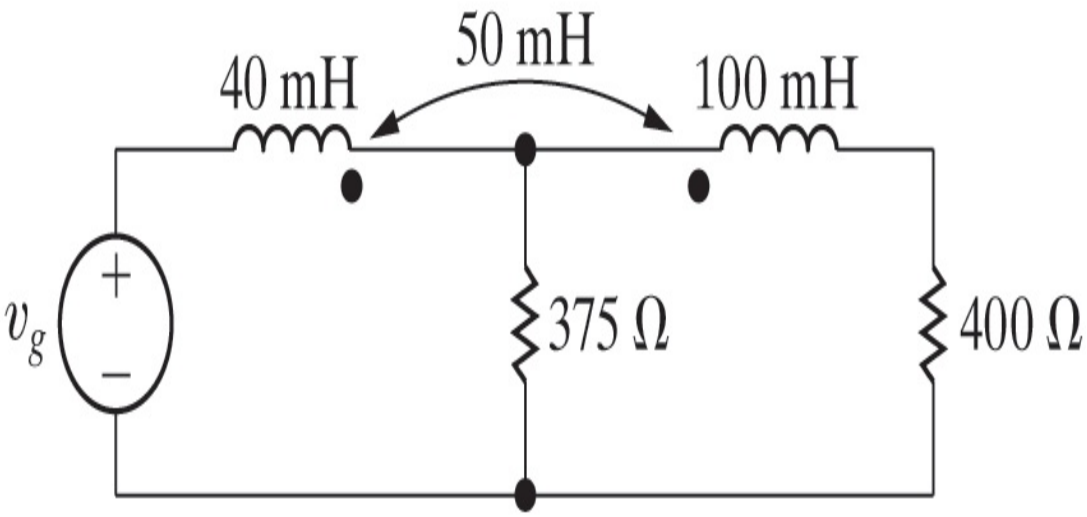
612.5 W.

2. 10.10

    1. a) Find the average power delivered to the 400 Ω resistor in the circuit shown if vg=248 cos 10,000t V.

    2. b) Find the average power delivered to the 375 Ω resistor.

    3. c) Find the power developed by the ideal voltage source. Check your result by showing that the power absorbed equals the power developed.



10.6-12 Full Alternative Text

Answer:

1. (a) 50 W;
2. (b) 49.2 W;
3. (c) 99.2 W, 50+49.2=99.2 W.

3. 10.11 Solve Example 10.12 if the polarity dot on the coil connected to terminal a is at the top.

   Answer:

   1. (a) 15 Ω;
   2. (b) 735 W.

4. 10.12 Solve Example 10.12 if the voltage source is reduced to 146 0° V rms and the turns ratio is reversed to 1:4.

   Answer:

   1. (a) 1460 Ω;

2. (b) 58.4 W.

SELF-CHECK: Also try Chapter Problems 10.56, 10.59, and 10.60.

Practical Perspective

Vampire Power

Vampire power, or standby power, may cost you more than you think. The average household has about 40 electrical products that draw power, even when they are turned off. Approximately 5% of typical residential power consumption can be attributed to standby power. Table 10.3 provides the power consumption for several different devices. Notice that when a device is "off" it is often still consuming power.

Table 10.3 Average power consumption of common electrical devices

Electrical device[+]	Power [W]*
Mobile phone charger	
Attached to phone, phone charging	3.68
Plugged into wall outlet but not into phone	0.26
Notebook computer AC adapter	
Attached to computer, computer charging	44.28
Attached to computer, computer sleeping	15.77
Attached to computer, computer off	8.9
Plugged into wall outlet but not into computer	4.42
DVD player	
On and playing	9.91
On and not playing	7.54
Off	1.55
Microwave oven	
Ready with door closed	3.08
Ready with door open	25.79
Cooking	1433.0
Inkjet multifunction printer	
On	9.16
Off	5.26

Table 10.3 Full Alternative Text

*Data in this table from Lawrence Berkeley National Laboratory report (http://standby.lbl.gov/standby.html).

[+]This value is the average of the power measured for many types of each device.

Consider a typical mobile phone charger. According to the values given in Table 10.3, when the charger is detached from the phone it consumes only a fraction of the power required when the charger is attached to the phone and the phone is charging. Suppose you charge your phone for three hours each day but leave the charger plugged into the wall outlet 24 hours a day. Recall that the electric company charges you based on the number of kilowatt-hours (kWh) you use in a given month. A device that uses 1000 W of power continuously over one hour has consumed 1 kWh. Let's calculate the number of kilowatt-hours used by the phone charger in one month:

$$P[\text{kWh}]=30[3(3.68)+21(0.26)]1000=1.8\ \text{kWh}$$

Now do the calculation again, this time assuming that you unplug the charger when it is not being used to charge the phone:

$$P[\text{kWh}]=30[3(3.68)+21(0)]1000=0.33\ \text{kWh}$$

Keeping the charger plugged in when you are not using it consumes more than 5 times the power needed to charge your phone every day. You can therefore minimize the cost of vampire power by unplugging electrical devices if they are not being used.

Why does the phone charger consume power when not plugged into the phone? The electronic circuitry in your phone uses 5 V(dc) sources to supply power. The phone charger must transform the 120 V(rms) signal supplied by the wall outlet into a signal that can be used to charge the phone. Phone chargers can use linear transformers, together with other circuitry, to output the voltage needed by the phone.

Consider the circuit in Fig. 10.29. The linear transformer is part of the circuitry used to reduce the voltage supplied by the source to the level required by the phone. The additional components needed to complete this task are not shown in the circuit. When the phone is unplugged from the circuit in Fig. 10.29, but the circuit is still connected to the 120 V(rms) source, there is still a path for the current, as shown in Fig. 10.30. The current

is

Figure 10.29 A linear transformer used in a phone charger.

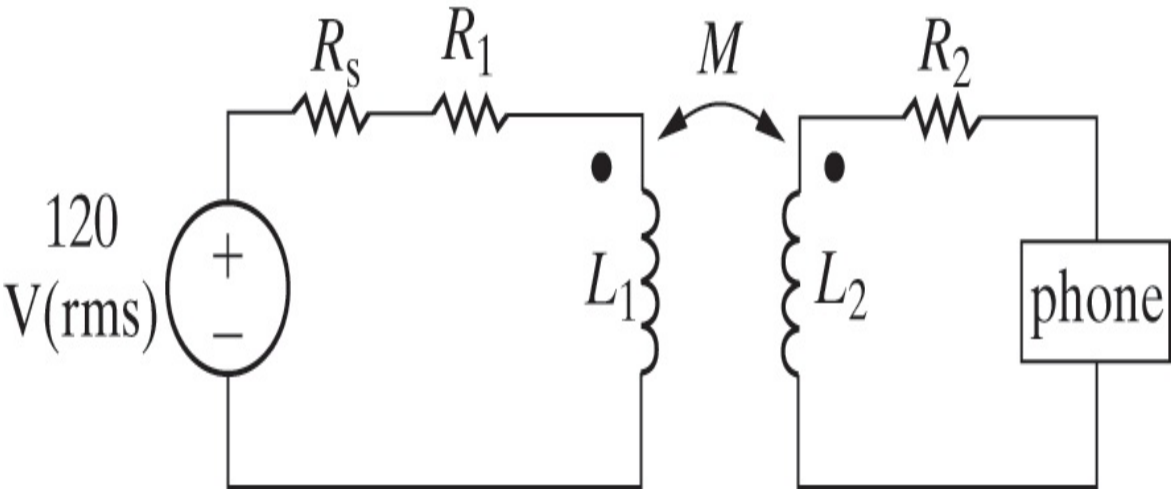


Figure 10.29 Full Alternative Text

Figure 10.30 The phone charger circuit when the phone is not connected.

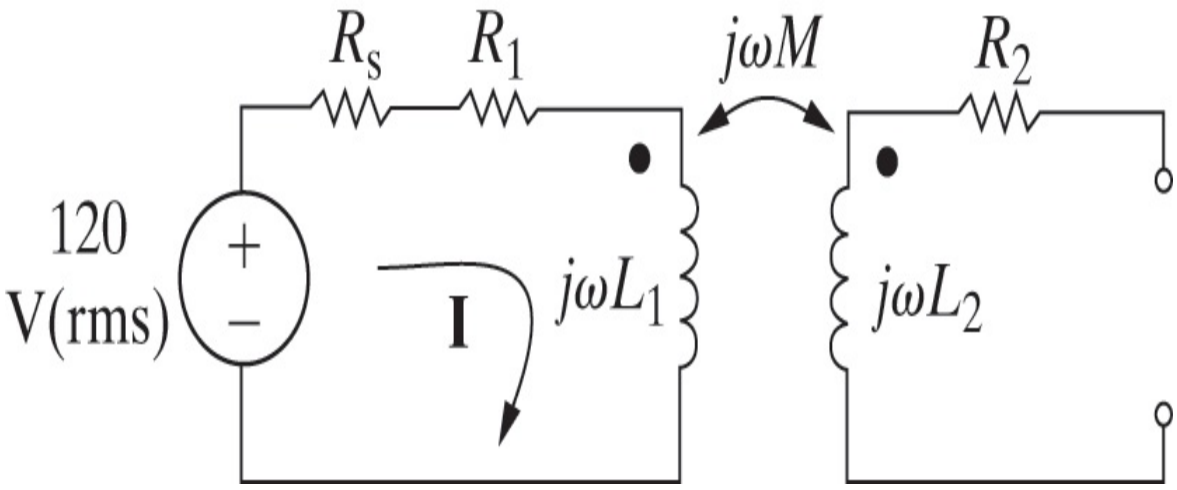


Figure 10.30 Full Alternative Text
I=120Rs+R1+jωL1.

The real power, delivered by the voltage source and supplied to the resistors, is

P=(Rs+R1)|I|2.

This is the vampire power being consumed by the phone charger even when it is not connected to the phone.

SELF-CHECK: Assess you understanding of this Practical Perspective by trying Chapter Problems 10.67–10.69.

Summary

- **Instantaneous power** is the product of the instantaneous terminal voltage and current, or $p=\pm vi$. The positive sign is used when the reference direction for the current is from the positive to the negative reference polarity of the voltage. The frequency of the instantaneous power is twice the frequency of the voltage (or current). (See page 376.)

- **Average power** is the average value of the instantaneous power over one period. It is the power converted from electric to nonelectric form and vice versa, so it is also called real power. Average power is given by

  $$P=12VmIm\cos(\theta v-\theta i)$$
  $$=VrmsIrms\cos(\theta v-\theta i).$$

  (See page 377.)

- **Reactive power** is the electric power exchanged between the magnetic field of an inductor and the source that drives it or between the electric field of a capacitor and the source that drives it. Reactive power is never converted to nonelectric power. Reactive power is given by

  $$Q=12VmIm\sin(\theta v-\theta i)$$
  $$=VrmsIrms\sin(\theta v-\theta i).$$

  Both average power and reactive power can be expressed in terms of either peak (Vm, Im) or rms (Vrms,Irms) current and voltage. RMS values, also called *effective values*, are widely used in both household and industrial applications. (See page 378.)

- The **power factor** is the cosine of the phase angle between the voltage and the current:

  $$pf=\cos(\theta v-\theta i).$$

  The terms *lagging* and *leading*, added to the description of the power

factor, indicate whether the current is lagging or leading the voltage and thus whether the load is inductive or capacitive. (See page 379.)

- The **reactive factor** is the sine of the phase angle between the voltage and the current:

  rf=sin(θv−θi).

  (See page 379.)

- **Complex power** is the complex sum of the real and reactive powers, or

  S = P+j Q = 1 2 V I * = V rms I rms * 2 = | I rms | 2 Z= | V rms | 2 Z * .

  (See page 384.)

- **Apparent power** is the magnitude of the complex power:

  |S|=P2+Q2.

  (See page 385.)

- The **watt** is used as the unit for both instantaneous and real power. The **var** (volt amp reactive, or VAR) is used as the unit for reactive power. The **volt-amp** (VA) is used as the unit for complex and apparent power. (See page 385.)

- **Maximum power transfer** occurs in circuits operating in the sinusoidal steady state when the load impedance is the conjugate of the Thévenin impedance as viewed from the terminals of the load impedance. (See page 393.)

Problems

Sections 10.1–10.2

1. 10.1 The following sets of values for v and i pertain to the circuit seen in Fig. 10.1. For each set of values, calculate P and Q and state whether the circuit inside the box is absorbing or delivering (1) average power and (2) magnetizing vars.

    1. a) $v=100\cos(\omega t+50°)$ V,$i=10\cos(\omega t+15°)$ A.

    2. b) $v=40\cos(\omega t-15°)$ V,$i=5\cos(\omega t+60°)$ A.

    3. c) $v=400\cos(\omega t+30°)$ V,$i=10\sin(\omega t+240°)$ A.

    4. d) $v=200\sin(\omega t+250°)$ V,$i=5\cos(\omega t+40°)$ A.

2. 10.2

    1. a) A college student wakes up on a warm day. The central air conditioning is on, and the room feels comfortable. She turns on the dishwasher, takes some milk out of the old refrigerator, and puts some oatmeal in the microwave oven to cook. If all of these appliances in her dorm room are supplied by a 120 V(rms) branch circuit protected by a 50 A(rms) circuit breaker, will the breaker interrupt her morning?

    2. b) Her roommate wakes up and moves wet clothes from the washer to the dryer. Now does the circuit breaker interrupt their morning?

3. 10.3 Show that the maximum value of the instantaneous power given by Eq. 10.3 is P+P2+Q2 and that the minimum value is P−P2+Q2.

4. 10.4 A load consisting of a 480 Ω resistor in parallel with a (5/9) μF capacitor is connected across the terminals of a sinusoidal voltage

source vg, where vg=240cos 5000t V.

1. a) What is the peak value of the instantaneous power delivered by the source?
2. b) What is the peak value of the instantaneous power absorbed by the source?
3. c) What is the average power delivered to the load?
4. d) What is the reactive power delivered to the load?
5. e) Does the load absorb or generate magnetizing vars?
6. f) What is the power factor of the load?
7. g) What is the reactive factor of the load?

5. 10.5
    1. a) Calculate the real and reactive power associated with each circuit element in the circuit in Fig. P9.60.
    2. b) Verify that the average power generated equals the average power absorbed.
    3. c) Verify that the magnetizing vars generated equal the magnetizing vars absorbed.
6. 10.6 Repeat Problem 10.5 for the circuit shown in Fig. P9.64.
7. 10.7 PSPICEMULTISIM Find the average power delivered by the ideal current source in the circuit in Fig. P10.7 if ig=40cos 1250t mA.

Figure P10.7

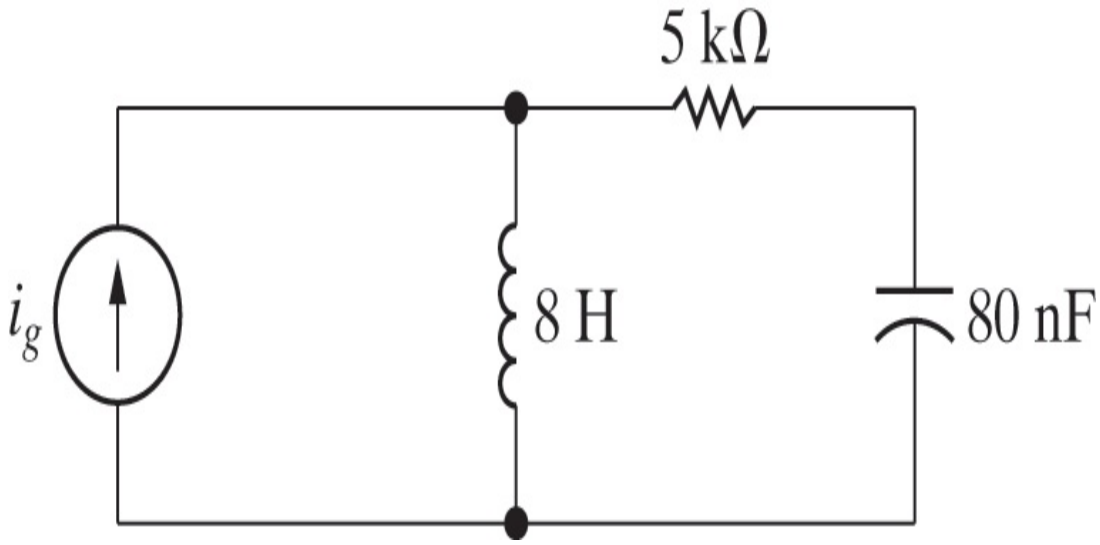


Figure P10.7 Full Alternative Text

8. 10.8 PSPICEMULTISIM The op amp in the circuit shown in Fig. P10.8 is ideal. Calculate the average power delivered to the 1 kΩ resistor when vg=cos 1000t V.

Figure P10.8

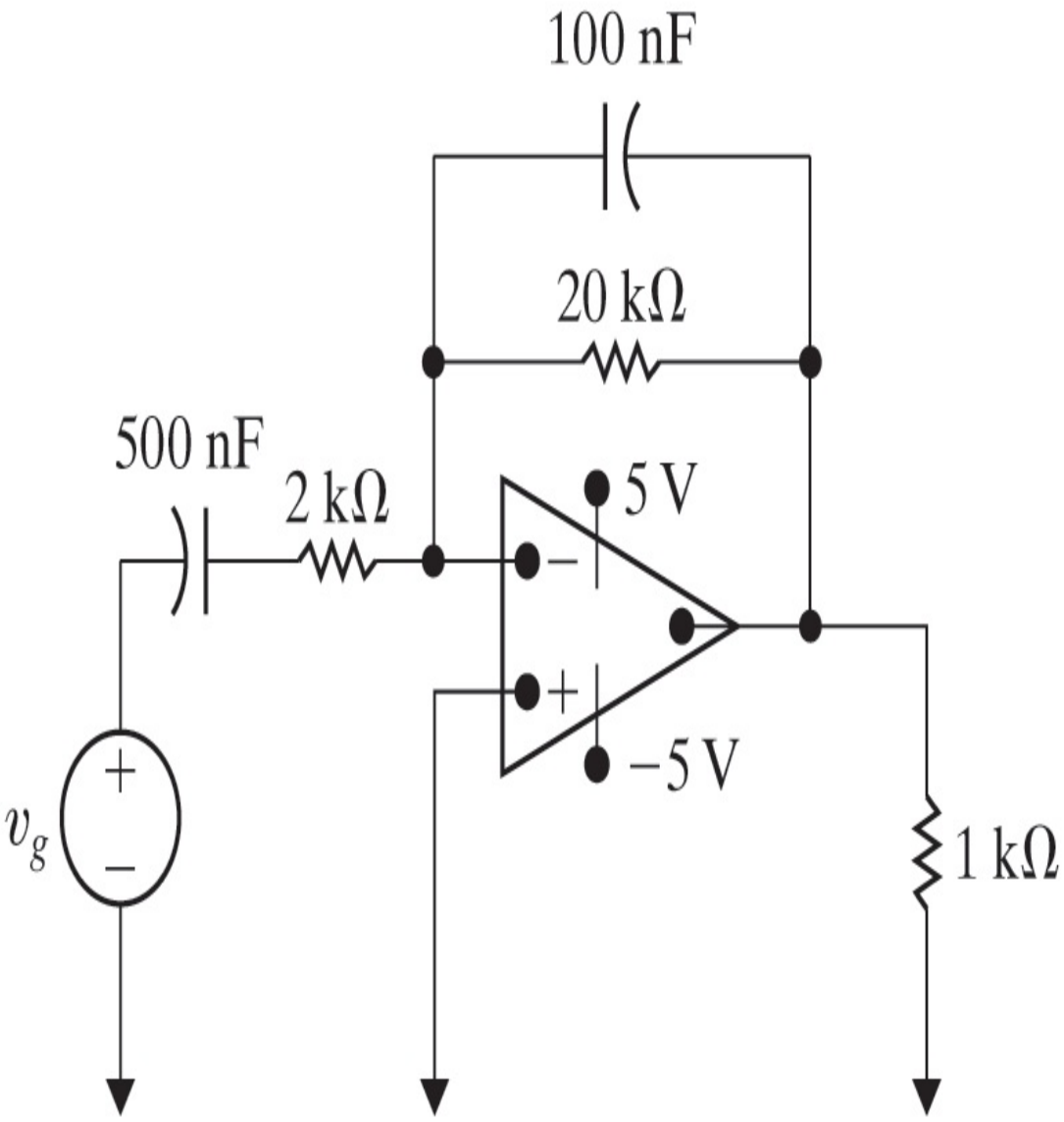


Figure P10.8 Full Alternative Text

9. 10.9 PSPICEMULTISIM Find the average power dissipated in the 40 Ω resistor in the circuit seen in Fig. P10.9 if ig=4cos 105t A.

Figure P10.9

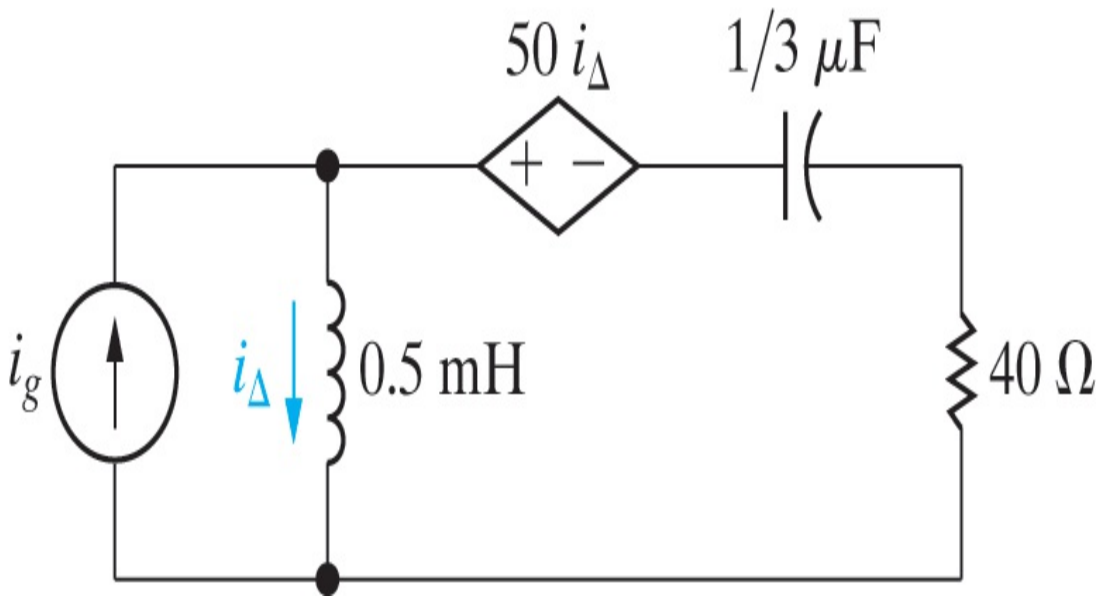


Figure P10.9 Full Alternative Text

10. 10.10 The load impedance in Fig. P10.10 absorbs 2.5 kW and generates 5 kVAR. The sinusoidal voltage source develops 7.5 kW.

    1. a) Find the values of inductive line reactance that will satisfy these constraints.

    2. b) For each value of line reactance found in (a), show that the magnetizing vars developed equals the magnetizing vars absorbed.

Figure P10.10

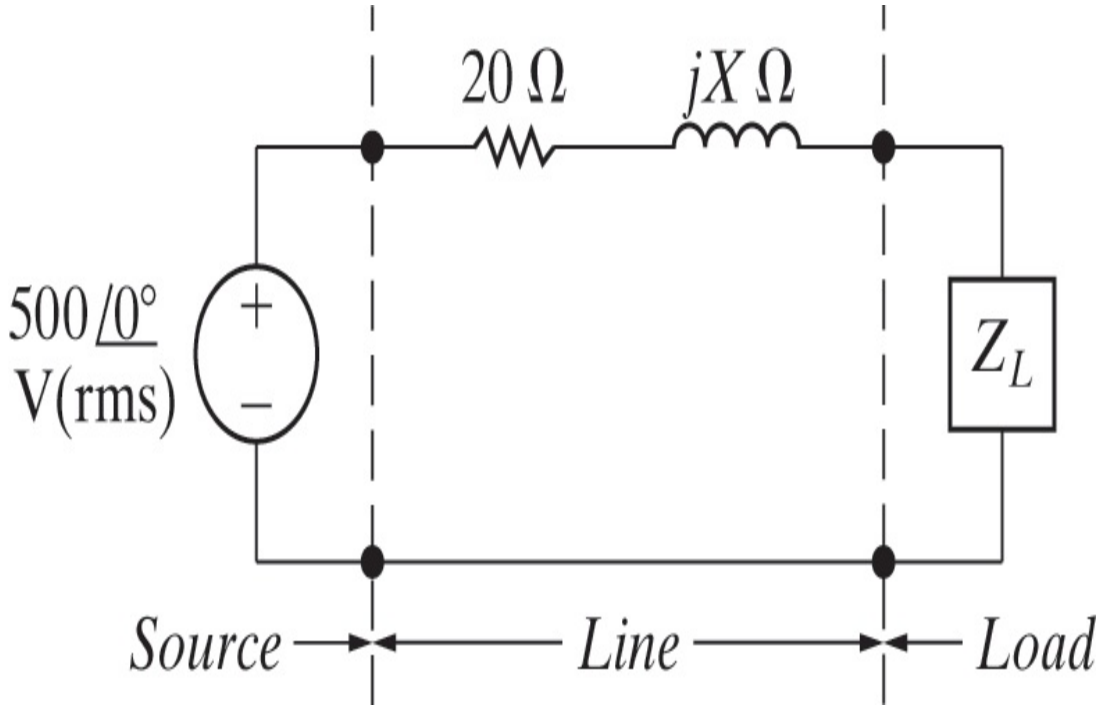


Figure P10.10 Full Alternative Text

Section 10.3

1. 10.11 Find the rms value of the periodic current shown in Fig. P10.11.

Figure P10.11

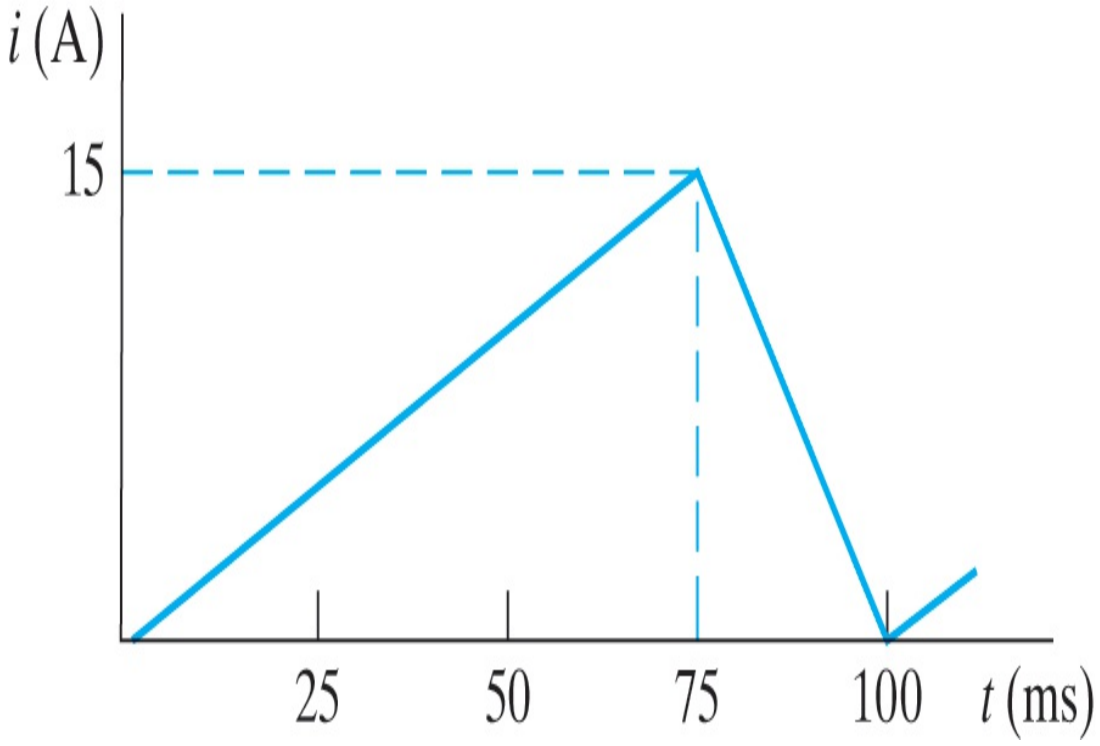


Figure P10.11 Full Alternative Text

2. 10.12 The periodic current shown in Fig. P10.11 dissipates an average power of 3 kW in a resistor. What is the value of the resistor?

3. 10.13

    1. a) Find the rms value of the periodic voltage shown in Fig. P10.13.

    2. b) Suppose the voltage in part (a) is applied to the terminals of a 40 Ω resistor. Calculate the average power dissipated by the resistor.

3. c) When the voltage in part (a) is applied to a different resistor, that resistor dissipates 10 mW of average power. What is the value of the resistor?

Figure P10.13

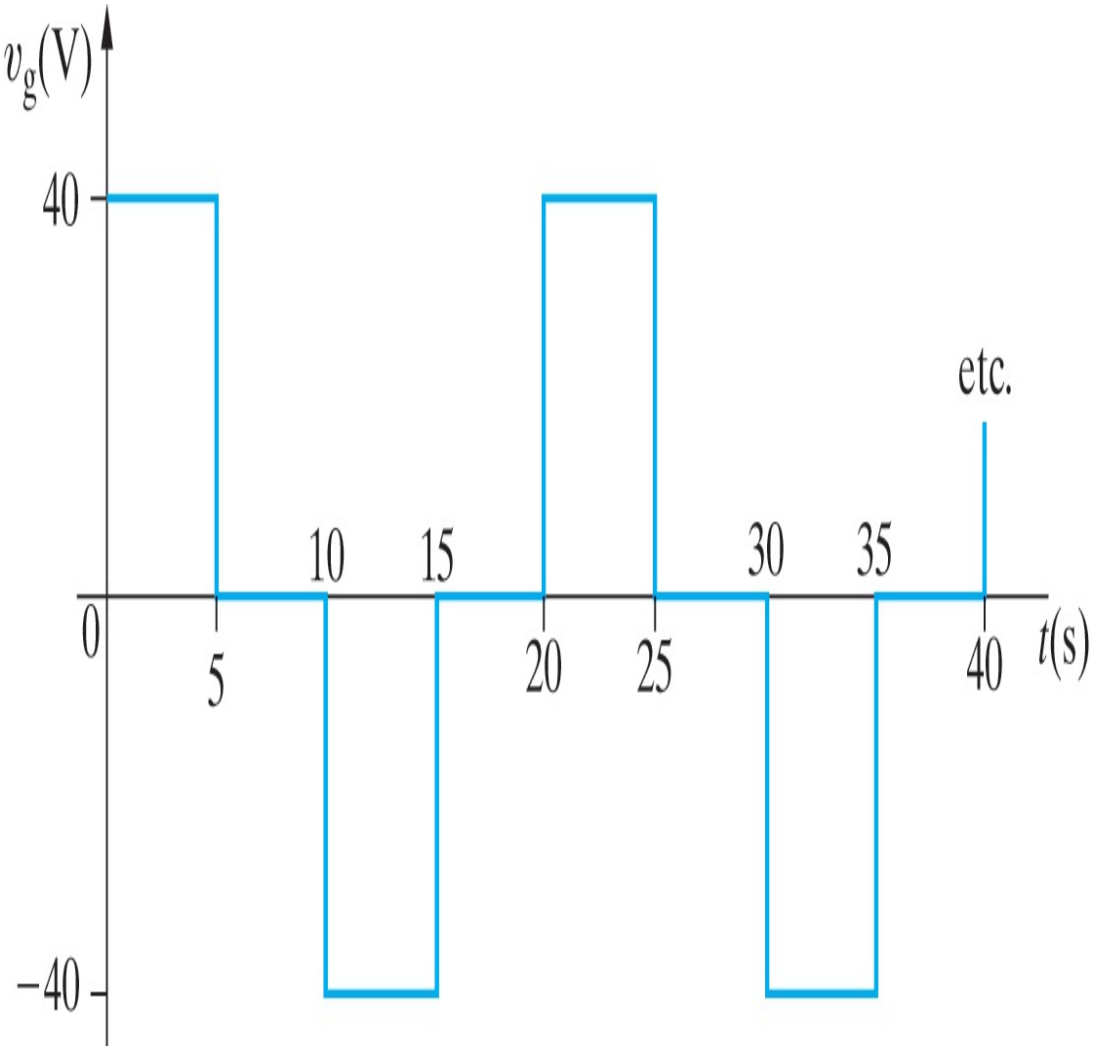


Figure P10.13 Full Alternative Text

4. 10.14

    1. a) Find the rms value of the periodic voltage shown in Fig. P10.14.

    2. b) If this voltage is applied to the terminals of a 2.25 Ω resistor,

what is the average power dissipated in the resistor?

Figure P10.14

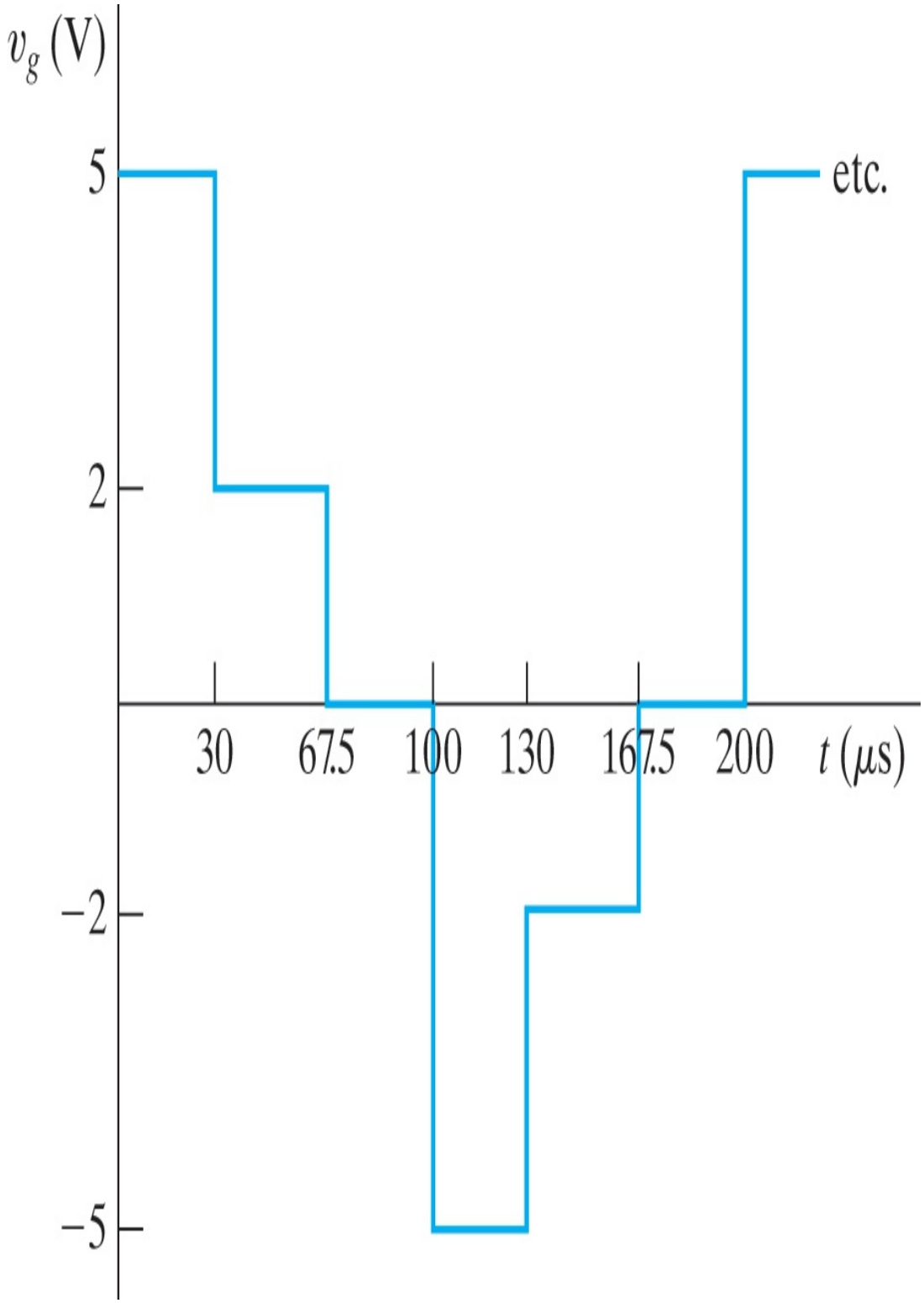


Figure P10.14 Full Alternative Text

5. 10.15

1. a) A personal computer with a monitor and keyboard requires 40 W at 115 V(rms). Calculate the rms value of the current carried by its power cord.

2. b) A laser printer for the personal computer in (a) is rated at 90 W at 115 V(rms). If this printer is plugged into the same wall outlet as the computer, what is the rms value of the current drawn from the outlet?

6. 10.16 A dc voltage equal to Vdc V is applied to a resistor of R Ω. A sinusoidal voltage equal to vs V is also applied to a resistor of R Ω. Show that the dc voltage will deliver the same amount of energy in T seconds (where T is the period of the sinusoidal voltage) as the sinusoidal voltage provided Vdc equals the rms value of vs. (Hint: Equate the two expressions for the energy delivered to the resistor.)

Sections 10.4–10.5

1. 10.17

    1. a) Find VL (rms) and θ for the circuit in Fig. P10.17 if the load absorbs 2500 VA at a lagging power factor of 0.8.

    2. b) Construct a phasor diagram of each solution obtained in (a).

Figure P10.17

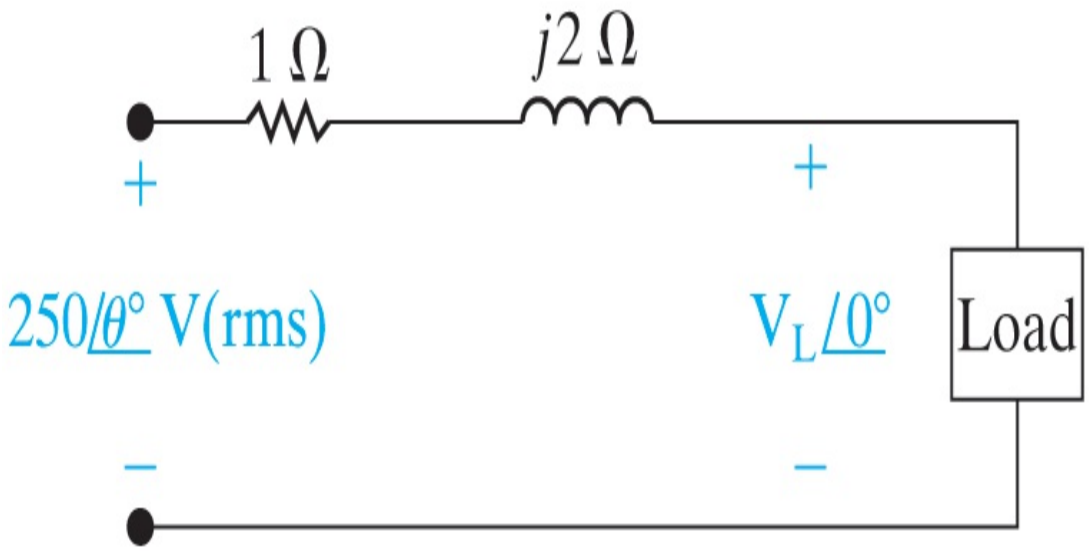


Figure P10.17 Full Alternative Text

2. 10.18 PSPICEMULTISIM

    1. a) Find the average power, the reactive power, and the apparent power supplied by the voltage source in the circuit in Fig. P10.18 if vg=40 cos 106t V.

    2. b) Check your answer in (a) by showing Pdev=∑Pabs.

    3. c) Check your answer in (a) by showing Qdev=∑Qabs.

Figure P10.18

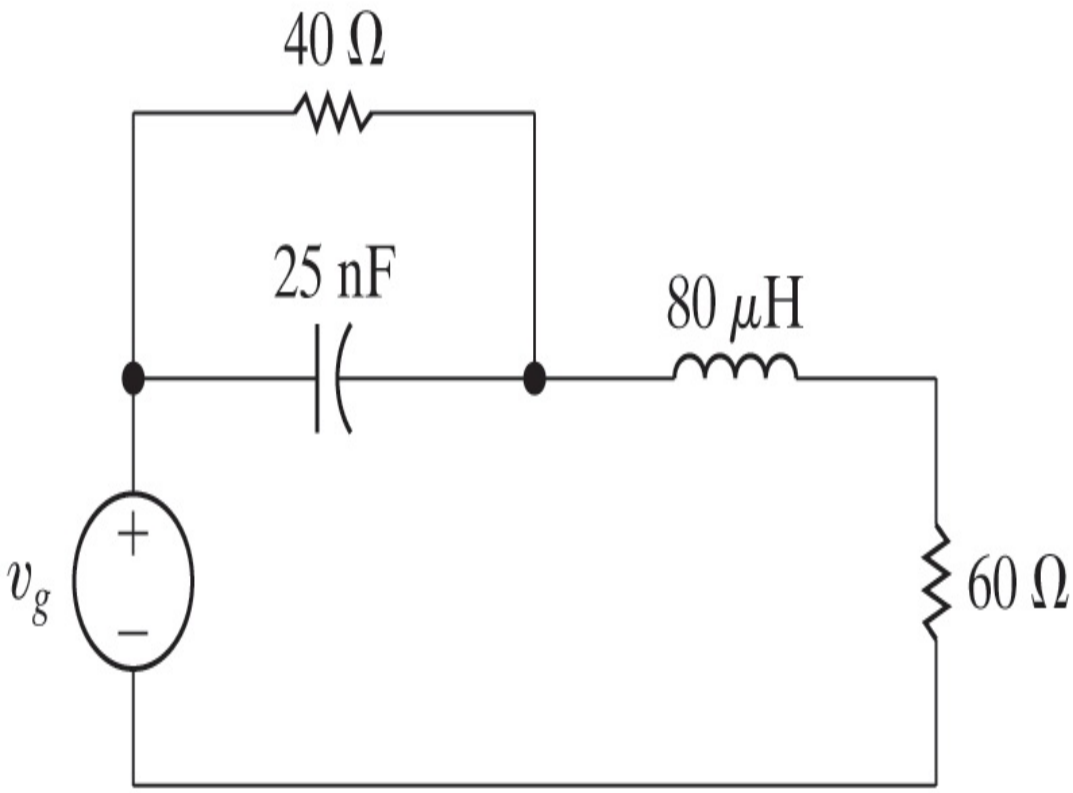


Figure P10.18 Full Alternative Text

3. 10.19 The voltage Vg in the frequency-domain circuit shown in Fig. P10.19 is 170 0° V (rms).

    1. a) Find the average and reactive power for the voltage source.
    2. b) Is the voltage source absorbing or delivering average power?
    3. c) Is the voltage source absorbing or delivering magnetizing vars?
    4. d) Find the average and reactive powers associated with each impedance branch in the circuit.
    5. e) Check the balance between delivered and absorbed average power.
    6. f) Check the balance between delivered and absorbed magnetizing vars.

Figure P10.19

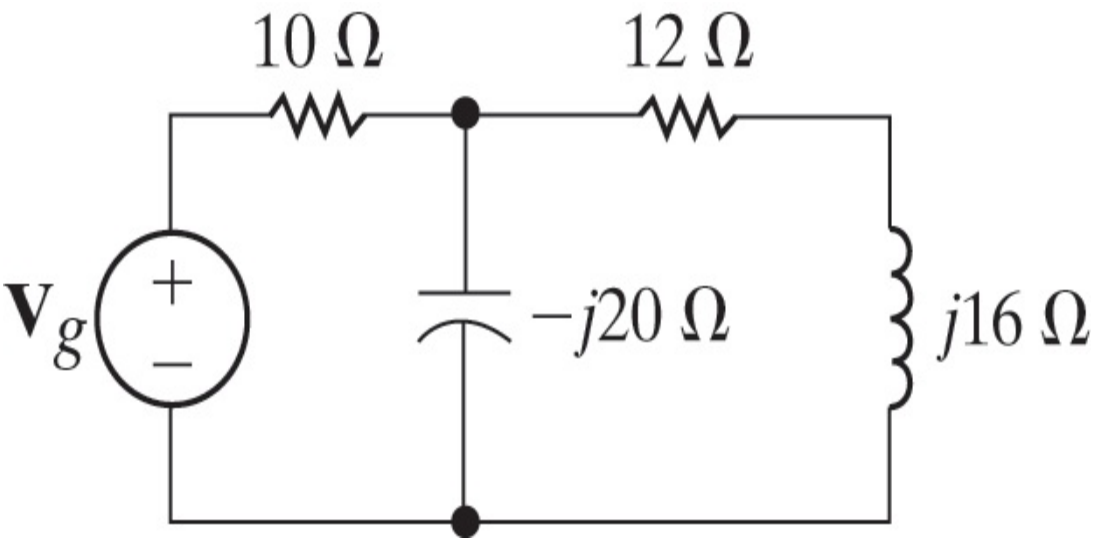


Figure P10.19 Full Alternative Text

4. 10.20 PSPICEMULTISIMFind the average power, the reactive power, and the apparent power absorbed by the load in the circuit in Fig. P10.20 if vg equals 150 cos 250t V.

Figure P10.20

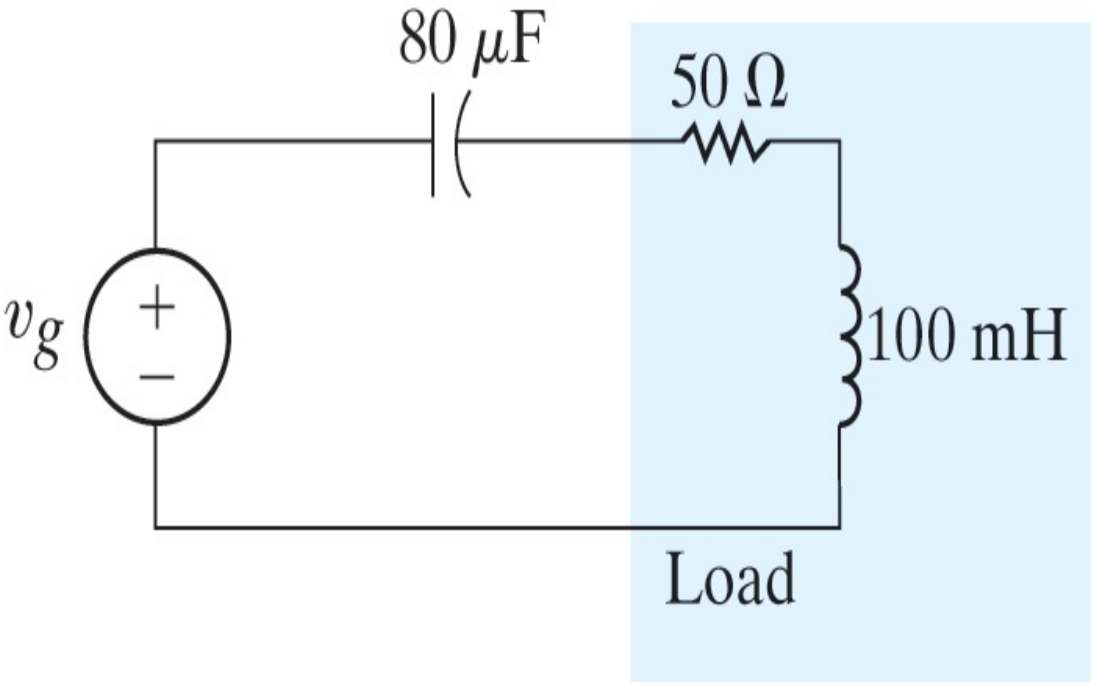


Figure P10.20 Full Alternative Text

5. 10.21 The two loads shown in Fig. P10.21 can be described as follows: Load 1 absorbs an average power of 60 kW and delivers 70 kVAR of reactive power; Load 2 has an impedance of (24+j7) Ω. The voltage at the terminals of the loads is 25002 cos 120π t V.

    1. a) Find the rms value of the source voltage.

    2. b) By how many microseconds is the load voltage out of phase with the source voltage?

    3. c) Does the load voltage lead or lag the source voltage?

Figure P10.21

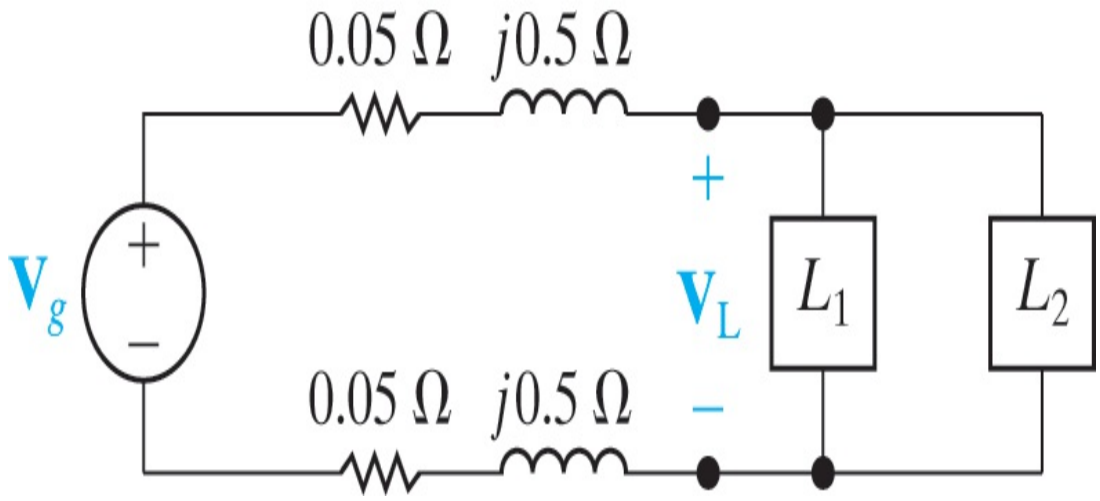


Figure P10.21 Full Alternative Text

6. 10.22 Two 125 V(rms) loads are connected in parallel. The two loads draw a total average power of 4500 W at a power factor of 0.96 leading. One of the loads draws 2700 W at a power factor of 0.8 lagging. What is the power factor of the other load?

7. 10.23 The three parallel loads in the circuit shown in Fig. 10.23 can be described as follows: Load 1 is absorbing an average power of 6 kW and delivering reactive power of 8 kvars; Load 2 is absorbing an average power of 9 kW and reactive power of 3 kvars; Load 3 is a 25 kΩ resistor in parallel with a capacitor whose reactance is −5 Ω. Find the rms magnitude and the phase angle of Vg if Vo=250 0° V(rms).

Figure P10.23

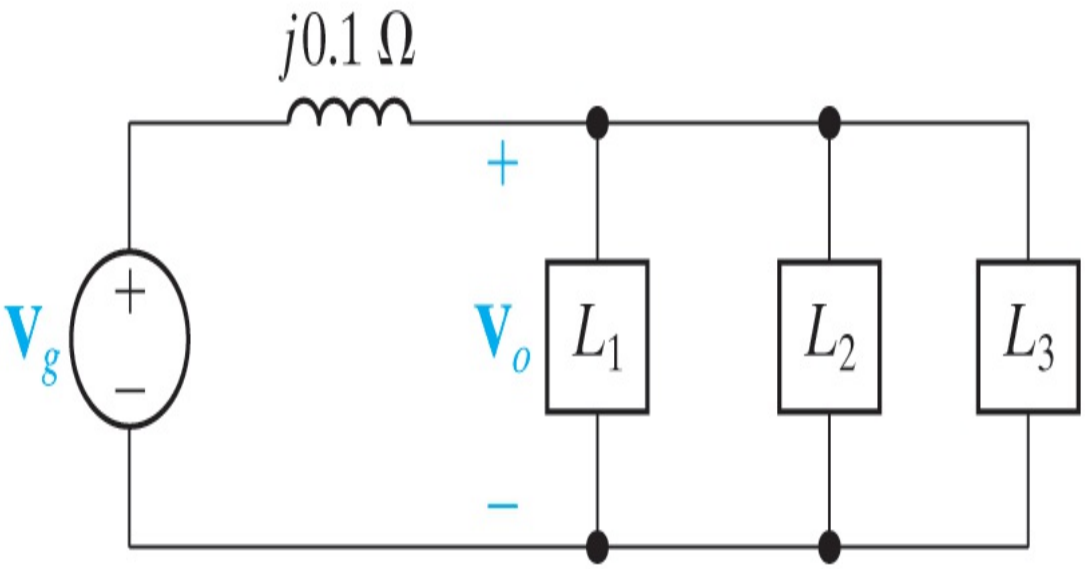


Figure P10.23 Full Alternative Text

8. 10.24 Three loads are connected in parallel across a 250 V(rms) line, as shown in Fig. P10.24. Load 1 absorbs 16 kW and 18 kVAR; Load 2 absorbs 10 kVA at 0.6 leading; Load 3 absorbs 8 kW at unity power factor.

    1. a) Find the impedance that is equivalent to the three parallel loads.

    2. b) Find the power factor of the equivalent load as seen from the line's input terminals.

Figure P10.24

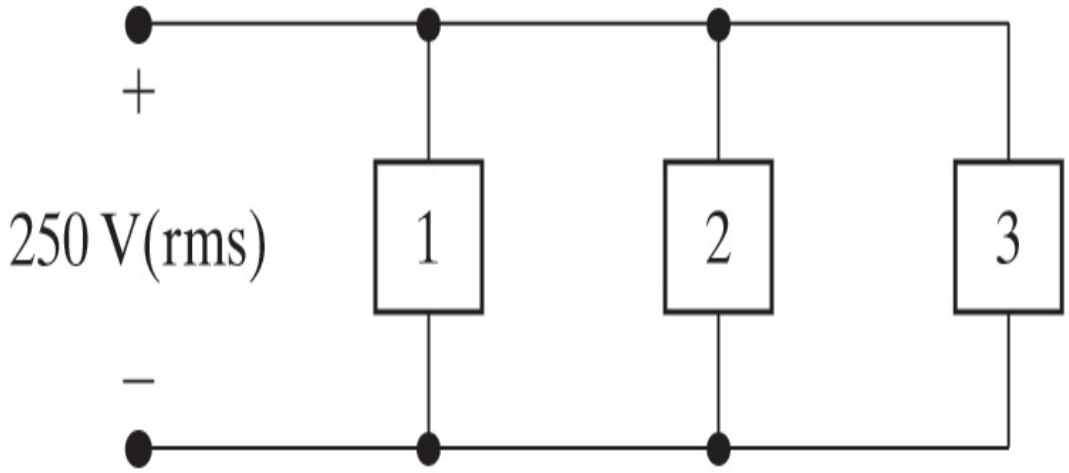


Figure P10.24 Full Alternative Text

9. 10.25 The three loads in Problem 10.24 are fed from a line having a series impedance 0.01+j0.08 Ω, as shown in Fig. P10.25.

    1. a) Calculate the rms value of the voltage (Vs) at the sending end of the line.

    2. b) Calculate the average and reactive powers associated with the line impedance.

    3. c) Calculate the average and reactive powers at the sending end of the line.

    4. d) Calculate the efficiency (η) of the line if the efficiency is defined as

    η=(Pload/Psending end)×100.

Figure P10.25

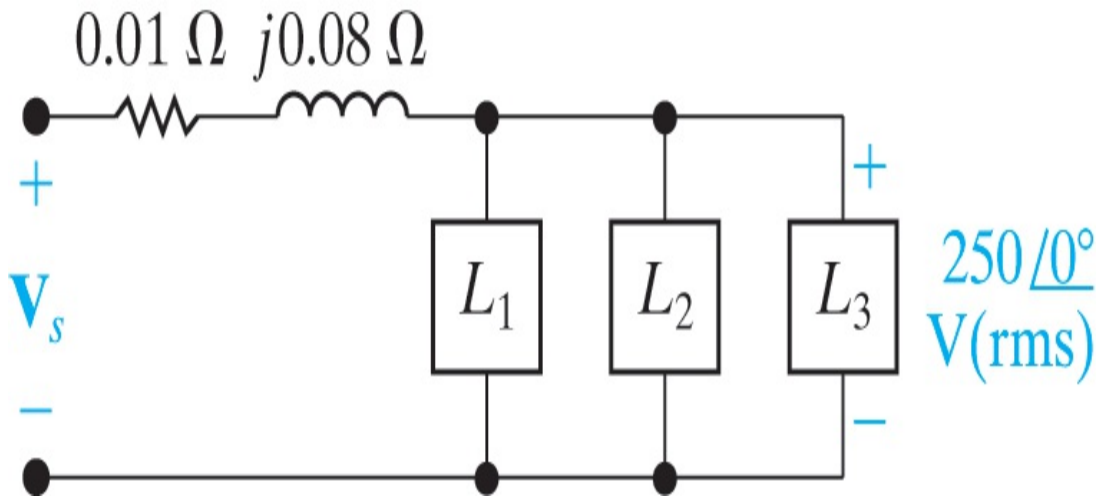


Figure P10.25 Full Alternative Text

10. 10.26 Consider the circuit described in Problem 9.78.

    1. a) What is the rms magnitude of the voltage across the load impedance?

    2. b) What percentage of the average power developed by the practical source is delivered to the load impedance?

11. 10.27 The three loads in the circuit in Fig. P10.27 can be described as follows: Load 1 is a 240 Ω resistor in series with an inductive reactance of 70 Ω; load 2 is a capacitive reactance of 120 Ω in series with a 160 Ω resistor; and load 3 is a 30 Ω resistor in series with a capacitive reactance of 40 Ω. The frequency of the voltage source is 60 Hz.

    1. a) Give the power factor and reactive factor of each load.

    2. b) Give the power factor and reactive factor of the composite load seen by the voltage source.

Figure P10.27

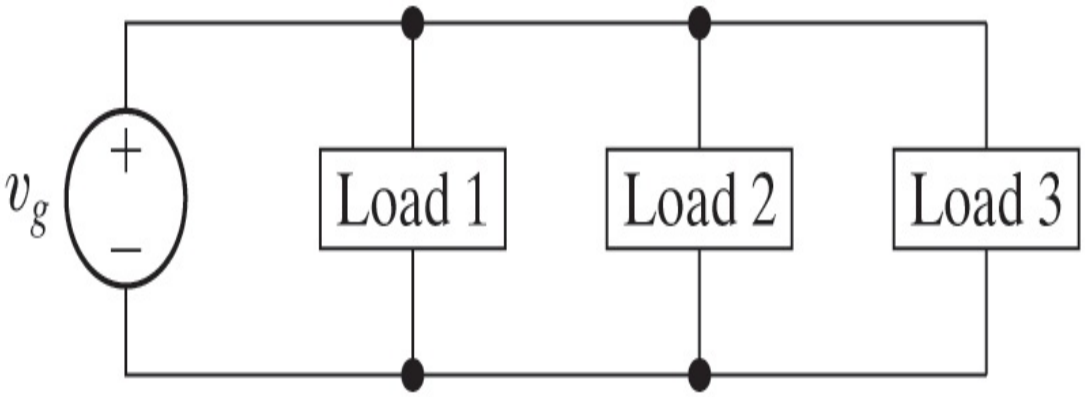


Figure P10.27 Full Alternative Text

12. 10.28 The three loads in the circuit seen in Fig. P10.28 are described as follows: Load 1 is absorbing 7.5 kW and 2.5 kVAR; Load 2 is absorbing 10 kVA at a power factor of 0.28 leading; Load 3 is a 12.5 Ω resistor in parallel with an inductance whose reactance is 50 Ω.

    1. a) Calculate the average power and the magnetizing reactive power delivered by each source if Vg1=Vg2=250 0° V (rms).

    2. b) Check your calculations by showing your results are consistent with the requirements

       ∑Pdev=∑Pabs;∑Pdev=∑Pabs.

Figure P10.28

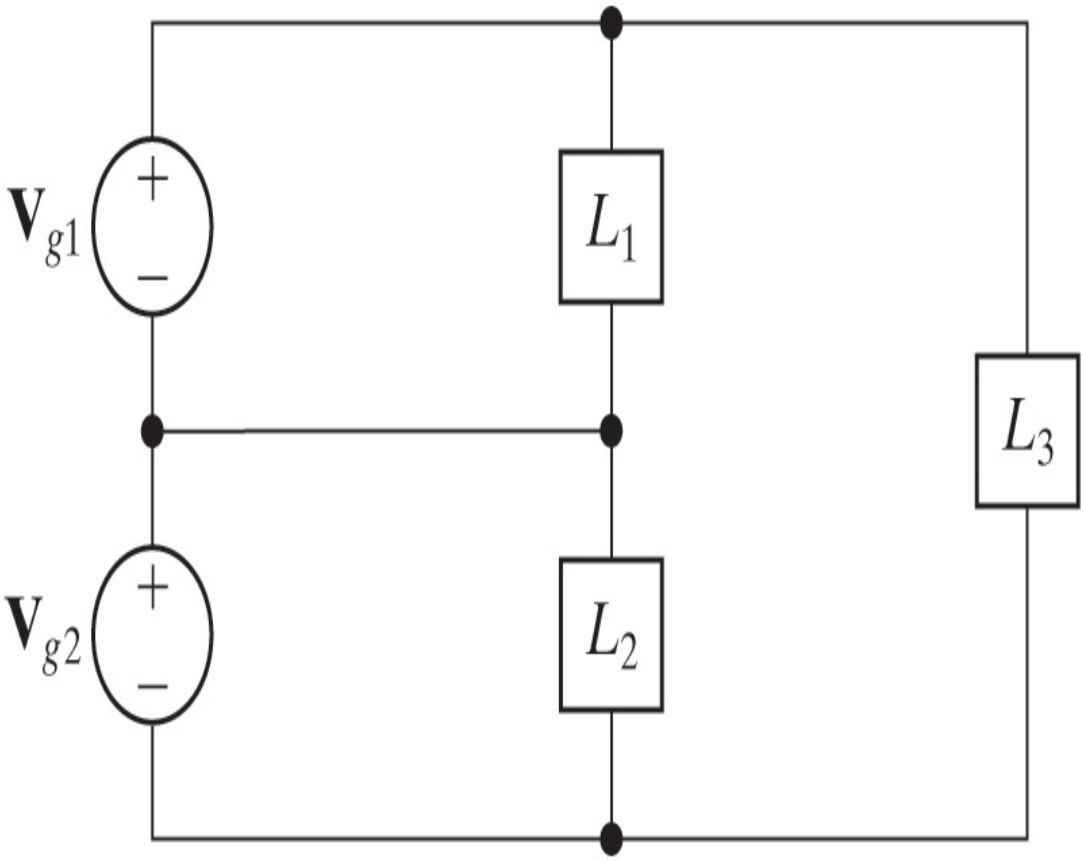


Figure P10.28 Full Alternative Text

13. 10.29 Suppose the circuit shown in Fig. P10.28 represents a residential distribution circuit in which the impedances of the service conductors are negligible and Vg1=Vg2=120 0° V (rms). The three loads in the circuit are L1 (a new refrigerator, a coffee maker, and a microwave oven); L2 (2 incandescent lamps, a humidifier, and a ceiling fan); and L3 (a clothes washer and a clothes dryer). Assume that all of these appliances are in operation at the same time. The service conductors are protected with 20 A circuit breakers. Will the service to this residence be interrupted? Why or why not?

14. 10.30 The three loads in the circuit seen in Fig. P10.30 are

    S1=4+j1 kVA,S2=5+j2 kVA,S3=10+j0 kVA.

    1. a) Calculate the complex power associated with each voltage source, Vg1 and Vg2.

2. b) Verify that the total real and reactive power delivered by the sources equals the total real and reactive power absorbed by the network.

Figure P10.30

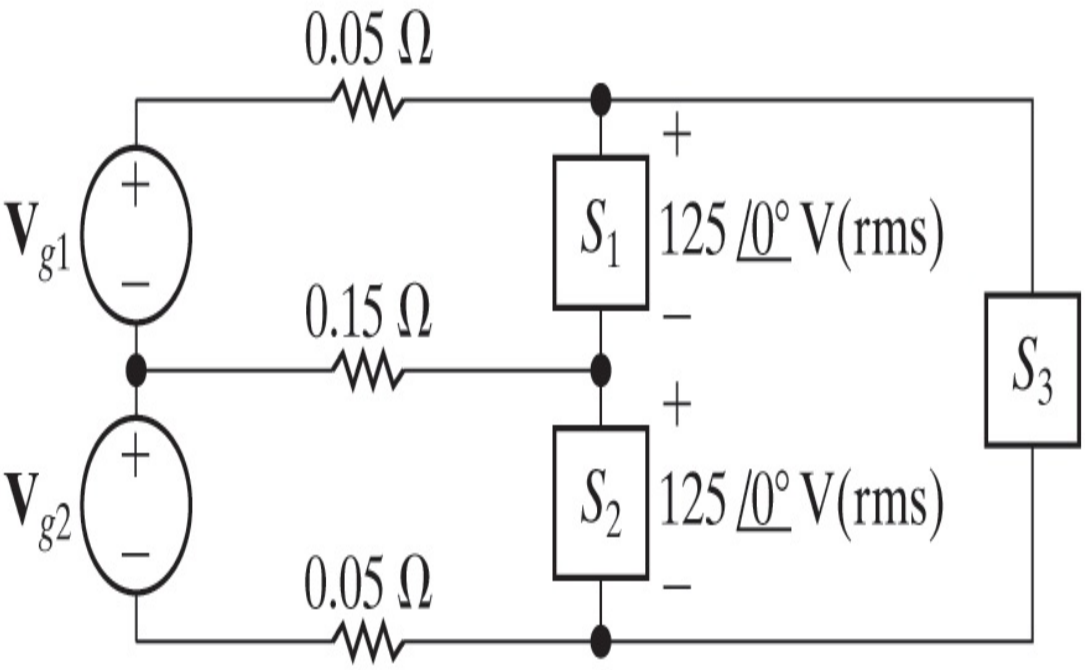


Figure P10.30 Full Alternative Text

15. 10.31 A group of small appliances on a 60 Hz system requires 20 kVA at 0.85 pf lagging when operated at 125 V(rms). The impedance of the feeder supplying the appliances is 0.01+j0.08 Ω. The voltage at the load end of the feeder is 125 V(rms).

    1. a) What is the rms magnitude of the voltage at the source end of the feeder?

    2. b) What is the average power loss in the feeder?

    3. c) What size capacitor (in microfarads) across the load end of the feeder is needed to improve the load power factor to unity?

4. d) After the capacitor is installed, what is the rms magnitude of the voltage at the source end of the feeder if the load voltage is maintained at 125 V(rms)?

5. e) What is the average power loss in the feeder for (d)?

16. 10.32

    1. a) Find the average power dissipated in the line in Fig. P10.32.

    2. b) Find the capacitive reactance that when connected in parallel with the load will make the load look purely resistive.

    3. c) What is the equivalent impedance of the load in (b)?

    4. d) Find the average power dissipated in the line when the capacitive reactance is connected across the load.

    5. e) Express the power loss in (d) as a percentage of the power loss found in (a).

Figure P10.32

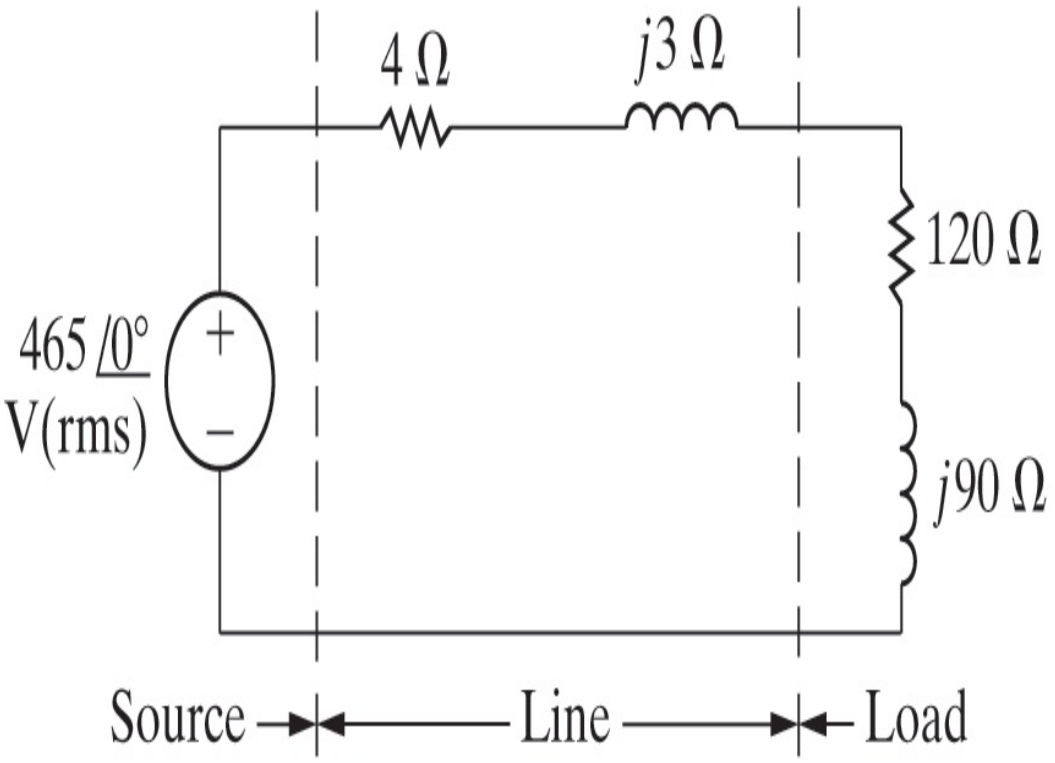


Figure P10.32 Full Alternative Text

17. 10.33 The steady-state voltage drop between the load and the sending end of the line seen in Fig. P10.33 is excessive. A capacitor is placed in parallel with the 192 kVA load and is adjusted until the steady-state voltage at the sending end of the line has the same magnitude as the voltage at the load end, that is, 4800 V(rms). The 192 kVA load is operating at a power factor of 0.8 lag. Calculate the size of the capacitor in microfarads if the circuit is operating at 60 Hz. In selecting the capacitor, keep in mind the need to keep the power loss in the line at a reasonable level.

Figure P10.33

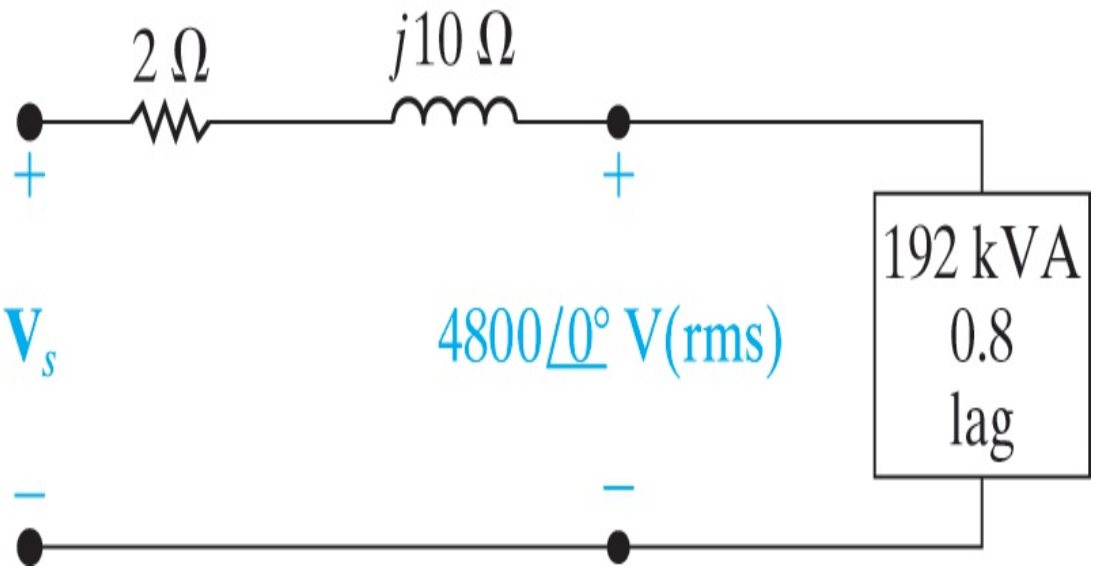


Figure P10.33 Full Alternative Text

18. 10.34 A factory has an electrical load of 1600 kW at a lagging power factor of 0.8. An additional variable power factor load is to be added to the factory. The new load will add 320 kW to the real power load of the factory. The power factor of the added load is to be adjusted so that the overall power factor of the factory is 0.96 lagging.

    1. a) Specify the reactive power associated with the added load.
    2. b) Does the added load absorb or deliver magnetizing vars?
    3. c) What is the power factor of the additional load?
    4. d) Assume that the voltage at the input to the factory is 2400 V(rms). What is the rms magnitude of the current into the factory before the variable power factor load is added?
    5. e) What is the rms magnitude of the current into the factory after the variable power factor load has been added?

19. 10.35 Assume the factory described in Problem 10.34 is fed from a line having an impedance of 0.05+j0.4 Ω. The voltage at the factory is maintained at 2400 V(rms).

1. a) Find the average power loss in the line before and after the load is added.

2. b) Find the magnitude of the voltage at the sending end of the line before and after the load is added.

20. 10.36

1. a) Find the six branch currents Ia−If in the circuit in Fig. P10.36.

2. b) Find the complex power in each branch of the circuit.

3. c) Check your calculations by verifying that the average power developed equals the average power dissipated.

4. d) Check your calculations by verifying that the magnetizing vars generated equal the magnetizing vars absorbed.

Figure P10.36

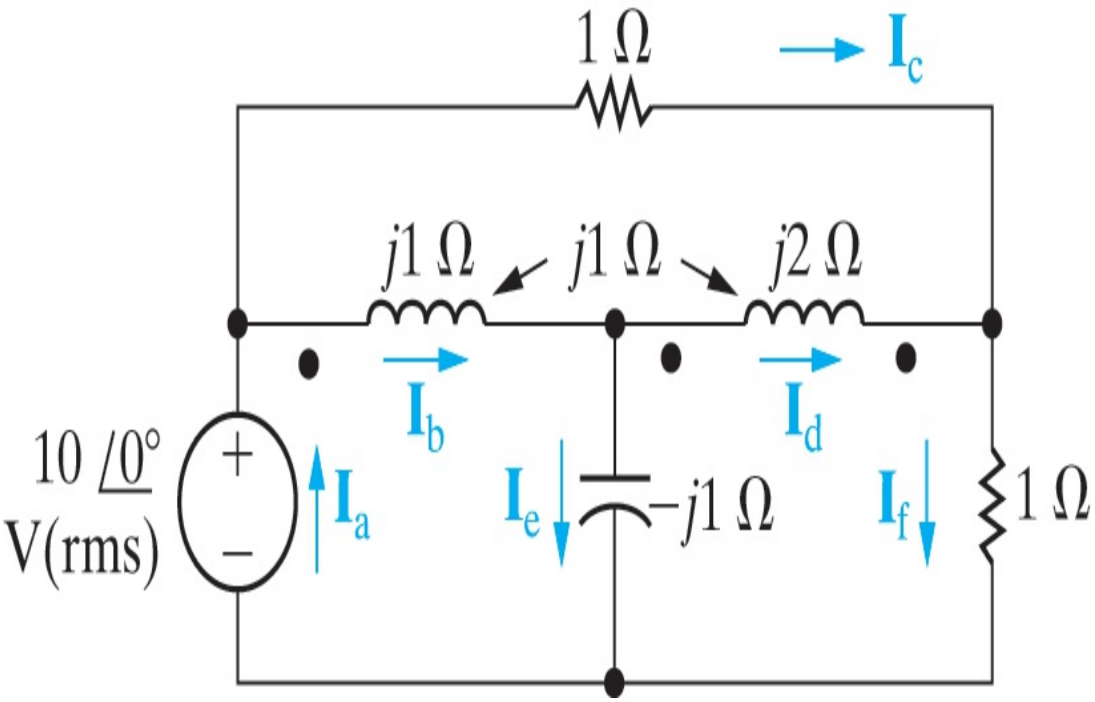


Figure P10.36 Full Alternative Text

21. 10.37

    1. a) Find the average power delivered to the 8 Ω resistor in the circuit in Fig. P10.37.

    2. b) Find the average power developed by the ideal sinusoidal voltage source.

    3. c) Find Zab.

    4. d) Show that the average power developed equals the average power dissipated.

Figure P10.37

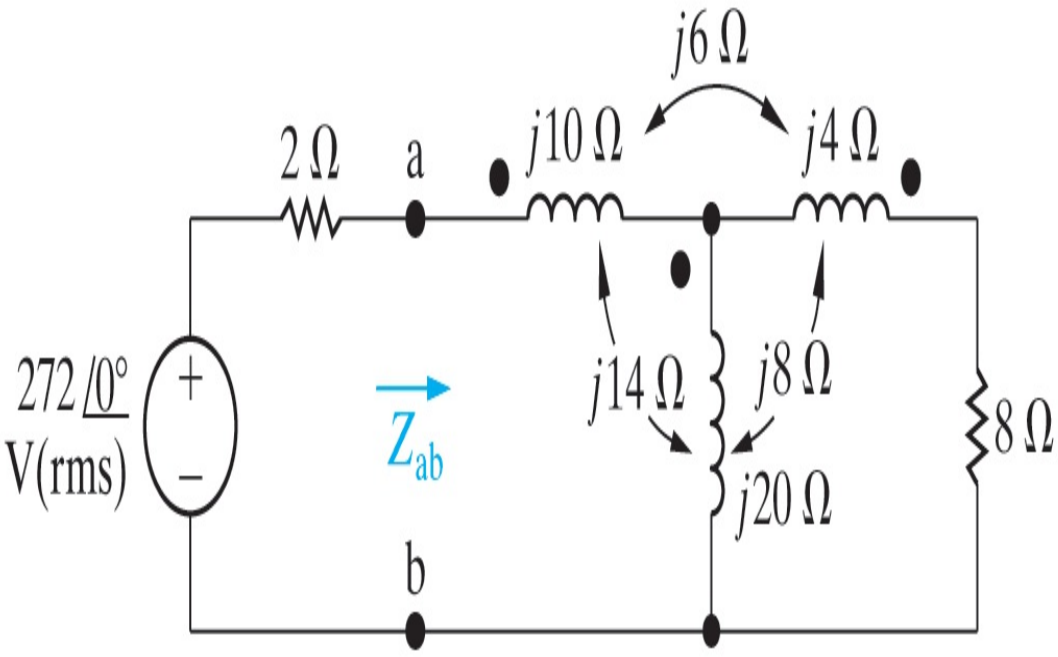


Figure P10.37 Full Alternative Text

22. 10.38

1. a) Find the average power delivered by the sinusoidal current source in the circuit of Fig. P10.38.

2. b) Find the average power delivered to the 20 Ω resistor.

Figure P10.38

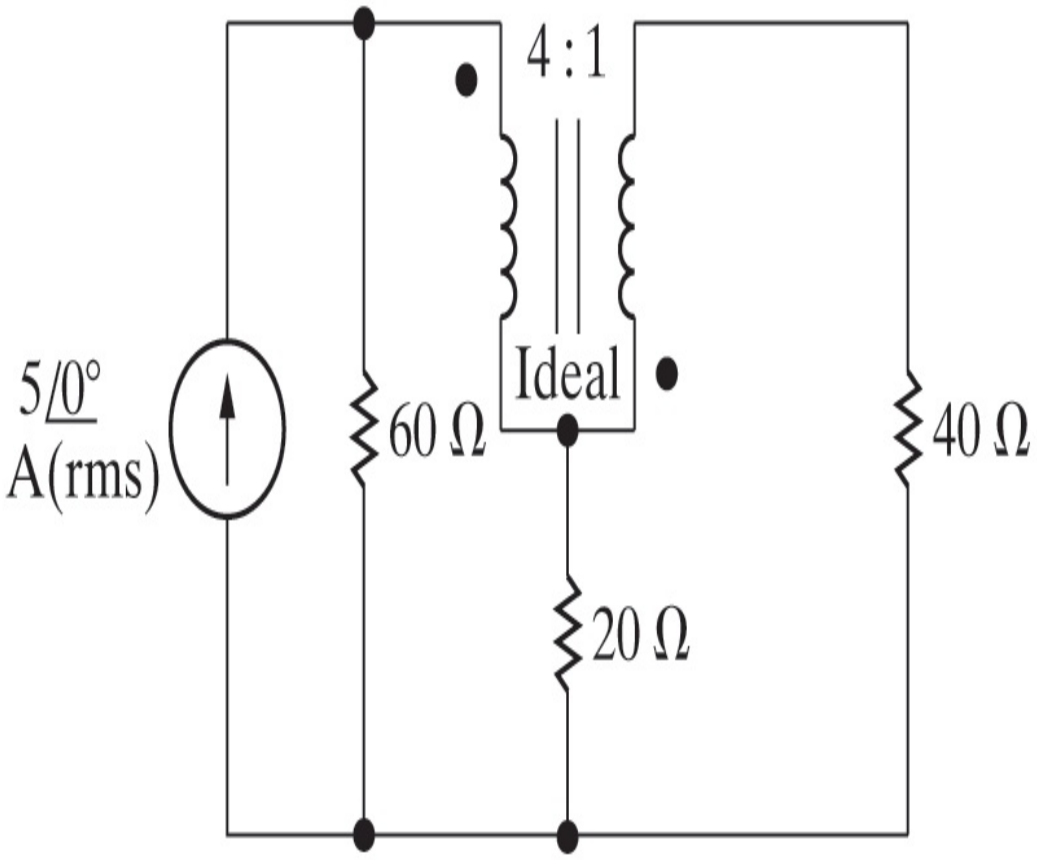


Figure P10.38 Full Alternative Text

23. 10.39

1. a) Find the average power dissipated in each resistor in the circuit in Fig. P10.39.

2. b) Check your answer by showing that the total power developed equals the total power absorbed.

Figure P10.39

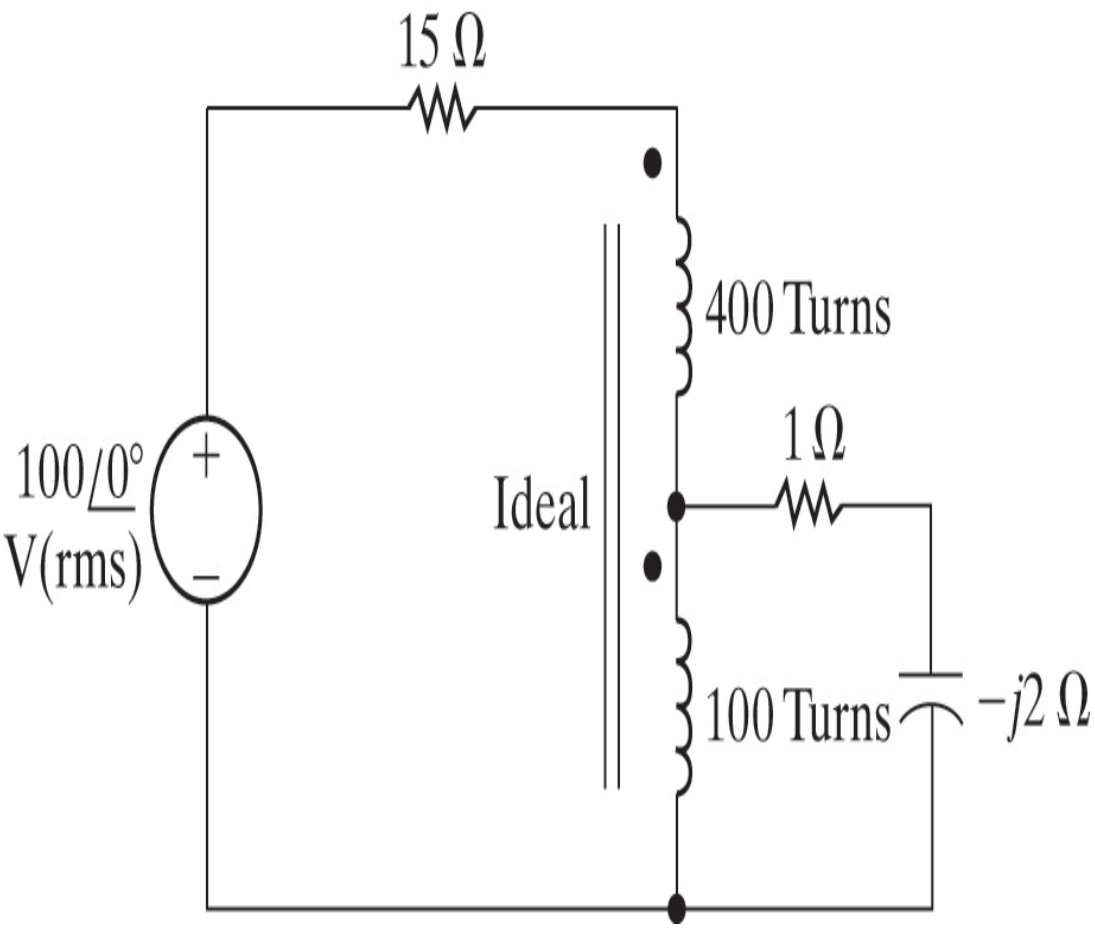


Figure P10.39 Full Alternative Text

24. 10.40 The sinusoidal voltage source in the circuit in Fig. P10.40 is developing an rms voltage of 2000 V. The 4 Ω load in the circuit is absorbing four times as much average power as the 25 Ω load. The two loads are matched to the sinusoidal source that has an internal impedance of 500 0° kΩ.

    1. a) Specify the numerical values of a1 and a2.

    2. b) Calculate the power delivered to the 25 Ω load.

    3. c) Calculate the rms value of the voltage across the 4 Ω resistor.

Figure P10.40

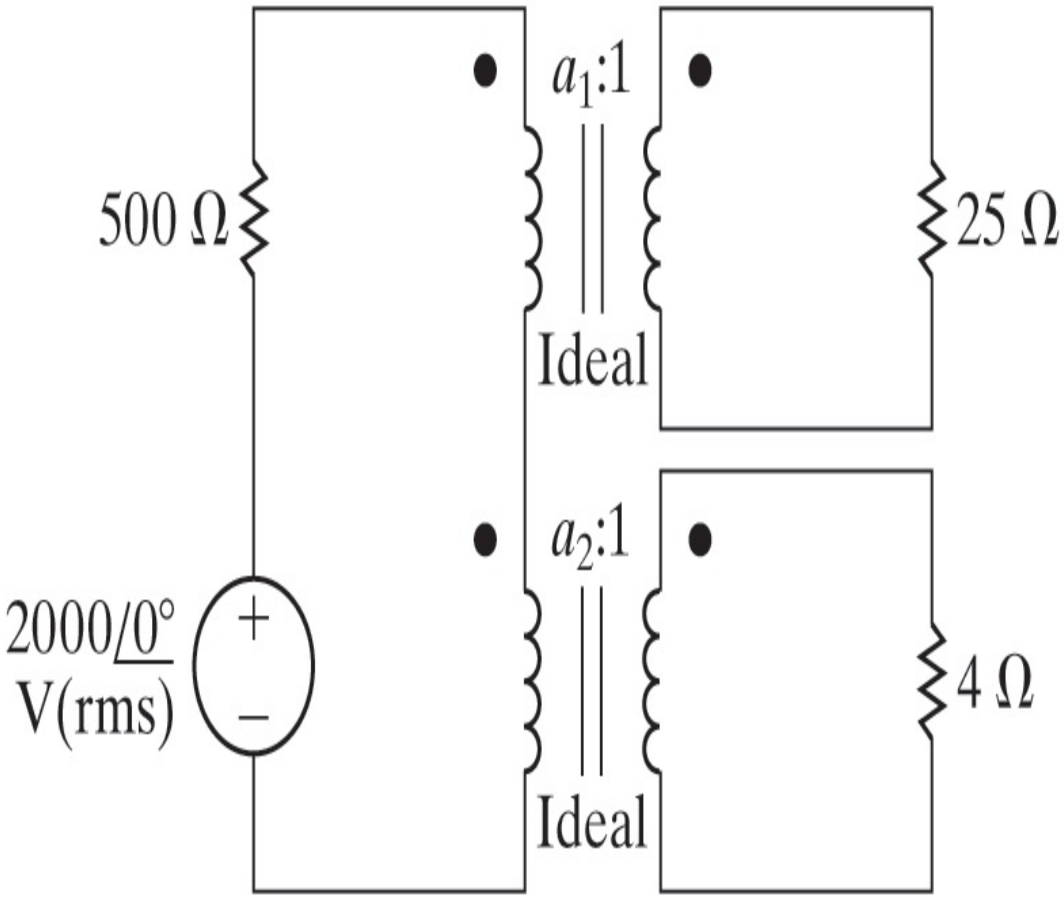


Figure P10.40 Full Alternative Text

Section 10.6

1. 10.41 Suppose an impedance equal to the conjugate of the Thévenin impedance is connected to the terminals c, d of the circuit shown in Fig. P9.75.

    1. a) Find the average power developed by the sinusoidal voltage source.

    2. b) What percentage of the power developed by the source is lost in the linear transformer?

2. 10.42 The phasor voltage Vab in the circuit shown in Fig. P10.42 is 240 0° V (rms) when no external load is connected to the terminals a, b. When a load having an impedance of 80−j60 Ω is connected across a, b, the value of Vab is 115.2+j33.6 V (rms).

    1. a) Find the impedance that should be connected across a, b for maximum average power transfer.

    2. b) Find the maximum average power transferred to the load of (a).

    3. c) Construct the impedance of part (a) using components from Appendix H if the source frequency is 1000 Hz.

Figure P10.42

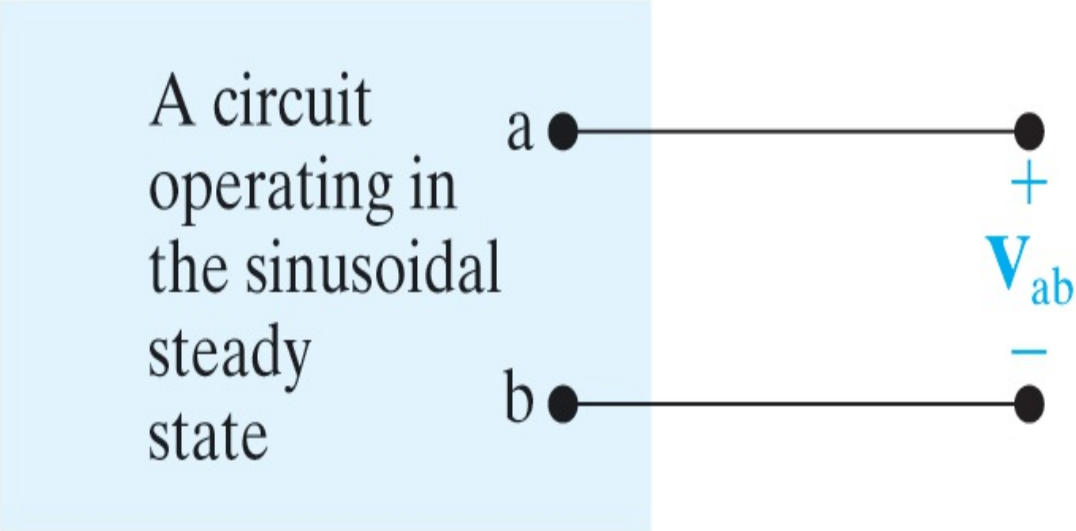


Figure P10.42 Full Alternative Text

3. 10.43

    1. a) Determine the load impedance for the circuit shown in Fig. P10.43 that will result in maximum average power being transferred to the load if ω=10 krad/s.

    2. b) Determine the maximum average power delivered to the load from part (a) if vg=120cos10,000t V.

    3. c) Repeat part (b) when ZL. consists of two components from Appendix H whose values yield a maximum average power closest to the value calculated in part (a).

Figure P10.43

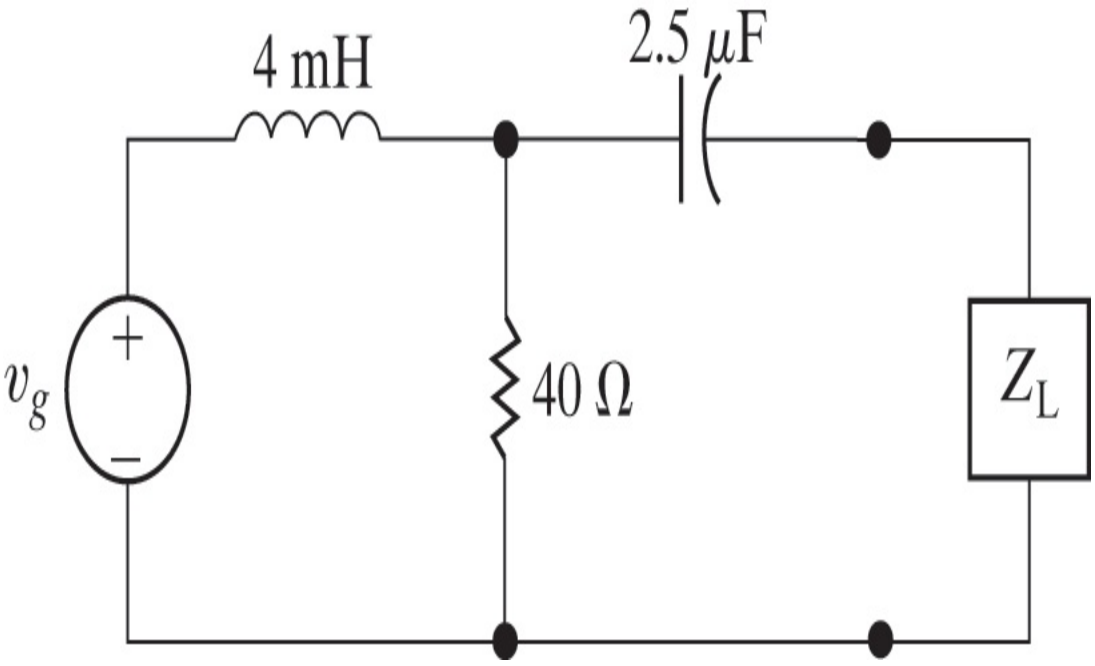


Figure P10.43 Full Alternative Text

4. 10.44 The load impedance ZL for the circuit shown in Fig. P10.44 is adjusted until maximum average power is delivered to ZL.

    1. a) Find the maximum average power delivered to ZL.

    2. b) What percentage of the total power developed in the circuit is delivered to ZL?

Figure P10.44

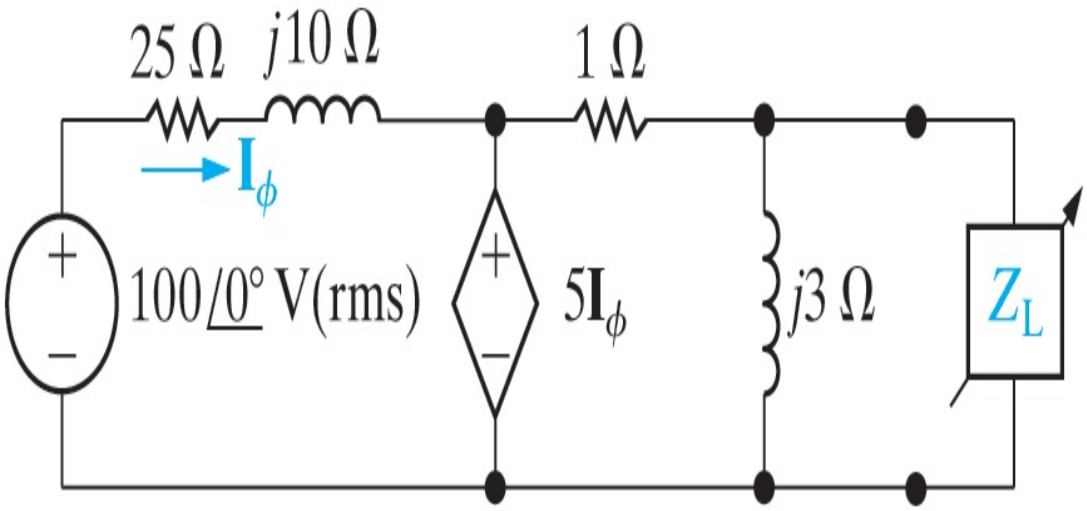


Figure P10.44 Full Alternative Text

5. 10.45 Prove that if only the magnitude of the load impedance can be varied, the most average power is transferred to the load when |ZL|=|ZTh|. (Hint: In deriving the expression for the load's average power, write the load impedance (ZL) in the form ZL=|ZL|cos θ +j|ZL|sin θ, and note that only |ZL| is variable.)

6. 10.46 The variable resistor in the circuit shown in Fig. P10.46 is adjusted until the average power it absorbs is maximum.

    1. a) Find R.

    2. b) Find the maximum average power.

    3. c) Find the resistor in Appendix H that would have the most average power delivered to it.

Figure P10.46

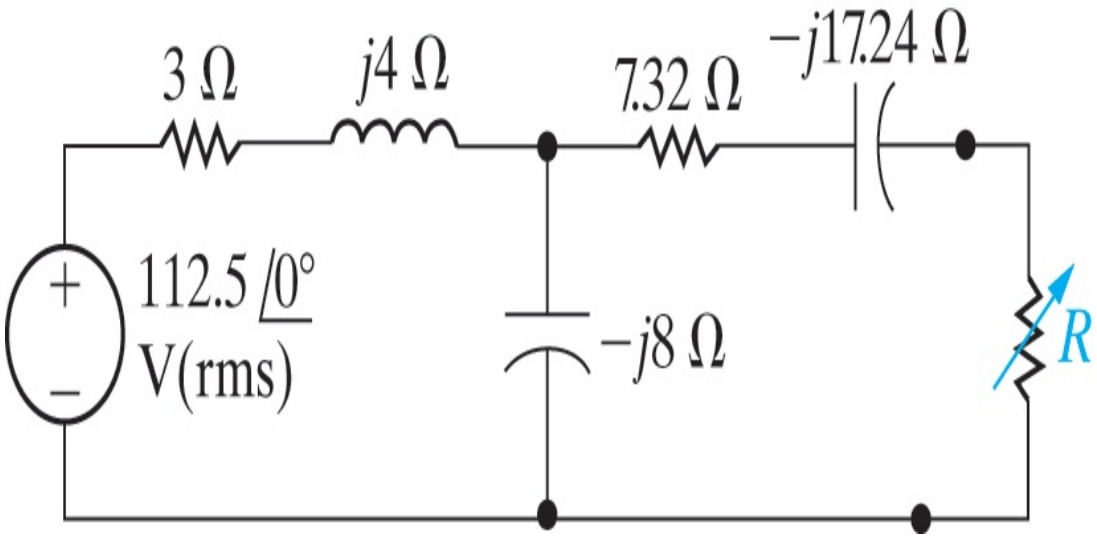


Figure P10.46 Full Alternative Text

7. 10.47 The variable resistor Ro in the circuit shown in Fig. P10.47 is adjusted until maximum average power is delivered to Ro.
    1. a) What is the value of Ro in ohms?
    2. b) Calculate the average power delivered to Ro.
    3. c) If Ro is replaced with a variable impedance Zo, what is the maximum average power that can be delivered to Zo?
    4. d) In (c), what percentage of the circuit's developed power is delivered to the load Zo?

Figure P10.47

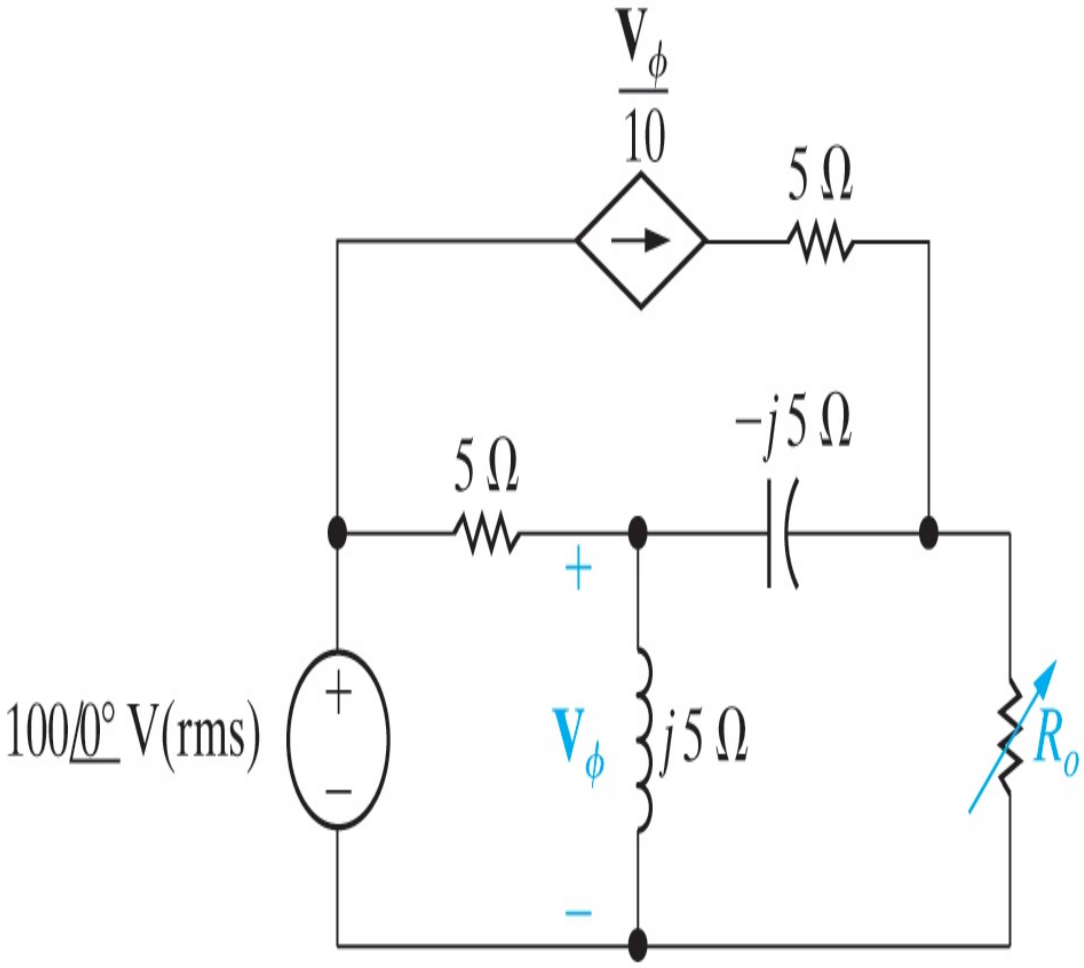


Figure P10.47 Full Alternative Text

8. 10.48 The sending-end voltage in the circuit seen in Fig. P10.48 is adjusted so that the load voltage is always 13,800 V(rms). The variable capacitor is adjusted until the average power dissipated in the line resistance is minimum.

    1. a) If the frequency of the sinusoidal source is 60 Hz, what is the value of the capacitance in microfarads?

    2. b) If the capacitor is removed from the circuit, what percentage increase in the magnitude of Vs is necessary to maintain 13,800 V(rms) at the load?

    3. c) If the capacitor is removed from the circuit, what is the percentage increase in line loss?

Figure P10.48

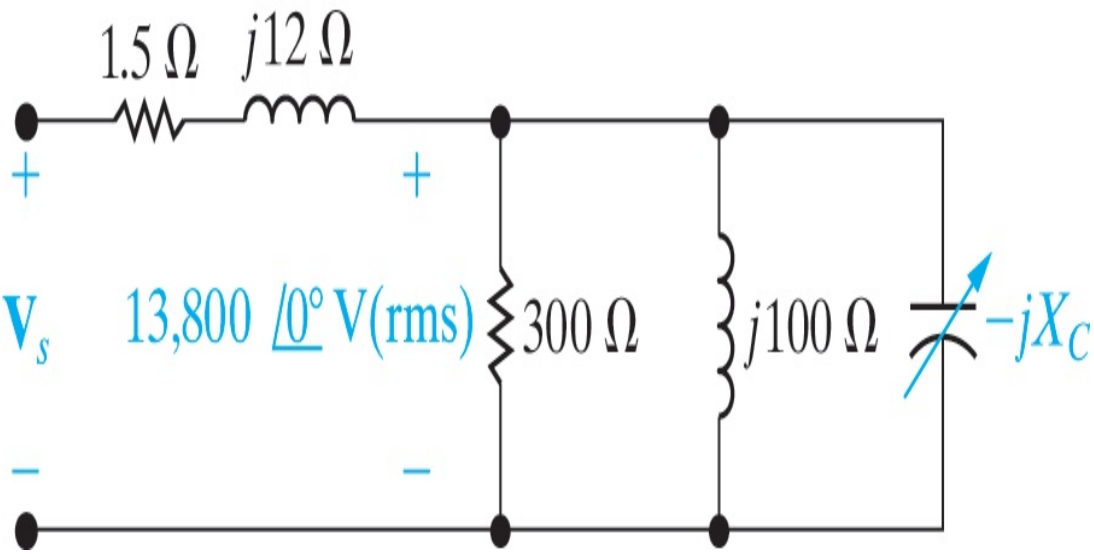


Figure P10.48 Full Alternative Text

9. 10.49 PSPICEMULTISIM The peak amplitude of the sinusoidal voltage source in the circuit shown in Fig. P10.49 is 180 V, and its frequency is 5000 rad/s. The load resistor can be varied from 0 to 4000 Ω, and the load capacitor can be varied from 0.1 μF to 0.5 μF.

    1. a) Calculate the average power delivered to the load when Ro=2000 Ω and Co=0.2 μF.

    2. b) Determine the settings of Ro and Co that will result in the most average power being transferred to Ro.

    3. c) What is the average power in (b)? Is it greater than the power in (a)?

    4. d) If there are no constraints on Ro and Co, what is the maximum average power that can be delivered to a load?

    5. e) What are the values of Ro and Co for the condition of (d)?

    6. f) Is the average power calculated in (d) larger than that calculated

in (c)?

Figure P10.49

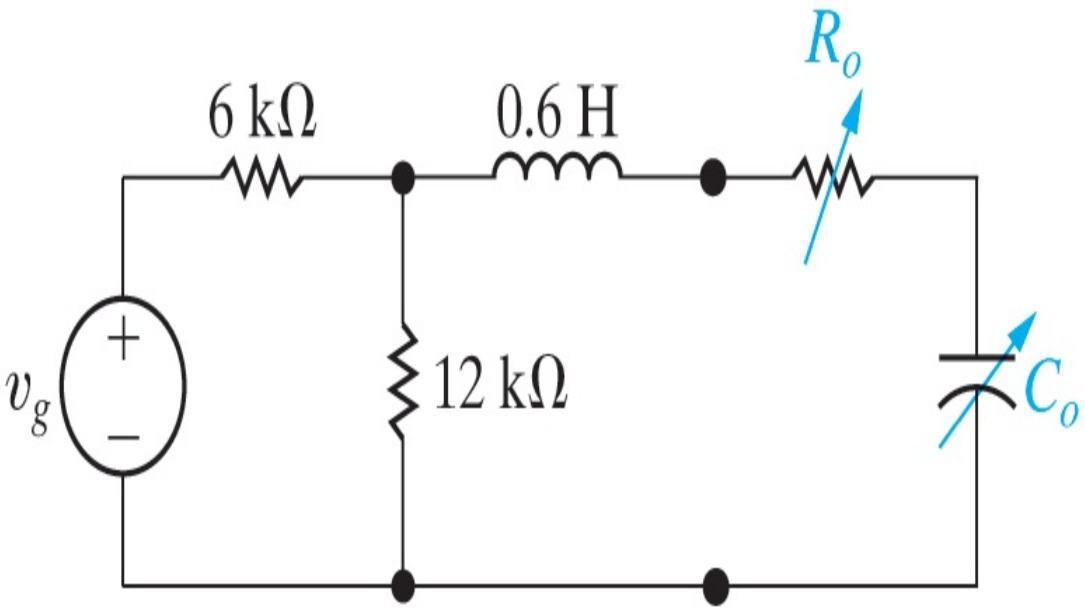


Figure P10.49 Full Alternative Text

10. 10.50 PSPICEMULTISIM

    1. a) Assume that Ro in Fig. P10.49 can be varied between 0 and 10 kΩ. Repeat (b) and (c) of Problem 10.49.

    2. b) Is the new average power calculated in (a) greater than that found in Problem 10.49(a)?

    3. c) Is the new average power calculated in (a) less than that found in 10.49(d)?

11. 10.51 PSPICEMULTISIM

    1. a) Find the steady-state expression for the currents ig and iL in the circuit in Fig. P10.51 when vg=400cos 400t V.

2. b) Find the coefficient of coupling.
3. c) Find the energy stored in the magnetically coupled coils at t=1.25π ms and t=2.5π ms.
4. d) Find the power delivered to the 375 Ω resistor.
5. e) If the 375 Ω resistor is replaced by a variable resistor RL, what value of RL will yield maximum average power transfer to RL?
6. f) What is the maximum average power in (e)?
7. g) Assume the 375 Ω resistor is replaced by a variable impedance ZL. What value of ZL will result in maximum average power transfer to ZL?
8. h) What is the maximum average power in (g)?

Figure P10.51

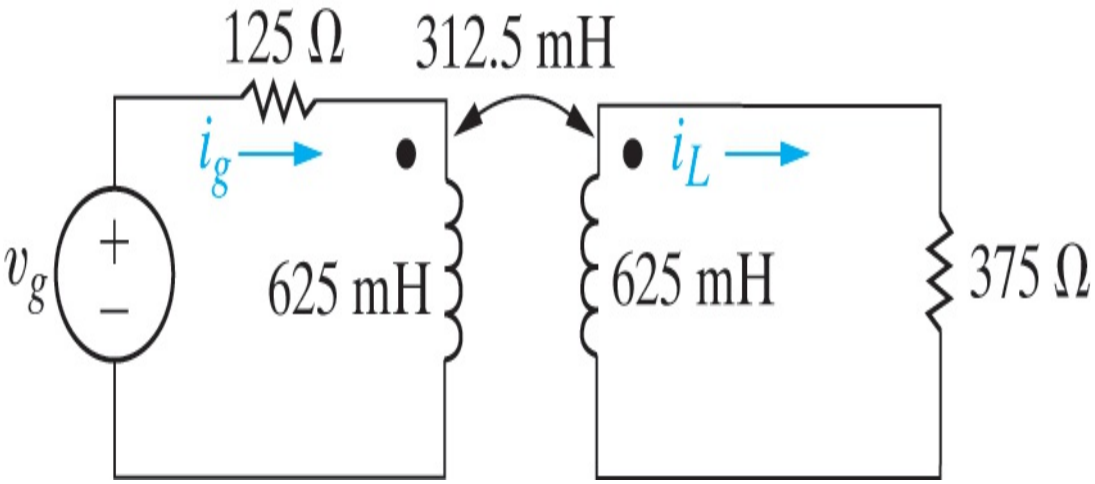


Figure P10.51 Full Alternative Text

12. 10.52 The values of the parameters in the circuit shown in Fig. P10.52 are L1=8 mH; L2=2 mH; k=0.75; Rg=1 Ω; and RL=7 Ω. If vg=542 cos 1000t V, find

1. a) the rms magnitude of vo;
2. b) the average power delivered to RL;
3. c) the percentage of the average power generated by the ideal voltage source that is delivered to RL.

Figure P10.52

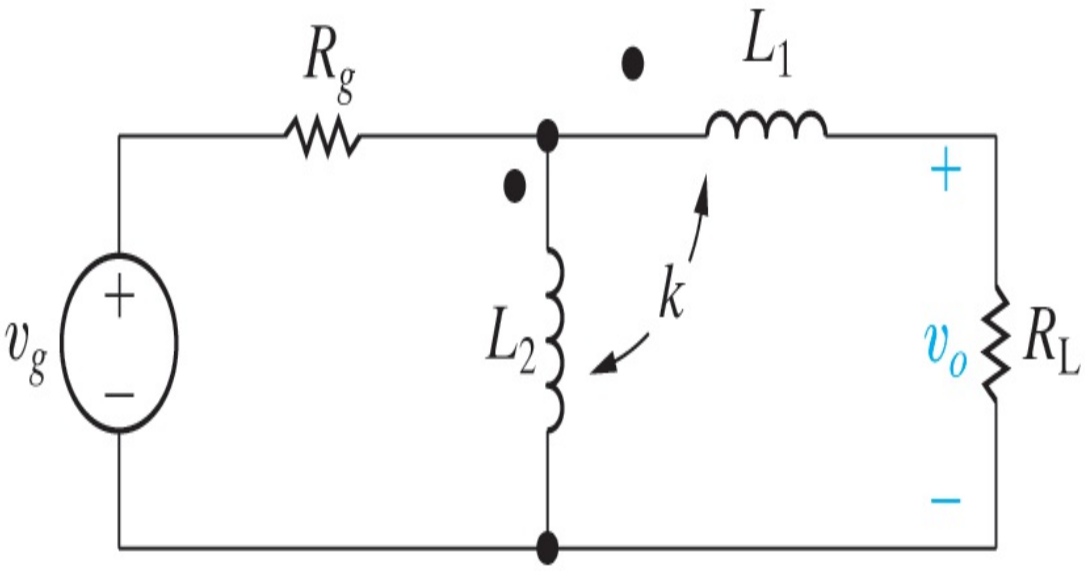


Figure P10.52 Full Alternative Text

13. 10.53 Assume the coefficient of coupling in the circuit in Fig. P10.52 is adjustable.
    1. a) Find the value of k that makes vo equal to zero.
    2. b) Find the power developed by the source when k has the value found in (a).
14. 10.54 Assume the load resistor (RL) in the circuit in Fig. P10.52 is adjustable.
    1. a) What value of RL will result in the maximum average power

being transferred to RL?

2. b) What is the value of the maximum power transferred?

15. 10.55 Find the impedance seen by the ideal voltage source in the circuit in Fig. P10.55 when Zo is adjusted for maximum average power transfer to Zo.

Figure P10.55

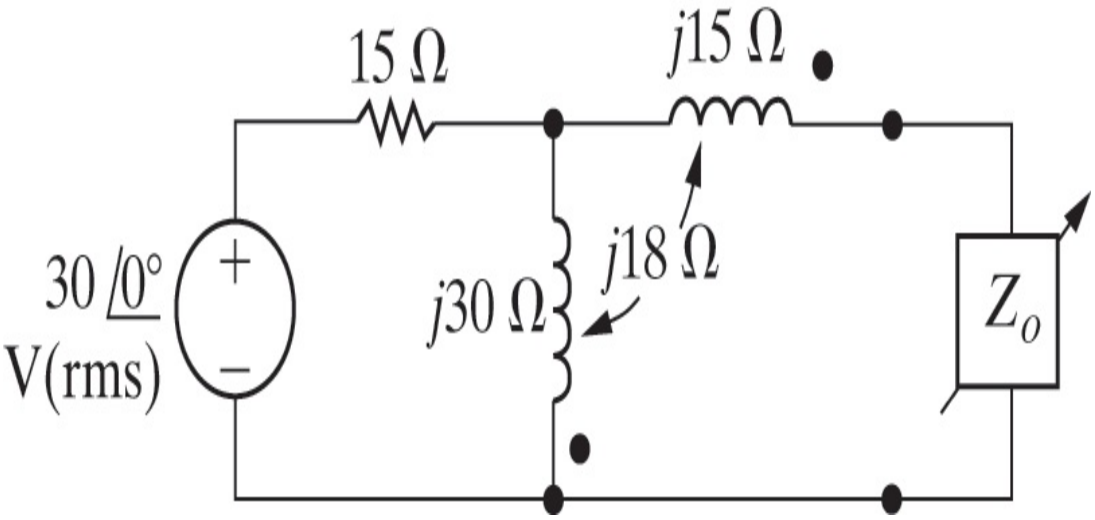


Figure P10.55 Full Alternative Text

16. 10.56 For the frequency-domain circuit in Fig. P10.56, calculate:

1. a) the rms magnitude of Vo;
2. b) the average power dissipated in the 9 Ω resistor;
3. c) the percentage of the average power generated by the ideal voltage source that is delivered to the 9 Ω load resistor.

Figure P10.56

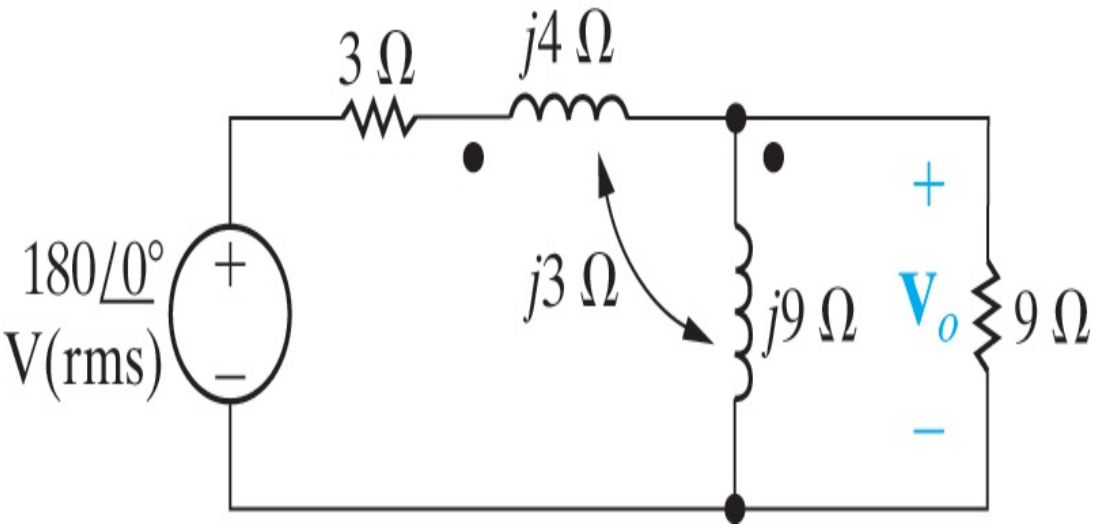


Figure P10.56 Full Alternative Text

17. 10.57 The 9 Ω resistor in the circuit in Fig. P10.56 is replaced with a variable impedance Zo. Assume Zo is adjusted for maximum average power transfer to Zo.

    1. a) What is the maximum average power that can be delivered to Zo?

    2. b) What is the average power developed by the ideal voltage source when maximum average power is delivered to Zo?

    3. c) Choose single components from Appendix H to form an impedance that dissipates average power closest to the value in part (a). Assume the source frequency is 60 Hz.

18. 10.58 The impedance ZL in the circuit in Fig. P10.58 is adjusted for maximum average power transfer to ZL. The internal impedance of the sinusoidal voltage source is 4+j7 Ω.

    1. a) What is the maximum average power delivered to ZL?

    2. b) What percentage of the average power delivered to the linear transformer is delivered to ZL?

Figure P10.58

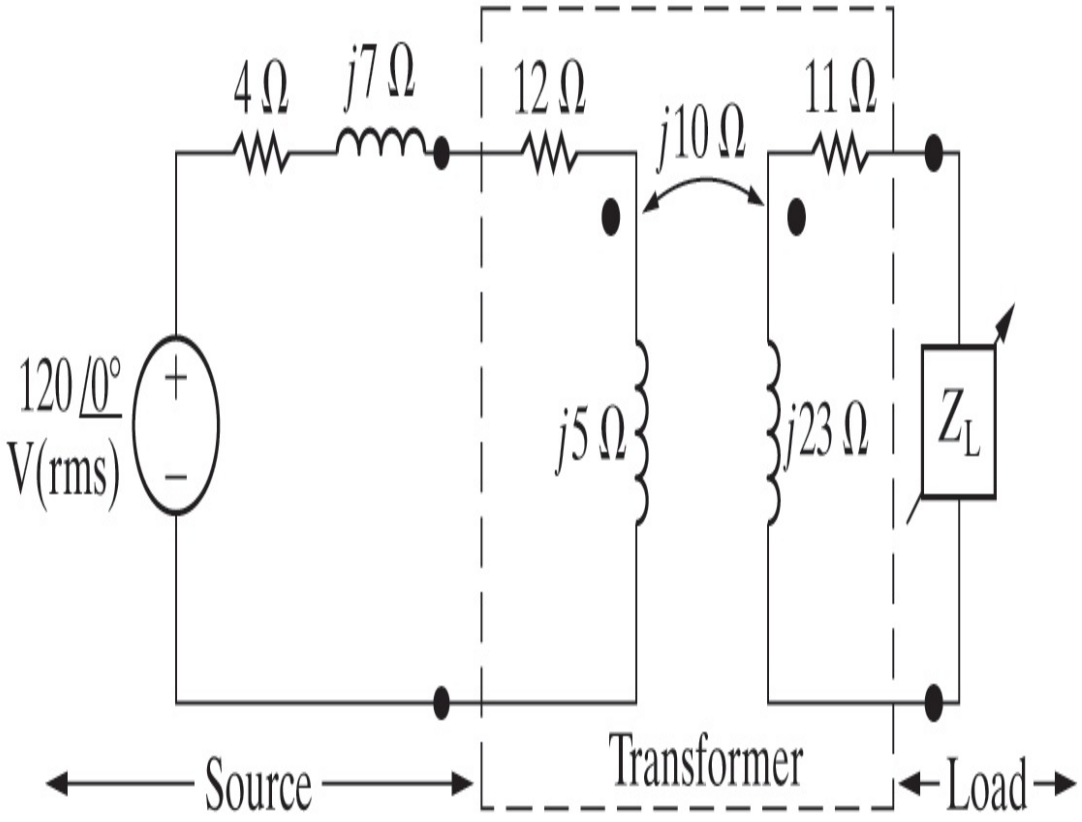


Figure P10.58 Full Alternative Text

19. 10.59 Find the average power delivered to the 5 kΩ resistor in the circuit of Fig. P10.59.

Figure P10.59

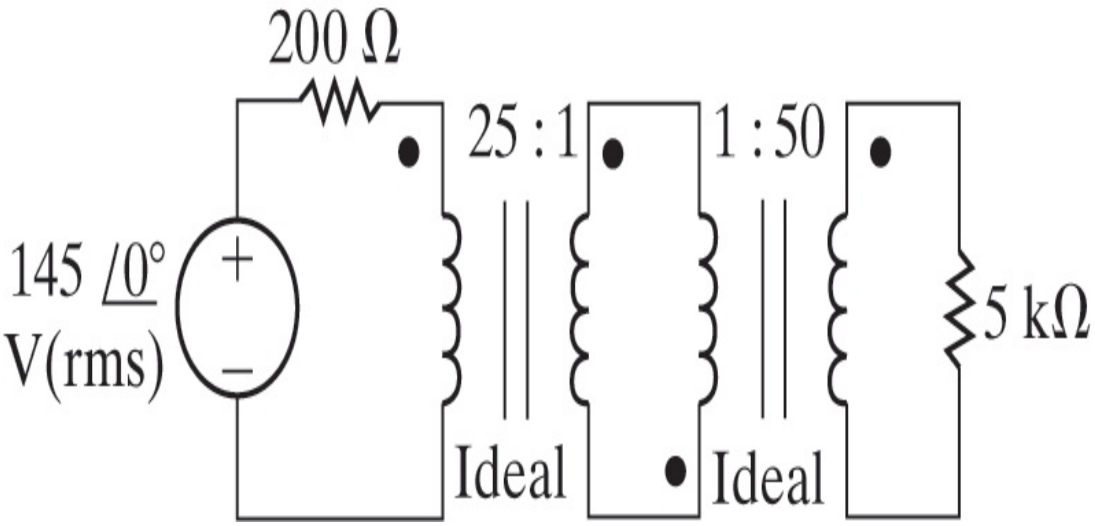


Figure P10.59 Full Alternative Text

20. 10.60 The ideal transformer connected to the 5 kΩ load in Problem 10.59 is replaced with an ideal transformer that has a turns ratio of 1:a.

    1. a) What value of a results in maximum average power being delivered to the 5 kΩ resistor?

    2. b) What is the maximum average power?

21. 10.61 The load impedance ZL in the circuit in Fig. P10.61 is adjusted until maximum average power is transferred to ZL.

    1. a) Specify the value of ZL if N1=3600 turns and N2=600 turns.

    2. b) Specify the values of IL and VL when ZL is absorbing maximum average power.

Figure P10.61

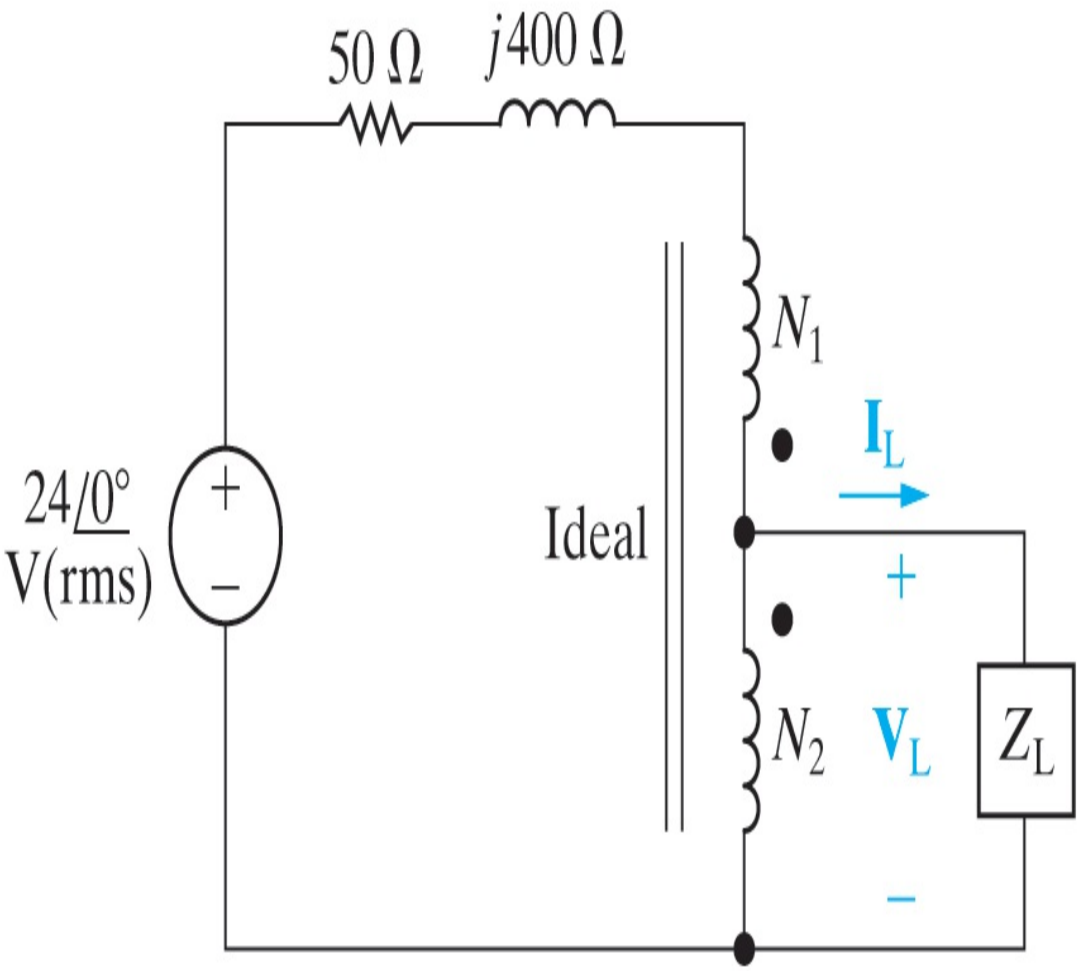


Figure P10.61 Full Alternative Text

22. 10.62

    1. a) Find the turns ratio N1/N2 for the ideal transformer in the circuit in Fig. P10.62 so that maximum average power is delivered to the 400 Ω load.

    2. b) Find the average power delivered to the 400 Ω load.

    3. c) Find the voltage V1.

    4. d) What percentage of the power developed by the ideal current source is delivered to the 400 Ω resistor?

Figure P10.62

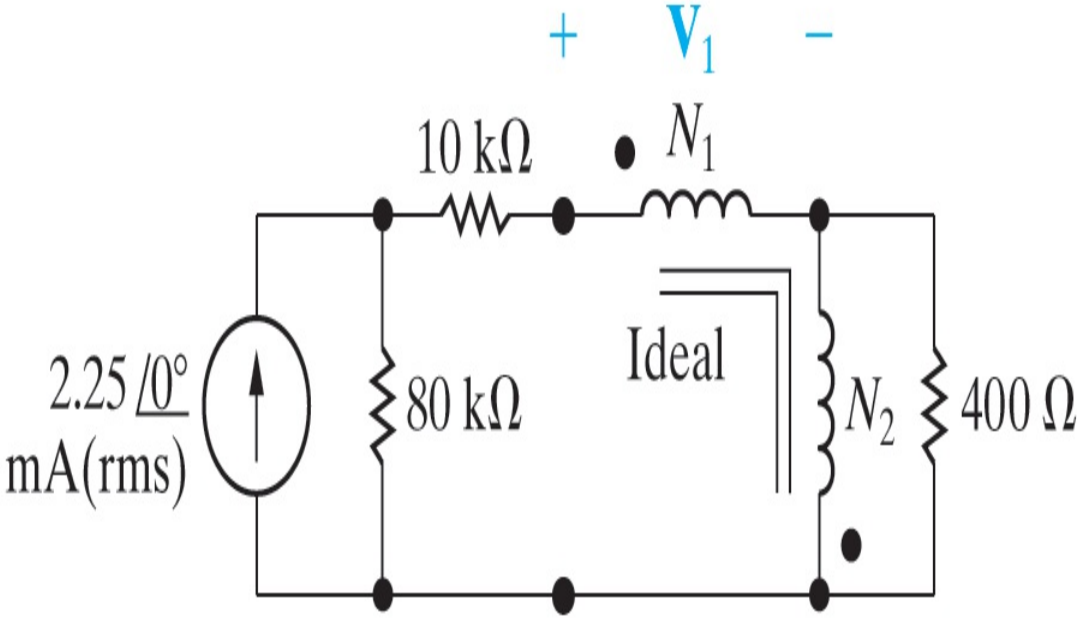


Figure P10.62 Full Alternative Text

23. 10.63 The sinusoidal voltage source in the circuit in Fig. P10.63 is operating at a frequency of 20 krad/s. The variable capacitive reactance in the circuit is adjusted until the average power delivered to the 100 Ω resistor is as large as possible.

    1. a) Find the value of C in microfarads.

    2. b) When C has the value found in (a), what is the average power delivered to the 100 Ω resistor?

    3. c) Replace the 100 Ω resistor with a variable resistor Ro. Specify the value of Ro so that maximum average power is delivered to Ro.

    4. d) What is the maximum average power that can be delivered to Ro?

Figure P10.63

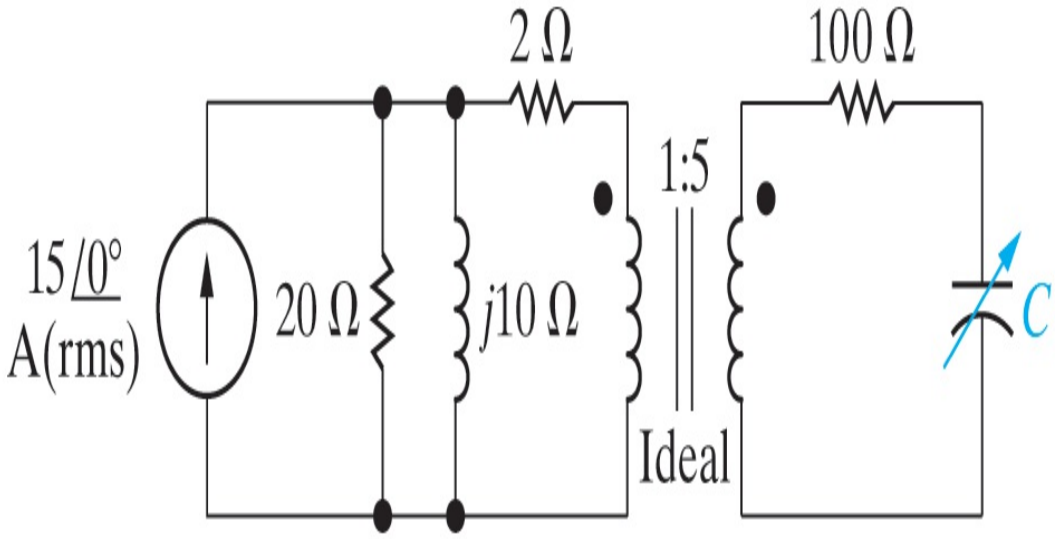


Figure P10.63 Full Alternative Text

24. 10.64 PSPICEMULTISIM The variable load resistor RL in the circuit shown in Fig. P10.64 is adjusted for maximum average power transfer to RL.

    1. a) Find the maximum average power.

    2. b) What percentage of the average power developed by the ideal voltage source is delivered to RL when RL is absorbing maximum average power?

    3. c) Test your solution by showing that the power developed by the ideal voltage source equals the power dissipated in the circuit.

Figure P10.64

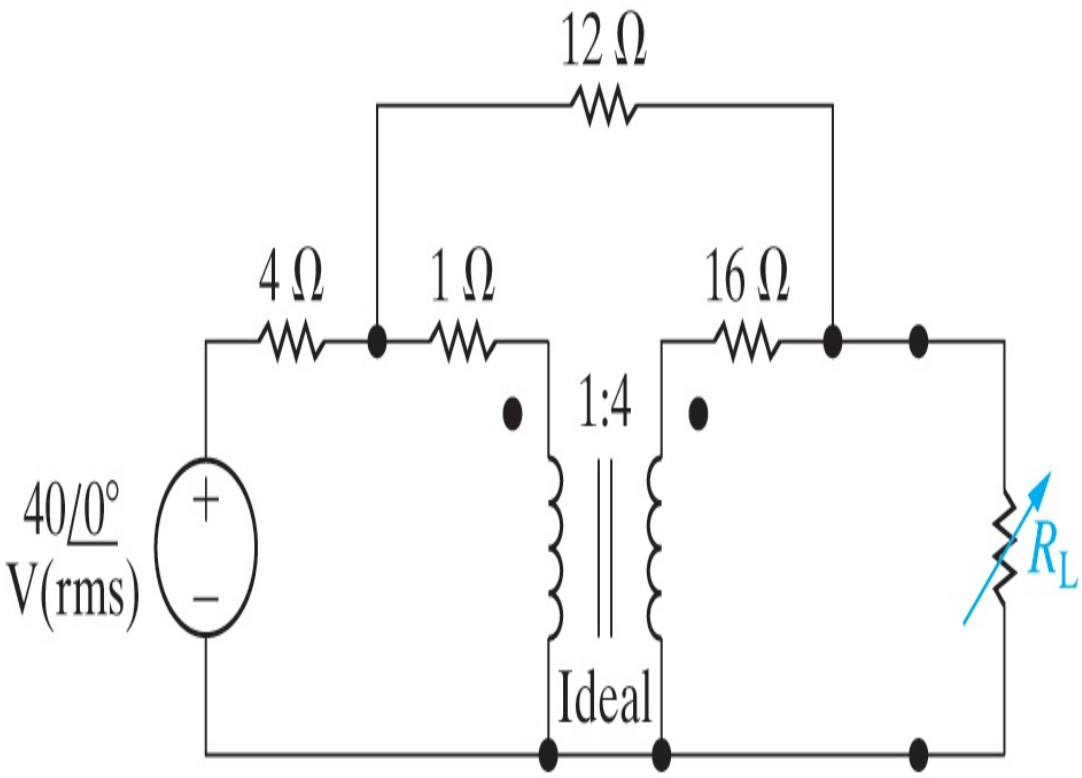


Figure P10.64 Full Alternative Text

25. 10.65 PSPICEMULTISIM Repeat Problem 10.64 for the circuit shown in Fig. P10.65.

Figure P10.65

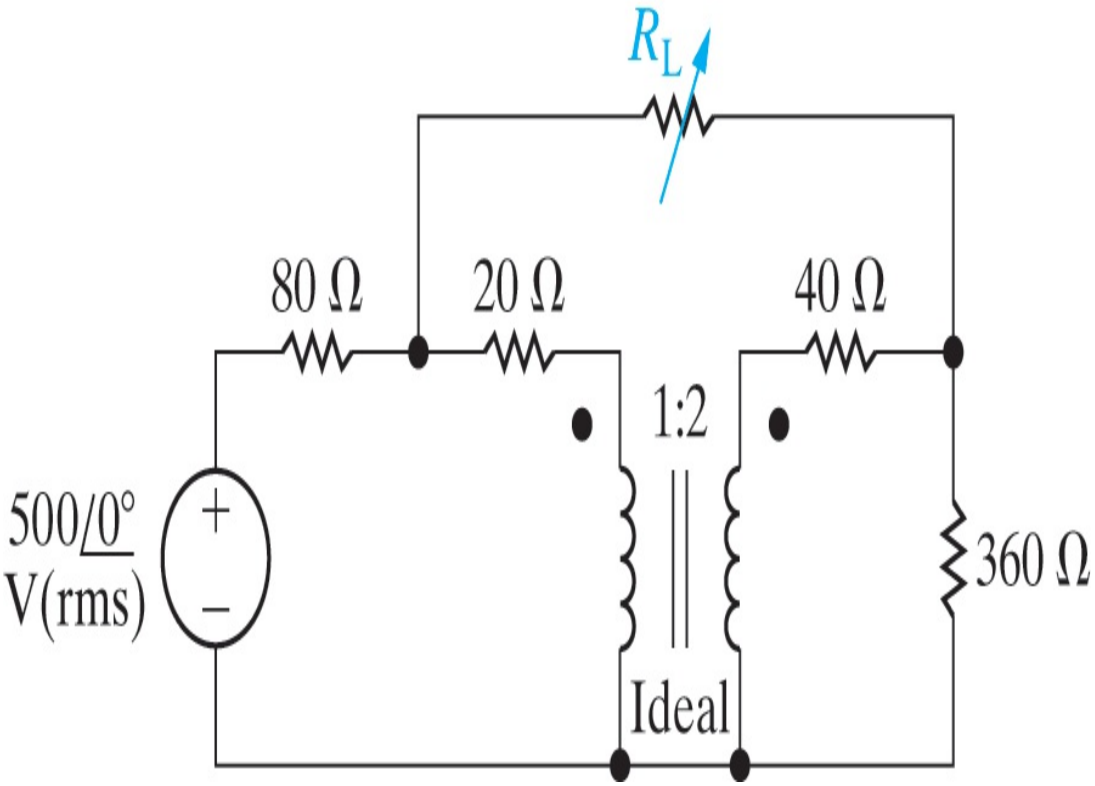


Figure P10.65 Full Alternative Text

26. 10.66

    1. a) If N1 equals 1000 turns, how many turns should be placed on the N2 winding of the ideal transformer in the circuit seen in Fig. P10.66 so that maximum average power is delivered to the 6800 Ω load?

    2. b) Find the average power delivered to the 6800 Ω resistor.

    3. c) What percentage of the average power delivered by the ideal voltage source is dissipated in the linear transformer?

Figure P10.66

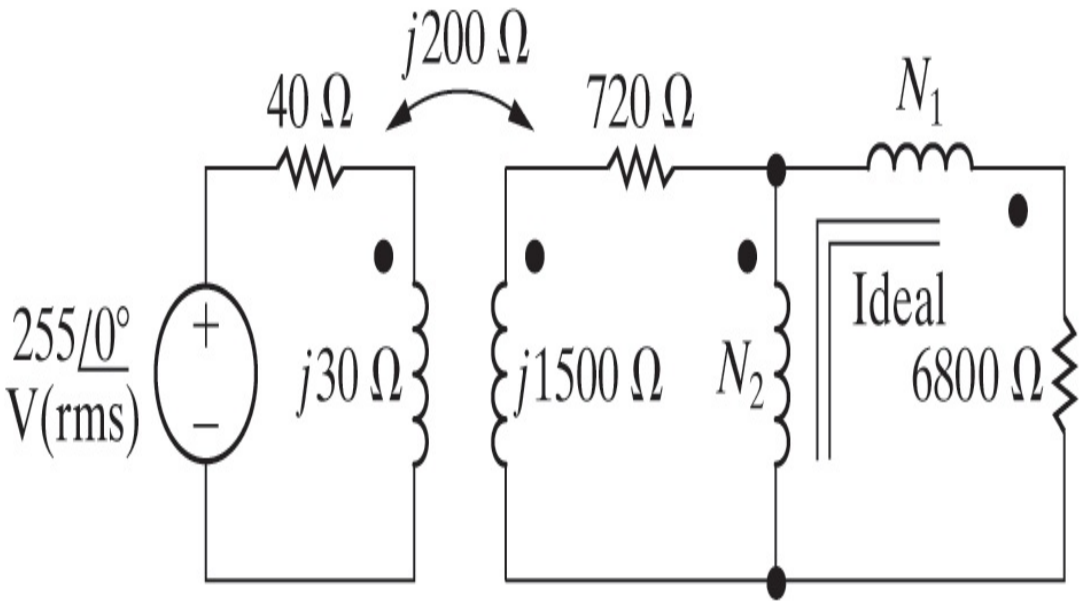


Figure P10.66 Full Alternative Text

Sections 10.1–10.6

1. 10.67 PRACTICALPERSPECTIVE_

    1. a) Use the values in Table 10.3 to calculate the number of kilowatt-hours consumed in one month by a notebook computer AC adapter if every day the computer is charging for 5 hours and sleeping for 19 hours..

    2. b) Repeat the calculation in part (a) assuming that the computer is charging for 5 hours and off for 19 hours.

    3. c) Repeat the calculation in part (a) assuming that the computer is charging for 5 hours and disconnected from the AC adapter for 19 hours, but the AC adapter remains plugged into the wall outlet.

    4. d) Repeat the calculation in part (a) assuming that the computer is charging for 5 hours and the AC adapter is unplugged from the wall outlet for 19 hours.

2. 10.68 PRACTICALPERSPECTIVE_

    1. a) Suppose you use your microwave oven for 12 minutes each day. The remaining time, the oven is ready with the door closed. Use the values in Table 10.3 to calculate the total number of kilowatt-hours used by the microwave oven in one month.

    2. b) What percentage of the power used by the microwave oven in one month is consumed when the oven is ready with the door closed?

3. 10.69 PRACTICALPERSPECTIVE_ Determine the amount of power, in watts, consumed by the transformer in Fig. 10.30. Assume that the voltage source is ideal (Rs=0 Ω), R1=5 Ω, and L1=250 mH. The frequency of the 120 V(rms) source is 60 Hz.

4. 10.70 PRACTICALPERSPECTIVE_ Repeat Problem 10.69, but assume that the linear transformer has been improved so that Rs=50 mΩ. All other values are unchanged.

5. 10.71 PRACTICALPERSPECTIVE_ Repeat Problem 10.69 assuming that the linear transformer in Fig. 10.30 has been replaced by an ideal transformer with a turns ratio of 30:1. (Hint: you shouldn't need to make any calculations to determine the amount of power consumed.)

Chapter 11 Balanced Three-Phase Circuits

Chapter Contents

1. 11.1 Balanced Three-Phase Voltages
2. 11.2 Three-Phase Voltage Sources
3. 11.3 Analysis of the Wye-Wye Circuit
4. 11.4 Analysis of the Wye-Delta Circuit
5. 11.5 Power Calculations in Balanced Three-Phase Circuits
6. 11.6 Measuring Average Power in Three-Phase Circuits

Chapter Objectives

1. Know how to analyze a balanced, three-phase wye-wye connected circuit.

2. Know how to analyze a balanced, three-phase wye-delta connected circuit.

3. Be able to calculate power (average, reactive, and complex) in any three-phase circuit.

We use three-phase circuits to generate, transmit, distribute, and consume large blocks of electric power. The comprehensive analysis of such systems is a field of study in its own right; we cannot cover it in a single chapter. Fortunately, an understanding of the steady-state sinusoidal behavior of balanced three-phase circuits is sufficient for engineers who do not specialize in power systems. We analyze balanced three-phase circuits using several shortcuts based on circuit-analysis techniques discussed in earlier chapters.

The basic structure of a three-phase system consists of voltage sources connected to loads by means of transformers and transmission lines. To analyze such a circuit, we can reduce it to a voltage source connected to a load via a line. The omission of the transformer simplifies the discussion without jeopardizing a basic understanding of the three-phase system. Figure 11.1 on page 414 shows a basic circuit. A defining characteristic of a balanced three-phase circuit is that it contains a set of balanced three-phase voltages at its source. We begin by considering these voltages, and then we move to the voltage and current relationships for the Y-Y and Y-Δ circuits. After considering voltage and current in such circuits, we conclude with sections on power and power measurement.

Figure 11.1 A basic three-phase circuit.

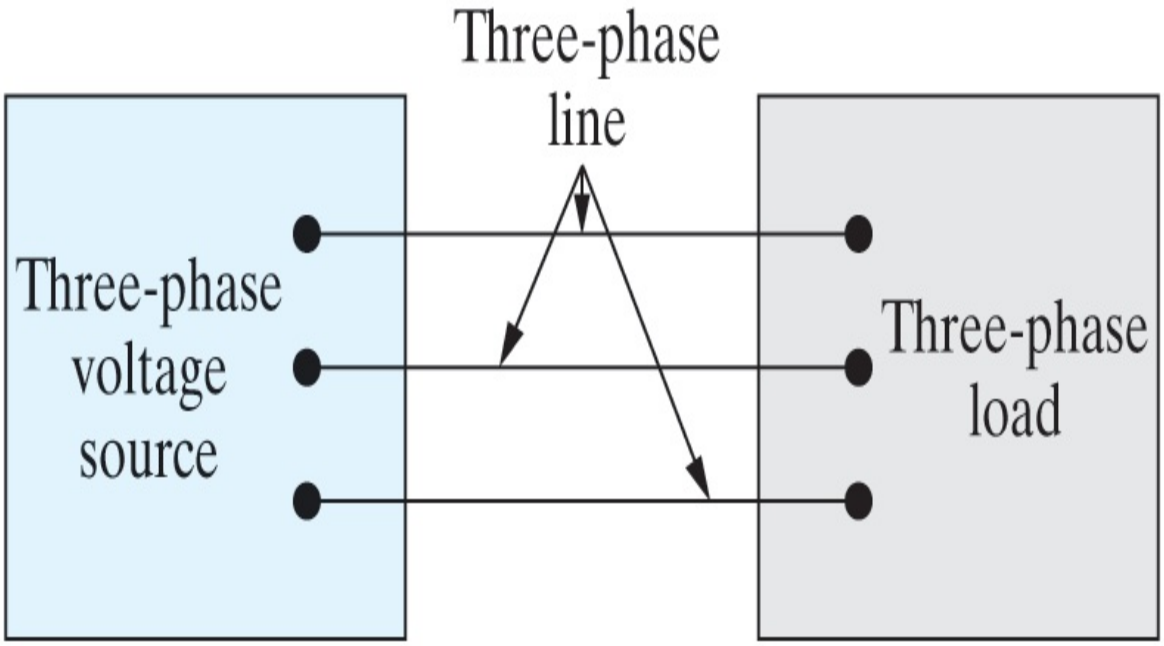


Figure 11.1 Full Alternative Text

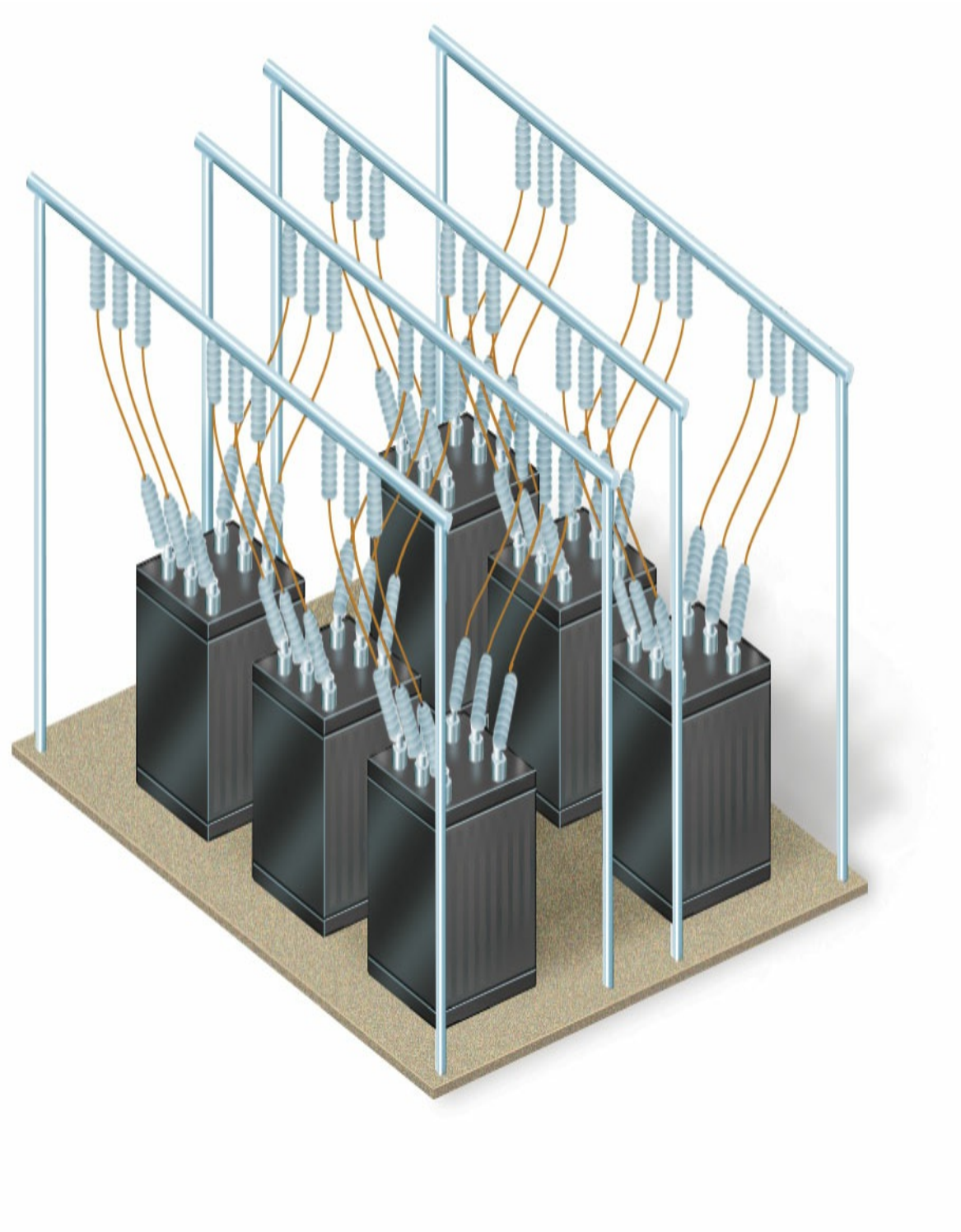
Practical Perspective

Transmission and Distribution of Electric Power

In this chapter, we introduce circuits that are designed to handle large blocks of electric power. These are the circuits used to transport electric power from generating plants to both industrial and residential customers. We introduced a very basic residential customer circuit in the design perspective for Chapter 1. Now we introduce the type of circuit that delivers electric power to an entire residential subdivision.

One of the requirements imposed on electric utilities is to maintain the rms voltage level at the customer's premises. Whether lightly loaded, as at 3:00 a.m., or heavily loaded, as at midafternoon on a hot, humid day, the utility must supply the same rms voltage. To satisfy this requirement, utility systems

place capacitors at strategic locations in the distribution network. The capacitors supply magnetizing vars close to the loads requiring them, adjusting the power factor of the load. We illustrate this concept after we have analyzed balanced three-phase circuits.



Douglas Sacha/Moment/Getty Images

11.1 Balanced Three-Phase Voltages

A set of balanced three-phase voltages consists of three sinusoidal voltages that have identical amplitudes and frequencies but are out of phase with each other by exactly 120°. We refer to the three phases as a, b, and c, and use the a-phase as the reference phase. The three voltages are referred to as the **a-phase voltage**, the **b-phase voltage**, and the **c-phase voltage**.

Only two possible phase relationships can exist between the a-phase voltage and the b- and c-phase voltages. One possibility is for the b-phase voltage to lag the a-phase voltage by 120°, in which case the c-phase voltage must lead the a-phase voltage by 120°. This phase relationship is known as the **abc** (or **positive**) **phase sequence**. Using phasor notation, we see that the abc phase sequence is

$$V_a = V_m \angle 0°, \quad V_b = V_m \angle -120°, \quad V_c = V_m \angle +120°. \tag{11.1}$$

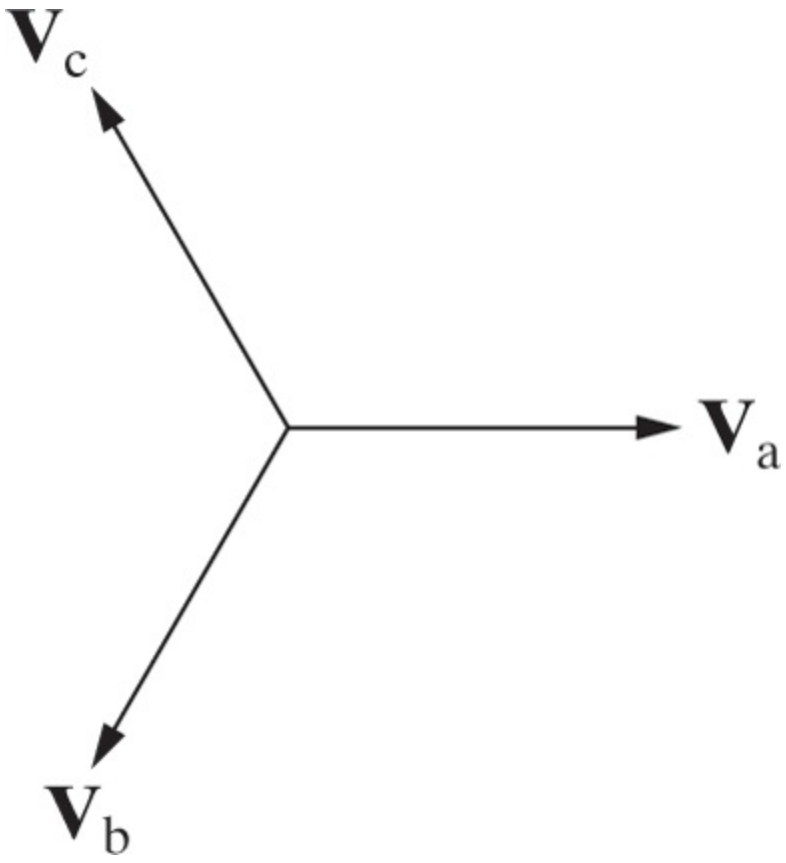
The only other possibility is for the b-phase voltage to lead the a-phase voltage by 120°, in which case the c-phase voltage must lag the a-phase voltage by 120°. This phase relationship is known as the **acb** (or **negative**) **phase sequence**. In phasor notation, the acb phase sequence is

$$V_a = V_m \angle 0°, \quad V_b = V_m \angle +120°, \quad V_c = V_m \angle -120°. \tag{11.2}$$

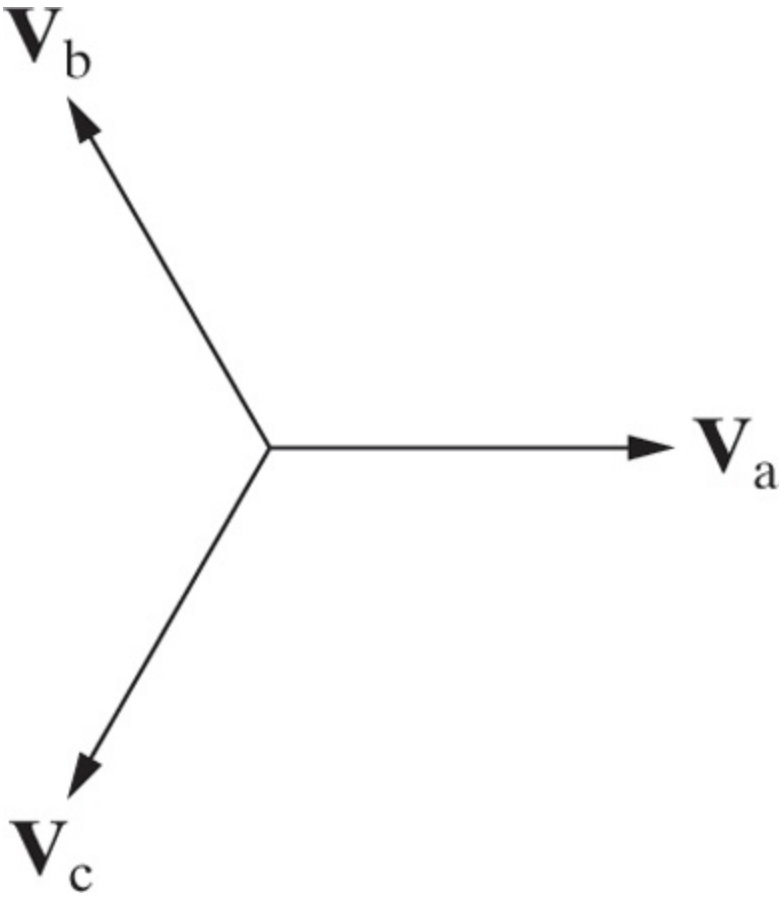
Figure 11.2 shows the phasor diagrams of the voltages in Eqs. 11.1 and 11.2. The phase sequence is the clockwise order of the subscripts around the diagram, starting from Va. You can use the phasor diagrams to show that the sum of the three phasor voltages in a balanced set is zero. This important characteristic can also be derived from Eq. 11.1 or 11.2 to give

Figure 11.2 Phasor diagrams of a balanced set of three-phase

voltages. (a) The abc (positive) sequence. (b) The acb (negative) sequence.



11.1-2 Full Alternative Text



11.1-2 Full Alternative Text

Va + Vb + Vc=0.

Because the sum of the phasor voltages is zero, the sum of the instantaneous voltages also is zero; that is,

va + vb + vc=0.

Now that we know the nature of a balanced set of three-phase voltages, we can state the first of the analytical shortcuts alluded to in the introduction to this chapter: if we know the phase sequence and one voltage in the set, we know the entire set. Thus, for a balanced three-phase system, once we determine the voltages (or currents) in one phase, we can use the 120° phase angle difference and the phase sequence to find the voltages or currents in the remaining two phases.

SELF-CHECK: *Assess your understanding of three-phase voltages by trying*

Chapter Problems 11.1 and 11.2.

11.2 Three-Phase Voltage Sources

A three-phase voltage source is a generator with three separate windings distributed around the periphery of the stator. Each winding comprises one phase of the generator. The rotor of the generator is an electromagnet driven at synchronous speed by a prime mover, such as a steam or gas turbine. Rotation of the electromagnet induces a sinusoidal voltage in each winding. The phase windings are designed so that the sinusoidal voltages induced in them are equal in amplitude and out of phase with each other by 120°. The phase windings are stationary with respect to the rotating electromagnet, so the frequency of the voltage induced in each winding is the same. Figure 11.3 shows a sketch of a two-pole three-phase source.

Figure 11.3 A sketch of a three-phase voltage source.

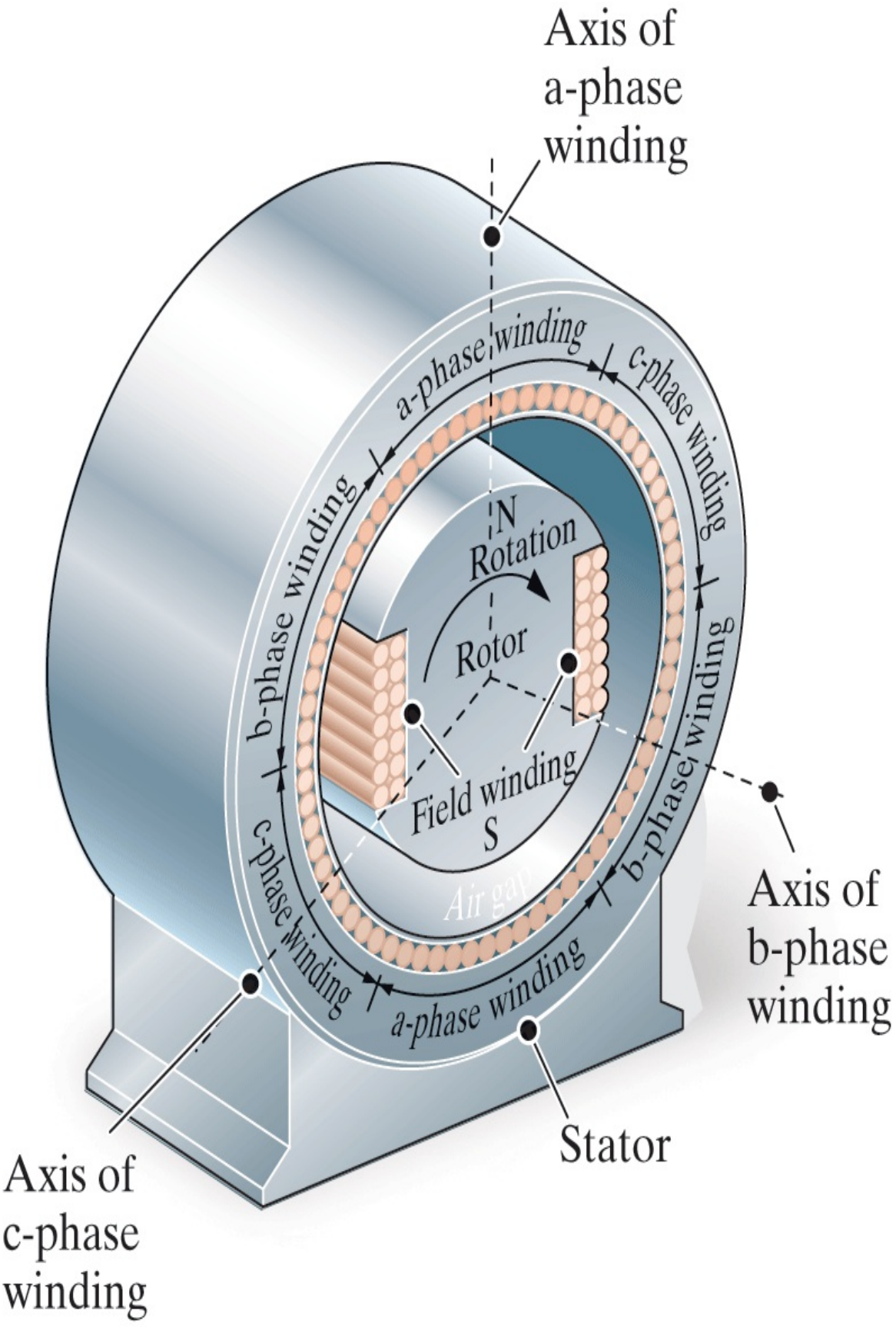
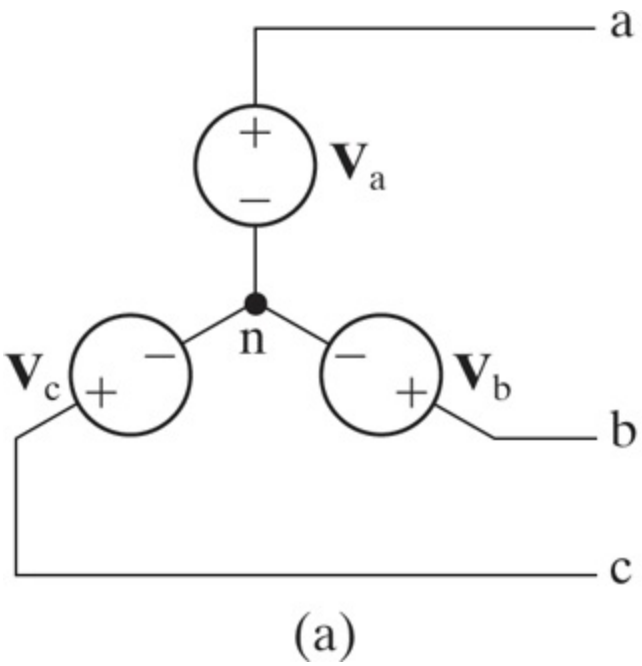

Axis of
a-phase
winding
a-phase winding
c-phase winding
b-phase winding
N
Rotation
Rotor
Field winding
S
Air gap
c-phase winding
b-phase winding
a-phase winding
Axis of
b-phase
winding
Stator
Axis of
c-phase
winding

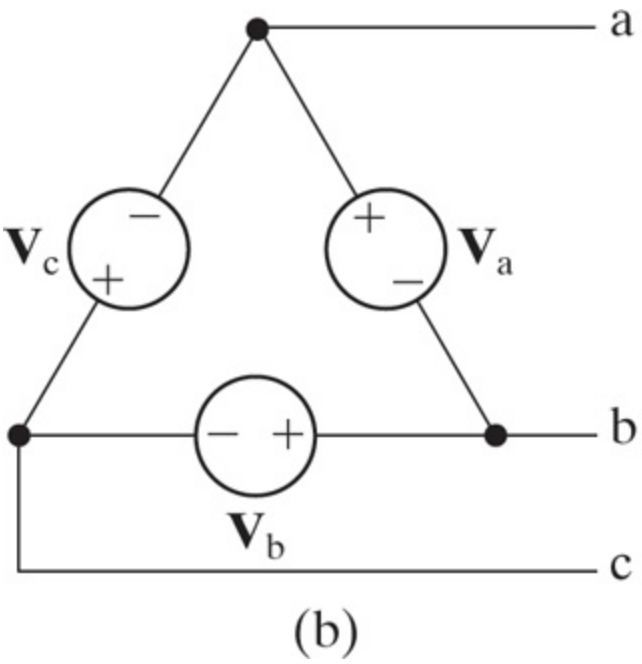
Figure 11.3 Full Alternative Text

There are two ways of interconnecting the separate phase windings to form a three-phase source: as a wye (Y) or as a delta (Δ). Figure 11.4 shows both, with ideal voltage sources modeling the phase windings of the three-phase generator. The common terminal in the Y-connected source, labeled n in Fig. 11.4(a), is called the **neutral terminal** of the source, which may or may not be available for external connections.

Figure 11.4 The two basic connections of an ideal three-phase source. (a) A Y-connected source. (b) A Δ-connected source.



11.2-2 Full Alternative Text

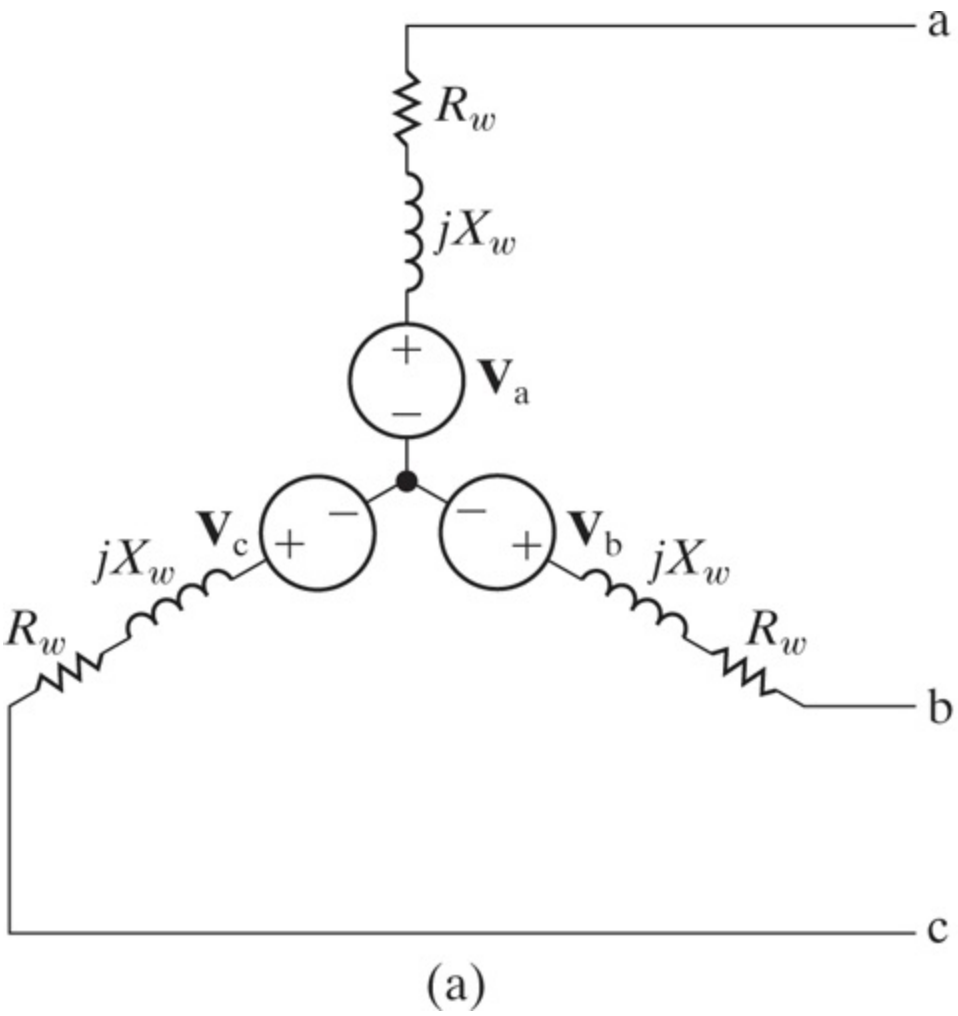


11.2-2 Full Alternative Text

When constructing a circuit model of a three-phase generator, we need to consider the impedance of each phase winding. Sometimes it is so small (compared with other impedances in the circuit) that we don't include it in the generator model; the model then consists solely of ideal voltage sources, as in Fig. 11.4. If the impedance of each phase winding is not negligible, we connect an inductive winding impedance in series with an ideal sinusoidal voltage source in each phase. Because the winding construction in each phase is the same, we make the winding impedances in each phase identical. Figure 11.5 shows two models of a three-phase generator, one using a Y connection and the other using a Δ connection. In both models, Rw is the winding resistance, and Xw is the inductive reactance of the winding.

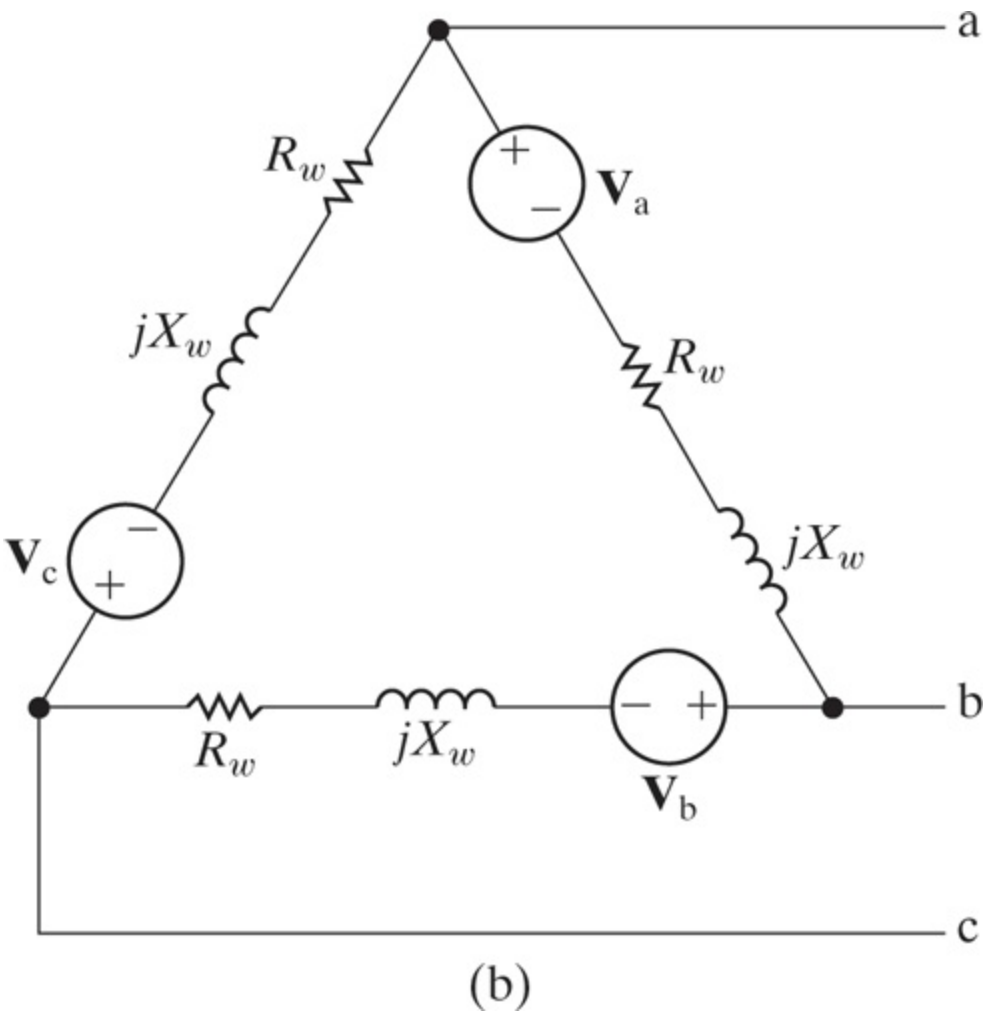
Figure 11.5 A model of a three-phase source with winding impedance: (a) a Y-connected source; and (b) a Δ-connected

source.



(a)

[11.2-2 Full Alternative Text]



(b)

11.2-2 Full Alternative Text

Because both three-phase sources and three-phase loads can be either Y-connected or Δ-connected, the basic circuit in Fig. 11.1 represents four different configurations:

Source	Load
Y	Y
Y	Δ
Δ	Y
Δ	Δ

We begin by analyzing the Y-Y circuit. The remaining three configurations can be reduced to a Y-Y equivalent circuit, so analysis of the Y-Y circuit is the key to solving all balanced three-phase arrangements. We then illustrate how to transform a Y-Δ circuit to a Y-Y circuit and leave the analysis of the Δ-Y and Δ-Δ circuits to you in the Problems.

11.3 Analysis of the Wye-Wye Circuit

Figure 11.6 illustrates a general Y-Y circuit, in which we included a fourth conductor that connects the source neutral to the load neutral. A fourth conductor is possible only in the Y-Y arrangement. For convenience, we transformed the Y connections into “tipped-over tees.” In Fig. 11.6, Zga, Zgb, and Zgc represent the internal impedance associated with each phase winding of the voltage generator; Zl1, Zlb and Zlc represent the impedance of the lines connecting a phase of the source to a phase of the load; Z0 is the impedance of the neutral conductor connecting the source neutral to the load neutral; and ZA, ZB, and ZC represent the impedance of each phase of the load.

Figure 11.6 A three-phase Y-Y system.

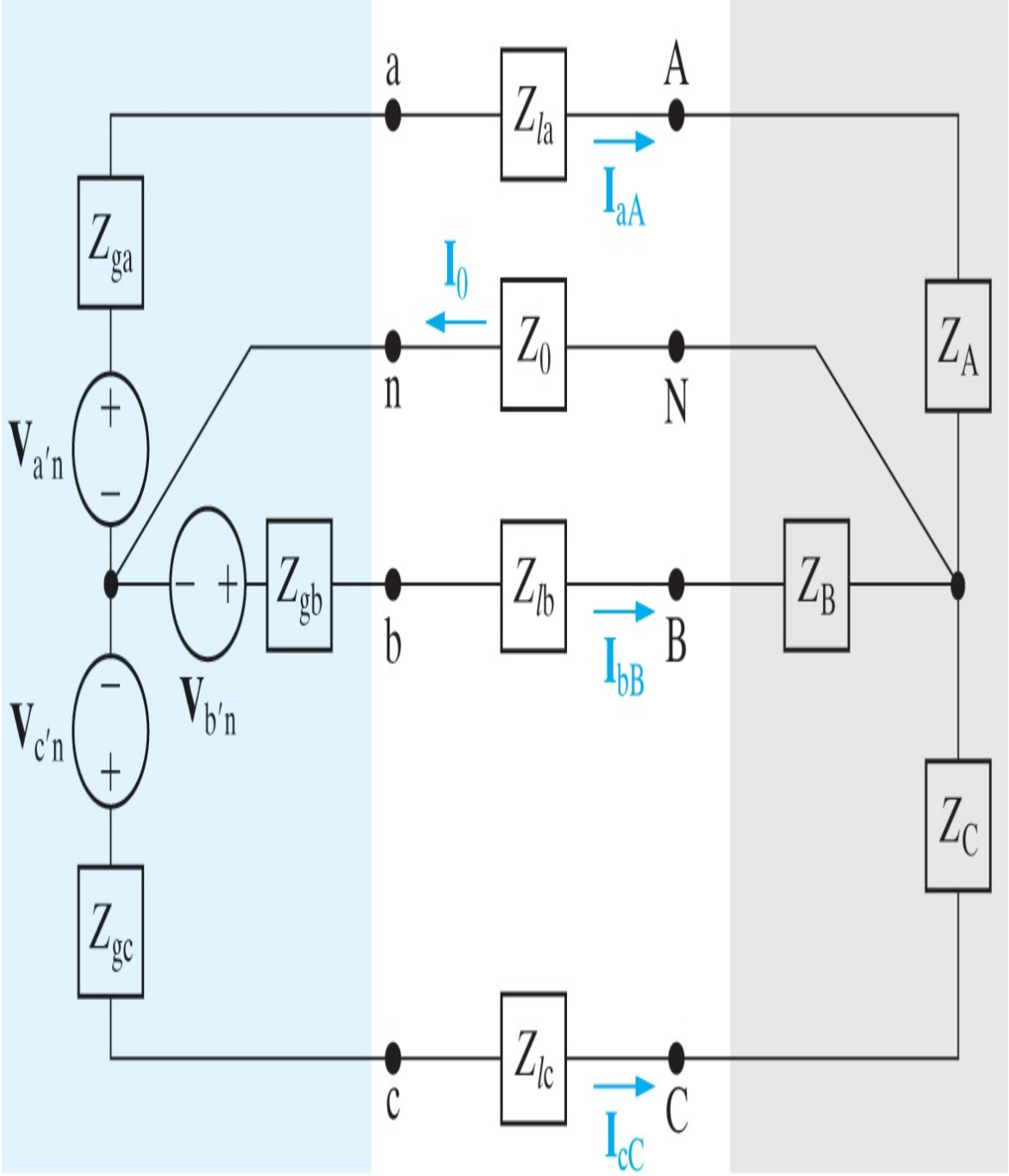


Figure 11.6 Full Alternative Text

We can describe this circuit with a single KCL equation. Using the source neutral as the reference node and letting VN denote the node voltage between the nodes N and n, we get

VNZ0 + VN−Va′nZA + Zla + Zga + VN−Vb′nZB + Zlb + Zgb + VN−Vc

′nZC + Zlc + Zgc=0.

What happens to the KCL equation if the three-phase circuit in Fig. 11.6 is balanced? To answer this question, we must formally define a balanced three-phase circuit. A three-phase circuit is balanced if it satisfies the following criteria:

Conditions for a Balanced Three-Phase Circuit

1. The voltage sources form a balanced three-phase set.
2. The impedance of each phase of the voltage source is the same.
3. The impedance of each line is the same.
4. The impedance of each phase of the load is the same.

There is no restriction on the impedance of the neutral conductor; its value has no effect on whether the system is balanced.

The circuit in Fig. 11.6 is balanced if

1. Va′n, Vb′n, and Vc′n are a set of balanced three-phase voltages.
2. Zga=Zgb=Zgc.
3. Zla=Zlb=Zlc.
4. ZA=ZB=ZC.

If the circuit in Fig. 11.6 is balanced, we can rewrite the KCL equation at VN as

VN(1Z0 + 3Zϕ)=Va′n + Vb′n + Vc′nZϕ,(11.3)

where

$Z\varphi = ZA + Zla + Zga = ZB + Zlb + Zgb = ZC + Zlc + Zgc.$

The right-hand side of Eq. 11.3 is zero because by hypothesis the numerator is a set of balanced three-phase voltages and $Z\phi$ is not zero. The only value of VN that satisfies Eq. 11.3 is zero. Therefore, for a balanced three-phase circuit,

$VN = 0.$ (11.4)

When a three-phase circuit is balanced, VN is zero, so the voltage between the source neutral, n, and the load neutral, N, is zero. Consequently, the current in the neutral conductor is zero, so we can either remove the neutral conductor from a balanced Y-Y configuration (I0=0) or replace it with a short circuit between the nodes n and N (VN=0). Both equivalents are convenient to use when modeling balanced three-phase circuits.

If the three-phase circuit in Fig. 11.6 is balanced, the three line currents are

$IaA = \frac{Va'n - VN}{ZA + Zla + Zga} = \frac{Va'n}{Z\phi}, \quad IbB = \frac{Vb'n - VN}{ZB + Zlb + Zgb} = \frac{Vb'n}{Z\phi}, \quad IcC = \frac{Vc'n - VN}{ZC + Zlc + Zgc} = \frac{Vc'n}{Z\phi}.$ (11.5)

From these equations, we see that the three line currents form a balanced set; that is, the current in each line is equal in amplitude and frequency and is 120° out of phase with the other two line currents. Thus, if we calculate the current IaA and we know the phase sequence, we have a shortcut for finding IbB and IcC. This is the same shortcut used to find the b- and c-phase source voltages from the a-phase source voltage.

Terminology

We use Fig. 11.6 to define some important terms.

- **Line voltage** is the voltage across any pair of lines. In Fig. 11.6, the three line voltages at the load are VAB, VBC, and VCA.

- **Phase voltage** is the voltage across a single phase. In Fig. 11.6, the three phase voltages at the load are VAN, VBN, and VCN.
- **Line current** is the current in a single line. In Fig. 11.6, the three line currents are IaA, IbB, and IcC.
- **Phase current** is the current in a single phase. In Fig. 11.6, the three phase currents for the load are IAN, IBN, and ICN.

From Fig. 11.6 you can see that when the load is Y-connected; the line current and phase current in each phase are identical, but the line voltage and phase voltage in each phase are different.

The Greek letter phi (ϕ) is widely used in the literature to denote a per-phase quantity. Thus, $V\phi$, $I\phi$, $Z\phi$, $P\phi$, and $Q\phi$ are interpreted as voltage-per-phase, current-per-phase, impedance-per-phase, real power-per-phase, and reactive power-per-phase, respectively.

Since three-phase systems are designed to handle large blocks of electric power, all voltage and current specifications are rms values. When voltage ratings are given, they refer specifically to the rating of the line voltage. Thus, when a three-phase transmission line is rated at 345 kV, the value of the line-to-line voltage is 345,000 V(rms). In this chapter, we express all voltages and currents as rms values.

Constructing a Single-Phase Equivalent Circuit

We can use Eq. 11.5 to construct an equivalent circuit for the a-phase of the balanced Y-Y circuit. From this equation, the line current in the a-phase is the voltage generated in the a-phase winding of the generator divided by the total impedance in the a-phase of the circuit. Thus, Eq. 11.5 describes the circuit shown in Fig. 11.7, in which the neutral conductor has been replaced by a short circuit. The circuit in Fig. 11.7 is the **single-phase equivalent circuit** for a balanced three-phase circuit. Once we solve this circuit, we can write

down the voltages and currents in the other two phases, using the relationships among the phases. Thus, drawing a single-phase equivalent circuit is an important first step in analyzing a balanced three-phase circuit.

Figure 11.7 A single-phase equivalent circuit.

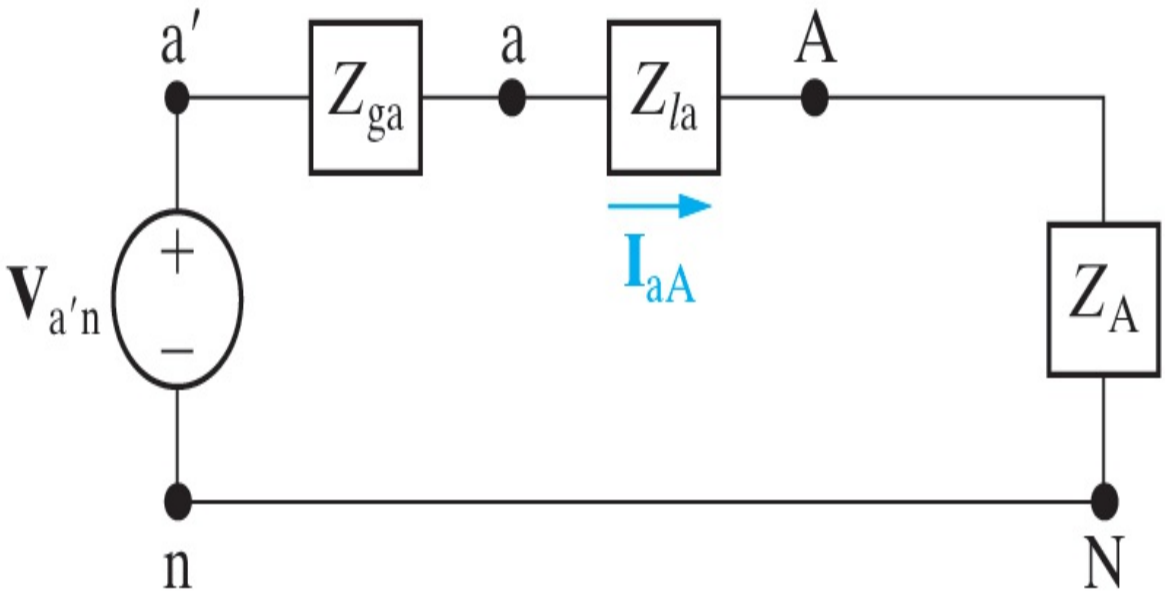


Figure 11.7 Full Alternative Text

Note that the current in the neutral conductor in Fig. 11.7 is IaA. This is not the same as the current in the neutral conductor of the balanced three-phase circuit, which is

Io=IaA + IbB + IcC.

Thus, the circuit shown in Fig. 11.7 gives the correct value of the line current but only the a-phase component of the neutral current. In a balanced three-phase circuit, the line currents form a balanced three-phase set, and Io=0.

Once we know the line current (IaA) in Fig. 11.7, we know the phase current

(IAN) because they are equal. We can also use Ohm's law to calculate the phase voltage (VAN) from the phase current and the load impedance. But how do we calculate the line voltage, VAB? The line voltage is not even shown in the single-phase equivalent circuit in Fig. 11.7. In a Y-connected load, the line voltages and the phase voltages are related. Using Fig. 11.8, we can describe those relationships using KVL:

Figure 11.8 Line-to-line and line-to-neutral voltages.

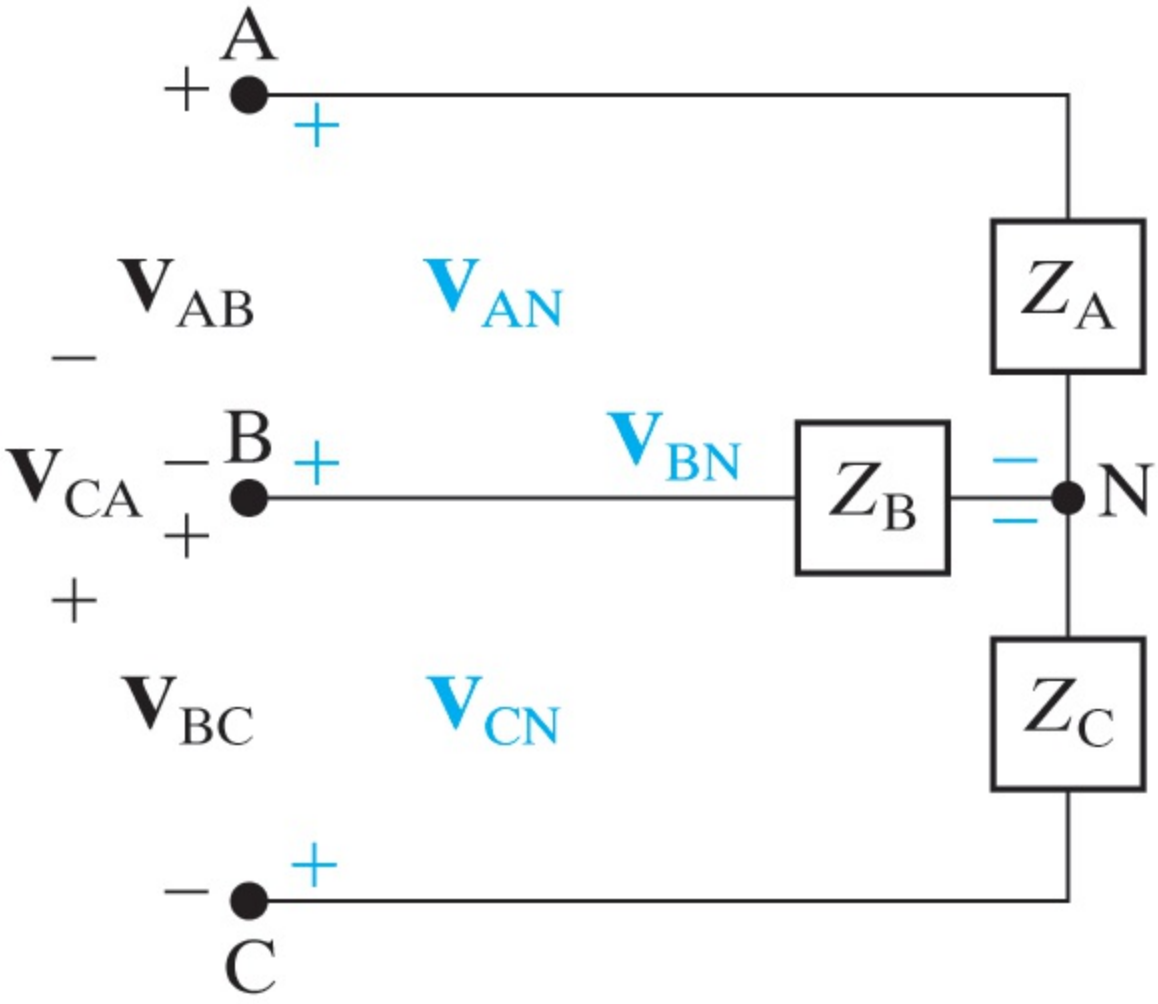


Figure 11.8 Full Alternative Text

$V_{AB} = V_{AN} - V_{BN}$, $V_{BC} = V_{BN} - V_{CN}$, $V_{CA} = V_{CN} - V_{AN}$.

To derive the relationship between the line voltages and the phase voltages, we assume a positive, or abc, sequence. Using the phase voltage in the a-phase as the reference,

$V_{AN} = V_\phi \angle 0°$, $V_{BN} = V_\phi \angle -120°$, $V_{CN} = V_\phi \angle +120°$,

where V_ϕ represents the magnitude of the phase voltage. Substituting the phase voltages into the equations for the line voltages yields

$$V_{AB} = V_\phi \angle 0° - V_\phi \angle -120° = 3V_\phi \angle 30°, \quad (11.6)$$
$$V_{BC} = V_\phi \angle -120° - V_\phi \angle 120° = 3V_\phi \angle -90°, \quad (11.7)$$
$$V_{CA} = V_\phi \angle 120° - V_\phi \angle 0° = 3V_\phi \angle 150°. \quad (11.8)$$

Equations 11.6–11.8 reveal that

1. The magnitude of the line voltage is 3 times the magnitude of the phase voltage.

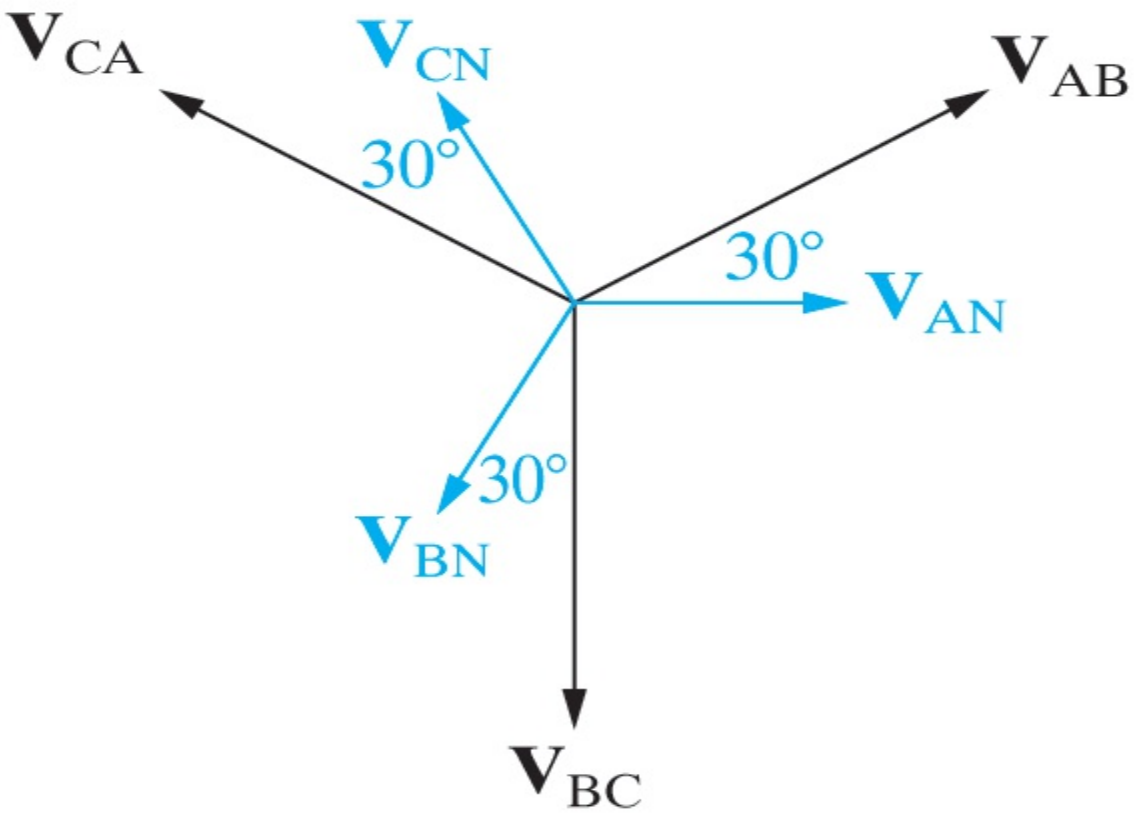
2. The line voltages form a balanced three-phase set.

3. The set of line voltages leads the set of phase voltages by 30°

We leave it to you to demonstrate that for a negative sequence, the only change is that the set of line voltages lags the set of phase voltages by 30°. The phasor diagrams shown in Fig. 11.9 summarize these observations. Here, again, is a shortcut in the analysis of a balanced system: If you know any phase voltage in the circuit, say V_{BN}, you can determine the corresponding line voltage, which is V_{BC}, and vice versa.

Figure 11.9 Phasor diagrams showing the relationship between line-to-line and line-to-

neutral voltages in a balanced system. (a) The abc sequence. (b) The acb sequence.


$\mathbf{V}_{CA}$
$\mathbf{V}_{CN}$
$\mathbf{V}_{AB}$
30°
30°
$\mathbf{V}_{AN}$
30°
$\mathbf{V}_{BN}$
$\mathbf{V}_{BC}$

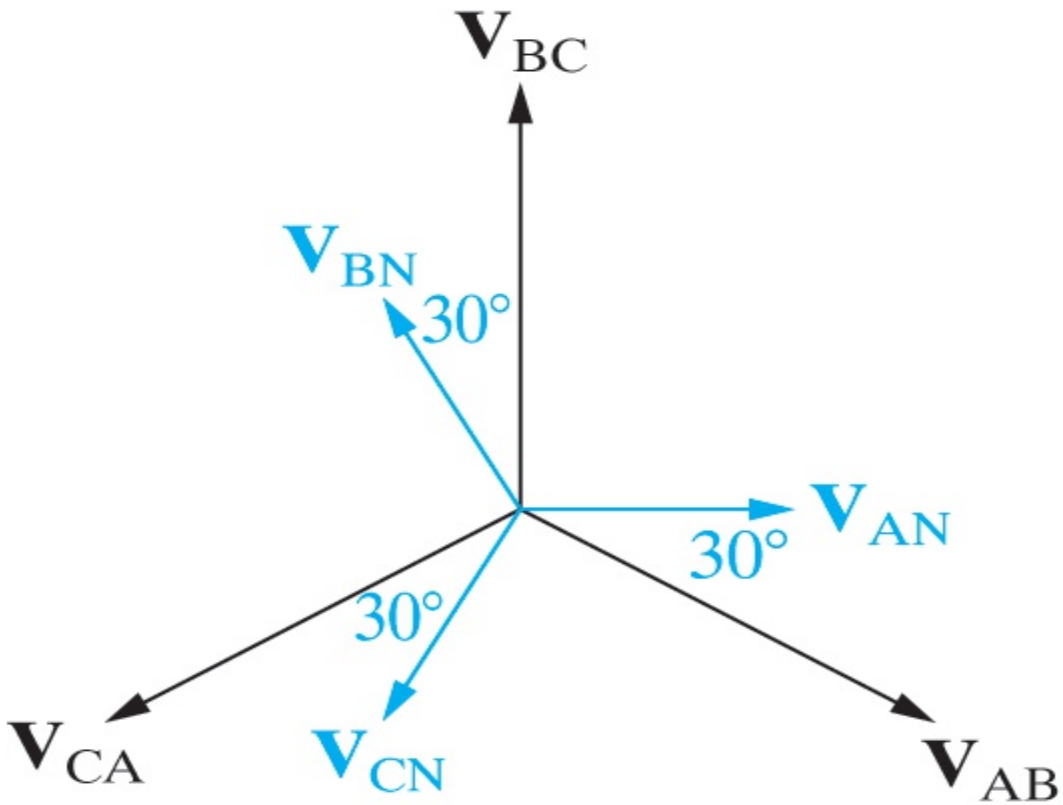

$\mathbf{V}_{BC}$
$\mathbf{V}_{BN}$
30°
$\mathbf{V}_{AN}$
30°
30°
$\mathbf{V}_{CA}$
$\mathbf{V}_{CN}$
$\mathbf{V}_{AB}$

Figure 11.9 Full Alternative Text

Example 11.1 shows how to use a single-phase equivalent circuit to solve a balanced three-phase Y-Y circuit.

Example 11.1 Analyzing a Wye-Wye Circuit

A balanced three-phase Y-connected generator with positive sequence has an impedance of 0.2 + j0.5 Ω/ϕ and an internal voltage of 120 V/ϕ. The generator feeds a balanced three-phase Y-connected load having an impedance of 39 + j28 Ω/ϕ. The impedance of the line connecting the generator to the load is 0.8 + j1.5 Ω/ϕ. The internal voltage of the generator in the a-phase is the reference phasor.

1. Construct the a-phase equivalent circuit of the system.
2. Calculate the three line currents IaA, IbB, and IcC.
3. Calculate the three phase voltages at the load, VAN, VBN, and VCN.
4. Calculate the line voltagesVAB, VBC, and VCA at the terminals of the load.
5. Calculate the phase voltages at the terminals of the generator, Van, Vbn, and Vcn.
6. Calculate the line voltages Vab, Vbc, and Vca at the terminals of the generator.
7. Repeat (a)–(f) for a negative phase sequence.

Solution

1. Figure 11.10 shows the single-phase equivalent circuit.

Figure 11.10 The single-phase equivalent circuit for Example 11.1.

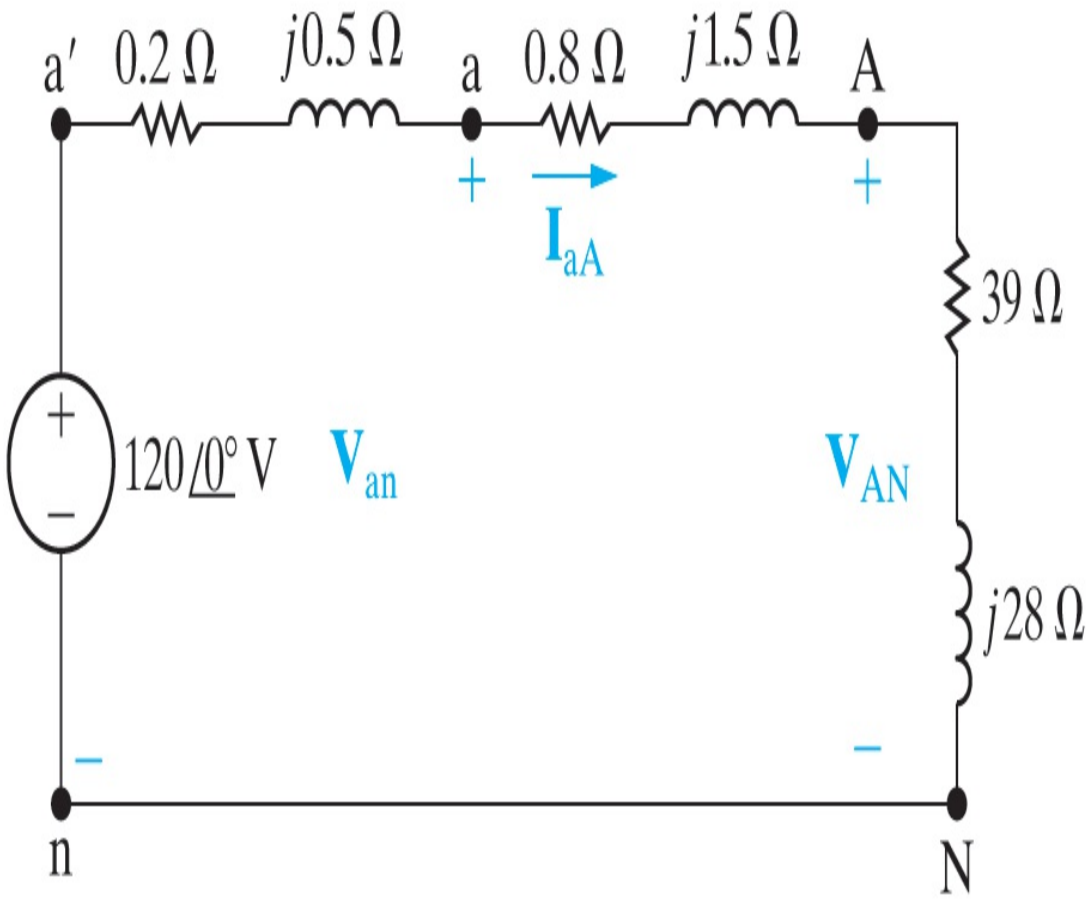


Figure 11.10 Full Alternative Text

2. The a-phase line current is the voltage in the a-phase divided by the total impedance in the a-phase:

   I aA = 120 0° (0.2 + 0.8 + 39) + j(0.5 + 1.5 + 28) = 120 0° 40 + j30 = 2.4 −36.87° A.

For a positive phase sequence,

$I_{bB} = 2.4 \angle -156.87°$ A, $I_{cC} = 2.4 \angle 83.13°$ A.

3. The line current and the phase current in the a-phase are equal. The phase voltage across the load in the a-phase is the product of the single-phase impedance of the load and the phase current in the load:

   $V_{AN} = (39 + j28)(2.4 \angle -36.87°) = 115.22 \angle -1.19°$ V.

   For a positive phase sequence,

   $V_{BN} = 115.22 \angle -121.19°$ V,
   $V_{CN} = 115.22 \angle 118.81°$ V.

4. For a positive phase sequence, the magnitude of the line voltages is 3 times the magnitude of the phase voltages, and the line voltages lead the phase voltages by 30°. Thus

   $V_{AB} = (\sqrt{3} \angle 30°) V_{AN} = 199.58 \angle 28.81°$ V, $V_{BC} = 199.58 \angle -91.19°$ V, $V_{CA} = 199.58 \angle 148.81°$ V.

5. The phase voltage of the source in the a-phase is the voltage of the ideal source minus the voltage across the source impedance. Therefore,

   $V_{an} = 120 - (0.2 + j0.5)(2.4 \angle -36.87°) = 120 - 1.29 \angle 31.33° = 118.90 - j0.67 = 118.90 \angle -0.32°$ V.

   For a positive phase sequence,

   $V_{bn} = 118.90 \angle -120.32°$ V, $V_{cn} = 118.90 \angle 119.68°$ V.

6. The line voltages at the source terminals are

   $V_{ab} = (\sqrt{3} \angle 30°) V_{an} = 205.94 \angle 29.68°$ V, $V_{bc} = 205.94 \angle -90.32°$ V, $V_{ca} = 205.94 \angle 149.68°$ V.

7. Changing the phase sequence has no effect on the single-phase equivalent circuit. The three line currents are

I aA = 2.4 ∠−36.87° A, I bB = 2.4 ∠83.13° A, I cC = 2.4 ∠−156.87° A.

The phase voltages at the load are

V AN = 115.22 ∠−1.19° V, V BN = 115.22 ∠118.81° V, V CN = 115.22 ∠−121.19° V.

For a negative phase sequence, the line voltages lag the phase voltages by 30°:

V AB = (3 ∠30°) V AN = 199.58 ∠−31.19° V, V BC = 199.58 ∠88.81° V, V CA = 199.58 ∠−151.19° V.

The phase voltages at the terminals of the generator are

V an = 118.90 ∠−0.32° V, V bn = 118.90 ∠119.68° V, V cn = 118.90 ∠−120.32° V.

The line voltages at the terminals of the generator are

V ab = (3 ∠−30°) V an = 205.94 ∠−30.32° V, V bc = 205.94 ∠89.68° V, V ca = 205.94 ∠−150.32° V.

Assessment Problems

Objective 1—Know how to analyze a balanced, three-phase wye-wye circuit

1. 11.1 The voltage from A to N in a balanced three-phase circuit is 240 ∠−30° V. If the phase sequence is positive, what is the value of VBC?

   Answer:

   415.69 ∠−120°V.

2. 11.2 The c-phase voltage of a balanced three-phase Y-connected system is 450 −25° V. If the phase sequence is negative, what is the value of VAB?

   Answer:

   779.42 65° V.

3. 11.3 The phase voltage at the terminals of a balanced three-phase Y-connected load is 2400 V. The load has an impedance of 16 + j12 Ω/ϕ and is fed from a line having an impedance of 0.10 + j0.80 Ω/ϕ. The Y-connected source at the sending end of the line has a phase sequence of acb and an internal impedance of 0.02 + j0.16 Ω/ϕ. Use the a-phase voltage at the load as the reference and calculate (a) the line currents IaA, IbB, and IcC; (b) the line voltages at the source, Vab, Vbc, and Vca; and (c) the internal phase-to-neutral voltages at the source, Va′n, Vb′n, and Vc′n.

   Answer:

   1. (a) IaA=120 −36.87° A,IbB=120 83.13° A, andIcC=120 −156.87° A;

   2. (b) V ab = 4275.02 −28.38° V, V bc = 4275.02 91.62° V, and V ca = 4275.02 −148.38° V;

   3. (c) V a'n = 2482.05 − 1.93° V, V b'n = 2482.05 121.93° V, and V c'n = 2482.05 −118.07° V.

SELF-CHECK: *Also try Chapter Problems 11.9, 11.11, and 11.12.*

11.4 Analysis of the Wye-Delta Circuit

A balanced, three-phase, Y-Δ circuit is shown in Fig. 11.11. From this circuit you can see that the line voltage and the phase voltage in each phase of the load are the same. For example, VAN=VAB. But the line current and the phase current in each phase of the load are not the same. This is an important difference when comparing the Y-Δ circuit with the Y-Y circuit, where the line and phase *currents* are the same in each phase of the load but the line and phase *voltages* are different.

Figure 11.11 A balanced three-phase Y-Δ system.

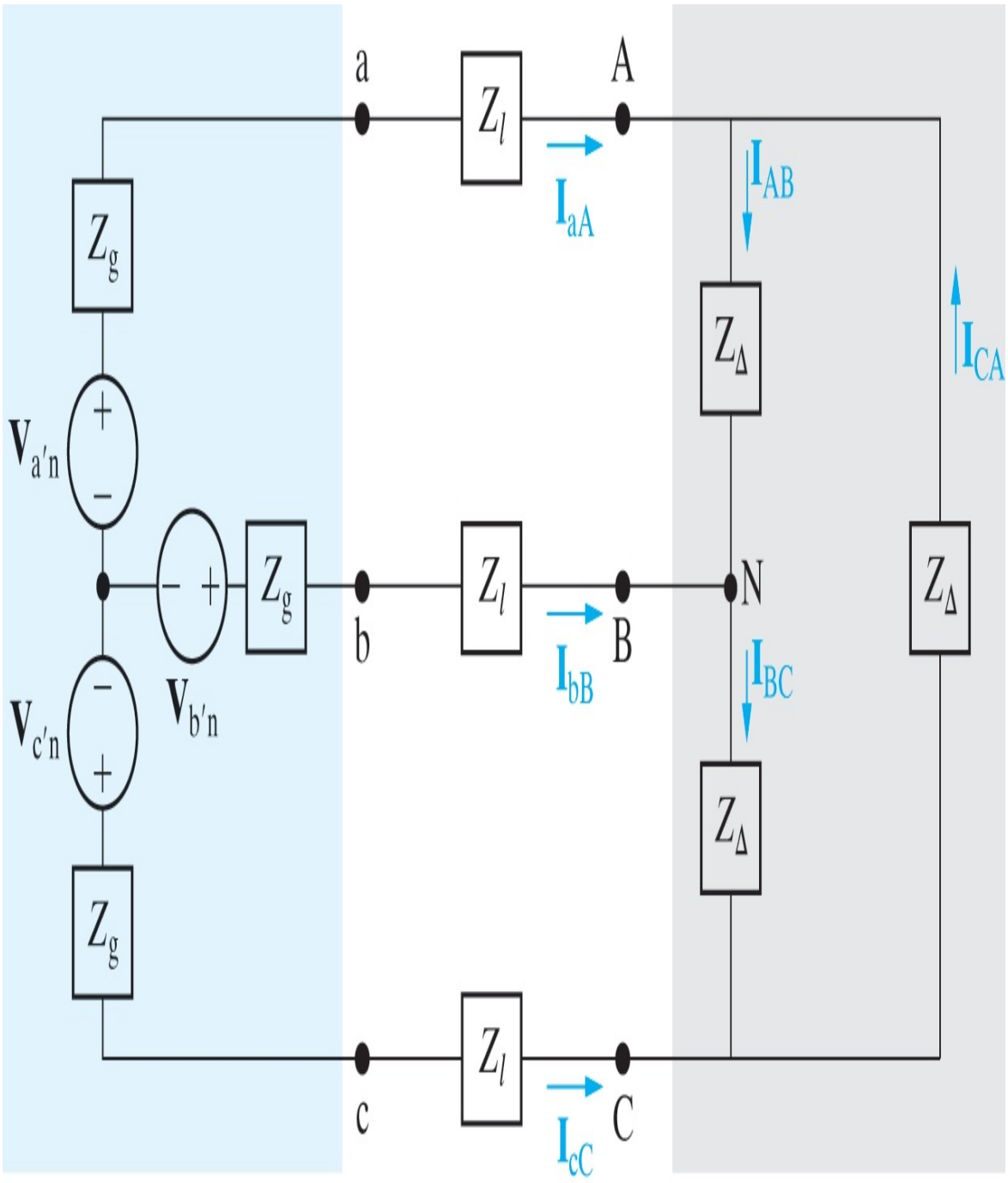


Figure 11.11 Full Alternative Text

When the load in a three-phase circuit is connected in a delta, it can be transformed into a wye by using the delta-to-wye transformation discussed in Section 9.6. When the load is balanced, the impedance of each leg of the wye is one-third the impedance of each leg of the delta, or

Relationship between Delta-Connected and Wye-Connected Impedances

ZY=ZΔ3, (11.9)

which follows directly from Eqs. 9.21–9.23. After the Δ load has been replaced by its Y equivalent, the a-phase can be modeled by the single-phase equivalent circuit shown in Fig. 11.12.

Figure 11.12 A single-phase equivalent circuit.

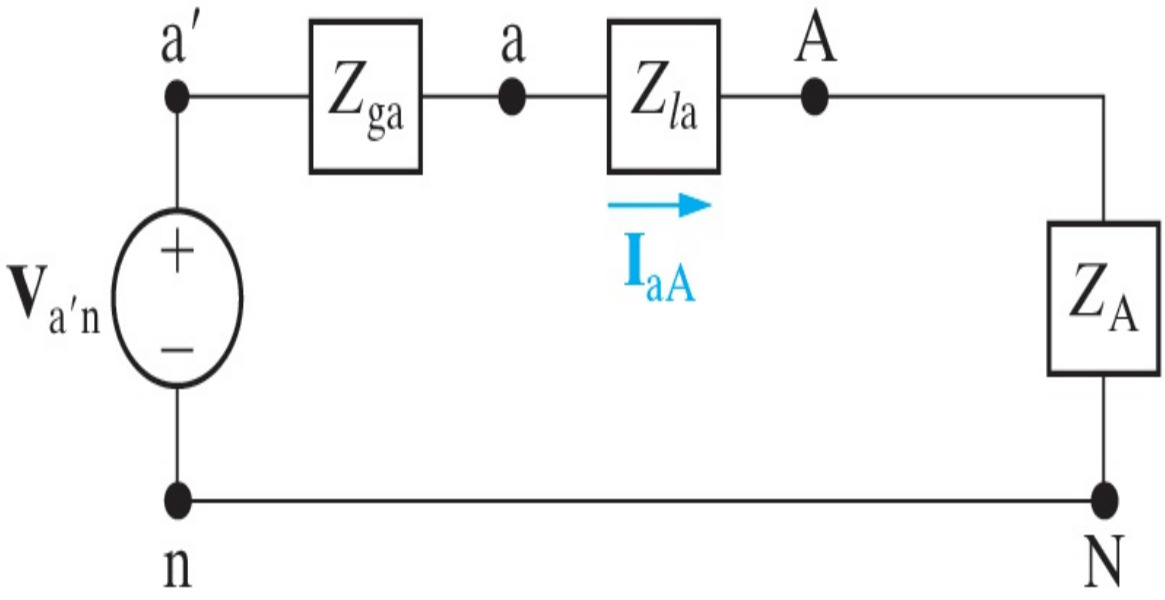


Figure 11.12 Full Alternative Text

We use this circuit to calculate the line currents, and we then use the line currents to find the currents in each phase of the original Δ-connected load.

The relationship between the line currents and the phase currents in each phase of the delta can be derived using the circuit shown in Fig. 11.11. We assume a positive phase sequence and let Iϕ represent the magnitude of the phase current. Then

$$I_{AB} = I_\phi \angle 0^\circ, \quad I_{BC} = I_\phi \angle -120^\circ, \quad I_{CA} = I_\phi \angle 120^\circ.$$

In writing these equations, we arbitrarily selected IAB as the reference phasor.

We can write the line currents in terms of the phase currents by applying KCL at the nodes labeled A, B, and C in Fig. 11.11:

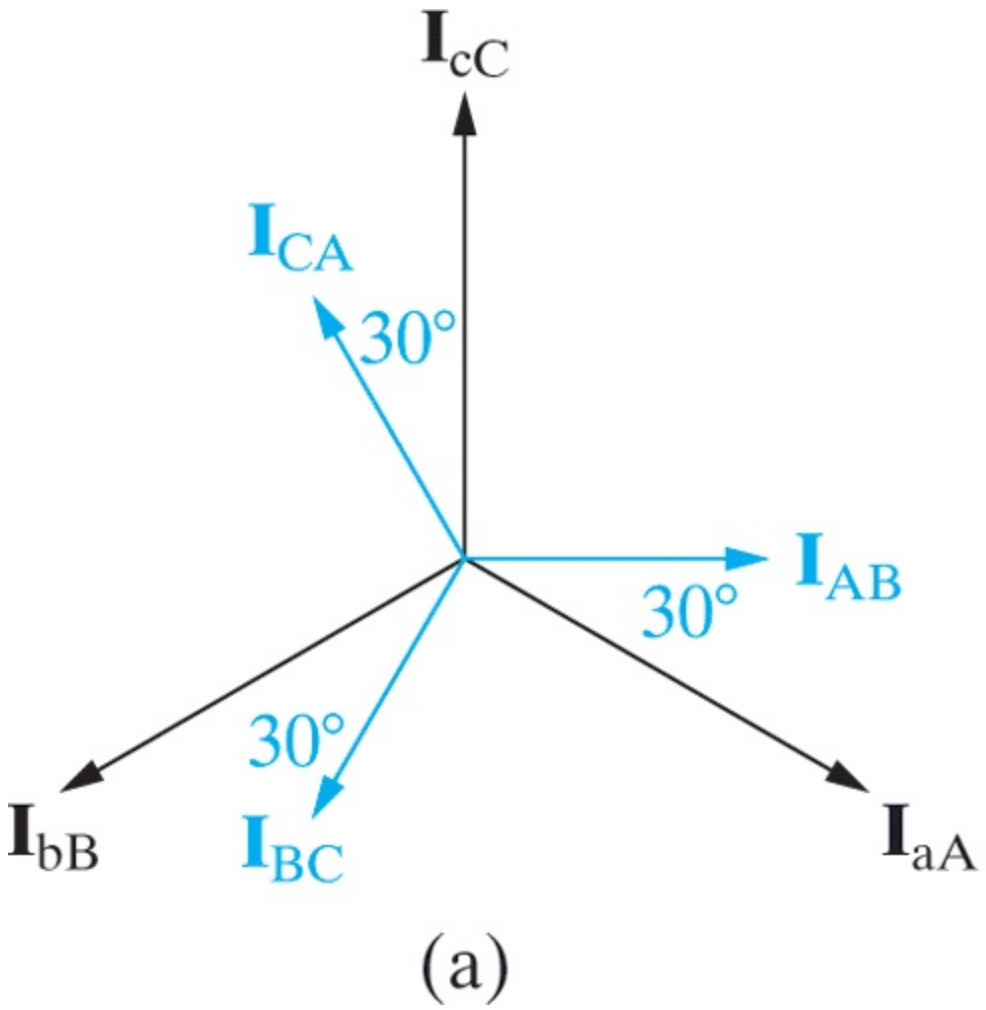
$$I_{aA} = I_{AB} - I_{CA} = I_\phi \angle 0^\circ - I_\phi \angle 120^\circ = 3 I_\phi \angle -30^\circ, \quad (11.10)$$
$$I_{bB} = I_{BC} - I_{AB} = I_\phi \angle -120^\circ - I_\phi \angle 0^\circ = 3 I_\phi \angle -150^\circ, \quad (11.11)$$
$$I_{cC} = I_{CA} - I_{BC} = I_\phi \angle 120^\circ - I_\phi \angle -120^\circ = 3 I_\phi \angle 90^\circ. \quad (11.12)$$

Comparing the line currents and the phase currents reveals that the magnitude of the line currents is 3 times the magnitude of the phase currents and that the set of line currents lags the set of phase currents by 30°.

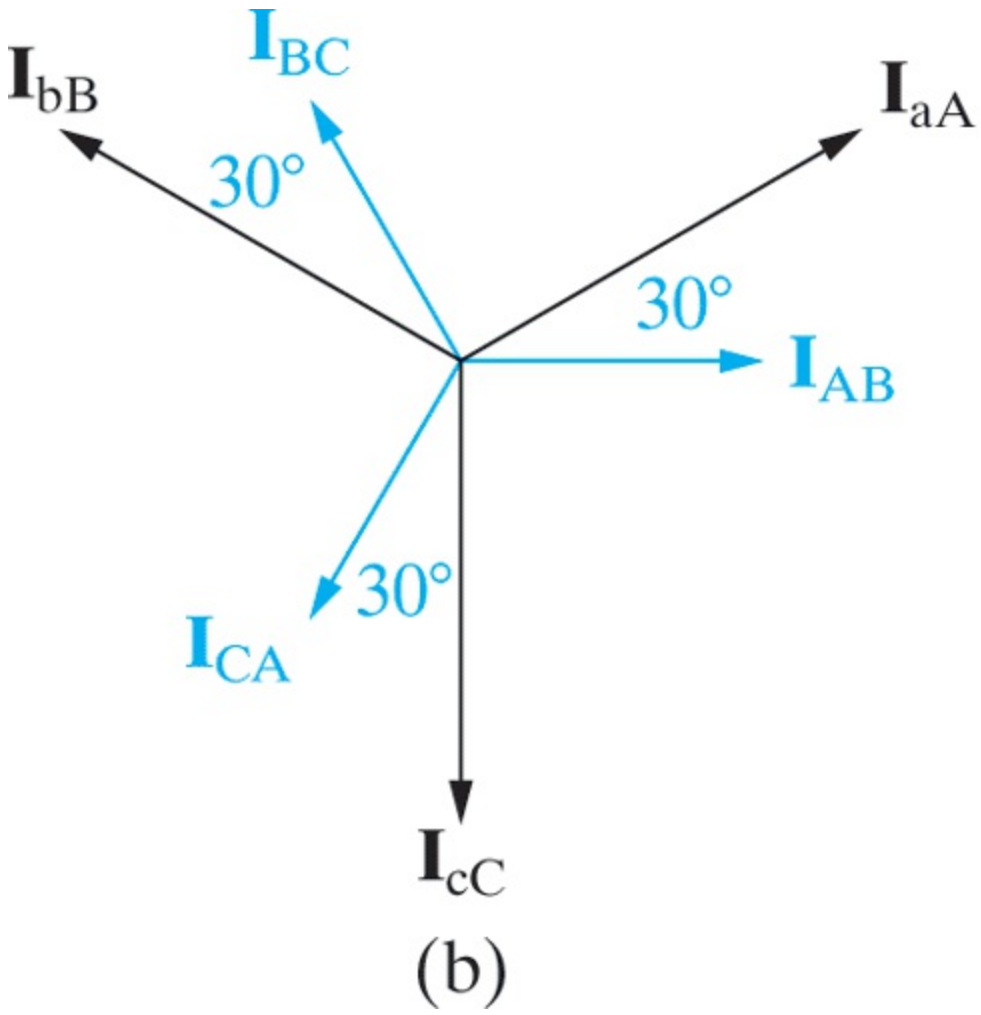
We leave it to you to verify that, for a negative phase sequence, the line currents are still 3 times larger than the phase currents, but they lead the phase currents by 30°. Thus, we have a shortcut for calculating line currents from phase currents (or vice versa) for a balanced three-phase Δ-connected load. Figure 11.13 summarizes this shortcut graphically. Example 11.2 analyzes a balanced three-phase circuit having a Y-connected source and a Δ-connected load, using a single-phase equivalent circuit.

Figure 11.13 Phasor diagrams showing the relationship between line currents and phase currents in a Δ-

connected load. (a) The positive sequence. (b) The negative sequence.



11.4-3 Full Alternative Text



11.4-3 Full Alternative Text

Example 11.2 Analyzing a Wye-Delta Circuit

The Y-connected source in Example 11.1 feeds a Δ-connected load through a distribution line having an impedance of 0.3 + j0.9 Ω/ϕ. The load impedance is 118.5 + j85.8 Ω/ϕ. Use the internal voltage of the generator in the a-phase as the reference.

1. Construct a single-phase equivalent circuit of the three-phase system.

2. Calculate the line currents IaA, IbB, and IcC.
3. Calculate the phase voltages at the load terminals.
4. Calculate the phase currents of the load.
5. Calculate the line voltages at the source terminals.

Solution

1. Figure 11.14 shows the single-phase equivalent circuit. The load impedance of the Y equivalent is

Figure 11.14 The single-phase equivalent circuit for Example 11.2.

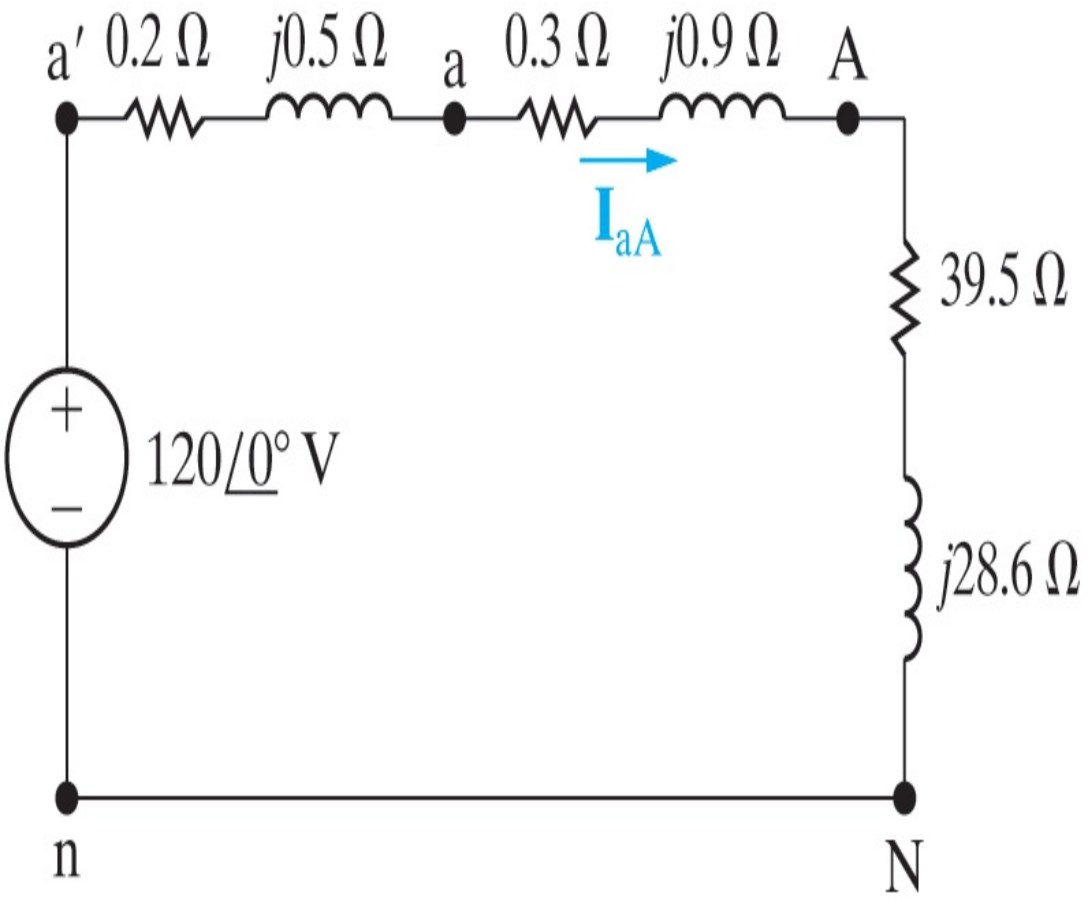


Figure 11.14 Full Alternative Text

$118.5 + j85.83 = 39.5 + j28.6\ \Omega/\phi$.

2. The a-phase line current is the source voltage in the a-phase divided by the sum of the impedances in the a-phase:

   $I_{aA} = 120\ \underline{0°}\ /(0.2 + 0.3 + 39.5) + j(0.5 + 0.9 + 28.6) = 120\ \underline{0°}\ /40 + j30 = 2.4\ \underline{-36.87°}$ A.

   Hence

   $I_{bB} = 2.4\ \underline{-156.87°}$ A, $I_{cC} = 2.4\ \underline{83.13°}$ A.

3. Because the load is Δ connected, the phase voltages are the same as the line voltages. To calculate the line voltages, we first calculate VAN using the single-phase equivalent circuit in Fig. 11.14:

V AN = (39.5 + j28.6)(2.4 −36.87°) = 117.04 −0.96° V.

Because the phase sequence is positive, the line voltage VAB is

V AB = (3 30°) V AN = 202.72 29.04° V.

Therefore

V BC = 202.72 −90.96° V, V CA = 202.72 149.04° V.

4. The phase currents of the load can be calculated directly from the line currents:

   I AB = (1 3 30°) I aA = 1.39 −6.87° A.

   Once we know IAB, we also know the other load phase currents:

   I BC = 1.39 −126.87° A, I CA = 1.39 113.13° A.

   Note that we can check the calculation of IAB by using the previously calculated VAB and the impedance of the Δ-connected load; that is,

   I AB = V AB Z ϕ = 202.72 29.04° 118.5 + j85.8 = 1.39 −6.87° A.

5. To calculate the line voltage at the terminals of the source, we first calculate Van. Figure 11.14 shows that Van is the voltage drop across the line impedance plus the load impedance, so

   V an = (39.8 + j29.5)(2.4 −36.87°) = 118.90 −0.32° V.

   The line voltage Vab is

   V ab = (3 30°) V an , = 205.94 29.68° V.

   Therefore

   V bc = 205.94 −90.32° V, V ca = 205.94 149.68° V.

Assessment Problems

Objective 2—Know how to analyze a balanced, three-phase wye-delta connected circuit

1. 11.4 The current ICA in a balanced three-phase Δ-connected load is 8 −15° A. If the phase sequence is positive, what is the value of IcC?

   Answer: 13.86 −45° A.

2. 11.5 A balanced three-phase Δ-connected load is fed from a balanced three-phase circuit. The reference for the b-phase line current is toward the load. The value of the current in the b-phase is 12 65° A. If the phase sequence is negative, what is the value of IAB?

   Answer:

   6.93 −85° A.

3. 11.6 The line voltage VAB at the terminals of a balanced three-phase Δ-connected load is 4160 0° V. The line current IaA is 69.28 −10° A.

   1. a) Calculate the per-phase impedance of the load if the phase sequence is positive.

   2. b) Repeat (a) for a negative phase sequence.

   Answer:

   1. (a) 104 −20° Ω;

   2. (b) 104 +40° Ω.

4. 11.7 The line voltage at the terminals of a balanced Δ-connected load is 110 V. Each phase of the load consists of a 3.667 Ω resistor in parallel with a 2.75 Ω inductive impedance. What is the magnitude of the current

in the line feeding the load?

Answer:

86.60 A.

SELF-CHECK: *Also try Chapter Problems 11.14–11.16.*

11.5 Power Calculations in Balanced Three-Phase Circuits

So far, we have analyzed balanced three-phase circuits using a single-phase equivalent circuit to determine line currents, phase currents, line voltages, and phase voltages. We now calculate power for balanced three-phase circuits. We begin by considering the average power delivered to a balanced Y-connected load.

Average Power in a Balanced Wye Load

Figure 11.15 shows a Y-connected load, along with its pertinent currents and voltages. We calculate the average power associated with any one phase by using the techniques introduced in Chapter 10. Using Eq. 10.9, we express the average power associated with the a-phase as

Figure 11.15 A balanced Y load used to introduce average power calculations in three-phase circuits.

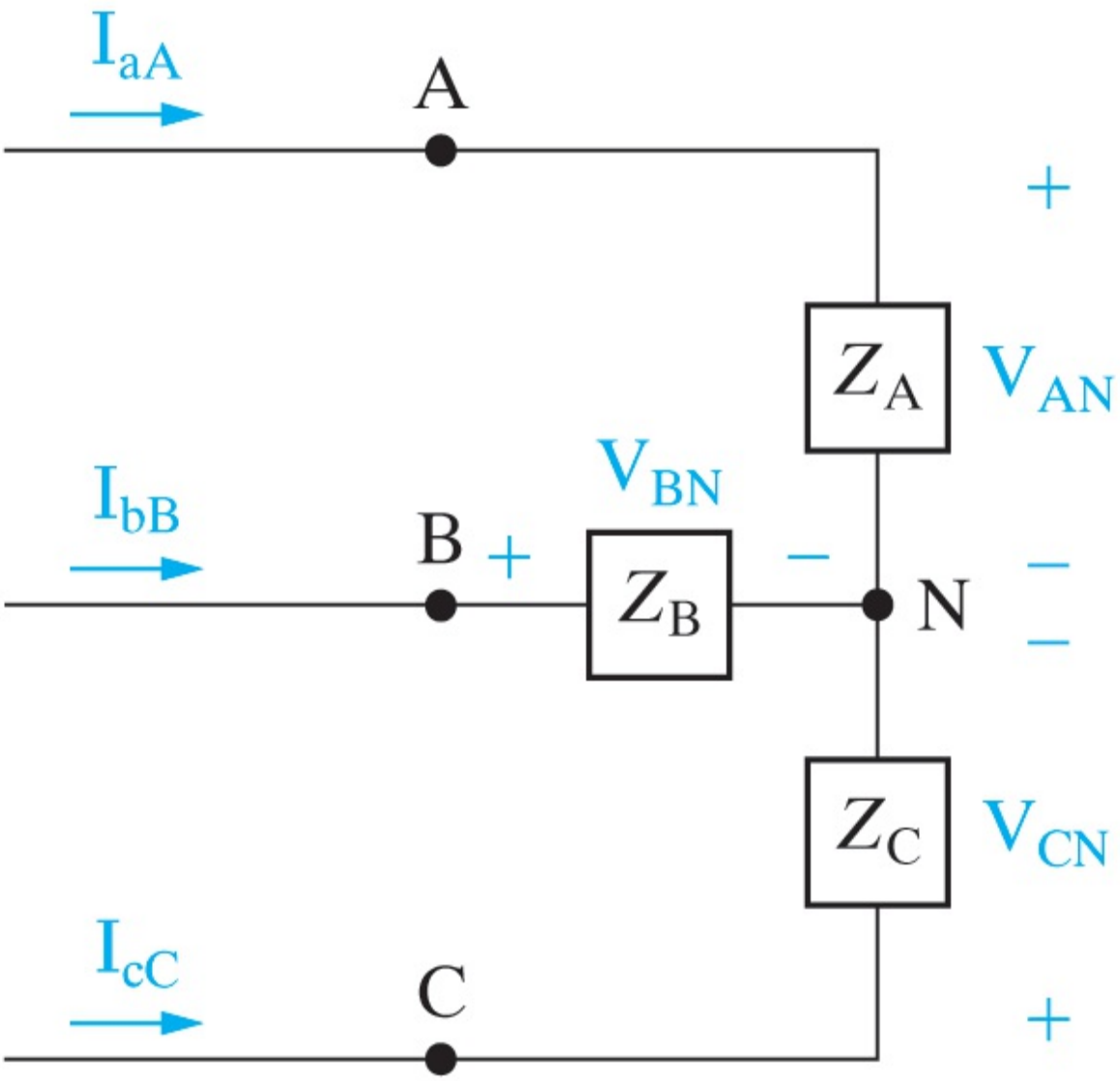


Figure 11.15 Full Alternative Text

$$P_A = |V_{AN}||I_{aA}| \cos(\theta_{vA} - \theta_{iA}), \quad (11.13)$$

where θ_{vA} and θ_{iA} denote the phase angles of V_{AN} and I_{aA}, respectively. Using the notation introduced in Eq. 11.13, we can find the power associated with the b- and c-phases:

$$P_B = |V_{BN}||I_{bB}| \cos(\theta_{vB} - \theta_{iB}), \quad (11.14)$$
$$P_C = |V_{CN}||I_{cC}| \cos(\theta_{vC} - \theta_{iC}). \quad (11.15)$$

In Eqs. 11.13–11.15, all phasor currents and voltages use the rms value of the sinusoidal function they represent.

In a balanced three-phase system, the magnitude of each line-to-neutral voltage is the same, as is the magnitude of each phase current. The argument of the cosine functions is also the same for all three phases. We introduce the following notation to take advantage of these observations:

$$V_\phi = |V_{AN}| = |V_{BN}| = |V_{CN}|, \quad I_\phi = |I_{aA}| = |I_{bB}| = |I_{cC}|,$$

and

$$\theta_\phi = \theta_{vA} - \theta_{iA} = \theta_{vB} - \theta_{iB} = \theta_{vC} - \theta_{iC}.$$

Moreover, for a balanced system, the power delivered to each phase of the load is the same, so

$$P_A = P_B = P_C = P_\phi = V_\phi I_\phi \cos\theta_\phi,$$

where P_ϕ represents the average power per phase. The total average power delivered to the balanced Y-connected load is simply three times the power per phase, or

$$P_T = 3P_\phi = 3V_\phi I_\phi \cos\theta_\phi. \quad (11.16)$$

Expressing the total power in terms of the rms magnitudes of the line voltage and current is also desirable. If we let V_L and I_L represent the rms magnitudes of the line voltage and current, respectively, we can modify Eq. 11.16 as follows:

Total Real Power in a Balanced Three-Phase Load

$$P_T = 3\left(\frac{V_L}{\sqrt{3}}\right) I_L \cos\theta_\phi = \sqrt{3} V_L I_L \cos\theta_\phi. \quad (11.17)$$

In deriving Eq. 11.17, we recognized that, for a balanced Y-connected load, the magnitude of the phase voltage is the magnitude of the line voltage divided by 3, and that the magnitude of the line current is equal to the magnitude of the phase current. When using Eq. 11.17 to calculate the total

power delivered to the load, remember that $\theta\phi$ is the phase angle between the phase voltage and current.

Reactive and Complex Power in a Balanced Wye Load

We can also calculate the reactive power and complex power associated with any phase of a Y-connected load using the techniques introduced in Chapter 10. For a balanced load, the expressions for the reactive power are

$$Q\phi = V\phi I\phi \sin\theta\phi,$$

Total Reactive Power in a Balanced Three-Phase Load

$$QT = 3Q\phi = 3VLIL \sin\theta\phi. \quad (11.18)$$

Use Eq. 10.13 to express the complex power of any phase. For a balanced Y-connected load,

$$S\phi = VANIaA^* = VBNIbB^* = VCNIcC^* = V\phi I\phi^*,$$

where $V\phi$ and $I\phi$ represent a phase voltage and current for the same phase. Thus, in general,

$$S\phi = P\phi + jQ\phi = V\phi I\phi^*,$$

Total Complex Power in a Balanced Three-Phase Load

$$S_T = 3S_\phi = 3V_L I_L \theta_\phi^\circ. \quad (11.19)$$

Power Calculations in a Balanced Delta Load

If the load is Δ-connected, the calculation of power—reactive or complex—is basically the same as that for a Y-connected load. Figure 11.16 shows a Δ-connected load, along with its pertinent currents and voltages. The power associated with each phase is

Figure 11.16 A Δ-connected load used to discuss power calculations.

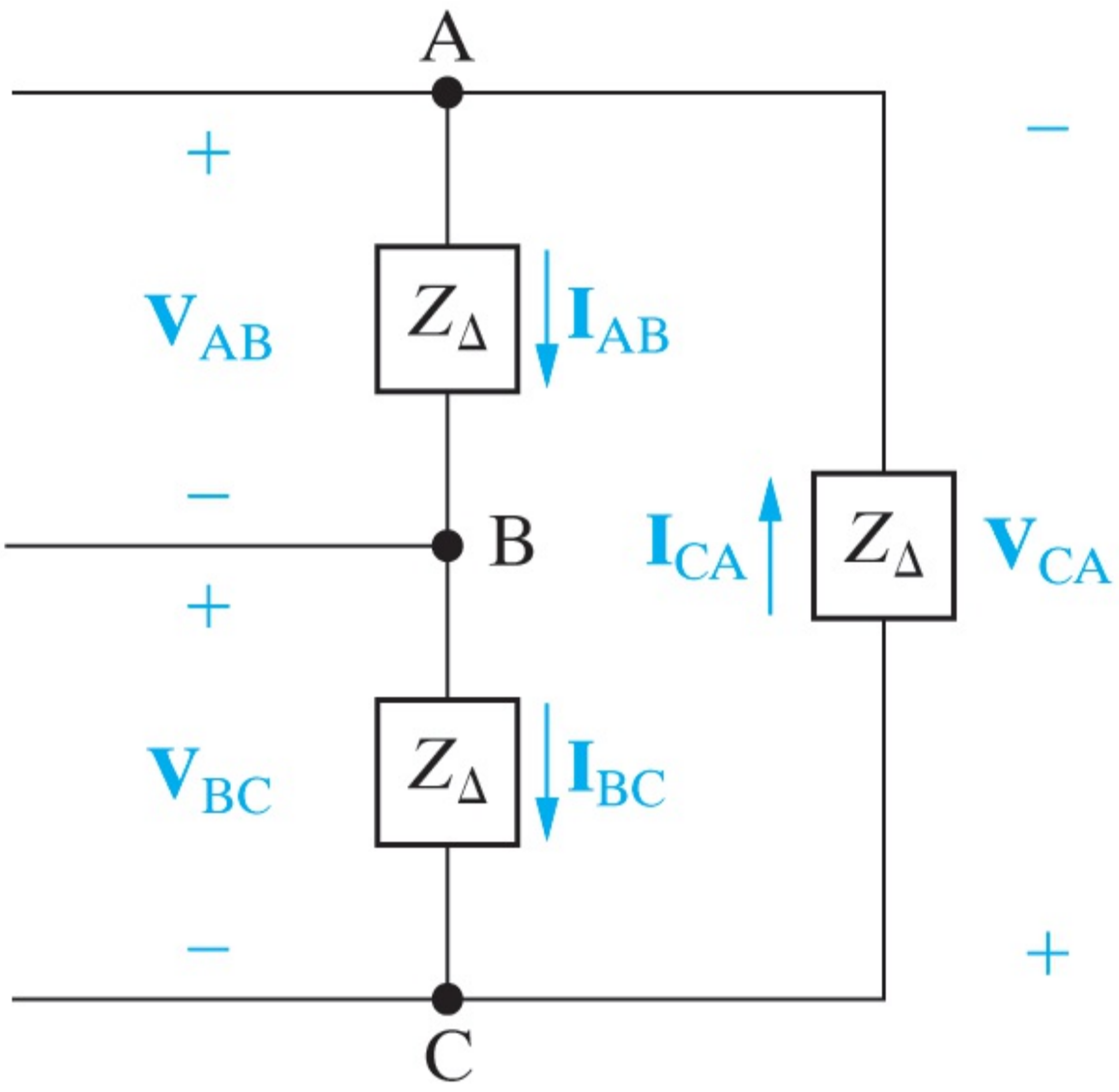


Figure 11.16 Full Alternative Text

P A = | V AB || I AB |cos (θ vAB − θ iAB), P B = | V BC || I BC | cos (θ vBC − θ iBC), P C = | V CA || I CA | cos (θ vCA − θ iCA).

For a balanced load,

| V AB | = | V BC |=| V CA |= V ϕ , | I AB | = | I BC |=| I CA |= I ϕ , θ vAB − θ iAB = θ vBC − θ iBC = θ vCA − θ iCA = θ ϕ ,

and

PA=PB=PC=Pϕ=VϕIϕcosθϕ.

Thus, whether you have a balanced Y- or Δ-connected load, the average power per phase is equal to the product of the rms magnitude of the phase voltage, the rms magnitude of the phase current, and the cosine of the angle between the phase voltage and current.

The total average power delivered to a balanced Δ-connected load is

$$P_T = 3P_\phi = 3V_\phi I_\phi \cos\theta_\phi = 3V_L(I_L\sqrt{3})\cos\theta_\phi = 3V_L I_L \cos\theta_\phi.$$

Note that this equation is the same as Eq. 11.17. The expressions for reactive power and complex power also are the same as those developed for the Y load:

$$Q_\phi = V_\phi I_\phi \sin\theta_\phi ; \; Q_T = 3Q_\phi = 3V_\phi I_\phi \sin\theta_\phi ; \; S_\phi = P_\phi + jQ_\phi = V_\phi I_\phi^* ; \; S_T = 3S_\phi = 3V_L I_L \theta_\phi .$$

Instantaneous Power in Three-Phase Circuits

Although we are primarily interested in average, reactive, and complex power calculations, computing the total instantaneous power is also important. In a balanced three-phase circuit, this power has an interesting property: it is invariant with time! Thus, the torque developed at the shaft of a three-phase motor is constant, which in turn means less vibration in machinery powered by three-phase motors.

Let the instantaneous line-to-neutral voltage v_{AN} be the reference, and, as before, θ_ϕ is the phase angle $\theta_{vA} - \theta_{iA}$. Then, for a positive phase sequence, the instantaneous power in each phase is

$$p_A = v_{AN} i_{aA} = V_m I_m \cos\omega t \cos(\omega t - \theta_\phi), \; p_B = v_{BN} i_{bB} = V_m I_m \cos(\omega t - 120°)\cos(\omega t - \theta_\phi - 120°), \; p_C = v_{CN} i_{cC} = V_m I_m \cos(\omega t + 120°)\cos(\omega t - \theta_\phi + 120°),$$

where V_m and I_m represent the maximum amplitude of the phase voltage and

line current, respectively. The total instantaneous power is the sum of the instantaneous phase powers, which reduces to $1.5V_mI_m\cos\theta_\phi$; that is,

$$p_T = p_A + p_B + p_C = 1.5V_mI_m\cos\theta_\phi.$$

Note that this result is consistent with Eq. 11.16 since $V_m = 2V_\phi$ and $I_m = 2I_\phi$ (see Problem 11.31).

Examples 11.3–11.5 illustrate power calculations in balanced three-phase circuits.

Example 11.3 Calculating Power in a Three-Phase Wye-Wye Circuit

1. Calculate the average power per phase delivered to the Y-connected load of Example 11.1.
2. Calculate the total average power delivered to the load.
3. Calculate the total average power lost in the line.
4. Calculate the total average power lost in the generator.
5. Calculate the total number of magnetizing vars absorbed by the load.
6. Calculate the total complex power delivered by the source.

Solution

1. From Example 11.1, $V_\phi = 115.22$ V, $I_\phi = 2.4$ A, and $\theta_\phi = -1.19 - (-36.87) = 35.68°$. Therefore

   $$P_\phi = (115.22)(2.4)\cos 35.68° = 224.64 \text{ W}.$$

The power per phase may also be calculated from $I_\phi^2 R_\phi$, or

$P_\phi = (2.4)^2(39) = 224.64$ W.

2. The total average power delivered to the load is $P_T = 3P_\phi = 673.92$ W. We calculated the line voltage in Example 11.1, so we can also use Eq. 11.17:

   $P_T = 3\,(199.58)(2.4)\cos 35.68° = 673.92$ W.

3. The total power lost in the line is

   $P_{line} = 3(2.4)^2(0.8) = 13.824$ W.

4. The total internal power lost in the generator is

   $P_{gen} = 3(2.4)^2(0.2) = 3.456$ W.

5. The total number of magnetizing vars absorbed by the load is

   $Q_T = 3\,(199.58)(2.4)\sin 35.68° = 483.84$ VAR.

6. The total complex power associated with the source is

   $S_T = 3\,S_\phi = -3(120)(2.4)\ \ 36.87° = -691.20 - j518.40$ VA.

   The minus sign indicates that the average power and magnetizing reactive power are being delivered to the circuit. We check this result by calculating the total average and reactive power absorbed by the circuit:

   $P = 673.92 + 13.824 + 3.456 = 691.20$ W (check), $Q = 483.84 + 3\,(2.4)^2\,(1.5) + 3\,(2.4)^2\,(0.5) = 483.84 + 25.92 + 8.64 = 518.40$ VAR(check).

Example 11.4 Calculating Power in a Three-Phase Wye-Delta Circuit

1. Calculate the total complex power delivered to the Δ-connected load of Example 11.2.

2. What percentage of the average power at the sending end of the line is delivered to the load?

Solution

1. Using the a-phase values from the solution of Example 11.2, we obtain

   $V_\phi = V_{AB} = 202.72 \angle 29.04°$ V, $I_\phi = I_{AB} = 1.39 \angle -6.87°$ A.

   Using Eq. 11.19, we have

   $S_T = 3(202.72 \angle 29.04°)(1.39 \angle 6.87°) = 682.56 + j494.21$ VA.

2. The total average power at the sending end of the distribution line equals the total average power delivered to the load plus the total average power lost in the line; therefore

   $P_{input} = 682.56 + 3(2.4)^2(0.3) = 687.74$ W.

   The fraction of the average power reaching the load is 682.56/687.74, or 99.25%. Nearly 100% of the average power at the input is delivered to the load because the resistance of the line is quite small compared to the load resistance.

Example 11.5 Calculating Three-Phase Power with an Unspecified Load

A balanced three-phase load requires 480 kW at a lagging power factor of 0.8. The load is fed from a line having an impedance of $0.005 + j0.025\ \Omega/\phi$.

The line voltage at the terminals of the load is 600 V.

1. Construct a single-phase equivalent circuit of the system.
2. Calculate the magnitude of the line current.
3. Calculate the magnitude of the line voltage at the sending end of the line.
4. Calculate the power factor at the sending end of the line.

Solution

1. Figure 11.17 shows the single-phase equivalent circuit. We arbitrarily selected the line-to-neutral voltage at the load as the reference.

Figure 11.17 The single-phase equivalent circuit for Example 11.5.

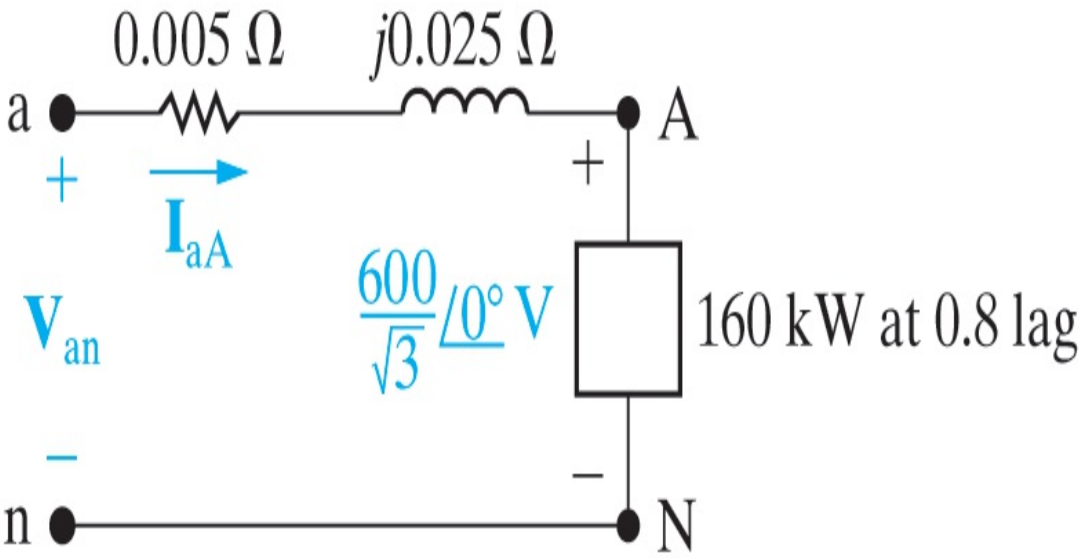


Figure 11.17 Full Alternative Text

2. The line current IaA* appears in the equation for the complex power of the load:

   (6003)IaA*=(160 + j120)103 VA.

   Solving for IaA* we get

   IaA*=577.35 36.87° A.

   Therefore,

   I aA =577.35 −36.87° A.

   The magnitude of the line current is the magnitude of IaA, so IL=577.35 A.

   We obtain an alternative solution for IL from the expression

   P T = 3 V L I L cos θ p = 3 (600) I L (0.8) = 480,000 W; I L = 480,000 3 (600)(0.8) = 1000 3 = 577.35 A.

3. To calculate the magnitude of the line voltage at the sending end, we first calculate Van. From Fig. 11.17,

   V an = V AN + Z ℓ I aA = 600 3 + (0.005 + j0.025)(577.35 −36.87°) = 357.51 1.57° V.

   Thus

   V L = 3 | V an | = 619.23 V.

4. The power factor at the sending end of the line is the cosine of the phase angle between Van. and I aA :

   pf = cos [1.57° − (−36.87°)] aA = cos 38.44° = 0.783 lagging.

   An alternative method for calculating the power factor is to first

calculate the complex power at the sending end of the line:

$S_\phi = (160 + j120)\ 10\ 3\ +\ (577.35)\ 2\ (0.005 + j0.025) = 161.67 + j128.33\ \text{kVA} = 206.41\ \ 38.44°\ \ \text{kVA}.$

The power factor is

$pf = \cos 38.44° = 0.783\ \text{lagging}.$

Finally, if we calculate the total complex power at the sending end, after first calculating the magnitude of the line current, we can use this value to calculate VL. That is,

$3\ V_L\ I_L = 3(206.41) \times\ 10\ 3\ ,\ V_L = 3(206.41) \times\ 10\ 3\ 3\ (577.35)\ , = 619.23\ \text{V}.$

Assessment Problems

Objective 3—Be able to calculate power (average, reactive, and complex) in any three-phase circuit

1. 11.8 The three-phase average power rating of the central processing unit (CPU) on a mainframe digital computer is 22,659 W. The three-phase line supplying the computer has a line voltage rating of 208 V. The line current is 73.8 A. The computer absorbs magnetizing VARs.

    1. a) Calculate the total magnetizing reactive power absorbed by the CPU.

    2. b) Calculate the power factor.

    Answer:

    1. (a) 13,909.50 VAR;

    2. (b) 0.852 lagging.

2. 11.9 The complex power associated with each phase of a balanced load is 144 + j192 kVA. The line voltage at the terminals of the load is 2450 V.

    1. a) What is the magnitude of the line current feeding the load?

    2. b) The load is delta connected, and the impedance of each phase consists of a resistance in parallel with a reactance. Calculate R and X.

    3. c) The load is wye connected, and the impedance of each phase consists of a resistance in series with a reactance. Calculate R and X.

    Answer:

    1. (a) 169.67 A;

    2. (b) R=41.68 Ω, X=31.26 Ω;

    3. (c) R=5 Ω, X=6.67 Ω.

SELF-CHECK: *Also try Chapter Problems 11.25 and 11.27*.

11.6 Measuring Average Power in Three-Phase Circuits

The instrument used to measure power in three-phase circuits is the electrodynamometer wattmeter. It contains two coils. One coil, called the **current coil**, is stationary and is designed to carry a current proportional to the load current. The second coil, called the **potential coil**, is movable and carries a current proportional to the load voltage. The important features of the wattmeter are shown in Fig. 11.18.

Figure 11.18 The key features of the electrodynamometer wattmeter.

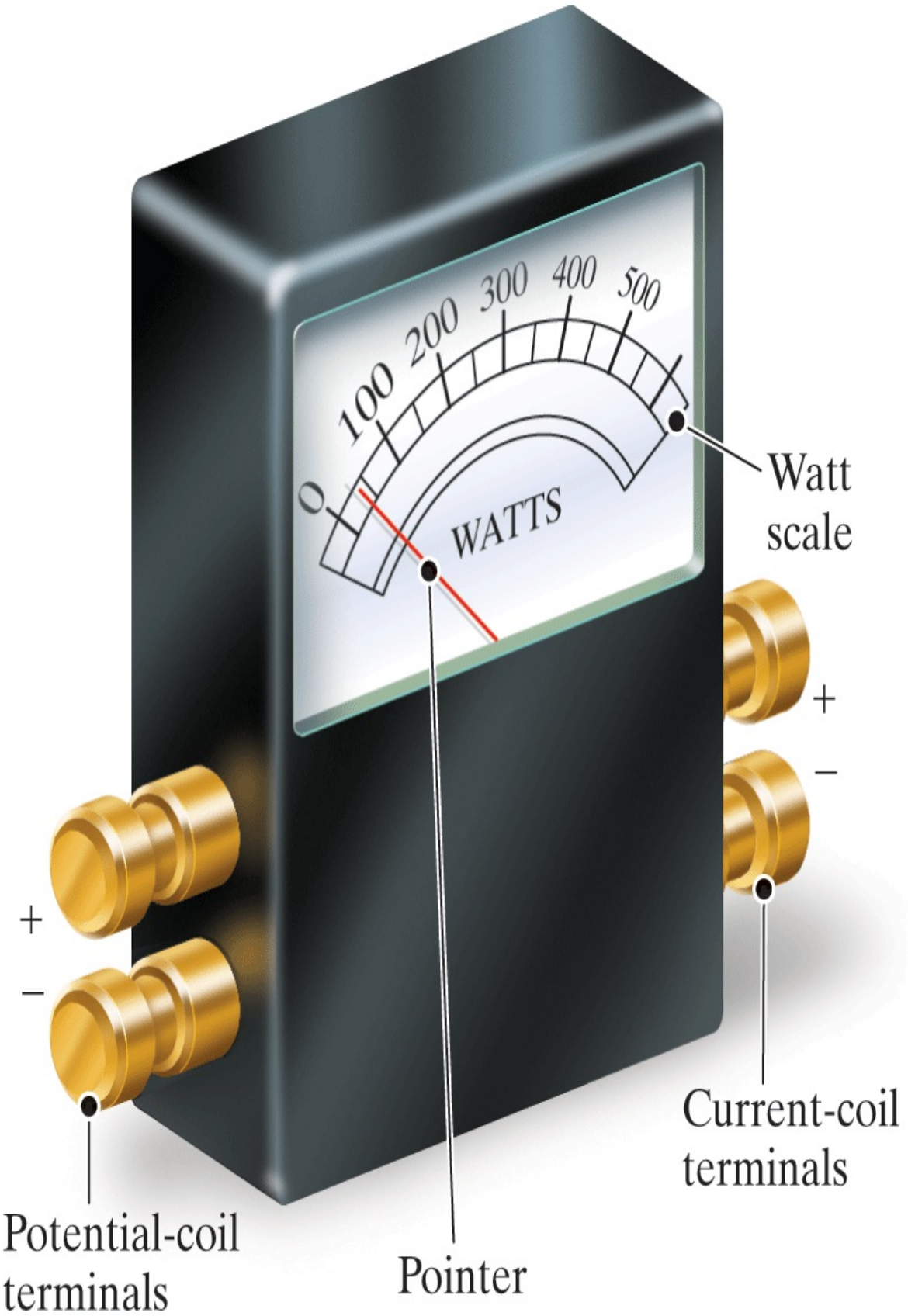

0
100
200
300
400
500
WATTS
Watt scale
+
−
+
−
Potential-coil terminals
Pointer
Current-coil terminals

Figure 11.18 Full Alternative Text

The average deflection of the pointer attached to the movable coil is proportional to the product of the rms current in the current coil, the rms voltage impressed on the potential coil, and the cosine of the phase angle between the voltage and current. The pointer deflects in a direction that depends on the instantaneous polarity of the current-coil current and the potential-coil voltage. Therefore, each coil has one terminal with a polarity mark—usually a plus sign—but sometimes the double polarity mark ± is used. The wattmeter deflects upscale when (1) the polarity-marked terminal of the current coil is toward the source, and (2) the polarity-marked terminal of the potential coil is connected to the same line in which the current coil has been inserted.

The Two-Wattmeter Method

Consider a general network inside a box, with power supplied by n conducting lines. Such a system is shown in Fig. 11.19. If we wish to measure the total power at the terminals of the box, we need to know n−1 currents and voltages. This follows because if we choose one terminal as a reference, there are only n−1 independent voltages. Likewise, only n−1 independent currents can exist in the n conductors entering the box. Thus, the total power is the sum of n−1 product terms; that is, $p = v_1 i_1 + v_2 i_2 + \cdots + v_{n-1} i_{n-1}$.

Figure 11.19 A general circuit whose power is supplied by n conductors.

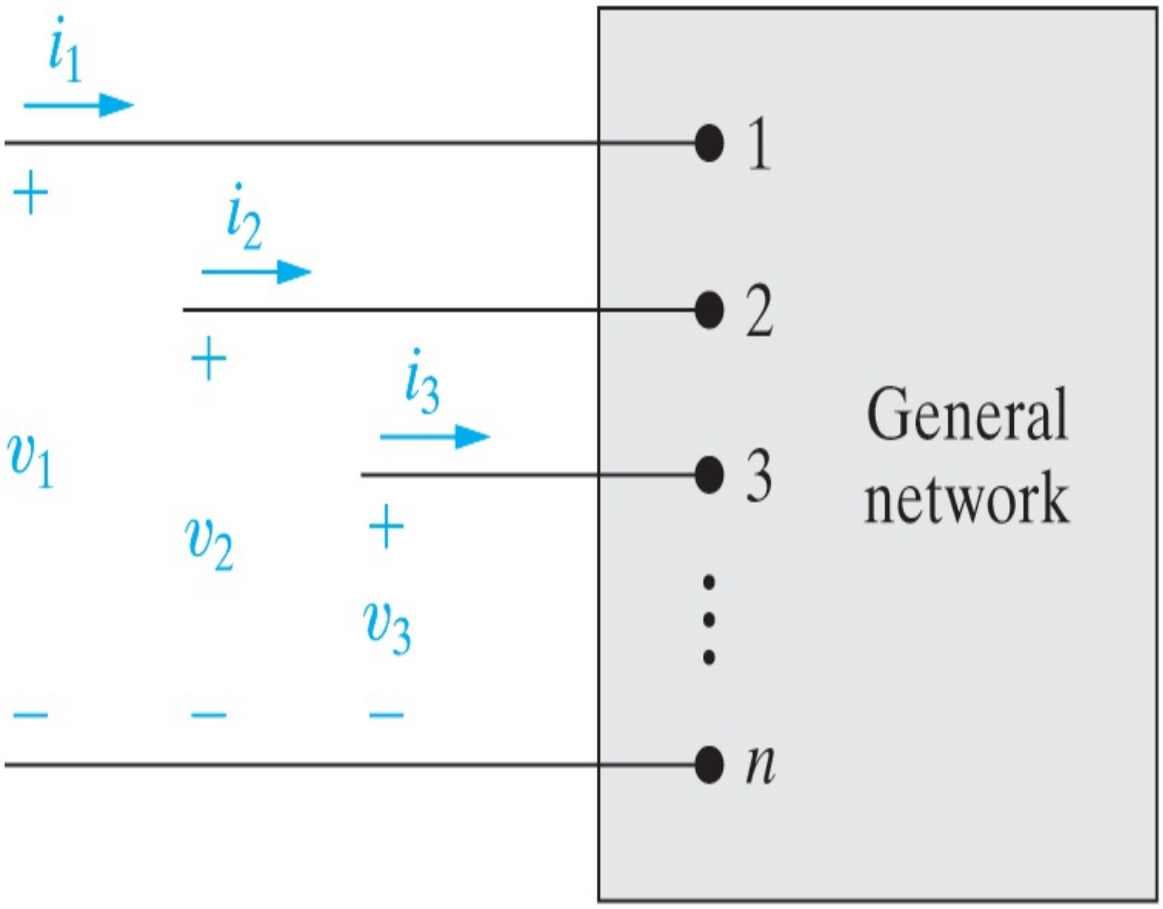


Figure 11.19 Full Alternative Text

Applying this general observation, we can see that for a three-conductor circuit, whether balanced or not, we need only two wattmeters to measure the total power. For a four-conductor circuit, we need three wattmeters if the three-phase circuit is unbalanced, but only two wattmeters if it is balanced, because in the latter case there is no current in the neutral line. Thus, only two wattmeters are needed to measure the total average power in any balanced three-phase system.

The two-wattmeter method determines the magnitude and algebraic sign of the average power indicated by each wattmeter. We can describe the basic problem using the circuit shown in Fig. 11.20, where the two wattmeters are indicated by the shaded boxes and labeled W1 and W2. The coil notations cc and pc stand for current coil and potential coil, respectively. The current coils of the wattmeters are inserted in lines aA and cC, making line bB the reference line for the two potential coils. The load is Y-connected, and its

per-phase impedance is $Z\phi = |Z| \ \theta$. This is a general representation, as any Δ-connected load can be represented by its Y equivalent; furthermore, for the balanced case, the impedance angle θ is unaffected by the Δ-to-Y transformation.

Figure 11.20 A circuit used to analyze the two-wattmeter method of measuring average power delivered to a balanced load.

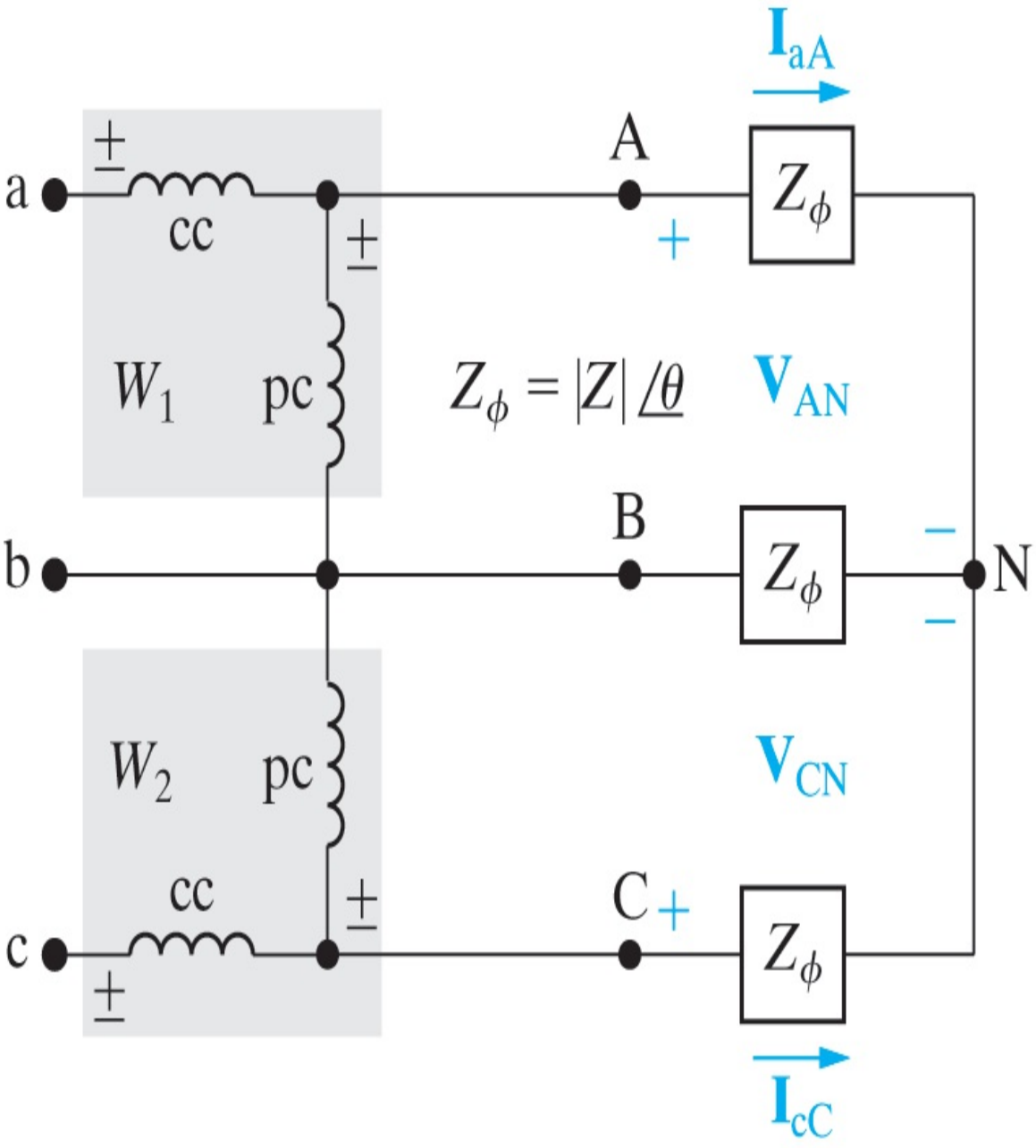


Figure 11.20 Full Alternative Text

We now develop general equations for the readings of the two wattmeters, making the following assumptions.

- The current drawn by the potential coil of the wattmeter is negligible compared with the line current measured by the current coil.

- The loads can be modeled by passive circuit elements, so the phase angle of the load impedance (θ in Fig. 11.20) lies between $-90°$ (pure capacitance) and $+90°$ (pure inductance).

- The phase sequence is positive.

From our introductory discussion of the average deflection of the wattmeter and the placement of wattmeter 1 in Fig. 11.20, we note that the wattmeter reading, W1, is

$$W1=|VAB||IaA| \cos \theta 1 \quad (11.20)$$
$$=VLIL \cos \theta 1.$$

It follows that

$$W2=|VCB||IcC| \cos \theta 2 \quad (11.21)$$
$$=VLIL \cos \theta 2.$$

In Eq. 11.20, $\theta 1$ is the phase angle between VAB and IaA, and in Eq. 11.21, $\theta 2$ is the phase angle between VCB and IcC.

To calculate W1 and W2, we express $\theta 1$ and $\theta 2$ in terms of the impedance angle θ, which is also the same as the phase angle between the phase voltage and current. For a positive phase sequence,

$$\theta 1=\theta + 30°=\theta\phi + 30°, \quad (11.22)$$
$$\theta 2=\theta - 30°=\theta\phi - 30°. \quad (11.23)$$

The derivation of Eqs. 11.22 and 11.23 is left as an exercise (see Problem 11.46). When we substitute Eqs. 11.22 and 11.23 into Eqs. 11.20 and 11.21, respectively, we get

$$W1 = V_L I_L \cos(\theta_\phi + 30°), \quad W2 = V_L I_L \cos(\theta_\phi - 30°).$$

To find the total power, we add W1 and W2; thus

$$P_T = W1 + W2 = 2 V_L I_L \cos\theta_\phi \cos 30° = \sqrt{3} V_L I_L \cos\theta_\phi,$$

which is the expression for the total average power in a three-phase circuit,

given in Eq. 11.17. Therefore, we have confirmed that the sum of the two wattmeter readings is the total average power.

A closer look at the expressions for W1 and W2 reveals the following about the readings of the two wattmeters:

1. If the power factor is greater than 0.5, both wattmeters read positive.
2. If the power factor equals 0.5, one wattmeter reads zero.
3. If the power factor is less than 0.5, one wattmeter reads negative.
4. Reversing the phase sequence will interchange the readings on the two wattmeters.

Example 11.6 and Problems 11.41–11.52 illustrate these observations.

Example 11.6 Computing Wattmeter Readings in Three-Phase Circuits

Calculate the reading of each wattmeter in the circuit in Fig. 11.20 if the phase voltage at the load is 120 V and

1. $Z\phi = 8 + j6\ \Omega$;
2. $Z\phi = 8 - j6\ \Omega$;
3. $Z\phi = 5 + j5\ 3\ \Omega$; and
4. $Z\phi = 10\ \angle -75^\circ\ \Omega$.
5. Verify for (a)–(d) that the sum of the wattmeter readings equals the total power delivered to the load.

Solution

1. $Z_\phi = 10\ \angle 36.87°\ \Omega$, $V_L = 120\sqrt{3}$ V, $I_L = 120/10 = 12$ A. $W_1 = (120\sqrt{3})(12)\cos(36.87° + 30°) = 979.75$ W, $W_2 = (120\sqrt{3})(12)\cos(36.87° - 30°) = 2476.25$ W.

   The power factor is $\cos 36.87^\circ = 0.8$, so as expected, both wattmeter readings are positive.

2. $Z_\phi = 10\ \angle -36.87°\ \Omega$, $V_L = 120\sqrt{3}$ V, $I_L = 120/10 = 12$ A. $W_1 = (120\sqrt{3})(12)\cos(-36.87° + 30°) = 2476.25$ W, $W_2 = (120\sqrt{3})(12)\cos(-36.87° - 30°) = 979.75$ W.

   The power factor is $\cos -36.87^\circ = 0.8$, so as expected, both wattmeter readings are positive. But the readings on the wattmeters are interchanged when compared to the results of part (a) because the sign of the power factor angle changed, which has the same impact on the wattmeter readings as a change in phase sequence.

3. $Z_\phi = 5(1 + j\sqrt{3}) = 10\ \angle 60°\ \Omega$, $V_L = 120\sqrt{3}$ V, and $I_L = 12$ A. $W_1 = (120\sqrt{3})(12)\cos(60° + 30°) = 0$, $W_2 = (120\sqrt{3})(12)\cos(60° - 30°) = 2160$ W.

   The power factor is $\cos 60^\circ = 0.5$, so as expected, one of the wattmeter readings is zero.

4. $Z_\phi = 10\ \angle -75°\ \Omega$, $V_L = 120\sqrt{3}$ V, $I_L = 12$ A. $W_1 = (120\sqrt{3})(12)\cos(-75° + 30°) = 1763.63$ W, $W_2 = (120\sqrt{3})(12)\cos(-75° - 30°) = -645.53$ W.

   The power factor is $\cos -75^\circ = 0.26$, so as expected, one wattmeter reading is negative.

5. For each load impedance value, the real power delivered to a single phase of the load is $I_L^2 R_L$, where R_L is the resistive impedance of the load. Since the three-phase circuit is balanced, the total real power delivered to the three-phase load is $3I_L^2 R_L$.

For the impedance in part (a),

$P_T = 3(12)^2(8) = 3456$ W, $W_1 + W_2 = 979.75 + 2476.25 = 3456$ W.

For the impedance in part (b),

$P_T = 3(12)^2(8) = 3456$ W, $W_1 + W_2 = 2476.25 + 979.75 = 3456$ W.

For the impedance in part (c),

$P_T = 3(12)^2(5) = 2160$ W, $W_1 + W_2 = 0 + 2160 = 2160$ W.

For the impedance in part (d),

$Z_\varphi = 2.5882 - j9.6593\ \Omega$ so $P_T = 3(12)^2(2.5882) = 1118.10$ W, $W_1 + W_2 = 1763.63 - 645.53 = 1118.10$ W.

SELF-CHECK: *Assess your understanding of the two-wattmeter method by trying Chapter Problems 11.41 and 11.45.*

Practical Perspective

Transmission and Distribution of Electric Power

At the start of this chapter we noted that utilities must maintain the rms voltage level at their customer's premises. Although the acceptable deviation from a nominal level may vary among different utilities, we will assume that the tolerance is ±5%. Thus, a nominal rms voltage of 120 V could range from 114 V to 126 V. We also pointed out that strategically located capacitors can be used to support voltage levels.

The circuit shown in Fig. 11.21 represents a substation in a municipal system.

We assume that the system is balanced, the line-to-line voltage at the substation is 13.8 kV, the impedance of the distribution line is 0.6+j4.8Ω/φ, and the load at the substation at 3:00 p.m. on a hot, humid day in July is 3.6 MW and 3.6 magnetizing MVAR.

Figure 11.21 A substation connected to a power plant via a three-phase line.

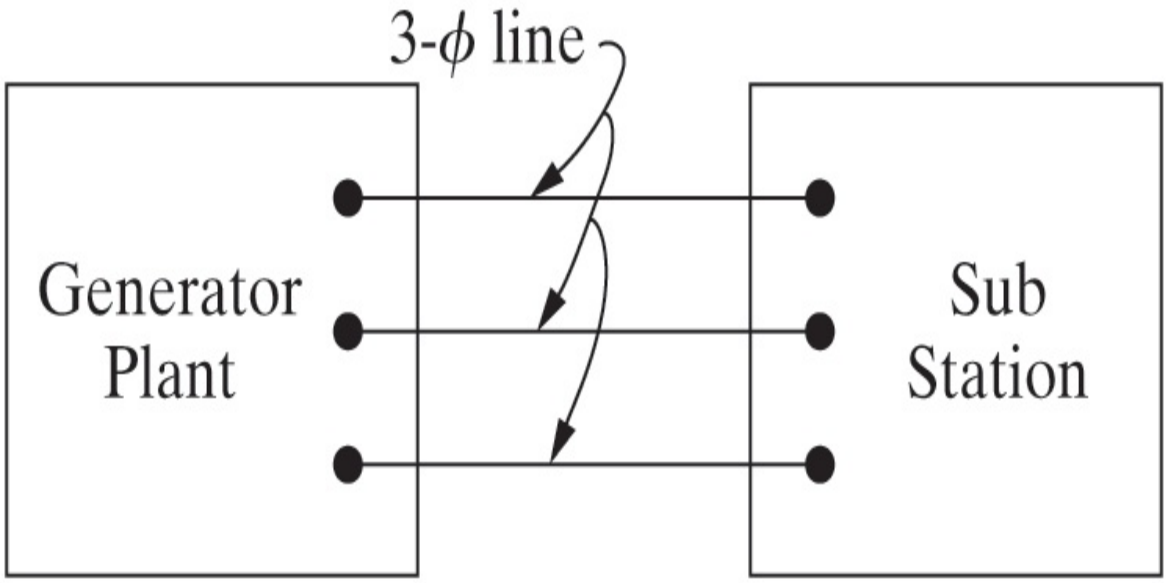


Figure 11.21 Full Alternative Text

Using the line-to-neutral voltage at the substation as a reference, the single-phase equivalent circuit for the system in Fig. 11.21 is shown in Fig. 11.22. The line current can be calculated from the expression for the complex power at the substation. Thus,

Figure 11.22 A single phase

equivalent circuit for the system in Fig. 11.21.

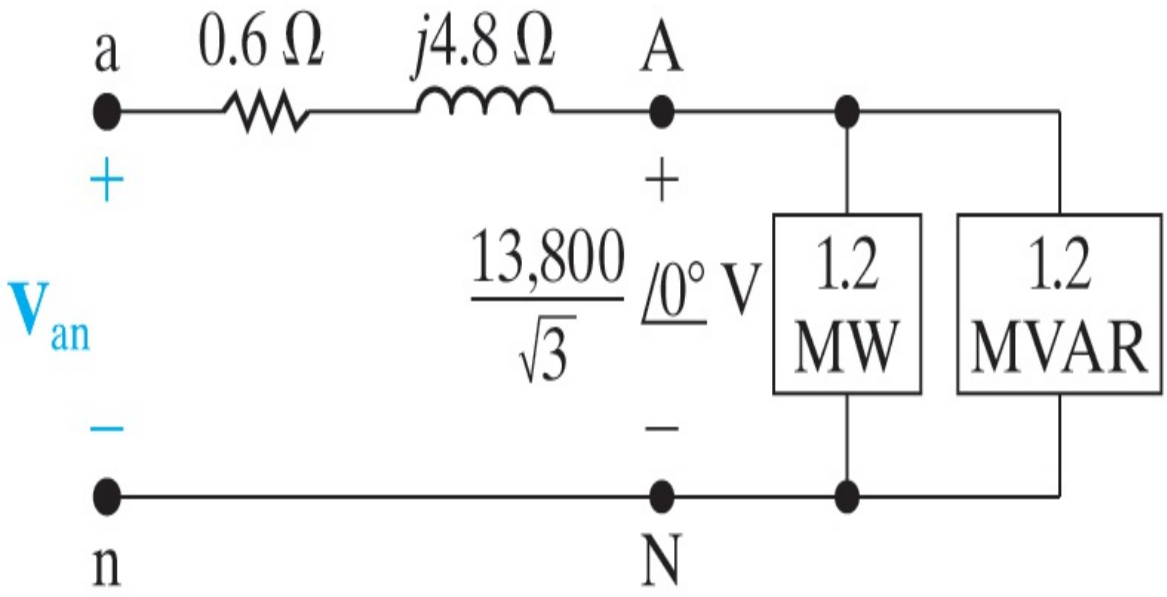


Figure 11.22 Full Alternative Text
13,8003IaA*=(1.2 + j1.2)106.

It follows that

IaA*=150.61 + j150.61 A

or

IaA=150.61 − j150.61 A.

The line-to-neutral voltage at the generating plant is the voltage drop across the load, VAN, plus the voltage across the transmission line, so

V an = 13,800 3 0° + (0.6 + j4.8)(150.61 − j150.61) =
8780.74 + j632.58=8803.50 4.12° V.

Therefore, the magnitude of the line voltage at the generating plant is

|Vab|=3(8803.50)=15,248.11 V.

We are assuming the utility is required to keep the voltage level within ±5% of the nominal value. This means the magnitude of the line-to-line voltage at the power plant should not exceed 14.5 kV nor be less than 13.1 kV. The magnitude of the line voltage at the generating plant exceeds 14.5 kV, so could cause problems for customers.

To address this problem, connect a capacitor bank to the substation bus that supplies the magnetizing vars required by the load. Now the generator does not need to supply this reactive power, the load has a unity power factor, and the line current IaA becomes

IaA=150.61 + j0 A.

Therefore, the voltage at the generating plant necessary to maintain a line-to-line voltage of 13,800 V at the substation is

V an = 13,800 3 0° + (0.6 + j4.8)(150.61 − j0) = 8057.80 + j722.94=8090.17 5.13° V.

Hence

|Vab|=3(8090.17)=14,012.58 V.

This voltage level falls within the allowable range of 13.1 kV to 14.5 kV.

SELF-CHECK: *Assess your understanding of this Practical Perspective by trying Chapter Problems 11.53(a)–(b) and 11.54, 11.57, and 11.58.*

Summary

- A set of balanced three-phase voltages consists of three sinusoidal voltages that have identical amplitudes and frequencies but are out of phase with each other by exactly 120°. For the abc (or positive) phase sequence,

  $V_a = V_m \angle 0°, \quad V_b = V_m \angle -120°, \quad V_c = V_m \angle 120°.$

  For the acb (or negative) phase sequence,

  $V_a = V_m \angle 0°, \quad V_b = V_m \angle 120°, \quad V_c = V_m \angle -120°.$

  (See 414)

- Three-phase systems can be unbalanced or balanced. A three-phase system is balanced when the following conditions are satisfied:

  1. The voltage sources form a balanced three-phase set.
  2. The impedance of each phase of the voltage source is the same.
  3. The impedance of each line is the same.
  4. The impedance of each phase of the load is the same.

  (See page 417.)

- A **single-phase equivalent circuit** is used to calculate the line current and the phase voltage in one phase of the Y-Y structure. The a-phase is normally chosen for this purpose. If the structure is not Y-Y, any Δ connections should be transformed into equivalent Y connections before creating a single-phase equivalent circuit. (See 418.)

- Once we know the line current and phase voltage in the a-phase equivalent circuit, we can use analytical shortcuts to find any current or

voltage in a balanced three-phase circuit, based on the following facts:

  - The b- and c-phase currents and voltages are identical to the a-phase current and voltage except for a 120° shift in phase. The direction of the phase shift depends on the phase sequence.
  - The set of line voltages is out of phase with the set of phase voltages by ±30°. The plus sign corresponds to the positive phase sequence, while the minus sign corresponds to the negative phase sequence.
  - In a Y-Y circuit, the magnitude of a line voltage is 3 times the magnitude of a phase voltage.
  - The set of line currents is out of phase with the set of phase currents in Δ-connected sources and loads by ∓30°. The minus sign corresponds to the positive phase sequence, while the plus sign corresponds to the negative phase sequence.
  - The magnitude of a line current is 3 times the magnitude of a phase current in a Δ-connected source or load.

  (See pages 419 and 423.)

- The techniques for calculating per-phase average power, reactive power, and complex power are identical to those introduced in Chapter 10. (See page 425.)
- The total real, reactive, and complex power can be determined either by multiplying the corresponding per-phase quantity by 3 or by using the expressions based on line current and line voltage, as given by Eqs. 11.17, 11.18, and 11.19. (See 426.)
- The total instantaneous power in a balanced three-phase circuit is constant and equals 1.5VmImpf. (See 427.)
- A wattmeter measures the average power delivered to a load by using a current coil connected in series with the load and a potential coil

connected in parallel with the load. (See 430.)

- The total average power in a balanced three-phase circuit can be measured by summing the readings of two wattmeters connected in two different phases of the circuit. (See 432.)

Problems

All phasor voltages in the following Problems are stated in terms of the rms value.

Section 11.1

1. 11.1 What is the phase sequence of each of the following sets of voltages?

    1. a) $v_a = 180 \cos(\omega t + 27^\circ)$ V, $v_b = 180 \cos(\omega t + 147^\circ)$ V, $v_c = 180 \cos(\omega t - 93^\circ)$ V.

    2. b) $v_a = 4160 \cos(\omega t - 18^\circ)$ V, $v_b = 4160 \cos(\omega t - 138^\circ)$ V, $v_c = 4160 \cos(\omega t + 102^\circ)$ V.

2. 11.2 PSPICEMULTISIM For each set of voltages, state whether or not the voltages form a balanced three-phase set. If the set is balanced, state whether the phase sequence is positive or negative. If the set is not balanced, explain why.

    1. a) $v_a = 180 \cos 377t$ V, $v_b = 180 \cos(377t - 120^\circ)$ V, $v_c = 180 \cos(377t - 240^\circ)$ V.

    2. b) $v_a = 180 \sin 377t$ V, $v_b = 180 \sin(377t + 120^\circ)$ V, $v_c = 180 \sin(377t - 120^\circ)$ V.

    3. c) $v_a = -400 \sin 377t$ V, $v_b = 400 \sin(377t + 210^\circ)$ V, $v_c = 400 \cos(377t - 30^\circ)$ V.

    4. d) $v_a = 200 \cos(2000t + 30^\circ)$ V, $v_b = 201 \cos(2000t + 150^\circ)$ V, $v_c = 200 \cos(2000t + 270^\circ)$ V.

    5. e) $v_a = 208 \cos(630t + 42^\circ)$ V, $v_b = 208 \cos(630t - 78^\circ)$ V, $v_c = 208 \cos(630t - 201^\circ)$ V.

    6. f) $v_a = 240 \cos 377t$ V, $v_b = 240 \cos(377t - 120^\circ)$ V, $v_c = 240 \cos(397t + 120^\circ)$ V.

3. 11.3 Verify that the sum of the three voltage phasors is zero for either Eq. 11.1 or Eq. 11.2.

Section 11.2

1. 11.4 Refer to the circuit in Fig. 11.5(b). Assume that there are no external connections to the terminals a, b, c. Assume further that the three windings are from a three-phase generator whose voltages are those described in Problem 11.2(b). Determine the current circulating in the Δ-connected generator.

2. 11.5 Repeat Problem 11.4 but assume that the three-phase voltages are those described in Problem 11.2(c).

Section 11.3

1. 11.6 PSPICEMULTISIM

    1. a) Is the circuit in Fig. P11.6 a balanced or unbalanced three-phase system? Explain.

    2. b) Find I0.

Figure P11.6

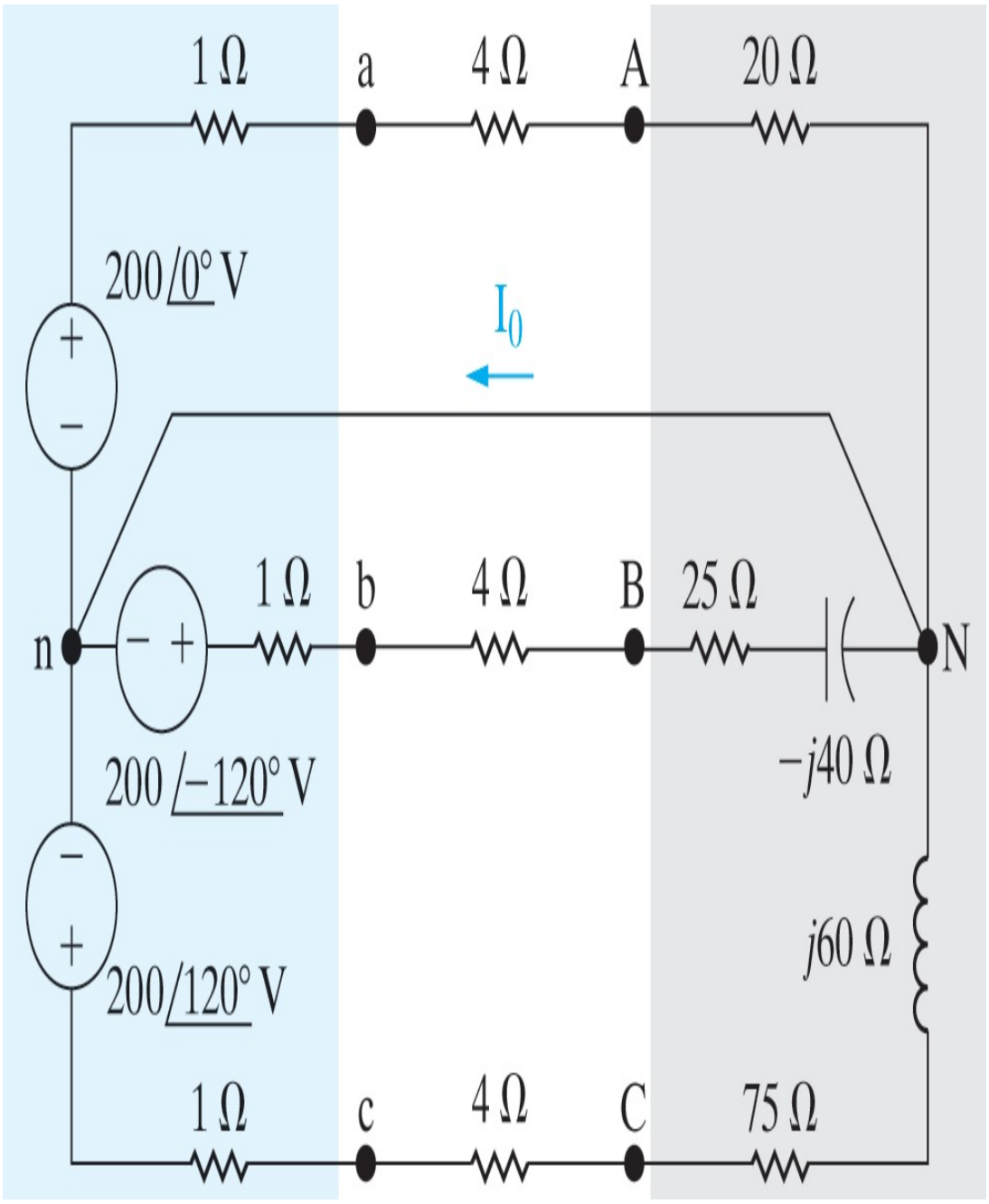


Figure P11.6 Full Alternative Text

2. 11.7 PSPICEMULTISIM

    1. a) Find I0 in the circuit in Fig. P11.7.

    2. b) Find VAN.

3. c) Find VAB.
4. d) Is the circuit a balanced or unbalanced three-phase system?

Figure P11.7

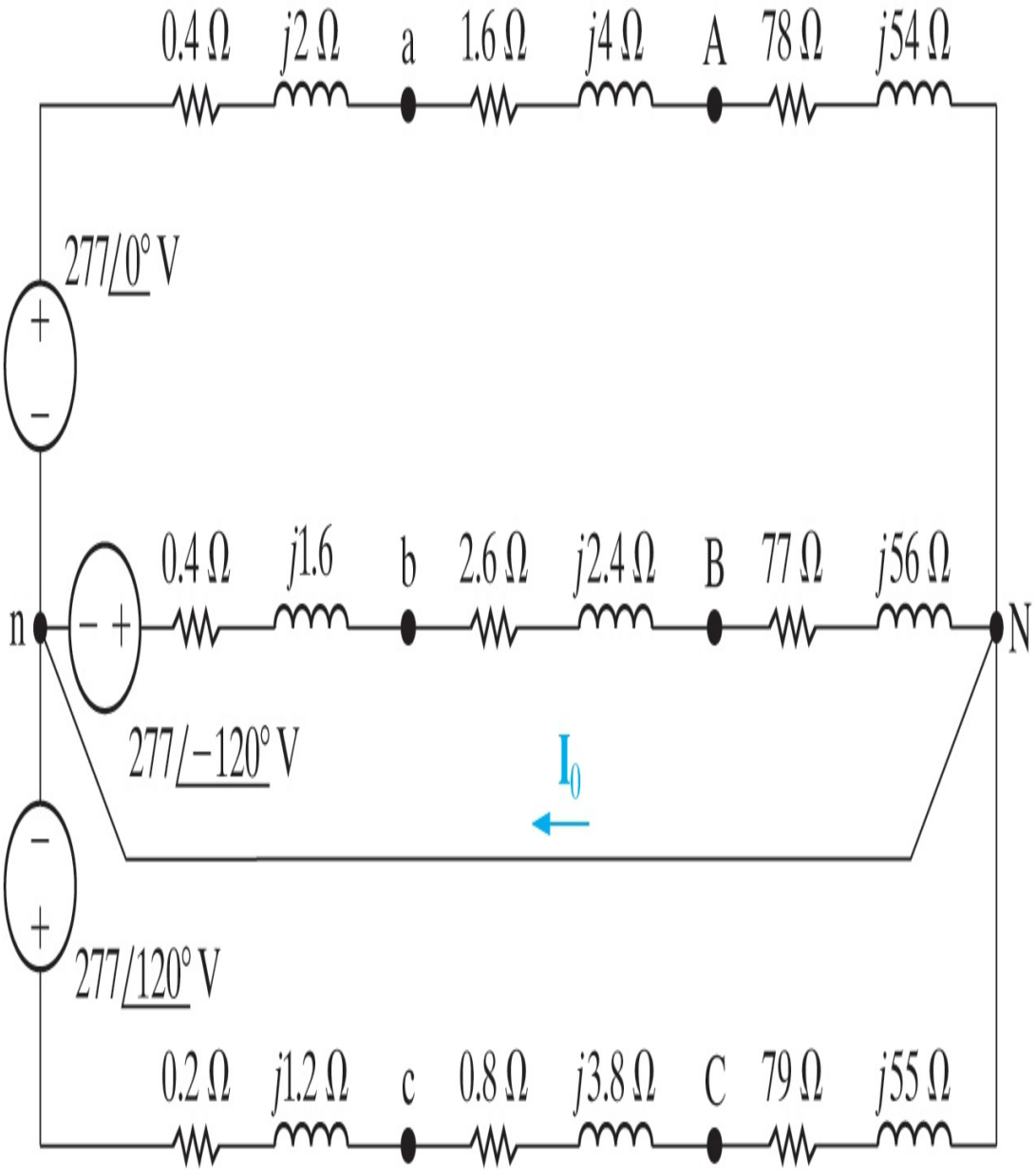


Figure P11.7 Full Alternative Text

3. 11.8 Find the rms value of I0 in the unbalanced three-phase circuit seen in Fig. P11.8.

Figure P11.8

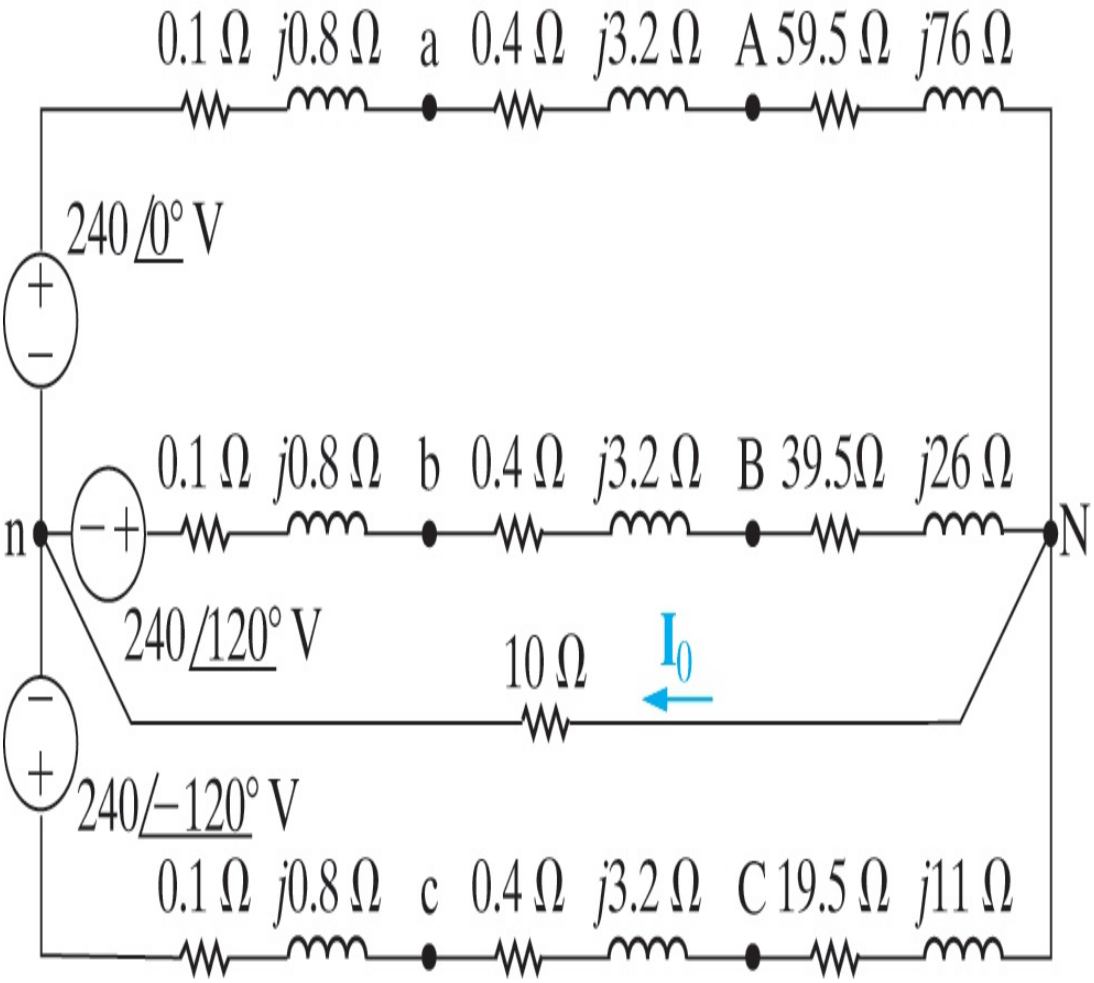


Figure P11.8 Full Alternative Text

4. 11.9 The time-domain expressions for three line-to-neutral voltages at the terminals of a Y-connected load are

   v AN = 169.71 cos (ωt+ 26 °) V, v BN = 169.71 cos (ωt− 94 °) V, v CN = 169.71 cos (ωt− 94 °) V,

   What are the time-domain expressions for the three line-to-line voltages

vAB, vBC, and vCA?

5. 11.10 A balanced three-phase circuit has the following characteristics:
    - Y-Y connected;
    - The line voltage at the source is $V_{ab} = 110\sqrt{3}\angle -60°$ V;
    - The phase sequence is positive;
    - The line impedance is 3+j2 Ω/ϕ;
    - The load impedance is 37+j28 Ω/ϕ;

    1. a) Draw the single phase equivalent circuit for the a-phase.
    2. b) Calculate the line current in the a-phase.
    3. c) Calculate the line voltage at the load in the a-phase.

6. 11.11 The magnitude of the line voltage at the terminals of a balanced Y-connected load is 660 V. The load impedance is 30.48+j22.86 Ω/ϕ. The load is fed from a line that has an impedance of 0.25+j2 Ω/ϕ.

    1. a) What is the magnitude of the line current?
    2. b) What is the magnitude of the line voltage at the source?

7. 11.12 The magnitude of the phase voltage of an ideal balanced three-phase Y-connected source is 125 V. The source is connected to a balanced Y-connected load by a distribution line that has an impedance of 0.1+j0.8 Ω/ϕ. The load impedance is 19.9+j14.2 Ω/ϕ. The phase sequence of the source is acb. Use the a-phase voltage of the source as the reference. Specify the magnitude and phase angle of the following quantities: (a) the three line currents, (b) the three line voltages at the source, (c) the three phase voltages at the load, and (d) the three line voltages at the load.

Section 11.4

1. 11.13 A balanced Δ-connected load has an impedance of 60+j45 Ω/ϕ. The load is fed through a line having an impedance of 0.8+j0.6 Ω/ϕ. The phase voltage at the terminals of the load is 480 V. The phase sequence is positive. Use VAB as the reference.
    1. a) Calculate the three phase currents of the load.
    2. b) Calculate the three line currents.
    3. c) Calculate the three line voltages at the sending end of the line.
2. 11.14 A balanced, three-phase circuit is characterized as follows:
    - Y-Δ connected;
    - Source voltage in the b-phase is 150 135° V;
    - Source phase sequence is acb;
    - Line impedance is 2+j3 Ω/ϕ;
    - Load impedance is 129+j171 Ω/ϕ.

    1. a) Draw the single phase equivalent for the a-phase.
    2. b) Calculate the a-phase line current.
    3. c) Calculate the a-phase line voltage for the three-phase load.
3. 11.15 An acb sequence balanced three-phase Y-connected source supplies power to a balanced, three-phase Δ-connected load with an impedance of 12+j9 Ω/ϕ. The source voltage in the b-phase is 240 -50° V. The line impedance is 1+j1 Ω/ϕ. Draw the single phase equivalent circuit for the a-phase and use it to find the current in the a-phase of the

load.

4. 11.16 In a balanced three-phase system, the source is a balanced Y with an abc phase sequence and a line voltage V ab =208 50 ° V. The load is a balanced Y in parallel with a balanced Δ. The phase impedance of the Y is 4+j3 Ω/ϕ and the phase impedance of the Δ is 3−j9 Ω/ϕ. The line impedance is 1.4+j0.8 Ω/ϕ. Draw the single phase equivalent circuit and use it to calculate the line voltage at the load in the a-phase.

5. 11.17 A balanced Y-connected load having an impedance of 72+j21 Ω/ϕ is connected in parallel with a balanced Δ-connected load having an impedance of 1500° Ω/ϕ. The parallel loads are fed from a line having an impedance of j1 Ω/ϕ. The magnitude of the line-to-line voltage of the Y-load is 7650 V.

    1. a) Calculate the magnitude of the current in the line feeding the loads.

    2. b) Calculate the magnitude of the phase current in the Y-connected load.

    3. c) Calculate the magnitude of the phase current in the Δ-connected load.

    4. d) Calculate the magnitude of the line voltage at the sending end of the line.

6. 11.18 A three-phase Δ-connected generator has an internal impedance of 9+j90 m Ω/ϕ. When the load is removed from the generator, the magnitude of the terminal voltage is 13,800 V. The generator feeds a Δ-connected load through a transmission line with an impedance of 20+j180 m Ω/ϕ. The per-phase impedance of the load is 7.056+j3.417 Ω.

    1. a) Construct a single-phase equivalent circuit.

    2. b) Calculate the magnitude of the line current.

3. c) Calculate the magnitude of the line voltage at the terminals of the load.

4. d) Calculate the magnitude of the line voltage at the terminals of the source.

5. e) Calculate the magnitude of the phase current in the load.

6. f) Calculate the magnitude of the phase current in the source.

7. 11.19 The impedance Z in the balanced three-phase circuit in Fig. P11.19 is 160+j120 Ω. Find

    1. a) IAB, IBC, and ICA,

    2. b) IaA, IbB, and IcC,

    3. c) Iba, Icb, and Iac.

Figure P11.19

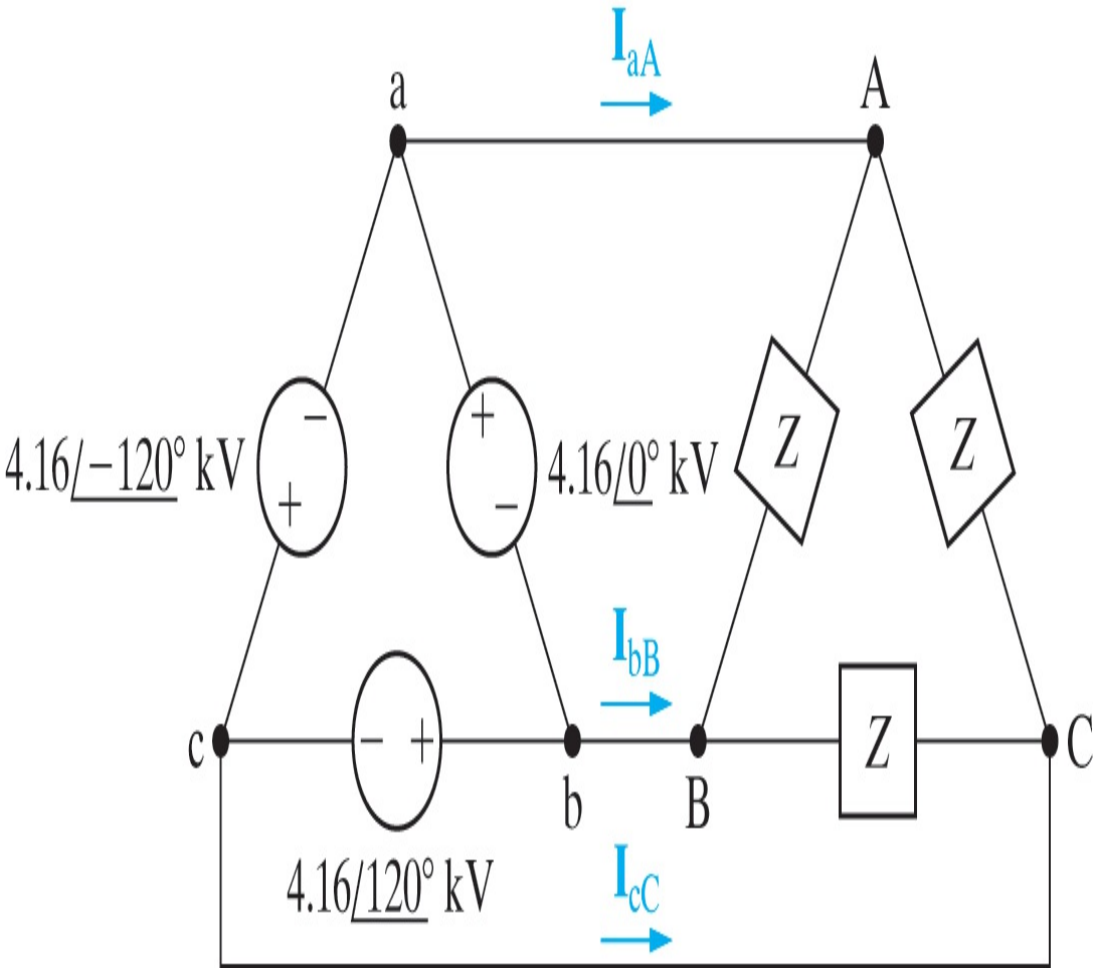


Figure P11.19 Full Alternative Text

8. 11.20 PSPICEMULTISIM For the circuit shown in Fig. P11.20, find
    1. a) the phase currents IAB, IBC, and ICA,
    2. b) the line currents IaA, IbB, and IcC

when Z1=2.4−j0.7 Ω, Z2=8+j6 Ω, and Z3=20+j0 Ω.

Figure P11.20

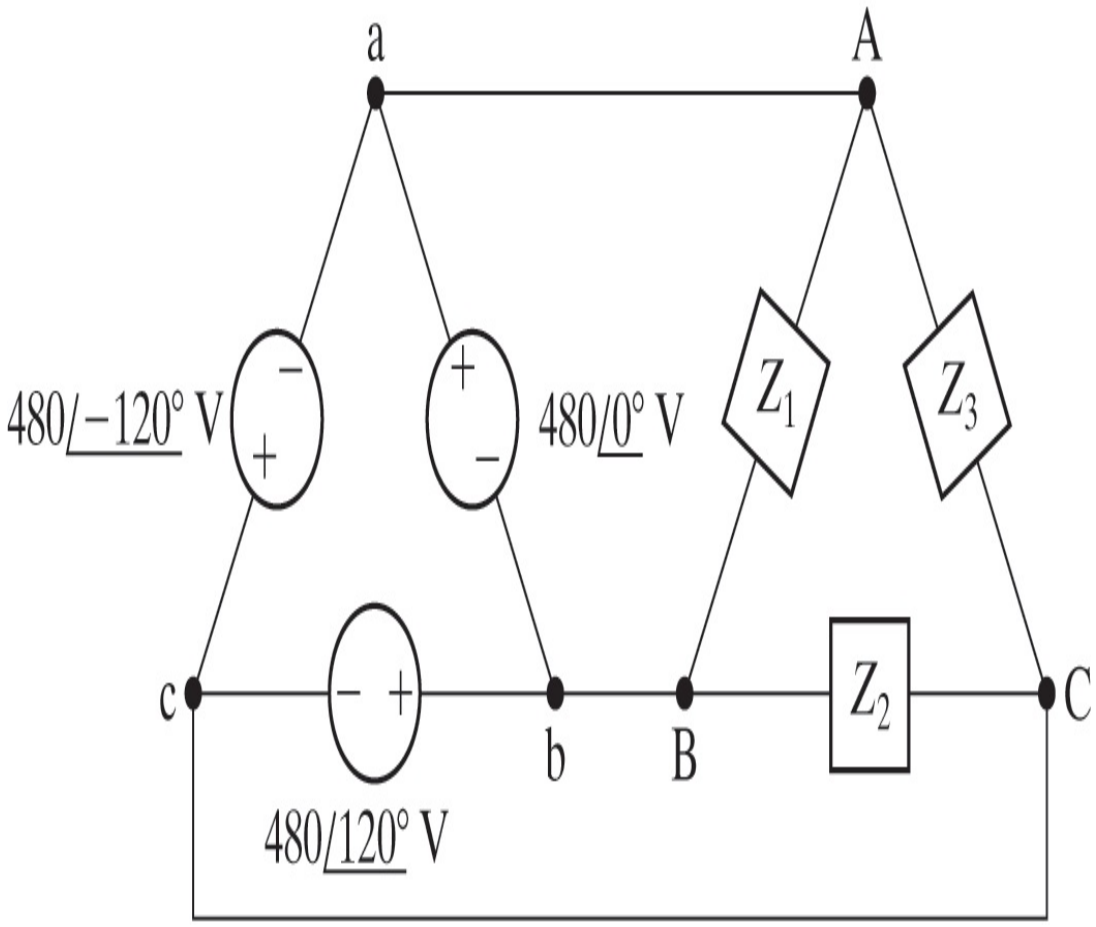


Figure P11.20 Full Alternative Text

9. 11.21 A balanced three-phase Δ-connected source is shown in Fig. P11.21.

    1. a) Find the Y-connected equivalent circuit.

    2. b) Show that the Y-connected equivalent circuit delivers the same open-circuit voltage as the original Δ-connected source.

    3. c) Apply an external short circuit to the terminals A, B, and C. Use the Δ-connected source to find the three line currents IaA, IbB, and IcC.

    4. d) Repeat (c) but use the Y-equivalent source to find the three line currents.

Figure P11.21

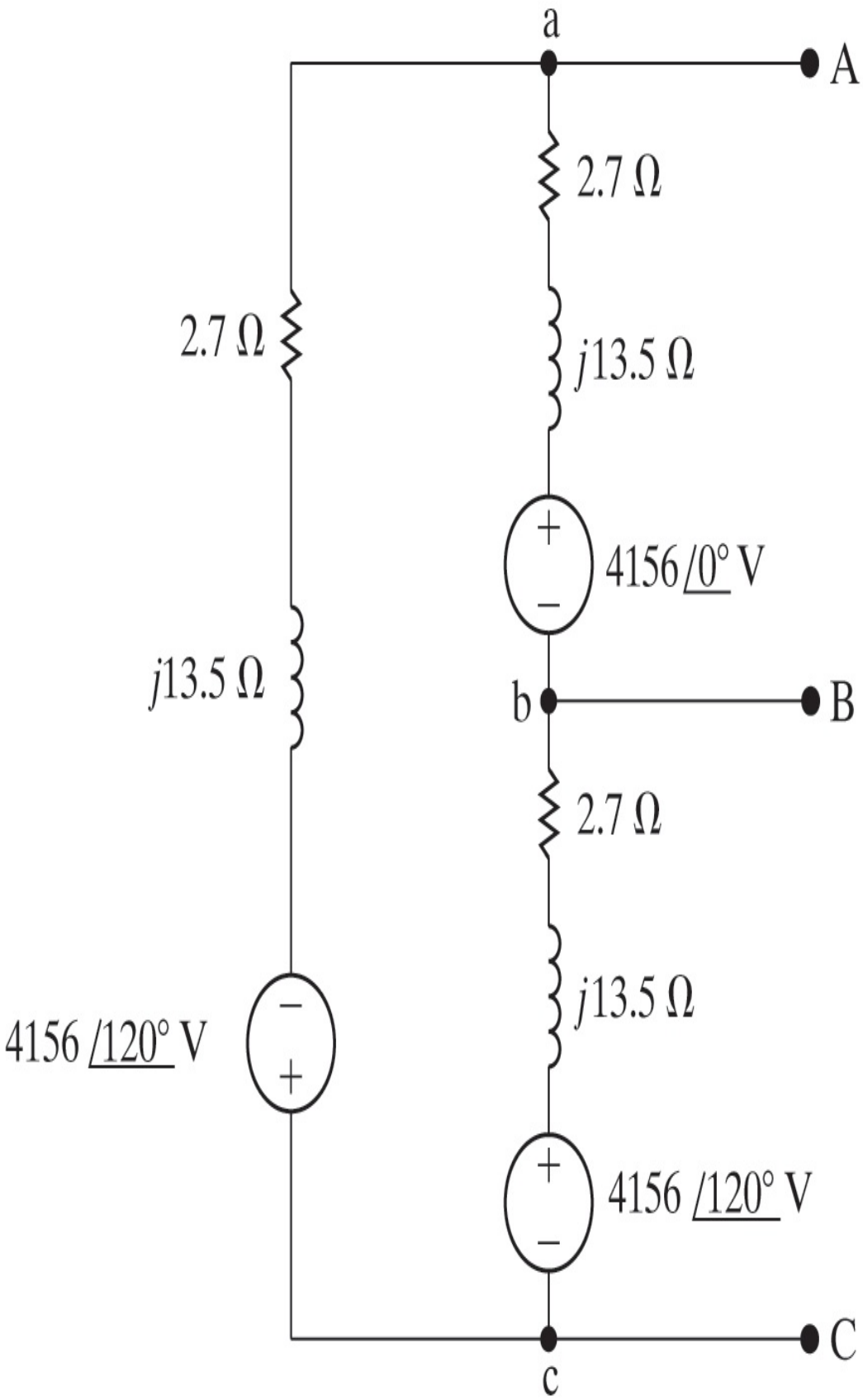

a
A
2.7 Ω
2.7 Ω
j13.5 Ω
4156 /0° V
j13.5 Ω
b
B
2.7 Ω
j13.5 Ω
4156 /120° V
4156 /120° V
C
c

Figure P11.21 Full Alternative Text

10. 11.22 The Δ-connected source of Problem 11.21 is connected to a Y-connected load by means of a balanced three-phase distribution line. The load impedance is 1910−j636 Ω/ϕ. and the line impedance is 9.1+j71.5 Ω/ϕ.

    1. a) Construct a single-phase equivalent circuit for the system.

    2. b) Determine the magnitude of the line voltage at the terminals of the load.

    3. c) Determine the magnitude of the phase current in the Δ-source.

    4. d) Determine the magnitude of the line voltage at the terminals of the source.

Section 11.5

1. 11.23

    1. a) Find the rms magnitude and the phase angle of ICA in the circuit shown in Fig. P11.23.

    2. b) What percent of the average power delivered by the three-phase source is dissipated in the three-phase load?

Figure P11.23

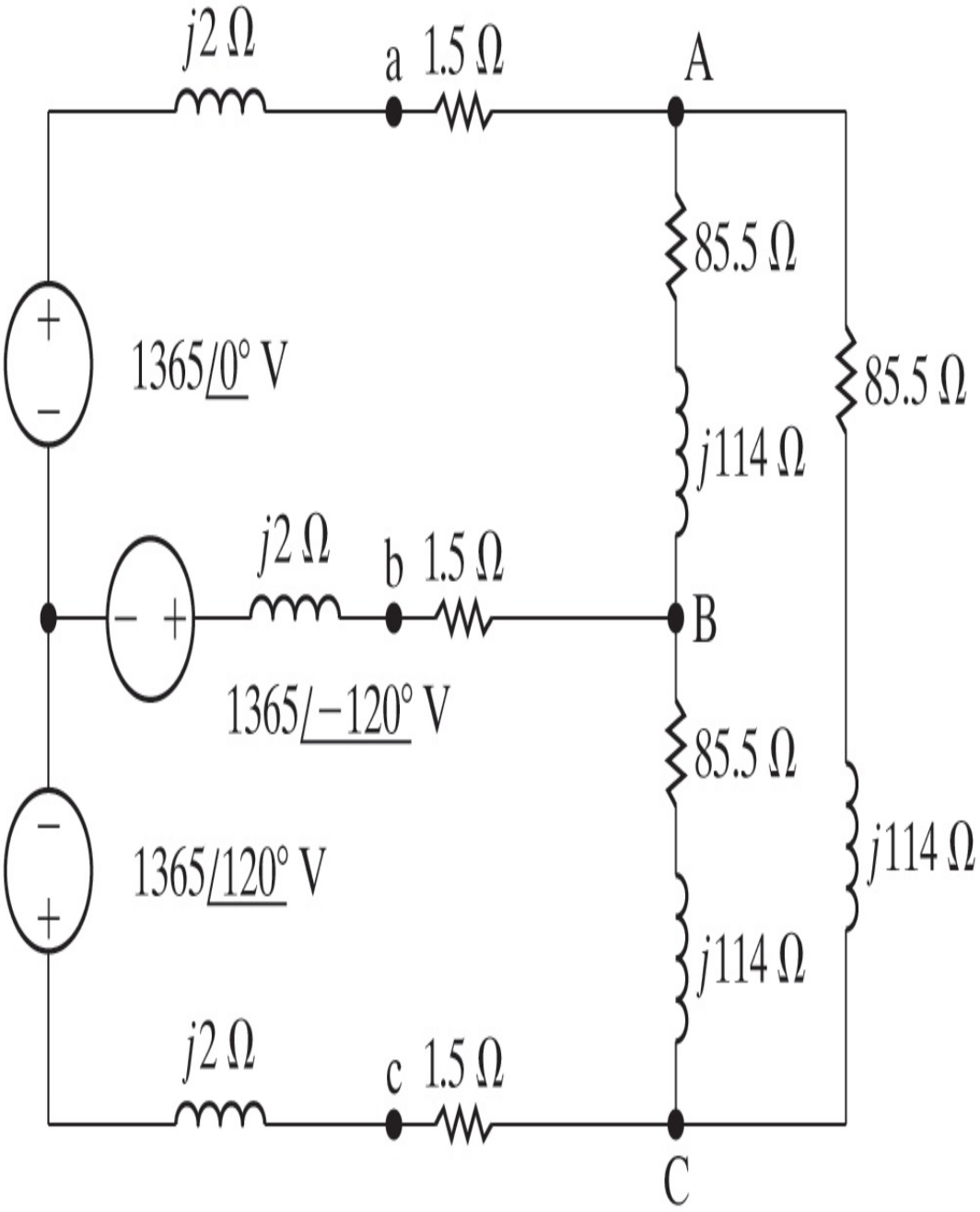


Figure P11.23 Full Alternative Text

2. 11.24 A balanced three-phase source is supplying 60 kVA at 0.6 lagging to two balanced Y-connected parallel loads. The distribution line connecting the source to the load has negligible impedance. Load 1 is purely resistive and absorbs 30 kW. Find the per-phase impedance of Load 2 if the line voltage is 1203 V and the impedance components are

in series.

3. 11.25 In a balanced three-phase system, the source has an abc sequence, is Y-connected, and Van=12020° V. The source feeds two loads, both of which are Y-connected. The impedance of load 1 is 8+j6 Ω/ϕ. The complex power for the a-phase of load 2 is 60036° VA. Find the total complex power supplied by the source.

4. 11.26 The line-to-neutral voltage at the terminals of the balanced three-phase load in the circuit shown in Fig. P11.26 is 1200 V. At this voltage, the load is absorbing 500 kVA at 0.96 pf lag.

    1. a) Using VAN as the reference, find Ina in polar form.

    2. b) Calculate the complex power associated with the ideal three-phase source.

    3. c) Check that the total average power delivered equals the total average power absorbed.

    4. d) Check that the total magnetizing reactive power delivered equals the total magnetizing reactive power absorbed.

Figure P11.26

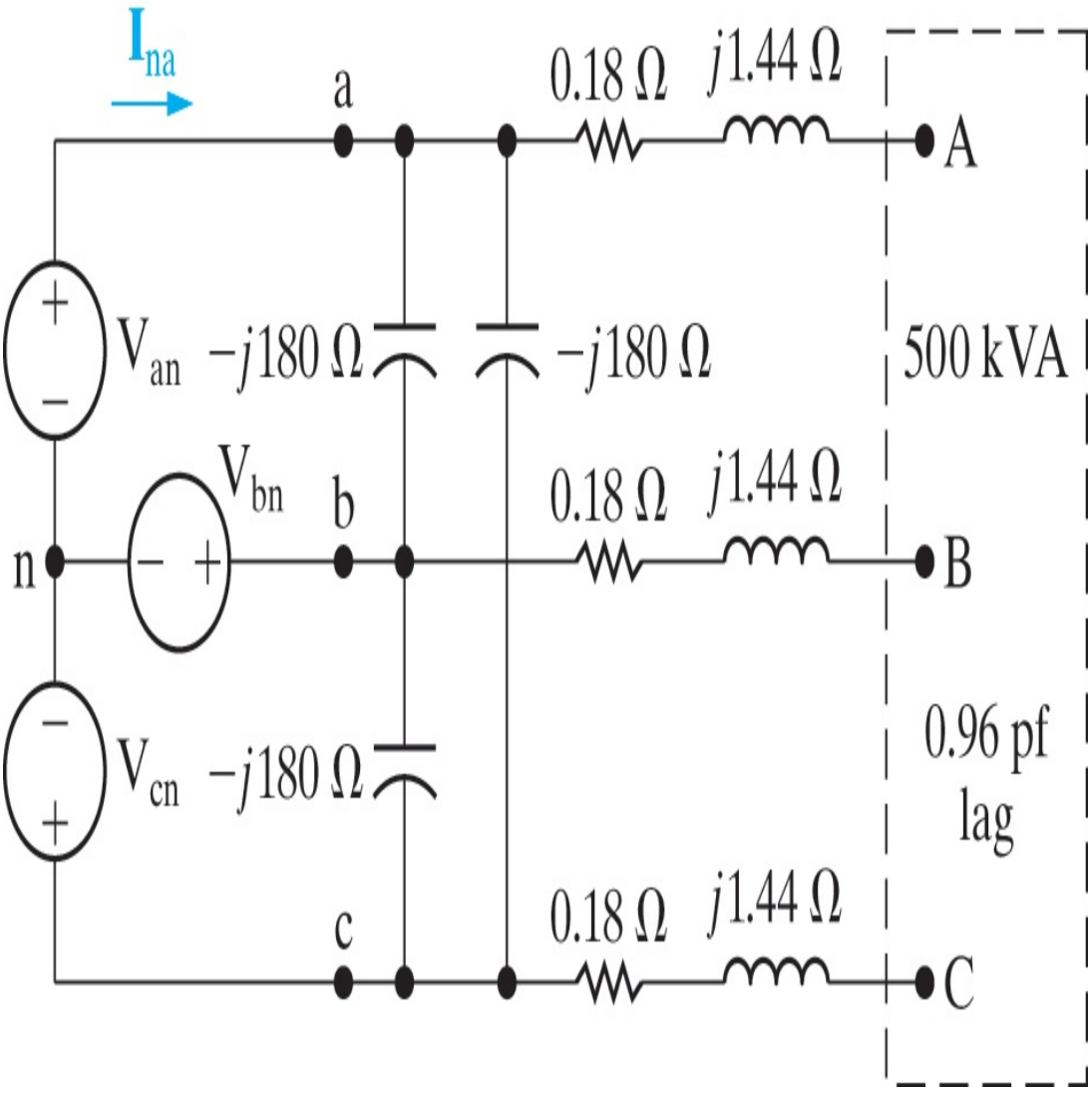


Figure P11.26 Full Alternative Text

5. 11.27 A three-phase positive sequence Y-connected source supplies 14 kVA with a power factor of 0.75 lagging to a parallel combination of a Y-connected load and a Δ-connected load. The Y-connected load uses 9 kVA at a power factor of 0.6 lagging and has an a-phase current of $10\angle -30^\circ$ A.

    1. a) Find the complex power per phase of the Δ-connected load.

    2. b) Find the magnitude of the line voltage.

6. 11.28 A balanced three-phase distribution line has an impedance of

1+j8 Ω/ϕ. This line is used to supply three balanced three-phase loads that are connected in parallel. The three loads are L1=120 kVA at 0.96 pf leading, L2=180 kVA at 0.80 pf lagging, and L3=100.8 kW and 15.6 kVAR (magnetizing). The magnitude of the line voltage at the terminals of the loads is 24003 V.

1. a) What is the magnitude of the line voltage at the sending end of the line?

2. b) What is the percent efficiency of the distribution line with respect to average power?

7. 11.29 The three tools described below are part of a university's machine shop. Each piece of equipment is a balanced three-phase load rated at 220 V. Calculate (a) the magnitude of the line current supplying these three tools and (b) the power factor of the combined load.

   - Drill press: 10.2 kVA at 0.87 pf lag.
   - Lathe: 4.2 kW at 0.91 pf lag.
   - Band saw: line current 36.8 A, 7.25 kVAR.

8. 11.30 Calculate the complex power in each phase of the unbalanced load in Problem 11.20.

9. 11.31 Show that the total instantaneous power in a balanced three-phase circuit is constant and equal to $1.5V_mI_m\cos\theta\phi$, where Vm and Im represent the maximum amplitudes of the phase voltage and phase current, respectively.

10. 11.32 The total apparent power supplied in a balanced, three-phase Y-Δ system is 4800 VA. The line voltage is 240 V. If the line impedance is negligible and the power factor angle of the load is -50°, determine the impedance of the load.

11. 11.33 A balanced three-phase load absorbs 96 kVA at a lagging power factor of 0.8 when the line voltage at the terminals of the load is 480 V.

Find four equivalent circuits that can be used to model this load.

12. 11.34 At full load, a commercially available 100 hp, three-phase induction motor operates at an efficiency of 97% and a power factor of 0.88 lag. The motor is supplied from a three-phase outlet with a line-voltage rating of 208 V.

    1. a) What is the magnitude of the line current drawn from the 208 V outlet? (1 hp=746 W.)

    2. b) Calculate the reactive power supplied to the motor.

13. 11.35 A three-phase line has an impedance of 0.1+j0.8 Ω/ϕ. The line feeds two balanced three-phase loads connected in parallel. The first load is absorbing a total of 630 kW and absorbing 840 kVAR magnetizing vars. The second load is Y-connected and has an impedance of 15.36−j4.48 Ω/ϕ. The line-to-neutral voltage at the load end of the line is 4000 V. What is the magnitude of the line voltage at the source end of the line?

14. 11.36 Three balanced three-phase loads are connected in parallel. Load 1 is Y-connected with an impedance of 400+j300 Ω/ϕ; load 2 is Δ-connected with an impedance of 2400+j1800 Ω/ϕ; and load 3 is 172.8+j2203.2 kVA. The loads are fed from a distribution line with an impedance of 2+j16 Ω/ϕ. The magnitude of the line-to-neutral voltage at the load end of the line is 243 kV.

    1. a) Calculate the total complex power at the sending end of the line.

    2. b) What percentage of the average power at the sending end of the line is delivered to the loads?

15. 11.37 The output of the balanced positive-sequence three-phase source in Fig. P11.37 is 41.6 kVA at a lagging power factor of 0.707. The line voltage at the source is 240 V.

    1. a) Find the magnitude of the line voltage at the load.

2. b) Find the total complex power at the terminals of the load.

Figure P11.37

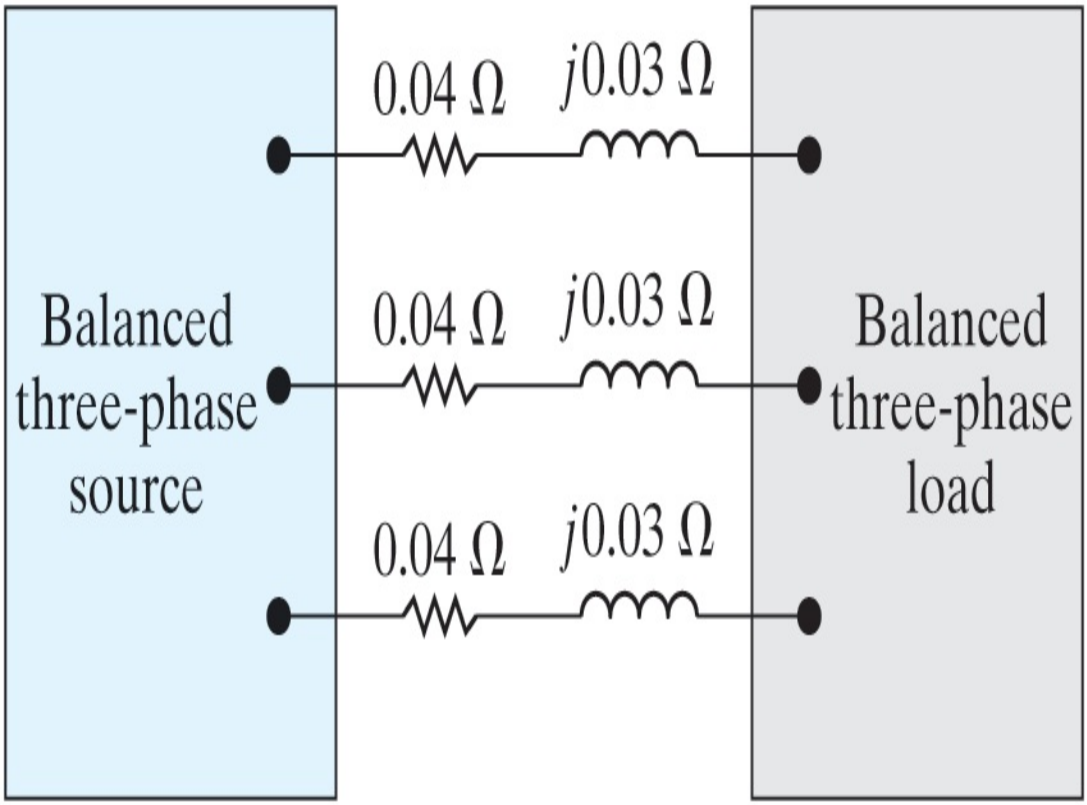


Figure P11.37 Full Alternative Text

16. 11.38 A balanced three-phase source is supplying 60 kVA at 0.96 pf leading to two balanced Δ-connected parallel loads. The distribution line connecting the source to the load has negligible impedance. Load 1 is purely resistive and absorbs 45 kW.

    1. a) Determine the types of components and their impedances in each phase of load 2 if the line voltage is 630 V and the impedance components are in series.

    2. b) Repeat (a) with the impedance components in parallel.

17. 11.39 The total power delivered to a balanced three-phase load when

operating at a line voltage of 24003 V is 720 kW at a lagging power factor of 0.8. The impedance of the distribution line supplying the load is 0.8+j6.4 Ω/ϕ. Under these operating conditions, the drop in the magnitude of the line voltage between the sending end and the load end of the line is excessive. To compensate, a bank of Y-connected capacitors is placed in parallel with the load. The capacitor bank is designed to furnish 576 kVAR of magnetizing reactive power when operated at a line voltage of 24003 V.

1. a) What is the magnitude of the voltage at the sending end of the line when the load is operating at a line voltage of 24003 V and the capacitor bank is disconnected?

2. b) Repeat (a) with the capacitor bank connected.

3. c) What is the average power efficiency of the line in (a)?

4. d) What is the average power efficiency in (b)?

5. e) If the system is operating at a frequency of 60 Hz, what is the size of each capacitor in microfarads?

18. 11.40 A balanced bank of delta-connected capacitors is connected in parallel with the load described in Assessment Problem 11.9. The effect is to place a capacitor in parallel with the load in each phase. The line voltage at the terminals of the load thus remains at 2450 V. The circuit is operating at a frequency of 60 Hz. The capacitors are adjusted so that the magnitude of the line current feeding the parallel combination of the load and capacitor bank is at its minimum.

    1. a) What is the size of each capacitor in microfarads?

    2. b) Repeat (a) for wye-connected capacitors.

    3. c) What is the magnitude of the line current?

Section 11.6

1. 11.41 The two-wattmeter method is used to measure the power at the load end of the line in Example 11.1. Calculate the reading of each wattmeter.

2. 11.42 The wattmeters in the circuit in Fig. 11.20 read as follows: W1=40,823.09 W, and W2=103,176.91 W. The magnitude of the line voltage is 24003 V. The phase sequence is positive. Find Zϕ.

3. 11.43 In the balanced three-phase circuit shown in Fig. P11.43, the current coil of the wattmeter is connected in line aA, and the potential coil of the wattmeter is connected across lines b and c. Show that the wattmeter reading multiplied by 3 equals the total reactive power associated with the load. The phase sequence is positive.

Figure P11.43

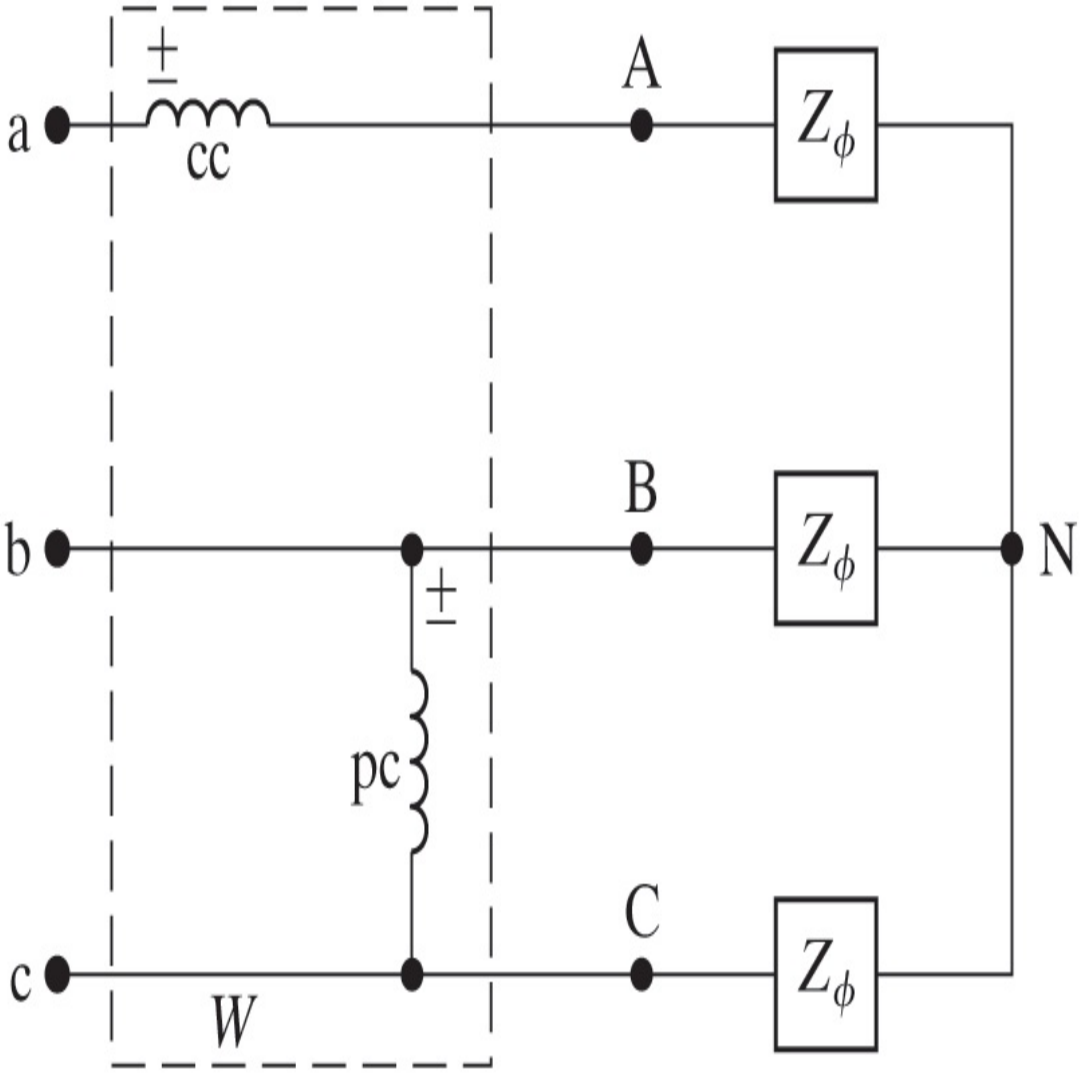


Figure P11.43 Full Alternative Text

4. 11.44 The line-to-neutral voltage in the circuit in Fig. P11.43 is 680 V, the phase sequence is positive, and the load impedance is 16+j12 Ω/ϕ.
    1. a) Calculate the wattmeter reading.
    2. b) Calculate the total reactive power associated with the load.
5. 11.45 The two wattmeters in Fig. 11.20 can be used to compute the total reactive power of the load.
    1. a) Prove this statement by showing that $3(W2-W1)=3V_L I_L \sin\theta_\phi$.

2. b) Compute the total reactive power from the wattmeter readings for each of the loads in Example 11.6. Check your computations by calculating the total reactive power directly from the given voltage and impedance.

6. 11.46 Derive Eqs. 11.22 and 11.23.

7. 11.47

    1. a) Calculate the complex power associated with each phase of the balanced load in Problem 11.19.

    2. b) If the two-wattmeter method is used to measure the average power delivered to the load, specify the reading of each meter.

8. 11.48 The two-wattmeter method is used to measure the power delivered to the unbalanced load in Problem 11.20. The current coil of wattmeter 1 is placed in line aA and that of wattmeter 2 is placed in line bB.

    1. a) Calculate the reading of wattmeter 1.

    2. b) Calculate the reading of wattmeter 2.

    3. c) Show that the sum of the two wattmeter readings equals the total power delivered to the unbalanced load.

9. 11.49 The balanced three-phase load shown in Fig. P11.49 is fed from a balanced, positive-sequence, three-phase Y-connected source. The impedance of the line connecting the source to the load is negligible. The line-to-neutral voltage of the source is 7200 V.

    1. a) Find the reading of the wattmeter in watts.

    2. b) Explain how you would connect a second wattmeter in the circuit so that the two wattmeters would measure the total power.

    3. c) Calculate the reading of the second wattmeter.

    4. d) Verify that the sum of the two wattmeter readings equals the

total average power delivered to the load.

Figure P11.49

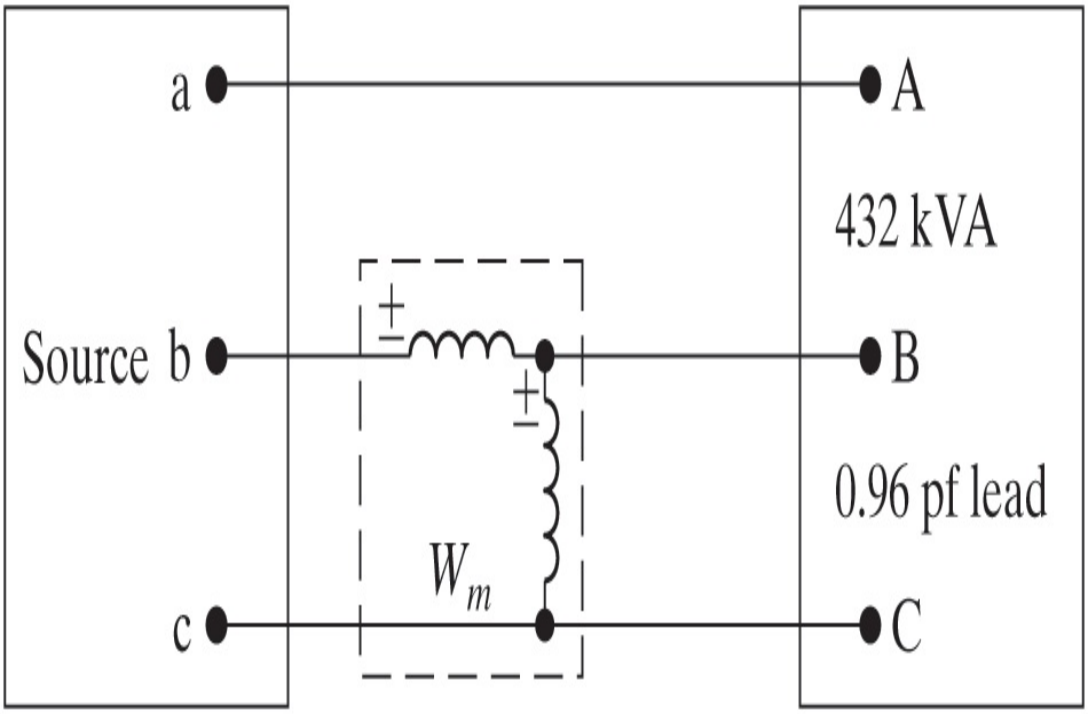


Figure P11.49 Full Alternative Text

10. 11.50

    1. a) Calculate the reading of each wattmeter in the circuit shown in Fig. P11.50. The value of Zϕ is 40 −30° Ω.

    2. b) Verify that the sum of the wattmeter readings equals the total average power delivered to the Δ-connected load.

Figure P11.50

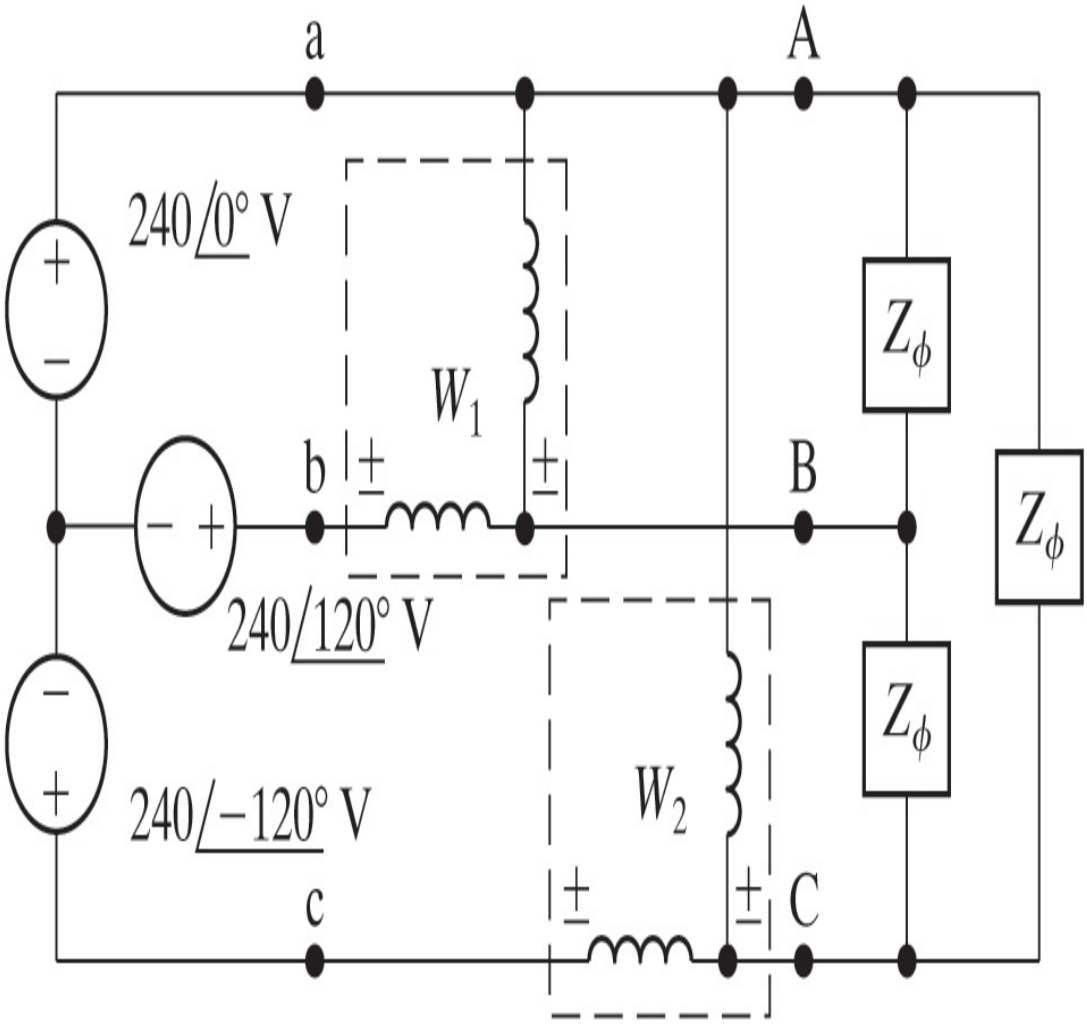


Figure P11.50 Full Alternative Text

11. 11.51

    1. a) Calculate the reading of each wattmeter in the circuit shown in Fig. P11.51 when Z=13.44+j46.08 Ω.

    2. b) Check that the sum of the two wattmeter readings equals the total power delivered to the load.

    3. c) Check that 3(W1−W2) equals the total magnetizing vars delivered to the load.

Figure P11.51

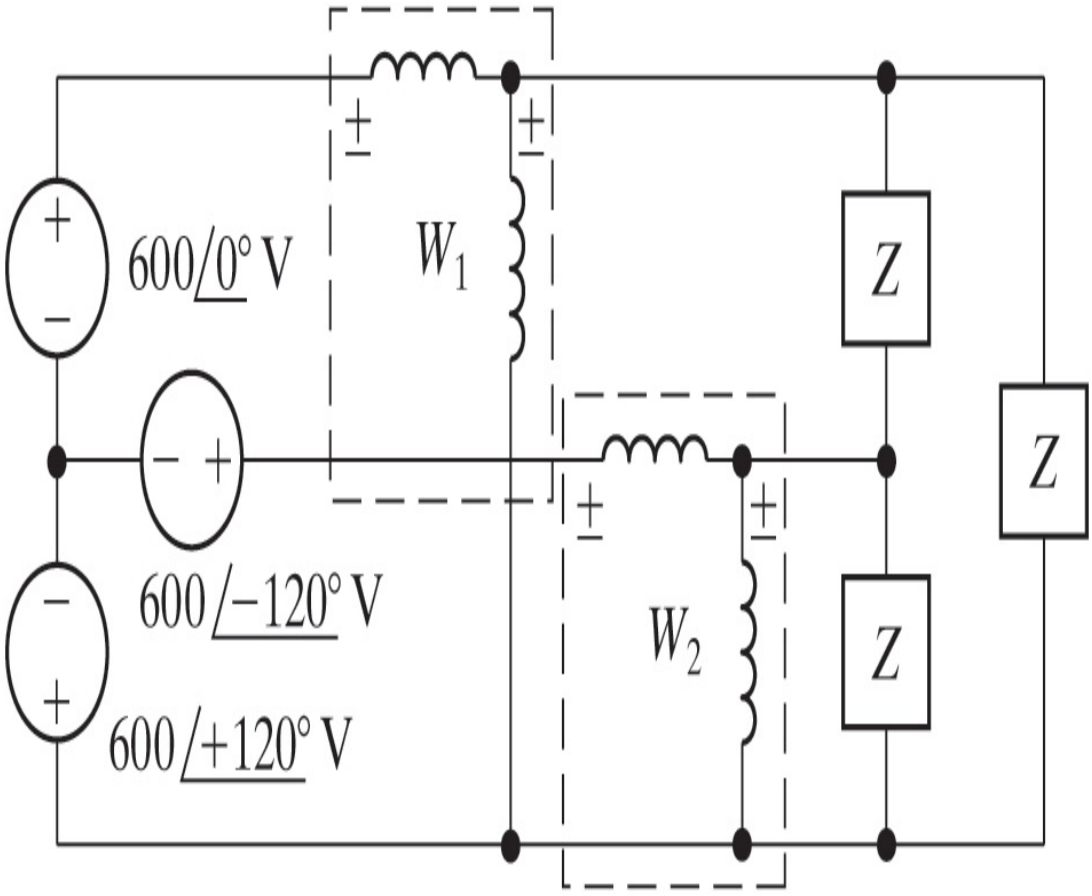


Figure P11.51 Full Alternative Text

12. 11.52

    1. a) Find the reading of each wattmeter in the circuit shown in Fig. P11.52 if ZA=20 30° Ω, ZB=60 0° Ω, and ZC=40 −30° Ω.

    2. b) Show that the sum of the wattmeter readings equals the total average power delivered to the unbalanced three-phase load.

Figure P11.52

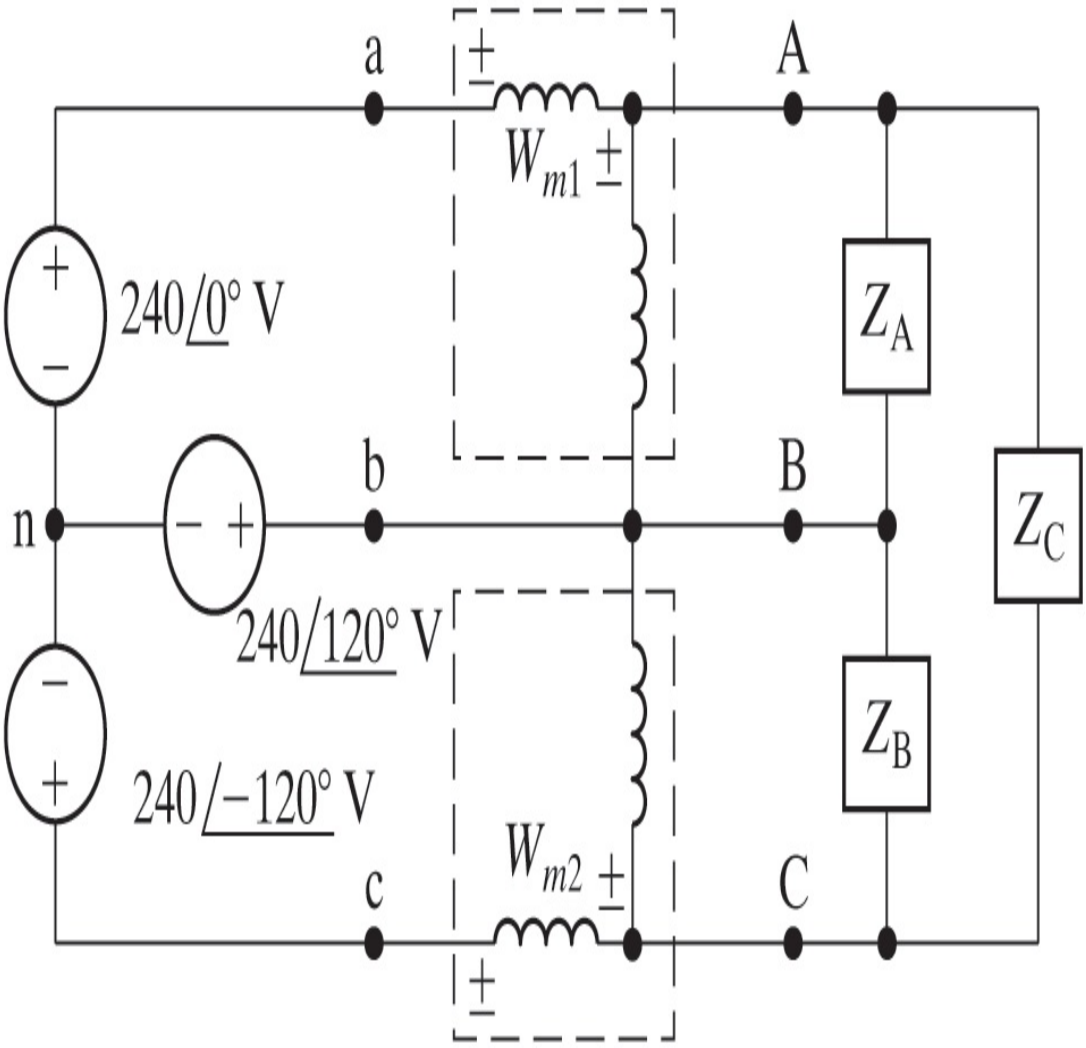


Figure P11.52 Full Alternative Text

Sections 11.1–11.6

1. 11.53 Refer to the Practical Perspective example:
    1. a) Construct a power triangle for the substation load before the capacitors are connected to the bus.
    2. b) Repeat (a) after the capacitors are connected to the bus.
    3. c) Using the line-to-neutral voltage at the substation as a reference,

construct a phasor diagram that depicts the relationship between VAN and Van before the capacitors are added.

4. d) Assume a positive phase sequence and construct a phasor diagram that depicts the relationship between VAB and Vab.

2. 11.54 PRACTICALPERSPECTIVE_ Refer to the Practical Perspective example. Assume the frequency of the utility is 60 Hz.

    1. a) What is the μF rating of each capacitor if the capacitors are delta-connected?

    2. b) What is the μF rating of each capacitor if the capacitors are wye-connected?

3. 11.55 PRACTICALPERSPECTIVE_ Choose a single capacitor from Appendix H that is closest to the μF rating of the wye-connected capacitor from Problem 11.54(b).

    1. a) How much reactive power will a capacitor bank using this new value supply?

    2. b) What line-to-line voltage at the generating plant will be required when this new capacitor bank is connected to the substation bus?

4. 11.56 PRACTICALPERSPECTIVE_ Choose a single capacitor from Appendix H that is closest to the μF rating of the delta-connected capacitor from Problem 11.54(a).

    1. a) How much reactive power will a capacitor bank using this new value supply?

    2. b) What line-to-line voltage at the generating plant will be required when this new capacitor bank is connected to the substation bus?

5. 11.57 PRACTICALPERSPECTIVE_ In the Practical Perspective example, what happens to the voltage level at the generating plant if the substation is maintained at 13.8 kV, the substation load is removed, and the added capacitor bank remains connected?

6. 11.58 PRACTICALPERSPECTIVE_ In the Practical Perspective example, calculate the total line loss in kW before and after the capacitors are connected to the substation bus.

7. 11.59 PRACTICALPERSPECTIVE_ Assume the load on the substation bus in the Practical Perspective example drops to 240 kW and 600 magnetizing kVAR. Also assume the capacitors remain connected to the substation.

    1. a) What is the magnitude of the line-to-line voltage at the generating plant that is required to maintain a line-to-line voltage of 13.8 kV at the substation?

    2. b) Will this power plant voltage level cause problems for other customers?

8. 11.60 PRACTICALPERSPECTIVE_ Assume in Problem 11.59 that when the load drops to 240 kW and 600 magnetizing kVAR the capacitor bank at the substation is disconnected. Also assume that the line-to-line voltage at the substation is maintained at 13.8 kV.

    1. a) What is the magnitude of the line-to-line voltage at the generating plant?

    2. b) Is the voltage level found in (a) within the acceptable range of variation?

    3. c) What is the total line loss in kW when the capacitors stay on line after the load drops to 240+j600 kVA?

    4. d) What is the total line loss in kW when the capacitors are removed after the load drops to 240+j600 kVA?

    5. e) Based on your calculations, would you recommend disconnecting the capacitors after the load drops to 240+j600 kVA? Explain.

Chapter 12 Introduction to the Laplace Transform

Chapter Contents

1. 12.1 Definition of the Laplace Transform
2. 12.2 The Step Function
3. 12.3 The Impulse Function
4. 12.4 Functional Transforms
5. 12.5 Operational Transforms
6. 12.6 Applying the Laplace Transform
7. 12.7 Inverse Transforms
8. 12.8 Poles and Zeros of F(s)
9. 12.9 Initial- and Final-Value Theorems

Chapter Objectives

1. Be able to calculate the Laplace transform of a function using the definition of Laplace transform, the Laplace transform table, and/or a table of operational transforms.

2. Be able to calculate the inverse Laplace transform using partial fraction expansion and the Laplace transform table.

3. Be able to find and plot the poles and zeros for a rational function in the s-domain.

4. Understand and know how to use the initial-value theorem and the final-value theorem.

We now introduce the Laplace transform, which forms the basis of a powerful technique that is widely used to analyze linear, lumped-parameter circuits. We need the Laplace transform analysis technique because we can use it to solve the following types of problems.

- Finding the transient behavior of circuits whose describing equations consist of more than a single node-voltage or mesh-current differential equation. In other words, analyzing multiple-node and multiple-mesh circuits that are described by sets of linear differential equations.

- Determining the transient response of circuits whose signal sources vary in ways more complicated than the simple dc level jumps considered in Chapters 7 and 8.

- Calculating the transfer function for a circuit and using it to find the steady-state sinusoidal response of that circuit when the frequency of the sinusoidal source is varied. We discuss the transfer function in Chapter 13.

- Relating the time-domain behavior of a circuit to its frequency-domain behavior, in a systematic fashion.

In this chapter, we define the Laplace transform, discuss its pertinent characteristics, and present a systematic method for transforming expressions from the frequency domain to the time domain.

Practical Perspective

Transient Effects

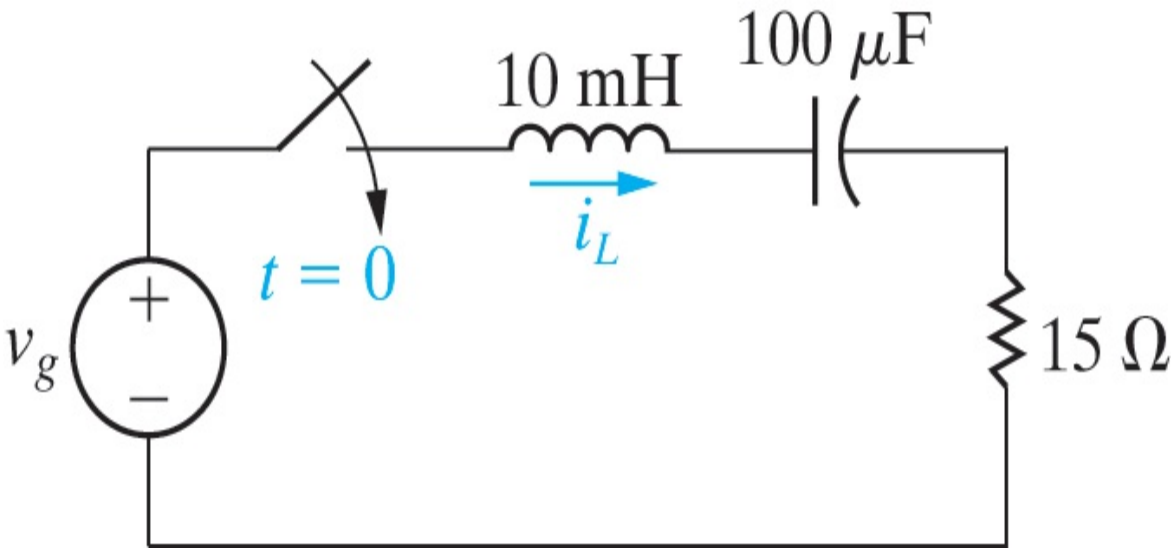
As we learned in Chapter 9, power delivered from electrical wall outlets in the United States can be modeled as a sinusoidal voltage or current source, where the frequency of the sinusoid is 60 Hz. We used the phasor transform, introduced in Chapter 9, to find the steady-state response of a circuit to a sinusoidal source.

But in many cases, we need to consider the complete response of a circuit to a sinusoidal source. Remember that the complete response has two parts—the steady-state response that takes the same form as the input to the circuit, and the transient response that decays to zero as time progresses. When a circuit's source is a 60 Hz sinusoid, the steady-state response is also a 60 Hz sinusoid whose magnitude and phase angle can be calculated using phasor circuit analysis. The transient response depends on the components that make up the circuit, the values of those components, and the way the components are interconnected. Once the source is switched into the circuit, the voltage and current for every circuit component are the sum of a transient expression and a steady-state expression.

The transient part of the voltage and current eventually decays to zero. But, initially, the sum of the transient part and the steady-state part might exceed the voltage or current rating of the circuit component. This is why it is important to determine the complete response of a circuit. The Laplace transform techniques introduced in this chapter can be used to find the complete response of a circuit to a sinusoidal source.

Consider the RLC circuit shown below, comprised of components from Appendix H and powered by a 60 Hz sinusoidal source. As detailed in

Appendix H, the 10 mH inductor has a current rating of 40 mA. The amplitude of the sinusoidal source has been chosen so that this rating is not exceeded in the steady state. Once we have presented the Laplace transform method, we will be able to determine whether this current rating is exceeded when the source is first switched on and both the transient and steady-state components of the inductor current exist.



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12.1-1 Full Alternative Text

12.1 Definition of the Laplace Transform

The **Laplace transform** of a function is given by the expression

Laplace Transform

$$L\{f(t)\}=\int_0^\infty f(t)e^{-st}\,dt, \quad (12.1)$$

where the symbol $L\{f(t)\}$ is read “the Laplace transform of f(t).”

The Laplace transform of f(t) is also denoted F(s); that is,

$$F(s)=L\{f(t)\}. \quad (12.2)$$

This notation emphasizes that when the integral in Eq. 12.1 has been evaluated, the resulting expression is a function of s. In our applications, t represents the time domain and s represents the frequency domain. Note that the dimension of s must be reciprocal time, or frequency, because the exponent of e in the integral of Eq. 12.1 must be dimensionless. The Laplace transform transforms the problem from the time domain to the frequency domain. Once we solve the problem in the frequency domain, we inverse-transform the solution back to the time domain.

Recall that the phasor is also a transform. As we know from Chapter 9, it converts a sinusoidal signal into a complex number for easier, algebraic computation of circuit values. After determining the phasor value of a signal, we transform it back to its time-domain expression. Both the Laplace transform and the phasor transform exhibit an essential feature of mathematical transforms: They create a new domain to make the mathematical manipulations easier. After finding the unknown in the new domain, we inverse-transform it back to the original domain.

In circuit analysis, we use the Laplace transform to transform a set of integrodifferential equations in the time domain to a set of algebraic equations in the frequency domain. We can therefore find the solution for an unknown quantity by solving a set of algebraic equations.

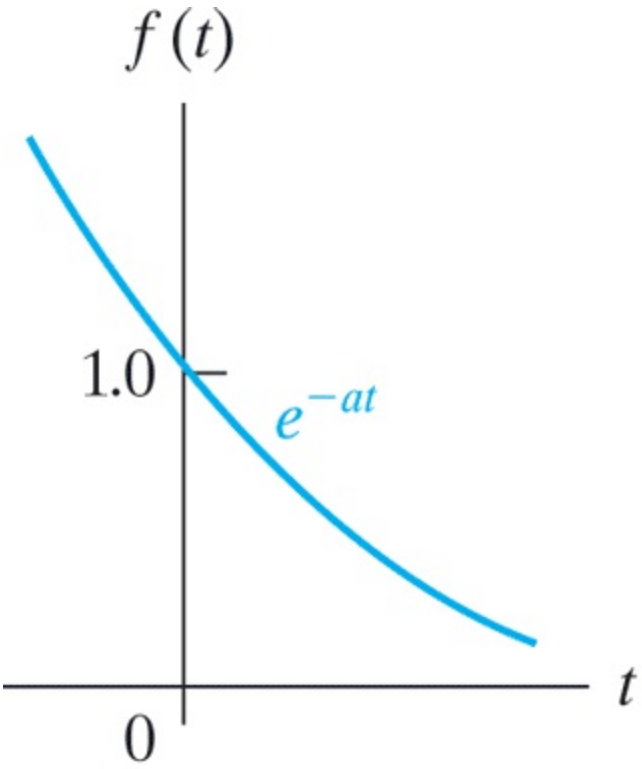
Before we illustrate some of the important properties of the Laplace transform, some general comments are in order. First, note that the integral in Eq. 12.1 is improper because the upper limit is infinite. Thus, we are confronted immediately with the question of whether the integral converges. In other words, does a given f(t) have a Laplace transform? Obviously, the functions of primary interest in engineering analysis have Laplace transforms; otherwise we would not be interested in the transform. In linear circuit analysis, we excite circuits with sources that have Laplace transforms. Excitation functions such as tt or et2, which do not have Laplace transforms, are of no interest here.

Second, because the lower limit on the integral is zero, the Laplace transform ignores f(t) for negative values of t. That is, F(s) is determined by the behavior of f(t) only for positive values of t. To emphasize that the lower limit is zero, Eq. 12.1 is frequently referred to as the **one-sided**, or **unilateral, Laplace transform**. In the two-sided, or bilateral, Laplace transform, the lower limit is $-\infty$. We do not use the bilateral form here; hence, F(s) is understood to be the one-sided transform.

The Laplace transform's lower limit creates a concern: what happens when f(t) has a discontinuity at the origin? If f(t) is continuous at the origin, like the function in Fig. 12.1(a), f(0) is not ambiguous. However, if f(t) has a finite discontinuity at the origin, like the function in Fig. 12.1(b), should the Laplace transform integral include or exclude the discontinuity? That is, should we make the lower limit 0− and include the discontinuity, or choose 0+as the lower limit and exclude the discontinuity? (We use the notation 0− and 0+ to denote values of t just to the left and right of the origin, respectively.) Actually, we may choose either as long as we are consistent. For reasons we explain later, we choose 0− as the lower limit.

Figure 12.1 A continuous and

discontinuous function at the origin. (a) f(t) is continuous at the origin. (b) f(t) is discontinuous at the origin.



(a)

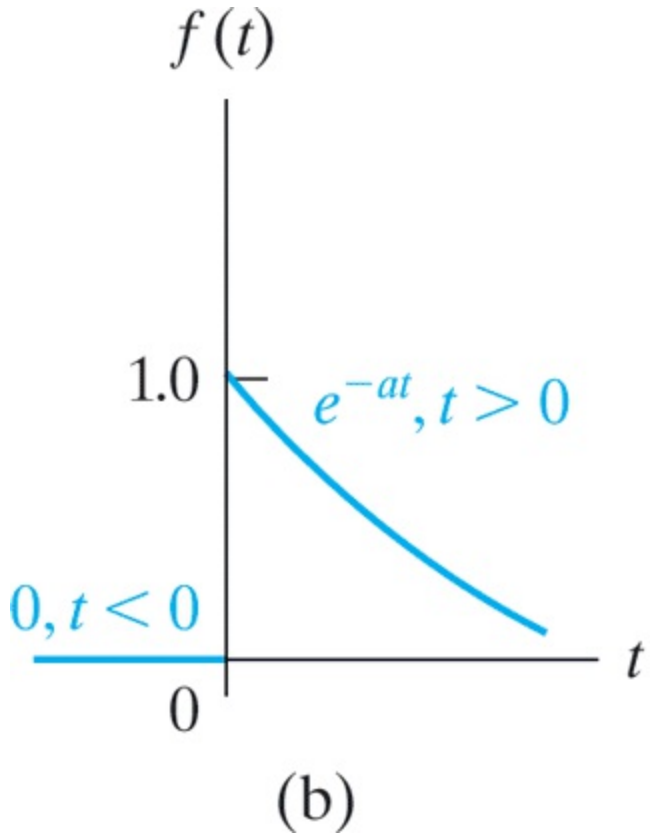


Figure 12.1 Full Alternative Text

Because we are using 0− as the lower limit, we note that the integral from 0− to 0+ is zero, except when the discontinuity at the origin is an impulse function. We discuss this situation in Section 12.3. The two functions shown in Fig. 12.1 have the same unilateral Laplace transform because there is no impulse function at the origin for the function in Fig. 12.1(b).

The one-sided Laplace transform ignores f(t) for t<0−. We use initial conditions to account for what happens prior to 0−. Thus, the Laplace transform predicts the response to a function that begins after initial conditions have been established.

In the discussion that follows, we divide the Laplace transforms into two types: functional transforms and operational transforms. A **functional transform** is the Laplace transform of a specific function, such as sinωt, t, e −at, and so on. An **operational transform** defines a general mathematical property of the Laplace transform, such as finding the transform of the derivative of f(t). Before considering functional and operational transforms,

we introduce the step and impulse functions.

12.2 The Step Function

When a circuit contains a switch, a change in the switch position creates abrupt changes in currents and voltages, as we have seen in previous chapters. An abrupt change is represented mathematically as a discontinuity, which can occur at any instant in time. We represent such discontinuities in the functions that describe the currents and voltages using step and impulse functions.

Figure 12.2 illustrates the step function.The symbol for the step function is *Ku*(t). The mathematical definition of the **step function** is

Figure 12.2 The step function.

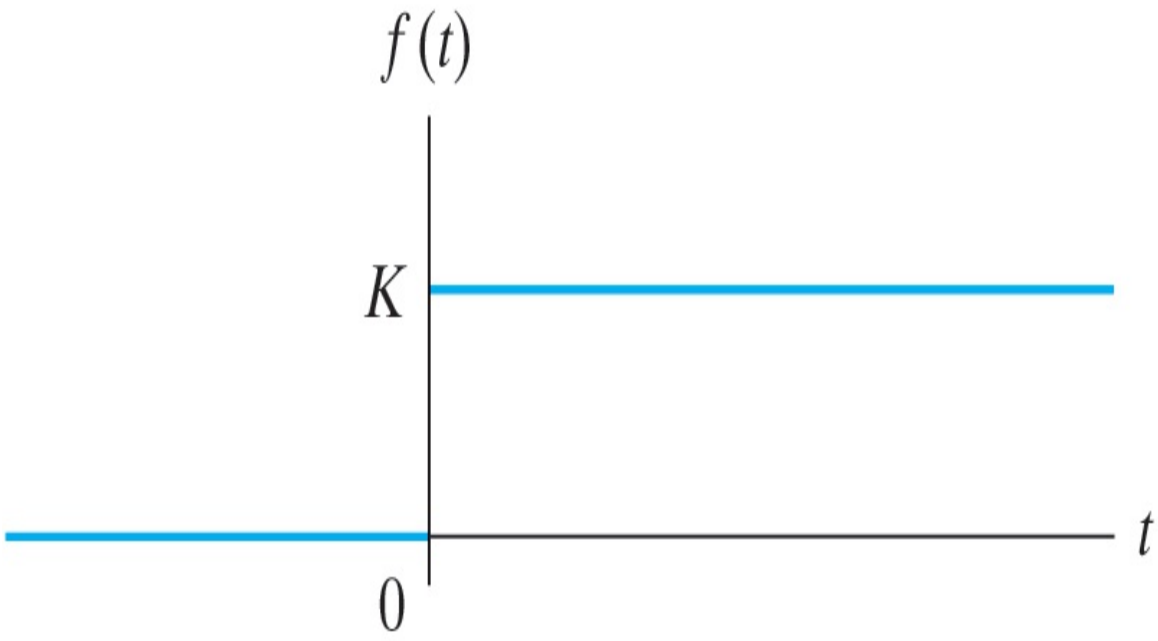


Figure 12.2 Full Alternative Text

Ku(t)=0, t<0,Ku(t)=K, t>0. (12.3)

If K is 1, the function in Eq. 12.3 is the **unit step**, u(t).

The step function is not defined at t=0. If we need to define the transition between 0− and 0+, we assume that it is linear and that

Ku(0)=0.5K.

As before, 0− and 0+ represent symmetric points arbitrarily close to the left and right of the origin. Figure 12.3 illustrates the linear transition from 0− to 0+.

Figure 12.3 The linear approximation to the step function.

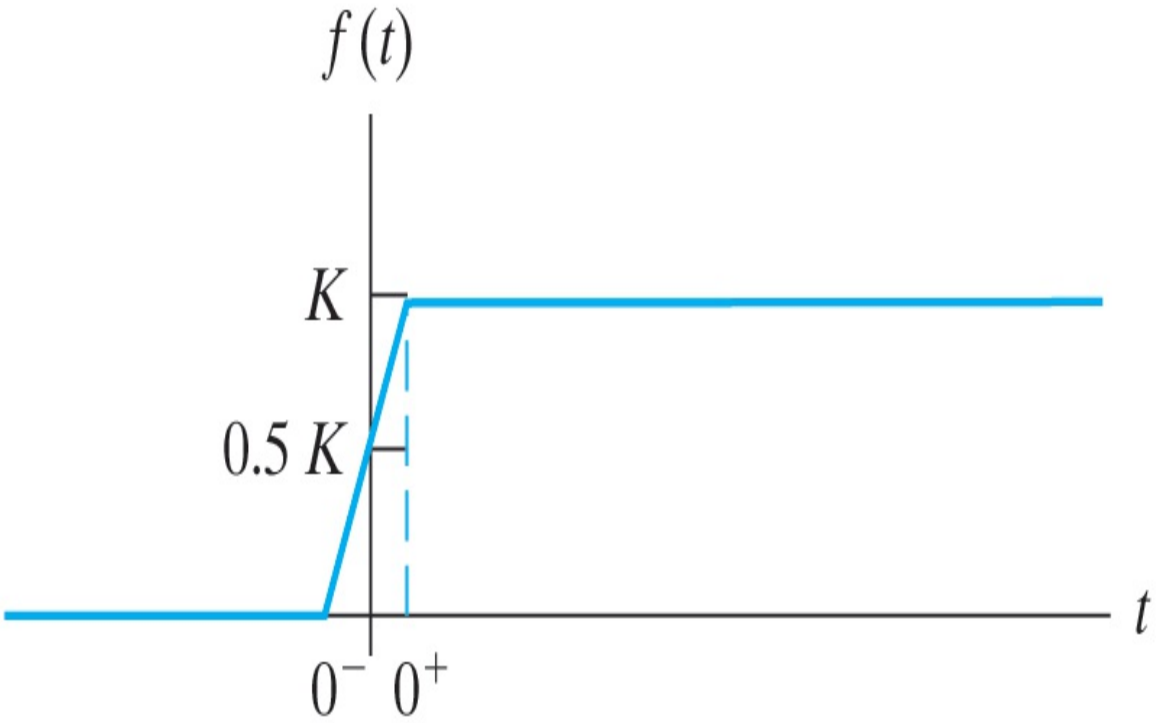


Figure 12.3 Full Alternative Text

A discontinuity may occur at some time other than t=0; for example, it may occur in sequential switching. A step that occurs at t=a is represented by the

function Ku(t−a). Thus

Ku(t−a)=0, t<a,Ku(t−a)=K, t>a.

If a>0, the step occurs to the right of the origin, and if a<0, the step occurs to the left of the origin. The step function's value is 0 when the argument t−a is negative, and it is K when the argument is positive. Figure 12.4 illustrates a step that occurs at t=a.

Figure 12.4 A step function occurring at t=a when a>0.

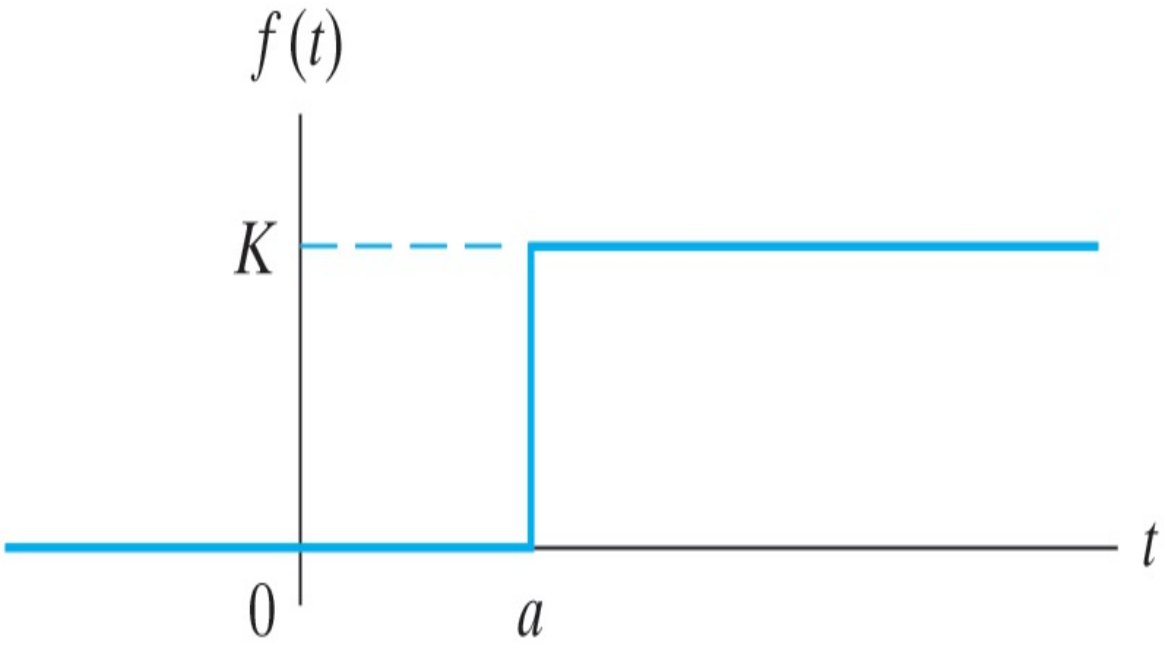


Figure 12.4 Full Alternative Text

A step function equal to K for t<a is written as Ku(a−t). Thus

Ku(a−t)=K, t<a,Ku(a−t)=0, t<a,

The discontinuity is to the left of the origin when a<0. This type of step is shown in Fig. 12.5.

Figure 12.5 A step function Ku(a−t) for a>0.

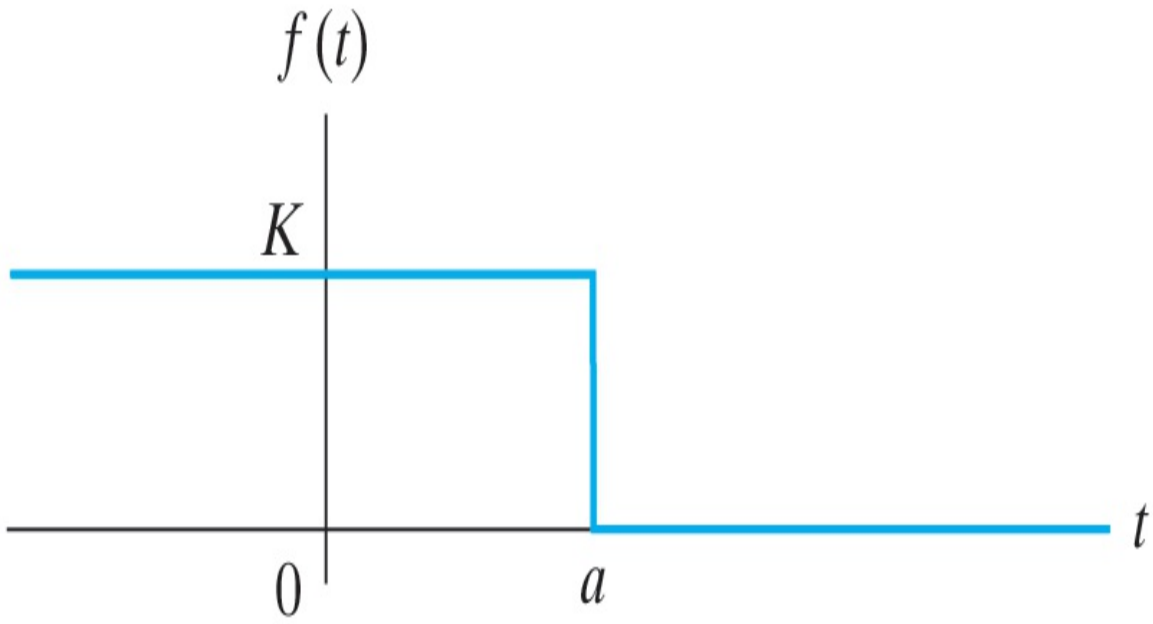


Figure 12.5 Full Alternative Text

We can add two step functions to create a function that describes a finite-width pulse. For example, the function K[u(t−1) − u(t−3)] has the value K for 1<t<3 and the value 0 everywhere else, so it is a finite-width pulse of height K initiated at t=1 and terminated at t=3. In defining this pulse, think of the step function u(t−1) as “turning on” the constant value K at t=1, and the step function −u(t−3) as “turning off” the constant value K at t=3. We use step functions to turn on and turn off linear functions at desired times in Example 12.1.

Example 12.1 Using Step Functions to Represent a Function of Finite

Duration

Use step functions to write an expression for the function illustrated in Fig. 12.6.

Figure 12.6 The function for Example 12.1.

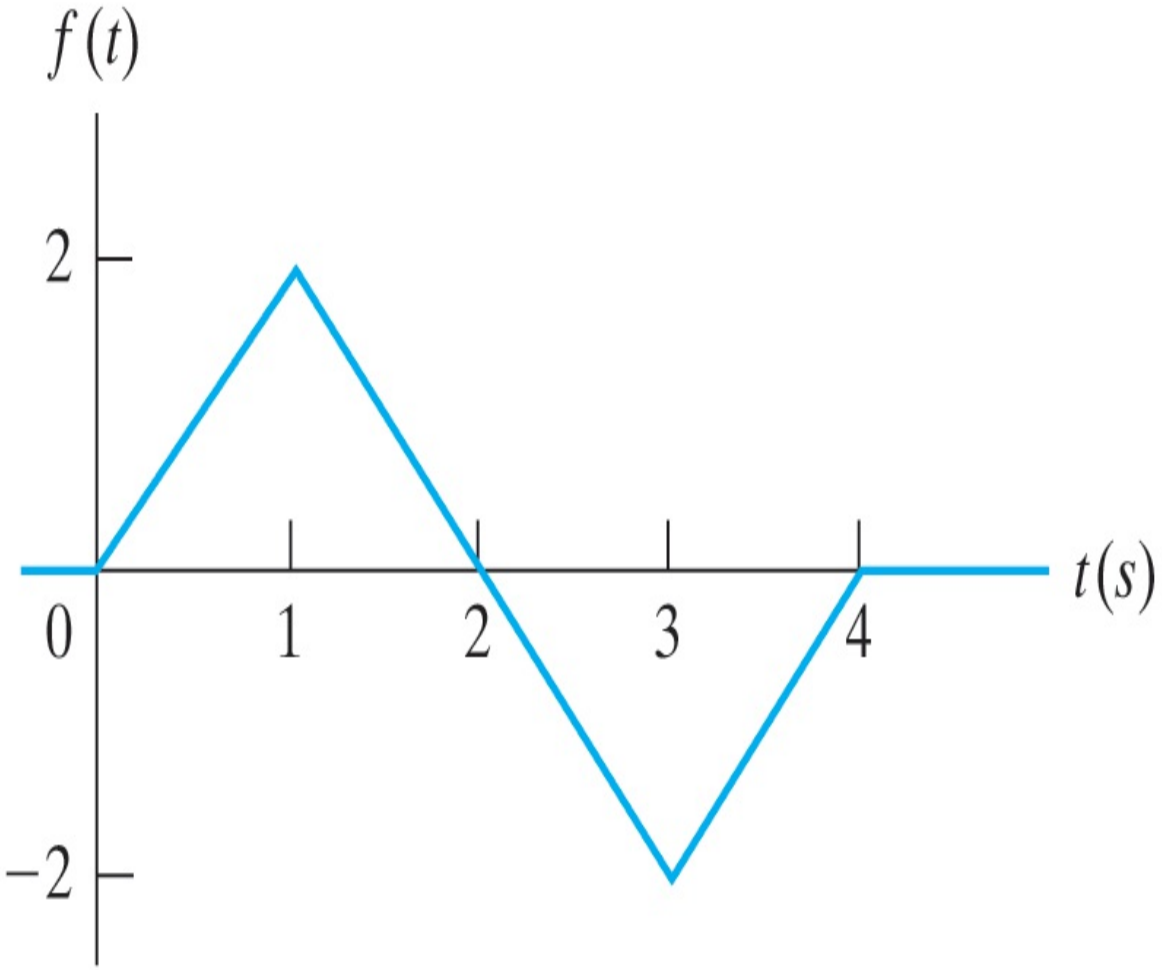


Figure 12.6 Full Alternative Text

Solution

The function shown in Fig. 12.6 is made up of linear segments, so it is a piecewise linear function. You have probably seen such functions defined using a different functional form for each of its time segments. For this function, the piecewise-linear definition is

=0,t≤0;=2t,0s≤t≤1s;f(t)=−2t+4,1s≤t≤3s;=2t−8,3s≤t≤4s;=0,t≥4s.

We can construct a single continuous definition for this function using step functions to initiate and terminate each linear segment at the proper times. In other words, we use the step function to turn on and turn off the three nonzero pieces of the function:

+2t, on at t=0, off at t=1;−2t+4, on at t=1, off at t=3;+2t
−8, on at t=3, off at t=4.

These straight line segments and their equations are shown in Fig. 12.7. The expression for f(t), valid for all values of t, is

Figure 12.7 Definition of the three line segments turned on and off with step functions to form the function shown in Fig. 12.6.

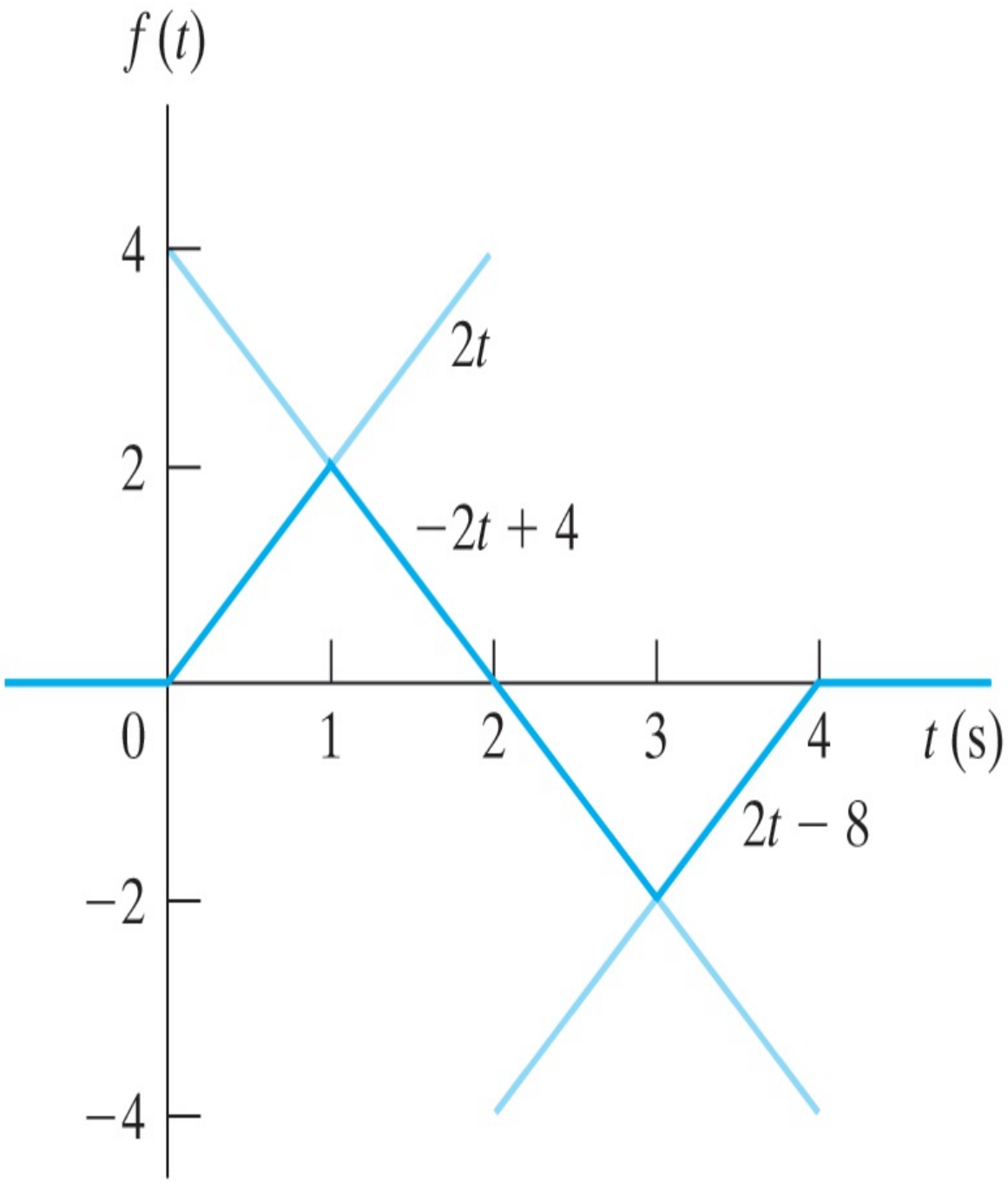


Figure 12.7 Full Alternative Text
f(t)=2t[u(t)−u(t−1)]+(−2t+4)[u(t−1)− u(t−3) + (2t−8)[u(t−3)−u(t−4)].

SELF-CHECK: Assess your understanding of step functions by trying Chapter Problems 12.1 and 12.2.

12.3 The Impulse Function

An **impulse** is a signal of infinite amplitude and zero duration. Such signals don't exist in nature, but some circuit signals come very close to approximating this definition, so a mathematical model of an impulse is useful. For example, the impulse function[1] enables us to define the derivative at a discontinuity, such as the one in the function of Fig. 12.1(b), and thus to define the Laplace transform of that derivative. Also, voltage and current impulses occur in circuit analysis either because of a switching operation or because the circuit is excited by an impulsive source. We will analyze these situations in Chapter 13, but here we define the impulse function generally.

[1]The impulse function is also known as the Dirac delta function.

Describing the Impulse Function

We describe the impulse function by considering how we would define the derivative of the function in Fig. 12.1(b) at its discontinuity. First, we assume that the function varies linearly across the discontinuity, as shown in Fig. 12.8. In this figure, note that as $\epsilon \to 0$, an abrupt discontinuity occurs at the origin. When we differentiate the function, the derivative between $-\epsilon$ and $+\epsilon$ is constant, with a value of $1/2\epsilon$. For $t>\epsilon$, the derivative is $-ae^{-a(t-\varepsilon)}$. Figure 12.9 shows these observations graphically. As ϵ approaches zero, the value of $f'(t)$ between $\pm\epsilon$ approaches infinity. At the same time, the duration of this large value is approaching zero. Furthermore, the area under $f'(t)$ between $\pm\epsilon$ remains constant as $\epsilon \to 0$. In this example, the area is unity. As ϵ approaches zero, we say that the function between $\pm\epsilon$ approaches a **unit impulse function**, denoted $\Delta(t)$. Thus, the derivative of f(t)at the origin approaches a unit impulse function as ϵ approaches zero, or

Figure 12.8 A magnified view

of the discontinuity in Fig. 12.1(b), assuming a linear transition between $-\epsilon$ and $+\epsilon$.

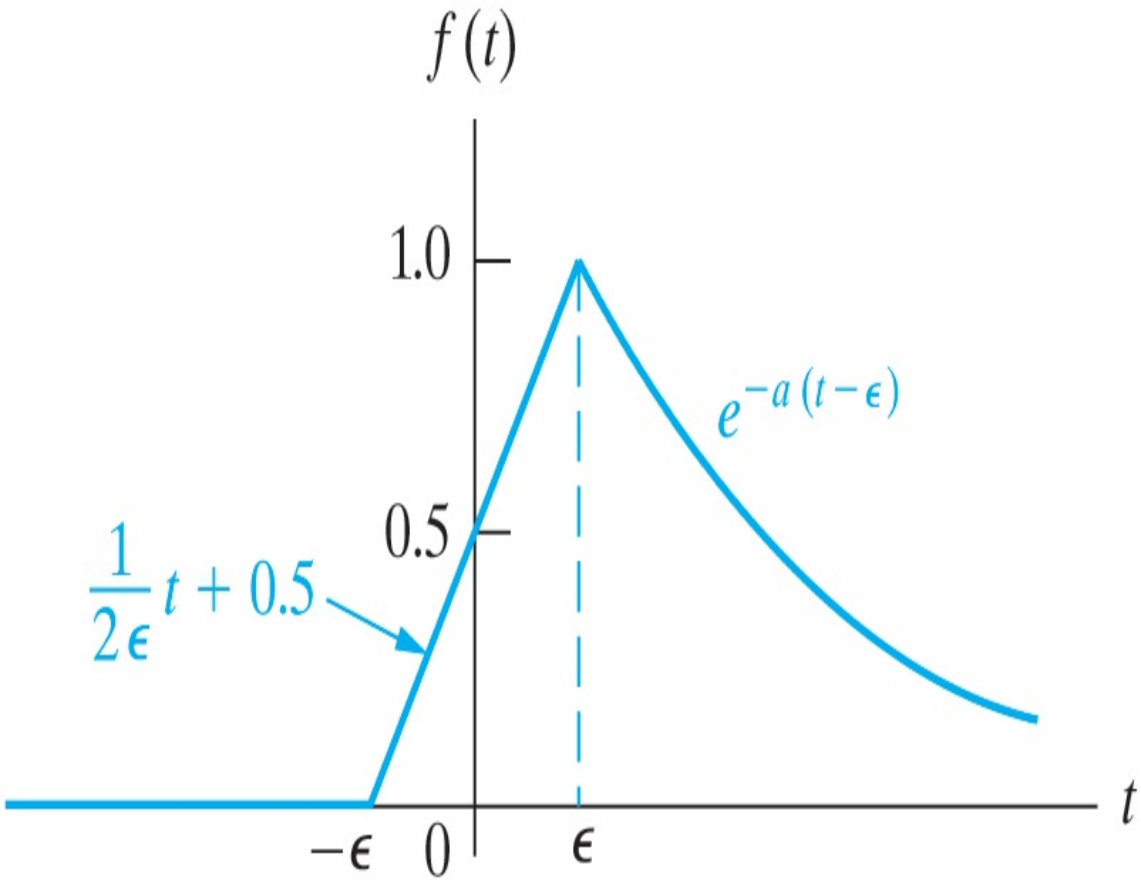


Figure 12.8 Full Alternative Text

Figure 12.9 The derivative of the function shown in Fig. 12.8.

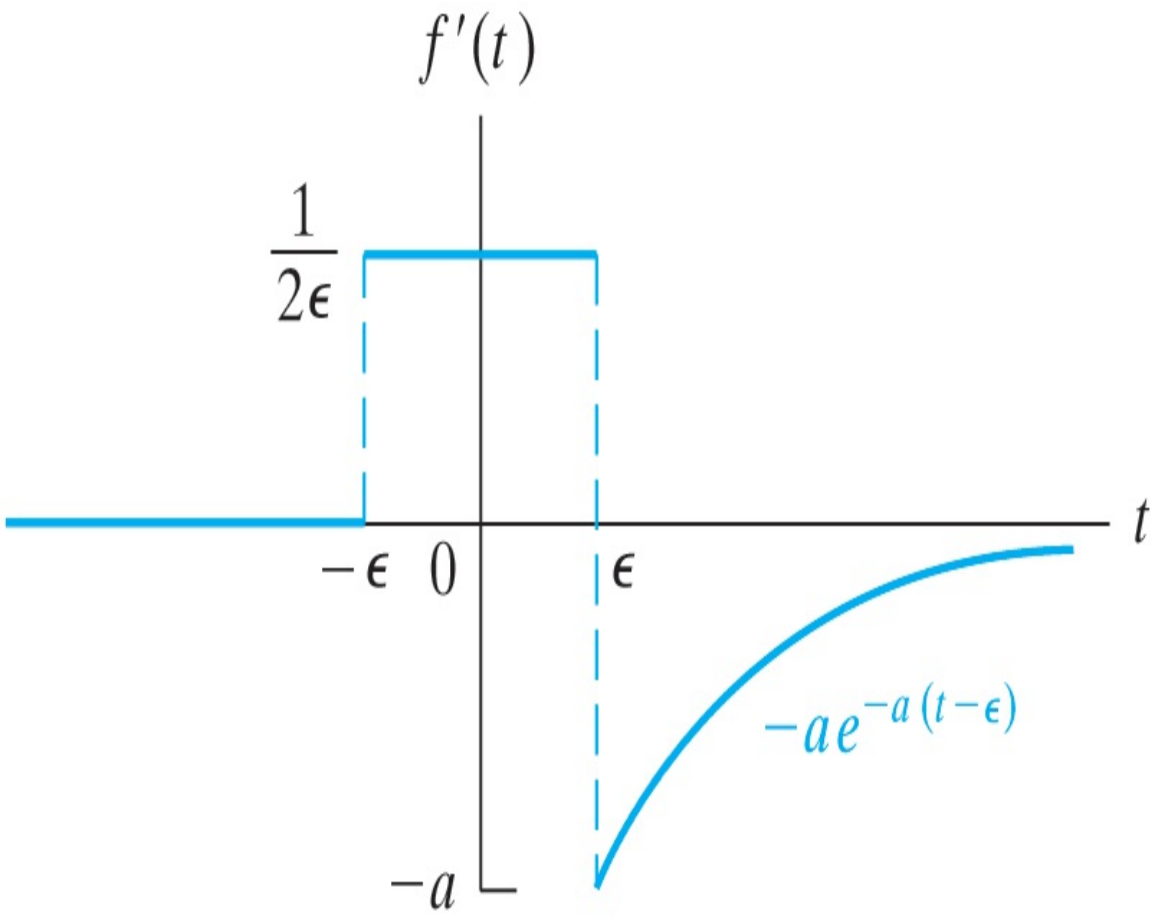


Figure 12.9 Full Alternative Text

$f'(0) \rightarrow \Delta(t)$ as $\epsilon \rightarrow 0$.

If the area under the impulse function curve is other than unity, the impulse function is denoted $K\Delta(t)$, where K is the area. K is often referred to as the **strength** of the impulse function.

To summarize, an impulse function is created from a variable-parameter function whose parameter approaches zero. The variable-parameter function must exhibit the following three characteristics as the parameter approaches zero:

1. The amplitude of the function approaches infinity.
2. The duration of the function approaches zero.

3. The area under the variable-parameter function is constant as the parameter changes.

Many different variable-parameter functions have these three characteristics. In Fig. 12.8, we used a linear function $f(t)=0.5t/\epsilon +0.5$. Another example of a variable-parameter function with the three characteristics is the exponential function:

$f(t)=K2\epsilon e^{-|t|/\epsilon}$.

As ϵ approaches zero, the function becomes infinite at the origin and at the same time decays to zero in an infinitesimal length of time.

Figure 12.10 illustrates $f(t)$ as $\epsilon \rightarrow 0$. An impulse function is created as $\varepsilon \rightarrow 0$ if the area under the function is independent of ϵ. Thus,

Figure 12.10 A variable-parameter function used to generate an impulse function.

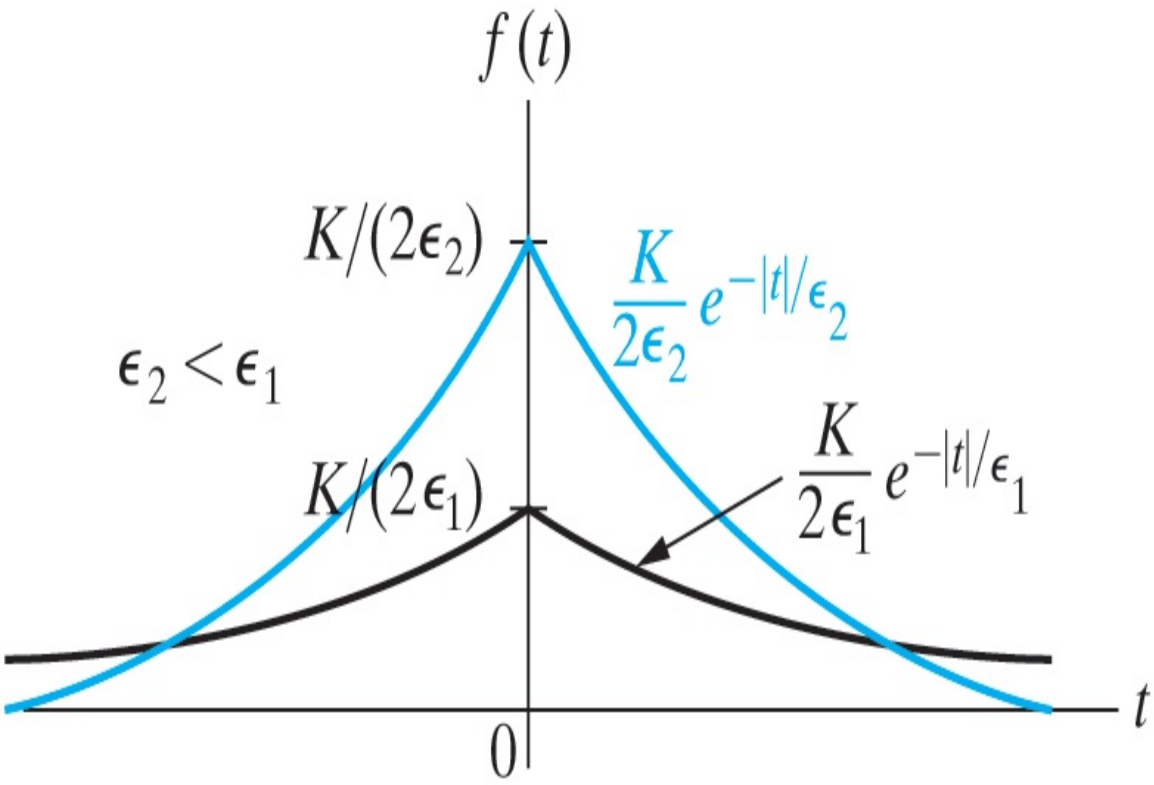


Figure 12.10 Full Alternative Text

Area=∫−∞0K2ϵet/ϵdt + ∫0∞K2ϵe−t/ϵdt=K2ϵ · et/ϵ1/ϵ|−∞0 + K2ϵ · e−t/ϵ−1/ϵ|0∞=K2 + K2=K,

so the area under the curve is constant and equal to K units. Therefore, as $\epsilon \to 0$, $f(t) \to K\Delta(t)$.

Defining the Impulse Function and Its Sifting Property

Mathematically, the **impulse function** is defined as

∫−∞∞KΔ(t)dt=K; (12.4)

Δ(t)=0,t≠0. (12.5)

Equation 12.4 states that the area under the impulse function is constant. This

area represents the strength of the impulse. Equation 12.5 states that the impulse is zero everywhere except at t=0. An impulse that occurs at t=a is denoted KΔ(t−a). The graphic symbol for the impulse function is an arrow. The strength of the impulse is given parenthetically next to the head of the arrow. Figure 12.11 shows the impulses KΔ(t) and KΔ(t−a).

Figure 12.11 A graphic representation of the impulse KΔ(t) and KΔ(t−a).

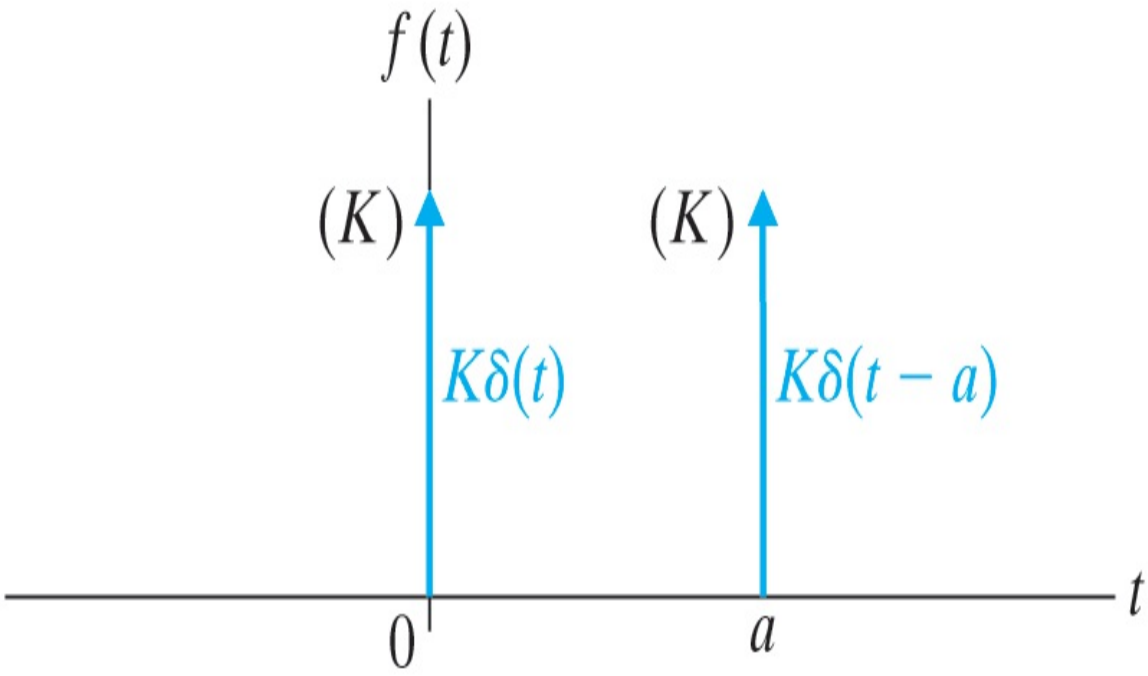


Figure 12.11 Full Alternative Text

An important property of the impulse function is the **sifting property**, which is expressed as

∫−∞∞f(t)Δ(t−a)dt=f(a), (12.6)

where we assume the function f(t) is continuous at t=a, the location of the

impulse. Equation 12.6 shows that the impulse function sifts out all values of f(t) except the one at t=a. Equation 12.6 follows from Eqs. 12.4 and 12.5, noting that $\Delta(t-a)$ is zero everywhere except at t=a, and hence the integral can be written

$$\int_{-\infty}^{\infty} f(t)\Delta(t-a)\,dt = \int_{a-\epsilon}^{a+\epsilon} f(t)\Delta(t-a)\,dt.$$

But because f(t) is continuous at a, it takes on the value f(a) as $t \to a$, so

$$\int_{a-\epsilon}^{a+\epsilon} f(a)\Delta(t-a)\,dt = f(a)\int_{a-\epsilon}^{a+\epsilon} \Delta(t-a)\,dt = f(a).$$

Laplace Transform and Derivatives of the Impulse Function

We use the sifting property of the impulse function to find its Laplace transform:

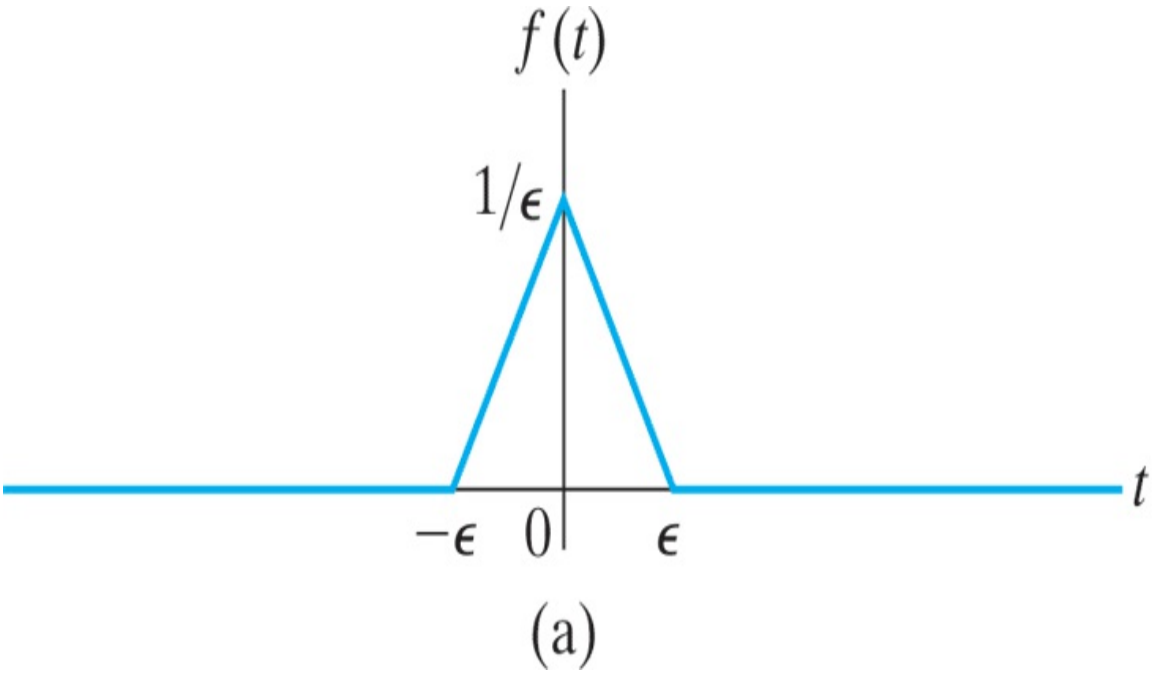
$$\mathcal{L}\{\Delta(t)\} = \int_{0-}^{\infty} \Delta(t)\, e^{-st}\,dt = e^{-s(0)} = 1,$$

which is an important Laplace transform. We can also define the derivatives of the impulse function and the Laplace transform of these derivatives. We discuss the first derivative, along with its transform, and then state the result for the higher-order derivatives.

The function illustrated in Fig. 12.12(a) generates an impulse function as $\epsilon \to 0$. Figure 12.12(b) shows the derivative of this impulse-generating function, which is defined as the derivative of the impulse $[\Delta'(t)]$ as $\epsilon \to 0$. The derivative of the impulse function sometimes is referred to as a moment function, or unit doublet.

Figure 12.12 The first derivative of the impulse

function. (a) The impulse-generating function used to define the first derivative of the impulse. (b) The first derivative of the impulse-generating function that approaches $\Delta'(t)$ as $\epsilon \to 0$.



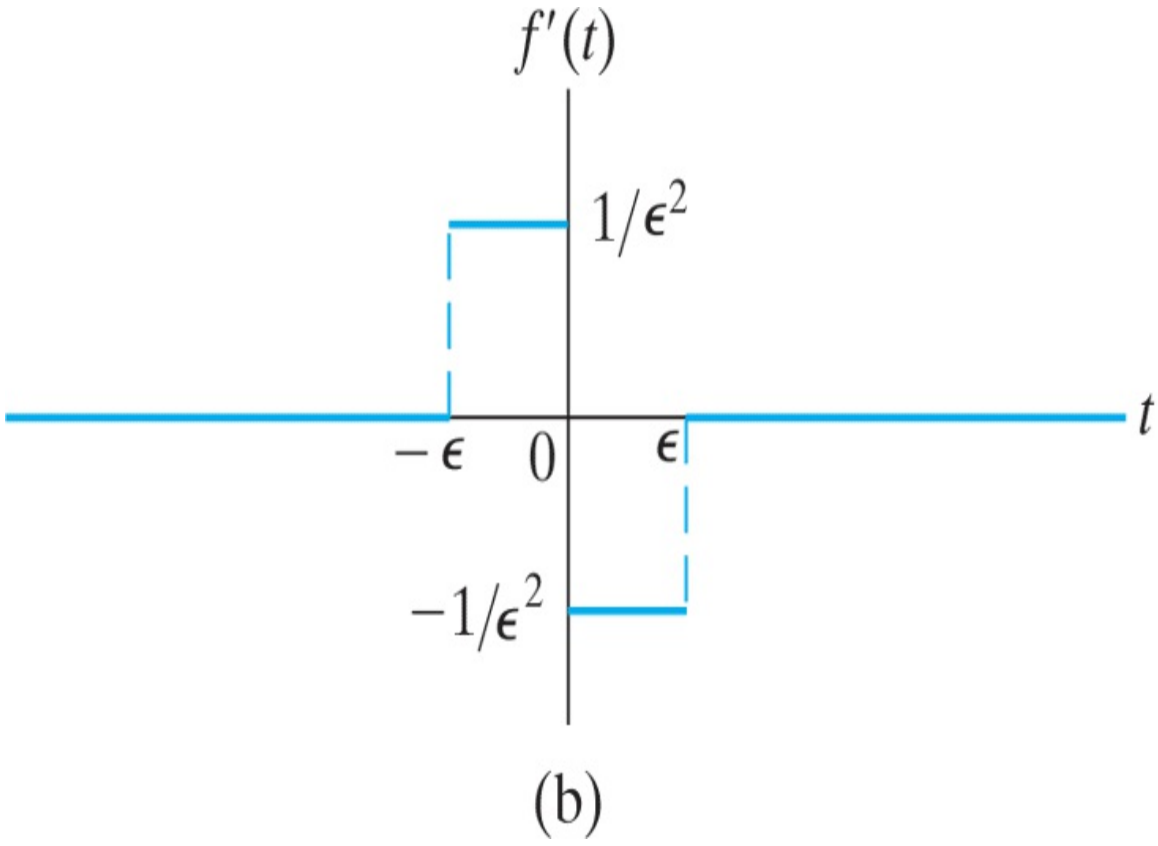


Figure 12.12 Full Alternative Text

To find the Laplace transform of Δ′(t), we apply the defining integral to the function shown in Fig. 12.12(b) and, after integrating, let ϵ→0. Then

$$\mathcal{L}\{\Delta'(t)\}=\lim_{\epsilon\to 0}\left[\int_{-\epsilon}^{0^-} 1\epsilon 2e^{-st}\,dt + \int_{0^+}^{\epsilon}(-1\epsilon 2)e^{-st}dt\right]=\lim_{\epsilon\to 0}es\epsilon +e-s\epsilon-2s\epsilon 2=\lim_{\epsilon\to 0}ses\epsilon-se-s\epsilon 2\epsilon s=\lim_{\epsilon\to 0}s2es\epsilon+s2e-s\epsilon 2s=s.$$

We used l'Hôpital's rule twice in this derivation, to evaluate the indeterminate form 0/0.

Higher-order derivatives can be generated in a similar manner (see Problem 12.6), and the defining integral can then be used to find the Laplace transforms. The Laplace transform of the nth derivative of the impulse function is

$$L\{\Delta(n)(t)\}=sn.$$

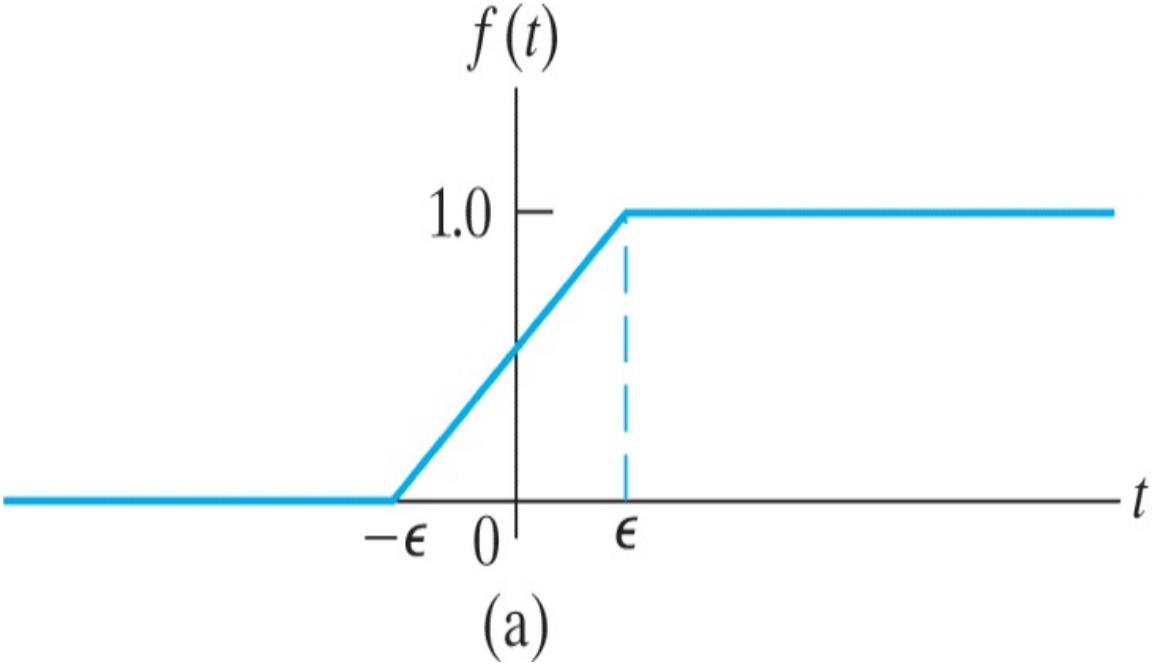
Finally, note that the derivative of a step function is an impulse function; that

is,

$\Delta(t)=du(t)dt.$

Figure 12.13 depicts the relationship between the impulse function and the step function. The function shown in Fig. 12.13(a) approaches a unit step function as $\epsilon \to 0$. The function shown in Fig. 12.13(b)—the derivative of the function in Fig. 12.13(a)—approaches a unit impulse as $\epsilon \to 0$.

Figure 12.13 The impulse function as the derivative of the step function: (a) $f(t) \to u(t)$ as $\epsilon \to 0$; and (b) $f'(t) \to \Delta(t)$ as $\epsilon \to 0$.



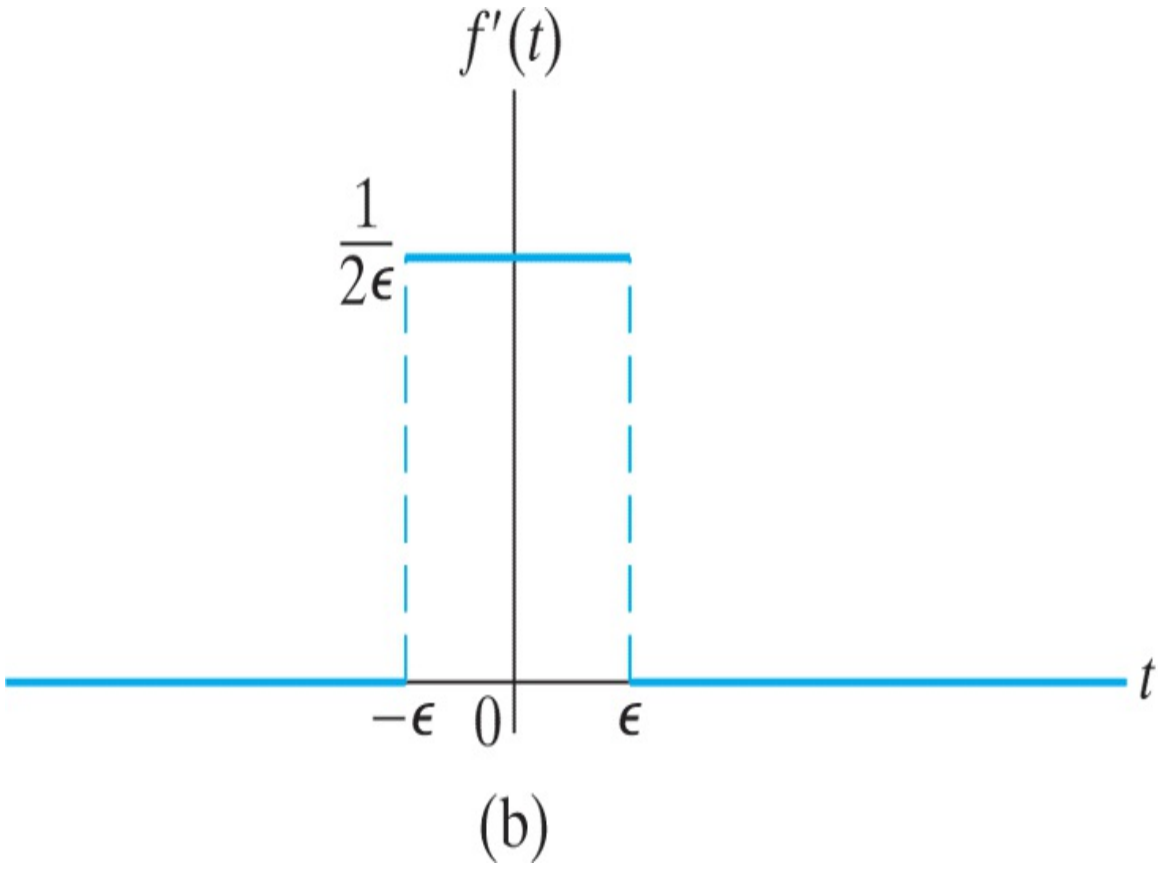


Figure 12.13 Full Alternative Text

The impulse function is an extremely useful concept in circuit analysis, and we say more about it in the following chapters. We introduced the concept here so that we can include discontinuities at the origin in our definition of the Laplace transform.

SELF-CHECK: Assess your understanding of the impulse function by trying Chapter Problems 12.8 and 12.10.

12.4 Functional Transforms

The Laplace transform of a specified function of t is called a functional transform. Because we are using the unilateral, or one-sided, Laplace transform, we define all functions to be zero for $t<0-$.

We derived one functional transform pair in Section 12.3, where we showed that the Laplace transform of the unit impulse function equals 1. Next, we find the Laplace transform of the unit step function at the origin, where

$$\mathcal{L}\{ u(t) \}= \int 0\infty f(t)e-st\, dt= \int 0+\infty 1e-stdt=e-st-s|0+\infty=1s.$$

Thus, the Laplace transform of the unit step function is 1/s.

The Laplace transform of the decaying exponential function shown in Fig. 12.14 is

Figure 12.14 A decaying exponential function.

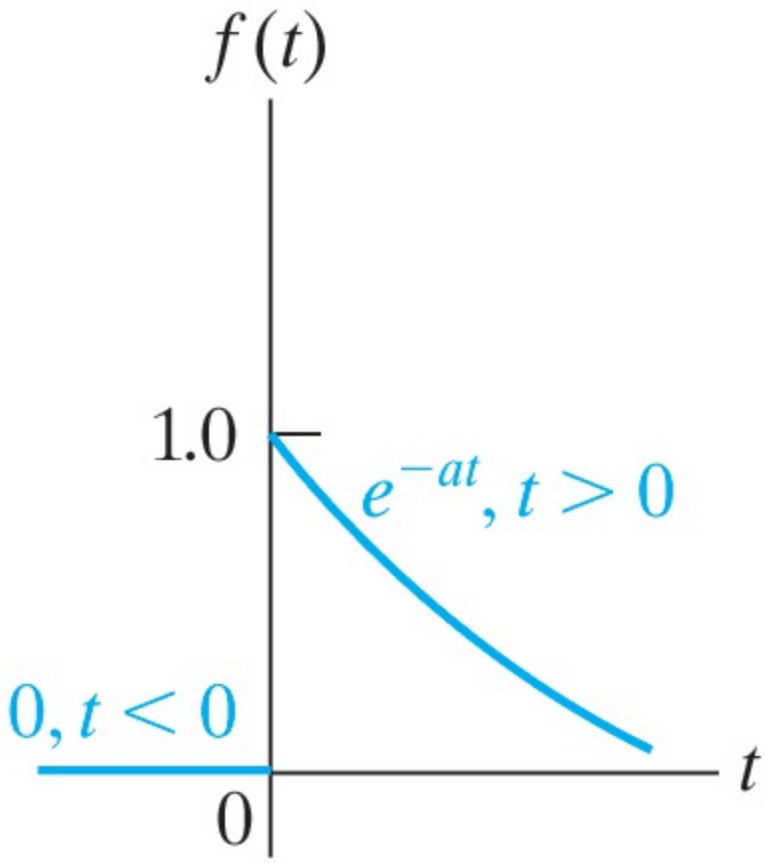


Figure 12.14 Full Alternative Text
L {e−at}=∫0+∞e−ate−stdt=∫0+∞e−(a+s)tdt=1s + a.

In deriving the Laplace transforms of the unit step function and the decaying exponential function, we used the fact that integration across the discontinuity at the origin is zero.

Let’s find the Laplace transform of the sinusoidal function shown in Fig. 12.15. The expression for f(t) for t>0− is sinωt; hence, the Laplace transform is

Figure 12.15 A sinusoidal function for .

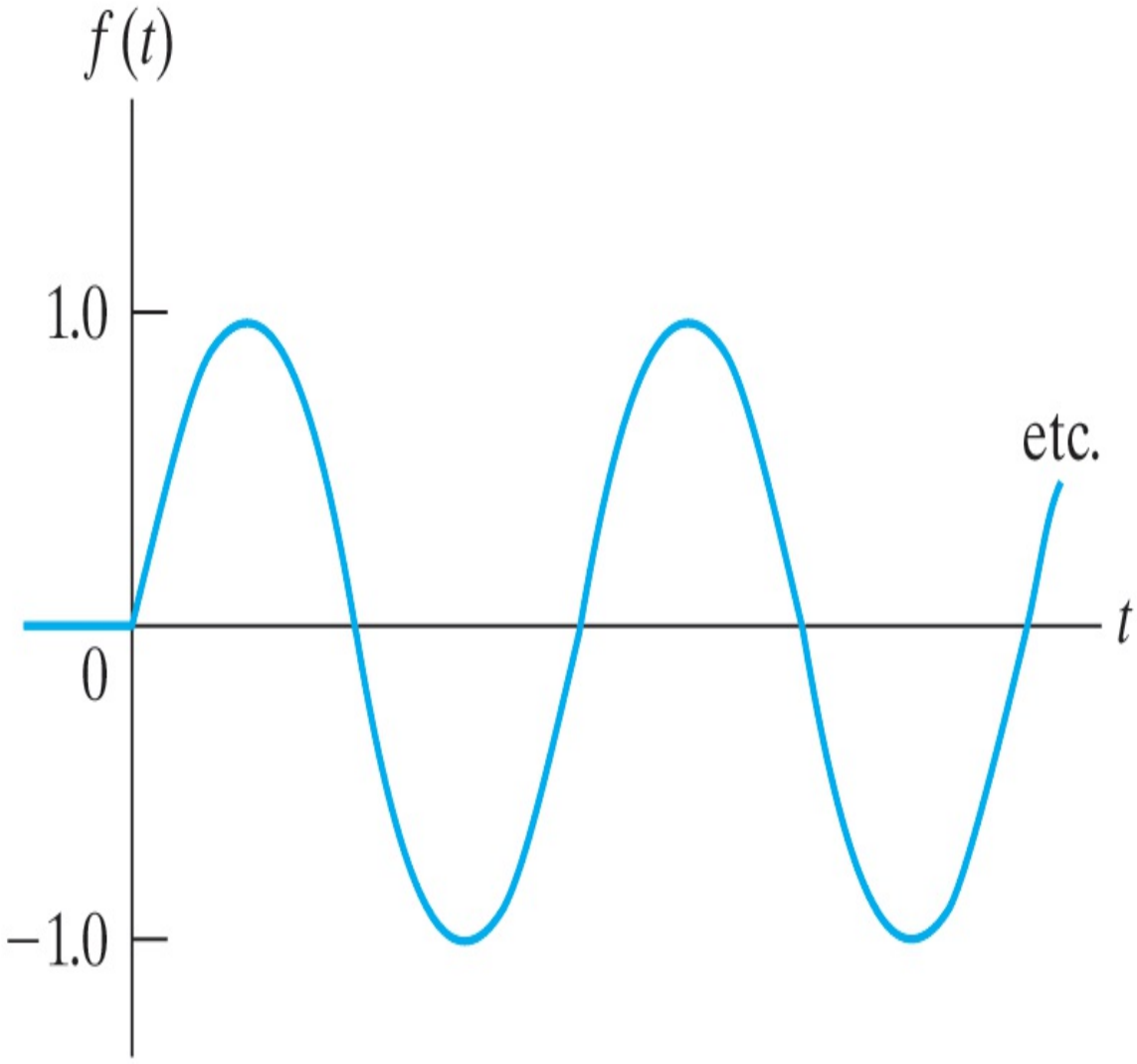


Figure 12.15 Full Alternative Text

$\mathcal{L}\{\ \sin\omega t\ \}=\int 0-\infty(\sin\ \omega t)e-st\ dt=\int 0-\infty(ej\omega t-e-j\omega t2j)e-st\ dt=\int 0-\infty e-(s-j\omega)t\ -e-(s+j\omega)t2j\ dt=12j\ (1s-j\omega\ -\ 1s+j\omega)=\omega s2+\omega 2.$

Table 12.1 gives an abbreviated list of functional Laplace transform pairs. It includes the functions of most interest in an introductory course on circuit applications.

Table 12.1 An Abbreviated List of Laplace Transform Pairs

Type	$f(t)$ $(t > 0^-)$	$F(s)$
(impulse)	$\delta(t)$	1
(step)	$u(t)$	$\frac{1}{s}$
(ramp)	t	$\frac{1}{s^2}$
(exponential)	e^{-at}	$\frac{1}{s+a}$
(sine)	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
(cosine)	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
(damped ramp)	te^{-at}	$\frac{1}{(s+a)^2}$
(damped sine)	$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
(damped cosine)	$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

Table 12.1 Full Alternative Text

Assessment Problem

Objective 1–Be able to calculate the Laplace transform of a function using the definition of Laplace transform

1. 12.1 Use the defining integral to

1. find the Laplace transform of cosh βt;
2. find the Laplace transform of sinh βt.

Answer:

1. s/(s2−β2);
2. β/(s2−β2).

SELF-CHECK: Also try Chapter Problem 12.20.

12.5 Operational Transforms

Operational Laplace transforms define how mathematical operations performed on f(t) affect its Laplace transform, F(s). Operational transforms also define how mathematical operations performed on F(s) affect its corresponding time-domain function f(t). The operations we consider include (1) multiplication by a constant; (2) addition and subtraction; (3) differentiation; (4) integration; (5) translation in the time domain; (6) translation in the frequency domain; and (7) scale changing.

Multiplication by a Constant

From the defining integral, if

$L\{f(t)\}=F(s)$,

then

$L\{Kf(t)\}=KF(s)$.

Thus, multiplication of f(t) by a constant corresponds to multiplying F(s) by the same constant.

Addition and Subtraction

Addition and subtraction in the time domain translates into addition and subtraction in the frequency domain. Thus if

$\mathcal{L}\{f1(t)\}=F1(s), \mathcal{L}\{f2(t)\}=F2(s), \mathcal{L}\{f3(t)\}=F3(s)$,

then

$L\{f1(t)+f2(t)-f3(t)\}=F1(s)+F2(s)-F3(s)$.

Use the defining integral to derive this operation transform by recognizing that the integral of a sum of functions equals the sum of each function’s integral.

Differentiation

We use the definition of the Laplace transform to find the operational transform for differentiation in the time domain:

$$\mathcal{L}\{ df(t)dt \}=\int 0-\infty[df(t)dt]e-st\ dt.$$

We integrate by parts to evaluate this integral. Let $u=e-st$ and $dv=[df(t)/dt]\ dt$ to give

$$\mathcal{L}\{ df(t)dt \}=e-stf(t)|0-\infty - \int 0-\infty f(t)(-se-stdt).$$

Because we are assuming that f(t) has a Laplace transform, evaluating $e-stf(t)$ at $t=\infty$ gives zero. We complete the evaluation of the integral to get

$$-f(0-) + s\int 0-\infty f(t)e-stdt=sF(s)-f(0-).$$

Thus, the Laplace transform of the derivative of f(t) is

$$L\{df(t)dt\}=sF(s)-f(0-).$$

This important result shows that differentiation in the time domain transforms to an algebraic operation in the s domain.

Use the Laplace transform of the first derivative of f(t) to find the Laplace transform of higher-order derivatives. For example, to find the Laplace transform of the second derivative of f(t), we first let

$$g(t)=df(t)dt.$$

Now we find the Laplace transform of g(t):

$$G(s)=sF(s)-f(0-).$$

But because

$$dg(t)dt=d2f(t)dt2,$$

we know

$$L\{dg(t)dt\}=L\{d2f(t)dt2\}=sG(s)-g(0-).$$

Therefore,

$$L\{d2f(t)dt2\}=s2F(s)-sf(0-)-df(0-)dt.$$

We find the Laplace transform of the nth derivative by successively applying the preceding process, which leads to the general result

$$L\{dnf(t)dtn\}=snF(s)-sn-1f(0-)-sn-2df(0-)dt$$
$$-sn-3d2f(0-)dt2-\cdots-dn-1f(0-)dtn-1.$$

Integration

We find the Laplace transform of the integral of f(t) by again applying the defining integral:

$$\mathcal{L}\{\int 0-tf(x)\,dx\}=\int 0-\infty[\int 0-tf(x)\,dx]\,e-st\,dt.$$

Integrate by parts to evaluate the integral on the right-hand side of this expression. Let

$$u=\int 0-tf(x)\,dx,dv=e-st\,dt.$$

Then

$$du=f(t)\,dt,v=-e-sts.$$

The integration-by-parts formula yields

$$\mathcal{L}\{\int 0-tf(x)dx\}=-e-sts\int 0-tf(x)dx|0-\infty+\int 0-\infty e-stsf(t)\,dt.$$

The first term on the right-hand side is zero at both the upper and lower limits. The value at the lower limit is zero because both limits on the integral are the same. The value at the upper limit is zero because $e^{-st} \to 0$ as $t \to \infty$. The second term on the right-hand side is F(s)/s; therefore

$$L\{\int_0^{-t} f(x)\,dx\} = F(s)s,$$

which reveals that integration in the time domain transforms to multiplication by 1/s in the s domain. We have therefore demonstrated that the Laplace transform translates a set of integrodifferential equations into a set of algebraic equations.

Translation in the Time Domain

If we start with any function f(t) u(t), we can represent the same function, translated in time by the constant a, as f(t−a) u(t−a).[2] To find the Laplace transform of f(t−a) u(t−a), start with the defining integral:

[2]Note that throughout we multiply any arbitrary function f(t) by the unit step function u(t) to ensure that the resulting function is defined for all positive time.

$$\mathcal{L}\{\ (t-a)\,u(t-a)\ \} = \int_{0^-}^{\infty} u(t-a) f(t-a) e^{-st}\,dt = \int_{a}^{\infty} f(t-a) e^{-st}\,dt.$$

In writing this equation, we took advantage of u(t−a)=0 for t<a. Now change the variable of integration by letting x=t−a. Then x=0 when t=a, x=∞ when t=∞ and dx=dt. Rewrite the integral as

$$\mathcal{L}\{\ f(t-a)\,u(t-a)\ \} = \int_0^{\infty} f(x) e^{-s(x+a)}\,dx = e^{-sa}\int_0^{\infty} f(x) e^{-sx}\,dx = e^{-as}F(s).$$

Thus

$$L\ \{f(t-a)\,u(t-a)\} = e^{-as}F(s), \quad a>0.$$

Translation in the time domain corresponds to multiplication by an exponential in the frequency domain.

For example, knowing that

$$L\{tu(t)\}=1s2,$$

we can use this operational transform to find the Laplace transform of $(t-a)\,u(t-a)$:

$$L\{(t-a)\,u(t-a)\}=e-ass2.$$

Translation in the Frequency Domain

Translation in the frequency domain corresponds to multiplication by an exponential in the time domain:

$$L\{e-atf(t)\}=F(s+a),$$

which follows from the defining integral. Problem 12.15 asks you to derive this result.

We can use this operational transform to derive new transform pairs. Thus, knowing that

$$L\{\cos \omega t\}=ss2+\omega 2,$$

we use the effect of translation in the frequency domain to deduce that

$$L\{e-at \cos \omega t\}=s+a(s+a)2+\omega 2.$$

Scale Changing

The scale-change property gives the relationship between f(t) and F(s) when the time variable is multiplied by a positive constant:

L {f(at)}=1aF(sa), a>0.

The derivation is left to Problem 12.16. The scale-change property is particularly useful in experimental work, when time-scale changes are made to assist in building a model of a system.

We use this operational transform to formulate new transform pairs. Thus, knowing that

L {cos t}=ss2+1,

we use the effect of scale changing to show that

L {cos ωt}=1ω s/ω(s/ω)2 + 1=ss2 + ω2.

Table 12.2 lists these operational transforms.

Table 12.2 An Abbreviated List of Operational Transforms

Operation	$f(t)$	$F(s)$
Multiplication by a constant	$Kf(t)$	$KF(s)$
Addition/subtraction	$f_1(t) + f_2(t) - f_3(t) + \cdots$	$F_1(s) + F_2(s) - F_3(s) + \cdots$
First derivative (time)	$\frac{df(t)}{dt}$	$sF(s) - f(0^-)$
Second derivative (time)	$\frac{d^2f(t)}{dt^2}$	$s^2F(s) - sf(0^-) - \frac{df(0^-)}{dt}$
nth derivative (time)	$\frac{d^nf(t)}{dt^n}$	$s^nF(s) - s^{n-1}f(0^-) - s^{n-2}\frac{df(0^-)}{dt} - s^{n-3}\frac{df^2(0^-)}{dt^2} - \cdots - \frac{d^{n-1}f(0^-)}{dt^{n-1}}$
Time integral	$\int_0^t f(x)\,dx$	$\frac{F(s)}{s}$
Translation in time	$f(t-a)u(t-a), a > 0$	$e^{-as}F(s)$
Translation in frequency	$e^{-at}f(t)$	$F(s+a)$
Scale changing	$f(at), a > 0$	$\frac{1}{a}F\left(\frac{s}{a}\right)$
First derivative (s)	$tf(t)$	$-\frac{dF(s)}{ds}$
nth derivative (s)	$t^nf(t)$	$(-1)^n\frac{d^nF(s)}{ds^n}$
s integral	$\frac{f(t)}{t}$	$\int_s^\infty F(u)\,du$

Table 12.2 Full Alternative Text

Assessment Problem

Objective 1–Be able to calculate the Laplace transform of a function using the Laplace transform table or a table of operational transforms

1. 12.2 Use the appropriate operational transform from Table 12.2 to find the Laplace transform of each function:

    1. t2e−at;

    2. ddt (e−atsinh βt);

    3. t cos ωt.

    Answer:

    1. 2(s+a)3;

    2. βs(s+a)2−β2;

    3. s2−ω2(s2+ω2)2.

SELF-CHECK: Also try Chapter Problems 12.18 and 12.22.

12.6 Applying the Laplace Transform

We now use the Laplace transform to solve the ordinary integrodifferential equations that describe the behavior of a lumped-parameter circuit, such as the one shown in Fig. 12.16. We assume that no initial energy is stored in the circuit at the instant the switch is opened. The problem is to find the time-domain expression for v(t) when t≥0.

Figure 12.16 A parallel RLC circuit.

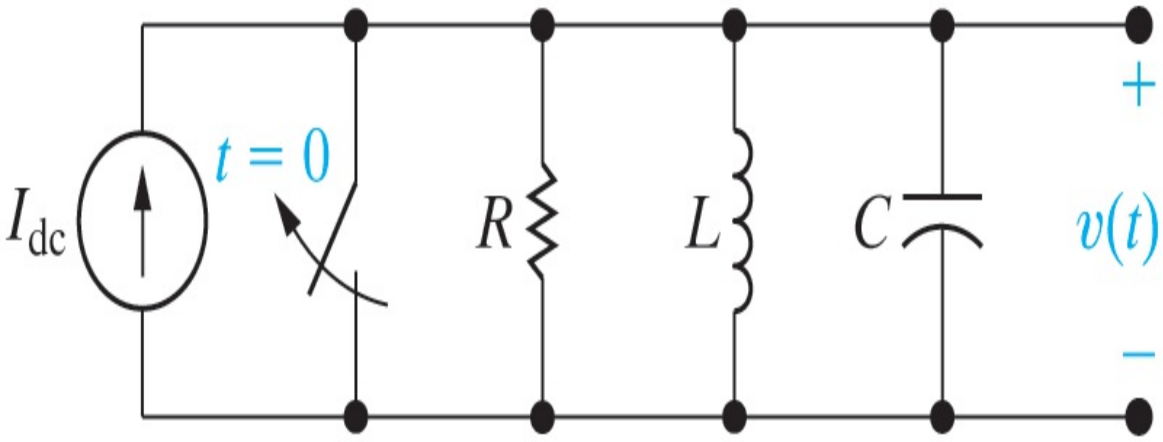


Figure 12.16 Full Alternative Text

We begin by writing the integrodifferential equation that v(t) must satisfy, using a single KCL equation to describe the circuit. Summing the currents away from the top node in the circuit gives:

v(t)R + 1ℒ∫0tv(x) dx+ Cdv(t)dt=Idcu(t).

Note that we represented the switch opening at t=0 with the product of the

source current and the unit step function, Idcu(t).

We transform the KCL equation into the s-domain using four operational transforms (multiplication by a constant, addition, integration, and differentiation) and one functional transform (unit step) to get

$$V(s)R + 1LV(s)s + C[sV(s) - v(0-)] = Idc(1s).$$

The result is an algebraic equation with one unknown variable, V(s). We are assuming that the circuit parameters R, L, and C, as well as the source current Idc, are known; the initial voltage on the capacitor v(0−) is zero because the initial energy stored in the circuit is zero. Thus, we have reduced the problem to solving an algebraic equation.

Solving for V(s) gives

$$V(s)(1R+1sL+sC) = Idcs, V(s) = Idc/Cs2+(1/RC)s+(1/LC).$$

To find v(t), we must inverse-transform the expression for V(s). We denote this inverse operation as

$$v(t) = L-1\{V(s)\}.$$

The inverse transform, which takes the solution from the s-domain to the time domain, is the subject of Section 12.7. In that section, we also present a final, critical step: checking the validity of the resulting time-domain expression. This final step is not unique to the Laplace transform; it is always a good idea to test any derived solution to be sure it makes sense in terms of known system behavior.

Before continuing, we simplify the notation by dropping the parenthetical t in time-domain expressions and the parenthetical s in frequency-domain expressions. We use lowercase letters for all time-domain variables, and we represent the corresponding s-domain variables with uppercase letters. Thus

$$\mathcal{L}\{ v \} = V \quad \text{or } v = \mathcal{L}-1\{ V \}, \mathcal{L}\{ i \} = I \quad \text{ori} = \mathcal{L}-1\{ I \}, \mathcal{L}\{ f \} = F \quad \text{orf} = \mathcal{L}-1\{ F \},$$

and so on.

Example 12.2 supplies component values for the circuit in Fig, 12.16 and uses Laplace transforms to predict the circuit's output voltage.

Example 12.2 Using Laplace Transforms to Predict a Circuit's Response

Suppose for the circuit in Fig. 12.16, Idc=24 mA, R=400Ω, L=25 mH, and C=25 nF. There is no energy stored in the circuit when the switch opens at t=0. Find the Laplace transform of v(t) and use it to predict the functional form of v(t).

Solution

Using the expression for V(s) found for the circuit in Fig. 12.16,

V(s)=Idc/Cs2+(1/RC)s+(1/ℒC)=0.024/(25×10−9)s2+[1/(400)(25×10−9)]s+[1/(0.025)(25×10−9)]=96×104s2+105s+16×108

The expression for V(s) is not a familiar functional Laplace transform, so we cannot use Tables 12.1 or 12.2 to find its inverse transform. But note that we can rewrite the denominator of V(s) as the product of two factors:

V(s)=96×104(s+20,000)(s+80,000).

Now, recognize that we can write V(s) as the sum of two terms, with each factor appearing in the denominator of one term:

V(s)=K1(s+20,000)+K2(s+80,000).

Each of these terms looks like a familiar Laplace transform. Using the transform for the exponential function (Table 12.1) and the operational

transforms for multiplication by a constant and addition (Table 12.2) we predict that v(t) is the sum of two exponential terms in the form $e^{-20,000t}$ and $e^{-80,000t}$.

SELF-CHECK: Assess your understanding of this material by trying Chapter Problem 12.26.

12.7 Inverse Transforms

The expression for V(s) in Example 12.2 is a **rational** function of s. This means we can write V(s) as a ratio of two polynomials in s where only integer powers of s appear in the polynomials. For linear, lumped-parameter circuits with constant component values, the s-domain expressions for the unknown voltages and currents are always rational functions of s. (For verification, work Problems 12.27–12.31.) If we can inverse-transform rational functions of s, we can find the time-domain expressions for the voltages and currents. This section presents a straightforward and systematic technique for finding the inverse transform of a rational function.

In general, we need to find the inverse transform of a function that has the form

$$F(s)=\frac{N(s)}{D(s)}=\frac{a_n s^n+a_{n-1}s^{n-1}+\cdots+a_1 s+a_0}{b_m s^m+b_{m-1}s^{m-1}+\cdots+b_1 s+b_0}. \quad (12.7)$$

The coefficients a and b are real constants, and the exponents m and n are positive integers. The ratio N(s)/D(s) is called a **proper rational function** if $m>n$, and an **improper rational function** if $m\leq n$. Only a proper rational function can be expanded as a sum of partial fractions. This restriction poses no problem, as we show at the end of this section.

Partial Fraction Expansion: Proper Rational Functions

A proper rational function is expanded into a sum of partial fractions by writing a term or a series of terms for each root of D(s). Thus, D(s) must be factored before we construct a partial fraction expansion. For each distinct root of D(s), a single term appears in the sum of partial fractions. For each multiple root of D(s) of multiplicity r, the sum of partial fractions contains r terms. For example, in the rational function

s+6s(s+3)(s+1)2,

the denominator has four roots. Two of these roots are distinct—namely, at s=0 and s=−3. A multiple root of multiplicity 2 occurs at s=−1. Thus, the partial fraction expansion takes the form

s+6s(s+3)(s+1)2≡K1s+K2s+3+K3(s+1)2+K4s+1.

To find the inverse transform from the sum of partial fractions, we identify the f(t) corresponding to each term in the sum using the functional and operational transform tables. Use Tables 12.1 and 12.2 to verify that

L−1{K1s+K2s+3+K3(s+1)2+K4s+1} =(K1+K2e−3t+K3te−t+K4e−t)u(t).

Now we need to find the numerator coefficients (K1,K2,K3,…) that appear in each partial fraction term. There are only four different types of partial fraction terms because the roots of D(s) can be (1) real and distinct; (2) complex and distinct; (3) real and repeated; or (4) complex and repeated. We develop a technique to determine the numerator coefficient for each type of partial fraction term. Before doing so, a few general comments are in order.

We used the identity sign ≡ in the partial fraction expansion to emphasize that expanding a rational function into a sum of partial fractions establishes an identical equation. This means that both sides of the equation must be the same for all values of the variable s. Also, the identity relationship must hold when the same mathematical operation is applied to both sides. These observations will help us calculate the coefficient values.

Before creating a partial fraction expansion, you should verify that the rational function is proper. This check is important because the procedure for finding the partial fraction coefficients will not prevent you from generating invalid results if the rational function is improper. We present a procedure for checking the coefficients, but you can avoid wasted effort by always asking, “Is F(s) a proper rational function?”

Partial Fraction Expansion: Distinct

Real Roots of D(s)

Let’s find the partial fraction expansion of a proper rational function whose denominator has distinct real roots. For example,

$$F(s)=N(s)(s+p1)(s+p2)(s+p3)\equiv K1(s+p1)+K2(s+p2)+K3(s+p3).$$

To find K1 for the first partial fraction term, we multiply both sides of the identity by the denominator beneath K1 to get

$$N(s)(s+p2)(s+p3)\equiv K1+K2(s+p1)(s+p2)+K3(s+p1)(s+p3).$$

Then when we evaluate both sides of the identity for $s=-p1$, which is the root of the partial fraction term whose coefficient is K1:

$$N(s)(s+p2)(s+p3)|_{s=-p1} \equiv K1+K2(s+p1)(s+p2)|_{s=-p1}+K3(s+p1)(s+p3)|_{s=-p1}=K1.$$

The right-hand side is always the desired K, and the left-hand side is always its numerical value. To find K2 and K3, repeat the steps used to find K1.

Example 12.3 illustrates this process.

Example 12.3 Finding the Inverse Laplace Transform when F(s) has Distinct Real Roots

Use partial fraction expansion to find the inverse Laplace transform of

$$F(s)=96(s+5)(s+12)s(s+8)(s+6).$$

Solution

The partial fraction expansion of F(s) is

$$F(s)=96(s+5)(s+12)s(s+8)(s+6)\equiv K1s+K2(s+8)+K3(s+6).$$

To find the value of K1, we multiply both sides by s and then evaluate both sides at s=0:

$$96(s+5)(s+12)(s+8)(s+6)|s=0\equiv K1+K2ss+8|s=0+K3ss+6|s=0,$$

or

$$96(5)(12)8(6)=K1=120.$$

To find the value of K2, we multiply both sides by s+8 and then evaluate both sides at s=−8:

$$96(s+5)(s+12)s(s+6)|s=-8 \equiv K1(s+8)s|s=-8+K2+K3(s+8)(s+6)|s=-8,$$

or

$$96(-3)(4)(-8)(-2)=K2=-72.$$

Then K3 is

$$96(s+5)(s+12)s(s+8)|s=-6=K3=48.$$

Therefore,

$$96(s+5)(s+12)s(s+8)(s+6)=120s-72(s+8)+48(s+6).$$

It is a good idea to test this result. While the choice of test values is completely open, we choose values that are easy to verify. For example, testing at either −5 or −12 is convenient because in both cases the left-hand side reduces to zero. Choosing −5 yields

$$120-5-723+481=-24-24+48=0,$$

whereas testing −12 gives

120−12−72−4+48−6=−10+18−8=0.

Now find the inverse transform of F(s) using the tables of functional and operational transforms (Tables 12.1 and 12.2):

L−1{96(s+5)(s+12)s(s+8)(s+6)}=(120−78e−8t+48e−6t) u(t).

Assessment Problems

Objective 2–Be able to calculate the inverse Laplace transform using partial fraction expansion and the Laplace transform table

1. 12.3 Find f(t) if

   F(s)=6s2+26s+26(s+1)(s+2)(s+3).

   Answer:

   f(t)=(3e−t+2e−2t+e−3t) u(t).

2. 12.4 Find f(t) if

   F(s)=7s2+63s+134(s+3)(s+4)(s+5).

   Answer:

   f(t)=(4e−3t+6e−4t−3e−5t) u(t).

3. 12.5 Find v(t) for the circuit analyzed in Example 12.2 and shown in Fig. 12.16.

   Answer:

   v(t)=(16e−20,000t−16e−80,000t) u(t) V.

SELF-CHECK: Also try Chapter Problems 12.40(a) and (b).

Partial Fraction Expansion: Distinct Complex Roots of D(s)

We begin by noting that if F(s) describes the Laplace transform of a voltage or current in a physically realizable circuit, the factors of D(s) that have complex roots will always come in conjugate pairs. Let's assume that

$$F(s)=A(s+z1)(s+\alpha-j\beta)(s+\alpha+j\beta)\equiv K1(s+\alpha-j\beta)+K2(s+\alpha+j\beta).$$

Here we assume that z1 is a real number.

We find the unknown coefficients, K1 and K2, using the same technique we employed when the factors of D(s) were real and distinct. As you will see, the only difference is that the algebra involves complex numbers. To find K1, we multiply both sides of the identity by the denominator beneath K1 to get

$$A(s+z1)(s+\alpha+j\beta)\equiv K1+K2(s+\alpha-j\beta)(s+\alpha+j\beta).$$

Then when we evaluate both sides of the identity for $s=-\alpha+j\beta$, which is the root of the partial fraction term whose coefficient is K1:

$$A(s+z1)(s+\alpha+j\beta)|s=-\alpha+j\beta\equiv K1+K2(s+\alpha-j\beta)(s+\alpha+j\beta)|s=-\alpha+j\beta=K1.$$

We evaluate the left-hand side of the identity to get an expression for K1:

$$K1=A(-\alpha+z1+j\beta)(j2\beta)=A2+j(\alpha-z12\beta).$$

As expected, K1 is a complex number. We repeat the steps used to find K1 when finding the expression for K2:

$$A(s+z1)(s+\alpha-j\beta)=K1(s+\alpha+j\beta)(s+\alpha-j\beta)+K2;$$
$$A(s+z1)(s+\alpha-j\beta)|s=-\alpha-j\beta=K1(s+\alpha+j\beta)(s+\alpha-j\beta)|s=-\alpha-j\beta=K2;$$
$$K2=A(-\alpha+z1-j\beta)(-j2\beta)=A2-j(\alpha-z12\beta).$$

Compare the expression for K1 with the expression for K2; K1 and K2 are conjugates. This will always be the case when D(s) has complex roots, so we

need only calculate one coefficient, since the other is its conjugate. We can use the following general polar form for K1 and K2:

$K1=|K|e^{j\theta}$ and $K2=|K|e^{-j\theta}$.

Using the polar form for the general coefficients K1 and K2, we can find the inverse Laplace transform of F(s), with the help of the functional and operational transform tables (Tables 12.1 and 12.2) and Euler's identity:

$$\mathcal{L}^{-1}\{ |K|e^{j\theta}/(s+\alpha-j\beta)+|K|e^{-j\theta}/(s+\alpha+j\beta) \}=|K|e^{j\theta}e^{-(\alpha-j\beta)t}+|K|e^{-j\theta}e^{-(\alpha+j\beta)t}=|K|e^{-\alpha t}e^{j(\beta t+\theta)}+|K|e^{-\alpha t}e^{-j(\beta t+\theta)}=|K|e^{-\alpha t}[\cos(\beta t+\theta)+j\sin(\beta t+\theta)]+|K|e^{-\alpha t}[\cos(\beta t+\theta)-j\sin(\beta t+\theta)]=2|K|e^{-\alpha t}\cos(\beta t+\theta).$$

Because distinct complex roots appear frequently in lumped-parameter linear circuit analysis, we summarize these results with a new transform pair. Whenever D(s) contains distinct complex roots—that is, factors of the form $(s+\alpha-j\beta)(s+\alpha+j\beta)$—a pair of terms of the form

$$\frac{K}{s+\alpha-j\beta}+\frac{K^*}{s+\alpha+j\beta}$$

appears in the partial fraction expansion, where the partial fraction coefficient is, in general, a complex number. In polar form,

$$K=|K|e^{j\theta}=|K|\angle\theta, \quad (12.8)$$

where |K| denotes the magnitude of the complex coefficient. Then

$$K^*=|K|e^{-j\theta}=|K|\angle-\theta. \quad (12.9)$$

The pair of complex conjugate partial fraction terms always inverse-transforms as

$$\mathcal{L}^{-1}\left\{\frac{K}{s+\alpha-j\beta}+\frac{K^*}{s+\alpha+j\beta}\right\} = 2|K|e^{-\alpha t}\cos(\beta t+\theta). \quad (12.10)$$

In applying Eq. 12.10, it is important to note that K is defined as the coefficient associated with the denominator $s+\alpha-j\beta$, and K* is defined as the coefficient associated with the denominator $s+\alpha+j\beta$.

Example 12.4 finds the inverse Laplace transform of an s-domain function with distinct complex roots.

Example 12.4 Finding the Inverse Laplace Transform when F(s) has Distinct Complex Roots

Use partial fraction expansion to find the inverse Laplace transform of

F(s)=100(s+3)(s+6)(s2+6s+25).

Solution

We begin by noting that F(s) is a proper rational function. Next we find the roots of the quadratic term s2+6s+25:

s2+6s+25=(s+3−j4)(s+3+j4).

With the denominator in factored form, we create the partial fraction expansion:

100(s+3)(s+6)(s2+6s+25)≡ K1s+6+K2s+3−j4+K2*s+3+j4.

Find K1 and K2 using the same process employed in Example 12.3:

K1=100(s+3)s2+6s+25|s=−6=100(−3)25=−12,K2=100(s+3)(s+6)(s+3+j4)|s=−3+j4=100(j4)(3+j4)(j8)=6−j8=10e−j53.13°.

Thus

100(s+3)(s+6)(s2+6s+25) ≡
−12s+6+10 −53.13°s+3−j4+10 53.13°s+3+j4.

Before inverse-transforming the terms in the partial fraction expansion, we check the expansion numerically. We test using s=−3 because the left-hand side reduces to zero at this value:

F(s)=−123+10 −53.13°−j4+10 53.13°j4=−4+2.5 36.87°_+2.5 −36.87°_= −4+2.0+j1.5+2.0−j1.5=0.

Finally, perform the inverse-transform, using the functional and operational transform tables (Tables 12.1 and 12.2) and the new transform pair in Eq. 12.10:

ℒ−1{ 100(s+3)(s+6)(s2+6s+25) } =[−12e−6t+20e−3t cos (4t−53.13°)]u(t).

Assessment Problems

Objective 2–Be able to calculate the inverse Laplace transform using partial fraction expansion and the Laplace transform table

1. 12.6 Find f(t) if

   F(s)=10(s2+119)(s+5)(s2+10s+169).

   Answer:

   f(t)=(10e−5t−8.33e−5t sin 12t) u(t).

2. 12.7 Suppose for the circuit in Fig. 12.16, Idc=24 mA, R=625 Ω, L=25 mH, and C=25 nF. These are the same values used in Example 12.2 except for the value of R. There is no energy stored in the circuit when the switch opens at t=0.

   1. Find V(s), the Laplace transform of v(t).

   2. Find v(t) by finding the inverse transform of the partial fraction expansion of V(s).

Answer:

1. V(s)=96×104s2+64,000s+16×108;
2. v(t)=40e−32,000tcos(24,000t−90°) u(t) V.

SELF-CHECK: Also try Chapter Problems 12.41(c) and (d).

Partial Fraction Expansion: Repeated Real Roots of D(s)

We describe two methods for finding the partial fraction coefficients for terms generated by a multiple root with multiplicity r. Method A requires you to solve several simultaneous algebraic equations. Method B is a modification of the process we have been using to find the coefficients for the partial fraction terms associated with distinct roots. Both methods begin with the identity relating the original s-domain function, which must be a proper rational function, and its partial fraction expansion:

$$N(s)D(s)≡K1(s+p)r+K2(s+p)r−1+⋯+Kr(s+p)$$

Method A

1. Combine all terms in the partial fraction expansion over the common denominator, D(s). Call the numerator of this new function N1(s).
2. Collect all of the terms in the numerator N1(s) according to their power of s. The coefficient of each power of s will include some of the unknown partial fraction coefficients.
3. Equate the coefficient of each power of s in N1(s) with the coefficient of the corresponding power of s in N(s). The result is a collection of simultaneous equations whose unknowns are the partial fraction coefficients.
4. Solve the simultaneous equations to find the partial fraction coefficients.

Method B

1. Multiply both sides of the identity defining the partial fraction expansion by the multiple root raised to its rth power. Call the resulting identity Ir.

2. Find K in the numerator of the factor raised to the rth power by evaluating both sides of Ir at the multiple root.

3. To find K in the numerator of the factor raised to the (r–1) power, differentiate both sides of Ir with respect to s. Call the resulting identity I(r−1). Evaluate both sides of I(r−1) at the multiple root. The right-hand side is always the desired K, and the left-hand side is always its numerical value.

4. Repeat Step 3 to find the remaining partial fraction coefficients by differentiating I(r−1) to get I(r−2) and so on. In total, you will have differentiated Ir(r–1) times.

Example 12.5 uses Method A to find the inverse Laplace transform for an s-domain function with repeated real roots.

Example 12.5 Finding the Inverse Laplace Transform when F(s) has Repeated Real Roots

Use partial fraction expansion to find the inverse Laplace transform of

$F(s)=100(s+25)s(s+5)3$.

Solution

We begin by noting that F(s) is a proper rational function. Next we find partial fraction expansion of F(s):

$$100(s+25)s(s+5)3\equiv K1s+K2(s+5)3+K3(s+5)2+K4s+5.$$

We will use Method A to find the partial fraction coefficients. Begin by multiplying the numerator and denominator of each term in the partial fraction expansion with an expression that creates a denominator of s(s+5)3 in each term:

$$100(s+25)s(s+5)3\equiv K1(s+5)3s(s+5)3+K2ss(s+5)3 \quad +K3s(s+5)s$$

Combine the terms on the right-hand side over their common denominator, and expand the resulting numerator by collectiong the coefficients for each power of s. The numerator on the right-hand side is

$$(K1+K4)s3+(15K1+K3+10K4)s2 \quad +(35K1+K2+5K3+25K4)s \quad + (125K1).$$

Equate the coefficients of each power of s in the numerators on the right-hand side and left-hand side to create four simultaneous equations:

$$K1+K4=0;15K1+K3+10K4=0;35K1+K2+5K3+25K4=100;125K1=2500.$$

Solve the simultaneous equations to find

$$K1=20;K2=-400;K3=-100;K4=-20.$$

Therefore, the partial fraction expansion is

$$100(s+25)s(s+5)3\equiv 20s-400(s+5)3-100(s+5)2-20s+5.$$

At this point, we can check our expansion by testing both sides at s=−25; for this value of s, both sides should equal zero. The result of evaluating the partial fraction expansion at s=−25 is

$$20-25-400(-20)3-100(-20)2-20(-20)=0.$$

Use the functional and operational transform tables (Tables 12.1 and 12.2) to transform each term in the partial fraction expansion. Thus, the inverse transform of F(s) is

L−1{100(s+25)s(s+5)3}=[20−200t2e−5t−100te−5t−20e−5t]u(t).

Assessment Problems

Objective 2–Be able to calculate the inverse Laplace transform using partial fraction expansion and the Laplace transform table

1. 12.8 Find f(t) if

   F(s)=(4s2+7s+1)s(s+1)2.

   Answer:

   f(t)=(1+2te−t+3e−t) u(t).

2. 12.9 Suppose for the circuit in Fig. 12.16, Idc=24 mA, R=500 Ω, L=25 mH, and C=25 nF. These are the same values used in Example 12.2 and Assessment Problem 12.7 except for the value of R. There is no energy stored in the circuit when the switch opens at t=0.

   1. Find V(s), the Laplace transform of v(t).

   2. Find v(t) by finding the inverse transform of the partial fraction expansion of V(s).

   Answer:

   1. V(s)=96×104s2+80,000s+16×108;

   2. v(t)=96×104te−40,000tu(t) V.

SELF-CHECK: Also try Chapter Problems 12.42(b) and (c).

Partial Fraction Expansion: Repeated Complex Roots of D(s)

We can find the coefficients of the partial fraction terms corresponding to repeated complex roots using either Method A or Method B. The algebra involves complex numbers. Recall that complex roots always appear in conjugate pairs and that the coefficients associated with a conjugate pair are also conjugates, so that only half the Ks need to be evaluated. We illustrate the process using Method B in Example 12.6.

Example 12.6 Finding the Inverse Laplace Transform when F(s) has Repeated Complex Roots

Use partial fraction expansion to find the inverse Laplace transform of

F(s)=768(s2+6s+25)2.

Solution

After factoring the denominator polynomial, we write

F(s)=768(s+3−j4)2(s+3+j4)2≡K1(s+3−j4)2+K2s+3−j4+K1*(s+3+j4)2+K2*s+3+j4.

Now we need to evaluate only K1 and K2, because K1* and K2* are conjugate values. We use Method B to find these two partial fraction coefficients.

The value of K1 is

K1=768(s+3+j4)2|s=−3+j4=768(j8)2=−12.

To find the value of K2, multiply F(s) by (s+3–j4)2, find the first derivative of the result with respect to s, and evaluate for s=–3+j4:

$K2=dds[\ 768(s+3+j4)2\ ]s=-3+j4=-2(768)(s+3+j4)3|s=-3+j4=-2(768)(j8)3-j3=3\ -90°.$

From the values for K1 and K2, $K1^*=-12$, and $K2^*=j3=3\ 90°$. Group the partial fraction expansion by conjugate terms to obtain

$$F(s)=[\ -12(s+3-j4)2+-12(s+3+j4)2\ ]\ +\ [3\ \angle\ -90°_(s+3-j4)\ +\ 3\ \angle\ 90°_(s+3+j4)].$$

Inverse-transform F(s) by applying the functional and operational transforms (Tables 12.1 and 12.2) to the terms in the partial fraction expansion. The result is $f(t)=[-24te-3t\cos 4t+6e-3t\cos(4t-90°)]u(t)$.

Assessment Problem

Objective 2–Be able to calculate the inverse Laplace transform using partial fraction expansion and the Laplace transform table

1. 12.10 Find f(t) if

   $F(s)=40(s2+4s+5)2.$

   Answer:

   $f(t)=(-20te-2t\cos t+20e-2t\sin t)\ u(t).$

SELF-CHECK: Also try Chapter Problem 12.43(b).

Note that if F(s) has a real root a of multiplicity r in its denominator, the partial fraction expansion has a term of the form

$K(s+a)r.$

The inverse transform of this term is

$$L-1\{K(s+a)r\}=Ktr-1e-at(r-1)!u(t).\ (12.11)$$

If F(s) has a complex root of α+jβ of multiplicity r in its denominator, the partial fraction expansion has a conjugate pair of terms in the form

K(s+α−jβ)r+K*(s+α+jβ)r.

The inverse transform of this pair is

ℒ−1{ | K |θ(s+α−jβ)r+| K |−θ(s+α+jβ)r } =[2| K |tr−1(r−1)! e−αt cos (βt+θ)] u(t). (12.12)

Equations 12.11 and 12.12 are the key to finding the inverse transform for any partial fraction expansion with repeated roots. One further note regarding these two equations: In circuit analysis problems, r is seldom greater than 2. Therefore, the inverse transform of a rational function can be handled with four transform pairs. Table 12.3 lists these pairs.

Table 12.3 Four Useful Transform Pairs

Pair Number	Nature of Roots	$F(s)$	$f(t)$
1	Distinct real	$\frac{K}{s+a}$	$Ke^{-at}u(t)$
2	Repeated real	$\frac{K}{(s+a)^2}$	$Kte^{-at}u(t)$
3	Distinct complex	$\frac{K}{s+\alpha-j\beta}+\frac{K^*}{s+\alpha+j\beta}$	$2\lvert K\rvert e^{-\alpha t}\cos(\beta t+\theta)u(t)$
4	Repeated complex	$\frac{K}{(s+\alpha-j\beta)^2}+\frac{K^*}{(s+\alpha+j\beta)^2}$	$2t\lvert K\rvert e^{-\alpha t}\cos(\beta t+\theta)u(t)$

Table 12.3 Full Alternative Text

Note: In pairs 1 and 2, *K* is a real quantity, whereas in pairs 3 and 4, *K* is the

complex quantity | K |θ.

Partial Fraction Expansion: Improper Rational Functions

An improper rational function can always be written as the sum of a polynomial and a proper rational function. The polynomial is then inverse-transformed into impulse functions and derivatives of impulse functions, while the proper rational function is inverse-transformed by the techniques outlined in this section.

We illustrate the procedure in Example 12.7.

Example 12.7 Finding the Inverse Laplace Transform of an Improper Rational Function

Use partial fraction expansion to find the inverse Laplace transform of

F(s)=s4+13s3+66s2+200s+300s2+9s+20.

Solution

The order of the numerator is 4, while the order of the denominator is 2, so F(s) is an improper rational function. To write it as the sum of a polynomial and a proper rational function, divide the denominator into the numerator until the remainder is a proper rational function. The result is

F(s)=
(s2+9s+20)s2+4s+10 s4+13s3+66s2+200s+300=s2+4s+10+

Next, expand the proper rational function into a sum of partial fractions: $\frac{30s+100}{s^2+9s+20} = \frac{30s+100}{(s+4)(s+5)} \equiv \frac{-20}{s+4} + \frac{50}{s+5}$.

Replace the proper rational function in F(s) with the partial fraction expansion to get

$$F(s) = s^2 + 4s + 10 - \frac{20}{s+4} + \frac{50}{s+5}.$$

Using the tables of functional and operational transforms (Tables 12.1 and 12.2) we can now inverse-transform F(s). Hence

$$f(t) = \frac{d^2\Delta(t)}{dt^2} + 4\frac{d\Delta(t)}{dt} + 10\Delta(t) - (20e^{-4t} - 50e^{-5t})\, u(t).$$

Assessment Problems

Objective 2–Be able to calculate the inverse Laplace transform using partial fraction expansion and the Laplace transform table

1. 12.11 Find f(t) if

   $$F(s) = \frac{(5s^2+29s+32)}{(s+2)(s+4)}.$$

   Answer:

   $$5\Delta(t) - (3e^{-2t} - 2e^{-4t})\, u(t).$$

2. 12.12 Find f(t) if

   $$F(s) = \frac{(2s^3+8s^2+2s-4)}{(s^2+5s+4)}.$$

   Answer:

   $$2\frac{d\Delta(t)}{dt} - 2\Delta(t) + 4e^{-4t}\, u(t).$$

SELF-CHECK: Also try Chapter Problem 12.43(d).

12.8 Poles and Zeros of F(s)

The rational function of Eq. 12.7 can also be expressed as the ratio of two factored polynomials. In other words, we can write F(s) as

$$F(s)=K(s+z1)(s+z2)\cdots(s+zn)(s+p1)(s+p2)\cdots(s+pm), \quad (12.13)$$

where K is the constant an/bm.

The roots of the denominator polynomial, that is, −p1, −p2, −p3, …, −pm, are called the **poles of F(s)**; they are the values of s at which F(s) becomes infinitely large. The roots of the numerator polynomial, that is, −z1, −z2, −z3, …, −zn are called the **zeros of F(s)**; they are the values of s at which F(s) becomes zero. We can visualize the poles and zeros of F(s) as points on a complex s plane. In the complex s plane, we use the horizontal axis to plot the real values of s and the vertical axis to plot the imaginary values of s. Example 12.8 finds the poles and zeros for two different functions of s and plots the locations of the poles and zeros on the complex plane.

Example 12.8 Finding and Plotting the Poles and Zeros of an s-Domain Function

1. Suppose F1(s) is given by

   $$F1(s)=40s3+440s2+2200s+50004s4+88s3+880s2+4000s.$$

   Find the poles and zeros of F1(s) and plot them on the complex s-plane.

2. Suppose F2(s) is given by

   $$F2(s)=8s2+120s+4002s4+20s3+70s2+100s+48.$$

Find the poles and zeros of F2(s) and plot them on the complex s-plane.

Solution

1. Begin by factoring out a constant in the numerator and denominator of F1(s). Then factor the numerator and denominator polynomials. The result is

   F1(s)=40(s3+11s2+55s+125)4(s4+22s3+220s2+1000s)=10(s+5)(s+3-j4)(s+3+j4)s(s+10)(s+6-j8)(s+6+j8).

   The poles of F1(s) are at 0, −10, −6+j8, and −6−j8. The zeros are at −5, −3+j4, and −3−j4. Figure 12.17 shows the poles and zeros plotted on the s plane, where X's represent poles and O's represent zeros.

Figure 12.17 Plotting poles and zeros on the s plane for F1(s) in Example 12.8.

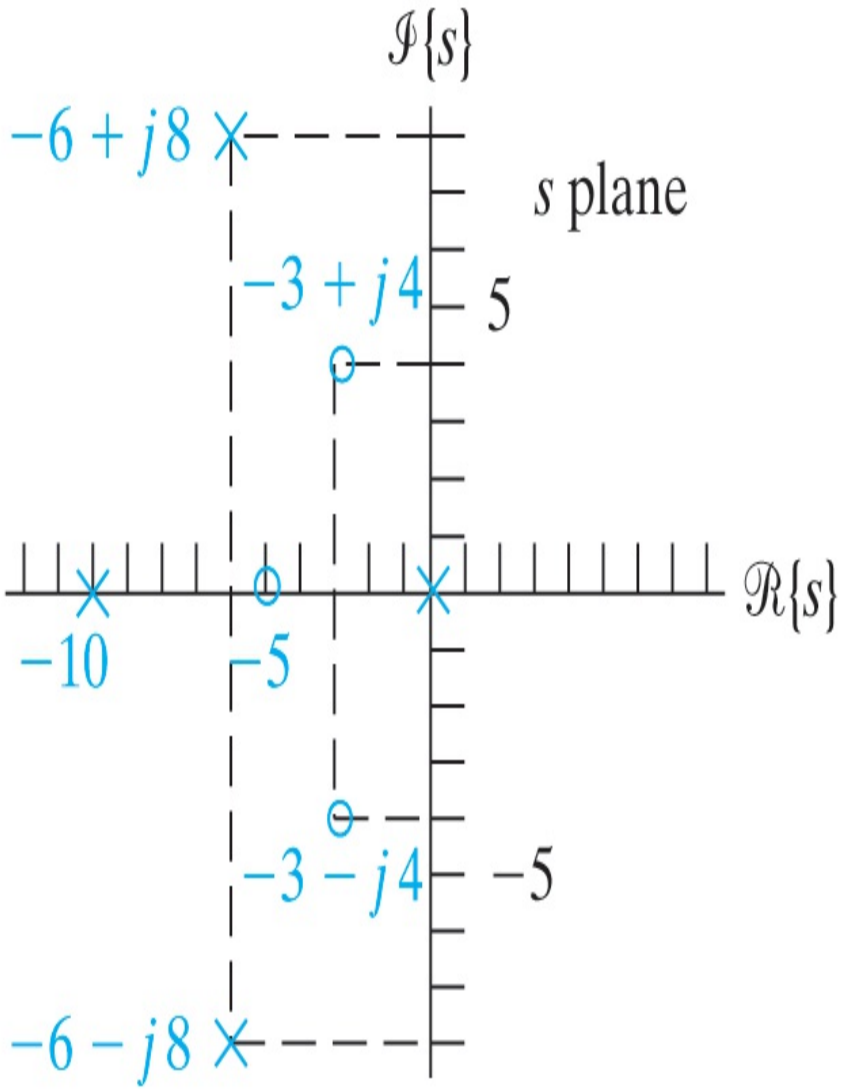


Figure 12.17 Full Alternative Text

2. Factor out a constant in the numerator and denominator of F2(s). Then factor the numerator and denominator polynomials to give

F2(s)=8(s2+15s+50)2(s4+10s3+35s2+50s+24)=4(s+5)(s+10)(s+1)(s+2)(s+3)(s+4).

The zeros of F2(s) are −5 and −10. The poles of F2(s) are −1, −2, −3, and −4. They are plotted in the complex s-plane in Fig. 12.18. Note that F2(s) also has a second-order zero at infinity because for large values of s the function reduces to 4/s2, and F2(s)=0 when s=∞. In general, F(s) can have either an rth-order pole or an rth-order zero at infinity. In this text, we are interested in

the poles and zeros located in the finite s plane. Therefore, when we refer to the poles and zeros of a rational function of s, we are referring to the finite poles and zeros.

Figure 12.18 Plotting poles and zeros on the s plane for F2(s) in Example 10.8.

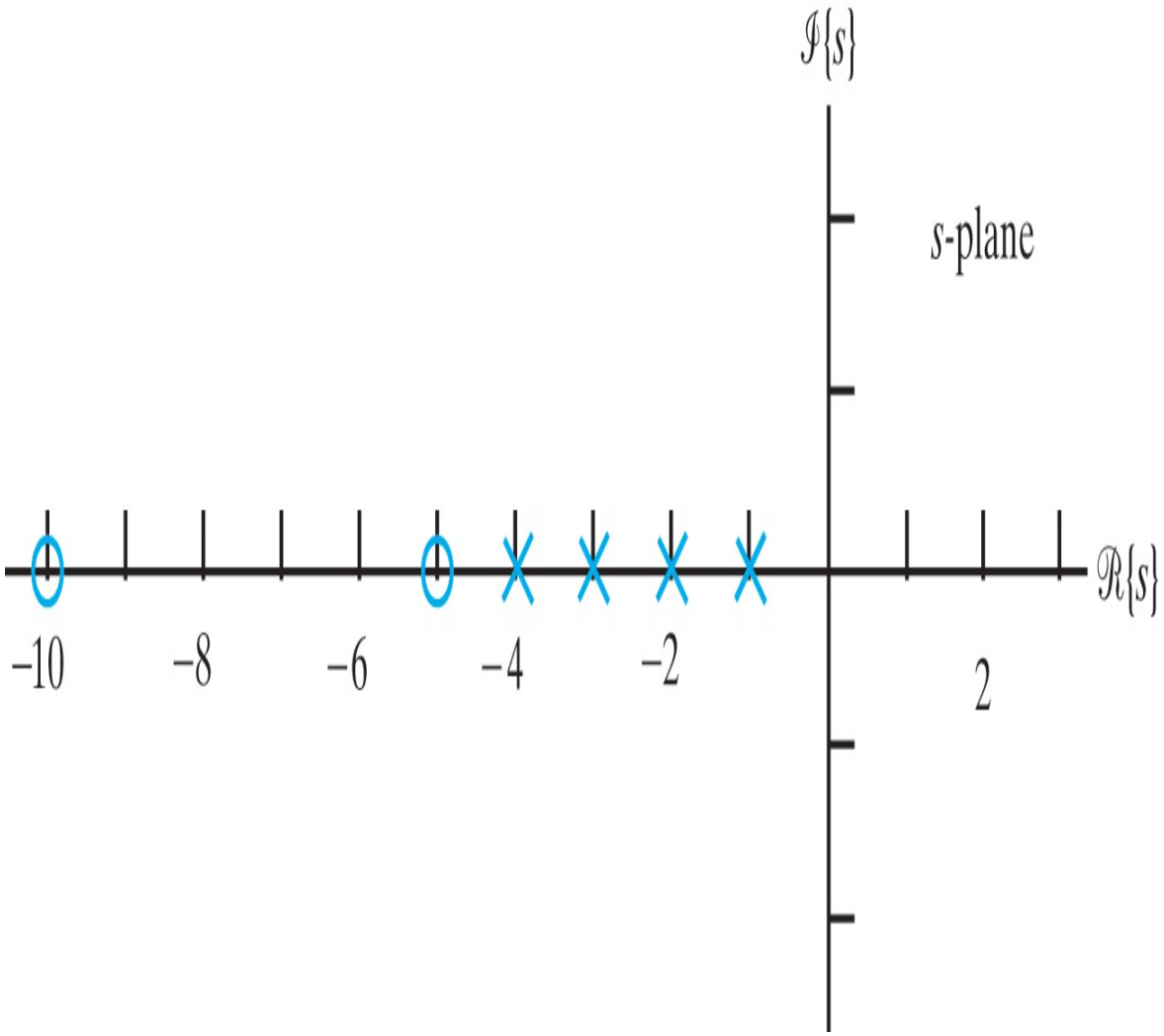


Figure 12.18 Full Alternative Text

Assessment Problem

Objective 3–Be able to find and plot the poles and zeros for a rational function in the s-domain

1. 12.13 Find the poles and zeros for the following rational functions of s.
    1. F(s)=20s2+80s+100s3+10s2+21s;
    2. F(s)=10s2+20s+102s3+28s2+258s+712;
    3. F(s)=125s5s4+90s3+670s2+2360s+3400;

    Answer:

    1. Zeros at −2+j and −2−j, poles at 0, −3, and −7;
    2. Two zeros at −1, poles at −4, −5+j8, and −5−j8;
    3. Zero at 0, poles at −5+j3, −5−j3, −4+j2, and −4−j2.

SELF-CHECK: *Also try Chapter Problem 12.46*.

12.9 Initial- and Final-Value Theorems

The initial- and final-value theorems enable us to determine the behavior of f(t) at 0 and ∞ from F(s). Hence, we can check the initial and final values of f(t) to see if they conform to known circuit behavior, before actually finding the inverse transform of F(s).

The initial-value theorem states that

Initial-Value Theorem

$$\lim_{t \to 0+} f(t) = \lim_{s \to \infty} sF(s), \quad (12.14)$$

and the final-value theorem states that

Final-Value Theorem

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s). \quad (12.15)$$

The initial-value theorem assumes that f(t) does not have an impulse function at the origin. The final-value theorem is valid only if the poles of F(s), except for a first-order pole at the origin, lie in the left half of the s plane.

To prove Eq. 12.14, we start with the operational transform of the first derivative:

$$\mathcal{L}\{ \frac{df}{dt} \} = sF(s) - f(0-) = \int_{0-}^{\infty} \frac{df}{dt} e^{-st} dt.$$

Now we take the limit as $s \to \infty$:

$$\lim_{s \to \infty} [sF(s) - f(0-)] = \lim_{s \to \infty} \int_{0-}^{\infty} \frac{df}{dt} e^{-st} dt. \quad (12.16)$$

Observe that the right-hand side of Eq. 12.16 can be written as

$$\lim_{s\to\infty}\left(\int_{0-}^{0+}\frac{df}{dt}e^{0}dt+\int_{0+}^{\infty}\frac{df}{dt}e^{-st}dt\right).$$

As $s\to\infty$, $(df/dt)e^{-st}\to 0$; hence, the second integral vanishes in the limit. The first integral reduces to $f(0+)-f(0-)$, which is independent of s. Thus, the right-hand side of Eq. 12.16 becomes

$$\lim_{s\to\infty}\int_{0-}^{\infty}\frac{df}{dt}e^{-st}dt=f(0+)-f(0-).$$

Because $f(0-)$ is independent of s, the left-hand side of Eq. 12.16 can be written as

$$\lim_{s\to\infty}[sF(s)-f(0-)]=\lim_{s\to 0}[sF(s)]-f(0-).$$

Therefore,

$$\lim_{s\to\infty}sF(s)=f(0+)=\lim_{t\to 0+}f(t),$$

which completes the proof of the initial-value theorem.

The proof of the final-value theorem also starts with the operational transform of the first derivative. Here we take the limit as $s\to 0$:

$$\lim_{s\to 0}[sF(s)-f(0-)]=\lim_{s\to 0}\left(\int_{0-}^{\infty}\frac{df}{dt}e^{-st}dt\right). \quad (12.17)$$

The integration is with respect to t and the limit operation is with respect to s, so the right-hand side of Eq. 12.17 reduces to

$$\lim_{s\to 0}\left(\int_{0-}^{\infty}\frac{df}{dt}e^{-st}dt\right)=\int_{0-}^{\infty}\frac{df}{dt}dt.$$

Because the upper limit on the integral is infinite, this integral may also be written as a limit process:

$$\int_{0-}^{\infty}\frac{df}{dt}\,dt=\lim_{t\to\infty}\int_{0-}^{t}\frac{df}{dy}dy,$$

where we use y as the symbol of integration to avoid confusion with the upper limit on the integral. Carrying out the integration on the right-hand side gives

$\lim_{t \to \infty}[f(t) - f(0-)] = \lim_{t \to \infty}[f(t)] - f(0-)$.

Substituting this expression into the right-hand side of Eq. 12.17 gives

$\lim_{s \to 0}[sF(s)] - f(0-) = \lim_{t \to \infty}[f(t)] - f(0-)$.

Since $f(0-)$ cancels, we get

$\lim_{s \to 0} sF(s) = \lim_{t \to \infty} f(t)$,

which completes the proof of the final value theorem.

The final-value theorem is useful only if $f(\infty)$ exists. This condition is true only if all the poles of $F(s)$, except for a single pole at the origin, lie in the left half of the s plane.

Example 12.9 applies the initial- and final-value theorems to the s-domain function from Example 12.2.

Example 12.9 Applying the Initial- and Final-Value Theorems

Suppose for the circuit in Fig. 12.16, $I_{dc} = 24$ mA, $R = 400\ \Omega$, $L = 25$ mH, and $C = 25$ nF. There is no energy stored in the circuit when the switch opens at $t = 0$. In Example 12.2, we found the Laplace transform of $v(t)$ is

$V(s) = \frac{96 \times 10^4 s^2 + 10^5 s + 16 \times 10^8}{}$.

Use the initial- and final-value theorems to predict the initial and final values of $v(t)$ and verify that $V(s)$ correctly predicts the values of $v(0+)$ and $v(\infty)$ from the circuit.

Solution

From the initial-value theorem,

$\lim_{s \to \infty} sV(s) = \lim_{s \to \infty} \frac{96 \times 104s}{s^2 + 105s + 16 \times 108}$.

To evaluate the limit on the right-hand side, divide numerator and denominator by the highest power of s in the denominator, in this case s^2, and find the limit as $1/s \to 0$:

$$\lim_{s \to \infty} \frac{96 \times 104s}{s^2 + 105s + 16 \times 108}$$
$$= \lim_{1/s \to 0} \frac{96 \times 104(1/s)}{1 + 105(1/s) + 16 \times 108(1/s)^2} = \frac{0}{1 + 0 + 0} = 0 = \lim_{t \to 0} v(t) = v(0$$

Since the problem states that there is no energy stored in the circuit prior to the switch opening at t=0, we have confirmed that the initial voltage is zero.

Before applying the final-value theorem, find the poles of V(s). They are −20,000 and −80,000, so both lie in the left-half complex plane, and we can use the final-value theorem to get

$\lim_{s \to 0} sV(s) = \lim_{s \to 0} \frac{96 \times 104s}{s^2 + 105s + 16 \times 108} = 0$.

As we expected from the circuit, as $t \to \infty$, the final-value theorem gives $v(\infty) = 0$. Thus, V(s) correctly predicts the initial and final values of v(t).

Assessment Problem

Objective 4–Understand and know how to use the initial-value theorem and the final-value theorem

1. 12.14 Use the initial- and final-value theorems to find the initial and final values of f(t) in Assessment Problems 12.4, 12.8, and 12.10.

   Answer:

   7, 0; 4, 1; and 0, 0.

SELF-CHECK: Also try Chapter Problem 12.52.

Practical Perspective

Transient Effects

The circuit introduced in the Practical Perspective at the beginning of the chapter is repeated in Fig. 12.19 with the switch closed and the chosen sinusoidal source.

Figure 12.19 A series RLC circuit with a 60 Hz sinusoidal source.

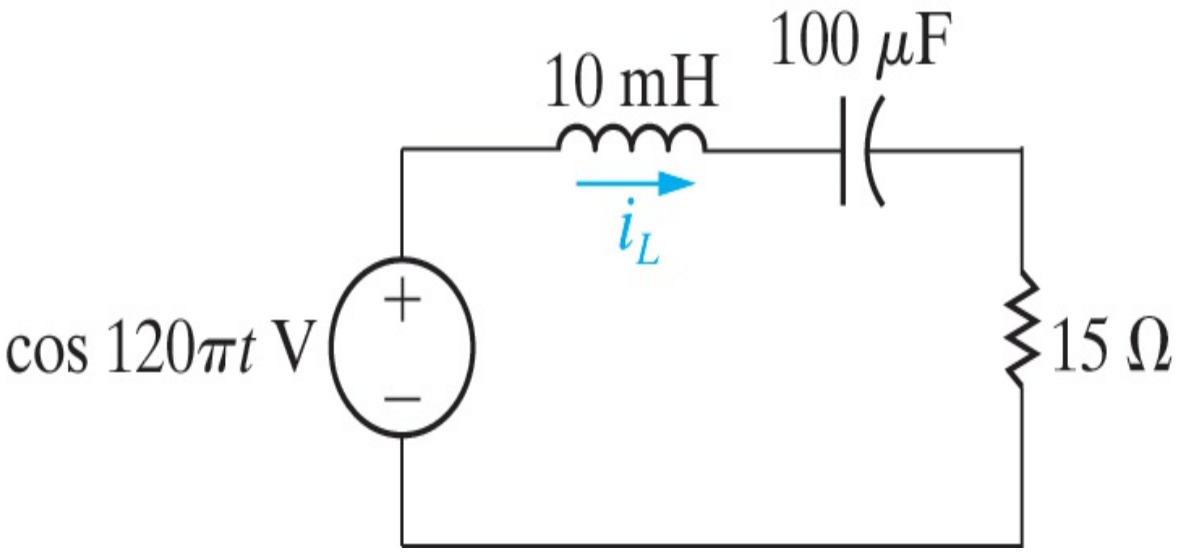


Figure 12.19 Full Alternative Text

We use Laplace methods to determine the complete response of the inductor current, iL(t). To begin, use KVL to sum the voltage drops around the circuit, in the clockwise direction:

$$15 i_L(t) + 0.01 \frac{di_L}{dt} + 1100 \times 10^{-6} \int_0^t i_L(x)dx = \cos 120\pi t$$

Using Tables 12.1 and 12.2, we find the Laplace transform of the KVL equation:

$$15 I_L(s) + 0.01 s I_L(s) + 104 I_L s = \frac{s}{s^2 + (120\pi)^2}$$

Solve this equation for IL(s):

$$I_L(s) = \frac{100s^2}{[s^2 + 1500s + 10^6][s^2 + (120\pi^2)]}.$$

Note that the expression for IL has two complex conjugate pairs of poles, so the partial fraction expansion of IL(s) will have four terms:

$$I_L(s) = \frac{K1}{(s + 750 - j661.44)} + \frac{K1^*}{(s + 750 + j661.44)} + \frac{K2}{(s - j120\pi)} + \frac{K2^*}{(s + j120\pi)}.$$

Determine the values of K1andK2:

$$K1 = \frac{100s^2}{[\, s + 750s + j661.44\,][\, s^2 + (120\pi)^2\,]}\Big|_{s = -750 + j661.44} = 0.07357 \angle -97.89°, K2 = \frac{100s^2}{[s^2 + 1500s + 10^6][s + j120\pi]}\Big|_{s = j120\pi} = 0.01834 \angle 56.61°.$$

Therefore, the s-domain expression for the inductor current is

$$I_L = \frac{0.07357 \angle -97.89°}{(s + 750 - j661.44)} + \frac{0.07357 \angle 97.89°}{(s + 750 + j661.44)} + \frac{0.01834 \angle 56.61°}{(s - j120\pi)} + \frac{0.01834 \angle -56.61°}{(s + j120\pi)}.$$

Finally, we use Table 12.3 to calculate the inverse Laplace transform and find iL:

$$i_L = 147.14 e^{-750t} \cos(661.44t - 97.89°) + 36.69 \cos(120\pi t + 56.61°)\ \text{mA}.$$

The first term in the inductor current is the transient response, which will decay to zero in about 7 ms. The second term in the inductor current is the steady-state response, which has the same frequency as the 60 Hz sinusoidal source and will persist as long as this source is connected in the circuit. Note

that the amplitude of the steady-state response is 36.69 mA, which is less than the 40 mA current rating of the inductor. But the transient response has an initial amplitude of 147.14 mA, far greater than the 40 mA current rating. Calculate the value of the inductor current at t=0:

$$i_L(0) = 147.14(1)\cos(-97.89°) + 36.69\cos(56.61°) = -6.21\mu A.$$

Clearly, the transient part of the response does not cause the inductor current to exceed its rating initially. But we need a plot of the complete response to determine whether or not the current rating is ever exceeded, as shown in Fig. 12.20.

Figure 12.20 Plot of the inductor current for the circuit in Fig. 12.19.

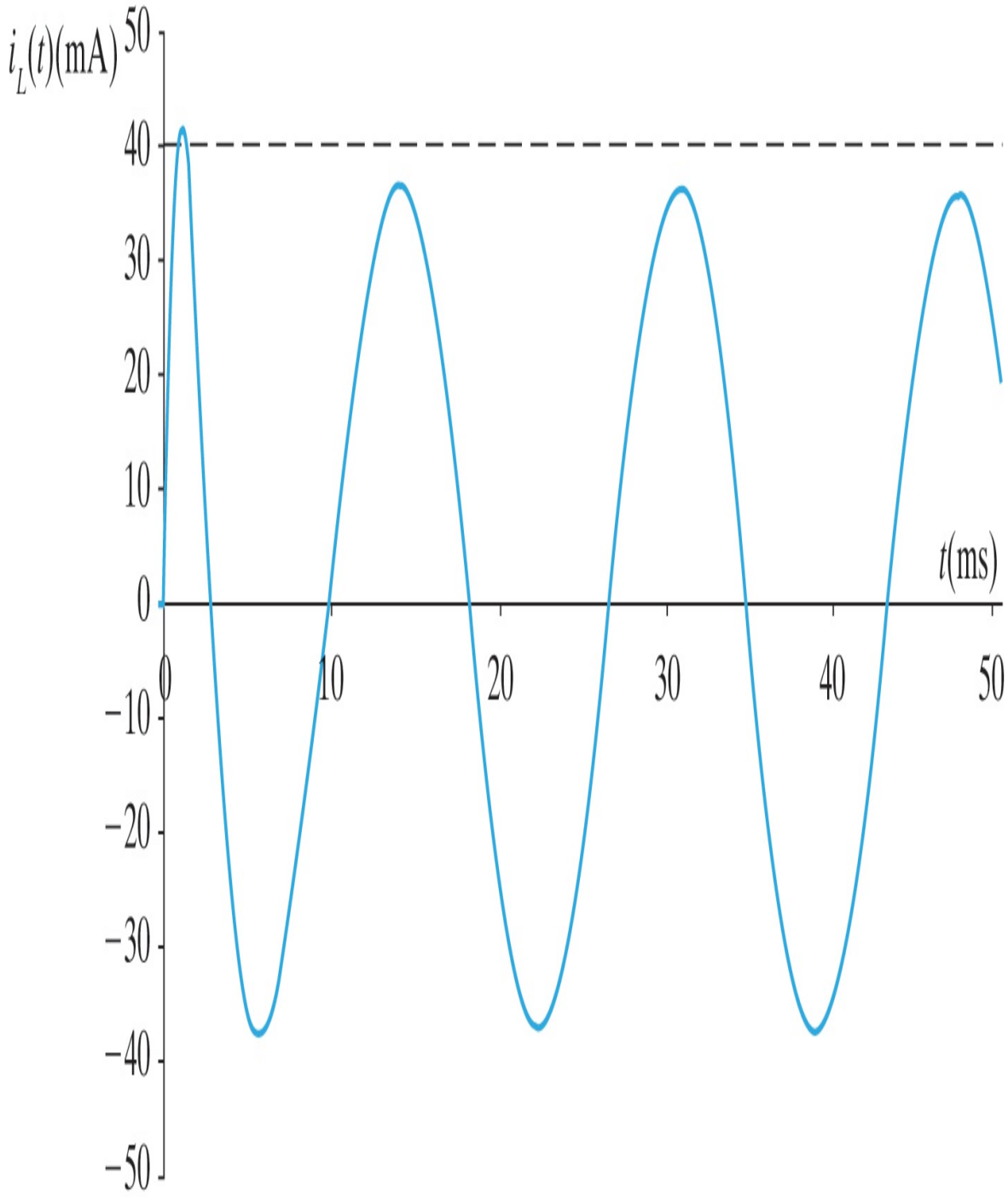


Figure 12.20 Full Alternative Text

The plot suggests we check the value of the inductor current at 1 ms:

iL(0.001)=147.14e−0.75 cos (−59.99°)+36.69 cos (78.21°)=42.4 mA

Thus, the current rating is exceeded in the inductor, at least momentarily. If we determine that we never want to exceed the current rating, we should reduce the magnitude of the sinusoidal source. This example illustrates the importance of considering the complete response of a circuit to a sinusoidal input, even if we are satisfied with the steady-state response.

SELF-CHECK: Access your understanding of the Practical Perspective by trying Chapter Problems 12.57 and 12.58.

Summary

- The **Laplace transform** is a tool for converting time-domain equations into frequency-domain equations, according to the following general definition:

  $$L\{ f(t) \}=\int_0^\infty f(t)e^{-st}\,dt=F(s),$$

  where f(t) is the time-domain expression and F(s) is the frequency-domain expression. (See page 446.)

- The **step function** Ku(t) describes a function that experiences a discontinuity from one constant level to another at some point in time. K is the magnitude of the jump; if K=1, Ku(t) is the **unit step function**. (See page 447.)

- The **impulse function** $K\Delta(t)$ is defined as

  $$\int_{-\infty}^{\infty}K\Delta(t)dt=K, \qquad \Delta(t)=0,\ t\neq 0.$$

  K is the strength of the impulse; if K=1, $K\Delta(t)$ is the **unit impulse function**. (See page 449.)

- A **functional transform** is the Laplace transform of a specific function. Important functional transform pairs are summarized in Table 12.1. (See page 453.)

- **Operational transforms** define the general mathematical properties of the Laplace transform. Important operational transform pairs are summarized in Table 12.2. (See page 458.)

- In linear lumped-parameter circuits, F(s) is a rational function of s. (See page 460.)

- If F(s) is a proper rational function, the inverse transform is found by a partial fraction expansion. (See page 461.)

- If F(s) is an improper rational function, it can be inverse-transformed by first expanding it into a sum of a polynomial and a proper rational function. (See page 469.)

- F(s) can be expressed as the ratio of two factored polynomials. The roots of the denominator are called **poles** and are plotted as Xs on the complex s plane. The roots of the numerator are called **zeros** and are plotted as 0s on the complex s plane. (See page 470.)

- The initial-value theorem states that

  $\lim_{t \to 0+} f(t) = \lim_{s \to \infty} sF(s)$.

  The theorem assumes that f(t) contains no impulse functions. (See page 472.)

- The final-value theorem states that

  $\lim_{t \to \infty} f(t) = \lim_{s \to 0+} sF(s)$.

  The theorem is valid only if the poles of F(s), except for a first-order pole at the origin, lie in the left half of the s plane. (See page 472.)

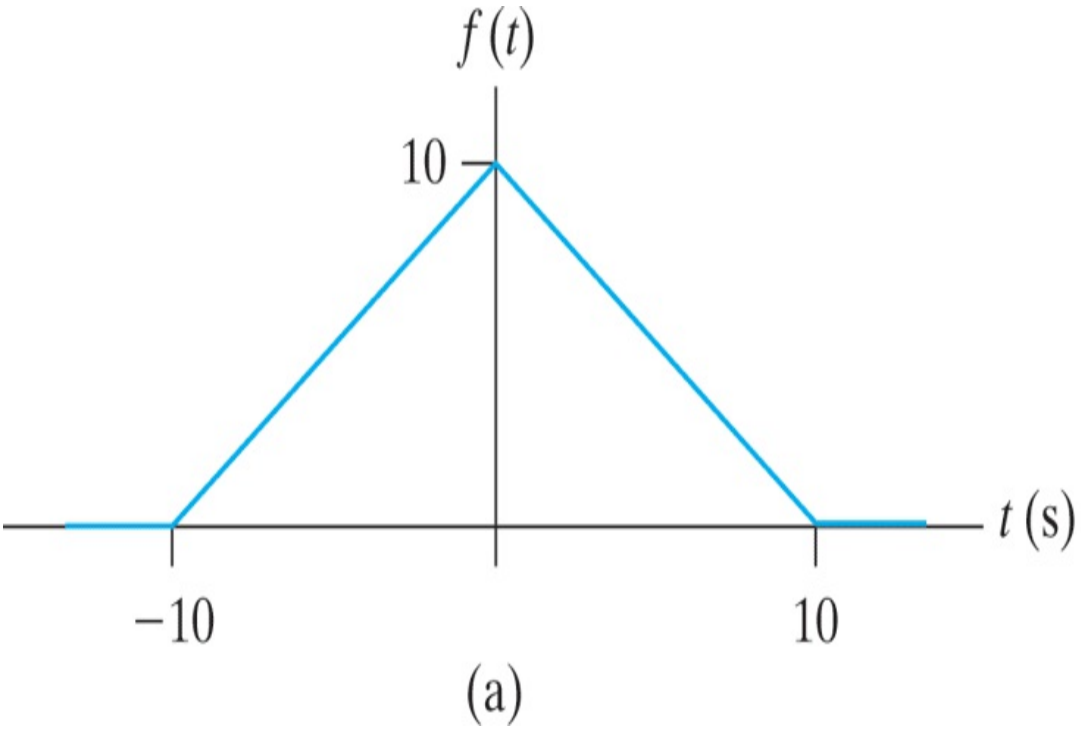
- The initial- and final-value theorems allow us to predict the initial and final values of f(t) from an s-domain expression. (See page 474.)

Problems

Section 12.2

1. 12.1 Use step functions to write the expression for each of the functions shown in Fig. P12.1.

Figure P12.1



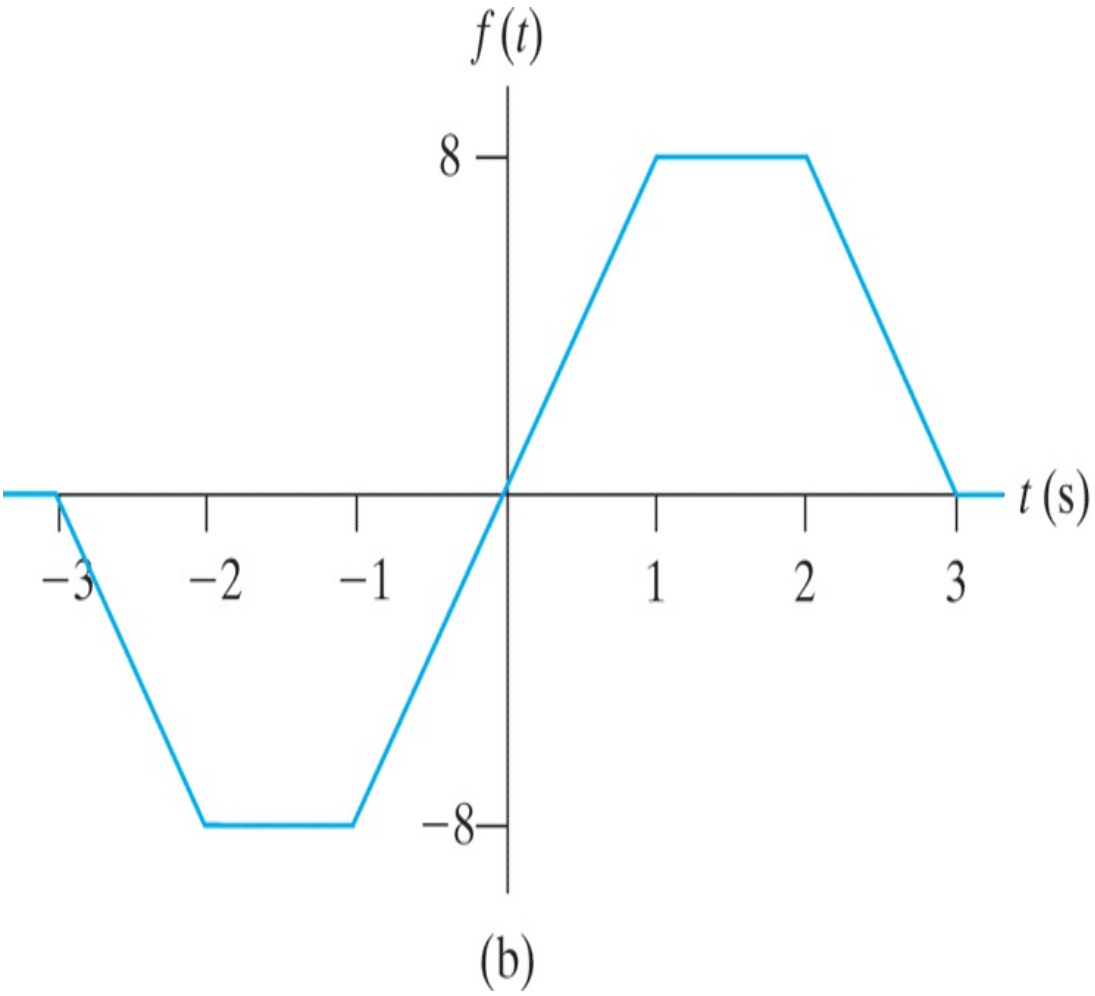
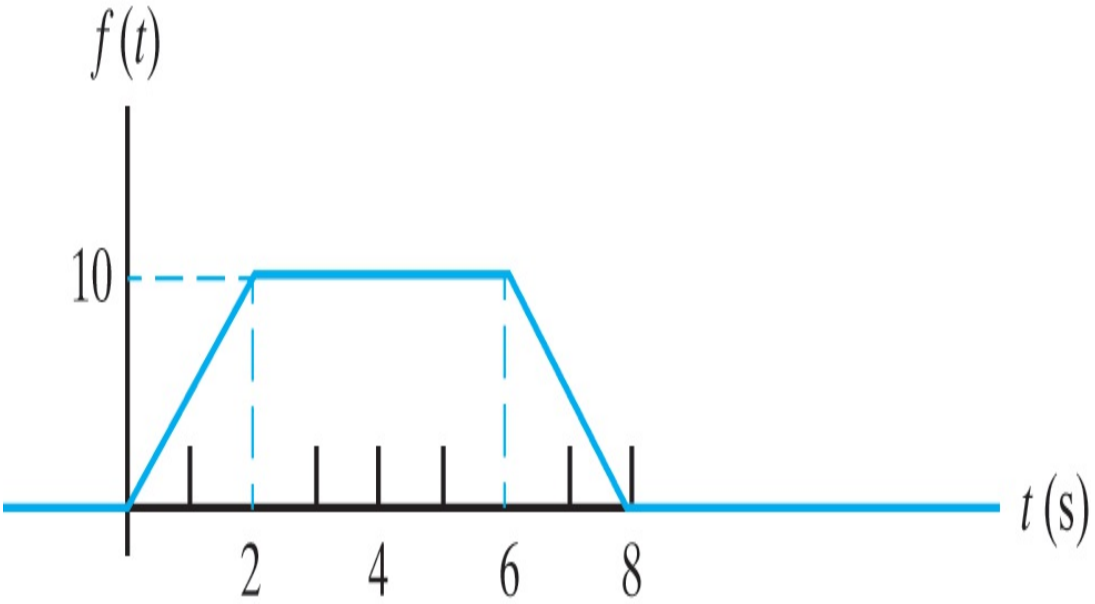


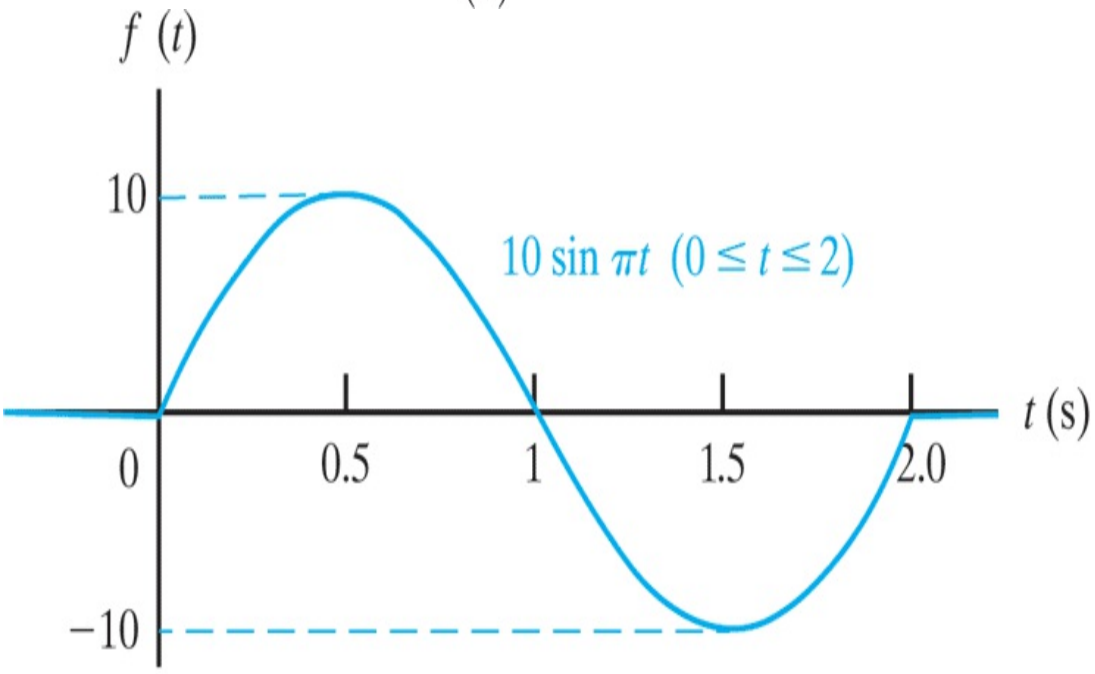
Figure P12.1 Full Alternative Text

2. 12.2 Use step functions to write the expression for each function shown in Fig. P12.2.

Figure P12.2



(a)



(b)

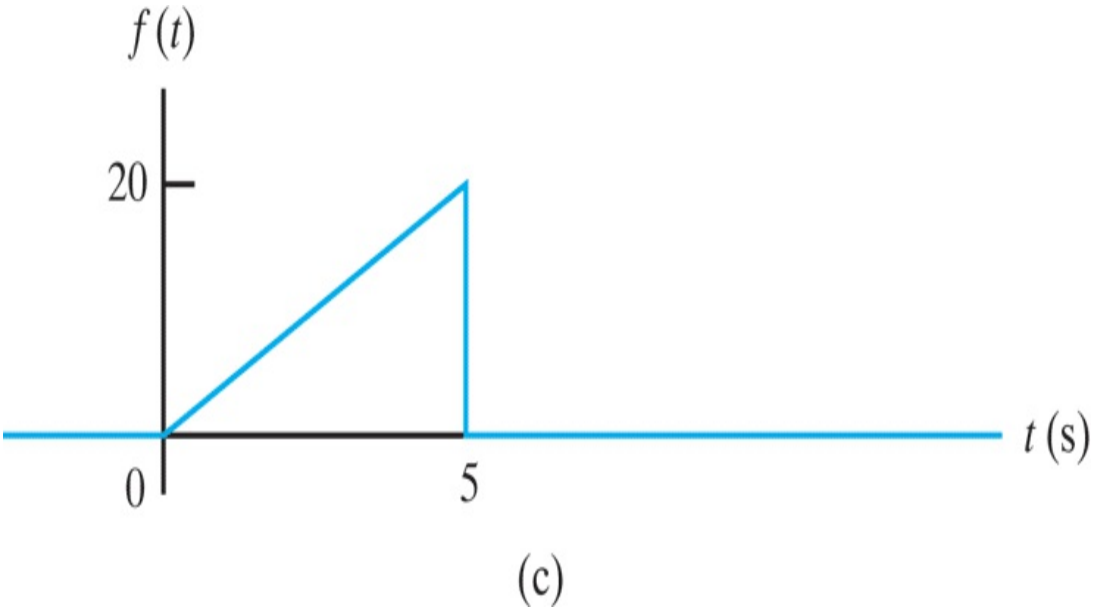


Figure P12.2 Full Alternative Text

3. 12.3 Make a sketch of f(t) for −15 s≤t≤35 s when f(t) is given by the following expression:

   f(t)=(100+10t)u(t+10)−(50+10t)u(t+5)+(50−10t)u(t−5)−(150−10t)u(t−15)+(10t−250)u(t−25)−(10t−300)u(t−3)

4. 12.4 Step functions can be used to define a *window* function. Thusu(t−1)−u(t−4) defines a window 1 unit high and 3 units wide located on the time axis between 1 and 4.

   A function f(t) is defined as follows:

   f(t)=0,t≤0;=−20t,0≤t≤1 s=
   −20,1 s≤t≤2 s=2 cosπ2t,2 s≤t≤4 s;=100−20t,4 s≤t≤5 s=0,5 s≤t<∞.

   1. Sketch f(t) over the interval −1 s≤t≤6 s.

   2. Use the concept of the window function to write an expression for f(t).

Section 12.3

1. 12.5 Explain why the following function generates an impulse function as $\epsilon \to 0$:

   $f(t)=\epsilon/\pi\epsilon 2+t2,-\infty\leq t\leq\infty$.

2. 12.6

   1. Find the area under the function shown in Fig. 12.12(a).

   2. What is the duration of the function when $\epsilon=0$?

   3. What is the magnitude of f(0) when $\epsilon=0$?

3. 12.7 The triangular pulses shown in Fig. P12.7 are equivalent to the rectangular pulses in Fig. 12.12(b), because they both enclose the same area $(1/\epsilon)$ and they both approach infinity proportional to $1/\epsilon 2$ as $\epsilon \to 0$. Use this triangular-pulse representation for $\Delta'(t)$ to find the Laplace transform of $\Delta''(t)$.

Figure P12.7

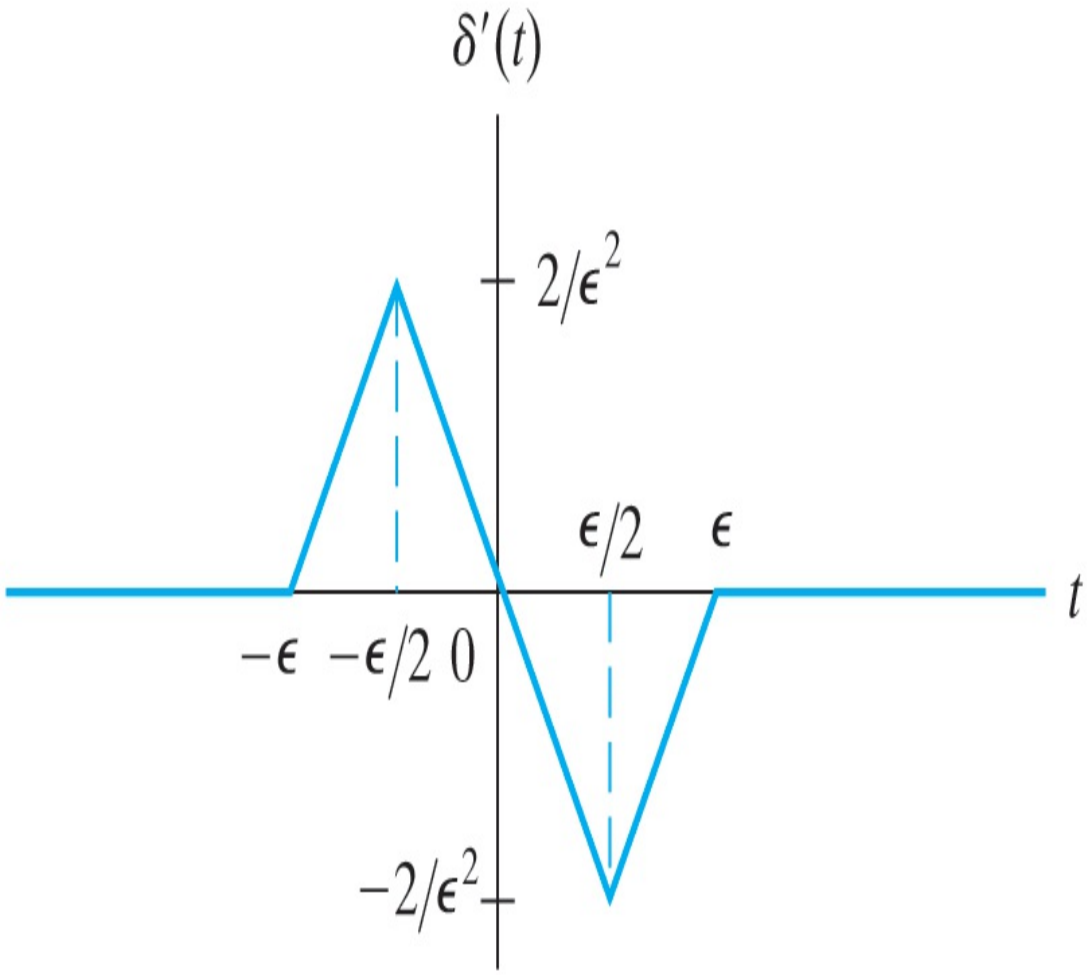


Figure P12.7 Full Alternative Text

4. 12.8 Evaluate the following integrals:
    1. $I=\int -13(t3+2)[\Delta(t)+8\Delta(t-1)]dt.$
    2. $I=\int -22t2[\Delta(t)+\Delta(t+1.5)+\Delta(t-3)]dt.$
5. 12.9 In Section 12.3, we used the sifting property of the impulse function to show that $L\{\Delta(t)\}=1$. Show that we can obtain the same result by finding the Laplace transform of the rectangular pulse that exists between $\pm\epsilon$ in Fig. 12.9 and then finding the limit of this transform as $\epsilon \to 0$.
6. 12.10 Find f(t) if

$f(t)=12\pi\int_{-\infty}^{\infty}F(\omega)e^{j\omega t}\,d\omega,$

and

$F(\omega)=\frac{4+j\omega}{9+j\omega}\pi\Delta(\omega).$

7. 12.11 Show that

   $L\{\Delta^{(n)}(t)\}=s^n.$

8. 12.12

   1. Show that

      $\int_{-\infty}^{\infty}f(t)g'(t-a)\,dt=-f'(a).$

      (Hint: Integrate by parts.)

   2. Use the formula in (a) to show that

      $L\{\Delta'(t)\}=s.$

Sections 12.4–12.5

1. 12.13 Find the Laplace transform of each of the following functions:

   1. $f(t) = 20e^{-500(t-10)}u(t-10).$

   2. $f(t)=(5t+20)[u(t+4)-u(t+2)]-5t[u(t+2)-u(t-2)]+(5t-20)[u(t-2)-u(t-4)].$

2. 12.14

   1. Find the Laplace transform of the function illustrated in Fig. P12.14.

   2. Find the Laplace transform of the first derivative of the function illustrated in Fig. P12.14.

3. Find the Laplace transform of the second derivative of the function illustrated in Fig. P12.14.

Figure P12.14

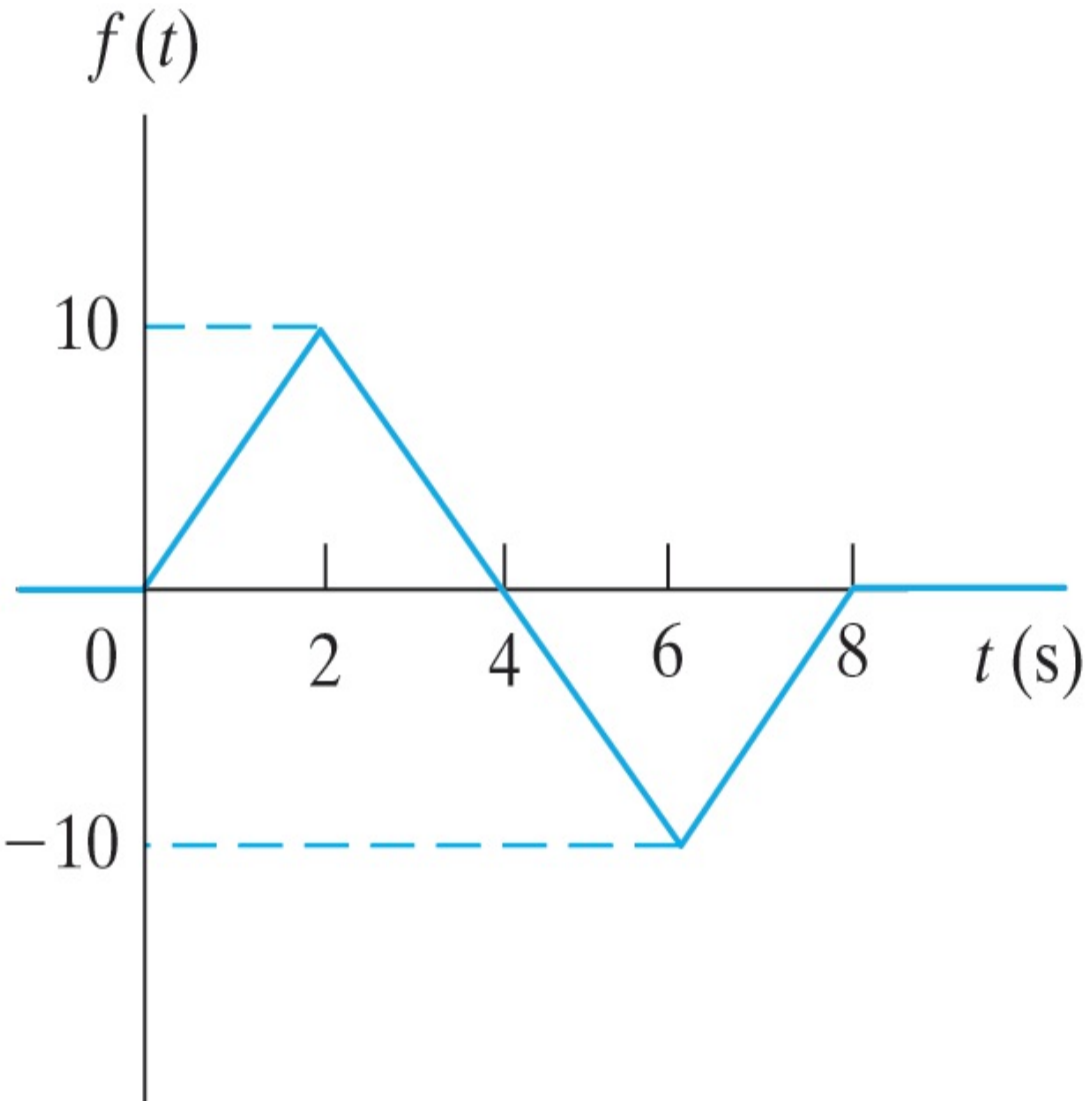


Figure P12.14 Full Alternative Text

3. 12.15 Show that

   L{e−atf(t)}=F(s+a).

4. 12.16 Show that

   $L\{f(at)\}=1aF(sa)$.

5. 12.17

   1. Find the Laplace transform of te^{-at}.

   2. Use the first derivative (time) operational transform given in Table 12.2 to find the Laplace transform of $\frac{d}{dt}(te^{-at})$.

   3. c) Check your result in part (b) by first differentiating and then transforming the resulting expression.

6. 12.18

   1. Find $L\{\int_0^{-} te^{-ax}dx\}$.

   2. Check the results of (a) by first integrating and then transforming.

7. 12.19

   1. Find the Laplace transform of

      $\int_0^{-} tx\ dx$

      by first integrating and then transforming.

   2. Check the result obtained in (a) by using the time integral operational transform given in Table 12.2.

8. 12.20 Find the Laplace transform of each of the following functions:

   1. $f(t)=te^{-at}$;

   2. $f(t)=\sin \omega t$;

   3. $f(t)=\sin (\omega t+\theta)$;

4. f(t)=t;
5. f(t)=cosh(t+θ).

(Hint: See Assessment Problem 12.1.)

9. 12.21
    1. Given that F(s)=L{f(t)}, show that

       −d F(s)ds=L{t f(t)}.
    2. Show that

       (−1)ndnF(s)dsn=L{tnf(t)}.
    3. Use the result of (b) to find L{t5}, L{tsinβt}, and L{te−1cosht}.
10. 12.22
    1. Find L{ddtsinωt}.
    2. Find L{ddtcosωt}.
    3. Find L{d3dt3t2u(t)}.
    4. Check the results of parts (a), (b), and (c) by first differentiating and then transforming.
11. 12.23 Find the Laplace transform (when $\epsilon \to 0$) of the derivative of the exponential function illustrated in Fig. 12.8, using each of the following two methods:
    1. First differentiate the function and then find the transform of the resulting function.
    2. Use the first derivative (time) operational transform given in Table 12.2.

12. 12.24 Find the Laplace transform for (a) and (b).

    1. $f(t)=\frac{d}{dt}(e^{-at}\cos\omega t)$.

    2. $f(t)=\int_0^{-t} e^{-ax}\sin\omega x\, dx$.

    3. Verify the results obtained in (a) and (b) by first carrying out the indicated mathematical operation and then finding the Laplace transform.

13. 12.25

    1. Show that if $F(s)=L\{f(t)\}$, and $\{f(t)/t\}$ is Laplace-transformable, then

       $\int_s^{\infty} F(u)du=L\{f(t)t\}$.

       (Hint: Use the defining integral to write

       $\int_s^{\infty} F(u)du=\int_s^{\infty}(\int_0^{-\infty} f(t)e^{-ut}dt)\, du$

       and then reverse the order of integration.)

    2. Starting with the Laplace transform of $t\sin\beta t$, from Problem 12.21(c), use the operational transform given in (a) of this problem to find $L\{\sin\beta t\}$.

Section 12.6

1. 12.26 In the circuit shown in Fig. 12.16, the dc current source is replaced with a sinusoidal source that delivers a current of 1.2cost A. The circuit components are R=1 Ω, C=625 mF, and L=1.6 H. Find the numerical expression for V(s).

2. 12.27 There is no energy stored in the circuit shown in Fig. P12.27 at the time the switch is opened.

    1. In Section 12.6, we derived the integrodifferential equation that governs the behavior of the voltage vo.

    2. We also showed that the Laplace transform of vo is

       Vo(s)=Idc/Cs2+(1/RC)s+(1/LC).

    3. Use Vo(s) to show that the Laplace transform of io is

       Io(s)=sIdcs2+(1/RC)s+(1/LC).

Figure P12.27

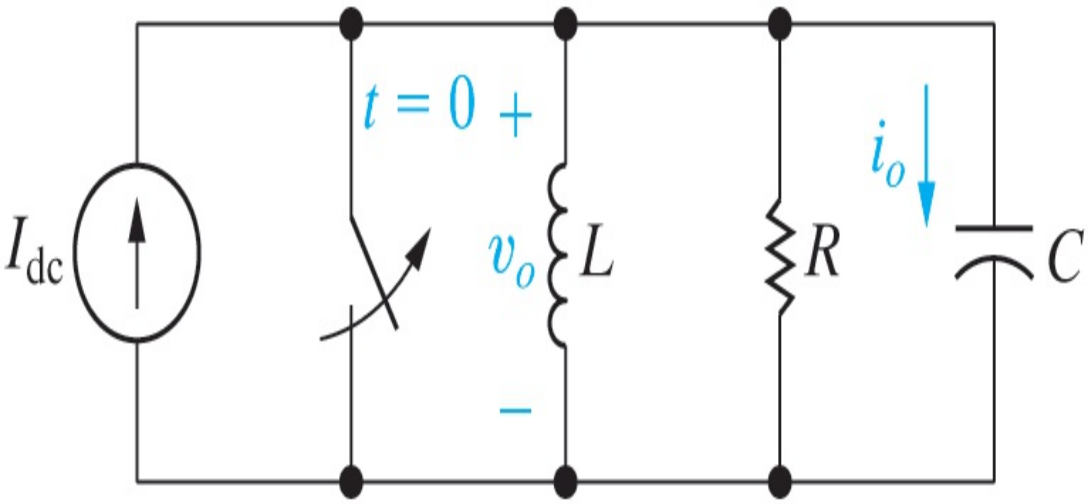


Figure P12.27 Full Alternative Text

3. 12.28 The switch in the circuit in Fig. P12.28 has been in position a for a long time. At t=0, the switch moves instantaneously to position b.

    1. Derive the integrodifferential equation that governs the behavior of the current io for t≥0+.

    2. Show that

       Io(s)=Idc[s+(1/RC)][s2+(1/RC)s+(1/LC)].

Figure P12.28

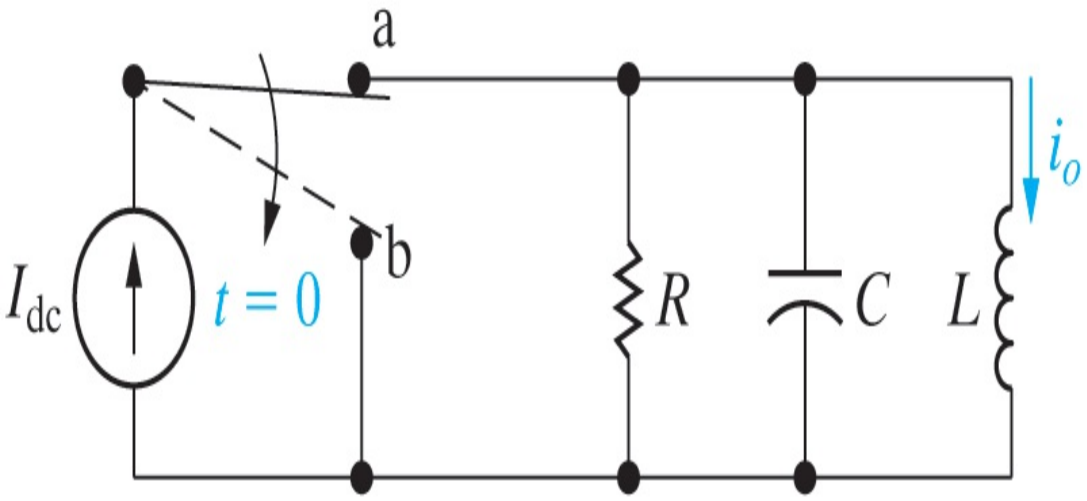


Figure P12.28 Full Alternative Text

4. 12.29 The switch in the circuit in Fig. P12.29 has been in position a for a long time. At t=0, the switch moves instantaneously to position b.

    1. Derive the integrodifferential equation that governs the behavior of the voltage vo for t≥0+.

    2. Show that

Vo(s)=Vdc[s+(R/L)][s2+(R/L)s+(1/LC)].

Figure P12.29

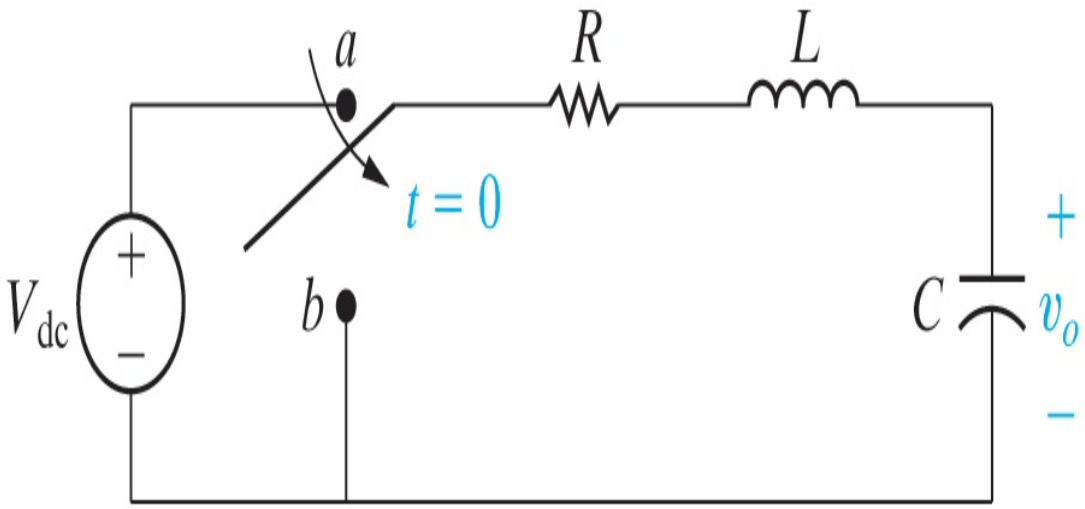


Figure P12.29 Full Alternative Text

5. 12.30 The switch in the circuit in Fig. P12.30 has been open for a long time. At t=0, the switch closes.

    1. Derive the integrodifferential equation that governs the behavior of the voltage vo for t≥0.

    2. Show that

       Vo(s)=Vdc/RCs2+(1/RC)s+(1/LC).

    3. Show that

       Io(s)=Vdc/RLCs[s2+(1/RC)s+(1/LC)].

Figure P12.30

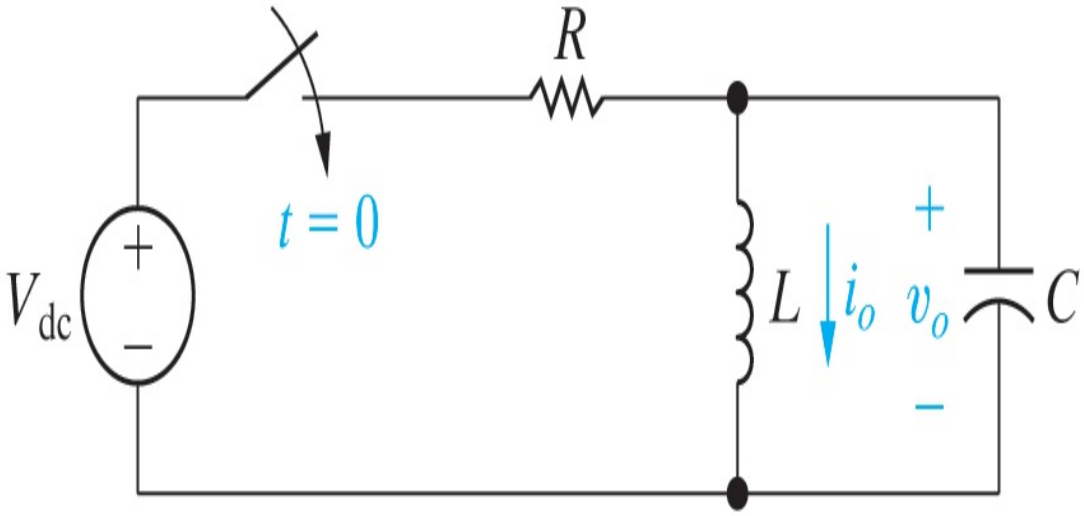


Figure P12.30 Full Alternative Text

6. 12.31 There is no energy stored in the circuit shown in Fig. P12.31 at the time the switch is opened.

    1. Derive the integrodifferential equations that govern the behavior of the node voltages v1 and v2.

    2. Show that

       V2(s)=sIg(s)C[s2+(R/L)s+(1/LC)].

Figure P12.31

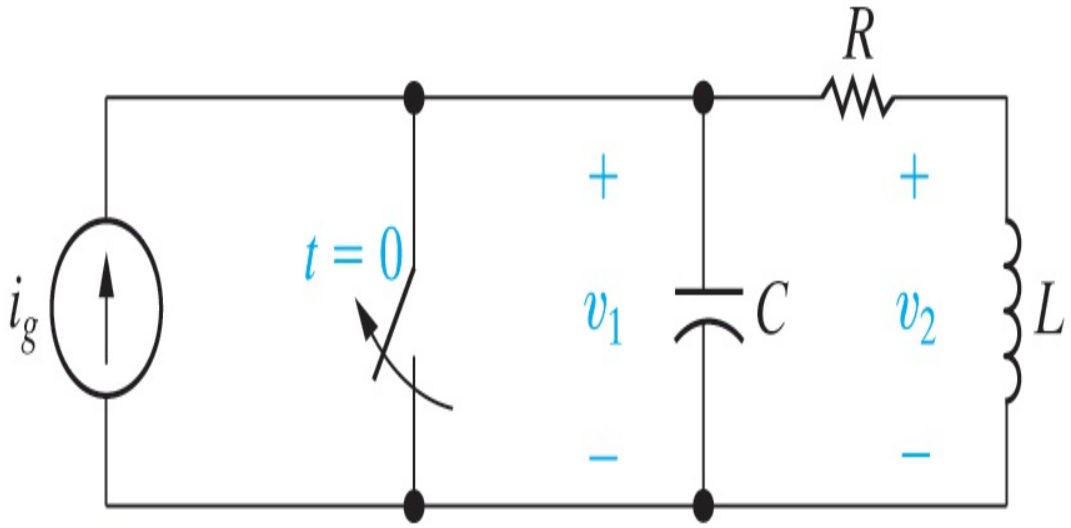


Figure P12.31 Full Alternative Text

7. 12.32 PSPICEMULTISIM

    1. Write the two simultaneous differential equations that describe the circuit shown in Fig. P12.32 in terms of the mesh currents i1 and i2.

    2. Laplace-transform the equations derived in (a). Assume that the initial energy stored in the circuit is zero.

    3. Solve the equations in (b) for I1(s) and I2(s).

Figure P12.32

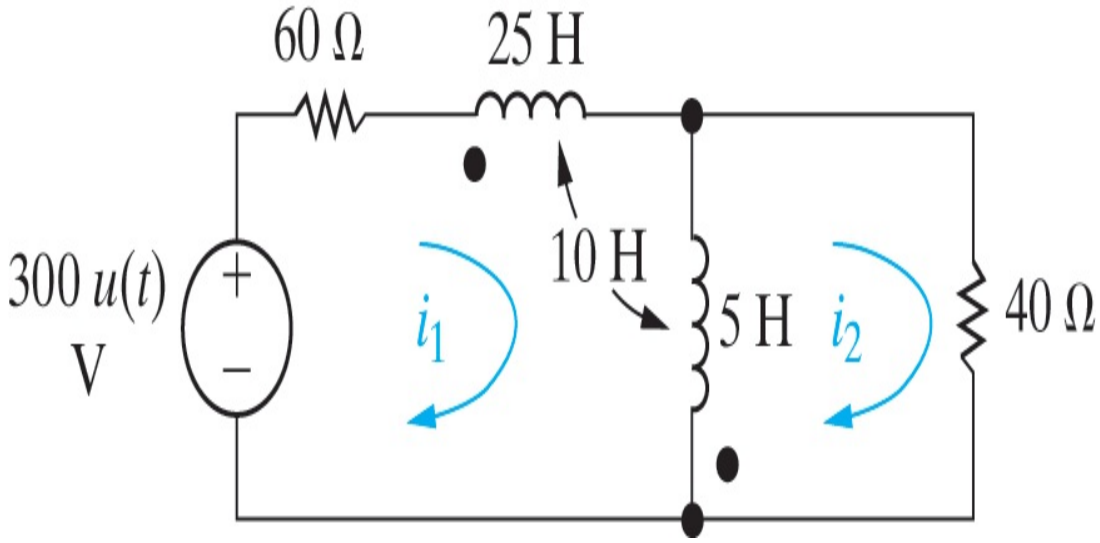


Figure P12.32 Full Alternative Text

Section 12.7

1. 12.33 PSPICEMULTISIM Find v(t) in Problem 12.26.

2. 12.34 PSPICEMULTISIM The parameter values for the circuit in Fig. P12.27 are as follows: R=1 kΩ, L=12.5 H, C=2 μF, and Idc=30 mA.

    1. Find vo(t) for t≥0.

    2. Find io(t) for t≥0.

    3. Does your solution for io(t) make sense when t=0? Explain.

3. 12.35 PSPICEMULTISIM The circuit parameters in the circuit in Fig. P12.28 are R=500 Ω, L=250 mH, and C=250 nF. If Idc=5 mA, find io(t) for t≥0.

4. 12.36 The circuit parameters in the circuit in Fig. P12.29 are R=4 kΩ, L=400 mH, and C=156.25 nF. If Vdc=120 V, find vo(t) for t≥0.

5. 12.37 PSPICEMULTISIM The circuit parameters in the circuit in Fig. P12.30 are R=5 kΩ; L=200 mH; and C=100 nF. If Vdc is 35 V, find

    1. vo(t) for t≥0

    2. io(t) for t≥0

6. 12.38 PSPICEMULTISIM The circuit parameters in the circuit in Fig. P12.31 are R=1600 Ω; L=200 mH; and C=200 nF. If ig(t)=6 mA, find v2(t).

7. 12.39 Use the results from Problem 12.32 and the circuit shown in Fig P12.32 to

    1. Find i1(t) and i2(t).

2. Find i1(∞) and i2(∞).
3. Do the solutions for i1 and i2 make sense? Explain.

8. 12.40 Find f(t) for each of the following functions:
    1. F(s)=8s2+37s+32(s+1)(s+2)(s+4).
    2. F(s)=8s3+89s2+311s+300s(s+2)(s2+8s+15).
    3. F(s)=22s2+60s+58(s+1)(s2+4s+5).
    4. F(s)=250(s+7)(s+14)s(s2+14s+50).
9. 12.41 Find f(t) for each of the following functions.
    1. F(s)=280s2+14s+245.
    2. F(s)=−s2+52s+445s(s2+10s+89).
    3. F(s)=14s2+56s+152(s+6)(s2+4s+20).
    4. F(s)=8(s+1)2(s2+10s+34)(s2+8s+20).
10. 12.42 Find f(t) for each of the following functions.
    1. F(s)=320s2(s+8).
    2. F(s)=80(s+3)s(s+2)2.
    3. F(s)=60(s+5)(s+1)2(s2+6s+25).
    4. F(s)=25(s+4)2s2(s+5)2.
11. 12.43 Find f(t) for each of the following functions.
    1. F(s)=135s(s+3)3.
    2. F(s)=10(s+2)2(s2+2s+2)2.

3. F(s)=25s2+395s+1494s2+15s+54
4. F(s)=5s3+20s2−49s−108s2+7s+10

12. 12.44 Derive the transform pair given by Eq. 12.10.
13. 12.45
    1. Derive the transform pair given by Eq. 12.11.
    2. Derive the transform pair given by Eq. 12.12.

Sections 12.8–12.9

1. 12.46 Find the poles and zeros for the s-domain functions in Problems 12.42(b) and 12.42(c).
2. 12.47 Find the poles and zeros for the s-domain functions in Problems 12.43(a) and 12.43(b).
3. 12.48
    1. Use the initial-value theorem to find the initial value of v in Problem 12.26.
    2. Can the final-value theorem be used to find the steady-state value of v? Why or why not?
4. 12.49 Use the initial- and final-value theorems to check the initial and final values of the current and voltage in Problem 12.27.
5. 12.50 Use the initial- and final-value theorems to check the initial and final values of the current in Problem 12.28.
6. 12.51 Use the initial- and final-value theorems to check the initial and final values of the current and voltage in Problem 12.30.

7. 12.52 Apply the initial- and final-value theorems to each transform pair in Problem 12.40.

8. 12.53 Apply the initial- and final-value theorems to each transform pair in Problem 12.41.

9. 12.54 Apply the initial- and final-value theorems to each transform pair in Problem 12.42.

10. 12.55 Apply the initial- and final-value theorems to each transform pair in Problem 12.43.

Sections 12.1–12.9

1. 12.56 PRACTICALPERSPECTIVE

    1. Use phasor circuit analysis techniques from Chapter 9 to determine the steady-state expression for the inductor current in Fig. 12.19.

    2. How does your result in part (a) compare to the complete response for inductor current calculated in the Practical Perspective?

2. 12.57 PRACTICALPERSPECTIVE Find the maximum magnitude of the sinusoidal source in Fig. 12.19 such that the complete response of the inductor current does not exceed the 40 mA current rating at t=1 ms.

3. 12.58 PRACTICALPERSPECTIVE Suppose the input to the circuit in Fig 12.19 is a damped ramp of the form Kte^{-100t} V. Find the largest value of K such that the inductor current does not exceed the 40 mA current rating.

Chapter 13 The Laplace Transform in Circuit Analysis

Chapter Contents

1. 13.1 Circuit Elements in the s Domain
2. 13.2 Circuit Analysis in the s Domain
3. 13.3 Applications
4. 13.4 The Transfer Function
5. 13.5 The Transfer Function in Partial Fraction Expansions
6. 13.6 The Transfer Function and the Convolution Integral
7. 13.7 The Transfer Function and the Steady-State Sinusoidal Response
8. 13.8 The Impulse Function in Circuit Analysis

Chapter Objectives

1. Be able to transform a circuit into the s domain using Laplace transforms; be sure you understand how to represent the initial conditions on energy-storage elements in the s domain.

2. Know how to analyze a circuit in the s domain and be able to transform an s-domain solution back to the time domain.

3. Understand the definition and significance of the transfer function and be able to calculate the transfer function for a circuit using s-domain techniques.

4. Know how to use a circuit's transfer function to calculate the circuit's unit impulse response, its unit step response, and its steady-state response to a sinusoidal input.

The **Laplace transform** has two characteristics that make it an attractive tool for circuit analysis.

- It transforms a set of linear constant-coefficient differential equations into a set of linear polynomial equations, which are easier to manipulate and solve.

- It automatically introduces into the polynomial equations the initial values of the current and voltage variables. Thus, initial conditions are an inherent part of the transform process. (This contrasts with the classical approach to the solution of differential equations, in which initial conditions are considered when the unknown coefficients are evaluated.)

We begin this chapter by showing how to avoid writing time-domain integrodifferential equations and transforming them into the s domain. In Section 13.1, we'll develop the s-domain circuit models for resistors, inductors, and capacitors. Then we can transform entire circuits into the s domain and write the s-domain equations directly. Section 13.2 reviews

Ohm's and Kirchhoff's laws for s-domain circuits. After establishing these fundamentals, we apply the Laplace transform method to a variety of circuit problems in Section 13.3, using the circuit analysis and simplification tools first introduced for resistive circuits.

After solving for the circuit response in the s domain, we inverse-transform back to the time domain, using partial fraction expansion (as demonstrated in Chapter 12). As before, checking the final time-domain equations in terms of the initial conditions and final values is an important step in the solution process.

We introduce a new concept, the transfer function, in Section 13.4. The transfer function for a circuit is the ratio of the Laplace transform of its output to the Laplace transform of its input. In Chapters 14 and 15, we'll use the transfer function in circuit design, but here we focus on using it for circuit analysis. We continue this chapter with a look at the role of partial fraction expansion (Section 13.5) and the convolution integral (Section 13.6) when using the transfer function in circuit analysis. We conclude with a discussion of the impulse function in circuit analysis.

Practical Perspective

Surge Suppressors

When using personal computers and other sensitive electronic equipment, we need to provide protection from voltage surges. These surges can occur during a lightning storm or just by switching an electrical device on or off. A commercially available surge suppressor is shown in the accompanying figure.

How can flipping a switch to turn on a light or turn off a hair dryer cause a voltage surge? At the end of this chapter, we will answer that question using Laplace transform techniques to analyze a circuit. We will illustrate how a voltage surge can be created by switching off a resistive load in a circuit operating in the sinusoidal steady state.

Virote Chuenwiset/Shutterstock.

Jhaz Photography/Shutterstock

13.1 Circuit Elements in the s Domain

A three-step procedure transforms each circuit element into an s-domain equivalent circuit.

1. Write the time-domain equation relating the terminal voltage to the terminal current.

2. Laplace transform the time-domain equation to generate an algebraic relationship between the s-domain current and voltage.

3. Construct a circuit model that satisfies the relationship between the s-domain current and voltage.

Note the s-domain dimensions: the s-domain voltage dimension is volt-seconds [V-s], the s-domain current dimension is ampere-seconds [A-s], and thus the s-domain voltage-to-current ratio dimension is volts per ampere, or ohms. An impedance in the s domain is measured in ohms, and an admittance is measured in siemens. We use the passive sign convention in all the derivations.

A Resistor in the s Domain

We begin with the resistor. From Ohm’s law,

$v=Ri.$

Because R is a constant, the Laplace transform of Ohm’s law is

$V=RI,$ (13.1)

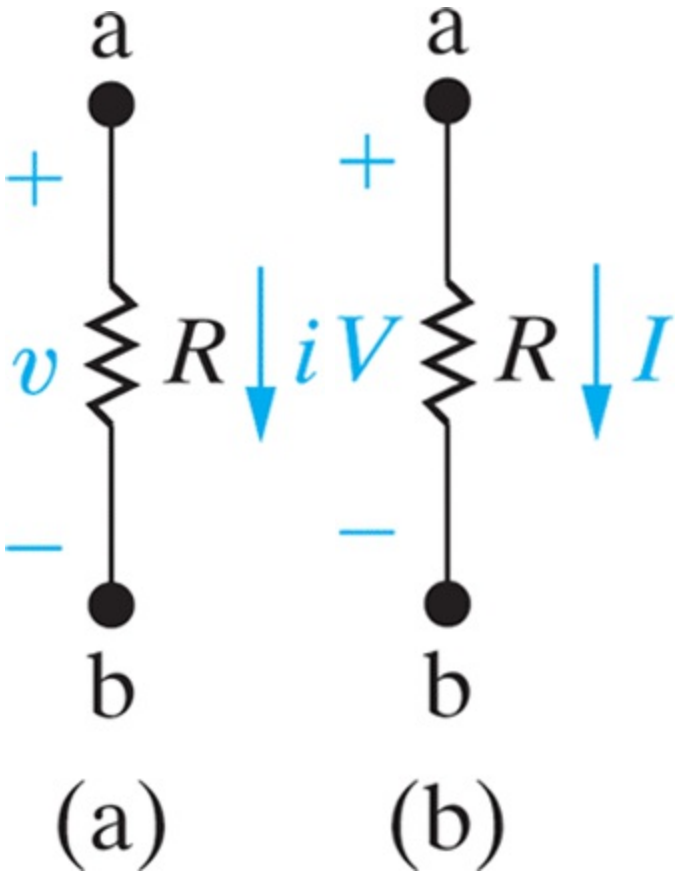
where

$V=L\{v\}$ and $I=L\{i\}.$

From Eq. 13.1 we see that the s-domain equivalent circuit of a resistor is a resistance of R ohms that carries a current of I ampere-seconds and has a terminal voltage of V volt-seconds.

Figure 13.1 shows the time- and frequency-domain circuits of the resistor. Note that going from the time domain to the frequency domain does not change the resistance element.

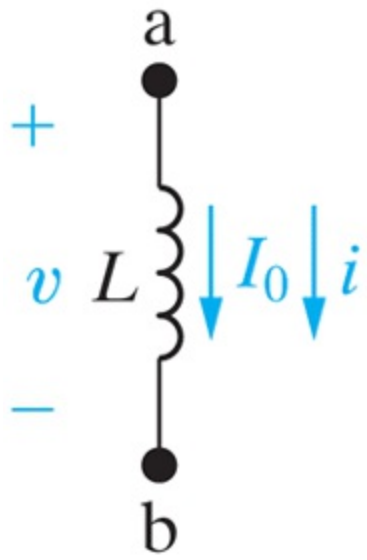
Figure 13.1 The resistor. (a) Time domain. (b) Frequency domain.



An Inductor in the s Domain

Figure 13.2 shows an inductor carrying an initial current of I0 amperes. The time-domain equation relating the terminal voltage to the terminal current is

Figure 13.2 An inductor of L henrys carrying an initial current of I0 amperes.



v=Ldidt.

The Laplace transform of the inductor equation gives

V=L[sI−i(0−)]=sLI−LI0. (13.2)

One circuit configuration that satisfies Eq. 13.2 is an impedance of sL ohms in series with an independent voltage source of LI0 volt-seconds, as shown in Fig. 13.3. Note that the polarity marks on the voltage source LI0 agree with the minus sign in Eq. 13.2. Note also that LI0 carries its own algebraic sign; that is, if the initial value of i is opposite to the reference direction for i, then I0 has a negative value.

Figure 13.3 The series equivalent circuit for an inductor of L henrys carrying an initial current of I0 amperes.

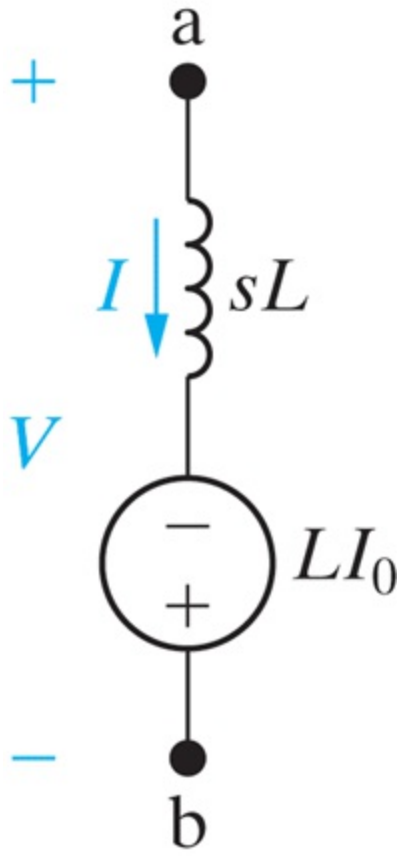


Figure 13.3 Full Alternative Text

We can also solve Eq. 13.2 for the current I and then construct the circuit to satisfy the resulting equation. The current I is given by

I=V+LI0sL=VsL+I0s.

The s-domain equivalent circuit that satisfies this equation is an impedance of sL ohms in parallel with an independent current source of I0/s ampere-seconds,

There are two other ways to construct the s domain circuit in Fig. 13.4: (1) find the Norton equivalent of the circuit shown in Fig. 13.3 or (2) start with the inductor current as a function of the inductor voltage in the time domain and then Laplace transform the resulting integral equation. We leave these two approaches to Problems 13.1 and 13.2.

Figure 13.4 The parallel equivalent circuit for an inductor of L henrys carrying an initial current of I0 amperes.

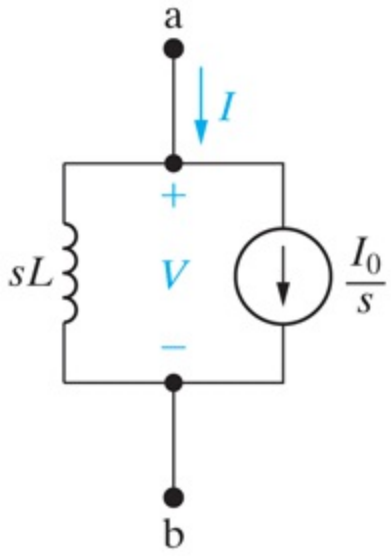
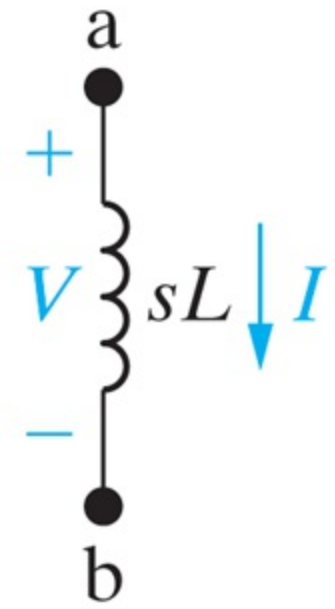


Figure 13.4 Full Alternative Text

If the initial energy stored in the inductor is zero, that is, if I0=0, the s-domain equivalent circuit is an inductor with an impedance of sL ohms. Figure 13.5 shows this circuit.

Figure 13.5 The s-domain circuit for an inductor when the initial current is zero.



A Capacitor in the s Domain

A capacitor with initial stored energy also has two s-domain equivalent circuits. Figure 13.6 shows a capacitor initially charged to V0 volts. The capacitor current is

Figure 13.6 A capacitor of C farads initially charged to V0

volts.

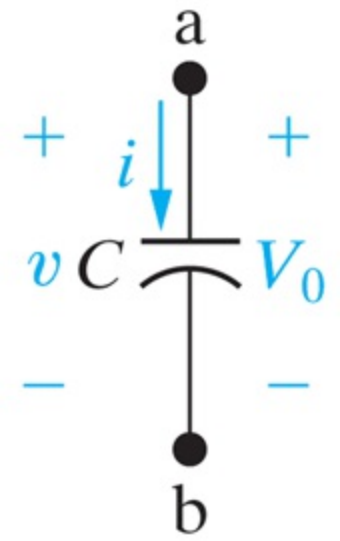


Figure 13.6 Full Alternative Text
i=Cdvdt.

Transforming the capacitor equation yields

I=C[sV−v(0−)]=sCV−CV0, (13.3)

so the s-domain current I is the sum of two branch currents. One branch contains an admittance of sC siemens, and the second branch contains an independent current source of CV0 ampere-seconds. Figure 13.7 shows this parallel equivalent circuit.

Figure 13.7 The parallel equivalent circuit for a capacitor initially charged to V0 volts.

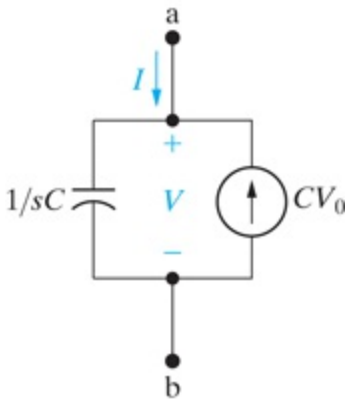


Figure 13.7 Full Alternative Text

To derive the other equivalent circuit for the charged capacitor, solve Eq. 13.3 for V:

V=(1sC)I+V0s.

Figure 13.8 shows the circuit that satisfies the equation for capacitor voltage, which is a series combination of an impedance and an independent voltage source.

Figure 13.8 The series equivalent circuit for a capacitor initially charged to V0 volts.

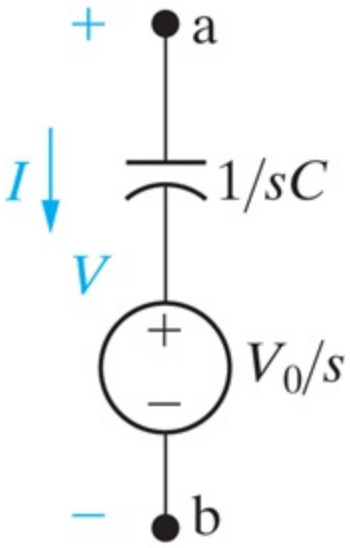
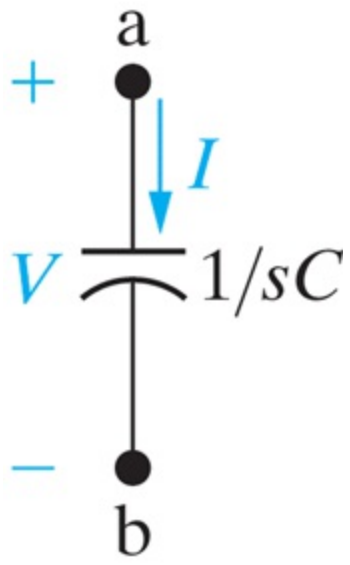


Figure 13.8 Full Alternative Text

In the equivalent circuits shown in Figs. 13.7 and 13.8, V0 carries its own algebraic sign. In other words, if the polarity of V0 is opposite to the reference polarity for v, V0 is a negative quantity. If the initial voltage on the capacitor is zero, both equivalent circuits reduce to an impedance of 1/sC ohms, as shown in Fig. 13.9.

Figure 13.9 The s-domain circuit for a capacitor when the initial voltage is zero.



In Chapter 9 we used the phasor transform to turn a time-domain circuit into a frequency-domain circuit. In this chapter, we use the s-domain equivalent circuits, summarized in Table 13.1, to transform a time-domain circuit into the s domain. When the time-domain circuit contains inductors and capacitors with initial stored energy, you need to decide whether to use the parallel or series s-domain equivalent circuit. With a little forethought and some experience, the best choice is often evident.

Table 13.1 Summary of the s-Domain Equivalent Circuits

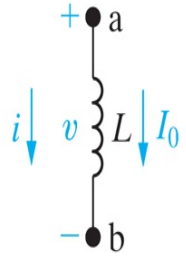
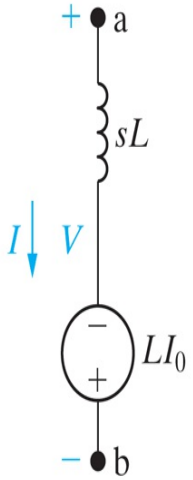
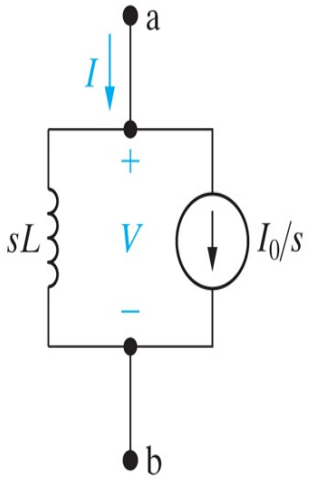
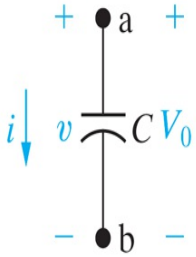
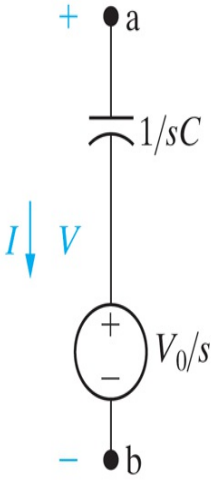
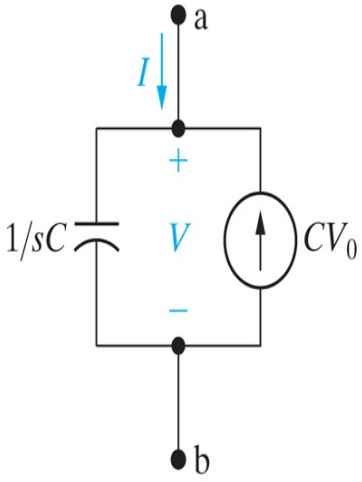
TIME DOMAIN	FREQUENCY DOMAIN
$v = Ri$	$V = RI$
 $v = L\,di/dt,$ $i = \frac{1}{L}\int_{0^-}^{t} v\,dx + I_0$	 $V = sLI - LI_0$  $I = \frac{V}{sL} + \frac{I_0}{s}$
 $i = C\,dv/dt,$ $v = \frac{1}{C}\int_{0^-}^{t} i\,dx + V_0$	 $V = \frac{I}{sC} + \frac{V_0}{s}$  $I = sCV - CV_0$

Table 13.1 Full Alternative Text

13.2 Circuit Analysis in the s Domain

Before presenting a method for using the s-domain equivalent circuits in analysis, we make some important observations.

- If no energy is stored in the inductor or capacitor, the relationship between the s-domain voltage and current for each passive element is:

Ohm’s law in the s-domain

$$V=ZI, \quad (13.4)$$

where Z is the s-domain impedance of the element. Thus, a resistor has an impedance of R ohms, an inductor has an impedance of sL ohms, and a capacitor has an impedance of 1/sC ohms. The relationship contained in Eq. 13.4 also appears in Figs. 13.1(b), 13.5, and 13.9. Equation 13.4 is known as Ohm’s law in the s domain. The reciprocal of the impedance is admittance. Therefore, the s-domain admittance of a resistor is 1/R siemens, an inductor has an admittance of 1/sL siemens, and a capacitor has an admittance of sC siemens.

- The rules for combining impedances and admittances in the s domain are the same as those for frequency-domain circuits. Thus, series-parallel simplifications and Δ-to-Y conversions also are applicable to s-domain analysis.

- Kirchhoff’s laws apply to s-domain currents and voltages because an operational Laplace transform states that the Laplace transform of a sum of time-domain functions equals the sum of the Laplace transforms of the individual functions (see Table 12.2). The algebraic sum of the currents at a node is zero in the time domain, so the algebraic sum of the Laplace-transformed currents is also zero. A similar statement holds for

the algebraic sum of the Laplace-transformed voltages around a closed path. The s-domain version of Kirchhoff's laws is

at every node in a circuit, alg $\sum I=0$, (13.5)
around every closed path in a circuit, alg $\sum V=0$. (13.6)

Therefore, because Ohm's law, KVL, and KCL hold in the s domain, all of the circuit analysis techniques developed in Chapters 2–4 for resistive circuits can be used to analyze circuits in the s-domain. These techniques include combining impedances in series and parallel to find equivalent impedances, voltage division and current division, the node-voltage method and the mesh-current method, source transformation, and Thévenin and Norton equivalent circuits. This leads us to the following step-by-step procedure for using Laplace transform techniques to analyze circuits.

1. Step 1 determines the initial current in each inductor and the initial voltage across each capacitor by analyzing the time-domain circuit for $t<0$.

2. Step 2 transforms each independent voltage or current source defined by time-domain functions into the s domain using the functional and operational transforms in Tables 12.1 and 12.2.

3. Step 3 transforms voltages and currents represented by time-domain symbols such as v(t) and i(t) into corresponding s-domain symbols such as V and I.

4. Step 4 transforms any remaining components in the time-domain circuit into the s domain using the circuits in Table 13.1. When inductors and capacitors have nonzero initial values, calculated in Step 1, these initial conditions are represented by independent sources in series or parallel with the component impedances.

5. Step 5 analyzes the resulting s-domain circuit using the techniques developed for resistive circuits in Chapters 2–4. The analysis produces s-domain voltages and currents that should each be represented as a ratio of two polynomials in s.

6. Step 6 applies the initial- and final-value theorems to the s-domain functions from Step 5 to check the values of the corresponding time-domain functions at t=0 and t=∞. Note that it might not be possible to apply one or both of these theorems, depending on the form of the s-domain function.

7. Step 7 represents each s-domain voltage and current of interest as a partial fraction expansion and then uses Table 12.3 to inverse-Laplace-transform the s-domain voltages and currents back to the time domain.

This analysis method yields the complete response to any circuit whose voltage and current sources have Laplace transforms. This method represents the most comprehensive circuit analysis technique presented in this text and is summarized in Analysis Method 13.1. Example 13.1 applies the first five steps in Analysis Method 13.1 to a time-domain circuit.

Laplace Transform Method

1. **Determine the initial conditions** for inductors and capacitors.

2. **Laplace-transform independent voltage and current functions** using Tables 12.1 and 12.2.

3. **Transform symbolic time-domain voltages and currents** into s-domain symbols.

4. **Transform remaining circuit components** into the s domain using Table 13.1.

5. **Analyze the s-domain circuit** using resistive circuit analysis techniques; represent the resulting s-domain voltages and currents as ratios of polynomials in s.

6. **Use the initial- and final-value theorems** to check the s-domain voltages and currents.

7. **Inverse-Laplace-transform the s-domain voltages and currents** using

partial fraction expansion and Table 12.3.

Analysis Method 13.1 Laplace-transform circuit analysis method.

Example 13.1 Transforming a Circuit into the s Domain

A 500 Ω resistor, a 16 mH inductor, and a 25 nF capacitor are connected in parallel.

1. Express the admittance of this parallel combination of elements as a rational function of s.

2. Compute the numerical values of the zeros and poles.

Solution

1. We use Analysis Method 13.1 to transform the three parallel-connected components from the time domain to the s domain.

    1. Step 1: There are no initial conditions for the inductor and capacitor.

    2. Step 2: There are no voltage or current sources.

    3. Step 3: There are no symbolic voltages or currents.

    4. Step 4: Because there are no initial conditions, each component is represented by its s-domain impedance:

        $Z_R = 500\ \Omega; Z_L = 0.016s\ \Omega; Z_C = 125 \times 10^{-9}s = 40 \times 10^{6}s\ \Omega.$

        The s-domain circuit is shown in Fig. 13.10.

Figure 13.10 The s domain circuit for Example 13.1.

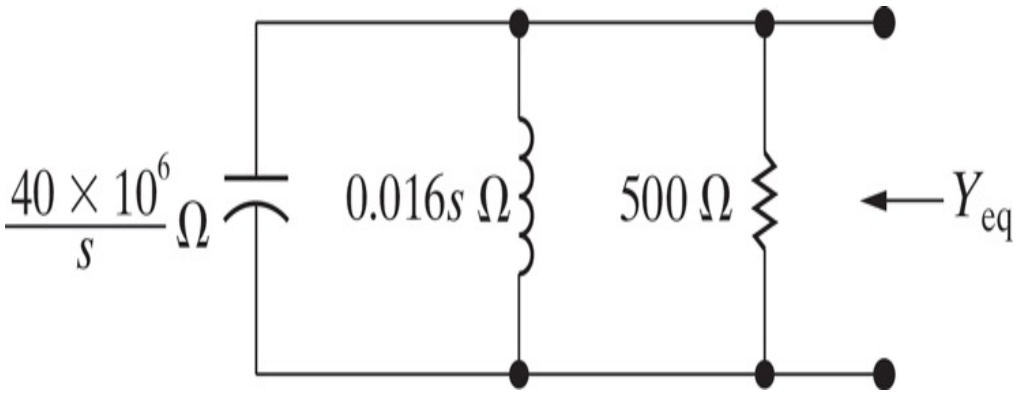


Figure 13.10 Full Alternative Text

5. Step 5: Find the equivalent admittance by adding the inverse of the three impedances:

   Yeq=1500+10.016s+s40×106=s2+80,000s+25×10840×106sS.

2. The numerator factors are (s+40,000+j30,000) and (s+40,000−j30,000). Therefore the zeros are −40,000+j30,000 and −40,000−j30,000. There is a pole at 0.

Assessment Problem

Objective 1—Be able to transform a circuit into the s domain using Laplace transforms

1. 13.1 The parallel circuit in Example 13.1 is placed in series with a 2000 Ω resistor.

   1. a) Express the impedance of this series combination as a rational function of s.

2. b) Compute the numerical values of the zeros and poles.

Answer:

1. (a) 2000(s + 50,000)2/(s2 + 80,000s + 25 × 108);
2. (b) −z1=−z2=−50,000;−p1=−40,000−j30,000,−p2= −40,000 + j30,000.

SELF-CHECK: Also try Chapter Problems 13.4 and 13.5.

13.3 Applications

We now use Analysis Method 13.1 to find the complete response of several example circuits. We start with two familiar circuits from Chapters 7 and 8 to show that the Laplace transform approach yields the same results found using the first- and second-order circuit analysis techniques. Example 13.2 solves an RC circuit, and Example 13.3 solves an RLC circuit.

Example 13.2 The Natural Response of an RC Circuit

The circuit in Fig. 13.11 was analyzed in Example 7.3 using first-order circuit analysis techniques. Use the Laplace transform method to find vo(t) for t≥0+.

Figure 13.11 The RC circuit for Example 13.2.

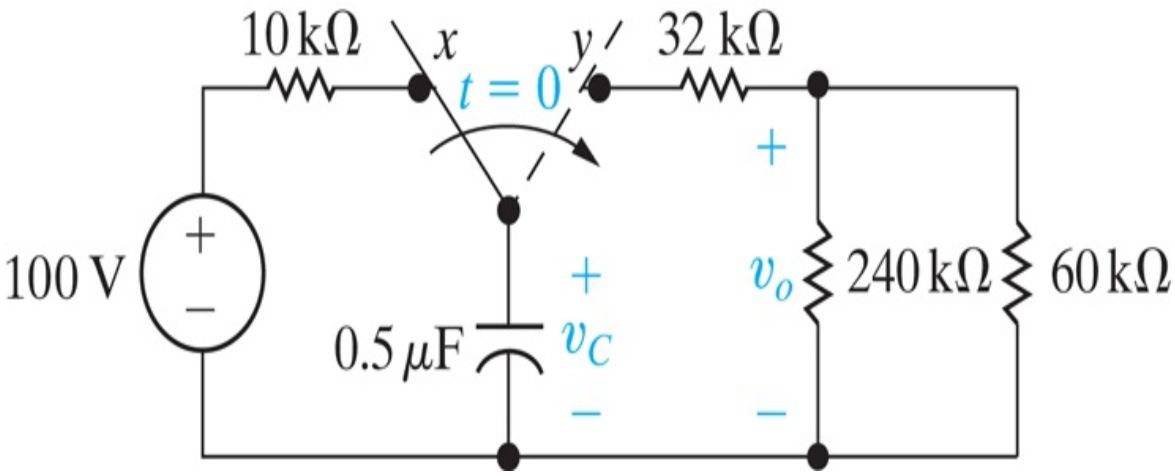


Figure 13.11 Full Alternative Text

Solution

Apply the Laplace transform method using Analysis Method 13.1.

1. Step 1: Determine the initial voltage across the capacitor by analyzing the circuit in Fig. 13.11 for t<0. Because the switch has been in position x for a long time, the capacitor behaves like an open circuit. The voltage across the open circuit V0=100 V.

2. Step 2: For t≥0, there are no voltage or current sources in the circuit, so we can skip this step.

3. Step 3: The voltage across the 240 kΩ resistor is represented in the s domain as V, as shown in Fig. 13.12.

4. Step 4: The impedance of the three resistors is their resistance. The impedance of the capacitor is

   $$Z_C = \frac{1}{sC} = \frac{1}{0.5\times10^{-6}s} = 2\times10^{6}s\ \Omega.$$

   Because the capacitor has an initial condition, we must decide whether to represent it using a series-connected voltage source or a parallel-connected current source, as shown in Table 13.1. Here we use the series-connected voltage source, whose value is V0/s=100/s V-s. The s-domain circuit that results from Steps 1–4 is shown in Fig. 13.12.

Figure 13.12 The circuit in Fig. 13.11 for t≥0 transformed into the s domain.

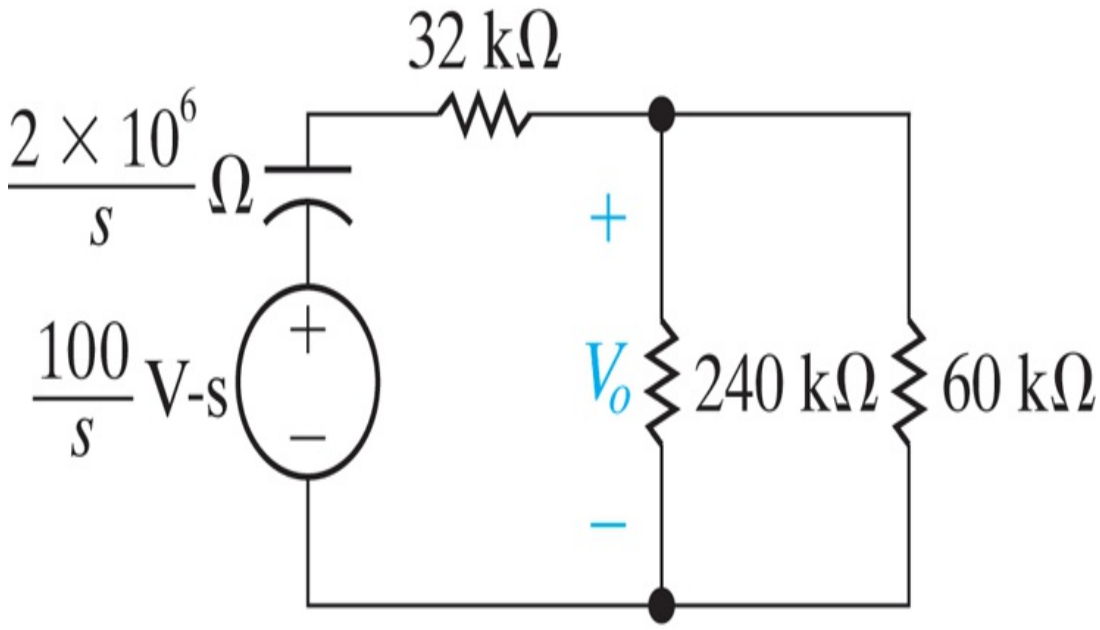


Figure 13.12 Full Alternative Text

5. Step 5: Begin by combining the parallel-connected 240 kΩ and 60 kΩ resistors into a single equivalent 48 kΩ resistor whose voltage is Vo. Now use voltage division to find Vo:

   Vo=48,000(2×106/s)+32,000+48,000(100s)=60s+25.

6. Step 6: Use the initial- and final-value theorems to show that the initial value of vo is 60 V and final value of vo is zero, as we expect from the circuit in Fig. 13.11.

   lims→∞sVo=lims→∞60ss+25=lim(1/s)→0601+(25/s)=601+0=60=limt→0vo(t);lims→0sVo=lims→060ss+25=60(0)0+2

7. Step 7: Since Vo is already a partial fraction, we can use the transforms in Table 12.3 to find vo:

   vo(t)=L−1{60s+25}=60e−25tu(t)V.

   This matches the voltage found in Example 7.3 using first-order circuit analysis methods.

Example 13.3 The Step Response of an RLC Circuit

Consider the circuit in Fig. 13.13, where the initial current in the inductor is 29 mA and the initial voltage across the capacitor is 50 V. This circuit was analyzed in Example 8.10 using second-order circuit analysis techniques. Use the Laplace transform method to find v(t) for t≥0.

Figure 13.13 The parallel RLC circuit for Example 13.3.

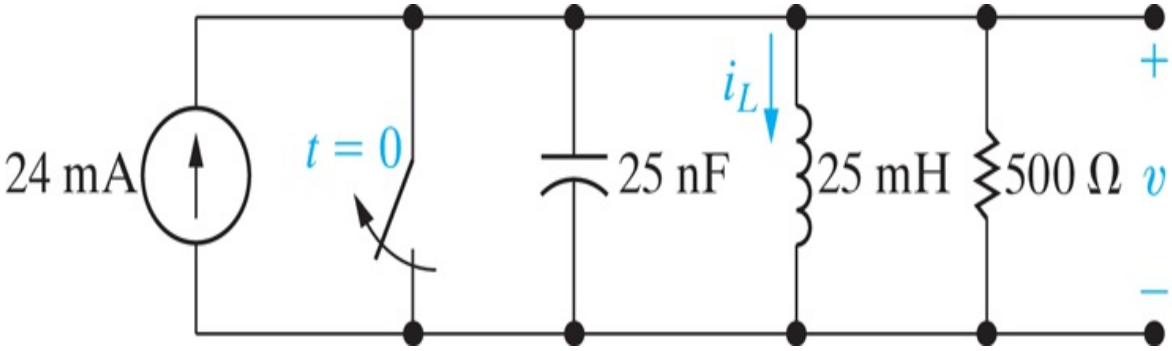


Figure 13.13 Full Alternative Text

Solution

Apply the Laplace transform method using Analysis Method 13.1.

1. Step 1: Both initial conditions are given in the problem statement, so we can skip this step.

2. Step 2: We can describe the parallel combination of the current source and the switch in Fig. 13.13 as 24u(t) mA. The Laplace transform of this function is 24/s mA-s, which is the current source value in the s-domain

circuit in Fig. 13.14.

3. Step 3: The voltage across the 240 kΩ resistor is represented in the s domain as V, as shown in Fig. 13.12.

4. Step 4: The impedance of the resistor is its resistance. The impedance of the inductor is

   $$Z_L = sL = 0.025s = s40\Omega$$

   and the impedance of the capacitor is

   $$Z_C = \frac{1}{sC} = 125\times10^{-9}s = 40\times10^{6}s\Omega.$$

   Both the inductor and capacitor have nonzero initial conditions. We must decide whether to represent them as series-connected voltage sources or as parallel-connected current sources, as shown in Table 13.1. Here we use parallel-connected current sources because the resulting circuit has only parallel-connected components. The value of the current source in parallel with the inductor is $I_0/s = 0.029/s$ A-s, with the current arrow directed down. The value of the current source in parallel with the capacitor is $CV_0 = (50)(25\times10^{-9}) = 1.25$ μ V-s with the current arrow directed up. The s-domain circuit that results from Steps 1–4 is shown in Fig. 13.14.

Figure 13.14 The circuit in Fig. 13.13 for t≥0, transformed into the s domain.

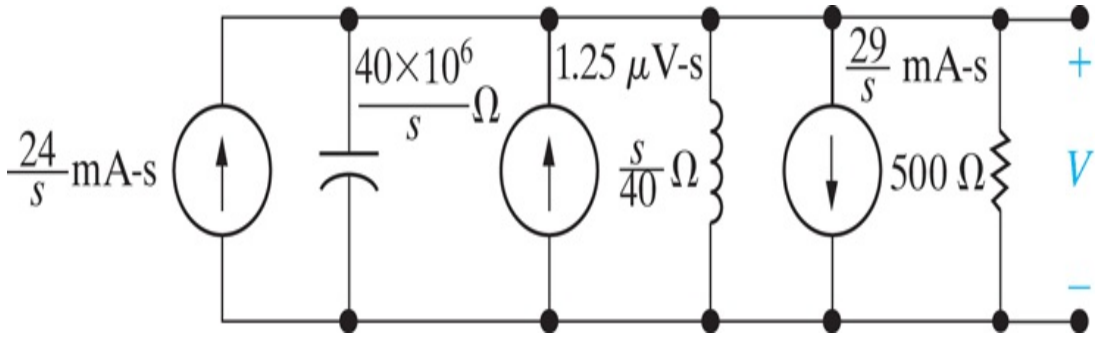


Figure 13.14 Full Alternative Text

5. Step 5: Begin by combining the three parallel-connected impedances into a single equivalent impedance:

   $Zeq=(s40\times106+40s+1500)-1=40\times106ss2+80,000s+16\times108\Omega.$

   To find the voltage across this equivalent impedance, multiply by the sum of the three parallel-connected currents:

   $V=Zeq(0.024s+1.25\times10-6-0.029s)=50s-20\times104s2+80,000s+16\times108V\text{-}s.$

6. Step 6: Use the initial and final value theorems to predict the initial and final values of v:

   $lims\rightarrow\infty sV=lims\rightarrow\infty50s2-20\times104ss2+80,000s+16\times108=lim(1/s)\rightarrow050\text{-}(0)16\times108=0=limt\rightarrow\infty v(t).$

   The initial-value theorem predicts the correct initial voltage from the problem statement, V0=50 V. To confirm the final value of the voltage, envision the circuit in Fig. 13.13 as $t\rightarrow\infty$. The inductor is behaving like a short circuit, and the capacitor is behaving like an open circuit in parallel with the short circuit. Therefore, the final value of capacitor voltage in the circuit is zero, as predicted by the final-value theorem.

7. Step 7: The partial fraction expansion of V is

   $V=50s-20\times104(s+40,000)2=-2.2\times106(s+40,000)2+50(s+40,000).$

Now use the transforms in Table 12.3 to find v:

$v(t)=L^{-1}\{-2.2\times106(s+40{,}000)2+50(s+40{,}000)\}=(-2.2\times106te^{-40{,}000t}+50e^{-40{,}000t})u(t)V.$

This matches the voltage found in Example 8.10 using second-order circuit analysis methods.

Assessment Problems

Objective 2—Know how to analyze a circuit in the s domain and be able to transform an s domain solution to the time domain

1. 13.2 The switch in the circuit shown has been in position a for a long time. At t=0, the switch is thrown to position b.

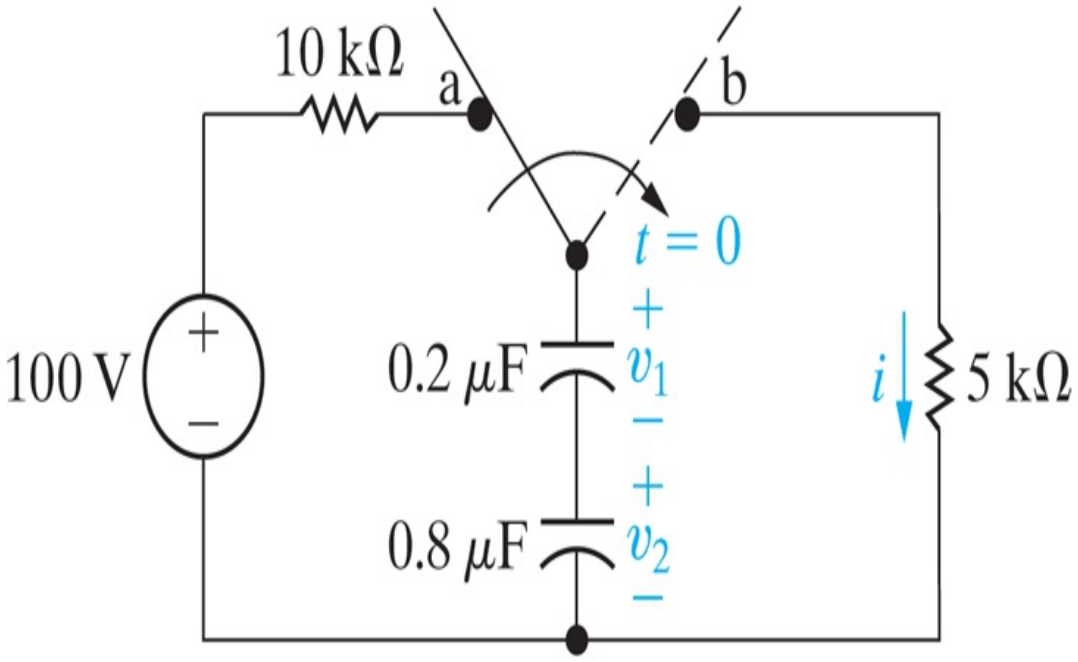
    1. a) Find I, V1, and V2 as rational functions of s.

    2. b) Find the time-domain expressions for i,v1, and v2.

    Answer:

    1. (a) $I=0.02/(s+1250), V1=80/(s+1250), V2=20/(s+1250)$;

    2. (b) $i=20e^{-1250t}u(t)$ mA, $v1=80e^{-1250t}u(t)$ V, $v2=20e^{-1250t}u(t)$ V.

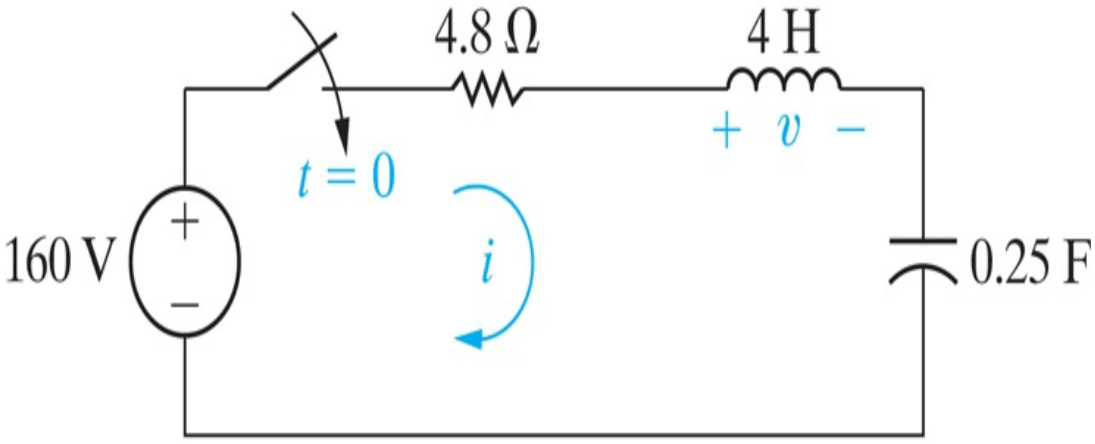


13.3-5 Full Alternative Text

2. 13.3 The energy stored in the circuit shown is zero at the time when the switch is closed.
    1. Find the s-domain expression for I.
    2. Find the time-domain expression for i when t>0.
    3. Find the s-domain expression for V.
    4. Find the time-domain expression for v when t>0.

    Answer:

    1. (a) I=40/(s2+1.2s+1);
    2. (b) i=(50e−0.6t sin0.8t)u(t)A;
    3. (c) V=160s/(s2+1.2s+1);
    4. (d) v=[200e−0.6t cos (0.8t + 36.87°)]u(t)V.



13.3-6 Full Alternative Text

SELF-CHECK: Also try Chapter Problems 13.9, 13.15, and 13.16.

Next, we analyze a circuit with a sinusoidal source using Laplace methods. Phasor methods, presented in Chapter 9, were used to analyze circuits with sinusoidal sources but have an important limitation—they only produce the steady-state response of the circuit, not the complete response. Recall that the complete response consists of both the steady-state and the transient response. Laplace methods do not have this limitation, so they produce a circuit's complete response to a sinusoidal source, as seen in Example 13.4.

Example 13.4 Analyzing a Circuit with a Sinusoidal Source

The circuit in Fig. 13.15 has no initial stored energy. At t=0, the switch closes and the circuit is driven by a sinusoidal source v(t)=15 cos 40,000t V. Use Laplace methods to find i(t) for t≥0.

Figure 13.15 The circuit for Example 13.4.

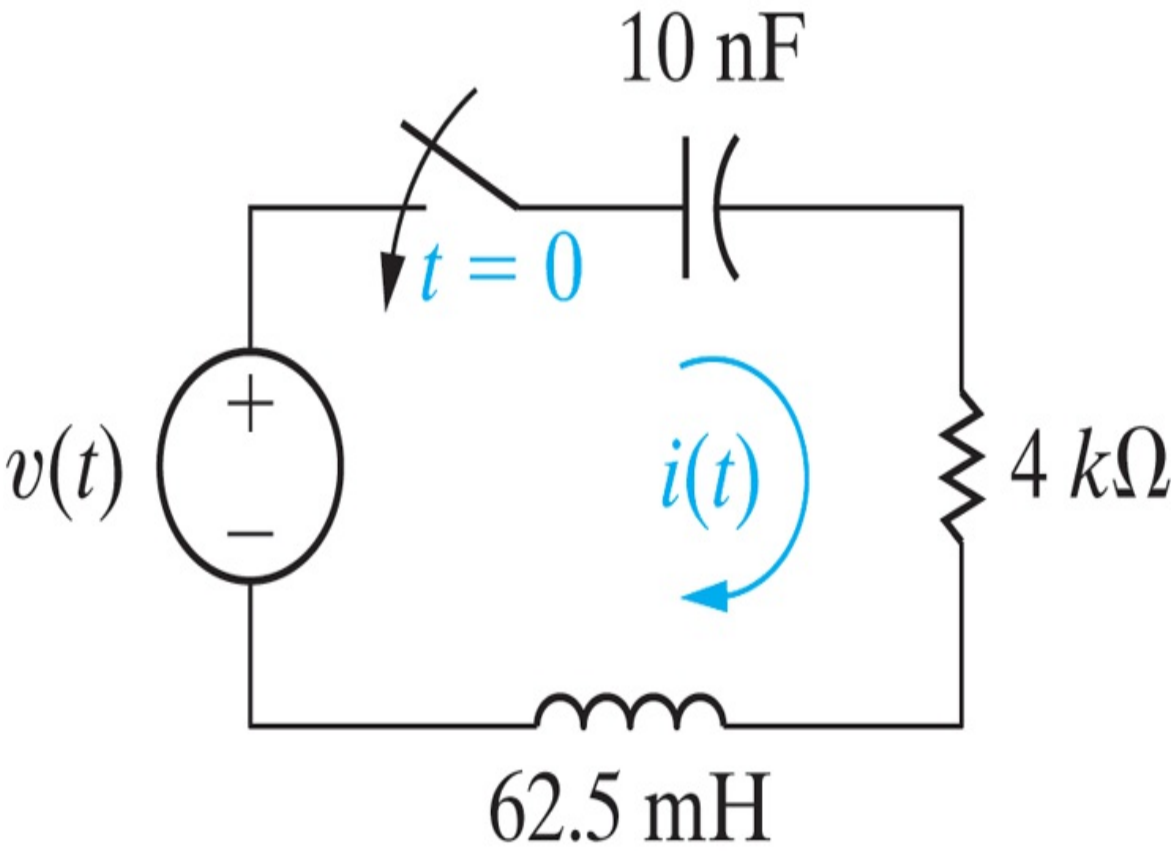


Figure 13.15 Full Alternative Text

Solution

Apply the Laplace transform method using Analysis Method 13.1.

1. Step 1: There is no initial stored energy, so both initial conditions are zero.

2. Step 2: Using the functional and operational transform tables (Tables 12.1 and 12.2), we see that the Laplace transform of v(t) is

   V=L{15 cos 40,000t}=15ss2+40,0002V-s.

   This is the value of the voltage source for the s-domain circuit in Fig. 13.16.

3. Step 3: The current is represented in the s domain as I, as shown in Fig. 13.16.

4. Step 4: The impedance of the resistor is its resistance. The impedance of the inductor is

   $Z_L = sL = 0.0625s = s16\Omega$

   and the impedance of the capacitor is

   $Z_C = \frac{1}{sC} = \frac{1}{10 \times 10^{-9} s} = \frac{10^8}{s}\ \Omega$.

   The *s*-domain circuit that results from Steps 1–4 is shown in Fig. 13.16.

Figure 13.16 The circuit in Fig 13.16 for $t \geq 0$, transformed into the s domain.

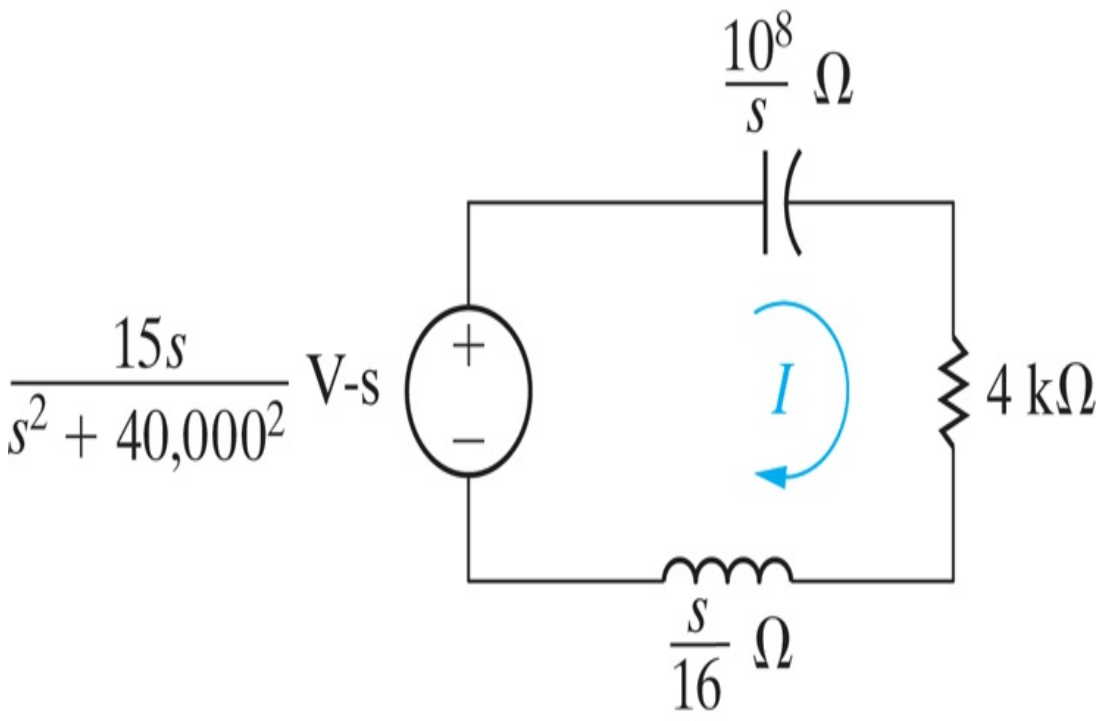


Figure 13.16 Full Alternative Text

5. Step 5: To find the current I, divide the source voltage by the sum of the impedances:

   I=15ss2+40,00024000+108s+s16=240s2(s2+40,0002)(s2+64,000s+16×108)A-s.

6. Step 6: Use the initial value theorem to predict the initial value of i:

   lim x→∞ sI= lim s→ ∞ 240 S 3 (S 2 +40, 000 2)(s 2 +64,000s+16× 10 8) = lim (1/s)→0 240(1 s) [1+ (40,000 s) 2][1+(60,000 s)+(16× 10 8 s 2)] = 0 (1)(1) =0= lim t→0 i(t).

   The initial-value theorem predicts correctly that the initial current is zero. We cannot use the final-value theorem to predict the final current because I has two poles on the imaginary axis at ±j40,000 rad/s.

7. Step 7: The partial fraction expansion of I is

   I=240s2(s2+40,0002)(s2+64,000s+16×108)=K1s

−j40,000+K1*s+j40,000+K2s+32,000−j24,000+K2*s+32,000+j24,000.

Finding K1 and K2,

K1=240s2(s+j40,000)
(s2+64,000s+16×108)|s=j40,000=1.875×10−3;K2=240s2(s2+40,0002)
(s+32,000+j24000)|s=−32,000+j40,000=3.125×10−3−126.87°.

Now use the transforms in Table 12.3 to find v:

i(t)=L−1{ 1.875×10−3s
−j40,000+1.875×10−3s+j40,000+3.125×10−3−126.87°s+32,000−j24,00(
}=[3.75 cos 40,000t+6.25e−32,000t cos(24,000t−126.87°)]u(t)mA.

The first term in the expression for i(t) is the steady-state response. Its frequency matches the frequency of the source, and this term persists for all time. You should use the phasor methods from Chapter 9 to verify this result. The second term in the expression for i(t) is the transient response, or the natural response. Note that it decays to zero as $t \to \infty$. This part of the response is independent of the voltage source and is based only on the passive component values and their interconnections.

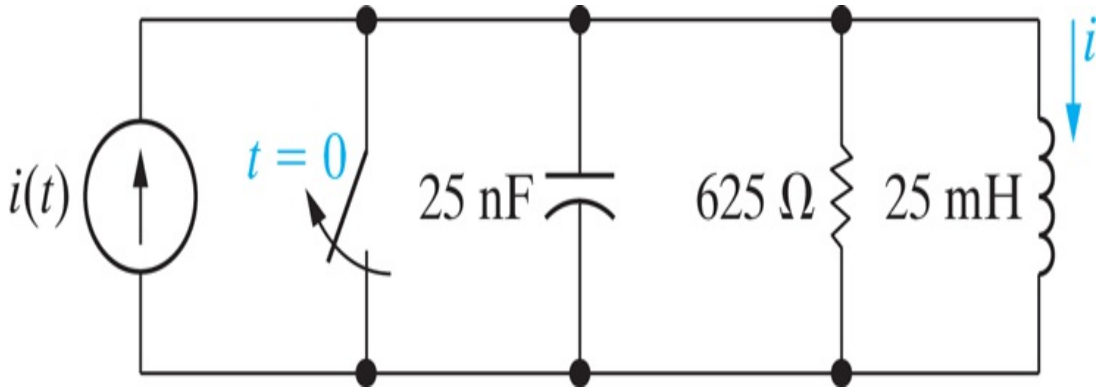
Assessment Problem

Objective 2—Know how to analyze a circuit in the s domain and be able to transform an s domain solution to the time domain

1. 13.4 The energy stored in the circuit shown is zero at the time when the switch is opened. The current source is 24 cos 40,000t mA. Find the voltage v(t) for $t \geq 0$.

   Answer:

   (15 sin 40,000t−25e−32,000t sin 24,000t)u(t) mA.



13.3-7 Full Alternative Text

SELF-CHECK: Also try Chapter Problem 13.23.

Until now, we avoided analyzing circuits with inductors and capacitors that have two or more meshes, or three or more essential nodes. Such circuits are described by two or more simultaneous differential equations, and the techniques for solving these systems of equations are beyond the scope of this text. However, using Laplace techniques, we can transform a circuit into the s domain and write a set of simultaneous algebraic equations, whose solution is much more manageable. Example 13.5 illustrates this by solving a circuit with two meshes.

Example 13.5 Analyzing a Circuit with Multiple Meshes

The circuit in Fig. 13.17 has no initial stored energy. At t=0, the switch closes. Use Laplace methods to find i1(t) and i2(t) for t≥0.

Figure 13.17 A multiple-mesh RL circuit.

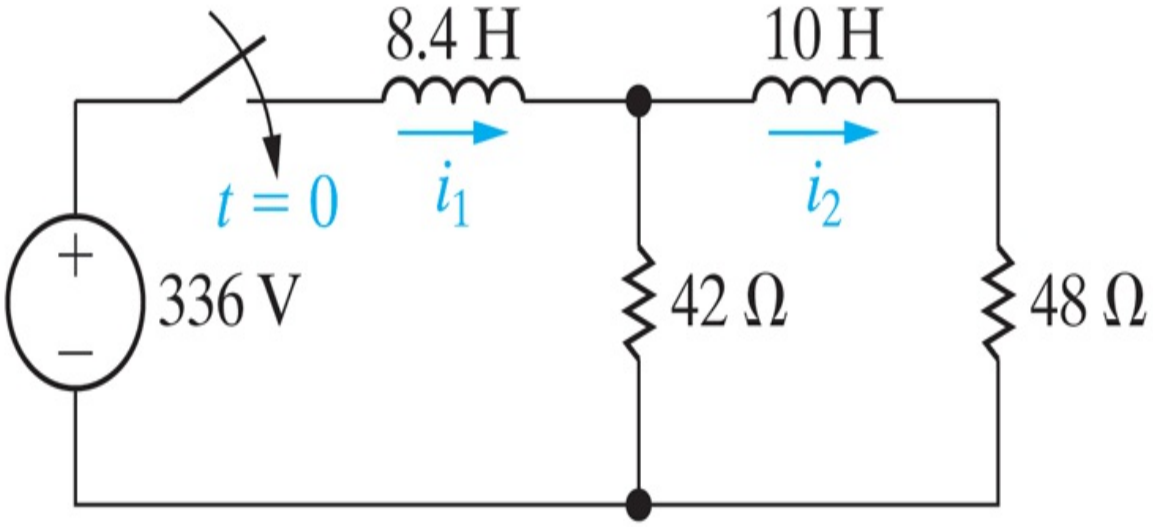


Figure 13.17 Full Alternative Text

Solution

Apply the Laplace transform method using Analysis Method 13.1.

1. Step 1: There is no initial stored energy, so both initial conditions are zero.

2. Step 2: The series connection of the 336 V dc voltage source and the switch can be described in the time domain as 336u(t) V. Using the functional and operational transform tables (Tables 12.1 and 12.2), the Laplace transform of this voltage is

   L{336u(t)}=336sV-s.

   This is the value of the voltage source for the s-domain circuit in Fig. 13.18.

3. Step 3: The currents are represented in the s domain as I1 and I2, as shown in Fig. 13.18.

4. Step 4: The impedance of the resistors is their resistance. The impedance

of the inductors is

$ZL1=sL1=8.4s\ \Omega; ZL2=sL2=10s\ \Omega.$

The s-domain circuit that results from Steps 1–4 is shown in Fig. 13.18.

Figure 13.18 The s-domain equivalent circuit for the circuit shown in Fig. 13.17.

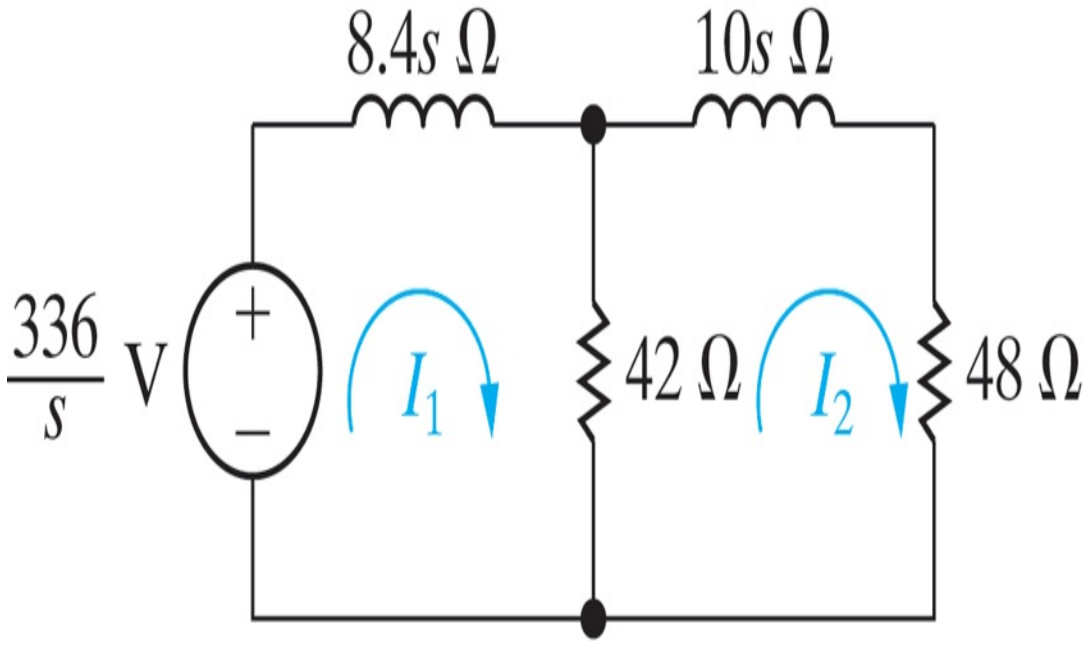


Figure 13.18 Full Alternative Text

5. Step 5: The two KVL equations are

   $336s=(42+8.4s)I1-42I2, 0=-42I1+(90+10s)I2.$

   Using Cramer's method, we get

$\Delta = \begin{vmatrix} 42+8.4s & -42 \\ -42 & 90+10s \end{vmatrix} = 84(s^2+14s+24) = 84(s+2)(s+12), N1 = \begin{vmatrix} 336/s & -42 \\ 0 & 90+10s \end{vmatrix} = 3360(s+9)s, N2 = \begin{vmatrix} 42+8.4s & 336/s \\ -42 & 0 \end{vmatrix} = 14{,}112s.$

Therefore,

$I1 = N1\Delta = 40(s+9)s(s+2)(s+12), I2 = N2\Delta = 168s(s+2)(s+12).$

6. Step 6: Use the initial-value theorem to predict the initial values of i1 and i2:

$\lim_{s\to\infty} sI1 = \lim_{s\to\infty} 40(s+9)(s+2)(s+12) = \lim(1/s)\to 0\, 40[\ (1/s)+9(1/s)2\ ](1+(2/s))(1+(12/s)) = 0(1)(1) = 0 = \lim_{t\to 0} i1(t); \lim_{s\to\infty} sI2 = \lim_{s\to\infty} 168(s+2)(s+12) = \lim(1/s)\to 0\, 168(1/s)2(1+(2/s))(1+(12/s)) = 0(1)(1) = 0 = \lim_{t\to 0} i2(t)$

The initial-value theorem predicts correctly that the initial currents are zero. Now use the final-value theorem to predict the values of i1 and i2 as $t \to \infty$:

$\lim_{s\to 0} sI1 = \lim_{s\to 0} 40(s+9)(s+2)(s+12) = 40(9)(2)(12) = 15A = \lim_{t\to\infty} i1(t); \lim_{s\to 0} sI2 = \lim_{s\to 0} 168(s+2)(s+12) = 168(2)(12) = 7A = \lim_{t\to\infty} i2(t).$

To verify these values, consider the circuit in Fig. 13.16 as $t \to \infty$. The inductors now behave like short circuits, as shown in Fig. 13.19.

Figure 13.19 The circuit in Fig 13.17 as $t \to \infty$

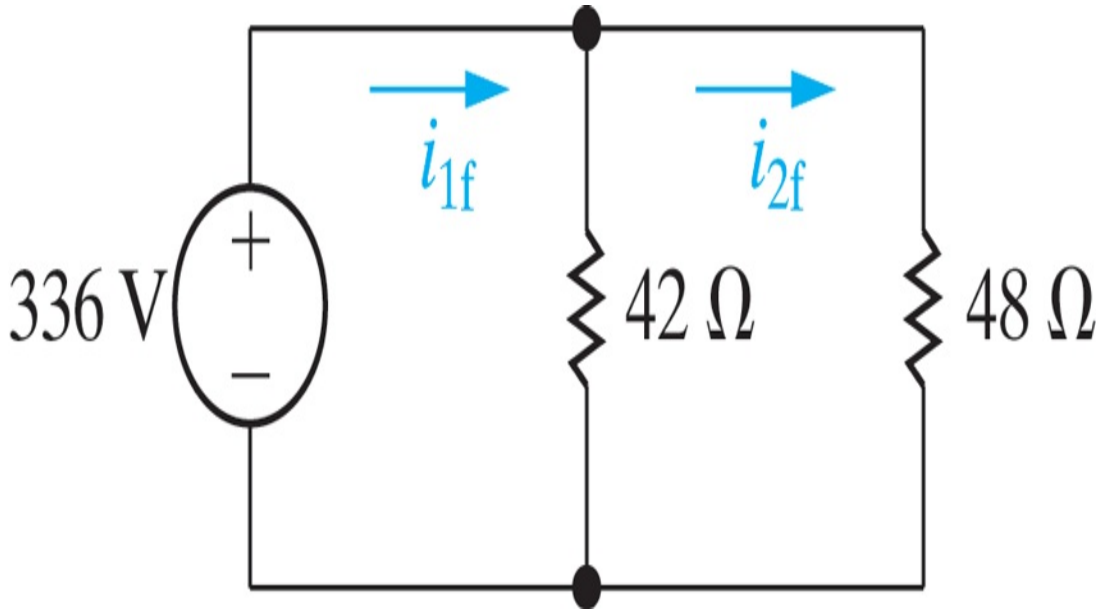


Figure 13.19 Full Alternative Text

The two KVL equations that describe this circuit are

$42i1f-42i2f=336; -42i1f+90i2f=0.$

Solving, we find the final value of i1 is 15 A and the final value of i2 is 7 A, as predicted by the final value theorem.

7. Step 7: Expanding I1 and I2 into a sum of partial fractions gives

   $I1=15s-14s+2-1s+12, I2=7s-8.4s+2+1.4s+12.$

   We obtain the expressions for i1 and i2 by inverse- transforming I1 and I2, using Table 12.3:

   $i1=(15-14e-2t-e-12t)u(t)$ A, $i2=(7-8.4e-2t+1.4e-12t)u(t)$ A.

   One final test involves calculating the voltage drop across the 42 Ω resistor using three different methods. From the circuit, the voltage across the 42 Ω resistor (positive at the top) is

   $v=42(i1-i2)=336-8.4di1dt=48i2+10di2dt.$

You should verify that regardless of which expression is used, the voltage is

$v=(336-235.2e^{-2t}-100.80e^{-12t})u(t)V$.

We are thus confident that the solutions for i1 and i2 are correct.

Assessment Problem

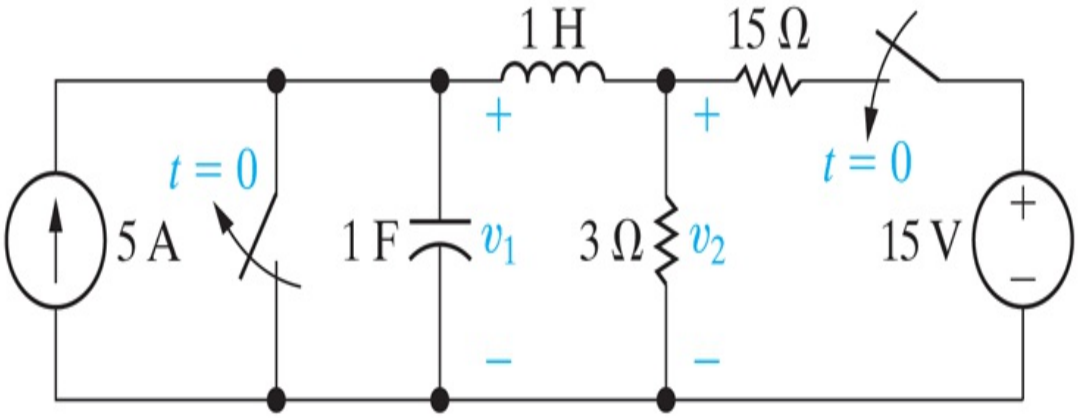
Objective 2—Know how to analyze a circuit in the s domain and be able to transform an s-domain solution to the time domain

1. 13.5 The dc current and dc voltage sources are applied simultaneously to the circuit shown. No energy is stored in the circuit at the instant of application.

    1. a) Derive the s-domain expressions for V1 and V2.
    2. b) For t>0, derive the time-domain expressions for v1 and v2.
    3. c) Calculate v1(0+) and v2(0+).
    4. d) Compute the steady-state values of v1 and v2.

    Answer:

    1. (a) $V1=[5(s+3)]/[s(s+0.5)(s+2)]$,$V2=[2.5(s2+6)]/[s(s+0.5)(s+2)]$;
    2. (b) $v1=(15-503e^{-0.5t}+53e^{-2t})u(t)V$,$v2=(15-1256e^{-0.5t}+253e^{-2t})u(t)V$;
    3. (c) v1(0+)=0, v2(0+)=2.5V;
    4. (d) v1=v2=15V.



13.3-8 Full Alternative Text

SELF-CHECK: Also try Chapter Problems 13.19 and 13.20.

We can use Thévenin and Norton equivalents to simplify circuits in the s domain. When one part of the circuit changes frequently while another part remains constant, we can find a simpler equivalent for the constant part of the circuit that makes analysis of the entire circuit easier. Example 13.6 creates a Thévenin equivalent to simplify part of an s-domain circuit.

Example 13.6 Creating a Thévenin Equivalent in the s Domain

We want to find the capacitor current, iC, for the circuit in Fig. 13.20. The circuit has no initial stored energy, and at t=0, the switch closes. Find the Thévenin equivalent for the circuit to the left of the terminals a and b in the s domain, using Laplace methods. Then analyze the simplified circuit to find iC(t) for t≥0.

Figure 13.20 The circuit for Example 13.6.

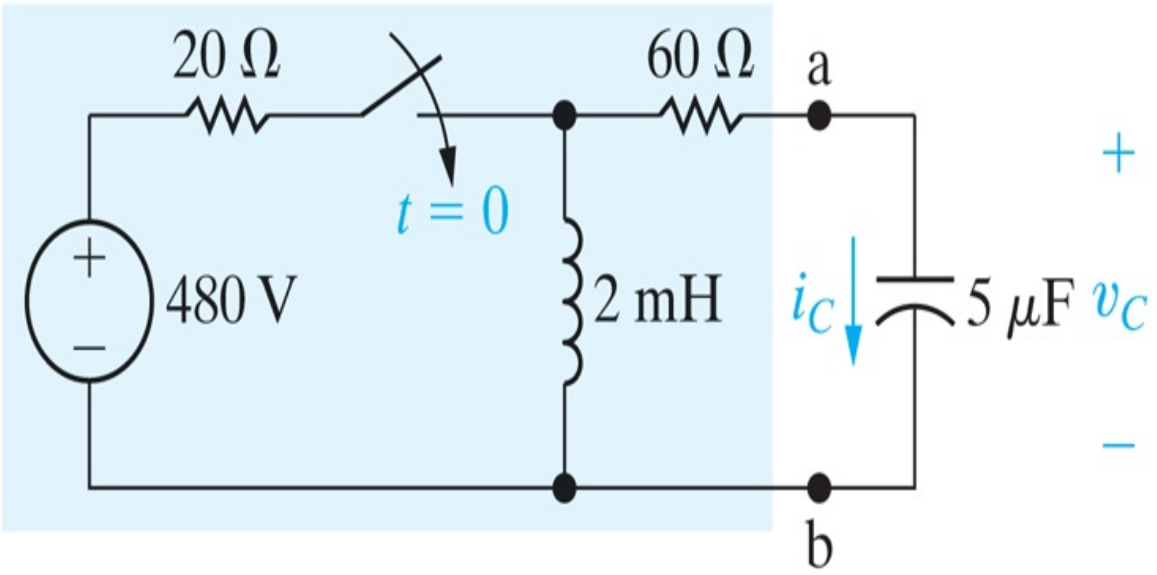


Figure 13.20 Full Alternative Text

Solution

Apply the Laplace transform method using Analysis Method 13.1.

1. Step 1: There is no initial stored energy, so both initial conditions are zero.

2. Step 2: The series connection of the 480 V dc voltage source and the switch can be described in the time domain as 480u(t) V. Using the functional and operational transform tables (Tables 12.1 and 12.2), the Laplace transform of this voltage is

   L{480u(t)}=480sV-s.

   This is the value of the voltage source for the s-domain circuit in Fig. 13.21.

3. Step 3: The capacitor current and voltage are represented in the s domain as IC and VC, as shown in Fig. 13.21.

4. Step 4: The impedance of the resistors is their resistance. The impedances of the inductor and capacitor are

   $Z_L = sL = 0.002s = s500\Omega; Z_C = \frac{1}{sC} = \frac{1}{5 \times 10^{-6}s} = \frac{2 \times 10^5}{s}\Omega.$

   The s-domain circuit that results from Steps 1–4 is shown in Fig. 13.21.

Figure 13.21 The Laplace transform of the circuit shown in Fig. 13.20.

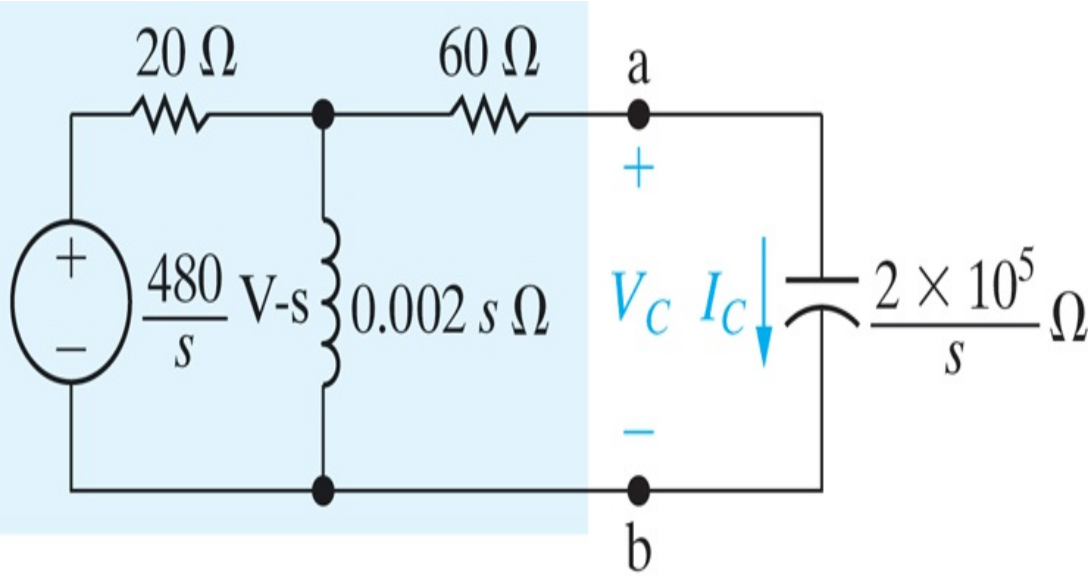


Figure 13.21 Full Alternative Text

5. Step 5: The Thévenin voltage is the open-circuit voltage across terminals a and b. Under open-circuit conditions, there is no voltage across the 60 Ω resistor. Using voltage division,

   $V_{Th} = \frac{(480/s)(0.002s)}{20 + 0.002s} = \frac{480}{s + 10^4}.$

   The Thévenin impedance seen from terminals a and b equals the 60 Ω resistor in series with the parallel combination of the 20 Ω resistor and

the inductive impedance. Thus

ZTh=60+0.002s(20)20+0.002s=80(s+7500)s+104.

Using the Thévenin equivalent, we reduce the circuit shown in Fig. 13.21 to the one shown in Fig. 13.22.

Figure 13.22 A simplified version of the circuit shown in Fig. 13.21, using a Thévenin equivalent.

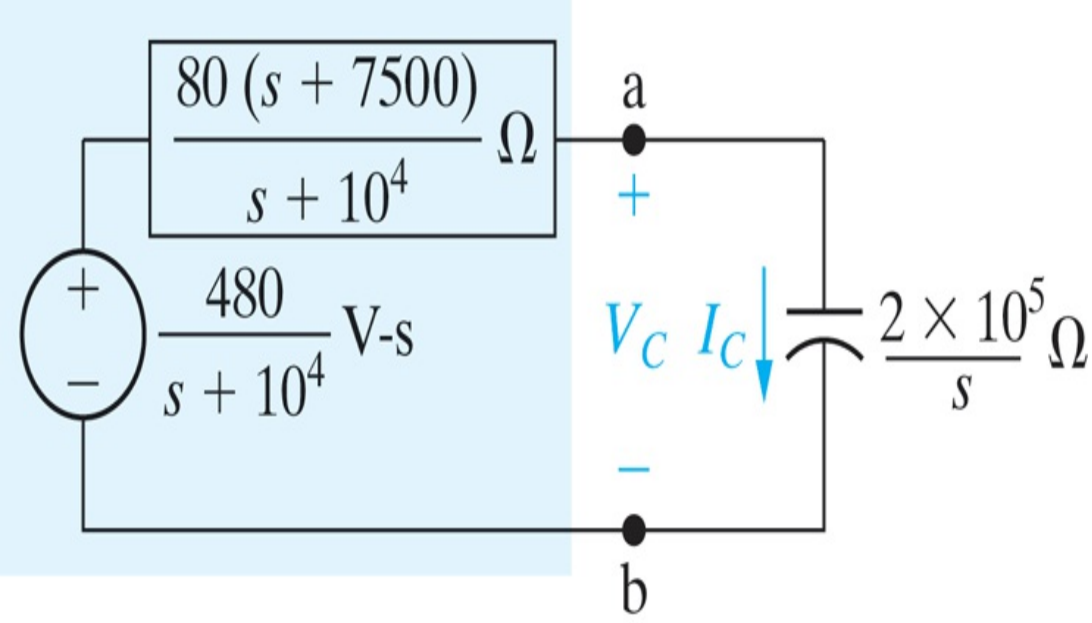


Figure 13.22 Full Alternative Text
In this circuit, the capacitor current IC equals the Thévenin voltage divided by the total series impedance. Thus, IC=480s+10480(s+7500)s+104+2×105s=6s(s+5000)2.

6. Step 6: Use the initial- and final-value theorems to predict the initial and

final values of iC:

$\lim_{s\to\infty} sI_C = \lim_{s\to\infty} \frac{6s^2}{(s+5000)^2} = \lim \frac{(1/s)\to 0}{(1+(5000/s))^2} = \frac{6(1)^2}{=6A} = \lim_{t\to 0} i_C(t); \lim_{s\to 0} sI_C = \lim_{s\to 0} \frac{6s^2}{(s+5000)^2} = ($

Let's calculate the initial capacitor current from the circuit in Fig. 13.20. The initial inductor current is zero and the initial capacitor voltage is zero, so the initial capacitor current is 480/(20+60) or 6 A, which agrees with the prediction of the initial-value theorem. The final value of the capacitor current is zero because as $t\to\infty$ in the circuit shown in Fig. 13.20, the capacitor behaves like an open circuit. This final capacitor current also agrees with the prediction of the final-value theorem.

7. Step 7: The partial fraction expansion of IC is

$$I_C = \frac{-30{,}000}{(s+5000)^2} + \frac{6}{s+5000},$$

and its inverse transform is

$$i_C = (-30{,}000te^{-5000t} + 6e^{-5000t})u(t) \text{ A}.$$

Suppose we also want to find the voltage drop across the capacitor, vC. Once we know iC, we could find vC by integration in the time domain:

$$v_C = 2\times 10^5 \int_0^{-t} (6 - 30{,}000x)e^{-5000x}\, dx.$$

Although the integration is not difficult, we can avoid it altogether by first finding the s-domain expression for Vc and then using the inverse transform to find vC. Thus

$$V_C = \frac{1}{sC} I_C = \frac{2\times 10^5}{s} \frac{6s}{(s+5000)^2} = \frac{12\times 10^5}{(s+500)^2},$$

and

$$v_C = 12\times 10^5 te^{-5000t} u(t) \text{ V}.$$

You can explore this circuit's behavior further in Problem 13.35.

Assessment Problem

Objective 2—Know how to analyze a circuit in the s domain and be able to transform an s-domain solution to the time domain

1. 13.6 The initial charge on the capacitor in the circuit shown is zero.

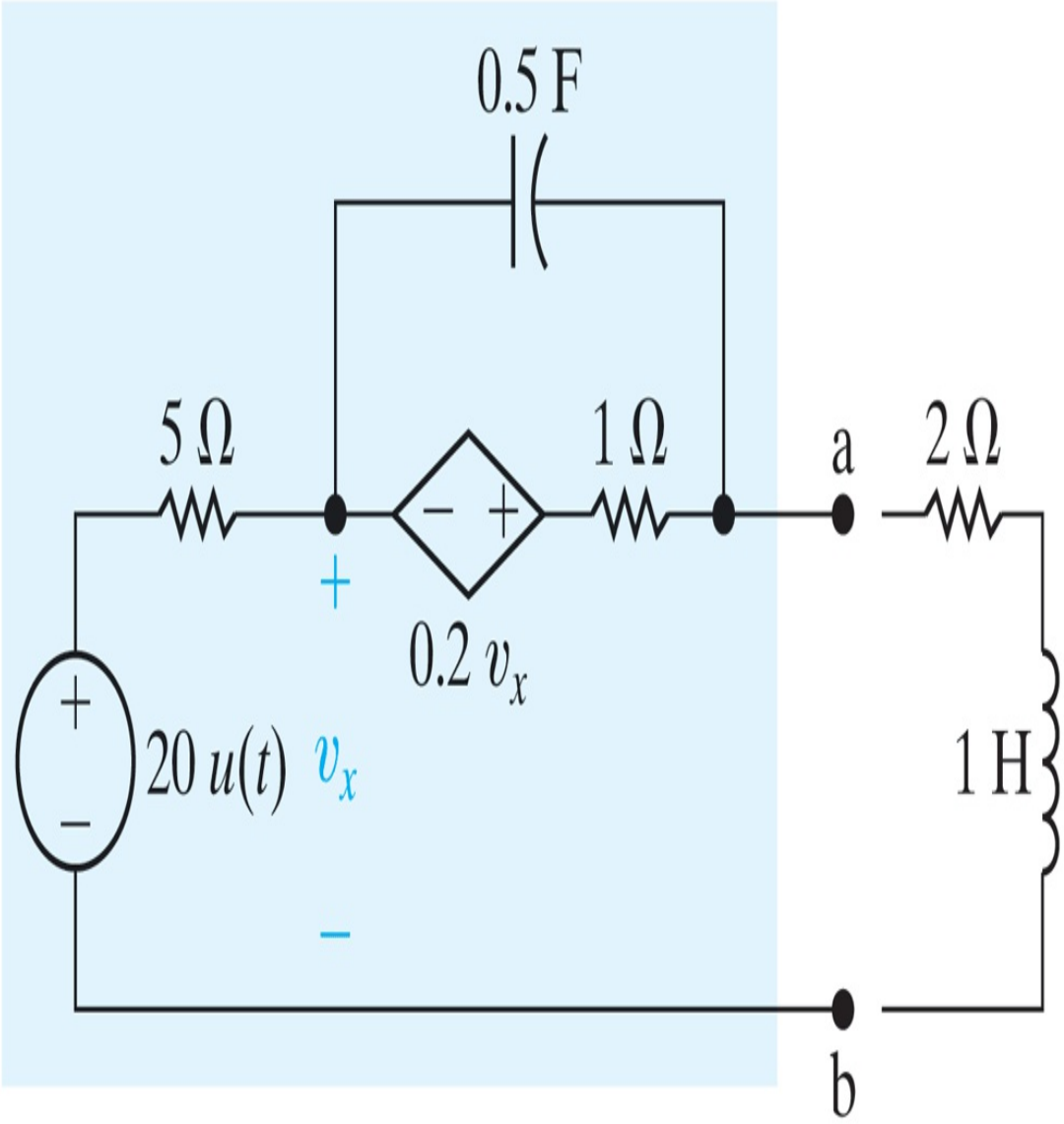
    1. a) Find the s-domain Thévenin equivalent circuit with respect to terminals a and b.

    2. b) Find the s-domain expression for the current that the circuit delivers to a load consisting of a 1 H inductor in series with a 2 Ω resistor.

    Answer:

    1. (a) VTh=Vab=[20(s+2.4)]/[s(s+2)],ZTh=5(s+2.8)/(s+2);

    2. (b) Iab=[20(s+2.4)]/[s(s+3)(s+6)].



13.3-9 Full Alternative Text

SELF-CHECK: Also try Chapter Problem 13.37.

We can also use Laplace transform methods to analyze circuits with mutually coupled coils. Example 13.7 illustrates this process.

Example 13.7 Analyzing a Circuit with Mutual Inductance

Consider the circuit in Fig. 13.23. The make-before-break switch has been in position a for a long time. At t=0, the switch moves instantaneously to position b. Use Laplace methods to find i2(t) for t≥0.

Figure 13.23 The circuit for Example 13.7, containing magnetically coupled coils.

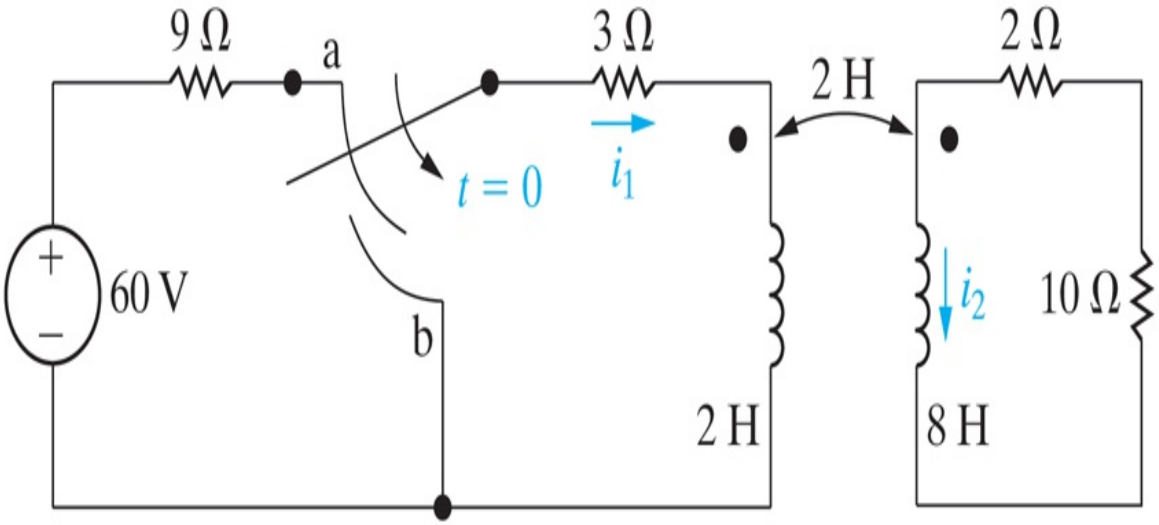


Figure 13.23 Full Alternative Text

Solution

Apply the Laplace transform method using Analysis Method 13.1.

1. Step 1: Because the switch has been in position a for a long time, both inductors behave like short circuits. The current in the 2 H inductor is 60/(9+3)=5 A, and the current in the 8 H inductor is zero. Therefore, i1(0−)=5 A and i2(0−)=0.

2. Step 2: For t≥0, there is no independent source in the circuit, so we can skip this step.

3. Step 3: The currents in the two coils are represented in the s domain as I1 and I2.

4. Step 4: Before calculating the impedance of the resistors and inductors, we replace the magnetically coupled coils with a T-equivalent circuit.[1] Figure 13.24 shows the new circuit.

   [1] See Appendix H.

Figure 13.24 The circuit shown in Fig. 13.23, with the magnetically coupled coils replaced by a T-equivalent circuit.

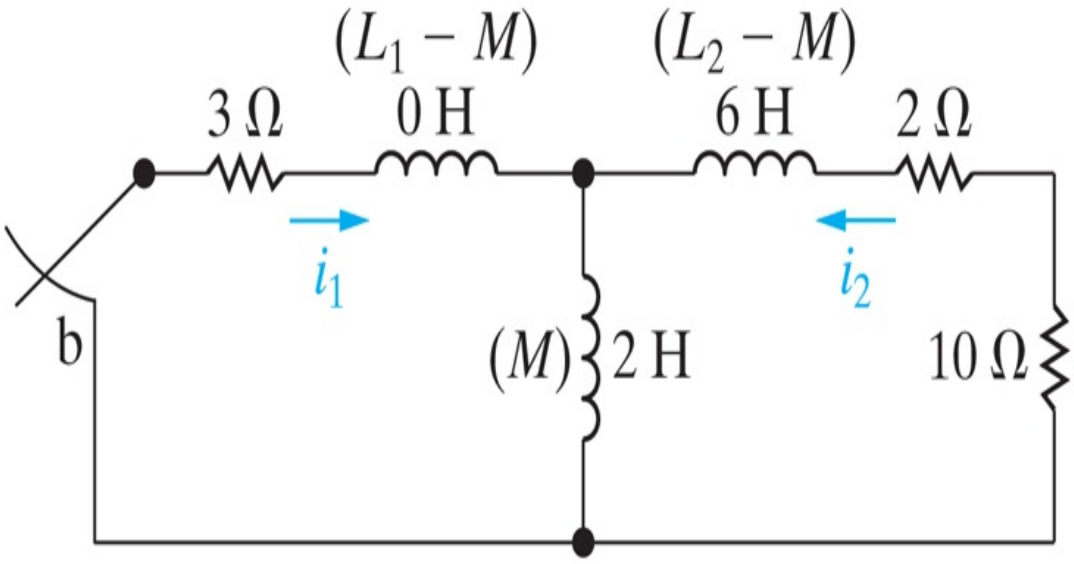


Figure 13.24 Full Alternative Text

The impedance of the resistors is their resistance, and the impedance of the inductors is sL. Because we plan to use the mesh-current method in the s domain, we use the series-equivalent circuit for inductors carrying initial current. We place a voltage source in the vertical leg of the tee, to represent the initial value of the current in that vertical leg, which is i1(0−) + i2(0−), or 5 A. The s-domain circuit resulting from Steps 1–4 is shown in Fig. 13.25.

Figure 13.25 The s-domain equivalent circuit for the circuit shown in Fig. 13.24.

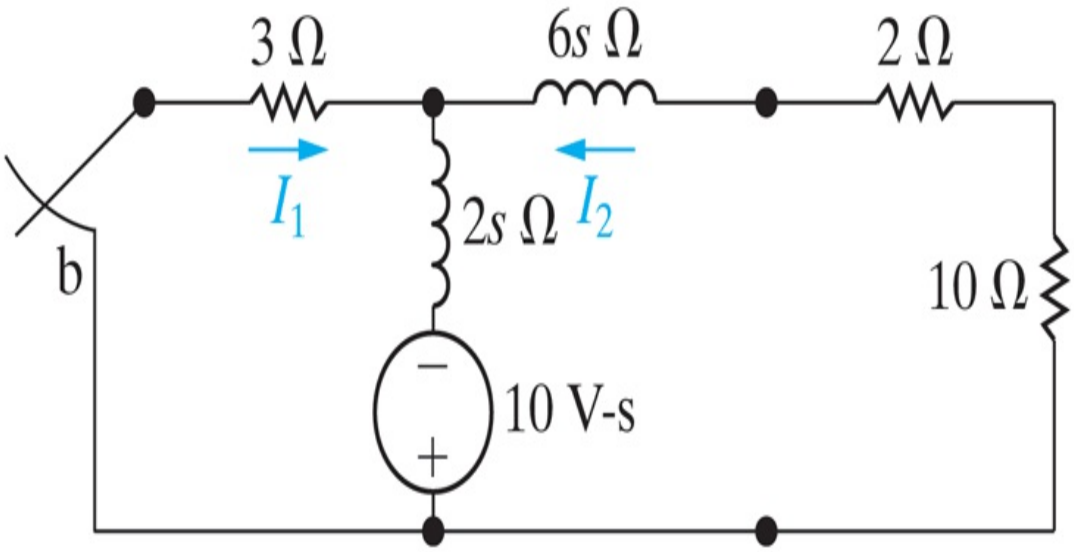


Figure 13.25 Full Alternative Text

5. Step 5: Write the two s-domain mesh equations for the circuit in Fig. 13.25:

(3+2s)I1+2sI2=10;2sI1+(12+8s)I2=10.

Solving for I2 yields

$$I2=2.5(s+1)(s+3).$$

6. Step 6: Use the initial- and final-value theorems to predict the initial and final values of i2:

$$\lim_{s\to\infty} sI2=\lim_{s\to\infty} 2.5s(s+1)(s+3)=\lim(1/s)\to 0\, 2.5(1/s)(1+(1/s))(1+(3/s))=0(1)(1)=0=\lim_{t\to 0} i2(t);\lim_{s\to 0} sI2=\lim_{s\to 0} 2.5s(s+1)(s+3)=0(1)(3)=0=\lim_{t\to\infty} i2(t).$$

   The initial- and final-value theorems correctly predict the initial and final values of the current i2 for the circuit in Fig. 13.23.

7. Step 7: The partial fraction expansion of I2 is

$$I2=1.25s+1-1.25s+3.$$

   Using Table 12.3, the inverse Laplace transform of I2 is

$$i2=(1.25e-t-1.25e-3t)u(t)\ \text{A}.$$

   Figure 13.26 shows a plot of i2 versus t. This response makes sense in terms of the known physical behavior of the magnetically coupled coils. A current can exist in the L2 inductor only if there is a time-varying current in the L1 inductor. As i1 decreases from its initial value of 5 A, i2 increases from zero and then approaches zero as i1 approaches zero.

Figure 13.26 The plot of i2 versus t for the circuit shown in Fig. 13.23.

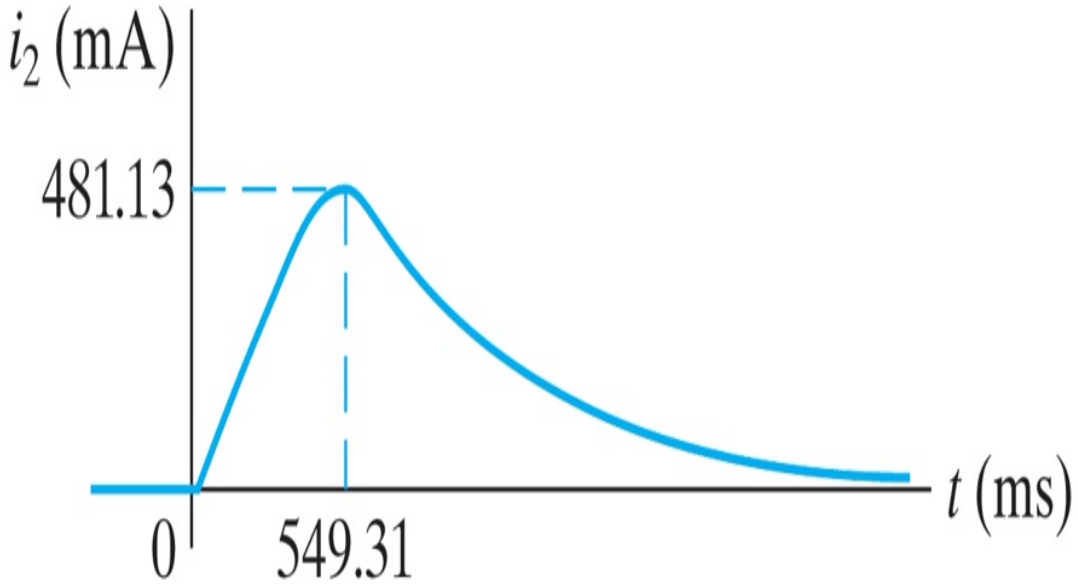


Figure 13.26 Full Alternative Text

Assessment Problem

Objective 2—Know how to analyze a circuit in the s domain and be able to transform an s-domain solution to the time domain

1. 13.7

    1. a) Using the results from Example 13.7 for the circuit in Fig. 13.23, verify that i2 reaches a peak value of 481.13 mA at t=549.31 ms.

    2. b) Find i1, for t>0, for the circuit shown in Fig. 13.23.

    3. c) Compute di1/dt when i2 is at its peak value.

    4. d) Express i2 as a function of di1/dt when i2 is at its peak value.

    5. e) Use the results obtained in (c) and (d) to calculate the peak value of i2.

    Answer:

1. (a) di2/dt=0 when t = 0.5 ln 3 = 549.31 ms;
2. (b) i1=2.5(e−t + e−3t)u(t)A;
3. (c) −2.89 A/s;
4. (d) i2=−(M12)di1dt;
5. (e) 481.13 mA.

SELF-CHECK: Also try Chapter Problems 13.38 and 13.39.

Because we are analyzing linear lumped-parameter circuits, we can use superposition to divide the response into components that can be identified with particular sources and initial conditions. We need to identify these components in order to use the transfer function, which we introduce in the next section. Example 13.8 illustrates superposition in the s domain by revisiting the circuit from Example 13.3 and separating its output voltage, v, into three components, two associated with the circuit's initial conditions and one associated with the circuit's independent source.

Example 13.8 Applying Superposition in the s Domain

Repeat the analysis of the s-domain circuit for Example 13.3, shown in Fig. 13.14. Use superposition to separate the time-domain voltage v into three components, each one being the response to one of the three sources in Fig. 13.14.

Solution

Since each source in Fig. 13.14 is a current source, we will use Ohm's law to find each component of V, using the equivalent impedance of the three parallel-connected impedances. This equivalent impedance, calculated in

Example 13.3, is

$$Z_{eq}=\frac{40\times 10^{6}s}{s^{2}+80{,}000s+16\times 10^{8}}\Omega.$$

Define V' as the component of V in Fig. 13.14 due to the initial capacitor energy. Find V' using the circuit in Fig. 13.27, where the only source is the one that represents the initial condition for the capacitor.

Figure 13.27 The response of the circuit in Fig. 13.14 due to the initial capacitor voltage.

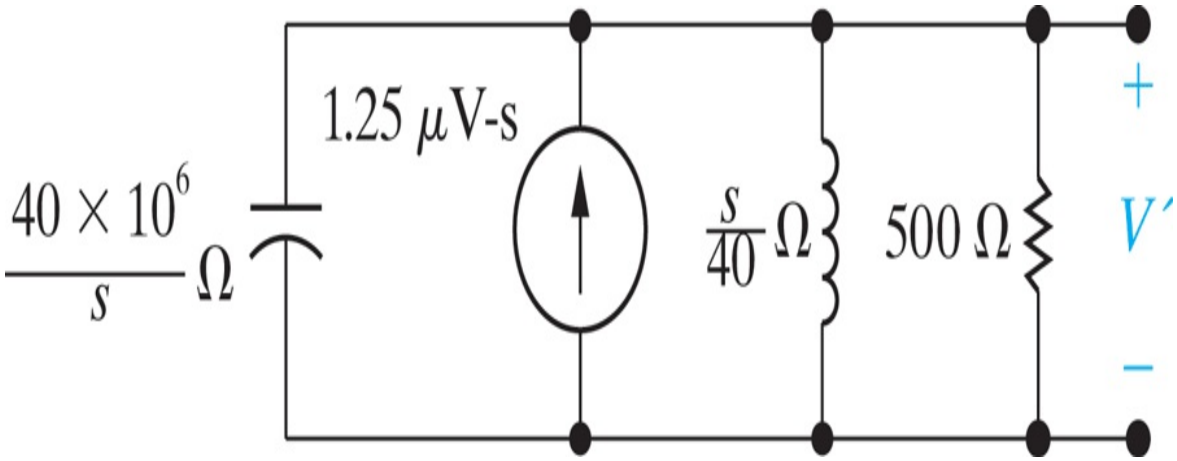


Figure 13.27 Full Alternative Text

$$V'=Z_{eq}(1.25\times 10^{-6})=\frac{50s}{s^{2}+80{,}000s+16\times 10^{8}}.$$

The partial fraction expansion of V' is

$$V'=\frac{-2\times 10^{6}}{(s+40{,}000)^{2}}+\frac{50}{(s+40{,}000)}.$$

Use Table 12.3 to find the inverse Laplace transform of V':

$$v'=L^{-1}\{V'\}=(50e^{-40{,}000t}-2\times 10^{6}te^{-40{,}000t})u(t)V.$$

Therefore, v' is the component of v that is due only to the initial energy of the

capacitor in the circuit of Fig. 13.13.

Next, define V″ as the component of V in Fig. 13.14 due to the initial inductor energy. Find V″ using the circuit in Fig. 13.28, where the only source is the one that represents the initial condition for the inductor.

Figure 13.28 The response of the circuit in Fig. 13.14 due to the initial inductor current.

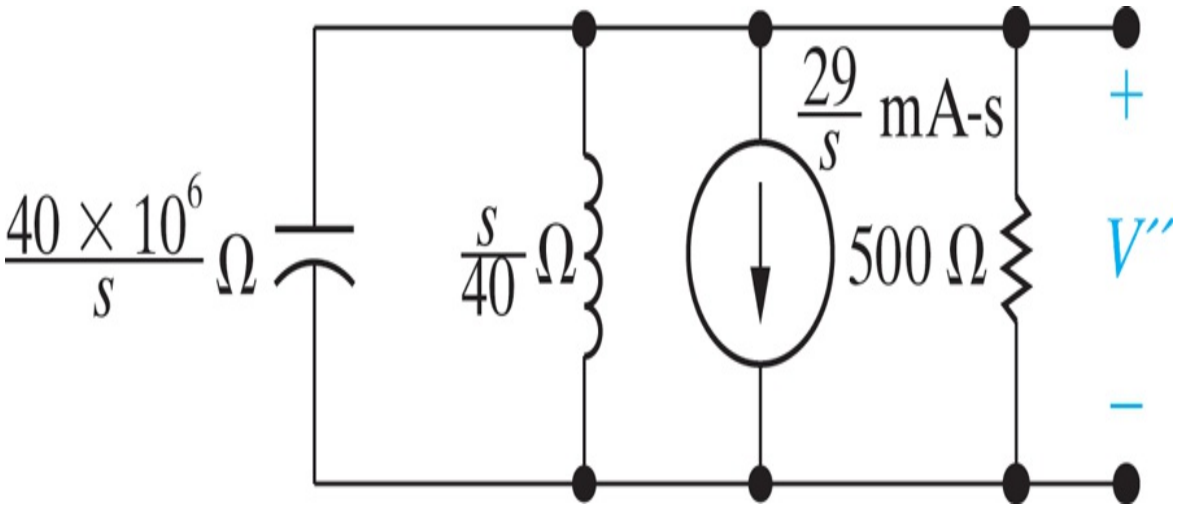


Figure 13.28 Full Alternative Text

V″=Zeq(−0.029s)=−1.16×106s2+80,000s+16×108.

The partial fraction expansion of V″ is

V″=−1.16×106(s+40,000)2.

Use Table 12.3 to find the inverse Laplace transform of V″:

v″=L−1{V″}=(−1.16×106te−40,000t)u(t)V.

Therefore, v″ is the component of v that is due only to the initial energy of the inductor in the circuit of Fig. 13.13.

Finally, define V‴ as the component of V in Fig. 13.14 due to the 24 mA dc current source. Find V‴ using the circuit in Fig. 13.29, where the only source is the one that represents the 24 mA dc current source.

Figure 13.29 The response of the circuit in Fig. 13.14 due to the independent dc current source.

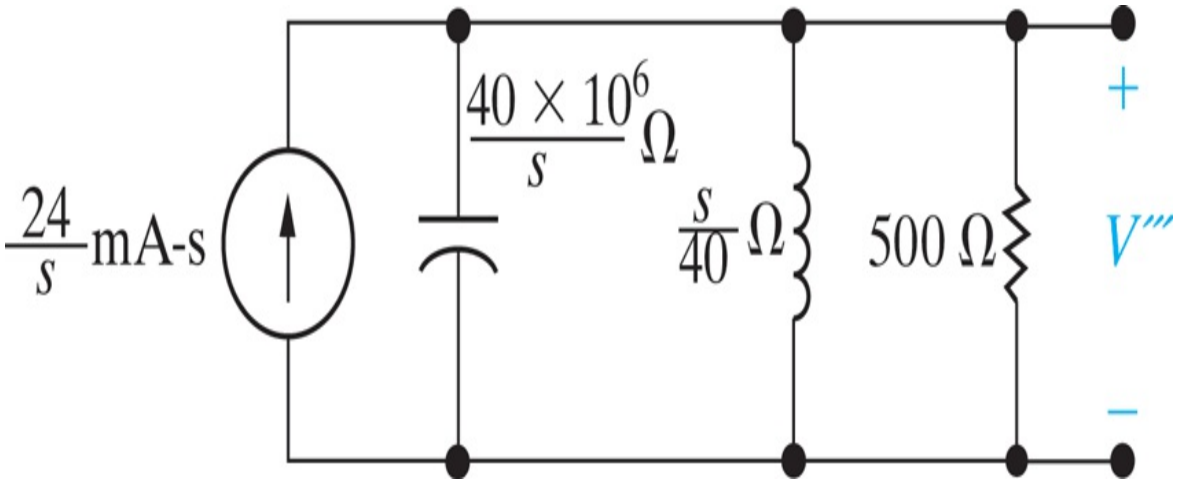


Figure 13.29 Full Alternative Text

V‴=Zeq(0.024s)=0.96×106s2+80,000s+16×108.

The partial fraction expansion of V‴ is

V‴=0.96×106(s+40,000)2.

Use Table 12.3 to find the inverse Laplace transform of V‴:

v‴=L−1{V‴}=(0.96×106te−40,000t)u(t)V.

Therefore, v‴ is the component of v that is due only to the 24 mA current

source in the circuit of Fig. 13.13.

The voltage v in the circuit of Fig. 13.13 is the sum of the three component voltages we just found:

$v=v'+v''+v'''=(50e-40{,}000t-2.2\times106te-40{,}000t)u(t)V.$

This is the result found in Example 13.3.

Assessment Problem

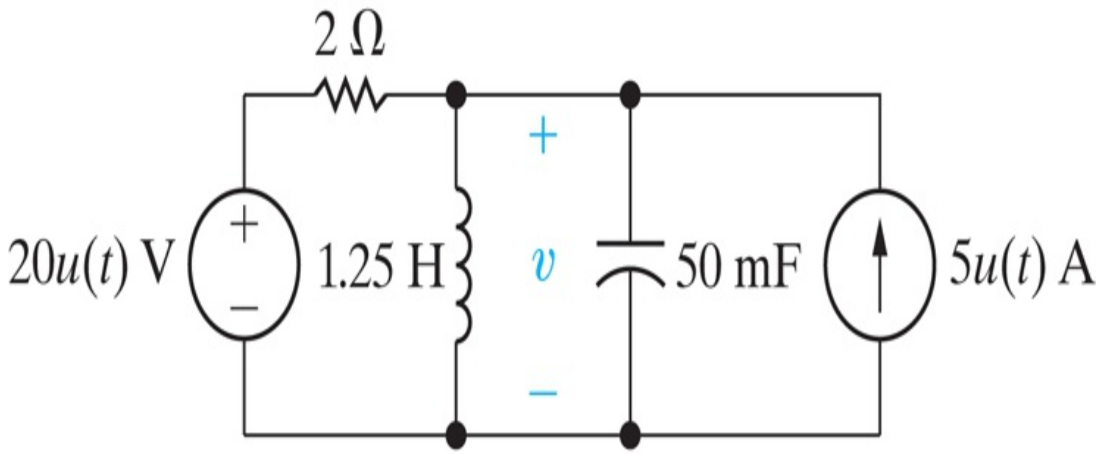
Objective 2—Know how to analyze a circuit in the s domain and be able to transform an s-domain solution to the time domain

1. 13.8 The energy stored in the circuit shown is zero at the instant the two sources are turned on.

    1. a) Find the component of v for t>0 owing to the voltage source.
    2. b) Find the component of v for t>0 owing to the current source.
    3. c) Find the expression for v when t>0.

    Answer:

    1. (a) $[(100/3)e-2t-(100/3)e-8t]u(t)V$;
    2. (b) $[(50/3)e-2t-(50/3)e-8t]u(t)$ V;
    3. (c) $[50e-2t-50e-8t]u(t)V$.



13.3-10 Full Alternative Text

SELF-CHECK: Also try Chapter Problem 13.44.

13.4 The Transfer Function

The **transfer function** is defined as the s-domain ratio of the Laplace transform of the output (response) to the Laplace transform of the input (source). As we will see, the transfer function characterizes a circuit's behavior in a single s-domain expression, without revealing what components make up the circuit or how those components are interconnected. In computing the transfer function, we only consider circuits where all initial conditions are zero. If a circuit has multiple independent sources, we can find the transfer function for each source and use superposition to find the response to all sources.

The transfer function is

Definition of a Transfer Function

$$H(s)=Y(s)X(s),(13.7)$$

where Y(s) is the Laplace transform of the output signal, and X(s) is the Laplace transform of the input signal. Note that the transfer function depends on what is defined as the output signal. Consider, for example, the series circuit shown in Fig. 13.30. If the current is defined as the output signal of the circuit,

Figure 13.30 A series RLC circuit.

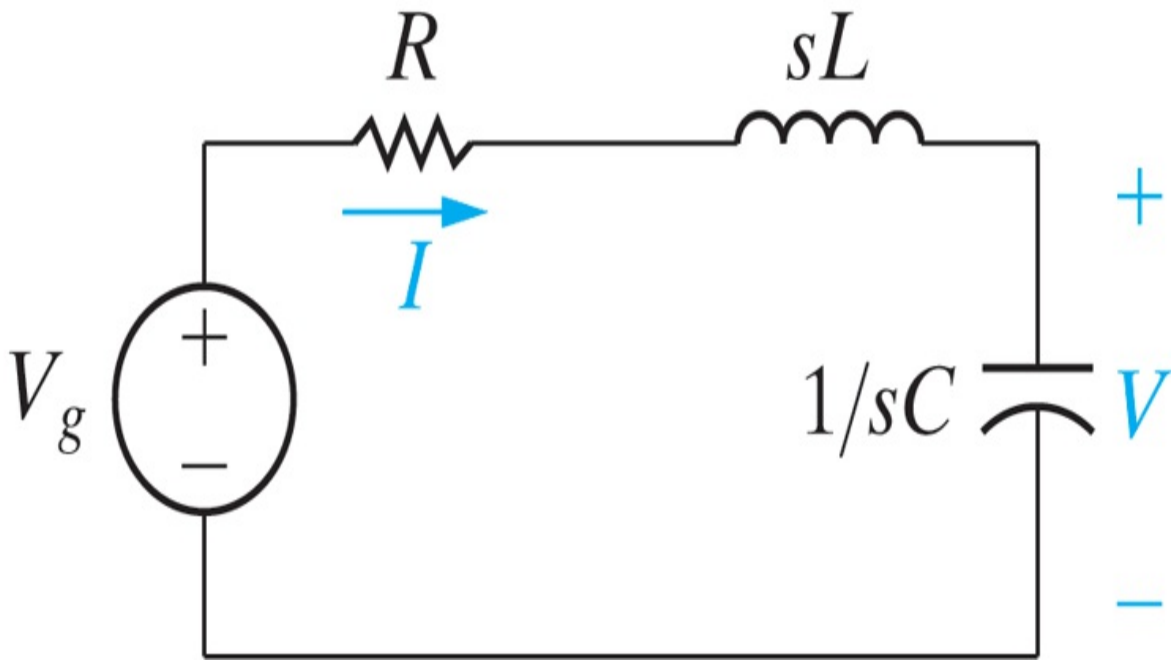


Figure 13.30 Full Alternative Text
H(s)=IVg=1R+sL+1/sC=sCs2LC+RCs+1.

In deriving H(s), we recognized that I corresponds to the output Y(s) and Vg corresponds to the input X(s).

If, instead, the capacitor voltage is defined as the output signal of the circuit shown in Fig. 13.30, then the transfer function is

H(s)=VVg=1/sCR+sL+1/sC=1s2LC+RCs+1.

Thus, because circuits may have multiple sources and because the definition of the output signal of interest can vary, a single circuit can generate many transfer functions. Remember that when multiple sources are involved, no single transfer function can represent the total output; transfer functions associated with each source must be combined using superposition to yield the total response. Example 13.9 illustrates the computation of a transfer function for known numerical values of R, L, and C.

Example 13.9 Deriving the Transfer

Function of a Circuit

The voltage source vg drives the circuit shown in Fig. 13.31. The output signal is the voltage across the capacitor, vo.

Figure 13.31 The circuit for Example 13.9.

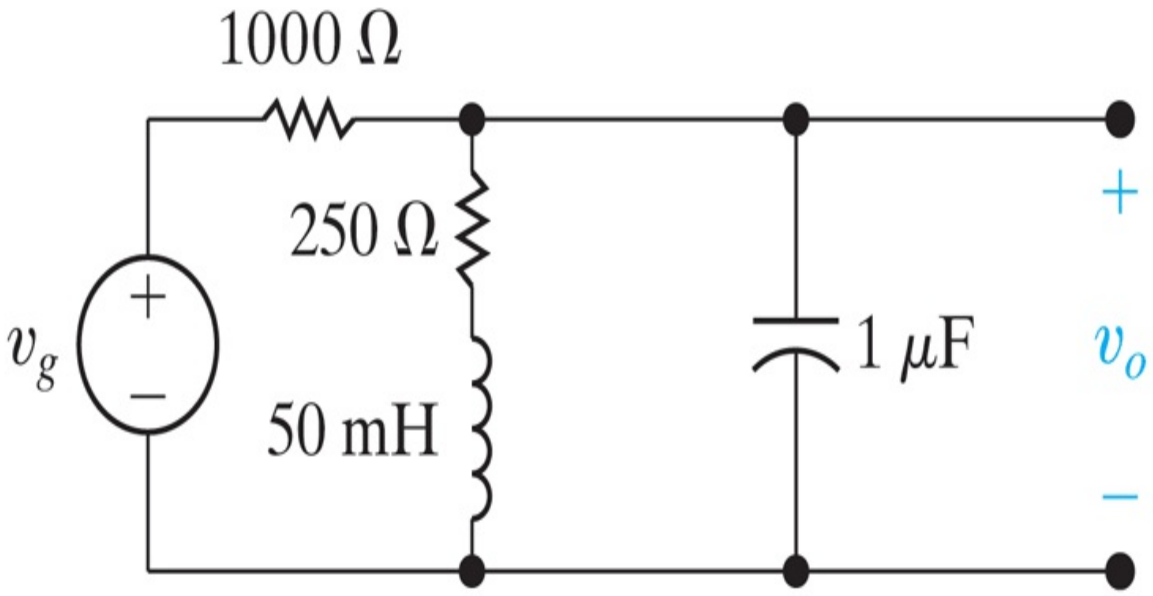


Figure 13.31 Full Alternative Text

1. Find the transfer function for this circuit.
2. Calculate the numerical values for the poles and zeros of the transfer function.

Solution

1. Use Analysis Method 13.1 to construct and analyze the s-domain circuit.

1. Step 1: The circuit in Fig. 13.31 has no initial stored energy, an assumption we always make when calculating a circuit's transfer function. Therefore, we can skip this step.

2. Step 2: There are no independent sources described by a time-domain function in this circuit, so we can skip this step.

3. Step 3: Represent the source and output voltages using Vg and Vo, respectively, as shown in Fig. 13.32.

4. Step 4: The impedance of the resistors is their resistance. The impedances of the inductor and capacitor are

   $$Z_L = sL = 0.05s\ \Omega; Z_C = \frac{1}{sC} = \frac{1}{s(10^{-6})} = 106s\ \Omega.$$

   The s-domain circuit resulting from Steps 1–4 is shown in Fig. 13.32.

Figure 13.32 The s-domain equivalent circuit for the circuit shown in Fig. 13.31.

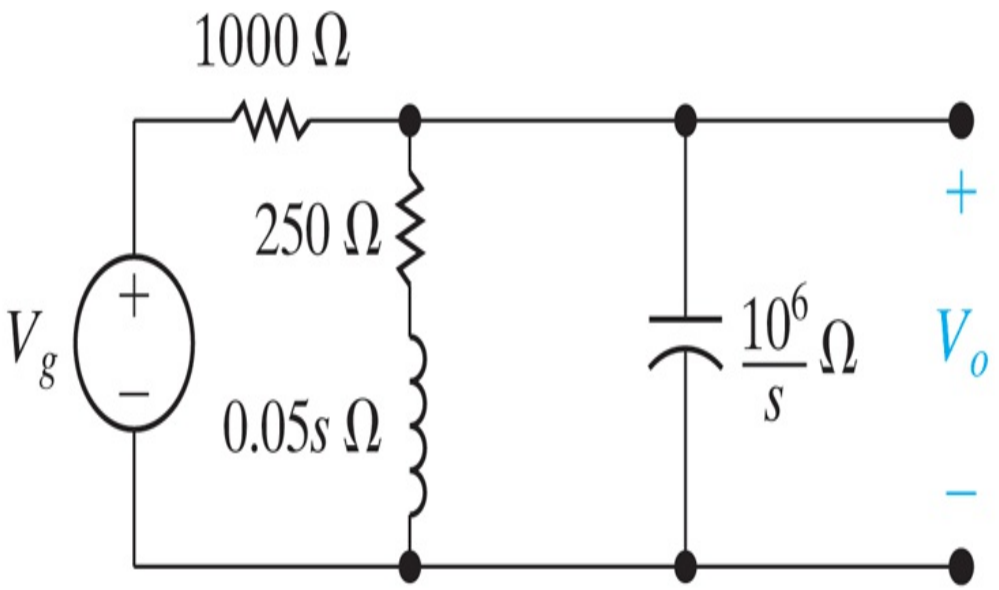


Figure 13.32 Full Alternative Text

5. Step 5: From the problem statement, the transfer function is the ratio of Vo/Vg. Writing a node-voltage equation at the upper node and summing the currents leaving the node gives

   Vo−Vg1000 + Vo250 + 0.05s=Vos106=0.

   Solving for Vo yields

   Vo=1000(s + 5000)Vgs2 + 6000s + 25 × 106.

   Hence, the transfer function is

   H(s)=VoVg=1000(s+5000)s2+6000s+25×106.

2. The poles of H(s) are the roots of the denominator polynomial. Therefore

   −p1=−3000−j4000,
   −p2=−3000+j4000.

   The zeros of H(s) are the roots of the numerator polynomial; thus, H(s) has a zero at

−z1=−5000.

Assessment Problem

Objective 3—Understand the definition and significance of the transfer function; be able to derive a transfer function

1. 13.9

    1. a) Derive the numerical expression for the transfer function Vo/Ig for the circuit shown.

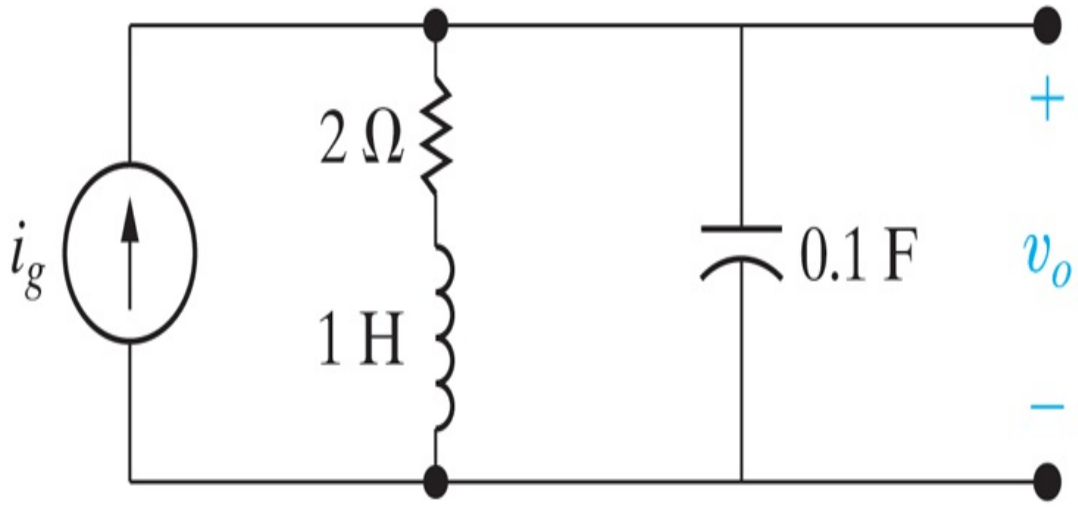
    2. b) Give the numerical value of each pole and zero of H(s).

    Answer:

    1. (a) H(s)=10(s+2)/(s2+2s+10);

    2. (b) −p1=−1+j3, −p2=−1−j3,

        −z=−2.



13.4-11 Full Alternative Text

SELF-CHECK: Also try Chapter Problem 13.51.

The Location of Poles and Zeros of H(s)

For linear lumped-parameter circuits, H(s) is always a rational function of s. Complex poles and zeros always occur in conjugate pairs. The poles of H(s) must lie in the left half of the s plane if the response to a bounded source (one whose values lie within some finite bounds) is to be bounded. The zeros of H(s) may lie in either the right half or the left half of the s plane.

With these general characteristics in mind, we next discuss the role that H(s) plays in determining the circuit's output.

13.5 The Transfer Function in Partial Fraction Expansions

Using the definition of a circuit's transfer function (Eq. 13.7), we can find the circuit's output by multiplying the transfer function and the circuit's input:

$$Y(s)=H(s)X(s). \quad (13.8)$$

We have already noted that H(s) is a rational function of s; X(s) also is a rational function of s for the time-domain functions of most interest in circuit analysis (see Table 12.1).

Expanding the right-hand side of Eq. 13.8 into a sum of partial fractions produces a term for each pole of H(s) and X(s). The terms generated by the poles of H(s) correspond to the transient components of the total response. The terms generated by the poles of X(s) correspond to the steady-state components of the response, which exist after the transient components have become negligible. Example 13.10 illustrates these general observations.

Example 13.10 Analyzing the Transfer Function of a Circuit

The circuit in Example 13.9 (Fig. 13.31) is driven by a voltage source whose voltage increases linearly with time, namely, $v_g=50tu(t)$ V.

1. Use the transfer function to find v_o.
2. Identify the transient component of the response.
3. Identify the steady-state component of the response.
4. Sketch v_o versus t for $0\le t\le 1.5$ ms.

Solution

1. From Example 13.9,

   $$H(s)=\frac{1000(s+5000)}{s^2+6000s+25\times 10^6}.$$

   The Laplace transform of the source voltage is 50/s2; therefore, the s-domain expression for the output voltage is

   $$V_o=\frac{1000(s+5000)}{(s^2+6000s+25\times 10^6)}\,\frac{50}{s^2}.$$

   The partial fraction expansion of Vo is

   $$V_o=\frac{K_1}{s+3000-j4000}+\frac{K_1^*}{s+3000+j4000}+\frac{K_2}{s^2}+\frac{K_3}{s}.$$

   We evaluate the coefficients K1,K2, and K3 by using the techniques described in Section 12.7:

   $$K_1=55\times 10^{-4}\ \angle 79.70^\circ;K_1^*=55\times 10^{-4}\ \angle -79.70^\circ,K_2=10,K_3=-4\times 10^{-4}.$$

   Using Table 12.3, the time-domain expression for vo is

   $$v_o=[105\times 10^{-4}e^{-3000t}\cos(4000t+79.70^\circ)+10t-4\times 10^{-4}]u(t)\text{V}.$$

2. The transient component of vo is

   $$105\times 10^{-4}e^{-3000t}\cos(4000t+79.70^\circ).$$

   Note that this term is generated by the poles (−3000+j4000) and (−3000−j4000) of the transfer function.

3. The steady-state component of the response is

   $$(10t-4\times 10^{-4})u(t)\ \text{V}.$$

   These two terms are generated by the second-order pole (K/s2) of the input voltage.

4. Figure 13.33 shows a sketch of vo versus t. Note that the deviation from the steady-state solution $10{,}000t - 0.4\text{mV}$ is imperceptible after approximately 1 ms.

Figure 13.33 The graph of vo versus t for Example 13.10.

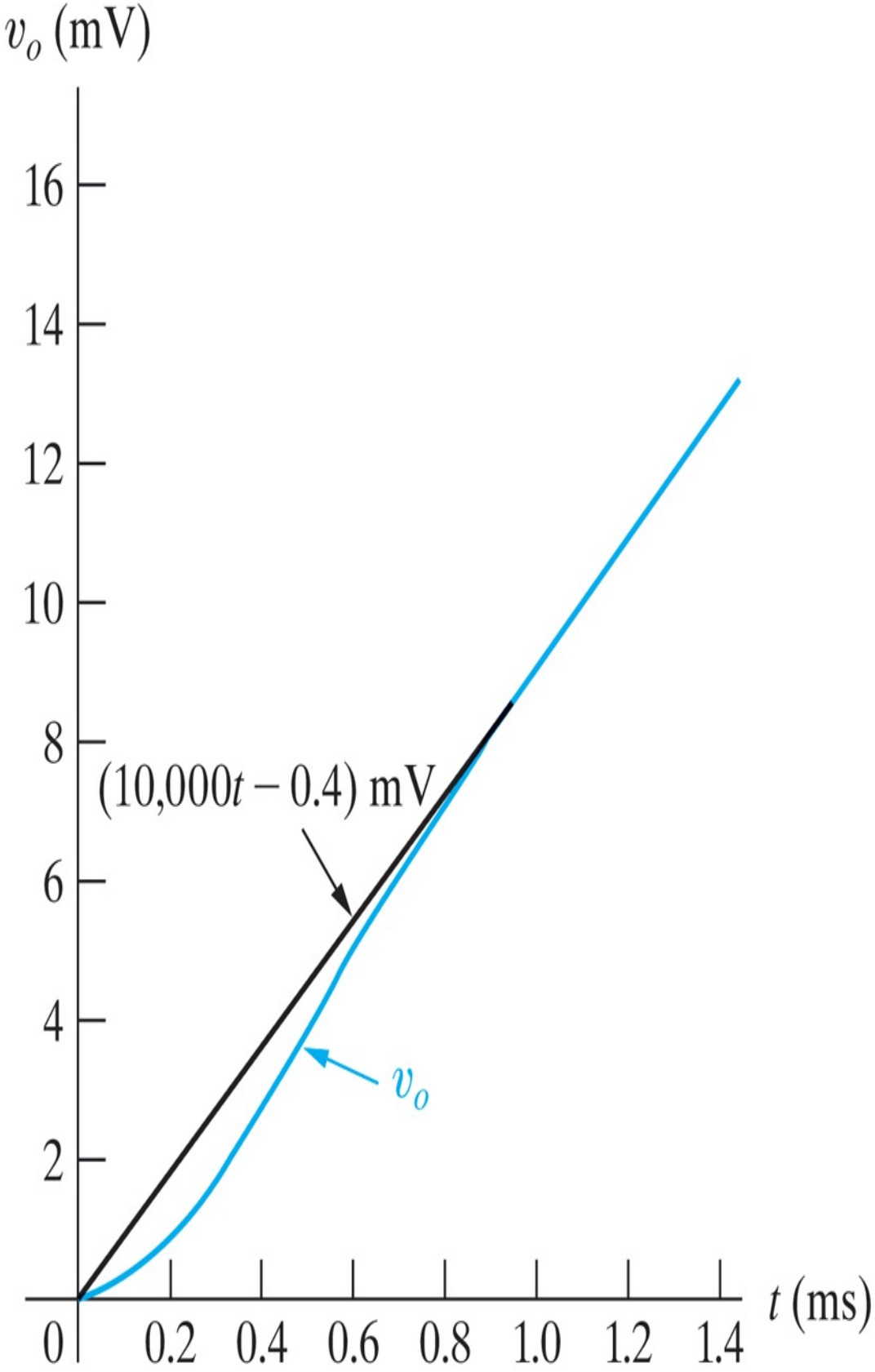

v_o (mV)
16
14
12
10
8
6
4
2
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
t (ms)
$(10{,}000t - 0.4)$ mV
v_o

Figure 13.33 Full Alternative Text

Observations on the Use of H(s) in Circuit Analysis

Example 13.10 related the components that make up a circuit's response to the poles of the transfer function, H(s), and the poles of the circuit's input in the s domain, using a partial fraction expansion. However, the example raises questions about driving a circuit with an increasing ramp voltage that generates an increasing ramp response. Eventually, excessive voltage will cause the circuit components to fail, and then our linear model is invalid. But some practical applications have input ramp functions that increase to some maximum value over a finite time interval, so the response to a ramp input is important. If the time it takes for the ramp to reach its maximum value is long compared with the time constants of the circuit, the solution assuming an unbounded ramp is valid for this finite time interval.

Here are some additional observations about a circuit's transfer function, defined in Eq. 13.7.

- If the circuit's input is delayed by a seconds,

  $$\mathcal{L}\{x(t-a)u(t-a)\}=e^{-as}X(s),$$

  then from Eq. 13.8, the circuit's output is

  $$Y(s)=H(s)X(s)e^{-as}.$$

  If $y(t)=\mathcal{L}^{-1}\{H(s)X(s)\}$, then,

  $$y(t-a)u(t-a)=\mathcal{L}^{-1}\{H(s)X(s)e^{-as}\}.$$

  Therefore, delaying the input by a seconds delays the output by a seconds. A circuit that exhibits this characteristic is **time invariant**.

- If the circuit's input is a unit impulse, the circuit's output equals the inverse transform of the transfer function. Thus, if

  $$x(t)=\delta(t), \quad \text{then} X(s)=1$$

and

$Y(s)=H(s)$.

Hence,

$y(t)=h(t)$,

so the inverse transform of the transfer function equals the unit impulse response of the circuit.

- A circuit's unit impulse response is also its natural response because applying an impulsive source is equivalent to instantaneously storing energy in the circuit (see Section 13.8). The subsequent release of this stored energy is the circuit's natural response (see Problem 13.84).

- A circuit's unit impulse response, h(t), contains enough information to compute the response to any source that drives the circuit. We can extract a circuit's response to an arbitrary source from the circuit's unit impulse response, using the convolution integral. This technique is demonstrated in the next section.

Assessment Problems

Objective 4—Know how to use a circuit's transfer function to calculate the circuit's impulse response, unit step response, and steady-state response to sinusoidal input

1. 13.10 Find (a) the unit step and (b) the unit impulse response of the circuit shown in Assessment Problem 13.9.

   Answer:

   1. (a) $[2 + (10/3)e^{-t}\cos(3t-126.87°)]u(t)$ V;

   2. (b) $10.54e^{-t}\cos(3t-18.43°)u(t)$V.

2. 13.11 The unit impulse response of a circuit is

   $v_o(t)=10{,}000e^{-70t}\cos(240t+\theta)V$,

   where $\tan\theta = 7\ 24$.

   1. Find the transfer function of the circuit.
   2. Find the unit step response of the circuit.

   Answer:

   1. (a) 9600s/(s2+140s+62,500);
   2. (b) [40e−70tsin 240t]u(t) V.

SELF-CHECK: Also try Chapter Problems 13.55 and 13.56.

13.6 The Transfer Function and the Convolution Integral

The **convolution integral** relates the output y(t) of a linear time-invariant circuit to the input x(t) of the circuit and the circuit's impulse response h(t). The integral relationship can be expressed in two ways:

Convolution Integral

$$y(t)=\int_{-\infty}^{\infty}h(\lambda)x(t-\lambda)\,d\lambda=\int_{-\infty}^{\infty}h(t-\lambda)x(\lambda)\,d\lambda. \quad (13.9)$$

We are interested in the convolution integral for several reasons.

- We can find a circuit's output for any input by working entirely in the time domain. This is beneficial when x(t) and h(t) are known only through experimental data. In such cases, using Laplace transform methods may be awkward or even impossible, as we would need to compute the Laplace transform of experimental data.

- The convolution integral introduces the concepts of memory and the weighting function into analysis. The concept of memory enables us to predict, to some degree, how closely the output waveform replicates the input waveform, using the impulse response (or the weighting function) h(t).

- The convolution integral provides a formal procedure for finding the inverse transform of products of Laplace transforms.

To derive Eq. 13.9, we assume that the circuit is linear and time invariant. Because the circuit is linear, the principle of superposition is valid, and because it is time invariant, the response delay and the input delay are the same. Consider Fig. 13.34, in which the block containing h(t) represents any linear time-invariant circuit whose impulse response is known, x(t) represents

the input signal and y(t) represents the output signal. We also assume the following:

Figure 13.34 A block diagram of a general circuit.

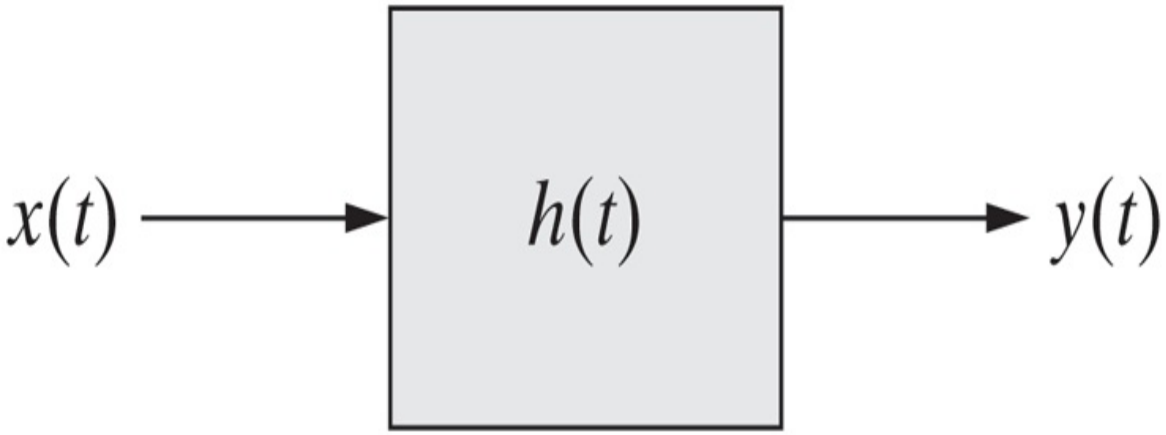
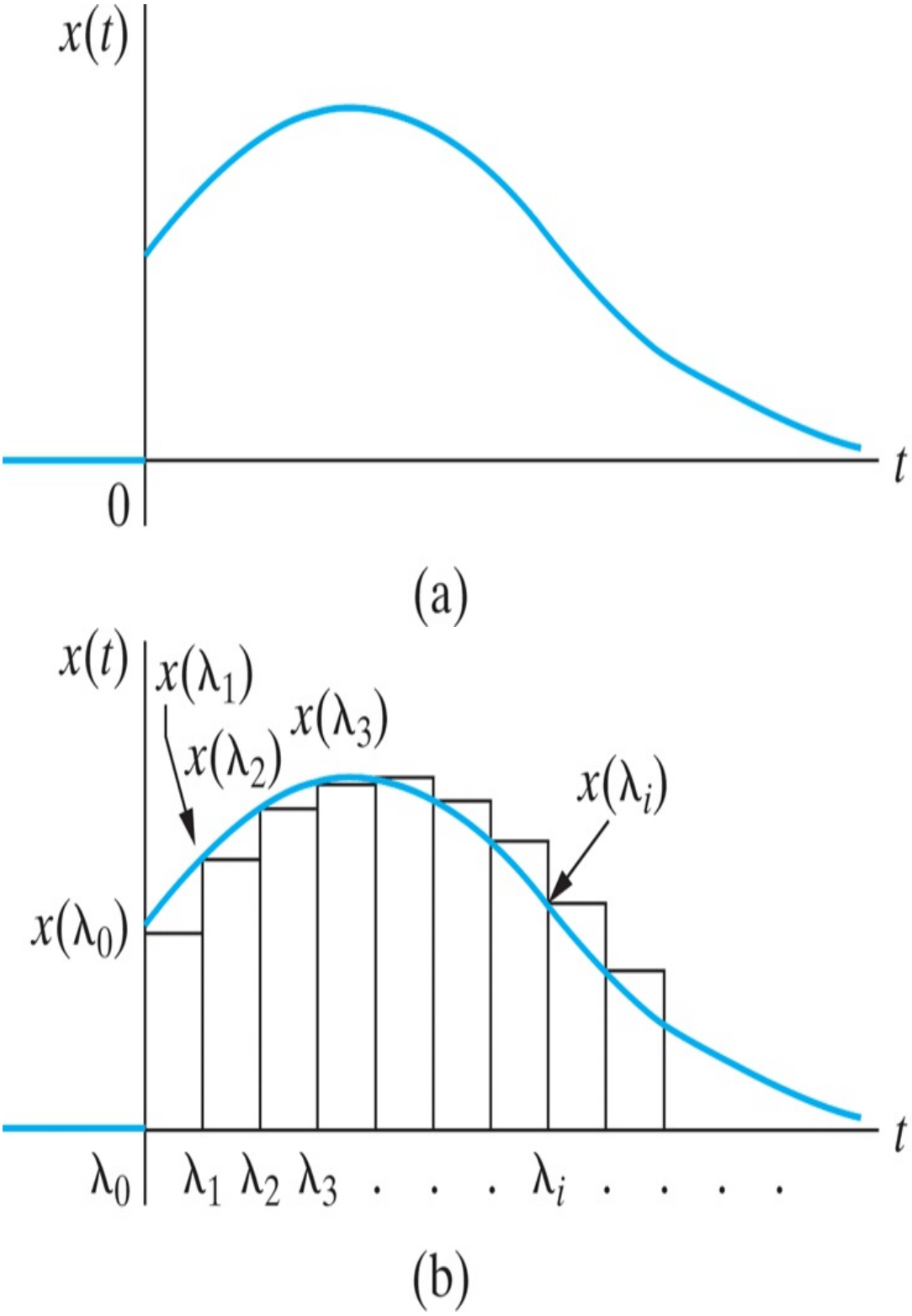


Figure 13.34 Full Alternative Text

- x(t) is the general signal shown in Fig. 13.35(a).

Figure 13.35 The excitation signal of x(t). (a) A general excitation signal. (b) Approximating x(t) with a series of pulses. (c) Approximating x(t) with a

series of impulses.



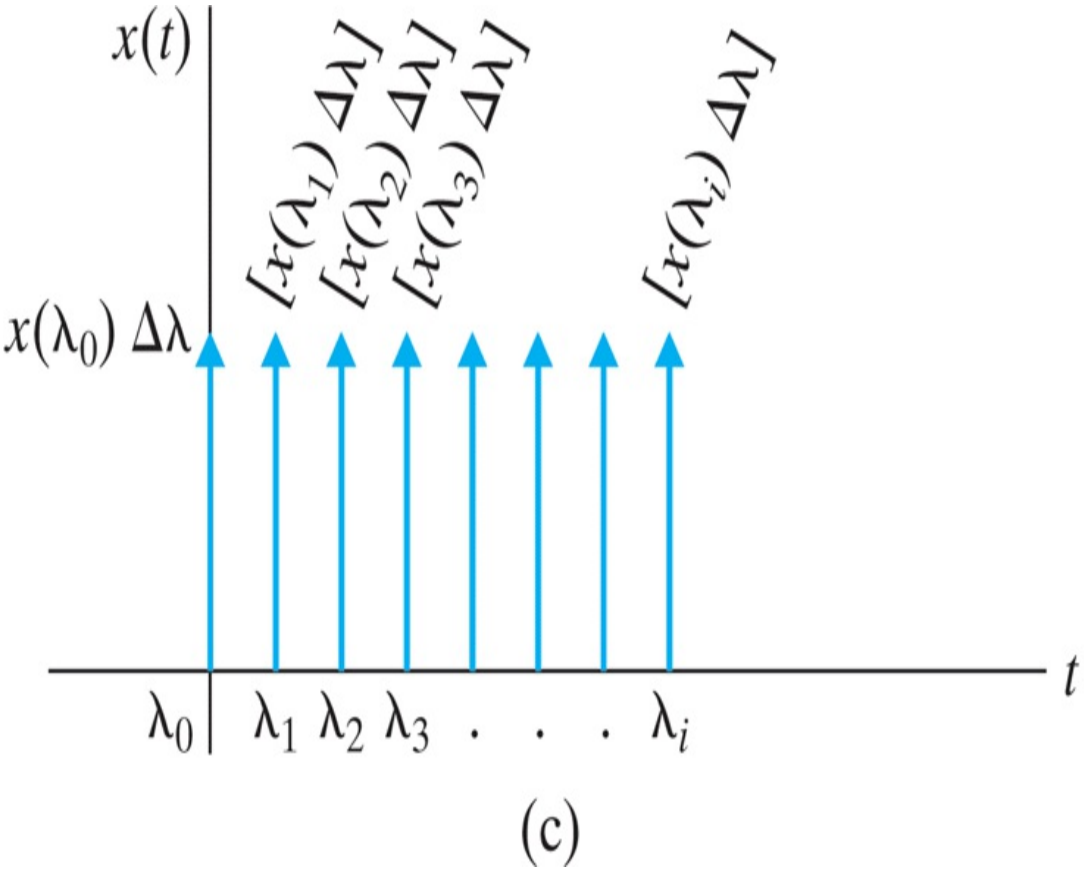


Figure 13.35 Full Alternative Text

- x(t)=0 for t<0−. Once we derive the convolution integral assuming x(t)=0 for t<0−, extending the integral to include excitation functions that exist for all time is straightforward.
- A discontinuity in x(t) at the origin (between 0− and 0+) is permitted.

Begin by approximating x(t) with a series of rectangular pulses of uniform width Δλ, as shown in Fig. 13.35(b). Thus

$$x(t)=x0(t)+x1(t)+\cdots+xi(t)+\cdots,$$

where xi(t) is a rectangular pulse that equals x(λi) between λi and λi+1 and is zero elsewhere. Note that the ith pulse can be expressed using step functions; that is,

$$xi(t)=x(\lambda i)\{u(t-\lambda i)-u[t-(\lambda i+\Delta\lambda)]\}.$$

Continue to approximate x(t) by making the pulse width Δλ so small that we can approximate the ith component using an impulse function of strength x(λi)Δλ. Figure 13.35(c) shows this impulse representation, where the brackets beside each arrow represent the impulse strength. The impulse representation of x(t) is

$$x(t)=x(\lambda 0)\Delta\lambda\delta(t-\lambda 0)+x(\lambda 1)\Delta\lambda\delta(t-\lambda 1)+\cdots \quad +x(\lambda i)\Delta\lambda\delta(t-\lambda i)+\cdots$$

.

When we represent x(t) with a series of impulse functions (which occur at equally spaced intervals of time, that is, at λ0, λ1, λ2, …), the output function y(t) is the sum of uniformly delayed impulse responses. The strength of each response depends on the strength of the impulse driving the circuit. For example, let's assume that the unit impulse response of the circuit represented by the box in Fig. 13.34 is the exponential decay function shown in Fig. 13.36(a). Then the approximation of y(t) is the sum of the impulse responses shown in Fig. 13.36(b).

Figure 13.36 The approximation of y(t). (a) The impulse response of the box shown in Fig. 13.34. (b) Summing the impulse responses.

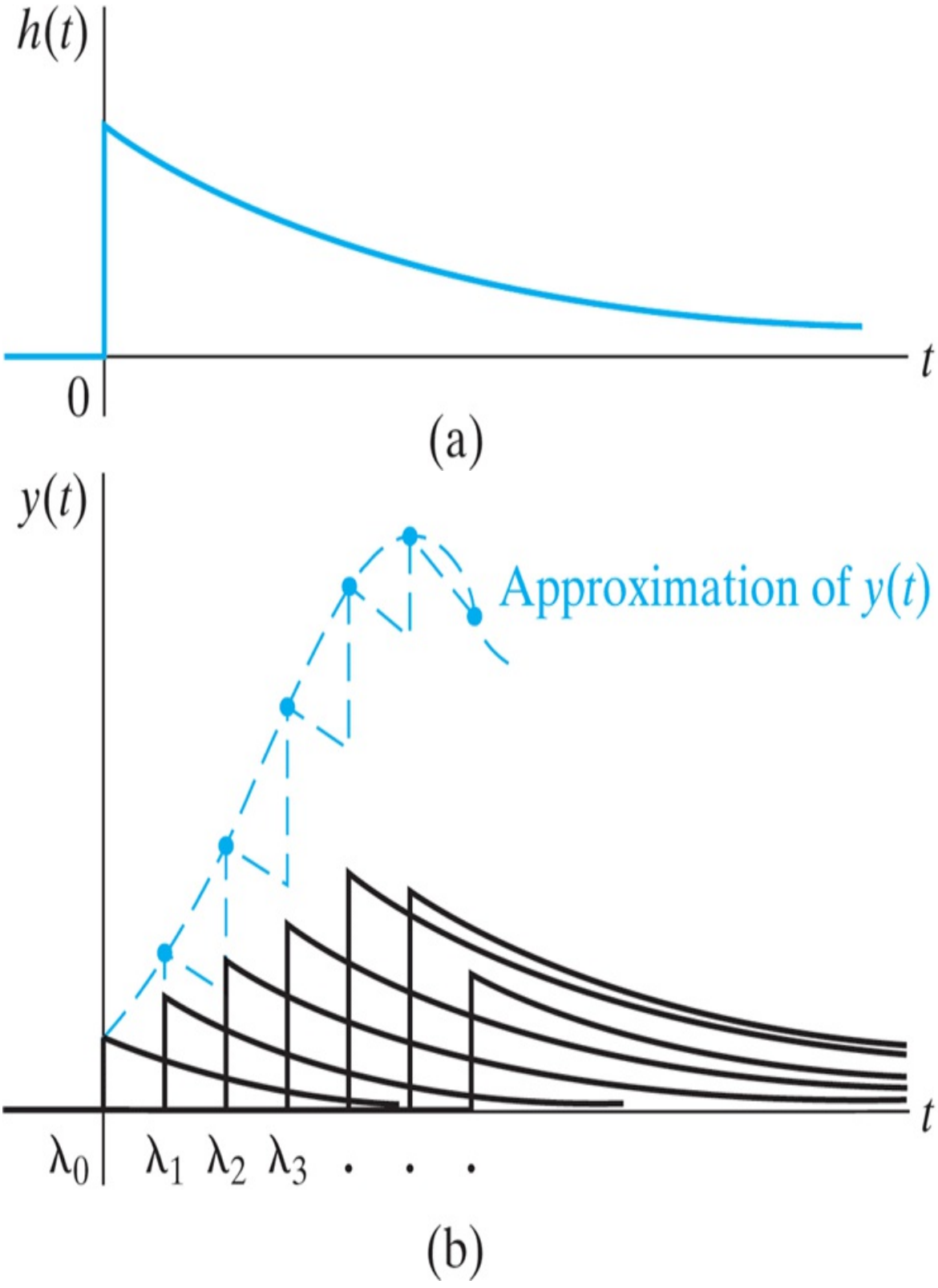


Figure 13.36 Full Alternative Text

Analytically, the expression for y(t) is

$$y(t)=x(\lambda 0)\Delta\lambda h(t-\lambda 0)+x(\lambda 1)\Delta\lambda h(t-\lambda 1)+x(\lambda 2)\Delta\lambda h(t-\lambda 2)+\cdots+x(\lambda i)\Delta\lambda h(t-\lambda i)+\cdots .$$

As $\Delta\lambda \rightarrow 0$, the sum approaches a continuous integral, or

$$\sum i=\infty\infty x(\lambda i)h(t-\lambda i)\Delta\lambda \rightarrow \int 0\infty x(\lambda)h(t-\lambda)\, d\lambda.$$

Therefore,

$$y(t)=\int 0\infty x(\lambda)h(t-\lambda)\, d\lambda.$$

If x(t) exists over all time, then the lower limit on the integral is $-\infty$. Thus, in general,

$$y(t)=\int -\infty\infty x(\lambda)h(t-\lambda)\, d\lambda,$$

which is the second form of the convolution integral given in Eq. 13.9.

To derive the first form of the integral in Eq. 13.9, we change the variable of integration in the second form of the integral. Let $u=t-\lambda$, and then $du=-d\lambda$, $u=-\infty$ when $\lambda=\infty$, and $u=+\infty$ when $\lambda=-\infty$. Now we can write

$$y(t)=\int \infty -\infty x(t-u)h(u)(-du),$$

or

$$y(t)=\int -\infty\infty x(t-u)h(u)(du).$$

Since u is a symbol of integration, this integral is equivalent to the first form of the convolution integral, Eq. 13.9.

The convolution integral relating y(t), h(t), and x(t), (Eq. 13.9), is often written using a shorthand notation:

$$y(t)=h(t)*x(t)=x(t)*h(t), \quad (13.10)$$

where the asterisk represents the integral relationship between h(t) and x(t).

Thus, h(t)*x(t) is read as “h(t) is convolved with x(t)” and implies that

$$h(t)*x(t)=\int_{-\infty}^{\infty}h(\lambda)x(t-\lambda)\,d\lambda,$$

whereas x(t)*h(t) is read as “x(t) is convolved with h(t)” and implies that

$$x(t)*h(t)=\int_{-\infty}^{\infty}x(\lambda)h(t-\lambda)\,d\lambda.$$

The integrals in Eq. 13.9 give the most general relationship for the convolution of two functions. However, when we apply the convolution integral, we can change the lower limit to zero and the upper limit to t. Then we can write Eq. 13.9 as

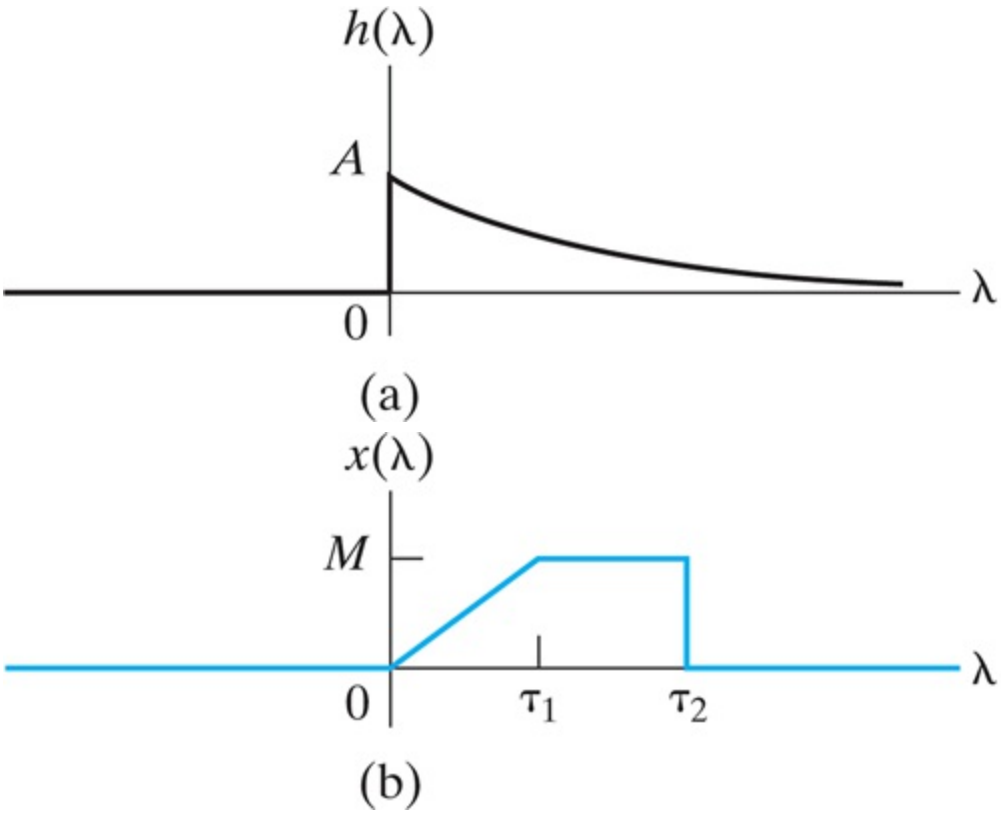
$$y(t)=\int_{0}^{t}h(\lambda)x(t-\lambda)\,d\lambda=\int_{0}^{t}x(\lambda)h(t-\lambda)\,d\lambda.$$

We change the limits for two reasons. First, for physically realizable circuits, h(t) is zero for $t<0$ because there is no impulse response before you apply an impulse. Second, we start measuring time at the instant we turn on the input x(t); therefore $x(t)=0$ for $t<0-$.

A graphic interpretation of the convolution integrals (Eq. 13.9) helps us use convolution as a computational tool. We begin with the first integral and assume that the circuit’s impulse response is the exponential decay function shown in Fig. 13.37(a) and its input function has the waveform shown in Fig. 13.37(b). In each of the plots, we replace t with λ, the symbol of integration. Replacing λ with −λ folds the excitation function over the vertical axis, as shown in Fig 13.37(d). The folding operation explains why Eq. 13.9 is called the *convolution* integral. Replacing −λ with t−λ slides the folded function to the right, as shown in Fig. 13.37(d).

Figure 13.37 A graphic interpretation of the convolution integral $\int_{0}^{t}h(\lambda)x(t$

−λ) dλ. (a) The impulse response. (b) The excitation function. (c) The folded excitation function. (d) The folded excitation function displaced t units. (e) The product h(λ)x(t−λ).



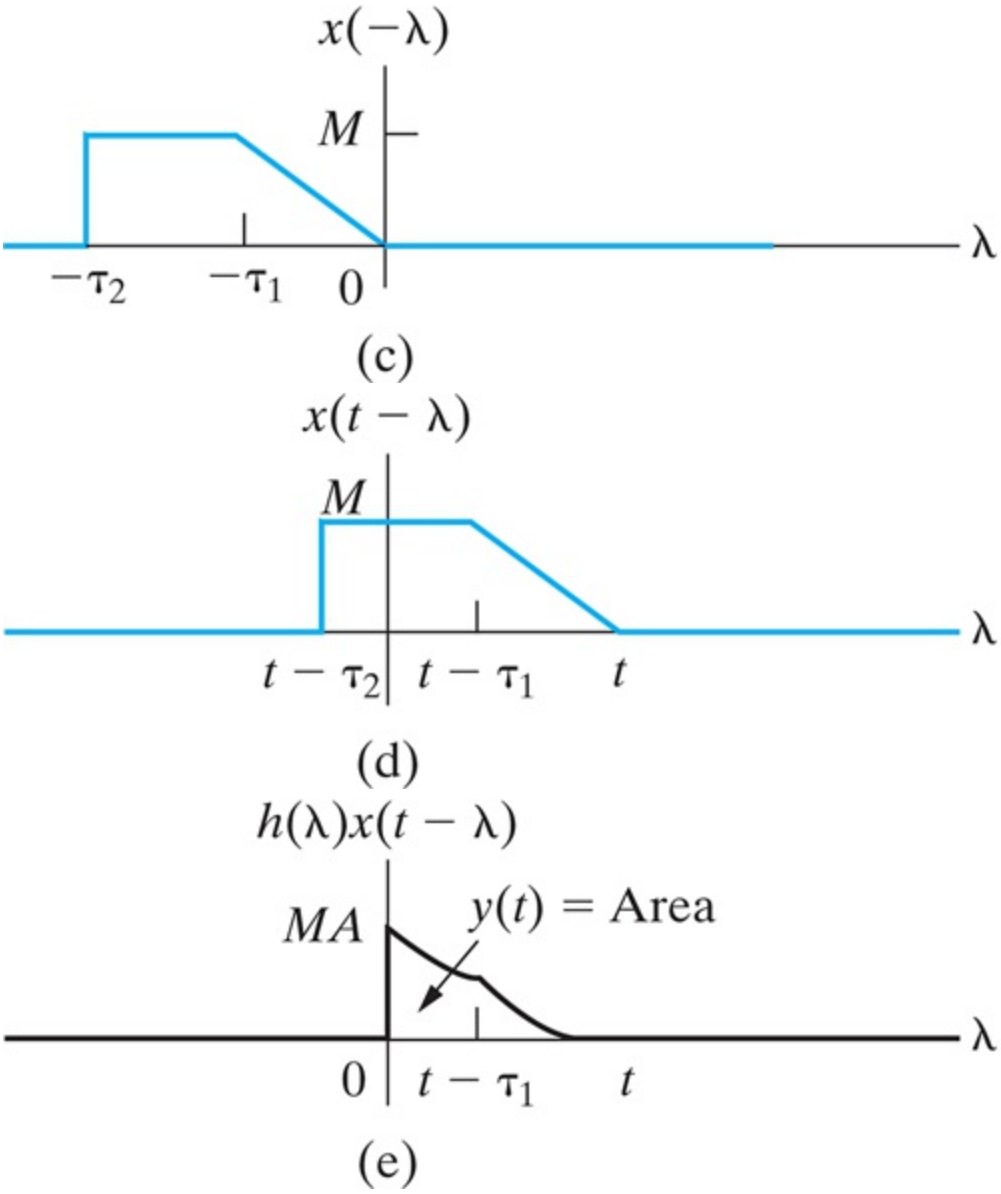
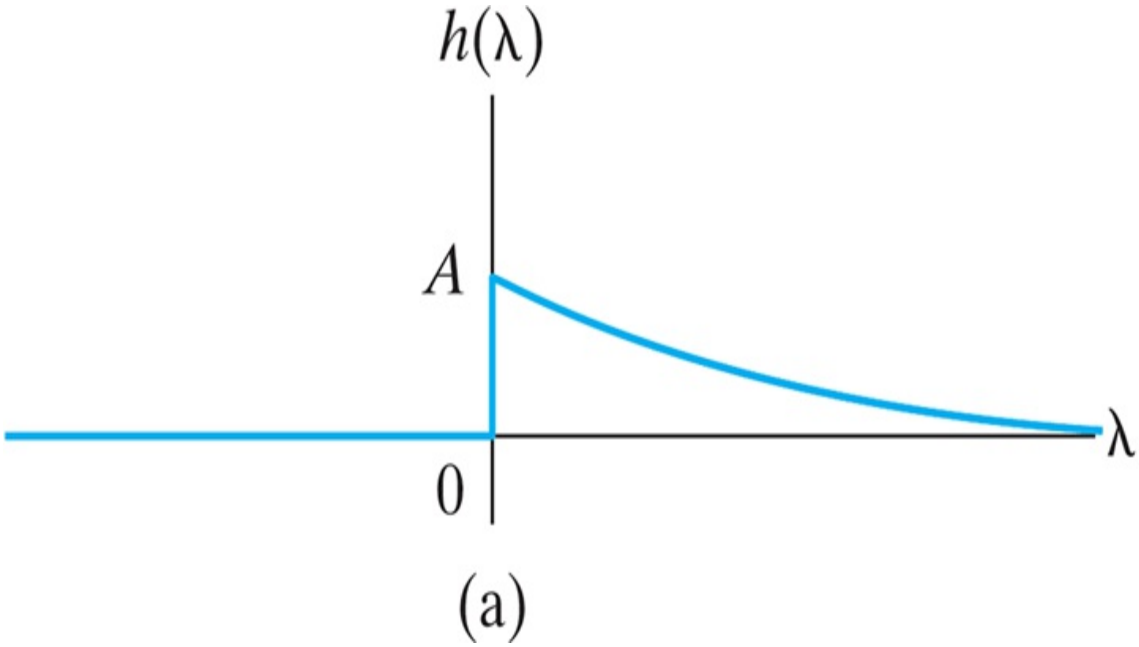


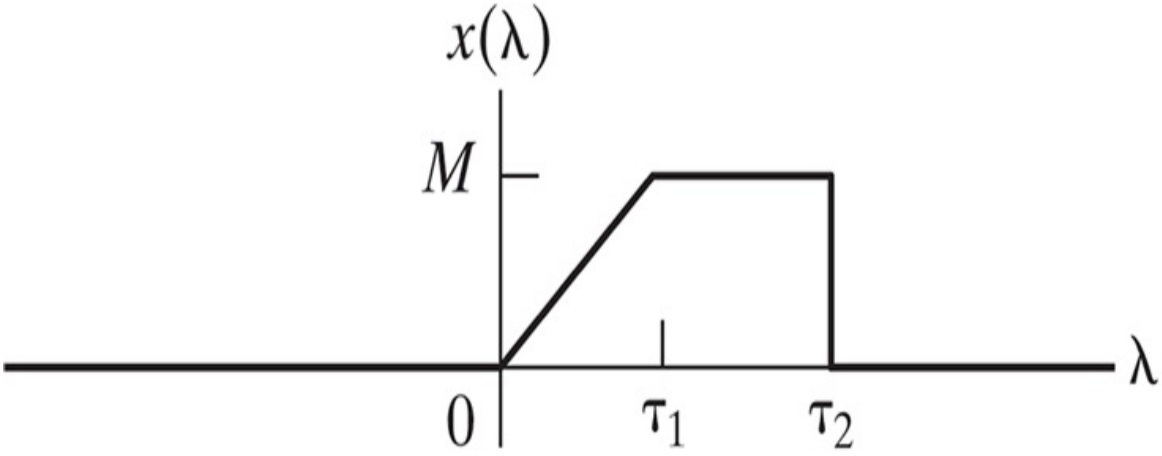
Figure 13.37 Full Alternative Text

At any specified value of t, the output function y(t) is the area under the product function h(λ)x(t−λ), as shown in Fig. 13.37(e). This plot explains why we can set the lower limit on the convolution integral to zero and the upper limit to t. For λ<0, the product h(λ)x(t−λ) is zero because h(λ) is zero. For λ>t, the product h(λ)x(t−λ) is zero because x(t−λ) is zero.

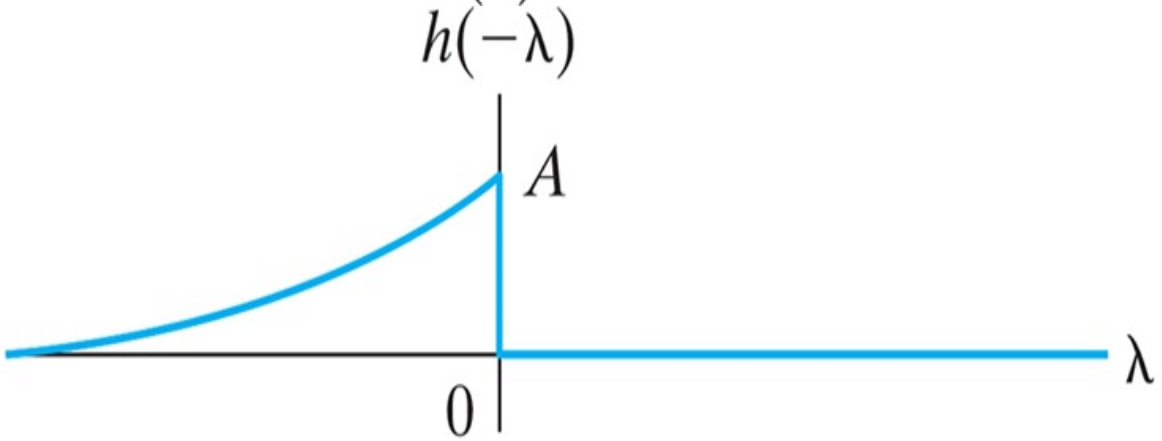
Figure 13.38 shows the second form of the convolution integral. Once again, the product function in Fig. 13.38(e) explains why we use zero for the lower limit and t for the upper limit.

Figure 13.38 A graphic interpretation of the convolution integral ∫0th(t −λ)x(λ) dλ. (a) The impulse response. (b) The excitation function. (c) The folded impulse response. (d) The folded impulse response displaced t units. (e) The product h(t −λ)x(λ).

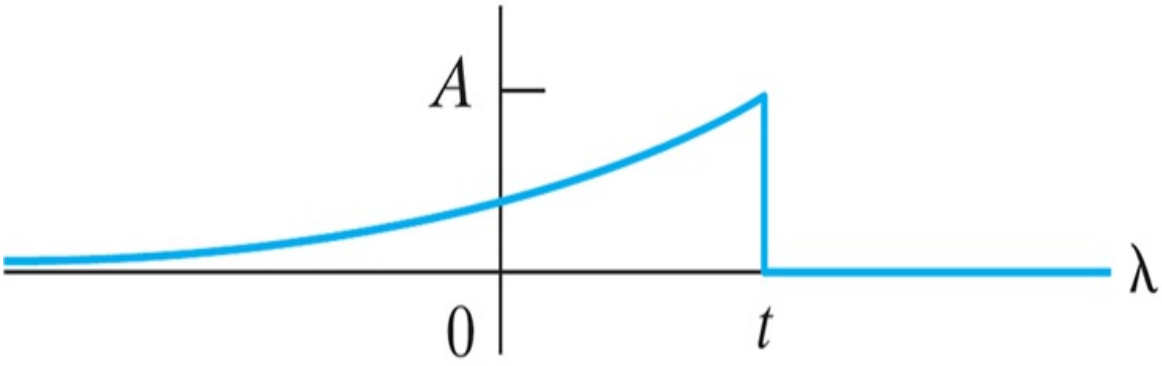



$x(\lambda)$
M
λ
0
τ_1
τ_2

(b)


$h(-\lambda)$
A
λ
0

(c)


$h(t-\lambda)$
A
λ
0
t

(d)

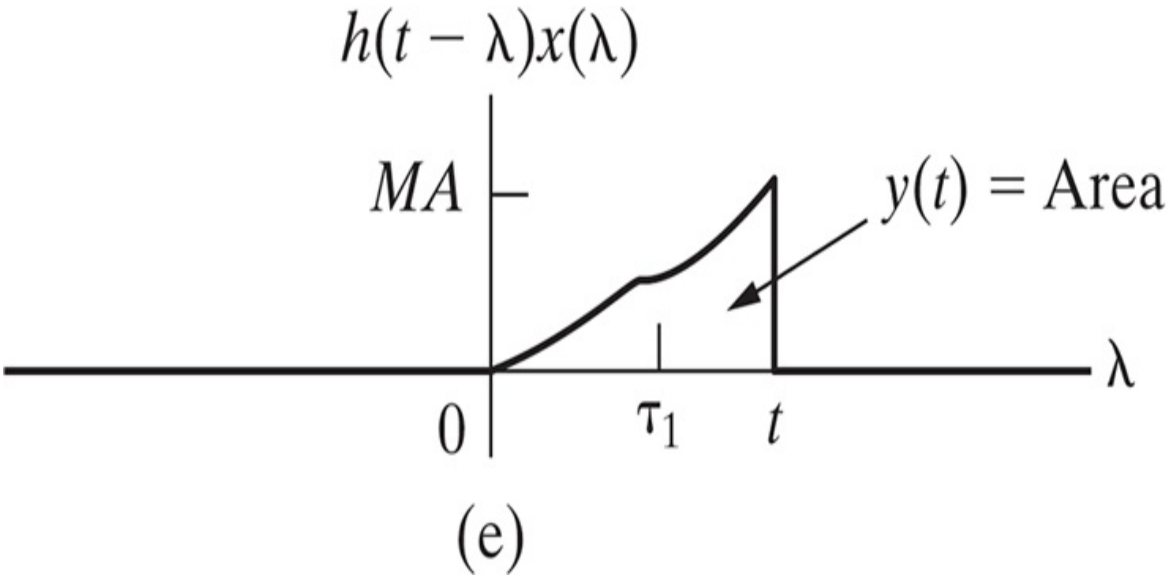


Figure 13.38 Full Alternative Text

Example 13.11 illustrates how to use the convolution integral and the unit impulse response to find a circuit's output in response to a given input.

Example 13.11 Using the Convolution Integral to Find an Output Signal

The input voltage vi for the circuit shown in Fig. 13.39(a) is shown in Fig. 13.39(b).

Figure 13.39 The circuit and excitation voltage for Example 13.11. (a) The circuit. (b) The input voltage.

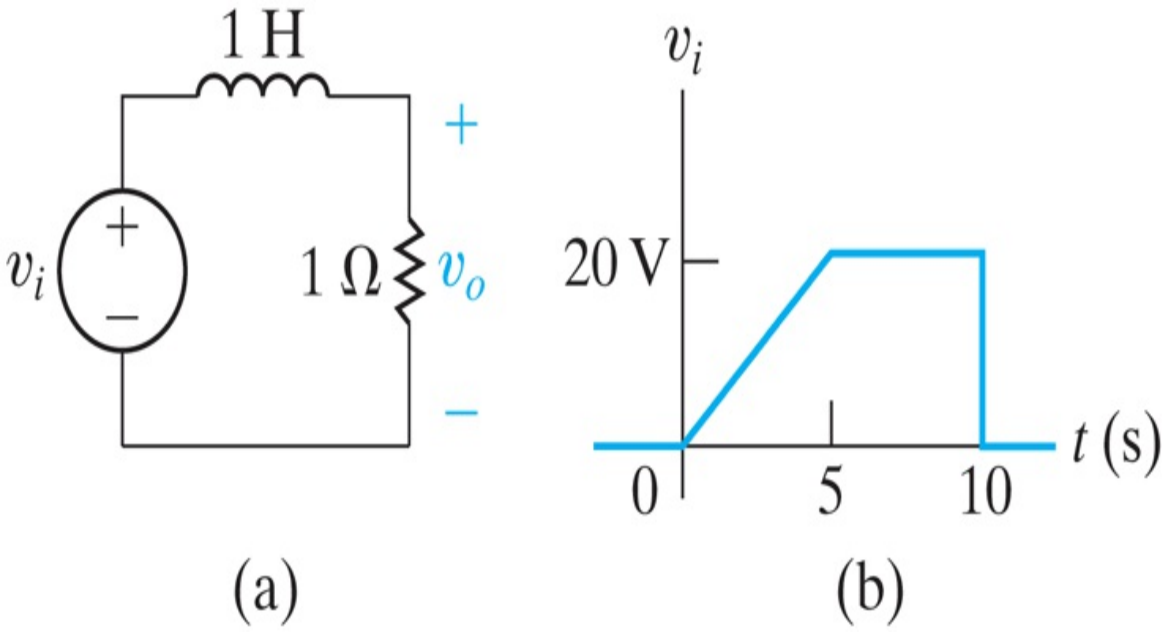


Figure 13.39 Full Alternative Text

1. Use the convolution integral to find vo.
2. Plot vo over the range of 0≤t≤15 s.

Solution

1. Begin by finding the unit impulse response of the circuit. The s-domain equivalent of the circuit in Fig. 13.39(a) replaces the inductor with an impedance of s Ω and the two voltages with Vo and Vi. Using voltage division,

   Vo=1s+1Vi.

   When vi is a unit impulse function δ(t), Vi=1 and H(s)=Vo/ Vi=1/(s+1). Then,

   vo=h(t)=e−tu(t),

   from which

$h(\lambda)=e^{-\lambda}u(\lambda)$.

Using the first form of the convolution integral in Eq. 13.9, we construct the impulse response and folded input function shown in Fig. 13.40, which helps us select the convolution integral limits. As we slide the folded input function to the right, we break the integration into three intervals: $0 \le t \le 5$; $5 \le t \le 10$; and $10 \le t \le \infty$. The integration intervals correspond to points where the input function's definition changes. Figure 13.41 depicts the position of the folded input function for each interval.

Figure 13.40 The impulse response and the folded input function for Example 13.11.

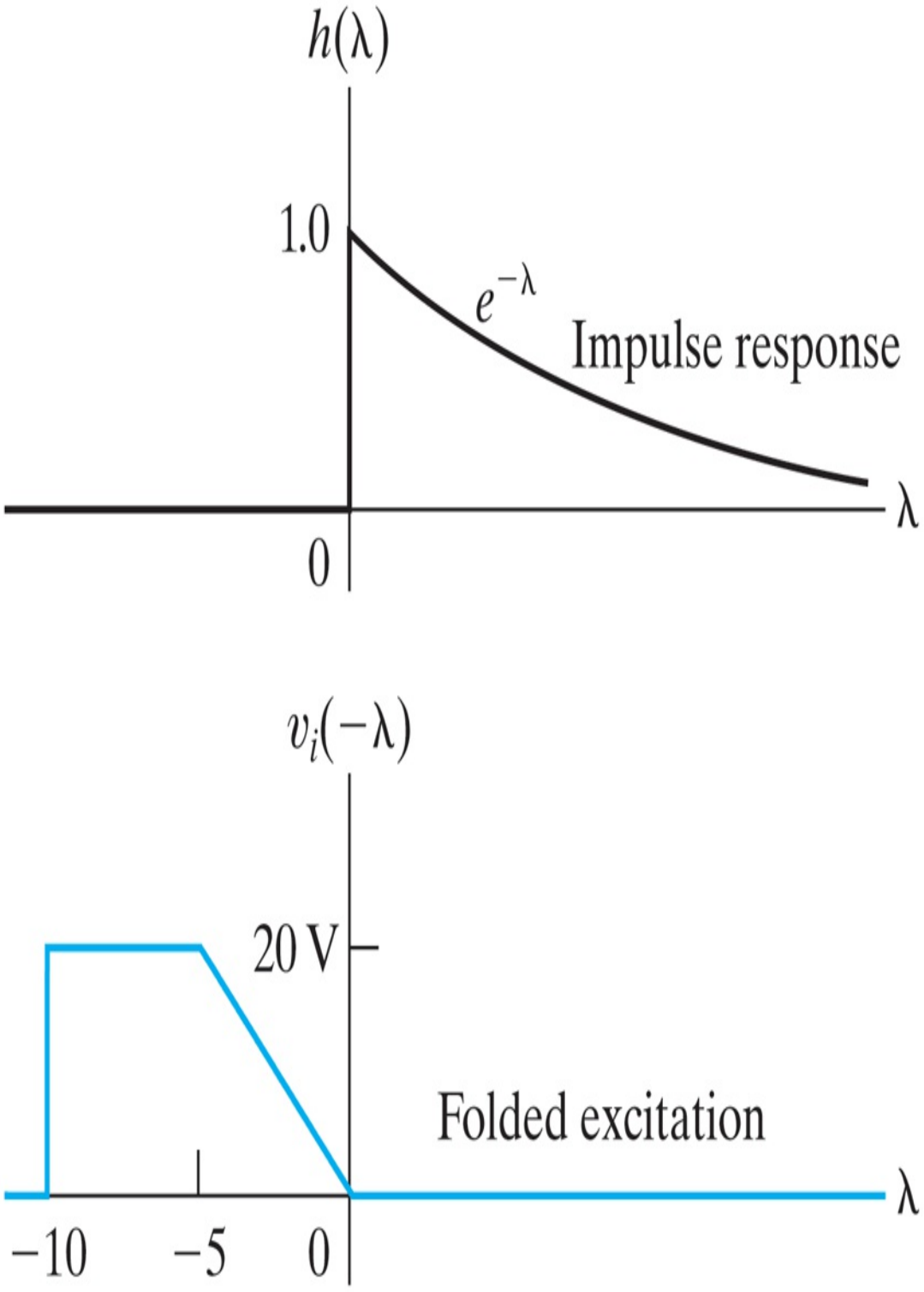
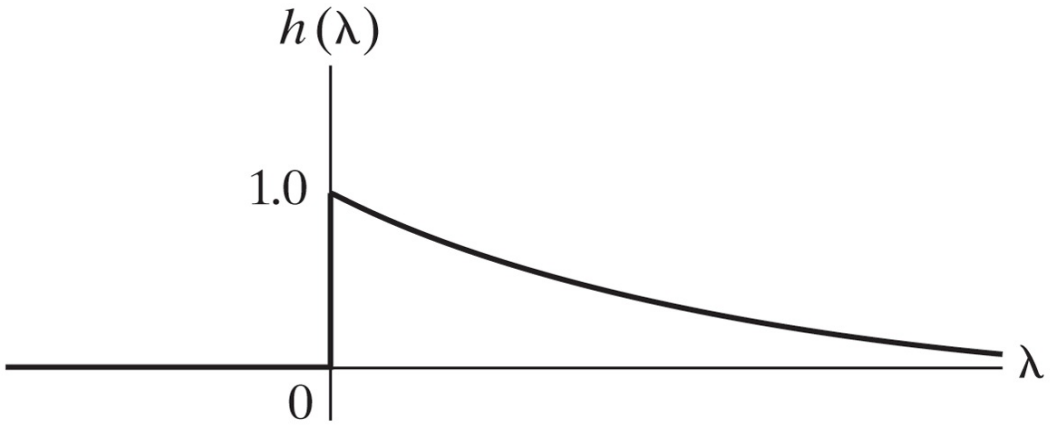
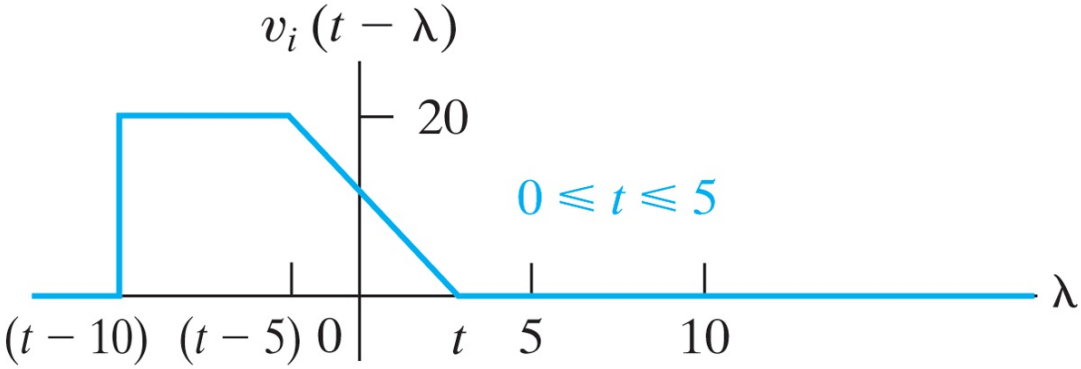
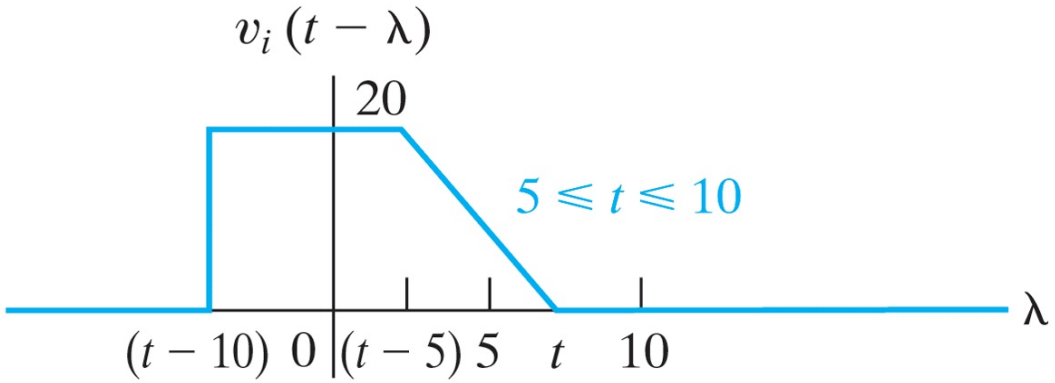


Figure 13.40 Full Alternative Text

Figure 13.41 The displacement $v_i(t-\lambda)$ of for three different time intervals.


$h(\lambda)$
1.0
0
λ


$v_i(t-\lambda)$
20
$0 \leqslant t \leqslant 5$
λ
$(t-10)$
$(t-5)$
0
t
5
10


$v_i(t-\lambda)$
20
$5 \leqslant t \leqslant 10$
λ
$(t-10)$
0
$(t-5)$
5
t
10

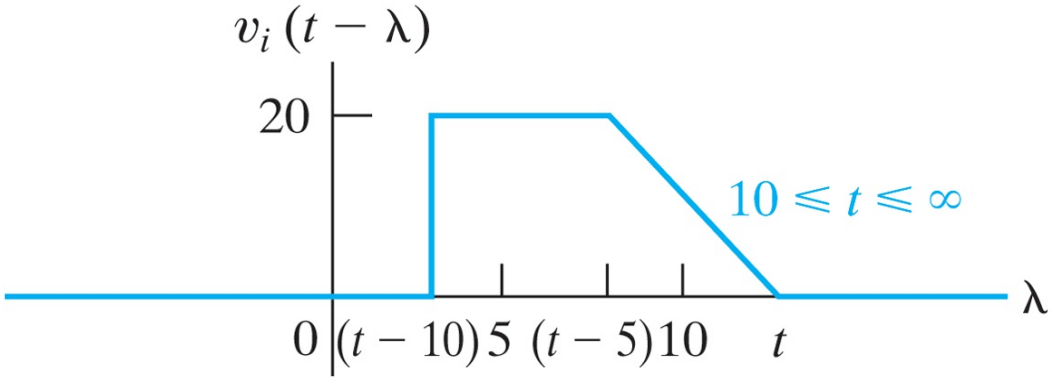

$v_i(t-\lambda)$
20
$10 \leqslant t \leqslant \infty$
λ
0
$(t-10)$
5
$(t-5)$
10
t

Figure 13.41 Full Alternative Text

The expression for vi in the time interval $0 \le t \le 5$ is

$v_i = 4t, \quad 0 \le t \le 5$ s.

Hence, the expression for the folded input function in the interval $t - 5 \le \lambda \le t$ is

$v_i(t-\lambda) = 4(t-\lambda), \quad t-5 \le \lambda \le t.$

In the other two time intervals, vi is a constant.

We can now set up the three convolution integrals to find vo. For $0 \le t \le 5$ s :

$v_o = \int_0^t 4(t-\lambda)e^{-\lambda}\, d\lambda = 4(e^{-t}+t-1)$ V.

For $5 \le t \le 10$ s,

$v_o = \int_0^{t-5} 20e^{-\lambda}\, d\lambda + \int_{t-5}^{t} 4(t-\lambda)e^{-\lambda}\, d\lambda = 4(5+e^{-t}-e^{-(t-5)})$ V.

For $10 \le t \le \infty$ s,

$v_o = \int_{t-10}^{t-5} 20e^{-\lambda}\, d\lambda + \int_{t-5}^{t} 4(t-\lambda)e^{-\lambda}\, d\lambda = 4(e^{-t}-e^{-(t-5)}+5e^{-(t-10)})$ V.

2. Values for vo are shown graphically in Fig. 13.42.

Figure 13.42 The voltage response versus time for Example 13.11.

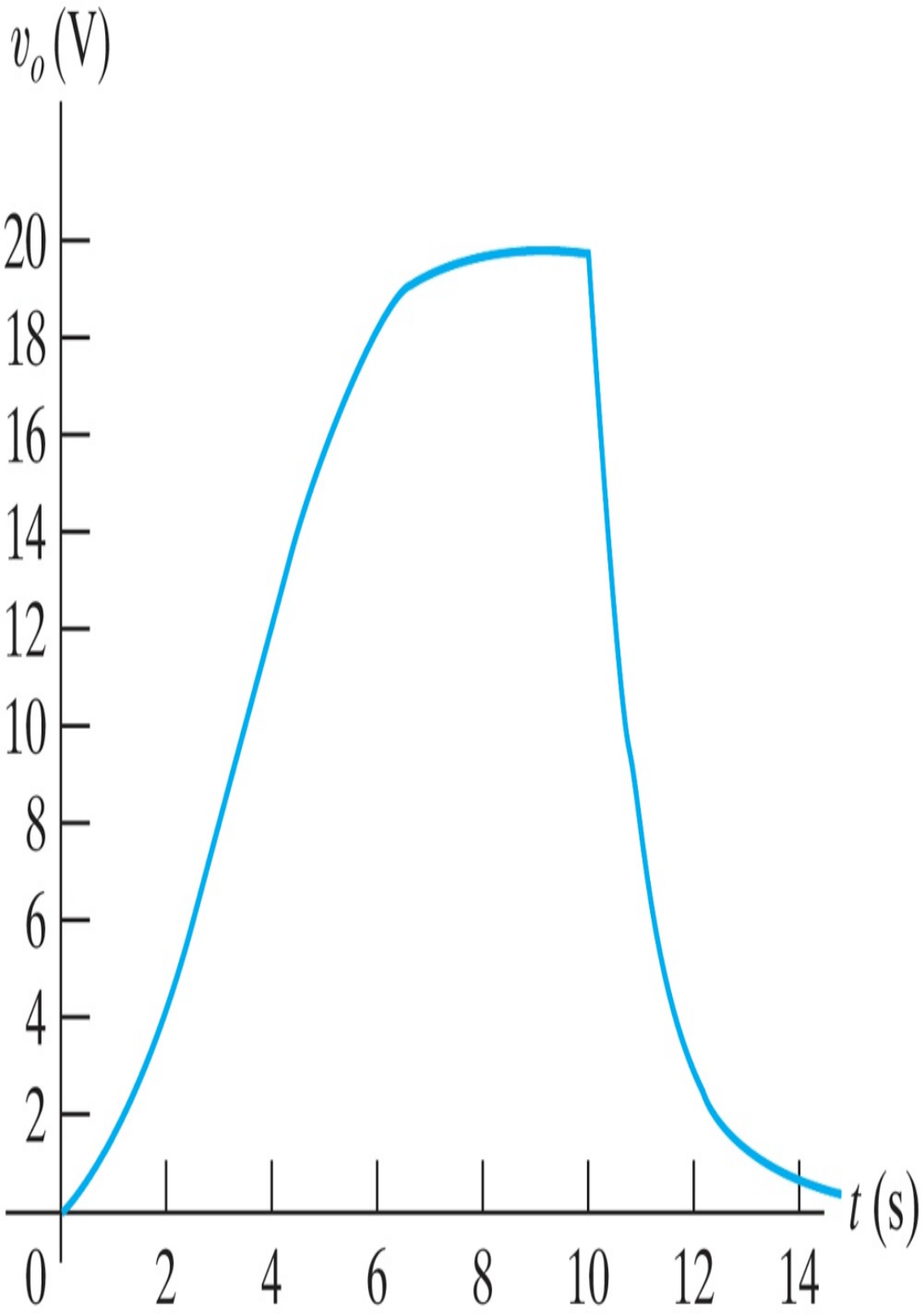


Figure 13.42 Full Alternative Text

SELF-CHECK: Assess your understanding of convolution by trying Chapter Problems 13.61 and 13.62.

The Concepts of Memory and the Weighting Function

We mentioned at the beginning of this section that the convolution integral introduces the concepts of memory and the weighting function into circuit analysis. The graphical interpretation of the convolution integral is the easiest way to understand these concepts. We can view the folding and sliding of a circuit's input function on a time scale divided into past, present, and future time intervals. The vertical axis represents the present value of x(t), with past values to the right of the vertical axis and future values to the left. Figure 13.43 shows this description of x(t), using the input function from Example 13.11.

Figure 13.43 The past, present, and future values of the input function.

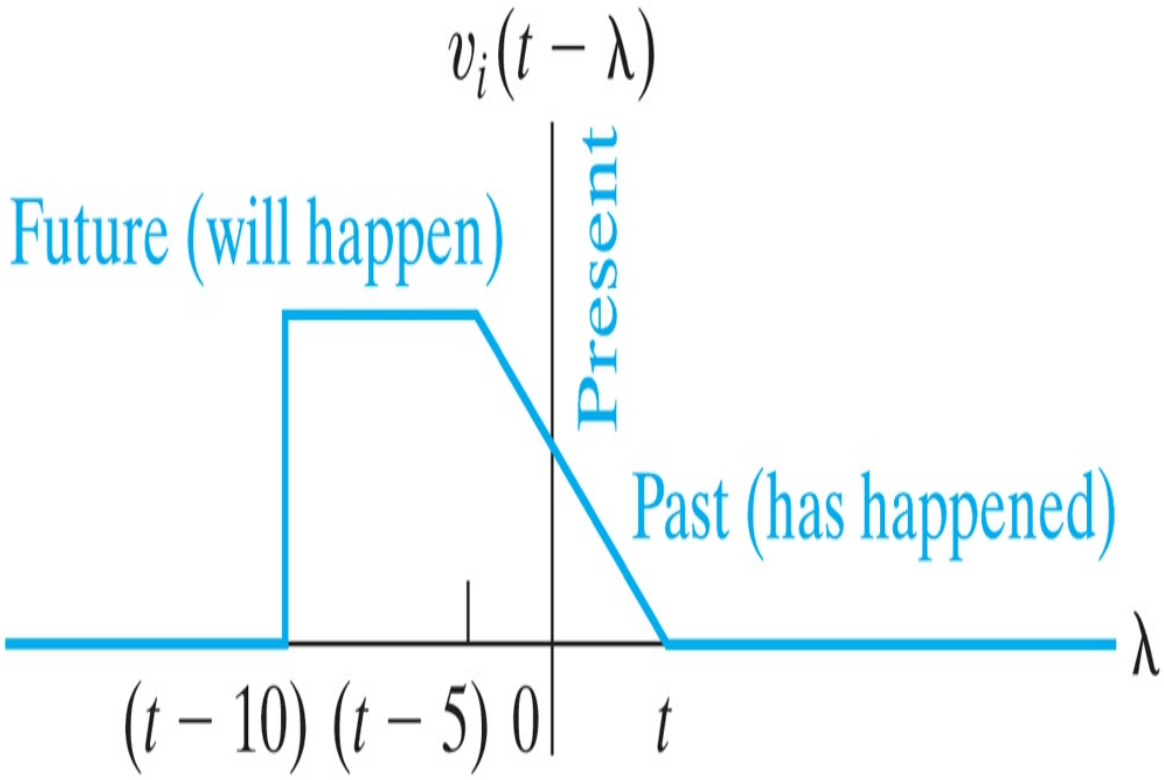


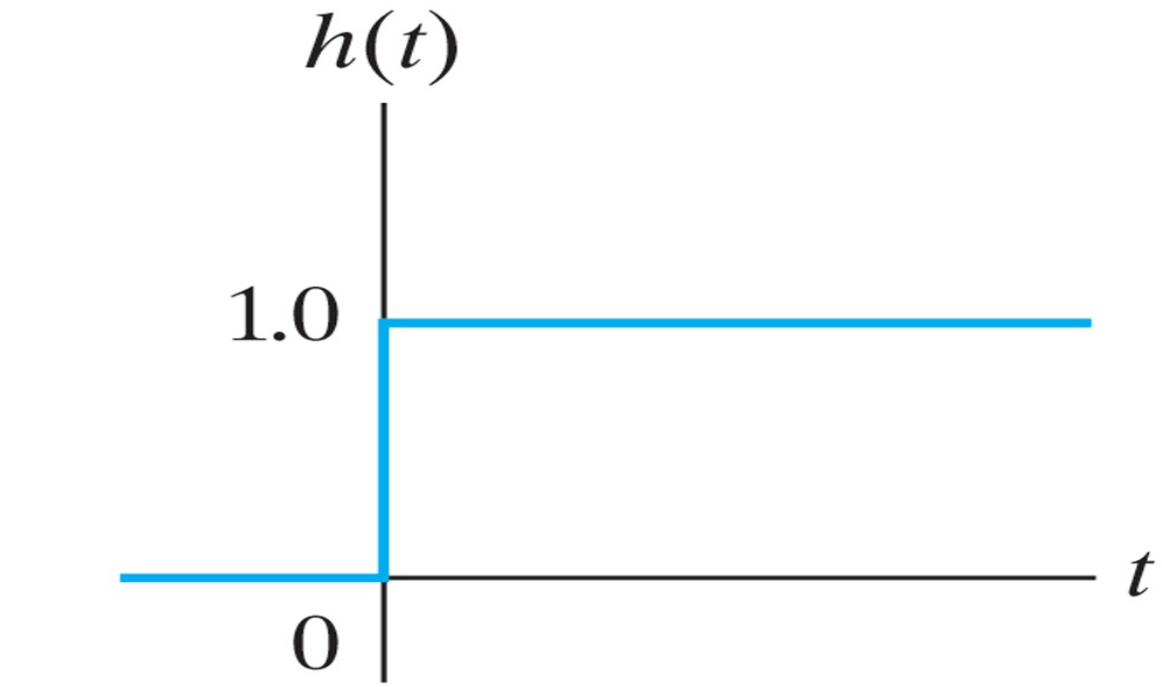
Figure 13.43 Full Alternative Text

When we combine the past, present, and future views of x(t−τ) with the impulse response of the circuit, we see that the impulse response weights x(t) according to present and past values. For example, Fig. 13.41 shows that the impulse response in Example 13.11 gives less weight to past values of x(t) than to the present value of x(t). In other words, the circuit "remembers" less and less about past input values. Therefore, in Fig. 13.42, vo quickly approaches zero when the present value of the input is zero (that is, when t>10 s). Since the present value of the input receives more weight than the past values, the output quickly approaches the present value of the input.

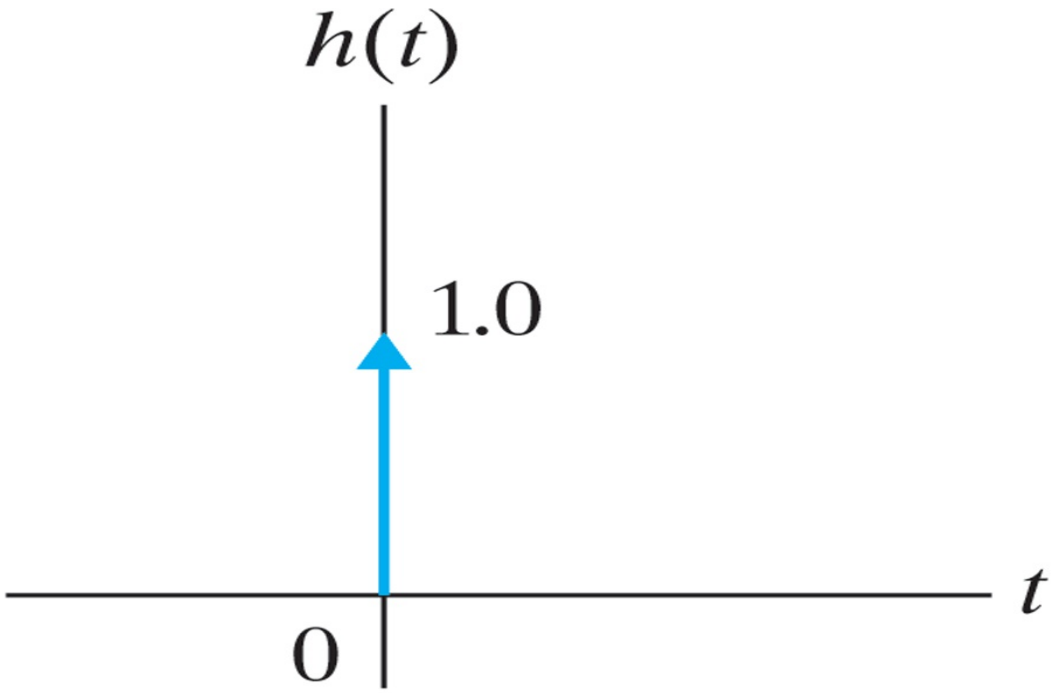
Because the convolution integral uses the product of x(t−λ) and h(λ), the impulse response is considered the circuit's **weighting function**. The weighting function, in turn, determines how much memory the circuit has. **Memory** represents how accurately the circuit's response matches its input. For example, if the impulse response, or weighting function, is flat, as shown in Fig. 13.44(a), it gives equal weight to all values of x(t), past and present. Such a circuit has a perfect memory. However, if the impulse response is an

impulse function, as shown in Fig. 13.44(b), it gives no weight to past values of x(t) and the circuit has no memory.

Figure 13.44 Weighting functions. (a) Perfect memory. (b) No memory.



(a)



(b)

Figure 13.44 Full Alternative Text

The more memory a circuit has, the more distortion exists between the circuit's input waveform and its output. We can demonstrate this by assuming that the circuit has no memory, that is, $h(t)=A\delta(t)$, and then noting from the convolution integral that

$$y(t) = \int_0^t h(\lambda)x(t-\lambda)\,d\lambda = \int_0^t A\delta(\lambda)x(t-\lambda)\,d\lambda = Ax(t).$$

The expression for $y(t)$ shows that, if the circuit has no memory, the output is a scaled replica of the input.

The circuit shown in Example 13.11 illustrates the distortion between input and output for a circuit that has some memory. This distortion is clear when we plot the input and output waveforms on the same graph, as in Fig. 13.45.

Figure 13.45 The input and output waveforms for Example 13.11.

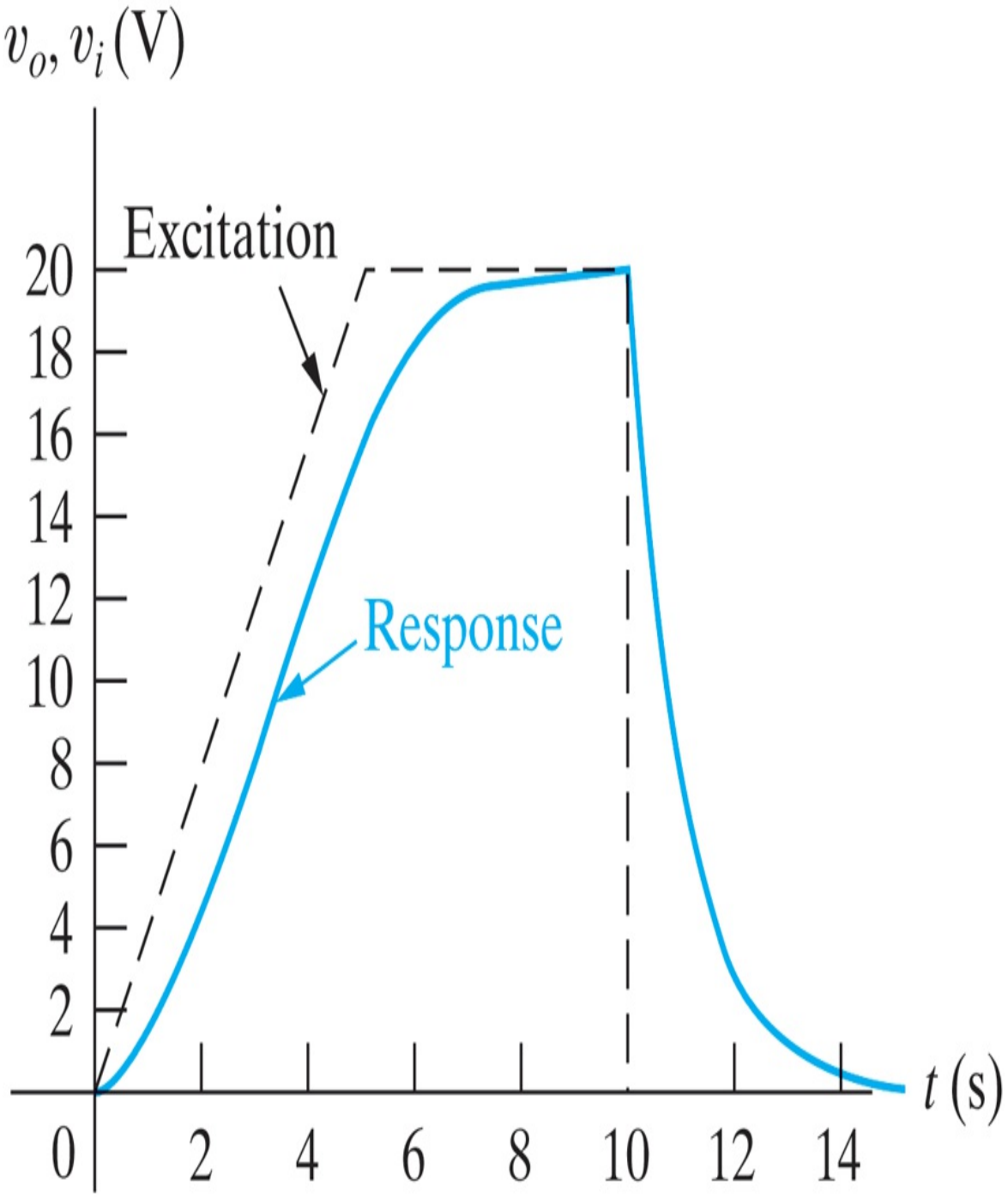


Figure 13.45 Full Alternative Text

13.7 The Transfer Function and the Steady-State Sinusoidal Response

Once we have computed a circuit's transfer function, we can use it to find the steady-state response to a sinusoidal input. To show this, we assume that

$$x(t)=A\cos(\omega t+\phi),$$

and then we use Eq. 13.8 to find the steady-state solution of y(t). To find the Laplace transform of x(t), we first write x(t) as

$$x(t)=A\cos\omega t\cos\phi-A\sin\omega t\sin\phi,$$

from which

$$X(s)=(A\cos\phi)s\,s^2+\omega^2-(A\sin\phi)\omega\,s^2+\omega^2=A(s\cos\phi-\omega\sin\phi)\,s^2+\omega^2.$$

Substituting the expression for X(s) into Eq. 13.8 gives the s-domain expression for the response:

$$Y(s)=H(s)A(s\cos\phi-\omega\sin\phi)s2+\omega2.$$

Think about the partial fraction expansion of Y(s). The number of terms in the expansion depends on the number of poles of H(s). Because H(s) is not specified, we write the expansion of Y(s) as

$$Y(s)=K\,1\,s-j\omega+K\,1\,*\,s+j\omega+\sum\text{ terms generated by the poles of H(s).}$$

The first two terms in the partial fraction expansion correspond to the complex conjugate poles of the sinusoidal input because $s2+\omega2=(s-j\omega)(s+j\omega)$. But the terms corresponding to the poles of H(s) do not contribute to the steady-state response of y(t), because all these poles lie in the left half of the s plane. Consequently, the corresponding time-domain terms approach zero as t increases. Thus, only the first two partial fraction terms determine

the steady-state response, so we only have to calculate a single partial fraction coefficient, K1:

K 1 = H(s)A(s cos ϕ−ω sin ϕ) s+jω | s=jω = H(jω)A(jω cos ϕ−ω sin ϕ) 2jω = H(jω)A(cos ϕ+j sin ϕ) 2 = 1 2 H(jω)A e jϕ .

In general, H(jω) is a complex number, which we can write in polar form as

H(jω)=|H(jω)|ejθ(ω).

We see that the transfer function's magnitude, |H(jω)|, and its phase angle, θ(ω), vary with the frequency ω. Substituting the polar form for H(jω) into the equation for K1 and simplifying, we see that the expression for K1 becomes

K1=A2|H(jω)|ej[θ(ω)+ϕ].

We find the steady-state component for y(t) by inverse-transforming the first two terms in the partial fraction expansion of Y(s), ignoring the terms generated by the poles of H(s). Thus

Sinusoidal Steady-State Response computed Using a Transfer Function

yss(t)=A|H(jω)| cos [ωt+ϕ+θ(ω)],(13.11)

which tells us how to find a circuit's steady-state response to a sinusoidal input using the circuit's transfer function:

- Determine the input sinusoid's magnitude, A, frequency, ω, and phase angle, ϕ
- Evaluate the circuit's transfer function, H(s), for s=jω.

- Transform H(jω) into polar form, with a magnitude |H(jω)| and a phase angle θ.
- Write the steady-state output as a cosine with the amplitude A|H(jω)|, a phase angle of ϕ+θ, and a frequency of ω.

Example 13.12 uses a circuit's transfer function to find its sinusoidal steady-state response.

Example 13.12 Using the Transfer Function to Find the Steady-State Sinusoidal Response

The circuit from Example 13.9 is shown in Fig. 13.46.

The sinusoidal source voltage is

$\upsilon_g = 120\cos(5000t+30°)\text{V}$.

Find the steady-state expression for vo.

Figure 13.46 The circuit for Example 13.12.

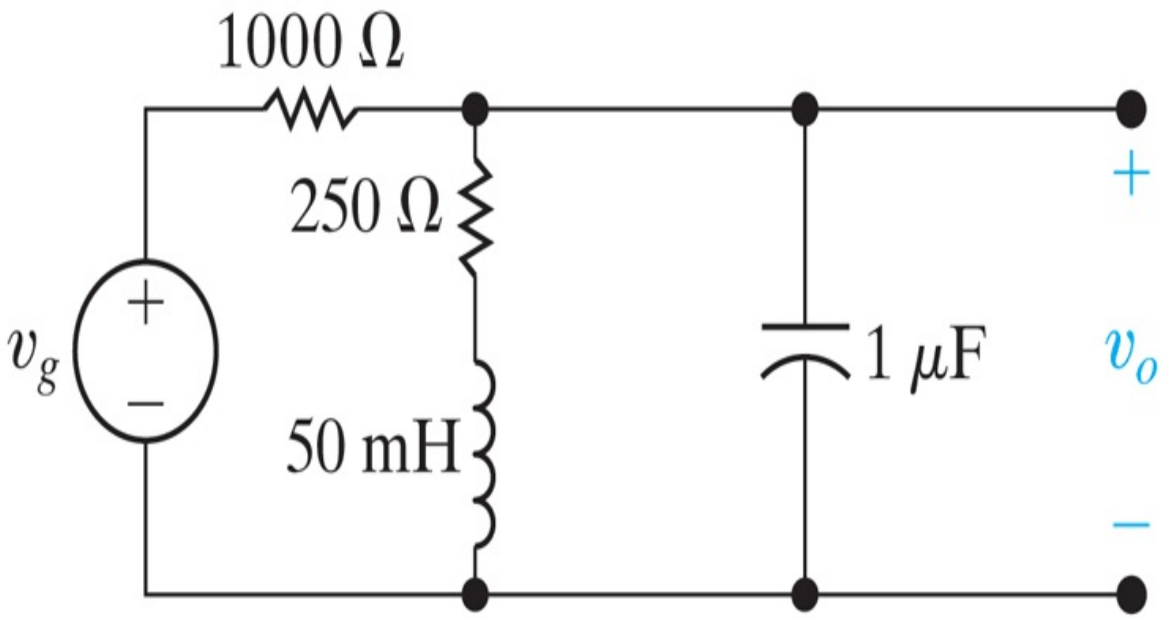


Figure 13.46 Full Alternative Text

Solution

From Example 13.9,

H(s)=1000(s+5000)s2+6000s+25×106.

The frequency of the voltage source is 5000 rad/s, so we evaluate H(j5000):

H(j5000) = 1000(5000+j5000) −25× 10 6 +j5000(6000)+25× 10 6 = 1+j1 j6 = 1−j1 6 = 2 6 −45° .

Then, from Eq. 13.11,

v o ss = (120) 2 6 cos (5000t+30°−45°) = 20 2 cos (5000t−15°) V.

The relationship between H(s) and H(jω) provides a link between the time domain and the frequency domain. Using Eq. 13.11, we can find the sinusoidal steady-state response of a circuit by evaluating H(jω). Theoretically, we can reverse the process; instead of using H(s) to find H(jω), we can use H(jω) to find H(s). To do so, we determine H(jω) experimentally

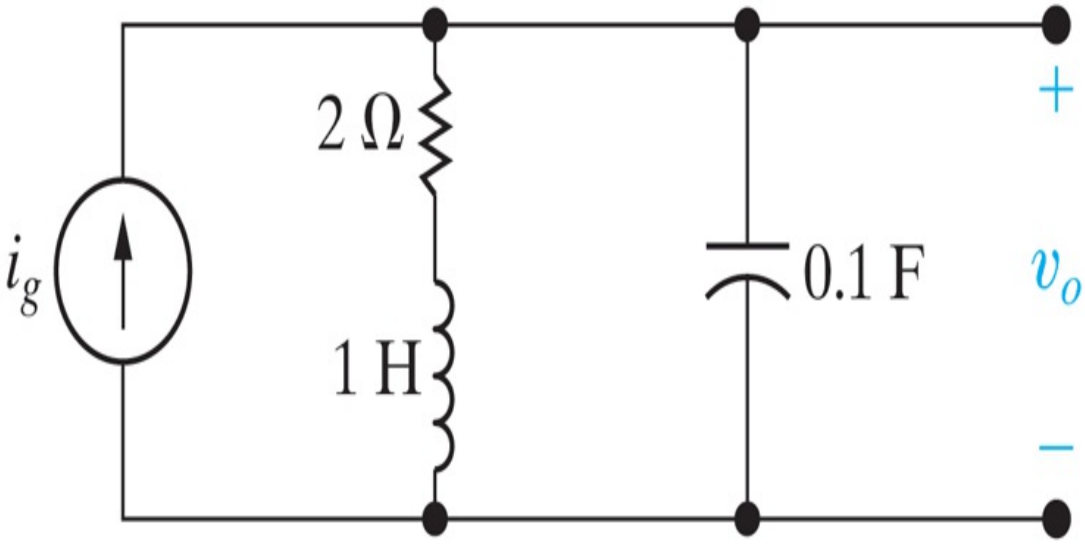
and then construct H(s) from the data. Once we know H(s), we can find the circuit's response to other inputs. While this experimental approach is not always possible, in some cases it does provide a way to find H(s) for a circuit whose components and their values are unknown.

The transfer function is also used to find a circuit's frequency response, a concept we introduce in the next chapter.

Assessment Problems

Objective 4—Know how to use a circuit's transfer function to calculate the circuit's impulse response, unit step response, and steady-state response to sinusoidal input

1. 13.12 The current source in the circuit shown is delivering 10 cos 4t A. Use the transfer function to compute the steady-state expression for vo.



13.7-12 Full Alternative Text

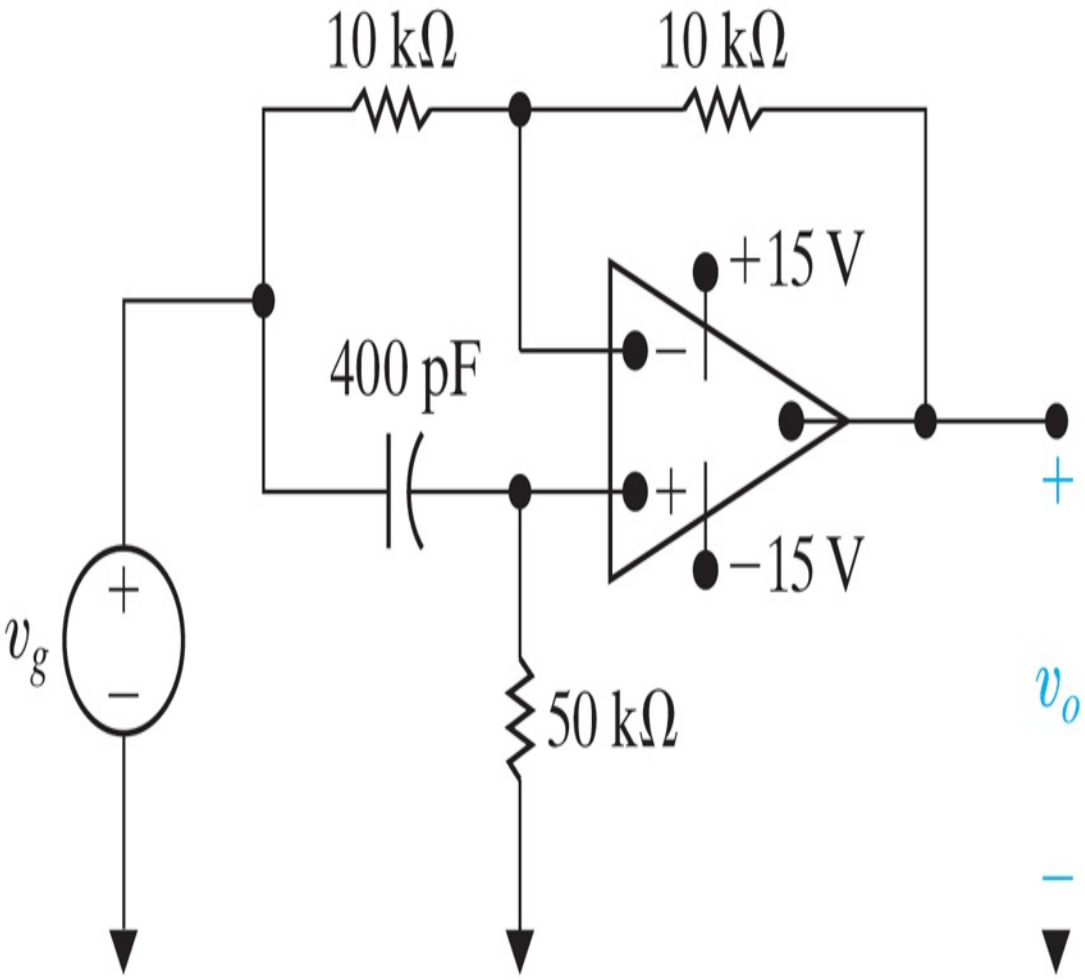
Answer:

44.7cos(4t−63.43°) V.

2. 13.13

1. a) For the circuit shown, find the steady-state expression for vo when vg=10 cos 50,000t V.

2. b) Replace the 50 kΩ resistor with a variable resistor and compute the value of resistance necessary to cause vo to lead vg by 120°.



13.7-13 Full Alternative Text

Answer:

1. (a) 10 cos (50,000t + 90°) V;

2. (b) 28,867.51 Ω.

SELF-CHECK: Also try Chapter Problems 13.79 and 13.82.

13.8 The Impulse Function in Circuit Analysis

Circuit analysis uses impulse functions to represent a switching operation or because a circuit's input is an impulsive source. We can use Laplace transform methods to determine the impulsive currents and voltages created during switching, and to find a circuit's response to an impulsive source. To begin, we create an impulse function with a switching operation.

Switching Operations

We use two different circuits to create an impulse function with a switching operation: a capacitor circuit, and a series inductor circuit.

Capacitor Circuit

In the circuit shown in Fig. 13.47, the capacitor C1 is charged to an initial voltage of V0 at the time the switch is closed. The initial charge on C2 is zero. We want to find the expression for i(t) as $R \rightarrow 0$. Figure 13.48 shows the s-domain equivalent circuit, and we use Ohm's law to find the s-domain current I:

Figure 13.47 A circuit showing the creation of an impulsive current.

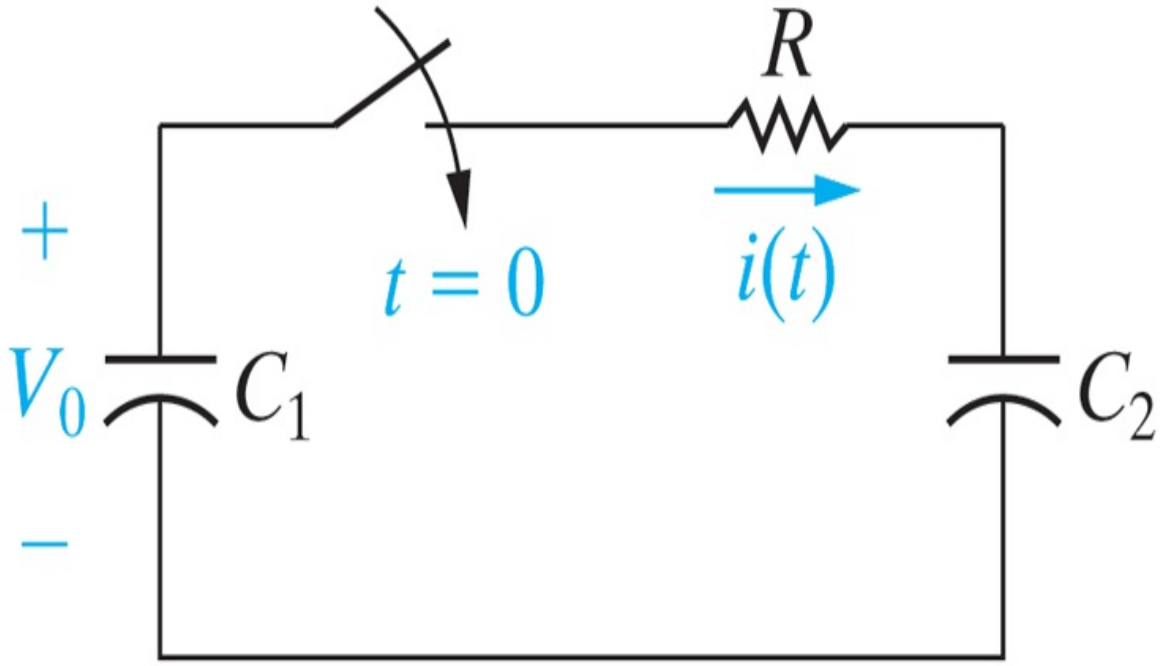


Figure 13.47 Full Alternative Text

Figure 13.48 The s-domain equivalent circuit for the circuit shown in Fig. 13.47.

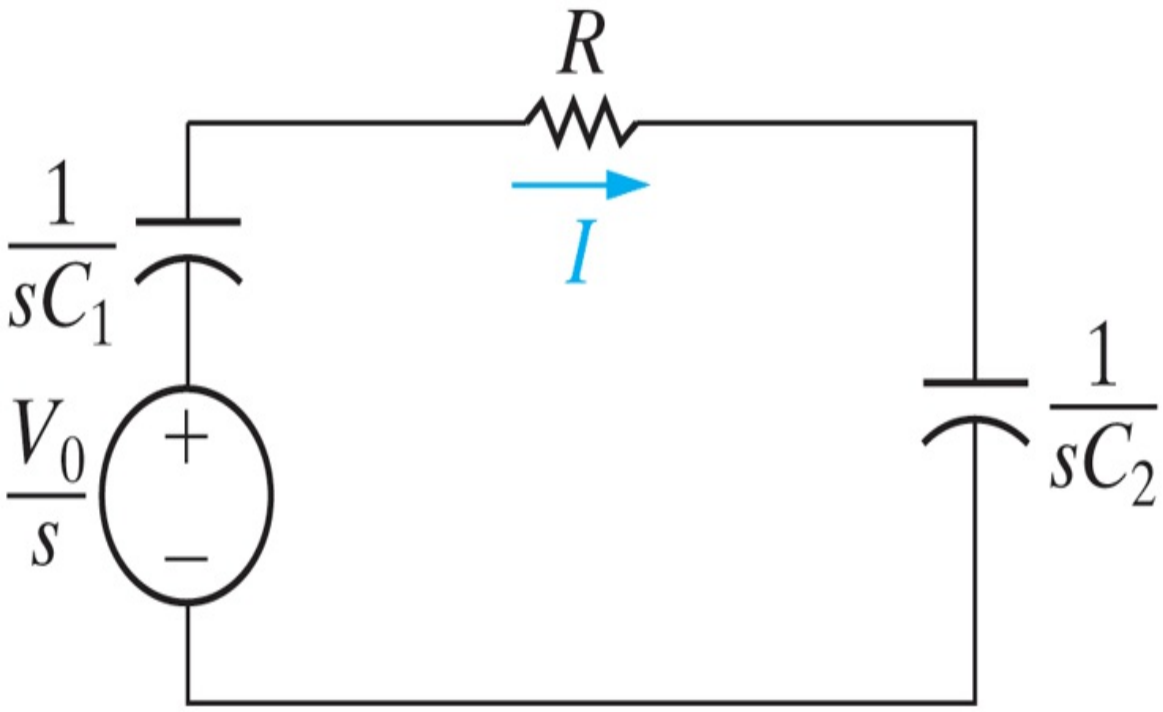


Figure 13.48 Full Alternative Text

I = V 0 /s R+(1/s C 1)+(1/s C 2) = V 0 /R s+(1/R C e) ,

where the equivalent capacitance C1C2/(C1+C2) is replaced by Ce.

We inverse-transform the expression for I using Table 12.3 to get

i=(V0Re−t/RCe) u(t),

which indicates that as R decreases, the initial current (V0/R) increases and the time constant (RCe) decreases. Thus, as R gets smaller, the current starts from a larger initial value and then drops off more rapidly. Figure 13.49 shows these characteristics of i.

Figure 13.49 The plot of i(t) versus t for two different values of R.

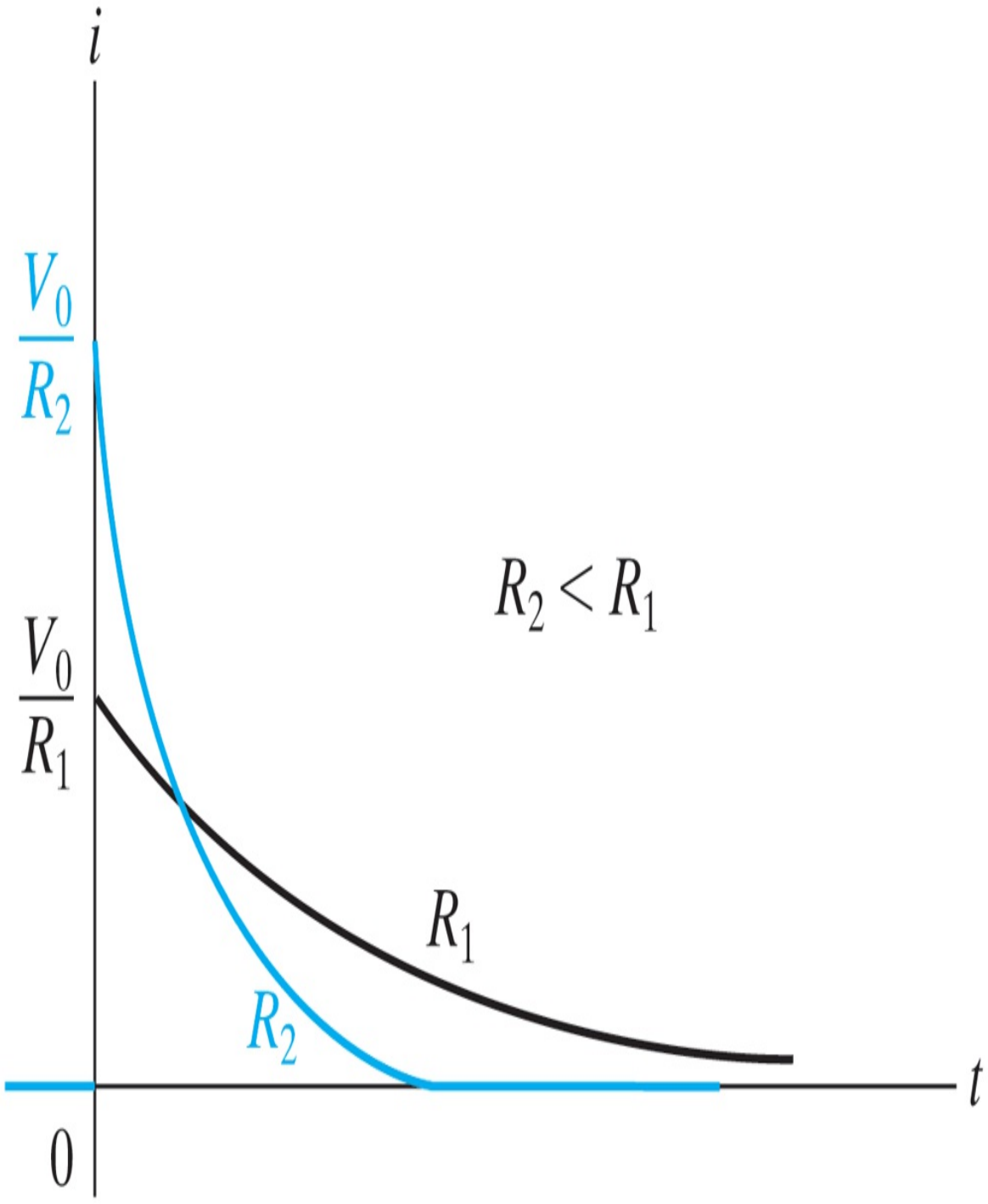


Figure 13.49 Full Alternative Text

As R approaches zero, the initial value of i approaches infinity and the duration of i approaches zero. If the area under the current function is independent of R, i approaches an impulse function. Physically, the total area

under the i versus t curve represents the total charge transferred to C2 after the switch is closed. Thus

$$\text{Area}=q=\int_{0^-}^{\infty} V_0 R e^{-t/RC} dt = V_0 C e,$$

so the total charge transferred to C2 equals V_0Ce coulombs and is independent of R. Thus, as R approaches zero, the current approaches an impulse with strength V_0Ce; that is,

$$i \rightarrow V_0 C e \delta(t).$$

The physical interpretation of this expression says that when R=0, a finite amount of charge is transferred to C2 instantaneously. Set R=0 in the circuit of Fig. 13.47 to see why we get an instantaneous transfer of charge. With R=0, closing the switch creates a contradiction: we apply a voltage across a capacitor whose initial voltage is zero. The capacitor voltage can change instantaneously only if there is an instantaneous transfer of charge. When the switch is closed, the voltage across C2 does not jump to V0 but to its final value of

$$v_2 = \frac{C_1 V_0}{C_1 + C_2}.$$

We leave the derivation of v2 to you (see Problem 13.83).

If we set R=0 at the outset, Laplace transform analysis predicts the impulsive current response. Thus,

$$I = \frac{V_0/s}{(1/sC_1)+(1/sC_2)} = \frac{C_1 C_2 V_0}{C_1 + C_2} = CeV_0.$$

In writing this equation for I, we use the capacitor voltages at $t=0^-$. The inverse transform of I, which is constant, is the constant times the impulse function; therefore,

$$i = CeV_0\delta(t).$$

Example 13.13 uses Laplace methods to analyze an inductor circuit whose output contains an impulse.

Example 13.13 A Series Inductor Circuit with an Impulsive Response

The switch in the circuit shown in Fig. 13.50 has been closed for a long time. At t=0 it opens. Use Laplace methods to find the output voltage, vo, and the current in the 3 H inductor, i1.

Figure 13.50 The circuit for Example 13.13.

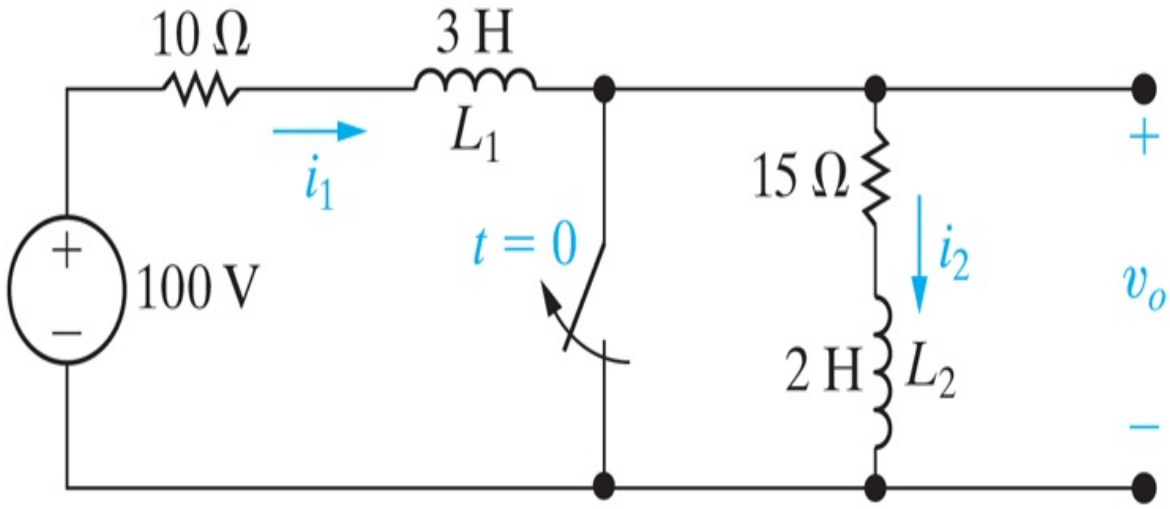


Figure 13.50 Full Alternative Text

Solution

Use Analysis Method 13.1 to construct and analyze the s-domain circuit.

1. Step 1: For t<0, the switch is closed, and the inductors behave like short circuits. The current in the 2 H inductor is zero, since there is no source in that part of the circuit. The current in the 3 H inductor is

100/10=10 A.

2. Step 2: The combination of the dc voltage source and the switch is defined by the time-domain function 100u(t). The Laplace transform of this function is 100/s V-s, which labels the voltage source in the s-domain circuit in Fig. 13.51.

3. Step 3: Represent the output voltage and current using Vo and I1, respectively, as shown in Fig. 13.51. Note that in the s-domain circuit, the current in both inductors is the same.

4. Step 4: The impedance of the resistors is their resistance. The impedance of the inductors is sL. We also need to represent the nonzero initial current in the 3 H inductor using either a series-connected voltage source or a parallel-connected current source. Here, we choose the voltage source, so the resulting s-domain circuit has a single mesh. The voltage source has the value LI0=3(10)=30 V-s. The s-domain circuit resulting from Steps 1–4 is shown in Fig. 13.51.

Figure 13.51 The s-domain equivalent circuit for the circuit shown in Fig. 13.50.

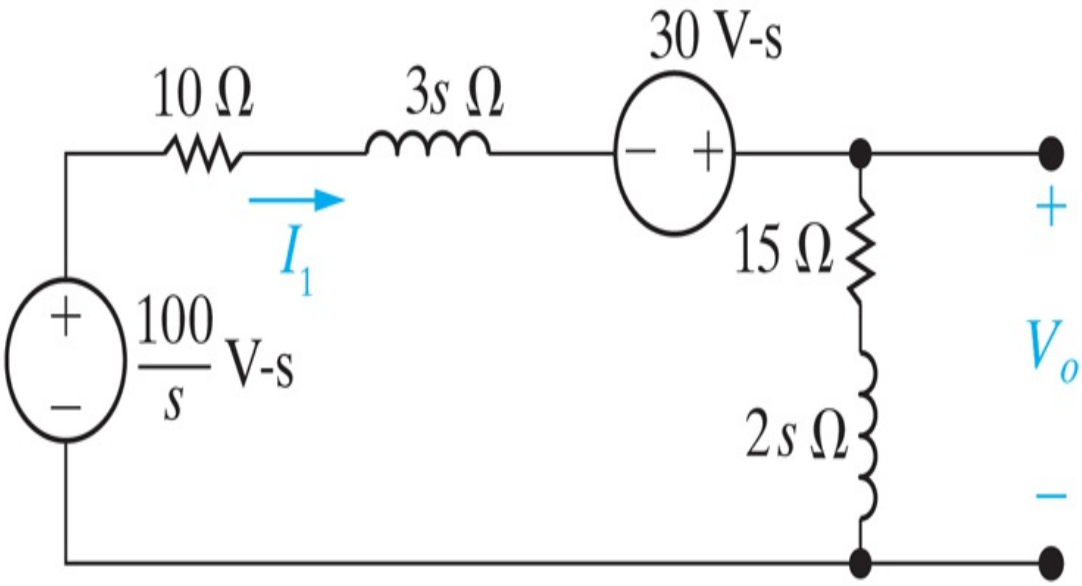


Figure 13.51 Full Alternative Text

5. Step 5: From the circuit in Fig. 13.51, we find Vo using voltage division:

   Vo=2s+155s+25(100s+30)=12s2+130s+300s2+5s.

   Now use Ohm's law to find I1:

   I1=(100/s)+305s+25=6s+20s(s+5).

6. Step 6: We will discuss initial and final values in this circuit after we find the output voltage and current, so for now we skip this step.

7. Step 7: The expression for Vo is an improper rational function, so before we find its partial fraction expansion we must divide numerator by denominator to get

   Vo=12s2+130s+300s2+5s=12+70s+300s(s+5).

   The second term is a proper rational function of s, so we can find the partial fraction expansion:

   Vo=12+60s+10(s+5).

Using Tables 12.1–12.3, we can inverse-Laplace-transform Vo to get

$$v_o = 12\delta(t) + (60 + 10e^{-5t})u(t) V.$$

The partial fraction expansion for I1 is

$$I_1 = \frac{6s + 20}{s(s+5)} = \frac{4}{s} + \frac{2}{(s+5)}.$$

Using Table 12.3, we can inverse-Laplace-transform I1 to get

$$i_1 = (4 + 2e^{-5t})u(t) A.$$

Do the solutions for the output voltage and current in the circuit of Fig. 13.50 make sense? Before the switch is opened, the current in the 3 H inductor is 10 A, and the current in the 2 H inductor is 0 A. From the equation for i1 derived in Example 13.13, we know that at $t = 0+$, the current in both inductors is 6 A. This means the current in the 3 H inductor changes instantaneously from 10 to 6 A, while the current in the 2 H inductor changes instantaneously from 0 to 6 A. Then, the current decreases exponentially from 6 A to a final value of 4 A. This final value is easily verified from the circuit; that is, it should equal 100/25, or 4 A. Figure 13.52 shows these characteristics of i1 and i2.

Figure 13.52 The inductor currents versus t for the circuit shown in Fig. 13.50.

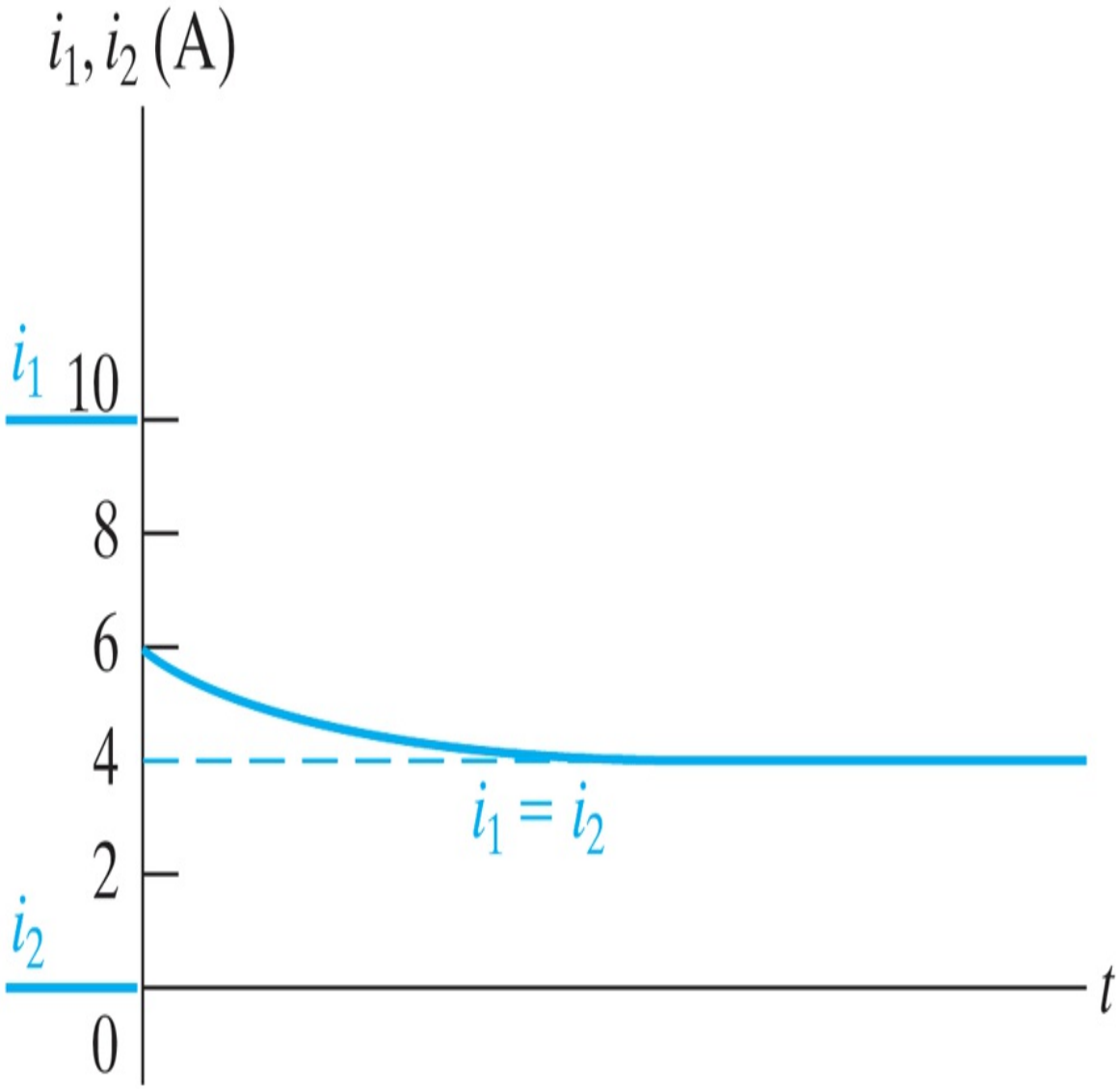


Figure 13.52 Full Alternative Text

Do these instantaneous jumps in the inductor current make sense in terms of the physical behavior of the circuit? First, note that when the switch opens in Fig. 13.50, the two inductors are in series. Any impulsive voltage appearing across the 3 H inductor must be exactly balanced by an impulsive voltage across the 2 H inductor because the sum of the impulsive voltages around a closed path must equal zero. Faraday's law states that the induced voltage is proportional to the change in flux linkage ($v=d\lambda/dt$). Therefore, the change in flux linkage must sum to zero. In other words, the total flux linkage immediately after switching is the same as that before switching. For the circuit here, the flux linkage before switching is

$\lambda = L_1 i_1 + L_2 i_2 = 3(10) + 2(0) = 30$ Wb-turns.

Immediately after switching, it is

$\lambda = (L_1 + L_2) i(0+) = 5i(0+)$.

Therefore,

$i_1(0+) = 30/5 = 6$ A.

Thus, the solution for i1 in Example 13.13 agrees with the principle of the conservation of flux linkage.

We now test the validity of vo from Example 13.13. First, we check the impulsive term $12\delta(t)$. The instantaneous jump of i2 from 0 to 6 A at t=0 means the derivative of i2 contains the impulse $6\delta(t)$. This current impulse results in a $12\delta(t)$ impulse in the voltage across the 2 H inductor. For $t>0$, $di_2/dt = di_1/dt = -10e^{-5t}$ A/s. The voltage vo is the sum of the voltage across the 15 Ω resistor and the voltage across the 2 H inductor:

$$v_o = 15(4+2e^{-5t}) + 2(-10e^{-5t}) = (60+10e^{-5t})u(t) \text{ V}.$$

This expression agrees with the last two terms of vo from Example 13.13. Thus, the expression for vo does make sense in terms of known circuit behavior.

We can also check the instantaneous drop from 10 to 6 A in the current i1. This drop means the derivative of i1 contains the impulse $-4\delta(t)$. Therefore, the voltage across L1 includes the impulse $-12\delta(t)$. This impulse exactly balances the impulse included in the voltage across L2; that is, the sum of the impulsive voltages around a closed path equals zero.

Impulsive Sources

Impulse functions can appear in circuit sources as well as circuit outputs. These sources are called **impulsive sources**. An impulsive source creates a finite amount of energy in the circuit instantaneously. A mechanical analogy is striking a bell with an impulsive clapper blow. After the energy has been transferred to the bell, the natural response of the bell determines the tone

emitted (that is, the frequency of the resulting sound waves) and the tone's duration.

In the circuit shown in Fig. 13.53, an impulsive voltage source with a strength of V0 volt-seconds is applied to a series connection of a resistor and an inductor. When the voltage source is applied, the initial energy in the inductor is zero, so the initial current is zero. There is no voltage drop across R, so the impulsive voltage source appears directly across L. The impulsive voltage across the inductor creates an instantaneous current. The current is

Figure 13.53 An RL circuit excited by an impulsive voltage source.

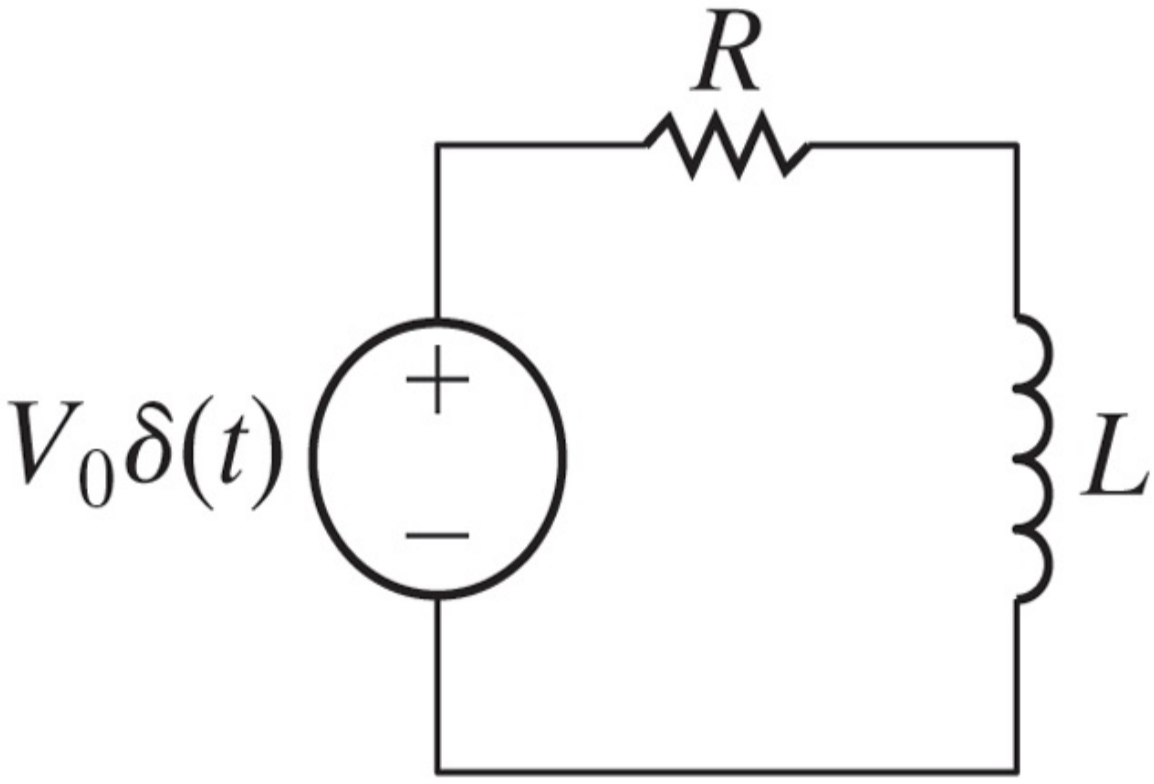


Figure 13.53 Full Alternative Text

$$i = \frac{1}{L}\int_{0^-}^{t} V_0\,\delta(x)\,dx.$$

The integral of $\delta(t)$ over any interval that includes zero is 1, so

$$i(0^+) = \frac{V_0}{L}\ \text{A}.$$

Thus, in an infinitesimal moment, the impulsive voltage source has stored energy in the inductor, given by

$$w = \frac{1}{2}L\left(\frac{V_0}{L}\right)^2 = \frac{1}{2}\frac{V_0^2}{L}\ \text{J}.$$

The initial current V_0/L now decays to zero due to the natural response of the circuit; that is,

$$i = \frac{V_0}{L}e^{-t/\tau}u(t),$$

where $\tau = L/R$. Remember from Chapter 7 that the natural response occurs as passive elements release or store energy. When a circuit is driven by an impulsive source, the total response is completely defined by the natural response; the duration of the impulsive source is so infinitesimal that it does not create a forced response.

We can also derive the inductor current using the Laplace transform method. Figure 13.54 shows the s-domain equivalent of the circuit in Fig. 13.53. From this circuit,

Figure 13.54 The s-domain equivalent circuit for the circuit shown in Fig. 13.53.

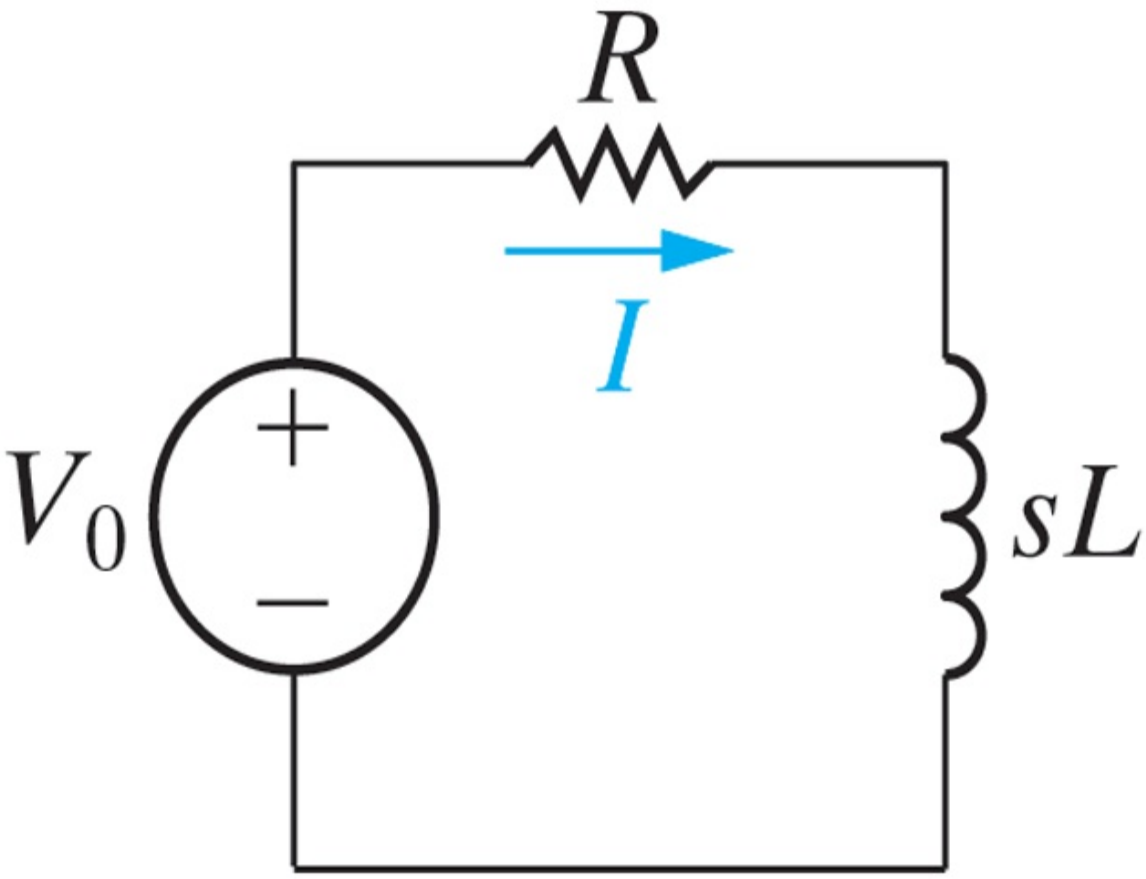


Figure 13.54 Full Alternative Text

$I=\frac{V_0}{R+sL}=\frac{V_0/L}{s+(R/L)}$,

The inverse Laplace transform is

$i=V_0Le^{-(R/L)t}=V_0Le^{-t/\tau}u(t)$.

Thus, the Laplace transform method gives the correct solution for $i \geq 0+$.

Finally, we consider the case in which internally generated impulses and externally applied impulses occur simultaneously. As we see in Example 13.14, the Laplace transform method automatically ensures the correct solution for $t>0+$ if inductor currents and capacitor voltages at $t=0-$ are used in constructing the s-domain equivalent circuit and if externally applied impulses are represented by their transforms.

Example 13.14 A Circuit with Both Internally Generated and Externally Applied Impulses

The switch in the circuit shown in Fig. 13.55 has been closed for a long time. At t=0 it opens. Use Laplace methods to find the output voltage, vo, and the current in the 3 H inductor, i1.

Figure 13.55 The circuit shown in Fig. 13.50 with an impulsive voltage source added in series with the 100 V source.

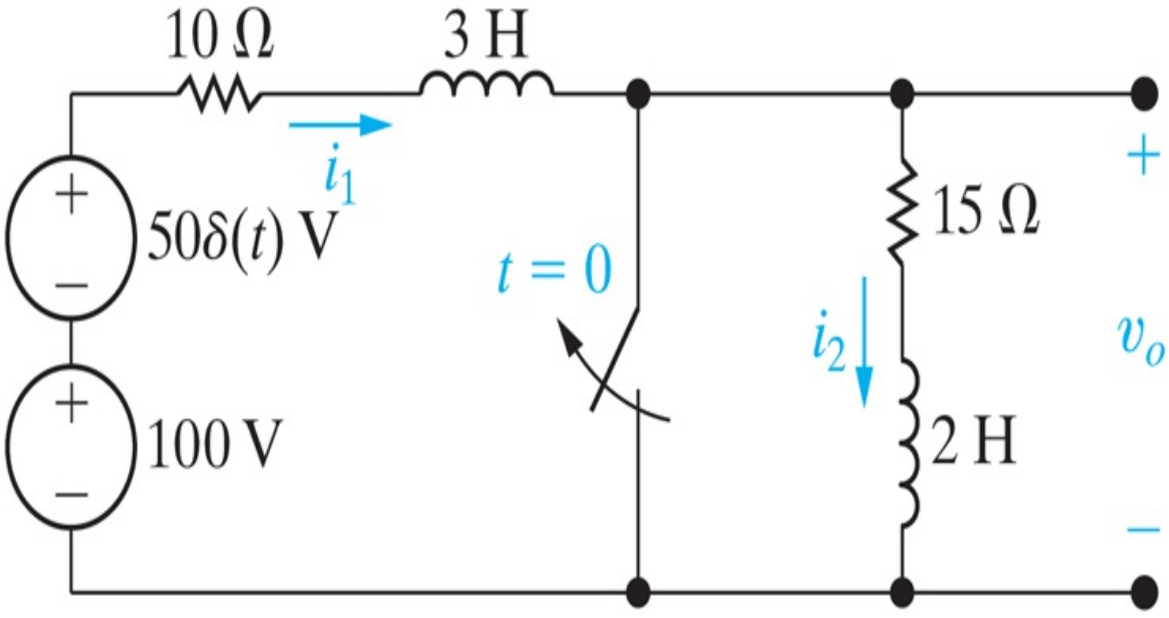


Figure 13.55 Full Alternative Text

Solution

Note that the circuit in Fig. 13.55 was created from the circuit for Example 13.13 (Fig. 13.50) by adding an impulsive voltage source of 50δ(t) in series with the 100 V source. In Example 13.13, we found the s-domain circuit that corresponds to the time-domain circuit in Fig. 13.50. The s-domain circuit is in Fig. 13.51. To find the s-domain circuit corresponding to the circuit in Fig. 13.55, we add another voltage source in series, whose value is L{50δ(t)}=50 V-s. The s-domain equivalent circuit is shown in Fig. 13.56.

Figure 13.56 The s-domain equivalent circuit for the circuit shown in Fig. 13.55.

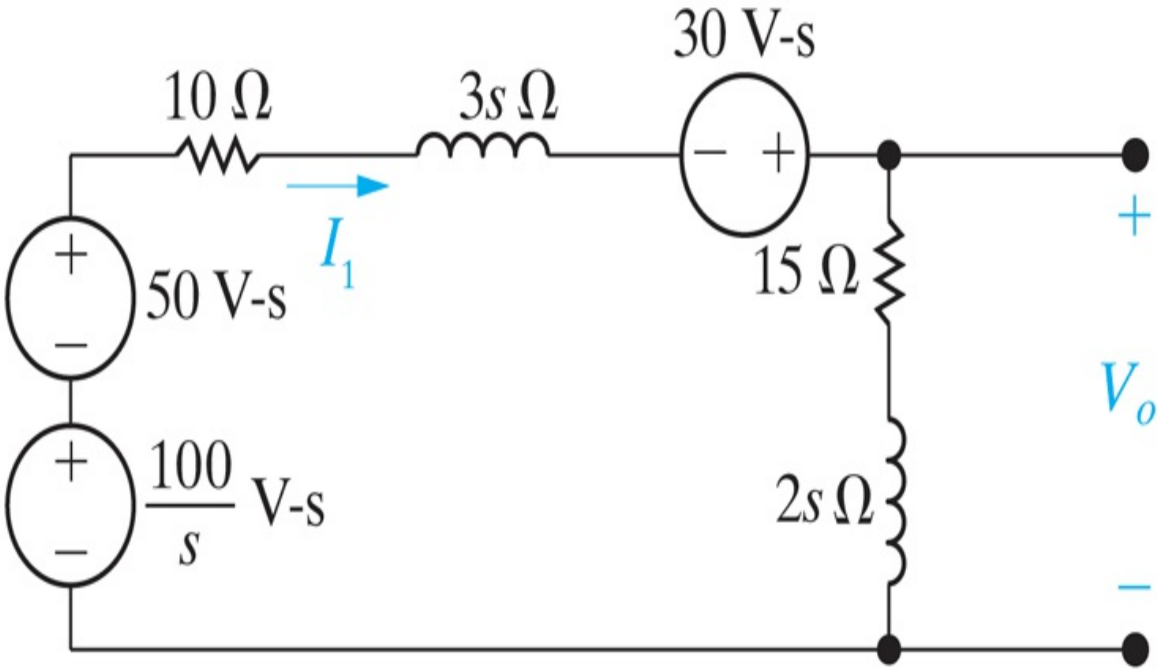


Figure 13.56 Full Alternative Text

From this circuit, the expression for I1 is

$$I1=(100/s)+50+305s+25=16s+20s(s+5).$$

The partial fraction expansion of I1 is

$$I1=4s+12s+5,$$

and the inverse Laplace transform of I1 is

$$i1=(4+12e-5t)u(t)A.$$

The expression for Vo is

$$Vo=(2s+15)I1=32s2+280s+300s(s+5),$$

which is an improper rational function. Before we can find the partial fraction expansion, we divide the numerator of Vo by its denominator to get

$$Vo=32+120s+300s(s+5)$$

.

The partial fraction expansion of Vo is then

$$Vo=32+60s+60s+5$$

and the inverse Laplace transform of Vo is

$$vo=32\delta(t)+(60+60e-5t)u(t)V.$$

As before, we test the results of Example 13.14 to see whether they make sense. From the expression for i1, we see that the current in L1 and L2 is 16 A at t=0+. In Example 13.13, at the instant the switch opens, i1 decreases from 10 to 6 A and i2 increases from 0 to 6 A. Superimposed on these changes is a 10 A current in L1 and L2, due to the impulsive voltage source; that is,

$$i1=13+2\int 0-t50\delta(x)dx=10 \text{ A}.$$

Therefore, i1 increases suddenly from 10 to 16 A, while i2 increases suddenly from 0 to 16 A. The final value of both currents is 4 A. Figure 13.57

shows i1 and i2 graphically.

Figure 13.57 The inductor currents versus t for the circuit shown in Fig. 13.55.

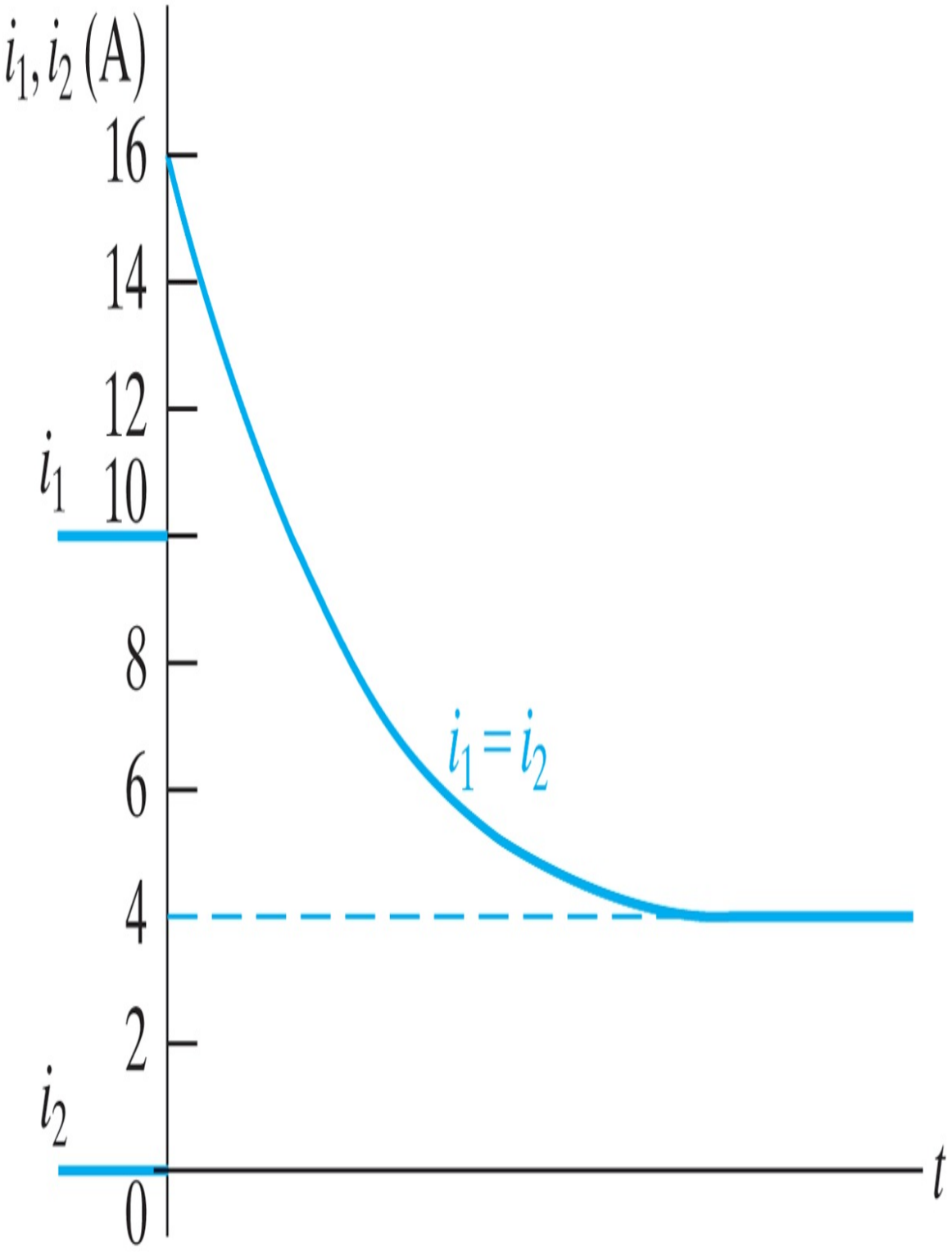


Figure 13.57 Full Alternative Text

We can also find the abrupt changes in i1 and i2 without using superposition.

The sum of the impulsive voltages across the 3 H and 2 H inductors equals $50\delta(t)$. Thus, the change in flux linkage must sum to 50; that is,

$$\Delta\lambda 1+\Delta\lambda 2=50.$$

Because $\lambda=Li$, we get

$$3\Delta i1+2\Delta i2=50.$$

But because i1 and i2 must be equal after the switch opens,

$$i1(0-)+\Delta i1=i2(0-)+\Delta i2.$$

Thus,

$$10+\Delta i1=0+\Delta i2.$$

Solving for $\Delta i1$ and $\Delta i2$ yields

$$\Delta i 1 = 6 \text{ A}, \Delta i 2 = 16 \text{ A}.$$

These expressions agree with the previous check.

Figure 13.57 also indicates that the derivatives of i1 and i2 will contain an impulse at t=0. Specifically, the derivative of i1 will have an impulse of $6\delta(t)$, and the derivative of i2 will have an impulse of $16\delta(t)$. Figure 13.58 illustrates the derivatives of i1 and i2.

Figure 13.58 The derivatives of i1 and i2.

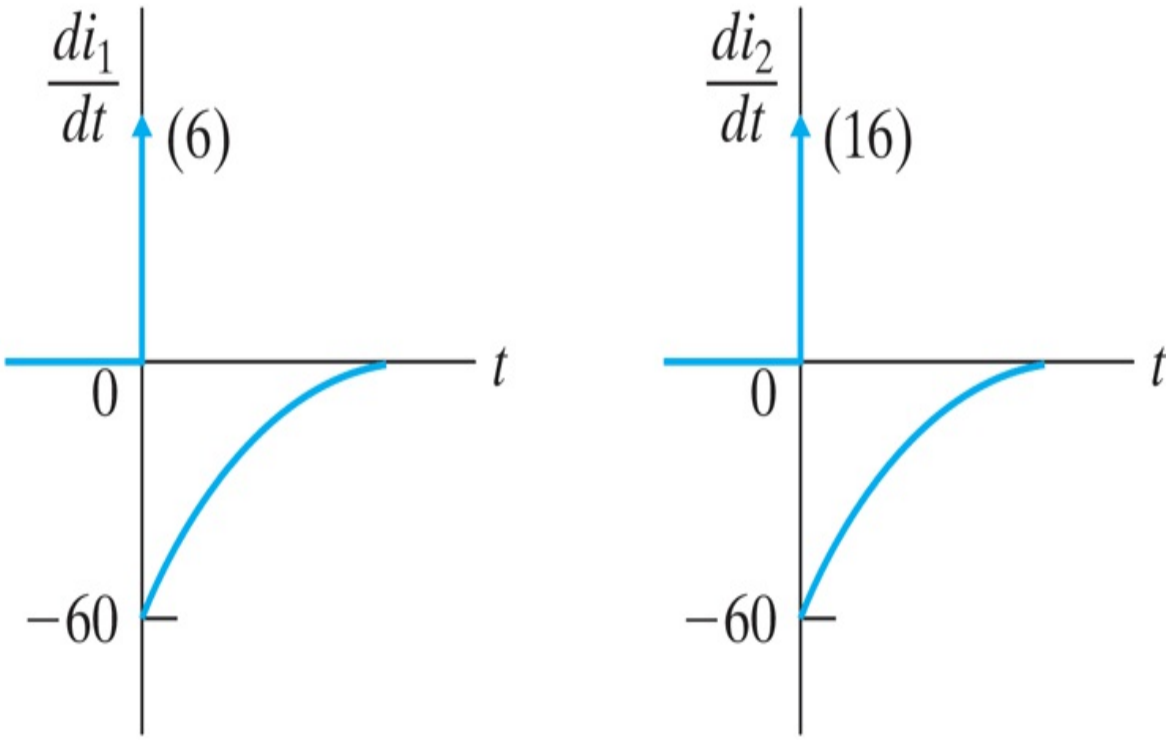


Figure 13.58 Full Alternative Text

Now let’s turn to the expression for the output voltage, vo, found in Example 13.14. The impulsive component 32δ(t) agrees with the impulse 16δ(t) that characterizes di2/dt at the origin. The term (60e−5t+60) agrees with the fact that for t>0+,

vo=15i2+2di2dt=15i1+2di1dt.

We test the impulsive component of di1/dt by noting that it produces an impulsive voltage of (3)6δ(t), or 18δ(t), across L1. This voltage, added to 32δ(t) across L2, gives us 50δ(t). Thus, the algebraic sum of the impulsive voltages around the mesh is zero.

To summarize, the Laplace transform will correctly predict the impulsive currents and voltages created by switching. However, the s-domain equivalent circuits must be based on initial conditions at t=0−, that is, on the initial conditions that exist prior to the switching. The Laplace transform will correctly predict the response to impulsive input sources by representing these sources in the s domain by their Laplace transforms.

SELF-CHECK: Assess your understanding of the impulse function in circuit analysis by trying Chapter Problems 13.90 and 13.91.

Practical Perspective

Surge Suppressors

As mentioned at the beginning of this chapter, voltage surges can occur in a circuit that is operating in the sinusoidal steady state. We use Laplace transform methods to see how a voltage surge is created between the line and neutral conductors of a household circuit when a load is switched off during sinusoidal steady-state operation.

Consider the circuit shown in Fig. 13.59, which models a household circuit with three loads, one of which is switched off at t=0. To simplify the analysis, we assume that the line-to-neutral voltage, V0, is 120 0°V , a standard household voltage, and that when the load is switched off at t=0, the value of Vg does not change. After the switch is opened, we can construct the s-domain circuit, as shown in Fig. 13.60. Note that because the phase angle of the voltage across the inductive load is 0°, the initial current through the inductive load is 0. Therefore, only the line inductance has a nonzero initial condition, which is modeled in the s-domain circuit as a voltage source with the value LlI0, as seen in Fig. 13.60.

Figure 13.59 Circuit used to introduce a switching surge voltage.

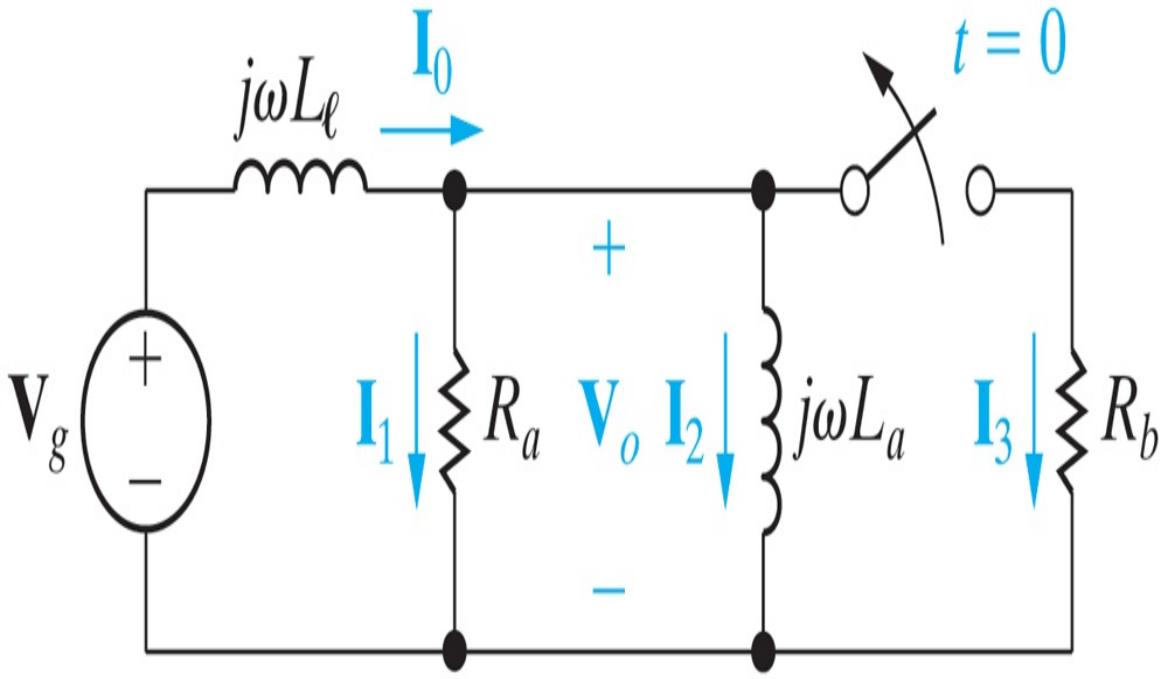


Figure 13.59 Full Alternative Text

Figure 13.60 The circuit in Fig. 13.59, transformed to the s-domain.

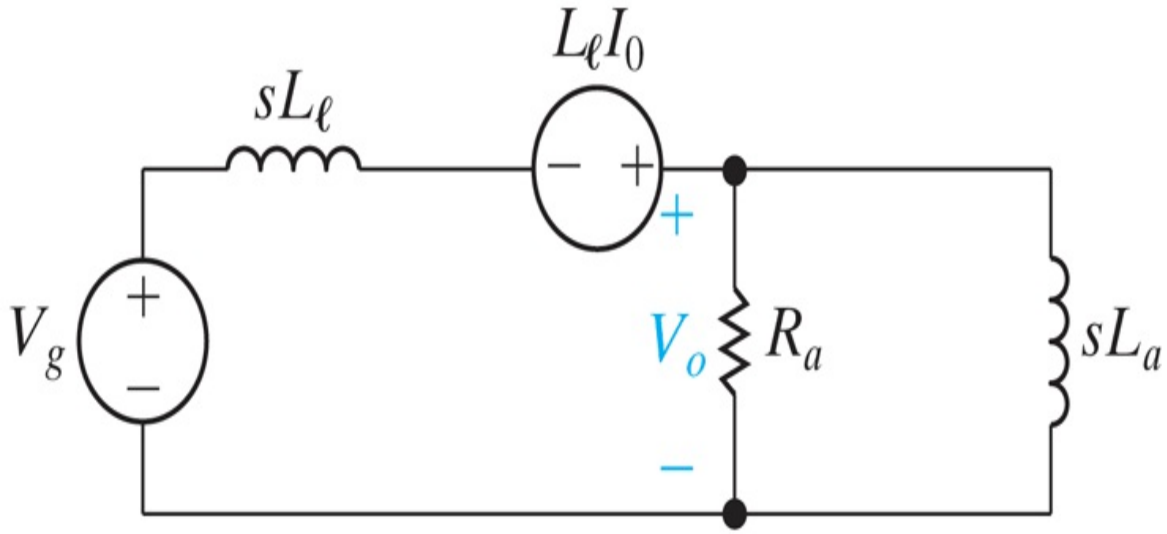


Figure 13.60 Full Alternative Text

Just before the switch is opened at t=0, each of the loads has a steady-state sinusoidal voltage with a peak magnitude of 1202=169.7 V. All of the current in the line from the voltage source divides among the three loads. When the switch is opened at t=0, all of the current in the line appears in the remaining resistive load because the inductive load current is 0 at t=0 and the inductor current cannot change instantaneously. Therefore, the voltage drop across the remaining loads can experience a surge as the line current is directed through the resistive load.

For example, if the initial current in the line is 25 A (rms) and the impedance of the resistive load is12 Ω, the voltage drop across the resistor surges from 169.7 V to (25)(2)(12)=424.3 V when the switch is opened. If the resistive load cannot handle this amount of voltage, it needs to be protected with a surge suppressor such as the one shown at the beginning of the chapter.

SELF-CHECK: Assess your understanding of this Practical Perspective by trying Chapter Problems 13.94 and 13.95.

Summary

- We can represent each of the circuit elements as an s-domain equivalent circuit by Laplace-transforming the voltage-current equation for each element:
  - Resistor: V=RI
  - Inductor: V=sLI−LI0
  - Capacitor: V=(1/sC)I+Vo/s

  In these equations, V=L{v}, I=L{i}, I0 is the initial current through the inductor, and V0 is the initial voltage across the capacitor. (See pages 484–485.)

- Using Steps 1–4 in Analysis Method 13.1, we can transform the time-domain circuit into the s domain. Table 13.1 summarizes the equivalent circuits for resistors, inductors, and capacitors in the s domain. (See page 486.)

- Solve the s-domain equivalent circuit in Analysis Method 13.1, Step 5, by writing algebraic equations using the circuit analysis techniques from resistive circuits. Step 6 checks the resulting s-domain voltages and currents using the initial- and final-value theorems, where possible. Step 7 finds the partial fraction expansion for the s-domain voltages and currents, and uses Table 12.3 to find the inverse Laplace transforms. (See page 487.)

- Circuit analysis in the s domain is particularly advantageous for solving transient response problems in linear lumped parameter circuits when initial conditions are known. It is also useful for problems involving multiple simultaneous mesh-current or node-voltage equations because it reduces problems to algebraic rather than differential equations. (See page 489.)

- The **transfer function** is the s-domain ratio of a circuit's output to its input. It is defined as

  $H(s)=Y(s)X(s)$,

  where Y(s) is the Laplace transform of the output signal, and X(s) is the Laplace transform of the input signal. (See page 500.)

- The partial fraction expansion of the product H(s)X(s) yields a term for each pole of H(s) and X(s). The H(s) terms correspond to the transient component of the total response, while the X(s) terms correspond to the steady-state component. (See page 503.)

- If a circuit is driven by a unit impulse, $x(t)=\delta(t)$, then the response of the circuit equals the inverse Laplace transform of the transfer function, $y(t)=L-1\{H(s)\}=h(t)$. (See pages 504–505.)

- When a circuit is **time-invariant**, delaying the input by a seconds delays the output by a seconds. (See page 504.)

- The output of a circuit, y(t), can be computed by convolving the input, x(t), with the impulse response of the circuit, h(t) :

  $y(t) = h(t) * x(t) = \int 0\ t\ h(\lambda)x(t-\lambda)\ d\lambda = x(t) * h(t) = \int 0\ t\ x(\lambda)h(t-\lambda)\ d\lambda.$

  A graphical interpretation of the convolution integral usually helps you to compute y(t). (See pages 505 and 508.)

- We can use the transfer function of a circuit to compute its steady-state response to a sinusoidal source. To do so, make the substitution $s=j\omega$ in H(s) and represent the resulting complex number as a magnitude and phase angle. If

  $x(t) = A\cos(\omega t + \phi),\ H(j\omega) = |\ H(j\omega)\ |\ e\ j\theta(\omega)\ ,$

  then

  $yss(t)=A|H(j\omega)| \cos[\omega t + \phi + \theta(\omega)].$

(See page 512.)

- Laplace transform analysis correctly predicts impulsive currents and voltages arising from switching and impulsive sources. You must ensure that the s-domain equivalent circuits are based on initial conditions prior to the switching, that is, at $t=0^-$. (See page 520.)

Problems

Section 13.1

1. 13.1 Derive the s-domain equivalent circuit shown in Fig. 13.4 by expressing the inductor current i as a function of the terminal voltage v and then finding the Laplace transform of this time-domain integral equation.

2. 13.2 Find the Norton equivalent of the circuit shown in Fig. 13.3.

3. 13.3 Find the Thévenin equivalent of the circuit shown in Fig. 13.7.

Section 13.2

1. 13.4 A 400 Ω resistor, a 2.5 mH inductor, and a 40 nF capacitor are in series.
    1. a) Express the s-domain impedance of this series combination as a rational function.
    2. b) Give the numerical value of the poles and zeros of the impedance.
2. 13.5 An 2 kΩ resistor, a 6.25 H inductor, and a 250 nF capacitor are in parallel.
    1. a) Express the s-domain impedance of this parallel combination as a rational function.
    2. b) Give the numerical values of the poles and zeros of the impedance.
3. 13.6 A 250 Ω resistor is in series with an 80 mH inductor. This series combination is in parallel with a 500 nF capacitor.
    1. a) Express the equivalent s-domain impedance of these parallel branches as a rational function.
    2. b) Determine the numerical values of the poles and zeros.
4. 13.7 Find the poles and zeros of the impedance seen looking into the terminals a,b of the circuit shown in Fig. P13.7.

Figure P13.7

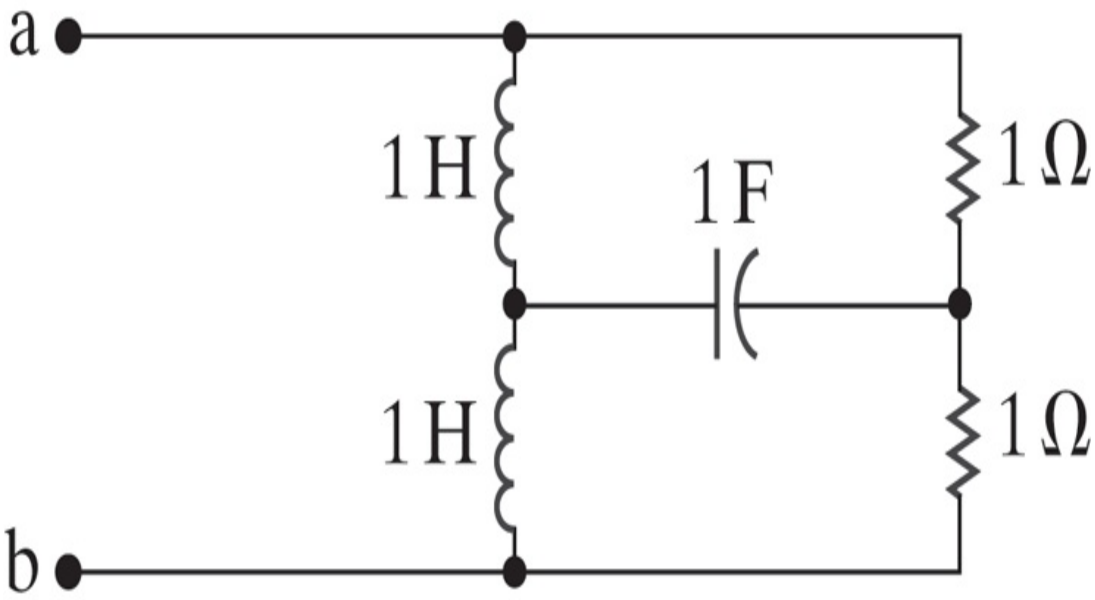


Figure P13.7 Full Alternative Text

5. 13.8 Find the poles and zeros of the impedance seen looking into the terminals a,b of the circuit shown in Fig. P13.8.

Figure P13.8

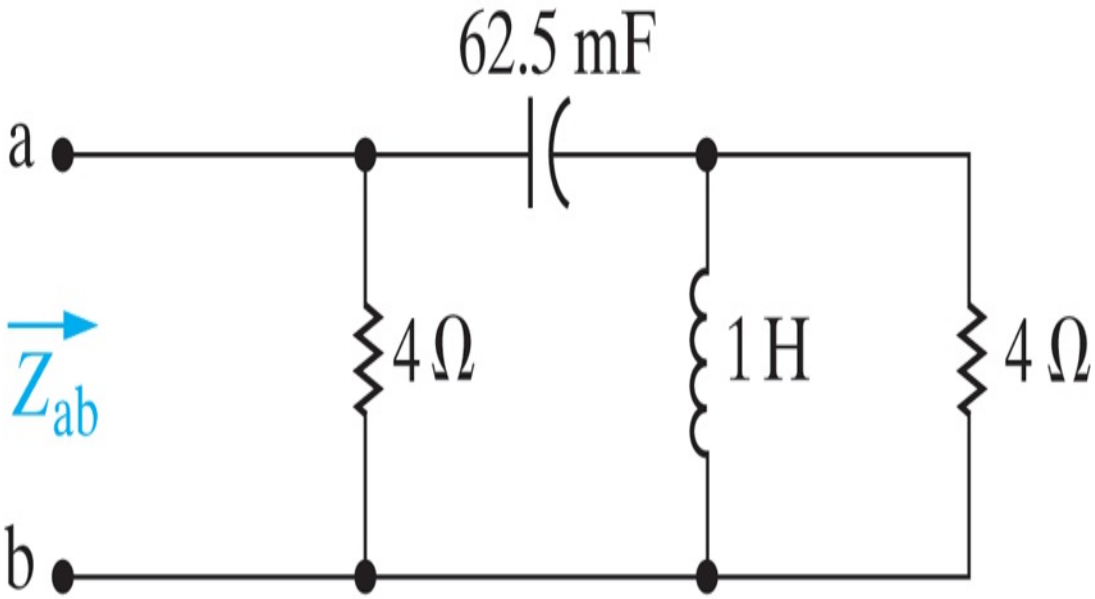


Figure P13.8 Full Alternative Text

Section 13.3

1. 13.9 The switch in the circuit shown in Fig. P13.9 has been in position x for a long time. At t=0, the switch moves instantaneously to position y.
    1. a) Construct an s-domain circuit for t>0.
    2. b) Find Vo.
    3. c) Find vo.

Figure P13.9

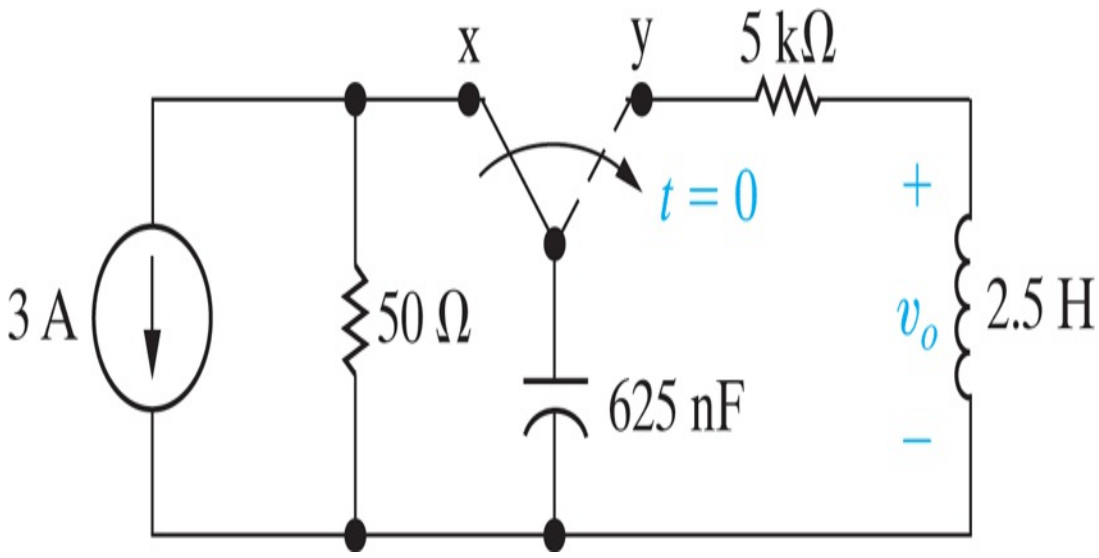


Figure P13.9 Full Alternative Text

2. 13.10 The switch in the circuit in Fig. P13.10 has been closed for a long time. At t=0, the switch is opened.
    1. a) Find io for t≥0.
    2. b) Find vo for t≥0.

Figure P13.10

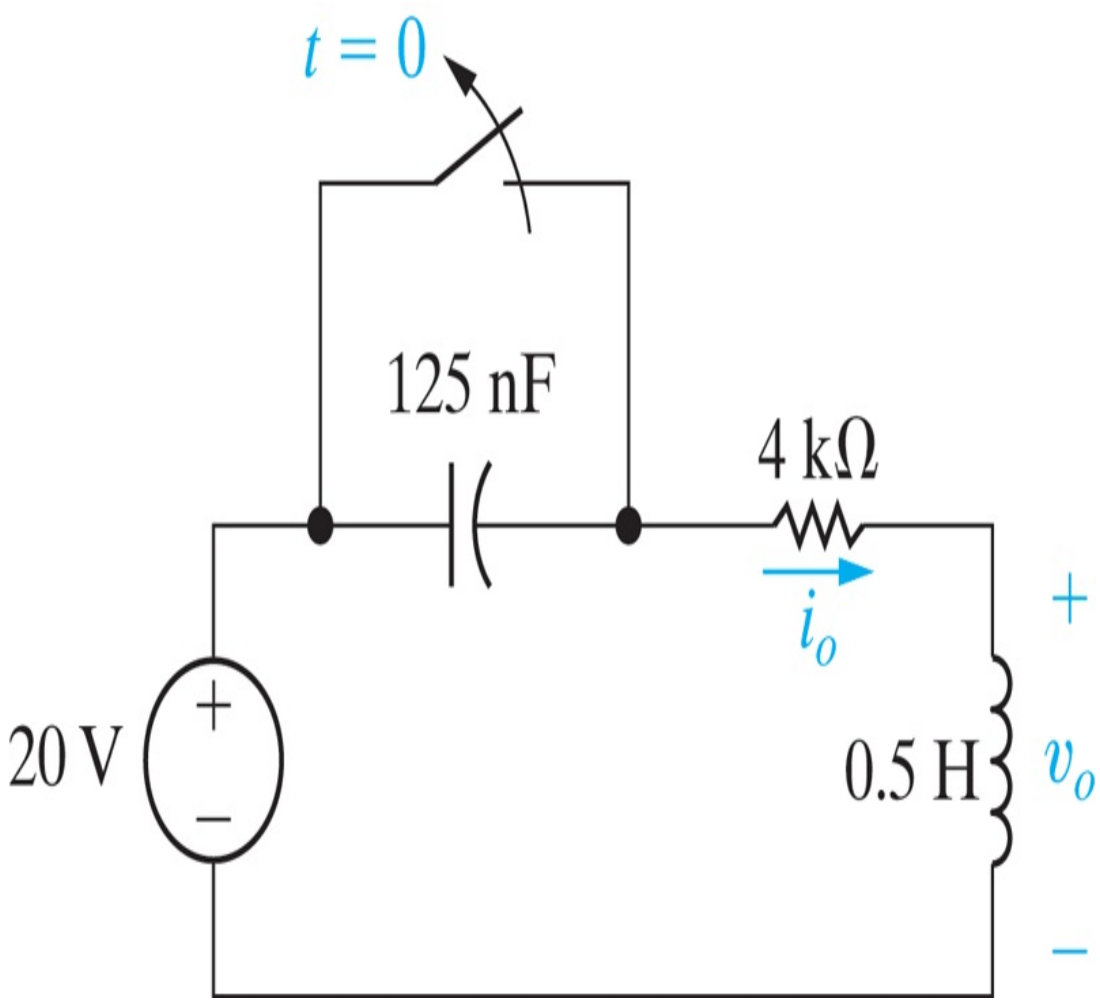


Figure P13.10 Full Alternative Text

3. 13.11 Find Vo and vo in the circuit shown in Fig. P13.11 if the initial energy is zero and the switch is closed at t=0.

Figure P13.11

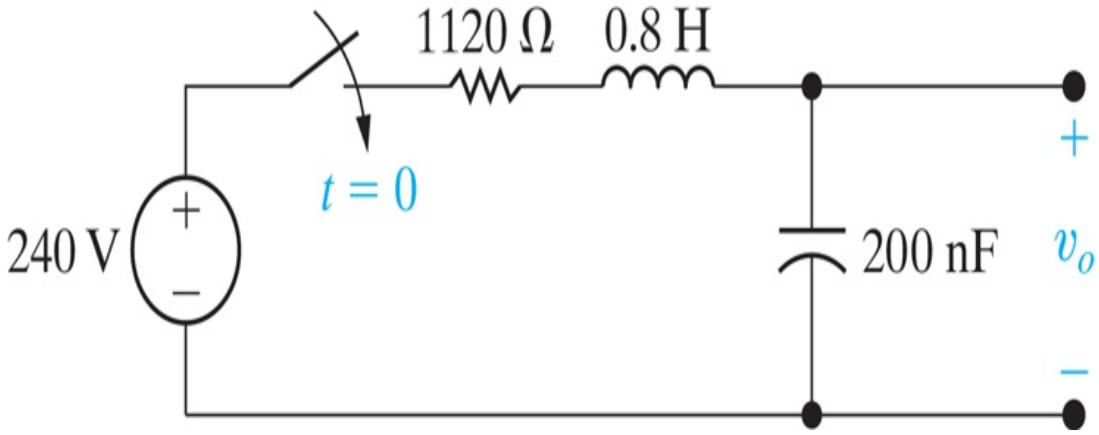


Figure P13.11 Full Alternative Text

4. 13.12 Repeat Problem 13.11 if the initial voltage on the capacitor is 72 V positive at the lower terminal.

5. 13.13

    1. a) Find the s-domain expression for Vo in the circuit in Fig. P13.13.

    2. b) Use the s-domain expression derived in (a) to predict the initial- and final-values of vo.

    3. c) Find the time-domain expression for vo.

Figure P13.13

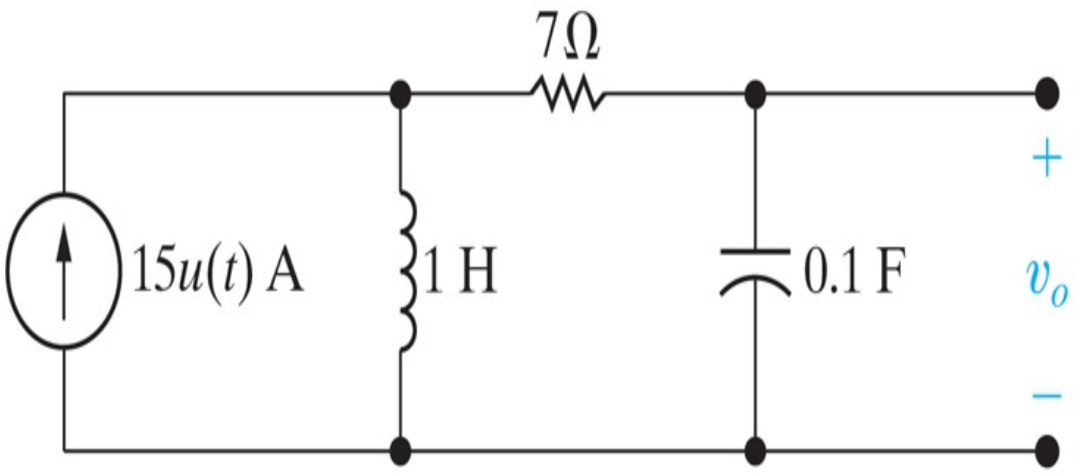


Figure P13.13 Full Alternative Text

6. 13.14 Find the time-domain expression for the current in the inductor in Fig. P13.13. Assume the reference direction for iL is down.

7. 13.15 The switch in the circuit in Fig. P13.15 has been in position a for a long time. At t=0, the switch moves instantaneously to position b.

    1. a) Construct the s-domain circuit for t>0.
    2. b) Find Vo.
    3. c) Find IL.
    4. d) Find vo for t>0.
    5. e) Find iL for t≥0.

Figure P13.15

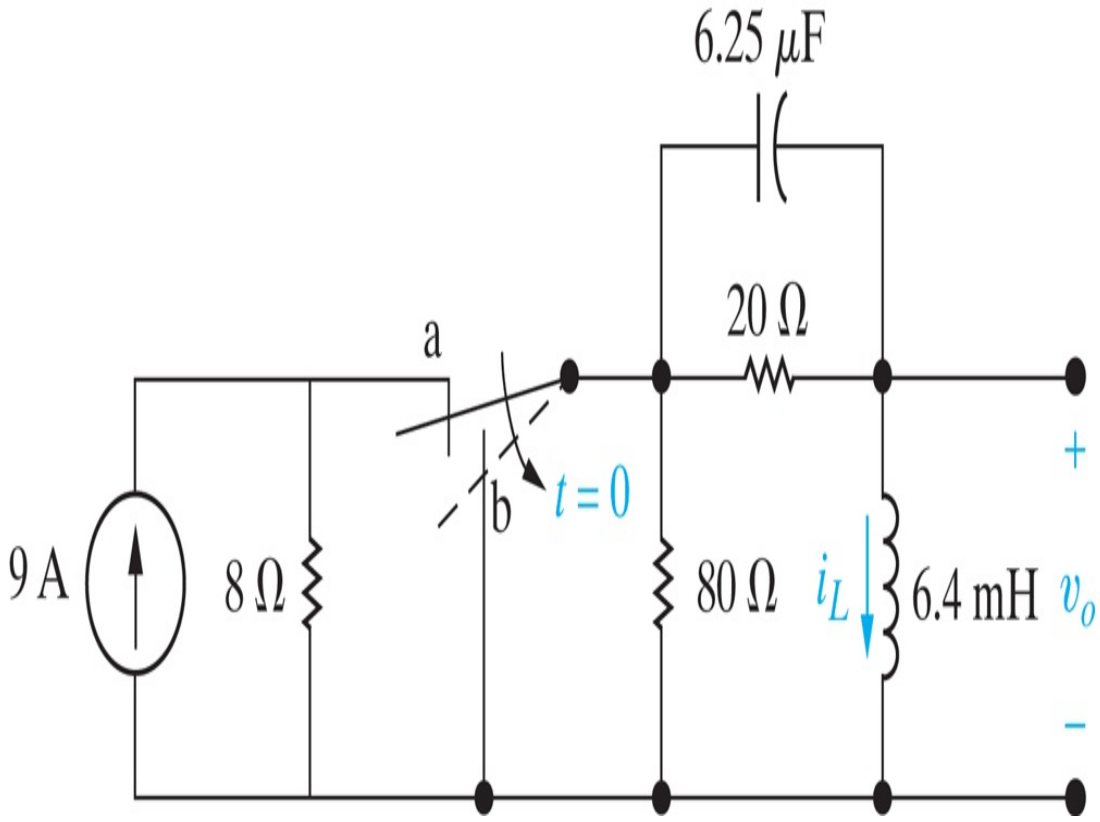


Figure P13.15 Full Alternative Text

8. 13.16 The switch in the circuit in Fig. P13.16 has been in position a for a long time. At t=0, it moves instantaneously from a to b.

    1. a) Construct the s-domain circuit for t>0.
    2. b) Find Vo(s).
    3. c) Find vo(t) for t≥0.

Figure P13.16

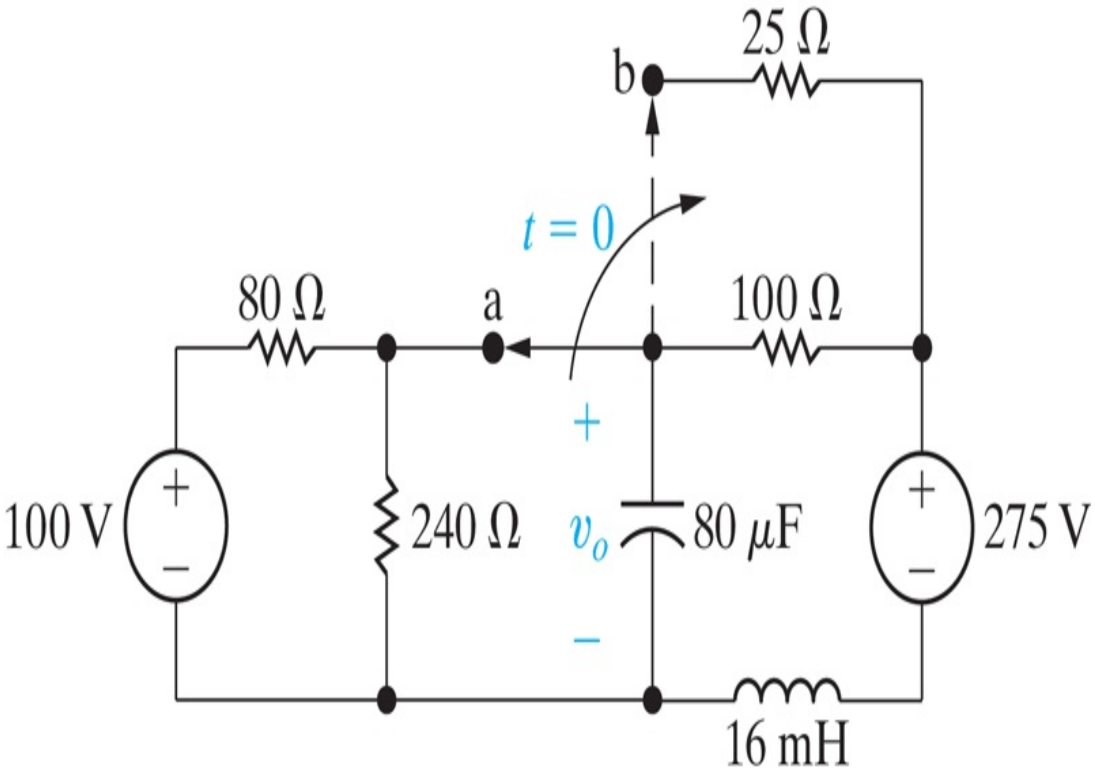


Figure P13.16 Full Alternative Text

9. 13.17 The make-before-break switch in the circuit in Fig. P13.17 has been in position a for a long time. At t=0, it moves instantaneously to position b. Find io for t≥0.

Figure P13.17

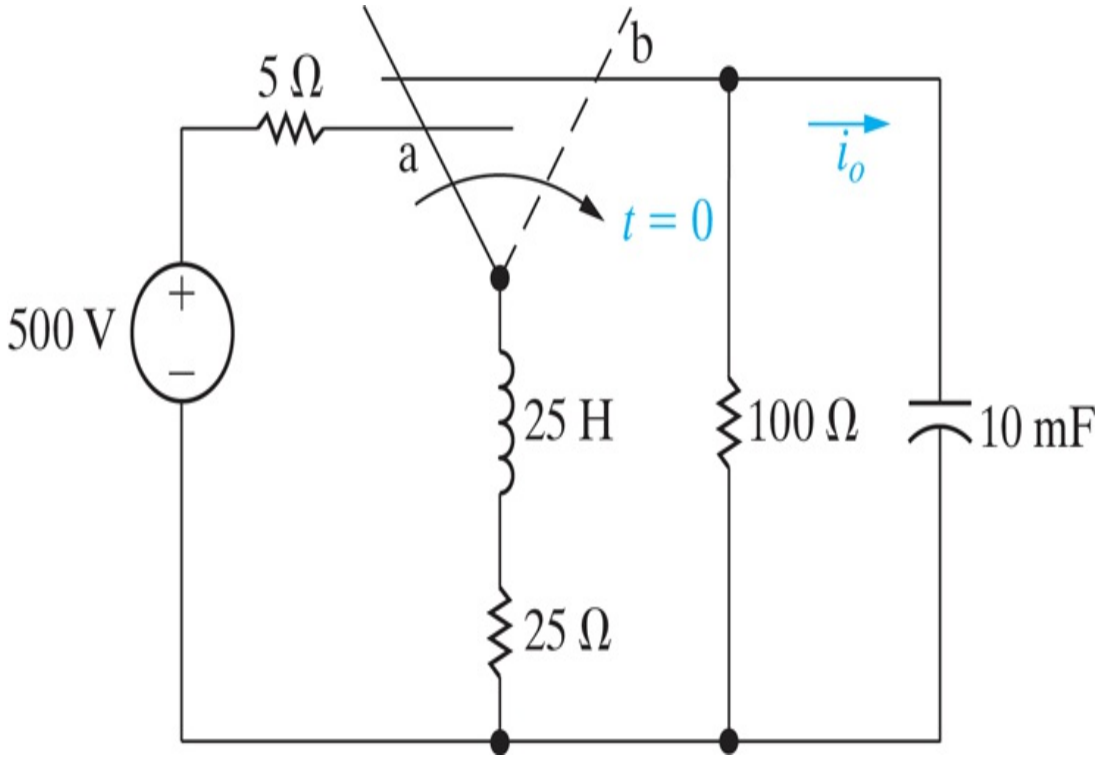


Figure P13.17 Full Alternative Text

10. 13.18 The switch in the circuit in Fig. P13.18 has been closed for a long time before opening at t=0.

    1. a) Construct the s-domain equivalent circuit for t>0.
    2. b) Find Io.
    3. c) Find io for t≥0.

Figure P13.18

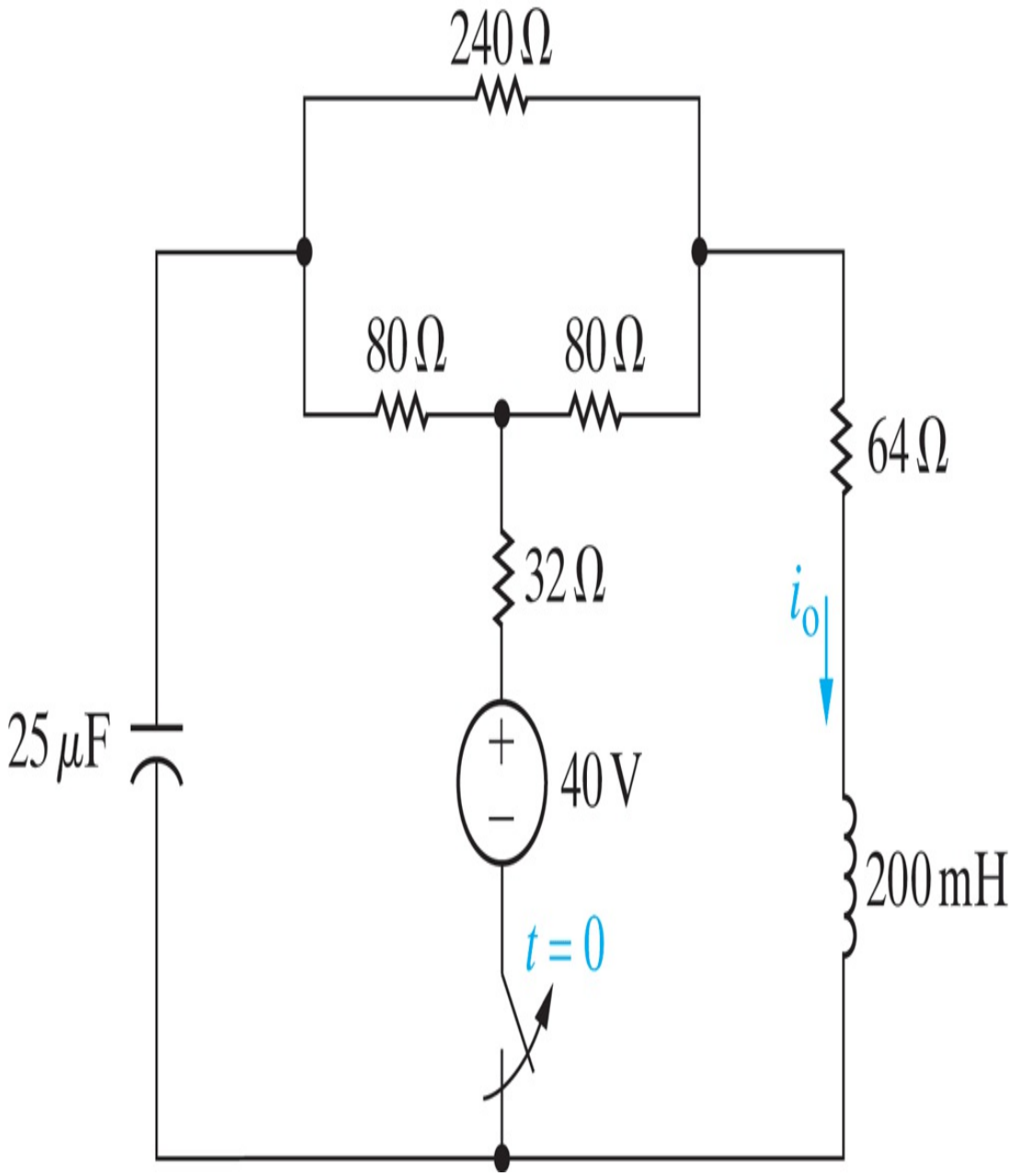


Figure P13.18 Full Alternative Text

11. 13.19 There is no energy stored in the circuit in Fig. P13.19 at the time the sources are energized.

    1. a) Find I1(s) and I2(s).

    2. b) Use the initial- and final-value theorems to check the initial- and final-values of i1(t) and i2(t).

3. c) Find i1(t) and i2(t) for t≥0.

Figure P13.19

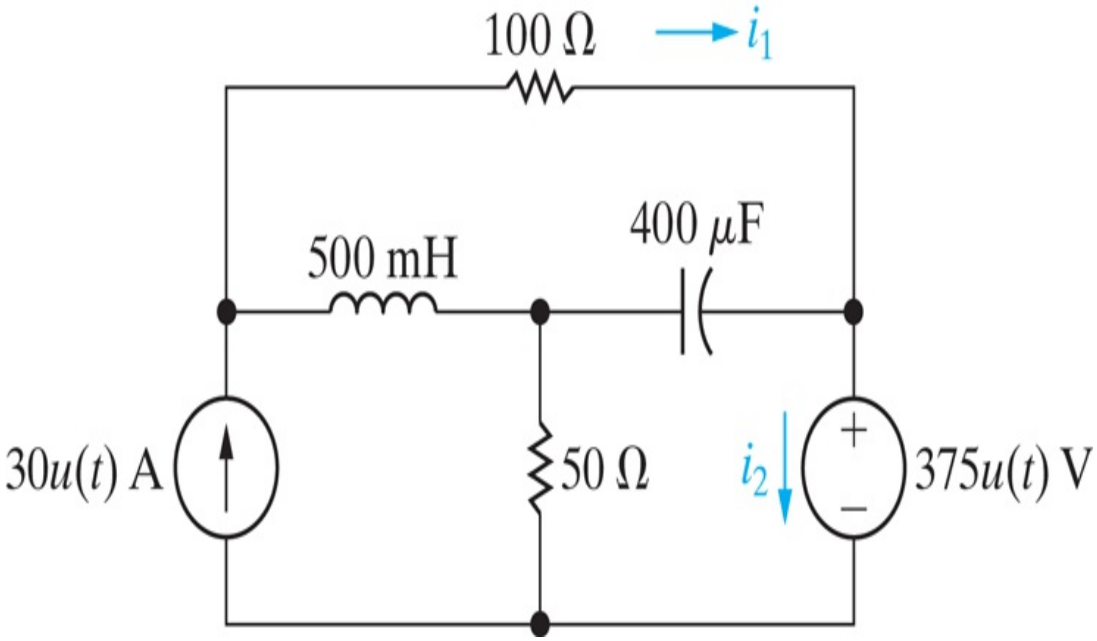


Figure P13.19 Full Alternative Text

12. 13.20 There is no energy stored in the circuit in Fig. P13.20 at t=0−.

    1. a) Use the mesh current method to find io.
    2. b) Find the time domain expression for vo.
    3. c) Do your answers in (a) and (b) make sense in terms of known circuit behavior? Explain.

Figure P13.20

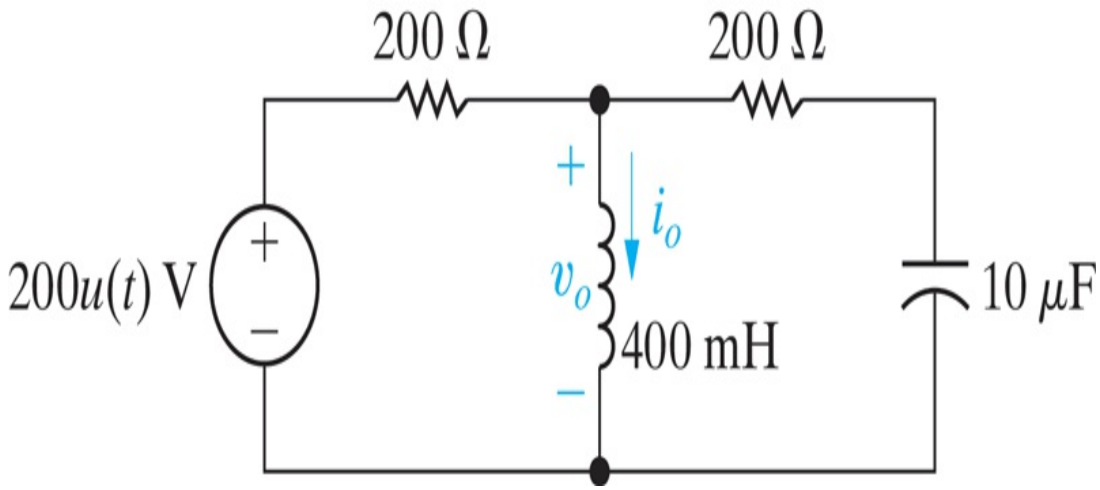


Figure P13.20 Full Alternative Text

13. 13.21 There is no energy stored in the circuit in Fig. P13.21 at t=0−.
    1. a) Find Vo.
    2. b) Find vo.
    3. c) Does your solution for vo make sense in terms of known circuit behavior? Explain.

Figure P13.21

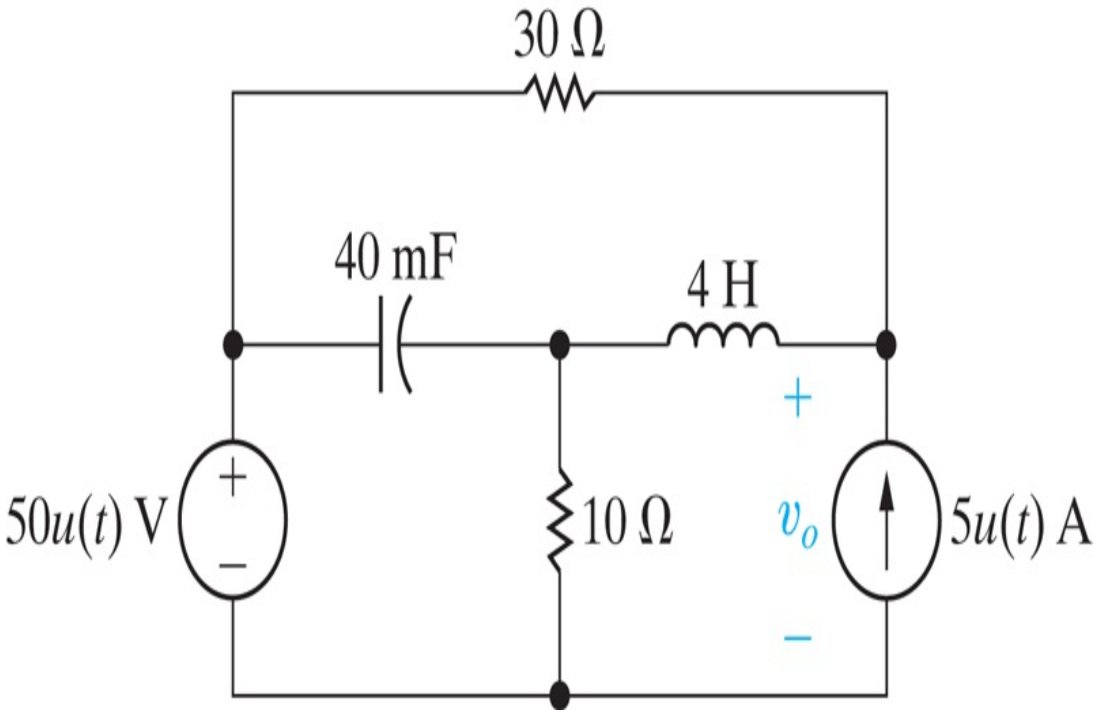


Figure P13.21 Full Alternative Text

14. 13.22 There is no energy stored in the circuit in Fig. P13.22 at the time the voltage source is turned on, and vg=325u(t)V.

    1. a) Find Vo and Io.
    2. b) Find vo and io.
    3. c) Do the solutions for vo and io make sense in terms of known circuit behavior? Explain.

Figure P13.22

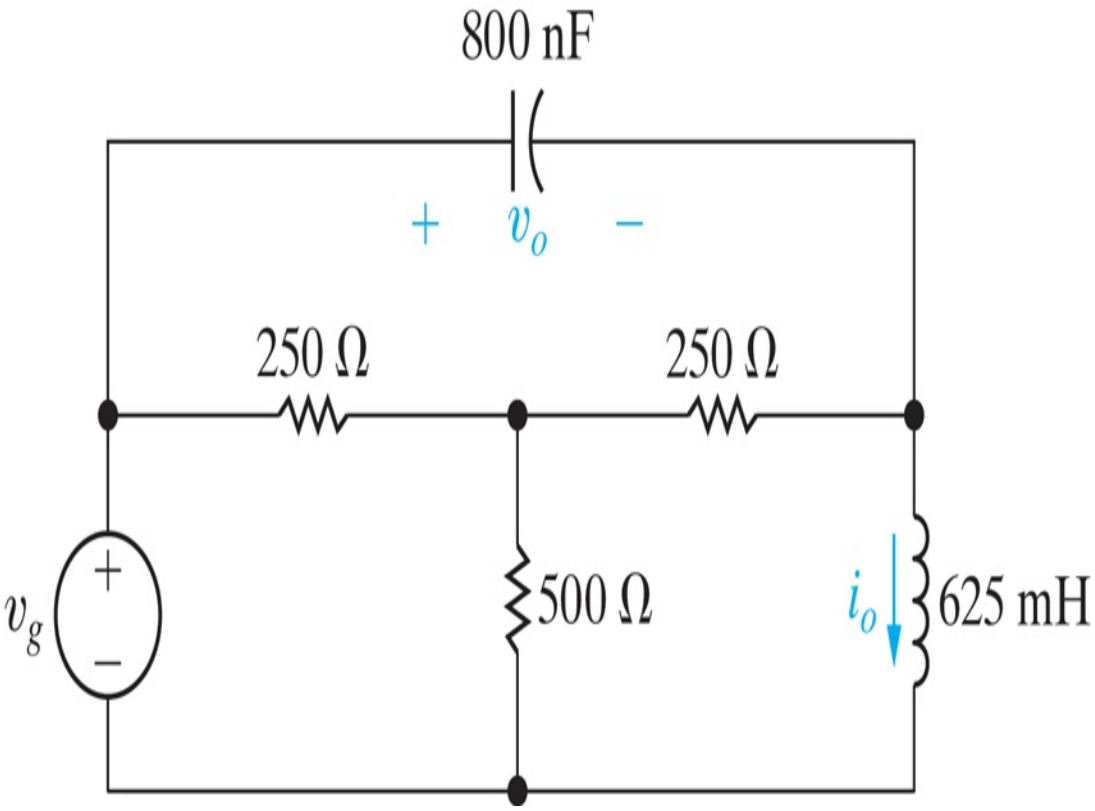


Figure P13.22 Full Alternative Text

15. 13.23 There is no initial energy in the circuit in Fig. P13.23 before the switch closes at t=0. Find io(t) for t≥0.

Figure P13.23

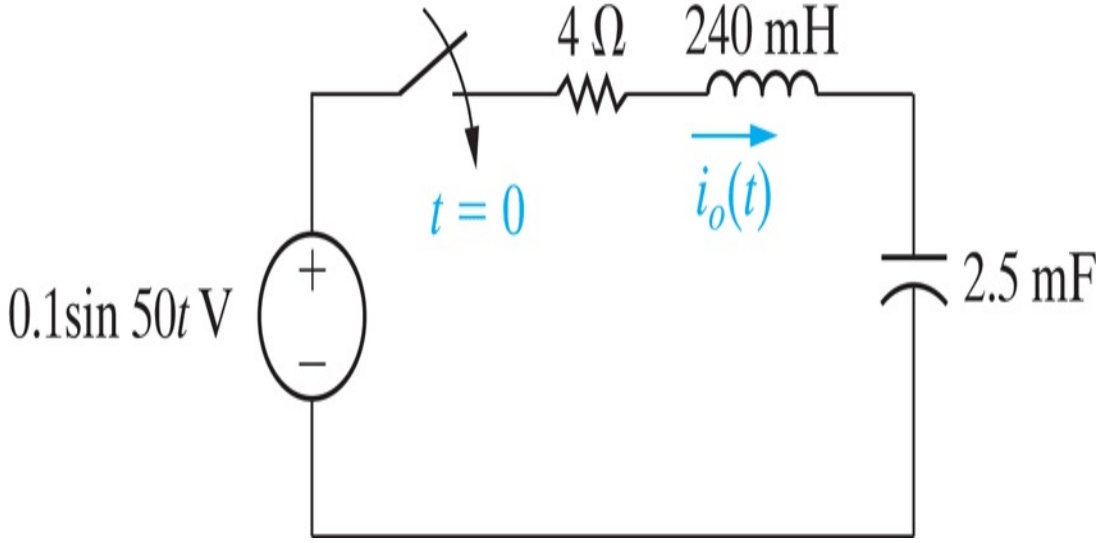


Figure P13.23 Full Alternative Text

16. 13.24 There is no initial energy in the circuit in Fig. P13.24 before the switch closes at t=0. Find vo(t) for t≥0.

Figure P13.24

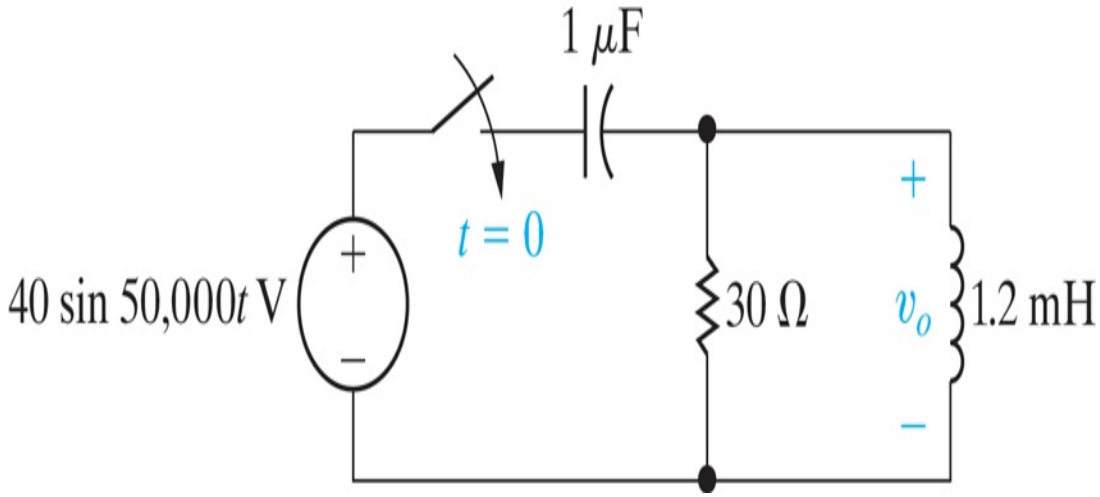


Figure P13.24 Full Alternative Text

17. 13.25 There is no energy stored in the circuit in Fig. P13.25 at the time the switch is closed.

1. a) Find vo for t≥0.

2. b) Does your solution make sense in terms of known circuit behavior? Explain.

Figure P13.25

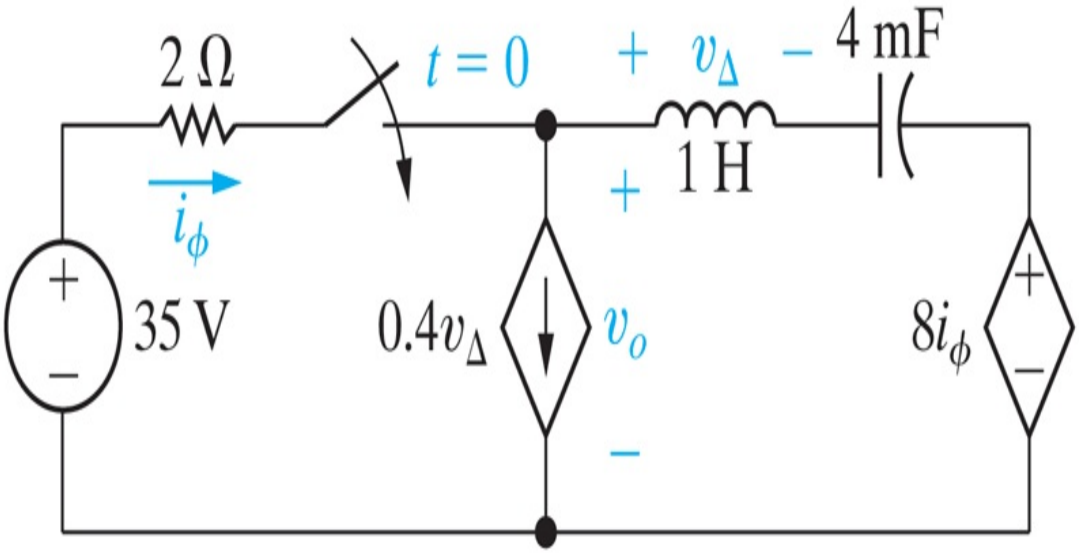


Figure P13.25 Full Alternative Text

18. 13.26 Find vo in the circuit shown in Fig. P13.26 if ig=5u(t) mA. There is no energy stored in the circuit at t=0.

Figure P13.26

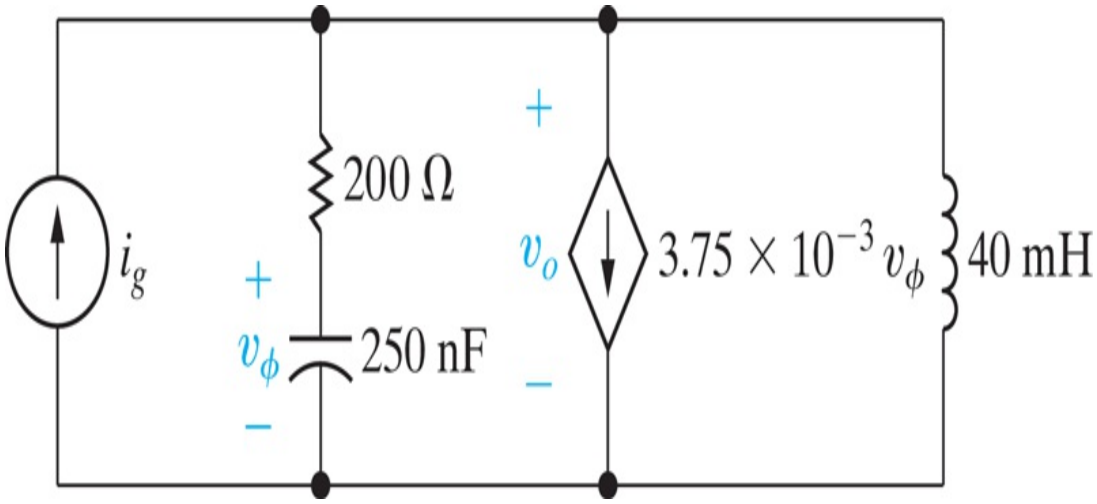


Figure P13.26 Full Alternative Text

19. 13.27 The initial energy in the circuit in Fig. P13.27 is zero. The ideal voltage source is 120u(t)V.

    1. a) Find I(s).
    2. b) Use the initial- and final-value theorems to find i(0+) and i(∞).
    3. c) Do the values obtained in (b) agree with known circuit behavior? Explain.
    4. d) Find i(t).

Figure P13.27

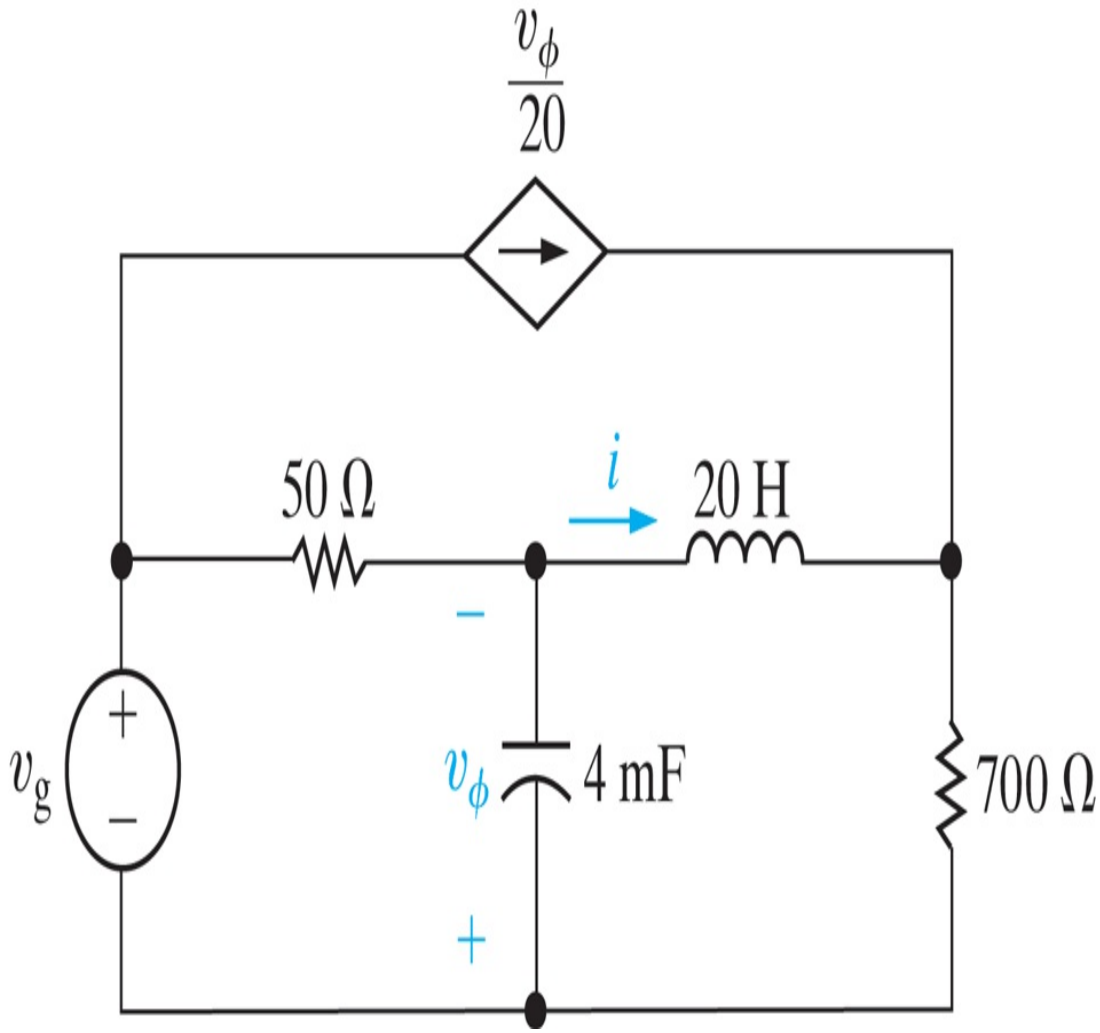


Figure P13.27 Full Alternative Text

20. 13.28 The switch in the circuit in Fig. P13.28 has been closed for a long time before opening at t=0. Find vo for t≥0.

Figure P13.28

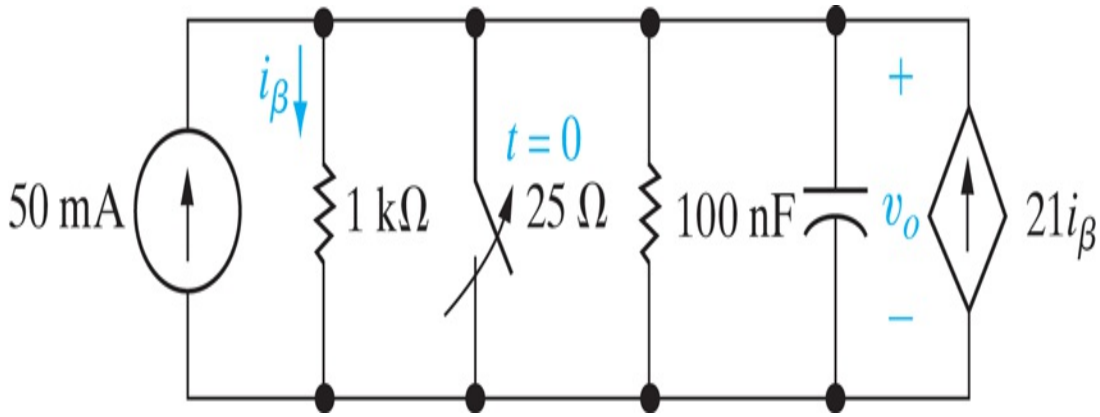


Figure P13.28 Full Alternative Text

21. 13.29 The switch in the circuit seen in Fig. P13.29 has been in position a for a long time. At t=0, it moves instantaneously to position b.

    1. a) Find Vo.
    2. b) Find vo.

Figure P13.29

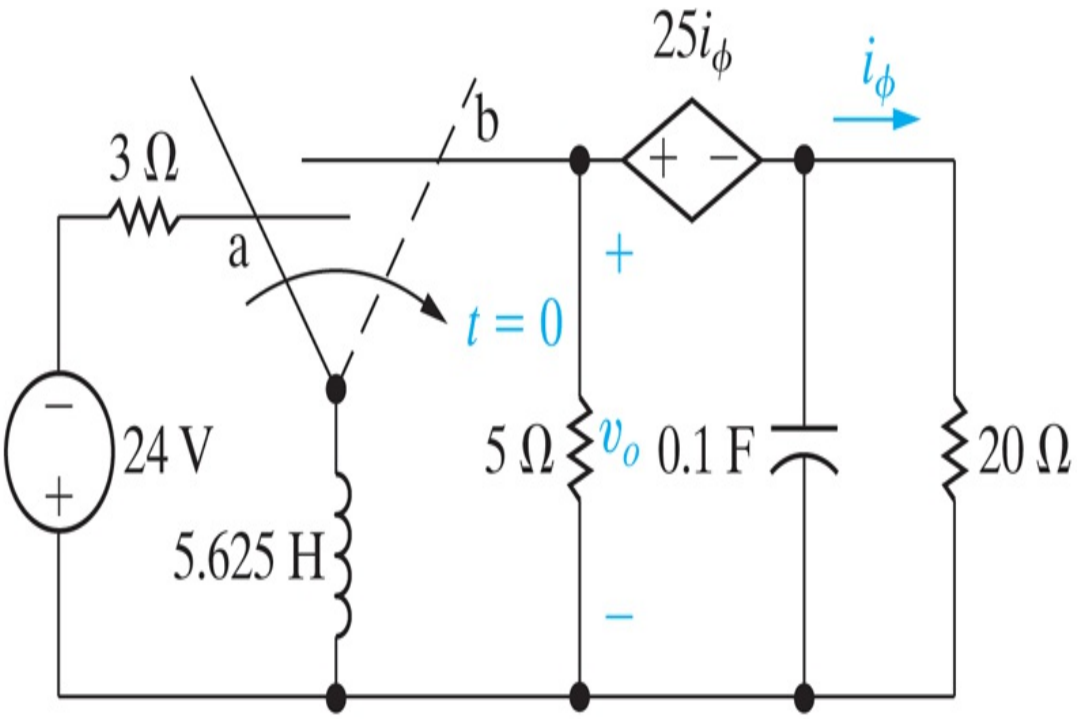


Figure P13.29 Full Alternative Text

22. 13.30 There is no energy stored in the circuit in Fig. P13.30 at the time the current source turns on. Given that ig=100u(t) A:

    1. a) Find Io(s).
    2. b) Use the initial- and final-value theorems to find io(0+) and io(∞).
    3. c) Determine if the results obtained in (b) agree with known circuit behavior.
    4. d) Find io(t).

Figure P13.30

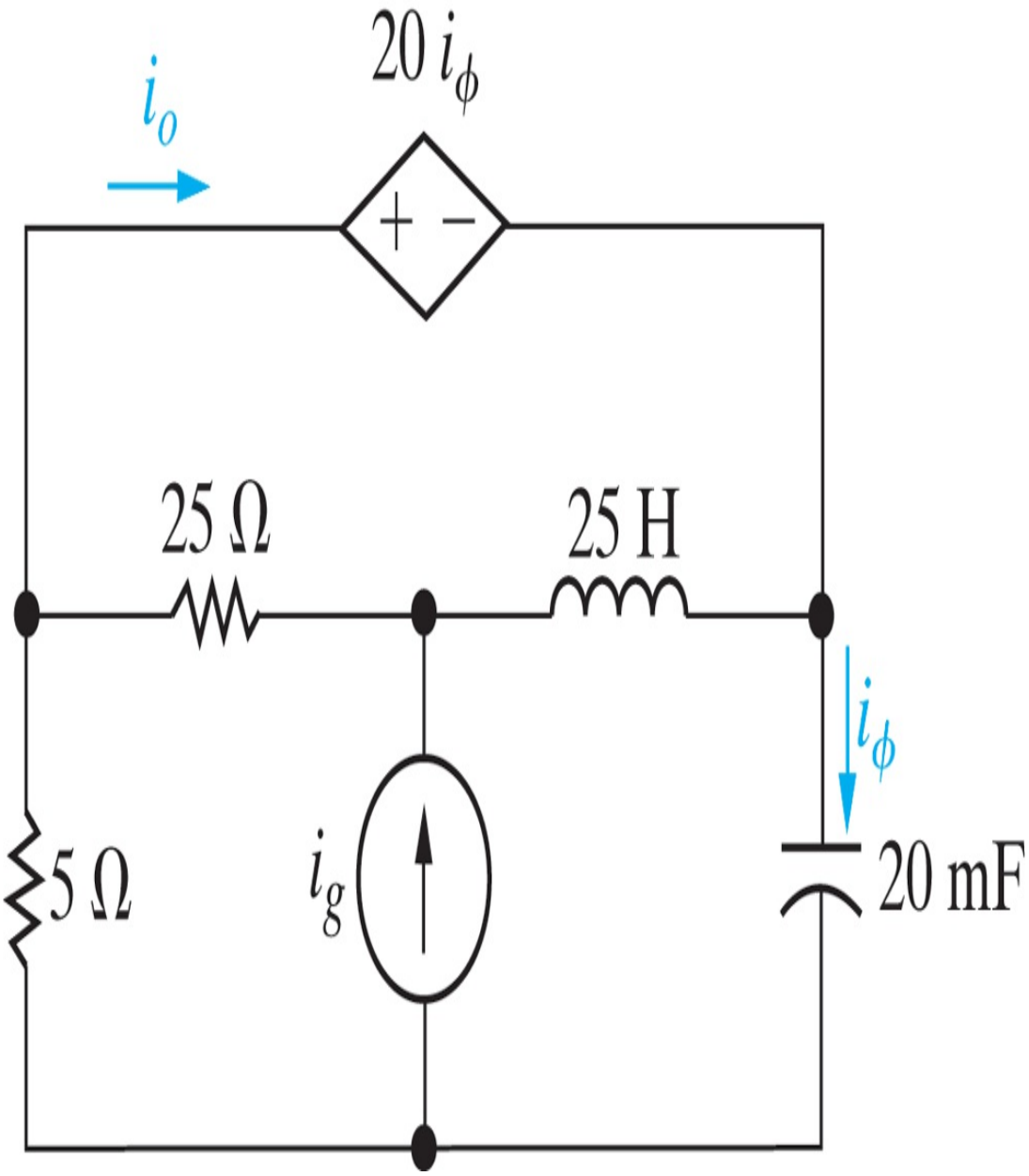


Figure P13.30 Full Alternative Text

23. 13.31 There is no energy stored in the capacitors in the circuit in Fig. P13.31 at the time the switch is closed.

    1. a) Construct the s-domain circuit for t>0.
    2. b) Find I1, V1, and V2.
    3. c) Find i1, v1, and v2.

4. d) Do your answers for i1, v1, and v2 make sense in terms of known circuit behavior? Explain.

Figure P13.31

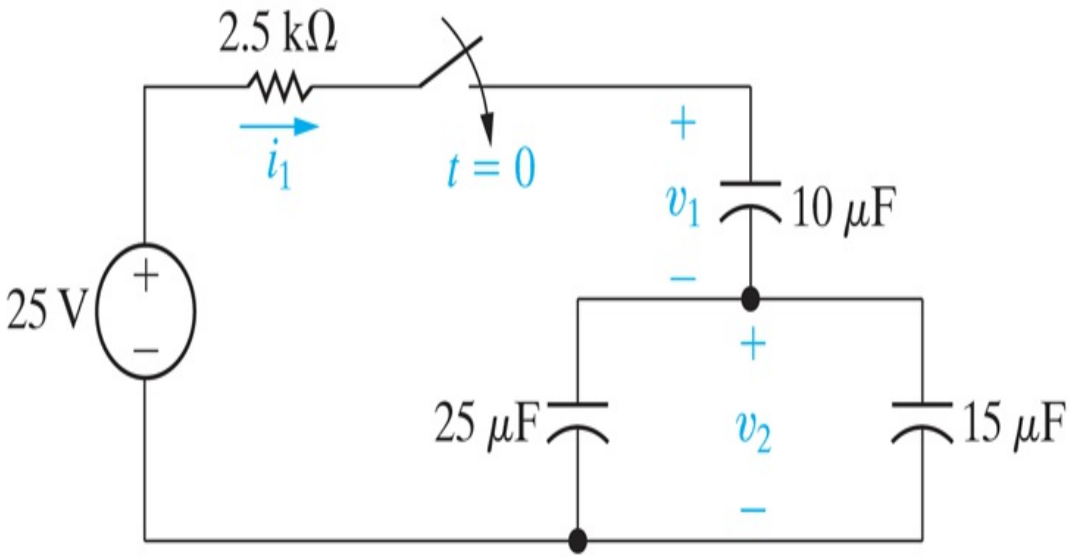


Figure P13.31 Full Alternative Text

24. 13.32 The switch in the circuit seen in Fig. P13.32 has been in position a for a long time before moving instantaneously to position b at t=0.

    1. a) Construct the s-domain equivalent circuit for t>0.
    2. b) Find V1 and v1.
    3. c) Find V2 and v2.

Figure P13.32

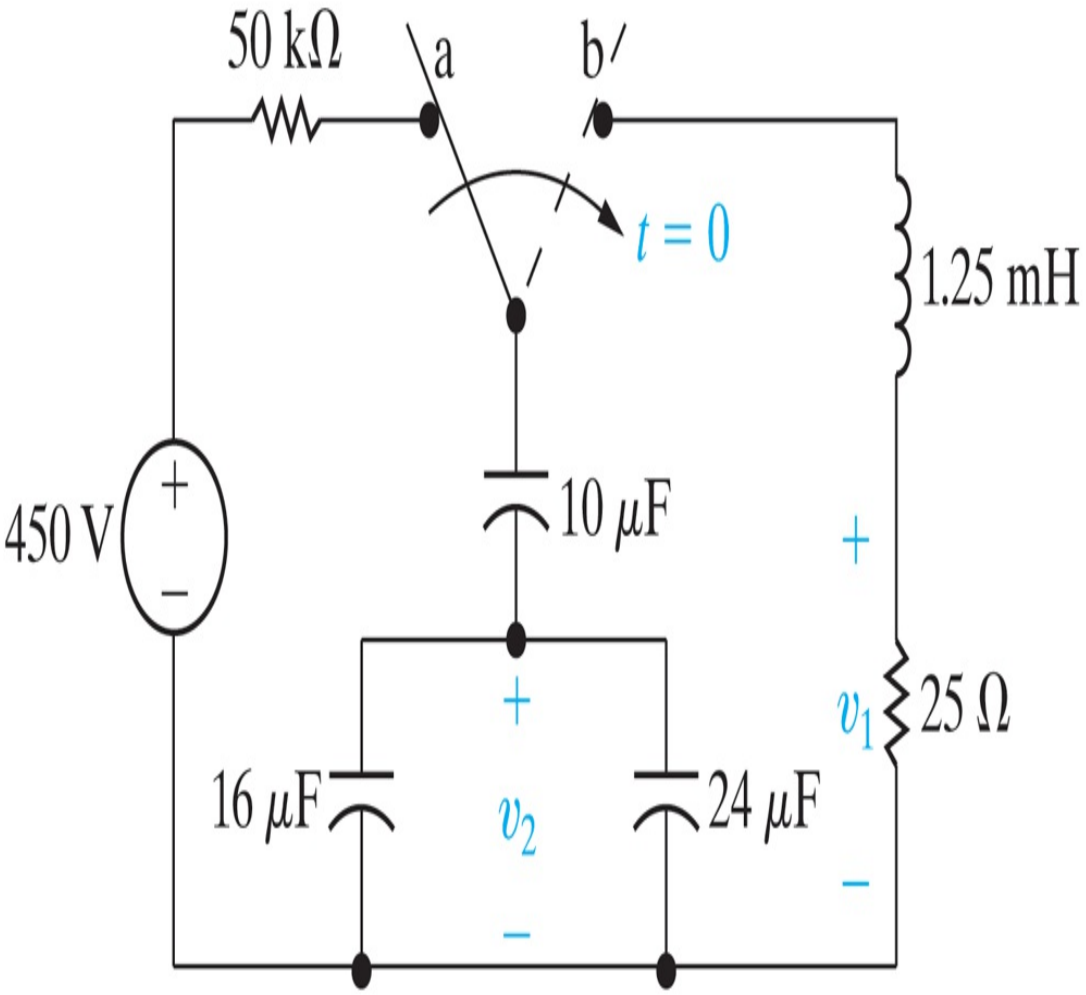


Figure P13.32 Full Alternative Text

25. 13.33 There is no energy stored in the circuit in Fig. P13.33 at the time the current source is energized.

    1. a) Find Ia and Ib.
    2. b) Find ia and ib.
    3. c) Find Va, Vb, and Ic.
    4. d) Find va, vb, and vc.
    5. e) Assume a capacitor will break down whenever its terminal

voltage is 1000 V. How long after the current source turns on will one of the capacitors break down?

Figure P13.33

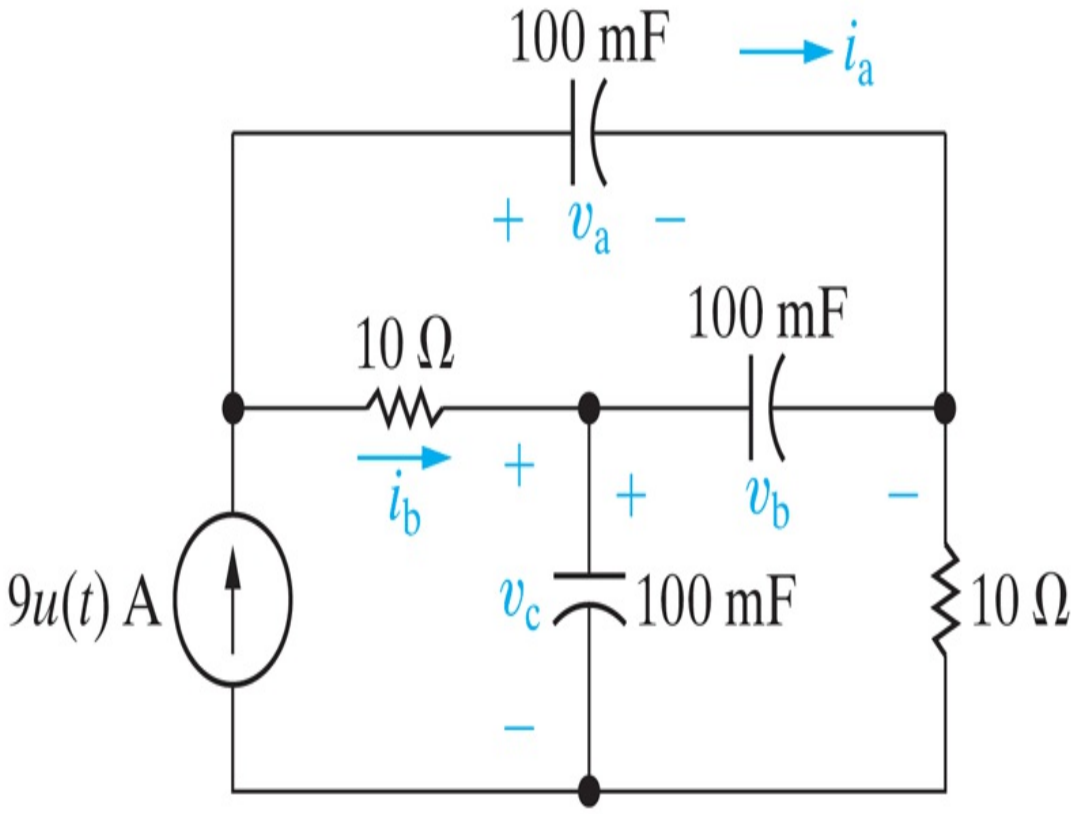


Figure P13.33 Full Alternative Text

26. 13.34 The switch in the circuit shown in Fig. P13.34 has been open for a long time. The voltage of the sinusoidal source is vg=Vm sin (ωt+ϕ). The switch closes at t=0. Note that the angle ϕ in the voltage expression determines the value of the voltage at the moment when the switch closes, that is, vg(0)=Vm sin ϕ.

    1. a) Use the Laplace transform method to find i for t>0.

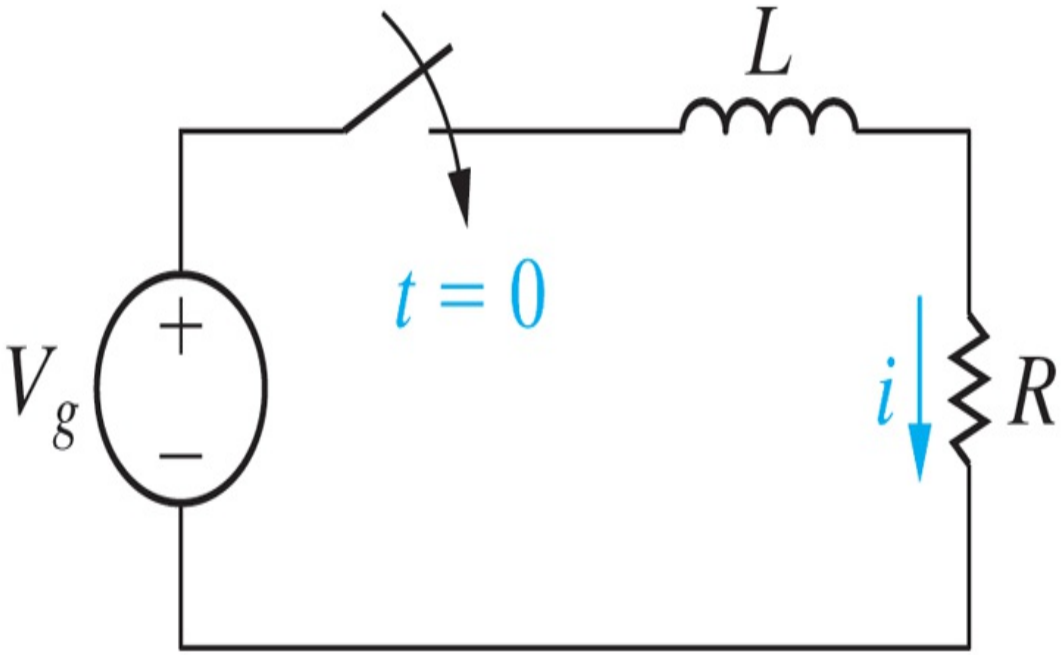
    2. b) Using the expression derived in (a), write the expression for the current after the switch has been closed for a long time.

3. c) Using the expression derived in (a), write the expression for the transient component of i.

4. d) Find the steady-state expression for i using the phasor method. Verify that your expression is equivalent to that obtained in (b).

5. e) Specify the value of ϕ so that the circuit passes directly into steady-state operation when the switch is closed.

Figure P13.34



27. 13.35 Using the expression for capacitor voltage found in Example 13.6, show that the capacitor current in Fig. 13.20 is positive for 0<t<200 μs and negative for t>200 μs. Also show that at 200 μs, the current is zero and that this corresponds to when dvC/dt is zero.

28. 13.36 There is no energy stored in the circuit in Fig. P13.36 at the time the voltage source is energized.

    1. a) Find Vo, Io, and IL.

2. b) Find vo, io, and iL for t≥0.

Figure P13.36

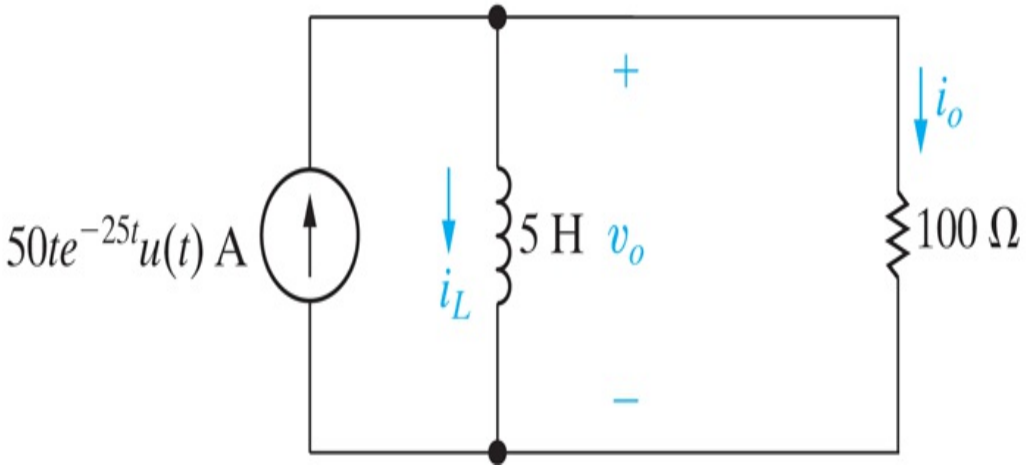


Figure P13.36 Full Alternative Text

29. 13.37 The two switches in the circuit shown in Fig. P13.37 operate simultaneously. There is no energy stored in the circuit at the instant the switches close. Find i(t) for t≥0+ by first finding the s-domain Thévenin equivalent of the circuit to the left of the terminals a, b.

Figure P13.37

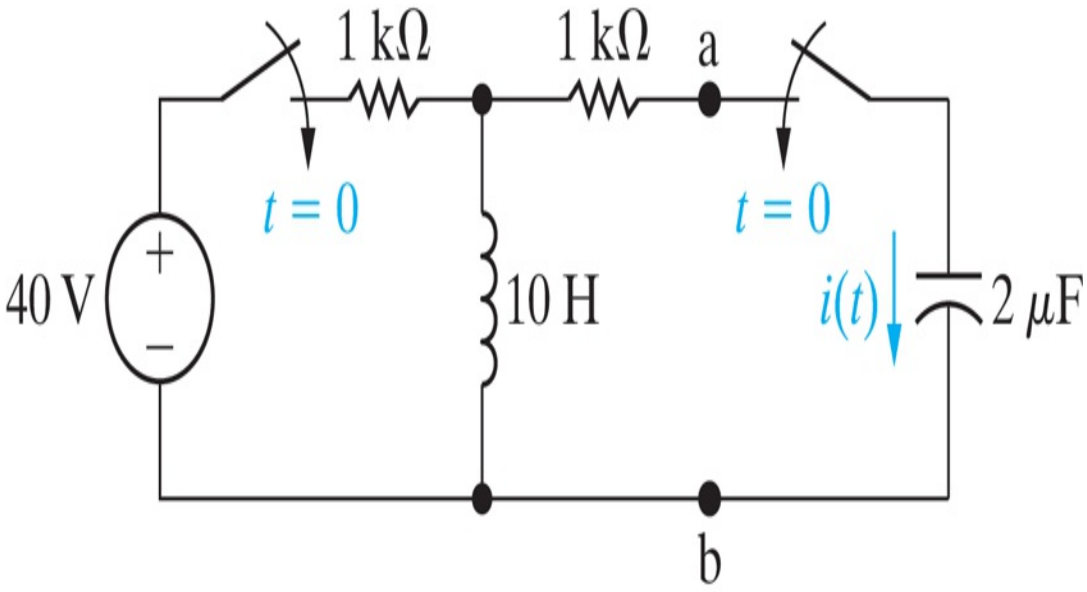


Figure P13.37 Full Alternative Text

30. 13.38 There is no energy stored in the circuit in Fig. P13.38 at the time the switch is closed.

    1. a) Find Vo.
    2. b) Use the initial- and final-value theorems to find i1(0+) and i1(∞).
    3. c) Find vo.

Figure P13.38

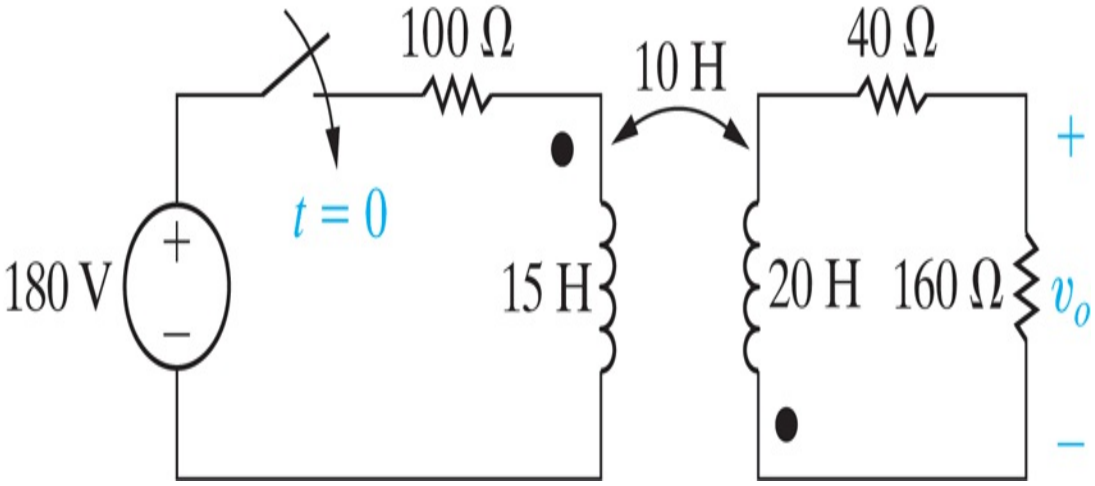


Figure P13.38 Full Alternative Text

31. 13.39 Find vo in the circuit in Fig. P13.38 if the polarity dot on the 20 H coil is at the top.

32. 13.40 The magnetically coupled coils in the circuit seen in Fig. P13.40 carry initial currents of 15 and 10 A, as shown.

    1. a) Find the initial energy stored in the circuit.
    2. b) Find I1 and I2.
    3. c) Find i1 and i2.
    4. d) Find the total energy dissipated in the 120 and 270 Ω resistors.
    5. e) Repeat (a)–(d), with the dot on the 18 H inductor at the lower terminal.

Figure P13.40

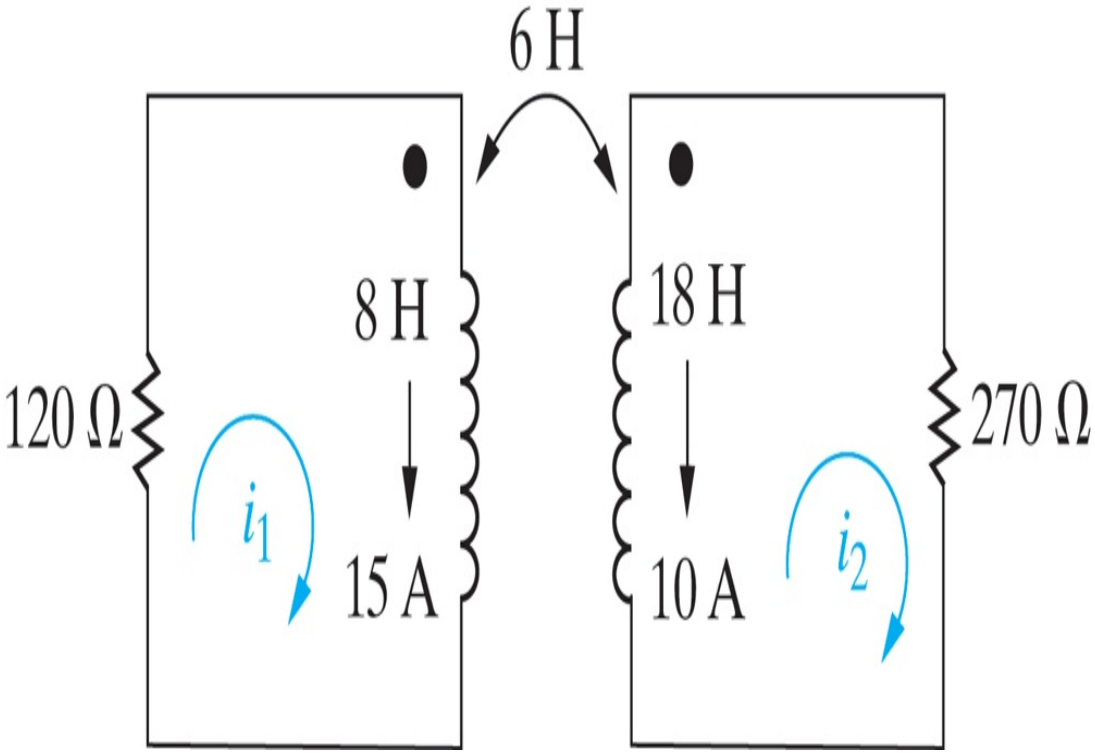


Figure P13.40 Full Alternative Text

33. 13.41 In the circuit in Fig. P13.41, switch 1 closes at t=0, and the make-before-break switch moves instantaneously from position a to position b.

    1. a) Construct the s-domain equivalent circuit for t>0.
    2. b) Find I1.
    3. c) Use the initial- and final-value theorems to check the initial and final values of i1.
    4. d) Find i1 for t≥0+.

Figure P13.41

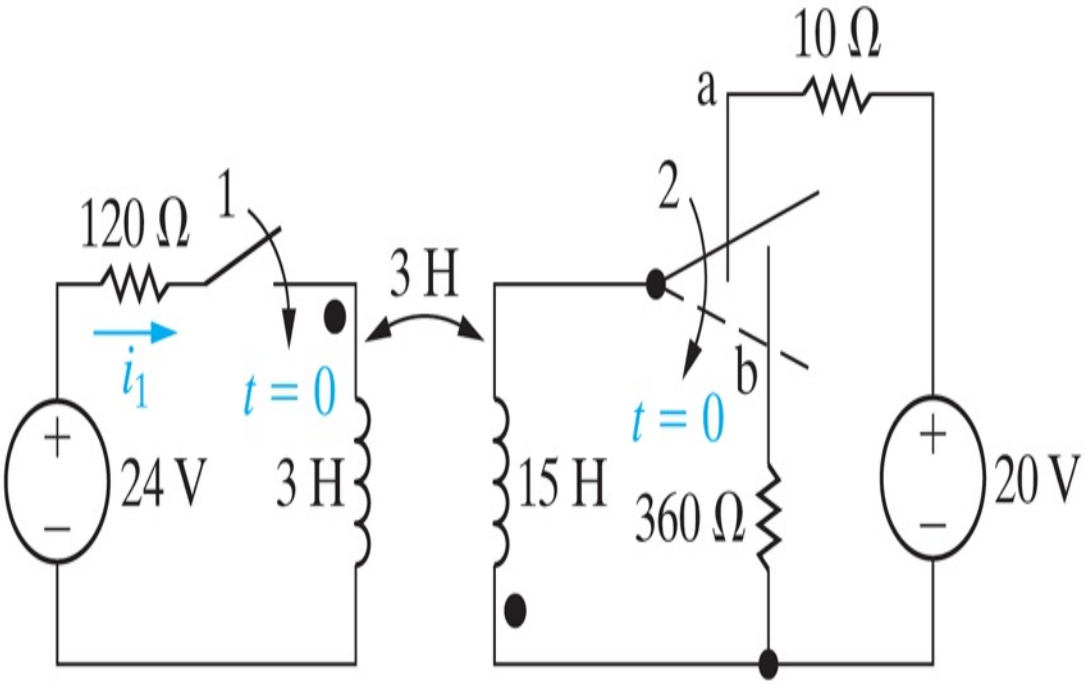


Figure P13.41 Full Alternative Text

34. 13.42 The make-before-break switch in the circuit seen in Fig. P13.42 has been in position a for a long time. At t=0, it moves instantaneously to position b. Find io for t≥0.

Figure P13.42

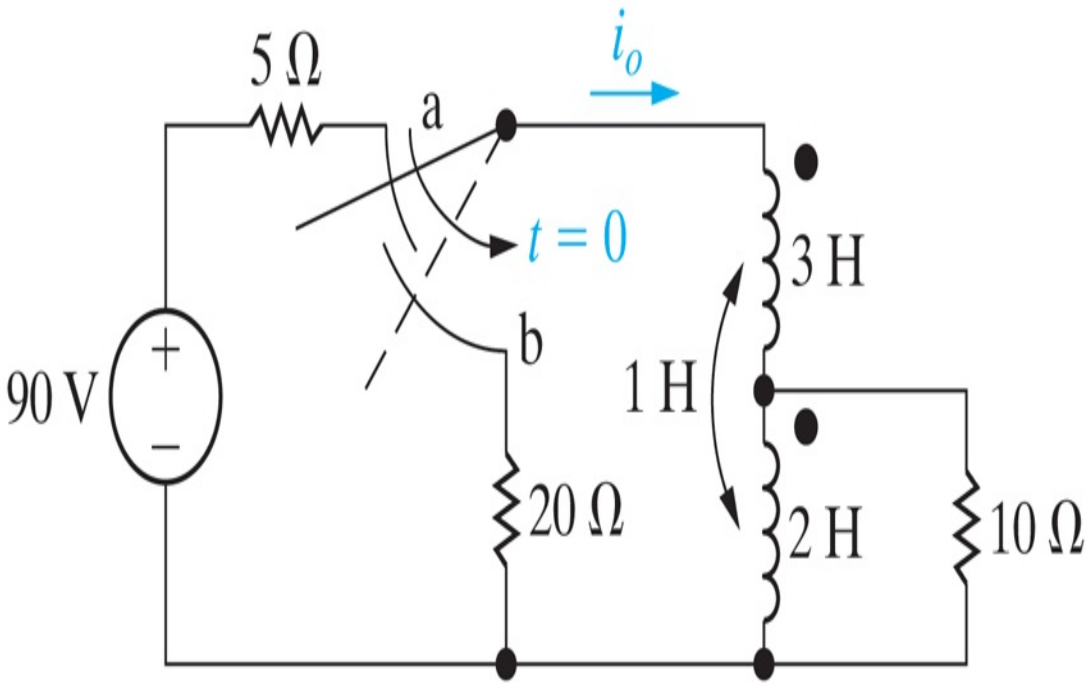


Figure P13.42 Full Alternative Text

35. 13.43 The switch in the circuit seen in Fig. P13.43 has been closed for a long time before opening at t=0. Use the Laplace transform method of analysis to find vo.

Figure P13.43

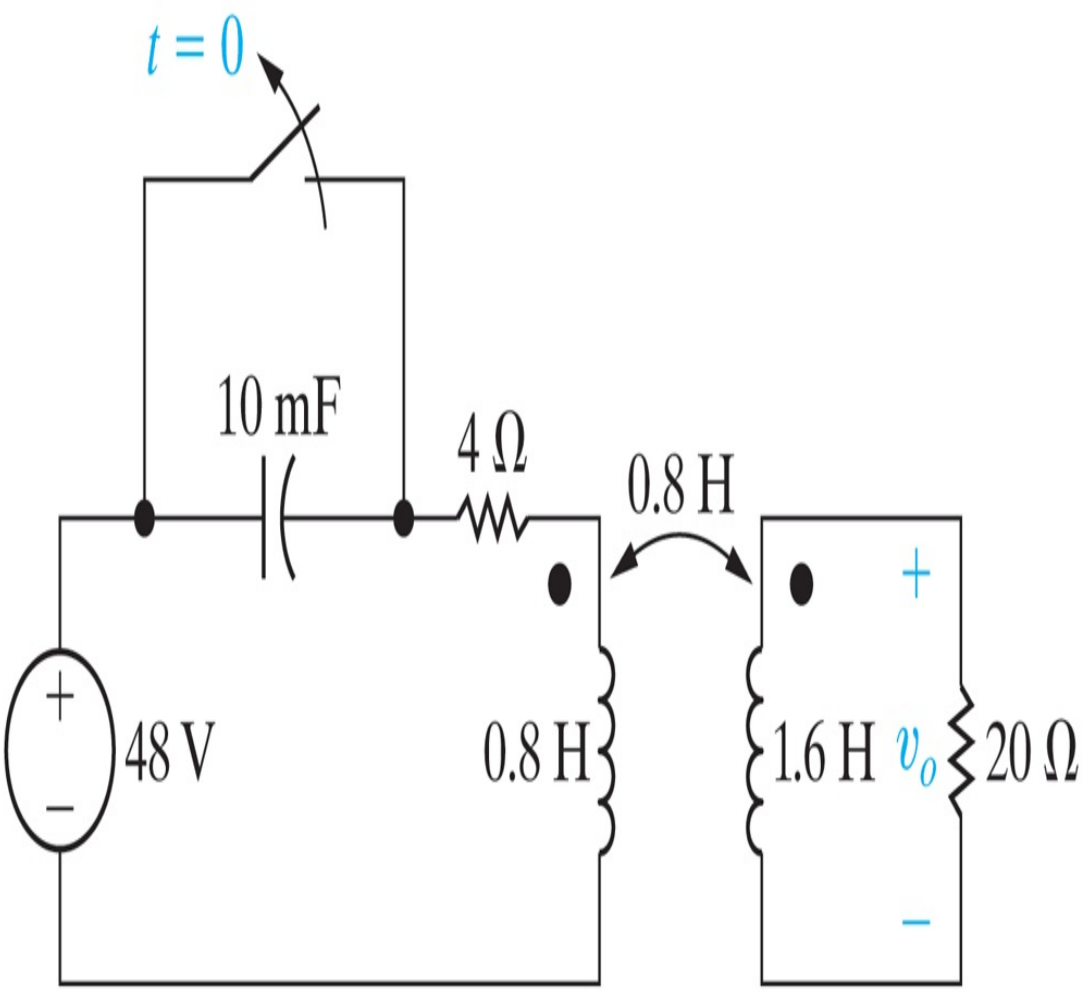


Figure P13.43 Full Alternative Text

36. 13.44 There is no energy stored in the circuit seen in Fig. P13.44 at the time the two sources are energized.

    1. a) Use the principle of superposition to find Vo.
    2. b) Find vo for t>0.

Figure P13.44

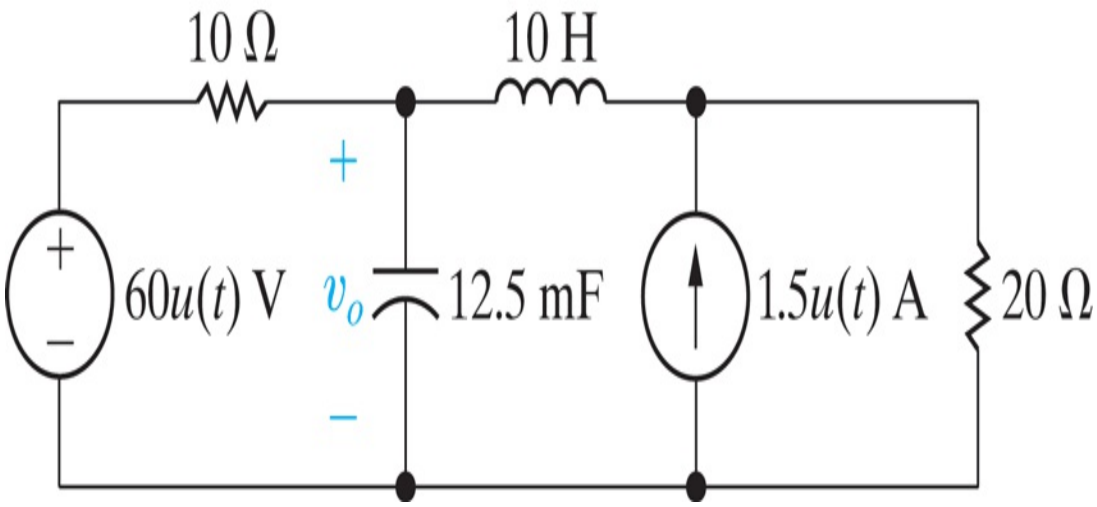


Figure P13.44 Full Alternative Text

37. 13.45 The op amp in the circuit seen in Fig. P13.45 is ideal. There is no energy stored in the capacitors at the time the circuit is energized. Determine (a) Vo, (b) vo, and (c) how long it takes to saturate the operational amplifier.

Figure P13.45

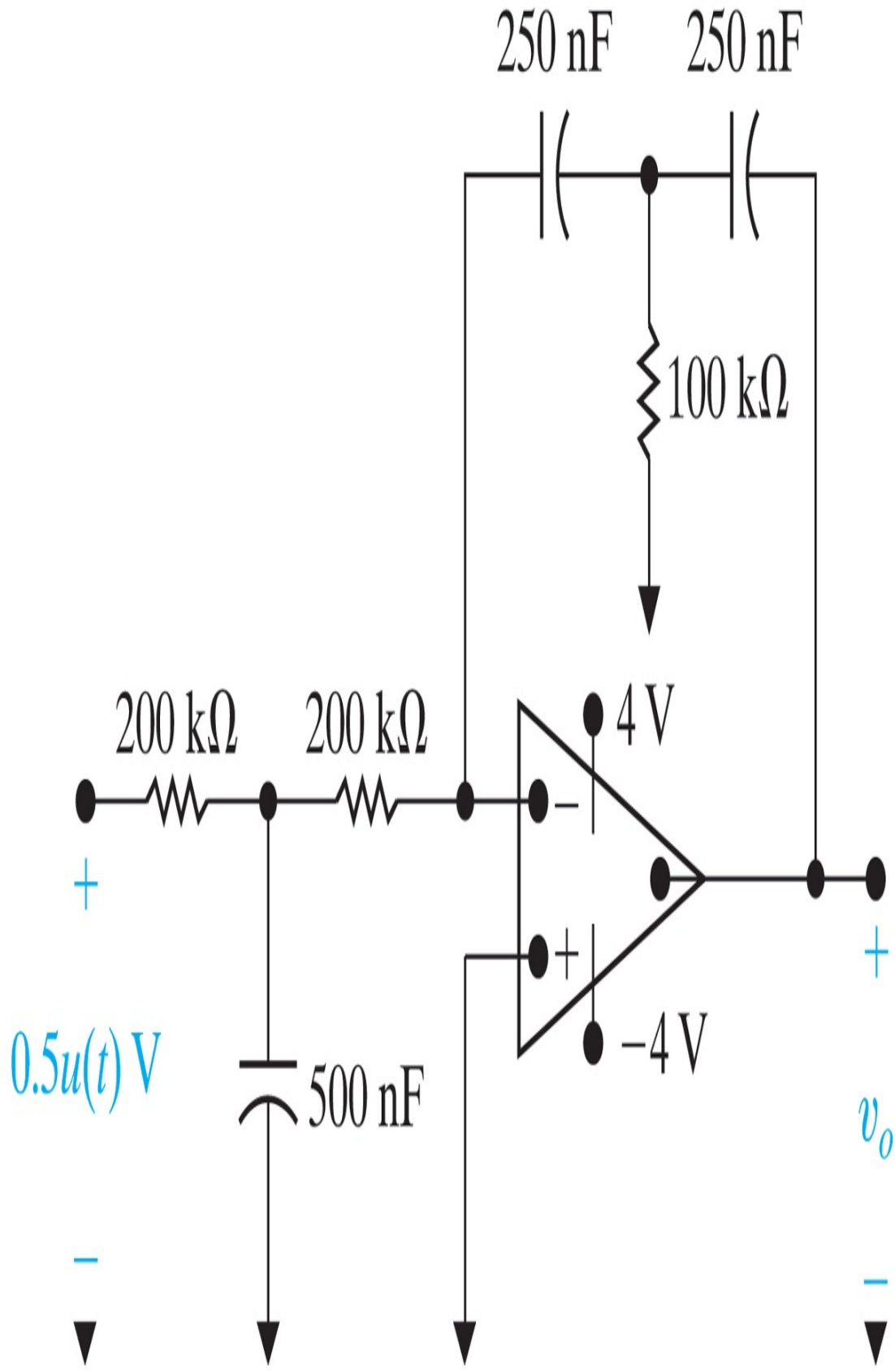

250 nF
250 nF
100 kΩ
200 kΩ
200 kΩ
4 V
−4 V
+
−
0.5u(t) V
500 nF
v_o

Figure P13.45 Full Alternative Text

38. 13.46 The op amp in the circuit shown in Fig. P13.46 is ideal. There is no energy stored in the circuit at the time it is energized. If vg=20,000tu(t)V, find (a) Vo, (b) vo, (c) how long it takes to saturate the operational amplifier, and (d) how small the rate of increase in vg must be to prevent saturation.

Figure P13.46

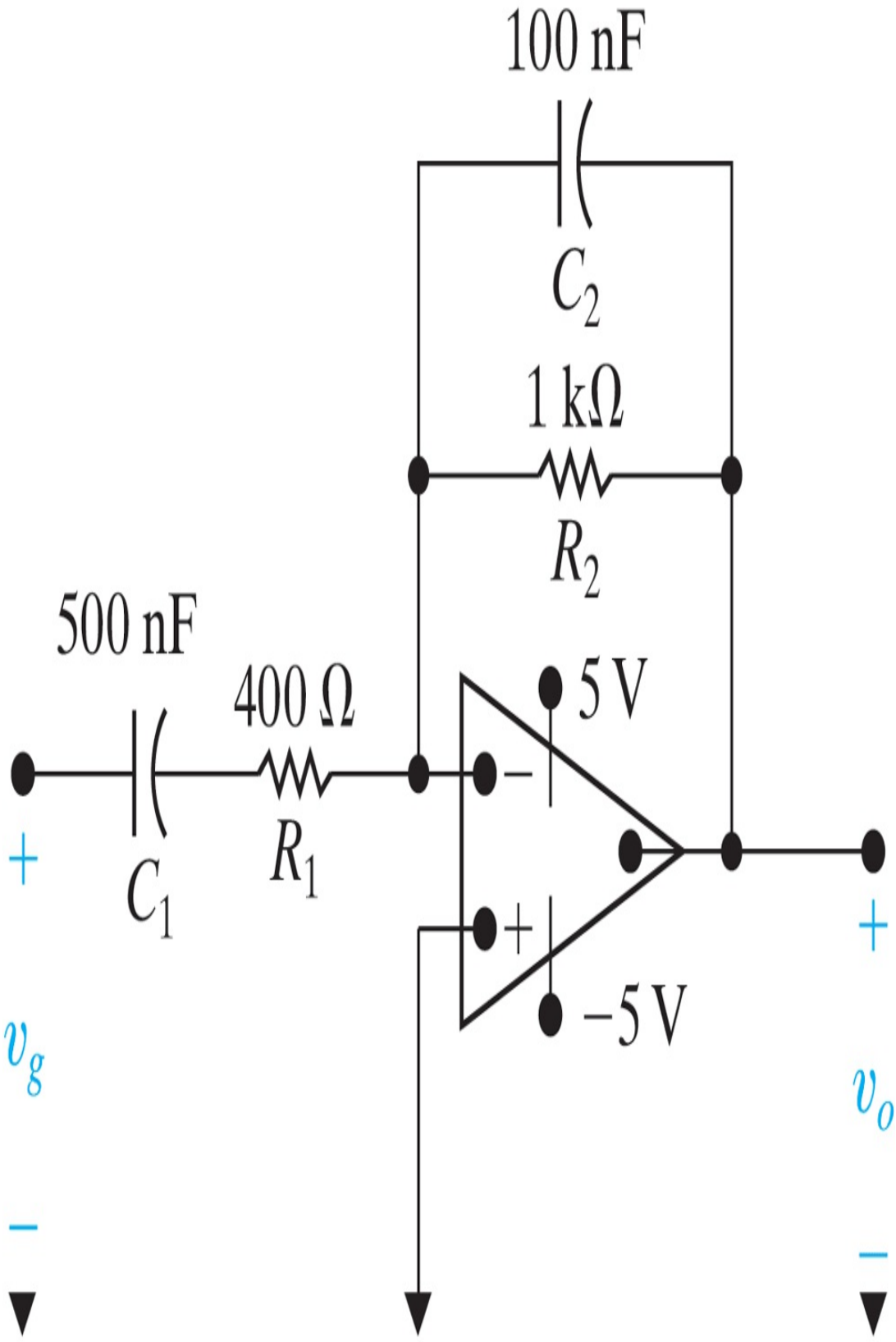

100 nF
C_2
1 kΩ
R_2
500 nF
400 Ω
5 V
C_1
R_1
+
v_g
−
−5 V
+
v_o
−

Figure P13.46 Full Alternative Text

39. 13.47 The op amp in the circuit shown in Fig. P13.47 is ideal. There is no energy stored in the capacitors at the instant the circuit is energized.

    1. a) Find vo if vg1=40u(t) V and vg2=16u(t) V.

    2. b) How many milliseconds after the two voltage sources are turned on does the op amp saturate?

Figure P13.47

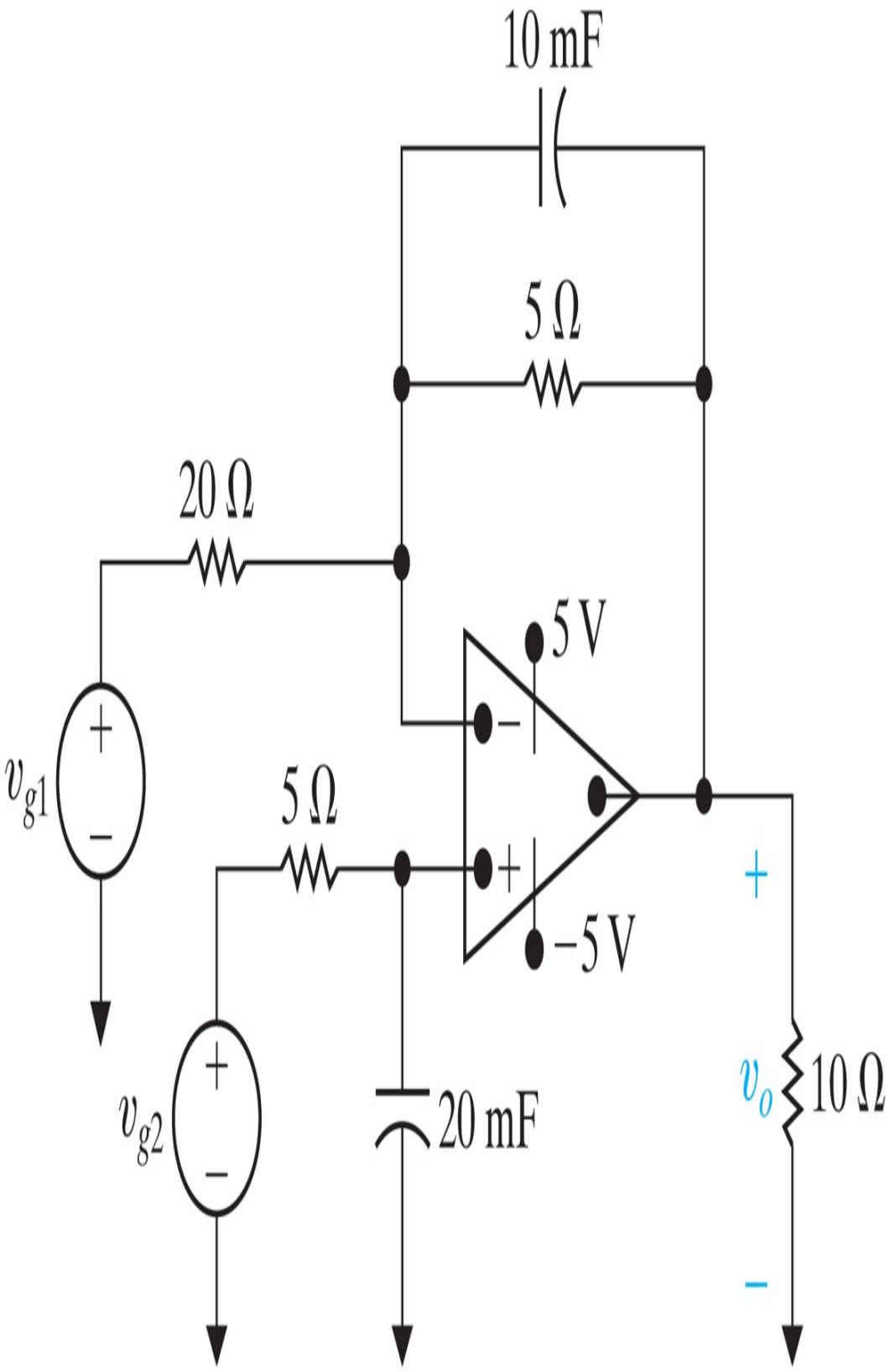

10 mF
5 Ω
20 Ω
5 V
v_{g1}
5 Ω
+
−
−5 V
v_{g2}
20 mF
v_o
10 Ω
+
−

Figure P13.47 Full Alternative Text

40. 13.48 Find vo(t) in the circuit shown in Fig. P13.48 if the ideal op amp operates within its linear range and vg=4.8u(t)V.

Figure P13.48

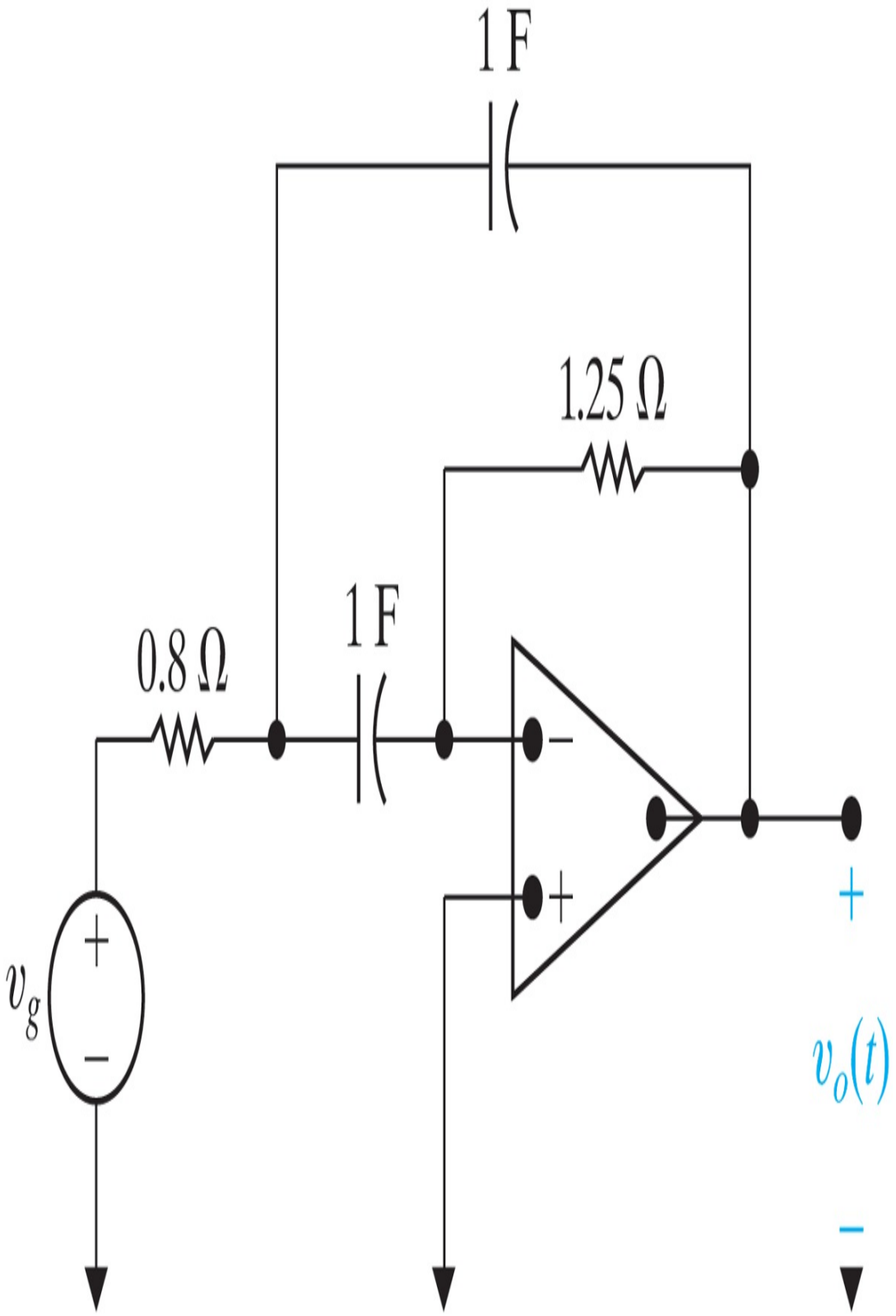


Figure P13.48 Full Alternative Text

Sections 13.4–13.5

1. 13.49

    1. a) Find the transfer function H(s)=Vo/Vi for the circuit shown in Fig. P13.49(a).

    2. b) Find the transfer function H(s)=Vo/Vi for the circuit shown in Fig. P13.49(b).

    3. c) Create two different circuits that have the transfer function H(s)=Vo/Vi=s/(s+10,000). Use components selected from Appendix H and Figs. P13.49(a) and (b).

Figure P13.49

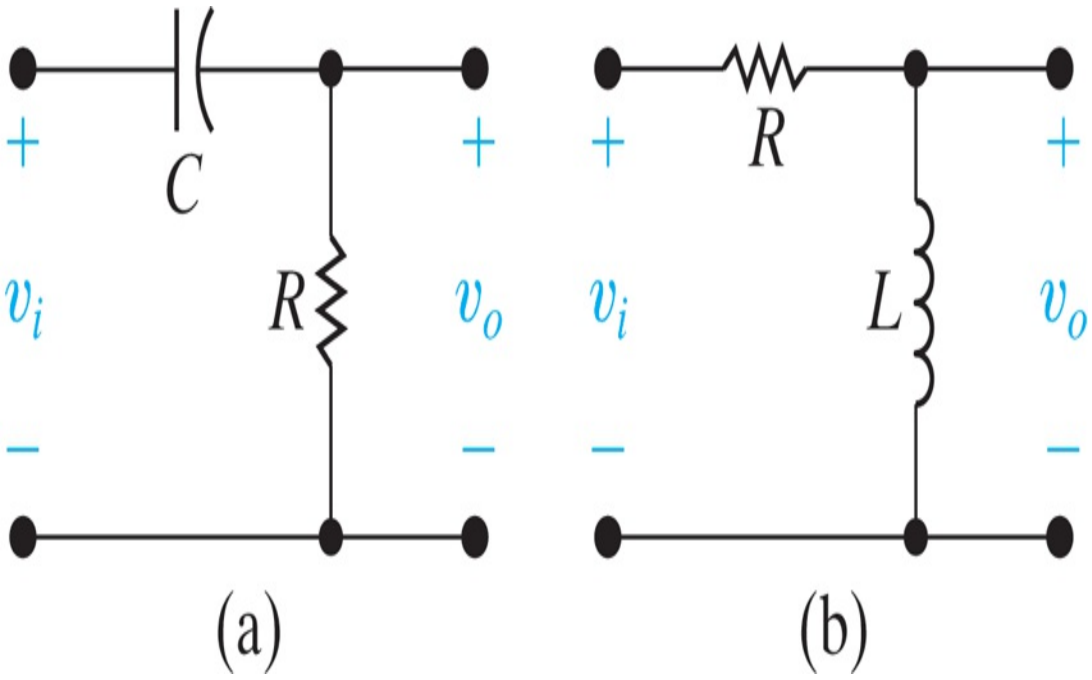


Figure P13.49 Full Alternative Text

2. 13.50

1. a) Find the transfer function H(s)=Vo/Vi for the circuit shown in Fig. P13.50(a).

2. b) Find the transfer function H(s)=Vo/Vi for the circuit shown in Fig. P13.50(b).

3. c) Create two different circuits that have the transfer function H(s)=Vo/Vi=1000/(s+1000). Use components selected from Appendix H and Figs. P13.50(a) and (b).

Figure P13.50

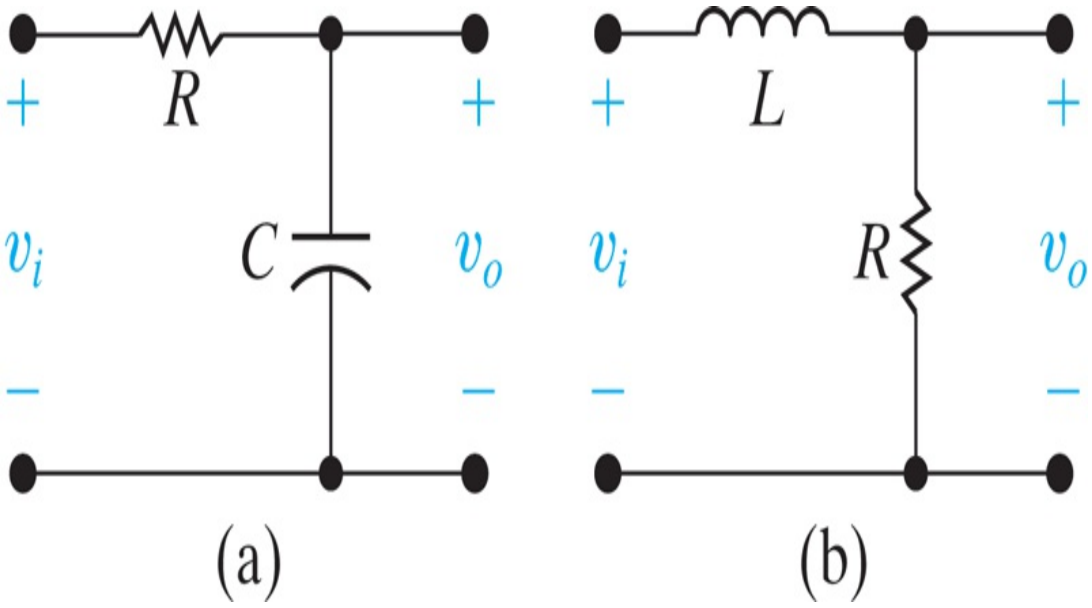
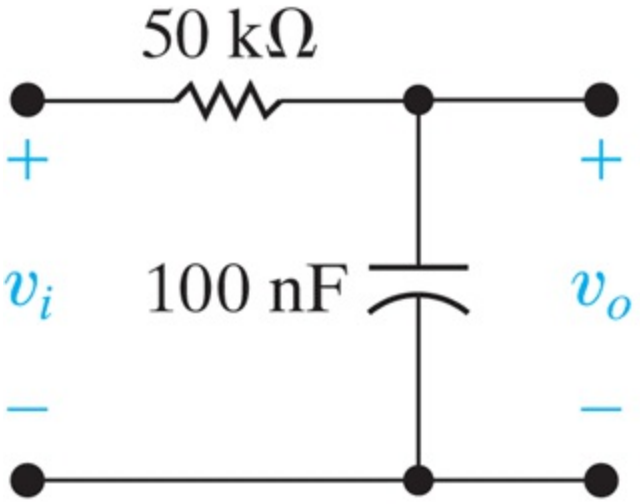


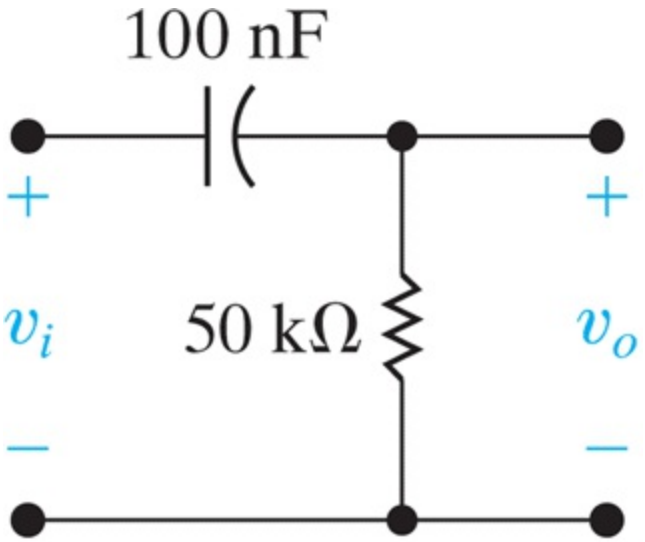
Figure P13.50 Full Alternative Text

3. 13.51 Find the numerical expression for the transfer function (Vo/Vi) of each circuit in Fig. P13.51 and give the numerical value of the poles and zeros of each transfer function.

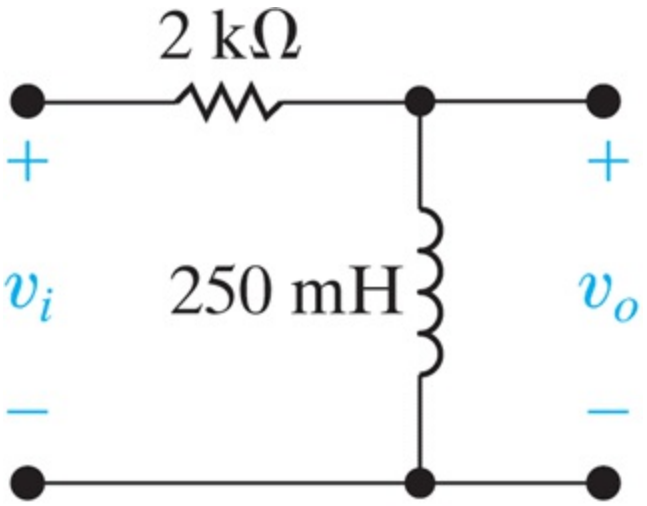
Figure P13.51



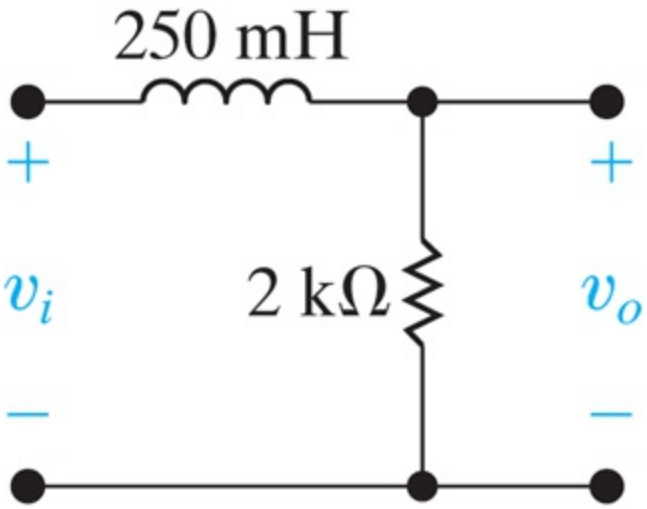
(a)



(b)



(c)



(d)

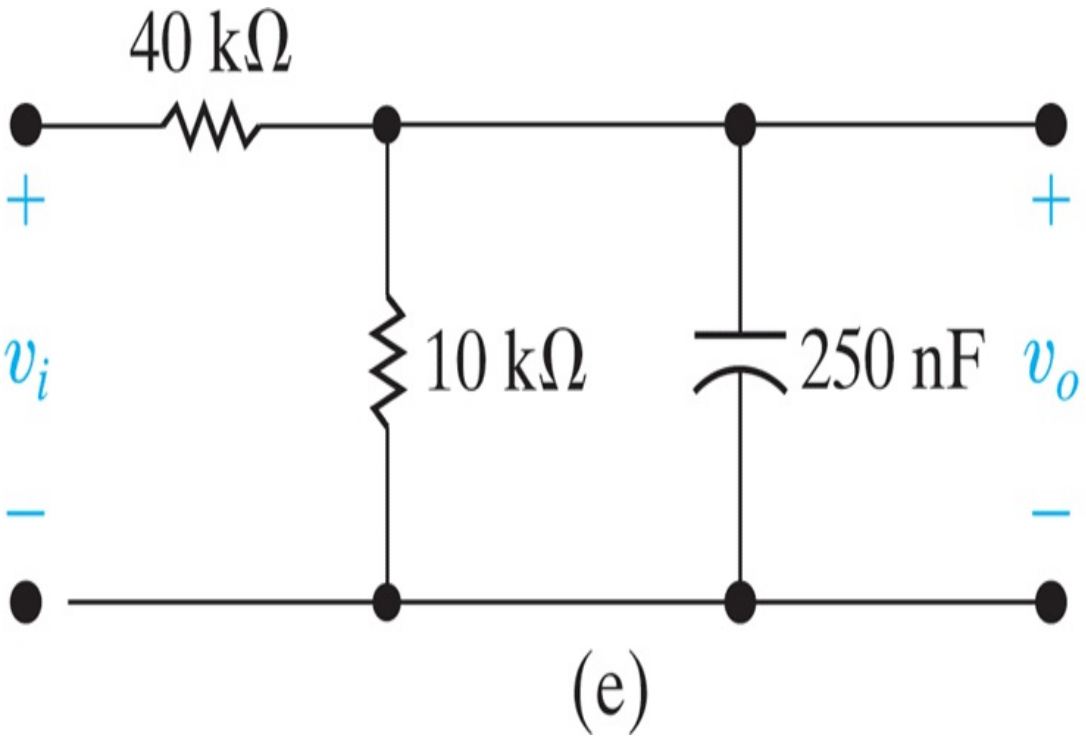


Figure P13.51 Full Alternative Text

4. 13.52

    1. a) Find the transfer function H(s)=Vo/Vi for the circuit shown in Fig. P13.52. Identify the poles and zeros for this transfer function.

    2. b) Find three components from Appendix H which when used in the circuit of Fig. P13.52 will result in a transfer function with two poles that are distinct real numbers. Calculate the values of the poles.

    3. c) Find three components from Appendix H which when used in the circuit of Fig. P13.52 will result in a transfer function with two poles, both with the same value. Calculate the value of the poles.

    4. d) Find three components from Appendix H which when used in the circuit of Fig. P13.52 will result in a transfer function with two poles that are complex conjugate complex numbers. Calculate the values of the poles.

Figure P13.52

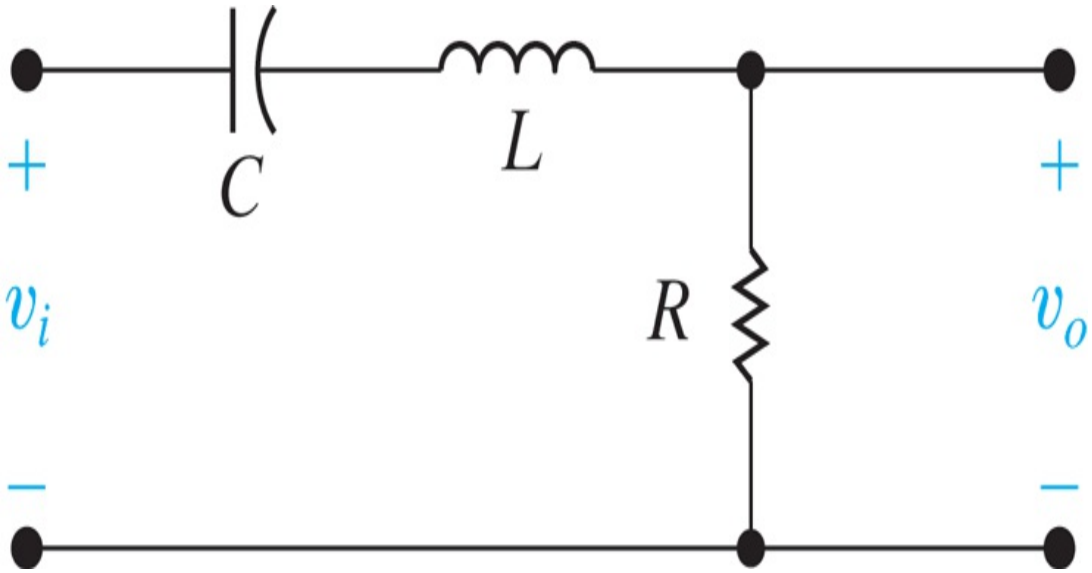


Figure P13.52 Full Alternative Text

5. 13.53 Repeat Problem 13.52 for the circuit in Fig. P13.53. In parts (b)—(d), let L=10 mH and C=1 μF, and use one or two resistors from Appendix H.

Figure P13.53

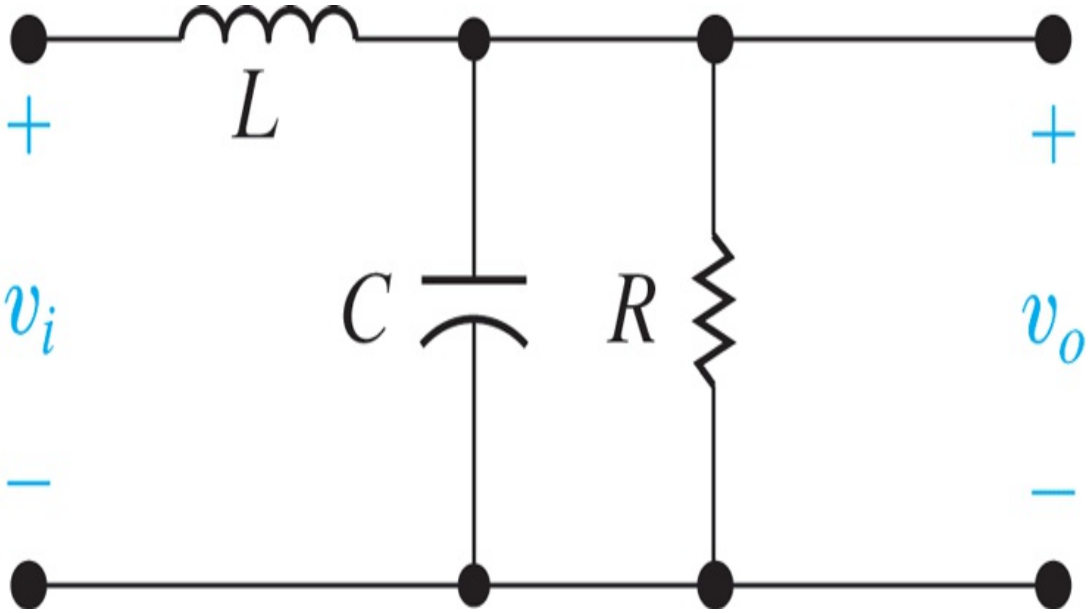


Figure P13.53 Full Alternative Text

6. 13.54

    1. a) Find the numerical expression for the transfer function H(s)=Vo/Vi for the circuit in Fig. P13.54.

    2. b) Give the numerical value of each pole and zero of H(s).

Figure P13.54

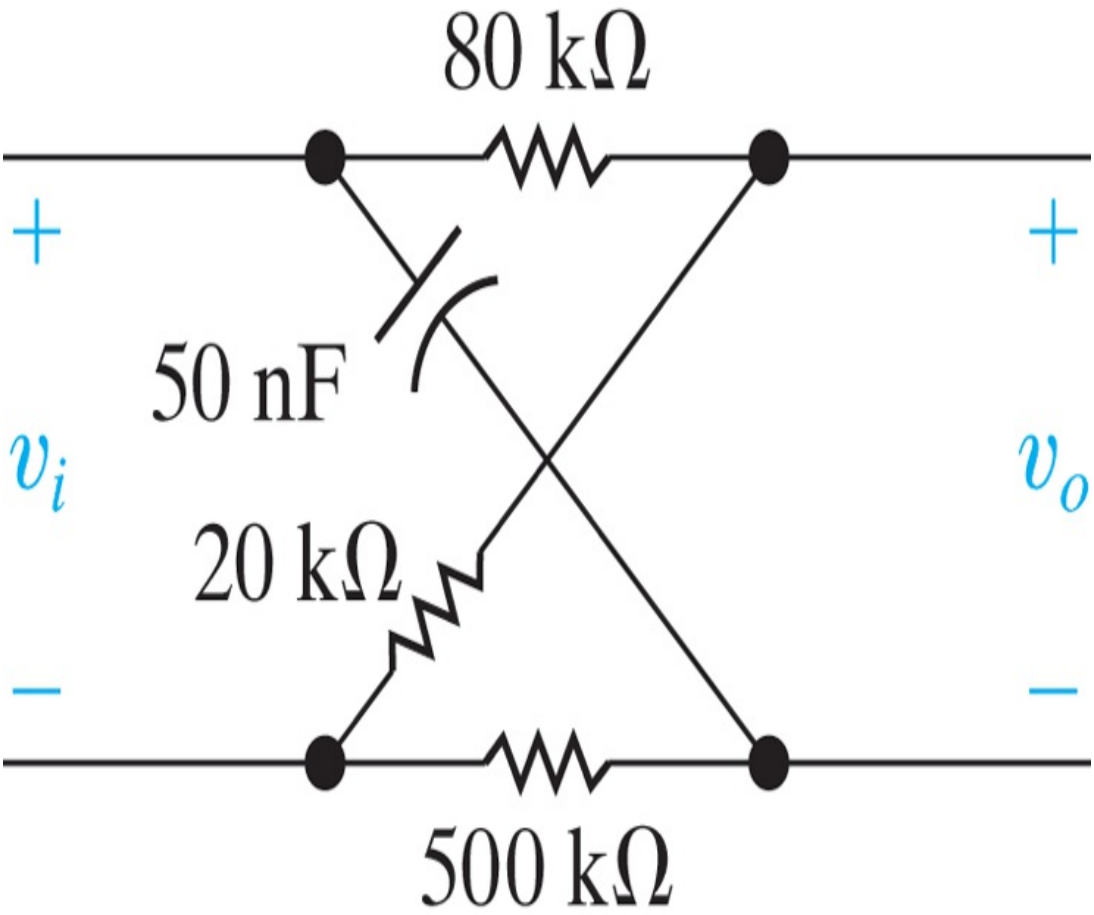


Figure P13.54 Full Alternative Text

7. 13.55 The operational amplifier in the circuit in Fig. P13.55 is ideal.

    1. a) Derive the numerical expression of the transfer function H(s)=Vo/Vg for the circuit in Fig. P13.55.

    2. b) Give the numerical value of each pole and zero of H(s).

    3. c) Use the transfer function to find the unit impulse response for the circuit in Fig. P13.55.

Figure P13.55

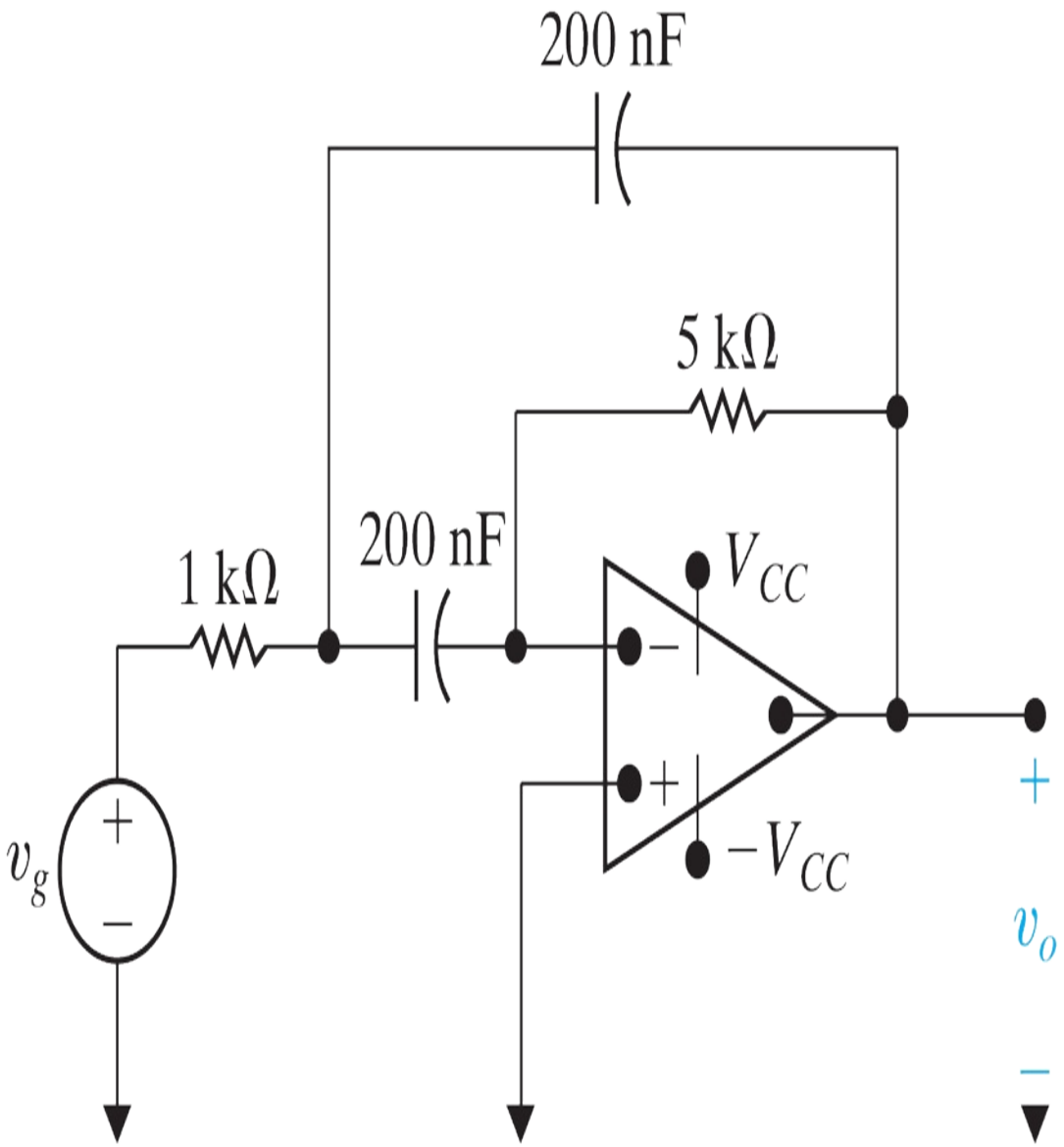


Figure P13.55 Full Alternative Text

8. 13.56 The operational amplifier in the circuit in Fig. P13.56 is ideal.

    1. a) Find the numerical expression for the transfer function H(s)=Vo/Vg.

    2. b) Give the numerical value of each zero and pole of H(s).

    3. c) Use the transfer function to find the unit step response for the circuit in Fig. P13.56.

Figure P13.56

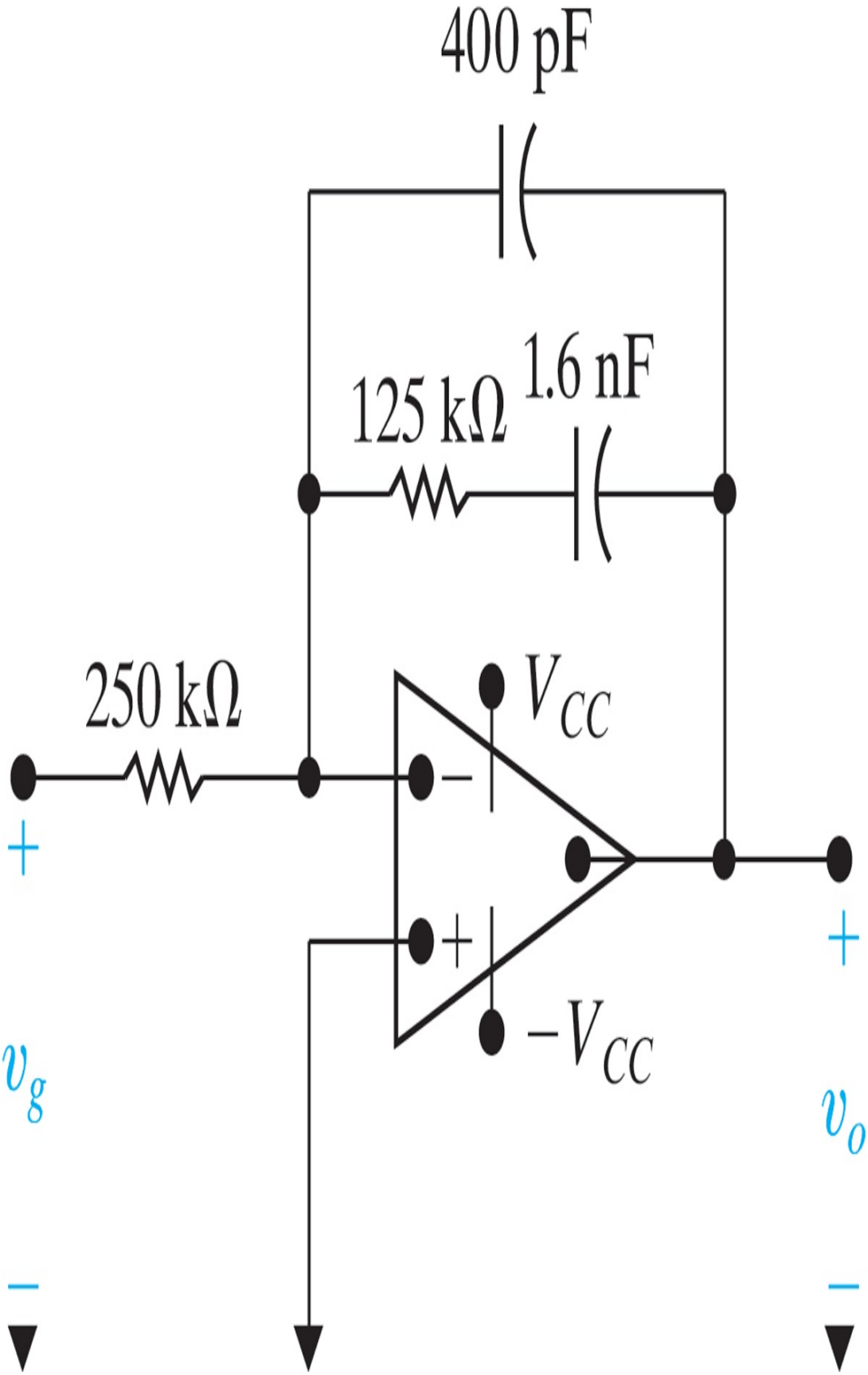

400 pF
125 kΩ
1.6 nF
250 kΩ
V_{CC}
$-V_{CC}$
+
−
v_g
v_o

Figure P13.56 Full Alternative Text

9. 13.57 The operational amplifier in the circuit in Fig. P13.57 is ideal.
    1. a) Find the numerical expression for the transfer function H(s)=Vo/Vg.
    2. b) Give the numerical value of each zero and pole of H(s).
    3. c) Use the transfer function to find the unit impulse response and unit step response for the circuit in Fig. P13.57.

Figure P13.57

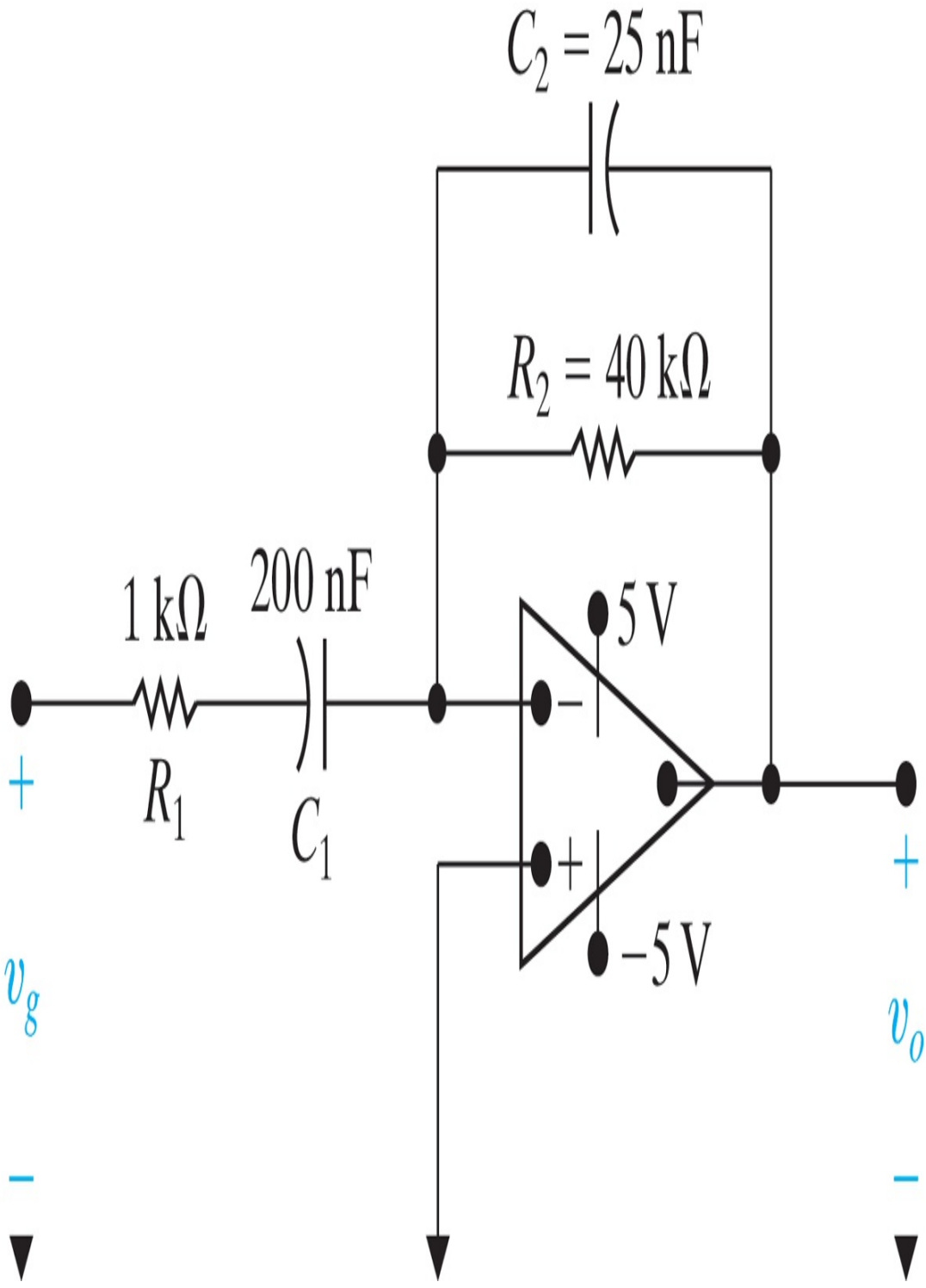


Figure P13.57 Full Alternative Text

10. 13.58 There is no energy stored in the circuit in Fig. P13.58 at the time the switch is opened. The sinusoidal current source is generating the

signal 100cos10,000t mA. The response signal is the current io.

1. a) Find the transfer function Io/Ig.
2. b) Find Io(s).
3. c) Describe the nature of the transient component of io(t) without solving for io(t).
4. d) Describe the nature of the steady-state component of io(t) without solving for io(t).
5. e) Verify the observations made in (c) and (d) by finding io(t).

Figure P13.58

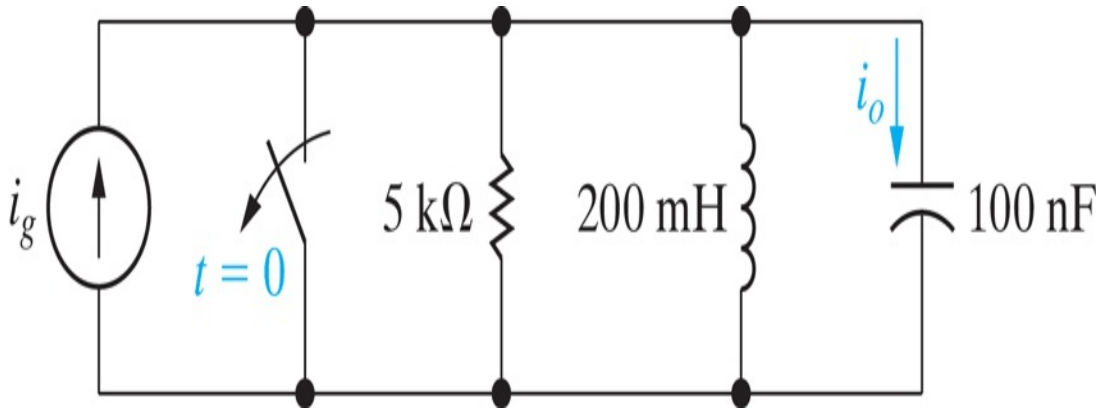


Figure P13.58 Full Alternative Text

11. 13.59

1. a) Find the transfer function Io/Ig as a function of μ for the circuit seen in Fig. P13.59.
2. b) Find the largest value of μ that will produce a bounded output signal for a bounded input signal.
3. c) Find io for μ=−3, 0, 4, 5, and 6 if ig=5u(t) A.

Figure P13.59

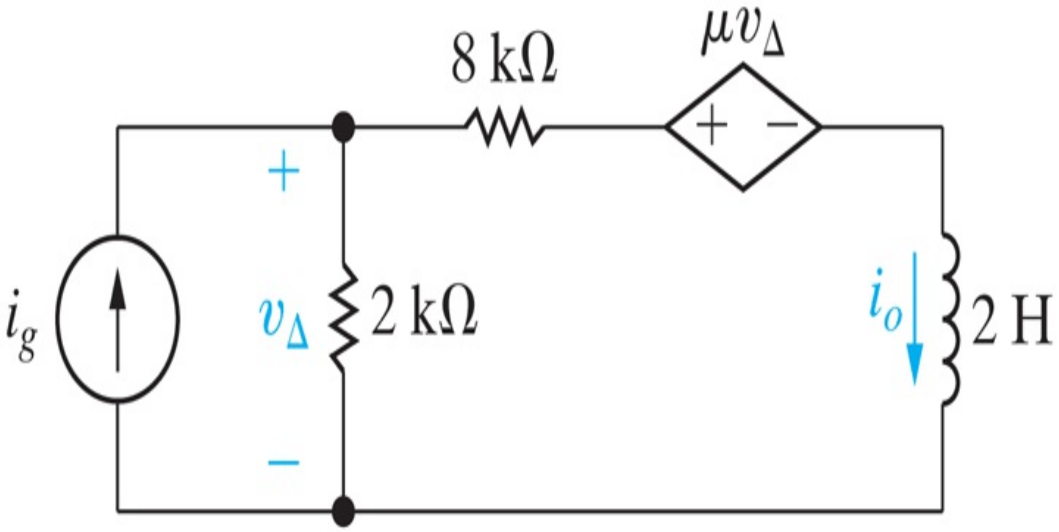


Figure P13.59 Full Alternative Text

12. 13.60 In the circuit of Fig. P13.60 io is the output signal and vg is the input signal. Find the poles and zeros of the transfer function, assuming there is no initial energy stored in the linear transformer or in the capacitor.

Figure P13.60

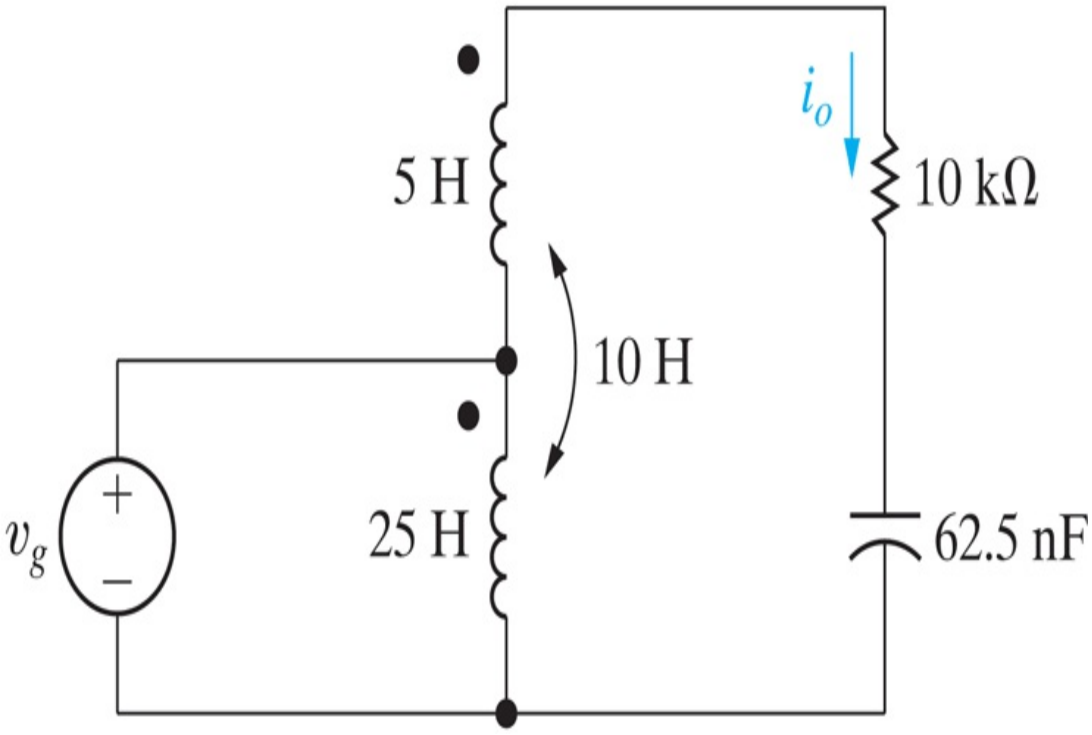


Figure P13.60 Full Alternative Text

Section 13.6

1. 13.61 A rectangular voltage pulse vi[u(t)−u(t−1)] V is applied to the circuit in Fig. P13.61. Use the convolution integral to find vo.

Figure P13.61

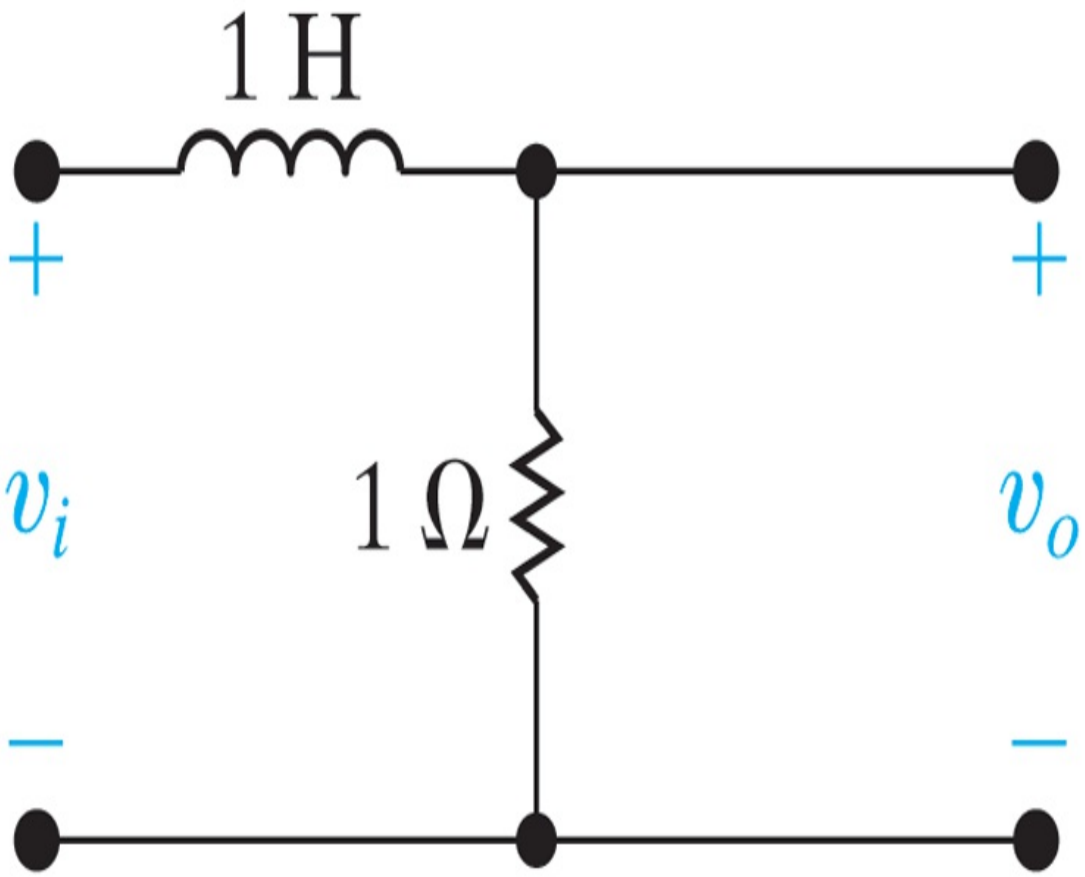


Figure P13.61 Full Alternative Text

2. 13.62 Interchange the inductor and resistor in Problem 13.61 and again use the convolution integral to find vo.

3. 13.63

1. a) Given y(t)=h(t)*x(t), find y(t) when h(t) and x(t) are the rectangular pulses shown in Fig. P13.63(a).

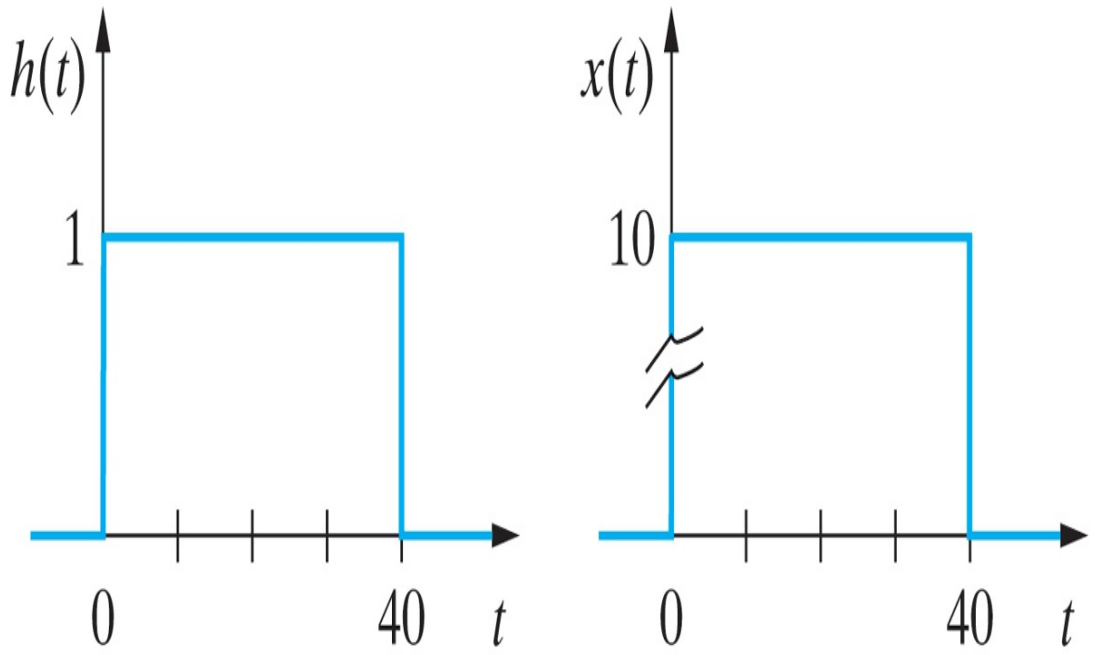
2. b) Repeat (a) when h(t) changes to the rectangular pulse shown in Fig. P13.63(b).

3. c) Repeat (a) when h(t) changes to the rectangular pulse shown in Fig. P13.63(c).

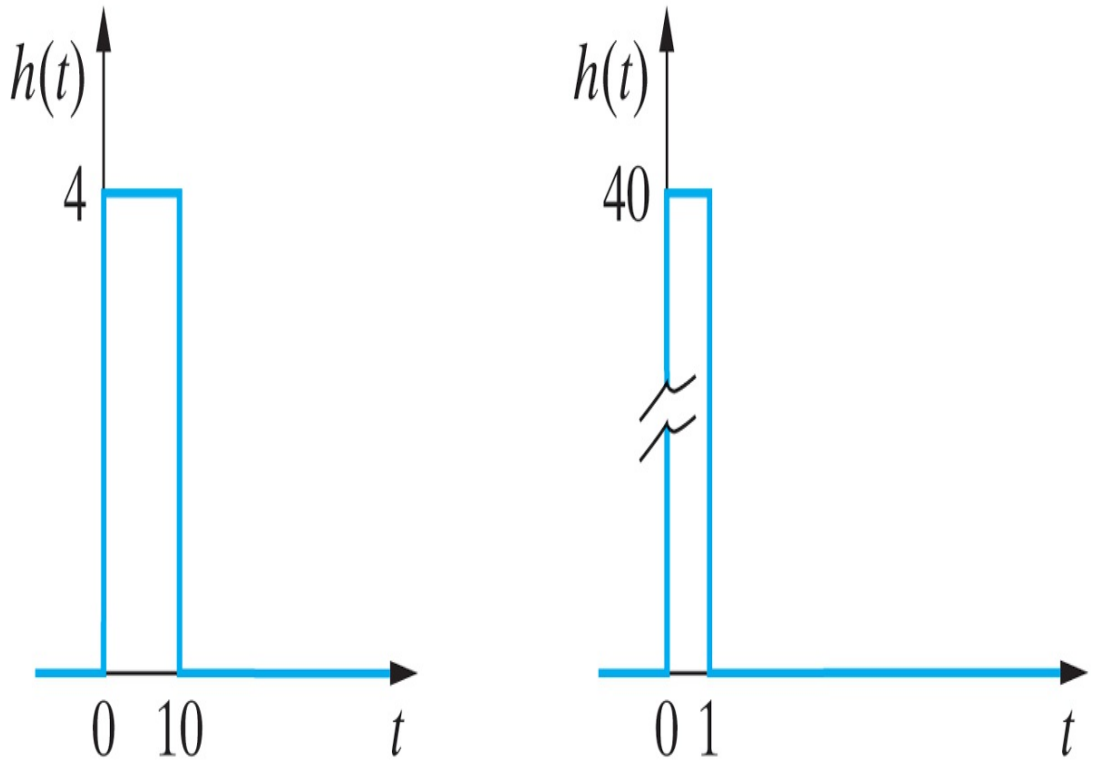
4. d) Sketch y(t) versus t for (a)–(c) on a single graph.

5. e) Do the sketches in (d) make sense? Explain.

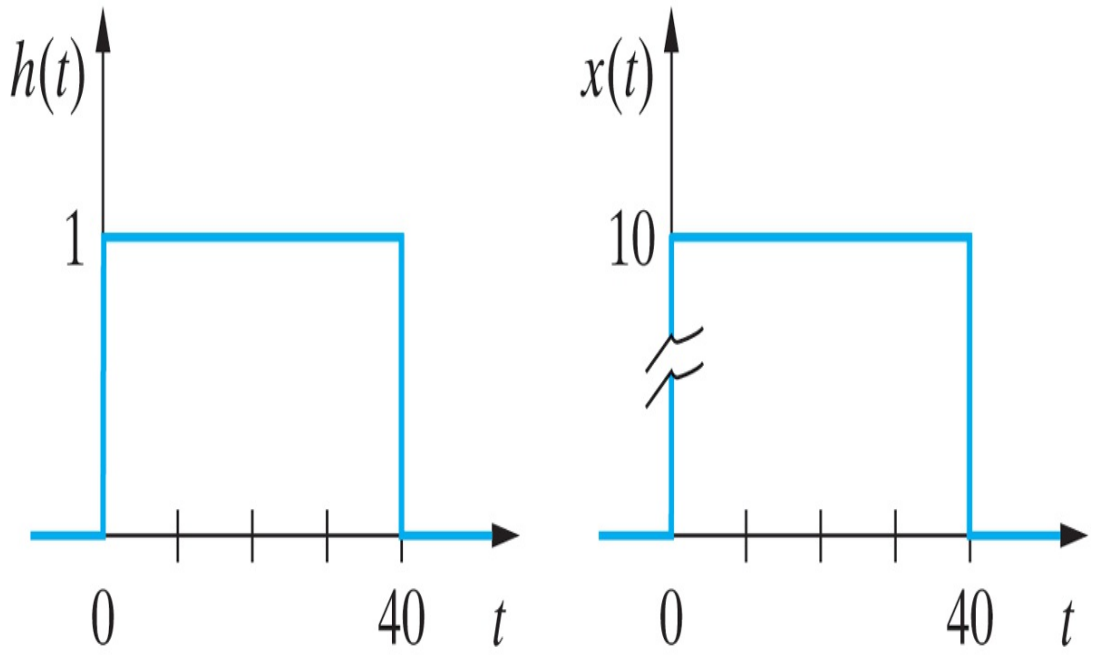
Figure P13.63



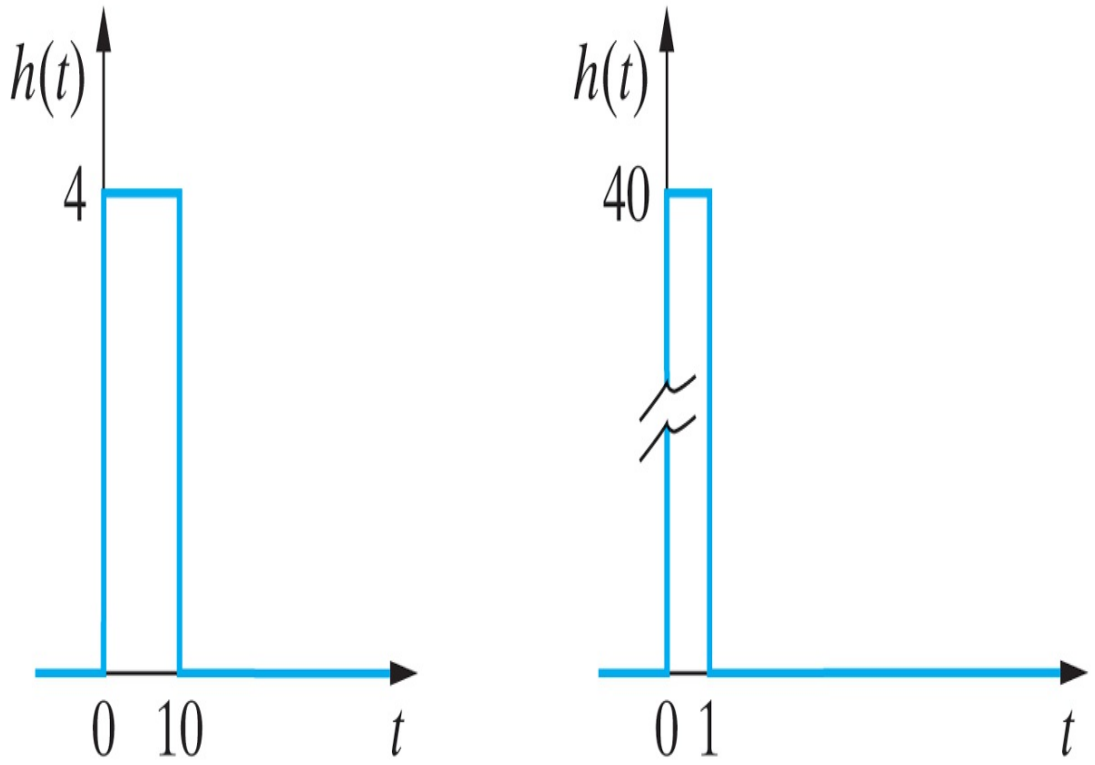
(a)



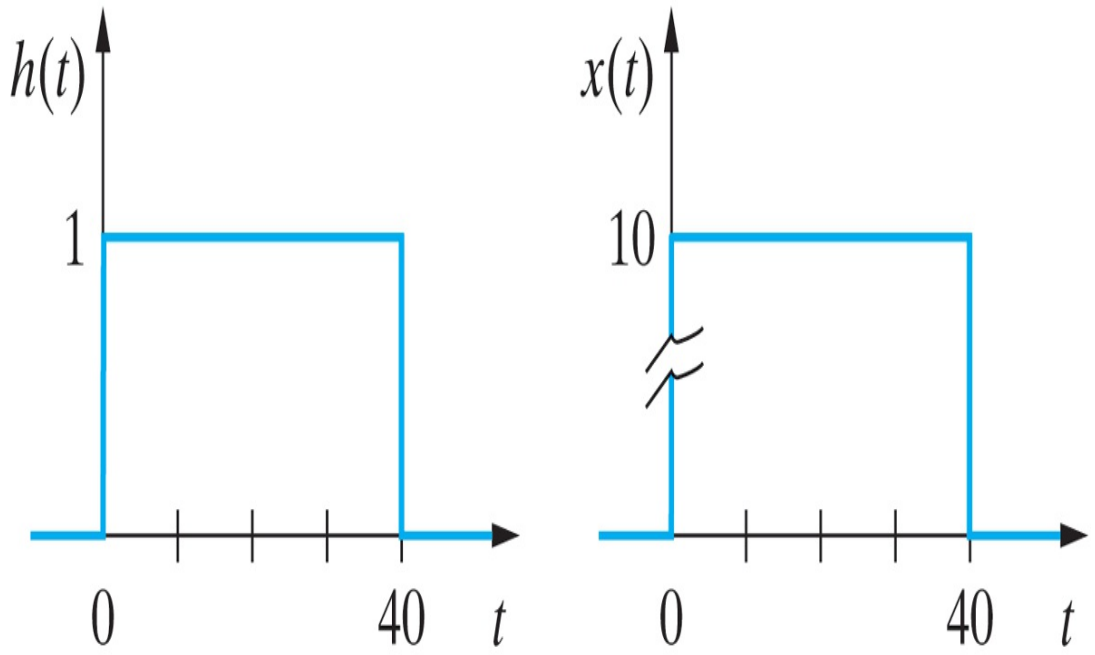
(b) (c)



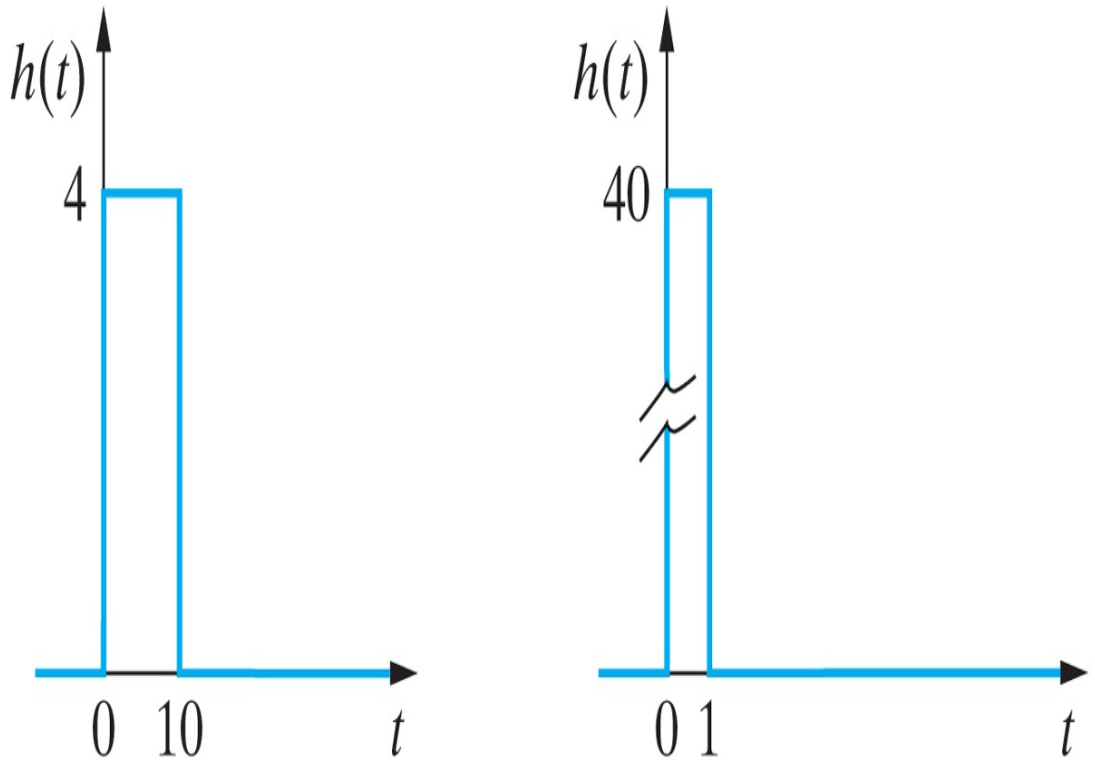
(a)



(b) (c)



(a)



(b) (c)

Figure P13.63 Full Alternative Text

4. 13.64 Assume the voltage impulse response of a circuit can be modeled by the triangular waveform shown in Fig. P13.64. The voltage input signal to this circuit is the step function 10u(t) V.

    1. a) Use the convolution integral to derive the expressions for the output voltage.

    2. b) Sketch the output voltage over the interval 0 to 15 s.

    3. c) Repeat parts (a) and (b) if the area under the voltage impulse response stays the same but the width of the impulse response narrows to 4 s.

    4. d) Which output waveform is closer to replicating the input waveform: (b) or (c)? Explain.

Figure P13.64

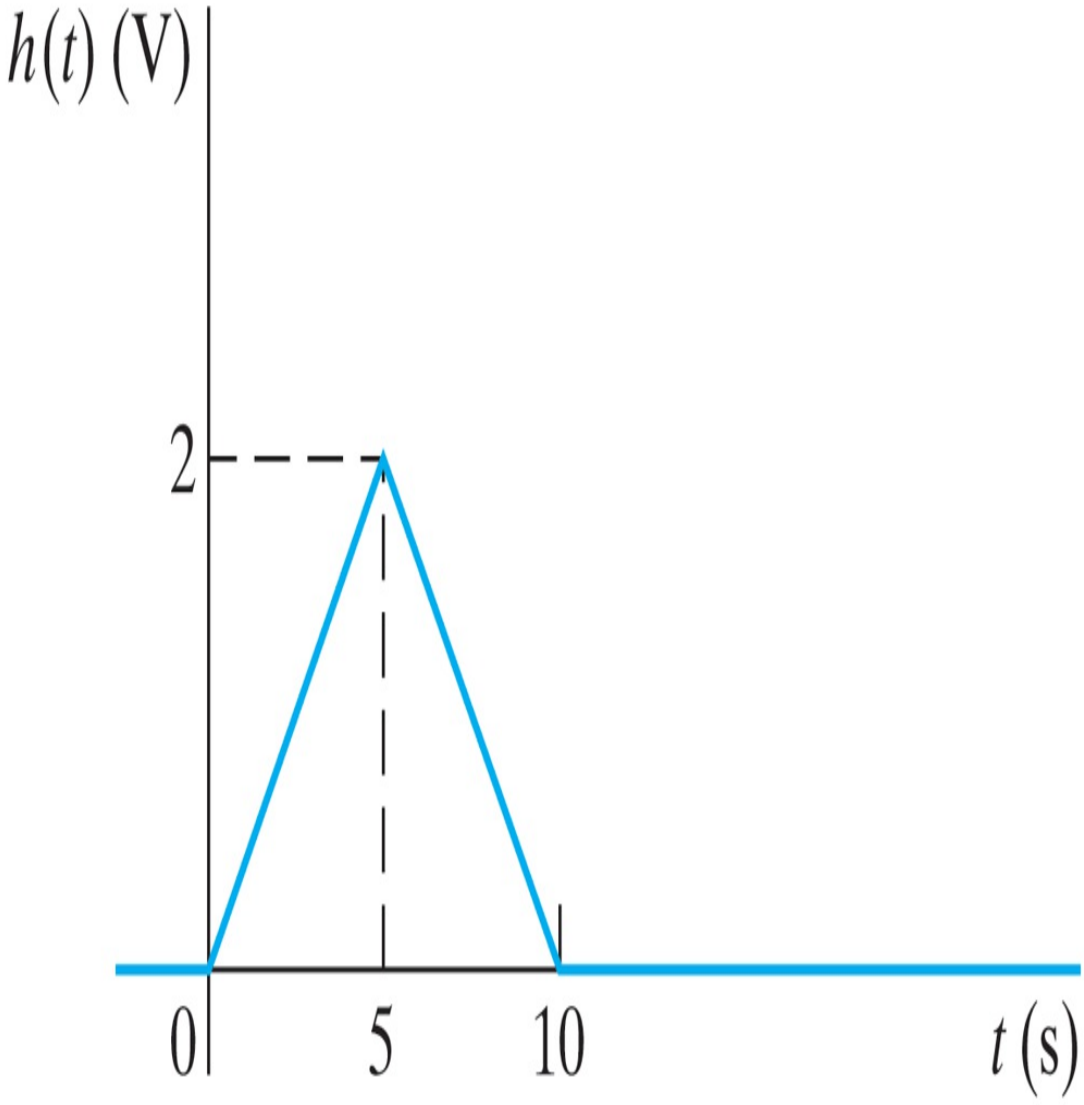


Figure P13.64 Full Alternative Text

5. 13.65

    1. a) Find h(t)*x(t) when h(t) and x(t) are the rectangular pulses shown in Fig. P13.65(a).

    2. b) Repeat (a) when x(t) changes to the rectangular pulse shown in Fig. P13.65(b).

    3. c) Repeat (a) when h(t) changes to the rectangular pulse shown in Fig. P13.65(c).

Figure P13.65

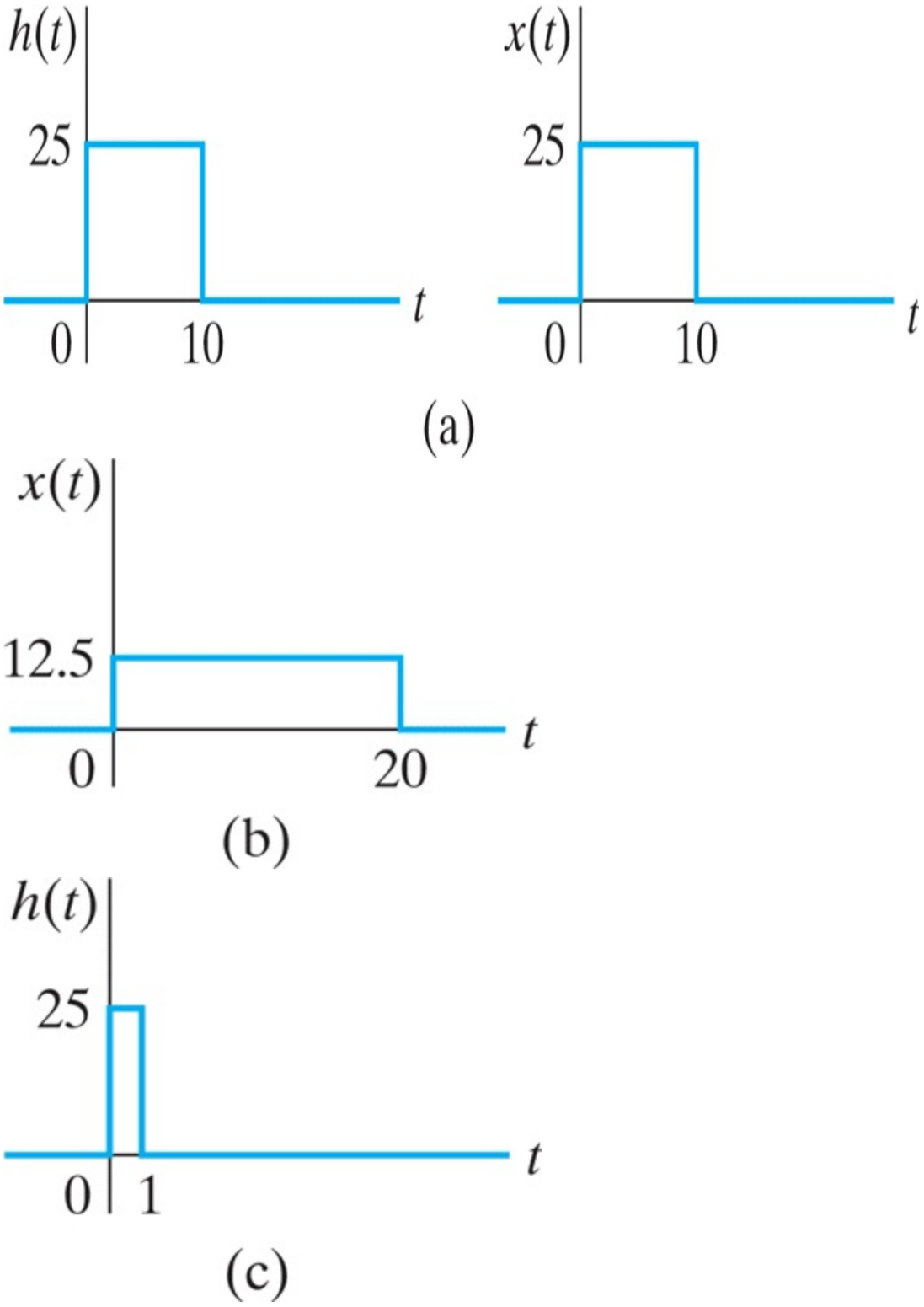


Figure P13.65 Full Alternative Text

6. 13.66

    1. a) Use the convolution integral to find the output voltage of the circuit in Fig. P13.51(a) if the input voltage is the rectangular pulse shown in Fig. P13.66.

    2. b) Sketch vo(t) versus t for the time interval 0≤t≤100 ms.

Figure P13.66

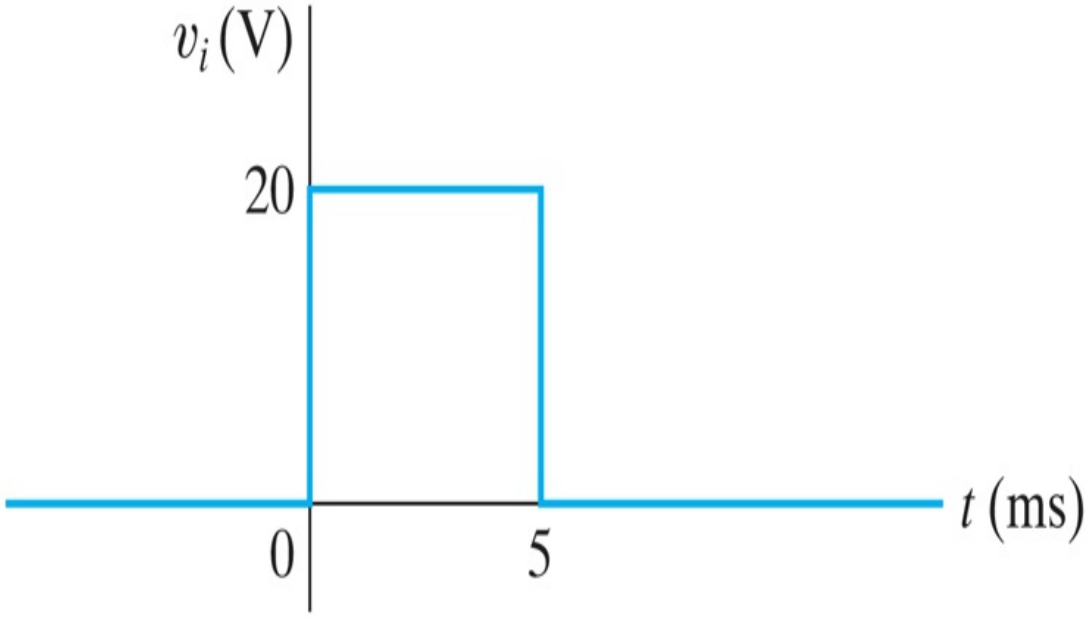


Figure P13.66 Full Alternative Text

7. 13.67

    1. a) Repeat Problem 13.66, given that the resistor in the circuit in Fig. P13.51(a) is decreased to 5 kΩ.

    2. b) Does decreasing the resistor increase or decrease the memory of the circuit?

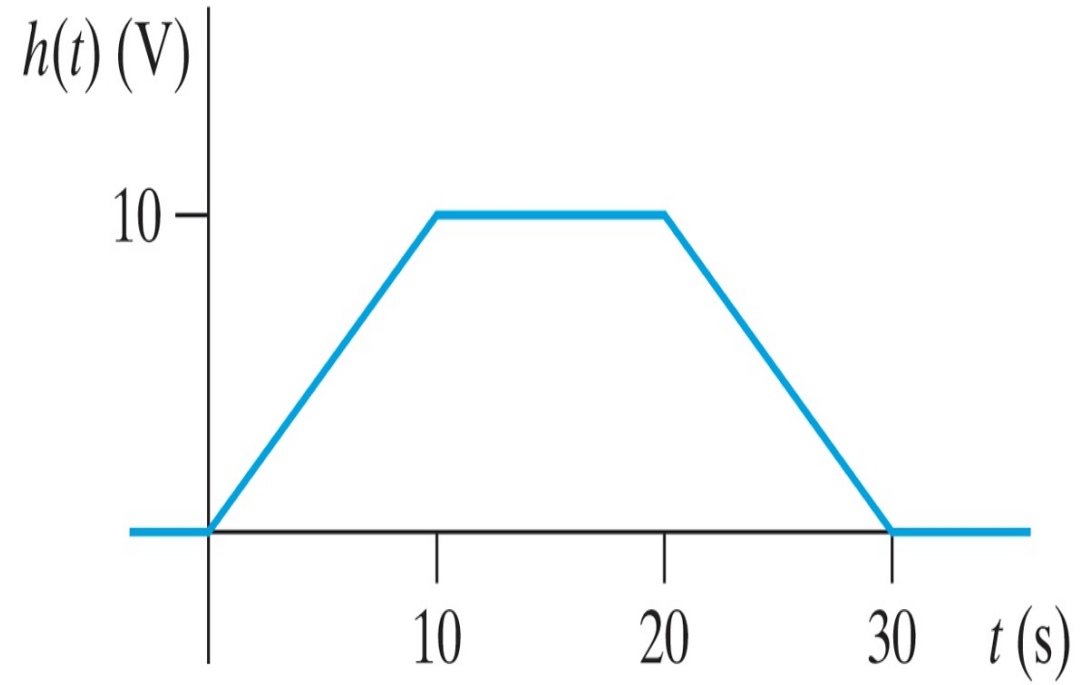
3. c) Which circuit comes closer to transmitting a replica of the input voltage?

8. 13.68 The voltage impulse response of a circuit is shown in Fig. P13.68(a). The input signal to the circuit is the rectangular voltage pulse shown in Fig. P13.68(b).

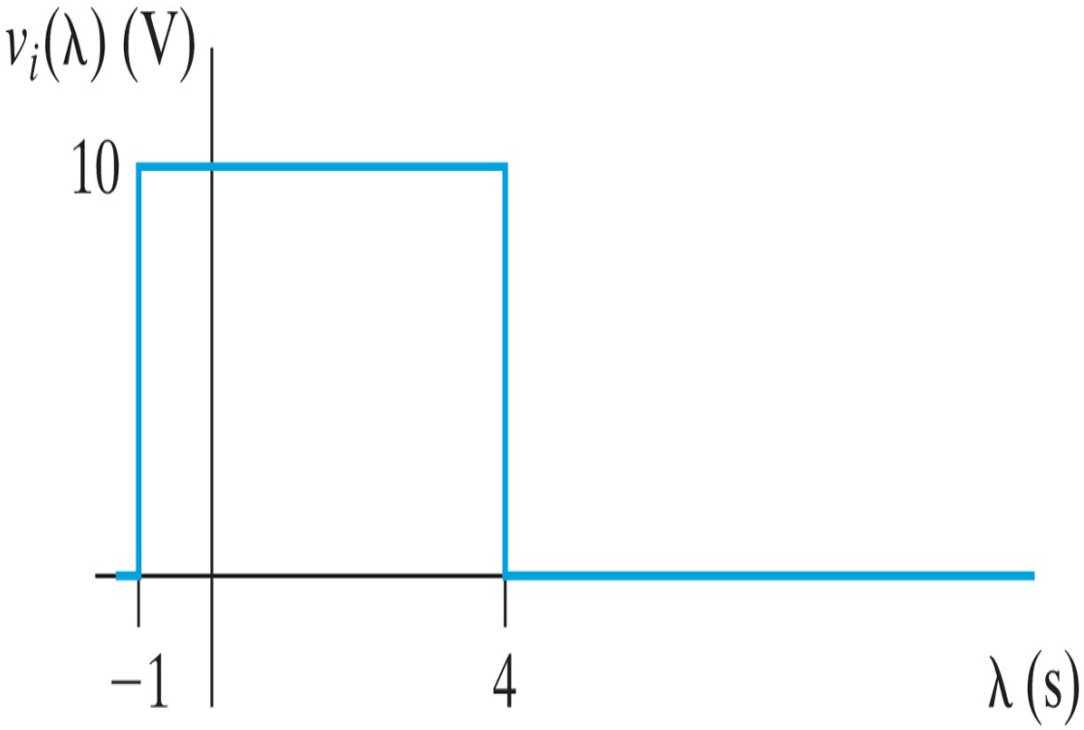
    1. a) Derive the equations for the output voltage. Note the range of time for which each equation is applicable.

    2. b) Sketch vo for $-1 \leq t \leq 34$ s.

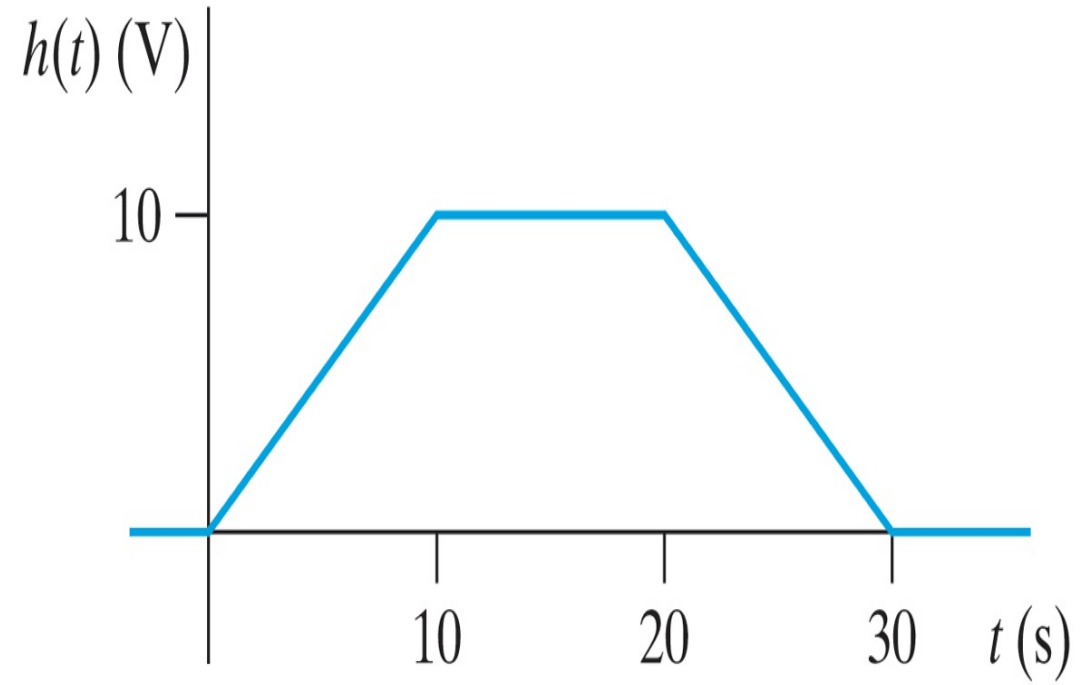
Figure P13.68



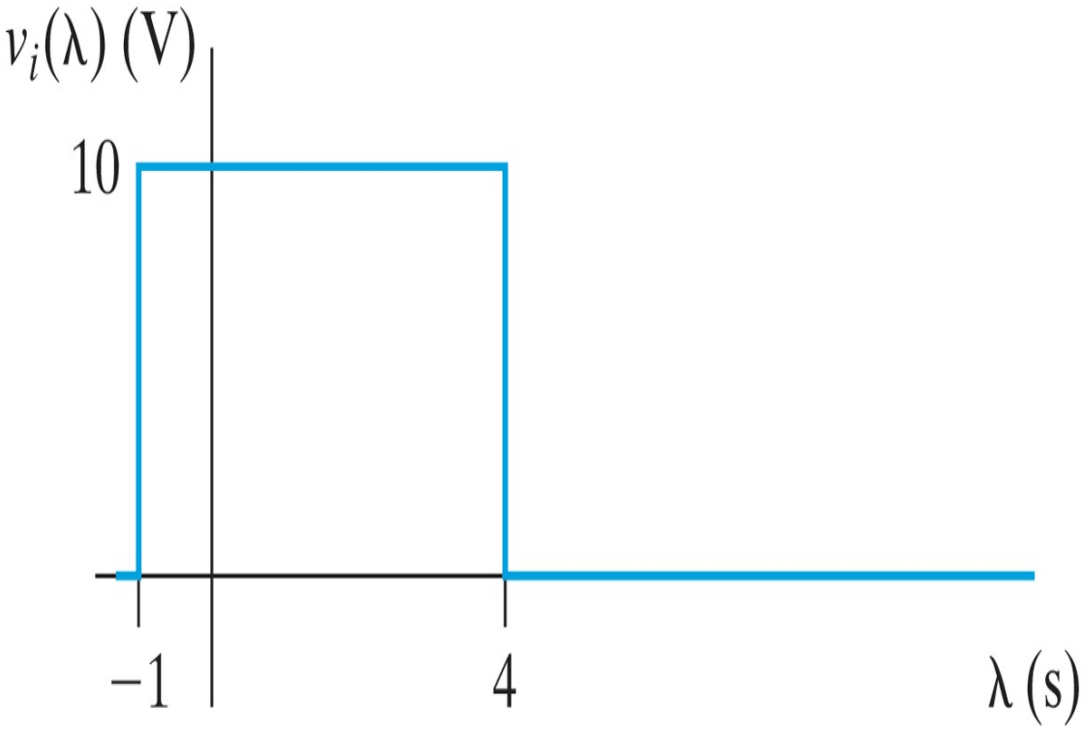
(a)



(b)



(a)



(b)

Figure P13.68 Full Alternative Text

9. 13.69 Use the convolution integral to find vo in the circuit seen in Fig. P13.69 if vi=75u(t) V.

Figure P13.69

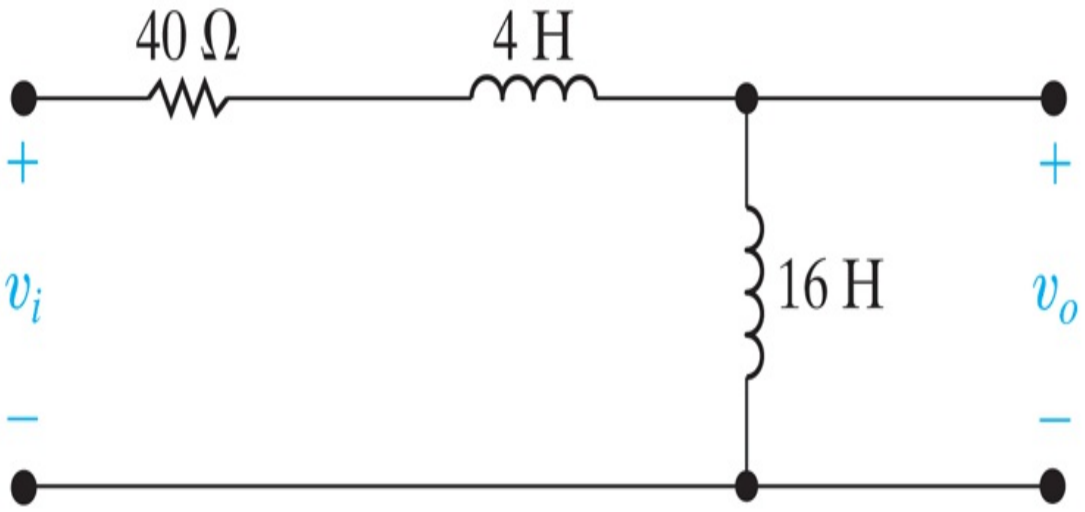


Figure P13.69 Full Alternative Text

10. 13.70 The input voltage in the circuit seen in Fig. P13.70 is

Figure P13.70

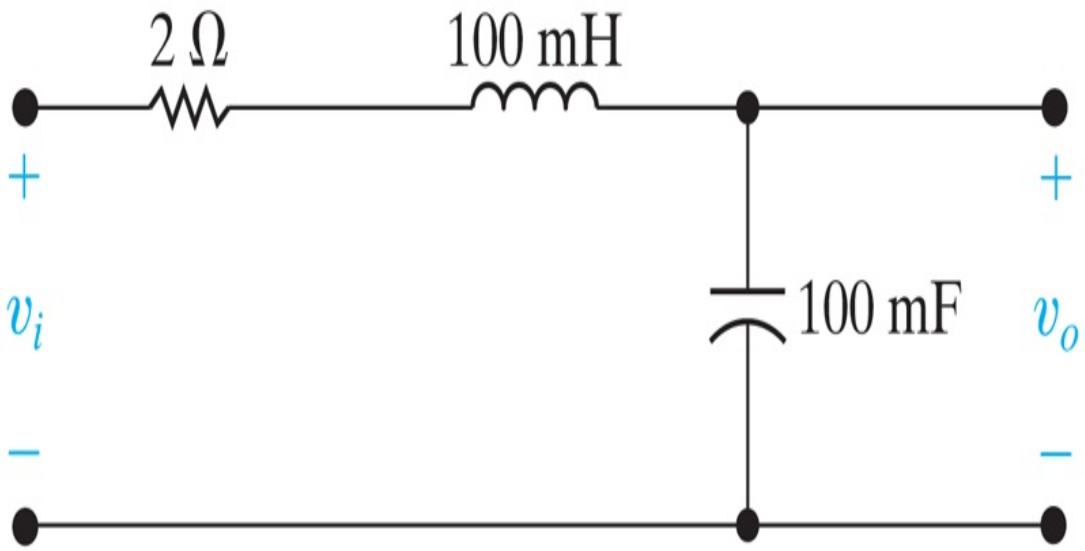


Figure P13.70 Full Alternative Text
vi=5[u(t)−u(t−0.5)] V.

1. a) Use the convolution integral to find vo.
2. b) Sketch vo for 0≤t≤1 s.

11. 13.71

    1. a) Find the impulse response of the circuit shown in Fig. P13.71(a) if vg is the input signal and vo is the output signal.
    2. b) Given that vg has the waveform shown in Fig. P13.71(b), use the convolution integral to find vo.
    3. c) Does vo have the same waveform as vg? Why or why not?

Figure P13.71

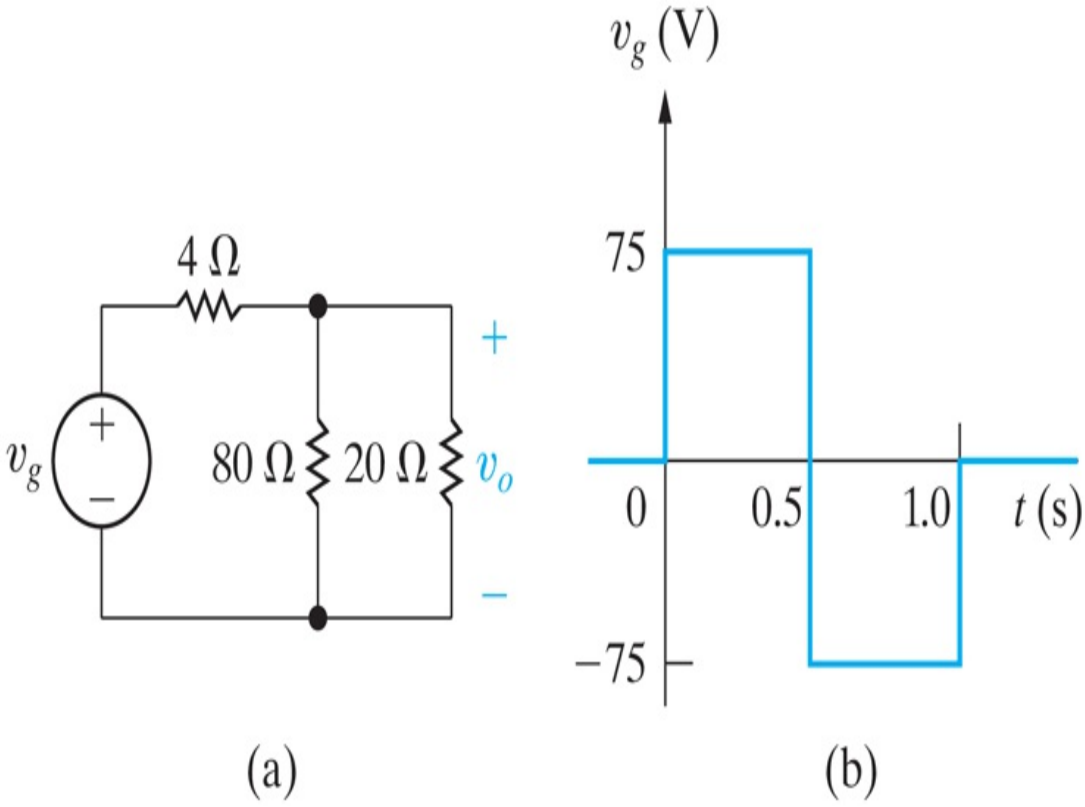


Figure P13.71 Full Alternative Text

12. 13.72

    1. a) Find the impulse response of the circuit seen in Fig. P13.72 if vg is the input signal and vo is the output signal.

    2. b) Assume that the voltage source has the waveform shown in Fig. P13.71(b). Use the convolution integral to find vo.

    3. c) Sketch vo for 0≤t≤2 s.

    4. d) Does vo have the same waveform as vg? Why or why not?

Figure P13.72

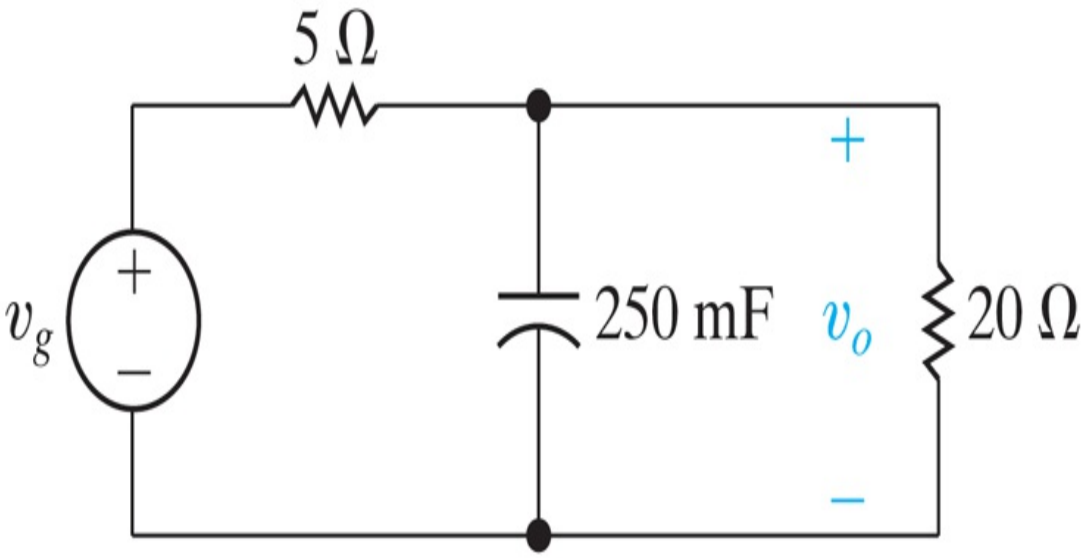


Figure P13.72 Full Alternative Text

13. 13.73

    1. a) Assume the voltage impulse response of a circuit is

       h(t)={0,t<0;10e−4t,t≥0.

       Use the convolution integral to find the output voltage if the input signal is 10u(t) V.

    2. b) Repeat (a) if the voltage impulse response is

       h(t)={0,t<0;10(1−2t),0≤t≤0.5 s;0,t≥0.5 s.

    3. c) Plot the output voltage versus time for (a) and (b) for 0≤t≤1 s.

14. 13.74

    1. a) Use the convolution integral to find vo in the circuit in Fig. P13.74(a) if ig is the pulse shown in Fig. P13.74(b).

    2. b) Use the convolution integral to find io.

    3. c) Show that your solutions for vo and io are consistent by

calculating vo and io at 100− ms, 100+ ms, 200− ms, and 200+ ms.

Figure P13.74

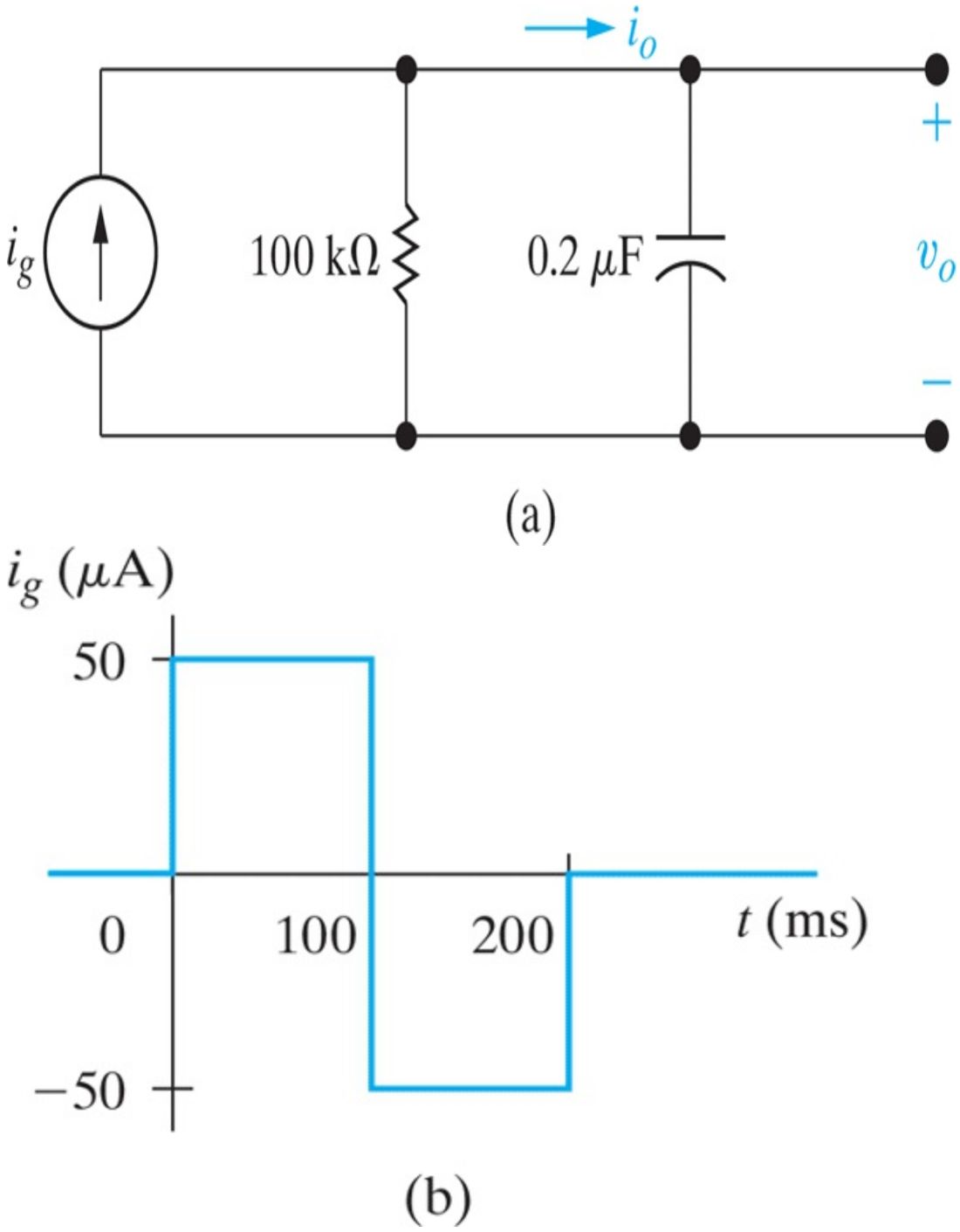


Figure P13.74 Full Alternative Text

15. 13.75 The sinusoidal voltage pulse shown in Fig. P13.75(a) is applied to the circuit shown in Fig. P13.75(b). Use the convolution integral to find the value of vo at t=2.2 s.

Figure P13.75

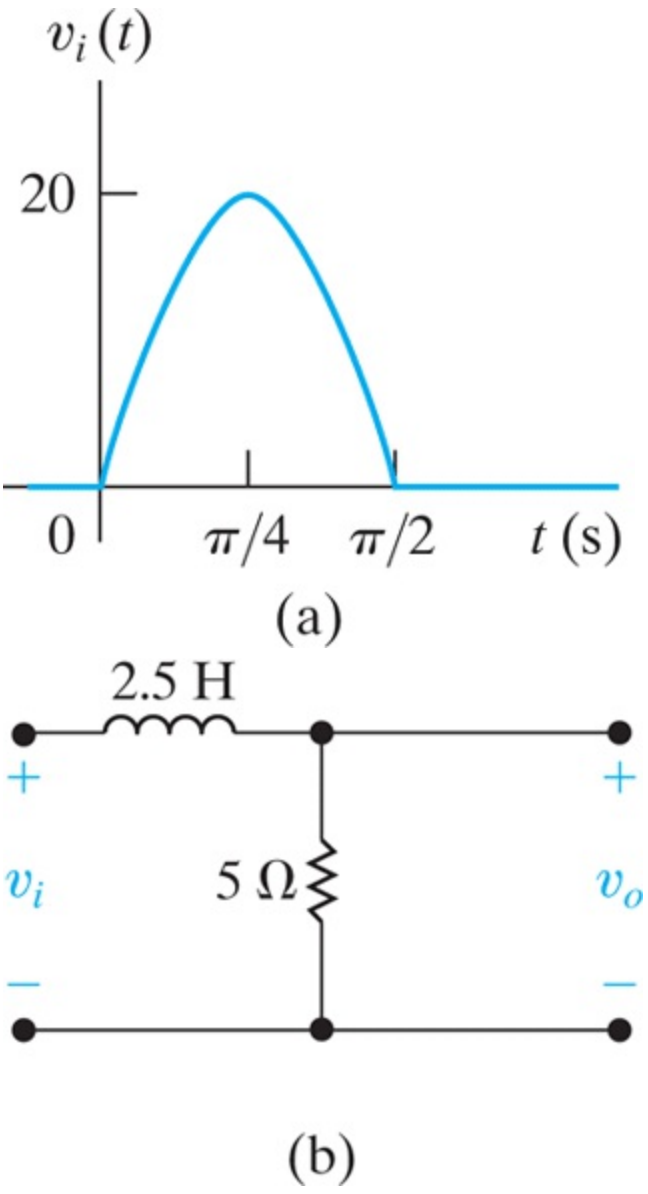
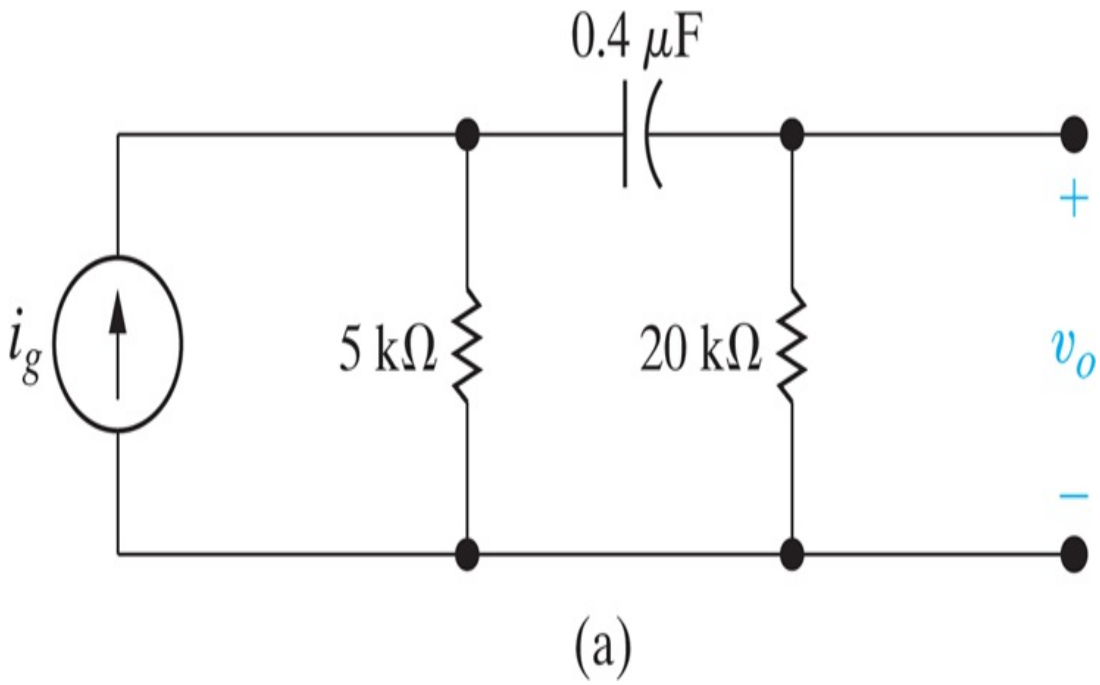


Figure P13.75 Full Alternative Text

16. 13.76 The current source in the circuit shown in Fig. P13.76(a) is generating the waveform shown in Fig. P13.76(b). Use the convolution integral to find vo at t=5 ms.

Figure P13.76



(a)

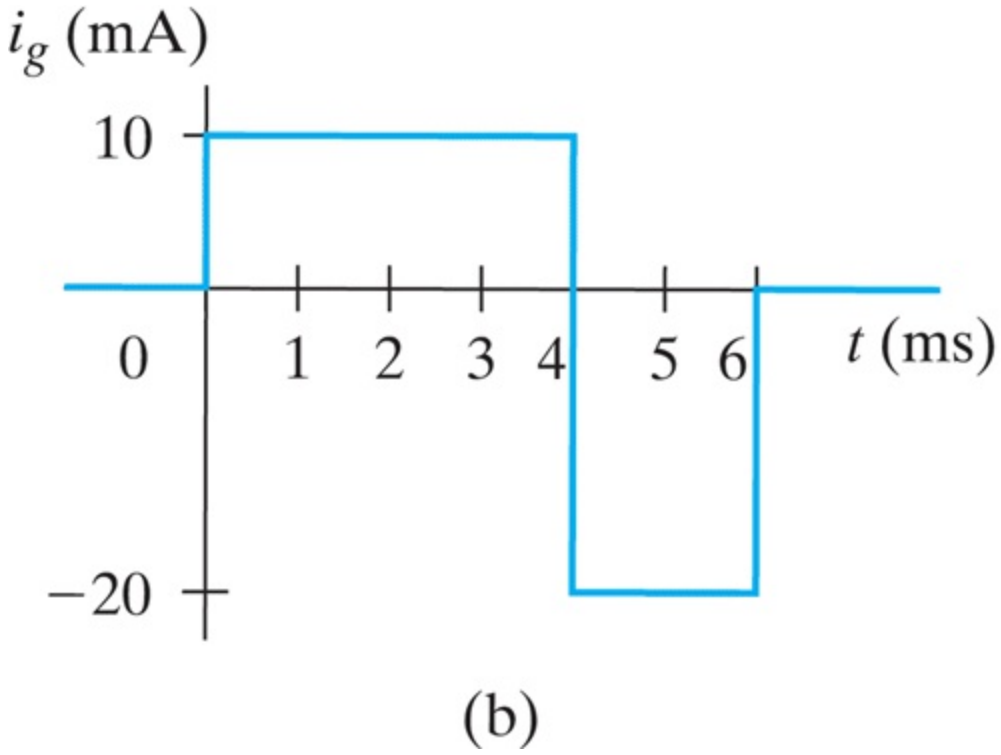


Figure P13.76 Full Alternative Text

17. 13.77

    1. a) Show that if y(t)=h(t)*x(t), then Y(s)=H(x)X(s).
    2. b) Use the result given in (a) to find f(t) if

       F(s)=as(s+a)2.

Section 13.7

1. 13.78 The transfer function for a linear time-invariant circuit is

   H(s)=IoIg=125(s+400)s(s2+200s+104).

   If ig=80 cos 500t A, what is the steady-state expression for io?

2. 13.79 The transfer function for a linear time-invariant circuit is

   H(s)=VoVg=4(s+3)s2+8s+41.

   If vg=40 cos 3t V, what is the steady-state expression for vo?

3. 13.80 When an input voltage of 30u(t) V is applied to a circuit, the response is known to be

   vo=(50e−8000t−20e−5000t)u(t) V.

   What will the steady-state response be if vg=120 cos 6000 t V?

4. 13.81 The op amp in the circuit seen in Fig. P13.81 is ideal.

   1. a) Find the transfer function Vo/Vg.
   2. b) Find vo if vg=0.6 u(t) V.
   3. c) Find the steady-state expression for vo if vg=2 cos 10,000t V.

Figure P13.81

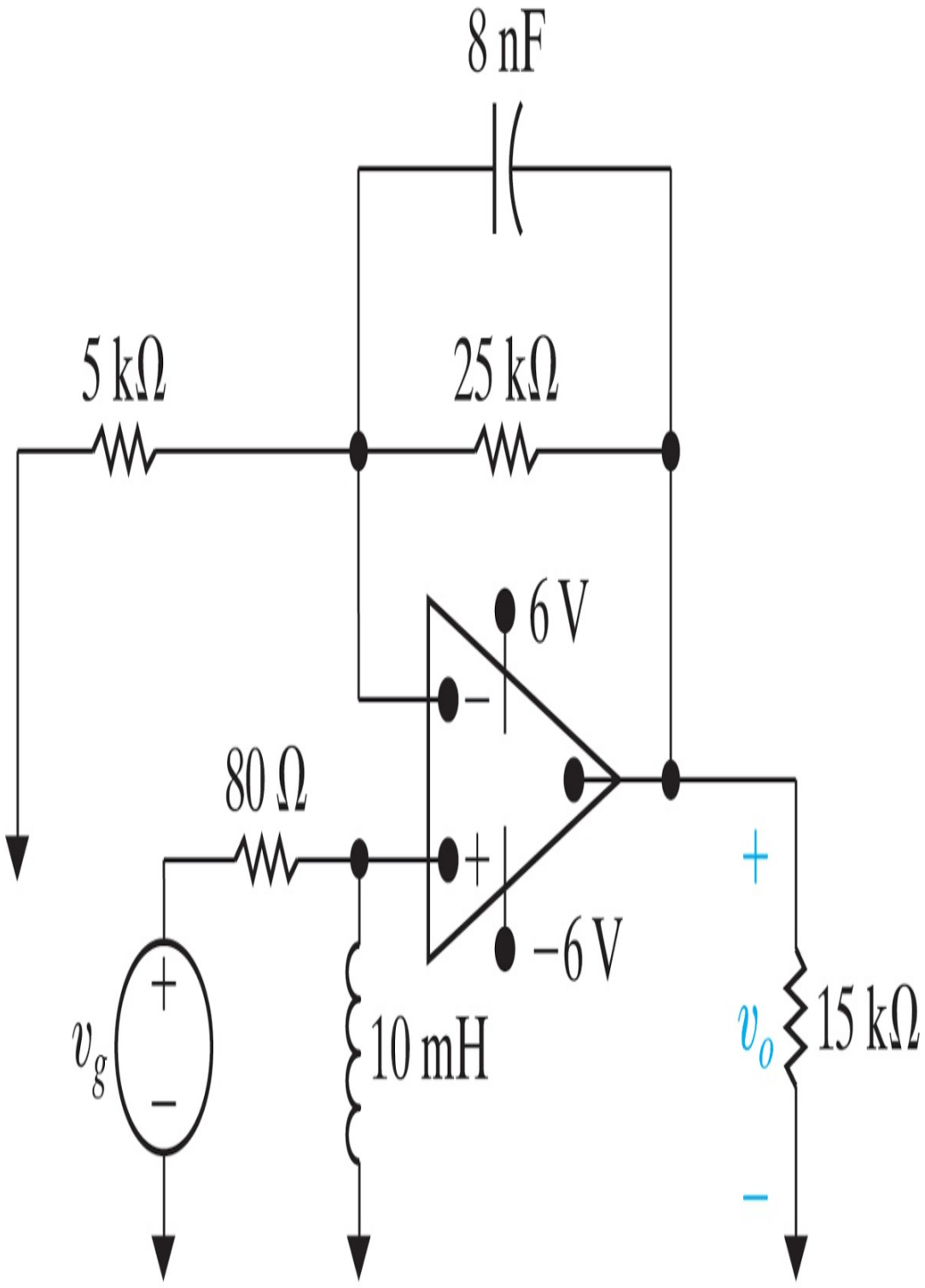


Figure P13.81 Full Alternative Text

5. 13.82 The operational amplifier in the circuit seen in Fig. P13.82 is ideal

and is operating within its linear region.

1. a) Calculate the transfer function Vo/Vg.
2. b) If vg=2 cos 400t V, what is the steady-state expression for vo?

Figure P13.82

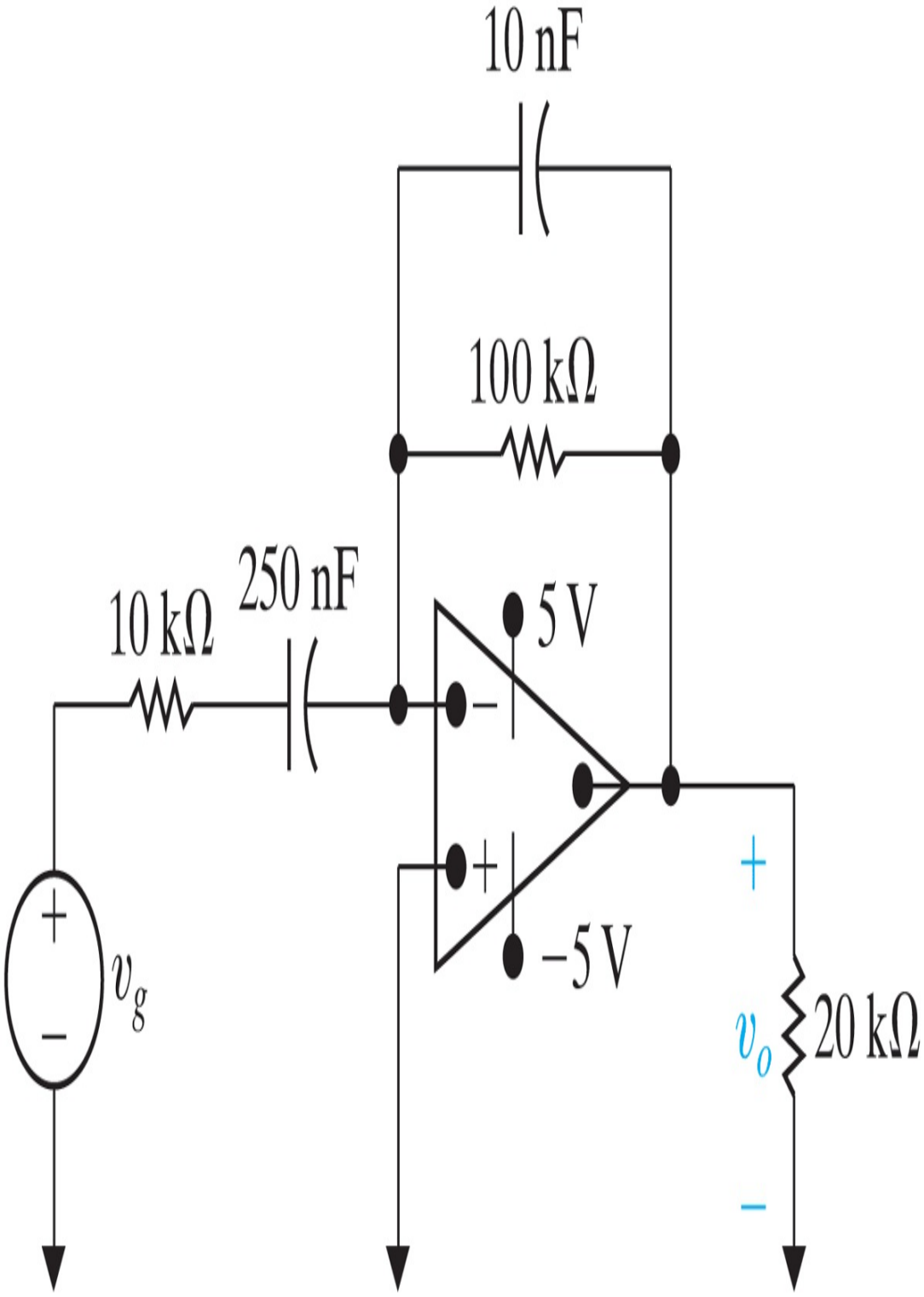


Figure P13.82 Full Alternative Text

Section 13.8

1. 13.83 Show that after V0Ce coulombs are transferred from C1 to C2 in the circuit shown in Fig. 13.47 (with [&R|=|0&]), the voltage across each capacitor is C1V0/(C1+C2). (*Hint:* Use the conservation-of-charge principle.)

2. 13.84 The voltage source in the circuit in Example 13.9 is changed to a unit impulse; that is, vg=δ(t).

    1. a) How much energy does the impulsive voltage source store in the capacitor?

    2. b) How much energy does it store in the inductor?

    3. c) Use the transfer function to find vo(t).

    4. d) Show that the response found in (c) is identical to the response generated by first charging the capacitor to 1000 V and then releasing the charge to the circuit, as shown in Fig. P13.84.

Figure P13.84

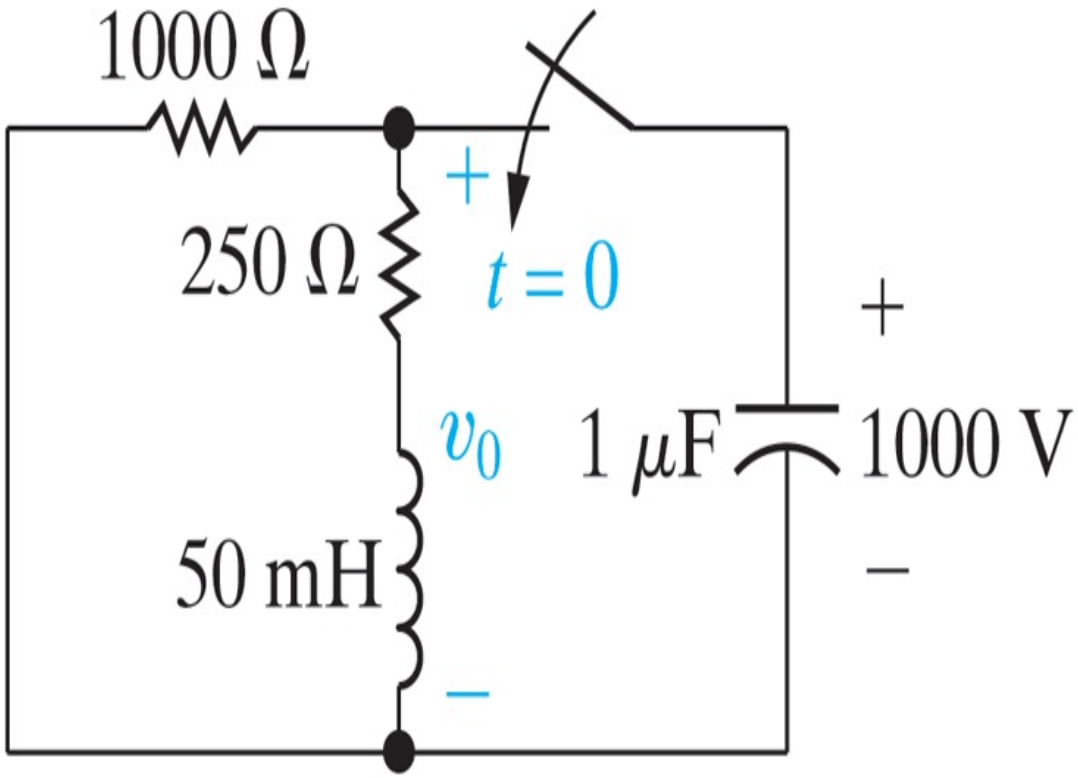


Figure P13.84 Full Alternative Text

3. 13.85 There is no energy stored in the circuit in Fig. P13.85 at the time the impulsive voltage is applied.

    1. a) Find vo(t) for t≥0.

    2. b) Does your solution make sense in terms of known circuit behavior? Explain.

Figure P13.85

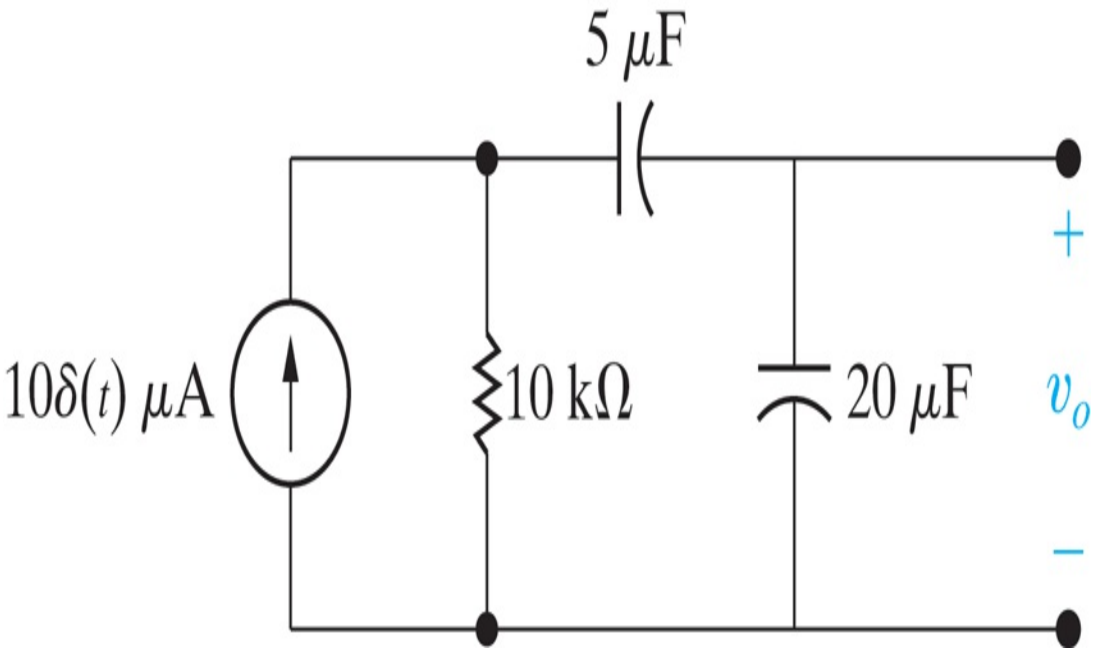


Figure P13.85 Full Alternative Text

4. 13.86 The inductor L1 in the circuit shown in Fig. P13.86 is carrying an initial current of ρ A at the instant the switch opens. Find (a) v(t); (b) i1(t); (c) i2(t); and (d) λ(t), where λ(t) is the total flux linkage in the circuit.

Figure P13.86

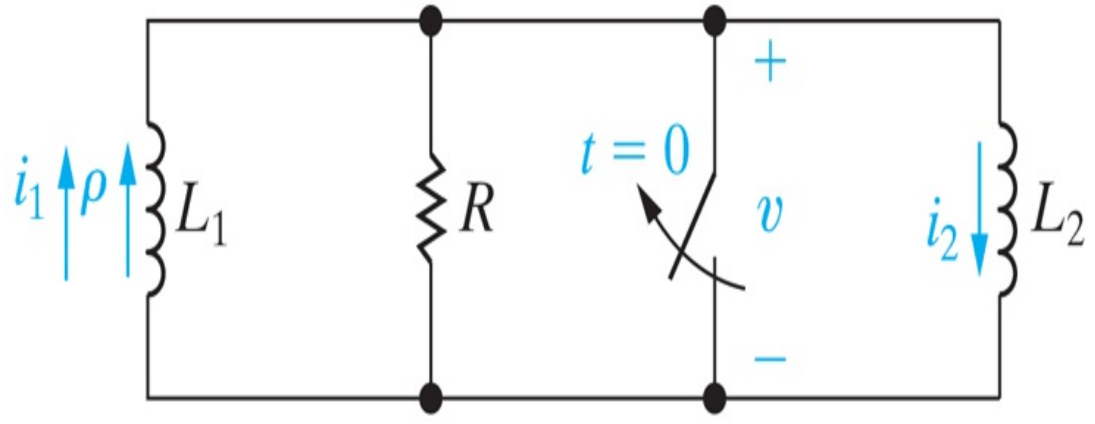


Figure P13.86 Full Alternative Text

5. 13.87

    1. a) Let R→∞ in the circuit shown in Fig. P13.86, and use the solutions derived in Problem 13.86 to find v(t), i1(t), and i2(t).

    2. b) Let R=∞ in the circuit shown in Fig. P13.86 and use the Laplace transform method to find v(t), i1(t), and i2(t).

6. 13.88 There is no energy stored in the circuit in Fig. P13.88 at the time the impulse voltage is applied.

    1. a) Find i1 for t≥0+.

    2. b) Find i2 for t≥0+.

    3. c) Find vo for t≥0+.

    4. d) Do your solutions for i1, i2, and vo make sense in terms of known circuit behavior? Explain.

Figure P13.88

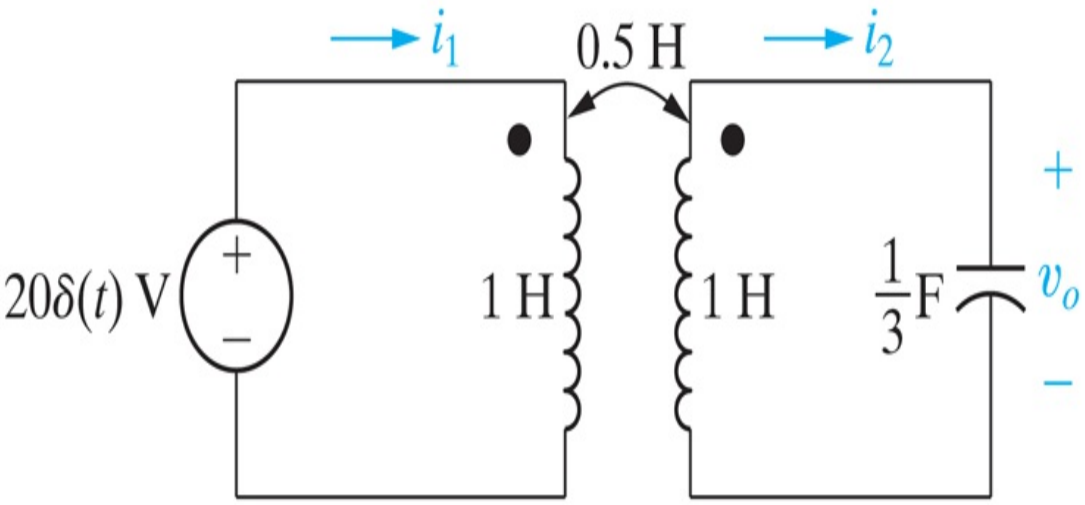


Figure P13.88 Full Alternative Text

7. 13.89 There is no energy stored in the circuit in Fig. P13.89 at the time

the impulsive current is applied.

1. a) Find vo for t≥0+.
2. b) Does your solution make sense in terms of known circuit behavior? Explain.

Figure P13.89

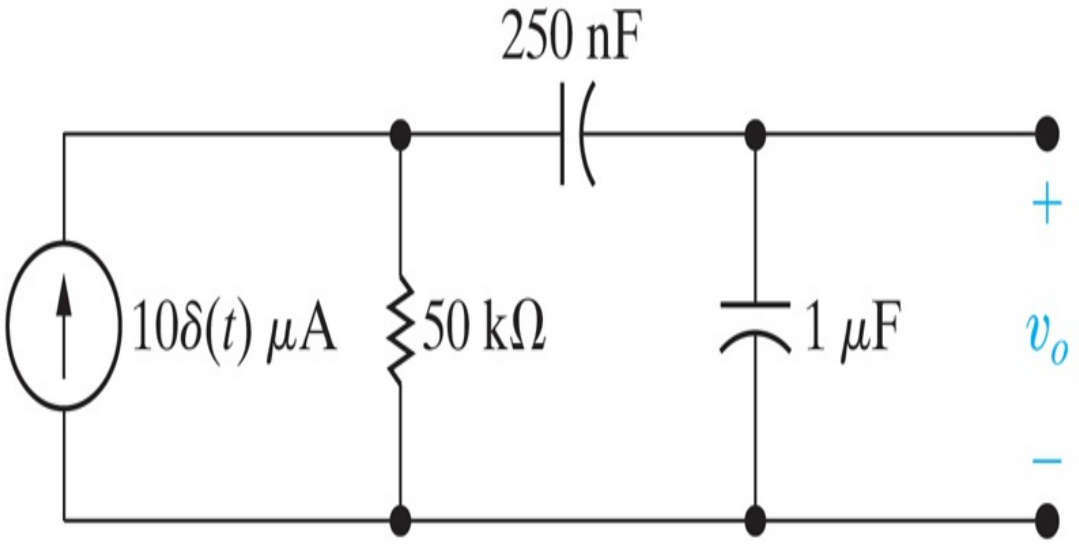


Figure P13.89 Full Alternative Text

8. 13.90 The switch in the circuit in Fig. P13.90 has been in position a for a long time. At t=0, the switch moves to position b. Compute (a) v1(0−); (b) v2(0−); (c) v3(0−); (d) i(t); (e) v1(0+); (f) v2(0+); and (g) v3(0+).

Figure P13.90

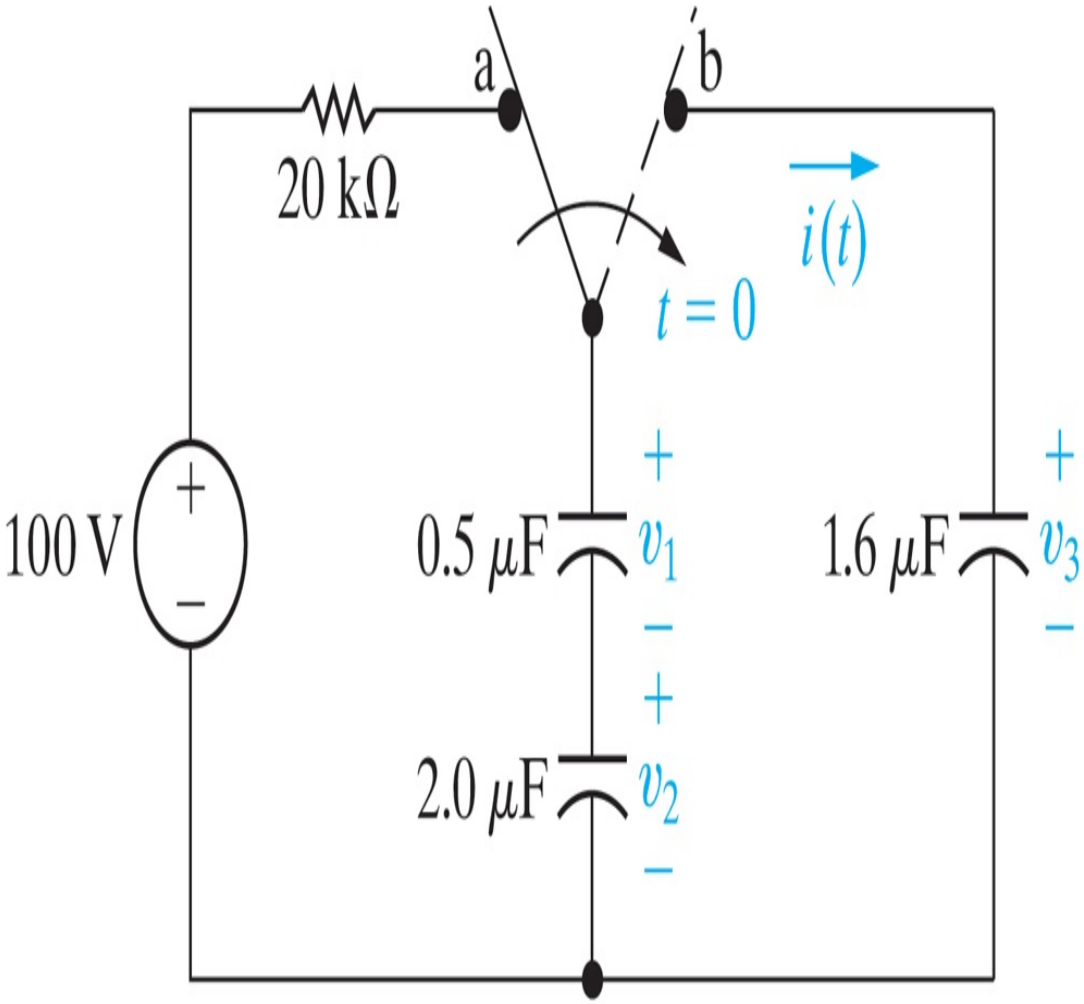


Figure P13.90 Full Alternative Text

9. 13.91 The switch in the circuit in Fig. P13.91 has been closed for a long time. The switch opens at t=0. Compute (a) i1(0−); (b) i1(0+); (c) i2(0−); (d) i2(0+); (e) i1(t); (f) i2(t); and (g) v(t).

Figure P13.91

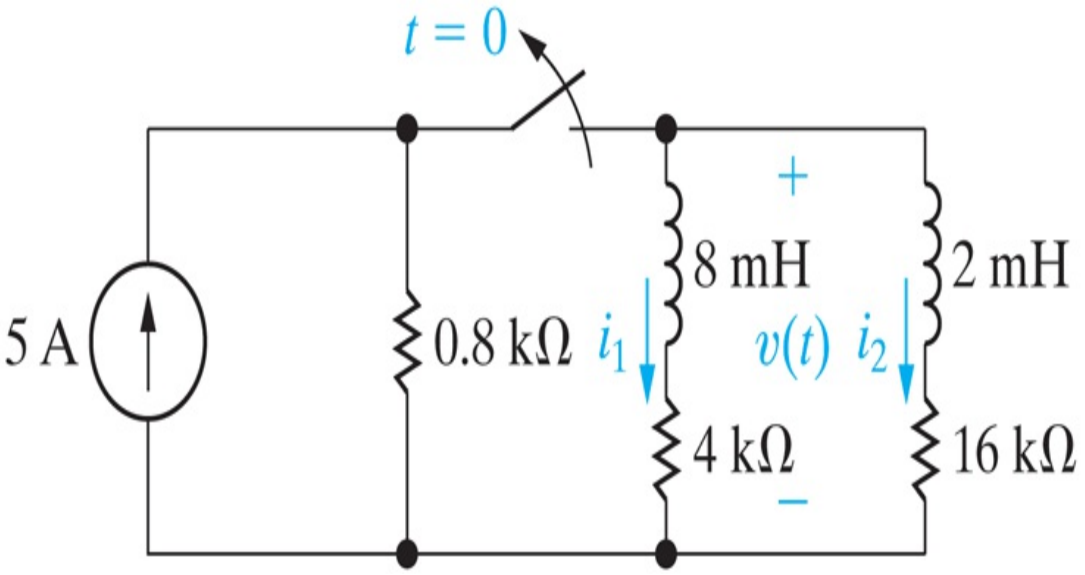


Figure P13.91 Full Alternative Text

10. 13.92 The parallel combination of R2 and C2 in the circuit shown in Fig. P13.92 represents the input circuit to a cathode-ray oscilloscope (CRO). The parallel combination of R1 and C1 is a circuit model of a compensating lead that is used to connect the CRO to the source. There is no energy stored in C1 or C2 at the time when the 10 V source is connected to the CRO via the compensating lead. The circuit values are C1=4 pF, C2=16 pF, R1=1.25 MΩ, and R2=5 MΩ.

    1. a) Find vo.
    2. b) Find io.
    3. c) Repeat (a) and (b) given C1 is changed to 64 pF.

Figure P13.92

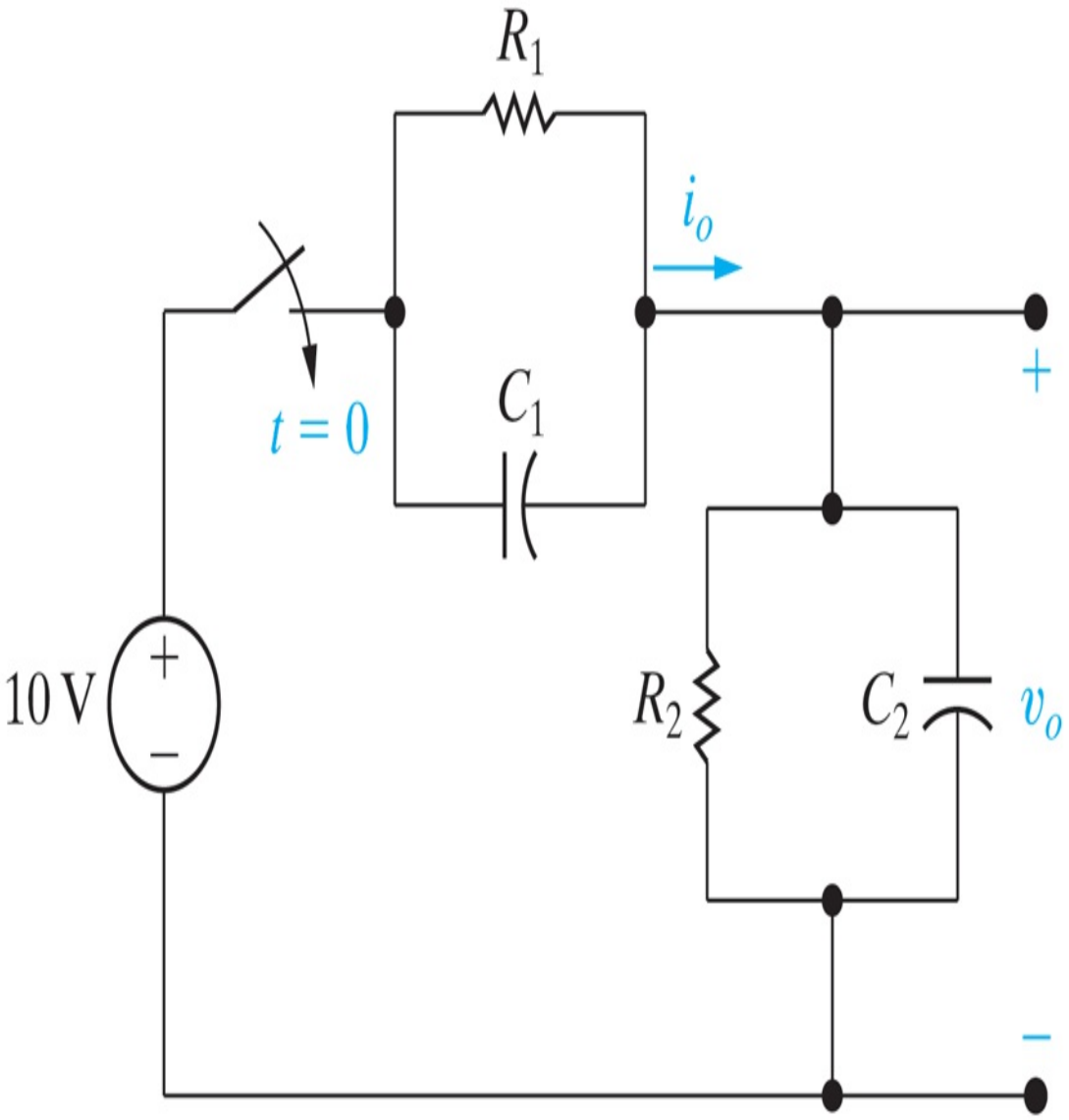


Figure P13.92 Full Alternative Text

11. 13.93 Show that if R1C1=R2C2 in the circuit shown in Fig. P13.92, vo will be a scaled replica of the source voltage.

Sections 13.1–13.8

1. 13.94 Assume the line-to-neutral voltage Vo in the 60 Hz circuit of Fig. 13.59 is 1200°V(rms). Load Ra is absorbing 1200 W; load Rb is absorbing 1800 W; and the inductive load is absorbing 350 magnetizing

VAR. The inductive reactance of the line is 1 Ω. Assume Vg does not change after the switch opens.

1. a) Calculate the initial value of i2(t) and io(t).

2. b) Find Vo, vo(t), and vo(0+) using the s-domain circuit of Fig. 13.60.

3. c) Test the steady-state component of vo using phasor domain analysis.

4. d) Using a computer program of your choice, plot vo vs. t for $0 \le t \le 20$ ms.

2. 13.95 Assume the switch in the circuit in Fig. 13.59 opens at the instant the sinusoidal steady-state voltage vo is zero and going positive, i.e., vo=1202 sin 120πt V.

    1. a) Find vo(t) for $t \ge 0$.

    2. b) Using a computer program of your choice, plot vo(t) vs. t for $0 \le t \le 20$ ms.

    3. c) Compare the disturbance in the voltage in part (a) with that obtained in part (c) of Problem 13.94.

3. 13.96 The purpose of this problem is to show that the line-to-neutral voltage in the circuit in Fig. 13.59 can go directly into steady state if the load Rb is disconnected from the circuit at precisely the right time. Let $v_o = V_m \cos(120\pi t - \theta^\circ)$ V, where Vm=1202. Assume vg does not change after Rb is disconnected.

    1. a) Find the value of θ (in degrees) so that vo goes directly into steady-state operation when the load Rb is disconnected.

    2. b) For the value of θ found in part (a), find vo(t) for $t \ge 0$.

    3. c) Using a computer program of your choice, plot on a single graph, for $-10 \text{ ms} \le t \le 10$ ms, vo(t) before and after load Rb is

disconnected.

Chapter 14 Introduction to Frequency Selective Circuits

Chapter Contents

1. 14.1 Some Preliminaries
2. 14.2 Low-Pass Filters
3. 14.3 High-Pass Filters
4. 14.4 Bandpass Filters
5. 14.5 Bandreject Filters

Chapter Objectives

1. Know the RL and RC circuit configurations that act as low-pass filters and be able to design RL and RC circuit component values to meet a specified cutoff frequency.

2. Know the RL and RC circuit configurations that act as high-pass filters and be able to design RL and RC circuit component values to meet a specified cutoff frequency.

3. Know the RLC circuit configurations that act as bandpass filters, understand the definition of and relationship among the center frequency, cutoff frequencies, bandwidth, and quality factor of a bandpass filter, and be able to design RLC circuit component values to meet design specifications.

4. Know the RLC circuit configurations that act as bandreject filters, understand the definition of and relationship among the center frequency, cutoff frequencies, bandwidth, and quality factor of a band reject filter, and be able to design RLC circuit component values to meet design specifications.

Up to this point in our analysis of circuits with sinusoidal sources, the source frequency has been held constant. In this chapter, we analyze the effect of varying source frequency on circuit voltages and currents. The result of this analysis is the **frequency response** of a circuit.

We've seen in previous chapters that a circuit's response depends on the types of elements in the circuit, the way the elements are connected, and the element impedances. Varying the frequency of a sinusoidal source does not change the element types or their connections, but it does change the capacitor and inductor impedances because these impedances are functions of frequency.

We can design circuits whose output signals reside within a desired

frequency range, excluding all other frequencies that appear in the circuit's input. Such circuits are called **frequency-selective circuits**. Many devices that communicate via electric signals, such as telephones, radios, televisions, and satellites, employ frequency-selective circuits.

Frequency-selective circuits are also called **filters** because they filter out certain input signals on the basis of frequency. Note that no practical frequency-selective circuit can perfectly or completely filter out selected frequencies. Rather, filters **attenuate**—that is, weaken or lessen the effect of—any input signals with frequencies outside the desired frequency band. For example, your home stereo system may have a graphic equalizer. Each band in the graphic equalizer is a filter that amplifies sounds (audible frequencies) in the frequency range of the band and attenuates frequencies outside of that band. Thus, the graphic equalizer enables you to change the sound volume in each frequency band.

Figure 14.1 A circuit with voltage input and output.

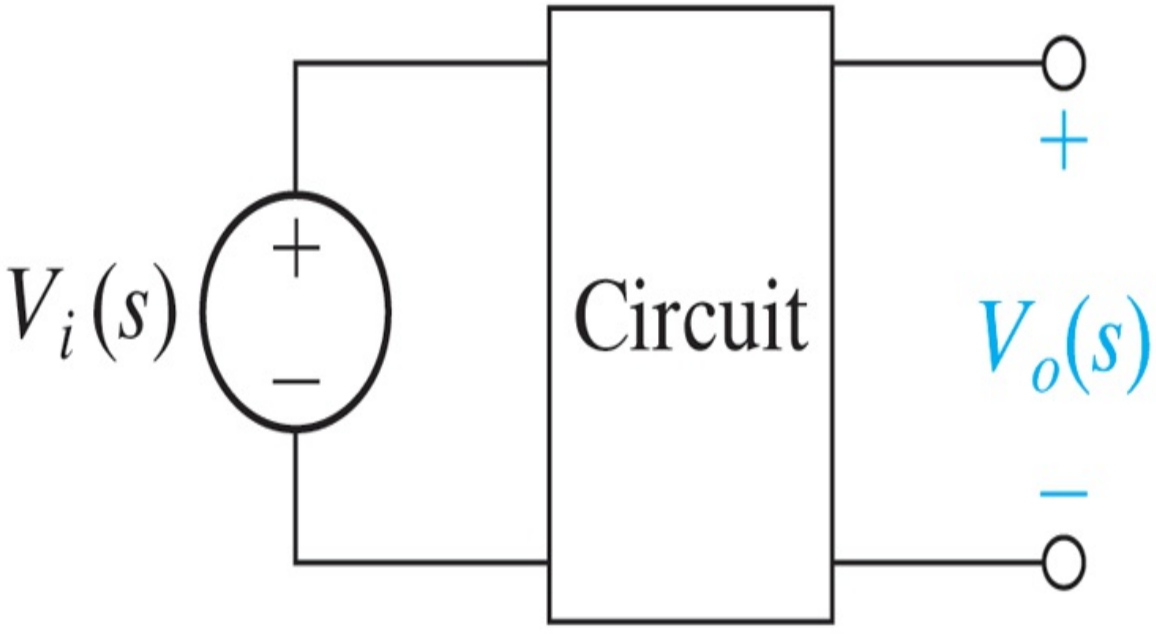


Figure 14.1 Full Alternative Text

We begin this chapter by analyzing circuits from each of the four major categories of filters: low pass, high pass, band pass, and band reject. The transfer function of a circuit is the starting point for the frequency response analysis. Pay close attention to the similarities among the transfer functions of circuits that perform the same filtering function. We will employ these similarities when designing filter circuits in Chapter 15.

Practical Perspective

Pushbutton Telephone Circuits

A pushbutton telephone produces tones that you hear when you press a button. You may have wondered about these tones. How are they used to tell the telephone system which button was pushed? Why are tones used at all? Why do the tones sound musical? How does the phone system tell the difference between button tones and the normal sounds of people talking or singing?

The telephone system was designed to handle audio signals—those with frequencies between 300 Hz and 3 kHz. Thus, all signals from the system to the user have to be audible—including the dial tone and the busy signal. Similarly, all signals from the user to the system have to be audible, including the signal that the user has pressed a button. It is important to distinguish button signals from the normal audio signal, so a dual-tone-multiple-frequency (DTMF) design is employed. When a number button is pressed, a unique pair of sinusoidal tones with very precise frequencies is sent by the phone to the telephone system. The DTMF frequency and timing specifications make it unlikely that a human voice could produce the exact tone pairs, even if the person were trying. In the central telephone facility, electric circuits monitor the audio signal, listening for the tone pairs that signal a number. In the Practical Perspective example at the end of the chapter, we will examine the design of the DTMF filters used to determine which button has been pushed.

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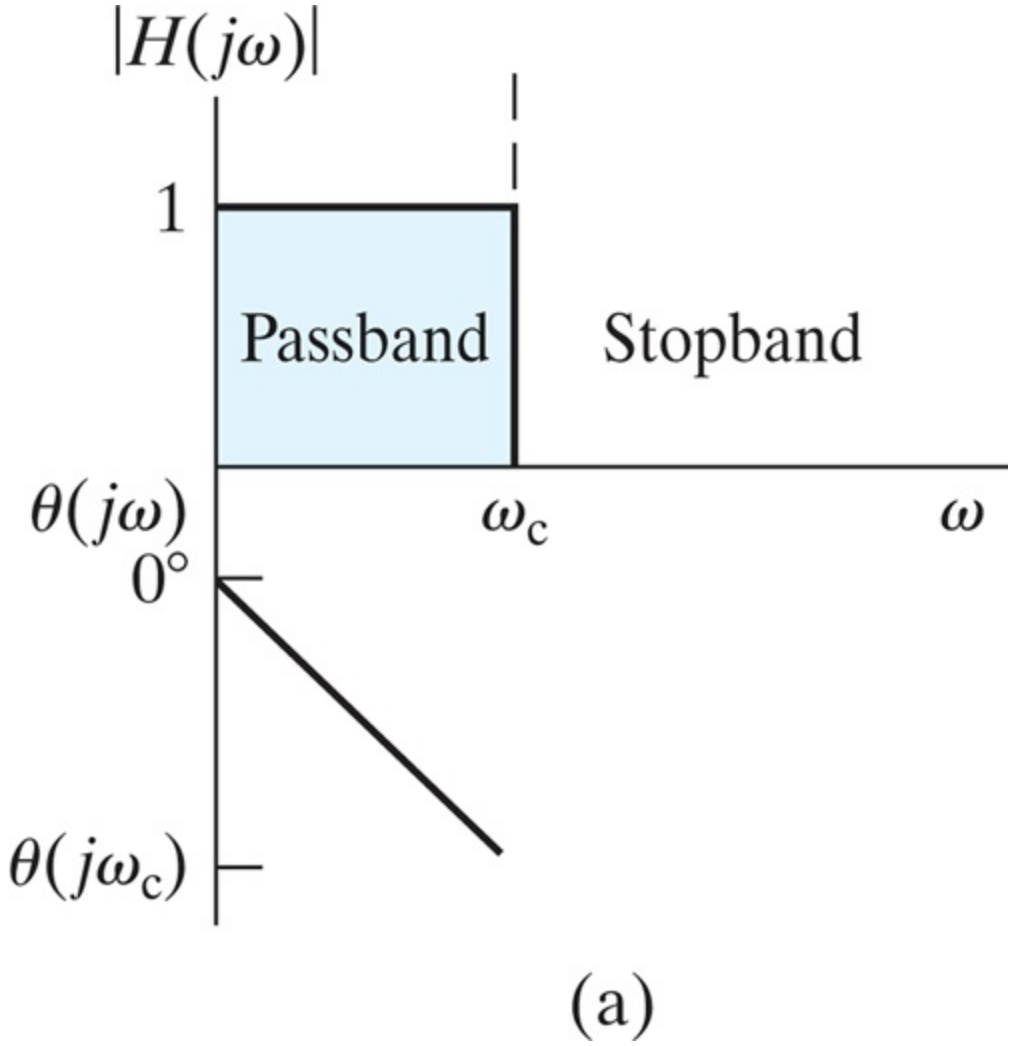
14.1 Some Preliminaries

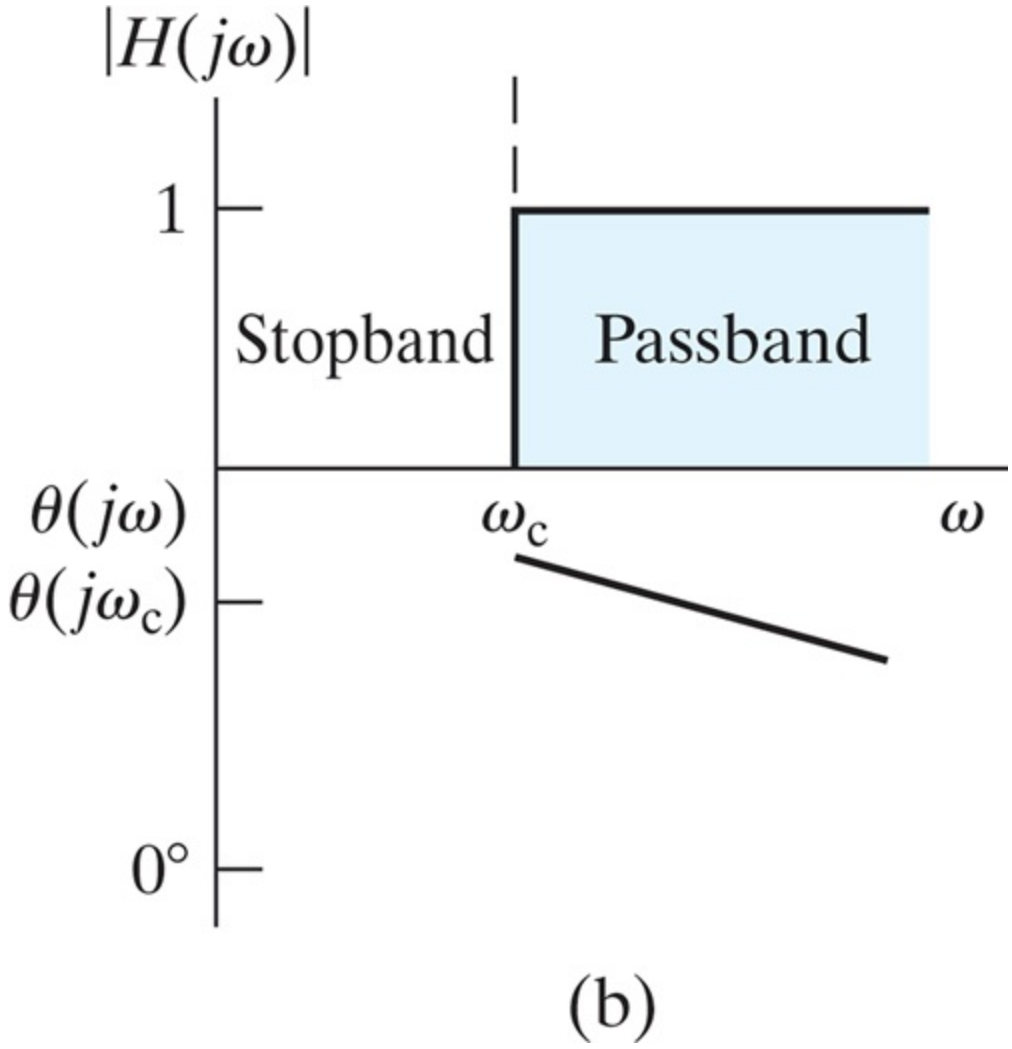
Recall from Section 13.7 that a circuit's transfer function provides an easy way to compute the steady-state response to a sinusoidal input. There, we considered only fixed-frequency sources. To study the frequency response of a circuit, we replace a fixed-frequency sinusoidal source with a varying-frequency sinusoidal source. The transfer function is still an immensely useful tool because the magnitude and phase of the output signal depend only on the magnitude and phase of the transfer function $H(j\omega)$, which vary as a function of the source frequency ω. Note that this method of analyzing the output of a circuit as its input frequency varies assumes that we can vary the input frequency without changing its magnitude or phase angle.

To further simplify this first look at frequency-selective circuits, we will also restrict our attention to cases where both the input and output signals are sinusoidal voltages, as illustrated in Fig. 14.1. Thus, the circuit's transfer function will be the ratio of the Laplace transform of the output voltage to the Laplace transform of the input voltage, or $H(s)=Vo(s) / Vi(s)$. We should keep in mind, however, that for a particular application, a current may be either the input signal or output signal of interest.

Figure 14.2 Ideal frequency response plots for (a) an ideal low-pass filter and (b) an ideal high-pass filter.



14.1-4 Full Alternative Text



14.1-4 Full Alternative Text

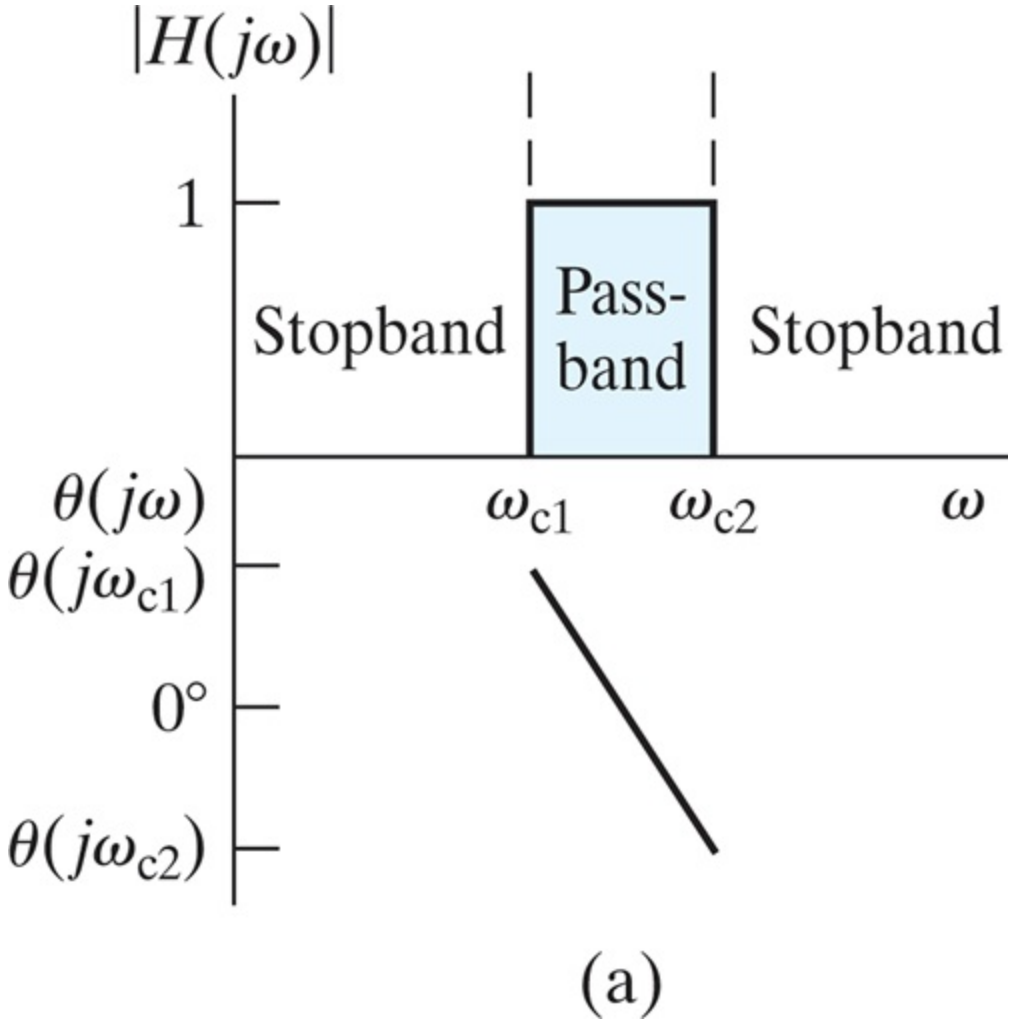
The signals passed from the input to the output fall within a band of frequencies called the **passband**. Input voltages outside this band have their magnitudes attenuated by the circuit and are thus effectively prevented from reaching the circuit's output. Frequencies not in a circuit's passband are in its **stopband**. Frequency-selective circuits are categorized by the location of the passband.

We can identify the type of frequency-selective circuit by examining its **frequency response plot**. A frequency response plot shows how a circuit's transfer function (both amplitude and phase) changes as the source frequency changes. A frequency response plot has two parts.

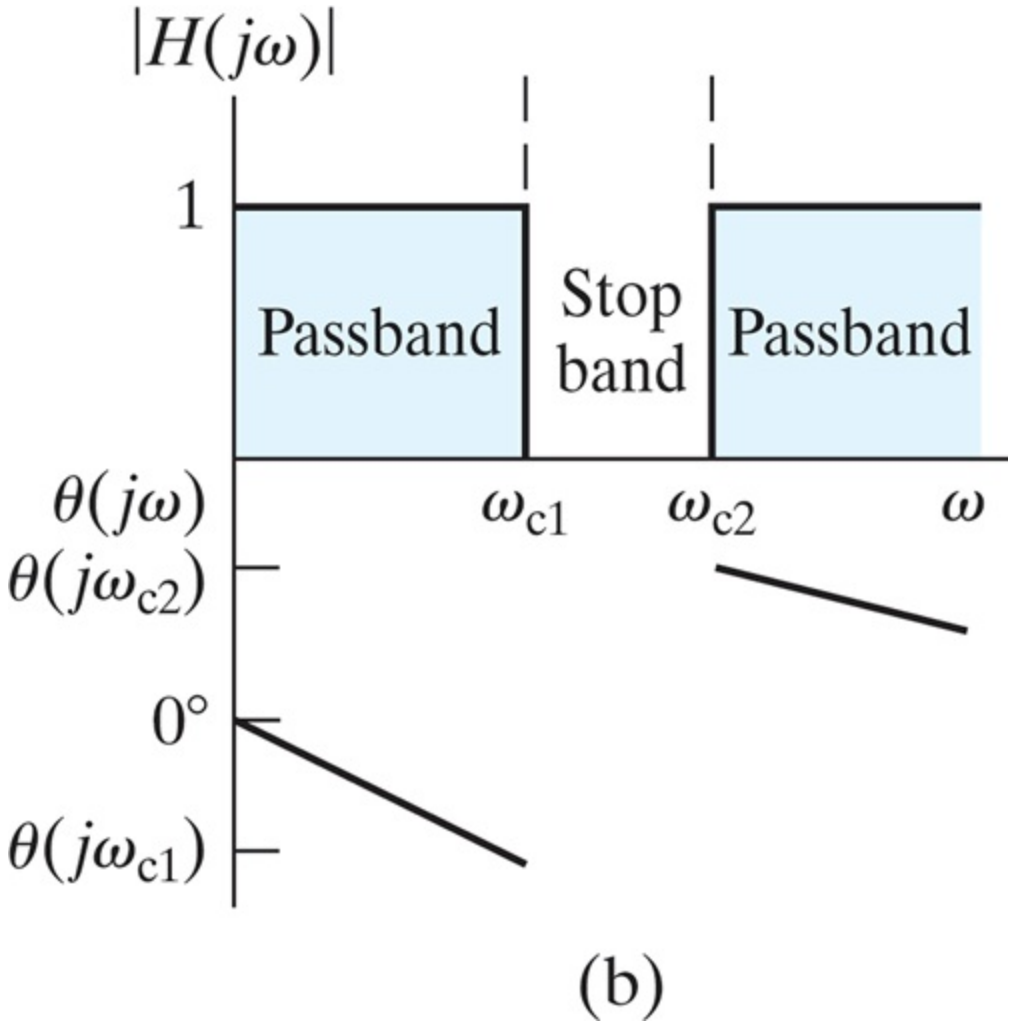
- A graph of $|H(j\omega)|$ versus frequency ω, called the **magnitude plot**.
- A graph of $\theta(j\omega)$ versus frequency ω, called the **phase angle plot**.

The ideal frequency response plots for the four major categories of filters are shown in Figs. 14.2 and Figure 14.3. Figure 14.2 illustrates the ideal plots for a low-pass and a high-pass filter, respectively. Both filters have one passband and one stopband, which are defined by the **cutoff frequency** that separates them. The names *low pass* and *high pass* are derived from the magnitude plots: a **low-pass filter** passes signals at frequencies lower than the cutoff frequency from the input to the output, and a **high-pass filter** passes signals at frequencies higher than the cutoff frequency. Thus, the terms *low* and *high* as used here do not refer to any absolute values of frequency, but rather to relative values with respect to the cutoff frequency.

Figure 14.3 Ideal frequency response plots for (a) an ideal bandpass filter and (b) an ideal bandreject filter.



14.1-4 Full Alternative Text



14.1-4 Full Alternative Text

Note from the graphs for both these filters (as well as those for the bandpass and bandreject filters in Fig. 14.3) that the phase angle plot for an ideal filter varies linearly in the passband. It is of no interest outside the passband because there the magnitude is zero. Linear phase variation is necessary to avoid phase distortion.

The two remaining categories of filters each have two cutoff frequencies. Figure 14.3(a) illustrates the ideal frequency response plot of a **bandpass filter**, which passes a source voltage to the output only when the source frequency is within the band defined by the two cutoff frequencies. Figure 14.3(b) shows the ideal plot of a **bandreject filter**, which passes a source voltage to the output only when the source frequency is outside the band

defined by the two cutoff frequencies. The bandreject filter thus rejects, or stops, the source voltage from reaching the output when its frequency is within the band defined by the cutoff frequencies.

In specifying a filter using any of the circuits from this chapter, it is important to note that the magnitude and phase angle characteristics are not independent. In other words, the characteristics of a circuit that result in a particular magnitude plot will also dictate the form of the phase angle plot and vice versa. For example, once we select a desired form for the magnitude response of a circuit, the phase angle response is also determined. Alternatively, if we select a desired form for the phase angle response, the magnitude response is also determined. Although there are some frequency-selective circuits for which the magnitude and phase angle behavior can be independently specified, these circuits are not presented here.

The next sections present examples of circuits from each of the four filter categories. These circuits represent only a few of the many circuits that act as filters. As you read, focus your attention on trying to identify what properties of a circuit determine its behavior as a filter. For example, you should note the various forms of the transfer functions that perform the same filtering function. Identifying the form of a filter's transfer function will ultimately help you in designing filter circuits for particular applications.

All of the filters in this chapter are **passive filters** because their behavior depends only on passive elements: resistors, capacitors, and inductors. Usually, the largest output amplitude a passive filter can achieve equals the input amplitude. The only passive filter described in this chapter that can amplify its output is the series RLC resonant filter. Also, if you place an impedance in series with the source or in parallel with the load, the maximum output amplitude will decrease. Because many practical filter applications require amplification (a ratio of output-to-input amplitude greater than 1), passive filters have some significant disadvantages. Many active filter circuits, introduced in Chapter 15, provide amplification, and thereby overcome this passive filter disadvantage.

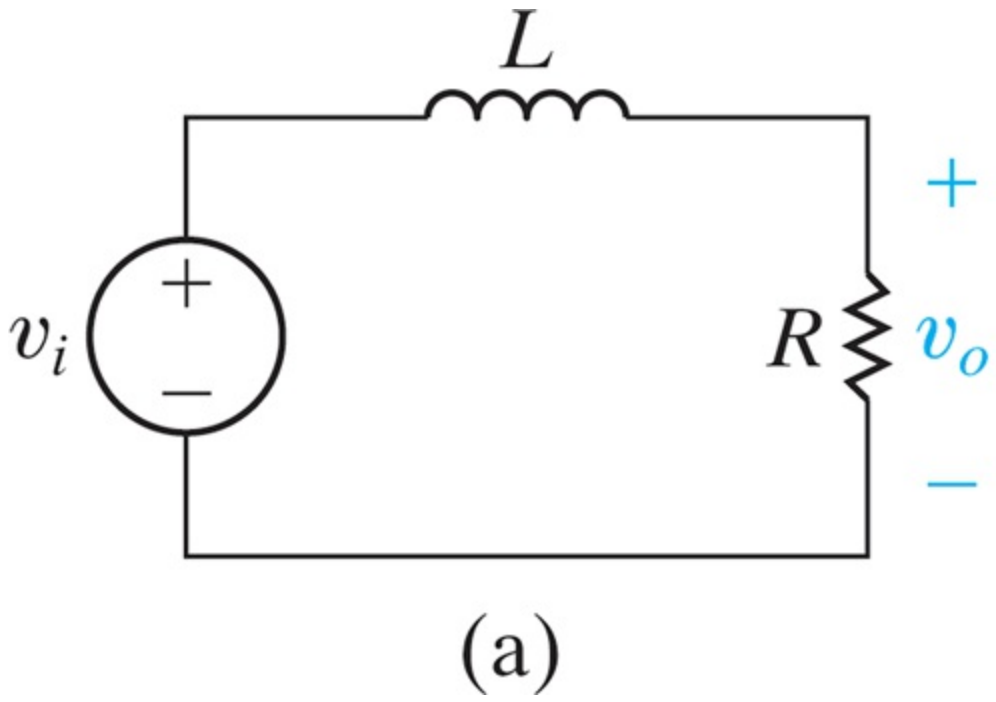
14.2 Low-Pass Filters

Two circuits that behave as low-pass filters are the series RL circuit and the series RC circuit. Let’s investigate the circuit characteristics that affect the cutoff frequency.

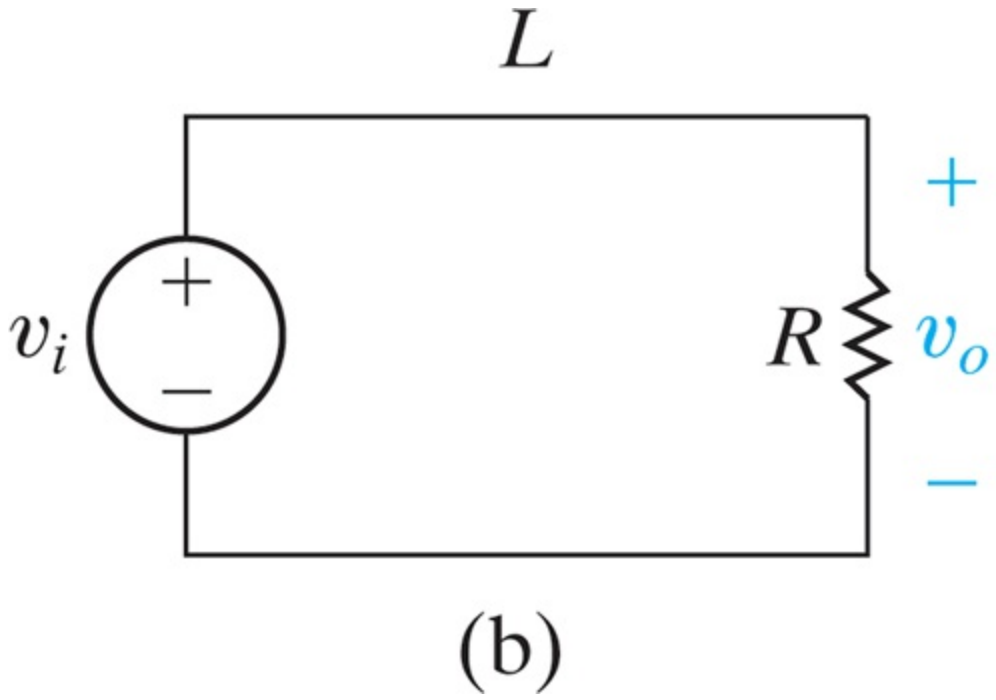
The Series RL Circuit—Qualitative Analysis

A series RL circuit is shown in Fig. 14.4(a). The circuit’s input is a sinusoidal voltage source with varying frequency. The circuit’s output is the voltage across the resistor. Suppose the source frequency starts very low and increases gradually. We know that the behavior of the ideal resistor does not change because its impedance is independent of frequency. But consider how the behavior of the inductor changes.

Figure 14.4 (a) A series RL low-pass filter. (b) The equivalent circuit at ω=0. and (c) The equivalent circuit at ω=∞



14.2-4 Full Alternative Text



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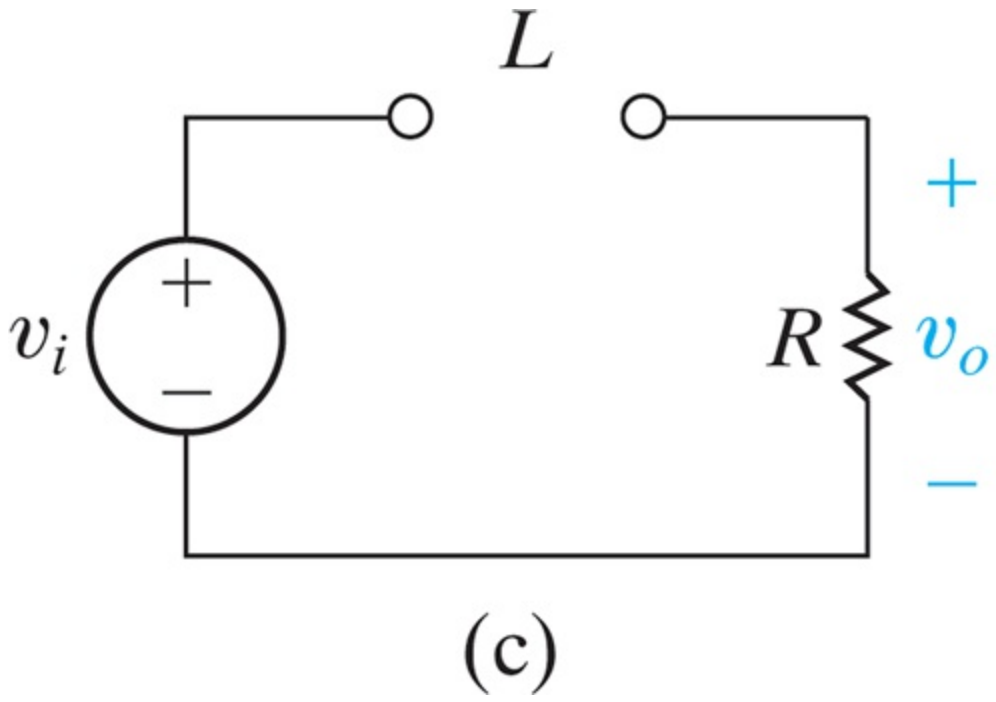


14.2-4 Full Alternative Text

Recall that the impedance of an inductor is jωL. At low frequencies, the inductor's impedance is very small compared with the resistor's impedance, and the inductor effectively functions as a short circuit. The term *low frequencies* thus refers to any frequencies for which ωL<<R. The equivalent circuit for ω=0 is shown in Fig. 14.4(b). In this equivalent circuit, the output voltage and the input voltage are equal both in magnitude and in phase angle.

As the frequency increases, the impedance of the inductor increases relative to that of the resistor. Increasing the inductor's impedance causes a corresponding increase in the magnitude of the voltage drop across the inductor and a corresponding decrease in the output voltage magnitude. Increasing the inductor's impedance also introduces a shift in phase angle between the inductor's voltage and current, resulting in a phase angle difference between the input and output voltage. The output voltage always lags the input voltage, and as the frequency increases, this phase lag approaches 90°.

At high frequencies, the inductor's impedance is very large compared with the resistor's impedance, and the inductor thus functions as an open circuit, effectively blocking the flow of current in the circuit. The term *high*

frequencies thus refers to any frequencies for which ωL>>R. The equivalent circuit for ω=∞ is shown in Fig. 14.4(c), where the output voltage magnitude is zero. The phase angle of the output voltage is 90° more negative than that of the input voltage, so the output lags the input by 90°.

Based on the behavior of the output voltage magnitude, this series RL circuit selectively passes low-frequency inputs to the output, and it blocks high-frequency inputs from reaching the output. The circuit's response to varying input frequency is shown in Fig. 14.5, and these plots represent the frequency response of the series RL circuit in Fig. 14.4(a). The upper plot shows how |H(jω)| varies with frequency. The lower plot shows how θ(jω) varies with frequency. Appendix E presents a method for constructing these plots.

Figure 14.5 The frequency response plot for the series RL circuit in Fig. 14.4(a).

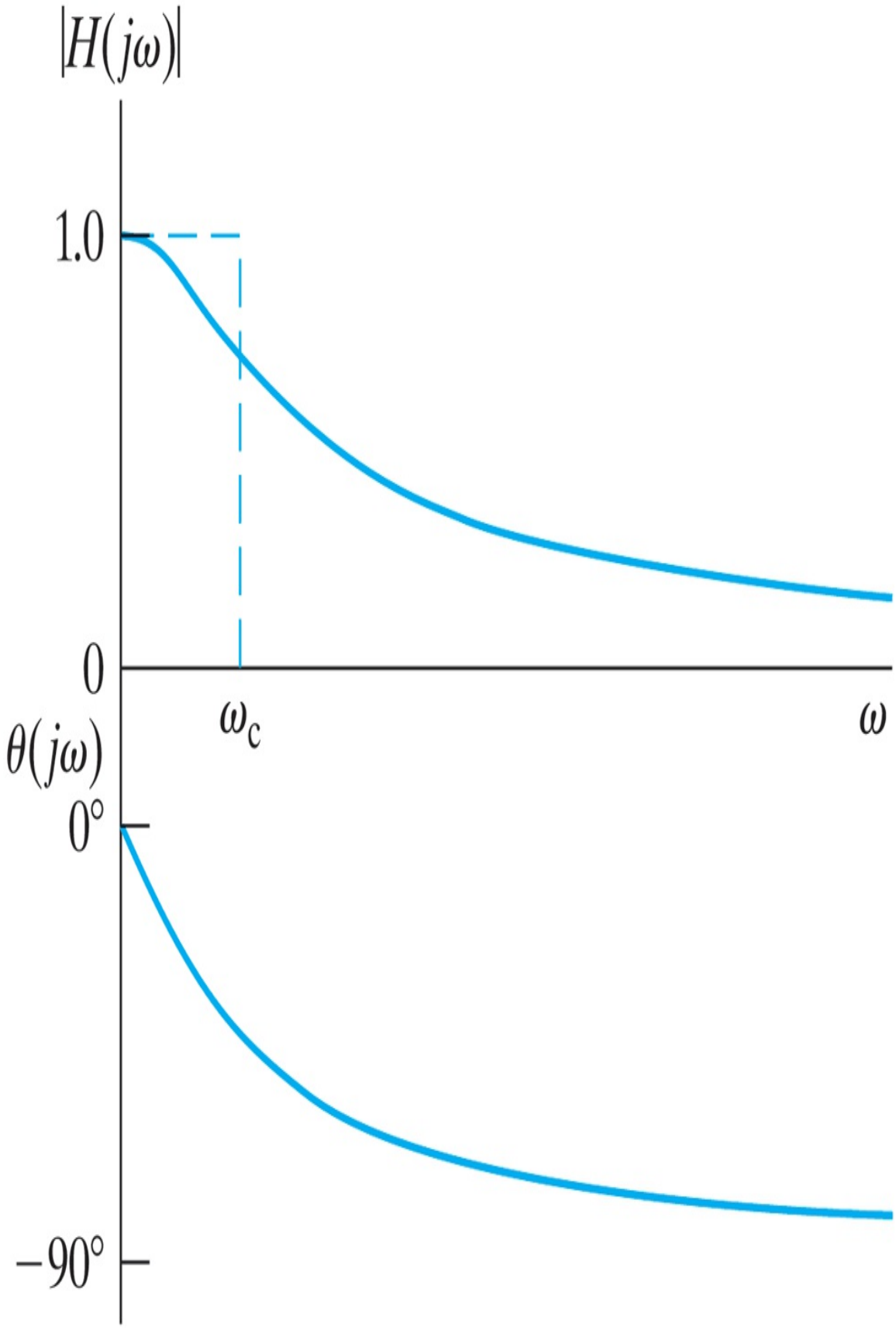

$|H(j\omega)|$
1.0
0
ω_c
ω
$\theta(j\omega)$
0°
−90°

Figure 14.5 Full Alternative Text

We have also superimposed the ideal magnitude plot for a low-pass filter from Fig. 14.2(a) on the magnitude plot of the RL filter in Fig. 14.5 (see the dashed line). There is an obvious difference between the magnitude plots of an ideal filter and an actual RL filter. The ideal filter exhibits a discontinuity in magnitude at the cutoff frequency, ωc, which creates an abrupt transition between the passband and the stopband. While this is, ideally, how we would like our filters to perform, it is not possible to use real components to construct a circuit with an abrupt transition in magnitude. Circuits acting as low-pass filters have a magnitude response that transitions gradually between the passband and the stopband. Hence, the magnitude plot of a real circuit requires us to define what we mean by the cutoff frequency, ωc.

Defining the Cutoff Frequency

We need to define the cutoff frequency, ωc, for realistic filter circuits because the magnitude plot does not allow us to identify a single frequency that divides the passband and the stopband. The definition for cutoff frequency widely used by electrical engineers is the frequency for which the transfer function magnitude is decreased by the factor 1/2 from its maximum value:

Cutoff Frequency Definition

|H(jωc)|=12Hmax, (14.1)

where Hmax is the maximum magnitude of the transfer function. It follows from Eq. 14.1 that the passband of a filter is defined as the range of frequencies for which the amplitude of the output voltage is at least 70.7% of the maximum possible amplitude.

Defining the cutoff frequency using the constant 1 / 2 is not an arbitrary choice. Examining another consequence of the cutoff frequency explains this choice. Recall from Section 10.5 that the average power delivered by any

circuit to a load is proportional to VL2, where VL is the amplitude of the voltage drop across the load:

P=12VL2R.

If the circuit has a sinusoidal voltage source, Vi(jω), then the load voltage is also a sinusoid, and its amplitude is a function of the frequency ω. We define Pmax as the value of the average power delivered to a load when the magnitude of the load voltage is maximum:

Pmax=12VLmax2R.

If we vary the frequency of the sinusoidal voltage source, Vi(jω), the load voltage is a maximum when the magnitude of the circuit's transfer function is also a maximum:

VLmax=Hmax|Vi|. (14.2)

Now consider what happens to the average power when the frequency of the voltage source is ωc. Using Eqs. 14.1 and 14.2, the magnitude of the load voltage at ωc is

|VL(jωc)|=|H(jωc)||Vi| =12Hmax|Vi| =12VLmax.

Now, compute the power delivered to the load at the cutoff frequency:

P(jωc)=12|VL2(jωc)|R =12(12VLmax)2R =12VLmax2/2R =Pmax2.

We see that, at the cutoff frequency ωc, the average power delivered by the circuit is one half the maximum average power. Thus, ωc is also called the **half-power frequency**. In the filter's passband, the average power delivered to a load is at least 50% of the maximum average power.

The Series RL Circuit—Quantitative Analysis

Now that we have defined the cutoff frequency for filter circuits, we can analyze the series RL circuit to find the relationship between the component values and the cutoff frequency for this low-pass filter. We begin by constructing the s-domain equivalent of the circuit in Fig. 14.4(a), assuming the initial conditions are zero. The resulting equivalent circuit is shown in Fig. 14.6. The voltage transfer function for this circuit is

Figure 14.6 The s-domain equivalent for the circuit in Fig. 14.4(a).

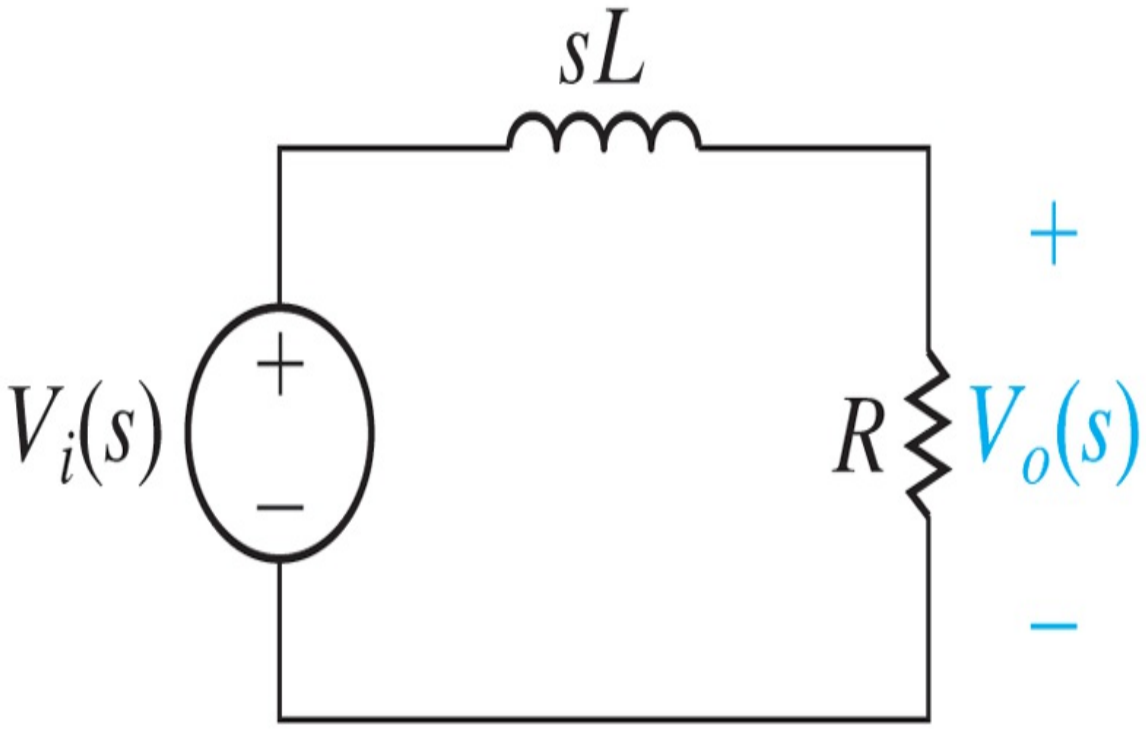


Figure 14.6 Full Alternative Text

H(s)=Vo(s)Vi(s)=R/Ls+R/L. (14.3)

To study the frequency response, we make the substitution s=jω in Eq. 14.3:

H(jω)=R/Ljω+R/L.

We can now separate H(jω) into two equations. The first defines the transfer function magnitude as a function of frequency; the second defines the transfer function phase angle as a function of frequency:

|H(jω)|=R/Lω2+(R/L)2, (14.4)
θ(jω)=−tan−1(ωLR).

Close examination of Eq. 14.4 provides the quantitative support for the magnitude plot shown in Fig. 14.5.

- When ω=0, the denominator and the numerator are equal and |H(j0)|=1. This means that at ω=0, the input voltage is passed to the output terminals without a change in the voltage magnitude.

- As the frequency increases, the numerator of Eq. 14.4 is unchanged, but the denominator gets larger. Thus, |H(jω)| decreases as the frequency increases, as shown in the plot in Fig. 14.5. Likewise, as the frequency increases, the phase angle changes from its dc value of 0°, becoming more negative.

- As ω→∞, the denominator of Eq. 14.4 approches infinity and |H(j∞)|=0, as seen in Fig. 14.5. As ω→∞, the phase angle approaches −90°, as seen from the phase angle plot in Fig. 14.5.

Using Eq. 14.4, we can compute the cutoff frequency, ωc. Remember that ωc is defined as the frequency at which |H(jωc)|=(1 / 2)Hmax. For the low-pass filter, Hmax=|H(j0)|=1, as seen in Fig. 14.5. Thus, for the circuit in Fig. 14.4(a),

|H(jωc)|=12|1|=R/Lωc2+(R/L)2.
.

Solving for ωc, we get

Cutoff Frequency for RL Filters

ωc=RL.(14.5)

Equation 14.5 provides an important result. The cutoff frequency, ωc, can be set to any desired value by appropriately selecting values for R and L. We can therefore design a low-pass filter with whatever cutoff frequency is needed. Example 14.1 demonstrates the design potential of Eq. 14.5.

Example 14.1 Designing a Low-Pass Filter

Electrocardiology is the study of the electric signals produced by the heart. These signals maintain the heart's rhythmic beat and are measured by an instrument called an electrocardiograph. This instrument must be capable of detecting periodic signals whose frequency is about 1 Hz (the normal heart rate is 72 beats per minute). The instrument must operate in the presence of sinusoidal noise consisting of signals from the surrounding electrical environment, whose fundamental frequency is 60 Hz—the frequency at which electric power is supplied.

Choose values for R and L in the circuit of Fig. 14.4(a) such that the resulting circuit could be used in an electrocardiograph to filter out any noise above 10 Hz and pass the electric signals from the heart at or near 1 Hz. Then compute the magnitude of Vo at 1 Hz, 10 Hz, and 60 Hz to see how well the filter performs.

Solution

The problem is to select values for R and L that yield a low-pass filter with a cutoff frequency of 10 Hz. From Eq. 14.5, we see that R and L cannot be specified independently to generate a value for ωc. Therefore, let's choose a commonly available value of L, 100 mH. Before we use Eq. 14.5 to compute the value of R needed to obtain the desired cutoff frequency, we need to convert the cutoff frequency from hertz to radians per second:

$$\omega c = 2\pi(10) = 20\pi \text{ rad / s.}$$

Now, solve for the value of R that, together with L=100mH, will yield a low-pass filter, with a cutoff frequency of 10 Hz:

$R=\omega cL=(20\pi)(100\times10-3)=6.28\Omega$.

We can compute the magnitude of Vo using the equation $|Vo|=|H(j\omega)|\cdot|Vi|$:

$|Vo(\omega)|=R/L\omega2+(R/L)2|Vi|=20\pi\omega2+400\pi2|Vi|$.

Table 14.1 summarizes the computed magnitude values for the frequencies 1 Hz, 10 Hz, and 60 Hz. As expected, the input and output voltages have the same magnitudes at the low frequency because the circuit is a low-pass filter. At the cutoff frequency, the output voltage magnitude has been reduced by 1/2 from the unity passband magnitude. At 60 Hz, the output voltage magnitude has been reduced by a factor of about 6, achieving the desired attenuation of the noise that could corrupt the signal the electrocardiograph is designed to measure.

Table 14.1 Input and Output Voltage Magnitudes for Several Frequencies

f(Hz)	$\lvert V_i\rvert$(V)	$\lvert V_o\rvert$(V)
1	1.0	0.995
10	1.0	0.707
60	1.0	0.164

A Series RC Circuit

The series RC circuit shown in Fig. 14.7 also behaves as a low-pass filter. We can verify this via the same qualitative analysis used previously. In fact, such a qualitative examination is an important problem-solving step that you should get in the habit of performing when analyzing filters. Doing so enables you to predict the filtering characteristics (low pass, high pass, etc.) and thus also predict the general form of the transfer function. If the calculated transfer function matches the qualitatively predicted form, you have an important accuracy check.

Figure 14.7 A series RC low-pass filter.

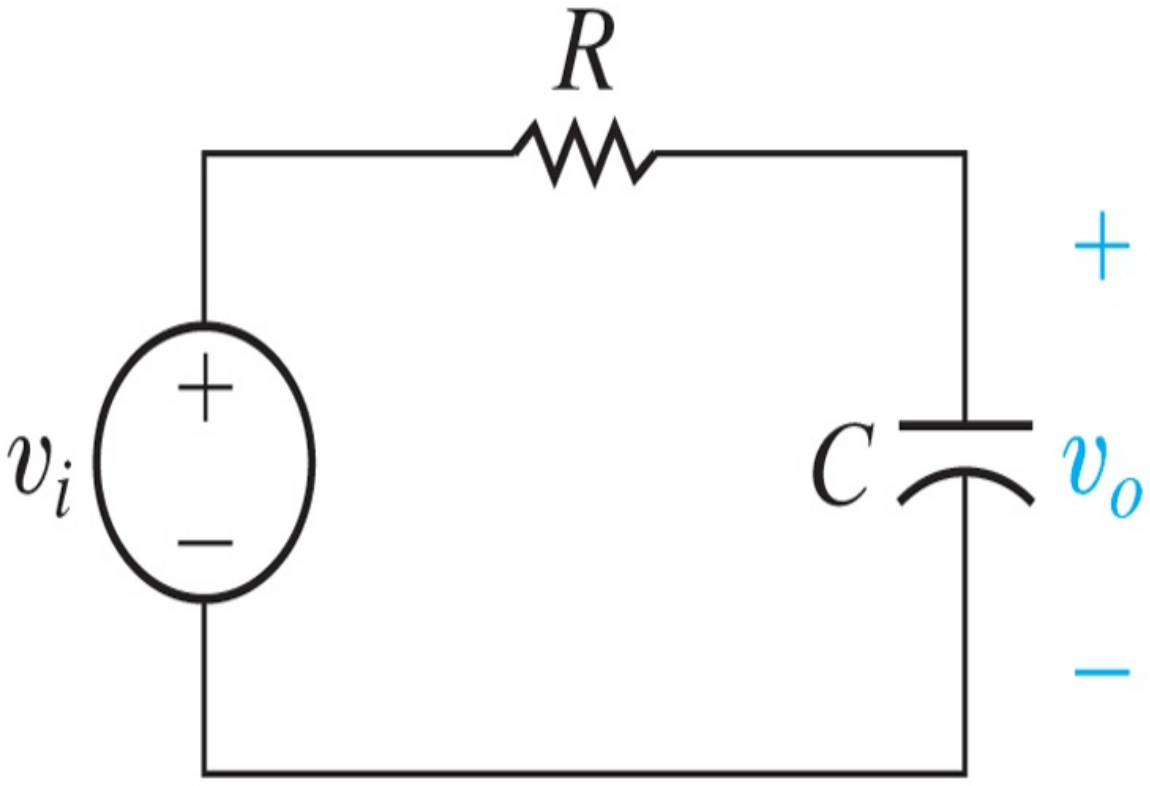


Figure 14.7 Full Alternative Text

Note that the circuit's output is defined as the voltage across the capacitor. We again use three frequency regions to determine how the series RC circuit

in Fig. 14.7 behaves:

- Zero frequency ($\omega=0$): The impedance of the capacitor is infinite, and the capacitor acts as an open circuit. The input and output voltages are thus the same.
- Frequencies increasing from zero: The impedance of the capacitor decreases relative to the impedance of the resistor, and the source voltage divides between the resistive impedance and the capacitive impedance. The output voltage is thus smaller than the source voltage.
- Infinite frequency ($\omega=\infty$): The impedance of the capacitor is zero, and the capacitor acts as a short circuit. The output voltage is thus zero.

Based on this analysis, the series RC circuit functions as a low-pass filter. Example 14.2 explores this circuit quantitatively.

Example 14.2 Designing a Series RC Low-Pass Filter

For the series RC circuit in Fig. 14.7:

1. Find the transfer function between the source voltage and the output voltage.
2. Determine an equation for the cutoff frequency in the series RC circuit.
3. Choose values for R and C that will yield a low-pass filter with a cutoff frequency of 3 kHz.

Solution

1. To derive an expression for the transfer function, we first construct the s-domain equivalent of the circuit in Fig. 14.7, as shown in Fig. 14.8.

Using s-domain voltage division on the equivalent circuit, we find

Figure 14.8 The s-domain equivalent for the circuit in Fig. 14.7.

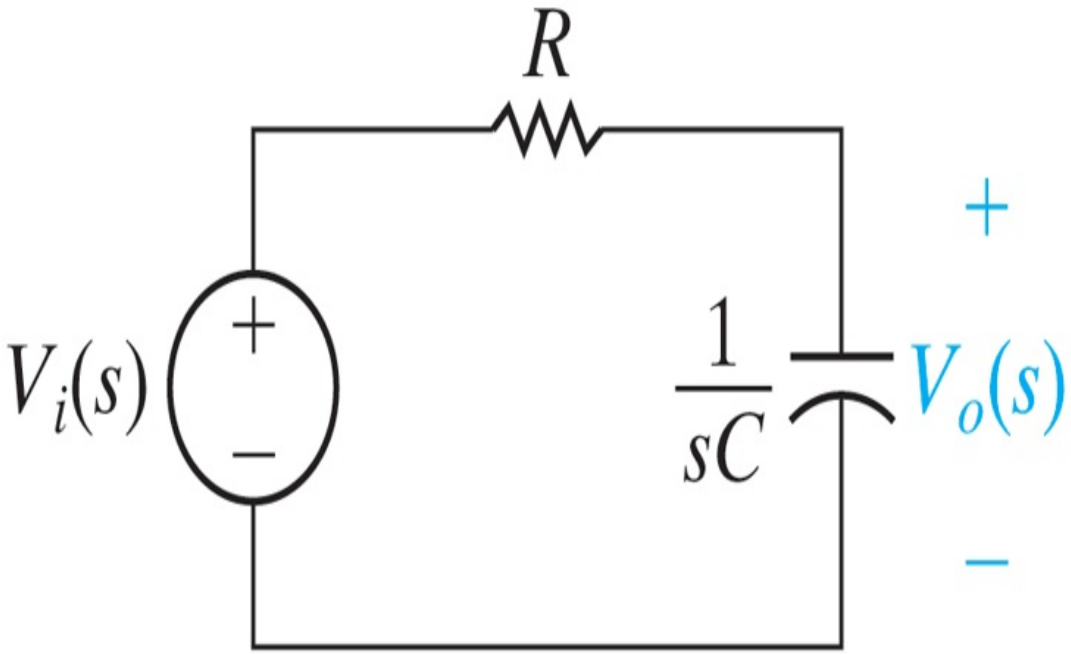


Figure 14.8 Full Alternative Text
H(s)=1RCs+1RC.

Now, substitute s=jω and compute the magnitude of the resulting expression:

|H(jω)|=1RCω2+ (1RC)2.

2. At the cutoff frequency ωc, |H(jω)| is equal to (1 / 2)Hmax. For a low-pass filter, Hmax=H(j0), and for the circuit in Fig. 14.8, H(j0)=1. We can then describe the relationship among the quantities R, C, and ωc:

$$|H(j\omega c)| = \frac{1}{2}(1) = \frac{1}{RC\omega c^2 + (\frac{1}{RC})^2}.$$

Solving this equation for ωc, we get

Cutoff Frequency for RC Filters

$$\omega c = \frac{1}{RC}.$$

3. From the results in (b), we see that the cutoff frequency is determined by the values of R and C. Because R and C cannot be determined independently, let's choose $C = 1\ \mu F$. Given a choice, we will usually specify a value for C first, rather than for R, because the number of available capacitor values is much smaller than the number of resistor values. Remember that we have to convert the specified cutoff frequency from 3 kHz to $(2\pi)(3)$ krad / s:

$$R = \frac{1}{\omega c C} = \frac{1}{(2\pi)(3\times 10^3)(1\times 10^{-6})} = 53.05\ \Omega.$$

Figure 14.9 summarizes the two low-pass filter circuits we have analyzed. Look carefully at the transfer functions and notice their similar form—they differ only in the terms that specify the cutoff frequency. We can therefore identify a general form for the transfer functions of these two low-pass filters:

Figure 14.9 Two low-pass filters, the series RL and the series RC, together with their transfer functions and cutoff frequencies.

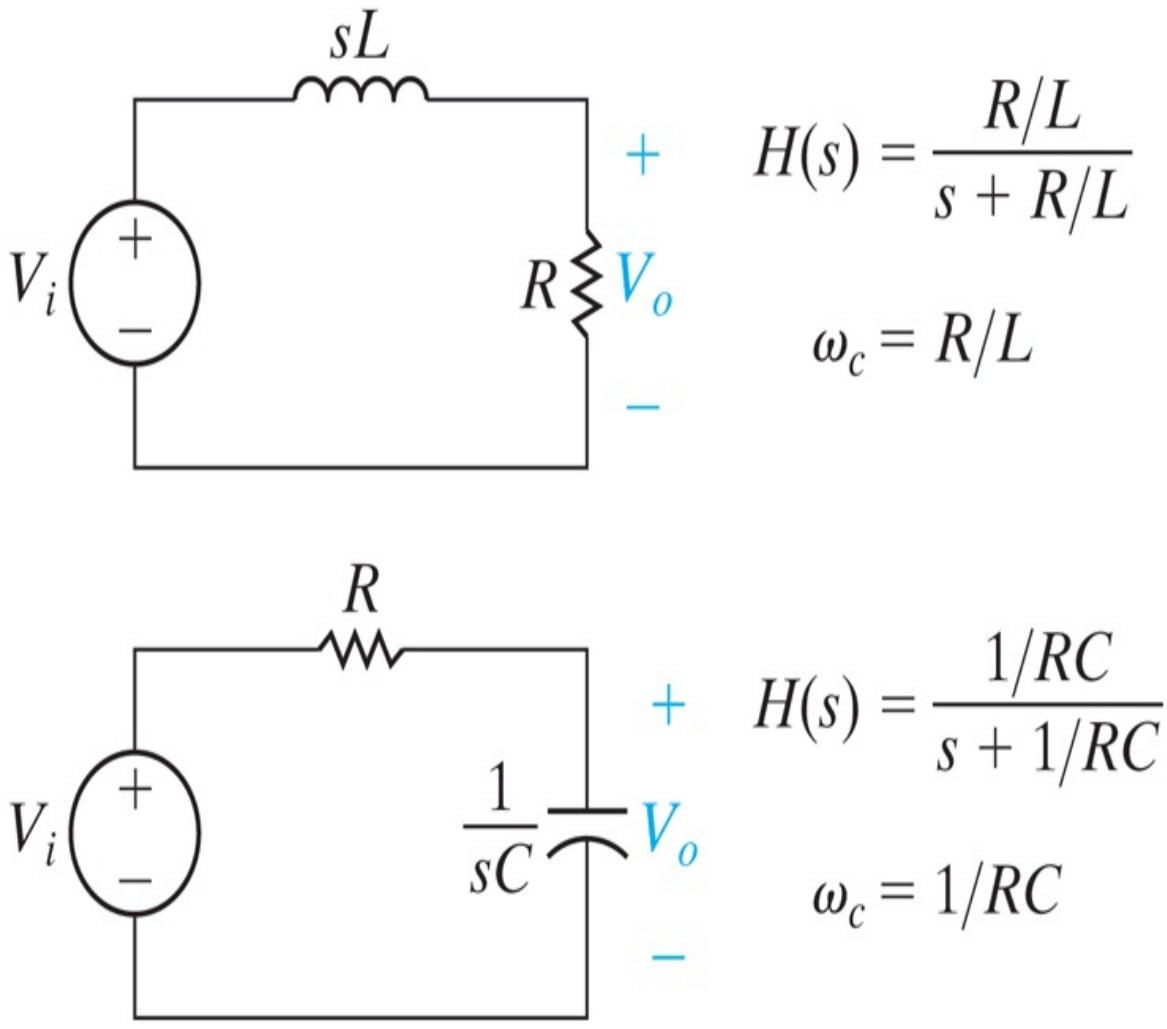


Figure 14.9 Full Alternative Text

Transfer Function for Low-Pass Filters

H(s)=ωcs+ωc.(14.6)

Any circuit with a voltage ratio described by Eq. 14.6 would behave as a low-pass filter with a cutoff frequency of ωc. The problems at the end of the chapter give you other examples of circuits with this voltage ratio.

Relating the Frequency Domain to the Time Domain

Finally, you might have noticed one other important relationship. Recall our discussion of the natural responses of the first-order RL and RC circuits in Chapter 7. An important parameter for these circuits is the time constant, τ, which characterizes the shape of the time response. For the RL circuit, the time constant has the value L / R (Eq. 7.3); for the RC circuit, the time constant is RC (Eq. 7.8). Compare the time constants to the cutoff frequencies for these circuits and notice that

$$\tau=1/\omega c. \quad (14.7)$$

This result is a direct consequence of the relationship between the time response of a circuit and its frequency response, as revealed by the Laplace transform. The discussion of memory and weighting as represented in the convolution integral of Section 13.6 shows that as $\omega c \rightarrow \infty$, the filter has no memory, and the output approaches a scaled replica of the input; that is, no filtering has occurred. As $\omega c \rightarrow 0$, the filter has increased memory, and the output voltage is a distortion of the input because filtering has occurred.

Assessment Problems

Objective 1—Know the RL and RC circuit configurations that act as low-pass filters

1. 14.1 A series RC low-pass filter requires a cutoff frequency of 8 kHz. Use $R=10k\Omega$ and compute the value of C required.

   Answer: 1.99 nF.

2. 14.2 A series RL low-pass filter with a cutoff frequency of 2 kHz is needed. Using $R=5k\Omega$, compute (a) L; (b) $|H(j\omega)|$ at 50 kHz; and (c) $\theta(j\omega)$ at 50 kHz.

Answer:

1. 0.40 H;
2. 0.04;
3. −87.71°.

SELF-CHECK: *Also try Chapter Problems 14.1 and 14.3.*

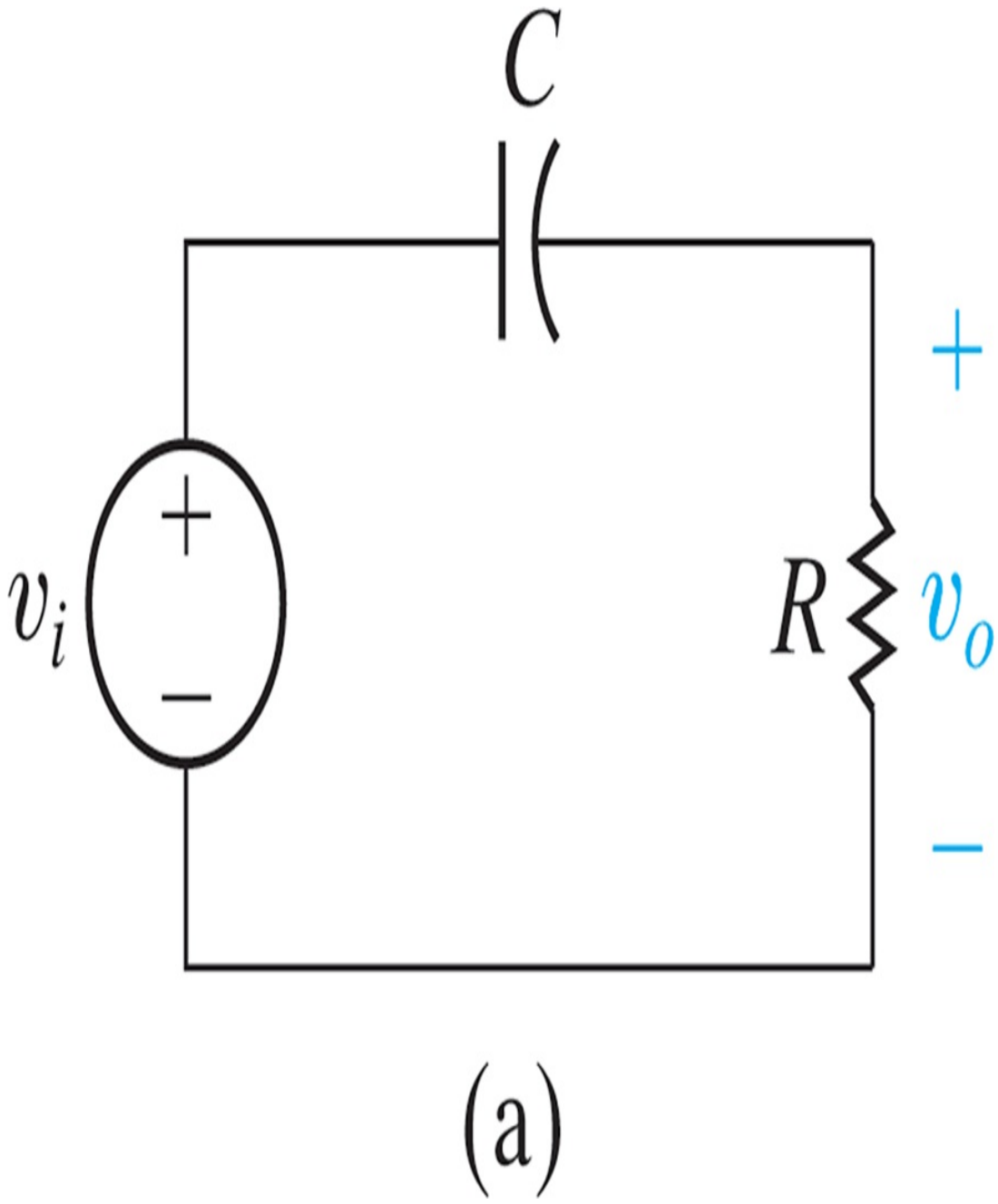
14.3 High-Pass Filters

Now we examine two circuits that function as high-pass filters. Once again, they are the series RL circuit and the series RC circuit. We will see that the same series circuit can act as either a low-pass or a high-pass filter, depending on where the output voltage is defined. We will also determine the relationship between the component values and the cutoff frequency of these filters.

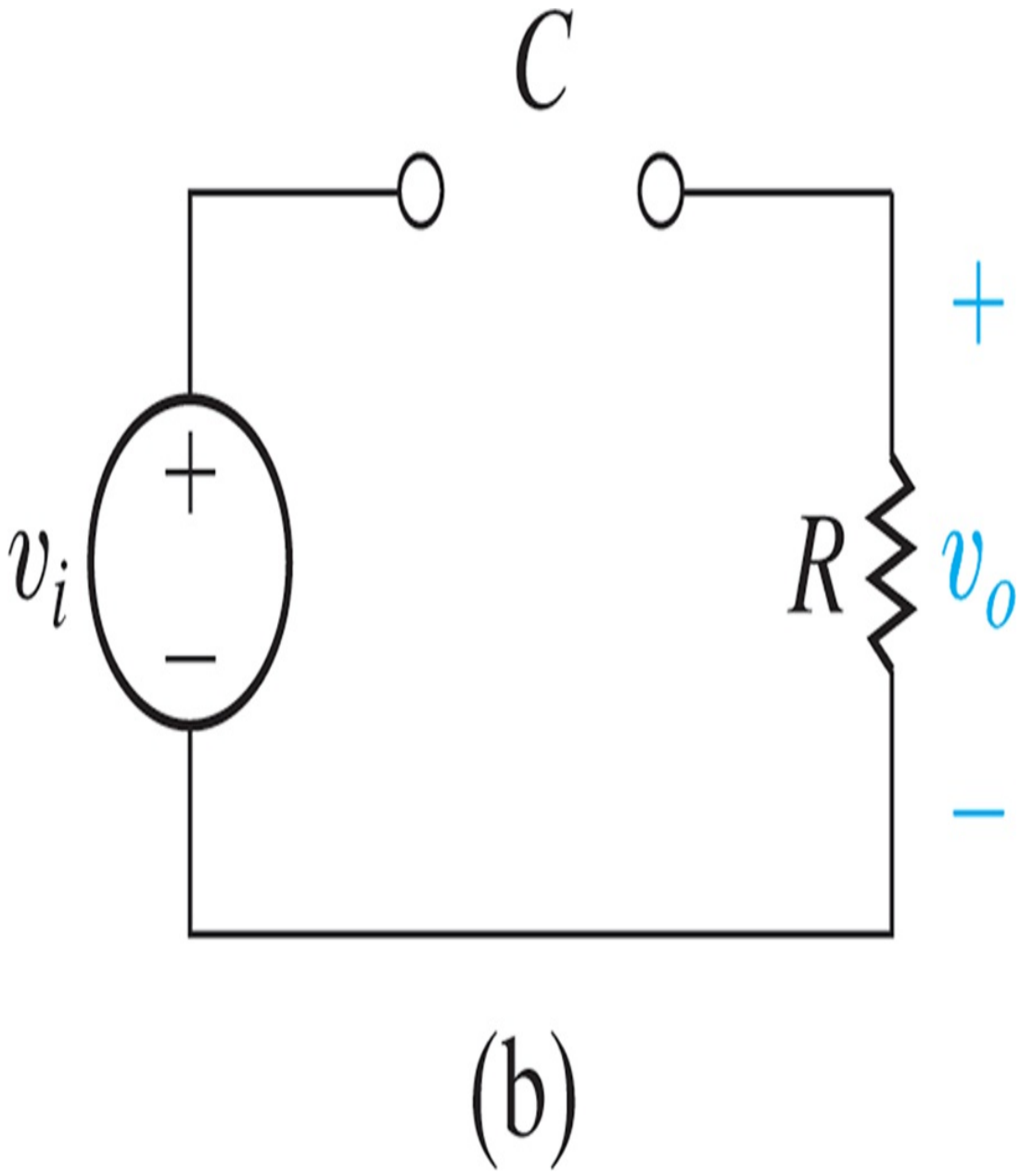
The Series RC Circuit—Qualitative Analysis

A series RC circuit is shown in Fig. 14.10(a). In contrast to its low-pass counterpart in Fig. 14.7, the output voltage here is defined across the resistor, not the capacitor. Because of this, the effect of the changing capacitive impedance is different than it was in the low-pass configuration.

Figure 14.10 (a) A series RC high-pass filter; (b) the equivalent circuit at $\omega=0$; and (c) the equivalent circuit at $\omega=\infty$.

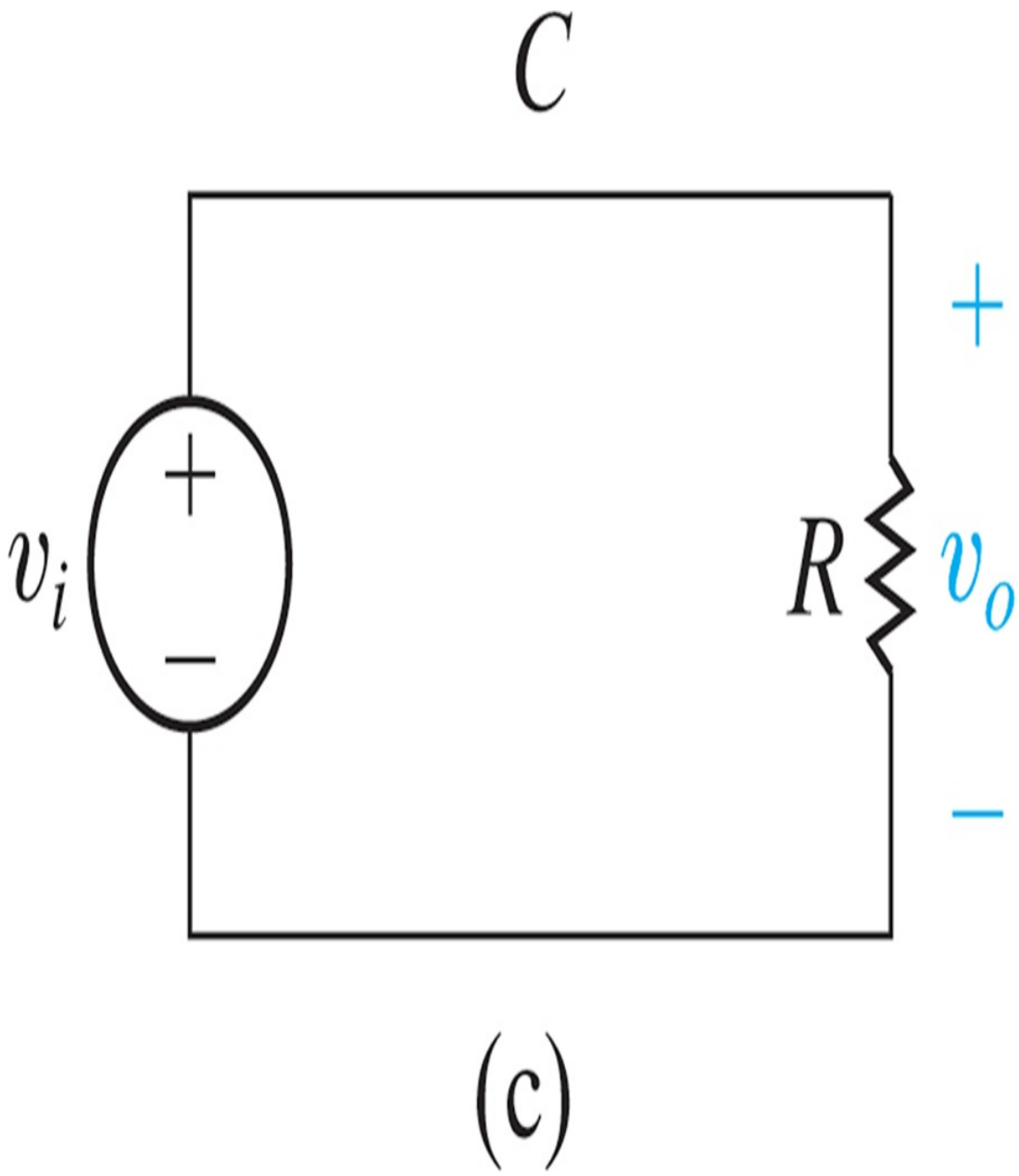


14.3-5 Full Alternative Text



(b)

14.3-5 Full Alternative Text



14.3-5 Full Alternative Text

At ω=0, the capacitor behaves like an open circuit, so there is no current in the resistor. This is illustrated in the equivalent circuit in Fig. 14.10(b). In this circuit, there is no voltage across the resistor and vo=0. The circuit filters out the low-frequency input voltage before it reaches the circuit's output.

As the frequency of the voltage source increases, the impedance of the capacitor decreases relative to the impedance of the resistor, and the source

voltage is now divided between the capacitor and the resistor. The output voltage magnitude thus begins to increase.

When the frequency of the source is infinite ($\omega=\infty$), the capacitor behaves as a short circuit, so the capacitor voltage is zero. This is illustrated in the equivalent circuit in Fig. 14.10(c). In this circuit, the input and output voltages are the same.

The phase angle difference between the input and output voltages also varies as the frequency of the source changes. For $\omega=\infty$, the output voltage is the same as the input voltage, so the phase angle difference is zero. As the frequency of the source decreases and the impedance of the capacitor increases, a phase shift is introduced between the voltage and the current in the capacitor. This creates a phase difference between the input and output voltages. The phase angle of the output voltage leads that of the source voltage. When $\omega=0$, this phase angle difference reaches its maximum of $+90°$.

Based on our qualitative analysis, we see that when the output is defined as the voltage across the resistor, the series RC circuit behaves as a high-pass filter. The components and connections are identical to the low-pass series RC circuit, but the choice of output is different. Thus, we have confirmed the earlier observation that the filtering characteristics of a circuit depend on the definition of the output as well as on circuit components, values, and connections.

Figure 14.11 shows the frequency response plot for the series RC high-pass filter. For reference, the dashed lines indicate the magnitude plot for an ideal high-pass filter. We now turn to a quantitative analysis of this same circuit.

Figure 14.11 The frequency response plot for the series RC circuit in Fig. 14.10(a).

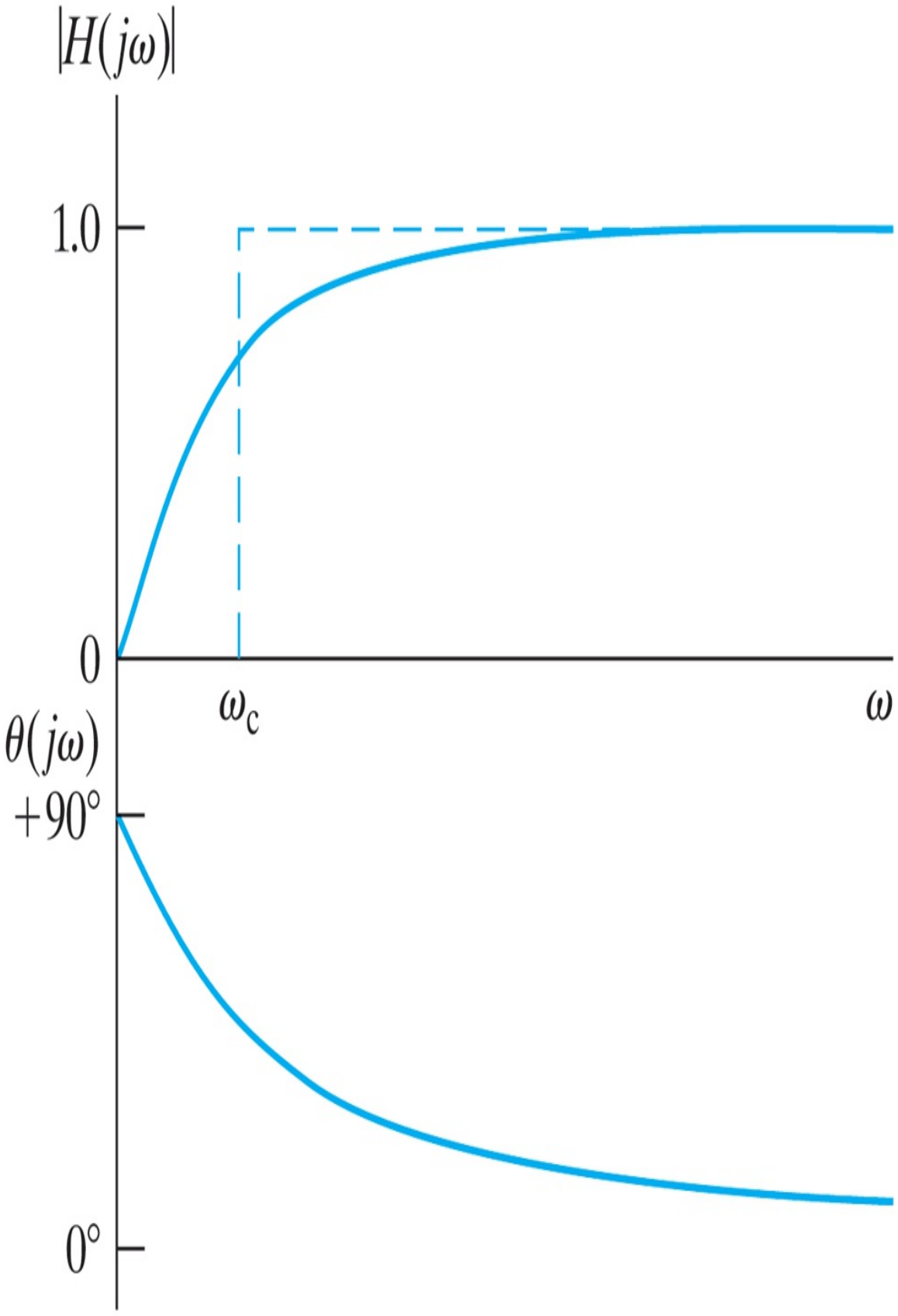

$|H(j\omega)|$
1.0
0
ω_c
ω
$\theta(j\omega)$
+90°
0°

Figure 14.11 Full Alternative Text

The Series RC Circuit—Quantitative Analysis

To begin, construct the s-domain equivalent of the circuit in Fig. 14.10(a), as shown in Fig. 14.12. We use voltage division to find the transfer function:

Figure 14.12 The s-domain equivalent of the circuit in Fig. 14.10(a).

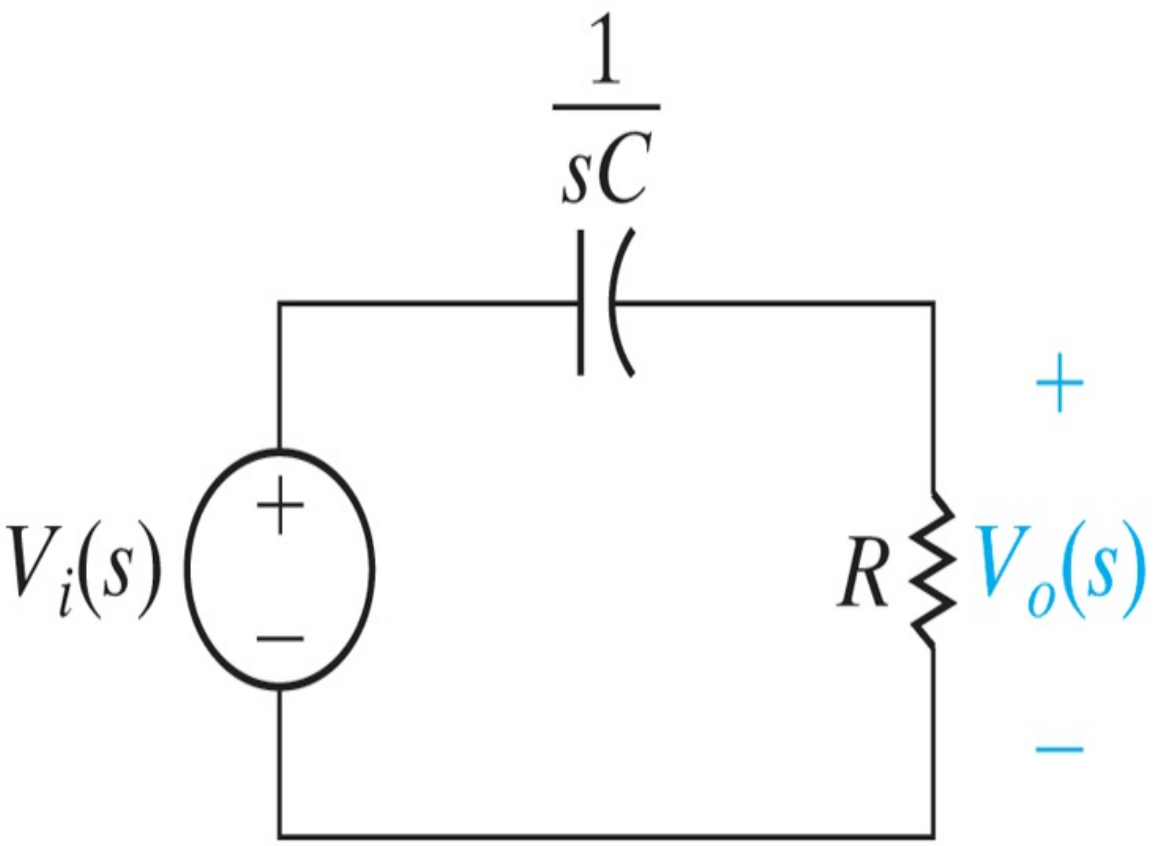


Figure 14.12 Full Alternative Text
H(s)=Vo(s)Vi(s)=ss+1/RC.

Making the substitution s=jω gives

H(jω)=jωjω+1/RC.

Next, we separate H(jω) into two equations. The first is the equation describing the magnitude of the transfer function; the second is the equation describing the phase angle of the transfer function:

|H(jω)|=ωω2+(1/RC)2, ω(jω)=90°-tan-1?RC. (14.8)

A close look at the equations for the magnitude and phase angle of the transfer function confirms the shape of the frequency response plot in Fig. 14.11. Using Eq. 14.8, we can calculate the cutoff frequency for the series RC high-pass filter. Recall that at the cutoff frequency, the magnitude of the transfer function is (1 / 2)Hmax. For a high-pass filter, Hmax=|H(jω)|

$\omega \to \infty = |H(j\infty)|$, as seen from Fig. 14.11. We can construct an equation for ωc by setting the left-hand side of Eq. 14.8 to $(1 / 2)|H(j\infty)|$, noting that for this series RC circuit, $|H(j\infty)|=1$:

$$12=\omega c\omega c2+(1/RC)2.$$

Solving for ωc, we get

$$\omega c=1RC. \quad (14.9)$$

Equation 14.9 presents a familiar result. The cutoff frequency for the series RC circuit has the value 1/RC, whether the circuit is configured as a low-pass filter in Fig. 14.7 or as a high-pass filter in Fig. 14.10(a). This is not a surprising result, as we have already discovered a connection between the cutoff frequency, ωc, and the time constant, τ, of a circuit.

Example 14.3 analyzes a series RL circuit, this time configured as a high-pass filter. Example 14.4 examines the effect of adding a load resistor at the output of the filter.

Example 14.3 Designing a Series RL High-Pass Filter

Show that the series RL circuit in Fig. 14.13 also acts like a high-pass filter:

Figure 14.13 The circuit for Example 14.3.

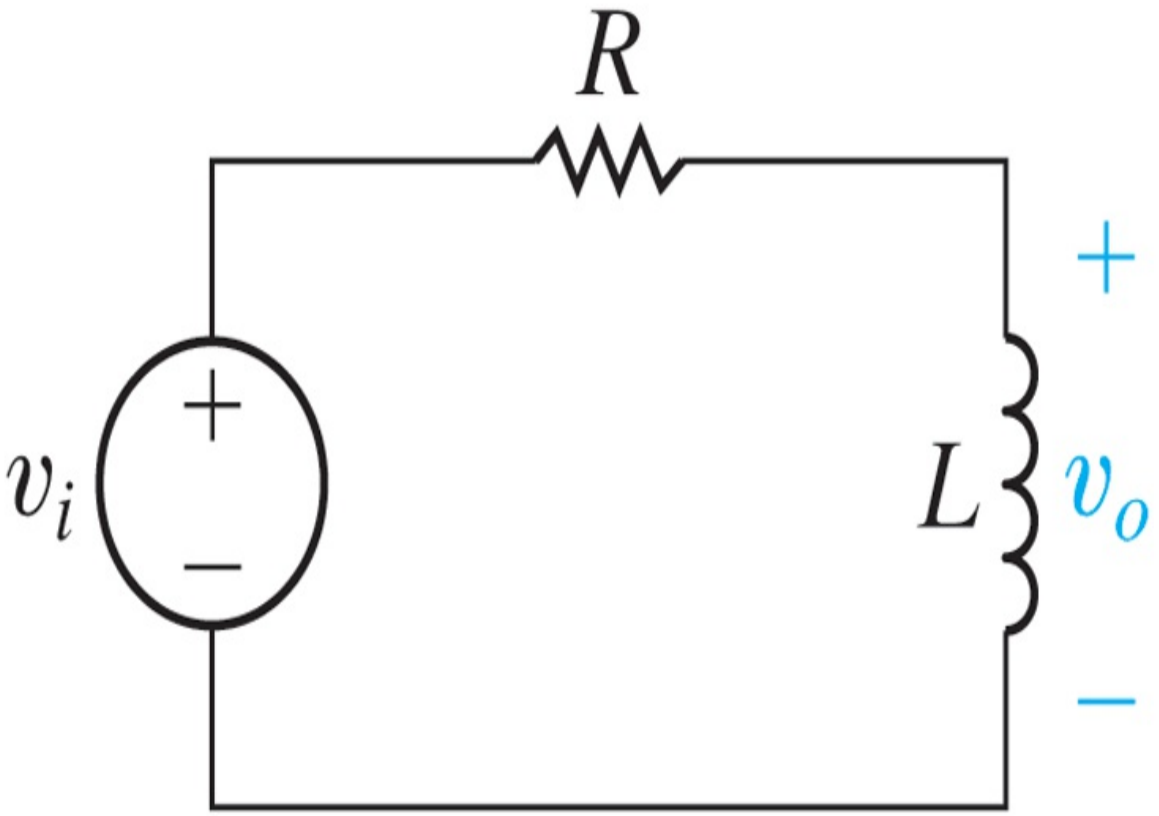


Figure 14.13 Full Alternative Text

1. Derive an expression for the circuit's transfer function.

2. Use the result from (a) to determine an equation for the cutoff frequency in the series RL circuit.

3. Choose values for R and L that will yield a high-pass filter with a cutoff frequency of 15 kHz.

Solution

1. Begin by constructing the s-domain equivalent of the series RL circuit, as shown in Fig. 14.14. Then use voltage division to find the transfer function:

Figure 14.14 The s-domain equivalent of the circuit in Fig. 14.13.

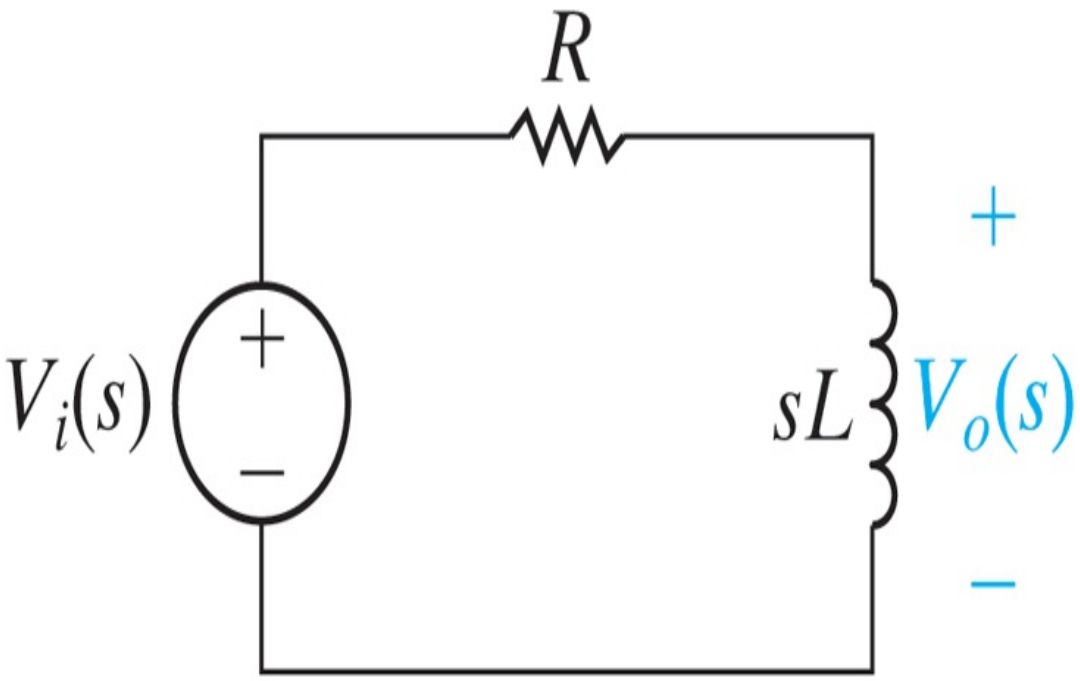


Figure 14.14 Full Alternative Text
H(s)=ss+R / L.

Making the substitution s=jω, we get

H(jω)=jωjω+R / L.

Notice that this equation has the same form as the equation for the series RC high-pass filter.

2. To find an equation for the cutoff frequency, first compute the magnitude of H(jω):

   |H(jω)|=ωω2+ (R / L)2.

Then, as before, we set the left-hand side of this equation to (1 / 2)Hmax, based on the definition of the cutoff frequency ωc. Remember that Hmax=|H(j∞)| for a high-pass filter, and for the series RL circuit, |H(j∞)|=1. We solve the resulting equation for the cutoff frequency:

12=ωcωc2 + (R / L)2,ωc=RL.

This is the same cutoff frequency we computed for the series RL low-pass filter.

3. Using the equation for ωc computed in (b), we find that it is not possible to specify values for R and L independently. Therefore, let's arbitrarily select a value of 500 Ω for R. Remember to convert the cutoff frequency to radians per second:

   L=Rωc=500(2π)(15,000)=5.31 mH.

Example 14.4 Loading the Series RL High-Pass Filter

Examine the effect of placing a load resistor in parallel with the inductor in the RL high-pass filter shown in Fig. 14.15:

Figure 14.15 The circuit for Example 14.4.

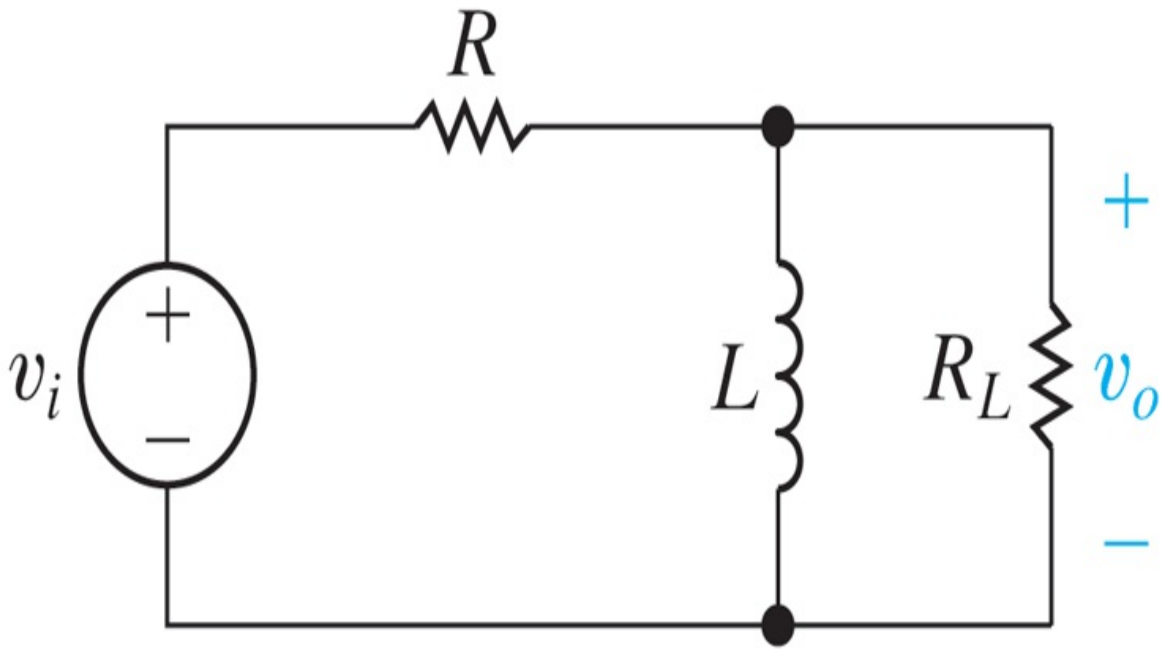


Figure 14.15 Full Alternative Text

1. Determine the transfer function for the circuit in Fig. 14.15.

2. Sketch the magnitude plot for the loaded RL high-pass filter, using the values for R and L from the circuit in Example 14.3(c) and letting RL=R. On the same graph, sketch the magnitude plot for the unloaded RL high-pass filter of Example 14.3(c).

Solution

1. Begin by transforming the circuit in Fig. 14.15 to the s-domain, as shown in Fig. 14.16. Use voltage division across the parallel combination of inductor and load resistor to compute the transfer function:

Figure 14.16 The s-domain equivalent of the circuit in

Fig. 14.15.

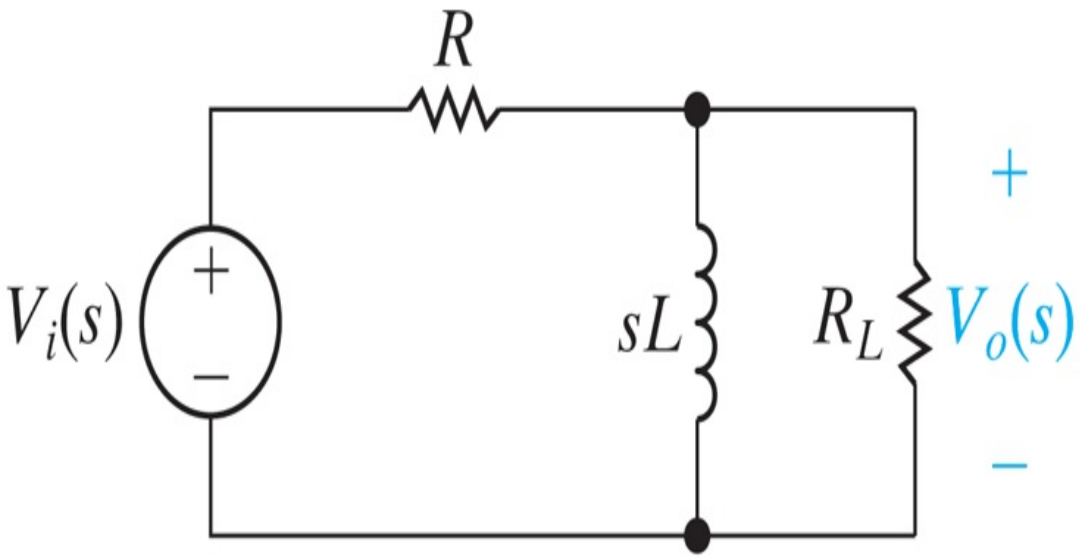


Figure 14.16 Full Alternative Text
H(s)=RLsLRL+sLR+RLsLRL+sL=(RLR+RL)ss+(RLR+RL)RL=Kss+ωc,

where

K=RLR+RL,ωc=KR / L.

Note that ωc is the cutoff frequency of the loaded filter.

2. For the unloaded RL high-pass filter from Example 14.3(c), the passband magnitude is 1, and the cutoff frequency is 15 kHz. For the loaded RL high-pass filter, R=RL=500 Ω, so K=1 / 2. Thus, for the loaded filter, the passband magnitude is (1)(1 / 2)=1 / 2, and the cutoff frequency is (15,000)(1 / 2)=7.5kHz. A sketch of the magnitude plots of the loaded and unloaded circuits is shown in Fig. 14.17.

Figure 14.17 The magnitude plots for the unloaded RL

high-pass filter of Fig 14.13 and the loaded RL high-pass filter of Fig. 14.15.

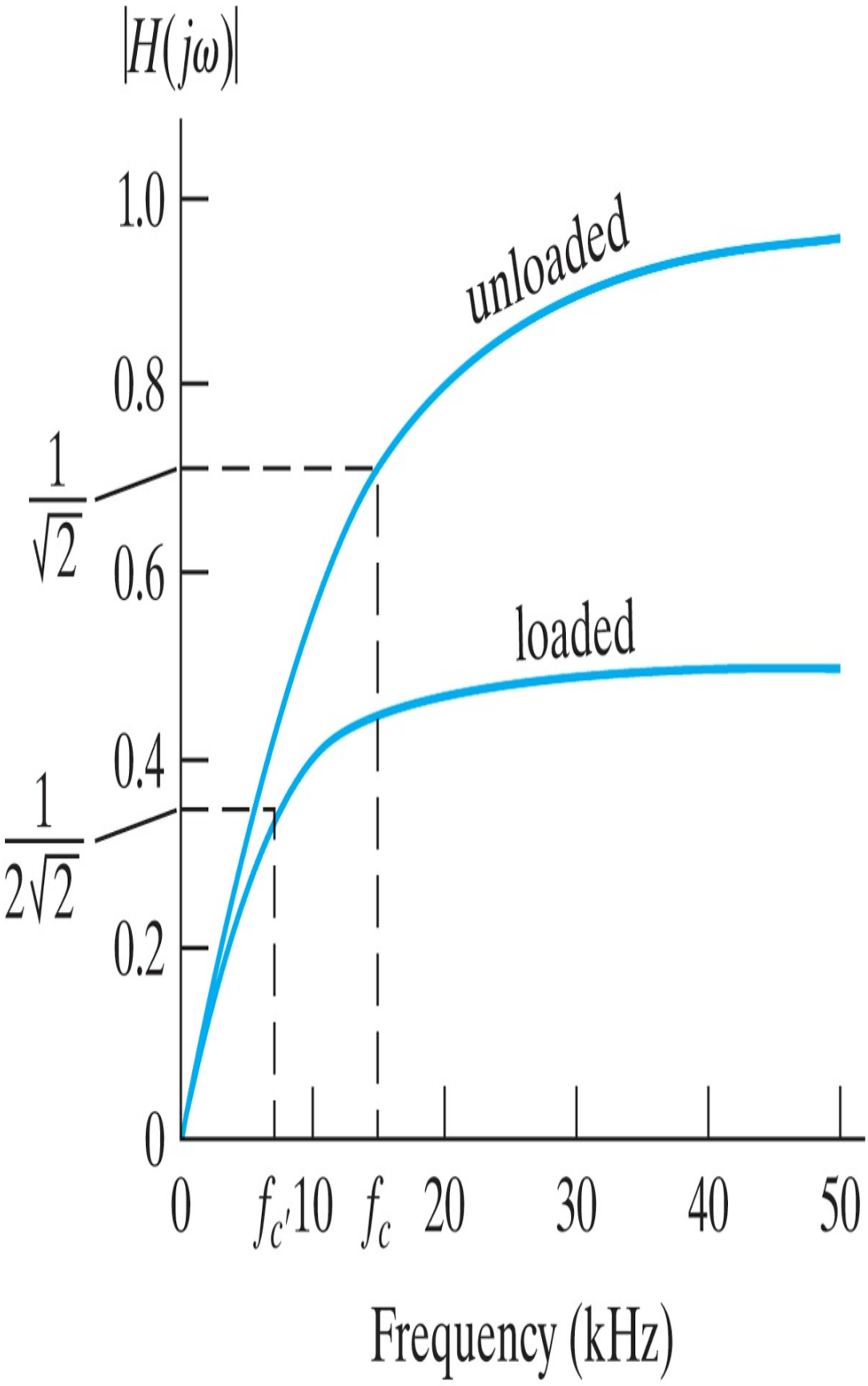

$|H(j\omega)|$
1.0
0.8
$\frac{1}{\sqrt{2}}$
0.6
0.4
$\frac{1}{2\sqrt{2}}$
0.2
0
unloaded
loaded
0
$f_{c'}$
10
f_c
20
30
40
50
Frequency (kHz)

Figure 14.17 Full Alternative Text

Let’s compare the transfer functions of the unloaded filter in Example 14.3 and the loaded filter in Example 14.4. Both transfer functions are in the form:

$$H(s)=Kss+K(R / L),$$

with $K=1$ for the unloaded filter and $K=RL / (R+RL)$ for the loaded filter. Note that the value of K for the loaded circuit reduces to the value of K for the unloaded circuit when $RL \rightarrow \infty$; that is, when there is no load resistor.

The cutoff frequencies for both filters can be seen directly from their transfer functions. In both cases, $\omega c=K(R / L)$, where $K=1$ for the unloaded circuit, and $K=RL / (R+RL)$ for the loaded circuit. Again, the cutoff frequency for the loaded circuit reduces to that of the unloaded circuit when $RL \rightarrow \infty$. Because $RL / (R+RL)< 1$, the effect of the load resistor is to reduce the passband magnitude by the factor K and to lower the cutoff frequency by the same factor.

We predicted these results at the beginning of this chapter. When the output voltage amplitude of a passive high-pass filter is maximum, it equals the amplitude of the filter’s input voltage. Placing a load across the filter, as we did in Example 14.4, decreases the output voltage amplitude. If we need to amplify signals in the passband, we must turn to active filters, such as those discussed in Chapter 15.

The effect of a load on a filter’s transfer function poses another dilemma in circuit design. We typically begin with a transfer function specification and then design a filter to meet that specification. We may or may not know what the load on the filter will be. Ideally, we want the filter’s transfer function to remain the same regardless of the load on it, but this is not possible for the passive filters presented here.

Figure 14.18 summarizes the high-pass filter circuits we have analyzed. Looking at the expressions for H(s), we see that they differ only in the denominator, which includes the cutoff frequency. As we did with the low-pass filters in Eq. 14.6, we state a general form for the transfer function of

these two high-pass filters:

Figure 14.18 Two high-pass filters, the series RC and the series RL, together with their transfer functions and cutoff frequencies.

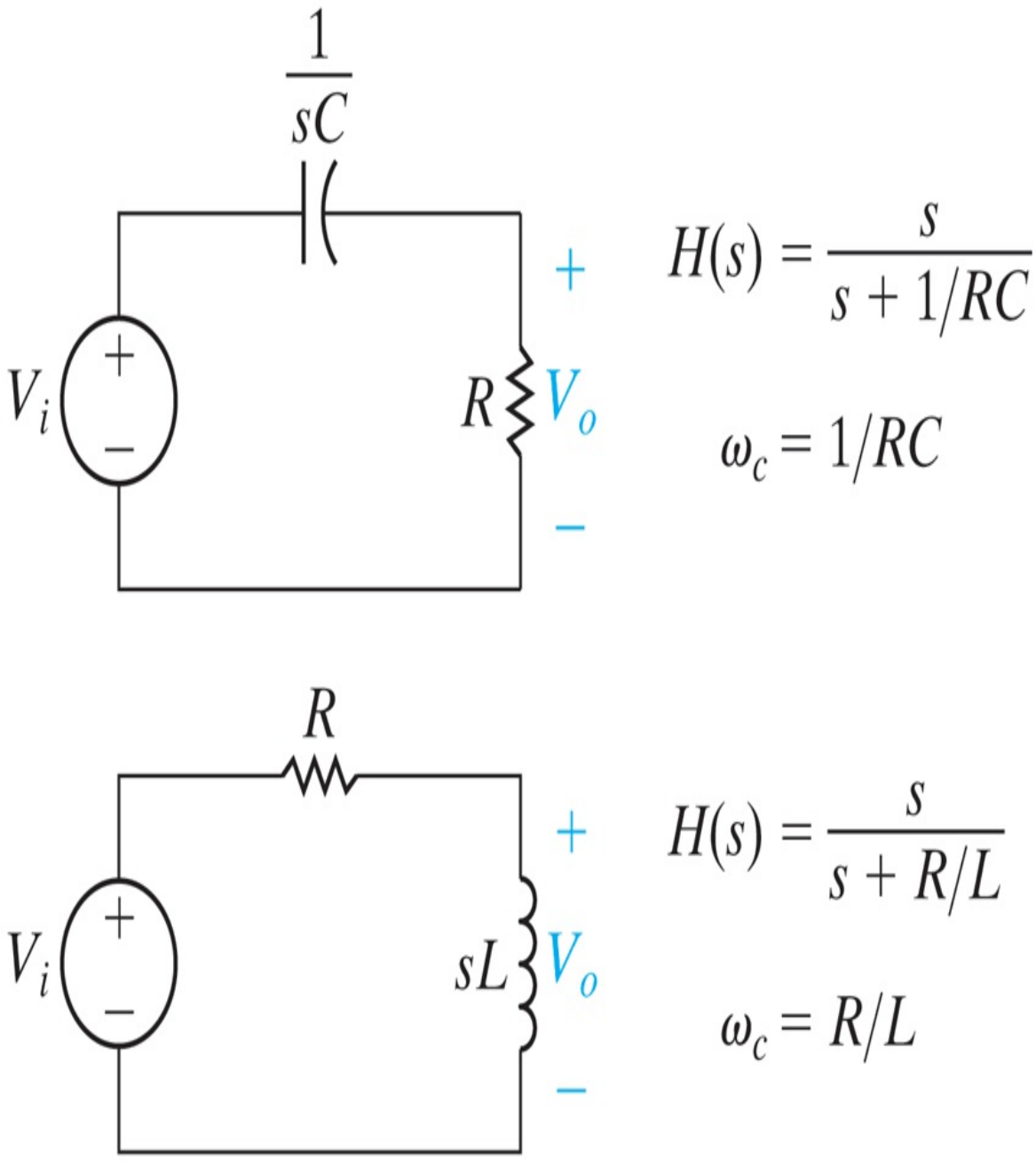


Figure 14.18 Full Alternative Text

Transfer Function for High-Pass Filters

$$H(s)=\frac{s}{s+\omega_c}. \quad (14.10)$$

Any circuit with a voltage ratio described by Eq. 14.10 would behave as a high-pass filter with a cutoff frequency of ω_c. The problems at the end of the chapter give you other examples of circuits with this voltage ratio.

We have drawn attention to another important relationship. We have discovered that a series RC circuit has the same cutoff frequency whether it is configured as a low-pass filter or as a high-pass filter. The same is true of a series RL circuit. Given the connection between the cutoff frequency of a filter circuit and its time constant, we should expect the cutoff frequency to be a characteristic parameter of the circuit whose value depends only on the circuit components, their values, and the way they are connected.

Assessment Problems

Objective 2—Know the RL and RC circuit configurations that act as high-pass filters

1. 14.3 A series RL high-pass filter has $R=5k\Omega$ and $L=3.5mH$. What is ω_c for this filter?

   Answer: $1.43Mrad/s$.

2. 14.4 A series RC high-pass filter has $C=1\mu F$. Compute the cutoff frequency for the following values of R: (a) 100 Ω; (b) 5kΩ; and (c) 30kΩ.

   Answer:

   1. $10krad/s$;
   2. $200rad/s$;
   3. $33.33rad/s$.

3. 14.5 Compute the transfer function of a series RC low-pass filter that

has a load resistor RL in parallel with its capacitor.

Answer: H(s)=1RCs+1KRC, where K=RLR+RL.

SELF-CHECK: *Also try Chapter Problems 14.13 and 14.17.*

14.4 Bandpass Filters

The next filters we examine are those that pass voltages within a band of frequencies to the output while filtering out voltages at frequencies outside this band. These filters are somewhat more complicated than the low-pass and high-pass filters of the previous sections. As we have already seen in Fig. 14.3(a), ideal bandpass filters have two cutoff frequencies, ωc1 and ωc2, which identify the passband. For realistic bandpass filters, these cutoff frequencies are again defined as the frequencies for which the magnitude of the transfer function equals (1 / 2)Hmax.

Center Frequency, Bandwidth, and Quality Factor

Besides the cutoff frequencies ωc1 and ωc2, three other important parameters characterize a bandpass filter. The first is the **center frequency**, ωo, defined as the frequency for which a circuit's transfer function is purely real. Another name for the center frequency is the **resonant frequency**. This is the same name given to the frequency that characterizes the natural response of the second-order circuits in Chapter 8 because they are the same frequencies! When a circuit is driven at the resonant frequency, we say that the circuit is *in resonance* because the frequency of the forcing function is the same as the natural frequency of the circuit. The center frequency is the geometric center of the passband; that is, ωo=ωc1ωc2. For bandpass filters, the magnitude of the transfer function is maximum at the center frequency (Hmax=|H(jωo)|).

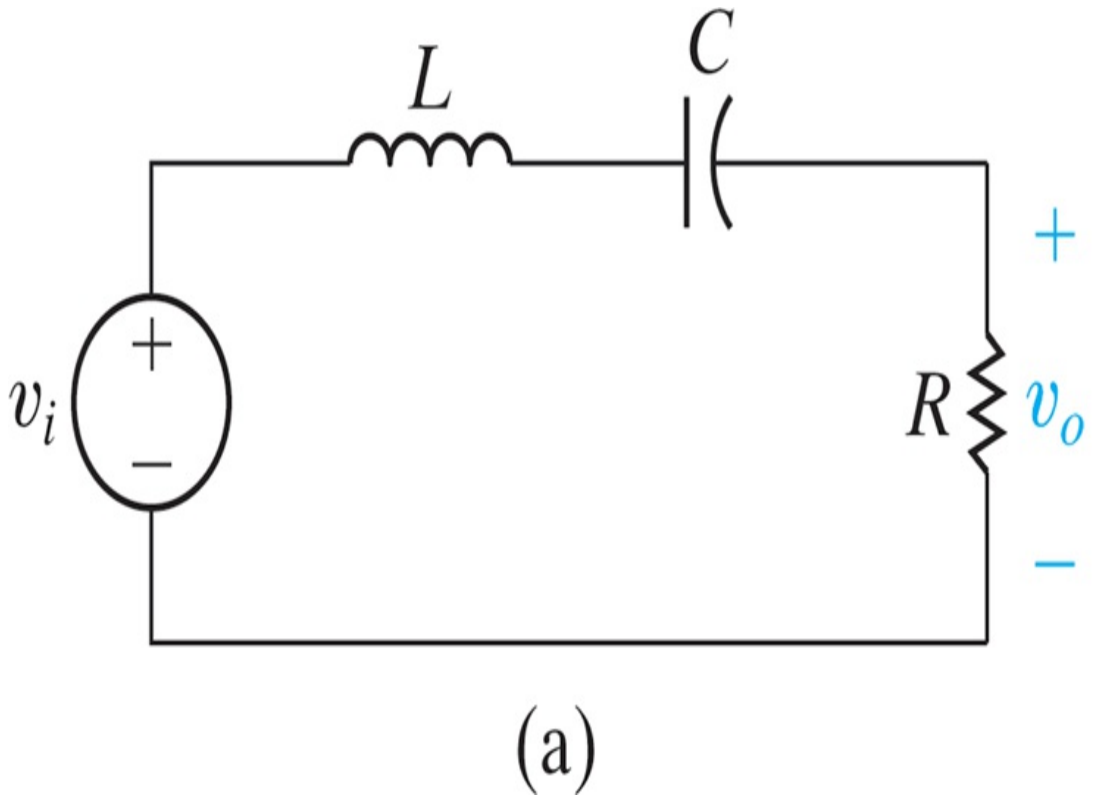
The second parameter is the **bandwidth**, β, which is the width of the passband. The final parameter is the **quality factor Q**, which is the ratio of the center frequency to the bandwidth. The quality factor describes the width of the passband, independent of its location on the frequency axis. It also describes the shape of the magnitude plot, independent of frequency.

Although five different parameters characterize bandpass filters—ωc1, ωc2, ωo, β, and Q—only two of the five can be specified independently. That is, once we specify any two of these parameter values, the other three can be calculated from the dependent relationships among them. We explore these relationships next, as we derive expressions for the five characteristic parameters in terms of the component values for two RLC circuits that act as bandpass filters.

The Series RLC Circuit—Qualitative Analysis

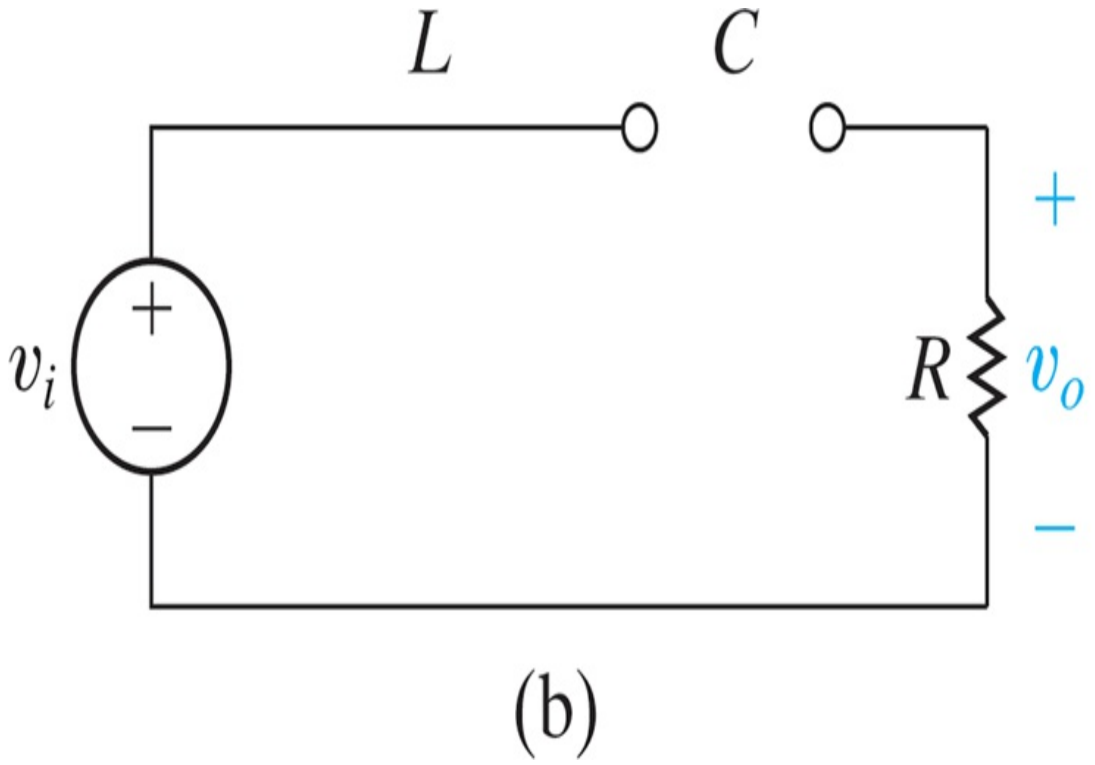
Figure 14.19(a) depicts a series RLC circuit. We want to consider the effect of changing the source frequency on the magnitude of the output voltage. As before, changes to the source frequency result in changes to the impedance of the capacitor and the inductor. This time, the qualitative analysis is somewhat more complicated because the circuit has both an inductor and a capacitor.

Figure 14.19 (a) A series RLC bandpass filter; (b) the equivalent circuit for ω=0;; and (c) the equivalent circuit for ω=∞..



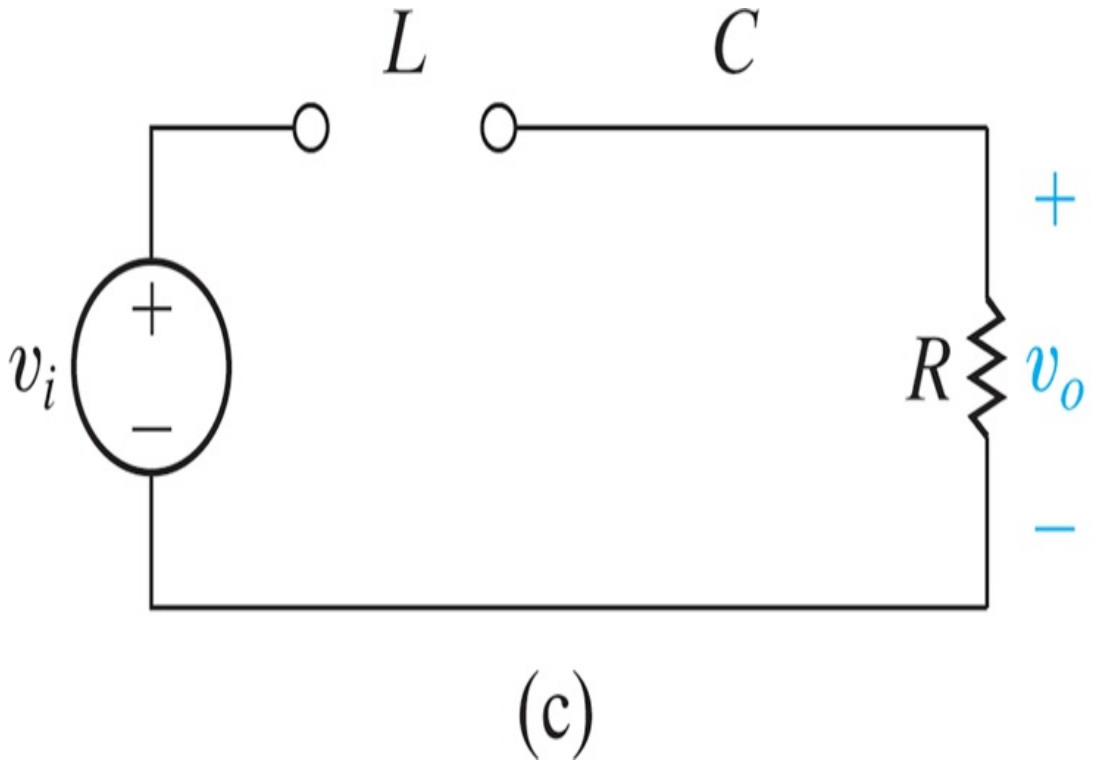
(a)

14.4-5 Full Alternative Text



(b)

14.4-5 Full Alternative Text



14.4-5 Full Alternative Text

At ω=0, the capacitor behaves like an open circuit, and the inductor behaves like a short circuit. The equivalent circuit is shown in Fig. 14.19(b). The open circuit representing the capacitor impedance prevents current from reaching the resistor, and the resulting output voltage is zero.

At $\omega \to \infty$, the capacitor behaves like a short circuit, and the inductor behaves like an open circuit. The equivalent circuit is shown in Fig. 14.19(c). The inductor now prevents current from reaching the resistor, and again the output voltage is zero.

But what happens in the frequency region between ω=0 and ω=∞? Between these two extremes, both the capacitor and the inductor have finite impedances. In this region, the source voltage divides among the capacitor, inductor, and resistor. Remember that the capacitor impedance is negative, whereas the inductor impedance is positive. Thus, at some frequency, the capacitor impedance and the inductor impedance have equal magnitudes and

opposite signs; the two impedances cancel out, so the output voltage equals the source voltage. This special frequency is the center frequency, ωo. On either side of ωo, the output voltage is less than the source voltage. Note that at ωo, the series combination of the inductor and capacitor behaves like a short circuit.

The frequency response plot for the circuit in Fig. 14.19(a) is shown in Fig. 14.20. The ideal bandpass filter magnitude plot is overlaid (as a dashed line) on the transfer function magnitude plot for comparison.

Figure 14.20 The frequency response plot for the series RLC bandpass filter circuit in Fig. 14.19.

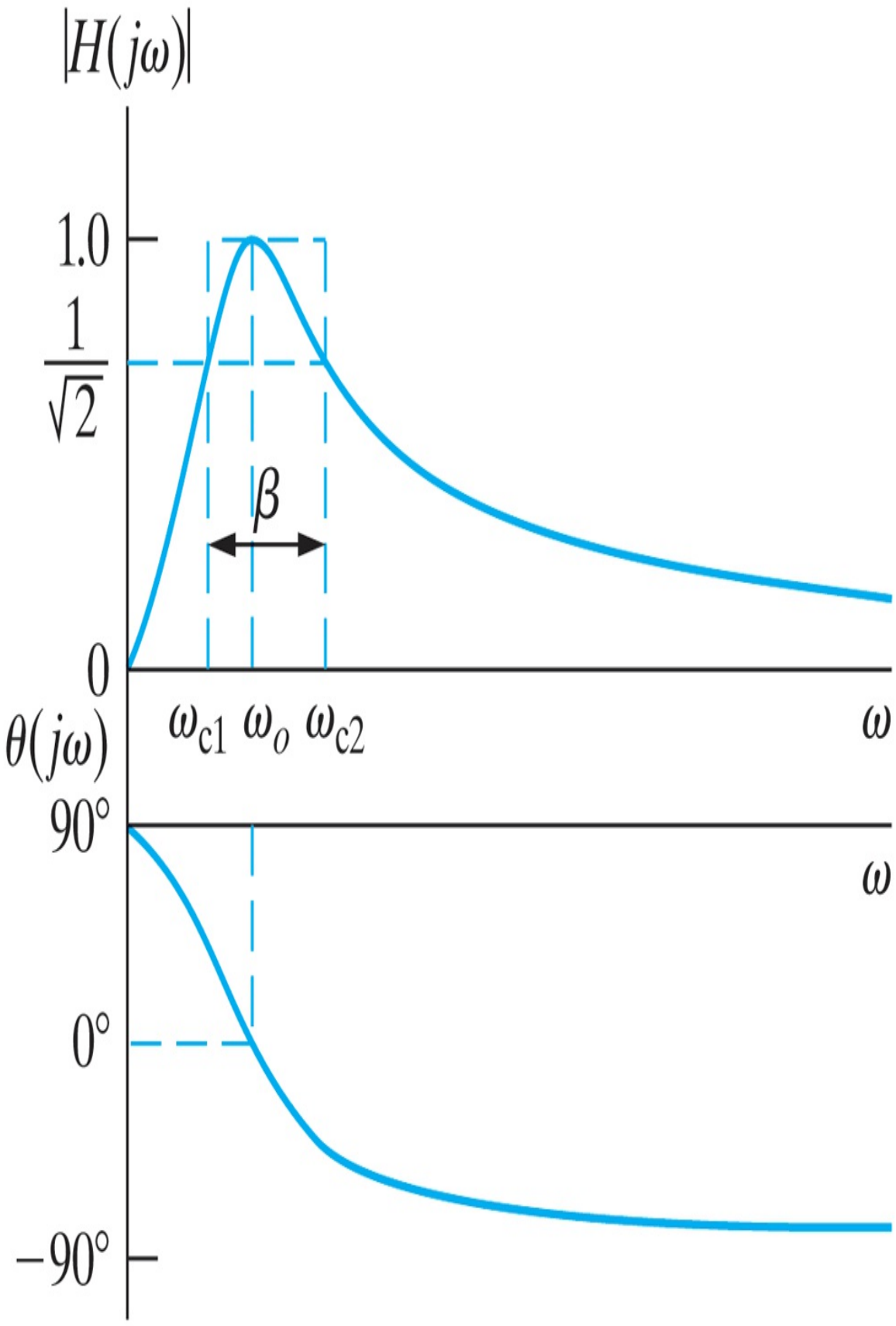

$|H(j\omega)|$
1.0
$\frac{1}{\sqrt{2}}$
β
0
ω_{c1} ω_o ω_{c2}
ω
$\theta(j\omega)$
90°
ω
0°
−90°

Figure 14.20 Full Alternative Text

Now consider what happens to the transfer function phase angle. At the center frequency, ωo, the phase angles of the input and output voltages are equal, so the phase angle of the transfer function is zero. As the frequency decreases, the capacitor phase angle is larger than the inductor phase angle. Because the capacitor contributes positive phase shift, the transfer function phase angle is positive. At very low frequencies, the transfer function phase angle is +90°.

Conversely, if the frequency increases from the center frequency, the inductor phase angle is larger than the capacitor phase angle. The inductor contributes negative phase shift, so the transfer function phase angle is negative. At very high frequencies, the transfer function phase angle is−90°. The plot of the transfer function phase angle is shown in Fig. 14.20.

The Series RLC Circuit—Quantitative Analysis

We begin by drawing the s-domain equivalent for the series RLC circuit, as shown in Fig. 14.21. Using voltage division, we find that the transfer function equation is

Figure 14.21 The s-domain equivalent for the circuit in Fig. 14.19(a).

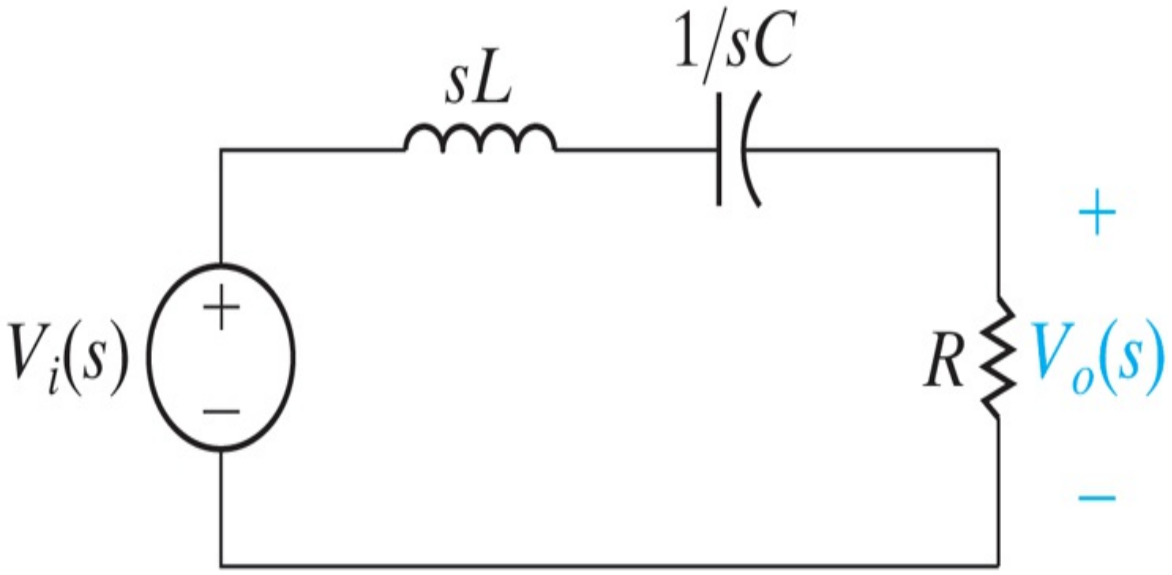


Figure 14.21 Full Alternative Text

H(s)=(R/L)ss2+(R/L)s+(1/LC). (14.11)

As before, we substitute s=jω into Eq. 14.11 and produce the equations for the magnitude and the phase angle of the transfer function:

|H(jω)| =ω(R/L)[(1/LC)−ω2]2+[ω(R/L)]2, (14.12)
θ(jω)=90° −tan−1[ω(R/L)(1/LC)−ω2].

Now calculate the five parameters that characterize this RLC bandpass filter. Recall that the center frequency, ωo, is defined as the frequency for which the circuit's transfer function is purely real. The transfer function for the RLC circuit in Fig. 14.21 will be real when the capacitor and inductor impedances sum to zero:

jωoL+1jωoC=0.

Solving for ωo, we get

Center Frequency

ωo=1LC.(14.13)

Next, calculate the cutoff frequencies, ωc1 and ωc2. Remember that at the cutoff frequencies, the magnitude of the transfer function is (1 / 2)Hmax. Because Hmax=|H(jωo)|, we find Hmax by substituting Eq. 14.13 into Eq. 14.12:

Hmax=|H(jωo)|=ωo(R / L)[(1 / LC)−ωo2]2 + (ωoR / L)2=(1 / LC)(R / L)
[(1 / LC)−(1 / LC)]2 + [(1 / LC)(R / L)]2=1.

Now set the left-hand side of Eq. 14.12 to (1 / 2)Hmax (which equals 1 / 2) and prepare to solve for ωc:

12=ωc(R / L)[(1 / LC)−ωc2]2 + (ωcR / L)2=1[(ωcL/R)−(1/ωcRC)]2+1.

We can equate the denominators of the two sides of this expression and simplify to get

±1=ωcLR−1ωcRC.

Rearrange to get the following quadratic equation:

ωc2±RLωc−1LC=0.

The solution of the quadratic equation yields four values for the cutoff frequency. Only two of these values are positive and have physical significance; they identify the passband of this filter:

Cutoff Frequencies, Series RLC Filters

ωc1=−R2L+(R2L)2+(1LC),(14.14)
ωc2=R2L + (R2L)2+ (1LC).(14.15)

We can use Eqs. 14.14 and 14.15 to confirm that the center frequency, ωo, is the geometric mean of the two cutoff frequencies:

Relationship Between Center Frequency and Cutoff Frequencies

$$\omega_o=\omega_{c1}\cdot\omega_{c2}$$
$$=[-R2L+(R2L)2+(1LC)][R2L+(R2L)2+(1LC)]=1LC.$$

Recall that the bandwidth of a bandpass filter is defined as the difference between the two cutoff frequencies. Because $\omega_{c2}>\omega_{c1}$ we can compute the bandwidth by subtracting Eq. 14.14 from Eq. 14.15:

Relationship Between Bandwidth and Cutoff Frequencies

$$\beta=\omega_{c2}-\omega_{c1}$$
$$=[R2L+(R2L)2+(1LC)]-[-R2L+(R2L)2+(1LC)],$$

so

Bandwidth, Series RLC Filters

$$\beta=RL. \quad (14.16)$$

The quality factor, the last of the five characteristic parameters, is defined as the ratio of center frequency to bandwidth. Using Eqs. 14.13 and 14.16:

Relationship Among Quality Factor, Center Frequency and Bandwidth

$$Q=\omega_o\beta$$

=1 / LC(R / L),

so

Quality Factor, Series RLC Filters

Q=LR2C.(14.17)

We now have five parameters that characterize the series RLC bandpass filter: two cutoff frequencies, ωc1 and ωc2, which delimit the passband; the center frequency, ωo, at which the magnitude of the transfer function is maximum; the bandwidth, β, a measure of the width of the passband; and the quality factor, Q, a second measure of passband width. Remember, only two of these parameters can be specified independently in a design. We have already observed that the quality factor is the ratio of the center frequency to the bandwidth. We can also rewrite the equations for the cutoff frequencies in terms of the center frequency and the bandwidth:

ωc1=−β2+(β2)2+ωo2,ωc2=β2+(β2)2+ωo2.

Alternative forms for these equations express the cutoff frequencies in terms of the quality factor and the center frequency (see Problem 14.18):

ωc1=ωo·[−12Q+1+(12Q)2],ωc2=ωo·[12Q+1+(12Q)2].

Examples 14.5, 14.6, and 14.7 illustrate the design of bandpass filters, introduce another RLC circuit that behaves as a bandpass filter, and examine the effects of source resistance on the characteristic parameters of a series RLC bandpass filter.

Example 14.5 Designing a Bandpass Filter

A graphic equalizer is an audio amplifier that allows you to select different

levels of amplification within different frequency regions. Using the series RLC circuit in Fig. 14.19(a), choose values for R, L, and C that yield a bandpass circuit able to select inputs within the 1 kHz–10 kHz frequency band. Such a circuit might be used in a graphic equalizer to select this frequency band from the larger audio band (generally 0–20 kHz) prior to amplification.

Solution

We need to compute values for R, L, and C that produce a bandpass filter with cutoff frequencies of 1 kHz and 10 kHz. There are many possible approaches to a solution. For instance, we could use Eqs. 14.14 and 14.15, which specify $\omega c1$ and $\omega c2$ in terms of R, L, and C. Because of the form of these equations, the algebraic manipulations might get complicated. Instead, we will calculate the center frequency, ωo, from the cutoff frequencies and then use Eq. 14.13 to compute L and C from ωo. Next we will calculate the bandwidth, β, from the cutoff frequencies and finally, use Eq. 14.16 to compute R from β. While this approach involves several computational steps, each calculation is fairly simple.

Any approach we choose will provide only two equations—insufficient to solve for the three unknowns—because of the dependencies among the bandpass filter characteristics. Thus, we need to select a value for either R, L, or C and use the two equations we've chosen to calculate the remaining component values. Here, we arbitrarily choose 1μF as the capacitor value.

We compute the center frequency as the geometric mean of the cutoff frequencies:

$$fo=fc1fc2=(1000)(10{,}000)=3162.28\text{Hz}.$$

Next, use Eq. 14.13 to find L using C and the center frequency, which must be converted to radians/sec:

$$L=1\omega o2C=1[\ 2\pi(3162.28)\ ]2(10{-}6)=2.533\ \text{mH}.$$

The bandwidth is the difference between the two cutoff frequency values, so

$$\beta = \omega_{c2} - \omega_{c1} = 10{,}000 - 1000 = 9 \text{ kHz}.$$

Now convert the bandwidth to radians/sec and use Eq. 14.16 to calculate R:

$$R = \beta L = [2\pi(9000)](2.533 \times 10^{-3}) = 143.24\ \Omega.$$

To check whether these component values produce the bandpass filter we want, substitute them into Eqs. 14.14 and 14.15. We find that

$$\omega_{c1} = 6283.19 \text{rad/s (1000Hz)}, \omega_{c2} = 62{,}831.85 \text{rad/s (10,000 Hz)},$$

which are the cutoff frequencies specified for the filter.

This example reminds us that only two of the five bandpass filter parameters can be specified independently. The other three parameters can be computed from the two that are specified. In turn, these five parameter values depend on the three component values, R, L, and C, of which only two can be specified independently. It is almost always easiest to calculate the center frequency and bandwidth from whatever two parameters are specified, and then use Eqs. 14.13 and 14.16 to calculate the two unknown component values.

Example 14.6 Designing a Parallel RLC Bandpass Filter

1. Show that the RLC circuit in Fig. 14.22 is also a bandpass filter by deriving an expression for the transfer function H(s). Note that this circuit is a parallel RLC circuit with the parallel-connected current source and resistor source-transformed to a series-connected voltage source and resistor. This permits us to continue defining the filter transfer functions as ratios of output to input voltages.

Figure 14.22 The circuit for Example 14.6.

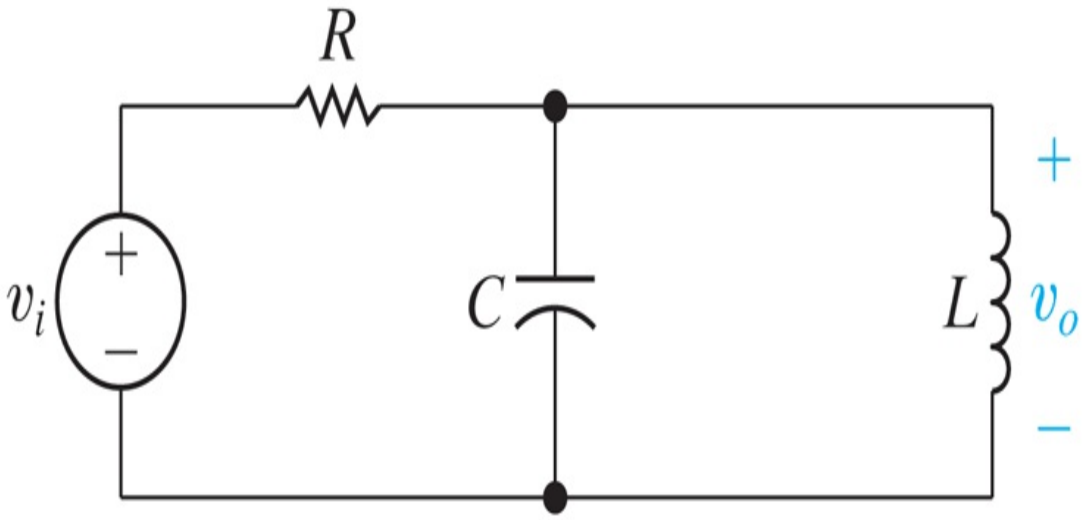


Figure 14.22 Full Alternative Text

1. Compute the center frequency, ωo.
2. Calculate the cutoff frequencies, ωc1 and ωc2, the bandwidth, β, and the quality factor, Q.
3. Compute values for R and L to yield a bandpass filter with a center frequency of 5 kHz and a bandwidth of 200 Hz, using a 5μF capacitor.

Solution

1. Begin by transforming the circuit in Fig. 14.22 to the s domain; the result is shown in Fig. 14.23. We can find the transfer function for the s-domain circuit using voltage division if we first compute the equivalent impedance of the parallel combination of L and C, identified as Zeq(s) in Fig. 14.23:

Figure 14.23 The s-domain equivalent of the circuit in Fig. 14.22.

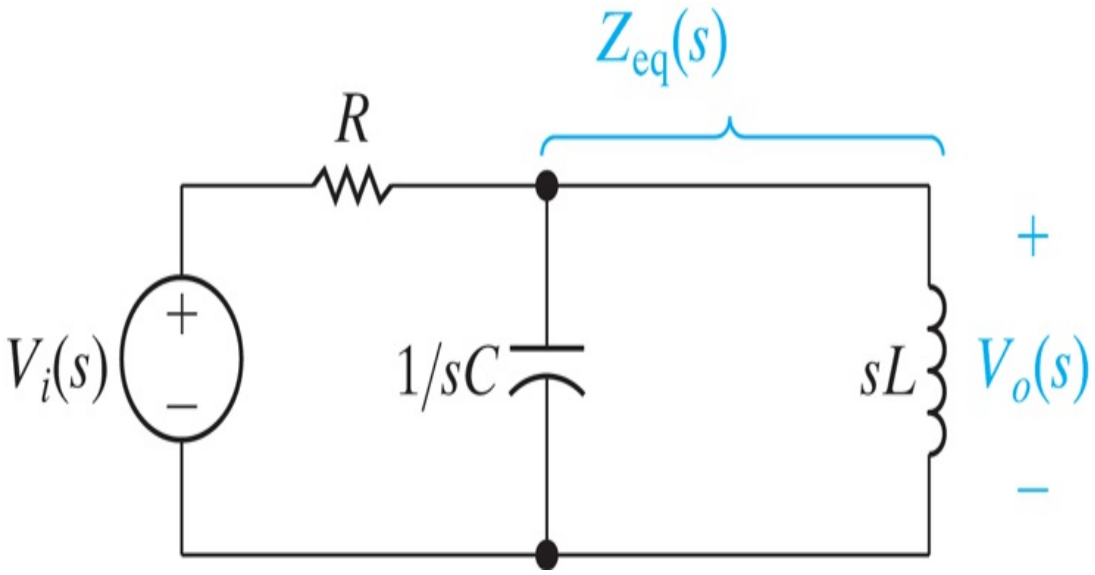


Figure 14.23 Full Alternative Text

Zeq=sL||1sC=(sL)(1/sC)sL+(1/sC)=sLs2LC+1.

Now,

H(s)=Vo(s)Vi(s)=ZeqR+Zeq=sRCs2+sRC+1LC.

2. To find the center frequency, we need to calculate the frequency for which the transfer function magnitude is maximum. Substituting s=jω in H(s),

   |H(jω)|=ωRC(1LC−ω2)2+ (ωRC)2=11+(ωRC−1ωLR)2.

   The magnitude of this transfer function is maximum when

   (1LC−ω2)2=0.

Thus,

$\omega o = 1LC$

and

$Hmax = |H(j\omega o)| = 1.$

3. At the cutoff frequencies, the magnitude of the transfer function is $(1 / 2)Hmax = 1 / 2$. Substituting this constant on the left-hand side of the magnitude equation and simplifying, we get

   $[\omega cRC - 1\omega cLR] = \pm 1.$

   Rearranging this equation once again produces two quadratic equations for the cutoff frequencies, with four solutions. Only two of them are positive and therefore have physical significance:

Cutoff Frequencies, Parallel RLC Filters

$\omega c1 = -12RC + (12RC)2 + 1LC, \omega c2 = 12RC + (12RC)2 + 1LC.$

We compute the bandwidth from the cutoff frequencies:

$\beta = \omega c2 - \omega c1$

Bandwidth, Parallel RLC Filters

$\beta = \omega c2 - \omega c1 = 1RC.$

Finally, use the definition of quality factor to calculate Q:

Quality Factor, Parallel RLC Filters

$Q=\omega_o\beta=R2CL$.

Notice that once again we can specify the cutoff frequencies for this bandpass filter in terms of its center frequency and bandwidth:

$\omega_{c1}=-\beta 2+(\beta 2)2+\omega_o 2$,
$\omega_{c2}=\beta 2+(\beta 2)2+\omega_o 2$.

4. Use the equation for bandwidth in (c) to compute a value for R, given $C=5\mu F$. Remember to convert the bandwidth to radians/sec:

   $R=1\beta C=1(2\pi)(200)(5\times 10-6)=159.15\ \Omega$.

   Using the value of capacitance and the equation for center frequency in (b), compute the inductor value:

   $L=1\omega_o 2C=1[2\pi(5000)]2(5\times 10-6)=202.64\ \mu H$.

Example 14.7 Determining Effect of a Nonideal Voltage Source on a RLC Bandpass Filter

For each of the bandpass filters we have constructed, we have always assumed an ideal voltage source, that is, a voltage source with no series resistance. When we design with a filter using values of R, L, and C whose equivalent impedance has a magnitude close to the actual impedance of the voltage source, it is not valid to assume the voltage source is ideal. In this example, we determine the effect of the nonzero source resistance Ri on the series RLC bandpass filter characteristics.

1. Determine the transfer function for the circuit in Fig. 14.24.

Figure 14.24 The circuit for Example 14.7.

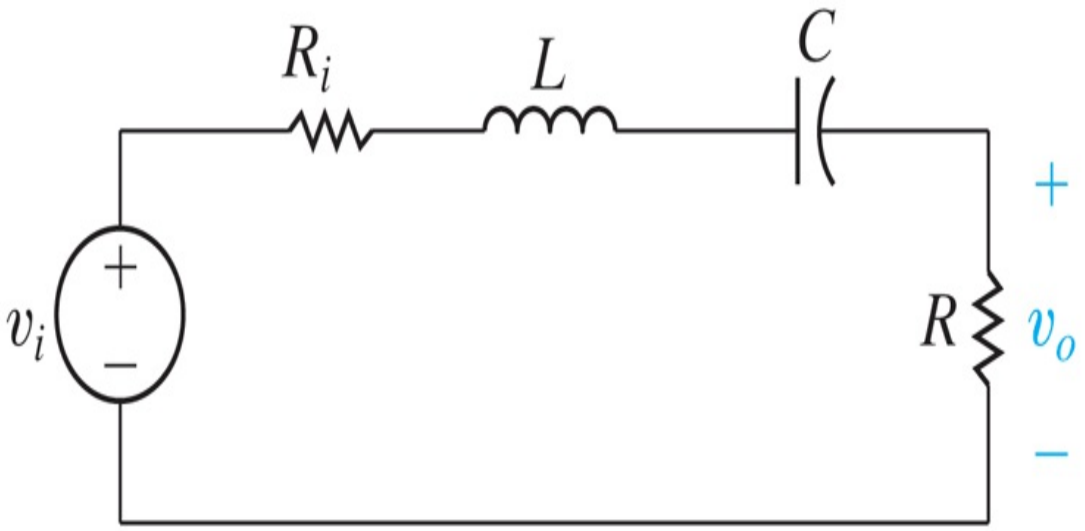


Figure 14.24 Full Alternative Text

2. Sketch the magnitude plot for the circuit in Fig. 14.24, using the values for R, L, and C from Example 14.5 and setting Ri=R. On the same graph, sketch the magnitude plot for the circuit in Example 14.5, where Ri=0.

Solution

1. Begin by transforming the circuit in Fig. 14.24 into the s domain, as shown in Fig. 14.25. Now construct the transfer function using voltage division:

Figure 14.25 The s-domain

equivalent of the circuit in [Fig. 14.24](#).

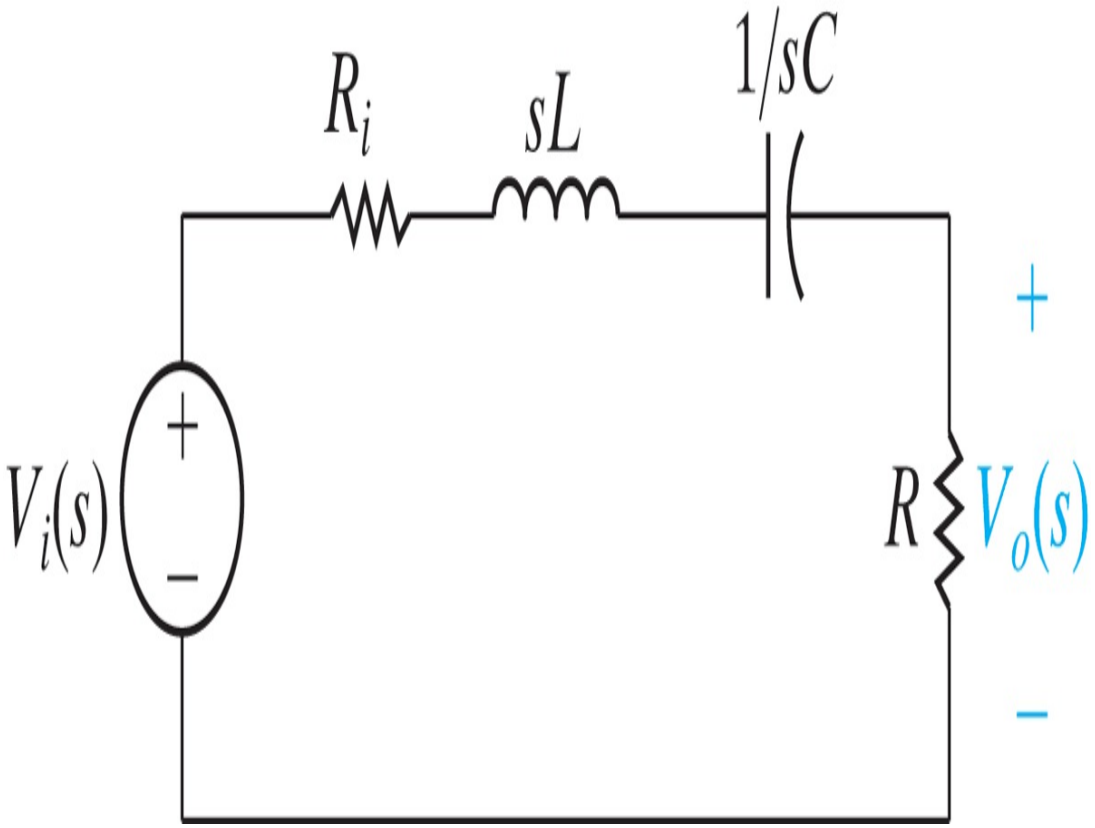


Figure 14.25 Full Alternative Text

H(s)=RLss2+(R+RiL)s+1LC.

Substitute s=jω and calculate the transfer function magnitude:

|H(jω)|=RLω(1LC−ω2)2+ (ωR+RiL)2.

The center frequency, ωo, is the frequency at which this transfer function magnitude is maximum, which is

ωo=1LC.

At the center frequency, the maximum magnitude is

$$H_{max} = |H(j\omega_o)| = \frac{R}{R_i + R}.$$

The cutoff frequencies can be computed by setting the transfer function magnitude equal to $(1 / 2)H_{max}$:

$$\omega_{c1} = -\frac{R + R_i}{2L} + \sqrt{\left(\frac{R + R_i}{2L}\right)^2 + \frac{1}{LC}}, \omega_{c2} = \frac{R + R_i}{2L} + \sqrt{\left(\frac{R + R_i}{2L}\right)^2 + \frac{1}{LC}}.$$

The bandwidth is calculated from the cutoff frequencies:

$$\beta = \omega_{c2} - \omega_{c1} = \frac{R + R_i}{L}.$$

Finally, the quality factor is computed from the center frequency and the bandwidth:

$$Q = \frac{\omega_o}{\beta} = \frac{1/\sqrt{LC}}{R + R_i}.$$

From this analysis, note that we can write the transfer function of the series RLC bandpass filter with nonzero source resistance as

$$H(s) = K\frac{\beta s}{s^2 + \beta s + \omega_o^2},$$

where

$$K = \frac{R}{R + R_i}.$$

Note that when $R_i = 0$, $K = 1$ and the transfer function is

$$H(s) = \frac{\beta s}{s^2 + \beta s + \omega_o^2}.$$

2. The circuit in Example 14.5 has a center frequency of 3162.28 Hz and a bandwidth of 9 kHz, and $H_{max} = 1$. If we use the same values for R, L, and C in the circuit in Fig. 14.24 and let $R_i = R$, then the center frequency remains at 3162.28 kHz, but $\beta = (R + R_i) / L = 18$ kHz, and $H_{max} = R / (R + R_i) = 1 / 2$. The transfer function magnitudes for these two bandpass filters are plotted on the same graph in Fig. 14.26.

Figure 14.26 The magnitude

plots for a series RLC bandpass filter with a zero source resistance and a nonzero source resistance.

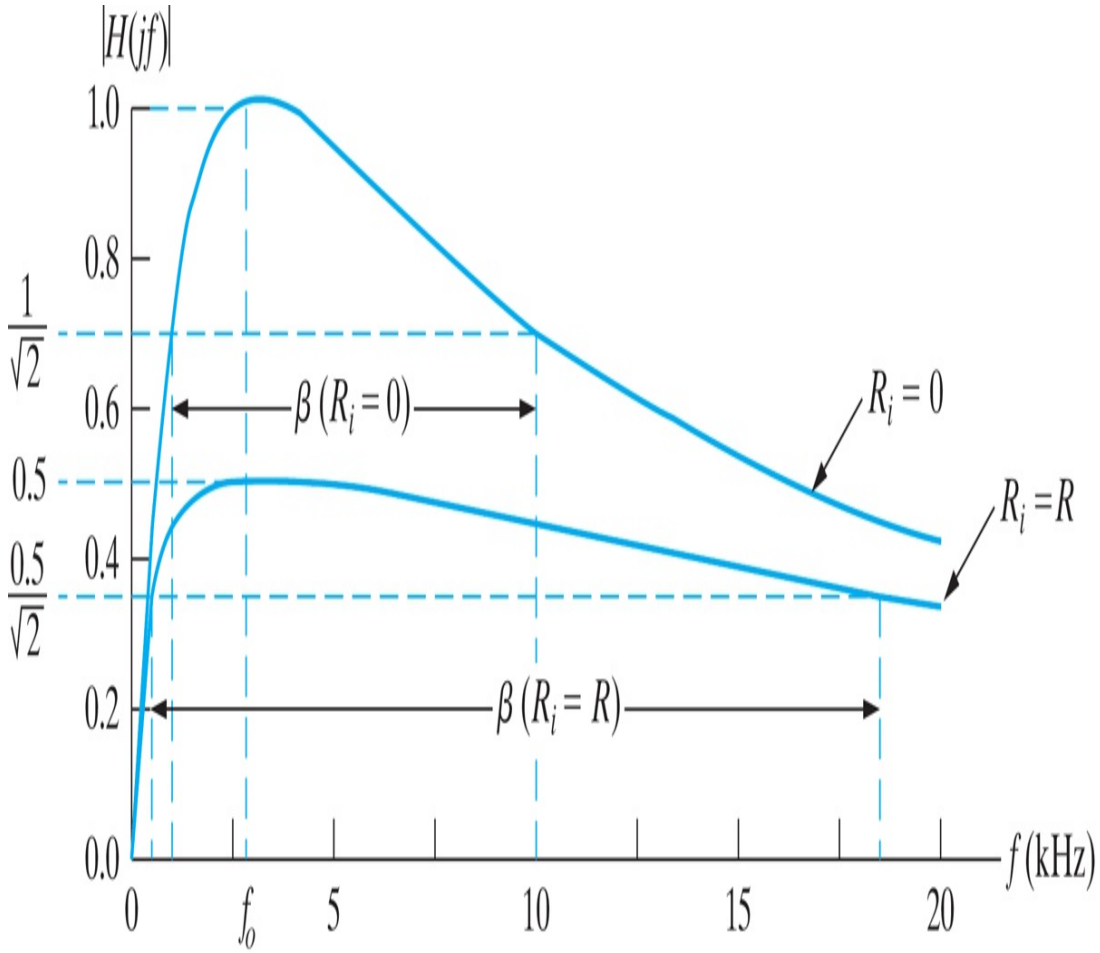


Figure 14.26 Full Alternative Text

If we compare the characteristic parameter values for the filter with Ri=0 to the values for the filter with Ri≠0, we see the following:

- The center frequencies are the same.

- The maximum transfer function magnitude for the filter with $R_i \neq 0$ is smaller than that for the filter with $R_i = 0$.

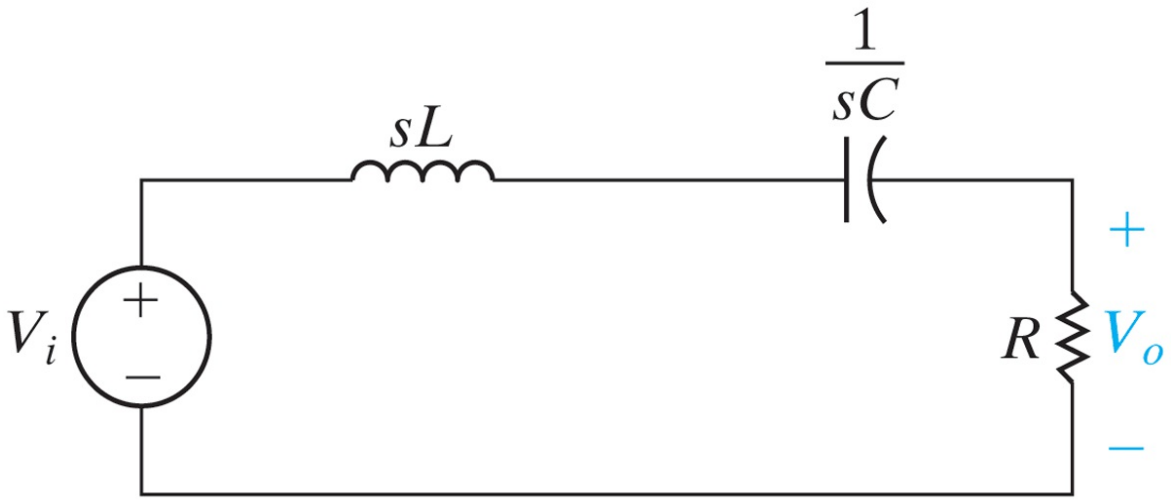
- The bandwidth for the filter with $R_i \neq 0$ is larger than that for the filter with $R_i = 0$. Thus, the cutoff frequencies and the quality factors for the two circuits are also different.

Adding a nonzero source resistance to a series RLC bandpass filter leaves the center frequency unchanged but widens the passband and reduces the passband magnitude.

Here we see the same design challenge we saw when adding a load resistor to the high-pass filter. We would like to design bandpass filters with filtering properties that are unchanged by the internal resistance of the voltage source. Unfortunately, this is not possible for filters constructed from passive elements. In Chapter 15, we will discover that active filters are insensitive to changes in source resistance and thus are better suited to designs in which this is an important issue.

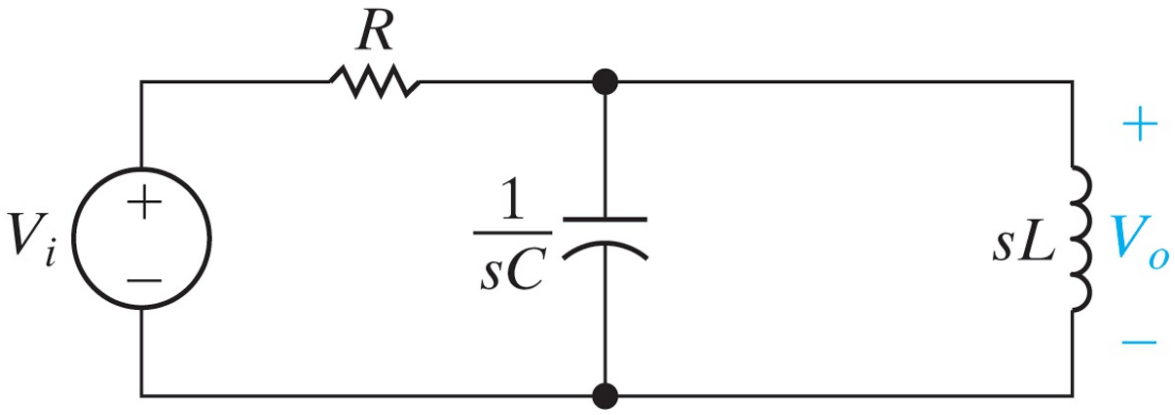
Figure 14.27 summarizes the two RLC bandpass filters we have studied. Note that the expressions for the circuit transfer functions have the same form. As we have done previously, we can create a general form for the transfer functions of these two bandpass filters:

Figure 14.27 Two RLC bandpass filters, together with equations for the transfer function, center frequency, and bandwidth of each.



$$H(s) = \frac{(R/L)s}{s^2 \quad (R/L)s + 1/LC}$$

$$\omega_o = \sqrt{1/LC} \qquad \beta = R/L$$



$$H(s) = \frac{s/RC}{s^2 + s/RC + 1/LC}$$

$$\omega_o = \sqrt{1/LC} \qquad \beta = 1/RC$$

Figure 14.27 Full Alternative Text

Transfer Function for Bandpass Filters

$$H(s)=\frac{\beta s}{s^2+\beta s+\omega_o^2}. \quad (14.18)$$

Any circuit with the transfer function in Eq. 14.18 acts as a bandpass filter with a center frequency ω_o and a bandwidth β.

In Example 14.7, we saw that the transfer function can also be written in the form

$$H(s)=\frac{K\beta s}{s^2+\beta s+\omega_o^2},$$

where the values for K and β depend on whether the series resistance of the voltage source is zero or nonzero.

Relating the Frequency Domain to the Time Domain

It should come as no surprise that the parameters characterizing the frequency response of RLC bandpass filters and the parameters characterizing the time response of RLC circuits are related. Consider the series RLC circuit in Fig. 14.19(a). In Chapter 8 we discovered that the natural response of this circuit is characterized by the neper frequency (α) and the resonant frequency (ω_o). These parameters were expressed in terms of the circuit components in Eqs. 8.29 and 8.30, which are repeated here for convenience:

$$\alpha=\frac{R}{2L}, \quad \omega_o=\frac{1}{\sqrt{LC}}.$$

We see that the same parameter ω_o is used to characterize both the time

response and the frequency response. That's why the center frequency is also called the resonant frequency. The bandwidth and the neper frequency are related by the equation

$\beta = 2\alpha$. (14.19)

Recall that the natural response of a series RLC circuit may be underdamped, overdamped, or critically damped. The transition from overdamped to underdamped occurs when $\omega_o^2 = \alpha^2$. Consider the relationship between α and β from Eq. 14.19 and the definition of the quality factor Q. The transition from an overdamped to an underdamped response occurs when $Q = 1/2$. Thus, a circuit whose frequency response contains a sharp peak at ω_o, indicating a high Q and a narrow bandwidth, will have an underdamped natural response. Conversely, a circuit whose frequency response has a broad bandwidth and a low Q will have an overdamped natural response.

Assessment Problems

Objective 3—Know the RLC circuit configurations that act as bandpass filters

1. 14.6 Using the circuit in Fig. 14.19(a), compute the values of R and L to give a bandpass filter with a center frequency of 12 kHz and a quality factor of 6. Use a 0.1 μF capacitor.

   Answer: L=1.76mH, R=22.10 Ω.

2. 14.7 Using the circuit in Fig. 14.22, compute the values of L and C to give a bandpass filter with a center frequency of 2 kHz and a bandwidth of 500 Hz. Use a 250 Ω resistor.

   Answer: L=4.97 mH, C=1.27 μF.

3. 14.8 Recalculate the component values for the circuit in Example 14.6(d) so that the frequency response of the resulting circuit is unchanged using a 0.2μF capacitor.

Answer: L=5.07 mH, R=3.98kΩ.

4. 14.9 Recalculate the component values for the circuit in Example 14.6(d) so that the quality factor of the resulting circuit is unchanged but the center frequency has been moved to 2 kHz. Use a 0.2μF capacitor.

   Answer: R=9.95kΩ, L=31.66mH.

SELF-CHECK: *Also try Chapter Problems 14.26 and 14.30*.

14.5 Bandreject Filters

We turn now to the last of the four filter categories—the bandreject filter. This filter passes source voltages outside the band between the two cutoff frequencies to the output (the passband) and attenuates source voltages before they reach the output at frequencies between the two cutoff frequencies (the stopband). Bandpass filters and bandreject filters thus perform complementary functions in the frequency domain.

Bandreject filters and bandpass filters have the same characteristic parameters: the two cutoff frequencies, the center frequency, the bandwidth, and the quality factor. Again, only two of these five parameters can be specified independently.

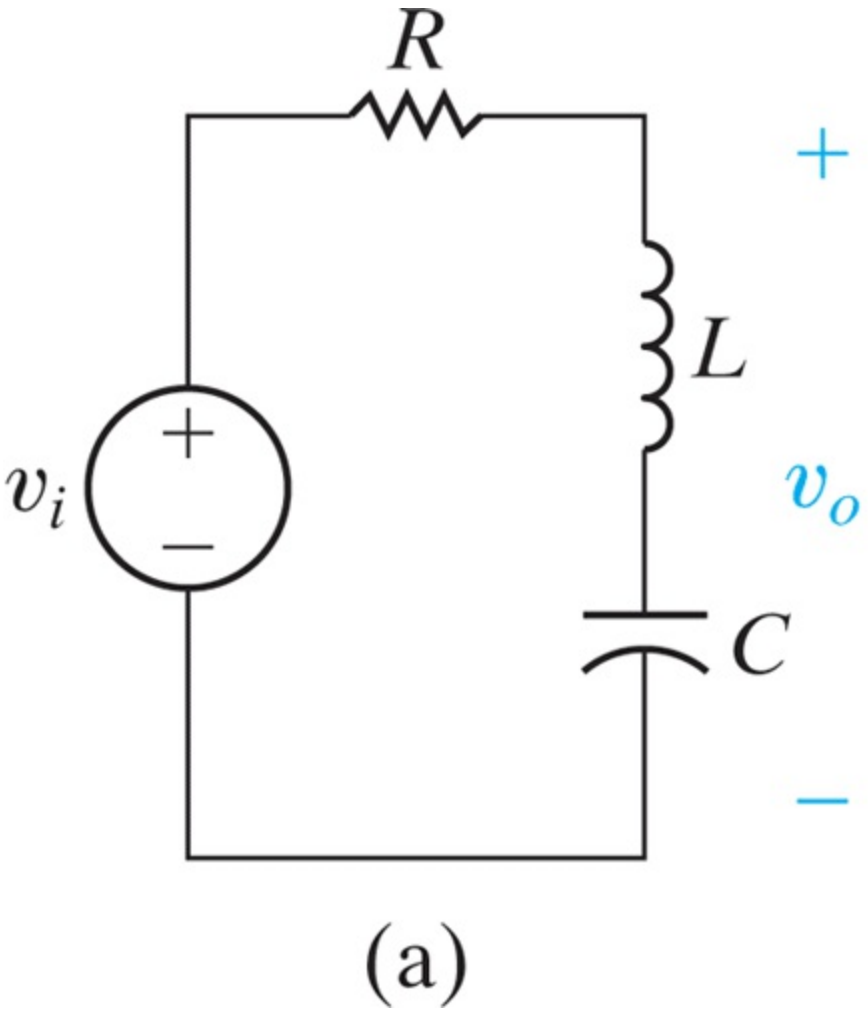
We examine two circuits that function as bandreject filters and then derive equations relating the circuit component values to the characteristic parameters for each circuit.

The Series RLC Circuit—Qualitative Analysis

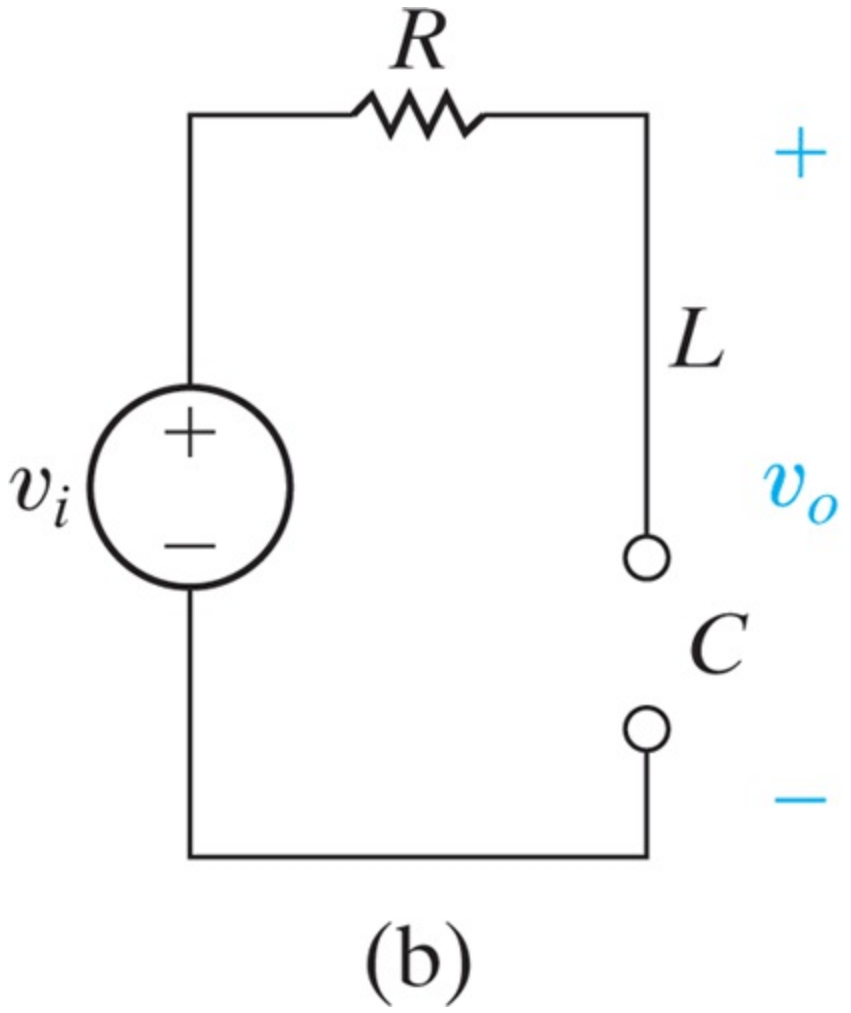
Figure 14.28(a) shows a series RLC circuit. Although the circuit components and connections are identical to those in the series RLC bandpass filter in Fig. 14.19(a), the output voltage is now defined across the inductor- capacitor pair. As we saw in the case of low- and high-pass filters, the same circuit may perform two different filtering functions, depending on the definition of the output voltage.

Figure 14.28 (a) A series RLC bandreject filter. (b) The

equivalent circuit for . (c) The equivalent circuit for .



14.5-5 Full Alternative Text



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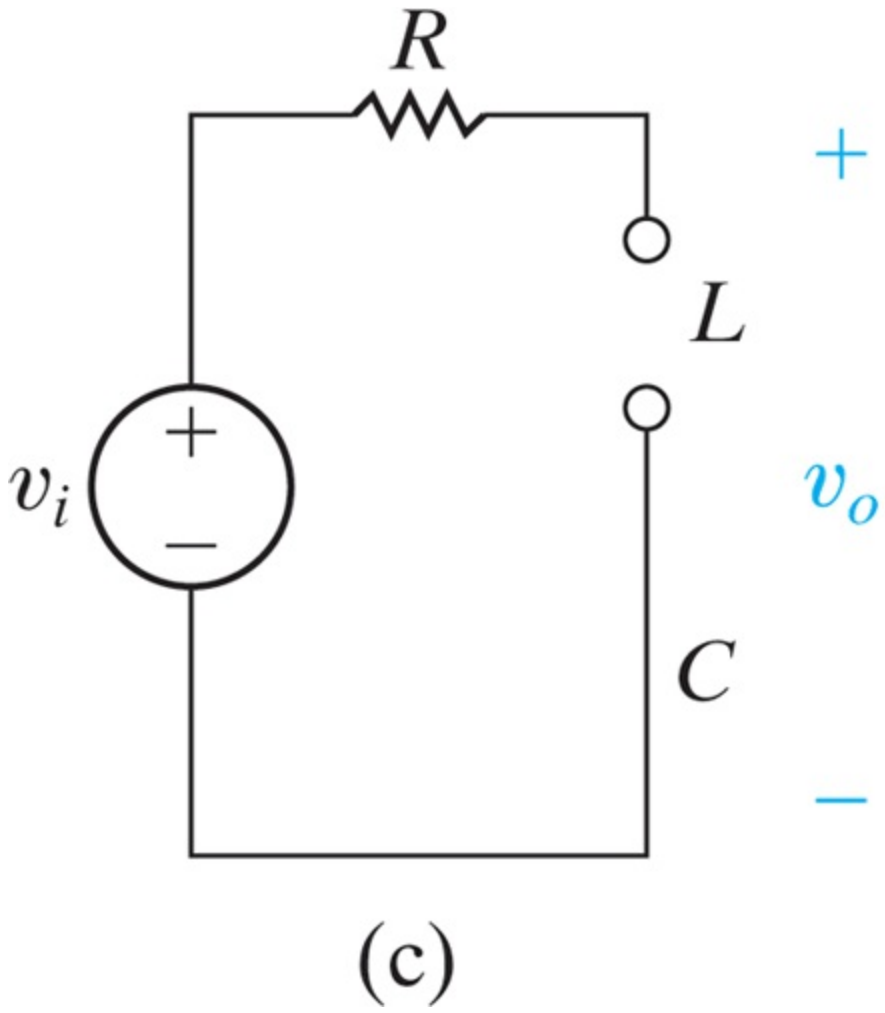


14.5-5 Full Alternative Text

We have already noted that at ω=0, the inductor behaves like a short circuit and the capacitor behaves like an open circuit, as shown in Fig, 14.28(b). At ω=∞, these roles switch, as shown in Fig. 14.28(c). In both equivalent circuits, the output voltage is defined across an open circuit, so the output and input voltages have the same magnitude. This series RLC bandreject filter circuit then has two passbands—one below the lower cutoff frequency and the other above the upper cutoff frequency.

Between these two passbands, both the inductor and the capacitor have finite impedances of opposite signs. As the frequency is increased from zero, the inductor impedance increases and the capacitor impedance decreases. Therefore, the phase shift between the input and the output approaches −90° as ωL approaches 1 / ωC. As soon as ωL exceeds 1 / ωC, the phase shift

jumps to +90° and then approaches zero as ω continues to increase.

At some frequency between the two passbands, the impedances of the inductor and capacitor are equal but have opposite signs, so their sum is zero. Thus, at this frequency, the series combination of the inductor and capacitor acts like a short circuit, and the output voltage magnitude is zero. This is the center frequency of the series RLC bandreject filter.

Figure 14.29 presents a sketch of the frequency response for the series RLC bandreject filter from Fig. 14.28(a). Note that the magnitude plot is overlaid with that of the ideal bandreject filter from Fig. 14.3(b). Our qualitative analysis has confirmed the shape of the magnitude and phase angle plots. We now turn to a quantitative analysis of the circuit to confirm this frequency response and to compute values for the parameters that characterize this response.

Figure 14.29 The frequency response plot for the series RLC bandreject filter circuit in Fig. 14.28(a).

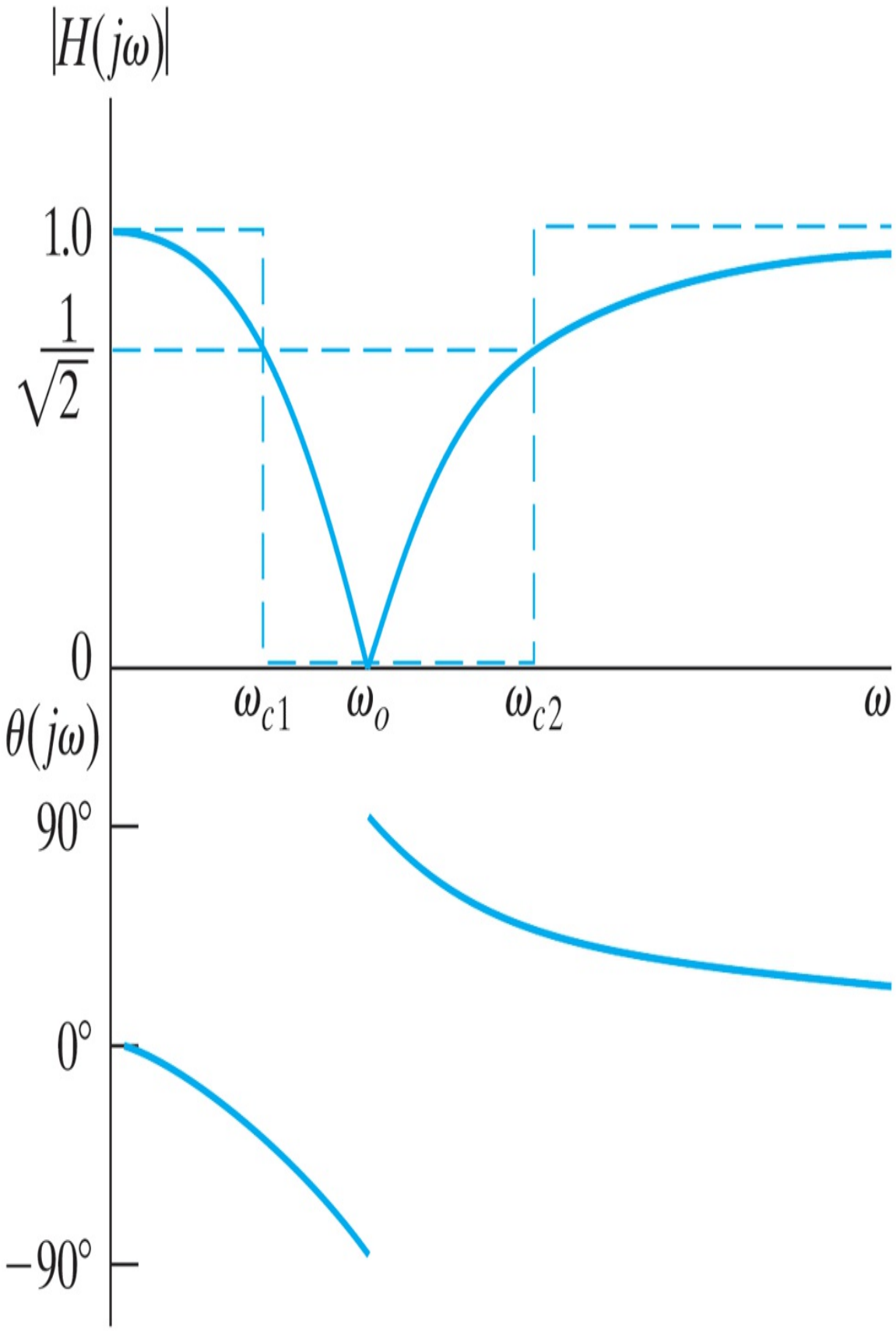

$|H(j\omega)|$
1.0
$\frac{1}{\sqrt{2}}$
0
ω_{c1}
ω_o
ω_{c2}
ω
$\theta(j\omega)$
90°
0°
−90°

Figure 14.29 Full Alternative Text

The Series RLC Circuit—Quantitative Analysis

After transforming to the s domain, as shown in Fig. 14.30, we use voltage division to find the transfer function equation:

Figure 14.30 The s-domain equivalent of the circuit in Fig. 14.28(a).

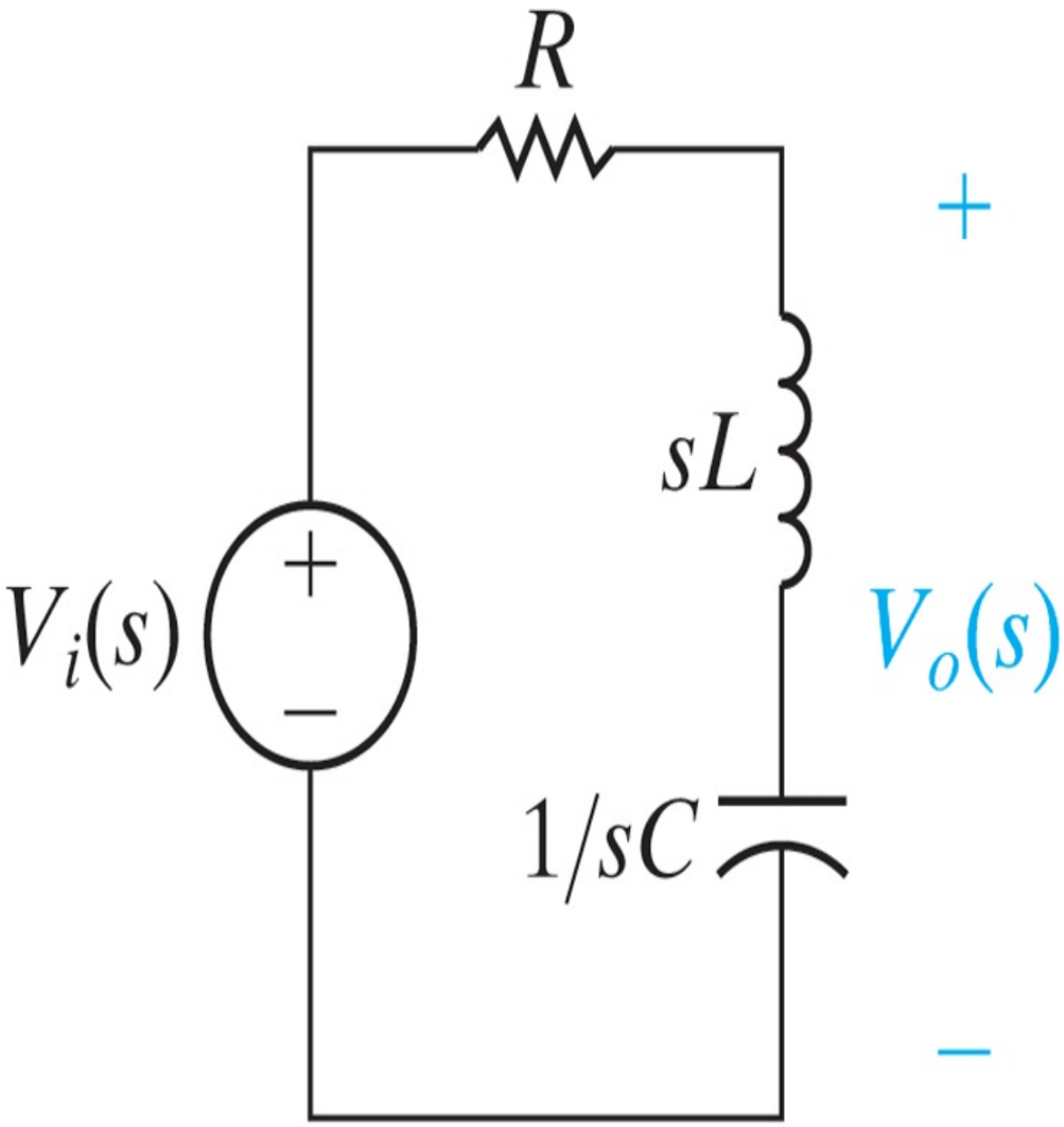


Figure 14.30 Full Alternative Text

H(s)=sL+1sCR+sL+1sC=s2+1LCs2+RLs+1LC.

Substituting jω for s in H(s), we generate equations for the transfer function magnitude and the phase angle:

|H(jω)| =|1LC−ω2|(1LC−ω2)2+(ωRL)2,θ(jω)=−tan−1(ωRL1LC−ω2).

Note that the equations for the transfer function magnitude and phase angle

confirm the frequency response shape pictured in Fig. 14.29, which we developed from the qualitative analysis.

We use the circuit in Fig. 14.30 to calculate the center frequency. For the bandreject filter, the center frequency is still defined as the frequency for which the sum of the impedances of the capacitor and inductor is zero. In the bandpass filter, the magnitude at the center frequency was a maximum, but in the bandreject filter, this magnitude is a minimum. This is because in the bandreject filter, the center frequency is in the stopband, not in the passband. Because the sum of the capacitor and inductor impedances is zero at the center frequency,

$$\omega o=1LC.$$

Substituting 1/LC for ωo in the equation for the transfer function magnitude shows that $|H(j\omega o)|=0$.

The cutoff frequencies, the bandwidth, and the quality factor are defined and calculated for the bandreject filter and the bandpass filter in exactly the same way. Note that for the bandreject filter, $Hmax=|H(j0)|=|H(j\infty)|$, and for the series RLC bandreject filter in Fig. 14.28(a), Hmax=1. Thus,

$$\omega c1=-R2L+(R2L)2+1LC,\omega c2=R2L+(R2L)2+1LC.$$

These equations are the same as Eqs. 14.14 and 14.15.

Use the cutoff frequencies to generate an expression for the bandwidth, β:

$$\beta=R/L.$$

This equation is the same as Eq. 14.16.

Finally, the center frequency and the bandwidth produce an equation for the quality factor, Q:

$$Q=LR2C.$$

This equation and Eq. 14.17 are the same.

Again, we can represent the expressions for the two cutoff frequencies in terms of the bandwidth and center frequency, as we did for the bandpass filter:

$$\omega_{c1} = -\frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2}, \quad \omega_{c2} = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2}.$$

Alternative forms for these equations express the cutoff frequencies in terms of the quality factor and the center frequency:

$$\omega_{c1} = \omega_o \cdot \left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right], \quad \omega_{c2} = \omega_o \cdot \left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right].$$

Example 14.8 presents the design of a series RLC bandreject filter.

Example 14.8 Designing a Series RLC Bandreject Filter

Using the series RLC circuit in Fig. 14.28(a), compute the component values that yield a bandreject filter with a bandwidth of 250 Hz and a center frequency of 750 Hz. Use a 100 nF capacitor. Compute values for R, L, ω_{c1}, ω_{c2}, and Q.

Solution

We begin by using the definition of quality factor to compute its value for this filter:

$$Q = \omega_o / \beta = 3.$$

Use Eq. 14.13 to compute L, remembering to convert ω_o to radians per second:

$$L = \frac{1}{\omega_o^2 C} = \frac{1}{[2\pi(750)]^2 (100 \times 10^{-9})} = 450 \text{ mH}.$$

Use Eq. 14.16 to calculate R:

$R=\beta L=2\pi(250)(450\times10-3)=707\ \Omega.$

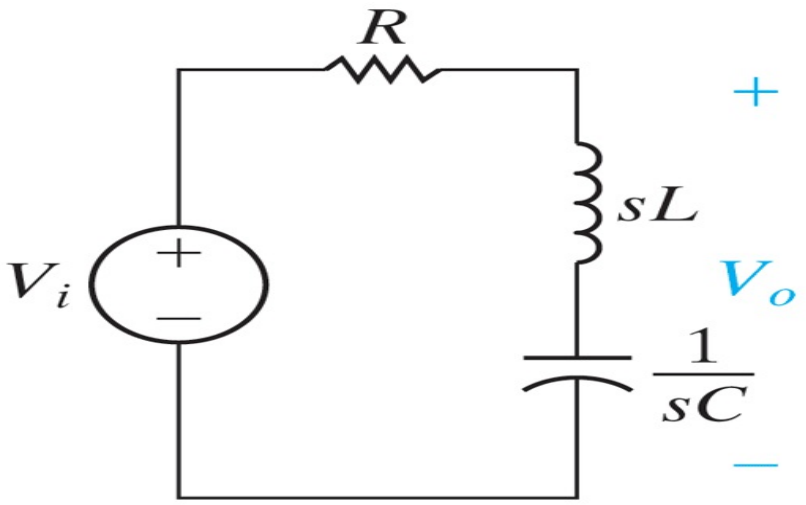
The values for the center frequency and bandwidth can be used to compute the two cutoff frequencies:

$\omega c1=-\beta 2+(\beta 2)2+\ \omega o2=3992.0\ \text{rad}/\text{s},\omega c2=\beta 2+(\beta 2)2+\ \omega o2=5562.8\ \text{rad}/\text{s}.$

The cutoff frequencies are at 635.3 Hz and 885.3 Hz. Their difference is $885.3-635.3=250\text{Hz}$, confirming the specified bandwidth. The geometric mean is $(635.3)(885.3)=750\text{Hz}$, confirming the specified center frequency.

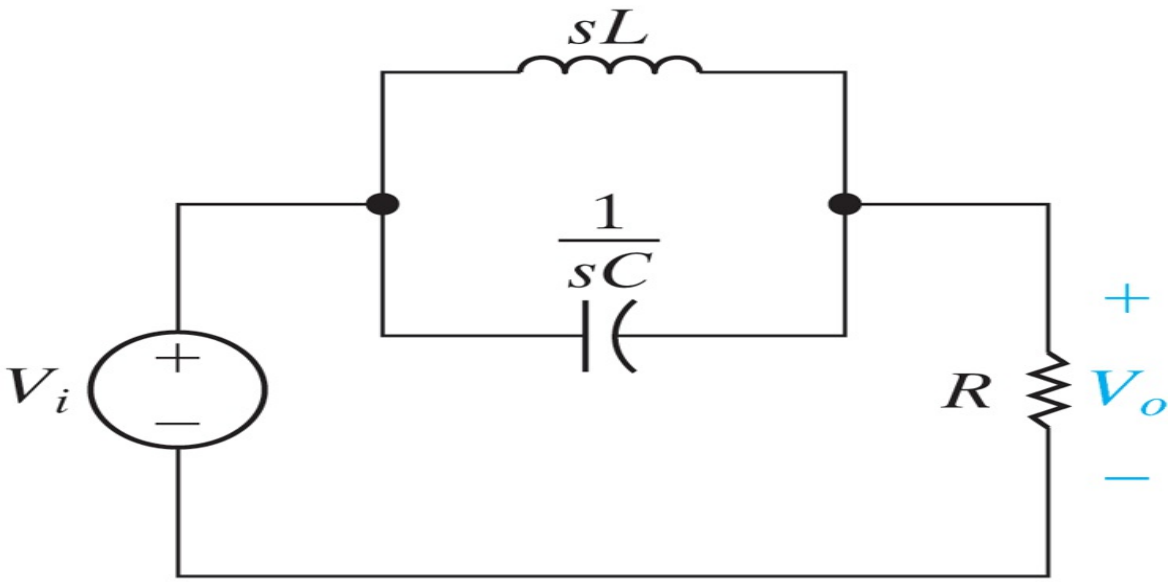
As you might suspect by now, another configuration that produces a bandreject filter is a parallel RLC circuit. The analysis details of the parallel RLC circuit are left to Problem 14.40, and the results are summarized in Fig. 14.31, along with the series RLC bandreject filter. As we did for other categories of filters, we can state a general form for the transfer functions of bandreject filters, replacing the constant terms with β and ωo:

Figure 14.31 Two RLC bandreject filters, together with equations for the transfer function, center frequency, and bandwidth of each.



$$H(s) = \frac{s^2 + 1/LC}{s^2 + (R/L)s + 1/LC}$$

$$\omega_o = \sqrt{1/LC} \qquad \beta = R/L$$



$$H(s) = \frac{s^2 + 1/LC}{s^2 + s/RC + 1/LC}$$

$$\omega_o = \sqrt{1/LC} \qquad \beta = 1/RC$$

Figure 14.31 Full Alternative Text

Transfer Function for Bandreject Filters

$$H(s)=\frac{s^2+\omega_o^2}{s^2+\beta s+\omega_o^2}. \quad (14.20)$$

Equation 14.20 is useful in filter design because any circuit with a transfer function in this form can be used as a bandreject filter.

Assessment Problems

Objective 4—Know the RLC circuit configurations that act as bandreject filters

1. 14.10 Design the component values for the series RLC bandreject filter shown in Fig. 14.28(a) so that the center frequency is 4 kHz and the quality factor is 5. Use a 500 nF capacitor.

   Answer: L=3.17 mH,

   R=15.92 Ω.

2. 14.11 Recompute the component values for Assessment Problem 14.10 to achieve a bandreject filter with a center frequency of 20 kHz. The filter has a 100 Ω resistor. The quality factor remains at 5.

   Answer: L=3.98 mH,

   C=15.92 nF.

SELF-CHECK: *Also try Chapter Problems 14.41 and 14.42.*

Practical Perspective

Pushbutton Telephone Circuits

In the Practical Perspective at the start of this chapter, we described the dual-tone-multiple-frequency (DTMF) system used to signal that a button has been pushed on a pushbutton telephone. A key element of the DTMF system is the DTMF receiver—a circuit that decodes the tones produced by pushing a button and determines which button was pushed.

In order to design a DTMF receiver, we need a better understanding of the DTMF system. As you can see from Fig. 14.32, the buttons on the telephone are organized into rows and columns. The pair of tones generated by pushing a button depends on the button's row and column. The button's row determines its low-frequency tone, and the button's column determines its high-frequency tone.[1] For example, pressing the "6" button produces sinusoidal tones with the frequencies 770Hz and 1477Hz.

1A fourth high-frequency tone is reserved at 1633 Hz. This tone is used infrequently and is not produced by a standard 12-button telephone.

Figure 14.32 Tones generated by the rows and columns of telephone pushbuttons.

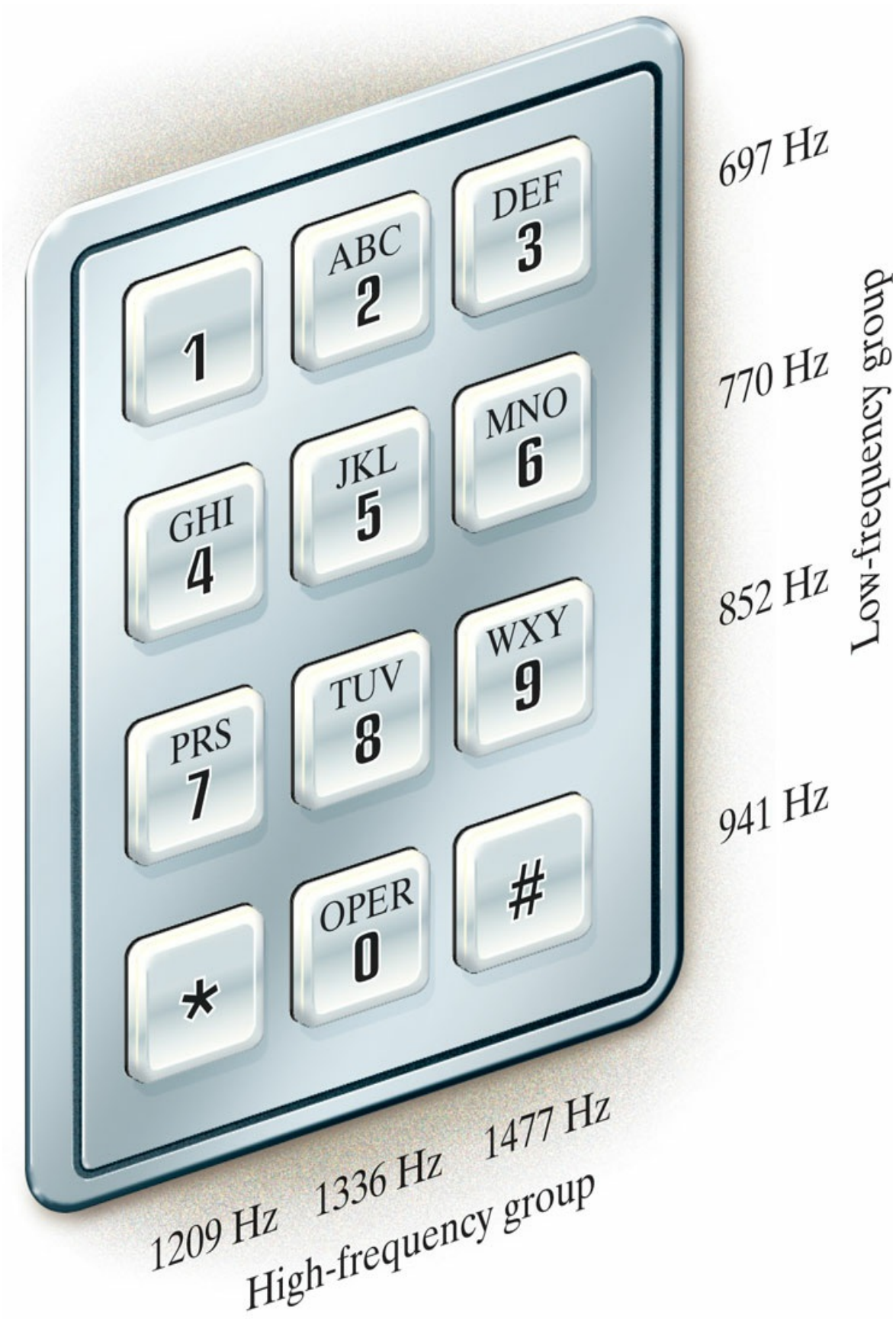

1
ABC 2
DEF 3
GHI 4
JKL 5
MNO 6
PRS 7
TUV 8
WXY 9
*
OPER 0
#
697 Hz
770 Hz
852 Hz
941 Hz
Low-frequency group
1209 Hz
1336 Hz
1477 Hz
High-frequency group

Figure 14.32 Full Alternative Text

At the telephone switching facility, bandpass filters in the DTMF receiver first detect whether tones from both the low-frequency and high-frequency groups are simultaneously present. This test rejects many extraneous audio signals that are not DTMF. If tones are present in both bands, other filters are used to select among the possible tones in each band so that the frequencies can be decoded, identifying a unique button. Additional tests are performed to prevent false button detection. For example, only one tone per frequency band is allowed; the high- and low-band frequencies must start and stop within a few milliseconds of one another to be considered valid; and the high- and low-band signal amplitudes must be sufficiently close to each other.

You may wonder why bandpass filters are used instead of a high-pass filter for the high-frequency group of DTMF tones and a low-pass filter for the low-frequency group of DTMF tones. The reason is that the telephone system uses frequencies outside of the 300 −3kHz band for other signaling purposes, such as ringing the phone’s bell. Bandpass filters prevent the DTMF receiver from erroneously detecting these other signals.

SELF-CHECK: *Assess your understanding of this Practical Perspective by trying Chapter Problems 14.51–14.53.*

Summary

- A **frequency selective circuit**, or **filter**, enables signals at certain frequencies to reach the output, and it attenuates signals at other frequencies to prevent them from reaching the output. The **passband** contains the frequencies of those signals that are passed; the **stopband** contains the frequencies of those signals that are attenuated. (See page 538.)

- The **cutoff frequency**, ωc, separates frequencies in the stopband from frequencies in the passband. At the cutoff frequency, the magnitude of the transfer function equals $(1 / 2)Hmax$. (See page 540.)

- A **low-pass filter** passes voltages at frequencies below ωc and attenuates frequencies above ωc. Any circuit with the transfer function

  $$H(s)=\omega cs+\omega c$$

  functions as a low-pass filter. (See page 545.)

- A **high-pass filter** passes voltages at frequencies above ωc and attenuates voltages at frequencies below ωc. Any circuit with the transfer function

  $$H(s)=ss+\omega c$$

  functions as a high-pass filter. (See page 549.)

- Bandpass filters and bandreject filters each have two cutoff frequencies, $\omega c1$ and $\omega c2$. These filters are further characterized by their **center frequency** (ωo), **bandwidth** (β), and **quality factor** (Q). These quantities are defined as

  $$\omega o=\omega c1 \cdot \omega c2, \beta=\omega c2-\omega c1, Q=\omega o / \beta.$$

  (See pages 553 and 554.)

- A **bandpass filter** passes voltages at frequencies within the passband, which is between ωc1 and ωc2. It attenuates frequencies outside of the passband. Any circuit with the transfer function

  H(s)=βSs2+βS+ωo2

  functions as a bandpass filter. (See page 559.)

- A **bandreject filter** attenuates voltages at frequencies within the stopband, which is between ωc1 and ωc2. It passes frequencies outside of the stopband. Any circuit with the transfer function

  H(s)=s2+ ωo2s2+βS+ ωo2

  functions as a bandreject filter. (See page 563.)

- Adding a load to the output of a passive filter changes its filtering properties by altering the location and magnitude of the passband. Replacing an ideal voltage source with one whose source resistance is nonzero also changes the filtering properties of the rest of the circuit, again by altering the location and magnitude of the passband. (See page 548 and 557.)

Problems

Section 14.2

1. 14.1

    1. a) Find the cutoff frequency in hertz for the RL filter shown in Fig. P14.1.

    2. b) Calculate H(jω) at ωc, 0.2ωc, and 5ωc.

    3. c) If vi=10 cos ωt V, write the steady-state expression for vo when ω=ωc, ω=0.2ωc, and ω=5ωc.

    ## Figure P14.1

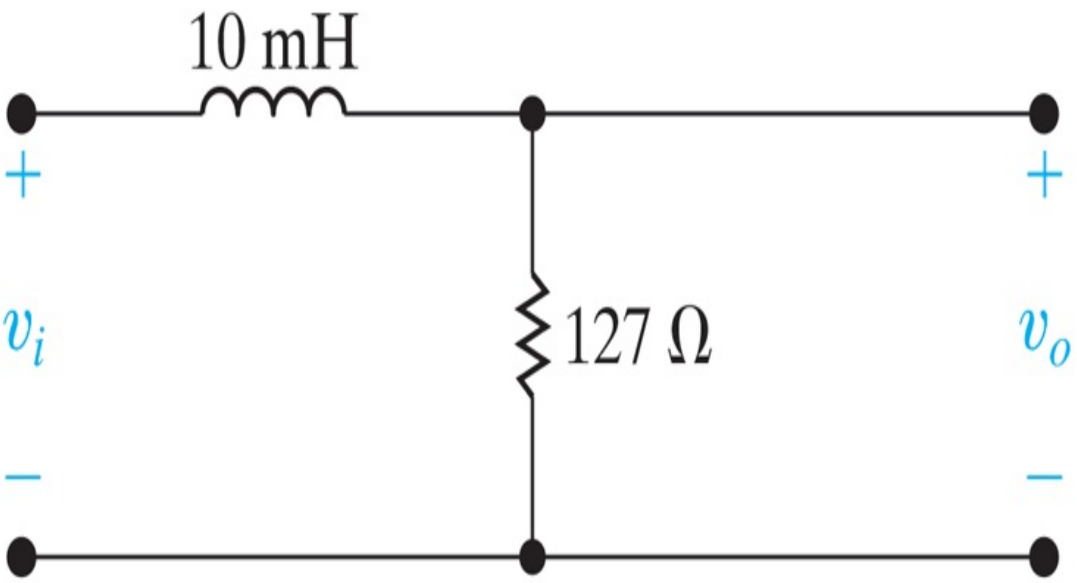
    

    Figure P14.1 Full Alternative Text

2. 14.2 Consider the low-pass filter in Fig. P14.2, which has a load resistor RL.

    1. a) What is the transfer function of the unloaded filter?

2. b) What is the transfer function of the loaded filter?

3. c) Compare the transfer function of the unloaded filter in part (a) and the transfer function of the loaded filter in part (b). Are the cutoff frequencies different? Are the passband gains different?

4. d) If R=330 Ω and L=10 mH, what is the cutoff frequency of the unloaded filter in rad/s?

5. e) What is the smallest value of load resistance that can be used with the filter components in part (d) so that the cutoff frequency of the resulting filter is no more than 5% different from the unloaded filter?

Figure P14.2

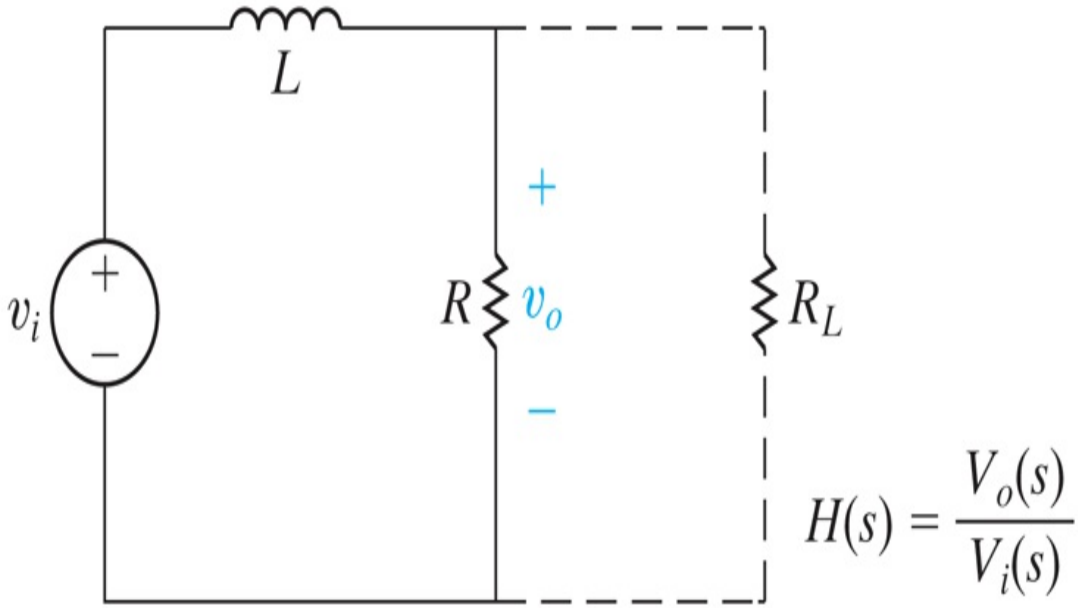


Figure P14.2 Full Alternative Text

3. 14.3 DESIGNPROBLEM_ PSPICEMULTISIM Use a 5 mH inductor to design a low-pass, RL, passive filter with a cutoff frequency of 1 kHz.

    1. a) Specify the value of the resistor.

2. b) A load having a resistance of 270 Ω is connected across the output terminals of the filter. What is the corner, or cutoff, frequency of the loaded filter in hertz?

3. c) If you must use a single resistor from Appendix H for part (a), what resistor should you use? What is the resulting cutoff frequency of the filter?

4. 14.4 A resistor, denoted as Rl, is added in series with the inductor in the circuit in Fig. 14.4(a). The new low-pass filter circuit is shown in Fig. P14.4.

    1. a) Derive the expression for H(s) where H(s)=Vo/Vi.

    2. b) At what frequency will the magnitude of H(jω) be maximum?

    3. c) What is the maximum value of the magnitude of H(jω)?

    4. d) At what frequency will the magnitude of H(jω) equal its maximum value divided by 2?

    5. e) Assume a resistance of 75 Ω is added in series with the 10 mH inductor in the circuit in Fig. P14.1. Find ωc, H(j0), H(jωc), H(j0.2ωc), and H(j5ωc).

Figure P14.4

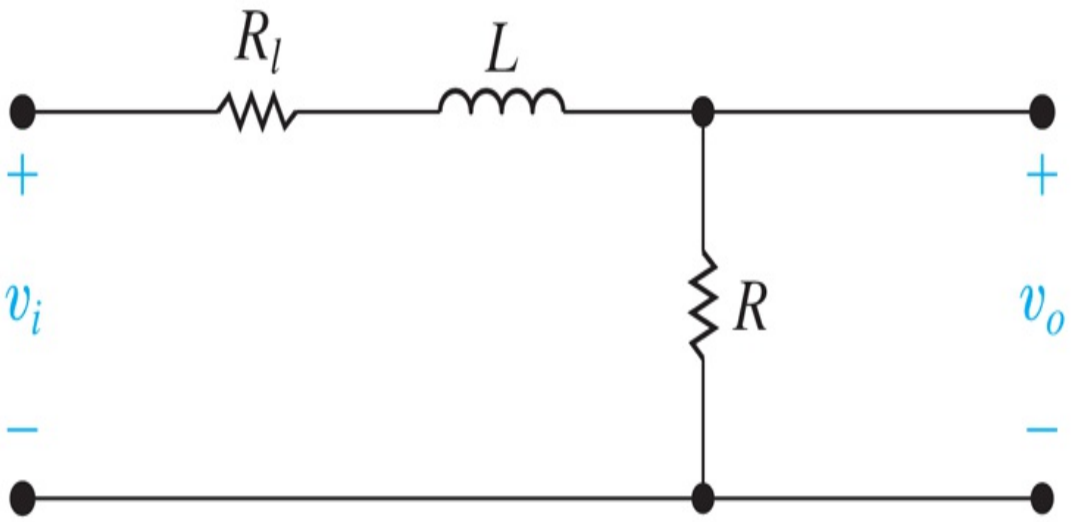


Figure P14.4 Full Alternative Text

5. 14.5 Use a 250 Ω resistor to design a low-pass passive filter with a cutoff frequency of 8 krad/s.

    1. a) Specify the value of the filter's inductor.

    2. b) A load resistor of 270 Ω is connected across the output terminals of the filter. What is the cutoff frequency of the loaded filter, in rad/s?

6. 14.6 Consider the low-pass filter designed in Problem 14.5.

    1. a) Assume the cutoff frequency cannot decrease by more than 10% from the specified value, 8 krad/s. What is the smallest value of load resistance that can be connected across the output terminals of the filter?

    2. b) If the resistor found in part (a) is connected across the output terminals of the filter, what is the magnitude of H(jω) when ω=0?

7. 14.7

    1. a) Find the cutoff frequency (in hertz) of the low-pass filter shown in Fig. P14.7.

2. b) Calculate H(jω) at ωc, 0.1ωc, and 10ωc.
3. c) If vi=200 cos ωt mV, write the steady-state expression for vo when ω=ωc, 0.1ωc, and 10ωc.

Figure P14.7

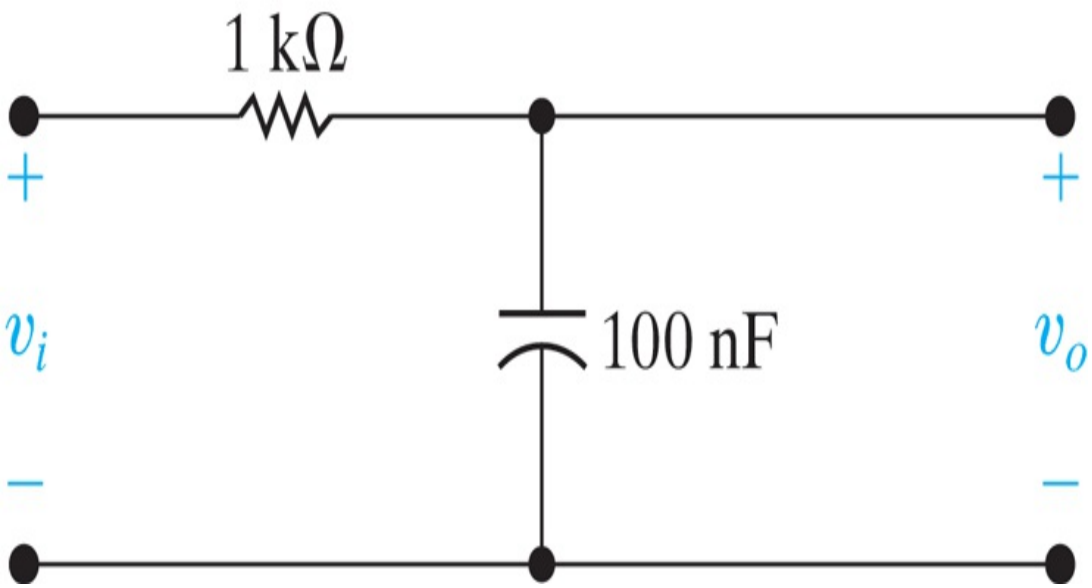


Figure P14.7 Full Alternative Text

8. 14.8 A resistor denoted as RL is connected in parallel with the capacitor in the circuit in Fig. 14.7. The loaded low-pass filter circuit is shown in Fig. P14.8.
    1. a) Derive the expression for the voltage transfer function Vo/Vi.
    2. b) At what frequency will the magnitude of H(jω) be maximum?
    3. c) What is the maximum value of the magnitude of H(jω)?
    4. d) At what frequency will the magnitude of H(jω) equal its maximum value divided by 2?
    5. e) Assume a resistance of 10 kΩ is added in parallel with the 100

nF capacitor in the circuit in Fig. P14.7. Find ωc, H(j0), H(jωc), H(j0.1ωc), and H(j10ωc).

Figure P14.8

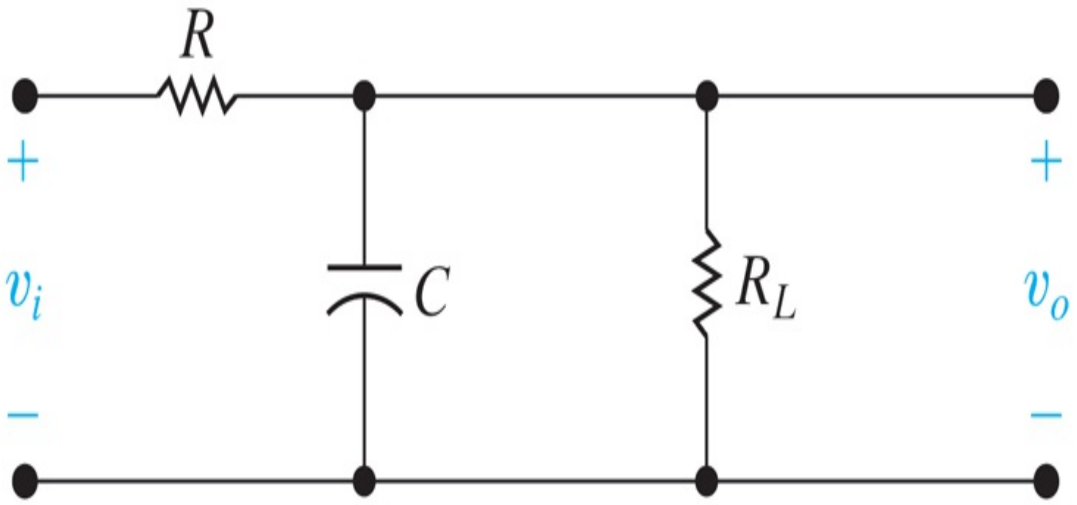


Figure P14.8 Full Alternative Text

9. 14.9 DESIGNPROBLEM_PSPICEMULTISIMUse a 500 nF capacitor to design a low-pass passive filter with a cutoff frequency of 50 krad/s.

    1. a) Specify the cutoff frequency in hertz.

    2. b) Specify the value of the filter resistor.

    3. c) Assume the cutoff frequency cannot increase by more than 5%. What is the smallest value of load resistance that can be connected across the output terminals of the filter?

    4. d) If the resistor found in (c) is connected across the output terminals, what is the magnitude of H(jω) when ω=0?

10. 14.10 Design a passive RC low pass filter (see Fig. 14.7) with a cutoff frequency of 100 Hz using a 4.7 μF capacitor.

    1. a) What is the cutoff frequency in rad/s?

2. b) What is the value of the resistor?

3. c) Draw your circuit, labeling the component values and output voltage.

4. d) What is the transfer function of the filter in part (c)?

5. e) If the filter in part (c) is loaded with a resistor whose value is the same as the resistor part (b), what is the transfer function of this loaded filter?

6. f) What is the cutoff frequency of the loaded filter from part (e)?

7. g) What is the gain in the pass band of the loaded filter from part (e)?

Section 14.3

1. 14.11 Consider the circuit shown in Fig. P14.11.
    1. a) What is the transfer function, H(s)=Vo(s)/Vi(s), of this filter?
    2. b) What is the cutoff frequency of this filter?
    3. c) What is the magnitude of the filter's transfer function at s=jωc?

Figure P14.11

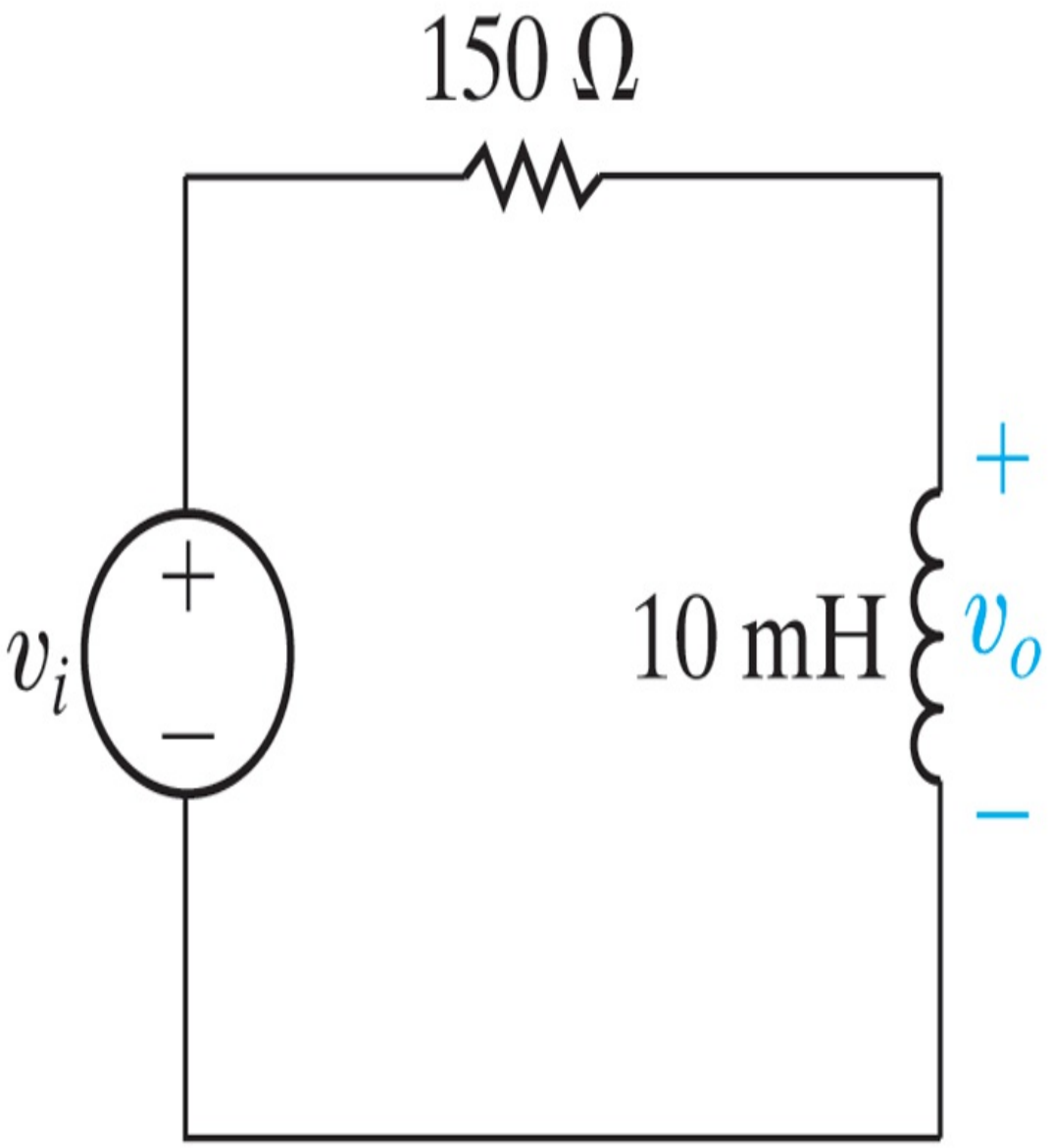


Figure P14.11 Full Alternative Text

2. 14.12 Suppose a 150 Ω load resistor is attached to the filter in Fig. P14.11.
    1. a) What is the transfer function, H(s)=Vo(s)/Vi(s), of this filter?
    2. b) What is the cutoff frequency of this filter?
    3. c) How does the cutoff frequency of the loaded filter compare with the cutoff frequency of the unloaded filter in Fig. P14.11?

4. d) What else is different for these two filters?

3. 14.13 DESIGNPROBLEM_PSPICEMULTISIMUsing a 5 mH inductor, design a high-pass, RL, passive filter with a cutoff frequency of 25 krad/s.

    1. a) Specify the value of the resistance.

    2. b) Assume the filter is connected to a pure resistive load. The cutoff frequency is not to drop below 24 krad/s. What is the smallest load resistor that can be connected across the output terminals of the filter?

4. 14.14 Design a passive RC high pass filter (see Fig. 14.10[a]) with a cutoff frequency of 500 Hz using a 220 pF capacitor.

    1. a) What is the cutoff frequency in rad/s?

    2. b) What is the value of the resistor?

    3. c) Draw your circuit, labeling the component values and output voltage.

    4. d) What is the transfer function of the filter in part (c)?

    5. e) If the filter in part (c) is loaded with a resistor whose value is the same as the resistor in (b), what is the transfer function of this loaded filter?

    6. f) What is the cutoff frequency of the loaded filter from part (e)?

    7. g) What is the gain in the pass band of the loaded filter from part (e)?

5. 14.15

    1. a) Find the cutoff frequency (in hertz) for the high-pass filter shown in Fig. P14.15.

2. b) Find H(jω) at ωc, 0.2ωc, and 5ωc.
3. c) If v1=0.5 cos ωt V, write the steady-state expression for vo when ω=ωc, ω=0.2ωc, and ω=5ωc.

Figure P14.15

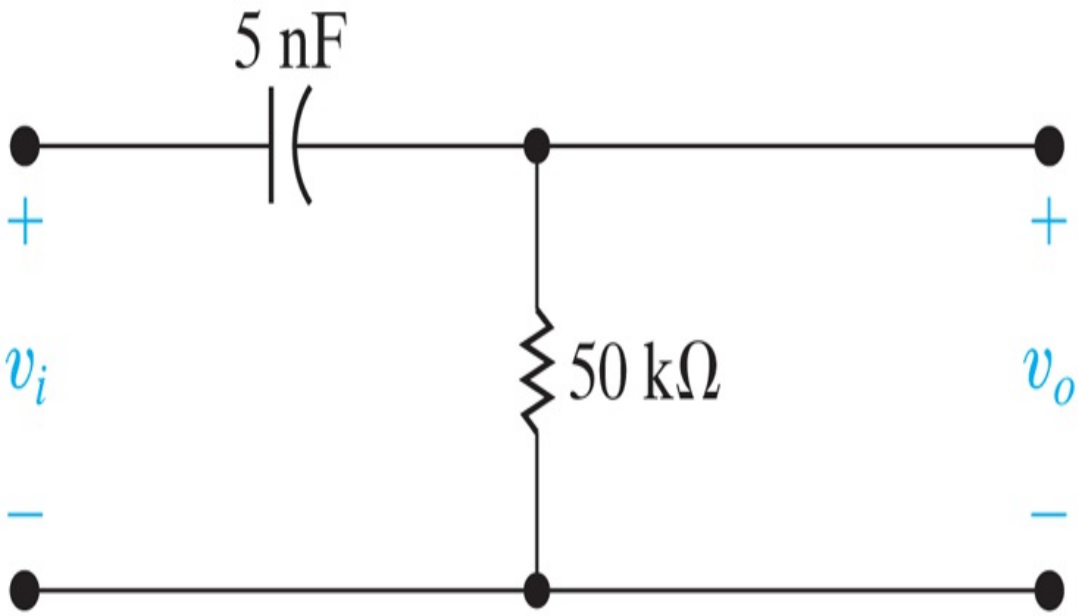


Figure P14.15 Full Alternative Text

6. 14.16 A resistor, Rc, is connected in series with the capacitor in the circuit in Fig. 14.10(a). The new high-pass filter circuit is shown in Fig. P14.16.
    1. a) Derive the expression for H(s) where H(s)=Vo/Vi.
    2. b) At what frequency will the magnitude of H(jω) be maximum?
    3. c) What is the maximum value of the magnitude of H(jω)?
    4. d) At what frequency will the magnitude of H(jω) equal its

maximum value divided by 2?

5. e) Assume a resistance of 12.5 kΩ is connected in series with the 5 nF capacitor in the circuit in Fig. P14.15. Calculate ωc, H(jωc), H(j0.2ωc), and H(j5ωc).

Figure P14.16

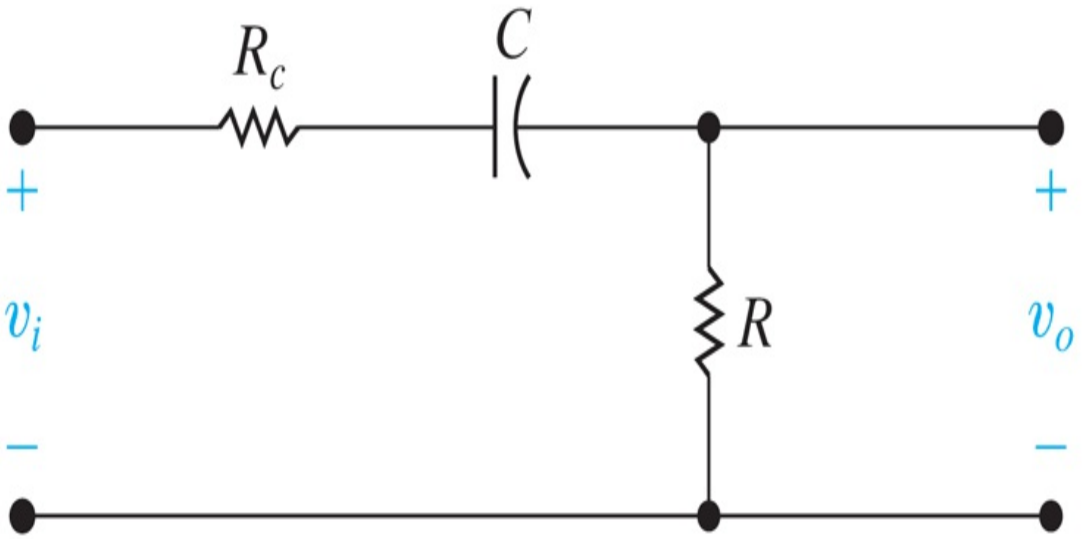


Figure P14.16 Full Alternative Text

7. 14.17 DESIGNPROBLEM_Using a 100 nF capacitor, design a high-pass passive filter with a cutoff frequency of 300 Hz.

    1. a) Specify the value of R in kilohms.

    2. b) A 47 kΩ resistor is connected across the output terminals of the filter. What is the cutoff frequency, in hertz, of the loaded filter?

Section 14.4

1. 14.18

    1. a) Using the relationship between the bandwidth and the cutoff frequencies and the relationship between the center frequency and the cutoff frequencies, show that

       $\omega c1 = -\beta 2 + (\beta 2)2 + \omega o2$; $\omega c2 = \beta 2 + (\beta 2)2 + \omega o2$.

    2. b) Using the expressions for the two cutoff frequencies in terms of the center frequency and the bandwidth given in part (a), show that

       $\omega c1 = \omega o \cdot [-12Q + 1 + (12Q)2], \omega c2 = \omega o.[12Q + 1 + (12Q)2]$.

2. 14.19 A bandpass filter has a center, or resonant, frequency of 50 krad/s and a quality factor of 4. Find the bandwidth, the upper cutoff frequency, and the lower cutoff frequency. Express all answers in kilohertz.

3. 14.20 Calculate the center frequency, the bandwidth, and the quality factor of a bandpass filter that has an upper cutoff frequency of 121 krad/s and a lower cutoff frequency of 100 krad/s.

4. 14.21 Design a series RLC bandpass filter (see Fig. 14.19[a]) with a quality of 2 and a center frequency of 8 kHz, using a 5 nF capacitor.

    1. a) Draw your circuit, labeling the component values and output voltage.

    2. b) For the filter in part (a), calculate the bandwidth and the values of the two cutoff frequencies.

5. 14.22 The input to the series RLC bandpass filter designed in Problem 14.21 is $20\cos\omega t$ V. Find the voltage drop across the resistor when (a) $\omega = \omega o$; (b) $\omega = \omega c1$; (c) $\omega = \omega c2$; (d) $\omega = 0.1\omega o$; (e) $\omega = 10\omega o$.

6. 14.23 The input to the series RLC bandpass filter designed in Problem 14.21 is 20cosωt V. Find the voltage drop across the series combination of the inductor and capacitor when (a) ω=ωo; (b) ω=ωc1; (c) ω=ωc2; (d) ω=0.1ωo; (e) ω=10ωo.

7. 14.24 PSPICEMULTISIMFor the bandpass filter shown in Fig. P14.24, calculate the following: (a) fo; (b) Q; (c) fc1; (d) fc2; and (e) β.

Figure P14.24

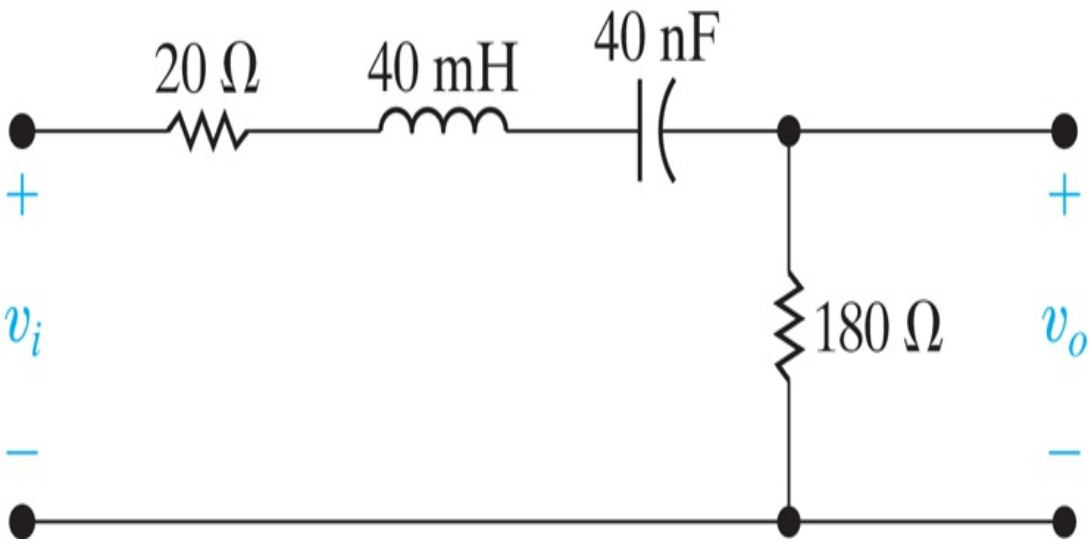


Figure P14.24 Full Alternative Text

8. 14.25 The input voltage in the circuit in Fig. P14.24 is 0.5 cos ωt V. Calculate the output voltage when (a) ω=ωo; (b) ω=ωc1; and (c) ω=ωc2.

9. 14.26 DESIGNPROBLEM_PSPICEMULTISIMUsing a 50 nF capacitor in the bandpass circuit shown in Fig. 14.22, design a filter with a quality factor of 5 and a center frequency of 20 krad/s.

    1. a) Specify the numerical values of R and L.

    2. b) Calculate the upper and lower cutoff frequencies in kilohertz.

    3. c) Calculate the bandwidth in hertz.

10. 14.27 Design a series RLC bandpass filter using only three components from Appendix H that comes closest to meeting the filter specifications in Problem 14.26.

    1. a) Draw your filter, labeling all component values and the input and output voltages.

    2. b) Calculate the percent error in this new filter's center frequency and quality factor when compared to the values specified in Problem 14.26.

11. 14.28 DESIGNPROBLEM_PSPICEMULTISIMUse a 5 nF capacitor to design a series RLC bandpass filter, as shown at the top of Fig. 14.27. The center frequency of the filter is 8 kHz, and the quality factor is 2.

    1. a) Specify the values of R and L.

    2. b) What is the lower cutoff frequency in kilohertz?

    3. c) What is the upper cutoff frequency in kilohertz?

    4. d) What is the bandwidth of the filter in kilohertz?

12. 14.29 Design a series RLC bandpass filter using only three components from Appendix H that comes closest to meeting the filter specifications in Problem 14.28.

    1. a) Draw your filter, labeling all component values and the input and output voltages.

    2. b) Calculate the percent error in this new filter's center frequency and quality factor when compared to the values specified in Problem 14.28.

13. 14.30 PSPICEMULTISIMFor the bandpass filter shown in Fig. P14.30, find (a) ωo, (b) fo, (c) Q, (d) $\omega c1$, (e) fc1, (f) $\omega c2$, (g) fc2, and (h) β.

Figure P14.30

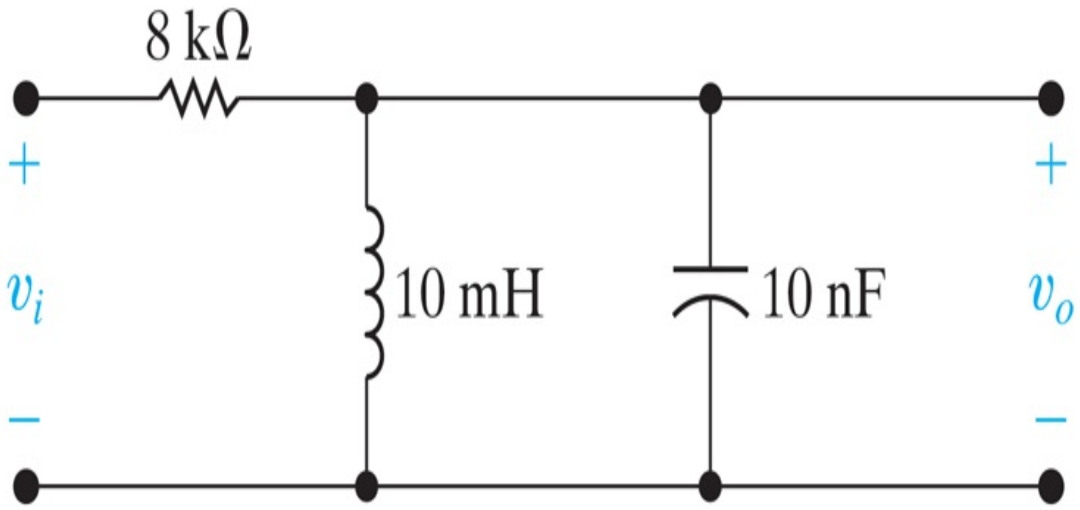


Figure P14.30 Full Alternative Text

14. 14.31 PSPICEMULTISIMConsider the circuit shown in Fig. P14.31.
    1. a) Find ωo.
    2. b) Find β.
    3. c) Find Q.
    4. d) Find the steady-state expression for vo when vi=250 cos ωot mV.
    5. e) Show that if RL is expressed in kilohms the Q of the circuit in Fig. P14.31 is

       Q=201+100/RL
    6. f) Plot Q versus RL for 20 kΩ≤RL≤2 MΩ.

Figure P14.31

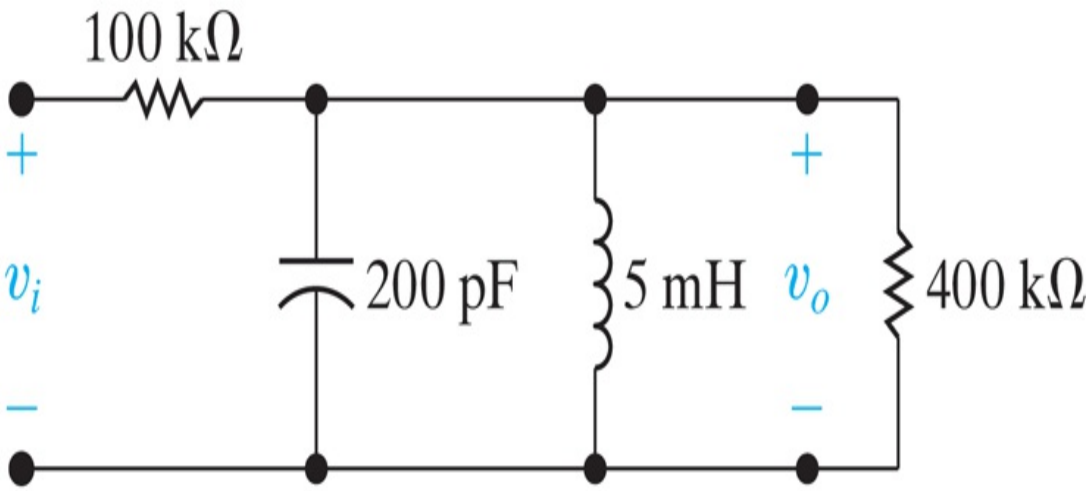


Figure P14.31 Full Alternative Text

15. 14.32 A block diagram of a system consisting of a sinusoidal voltage source, a series RLC bandpass filter, and a load is shown in Fig. P14.32. The internal impedance of the sinusoidal source is 80+j0 Ω, and the impedance of the load is 480+j0 Ω.

Figure P14.32

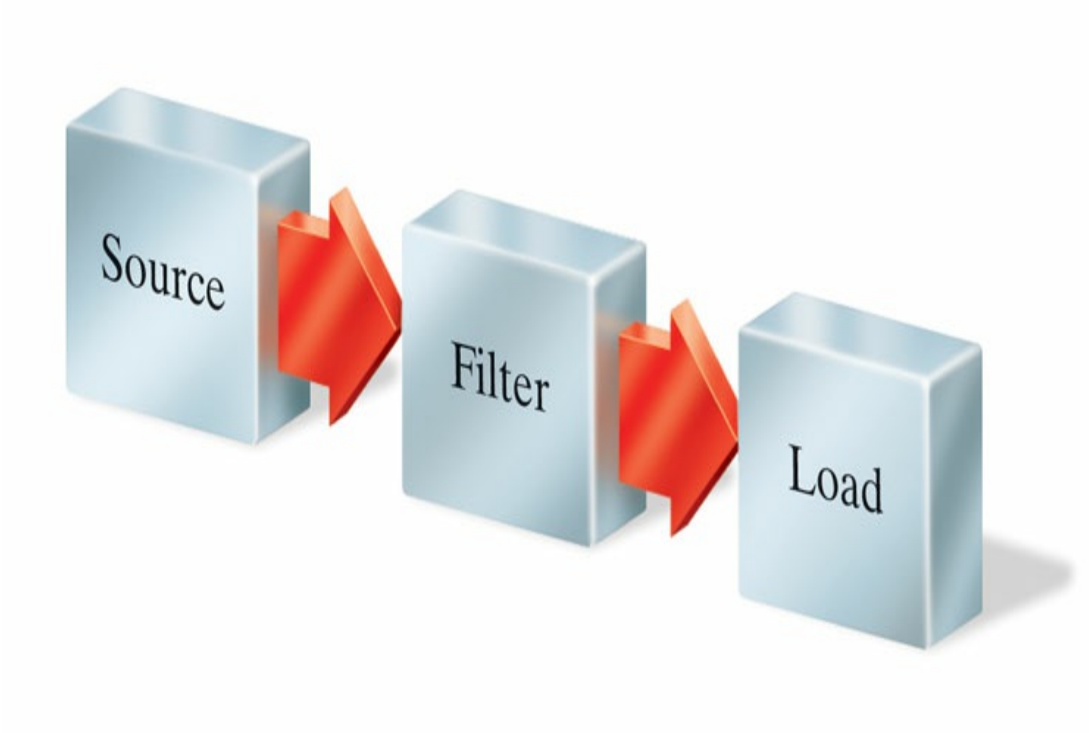


Figure P14.32 Full Alternative Text

The RLC series bandpass filter has a 20 nF capacitor, a center frequency of 50 krad/s, and a quality factor of 6.25.

1. a) Draw a circuit diagram of the system.
2. b) Specify the numerical values of L and R for the filter section of the system.
3. c) What is the quality factor of the interconnected system?
4. d) What is the bandwidth (in hertz) of the interconnected system?

16. 14.33 The purpose of this problem is to investigate how a resistive load connected across the output terminals of the bandpass filter shown in Fig. 14.19 affects the quality factor and hence the bandwidth of the filtering system. The loaded filter circuit is shown in Fig. P14.33.

    1. a) Calculate the transfer function Vo/Vi for the circuit shown in Fig. P14.33.

2. b) What is the expression for the bandwidth of the system?
3. c) What is the expression for the loaded bandwidth (βL) as a function of the unloaded bandwidth (βU)?
4. d) What is the expression for the quality factor of the system?
5. e) What is the expression for the loaded quality factor (QL) as a function of the unloaded quality factor (QU)?
6. f) What are the expressions for the cutoff frequencies ωc1 and ωc2?

Figure P14.33

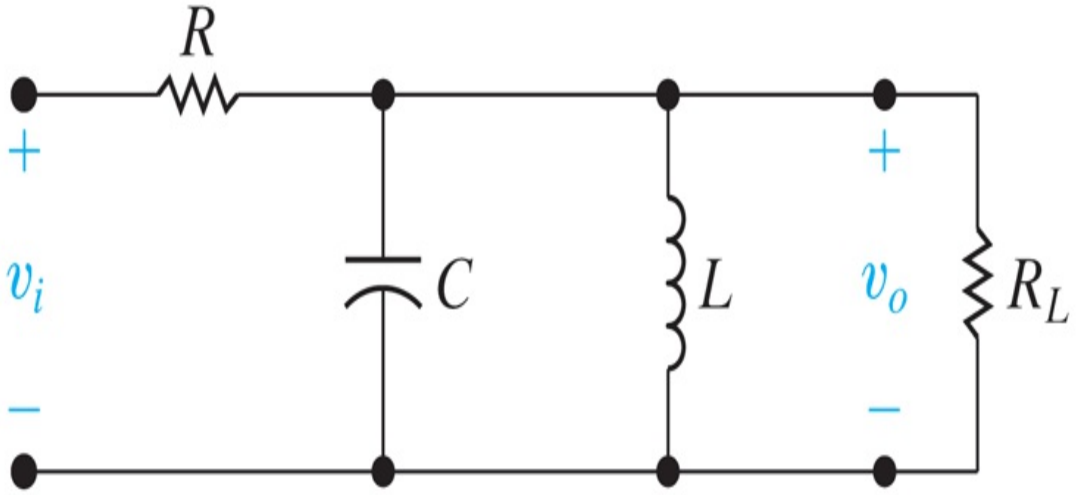


Figure P14.33 Full Alternative Text

17. 14.34 The parameters in the circuit in Fig. P14.33 are R=2.4 kΩ, C=50 pF, and L=2 μH. The quality factor of the circuit is not to drop below 7.5. What is the smallest permissible value of the load resistor RL?

Section 14.5

1. 14.35 For the bandreject filter in Fig. P14.35, calculate (a) ωo; (b) fo; (c) Q; (d) β in hertz; (e) ωc1; (f) fc1; (g) ωc2; and (h) fc2.

Figure P14.35

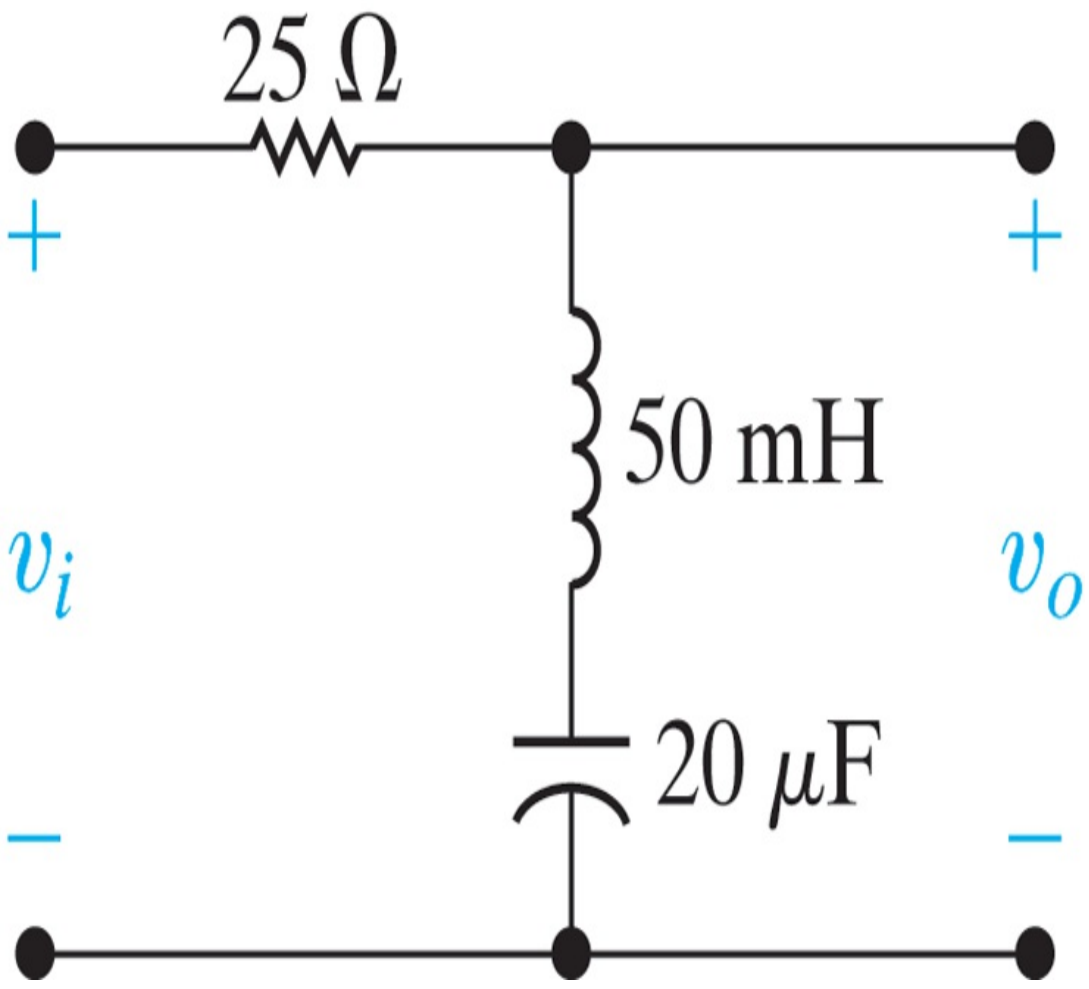


Figure P14.35 Full Alternative Text

2. 14.36 For the bandreject filter in Fig. P14.35,

1. a) Find H(jω) at ωo, ωc1, ωc2, 0.125ωo, and 8ωo.

2. b) If vi=5cosωt V, write the steady-state expression for vo when ω=ωo, ω=ωc1, ω=ωc2, ω=0.125ωo, and ω=8ωo.

3. 14.37 Design an RLC bandreject filter (see Fig. 14.28[a]) with a quality of 5 and a center frequency of 20 krad/s, using a 50 nF capacitor.

    1. a) Draw your circuit, labeling the component values and output voltage.

    2. b) For the filter in part (a), calculate the bandwidth and the values of the two cutoff frequencies.

4. 14.38 The input to the RLC bandreject filter designed in Problem 14.37 is 50 cosωt V. Find the voltage drop across the series combination of the inductor and capacitor when (a) ω=ωo; (b) ω=ωc1; (c) ω=ωc2; (d) ω=0.1ωo; (e) ω=10ωo.

5. 14.39 The input to the RLC bandreject filter designed in Problem 14.37 is 50cosωt V. Find the voltage drop across the resistor when (a) ω=ωo; (b) ω=ωc1; (c) ω=ωc2; (d) ω=0.1ωo; (e) ω=10ωo.

6. 14.40

    1. a) Show (via a qualitative analysis) that the circuit in Fig. P14.40 is a bandreject filter.

    2. b) Support the qualitative analysis of (a) by finding the voltage transfer function of the filter.

    3. c) Derive the expression for the center frequency of the filter.

    4. d) Derive the expressions for the cutoff frequencies ωc1 and ωc2.

    5. e) What is the expression for the bandwidth of the filter?

    6. f) What is the expression for the quality factor of the circuit?

Figure P14.40

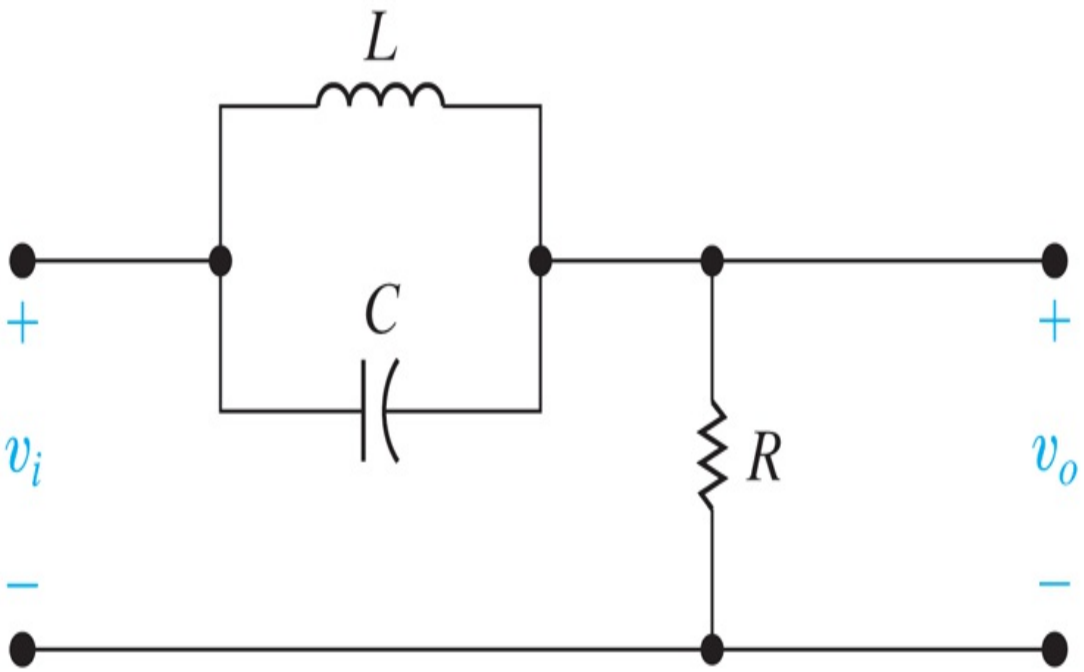


Figure P14.40 Full Alternative Text

7. 14.41 PSPICEMULTISIM For the bandreject filter in Fig. P14.41, calculate (a) ωo; (b) fo; (c) Q; (d) ωc1; (e) fc1; (f) ωc2; (g) fc2; and (h) β in kilohertz.

Figure P14.41

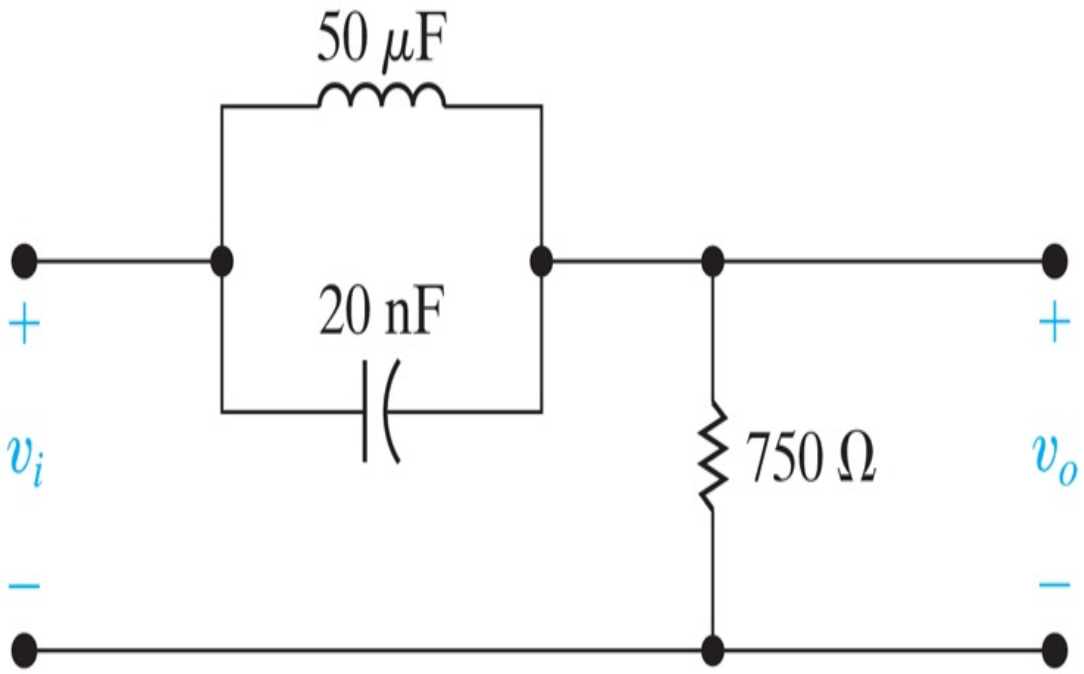


Figure P14.41 Full Alternative Text

8. 14.42 DESIGNPROBLEM_PSPICEMULTISIM Use a 500 nF capacitor to design a bandreject filter, as shown in Fig. P14.42. The filter has a center frequency of 4 kHz and a quality factor of 5.

    1. a) Specify the numerical values of R and L.
    2. b) Calculate the upper and lower corner, or cutoff, frequencies in kilohertz.
    3. c) Calculate the filter bandwidth in kilohertz.

Figure P14.42

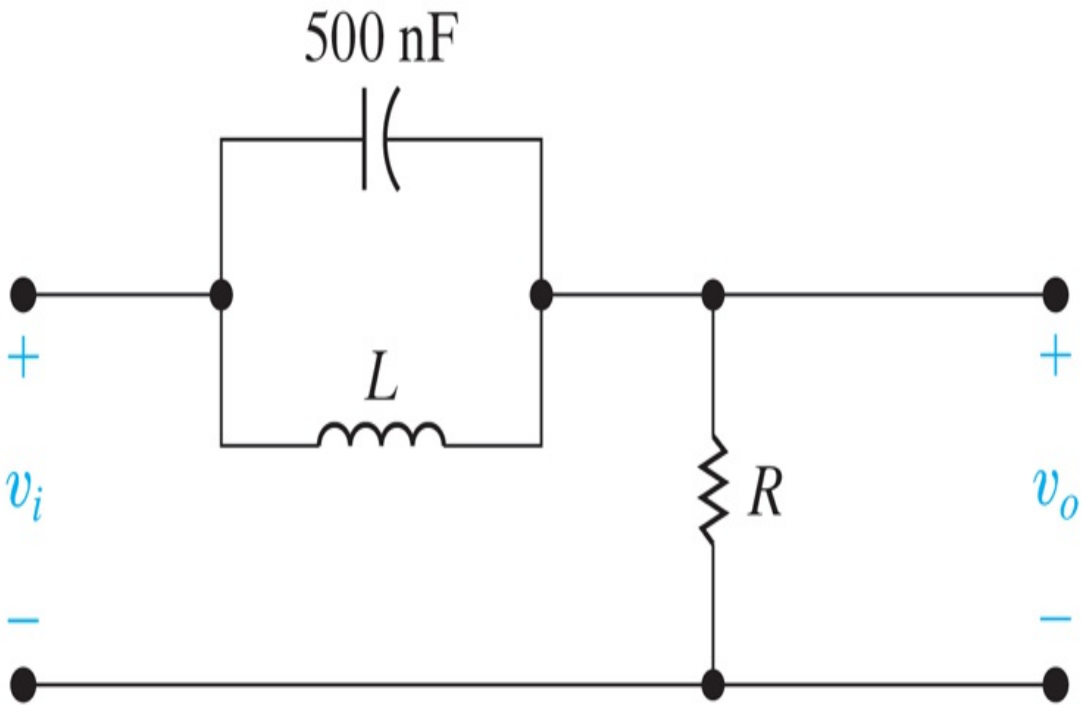


Figure P14.42 Full Alternative Text

9. 14.43 PSPICEMULTISIM Assume the bandreject filter in Problem 14.42 is loaded with a 1 kΩ resistor.
    1. a) What is the quality factor of the loaded circuit?
    2. b) What is the bandwidth (in kilohertz) of the loaded circuit?
    3. c) What is the upper cutoff frequency in kilohertz?
    4. d) What is the lower cutoff frequency in kilohertz?
10. 14.44 Design a parallel RLC bandreject filter using only three components from Appendix H that comes closest to meeting the filter specifications in Problem 14.42.
    1. a) Draw your filter, labeling all component values and the input and output voltages.
    2. b) Calculate the percent error in this new filter's center frequency and quality factor when compared to the values specified in

Problem 14.42.

11. 14.45 The purpose of this problem is to investigate how a resistive load connected across the output terminals of the bandreject filter shown in Fig. 14.28(a) affects the behavior of the filter. The loaded filter circuit is shown in Fig. P14.45.

    1. a) Find the voltage transfer function Vo/Vi.
    2. b) What is the expression for the center frequency?
    3. c) What is the expression for the bandwidth?
    4. d) What is the expression for the quality factor?
    5. e) Evaluate H(jωo).
    6. f) Evaluate H(j0).
    7. g) Evaluate H(j∞).
    8. h) What are the expressions for the corner frequencies ωc1 and ωc2?

Figure P14.45

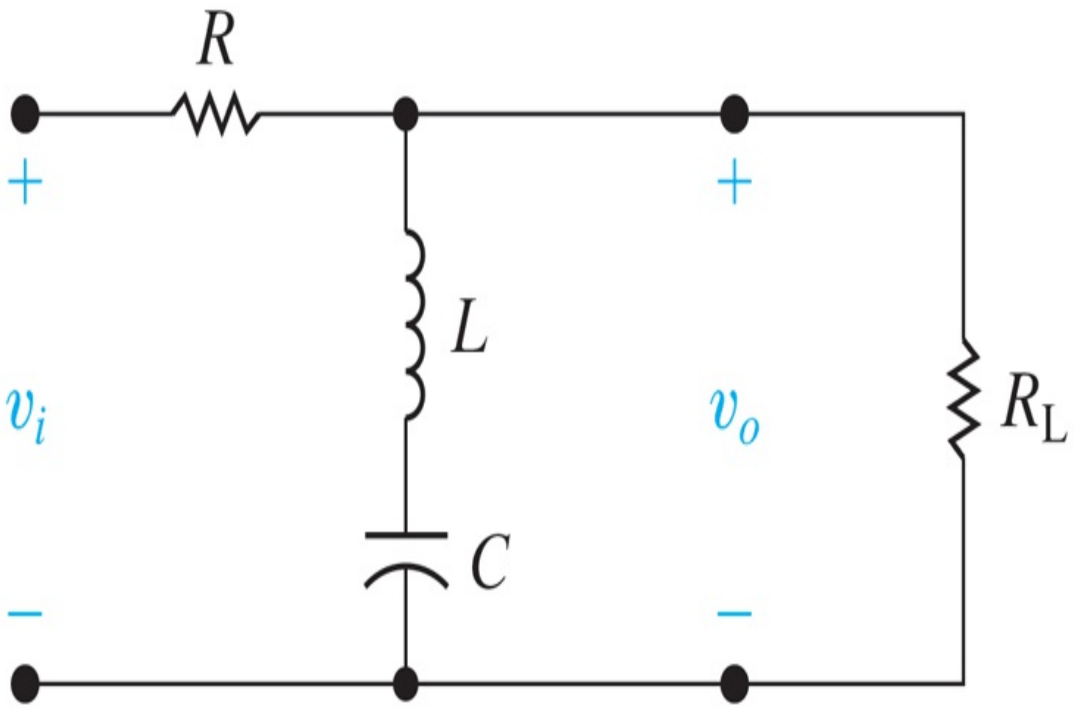


Figure P14.45 Full Alternative Text

12. 14.46 PSPICEMULTISIM The parameters in the circuit in Fig. P14.45 are R=30 Ω, L=1 μH, C=4 pF, and RL=150 Ω.

    1. a) Find ωo, β (in kilohertz), and Q.
    2. b) Find H(j0) and H(j∞).
    3. c) Find fc2 and fc1.
    4. d) Show that if RL is expressed in ohms the Q of the circuit is

       Q=503[1+(30/RL)].
    5. e) Plot Q versus RL for 10 Ω≤RL≤300 Ω.

13. 14.47 PSPICEMULTISIM A 500 Ω load is added to the bandreject filter shown in Fig. P14.42. The center frequency of the filter is 25 krad/s, and the capacitor is 25 nF. At very low and very high frequencies, the amplitude of the sinusoidal output voltage should be at least 90% of the

amplitude of the sinusoidal input voltage.

1. a) Specify the numerical values of R and L.

2. b) What is the quality factor of the circuit?

Sections 14.1–14.5

1. 14.48 Consider the following voltage transfer function:

   H(s)=VoVi
   =1010s2+50,000+1010.

   1. a) At what frequencies (in radians per second) is the magnitude of the transfer function equal to unity?

   2. b) At what frequency is the magnitude of the transfer function maximum?

   3. c) What is the maximum value of the transfer function magnitude?

2. 14.49 Consider the series RLC circuit shown in Fig. P14.49. When the output is the voltage across the resistor, we know this circuit is a bandpass filter. When the output is the voltage across the series combination of the inductor and capacitor, we know this circuit is a bandreject filter. This problem investigates the behavior of this circuit when the output is across the inductor.

   1. a) Find the transfer function, H(s)=Vo(s)/Vi(s) when Vo(s) is the voltage across the inductor.

   2. b) Find the magnitude of the transfer function in part (a) for very low frequencies.

   3. c) Find the magnitude of the transfer function in part (a) for very high frequencies.

4. d) Based on your answers in parts (b) and (c), what type of filter is this?

5. e) Suppose R=600 Ω, L=400 mH, C=2.5 μF. Calculate the cutoff frequency of this filter, that is, the frequency at which the magnitude of the transfer function is 1/2.

Figure P14.49

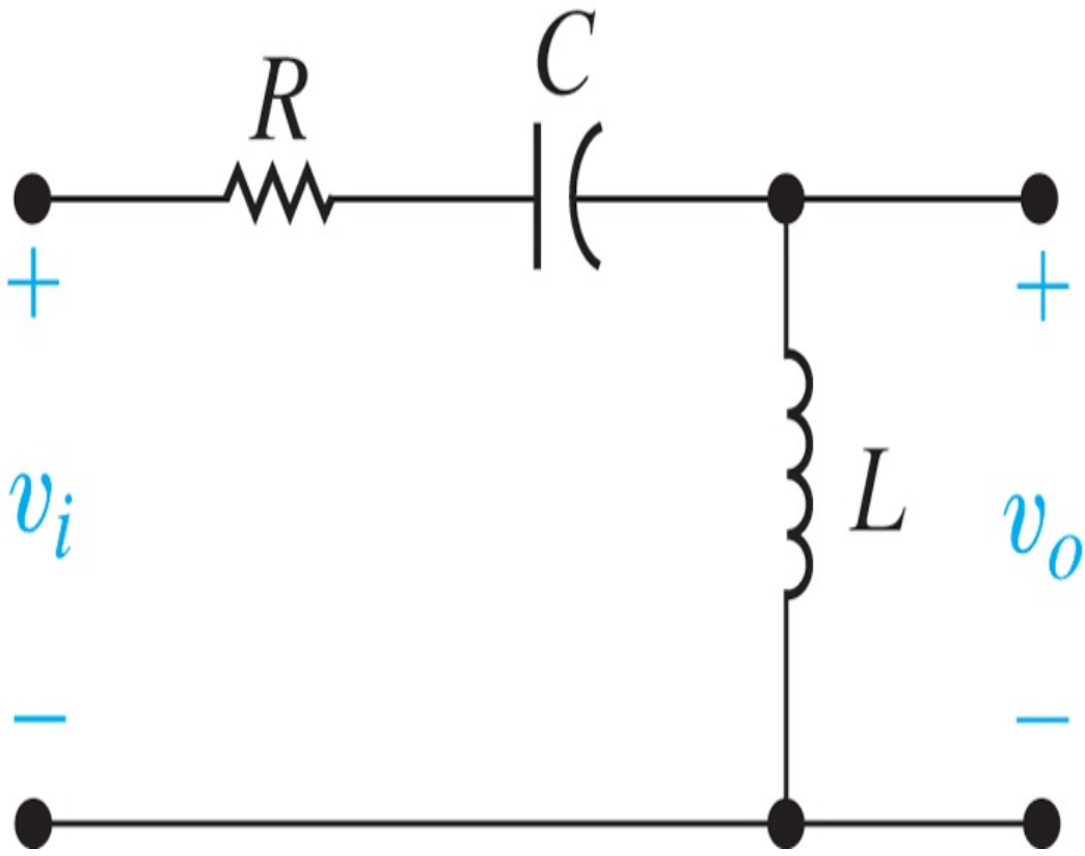


Figure P14.49 Full Alternative Text

3. 14.50 Repeat Problem 14.49 for the circuit shown in Fig. P14.50. Note that the output voltage is now the voltage across the capacitor.

Figure P14.50

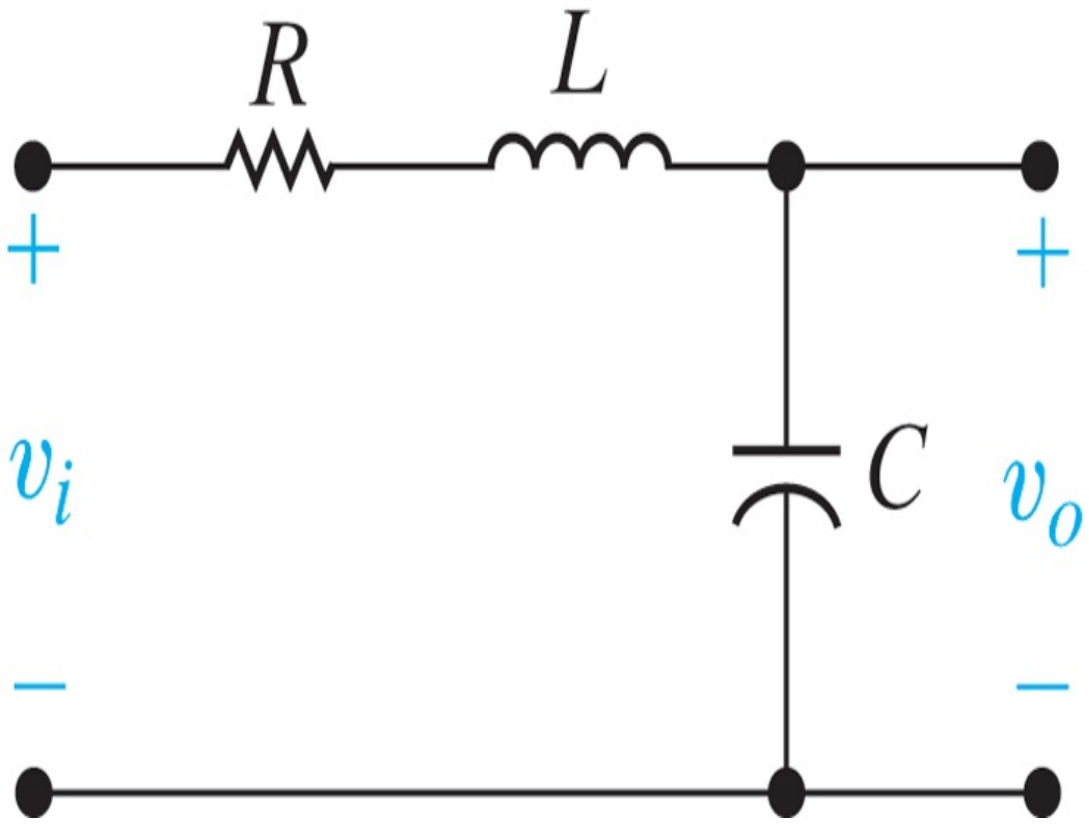


Figure P14.50 Full Alternative Text

4. 14.51 PRACTICALPERSPECTIVE_DESIGNPROBLEM_ Design a series RLC bandpass filter (see Fig. 14.27) for detecting the low-frequency tone generated by pushing a telephone button as shown in Fig. 14.32.

    1. a) Calculate the values of L and C that place the cutoff frequencies at the edges of the DTMF low-frequency band. Note that the resistance in standard telephone circuits is always R=600 Ω.

    2. b) What is the output amplitude of this circuit at each of the low-band frequencies, relative to the peak amplitude of the bandpass filter?

    3. c) What is the output amplitude of this circuit at the lowest of the high-band frequencies?

5. 14.52 PRACTICALPERSPECTIVE_DESIGNPROBLEM_ Design a

DTMF high-band bandpass filter similar to the low-band filter design in Problem 14.51. Be sure to include the fourth high-frequency tone, 1633 Hz, in your design. What is the response amplitude of your filter to the highest of the low-frequency DTMF tones?

6. 14.53 PRACTICALPERSPECTIVE_DESIGNPROBLEM_ The 20 Hz signal that rings a telephone's bell has to have a very large amplitude to produce a loud enough bell signal. How much larger can the ringing signal amplitude be, relative to the low-band DTMF signal, so that the response of the filter in Problem 14.51 is no more than half as large as the largest of the DTMF tones?

Chapter 15 Active Filter Circuits

Chapter Contents

1. 15.1 First-Order Low-Pass and High-Pass Filters
2. 15.2 Scaling
3. 15.3 Op Amp Bandpass and Bandreject Filters
4. 15.4 Higher-Order Op Amp Filters
5. 15.5 Narrowband Bandpass and Bandreject Filters

Chapter Objectives

1. Know the op amp circuits that behave as first-order low-pass and high-pass filters and be able to calculate component values for these circuits to meet specifications of cutoff frequency and passband gain.

2. Be able to design filter circuits starting with a prototype circuit and use scaling to achieve desired frequency response characteristics and component values.

3. Understand how to use cascaded first- and second-order Butterworth filters to implement low-pass, high-pass, bandpass, and bandreject filters of any order.

4. Be able to use the design equations to calculate component values for prototype narrowband bandpass, and narrowband bandreject filters to meet desired filter specifications.

Active filter circuits, which employ op amps, can be configured as low-pass filters, high-pass filters, bandpass filters and bandreject filters, just like the passive circuits we analyzed in Chapter 14. Active filter circuits have several advantages over their passive counterparts.

- Active circuits can produce bandpass and bandreject filters without using inductors. This is advantageous because inductors are usually large, heavy, and costly, and they can introduce electromagnetic field effects that compromise the desired frequency response characteristics.

- Active filters provide control over passband amplification that is not available in passive filter circuits. Examine the transfer functions of all the filter circuits from Chapter 14 and you will notice that the maximum magnitude does not exceed 1. This is not surprising, since most of the transfer functions in Chapter 14 were derived using voltage division. Active filters permit us to specify both the filtering characteristics and a passband gain, an advantage over passive filters.

- Active filters can have resistive loads at their outputs whose presence *does not* alter the filter characteristics due to the properties of ideal op amps. This is another advantage of active filters, as both the cutoff frequency and the passband magnitude of passive filters can change when a resistive load is added at the output.

Thus, we can implement filter designs using active circuits when physical size, passband amplification, and load variation are important parameters in the design specifications.

In this chapter, we analyze a few of the many filter circuits that use op amps. You will see how these op amp circuits overcome the disadvantages of passive filter circuits. We will also combine basic op amp filter circuits to achieve specific frequency responses and to attain a more nearly ideal filter response. Note that throughout this chapter we assume that every op amp behaves as an ideal op amp.

Practical Perspective

Bass Volume Control

The circuits we analyze in this chapter are frequency selective, which means that the circuit's behavior depends on its sinusoidal input frequency. These filter circuits play important roles in many audioelectronic applications.

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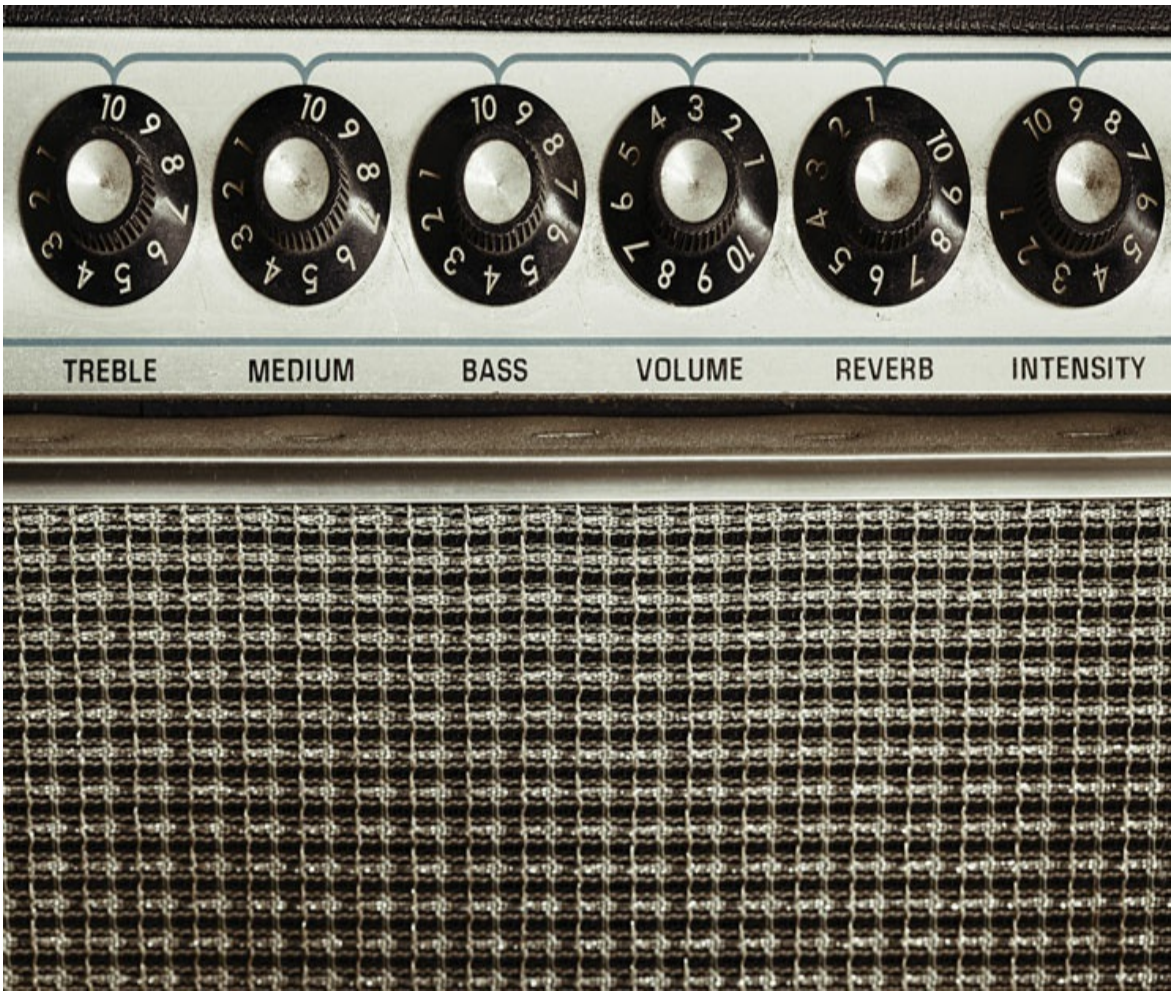
Audioelectronic systems such as radios, CD players, and home stereo systems often provide separate volume controls labeled “treble” and “bass.” These controls allow you to set the volume of high-frequency audio signals (“treble”) and the volume of low-frequency audio signals (“bass”) independently. Adjusting the amplification (boost) or attenuation (cut) in these two frequency bands allows you to customize the sound with more precision than you get using a single volume control.



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The Practical Perspective example at the end of this chapter presents a bass volume control circuit composed of a single op amp together with resistors and capacitors. An adjustable resistor controls the amplification in the bass frequency range.

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15.1 First-Order Low-Pass and High-Pass Filters

Consider the circuit in Fig. 15.1. When the frequency of the source is varied, only the impedance of the capacitor is affected. At very low frequencies, the capacitor acts like an open circuit, and the op amp circuit acts like an inverting amplifier with a gain of −R2 / R1. At very high frequencies, the capacitor acts like a short circuit that connects the op amp's output to ground. The op amp circuit in Fig. 15.1 thus functions as a low-pass filter with a passband gain of −R2 / R1.

Figure 15.1 A first-order low-pass filter.

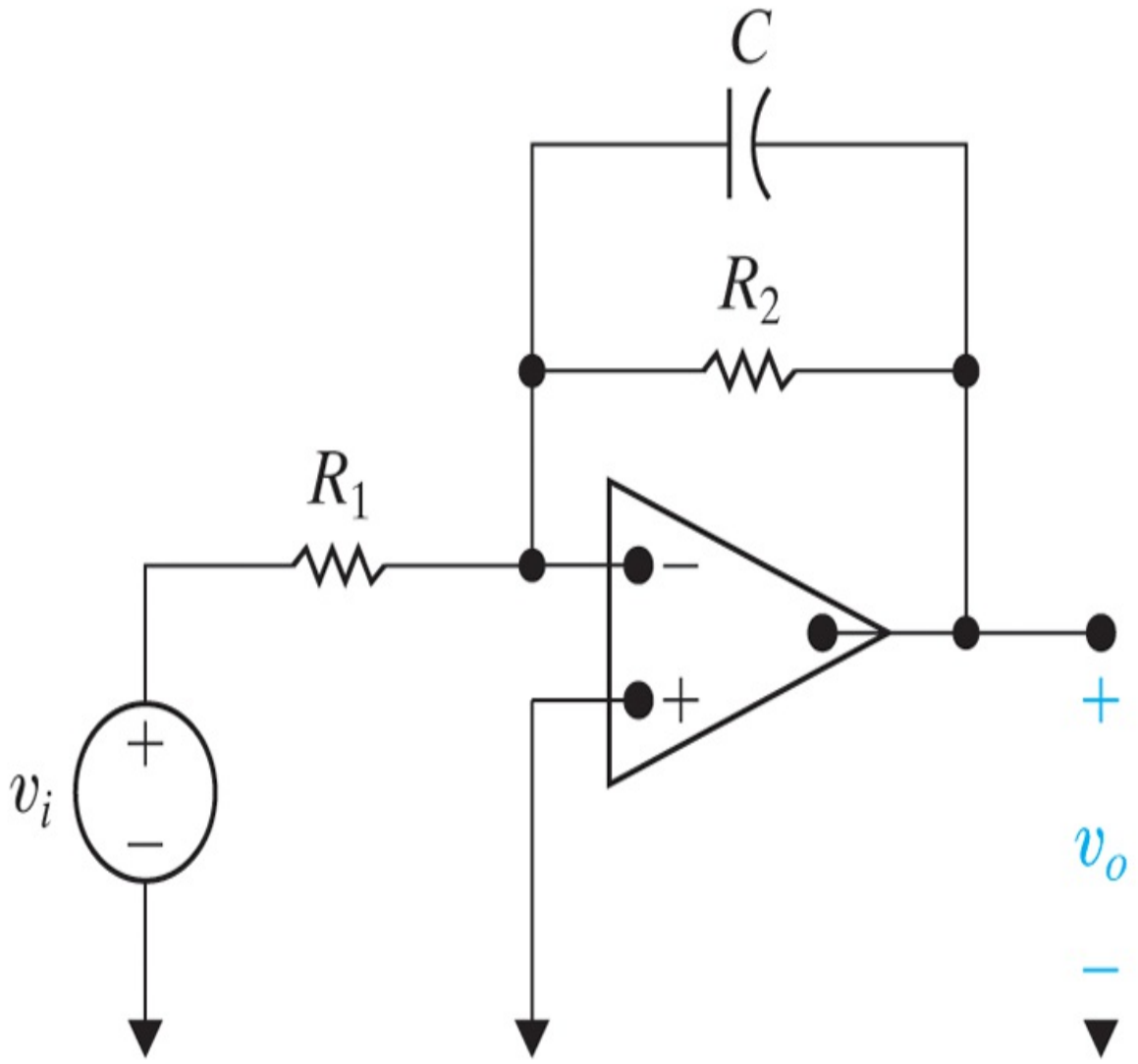


Figure 15.1 Full Alternative Text

To confirm this qualitative assessment, we compute the transfer function H(s)=Vo(s) / Vi(s). Note that the circuit in Fig. 15.1 has the general form of the circuit shown in Fig. 15.2, where the impedance in the input path (Zi) is the resistor R1, and the impedance in the feedback path (Zf) is the parallel combination of the resistor R2 and the capacitor impedance 1/sC.

Figure 15.2 A general op amp circuit.

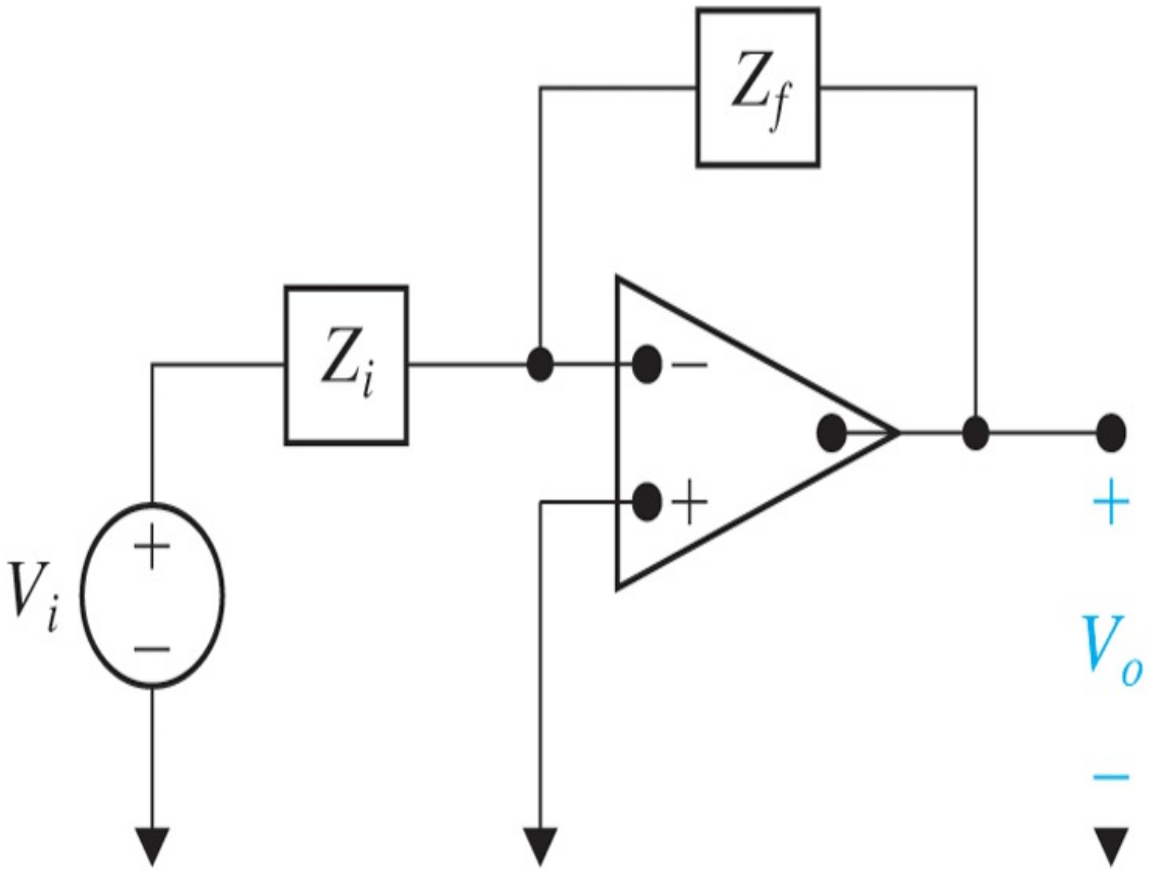


Figure 15.2 Full Alternative Text

The circuit in Fig. 15.2 has the same configuration as the inverting amplifier circuit from Chapter 5, so its transfer function is −Zf / Zi. Therefore, the transfer function for the circuit in Fig. 15.1 is

$$H(s)=-\frac{Z_f}{Z_i}=-\frac{R_2\,\|\,(1/sC)}{R_1}=-\frac{K\omega_c}{s+\omega_c}, \quad (15.1)$$

where

$$K=\frac{R_2}{R_1}, \quad (15.2)$$

and

$$\omega_c=\frac{1}{R_2C}. \quad (15.3)$$

Note that Eq. 15.1 has the same form as the general equation for low-pass filters given in Eq. 14.6, with an important difference: the gain in the

passband, K, is set by the ratio R2 / R1. The op amp low-pass filter thus permits the passband gain and the cutoff frequency to be specified independently.

A Note About Frequency Response Plots

Frequency response plots, introduced in Chapter 14, provide valuable insight into the way a filter circuit functions. Thus, we make extensive use of frequency response plots in this chapter, too. The frequency response plots in Chapter 14 have two components—a plot of the transfer function magnitude versus frequency and a plot of the transfer function phase angle, in degrees, versus frequency. The two plots are usually stacked on top of one another so that they can share the same frequency axis.

In this chapter, we use a special type of frequency response plot called the **Bode plot**. Bode plots are discussed in detail in Appendix E, which includes information about how to construct these plots by hand. You will probably use a computer to construct Bode plots, so here we summarize the special features of these plots. Bode plots differ from the frequency response plots in Chapter 14 in two important ways.

- A Bode plot uses a logarithmic axis for the frequency values instead of a linear axis. This permits us to plot a wider range of frequencies of interest. Normally, we plot three or four decades of frequencies, say from 102 rad / s to 106 rad / s, or 1 kHz to 1 MHz, choosing the frequency range where the transfer function characteristics are changing. If we plot both the magnitude and phase angle plots, they share the frequency axis.

- The Bode magnitude is plotted in decibels (dB) versus the log of the frequency, instead of plotting the absolute magnitude versus linear frequency. The decibel is discussed in Appendix D. Briefly, if the magnitude of the transfer function is $|H(j\omega)|$, its value in dB is given by

$A_{dB} = 20 \log_{10} |H(j\omega)|$.

Note that although $|H(j\omega)|$ is an unsigned quantity, A_{dB} is a signed quantity. When $A_{dB} = 0$, the transfer function magnitude is 1, since $20 \log_{10}(1) = 0$. When $A_{dB} < 0$, the transfer function magnitude is between 0 and 1, and when $A_{dB} > 0$, the transfer function magnitude is greater than 1. Finally, note that

$20 \log_{10} |1 / 2| = -3$ dB.

Recall that at a filter's cutoff frequency the transfer function's magnitude has been reduced from its maximum value by $1 / 2$. Translating this definition to magnitude in dB, we find that a transfer function's magnitude at its cutoff frequency has been reduced from its maximum magnitude by 3 dB. For example, if the magnitude of a low-pass filter in its passband is 26 dB, the magnitude used to find the cutoff frequency is $26 - 3 = 23$ dB.

Example 15.1 illustrates the design of a first-order low-pass filter to meet desired specifications of passband gain and cutoff frequency; it also illustrates a Bode magnitude plot of the filter's transfer function.

Example 15.1 Designing a Low-Pass Op Amp Filter

Using the circuit shown in Fig. 15.1, calculate values for C and R2 that, together with $R1 = 1\Omega$, produce a low-pass filter having a gain of 1 in the passband and a cutoff frequency of 1 rad / s. Find the transfer function for this filter and use it to sketch a Bode magnitude plot of the filter's frequency response.

Solution

Equation 15.2 gives the passband gain in terms of R1 and R2, so we use it to calculate the required value of R2:

$R2 = KR1 = (1)(1) = 1\Omega$.

Now use Eq. 15.3 to calculate C and satisfy the cutoff frequency specification:

$C = \frac{1}{R2\omega c} = \frac{1}{(1)(1)} = 1F$.

The transfer function for the low-pass filter is given by Eq. 15.1:

$H(s) = \frac{-K\omega c}{s + \omega c} = \frac{-1}{s + 1}$.

The Bode plot of $|H(j\omega)|$ is shown in Fig. 15.3. We have just designed a **prototype** low-pass op amp filter. It uses a resistor value of 1Ω and a capacitor value of 1 F, and it has a cutoff frequency of 1 rad / s. As we shall see in the next section, prototype filters provide a useful starting point for the design of filters that use more realistic component values to achieve a desired frequency response.

Figure 15.3 The Bode magnitude plot of the low-pass filter from Example 15.1.

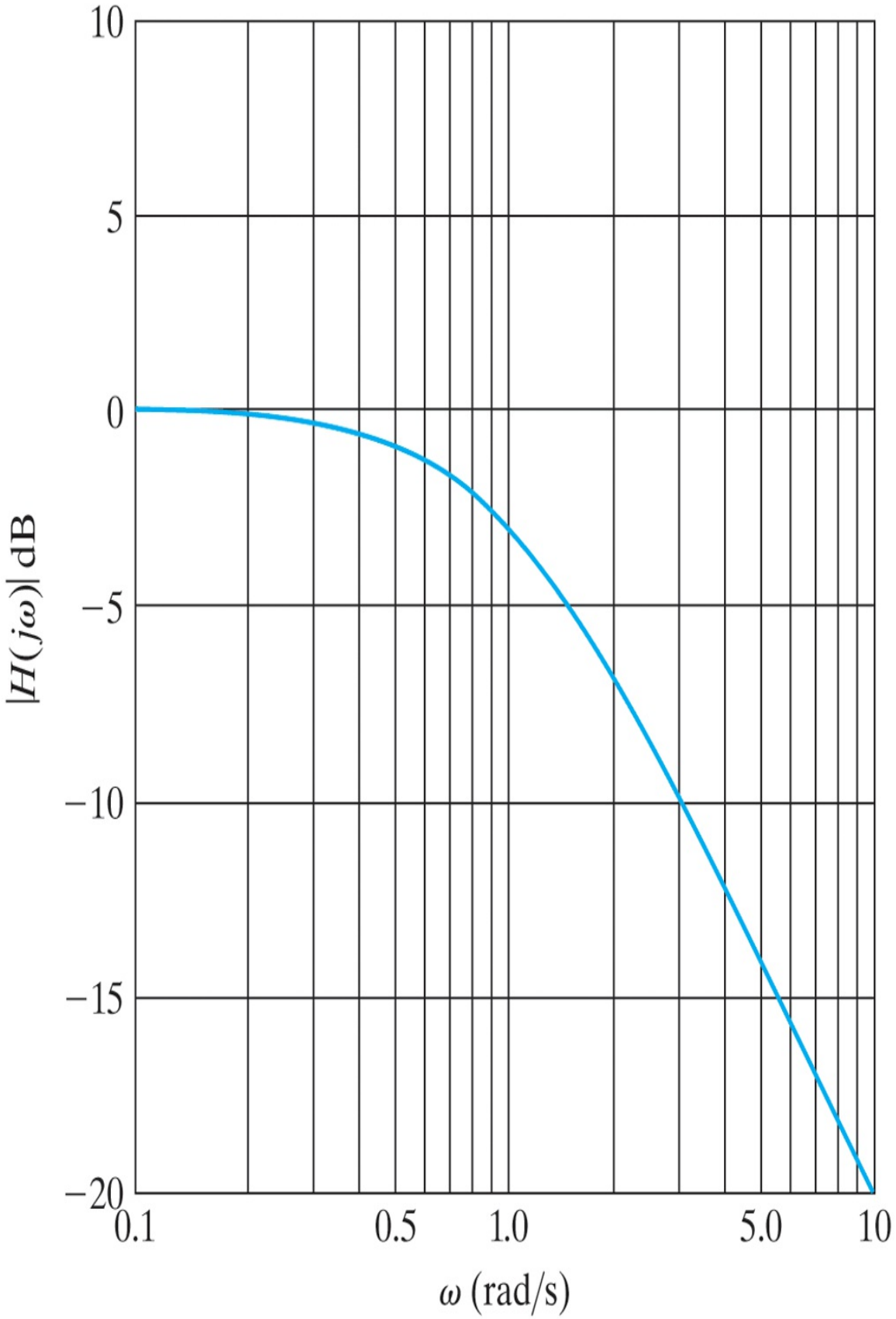

10
5
0
−5
−10
−15
−20
0.1
0.5
1.0
5.0
10
|H(jω)| dB
ω (rad/s)

Figure 15.3 Full Alternative Text

You may have recognized the circuit in Fig. 15.1 as the integrating amplifier circuit introduced in Chapter 7. They are indeed the same circuit, so integrating in the time domain corresponds to low-pass filtering in the frequency domain. This relationship between integration and low-pass filtering is also evident from the operational Laplace transform for integration derived in Chapter 12.

The circuit in Fig. 15.4 is an active first-order high-pass filter. This circuit also has the general form of the circuit in Fig. 15.2, only now the impedance in the input path is the series combination of R1 and 1/sC , and the impedance in the feedback path is the resistor R2. The transfer function for the circuit in Fig 15.4 is thus

Figure 15.4 A first-order high-pass filter.

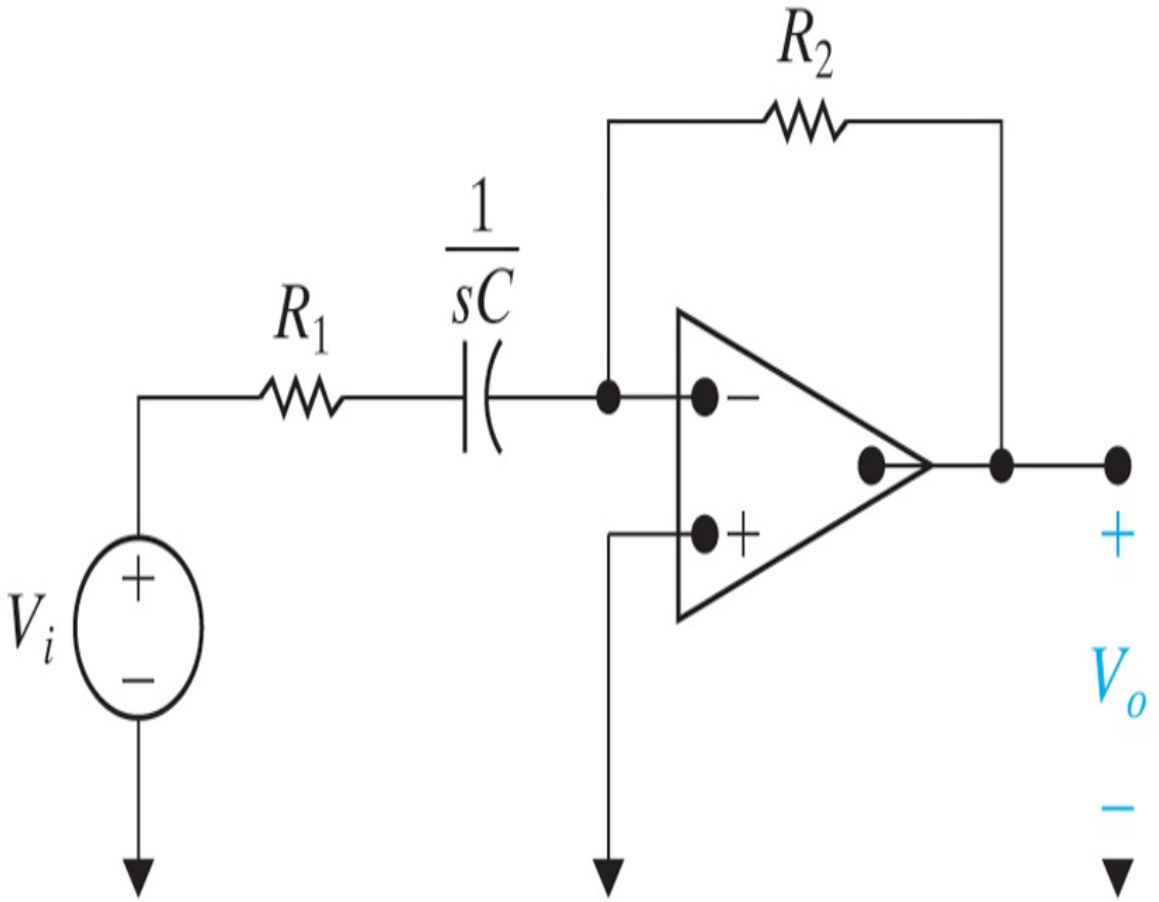


Figure 15.4 Full Alternative Text

$$H(s) = -\frac{Z_f}{Z_i} = -\frac{R_2}{R_1 + \frac{1}{sC}} = -\frac{Ks}{s + \omega_c}, \quad (15.4)$$

where

$$K = \frac{R_2}{R_1}, \quad (15.5)$$

and

$$\omega_c = \frac{1}{R_1 C}. \quad (15.6)$$

The form of the transfer function given in Eq. 15.4 is the same as that in Eq. 14.10, the equation for passive high-pass filters. Again, the active filter can have a passband gain greater than 1.

Example 15.2 designs an active high-pass filter that must meet frequency response specifications from a Bode plot.

Example 15.2 Designing a High-Pass Op Amp Filter

Figure 15.5 shows the Bode magnitude plot of a high-pass filter. Using the active high-pass filter circuit in Fig. 15.4, calculate values of R1 and R2 that produce the desired magnitude response. Use a 0.1 μF capacitor. If a 10 kΩ load resistor is added to this filter, how will the magnitude response change?

Figure 15.5 The Bode magnitude plot of the high-pass filter for Example 15.2.

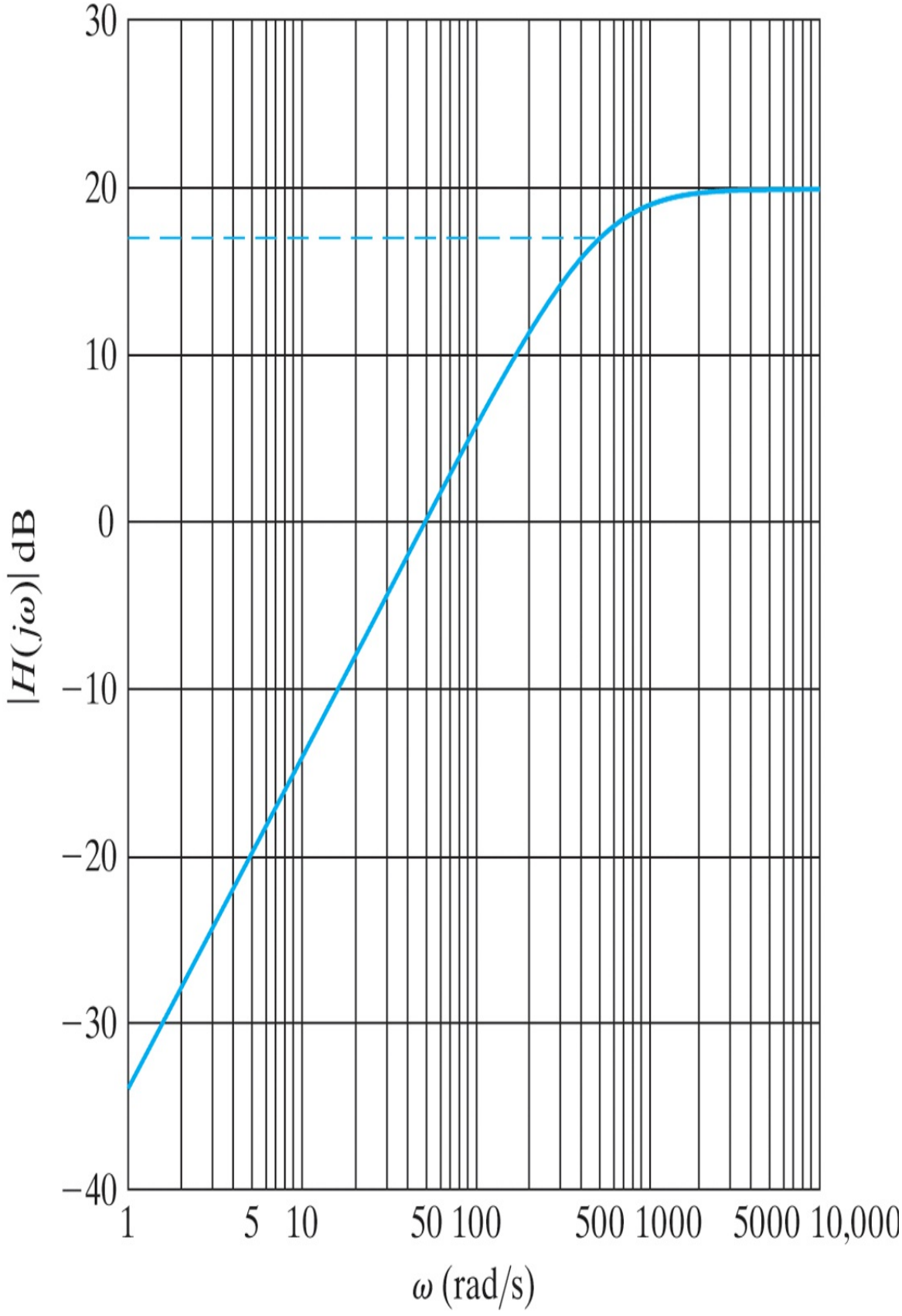

30
20
10
0
−10
−20
−30
−40
|H(jω)| dB
1
5
10
50
100
500
1000
5000
10,000
ω (rad/s)

Figure 15.5 Full Alternative Text

Solution

Begin by writing a transfer function that has the magnitude plot shown in Fig. 15.5. To do this, note that the gain in the passband is 20 dB; therefore, K=10. Also note that the 3 dB point is 500 rad/s, which must be the filter's cutoff frequency. Equation 15.4 is the transfer function for a high-pass filter, so the transfer function that has the magnitude response shown in Fig. 15.5 is given by

$$H(s)=\frac{-10s}{s+500}.$$

Next, equate this transfer function with Eq. 15.4:

$$H(s)=\frac{-10s}{s+500}=\frac{-(R2\,/\,R1)s}{s+(1\,/\,R1C)}.$$

Equating the numerators and denominators and then simplifying, we get two equations:

$$10=R2R1,\ 500=1R1C.$$

Using the specified value of C (0.1 μF), we find

$$R1=20k\Omega,\ R2=200\ k\Omega.$$

The circuit is shown in Fig. 15.6.

Figure 15.6 The high-pass filter for Example 15.2.

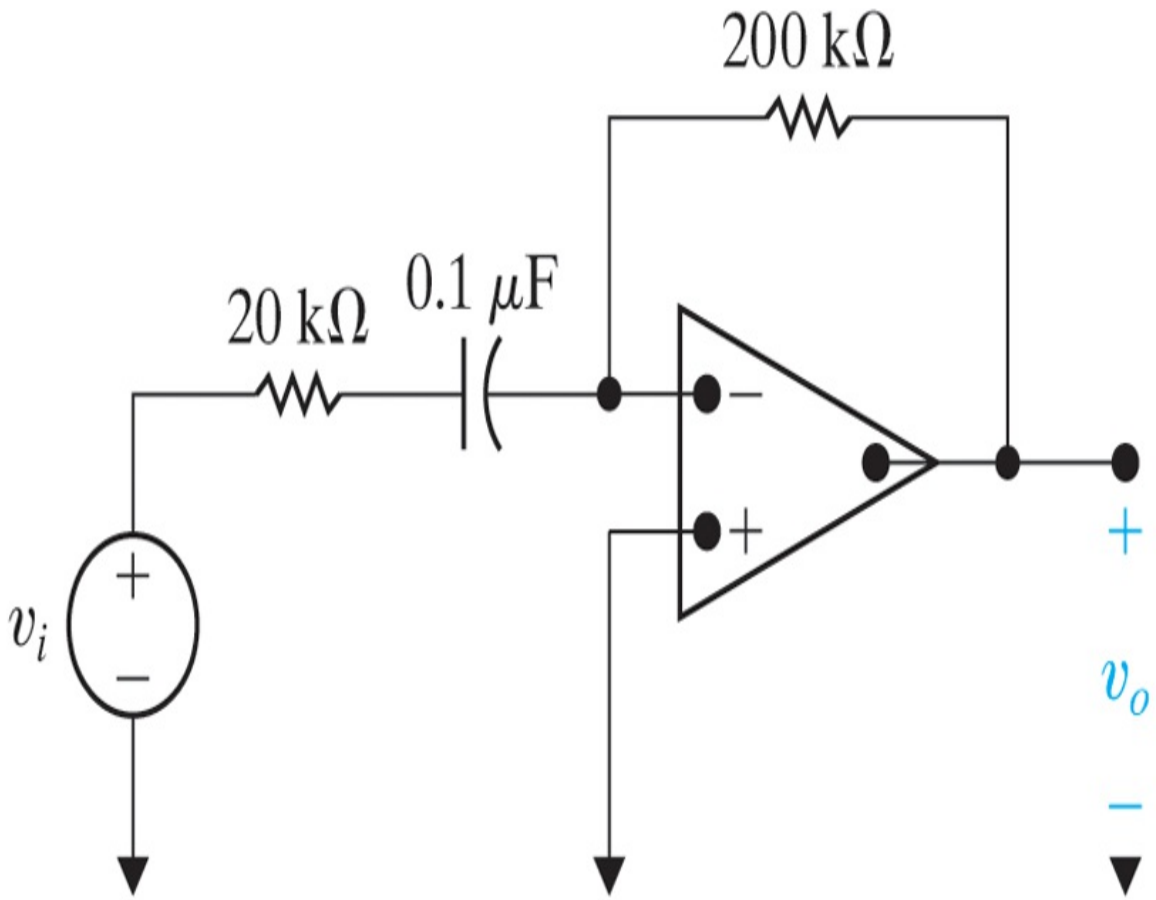


Figure 15.6 Full Alternative Text

Because we have made the assumption that the op amp is ideal, adding any load resistor, regardless of its resistance, has no effect on the filter circuit. Thus, the magnitude response of a high-pass filter with a load resistor is the same as that of a high-pass filter with no load resistor, which is depicted in Fig. 15.5.

Assessment Problems

Objective 1—Know the op amp circuits that behave as first-order low-pass and high-pass filters and be able to calculate their component values

1. 15.1 Compute the values for R2 and C that yield a high-pass filter with a passband gain of 1 and a cutoff frequency of 1 rad / s if R1 is 1Ω. (*Note:*

This is the prototype high-pass filter.)

Answer: R2=1Ω,C=1F.

2. 15.2 Compute the resistor values needed for the low-pass filter circuit in Fig. 15.1 to produce the transfer function

   $H(s)=-20{,}000s+5000$.

   Use a 5 μF capacitor.

   Answer: R1=10Ω,R2=40Ω.

SELF-CHECK: Also try Chapter Problems 15.1 and 15.8.

15.2 Scaling

When designing and analyzing filter circuits, it is convenient to use component values like 1Ω, 1 H, and 1 F because any computations required are simple. Unfortunately, these values are unrealistic for specifying practical components, and they result in filters with undesirable characteristics, like a cutoff frequency of 1 rad/s. But we can use **scaling** to transform the convenient component values into realistic values and transform undesirable filter characteristics into desirable ones. There are two types of scaling: magnitude and frequency.

Magnitude Scaling

We scale a circuit in magnitude by multiplying the impedance at a given frequency by the scale factor km. Thus, we multiply all resistors and inductors by km and all capacitors by 1 / km. If we let unprimed variables represent the initial values of the parameters, and we let primed variables represent the scaled values of the variables, we have

$R' = kmR, \quad L' = kmL, \quad \text{and} \quad C' = C/km.$

Note that km is by definition a positive real number that can be either less than or greater than 1.

Frequency Scaling

We scale a circuit in frequency by changing the circuit's component values so that at the new frequency, the impedance of each element is the same as it was at the original frequency. Because resistive impedance is independent of frequency, resistors are unaffected by frequency scaling. If we let kf denote the frequency scale factor, inductor and capacitor values are both multiplied by 1 / kf. Thus, for frequency scaling,

$R'=R$, $L'=L/kf$, and $C'=C/kf$.

The frequency scale factor kf is also a positive real number that can be less than or greater than 1.

A circuit can be scaled simultaneously in both magnitude and frequency. The scaled values (primed) in terms of the original values (unprimed) are

Component Scale Factors

$R' = k_m R$, $L' = k_m k_f L$, $C' = 1 k_m k_f C$. (15.7)

The Use of Scaling in the Design of Filters

When designing filters, follow these steps to use scaling:

- Select the cutoff frequency, ωc, to be 1 rad/s (if you are designing low- or high-pass filters), or select the center frequency, ωo, to be 1 rad/s (if you are designing bandpass or bandreject filters).
- Select a 1 F capacitor and calculate the values of the resistors needed to give the desired passband gain (if you are designing an active filter) and the 1 rad/s cutoff or center frequency.
- Use scaling to compute more realistic component values that give the desired cutoff or center frequency.

Example 15.3 illustrates the scaling process in general, and Example 15.4 illustrates the use of scaling in the design of a low-pass filter.

Example 15.3 Scaling a Series RLC

Filter

The passive series RLC filter shown in Fig. 15.7 has a center frequency of 1 / LC=1 rad/s, a bandwidth of R / L=1 rad/s, and thus a quality factor of 1. Use scaling to compute new values of R and L that yield a circuit with the same quality factor but with a center frequency of 500 Hz. Use a 2 μF capacitor.

Figure 15.7 The series RLC circuit for Example 15.3.

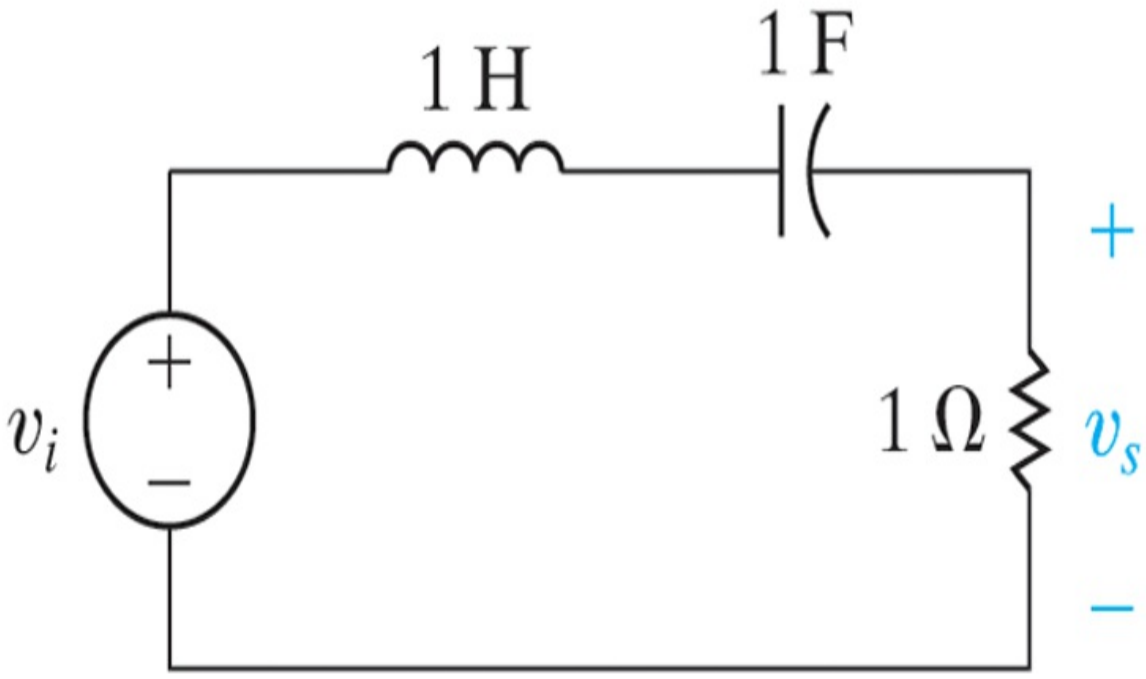


Figure 15.7 Full Alternative Text

Solution

Compute the frequency scale factor that will shift the center frequency from 1 rad/s to 500 Hz. Remember, the unprimed variables represent values before scaling, whereas the primed variables represent values after scaling.

$$kf=\omega'o\omega o=2\pi(500)1=3141.59.$$

Now, use Eq. 15.7 to compute the magnitude scale factor that, together with the frequency scale factor, will yield a capacitor value of 2 μF:

$$km=1kfCC'=1(3141.59)(2\times10-6)=159.155.$$

Use Eq. 15.7 again to compute the magnitude- and frequency-scaled values of R and L:

$$R'=kmR=159.155\Omega,$$
$$L'=kmkfL=50.66mH.$$

With these component values, the center frequency of the series RLC circuit is

$$1 / LC=3141.61 \text{ rad/s or } 500Hz,$$

and the bandwidth is

$$R / L=3141.61 \text{ rad/s or } 500Hz;$$

thus, the quality factor is still 1.

Example 15.4 Scaling a Prototype Low-Pass Op Amp Filter

Use the prototype low-pass op amp filter from Example 15.1, along with magnitude and frequency scaling, to compute the resistor values for a low-pass filter with a gain of 5, a cutoff frequency of 1000 Hz, and a feedback capacitor of 10 nF. Construct a Bode plot of the resulting transfer function’s magnitude.

Solution

To begin, use frequency scaling to place the cutoff frequency at 1000 Hz:

$$kf=\omega'c / \omega c=2\pi(1000)/1=6283.185,$$

where the primed variable has the new value and the unprimed variable has the old value of the cutoff frequency. Then compute the magnitude scale factor that, together with kf=6283.185, will scale the capacitor to 10 nF:

$$km=1kfCC'=1(6283.185)(10-8)=15,915.5.$$

Since resistors are scaled only by using magnitude scaling,

$$R'1=R'2=kmR=(15,915.5)(1)=15,915.5\Omega.$$

Finally, we need to meet the passband gain specification. We can adjust the values of either R′1 or R′2 because K=R′2/R′1. If we adjust R′2, we will change the cutoff frequency because $\omega'c=1 / R'2C'$. Therefore, we can adjust the value of R′1 to alter only the passband gain:

$$R'1=R'2 / K=(15,915.5) / (5)=3183.1\Omega.$$

The final component values are

$$R'1=3183.1\Omega,\ R'2=15,915.5\Omega,\ C'=0.01\mu F.$$

The transfer function of the filter is given by

$$H(s)=-31,415.93s+6283.185.$$

The Bode plot of this transfer function's magnitude is shown in Fig. 15.8.

Figure 15.8 The Bode magnitude plot of the low-pass

filter from Example 15.4.

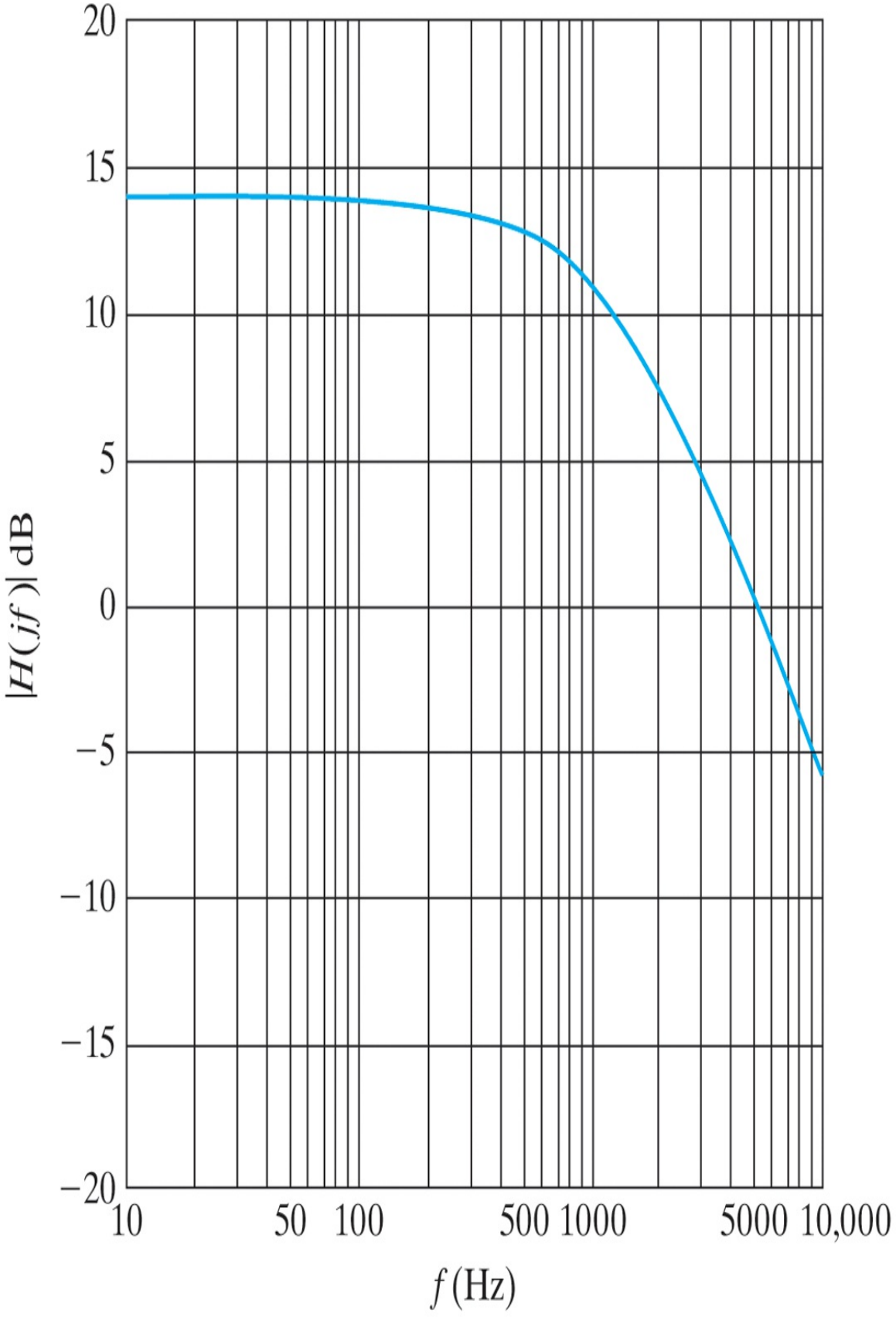

20
15
10
5
0
−5
−10
−15
−20
|H(jf)| dB
10
50
100
500
1000
5000
10,000
f (Hz)

Figure 15.8 Full Alternative Text

Assessment Problem

Objective 2—Be able to design filter circuits starting with a prototype and use scaling to achieve desired frequency response and component values

1. 15.3 What magnitude and frequency scale factors will transform the prototype high-pass filter into a high-pass filter with a 0.5 μF capacitor and a cutoff frequency of 10 kHz?

   Answer: kf=62,831.85, km=31.831.

SELF-CHECK: Also try Chapter Problems 15.15 and 15.16.

15.3 Op Amp Bandpass and Bandreject Filters

We now analyze and design op amp circuits that act as bandpass and bandreject filters. Our initial approach is motivated by the Bode plot construction shown in Fig. 15.9. We can see from the plot that the bandpass filter consists of three separate elements:

- A unity-gain low-pass filter whose cutoff frequency is ωc2, the larger of the two cutoff frequencies.
- A unity-gain high-pass filter whose cutoff frequency is ωc1, the smaller of the two cutoff frequencies.
- A gain component to provide the desired passband gain.

Figure 15.9 Constructing the Bode magnitude plot of a bandpass filter.

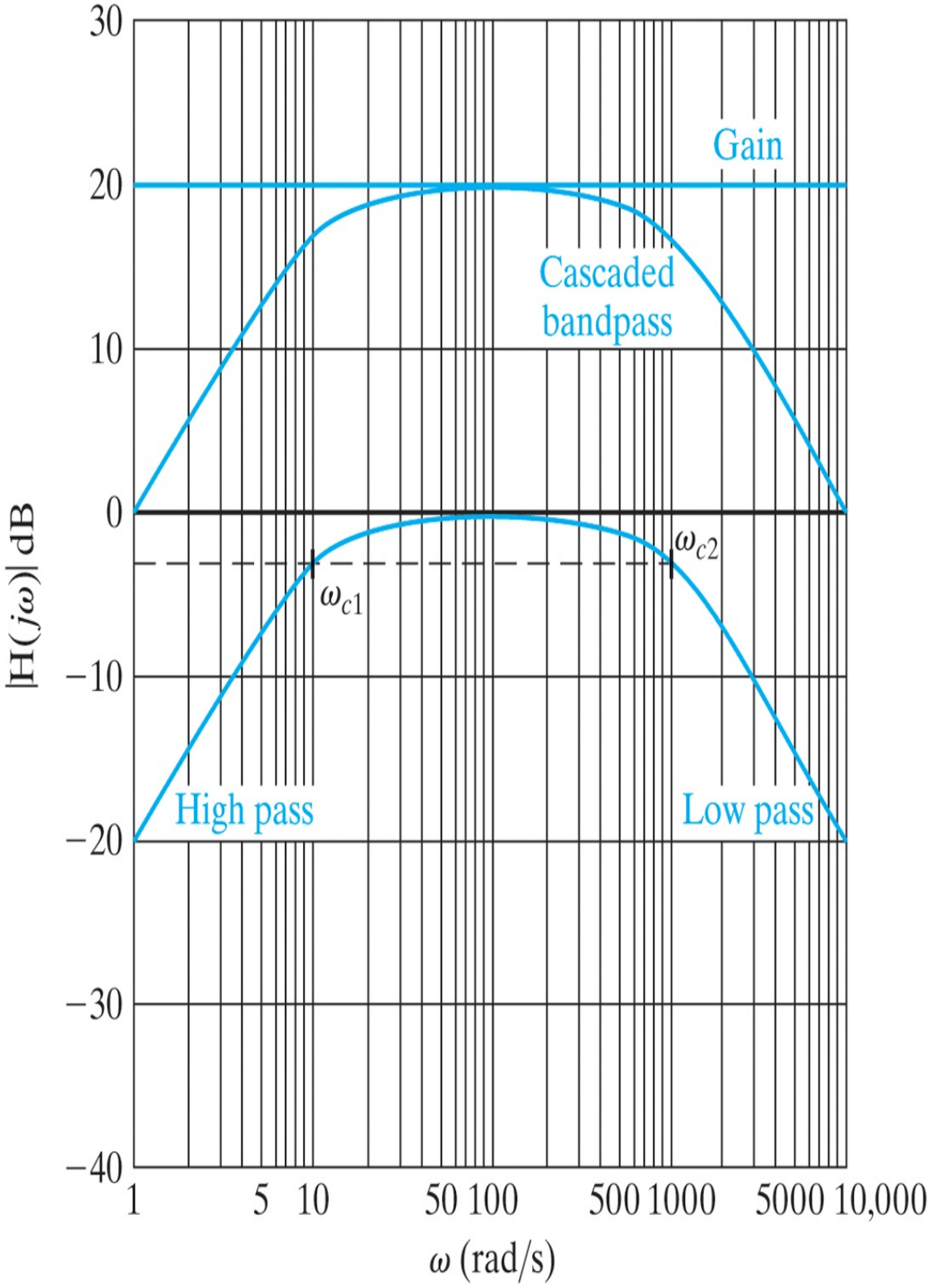

30
20
10
0
−10
−20
−30
−40
|H(jω)| dB
Gain
Cascaded
bandpass
ω_{c1}
ω_{c2}
High pass
Low pass
1
5
10
50
100
500
1000
5000
10,000
ω (rad/s)

Figure 15.9 Full Alternative Text

These three subcircuits are cascaded in series. The subcircuit transfer functions are multiplied to form the cascade transfer function, and the subcircuit magnitude plots are added to create the cascade magnitude plot.

This method of constructing an active bandpass filter and its corresponding magnitude response assumes that the lower cutoff frequency (ω_{c1}) is significantly smaller than the upper cutoff frequency (ω_{c2}). The resulting filter is called a **broadband** bandpass filter because the band of frequencies passed is wide. The formal definition of a broadband filter requires that the two cutoff frequencies satisfy the equation

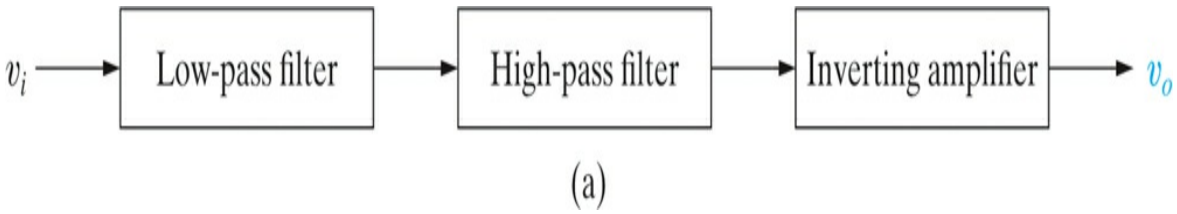
$$\omega_{c2}\omega_{c1} \geq 2.$$

As illustrated by the Bode plot construction in Fig. 15.9, we require that the magnitude of the high-pass filter be unity at the cutoff frequency of the low-pass filter and that the magnitude of the low-pass filter be unity at the cutoff frequency of the high-pass filter. Then the bandpass filter will have the cutoff frequencies specified by the low-pass and high-pass filters. We need to determine the relationship between ω_{c1} and ω_{c2} that will satisfy the requirements illustrated in Fig. 15.9.

We can construct a circuit consisting of three subcircuits by cascading a low-pass op amp filter, a high-pass op amp filter, and an inverting amplifier (see Section 5.3), as shown in Fig. 15.10(a). Figure 15.10(a) is called a **block diagram**. Each block represents a component or subcircuit, and the output of one block is the input to the next, in the direction indicated. We want to establish the relationship between ω_{c1} and ω_{c2} that will permit each subcircuit to be designed independently, without concern for the other subcircuits in the cascade. This reduces the bandpass filter design to the design of a unity-gain first-order low-pass filter, a unity-gain first-order high-pass filter, and an inverting amplifier, each of which is a simple circuit.

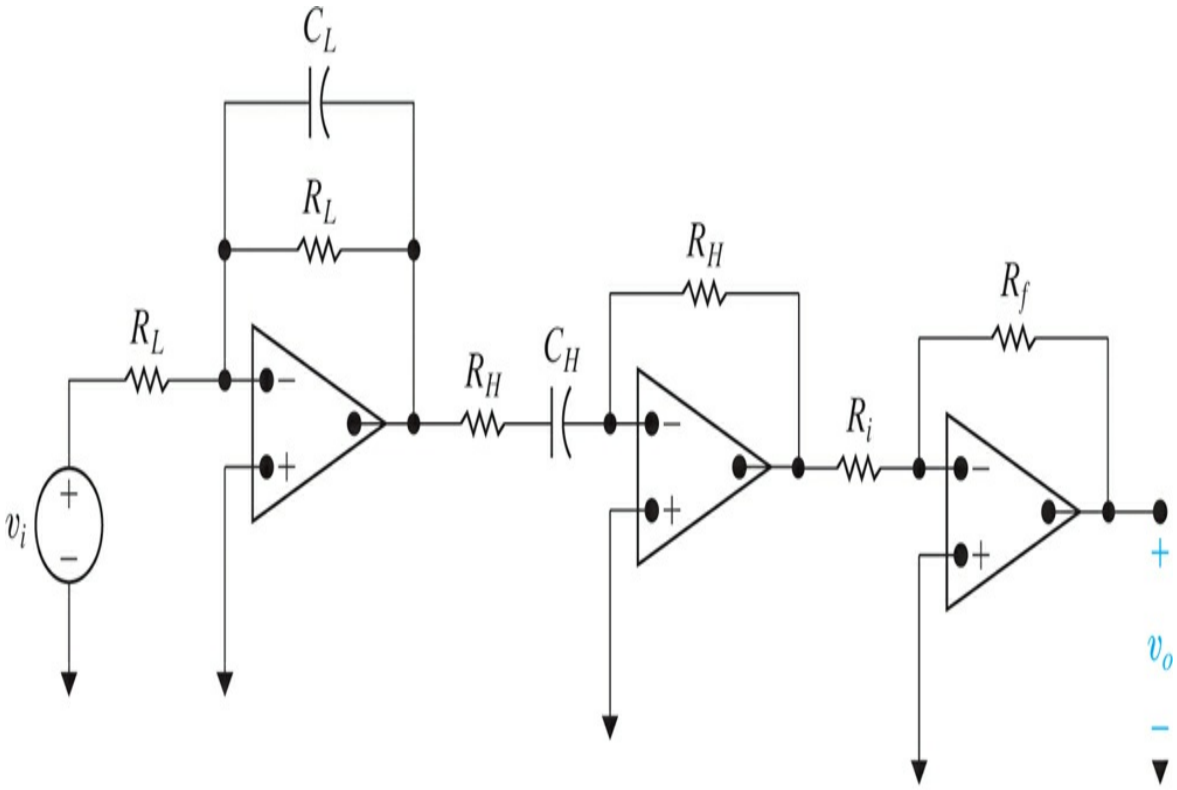
Figure 15.10 A cascaded op

amp bandpass filter. (a) The block diagram. (b) The circuit.



(a)

Figure 15.10 Full Alternative Text



(b)

15.3-5 Full Alternative Text

The transfer function of the cascaded bandpass filter is the product of the transfer functions of the three cascaded subcircuits:

$$H(s) = V_o V_i = (-\omega_{c2} s + \omega_{c2})(-s s + \omega_{c1})(-R_f R_i) = -K \omega_{c2} s (s + \omega_{c1})(s + \omega_{c2}) = -K \omega_{c2} s s^2 + (\omega_{c1} + \omega_{c2}) s + \omega_{c1} \omega_{c2} . \quad (15.8)$$

Equation 15.8 is not in the standard form for the transfer function of a bandpass filter discussed in Chapter 14, namely,

$$H_{BP} = \frac{\beta s}{s^2 + \beta s + \omega_o^2}.$$

In order to convert Eq. 15.8 into the form of the standard transfer function for a bandpass filter, we require that

$$\omega_{c2} \gg \omega_{c1}$$

When the upper cutoff frequency is much larger than the lower cutoff frequency,

$$(\omega_{c1} + \omega_{c2}) \approx \omega_{c2},$$

and the transfer function for the cascaded bandpass filter in Eq. 15.8 becomes

$$H(s) = \frac{-K \omega_{c2} s}{s^2 + \omega_{c2} s + \omega_{c1} \omega_{c2}}.$$

Thus, if the bandpass filter specifications include an upper cutoff frequency that is much larger than the lower cutoff frequency, we can design each subcircuit of the cascaded circuit independently. We compute the values of RL and CL in the low-pass filter to give us the desired upper cutoff frequency, ω_{c2}:

$$\omega_{c2} = \frac{1}{R_L C_L}.$$

We compute the values of RH and CH in the high-pass filter to give us the desired lower cutoff frequency,ω_{c1}:

$$\omega_{c1} = \frac{1}{R_H C_H}.$$

Now we compute the values of Ri and Rf in the inverting amplifier to provide the desired passband gain. To do this, we consider the magnitude of the bandpass filter's transfer function, evaluated at the center frequency, ω_o:

$| H(j \omega o) | = | -K \omega c2 (j \omega o) (j \omega o) 2 + \omega c2 (j \omega o)+ \omega c1 \omega c2 | = K \omega c2 \omega c2 = K.$

Recall from Chapter 5 that the gain of the inverting amplifier is Rf / Ri. Therefore,

$|H(j\omega o)|=RfRi.$

Any choice of resistors that satisfies this equation will produce the desired passband gain.

Example 15.5 illustrates the design process for the cascaded bandpass filter.

Example 15.5 Designing a Broadband Bandpass Op Amp Filter

Design a bandpass filter for a graphic equalizer to provide an amplification of 2 within the band of frequencies between 100 and 10,000 Hz. Use 0.2 μF capacitors.

Solution

We can design each subcircuit in the cascade and meet the specified cutoff frequency values only if the upper cutoff frequency is much larger than the lower cutoff frequency. In this case, $\omega c2=100\omega c1$, so we can say that $\omega c2 \gg \omega c1$. Begin with the low-pass filter, whose cutoff frequency is ωc2. From Eq. 15.3,

$\omega c2 = 1 R L C L =2\pi(10000), R L = 1 [2\pi(10000)](0.2\times 10 -6) \approx 80\Omega.$

Next, we turn to the high-pass filter, whose cutoff frequency is ωc2. From Eq. 15.6,

$\omega c1 = 1 R H C H = 2\pi(100), R H = 1 [2\pi(100)](0.2\times 10 -6) \approx 7958\Omega.$

Finally, we need the gain stage. Two resistors are required, so one of the resistors can be selected arbitrarily. Let’s select a 1 kΩ resistor for Ri. Then,

$R f = 2(1000) = 2000\Omega = 2k\Omega.$

The resulting circuit is shown in Fig. 15.11. We leave it to you to show that the magnitude of this circuit’s transfer function is reduced by 1 / 2 from its maximum value at both cutoff frequencies, verifying the validity of the assumption $\omega c2 \gg \omega c1$.

Figure 15.11 The cascaded op amp bandpass filter designed in Example 15.5.

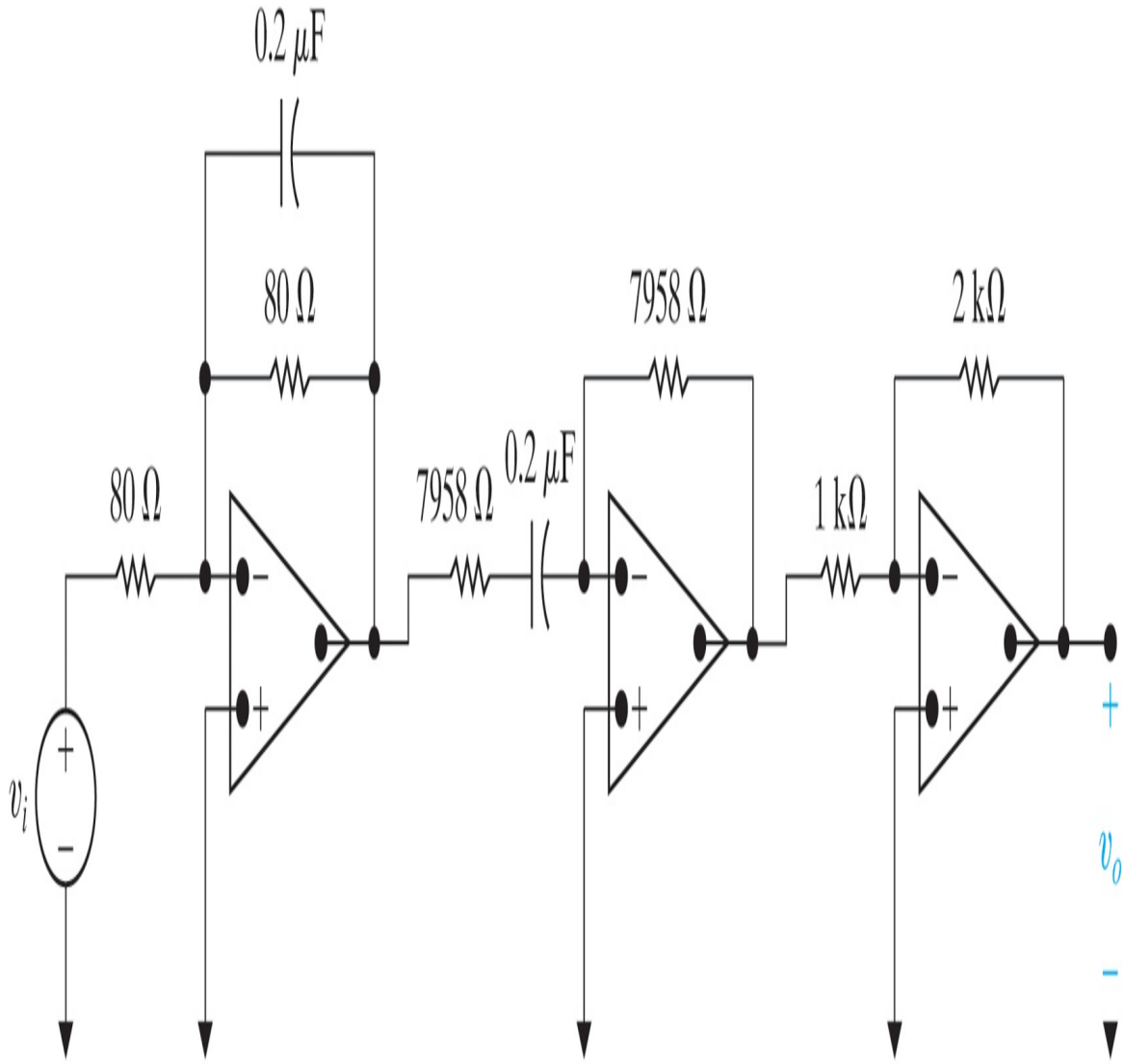


Figure 15.11 Full Alternative Text

We can use a subcircuit approach when designing op amp bandreject filters, too, as illustrated in Fig. 15.12. Like the bandpass filter, the bandreject filter consists of three separate elements.

Figure 15.12 Constructing the Bode magnitude plot of a

bandreject filter.

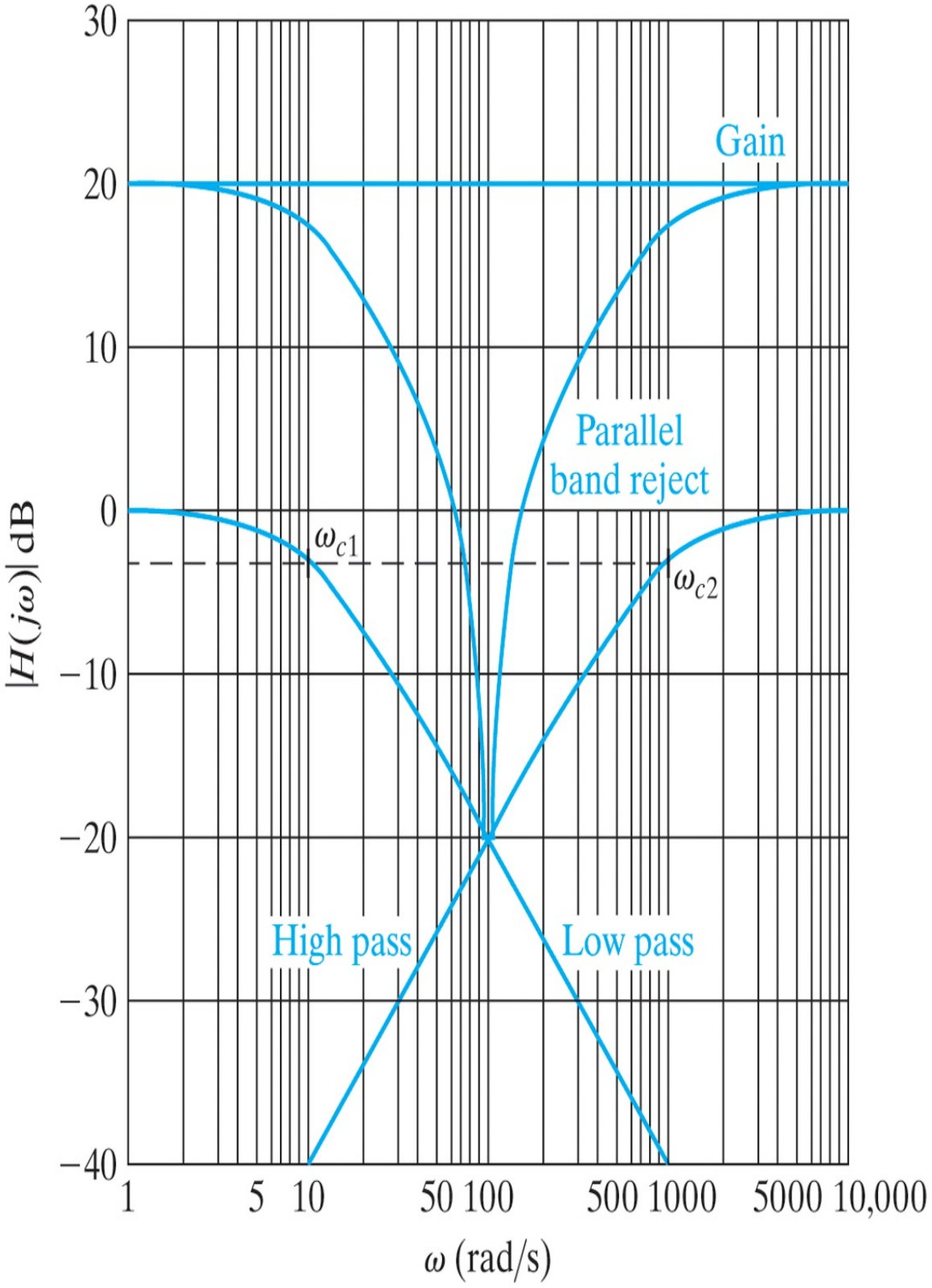

30
20
10
0
−10
−20
−30
−40
|H(jω)| dB
Gain
Parallel
band reject
ω_c1
ω_c2
High pass
Low pass
1
5
10
50
100
500
1000
5000
10,000
ω (rad/s)

Figure 15.12 Full Alternative Text

- A unity-gain low-pass filter with a cutoff frequency of ωc1, which is the smaller of the two cutoff frequencies.
- A unity-gain high-pass filter with a cutoff frequency of ωc2, which is the larger of the two cutoff frequencies.
- A summing amplifier that provides the desired gain in the passbands.

There are important differences between the three subcircuits that comprise the bandpass filter and those that comprise the bandreject filter. The cutoff frequencies of the low-pass and high-pass filters are obviously different—in the bandpass filter, the low-pass filter subcircuit has a cutoff frequency of ωc2, and in the bandreject filter, the low-pass filter subcircuit has a cutoff frequency of ωc1. The cutoff frequencies for the high-pass filter subcircuit are also reversed. The most important difference is that the three subcircuits in the bandreject filter cannot be cascaded in series because they do not combine additively on the Bode plot. Instead, the low-pass and high-pass filters act in parallel, and a summing amplifier combines their outputs and provides the passband gain. Figure 15.13 shows this design both in block diagram form and as a circuit.

Figure 15.13 A parallel op amp bandreject filter. (a) The block diagram. (b) The circuit.

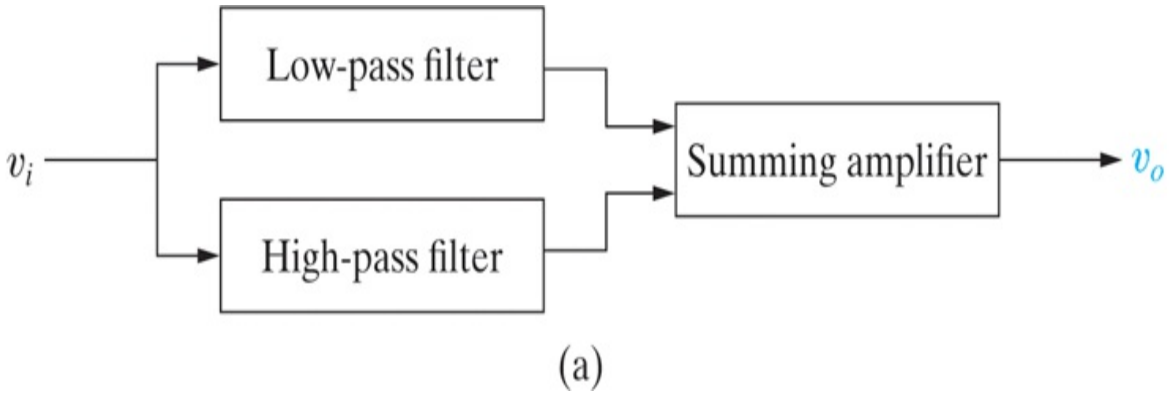
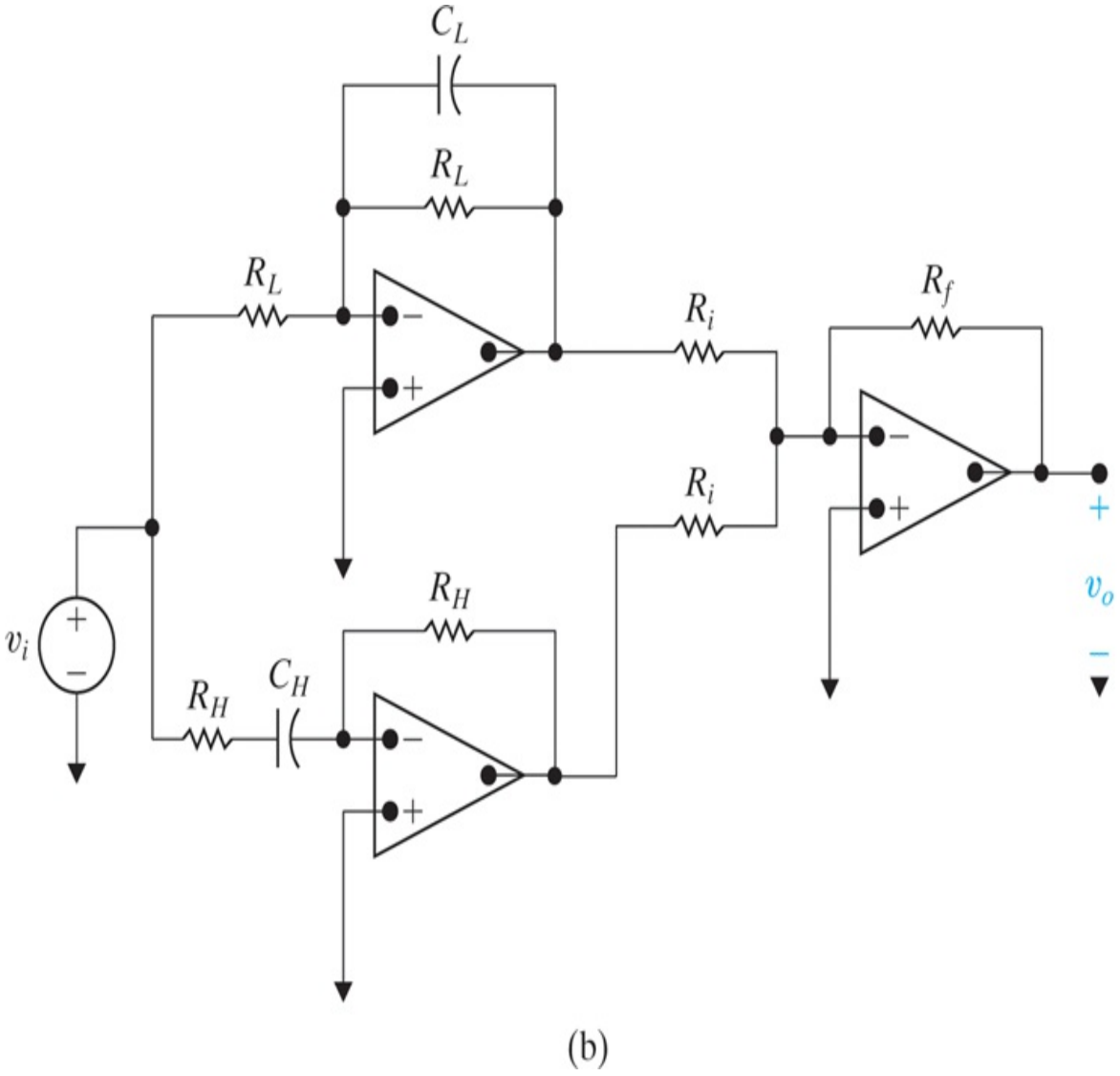


Figure 15.13 Full Alternative Text



15.3-5 Full Alternative Text

Again, it is assumed that the two cutoff frequencies for the bandreject filter are widely separated so that the resulting design is a broadband bandreject filter, and $\omega c2 \gg \omega c1$. Then each subcircuit in the parallel design can be created independently, and the cutoff frequency specifications will be satisfied. The transfer function of the resulting circuit is the sum of the low-pass and high-pass filter transfer functions. From Fig. 15.13(b),

$$H(s) = (-R_f R_i)[-\omega_{c1} s+\omega_{c1} + -s s+\omega_{c2}] = R_f R_i (\omega_{c1}(s+\omega_{c2})+s(s+\omega_{c1})(s+\omega_{c1})(s+\omega_{c2})) = R_f R_i (s^2+2\omega_{c1} s+\omega_{c1}\omega_{c2}(s+\omega_{c1})(s+\omega_{c2})). \quad (15.9)$$

As we saw in the cascaded bandpass filter design, the two cutoff frequencies for the transfer function in Eq. 15.9 are $\omega c1$ and $\omega c2$ only if $\omega c2 \gg \omega c1$. Then the cutoff frequencies are given by the equations

$$\omega_{c1} = 1 R_L C_L, \omega_{c2} = 1 R_H C_H .$$

In the two passbands (as $s \to 0$ and $s \to \infty$), the gain of the transfer function is Rf / Ri. Therefore,

$$K = R_f R_i.$$

As with the design of the cascaded bandpass filter, we have six unknowns and three equations. Typically, we choose a commercially available capacitor value for CL and CH. Then we use the equations for the cutoff frequencies of the low-pass and high-pass filters, calculating RL and RH to meet the specified cutoff frequencies. Finally, we choose a value for either Rf or Ri and then use the equation for K to compute the other resistance to meet the passband gain specification.

The magnitude of the transfer function in Eq. 15.9 at the center frequency, $\omega o = \omega c1 \omega c2$, is

$$|H(j\omega o)| = |R_f R_i ((j\omega o)^2+2\omega_{c1}(j\omega o)+\omega_{c1}\omega_{c2}(j\omega o)^2+(\omega_{c1}+\omega_{c2})(j\omega o)+\omega_{c1}\omega_{c2})| = R_f R_i 2\omega_{c1}\omega_{c1}+\omega_{c2} \approx R_f R_i 2\omega_{c1}\omega_{c2} .$$

If $\omega c2 \gg \omega c1$, then $|H(j\omega o)| \ll 2Rf/Ri$ (because $\omega c1/\omega c2 \ll 1$), so the magnitude

at the center frequency is much smaller than the passband magnitude. Thus, the bandreject filter successfully rejects frequencies near the center frequency but only if the specifications meet the requirements of a broadband filter.

Example 15.6 illustrates the design process for the parallel bandreject filter.

Example 15.6 Designing a Broadband Bandreject Op Amp Filter

Design a circuit based on the parallel bandreject op amp filter in Fig. 15.13(b). The Bode magnitude plot of the filter is shown in Fig. 15.14. Use 0.5 μF capacitors in your design.

Figure 15.14 The Bode magnitude plot for the circuit to be designed in Example 15.6.

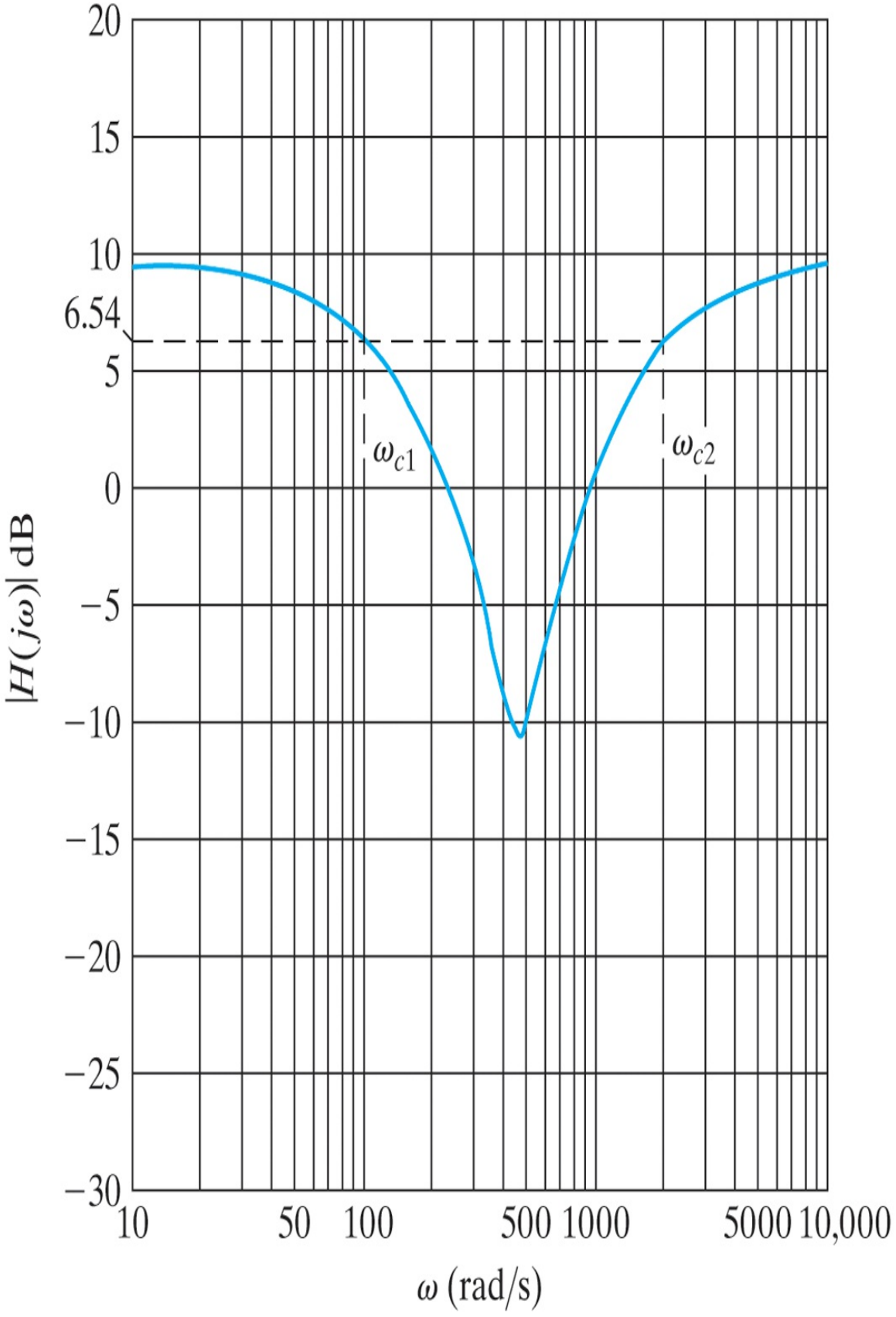

20
15
10
6.54
5
0
−5
−10
−15
−20
−25
−30
10
50
100
500
1000
5000
10,000
ω_{c1}
ω_{c2}
$|H(j\omega)|$ dB
ω (rad/s)

Figure 15.14 Full Alternative Text

Solution

From the Bode magnitude plot in Fig. 15.14, we see that the bandreject filter's cutoff frequencies are 100 rad/s and 2000 rad/s and its passband gain is 3. Thus, $\omega c2=20\omega c1$, so we make the assumption that $\omega c2 \gg \omega c1$. Let's begin with the prototype low-pass filter from Example 15.1 and use scaling to meet the specifications for cutoff frequency and capacitor value. The frequency scale factor kf is 100, which shifts the cutoff frequency from 1 rad/s to100 rad/s. The magnitude scale factor km is 20,000, which, together with the frequency scale factor, scales the capacitor from 1 F to 0.5 μF. Using these scale factors results in the following scaled component values:

$R_L = 20\ k\Omega$, $C_L = 0.5\ \mu F$.

The resulting cutoff frequency of the low-pass filter component is

$$\omega c1=1RLCL=1(20\times103)(0.5\times10-6)=100 \text{ rad/s}.$$

We use the same approach to design the high-pass filter, starting with the prototype high-pass op amp filter. Here, the frequency scale factor is $kf=2000$, shifting the cutoff frequency from 1 rad/s to 2000 rad/s . The magnitude scale factor is $km=1000$, which, together with the frequency scale factor, scales the capacitor from 1 F to 0.5 μF. The scaled component values are

$R_H = 1k\Omega$, $C_H = 0.5\mu F$.

Finally, because the cutoff frequencies are widely separated, we can use the ratio Rf / Ri to create the passband gain of 3. Let's choose $Ri=1\ k\Omega$, as we are already using that resistance for RH. Then $Rf=3\ k\Omega$, and $K=Rf/Ri=3000/1000=3$. The resulting active broadband bandreject filter circuit is shown in Fig. 15.15.

Figure 15.15 The resulting bandreject filter circuit designed in Example 15.6.

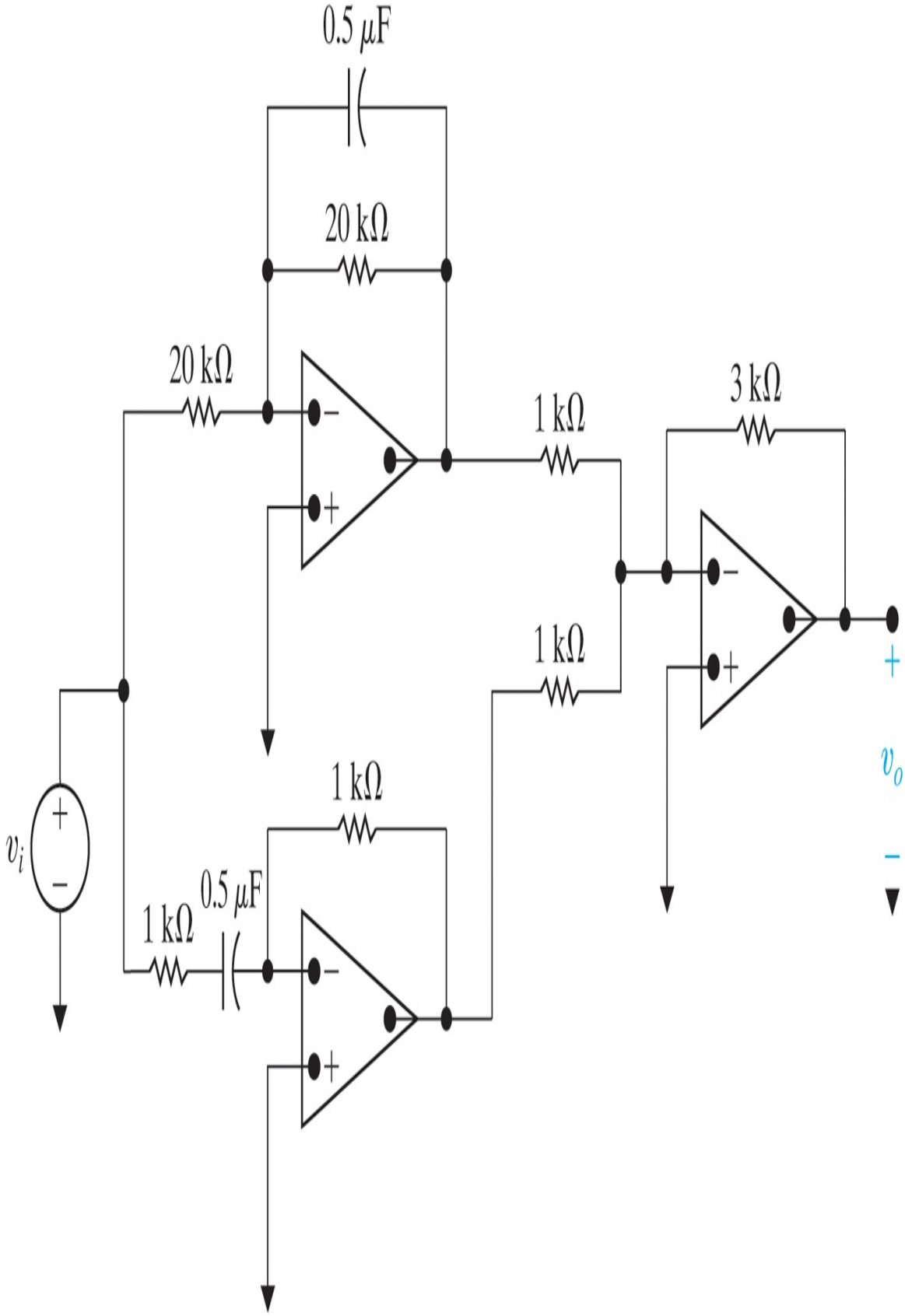

0.5 μF
20 kΩ
20 kΩ
1 kΩ
3 kΩ
1 kΩ
1 kΩ
0.5 μF
1 kΩ
v_i
+
v_o
−

Figure 15.15 Full Alternative Text

Now let’s check our assumption that $\omega_{c2} \gg \omega_{c1}$ by calculating the actual gain at the specified cutoff frequencies. We do this by making the substitutions $s=j100$ and $s=j2000$ into the transfer function for the parallel bandreject filter, Eq. 15.9, and calculating the resulting magnitude. You should verify that the magnitude at the specified cutoff frequencies is greater than the magnitude of $3/2=2.12$ that we expect. Therefore, our rejecting band is more narrow than specified in the problem statement.

SELF-CHECK: Assess your understanding of this material by trying Chapter Problems 15.30 and 15.31.

15.4 Higher-Order Op Amp Filters

None of the filter circuits we have examined so far, whether passive or active, are ideal. Remember from Chapter 14 that an ideal filter has a discontinuity at the cutoff frequency, which sharply divides the passband and the stopband. Although we cannot hope to construct a linear circuit with a discontinuous frequency response, we can construct circuits with a sharper, yet still continuous, transition at the cutoff frequency.

Cascading Identical Filters

How can we obtain a sharper transition between the passband and the stopband? One approach is suggested by the Bode magnitude plots in Fig. 15.16. This figure shows the Bode magnitude plots of a cascade of identical prototype low-pass filters and includes plots of a single filter, two in cascade, three in cascade, and four in cascade. You can see that the transition from the passband to the stopband becomes sharper as more filters are added to the cascade. The rules for constructing Bode plots (from Appendix E) tell us that with one filter, the transition occurs with an asymptotic slope of 20 decibels per decade (dB/dec). Because circuits in cascade are additive on a Bode magnitude plot, a cascade with two filters has a transition with an asymptotic slope of 20+20=40 dB / dec; for three filters, the asymptotic slope is 60 dB/dec, and for four filters, it is 80 dB/dec, as seen in Fig. 15.16.

Figure 15.16 The Bode magnitude plot of a cascade of identical prototype first-order filters.

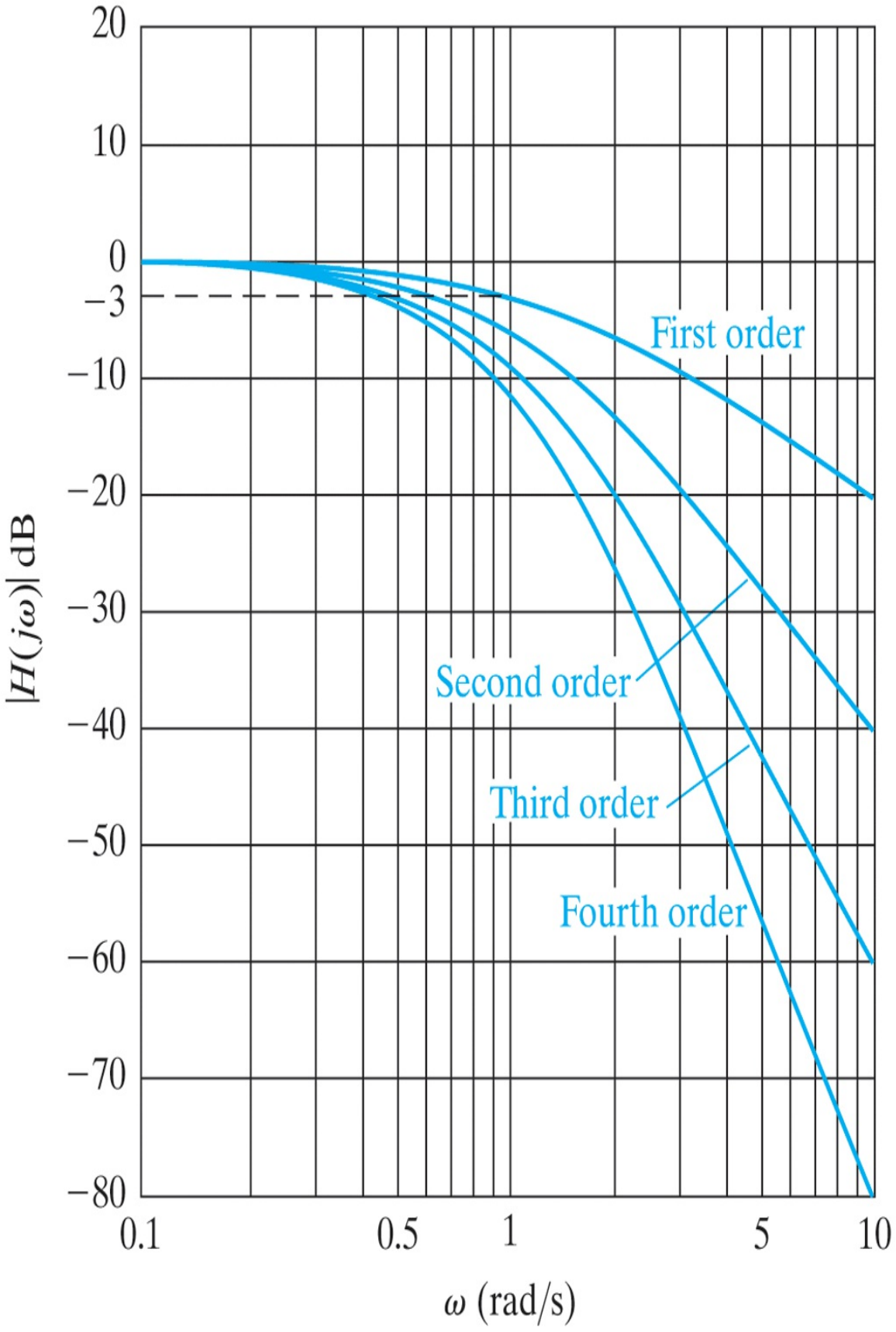
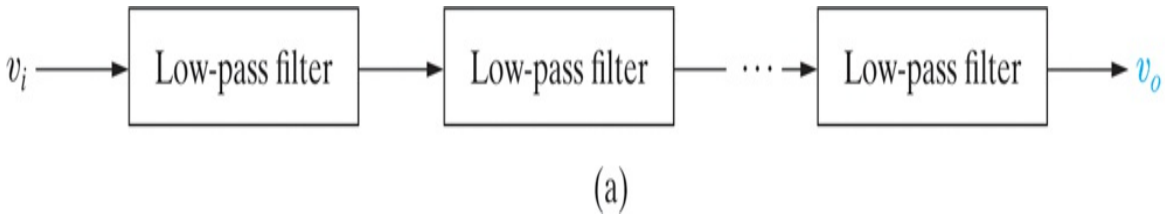

20
10
0
−3
−10
−20
−30
−40
−50
−60
−70
−80
|H(jω)| dB
First order
Second order
Third order
Fourth order
0.1
0.5
1
5
10
ω (rad/s)

Figure 15.16 Full Alternative Text

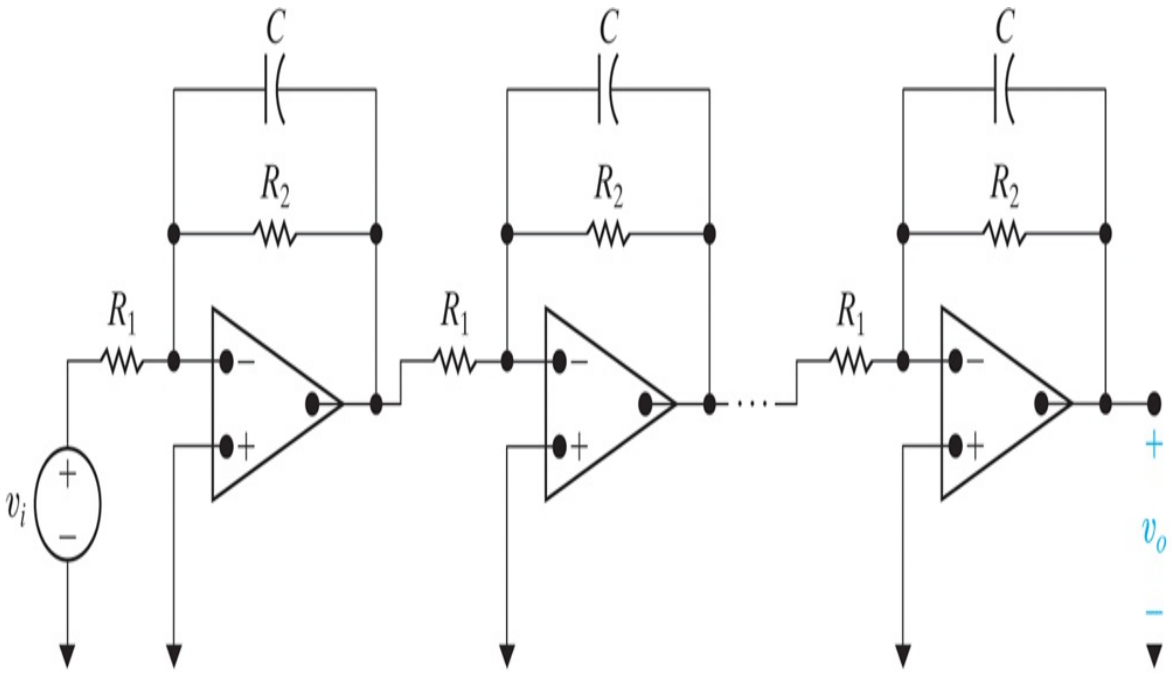
In general, an n-element cascade of identical low-pass filters will transition from the passband to the stopband with a slope of 20 ndB / dec. Figure 15.17 shows both the block diagram and the circuit diagram for such a cascade. We compute the transfer function for a cascade of n prototype low-pass filters by multiplying the individual transfer functions:

Figure 15.17 A cascade of identical unity-gain low-pass filters. (a) The block diagram. (b) The circuit.



(a)

Figure 15.17 Full Alternative Text



15.4-5 Full Alternative Text

$$H(s) = (-1\,s+1)(-1\,s+1)\cdots(-1\,s+1) = (-1)^n (s+1)^n \quad (15.10)$$

A cascade of first-order low-pass filters yields a higher-order filter. From Eq. 15.10, a cascade of n first-order filters produces an nth-order filter because the transfer function has n poles, and the filter has a final slope of 20n dB/dec in the transition band.

There is an important issue yet to be resolved, as you will see if you look closely at Fig. 15.16. As the order of the low-pass filter is increased by adding prototype low-pass filters to the cascade, the cutoff frequency changes. For example, in a cascade of two first-order low-pass filters, the magnitude of the resulting second-order filter at 1 rad/s, the cutoff frequency of the single prototype low-pass filer, is−6dB. The cutoff frequency of the second-order filter is not 1 rad/s. In fact, the cutoff frequency is less than 1 rad/s.

Therefore, we need to find the cutoff frequency of the nth-order filter formed in the cascade of n first-order filters. Then we can use frequency scaling to calculate component values that move the cutoff frequency to its specified

location. The cutoff frequency, ωcn, satisfies the equation |H(jω)| =1 / 2:

H(s) = (−1) n (s+1) n , | H(j ω cn) | = | 1 (j ω cn + 1) n |= 1 2 , 1 (ω cn 2 + 1) n = 1 2 , 1 ω cn 2 +1 = (1 2) 2/n , 2 n = ω cn 2 + 1, ω cn = 2 n −1. (15.11)

Let's use Eq. 15.11 to find the cutoff frequency of a fourth-order unity-gain low-pass filter constructed from a cascade of four prototype low-pass filters:

ωc4=24−1=0.435 rad/s.

Thus, we can design a fourth-order low-pass filter with the cutoff frequency ωc by starting with a fourth-order cascade consisting of prototype low-pass filters and then scaling the components by kf=ωc / 0.435.

We can build a higher-order low-pass filter with a nonunity gain by adding an inverting amplifier circuit to the cascade. Example 15.7 illustrates the design of a fourth-order low-pass filter with nonunity gain.

Example 15.7 Designing a Fourth-Order Low-Pass Active Filter

Design a fourth-order low-pass filter with a cutoff frequency of 500 Hz and a passband gain of 10. Use 1 μF capacitors. Sketch the Bode magnitude plot for this filter.

Solution

Our design cascades four prototype low-pass filters. We have already used Eq. 15.11 to calculate the cutoff frequency for the resulting fourth-order low-pass filter as 0.435 rad/s. A frequency scale factor of kf=7222.39 will scale the component values to give a 500 Hz cutoff frequency. A magnitude scale factor of km=138.46, together with the frequency scale factor, scales the capacitor value from 1 F to 1 μF. The scaled component values are thus

R=138.46Ω; C=1μF.

Add an inverting amplifier stage with a gain of Rf/Ri=10 to satisfy the passband gain specification. As usual, we can arbitrarily select one of the two resistor values. Because we are already using 138.46Ω resistors, let Ri=138.46Ω; then,

Rf=10Ri=1384.6Ω.

The circuit for this cascaded fourth-order low-pass filter is shown in Fig. 15.18. It has the transfer function

Figure 15.18 The cascade circuit for the fourth-order low-pass filter designed in Example 15.7.

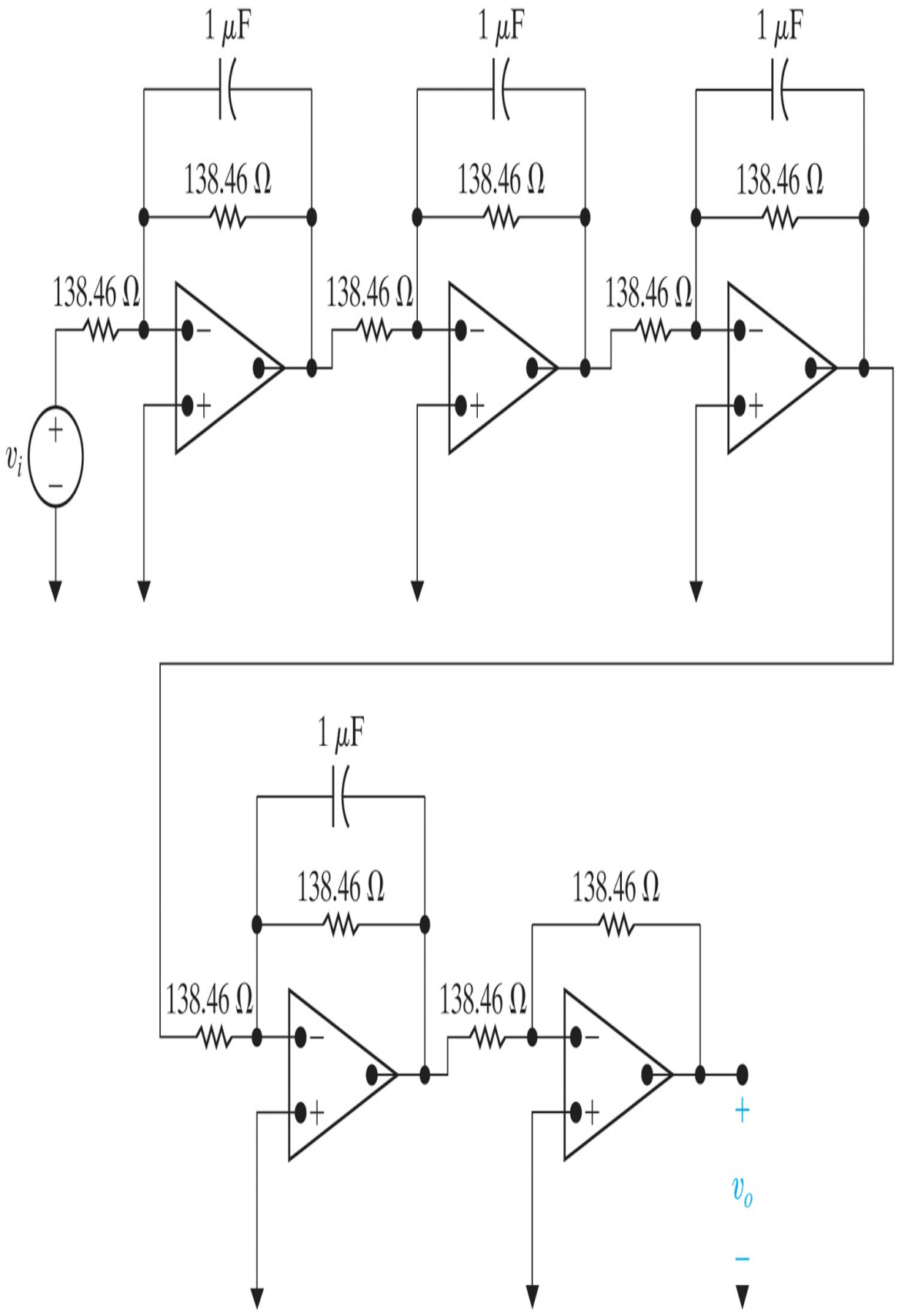

1 μF
1 μF
1 μF
138.46 Ω
138.46 Ω
138.46 Ω
138.46 Ω
138.46 Ω
138.46 Ω
v_i
1 μF
138.46 Ω
138.46 Ω
138.46 Ω
138.46 Ω
+
v_o
−

Figure 15.18 Full Alternative Text
H(s)=−10[7222.39s+7222.39]4.

The Bode magnitude plot for this transfer function is sketched in Fig. 15.19.

Figure 15.19 The Bode magnitude plot for the fourth-order low-pass filter designed in Example 15.7.

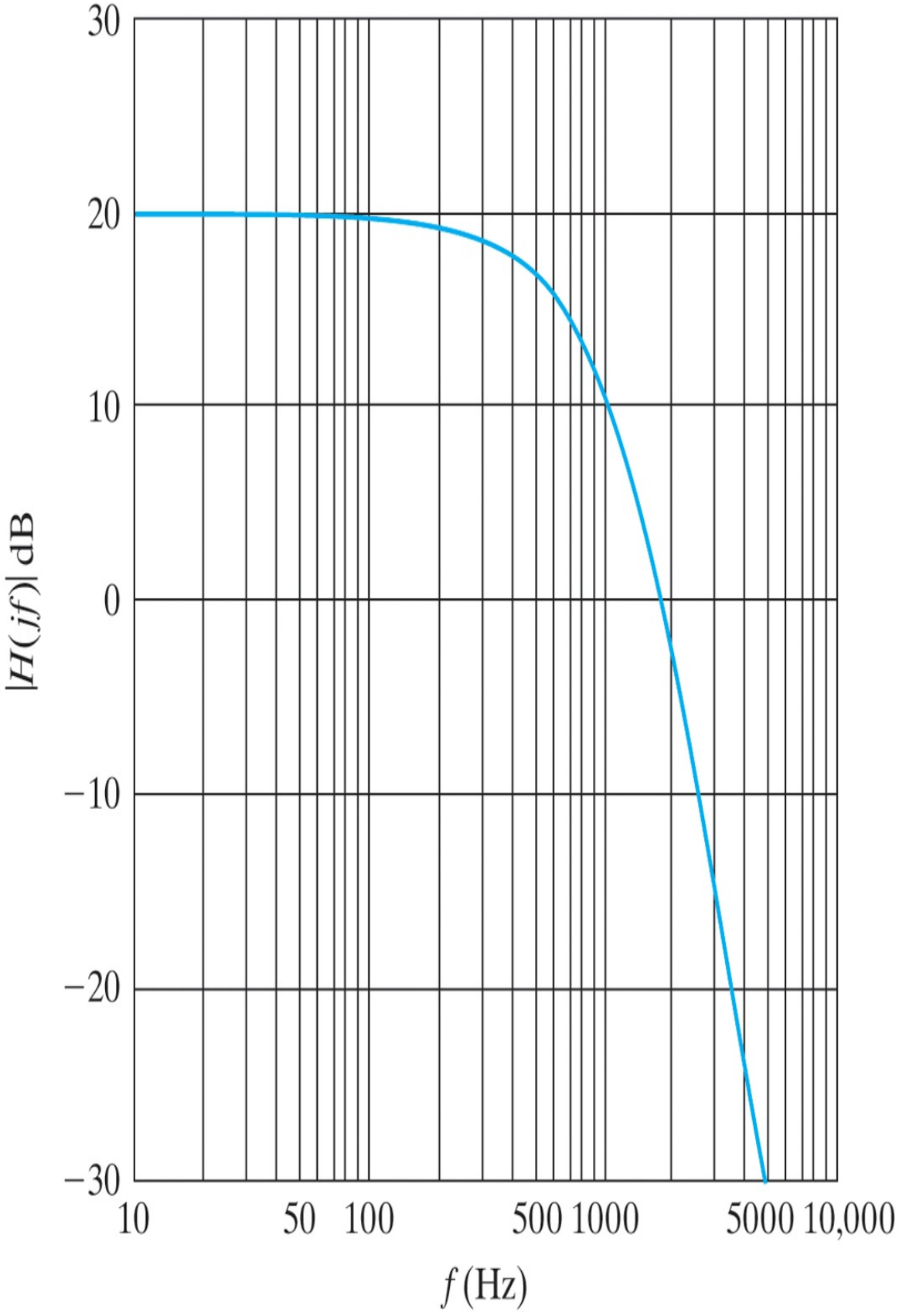

30
20
10
0
−10
−20
−30
|H(jf)| dB
10
50
100
500
1000
5000
10,000
f (Hz)

Figure 15.19 Full Alternative Text

By cascading identical low-pass filters, we can increase the asymptotic slope in the transition between passband and stopband, but our approach has a serious shortcoming: The gain of the filter is not constant between zero and the cutoff frequency ωc. Remember that in an ideal low-pass filter, the passband magnitude is 1 for all frequencies below the cutoff frequency. But in Fig. 15.16, we see that the passband magnitude is less than 1 (0 dB), even for frequencies much less than the cutoff frequency.

We can understand why the passband magnitude is not ideal by looking at the magnitude of the transfer function for a unity-gain low-pass nth-order cascade. Because

$$H(s)=\omega_{cn}^{n}(s+\omega_{cn})^{n},$$

the magnitude is

$$|H(j\omega)| = \omega_{cn}^{n}(\omega^2+\omega_{cn}^2)^{n} = 1((\omega/\omega_{cn})^2+1)^{n}.$$

As we can see from this expression, when $\omega \ll \omega_{cn}$, the denominator is approximately 1, and the magnitude of the transfer function is also nearly 1. But as $\omega \rightarrow \omega_{cn}$, the denominator becomes larger than 1, so the magnitude becomes smaller than 1. Because the cascade of low-pass filters results in this nonideal behavior in the passband, other approaches are used when designing higher-order filters. One such approach is examined next.

Butterworth Filters

A unity-gain **Butterworth low-pass filter** has a transfer function whose magnitude is

$$|H(j\omega)| = \frac{1}{1+(\omega/\omega_c)^{2n}}, \quad (15.12)$$

where n is an integer that denotes the order of the filter.1

[1]This filter was developed by the British engineer S. Butterworth and reported in *Wireless Engineering* 7 (1930): 536–541.

When studying Eq. 15.12, note the following:

- The cutoff frequency is ω_c rad/s for all values of n.
- If n is large enough, the denominator is always close to unity when $\omega < \omega_c$.
- In the expression for $|H(j\omega)|$, the exponent of ω / ω_c is always even.

This last observation is important because only even exponents exist in circuits with resistors, inductors, capacitors and op amps. (See Problem 15.33.)

Given an equation for the magnitude of the transfer function, how do we find H(s)? We can simplify the derivation for H(s) by using a prototype filter, so we set ω_c equal to 1 rad/s in Eq. 15.12. As before, we will use scaling to transform the prototype filter to a filter that meets the given specifications.

To find H(s), first note that if N is a complex quantity, then $|N|^2 = NN^*$, where N^* is the conjugate of N. It follows that

$$|H(j\omega)|^2 = H(j\omega)\, H(-j\omega).$$

But because $s = j\omega$, we can write

$$|H(j\omega)|^2 = H(s)\, H(-s).$$

Now observe that $s^2 = -\omega^2$. Thus,

$$|H(j\omega)|^2 = \frac{1}{1+\omega^{2n}} = \frac{1}{1+(\omega^2)^n} = \frac{1}{1+(-s^2)^n} = \frac{1}{1+(-1)^n s^{2n}},$$

or

$$H(s)\, H(-s) = \frac{1}{1+(-1)^n s^{2n}}.$$

Follow these steps to find H(s) for a given value of n:

1. Find the roots of the polynomial

   $1+(-1)^n s^{2n}=0$.

2. Assign the left-half plane roots to H(s) and the right-half plane roots to H(−s).

3. Combine terms in the denominator of H(s) to form first- and second-order factors.

Example 15.8 illustrates this process.

Example 15.8 Calculating Butterworth Transfer Functions

Find the Butterworth transfer functions for n=2 and n=3.

Solution

For n=2, we find the roots of the polynomial

$1+(-1)^2 s^4=0$.

Rearranging terms, we find

$s^4=-1=1\angle 180^\circ$.

Therefore, the four roots are

$s_1=1\angle 45^\circ=1/\sqrt{2}+j/\sqrt{2}$, $s_2=1\angle 135^\circ=-1/\sqrt{2}+j/\sqrt{2}$, $s_3=1\angle 225^\circ=-1/\sqrt{2}+-j/\sqrt{2}$, $s_4=1\angle 315^\circ=1/\sqrt{2}+-j/\sqrt{2}$.

Roots s2 and s3 are in the left-half plane. Thus,

$$H(s) = 1 \left(s+1/\sqrt{2} - j/\sqrt{2} \right)\left(s+1/\sqrt{2} + j/\sqrt{2} \right) = 1 \left(s^2 + 2s + 1 \right).$$

For n=3, we find the roots of the polynomial

$$1+(-1)3s6=0.$$

Rearranging terms,

$$s6=10^{\circ} = 1360^{\circ}.$$

Therefore, the six roots are

$$s_1 = 1\,0^{\circ} = 1,\ s_2 = 1\,60^{\circ} = 1/2 + j\sqrt{3}/2,\ s_3 = 1\,120^{\circ} = -1/2 + j\sqrt{3}/2,\ s_4 = 1\,180^{\circ} = -1 + j0,\ s_5 = 1\,240^{\circ} = -1/2 - j\sqrt{3}/2,\ s_6 = 1\,300^{\circ} = 1/2 - j\sqrt{3}/2.$$

Roots s3, s4, and s5 are in the left-half plane. Thus,

$$H(s) = 1 \left(s+1 \right)\left(s+1/2 - j\sqrt{3}/2 \right)\left(s+1/2 + j\sqrt{3}/2 \right) = 1 \left(s+1 \right)\left(s^2 + s + 1 \right).$$

Note that the roots of the Butterworth polynomial are always equally spaced around the unit circle in the s plane. To assist in the design of Butterworth filters, Table 15.1 lists the Butterworth polynomials up to n=8.

Table 15.1 Normalized (so that $\omega_c = 1$ rad/s) Butterworth Polynomials up to the Eighth Order

n	nth-Order Butterworth Polynomial
1	$(s+1)$
2	$(s^2+\sqrt{2}s+1)$
3	$(s+1)(s^2+s+1)$
4	$(s^2+0.765s+1)(s^2+1.848s+1)$
5	$(s+1)(s^2+0.618s+1)(s^2+1.618s+1)$
6	$(s^2+0.518s+1)(s^2+\sqrt{2}+1)(s^2+1.932s+1)$
7	$(s+1)(s^2+0.445s+1)(s^2+1.247s+1)(s^2+1.802s+1)$
8	$(s^2+0.390s+1)(s^2+1.111s+1)(s^2+1.6663s+1)(s^2+1.962s+1)$

Butterworth Filter Circuits

Now that we know how to specify the transfer function for a Butterworth filter circuit (either by calculating the poles of the transfer function directly or by using Table 15.1), we need to design a circuit with such a transfer function. Notice the form of the Butterworth polynomials in Table 15.1. They are the product of first- and second-order factors; therefore, we can construct a circuit whose transfer function has a Butterworth polynomial in its denominator by cascading active filter circuits, each of which provides one of the needed factors. Figure 15.20 presents a block diagram of a cascade whose transfer function has a fifth-order Butterworth polynomial in its denominator.

Figure 15.20 A cascade of first- and second-order circuits with the indicated transfer functions yielding a fifth-order low-pass

Butterworth filter with ωc=1 rad/s

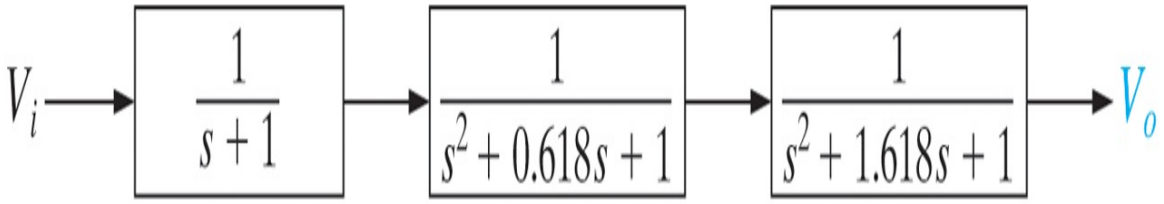


Figure 15.20 Full Alternative Text

All odd-order Butterworth polynomials include the factor (s+1), so all odd-order Butterworth filter circuits must have a subcircuit with the transfer function H(s)=1/(s+1). This is the transfer function of the prototype low-pass active filter from Fig. 15.1. So what remains is to find a circuit whose transfer function has the form H(s)=1/(s2+ b1s+1).

Such a circuit is shown in Fig. 15.21. To analyze this circuit, write the s-domain KCL equations at the noninverting terminal of the op amp and at the node labeled Va:

V a − V i R +(V a − V o)s C 1 + V a − V o R = 0, V o s C 2 + V o − V a R = 0.

Figure 15.21 A circuit that provides the second- order transfer function for the Butterworth filter cascade.

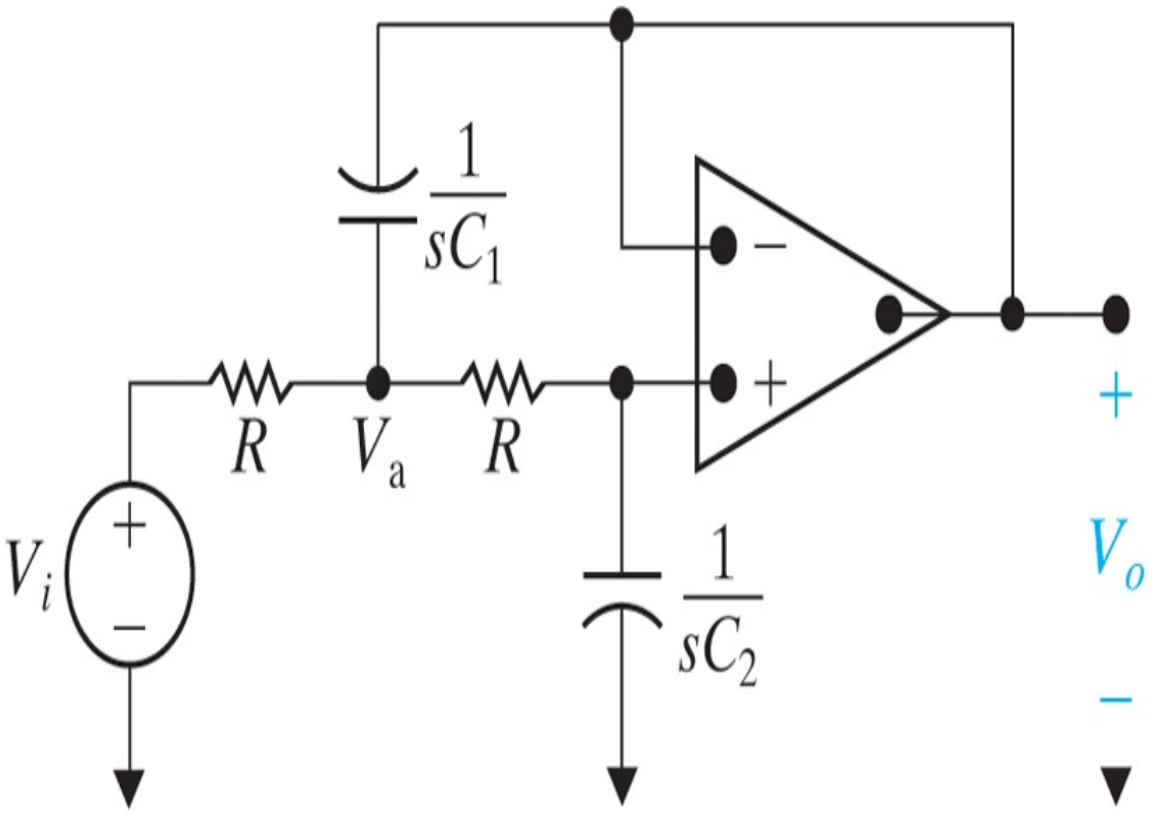


Figure 15.21 Full Alternative Text

Simplifying the two KCL equations yields

$(2+RC_1s)V_a-(1+RC_1s)V_o=V_i,$
$-V_a+(1+RC_2s)V_o=0.$

Use back-substitution to eliminate Va:

$(2+RC_1s)(1+RC_2s)V_o-(1+RC_1s)V_o=V_i.$

Then, rearrange this equation to write the transfer function for the circuit in Fig. 15.21:

$H(s)=\frac{V_o}{V_i}=\frac{1}{R^2C_1C_2s^2+2RC_1s+1R^2C_1C_2}.$

Finally, set R=1Ω; then

$H(s)=\frac{1}{C_1C_2s^2+2C_1s+1C_1C_2}.$ (15.13)

Equation 15.13 has the form required for the second-order circuit in the Butterworth cascade. To get a transfer function of the form

H(s)=1s2+b1s+1,

we use the circuit in Fig. 15.21 and choose capacitor values so that

b1=2C1and 1=1C1C2. (15.14)

We have thus outlined the procedure for designing an nth-order Butterworth low-pass filter circuit with a cutoff frequency of ωc=1 rad/s and a gain of 1 in the passband. We can use frequency scaling to calculate revised capacitor values that yield any specified cutoff frequency, and we can use magnitude scaling to provide more realistic or practical component values in our design. Cascading an inverting amplifier circuit provides a passband gain other than 1.

Example 15.9 illustrates this design process.

Example 15.9 Designing a Fourth-Order Low-Pass Butterworth Filter

Design a fourth-order Butterworth low-pass filter with a cutoff frequency of 500 Hz and a passband gain of 10. Use as many 1kΩ resistors as possible. Compare the Bode magnitude plot for this Butterworth filter with that of the identical cascade filter in Example 15.7.

Solution

From Table 15.1, we find that the fourth-order Butterworth polynomial is

(s2+0.765s+1)(s2+1.848s+1).

We will thus need a cascade of two second-order filters to get the fourth-

order transfer function and an inverting amplifier circuit to get a passband gain of 10. The circuit is shown in Fig. 15.22.

Figure 15.22 A fourth-order Butterworth filter with nonunity gain.

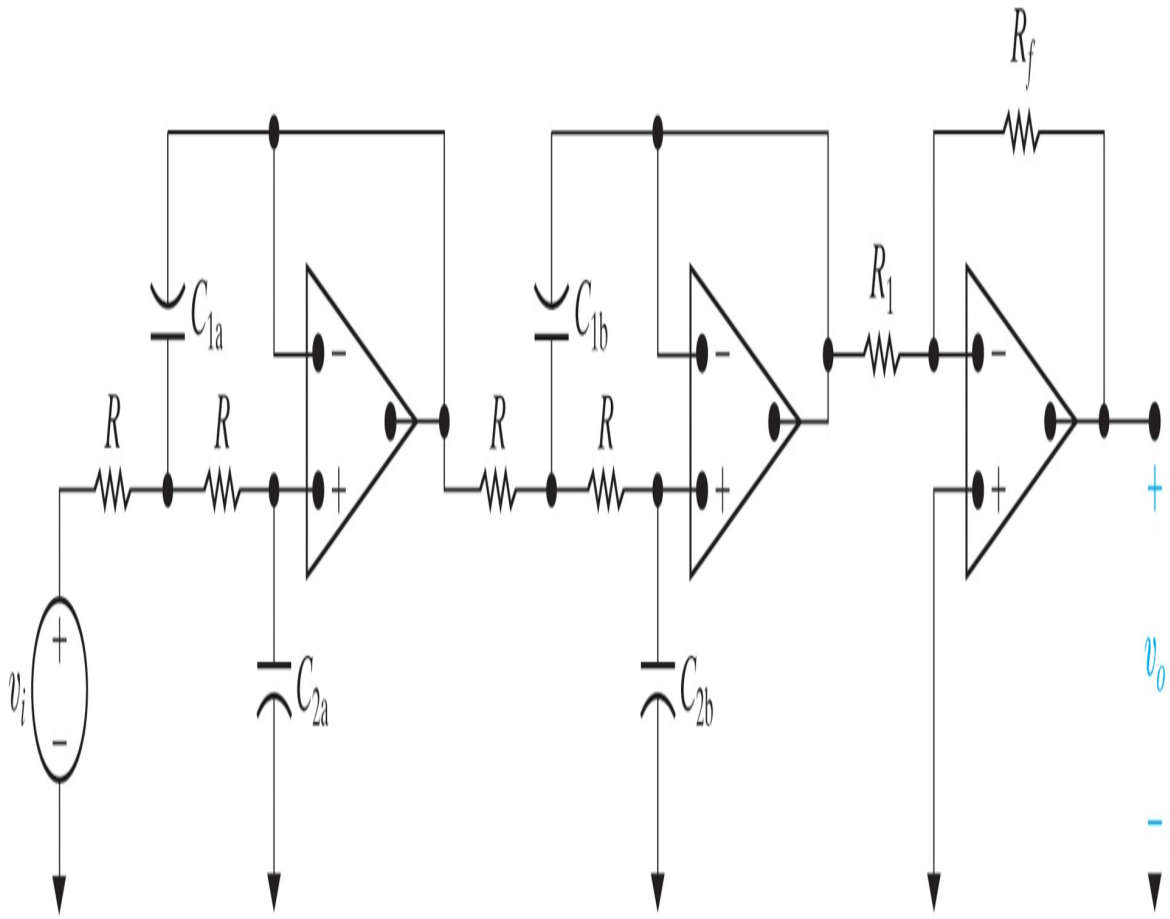


Figure 15.22 Full Alternative Text

The first stage of the cascade is a circuit whose transfer function has the polynomial (s2+0.765s+1) in its denominator. From Eq. 15.14,

c 1a = 2 0.765 = 2.61F, c 2a = 1 2.61 = 0.38F.

The second stage of the cascade is a circuit whose the transfer function has the polynomial (s2+1.848s+1) in its denominator. From Eq. 15.14,

C 1b = 2 1.848 = 1.08F, C 2b = 1 1.08 = 0.924F.

These values for C1a, C2b, C1b, and C2b yield a fourth-order Butterworth filter with a cutoff frequency of 1 rad/s. A frequency scale factor of kf=3141.6 will move the cutoff frequency to 500 Hz. A magnitude scale factor of km=1000 allows us to use 1 kΩ resistors in place of 1 Ω resistors. The resulting scaled component values are

R = 1 kΩ, C 1 = 831 nF, C 2 = 121 nF, C 3 = 344 nF, C 4 = 294 nF.

Finally, we need to specify the resistor values in the inverting amplifier stage to yield a passband gain of 10. Let R1=1 kΩ; then

Rf=10R1=10kΩ.

Figure 15.23 compares the magnitude responses of the fourth-order identical cascade filter from Example 15.7 and the Butterworth filter we just designed. Note that both filters provide a passband gain of 10 (20 dB) and a cutoff frequency of 500 Hz, but the Butterworth filter is closer to an ideal low-pass filter due to its flatter passband and steeper rolloff at the cutoff frequency. Thus, the Butterworth design is preferred to the identical cascade design.

Figure 15.23 A comparison of the magnitude responses for a fourth-order low-pass filter using the identical cascade and Butterworth designs.

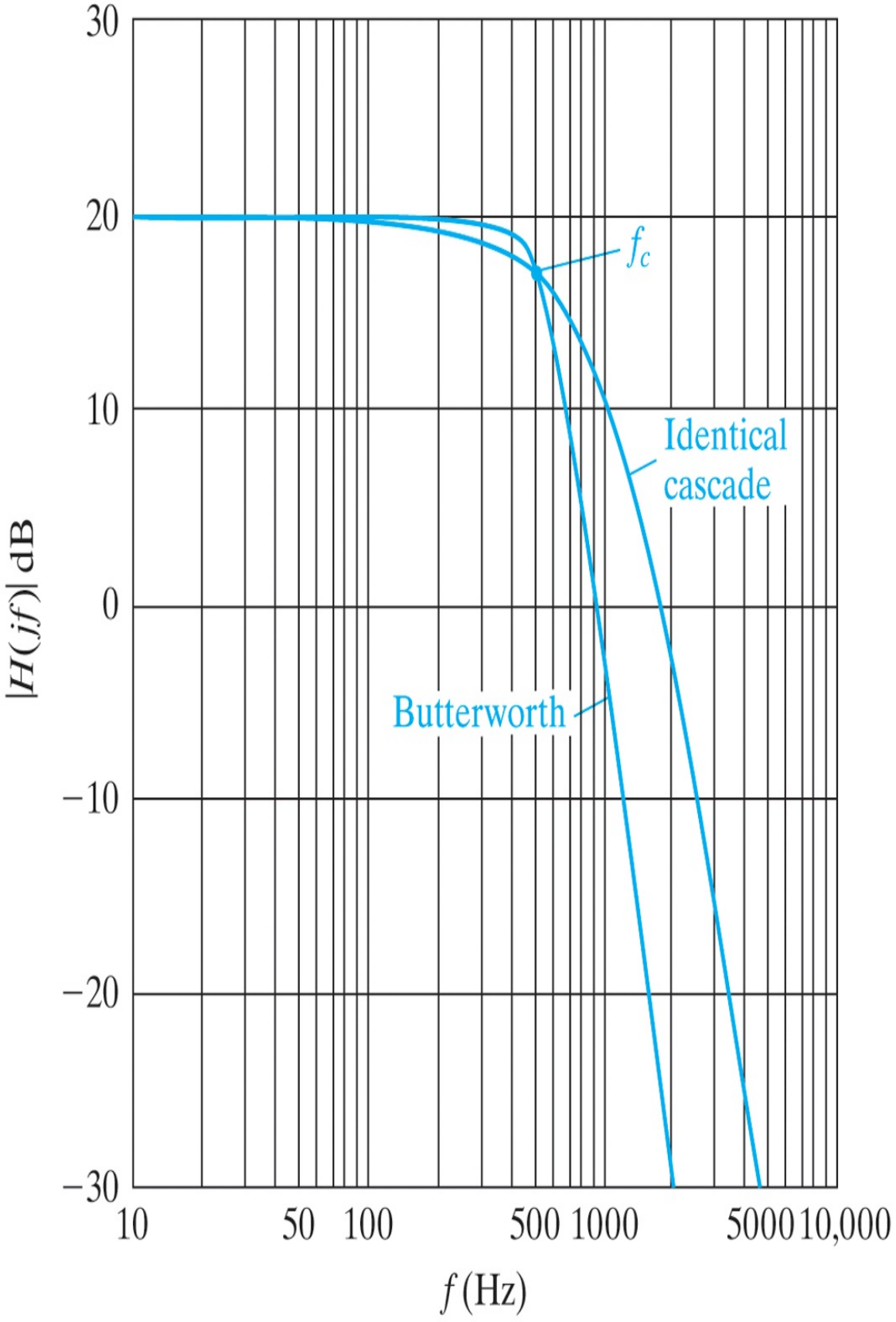

30
20
10
0
−10
−20
−30
|H(jf)| dB
10
50
100
500
1000
5000
10,000
f (Hz)
f_c
Identical cascade
Butterworth

Figure 15.23 Full Alternative Text

The Order of a Butterworth Filter

You have probably noticed that the higher the order of the Butterworth filter, the closer the transfer function magnitude is to that of an ideal low-pass filter. In other words, as n increases, the magnitude stays close to unity in the passband, the transition band narrows, and the magnitude stays close to zero in the stopband. At the same time, as the order increases, the number of circuit components increases. Thus, when designing a filter, determining the smallest value of n that will meet the filtering specifications is an important first step.

The filtering specifications for a low-pass filter usually define the abruptness of the transition region, as shown in Fig. 15.24. Once Ap, ωp, As, and ωs are specified, the order of the Butterworth filter can be determined.

Figure 15.24 Defining the transition region for a low-pass filter.

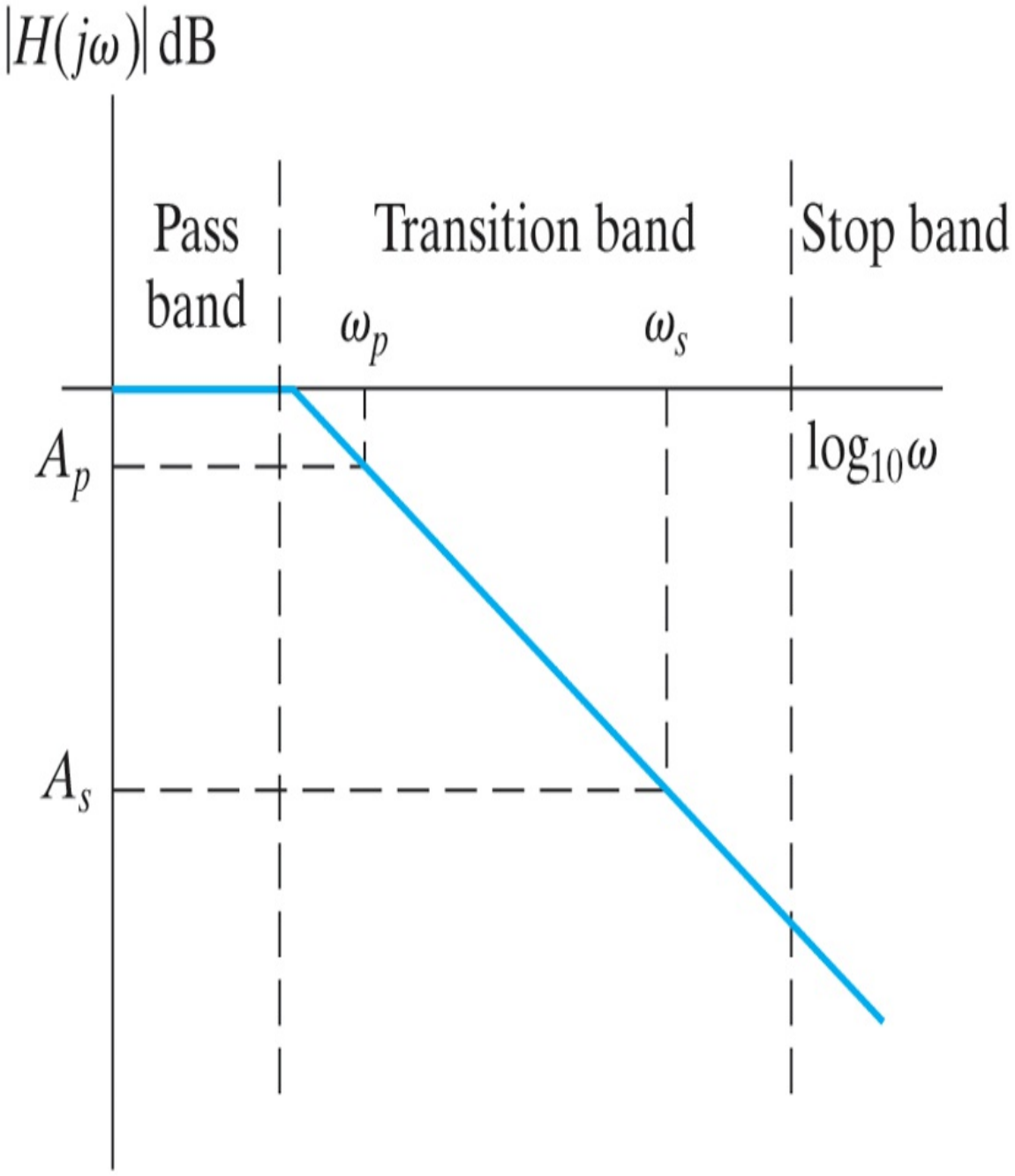


Figure 15.24 Full Alternative Text

For the Butterworth filter,

Ap=20 log1011+ωp2n=−10log10(1+ωp2n),
As=20 log1011+ωs2n=−10log10(1+ωs2n).

It follows from the definition of the logarithm that

$$10^{-0.1A_p}=1+\omega_p^{2n},$$
$$10^{-0.1A_s}=1+\omega_s^{2n}.$$

Now, solve for ω_p^n and ω_s^n and find the ratio $(\omega_s / \omega_p)^n$. We get

$$\left(\frac{\omega_s}{\omega_p}\right)^n=\frac{10^{-0.1A_s}-1}{10^{-0.1A_p}-1}=\frac{\sigma_s}{\sigma_p},$$

where the symbols σ_s and σ_p have been introduced for convenience.

From the expression for σ_s/σ_p we can write

$$n \log_{10}(\omega_s / \omega_p)=\log_{10}(\sigma_s / \sigma_p),$$

or

$$n=\frac{\log_{10}(\sigma_s/\sigma_p)}{\log_{10}(\omega_s/\omega_p)}. \quad (15.15)$$

We can simplify Eq. 15.15 if ω_p is the cutoff frequency because then A_p equals $-20\log_{10}2$, and $\sigma_p=1$. Hence

$$n=\frac{\log_{10}\sigma_s}{\log_{10}(\omega_s/\omega_p)}.$$

One further simplification is possible. We are using a Butterworth filter to achieve a steep transition region. Therefore, the filtering specification will make $10^{-0.1A_s} \gg 1$. Thus

$$\sigma_s \approx 10^{-0.05A_s},$$
$$\log_{10}\sigma_s \approx -0.05A_s.$$

Therefore, a good approximation for the calculation of n is

$$n=\frac{-0.05A_s}{\log_{10}(\omega_s/\omega_p)}. \quad (15.16)$$

Note that $\omega_s / \omega_p=f_s / f_p$, so we can work with frequencies specified in either radians per second or hertz to calculate n.

The order of the filter must be an integer; hence, in using either Eq. 15.15 or

Eq. 15.16, we select the nearest integer value greater than the result given by the equation. Examples 15.10 and 15.11 illustrate the use of Eqs. 15.15 and 15.16.

Example 15.10 Determining the Order of a Butterworth Filter

1. Determine the order of a Butterworth filter that has a cutoff frequency of 1000 Hz and a gain of no more than −50 dB at 6000 Hz.

2. What is the actual gain in dB at 6000 Hz?

Solution

1. Because the cutoff frequency is specified, we know σp=1. We also note from the specification that 10−0.1(−50) is much greater than 1. Hence, we can use Eq. 15.16 with confidence:

   n=(−0.05)(−50)log10(6000 / 1000)=3.21.

   Therefore, we need a fourth-order Butterworth filter.

2. We can use Eq. 15.12 to calculate the actual gain at 6000 Hz. The gain in decibels will be

   K=20 log10(11+68)=−62.25dB.

Example 15.11 An Alternate Approach to Determining the Order of a Butterworth Filter

1. Determine the order of a Butterworth filter whose magnitude is 10 dB less than the passband magnitude at 500 Hz and at least 60 dB less than the passband magnitude at 5000 Hz.

2. Determine the cutoff frequency of the filter (in hertz).

3. What is the actual gain of the filter (in decibels) at 5000 Hz?

Solution

1. Because the cutoff frequency is not given, we use Eq. 15.15 to determine the order of the filter:

   $\sigma p = 10 - 0.1(-10) - 1 = 3$, $\sigma s = 10 - 0.1(-60) - 1 \approx 1000$, $\omega s / \omega p = f s / f p = 5000 / 500 = 10$, $n = \log 10 (1000 / 3) \log 10 (10) = 2.52$.

   Therefore, we need a third-order Butterworth filter to meet the specifications.

2. Knowing that the gain at 500 Hz is −10 dB, we can determine the cutoff frequency. From Eq. 15.12 we can write

   $-10 \log 10[1+(\omega / \omega c)6]=-10$,

   where $\omega=1000\pi$ rad/s. Therefore

   $1+(\omega / \omega c)6=10$,

   and

   $\omega c = \omega 9 6 = 2178.26$ rad / s.

   It follows that

   fc=346.68 Hz.

3. The actual gain of the filter at 5000 Hz is

$$K = -10 \log 10 [\, 1 + (5000 / 346.68) 6 \,] = -69.54 \text{ dB}.$$

Butterworth High-Pass, Bandpass, and Bandreject Filters

An nth-order Butterworth high-pass filter has a transfer function with the nth-order Butterworth polynomial in the denominator, just like the nth-order Butterworth low-pass filter. But in the high-pass filter, the numerator of the transfer function is sn, whereas in the low-pass filter, the numerator is 1. Again, we use a cascade approach in designing the Butterworth high-pass filter. The first-order factor is achieved by including a prototype high-pass filter (Fig. 15.4, with R1=R2=1Ω, and C=1 F) in the cascade.

To produce the second-order factors in the Butterworth polynomial, we need a circuit with a transfer function of the form

H(s)=s2s2+b1s+ 1.

Such a circuit is shown in Fig. 15.25.

Figure 15.25 A second-order Butterworth high-pass filter circuit.

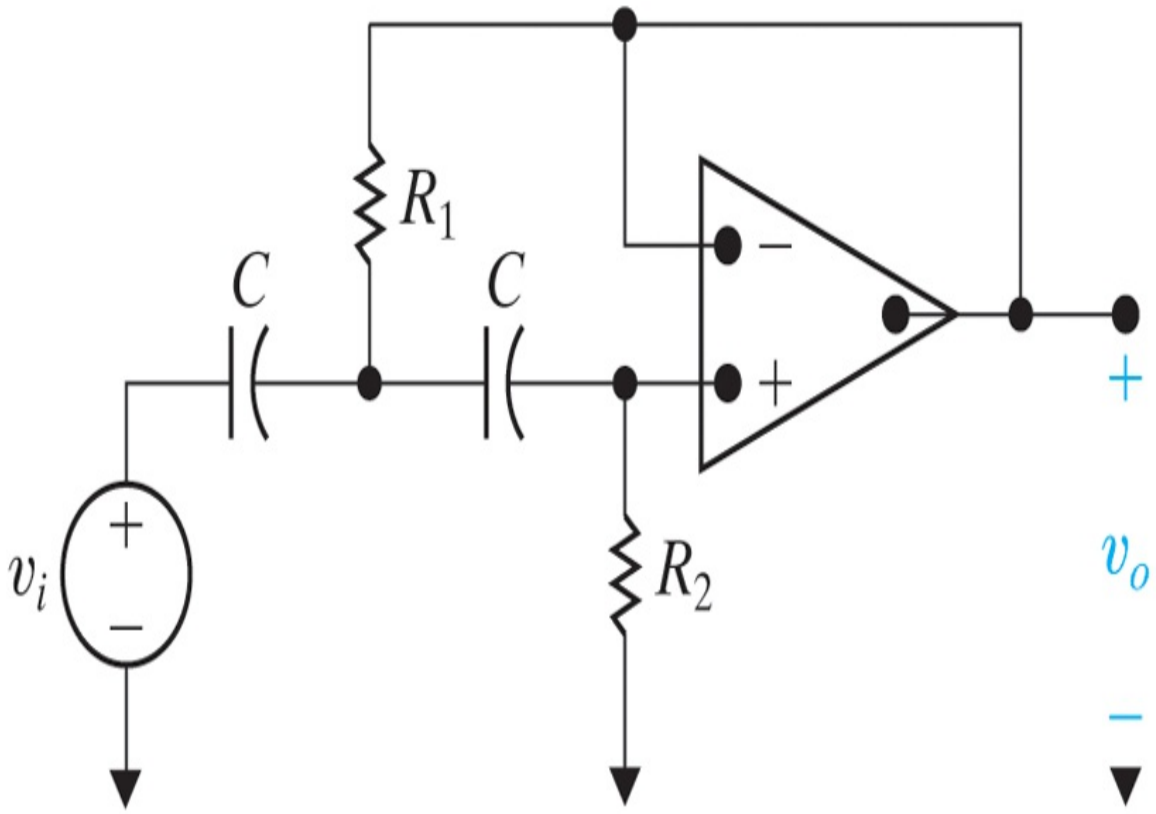


Figure 15.25 Full Alternative Text

This circuit has the transfer function

H(s)=VoVi=s2s2+2R2Cs+1R1R2C2.

Setting C=1 F yields

H(s)=s2s2+2R2s+1R1R2. (15.17)

Thus, we can realize any second-order factor in a Butterworth polynomial of the form (s2+b1s+1) by including in the cascade the second-order circuit in Fig. 15.25 with resistor values that satisfy Eq. 15.18:

b1=2R2and 1=1R1R2. (15.18)

At this point, we pause to make a couple of observations about the circuits in Figs. 15.21 and 15.25 and their prototype transfer functions 1 / (s2+ b1s+1) and s2 / (s2+ b1s+1).

- The high-pass circuit in Fig. 15.25 was obtained from the low-pass circuit in Fig. 15.21 by interchanging resistors and capacitors.

- The prototype transfer function of a high-pass filter can be obtained from that of a low-pass filter by replacing s in the low-pass expression with 1/s (see Problem 15.46).

These observations are important because they are true in general.

We can use frequency and magnitude scaling to design a Butterworth high-pass filter with practical component values and a cutoff frequency other than 1 rad / s. Adding an inverting amplifier to the cascade will accommodate designs with nonunity passband gains. The problems at the end of the chapter include several Butterworth high-pass filter designs.

Now that we can design both nth-order low-pass and high-pass Butterworth filters with arbitrary cutoff frequencies and passband gains, we can combine these filters in cascade (as we did in Section 15.3) to produce nth-order Butterworth bandpass filters. We can combine these filters in parallel with a summing amplifier (again, as we did in Section 15.3) to produce nth-order Butterworth bandreject filters. Example 15.12 illustrates the design of a Butterworth bandpass filter.

Example 15.12 Designing a Butterworth Bandpass Filter

Design the bandpass filter from Example 15.5 using a cascade of a fourth-order low-pass Butterworth filter, a fourth-order high-pass Butterworth filter, and an inverting amplifier. The filter should provide an amplification of 2 within the band of frequencies between 100 and 10,000 Hz and use as many 2 kΩ resistors and 0.2 μF capacitors as possible.

Solution

The circuit is shown in Fig. 15.26. The first two stages in the cascade form the fourth-order low-pass Butterworth filter, which should have a cutoff frequency of 10,000 Hz. We have already designed the prototype fourth-order low-pass Butterworth filter in Example 15.9, so R=1 Ω and

C 1a = 2.61 F, C 2a = 0.38 F, C 1b = 1.08 F, C 2b = 0.924 F.

Figure 15.26 The fourth-order Butterworth bandpass filter for Example 15.12.

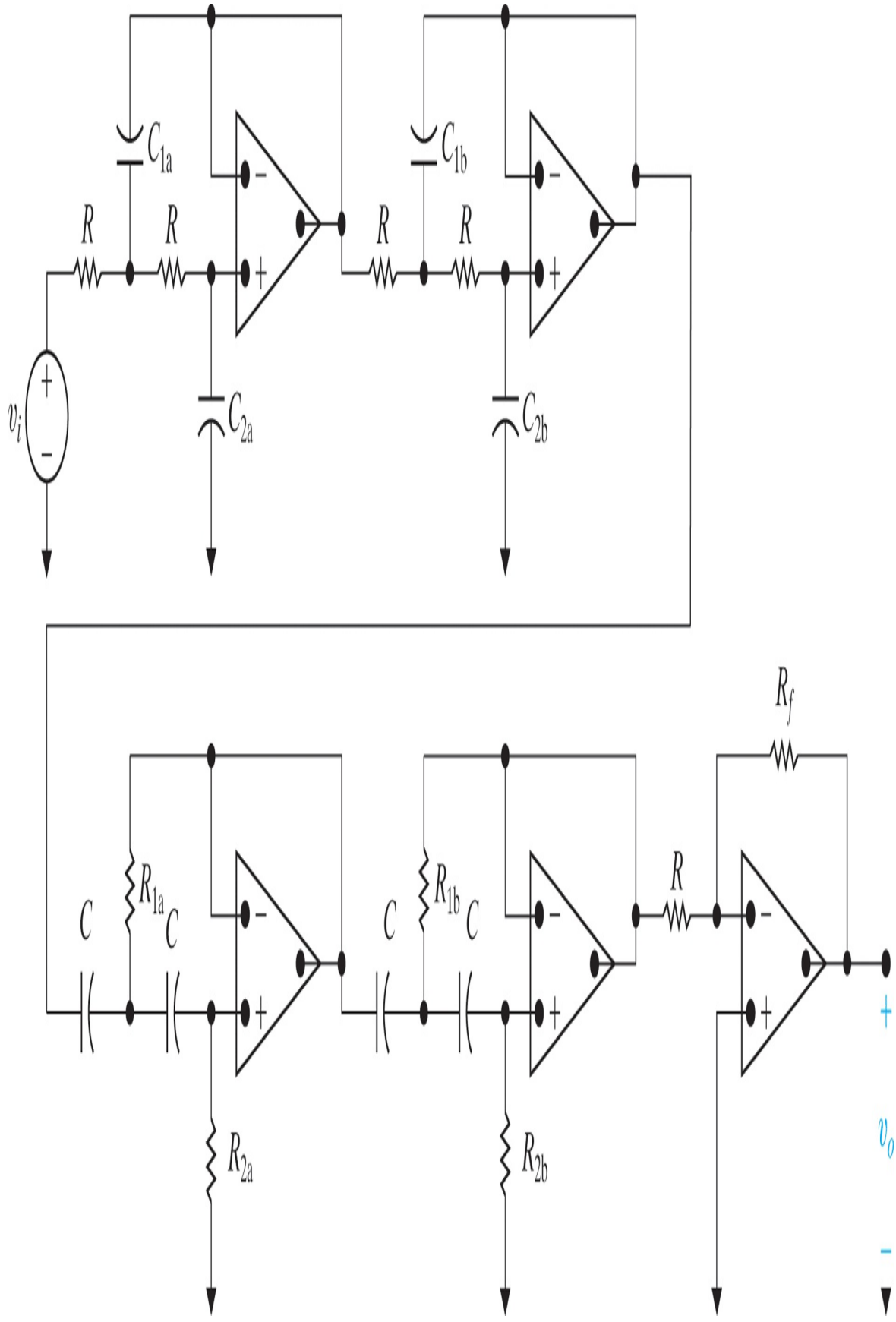

C_{1a}
C_{1b}
R
R
R
R
v_i
C_{2a}
C_{2b}
R_f
C
R_{1a}
C
C
R_{1b}
C
R
R_{2a}
R_{2b}
+
v_o
–

Figure 15.26 Full Alternative Text

This prototype low-pass filter has a cutoff frequency of 1 rad/s , so to move the cutoff frequency to 10,000 Hz we need a frequency scale factor kf=20,000π. To use 2 kΩ resistors, we need to scale the 1 Ω resistors using a magnitude scale factor km=2000. The scaled capacitor values are

C 1a = 20.77 nF, C 2a = 3.02 nF, C 1 b = 8.59 nF, C 2b = 7.35 nF.

The next two stages in the cascade form the fourth-order high-pass Butterworth filter, which should have a cutoff frequency of 100 Hz. To design the prototype filter, let C=1 F. The first stage of the prototype high-pass filter must have a transfer function whose denominator is the Butterworth polynomial (s2+0.765 s+1). From Eq. 15.18,

R 2a = 2 0.765 =2.61 Ω, R 1a = 1 R 2a = 1 2.61 =0.38 Ω.

The second stage of the prototype high-pass filter must have a transfer function whose denominator is the Butterworth polynomial (s2+1.848 s+1). From Eq. 15.18,

R 2b = 2 1.848 =1.08 Ω, R 1b = 1 R 2b = 1 1.08 =0.924 Ω.

This prototype high-pass filter has a cutoff frequency of 1 rad/s , so to move the cutoff frequency to 100 Hz, we need a frequency scale factor kf=200π. To use 0.2 μF capacitors, we need a magnitude scale factor km=7957.747. The scaled resistor values are

R 1a = 3023.94 Ω, R 2a = 20,769.72 Ω, R 1 b = 7352.96 Ω, R 2b = 8594.37 Ω.

The final stage of the cascade in Fig. 15.26 is the inverting amplifier, which must have a gain of 2. Let’s use R=2 kΩ, so Rf=2R=4 kΩ.

Assessment Problem

Objective 3—Understand how to use cascaded first- and second-order Butterworth filters

1. 15.4 For the circuit in Fig. 15.25, find values of R1 and R2 that yield a second-order prototype Butterworth high-pass filter.

   Answer:

   R1=0.707 Ω, R2=1.41 Ω.

SELF-CHECK: Also try Chapter Problems 15.34, 15.36, and 15.40.

15.5 Narrowband Bandpass and Bandreject Filters

We can only implement broadband or low-Q bandpass and bandreject filters when using the cascade and parallel subcircuit designs from the previous sections. This limitation is due principally to the form of the transfer functions for cascaded bandpass and parallel bandreject filters—they only have discrete real poles. The largest quality factor we can achieve with discrete real poles arises when the cutoff frequencies, and thus the pole locations, are the same. As an example, consider the transfer function for a bandpass filter created from a cascade of a first-order low-pass filter and a first-order high-pass filter, where both filters have the same cutoff frequency:

$$H(s) = (-\omega_c s + \omega_c)(-s s + \omega_c) = s\,\omega_c s^2 + 2\,\omega_c s + \omega_c^2 = 0.5\beta s\, s^2 + \beta s + \omega_c^2 .$$

This equation is in the standard form of the transfer function of a bandpass filter, and thus we can determine the bandwidth and center frequency directly:

$$\beta = 2\,\omega_c , \omega_o^2 = \omega_c^2 .$$

From the equations for bandwidth and center frequency and the definition of Q, we see that

$$Q = \frac{\omega_o}{\beta} = \frac{\omega_c}{2\omega_c} = \frac{1}{2}.$$

Thus, with discrete real poles, the highest quality bandpass filter (or bandreject filter) we can achieve has $Q = 1/2$.

To build active filters with high-quality factor values, we need an op amp circuit whose transfer function has complex conjugate poles. Figure 15.27 depicts one such circuit for us to analyze. At the inverting input of the op amp, write a KCL equation to get

Va1 / sC=−VoR3.

Figure 15.27 An active high-Q bandpass filter.

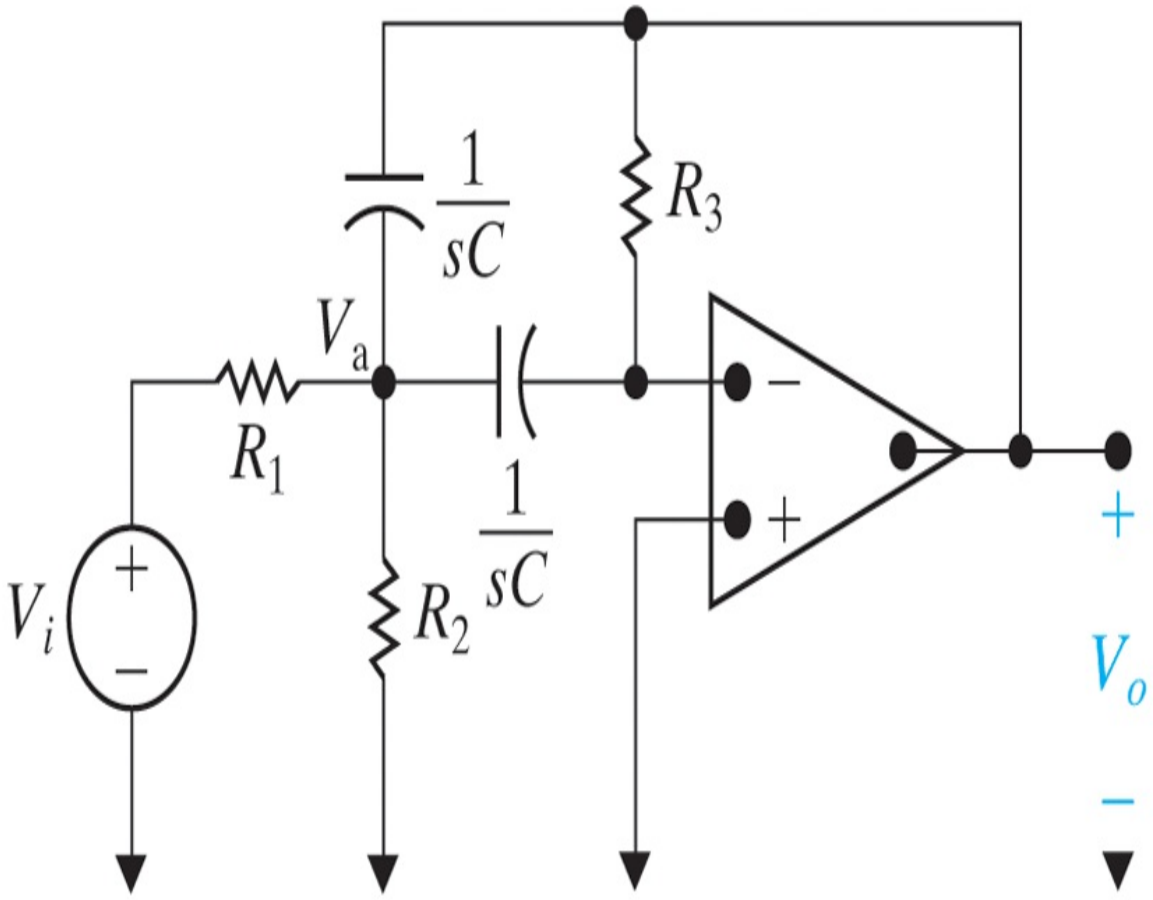


Figure 15.27 Full Alternative Text

Solving for Va,

Va=−VosR3C.

At the node labeled Va,, write a KCL equation to get

Vi−VaR1=Va−Vo1 / sC+Va1 / sC+VaR2.

Solving for Vi,

$$Vi=(1+2sR1C+R1/R2)Va-sR1CVo.$$

Substituting the equation for Va into the equation for Vi and then rearranging, we get an expression for the transfer function Vo / Vi:

$$H(s)=-sR1Cs2+2R3Cs+1ReqR3C2, \quad (15.19)$$

where

$$Req=R1 \,||\, R2=R1R2R1+R2.$$

Equation 15.19 is in the standard form for a bandpass filter transfer function; that is,

$$H(s)=-K\beta ss2+ \beta s+\omega o2,$$

so we can equate terms and solve for the values of the resistors that will achieve a specified center frequency (ωo), quality factor (Q), and passband gain (K):

$$\beta = 2 R 3 C ; K\beta = 1 R 1 C ; \omega o 2 = 1 R eq R 3 C 2 .$$

Let's define the prototype version of the circuit in Fig. 15.27 as a bandpass filter with ωo=1 rad/s and C=1 F. Then the expressions for R1, R2, and R3 can be given in terms of the desired quality factor and passband gain. You should verify (in Problem 15.57) that for the prototype circuit, the expressions for R1, R2, and R3 are

$$R 1 = Q / K, R 2 = Q / (2 Q 2 -K), R 3 = 2Q. \quad (15.20)$$

Use scaling to achieve the desired center frequency and to specify practical values for the circuit components. This design process is illustrated in Example 15.13.

Example 15.13 Designing a High-Q

Bandpass Filter

Design a bandpass filter, using the circuit in Fig. 15.27, that has a center frequency of 3000 Hz, a quality factor of 10, and a passband gain of 2. Use 0.01 μF capacitors in your design. Compute the transfer function of your circuit, and sketch a Bode plot of its magnitude response.

Solution

Since Q=10 and K=2, the values for R1, R2, and R3 in the prototype circuit, from Eq. 15.20, are

$R_1 = Q\,K = 10\,2 = 5\ \Omega$, $R_2 = Q\,2\,Q\,2 - K = 10\,200 - 2 = 10\,198\ \Omega$, $R_3 = 2Q = 2(10) = 20\ \Omega$.

The scaling factors are kf=6000π, to move the center frequency from 1 rad/s to 3000 Hz, and km=108/kf, so we can use 0.01 μF capacitors. After scaling,

$R_1 = 26.5\ \text{k}\Omega$, $R_2 = 268.0\ \Omega$, $R_3 = 106.1\ \text{k}\Omega$.

The circuit is shown in Fig. 15.28.

Figure 15.28 The high-Q bandpass filter designed in Example 15.13.

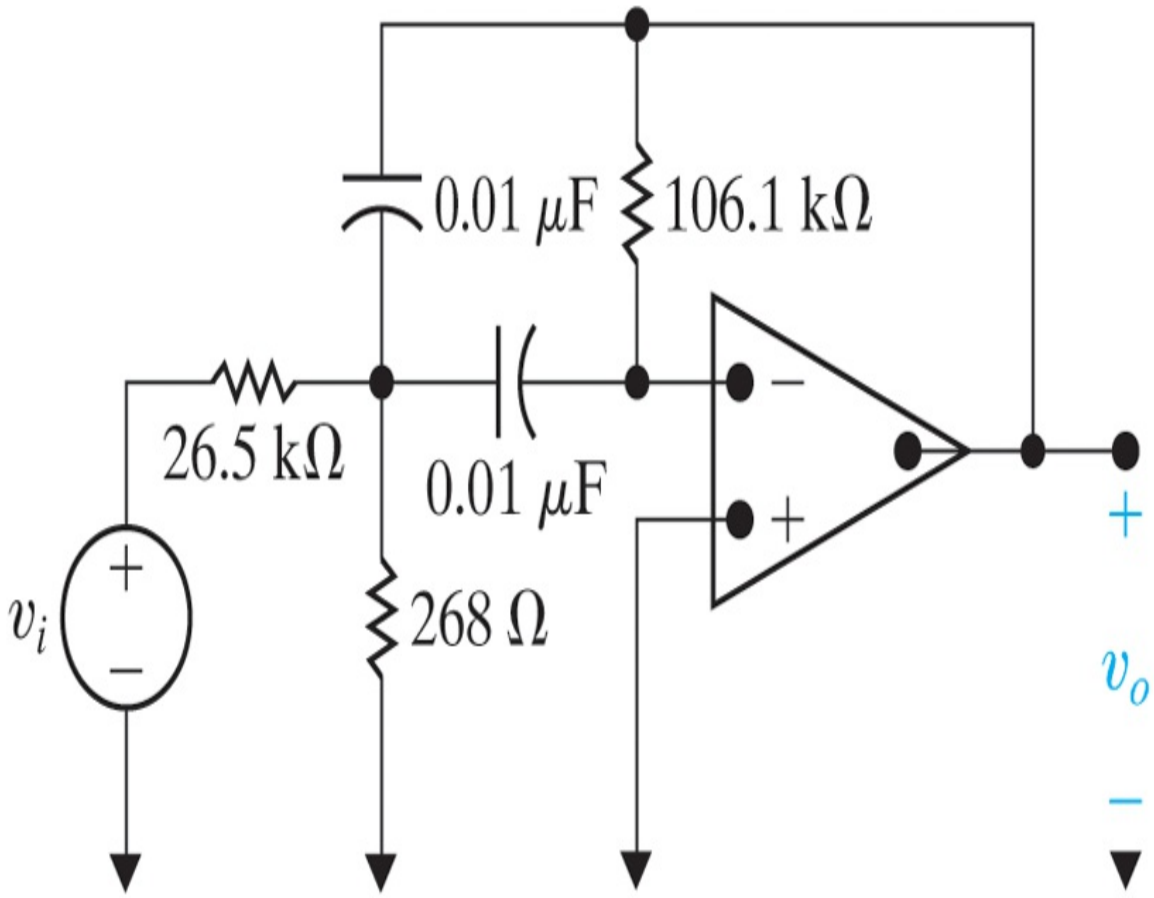


Figure 15.28 Full Alternative Text

Substituting the values of resistance and capacitance in Eq. 15.19 gives the transfer function for this circuit:

H(s)=−3770ss2+ 1885.0s+355 × 106.

It is easy to see that this transfer function meets the specification of the bandpass filter defined in the example. A Bode plot of its magnitude response is sketched in Fig. 15.29.

Figure 15.29 The Bode magnitude plot for the high-Q

bandpass filter designed in Example 15.13.

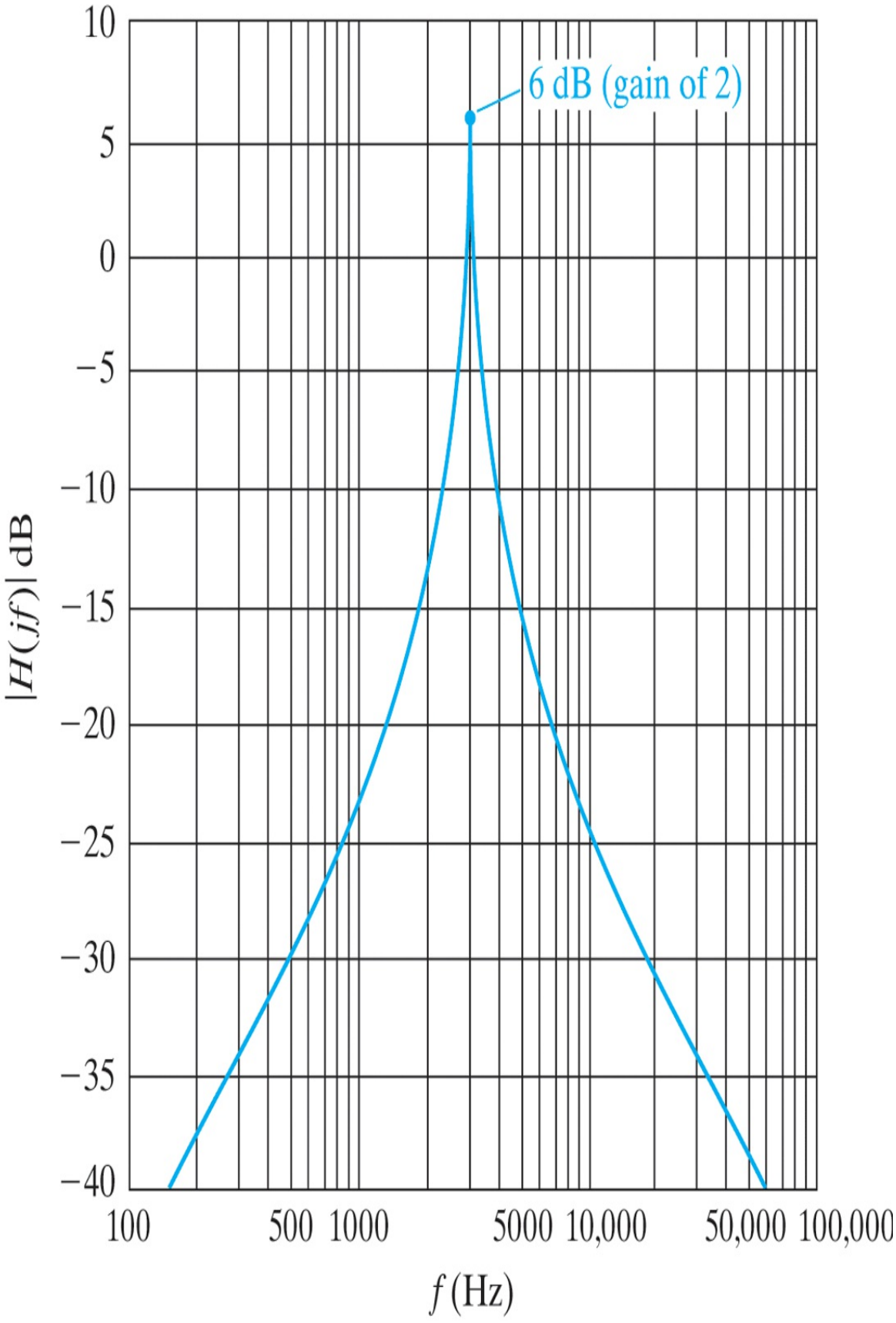

10
5
0
−5
−10
−15
−20
−25
−30
−35
−40
6 dB (gain of 2)
|H(jf)| dB
100
500
1000
5000
10,000
50,000
100,000
f (Hz)

Figure 15.29 Full Alternative Text

The parallel implementation of a bandreject filter that combines low-pass and high-pass filter components with a summing amplifier has the same low-Q restriction as the cascaded bandpass filter. The circuit in Fig. 15.30 is an active high-Q bandreject filter known as the **twin-T notch filter** because of the two T-shaped parts of the circuit at the nodes labeled a and b.

Figure 15.30 A high-Q active bandreject filter.

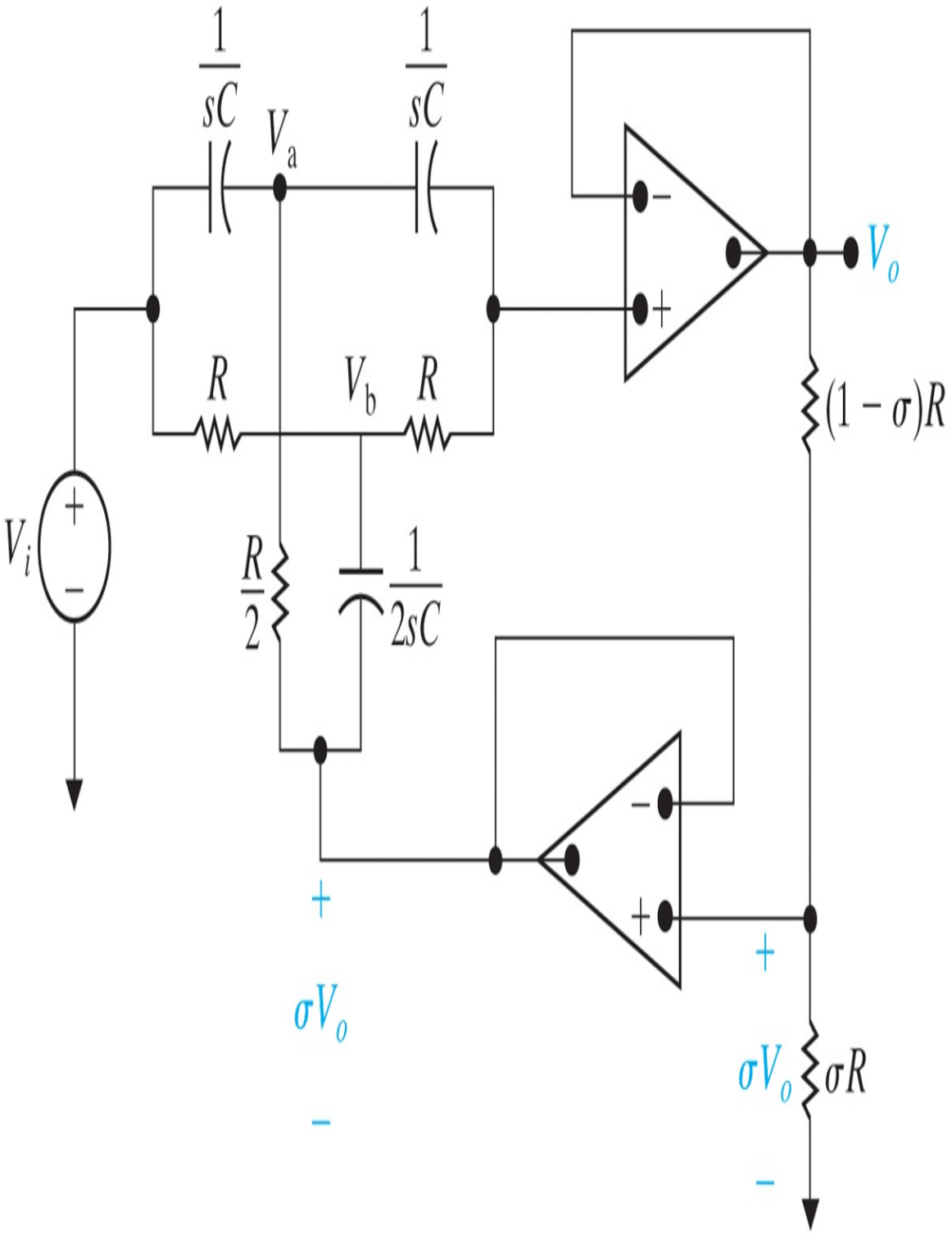


Figure 15.30 Full Alternative Text

We begin analyzing this circuit by writing a KCL equation at the node

labeled Va:

$$(Va-Vi)sC+(Va-Vo)sC+2(Va-\sigma Vo)R=0$$

or

$$Va[2sCR+2]-Vo[sCR+2\sigma]=sCRVi.$$

Writing a KCL equation at the node labeled Vb yields

$$Vb-ViR+Vb-VoR+(Vb-\sigma Vo)2sC=0$$

or

$$Vb[2+2RCs]-Vo[1+2\sigma RCs]=Vi.$$

Writing a KCL equation at the noninverting input terminal of the top op amp gives

$$(Vo-Va)sC+Vo-VbR=0$$

or

$$-sRCVa-Vb+(sRC+1)Vo=0.$$

From the three KCL equations, we can use Cramer's rule to solve for Vo:

$$Vo=\left|\begin{matrix}2(RCs+1) & 0 & sCR\\ Vi & 0 & 2(RCs+1)\\ Vi & -RCs & -1\end{matrix}\right| \Big/ \left|\begin{matrix}2(RCs+1) & 0 & -(RCs+2\sigma)\\ 0 & 2(RCs+1) & -(2\sigma RCs+1)\\ -RCs & -1 & RCs+1\end{matrix}\right| = (R^2C^2s^2+1)Vi \Big/ R^2C^2s^2+4RC(1-\sigma)s+1 .$$

Rearranging this equation for Vo, we can solve for the transfer function:

$$H(s)=VoVi=(s2+1R2C2)[s2+4(1-\sigma)RCs+1R2C2], \quad (15.21)$$

which is in the standard form for the transfer function of a bandreject filter:

$$H(s)=s2+\omega 02s2+\beta s+\omega 02.$$

Therefore,

$\omega_o^2 = \frac{1}{R^2C^2}$, $\beta = 4(1-\sigma)RC$.

In this circuit, we have three parameters (R, C, and σ) and two design constraints (ω_o and β). Thus, one parameter is chosen arbitrarily; it is usually the capacitor value because it normally has the fewest commercially available options. Once C is chosen,

$$R = \frac{1}{\omega_o C}, \quad \sigma = 1 - \frac{\beta}{4\omega_o} = 1 - \frac{1}{4Q}. \quad (15.22)$$

Example 15.14 illustrates the design of a high-Q active bandreject filter.

Example 15.14 Designing a High-Q Bandreject Filter

Design a high-Q active bandreject filter (based on the circuit in Fig. 15.30) with a center frequency of 5000 rad/s and a bandwidth of 1000 rad/s. Use 1 μF capacitors in your design.

Solution

In the bandreject prototype filter, $\omega_o = 1$ rad/s, $R = 1\Omega$, and $C = 1$ F. Once ω_o and Q are determined from the filter specifications, C can be chosen arbitrarily, and R and σ can be found from Eqs. 15.22. From the specifications, $\omega_o = 5000$ rad/s and $Q = 5$. Using Eqs. 15.22, we see that

$$R = \frac{1}{\omega_o C} = \frac{1}{(5000)(10^{-6})} = 200\Omega, \quad \sigma = 1 - \frac{1}{4Q} = 1 - \frac{1}{4(5)} = 0.95.$$

Therefore, we need resistors with the values 200Ω (R), 100Ω (R/2), 190Ω (σR), and 10Ω [(1−σ)R]. The final design is depicted in Fig. 15.31, and the Bode magnitude plot is shown in Fig. 15.32.

Figure 15.31 The high-Q active bandreject filter designed in Example 15.14.

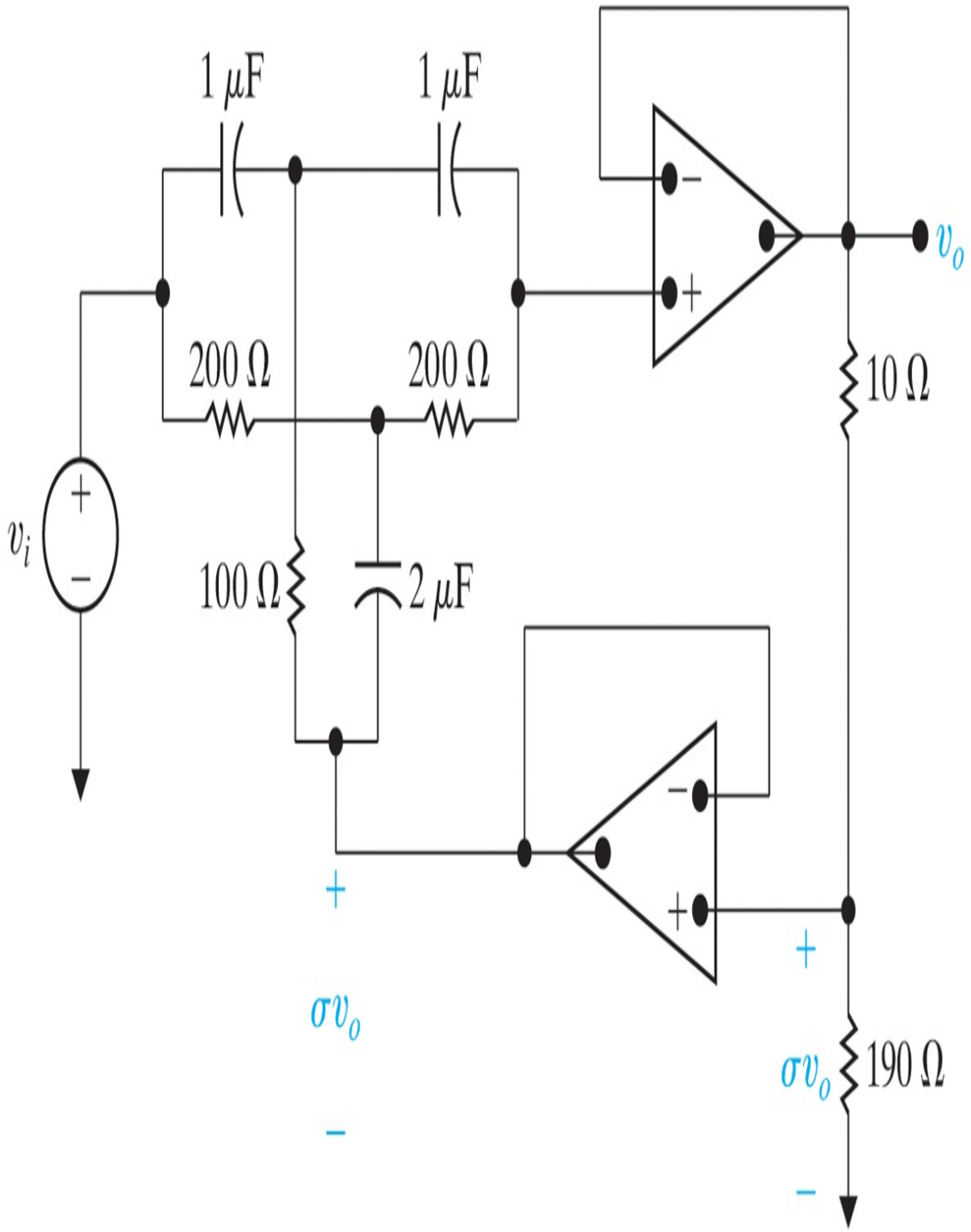


Figure 15.31 Full Alternative Text

Figure 15.32 The Bode magnitude plot for the high-Q active bandreject filter designed in Example 15.14.

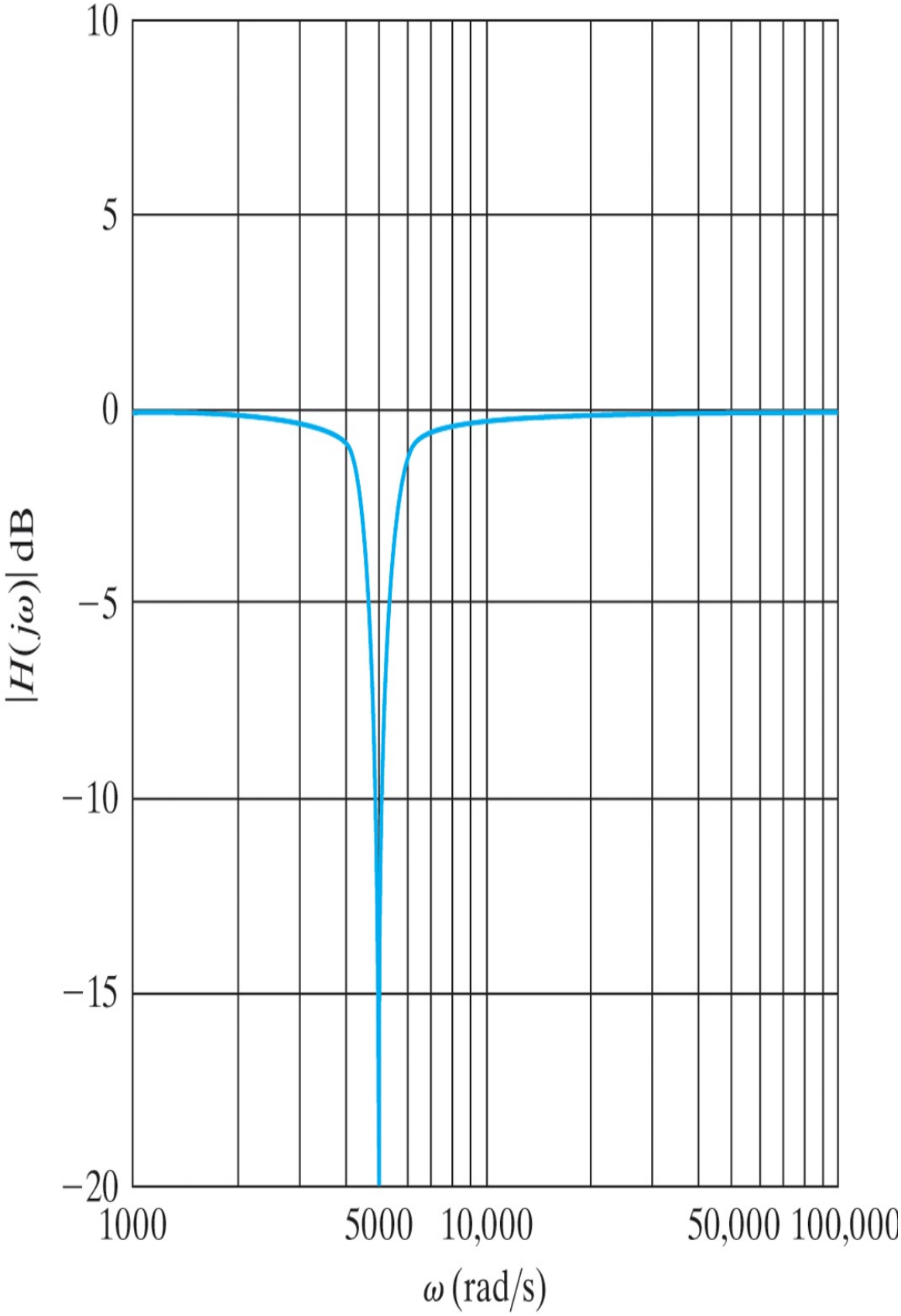

10
5
0
−5
−10
−15
−20
|H(jω)| dB
1000
5000
10,000
50,000
100,000
ω (rad/s)

Figure 15.32 Full Alternative Text

Assessment Problems

Objective 4—Be able to use design equations to calculate component values for prototype narrowband bandpass, and bandreject filters

1. 15.5 Design an active bandpass filter with Q=8, K=5, and ωo=1000 rad/s. Use 1 μF capacitors, and specify the values of all resistors.

   Answer:

   R1=1.6 kΩ, R2=65.04 Ω, R3=16 kΩ.

2. 15.6 Design an active unity-gain bandreject filter with ωo=1000 rad / s and Q=4. Use 2 μF capacitors in your design, and specify the values of R and σ.

   Answer:

   R=500Ω, σ=0.9375.

SELF-CHECK: Also try Chapter Problems 15.58 and 15.60.

Practical Perspective

Bass Volume Control

We now look at an op amp circuit that can be used to control audio signal amplification in the bass range. Signals in the audio range have frequencies from 20 Hz to20 kHz. The bass range includes frequencies up to 300 Hz. The volume control circuit and its frequency response are shown in Fig. 15.33.

We can select a desired frequency response curve from the family of curves in Fig. 15.33(b) by adjusting the potentiometer in Fig. 15.33(a).

Figure 15.33 (a) Bass volume control circuit; (b) Bass volume control circuit frequency response.

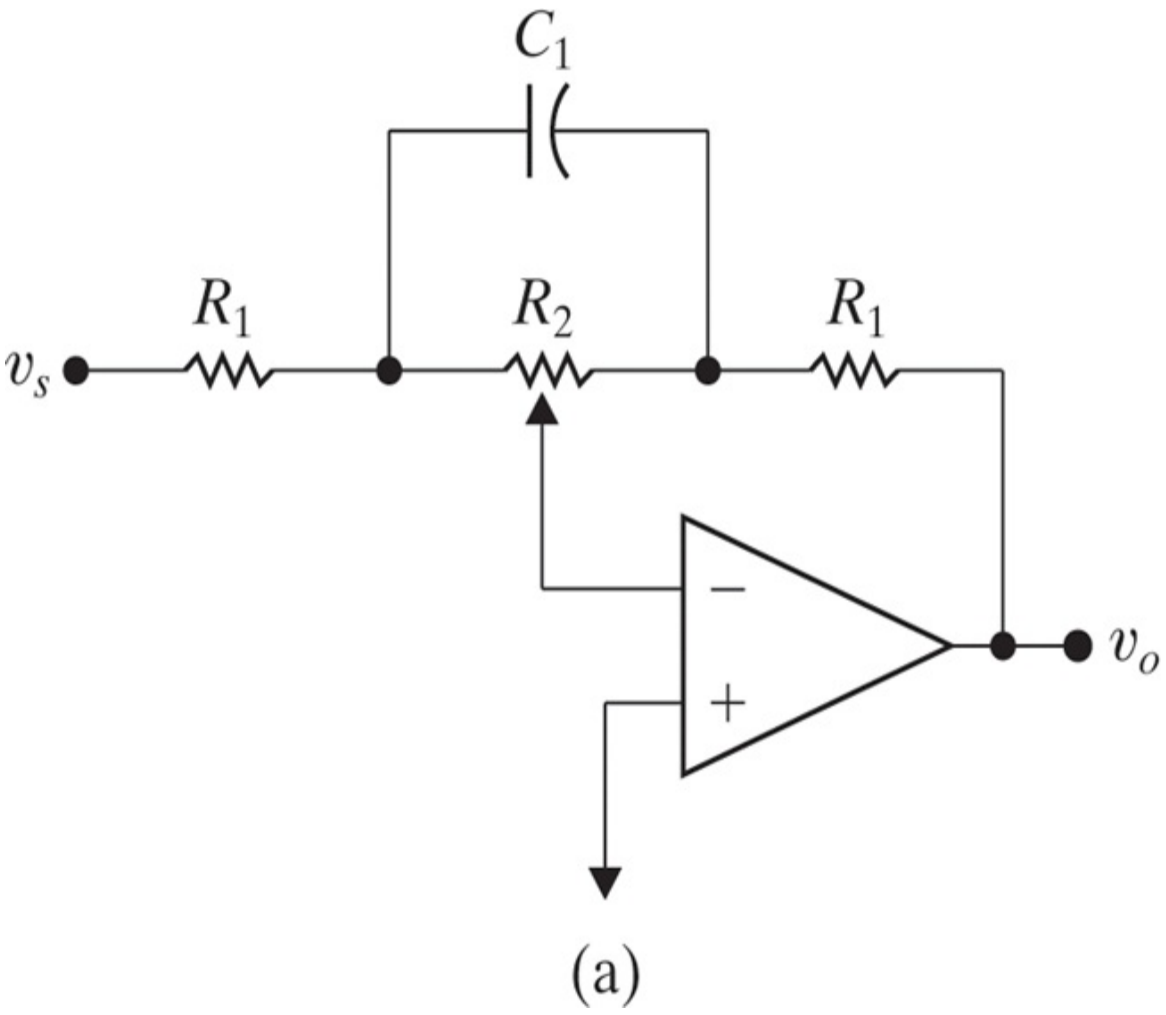
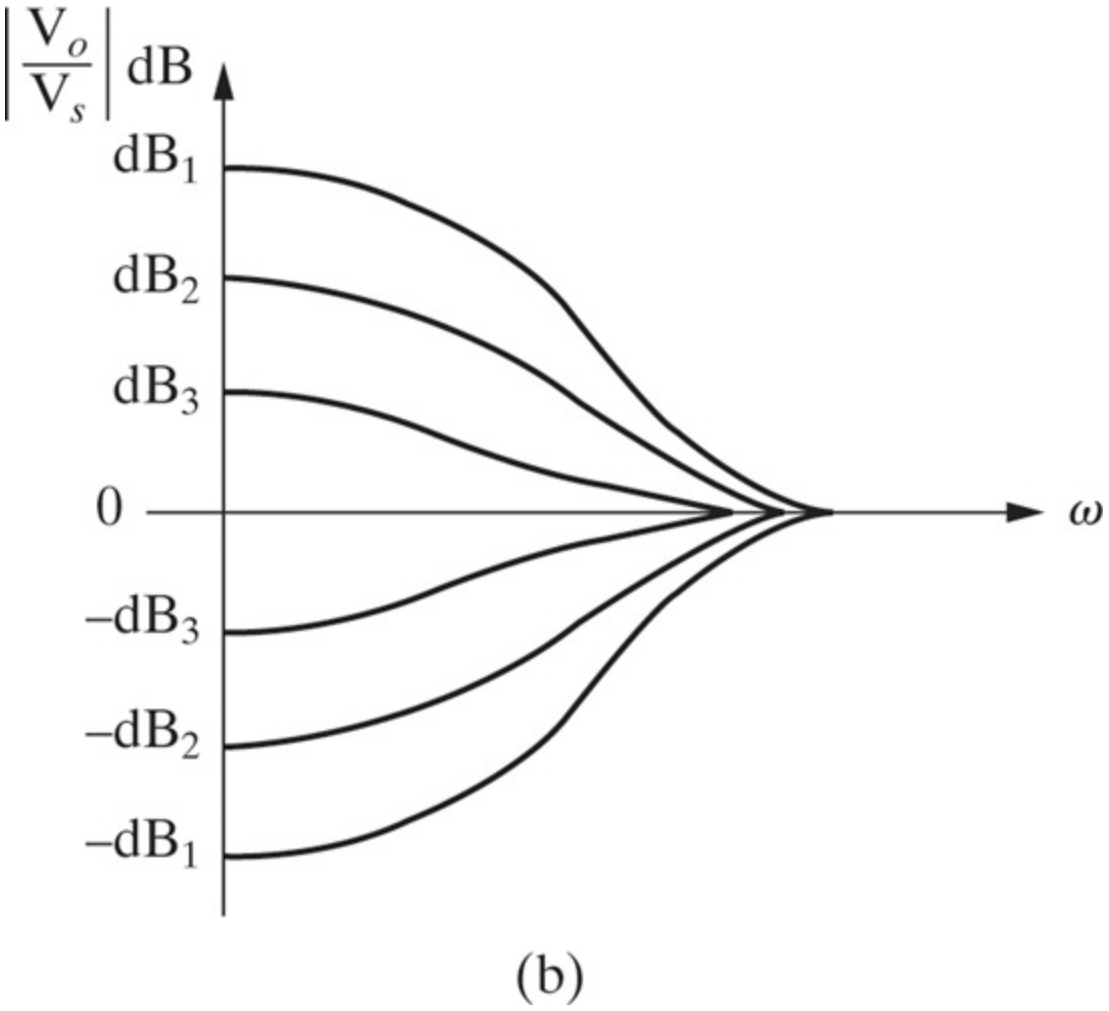


Figure 15.33 Full Alternative Text



(b)

15.5-6 Full Alternative Text

Study the frequency response curves in Fig. 15.33(b) and note the following.

- The gain in dB can be either positive or negative. If the gain is positive, a signal in the bass range is amplified or boosted. If the gain is negative, the signal is attenuated or cut.

- It is possible to select a response curve that has unity gain (zero dB) for all frequencies in the bass range. As we shall see, if the potentiometer is set at its midpoint, the circuit will have no effect on signals in the bass range.

- As the frequency increases, all frequency response curves approach zero dB or unity gain. Hence, the volume control circuit will have no effect

on signals in the upper end, or treble range, of the audio frequencies.

We need to find the transfer function, Vo / Vs, for the circuit in Fig. 15.33(a) in order to analyze the circuit's frequency response. To find the transfer function, we transform the circuit into the s domain as shown in Fig. 15.34. The node voltages Va and Vb have been labeled in the circuit to support node-voltage analysis. The position of the potentiometer is determined by the numerical value of α, as noted in Fig. 15.34.

Figure 15.34 The s-domain circuit for the bass volume control. Note that α determines the potentiometer setting, so $0 \leq \alpha \leq 1$.

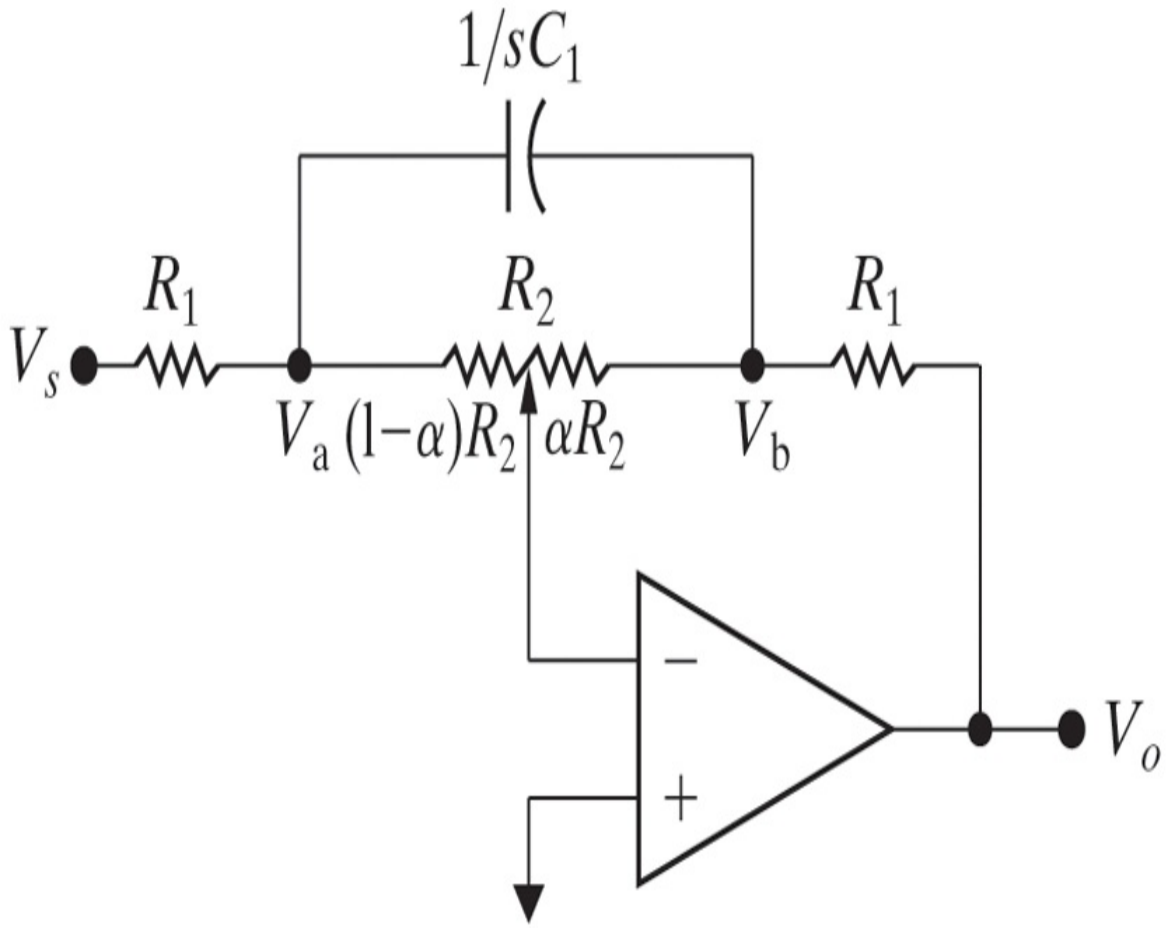


Figure 15.34 Full Alternative Text

To find the transfer function, we write the three KCL equations at the nodes labeled Va and Vb, and at the inverting input of the op amp:

V a (1−α) R 2 + V a − V s R 1 + (V a − V b)s C 1 = 0; V b α R 2 + (V b − V a)s C 1 + V b − V o R 1 = 0; V a (1−α) R 2 + V b α R 2 = 0.

Solve the three node-voltage equations to find Vo as a function of Vs and hence the transfer function H(s):

H(s)=VoVs=−(R1+ αR2+R1R2C1s)R1+(1−α)R2+R1R2C1s.

It follows directly that

H(jω)=−(R1+αR2+jωR1R2C1)[R1+(1−α)R2+jωR1R2C1].

Now let’s verify that this transfer function generates the family of frequency response curves depicted in Fig. 15.33(b). First note that when $\alpha=0.5$ the magnitude of $H(j\omega)$ is unity for all frequencies, that is,

$$|H(j\omega)|=|R1+0.5R2+j\omega R1R2C1||R1+ 0.5R2+j\omega R1R2C1|=1.$$

When $\omega=0$, we have

$$|H(j0)|=R1+\alpha R2R1+(1-\alpha)R2.$$

Observe that $|H(j0)|$ at $\alpha=1$ is the reciprocal of $|H(j0)|$ at $\alpha=0$; that is

$$|H(j0)|\alpha=1=R1+R2R1=1|H(j0)|\alpha=0.$$

In fact, this reciprocal relationship holds for all frequencies, not just $\omega=0$. For example, $\alpha=0.4$ and $\alpha=0.6$ are symmetric about $\alpha=0.5$ and

$$H(j\omega)\alpha=0.4=-(R1+0.4R2)+j\omega R1R2C1(R1+ 0.6R2)+j\omega R1R2C1$$

while

$$H(j\omega)\alpha=0.6=-(R1+0.6R2)+j\omega R1R2C1(R1+0.4R2)+j\omega R1R2C1.$$

Hence

$$H(j\omega)\alpha=0.4=1H(j\omega)\alpha=0.6.$$

The volume control circuit can either amplify or attenuate its input signal, depending on the value of α.

The numerical values of R1, R2, and C1 are based on two design specifications. The first specification is the passband amplification or attenuation in the bass range (as $\omega \rightarrow 0$). The second specification is the frequency at which this passband amplification or attenuation is changed by 3 dB. The component values that satisfy the specifications are calculated with α equal to either 1 or 0.

As we have already observed, the maximum gain will be $(R1+R2) / R1$, and the maximum attenuation will be$R1 / (R1+R2)$. If we assume

(R1+R2)/R1≫1, then the amplification (or attenuation) will differ by 3dB from its maximum value whenω=1 / R2C1. This can be seen by noting that

|H(j 1 R 2 C 1) | α=1 = | R 1 + R 2 +j R 1 | | R 1 +j R 1 | = | R 1 + R 2 R 1 | | 1+j1 | ≈ 1 2 (R 1 + R 2 R 1) and |H(j 1 R 2 C 1) | α=0 = | R 1 +j R 1 | | R 1 + R 2 +j R 1 | = | 1+j1 | | R 1 + R 2 R 1 +j1| ≈ 2 (R 1 R 1 + R 2).

SELF-CHECK: Assess your understanding of this Practical Perspective by trying Chapter Problems 15.61 and 15.62.

Summary

- Active filters consist of op amps, resistors, and capacitors. They can be configured as low-pass, high-pass, bandpass, and bandreject filters. They overcome many of the disadvantages associated with passive filters. (See page 572.)

- A **prototype low-pass filter** has component values of R1=R2=1Ω and C=1F, and it produces a unity passband gain and a cutoff frequency of 1rad / s. The **prototype high-pass filter** has the same component values and also produces a unity passband gain and a cutoff frequency of 1rad / s. (See pages 575 and 576.)

- **Magnitude scaling** can be used to alter component values without changing the frequency response of a circuit. For a magnitude scale factor of km, the scaled (primed) values of resistance, capacitance, and inductance are

  $R' = kmR$, $L' = kmL$, and $C' = C / km$.

  (See page 577.)

- **Frequency scaling** can be used to shift the frequency response of a circuit to another frequency region without changing the overall shape of the frequency response. For a frequency scale factor of kf, the scaled (primed) values of resistance, capacitance, and inductance are

  $R' = R$, $L' = L / kf$, and $C' = C / kf$.

  (See page 578.)

- Components can be scaled in both magnitude and frequency, with the scaled (primed) component values given by

  $R' = kmR$, $L' = (km / kf)L$, and $C' = C / (kmkf)$.

(See page 578)

- The design of active low-pass and high-pass filters can begin with a prototype filter circuit. Scaling can then be applied to shift the frequency response to the desired cutoff frequency, using component values that are commercially available. (See page 578.)

- An active broadband bandpass filter can be constructed using a cascade of a low-pass filter with the bandpass filter's upper cutoff frequency, a high-pass filter with the bandpass filter's lower cutoff frequency, and (optionally) an inverting amplifier gain stage to achieve nonunity gain in the passband. Bandpass filters implemented in this fashion must be broadband filters($\omega c2 \gg \omega c1$), so that each op amp circuit in the cascade can be specified independently. (See page 579.)

- An active broadband bandreject filter can be constructed using a parallel combination of a low-pass filter with the bandreject filter's lower cutoff frequency and a high-pass filter with the bandreject filter's upper cutoff frequency. The outputs are then fed into a summing amplifier, which can produce nonunity gain in the passband. Bandreject filters implemented in this way must be broadband filters($\omega c2 \gg \omega c1$), so that the low-pass and high-pass filter circuits can be designed independently. (See page 585.)

- Higher-order active filters have multiple poles in their transfer functions, resulting in a sharper transition from the passband to the stopband and thus a more nearly ideal frequency response. (See page 588.)

- The transfer function of an nth-order Butterworth low-pass filter with a cutoff frequency of 1rad / s can be determined from the equation

  $H(s)H(-s)=11+(-1)ns2n$

  by

  - finding the roots of the denominator polynomial
  - assigning the left-half plane roots to H(s)

    - writing the denominator of H(s) as a product of first- and second-order factors

(See page 591–592.)

- The fundamental problem in the design of a Butterworth filter is to determine the order of the filter. The filter specification usually defines the sharpness of the transition band in terms of the quantities Ap, ωp, As, and ωs. From these quantities, we calculate the smallest integer larger than the solution to either Eq. 15.15 or Eq. 15.16. (See page 596.)

- A cascade of second-order low-pass op amp filters (Fig. 15.21) with 1 Ω resistors and capacitor values chosen to produce each factor in the Butterworth polynomial will produce an even-order Butterworth low-pass filter. Adding a prototype low-pass op amp filter will produce an odd-order Butterworth low-pass filter. (See page 593.)

- A cascade of second-order high-pass op amp filters (Fig. 15.25) with 1 F capacitors and resistor values chosen to produce each factor in the Butterworth polynomial will produce an even-order Butterworth high-pass filter. Adding a prototype high-pass op amp filter will produce an odd-order Butterworth high-pass filter. (See page 598.)

- For both high- and low-pass Butterworth filters, frequency and magnitude scaling can be used to shift the cutoff frequency from 1rad / s and to include realistic component values in the design. Cascading an inverting amplifier will produce a nonunity passband gain. (See page 598.)

- Butterworth low-pass and high-pass filters can be cascaded to produce Butterworth bandpass filters of any order n. Butterworth low-pass and high-pass filters can be combined in parallel with a summing amplifier to produce a Butterworth bandreject filter of any order n. (See page 599.)

- If a high-Q, or narrowband, bandpass, or bandreject filter is needed, the cascade or parallel combination will not work. Instead, the circuits shown in Figs. 15.27 and 15.30 are used with the appropriate design

equations. Typically, capacitor values are chosen from those commercially available, and the design equations are used to specify the resistor values. (See page 600 and 603.)

Problems

Section 15.1

1. 15.1 DESIGNPROBLEM_

    1. Using the circuit in Fig. 15.1, design a low-pass filter with a passband gain of 5 dB and a cutoff frequency of 10 kHz. Assume a 50 nF capacitor is available.

    2. Draw the circuit diagram and label all components.

2. 15.2

    1. Using only three components from Appendix H, design a low-pass filter with a cutoff frequency and passband gain as close as possible to the specifications in Problem 15.1(a). Draw the circuit diagram and label all component values.

    2. Calculate the percent error in this new filter's cutoff frequency and passband gain when compared to the values that are specified in Problem 15.1(a).

3. 15.3 Design an op amp-based low-pass filter with a cutoff frequency of 500 Hz and a passband gain of 8 using a 5 μF capacitor.

    1. Draw your circuit, labeling the component values and output voltage.

    2. If the value of the feedback resistor in the filter is changed but the value of the resistor in the forward path is unchanged, what characteristic of the filter is changed?

4. 15.4 The low-pass filter designed in Problem 15.3 has an input of $2\cos\omega t$ V.

    1. Suppose the power supplies are ±Vcc. What is the smallest value of Vcc that will still cause the op amp to operate in its linear region?

2. Find the output voltage when ω=ωc.
3. Find the output voltage when ω=0.1ωc.
4. Find the output voltage when ω=10ωc.

5. 15.5 Find the transfer function Vo/Vi for the circuit shown in Fig. P15.5 if Zf is the equivalent impedance of the feedback circuit, Zi is the equivalent impedance of the input circuit, and the operational amplifier is ideal.

Figure P15.5

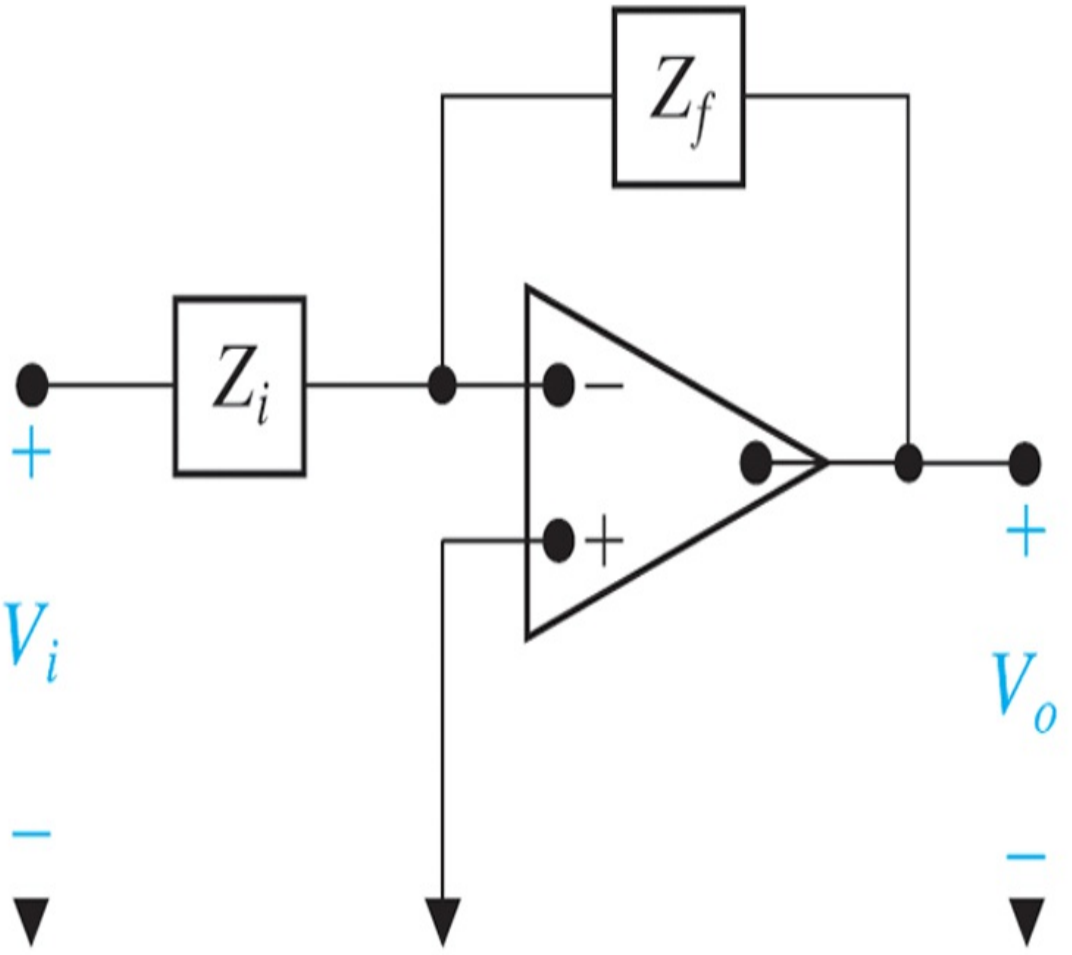


Figure P15.5 Full Alternative Text

6. 15.6

    1. Use the results of Problem 15.5 to find the transfer function of the circuit shown in Fig. P15.6.

    2. What is the gain of the circuit as $\omega \to 0$?

    3. What is the gain of the circuit as $\omega \to \infty$?

    4. Do your answers to (b) and (c) make sense in terms of known circuit behavior?

Figure P15.6

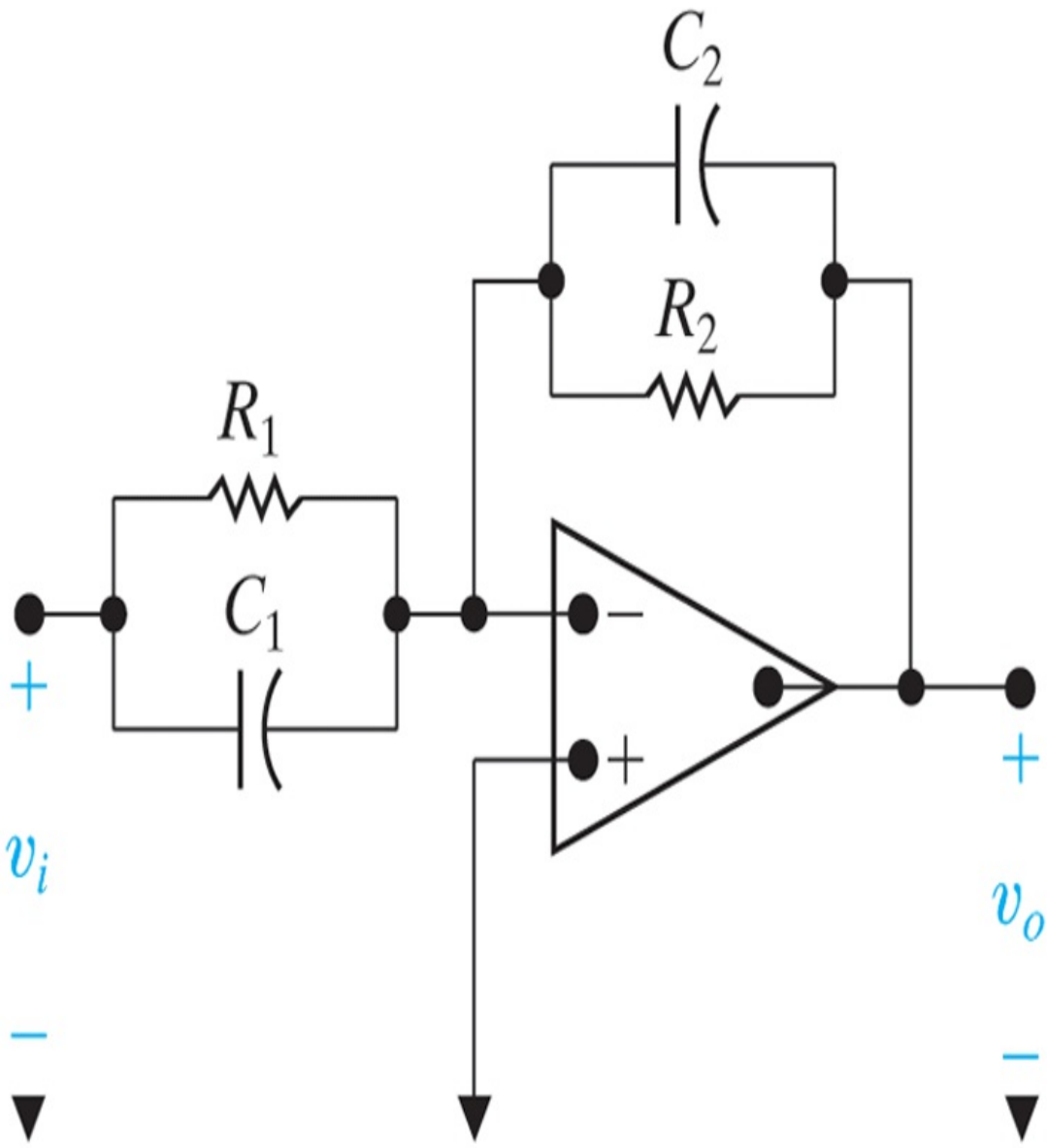


Figure P15.6 Full Alternative Text

7. 15.7 Repeat Problem 15.6, using the circuit shown in Fig. P15.7.

Figure P15.7

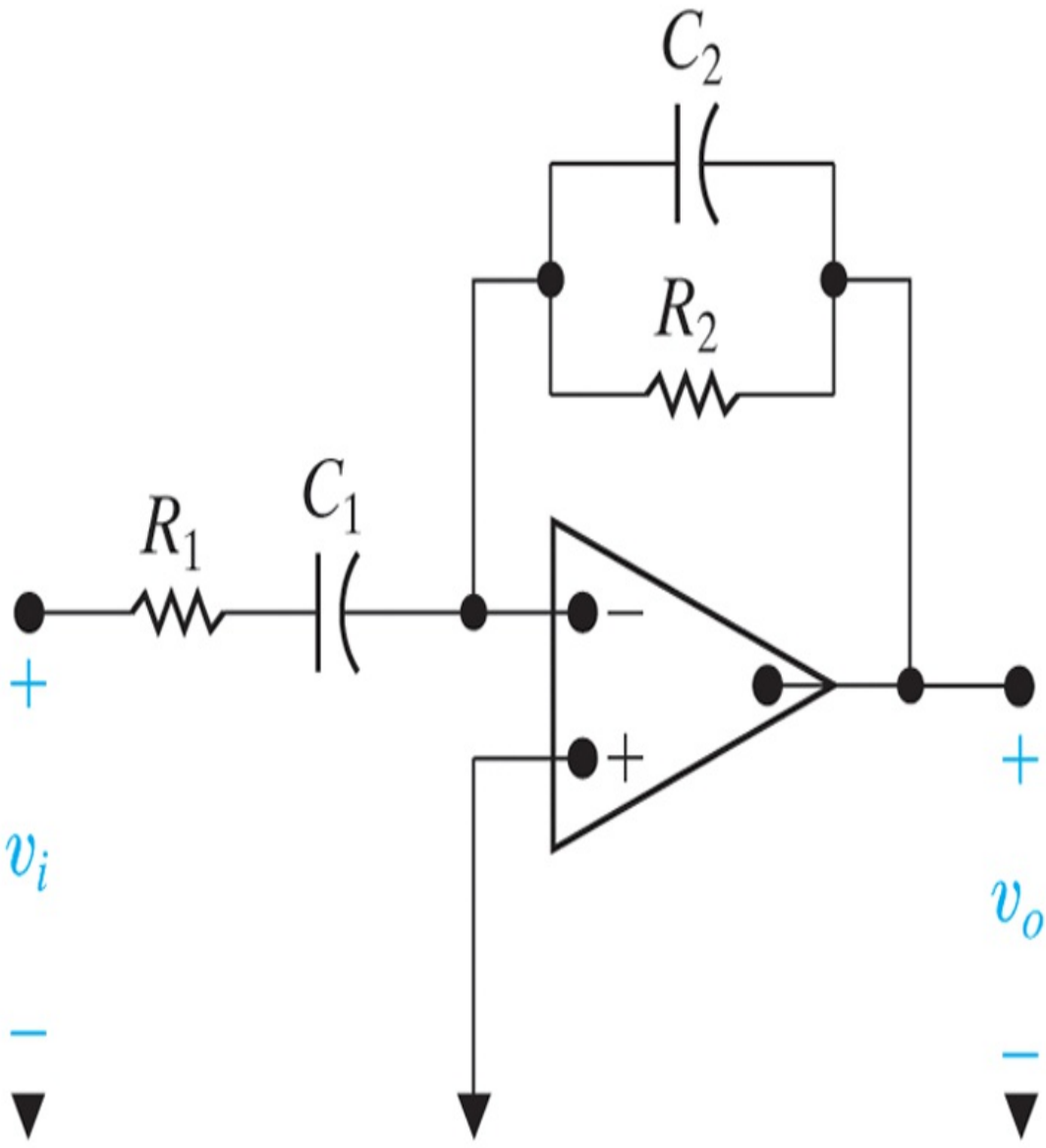


Figure P15.7 Full Alternative Text

8. 15.8 DESIGNPROBLEM_

    1. Use the circuit in Fig. 15.4 to design a high-pass filter with a cutoff frequency of 2.5 kHz and a passband gain of 10 dB. Use a 50 nF capacitor in the design.

    2. Draw the circuit diagram of the filter and label all the components.

9. 15.9 Using only three components from Appendix H, design a high-pass

filter with a cutoff frequency and passband gain as close as possible to the specifications in Problem 15.8.

1. Draw the circuit diagram and label all component values.

2. Calculate the percent error in this new filter's cutoff frequency and passband gain when compared to the values specified in Problem 15.8(a).

10. 15.10 Design an op amp-based high-pass filter with a cutoff frequency of 10 kHz and a passband gain of 4 using a 75 nF capacitor.

    1. Draw your circuit, labeling the component values and the output voltage.

    2. If the value of the feedback resistor in the filter is changed but the value of the resistor in the forward path is unchanged, what characteristic of the filter is changed?

11. 15.11 The input to the high-pass filter designed in Problem 15.10 is $4 \cos\omega t$ V.

    1. Suppose the power supplies are $\pm V_{cc}$. What is the smallest value of V_{cc} that will still cause the op amp to operate in its linear region?

    2. Find the output voltage when $\omega=\omega_c$.

    3. Find the output voltage when $\omega=0.1\omega_c$.

    4. Find the output voltage when $\omega=10\omega_c$.

Section 15.2

1. 15.12 The voltage transfer function for either high-pass prototype filter shown in Fig. P15.12 is

   H(s)=ss+1.

Figure P15.12

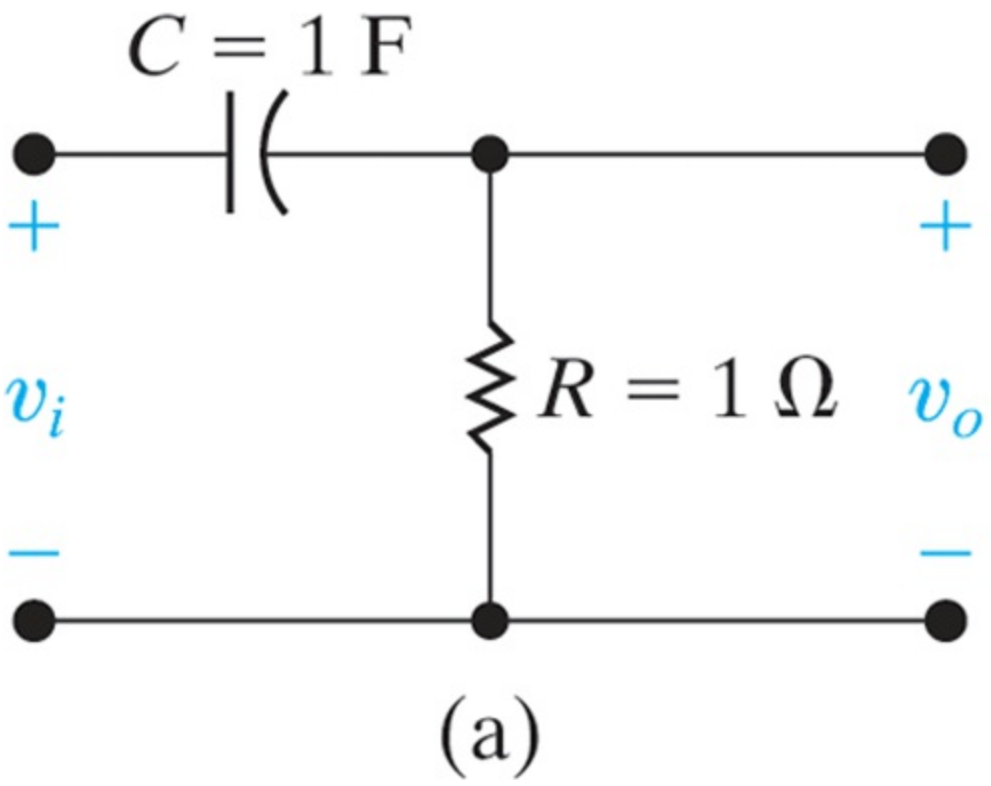
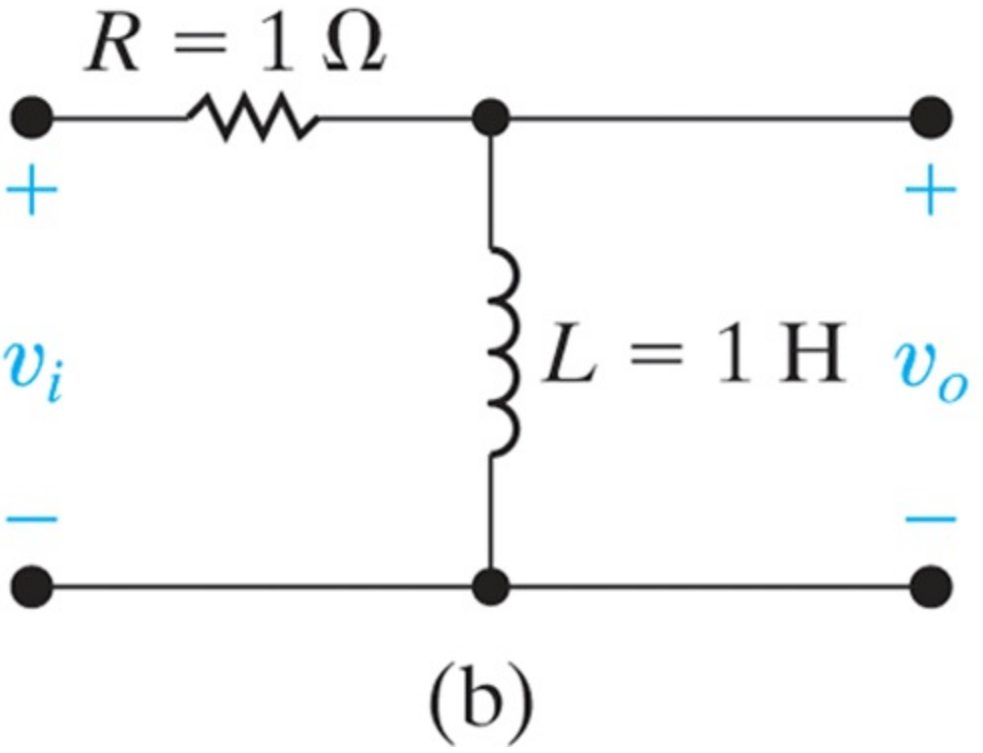


Figure P15.12 Full Alternative Text



15.2-6 Full Alternative Text

Show that if either circuit is scaled in both magnitude and frequency, the scaled transfer function is

H′(s)=(s/kf)(s/kf)+1.

2. 15.13 The voltage transfer function of either low-pass prototype filter shown in Fig. P15.13 is

   H(s)=1s+1.

Figure P15.13

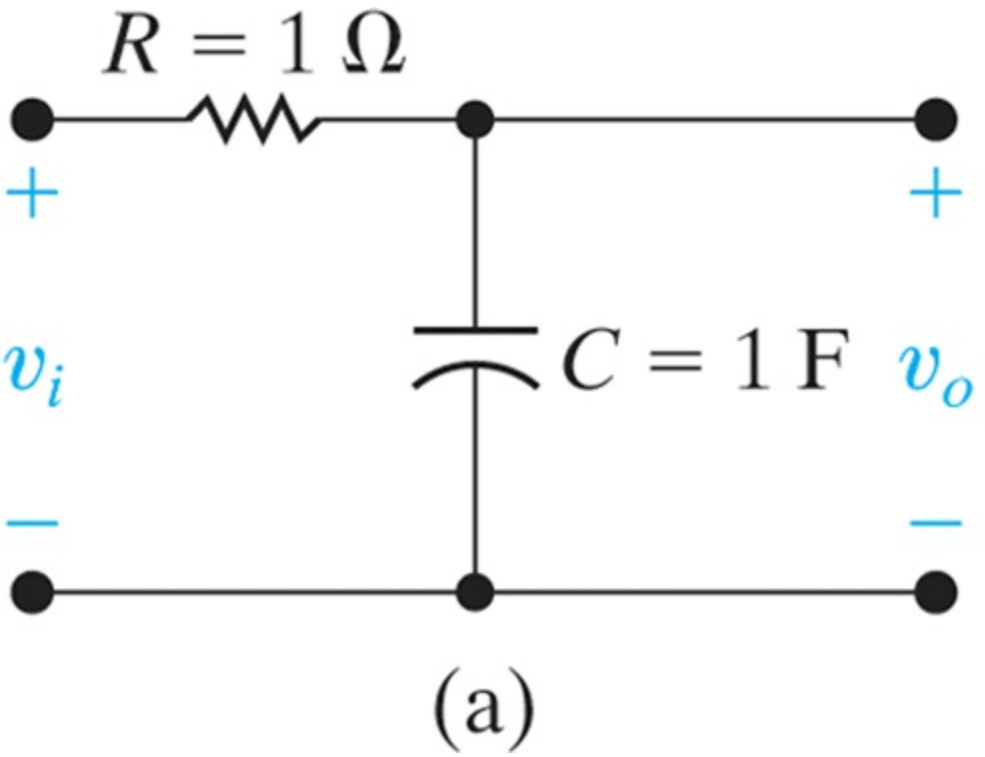
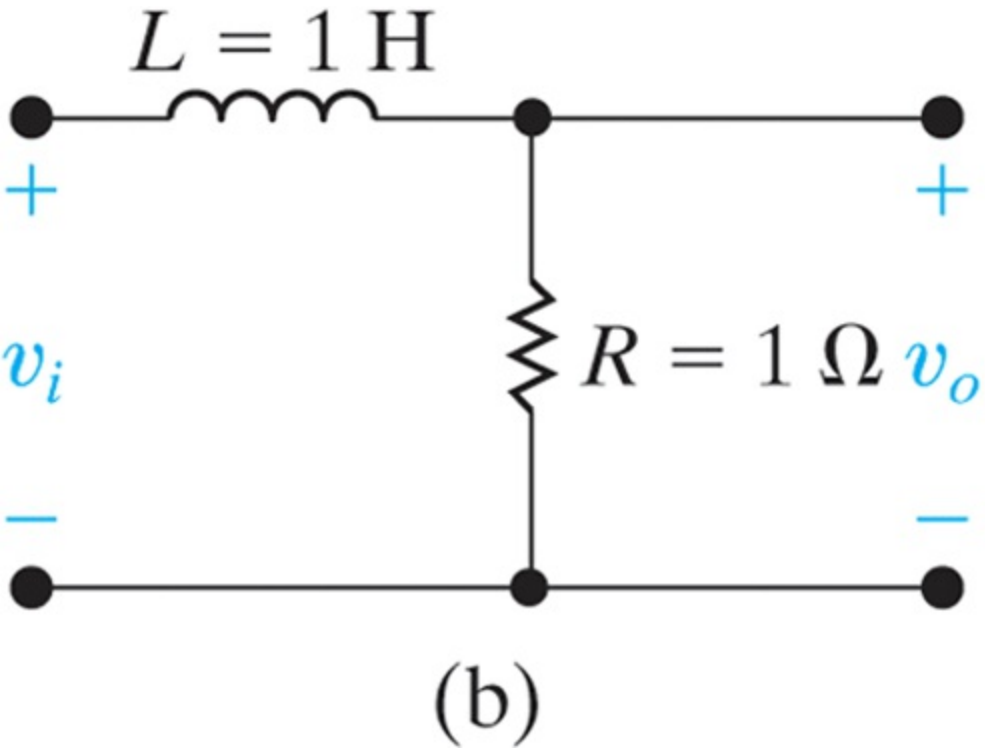


Figure P15.13 Full Alternative Text



15.2-6 Full Alternative Text

Show that if either circuit is scaled in both magnitude and frequency, the scaled transfer function is

H′(s)=1(s/kf)+1.

3. 15.14 The voltage transfer function of the prototype bandpass filter shown in Fig. P15.14 is

   H(s)=(1Q)ss2+(1Q)s+1.

Figure P15.14

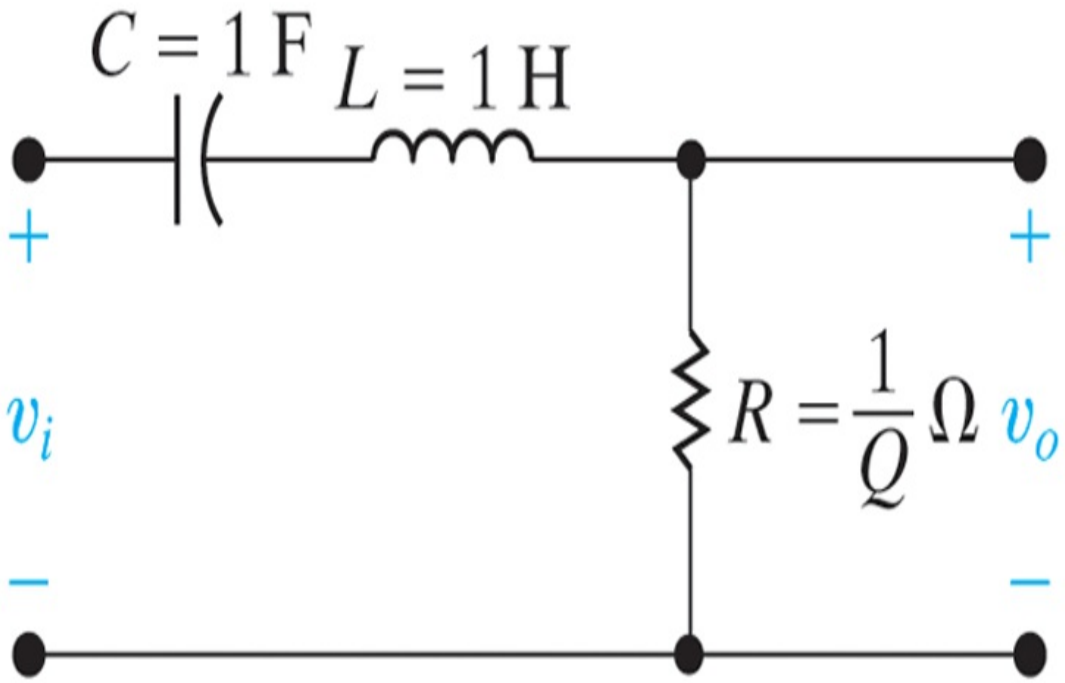


Figure P15.14 Full Alternative Text

Show that if the circuit is scaled in both magnitude and frequency, the scaled transfer function is

H′(s)=(1Q)(skf)(skf)2+(1Q)(skf)+1.

4. 15.15

1. Specify the component values for the prototype passive bandpass filter described in Problem 15.14 if the quality factor of the filter is 10.

2. Specify the component values for the scaled bandpass filter described in Problem 15.14 if the quality factor is 10; the center, or resonant, frequency is 75 krad/s; and the impedance at resonance is 2.5 kΩ.

3. Draw a circuit diagram of the scaled filter and label all the components.

5. 15.16 An alternative to the prototype bandpass filter illustrated in Fig. P15.14 is to make ωo=1 rad/s, R=1 Ω, and L=Q henrys.

    1. What is the value of C in the prototype filter circuit?

    2. What is the transfer function of the prototype filter?

    3. Use the alternative prototype circuit just described to design a passive bandpass filter that has a quality factor of 12, a center frequency of 40 krad/s, and an impedance of 50 kΩ at resonance.

    4. Draw a diagram of the scaled filter and label all the components.

    5. Use the results obtained in Problem 15.14 to write the transfer function of the scaled circuit.

6. 15.17 DESIGNPROBLEM_ The passive bandpass filter illustrated in Fig14.22. has two prototype circuits. In the first prototype circuit, ωo=1 rad/s, C=1 F, L=1 H, and R=Q ohms. In the second prototype circuit, ωo=1 rad/s, R=1 Ω, C=Q farads, and L=(1/Q) henrys.

    1. Use one of these prototype circuits (your choice) to design a passive bandpass filter that has a quality factor of 15 and a center frequency of 100 krad/s. The resistor R is 10 kΩ.

    2. Draw a circuit diagram of the scaled filter and label all components.

7. 15.18 The passive bandreject filter illustrated in Fig14.28(a). has the two prototype circuits shown in Fig. P15.18.

    1. Show that for both circuits, the transfer function is

       H(s)=s2+1s2+(1Q)s+1.

    2. Write the transfer function for a bandreject filter that has a center frequency of 10 krad/s and a quality factor of 8.

Figure P15.18

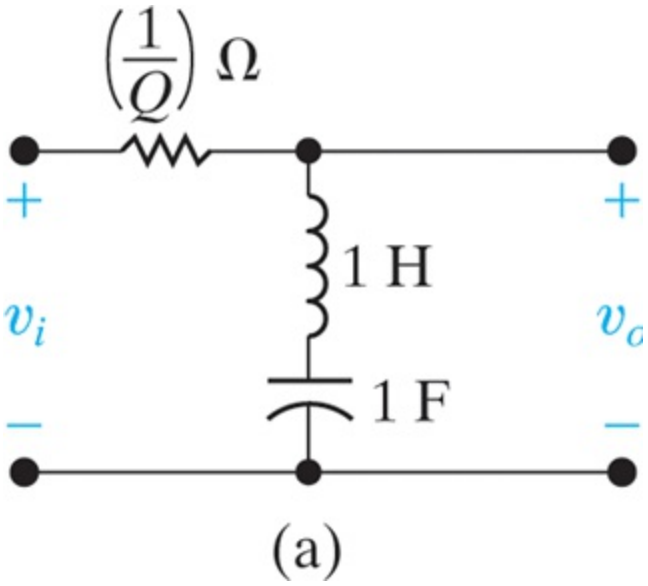
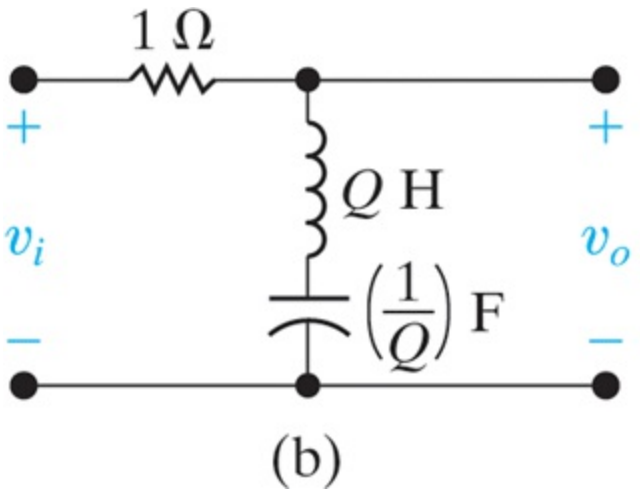


Figure P15.18 Full Alternative Text



15.2-6 Full Alternative Text

8. 15.19 The transfer function for the bandreject filter shown in Fig14.28(a). is

   H(s)=s2+(1LC)s2+(RL)s+(1LC).

   Show that if the circuit is scaled in both magnitude and frequency, the transfer function of the scaled circuit is equal to the transfer function of the unscaled circuit with s replaced by (s/kf), where kf is the frequency scale factor.

9. 15.20 Show that the observation made in Problem 15.19 with respect to the transfer function for the circuit in Fig14.28(a). also applies to the bandreject filter circuit (lower one) in Fig14.31. .

10. 15.21 The two prototype versions of the passive bandreject filter shown in Fig14.31. (lower circuit) are shown in Fig. P15.21(a) and (b).

Figure P15.21

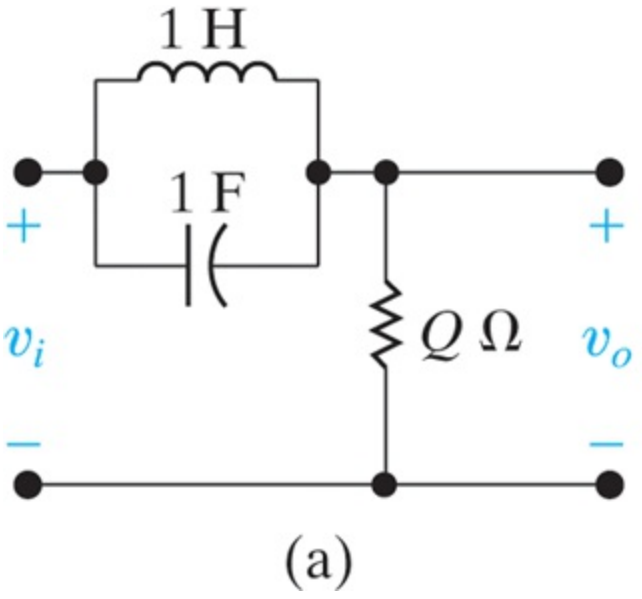
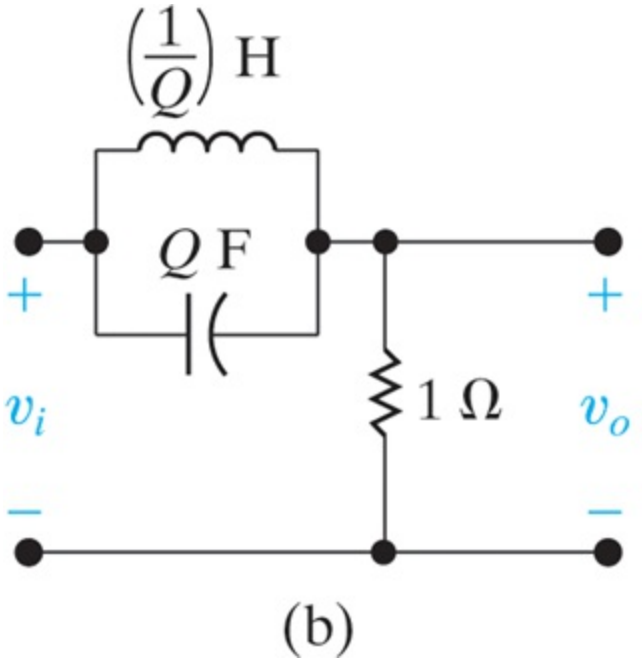


Figure P15.21 Full Alternative Text



15.2-6 Full Alternative Text

Show that the transfer function for either version is

H(s)=s2+1s2+(1Q)s+1.

11. 15.22 The circuit in Fig. P9.27 is scaled so that the 100 Ω resistor is

replaced by 500 Ω resistor and the 160 μH inductor is replaced by a 2 mH inductor.

1. What is the scaled value of the capacitor?

2. Find the frequency for which the impedance Zab is purely resistive for the scaled circuit.

3. How is the frequency you found in part (b) related to the frequency for which the impedance Zab is purely resistive in the un-scaled circuit?

12. 15.23 Scale the inductor and capacitor in Fig. P9.66 so that the magnitude and the phase angle of the output current do not change when the input frequency is changed from 20 krad/s to 500 rad/sec.

    1. What are the scaled values of the inductor and capacitor?

    2. What is the steady-state value of the output current, io, when the input current is 400 cos 500t mA?

13. 15.24 Scale the bandpass filter in Problem 14.30 so that the center frequency is 200 kHz and the quality factor is still 8, using a 2.5 nF capacitor. Determine the values of the resistor and the inductor, and the two cutoff frequencies of the scaled filter.

14. 15.25 Scale the bandreject filter in Problem 14.41 to get a center frequency of 50 krad[&*N*[-1%0]|sol|&]s, using a 200 μH inductor. Determine the values of the resistor, the capacitor, and the bandwidth of the scaled filter.

15. 15.26

    1. Show that if the low-pass filter circuit illustrated in Fig. 15.1 is scaled in both magnitude and frequency, the transfer function of the scaled circuit is the same as Eq. 15.1 with s replaced by s/kf, where kf is the frequency scale factor.

    2. In the prototype version of the low-pass filter circuit in Fig. 15.1,

ωc=1 rad/s, C=1 F, R2=1 Ω, and R1=1/K ohms. What is the transfer function of the prototype circuit?

3. Using the result obtained in (a), derive the transfer function of the scaled filter.

16. 15.27

1. Show that if the high-pass filter illustrated in Fig. 15.4 is scaled in both magnitude and frequency, the transfer function is the same as Eq. 15.4 with s replaced by s/kf, where kf is the frequency scale factor.

2. In the prototype version of the high-pass filter circuit in Fig. 15.4, ωc=1 rad/s, R1=1 Ω, C=1 F, and R2=K ohms. What is the transfer function of the prototype circuit?

3. Using the result in (a), derive the transfer function of the scaled filter.

Section 15.3

1. 15.28 DESIGNPROBLEM_PSPICEMULTISIM

    1. Using 100 nF capacitors, design an active broadband first-order bandpass filter that has a lower cutoff frequency of 1000 Hz, an upper cutoff frequency of 5000 Hz, and a passband gain of 0 dB. Use prototype versions of the low-pass and high-pass filters in the design process (see Problems 15.26 and 15.27).

    2. Write the transfer function for the scaled filter.

    3. Use the transfer function derived in part (b) to find H(jωo), where ωo is the center frequency of the filter.

    4. What is the passband gain (in decibels) of the filter at ωo?

    5. Using a computer program of your choice, make a Bode magnitude plot of the filter.

2. 15.29 DESIGNPROBLEM_PSPICEMULTISIM

    1. Using 10 nF capacitors, design an active broadband first-order bandreject filter with a lower cutoff frequency of 400 Hz, an upper cutoff frequency of 4000 Hz, and a passband gain of 0 dB. Use the prototype filter circuits introduced in Problems 15.26 and 15.27 in the design process.

    2. Draw the circuit diagram of the filter and label all of the components.

    3. What is the transfer function of the scaled filter?

    4. Evaluate the transfer function derived in (c) at the center frequency of the filter.

5. What is the gain (in decibels) at the center frequency?

6. Using a computer program of your choice, make a Bode magnitude plot of the filter transfer function.

3. 15.30 Design a bandpass filter, using a cascade connection, to give a center frequency of 500 Hz, a bandwidth of 4 kHz, and a passband gain of 4. Use 250 nF capacitors. Specify fc1, fc2, RL, and RH.

4. 15.31 Design a unity-gain parallel bandreject filter with a center frequency of 250 rad/s and a bandwidth of 2000 rad/s. Use 1 μF capacitors, and specify all resistor values.

5. 15.32 Show that the circuit in Fig. P15.32 behaves as a bandpass filter. (*Hint*—find the transfer function for this circuit and show that it has the same form as the transfer function for a bandpass filter. Use the result from Problem 15.1.)

    1. Find the center frequency, bandwidth and gain for this bandpass filter.

    2. Find the cutoff frequencies and the quality factor for this bandpass filter.

Figure P15.32

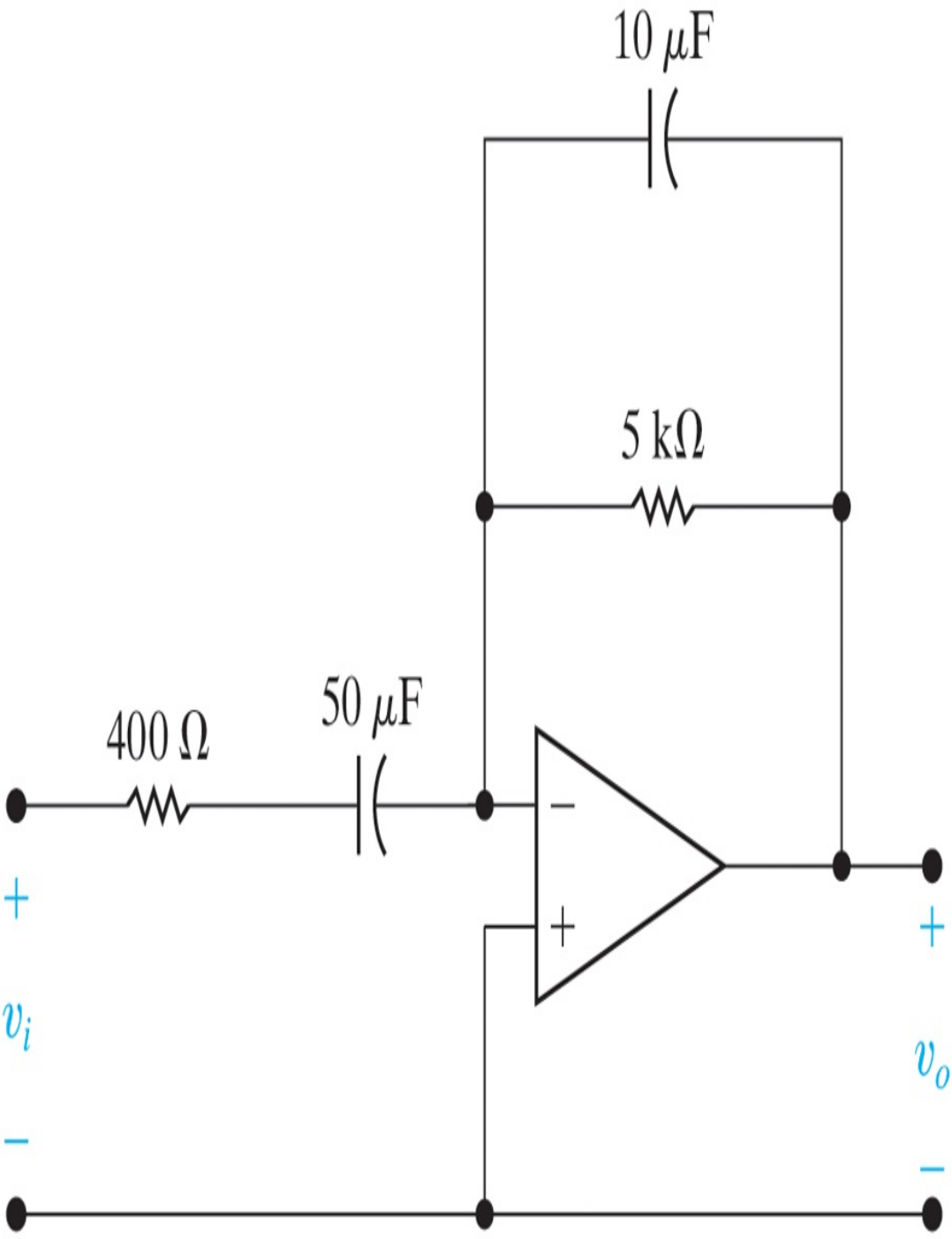


Figure P15.32 Full Alternative Text

6. 15.33 For circuits consisting of resistors, capacitors, inductors, and op amps, |H(jω)|2 involves only even powers of ω. To illustrate this, compute |H(jω)|2 for the three circuits in Fig. P15.33 when

H(s)=VoVi.

Figure P15.33

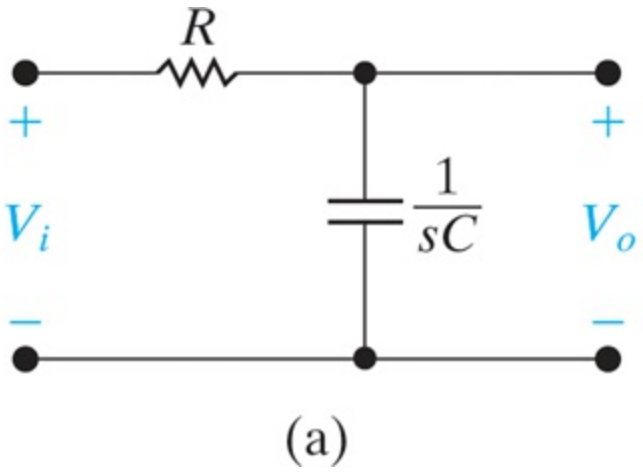
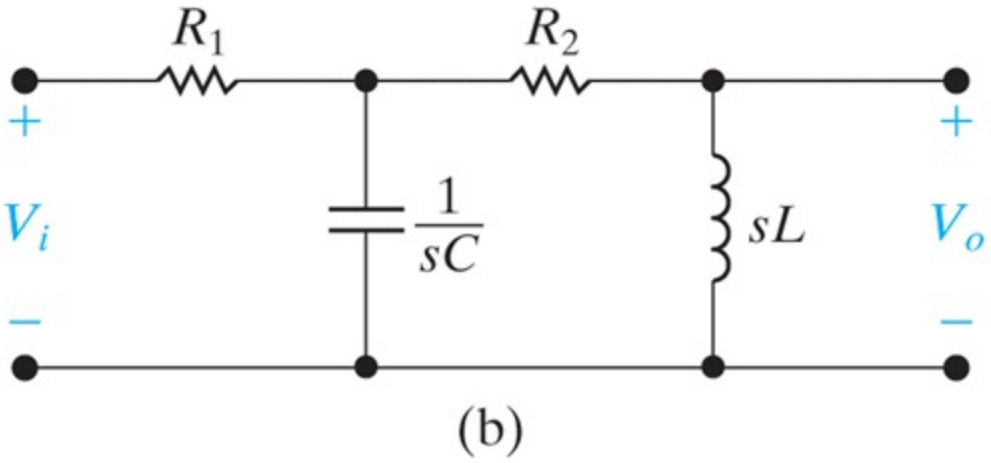
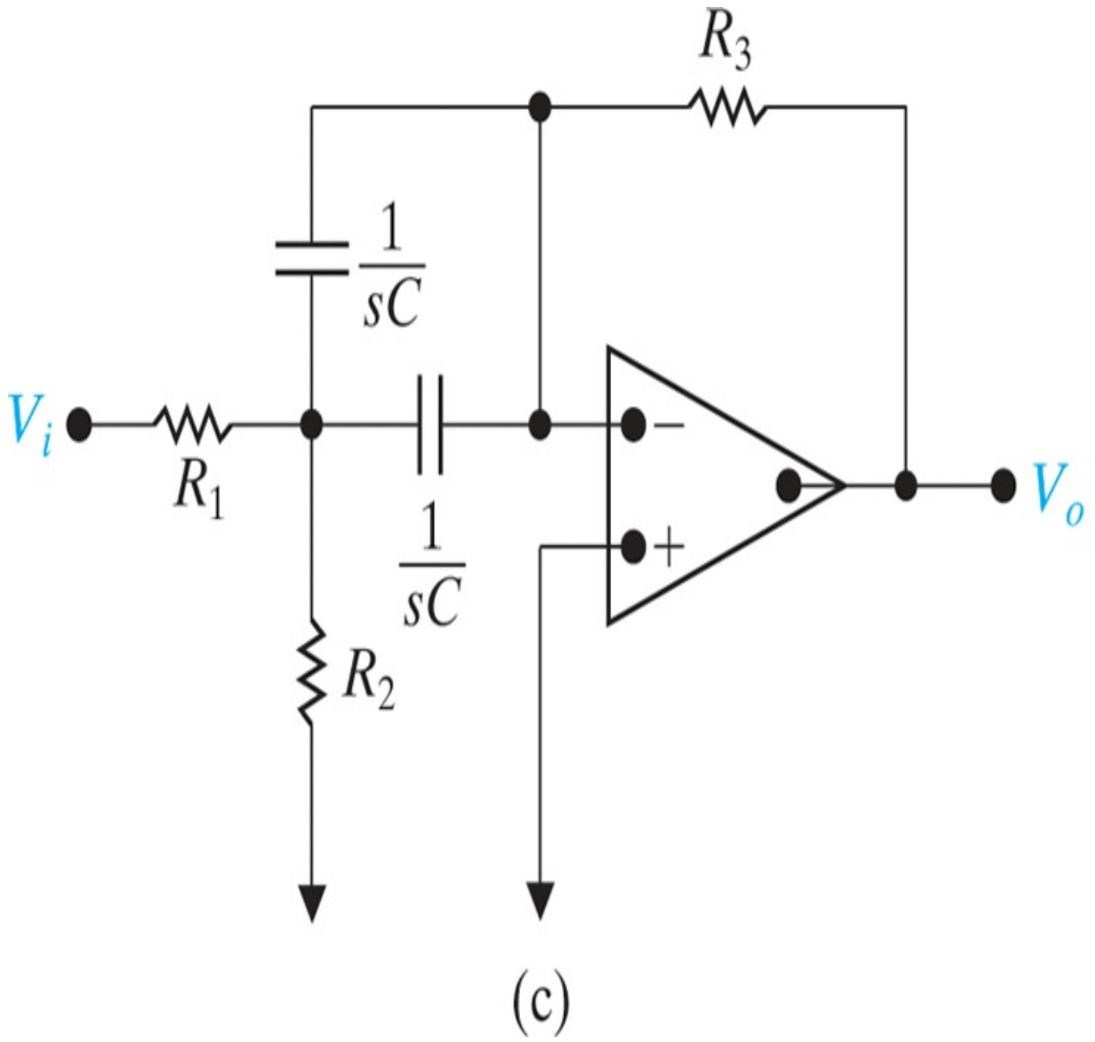


Figure P15.33 Full Alternative Text



15.2-6 Full Alternative Text



15.2-6 Full Alternative Text

Section 15.4

1. 15.34

    1. Determine the order of a low-pass Butterworth filter that has a cutoff frequency of 800 Hz and a gain of no more than −25 dB at 2500 Hz.

    2. What is the actual gain, in decibels, at 2500 Hz?

2. 15.35 The circuit in Fig. 15.21 has the transfer function

    $$H(s)=\frac{1}{R_2C_1C_2s^2+2RC_1s+1}\frac{1}{R_2C_1C_2}.$$

    Show that if the circuit in Fig. 15.21 is scaled in both magnitude and frequency, the transfer function of the scaled circuit is

    $$H'(s)=\frac{1}{R_2C_1C_2(sk_f)^2+2RC_1(sk_f)+1}\frac{1}{R_2C_1C_2}.$$

3. 15.36

    1. Write the transfer function for the prototype Butterworth filter using the filter order calculated in Problem 15.34(a).

    2. Use the filter specifications in Problem 15.34(a) to calculate the frequency scale factor, then use it to scale the transfer function in Problem 15.36(a). (Hint: see Problem 15.35.)

    3. Check the expression derived in part (b) by using it to calculate the gain (in decibels) at 2500 Hz. Compare your result with that found in Problem 15.34(b).

4. 15.37 DESIGNPROBLEM_

    1. Using 2.7 kΩ resistors and ideal op amps, design a circuit that will implement the low-pass Butterworth filter specified in Problem

15.34. The gain in the passband is one.

2. Construct the circuit diagram and label all component values.

5. 15.38 The purpose of this problem is to illustrate the advantage of an nth-order low-pass Butterworth filter over the cascade of n identical low-pass sections by calculating the slope (in decibels per decade) of each magnitude plot at the corner frequency ωc. To facilitate the calculation, let y represent the magnitude of the plot (in decibels), and let x=log10ω. Then calculate dy/dx at ωc for each plot.

    1. Show that at the corner frequency (ωc=1 rad/s) of an nth-order low-pass prototype Butterworth filter,

       dydx=−10n dB/dec.

    2. Show that for a cascade of n identical low-pass prototype sections, the slope at ωc is

       dydx=−20n(21/n−1)21/n dB/dec.

    3. Compute dy/dx for each type of filter for n=1, 2, 3, 4, and ∞.

    4. Discuss the significance of the results obtained in part (c).

6. 15.39 Verify the entries in Table 15.1 for n=5 and n=6.

7. 15.40 The circuit in Fig. 15.25 has the transfer function

   H(s)=s2s2+2R2Cs+1R1R2C2.

   Show that if the circuit is scaled in both magnitude and frequency, the transfer function of the scaled circuit is

   H′(s)=(skf)2(skf)2+2R2C(skf)+1R1R2C2.

   Hence the transfer function of a scaled circuit is obtained from the transfer function of an unscaled circuit by simply replacing s in the unscaled transfer function by s/kf, where kf is the frequency scaling

factor.

8. 15.41 DESIGNPROBLEM_

    1. Using 1 kΩ resistors and ideal op amps, design a low-pass unity-gain Butterworth filter that has a cutoff frequency of 8 kHz and is down at least 48 dB at 32 kHz.

    2. Draw a circuit diagram of the filter and label all the components.

9. 15.42 DESIGNPROBLEM_

    1. Using 10 nF capacitors and ideal op amps, design a high-pass unity-gain Butterworth filter with a cutoff frequency of 2 kHz and a gain of at least −48 dB at 500 Hz.

    2. Draw a circuit diagram of the filter and label all component values.

10. 15.43 The low-pass filter designed in Problem 15.41 is cascaded with the high-pass filter designed in Problem 15.42.

    1. Describe the type of filter formed by this interconnection.

    2. Specify the cutoff frequencies, the midfrequency, and the quality factor of the filter.

    3. Use the results of Problems 15.35 and 15.40 to derive the scaled transfer function of the filter.

    4. Check the derivation of (c) by using it to calculate $H(j\omega_o)$, where ω_o is the center frequency of the filter.

11. 15.44 DESIGNPROBLEM_

    1. Design a broadband Butterworth bandpass filter with a lower cutoff frequency of 500 Hz and an upper cutoff frequency of 4500 Hz. The passband gain of the filter is 20 dB. The gain should be down at least 20 dB at 200 Hz and 11.25 kHz. Use 15 nF capacitors in the high-pass circuit and 10 kΩ resistors in the low-pass circuit.

2. Draw a circuit diagram of the filter and label all the components.

12. 15.45

    1. Derive the transfer function for the filter designed in Problem 15.44.

    2. Using the expression derived in (a), find the gain (in decibels) at 200 Hz and 1500 Hz.

    3. Do the values obtained in part (b) satisfy the filtering specifications given in Problem 15.44?

13. 15.46 Derive the prototype transfer function for a sixth-order high-pass Butterworth filter by first writing the transfer function for a sixth-order prototype low-pass Butterworth filter and then replacing s by 1/s in the low-pass expression.

14. 15.47 The sixth-order Butterworth filter in Problem 15.46 is used in a system where the cutoff frequency is 25 krad/s.

    1. What is the scaled transfer function for the filter?

    2. Test your expression by finding the gain (in decibels) at the cutoff frequency.

15. 15.48 The purpose of this problem is to guide you through the analysis necessary to establish a design procedure for determining the circuit components in a filter circuit. The circuit to be analyzed is shown in Fig. P15.48.

    1. Analyze the circuit qualitatively and convince yourself that the circuit is a low-pass filter with a passband gain of R2/R1.

    2. Support your qualitative analysis by deriving the transfer function Vo/Vi. (Hint: In deriving the transfer function, represent the resistors with their equivalent conductances, that is, G1=1/R1, and so forth.) To make the transfer function useful in terms of the entries in Table 15.1, put it in the form

$H(s) = -Kbos^2 + b1s + bo$.

3. Now observe that we have five circuit components—R1, R2, R3, C1, and C2—and three transfer function constraints—K, b1, and bo. At first glance, it appears we have two free choices among the five components. However, when we investigate the relationships between the circuit components and the transfer function constraints, we see that if C2 is chosen, there is an upper limit on C1 in order for R2(G2) to be realizable. With this in mind, show that if C2=1 F, the three conductances are given by the expressions

   $G\,1 = K\,G\,2\ ;\ G\,3 = (\ b\ o\ G\,2\ )\ C\,1\ ;\ G\,3 = (\ b\ o\ G\,2\ )\ C\,1\ ;\ G\,2 = b\ 1 \pm b\ 1\ 2 - 4\ b\ o\ (1+K)\ C\,1\ 2(1+K)$.

   For G2 to be realizable,

   $C1 \leq b124bo(1+K)$.

4. Based on the results obtained in (c), outline the design procedure for selecting the circuit components once K, bo, and b1 are known.

Figure P15.48

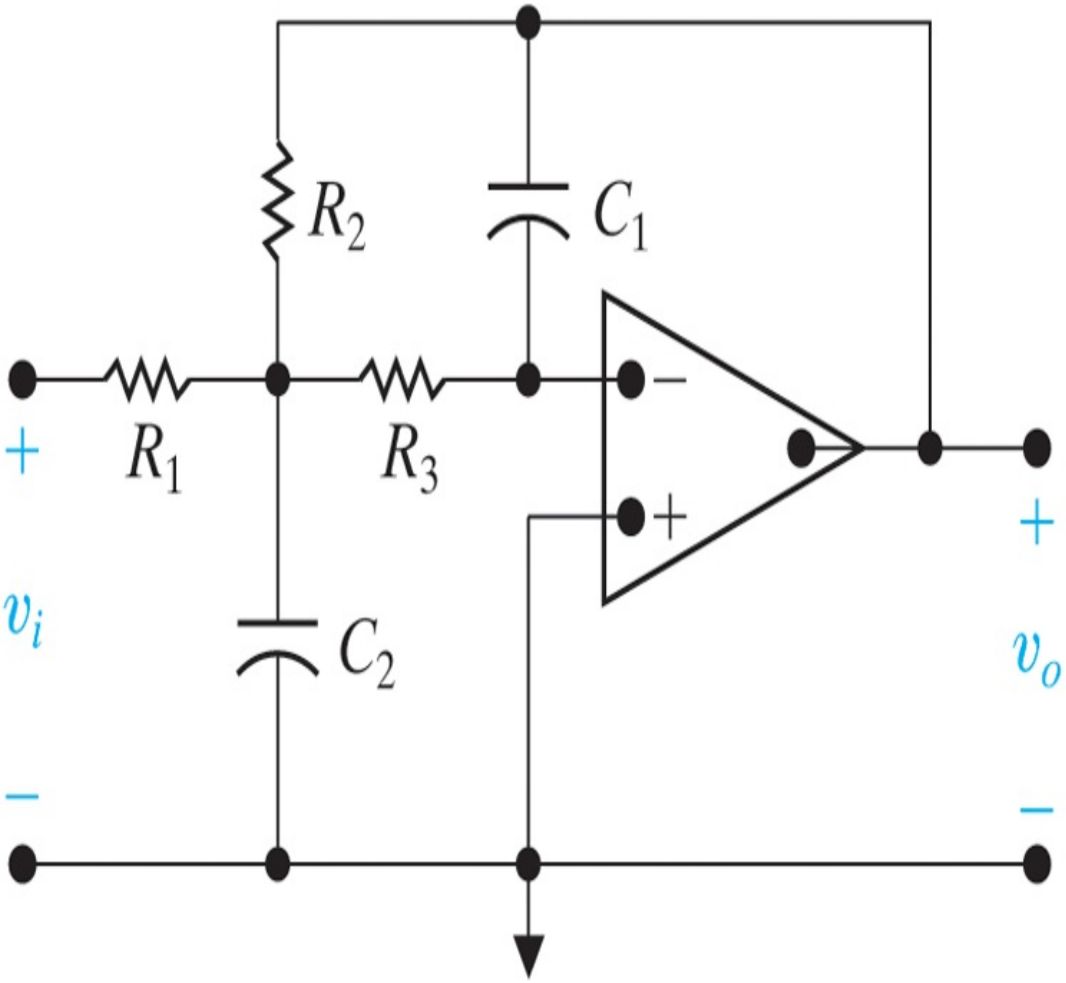


Figure P15.48 Full Alternative Text

16. 15.49 DESIGNPROBLEM_ Assume the circuit analyzed in Problem 15.48 is part of a third-order low-pass Butterworth filter having a passband gain of 4. (Hint: implement the gain of 4 in the second-order section of the filter.)

    1. If C2=1 F in the prototype second-order section, what is the upper limit on C1?

    2. If the limiting value of C1 is chosen, what are the prototype values of R1, R2, and R3?

    3. If the corner frequency of the filter is 2.5 kHz and C2 is chosen to be 10 nF, calculate the scaled values of C1, R1, R2, and R3.

4. Specify the scaled values of the resistors and the capacitor in the first-order section of the filter.

5. Construct a circuit diagram of the filter and label all the component values on the diagram.

17. 15.50 DESIGNPROBLEM_ Interchange the Rs and Cs in the circuit in Fig. P15.48; that is, replace R1 with C1, R2 with C2, R3 with C3, C1 with R1, and C2 with R2.

    1. Describe the type of filter implemented as a result of the interchange.

    2. Confirm the filter type described in (a) by deriving the transfer function Vo/Vi. Write the transfer function in a form that makes it compatible with Table 15.1.

    3. Set C2=C3=1 F and derive the expressions for C1, R1, and R2 in terms of K, b1, and bo. (See Problem 15.48 for the definition of b1 and bo.)

    4. Assume the filter described in (a) is used in a third-order Butterworth filter that has a passband gain of 8. With C2=C3=1 F, calculate the prototype values of C1, R1, and R2 in the second-order section of the filter.

18. 15.51 DESIGNPROBLEM_

    1. Use the circuits analyzed in Problems 15.48 and 15.50 to implement a broadband bandreject filter having a passband gain of 0 dB, a lower corner frequency of 400 Hz, an upper corner frequency of 6400 Hz, and an attenuation of at least 30 dB at both 1000 Hz and 2560 Hz. Use 10 nF capacitors whenever possible.

    2. Draw a circuit diagram of the filter and label all the components.

19. 15.52

    1. Derive the transfer function for the bandreject filter described in

Problem 15.51.

2. Use the transfer function derived in part (a) to find the attenuation (in decibels) at the center frequency of the filter.

20. 15.53 DESIGNPROBLEM_ The purpose of this problem is to develop the design equations for the circuit in Fig. P15.53. (See Problem 15.48 for suggestions on the development of design equations.)

    1. Based on a qualitative analysis, describe the type of filter implemented by the circuit.

    2. Verify the conclusion reached in (a) by deriving the transfer function Vo/Vi. Write the transfer function in a form that makes it compatible with the entries in Table 15.1.

    3. How many free choices are there in the selection of the circuit components?

    4. Derive the expressions for the conductances G1=1/R1 and G1=1/R2 in terms of C1, C2, and the coefficients bo and b1. (See Problem 15.48 for the definition of bo and b1.)

    5. Are there any restrictions on C1 or C2?

    6. Assume the circuit in Fig. P15.53 is used to design a fourth-order low-pass unity-gain Butterworth filter. Specify the prototype values of R1 and R2 in each second-order section if 1 F capacitors are used in the prototype circuit.

Figure P15.53

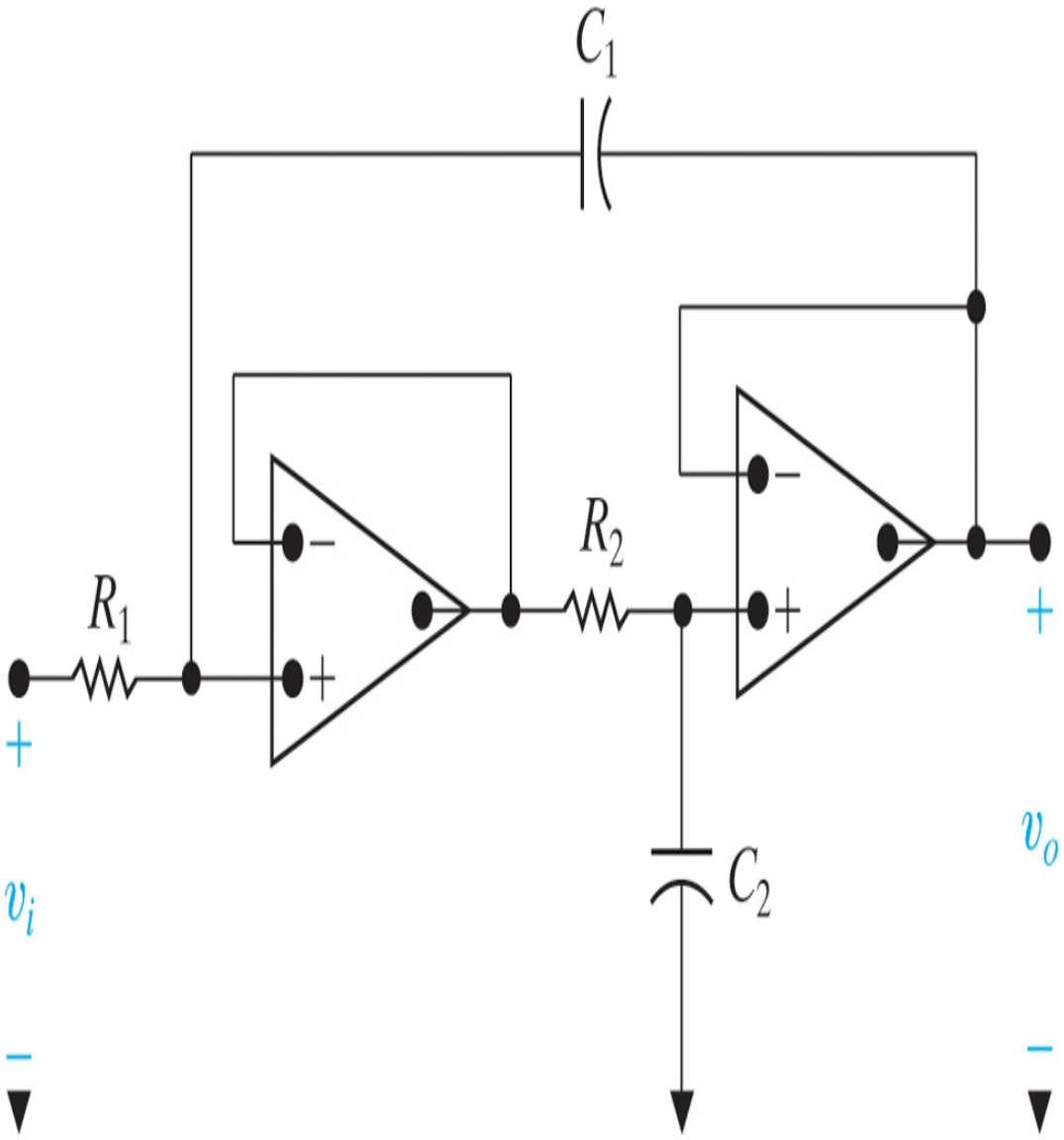


Figure P15.53 Full Alternative Text

21. 15.54 DESIGNPROBLEM_ The fourth-order low-pass unity-gain Butterworth filter in Problem 15.53 is used in a system where the cutoff frequency is 3 kHz. The filter has 4.7 nF capacitors.

    1. Specify the numerical values of R1 and R2 in each section of the filter.

    2. Draw a circuit diagram of the filter and label all the components.

22. 15.55 DESIGNPROBLEM_ Interchange the Rs and Cs in the circuit in Fig. P15.53, that is, replace R1 with C1, R2 with C2, and vice versa.

    1. Analyze the circuit qualitatively and predict the type of filter implemented by the circuit.

    2. Verify the conclusion reached in (a) by deriving the transfer function Vo/Vi. Write the transfer function in a form that makes it compatible with the entries in Table 15.1.

    3. How many free choices are there in the selection of the circuit components?

    4. Find R1 and R2 as functions of bo, b1, C1, and C2.

    5. Are there any restrictions on C1 and C2?

    6. Assume the circuit is used in a third-order Butterworth filter of the type found in (a). Specify the prototype values of R1 and R2 in the second-order section of the filter if C1=C2=1 F.

23. 15.56 DESIGNPROBLEM_

    1. The circuit in Problem 15.55 is used in a third-order high-pass unity-gain Butterworth filter that has a cutoff frequency of 5 kHz. Specify the values of R1 and R2 if 5 μF capacitors are available to construct the filter.

    2. Specify the values of resistance and capacitance in the first-order section of the filter.

    3. Draw the circuit diagram and label all the components.

    4. Give the numerical expression for the scaled transfer function of the filter.

    5. Use the scaled transfer function derived in (d) to find the gain in dB at the cutoff frequency.

Section 15.5

1. 15.57 Show that if ωo=1 rad/s and C=1 F in the circuit in Fig. 15.27, the prototype values of R1, R2, and R3 are

   R1=QK,R2=Q2Q2−K,R3=2Q.

2. 15.58 DESIGNPROBLEM_

   1. Use 20 nF capacitors in the circuit in Fig. 15.27 to design a bandpass filter with a quality factor of 16, a center frequency of 6.4 kHz, and a passband gain of 20 dB.

   2. Draw the circuit diagram of the filter and label all the components.

3. 15.59

   1. Show that the transfer function for a prototype narrow band bandreject filter is

      H(s)=s2+1s2+(1/Q)s+1.

   2. Use the result found in (a) to find the transfer function of the filter designed in Example 15.14.

4. 15.60 DESIGNPROBLEM_

   1. Using the circuit shown in Fig. 15.30, design a narrow-band bandreject filter having a center frequency of 1 kHz and a quality factor of 20. Base the design on C=15 nF.

   2. Draw the circuit diagram of the filter and label all component values on the diagram.

   3. What is the scaled transfer function of the filter?

Sections 15.1–15.5

1. 15.61 PRACTICALPERSPECTIVE_DESIGNPROBLEM_ Using the circuit in Fig. 15.33(a) design a volume control circuit to give a maximum gain of 20 dB and a gain of 17 dB at a frequency of 40 Hz. Use a 11.1 kΩ resistor and a 100 kΩ potentiometer. Test your design by calculating the maximum gain at ω=0 and the gain at ω=1/R2C1 using the selected values of R1, R2, and C1.

2. 15.62 PRACTICALPERSPECTIVE_DESIGNPROBLEM_ Use the circuit in Fig. 15.33(a) to design a bass volume control circuit that has a maximum gain of 13.98 dB that drops off 3 dB at 50 Hz.

3. 15.63 PRACTICALPERSPECTIVE_ Plot the maximum gain in decibels versus α when ω=0 for the circuit designed in Problem 15.61. Let α vary from 0 to 1 in increments of 0.1.

4. 15.64 PRACTICALPERSPECTIVE_

    1. Show that the circuits in Fig. P15.64(a) and (b) are equivalent.

    2. Show that the points labeled x and y in Fig. P15.64(b) are always at the same potential.

    3. Using the information in (a) and (b), show that the circuit in Fig. 15.34 can be drawn as shown in Fig. P15.64(c).

    4. Show that the circuit in Fig. P15.64(c) is in the form of the circuit in Fig. 15.2, where

        $$Z_i = R_1 + (1-\alpha)R_2 + \frac{R_1R_2C_1s}{1+R_2C_1s}, \quad Z_f = R_1 + \alpha R_2 + \frac{R_1R_2C_1s}{1+R_2C_1s}.$$

Figure P15.64

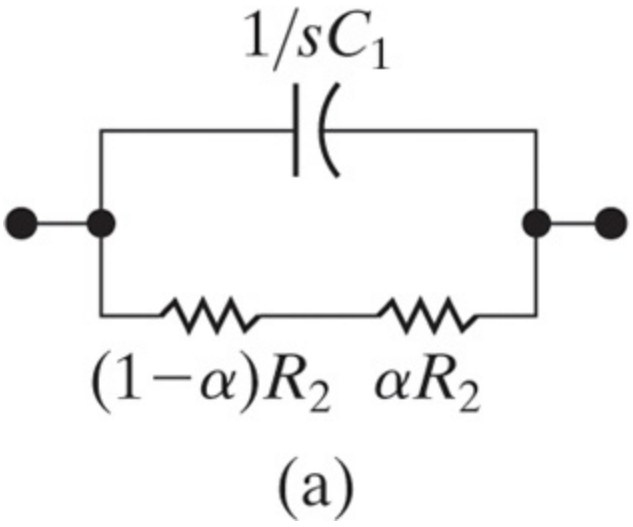
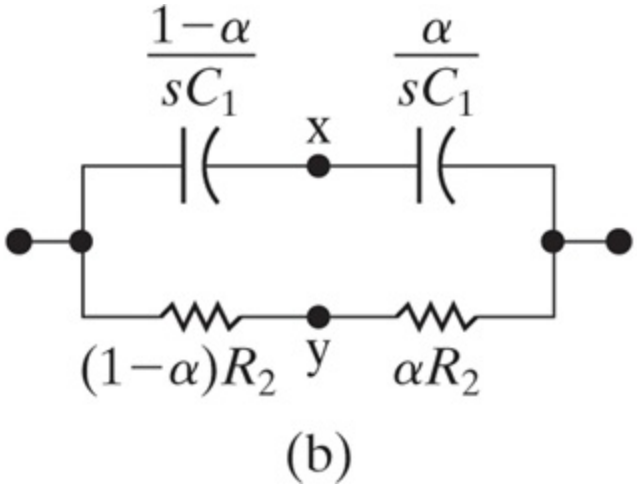
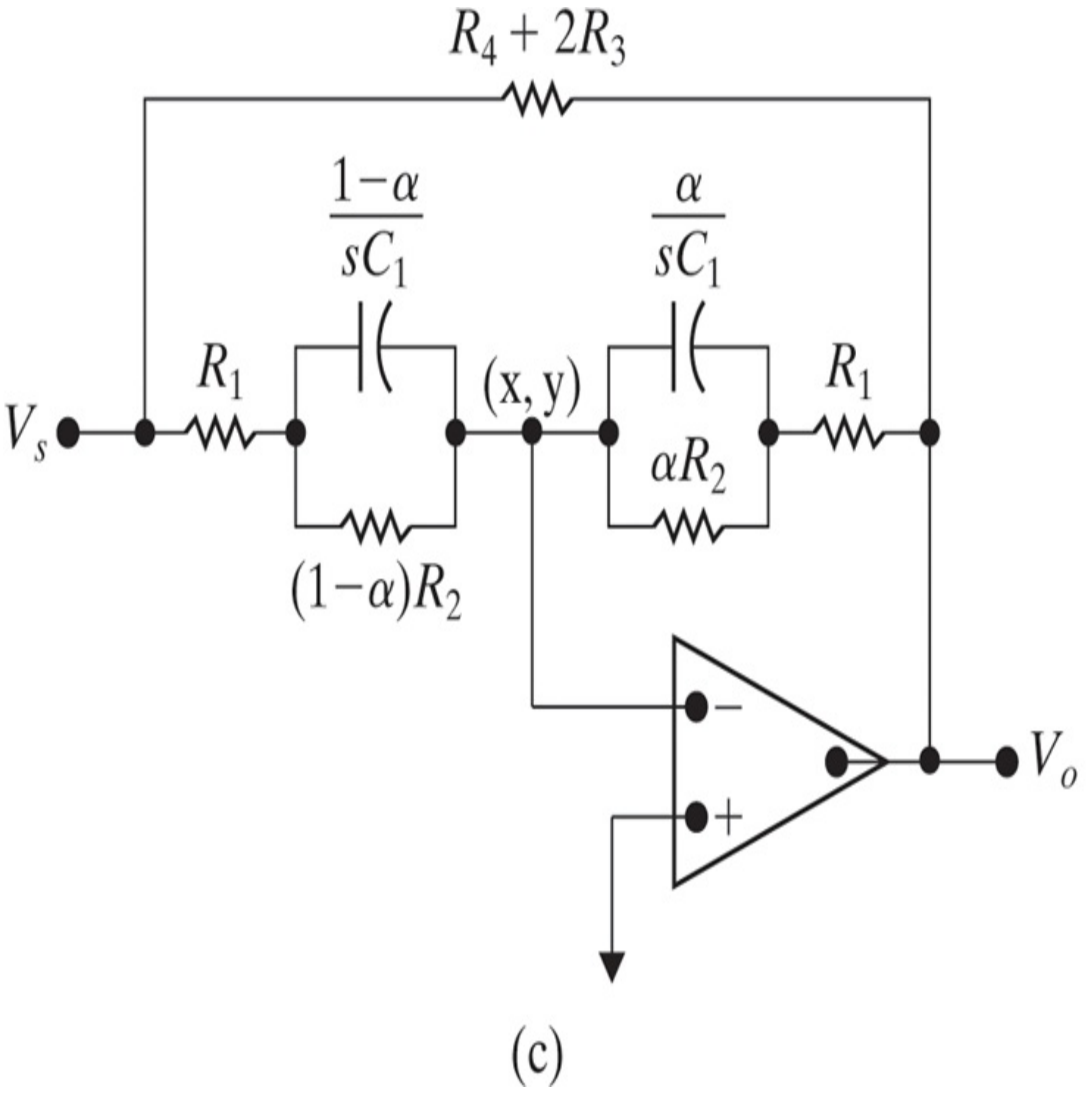


Figure P15.64 Full Alternative Text



15.2-6 Full Alternative Text



15.2-6 Full Alternative Text

5. 15.65 PRACTICALPERSPECTIVE_ An engineering project manager has received a proposal from a subordinate who claims the circuit shown in Fig. P15.65 could be used as a treble volume control circuit if R4≫R1+R3+2R2. The subordinate further claims that the voltage transfer function for the circuit is

   H(s)=VoVs
   =−{(2R3+R4)+[(1−β)R4+Ro](βR4+R3)C2s}{(2R3+R4)+[(1−β)R4+R3](βR4+Ro)C2s}

Figure P15.65

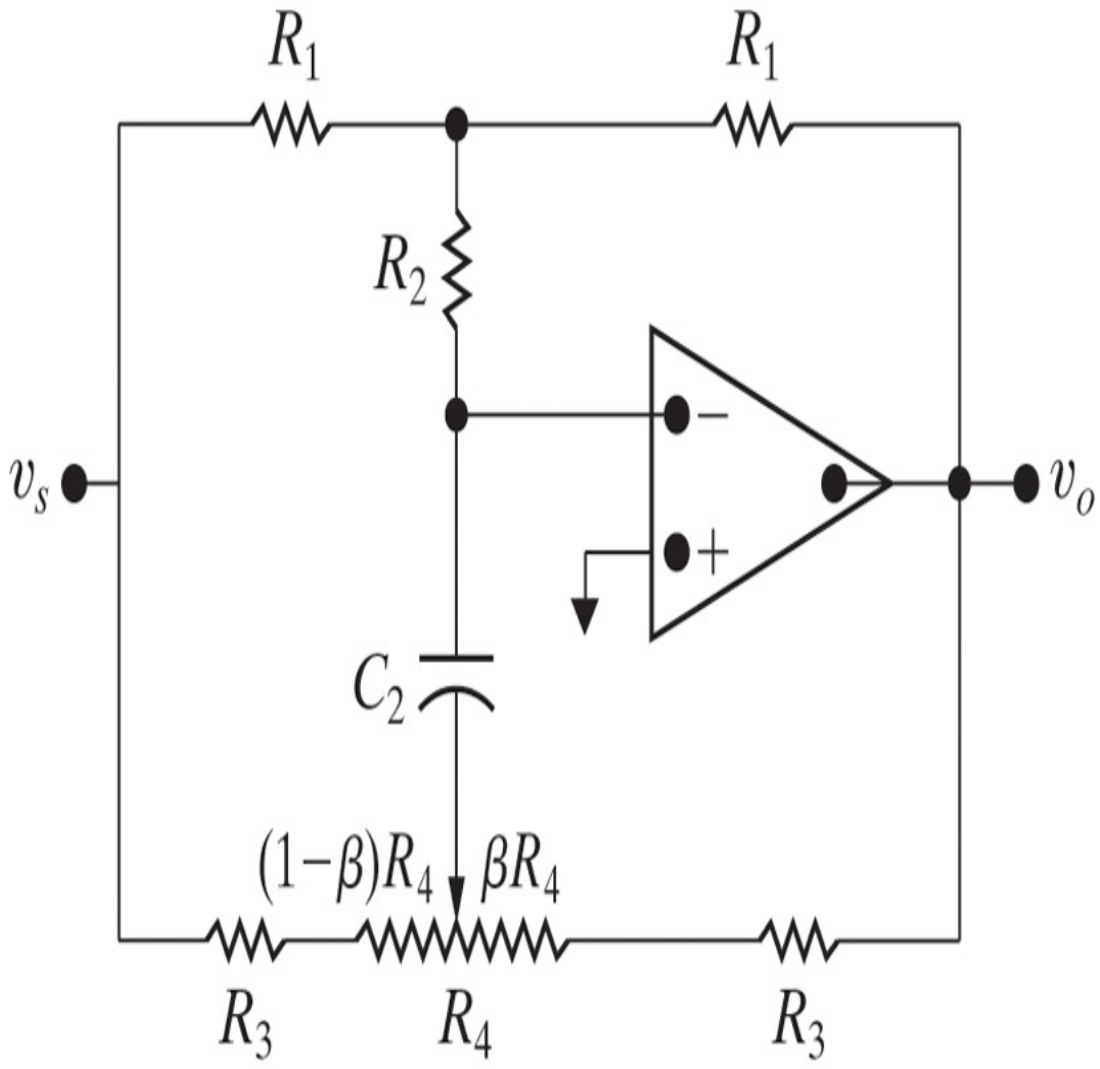


Figure P15.65 Full Alternative Text

where Ro=R1+R3+2R2. Fortunately the project engineer has an electrical engineering undergraduate student as an intern and therefore asks the student to check the subordinate's claim.

The student is asked to check the behavior of the transfer function as $\omega \rightarrow 0$; as $\omega \rightarrow \infty$; and the behavior when $\omega=\infty$ and β varies between 0 and 1. Based on your testing of the transfer function do you think the circuit could be used as a treble volume control? Explain.

6. 15.66 PRACTICALPERSPECTIVE_ In the circuit of Fig. P15.65 the component values are R1=R2=20 kΩ, R3=5.9 kΩ, R4=500 kΩ, and C2=2.7 nF.

    1. Calculate the maximum boost in decibels.

    2. Calculate the maximum cut in decibels.

    3. Is R4 significantly greater than Ro?

    4. When β=1, what is the boost in decibels when ω=1/R3C2?

    5. When β=0, what is the cut in decibels when ω=1/R3C2?

    6. Based on the results obtained in (d) and (e), what is the significance of the frequency 1/R3C2 when R4≫R0?

7. 15.67 PRACTICALPERSPECTIVE_ Using the component values given in Problem 15.66, plot the maximum gain in decibels versus β when ω is infinite. Let β vary from 0 to 1 in increments of 0.1.

Chapter 16 Fourier Series

Chapter Contents

1. 16.1 Fourier Series Analysis: An Overview
2. 16.2 The Fourier Coefficients
3. 16.3 The Effect of Symmetry on the Fourier Coefficients
4. 16.4 An Alternative Trigonometric Form of the Fourier Series
5. 16.5 An Application
6. 16.6 Average-Power Calculations with Periodic Functions
7. 16.7 The rms Value of a Periodic Function
8. 16.8 The Exponential Form of the Fourier Series
9. 16.9 Amplitude and Phase Spectra

Chapter Objectives

1. Be able to calculate the trigonometric form of the Fourier coefficients for a periodic waveform using the definition of the coefficients and the simplifications possible if the waveform exhibits one or more types of symmetry.

2. Know how to analyze a circuit's response to a periodic waveform using Fourier coefficients and superposition.

3. Be able to estimate the average power delivered to a resistor using a small number of Fourier coefficients.

4. Be able to calculate the exponential form of the Fourier coefficients for a periodic waveform and use them to generate magnitude and phase spectrum plots for that waveform.

In this chapter, we find the steady-state response of circuits to periodic, nonsinusoidal, inputs. A **periodic function** repeats itself every T seconds, so T is the period of the function. For example, the function plotted in Fig. 16.1 on 620 is a periodic waveform that is not a sinusoid.

Figure 16.1 A periodic waveform.

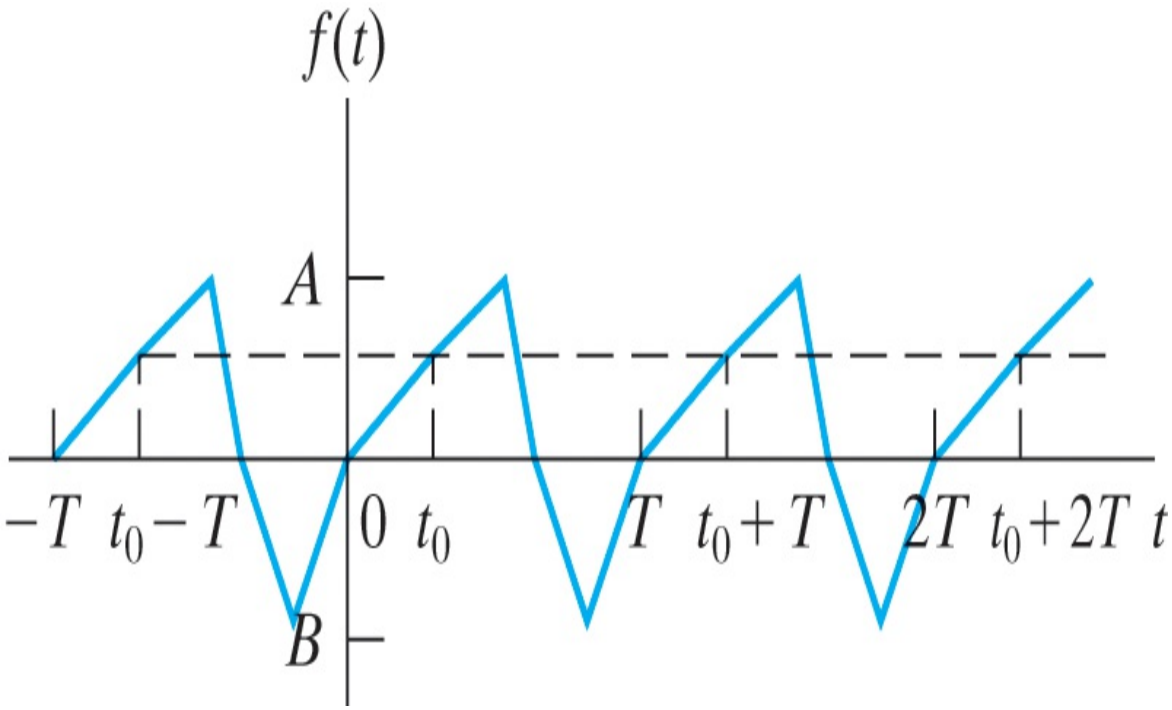


Figure 16.1 Full Alternative Text

A periodic function satisfies the relationship

$$f(t)=f(t\pm nT), \quad (16.1)$$

where n is an integer (1, 2, 3, …) and T is the period. The function shown in Fig. 16.1 is periodic because

$$f(t0)=f(t0-T)=f(t0+T)=f(t0+2T)=\cdots$$

for any arbitrarily chosen value of t0. Note that T is the smallest time interval that a periodic function may be shifted (either left or right) to produce a function that is identical to itself.

Why are we interested in the response of circuits to inputs that are periodic but not sinusoids? One reason is that many electrical sources of practical value generate such waveforms. Here are a few examples.

- A nonfiltered electronic rectifier driven from a sinusoidal source produces a rectified sine wave that is not sinusoidal but is periodic. Figures 16.2(a) and (b) on page 620 show the waveforms of the full-

wave and half-wave sinusoidal rectifiers, respectively.

- The sweep generator used to control the electron beam of a cathode-ray oscilloscope produces a periodic triangular wave like the one shown in Fig. 16.3 on page 620.

- Function generators, which are used to test equipment in a laboratory, are designed to produce nonsinusoidal periodic waveforms, including square waves, triangular waves, and rectangular waves. Figure 16.4 on 620 illustrates typical waveforms output by a function generator.

- A power generator is designed to produce a sinusoidal waveform but cannot produce a pure sine wave. Instead, it produces a distorted sinusoidal wave that is still periodic. We can use the analysis techniques in this chapter to determine the consequences of a circuit with a slightly distorted sinusoidal voltage.

- Any nonlinearity in an otherwise linear circuit creates a nonsinusoidal periodic function. The rectifier circuit mentioned earlier is one example of this phenomenon. Magnetic saturation, which occurs in both machines and transformers, is another example of a nonlinearity that generates a nonsinusoidal periodic function. An electronic clipping circuit, which uses transistor saturation, is yet another example.

Figure 16.2 Output waveforms of a nonfiltered sinusoidal rectifier. (a) Full-wave rectification. (b) Half-wave rectification.

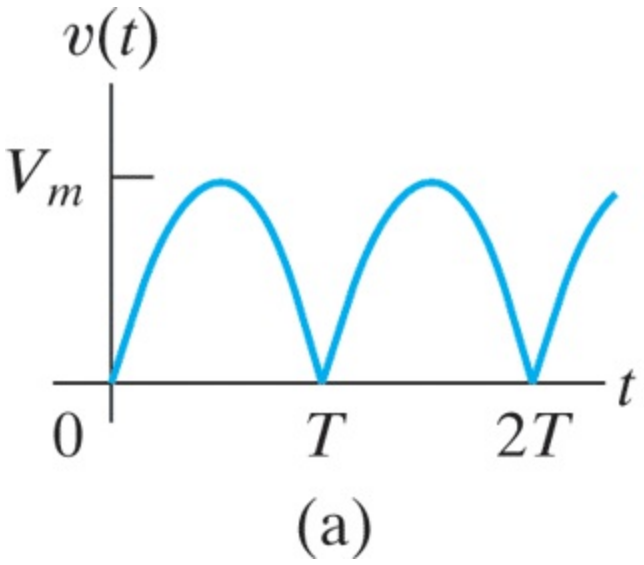
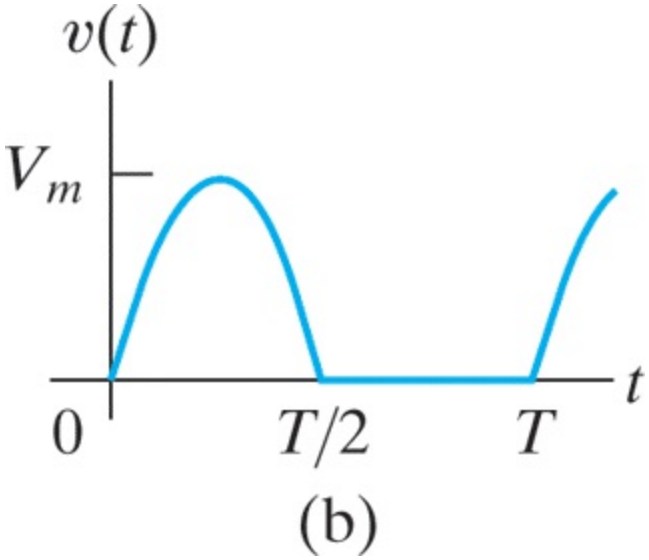


Figure 16.2 Full Alternative Text



16.2-1 Full Alternative Text

Figure 16.3 The triangular waveform of a cathode-ray oscilloscope sweep generator.

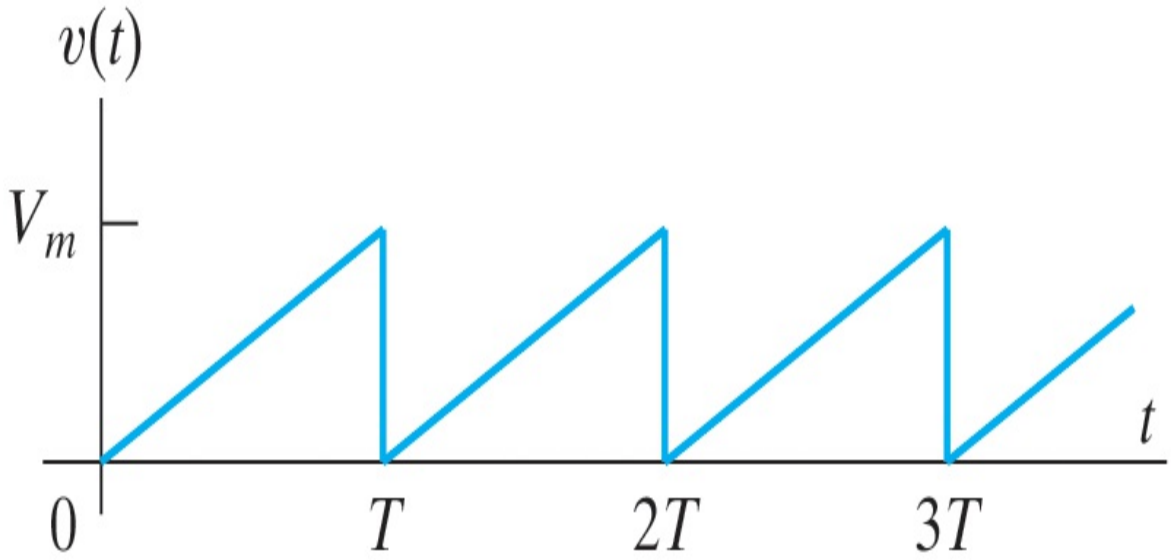


Figure 16.3 Full Alternative Text

Figure 16.4 Waveforms produced by function generators used in laboratory testing. (a) Square wave. (b) Triangular wave. (c) Rectangular pulse.

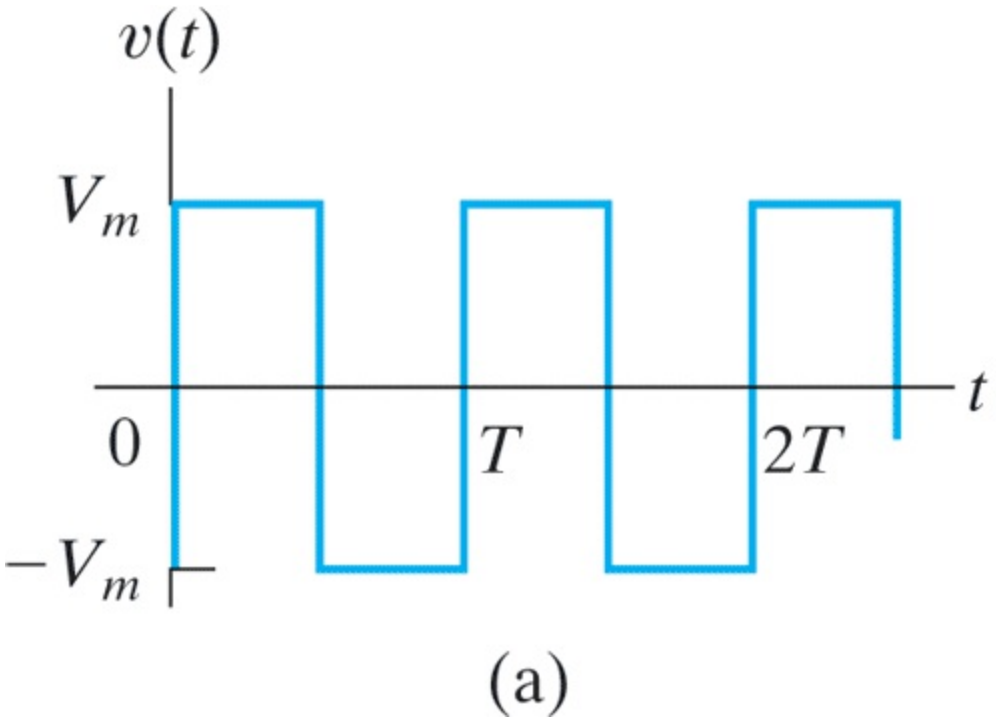
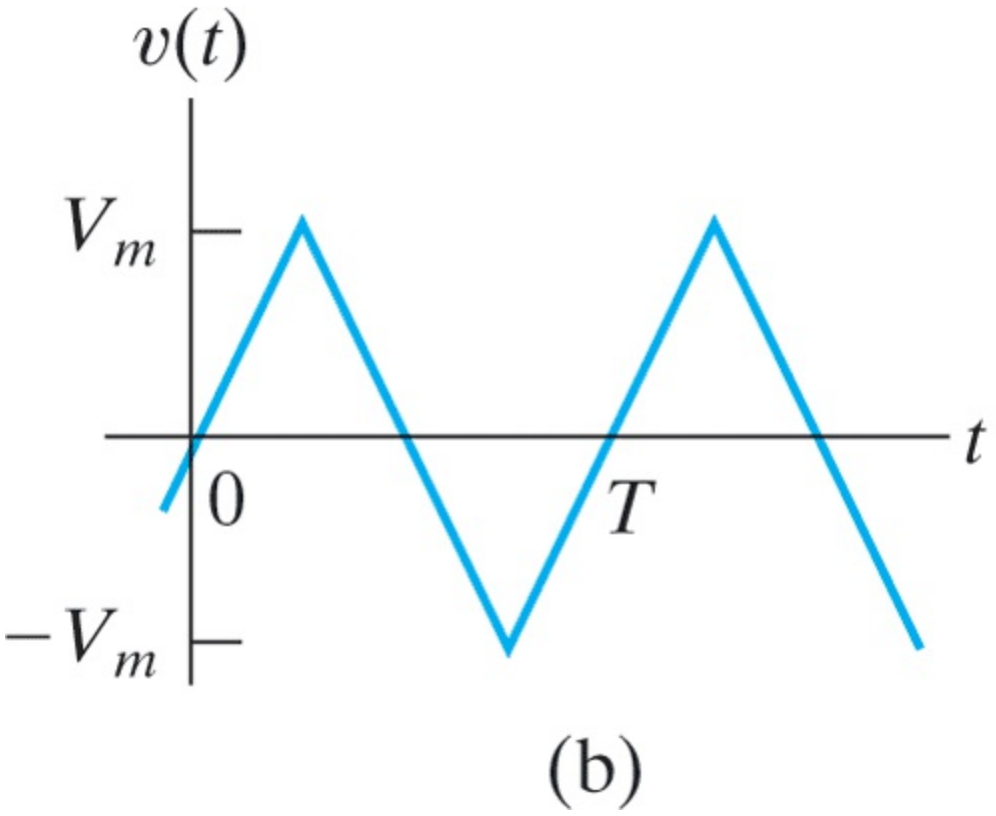
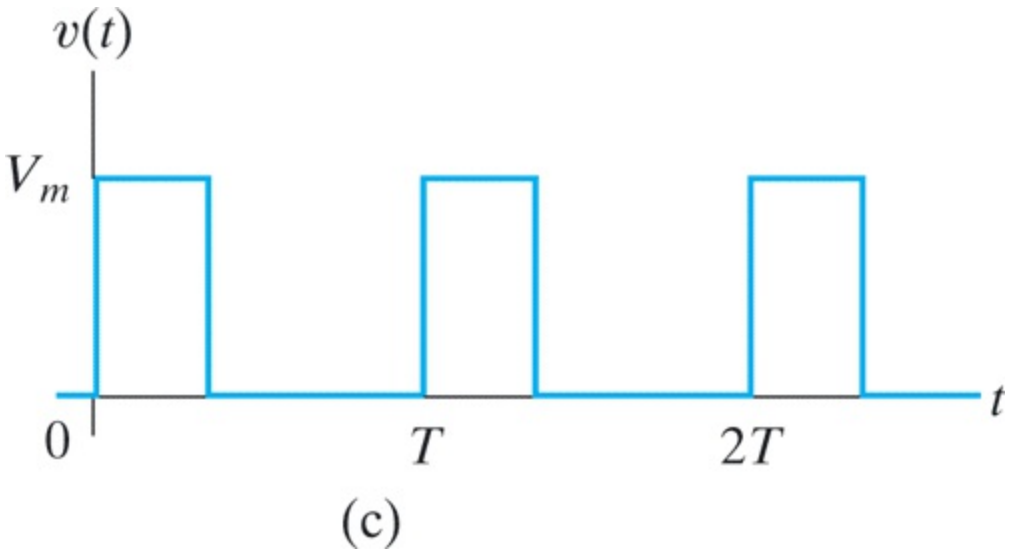


Figure 16.4 Full Alternative Text



16.2-1 Full Alternative Text



16.2-1 Full Alternative Text

Practical Perspective

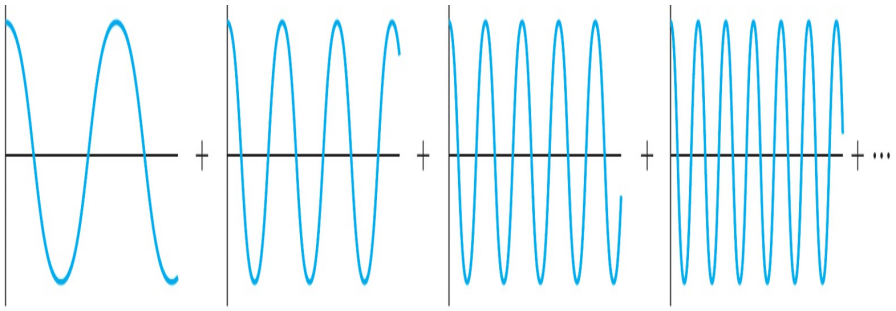
Active High-Q Filters

An important characteristic of bandpass and bandreject filters is the quality factor, Q, as we discovered in Chapters 14 and 15. The quality factor provides a measure of how selective the filter is at and near its center frequency. For example, an active bandpass filter with a large value of Q will amplify signals at or near its center frequency and will attenuate signals at all other frequencies. In contrast, a bandreject filter with a small value of Q will not effectively distinguish between signals at the center frequency and signals at frequencies quite different from the center frequency.

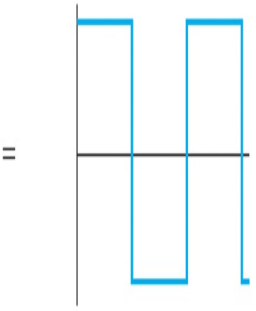
We can test the quality factor of a bandpass or bandreject filter using a periodic signal. For example, to test the quality factor of a bandpass filter, input a square wave whose frequency is the same as the center frequency of the bandpass filter and analyze the output. In this chapter, we learn that any periodic signal can be represented as a sum of sinusoids. The frequencies of the sinusoids include the frequency of the periodic signal and integer

multiples of that frequency. So the input square wave, whose frequency is ω, consists of a sum of sinusoids at the frequencies ω, 2ω, 3ω, and so on. If the bandpass filter has a high-quality factor, its output will be nearly sinusoidal because it filtered out all of the sinusoids that make up the square wave except the one at ω. If the filter has a low-quality factor, its output will still look like a square wave because the filter passed many of the sinusoids that make up the input square wave to the output. We present an example at the end of this chapter.

The sum of sinusoids whose frequencies are integer multiples of the fundamental frequency



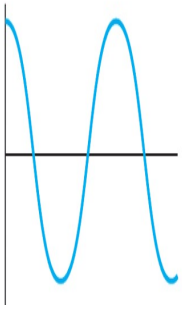
The sum is a square wave at the fundamental frequency



The center frequency of the filter is the fundamental frequency

High-*Q* Bandpass Filter

The filter extracts the sinusoid at the fundamental frequency from the square wave



16.2-1 Full Alternative Text

Nonsinusoidal periodic functions are also important when analyzing nonelectrical systems. Problems involving mechanical vibration, fluid flow, and heat flow all make use of periodic functions. In fact, the study and analysis of heat flow in a metal rod led the French mathematician Jean Baptiste Joseph Fourier (1768–1830) to the trigonometric series representation of a periodic function. This series bears his name and is the starting point for finding the steady-state response of a circuit to a periodic input.

16.1 Fourier Series Analysis: An Overview

A periodic function can be represented by an infinite sum of sine and cosine functions that are harmonically related, as Fourier discovered while investigating heat-flow problems. Specifically, the frequency of every trigonometric term in the infinite series is an integer multiple, or harmonic, of the fundamental frequency, ω0, of the periodic function. Thus, if f(t) is periodic, Fourier showed that it can be expressed as

Fourier Series Representation of a Periodic Function

$$f(t)=av+\sum_{n=1}^{\infty} an \cos n\omega ot+bn \sin n\omega 0t, \quad (16.2)$$

where n is the integer sequence 1, 2, 3, ….

In Eq. 16.2, av, an, and bn are known as the **Fourier coefficients** and are calculated from f(t). We discuss these calculations in Section 16.2. The term ω0 (which equals 2π/T) represents the **fundamental frequency** of the periodic function f(t). The integer multiples of ω0—that is, 2ω0, 3ω0, 4ω0, and so on—are known as the **harmonic frequencies** of f(t). Thus, 2ω0 is the second harmonic, 3ω0 is the third harmonic, and nω0 is the nth harmonic of f(t).

Before learning how to find a circuit's response to a periodic input using a Fourier series representation of that input, we first look at the process in general terms. We can express all the periodic functions of interest in circuit analysis using a Fourier series. But not every periodic function has a Fourier series representation. The conditions on a periodic function f(t) that ensure it can be expressed as a convergent Fourier series (known as **Dirichlet's**

conditions) are as follows.

1. f(t) is single-valued.
2. f(t) has a finite number of discontinuities in the periodic interval.
3. f(t) has a finite number of maxima and minima in the periodic interval.
4. The integral

   $\int_{t_0}^{t_0+T} |f(t)|\, dt$

   exists.

All periodic functions encountered in circuit analysis satisfy Dirichlet's conditions. These are **sufficient**, not **necessary** conditions. Thus, if f(t) meets these requirements, we know that we can express it as a Fourier series. However, if f(t) does not meet these requirements, we still may be able to express it as a Fourier series. The necessary conditions on f(t) are not known.

After we have determined f(t) and calculated the Fourier coefficients (av, an, and bn), we replace the periodic source with a dc source (av) and sinusoidal sources, all connected in series. Each source represents a term in the Fourier series representation of the periodic input. Then, we use superposition to calculate the response to each source. The sum of the individual responses gives us the total steady-state response. The steady-state response to each sinusoidal source is most easily found using the phasor methods of Chapter 9.

The procedure just outlined involves no new circuit analysis techniques. It produces the Fourier series representation of the steady-state response; consequently, the functional form of the response is unknown. Furthermore, the output waveform is expressed as an infinite sum, so we can only estimate its shape by adding a sufficient number of its terms together. While the Fourier series method for finding the steady-state response of a circuit to a periodic input does have some drawbacks, it introduces a way of thinking about a problem that is as important as getting quantitative results. In fact, the conceptual picture is even more important, in some respects, than the quantitative one.

16.2 The Fourier Coefficients

Using the definition of a periodic function over its fundamental period, we determine the Fourier coefficients from the relationships

Fourier Coefficients

$$a_v=\frac{1}{T}\int_{t_0}^{t_0+T} f(t)\,dt, \quad (16.3)$$
$$a_k=\frac{2}{T}\int_{t_0}^{t_0+T} f(t)\cos k\omega_0 t\,dt, \quad (16.4)$$
$$b_k=\frac{2}{T}\int_{t_0}^{t_0+T} f(t)\sin k\omega_0 t\,dt. \quad (16.5)$$

In Eqs. 16.4 and 16.5, the subscript k indicates the kth coefficient in the integer sequence 1, 2, 3, …. Note that av is the average value of f(t), ak is twice the average value of f(t) cos kω0t, and bk is twice the average value of f(t) sin kω0t.

We derive Eqs. 16.3–16.5 from Eq. 16.2 by recalling the following integral relationships, which hold when m and n are integers:

$$\int_{t_0}^{t_0+T}\sin m\omega_0 t\,dt=0, \text{for all m,} \int_{t_0}^{t_0+T}\cos m\omega_0 t\,dt=0, \text{for all m,} \int_{t_0}^{t_0+T}\cos m\omega$$

We leave you to verify these equations in Problem 16.5.

To derive Eq. 16.3, integrate both sides of Eq. 16.2 over one period:

$$\int_{t_0}^{t_0+T} f(t)\,dt=\int_{t_0}^{t_0+T}\left(a_v+\sum_{n=1}^{\infty} a_n\cos n\omega_0 t+b_n\sin n\omega_0 t\right)dt=\int_{t_0}^{t_0+T} a_v\,dt+\sum$$

Solving for av gives us Eq. 16.3.

To derive the expression for the kth value of an, we first multiply Eq. 16.2 by cos kω0t and then integrate both sides over one period of f(t) :

$$\int_{t_0}^{t_0+T} f(t)\cos k\omega_0 t\,dt=\int_{t_0}^{t_0+T} a_v\cos k\omega_0 t\,dt+\sum_{n=1}^{\infty}\int_{t_0}^{t_0+T}\left(a_n\cos n\omega_0 t\cos k\omega_0 t+b_n\sin n\omega_0 t\cos k\omega_0 t\right)dt$$

=0+ a k (T 2)+0.

Solving for ak yields the expression in Eq. 16.4.

We obtain the expression for the kth value of bn by first multiplying both sides of Eq. 16.2 by sin kω0t and then integrating each side over one period of f(t). You should complete the derivation of Eq. 16.5 in Problem 16.6. In Example 16.1 we use Eqs. 16.3–16.5 to find the Fourier coefficients for a specific periodic function.

Example 16.1 Finding the Fourier Series of a Triangular Waveform

Find the Fourier series for the periodic voltage shown in Fig. 16.5.

Figure 16.5 The periodic voltage for Example 16.1.

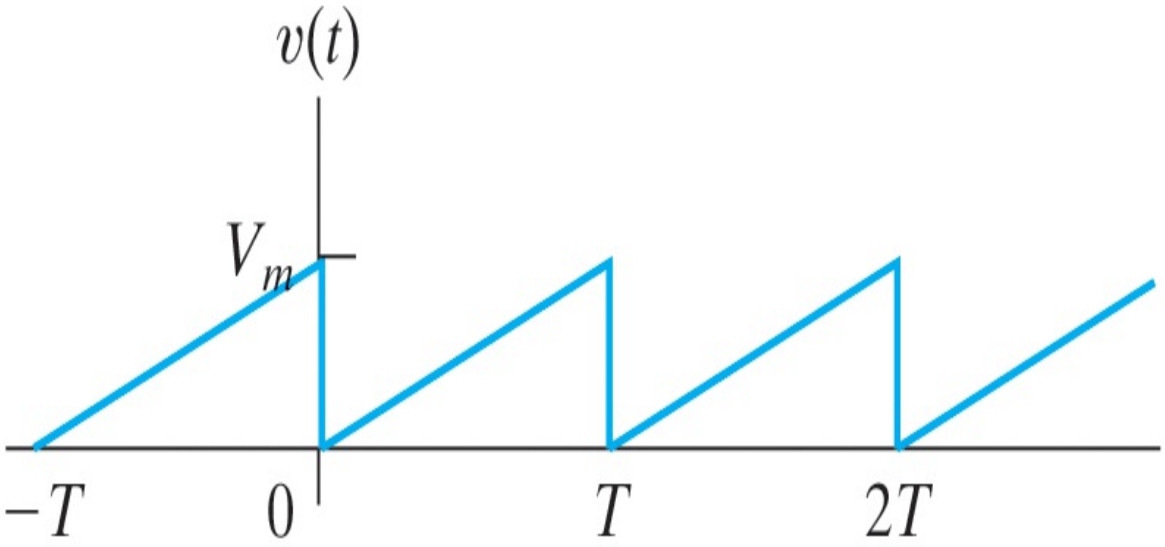


Figure 16.5 Full Alternative Text

Solution

To find av, ak, and bk using Eqs. 16.3–16.5, we must choose the starting time t0. For the periodic voltage of Fig. 16.5, the best choice for t0 is zero. Any other choice makes the required integrations more cumbersome. The expression for v(t) between 0 and T is

$$v(t)=(VmT)t.$$

The equation for av is

$$av=1T\int 0T(VmT)t\,dt=12Vm.$$

This is clearly the average value of the waveform in Fig. 16.5.

The equation for the kth value of an is

$$ak=2T\int 0T(VmT)t\cos k\omega 0t\,dt=2VmT2(1k2\omega 02\cos k\omega 0t+tk\omega 0\sin k\omega 0t)|0T=2\,1k2\omega 02\,(\cos 2\pi k-1)\,]=0 \quad \text{for all k.}$$

Notice that we used the relationship between T and ω0, T=2π/ω0, when evaluating the integral at its upper limit.

The equation for the kth value of bn is

$$bk=2T\int 0T(VmT)\,t\sin k\omega 0t\,dt=2VmT2\,(1k2\omega 02\sin k\omega 0t - tk\omega 0\cos k\omega 0t)|0T=2VmT2\,(0-Tk\omega 0\cos 2\pi k)=-Vm\pi k.$$

The Fourier series for v(t) is

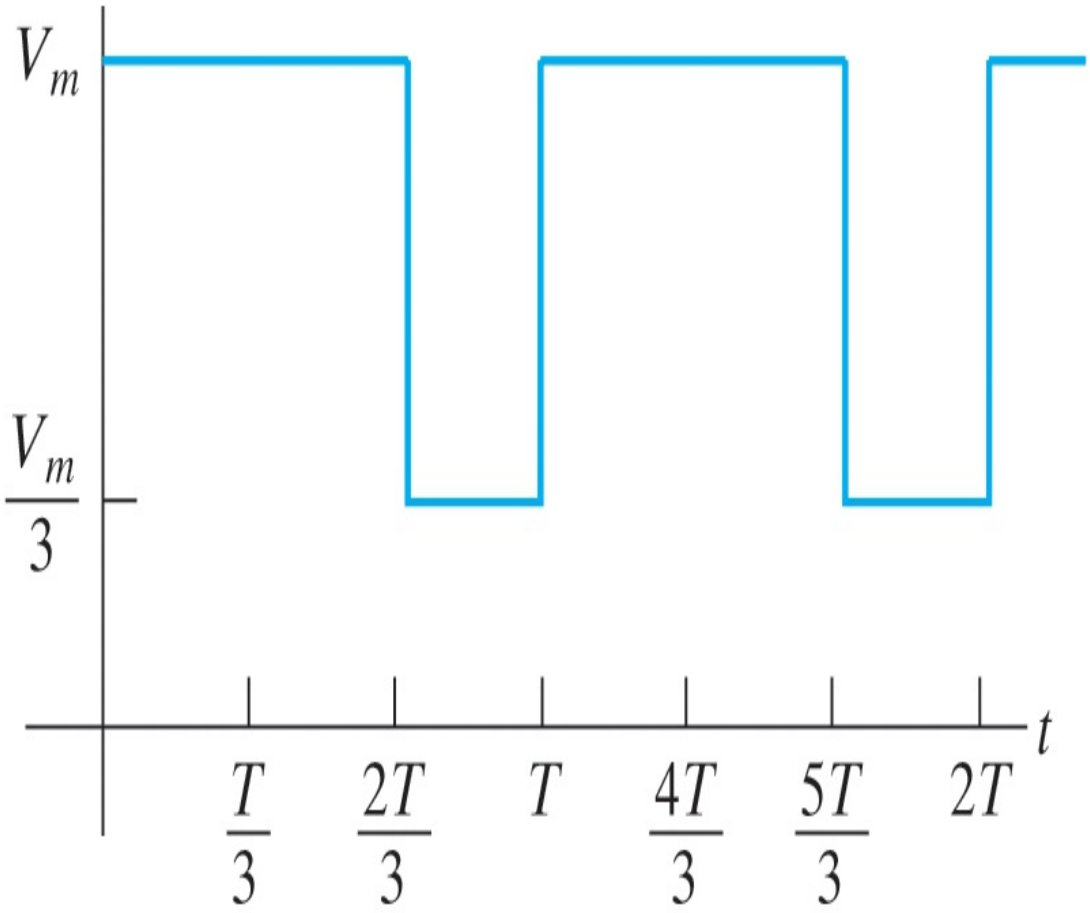
$$v(t)=Vm2-Vm\pi\sum n=1\infty 1n\sin n\omega 0t=Vm2-Vm\pi\sin\omega 0t-Vm2\pi\sin 2\omega 0t-Vm3\pi\sin 3\omega 0t-\cdots.$$

Assessment Problems

Objective 1—Be able to calculate the trigonometric form of the Fourier

coefficients for a periodic waveform

1. 16.1 Derive the expressions for av, ak, and bk for the periodic voltage function shown if Vm=9π V.



16.2-1 Full Alternative Text

Answer:

av=21.99 V,

ak=6k sin 4kπ3 V,

bk=6k (1−cos 4kπ3) V.

2. 16.2 Refer to Assessment Problem 16.1.

1. What is the average value of the periodic voltage?
2. Compute the numerical values of a1−a5 and b1−b5.
3. If T=125.66 ms, what is the fundamental frequency in radians per second?
4. What is the frequency of the third harmonic in hertz?
5. Write the Fourier series up to and including the fifth harmonic.

Answer:

1. 21.99 V;
2. −5.2 V, 2.6 V, 0 V, −1.3, and 1.04 V; 9 V, 4.5 V, 0 V, 2.25 V, and 1.8 V;
3. 50 rad/s;
4. 23.87 Hz;
5. v(t) = 21.99−5.2 cos 50t++9 sin 50t +2.6 cos 100t+4.5 sin 100t −1.3 cos 200t+2.25 sin 200t +1.04 cos 250t+1.8 sin 250t V.

SELF-CHECK: Also try Chapter Problems 16.1–16.3.

Finding the Fourier coefficients, in general, is tedious. Therefore, anything that simplifies the task is beneficial. Fortunately, when a periodic function possesses certain types of symmetry, we can find its Fourier coefficients with fewer computations. In Section 16.3, we discuss how symmetry affects the coefficients in a Fourier series.

16.3 The Effect of Symmetry on the Fourier Coefficients

Four types of symmetry make the task of evaluating the Fourier coefficients easier:

- even-function symmetry,
- odd-function symmetry,
- half-wave symmetry,
- quarter-wave symmetry.

The effect of each type of symmetry on the Fourier coefficients is discussed in the following sections.

Even-Function Symmetry

A function is defined as *even* if

Even Function

$$f(t)=f(-t). \quad (16.6)$$

Figure 16.6 illustrates an even periodic function. Functions that satisfy Eq. 16.6 are said to be even because polynomial functions with only even exponents possess this characteristic.

Figure 16.6 An even periodic

function, f(t)=f(−f).

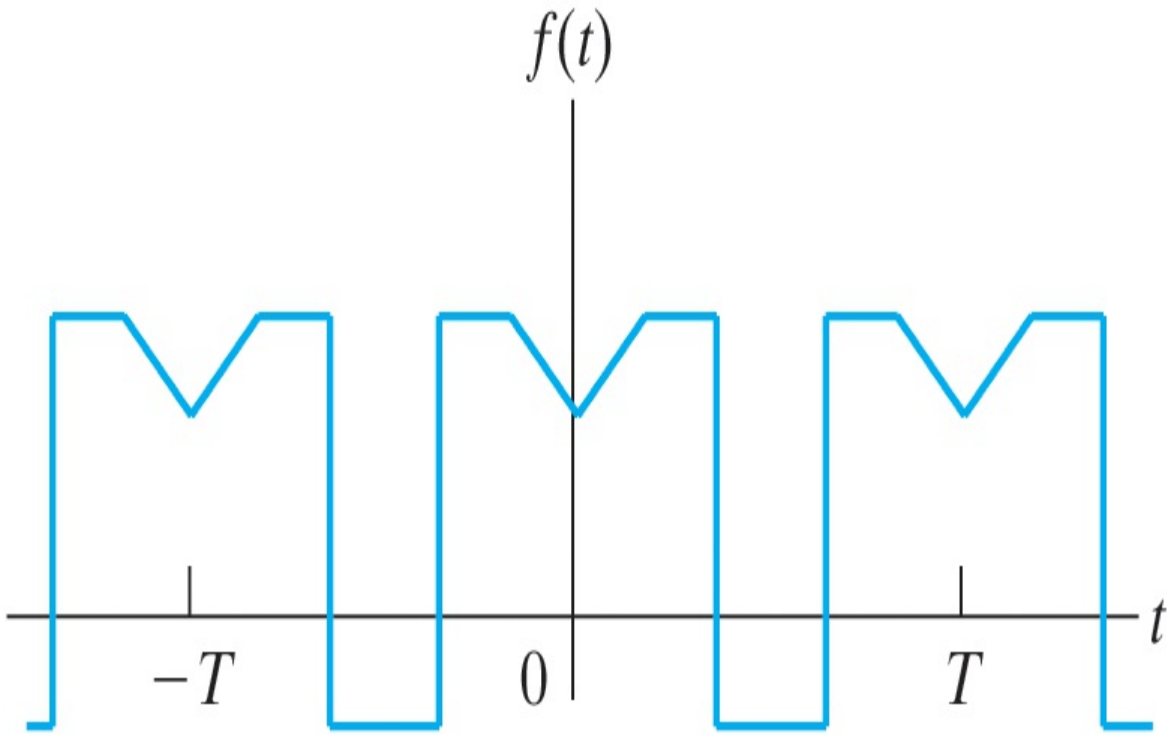


Figure 16.6 Full Alternative Text

For even periodic functions, the equations for the Fourier coefficients reduce to

av=2T∫0T/2f(t) dt, (16.7)
ak=4T∫0T/2f(t) cos kω0t dt, (16.8)
bk=0,for all k. (16.9)

Note that all the b coefficients are zero if the periodic function is even. This means that the Fourier series representation of an even periodic function consists only of the constant term and cosine terms—there are no sine terms. This is not surprising because the cosine function is even, but the sine function is not.

The derivations of Eqs. 16.7–16.9 follow directly from Eqs. 16.3–16.5. In each derivation, we select t0=−T/2 and then break the interval of integration

into the range from −T/2 to 0 and 0 to T/2, For example,

$$av=1T\int-T/2T/2f(t)\,dt=1T\int-T/20f(t)\,dt+1T\int0T/2f(t)\,dt.$$

Now we change the variable of integration in the first integral on the right-hand side of the equation for av. We let t=−x and note that f(t)=f(−x)=f(x) because the function is even. Note that x=T/2 when t=−T/2 and dt=−dx. Then

$$\int-T/20f(t)\,dt=\int T/20f(x)\,(-dx)=\int0T/2f(x)\,dx,$$

which shows that the integral from −T/2 to 0 is identical to the integral from 0 to T/2. This completes the derivation of Eq. 16.7.

The derivation of Eq. 16.8 proceeds along similar lines. Here,

$$a\ k = 2\ T \int 0\ -T/2\ f(t) \cos k\ \omega\ 0\ t\ dt + 2\ T \int T/2\ 0\ f(t) \cos k\ \omega\ 0\ t\ dt,$$

but

$$\int -T/2\ 0\ f(t) \cos k\ \omega\ 0\ t\ dt = \int T/2\ 0\ f(x) \cos (-k\ \omega\ 0\ x)(-dx) = \int 0\ T/2\ f(x) \cos k\ \omega\ 0\ x\ dx.$$

As before, the integral from −T/2 to 0 is the same as the integral from 0 to T/2. Replacing the first integral in the equation for ak with a copy of the second integral and simplifying yields Eq. 16.8.

All the b coefficients are zero when f(t) is an even periodic function because the integral from −T/2 to 0 is the negative of the integral from 0 to T/2; that is,

$$\int -T/2\ 0\ f(t) \sin k\ \omega\ 0\ t\ dt = \int T/2\ 0\ f(x) \sin (-k\ \omega\ 0\ x)(-dx) = \int 0\ T/2\ f(x) \sin k\ \omega\ 0\ x\ dx.$$

When we use Eqs. 16.7 and 16.8 to find the Fourier coefficients, the interval of integration must be between 0 and T/2.

Odd-Function Symmetry

A function is defined as odd if

Odd Function

$$f(t) = -f(-t). \quad (16.10)$$

Functions that satisfy Eq. 16.10 are said to be odd because polynomial functions with only odd exponents have this characteristic. Figure 16.7 shows an odd periodic function.

Figure 16.7 An odd periodic function f(t)=−f(−f).

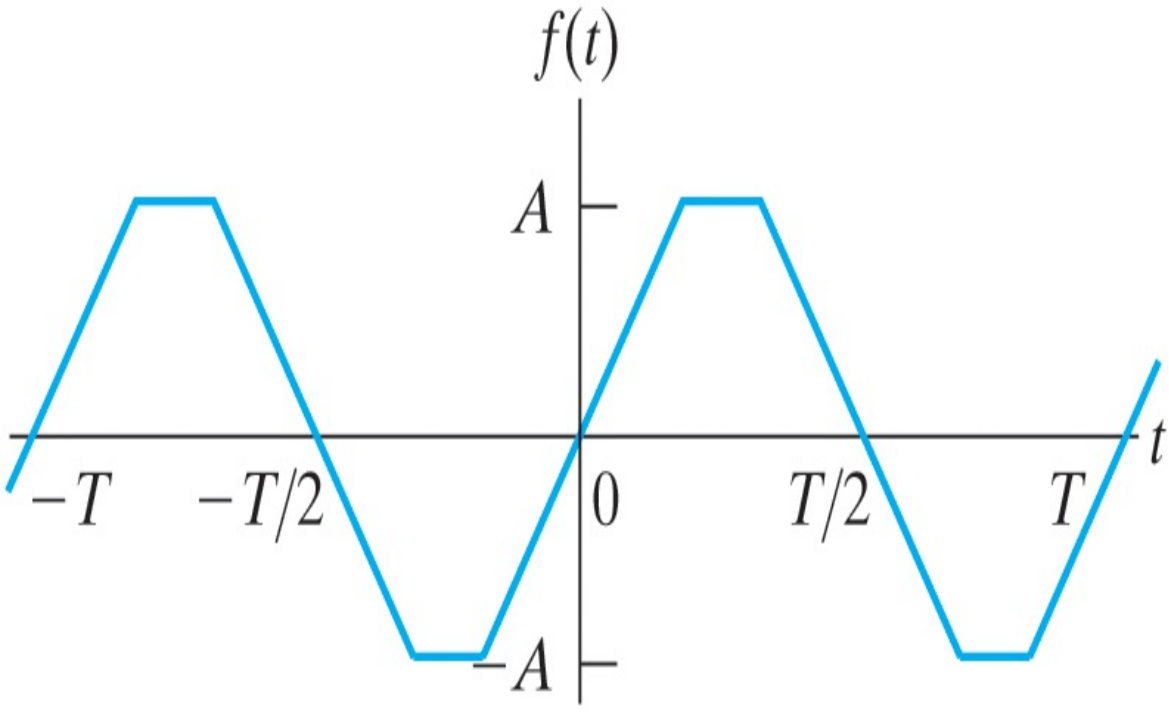


Figure 16.7 Full Alternative Text

The expressions for the Fourier coefficients are

$$a_v = 0; \quad (16.11)$$
$$a_k = 0, \text{for all } k; \quad (16.12)$$
$$b_k = \frac{4}{T}\int_0^{T/2} f(t) \sin k\omega_0 t \, dt. \quad (16.13)$$

Note that all the a coefficients are zero if the periodic function is odd. This means that the Fourier series representation of an odd periodic function consists only of sine terms; there are no cosine terms. This is not surprising because the sine function is odd, but the cosine function is even. It also means that the average value of an odd function is always zero because the constant term, av, is always zero.

We use the same process to derive Eqs. 16.11–16.13 that we used to derive Eqs. 16.7–16.9. We leave the derivations to you in Problem 16.7.

If a periodic function has even or odd symmetry, the symmetry can be destroyed by shifting the function along the time axis. This also means that if a periodic function lacks even or odd symmetry, it may be possible to shift the function along the time axis to create this symmetry. For example, the triangular function shown in Fig. 16.8(a) is neither even nor odd. However, we can make the function even by shifting it left, as shown in Fig. 16.8(b), or odd by shifting it right, as shown in Fig. 16.8(c).

Figure 16.8 How the choice of where can make a periodic function even, odd, or neither. (a) A periodic triangular wave that is neither even nor odd. (b) The triangular wave of (a) made even by shifting the

function along the t axis. (c) The triangular wave of (a) made odd by shifting the function along the t axis.

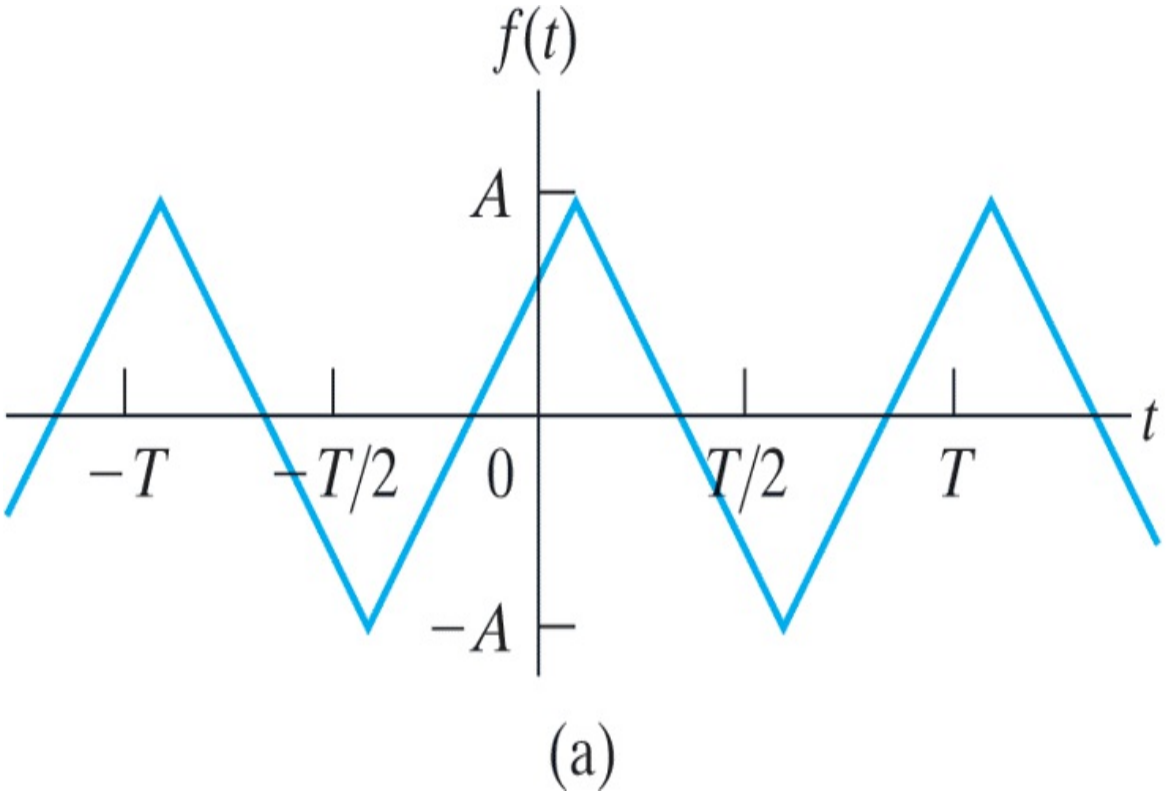
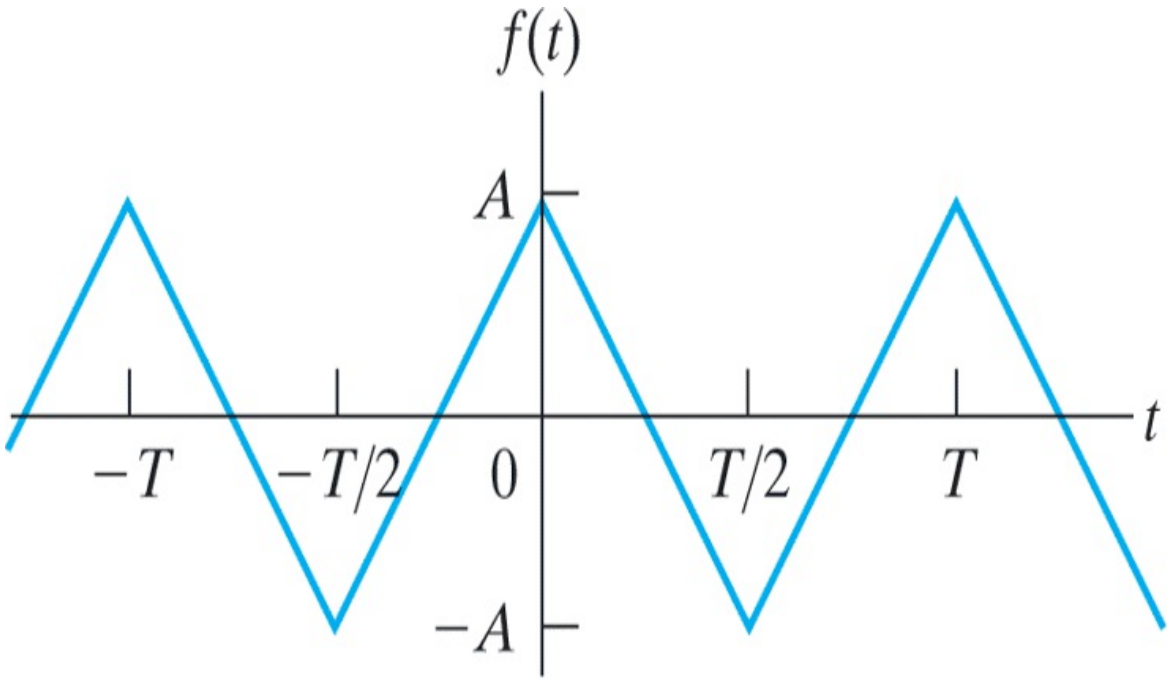
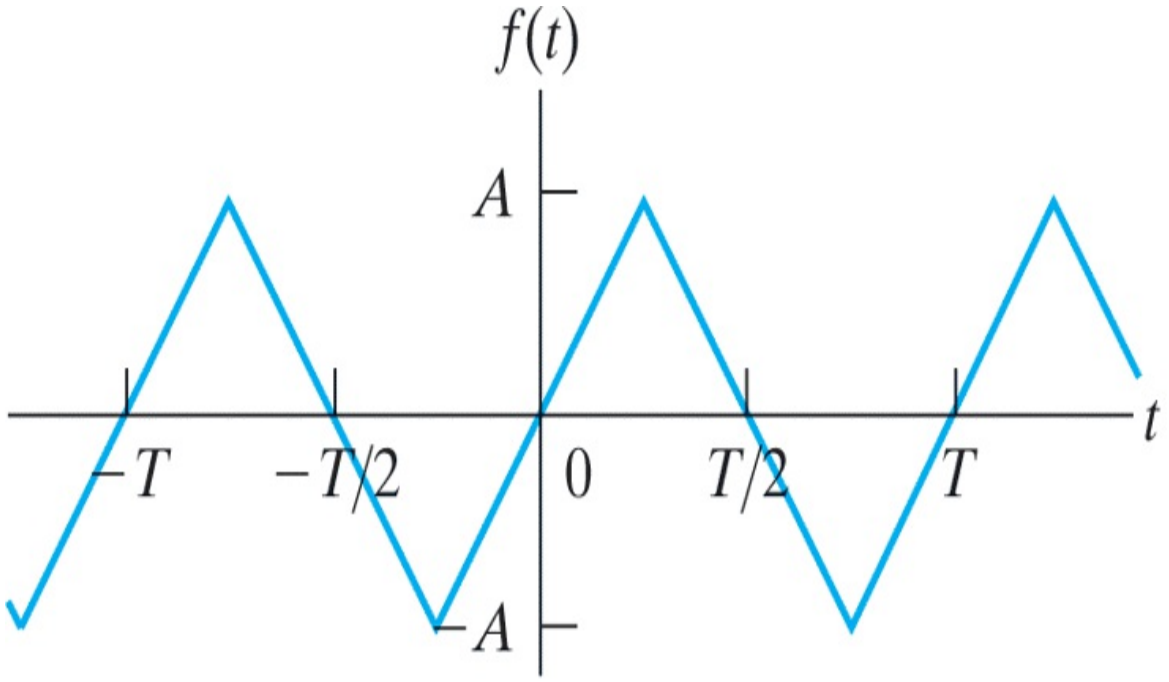


Figure 16.8 Full Alternative Text



(b)

16.3-2 Full Alternative Text



(c)

16.3-2 Full Alternative Text

Half-Wave Symmetry

A periodic function possesses half-wave symmetry if it satisfies the constraint

Half-Wave Symmetry

f(t)=−f(t−T/2).(16.14)

Equation 16.14 states that a periodic function has half-wave symmetry if, after it is shifted one-half period and inverted, it is identical to the original function. For example, the functions shown in Figs. 16.7 and 16.8 have half-wave symmetry, whereas those in Figs. 16.5 and 16.6 do not. Note that half-wave symmetry is not determined by where [&t|=|0&], as seen in Fig. 16.8.t=0.

If a periodic function has half-wave symmetry, both ak and bk are zero for even values of k. Moreover, av also is zero because the average value of a function with half-wave symmetry is zero. The expressions for the Fourier coefficients are

av=0, (16.15)
a k = 0 for k even; a k = 4 T ∫ 0 T/2 f(t) sin k ω 0 t dt, for k odd; (16.16)
b k = 0 for k even; b k = 4 T ∫ 0 T/2 f(t) sin k ω 0 t dt, for k odd. (16.17)

We derive Eqs. 16.15–16.17 by starting with Eqs. 16.3–16.5 and choosing the interval of integration as −T/2 to T/2. We then divide this range into the intervals −T/2. to 0 and 0 to T/2. For example, the derivation for ak is

a k = 2 T ∫ t 0 t 0 +T f(t) cos k ω 0 t dt = 2 T ∫ −T/2 T/2 f(t) cos k ω 0 t dt = 2 T ∫ −T/2 0 f(t) cos k ω 0 t dt + 2 T ∫ 0 T/2 f(t) cos k ω 0 t dt.

Now we change a variable in the first integral on the right-hand side of the

equation for ak. Specifically, we let

$$t=x-T/2.$$

Then

$$x=T/2, \quad \text{when } t=0; x=0, \quad \text{when } t=-T/2; dt=dx.$$

We rewrite the first integral as

$$\int_{-T/2}^{0} f(t)\cos k\omega_0 t\, dt=\int_{0}^{T/2} f(x-T/2)\cos k\omega_0(x-T/2)\, dx.$$

Note that

$$\cos k\omega_0(x-T/2)=\cos(k\omega_0 x-k\pi)=\cos k\pi \cos k\omega_0 x$$

and that, by hypothesis,

$$f(x-T/2)=-f(x).$$

Therefore,

$$\int_{-T/2}^{0} f(t)\cos k\omega_0 t\, dt=\int_{0}^{T/2}[-f(x)]\cos k\pi \cos k\omega_0 x\, dx.$$

Incorporating this integral into the equation for ak gives

$$a_k=2T(1-\cos k\pi)\int_{0}^{T/2} f(t)\cos k\omega_0 t\, dt.$$

But cos kπ is 1 when k is even and −1 when k is odd. Therefore, we get the expressions for ak given in Eqs. 16.16.

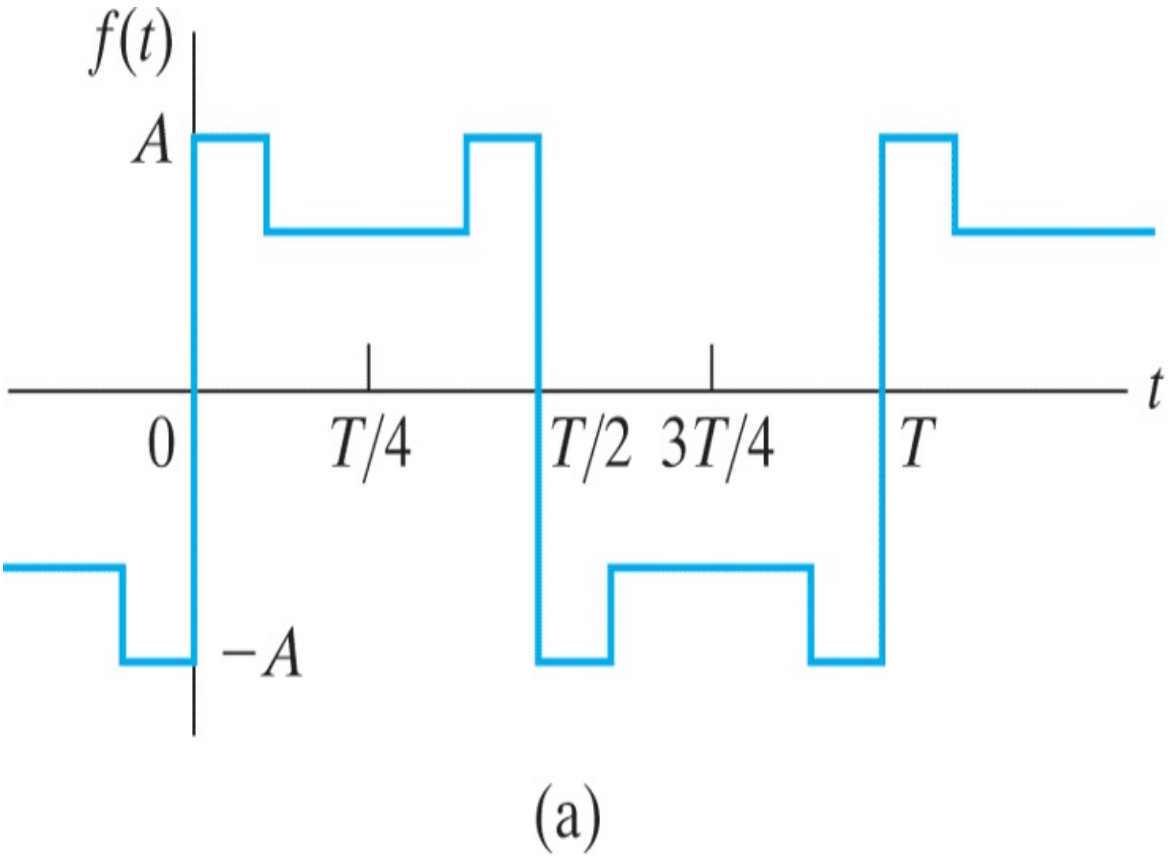
We leave it to you to verify that this same process can be used to derive Eqs. 16.17 (see Problem 16.8).

We summarize our observations by noting that the Fourier series representation of a periodic function with half-wave symmetry has zero average value and contains only odd harmonics.

Quarter-Wave Symmetry

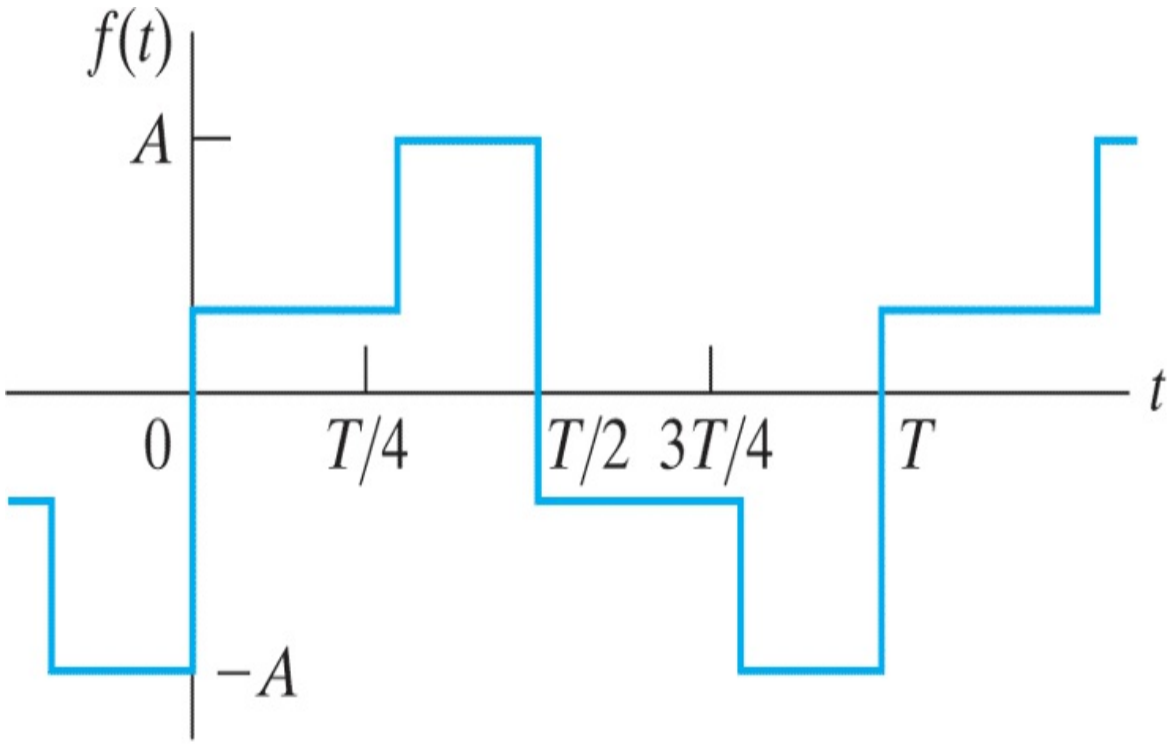
The term **quarter-wave symmetry** describes a periodic function that has half-wave symmetry and, in addition, symmetry about the midpoint of the positive and negative half-cycles. The function illustrated in Fig. 16.9(a) has quarter-wave symmetry about the midpoint of the positive and negative half-cycles. The function in Fig. 16.9(b) does not have quarter-wave symmetry, although it does have half-wave symmetry.

Figure 16.9 (a) A function that has quarter- wave symmetry. (b) A function that does not have quarterwave symmetry.



(a)

Figure 16.9 Full Alternative Text



(b)

16.3-2 Full Alternative Text

A periodic function that has quarter-wave symmetry can always be made either even or odd by the proper choice of the point where t=0. For example, the function shown in Fig. 16.9(a) is odd and can be made even by shifting the function T/4 units either right or left along the t axis. However, the function in Fig. 16.9(b) can never be made either even or odd.

To take advantage of quarter-wave symmetry when calculating the Fourier coefficients, you must choose the point where t=0 to make the function either even or odd. If the function is made even, then

av=0,because of half-wave symmetry;ak=0,for k even,because of half-wave symmetry;ak=8T∫0T/4f(t) cos kω0t dt, for k odd;bk=0,for all k,because (16.18)

Equations 16.18 result from the function's quarter-wave symmetry in addition to its being even. Remember that if a function has quarter-wave

symmetry, it also has half-wave symmetry, so we can eliminate av and ak for k even. Comparing the expression for ak, k odd, in Eqs. 16.18 with Eqs. 16.16, shows that when an even function also has quarter-wave symmetry, we can shorten the range of integration from 0 to T/2 to 0 to T/4. We leave the derivation of Eqs. 16.18 to you in Problem 16.9.

If the quarter-wave symmetric function is made odd,

av=0,because the function is odd;ak=0,for all k,because the function is odd;bk= wave symmetry;bk=8T∫0T/4f(t) sin kω0t dt,for k odd. (16.19)

Equations 16.19 are a direct consequence of the function being both odd and quarter-wave symmetric. Again, quarter-wave symmetry allows us to shorten the interval of integration from 0 to T/2 to 0 to T/4. We leave the derivation of Eqs. 16.19 to you in Problem 16.10.

Example 16.2 shows how symmetry simplifies the task of finding the Fourier coefficients.

Example 16.2 Finding the Fourier Series of a Periodic Function with Symmetry

Find the Fourier series representation for the current waveform shown in Fig. 16.10.

Figure 16.10 The periodic waveform for Example 16.2.

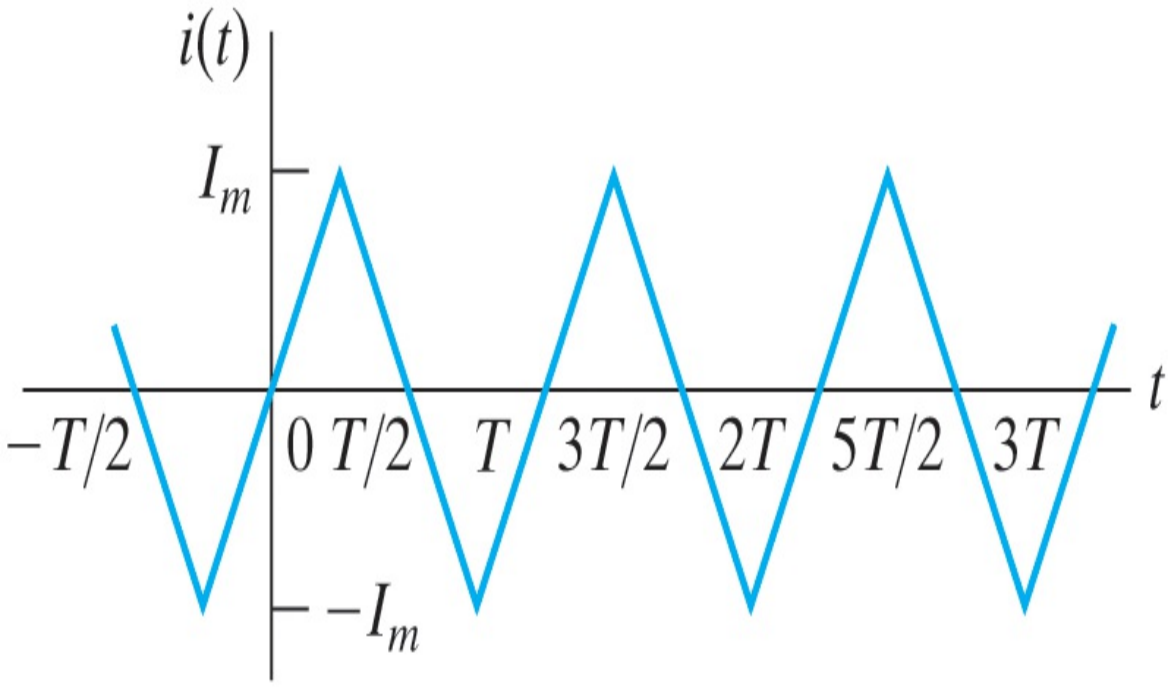


Figure 16.10 Full Alternative Text

Solution

We begin by looking for symmetry in the waveform. We find that the function is odd and has half-wave and quarter-wave symmetry. Because the function is odd, all the a coefficients are zero; that is, av=0 and ak=0 for all k. Because the function has half-wave symmetry, bk=0 for even values of k. Because the function has quarter-wave symmetry, the expression for bk for odd values of k is

bk=8T∫0T/4i(t) sin kω0t dt.

In the interval 0≤t≤T/4, the expression for i(t) is

i(t)=4ImTt.

Thus

bk=8T∫0T/44ImT t sin kω0t dt=32ImT2(sin kω0tk2ω02−t cos kω0tkω0|0T/4)=

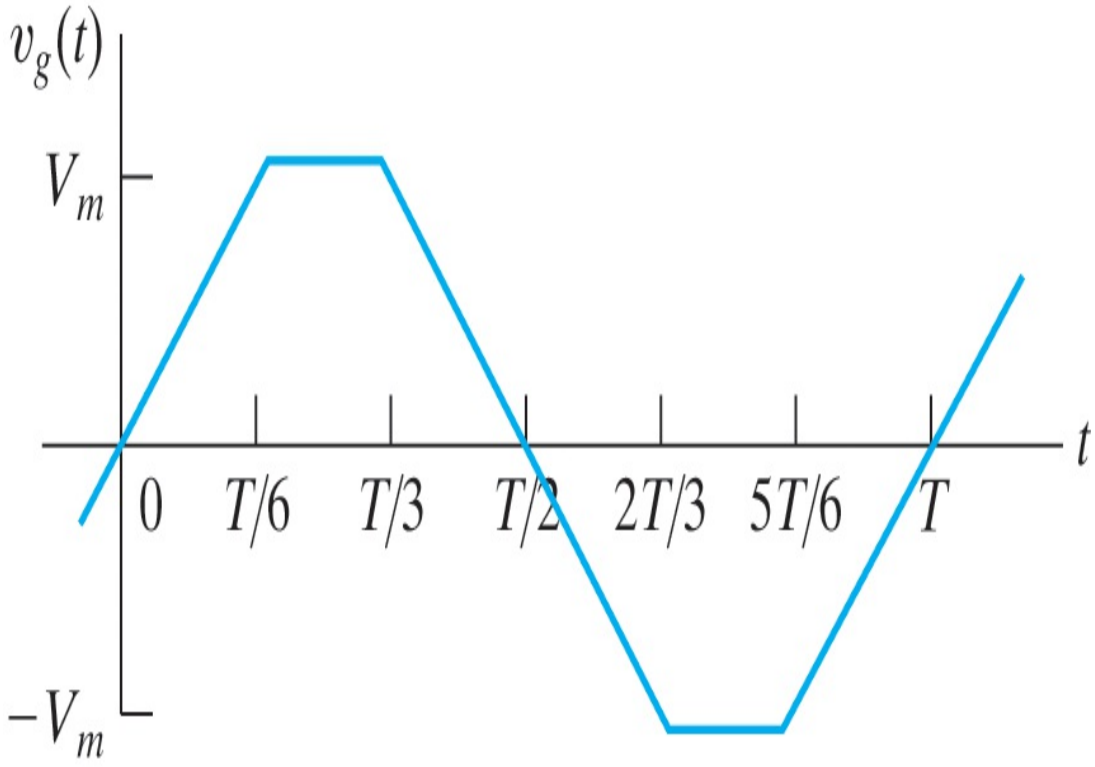
The Fourier series representation of i(t) is

$i(t) = 8 I_m \pi^2 \sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n^2} \sin \frac{n\pi}{2} \sin n\omega_0 t = \frac{8 I_m}{\pi^2} (\sin \omega_0 t - \frac{1}{9} \sin 3\omega_0 t + \frac{1}{25} \sin 5\omega_0 t - \frac{1}{49} \sin 7\omega_0 t + \cdots)$.

Assessment Problem

Objective 1—Be able to calculate the trigonometric form of the Fourier coefficients for a periodic waveform

1. 16.3 Derive the Fourier series for the periodic voltage shown.



16.3-2 Full Alternative Text

Answer:

$v_g(t) = \frac{12 V_m}{\pi^2} \sum_{n=1,3,5,\ldots}^{\infty} \frac{\sin (n\pi/3)}{n^2} \sin n\omega_0 t$.

SELF-CHECK: Also try Problems 16.11 and 16.12.

16.4 An Alternative Trigonometric Form of the Fourier Series

When analyzing a circuit with a periodic input voltage, we could replace the input voltage with a collection of series-connected voltage sources, where each source corresponds to a term in the Fourier series representation of the periodic voltage. This would mean that for every harmonic frequency, there is a source for the sine term and a source for the cosine term at that frequency.

Instead, to simplify the circuit and our analysis of it, we combine the sine and cosine terms at the same harmonic frequency into a single term. Then, we can transform the circuit into the phasor domain for each harmonic frequency, where the combined source is represented as a single phasor quantity. Thus, we write the Fourier series in Eq. 16.2 as

$$f(t)=a_v+\sum_{n=1}^{\infty}A_n\cos(n\omega_0 t-\theta_n), \quad (16.20)$$

where A_n and θ_n are defined as

$$a_n - jb_n = \sqrt{a_n^2 + b_n^2}\ \angle -\theta_n = A_n\ \angle -\theta_n . \quad (16.21)$$

We derive Eqs. 16.20 and 16.21 using the phasor method to add the cosine and sine terms in Eq. 16.2. We begin by expressing the sine functions as cosine functions; that is, we rewrite Eq. 16.2 as

$$f(t)=a_v+\sum_{n=1}^{\infty}a_n\cos n\omega_0 t+b_n\cos(n\omega_0 t-90^\circ).$$

Adding the terms under the summation sign by using phasors gives

$$\mathbf{P}\{a_n\cos n\omega_0 t\}=a_n\angle 0^\circ$$

and

$$\mathbf{P}\{b_n\cos(n\omega_0 t-90^\circ)\}=b_n\angle -90^\circ=-jb_n.$$

Then

$$P\{a_n \cos(n\omega_0 t + b_n \cos(n\omega_0 t - 90°)\} = a_n - jb_n = a_n^2 + b_n^2 - \theta_n = A_n - \theta_n.$$

The right-hand sides of this expression correspond to Eq. 16.21. When we inverse phasor-transform both sides of this expression, we get

$$a_n \cos n\omega_0 t + b_n \cos(n\omega_0 t - 90^\circ) = P - 1\{A_n - \theta_n\} = A_n \cos(n\omega_0 t - \theta_n).$$

After substituting the right-hand side for the argument of the summation in the expression for f(t), we get Eq. 16.20. If the periodic function is either even or odd, An reduces to either an (even) or bn (odd), and θn is either 0° (even) or 90° (odd).

Example 16.3 calculates the alternative form of the Fourier series for a specific periodic function.

Example 16.3 Calculating Forms of the Trigonometric Fourier Series for Periodic Voltage

1. Derive the expressions for ak and bk for the periodic function shown in Fig. 16.11.
2. Write the first four terms of the Fourier series representation of v(t) using the format of Eq. 16.20.

Figure 16.11 The periodic function for Example 16.3.

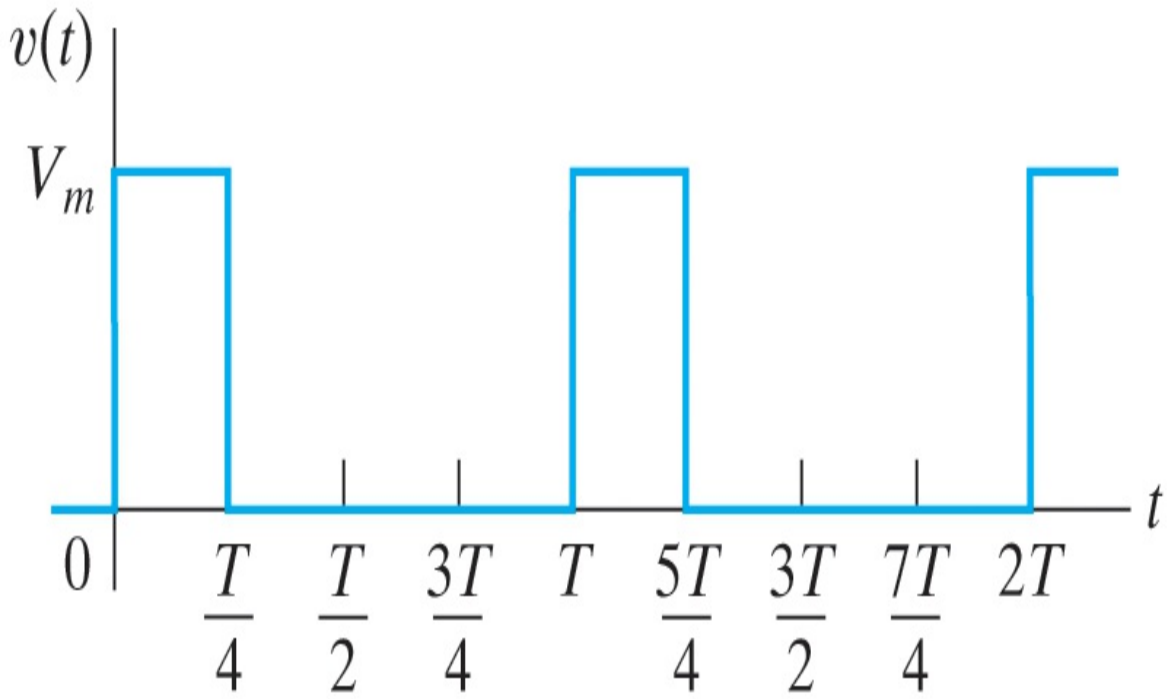


Figure 16.11 Full Alternative Text

Solution

1. The voltage v(t) is neither even nor odd, nor does it have half-wave symmetry. Therefore, we use Eqs. 16.4 and 16.5 to find ak and bk. Choosing t0 as zero, we obtain

   ak=2T[∫0T/4Vm cos kω0t dt+∫T/4T(0) cos kω0t dt]=2VmTsin kω0tkω0|0T

   and

   bk=2T∫0T/4Vm sin kω0t dt=2VmT(−cos kω0tkω0|0T/4)=Vmkπ (1−coskπ

2. The average value of v(t) is

   av=Vm(T/4)T=Vm4.

   The values of ak−jbk for k=1,2, and 3 are

$a1-jb1=\frac{Vm}{\pi}-j\frac{Vm}{\pi}=\frac{2Vm}{\pi}\angle -45^\circ$, $a2-jb2=0-j\frac{Vm}{\pi}=\frac{Vm}{\pi}\angle -90^\circ$, $a3-jb3=-\frac{Vm}{3\pi}-j\frac{Vm}{3\pi}=\frac{2Vm}{3\pi}\angle -135^\circ$.

Thus, the first four terms in the Fourier series representation of v(t) are

$v(t)=\frac{Vm}{4}+\frac{2Vm}{\pi}\cos(\omega 0t-45°)+\frac{Vm}{\pi}\cos(2\omega 0t-90°)+\frac{2Vm}{3\pi}\cos(3\omega 0t-135°)+\cdots$.

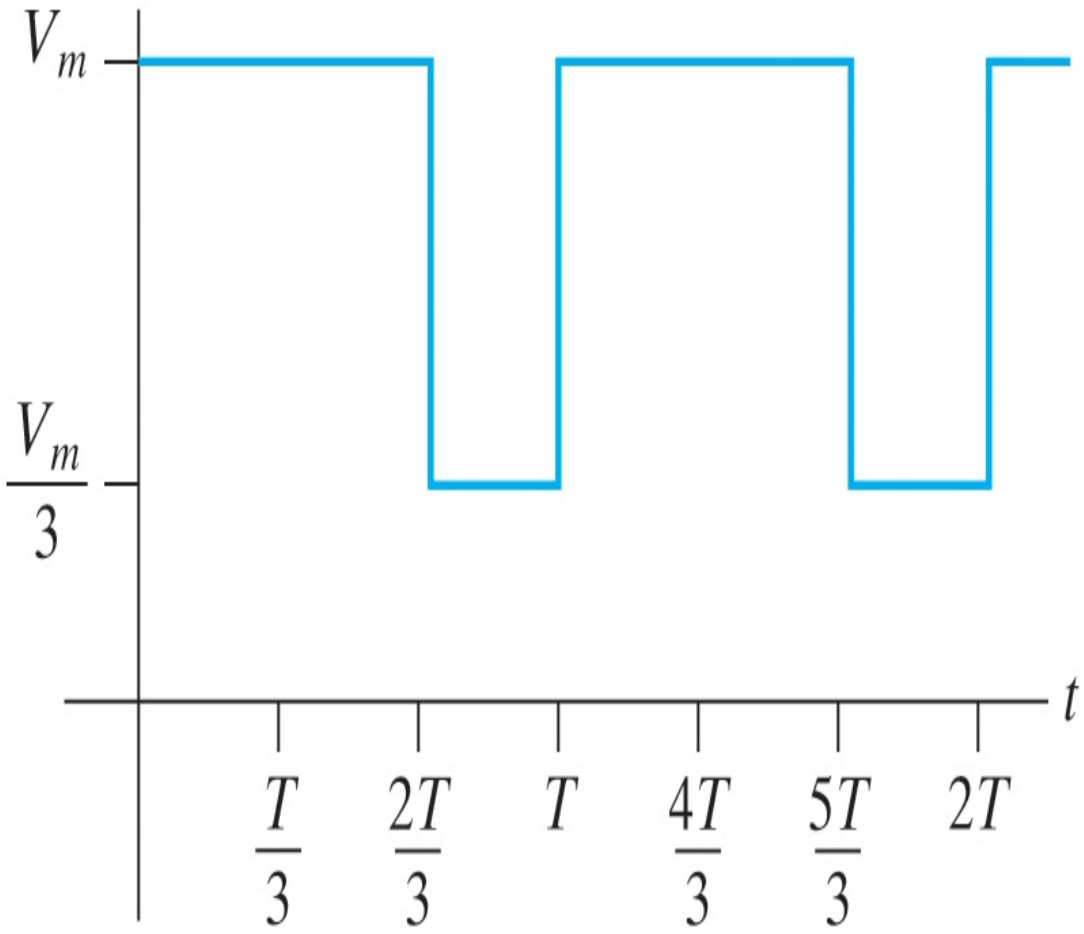
Assessment Problem

Objective 1—Be able to calculate the trigonometric form of the Fourier coefficients for a periodic waveform

1. 16.4

    1. Compute $A1-A5$ and $\theta 1-\theta 5$ for the periodic function shown if $Vm=9\pi$ V.

    2. Using the format of Eq. 16.20, write the Fourier series for v(t) up to and including the fifth harmonic assuming T=125.66 ms.



16.4-3 Full Alternative Text

Answer:

1. 10.4, 5.2, 0, 2.6, 2.1 V, and −120°, −60°, not defined, −120°, −60°;
2. v(t)=21.99+10.4 cos(50t−120°)+5.2 cos(100t−60°)+5.2 cos(100t −60°)+2.1 cos(250t−60°) V.

SELF-CHECK: Also try Problem 16.22.

16.5 An Application

Let’s use a Fourier series representation of a periodic input voltage to find the steady-state output voltage of a linear circuit, which is the RC circuit shown in Fig. 16.12(a). The circuit’s input voltage is the periodic square wave shown in Fig. 16.12(b). The output voltage is defined across the capacitor.

Figure 16.12 An RC circuit excited by a periodic voltage. (a) The RC circuit. (b) The square-wave voltage.

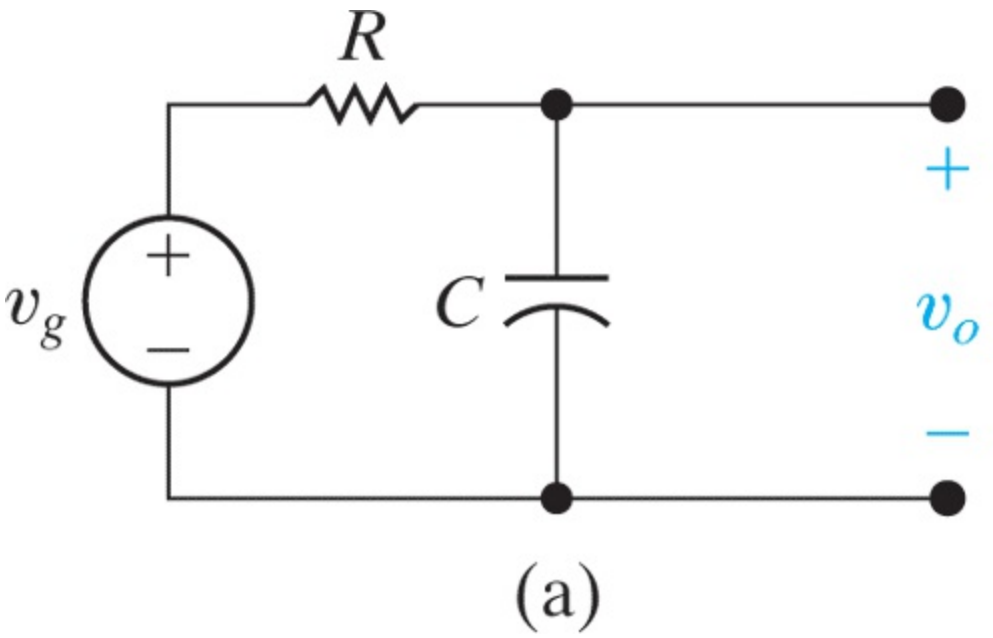
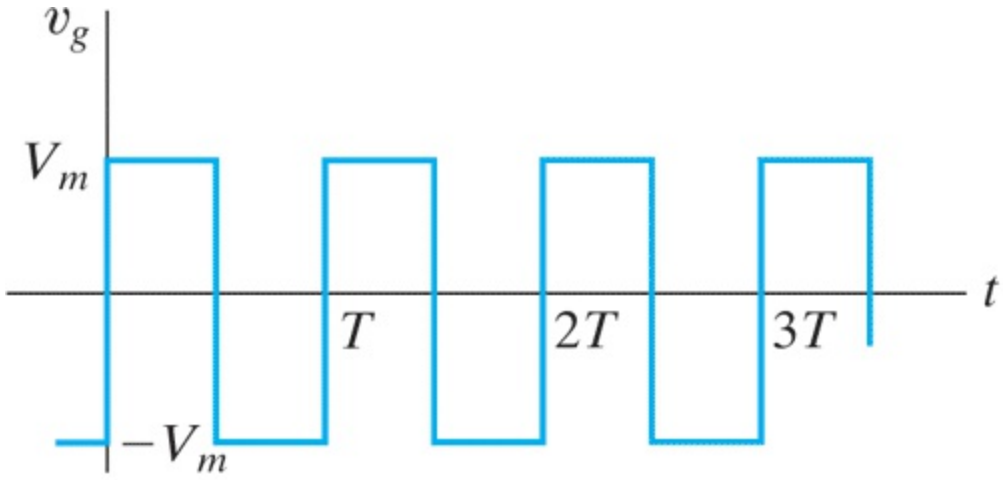


Figure 16.12 Full Alternative Text



(b)

16.5-4 Full Alternative Text

First, we represent the periodic voltage with its Fourier series. The waveform in Fig. 16.12(b) has odd, half-wave, and quarter-wave symmetry, so the only nonzero Fourier coefficients are the bk coefficients for odd values of k:

bk=8T∫0T/4Vmsin kω0t dt=4Vmπ k(k is odd).

Then the Fourier series representation of vg is

vg=4Vmπ∑n=1,3,5,…∞1nsin nω0t.

Writing the series in expanded form, we have

vg=4Vmπsin ω0t+4Vm3πsin 3ω0t+4Vm5πsin 5ω0t+4Vm7πsin7ω0t+⋯.

The voltage source, vg, in Fig. 16.12(a) can be replaced by infinitely many series-connected sinusoidal sources. Each source is a sine function whose frequency is an odd multiple of the square wave's frequency. We use the principle of superposition to find the contribution of each source to the output voltage.

For each sinusoidal source, the phasor-domain expression for the output voltage is

$$V_o = V_g \frac{1}{1 + j\omega RC}.$$

Since all of the voltage sources are expressed as sine functions, we interpret a phasor using the sine instead of the cosine. In other words, when we go from the phasor domain back to the time domain, we simply write the time-domain expressions as $\sin(\omega t + \theta)$ instead of $\cos(\omega t + \theta)$.

The phasor output voltage due to the sinusoidal source at the fundamental frequency is

$$V_{o1} = \frac{(4V_m/\pi) \angle -0^\circ}{1 + j\omega_0 RC}.$$

Writing V_{o1} in polar form gives

$$V_{o1} = \frac{(4V_m)}{\pi \sqrt{1 + \omega_0^2 R^2 C^2}} \angle -\beta_1,$$

where

$$\beta_1 = \tan^{-1} \omega_0 RC.$$

Inverse-phasor-transform V_{o1} to get the time-domain expression for the fundamental frequency component of v_o:

$$v_{o1} = \frac{4V_m}{\pi \sqrt{1 + \omega_0^2 R^2 C^2}} \sin(\omega_0 t - \beta_1).$$

We derive the third-harmonic component of the output voltage in a similar manner. The third-harmonic phasor output voltage is

$$V_{o3} = \frac{(4V_m/3\pi) \angle -0^\circ}{1 + j3\omega_0 RC} = \frac{4V_m}{3\pi \sqrt{1 + 9\omega_0^2 R^2 C^2}} \angle -\beta_3,$$

where

$$\beta_3 = \tan^{-1} 3\omega_0 RC.$$

The time-domain expression for the third-harmonic output voltage is

$$v_{o3} = \frac{4V_m}{3\pi \sqrt{1 + 9\omega_0^2 R^2 C^2}} \sin(3\omega_0 t - \beta_3).$$

Hence, the expression for the kth-harmonic component of the output voltage

is

$$v_{ok} = \frac{4V_m}{k\pi\sqrt{1+k^2\omega_0^2R^2C^2}} \sin(k\omega_0 t - \beta_k) \quad (k \text{ is odd}),$$

where

$$\beta_k = \tan^{-1} k\omega_0 RC \quad (k \text{ is odd}).$$

We now write down the Fourier series representation of the output voltage:

$$v_o(t) = \frac{4V_m}{\pi} \sum_{n=1,3,5,\ldots}^{\infty} \frac{\sin(n\omega_0 t - \beta_n)}{\sqrt{1+(n\omega_0 RC)^2}}. \quad (16.22)$$

We derived an analytic expression for the steady-state output, but the shape of vo(t), and its functional form, are not apparent from the equation. As we mentioned earlier, this is a disadvantage of the Fourier series approach.

The Fourier series representation of vo(t) in Eq. 16.22 does provide some information about the steady-state output voltage, if we focus on the frequency response of the circuit. For example, if C is large, $1/n\omega_0 C$ is small for the higher-order harmonics. Thus, the capacitor short circuits the high-frequency components of the input waveform, and the higher-order harmonics in the Fourier series representation of vo(t) are negligible compared to the lower-order harmonics. For large C, we can approximate vo(t) as

$$v_o \approx \frac{4V_m}{\pi\omega_0 RC} \sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n^2} \sin(n\omega_0 t - 90^\circ) \approx -\frac{4V_m}{\pi\omega_0 RC} \sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n^2} \cos n\omega_0 t. \quad (16.23)$$

Equation 16.23 shows that the amplitude of the harmonic in the output voltage is decreasing by $1/n^2$, while the amplitude of the harmonic in the input voltage decreases by $1/n$. If C is so large that only the fundamental component is significant, then to a first approximation

$$v_o(t) \approx -\frac{4V_m}{\pi\omega_0 RC} \cos\omega_0 t,$$

and Fourier analysis tells us that the square-wave input is deformed into a sinusoidal output.

Now let's see what happens as $C \to 0$. The circuit shows that vo and vg are the

same when C=0, because the capacitive branch looks like an open circuit at all frequencies. Equation 16.22 predicts the same result because, as $C \to 0$,

$$v_o = \frac{4V_m}{\pi} \sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n} \sin n\omega_0 t.$$

Therefore $v_o \to v_g$ as $C \to 0$.

Thus, Fourier series representation for vo(t) predicts that the output will be a highly distorted replica of the input waveform if C is large, and a reasonable replica if C is small. In Chapter 13, we looked at the distortion between the input and output in terms of how much memory the system weighting function had. In the frequency domain, we look at the distortion between the steady-state input and output in terms of how the amplitude and phase of each harmonic component are altered as it is transmitted through the circuit. When the circuit significantly alters the amplitude and phase relationships among the harmonics at the output relative to that at the input, the output is a distorted version of the input. Thus, in the frequency domain, we speak of amplitude distortion and phase distortion.

For the RC circuit in Fig. 16.12(a), amplitude distortion is present because the amplitudes of the input harmonics decrease as 1/n, whereas the amplitudes of the output harmonics decrease as

$$\frac{1}{n} \frac{1}{1 + (n\omega_0 RC)^2}.$$

This circuit also exhibits phase distortion because the phase angle of each input harmonic is zero, whereas that of the nth harmonic in the output signal is $-\tan^{-1} n\omega_0 RC$.

An Application of the Direct Approach to the Steady-State Response

For the simple RC circuit shown in Fig. 16.12(a), we can find the steady-state

response without using the Fourier series representation of the periodic input voltage. Doing this extra analysis here adds to our understanding of the Fourier series approach.

To find the steady-state expression for vo, we reason as follows. The square-wave input voltage alternates between charging the capacitor toward +Vm and −Vm. After the circuit reaches steady-state operation, this alternate charging becomes periodic. From our analysis of the first-order RC circuit in Chapter 7, we know that an abrupt change in the input voltage results in an output voltage that is exponential. Thus, the steady-state waveform of the capacitor voltage for the RC circuit shown in Fig. 16.12(a) looks like the waveform shown in Fig. 16.13.

Figure 16.13 The steady-state waveform of vo for the circuit in Figure 16.12(a).

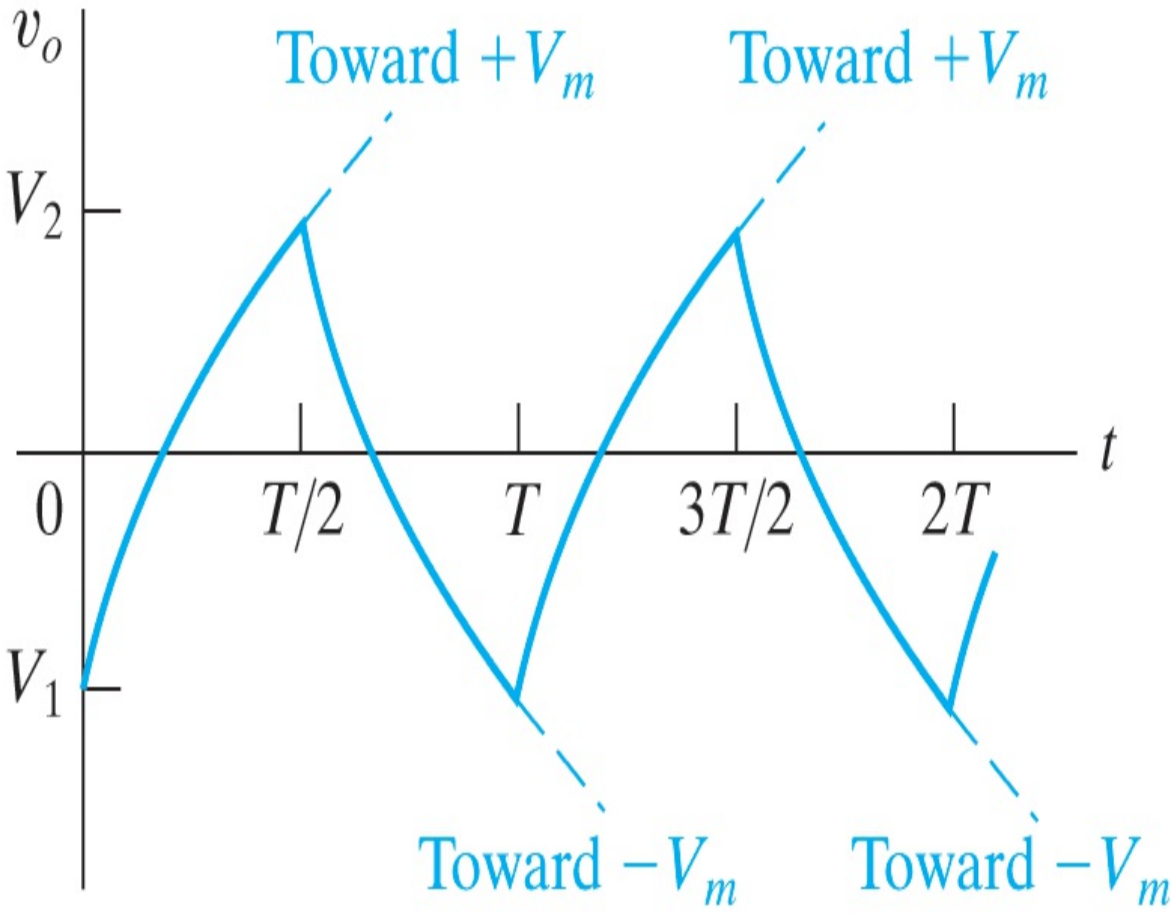


Figure 16.13 Full Alternative Text

The analytic expressions for vo(t) in the time intervals 0≤t≤T/2 and T/2≤t≤T are

vo=Vm+(V1−Vm)e−t/RC, 0≤t≤T/2;vo=−Vm+(V2+Vm)e−[t −(T/2)]/RC, T/2≤t≤T.

We derive these equations by using the methods of Chapter 7, as summarized by Eq. 7.23. The value of V2 is the value of vo(t) at the end of the interval0≤t≤T/2:

vo(T/2)=V2=Vm+(V1−Vm)e−T/2RC,

and the value of V1 is the value of vo(t) at the end of the interval T/2≤t≤T:

vo(T)=V1=−Vm+(V2+Vm)e−T/2RC.

Solving these equations for V1 and V2 yields

$$V2=-V1=Vm(1-e-T/2RC)1+e-T/2RC.$$

Substituting the expressions for V1 and V2 into the equations for vo(t)gives

$$vo=Vm-2Vm1+e-T/2RCe-t/RC, \quad 0\leq t\leq T/2, \quad (16.24)$$

and

$$vo=-Vm+2Vm1+e-T/2RCe-[t-(T/2)]/RC, \quad T/2\leq t\leq T. \quad (16.25)$$

Equations 16.24 and 16.25 indicate that vo(t) has half-wave symmetry, so the average value of vo is zero. This result agrees with the Fourier series solution for the steady-state response—namely, that because the input voltage has no dc component, the output voltage cannot have a dc component. Equations 16.24 and 16.25 also show the effect of changing the size of the capacitor. If C is small, the exponential functions quickly vanish, vo=Vm between 0 and T/2, and vo=−Vm between T/2 and T. In other words, vo → vg as C → 0. If C is large, the output waveform becomes triangular in shape, as Fig. 16.14 shows. Note that for large C, we may approximate the exponential terms e −t/RC and e−[t−(T/2)]/RC by the linear terms 1−(t/RC) and 1−{[t −(T/2)]/RC}, respectively. Equation 16.23 gives the Fourier series of this triangular waveform.

Figure 16.14 The effect of capacitor size on the steady-state response.

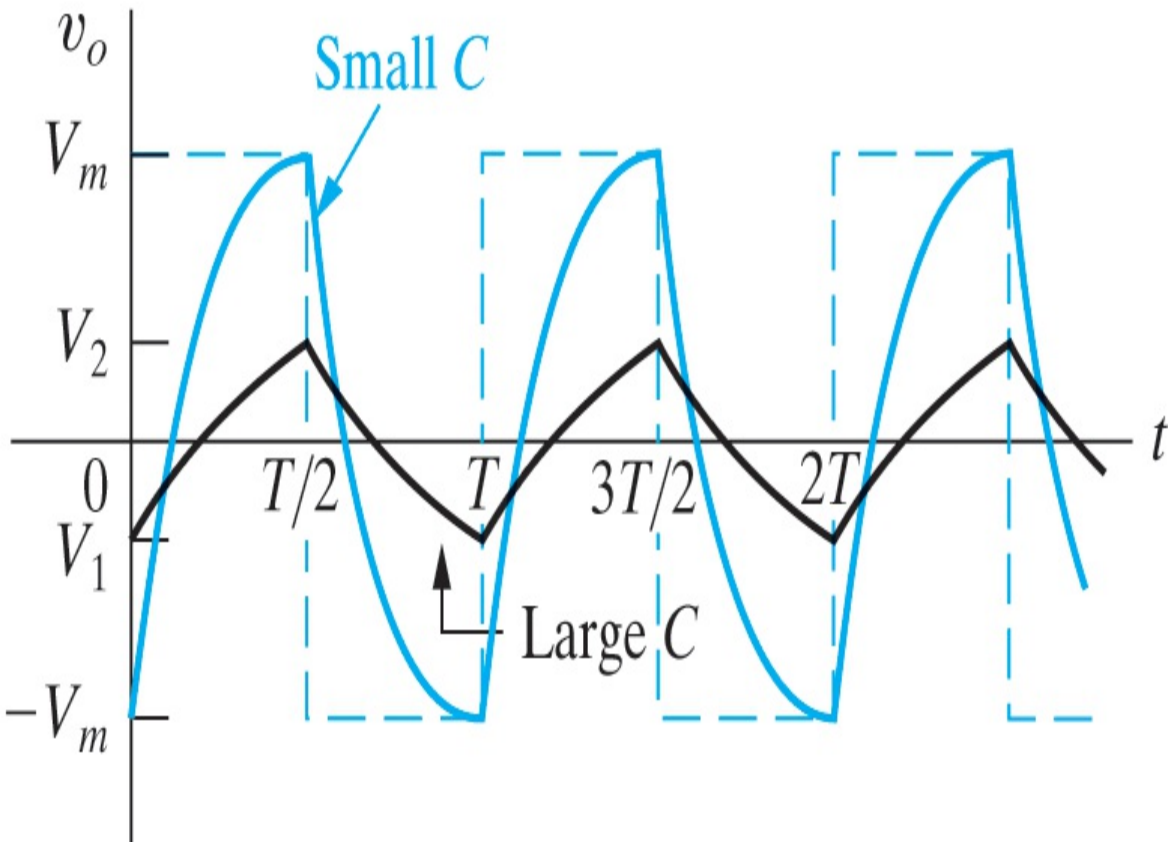


Figure 16.14 Full Alternative Text

Figure 16.14 summarizes the results. The dashed line in Fig. 16.14 is the input voltage, the solid colored line depicts the output voltage when C is small, and the solid black line depicts the output voltage when C is large.

Finally, we verify that the steady-state response of Eqs. 16.24 and 16.25 is equivalent to the Fourier series solution in Eq. 16.22. To do so, we derive the Fourier series representation of the periodic function described by Eqs. 16.24 and 16.25. We have already noted that the periodic voltage response has half-wave symmetry. Therefore, the Fourier series contains only odd harmonics. For k odd,

$$a_k = \frac{4}{T}\int_0^{T/2}\left(V_m - 2V_m e^{-t/RC}\frac{1}{1+e^{-T/2RC}}\right)\cos k\omega_0 t\, dt = -\frac{8RCV_m}{T}\left[\frac{1}{1+(k\omega_0 RC)^2}\right] \quad (k \text{ is odd}), \tag{16.26}$$

$$b_k = \frac{4}{T}\int_0^{T/2}\left(V_m - 2V_m e^{-t/RC}\frac{1}{1+e^{-T/2RC}}\right)\sin k\omega_0 t\, dt = \frac{4V_m}{k\pi} - \frac{8k\omega_0 V_m R^2C^2}{T}\left[\frac{1}{1+(k\omega_0 RC)^2}\right] \quad (k \text{ is odd}). \tag{16.27}$$

To show that Eqs. 16.26 and 16.27 are consistent with Eq. 16.22, we must prove that

$$a_k^2+b_k^2=\frac{4V_m}{k\pi}\frac{1}{1+(k\omega_0RC)^2}, \quad (16.28)$$

and that

$$\frac{a_k}{b_k}=-k\omega_0RC. \quad (16.29)$$

You can verify Eqs. 16.26–16.29 in Problems 16.23 and 16.24. Equations 16.28 and 16.29 are used with Eqs. 16.20 and 16.21 to derive the Fourier series expression in Eq. 16.22; we leave the details to you in Problem 16.25.

With this illustrative circuit, we showed how to use the Fourier series in conjunction with the principle of superposition to find the steady-state response to a periodic input voltage. Again, the principal shortcoming of the Fourier series approach is the difficulty of ascertaining the output waveform. However, by considering a circuit's frequency response, we can deduce a reasonable approximation of the steady-state response, using a finite number of terms in the Fourier series representation. (See Problems 16.29 and 16.30.) Example 16.4 provides another example of the Fourier series circuit analysis method.

Example 16.4 Finding the Response of an RLC Circuit to a Square-Wave Voltage

The square wave in Fig. 16.12(b) has a magnitude of 4πV and a frequency of 100/s. This periodic voltage is input to the circuit in Fig. 16.15.

Figure 16.15 The circuit for Example 16.4.

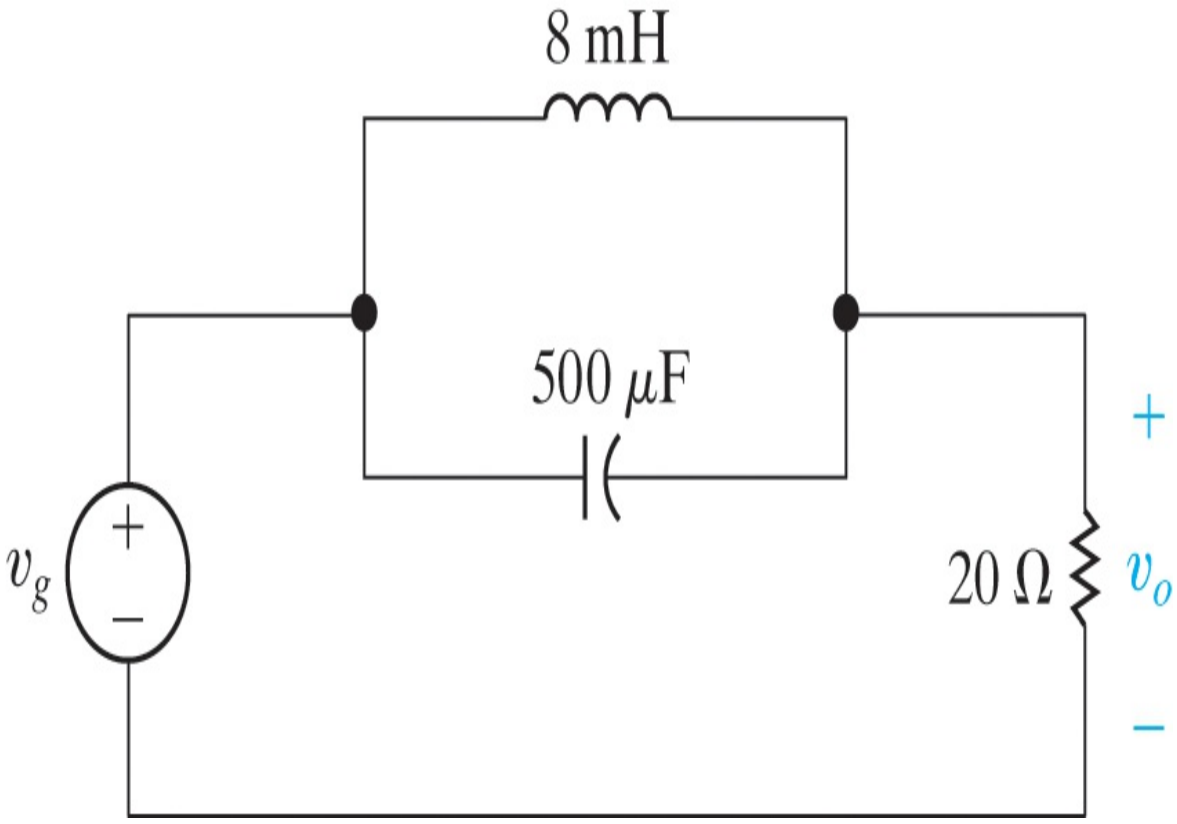


Figure 16.15 Full Alternative Text

1. Find the first five nonzero terms in the Fourier series representation of the square-wave input voltage.

2. Find the output voltage, vo, for the circuit in Fig. 16.15, using the five terms calculated in part (a).

3. Use the frequency response of the circuit in Fig. 16.15 to explain the output voltage terms you calculated in part (b).

Solution

1. The Fourier series representation of vg is

   vg=4Vmπ∑n=1,3,5,…
   ∞1nsinnωot16sin100t+5.33sin300t+3.2sin500t+2.29sin700t+1.78sin900t
   …V.

2. The circuit in Fig. 16.15 is a parallel RLC bandreject filter. From Fig. 14.31, the transfer function is

   $$H(s)=\frac{s^2+1/LC}{s^2+s/RC+1/LC}=\frac{s^2+25\times10^4}{s^2+100s+25\times10^4}.$$

   To find the terms in the Fourier series representation of vo, evaluate H(s) for $s=j\omega$, where ω is the frequency of the term in vg. Then, multiply by the phasor representation of that term in vg, and inverse-phasor-transform the result to find the corresponding term in vo. Note that we are using the sine function, instead of the cosine function for the phasor transform and its inverse.

   For $\omega=100$ rad/s:

   $$H(j100)=\frac{(j100)^2+25\times10^4}{(j100)^2+100(j100)+25\times10^4}=0.999\angle-2.4^\circ,\quad Vo(j100)=(0.998\angle-2.4^\circ)(16)=15.99\angle-2.4^\circ.$$

   For $\omega=300$ rad/s:

   $$H(j300)=\frac{(j300)^2+25\times10^4}{(j300)^2+100(j300)+25\times10^4}=0.983\angle-10.6^\circ,\quad Vo(j300)=(0.983\angle-10.6^\circ)(16)=5.24\angle-10.6^\circ.$$

   For $\omega=500$ rad/s:

   $$H(j500)=\frac{(j500)^2+25\times10^4}{(j500)^2+100(j500)+25\times10^4}=0,\quad Vo(j500)=(0)(3.2)=0.$$

   For $\omega=700$ rad/s:

   $$H(j700)=\frac{(j700)^2+25\times10^4}{(j700)^2+100(j700)+25\times10^4}=0.96\angle-16.26^\circ,\quad Vo(j700)=(0.96\angle-16.26^\circ)(2.29)=2.2\angle-16.26^\circ.$$

   For $\omega=900$ rad/s:

   $$H(j900)=\frac{(j900)^2+25\times10^4}{(j900)^2+100(j900)+25\times10^4}=0.99\angle-9.13^\circ,\quad Vo(j900)=$$

$(0.99-9.13^\circ)(1.78)=1.76-9.13^\circ$.

Therefore, the first five terms in the Fourier series representation of the steady-state output voltage are

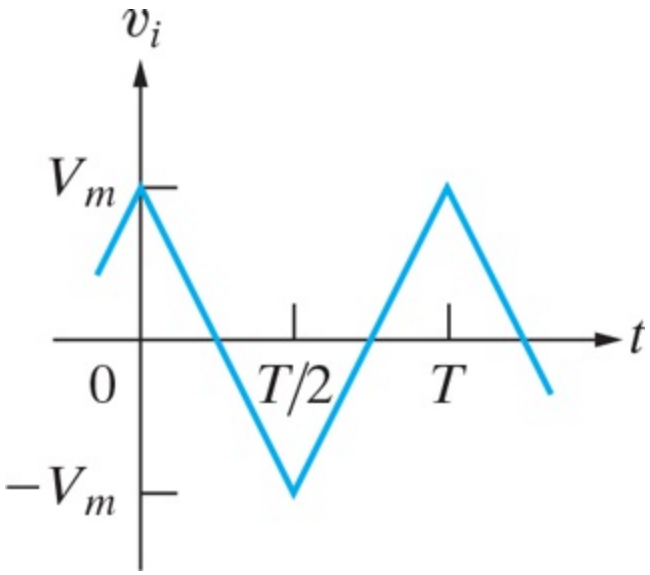
$v_o=15.99\sin(100t-2.4°)+5.24\sin(300t-10.6°)+0\sin 500t+2.2\sin(700t+16.26°)+1.76\sin(900t+9.13°)+\ldots V.$

3. As noted in part (a), the circuit in Fig. 16.15 is a bandreject filter. It has a center frequency of $1/LC=500$ rad/s, a bandwidth of $1/RC=100$ rad/s, and a quality of 5, so it is a selective filter. Its cutoff frequencies are 452.494 rad/s and 552.494 rad/s. The magnitude of v_o at 500 rad/s, the center frequency of the filter, is zero, exactly what we should expect from a bandreject filter. The magnitudes of v_o at the frequencies outside the passband defined by the cutoff frequencies are very close to the magnitudes of v_g at those frequencies. This is also the behavior we expect from a selective bandreject filter.

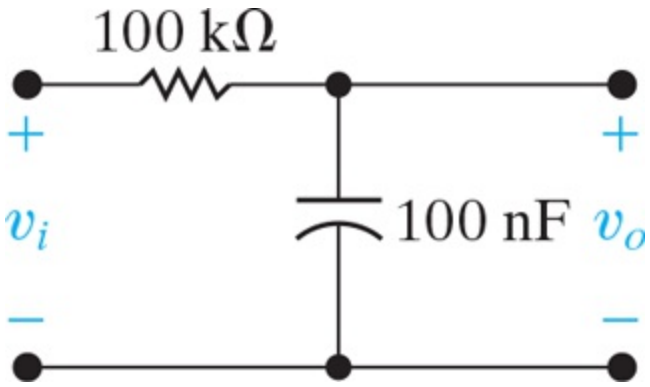
Assessment Problems

Objective 2—Know how to analyze a circuit's response to a periodic waveform

1. 16.5 The periodic triangular-wave voltage seen on the left is applied to the circuit shown on the right. Derive the first three nonzero terms in the Fourier series that represent the steady-state voltage v_o if $V_m=281.25\pi 2mV$ and the period of the input voltage is $200\pi ms$.



16.5-4 Full Alternative Text



16.5-5 Full Alternative Text

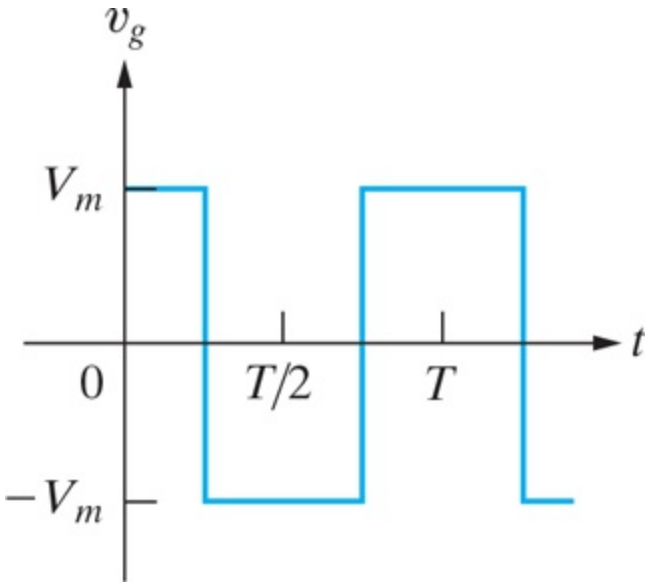
Answer:

2238.83 cos (10t − 5.71°)+239.46 cos (30t − 16.70°) + 80.50 cos(50t − 26.57°) + … mV

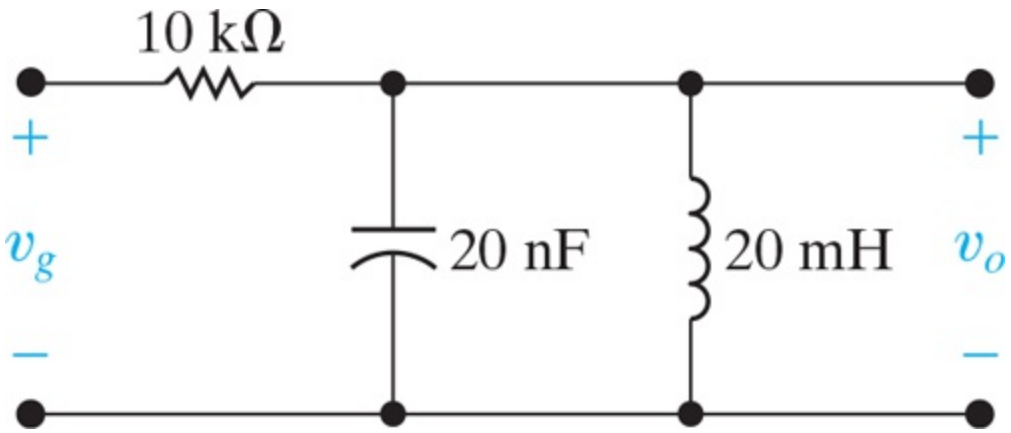
2. 16.6 The periodic square-wave shown on the top is applied to the circuit shown on the bottom.

    1. Derive the first four nonzero terms in the Fourier series that represents the steady-state voltage vo if Vm=210π V and the period of the input voltage is 0.2πms.

2. Which harmonic dominates the output voltage? Explain why.



16.5-6 Full Alternative Text



16.5-7 Full Alternative Text

Answer:

1. 17.5 cos (10,000t + 88.81°) + 26.14 cos (30,000t − 95.36°) + 168 cos (50,000t) + 17.32 cos (70,000t + 98.30°) + ··· V;

2. The fifth harmonic, at 10,000 rad/s, because the circuit is a bandpass filter with a center frequency of 50,000 rad/s and a quality factor of 10.

SELF-CHECK: Also try Problems 16.28 and 16.29.

16.6 Average-Power Calculations with Periodic Functions

Given the Fourier series representation of the voltage and current at a pair of terminals in a linear lumped-parameter circuit, we can express the average power at the terminals as a function of the harmonic voltages and currents. Using the trigonometric form of the Fourier series from Eq. 16.20, we find that the periodic voltage and current at the circuit's terminals are

$$v=V_{dc}+\sum_{n=1}^{\infty}V_n\cos(n\omega_0 t-\theta_{vn}),\quad i=I_{dc}+\sum_{n=1}^{\infty}I_n\cos(n\omega_0 t-\theta_{in}).$$

The notation used in the equations for voltage and current is defined as follows:

V_{dc}=the amplitude of the dc voltage component, V_n=the amplitude of thenth-harmonic voltage, θ_{vn}=the phase angle of thenth-harmonic voltage, I_{dc}=the amplitude of the dc current component, I_n=the amplitude of thenth-harmonic current, θ_{in}=the phase angle of thenth-harmonic current.

We assume that the current is in the direction of the voltage drop across the terminals, so, using the passive sign convention, the instantaneous power at the terminals is vi. The average power is

$$P=\frac{1}{T}\int_{t_0}^{t_0+T}p\,dt=\frac{1}{T}\int_{t_0}^{t_0+T}vi\,dt.$$

To find the expression for the average power, we substitute the expressions for voltage and current into the equation for average power and integrate. At first glance, this appears to be a formidable task because the product vi requires multiplying two infinite series. However, the only terms to survive integration are the products of voltage and current at the same frequency. (See Problem 16.5.) Therefore, the equation for average power reduces to

$$P=\frac{1}{T}V_{dc}I_{dc}t\Big|_{t_0}^{t_0+T}+\sum_{n=1}^{\infty}\frac{1}{T}\int_{t_0}^{t_0+T}V_nI_n\cos(n\omega_0 t-\theta_{vn})\cos(n\omega_0 t-\theta_{in})\,dt.$$

Now, using the trigonometric identity

$\cos\alpha\cos\beta = 12\cos(\alpha-\beta) + 12\cos(\alpha+\beta)$.

we simplify the expression for P to

$$P = V_{dc}I_{dc} + 1T\sum_{n=1}^{\infty} V_n I_n 2\int_{t0}^{t0+T}[\cos(\theta_{vn}-\theta_{in}) + \cos(2n\omega_0 t - \theta_{vn} - \theta_{in})]dt.$$

The second term under the integral sign integrates to zero, so

$$P = V_{dc}I_{dc} + \sum_{n=1}^{\infty} V_n I_n 2\cos(\theta_{vn}-\theta_{in}). \quad (16.30)$$

Equation 16.30 states that the total average power delivered by a periodic signal is the sum of the average powers associated with each harmonic voltage and current. Currents and voltages of different frequencies do not interact to produce average power. Example 16.5 computes the average power delivered by a periodic voltage.

Example 16.5 Calculating Average Power for a Circuit with a Periodic Voltage Source

Assume that the periodic square-wave voltage in Example 16.3 is applied across the terminals of a 15Ω resistor. The value of Vm is 60 V, and the value of T is 5 ms.

1. Write the first five nonzero terms of the Fourier series representation of v(t). Use the trigonometric form given in Eq. 16.20.
2. Calculate the average power associated with each term in (a).
3. Calculate the total average power delivered to the 15Ω resistor.
4. What percentage of the total power is delivered by the first five terms of the Fourier series?

Solution

1. The dc component of v(t) is

   av=(60)(T/4)T=15V.

   From Example 16.3 we have

   A1=2 60/π=27.01V,θ1=45°,A2=60/π=19.10V,θ2=90°,A3=202/π=9.00V,θ3=135°,A4=0,θ4=0°,A5=5.40V,θ5=45°,ω0=2πT=2π(1000)5=4

   Thus, using the first five nonzero terms of the Fourier series,

   v(t) = 15+27.01cos(400πt−45°) + 19.10cos(800πt−90°) + 9.00cos(1200πt−135°) + 5.40cos(2000πt−45°) + ··· V.

2. The voltage is applied to the terminals of a resistor, so we can find the power associated with each term as follows:

   Pdc=15215=15W,P1=12 27.01215=24.32W,P2=12 19.10215=12.16W,P3

3. To obtain the total average power delivered to the 15Ω resistor, we first calculate the rms value ofv(t):

   Vrms=(60)2(T/4)T=900=30V.

   The total average power delivered to the 15Ω resistor is

   PT=30215=60W.

4. The total power delivered by the first five nonzero terms is

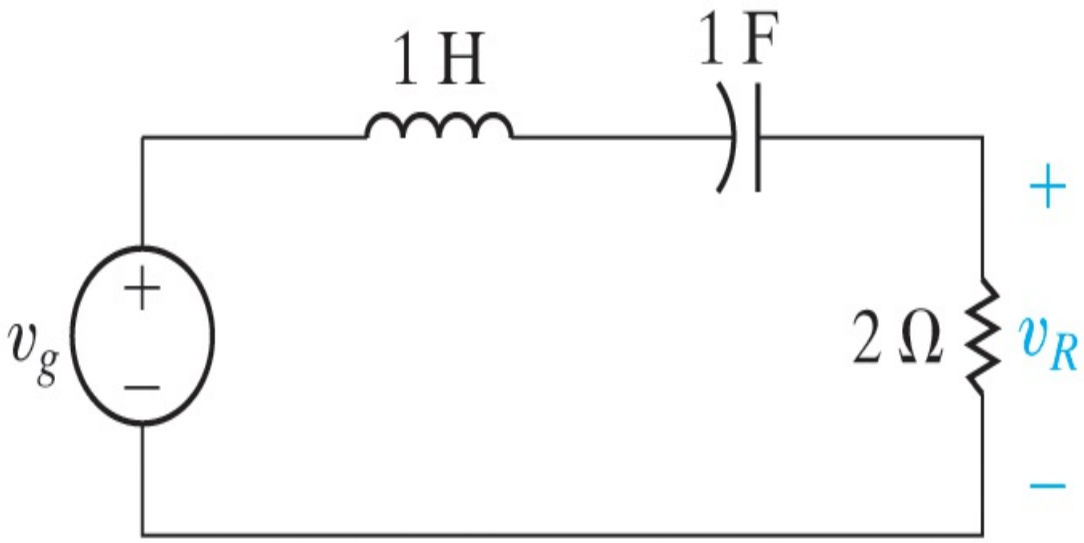
   P=Pdc+P1+P2+P3+P5=55.15W.

   This is (55.15/60) (100), or 91.92% of the total.

Assessment Problem

Objective 3—Be able to estimate the average power delivered to a resistor using a small number of Fourier coefficients

1. 16.7 The periodic voltage function in Assessment Problem 16.3 is applied to the circuit shown. If 12Vm=296.09 V and T=2094.4ms, estimate the average power delivered to the 2Ω resistor.



16.6-8 Full Alternative Text

Answer: 60.75 W.

SELF-CHECK: Also try ChapterProblems 16.34 and 16.35.

16.7 The rms Value of a Periodic Function

The rms value of a periodic function can be expressed in terms of the Fourier coefficients; by definition,

$$F_{rms}=\frac{1}{T}\int_{t_0}^{t_0+T} f(t)^2 dt.$$

Replacing f(t) with its Fourier series representation gives

$$F_{rms}=\frac{1}{T}\int_{t_0}^{t_0+T}\left[a_v+\sum_{n=1}^{\infty} A_n \cos(n\omega_0 t-\theta_n)\right]^2 dt.$$

The integral of the squared time function simplifies because the only terms whose integral over one period are not zero are the square of the dc term and the square of the terms at each frequency. Therefore,

$$F_{rms}=\frac{1}{T}\left(a_v^2 T+\sum_{n=1}^{\infty}\frac{T}{2}A_n^2\right)= a_v^2+\sum_{n=1}^{\infty}\frac{A_n^2}{2}= a_v^2+\sum_{n=1}^{\infty}\left(\frac{A_n}{\sqrt{2}}\right)^2. \quad (16.31)$$

We can use Eq. 16.31 to find the rms value of a periodic function by adding the square of the rms value of each harmonic to the square of the dc value and taking the square root of that sum. For example, let's assume that a periodic voltage is represented by the finite series

$$v = 10+30\cos(\omega_0 t-\theta_1)+20\cos(2\omega_0 t-\theta_2) + 5\cos(3\omega_0 t-\theta_3) + 2\cos(5\omega_0 t-\theta_5).$$

The rms value of this voltage is

$$V=\sqrt{10^2 + (30/\sqrt{2})^2 + (20/\sqrt{2})^2 + (5/\sqrt{2})^2 + (2/\sqrt{2})^2}=\sqrt{764.5}=27.65\text{V}.$$

Because the number of terms in a Fourier series representation of a periodic function is usually infinite, Eq. 16.31 yields an estimate of the actual rms value. We illustrate this result in Example 16.6.

Example 16.6 Estimating the rms Value of a Periodic Function

Use Eq. 16.31 to estimate the rms value of the voltage in Example 16.5.

Solution

From Example 16.5,

Vdc=15V,V1=27.01/2V, the rms value of the fundamental,V2=19.10/2V,

Therefore,

Vrms=152 + (27.012)2 + (19.102)2 + (9.002)2 + (5.402)2=28.76V.

From Example 16.5, the actual rms value is 30 V. We can get closer to this value by including more and more harmonics in Eq. 16.31. For example, if we include the harmonics through k=9, the estimated rms value is 29.32 V.

SELF-CHECK: Assess your understanding of this material by trying Chapter Problems 16.37 and 16.39.

16.8 The Exponential Form of the Fourier Series

The exponential form of the Fourier series is a concise representation of the series, given by

$$f(t)=\sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}, \quad (16.32)$$

where

$$C_n=\frac{1}{T}\int_{t_0}^{t_0+T} f(t)e^{-jn\omega_0 t}\,dt. \quad (16.33)$$

To derive Eqs. 16.32 and 16.33, start with Eq. 16.2 and replace the cosine and sine functions with their exponential equivalents:

$$\cos n\omega_0 t=\frac{e^{jn\omega_0 t}+e^{-jn\omega_0 t}}{2},\quad \sin n\omega_0 t=\frac{e^{jn\omega_0 t}-e^{-jn\omega_0 t}}{2j}.$$

Substituting the exponential equivalents into Eq. 16.2 gives

$$f(t)=a_v+\sum_{n=1}^{\infty}\frac{a_n}{2}\left(e^{jn\omega_0 t}+e^{-jn\omega_0 t}\right)+\frac{b_n}{2j}\left(e^{jn\omega_0 t}-e^{-jn\omega_0 t}\right)=a_v+\sum_{n=1}^{\infty}\left(\frac{a_n-jb_n}{2}\right)e^{jn\omega_0 t}+\left(\frac{a_n+jb_n}{2}\right)e^{-jn\omega_0 t}. \quad (16.34)$$

Now we define C_n as

$$C_n=\frac{1}{2}(a_n-jb_n)=\frac{A_n}{2}\angle -\theta_n,\quad n=1, 2, 3, \cdots.$$

From the definition of C_n,

$$C_n=\frac{1}{2}\left[\frac{2}{T}\int_{t_0}^{t_0+T} f(t)\cos n\omega_0 t\,dt-j\,\frac{2}{T}\int_{t_0}^{t_0+T} f(t)\sin n\omega_0 t\,dt\right]=\frac{1}{T}\int_{t_0}^{t_0+T} f(t)(\cos n\omega_0 t-j\sin n\omega_0 t)\,dt=\frac{1}{T}\int_{t_0}^{t_0+T} f(t)\,e^{-jn\omega_0 t}dt,$$

which completes the derivation of Eq. 16.33. To complete the derivation of Eq. 16.32, we first observe from Eq. 16.33 that

$$C_0=\frac{1}{T}\int_{t_0}^{t_0+T} f(t)dt=a_v.$$

Next we note that

$$C_{-n}=\frac{1}{T}\int_{t_0}^{t_0+T} f(t)e^{jn\omega_0 t}dt=C_n^*=\frac{1}{2}(a_n+jb_n).$$

Substituting the expressions for C0, Cn, and Cn* into Eq. 16.34 yields

$$f(t)=C_0+\sum_{n=1}^{\infty}(C_n e^{jn\omega_0 t}+C_n^* e^{-jn\omega_0 t})=\sum_{n=0}^{\infty}C_n e^{jn\omega_0 t}+\sum_{n=1}^{\infty}C_n^* e^{-jn\omega_0 t}.$$

Note that the second summation on the right-hand side is equivalent to summing $C_n e^{jn\omega_0 t}$ from −1 to −∞; that is,

$$\sum_{n=1}^{\infty}C_n^* e^{-jn\omega_0 t}=\sum_{n=-1}^{-\infty}C_n e^{jn\omega_0 t}.$$

Because the summation from −1 to −∞ is the same as the summation from −∞ to −1, we can rewrite f(t) as

$$f(t)=\sum_{n=0}^{\infty}C_n e^{jn\omega_0 t}+\sum_{-\infty}^{-1}C_n e^{jn\omega_0 t}=\sum_{-\infty}^{\infty}C_n e^{jn\omega_0 t},$$

which completes the derivation of Eq. 16.32.

We can also find the rms value of a periodic function using the complex Fourier coefficients. From Eqs. 16.21 and 16.31,

$$F_{rms}=\sqrt{\frac{a_v}{2}+\sum_{n=1}^{\infty}\frac{a_n^2+b_n^2}{2}},\quad |C_n|=\sqrt{\frac{a_n^2+b_n^2}{2}},\quad C_0^2=a_v^2.$$

Therefore:

$$F_{rms}=\sqrt{C_0^2+2\sum_{n=1}^{\infty}|C_n|^2}. \quad (16.35)$$

Example 16.7 finds the exponential Fourier series representation of a periodic function.

Example 16.7 Finding the Exponential Form of the Fourier

Series

Find the exponential Fourier series for the periodic voltage shown in Fig. 16.16.

Figure 16.16 The periodic voltage for Example 16.7.

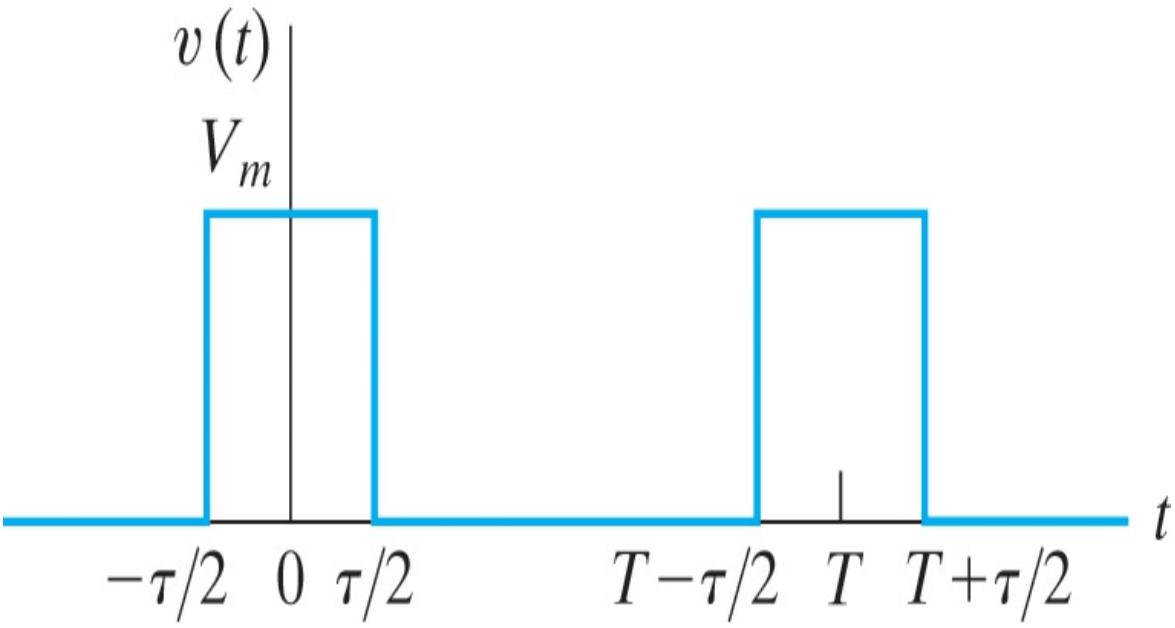


Figure 16.16 Full Alternative Text

Solution

Using −τ/2 as the starting point for the integration, we have, from Eq. 16.33,

$$C_n = \frac{1}{T}\int_{-\tau/2}^{\tau/2} V_m e^{-jn\omega_0 t}\,dt = \frac{V_m}{T}\left(\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\right)\Big|_{-\tau/2}^{\tau/2} = \frac{jV_m}{n\omega_0 T}\left(e^{-jn\omega_0\tau/2} - e^{jn\omega_0\tau/2}\right) = \frac{2V_m}{n\omega_0 T}\sin\frac{n\omega_0\tau}{2}.$$

Since v(t) has even symmetry, bn=0 for all n, and we expect Cn to be real. Moreover, the amplitude of Cn follows a (sinx)/x distribution, as indicated when we rewrite

$$C_n = \frac{V_m\tau}{T}\frac{\sin(n\omega_0\tau/2)}{n\omega_0\tau/2}.$$

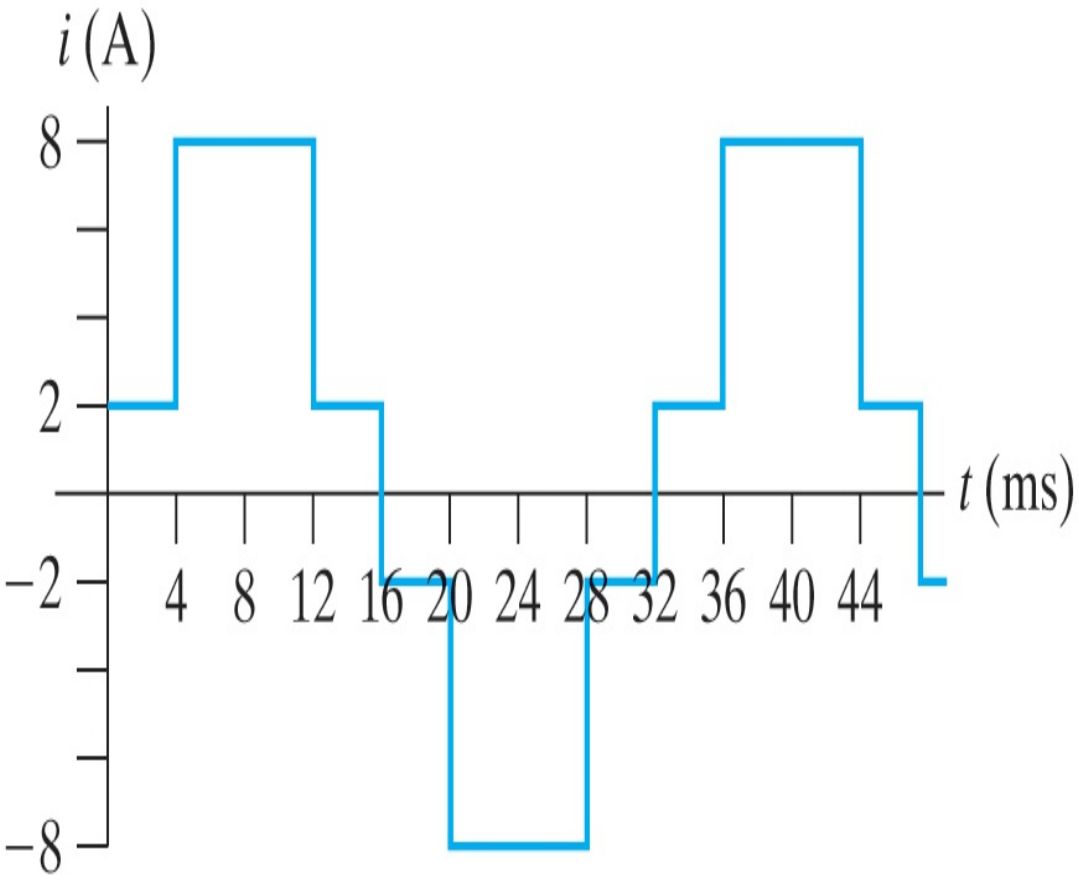
We say more about this subject in Section 16.9. The exponential Fourier series representation of v(t) is

$$v(t) = \sum_{n=-\infty}^{\infty}\left(\frac{V_m\tau}{T}\right)\frac{\sin(n\omega_0\tau/2)}{n\omega_0\tau/2}e^{jn\omega_0 t} = \left(\frac{V_m\tau}{T}\right)\sum_{n=-\infty}^{\infty}\frac{\sin(n\omega_0\tau/2)}{n\omega_0\tau/2}e^{jn\omega_0 t}.$$

Assessment Problems

Objective 4—Be able to calculate the exponential form of the Fourier coefficients for a periodic waveform

1. 16.8 Derive the expression for the Fourier coefficients Cn for the periodic function shown. Hint: Take advantage of symmetry by using the fact that $C_n = (a_n - jb_n)/2$.



16.8-9 Full Alternative Text

Answer:

Cn=−j4πn(1 + 3cosnπ4), n odd

2. 16.9

    1. a) Calculate the rms value of the periodic current in Assessment Problem 16.8.

    2. b) Using C1 − C11, estimate the rms value.

    3. c) What is the percentage of error in the value obtained in (b), based on the true value found in (a)?

    4. d) For this periodic function, could fewer terms be used to estimate the rms value and still insure the error is less than 1%?

Answer:

1. (a) 34A;
2. (b) 5.777 A;
3. (c) −0.93 %;
4. (d) yes; if C1 − C9 are used, the error is −0.98 %.

SELF-CHECK: *Also try Chapter Problems 16.44 and 16.45*.

16.9 Amplitude and Phase Spectra

As we have noted, we generally cannot visualize what a periodic function looks like in the time domain from its Fourier series representation. Nevertheless, we recognize that the Fourier coefficients characterize the periodic function completely.

We can describe a periodic function by plotting the amplitude and phase angle of each term in its Fourier series versus frequency. The plot of each term's amplitude versus the frequency is called the **amplitude spectrum** of f(t), and the plot of each term's phase angle versus the frequency is called the **phase spectrum** of f(t). Because the amplitude and phase angle are defined at discrete frequency values (that is, at $\omega 0$, $2\omega 0$, $3\omega 0$, …), these plots are also called **line spectra**.

Amplitude and phase spectra plots are based on either Eq. 16.20 (An and $-\theta n$) or Eq. 16.32 (Cn). We focus on Eq. 16.32 and leave the plots based on Eq. 16.20 to the problems. For example, consider the periodic voltage in Fig. 16.16. From Example 16.6,

$$Cn=VmτTsin\ (nω0τ/2)nω0τ/2,$$

From the expression for Cn, we see that the amplitude spectrum is bounded by the envelope of the |(sinx)/x| function. Figure 16.17 provides the plot of |(sinx)/x| versus x, where x is in radians. It shows that the function's value is zero whenever x is an integral multiple of π. Replacing $\omega 0$ with $2\pi/T$ in the argument of the sine function in the expression for Cn,

Figure 16.17 The plot of (sin x)/x versus x.

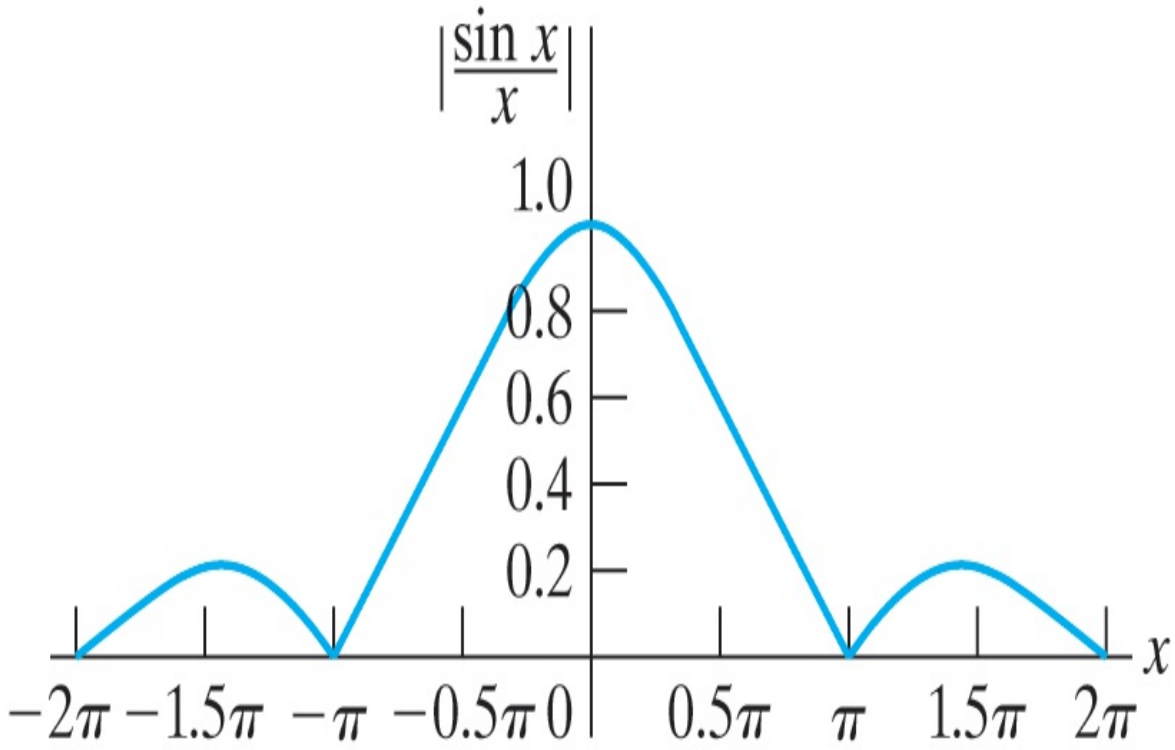


Figure 16.17 Full Alternative Text

nω0(τ2)=nπ τT=nπT/τ.

We can therefore deduce that the amplitude spectrum goes through zero whenever nτ/T is an integer. As the reciprocal of τ/T becomes an increasingly larger integer, the number of harmonics between every π radians increases. If nπ/T is not an integer, the amplitude spectrum still follows the |(sinx)/x| envelope. However, the envelope is not zero at an integral multiple of ω0. Because Cn is real for all n, the phase angle associated with Cn is either zero or 180°, depending on its algebraic sign.

Now, what happens to the amplitude and phase spectra if f(t) is shifted along the time axis? To find out, we shift the periodic voltage in Example 16.6 t0 units to the right. By hypothesis,

v(t)=∑n=−∞∞Cnejnω0t.

Therefore

v(t−t0)=∑n=−∞∞Cnejnω0(t−t0)=∑n=−∞Cne−jnω0t0ejnω0t,

which indicates that shifting the origin has no effect on the amplitude spectrum, because

$|C_n| = |C_n e^{-jn\omega_0 t_0}|$.

However, the phase spectrum has changed to $-(\theta_n + n\omega_0 t_0)$ rads.

Example 16.8 plots the amplitude and phase spectra for a specific instance of the periodic voltage in Example 16.7.

Example 16.8 Plotting the Amplitude and Phase Spectra for a Periodic Voltage

Suppose that for the periodic voltage in Fig. 16.16, $V_m = 5$ V, $T = 10$ ms, and $\tau = T/5$.

1. Plot the amplitude and phase spectra versus frequency (in Hz) for $-10 \le n \le +10$, using the expressions for C_n given by Eq. 16.32.

2. Repeat part (a) if the periodic voltage in Fig. 16.16 is shifted $\tau/2$ units to the right.

Solution

a)

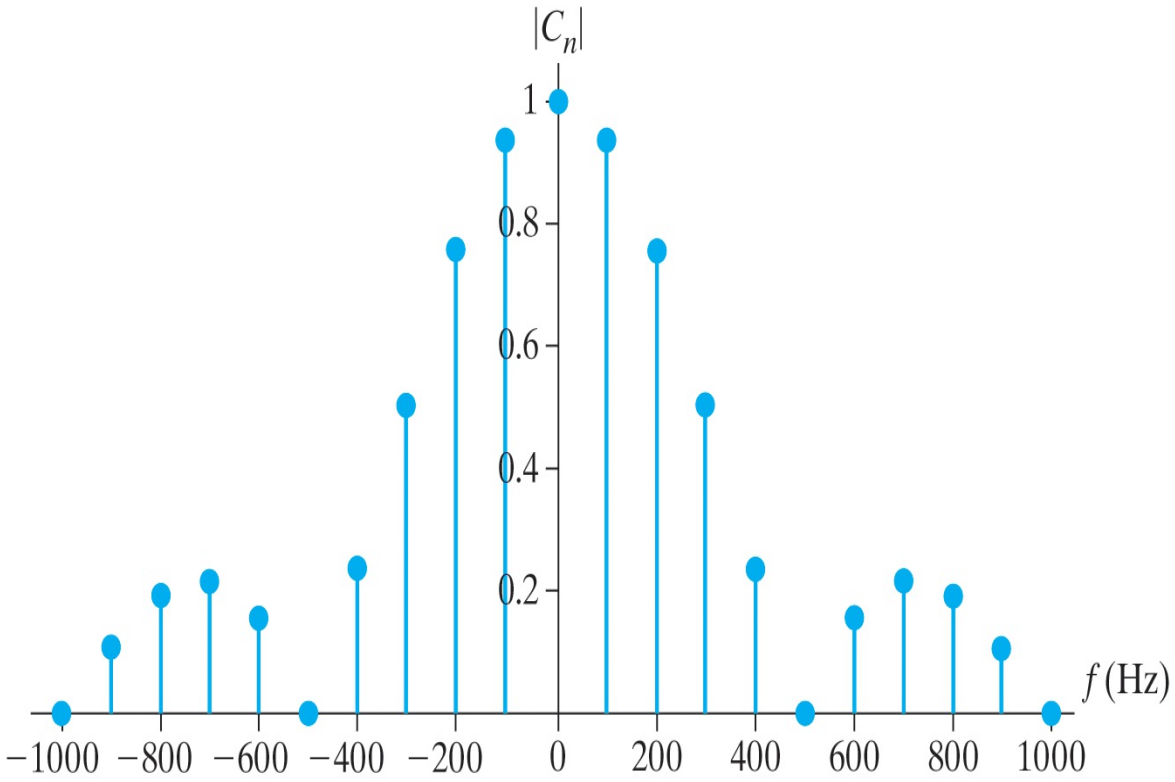
Substituting the values for V_m and t into the expression for C_n from Example 16.7 and simplifying, we get

$C_n = \sin(n\pi/5)n\pi/5$.

The values of C_n for n between –10 and +10 are tabulated at right. Since C_n

is a real number for all values of n, its magnitude is the absolute value of Cn and its phase angle is 0° if Cn is positive and 180° if Cn is negative. The amplitude and phase spectra are plotted in Fig. 16.18.

Figure 16.18 The amplitude and phase spectra for the periodic waveform in Fig. 16.16, with Vm=5 V, T=10 ms and τ=T/5


$|C_n|$
1
0.8
0.6
0.4
0.2
f (Hz)
−1000 −800 −600 −400 −200 0 200 400 600 800 1000

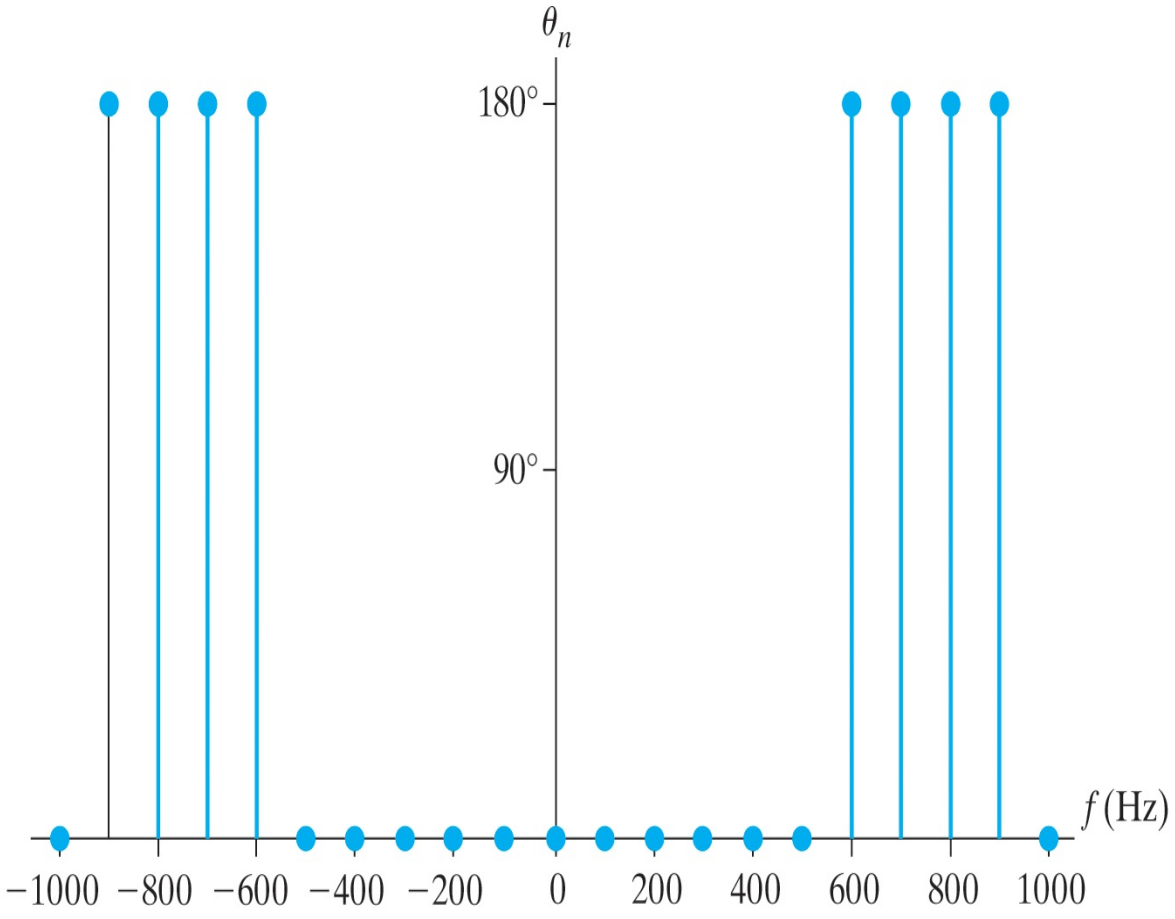

θ_n
180°
90°
f (Hz)
−1000 −800 −600 −400 −200 0 200 400 600 800 1000

Figure 16.18 Full Alternative Text

n	C_n	n	C_n
−10	0.000	1	0.935
−9	−0.104	2	0.757
−8	−0.189	3	0.505
−7	−0.216	4	0.234
−6	−0.156	5	0.000
−5	0.000	6	−0.156
−4	0.234	7	−0.216
−3	0.505	8	−0.189
−2	0.757	9	−0.104
−1	0.935	10	0.000
0	1.000		

b) When the periodic voltage in Fig. 16.16 is shifted to the right by τ/2, we know that the amplitude spectrum is unchanged. Since τ=T/5, the new phase

angle $\theta'n$ is

$\theta'n=-(\theta n+n\pi/5)$.

The plot of the amplitude spectra for this shifted periodic voltage is the same as the one shown in Fig. 16.18, while the plot of the phase spectra for the shifted period voltage is shown in Fig. 16.19, for $-10 \leq n \leq +10$.

Figure 16.19 The phase spectra for the periodic waveform in Fig. 16.16 , shifted to the right by τ/2, with Vm=5 V, T=10 ms and τ=T/5

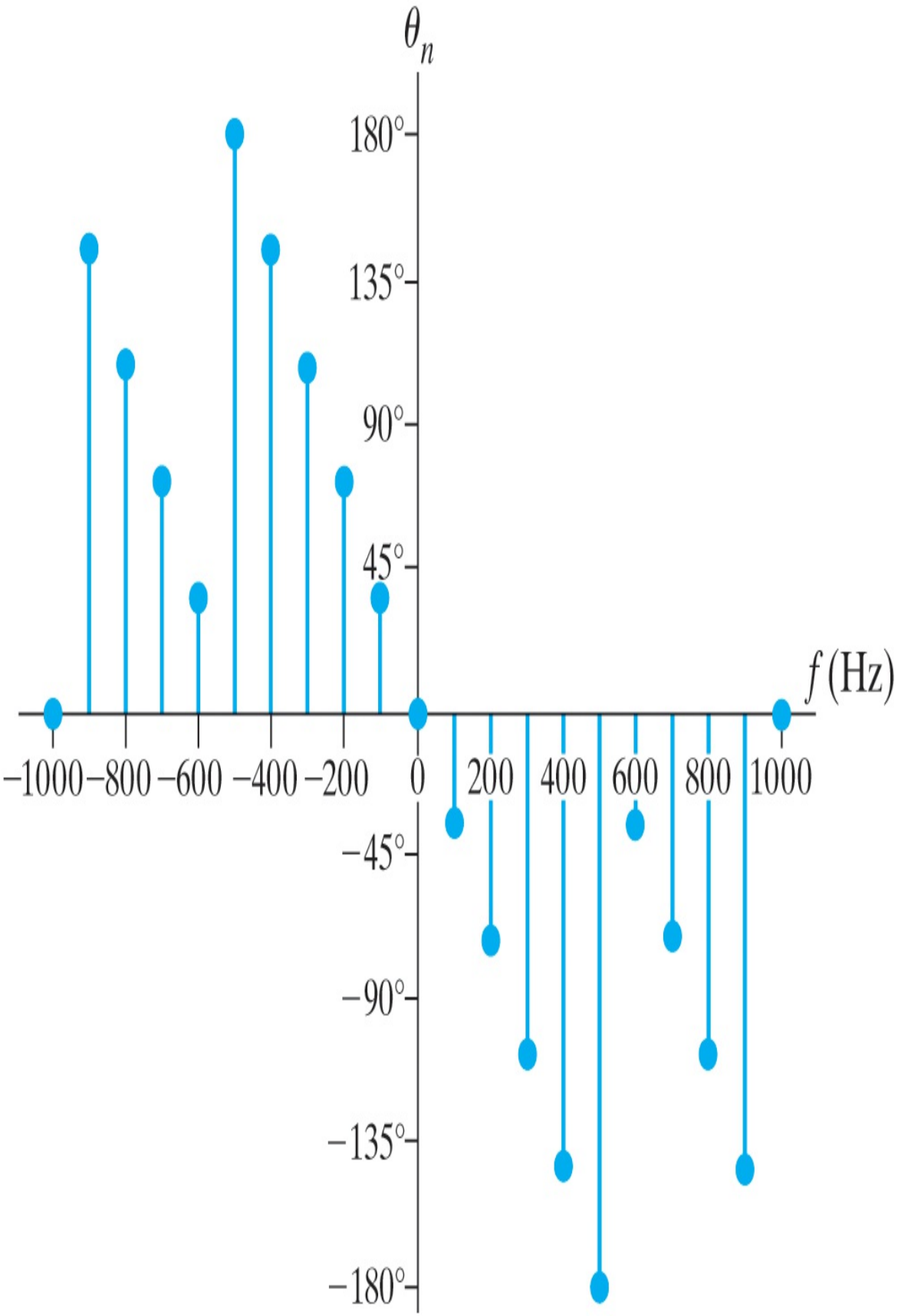

θ_n
180°
135°
90°
45°
−45°
−90°
−135°
−180°
f (Hz)
−1000
−800
−600
−400
−200
0
200
400
600
800
1000

Figure 16.19 Full Alternative Text

The periodic waveform in Fig. 16.16 is important because it provides an excellent way to illustrate the transition from the Fourier series representation of a periodic function to the Fourier transform representation of a nonperiodic function. We discuss the Fourier transform in Chapter 17.

Assessment Problem

Objective 4—Be able to calculate the exponential form of the Fourier coefficients for a periodic waveform

1. 16.10 The function in Assessment Problem 16.8 is shifted along the time axis 8 ms to the right. Write the exponential Fourier series for the periodic current.

   Answer:

   $i(t)=4\pi\sum n=-\infty(\text{odd})\infty 1n(1+3\cos n\pi 4)e-(j\pi/2)(n+1)ejn\omega 0t$ A.

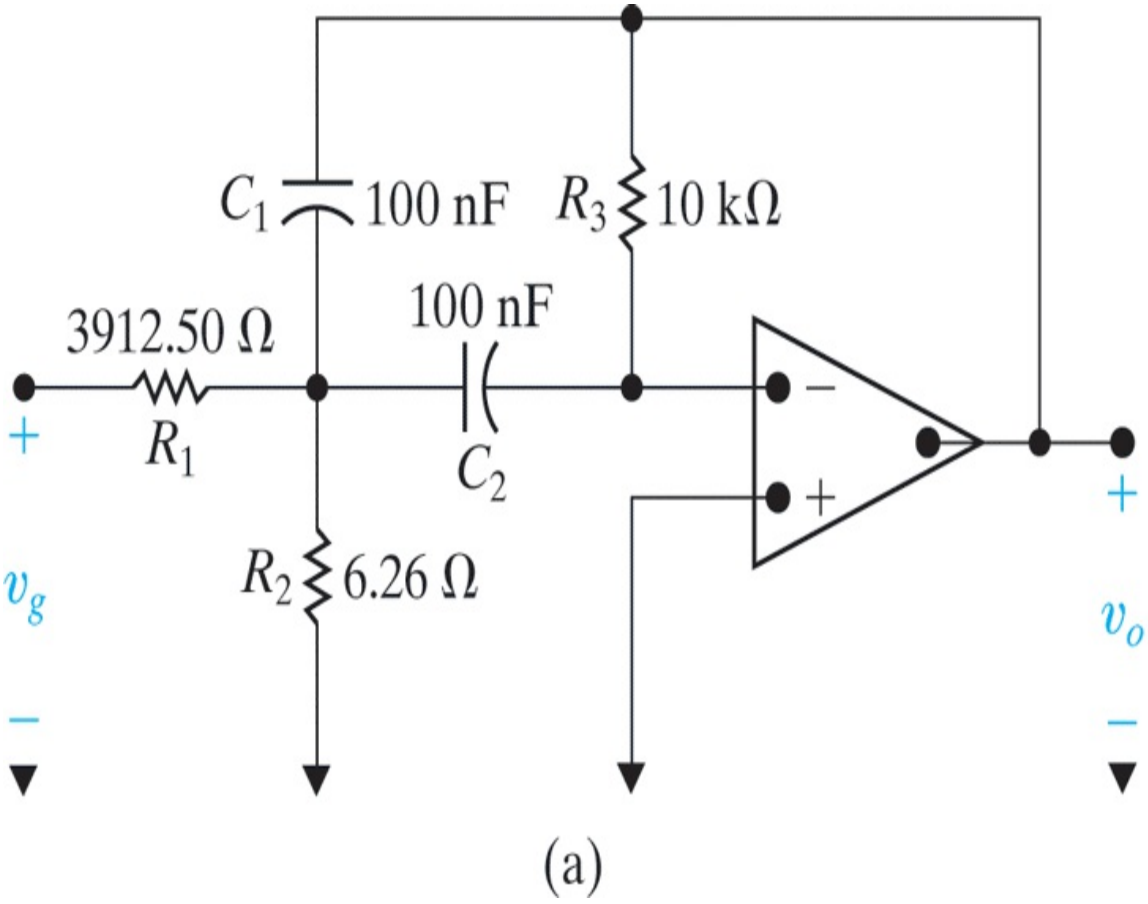
SELF-CHECK: *Also try Chapter Problems 16.49 and 16.50.*

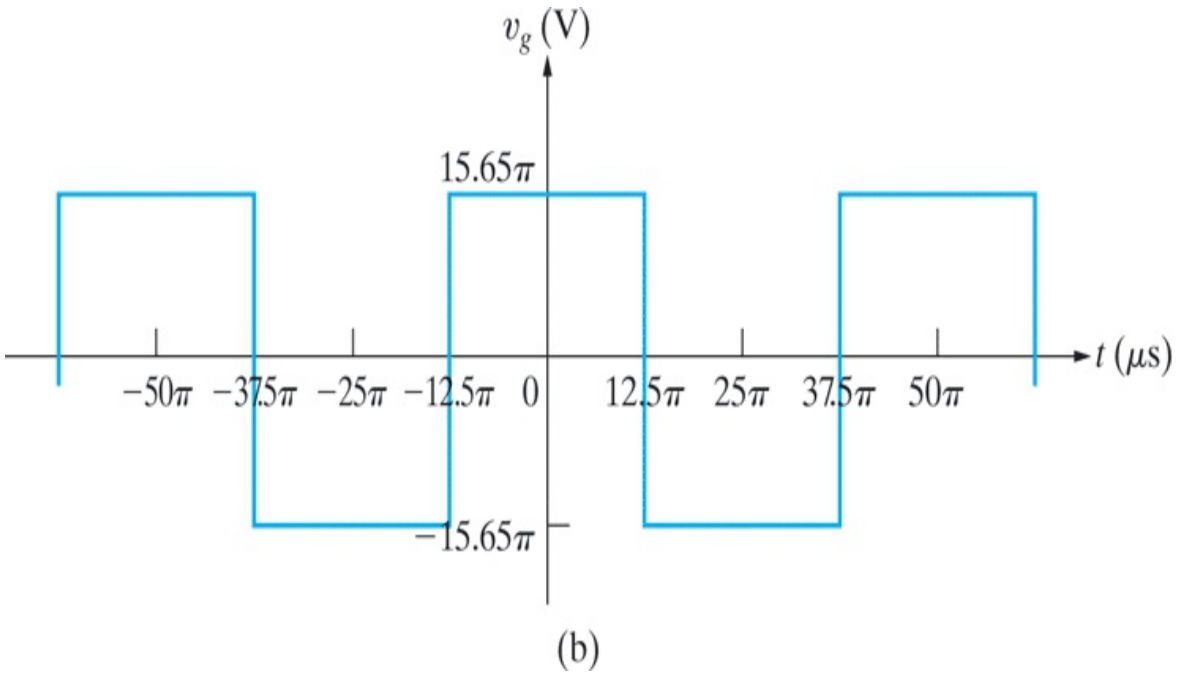
Practical Perspective

Active High-Q Filters

Consider the narrowband op amp bandpass filter shown in Fig. 16.20(a). The square-wave voltage shown in Fig. 16.20(b) is the input to the filter. We know that the square wave is composed of an infinite sum of sinusoids, one sinusoid at the same frequency as the square wave and all of the remaining sinusoids at integer multiples of that frequency. What effect will the filter have on this input sum of sinusoids?

Figure 16.20 (a) narrowband bandpass filter; (b) square wave input.





(b)

Figure 16.20 Full Alternative Text

The Fourier series representation of the square wave in Fig. 16.20(b) is given by

$$v_g(t)=\frac{4A}{\pi}\sum_{n=1,3,5,\ldots}^{\infty}\frac{1}{n}\sin\frac{n\pi}{2}\cos n\omega_0 t$$

where $A=15.65\pi$V. The first three terms of this Fourier series are given by

$$v_g(t)=62.6\cos\omega_0 t-20.87\cos 3\omega_0 t+12.52\cos 5\omega_0 t-\ldots V.$$

The period of the square wave is 50π μs, so the frequency of the square wave is 40,000 rad/s.

The transfer function for the bandpass filter in Fig. 16.20(a) is

$$H(s)=\frac{K\beta s}{s^2+\beta s+\omega_0^2}$$

where $K=400/313$, $\beta=2000$ rad/s, $\omega_0=40{,}000$ rad/s. This filter has a quality factor of $40{,}000/2000=20$. Note that the center frequency of the bandpass filter equals the frequency of the input square wave.

Multiply each term of the Fourier series representation of the square wave,

represented as a phasor, by the transfer function H(s) evaluated at the frequency of the term in the Fourier series to get the representation of the output voltage of the filter as a Fourier series:

$v_o(t) = -80\cos\omega_0 t - 0.5\cos 3\omega_0 t + 0.17\cos 5\omega_0 t + \cdots V.$

Notice the selective nature of the bandpass filter, which effectively amplifies the fundamental frequency component of the input square wave and attenuates all of the harmonic components.

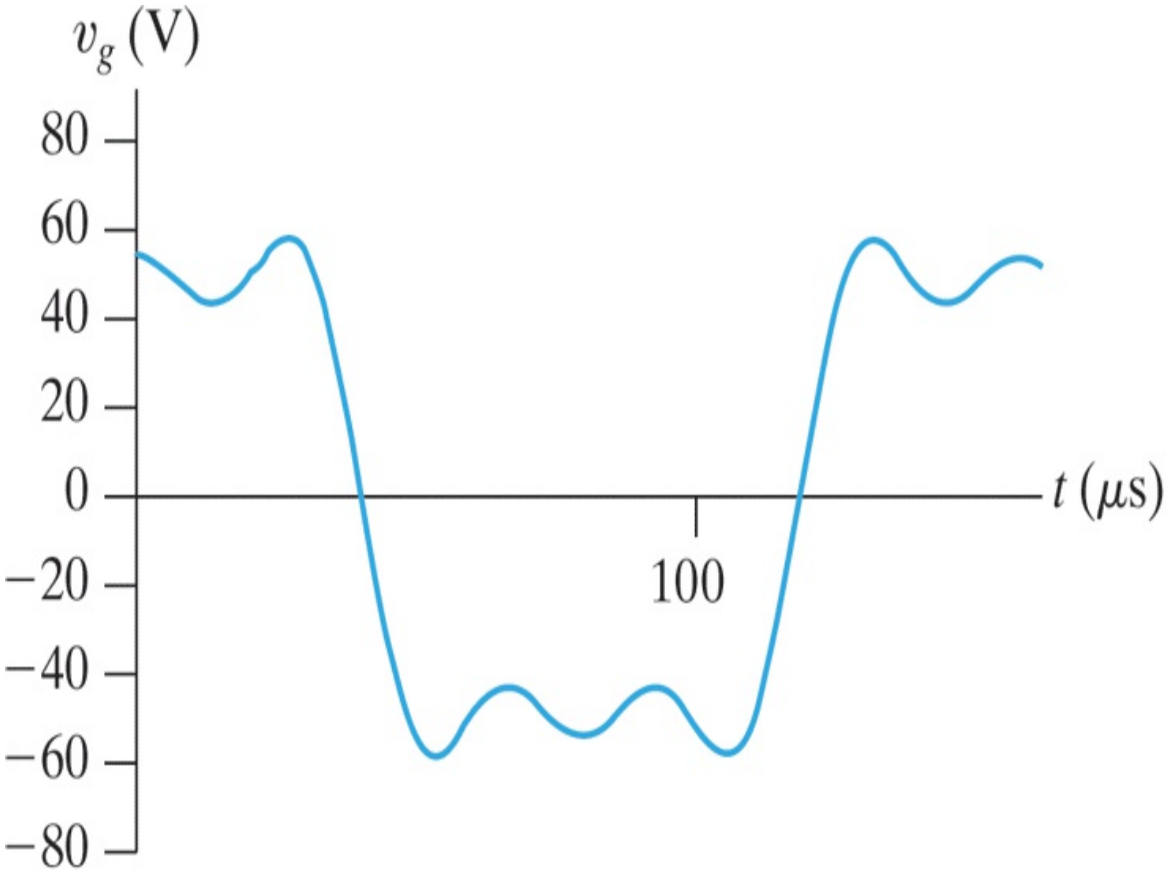
Now make the following changes to the bandpass filter of Fig. 16.20(a)—let $R1 = 391.25\Omega$, $R2 = 74.4\Omega$, $R3 = 1k\Omega$, and $C1 = C2 = 0.1\mu F$. The transfer function for the filter, H(s), has the same form given above, but now $K = 400/313$, $\beta = 20{,}000$ rad/s, $\omega_0 = 40{,}000$ rad/s. The passband gain and center frequency are unchanged, but the bandwidth has increased by a factor of 10. This makes the quality factor 2, and the resulting bandpass filter is less selective than the original filter. We can see this by looking at the output voltage of the filter as a Fourier series:

$v_o(t) = -80\cos\omega_0 t - 5\cos 3\omega_0 t + 1.63\cos 5\omega_0 t + \cdots V.$

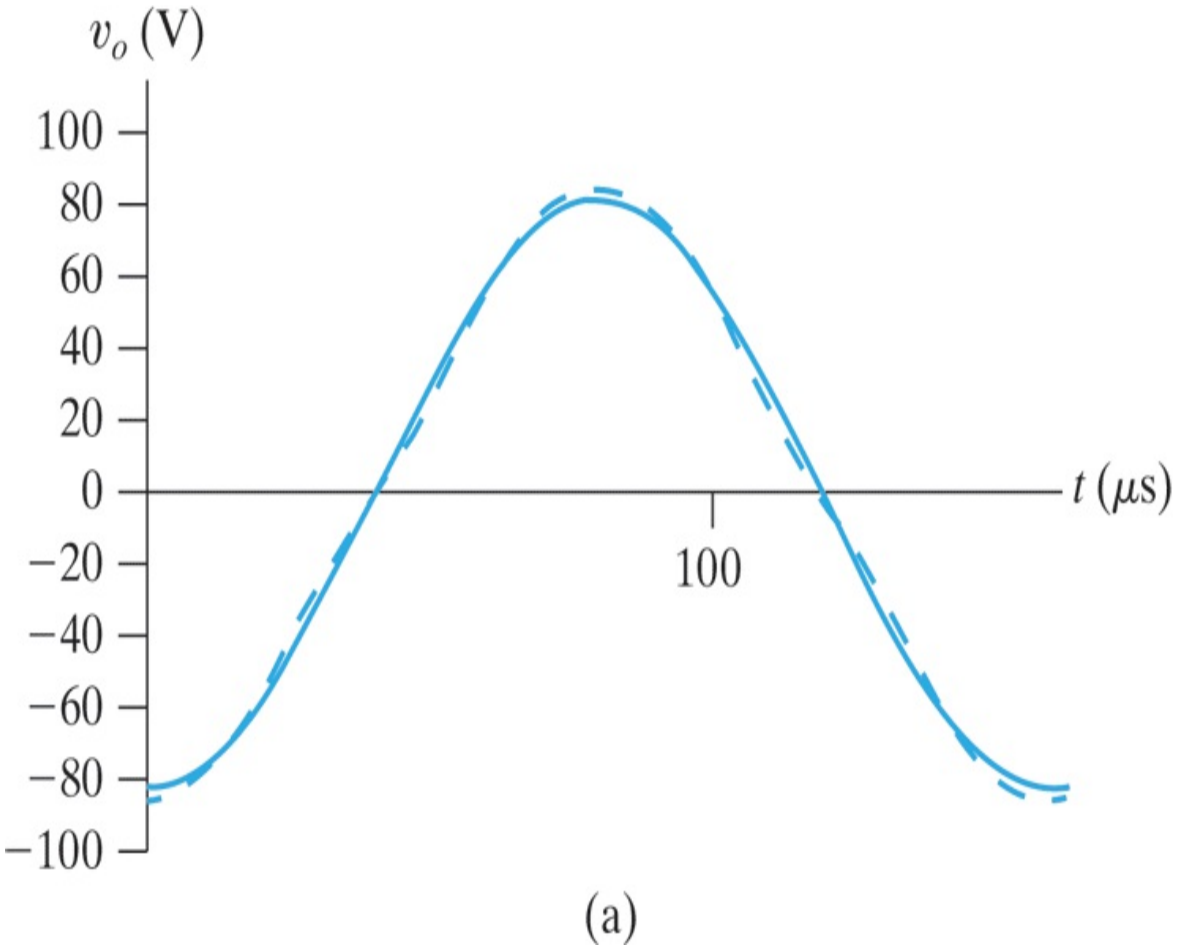
At the fundamental frequency, the output has the same amplification factor, but the higher harmonic components have not been attenuated as significantly as they were when the filter with Q=20 was used. Figure 16.21 plots the first three terms of the Fourier series representations of the input square wave and the resulting output waveforms for the two bandpass filters. Note the nearly perfect replication of a sinusoid in the solid line plot of Fig. 16.21(b) and the distortion that results when using a less selective filter in the dashed line plot of Fig. 16.21(b).

Figure 16.21 (a) The first three terms of the Fourier series of the square wave in Fig.

16.20(b); (b) the first three terms of the Fourier series of the output from the bandpass filter in Fig. 16.20(a), where (solid line); the first three terms of the Fourier series of the output from the bandpass filter in Fig. 16.20(a) with component values changed to give



(a)



(dashed line).

Figure 16.21 Full Alternative Text

SELF-CHECK: *Assess your understanding of the Practical Perspective by solving Chapter Problems 16.56 and 16.57.*

Summary

- A **periodic function** is a function that repeats itself every T seconds. A period is the smallest time interval (T) that a periodic function can be shifted to produce a function identical to itself. (See page 618.)

- The Fourier series is an infinite series used to represent a periodic function. The series consists of a constant term and infinitely many harmonically related cosine and sine terms. (See page 621.)

- The **fundamental frequency** is the frequency determined by the fundamental period ($f0=1/T$ or $\omega0=2\pi f0$). (See page 621.)

- The **harmonic frequency** is an integer multiple of the fundamental frequency. (See page 621.)

- The **Fourier coefficients** are the constant term and the coefficient of each cosine and sine term in the series. (See Eqs. 16.3–16.5.) (See page 622.)

- Five types of symmetry are used to simplify the computation of the Fourier coefficients:

    - *even,* in which all sine terms in the series are zero;

    - *odd,* in which all cosine terms and the constant term are zero;

    - *half-wave,* in which all even harmonics are zero;

    - *quarter-wave, half-wave, even,* in which the series contains only odd harmonic cosine terms;

    - *quarter-wave, half-wave, odd,* in which the series contains only odd harmonic sine terms.

  (See page 625.)

- In the alternative form of the Fourier series, each harmonic represented by the sum of a cosine and sine term is combined into a single term of the form $A_n \cos(n\omega_0 t - \theta_n)$. (See page 631.)

- For steady-state response, the Fourier series of the output signal is determined by first finding the output for each component of the input signal. The individual responses are added (superimposed) to form the Fourier series of the output signal. The response to the individual terms in the input series is found by either frequency domain or s-domain analysis. (See page 633.)

- The waveform of the output signal is difficult to obtain without the aid of a computer. Sometimes the frequency response (or filtering) characteristics of the circuit can be used to ascertain how closely the output waveform matches the input waveform. (See page 635.)

- Only harmonics of the same frequency interact to produce average power. The total average power is the sum of the average powers associated with each frequency. (See page 640.)

- The rms value of a periodic function can be estimated from the Fourier coefficients. (See Eqs. 16.31 and 16.35.) (See page 641.)

- The Fourier series of a periodic signal may also be written in exponential form by using Euler's identity to replace the cosine and sine terms with their exponential equivalents. (See page 642.)

- An amplitude spectrum plots the amplitudes of a function's Fourier series representation versus discrete frequencies. A phase spectrum plots the phase angles of a function's Fourier series representation versus discrete frequencies. These plots help to visualize the transformation of a circuit's input signal to its output signal. (See page 645.)

Problems

Sections 16.1–16.2

1. 16.1 For each of the periodic functions in Fig. P16.1, specify
    1. ωo in radians per second;
    2. fo in hertz;
    3. the value of av;
    4. the equations for ak and bk;
    5. v(t) as a Fourier series.

Figure P16.1

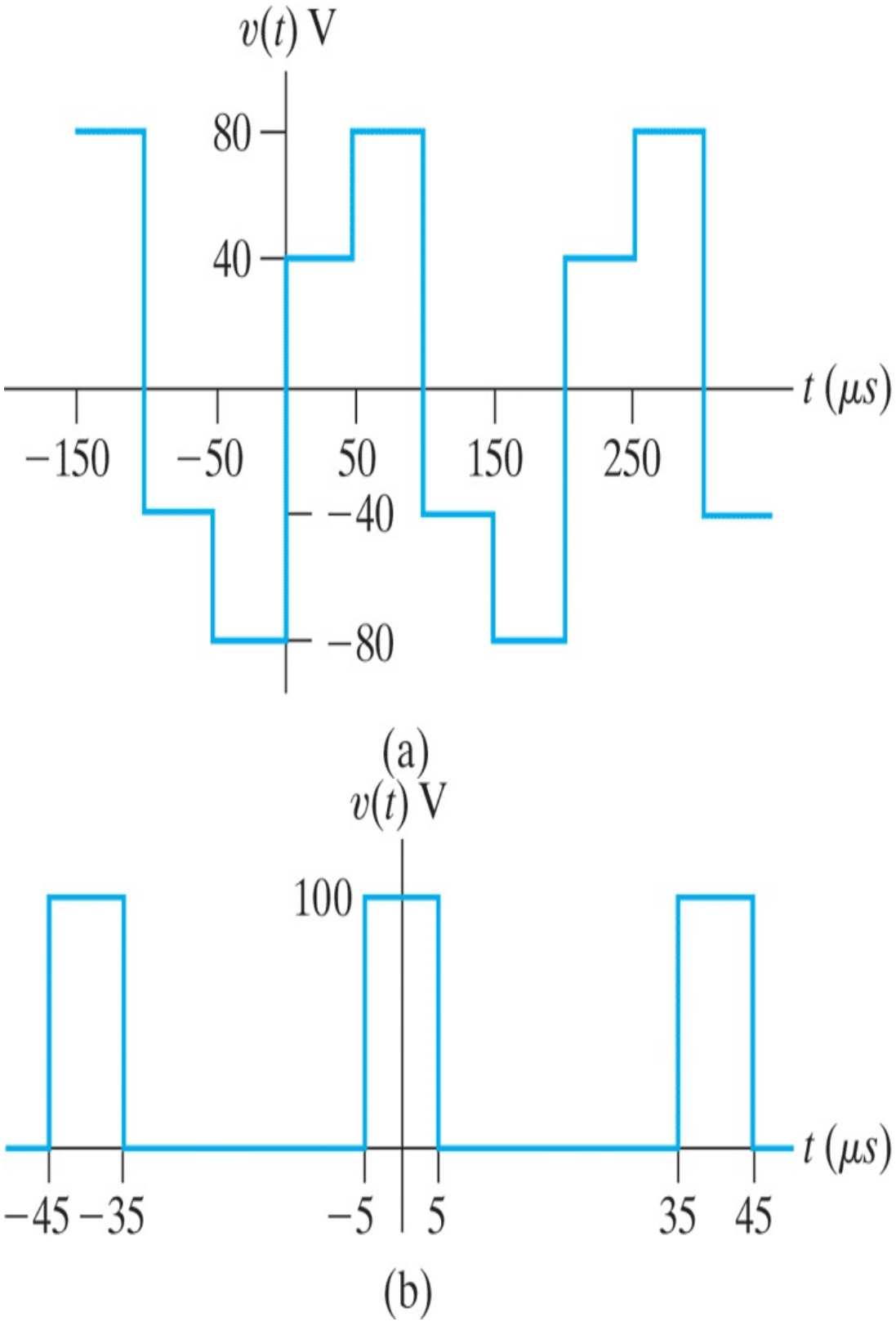


Figure P16.1 Full Alternative Text

2. 16.2 Derive the Fourier series for the periodic voltage shown in Fig. P16.2, given that

   $v(t) = 100\sin 2\pi T\ t\ V, 0 \le t \le T\ 2$; $v(t) = 60\sin 2\pi T\ (t - T/2)\ V, T\ 2 \le t \le T\ 2$.

Figure P16.2

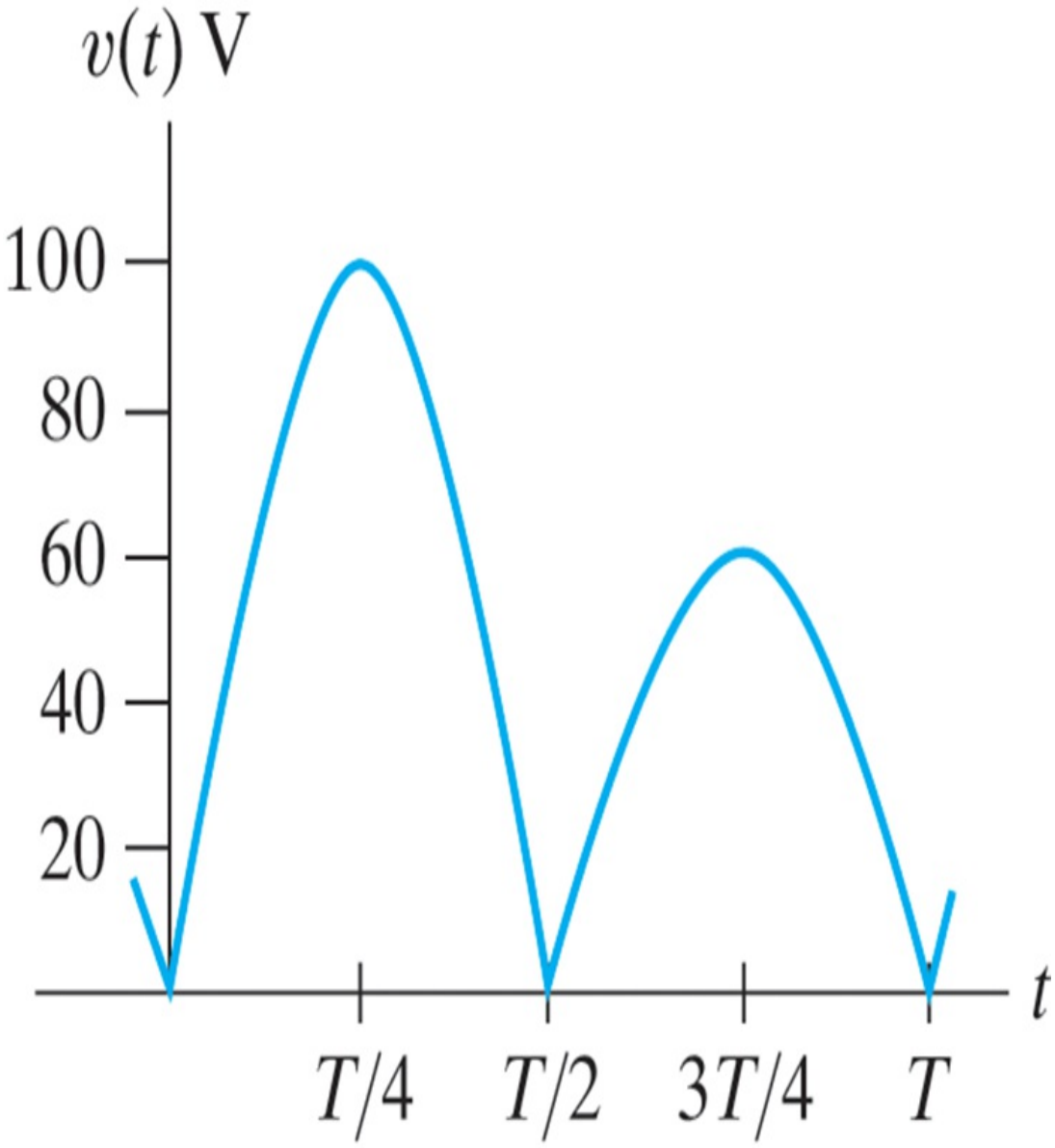
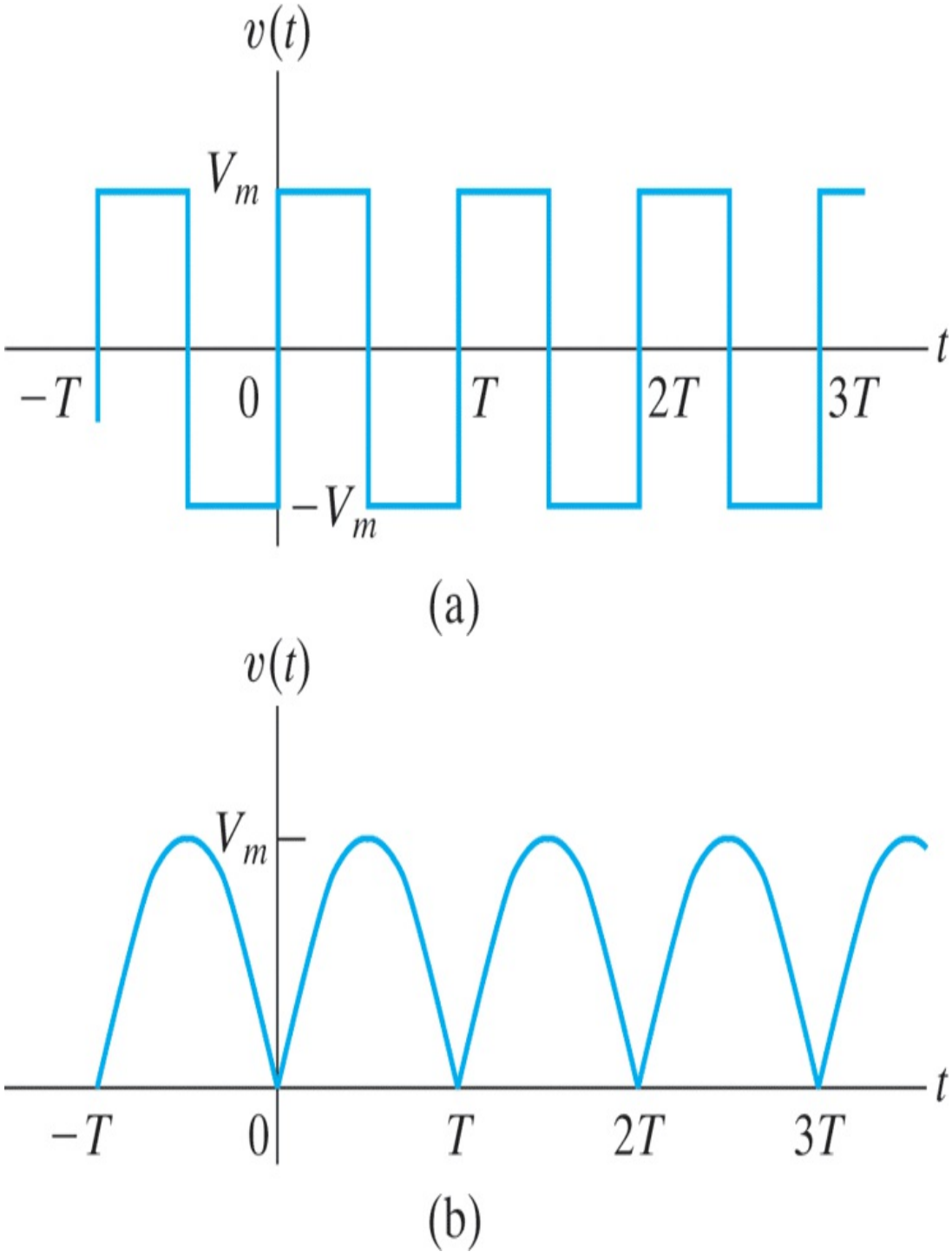


Figure P16.2 Full Alternative Text

3. 16.3 Find the Fourier series expressions for the periodic voltage functions shown in Fig. P16.3. Note that Fig. P16.3(a) illustrates the square wave; Fig. P16.3(b) illustrates the full-wave rectified sine wave, where $v(t)=V\sin(\pi/T)t$, $0 \le t \le T$; and Fig. P16.3(c) illustrates the half-

wave rectified sine wave, where $v(t)=V_m \sin(2\pi/T)t$, $0 \le t \le T/2$.

Figure P16.3



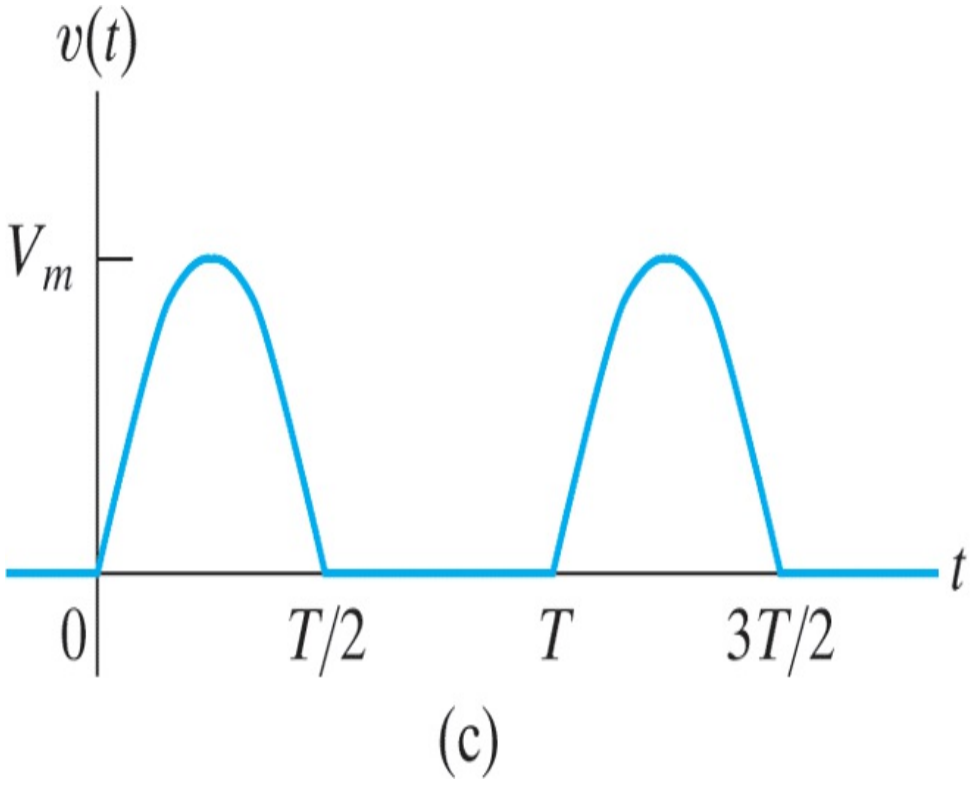


Figure P16.3 Full Alternative Text

4. 16.4 Derive the expressions for av, ak, and bk for the periodic voltage shown in Fig. P16.4 if Vm=100π V.

Figure P16.4

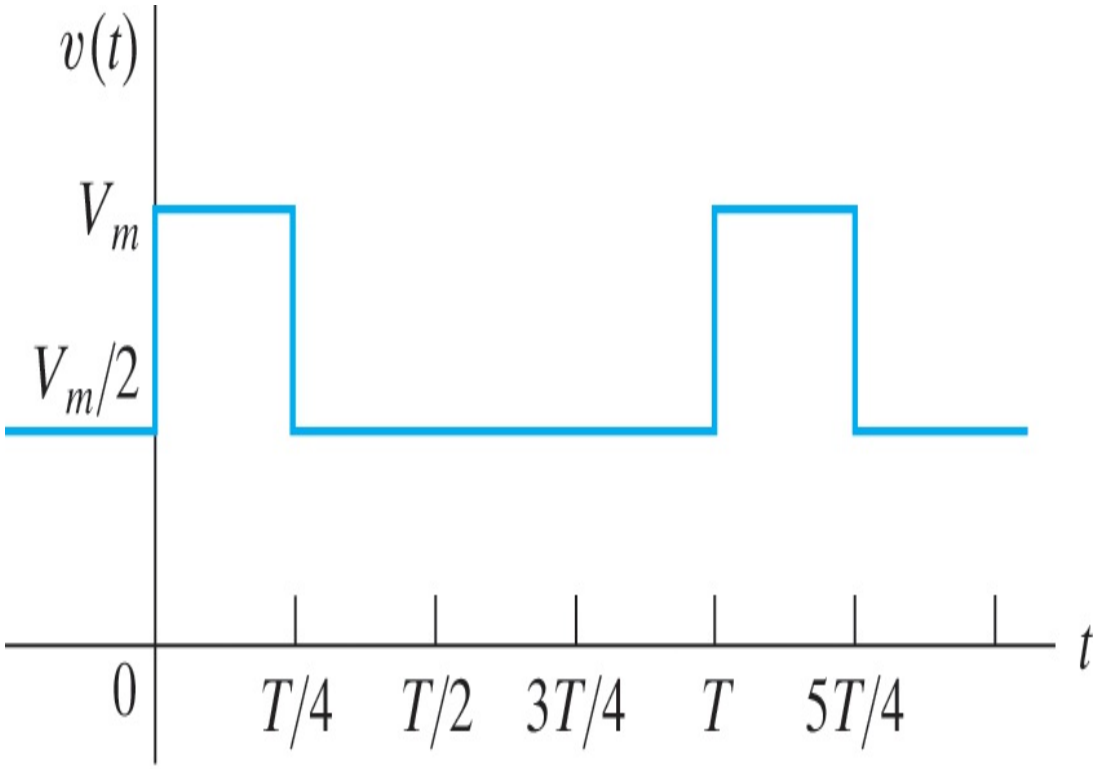


Figure P16.4 Full Alternative Text

5. 16.5

    1. a) Verify the following equations:

        ∫t0t0+Tsin mω0t dt=0,for all m,
        ∫t0t0+Tcos mω0t dt=0,for all m.

    2. b) Verify the following equation:

        ∫t0t0+Tcos mω0t sin nω0t dt=0,for all m and n.

        Hint: Use the trigonometric identity cosαsinβ=12sin(α+β)−12sin(α−β).

    3. c) Verify the following equation:

$$\int_{t_0}^{t_0+T} \sin m\omega_0 t \sin n\omega_0 t\, dt = 0, \text{for all } m \neq n, = \frac{T}{2}, \text{for } m = n.$$

Hint: Use the trigonometric identity $\sin\alpha\sin\beta = \frac{1}{2}\cos(\alpha-\beta) - \frac{1}{2}\cos(\alpha+\beta)$.

4. d) Verify the following equation:

$$\int_{t_0}^{t_0+T} \cos m\omega_0 t \cos n\omega_0 t\, dt = 0, \text{for all } m \neq n, = \frac{T}{2}, \text{for } m = n.$$

Hint: Use the trigonometric identity $\cos\alpha\cos\beta = \frac{1}{2}\cos(\alpha-\beta) + \frac{1}{2}\cos(\alpha+\beta)$.

6. 16.6 Derive Eq. 16.5.

Section 16.3

1. 16.7 Derive the expressions for the Fourier coefficients of an odd periodic function. Hint: Use the same technique as used in the text in deriving Eqs. 16.7–16.9.

2. 16.8 Show that if f(t)=–f(t−T/2), the Fourier coefficients bk are given by the expressions

   b k = 0 for k even; b k = 4 T ∫ 0 T/2 f(t) sink ω o t dt, =for k odd.

   Hint: Use the same technique as used in the text to derive Eqs, 16.16.

3. 16.9 Derive Eqs. 16.18. Hint: Start with Eq. 16.16 and divide the interval of integration into 0 to T/4 and T/4 to T/2. Note that because of evenness and quarter-wave symmetry, f(t)=–f(T/2−t) in the interval T/4≤t≤T/2. Let x=T/2−t in the second interval and integrate between 0 and T/4.

4. 16.10 Derive Eqs. 16.19. Follow the hint given in Problem 16.9 except that because of oddness and quarter- wave symmetry, f(t)=f(T/2−t) in the interval T/4≤t≤T/2.

5. 16.11 It is given that v(t)=20 sin π|t|V over the interval –1≤t≤1 s. The function then repeats itself.

   1. What is the fundamental frequency in radians per second?

   2. Is the function even?

   3. Is the function odd?

   4. Does the function have half-wave symmetry?

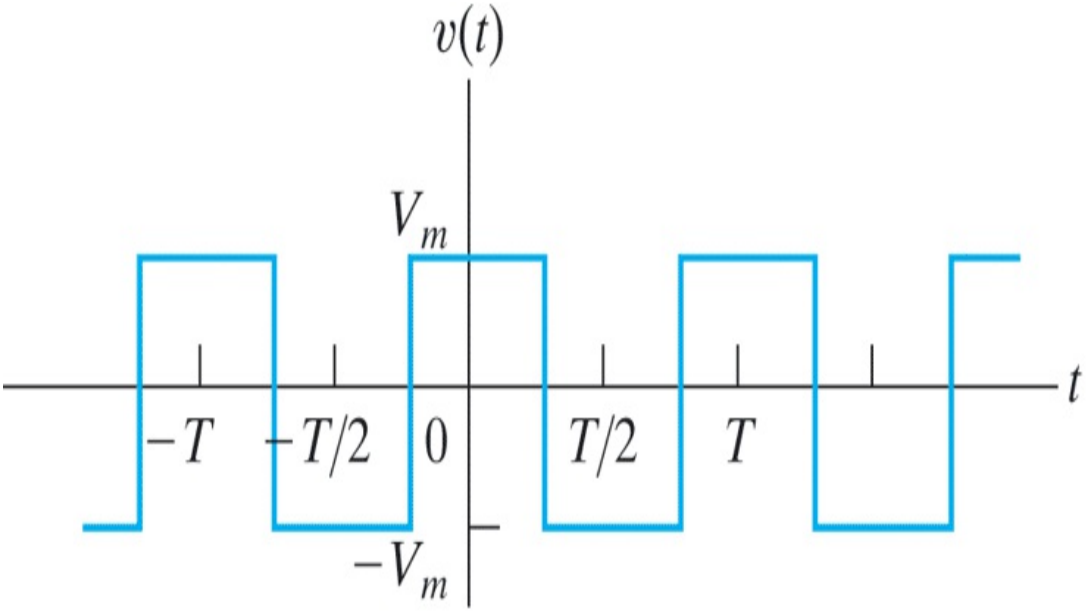
6. 16.12 One period of a periodic function is described by the following equations:

i(t) = 5t A, –2 ms≤t≤2 ms; i(t) = 10 mA, 2 ms≤t≤6 ms; i(t) = 0.04−5t A, 6 ms≤t≤10 ms; i(t) = −10 mA, 10 ms≤t≤14 ms.

1. What is the fundamental frequency in hertz?
2. Is the function even?
3. Is the function odd?
4. Does the function have half-wave symmetry?
5. Does the function have quarter-wave symmetry?
6. Give the numerical expressions for av, ak, and bk.

7. 16.13 Find the Fourier series of each periodic function shown in Fig. P16.13.

Figure P16.13



(a)

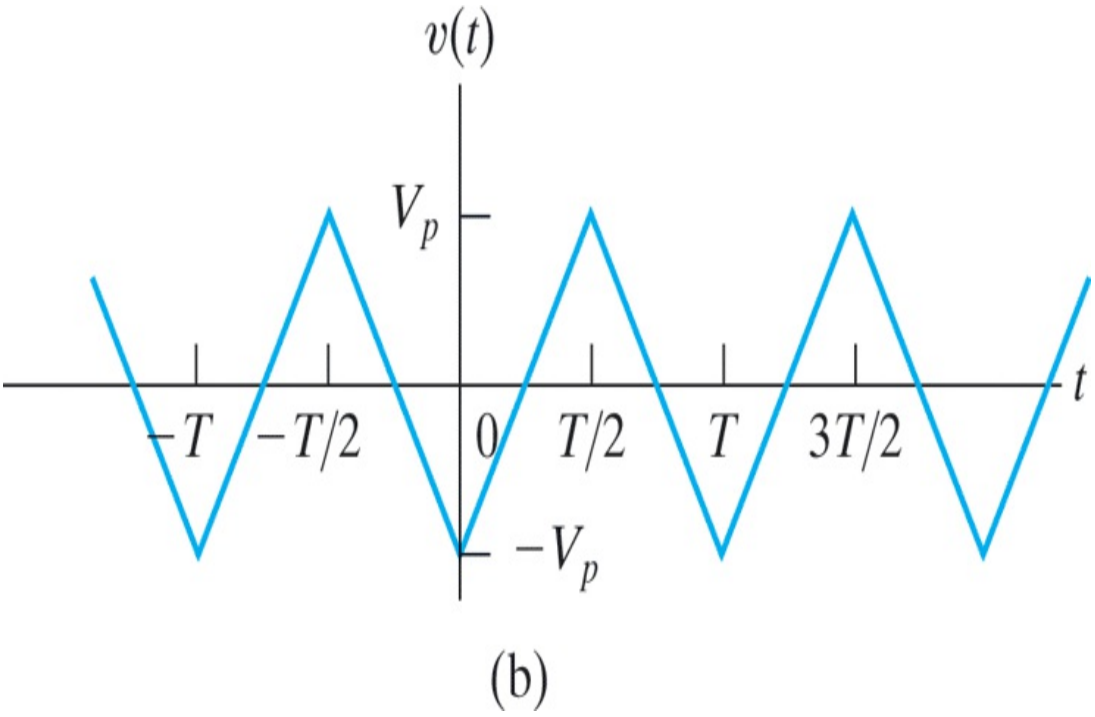


Figure P16.13 Full Alternative Text

8. 16.14 The periodic function shown in Fig. P16.14 is even and has both half-wave and quarter-wave symmetry.

    1. Sketch one full cycle of the function over the interval –T/4≤t≤3T/4.
    2. Derive the expression for the Fourier coefficients av, ak, and bk.
    3. Write the first three nonzero terms in the Fourier expansion of f(t).
    4. Use the first three nonzero terms to estimate f(T/4).

Figure P16.14

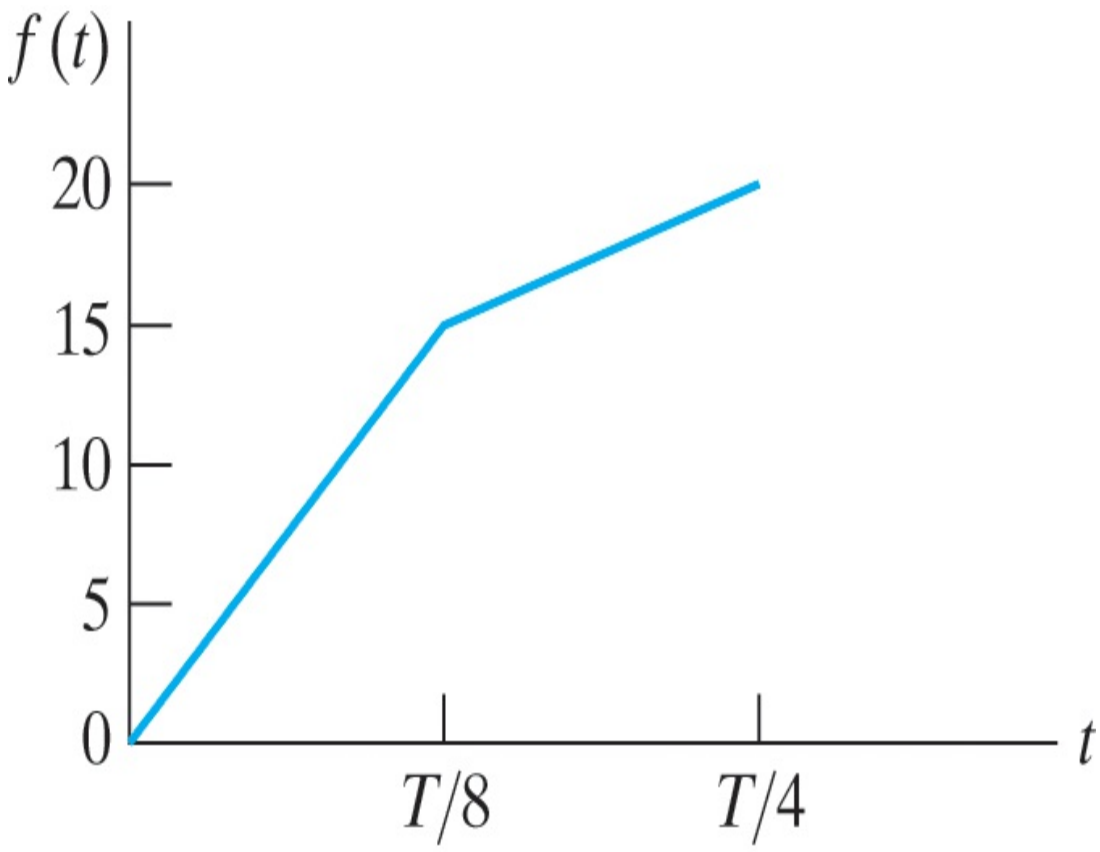


Figure P16.14 Full Alternative Text

9. 16.15 It is given that f(t)=4t2 over the interval –2<t<2 s.

    1. Construct a periodic function that satisfies this f(t) between −2 s and +2 s, has a period of 8 s, and has half-wave symmetry.

    2. Is the function even or odd?

    3. Does the function have quarter-wave symmetry?

    4. Derive the Fourier series for f(t).

    5. Write the Fourier series for f(t) if f(t) is shifted 2 s to the right.

10. 16.16 Repeat Problem 16.15 given that f(t)=t3 over the interval –2<t<2 s.

11. 16.17

    1. Derive the Fourier series for the periodic current shown in Fig. P16.17.

    2. Repeat (a) if the vertical reference axis is shifted T/2 units to the left.

Figure P16.17

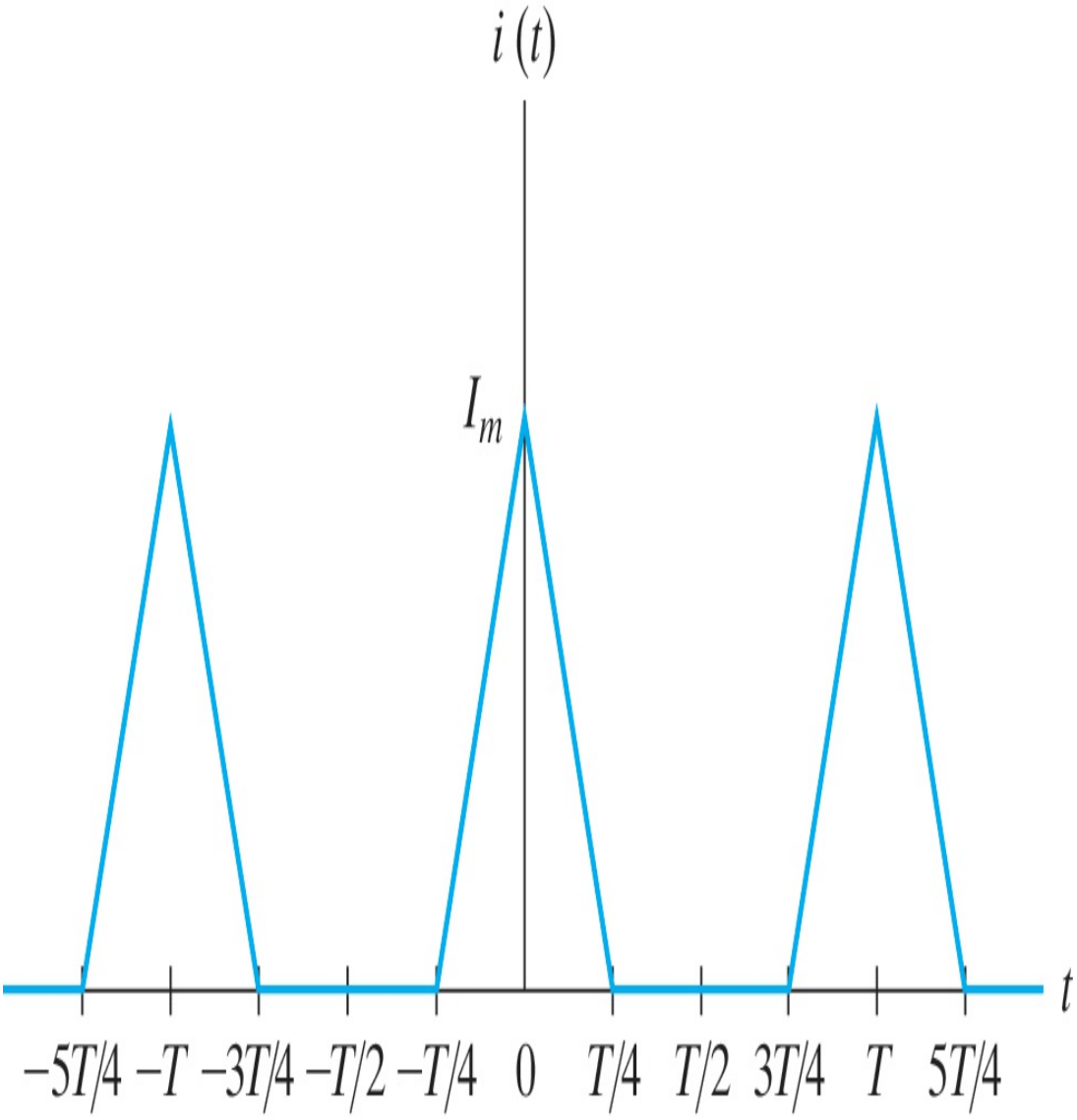
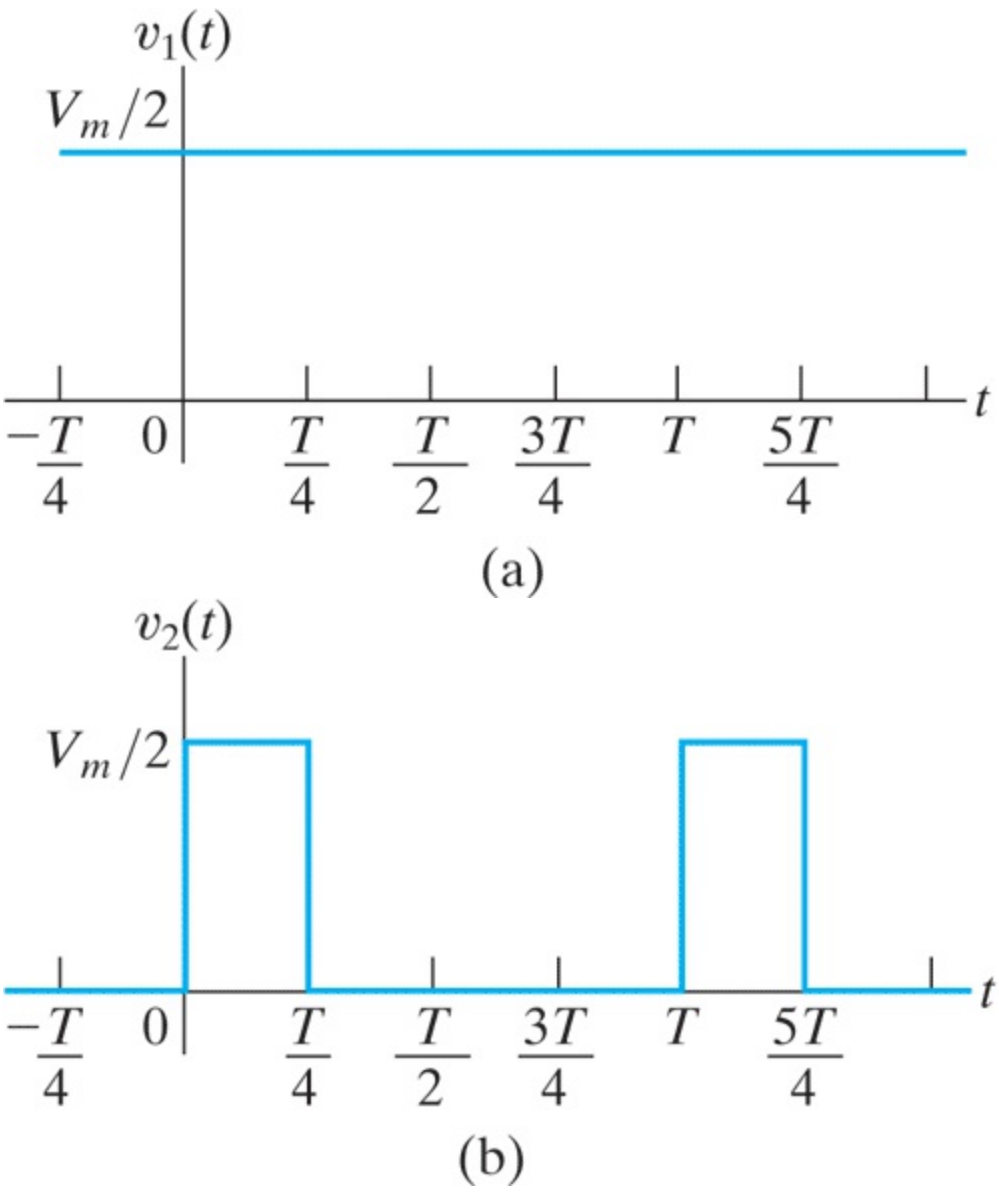


Figure P16.17 Full Alternative Text

12. 16.18 It is sometimes possible to use symmetry to find the Fourier coefficients, even though the original function is not symmetrical! With this thought in mind, consider the function in Fig P16.4. Observe that v(t) can be divided into the two functions illustrated in Fig. P16.18(a) and (b). Furthermore, we can make v2(t) an even function by shifting it T/8 units to the left. This is illustrated in Fig. P16.18(c). At this point we

note that v(t)=v1(t)+v2(t) and that the Fourier series of v1(t) is a single-term series consisting of Vm/2. To find the Fourier series of v2(t), we first find the Fourier series of v2(t+T/8) and then shift this series T/8 units to the right. Use the technique just outlined to verify the Fourier coefficients found in Problem 16.4.

Figure P16.18



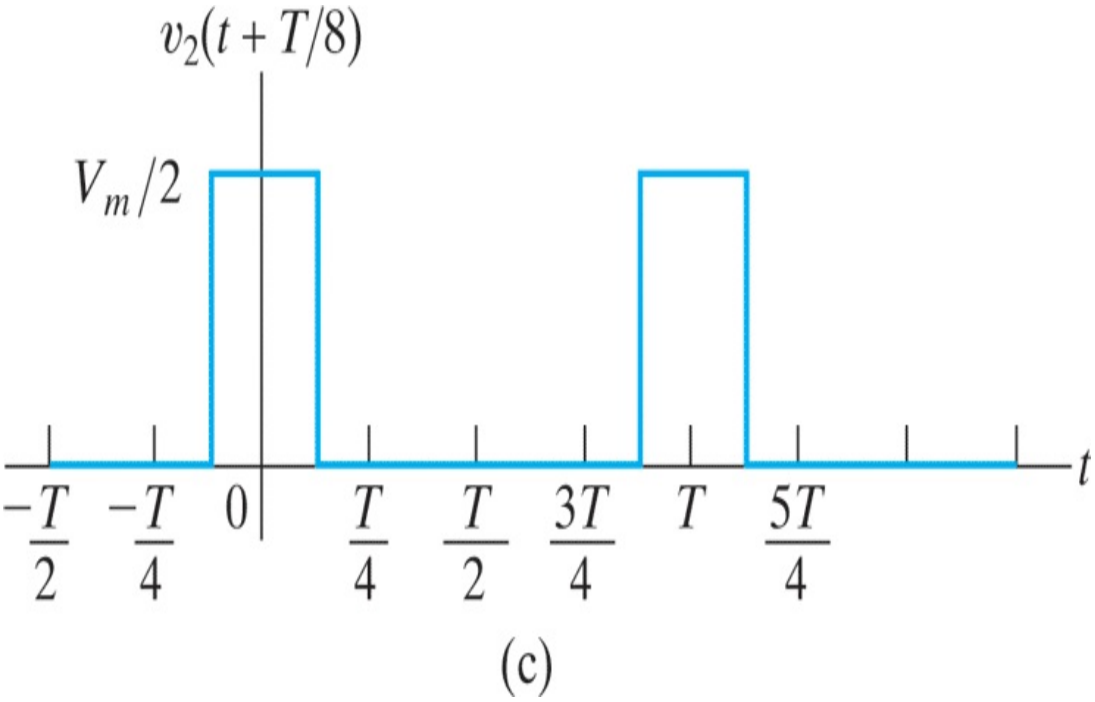


Figure P16.18 Full Alternative Text

Section 16.4

1. 16.19 Derive the Fourier series for the periodic function shown in Fig. P16.2, using the form of Eq. 16.20.

2. 16.20 Derive the Fourier series for the periodic function described in Problem 16.12, using the form of Eq. 16.20.

3. 16.21 Derive the Fourier series for the periodic function constructed in Problem 16.15, using the form of Eq. 16.20.

4. 16.22

    1. Derive the Fourier series for the periodic function shown in Fig. P16.22 when Im=5π2 A. Write the series in the form of Eq. 16.20.

    2. Use the first five nonzero terms to estimate i(T/4).

Figure P16.22

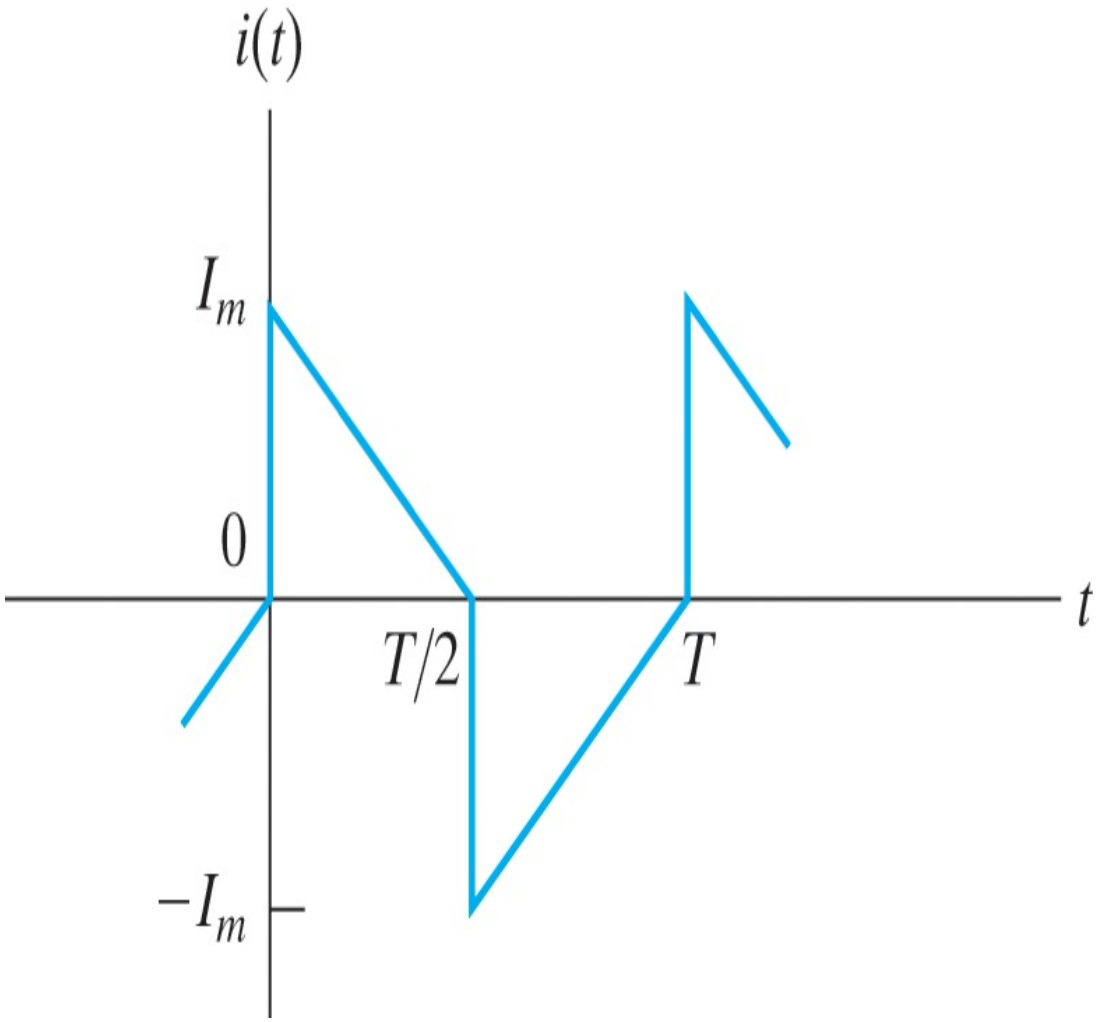


Figure P16.22 Full Alternative Text

Section 16.5

1. 16.23 Derive Eqs. 16.26 and 16.27.

2. 16.24

    1. Derive Eq. 16.28. Hint: Note that bk=4Vm/πk+kωoRCak. Use this expression for bk to find ak2+bk2 in terms of ak. Then use the expression for ak to derive Eq. 16.28.

    2. Derive Eq. 16.29.

3. 16.25 Show that when we combine Eqs. 16.28 and 16.29 with Eqs. 16.20 and 16.21, the result is Eq. 16.22. Hint: Note from the definition of βk that

    akbk=−tan βk,

    and from the definition of θk that

    tan θk=−cot βk.

    Now use the trigonometric identity

    tan x=cot(90−x)

    to show that θk=90+βk.

4. 16.26

    1. Show that for large values of C, Eq. 16.24 can be approximated by the expression

        vo(t)≈−VmT4RC+VmRCt.

        Note that this expression is the equation of the triangular wave for

0≤t≤T/2. Hints: (1) Let e−t/RC≈1−(t/RC) and e −T/2RC≈1−(T/2RC); (2) put the resulting expression over the common denominator 2−(T/2RC); (3) simplify the numerator; and (4) for large C, assume that T/2RC is much less than 2.

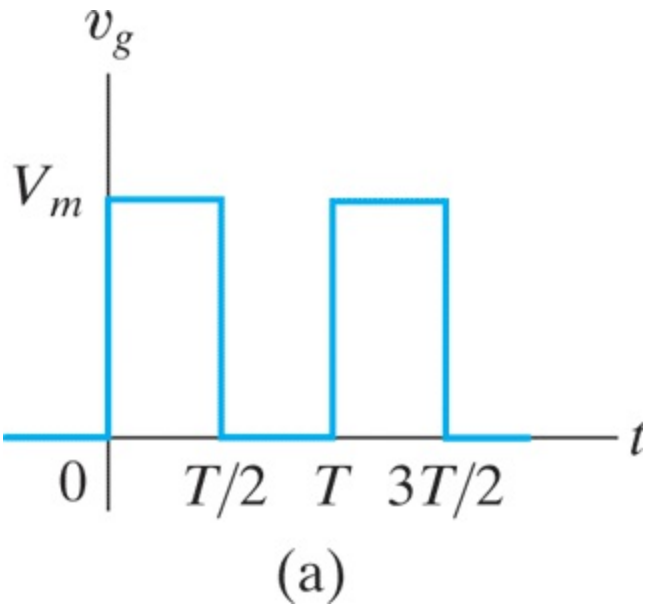
2. Substitute the peak value of the triangular wave into the solution for Problem 16.13 (see Fig. P16.13(b)) and show that the result is Eq. 16.23.

5. 16.27 PSPICEMULTISIM The square-wave voltage shown in Fig. P16.27(a) is applied to the circuit shown in Fig. P16.27(b).

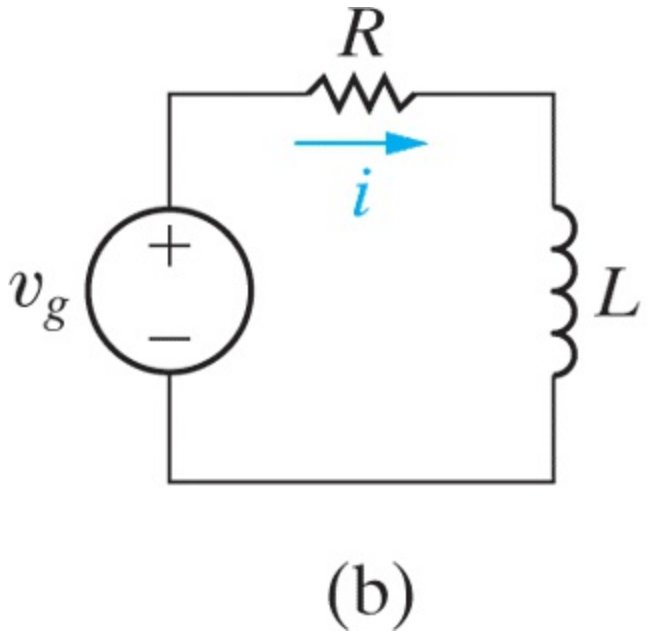
1. Find the Fourier series representation of the steady-state current i.

2. Find the steady-state expression for i by straightforward circuit analysis.

Figure P16.27



(a)



(b)

Figure P16.27 Full Alternative Text

6. 16.28 The periodic square-wave voltage seen in Fig. P16.28(a) is applied to the circuit shown in Fig. P16.28(b). Derive the first three nonzero terms in the Fourier series that represents the steady-state voltage vo if Vm=15π V and the period of the input voltage is 4π ms.

Figure P16.28

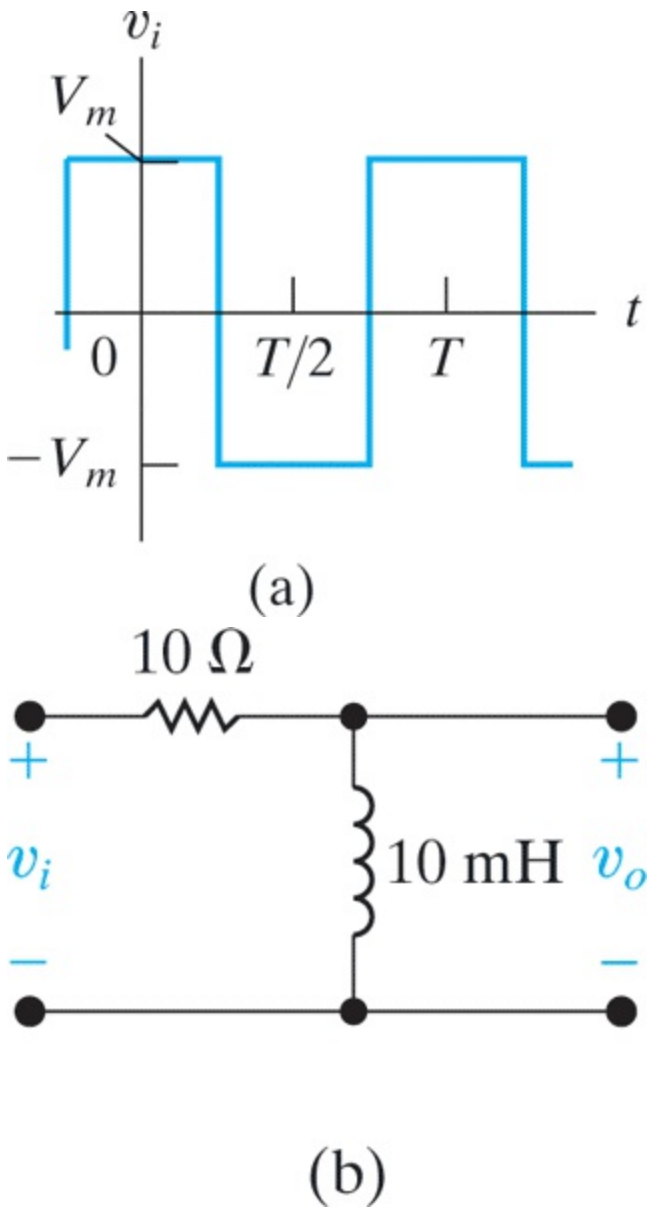


Figure P16.28 Full Alternative Text

7. 16.29 PSPICEMULTISIMThe periodic square-wave voltage shown in Fig P16.13(a) with Vm=10.5π V and T=π ms is applied to the circuit shown in Fig. P16.29.

    1. Derive the first four nonzero terms in the Fourier series that represents the steady-state voltage vo.

    2. Which frequency component in the input voltage is eliminated from the output voltage? Explain why.

Figure P16.29

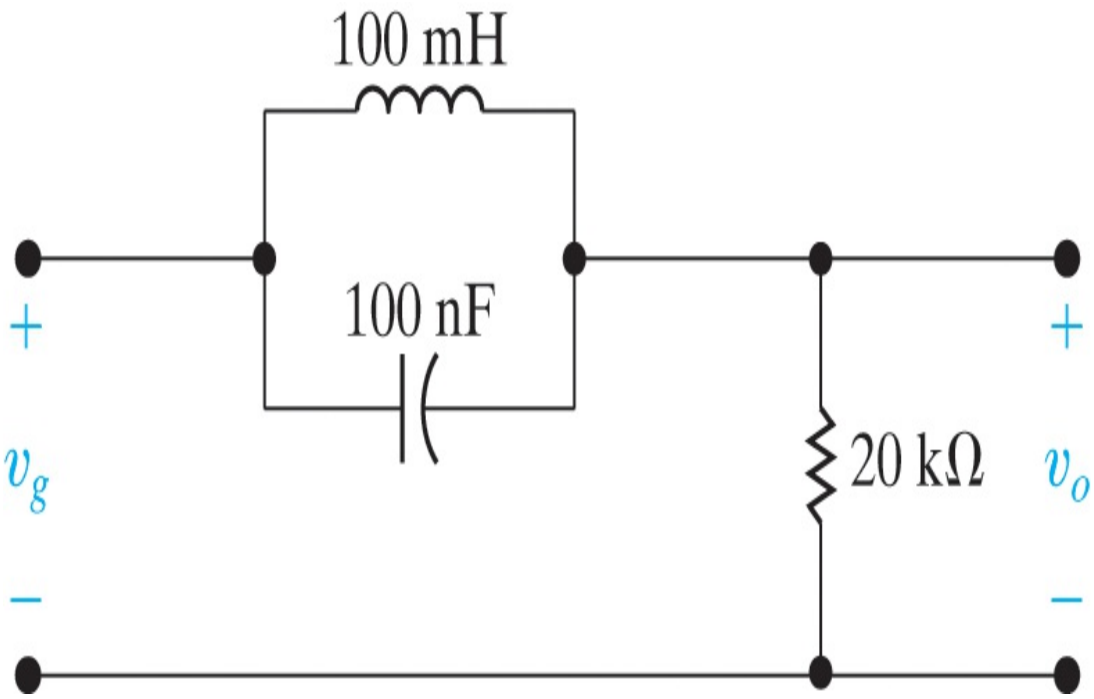


Figure P16.29 Full Alternative Text

8. 16.30 The full-wave rectified sine-wave voltage shown in Fig. P16.30(a) is applied to the circuit shown in Fig. P16.30(b).

    1. Find the first five nonzero terms in the Fourier series representation of io.

    2. Does your solution for io make sense? Explain.

Figure P16.30

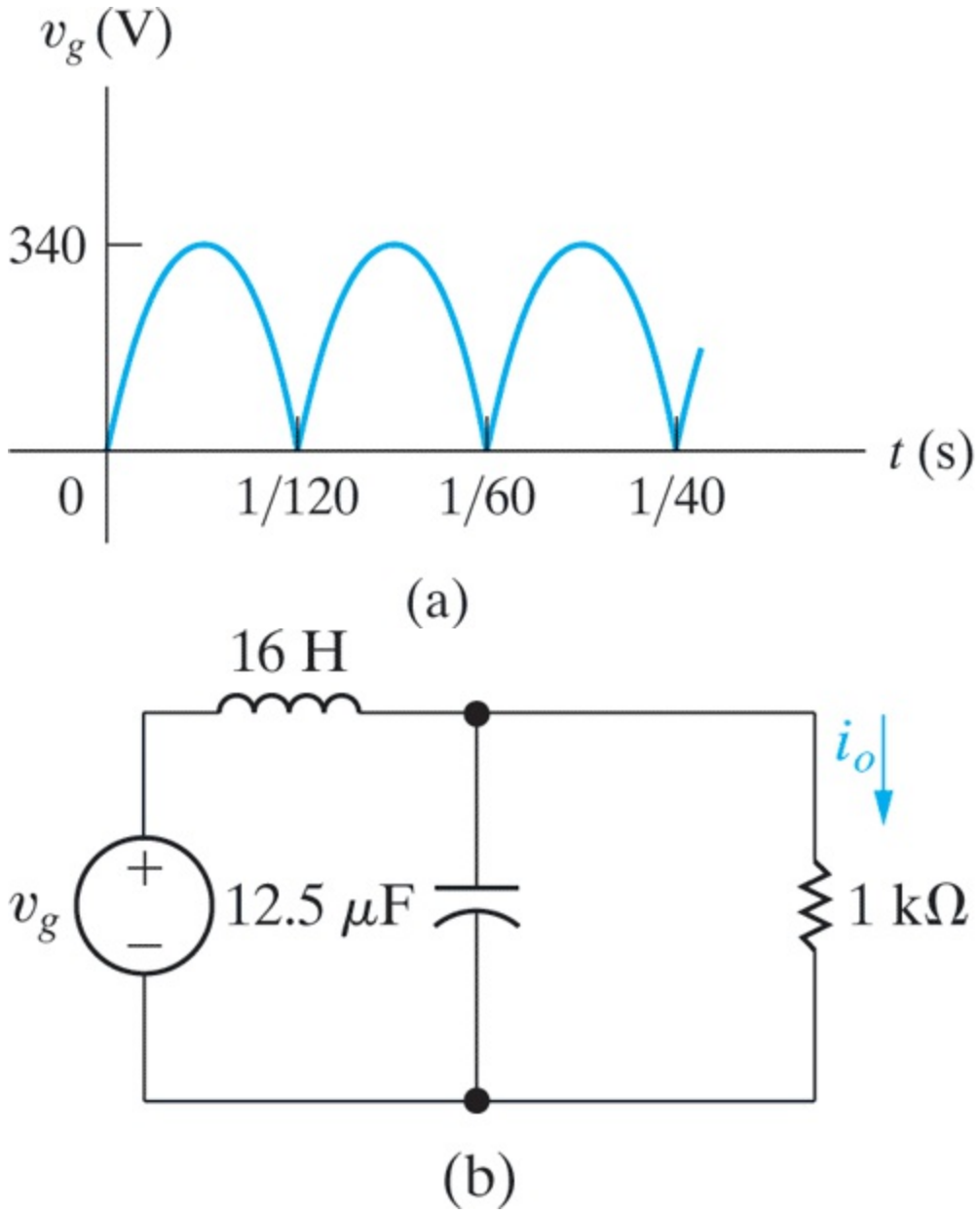


Figure P16.30 Full Alternative Text

9. 16.31 The periodic current described below is used to energize the circuit shown in Fig. P16.31. Write the time-domain expression for the third-harmonic component in the expression for vo.

   i g = 500t A, −2 ms≤t≤2 ms; = 1 A, 2 ms≤t≤8 ms; = 5−500t A, 8 ms≤t≤12 ms; = −1 A, 12 ms≤t≤18 ms.

Figure P16.31

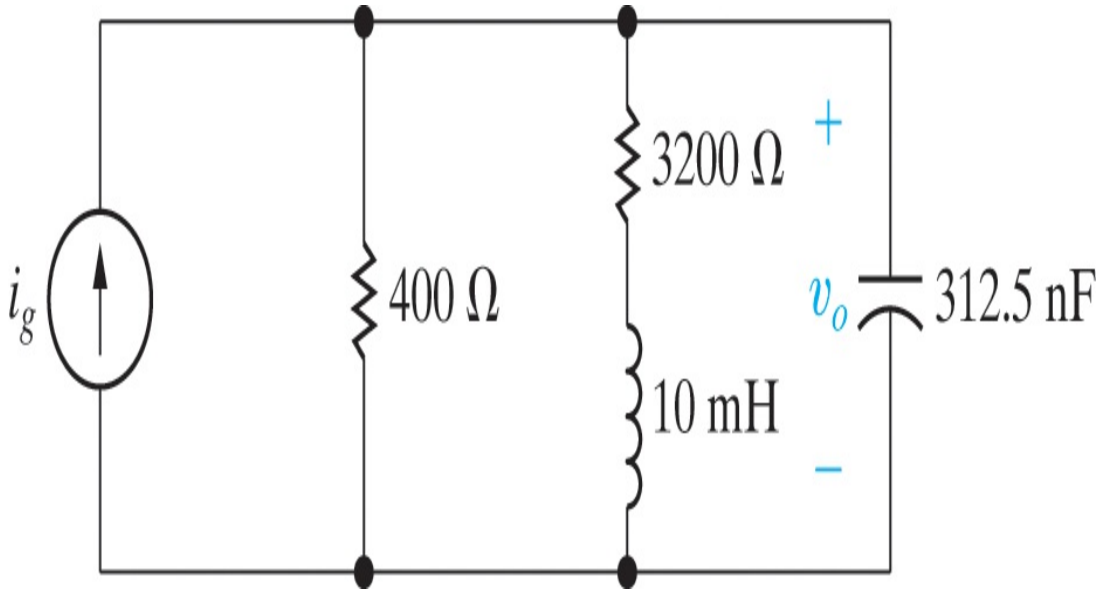


Figure P16.31 Full Alternative Text

10. 16.32 A periodic voltage having a period of 10π μs is given by the following Fourier series:

    vg=150∑n=1, 3, 5, …∞1n sin nπ2 cos nωot V.

    This periodic voltage is applied to the circuit shown in Fig. P16.32. Find the amplitude and phase angle of the components of vo that have frequencies of 3 and 5 [&~rom~Mrad|sol|s&].

Figure P16.32

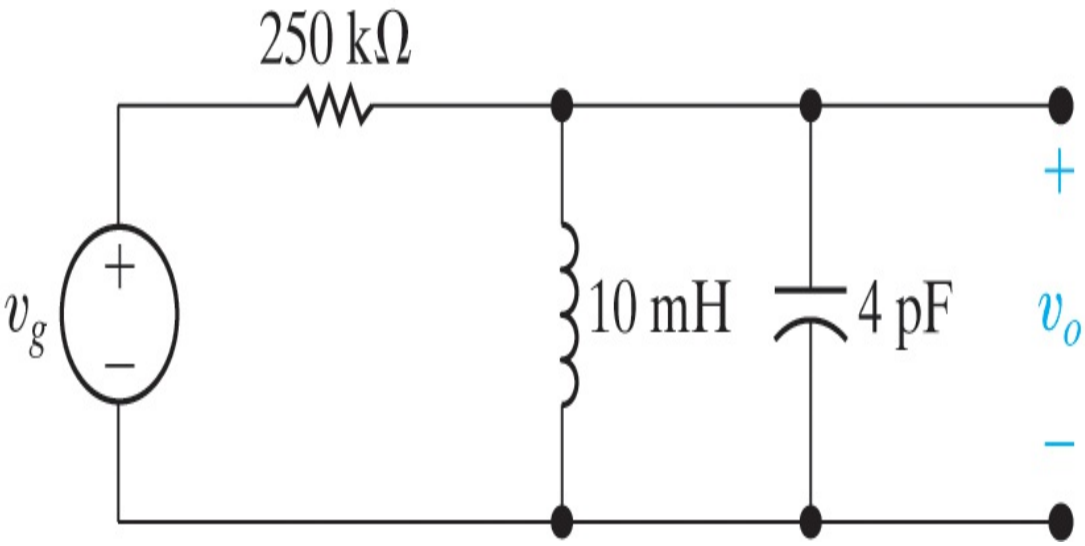


Figure P16.32 Full Alternative Text

Section 16.6

1. 16.33 The periodic current shown in Fig. P16.33 is applied to a 1 kΩ resistor.

    1. Use the first three nonzero terms in the Fourier series representation of i(t) to estimate the average power dissipated in the 1 kΩ resistor.

    2. Calculate the exact value of the average power dissipated in the 1 kΩ resistor.

    3. What is the percentage of error in the estimated value of the average power?

Figure P16.33

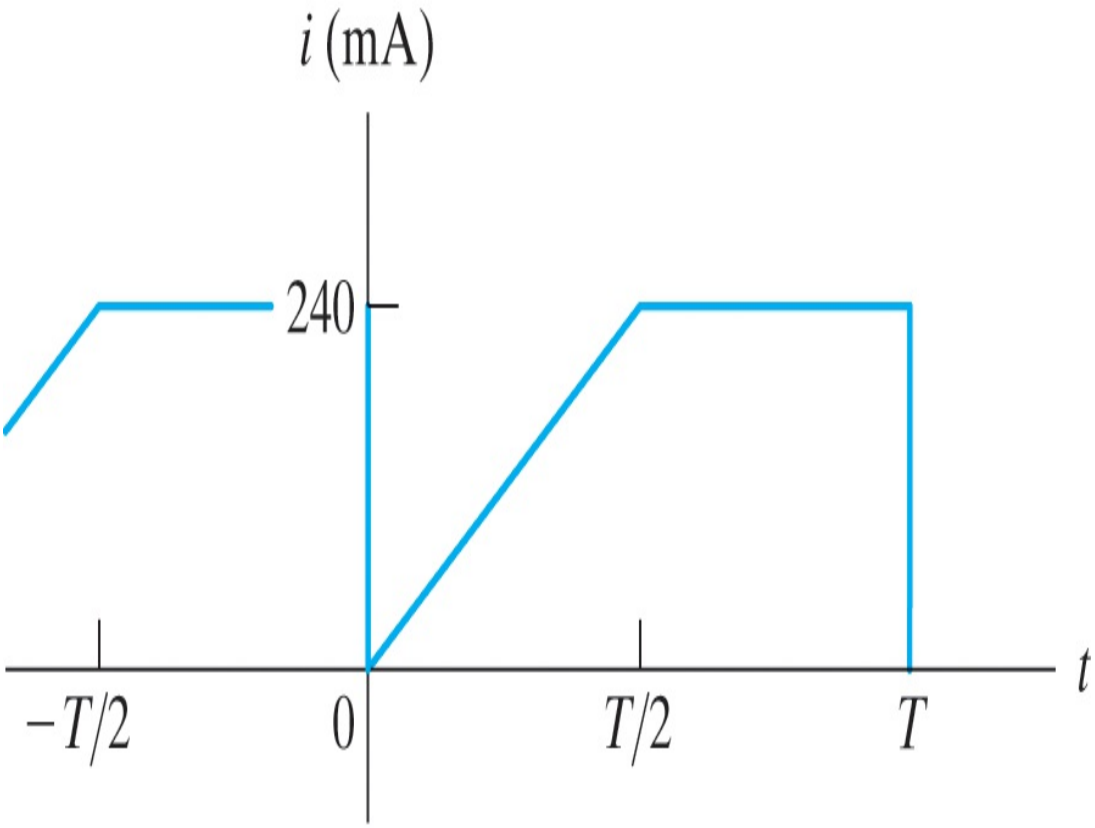


Figure P16.33 Full Alternative Text

2. 16.34 The periodic voltage across a 10 Ω resistor is shown in Fig. P16.34.

    1. Use the first three nonzero terms in the Fourier series representation of v(t) to estimate the average power dissipated in the 10 Ω resistor.

    2. Calculate the exact value of the average power dissipated in the 10 Ω resistor.

    3. What is the percentage error in the estimated value of the average power dissipated?

Figure P16.34

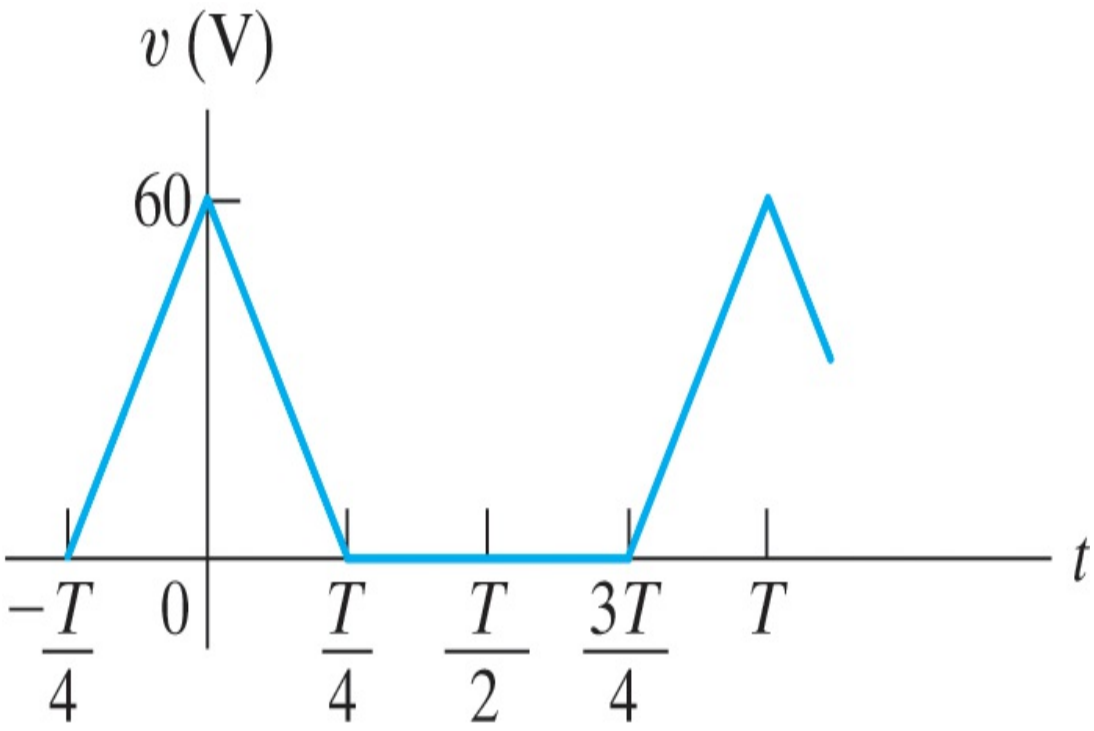


Figure P16.34 Full Alternative Text

3. 16.35 The triangular-wave voltage source, shown in Fig. P16.35(b), is applied to the circuit in Fig. P16.35(a). Estimate the average power delivered to the 502 Ω resistor when the circuit is in steady-state operation.

Figure P16.35

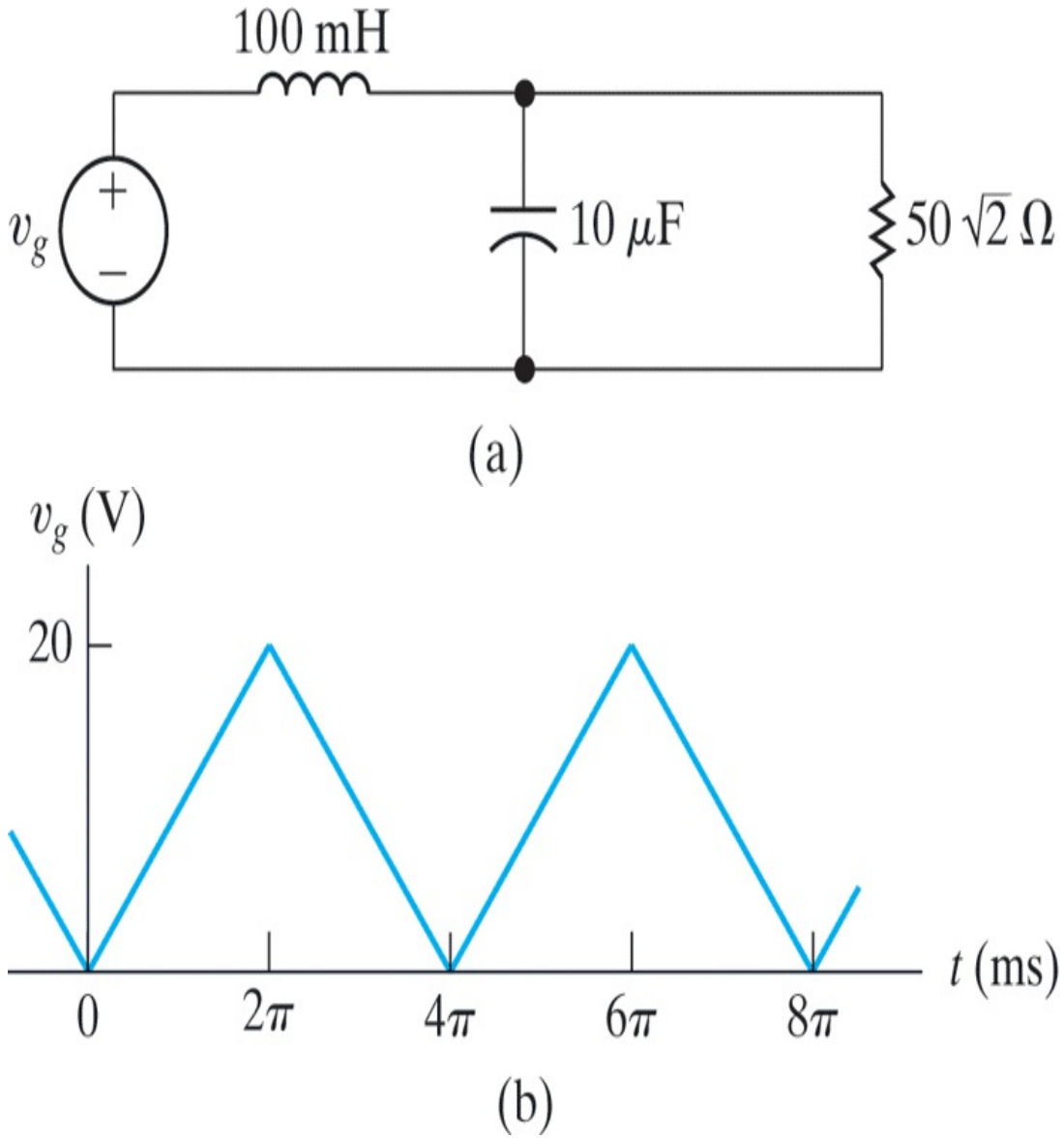


Figure P16.35 Full Alternative Text

Section 16.7

1. 16.36 The voltage and current at the terminals of a network are

   $v = 15+400\cos 500t+100\sin 1500t$ V, $i = 2+5\sin(500t+60^\circ)+3\sin(1500t-15^\circ)$ A.

   The current is in the direction of the voltage drop across the terminals.

   1. What is the average power at the terminals?
   2. What is the rms value of the voltage?
   3. What is the rms value of the current?

2. 16.37

   1. Find the rms value of the voltage shown in Fig. P16.37 for Vm=100 V. Note that the Fourier series for this periodic voltage was found in Assessment Problem 16.3.
   2. Estimate the rms value of the voltage, using the first three nonzero terms in the Fourier series representation of vg(t).

Figure P16.37

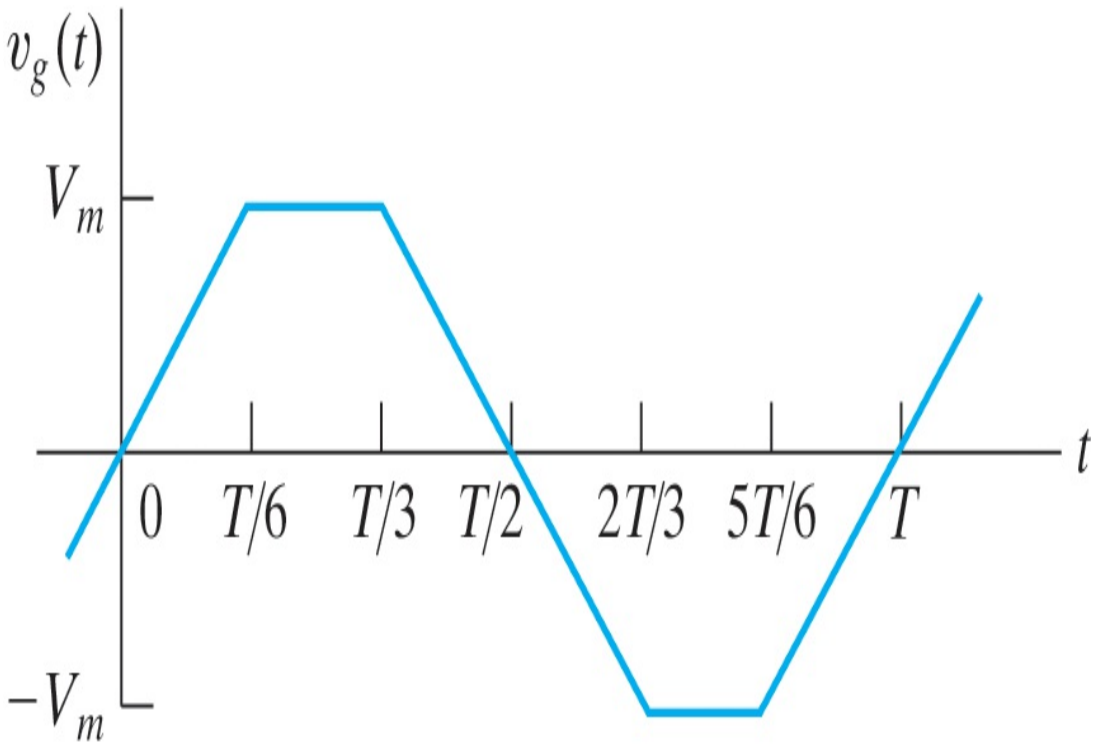


Figure P16.37 Full Alternative Text

3. 16.38

    1. Use the first four nonzero terms in the Fourier series approximation of the periodic voltage shown in Fig. P16.38 to estimate its rms value.

    2. Calculate the true rms value of the voltage.

    3. Calculate the percentage of error in the estimated value.

Figure P16.38

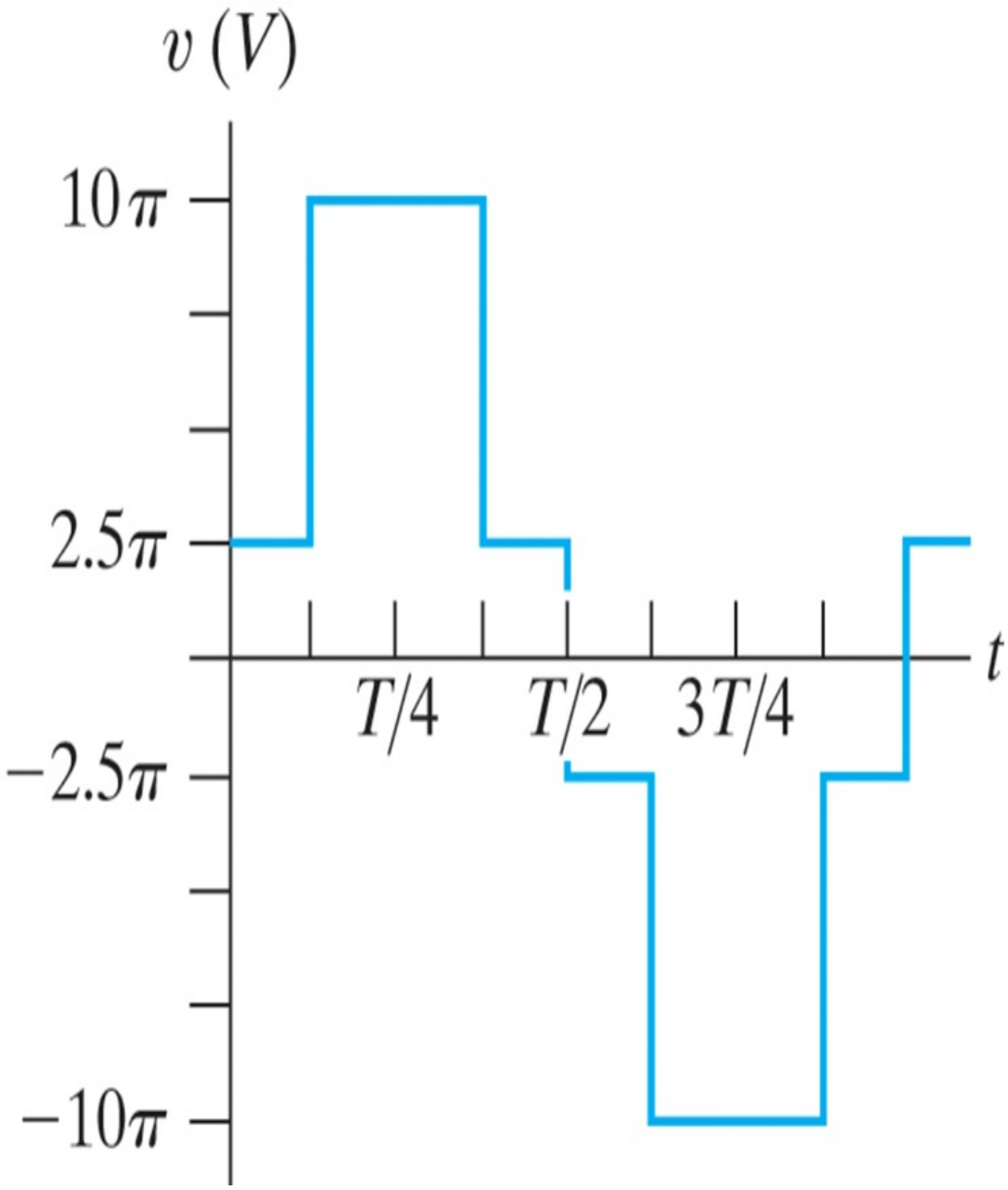


Figure P16.38 Full Alternative Text

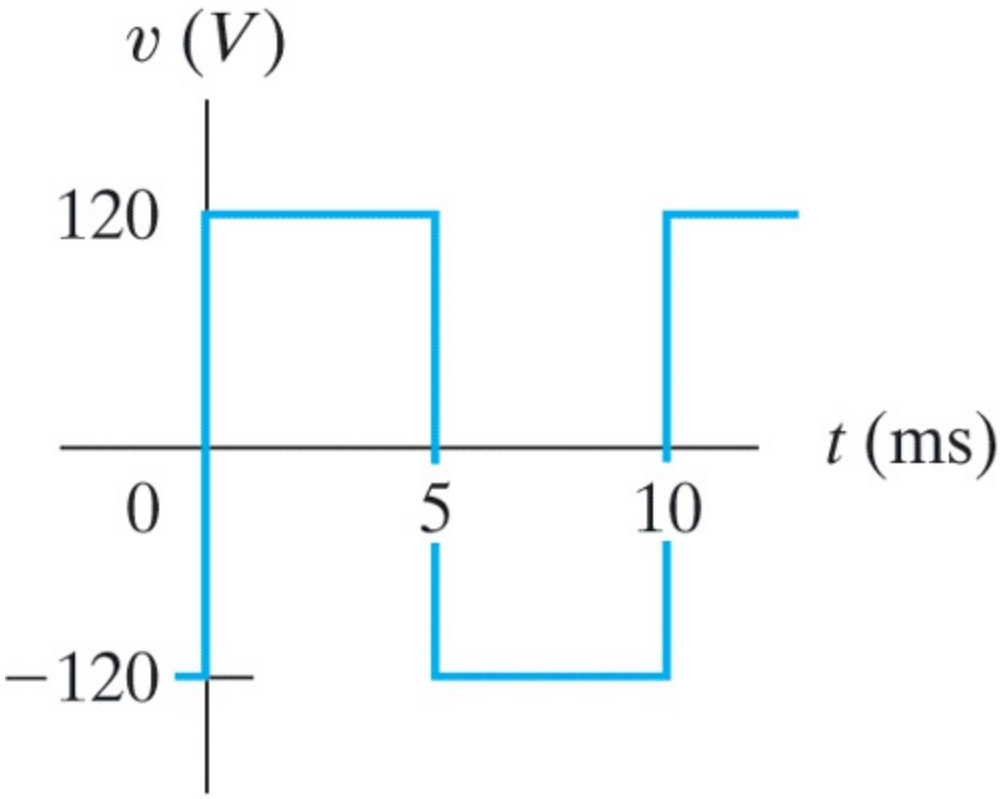
4. 16.39

    1. Estimate the rms value of the periodic square-wave voltage shown in Fig. P16.39(a) by using the first five nonzero terms in the Fourier series representation of v(t).

    2. Calculate the percentage of error in the estimation.

3. Repeat parts (a) and (b) if the periodic square-wave voltage is replaced by the periodic triangular voltage shown in Fig. P16.39(b).

Figure P16.39



(a)

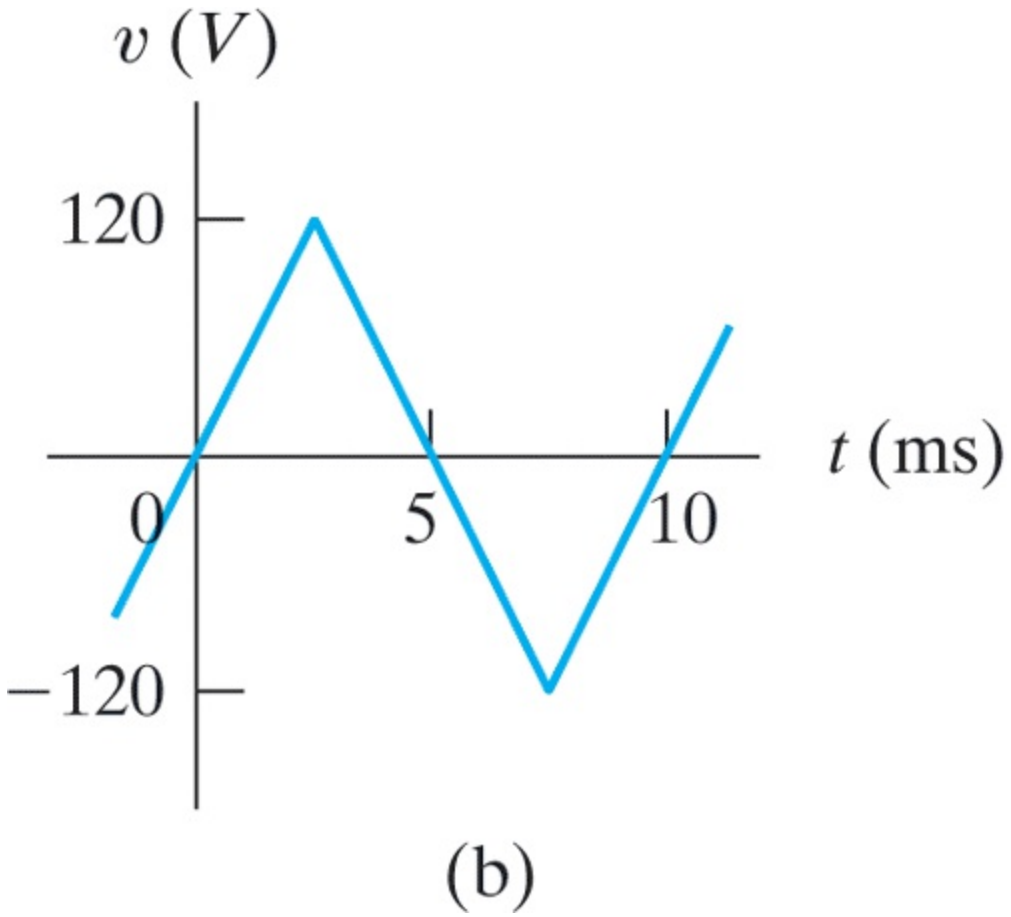


Figure P16.39 Full Alternative Text

5. 16.40

    1. Estimate the rms value of the full-wave rectified sinusoidal voltage shown in Fig. P16.40(a) by using the first three nonzero terms in the Fourier series representation of v(t).

    2. Calculate the percentage of error in the estimation.

    3. Repeat (a) and (b) if the full-wave rectified sinusoidal voltage is replaced by the half-wave rectified sinusoidal voltage shown in Fig. P16.40(b).

Figure P16.40

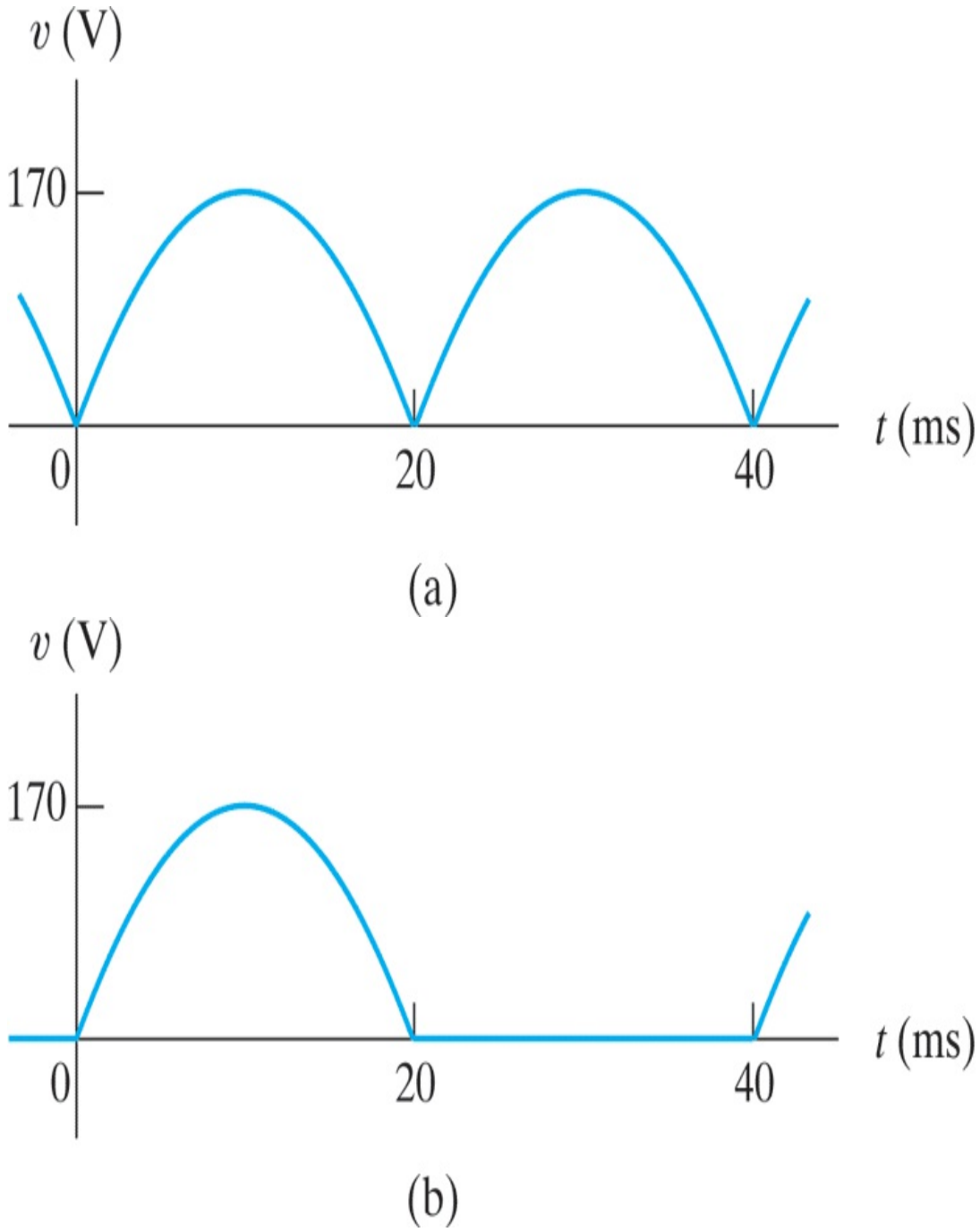


Figure P16.40 Full Alternative Text

6. 16.41 Assume the periodic function described in Problem 16.14 is a voltage v with a peak amplitude of 20 V.

    1. Find the rms value of the voltage.

2. If this voltage is applied to a 15 Ω resistor, what is the average power dissipated in the resistor?

3. If v is approximated by using just the fundamental frequency term of its Fourier series, what is the average power delivered to the 15 Ω resistor?

4. What is the percentage of error in the estimation of the power dissipated?

7. 16.42

    1. Derive the expressions for the Fourier coefficients for the periodic current shown in Fig. P16.42.

    2. Write the first four nonzero terms of the series using the alternative trigonometric form given by Eq. 16.20.

    3. Use the first four nonzero terms of the expression derived in (b) to estimate the rms value of ig.

    4. Find the exact rms value of ig.

    5. Calculate the percentage of error in the estimated rms value.

Figure P16.42

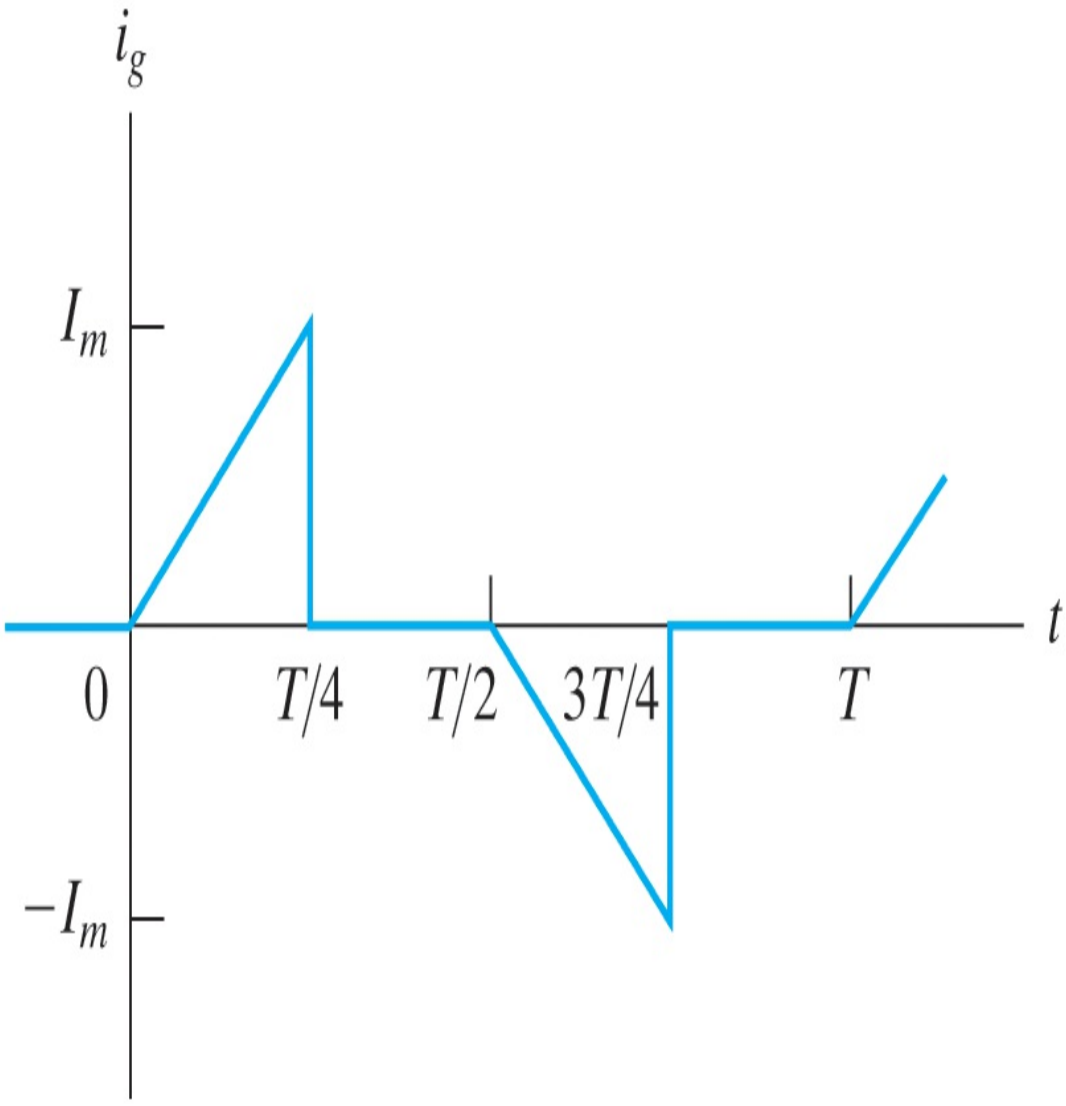
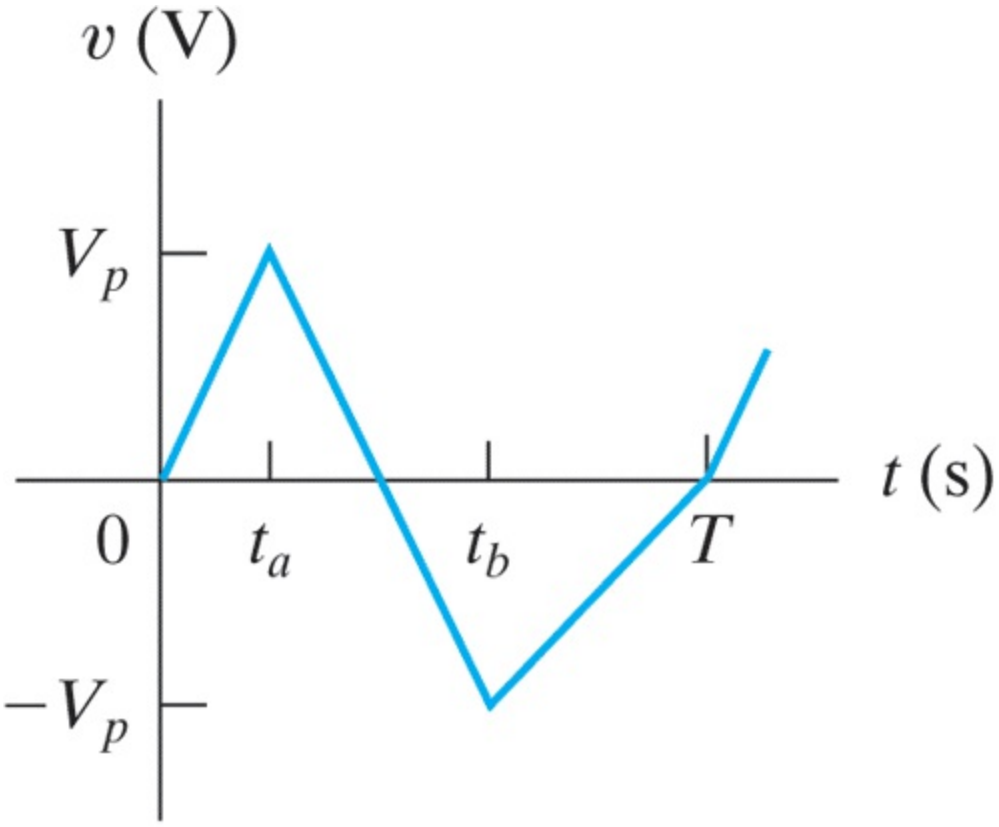


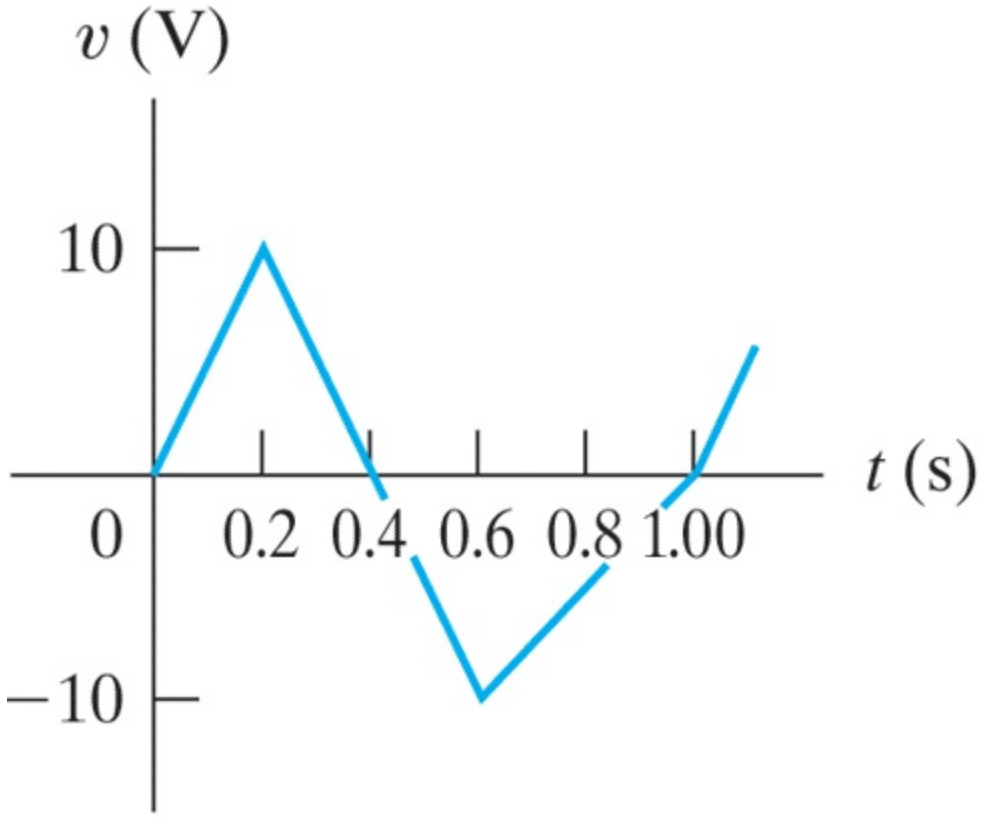
Figure P16.42 Full Alternative Text

8. 16.43 The rms value of any periodic triangular wave having the form depicted in Fig. P16.43(a) is independent of ta and tb. Note that for the function to be single valued, ta≤tb. The rms value is equal to Vp/3. Verify this observation by finding the rms value of the three waveforms depicted in Fig. P16.43(b)–(d).

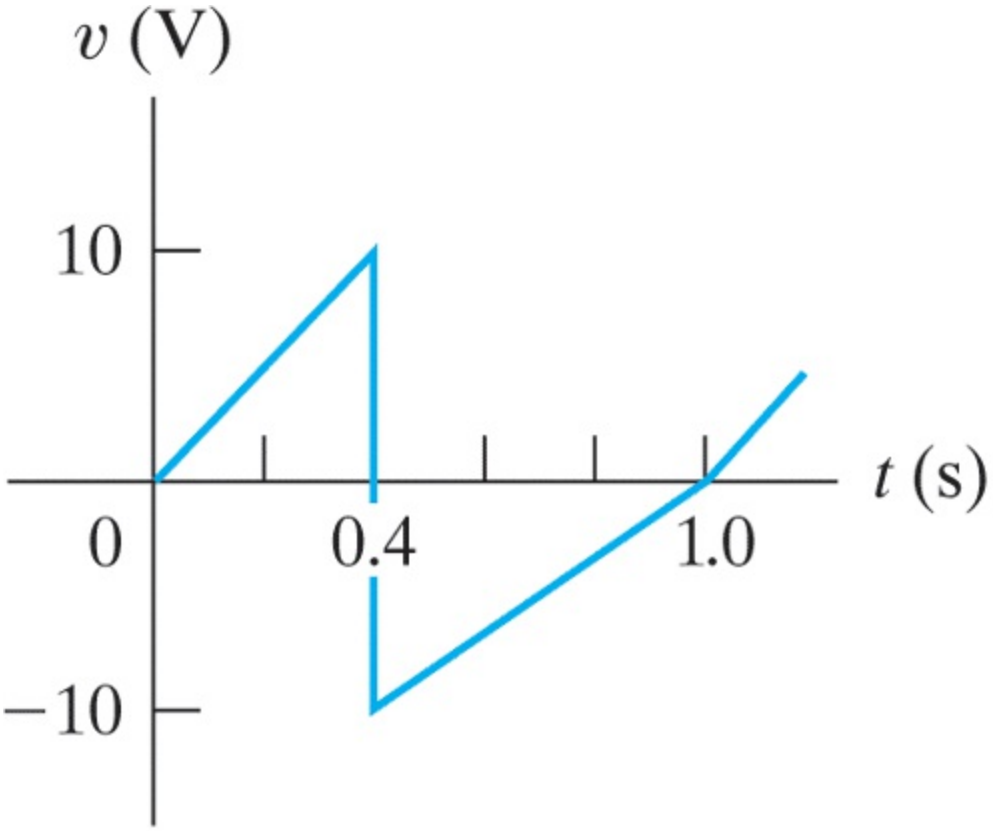
Figure P16.43



(a)



(b)



(c)

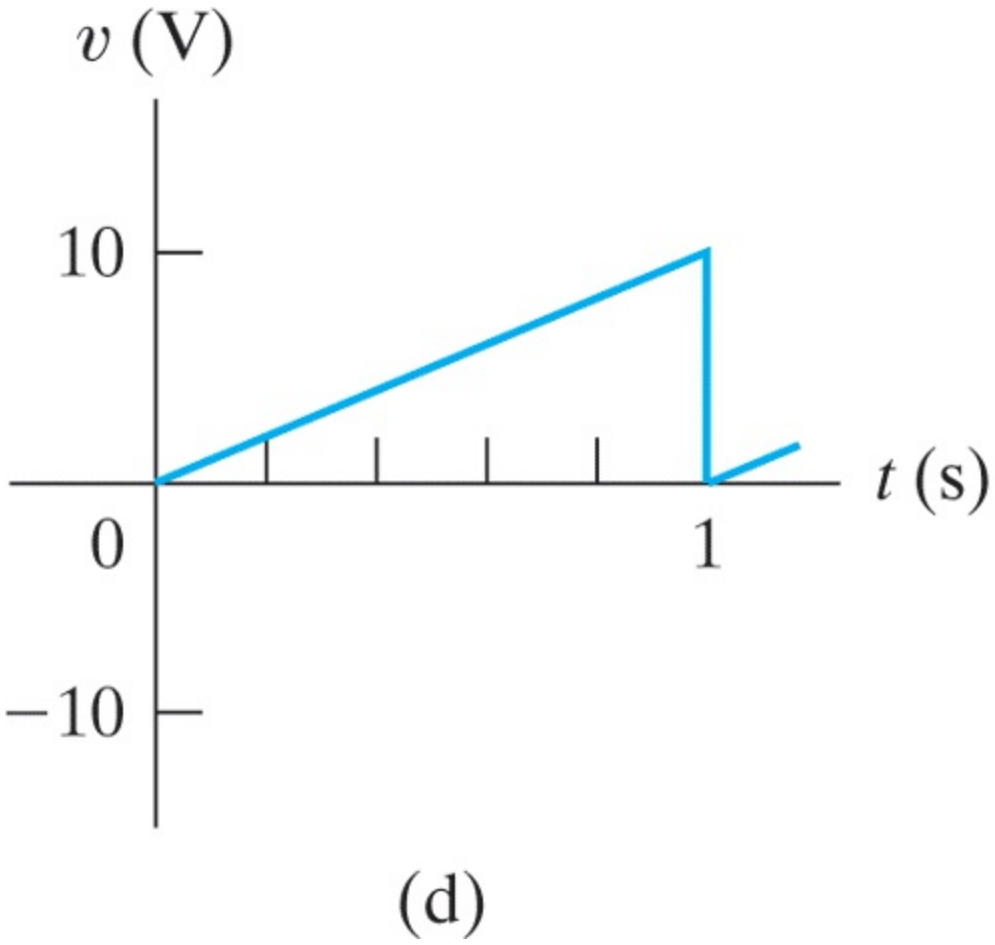


Figure P16.43 Full Alternative Text

Section 16.8

1. 16.44 Derive the expression for the complex Fourier coefficients for the periodic voltage shown in Fig. P16.44.

Figure P16.44

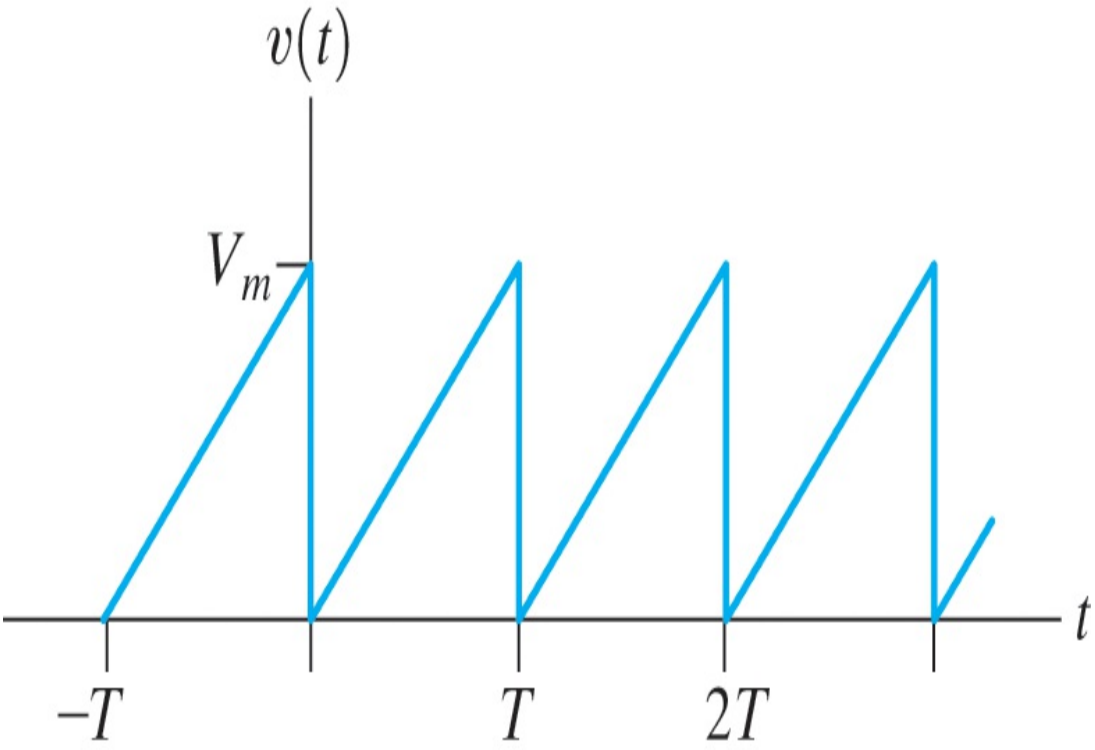


Figure P16.44 Full Alternative Text

2. 16.45

    1. The periodic voltage in Problem 16.44 is applied to a 10 Ω resistor. If Vm=120 V what is the average power delivered to the resistor?

    2. Assume v(t) is approximated by a truncated exponential form of the Fourier series consisting of the first seven nonzero terms, that is,

n=0, 1, 2, 3, 4, 5, 6, and 7. What is the rms value of the voltage, using this approximation?

3. If the approximation in part (b) is used to represent v, what is the percentage of error in the calculated power?

3. 16.46 Use the exponential form of the Fourier series to write an expression for the voltage shown in Fig. P16.46.

Figure P16.46

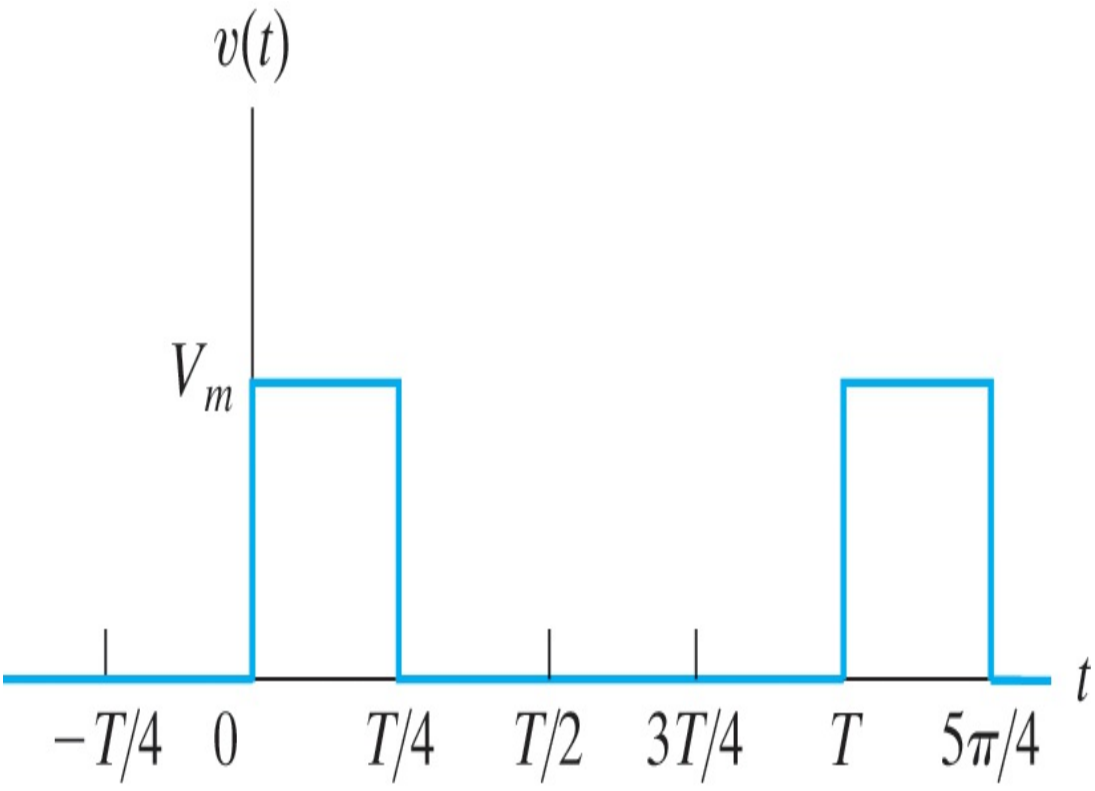


Figure P16.46 Full Alternative Text

4. 16.47 The periodic voltage source in the circuit shown in Fig. P16.47(a) has the waveform shown in Fig. P16.47(b).

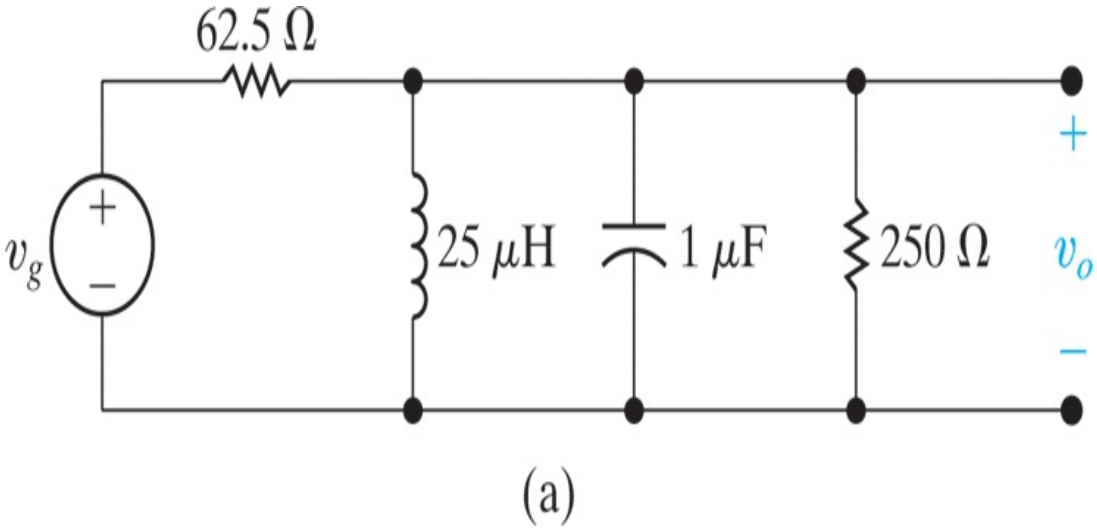
1. Derive the expression for Cn.

2. Find the values of the complex coefficients Co, C−1, C1, C −2, C2, C−3, C3, C−4, and C4 for the input voltage vg if Vm=54 V and T=10π μs.

3. Repeat (b) for vo.

4. Use the complex coefficients found in (c) to estimate the average power delivered to the 250 kΩ resistor.

Figure P16.47



(a)

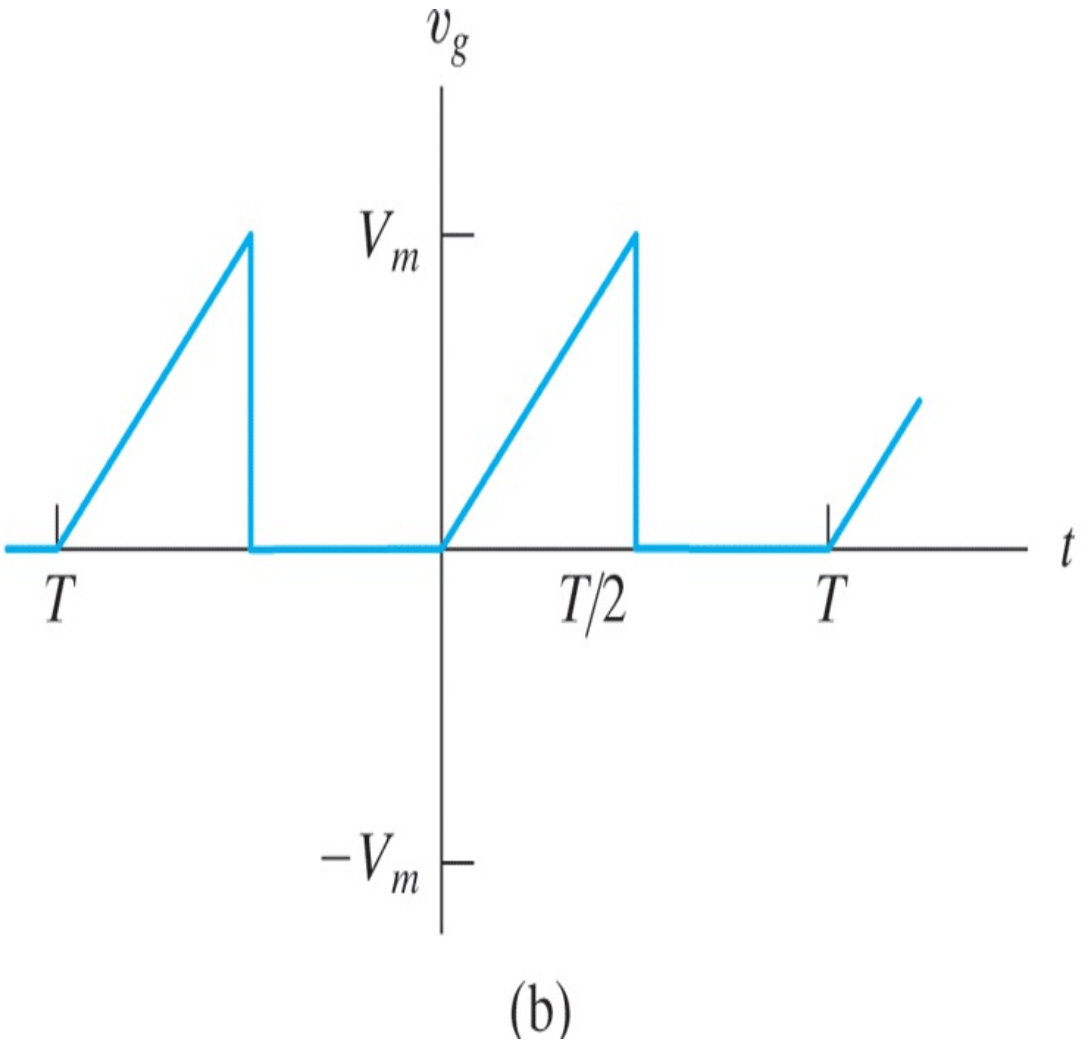


Figure P16.47 Full Alternative Text

5. 16.48

    1. Find the rms value of the periodic voltage in Fig. P16.47(b).

    2. Estimate the rms value of vg using the complex coefficients derived in Problem 16.47(b).

    3. What is the percentage of error in the estimated rms value of vg?

Section 16.9

1. 16.49

    1. Make an amplitude and phase plot, based on Eq. 16.20, for the periodic voltage in Example 16.3. Assume Vm is 40 V. Plot both amplitude and phase versus nωo, where n=0, 1, 2, 3, …

    2. Repeat (a), but base the plots on Eq. 16.32.

2. 16.50

    1. Make amplitude and phase plots for the periodic voltage in Problem 16.33, based on Eq. 16.20. Plot both amplitude and phase versus nωo where n=0, 1, 2, ….

    2. Repeat (a), but base the plots on Eq. 16.32.

3. 16.51 A periodic voltage is represented by a truncated Fourier series. The amplitude and phase spectra are shown in Fig. P16.51(a) and (b), respectively.

    1. Write an expression for the periodic voltage using the form given by Eq. 16.20.

    2. Is the voltage an even or odd function of t?

    3. Does the voltage have half-wave symmetry?

    4. Does the voltage have quarter-wave symmetry?

Figure P16.51

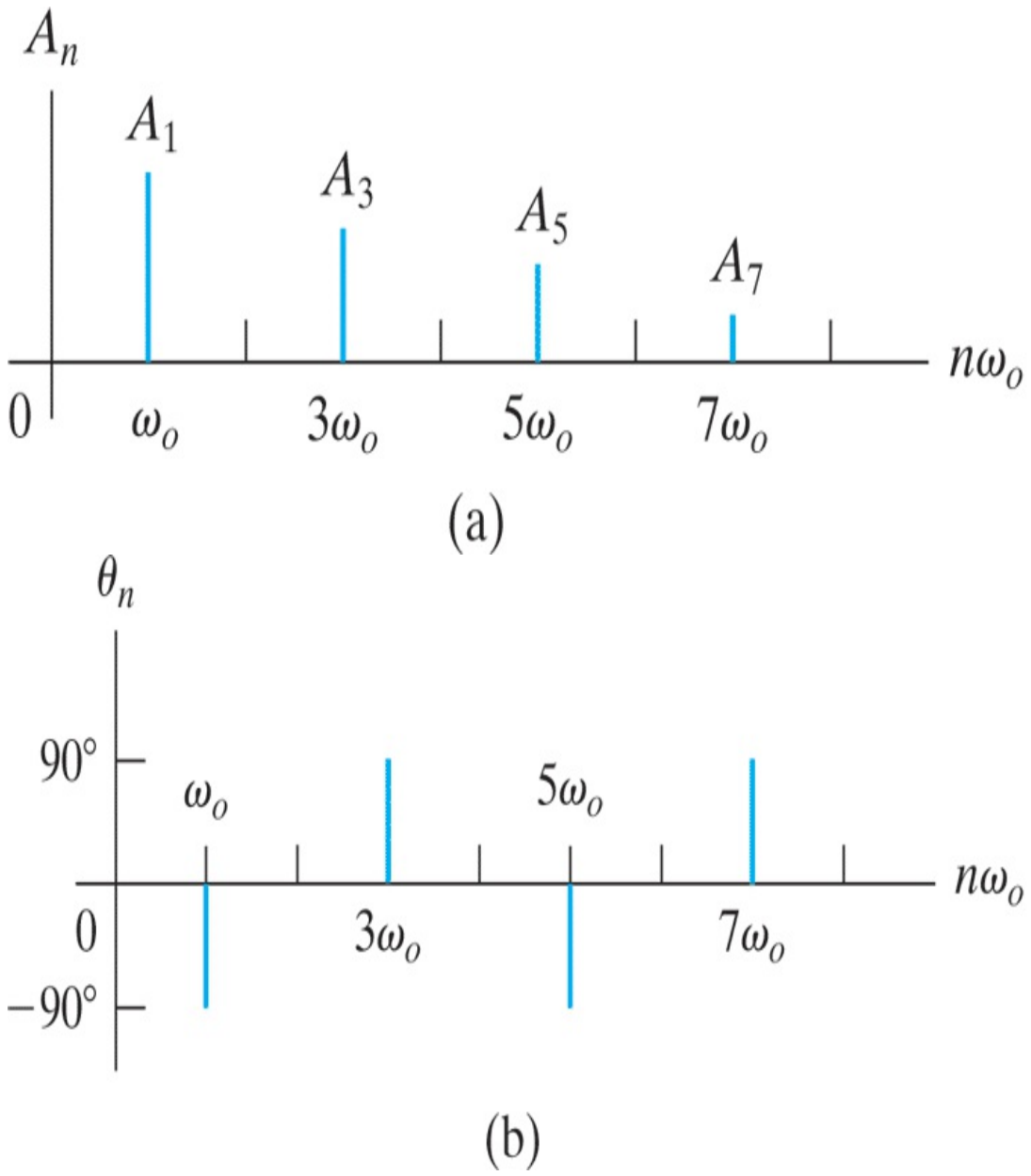


Figure P16.51 Full Alternative Text

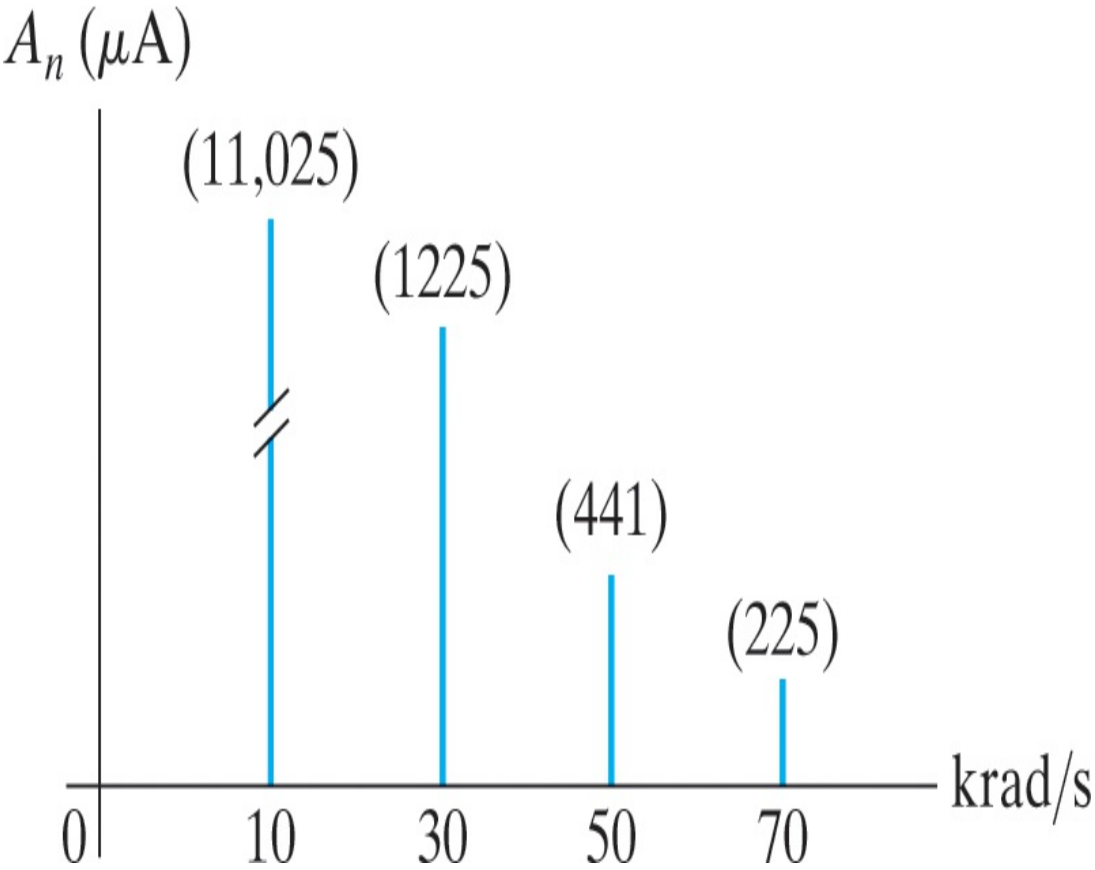
4. 16.52 A periodic function is represented by a Fourier series that has a finite number of terms. The amplitude and phase spectra are shown in Fig. P16.52(a) and (b), respectively.

    1. Write the expression for the periodic current using the form given by Eq. 16.20.

    2. Is the current an even or odd function of t?

3. Does the current have half-wave symmetry?
4. Calculate the rms value of the current in milliamperes.
5. Write the exponential form of the Fourier series.
6. Make the amplitude and phase spectra plots on the basis of the exponential series.

Figure P16.52



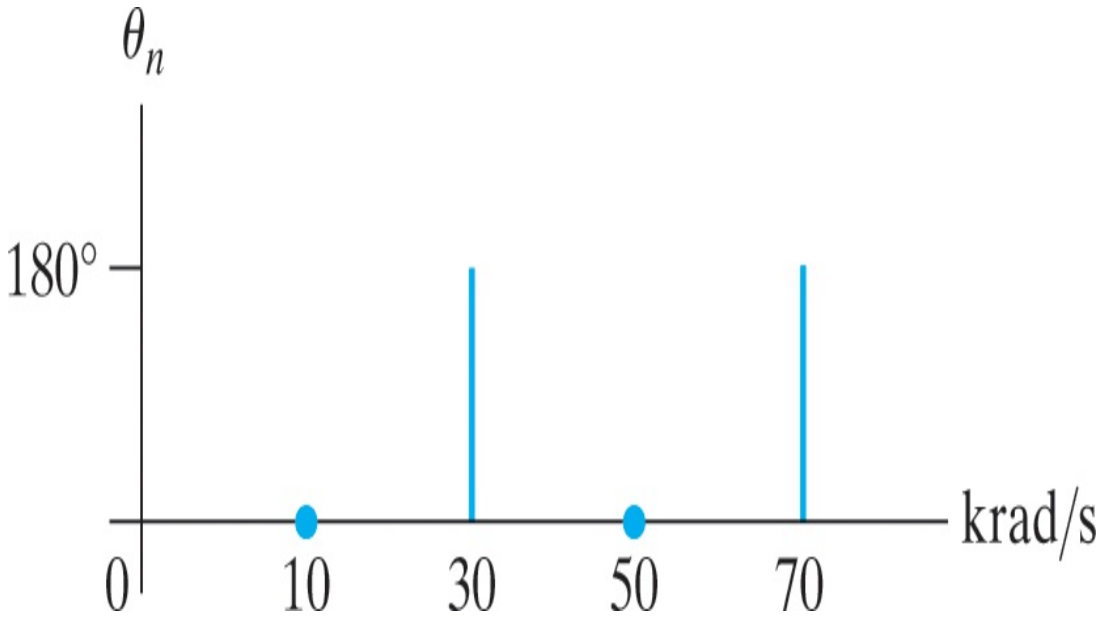


Figure P16.52 Full Alternative Text

Sections 16.1–16.9

1. 16.53 The input signal to a unity-gain third-order low-pass Butterworth filter is a half-wave rectified sinusoidal voltage. The corner frequency of the filter is 100 rad/s. The amplitude of the sinusoidal voltage is 54π V and its period is 5π ms. Write the first three terms of the Fourier series that represents the steady-state output voltage of the filter.

2. 16.54 The input signal to a unity-gain second-order low-pass Butterworth filter is the periodic triangular-wave voltage shown in Fig P16.54. The corner frequency of the filter is 2 krad/s. Write the first three terms of the Fourier series that represents the steady-state output voltage of the filter.

Figure P16.54

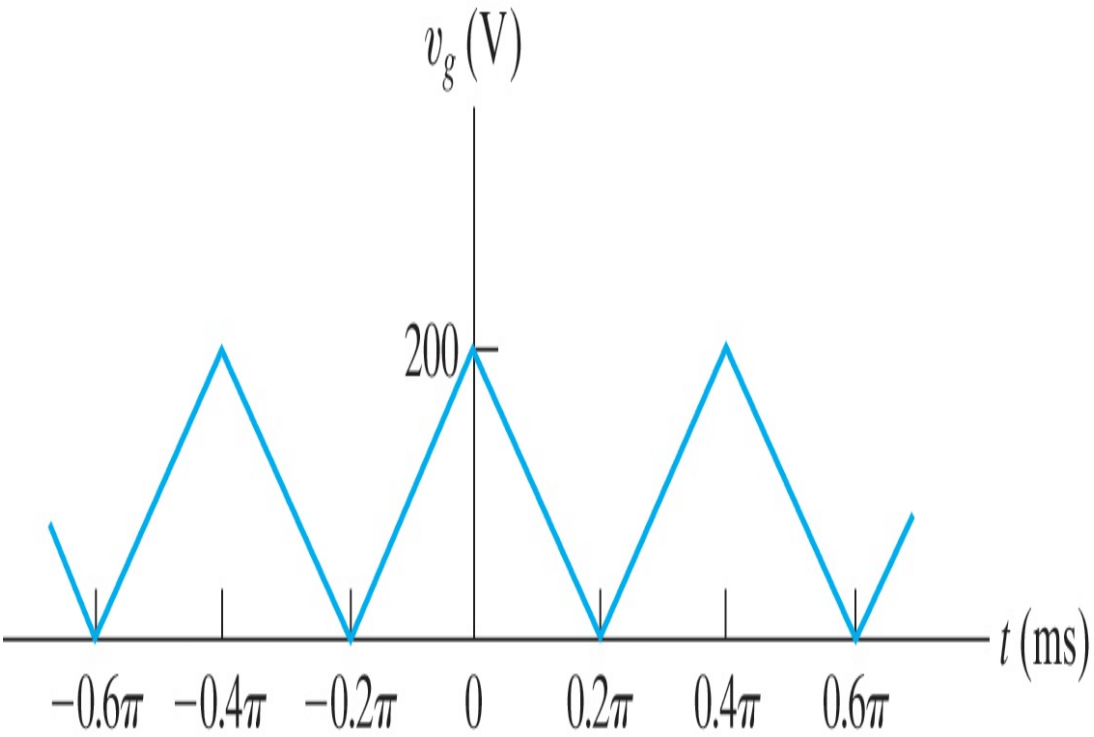


Figure P16.54 Full Alternative Text

3. 16.55 The input signal to a unity-gain second-order high-pass Butterworth filter is a full-wave rectified sine wave with an amplitude of 2.5π V and a fundamental frequency of 5000 rad/s. The corner frequency of the filter is 1 krad/s. Write the first two terms in the Fourier series that represents the steady-state output voltage of the filter.

4. 16.56 The transfer function (Vo/Vg) for the narrowband bandpass filter circuit in Fig. P16.56(a) is

   $H(s)=-K_o\beta s/(s^2+\beta s+\omega_o^2)$.

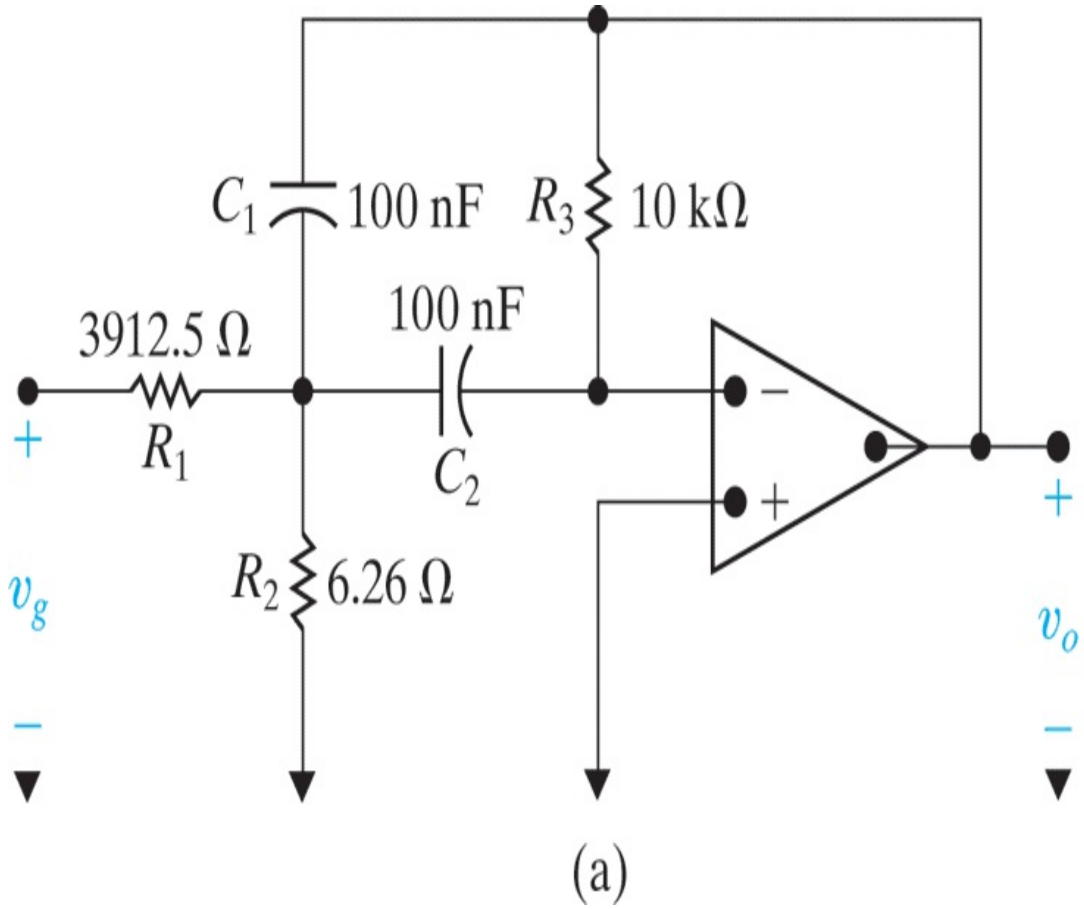
   1. Find Ko, β, and ωo2 as functions of the circuit parameters R1, R2, R3, C1, and C2.

   2. Write the first three terms in the Fourier series that represents vo if vg is the periodic voltage in Fig. P16.56(b).

   3. Predict the value of the quality factor for the filter by examining the result in part (b).

4. Calculate the quality factor for the filter using β and ωo and compare the value to your prediction in part (c).

Figure P16.56



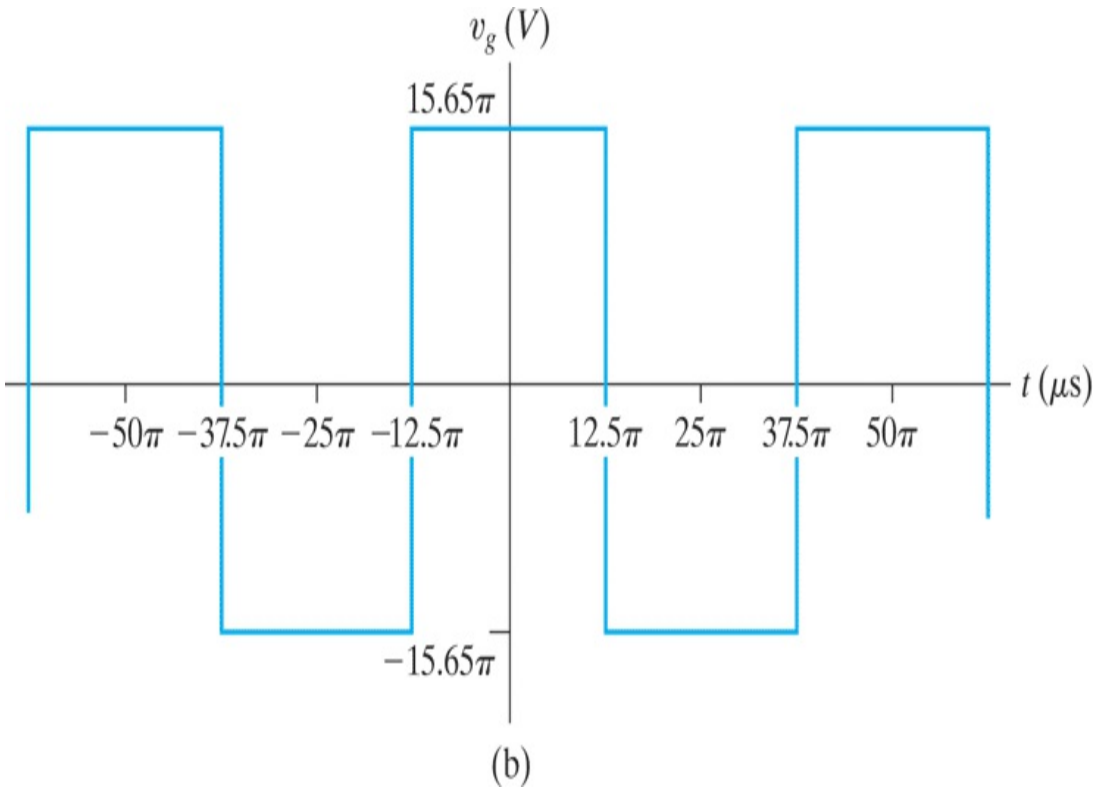


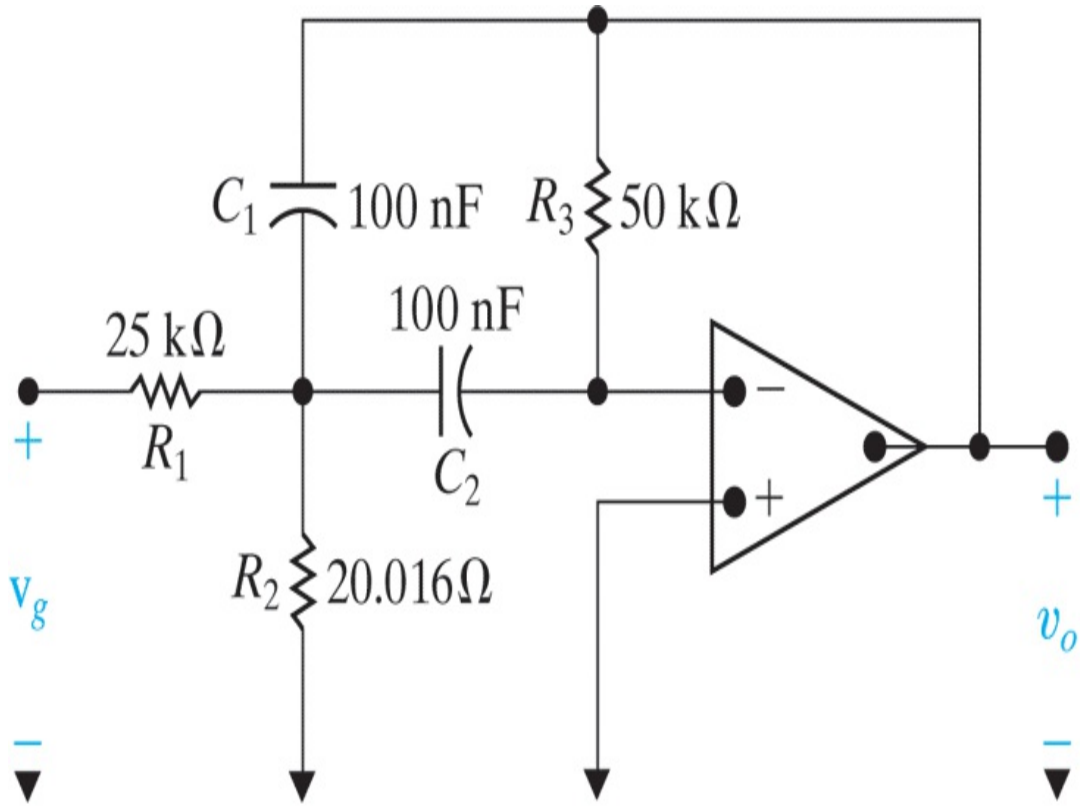
Figure P16.56 Full Alternative Text

5. 16.57

    1. Find the values for K, β, and ωo2 for the bandpass filter shown in Fig. P16.57(a).

    2. Find the first three terms in the Fourier series for vo in Fig. P16.57(a) if the input to the filter is the waveform shown in Fig. P16.57(b).

Figure P16.57



(a)

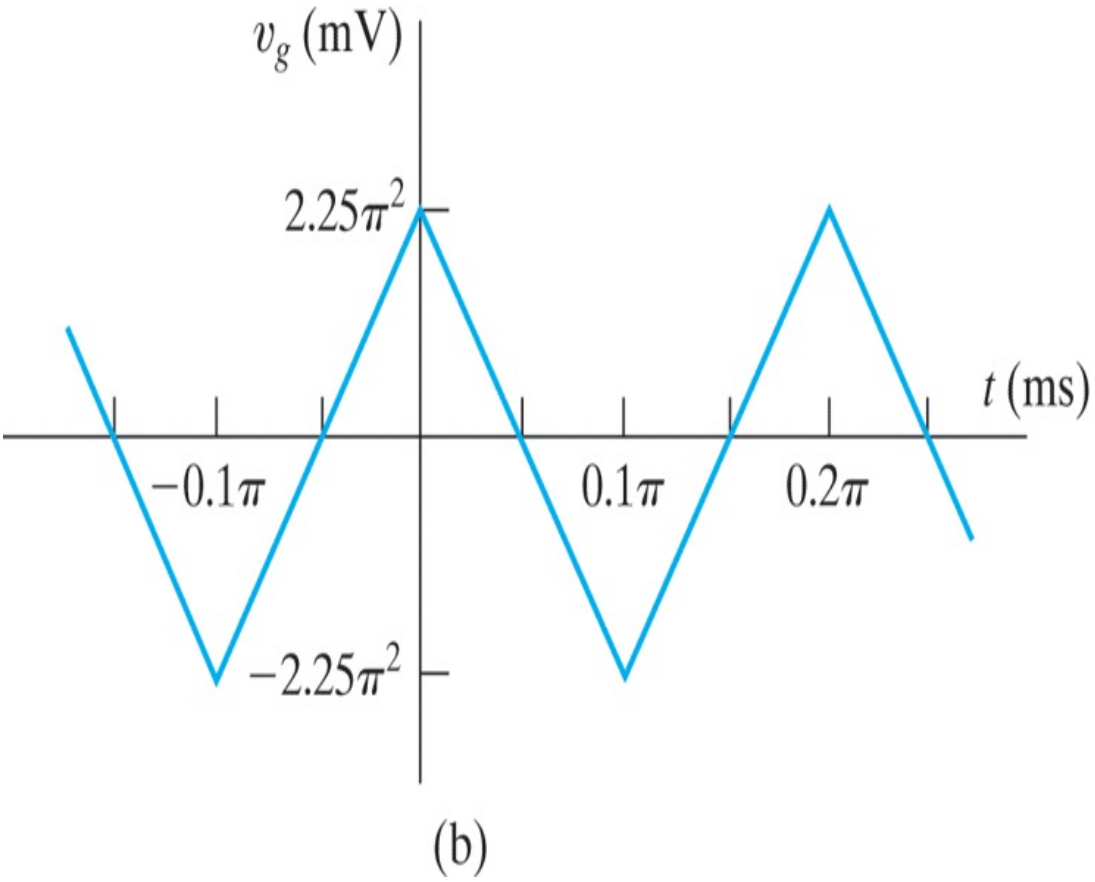


Figure P16.57 Full Alternative Text

Chapter 17 The Fourier Transform

Chapter Contents

1. 17.1 The Derivation of the Fourier Transform
2. 17.2 The Convergence of the Fourier Integral
3. 17.3 Using Laplace Transforms to Find Fourier Transforms
4. 17.4 Fourier Transforms in the Limit
5. 17.5 Some Mathematical Properties
6. 17.6 Operational Transforms
7. 17.7 Circuit Applications
8. 17.8 Parseval’s Theorem

Chapter Objectives

1. Be able to calculate the Fourier transform of a function using any or all of the following:
    - The definition of the Fourier transform;
    - Laplace transforms;
    - Mathematical properties of the Fourier transform;
    - Operational transforms.
2. Know how to use the Fourier transform to find the response of a circuit.
3. Understand Parseval's theorem and be able to use it to answer questions about the energy contained within specific frequency bands.

In Chapter 16, we represented periodic functions with a Fourier series. This series describes the periodic function using the frequency-domain attributes of amplitude and phase angle. The **Fourier transform** extends this frequency-domain representation to functions that are not periodic. But we have already transformed aperiodic functions from the time domain to the frequency domain, using the Laplace transform. You may wonder, then, why yet another type of transformation is necessary.

Strictly speaking, the Fourier transform is not a new transform. It is a special case of the bilateral Laplace transform, with the real part of the complex frequency set to zero. However, in terms of a physical interpretation, the Fourier transform is better viewed as a limiting case of a Fourier series. We present this point of view in Section 17.1, where we derive the Fourier transform equations.

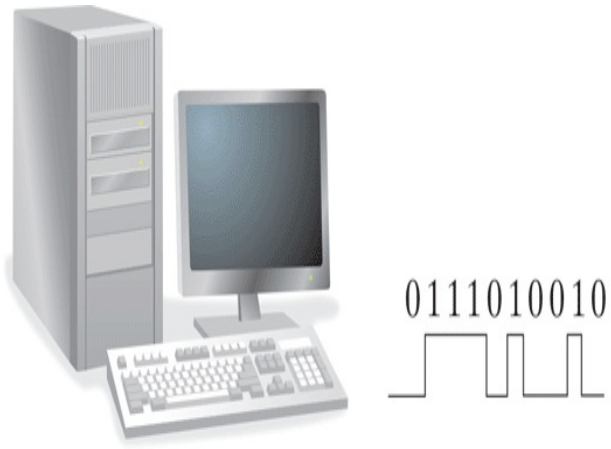
The Fourier transform is more useful than the Laplace transform in certain communications theory and signal-processing applications. Although we cannot pursue the Fourier transform in depth, its introduction here seems

appropriate while the ideas underlying the Laplace transform and the Fourier series are still fresh in your mind.

Practical Perspective

Filtering Digital Signals

It is common to use telephone lines to communicate information from one computer to another. As you may know, computers represent all information as collections of 1’’s and 0’’s. The value 1 is represented as a voltage, usually 5 V, and 0 is represented as 0 V, as shown below. The telephone line has a frequency response characteristic that is similar to a low-pass filter. We can use Fourier transforms to understand the effect of transmitting a digital value using a telephone line that behaves like a filter.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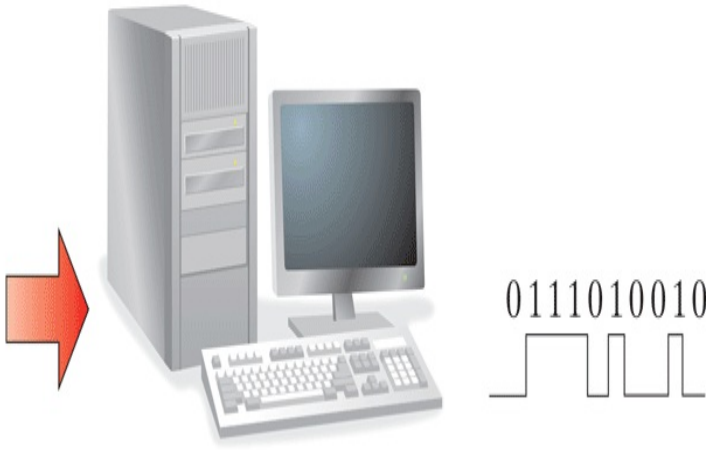
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17.1-1 Full Alternative Text

17.1 The Derivation of the Fourier Transform

We derive the Fourier transform, viewed as a limiting case of a Fourier series and using the exponential form of the series:

$$f(t)=\sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}, \quad (17.1)$$

where

$$C_n=\frac{1}{T}\int_{-T/2}^{T/2} f(t)e^{-jn\omega_0 t}\,dt. \quad (17.2)$$

In Eq. 17.2, we elected to start the integration at t0=-T/2.

If the period T increases without limit, f(t) transitions from a periodic to an aperiodic function. In other words, if T is infinite, f(t) never repeats itself and hence is aperiodic. As T increases, the separation between adjacent harmonic frequencies becomes smaller and smaller. In particular,

$$\Delta\omega=(n+1)\omega_0-n\omega_0=\omega_0=\frac{2\pi}{T},$$

and as T gets larger and larger, the incremental separation Δω approaches a differential separation dω. Therefore,

$$\frac{1}{T}\rightarrow\frac{d\omega}{2\pi} \quad \text{as} \quad T\rightarrow\infty.$$

As the period increases, the frequency is no longer a discrete variable but is instead a continuous variable, or

$$n\omega_0\rightarrow\omega \quad \text{as} \quad T\rightarrow\infty.$$

We can also see from Eq. 17.2 that, as the period increases, the Fourier coefficients Cn get smaller. In the limit, $C_n\rightarrow 0$ as $T\rightarrow\infty$. This result makes sense because we expect the Fourier coefficients to vanish when the function is no longer periodic. Note, however, the limiting value of the product CnT;

that is,

$$CnT \rightarrow \int_{-\infty}^{\infty} f(t)e^{-j\omega t}\,dt \quad \text{as} \quad T \rightarrow \infty.$$

In writing the limiting value of CnT, we replaced nω0 with ω. The integral represents the **Fourier transform** of f(t) and is denoted

Fourier Transform

$$F(\omega) = F\{f(t)\} = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}\,dt. \quad (17.3)$$

We obtain an explicit expression for the inverse Fourier transform by investigating the limiting form of Eq. 17.1 as $T \rightarrow \infty$. We begin by multiplying and dividing Eq. 17.1 by T:

$$f(t) = \sum_{n=-\infty}^{\infty} (CnT)e^{jn\omega 0t}\left(\frac{1}{T}\right).$$

As $T \rightarrow \infty$, the summation approaches integration, $CnT \rightarrow F(\omega)$, $n\omega 0 \rightarrow \omega$, and $1/T \rightarrow d\omega/2\pi$. Thus, in the limit, f(t) becomes

Inverse Fourier Transform

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}\,d\omega. \quad (17.4)$$

Equations 17.3 and 17.4 define the Fourier transform. Equation 17.3 transforms the time-domain expression f(t) into its corresponding frequency-domain expression F(ω). Equation 17.4 defines the inverse operation, transforming F(ω) intof(t).

Let's now derive the Fourier transform of the pulse shown in Fig. 17.1. Note that this pulse corresponds to the periodic voltage in Example 16.7 if we let $T \rightarrow \infty$. The Fourier transform of v(t) comes directly from Eq. 17.3:

$$V(\omega) = \int_{-\tau/2}^{\tau/2} Vme^{-j\omega t}\,dt = Vme^{-j\omega t}(-j\omega)\Big|_{-\tau/2}^{\tau/2} = Vm - j\omega(-2j\sin\omega\tau 2),$$

Figure 17.1 A voltage pulse.

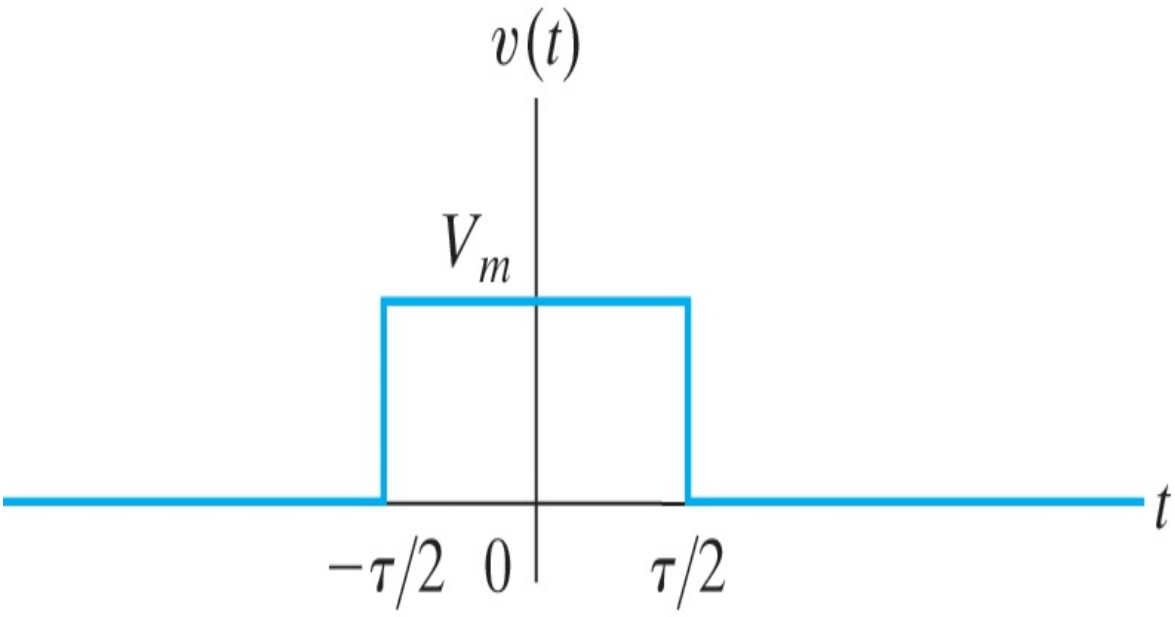


Figure 17.1 Full Alternative Text

which can be written in the form (sinx)/x by multiplying the numerator and denominator by τ/2. Then,

V(ω)=Vmτsinωτ/2ωτ/2. (17.5)

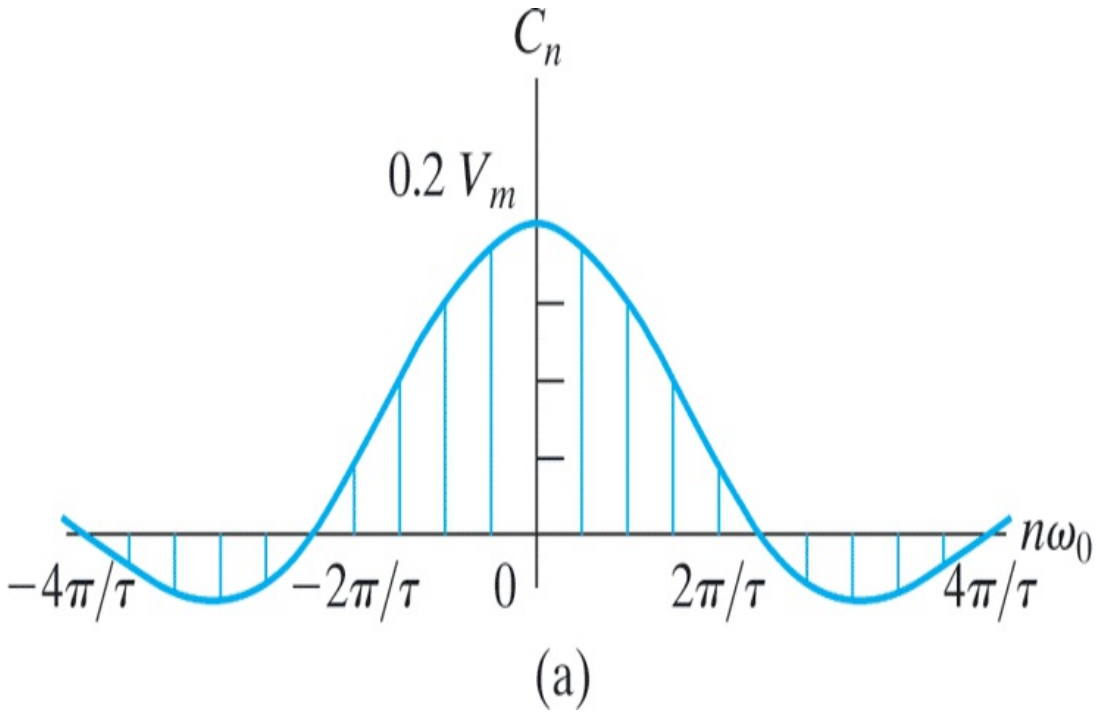
For the periodic voltage pulses in Example 16.7, the expression for the Fourier coefficients is

Cn=VmτTsin nω0τ/2nω0τ/2. (17.6)

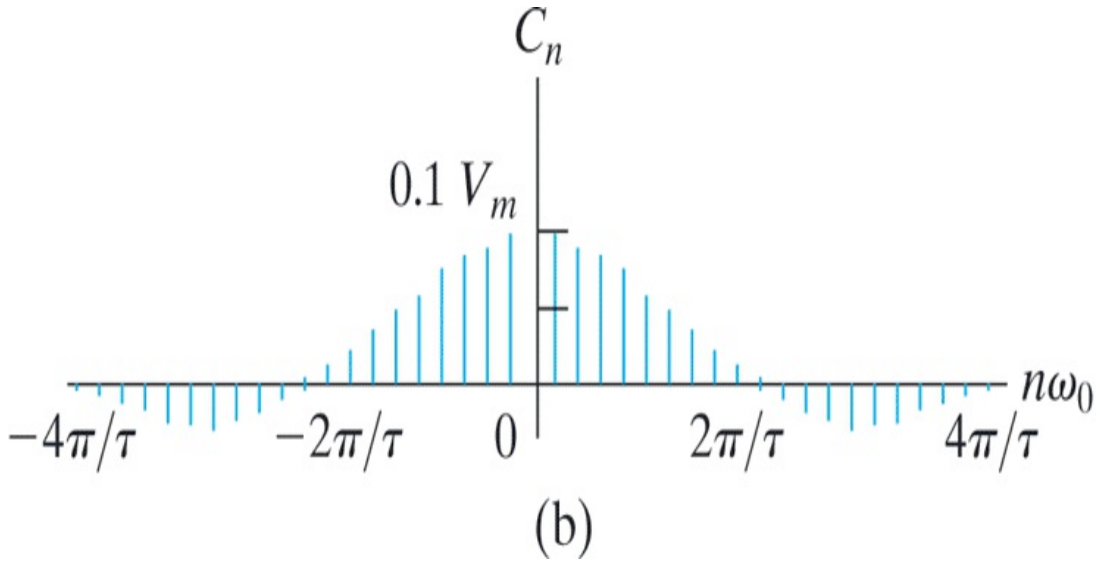
Compare Eqs. 17.5 and 17.6. Note that as the time-domain function goes from periodic to aperiodic, the amplitude spectrum goes from a discrete line spectrum to a continuous spectrum. Furthermore, the envelope of the line spectrum has the same shape as the continuous spectrum. Thus, as T increases, the lines in the spectrum get closer together and their amplitudes

get smaller, but their envelope doesn’’t change shape. The physical interpretation of the Fourier transform V(ω) is therefore a measure of the frequency content of v(t). Figure 17.2 illustrates these observations. The amplitude spectrum plot is based on the assumption that τ is constant and T is increasing.

Figure 17.2 Transition of the amplitude spectrum as f(t) goes from periodic to aperiodic. (a) Cn versus nω0, T/τ=5; (b) Cn versus nω0, T/τ=10; (c) V(ω) versus ω



17.1-2 Full Alternative Text



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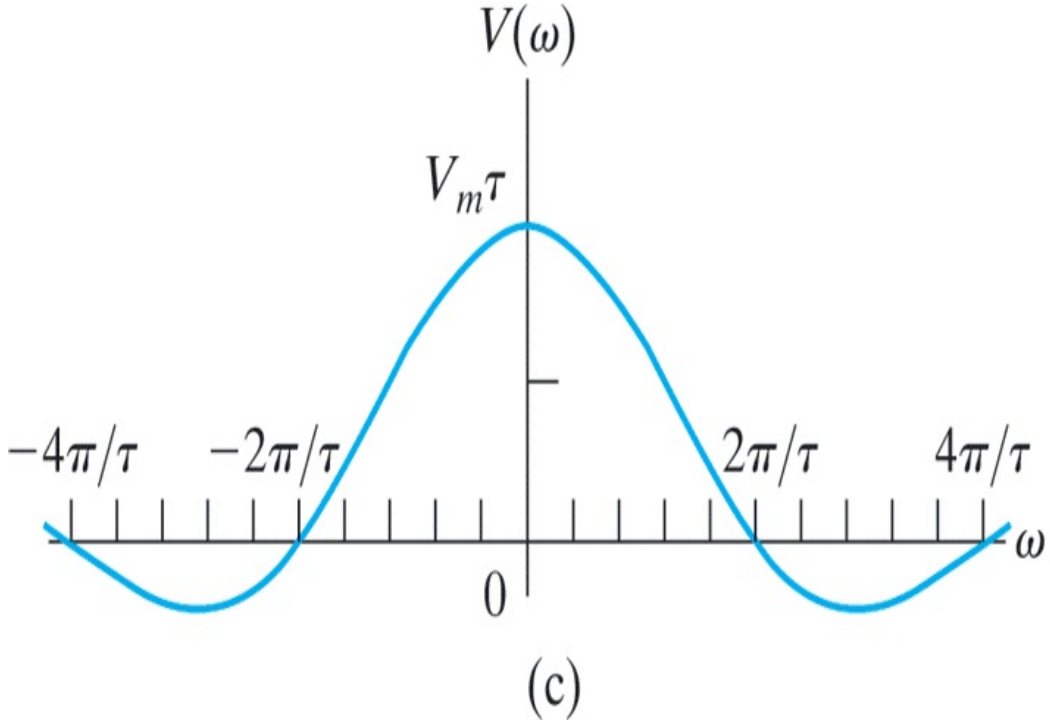


17.1-2 Full Alternative Text

17.2 The Convergence of the Fourier Integral

A function of time f(t) has a Fourier transform if the integral in Eq. 17.3 converges. If f(t) is a well-behaved function that differs from zero over a finite interval of time, convergence is no problem. *Well-behaved* implies that f(t) is single valued and encloses a finite area over the range of integration. In practical terms, all pulses of finite duration that interest us are well-behaved functions. The evaluation of the Fourier transform of the rectangular pulse discussed in Section 17.1 illustrates this point.

If f(t) is different from zero over an infinite interval, the convergence of the Fourier integral depends on the behavior of f(t) as $t \to \infty$. A single-valued function that is nonzero over an infinite interval has a Fourier transform if the integral

$$\int_{-\infty}^{\infty} |f(t)| \, dt$$

exists and if any discontinuities in f(t) are finite. An example is the decaying exponential function illustrated in Fig. 17.3. The Fourier transform of f(t) is

Figure 17.3 The decaying exponential function Ke-atu(t).

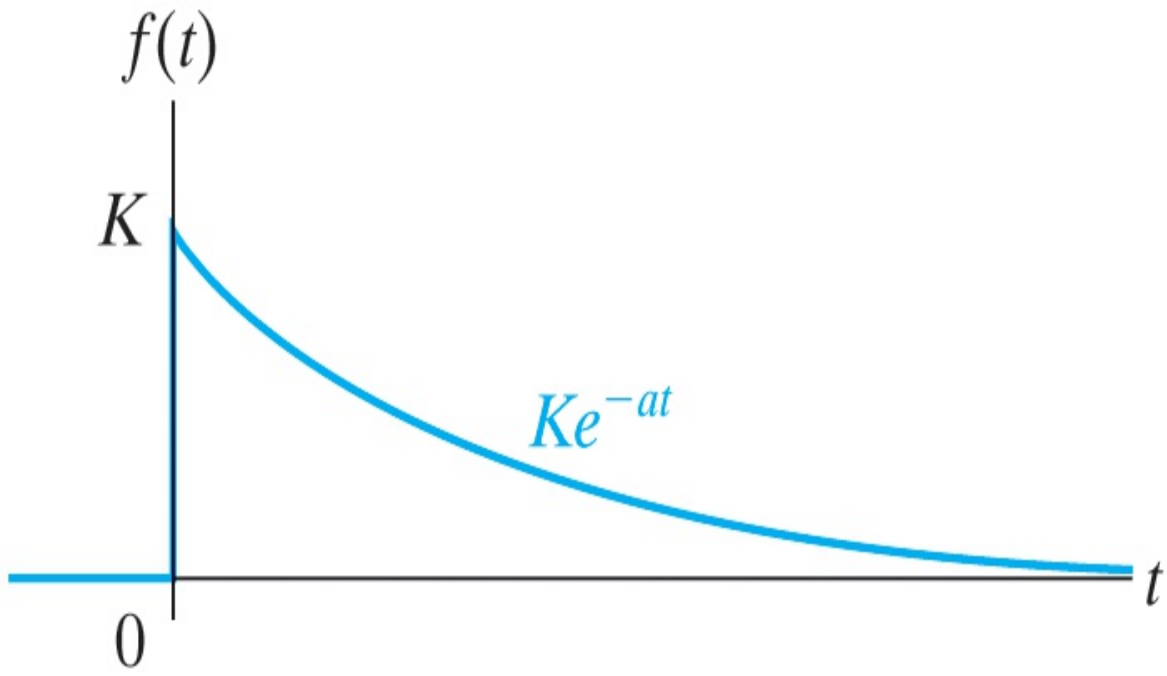


Figure 17.3 Full Alternative Text

$F(\omega)=\int_{-\infty}^{\infty} f(t)e^{-j\omega t}\,dt=\int_{0}^{\infty} Ke^{-at}e^{-j\omega t}\,dt=Ke^{-(a+j\omega)t}/-(a+j\omega)|_{0}^{\infty}=K/(a+j\omega)\,(0-1)=K/a+j\omega, \quad a>0.$

A third important group of functions has great practical interest, but these functions do not, in a strict sense, have Fourier transforms. For example, the integral in Eq. 17.3 doesn''t converge if f(t) is a constant. The same can be said if f(t) is a sinusoidal function, cos ω0t, or a step function, Ku(t). These functions are very important in circuit analysis, but, to include them in Fourier analysis, we must resort to the following mathematical manipulations.

- Create a function in the time domain that has a Fourier transform and at the same time can be made arbitrarily close to the function of interest. Call the approximating function f′(t).
- Find the Fourier transform of the approximating function, F′(ω), and then evaluate the limiting value of F′(ω) as f'(t) approaches f(t).
- Define the limiting value of F′(ω) as the Fourier transform of f(t).

Example 17.1 illustrates this three-step process.

Example 17.1 Finding the Fourier Transform of a Constant

Find the Fourier transform of a constant function, f(t)=A.

Solution

Approximate a constant with the exponential function

$f'(t)=Ae^{-\in|t|}, \quad \in>0.$

As $\in \to 0$, $f'(t) \to A$. Figure 17.4 shows the approximation graphically.

Figure 17.4 The approximation of a constant with an exponential function.

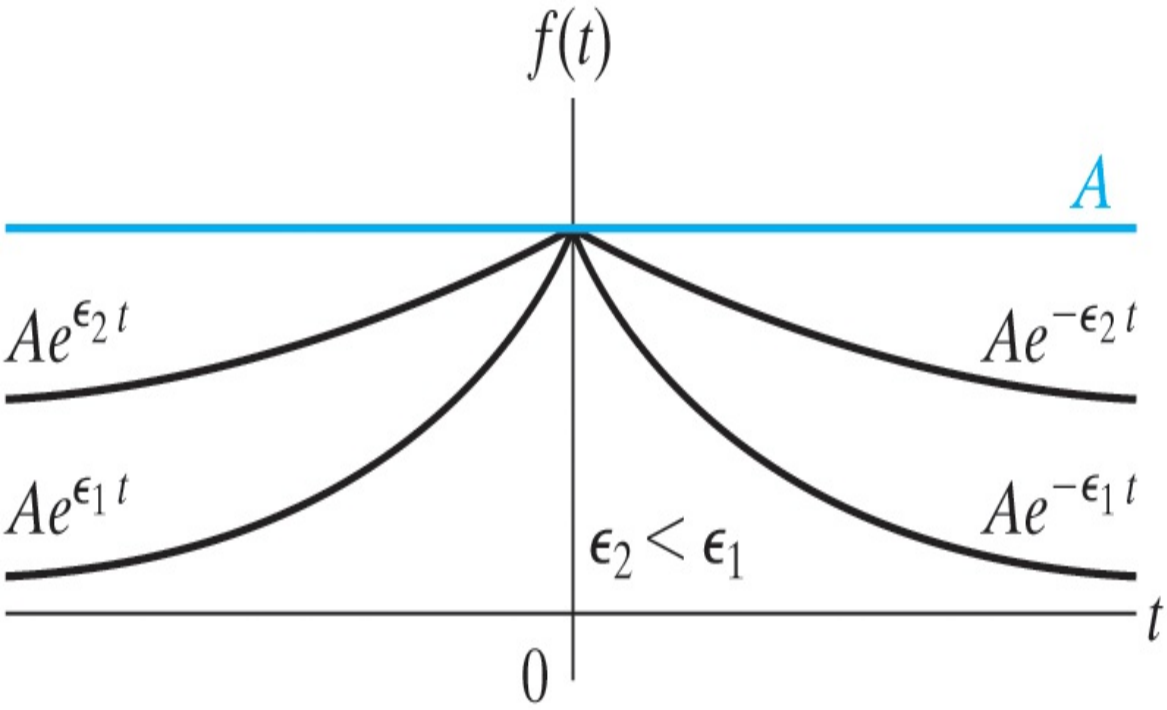


Figure 17.4 Full Alternative Text

The Fourier transform of f'(t) is

$F'(\omega)=\int_{-\infty}^{0} Ae^{\epsilon t}e^{-j\omega t}dt+\int_{0}^{\infty} Ae^{-\epsilon t}e^{-j\omega t}dt.$

Evaluating the integrals gives

$F'(\omega)=\frac{A}{\epsilon-j\omega}+\frac{A}{\epsilon+j\omega}=\frac{2\epsilon A}{\epsilon^2+\omega^2}.$

This function generates an impulse function at $\omega=0$ as $\epsilon \to 0$. You can verify this result by showing that

- $F'(\omega)$ approaches infinity at $\omega=0$ as $\epsilon \to 0$.
- The width of $F'(\omega)$ approaches zero as $\epsilon \to 0$.
- The area under $F'(\omega)$ is independent of ϵ.

The area under $F'(\omega)$ is the strength of the impulse and is

$\int_{-\infty}^{\infty}\frac{2\epsilon A}{\epsilon^2+\omega^2}d\omega=4\epsilon A\int_{0}^{\infty}\frac{d\omega}{\epsilon^2+\omega^2}=2\pi A.$

In the limit, f'(t) approaches a constant A, and F′(ω) approaches an impulse function 2πAΔ(ω). Therefore, the Fourier transform of a constant A is defined as 2πAΔ(ω), or

F{A}=2πAΔ(ω).

Assessment Problems

Objective 1——Be able to calculate the Fourier transform of a function

1. 17.1 Use the defining integral to find the Fourier transform of the following functions:

    1. f(t)=−A,−τ/2≤t<0;f(t)=A,0 < t ≤τ/2;f(t)=0elsewhere.

    2. f(t)=0,t<0;f(t)=te−at,t≥0,a>0.

    Answer:

    1. −j(2Aω)(1−cosωτ2);

    2. 1(a+jω)2.

2. 17.2 The Fourier transform of f(t) is given by

    F(ω)=0,−∞≤ω<−3 s;F(ω)=4,−3 s<ω<
    −2 s;F(ω)=1,−2 s<ω<2 s;F(ω)=4,2 s<ω<3 s;F(ω)=0,3 s<ω≤∞.

    Find f(t).

    Answer:

    f(t)=1πt(4 sin 3t−3 sin 2t).

SELF-CHECK: Also try Chapter Problems 17.1 and 17.2.

In Section 17.4, we say more about Fourier transforms defined through a

limit process. Before doing so, in Section 17.3, we use the Laplace transform to find the Fourier transform of functions for which the Fourier integral converges.

17.3 Using Laplace Transforms to Find Fourier Transforms

We can use a table of unilateral, or one-sided, Laplace transform pairs to find the Fourier transform for functions whose Fourier integral converges. The Fourier integral converges when the poles of F(s) lie in the left half of the s plane. Note that if F(s) has poles in the right half of the s plane or along the imaginary axis, f(t) does not satisfy the constraint that $\int_{-\infty}^{\infty} |f(t)|\,dt$ exists.

The following rules apply when using Laplace transforms to find the Fourier transforms of such functions.

1. If f(t) is zero for $t \leq 0-$, replace s by jω in the Laplace transform of f(t) to get the Fourier transform of f(t). Thus

   $$\mathcal{F}\{f(t)\} = \mathcal{L}\{f(t)\}_{s=j\omega} \quad (17.7)$$

2. A negative-time function is nonzero for negative values of time and zero for positive values of time. The Fourier transform of a negative-time function exists because the range of integration on the Fourier integral goes from -∞ to +∞. To find the Fourier transform of such a negative-time function, reflect the function over to the positive time domain and find its one-sided Laplace transform. Replace s with -jω in the Laplace transform to get the Fourier transform of the original time function. Therefore,

   $$F\{f(t)\} = L\{f(-t)\}_{s=-j\omega} \quad (17.8)$$

3. Functions that are nonzero over all time can be resolved into positive- and negative-time functions. We use Eqs. 17.7 and 17.8 to find the Fourier transform of the positive- and negative-time functions, respectively. The Fourier transform of the original function is the sum of the two transforms. Thus, if we let

$f+(t)=f(t)(\text{for } t>0), f-(t)=f(t)(\text{for } t<0),$

then

$f(t)=f+(t) + f-(t)$

and

$F\{ f(t) \}=F\{ f+(t) \}+F\{ f-(t) \}=L\{ f+(t) \}s=j\omega+L\{ f-(-t) \}s=-j\omega.$ (17.9)

If f(t) is even, Eq. 17.9 reduces to

$F\{f(t)\}=L\{f(t)\}s=j\omega+L\{f(t)\}s=-j\omega.$

If f(t) is odd, then Eq. 17.9 becomes

$F\{f(t)\}=L\{f(t)\}s=j\omega-L\{f(t)\}s=-j\omega.$

Example 17.2 uses the Laplace transform to find the Fourier transform.

Example 17.2 Finding the Fourier Transform from the Laplace Transform

Use the Laplace transform to find the Fourier transform for each of the following functions:

1. $f(t)=0, t\leq 0-; f(t)=e-at \cos \omega 0t, t\geq 0+.$
2. $f(t)=0, t \geq 0+; f(t)=eatcos \omega 0t, t \leq 0-.$
3. $f(t)=e-a|t|.$

Solution

1. Using Rule 1, find the Laplace transform of f(t) and substitute jω for s to get F(ω). Therefore,

   F{f(t)}=s+a(s+a)2+ω02|s=jω=jω+a(jω+a)2+ω02.

2. This is a negative-time function. Using Rule 2, reflect the function over to the positive-time domain to get

   f(−t)=0,(for t≤0−);f(−t)=e−atcosω0t,(for t≥0+).

   Both f(t) and its mirror image are plotted in Fig. 17.5. The Fourier transform of f(t) is

Figure 17.5 The reflection of a negative-time function over to the positive-time domain.

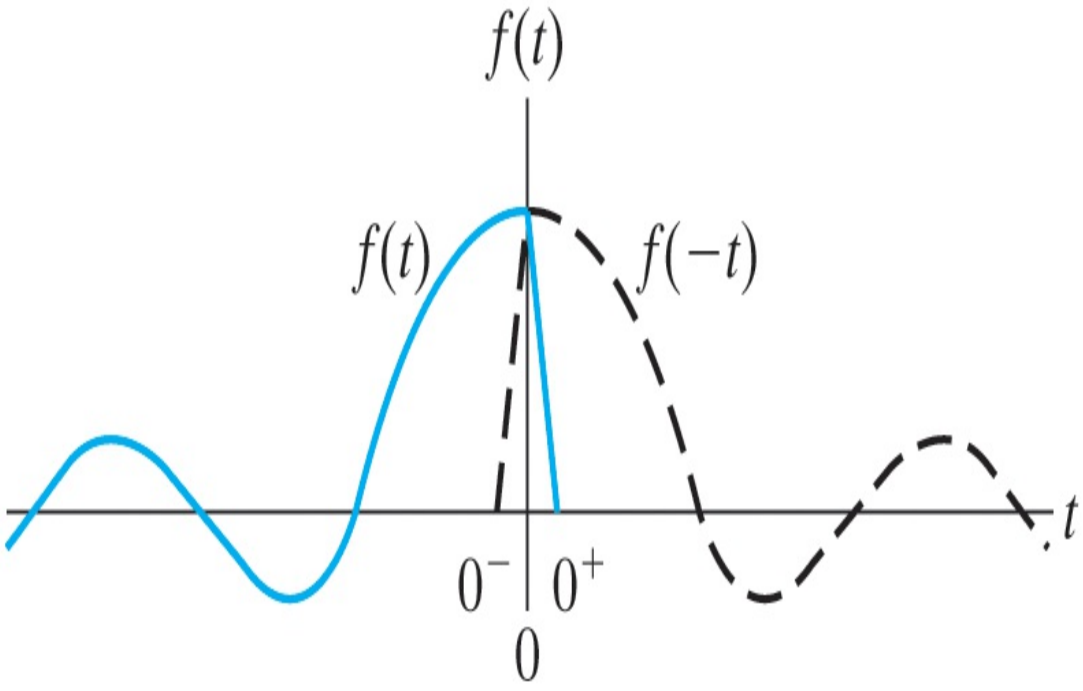


Figure 17.5 Full Alternative Text

$$F\{ f(t) \} = L\{ f(-t) \}_{s=-j\omega} = \frac{s+a}{(s+a)^2+\omega_0^2}\Big|_{s=-j\omega} = \frac{-j\omega+a}{(-j\omega+a)^2+\omega_0^2}.$$

3. This function is defined for all positive and negative time. Using Rule 3, we find that the positive- and negative-time functions are

   $$f_+(t) = e^{-at} \quad \text{and} \quad f_-(t) = e^{at}.$$

   Find the Laplace transform of $f_+(t)$ and $f_-(-t)$:

   $$L\{ f_+(t) \} = \frac{1}{s+a}, L\{ f_-(-t) \} = \frac{1}{s+a}.$$

   Therefore, from Eq. 17.9,

   $$F\{ e^{-a|\,t\,|} \} = \frac{1}{s+a}\Big|_{s=j\omega} + \frac{1}{s+a}\Big|_{s=-j\omega} = \frac{1}{j\omega+a} + \frac{1}{-j\omega+a} = \frac{2a}{\omega^2+a^2}.$$

Assessment Problem

Objective 1——Be able to calculate the Fourier transform of a function

1. 17.3 Find the Fourier transform of each function. In each case, a is a positive real constant.

   1. $f(t) = 0, t<0, f(t) = e^{-at}\sin \omega_0 t, t \geq 0.$
   2. $f(t) = 0, t>0, f(t) = -te^{at}, t \leq 0.$
   3. $f(t) = te^{-at}, t \geq 0, f(t) = te^{at}, t \leq 0.$

   Answer:

   1. $\frac{\omega_0}{(a+j\omega)^2+\omega_0^2};$
   2. $\frac{1}{(a-j\omega)^2};$
   3. $\frac{-j4a\omega}{(a^2+\omega^2)^2}.$

SELF-CHECK: Also try Chapter Problem 17.5.

17.4 Fourier Transforms in the Limit

As we pointed out in Section 17.2, the Fourier transforms of several practical functions must be defined by a limit process. We now return to these types of functions and develop their transforms.

The Fourier Transform of a Signum Function

We showed that the Fourier transform of a constant A is $2\pi A\Delta(\omega)$ in Example 17.1. Another function of interest is the signum function, defined as +1 for $t>0$ and -1 for $t<0$. The signum function is denoted sgn(t) and can be expressed in terms of unit step functions as

$$\text{sgn}(t)=u(t)-u(-t). \quad (17.10)$$

Figure 17.6 shows the function graphically.

Figure 17.6 The signum function.

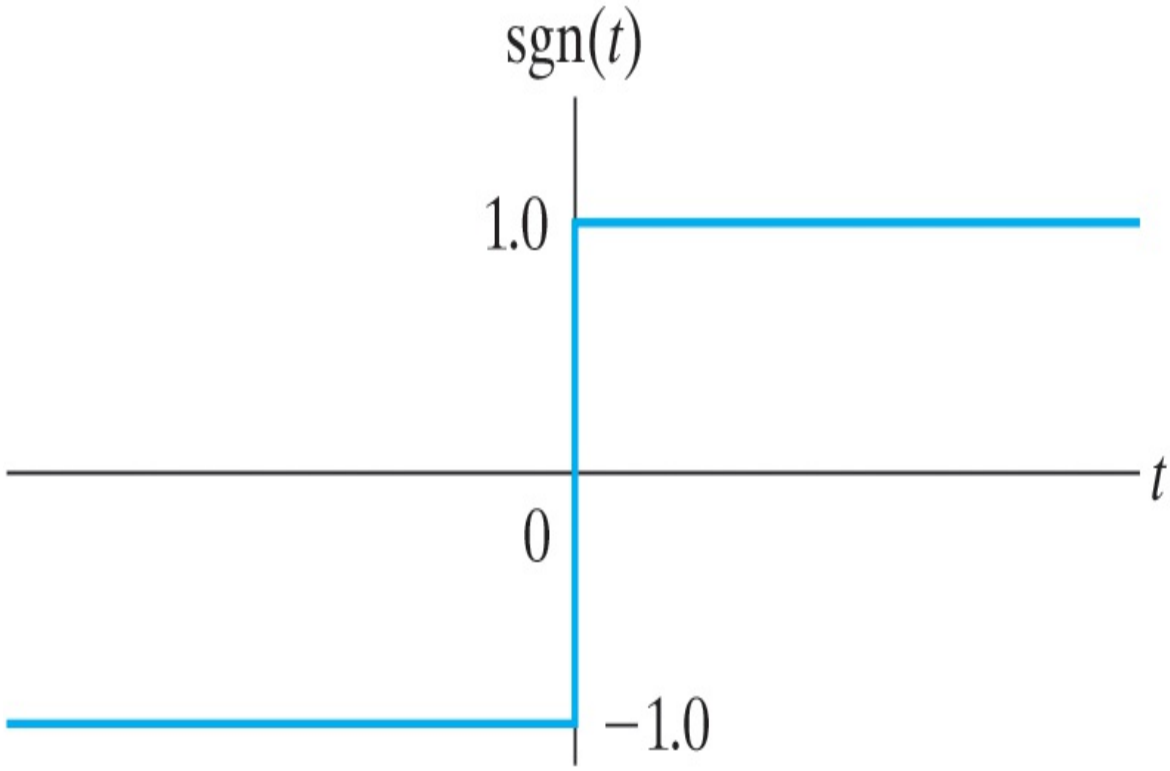


Figure 17.6 Full Alternative Text

To find the signum function’’s Fourier transform, create a function that approaches the signum function in the limit:

sgn (t)=lim∈→0[e-∈tu(t)-e∈tu(-t)], ∈>0.

The function inside the brackets, plotted in Fig. 17.7, has a Fourier transform because the Fourier integral converges. Since f(t) is an odd function, its Fourier transform is:

Figure 17.7 A function that approaches sgn (t) as ∈ approaches zero.

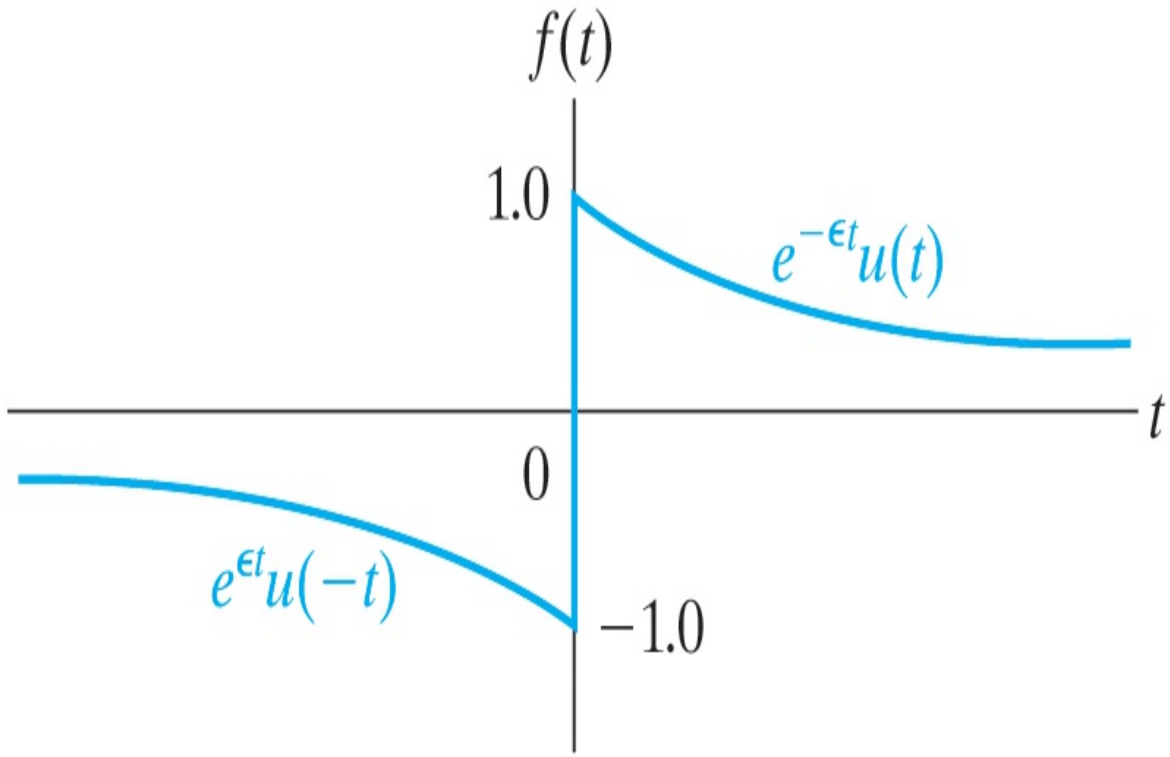


Figure 17.7 Full Alternative Text

F {f(t)}=1s + ∈|s=jω−1s + ∈|s=−jω=1jω + ∈−1−jω + ∈=−2jωω2+∈2.

As ∈ → 0, f(t) → sgn(t), and F {f(t)} → 2/jω. Therefore,

$\mathcal{F}${sgn (t)}=2jω.

The Fourier Transform of a Unit Step Function

To find the Fourier transform of a unit step function, we use the Fourier transforms of a constant and the signum function. Note that the unit step function can be expressed as

u(t)=12+12sgn (t).

Thus,

F { u(t) }=F { 12 }+F { 12sgn (t) }=πΔ(ω)+1jω.

The Fourier Transform of a Cosine Function

To find the Fourier transform of cos ω0t, we return to the inverse-transform integral of Eq. 17.4 and observe that if

F(ω)=2πΔ(ω-ω0),

then

f(t)=12π∫−∞∞[2πΔ(ω−ω0)]ejωt dω.

Using the sifting property of the impulse function, we simplify f(t) to

f(t)=ejω0t.

Therefore,

F {ejω0t}=2πΔ(ω-ω0).

We now use the Fourier transform of ejω0t to find the Fourier transform of cos ω0t because

cos ω0t=ejω0t+e-jω0t2.

Thus,

F { cos ω0t }=12(F { ejω0t }+F {e−jω0t })=12[2πΔ(ω−ω0)+2πΔ(ω +ω0)]=πΔ(ω−ω0)+πΔ(ω+ω0).

The Fourier transform of sin ω0t involves similar manipulation, which we leave for Problem 17.4. Table 17.1 presents a summary of the transform pairs of the important elementary functions.

Table 17.1 Fourier Transforms of Elementary Functions

Type	$f(t)$	$F(\omega)$
impulse	$\delta(t)$	1
constant	A	$2\pi A\delta(\omega)$
signum	$\text{sgn}(t)$	$2/j\omega$
step	$u(t)$	$\pi\delta(\omega) + 1/j\omega$
positive-time exponential	$e^{-at}u(t)$	$1/(a + j\omega), a > 0$
negative-time exponential	$e^{at}u(-t)$	$1/(a - j\omega), a > 0$
positive- and negative-time exponential	$e^{-a\|t\|}$	$2a/(a^2 + \omega^2), a > 0$
complex exponential	$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$
cosine	$\cos \omega_0 t$	$\pi[\delta(\omega + \omega_0) + \delta(\omega - \omega_0)]$
sine	$\sin \omega_0 t$	$j\pi[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$

We now turn to the properties of the Fourier transform that help us describe aperiodic time-domain behavior in terms of frequency-domain behavior.

17.5 Some Mathematical Properties

We begin by noting that F(ω) is a complex quantity and can be expressed in either rectangular or polar form. Thus, from the defining integral,

F(ω)=∫−∞∞f(t)e−jωtdt=∫−∞∞f(t)(cos ωt−j sin ωt) dt=∫−∞∞f(t) cos ωt dt−j∫−∞∞f(t) sin ωt dt.

Now we let

A(ω)=∫−∞∞f(t) cos ωt dtB(ω)=∫−∞∞f(t) sin ωt dt.

Thus, using the definitions for A(ω) and B(ω), we get

F(ω)=A(ω)+jB(ω)=| F(ω) |ejθ(ω).

We can make the following observations about F(ω):

- The real part of F(ω)—that— is, A(ω)—is an even function of ω, so A(ω)=A(-ω).
- The imaginary part of F(ω)——that is, B(ω)——is an odd function of ω, so B(ω)=-B(-ω).
- The magnitude of F(ω)—that— is, A2(ω)+B2(ω)—is— an even function of ω.
- The phase angle of F(ω)—that is, θ(ω)=tan-1B(ω)/A(ω)—is an odd function of ω.
- Replacing ω by -ω generates the conjugate of F(ω), so F(-ω)=F*(ω).

Hence, if f(t) is an even function, F(ω) is real, and if f(t) is an odd function, F(ω) is imaginary. If f(t) is even,

A(ω)=2∫0∞f(t)cos ωt dt (17.11)

and

$B(\omega)=0.$

If f(t) is an odd function,

$A(\omega)=0$

and

$B(\omega)=-2\int_0^\infty f(t)\sin \omega t\, dt.$

We leave the derivations for you as Problems 17.10 and 17.11.

If f(t) is an even function, its Fourier transform is an even function, and if f(t) is an odd function, its Fourier transform is an odd function. Moreover, if f(t) is an even function, from the inverse Fourier integral,

$$f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(\omega)e^{j\omega t}d\omega=\frac{1}{2\pi}\int_{-\infty}^{\infty}A(\omega)e^{j\omega t}d\omega=\frac{1}{2\pi}\int_{-\infty}^{\infty}A(\omega)(\cos\omega t+j\sin\omega t)d\omega=\frac{1}{2\pi}\int_{-\infty}^{\infty}A(\omega)\cos\omega t\, d\omega+0=\frac{2}{2\pi}\int_0^\infty A(\omega)\cos\omega t\, d\omega. \quad (17.12)$$

Now compare Eq. 17.12 with Eq. 17.11. Note that, except for a factor of $1/2\pi$, these two equations have the same form. Thus, the waveforms of A(ω) and f(t) become interchangeable if f(t) is an even function.

For example, we have already observed that a rectangular pulse in the time domain produces a frequency spectrum of the form $(\sin \omega)/\omega$. Specifically, Eq. 17.5 expresses the Fourier transform of the voltage pulse shown in Fig. 17.1. Hence, a rectangular pulse in the frequency domain must be generated by a time-domain function of the form $(\sin t)/t$. We can illustrate this transform by finding the time-domain function f(t) corresponding to the frequency spectrum shown in Fig. 17.8. From Eq. 17.12,

Figure 17.8 A rectangular frequency spectrum.

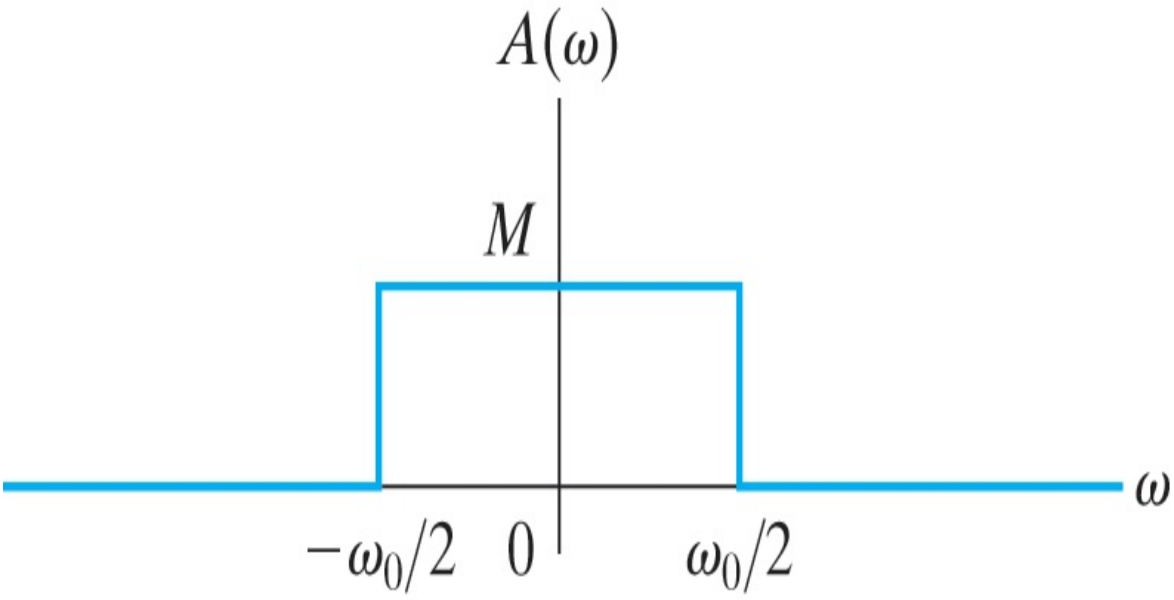


Figure 17.8 Full Alternative Text

f(t)=22π∫0ω0/2M cos ωt dω=2M2π(sin ωtt)|0ω0/2=12π(Msin ω0t/2t/2)=12π (M

We say more about the frequency spectrum of a rectangular pulse in the time domain versus the rectangular frequency spectrum of (sin t)/t after we introduce Parseval''s theorem.

17.6 Operational Transforms

Fourier transforms, like Laplace transforms, can be classified as functional and operational. So far, we have concentrated on the functional transforms. We now discuss some of the important operational Fourier transforms, which are similar to the operational Laplace transforms in Chapter 12. Hence, we leave their proofs to you as Problems 17.12–17.19.

Multiplication by a Constant

From the defining integral, if

$$F\{f(t)\}=F(\omega),$$

then

$$F\{Kf(t)\}=KF(\omega).$$

Thus, multiplication of f(t) by a constant corresponds to multiplying F(ω) by that same constant.

Addition (Subtraction)

Addition (subtraction) in the time domain translates into addition (subtraction) in the frequency domain. Thus, if

$$F\{f1(t)\}=F1(\omega), F\{f2(t)\}=F2(\omega), F\{f3(t)\}=F3(\omega),$$

then

$$F\{f1(t)-f2(t)+f3(t)\}=F1(\omega)-F2(\omega)+F3(\omega),$$

which is derived by substituting the algebraic sum of time-domain functions

into the defining integral.

Differentiation

The Fourier transform of the first derivative of f(t) is

$F\{df(t)dt\}=j\omega F(\omega)$.

The nth derivative of f(t) is

$F\{dnf(t)dtn\}=(j\omega)nF(\omega)$.

These equations are valid if f(t) is zero at $\pm\infty$.

Integration

If

$g(t)=\int_{-\infty}^{t} f(x)\,dx$,

then

$F\{g(t)\}=F(\omega)j\omega$.

This equation is valid if

$\int_{-\infty}^{\infty} f(x)\,dx=0$.

Scale Change

Dimensionally, time and frequency are reciprocals. Therefore, when time is expanded, frequency is compressed (and vice versa), as reflected in the functional transform

$$F\{f(at)\}=1aF(\omega a), \quad a>0.$$

Note that when $0<a<1.0$, time is expanded, whereas when $a>1.0$, time is compressed.

Translation in the Time Domain

Translating a function in the time domain alters the phase spectrum and leaves the amplitude spectrum unchanged. Thus

$$F\{f(t-a)\}=e-j\omega aF(\omega).$$

If a is positive in this equation, the time function is delayed (shifted to the right on the time axis), and if a is negative, the time function is advanced (shifted to the left on the time axis).

Translation in the Frequency Domain

Translation in the frequency domain corresponds to multiplication by the complex exponential in the time domain:

$$F\{ej\omega 0tf(t)\}=F(\omega-\omega 0).$$

Modulation

Amplitude modulation varies the amplitude of a sinusoidal carrier. If the carrier signal is f(t), the modulated carrier is $f(t)\cos \omega 0t$. The amplitude spectrum of the modulated carrier is one-half the amplitude spectrum of f(t) and is centered at $\pm\omega 0$, so,

$$F\{f(t)\cos \omega 0t\}=12F(\omega-\omega 0)+12F(\omega+\omega 0).$$

Convolution in the Time Domain

Convolution in the time domain corresponds to multiplication in the frequency domain. In other words,

$$y(t)=\int_{-\infty}^{\infty}x(\lambda)h(t-\lambda)\,d\lambda$$

becomes

$$F\{y(t)\}=Y(\omega)=X(\omega)H(\omega). \quad (17.13)$$

Equation 17.13 is important in Fourier transform applications because it states that the output's transform $Y(\omega)$ is the product of the input's transform $X(\omega)$ and the system function $H(\omega)$. We say more about this relationship in Section 17.7.

Convolution in the Frequency Domain

Convolution in the frequency domain corresponds to finding the Fourier transform of the product of two time functions. Thus, if

$$f(t)=f1(t)\,f2(t),$$

then

$$F(\omega)=12\pi\int_{-\infty}^{\infty}F1(u)F2(\omega-u)\,du.$$

Table 17.2 summarizes these 10 operational transforms and another operational transform that we introduce in Example 17.3.

Table 17.2 Operational

Transforms

$f(t)$	$F(\omega)$
$Kf(t)$	$KF(\omega)$
$f_1(t) - f_2(t) + f_3(t)$	$F_1(\omega) - F_2(\omega) + F_3(\omega)$
$d^n f(t)/dt^n$	$(j\omega)^n F(\omega)$
$\int_{-\infty}^{t} f(x)\,dx$	$F(\omega)/j\omega$
$f(at)$	$\frac{1}{a}F\left(\frac{\omega}{a}\right), \quad a > 0$
$f(t - a)$	$e^{-j\omega a}F(\omega)$
$e^{j\omega_0 t}f(t)$	$F(\omega - \omega_0)$
$f(t)\cos\omega_0 t$	$\frac{1}{2}F(\omega - \omega_0) + \frac{1}{2}F(\omega + \omega_0)$
$\int_{-\infty}^{\infty} x(\lambda)h(t - \lambda)\,d\lambda$	$X(\omega)H(\omega)$
$f_1(t)f_2(t)$	$\frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(u)F_2(\omega - u)\,du$
$t^n f(t)$	$(j)^n\frac{d^n F(\omega)}{d\omega^n}$

Table 17.2 Full Alternative Text

Example 17.3 Deriving an Operational Fourier Transform

Find the Fourier transform for the function f(t)=tnf(t), assuming that F(ω)=F {f(t)} is known.

Solution

From the definition of the Fourier transform in Eq. 17.3,

F(ω)=∫−∞∞f(t)e−jωtdt.

Take the derivative of both sides with respect to the frequency ω and simplify:

dF(ω)dω=∫−∞∞ddω[f(t)e−jωtdt]=−j∫−∞∞tf(t)e−jωtdt=−jF {tf(t) }.

Therefore,

F {tf(t)}=jdF(ω)dω.

Repeat this process, taking the second derivative of both sides of Eq. 17.3 with respect to ω:

d2F(ω)dω2=∫−∞∞d2dω2[f(t)e−jωtdt]=∫−∞∞(−jt)(−jt)f(t)e−jωtdt=(−j)2F { t2f(t) }.

Therefore,

F {t2f(t)}=(j)2d2F(ω)dω2.

Now take the nth derivative of both sides of Eq. 17.3 with respect to ?:

dnF(ω)dωn=∫−∞∞dndωn[f(t)e−jωtdt]=∫−∞∞(−jt)nf(t)e−jωtdt=(−j)nF { tnf(t) }.

Replacing (-j)n with 1/jn, we get

F {tnf(t)}=(j)ndnF(ω)dωn.

Therefore, the effect of multiplying a function by tn in the time domain corresponds to finding the nth derivative of the function''s Fourier transform with respect to ω.

Assessment Problems

Objective 1——Be able to calculate the Fourier transform of a function

1. 17.4 Suppose f(t) is defined as follows:

   f(t)=2Aττt + A,−τ2≤t≤0,f(t)=−2Aττt + A,0≤t≤τ2,f(t)=0,elsewhere.

   1. Find the second derivative of f(t).
   2. Find the Fourier transform of the second derivative.
   3. Use the result obtained in (b) to find the Fourier transform of the function in (a). (Hint: Use the operational transform of differentiation.)

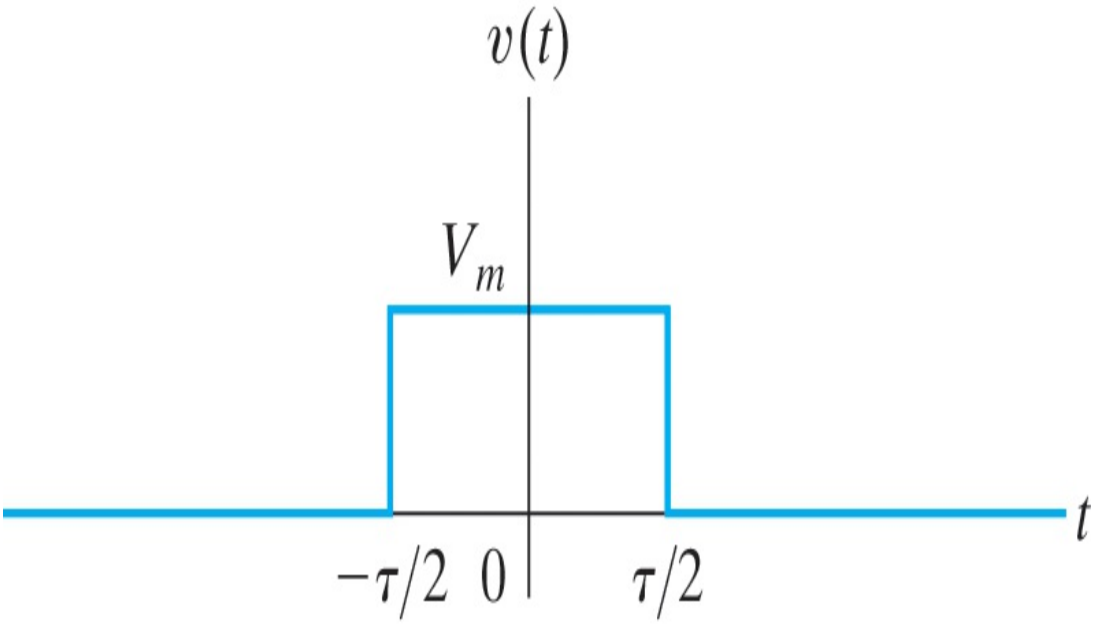
   Answer:

   1. 2AτΔ(t + τ2) − 4AτΔ(t)+ 2AτΔ(t−τ2);
   2. 4Aτ (cos ωτ2 - 1);
   3. 4Aω2τ (1-cos ωτ2).

2. 17.5 The rectangular pulse shown can be expressed as the difference between two step voltages; that is,

   v(t)=Vm u (t + τ2) - Vm u (t - τ2) V.

Use the operational transform for translation in the time domain to find the Fourier transform of v(t).



17.6-4 Full Alternative Text

Answer:

$V(\omega)=V_m\tau\sin(\omega\tau/2)(\omega\tau/2)$.

SELF-CHECK: Also try Chapter Problem 17.19.

17.7 Circuit Applications

The Laplace transform is used more often than the Fourier transform when finding a circuit’’s response to a nonperiodic input. This is because the Laplace transform integral converges for a wider range of input functions and it accommodates initial conditions. We can certainly use the Fourier transform to find a circuit’s response to an input using Eq. 17.13, which relates the transform of the output Y(ω) to the transform of the input X(ω) and the transfer function H(ω) of the circuit. Note that H(ω) is the familiar H(s), with s replaced by jω.

Example 17.4 uses the Fourier transform to find the response of a circuit.

Example 17.4 Using the Fourier Transform to Find the Transient Response

Use the Fourier transform to find io(t) in the circuit shown in Fig. 17.9. The current source ig(t) is the signum function 20 sgn(t) A.

Figure 17.9 The circuit for Example 17.4.

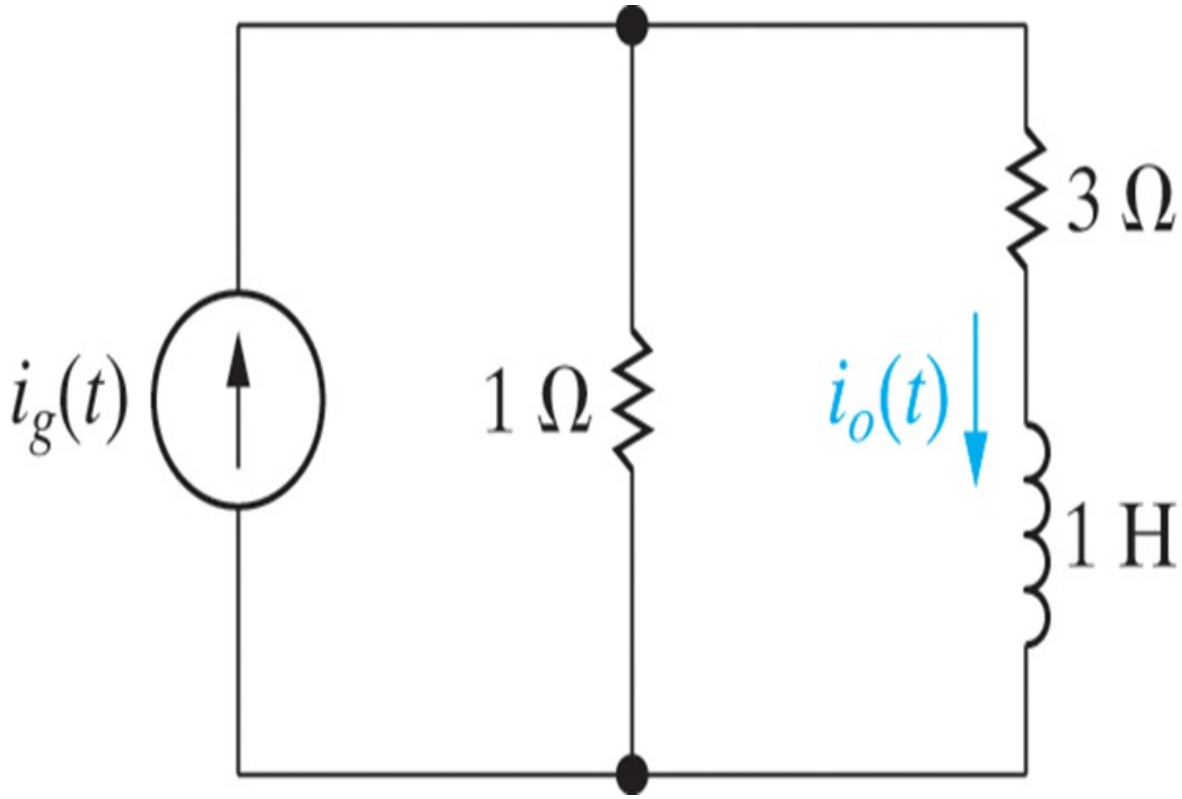


Figure 17.9 Full Alternative Text

Solution

The Fourier transform of the input is

$Ig(\omega)=F\{\ 20\ sgn(t)\ \}=20(2j\omega)=40j\omega.$

The transfer function of the circuit is the ratio of Io to Ig, which we can find using current division in the s domain:

$Io=1\|(3+s)3+sIg=14+sIg.$

Therefore,

$H(s)=14+s$

and

$H(\omega)=14+j\omega$.

The Fourier transform of io(t) is

$Io(\omega)=Ig(\omega)H(\omega)=40j\omega(4+j\omega)$.

Expanding Io(ω) into a sum of partial fractions yields

$Io(\omega)=K1j\omega+K24+j\omega$.

Evaluating K1 and K2 gives

$K1=404=10, K2=40-4=-10$.

Therefore

$Io(\omega)=10j\omega-104+j\omega$.

Find the output current in the time domain using the functional and operational Fourier transform tables, Tables 17.1 and 17.2. The result is

$io(t)=F-1[\ Io(\omega)\ ]=5\ sgn(t)-10e-4tu(t)$.

Figure 17.10 shows the response. Does the solution make sense in terms of known circuit behavior? The answer is yes, based on the following analysis. The current source delivers -20 A to the circuit between -∞ and 0, and because the current is constant, the inductor behaves like a short circuit over this time interval. Therefore, the resistance in each branch determines how the -20 A divides between the two branches. One-fourth of the -20 A appears in the io branch, so io is -5 A for t<0. When the current source jumps from -20 A to +20 A at t=0, io approaches its final value of +5 A exponentially. The equivalent resistance with respect to the inductor's terminals is 4 Ω, so the time constant of the exponential rise for t>0 is 14 s.

Figure 17.10 The plot of io(t) versus t.

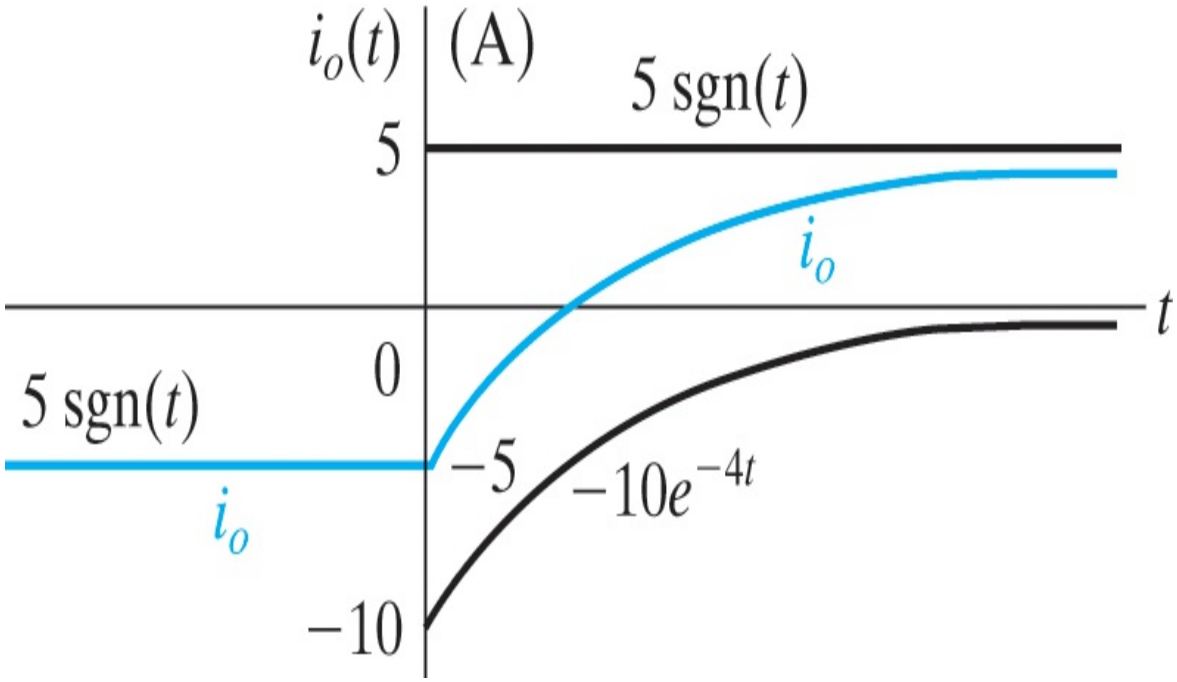


Figure 17.10 Full Alternative Text

An important characteristic of the Fourier transform is that it gives us the steady-state response to a sinusoidal input. There is no transient component in the response because the Fourier transform of cos ω0t assumes that the function exists over all time. Example 17.5 illustrates this feature.

Example 17.5 Using the Fourier Transform to Find the Sinusoidal Steady-State Response

Suppose the current source in the circuit in Example 17.4 (Fig. 17.9) changes to a sinusoid, given by

ig(t)=50cos 3t A.

Use the Fourier transform method to find io(t).

Solution

The transform of the input current is

$Ig(\omega)=50\pi[\Delta(\omega-3)+\Delta(\omega+3)]$.

As before, the transfer function of the circuit is

$H(\omega)=14 + j\omega$.

The transform of the current response then is

$Io(\omega)=50\pi\ \Delta(\omega-3) + \Delta(\omega+3)4 + j\omega$.

To find the inverse transform of $Io(\omega)$ we begin with the inverse Fourier transform intergral in Eq. 17.4, and then use the sifting property of the impulse function:

$io(t)=F-1\{\ Io(\omega)\ \}=50\pi 2\pi\int-\infty\infty[\ \Delta(\omega-3)+\Delta(\omega+3)4+j\omega\ ]e\ j\omega t\ d\omega=25\ (ej3t4+j3+e-j3t4-j3)=25\ (ej3te-j36.87°5+e-j3tej36.87°5)=5[\ 2\cos(3t-36.87°)\ ]=10\cos(3t-36.87°)A.$

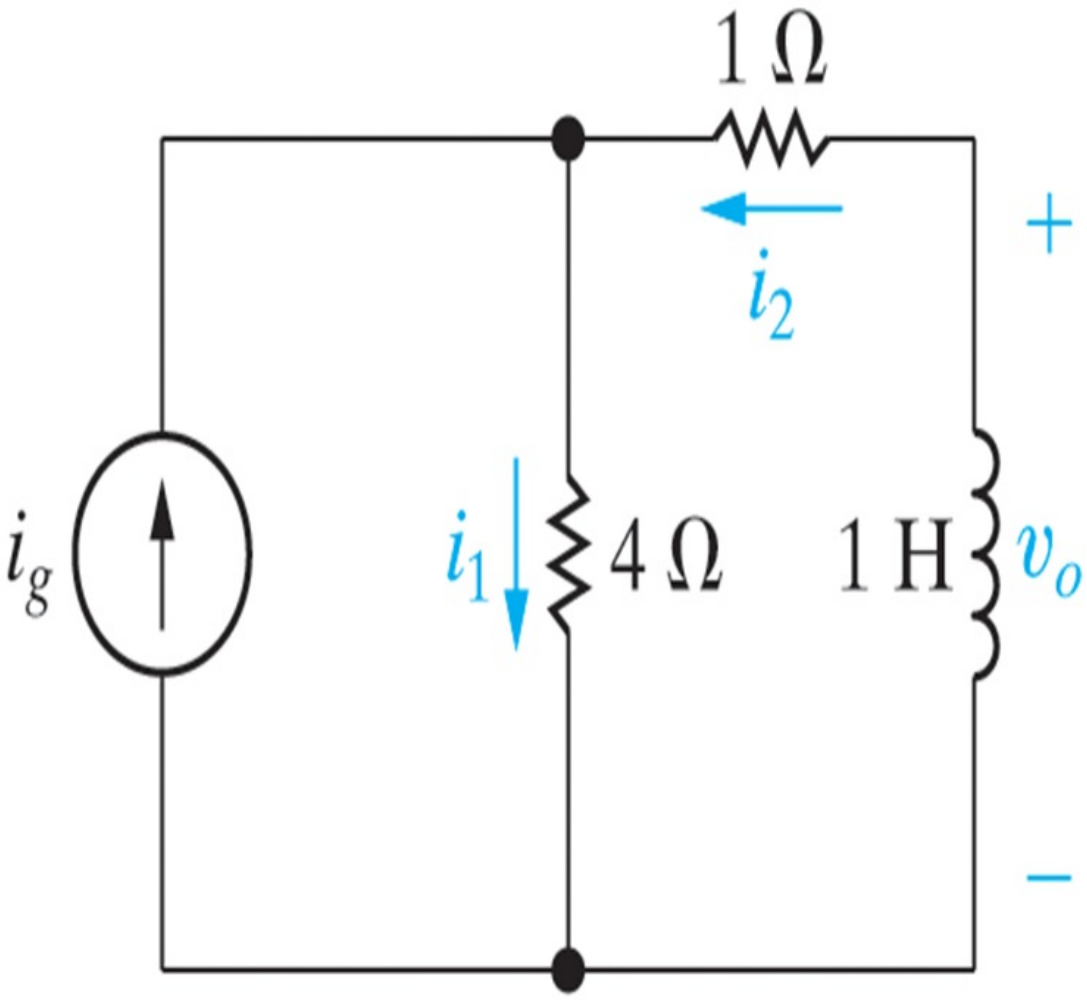
You should verify that the solution for io(t) is identical to that obtained by phasor analysis.

Assessment Problems

Objective 2—Know how to use the Fourier transform to find the response of a circuit

1. 17.6 The current source in the circuit shown delivers a current of 10sgn (t) A. The output is the voltage across the 1 H inductor. Compute
    1. Ig(ω);
    2. H(jω);

3. Vo(ω);
4. vo(t);
5. i1(0-);
6. i1(0+);
7. i2(0-);
8. i2(0+);
9. vo(0-); and
10. vo(0+).

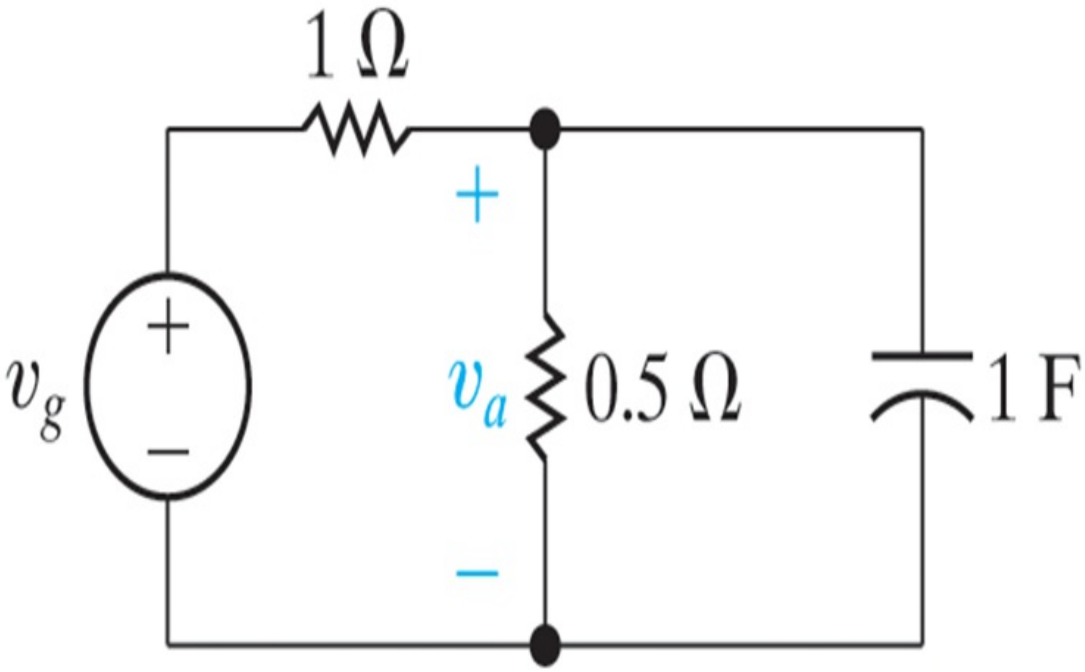


17.7-5 Full Alternative Text

Answer:

1. 20/jω;
2. 4jω/(5+jω);
3. 80/(5+jω);
4. 80e-5tu(t) V;
5. -2 A;
6. (f) 18 A;
7. (g) 8 A;
8. (h) 8 A;
9. (i) 0 V;
10. (j) 80 V.

2. 17.7 The voltage source in the circuit shown is generating the voltage vg=etu(-t) + u(t) V.
    1. Use the Fourier transform method to find va.
    2. Compute va(0-), va(0+), and va(∞).



17.7-6 Full Alternative Text

Answer:

1. va=14 etu(-t)-112 e-3tu(t)+16

   +16 sgn(t) V;

2. 14 V, 14 V, 13 V.

SELF-CHECK: Also try Chapter Problems 17.20, 17.28, and 17.30.

17.8 Parseval’’s Theorem

Parseval’’s theorem relates the energy of a time-domain function having finite energy to the Fourier transform of that function. Imagine that the time-domain function f(t) is either the voltage across or the current in a 1 Ω resistor. The energy of this function then is

$$W_{1\Omega}=\int_{-\infty}^{\infty} f^2(t)\, dt.$$

Parseval’’s theorem says that we can also calculate this energy using a frequency-domain integral:

$$W_{1\Omega}=\int_{-\infty}^{\infty} f^2(t)\, dt=\frac{1}{2\pi}\int_{-\infty}^{\infty} |F(\omega)|^2\, d\omega. \quad (17.14)$$

Therefore, we can calculate the 1 Ω energy associated with f(t) either by integrating the square of f(t) over all time or by integrating the square of the magnitude of the Fourier transform of f(t) over all frequencies and multiplying by 1/2π. Parseval’’s theorem is valid if both integrals exist.

The average power of time-domain signals with finite energy is zero when averaged over all time. Therefore, when comparing such signals, we use the energy content of the signals instead of their average power. Using a 1 Ω resistor when making the energy calculation is convenient and lets us compare the energy content of voltages and currents.

We begin deriving Eq. 17.14 by rewriting the left-hand side as f(t) times itself and then expressing one f(t) in terms of the inversion integral:

$$\int_{-\infty}^{\infty} f^2(t)\, dt=\int_{-\infty}^{\infty} f(t)\, f(t)\, dt=\int_{-\infty}^{\infty} f(t)\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}\, d\omega\right] dt.$$

We move f(t) inside the interior integral because the integration is with respect to ω, and then we factor the constant 1/2π outside both integrals. Then,

$$\int_{-\infty}^{\infty} f^2(t)\, dt=\frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} F(\omega)\, f(t)e^{j\omega t}\, d\omega\right] dt.$$

We reverse the order of integration and then factor F(ω) out of the integral with respect to t. Thus

$$\int_{-\infty}^{\infty} f^2(t)\, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\left[\int_{-\infty}^{\infty} f(t) e^{j\omega t}\, dt\right] d\omega.$$

The bracketed integral is F(-ω), so

$$\int_{-\infty}^{\infty} f^2(t)\, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\, F(-\omega)\, d\omega.$$

In Section 17.5, we noted that $F(-\omega)=F^*(\omega)$. Thus, the product $F(\omega)\, F(-\omega)$ is the magnitude of F(ω) squared, completing the derivation of Eq. 17.14. We also noted that $|F(\omega)|$ is an even function of ω. Therefore, we can also write Eq. 17.14 as

$$\int_{-\infty}^{\infty} f^2(t)\, dt = \frac{1}{\pi}\int_{0}^{\infty} |F(\omega)|^2\, d\omega. \quad (17.15)$$

Demonstrating Parseval''s Theorem

To demonstrate Eq. 17.15, suppose

$$f(t)=e^{-a|t|}.$$

Substituting this f(t) into the left-hand side of Eq. 17.15 and evaluating the left-hand side, we get

$$\int_{-\infty}^{\infty} e^{-2a|t|}\, dt = \int_{-\infty}^{0} e^{2at}\, dt + \int_{0}^{\infty} e^{-2at}\, dt = \frac{e^{2at}}{2a}\Big|_{-\infty}^{0} + \frac{e^{-2at}}{-2a}\Big|_{0}^{\infty} = \frac{1}{2a} + \frac{1}{2a} = \frac{1}{a}.$$

The Fourier transform of f(t) is

$$F(\omega)=\frac{2a}{a^2+\omega^2},$$

and therefore the right-hand side of Eq. 17.15 evaluates to

$$\frac{1}{\pi}\int_{0}^{\infty} \frac{4a^2}{(a^2+\omega^2)^2}\, d\omega = \frac{4a^2}{\pi}\frac{1}{2a^2}\left(\frac{\omega}{\omega^2+a^2}+\frac{1}{a}\tan^{-1}\frac{\omega}{a}\right)\Big|_{0}^{\infty} = \frac{2}{\pi}\left(0+\frac{\pi}{2a}-0-0\right)=\frac{1}{a}.$$

Note that both the left-hand side and the right-hand side of Eq. 17.15 evaluate to 1/a.

Interpreting Parseval’’s Theorem

A physical interpretation of Parseval’’s theorem tells us that the magnitude of the Fourier transform squared, | F(ω) |2, is an energy density (in joules per hertz). To see this, change the variable of integration on the right-hand side of Eq. 17.15, using ω=2πf. The result is

$$1\pi\int_0^\infty |F(2\pi f)|^2 2\pi\, df = 2\int_0^\infty |F(2\pi f)|^2\, df,$$

where | F(2πf) |2df is the energy in an infinitesimal band of frequencies (df), and the total 1 Ω energy associated with f(t) is the sum (integral) of | F(2πf) |2df over all frequencies.

We can also calculate the portion of the total energy for a specified frequency band. For example, the 1 Ω energy in the frequency band from ω1 to ω2 is

$$W_{1\Omega} = 1\pi\int_{\omega 1}^{\omega 2} |F(\omega)|^2\, d\omega. \quad (17.16)$$

Note that if we write the frequency-domain integral using

$$12\pi\int_{-\infty}^{\infty} |F(\omega)|^2\, d\omega$$

instead of

$$1\pi\int_0^\infty |F(\omega)|^2\, d\omega,$$

we can rewrite Eq. 17.16 as

$$W_{1\Omega} = 12\pi\int_{-\omega 2}^{-\omega 1} |F(\omega)|^2 d\omega + 12\pi\int_{\omega 2}^{\omega 1} |F(\omega)|^2 d\omega. \quad (17.17)$$

Figure 17.11 shows a graphical interpretation of Eq. 17.17.

Figure 17.11 The graphic

interpretation of Eq. 17.17.

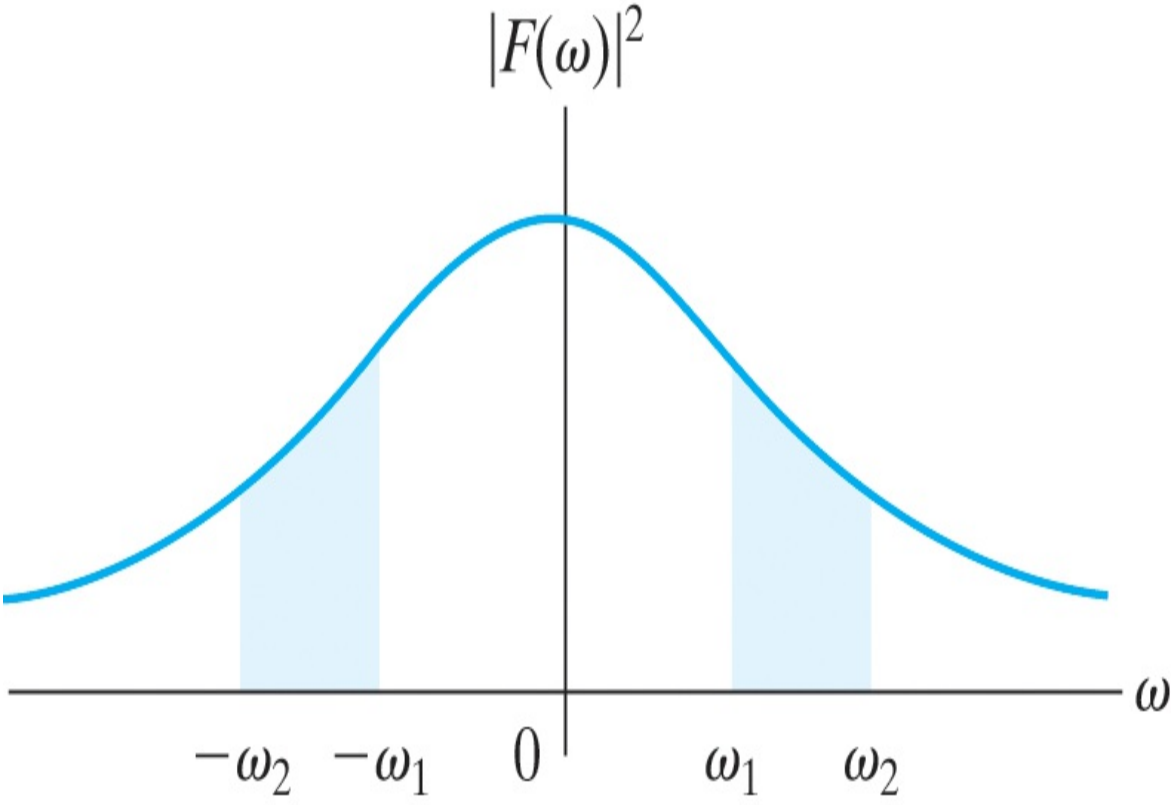


Figure 17.11 Full Alternative Text

Examples 17.6–17.9 illustrate calculations using Parseval''s theorem.

Example 17.6 Applying Parseval''s Theorem

The current in a 40 Ω resistor is

i=20e-2tu(t) A.

What percentage of the total energy dissipated in the resistor is associated with the frequency band 0≤ω≤23 rad/s?

Solution

The total energy dissipated in the 40 Ω resistor is

$$W_{40\Omega}=40\int_0^\infty 400e^{-4t}\,dt=16{,}000\ e^{-4t}-4|_0^\infty=4000\ \text{J}.$$

We can check this total energy calculation with Parseval''s theorem:

$$F(\omega)=\frac{20}{2+j\omega}.$$

Therefore

$$|\ F(\omega)\ |=\frac{20}{4+\omega^2}$$

and

$$W_{40\Omega}=40\pi\int_0^\infty \frac{400}{4+\omega^2}\,d\omega=16{,}000\pi\ (\tfrac{1}{2}\tan^{-1}\tfrac{\omega}{2}|_0^\infty)=8000\pi\ (\tfrac{\pi}{2})=4000\ \text{J}.$$

The energy associated with the frequency band $0\le\omega\le 23$ rad/s is

$$W_{40\Omega}=40\pi\int_0^{23}\frac{400}{4+\omega^2}\,d\omega=16{,}000\pi\ (\tfrac{1}{2}\tan^{-1}\tfrac{\omega}{2}|_0^{23})=8000\pi(\tfrac{\pi}{3})=80003\ \text{J}.$$

Hence, the percentage of the total energy associated with this range of frequencies is

$$\eta=8000/34000\times 100=66.67\%.$$

Example 17.7 Applying Parseval''s Theorem to an Ideal Bandpass Filter

The input voltage for an ideal bandpass filter is

v(t)=120e-24tu(t) V.

The filter passes all frequencies that lie between 24 and 48 rad/s, without attenuation, and completely rejects all frequencies outside this passband.

1. Sketch | V(ω) |2 for the filter input voltage.
2. Sketch | Vo(ω) |2 for the filter output voltage.
3. What percentage of the total 1 Ω energy content of the input voltage is available at the output?

Solution

1. The Fourier transform of the filter input voltage is

   V(ω)=12024+jω.

   Therefore

   | V(ω) |2=14,400576+ω2.

   Figure 17.12 shows the sketch of |V(ω)|2 versus ω.

Figure 17.12 |V(ω)|2 versus ω for Example 17.7.

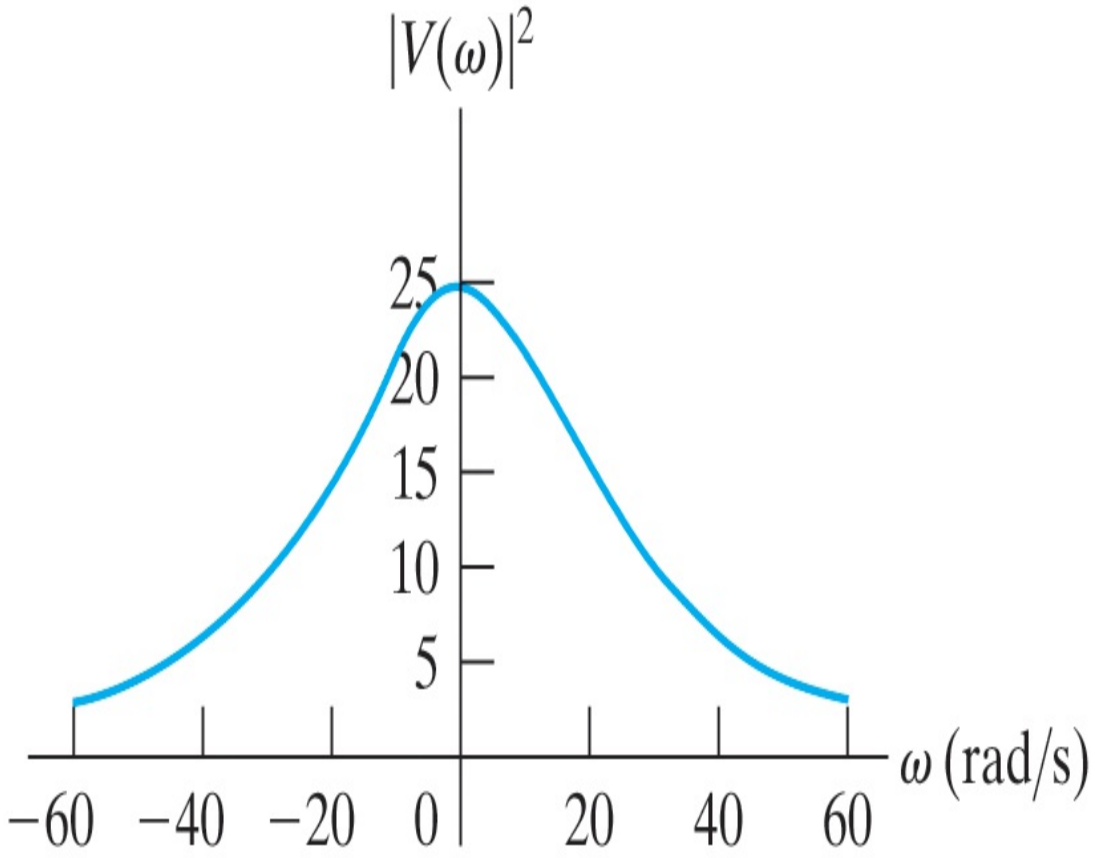


Figure 17.12 Full Alternative Text

2. The ideal bandpass filter rejects all frequencies outside the passband. The plot of |Vo(ω)|2 versus ω looks just like the plot in Fig. 17.12 between -48 and -24 rad/s and between 24 and 48 rad/s, and is zero for all other frequencies, as shown in Fig. 17.13.

Figure 17.13 |V0(ω)|2 versus ω for Example 17.7.

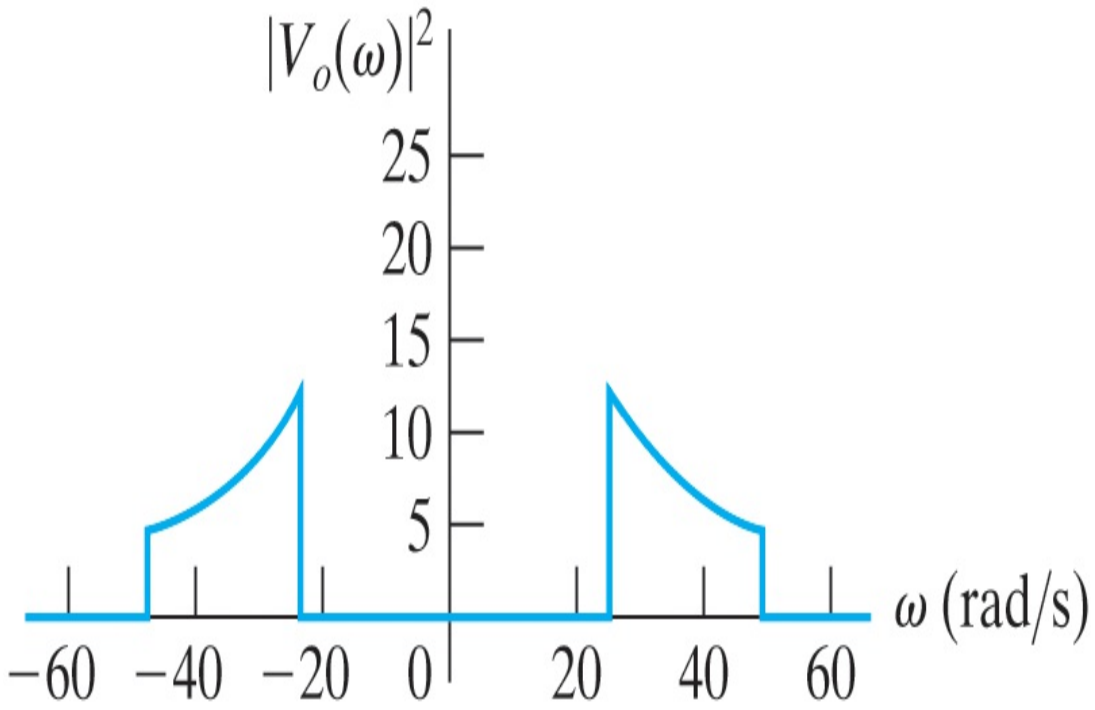


Figure 17.13 Full Alternative Text

3. The total 1 Ω energy for the input voltage is

   $W_i = \frac{1}{\pi}\int_0^{\infty} \frac{14{,}400}{576+\omega^2}\, d\omega = \frac{14{,}400}{\pi}\left(\frac{1}{24}\tan^{-1}\frac{\omega}{24}\Big|_0^{\infty}\right) = \frac{600}{\pi}\,\frac{\pi}{2} = 300$ J.

   The total 1 Ω energy for the filter's output is

   $W_o = \frac{1}{\pi}\int_{24}^{48} \frac{14{,}400}{576+\omega^2}\, d\omega = \frac{600}{\pi}\tan^{-1}\frac{\omega}{24}\Big|_{24}^{48} = \frac{600}{\pi}(\tan^{-1}2 - \tan^{-1}1) = \frac{600}{\pi}\left(\frac{\pi}{2.84} - \frac{\pi}{4}\right) = 61.45$ J.

   The percentage of the input energy available at the output is

   $\eta = \frac{61.45}{300}\times 100 = 20.48\%$.

Example 17.8 Applying Parseval''s

Theorem to a Low-Pass Filter

We can use Parseval’’s theorem to calculate the energy available at a filter’s output even if the time-domain expression for vo(t) is unknown. Suppose the input voltage to the low-pass RC filter circuit shown in Fig. 17.14 is

vi(t)=15e-5tu(t) V.

Figure 17.14 The low-pass RC filter for Example 17.8.

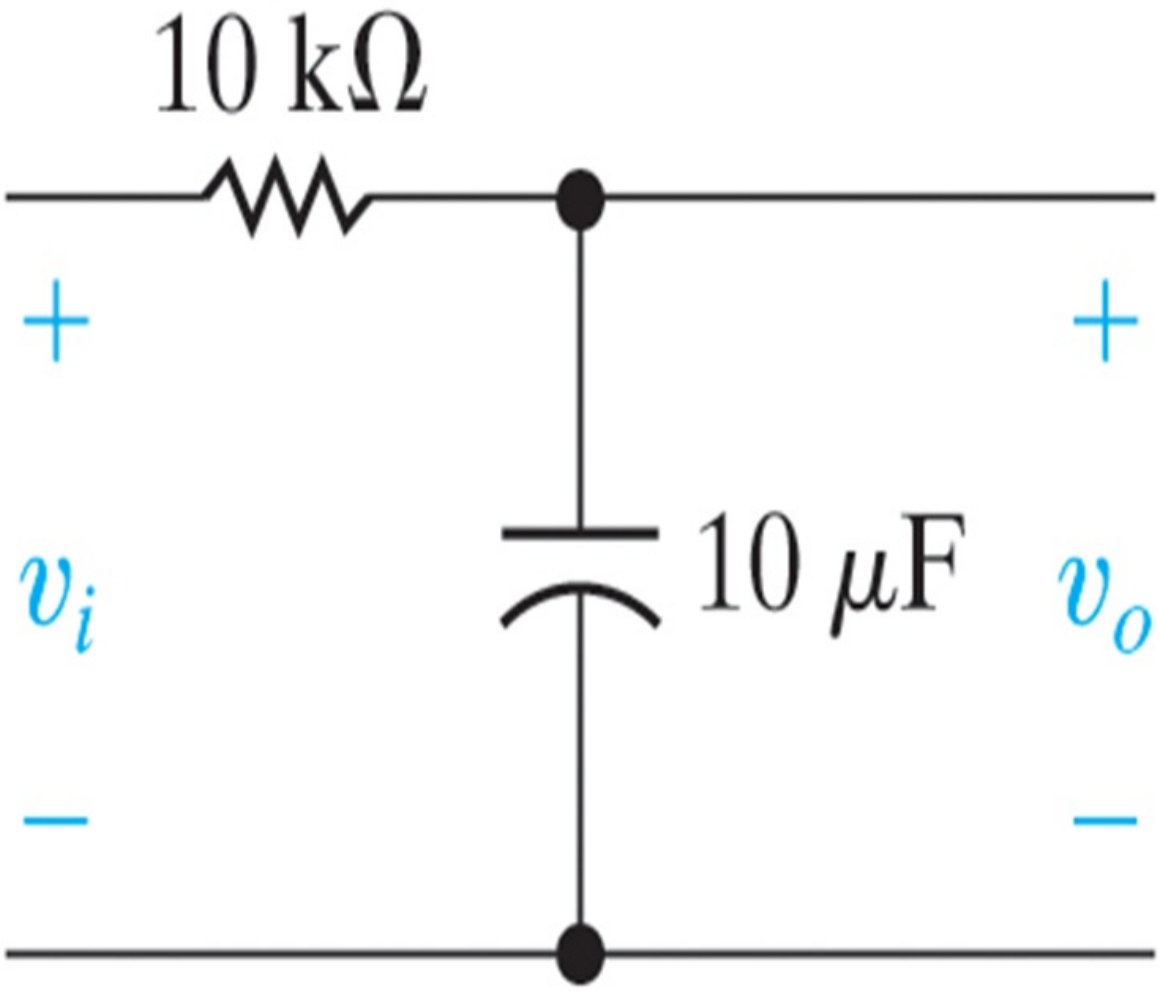


Figure 17.14 Full Alternative Text

1. What percentage of the input signal’’s 1 Ω energy is available in the output signal?

2. What percentage of the output energy is associated with the frequency range $0 \leq \omega \leq 10$ rad/s?

Solution

1. The 1 Ω energy in the input signal is

   $$W_i = \int_0^\infty (15e^{-5t})^2 \, dt = 225e^{-10t}/-10 \Big|_0^\infty = 22.5 J.$$

   The Fourier transform of the output voltage is

   $$V_o(\omega) = V_i(\omega) H(\omega),$$

   where

   $$V_i(\omega) = \frac{15}{5 + j\omega} \quad H(\omega) = \frac{1/RC}{1/RC + j\omega} = \frac{10}{10 + j\omega}.$$

   Hence

   $$V_o(\omega) = \frac{150}{(5 + j\omega)(10 + j\omega)} \quad |V_o(\omega)|^2 = \frac{22{,}500}{(25 + \omega^2)(100 + \omega^2)}.$$

   The 1 Ω energy available in the filter’’s output is

   $$W_o = \frac{1}{\pi} \int_0^\infty \frac{22{,}500}{(25 + \omega^2)(100 + \omega^2)} \, d\omega.$$

   We can evaluate the integral using a sum of partial fractions:

   $$\frac{22{,}500}{(25 + \omega^2)(100 + \omega^2)} = \frac{300}{25 + \omega^2} - \frac{300}{100 + \omega^2}.$$

   Then

   $$W_o = 300\pi \left\{ \int_0^\infty \frac{d\omega}{25 + \omega^2} - \int_0^\infty \frac{d\omega}{100 + \omega^2} \right\} = 300\pi \left[\frac{1}{5} \left(\frac{\pi}{2}\right) - \frac{1}{10} \left(\frac{\pi}{2}\right) \right] = 15 J.$$

The energy available in the output signal therefore is 66.67% of the energy available in the input signal; that is,

$\eta = 1522.5(100) = 66.67\%$.

2. The output energy associated with the frequency range $0 \le \omega \le 10$ rad/s is

   $W'_o = 300\pi\{ \int_0^{10} d\omega 25 + \omega 2 - \int_0^{10} d\omega 100 + \omega 2 \} = 300\pi$ (15 tan -1 105 $-$110 tan-1 1010) $= 30\pi$ ($2\pi 2.84 - \pi 4$) $= 13.64$ J.

   The total 1 Ω energy in the output signal is 15 J, so the percentage associated with the frequency range 0 to 10 rad/s is 90.97%.

Example 17.9 Calculating Energy Contained in a Rectangular Voltage Pulse

A voltage pulse, v(t), is shown in Fig. 17.15(a). Use Parseval''s theorem to calculate the fraction of the total energy associated with v(t) that lies in the frequency range $0 \le \omega \le 2\pi/\tau$. Recall from Section 17.1 that we found the Fourier transform of the voltage pulse to be

$V(\omega) = V_m \tau \sin \omega\tau/2 \omega\tau/2$.

Figure 17.15 The rectangular voltage pulse and its Fourier transform. (a) The rectangular voltage pulse. (b) The

Fourier transform of v(t)

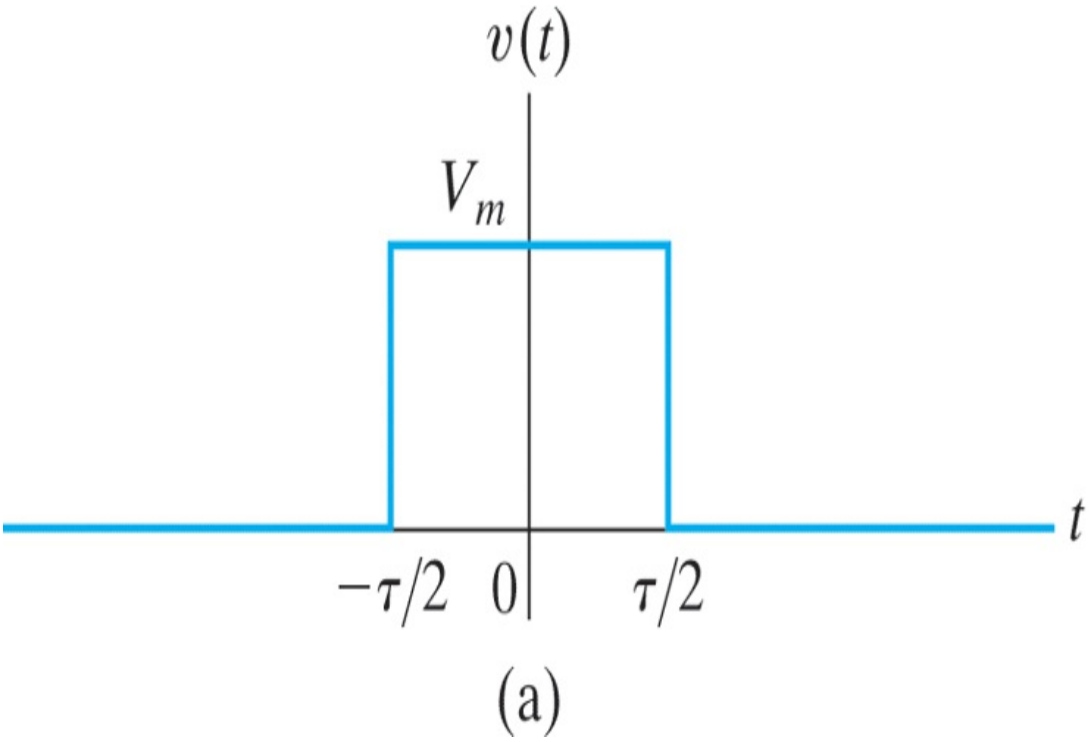
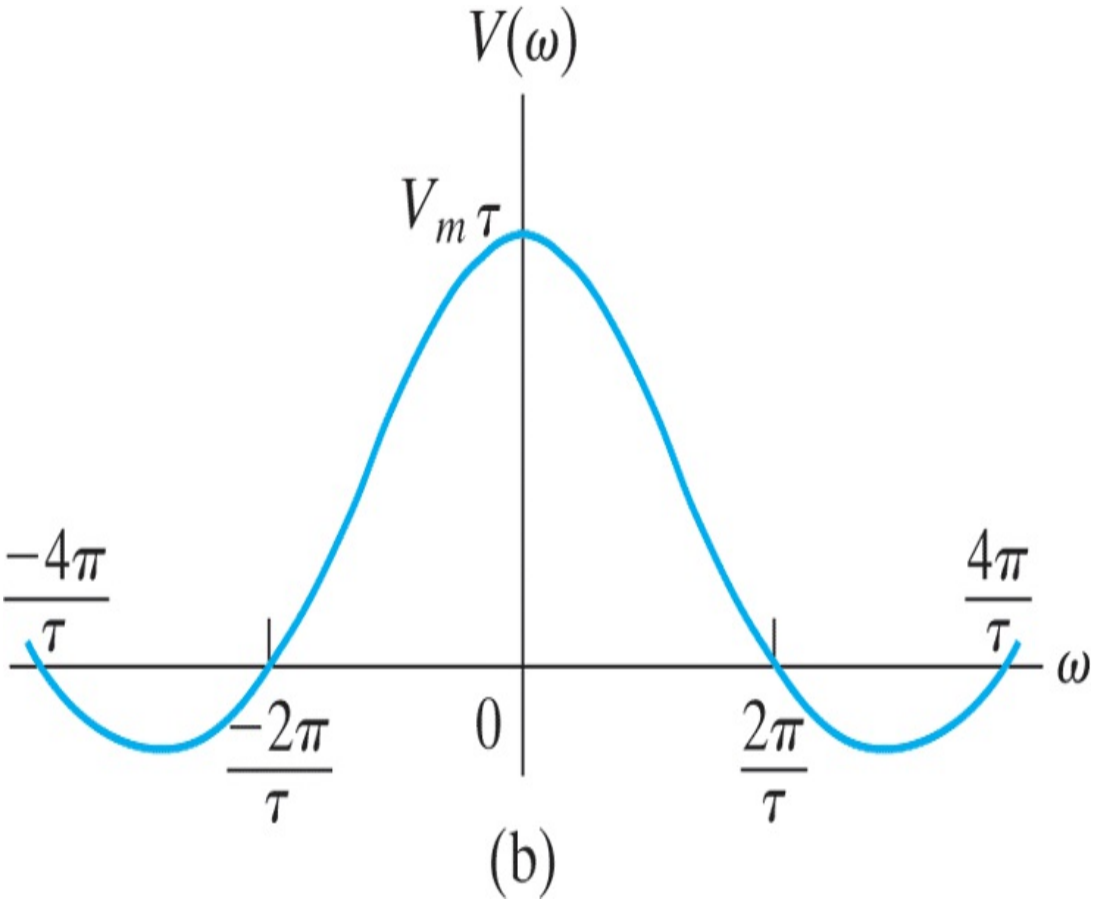


Figure 17.15 Full Alternative Text



17.8-7 Full Alternative Text

The Fourier transform of the voltage pulse is plotted in Fig. 17.15(b).

Solution

To begin, substitute the Fourier transform of the voltage pulse into Eq. 17.16:

W=1π∫02π/τVm2τ2sin2ωτ/2(ωτ/2)2 dω.

To evaluate this integral, let x=ωτ/2, so dx=(τ/2)dω. Note that when ω=2π/τ, x=π. Using these substitutions, the energy equation becomes

W=2Vm2ττπ∫0πsin2xx2 dx.

Now we can integrate by parts. Let u=sin2x and dv=dx/x2, so du=2 sin x cos

x dx=sin 2x dx and v=-1/x. Therefore,

$$\int_0^{\pi} \sin 2x x^2\, dx = -\sin 2x x\big|_0^{\pi} - \int_0^{\pi} -1x \sin 2x\, dx = 0 + \int_0^{\pi} \sin 2x x\, dx$$

and

$$W = 4V_m^2 \tau \pi \int_0^{\pi} \sin 2x 2x\, dx.$$

To evaluate this integral, make another substitution of variables to get the form sin y/y. Let y=2x so dy=2 dx, and y=2π when x=π. The resulting equation is

$$W = 2V_m^2 \tau \pi \int_0^{2\pi} \sin y y\, dy.$$

The integral's value can be found using an online calculator.[1] Its value is 1.41815, so

[1] http://www.wolframalpha.com/input/?i=integrate+sin+x+%2F+x+from+0+to+2*pi

$$W = 2V_m^2 \tau \pi (1.41815).$$

To find the total 1 Ω energy associated with v(t), use either the time-domain integral or the frequency-domain integral in Eq. 17.14. The total energy is

$$W_t = V_m^2 \tau.$$

The fraction of the total energy associated with the band of frequencies between 0 and 2π/τ is

$$\eta = \frac{W}{W_t} = \frac{2V_m^2 \tau (1.41815) \pi}{(V_m^2 \tau)} = 0.9028.$$

Therefore, approximately 90% of the energy associated with v(t) is contained in the dominant portion of the amplitude spectrum.

Note that the plots in Fig. 17.15 show that as the width of the voltage pulse (τ) becomes smaller, the dominant portion of the amplitude spectrum (that is, the spectrum from -2π/τ to 2π/τ) spreads out over a wider range of frequencies. This result agrees with our earlier comments about the

operational transform involving a scale change–when time is compressed, frequency is expanded and vice versa. To transmit a single rectangular pulse with reasonable fidelity, the bandwidth of the system must be at least wide enough to accommodate the dominant portion of the amplitude spectrum. Thus, the cutoff frequency should be at least $2\pi/\tau$ rad/s, or $1/\tau$ Hz.

Assessment Problems

Objective 3–Understand Parseval''s theorem and be able to use it

1. 17.8 The voltage across a 50 Ω resistor is

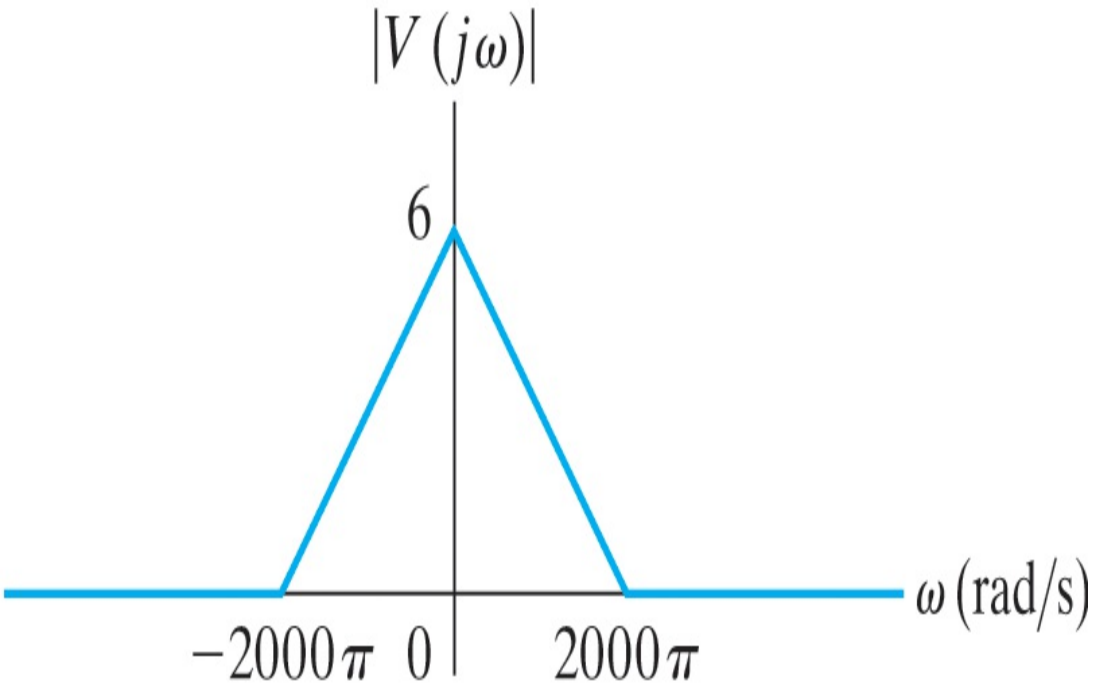
   $v=4te^{-t}u(t)$ V.

   What percentage of the total energy dissipated in the resistor can be associated with the frequency band $0\leq\omega\leq3$ rad/s?

   Answer:

   94.23%.

2. 17.9 Assume that the magnitude of the Fourier transform of v(t) is as shown. This voltage is applied to a 6 kΩ resistor. Calculate the total energy delivered to the resistor.



17.8-7 Full Alternative Text

Answer:

4 J.

SELF-CHECK: Also try Chapter Problem 17.40.

Practical Perspective

Filtering Digital Signals

To understand the effect of transmitting a digital signal on a telephone line, consider a pulse that represents a digital value of 1, as shown in Fig. 17.15(a), with Vm=5 V and τ=1 μs. The Fourier transform of this pulse is shown in Fig. 17.15(b), where the amplitude Vmτ=5 μV and the first positive zero-crossing on the frequency axis is 2π / τ=6.28 Mrad / s=1 MHz.

Note that the digital pulse representing the value 1 is ideally a sum of an

infinite number of frequency components. But the telephone line cannot transmit all of these frequency components. Typically, the line has a bandwidth of 10 MHz, meaning that it is capable of transmitting only those frequency components below 10 MHz. This causes the original pulse to be distorted once it is received by the computer on the other end of the telephone line, as seen in Fig. 17.16.

Figure 17.16 The effect of sending a square voltage pulse through a bandwidth- limited filter, causing distortion of the resulting output signal in the time domain.

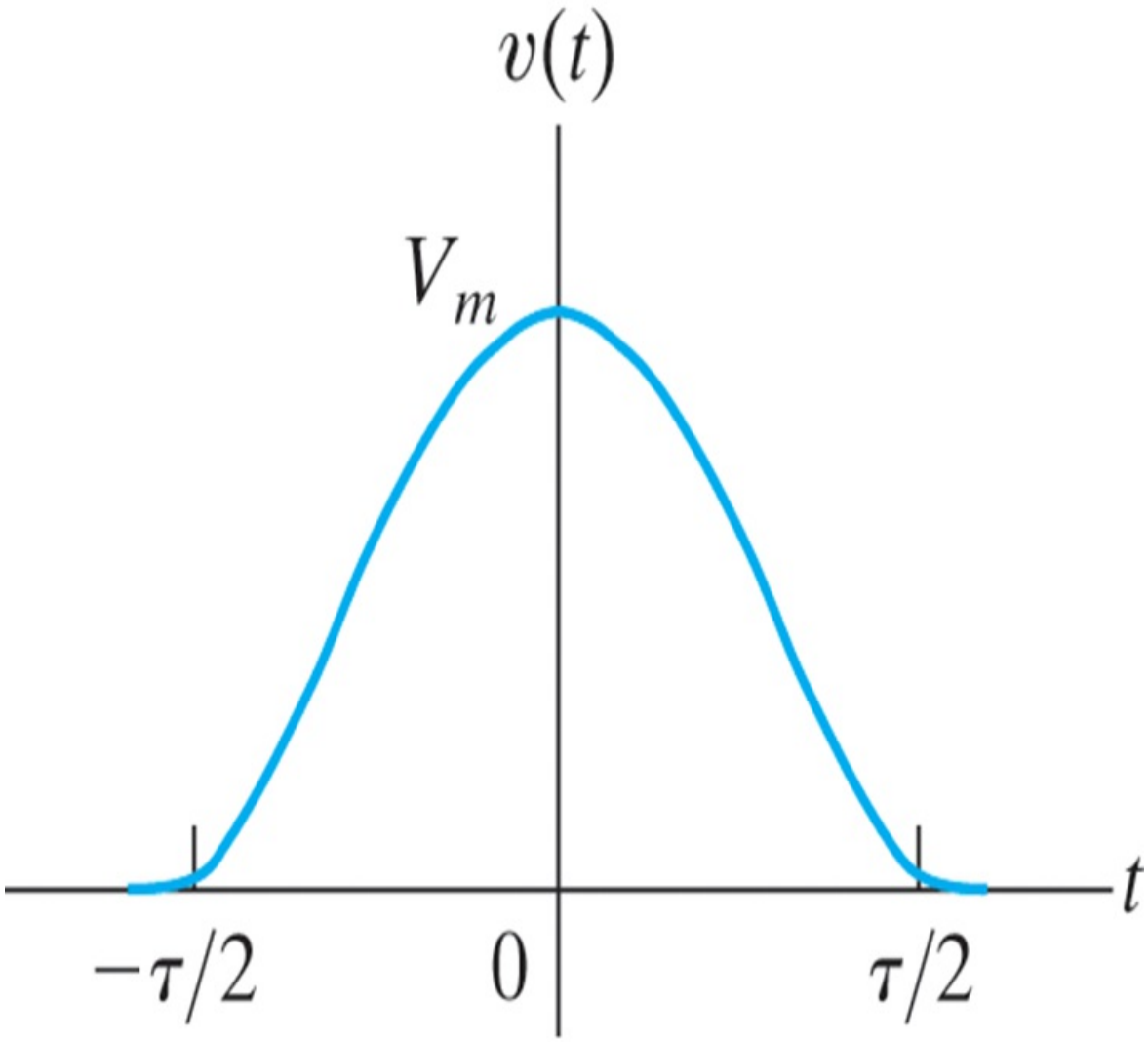


Figure 17.16 Full Alternative Text

Summary

- The **Fourier transform** gives a frequency-domain description of an aperiodic time-domain function. Depending on the nature of the time-domain signal, one of three approaches to finding its Fourier transform may be used:
    - If the time-domain signal is a well-behaved pulse of finite duration, the integral that defines the Fourier transform is used. (See page 662.)
    - If the one-sided Laplace transform of f(t) exists and all the poles of F(s) lie in the left half of the s plane, F(s) may be used to find F(ω). (See page 666.)
    - If f(t) is a constant, a signum function, a step function, or a sinusoidal function, the Fourier transform is found by using a limit process. (See page 668.)
- Functional and operational Fourier transforms that are useful in circuit analysis are tabulated in Tables 17.1 and 17.2. (See pages 670 and 675.)
- The Fourier transform accommodates both negative- time and positive-time functions and therefore is suited to problems whose signals start at t=-∞. In contrast, the unilateral Laplace transform is suited to problems with initial conditions and signals that exist for t>0. (See page 677.)
- The Fourier transform of a response signal y(t) is

  $$Y(\omega)=X(\omega)H(\omega),$$

  where X(ω) is the Fourier transform of the input signal x(t) and H(ω) is the transfer function H(s) evaluated at s=jω. (See page 677.)
- The magnitude of the Fourier transform squared is a measure of the energy density (joules per hertz) in the frequency domain (Parseval’’s

theorem). Thus, the Fourier transform permits us to associate a fraction of the total energy contained in f(t) with a specified band of frequencies. (See page 681.)

Problems

Sections 17.1 17.2

1. 17.1

    1. Find the Fourier transform of the function shown in Fig. P17.1.

    2. Find F(ω) when ω=0.

    3. Sketch |F(ω)| versus ω when A=10 and τ=0.1. Hint: Remember that |F(ω)| is an even function of ω.

Figure P17.1

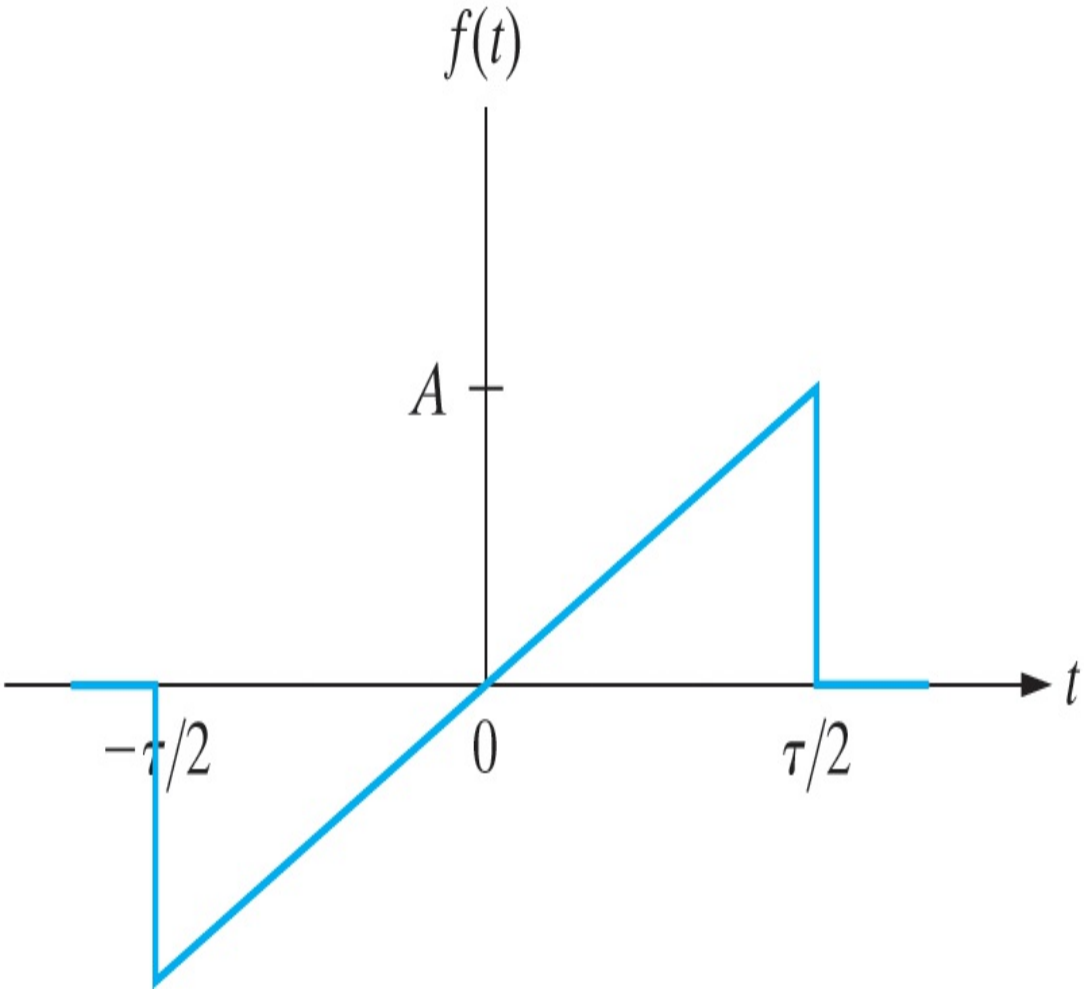


Figure P17.1 Full Alternative Text

2. 17.2 The Fourier transform of f(t) is shown in Fig. P17.2.

    1. Find f(t).
    2. Evaluate f(0).
    3. Sketch f(t) for −150 s≤t≤150 s when A=5π and ω0=100 rad/s. Hint: Remember that f(t) is even.

Figure P17.2

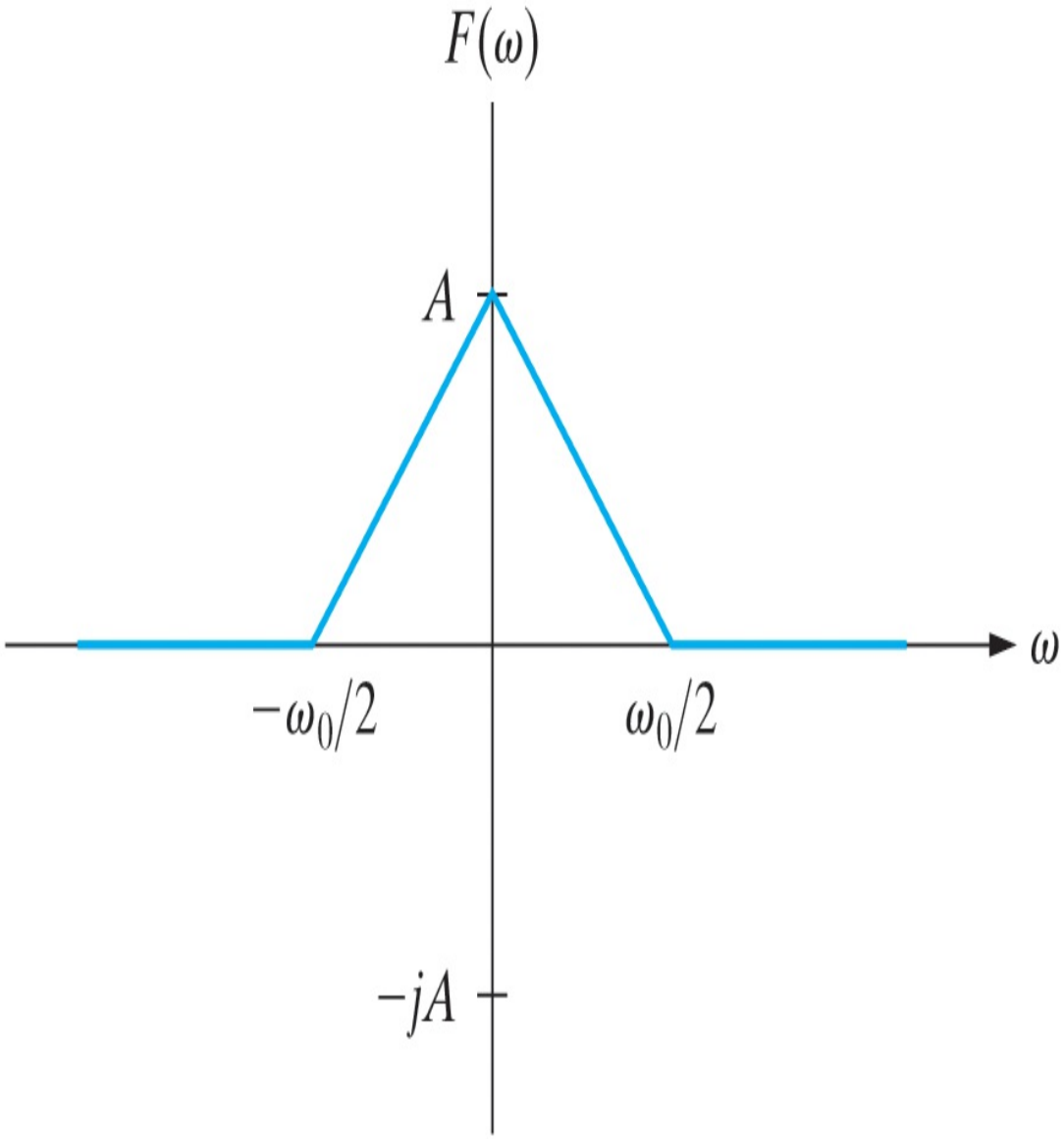


Figure P17.2 Full Alternative Text

3. 17.3 Use the defining integral to find the Fourier transform of the following functions:

    1. f(t)=Asinπ2t,−2≤t≤2;f(t)=0,elsewhere.

    2. f(t)=2Aττt+A,−τ2≤t≤0;f(t)=−2Aττt+A,0≤t≤τ2;f(t)=0,elsewhere.

Sections 17.3—17.5

1. 17.4 Derive F{sin ω0t}.

2. 17.5 Find the Fourier transform of each of the following functions. In all of the functions, a is a positive real constant and −∞≤t≤∞.

    1. f(t)=|t|e-a|t|;

    2. f(t)=t3e-a|t|;

    3. f(t)=e-a|t|cosω0t;

    4. f(t)=e-a|t|sinω0t;

    5. f(t)=Δ(t-t0).

3. 17.6 If f(t) is a real function of t, show that the inversion integral reduces to

    f(t)=12π∫−∞∞[A(ω)cos ωt−B(ω)sinωt] dω.

4. 17.7 If f(t) is a real, odd function of t, show that the inversion integral reduces to

    f(t)=−12π∫−∞∞B(ω)sinωt dω.

5. 17.8 Use the inversion integral (Eq. 17.4) to show that F-1{2/jω}=sgn(t). Hint: Use Problem 17.7.

6. 17.9 Find F{cosω0t} by using the approximating function

    f(t)=e-∈|t|cosω0t,

    where ∈ is a positive real constant.

7. 17.10 Show that if f(t) is an odd function,

$A(\omega)=0.B(\omega)=-2\int_0^\infty f(t)\sin\omega t\, dt.$

8. 17.11 Show that if f(t) is an even function,

   $A(\omega)=2\int_0^\infty f(t)\cos\omega t\, dt, B(\omega)=0.$

Section 17.6

1. 17.12
    1. Show that F{df(t)/dt}=jωF(ω), where F(ω)=F{f(t)}. Hint: Use the defining integral and integrate by parts.
    2. What is the restriction on f(t) if the result given in (a) is valid?
    3. Show that F{dnf(t)/dtn}=(jω)nF(ω), where F(ω)=F{f(t)}.
2. 17.13
    1. Show that

       F{ ∫−∞tf(x)dx }=F(ω)jω,

       where F(ω)=F{f(x)}. Hint: Use the defining integral and integrate by parts.
    2. What is the restriction on f(x) if the result given in (a) is valid?
    3. If f(x)=e-axu(x), can the operational transform in (a) be used? Explain.
3. 17.14
    1. Show that

       F{f(at)}=1aF(ωa), a>0.
    2. Given that f(at)=e-a|t| for a>0, sketch F(ω)=F{f(at)} for a=0.5, 1.0, and 2.0. Do your sketches reflect the observation that *compression* in the time domain corresponds to *stretching* in the frequency domain?

4. 17.15 Derive each of the following operational transforms:

    1. $F\{f(t-a)\}=e-j\omega aF(\omega)$;

    2. $F\{ej\omega 0tf(t)\}=F(\omega-\omega 0)$;

    3. $F\{f(t)\cos\omega 0t\}=12F(\omega-\omega 0)+12F(\omega+\omega 0)$.

5. 17.16 Given

    $y(t)=\int-\infty\infty x(\lambda)h(t-\lambda)\,d\lambda$,

    show that $Y(\omega)=F\{y(t)\}=X(\omega)H(\omega)$, where $X(\omega)=F\{x(t)\}$ and $H(\omega)=F\{h(t)\}$. Hint: Use the defining integral to write

    $F\{\ y(t)\ \}=\int-\infty\infty[\ \int-\infty\infty x(\lambda)h(t-\lambda)\,d\lambda\ ]e-j\omega t\,dt$.

    Next, reverse the order of integration and then make a change in the variable of integration; that is, let $u=t-\lambda$.

6. 17.17 Given $f(t)=f1(t)f2(t)$, show that $F(\omega)=(1/2\pi)\int-\infty\infty F1(u)F2(\omega-u)\,du$. Hint: First, use the defining integral to express $F(\omega)$ as

    $F(\omega)=\int-\infty\infty f1(t)f2(t)e-j\omega t\,dt$.

    Second, use the inversion integral to write

    $f1(t)=12\pi\int-\infty\infty F1(u)ej\omega t\,du$.

    Third, substitute the expression for f1(t) into the defining integral and then interchange the order of integration.

7. 17.18 Use the Fourier transform of tnf(t) to find each of the following Fourier transforms (assuming $a>0$):

    $F\{\ te-at\ u(t)\ \},F\{\ |\ t\ |e-a|\ t\ |\ \},F(te-a|\ t\ |\}$.

8. 17.19 Suppose that $f(t)=f1(t)f2(t)$, where

$f1(t)=\cos\omega 0t, f2(t)=1, -\tau/2<t<\tau/2; f2(t)=0, \text{elsewhere.}$

1. Use convolution in the frequency domain to find $F(\omega)$.

2. What happens to $F(\omega)$ as the width of $f2(t)$ increases so that $f(t)$ includes more and more cycles of $f1(t)$?

Section 17.7

1. 17.20 PSPICEMULTISIM

    1. Use the Fourier transform method to find io(t) in the circuit shown in Fig. P17.20 if vg=36sgn(t) V.

    2. Does your solution make sense in terms of known circuit behavior? Explain.

Figure P17.20

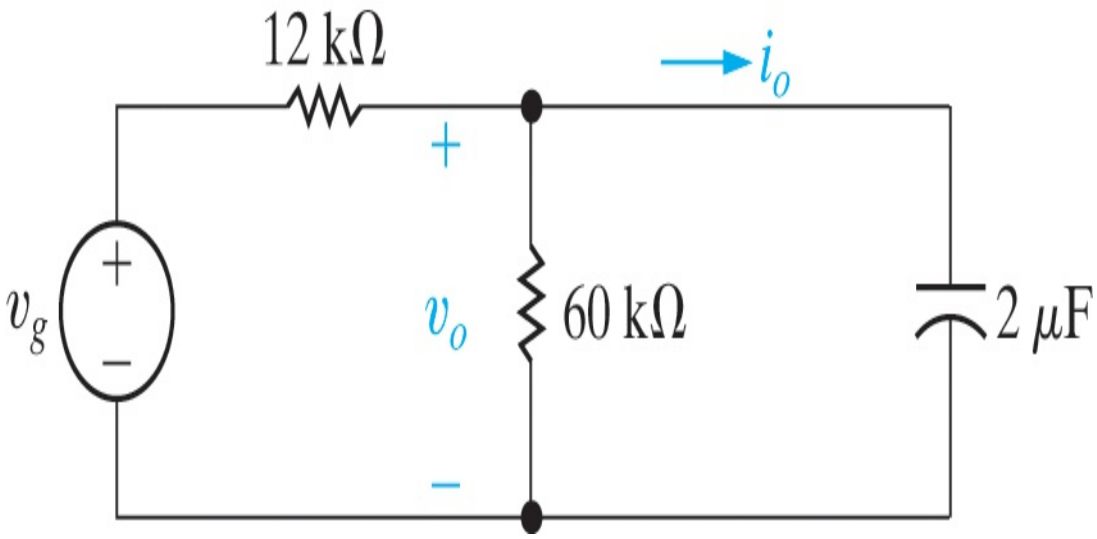


Figure P17.20 Full Alternative Text

2. 17.21 PSPICEMULTISIM Repeat Problem 17.20 except replace io(t) with vo(t).

3. 17.22 PSPICEMULTISIM

    1. Use the Fourier transform method to find vo(t) in the circuit shown in Fig. P17.22. The initial value of vo(t) is zero, and the source

voltage is 100u(t) V.

2. Sketch vo(t) versus t.

Figure P17.22

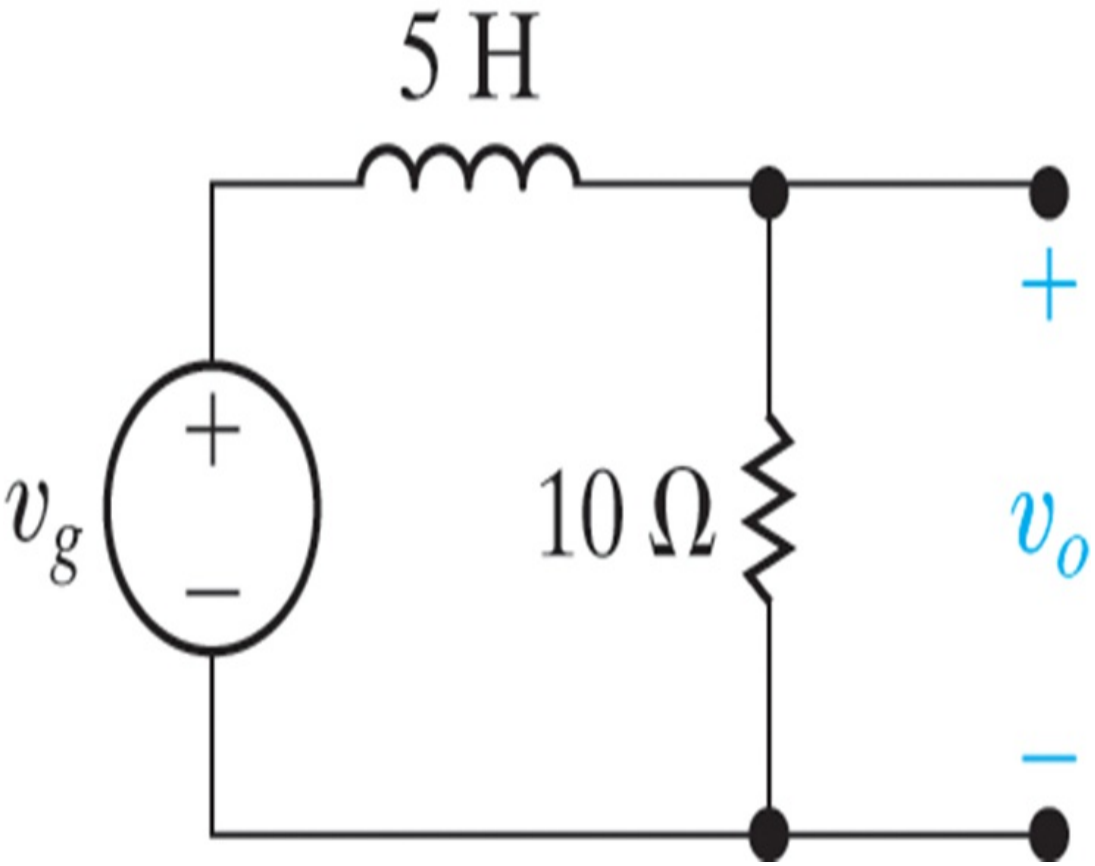


Figure P17.22 Full Alternative Text

4. 17.23 PSPICEMULTISIM Repeat Problem 17.22 if the input voltage (vg) is changed to 100 sgn(t).

5. 17.24 PSPICEMULTISIM

    1. Use the Fourier transform to find io in the circuit in Fig. P17.24 if ig=0.2 sgn(t) mA.

    2. Does your solution make sense in terms of known circuit behavior?

Explain.

Figure P17.24

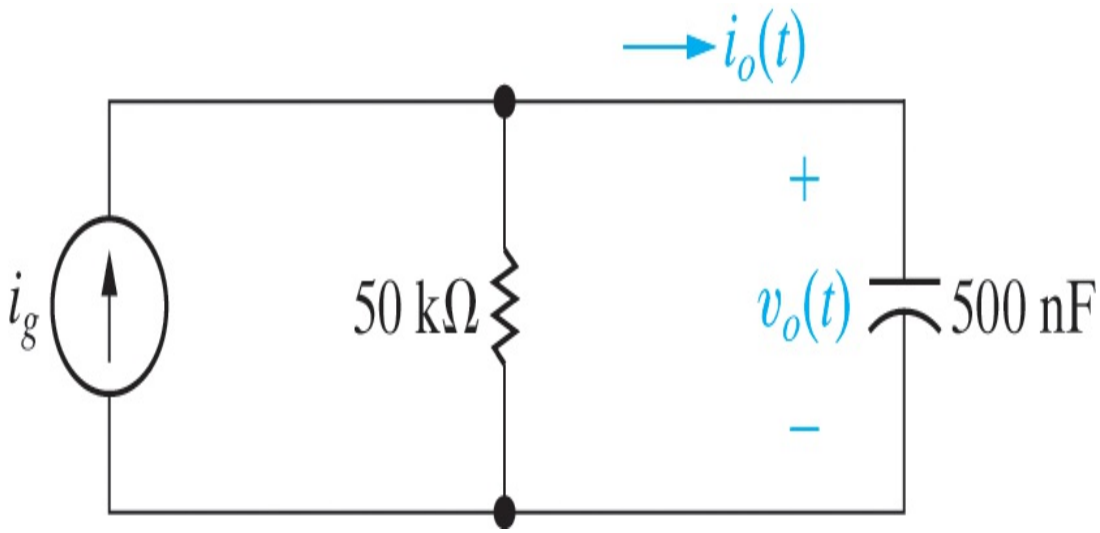


Figure P17.24 Full Alternative Text

6. 17.25 PSPICEMULTISIM Repeat Problem 17.24 except replace io with vo.

7. 17.26 PSPICEMULTISIM The voltage source in the circuit in Fig. P17.26 is given by the expression

   vg=3 sgn(t) V.

   1. Find vo(t).
   2. What is the value of vo(0-)?
   3. What is the value of vo(0+)?
   4. Use the Laplace transform method to find vo(t) for t>0+.
   5. Does the solution obtained in (d) agree with vo(t) for t>0+ from (a)?

Figure P17.26

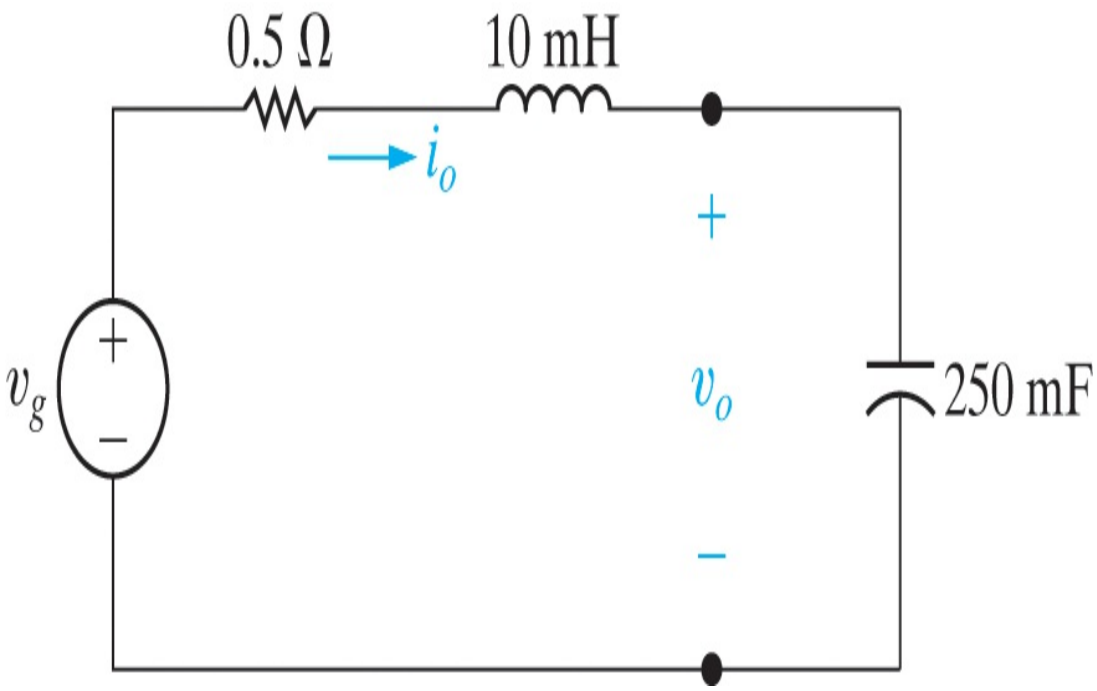


Figure P17.26 Full Alternative Text

8. 17.27 PSPICEMULTISIMRepeat Problem 17.26 except replace vo(t) with io(t).

9. 17.28 PSPICEMULTISIM

    1. Use the Fourier transform to find vo in the circuit in Fig. P17.28 if ig equals 3e-5|t| A

    2. Find vo(0-).

    3. Find vo(0+).

    4. Use the Laplace transform method to find vo for t≥0.

    5. Does the solution obtained in (d) agree with vo for t>0+ from (a)?

Figure P17.28

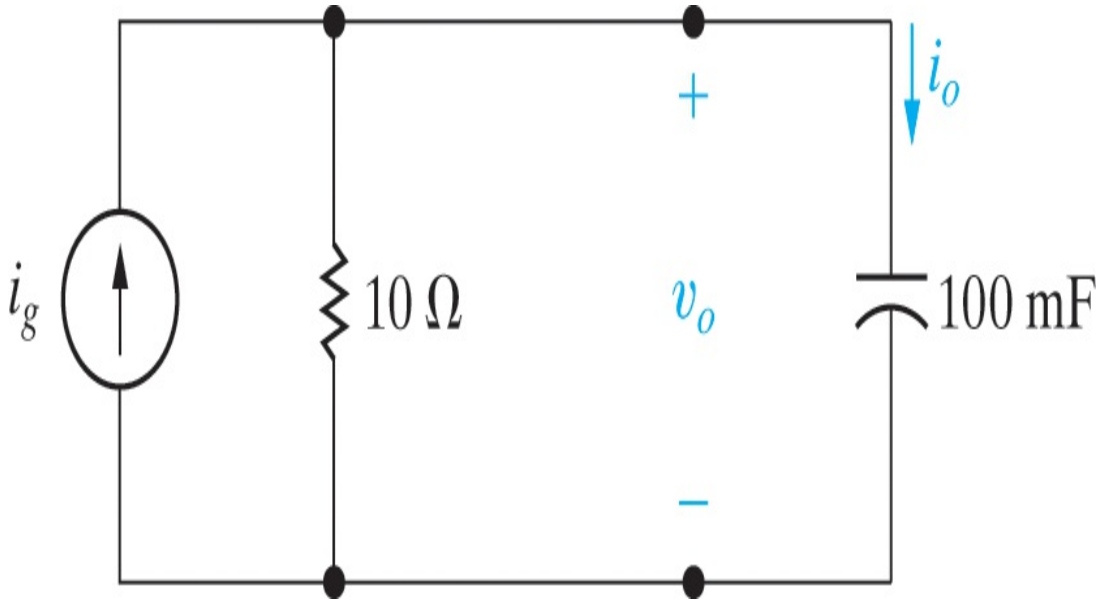


Figure P17.28 Full Alternative Text

10. 17.29 PSPICEMULTISIM

    1. Use the Fourier transform to find io in the circuit in Fig. P17.28 if ig equals 3e-5|t| A.
    2. Find io(0-).
    3. Find io(0+).
    4. Use the Laplace transform method to find io for t≥0.
    5. Does the solution obtained in (d) agree with io for t>0+ from (a)?

11. 17.30 PSPICEMULTISIM Use the Fourier transform method to find io in the circuit in Fig. P17.30 if vg=300 cos 5000t V.

Figure P17.30

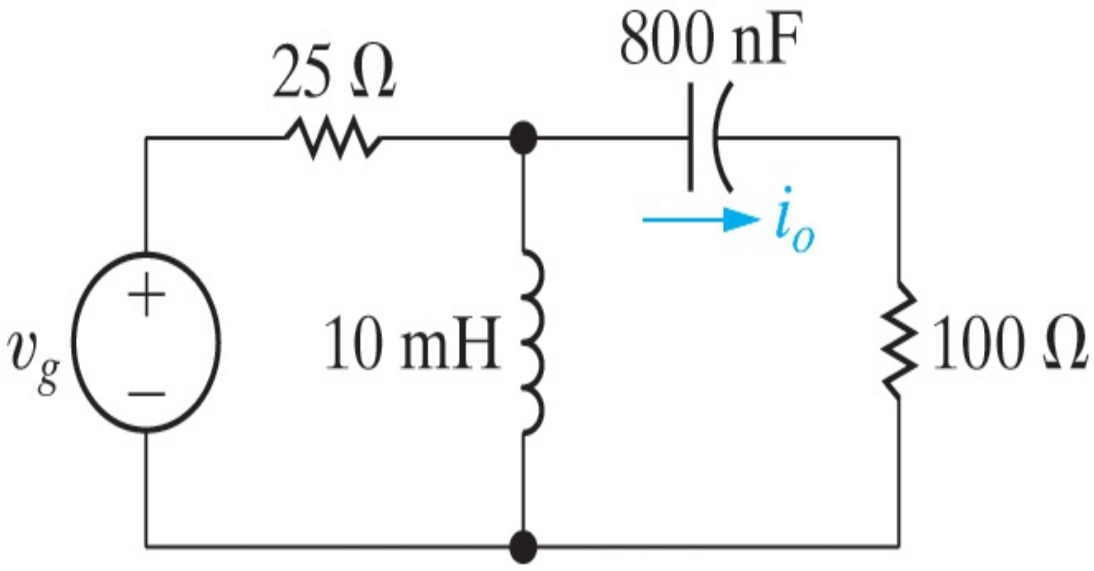


Figure P17.30 Full Alternative Text

12. 17.31 PSPICEMULTISIM

    1. Use the Fourier transform method to find io in the circuit in Fig. P17.31 if vg=125 cos 40,000t V.

    2. Check the answer obtained in (a) by finding the steady-state expression for io using phasor domain analysis.

Figure P17.31

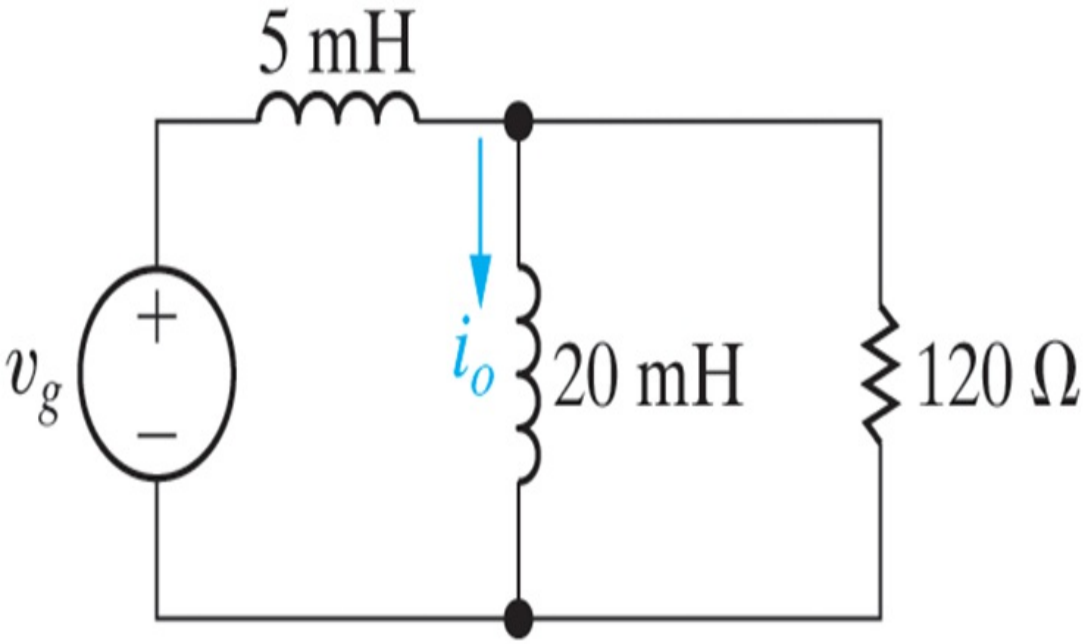


Figure P17.31 Full Alternative Text

13. 17.32 PSPICEMULTISIM

    1. Use the Fourier transform method to find vo in the circuit shown in Fig. P17.32. The voltage source generates the voltage

       vg=45e-500|t| V.

    2. Calculate vo(0-), vo(0+), and vo(∞).

    3. Find iL(0-); iL(0+); vC(0-); and vC(0+).

    4. Do the results in part (b) make sense in terms of known circuit behavior? Explain.

Figure P17.32

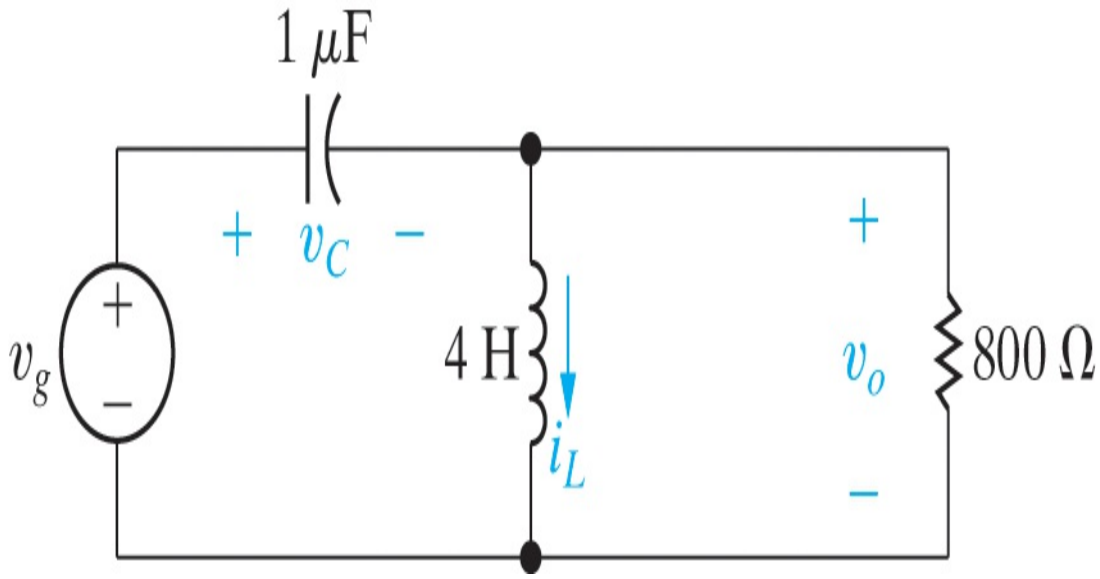


Figure P17.32 Full Alternative Text

14. 17.33 PSPICEMULTISIM The voltage source in the circuit in Fig. P17.33 is generating the signal

    vg=5 sgn(t)-5+30e-5tu(t) V.

    1. Find vo(0-) and vo(0+).
    2. Find io(0-) and io(0+).
    3. Find vo.

Figure P17.33

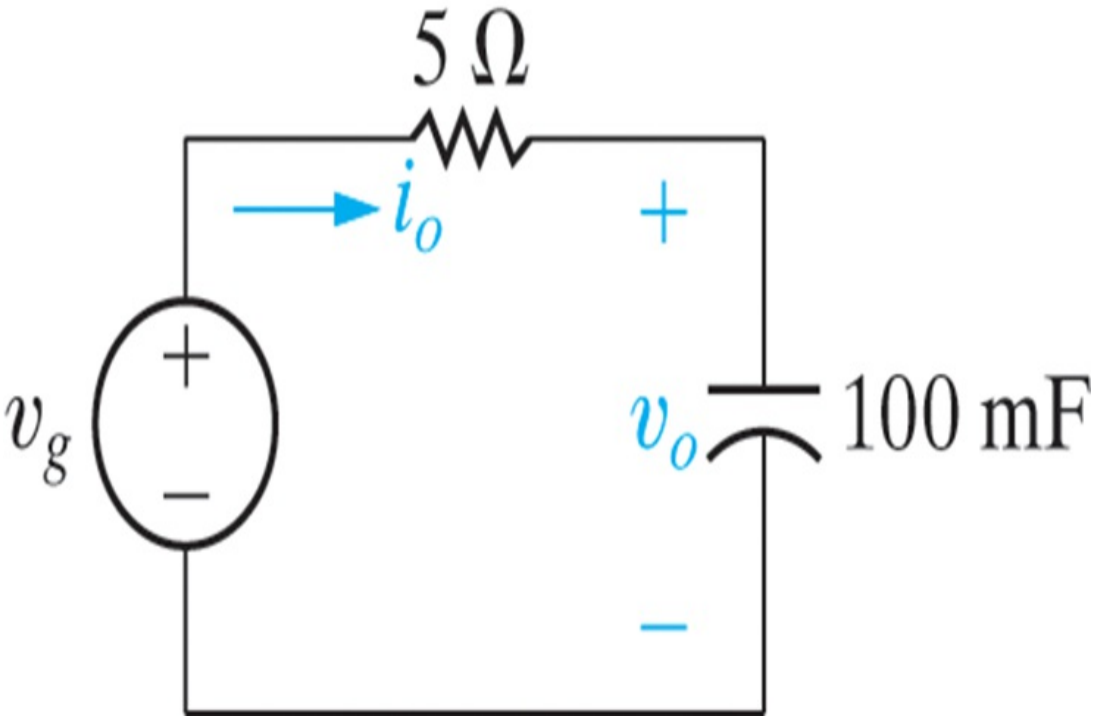


Figure P17.33 Full Alternative Text

15. 17.34 PSPICEMULTISIM

    1. Use the Fourier transform method to find vo in the circuit in Fig. P17.34 when

       vg=36e4tu(-t)-36e-4tu(t) V

    2. Find vo(0-).

    3. Find vo(0+).

Figure P17.34

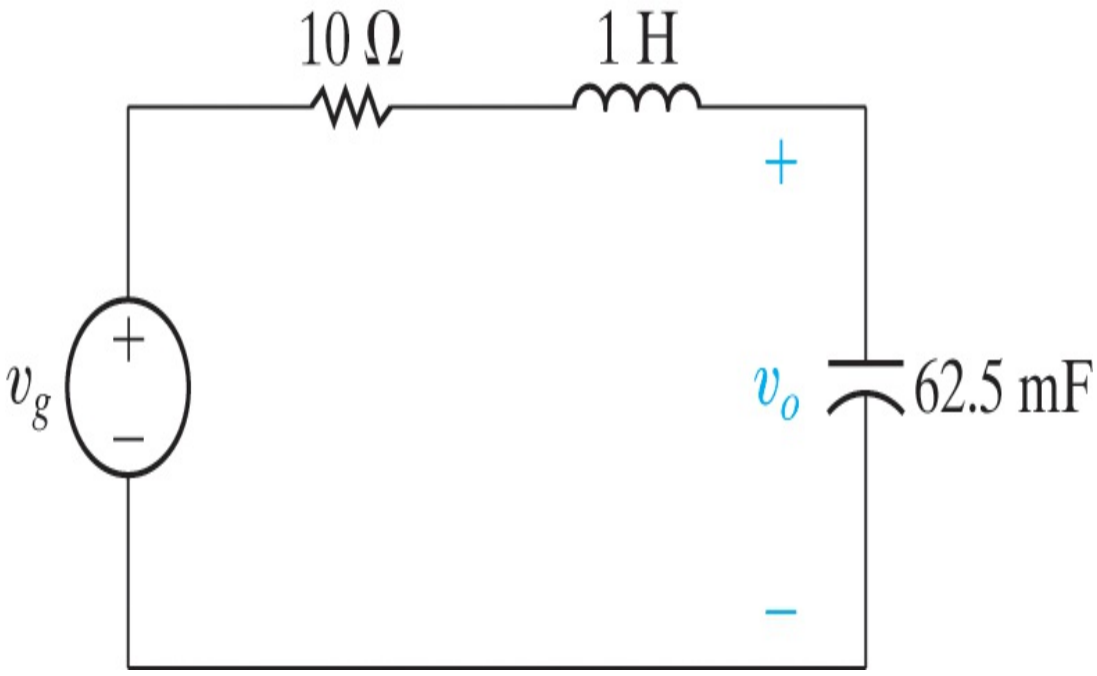


Figure P17.34 Full Alternative Text

16. 17.35 PSPICEMULTISIM

    1. Use the Fourier transform method to find vo in the circuit in Fig. P17.35 when

       ig=18e10tu(-t)-18e-10tu(t) A.

    2. Find vo(0-).

    3. Find vo(0+).

    4. Do the answers obtained in (b) and (c) make sense in terms of known circuit behavior? Explain.

Figure P17.35

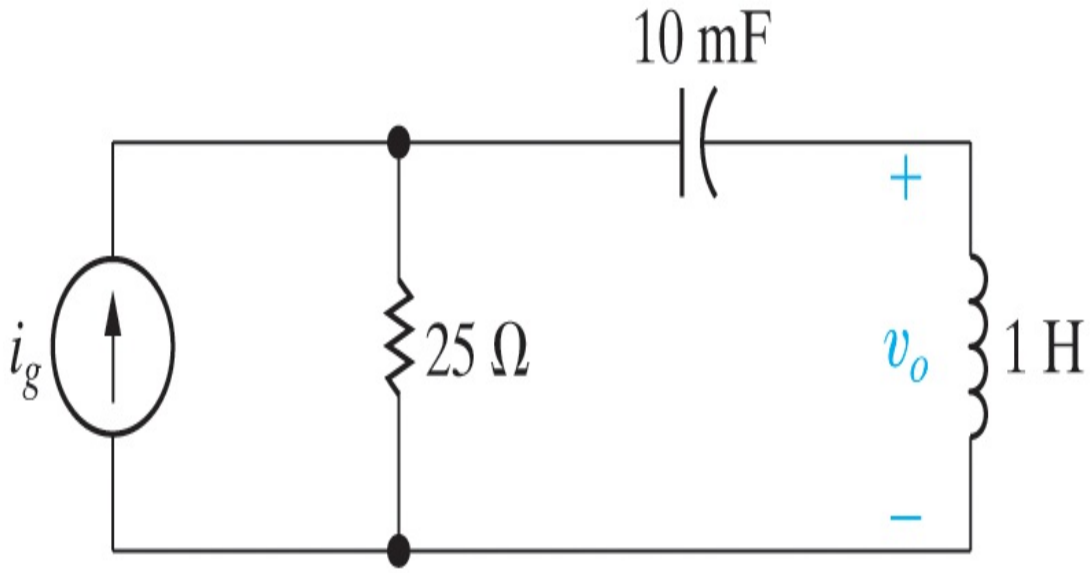


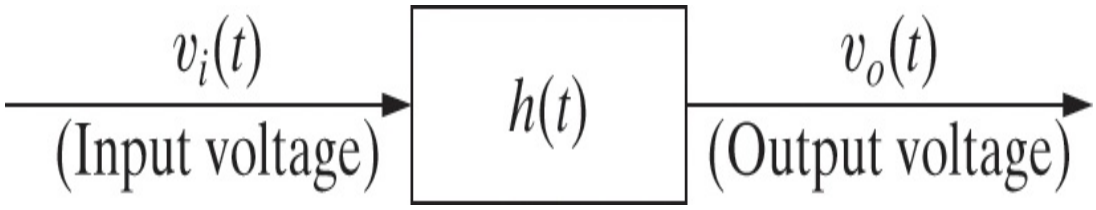
Figure P17.35 Full Alternative Text

17. 17.36 When the input voltage to the system shown in Fig. P17.36 is 15u(t) V, the output voltage is

    vo=[10+30e-20t-40e-30t]u(t) V.

    What is the output voltage if vi=15 sgn(t) V?

Figure P17.36



Section 17.8

1. 17.37 It is given that F(ω)=eωu(-ω)+e-ωu(?).
    1. Find f(t).
    2. Find the 1 Ω energy associated with f(t) via time-domain integration.
    3. Repeat (b) using frequency-domain integration.
    4. Find the value of ω1 if f(t) has 90% of the energy in the frequency band 0≤| ω |≤ω1.
2. 17.38 The circuit shown in Fig. P17.38 is driven by the current

    ig=12e-10tu(t) A.

    What percentage of the total 1 Ω energy content in the output current io lies in the frequency range 0≤| ω |≤100 rad/s?

Figure P17.38

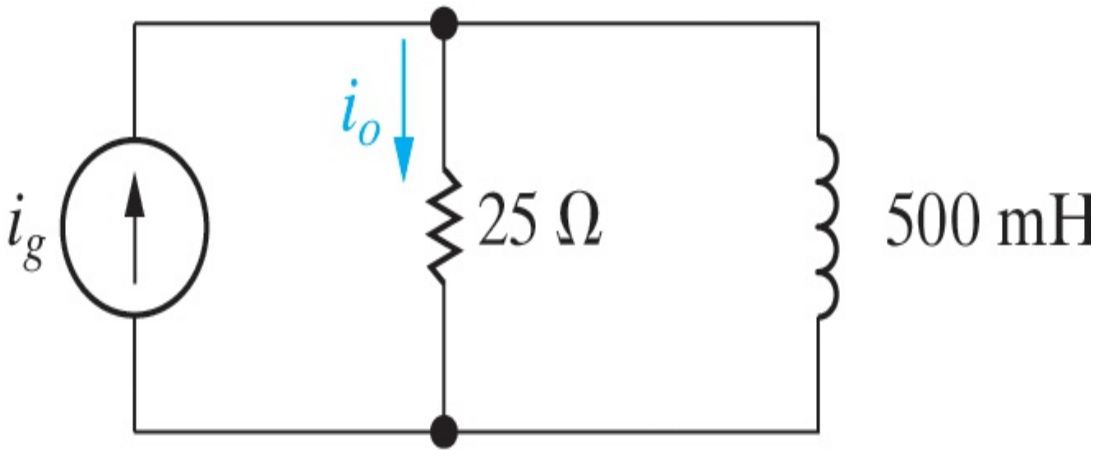


Figure P17.38 Full Alternative Text

3. 17.39 The input current signal in the circuit seen in Fig. P17.39 is

   ig=30e-2tu(t) μA, t=0+.

   What percentage of the total 1 Ω energy content in the output signal lies in the frequency range 0 to 4 rad/s?

Figure P17.39

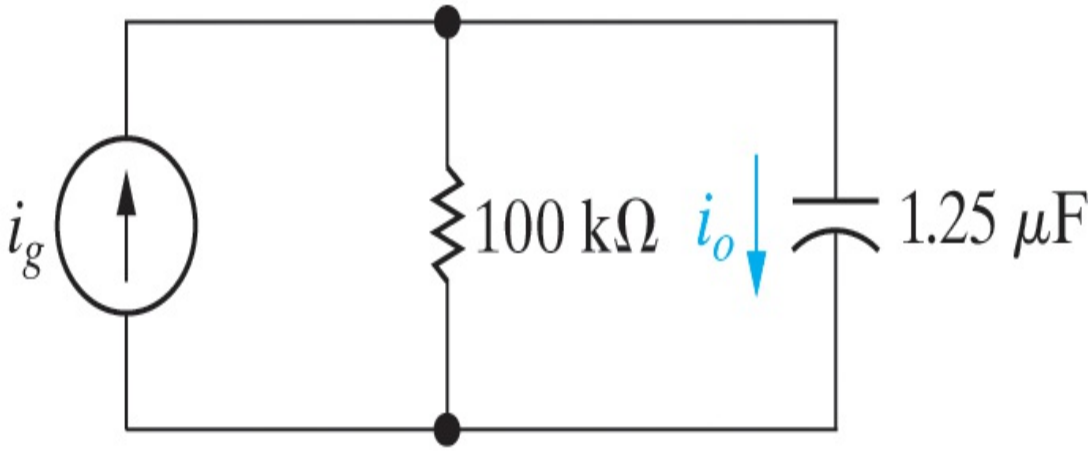


Figure P17.39 Full Alternative Text

4. 17.40 The input voltage in the circuit in Fig. P17.40 is vg=30e-|t| V.
   1. Find vo(t).
   2. Sketch |Vg(ω)| for −5≤ω≤5 rad/s.
   3. Sketch |Vo(ω)| for −5≤ω≤5 rad/s.
   4. Calculate the 1 Ω energy content of vg.
   5. Calculate the 1 Ω energy content of vo.
   6. What percentage of the 1 Ω energy content in vg lies in the

frequency range 0≤| ω |≤2 rad/s?

7. Repeat (f) for vo.

Figure P17.40

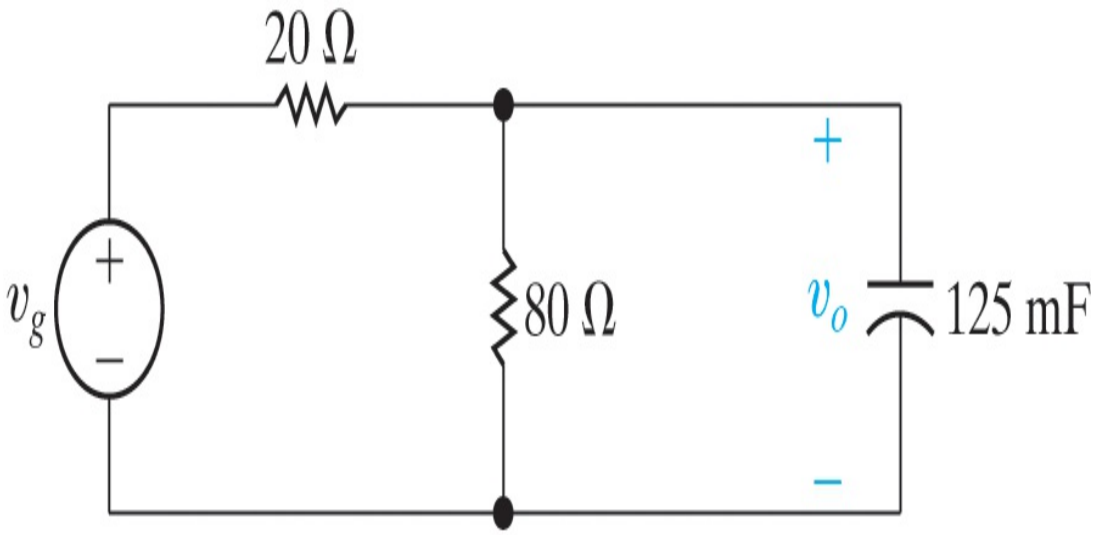


Figure P17.40 Full Alternative Text

5. 17.41 The amplitude spectrum of the input voltage to the high-pass RC filter in Fig. P17.41 is

   Vi(ω)=200| ω |,100 rad/s≤| ω |≤200 rad/s;Vi(ω)=0,elsewhere.

   1. Sketch |Vi(ω)|2 for −300≤ω≤300 rad/s.
   2. Sketch |Vo(ω)|2 for −300≤ω≤300 rad/s.
   3. Calculate the 1 Ω energy in the signal at the input of the filter.
   4. Calculate the 1 Ω energy in the signal at the output of the filter.

Figure P17.41

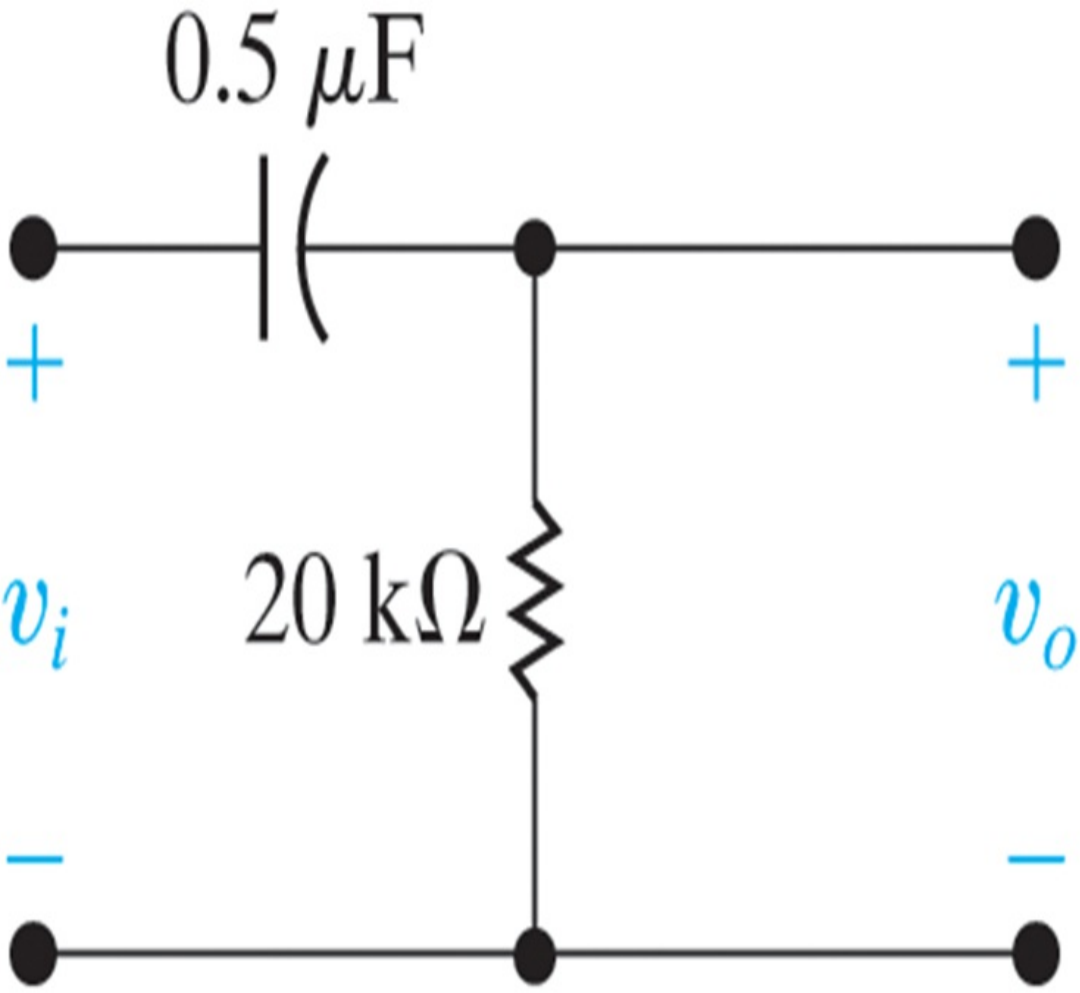


Figure P17.41 Full Alternative Text

6. 17.42 The input voltage to the high-pass RC filter circuit in Fig. P17.42 is

   vi(t)=Ae-atu(t).

   Let α denote the corner frequency of the filter, that is, a=1/RC.

   1. What percentage of the energy in the signal at the output of the filter is associated with the frequency band 0≤| ω |=α if α=a?

   2. Repeat (a), given that α=3a.

   3. Repeat (a), given that a=α/3.

Figure P17.42

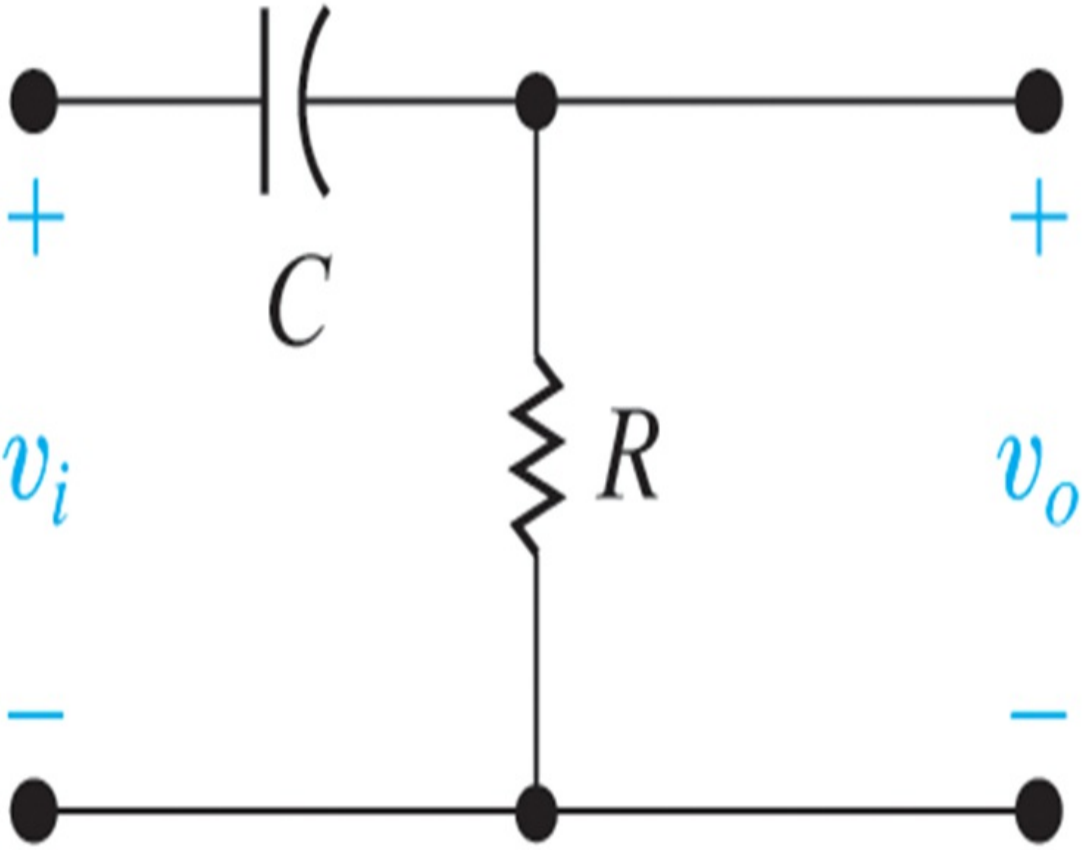


Figure P17.42 Full Alternative Text

Chapter 18 Two-Port Circuits

Chapter Contents

1. 18.1 The Terminal Equations
2. 18.2 The Two-Port Parameters
3. 18.3 Analysis of the Terminated Two-Port Circuit
4. 18.4 Interconnected Two-Port Circuits

Chapter Objectives

1. Be able to calculate any set of two-port parameters with any of the following methods:
    - Circuit analysis;
    - Measurements made on a circuit;
    - Converting from another set of two-port parameters using Table 18.1.
2. Be able to analyze a terminated two-port circuit to find currents, voltages, impedances, and ratios of interest using Table 18.2.
3. Know how to analyze a cascade interconnection of two-port circuits.

We have frequently focused on the behavior of a circuit at a specified pair of terminals. We introduced the Thévenin and Norton equivalent circuits to simplify circuit analysis relative to one pair of terminals. But in some electrical systems, a signal is fed into one pair of terminals, processed by the system, and extracted at a second pair of terminals. We can simplify the analysis of such systems using two pairs of terminals, each representing the points, or **ports**, where signals are either input or output.

In this chapter, we present circuits that have one input and one output port. Figure 18.1 on page 694 illustrates the basic two-port building block. We make several assumptions when using the two-port model in Fig. 18.1 to represent a circuit:

- There is no energy stored within the circuit.
- There are no independent sources within the circuit; dependent sources, however, are permitted.
- The current into a given port equals the current out of that port; that is,

$i1=i'1$ and $i2=i'2$.

- All external connections must be made to either the input port or the output port; no connections can be made between ports, that is, between terminals a and c, a and d, b and c, or b and d.

These assumptions limit the types of circuits that can be represented by a two-port model.

When we represent a circuit using a two-port model, we are only interested in the circuit's terminal variables (i1, v1, i2, and v2). We cannot use the two-port model to find the currents and voltages inside the circuit. We have already used such terminal behavior when analyzing operational amplifier circuits. In this chapter, we formalize that approach by introducing the two-port parameters.

Practical Perspective

Characterizing an Unknown Circuit

Up to this point, whenever we wanted to create a model of a circuit, we needed to know what types of components make up the circuit, the values of those components, and the interconnections among those components. But what if we want to model a circuit that is inside a "black box," where the components, their values, and their interconnections are hidden?

In this chapter, we will discover that by making two simple measurements on such a black box, we can create a model consisting of just four values—the two-port parameter model for the circuit. We can then use the two-port parameter model to predict the behavior of the circuit once we have attached a power source to one of its ports and a load to the other port.

In this example, suppose we have found a circuit, enclosed in a casing, with two wires extending from each side, as shown below. The casing is labeled "amplifier," and we want to determine whether or not it would be safe to use

this amplifier to connect a cd player modeled as a 2 V source to a speaker modeled as a 32 Ω resistor with a power rating of 100 W.

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18.1 The Terminal Equations

When we consider a circuit to be a two-port network, we want to relate the current and voltage at one port to the current and voltage at the other port. Figure 18.1 shows the reference polarities of the terminal voltages and the reference directions of the terminal currents. The references at each port are symmetric with respect to each other; that is, at each port the current is directed into the upper terminal, and at each port the voltage rises from the lower to the upper terminal.

Figure 18.1 The two-port building block.

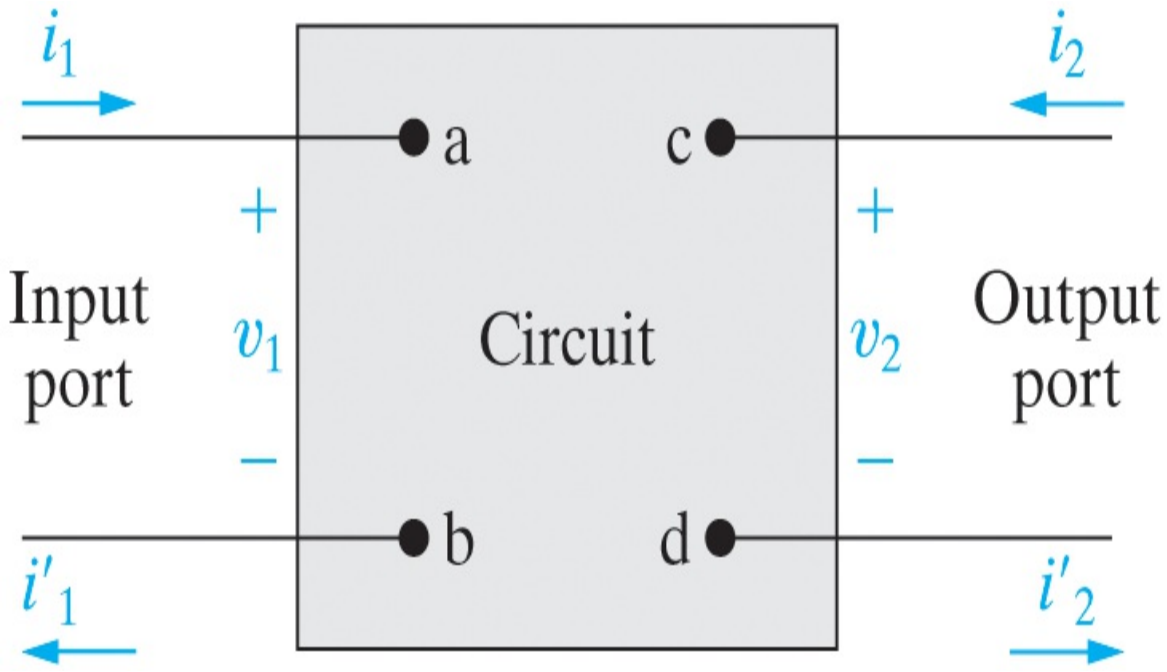


Figure 18.1 Full Alternative Text

The most general two-port network describes the circuit in the s domain. For purely resistive circuits, we can analyze the time-domain circuit to find its

two-port network description. Two-port networks used to find sinusoidal steady-state responses can be constructed by finding the appropriate s-domain expressions and then replacing s with jω, or by using phasor techniques directly. Here, we write all equations in the s domain; resistive networks and sinusoidal steady-state solutions become special cases. Figure 18.2 shows the basic building block in terms of the s-domain variables I1, V1, I2, and V2.

Figure 18.2 The s-domain two-port basic building block.

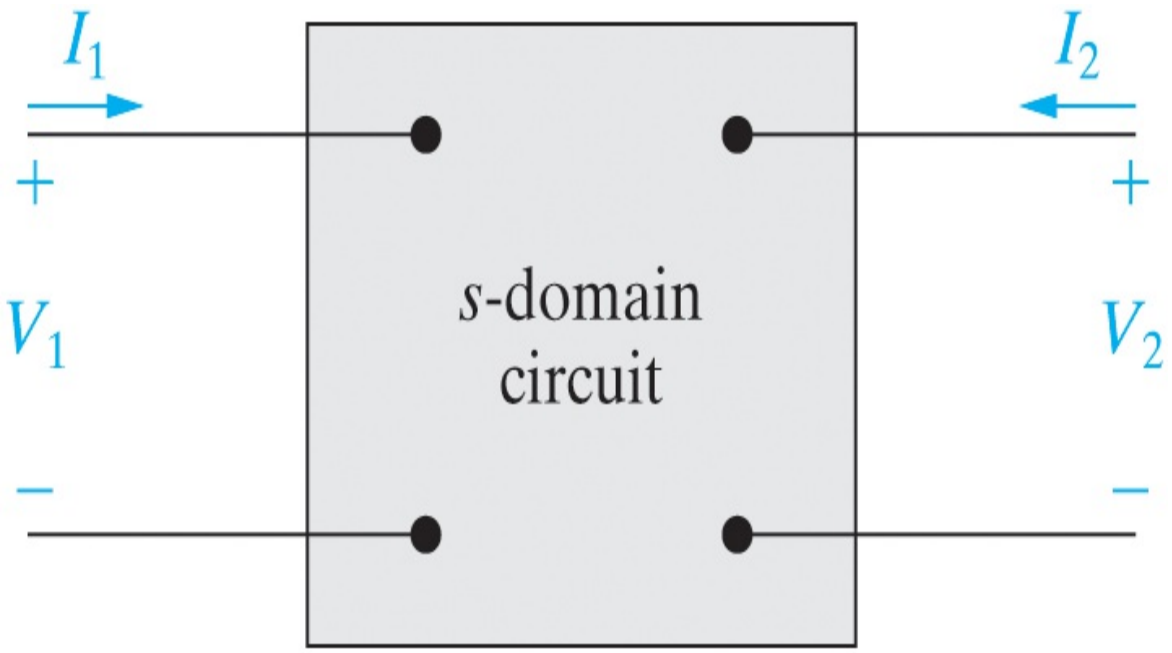


Figure 18.2 Full Alternative Text

Only two of the four terminal variables are independent. Thus, for any circuit, once we specify two of the variables, we can find the two remaining unknowns. For example, knowing V1 and V2 and the circuit within the box, we can find I1 and I2. The two-port network description consists of two simultaneous equations. However, there are six different ways in which to combine the four variables:

V1=z11I1+ z12I2,V2=z21I1+z22I2;(18.1)

$$I1=y11V1+y12V2, I2=y21V1+y22V2; \quad (18.2)$$
$$V1=a11V2-a12I2, I1=a21V2-a22I2; \quad (18.3)$$
$$V2=b11V1-b12I1, I2=b21V1-b22I1; \quad (18.4)$$
$$V1=h11I1+h12V22, I2=h21I1+h22V22; \quad (18.5)$$
$$I1=g11V1+g12I22, V2=g21V1+g22I22. \quad (18.6)$$

These six sets of equations may also be considered as three pairs of mutuallyinverse relations. The first set, Eqs. 18.1, gives the input and output voltages asfunctions of the input and output currents. The second set, Eqs. 18.2, gives the inverserelationship, that is, the input and output currents as functions of the inputand output voltages. Equations 18.3 and 18.4 areinverse relations, as are Eqs. 18.5 and 18.6.

The coefficients of the current and/or voltage variables on the right-handside of Eqs. 18.1–18.6 are called the **parameters** of thetwo-port circuit. Thus, when using Eqs. 18.1, we refer to the z parameters of the circuit. Similarly, we refer to the y parameters, the a parameters, the b parameters, the h parameters, and the g parameters of the network.

18.2 The Two-Port Parameters

Wecan determine the parameters for any circuit by computation or measurement. Thecomputations or measurements we need come from the parameter equations. Forexample, suppose we want to find the z parameters for a circuit. FromEqs. 18.1,

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \Omega, \quad (18.7)$$
$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \Omega, \quad (18.8)$$
$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \Omega, \quad (18.9)$$
$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \Omega. \quad (18.10)$$

Equations 18.7–18.10 revealthat the four z parameters can be described as follows:

- z11 is the impedance seen looking intoport 1 when port 2 is open.
- z12 is a transfer impedance. It is theratio of the port 1 voltage to the port 2 current when port 1 is open.
- z21 is a transfer impedance. It is theratio of the port 2 voltage to the port 1 current when port 2 is open.
- z22 is the impedance seen looking intoport 2 when port 1 is open.

Therefore, the impedance parametersmay be either calculated or measured by first opening port 2 and determining theratios V1/I1 and V2/I1, and then opening port 1 and determining the ratios V1/I2 and V2/I2.

Equations 18.7–18.10 showwhy the parameters in Eqs.18.1 are called the z parameters.Each parameter is the ratio of a voltage to a current and therefore is animpedance with the dimension of ohms.

We use the same process todetermine the remaining two-port parameters, which are either calculated ormeasured. Finding a two-port parameter requires that a port be either opened orshorted. The two-port parameters are

either impedances, admittances, ordimensionless ratios. The dimensionless ratios are either ratios of two voltagesor two currents. Equations18.11–18.15 summarize these observations.

$$y11=I1V1|V2=0S, y12=I1V2|V1=0S, y21=I2V1|V2=0S, y22=I2V2|V1=0S. \quad (18.11)$$

$$a11=V1V2|I2=0, a12=-V1I2|V2=0\Omega, a21=I1V2|I2=0S, a22=-I1I2|V2=0. \quad (18.12)$$

$$b11=V2V1|I1=0, b12=-V2I1|V1=0\Omega, b21=I2V1|I1=0S, b22=-I2I1|V1=0. \quad (18.13)$$

$$h11=V1I1|V2=0\Omega, h12=V1V2|I1=0, h21=I2I1|V2=0, h22=I2V2|I1=0S. \quad (18.14)$$

$$g11=I1V1|I2=0S, g12=I1I2|V1=0, g21=V2V1|I2=0, g22=V2I2|V1=0\Omega. \quad (18.15)$$

We also give descriptive names to the reciprocal sets of two-port parameters. The impedance and admittance parameters are grouped into the immittance parameters. An **immittance** is either an impedance or an admittance. The a and b parameters are called the **transmission** parameters because they describe the voltage and current at one end of the two-port network in terms of the voltage and current at the other end. The immittance and transmission parameters are the natural choices for relating the two-port variables because they relate either voltage to current variables or input to output variables. In contrast, the h and g parameters relate an input voltage and output current to an output voltage and input current. Therefore, the h and g parameters are called **hybrid** parameters.

Example 18.1 calculates the z parameters for a resistive circuit by analyzing the circuit. Example 18.2 illustrates how a set of measurements made at the terminals of a two-port circuit can be used to calculate the a parameters.

Example 18.1 Finding the z Parameters of a Two-Port Circuit

Find the z parameters for the circuit shown in Fig. 18.3.

Figure 18.3 The circuit for Example 18.1.

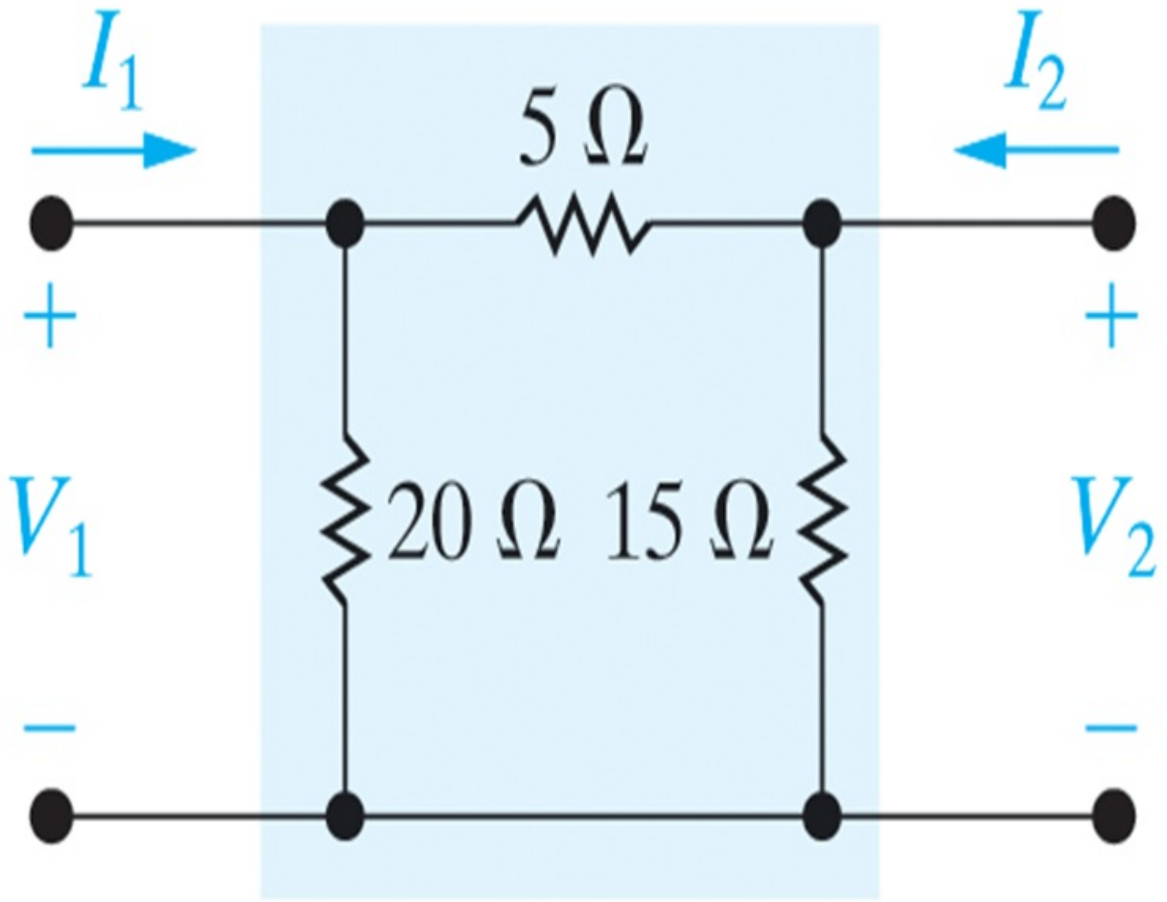


Figure 18.3 Full Alternative Text

Solution

The circuit is purely resistive, so the s-domain circuit is also purely resistive. There are many ways to find the two-port parameter values. Here, we note that with port 2 open, that is, I2=0, we can apply a 1 A current source at port 1 and use circuit analysis to find V1 and V2. With those two voltages, we can find z11 and z21. The circuit is shown in Fig. 18.4.

Figure 18.4 The circuit used to find z11 and z21 for the circuit in Fig. 18.3.

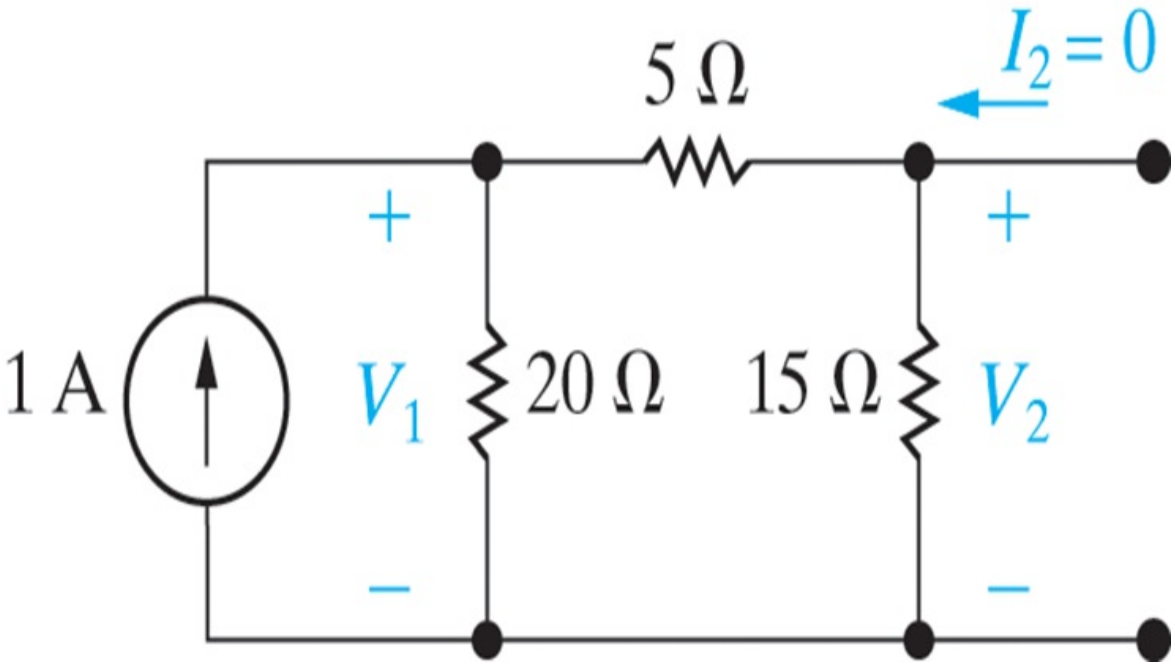


Figure 18.4 Full Alternative Text

To find V1 for the circuit in Fig. 18.4, find the equivalent resistance seen by the 1 A current source:

Zeq=20 ∥(5+15)=10Ω.

Then,

V1=Zeq(1)=10 V

and

z11=V1I1|I2=0=101=10Ω.

Now use voltage division to find V2 for the circuit in Fig. 18.4, noting that the voltage across the series combination of the 5 Ω and 15 Ω resistors is V1=10 V:

$$V2=1515+5V1=7.5\ V,$$

so

$$z21=V2I1|I2=0=7.51=7.5\Omega.$$

Next we leave port 1 open, that is, I1=0, apply a 1 A current source at port 2, and use circuit analysis to find V1 and V2. With those two voltages, we can find z22 and z12. The circuit is shown in Fig. 18.5. To find V2 for the circuit in Fig. 18.5, find the equivalent resistance seen by the 1 A current source:

$$Zeq=15\|(5+20)=9.375\Omega.$$

Figure 18.5 The circuit used to find z12 and z22 for the circuit in Fig. 18.3.

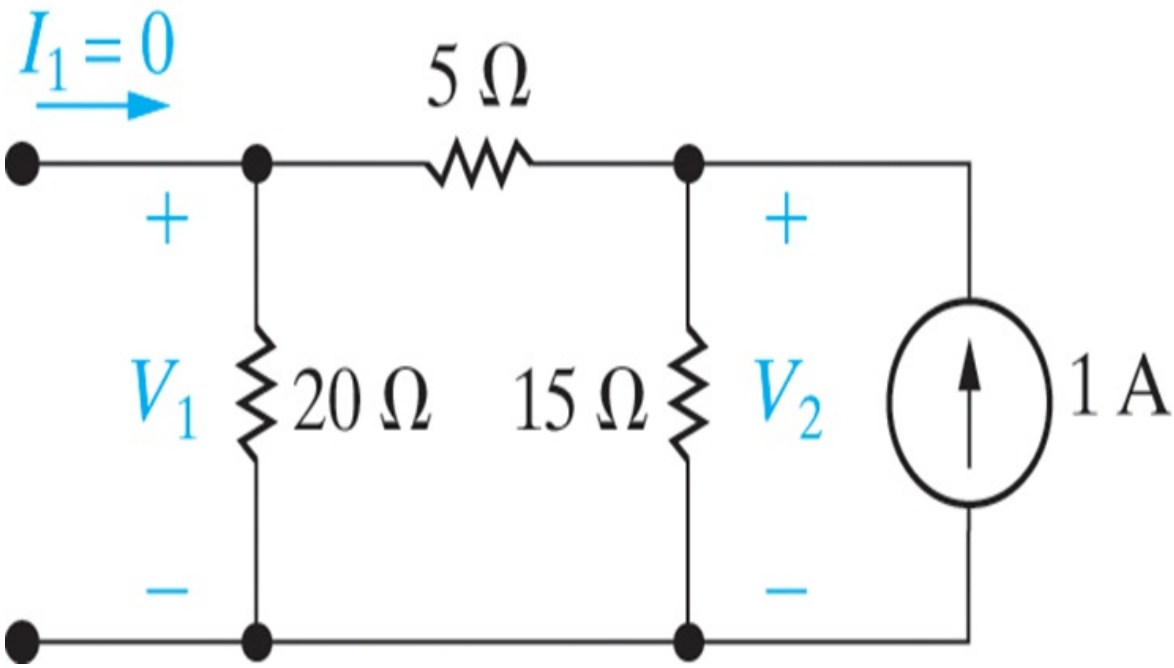


Figure 18.5 Full Alternative Text

Then,

V2=Zeq(1)=9.375 V

and

z22=V2I2|I1=0=9.3751=9.375Ω.

Now use voltage division to find V1 for the circuit in Fig. 18.5, noting that the voltage across the series combination of the 5 Ω and 20 Ω resistors is V2=9.375 V:

V1=2020+5V2=7.5 V,

so

z12=V1I2|I1=0=7.51=7.5Ω.

Example 18.2 Finding the a

Parameters from Measurements

A two-port circuit is operating in the sinusoidal steady state. With port 2 open, a voltage of 150 cos 4000t V is applied to port 1. Two measurements are made: the current into port 1 is 25 cos (4000t−45°) A, and the voltage across port 2 is 100 cos (4000t+15°) V. Then, port 2 is short-circuited, and a voltage of 30 cos 4000t V is applied to port 1. Two more measurements are made: the current into port 1 is 1.5 cos (4000t+30°) A, and the current into port 2 is 0.25 cos (4000t+ 150°) A. Find the a parameters that describe the sinusoidal steady-state behavior of the circuit.

Solution

Phasor-transforming the first set of measurements gives

$V1=150\ \ 0°\ V, I1=25\ \ -45°\ A, V2=100\ 15°\ V, I2=0\ A.$

From Eqs. 18.12,

$a11 = V1V2|I2=0=1500°100\ 15°=1.5\ −15°$, $a21 = I1V2|I2=0=25−45°10015°=0.25\ −60°S.$

Phasor-transforming the second set of measurements gives

$V1=300°\ V, I1=1.5\ 30°\ A, V2=0\ V, I2=0.25\ 150°\ A.$

Therefore

$a12=−V1I2|V2=0=−30\ 0°0.25\ 150°=120\ 30°\ Ω, a21=−I1I2|V2=0= −1.5\ 30°0.25\ 150°=6\ 60°.$

Assessment Problems

Objective 1—Be able to calculate any set of two-port parameters

1. 18.1 Find the y parameters for the circuit in Fig. 18.3.

   Answer:

   $y11=0.25\ S, y12=y21=-0.2\ S, y22=415 S$.

2. 18.2 Find the g and h parameters for the circuit in Fig. 18.3.

   Answer:

   $g11=0.1\ S;\ g12=-0.75;\ g21=0.75; g22=3.75\ \Omega;\ h11=4\ \Omega;\ h12=0.8; h21=-0.8;\ h22=0.1067\ S$.

3. 18.3 The following measurements were made on a two-port resistive circuit. With 50 mV applied to port 1 and port 2 open, the current into port 1 is 5 μA, and the voltage across port 2 is 200 mV. With port 1 short-circuited and 10 mV applied to port 2, the current into port 1 is 2 μA, and the current into port 2 is 0.5 μA. Find the g parameters of the network.

   Answer:

   $g11=0.1\ mS; g12=4; g21=4; g22=20\ k\Omega$.

SELF-CHECK: Also try Chapter Problems 18.2, 18.4, and 18.10.

Relationships Among the Two-Port Parameters

All six sets of two-port parameter equations relate to the same variables. Therefore, every set of two-port parameters must be related to every other set of these parameters. In other words, if we know one set of two-port parameters, we can derive all the other sets from the known set. The equations used to calculate any single two-port parameter from any set of two-port parameters are given in Table 18.1.

Table 18.1 Parameter Conversion Table

$$z_{11} = \frac{y_{22}}{\Delta y} = \frac{a_{11}}{a_{21}} = \frac{b_{22}}{b_{21}} = \frac{\Delta h}{h_{22}} = \frac{1}{g_{11}}$$

$$z_{12} = -\frac{y_{12}}{\Delta y} = \frac{\Delta a}{a_{21}} = \frac{1}{b_{21}} = \frac{h_{12}}{h_{22}} = -\frac{g_{12}}{g_{11}}$$

$$z_{21} = \frac{-y_{21}}{\Delta y} = \frac{1}{a_{21}} = \frac{\Delta b}{b_{21}} = -\frac{h_{21}}{h_{22}} = \frac{g_{21}}{g_{11}}$$

$$z_{22} = \frac{y_{11}}{\Delta y} = \frac{a_{22}}{a_{21}} = \frac{b_{11}}{b_{21}} = \frac{1}{h_{22}} = \frac{\Delta g}{g_{11}}$$

$$y_{11} = \frac{z_{22}}{\Delta z} = \frac{a_{22}}{a_{12}} = \frac{b_{11}}{b_{12}} = \frac{1}{h_{11}} = \frac{\Delta g}{g_{22}}$$

$$y_{12} = -\frac{z_{12}}{\Delta z} = -\frac{\Delta a}{a_{12}} = -\frac{1}{b_{12}} = -\frac{h_{12}}{h_{11}} = \frac{g_{12}}{g_{22}}$$

$$y_{21} = -\frac{z_{21}}{\Delta z} = -\frac{1}{a_{12}} = -\frac{\Delta b}{b_{12}} = \frac{h_{21}}{h_{11}} = -\frac{g_{21}}{g_{22}}$$

$$y_{22} = \frac{z_{11}}{\Delta z} = \frac{a_{11}}{a_{12}} = \frac{b_{22}}{b_{12}} = \frac{\Delta h}{h_{11}} = \frac{1}{g_{22}}$$

$$a_{11} = \frac{z_{11}}{z_{21}} = -\frac{y_{22}}{y_{21}} = \frac{b_{22}}{\Delta b} = -\frac{\Delta h}{h_{21}} = \frac{1}{g_{21}}$$

$$a_{12} = \frac{\Delta z}{z_{21}} = -\frac{1}{y_{21}} = \frac{b_{12}}{\Delta b} = -\frac{h_{11}}{h_{21}} = \frac{g_{22}}{g_{21}}$$

$$a_{21} = \frac{1}{z_{21}} = -\frac{\Delta y}{y_{21}} = \frac{b_{21}}{\Delta b} = -\frac{h_{22}}{h_{21}} = \frac{g_{11}}{g_{21}}$$

$$a_{22} = \frac{z_{22}}{z_{21}} = -\frac{y_{11}}{y_{21}} = \frac{b_{11}}{\Delta b} = -\frac{1}{h_{21}} = \frac{\Delta g}{g_{21}}$$

$$b_{11} = \frac{z_{22}}{z_{12}} = -\frac{y_{11}}{y_{12}} = \frac{a_{22}}{\Delta a} = \frac{1}{h_{12}} = -\frac{\Delta g}{g_{12}}$$

$$b_{12} = \frac{\Delta z}{z_{12}} = -\frac{1}{y_{12}} = \frac{a_{12}}{\Delta a} = \frac{h_{11}}{h_{12}} = -\frac{g_{22}}{g_{12}}$$

$$b_{21} = \frac{1}{z_{12}} = -\frac{\Delta y}{y_{12}} = \frac{a_{21}}{\Delta a} = \frac{h_{22}}{h_{12}} = -\frac{g_{11}}{g_{12}}$$

$$b_{22} = \frac{z_{11}}{z_{12}} = \frac{y_{22}}{y_{12}} = \frac{a_{11}}{\Delta a} = \frac{\Delta h}{h_{12}} = -\frac{1}{g_{12}}$$

$$h_{11} = \frac{\Delta z}{z_{22}} = \frac{1}{y_{11}} = \frac{a_{12}}{a_{22}} = \frac{b_{12}}{b_{11}} = \frac{g_{22}}{\Delta g}$$

$$h_{12} = \frac{z_{12}}{z_{22}} = -\frac{y_{12}}{y_{11}} = \frac{\Delta a}{a_{22}} = \frac{1}{b_{11}} = -\frac{g_{12}}{\Delta g}$$

$$h_{21} = -\frac{z_{21}}{z_{22}} = \frac{y_{21}}{y_{11}} = -\frac{1}{a_{22}} = -\frac{\Delta b}{b_{11}} = -\frac{g_{21}}{\Delta g}$$

$$h_{22} = \frac{1}{z_{22}} = \frac{\Delta y}{y_{11}} = \frac{a_{21}}{a_{22}} = \frac{b_{21}}{b_{11}} = \frac{g_{11}}{\Delta g}$$

$$g_{11} = \frac{1}{z_{11}} = \frac{\Delta y}{y_{22}} = \frac{a_{21}}{a_{11}} = \frac{b_{21}}{b_{22}} = \frac{h_{22}}{\Delta h}$$

$$g_{12} = -\frac{z_{12}}{z_{11}} = \frac{y_{12}}{y_{22}} = -\frac{\Delta a}{a_{11}} = -\frac{1}{b_{22}} = -\frac{h_{12}}{\Delta h}$$

$$g_{21} = \frac{z_{21}}{z_{11}} = -\frac{y_{21}}{y_{22}} = \frac{1}{a_{11}} = \frac{\Delta b}{b_{22}} = -\frac{h_{21}}{\Delta h}$$

$$g_{22} = \frac{\Delta z}{z_{11}} = \frac{1}{y_{22}} = \frac{a_{12}}{a_{11}} = \frac{b_{12}}{b_{22}} = \frac{h_{11}}{\Delta h}$$

$$\Delta z = z_{11}z_{22} - z_{12}z_{21}$$

$$\Delta y = y_{11}y_{22} - y_{12}y_{21}$$

$$\Delta a = a_{11}a_{22} - a_{12}a_{21}$$

$$\Delta b = b_{11}b_{22} - b_{12}b_{21}$$

$$\Delta h = h_{11}h_{22} - h_{12}h_{21}$$

$$\Delta g = g_{11}g_{22} - g_{12}g_{21}$$

We do not derive all the relationships listed in Table 18.1 because of the amount of algebra involved. To illustrate the general process, we derive the relationships between the z and y parameters and between the z and a parameters. To find the z parameters as functions of the y parameters, begin by solving Eqs. 18.2 for V1 and V2. We then compare the coefficients of I1 and I2 in the resulting expressions to the coefficients of I1 and I2 in Eqs. 18.1. From Eqs. 18.2,

$$V1=\begin{vmatrix} I1 & y12 \\ I2 & y22 \end{vmatrix} \Big/ \begin{vmatrix} y11 & y12 \\ y21 & y22 \end{vmatrix} = y22\Delta yI1 - y12\Delta yI2, V2=\begin{vmatrix} y11 & I1 \\ y21 & I2 \end{vmatrix} / \Delta y = -y21\Delta yI1 + y11\Delta yI2.$$

Comparing these expressions for V1 and V2 with Eqs. 18.1 shows

$$z11=y22\Delta y, z12=-y12\Delta y, z21=-y21\Delta y, z22=y11\Delta y.$$

To find the z parameters as functions of the a parameters, we rearrange Eqs. 18.3 in the form of Eqs. 18.1 and then compare coefficients. From the second equation in Eqs. 18.3,

$$V2=1a21I1+a22a21I2.$$

Therefore, substituting this expression for V2 into the first equation of Eqs. 18.3 yields

$$V1=a11a21I1+(a11a22a21-a12)I2.$$

Comparing these expressions for V1 and V2 with Eqs. 18.1 gives

$$z11=a11a21, z12=\Delta aa21. z21=1a21, z22=a22a21.$$

Example 18.3 illustrates the usefulness of the parameter conversion table.

Example 18.3 Finding h Parameters from Measurements and Table 18.1

Two sets of measurements are made on a two-port resistive circuit. The first set is made with port 2 open, and the second set is made with port 2 short-circuited. The results are as follows:

$$z_{11} = \frac{y_{22}}{\Delta y} = \frac{a_{11}}{a_{21}} = \frac{b_{22}}{b_{21}} = \frac{\Delta h}{h_{22}} = \frac{1}{g_{11}}$$

$$z_{12} = -\frac{y_{12}}{\Delta y} = \frac{\Delta a}{a_{21}} = \frac{1}{b_{21}} = \frac{h_{12}}{h_{22}} = -\frac{g_{12}}{g_{11}}$$

$$z_{21} = \frac{-y_{21}}{\Delta y} = \frac{1}{a_{21}} = \frac{\Delta b}{b_{21}} = -\frac{h_{21}}{h_{22}} = \frac{g_{21}}{g_{11}}$$

$$z_{22} = \frac{y_{11}}{\Delta y} = \frac{a_{22}}{a_{21}} = \frac{b_{11}}{b_{21}} = \frac{1}{h_{22}} = \frac{\Delta g}{g_{11}}$$

$$y_{11} = \frac{z_{22}}{\Delta z} = \frac{a_{22}}{a_{12}} = \frac{b_{11}}{b_{12}} = \frac{1}{h_{11}} = \frac{\Delta g}{g_{22}}$$

$$y_{12} = -\frac{z_{12}}{\Delta z} = -\frac{\Delta a}{a_{12}} = -\frac{1}{b_{12}} = -\frac{h_{12}}{h_{11}} = \frac{g_{12}}{g_{22}}$$

$$y_{21} = -\frac{z_{21}}{\Delta z} = -\frac{1}{a_{12}} = -\frac{\Delta b}{b_{12}} = \frac{h_{21}}{h_{11}} = -\frac{g_{21}}{g_{22}}$$

$$y_{22} = \frac{z_{11}}{\Delta z} = \frac{a_{11}}{a_{12}} = \frac{b_{22}}{b_{12}} = \frac{\Delta h}{h_{11}} = \frac{1}{g_{22}}$$

$$a_{11} = \frac{z_{11}}{z_{21}} = -\frac{y_{22}}{y_{21}} = \frac{b_{22}}{\Delta b} = -\frac{\Delta h}{h_{21}} = \frac{1}{g_{21}}$$

$$a_{12} = \frac{\Delta z}{z_{21}} = -\frac{1}{y_{21}} = \frac{b_{12}}{\Delta b} = -\frac{h_{11}}{h_{21}} = \frac{g_{22}}{g_{21}}$$

$$a_{21} = \frac{1}{z_{21}} = -\frac{\Delta y}{y_{21}} = \frac{b_{21}}{\Delta b} = -\frac{h_{22}}{h_{21}} = \frac{g_{11}}{g_{21}}$$

$$a_{22} = \frac{z_{22}}{z_{21}} = -\frac{y_{11}}{y_{21}} = \frac{b_{11}}{\Delta b} = -\frac{1}{h_{21}} = \frac{\Delta g}{g_{21}}$$

$$b_{11} = \frac{z_{22}}{z_{12}} = -\frac{y_{11}}{y_{12}} = \frac{a_{22}}{\Delta a} = \frac{1}{h_{12}} = -\frac{\Delta g}{g_{12}}$$

$$b_{12} = \frac{\Delta z}{z_{12}} = -\frac{1}{y_{12}} = \frac{a_{12}}{\Delta a} = \frac{h_{11}}{h_{12}} = -\frac{g_{22}}{g_{12}}$$

$$b_{21} = \frac{1}{z_{12}} = -\frac{\Delta y}{y_{12}} = \frac{a_{21}}{\Delta a} = \frac{h_{22}}{h_{12}} = -\frac{g_{11}}{g_{12}}$$

$$b_{22} = \frac{z_{11}}{z_{12}} = \frac{y_{22}}{y_{12}} = \frac{a_{11}}{\Delta a} = \frac{\Delta h}{h_{12}} = -\frac{1}{g_{12}}$$

$$h_{11} = \frac{\Delta z}{z_{22}} = \frac{1}{y_{11}} = \frac{a_{12}}{a_{22}} = \frac{b_{12}}{b_{11}} = \frac{g_{22}}{\Delta g}$$

$$h_{12} = \frac{z_{12}}{z_{22}} = -\frac{y_{12}}{y_{11}} = \frac{\Delta a}{a_{22}} = \frac{1}{b_{11}} = -\frac{g_{12}}{\Delta g}$$

$$h_{21} = -\frac{z_{21}}{z_{22}} = \frac{y_{21}}{y_{11}} = -\frac{1}{a_{22}} = -\frac{\Delta b}{b_{11}} = -\frac{g_{21}}{\Delta g}$$

$$h_{22} = \frac{1}{z_{22}} = \frac{\Delta y}{y_{11}} = \frac{a_{21}}{a_{22}} = \frac{b_{21}}{b_{11}} = \frac{g_{11}}{\Delta g}$$

$$g_{11} = \frac{1}{z_{11}} = \frac{\Delta y}{y_{22}} = \frac{a_{21}}{a_{11}} = \frac{b_{21}}{b_{22}} = \frac{h_{22}}{\Delta h}$$

$$g_{12} = -\frac{z_{12}}{z_{11}} = \frac{y_{12}}{y_{22}} = -\frac{\Delta a}{a_{11}} = -\frac{1}{b_{22}} = -\frac{h_{12}}{\Delta h}$$

$$g_{21} = \frac{z_{21}}{z_{11}} = -\frac{y_{21}}{y_{22}} = \frac{1}{a_{11}} = \frac{\Delta b}{b_{22}} = -\frac{h_{21}}{\Delta h}$$

$$g_{22} = \frac{\Delta z}{z_{11}} = \frac{1}{y_{22}} = \frac{a_{12}}{a_{11}} = \frac{b_{12}}{b_{22}} = \frac{h_{11}}{\Delta h}$$

$$\Delta z = z_{11}z_{22} - z_{12}z_{21}$$

$$\Delta y = y_{11}\,y_{22} - y_{12}\,y_{21}$$

$$\Delta a = a_{11}a_{22} - a_{12}a_{21}$$

$$\Delta b = b_{11}b_{22} - b_{12}b_{21}$$

$$\Delta h = h_{11}h_{22} - h_{12}h_{21}$$

$$\Delta g = g_{11}g_{22} - g_{12}g_{21}$$

Find the h parameters of the circuit.

Solution

We can find h11 and h21 directly from the short-circuit test:

$$h11=V1I1|V2=0=24\times10-320\times10-6=1.2\ k\Omega,h21=I2I1|V2=0=10-320\times10-6=5$$

The parameters h12 and h22 cannot be obtained directly from the open-circuit test. However, a check of Eqs. 18.7–18.15 indicates that the four a parameters can be derived from the test data. Therefore, h12 and h22 can be obtained through the conversion table. Specifically,

$$h12=\Delta aa22,h22=a21a22.$$

The a parameters are

$$a11=V1V2|I2=0=10\times10-3-40=-0.25\times10-3,a21=I1V2|I2=0=10\times10-6-40=-0.25\times10-6\ S,a12=-V1I2|V2=0=-24\times10-310-3=-24\ \Omega,a22=-I1I2|V2=0=-20\times10-610-3=-20\times10-3.$$

The numerical value of Δa is

$$\Delta a=a11a22-a12a21=5\times10-6-6\times10-6=-10-6.$$

Thus

$$h12=\Delta aa22=-10-6-20\times10-3=5\times10-5,h22=a21a22=-0.25\times10-6-20\times10-3=12.5\ \mu S.$$

Assessment Problem

Objective 1—Be able to calculate any set of two-port parameters

1. 18.4 The following measurements were made on a two-port resistive

circuit: With port 1 open, V2=15 V, V1=10 V, and I2=30 A; with port 1 short-circuited, V2=10 V, I2=4 A, and I1=−5 A. Calculate the z parameters.

Answer:

z11=(4 / 15) Ω;z12=(1 / 3) Ω;z21=−1.6 Ω;z22=0.5 Ω.

SELF-CHECK: Also try Chapter Problem 18.13.

Reciprocal Two-Port Circuits

A two-port circuit is **reciprocal** if interchanging an ideal voltage source at one port with an ideal ammeter at the other port produces the same ammeter reading. A two-port circuit is also reciprocal if interchanging an ideal current source at one port with an ideal voltmeter at the other port produces the same voltmeter reading. Relationships exist among reciprocal two-port parameters, as shown in Table 18.2. For a reciprocal two-port circuit, only three calculations or measurements are needed to determine a set of parameters.

Table 18.2 Two-Port Parameter Relationships for Reciprocal Circuits

Reciprocal Circuits

$$z_{12} = z_{21}$$

$$y_{12} = y_{21}$$

$$a_{11}a_{22} - a_{12}a_{21} = \Delta a = 1$$

$$b_{11}b_{22} - b_{12}b_{21} = \Delta b = 1$$

$$h_{12} = -h_{21}$$

$$g_{12} = -g_{21}$$

Reciprocal and Symmetric Circuits

$$z_{11} = z_{22}$$

$$y_{11} = y_{22}$$

$$a_{11} = a_{22}$$

$$b_{11} = b_{22}$$

$$h_{11}h_{22} - h_{12}h_{21} = \Delta h = 1$$

$$g_{11}\,g_{22} - g_{12}\,g_{21} = \Delta g = 1$$

A reciprocal two-port circuit is **symmetric** if its ports can be interchanged

without altering terminal current and voltage values. Figure 18.6 shows four examples of symmetric two-port circuits. In these circuits, additional relationships exist among the two-port parameters, as shown in Table 18.2. If a circuit is both reciprocal and symmetric, then only two calculations or measurements are necessary to determine all the two-port parameters. Example 18.4 determines whether a circuit is reciprocal and symmetric.

Figure 18.6 Four examples of symmetric two-port circuits. (a) A symmetric tee. (b) A symmetric pi. (c) A symmetric bridged tee. (d) A symmetric lattice.

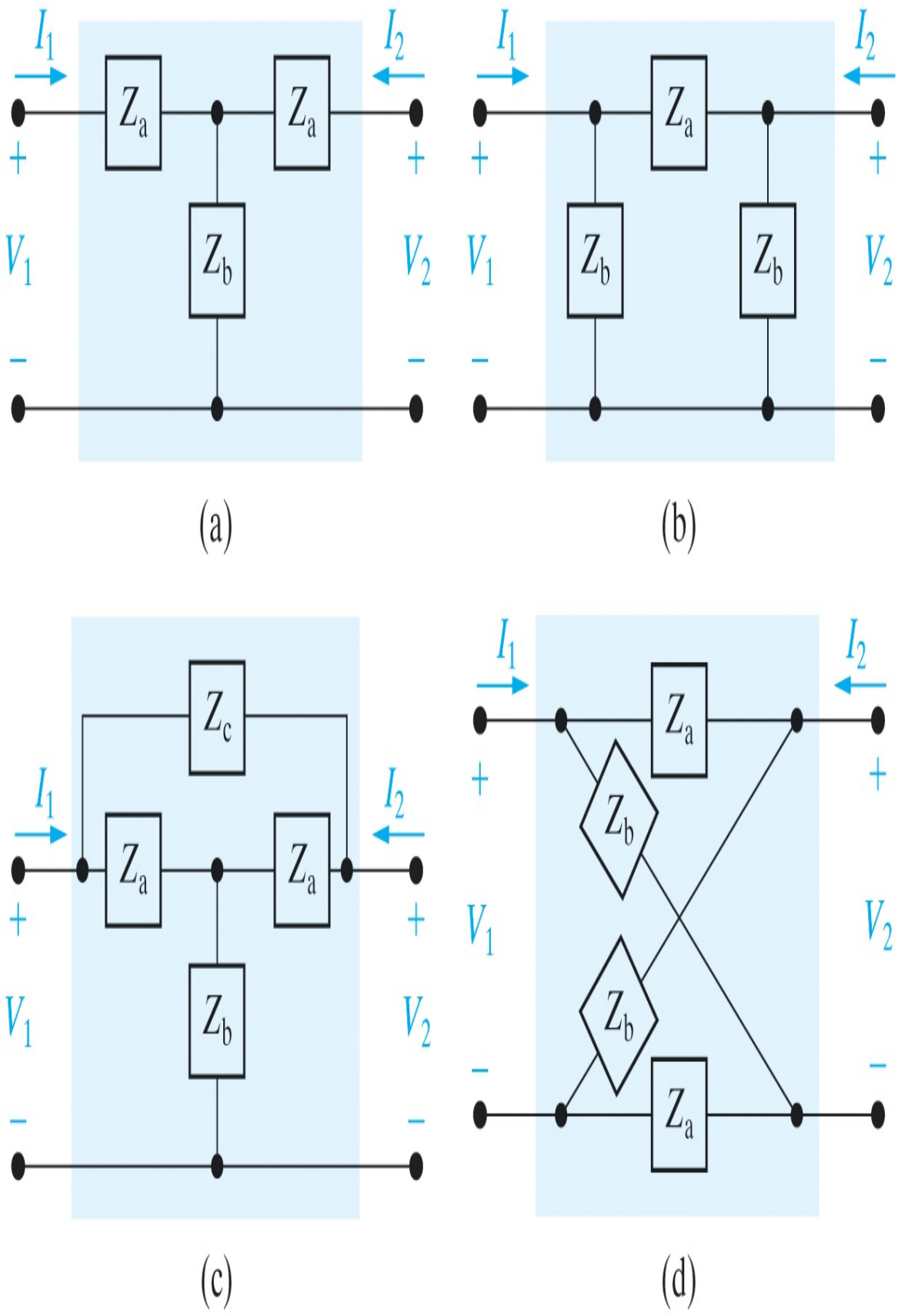

I_1
I_2
Z_a
Z_a
Z_b
+
V_1
−
+
V_2
−
(a)
I_1
I_2
Z_a
Z_b
Z_b
+
V_1
−
+
V_2
−
(b)
Z_c
I_1
I_2
Z_a
Z_a
Z_b
+
V_1
−
+
V_2
−
(c)
I_1
I_2
Z_a
Z_b
Z_b
Z_a
+
V_1
−
+
V_2
−
(d)

Figure 18.6 Full Alternative Text

Example 18.4 Determining Whether a Circuit Is Reciprocal and Symmetric

Consider the resistive circuit shown in Fig. 18.7.

Figure 18.7 The circuit for Example 18.4.

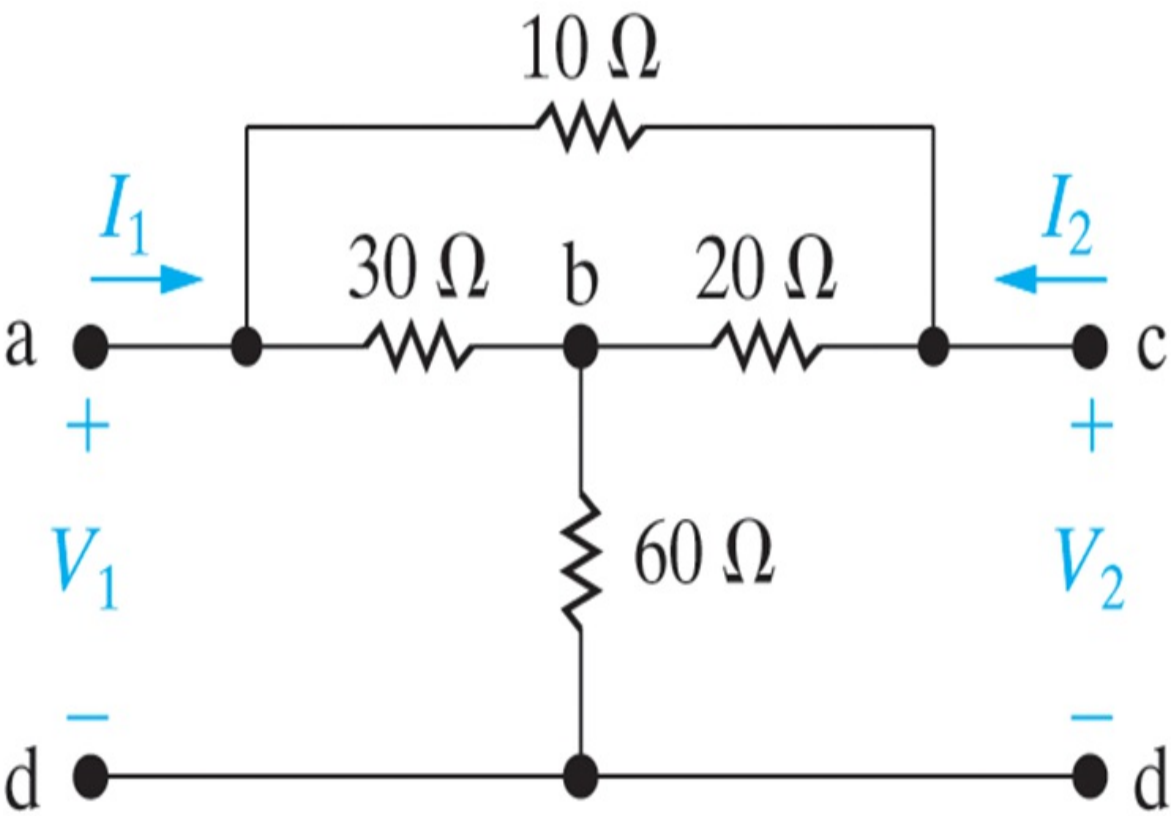


Figure 18.7 Full Alternative Text

1. Attach a 15 V source to the terminals a and d and calculate the reading on an ammeter attached between terminals c and d. Then attach a 15 V source to the terminals c and d and calculate the reading on an ammeter attached between terminals a and d. Is the circuit reciprocal?

2. Find the y parameters for the circuit and determine whether or not it is symmetric.

Solution

1. A 15 V source is attached between nodes a and d, and an ammeter is attached between terminals c and d. The resulting circuit is shown in Fig. 18.8. To find the ammeter current, begin by writing a KCL equation at node b to determine the voltage between nodes b and d, Vbd. Remember that an ideal ammeter behaves like a short circuit, so

   Vbd60+Vbd−1530+Vbd20=0,

Figure 18.8 The circuit in Fig. 18.7, with a voltage source and an ammeter attached.

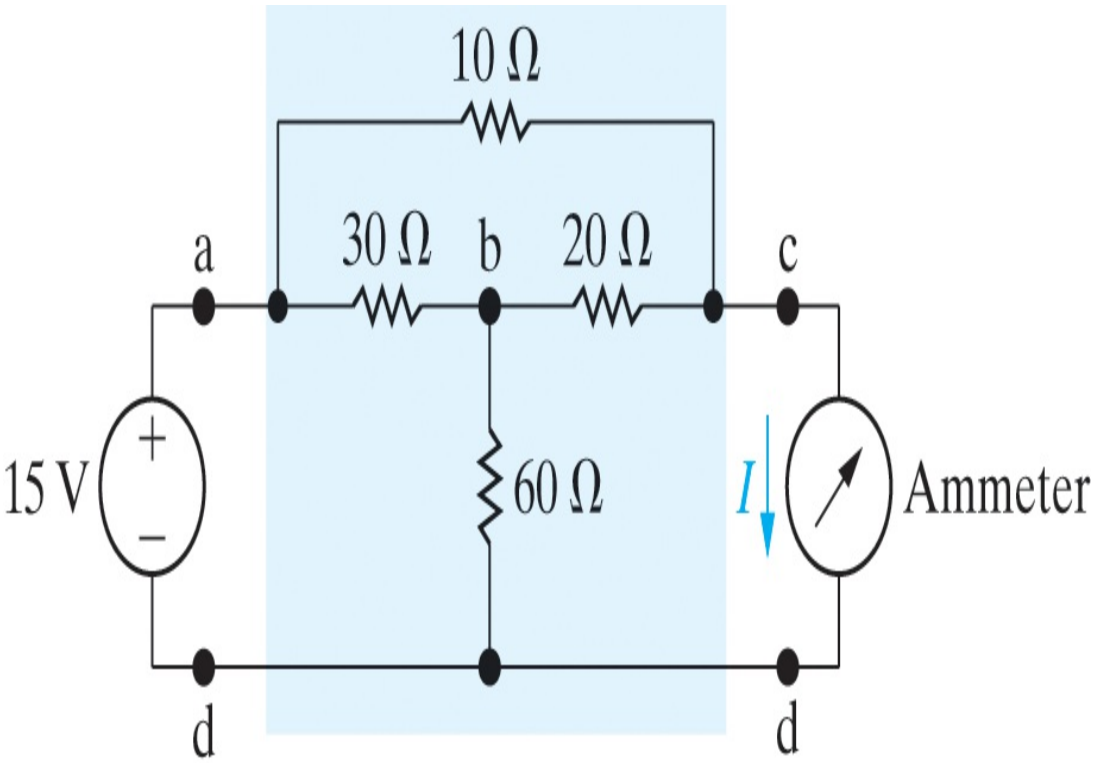


Figure 18.8 Full Alternative Text

and Vbd=5 V. Therefore, the current I in the ammeter is the sum of the current in the 10 Ω resistor and the current in the 20 Ω resistor:

I=520+1510=1.75 A.

Now interchange the voltage source and ammeter as shown in Fig. 18.9. Again, we write a KCL equation at node b to determine the voltage Vbd, remembering that the ideal ammeter behaves like a short circuit. Then,

Vbd60+Vbd30+Vbd−1520=0,

Figure 18.9 The circuit shown in Fig. 18.7, with the voltage source and ammeter

interchanged.

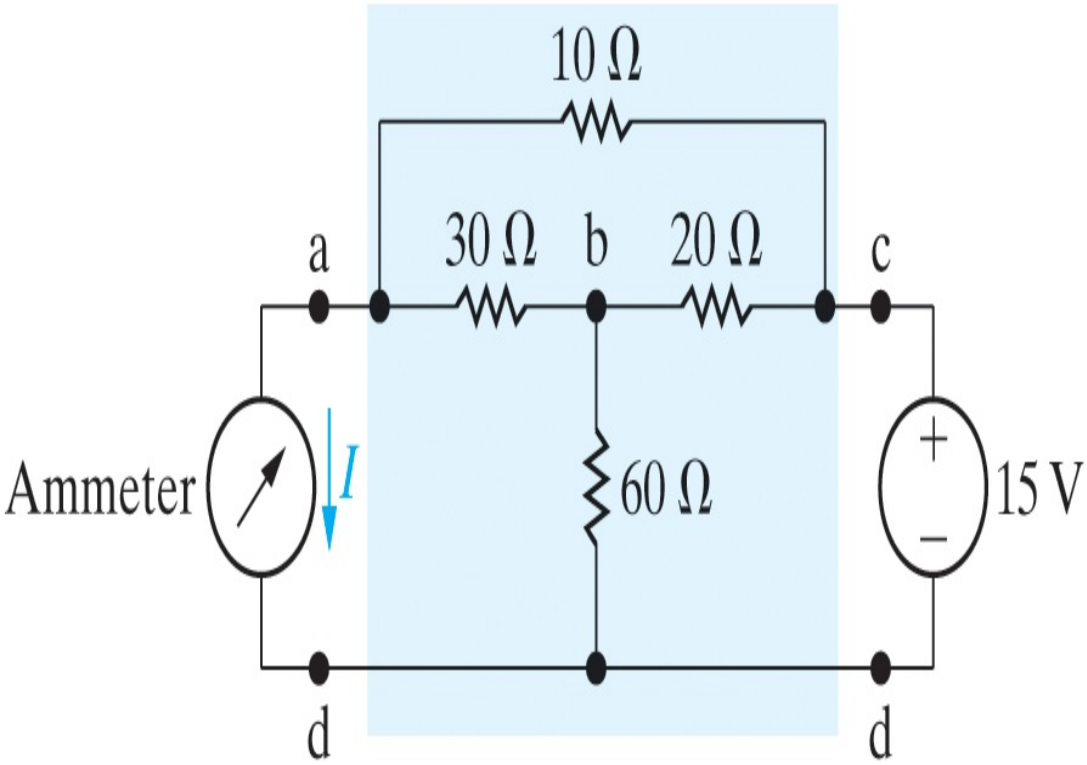


Figure 18.9 Full Alternative Text

so Vbd=7.5 V. The ammeter current I is the sum of the current in the 30 Ω resistor and the current in the 20 Ω resistor:

I=7.530+1510=1.75 A.

Since both ammeter readings are the same, the circuit in Fig. 18.7 is reciprocal.

2. We can use the analysis results for the circuit in Fig. 18.8 to find two of the y parameters. Attaching the ammeter between nodes c and d is the same as setting V2=0 in Fig. 18.7. Also, I2 in Fig. 18.7 equals −I in Fig. 18.8, and I1 in Fig. 18.7 is the sum of the currents in the 30 Ω and 20 Ω resistors in Fig. 18.8. Therefore,

   I2=−I=−1.75A,I1=15−530+1510=5530A,

so

$$y11 = I1V1|V2=0 = 55/3015 = 1199S, y21 = I2V1|V2=0 = -1.7515 = -760S.$$

We have already shown that the circuit in Fig. 18.7 is reciprocal, so $y12 = y21 = -7/60$ S.

We can use the analysis results for the circuit in Fig. 18.9 to find y22, because attaching the ammeter between nodes a and d is the same as setting V1=0 in Fig. 18.7. Also, I1 in Fig. 18.7 is the sum of the currents in the 10 Ω and 20 Ω resistors in Fig. 18.9. Therefore,

$$I1 = 15 - 7.520 + 1510 = 1.875 \text{ A}$$

so

$$y22 = I2V2|V1=0 = 1.87515 = 0.125 \text{ S}.$$

Note that $y11 \neq y22$, so the circuit in Fig. 18.7 is not symmetric.

Assessment Problem

Objective 1—Be able to calculate any set of two-port parameters

1. 18.5 The following measurements were made on a resistive two-port network that is symmetric and reciprocal: With port 2 open, V1=95 V and I1=5 A; with a short circuit across port 2, V1=11.52 V and I2= −2.72 A. Calculate the z parameters of the two-port network.

   Answer:

   $z11 = z22 = 19\Omega$, $z12 = z21 = 17\ \Omega$.

SELF-CHECK: Also try Chapter Problem 18.14.

18.3 Analysis of the Terminated Two-Port Circuit

A terminated two-port circuit usually has a source attached at port 1 and a load attached at port 2, as shown in the s-domain circuit of Fig. 18.10. In this circuit, Zg is the internal source impedance, Vg the internal source voltage, and ZL is the load impedance. To analyze this circuit, we find the terminal currents and voltages as functions of the two-port parameters, Vg, Zg, and ZL.

Figure 18.10 A terminated two-port model.

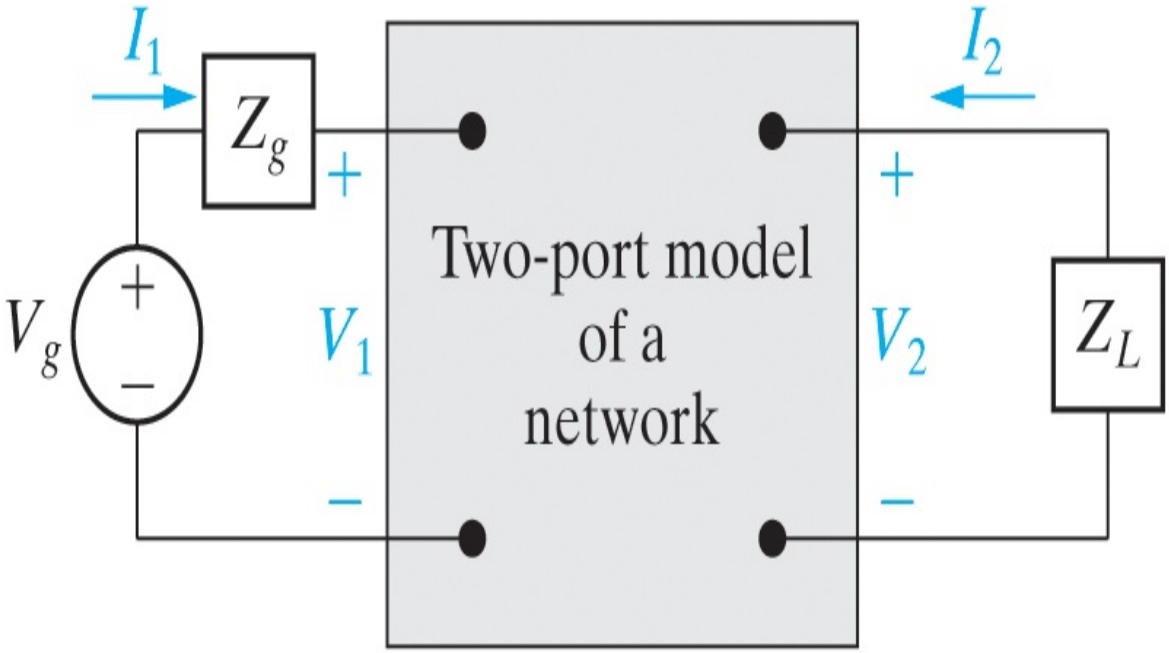


Figure 18.10 Full Alternative Text

Six characteristics of the terminated two-port circuit define its terminal

behavior:

- the input impedance Zin=V1/I1, or the admittance Yin=I1/V1;
- the output current I2;
- the Thévenin voltage and impedance (VTh, ZTh) with respect to port 2;
- the current gain I2/I1;
- the voltage gain V2/V1;
- the voltage gain V2/Vg.

The Six Characteristics in Terms of the z Parameters

To illustrate how these six characteristics are derived, we find their expressions using the z parameters of the two-port portion of the circuit. Table 18.3 summarizes the expressions involving the y, a, b, h, and g parameters.

Table 18.3 Terminated Two-Port Equations

z Parameters

$$Z_{\text{in}} = z_{11} - \frac{z_{12}z_{21}}{z_{22} + Z_L}$$

$$I_2 = \frac{-z_{21}V_g}{(z_{11} + Z_g)(z_{22} + Z_L) - z_{12}z_{21}}$$

$$V_{\text{Th}} = \frac{z_{21}}{z_{11} + Z_g} V_g$$

$$Z_{\text{Th}} = z_{22} - \frac{z_{12}z_{21}}{z_{11} + Z_g}$$

$$\frac{I_2}{I_1} = \frac{-z_{21}}{z_{22} + Z_L}$$

$$\frac{V_2}{V_1} = \frac{z_{21}Z_L}{z_{11}Z_L + \Delta z}$$

$$\frac{V_2}{V_g} = \frac{z_{21}Z_L}{(z_{11} + Z_g)(z_{22} + Z_L) - z_{12}z_{21}}$$

y Parameters

$$Y_{\text{in}} = y_{11} - \frac{y_{12}y_{21}Z_L}{1 + y_{22}Z_L}$$

$$I_2 = \frac{y_{21}V_g}{1 + y_{22}Z_L + y_{11}Z_g + \Delta y Z_g Z_L}$$

$$V_{\text{Th}} = \frac{-y_{21}V_g}{y_{22} + \Delta y Z_g}$$

$$Z_{\text{Th}} = \frac{1 + y_{11}Z_g}{y_{22} + \Delta y Z_g}$$

$$\frac{I_2}{I_1} = \frac{y_{21}}{y_{11} + \Delta y Z_L}$$

$$\frac{V_2}{V_1} = \frac{-y_{21}Z_L}{1 + y_{22}Z_L}$$

$$\frac{V_2}{V_g} = \frac{y_{21}Z_L}{y_{12}y_{21}Z_g Z_L - (1 + y_{11}Z_g)(1 + y_{22}Z_L)}$$

a Parameters

$$Z_{\text{in}} = \frac{a_{11}Z_L + a_{12}}{a_{21}Z_L + a_{22}}$$

$$I_2 = \frac{-V_g}{a_{11}Z_L + a_{12} + a_{21}Z_g Z_L + a_{22}Z_g}$$

$$V_{\text{Th}} = \frac{V_g}{a_{11} + a_{21}Z_g}$$

$$Z_{\text{Th}} = \frac{a_{12} + a_{22}Z_g}{a_{11} + a_{21}Z_g}$$

$$\frac{I_2}{I_1} = \frac{-1}{a_{21}Z_L + a_{22}}$$

$$\frac{V_2}{V_1} = \frac{Z_L}{a_{11}Z_L + a_{12}}$$

$$\frac{V_2}{V_g} = \frac{Z_L}{(a_{11} + a_{21}Z_g)Z_L + a_{12} + a_{22}Z_g}$$

b Parameters

$$Z_{\text{in}} = \frac{b_{22}Z_L + b_{12}}{b_{21}Z_L + b_{11}}$$

$$I_2 = \frac{-V_g \Delta b}{b_{11}Z_g + b_{21}Z_g Z_L + b_{22}Z_L + b_{12}}$$

$$V_{\text{Th}} = \frac{V_g \Delta b}{b_{22} + b_{21}Z_g}$$

$$Z_{\text{Th}} = \frac{b_{11}Z_g + b_{12}}{b_{21}Z_g + b_{22}}$$

$$\frac{I_2}{I_1} = \frac{-\Delta b}{b_{11} + b_{21}Z_L}$$

$$\frac{V_2}{V_1} = \frac{\Delta b Z_L}{b_{12} + b_{22}Z_L}$$

$$\frac{V_2}{V_g} = \frac{\Delta b Z_L}{b_{12} + b_{11}Z_g + b_{22}Z_L + b_{21}Z_g Z_L}$$

h Parameters

$$Z_{\text{in}} = h_{11} - \frac{h_{12}h_{21}Z_L}{1 + h_{22}Z_L}$$

$$I_2 = \frac{h_{21}V_g}{(1 + h_{22}Z_L)(h_{11} + Z_g) - h_{12}h_{21}Z_L}$$

$$V_{\text{Th}} = \frac{-h_{21}V_g}{h_{22}Z_g + \Delta h}$$

$$Z_{\text{Th}} = \frac{Z_g + h_{11}}{h_{22}Z_g + \Delta h}$$

$$\frac{I_2}{I_1} = \frac{h_{21}}{1 + h_{22}Z_L}$$

$$\frac{V_2}{V_1} = \frac{-h_{21}Z_L}{\Delta h Z_L + h_{11}}$$

$$\frac{V_2}{V_g} = \frac{-h_{21}Z_L}{(h_{11} + Z_g)(1 + h_{22}Z_L) - h_{12}h_{21}Z_L}$$

g Parameters

$$Y_{\text{in}} = g_{11} - \frac{g_{12}g_{21}}{g_{22} + Z_L}$$

$$I_2 = \frac{-g_{21}V_g}{(1 + g_{11}Z_g)(g_{22} + Z_L) - g_{12}g_{21}Z_g}$$

$$V_{\text{Th}} = \frac{g_{21}V_g}{1 + g_{11}Z_g}$$

$$Z_{\text{Th}} = g_{22} - \frac{g_{12}g_{21}Z_g}{1 + g_{11}Z_g}$$

$$\frac{I_2}{I_1} = \frac{-g_{21}}{g_{11}Z_L + \Delta g}$$

$$\frac{V_2}{V_1} = \frac{g_{21}Z_L}{g_{22} + Z_L}$$

$$\frac{V_2}{V_g} = \frac{g_{21}Z_L}{(1 + g_{11}Z_g)(g_{22} + Z_L) - g_{12}g_{21}Z_g}$$

We find the expression for each characteristic using one set of two-port equations, along with the two constraint equations imposed by the source applied at port 1 and the load applied at port 2. Using the z parameters, the circuit in Fig. 18.10 is described by the following four equations:

$$V1=z11I1+z12I2, \quad (18.16)$$
$$V2=z21I1+z22I2, \quad (18.17)$$
$$V1=Vg-I1Zg, \quad (18.18)$$
$$V2=-I2ZL. \quad (18.19)$$

Let's find the impedance seen looking into port 1, that is, $Zin=V1/I1$. In Eq. 18.17, replace V2 with $-I2ZL$ and solve the resulting expression for I2:

$$I2=-z21I1ZL+z22. \quad (18.20)$$

Then substitute this equation into Eq. 18.16 and solve for Zin:

$$Zin=z11-z12z21z22+ZL.$$

To find the output current I2, we first solve Eq. 18.16 for I1 after replacing V1 with the right-hand side of Eq. 18.18. The result is

$$I1=Vg-z12I2z11+Zg. \quad (18.21)$$

We now substitute Eq. 18.21 into Eq. 18.20 and solve the resulting equation for I2:

$$I2=-z21Vg(z11+Zg)(z22+ZL)-z12z21.$$

The Thévenin voltage with respect to port 2 equals V2 when I2=0. With I2=0, Eq. 18.17 reduces to

$$V2|I2=0=z21I1. \quad (18.22)$$

But when I2=0, Eq. 18.21 becomes $I1=Vg/(z11+Zg)$. Substituting this expression for I1 into Eq. 18.22 yields the open-circuit value of V2:

$$V2|I2=0=VTh=z21Zg+z11Vg.$$

The Thévenin, or output, impedance is the ratio V2/I2 when Vg is replaced by a short circuit. When Vg is zero, Eq. 18.18 reduces to

$$V1=-I1Zg. \quad (18.23)$$

Substituting Eq. 18.23 into Eq. 18.16 and solving for I1 gives

$$I1=-z12I2z11+Zg.$$

Now use this expression to replace I1 in Eq. 18.17, resulting in

$$V2I2|Vg=0=ZTh=z22-z12z21z11+Zg.$$

The current gain I2/I1 comes directly from Eq. 18.20:

$$I2I1=-z21ZL+z22.$$

To derive the expression for the voltage gain V2/V1, we start by replacing I2 in Eq. 18.17 with its value from Eq. 18.19; thus

$$V2=z21I1+z22(-V2ZL). \quad (18.24)$$

Next, solve Eq. 18.16 for I1 as a function of V1 and V2:

$$z11I1=V1-z12(-V2ZL)$$

or

$$I1=V1z11+z12V2z11ZL. \quad (18.25)$$

Finally, replace I1 in Eq. 18.24 with Eq. 18.25 and solve the resulting expression for V2/V1:

$$V2V1=z21ZLz11ZL+z11z22-z12z21=z21ZLz11ZL+\Delta z.$$

To derive the voltage ratio V2/Vg, we first combine Eqs. 18.16, 18.18, and 18.19 to find I1 as a function of V2 and Vg:

$$I1=z12V2ZL(z11+Zg)+Vgz11+Zg. \quad (18.26)$$

Then, starting with Eq. 18.17, use Eq. 18.26 to substitute for I1 and Eq. 18.19 to substitute for I2. The result is an expression involving only V2 and Vg:

$$V2=z21z12V2ZL(z11+Zg)+z21Vgz11+Zg−z22ZLV2,$$

which we can rearrange to get the desired voltage ratio:

$$V2Vg=z21ZL(z11+Zg)(z22+ZL)−z12z21.$$

Example 18.5 illustrates the usefulness of the relationships listed in Table 18.3.

Example 18.5 Analyzing a Terminated Two-Port Circuit

We describe the two-port circuit shown in Fig. 18.11 using its b parameters, whose values are

$$b11=−20,b12=−3000\ \Omega,b21=−2\ \text{mS},b22=−0.2.$$

1. Find the phasor voltage V2.
2. Find the average power delivered to the 5 kΩ load.
3. Find the average power delivered to the input port.
4. Find the load impedance for maximum average power transfer.
5. Find the maximum average power delivered to the load in (d).

Figure 18.11 The circuit for Example 18.5.

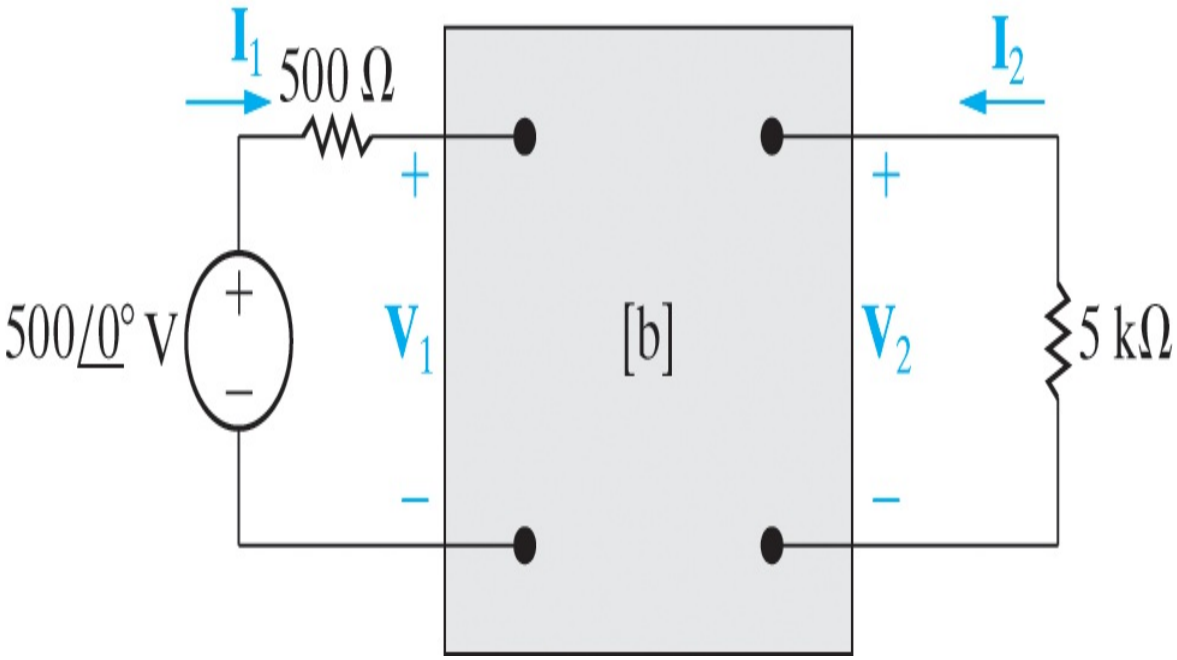


Figure 18.11 Full Alternative Text

Solution

1. To find V2, we have two choices from the entries in Table 18.3. We could find I2 and then find V2 from the relationship V2=−I2ZL, or we could find the voltage gain V2/Vg and calculate V2 from the gain. Let's use the latter approach. For the b-parameter values given, we have

   $\Delta b = b_{11}b_{22} - b_{12}b_{21} = (-20)(-0.2) - (-3000)(-0.002) = -2.$

   From Table 18.3,

   $\frac{V_2}{V_g} = \frac{\Delta b Z_L}{b_{12} + b_{11}Z_g + b_{22}Z_L + b_{21}Z_g Z_L} = \frac{(-2)(5000)}{-3000 + (-20)500 + (-0.2)5000 + [-0.002(500)(5000)]} = 1019.$

   Then,

   $V_2 = (1019)500 = 263.16^\circ \text{ V}.$

2. The average power delivered to the 5000Ω load is

$P_2 = |V_2|^2 2(5000) = 263.1622(5000) = 6.93$ W.

3. To find the average power delivered to the input port, we first find the input impedance Zin. From Table 18.3,

   $Z_{in} = b_{22}Z_L + b_{12}b_{21}Z_L + b_{11} = (-0.2)(5000) - 3000 - 2\times10^{-3}(5000) - 20 = 4003 = 133.33\ \Omega$.

   Now I1 follows directly:

   $I_1 = \frac{V_g}{Z_g + Z_{in}} = \frac{500}{500 + 133.33} = 789.47$ mA.

   The average power delivered to the input port is

   $P_1 = |I_1|^2 Z_{in} 2 = (0.78947)^2 (133.33)2 = 41.55$ W.

4. The load impedance for maximum power transfer equals the conjugate of the Thévenin impedance seen looking into port 2. From Table 18.3,

   $Z_{Th} = b_{11}Z_g + b_{12}b_{21}Z_g + b_{22} = (-20)(500) - 3000(-2\times10^{-3})(500) - 0.2 = 13{,}000 1.2 = 10{,}833.33\ \Omega$.

   Therefore $Z_L = Z_{Th}^* = 10{,}833.33\Omega$.

5. To find the maximum average power delivered to ZL, we first find V2 from the voltage-gain expression V2/Vg. When ZL is 10,833.33Ω, this gain is

   $\frac{V_2}{V_g} = \frac{\Delta b Z_L b_{12} + b_{11}Z_g + b_{22}Z_L + b_{21}Z_g Z_L}{} = (-2)(10{,}833.33)(-3000) + (-20)(500) + (-0.2)(10{,}833.33) + (-0.002)(500)(10{,}833.33) = 0.8333.$

   Thus

   $V_2 = (0.8333)(500) = 416.67$ V,

   and

   $P_{L(maximum)} = \frac{416.67^2}{10{,}833.33} = 8.01$ W.

Assessment Problem

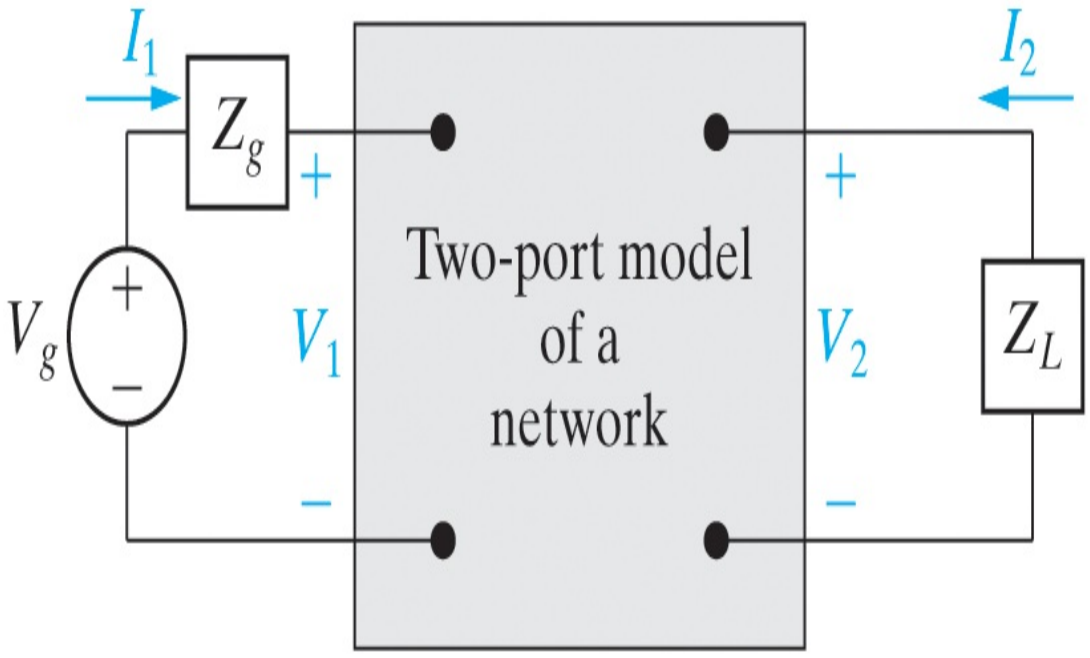
Objective 2—Be able to analyze a terminated two-port circuit to find currents, voltages, and ratios of interest

1. 18.6 The a parameters of the two-port network shown are a11=5×10−4, a12=10Ω, a21=10−6 S, and a22=−3×10−2. The network is driven by a sinusoidal voltage source having a maximum amplitude of 50 mV and an internal impedance of 100+j0Ω. It is terminated in a resistive load of 5 kΩ.

    1. Calculate the average power delivered to the load resistor.

    2. Calculate the load resistance for maximum average power.

    3. Calculate the maximum average power delivered to the resistor in (b).



18.3-6 Full Alternative Text

Answer:

1. (a) 62.5 mW;
2. (b) 70/6 kΩ;
3. (c) 74.4 mW.

SELF-CHECK: Also try Chapter Problems 18.29, 18.31, and 18.33.

18.4 Interconnected Two-Port Circuits

Synthesizing a large, complex system is usually made easier by first designing subsections of the system and then interconnecting the subsections. If the subsections are modeled by two-port circuits, we will need to analyze interconnected two-port circuits to complete the system design.

Two-port subsystem circuits can be interconnected in five ways: (1) in cascade, (2) in series, (3) in parallel, (4) in series-parallel, and (5) in parallel-series. Figure 18.12 depicts these five basic interconnections.

Figure 18.12 The five basic interconnections of two-port circuits. (a) Cascade. (b) Series. (c) Parallel. (d) Series-parallel. (e) Parallel-series.

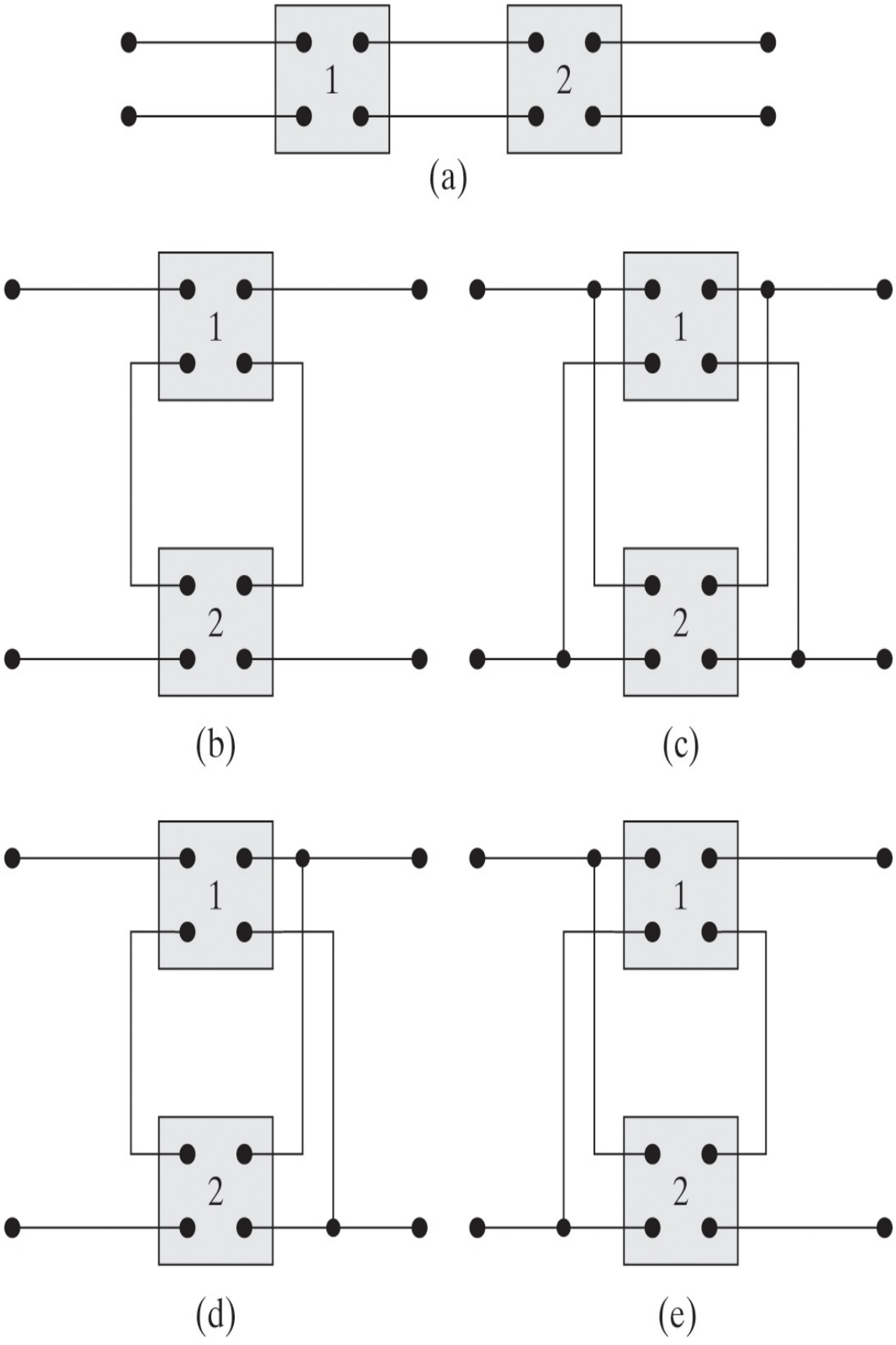

1
2
(a)
1
2
(b)
1
2
(c)
1
2
(d)
1
2
(e)

Figure 18.12 Full Alternative Text

We analyze and illustrate only the cascade connection in this section. The cascade connection is important because it occurs frequently in the modeling of large systems. The a parameters are best suited for describing the cascade connection. We analyze the cascade connection using the circuit shown in Fig. 18.13, where a single prime denotes a parameters in the first circuit and a double prime denotes a parameters in the second circuit. The output voltage and current of the first circuit are labeled V′2 and I′2, and the input voltage and current of the second circuit are labeled V′1 and I′1. The problem is to derive the a-parameter equations that relate V2 and I2 to V1 and I1. In other words, we want to construct the equations

V1=a11V2−a12I2,I1=a21V2−a22I2, (18.27)

Figure 18.13 A cascade connection.

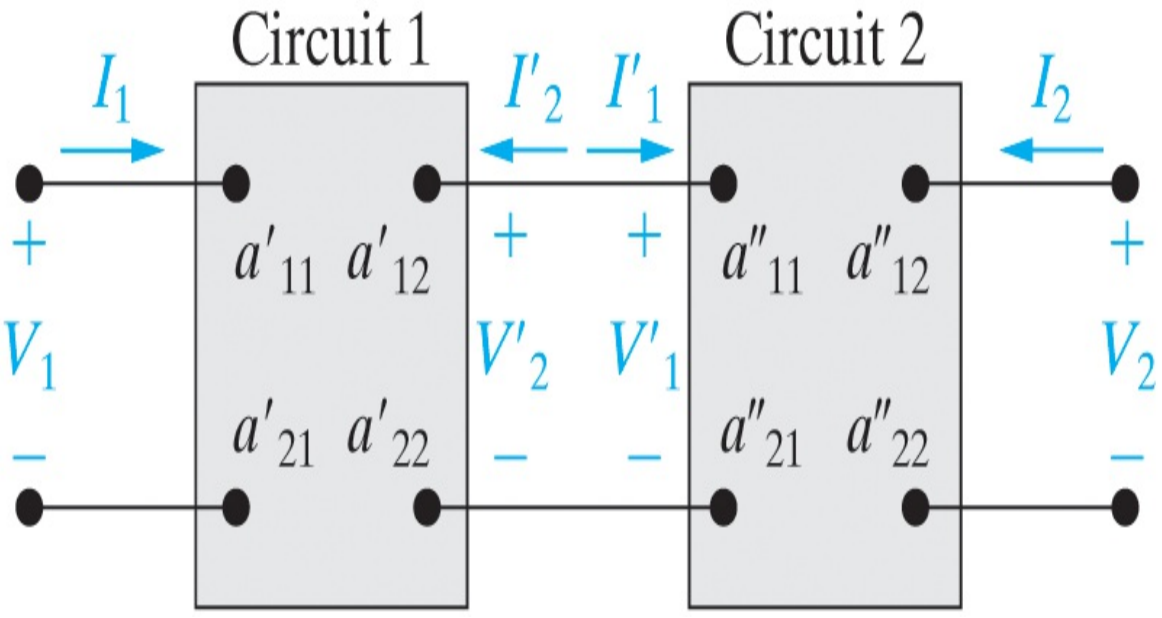


Figure 18.13 Full Alternative Text

where the a parameters are given explicitly in terms of the a parameters of the individual circuits.

We begin the derivation by noting from Fig. 18.13 that

$$V1=a'11V'2-a'12I'2, I1=a'21V'2-a'22I'2. \quad (18.28)$$

The interconnection means that $V'2=V'1$ and $I'2=-I'1$. Substituting these constraints into Eqs. 18.28 yields

$$V1=a'11V'1+a'12I'1, \quad (18.29)$$
$$I1=a'21V'1+a'22I'1.$$

The voltage $V'1$ and the current $I'1$ are related to V2 and I2 through the a parameters of the second circuit:

$$V'1=a''11V2-a''12I2, \quad (18.30)$$
$$I'1=a''21V2-a''22I2.$$

We substitute Eqs. 18.30 into Eqs. 18.29 to generate the relationships between V1, I1 and V2, I2:

$$V1=(a'11a''11+a'12a''21)V2-(a'11a''12+a'12a''22)I2, I1=(a'21a''11+a'22a''21)V2-(a'21a''12+a'22a''22)I2.$$

By comparing these expressions for V1 and I1 to Eqs. 18.27, we get the desired expressions for the a parameters of the interconnected networks, namely,

$$a11=a'11a''11+a'12a''21, \quad (18.31)$$
$$a12=a'11a''12+a'12a''22, \quad (18.32)$$
$$a21=a'21a''11+a'22a''21, \quad (18.33)$$
$$a22=a'21a''12+a'22a''22. \quad (18.34)$$

If more than two units are connected in cascade, the a parameters of the equivalent two-port circuit can be found by successively reducing the original set of two-port circuits one pair at a time.

Example 18.6 illustrates how to use Eqs. 18.31–18.34 to analyze a cascade

connection with two amplifier circuits.

Example 18.6 Analyzing Cascaded Two-Port Circuits

Two identical amplifiers are connected in cascade, as shown in Fig. 18.14. Each amplifier is described using its h parameters. The values are h11=1000Ω, h12=0.0015, h21=100, and h22=100 μS. Find the voltage gain V2/Vg.

Figure 18.14 The circuit for Example 18.6.

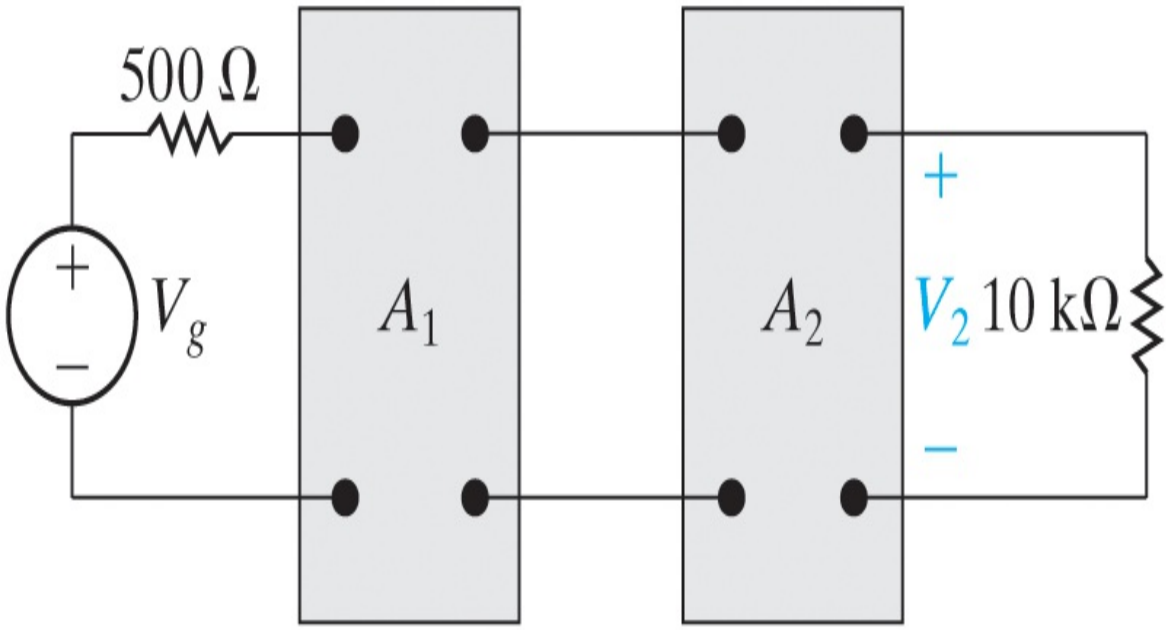


Figure 18.14 Full Alternative Text

Solution

The first step in finding V2/Vg is to convert from h parameters to a parameters. The amplifiers are identical, so one set of a parameters describes the amplifiers:

$$a'11=-\Delta hh21=+0.05100=5\times10-4, a'12=-h11h21=-1000100=-10\ \Omega, a'21=-h22h21=-100\times10-6100=-10-6\ S, a'22=-1h21=-1100=-10-2.$$

Next we use Eqs. 18.31–18.34 to compute the a parameters of the cascaded amplifiers:

$$a11=a'11a'11+a'12a'21=25\times10-8+(-10)(-10-6)=10.25\times10-6,$$
$$a12=a'11a'12+a'12a'22=(5\times10-4)(-10)+(-10)(-10-2)=0.095\ \Omega,$$
$$a21=a'21a'11+a'22a'21=(-10-6)(5\times10-4)+(-0.01)(-10-6)=9.5\times10-9\ S,$$
$$a22=a'21a'12+a'22a'22=(-10-6)(-10)+(-10-2)2=1.1\times10-4.$$

From Table 18.3,

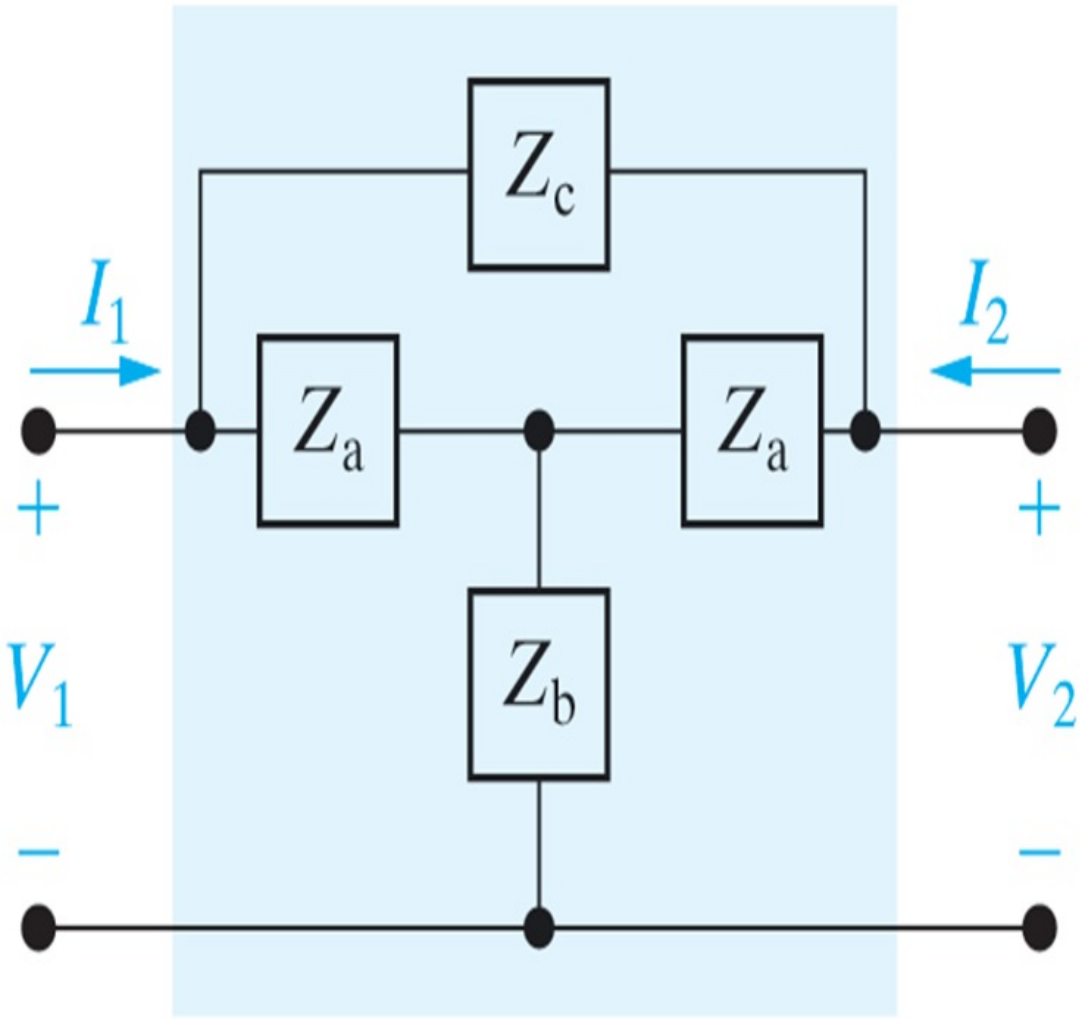
$$V2Vg=ZL(a11+a21Zg)ZL+a12+a22Zg=104[10.25\times10-6+9.5\times10-9(500)]104+0.095+1.1\times10-4(500)=1040.15+0.095+0.055$$

Thus, an input signal of 150 μV is amplified to an output signal of 5 V.

Assessment Problem

Objective 3—Know how to analyze a cascade interconnection of two-port circuits

1. 18.7 Each element in the symmetric bridged-tee circuit shown is a 15Ω resistor. Two of these bridged tees are connected in cascade between a dc voltage source and a resistive load. The dc voltage source has a no load voltage of 100 V and an internal resistance of 8Ω. The load resistor is adjusted until maximum power is delivered to the load. Calculate (a) the load resistance, (b) the load voltage, and (c) the load power.



18.4-7 Full Alternative Text

Answer:

1. 14.44Ω;
2. 16 V;
3. 17.73 W.

SELF-CHECK: Also try Chapter Problem 18.38.

Practical Perspective

Characterizing an Unknown Circuit

We make the following measurements to find the h parameters for our “black box” amplifier:

- With Port 1 open, apply 50 V at Port 2. Measure the voltage at Port 1 and the current at Port 2:

  V1=50 mV; I2=2.5 A.

- With Port 2 short-circuited, apply 2.5 mA at Port 1. Measure the voltage at Port 1 and the current at Port 2:

  V1=1.25 V; I2=3.75 A.

Calculate the h parameters according to Eqs. 18.14:

h11=V1I1|V2=0=1.250.0025=500 Ω;h12=V1V2|I1=0=0.0550=10−3;h21=I2I1|V2=0=3.750.0025=1500;h22=I2V

Now we use the terminated two-port equations to determine whether or not it is safe to attach a 2 V(rms) source with a 100 Ω internal impedance to Port 1 and use this source together with the amplifier to drive a speaker modeled as a 32 Ω resistance with a power rating of 100 W. Begin by finding the value of I2 from Table 18.3:

I2=h21Vg(1+h22ZL)(h11+Zg)−h12h21ZL=1500(2)[1+(0.05)(32)][500+100]−(1500)(10−3)(32)=1.98 A (rms).

Then, calculate the power delivered to the 32 Ω speaker:

P=RI22=(32)(1.98)2=126 W.

The amplifier would thus deliver 126 W to the speaker, which is rated at 100 W, so it would be better to use a different amplifier or buy a more powerful speaker.

SELF-CHECK: Also try Chapter Problem 18.46

Summary

- The **two-port model** is used to describe the performance of a circuit in terms of the voltage and current at its input and output ports. (See page 694.)
- The model is limited to circuits in which
  - no independent sources are inside the circuit between the ports;
  - no energy is stored inside the circuit between the ports;
  - the current into the port is equal to the current out of the port; and
  - no external connections exist between the input and output ports.

  (See page 692.)
- Two of the four terminal variables (V1, I1, V2, I2) are independent; therefore, only two simultaneous equations involving the four variables are needed to describe the circuit. (See page 694.)
- The six possible sets of simultaneous equations involving the four terminal variables are called the z-, y-, a-, b-, h-, and g-parameter equations. See Eqs. 18.1–18.6. (See page 694.)
- The equations are written in the s domain. The dc values of the parameters are obtained by setting s=0, and the sinusoidal steady-state values are obtained by setting s=jω. (See page 694.)
- Any set of two-port parameters can be calculated or measured by invoking appropriate short-circuit and open-circuit conditions at the input and output ports. See Eqs. 18.7–18.15. (See pages 695 and 696.)
- The relationships among the six sets of two-port parameters are given in Table 18.1. (See page 699.)

- A two-port circuit is **reciprocal** if interchanging an ideal voltage source at one port with an ideal ammeter at the other port produces the same ammeter reading. The effect of reciprocity on the two-port parameters is given in Table 18.2. (See page 701.)

- A reciprocal two-port circuit is **symmetric** if its ports can be interchanged without disturbing the values of the terminal currents and voltages. The added effect of symmetry on the two-port parameters is given in Table 18.2. (See page 701.)

- The performance of a two-port circuit connected to a Thévenin equivalent source and a load is summarized by the relationships given in Table 18.3. (See page 704.)

- Large networks can be divided into subnetworks by means of interconnected two-port models. The cascade connection was used in this chapter to illustrate the analysis of interconnected two-port circuits. (See page 709.)

Problems

Sections 18.1–18.2

1. 18.1 Find the h and g parameters for the circuit in Example 18.1.
2. 18.2 Find the z parameters for the circuit in Fig. P18.2.

Figure P18.2

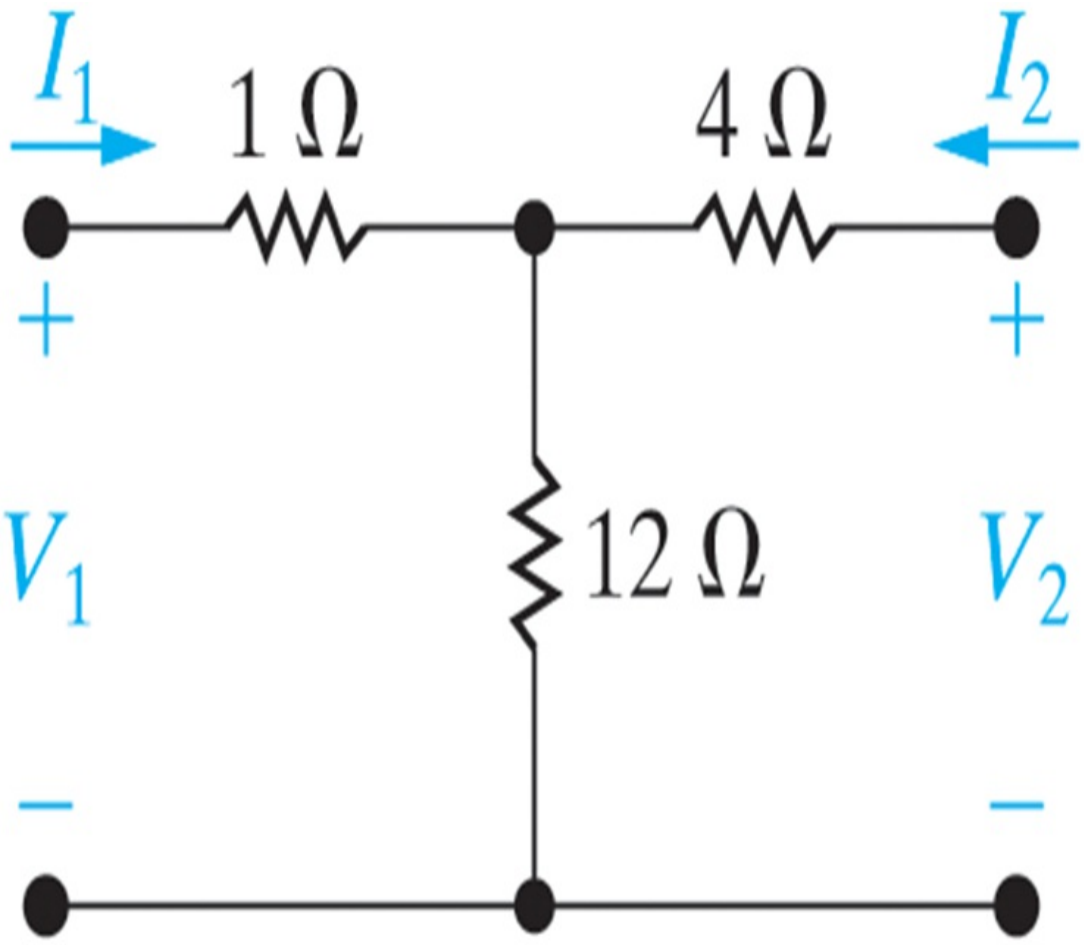


Figure P18.2 Full Alternative Text

3. 18.3 Use the results obtained in Problem 18.2 to calculate the y parameters for the circuit in Fig. P18.2.

4. 18.4 Find the b parameters for the circuit shown in Fig. P18.4.

Figure P18.4

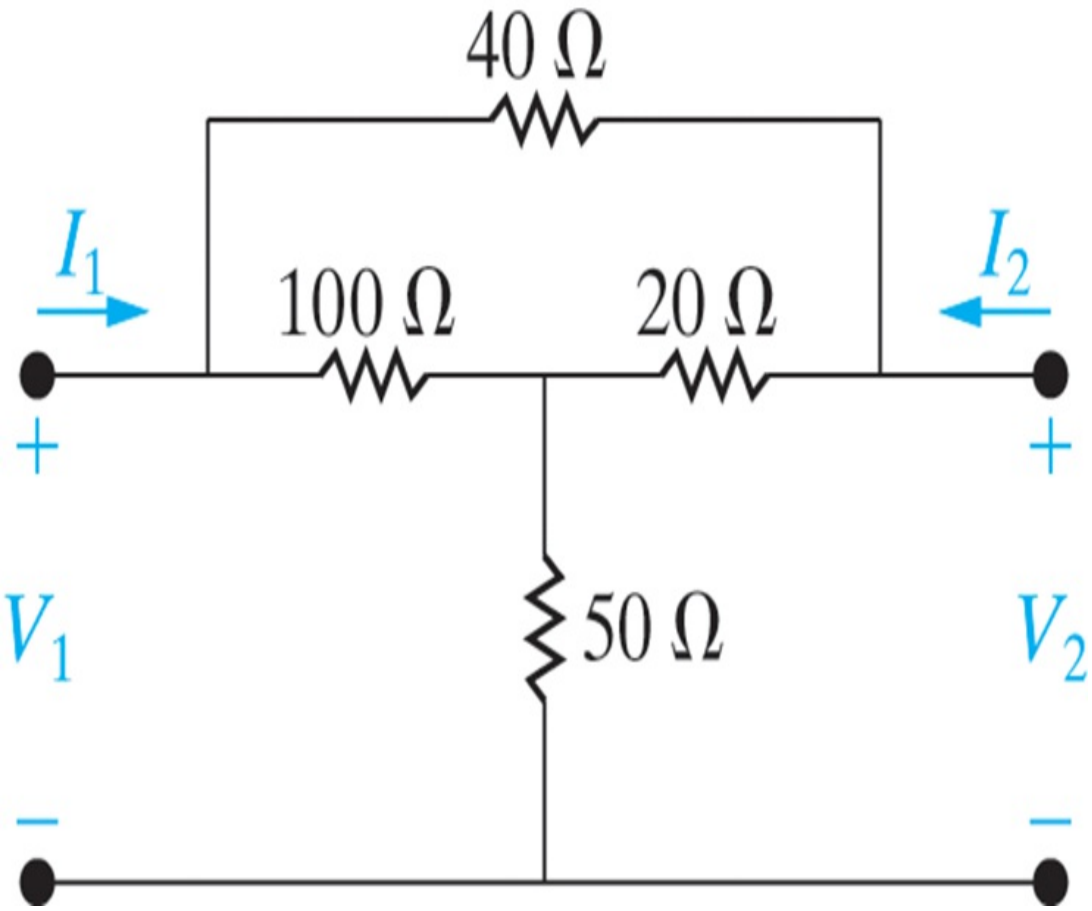


Figure P18.4 Full Alternative Text

5. 18.5 Find the y parameters for the circuit shown in Fig. P18.5.

Figure P18.5

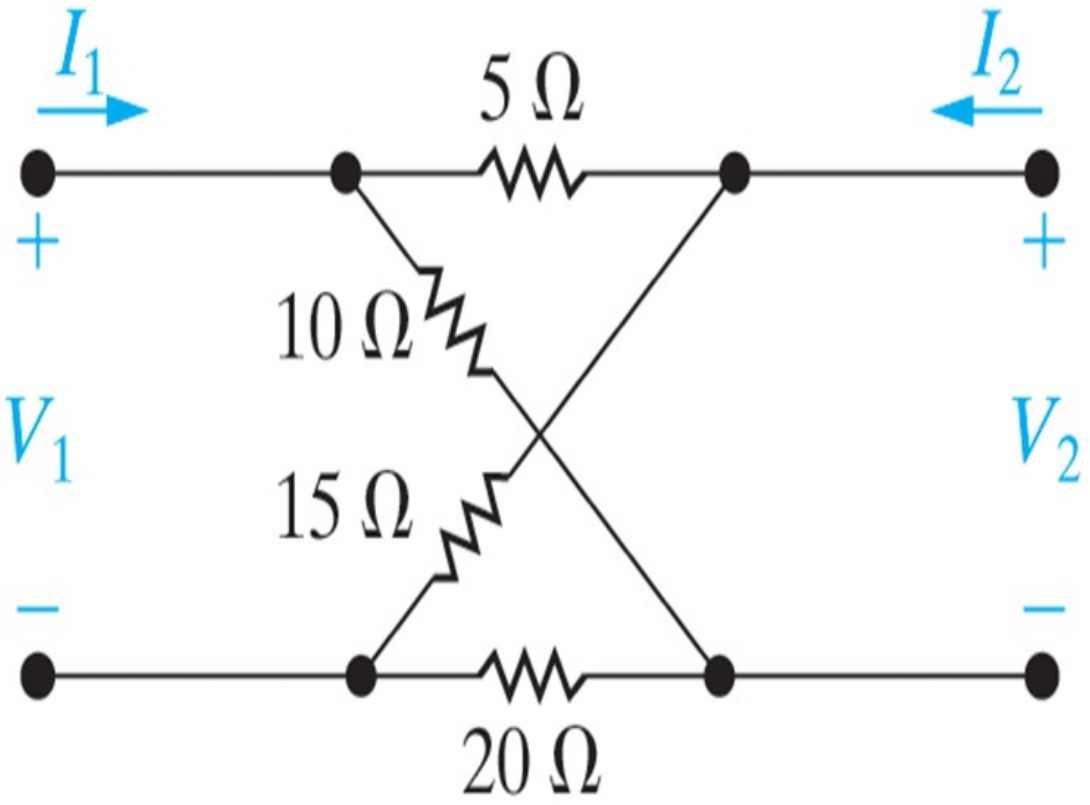


Figure P18.5 Full Alternative Text

6. 18.6 Find the g parameters for the circuit in Fig. P18.6.

Figure P18.6

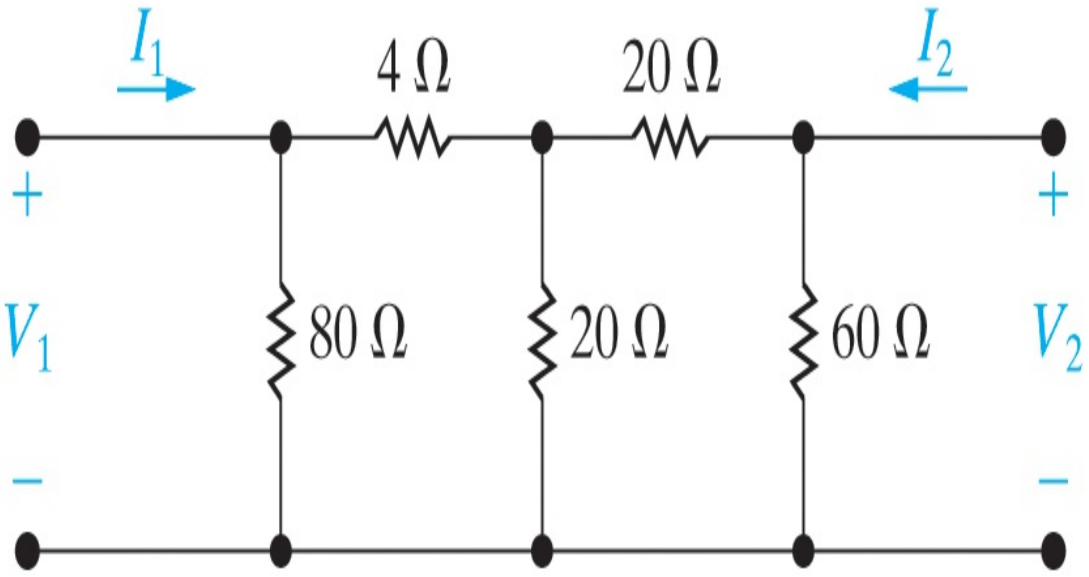


Figure P18.6 Full Alternative Text

7. 18.7 Select the values of R1, R2, and R3 in the circuit in Fig. P18.7 so that h11=4Ω, h12=0.8, h21=−0.8, and h22=0.14 S.

Figure P18.7

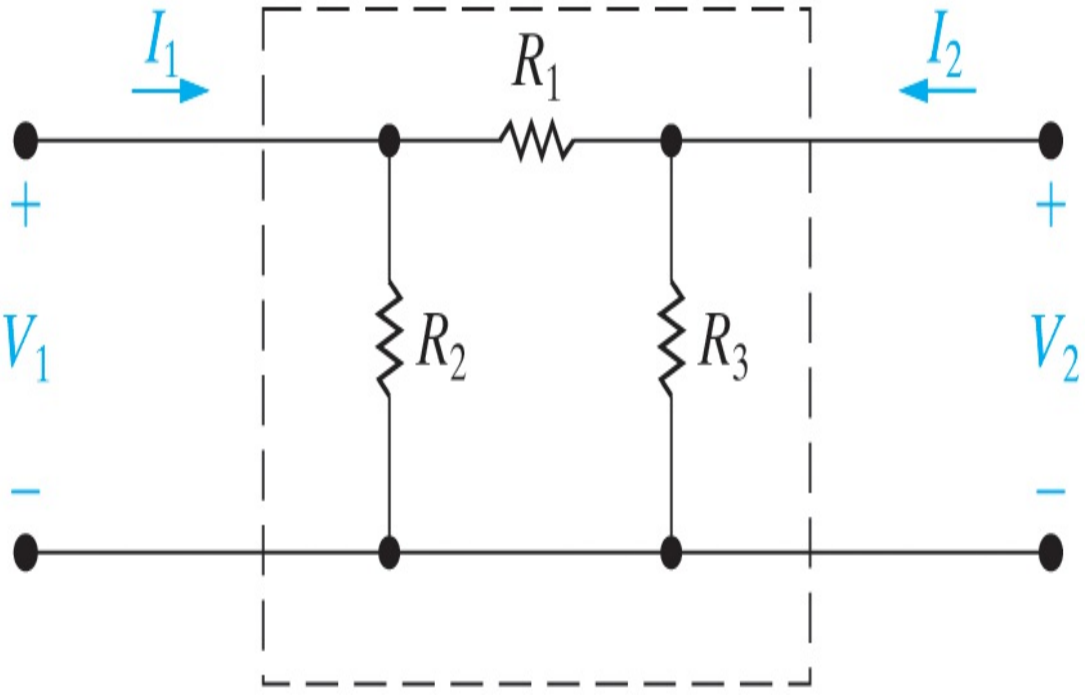


Figure P18.7 Full Alternative Text

8. 18.8 The operational amplifier in the circuit shown in Fig. P18.8 is ideal. Find the h parameters of the circuit.

Figure P18.8

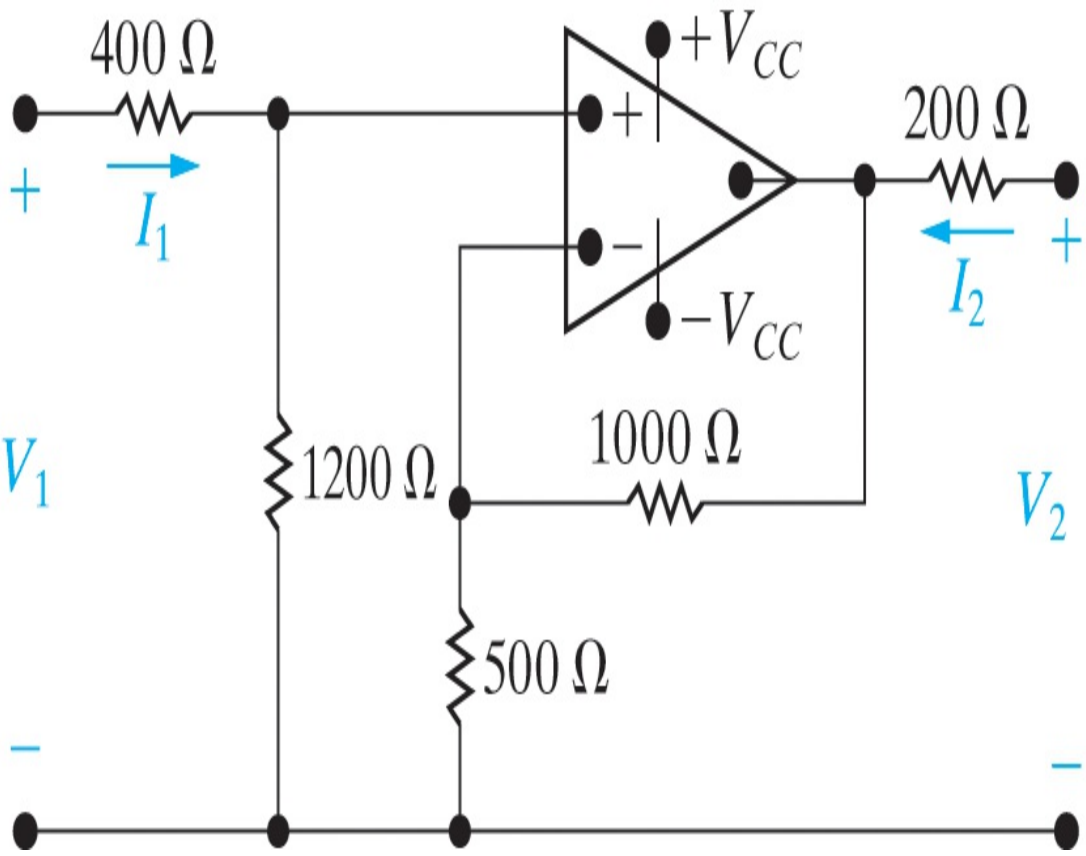


Figure P18.8 Full Alternative Text

9. 18.9 Find the g parameters for the operational amplifier circuit shown in Fig. P18.9.

Figure P18.9

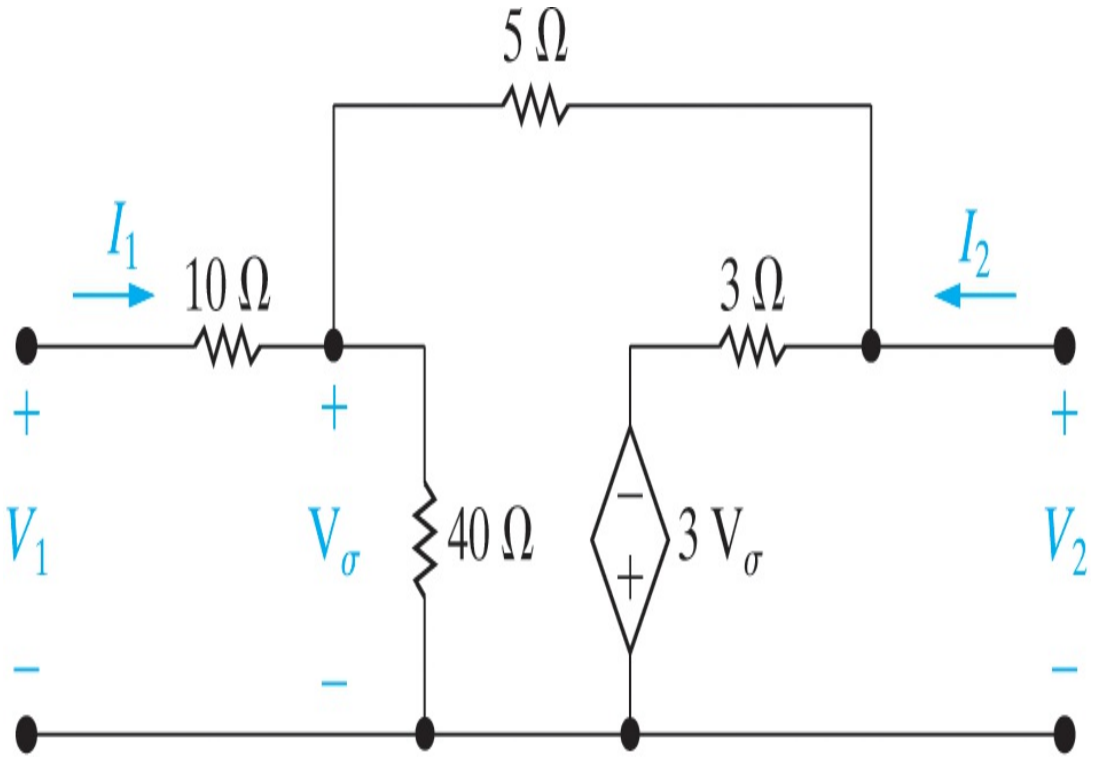


Figure P18.9 Full Alternative Text

10. 18.10 Find the a parameters for the circuit in Fig. P18.10.

Figure P18.10

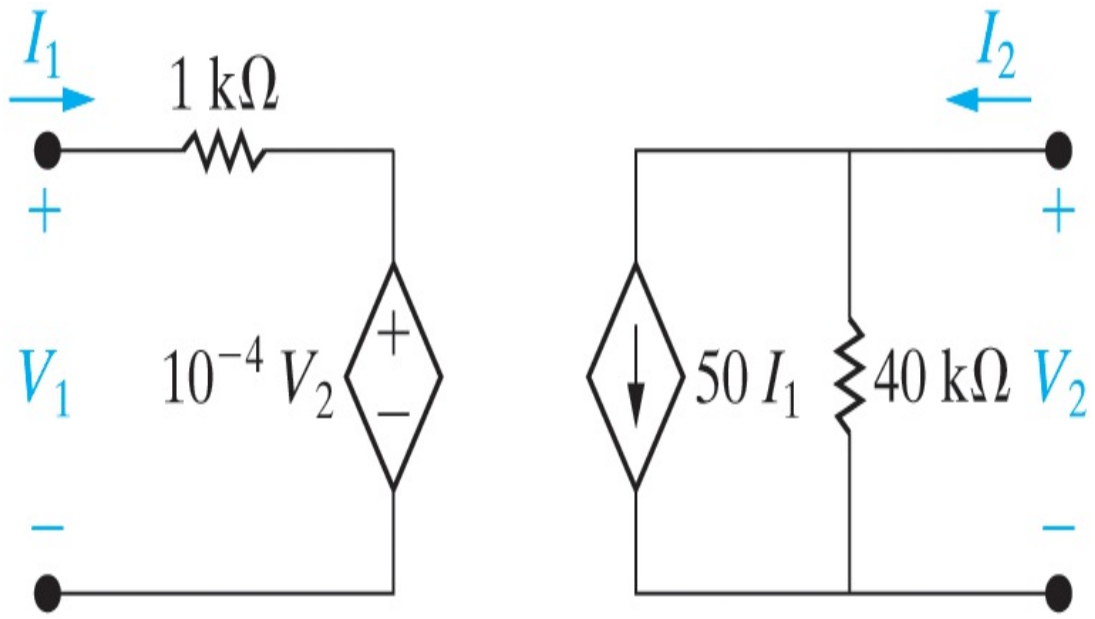


Figure P18.10 Full Alternative Text

11. 18.11 Use the results obtained in Problem 18.10 to calculate the g parameters of the circuit in Fig. P18.10.

12. 18.12 Find the h parameters of the two-port circuit shown in Fig. P18.12.

Figure P18.12

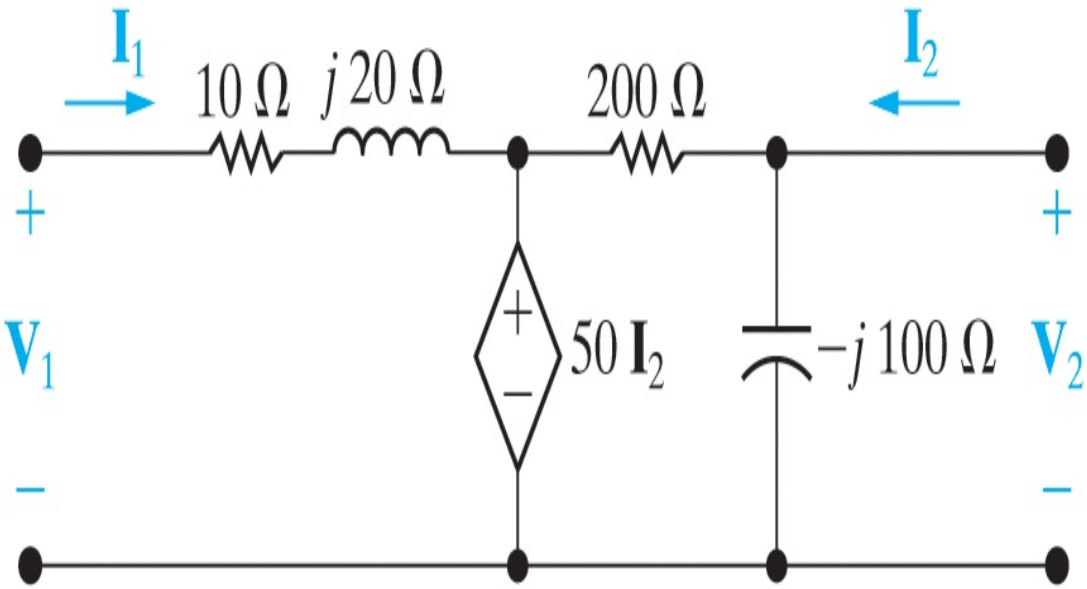


Figure P18.12 Full Alternative Text

13. 18.13 The following direct-current measurements were made on the two-port network shown in Fig. P18.13.

Figure P18.13

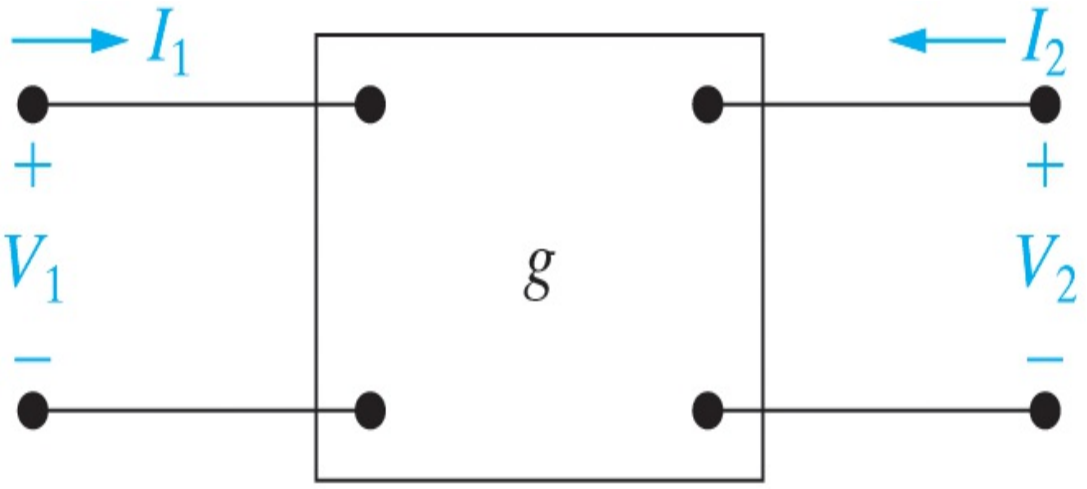


Figure P18.13 Full Alternative Text

Reciprocal Circuits

$$z_{12} = z_{21}$$

$$y_{12} = y_{21}$$

$$a_{11}a_{22} - a_{12}a_{21} = \Delta a = 1$$

$$b_{11}b_{22} - b_{12}b_{21} = \Delta b = 1$$

$$h_{12} = -h_{21}$$

$$g_{12} = -g_{21}$$

Reciprocal and Symmetric Circuits

$$z_{11} = z_{22}$$

$$y_{11} = y_{22}$$

$$a_{11} = a_{22}$$

$$b_{11} = b_{22}$$

$$h_{11}h_{22} - h_{12}h_{21} = \Delta h = 1$$

$$g_{11}\,g_{22} - g_{12}\,g_{21} = \Delta g = 1$$

Calculate the g parameters for the network.

14. 18.14

    1. Use the measurements given in Problem 18.13 to find the y parameters for the network.

    2. Check your calculations by finding the y parameters directly from the g parameters found in Problem 18.13.

15. 18.15 Derive the expressions for the h parameters as functions of the g parameters.

16. 18.16 Derive the expressions for the b parameters as functions of the h parameters.

17. 18.17 Derive the expressions for the g parameters as functions of the z parameters.

18. 18.18 Find the s-domain expressions for the a parameters of the two-port circuit shown in Fig. P18.18.

Figure P18.18

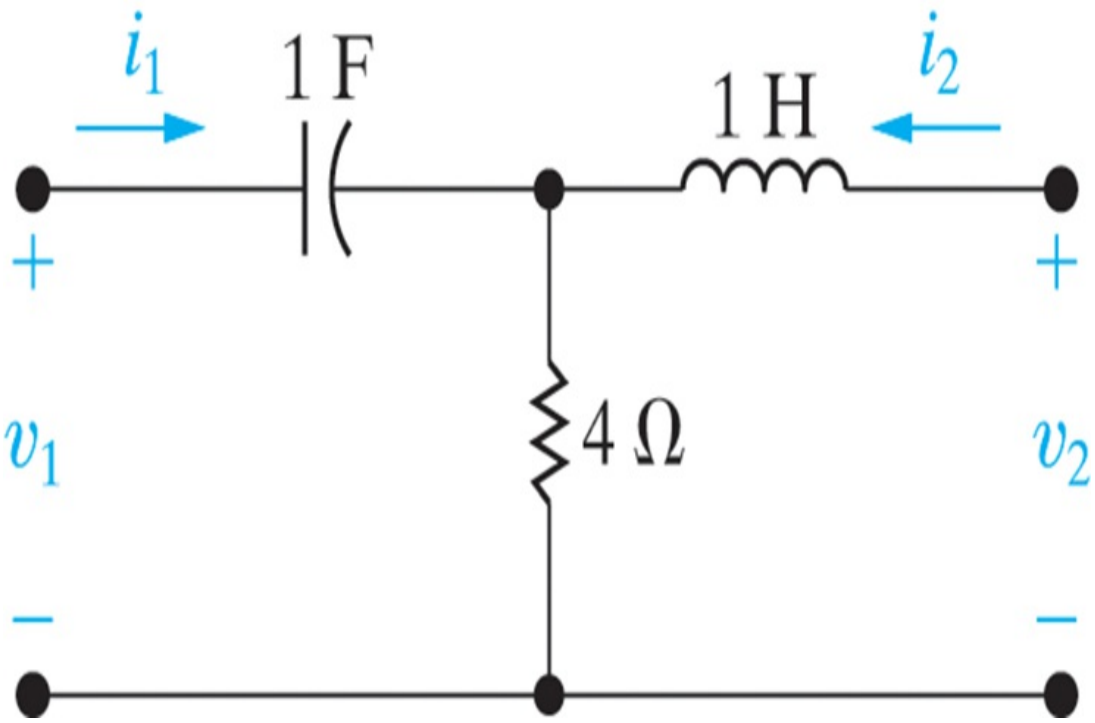


Figure P18.18 Full Alternative Text

19. 18.19 Find the s-domain expressions for the z parameters of the two-port circuit shown in Fig. P18.19.

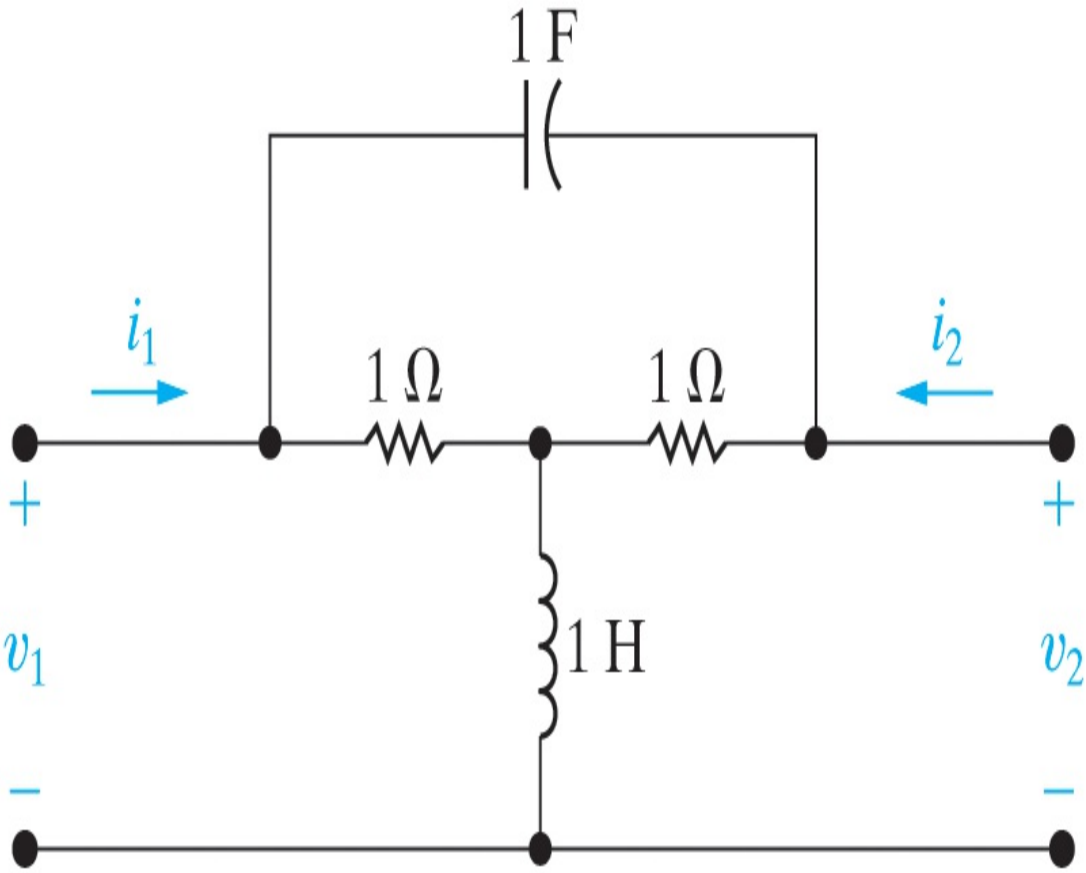


Figure P18.19

Figure P18.19 Full Alternative Text

20. 18.20

    1. Use the defining equations to find the s-domain expressions for the h parameters for the circuit in Fig. P18.20.

    2. Show that the results obtained in (a) agree with the h-parameter relationships for a reciprocal symmetric network.

Figure P18.20

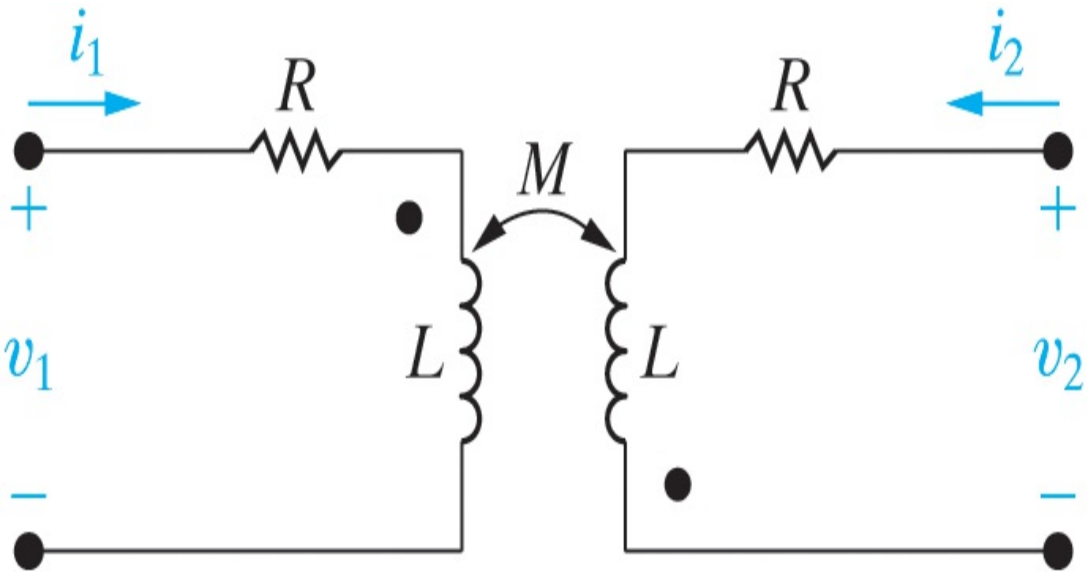


Figure P18.20 Full Alternative Text

21. 18.21 Find the frequency-domain values of the a parameters for the two-port circuit shown in Fig. P18.21.

Figure P18.21

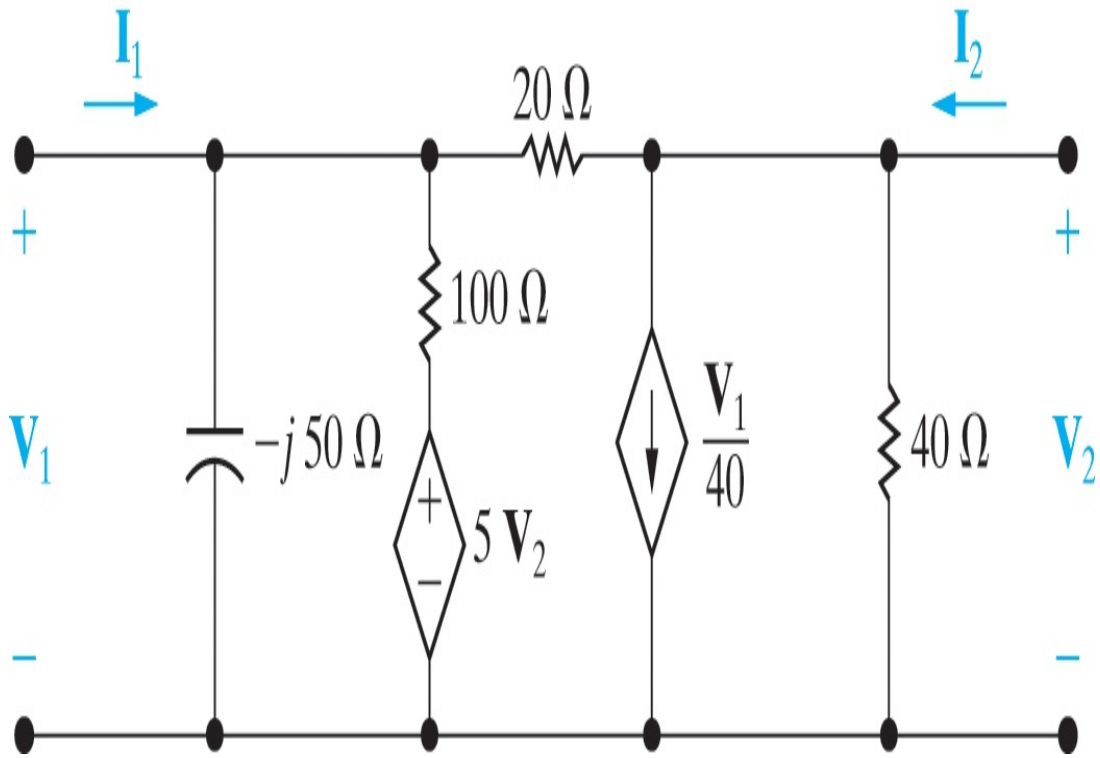


Figure P18.21 Full Alternative Text

22. 18.22 Find the h parameters for the two-port circuit shown in Fig. P18.21.

23. 18.23 Is the two-port circuit shown in Fig. P18.23 symmetric? Justify your answer.

Figure P18.23

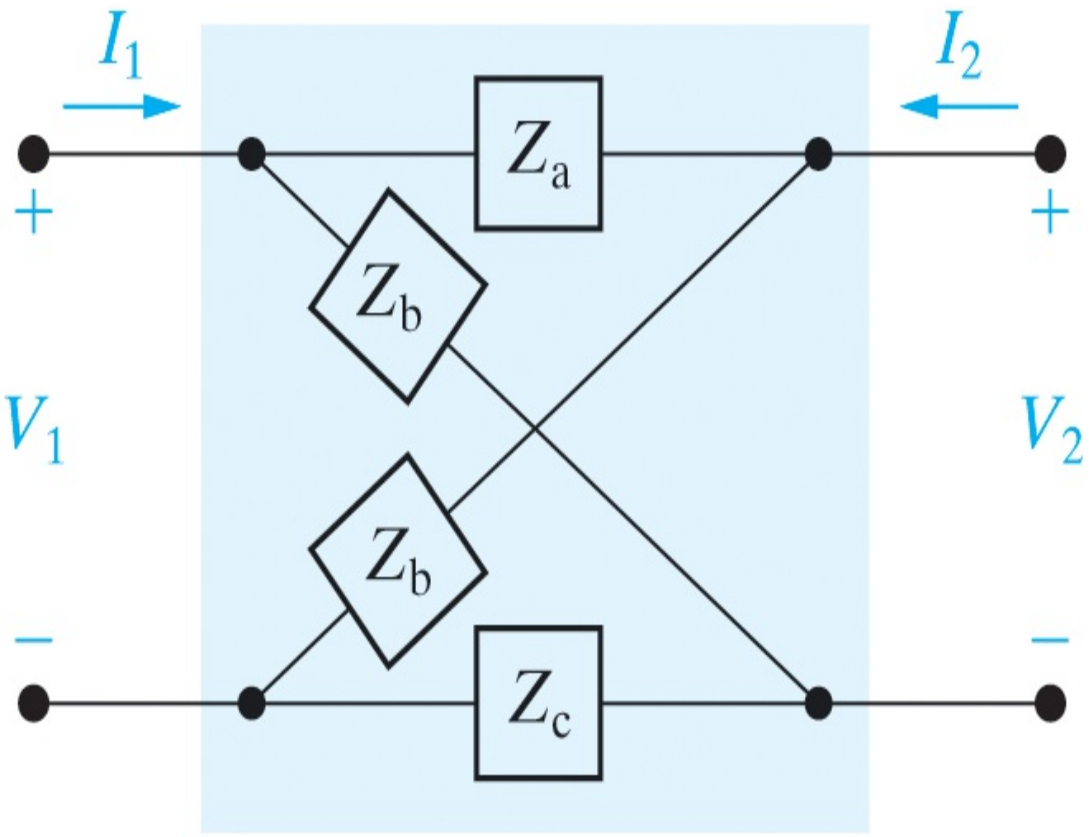


Figure P18.23 Full Alternative Text

Section 18.3

1. 18.24 Derive the expression for the input impedance, Zin=V1/I1 for the circuit in Fig. 18.10 in terms of the b parameters.

2. 18.25 Derive the expression for the current gain I2/I1 of the circuit in Fig. 18.10 in terms of the g parameters.

3. 18.26 Derive the expression for the voltage gain V2/V1 of the circuit in Fig. 18.10 in terms of the y parameters.

4. 18.27 Find the Thévenin equivalent circuit with respect to port 2 of the circuit in Fig. 18.10 in terms of the z parameters.

5. 18.28 Derive the expression for the voltage gain V2/Vg of the circuit in Fig. 18.10 in terms of the h parameters.

6. 18.29 The following dc measurements were made on the resistive network shown in Fig. P18.29.

Figure P18.29

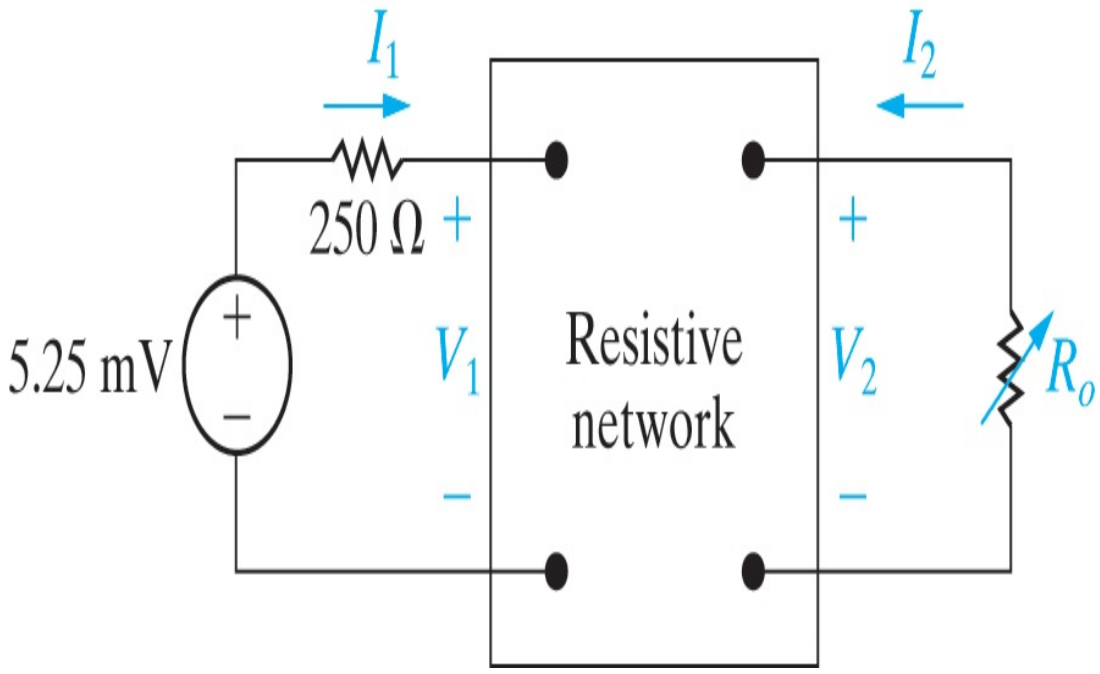


Figure P18.29 Full Alternative Text

z Parameters

$$Z_{\text{in}} = z_{11} - \frac{z_{12}z_{21}}{z_{22} + Z_L}$$

$$I_2 = \frac{-z_{21}V_g}{(z_{11} + Z_g)(z_{22} + Z_L) - z_{12}z_{21}}$$

$$V_{\text{Th}} = \frac{z_{21}}{z_{11} + Z_g} V_g$$

$$Z_{\text{Th}} = z_{22} - \frac{z_{12}z_{21}}{z_{11} + Z_g}$$

$$\frac{I_2}{I_1} = \frac{-z_{21}}{z_{22} + Z_L}$$

$$\frac{V_2}{V_1} = \frac{z_{21}Z_L}{z_{11}Z_L + \Delta z}$$

$$\frac{V_2}{V_g} = \frac{z_{21}Z_L}{(z_{11} + Z_g)(z_{22} + Z_L) - z_{12}z_{21}}$$

y Parameters

$$Y_{\text{in}} = y_{11} - \frac{y_{12}y_{21}Z_L}{1 + y_{22}Z_L}$$

$$I_2 = \frac{y_{21}V_g}{1 + y_{22}Z_L + y_{11}Z_g + \Delta y Z_g Z_L}$$

$$V_{\text{Th}} = \frac{-y_{21}V_g}{y_{22} + \Delta y Z_g}$$

$$Z_{\text{Th}} = \frac{1 + y_{11}Z_g}{y_{22} + \Delta y Z_g}$$

$$\frac{I_2}{I_1} = \frac{y_{21}}{y_{11} + \Delta y Z_L}$$

$$\frac{V_2}{V_1} = \frac{-y_{21}Z_L}{1 + y_{22}Z_L}$$

$$\frac{V_2}{V_g} = \frac{y_{21}Z_L}{y_{12}y_{21}Z_gZ_L - (1 + y_{11}Z_g)(1 + y_{22}Z_L)}$$

a Parameters

$$Z_{\text{in}} = \frac{a_{11}Z_L + a_{12}}{a_{21}Z_L + a_{22}}$$

$$I_2 = \frac{-V_g}{a_{11}Z_L + a_{12} + a_{21}Z_gZ_L + a_{22}Z_g}$$

$$V_{\text{Th}} = \frac{V_g}{a_{11} + a_{21}Z_g}$$

$$Z_{\text{Th}} = \frac{a_{12} + a_{22}Z_g}{a_{11} + a_{21}Z_g}$$

$$\frac{I_2}{I_1} = \frac{-1}{a_{21}Z_L + a_{22}}$$

$$\frac{V_2}{V_1} = \frac{Z_L}{a_{11}Z_L + a_{12}}$$

$$\frac{V_2}{V_g} = \frac{Z_L}{(a_{11} + a_{21}Z_g)Z_L + a_{12} + a_{22}Z_g}$$

b Parameters

$$Z_{\text{in}} = \frac{b_{22}Z_L + b_{12}}{b_{21}Z_L + b_{11}}$$

$$I_2 = \frac{-V_g\Delta b}{b_{11}Z_g + b_{21}Z_gZ_L + b_{22}Z_L + b_{12}}$$

$$V_{\text{Th}} = \frac{V_g\Delta b}{b_{22} + b_{21}Z_g}$$

$$Z_{\text{Th}} = \frac{b_{11}Z_g + b_{12}}{b_{21}Z_g + b_{22}}$$

$$\frac{I_2}{I_1} = \frac{-\Delta b}{b_{11} + b_{21}Z_L}$$

$$\frac{V_2}{V_1} = \frac{\Delta b Z_L}{b_{12} + b_{22}Z_L}$$

$$\frac{V_2}{V_g} = \frac{\Delta b Z_L}{b_{12} + b_{11}Z_g + b_{22}Z_L + b_{21}Z_gZ_L}$$

h Parameters

$$Z_{\text{in}} = h_{11} - \frac{h_{12}h_{21}Z_L}{1 + h_{22}Z_L}$$

$$I_2 = \frac{h_{21}V_g}{(1 + h_{22}Z_L)(h_{11} + Z_g) - h_{12}h_{21}Z_L}$$

$$V_{\text{Th}} = \frac{-h_{21}V_g}{h_{22}Z_g + \Delta h}$$

$$Z_{\text{Th}} = \frac{Z_g + h_{11}}{h_{22}Z_g + \Delta h}$$

$$\frac{I_2}{I_1} = \frac{h_{21}}{1 + h_{22}Z_L}$$

$$\frac{V_2}{V_1} = \frac{-h_{21}Z_L}{\Delta h Z_L + h_{11}}$$

$$\frac{V_2}{V_g} = \frac{-h_{21}Z_L}{(h_{11} + Z_g)(1 + h_{22}Z_L) - h_{12}h_{21}Z_L}$$

g Parameters

$$Y_{\text{in}} = g_{11} - \frac{g_{12}g_{21}}{g_{22} + Z_L}$$

$$I_2 = \frac{-g_{21}V_g}{(1 + g_{11}Z_g)(g_{22} + Z_L) - g_{12}g_{21}Z_g}$$

$$V_{\text{Th}} = \frac{g_{21}V_g}{1 + g_{11}Z_g}$$

$$Z_{\text{Th}} = g_{22} - \frac{g_{12}g_{21}Z_g}{1 + g_{11}Z_g}$$

$$\frac{I_2}{I_1} = \frac{-g_{21}}{g_{11}Z_L + \Delta g}$$

$$\frac{V_2}{V_1} = \frac{g_{21}Z_L}{g_{22} + Z_L}$$

$$\frac{V_2}{V_g} = \frac{g_{21}Z_L}{(1 + g_{11}Z_g)(g_{22} + Z_L) - g_{12}g_{21}Z_g}$$

A variable resistor Ro is connected across port 2 and adjusted for maximum power transfer to Ro. Find the maximum power.

7. 18.30 The b parameters for the two-port circuit in Fig. P18.30 are

   $b11=1+j13; b12=-1+j4\ \Omega; b21=13\ S; b22=1+j1.$

   The load impedance ZL is adjusted for maximum average power transfer to ZL. The ideal voltage source is generating a sinusoidal voltage of

   $vg=90\cos 8000t\ V.$

   1. Find the rms value of V2.
   2. Find the average power delivered to ZL.
   3. What percentage of the average power developed by the ideal voltage source is delivered by ZL?

Figure P18.30

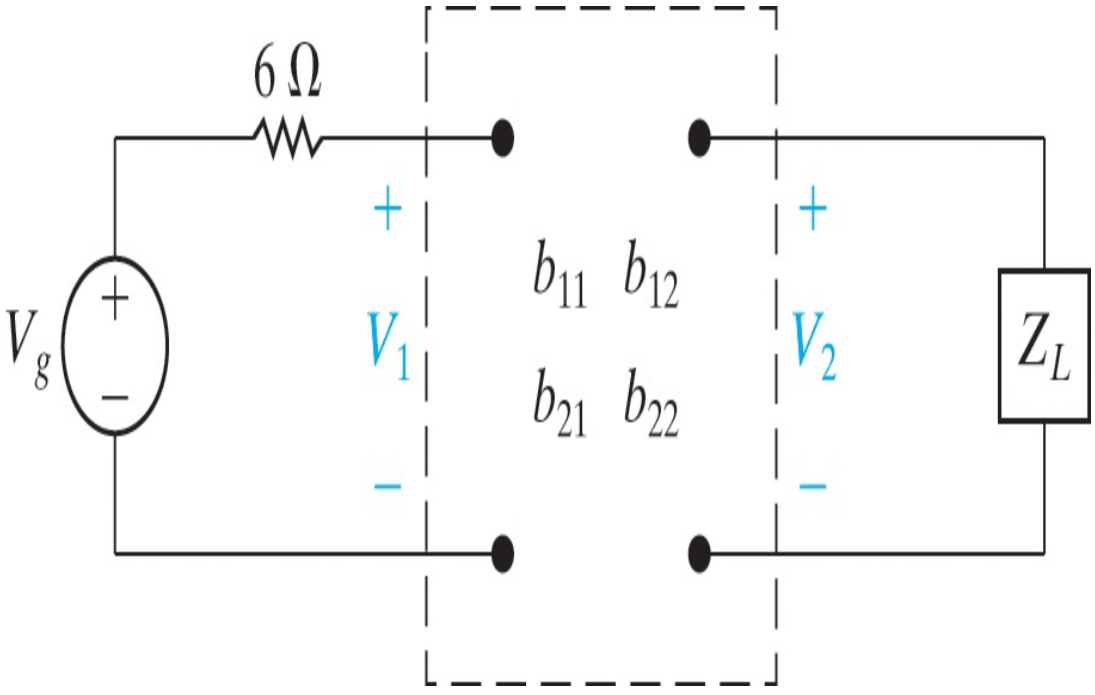


Figure P18.30 Full Alternative Text

8. 18.31 The h parameters for the two-port amplifier circuit in Fig. P18.31 are

   h11=500 Ω;h12=10−3;h21=50;h22=50 μS.

   The internal impedance of the source is 1500+j0 Ω, and the load impedance is 10,000+j0 Ω. The ideal voltage source is generating a voltage

   vg=250 cos 40,000t mV.

   1. Find the rms value of V2.
   2. Find the average power delivered to ZL.
   3. Find the average power developed by the ideal voltage source.

Figure P18.31

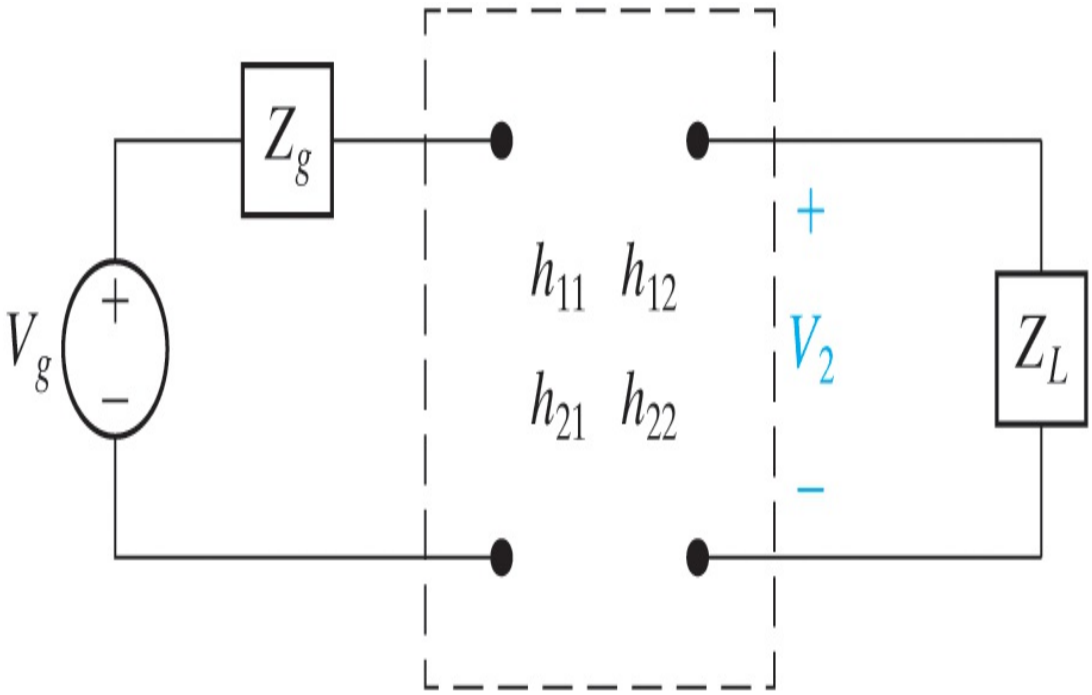


Figure P18.31 Full Alternative Text

9. 18.32 For the terminated two-port amplifier circuit in Fig. P18.31, find

    1. the value of ZL for maximum average power transfer to ZL.
    2. the maximum average power delivered to ZL.
    3. the average power developed by the ideal voltage source when maximum power is delivered to ZL.

10. 18.33 The b parameters of the amplifier in the circuit shown in Fig. P18.33 are

    b11=25;b12=1 kΩ;b21=−1.25 S;b22=−40.

    Find the ratio of the output power to that supplied by the ideal voltage source.

Figure P18.33

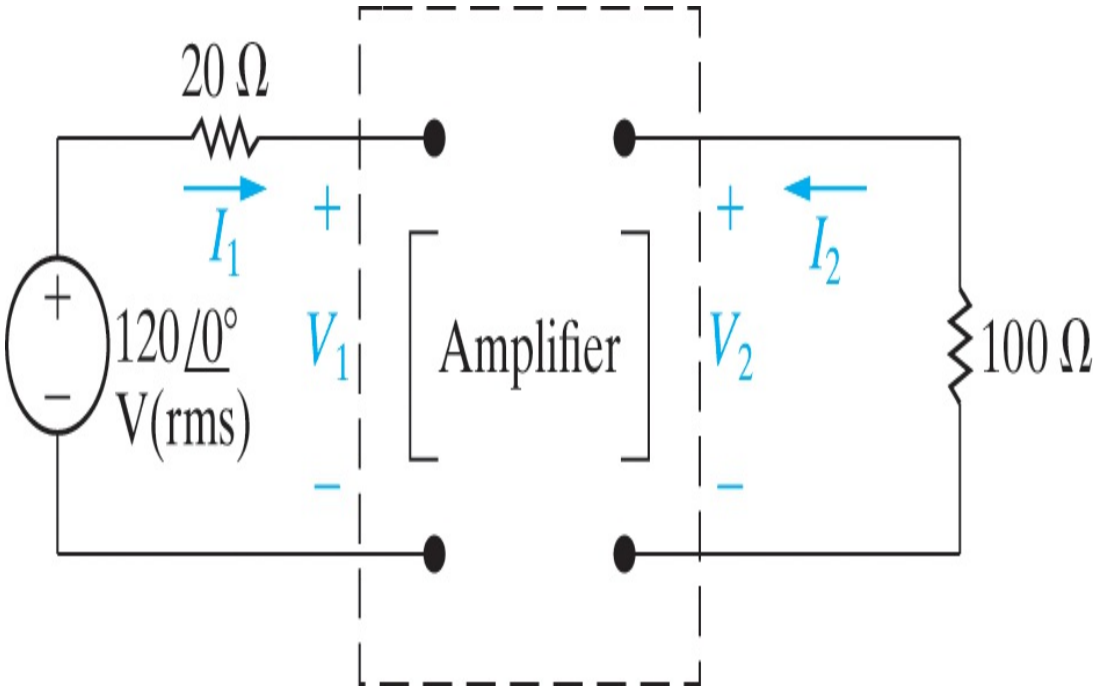


Figure P18.33 Full Alternative Text

11. 18.34 The linear transformer in the circuit shown in Fig. P18.34 has a coefficient of coupling of 0.75. The transformer is driven by a sinusoidal voltage source whose internal voltage is vg=260 cos 4000t V. The internal impedance of the source is 25+j0 Ω.

    1. Find the frequency-domain a parameters of the linear transformer.

    2. Use the a parameters to derive the Thévenin equivalent circuit with respect to the terminals of the load.

    3. Derive the steady-state time-domain expression for v2.

Figure P18.34

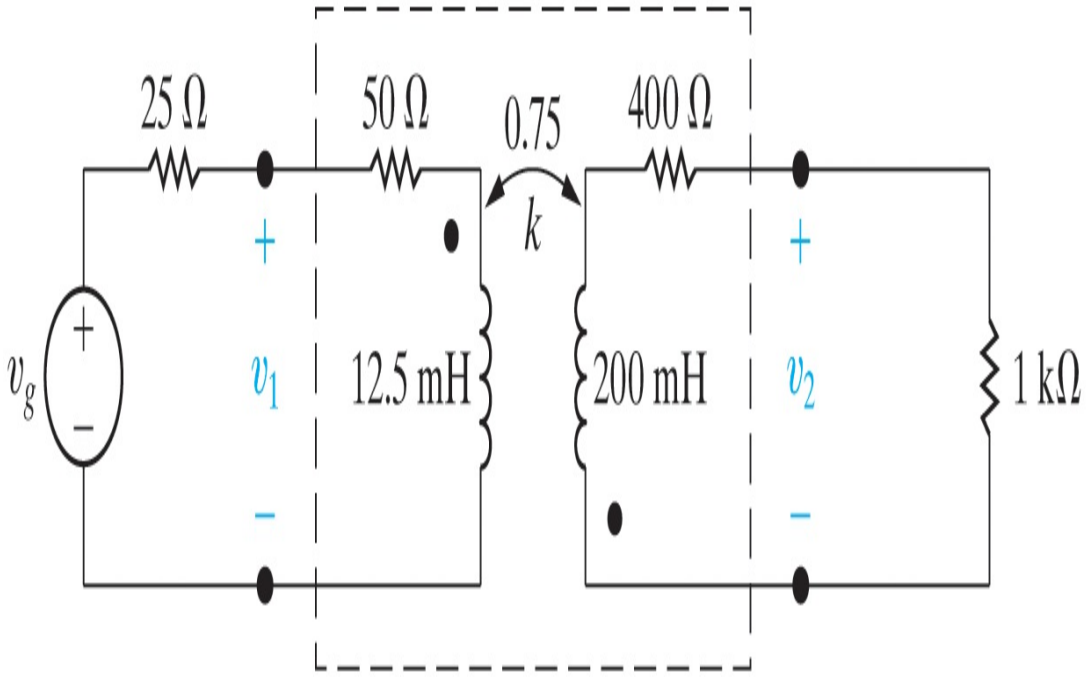


Figure P18.34 Full Alternative Text

12. 18.35 The following measurements were made on a resistive two-port network:

Condition 1 – create a short circuit at port 2 and apply 20 V to port 1:

Measurements: I1=1 A; I2=–1 A.

Condition 2 – create an open circuit at port 1 and apply 80 V to port 2:

Measurements: V1=400 V; I2=3 A.

Find the maximum power that this two-port circuit can deliver to a resistive load at port 2 when port 1 is driven by a 4 A dc current source with an internal resistance of 60 Ω.

13. 18.36

    1. Find the z parameters for the two-port network in Fig. P18.36.
    2. Find v2 for t>0 when vg=50u(t) V.

Figure P18.36

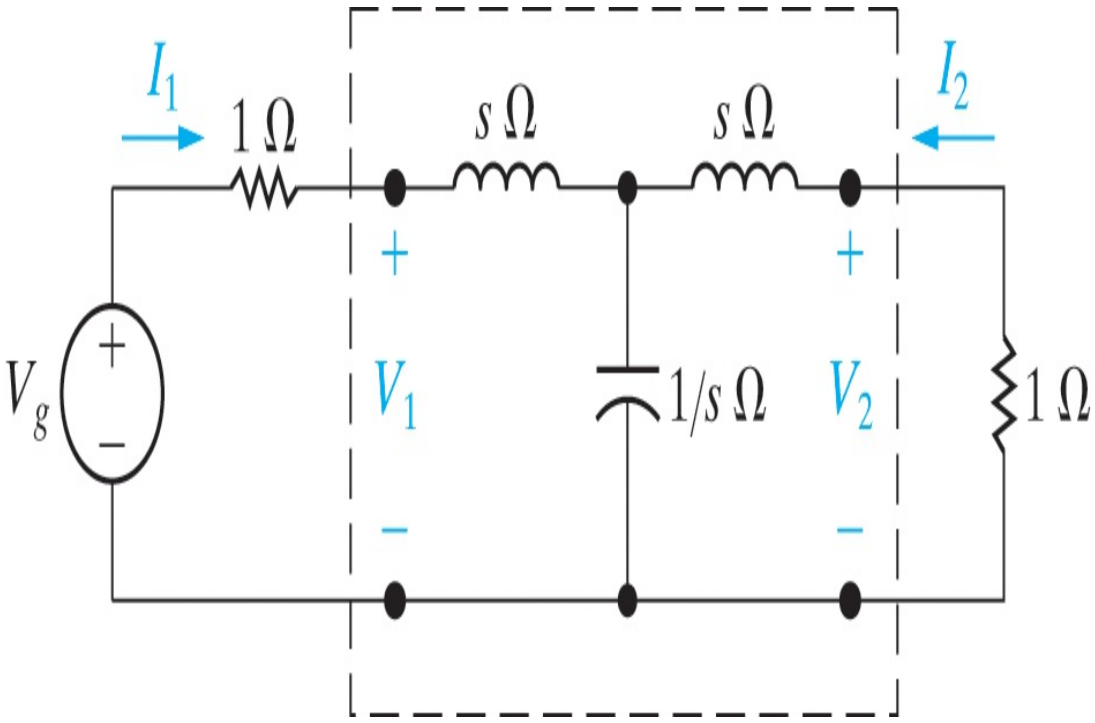


Figure P18.36 Full Alternative Text

14. 18.37

    1. Find the s-domain expressions for the h parameters of the circuit in Fig. P18.37.

    2. Port 2 in Fig. P18.37 is terminated in a resistance of 400 Ω, and port 1 is driven by a step voltage source v1(t)=30u(t) V. Find v2(t) for t>0 if C=200 nF and L=200 mH.

Figure P18.37

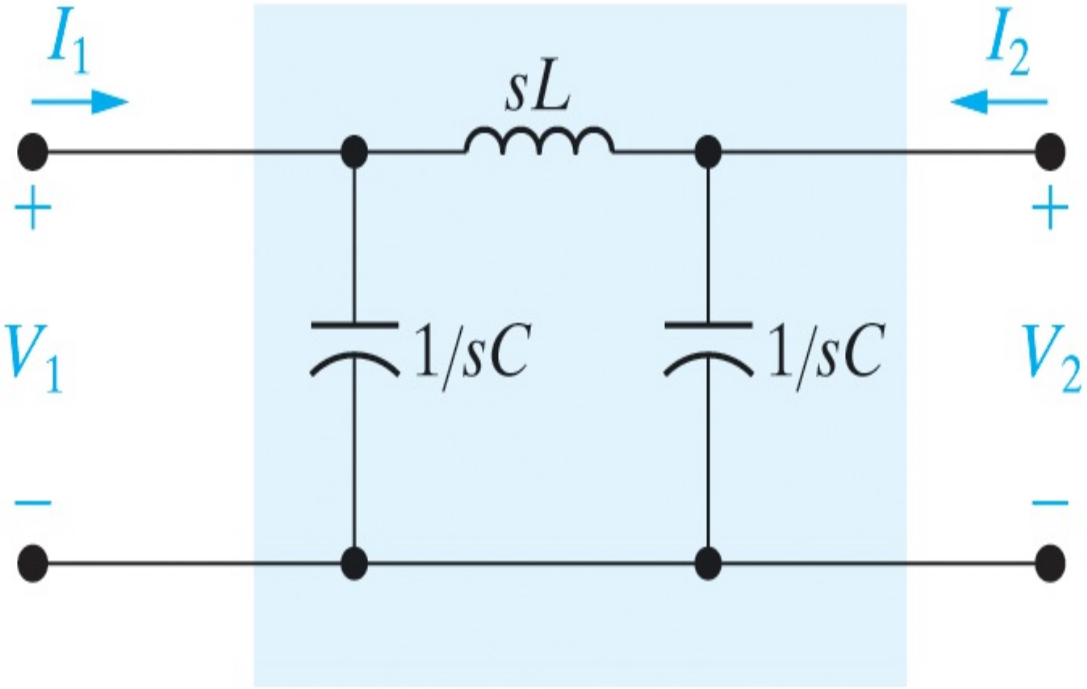


Figure P18.37 Full Alternative Text

Section 18.4

1. 18.38 The g and h parameters for the resistive two-ports in Fig. P18.38 are given by

   g11=335S;h11=5 kΩ;g12=207;h12=−0.2;g21=8007;h21=
   −4g22=507 kΩ;h22=200 μS;

   Calculate vo if vg=30 mV dc.

Figure P18.38

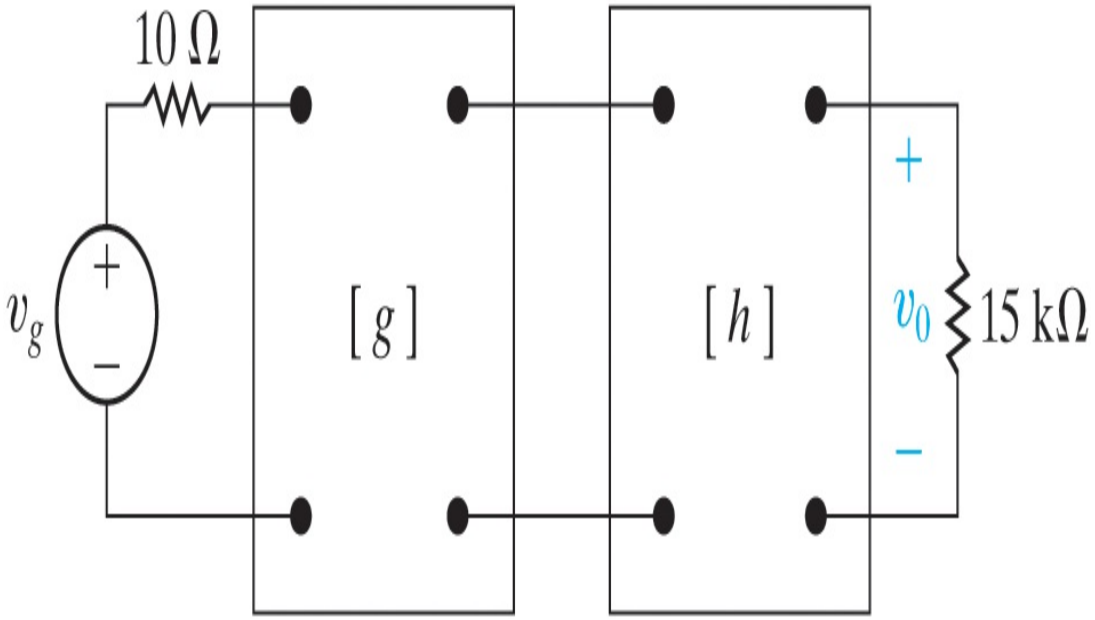


Figure P18.38 Full Alternative Text

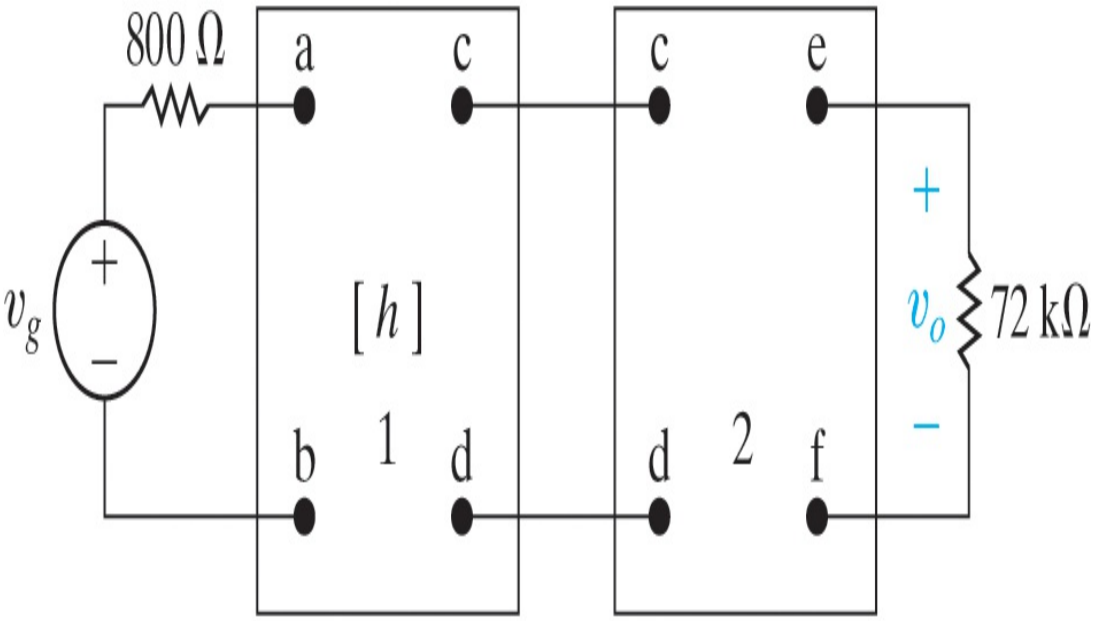
2. 18.39 The h parameters of the first two-port circuit in Fig. P18.39(a) are

   h11=1000 Ω;h12=5×10−4;h21=40;h22=25 μS.

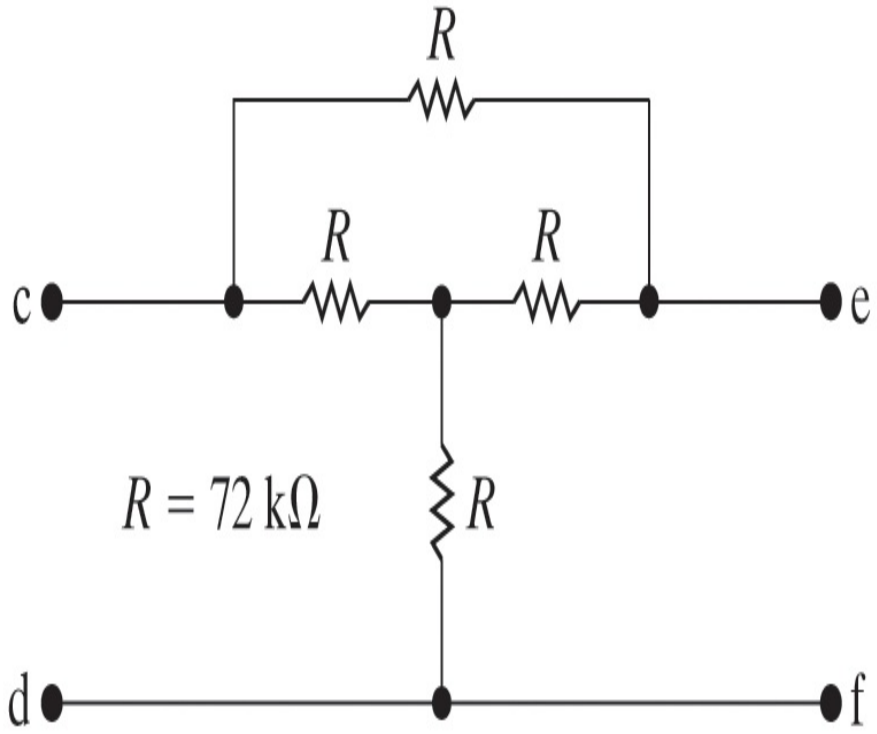
   The circuit in the second two-port circuit is shown in Fig. P18.39(b),

where R=72 kΩ. Find vo if vg=9 mV dc.

Figure P18.39



(a)



(b)

Figure P18.39 Full Alternative Text

3. 18.40 The networks A and B in the circuit in Fig. P18.40 are reciprocal and symmetric. For network A, it is known that a′11=2 and a′12=1 Ω.

    1. Find the a parameters of network B.

    2. Find V2 when Vg=750° V,

    Zg=10° Ω, and ZL=140° Ω.

Figure P18.40

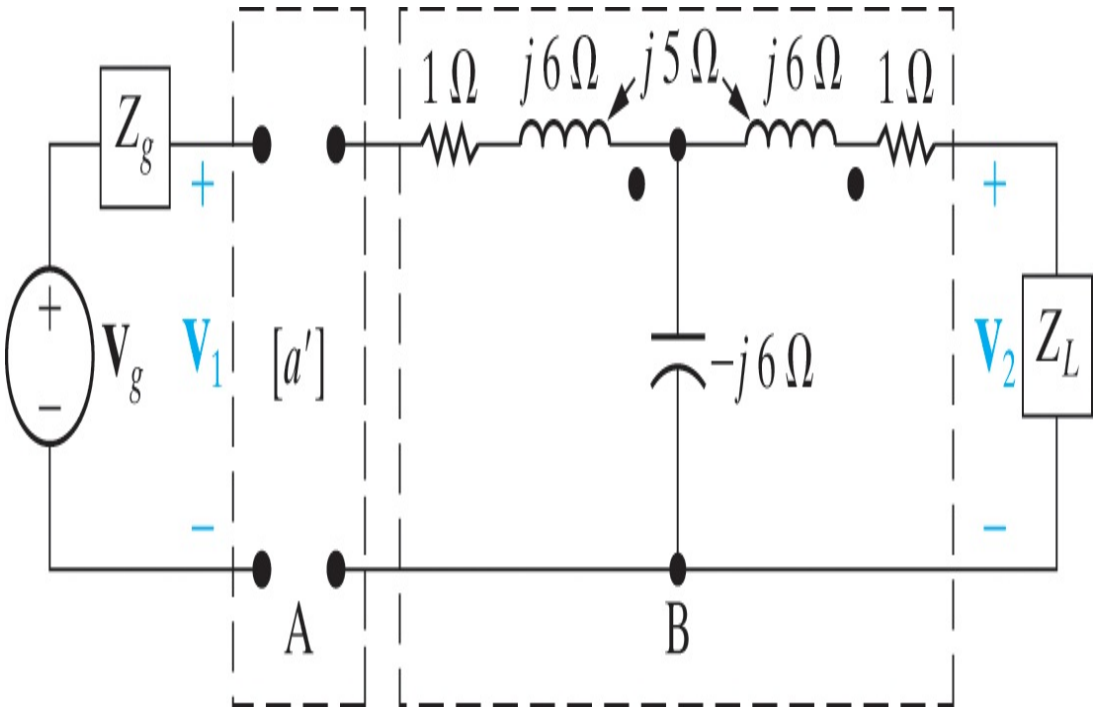


Figure P18.40 Full Alternative Text

Sections 18.1–18.4

1. 18.41

1. Show that the circuit in Fig. P18.41 is an equivalent circuit satisfied by the h-parameter equations.

2. Use the h-parameter equivalent circuit of (a) to find the voltage gain V2/Vg in the circuit in Fig. 18.14, using the h-parameter values given in Example 18.6.

Figure P18.41

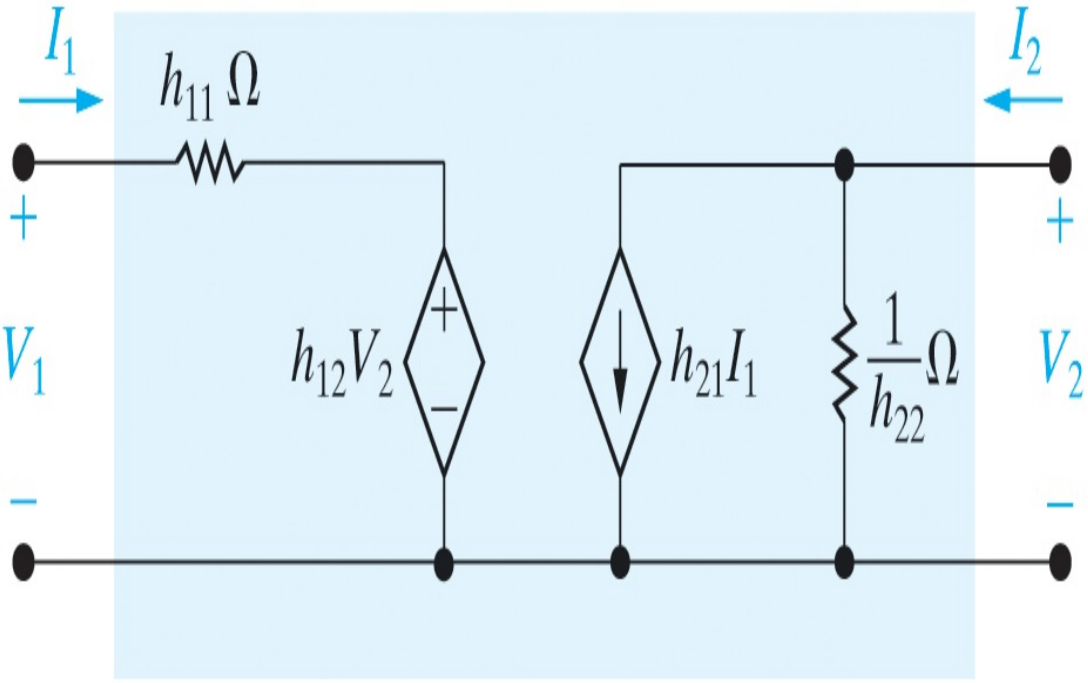


Figure P18.41 Full Alternative Text

2. 18.42

1. Show that the circuit in Fig. P18.42 is an equivalent circuit satisfied by the z-parameter equations.

2. Assume that the equivalent circuit in Fig. P18.42 is driven by a voltage source having an internal impedance of Zg ohms. Calculate the Thévenin equivalent circuit with respect to port 2. Check your results against the appropriate entries in Table 18.3.

Figure P18.42

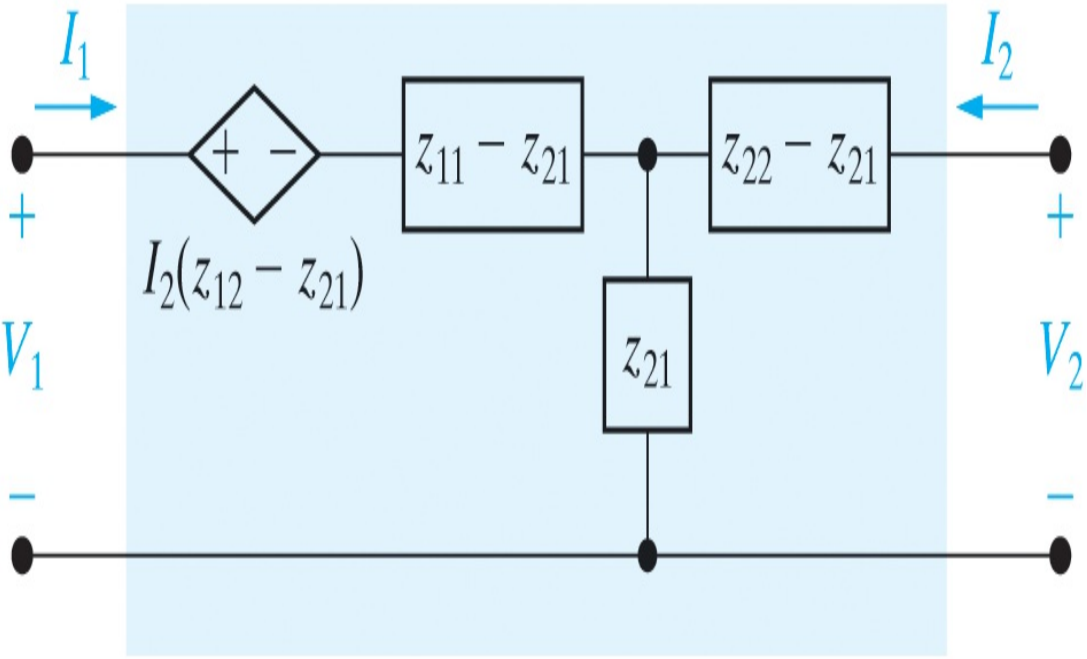


Figure P18.42 Full Alternative Text

3. 18.43

    1. Show that the circuit in Fig. P18.43 is also an equivalent circuit satisfied by the z-parameter equations.

    2. Assume that the equivalent circuit in Fig. P18.43 is terminated in an impedance of ZL ohms at port 2. Find the input impedance V1/I1. Check your results against the appropriate entry in Table 18.3.

Figure P18.43

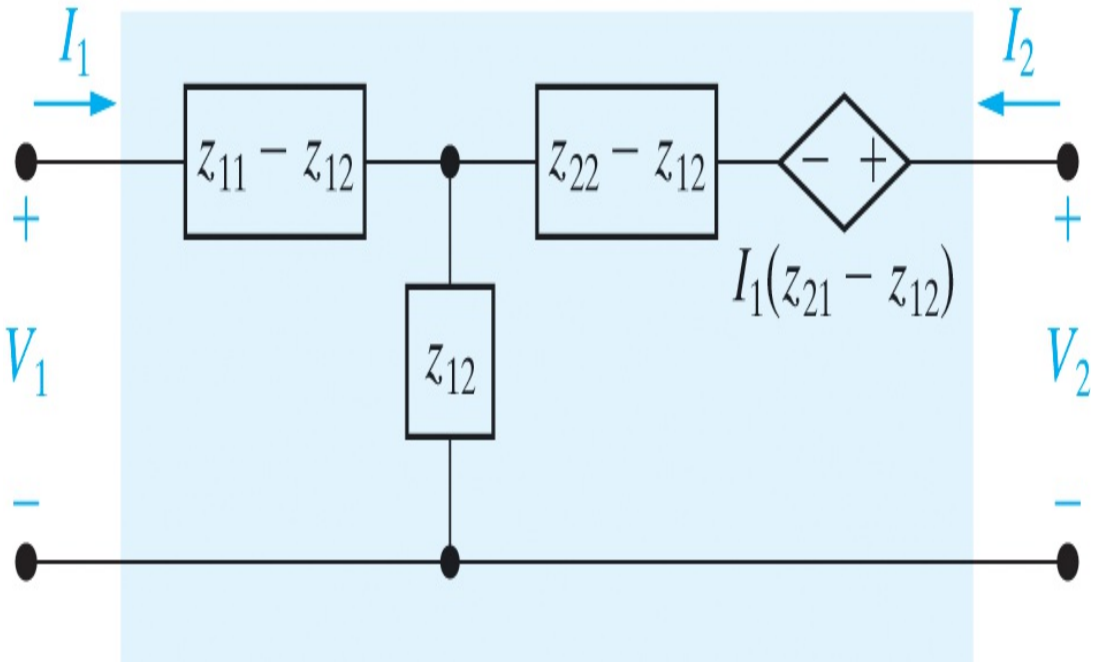


Figure P18.43 Full Alternative Text

4. 18.44

    1. Derive two equivalent circuits that are satisfied by the y-parameter equations. Hint: Start with Eqs. 18.2. Add and subtract y21V2 to the first equation of the set. Construct a circuit that satisfies the resulting set of equations, by thinking in terms of node voltages. Derive an alternative equivalent circuit by first altering the second equation in Eq. 18.2.

    2. b) Assume that port 1 is driven by a voltage source having an internal impedance Zg, and port 2 is loaded with an impedance ZL. Find the current gain I2/I1. Check your results against the appropriate entry in Table 18.3.

5. 18.45

    1. Derive the equivalent circuit satisfied by the g-parameter equations.

    2. Use the g-parameter equivalent circuit derived in part (a) and the h-parameter equivalent circuit derived in Problem 18.41 to solve for the output voltage vo in Problem 18.38.

6. 18.46

   PRACTICALPERSPECTIVE_

   1. What conditions and measurements will allow you to calculate the b parameters for the “black box” amplifier described in the Practical Perspective?

   2. What measurements will be made if the resulting b parameters are equivalent to the h parameters calculated in the Practical Perspective?

7. 18.47 PRACTICAL PERSPECTIVE_ Repeat Problem 18.46 for the y parameters.

Appendix A The Solution of Linear Simultaneous Equations

Circuit analysis frequently requires us to solve a set of linear simultaneous equations. We present several different solution methods here—some that employ engineering calculators or personal computers, and others that require just a pencil and paper. Most of the methods begin by placing the system of equations in matrix form. You should review matrices and matrix arithmetic, topics found in most intermediate-level algebra texts.

A.1 Preliminary Steps

To solve a set of simultaneous equations, we begin by organizing the equations into a standard form. To do this, collect all terms containing an unknown variable on the left-hand side of each equation and place all constants on the right-hand side. Then, arrange the equations in a vertical stack such that each variable occupies the same horizontal position in every equation. For example, in Eqs. A.1, the variables i1, i2, and i3 occupy the first, second, and third position, respectively, on the left-hand side of each equation:

$$21i1-\ 9i2-12i3=-33,-3i1+6i2-\ 2i3=3,-8i1-4i2+22i3=50. \quad (A.1)$$

Once the equations are in this standard form, you can write the equations using matrix notation as

$$[21-9-12\ -36-2\ -8-422][i1i2i3]=[\ -33350].$$

We can abbreviate the set of equations in matrix form as AX=B, where A is the matrix of coefficients that multiply the variables, X is the vector of the variables, and B is the vector of constants from the right-hand side of the equations. When written in this abbreviated form, we can solve for the vector of unknowns by finding the inverse of the A matrix and multiplying it by the B vector:

$$X=A^{-1}B.$$

If any equation is missing one or more variables, the missing variables can be inserted by making their coefficients zero. Thus, Eqs. A.2 are written in standard form as shown by Eqs. A.3:

$$2\upsilon1-\upsilon2=4,4\upsilon2+3\upsilon3=16,7\upsilon1+2\upsilon3=5; \quad (A.2)$$
$$2\upsilon1-\ \upsilon2+0\upsilon3=4,0\upsilon1+4\upsilon2+3\upsilon3=16,7\upsilon1+0\upsilon2+2\upsilon3=5.$$

Equation A.3 is written using matrix form as

[3−10043702][υ1υ2υ3]=[4165].

A.2 Calculator and Computer Methods

Most calculators recommended for engineering students can solve a set of simultaneous algebraic equations. Because there are many different calculators, it is impractical to provide directions or an example here. Instead, we provide the general steps for using your calculator to solve equations; you should refer to the manual for your specific calculator or search for instructions on the web.

1. Input the problem dimension by specifying the number of unknowns.

2. Create the A matrix, which is the matrix of coefficients that multiply the unknowns on the left-hand sides of the equations.

3. Create the B array, which is the array of constants on the right-hand sides of the equations.

4. Use the “solve” function (which has different names for different calculators) for the matrix A and the array B, to calculate the array X that contains the values of the unknowns.

Most calculators can solve simultaneous equations that have real number coefficients and complex number coefficients. Some can even solve simultaneous equations whose coefficients include symbols.

There are many different computer programs that can solve a set of algebraic equations. We present two examples: Excel, the spreadsheet application, and MATLAB, the matrix-based programming language. You should explore the options available and pick the software that works best for you.

Using Excel

Figure A.1 uses Excel to solve the simultaneous equations given in Eqs. A.1. Note that Excel can only solve simultaneous equations whose coefficients are real numbers. To begin, enter the A matrix in a square collection of cells and enter the B vector in a column of cells. You can label the matrix and vector, as shown in the figure, but this is not required. Then highlight a column of cells for the vector X, which will contain the values of the unknowns. The number of cells in this column must equal the number of unknowns. Type the following function in the function box:

= MMULT(MINVERSE(start_cell, end_cell),b1:bn)

Figure A.1 Using Excel to solve the simultaneous equations in Eqs. A.1.

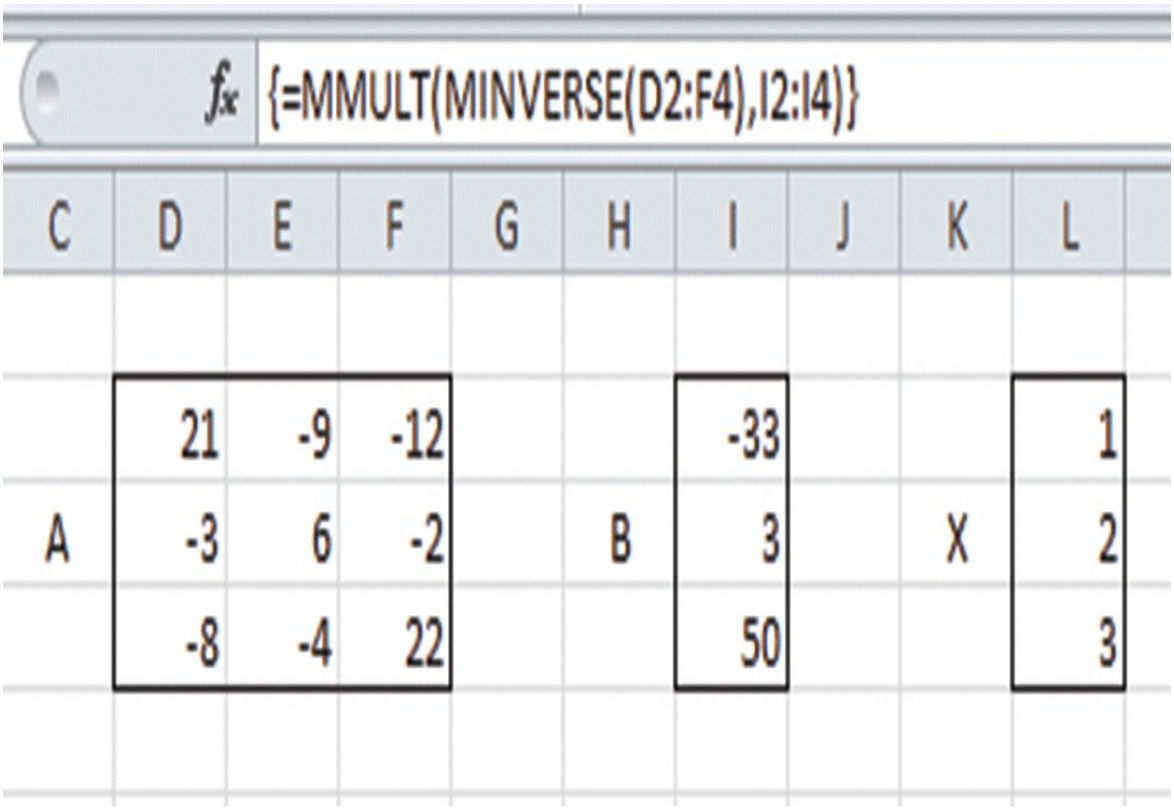


Figure A.1 Full Alternative Text

and simultaneously press the Ctrl-Shift-Enter keys. This will enter the values for the X vector into the highlighted cells and also will surround the function with braces, as shown in Fig. A.1. The function MINVERSE(start_cell, end_cell) calculates the inverse of a matrix. The matrix values occupy square collection of cells whose upper left cell is start_cell and whose lower right cell is end_cell. The MMULT(R, S) multiplies two matrices, R and S, supplied as arguments.

Using MATLAB

There are several ways to solve a system of simultaneous equations using MATLAB. Figure A.2 illustrates one method to solve Eqs. A.1. It begins by defining the three unknown variables and the three simultaneous equations. Then, the function equationsToMatrix(eq, var) constructs the A matrix and the B vector from the equations supplied in the function's first argument, using the variables specified in the function's second argument. Finally, the function linsolve(A, B) calculates the values of the unknowns by inverting the matrix supplied in the function's first argument and then multiplying by the vector supplied in the function's second argument. Note that, unlike Excel, the simultaneous equations specified in MatLab can have complex numbers as coefficients on the left-hand side and constants on the right-hand side.

Figure A.2 Using MatLab to solve the simultaneous equations in Eqs. Eqs. A.1.

```
>> syms  i1 i2 i3
>> eq1=21*i1 - 9*i2 - 12*i3 == -33;
>> eq2=-3*i1 + 6*i2 - 2*i3 ==3;
>> eq3=-8*i1 - 4*i2 + 22*i3 == 50;
>> [A,B] = equationsToMatrix([eq1, eq2, eq3], [i1, i2, i3])

A =

[ 21, -9, -12]
[ -3,  6,  -2]
[ -8, -4, -22]


B =

 -33
   3
  50

>> X = linsolve(A,B)

X =

 1
 2
 3
```

Figure A.2 Full Alternative Text

A.3 Paper-and-Pencil Methods

We present two methods that do not require an engineering calculator or computer software: back-substitution and Cramer's method. Both methods are easy to use when solving two or three simultaneous equations. If you have four or more simultaneous equations, you should solve them with your calculator or computer because the paper-and-pencil methods are quite complicated. Both back-substitution and Cramer's method work for equations with real numbers, with complex numbers, or even with symbols as coefficients and constants.

Back-Substitution

The back-substitution method picks one equation and solves it for one unknown in terms of the remaining unknowns. The solution is used to eliminate that unknown in the remaining equations. This process is repeated until only one equation and one unknown remain. To illustrate, we will solve Eqs. A.2 using back-substitution. Begin by solving the third equation for $\upsilon 3$, to get

$$\upsilon 3=2.5-3.5\upsilon 1.$$

Now eliminate $\upsilon 3$ in the remaining equations:

$$2\upsilon 1-\upsilon 2=4,$$
$$4\upsilon 2+3(2.5-3.5\upsilon 1)=16.$$

Next, solve the first of the two remaining equations for $\upsilon 2$, to get

$$\upsilon 2=2\upsilon 1-4.$$

Eliminate $\upsilon 2$ in the other equation:

$$4(2\upsilon 1-4)+3(2.5-3.5\upsilon 1)=16.$$

Simplify and solve for υ1:

$-2.5\upsilon_1 = 24.5 \quad \text{so} \quad \upsilon_1 = 24.5 - 2.5 = -9.8$ V.

Finally, use this value for υ1 to find the remaining unknowns:

$\upsilon_2 = 2\upsilon_1 - 4 = 2(-9.8) - 4 = -23.6$ V, $\upsilon_3 = 2.5 - 3.5\upsilon_1 = 2.5 - 3.5(-9.8) = 36.8$ V.

There is another way to solve a set of simultaneous equations with the back-substitution method. Begin by picking any two equations, and multiply one or both equations by a different constant such that when the resulting equations are added together, one of the unknowns is eliminated. For example, consider the first two equations in Eqs. A.3. If we multiply the first equation by the constant 4 and add the resulting equation to the second equation, we eliminate υ2:

$8\upsilon_1 - 4\upsilon_2 + 0\upsilon_3 = 16 + 0\upsilon_1 + 4\upsilon_2 + 3\upsilon_3 = 16 _ 8\upsilon_1 + 0\upsilon_2 + 3\upsilon_3 = 32.$

Now multiply this new equation by 2, and multiply the third equation in Eqs. A.3 by −3. Then add the two equations together to eliminate υ3:

$16\upsilon_1 + 6\upsilon_3 = 64 + -21\upsilon_1 + -6\upsilon_3 = -15 _ -5\upsilon_1 + 0\upsilon_3 = 49.$

Therefore,

$\upsilon_1 = 49 - 5 = -9.8$ V.

We can substitute the value of υ1 back into the first equation in Eqs. A.3 to find υ2 and then substitute the value of υ2 back into the second equation in Eqs. A.3 to find υ3. You should complete these final steps to verify the values.

Cramer's Method

We can also use **Cramer's method** to solve a set of simultaneous equations. The value of each unknown variable is the ratio of two determinants. If we let N, with an appropriate subscript, represent the numerator determinant and Δ

represent the denominator determinant, then the kth unknown xk is

$$x_k = N_k \Delta. \quad \text{(A.4)}$$

The denominator determinant Δ is the same for every unknown variable and is called the **characteristic determinant** of the set of equations. The numerator determinant Nk varies with each unknown.

The characteristic determinant is the determinant of the A matrix. For example, the characteristic determinant of Eqs. A.3 is

$$\Delta = \begin{bmatrix} 2 & -1 & 0 \\ 0 & 4 & 3 \\ 7 & 0 & 2 \end{bmatrix}.$$

To find the determinant, rewrite the first two columns to the right of the determinant to get

$$\Delta = \begin{bmatrix} 2 & -1 & 0 \\ 0 & 4 & 3 \\ 7 & 0 & 2 \end{bmatrix} \quad \begin{matrix} 2 & -1 \\ 0 & 4 \\ 7 & 0 \end{matrix}$$
.

There are now five columns. Sum the products of the left-to-right diagonals for the first three columns; then subtract the sum of products of the right-to-left diagonals for the last three columns:

$$\Delta = (2 \cdot 4 \cdot 2) + (-1 \cdot 3 \cdot 7) + (0 \cdot 0 \cdot 0) - (0 \cdot 4 \cdot 7) - (2 \cdot 3 \cdot 0) - (-1 \cdot 0 \cdot 2) = 16 - 21 + 0 - 0 - 0 - 0 = -5.$$

Note that this shortcut method for finding a determinant works only for square matrices of dimension 3. To find the determinant for a matrix whose dimension is larger than 3, consult a reference on determinants.

To construct the numerator determinant Nk, replace the kth column in the characteristic determinant with the values in the B vector. For example, the numerator determinants for evaluating υ1, υ2, and υ3 in Eqs. A.3 are

$$N_1 = \begin{bmatrix} 4 & -1 & 0 \\ 16 & 4 & 3 \\ 5 & 0 & 2 \end{bmatrix} = (4 \cdot 4 \cdot 2) + (-1 \cdot 3 \cdot 5) + (0 \cdot 16 \cdot 0) - (0 \cdot 4 \cdot 5) - (4 \cdot 3 \cdot 0) - (-1 \cdot 16 \cdot 2) = 49,$$

$$N_2 = \begin{bmatrix} 2 & 4 & 0 \\ 0 & 16 & 3 \\ 7 & 5 & 2 \end{bmatrix} = (2 \cdot 16 \cdot 2) + (4 \cdot 3 \cdot 7) + (0 \cdot 0 \cdot 5) - (0 \cdot 16 \cdot 7) - (2 \cdot 3 \cdot 5) - (4 \cdot 0 \cdot 2) = 118,$$

and

$$N3=[2-140416705]=(2\cdot 4\cdot 5)+(-1\cdot 16\cdot 7)+$$
$$(4\cdot 0\cdot 0)-(4\cdot 4\cdot 7)-(2\cdot 16\cdot 0)-(-1\cdot 0\cdot 5)=-184.$$

Using Cramer's method in Eq. A.4, we can solve for υ1, υ2, and υ3:

$$\upsilon 1=N1\Delta=49\ -5=\ -9.8\ \text{V},\upsilon 2=N2\Delta=118\ -5=\ -23.6\ \text{V},$$

and

$$\upsilon 3=N3\Delta=\ -184\ -5=36.8\ \text{V}.$$

A.4 Applications

The following examples demonstrate the various techniques for solving a system of simultaneous equations generated from circuit analysis.

Example A.1

Use Cramer's method to solve for the node voltages υ1 and υ2 in Eqs. 4.1 and 4.2.

Solution

The first step is to rewrite Eqs. 4.1 and 4.2 in standard form. Collecting the coefficients of υ1 and υ2 on the left-hand side and moving the constant terms to the right-hand side of the equations gives us

$$1.7\upsilon 1-0.5\upsilon 2=10, -0.5\upsilon 1+0.6\upsilon 2=\ \ 2.$$

Rewriting this set of equations in AX=B format gives us

$$[1.7 -0.5\ -0.5 0.6][\upsilon 1 \upsilon 2]=[10 2].$$

Using Cramer's method (Eq. A.4), we can write expressions for the unknown voltages:

$$\upsilon 1=|10 -0.5 2 0.6||1.7 -0.5\ -0.5 0.6|,$$
$$\upsilon 2=|1.7 10\ -0.5 2||1.7 -0.5\ -0.5 0.6|.$$

The shortcut for calculating the determinant for a matrix of dimension 3 does not work for matrices of dimension 2. Instead, there is a different shortcut for these matrices. Starting at the top of the first column, find the product along the left-to-right diagonal. Then subtract the product along the right-to-left diagonal that starts at the top of the second column. Thus,

υ 1 = | 10 −0.5 2 0.6 | | 1.7 −0.5 −0.5 0.6 | = (10)(0.6)−(−0.5)(2) (1.7)(0.6)−(−0.5)(−0.5) = 6+1 1.02−0.25 =9.09 V, υ 2 = | 1.7 10 −0.5 2 | | 1.7 −0.5 −0.5 0.6 | = (1.7)(2)−(10)(−0.5) (1.7)(0.6)−(−0.5)(−0.5) = 3.4+5 1.02−0.25 =10.91 V.

Example A.2

Use Excel to find the three mesh currents in the circuit in Fig. 4.24.

Solution

The equations that describe the circuit in Fig. 4.24 were derived in Example 4.7. There are three KVL equations:

5(i1−i2)+20(i1−i3)−50=0,5(i2−i1)+1i2+4(i2−i3)=0,20(i3−i1)+4(i3−i2)+15io=

There is also a dependent source constraint equation:

io=i1−i3.

Putting these four equations in standard form, we get

25i1−5i2−20i3+0io=50, −5i1+10i2−4i3+0io=0,
−20i1−4i2+24i3+15io=0,i1−0i2−i3−io=0.

Rewriting this set of equations in AX=B format gives us

[25−5−200 −510−40 −20−4241510−1−1][i1i2i3io]=[50000].

The Excel solution is shown in Fig. A.3. The mesh currents are i1=29.6 A, i2=26 A, and i3=28 A, and the controlling current for the dependent source is io=1.6 A.

Figure A.3 Using Excel to solve

the simultaneous equations in Example A.2.

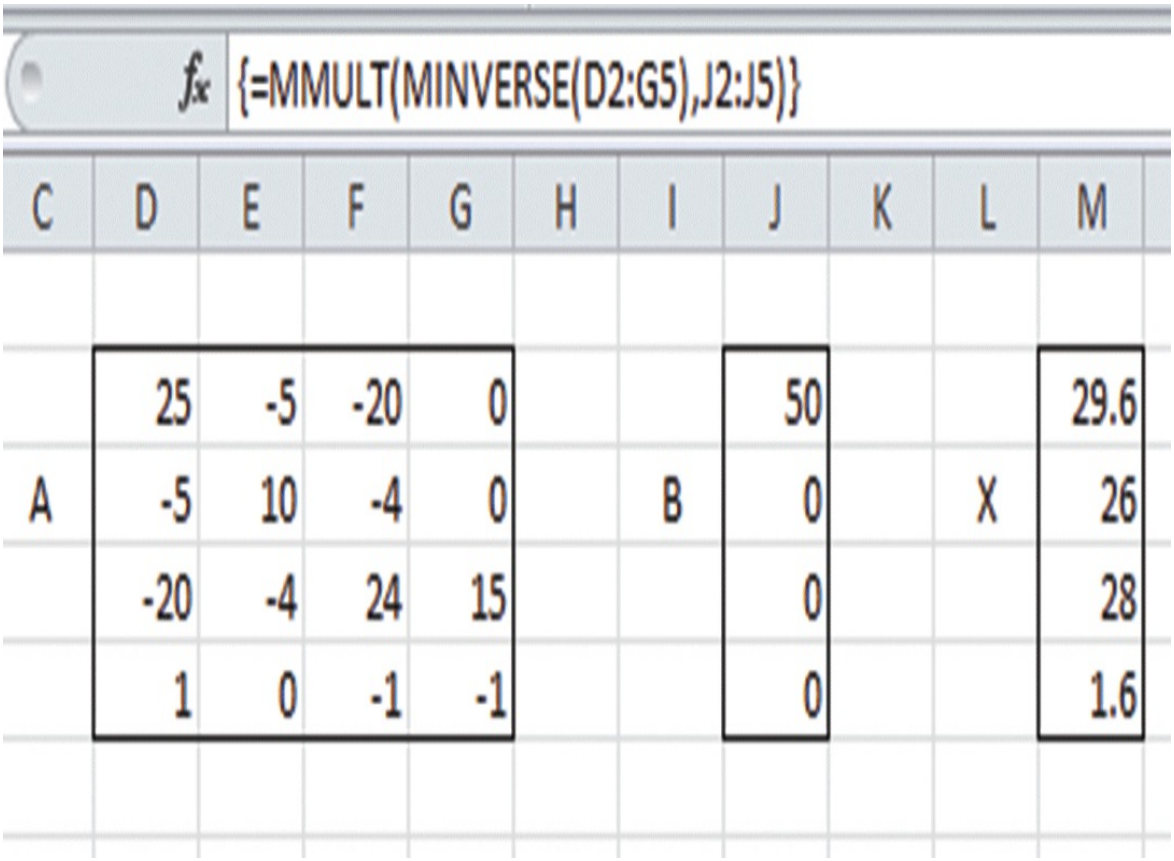


Figure A.3 Full Alternative Text

Example A.3

Use MatLab to find the phasor mesh currents I1 and I2 in the circuit in Fig. 9.40.

Solution

Summing the voltages around mesh 1 generates the equation

$(1+j2)I1+(12-j16)(I1-I2)=150\ 0°$.

Summing the voltages around mesh 2 produces the equation

$(12-j16)(I2-I1)+(1+j3)I2+39Ix=0.$

The current controlling the dependent voltage source is

$Ix=(I1-I2).$

Converting these three equations into standard form, we get

$(13+j14)I1+(-12+j16)I2+0Ix=150,(-12+j16)I1+$
$(13+j13)I2+39Ix=0,I1-I2-Ix=0.$

The MatLab commands used to solve this set of simultaneous equations are shown in Fig. A.4. Note the use of the complex(a, b) function to construct the complex coefficients, where a is the real part of the coefficient and b is the imaginary part of the coefficient. MatLab gives us the solution

Figure A.4 Using MatLab to solve the simultaneous equations in Example A.3.

```
>> syms  i1 i2 ix
>> eq1 = complex(13,-14)*i1 + complex(-12,16)*i2 == 150;
>> eq2 = complex(-12,16)*i1 + complex(13,-13)*i2 + 39*ix == 0;
>> eq3 = i1 - i2 - ix ==0,
>> [A,B] = equationsToMatrix([eq1, eq2, eq3], [i1, i2, ix])

A =

[   13 - 14i, - 12 + 16i,  0]
[  -12 + 16i,   13 - 13i, 39]
[          1,         -1, -1]


B =

  150
    0
    0

>> X = linsolve(A,B)

X =

  - 26 - 52i
  - 24 - 58i
  - 2 +   6i
```

Figure A.4 Full Alternative Text
$I1=(-26-j52)=58.14\ -116.57°\ \ A, I2=(-24-j58)=62.77-122.48°$ A.

In the first three examples, the matrix elements have been numbers— real numbers in Examples A.1 and A.2, and complex numbers in Example A.3. It is also possible for the elements to be symbols. Example A.4 illustrates the use of back substitution in a circuit problem where the elements in the coefficient matrix are symbols.

Example A.4

Use back-substitution to derive expressions for the node voltages V1 and V2 in the circuit in Fig. A.5.

Figure A.5 The circuit for Example A.4.

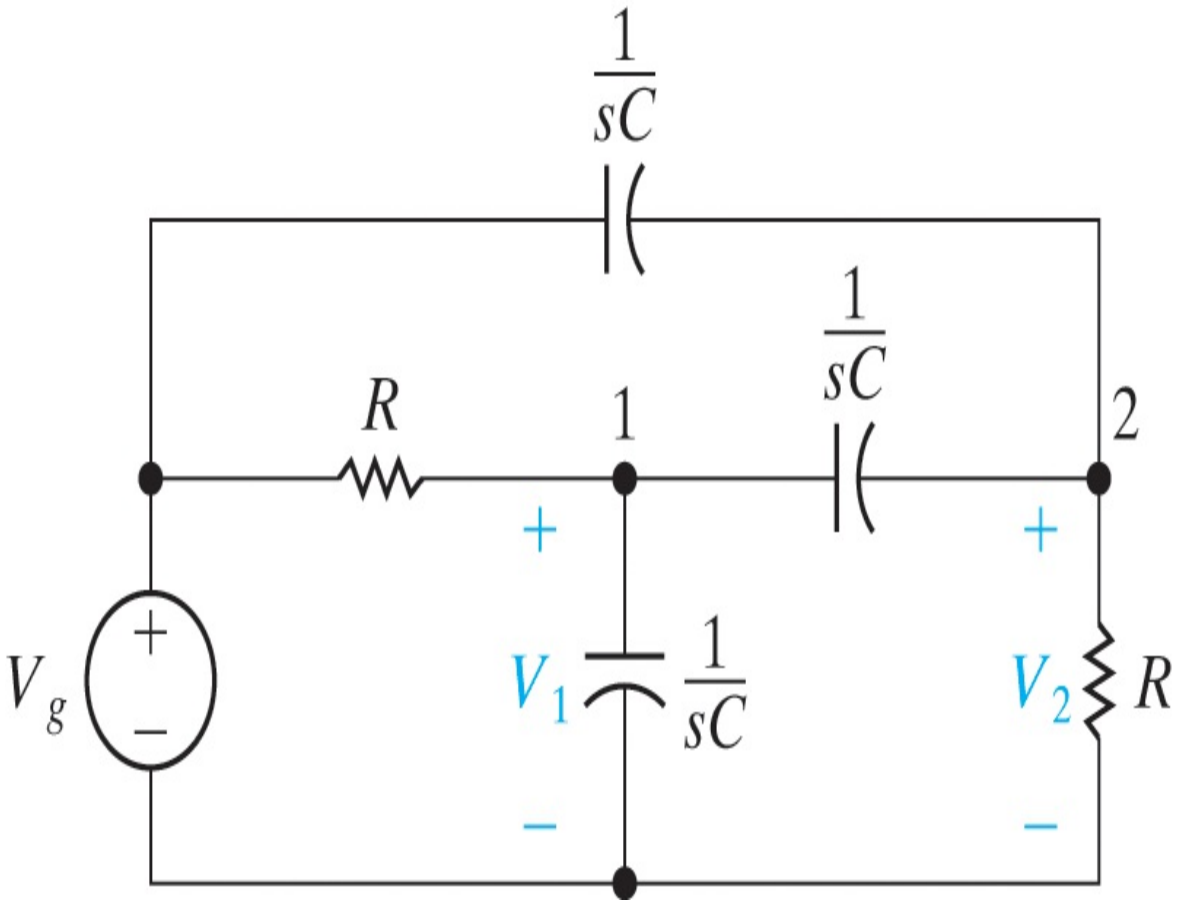


Figure A.5 Full Alternative Text

Solution

Summing the currents leaving nodes 1 and 2 generates the following set of equations:

V1−VgR+V1sC+(V1−V2)sC =0,V2R+(V2−V1)sC+(V2−Vg)sC =0.

Let G=1/R and collect the coefficients of V1 and V2 to get

(G+2sC)V1−sCV2 =GVg,−sCV1+(G+2sC)V2 =sCVg.

Solve the first equation for V2 to get

V2=(G+2sC)sC V1−GsC Vg.

Substitute this expression into the second equation to eliminate V2:

$-sCV1+(G+2sC)[(G+2sC)sC\ V1-GsC\ Vg]=sCVg.$

Rearrange and simplify this equation to find the expression for V1:

$(G+2sC)2-(sC)2sC\ V1=G(G+2sC)+(sC)2sC\ Vg,$

so

$V\ 1 = G(G+2sC)+ (sC) 2 (G+2sC) 2 - (sC) 2 \quad V\ g = (G\ 2 +2sCG+ s\ 2\ C\ 2) (G\ 2 +4sCG+3\ s\ 2\ C\ 2) \quad V\ g\ .$

Finally, substitute this expression for V1 into the equation for V2; rearrange and simplify to find the expression for V2:

$V2=(G+2sC)sC\ [(G2+2sCG+s2C2)(G2+4sCG+3s2C2)\ Vg]-GsC\ Vg,$

so

$V2=[(G+2sC)(G2+2sCG+s2C2)-G(G2+4sCG+3s2C2)sC(G2+4sCG+3s2C2)]Vg=2sC(G+s$
$(G2+4sCG+3s2C2)\ Vg.$

Appendix B Complex Numbers

Complex numbers allow us to find the square root of negative numbers. For example, consider the equation $x2+8x+41=0$. From the quadratic formula, we know that the two values of x that satisfy this equation are

$$x1,2=−8±82−4(41)2=−4±−25.$$

Therefore, this equation has no solution in a number system that excludes complex numbers. Complex numbers, and the ability to manipulate them algebraically, are extremely useful in circuit analysis.

B.1 Notation

There are two ways to designate a complex number: with the cartesian, or rectangular, form or with the polar, or trigonometric, form. In rectangular form, a complex number is written as a sum of a real component and an imaginary component; hence

$$n=a+jb,$$

where a is the real component, b is the imaginary component, and j is, by definition, -1.[1]

[1] You may be more familiar with the notation $i=-1$. In electrical engineering, i is used as the symbol for current, and hence in electrical engineering literature, j is used to denote -1.

In **polar form**, a complex number is written in terms of its magnitude (or modulus) and angle (or argument); hence

$$n=ce^{j\theta}$$

where c is the magnitude, θ is the angle, e is the base of the natural logarithm, and, as before, $j=-1$. The symbol $\theta°$ is frequently used in place of $e^{j\theta}$, so the polar form can also be written as

$$n=c\ \theta° .$$

When we write a complex number in polar form, we typically use the angle notation. But when we perform mathematical operations using complex numbers in polar form, we use the exponential notation because the rules for manipulating an exponential quantity are well known. For example, because $(y^x)^n=y^{xn}$, then $(e^{j\theta})^n=e^{jn\theta}$; because $y^{-x}=1/y^x$, then $e^{-j\theta}=1/e^{j\theta}$; and so forth.

Since there are two ways of expressing the same complex number, we need to relate one form to the other. The transition from polar to rectangular form

makes use of Euler’s identity:

$e \pm j\theta = \cos\theta \pm j \sin\theta.$

A complex number in polar form can be put in rectangular form by writing

$cej\theta = c(\cos\theta + jc \sin\theta) = c(\cos\theta + jc \sin\theta) = a + jb.$

The transition from rectangular to polar form uses right triangle geometry, namely,

$a + jb = (a2+b2)ej\theta = cej\theta,$

where

$\tan\theta = b/a.$

The expression for $\tan\theta$ does not specify the quadrant where the angle θ is located. We can determine the location of θ using a graphical representation of the complex number.

B.2 The Graphical Representation of a Complex Number

A complex number is represented graphically on a complex-number plane, where the horizontal axis represents the real component and the vertical axis represents the imaginary component. The angle of the complex number is measured counterclockwise from the positive real axis. The plot of the complex number n=a + jb=c θ° , assuming that a and b are both positive, is shown in Fig. B.1.

Figure B.1 The graphical representation of a + jb when a and b are both positive.

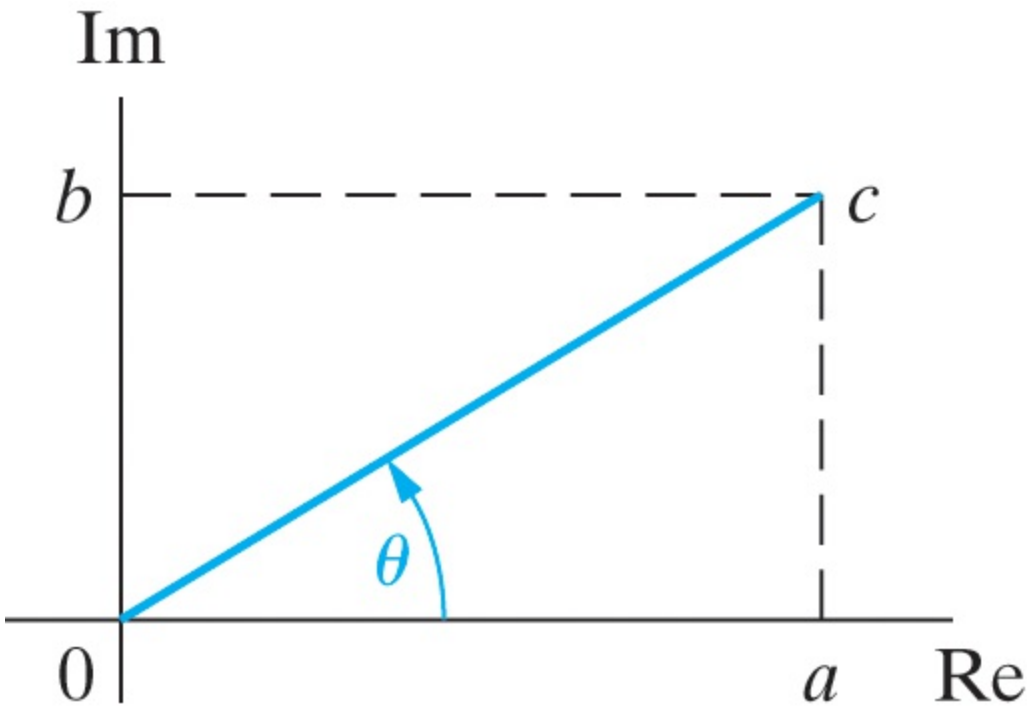


Figure B.1 Full Alternative Text

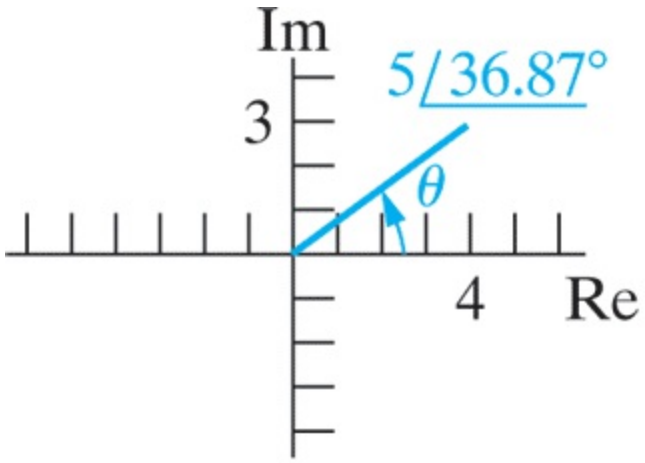
This plot makes very clear the relationship between the rectangular and polar forms. Any point in the complex-number plane is uniquely defined by giving either its distance from each axis (that is, a and b) or its radial distance from the origin (c) and the angle with respect to the positive real axis, θ.

It follows from Fig. B.1 that θ is in the first quadrant when a and b are both positive, in the second quadrant when a is negative and b is positive, in the third quadrant when a and b are both negative, and in the fourth quadrant when a is positive and b is negative. These observations are illustrated in Fig. B.2, where we have plotted 4+j3, −4+j3, −4−j3, and 4−j3.

Note that we can also specify θ as a clockwise angle from the positive real axis. Thus, in Fig. B.2(c), we could also designate −4−j3 as $5\ \angle -143.13°$. In Fig. B.2(d), we observe that $5\ \angle 323.13° = 5\ \angle -36.87°$. It is customary to express θ in terms of negative values when θ lies in the third or fourth quadrant, so that $-180^\circ \leq \theta \leq 180^\circ$.

The **conjugate of a complex number** is formed by reversing the sign of its imaginary component. Thus, the conjugate of a+jb is a−jb, and the conjugate of −a+jb is −a−jb. When we write a complex number in polar form, we construct its conjugate by reversing the sign of the angle θ. Therefore, the conjugate of $c\ \angle \theta°$ is $c\ \angle -\theta°$. The conjugate of a complex number is designated with an asterisk, so n* is understood to be the conjugate of n. Figure B.3 shows two complex numbers and their conjugates plotted on the complex-number plane. Note that conjugation reflects a complex number about the real axis.

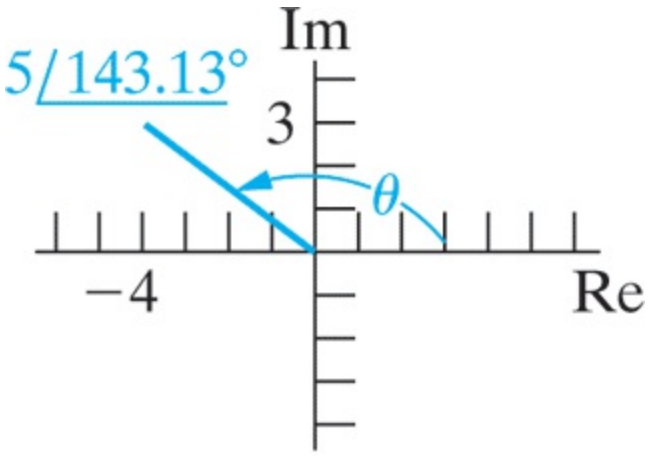
Figure B.2 The graphical representation of four complex numbers.



$$4 + j3 = 5\angle 36.87°$$

(a)

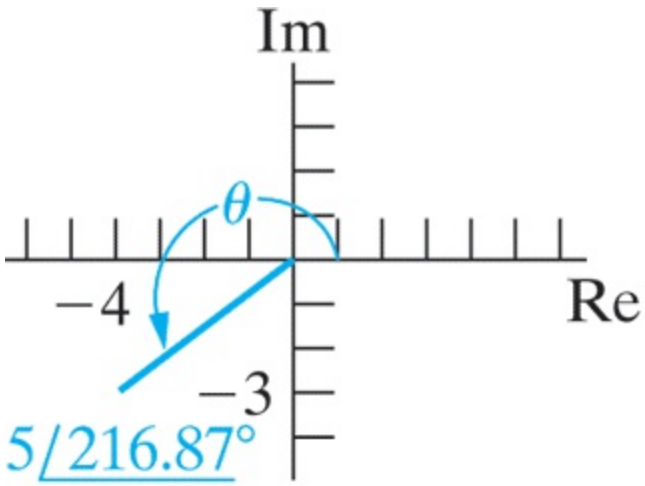
Figure B.2 Full Alternative Text



$$-4 + j3 = 5\angle 143.13°$$

(b)

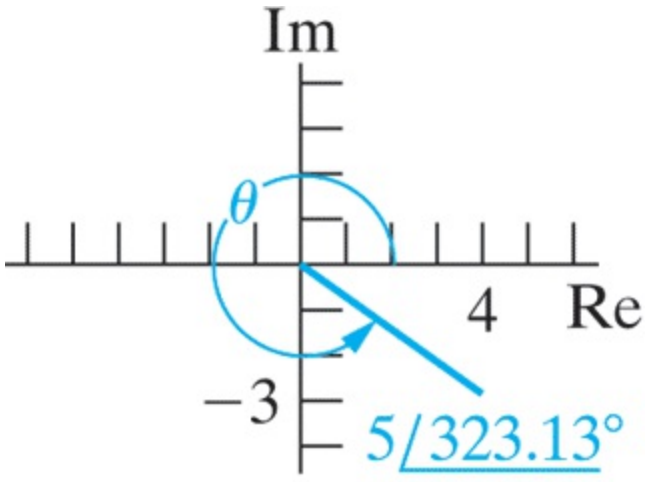
B.2-1 Full Alternative Text



$$-4-j3 = 5\angle 216.87°$$

(c)

B.2-1 Full Alternative Text



$$4-j3 = 5\angle 323.13°$$

(d)

B.2-1 Full Alternative Text

Figure B.3 The complex

numbers n 1 and n 2 and their conjugates n 1 * and n 2 * .

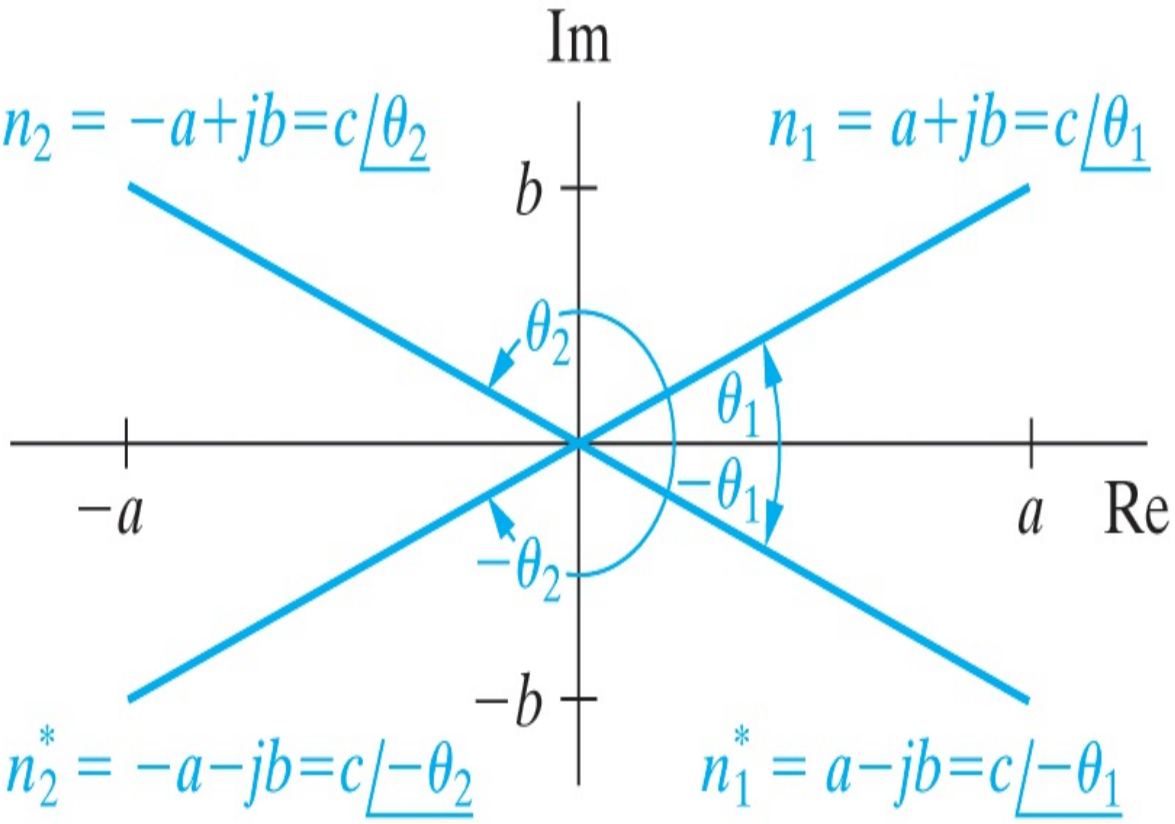


Figure B.3 Full Alternative Text

B.3 Arithmetic Operations

Addition (Subtraction)

When adding or subtracting complex numbers, we express the numbers in rectangular form. Addition involves adding the real parts of the complex numbers to form the real part of the sum and adding the imaginary parts to form the imaginary part of the sum. Thus, if we are given

$n1=8+j16$

and

$n2=12-j3,$

then

$n1+n2=(8+12)+j(16-3)=20+j13.$

Subtraction follows the same rule. Thus

$n2-n1=(12-8)+j(-3-16)=4-j19.$

If the numbers to be added or subtracted are given in polar form, they are first converted to rectangular form. For example, if

$n\ 1\ =10\ \ 53.13°$

and

$n\ 2\ =5\ \ -135°\ ,$

then

$n1\ +\ n2=6+j8-3.535-j3.535=$

(6−3.535)+j(8−3.535)=2.465 + j4.465=5.10 61.10°,

and

n1−n2=6+j8−(−3.535−j3.535)=9.535+j11.535=14.966 50.42°.

Multiplication (Division)

When multiplying or dividing complex numbers, the numbers can be written in either rectangular or polar form. In most cases, polar form is more convenient. As an example, let’s find the product n1n2 when n1=8+j10 and n2=5−j4. Using rectangular form, we have

n1n2=(8+j10)(5−j4)=40−j32+j50+40=80+j18=82 12.68°.

If we use polar form, we find the product of two complex numbers by multiplying their magnitudes and adding their angles. For example,

n1n2=(12.81 51.34°)(6.40−38.66°)=82 12.68°=80+j18.

The first step in dividing two complex numbers in rectangular form is to multiply the numerator and denominator by the conjugate of the denominator. This makes the denominator a real number. We then divide the real number into the new numerator. As an example, let’s calculate n1/n2, where n1=6+j3 and n2=3−j1. We have

n 1 n 2 = 6+j3 3−j1 = (6+j3)(3+j1) (3−j1)(3+j1) = 18+j6+j9−3 9+1 = 15+j15 10 =1.5+j1.5 = 2.12   45° .

In polar form, we calculate n1/n2 by dividing the magnitudes and subtracting the angles. For example,

n1n2=6.71 26.57°3.16 −18.43°=2.12 45°=1.5+j1.5.

B.4 Useful Identities

In working with complex numbers and quantities, the following identities are very useful:

$\pm j2 = \pm 1, (-j)(j) = 1, j = 1 - j, e^{\pm j\pi} = -1, e^{\pm j\pi/2} = \pm j.$

Given that $n = a + jb = c \angle \theta°$, it follows that

$n\, n^* = a^2 + b^2 = c^2$, $n + n^* = 2a$, $n - n^* = j2b$, $n / n^* = 1 \angle 2\theta°$.

B.5 The Integer Power of a Complex Number

To raise a complex number to an integer power k, begin by expressing the complex number in polar form. Then, to find the kth power of a complex number, raise its magnitude to the kth power and multiply its angle by k. Thus

$$n^k = (a+jb)^k = (c\, e^{j\theta})^k = c^k e^{jk\theta} = c^k (\cos k\theta + j \sin k\theta).$$

For example,

$$(2\, e^{j12^\circ})^5 = 2^5 e^{j60^\circ} = 32\, e^{j60^\circ} = 16+j27.71,$$

and

$$(3+j4)^4 = (5\, e^{j53.13^\circ})^4 = 5^4 e^{j212.52^\circ} = 625\, e^{j212.52^\circ} = -527-j336.$$

B.6 The Roots of a Complex Number

To find the kth root of a complex number, we solve the equation

$$x^k - ce^{j\theta} = 0,$$

which is an equation of the kth degree and therefore has k roots.

To find the k roots ($x_1, x_2, \ldots, x_k$), we first note that

$$ce^{j\theta} = ce^{j(\theta + 2\pi)} = ce^{j(\theta + 4\pi)} = \cdots.$$

It follows that

$$x_1 = (ce^{j\theta})^{1/k} = c^{1/k} e^{j\theta/k}, \quad x_2 = [ce^{j(\theta+2\pi)}]^{1/k} = c^{1/k} e^{j(\theta+2\pi)/k}, \quad x_3 = [ce^{j(\theta+4\pi)}]^{1/k} = c^{1/k} e^{j(\theta+4\pi)/k},$$
$$\vdots$$

We continue this process until the roots start repeating. This will happen when the multiple of π is equal to $2k$. For example, let's find the four roots of $81e^{j60°}$. We have

$$x_1 = 81^{1/4} e^{j60/4} = 3e^{j15°}, \quad x_2 = 81^{1/4} e^{j(60+360)/4} = 3e^{j105°}, \quad x_3 = 81^{1/4} e^{j(60+720)/4} = 3e^{j195°}, \quad x_4 = 81^{1/4} e^{j(60+1080)/4} = 3e^{j285°}, \quad x_5 = 81^{1/4} e^{j(60+1440)/4} = 3e^{j375°} = 3e^{j15°}.$$

Here, x_5 is the same as x_1, so the roots have started to repeat. Therefore, we know the four roots of $81e^{j60°}$ are the values given by x_1, x_2, x_3, and x_4.

Note that the roots of a complex number lie on a circle in the complex-number plane. The radius of the circle is $c^{1/k}$. The roots are uniformly distributed around the circle, and the angle between adjacent roots is $2\pi/k$ radians or $360/k$ degrees. The four roots of $81e^{j60°}$ are plotted in Fig. B.4.

Figure B.4 The four roots of 81ej60°

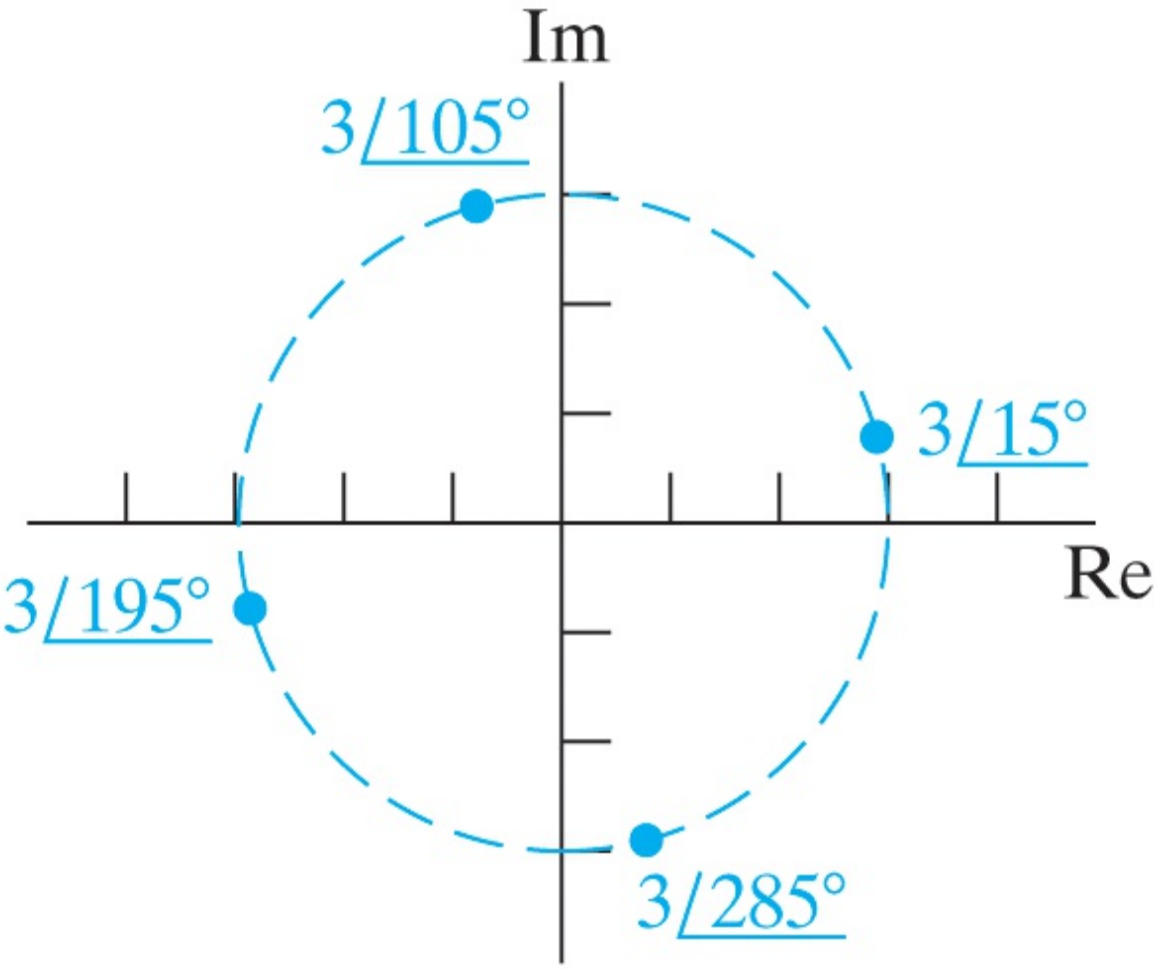


Figure B.4 Full Alternative Text

Appendix C More on Magnetically Coupled Coils and Ideal Transformers

C.1 Equivalent Circuits for Magnetically Coupled Coils

It is sometimes convenient to model magnetically coupled coils with an equivalent circuit that does not include magnetic coupling. Consider the two magnetically coupled coils shown in Fig. C.1. The resistances R1 and R2 represent the winding resistance of each coil. The goal is to replace the magnetically coupled coils inside the shaded area with a set of inductors that are not magnetically coupled. Before deriving the equivalent circuits, we must point out an important restriction: The voltage between terminals b and d must be zero. In other words, we must be able to short together terminals b and d without disturbing the voltages and currents in the original circuit. This restriction is imposed because, while the equivalent circuits we develop have four terminals, two of those four terminals are shorted together. Thus, the same requirement is placed on the original circuits.

Figure C.1 The circuit used to develop an equivalent circuit for magnetically coupled coils.

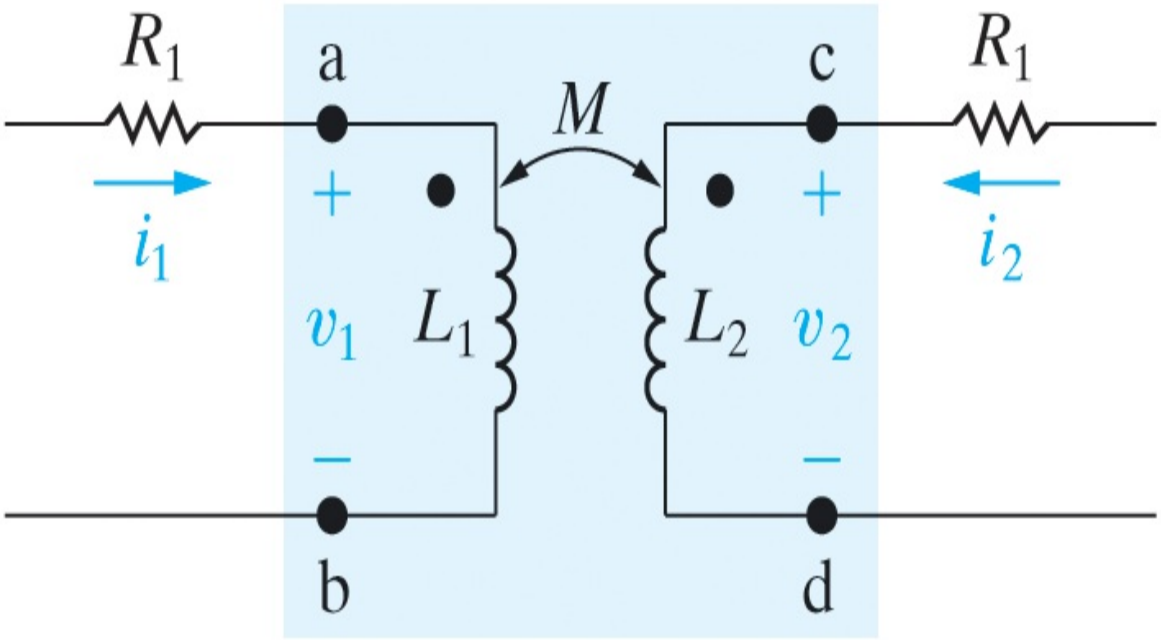


Figure C.1 Full Alternative Text

We begin by writing the two equations that relate the terminal voltages v1 and v2 to the terminal currents i1 and i2. For the given references and polarity dots,

v1=L1di1dt + Mdi2dt(C.1)

and

v2=Mdi1dt + L2di2dt.(C.2)

The T-Equivalent Circuit

How can we arrange uncoupled inductors in a circuit that can be described by Eqs. C.1 and C.2? If we regard Eqs. C.1 and C.2 as mesh-current equations with i1 and i2 as the mesh currents, we need one mesh with a total inductance of L1 and a second mesh with a total inductance of L2. Also, the two meshes must have a common inductance of M. The T-equivalent circuit shown in Fig. C.2 satisfies these requirements.

Figure C.2 The T-equivalent circuit for the magnetically coupled coils of Fig. C.2.

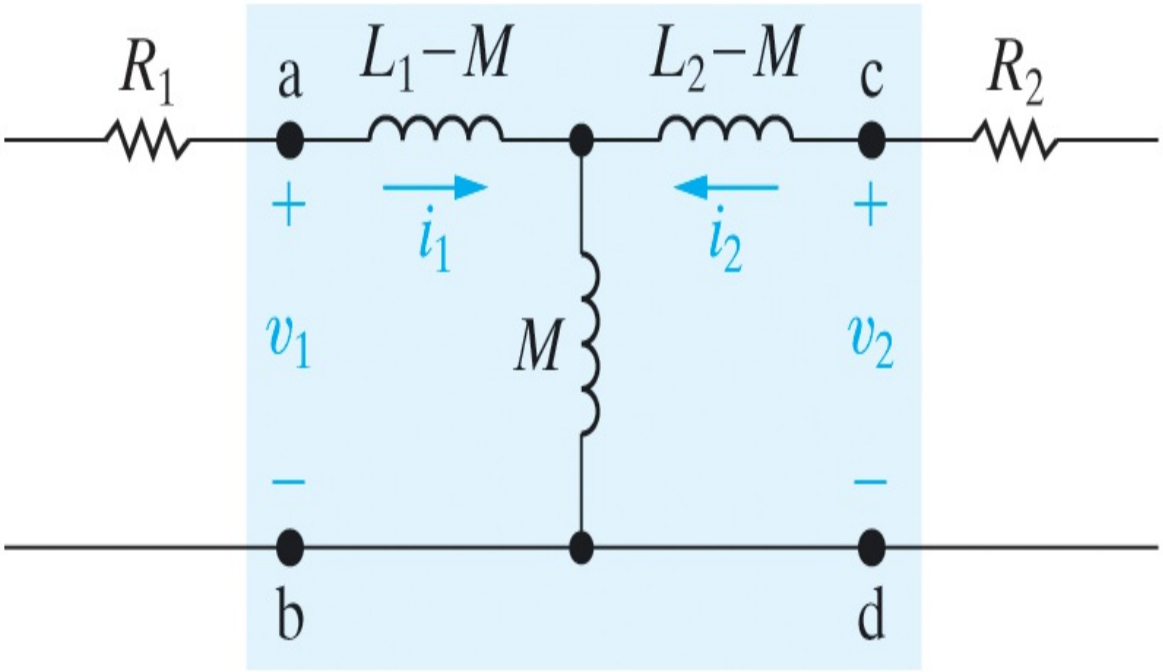


Figure C.2 Full Alternative Text

You should verify that the equations relating v1 and v2 to i1 and i2 reduce to Eqs. C.1 and C.2. Note the absence of magnetic coupling between the inductors and the zero voltage between b and d.

The π-Equivalent Circuit

We can derive a π-equivalent circuit for the magnetically coupled coils shown in Fig. C.1 by solving Eqs. C.1 and C.2 for the derivatives di1/dt and di2/dt. We treat the resulting expressions as a pair of node-voltage equations. Find di1/dt and di2/dt using Cramer's method:

di1dt=|v1Mv2L2||L1MML2|=L2L1L2−M2v1−ML1L2−M2v2;(C.3)
di2dt=|L1v1Mv2|L1L2−M2=−ML1L2−M2v1+L1L1L2−M2v2.(C.4)

Now we solve for i1 and i2 by multiplying both sides of Eqs. C.3 and C.4 by dt and then integrating:

i1=i1(0) + L2L1L2−M2∫0tv1 dτ − ML1L2 − M2∫0tv2 dτ(C.5)

and

i2=i2(0)−ML1L2−M2∫0tv1 dτ + L1L1L2−M2∫0tv2 dτ.(C.6)

If we regard v1 and v2 as node voltages, Eqs. C.5 and C.6 describe a circuit like the one shown in Fig. C.3.

Figure C.3 The circuit used to derive the -equivalent circuit for magnetically coupled coils.

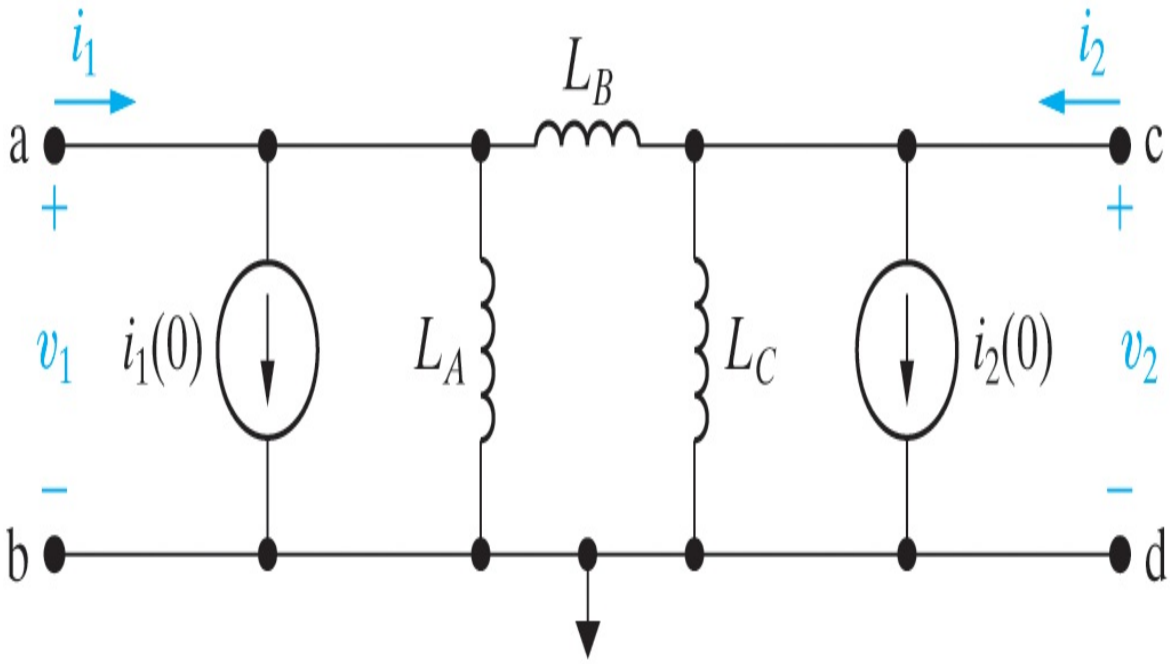


Figure C.3 Full Alternative Text

To find LA, LB, and LC as functions of L1, L2, and M, write the equations for i1 and i2 in Fig. C.3 and compare them with Eqs. C.5 and C.6. Thus

$$i1=i1(0) + 1LA\int_0^t v1\,d\tau + 1LB\int_0^t (v1-v2)\,d\tau = i1(0) + (1LA + 1LB)\int_0^t v1\,d\tau - 1LB\int_0^t v2\,d\tau$$

and

$$i2=i2(0) + 1LC\int_0^t v2\,d\tau + 1LB\int_0^t (v1-v2)\,d\tau = i2(0) + 1LB\int_0^t v1\,d\tau + (1LB + 1LC$$

Then

$$1LB=ML1L2-M2,$$
$$1LA=L2-ML1L2-M2,$$
$$1LC=L1-ML1L2-M2.$$

Incorporate the expressions for LA, LB, and LC into the circuit shown in Fig. C.3 to get the π-equivalent circuit for the magnetically coupled coils shown in Fig. C.1. The result is shown in Fig. C.4.

Figure C.4 The π-equivalent circuit for the magnetically coupled coils of Fig. C.1.

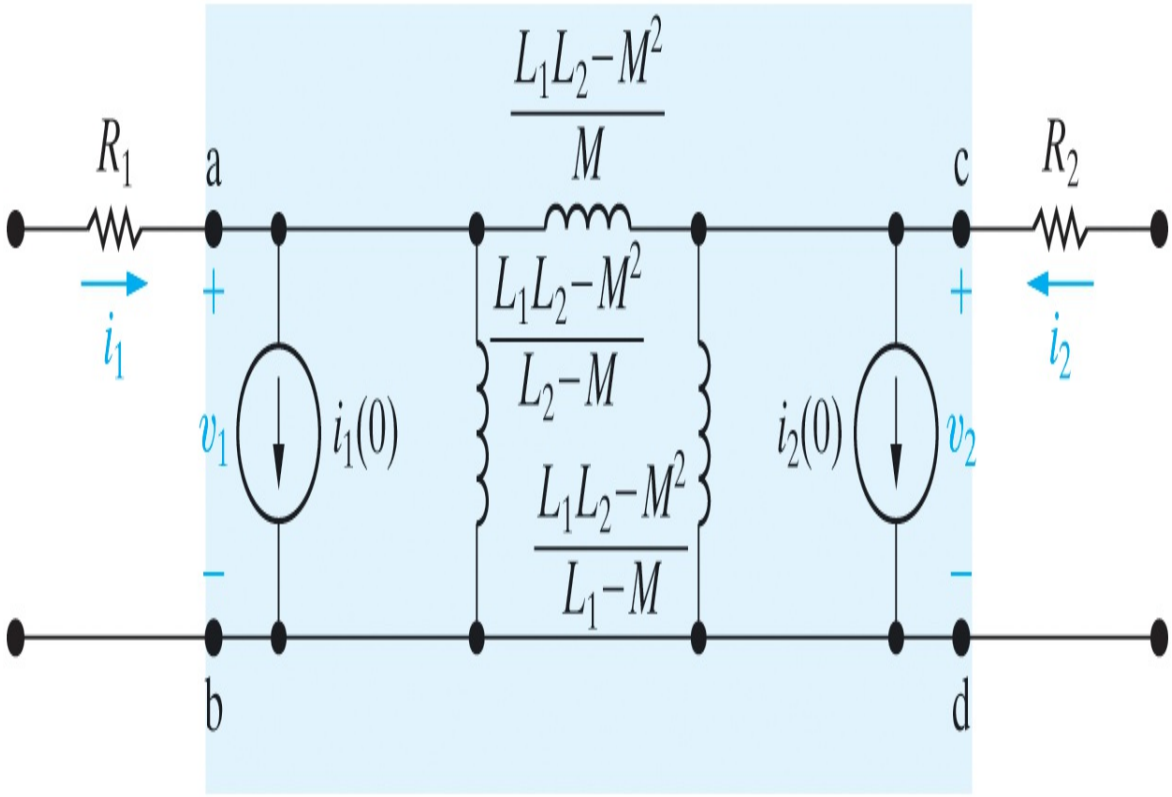


Figure C.4 Full Alternative Text

Note that the initial values of i1 and i2 are explicit in the π-equivalent circuit but implicit in the T-equivalent circuit. If we want to find the sinusoidal steady-state behavior of circuits containing mutual inductance, we can assume that the initial values of i1 and i2 are zero. We can thus eliminate the current sources in the π-equivalent circuit, and the circuit shown in Fig. C.4 simplifies to the one shown in Fig. C.5.

Figure C.5 The π-equivalent circuit used for sinusoidal steady-state analysis.

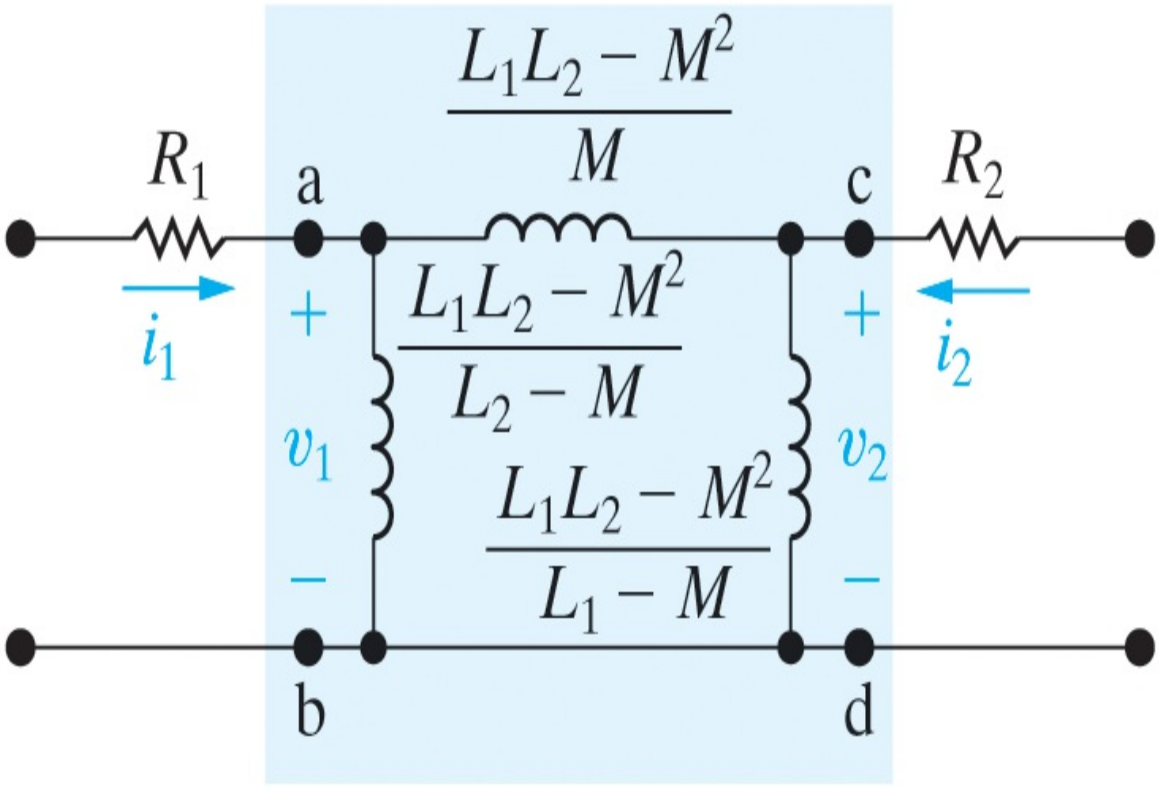


Figure C.5 Full Alternative Text

The mutual inductance carries its own algebraic sign in the T- and π-equivalent circuits. In other words, if the magnetic polarity of the coupled coils is reversed from that given in Fig. C.1, the algebraic sign of M reverses. A reversal in magnetic polarity requires moving one polarity dot without changing the reference polarities of the terminal currents and voltages.

Example C.1 illustrates the application of the T-equivalent circuit.

Example C.1

1. Replace the magnetically coupled coils shown in Fig. C.6 with a T-equivalent circuit. Then find the phasor currents I1 and I2. The source frequency is 400 rad/s.

Figure C.6 The frequency-domain equivalent circuit for Example C.1.

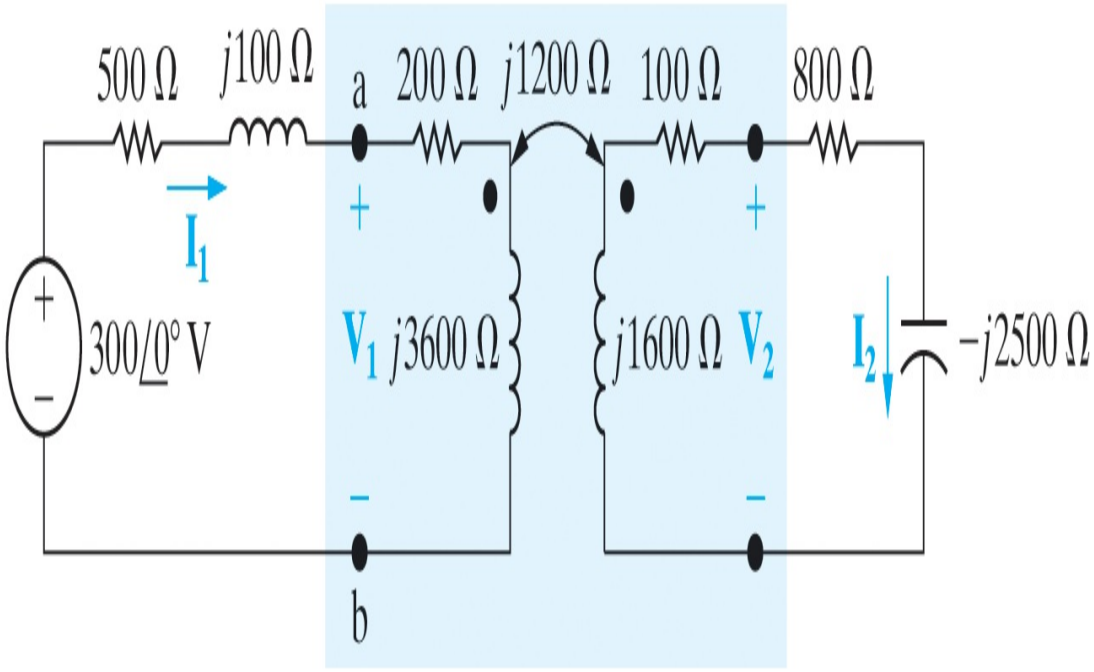


Figure C.6 Full Alternative Text

2. Repeat (a), but with the polarity dot on the secondary winding moved to the lower terminal.

Solution

1. For the polarity dots shown in Fig. C.6, M has a value of +3 H in the T-equivalent circuit. Therefore, the three inductances in the equivalent circuit are

   L1−M=9−3=6H;L2−M=4−3=1H;M=3H.

Figure C.7 shows the T-equivalent circuit, and Fig. C.8 shows the frequency-domain equivalent circuit at a frequency of 400 rad/s.

Figure C.7 The T-equivalent circuit for the magnetically coupled coils in Example C.1.

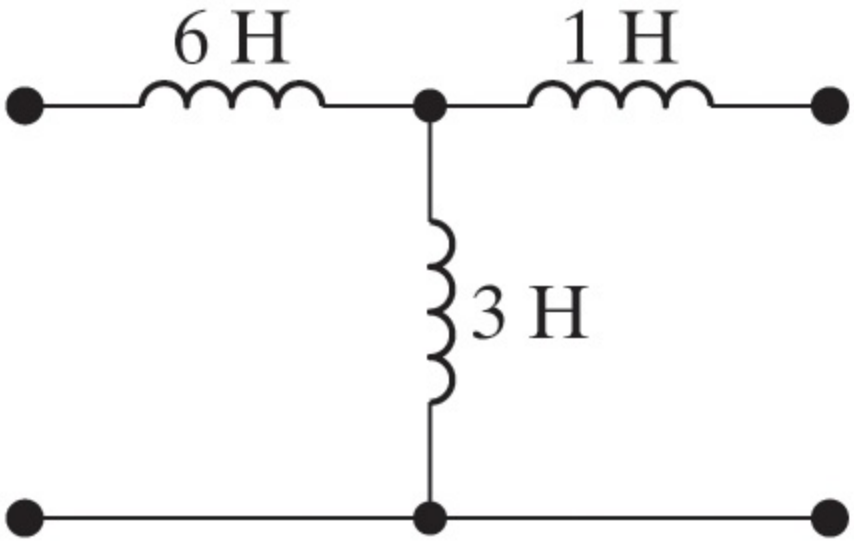


Figure C.7 Full Alternative Text

Figure C.8 The frequency-domain model of the equivalent circuit at 400rad/s.

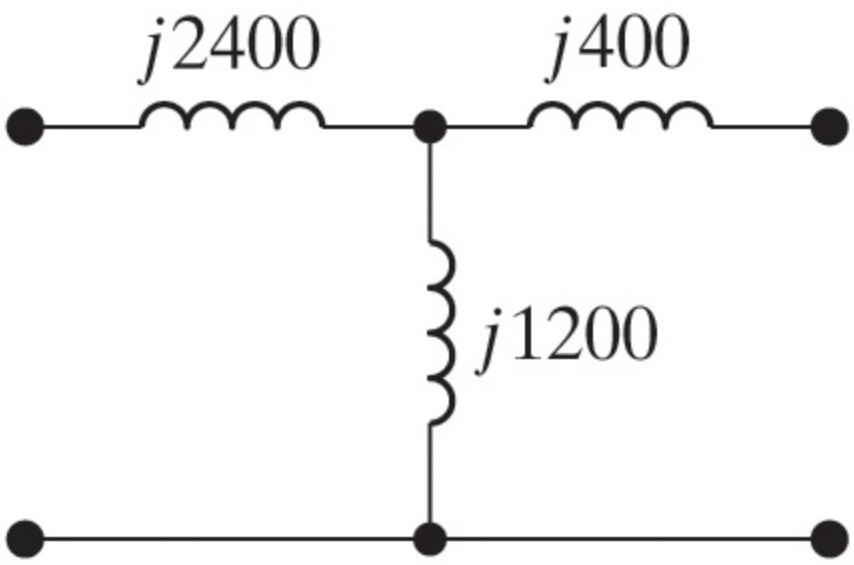


Figure C.8 Full Alternative Text

Figure C.9 shows the original frequency- domain circuit with the magnetically coupled coils replaced by the T-equivalent circuit in Fig. C.8. To find the phasor currents I1 and I2, we first find the node voltage across the 1200 Ω inductive reactance. If we use the lower node as the reference, the single KCL equation is

Figure C.9 The circuit of Fig. C.6, with the magnetically coupled coils replaced by their T-equivalent circuit.

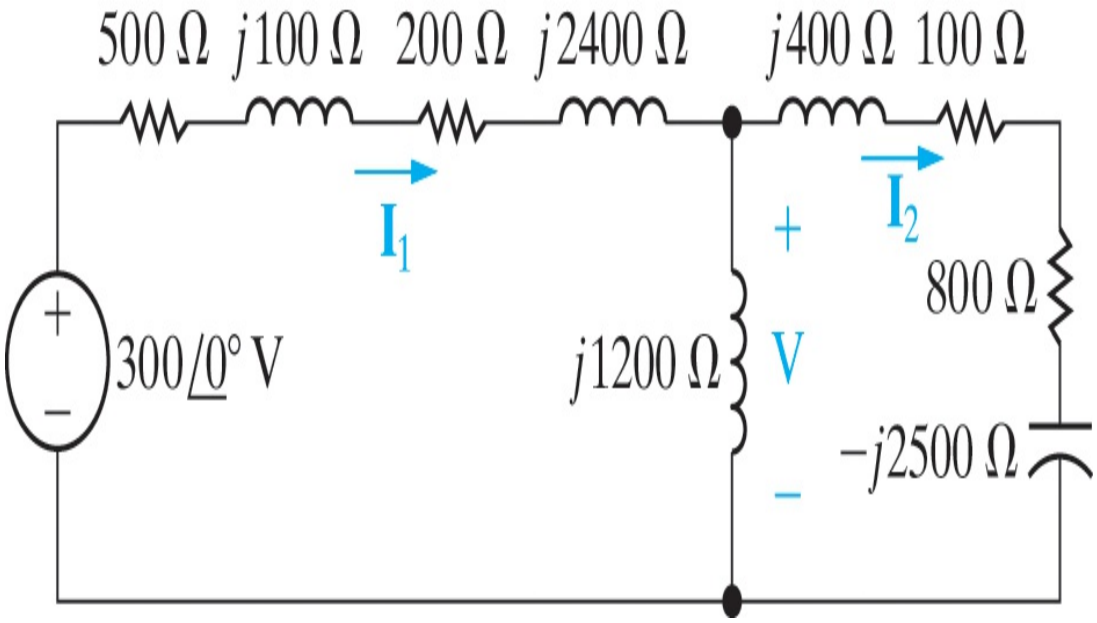


Figure C.9 Full Alternative Text

V−300700+j2500+Vj1200+V900−j2100=0.

Solving for V yields

V=136−j8=136.24 −3.37° V.

Then

I1=300−(136−j8)700+j2500=63.25 −71.57° mA

and

I2=136−j8900−j2100=59.63 63.43° mA.

2. When the polarity dot is moved to the lower terminal of the secondary coil, M has a value of −3H in the T-equivalent circuit. Before analyzing the new T-equivalent circuit, we note that reversing the algebraic sign of M has no effect on the solution for I1 and shifts I2 by 180°. Therefore, we anticipate that

   I1=63.25 −71.57° mA

   and

$I_2 = 59.63 \angle -116.57°$ mA.

Now let's analyze the new T-equivalent circuit. With $M = -3$H, the three inductances in the equivalent circuit are

$L_1 - M = 9 - (-3) = 12$H; $L_2 - M = 4 - (-3) = 7$H; $M = -3$H.

At an operating frequency of 400 rad/s, the frequency-domain equivalent circuit requires two inductors and a capacitor, as shown in Fig. C.10.

Figure C.10 The frequency-domain equivalent circuit for $M = -3$ H and $\omega = 400$rad/s

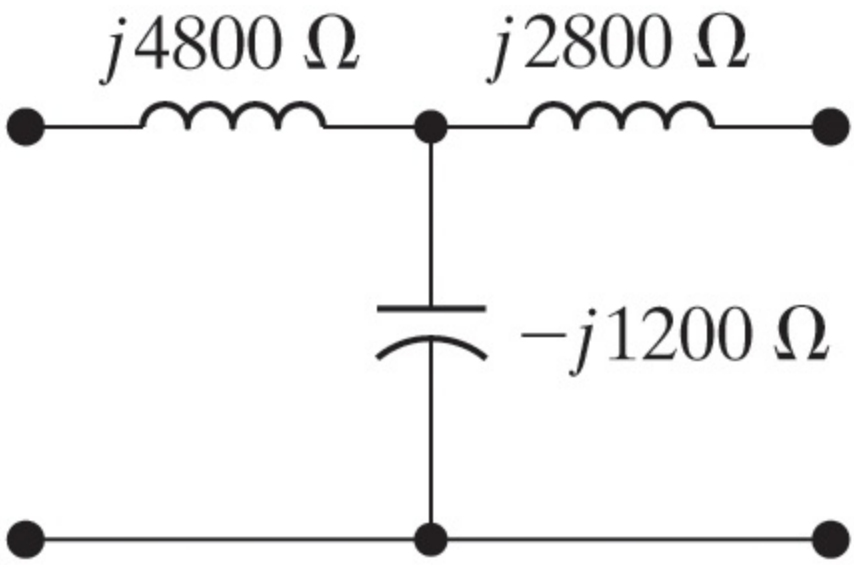


Figure C.10 Full Alternative Text

The resulting frequency-domain circuit for the original system appears in Fig. C.11. As before, we first find the voltage Across The Center Branch, Which contains a capacitive reactance of −j1200 Ω. If we use the lower node as the reference, the KCL equation is

Figure C.11 The frequency-domain equivalent circuit for Example C.1(b).

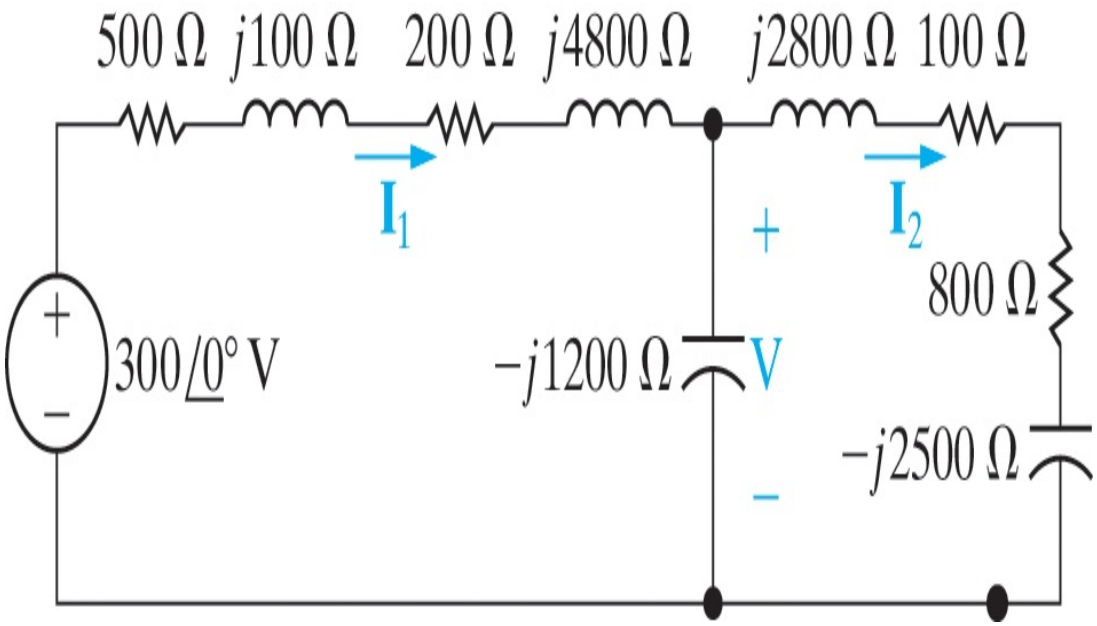


Figure C.11 Full Alternative Text

V−300700+j4900+V−j1200+V900+j300=0.

Solving for **V** gives

V=−8−j56=56.57 −98.13° V.

Then

I1=300−(−8−j56)700+j4900=63.25 −71.57° mA.

and

I2=−8−j56900+j300=59.63 −116.57° mA.

C.2 The Need for Ideal Transformers in the Equivalent Circuits

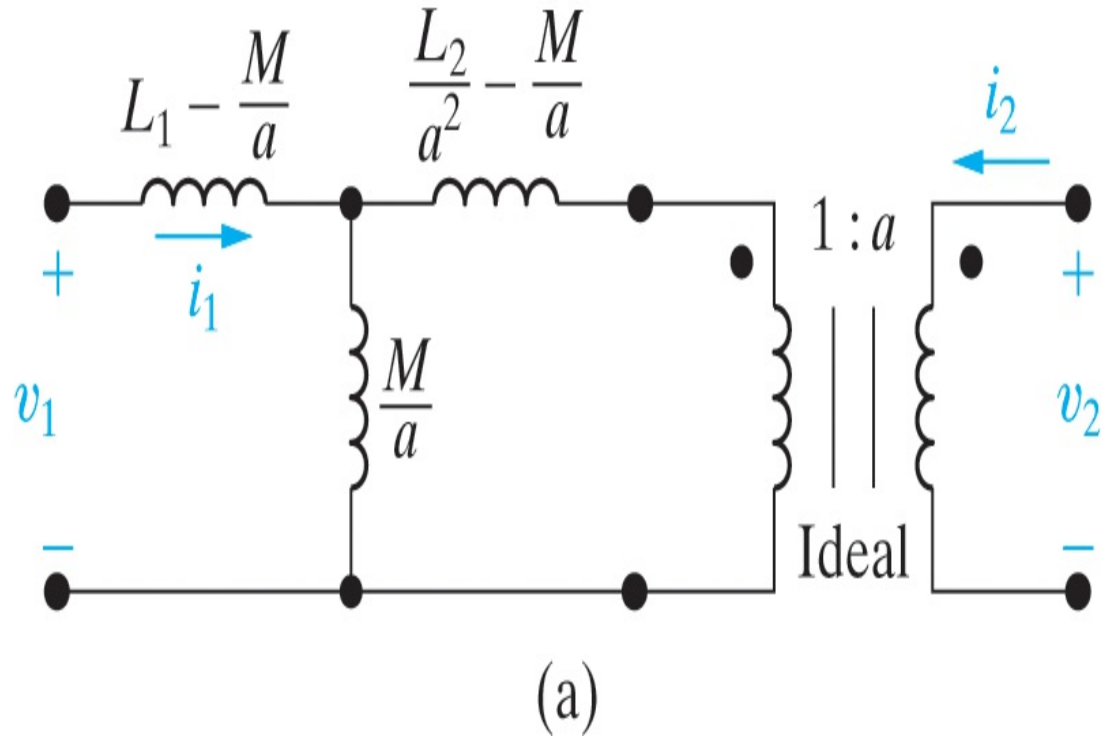
The inductors in the T- and π-equivalent circuits of magnetically coupled coils can have negative values. For example, if L1=3mH, L2=12mH, and M=5mH, the T-equivalent circuit requires an inductor of −2mH, and the π-equivalent circuit requires an inductor of −5.5mH. These negative inductance values do not create a problem if you use the equivalent circuits in computations. However, if you want to build the equivalent circuits using circuit components, the negative reactance can only be achieved using capacitors. But whenever the frequency of the sinusoidal source changes, you must change the capacitor used to generate the negative reactance. For example, at a frequency of 50 krad/s, a −2mH inductor has an impedance of −j100 Ω. This impedance can be modeled with a 0.2 μF capacitor. If the frequency changes to 25 krad/s, the −2mH inductive impedance changes to −j50 Ω. and we need a 0.8 μF capacitor. If the frequency is varied continuously, using a capacitor to simulate negative inductance is impractical.

Instead of using capacitors to create negative reactance, you can include an ideal transformer in the equivalent circuit. This doesn't completely solve the problem because ideal transformers can only be approximated. However, in some situations the approximation is satisfactory, so knowing how to use an ideal transformer in the T- and π-equivalent circuits of magnetically coupled coils is an important tool.

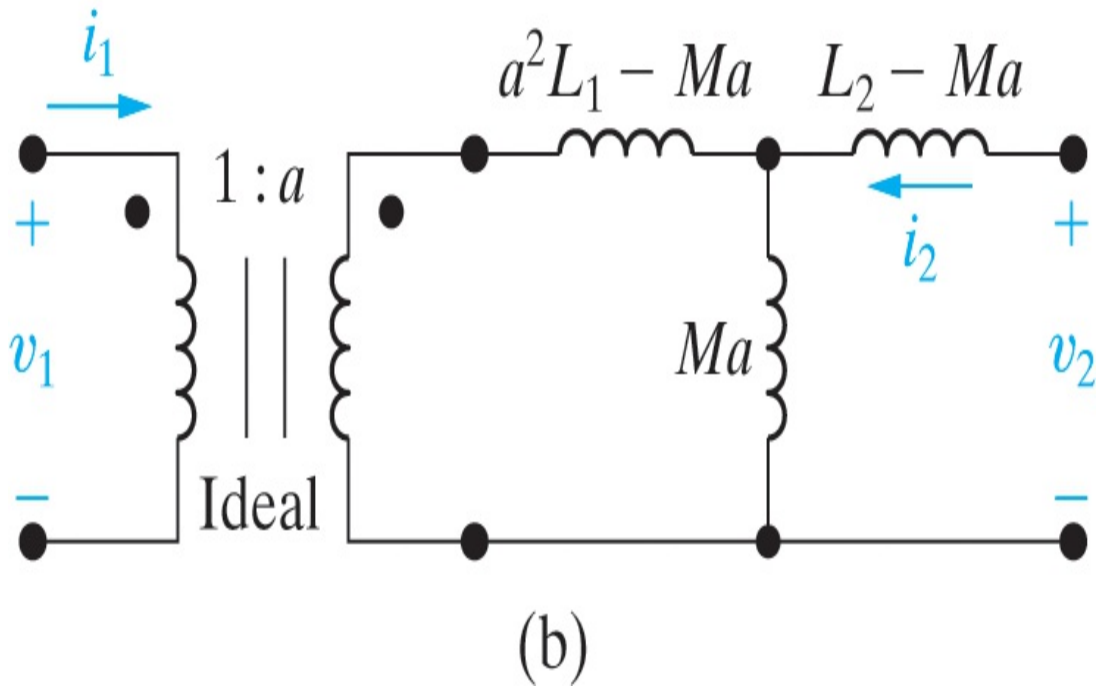
An ideal transformer can be used in two different ways in either the T-equivalent or the π-equivalent circuit. Figure C.12 shows the two arrangements for each type of equivalent circuit. To verify any of the equivalent circuits in Fig. C.12, we must show that the equations relating v1 and v2 to di1/dt and di2/dt are identical to Eqs. C.1 and C.2. To demonstrate, we validate the circuit shown in Fig. C.12(a); we leave it to you to verify the

circuits in Figs. C.12(b), (c), and (d).

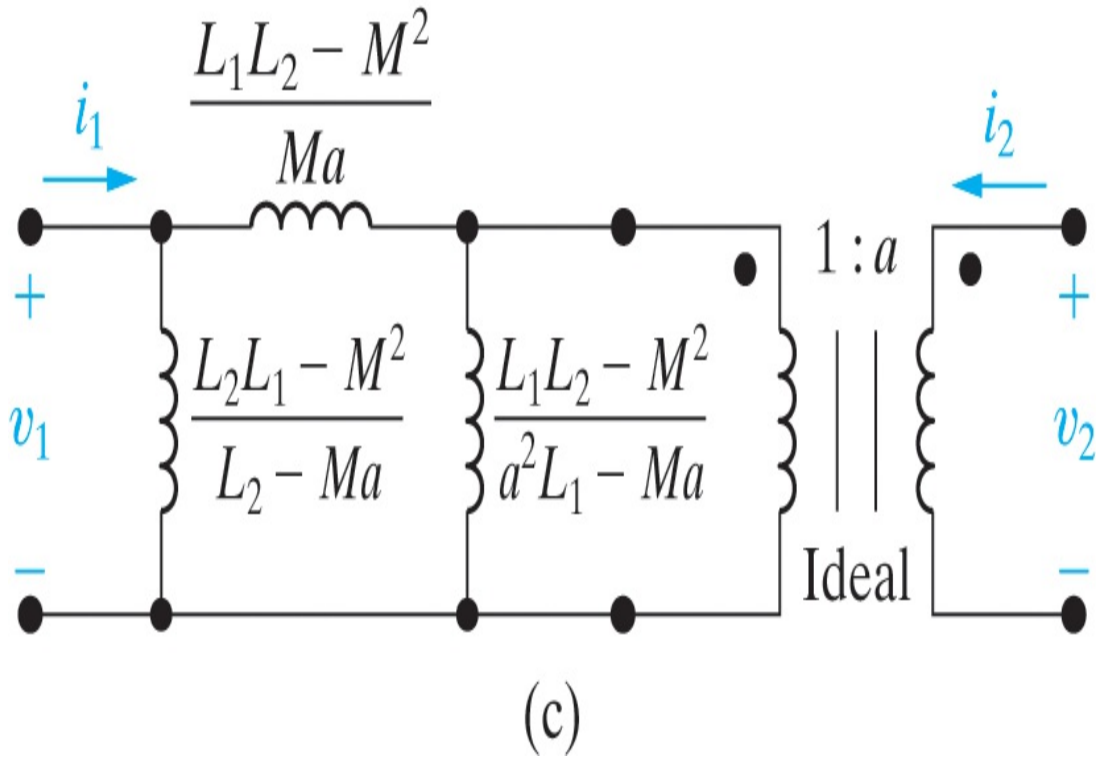
Figure C.12 The four ways of using an ideal transformer in the T- and π-equivalent circuit for magnetically coupled coils.



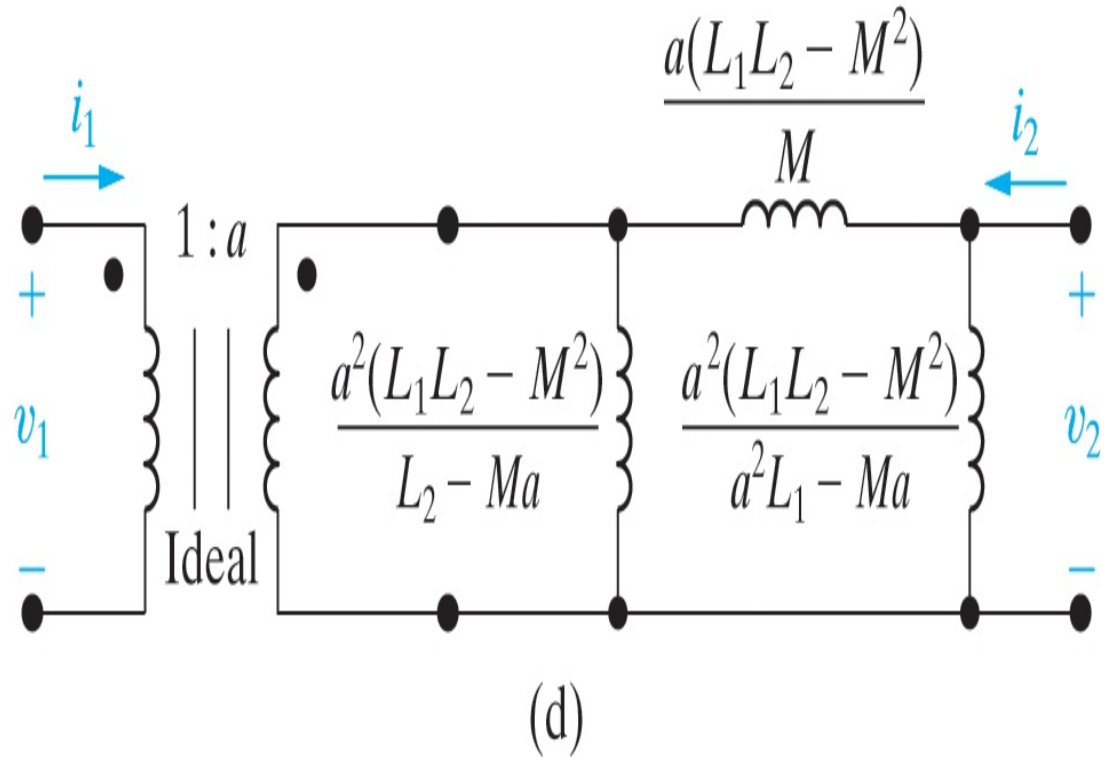
C.2-1 Full Alternative Text



C.2-1 Full Alternative Text



C.2-1 Full Alternative Text



C.2-1 Full Alternative Text

We redrew the circuit shown in Fig. C.12(a) as Fig. C.13, adding the variables i0 and v0 to aid the discussion. From this circuit,

Figure C.13 The circuit of Fig. C.12(a) with i0 and v0 defined.

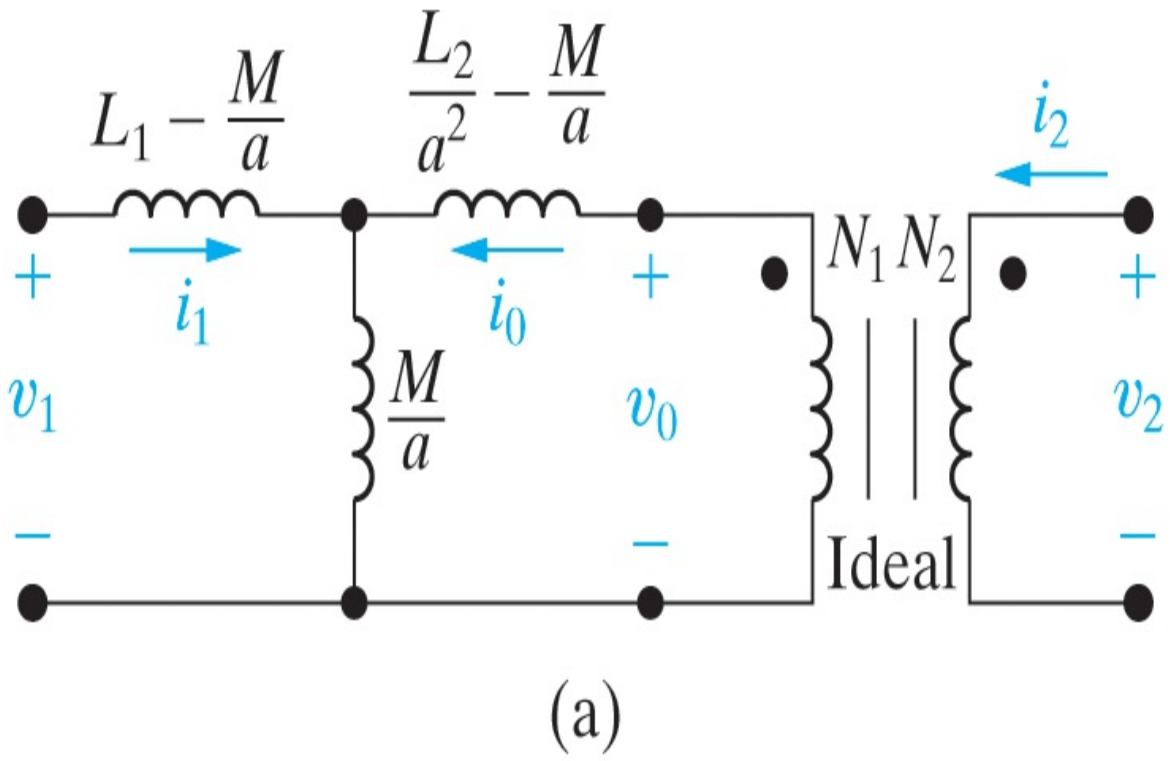


Figure C.13 Full Alternative Text

v1=(L1 − Ma) di1dt + Ma ddt (i1 + i0)

and

v0=(L2a2 − Ma) di0dt + Ma ddt (i0 + i1).

The ideal transformer imposes constraints on v0 and i0 :

v0=v2a;
i0=ai2.

Substituting the constraint equations into the expressions for v1 and v0 from the circuit gives

v1=L1di1dt + Ma ddt(ai2)

and

v2a=L2a2 ddt(ai2) + Ma di1dt.

Simplifying, we get,

$v1=L1di1dt + Mdi2dt$

and

$v2=Mdi1dt + L2di2dt.$

These expressions for v1 and v2 are identical to Eqs. C.1 and C.2. Thus, the circuit shown in Fig. C.13 is equivalent to the magnetically coupled coils shown inside the box in Fig. C.1 because the terminal behavior of these two circuits is the same.

When we showed that the circuit in Fig. C.13 is equivalent to the magnetically coupled coils in Fig. C.1, we placed no restrictions on the turns ratio a. Therefore, an infinite number of equivalent circuits are possible. However, we can always find a turns ratio that makes all of the inductances positive. Two values of a are of particular interest:

$a=ML1,$ (C.7)

and

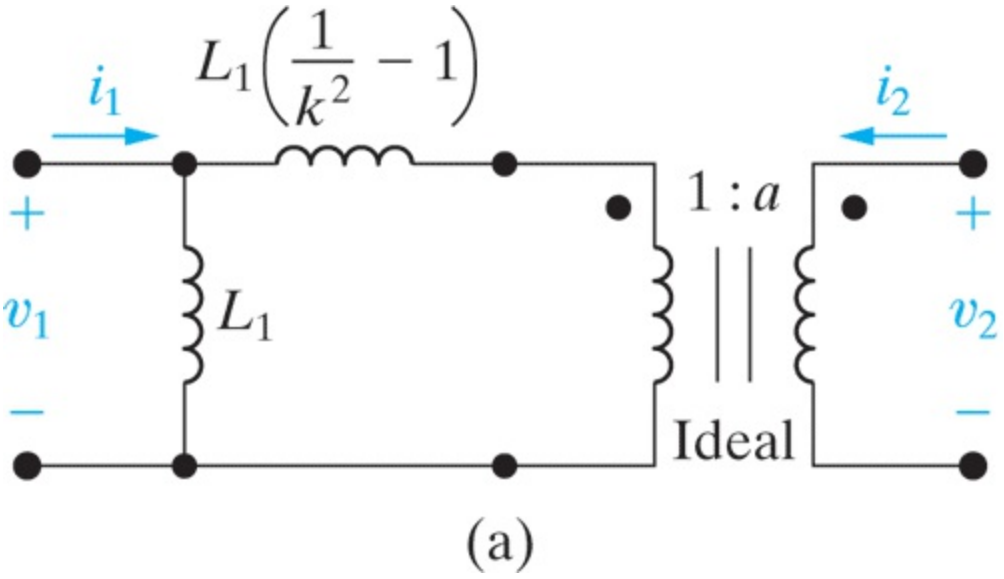
$a=L2M,$ (C.8)

The value of a given by Eq. C.7 eliminates the inductances $L1 − M/a$ and $a2L1−aM$ from the T-equivalent circuits and the inductances $(L1L2 − M2)/(a2L1 − aM)$ and $a2(L1L2−M2)/(a2L1−aM)$ from the π-equivalent circuits. The value of a given by Eq. C.8 eliminates the inductances $(L2/a2) − (M/a)$ and $L2−aM$ from the T-equivalent circuits and the inductances $(L1L2−M2)/(L2−aM)$ and $a2(L1L2−M2)/(L2−aM)$ from the π-equivalent circuits.

Also note that when $a=M/L1$, the circuits in Figs. C.12(a) and (c) are identical, and when $a=L2/M$, the circuits in Figs. C.12(b) and (d) are identical. Figures C.14 and C.15 summarize these observations. We can use the relationship $M=kL1L2$ to derive the inductor values in Figs. C.14 and C.15 as functions of the self-inductances L1 and L2, and the coupling

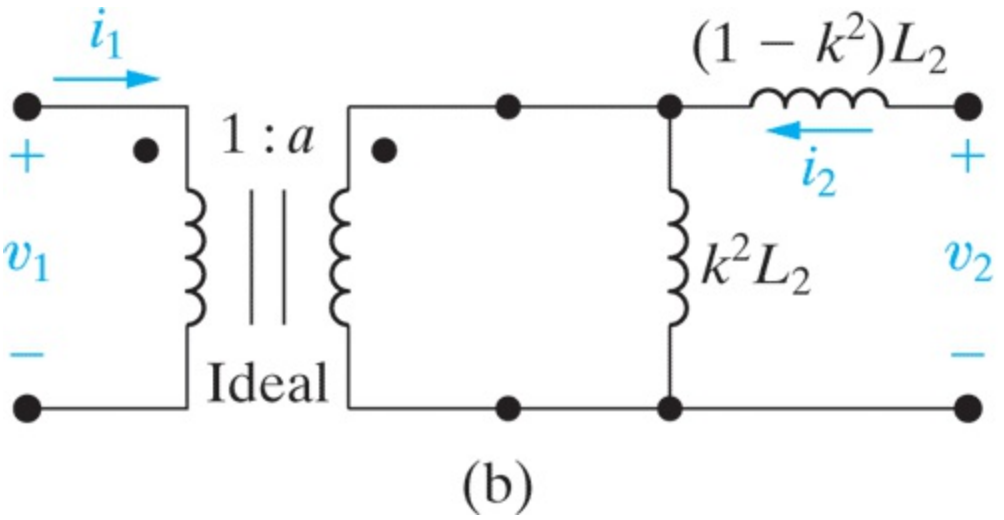
coefficient k. Then, the values of a given by Eqs. C.7 and C.8 will reduce the number of inductances needed in the equivalent circuit and guarantee that all the inductances will be positive.

Figure C.14 Two equivalent circuits when a = M/L1.



(a)

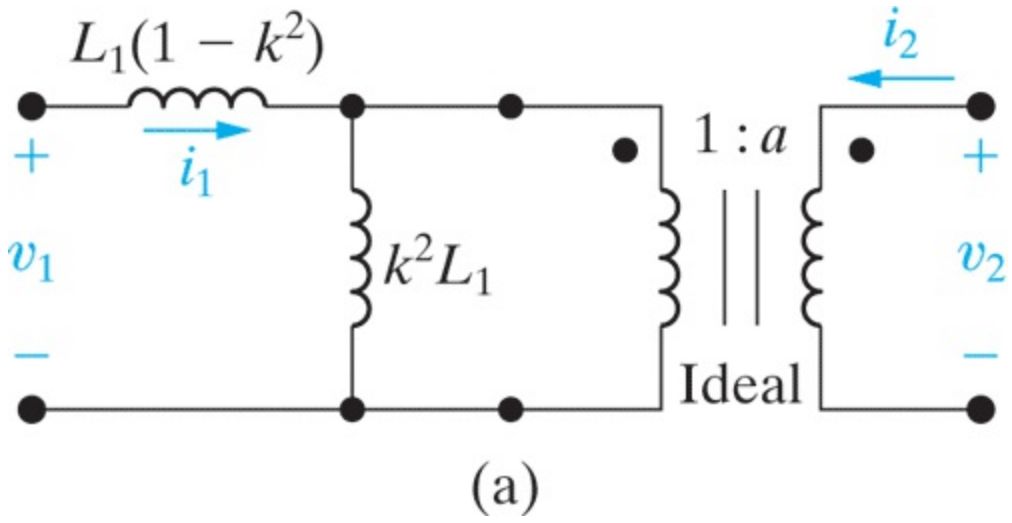
C.2-1 Full Alternative Text



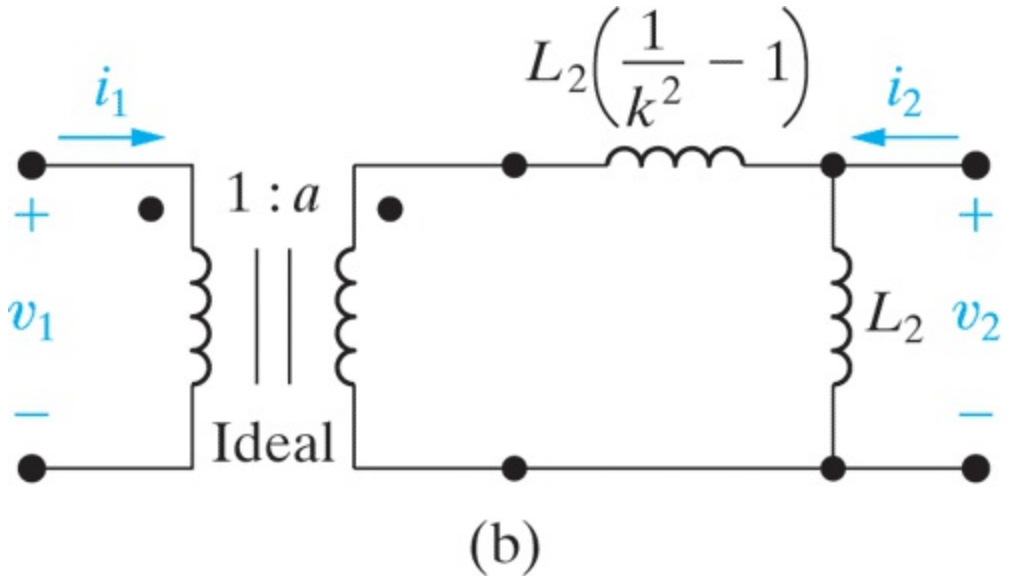
(b)

C.2-1 Full Alternative Text

Figure C.15 Two equivalent circuits when a = L2/M.



C.2-1 Full Alternative Text



C.2-1 Full Alternative Text

The values of a given by Eqs. C.7 and C.8 can be determined experimentally from the magnetically coupled coils. To find the ratio M/L1, drive the coil with N1 turns using a sinusoidal voltage source and leave the N2 coil open.

Use a source frequency that guarantees ωL1≫R1. Figure C.16 shows the resulting circuit.

Figure C.16 Experimental determination of the ratio M/L1.

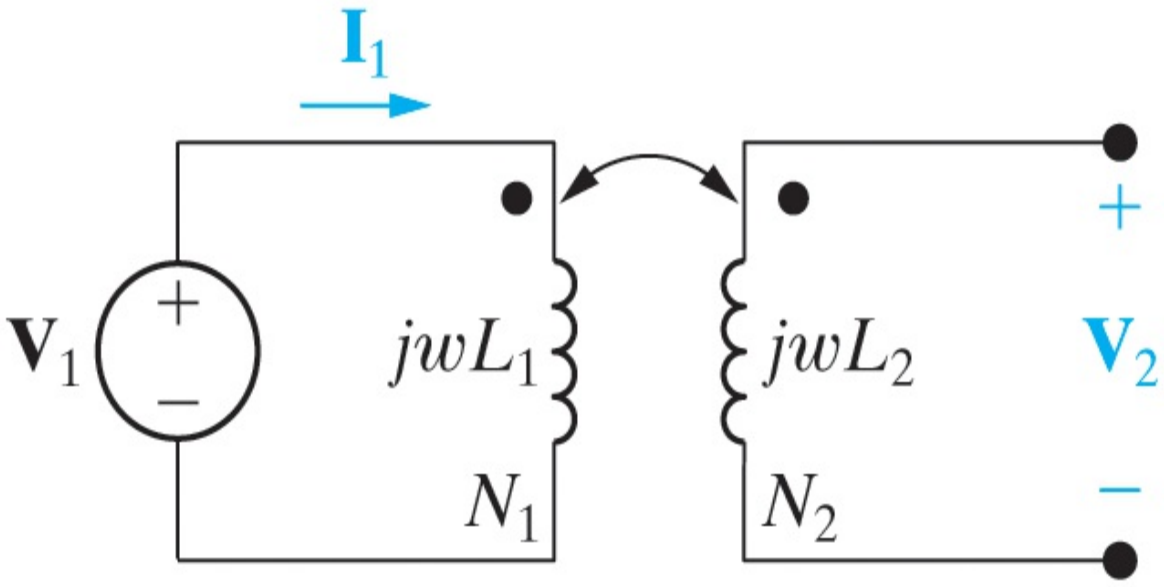


Figure C.16 Full Alternative Text

Because the N2 coil is open,

V2=jωMI1.

Since ωL1≫R1, the current I1 is

I1=V1jωL1.

Substituting the expression for I1 into the equation for V2 and rearranging yields

(V2V1)I2=0=ML1.

Thus, the ratio M/L1 equals the ratio of the terminal voltages when I2=0.

To experimentally determine the ratio L2/M, reverse the procedure; that is, coil 2 is energized and coil 1 is left open. Then

L2M=(V2V1)I1=0.

Example C.2 illustrates how to replace magnetically coupled coils with an equivalent circuit that includes an ideal transformer and positive inductor values.

Example C.2

Consider the circuit we analyzed in part (b) of Example C.1, which replaced the magnetically coupled coils in Fig. C.6 (with the polarity dot on the secondary winding moved to the bottom terminal) with a T-equivalent circuit. The equivalent circuit is shown in Fig. C.11 and must use a capacitor to represent the negative reactance. Repeat the analysis in Example C.1(b). but now replace the magnetically coupled coils with the equivalent circuit in Fig. C.14(a).

Solution

From Example C.1(b) we know that L1=9 H, L2=4 H, and M=−3 H. We need to calculate the coupling coefficient k, the turns ratio a, and the inductor values for the equivalent circuit in Fig. C.14(a):

k=ML1L2=3(9)(4)=0.5,
a=ML1=−39=−13,
L1(1k2−1)=9(10.52−1)=27H.

Note that in calculating the coupling coefficient, we ignore the sign of the mutual inductance because this sign represents the location of the polarity marks and has no effect on the amount of coupling between the coils.

Using the values we calculated, we can replace the magnetically coupled coils in Fig. C.6 with the equivalent circuit in Fig. C.14(a). Remember that the source frequency is 400 rad/sec. The result is the circuit in Fig. C.17. We have defined the currents and voltages for the ideal transformer, so we can write the equations for this circuit. There are three meshes, so we will write three KVL equations using the mesh currents I1, Ia, and Ib. Then we use the dot convention for ideal transformers to write the equations relating the phasor currents Ia and Ib, and the phasor voltages Va and Vb:

Figure C.17 The circuit of Fig. C.6, with the magnetically coupled coils replaced by the equivalent circuit in Fig. C.14(a)

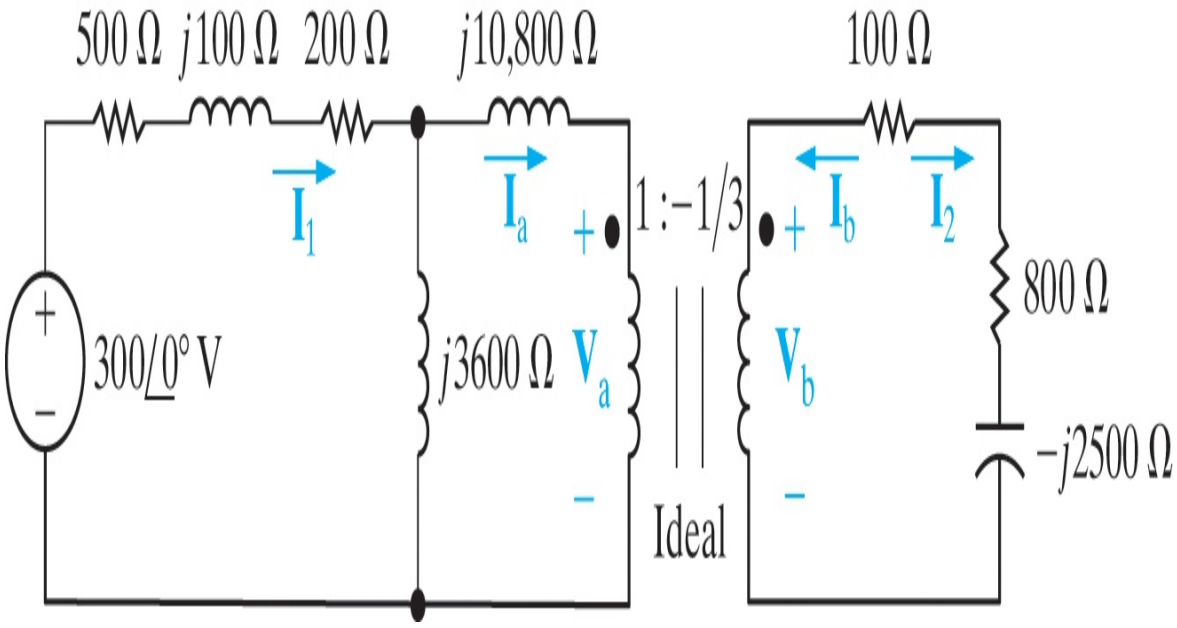


Figure C.17 Full Alternative Text

(700+j3700)I1−j3600Ia−300=0,−j3600I1−j14,400Ia+Va=0,

$(900-j2500)Ib+Vb=0, Va=Vb-(1/3), Ia=13Ib.$

Solving for I1 and Ib,

$I1=0.02-j0.06=63.25\angle-71.57°mA, Ib=0.0267+j0.0533=59.63\angle63.43°mA.$

Because $I2=-Ib$,

$I2=-0.0267-j0.0533=59.63\angle-116.57°mA$.

The values for I1 and I2 match those found in part (b) of Example C.1.

Appendix D The Decibel

The decibel was introduced by telephone engineers to characterize the power loss across the cascaded circuits used to transmit telephone signals. Figure D.1 defines the problem.

Figure D.1 Three cascaded circuits.

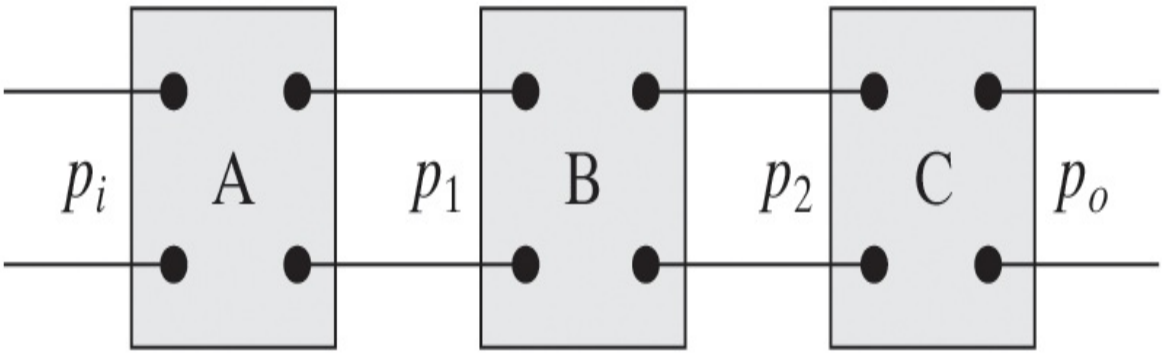


Figure D.1 Full Alternative Text

There, pi is the power input to the system, p1 is the power output of circuit A, p2 is the power output of circuit B, and po is the power output of the system. The power gain of each circuit is the ratio of the power out to the power in. Thus

σA=p1pi, σB=p2p1, and σC=pop2.

The overall power gain of the system is the product of the individual gains, or

popi=p1pi p2p1 pop2=σAσBσC.

We can replace the multiplication of power ratios with addition if we use the logarithm; that is,

log10popi=log10σA+log10σB+log10σC.

The log of a power ratio was named the **bel**, in honor of Alexander Graham Bell. Thus, we calculate the overall power gain, in bels, by summing the power gains, also in bels, of each segment of the transmission system. In practice, the bel is an inconveniently large quantity. One-tenth of a bel is a more useful measure of power gain—hence the **decibel**. The number of decibels equals 10 times the number of bels, so

Number of decibels=10 log10popi.

When we use the decibel as a measure of power ratios, in some situations the resistance seen looking into the circuit equals the resistance loading the circuit, as illustrated in Fig. D.2. When the input resistance equals the load resistance, we can convert the power ratio to either a voltage ratio or a current ratio:

p o p i = υ out 2 / R L υ in 2 / R in = (υ out υ in) 2

or

p o p i = i out 2 / R L i in 2 / R in = (i out i in) 2

Figure D.2 A circuit in which the input resistance equals the load resistance.

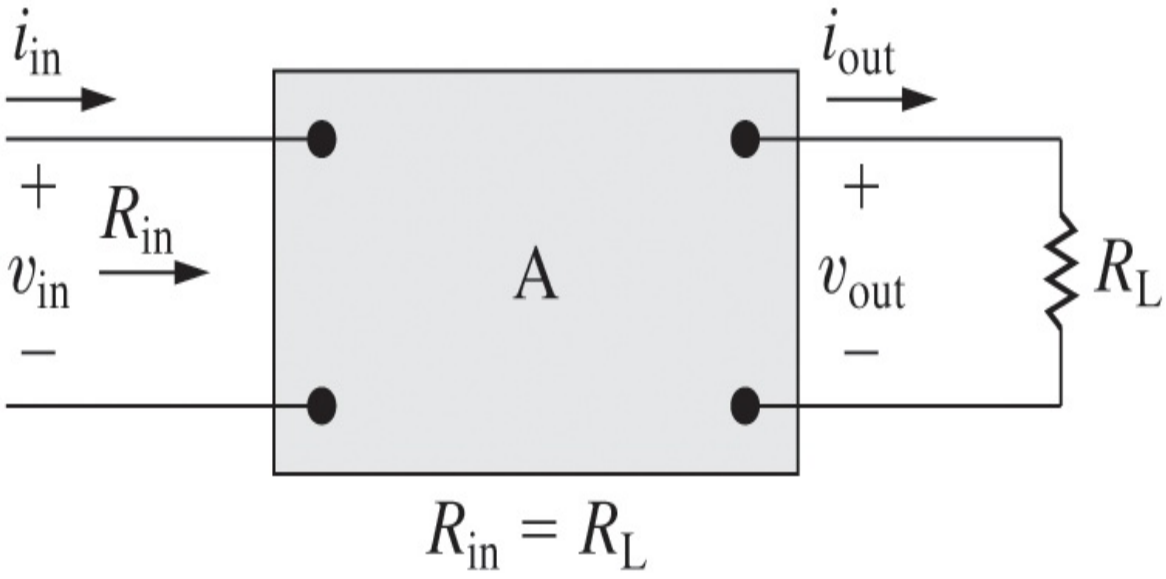


Figure D.2 Full Alternative Text

These equations show that the number of decibels becomes

Number of decibels=20 log10υoutυin =20 log10ioutiin. (D.1)

The definition of the decibel used in Bode diagrams (see Appendix E) is borrowed from the results expressed by Eq. D.1, since these results apply to any transfer function involving a voltage ratio, a current ratio, a voltage-to-current ratio, or a current-to-voltage ratio. You should keep the original definition of the decibel firmly in mind because it is of fundamental importance in many engineering applications.

When you are working with transfer function amplitudes expressed in decibels, having a table that translates the decibel value to the actual value of the output/input ratio is helpful. Table D.1 gives some useful pairs. The ratio corresponding to a negative decibel value is the reciprocal of the positive ratio. For example, −3 dB corresponds to an output/input ratio of 1/1.41, or 0.707. Interestingly, −3 dB corresponds to the half-power frequencies of the filter circuits discussed in Chapters 14 and 15.

Table D.1 Some dB-Ratio Pairs

dB	Ratio	dB	Ratio
0	1.00	30	31.62
3	1.41	40	100.00
6	2.00	60	10^3
10	3.16	80	10^4
15	5.62	100	10^5
20	10.00	120	10^6

The decibel is also used as a unit of power when it expresses the ratio of a known power to a reference power. Usually, the reference power is 1 mW, and the power unit is written dBm, which stands for “decibels relative to one milliwatt.” For example, a power of 20 mW corresponds to ±13 dBm.

AC voltmeters commonly provide dBm readings that assume not only a 1 mW reference power but also a 600 Ω reference resistance (a value commonly used in telephone systems). Since a power of 1 mW in 600 Ω corresponds to 0.7746 V (rms), that voltage is read as 0 dBm on the meter. For analog meters, there usually is exactly a 10 dB difference between adjacent ranges. Although the scales may be marked 0.1, 0.3, 1, 3, 10, and so on, in fact 3.16 V on the 3 V scale lines up with 1 V on the 1 V scale.

Some voltmeters provide a switch to choose a reference resistance (50, 135, 600, or 900 Ω) or to select dBm or dBV (decibels relative to one volt).

Appendix E Bode Diagrams

The frequency response plot is a very important tool for analyzing a circuit's behavior. Up to this point, we have shown qualitative sketches of the frequency response without discussing how to create these plots. The best way to generate and plot the amplitude and phase data is to use a computer; we can rely on it to give us accurate numerical plots of $|H(j\omega)|$ and $\theta(j\omega)$ versus ω. But we can create preliminary sketches of the frequency response using Bode diagrams.

A Bode diagram is a graphical technique that approximates a system's frequency response. These diagrams are named to recognize the pioneering work of H. W. Bode.[1] They are most useful for systems whose transfer function poles and zeros are reasonably well separated.

[1]See H. W. Bode, *Network Analysis and Feedback Design* (New York: Van Nostrand, 1945).

A Bode diagram consists of two separate plots: One shows how the transfer function amplitude varies with frequency, and the other shows how the transfer function phase angle varies with frequency. The Bode diagram plots are constructed using semilog graph paper to accommodate a wide range of frequencies. In both the amplitude and phase plots, the frequency is plotted on the horizontal log scale. The amplitude and phase angle are plotted on the linear vertical scale.

E.1 Real, First-Order Poles and Zeros

First, we consider a transfer function, H(s), with poles and zeros that are real and distinct. We introduce the procedure for constructing a Bode diagram using

$$H(s)=K(s+z1)s(s+p1),$$

from which

$$H(j\omega)=K(j\omega+z1)j\omega(j\omega+p1).$$

To begin, put the expression for H(jω) in a **standard form**, which we derive by dividing out the poles and zeros:

$$H(j\omega)=Kz1(1+j\omega/z1)p1(j\omega)(1+j\omega/p1).$$

Next we let Ko equal Kz1/p1, and express H(jω) in polar form:

$$H(j\omega)= K o \mid 1+j\omega/ z 1 \mid \psi 1 (\mid \omega \mid 90°)(\mid 1+j\omega/ p 1 \mid \beta 1)$$

$$= K o \mid 1+j\omega/ z 1 \mid \mid \omega \parallel 1+j\omega/ p 1 \mid (\psi 1 -90°- \beta 1) .$$

Therefore,

$$|H(j\omega)|=Ko|1+j\omega/z1|\omega|1+j\omega/p1|,(E.1)$$
$$\theta(j\omega)=\psi1-90°-\beta1.(E.2)$$

By definition, the phase angles ψ1 and β1 are

$$\psi1= \tan^{-1}\omega/z1;\beta1= \tan^{-1}\omega/p1.$$

The Bode diagram includes the plots of Eq. E.1 (amplitude) and Eq. E.2 (phase) as functions ofω.

E.2 Straight-Line Amplitude Plots

Notice from Eq. E.1 that the amplitude plot requires us to multiply and divide values associated with the poles and zeros of H(s). We can transform the multiplication and division into addition and subtraction if we express $|H(j\omega)|$ as a logarithmic value: the decibel (dB).[2] The amplitude of $H(j\omega)$ in decibels is

[2]See Appendix D for more information regarding the decibel.

$$A_{dB}=20\log_{10}|H(j\omega)|.$$

Expressing Eq. E.1 in terms of decibels gives

$$A_{dB} = 20\log_{10} K_o \left|1+j\omega/z_1\right| \omega \left|1+j\omega/p_1\right| = 20\log_{10} K_o + 20\log_{10}\left|1+j\omega/z_1\right| - 20\log_{10}\omega - 20\log_{10}\left|1+j\omega/p_1\right|. \quad (E.3)$$

To construct a plot of Eq. E.3 versus frequency ω, we plot each term in the equation separately and then combine the separate plots graphically. The terms in Eq. E.3 are easy to plot because they can be approximated by straight lines.

The plot of $20\log_{10}K_o$ is a horizontal straight line because K_o is not a function of frequency. The value of this term is positive for $K_o>1$, zero for $K_o=1$, and negative for $K_o<1$.

Two straight lines approximate the plot of $20\log_{10}|1+j\omega/z_1|$. For small values of ω, the magnitude $|1+j\omega/z_1|$ is approximately 1, and therefore

$$20\log_{10}|1+j\omega/z_1| \to 0 \quad \text{as } \omega \to 0.$$

For large values of ω, the magnitude $|1 + j\omega/z_1|$ is approximately ω/z_1, and therefore

$$20\log_{10}|1+j\omega/z_1| \to 20\log_{10}(\omega/z_1) \text{ as } \omega \to \infty.$$

On a log frequency scale, 20 log10(ω/z1) is a straight line with a slope of 20 dB/decade (a decade is a 10-to-1 change in frequency). This straight line intersects the 0 dB axis at ω=z1. This value of ω is called the **corner frequency**. Thus, two straight lines can approximate the amplitude plot of a first-order zero, as shown in Fig. E.1.

Figure E.1 A straight-line approximation of the amplitude plot of a first-order zero.

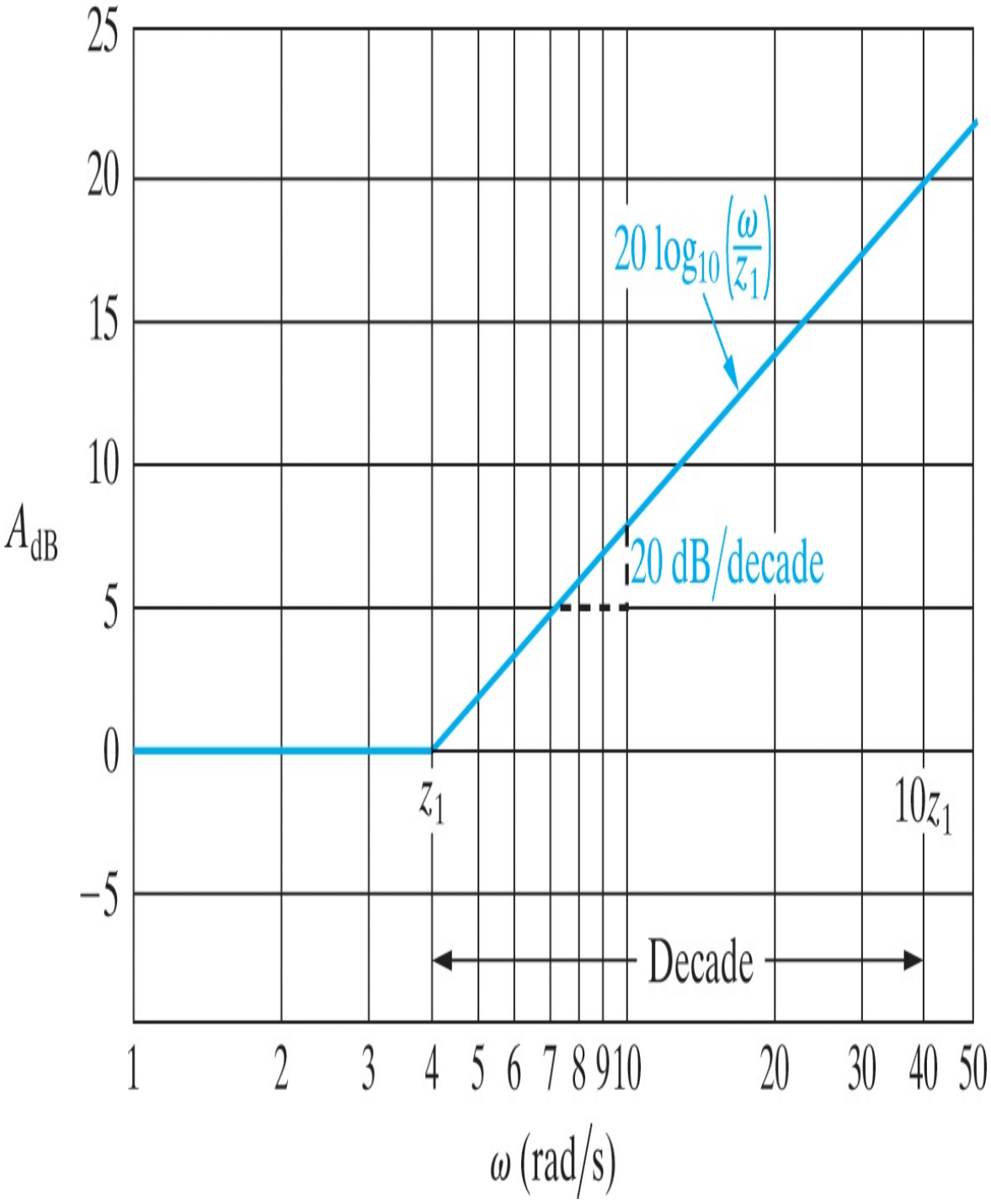


Figure E.1 Full Alternative Text

The plot of −20 log10ω is a straight line having a slope of −20 dB/decade that intersects the 0 dB axis at ω=1. Two straight lines approximate the plot of −20 log10|1+jω/p1| and intersect on the 0 dB axis at ω=p1. For large values

of ω, the straight line 20 log10(ω/p1) has a slope of −20 dB/decade. Figure E.2 shows the straight-line approximation of the amplitude plot for a first-order pole.

Figure E.2 A straight-line approximation of the amplitude plot of a first-order pole.

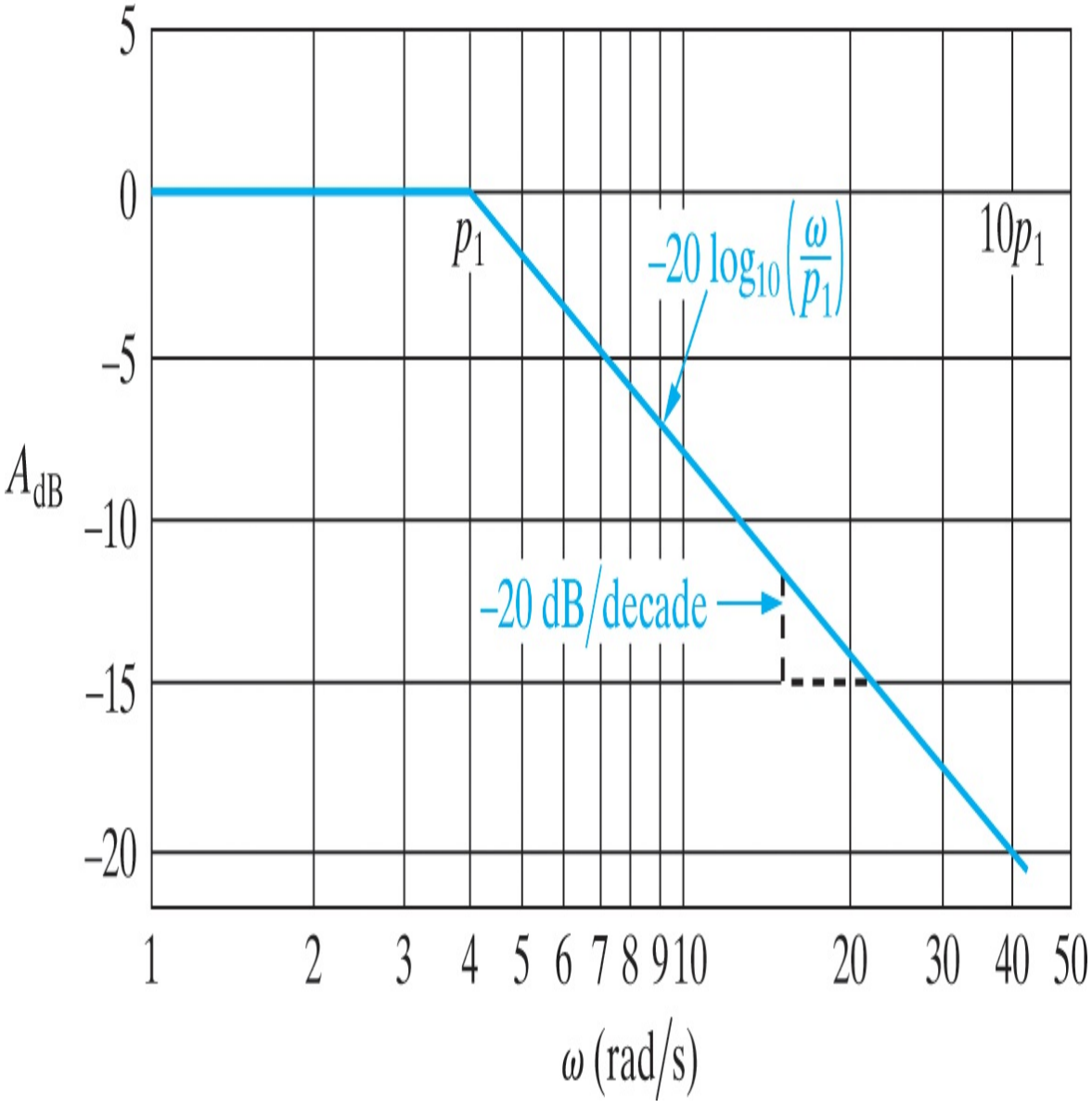


Figure E.2 Full Alternative Text

Figure E.3 shows a plot of Eq. E.3 for Ko=10, z1=0.1 rad/s, and p1=5 rad/s. Each term in Eq. E.3 is also plotted in Fig. E.3, so you can verify that the individual terms sum to create the plot of the transfer function's amplitude, labeled 20 log10|H(jω)|.

Figure E.3 A straight-line approximation of the amplitude plot for Eq. E.3.

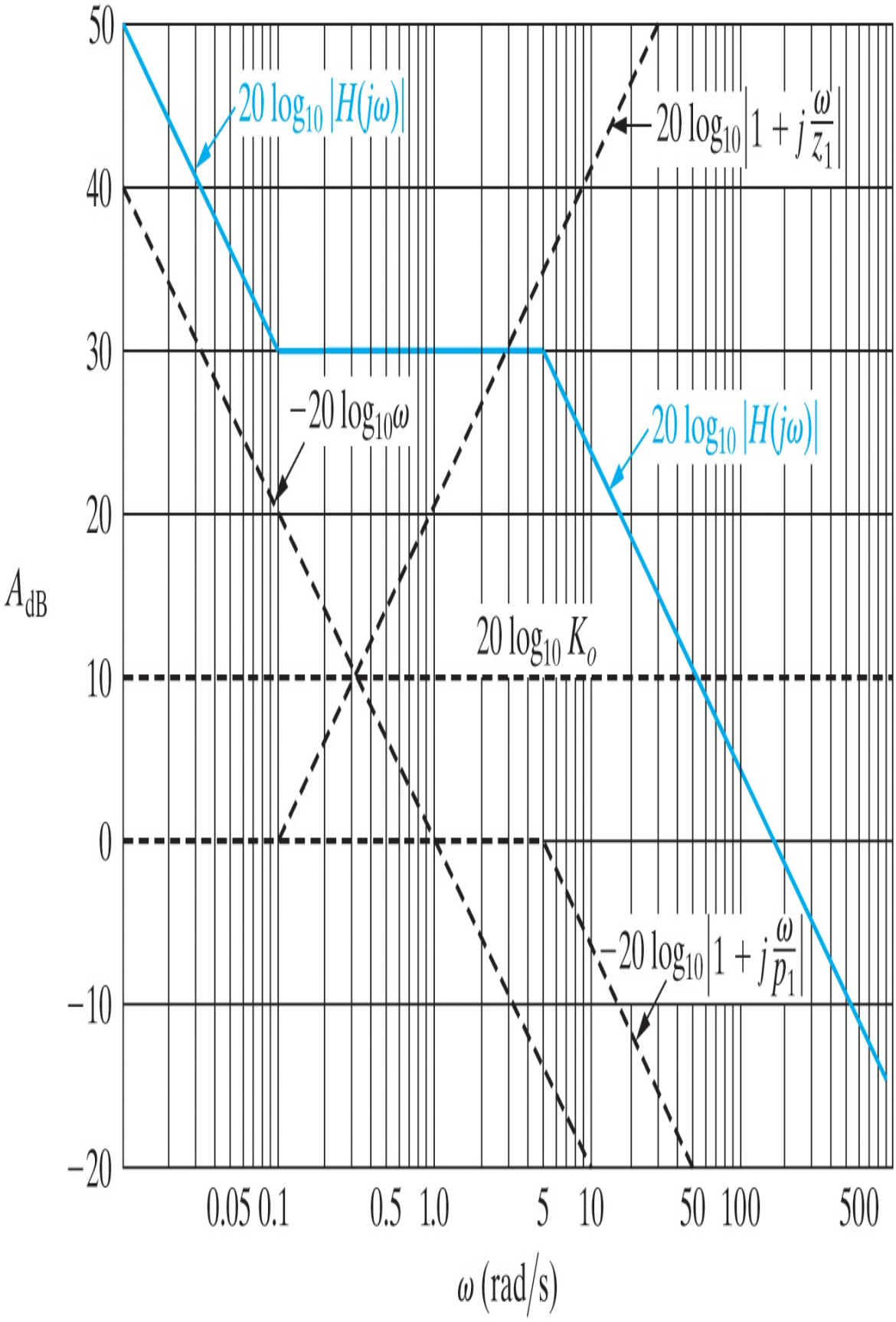

50
40
30
20
10
0
−10
−20
A_{dB}
$20 \log_{10} |H(j\omega)|$
$-20 \log_{10} \left|1 + j\frac{\omega}{z_1}\right|$
$-20 \log_{10}\omega$
$20 \log_{10} |H(j\omega)|$
$20 \log_{10} K_o$
$-20 \log_{10} \left|1 + j\frac{\omega}{p_1}\right|$
0.05 0.1 0.5 1.0 5 10 50 100 500
ω (rad/s)

Figure E.3 Full Alternative Text

Example E.1 constructs a straight-line amplitude plot for a transfer function with first-order poles and zeros.

Example E.1

For the circuit in Fig. E.4:

Figure E.4 The circuit for Example E.1.

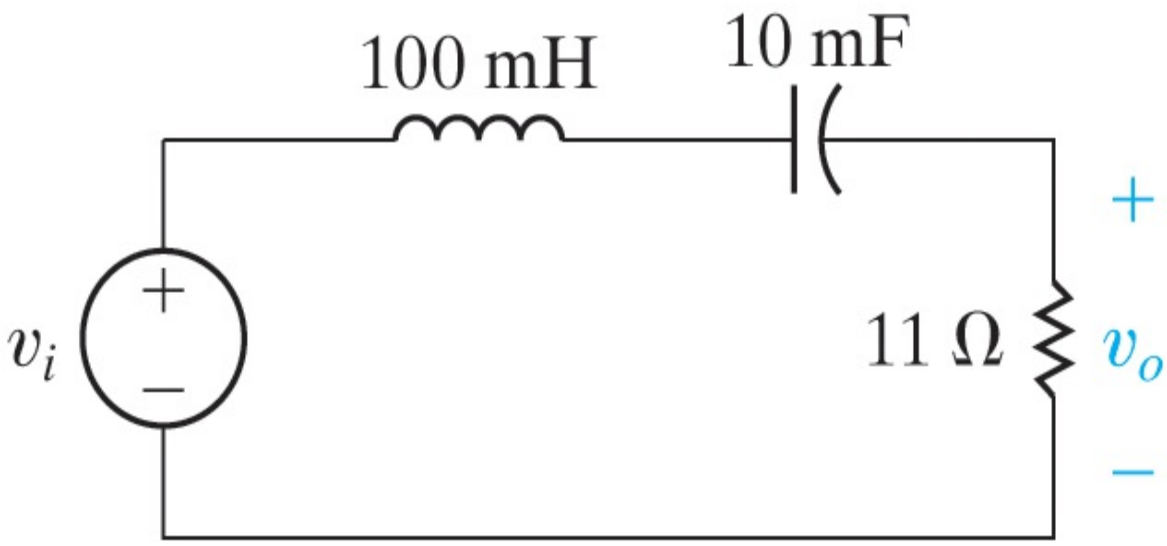


Figure E.4 Full Alternative Text

1. Find the transfer function, H(s)=Vo(s)/Vi(s).
2. Construct a straight-line approximation of the Bode amplitude plot.
3. Calculate 20 log10|H(jω)| at ω=50 rad/s and ω=1000 rad/s.Identify these two values on the plot you constructed in (b).

4. If υi(t)=5 cos (500t+15°) V, use the Bode plot you constructed to predict the amplitude of υo(t) in the steady state.

Solution

1. Transform the circuit in Fig. E.4 into the s-domain and then use voltage division to get

   H(s)=(R/L)ss2+(R/L)s+1LC .

   Substituting the numerical values from the circuit, we get

   H(s)=110ss2+110s+1000=110s(s+10)(s+100).

2. Write H(jω) in standard form:

   H(jω)=0.11jω[1+j(ω/10)][1+j(ω/100)].

   The expression for the amplitude of H(jω) in decibels is

   $$A\ dB = 20\ \log 10\ |H(j\omega)| = 20\ \log 10\ 0.11 + 20\ \log 10\ |j\omega| - 20\ \log 10\ |1+j\ \omega\ 10| - 20\ \log 10\ |1+j\ \omega\ 100|.$$

   Figure E.5 shows the straight-line plot. Each term contributing to the overall amplitude is also plotted and identified.

Figure E.5 The straight-line amplitude plot for the transfer function of the circuit in Figure E.4.

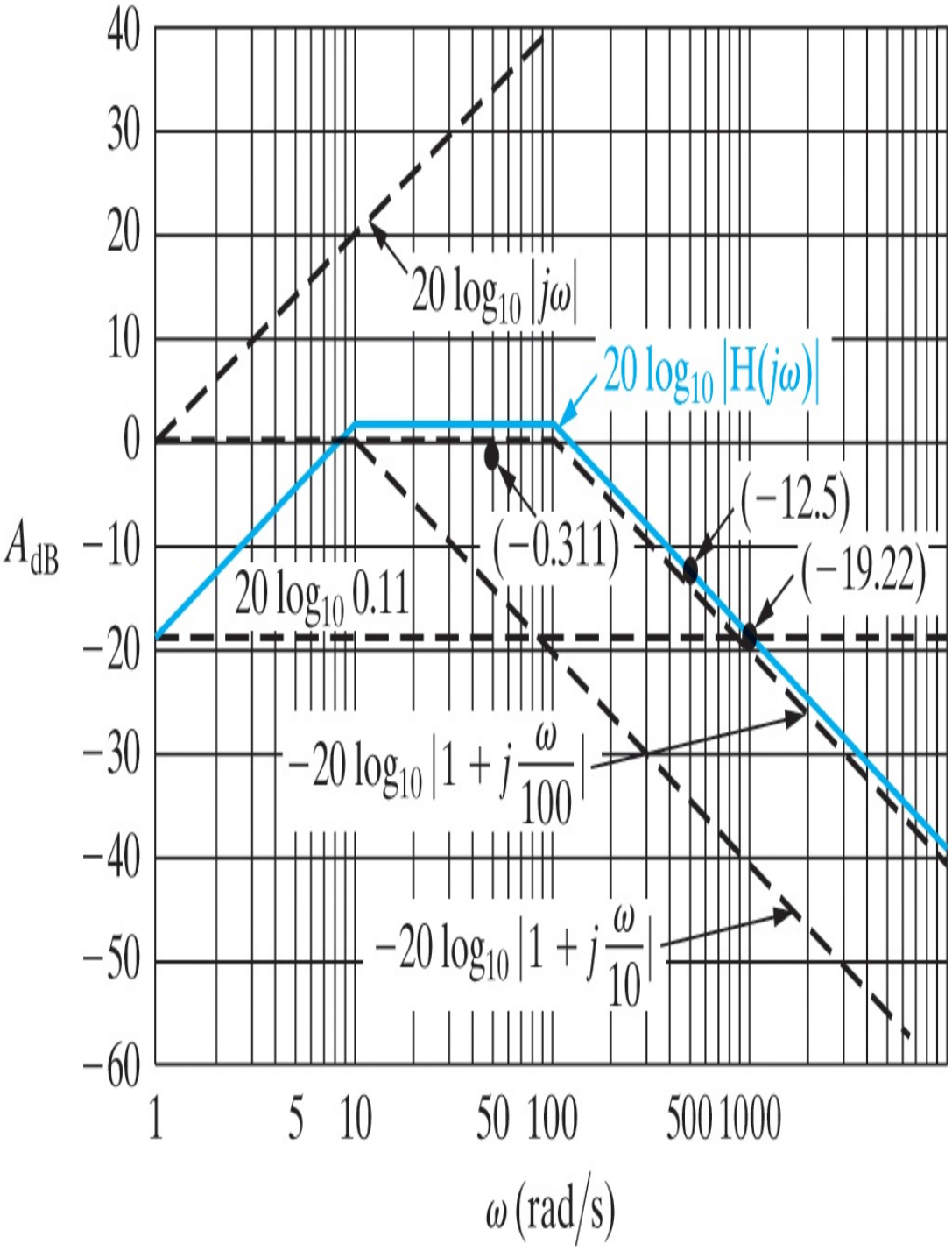


Figure E.5 Full Alternative Text

3. We have

H(j50)=0.11(j50)(1+j5)(1+j0.5) =0.9648 −15.25°

, $20 \log_{10}|H(j50)| = 20 \log_{10} 0.9648 = -0.311$ dB;
$H(j1000) = 0.11(j1000)(1+j100)(1+j10) = 0.1094 \angle -83.72°$; $20 \log_{10} 0.1094 = -19.22$ dB.

These two magnitude values are plotted in Fig. E.5.

4. The frequency of vi is 500 rad/sec. As we can see from the Bode plot in Fig. E.5, the value of AdB at $\omega = 500$ rad/s is approximately −12.5 dB. Therefore,

   $|H(j500)| = 10^{(-12.5/20)} = 0.24$

   and

   $V_o = |H(j500)| V_i = (0.24)(5) = 1.19$ V.

   To calculate the actual value of $|H(j\omega)|$, substitute $\omega = 500$ into the equation for $|H(j\omega)|$:

   $H(j500) = 0.11(j500)(1+j50)(1+j5) = 0.22 \angle -77.54°$.

   Thus, the actual output voltage magnitude for the specified signal source at a frequency of 500 rad/s is

   $V_o = |H(j500)| V_i = (0.22)(5) = 1.1$ V.

E.3 More Accurate Amplitude Plots

We can make the straight-line plots for first-order poles and zeros more accurate by correcting the amplitude values at the corner frequency, one-half the corner frequency and twice the corner frequency. At the corner frequency, the actual value in decibels is

$$A_{dBc} = \pm 20 \log_{10}|1+j1| = \pm 20 \log_{10} 2 \approx \pm 3 \text{ dB}.$$

The actual value at one-half the corner frequency is

$$A_{dBc/2} = \pm 20 \log_{10}|1+j\tfrac{1}{2}| = \pm 20 \log_{10}\tfrac{5}{4} \approx \pm 1 \text{ dB}.$$

At twice the corner frequency, the actual value in decibels is

$$A_{dB2c} = \pm 20 \log_{10}|1 + j2| = \pm 20 \log_{10} 5 \approx \pm 7 \text{ dB}.$$

In these three equations, the plus sign applies to a first-order zero, and the minus sign applies to a first-order pole. The straight-line approximation of the amplitude plot is 0 dB at both the corner frequency and at one-half the corner frequency, and is ±6 dB at twice the corner frequency. Hence, the corrections are ±3 dB at the corner frequency and ±1 dB at both one-half the corner frequency and twice the corner frequency. Figure E.6 summarizes these corrections.

Figure E.6 Corrected amplitude plots for a first-order zero and pole.

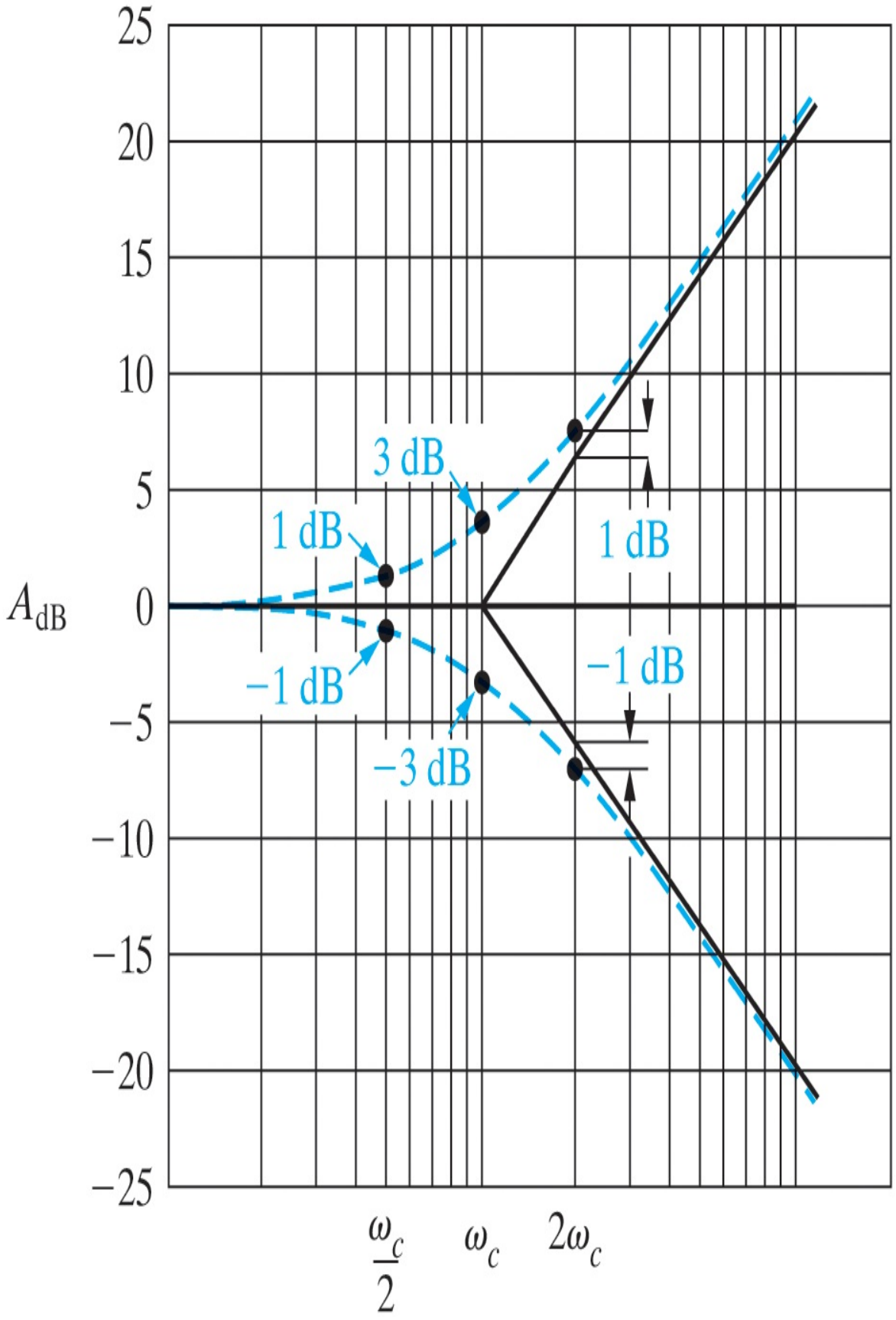

25
20
15
10
5
0
−5
−10
−15
−20
−25
A_{dB}
3 dB
1 dB
1 dB
−1 dB
−1 dB
−3 dB
$\frac{\omega_c}{2}$
ω_c
$2\omega_c$

Figure E.6 Full Alternative Text

If the poles and zeros of H(s) are widely separated, you can include these corrections into the overall amplitude plot and produce a reasonably accurate curve. However, if the poles and zeros are close together, the overlapping corrections are difficult to evaluate, so estimating the amplitude characteristic using the uncorrected straight-line plot is a better choice. Use a computer to create an accurate amplitude plot in the frequency range of interest.

E.4 Straight-Line Phase Angle Plots

We can also use straight-line approximations to plot the transfer function phase angle versus frequency. Using the transfer function phase angle in Eq. E.2, we know that the phase angle associated with the constant Ko is zero, and the phase angle associated with a first-order zero or pole at the origin is a constant ±90°. For a first-order zero or pole not at the origin, the straight-line approximations are as follows:

- For frequencies less than one-tenth the corner frequency, the phase angle is approximated by zero.
- For frequencies greater than 10 times the corner frequency, the phase angle is approximated by ±90°.
- Between one-tenth the corner frequency and 10 times the corner frequency, the phase angle plot is a straight line with the value 0° at one-tenth the corner frequency, ±45° at the corner frequency, and ±90° at 10 times the corner frequency.

Throughout, the plus sign applies to the first-order zero and the minus sign to the first-order pole. Figure E.7 depicts the straight-line approximation for a first-order zero and pole. The dashed curves show the exact plot of the phase angle versus frequency. Note how closely the straight-line plot approximates the actual phase angle plot. The maximum deviation between the straight-line plot and the actual plot is approximately 6°.

Figure E.7 Phase angle plots for a first-order zero and pole.

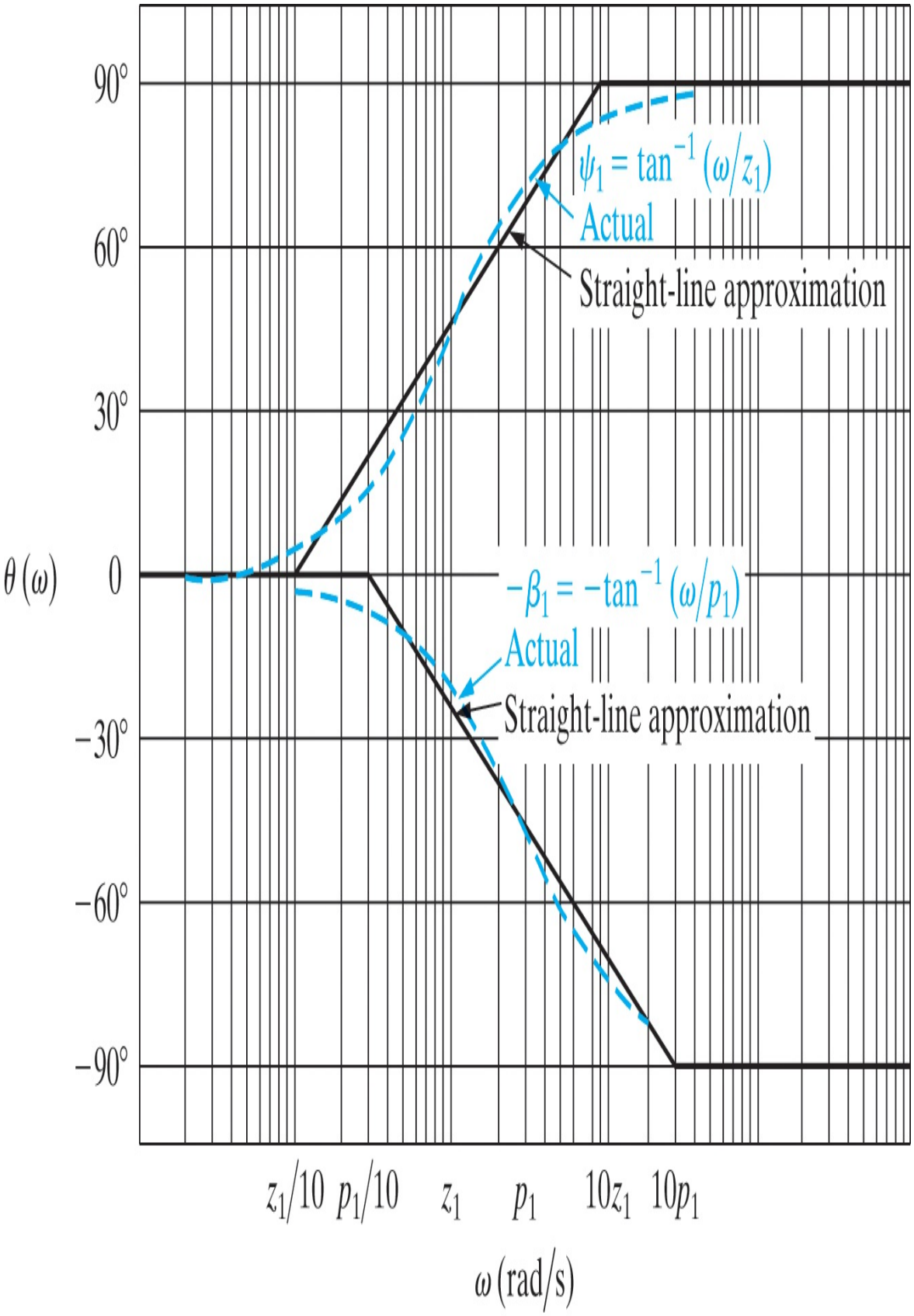

90°
60°
30°
0
−30°
−60°
−90°
θ(ω)
ψ1 = tan−1 (ω/z1)
Actual
Straight-line approximation
−β1 = −tan−1 (ω/p1)
Actual
Straight-line approximation
z1/10
p1/10
z1
p1
10z1
10p1
ω (rad/s)

Figure E.7 Full Alternative Text

We construct a phase angle plot using a straight-line approximation in Example E.2.

Example E.2

1. Make a straight-line phase angle plot for the transfer function in Example E.1.

2. Compute the phase angle $\theta(j\omega)$ at $\omega=50$, 500, and 1000 rad/s. Add these phase angle values to the plot you constructed in (a).

3. Using the results from Example E.1(e) and Example E.2(b), compute the steady-state output voltage if the source voltage is given by $\upsilon i(t)=10\ \cos(500t-25°)$ V.

Solution

1. From Example E.1,

   $H(j\omega)=0.11(j\omega)[1+j(\omega/10)]$
   $[1+j(\omega/100)]=0.11|j\omega||1+j(\omega/10)||1+j(\omega/100)|\quad(\psi 1-\beta 1-\beta 2)$.

   Therefore,

   $\theta(j\omega)=\psi 1-\beta 1-\beta 2$,

   where

   $\psi 1 = 90°$, $\beta 1 = \tan^{-1}(\omega/10)$, and $\beta 2 = \tan^{-1}(\omega/100)$.

   We construct the straight-line approximation of $\theta(j\omega)$ by adding $\psi 1=90°$ and the straight-line approximations for $-\beta 1$ and $-\beta 2$, all of which are depicted in Fig. E.8.

Figure E.8 A straight-line approximation of θ(ω) for Example E.2.

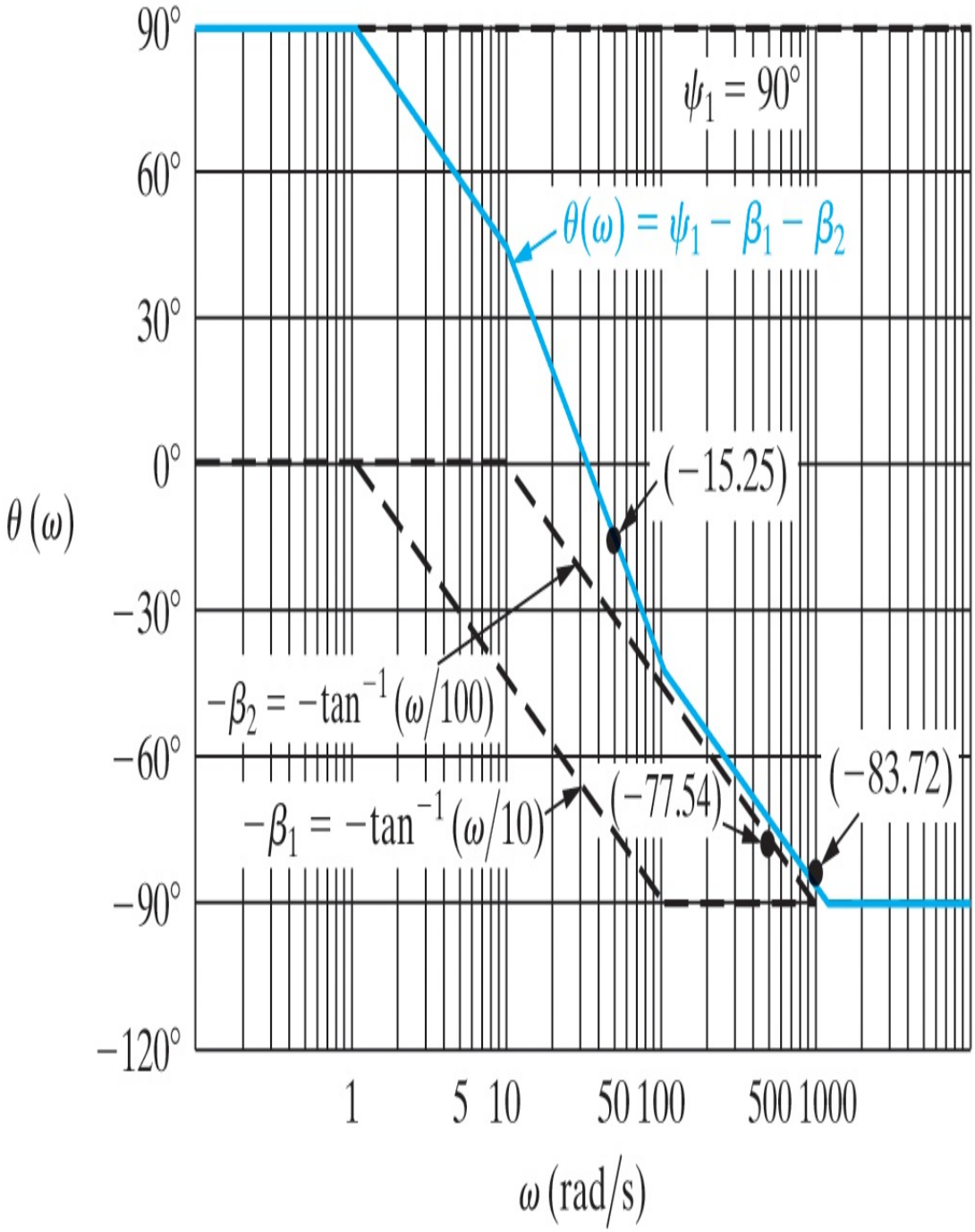


Figure E.8 Full Alternative Text

2. From the transfer function we have

   $H(j50)=0.96\angle -15.25°$, $H(j500)=0.22\angle -77.54°$, $H(j1000)=0.11\angle -83.72°$.

   Thus,

   $\theta(j50)= -15.25°, \theta(j500)= -77.54°,$

   and

   $\theta(j1000)= -83.72°.$

   These three phase angles are identified in Fig. E.8.

3. The frequency of the input voltage is 500 rad/sec. Therefore,

   $V_o=|H(j500)|V_i =(0.22)(10) =2.2$ V,

   and

   $\theta_o=\theta(j500)+\theta_i = -77.54°-25° = -102.54°.$

   Thus,

   $\upsilon_o(t)=2.2\cos(500t-102.54°)$ V.

E.5 Bode Diagrams: Complex Poles and Zeros

We now consider how to construct a Bode diagram when the transfer function has complex poles and zeros. Let’s focus on constructing the amplitude and phase angle plots for a transfer function with a pair of complex poles. Once you understand the rules for handling complex poles, you can apply these rules, with minor adjustments, to create plots for a pair of complex zeros.

The complex poles and zeros of H(s) always appear in conjugate pairs. When H(s) has complex poles, the first step in constructing a Bode diagram for H(s) is to combine the conjugate pair into a single quadratic term. Thus, for

$$H(s)=K(s+\alpha-j\beta)(s+\alpha+j\beta),$$

we first rewrite the product $(s+\alpha-j\beta)(s+\alpha+j\beta)$ as

$$(s+\alpha)2+\beta2=s2+2\alpha s+\alpha2+\beta2.$$

When making Bode diagrams, we write the quadratic term in a more convenient form:

$$s2+2\alpha s+\alpha2+\beta2=s2+2\zeta\omega ns+\omega n2.$$

Comparing the two forms shows that

$$\omega n2=\alpha2+\beta2$$

and

$$\zeta\omega n=\alpha.$$

The term ωn is the corner frequency of the quadratic term, and ζ is the damping coefficient of the quadratic term. If $\zeta<1$, the roots of the quadratic

term are complex, and if $\zeta \geq 1$, the roots are real. Assuming that $\zeta < 1$, we rewrite the transfer function as

$$H(s) = Ks2 + 2\zeta\omega ns + \omega n2.$$

We then write the transfer function in standard form by dividing through by $\omega n2$, so

$$H(s) = K\omega n2 \ \ 11 + (s/\omega n)2 + 2\zeta(s/\omega n),$$

from which

$$H(j\omega) = Ko1 - (\omega 2/\omega n2) + j(2\zeta\omega/\omega n),$$

where

$$Ko = K\omega n2.$$

We replace the ratio $\omega/\omega n$ by a new variable, u. Then

$$H(j\omega) = Ko1 - u2 \ + j2\zeta u.$$

Now we write $H(j\omega)$ in polar form:

$$H(j\omega) = Ko|(1 - u2) \ + j2\zeta u| \angle \beta 1_,$$

from which

$$AdB = 20 \log 10|H(j\omega)| = 20 \log 10Ko - 20 \log 10|(1 - u2) + j2\zeta u|,$$

and

$$\theta(j\omega) = -\beta 1 = -\tan \ -12\zeta u1 - u2.$$

E.6 Straight-Line Amplitude Plots for Complex Poles

The expression $-20 \log_{10}|1 - u^2 + j2\zeta u|$ represents the quadratic term's contribution to the amplitude of $H(j\omega)$. Because $u=\omega/\omega_n$, $u \to 0$ as $\omega \to 0$, and $u \to \infty$ as $\omega \to \infty$. To see how the term behaves as ω ranges from 0 to∞, we note that

$$-20 \log_{10}|(1 - u^2)+j2\zeta u| = -20 \log_{10}(1 - u^2)^2 + 4\zeta^2 u^2 \quad \text{(E.4)}$$
$$= -10 \log_{10}[u^4 + 2u^2(2\zeta^2 - 1) + 1].$$

Then, as $u \to 0$,

$$-10 \log_{10}[u^4+2u^2(2\zeta^2-1)+1] \to 0,$$

and as $u \to \infty$,

$$-10 \log_{10}[u^4+2u^2(2\zeta^2-1)+1] \to -40 \log_{10} u.$$

From these limiting expressions, we see that the approximate amplitude plot consists of two straight lines. For $\omega<\omega_n$, the straight line lies along the 0 dB axis, and for $\omega>\omega_n$, the straight line has a slope of −40dB/decade. These two straight lines join on the 0 dB axis at $u=1$ or $\omega=\omega_n$. Figure E.9 shows the straight-line approximation for a quadratic term with $\zeta<1$.

Figure E.9 The amplitude plot for a pair of complex poles.

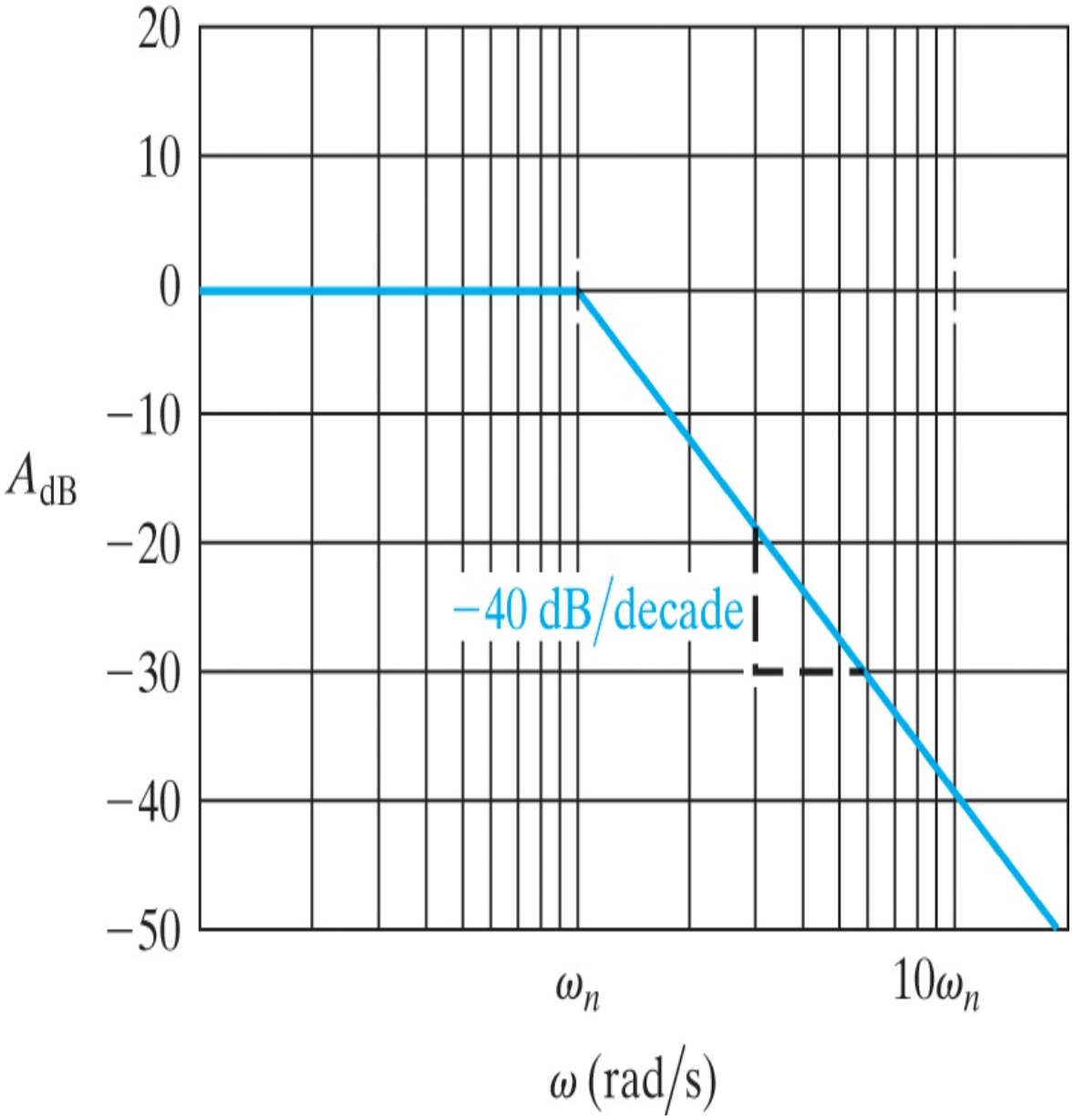


Figure E.9 Full Alternative Text

E.7 Correcting Straight-Line Amplitude Plots for Complex Poles

Corrections to the straight-line amplitude plot for a pair of complex poles depend on the damping coefficient ζ. Figure E.10 shows the effect of ζ on the amplitude plot. Note that when ζ is very small, a large peak in the amplitude occurs in the neighborhood of the corner frequency ωn(u=1). When ζ≥1/2, the corrected amplitude plot lies entirely below the straight-line approximation. The straight-line amplitude plot can be corrected by locating four points on the actual curve. These four points correspond to (1) one-half the corner frequency, (2) the frequency at which the amplitude reaches its peak value, (3) the corner frequency, and (4) the frequency at which the amplitude is zero.

Figure E.10 The effect of ζ on the amplitude plot.

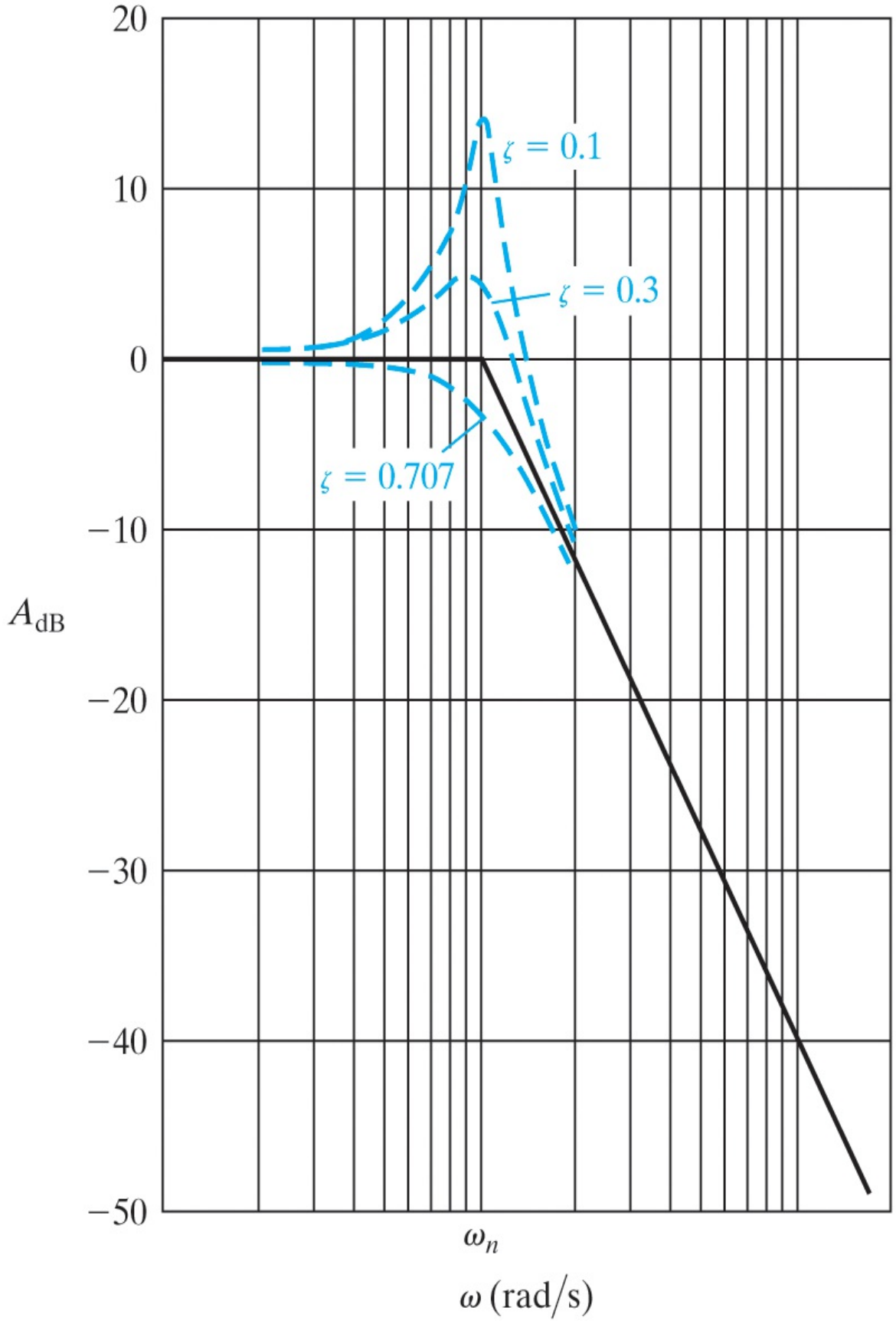

20
10
0
−10
−20
−30
−40
−50
A_{dB}
$\zeta = 0.1$
$\zeta = 0.3$
$\zeta = 0.707$
ω_n
ω (rad/s)

Figure E.10 Full Alternative Text

Figure E.11 Four points on the corrected amplitude plot for a pair of complex poles.

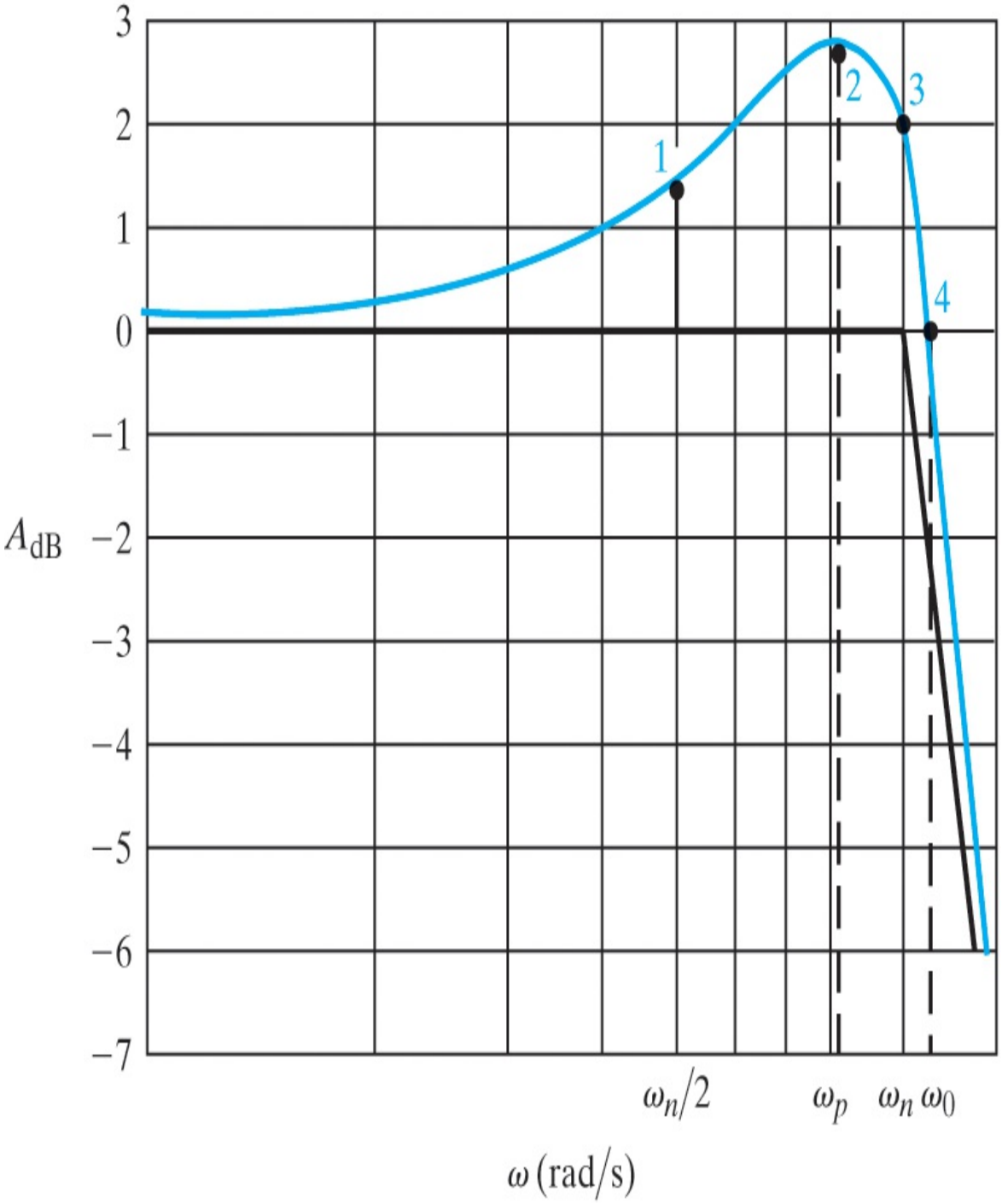


Figure E.11 Full Alternative Text

At one-half the corner frequency (point 1), the actual amplitude is

AdB(ωn/2)= −10 log10(ζ2+0.5625). (E.5)

The amplitude peaks (point 2) at a frequency of

$$\omega_p = \omega_n \sqrt{1 - 2\zeta^2}, \quad \text{(E.6)}$$

and it has a peak amplitude of

$$A_{dB}(\omega_p) = -10 \log_{10}[4\zeta^2(1-\zeta^2)]. \quad \text{(E.7)}$$

At the corner frequency (point 3), the actual amplitude is

$$A_{dB}(\omega_n) = -20 \log_{10} 2\zeta. \quad \text{(E.8)}$$

The corrected amplitude plot crosses the 0 dB axis (point 4) at

$$\omega_o = \omega_n \sqrt{2(1-2\zeta^2)} = 2\omega_p. \quad \text{(E.9)}$$

Figure E.11 shows these four points. Evaluating Eq. E.4 at u=0.5 and u=1.0, respectively, yields Eqs. E.5 and E.8. Equation E.9 results from finding the value of u that makes $u^4 + 2u^2(2\zeta^2 - 1) + 1 = 1$. To derive Eq. E.6, differentiate Eq. E.4 with respect to u and find the value of u where the derivative is zero. To derive E.7, find the value of u for the frequency in Eq. E.6, then evaluate Eq. E.4 at that value of u.

Example E.3 illustrates the amplitude plot for a transfer function with a pair of complex poles.

Example E.3

Compute the transfer function for the circuit shown in Fig. E.12.

Figure E.12 The circuit for Example E.3.

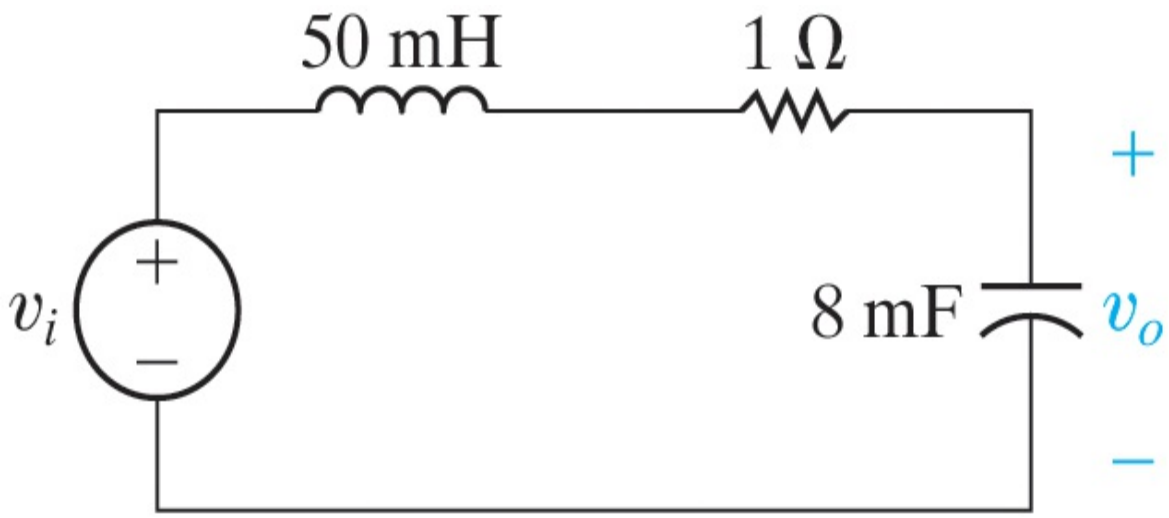


Figure E.12 Full Alternative Text

Transform the circuit in Fig. E.12 to the s- domain and then use voltage division to get

H(s)=1LC s2+(RL)s+1LC .

Substituting the component values,

H(s)=2500s2+20s+2500.

1. Find the value of the corner frequency in radians per second.
2. Find the value of Ko.
3. Find the value of the damping coefficient.
4. Make a straight-line amplitude plot for frequencies from 10 to 500 rad/s.
5. Calculate the actual amplitude in decibels at ωn/2, ωp, ωn, and ωo. Use these values to correct the plot in (d).
6. Using the corrected amplitude plot, describe the type of filter represented by the circuit in Fig. E.12 and estimate its cutoff frequency, ωc.

Solution

1. From the expression for H(s), ωn2=2500; therefore, ωn=50 rad/s.
2. By definition, Ko is 2500/ωn2, or 1.
3. The coefficient of s in the denominator equals 2ζωn; therefore ζ=202ωn=0.20.
4. See Fig. E.13.

Figure E.13 The amplitude plot for Example E.3.

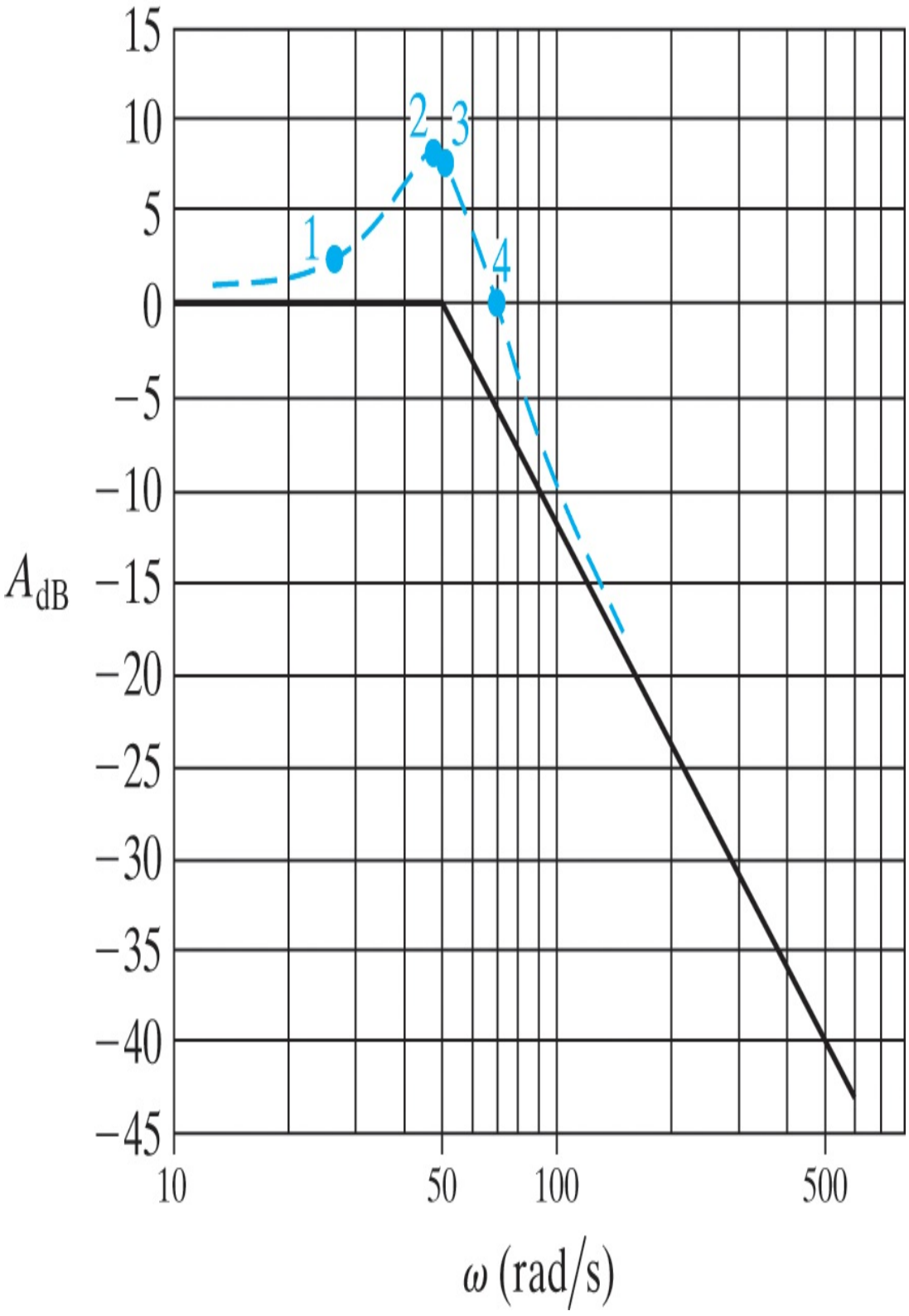


Figure E.13 Full Alternative Text

5. The actual amplitudes are

   $A_{dB}(\omega_n/2) = -10 \log_{10}(0.6025) = 2.2$ dB, $\omega_p = 500.92 = 47.96$ rad/s, $A_{dB}(\omega_p) = -10 \log_{10}(0.16)(0.96) = 8.14$ dB, $A_{dB}(\omega_n) = -20 \log_{10}(0.4) = 7.96$ dB, $\omega_o = 2\omega_p = 67.82$ rad/s, $A_{dB}(\omega_o) = 0$ dB.

   Figure E.13 shows these four points, identified as follows:

   - Point 1 has the coordinates (25 rad/s, 2.2 dB),
   - Point 2 has the coordinates (47.96 rad/s, 8.14 dB),
   - Point 3 has the coordinates (50 rad/s, 7.96 dB), and
   - Point 4 has the coordinates (67.82 rad/s, 0 dB).

   Figure E.13 also shows the corrected plot, which is the dashed line that passes through these four points.

6. The amplitude plot in Fig. E.13 identifies the circuit as a low-pass filter. At the cutoff frequency, the magnitude of the transfer function, $|H(j\omega_c)|$, is 3 dB less than the maximum magnitude. From the corrected plot, the cutoff frequency appears to be about 55 rad/s, almost the same as that predicted by the straight-line Bode diagram.

E.8 Phase Angle Plots for Complex Poles

The phase angle plot for a pair of complex poles is a plot of $\theta(j\omega) = -\tan^{-1}[2\zeta u/(1-u2)]$. The phase angle is zero at zero frequency and is −90° at the corner frequency. It approaches −180° as ω (and, therefore, u) becomes large. As in the case of the amplitude plot, ζ determines the exact shape of the phase angle plot. Figure E.14 shows the effect of ζ on the phase angle plot.

Figure E.14 The effect of ζ on the phase angle plot.

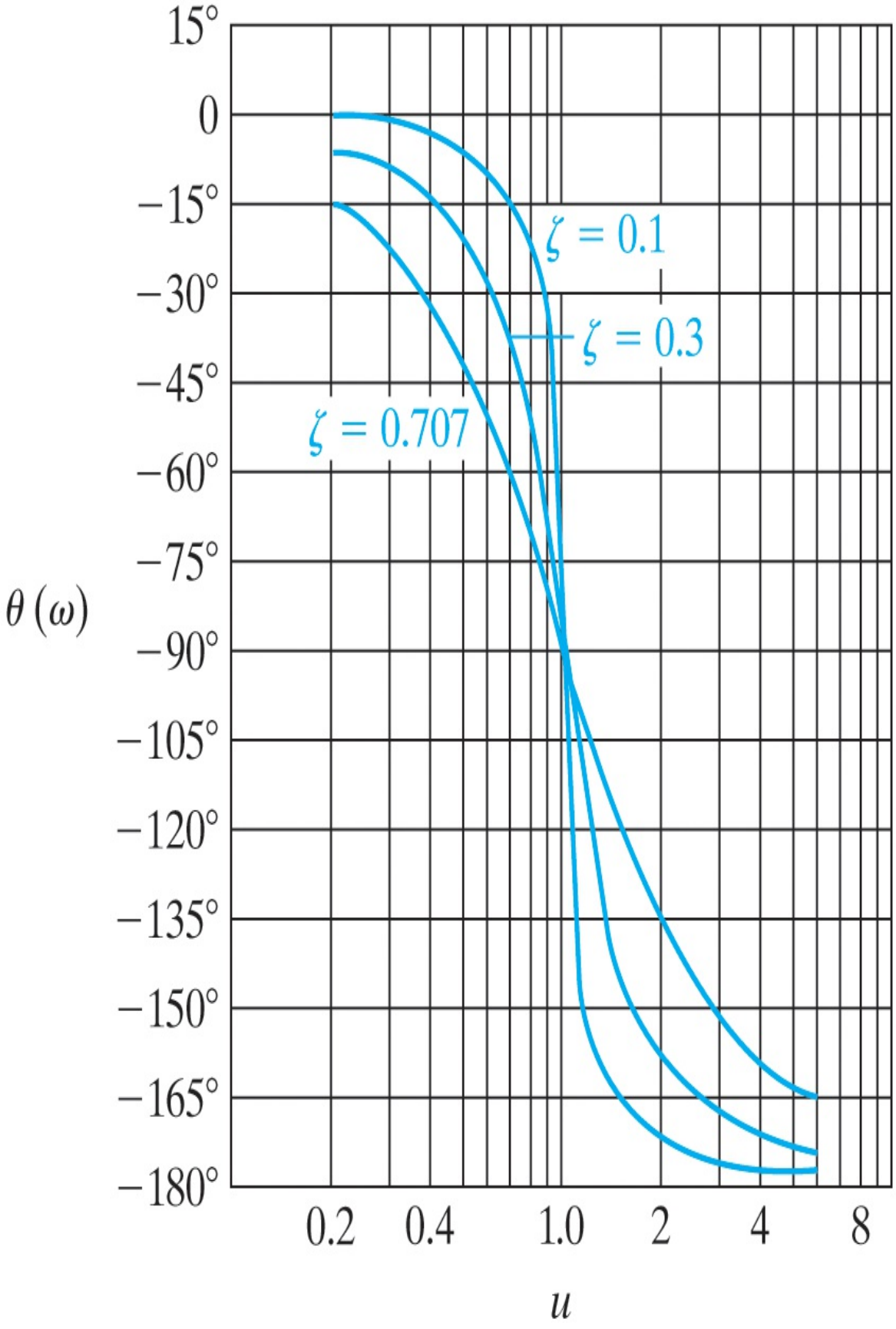

15°
0
−15°
−30°
−45°
−60°
−75°
−90°
−105°
−120°
−135°
−150°
−165°
−180°
θ (ω)
ζ = 0.1
ζ = 0.3
ζ = 0.707
0.2
0.4
1.0
2
4
8
u

Figure E.14 Full Alternative Text

We can make a straight-line approximation of the phase angle plot for a pair of complex poles. We do so by drawing three line segments:

- For $\omega \leq (4.81 - \zeta)\omega_n$, draw a horizontal line at 0°;
- For $\omega \geq (4.81\zeta)\omega_n$, draw a horizontal line at –180°;
- For $(4.81 - \zeta)\omega_n \leq \omega \leq (4.81\zeta)\omega_n$, draw a straight line connecting the 0° phase angle at $(4.81 - \zeta)\omega_n$ to the –180° phase angle at $(4.81\zeta)\omega_n$. This line passes through –90° at ω_n and has a slope of $-132/\zeta$ degrees/decade ($-2.3/\zeta$ rad/decade).

Figure E.15 depicts the straight-line approximation for $\zeta = 0.3$ and shows the actual phase angle plot. We see that the straight line is a good approximation of the actual curve near the corner frequency, but the error is quite large near the two points where the straight lines intersect. In Example E.4, we summarize our discussion of Bode diagrams.

Figure E.15 A straight-line approximation of the phase angle for a pair of complex poles.

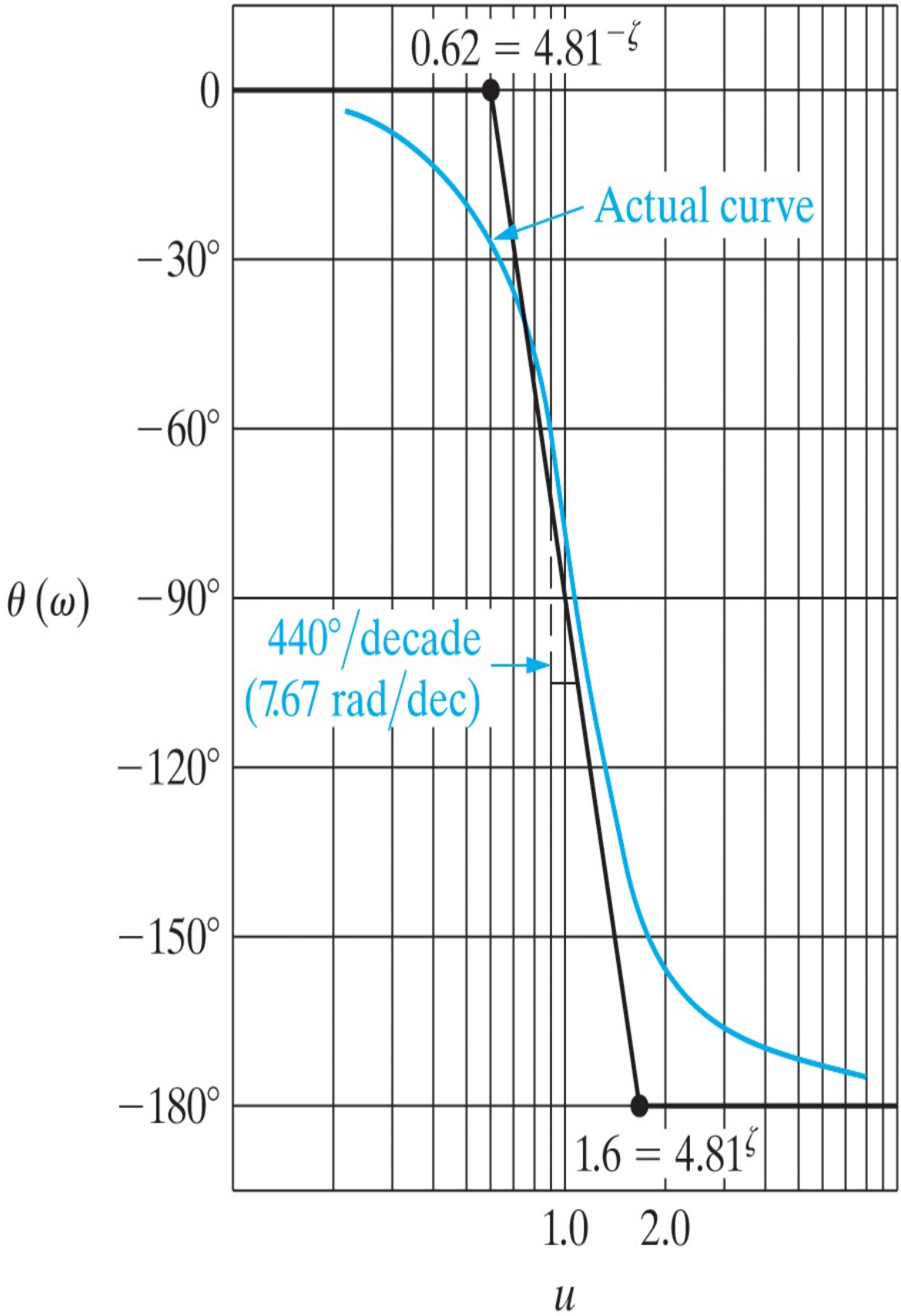

$0.62 = 4.81^{-\zeta}$
0
−30°
−60°
−90°
−120°
−150°
−180°
$\theta(\omega)$
Actual curve
440°/decade
(7.67 rad/dec)
$1.6 = 4.81^{\zeta}$
1.0
2.0
u

Figure E.15 Full Alternative Text

Example E.4

1. Compute the transfer function, H(s), for the circuit shown in Fig. E.16.

Figure E.16 The circuit for Example E.4.

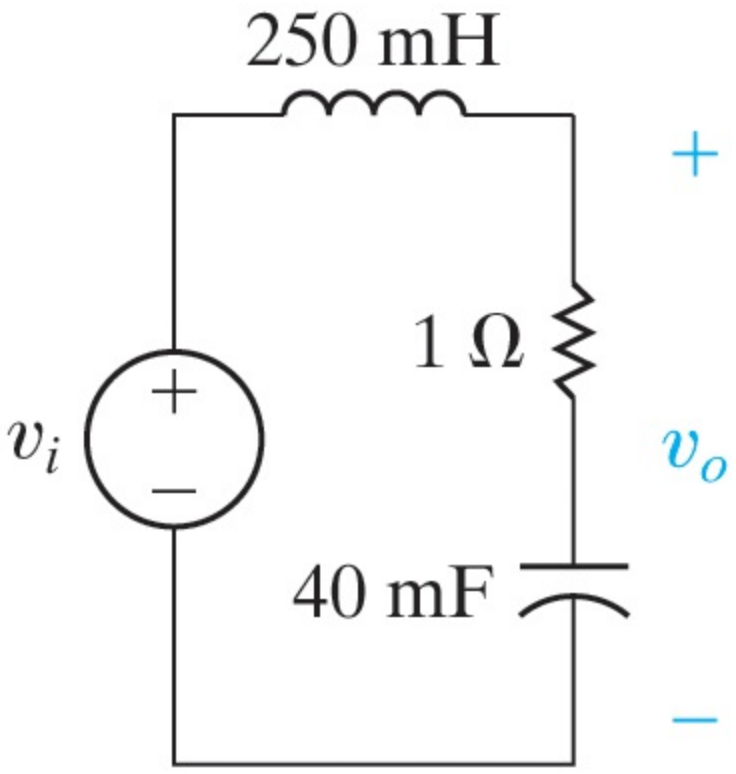


Figure E.16 Full Alternative Text

2. Make a straight-line amplitude plot of 20 log10|H(jω)|.

3. Use the straight-line amplitude plot to determine the type of filter represented by this circuit and then estimate its cutoff frequency.

4. Find the actual cutoff frequency.
5. Make a straight-line phase angle plot of H(jω).
6. Using the plot in (e), find the phase angle at the estimated cutoff frequency found in (c).
7. Find the phase angle at the actual cutoff frequency found in (d).

Solution

1. Transform the circuit in Fig. E.16 to the s-domain and then use voltage division to get

   H(s)=RL s+1LC s2+RL s+1LC .

   Substituting the component values from the circuit gives

   H(s)=4(s+25)s2+4s+100.

2. The first step in making Bode diagrams is to put H(jω) in standard form. Because H(s) contains a quadratic factor, we first check the value of ζ. We find that ζ=0.2 and ωn=10, so

   H(s)=s/25 +11 +(s/10)2 +0.4(s/10),

   from which

   H(jω)= | 1 +jω/25 | ψ 1 | 1− (ω/10) 2 +j0.4ω/10 | β 1 .

   Note that for the quadratic term, u=ω/10. The amplitude of H(jω) in decibels is

   A dB = 20 log 10 | 1+jω/25 | −20 log 10 [| 1− (ω 10) 2 + j0.4(ω 10) |],

   and the phase angle is

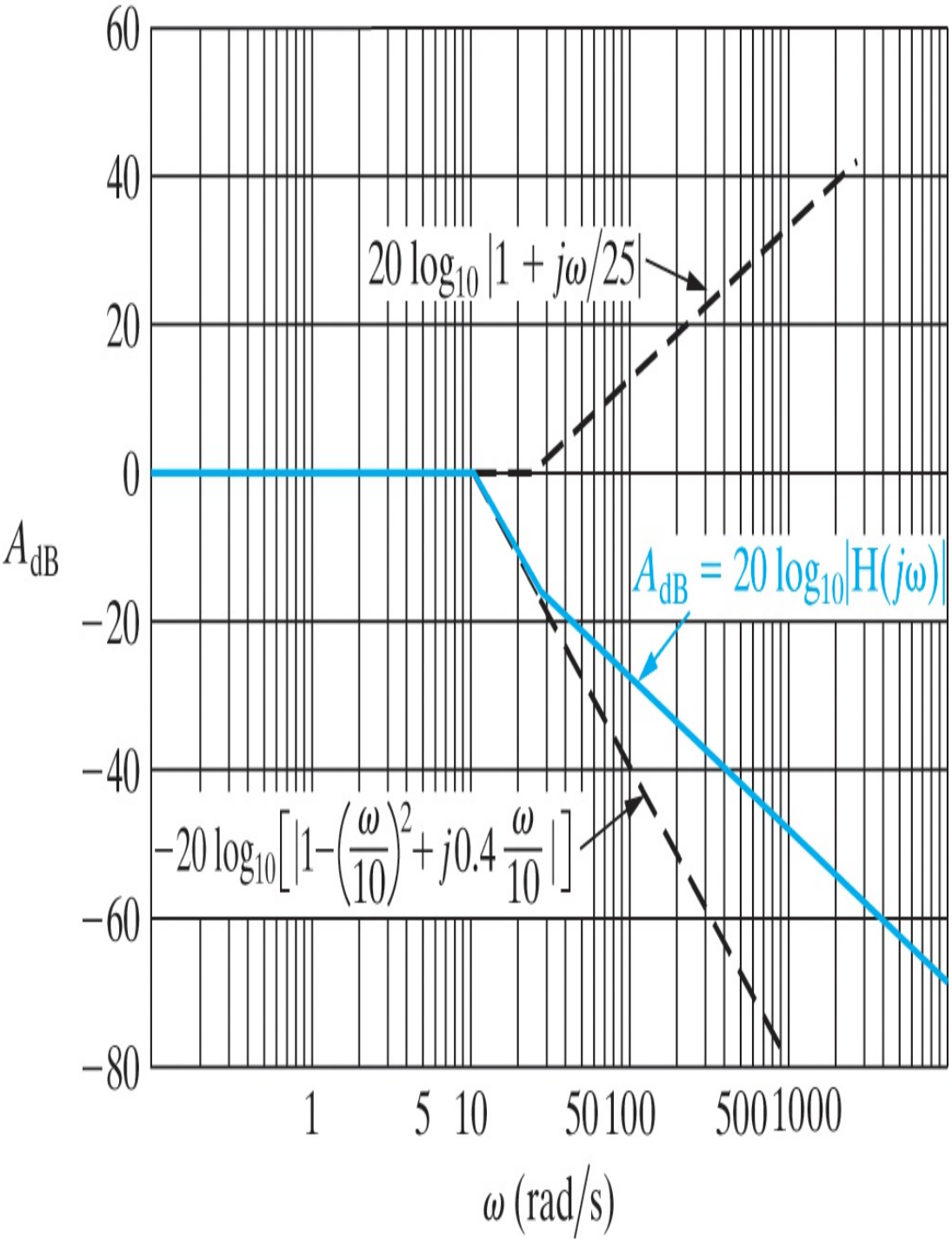
$$\theta(j\omega) = \psi 1 - \beta 1,$$

where

$$\psi 1 = \tan^{-1}(\omega/25), \beta 1 = \tan^{-1} 0.4(\omega/10)1-(\omega/10)2.$$

Figure E.17 shows the amplitude plot and includes the plots of the two terms that make up AdB.

Figure E.17 The amplitude plot for Example E.4.



The X-axis plots frequency omega in rad per second, and marks 1, 5, 10, 50, 100, 500, and 1000. The Y-axis plots Amplitude A in dB, ranging from minus 80 to 60 in increments of 20. A grid of horizontal and vertical lines spaced unevenly is drawn all over this graph. The graph shows

plot of 20 log 10 mod 1 plus j omega over 25 as a broken line that runs horizontally from (0, 10) to (0, 25), and then moves with an upward slope to reach (2000, 40).

A second plot of minus 20 log 10 mod 1 minus (omega over 10) squared plus j 0.4 multiplied by omega over 10| is shown as a broken line that moves with a downward slope from (0, 10) to (1000, minus 80).

The Amplitude plot A sub dB = 20 log 10 |H(j omega)| is shown as a colored line that runs horizontally from the origin to (0, 10), then moves with a downward slope to (30, minus 18), and then continues downward with a slightly less steeper slope to reach (500, minus 40).

3. The straight-line amplitude plot in Fig. E.17 indicates that the circuit is a low-pass filter. At the cutoff frequency, the amplitude of H(jω) is 3 dB less than the amplitude in the passband. From the plot, we predict that the cutoff frequency is approximately 13 rad/s.

4. To solve for the actual cutoff frequency, replace s with jω in H(s), compute the expression for |H(jω)|, set |H(jωc)|=(1/2)Hmax=1/2, and solve for ωc. First,

   H(jω)=4(jω)+100(jω)2+4(jω)+100.

   Then,

   |H(jωc)|=(4ωc)2+1002(100−ωc2)2+(4ωc)2=12.

   Solving for ωc gives us

   ωc=16 rad/s.

5. Figure E.18 shows the phase angle plot, as well as plots of the two angles −ψ1 and −β1. Note that the straight-line segment of θ(ω) between 1.0 and 2.5 rad/s does not have the same slope as the segment between 2.5 and 100 rad/s.

Figure E.18 The phase angle plot for Example E.4.

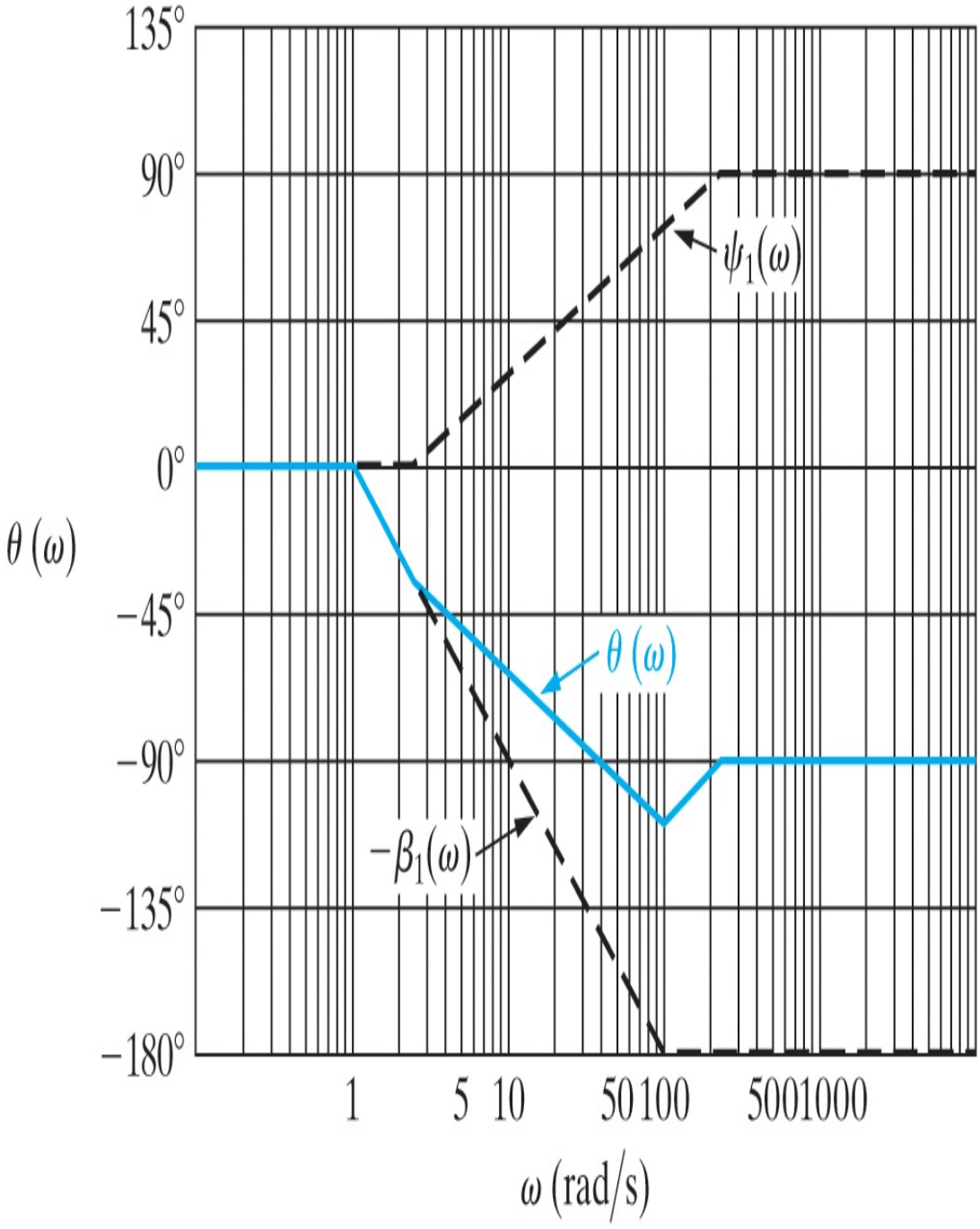


Figure E.18 Full Alternative Text

6. From the phase angle plot in Fig. E.18, we estimate the phase angle at the estimated cutoff frequency of 13 rad/s to be −65°.

7. We can compute the exact phase angle at the actual cutoff frequency by substituting s=j16 into the transfer function H(s):

   H(j16)=4(j16+25)(j16)2+4(j16)+100.

   Computing the phase angle, we see

   θ(jωc)=θ(j16)= −125.0°.

   Note the large error in the predicted angle. In general, straight-line phase angle plots do not give satisfactory results in the frequency band where the phase angle is changing. The straight-line phase angle plot is useful only in predicting the general behavior of the phase angle, not in estimating actual phase angle values at particular frequencies.

Appendix F An Abbreviated Table of Trigonometric Identities

1. $\sin(\alpha \pm \beta) = \sin\alpha \cos\beta \pm \cos\alpha \sin\beta$
2. $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha \sin\beta$
3. $\sin\alpha + \sin\beta = 2 \sin \frac{\alpha+\beta}{2} \cos \frac{\alpha-\beta}{2}$
4. $\sin\alpha - \sin\beta = 2 \cos\left(\frac{\alpha+\beta}{2}\right) \sin\left(\frac{\alpha-\beta}{2}\right)$
5. $\cos\alpha + \cos\beta = 2 \cos\left(\frac{\alpha+\beta}{2}\right) \cos\left(\frac{\alpha-\beta}{2}\right)$
6. $\cos\alpha - \cos\beta = -2 \sin\left(\frac{\alpha+\beta}{2}\right) \sin\left(\frac{\alpha-\beta}{2}\right)$
7. $2 \sin\alpha \sin\beta = \cos(\alpha-\beta) - \cos(\alpha+\beta)$
8. $2 \cos\alpha \cos\beta = \cos(\alpha-\beta) + \cos(\alpha+\beta)$
9. $2 \sin\alpha \cos\beta = \sin(\alpha+\beta) + \sin(\alpha-\beta)$
10. $\sin 2\alpha = 2 \sin\alpha \cos\alpha$
11. $\cos 2\alpha = 2 \cos^2\alpha - 1 = 1 - 2 \sin^2\alpha$
12. $\cos^2\alpha = \frac{1}{2} + \frac{1}{2} \cos 2\alpha$
13. $\sin^2\alpha = \frac{1}{2} - \frac{1}{2} \cos 2\alpha$
14. $\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha \tan\beta}$
15. $\tan 2\alpha = \frac{2 \tan\alpha}{1 - \tan^2\alpha}$

Appendix G An Abbreviated Table of Integrals

1. $\int xe^{ax}dx = \frac{e^{ax}}{a^2}(ax-1)$
2. $\int x^2 e^{ax}\,dx = \frac{e^{ax}}{a^3}(a^2x^2-2ax+2)$
3. $\int x \sin ax\,dx = \frac{1}{a^2}\sin ax - \frac{x}{a}\cos ax$
4. $\int x \cos ax\,dx = \frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax$
5. $\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$
6. $\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$
7. $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$
8. $\int \frac{dx}{(x^2+a^2)^2} = \frac{1}{2a^2}\left(\frac{x}{x^2+a^2} + \frac{1}{a}\tan^{-1}\frac{x}{a}\right)$
9. $\int \sin ax \sin bx\,dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)}, a^2 \neq b^2$
10. $\int \cos ax \cos bx\,dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)}, a^2 \neq b$
11. $\int \sin ax \cos bx\,dx = -\frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)}, a^2 \neq b^2$
12. $\int \sin^2 ax\,dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$
13. $\int \cos^2 ax\,dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$
14. $\int_0^\infty \frac{dx}{a^2+x^2} = \begin{cases} \frac{\pi}{2}, & a>0 \\ 0, & a=0 \\ -\frac{\pi}{2}, & a<0 \end{cases}$
15. $\int_0^\infty \frac{\sin ax}{x}dx = \begin{cases} \frac{\pi}{2}, & a>0 \\ -\frac{\pi}{2}, & a<0 \end{cases}$
16. $\int x^2 \sin ax\,dx = \frac{2x}{a^2}\sin ax - \frac{a^2x^2-2}{a^3}\cos ax$
17. $\int x^2 \cos ax\,dx = \frac{2x}{a^2}\cos ax + \frac{a^2x^2-2}{a^3}\sin ax$

18. $\int e^{ax} \sin^2 bx \, dx = \frac{e^{ax}}{a^2+4b^2}\left[(a \sin bx - 2b \cos bx) \sin bx + \frac{2b^2}{a}\right]$

19. $\int e^{ax} \cos^2 bx \, dx = \frac{e^{ax}}{a^2+4b^2}\left[(a \cos bx + 2b \sin bx) \cos bx + \frac{2b^2}{a}\right]$

Appendix H Common Standard Component Values

Resistors (5% tolerance)[Ω]

10	100	1.0 k	10 k	100 k	1.0 M
	120	1.2 k	12 k	120 k	
15	150	1.5 k	15 k	150 k	1.5 M
	180	1.8 k	18 k	180 k	
22	220	2.2 k	22 k	220 k	2.2 M
	270	2.7 k	27 k	270 k	
33	330	3.3 k	33 k	330 k	3.3 M
	390	3.9 k	39 k	390 k	
47	470	4.7 k	47 k	470 k	4.7 M
	560	5.6 k	56 k	560 k	
68	680	6.8 k	68 k	680 k	6.8 M

Capacitors

10 pF	22 pF	47 pF
100 pF	220 pF	470 pF
0.001 μF	0.0022 μF	0.0047 μF
0.01 μF	0.022 μF	0.047 μF
0.1 μF	0.22 μF	0.47 μF
1 μF	2.2 μF	4.7 μF
10 μF	22 μF	47 μF
100 μF	220 μF	470 μF

Inductors

Value	Current Rating
10 μH	3 A
100 μH	0.91 A
1 mH	0.15 A
10 mH	0.04 A

Answers to Selected Problems

Chapter 1

1. 1.3
    1. 3.456 s
    2. 175,000 cell lengths/week
2. 1.6 104.4 gigawatt-hours
3. 1.15
    1. −100 W; power is being delivered by the box
    2. Leaving
    3. Gain
4. 1.18
    1. 937.5 mW
    2. 1.875 mJ
5. 1.25
    1. 5.39 W
    2. 9 J
6. 1.34 $\Sigma P_{del}=\Sigma P_{abs}=2280$ W

Chapter 2

1. 2.7
    1. 20 V
    2. 8 W (absorbed)
2. 2.11
    1. −16 mA
    2. 640 mW
    3. 16 mA; 640 mW
3. 2.16 100 Ω resistor
4. 2.17
    1. 1.2 A, 0.3 A
    2. 120 V
    3. ΣPdev=ΣPdiss=180 W
5. 2.29
    1. 10 A in parallel with 5 Ω
    2. 80 W
6. 2.33 15 V, 1.4167 W
7. 2.42 1800 W, which is 1/2 the power for the circuit in Fig. 2.41

Chapter 3

1. 3.2
    1. 21.2 Ω, 10 kΩ, 1600 Ω, 120 Ω
    2. 4.72 W, 90 mW, 25 µW, 108 mW
2. 3.13
    1. 15 V
    2. 1.8 W, 450 mW
    3. 3600 Ω, 900 Ω
3. 3.14
    1. 1200 Ω, 300 Ω
    2. 1 W
4. 3.27
    1. 150 mA
    2. 5.4 V
    3. 3.6 V
    4. 1 V
5. 3.33 7.5 A
6. 3.37
    1. 49,980 Ω

    2. 4980 Ω
    3. 230 Ω
    4. 5 Ω
7. 3.51
    1. 900 Ω
    2. 25 mA
    3. 800 Ω, 180 mW
    4. 900 Ω, 90 mW
8. 3.60 2.4 A, 72.576 W
9. 3.63
    1. 80 Ω
    2. 279 W
10. 3.73
    1. 0.2, 0.75
    2. 384, 200

Chapter 4

1. 4.2
    1. 8
    2. 3
    3. 4
    4. Top-most and left-most meshes cannot be used, as they both contain current sources
2. 4.5
    1. $i4 + i6 - i5 - I = 0$
    2. derivation
3. 4.10 120 V, 96 V
4. 4.12
    1. 8 A, 2 A, 6 A, 2 A, 4 A
    2. 360 W
5. 4.17 750 W
6. 4.27
    1. −37.5 V, 75 W
    2. −37.5 V, 75 W
    3. Part (b), fewer equations

7. 4.36
    1. 0.1 A, 0.3 A, 0.2 A
    2. 0.38 A, 0.02 A, −0.36 A
8. 4.41 2700 W
9. 4.46
    1. 2 mA
    2. 304 mW
    3. 0.9 mW
10. 4.50 740 W
11. 4.56
    1. Mesh current method
    2. 4 mW
    3. No
    4. 200 mW
12. 4.62
    1. −0.85 A
    2. −0.85 A
13. 4.68 8 mA down in parallel with 10 kΩ
14. 4.72
    1. 51.3 V

    2. −5%

15. 4.79 150 Ω
16. 4.87 2.5 Ω and 22.5 Ω
17. 4.92
    1. 50 V
    2. 250 W
18. 4.105 39.583 V, 102.5 V

Chapter 5

1. 5.3 −1 mA
2. 5.10
    1. $0 \le \sigma \le 0.40$
    2. 556.25 µA
3. 5.14 $0 \le R_f \le 75\ k\Omega$
4. 5.20
    1. 7.56 V
    2. $-3.97\ V \le v_g \le 3.97\ V$
    3. 35 kΩ
5. 5.27
    1. −1.53 mV
    2. 33.5 kΩ
    3. 80 kΩ
6. 5.31
    1. 16 V
    2. $-4.2\ V \le v_b \le 3.8\ V$
7. 5.33 $19.93\ k\Omega \le R_x \le 20.07\ k\Omega$
8. 5.44

1. −19.9844
2. 736.1 µV
3. 5003.68 Ω
4. −20, 0, 5000 Ω

9. 5.49
    1. 2 kΩ
    2. 12 mΩ

Chapter 6

1. 6.3
    1. i=0, t≤0

       i=50t A, 0≤t≤5 ms

       i=(5−50t) A, 5 ms≤t≤10 ms

       i=0, t≥10 ms

    2. v=0, t≤0

       v=1 V, 0≤t≤5 ms

       v= −1 V, 5 ms≤t≤10 ms

       v=0, t≥10 ms

       p=0, t≤0

       p=50t W, 0≤t≤5 ms

       p=(50t−0.5) W, 5 ms≤t≤10 ms

       p=0, t≥10 ms

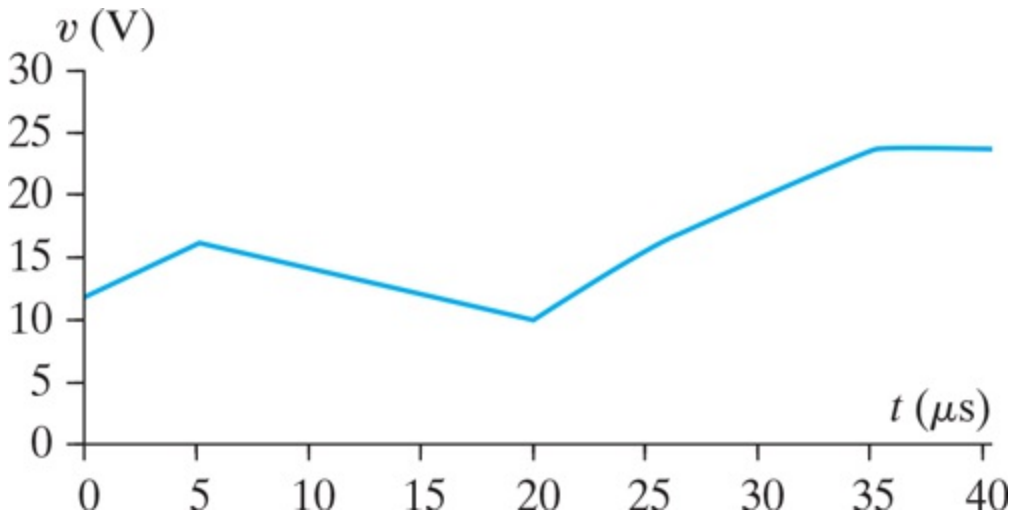
       w=0, t≤0

       w=25t2 J, 0≤t≤5 ms

       w=(25t2−0.5t+ 0.0025) J, 5 ms≤t≤10 ms

       w=0, t≥10 ms

2. 6.21

1. (8×105t+12) V
2. (−4×105t+18) V
3. (12×105t−14) V
4. (8×105t−4) V
5. 24 V
6. 

3. 6.30 5 nF with an initial voltage drop of 15 V; 10 μF with an initial voltage drop of 25 V
4. 6.31
    1. 10e−t V, t≥0
    2. (6.67e−t−2.67) V, t≥0
    3. (3.33e−t+2.67) V, t≥0
    4. 100 μJ
    5. 132 μJ
    6. 32 μJ

7. 100+32=132 μJ

5. 6.41
    1. 0.2di2dt+10i2= −0.5digdt
    2. 0.2di2dt+10i2=5e −10t and −0.5digdt=5e −10t
    3. (−53.125e−10t+6.25e −50t) V, t≥0
    4. −46.875 V
6. 6.48
    1. 0.95
    2. 24 mH
    3. 2.5
7. 6.49 0.8 nWb/A, 1.2 nWb/A
8. 6.53
    1. (2.1, 4.3); (3.2, 2.5); (2.1, 2.5); (3.2, 4.3)
    2. Zoom in
    3. Zoom out

Chapter 7

1. 7.2
    1. 0.5 A
    2. 2 ms
    3. $0.5e^{-500t}$ A, $t \geq 0$; $-80e^{-500t}$ V, $t \geq 0+$; $-35e^{-500t}$ V, $t \geq 0+$
    4. 35.6%
2. 7.4
    1. 0 A, 100 mA, 0 V
    2. 400 mA, 100 mA, –20 V
    3. 500 mA, 0 A, 0 V
    4. $0.1e^{-4000t}$ A
    5. $(0.5-0.1e^{-4000t})$ A
    6. $-20e^{-4000t}$ V
3. 7.18
    1. $-10e^{-5000t}$ A, $t \geq 0$
    2. 80 mJ
    3. 1.498τ
4. 7.22
    1. $59.4e^{-1000t}$ V, $t \geq 0$

2. 9.9e −1000t mA, t≥0+

5. 7.27
    1. 4 kΩ
    2. 10 μF
    3. 40 ms
    4. 11.52 mJ
    5. 10.95 mJ
6. 7.38
    1. (−0.8+2.4e −4000t) A, t≥0; (41.6+19.2e −4000t) V, t≥0 +
    2. −48 V, 60.8 V
7. 7.46 17.33 ms
8. 7.55
    1. 90 V
    2. −60 V
    3. 1000 μs
    4. 916.3 μs
9. 7.59 (−100+130e −200t) V, t≥0
10. 7.63 3.67 ms
11. 7.69
    1. (6−6e −10t) A, t≥0

2. 100e −10t V, t≥0+
3. (7−7e −10t) A, t≥0
4. (−1+e −10t) A, t≥0
5. Yes

12. 7.73 −5.013 V
13. 7.78 10 V, 0≤t≤50 ms; 10e −1000(t−0.05) V, t≥50 ms
14. 7.87 83.09 ms
15. 7.94
    1. 1RC∫0t(υb−υa)dy
    2. Output is the integral of the difference between vb and va, scaled by a factor of 1/RC
    3. 120 ms
16. 7.105 73 beats per minute

Chapter 8

1. 8.3 (75,000t+15)e −10,000t V, t≥0
2. 8.9
    1. 1 kΩ, 1 μF, 6000 V/s, 8 V
    2. (−3000t+2)e −500t mA, t≥0+
3. 8.14 (5e −2000t+10e −8000t) V, t≥0
4. 8.16 (15e −2500t cos 3122.5t+7.21e −2500t sin 3122.5t) V, t≥0
5. 8.33 (2+0.33e −400t−1.33e −1600t) A, t≥0
6. 8.41 8 kΩ, 2 H, 7.5 mA, 0
7. 8.51 (20−10,000te −500t−20e −500t) V, t≥0
8. 8.56
    1. 75e −37,500t sin 50,000t V
    2. 18.55 μs
    3. 29.93 V
    4. 601.08e −3750t sin 62,387.4t V, 24.22 μs, 547.92 V
9. 8.63
    1. 0≤t≤0.5 − s: υo1= −1.6t V, υo=10t2 V

       0.5+ s≤t≤tsat: υo1=(0.8t−12) V, υo=(−5t2+15t−3.75) V
    2. 3.5 s

10. 8.64 $0 \leq t \leq 0.5$ s: $\upsilon_{o1} = (-0.8e^{-t} + 0.8e^{-2t})$ V, $\upsilon_o = (10 - 20e^{-t} + 10e^{-2t})$ V $t \geq 0.5$ s: $\upsilon_{o1} = (0.4 - 0.91e^{-2(t-0.5)})$ V, $\upsilon_o = (-5 + 19.42e^{-(t-0.5)} - 12.87e^{-2(t-0.5)})$ V

11. 8.67

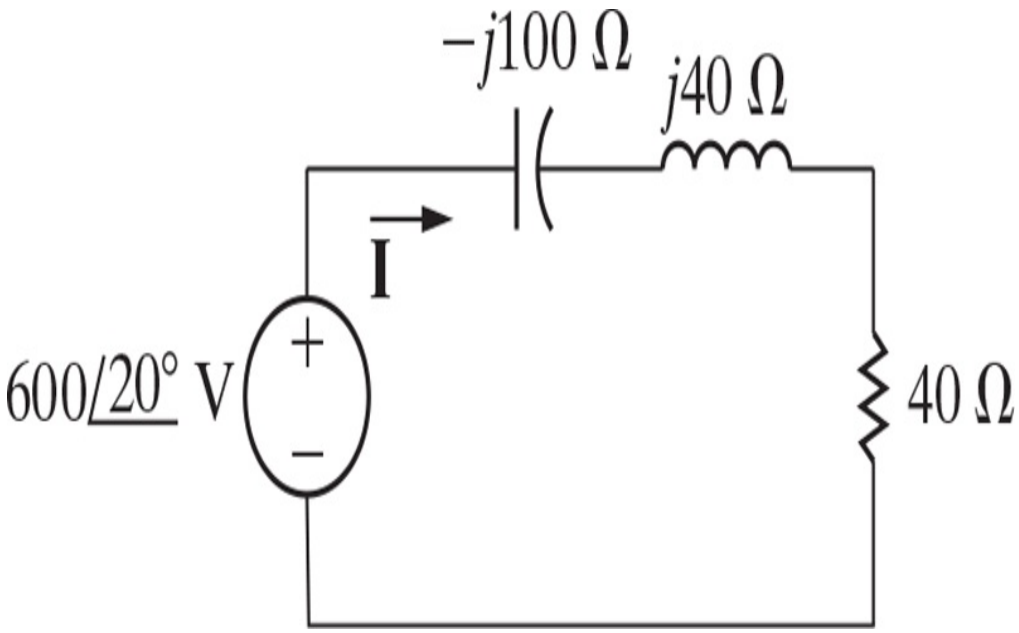
    1. 6.33 pF

    2. $5.03 \sin(4\pi \times 109t)$ V, $t \geq 0$

Chapter 9

1. 9.1
    1. 40 V
    2. 50 Hz
    3. 314.159 rad/s
    4. 1.05 rad
    5. 60°
    6. 20 ms
    7. 6.67 ms
    8. 40 cos 100πt V
    9. 8.33 ms
2. 9.10
    1. [−195.72e −1066.67t+200 cos(800t−11.87°)] mA, t≥0
    2. transient: −195.72e −1066.67t mA; steady-state: 200 cos (800t −11.87°) mA
    3. 28.39 mA
    4. 200 mA, 800 rad/s, −11.87°
    5. Current lags voltage by 36.87°
3. 9.11

1. 111.8 cos(500t−3.43°)
2. 102.99 cos(377t+40.29°)
3. 161.57 cos(100t−29.96°)
4. 0

4. 9.13
    1. 400 Hz
    2. −90°
    3. 5 Ω
    4. 1.99 mH
    5. j5 Ω
5. 9.15
    1. 

    2. 8.3276.31° A

3. 8.32 cos(8000t+76.31°) A

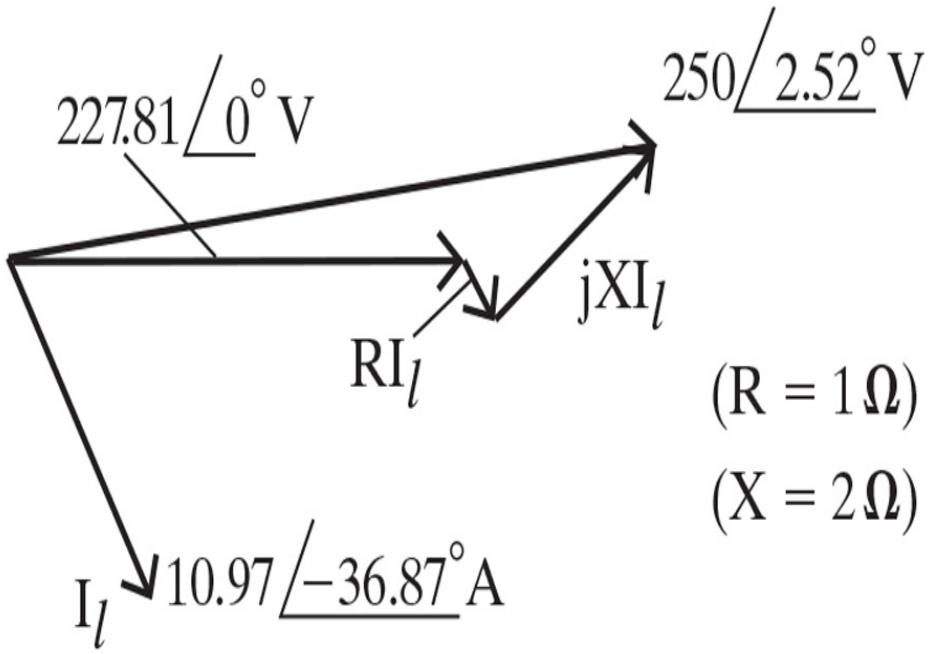
6. 9.17
    1. 160+j120 mS
    2. 160 mS
    3. 120 mS
    4. 10 A
7. 9.28 42.43 sin(50,000t+45°) V
8. 9.36 4000 rad/s
9. 9.42 2/3 Ω
10. 9.48 (6+j4) A in parallel with (−20+j20) Ω
11. 9.55 188.43 −42.88° V
12. 9.64 178.89 cos(5000t+153.43°) V
13. 9.77
    1. 0.3536
    2. 2 A
14. 9.79 1250 −45° Ω
15. 9.83
    1. (247+j7.25) V
    2. −j32 Ω, (241+j8) V
    3. −26.90 Ω

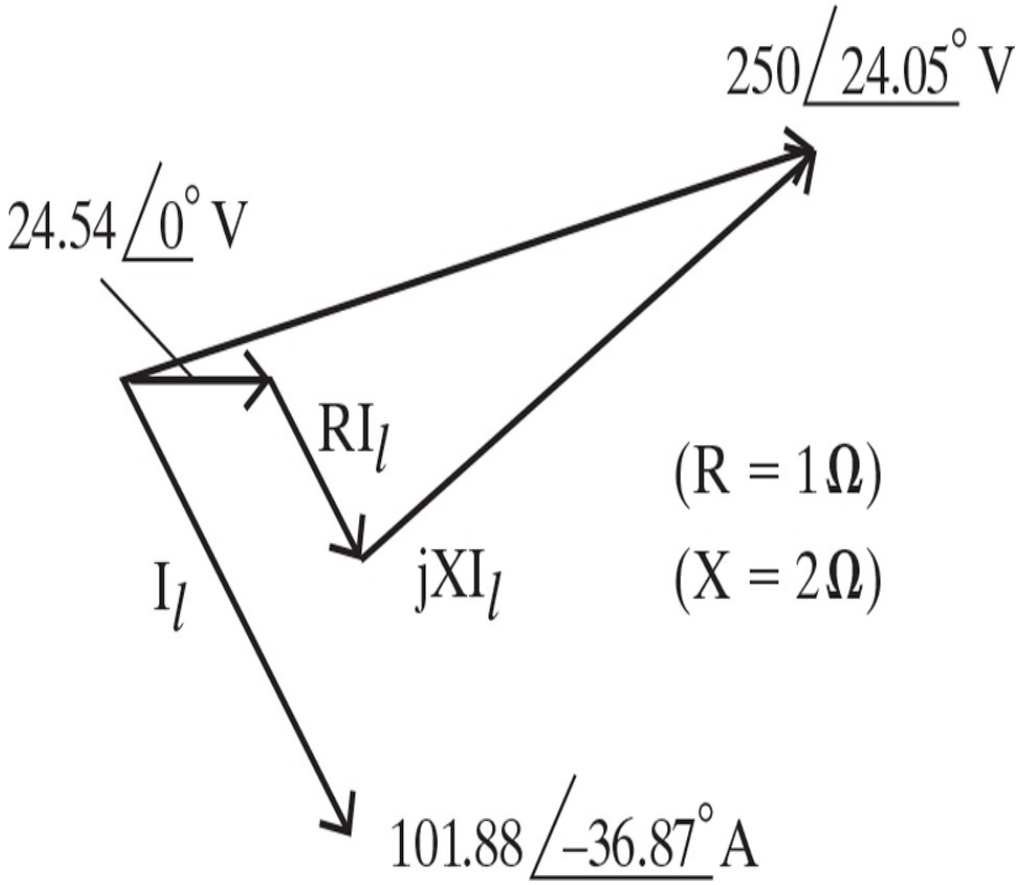
16. 9.87

    1. 0 A
    2. 0.4360° A
    3. Yes

Chapter 10

1. 10.1
    1. 409.58 W(abs), 286.79 var(abs)
    2. 103.53 W(abs), −386.37 var(del)
    3. −1000 W(del), −1732.05 var (del)
    4. −250 W(del), 433.01 var(abs)
2. 10.2
    1. No [40.1 A(rms)]
    2. Yes [66.2 A(rms)]
3. 10.8 5 mW
4. 10.14
    1. 3 V(rms)
    2. 4 W
5. 10.17
    1. 227.812.52° V(rms); 24.5424.05° V(rms)
    2.


$250\angle 2.52^\circ$ V
$227.81\angle 0^\circ$ V
jXI_l
RI_l
$(R = 1\,\Omega)$
$(X = 2\,\Omega)$
I_l
$10.97\angle -36.87^\circ$ A


$250\angle 24.05^\circ$ V
$24.54\angle 0^\circ$ V
RI_l
$(R = 1\,\Omega)$
jXI_l
$(X = 2\,\Omega)$
I_l
$101.88\angle -36.87^\circ$ A

6. 10.27
    1. 0.96 (lag), 0.28; 0.6 (lead), −0.8; 0.8 (lead), −0.6
    2. 0.74 (lead), −0.67
7. 10.43
    1. (20+j20) Ω
    2. 45 W
    3. R=22 Ω and L=1 mH gives 42.5 W
8. 10.49
    1. 360 mW
    2. 4000 Ω, 0.1 μF
    3. 443.1 mW
    4. 450 mW
    5. 4000 Ω, 66.67 nF
    6. Yes
9. 10.52
    1. −210° V(rms)
    2. 63 W
    3. 9.72%
10. 10.60
    1. 125

2. 26.28125 W

11. 10.67

    1. 15.63 kWh
    2. 11.72 kWh
    3. 9.16 kWh
    4. 6.64 kWh

Chapter 11

1. 11.1
    1. acb
    2. abc
2. 11.11
    1. 10 A(rms)
    2. 669.3 V(rms)
3. 11.12
    1. 5 −36.87° A(rms), 583.13° A(rms), 5 −156.87° A(rms)
    2. 216.51 −30° V(rms), 216.5190° V(rms), 216.51 −150° V(rms)
    3. 122.23 −1.36° V(rms), 122.23118.64° V(rms), 122.23 −121.36° V(r
    4. 211.72 −31.36° V(rms), 211.7288.64° V(rms), 211.72 −151.36° V(r
4. 11.15 21.64121.34° A(rms)
5. 11.16 159.529.34° V(rms)
6. 11.25 612036.61° VA
7. 11.27
    1. 1833.4622° VA
    2. 519.62 V(rms)
8. 11.35 6990.62 V(rms)

9. 11.45
    1. proof
    2. 2592 var, −2592 var, 3741.23 var, −4172.80 var
10. 11.54
    1. 16.71 μF
    2. 50.14 μF

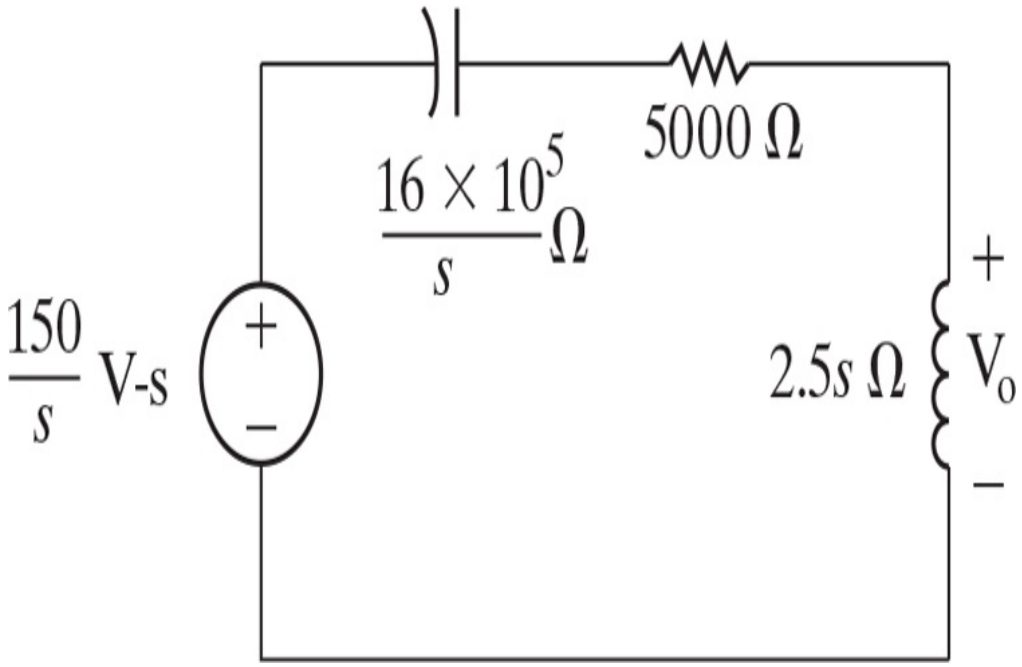
Chapter 12

1. 12.1
    1. (t+10)u(t+10)−2tu(t)+(t−10)u(t−10)
    2. −8(t+3)u(t+3)+8(t+2)u(t+2)+8(t+1)u(t+1)−8(t−1)u(t−1)−8(t−2)u(t−2)+8(t−3)u(t−3)
2. 12.10 2/9
3. 12.20
    1. 1(s+a)2
    2. ωs2+ω2
    3. ω cos θ+s sin θs2+ω2
    4. 1s2
    5. sinh θ+s cosh θ(s2−1)
4. 12.22
    1. ωss2+ω2
    2. −ω2s2+ω2
    3. 2
    4. Check
5. 12.26 1.92s2(s2+1.6s+1)(s2+1)
6. 12.40

1. [e −t+5e −2t+2e −4t]u(t)
2. [10+5e −2t−8e −3t+e −5t]u(t)

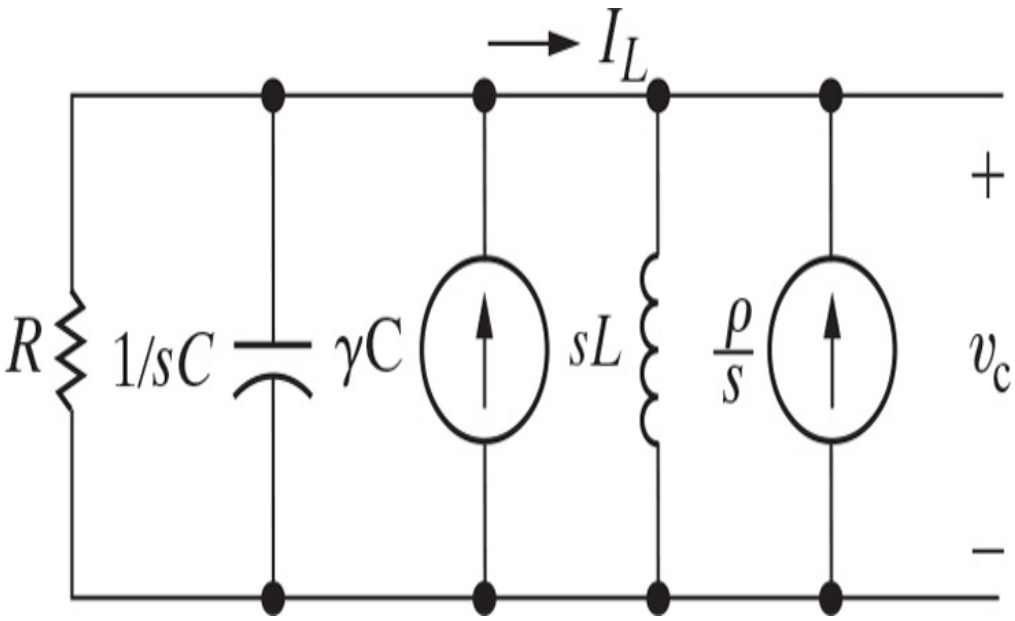
7. 12.41
    1. [10e −6t+5.66e −2t cos(4t+45°)]u(t)
    2. [9.25e −5t cos(3t−40.05°)+7.21e −4t cos(2t+168.93°)]u(t)
8. 12.42
    1. [60−40te−2t−60e−2t]u(t)
    2. [12te−t+0.6e−t+3.35e−3tcos(4t+100.305°)]u(t)
9. 12.43
    1. [10te−t cos(t−90°)+10e−t cos(t−90°)]u(t)
    2. 5Δ′(t)−15Δ(t)+[10e−2t−4e−5t]u(t)
10. 12.46 zero at −3 rad/s, poles at 0 and two at −2 rad/s; zero at −5 rad/s, poles at (−3+j4) rad/s, (−3−j4) rad/s and two at −1 rad/s
11. 12.52
    1. 8, 0
    2. 8, 10
    3. 22, 0
    4. 250, 490
12. 12.57 0.947

Chapter 13

1. 13.4
    1. 0.025(s2+16×104s+1010)s
    2. zeros at (−80,000+j60,000) rad/s and (−80,000−j60,000) rad/s, pole at 0
2. 13.5
    1. 4×106ss2+2000s+64×104
    2. zero at 0, poles at −400 rad/s and −1600 rad/s
3. 13.9
    1. 

    2. −150(s+400)(s+1600)
    3. [50e−400t−200e−1600t]u(t) V

4. 13.15

    1. 

    where R=20 Ω, C=6.25 μF, γ= −48 V, L=6.4 mH, ρ = −2.4 A

    2. −48(s+8000)s2+8000s+25×106
    3. 2.4(s+4875)s2+8000s+25×106
    4. [80e −4000t cos(3000t+126.87°)]u(t) V
    5. [2.5e −4000t cos(3000t−16.26°)]u(t) A

5. 13.20

    1. [1−1e−250tcos 250t]u(t) A
    2. [141.42e −250t cos(250t−45°)]u(t) V
    3. Yes

6. 13.23 [17.68 cos(50t−135°)+18.02e −8.33t cos(39.965t+46.194°)]u(t) mA

7. 13.37 [63.25e−150t cos(50t+71.57°)]u(t) mA

8. 13.39 [96e−5t−96e−20t]u(t) V
9. 13.44
    1. 480(s+2.5)s(s+4)(s+6)
    2. [50+90e−4t−140e−6t]u(t) V
10. 13.51
    1. 200(s+200); no zeros, pole at −200 rad/s
    2. s(s+200); zero at 0, pole at −200 rad/s
    3. s(s+8000); zero at 0, pole at −8000 rad/s
    4. 8000(s+8000); no zeros, pole at −8000 rad/s
    5. 100(s+500); no zeros, pole at −500 rad/s
11. 13.56
    1. −104(s+5000)s(s+25,000)
    2. zero at −5000 rad/s, poles at 0 and −25,000 rad/s
    3. [−2000t−0.32+0.32e−25,000t]u(t) V
12. 13.62e −t V, 0≤t≤1 s; (1−e)e −t V, 1 s≤t ≤ ∞
13. 13.79 16.97cos(3t+8.13°) V
14. 13.91
    1. 0.8 A
    2. 0.6 A
    3. 0.2 A

4. −0.6 A
5. 0.6e−2×106t u(t) A
6. −0.6e−2×106t u(t) A
7. [−1.6×10−3Δ(t)−7200e−2×106t u(t)] V

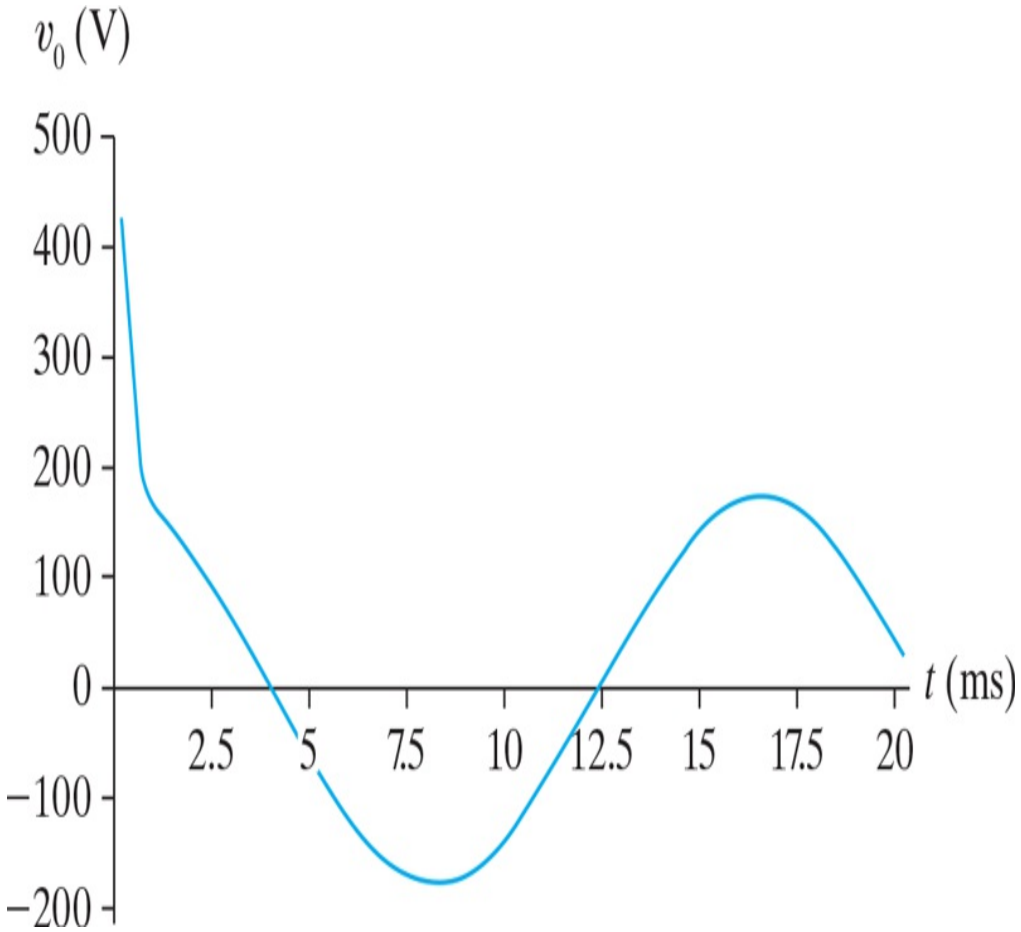
15. 13.94

    1. 0A, 252 A
    2. 1449π(122.922s−3000π2)(s+1475π)
       (s2+14,400π2) + 3002(s+1475π),

       [178.822e −1475πt+122.062 cos(120πt+6.85°)] V,

       3002 V
    3. 122.062 cos(120πt+6.85°) V

4.



Chapter 14

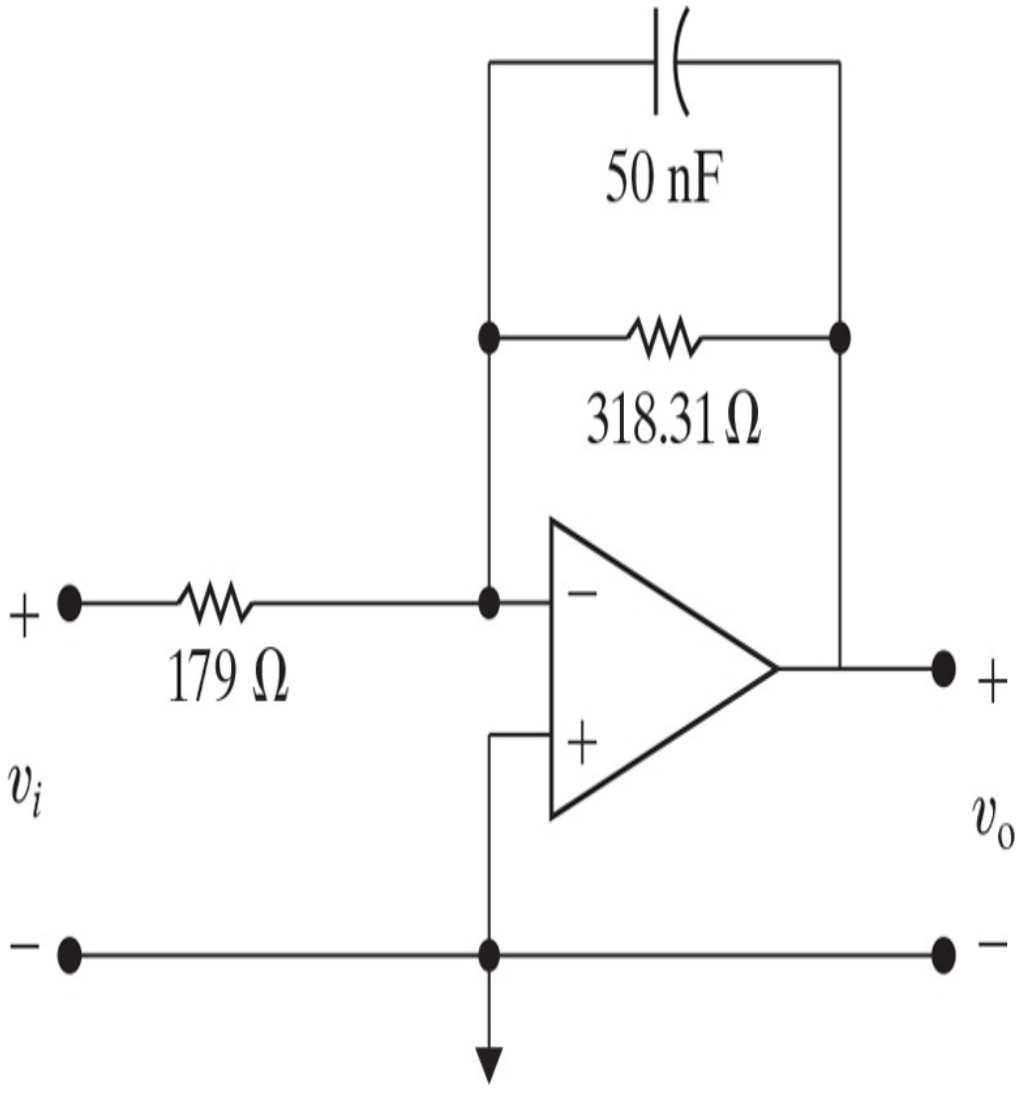
1. 14.1
    1. 2021.27 Hz
    2. 0.7071 −45°, 0.981 −11.31°,0.196 −78.69°
    3. 7.07 cos(12,700t−45°) V, 9.81 cos(2540t−11.31°) V
       1.96 cos(63,500t−78.69°) V
2. 14.3
    1. 31.42 Ω
    2. 895.77 Hz
    3. With 33 Ω resistor, 1050.4 Hz
3. 14.13
    1. 125 Ω
    2. 3 kΩ
4. 14.17
    1. 5.305 kΩ
    2. 333.86 Hz
5. 14.26
    1. 5 kΩ, 50 mH
    2. 3.52 kHz, 2.88 kHz

3. 636.62 Hz

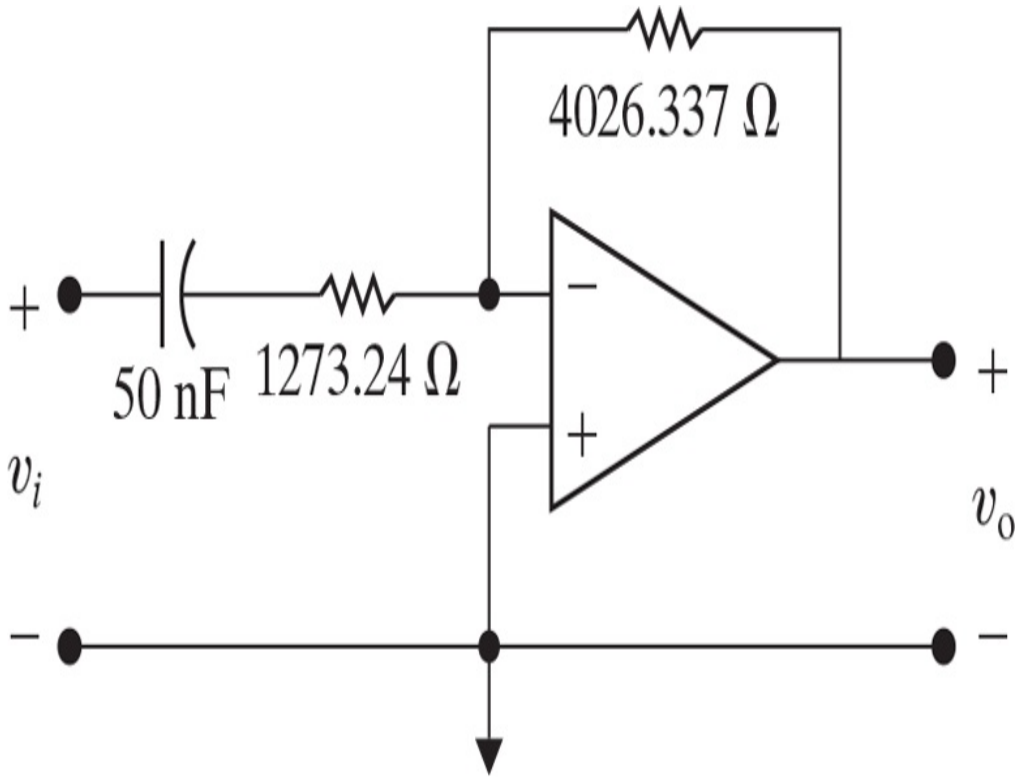
6. 14.34 4 kΩ
7. 14.42
    1. 397.89 Ω, 3.17 mH
    2. 4.42 kHz, 3.62 kHz
    3. 800 Hz
8. 14.51
    1. 0.39 H, 0.1 μF
    2. |V697 Hz|=|V941 Hz|=0.707|Vpeak|;

       |V770 Hz|=|V852 Hz|=0.948|Vpeak|
    3. 0.344|Vpeak|

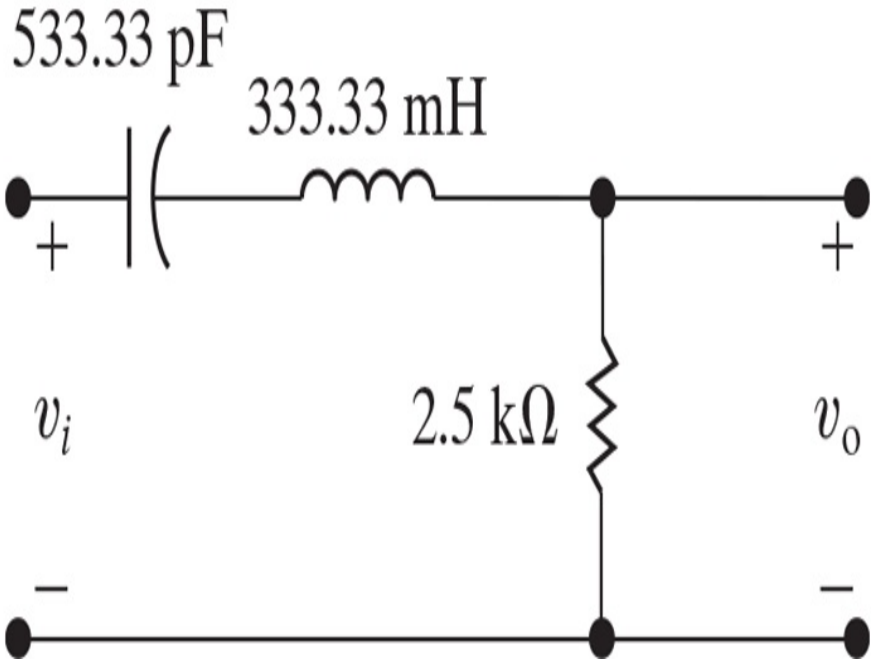
Chapter 15

1. 15.1
    1. 179 Ω, 318.31 Ω
    2. 

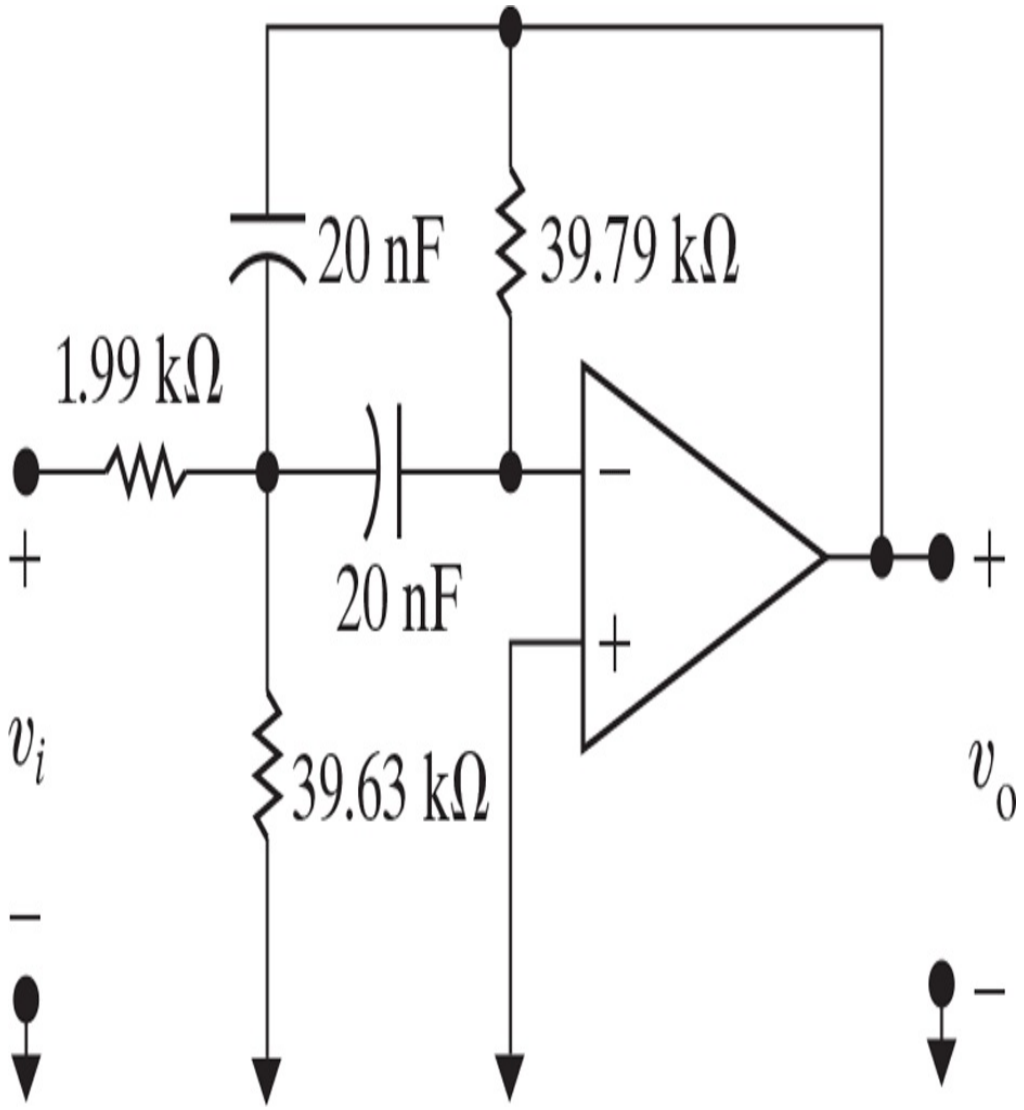
2. 15.8
    1. 1273.24 Ω, 4026.337 Ω

2. 

3. 15.15
    1. 1H, 0.1 Ω,1 F
    2. 333.33 mH, 2.5 kΩ, 533.33 pF

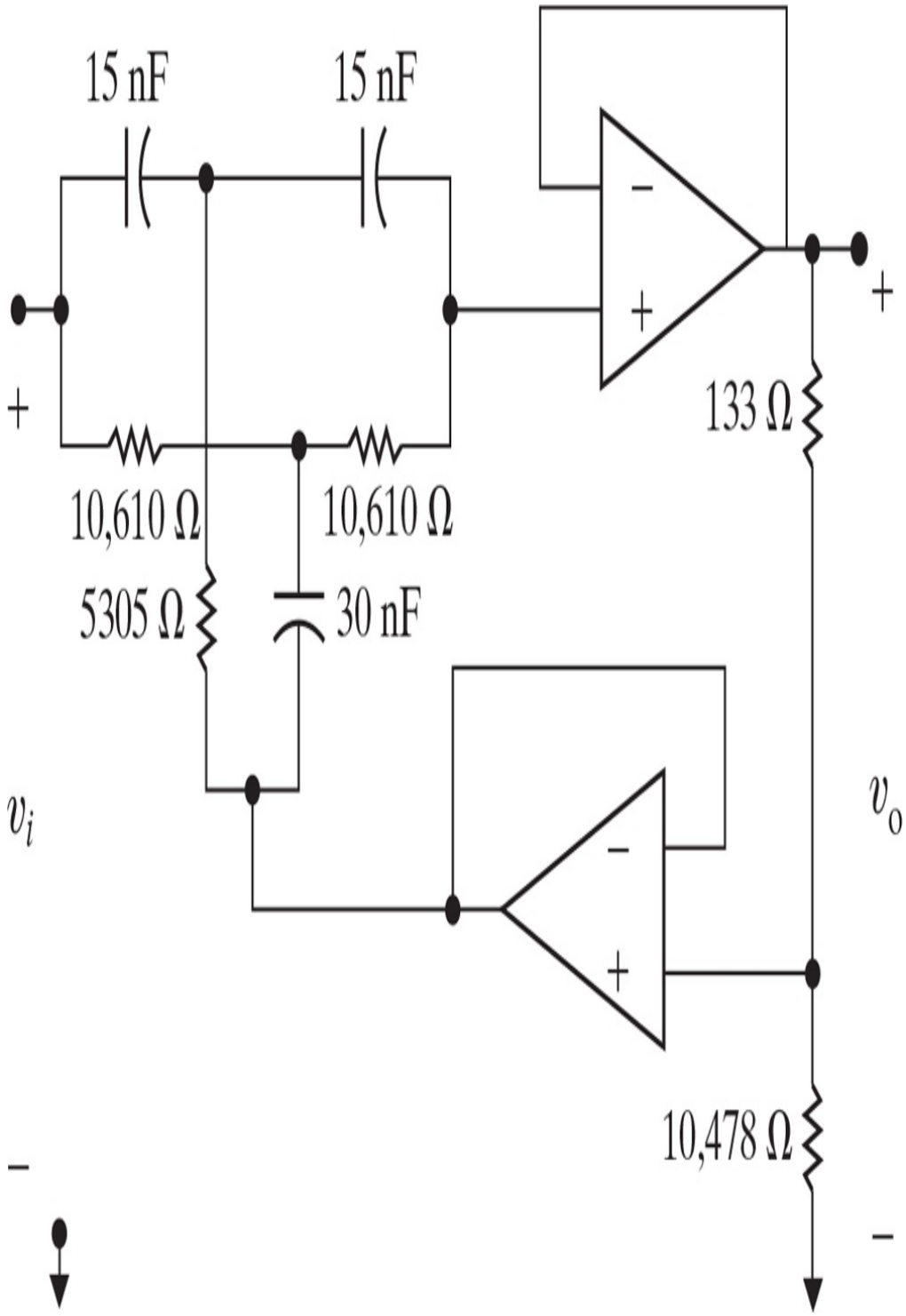
3. 

4. 15.23
    1. 2.4 H, 5 µF
    2. 0.57 cos(500t+15.14°) A
5. 15.30 61.553 Hz, 4061.553 Hz, 156.7 Ω, 10.34 kΩ
6. 15.31 RL=32.49 kΩ, RH=492.4 Ω, Ri=Rf=1 kΩ
7. 15.34
    1. 3
    2. −29.7 dB
8. 15.58
    1. R1=1.99 kΩ, R2=39.63 kΩ, R3=39.79 kΩ

2. 

9. 15.60

    1. See component values in part (b)

2. 

3. s2+4×106π2s2+100πs+4×106π2

10. [15.62 R1=100 kΩ, R2=400 kΩ, C1=7.96 nF]

Chapter 16

1. 16.1
    1. 31,415.93 rad/s, 3978.87 rad/s
    2. 5 kHz, 25 kHz
    3. 0, 25 V
    4. ava=0; for k even, aka=0, bka=0;

       for k odd, aka= −80πk sinπk2, bka=240πk;

       avb=25; bkb=0 for all k;

       akb=200πk sinπk4 for all k
    5. 80π∑n=1, 3, 5,…∞(−1n sinnπ2 cos nω0t+3n sin nω0t) V;

       [25+200π∑n=1∞(1n sinnπ4 cos nω0t)] V
2. 16.2 [160π+20 sin ω0t−320π∑n=2, 4, 6,…∞cos nω0tn2−1] V
3. 16.11
    1. 2π rad/s
    2. yes
    3. no
    4. no
4. 16.22
    1. 10∑n=1, 3, 5,…∞(nπ)2+4n2 cos(nω0t−θn) A, θn=tan−1nπ2

2. 26.23 A

5. 16.29

    1. [41.998 cos(2000t−0.60°)+13.985 cos(6000t −177.32°)+5.984 cos(14,000t+184.17°)] V

    2. Fifth harmonic

6. 16.35 1.85 W

7. 16.39

    1. 117.55 V(rms)

    2. −2.04%

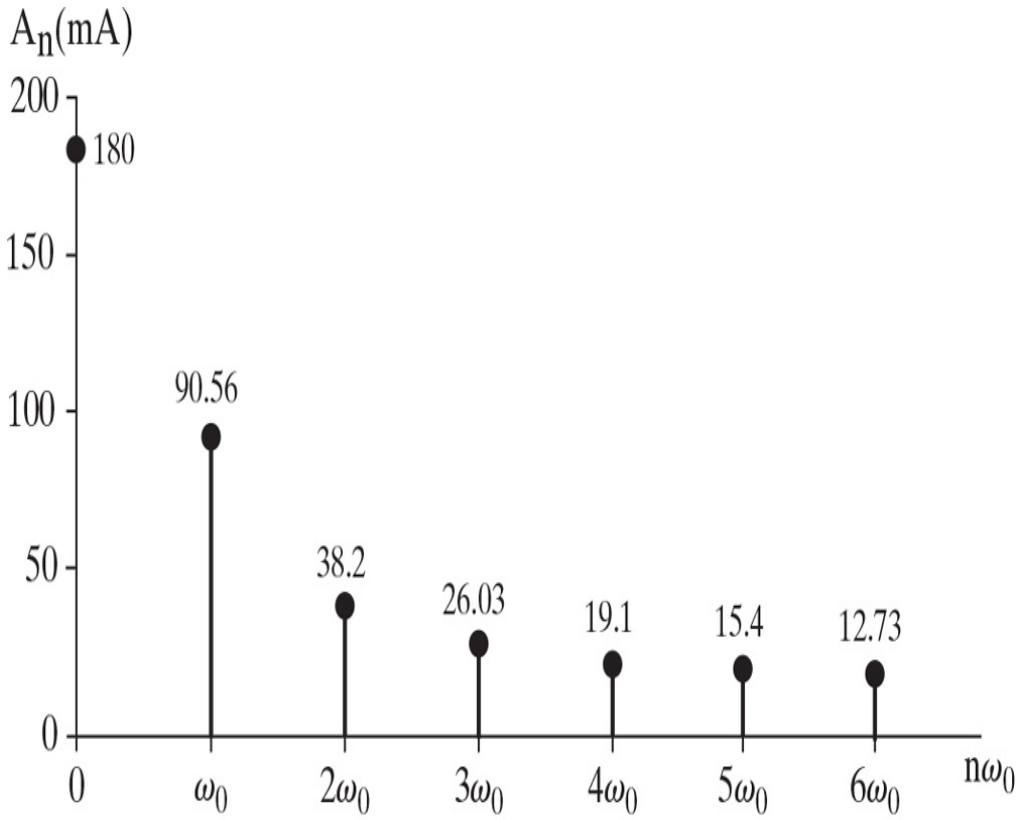
    3. 69.2765 V(rms), −0.0081%

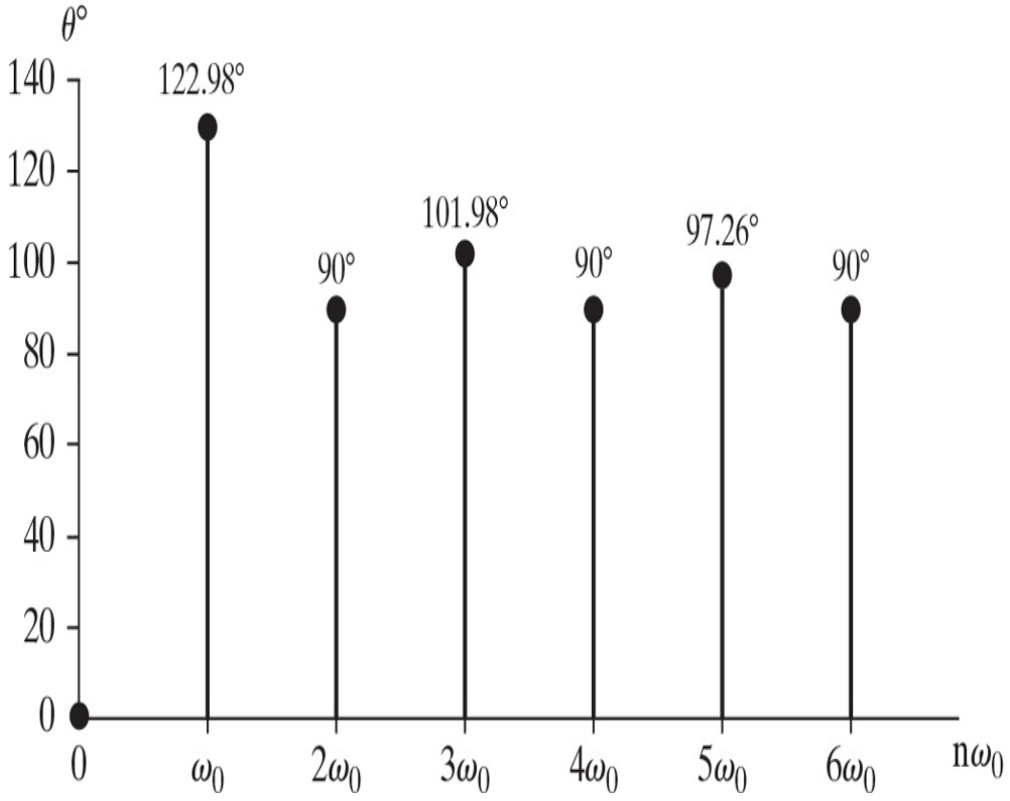
8. 16.44 $C0=Vm2$, $Cn=jVm2n\pi$

9. for n=±1, ±2, ±3,…

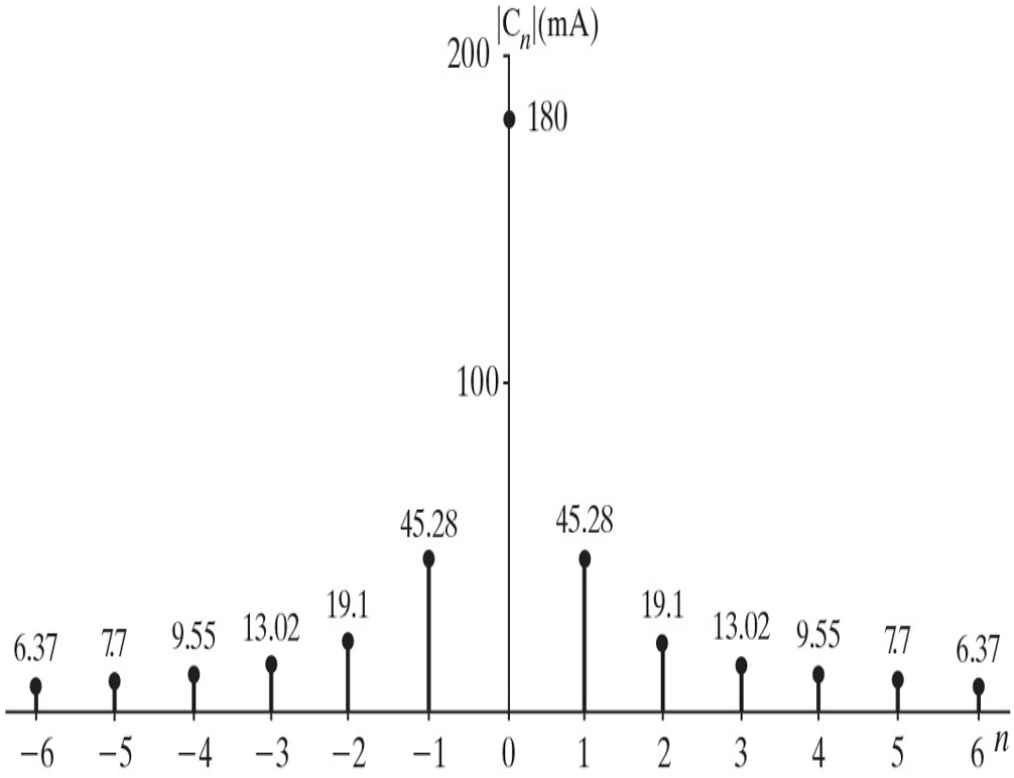
10. 16.50

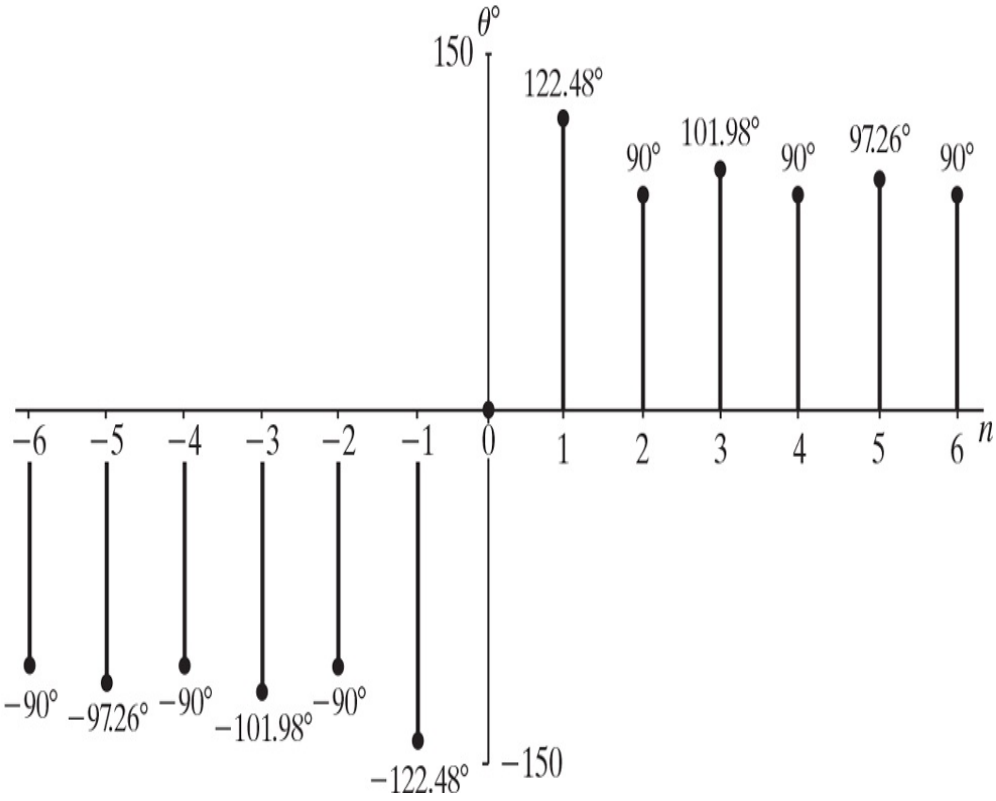
    1.


A_n(mA)
200
150
100
50
0
180
90.56
38.2
26.03
19.1
15.4
12.73
0
ω_0
$2\omega_0$
$3\omega_0$
$4\omega_0$
$5\omega_0$
$6\omega_0$
$n\omega_0$


$\theta°$
140
120
100
80
60
40
20
0
122.98°
90°
101.98°
90°
97.26°
90°
0
ω_0
$2\omega_0$
$3\omega_0$
$4\omega_0$
$5\omega_0$
$6\omega_0$
$n\omega_0$

2.


$|C_n|$(mA)
200
180
100
45.28
45.28
19.1
19.1
13.02
13.02
9.55
9.55
7.7
7.7
6.37
6.37
−6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 n

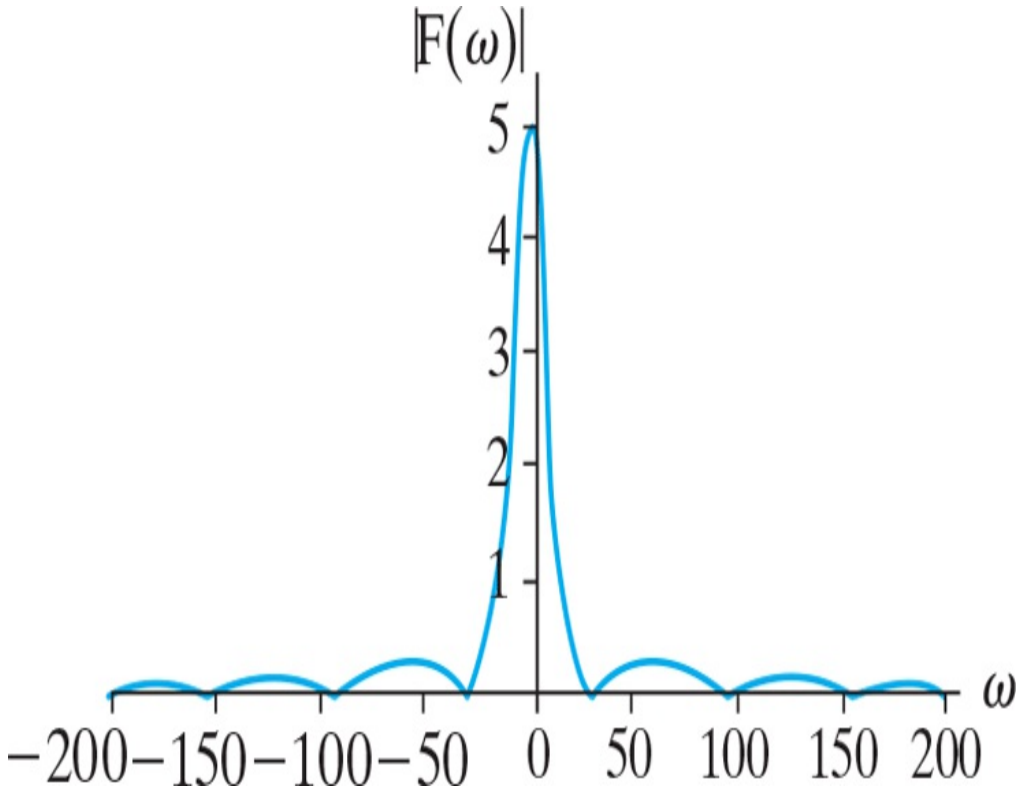

θ°
150
122.48°
90°
101.98°
90°
97.26°
90°
−6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 n
−90°
−97.26°
−90°
−101.98°
−90°
−122.48°
−150

11. 16.57

    1. 400313, 2000 rad/s, 16×108 rad2/sec2

    2. [−80 cos ω0t−0.50 cos(3ω0t+91.07°) + 0.17 cos(5ω0t+90.60°)] V

Chapter 17

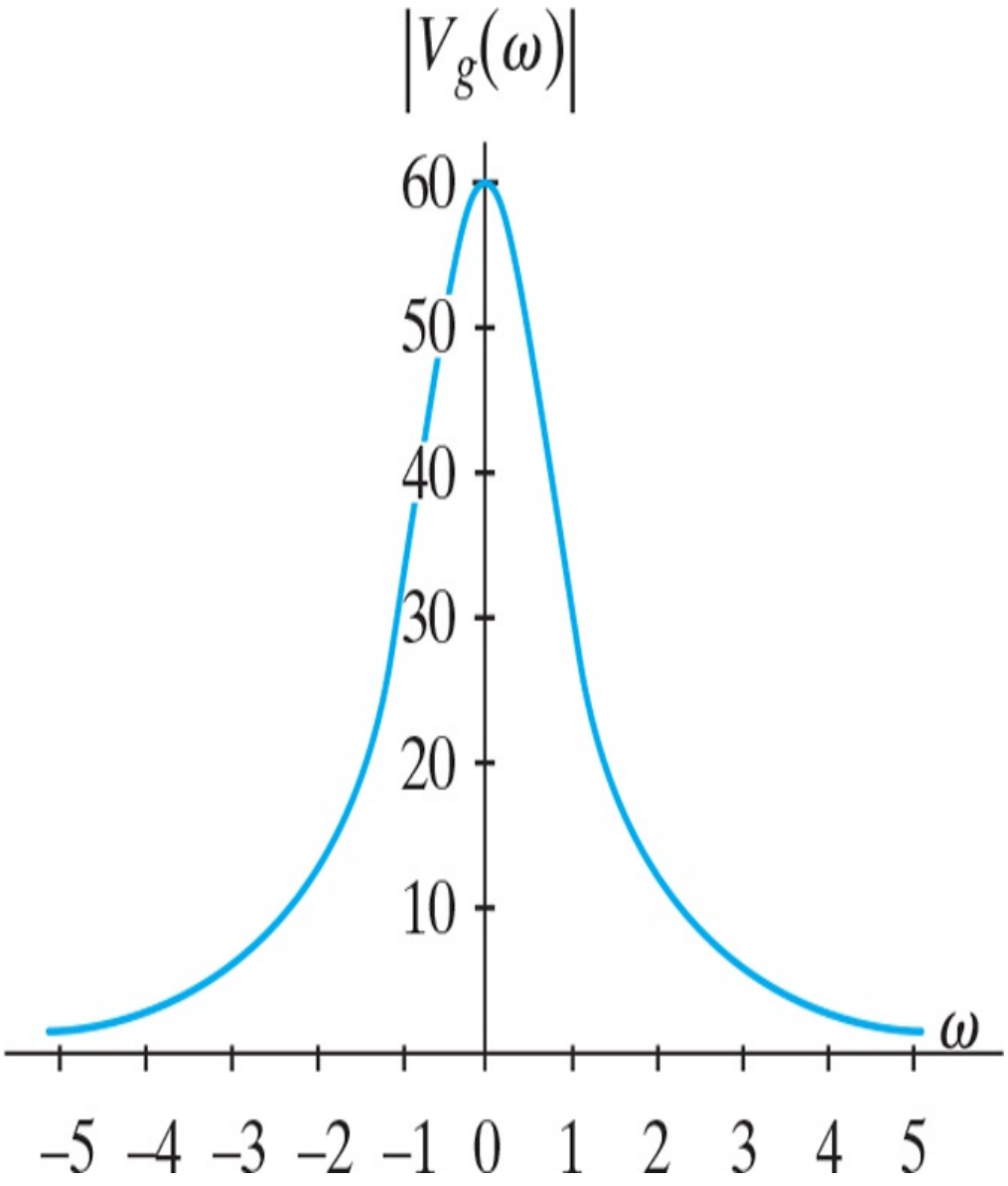
1. 17.1
    1. j2Aτ[ωτ cos(ωτ/2)−2 sin(ωτ/2)ω2]
    2. 0
    3. 

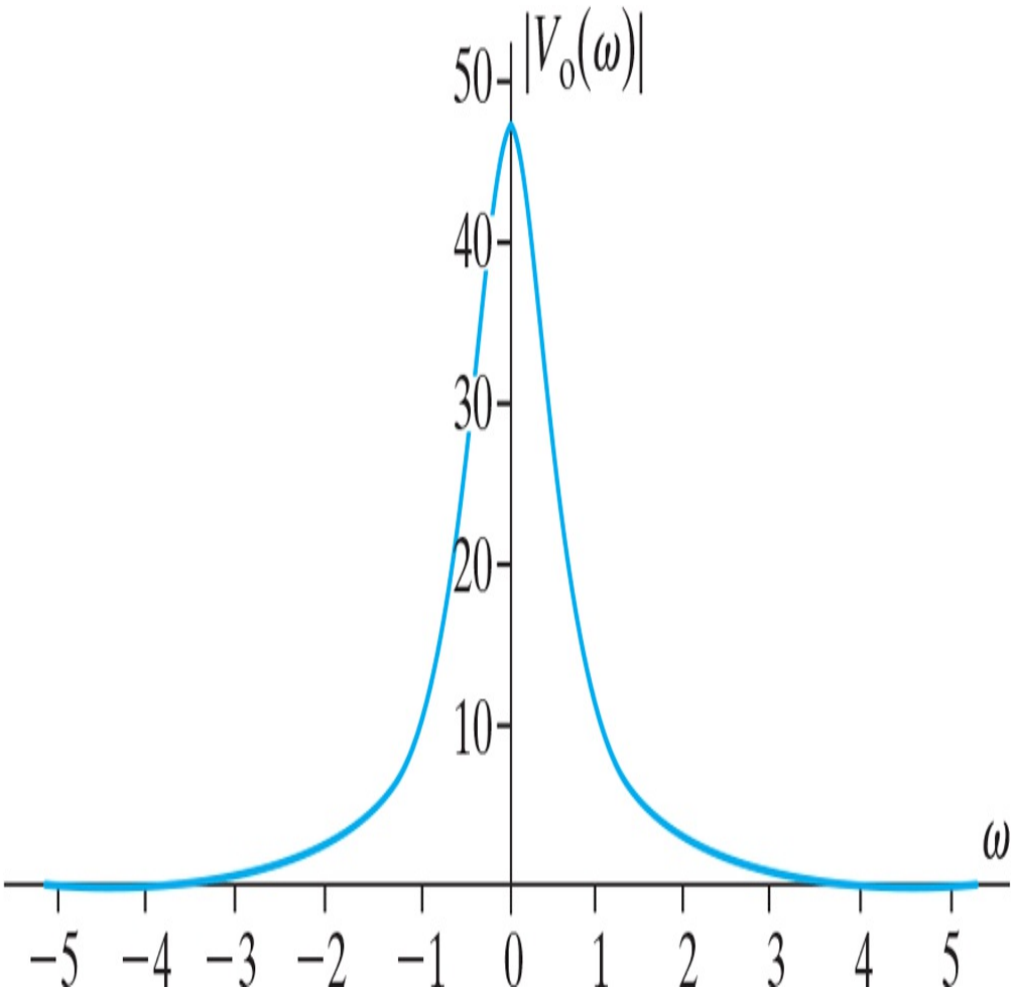
2. 17.5
    1. 2(a2−ω2)(a2+ω2)2
    2. −j48aω (a2−ω2)(a2+ω2)4
    3. aa2+(ω−ω0)2+aa2+(ω+ω0)2
    4. −jaa2+(ω−ω0)2+jaa2+(ω+ω0)2

5. $e^{-j\omega t_0}$

3. 17.19
    1. $\tau_2 \cdot \sin[(\omega+\omega_0)(\tau/2)](\omega+\omega_0)(\tau/2)+\tau_2 \cdot \sin[(\omega-\omega_0)(\tau/2)](\omega-\omega_0)(\tau/2)$
    2. $F(\omega) \rightarrow \pi[\Delta(\omega-\omega_0)+\Delta(\omega+\omega_0)]$
4. 17.28
    1. $[12.5e^{-t}-7.5e^{-5t}]u(t)+5e^{5t}u(-t)$ V
    2. 5 V
    3. 5 V
    4. $[12.5e^{-t}-7.5e^{-5t}]u(t)$ V
    5. Yes
5. 17.30 1cos(5000t+90°) A
6. 17.40
    1. $[(-24e^{-t}+32e^{-5t/2})u(t)+8etu(-t)]$ V

2.



3. 

4. 900 J

5. 320 J

6. 95.95%

7. 99.75%

Chapter 18

1. 18.2 z11=13 Ω; z12=12 Ω; z21=12 Ω; z22=16 Ω
2. 18.10 a11=−4×10 −4; a12= −20 Ω; a21= −0.5 μS; a22= −0.02
3. 18.13 g11=12.5 μS; g12=1.5;Ωg21= −250; g22=50 MΩ
4. 18.14
    1. y11=20 μS; y12=30 nS;y21=5 μS; y22=20 nS
    2. y11=20 μS; y12=30 nS;y21=5 μS; y22=20 nS
5. 18.21 a11=3; a12=40 Ω; a21=80+j60 mS;Ωa22=2.4+j0.8
6. 18.31
    1. 35.36 V(rms)
    2. 125 mW
    3. 18.75 μW
7. 18.33 12.5
8. 18.38 3750 V

Index

A

- a-, b-, and c-phase voltage references, 414
- ac circuits, balancing power in, 391–392
- Active circuit elements, 30
- Active filter circuits, 572–617
  - advantages of, 572, 608
  - bandpass filters, 580–583, 598–599, 600–602, 608–609
  - bandreject filters, 583–587, 598, 602–605, 608–609
  - bass volume control, 573, 605–607
  - block diagram for, 581
  - Bode plots for, 574–575, 580, 583–584, 587–588, 602, 605
  - broadband (low-Q) filters, 581–587, 608
  - Butterworth filters, 591–600, 608–609
  - cascading filters, 581–583, 587–591, 608–609
  - first-order, 574–577, 608
  - fourth-order, 589–590, 594–595
  - higher-order, 587–600, 608
  - high-pass filters, 576–577, 598, 608
  - low-pass filters, 574–575, 579, 587–591, 593–595, 608

    - narrowband (high-Q) filters, 600–605, 609
    - op amp filters, 575–577, 579, 580–600, 608
    - prototypes, 575–577, 608
    - scaling, 577–580, 608
- Active high-Q filter response, 619, 647–649
- Addition/subtraction operations, 454, 673, 729
- Admittance (Y), 336, 361
- Ammeter, 70, 71–72, 80
- Amplifier circuit analysis, 47, 102–103, 111, 125–126, 252–254, 256, 303–308, 310. See also Operational amplifiers
    - cascading connections, 303–308, 310
    - integrating-amplifier, 252–254, 256, 303–308, 310
    - Kirchhoff’s laws for, 47
    - mesh-current method, 111
    - node-voltage method, 102–103
    - Ohm’s law for, 47
    - resistor-inductor-capacitor (RLC) circuits for, 303–308, 310
    - Thévenin equivalent circuits for, 125–126
- Amplitude plots, 744–748, 751–753
    - accuracy of, 747–748
    - complex poles, 751–753

    - corrections to, 752–753
    - straight-line, 744–748, 751–753
- Amplitude spectrum, 645–647, 650, 663–664
    - Fourier series and, 645–647, 650
    - Fourier transform transition of, 663–664
- Amplitude, 320
- Analog meters, 71–72, 80
- Analysis methods, 97, 99, 102, 105, 107, 110, 154, 224, 230, 234, 238, 242, 280, 283, 287, 292, 299, 487
    - analyzing a circuit with an ideal op amp, 154
    - basics version of the mesh-current method, 105
    - basic version of the node-voltage method, 97
    - complete form of the mesh-current method, 110
    - complete form of the node-voltage method , 102
    - finding the RC natural response, 230
    - finding the RC step response, 238
    - finding the RL and RC natural and step response, 242
    - finding the RL natural response, 224
    - finding the RL step response, 234
    - Laplace-transform circuit analysis method, 487
    - modified step 3 for the mesh-current method, 107

    - modified step 3 for the node-voltage method, 99
    - natural response of an overdamped or underdamped parallel RLC circuit, 283
    - natural response of an overdamped parallel RLC circuit, 280
    - natural response of parallel RLC circuits, 287
    - natural response of series RLC circuits, 299
    - step response of parallel RLC circuits, 292
- Angular frequency (ω), 320
- Appliance power ratings, 381–382
- Artificial pacemaker design, 221, 255
- Attenuation, 536
- Average power (P), 377–384, 389, 394, 401, 425–426, 430–433, 435, 639–641, 650
    - absorbed, 394
    - appliance power ratings, 381–382
    - balanced three-phase circuits, 425–426, 430–433, 435
    - calculations for, 377–382, 389, 401, 425–426, 639–641
    - Fourier series applications, 639–641, 650
    - maximum power transfer (Pmax), 394
    - measurement of, 430–433, 435
    - periodic functions (f(t)) and, 639–641, 650

- power factor (pf) for, 379, 401
- root-mean-square (rms) value for, 382–384
- sinusoidal steady-state circuits, 377–382, 389, 394, 401
- wye (Y) loads, 425–426

B

- Back-substitution method, 721–722, 726
- Balanced three-phase circuits, 412, 416–417, 435. *See also* Three-phase circuits
- Bandpass filters, 538–539, 550–560, 565, 580–583, 598–599, 600–602, 608–609, 682–683
  - active, 580–583, 598–599, 600–602, 608
  - bandwidth (β) for, 550, 553–554, 556, 565
  - block diagram for, 581
  - Bode plot for, 580, 601
  - broadband, 581–583, 608
  - Butterworth, 598–599, 608
  - cascaded, 581–583
  - center frequency (ωo) for, 550, 552–554, 565
  - cutoff frequency (ωc) for, 552–553, 556
  - design of, 555–557
  - frequency and time domain relationships, 559–560
  - frequency of, 538–539
  - frequency response plot, 539
  - narrowband, 600–602, 609

    - op amp filters, 580–583, 598–599, 608
    - parallel RLC circuit, 555–557
    - parameters of, 550
    - Parseval’s theorem for, 682–683
    - qualitative analysis, 551
    - quality factor (Q) for, 550, 554, 556
    - quantitative analysis, 551–554
    - resonant frequency (ωo) for, 550
    - series RLC circuit, 551–555, 557–559
    - transfer function (H(s)) for, 559
- Bandreject filter, 538–539, 560–563, 565, 583–587, 598, 602–605, 608–609
    - active, 583–587, 598, 602–605, 608–609
    - Bode plots for, 584, 605
    - broadband, 583–587, 608
    - Butterworth, 598
    - center frequency (ωo) for, 562
    - cutoff frequency (ωc) for, 562
    - design of, 562–563
    - frequency of, 538–539
    - frequency response plot, 539

    - narrowband, 602–605, 609
    - op amp filters, 583–587, 608
    - parallel RLC circuit, 563
    - qualitative analysis, 560–561
    - quantitative analysis, 561–562
    - series RLC circuit, 560–563
    - transfer function (H(s)) for, 561, 563
- Bandwidth (β), 550, 553–554, 556, 562, 565
- Bass volume control circuit analysis, 573, 605–607
- Bilateral configurations, 115–116
- Black box concept, 60
- Block diagrams, 581
- Bode plots (diagrams), 574–576, 580, 583–584, 587–588, 602, 605, 743–756
    - active filter analysis using, 574–576, 580, 583–584, 588
    - amplitude plots, 744–748, 751–753
    - bandpass filters, 580, 602
    - bandreject filters, 583–584, 605
    - broadband (low-Q) filters, 581, 583–584
    - cascading filters, 587–588
    - complex poles, 750–756

    - complex zeros, 750–751
    - corner frequency for, 747–748
    - high-pass filters, 576
    - low-pass filters, 574–576, 588
    - magnitude plots, 587–588
    - narrowband (high-Q) filters, 602, 605
    - phase angle plots, 748–750, 754–756
    - real, first-order poles and zeros, 743–744
    - straight-line plots, 744–756
- Branches, 94–96
- Broadband (low-Q) filters, 581–587, 608
    - bandpass, 581–583, 608
    - bandreject, 583–587, 608
    - Bode plots for, 581, 583–584
- Butterworth filters, 591–600, 608–609
    - bandpass, 598–599, 608
    - bandreject, 598, 608
    - cascading, 591–600, 608–609
    - circuit analysis, 591, 593–595
    - high-pass, 598, 608

- low-pass, 591, 593–595
- order of, 596–597
- transfer functions (H(s)) for, 592–593
- transition region, 596–597

C

- Capacitance (C), 183, 189, 195–198, 209–210, 211
  - equivalent, 196–197
  - series–parallel combinations, 195–198
  - touch screens values of, 183, 209–210
- Capacitive circuits, power for, 379
- Capacitors, 182, 189–193, 195–196, 198–199, 211, 329–330, 361, 485–486, 514–515, 521, 760
  - circuit analysis of, 189–193, 514–515
  - circuit component values, 760
  - circuit parameter of, 189
  - current to voltage (i–v) relationships, 190
  - displacement current, 189–190
  - duality (symmetry) of, 198–199, 211
  - energy in, 182, 191, 211
  - equivalent circuits for, 196–197, 485–486, 521
  - impulse function (KΔ(t)) for, 514–515
  - in series, 195
  - Laplace transform method for, 485–486, 514–515, 521

    - multiple, 196–197
    - parallel, 195–196
    - passive behavior of, 182, 211
    - phasor relationships, 329–330, 361
    - power in, 182, 190–191, 211
    - s domain representation, 485–486, 521
    - symbols for, 189
    - voltage to current (v–i) relationships, 190, 329–330, 361
- Cascading connections, 303–308, 310, 581–583, 587–600, 608–609, 708–711
    - active filters, 581–583, 587–600, 608–609
    - bandpass filters, 581–583
    - Bode magnitude plot for, 588
    - Butterworth filters, 591–600, 608–609
    - fifth-order, 593
    - fourth-order filters, 589–590
    - high-pass filters, 598, 608
    - identical first-order filters, 587–591
    - integrating-amplifiers, 303–308, 310
    - low-pass filters, 587–591, 593–595, 608
    - op amp filters, 587–591

    - two-port circuits, 708–711
- Center frequency (ωo), 550, 552–554, 562, 565
- Characteristic determinant, 722–723
- Characteristic equations, 275–276, 293, 297, 299–301, 309
    - parallel RLC circuits, 275–276, 293, 309
    - series RLC circuits, 297, 299–301
- Circuit analysis, 11–15, 40–48, 68–70, 75–80, 92–149, 150–181, 220, 224, 230, 234, 238, 241–242, 246–254, 256, 273, 308–309, 318–373, 482–535, 536–571, 573, 605–607, 677–679, 681–685
    - amplifiers, 47, 102–103, 111, 125–126
    - bass volume control, 573, 605–607
    - circuit design and, 11–12
    - circuit models for, 13
    - clock for computer timing, 273, 308–309
    - component models for, 13
    - current division, 69–70, 79–80, 336
    - delta-to-wye (Δ-to-Y) transformations, 75–78, 80, 338–340
    - dependent sources and, 45–48, 98–100, 107–108, 122
    - Fourier transforms for, 677–679, 681–685
    - frequency-selective circuits, 536–571
    - household distribution, 319, 359–360

- ideal basic circuit element, 14
- ideal circuit components, 11–12
- integrating-amplifiers, 252–254, 256
- Kirchhoff’s laws for, 40–44, 487
- Laplace transform in, 482–535
- Laplace transform method for, 487–488, 521
- maximum power transfer, 92, 126–128, 135
- mesh-current method, 92, 104–115, 134–135, 354–347
- natural response method, 220, 224, 230, 241–242, 256
- node-voltage method, 92, 96–104, 112–115, 134–135, 344–345
- nonplanar circuits for, 94
- Norton equivalent circuits for, 92, 120–123, 135
- Ohm’s law for, 42–43, 486
- operational amplifiers (op amps), 150–181
- passive sign convention for, 14–15
- physical prototypes, 12
- planar circuits for, 94
- resistive circuits, 68–70, 79–80, 93, 115–118, 131–134
- s domain, 486–488, 521
- sensitivity analysis, 93, 131–134

    - sequential switching and, 246–250, 256
    - simultaneous equations for, 94–96
    - sinusoidal steady-state, 318–373
    - source transformation, 92, 115–118, 121, 134, 340–343
    - step response method, 220, 234, 238, 241–242, 256
    - superposition, 92, 129–131, 135
    - surge suppressors and, 483, 520
    - terminal behavior and, 118–126, 152–156
    - terminals, 118–126
    - Thévenin equivalent circuits for, 92, 118–126, 135
    - transformers, 347–356, 361
    - voltage division, 68–70, 79, 334
- Circuit components, 11–13, 37, 324, 578, 760
    - electrical behavior of, 37
    - ideal, 11–12
    - models, 13, 37
    - scale factors, 578
    - steady-state current, 324
    - transient current, 324
    - values, 760

- Circuit elements, 26–57, 194–209, 211, 484–486, 521
  - active, 30
  - capacitance (C), 195–198, 211, 485–486, 521
  - current sources, 28–31, 50
  - dependent sources, 28, 31, 45–48, 50
  - electrical radiator examples of, 27, 48–49
  - electrical sources, 28
  - equivalent circuits in s domain, 485–486, 521
  - in series, 41, 50
  - inductance (L), 194–209, 211, 484–485, 521
  - Kirchhoff’s laws for, 39–44, 45–47, 50
  - Laplace transform method and, 484–487
  - loops (closed path), 40, 50
  - model construction, 36–38
  - nodes, 39, 50
  - Ohm’s law for, 32–33, 42–43, 45–47, 50, 484
  - passive, 30
  - resistance (R), 32–35, 484, 521
  - s domain representation, 484–486, 521
  - symbols for, 28–29

    - voltage sources, 28–31, 50
- Circuit models, 13, 27, 32–38, 47–49, 154–155, 167–170, 172–173, 692, 712
    - advantage of, 13
    - amplifier, 47
    - approximation for, 38
    - construction of, 36–38, 43–44
    - electrical behavior of components, 37
    - electrical effects from, 37
    - electrical radiator, 27, 48–49
    - flashlight (electrical system), 36–38, 39–41
    - operational amplifiers (op amps), 154–155, 167–170, 172–173
    - resistors, 32–35
    - terminal measurements for, 38, 43–44
    - two-port, 692, 712
- Circuit theory, 7–8
- Circuits, 2–25, 32–35, 58–91, 445, 474–476, 483, 520, 537, 564
    - analysis of, 11–12
    - current (i), 12–13, 15–17, 33
    - current-divider, 67, 79
    - delta-to-wye (Δ-to-Y) equivalent, 75–78, 80

- electrical charge, 12–13
- electrical effects, 7
- electrical engineering and, 2–9
- energy and, 15–17
- frequency (f) and, 7
- ideal basic element, 14–15
- International System of Units (SI) for, 9–11
- lumped-parameter systems, 7–8
- magnetic coupling, 7
- net charge, 7
- open, 37
- passive sign convention for, 14–17
- power, 3, 15–20, 33–34
- pushbutton telephone, 537, 564
- resistive, 32–35, 58–91
- short, 37
- surge suppressors for, 483, 520
- transient effects on, 445, 474–476
- voltage (v), 12–13, 15–17, 34
- voltage-divider, 64–66, 79

    - Wheatstone bridge, 73–75, 80
- Clock for computer timing, circuit analysis of, 273, 308–309
- Closed path (loop), 40, 50
- Coefficient of coupling, 207–208, 211
- Common mode input, 164
- Common mode rejection ratio (CMRR), 165–167, 173
- Communication systems, 4–5
- Complex numbers, 727–732
    - addition/subtraction of, 729
    - arithmetic operations, 729–730
    - graphical representation of, 728–729
    - identities for, 730–731
    - integer power of, 731
    - multiplication/division of, 729
    - notation for, 727–728
    - polar form, 727–728
    - rectangular form, 727–728
    - roots of, 731–732
- Complex power, 384–393, 401, 426–427
    - apparent power (magnitude), 385, 401

    - balanced three-phase circuits, 426–427
    - balancing power in ac circuits, 391–392
    - calculations for, 384–393, 401
    - defined, 384
    - delta (Δ) loads, 426–427
    - parallel loads and, 390–391
    - phasors for, 387–388
    - power calculations for, 384–393, 401, 426–427
    - power triangle relationship, 385
    - units for, 384–385, 401
    - wye (Y) loads, 426
- Computer systems, 5
- Condition of equivalence, 116
- Conductance (G), 33, 336
- Continuous functions, 446–447
- Control systems, 5
- Convolution, 505–511, 521, 674–675
    - Fourier operational transforms, 674–675
    - frequency domain, 675
    - integral, 505–511, 521, 675

    - memory, concept of using, 510–511
    - output signal from, 509–510
    - time domain, 674
    - transfer function (H(s)) in, 505–511, 521
    - weighting function and, 510
- Cosine functions, 631–632, 650, 669–670
- Cramer’s method, 722–724
- Critically damped response, 286–289, 291–293, 297–299, 300–301, 309
    - natural response, 286–289, 297–300, 309
    - parallel RLC circuits, 286–289, 291–293, 309
    - series RLC circuits, 297, 297–299, 301
    - step response, 291–293, 300–301
- Current (i), 12–17, 33, 39–40, 41, 50, 58, 70–73, 80, 152–156, 172, 184–186, 189–193, 222–224, 230, 289–290, 318, 320–321, 324, 327–333, 353–355, 417, 418–419, 422–423, 435, 639–641
    - balanced three-phase circuits, 417, 418–419, 422–423, 435
    - capacitors, 189–193
    - defined, 13
    - displacement, 189–190
    - dot convention for, 354–355
    - electric charge and, 12–14

- Fourier series applications, 639–641,
- frequency domain and, 327–333, 353–355
- ideal transformer ratios, 353–355
- inductor relationships, 184–186
- initial inductor (I0), 223
- input constraint, 154
- Kirchhoff’s current law (KCL), 39–40, 41, 50, 332–333
- line, 417, 418–419, 422–423, 435
- measurement of, 58, 70–73, 80
- natural response and, 222–224, 230
- Ohm’s law for, 33, 50
- op-amp terminals, 152–156, 172
- periodic, 639–641
- phase, 418–419, 422–423, 435
- polarity of, 354–355
- polarity reference, 14
- power and energy relationship to, 15–17
- resistor power in terms of, 33
- resistor-capacitor (RC) circuit expression, 230
- resistor-inductor (RL) circuit expression, 222–224

    - sinusoidal source, 320–321
    - steady-state analysis and behavior of, 318, 324, 327–333, 353–355
    - steady-state component, 324
    - step response inductor, 289–290
    - transient component, 324
- Current coil, 430, 435
- Current-divider circuit, 67, 79
- Current division, 69–70, 79–80, 336
    - frequency domain, 336
    - resistive circuit analysis, 69–70, 79–80
- Current sources, 28–31, 50
- Current to voltage (i–v) relationships, 185–186, 190
- Cutoff frequency (ωc), 540–542, 544, 552–553, 556, 562, 564
    - bandpass filters, 552–553, 556
    - bandreject filter, 562
    - bandwidth (β) relationship to, 553
    - center frequency (ωo) relationship to, 553
    - defined, 540–542, 564
    - half-power frequency, 541
    - low-pass filters, 540–542, 544

- RL circuit filters, 544
- RLC circuit filters, 553, 556

D

- d’Arsonval meter movement, 71
- Damped radian frequency (ωd), 282, 293, 299, 301
- Decaying exponential function, 452
- Decibels (dB), 741–742
- Delta (Δ) interconnection, 76
- Delta (Δ) loads, 426–427
- Delta-to-wye (Δ-to-Y) transformations, 75–78, 80, 338–340
  - equivalent circuits, 75–78, 80
  - frequency domain, 338–340
- Dependent sources, 28, 31, 45–48, 98–100, 107–108, 122, 130–131
  - analysis of circuits with, 45–48, 98–100, 107–108, 130–131
  - circuit elements as, 28, 50
  - interconnections of, 31
  - mesh-current method for, 107–108
  - node-voltage method for, 98–100
  - Ohm’s law for, 45–47
  - superposition for, 130–131
  - Thévenin equivalent circuits of, 122

- Derived units, 10
- Difference-amplifier circuit, 162–167, 173
  - common mode input, 164
  - common mode rejection ratio (CMRR) for, 165–167, 173
  - differential mode input, 164
  - ideal op-amp model for, 162–167, 173
  - negative feedback in, 162
- Differential mode input, 164
- Differentiation, operational transforms for, 454–455, 673
- Digital meters, 73, 80
- Digital signal filtering, 661, 685–686
- Direct approach for Fourier series, 635–637
- Dirichlet’s conditions, 621
- Discontinuities of circuits, 447–450, 476. See also Impulse function; Step function
- Discontinuous functions, 446–447
- Displacement current, 189–190
- Domain translation, Laplace transform for, 456–457
- Dot convention, 199–201, 211, 354–355
  - ideal transformers, 354–355
  - mutual inductance, 199–201, 211

    - polarity and, 199–201, 211, 354–355
    - procedure for determining, 200–201
- Duality, 104, 198–199, 211. See also Symmetry
- Dual-tone multiple-frequency (DTMF) design, 537
- Dynamo, 28

E

- Effective value, 383
- Efficiency, power system optimization for, 126
- Electric power transmission and distribution, 413, 433–435
- Electrical charge, 12–13
- Electrical engineering, 2–9
  - balancing power, 3
  - circuit theory for, 7–8
  - communication systems, 4–5
  - computer systems, 5
  - control systems, 5
  - interaction of systems, 6
  - power systems, 5
  - problem-solving strategy, 8–9
  - profession of, 2
  - signal-processing systems, 5
- Electrical radiator circuits, 27, 48–49
- Electrical sources, 28. See also Sources
- Electrodynamic wattmeter, 430–433, 435

- Energy, 15–17, 182, 187–189, 191, 207–209, 211, 226, 230, 374, 679–685, 686
  - capacitors and, 182, 191, 211
  - inductors and, 182, 187–189, 211
  - mutual inductance and, 207–209, 211
  - natural response and, 226, 230
  - power and, 15–17
  - Parseval’s theorem for, 679–685, 686
  - power calculations for delivery of, 374
  - resistor-capacitor (RC) circuit expression, 230
  - resistor-inductor (RL) circuit expression, 226
  - storage in magnetically coupled coils, 207–209, 211
- Equivalent capacitance (Ceq), 197
- Equivalent circuits, 61–62, 194–197, 418–421, 435, 484–486, 521, 733–740
  - capacitance (Ceq), 196–197, 485–486, 521
  - ideal transformers in, 737–740
  - inductance (Leq), 194–195, 196, 484–485, 521
  - magnetically coupled coils and, 733–737
  - π-equivalent circuit, 734–735
  - resistance (Req), 61–62, 484, 521

    - s domain representation, 484–486, 521
    - single-phase, 418–421, 435
    - T-equivalent circuit, 733
- Equivalent inductance (Leq), 196
- Essential branches and nodes, 94–96
- Even periodic function, 625–626
- Exponential form of Fourier series, 642–644, 650

F

- Faraday’s law, 203–204
- Feedback, see Negative feedback
- Feedback resistors, 305–308, 310
- Filters, 536, 538–539. See also Active filter circuits; Frequency; Frequency-selective circuits
- Final-value theorem, 472–474, 477
- First-order active filters, 574–577, 608
    - Bode plots for, 574–576
    - frequency response plots for, 574–575
    - high-pass filters, 576–577, 608
    - low-pass filters, 574–575, 608
    - op amp filter design, 575, 576–577
    - prototypes, 575–577, 608
- First-order circuits, 220, 222, 256. See also Resistor-capacitor (RC) circuits; Resistor-inductor (RL) circuits
- Flashlight (electrical system) circuit model, 36–38, 39–41
- Fourier coefficients, 621–630, 649
    - even-function symmetry, 625–626
    - Fourier series of periodic function found with, 630

    - Fourier series of triangular waveform using, 623–624
    - half-wave symmetry, 627–628
    - odd-function symmetry, 626–627
    - periodic functions (f(t)) for, 621–630
    - quarter-wave symmetry, 628–629
    - symmetry effects on, 625–630, 649
    - trigonometric form of, 622–624
- Fourier series, 618–659
    - active high-Q filter response, 619, 647–649
    - amplitude spectrum, 645–647, 650
    - average-power calculations, 639–641, 650
    - direct approach, 635–637
    - exponential form of, 642–644, 650
    - Fourier coefficients, 621–630, 649
    - fundamental frequency (ωo), 621, 649
    - harmonic frequency, 621, 649
    - periodic functions (f(t)), 618, 620–622, 639–642, 649
    - periodic response and, 618, 620
    - periodic voltage applications, 631–637, 639–641, 645–647
    - phase spectrum, 645–647, 650

    - phasor domain circuit transformation, 631–632, 650
    - quality factor (Q), 619
    - RC circuit application, 633–637
    - root-mean-square (rms) value, 641–642, 650
    - sine and cosine terms for, 631–632, 650
    - steady-state response and, 621–622, 633–638, 650
    - symmetry functions, 625–630, 649
    - waveforms, 618, 620, 623–629, 633, 635–636, 650
- Fourier transform, 660–691
    - amplitude spectrum, 663–664
    - circuit applications, 677–679, 681–685
    - convergence of integral, 664–666
    - derivation of, 662–664
    - digital signal filtering, 661, 685–686
    - frequency-domain (F(ω)), 662–663, 671–672
    - frequency-domain representation, 660, 686
    - integrals used in, 662, 664–666, 671–672
    - inverse, 663
    - Laplace transforms for, 666–668
    - limiting values, 662–663, 665–670

    - mathematical properties of, 671–672
    - operational transforms, 672–676
    - Parseval’s theorem for, 679–685, 686
    - periodic to aperiodic transition, 660, 663, 686
    - periodic voltage pulse from, 663
    - steady-state response from, 678
    - time-domain (f(t)), 662–663, 671–672
    - transient response from, 677–678
- Fourth-order filters, 589–590, 594–595
- Frequency (ω), 324, 361, 538–542, 544, 550, 552–554, 564–565, 621, 649
    - bandpass filter, 538–539
    - bandreject filter, 538–539
    - center (ωo), 550, 552–554, 565
    - cutoff (ωc), 540–542, 544, 552–553, 562, 564
    - fundamental (ωo), 621, 649
    - half-power, 541
    - harmonic, 621, 649
    - high-pass filter, 538
    - infinite, 544
    - low-pass filter, 538–542, 544

    - passband, 538, 564
    - passive filters, 539, 549, 559, 565
    - resonant (ωo), 550
    - steady-state response, 324, 361
    - stopband, 538, 564
    - zero, 544
- Frequency domain, 327–357, 361, 446, 457, 460–474, 476–477, 545, 559–560, 660, 662–663, 671–672, 674–675, 686
    - bandpass filters, 559–560
    - convolution in, 675
    - current division in, 336
    - delta-to-wye (Δ-to-Y) transformations, 338–340
    - final-value theorem for, 472–474, 477
    - Fourier transform (F(ω)), 660, 662–663, 671–672, 674–675, 686
    - frequency-selective circuit analysis, 545, 559–560
    - impedance (Z) and, 330–331, 333–338, 340–343, 361
    - initial-value theorem for, 472–474, 477
    - inverse Laplace transforms for, 460–470, 476
    - Kirchhoff’s laws in, 332–333
    - Laplace transform (F(s)), 446, 457, 460–474, 476–477
    - low-pass filters, 545

    - node-voltage method in, 344–345
    - Norton equivalent circuits for, 340–341
    - operational Laplace transform for, 457
    - operational transforms, 674–675
    - parallel impedance combination, 335, 337
    - partial fraction expansion, 461–470
    - passive circuit elements in, 327–331
    - phasor transforms and, 325, 327–331
    - poles of F(s), 470–472, 476
    - rational functions (F(s)) and, 460–470
    - series impedance combination, 334–335, 337
    - source transformations for, 340–343
    - steady-state circuit analysis in, 327–357, 361
    - Thévenin equivalent circuits for, 340–341, 343
    - time domain (t) relationships, 446, 472–474, 477, 545
    - translation in, 457, 674
    - voltage division in, 334
    - voltage to current (v–i) relationships in, 330–331
    - zeros of F(s), 470–472, 476
- Frequency of lumped-parameter systems, 7

- Frequency response, 536
- Frequency response plots, 538–539, 574–576, 580, 584
  - bandpass filter, 539
  - bandreject filter, 539
  - Bode plots, 574–576, 580, 584
  - first-order active filters, 574–576
  - high-pass filter, 538, 576
  - ideal, 538–539
  - low-pass filter, 538, 575
  - magnitude plot, 538
  - phase angle plot, 538
- Frequency scaling, 578, 608
- Frequency-selective circuits, 536–571
  - attenuation of, 536
  - bandpass filters, 550–560, 565
  - bandreject filters, 560–563, 565
  - center frequency (ωo) for, 550, 552–554, 565
  - cutoff frequency (ωc) for, 540–542, 544, 552–553, 556, 562, 564
  - filter categories, 538–539
  - filters as, 536, 564

    - frequency and time domain relationships, 545, 559–560
    - high-pass filters, 545–550, 564–565
    - low-pass filters, 539–545, 564
    - parallel RLC, 555–557, 563
    - pushbutton telephone circuits, 537, 564
    - qualitative analysis, 539–541, 544, 546, 551, 560–561
    - quantitative analysis, 542–543, 546–547, 551–554, 561–562
    - series RC, 543–544, 546–547
    - series RL, 539–540, 542–543, 547–548
    - series RLC, 551–555, 557–559, 560–563
    - transfer function (H(s)) for, 545, 549, 559, 561, 563
- Functional Laplace transforms (t), 447, 452–453, 476
- Fundamental frequency (ω0), 621, 649

G

- Gain, 153
- Galvanometer, 73
- Generator, 28
- Graphical representation of complex numbers, 728–729

H

- Half-power frequency, 541
- Half-wave periodic function, 627–628
- Harmonic frequency, 621, 649
- High-pass filters, 538, 545–550, 564–565, 576–577, 598, 608
  - active, 576–577, 598, 608
  - Bode plot for, 576
  - Butterworth, 598, 608
  - cascading, 598, 608
  - design of, 547–548, 576–577
  - first-order, 576–577, 608
  - frequency of, 538
  - frequency response plots for, 538, 576
  - frequency-selective circuit analysis, 538, 545–550, 564–565
  - loading, 548
  - op amp filter design, 576–577
  - prototypes, 576–577, 608
  - qualitative analysis, 546
  - quantitative analysis, 546–547

    - second-order, 598, 608
    - series RC circuits, 546–547
    - series RL circuits, 547–548
    - transfer function (H(s)) for, 549, 565
- Household appliance ratings, 381–382
- Household distribution circuit, 319, 359–360
- Hybrid parameters, 696, 697

I

- Ideal basic circuit element, 14–15
- Ideal transformers, 351–356, 361, 397–398, 737–740
  - current (i) ratios, 353–355
  - dot convention for, 354–355
  - equivalent circuits with, 737–740
  - frequency domain analysis, 351–356
  - impedance matching using, 356
  - limiting values of, 351–353
  - maximum power transfer in, 397–398
  - polarity of voltage and current, 354–355
  - steady-state analysis of, 351–356, 361
  - symbol for, 354
  - voltage (v) ratios, 353–355
- Ideal versus realistic op-amp models, 173, 14–15
- Identities for complex numbers, 730–731
- Immitance, 696
- Impedance (Z), 330–331, 333–338, 340–343, 348–349, 361, 393–397, 422

- admittance (Y) and, 336, 361
- balanced three-phase circuits, 422
- conductance (G) and, 336
- current division and, 336
- defined, 330
- frequency domain simplifications, 330–331, 333–338, 340–343, 361
- linear transformer circuit analysis using, 348–349, 361
- maximum power transfer conditions and restrictions from, 393–397
- parallel combination, 335, 337
- passive circuit elements, 330–331
- phasors and, 330–331
- reactance and, 331
- reflected (Zr), 348–349, 361
- self-, 348
- series combination, 334–335, 337
- source transformations using, 340–343
- steady-state analysis using, 330–331, 333–338, 361, 348–349
- susceptance (B) and, 336
- voltage division and, 334
- voltage to current (v–i) relationships and, 330–331

    - wye- and delta-connected relationships, 422
- Impedance matching, 356
- Improper rational functions, 460, 469–470
- Impulse function (KΔ(t)), 449–451, 476, 514–520, 521
    - capacitor circuit analysis, 514–515
    - circuit analysis using, 514–520, 521
    - derivatives of, 451
    - discontinuities of circuits and, 449–450
    - impulsive sources, 517–520
    - inductor circuit analysis, 515–517
    - Laplace transform method and, 514–520, 521
    - Laplace transform of, 450–451
    - sifting property, 450–451
    - strength (K) of, 449, 476
    - switching operations, 514–517
    - unit impulse function (Δ(t)), 449, 476
    - variable parameter function, 449–450
    - voltage drop and, 516–517
- Independent sources, 28, 31, 50
- Induced voltage, 199–201, 203–204, 211

- Inductance (L), 184, 194–209, 211, 347–348, 351–352
  - circuit parameter of, 184
  - equivalent, 194–195, 196
  - mutual, 182, 199–209, 211
  - series–parallel combinations, 194–198
  - self-, 199–200, 203–204, 206–207, 211, 347–348, 351–352
  - steady-state transformer analysis and, 347–348, 351–352
- Inductive circuits, power for, 378–379
- Inductor current, 289–290
- Inductors, 182, 184–189, 194–195, 198–199, 211, 328–329, 361, 484–485, 515–517, 521, 760
  - circuit analysis of, 184–189, 515–517
  - circuit component values, 760
  - current to voltage (i–v) relationships, 185–186
  - duality (symmetry) of, 198–199, 211
  - energy in, 182, 187–189, 211
  - equivalent circuits for, 194–195, 196, 484–485, 521
  - impulse function ($K\Delta(t)$) for, 515–517
  - in series, 194
  - Laplace transform method for, 484–485, 515–517, 521
  - magnetic field and, 184

    - multiple, 194–195, 196
    - parallel, 194–195
    - passive behavior of, 182, 211
    - phasor relationships, 328–329, 361
    - power in, 182, 186–187, 211
    - s domain representation, 484–485, 521
    - symbols for, 184
    - voltage to current (v–i) relationships, 184–185, 328–329, 361
- Infinite frequency, 544
- Initial-value theorem, 472–474, 477
- Input constraints, 153–154, 172
- In-series circuit elements, 41, 50. See also Series-connected circuits
- Instantaneous power, 376–377, 378, 401, 427–428
    - balanced three-phase circuits, 427–428
    - calculations for, 376–377, 378, 401, 427–428
    - sinusoidal steady-state circuits, 376–377, 378, 401
    - time-invariant, 427–428
- Instantaneous real power, 378
- Integer power of complex numbers, 731
- Integrals, 446, 455–456, 505–511, 521, 662–663, 664–666, 671–672, 758–759

    - convergence of, 664–666
    - convolution, 505–511, 521
    - Fourier transforms, 662, 664–666, 671–672
    - frequency-domain function (F(ω)) and, 671–672
    - Laplace transforms, 446, 455–456
    - table of, 758–759
    - time-domain function (F(t)) and, 671–672
    - transfer function (H(s)) use of, 505–511, 521
- Integrating-amplifier, 252–254, 256, 303–308, 310
    - cascading connections, 303–308, 310
    - feedback resistors and, 305–308, 310
    - first-order circuit analysis, 252–254, 256
    - second-order circuit analysis, 303–308, 310
    - step response of, 305
- Integration, operational transforms for, 673
- Interconnections, 30–31, 36–37, 58, 60–64, 75–78, 79–80
    - circuit model creation for, 36–37
    - delta (Δ), 76
    - delta-to-wye (Δ-to-Y) equivalent circuits, 75–78, 80
    - dependent sources, 31

    - independent sources, 31
    - parallel-connected resistors, 58, 61–64, 79
    - pi (π), 76
    - pi-to-tee (π-to-T) equivalent circuits, 75–78, 80
    - resistive circuits, 58, 60–64, 75–78, 79–80
    - series-connected resistors, 58, 60, 79
    - series–parallel simplification, 62–63
    - tee (T), 76
    - testing ideal sources, 30–31
    - wye (Y), 76
- International System of Units (SI), 9–11
- Inverse Fourier transform, 663
- Inverse Laplace transforms, 460–470, 476
    - distinct complex roots of D(s), 463–465
    - distinct roots of D(s), 461–462
    - improper rational functions, 460, 469–470
    - partial fraction expansion, 461–470
    - proper rational functions, 461–469
    - rational functions (F(s)) and, 460–470
    - repeated complex roots of D(s), 468–468

    - repeated real roots of D(s), 466–467
    - s-domain and, 460–470, 476
    - transform pairs for, 469
- Inverse phasor transform, 326
- Inverting-amplifier circuit, 156–158, 168, 172–173
    - ideal op-amp model for, 156–158, 172
    - negative feedback in, 157
    - realistic op-amp model for, 168, 173

K

- Kirchhoff’s laws, 39–44, 45–47, 50, 60–61, 94–95, 332–333, 487
  - amplifier circuit analysis using, 47
  - circuit analysis using, 40–47, 50
  - current law (KCL), 39–40, 41, 50, 60–61, 332–333
  - dependent sources and, 45–47
  - frequency domain and, 332–333
  - loop (closed path) for, 40
  - nodes for, 39
  - Ohm’s law and, 42–43
  - parallel-connected circuits and, 61
  - s domain use of, 487
  - series-connected (in-series) circuits and, 60
  - simultaneous equations from, 94–95
  - steady-state analysis using, 332–333
  - unknown voltage found from, 46
  - voltage law (KVL), 40, 42, 50, 60, 332–333

L

- Lagging/leading power factors, 379, 401
- Laplace transform method, 482–535
  - circuit analysis in s domain, 486–488
  - circuit analysis using, 482–535
  - circuit elements in s domain, 484–486
  - convolution integral and, 505–511, 521
  - equivalent circuits for time and frequency domains, 485–486
  - impulse function and, 514–520, 521
  - impulsive sources and, 517–520
  - Kirchhoff’s laws in s domain, 487
  - multiple mesh circuit analysis, 493–494
  - mutual inductance circuit analysis, 497–498
  - natural response using, 489
  - Ohm’s law in s domain, 484, 486
  - partial fraction expansions for, 502–505
  - procedure for, 487–488, 521
  - RC circuit analysis, 489
  - RLC circuit analysis, 489–490

    - s-domain applications, 495–496, 499–500
    - s-domain equivalent circuits, 485–486
    - sinusoidal source circuit analysis, 491–492
    - sinusoidal steady-state response and, 511–513, 521
    - step response using, 489–490
    - superposition applications, 499–500
    - surge suppressor analysis, 483, 520
    - Thévenin equivalent circuit from, 495–496
    - time-invariant circuits, 504–505, 521
    - transfer function (H(s)) and, 500–513, 521
    - voltage to current (v-i) equations for, 484–485, 521
- Laplace transform, 444–481, 666–668
    - applications of, 458–460
    - continuous and discontinuous functions, 446–447
    - defined, 446
    - final-value theorem for, 472–474, 477
    - Fourier transforms from, 666–668
    - frequency domain (F(s)), 446, 452–458, 460–474, 476–477
    - functional transforms, 447, 452–453, 476
    - impulse function (KΔ(t)), 449–451, 476

    - initial-value theorem for, 472–474, 477
    - integral of, 446, 455–456
    - inverse transforms, 460–470, 476
    - lumped-parameter circuits and, 458–460, 476
    - operational transforms, 447, 453–458, 476
    - partial fraction expansion for, 460–470
    - poles of F(s), 470–472, 476
    - problem-solving uses, 444
    - s-domain, 446, 452–458, 460–474, 476–477
    - step function (Ku(t)), 447–448, 476
    - time domain (f(t)), 446–458, 472–474, 476–477
    - transform pairs, 452–453, 469
    - transient effects on circuits, 445, 474–476
    - unilateral (one-sided) behavior of, 446–447
    - unit impulse function (Δ(t)), 449, 476
    - unit step function (u(t)), 447, 476
    - zeros of F(s), 470–472, 476
- Limiting values, 662–663, 665–670
    - cosine functions, 669–670
    - elementary functions, 670

    - Fourier transforms derived using, 662–663
    - Fourier transforms of, 665–670
    - signum functions, 669
    - unit step function, 669
- Line current, 417, 418–419, 422–423, 435
- Line spectra, 645
- Line voltage, 417, 418–419, 435
- Linear simultaneous equations, 718–726. See also Simultaneous equations
- Linear transformer circuits, 347–351, 361
    - frequency domain analysis of, 347–351
    - reflected impedance (Zr), 348–349, 361
    - self-impedance, 348
    - steady-state analysis, 347–351, 361
    - winding (primary and secondary), 347
- Loads, 127–128, 390–391, 393–397, 425–430
    - balanced three-phase circuits, 425–430
    - delta (Δ), 426–427
    - impedance (Z) conditions and restrictions, 393–397
    - maximum power transfer, 127–128, 393–397
    - parallel, 390–391

    - power calculations for, 390–391, 393–397, 425–430
    - resistive, 127–128
    - unspecified, 429–430
    - wye (Y), 425–426
- Loop (closed path), 40, 50, 94
- Low-pass filters, 538, 539–545, 564, 574–575, 587–591, 593–595, 608, 683–684
    - active, 574–575, 587–591, 593–595, 608
    - Bode plots for, 574–576, 588
    - Butterworth, 593–595, 608
    - cascading connections, 587–591, 593–595, 608
    - cutoff frequency (ωc) for, 540–542, 544
    - design of, 543, 544, 575
    - first-order active, 574–575, 608
    - fourth-order active, 589–590, 594–595
    - frequency and time domain relationships, 545
    - frequency of, 538
    - frequency response plots for, 538, 574–576
    - frequency-selective circuit analysis, 539–545, 564
    - half-power frequency, 541
    - infinite frequency, 544

    - op amp filter design, 575
    - Parseval’s theorem for, 683–684
    - prototype, 575, 608
    - qualitative analysis, 539–541, 544
    - quantitative analysis, 542–543
    - series RC circuits, 543–544
    - series RL circuits, 539–540, 542–543
    - transfer function (H(s)) for, 545, 564
    - zero frequency, 544
- Lumped-parameter circuits, 7–8, 458–460, 476
    - frequency (f) of, 7
    - Laplace transform for, 458–460, 476
    - systems of, 7–8

M

- Magnetic coupling, 7
- Magnetic fields, inductors and, 184
- Magnetically coupled coils, 207–209, 211, 245–246, 733–737
  - energy storage in, 207–209, 211
  - equivalent circuits for, 733–737
  - mutual inductance and, 207–209, 211
  - π-equivalent circuit, 734–735
  - step response of circuit with, 245–246
  - T-equivalent circuit, 733
- Magnitude plot, 538
- Magnitude scaling, 577–578, 608
- Maximum power transfer
- Maximum power transfer (Pmax), 92, 126–128, 135, 393–399, 401
  - average power absorbed, 394
  - circuit analysis for, 92, 126–128, 135
  - ideal transformer analysis, 397–398
  - impedance (Z) conditions and restrictions, 393–397
  - power calculations for, 393–399, 401

    - resistive loads, 127–128
    - sinusoidal steady-state analysis of, 393–399, 401
    - system optimization and, 126
    - with load restrictions, 396
    - without load restrictions, 395
- Measurement, 9–11, 58, 70–73, 80, 430–433, 435, 697, 700, 741–742
    - ammeter for, 70, 71–72, 80
    - analog meters for, 71–72, 80
    - current, 58, 70–73, 80
    - d’Arsonval meter movement, 71
    - decibels (dB), 741–742
    - digital meters for, 73, 80
    - electrodynamic wattmeter for, 430–433, 435
    - International System of Units (SI), 9–11
    - power, 430–433, 741–742
    - resistance, 73–75, 80
    - three-phase circuits, 430–433,
    - two-port circuit parameters from, 697, 700
    - unit prefixes, 10–11
    - voltage, 58, 70–73, 80

    - voltmeter for, 70, 72, 80
    - Wheatstone bridge, 73–75, 80
- Memory, concept of using convolution integral, 510–511
- Mesh, 94
- Mesh circuit analysis, Laplace transform method for, 493–494
- Mesh current, 104–105
- Mesh-current method, 92, 104–115, 134–135, 199–202, 345–347
    - amplifier circuit analysis, 111
    - circuit analysis process, 92, 104–115, 134–135
    - dependent sources and, 107–108
    - duality of, 104
    - frequency-domain circuit analysis, 345–347
    - mutual inductance and, 199–202
    - node-voltage method compared to, 112–115
    - special cases for, 108–112
    - steady-state circuit analysis, 345–347
    - supermesh and, 109–110
- Modulation, 674
- Motor, 28
- Multiplication operations, 453, 673, 730

- Mutual inductance, 182, 199–209, 211, 497–498
  - coefficient of coupling for, 207–208, 211
  - concept of, 204–206
  - dot convention, 199–201, 211
  - energy storage in magnetically coupled coils, 207–209, 211
  - Laplace transform method for, 497–498
  - mesh-current method for, 199–202
  - polarity of induced voltages, 199–201, 204, 211
  - procedure for determining dot markings, 200–201
  - s domain circuit, 497–498
  - self-inductance and, 199–200, 203–204, 206–207, 211

N

- Narrowband (high-Q) filters, 600–605, 609
  - bandpass, 600–602, 608
  - bandreject, 602–605, 609
  - Bode plots for, 602, 605
- Natural response, 220, 222–232, 241–246, 256, 274–289, 296–303, 308–310, 489
  - characteristic equation for, 275–276, 297, 299, 309
  - circuit phase analysis using, 220
  - clock analysis for computer timing, 273, 308–309
  - critically damped response, 286–289, 297–300, 309
  - current (i) expression for, 222–224, 230
  - damped radian frequency (ωd), 282, 293, 299–300
  - energy (w) expression for, 226, 230
  - general solution for, 241–246, 256
  - initial inductor current (I0), 223
  - Laplace transform method for, 489
  - method for, 224, 230, 241–242, 256, 280, 283, 298–299
  - Neper frequency (α), 276, 297, 299, 309

    - overdamped response, 279–282, 297–300, 309
    - parallel RLC circuits, 272, 274–296, 309
    - power (p) expression for, 226, 230
    - resistor-capacitor (RC) circuits, 220, 228–232, 241–246, 256, 489
    - resistor-inductor (RL) circuits, 220, 222–228, 241–246, 256
    - resistor-inductor-capacitor (RLC) circuits, 274–289, 296–300, 302, 308–310
    - resonant radian frequency (ω0), 276, 293, 297, 299
    - series RLC circuits, 272, 296–300, 302, 310
    - steady-state response for, 227
    - symbols for, 274
    - time constant (τ), 223–224, 226–227, 229, 256
    - transient response of, 227
    - underdamped response, 282–286, 297–300, 309
    - voltage (v) expressions for, 226, 229, 274
- Negative feedback, 153–155, 157, 158, 160, 162, 172
    - difference-amplifier circuit, 162
    - input voltage constraints and, 153–154
    - inverting-amplifier circuit, 157
    - op-amp circuit analysis and, 153–155
    - summing-amplifier circuit, 158

    - voltage constraint and, 153–154, 172
- Neper frequency (α), 276, 293, 297, 299, 301, 309
    - parallel RLC circuits, 276, 293, 309
    - series RLC circuits, 297, 299, 301
- Net charge, 7
- Neutral terminal, 415
- Node voltage, 97
- Node-voltage method, 92, 96–104, 112–115, 134–135, 344–345
    - amplifier circuit analysis, 102–103
    - circuit analysis process, 96–98, 101–102, 134–135
    - dependent sources and, 98–100
    - duality of, 104
    - frequency-domain circuit analysis, 344–345
    - mesh-current method compared to, 112–115
    - special cases for, 100–104
    - steady-state circuit analysis, 344–345
    - supernodes and, 101–102
- Nodes, 39, 50, 94–96
- Noninverting-amplifier circuit, 159–162, 168–170, 172–173
    - ideal op-amp model for, 159–162, 172

    - negative feedback in, 160
    - realistic op-amp model for, 168–170, 173
- Nonplanar circuits, 94
- Norton equivalent circuits, 92, 120–123, 135, 340–341
    - frequency domain simplification, 340–341
    - impedance (Z) in, 340–341
    - source transformation, 92, 120–123, 135, 340–341
    - terminal circuit simplification using, 92, 120–123, 135

O

- Odd periodic function, 626–627
- Ohm’s law, 32–33, 42–43, 45–47, 50, 61, 484, 486
  - amplifier circuit analysis using, 47
  - circuit analysis using, 42–43, 45–47, 50
  - dependent sources and, 45–47
  - electrical resistance and, 32–33, 50
  - Kirchhoff’s law and, 42–43
  - parallel-connected circuits and, 61
  - s domain use of, 484, 486
- Op amp filters, 575–577, 579, 580–600, 608
  - bandpass, 580–583, 598–599, 608
  - bandreject, 583–587, 608
  - Butterworth, 591–600
  - cascading, 587–591
  - design of, 575, 576–577
  - first-order active, 575, 576–577, 587–591
  - higher-order active, 587–600
  - high-pass, 576–577

    - low-pass, 575, 579
    - scaling, 579
- Open circuit, 37
- Open-loop operation, 157
- Operational amplifiers (op amps), 150–181. See also Op amp filters
    - common mode rejection ratio (CMRR) for, 165–167, 173
    - current (i), 152–156, 172
    - difference-amplifier circuit, 162–167, 173
    - gain, 153
    - ideal model, analysis of, 154–155, 172–173
    - input constraints, 153–154, 172
    - inverting-amplifier circuit, 156–158, 168, 172
    - negative feedback in, 153–154, 157, 158, 160, 162, 172
    - noninverting-amplifier circuit, 159–162, 168–170, 172
    - open-loop operation, 157
    - realistic models, analysis of, 167–170, 173
    - strain gage analysis, 151, 171–172
    - summing-amplifier circuit, 158–159, 172
    - symbols for, 152
    - terminals, 152–156, 172

    - transducers, 151, 171–172
    - voltage (v), 152–156, 172
- Operational transforms, 447, 453–458, 476, 672–676
    - addition and subtraction, 454, 673
    - convolution in frequency domain, 675
    - convolution in time domain, 674
    - defined, 447, 476
    - differentiation, 454–455, 673
    - Fourier, 672–676
    - integration, 455–456, 673
    - Laplace, 447, 453–458, 476
    - modulation, 674
    - multiplication by a constant, 453, 673
    - scale changing, 457, 674
    - translation in frequency domain, 457, 674
    - translation in time domain, 456–457, 674
- Overdamped response, 279–282, 291–293, 297–299, 300–301, 309
    - natural response, 279–282, 297–300, 309
    - parallel RLC circuits, 279–282, 291–293, 309
    - series RLC circuits, 297–299, 301

- step response, 291–293, 300–301

P

- Parallel-connected circuits, 58, 61–64, 79, 194–196, 335, 337, 390–391, 708–709.See also Parallel RLC circuits
    - capacitors, 195–196
    - circuit elements, 61
    - combining, 61–62
    - frequency domain, 335, 337
    - impedance (Z) combined in, 335, 337
    - inductors, 194–195
    - Kirchhoff’s current law for, 61
    - Ohm’s law for, 61
    - power calculations for, 390–391
    - resistors, 58, 61–64, 79
    - series–parallel simplification, 62–63
    - two-port, 708–709
- Parallel RLC circuits, 272, 274–296, 309, 555–557, 563
    - bandpass filters, 555–557
    - bandreject filters, 563
    - bandwidth (β), 556

    - characteristic equation for, 275–276, 293, 309
    - critically damped voltage response, 286–289, 291–293, 309
    - cutoff frequency (ωc), 556
    - damped radian frequency (ωd), 282, 293
    - frequency-selective circuits, 555–557, 563
    - natural response of, 274–289, 309
    - Neper frequency (α), 276, 293, 309
    - overdamped response, 279–282, 291–293, 309
    - parameters of, 276–277
    - quality factor (Q), 556
    - resonant radian frequency (ω0), 276, 293
    - second-order differential equations for, 274–277
    - step response of, 289–296, 309
    - symbols for, 274
    - underdamped response, 282–286, 291–293, 309
- Parasitic resistance, 37
- Parseval’s theorem, 679–685, 686
    - bandpass filter application, 662–663
    - energy calculations using, 679–685, 686
    - Fourier transform time-domain functions, 679–685, 686

    - graphic interpretation of, 681
    - low-pass filter application, 683–684
    - rectangular voltage pulse application, 684–685
- Partial fraction expansion, 461–470, 502–505
    - distinct complex roots of D(s), 463–465
    - distinct roots of D(s), 461–462
    - improper rational functions, 460, 469–470
    - inverse Laplace transforms and, 461–470
    - proper rational functions, 461–469
    - repeated complex roots of D(s), 468–468
    - repeated real roots of D(s), 466–467
    - s domain use of, 461–470, 502–505
    - transfer function (H(s)) in, 502–505
    - transform pairs for, 469
- Passband frequency, 538, 564
- Passive circuit elements, 30, 182, 211, 327–331, 361
    - capacitors, 182, 211, 329–330, 361
    - defined, 30
    - frequency domain, 327–331
    - impedance (Z) and, 330–331

    - inductors, 182, 211
    - phasor transforms and, 325, 327–331
    - reactance and, 331
    - resistors, 327–328, 361
    - voltage to current (v–i) relationships in, 330–331
- Passive filters, 539, 549, 559, 565
- Passive sign convention, 14–17
- Period of time (T), 320
- Periodic current, average-power calculations and, 639–641
- Periodic functions (f(t)), 618, 620–630, 639–642, 649–650
    - average power calculations with, 639–641, 650
    - defined, 618, 649
    - Dirichlet’s conditions, 621
    - even, 625–626
    - Fourier coefficients and, 621–623, 649
    - Fourier series of found with symmetry, 630
    - Fourier series representation, 621–622, 639–641
    - fundamental frequency (ω0), 621, 649
    - half, 627–628
    - harmonic frequency, 621, 649

    - odd, 626–627
    - periodic voltage and, 639–641
    - quarter, 628–629
    - root-mean-square (rms) value of, 641–642, 650
    - steady-state response from, 621–622
    - symmetry effects, 625–630, 649
    - waveforms, 618, 620–629
- Periodic response, 618, 620
- Periodic to aperiodic transition, 660, 663, 686
- Periodic voltage, 631–637, 639–641, 645–647, 663
    - amplitude spectra for, 645–647
    - average-power calculations expressed from, 639–641
    - Fourier series applications, 631–637, 639–641, 645–647
    - inverse Fourier transform and, 663
    - phase spectra for, 645–647
    - phasor domain circuit transformation and, 631–632
    - sine and cosine terms for, 631–632
    - steady-state response and, 633–637
    - waveforms, 633, 635–636
- Periodic waveforms, 618, 620

- Phase angle (ϕ) of, 320
- Phase angle plots, 538, 748–750, 754–756
    - complex poles, 754–756
    - frequency response and, 538
    - straight-line, 748–750, 754–756
- Phase current, 418–419, 422–423, 435
- Phase sequences for three-phase circuits, 414, 435
- Phase spectrum, 645–647, 650
- Phase voltage, 418–419, 435
- Phase windings, 415
- Phasor diagrams, 357–359, 414, 419
- Phasor transform, 325–333
    - frequency domain and, 325, 327–331
    - inverse, 326
    - phasor representation as, 325
    - voltage to current (v–i) relationships, 327–331, 361
- Phasors, 324–331, 387–388, 631–632, 650. See also Phasor transforms
    - capacitor voltage to current (v–i) relationships, 329–330, 361
    - complex power calculations using, 387–388
    - concept of, 324–325

    - Fourier series transformation to phasor domain, 631–632, 650
    - impedance (Z) and, 330–331
    - inductor voltage to current (v–i) relationships, 328–329, 361
    - reactance and, 331
    - representation, 325
    - resistor voltage to current (v–i) relationships, 327–328, 361
    - sinusoidal functions and, 324–327
    - steady-state analysis using, 324–330
- Pi (π-equivalent circuit, 734–735
- Pi (π) interconnection, 76
- Pi-to-tee (π-to-T) equivalent circuits, 75–78, 80
- Planar circuits, 94
- Polar form of complex numbers, 727–728
- Polarity, 14, 16–17, 199–201, 204, 211, 354–355
    - arrows for reference of, 14, 16
    - coil current and voltage, 354–355
    - dot convention for, 199–201, 211, 354–355
    - ideal transformers, 354–355
    - induced voltages, 199–201, 204, 211
    - mutual inductance, 199–201, 211

    - power reference, 16–17
    - self-inductance, 204
    - voltage and current references, 14
- Poles, 470–472, 476, 502, 743–744, 750–756
    - amplitude plots, 750–753
    - complex, 750–756
    - frequency domain (F(s)), 470–472, 476
    - phase angle plots, 754–756
    - real, first-order, 743–744
    - transfer functions (H(s)), 502
- Ports, 692
- Potential coil, 430, 435
- Power, 3, 15–20, 33–34, 126–128, 135, 182, 186–187, 190–191, 211, 226, 230, 378–379, 384–385, 391–392, 401, 413, 427–428, 433–435
    - ac circuits, 391–392
    - algebraic sign of, 16–17
    - balance of in circuits, 3, 19–20
    - balanced three-phase circuits, 413, 433–435
    - capacitive circuits, 379
    - capacitors and, 182, 190–191, 211
    - current and voltage relationship to, 15–17

    - defined, 16
    - electric transmission and distribution, 413, 433–435
    - energy and, 15–17
    - inductive circuits, 378–379
    - inductors and, 182, 186–187, 211
    - maximum power transfer, 126–128, 135
    - natural response and, 226, 230
    - passive sign convention for, 16–17
    - polarity reference, 16–17
    - resistive circuits, 378
    - resistive load transfer, 127–128
    - resistor-capacitor (RC) circuit expression, 230
    - resistor-inductor (RL) circuit expression, 226
    - resistors, 33–34
    - time-invariant, 427–428
    - units for, 379, 384–385, 401
- Power calculations, 374–411, 425–430, 435
    - apparent power, 385, 401
    - appliance ratings for, 381–382
    - average power (P), 377–384, 389, 394, 401, 425–426

- balanced three-phase circuits, 425–430, 435
- balancing power in ac circuits, 391–392
- complex power, 384–393, 401, 426–427
- delta (Δ) loads, 426–427
- energy delivery and, 374
- instantaneous power, 376–377, 378, 401, 427–428
- lagging/leading factors for, 379, 401
- maximum power transfer (Pmax), 393–399, 401
- parallel loads and, 390–391
- phasors for, 387–388
- power factor (pf) for, 379, 401
- reactive factor (rf) for, 379, 401
- reactive power (Q), 377–382, 389, 401, 426–427
- root-mean-square (rms) value for, 382–384
- sinusoidal steady-state analysis, 374–411
- standby (vampire) power, 375, 399–400
- unspecified loads, 429–430
- wye (Y) loads, 425–426
- wye-delta (Y-Δ) circuits, 428–429
- wye-wye (Y-Y) circuits, 428

- Power consumption, 399
- Power equation, 16
- Power factor (pf), 379, 401
- Power measurement, 430–433, 435, 741–742
  - balanced three-phase circuits, 430–433, 435
  - bels, 741
  - decibels (dB), 741–742
  - electrodynamic wattmeter for, 430–433, 435
  - current coil, 430, 435
  - potential coil, 430, 435
  - power gain, 741–742
  - two-wattmeter method, 431–432, 435
  - wattmeter reading calculations, 432–433
- Power systems, 5, 319, 359–360
- Power triangle, 385
- Problem-solving strategy, 8–9
- Proper rational functions, 460–469. See also Partial fraction expansion
- Prototypes, 12, 575–577, 608
- Pushbutton telephone circuits, 537, 564

Q

- Qualitative analysis, 539–541, 544, 546, 551, 560–561
  - bandpass filters, 551
  - bandreject filters, 560–561
  - high-pass filters, 546
  - low-pass filters, 539–541, 544
- Quality factor (Q), 550, 554, 562, 556, 619
- Quantitative analysis, 542–543, 546–547, 551–554, 561–562
  - bandpass filters, 551–554
  - bandreject filters, 561–562
  - high-pass filters, 546–547
  - low-pass filters, 542–543
- Quarter-wave periodic function, 628–629

R

- Rational functions (F(s)), 460–470
  - improper, 460, 469–470
  - inverse Laplace transforms and, 469–470
  - proper, 461–469
  - transform pairs for, 469
- Reactance, 331
- Reactive factor (rf), 379, 401
- Reactive power (Q), 377–382, 389, 401, 426–427
  - balanced three-phase circuits, 426–427
  - calculations for, 377–382, 389, 401, 426–427
  - delta (Δ) loads, 426–427
  - sinusoidal steady-state analysis, 377–382, 389
  - wye (Y) loads, 426
- Reciprocal two-port circuits, 701–702, 712
- Rectangular form of complex numbers, 727–728
- Rectangular waveforms, 620
- Reflected impedance (Zr), 348–349, 361
- Resistance (R), 32–35, 37, 50, 61–62, 73–75, 80

    - conductance (G) and, 33
    - equivalent (Req), 61–62
    - measurement of, 73–75, 80
    - Ohm’s law and, 32–33, 50
    - parasitic, 37
    - resistors as models of, 32–35
    - Wheatstone bridge circuit for, 73–75, 80
- Resistive circuits, 58–91, 115–118, 121, 134, 378
    - analysis of, 68–70, 79–80, 115–118, 121, 134
    - current-divider circuit, 67, 79
    - current division, 69–70, 79–80
    - delta-to-wye (Δ-to-Y) equivalent circuits, 75–78, 80
    - interconnections, 58, 60–64, 75–78
    - measurement of voltage and current, 58, 70–73, 80
    - parallel connections, 58, 61–64, 79
    - pi-to-tee (π-to-T) equivalent circuits, 75–78, 80
    - power for, 378
    - resistor value measurements, 73–75, 80
    - series connections, 58, 60, 79
    - series–parallel simplification, 62–63

    - source transformation, 115–118, 121, 134
    - touch screens, 59, 78–80
    - voltage-divider circuit, 64–66, 79
    - voltage division, 68–70, 79
    - Wheatstone bridge, 73–75, 80
- Resistive loads, 127–128
- Resistor-capacitor (RC) circuits, 220–222, 228–232, 238–246, 249–255, 489, 543–544, 546–547, 633–637
    - analysis phases for, 220
    - artificial pacemaker design, 221, 255
    - current (i) expression, 230
    - cutoff frequency (ωc), 544
    - energy (w) expression, 230
    - first-order circuits as, 220, 222, 256
    - Fourier series application, 633–637
    - frequency-selective analysis of, 543–544, 546–547
    - general solution for, 241–246, 256
    - high-pass filters, 546–547
    - integrating-amplifier circuit analysis, 252–254, 256
    - Laplace transform method for, 489
    - low-pass filters, 543–544

    - natural response of, 220, 228–232, 241–246, 256, 489
    - periodic voltage in, 633–637
    - power (p) expression, 230
    - sequential switching, 246, 249–250, 256
    - steady state response, 633–637
    - step response of, 220, 238–246, 256
    - time constant (τ), 229, 256
    - unbounded response, 250–251, 256
    - voltage (v) expression, 229
- Resistor-inductor (RL) circuits, 220–228, 233–237, 241–251, 256, 539–540, 542–543, 547–548
    - analysis phases for, 220
    - current (i) expression, 222–224
    - cutoff frequency (ωc), 542
    - energy (w) expression, 226
    - first-order circuits as, 220, 222, 256
    - frequency-selective analysis of, 539–540, 542–543, 547–548
    - general solution for, 241–246, 256
    - high-pass filters, 547–548
    - low-pass filters, 539–540, 542–543
    - natural response of, 220, 222–228, 241–246, 256

    - power (p) expression, 226
    - qualitative analysis, 546
    - quantitative analysis, 546–547
    - sequential switching, 246–249, 256
    - step response of, 220, 233–237, 241–246, 256
    - time constant (τ), 223–224, 226–227, 256
    - unbounded response, 250–251, 256
    - voltage (v) expression, 226
- Resistor-inductor-capacitor (RLC) circuits, 272–317, 489–490, 551–559, 560–563, 637–638
    - bandwidth (β), 553–554, 556
    - center frequency (ωo), 552–554
    - characteristic equations for, 275–276, 293, 297, 299, 301, 309
    - clock for computer timing, 273, 308–309
    - critically damped voltage response, 286–289, 291–293, 298–299, 300–301, 309
    - cutoff frequency (ωc), 553, 556
    - damped radian frequency (ωd), 282, 293, 301
    - direct approach for, 291–292, 637–638
    - Fourier series approach for, 637–638
    - frequency-selective circuit analysis, 551–559, 560–563

- indirect approach for, 290–291
- inductor current for, 289–290
- integrating amplifiers in cascade, 303–308, 310
- Laplace transform method for, 489–490
- natural response of, 274–289, 296–300, 302, 308–310
- Neper frequency (α), 276, 293, 297, 301, 309
- overdamped response, 279–282, 291–293, 298, 300–301, 309
- parallel, 272, 274–296, 309, 555–557, 563
- quality factor (Q) for, 554, 556
- resonant radian frequency (ω0), 276, 293, 297, 301
- second-order differential equations for, 274–277
- series-connected, 272, 274, 296–303, 310, 551–555, 557–559, 560–563
- square-wave voltage and, 637–638
- steady-state response of, 637–638
- step response of, 289–296, 300–303, 305, 309–310, 489–490
- symbols for, 274, 289
- timing signals, 273
- underdamped voltage response, 282–286, 291–293, 298–299, 300–301, 309
- voltage expressions for, 274

- Resistors, 32–35, 50, 61–64, 93, 131–134, 327–328, 361, 484, 521, 760
  - circuit component values, 760
  - conductance (G) and, 33
  - equivalent circuits for, 61–62, 484, 521
  - multiple, 61–62
  - Ohm’s law for, 32–33, 50
  - phasor relationships, 327–328, 361
  - power in terms of current, 33
  - power in terms of voltage, 34
  - resistance (R) models, 32–35
  - s domain representation, 484, 521
  - sensitivity analysis of, 93, 131–134
  - series–parallel simplification, 62–63
  - signals in phase, 328
  - voltage to current (v–i) relationships, 327–328, 361
- Resonant frequency (ωo), 550
- Resonant radian frequency (ω0), 276, 293, 297, 299, 301
  - parallel RLC circuits, 276, 293
  - series RLC circuits, 297, 299, 301
- Response, 220, 222–246, 241–246, 250–251, 256, 272–317, 323–324, 361, 504–513, 521, 677–678

- critically damped, 286–289, 291–293, 297–299, 300–301, 309
- damped radian frequency (ωd), 282, 293, 299, 301
- Fourier transforms for, 677–678
- general solution for, 241–246, 256
- memory, concept of, 510–511
- natural, 220, 222–232, 241–246, 256, 274–289, 296–300, 302, 308–310
- overdamped, 279–282, 291–293, 297–299, 300–301, 309
- resistor-capacitor (RC) circuits, 220, 228–232, 238–246, 250–251, 256
- resistor-inductor (RL) circuits, 220, 222–228, 233–237, 241–251, 256
- resistor-inductor-capacitor (RLC) circuits, 272–317
- sinusoidal, 323–324, 361, 504–513, 521
- steady-state analysis of, 318, 323–324, 361
- steady-state, 227, 318, 323–324, 361, 504–513, 521, 678
- step, 220, 233–246, 256, 289–296, 300–303, 309–310
- transfer function (H(s)) and, 504–513, 521
- transient, 227, 677–678
- unbounded, 250–251, 256
- underdamped, 282–286, 291–293, 297–299, 300–301, 309

    - unit impulse (h(t)), 504–511
    - weighting function for, 510
- Root-mean-square (rms) value, 321, 382–384, 641–642, 650
    - effective value as, 383
    - periodic functions (f(t)), 641–642, 650
    - power calculations using, 382–384
    - sinusoidal sources and, 321
- Roots of complex numbers, 731–732

S

- s domain, 446, 452–458, 460–474, 476–477, 482–535. See also Frequency domain
  - circuit analysis in, 486–488
  - circuit elements in, 484–486
  - final-value theorem for, 472–474, 477
  - initial-value theorem for, 472–474, 477
  - inverse Laplace transforms for, 460–470, 476
  - Kirchhoff’s laws in, 487
  - Laplace transform method applications, 482–535
  - Laplace transform (F(s)) of, 460–474, 476–477
  - mutual inductance circuit in, 497–498
  - Ohm’s law in, 484, 486
  - operational transforms for, 447, 453–458, 476
  - partial fraction expansion, 461–470, 502–505
  - poles of F(s), 470–472, 476, 502
  - rational functions (F(s)) and, 460–470
  - superposition applications in, 499–500
  - Thévenin equivalent circuit in, 495–496

    - time domain (t) relationships, 446, 453–458, 472–474, 477
    - transfer function (H(s)), 500–513, 521
    - transform pairs, 452–453, 469
    - zeros of F(s), 470–472, 476, 502
- Scale change, 457, 674
- Scaling, 577–580, 608
    - circuit component scale factors, 578
    - filter design using, 578
    - frequency, 578, 608
    - low-pass op-amp filter, 579
    - magnitude, 577–578, 608
    - series RLC filter, 578–579
- Second-order circuits, 274. See also Resistor-inductor-capacitor (RLC) circuits
- Second-order filters, 598
- Self-impedance, 348
- Self-inductance, 199–200, 203–204, 206–207, 211, 347–348, 351–352
    - Faraday’s law for, 203–204
    - mutual inductance and, 199–200, 206–207, 211
    - polarity of induced voltages, 199–201, 204, 211
    - steady-state transformer analysis and, 347–348, 351–352

    - voltage drop, 199
- Sensitivity analysis of resistors, 93, 131–134
- Sequential switching, 246–250, 256
    - circuit analysis and, 246–250, 256
    - defined, 246
    - resistor-capacitor (RC) circuits with, 246, 249–250, 256
    - resistor-inductor (RL) circuits with, 246–249, 256
- Series-connected (in-series) circuits, 41, 50, 58, 60, 79, 194–195, 334–335, 337, 539–540, 542–544, 546–548, 708–709. See also Series RLC circuits
    - black box concept, 60
    - capacitors, 195
    - circuit elements, 41, 50
    - combining, 60
    - frequency domain, 334–335, 337
    - frequency-selective circuits, 539–540, 542–544, 546–548
    - high-pass filters, 546–548
    - impedances (Z) combined in, 334–335, 337
    - inductors, 194
    - Kirchhoff’s laws for, 60
    - low-pass filters, 539–540, 542–544

    - resistors, 58, 60, 79
    - two-port circuits, 708–709
- Series–parallel connections, 62–63, 708–709
    - simplification of, 62–63
    - two-port circuits, 708–709
- Series RLC circuits, 272, 274, 296–303, 310, 551–555, 557–563, 578–579
    - bandpass filters, 551–555, 557–559
    - bandreject filter, 560–563
    - bandwidth (β), 553, 562
    - center frequency (ωo), 552, 562
    - characteristic equation for, 297, 299–301
    - critically damped response, 297, 297–299, 301
    - cutoff frequency (ωc), 552–553, 562
    - frequency-selective circuits, 551–555, 557–563
    - natural response of, 272, 274, 296–303, 310
    - Neper frequency (α), 297, 299, 301
    - overdamped response, 297–299, 301
    - quality factor (Q), 554, 562
    - resonant radian frequency (ω0), 297, 299, 301
    - scaling, 578–579

    - step response of, 300–303, 310
    - symbols for, 274
    - underdamped response, 297–299, 301
- Short circuit, 37
- Sifting property, 450–451
- Signal-processing systems, 5
- Signals in phase, 328
- Signum functions, 669
- Simplification techniques, 62–63, 75–78, 115–118, 121
    - delta-to-wye (Δ-to-Y) transformation, 75–78
    - series–parallel simplification, 62–63
    - source transformation, 115–118, 121
- Simultaneous equations, 94–96, 718–726
    - applications of, 723–726
    - back-substitution method for, 721–722, 726
    - calculator and computer methods for, 719–721, 724–726
    - characteristic determinant of, 722–723
    - circuit analysis using, 94–96
    - Cramer’s method for, 722–724
    - essential nodes and branches for, 95–96

    - Kirchhoff’s laws for, 94–95
    - linear, 718–726
    - number of, 94–96
    - solution of, 718–726
- Sine functions, 631–632, 650
- Single-phase equivalent circuits, 418–421, 435
- Sinusoidal circuits, 318–373, 374–411
    - power calculations, 374–411
    - steady-state analysis, 318–373
- Sinusoidal function, 452
- Sinusoidal rectifiers, 618, 620
- Sinusoidal response, 323–324, 361, 511–513, 521
    - frequency (ω) of, 324, 361
    - steady-state analysis of, 323–324, 361
    - steady-state current component, 324
    - steady-state solution characteristics, 324
    - transient current component, 324
    - transfer function (H(s)) and, 511–513, 521
- Sinusoidal sources, 318–323, 361, 491–492
    - amplitude of, 320

    - angular frequency (ω), 320
    - current behavior and, 318
    - current (i), 320–321
    - Laplace transform method for, 491–492
    - period of time (T), 320
    - phase angle (ϕ) of, 320
    - root-mean-square (rms) value, 321–323
    - steady-state analysis and, 318–323, 361
    - steady-state response from, 319, 323–324
    - voltage (v), 320–322
- Source transformation, 92, 115–118, 121, 134, 340–343
    - bilateral configurations, 115–116
    - condition of equivalence for, 116
    - defined, 115
    - frequency-domain circuit simplification, 340–343
    - impedance (Z) for, 340–343
    - Norton equivalent circuits from, 121, 340–341
    - resistive circuit simplification, 92, 115–118, 121, 134
    - steady-state circuit analysis, 340–343
    - Thévenin equivalent circuits from, 121, 340–341, 343

- Sources, 28–31, 45–48, 318–323, 361, 491–492, 517–520
  - active circuit elements, 30
  - current, 28–31, 50
  - dependent, 28, 31, 45–48, 50
  - direct current (dc), 30
  - electrical, 28
  - impulsive, 517–520
  - ideal, 28–31, 50
  - independent, 28, 31, 50
  - interconnections of, 30–31
  - Laplace transform method for, 517–520
  - passive circuit elements, 30
  - sinusoidal, 318–323, 361, 491–492
  - symbols for, 28–29
  - voltage, 28–31, 50
- Square-wave voltage, 633, 637–638
- Square waveforms, 620, 633
- Standby (vampire) power analysis, 375, 399–400
- Steady-state analysis, 318–373
  - admittance (Y), 336, 361

- challenges of, 318
- delta-to-wye (Δ-to-Y) transformations, 338–340
- frequency domain of, 327–357
- household distribution circuit, 319, 359–360
- impedance (Z) for, 330–331, 333–338, 340–343, 361
- Kirchhoff’s laws for, 332–333
- mesh-current method, 345–347
- node-voltage method for, 344–345
- Norton equivalent circuit for, 340–342
- parallel impedances, 335–338
- passive circuit elements, 327–331
- phasor diagrams for, 357–359
- phasor transforms for, 325–333
- phasors, 324–331
- responses, 227, 323–324
- series impedances, 334–335
- sinusoidal sources for, 318–323, 361
- source transformations for, 340–343
- Thévenin equivalent circuit for, 340–343
- transformers, 347–356, 361

    - voltage to current (v–i) relationships, 327–331, 361
- Steady-state current component, 324
- Steady-state response, 227, 318, 323–324, 361, 511–513, 521, 621–622, 633–638, 650, 678
    - direct approach to, 635–638, 650
    - Fourier series approach for, 621–622, 633–638, 650
    - Fourier transforms for, 678
    - periodic functions used for, 621–622
    - periodic voltage and, 633–637
    - RC circuit periodic voltage response, 633–637
    - RLC circuit square-wave voltage response, 637–638
    - sinusoidal analysis conditions, 318
    - sinusoidal sources of, 318, 323–324
    - square-wave voltage and, 637–638
    - time constant (τ) and, 227
    - transfer function (H(s)) and, 511–513, 521
    - waveforms of, 635–636
- Step function (Ku(t)), 447–448, 476
    - discontinuities of circuits and, 447–448, 476
    - finite duration representation, 448
    - unit step function (u(t)), 447, 476

- Step response, 220, 233–246, 256, 289–296, 300–303, 305, 309–310, 489–490
  - characteristic equation for, 293, 301
  - circuit analysis using, 220
  - comparison of RC and RL circuits, 241
  - critically damped response, 291–293, 300–301
  - damped radian frequency (ωd), 293, 301
  - direct approach for, 291–292
  - general solution for, 241–246, 256
  - indirect approach for, 290–291
  - inductor current for, 289–290
  - inductor voltage versus time, 236–237
  - integrating-amplifier analysis of, 305
  - magnetically coupled coils and, 245–246
  - Laplace transform method for, 489–490
  - method for, 234, 238, 241–242, 256
  - Neper frequency (α), 293, 301
  - overdamped response, 291–293, 300–301
  - parallel, 272, 289–296
  - resistor-capacitor (RC) circuits, 220, 238–246, 256
  - resistor-inductor (RL) circuits, 220, 233–237, 241–246, 256

    - resistor-inductor-capacitor (RLC) circuits, 289–296, 300–303, 309–310, 489–490
    - resonant radian frequency (ω0), 293, 301
    - series-connected, 272, 300–303
    - symbols for, 274, 289
    - underdamped response, 291–293, 300–301
- Stopband frequency, 538, 564
- Straight-line plots, 744–756. See also Amplitude plots; Phase angle plots
- Strain gages, op-amp circuit analysis for, 151, 171–172
- Strength (K) of impulse function, 449, 476
- Summing-amplifier circuit, 158–159, 172
- Supermesh, 109–110
- Supernodes, 101–102
- Superposition, 92, 129–131, 135, 499–500
    - circuit analysis using, 92, 129–131, 135
    - dependent sources and, 130–131
    - Laplace transform method using, 499–500
    - s domain applications, 499–500
- Surge suppressor analysis, 483, 520
- Susceptance (B), 336
- Switching operations, impulse function (KΔ(t)) for, 514–517

- Symmetric two-port circuits, 701–702, 712–713
- Symmetry, 104, 198–199, 211, 625–630, 649
    - capacitors, 198–199, 211
    - duality as, 104, 198
    - even-function, 625–626
    - Fourier coefficient, effects on, 625–630, 649
    - Fourier series of periodic function found with, 630
    - half-wave, 627–628
    - inductors, 198–199, 211
    - odd-function, 626–627
    - quarter-wave, 628–629

T

- T-equivalent circuit, 733
- Tee (T) interconnection, 76
- Terminals, 38, 43–44, 118–126, 152–156, 172, 692–717
  - circuit behavior and, 118–126, 152–156
  - current of, 152–156, 172
  - measurements for circuit construction, 38, 43–44
  - negative feedback and, 153–154, 172
  - Norton equivalent circuits and, 120–123
  - operational amplifier (op amp), 152–156, 172
  - ports, 692
  - symbols for, 152
  - Thévenin equivalent circuits and, 118–126
  - two-port circuits, 692–717
- Terminals, 38, 43–44, 118–126, 152–156, 172, 692–717
- Terminated two-port circuits, 703–708
- Thévenin equivalent circuits, 92, 118–126, 135, 340–341, 343, 495–496
  - amplifier circuit analysis using, 125–126
  - dependent sources and, 122

    - impedance (Z) in, 340–341
    - frequency-domain circuit simplification, 340–341,343
    - Laplace transform method for, 495–496
    - resistance directly from circuit, 123–126
    - s domain, 495–496
    - source transformation for, 121, 340–341, 343
    - terminal circuit simplification using, 92, 118–123, 135
- voltage of, 152–156, 172
- Three-phase circuits, 412–443
    - a-, b-, and c-phase voltage references, 414
    - average power measurement in, 430–433, 435
    - balanced conditions, 412, 416–417, 435
    - basic circuit use and characteristics, 412, 414
    - delta (Δ) loads, 426–427
    - electric power transmission and distribution, 413, 433–435
    - impedance relationships, 422
    - instantaneous power in, 427–428
    - line current, 417, 418–419, 422–423, 435
    - line voltage, 417, 418–419, 435
    - neutral terminal for, 415

    - phase current, 418–419, 422–423, 435
    - phase sequences, 414, 435
    - phase voltage, 418–419, 435
    - phase windings, 415
    - phasor diagrams for, 414, 419
    - power calculations in, 425–430, 435
    - single-phase equivalent circuit for, 418–421, 435
    - unspecified loads, 429–430
    - voltage sources, 415
    - wye (Y) loads, 425–426
    - wye-delta (Y-Δ) circuit analysis, 422–425, 428–429
    - wye-wye (Y-Y) circuit analysis, 416–421, 428
- Time constant (τ), 223–224, 226–227, 229, 256
    - resistor-capacitor (RC) circuits, 229, 256
    - resistor-inductor (RL) circuits, 223–224, 226–227, 256
    - significance of, 226–227
    - steady-state response and, 227
    - transient response and, 227
- Time domain (t), 446–458, 472–474, 476–477, 545, 559–560, 662–663, 671–672, 674, 679–685, 686
    - bandpass filters, 559–560

    - convolution in, 674
    - final-value theorem for, 472–474, 477
    - Fourier transform (f(t)), 662–663, 671–672, 674, 679–685, 686
    - frequency domain (s) relationships, 446, 472–474, 477, 545, 559–560
    - functional transforms, 447, 452–453, 476
    - impulse function (KΔ(t)), 449–451, 476
    - initial-value theorem for, 472–474, 477
    - Laplace transform (f(t)), 446–447, 453–458, 472–474, 476–477
    - low-pass filters, 545
    - operational transforms for, 447, 453–458, 476, 674
    - Parseval’s theorem, 679–685, 686
    - sifting property and, 450–451
    - step function (Ku(t)), 447–448, 476
    - transform pairs, 452–453, 469
    - translation in, 674
    - unit impulse function (Δ(t)), 449, 476
    - unit step function (u(t)), 447, 476
- Time-invariant circuits, 504–505, 521
- Time-invariant instantaneous power, 427–428
- Timing signals, 273

- Touch screens, 59, 78–80, 183, 209–210
  - capacitance of, 183, 209–210
  - resistive circuits of, 59, 78–80
- Transducers (strain gages), 151, 171–172
- Transfer function (H(s)), 500–513, 521, 545, 549, 559, 561, 563, 564–565, 592–593
  - bandpass filters, 559, 565
  - bandreject filters, 561, 563, 565
  - Butterworth filters, 592–593
  - circuit analysis and, 501–502, 504–505
  - convolution integral and, 505–511, 521
  - defined, 500–501
  - frequency-selective circuit analysis using, 545, 549, 559, 564–565
  - high-pass filters, 549, 565
  - Laplace transform method for, 500–513, 521
  - low-pass filters, 545, 564
  - partial fraction expansion and, 461–470, 502–505
  - poles of, 502
  - sinusoidal steady-state response and, 511–513, 521
  - time-invariant circuits, 504–505, 521
  - unit impulse response (h(t)) and, 504–511

    - weighting function, 510
    - zeros of, 502
- Transform pairs, 452–453, 469
- Transformers, 347–356, 361, 737–740
    - current (i) ratios, 353–355
    - dot convention for, 354–355
    - equivalent circuits with, 737–740
    - frequency domain analysis of, 347–356
    - ideal, 351–356, 361, 737–740
    - impedance matching, 356
    - limiting values of, 351–353
    - linear circuits, 347–351, 361
    - polarity of voltage and current, 354–355
    - reflected impedance (Zr), 348–349, 361
    - self-impedance of, 348
    - self-inductance of, 347–348, 351–352
    - steady-state analysis of, 347–356, 361
    - steady-state analysis, 347–356, 361
    - voltage (v) ratios, 353–355
    - winding (primary and secondary), 347

- Transient current component, 324
- Transient response, 227, 677–678
- Transition region, 596–597
- Transmission parameters, 696, 700
- Triangular waveforms, 618, 620, 623–624
- Trigonometric identities, 757
- Twin-T notch filter, 602
- Two-port circuits, 692–717
  - cascaded, 708–711
  - conversion table for parameters, 699
  - hybrid parameters, 696, 697
  - immitance, 696
  - interconnected, 708–711, 713
  - measurements for parameters of, 697, 700
  - model assumptions, 692, 712
  - parallel, 708–709
  - parameters for, 695–703, 712
  - reciprocal, 701–702, 712
  - relationships among, 698–699, 701
  - series-connected, 708–709

    - series-parallel, 708–709
    - symmetric, 701–702, 712–713
    - terminal equations for, 694, 712
    - terminated, 703–708
    - transmission parameters, 696, 700
    - unknown circuit characterization, 693, 711–712
    - z parameters, 696–697, 703–706
- Two-wattmeter method, 431–432, 435

U

- Unbounded response, 250–251, 256
- Underdamped response, 282–286, 291–293, 297–299, 300–301, 309
  - natural response, 282–286, 297–300, 309
  - parallel RLC circuits, 282–286, 291–293, 309
  - series RLC circuits, 297–299, 301
  - step response, 291–293, 300–301
- Unilateral (one-sided) Laplace transform, 446–447
- Unit impulse function (Δ(t)), 449, 476
- Unit impulse response (h(t)), 504–511
- Unit prefixes, 10–11
- Unit step function (u(t)), 447, 452, 476, 669
- Unknown circuit characterization, 693, 711–712

V

- Vampire (standby) power analysis, 375, 399–400
- Volt-amp reactive (VAR), unit of, 379, 385, 401
- Volt-amps (VA), unit of, 384–385, 401
- Voltage (v), 12–17, 34, 40, 42, 46, 50, 58, 70–73, 80, 152–156, 172, 184–186, 199–201, 203–204, 211, 226, 229, 274, 320–322, 332–333, 354–355, 414, 417–419, 435, 631–641, 645–647, 684–685
    - a-, b-, and c-phase references, 414
    - balanced three-phase circuits, 414, 417–419, 435
    - defined, 13
    - dot convention for, 354–355
    - electric charge and, 12–14
    - Fourier series applications, 631–641, 645–647
    - frequency domain, 332–333, 354–355
    - ideal transformer ratios, 353–355
    - induced, 203–204
    - inductor relationships, 184–186
    - input constraint, 153–154
    - Kirchhoff’s voltage law (KVL), 40, 42, 50, 332–333
    - line, 417, 418–419, 435

- measurement of, 58, 70–73, 80
- mutual inductance and, 199–201, 204, 211
- natural response expressions for, 226, 229, 274
- negative feedback and, 153–154, 172
- Ohm’s law for, 33, 50
- op-amp terminals, 152–156, 172
- Parseval’s theorem for, 684–685
- phase, 418–419, 435
- phasor notation for, 414
- periodic, 631–637, 639–641, 645–647
- polarity of, 199–201, 204, 211, 354–355
- polarity reference, 14
- power and energy relationship to, 15–17
- resistor power in terms of, 34
- resistor-capacitor (RC) circuits, 229
- resistor-inductor (RL) circuits, 226
- resistor-inductor-capacitor (RLC) circuits, 274, 637–638
- sinusoidal source, 320–322
- square-wave, 633, 637–638
- steady-state analysis and, 320–322, 332–333, 354–355

    - unknown found using Kirchhoff’s laws, 46
- Voltage-divider circuit, 64–66, 79
- Voltage division, 68–70, 79, 334
- Voltage drop, 199, 516–517
- Voltage sources, 28–31, 50, 415
- Voltage to current (v–i) relationships, 184–185, 190, 327–330, 361, 484–485, 521
    - capacitors, 190, 329–330, 361
    - circuit analysis and, 184–185, 190
    - inductors, 184–185, 328–329, 361
    - Laplace transform method using, 484–485, 521
    - phasor domain of, 327–330, 361
    - resistors, 327–328, 361
    - steady-state analysis and, 327–330, 361
- Voltmeter, 70, 72, 80

W

- Watt (W), unit of, 384–385, 401
- Waveforms, 618, 620, 623–629, 633, 635–636, 650
  - even periodic function, 625–626
  - half-wave periodic function, 627–628
  - odd periodic function, 626–627
  - periodic functions for, 618, 620
  - periodic voltage, 633, 635–636
  - periodic, 618, 620
  - quarter-wave periodic function, 628–629
  - rectangular, 620
  - sinusoidal rectifiers, 618, 620
  - square, 620, 633
  - square-wave voltage, 633
  - steady-state response, 635–636
  - symmetry of periodic functions represented as, 625–629
  - triangular, 618, 620, 623–624
- Wavelength (λ), 7
- Weighting function, 510

- Wheatstone bridge, 73–75, 80
- Winding (primary and secondary), 347, 415
- Wye (Y) interconnection, 76
- Wye (Y) loads, 425–426
- Wye-delta (Y-Δ) circuits, 422–425, 428–429
    - analysis of balanced, 422–425
    - power calculations for, 428–429
- Wye-wye (Y-Y) circuits, 416–421, 428
    - analysis of balanced, 416–421
    - power calculations for, 428

Z

- z parameters, two-port circuits, 696–697, 703–706
- Zero frequency, 544
- Zeros, 470–472, 476, 502, 743–744, 750–751
  - Bode plots and, 743–744, 750–751
  - complex, 750–751
  - frequency domain (F(s)), 470–472, 476
  - real, first-order, 743–744
  - transfer functions (H(s)), 502

Natural Response of a Series RLC circuits

1. **Determine the initial capacitor voltage (V0) and inductor current (I0)** from the circuit.

2. **Determine the values of** α **and** $\omega 0$ using the equations in Table 8.4.

3. **If** $\alpha 2 > \omega 02$, **the response is overdamped** and $i(t)=A1es1t+A2es2t, t \geq 0$;

   If $\alpha 2 < \omega 02$ **the response is underdamped** and $i(t)=B1e\alpha t\cos\omega dt+B2e\alpha t\sin\omega dt, t \geq 0$;

   If $\alpha 2=\omega 02$, **the response is critically damped** and $i(t)=D1te-\alpha t+D2e-\alpha t, t \geq 0$

4. **If the response is overdamped, calculate s1 and s2** using the equations in Table 8.4;

   If the response is underdamped, calculate ωd using the equation in Table 8.4.

5. **If the response is overdamped, calculate A1 and A2** by simultaneously solving the equations in Table 8.4;

   If the response is underdamped, calculate B1 and B2 by simultaneously solving the equations in Table 8.4;

   If the response is critically damped, calculate D1 and D2 by simultaneously solving the equations in Table 8.4.

6. **Write the equation for i(t) from Step 3** using the results from Steps 4 and 5; find any desired component voltages.

Analysis Method 8.5 The natural response of series RLC circuits.

Table 8.4 Equations for analyzing the natural response of series RLC circuits

Characteristic equation	$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$
Neper, resonant, and damped frequencies	$\alpha = \frac{R}{2L} \quad \omega_0 = \sqrt{\frac{1}{LC}} \quad \omega_d = \sqrt{\omega_0^2 - \alpha^2}$
Roots of the characteristic equation	$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}, \quad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$
$\alpha^2 > \omega_0^2$: overdamped	$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}, t \geq 0$
	$i(0^+) = A_1 + A_2 = I_0$
	$\frac{di(0^+)}{dt} = s_1 A_1 + s_2 A_2 = \frac{1}{L}(-RI_0 - V_0)$
$\alpha^2 < \omega_0^2$: underdamped	$i(t) = B_1 e^{-\alpha t} \cos \omega_d t + B_2 e^{-\alpha t} \sin \omega_d t, \quad t \geq 0$
	$i(0^+) = B_1 = I_0$
	$\frac{di(0^+)}{dt} = -\alpha B_1 + \omega_d B_2 = \frac{1}{L}(-RI_0 - V_0)$
$\alpha^2 = \omega_0^2$: critically damped	$i(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}, \quad t \geq 0$
	$i(0^+) = D_2 = I_0$
	$\frac{di(0^+)}{dt} = D_1 - \alpha D_2 = \frac{1}{L}(-RI_0 - V_0)$

(Note that the equations in the last three rows assume that the

reference direction for the current in every component is in the direction of the reference voltage drop across that component.)

Step Response of a Series RLC circuits

1. **Determine the initial capacitor voltage (V0), the initial inductor current (I0), and the final capacitor voltage (Vf)** from the circuit.

2. **Determine the values of** α **and** $\omega 0$ using the equations in Table 8.5.

3. **If** $\alpha 2 > \omega 02$, **the response is overdamped** and $vC(t) = Vf + A'1es1t + A'2es2t$, $t \geq 0+$;

   If $\alpha 2 < \omega 02$, **the response is underdamped** and $vC(t) = Vf + B'1e^{-\alpha t}\cos\omega dt + B'2e^{-\alpha t}\sin\omega dt$, $t \geq 0+$;

   If $\alpha 2 = \omega 02$, **the response is critically damped** and $vC(t) = Vf + D'1te^{-\alpha t} + D'2e^{-\alpha t}$, $t \geq 0+$

4. **If the response is overdamped, calculate s1 and s2** using the equations in Table 8.5;

   If the response is underdamped, calculate ωd using the equation in Table 8.5.

5. **If the response is overdamped, calculate** $A1'$ **and** $A2'$ by simultaneously solving the equations in Table 8.5;

   If the response is underdamped, calculate $B1'$ **and** $B2'$ by simultaneously solving the equations in Table 8.5;

   If the response is critically damped, calculate $D1'$ **and** $D2'$ by simultaneously solving the equations in Table 8.5.

6. **Write the equation for vC(t) from Step 3** using the results from Steps

4 and 5; find the inductor voltage and any desired branch currents.

Table 8.5 Equations for analyzing the step response of series RLC circuits

Characteristic equation	$s^2 + \frac{R}{L}s + \frac{1}{LC} = \frac{V}{LC}$
Neper, resonant, and damped frequencies	$\alpha = \frac{R}{2L} \quad \omega_0 = \sqrt{\frac{1}{LC}} \quad \omega_d = \sqrt{\omega_0^2 - \alpha^2}$
Roots of the characteristic equation	$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}, \quad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$
$\alpha^2 > \omega_0^2$: overdamped	$v_C(t) = V_f + A_1' e^{s_1 t} + A_2' e^{s_2 t}, t \geq 0$
	$v_C(0^+) = V_f + A_1' + A_2' = V_0$
	$\frac{dv_C(0^+)}{dt} = s_1 A_1' + s_2 A_2' = \frac{I_0}{C}$
$\alpha^2 < \omega_0^2$: underdamped	$v_C(t) = V_f + B_1' e^{-\alpha t} \cos \omega_d t + B_2' e^{-\alpha t} \sin \omega_d t, t \geq 0$
	$v_C(0^+) = V_f + B_1' = V_0$
	$\frac{dv_C(0^+)}{dt} = -\alpha B_1' + \omega_d B_2' = \frac{I_0}{C}$
$\alpha^2 = \omega_0^2$: critically damped	$v_C(t) = V_f + D_1' t e^{-\alpha t} + D_2' e^{-\alpha t}, t \geq 0$
	$v_C(0^+) = V_f + D_2' = V_0$
	$\frac{dv_C(0^+)}{dt} = D_1' - \alpha D_2' = \frac{I_0}{C}$

(Note that the equations in the last three rows assume that the reference direction for the current in every component is in the direction of the reference voltage drop across that component.)

Table 12.3 Four Useful Transform Pairs

Pair Number	Nature of Roots	$F(s)$	$f(t)$
1	Distinct real	$\frac{K}{s+a}$	$Ke^{-at}u(t)$
2	Repeated real	$\frac{K}{(s+a)^2}$	$Kte^{-at}u(t)$
3	Distinct complex	$\frac{K}{s+\alpha-j\beta}+\frac{K^*}{s+\alpha+j\beta}$	$2\lvert K\rvert e^{-\alpha t}\cos(\beta t+\theta)u(t)$
4	Repeated complex	$\frac{K}{(s+\alpha-j\beta)^2}+\frac{K^*}{(s+\alpha+j\beta)^2}$	$2t\lvert K\rvert e^{-\alpha t}\cos(\beta t+\theta)u(t)$

Note: In pairs 1 and 2, *K* is a real quantity, whereas in pairs 3 and 4, *K* is the complex quantity | K |θ.

Laplace Transform Method

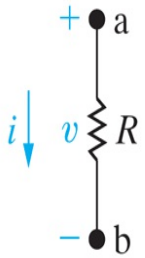
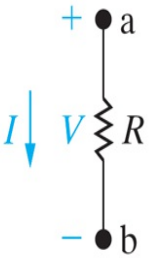
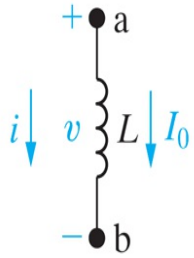
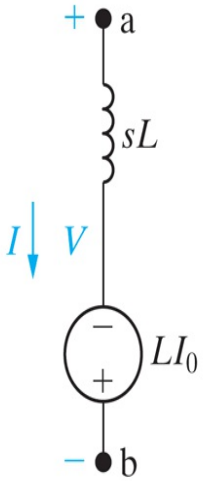
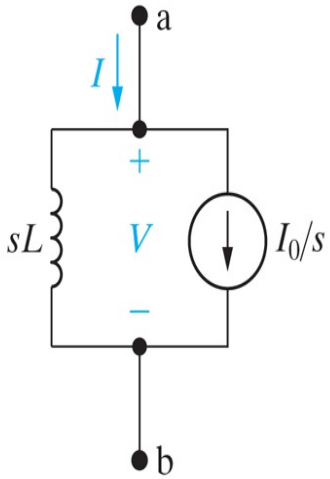
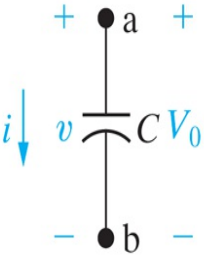
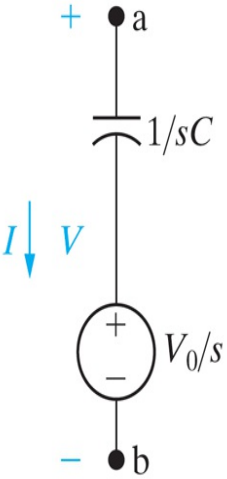
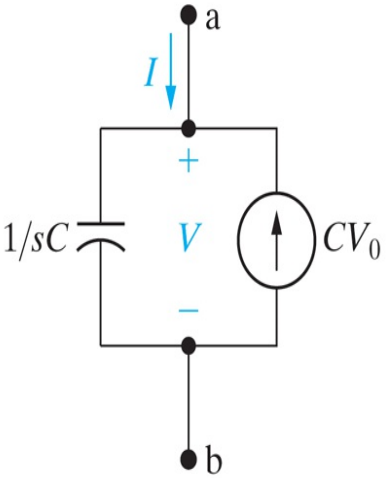
1. **Determine the initial conditions** for inductors and capacitors.
2. **Laplace-transform independent voltage and current functions** using Tables 12.1 and 12.2.
3. **Transform symbolic time-domain voltages and currents** into s-domain symbols.
4. **Transform remaining circuit components** into the s domain using Table 13.1.
5. **Analyze the s-domain circuit** using resistive circuit analysis techniques; represent the resulting s-domain voltages and currents as

ratios of polynomials in s.

6. **Use the initial- and final-value theorems** to check the s-domain voltages and currents.

7. **Inverse-Laplace-transform the s-domain voltages and currents** using partial fraction expansion and Table 12.3.

Table 13.1 Summary of the s-Domain Equivalent Circuits

TIME DOMAIN	FREQUENCY DOMAIN
 $v = Ri$	 $V = RI$
 $v = L\,di/dt,$ $i = \frac{1}{L}\int_{0^-}^{t} v\,dx + I_0$	 $V = sLI - LI_0$  $I = \frac{V}{sL} + \frac{I_0}{s}$
 $i = C\,dv/dt,$ $v = \frac{1}{C}\int_{0^-}^{t} i\,dx + V_0$	 $V = \frac{I}{sC} + \frac{V_0}{s}$  $I = sCV - CV_0$

Contents

1. Electric Circuits
2. Electric Circuits
3. Brief Contents
4. Contents
5. List of Examples
6. List of Tables
7. List of Analysis Methods
8. Combine this...
9. With the Power of Mastering Engineering for Electric Circuits 11/e
10. Empower each learner
11. Preface
    1. Why This Edition?
    2. Hallmark Features
    3. Resources For Students
    4. Resources for Instructors
    5. Prerequisites
    6. Course Options
12. Electric Circuits
13. Chapter 1 Circuit Variables
    1. Chapter Contents
    2. Chapter Objectives
    3. 1.1 Electrical Engineering: An Overview
        1. Circuit Theory
        2. Problem Solving
    4. 1.2 The International System of Units
    5. 1.3 Circuit Analysis: An Overview
    6. 1.4 Voltage and Current
    7. 1.5 The Ideal Basic Circuit Element
    8. 1.6 Power and Energy
    9. Summary
    10. Problems
        1. Section 1.2
        2. Section 1.4

3. Sections 1.5–1.6

14. Chapter 2 Circuit Elements
    1. Chapter Contents
    2. Chapter Objectives
    3. 2.1 Voltage and Current Sources
    4. 2.2 Electrical Resistance (Ohm's Law)
    5. 2.3 Constructing a Circuit Model
    6. 2.4 Kirchhoff's Laws
        1. Kirchhoff's Current Law
        2. Kirchhoff's Voltage Law
    7. 2.5 Analyzing a Circuit Containing Dependent Sources
    8. Summary
    9. Problems
        1. Section 2.1
        2. Sections 2.2–2.3
        3. Section 2.4
        4. Sections 2.1–2.5
15. Chapter 3 Simple Resistive Circuits
    1. Chapter Contents
    2. Chapter Objectives
    3. 3.1 Resistors in Series
    4. 3.2 Resistors in Parallel
    5. 3.3 The Voltage-Divider and Current-Divider Circuits
        1. The Voltage-Divider Circuit
        2. The Current-Divider Circuit
    6. 3.4 Voltage Division and Current Division
        1. Voltage Division
        2. Current Division
    7. 3.5 Measuring Voltage and Current
        1. Analog Meters
        2. Digital Meters
    8. 3.6 Measuring Resistance—The Wheatstone Bridge
    9. 3.7 Delta-to-Wye (Pi-to-Tee) Equivalent Circuits
    10. Summary
    11. Problems
        1. Sections 3.1–3.2
        2. Section 3.3

        3. Section 3.4
        4. Section 3.5
        5. Section 3.6
        6. Section 3.7
        7. Sections 3.1–3.7
16. Chapter 4 Techniques of Circuit Analysis
    1. Chapter Contents
    2. Chapter Objectives
    3. 4.1 Terminology
        1. Describing a Circuit—The Vocabulary
        2. Simultaneous Equations—How Many?
    4. 4.2 Introduction to the Node-Voltage Method
    5. 4.3 The Node-Voltage Method and Dependent Sources
    6. 4.4 The Node-Voltage Method: Some Special Cases
        1. The Concept of a Supernode
    7. 4.5 Introduction to the Mesh-Current Method
        1. Applying the Mesh-Current Method
    8. 4.6 The Mesh-Current Method and Dependent Sources
    9. 4.7 The Mesh-Current Method: Some Special Cases
        1. The Concept of a Supermesh
    10. 4.8 The Node-Voltage Method Versus the Mesh-Current Method
    11. 4.9 Source Transformations
    12. 4.10 Thévenin and Norton Equivalents
        1. The Thévenin Equivalent
        2. The Norton Equivalent
        3. Using Source Transformations
    13. 4.11 More on Deriving the Thévenin Equivalent
    14. 4.12 Maximum Power Transfer
    15. 4.13 Superposition
    16. Summary
    17. Problems
        1. Section 4.1
        2. Section 4.2
        3. Section 4.3
        4. Section 4.4
        5. Section 4.5
        6. Section 4.6

7. Section 4.7
8. Section 4.8
9. Section 4.9
10. Section 4.10
11. Section 4.11
12. Section 4.12
13. Section 4.13
14. Sections 4.1–4.13

17. Chapter 5 The Operational Amplifier
    1. Chapter Contents
    2. Chapter Objectives
    3. 5.1 Operational Amplifier Terminals
    4. 5.2 Terminal Voltages and Currents
        1. OP AMP Input Voltage Constraints
        2. OP AMP Input Current Constraints
    5. 5.3 The Inverting-Amplifier Circuit
    6. 5.4 The Summing-Amplifier Circuit
    7. 5.5 The Noninverting-Amplifier Circuit
    8. 5.6 The Difference-Amplifier Circuit
        1. The Difference Amplifier—Another Perspective
        2. Measuring Difference-Amplifier Performance—The Common Mode Rejection Ratio
    9. 5.7 A More Realistic Model for the Operational Amplifier
        1. Analyzing an Inverting-Amplifier Circuit Using a More Realistic Op Amp Model
        2. Analyzing a Noninverting-Amplifier Circuit Using a More Realistic Op Amp Model
    10. Summary
    11. Problems
        1. Sections 5.1–5.2
        2. Section 5.3
        3. Section 5.4
        4. Section 5.5
        5. Section 5.6
        6. Sections 5.1–5.6
        7. Section 5.7
        8. Sections 5.1–5.7

18. Chapter 6 Inductance, Capacitance, and Mutual Inductance
    1. Chapter Contents
    2. Chapter Objectives
    3. 6.1 The Inductor
        1. Current in an Inductor in Terms of the Voltage Across the Inductor
        2. Power and Energy in the Inductor
    4. 6.2 The Capacitor
    5. 6.3 Series-Parallel Combinations of Inductance and Capacitance
        1. Inductors in Series and Parallel
        2. Capacitors in Series and Parallel
        3. Inductor and Capacitor Symmetry
    6. 6.4 Mutual Inductance
        1. The Procedure for Determining Dot Markings
    7. 6.5 A Closer Look at Mutual Inductance
        1. A Review of Self-Inductance
        2. The Concept of Mutual Inductance
        3. Mutual Inductance in Terms of Self-Inductance
        4. Energy Calculations
    8. Summary
    9. Problems
        1. Section 6.1
        2. Section 6.2
        3. Section 6.3
        4. Section 6.4
        5. Section 6.5
        6. Sections 6.1–6.5
19. Chapter 7 Response of First-Order RL and RC Circuits
    1. Chapter Contents
    2. Chapter Objectives
    3. 7.1 The Natural Response of an *RL* Circuit
        1. Deriving the Expression for the Current
        2. Deriving the Expressions for Voltage, Power, and Energy
        3. The Significance of the Time Constant
    4. 7.2 The Natural Response of an RC Circuit
        1. Deriving the Expression for the Voltage
    5. 7.3 The Step Response of RL and RC Circuits

        1. The Step Response of an RL Circuit
        2. Observations on the Step Response of an RL Circuit
        3. The Step Response of an RC Circuit
    6. 7.4 A General Solution for Step and Natural Responses
    7. 7.5 Sequential Switching
    8. 7.6 Unbounded Response
    9. 7.7 The Integrating Amplifier
    10. Summary
    11. Problems
        1. Section 7.1
        2. Section 7.2
        3. Section 7.3
        4. Section 7.4
        5. Section 7.5
        6. Section 7.6
        7. Section 7.7
        8. Sections 7.1–7.7
20. Chapter 8 Natural and Step Responses of RLC Circuits
    1. Chapter Contents
    2. Chapter Objectives
    3. 8.1 Introduction to the Natural Response of a Parallel RLC Circuit
        1. The General Solution of the Second-Order Differential Equation
    4. 8.2 The Forms of the Natural Response of a Parallel RLC Circuit
        1. The Overdamped Response
        2. The Underdamped Voltage Response
        3. Characteristics of the Underdamped Response
        4. The Critically Damped Voltage Response
    5. 8.3 The Step Response of a Parallel RLC Circuit
        1. The Indirect Approach
        2. The Direct Approach
    6. 8.4 The Natural and Step Response of a Series RLC Circuit
    7. 8.5 A Circuit with Two Integrating Amplifiers
        1. Two Integrating Amplifiers with Feedback Resistors
    8. Summary
    9. Problems
        1. Sections 8.1–8.2

    2. Section 8.3
    3. Section 8.4
    4. Section 8.5
    5. Sections 8.18.5
21. Chapter 9 Sinusoidal Steady-State Analysis
    1. Chapter Contents
    2. Chapter Objectives
    3. 9.1 The Sinusoidal Source
    4. 9.2 The Sinusoidal Response
    5. 9.3 The Phasor
        1. Inverse Phasor Transform
    6. 9.4 The Passive Circuit Elements in the Frequency Domain
        1. The V-I Relationship for a Resistor
        2. The V-I Relationship for an Inductor
        3. The V-I Relationship for a Capacitor
        4. Impedance and Reactance
    7. 9.5 Kirchhoff's Laws in the Frequency Domain
        1. Kirchhoff's Voltage Law in the Frequency Domain
        2. Kirchhoff's Current Law in the Frequency Domain
    8. 9.6 Series, Parallel, and Delta-to-Wye Simplifications
        1. Combining Impedances in Series
        2. Combining Impedances in Parallel
        3. Delta-to-Wye Transformations
    9. 9.7 Source Transformations and Thévenin–Norton Equivalent Circuits
    10. 9.8 The Node-Voltage Method
    11. 9.9 The Mesh-Current Method
    12. 9.10 The Transformer
        1. The Analysis of a Linear Transformer Circuit
        2. Reflected Impedance
    13. 9.11 The Ideal Transformer
        1. Exploring Limiting Values
        2. Determining the Voltage and Current Ratios
        3. Determining the Polarity of the Voltage and Current Ratios
        4. Using an Ideal Transformer for Impedance Matching
    14. 9.12 Phasor Diagrams
    15. Summary

16. Problems
    1. Section 9.1
    2. Section 9.2
    3. Sections 9.3-9.4
    4. Sections 9.5 and 9.6
    5. Section 9.7
    6. Section 9.8
    7. Section 9.9
    8. Sections 9.5-9.9
    9. Section 9.10
    10. Section 9.11
    11. Section 9.12
    12. Section 9.1-9.12
22. Chapter 10 Sinusoidal Steady-State Power Calculations
    1. Chapter Contents
    2. Chapter Objectives
    3. 10.1 Instantaneous Power
    4. 10.2 Average and Reactive Power
        1. Power for Purely Resistive Circuits
        2. Power for Purely Inductive Circuits
        3. Power for Purely Capacitive Circuits
        4. The Power Factor
        5. Appliance Ratings
    5. 10.3 The rms Value and Power Calculations
    6. 10.4 Complex Power
    7. 10.5 Power Calculations
        1. Alternate Forms for Complex Power
    8. 10.6 Maximum Power Transfer
        1. The Maximum Average Power Absorbed
        2. Maximum Power Transfer When ZL is Restricted
    9. Summary
    10. Problems
        1. Sections 10.1–10.2
        2. Section 10.3
        3. Sections 10.4–10.5
        4. Section 10.6
        5. Sections 10.1–10.6

23. Chapter 11 Balanced Three-Phase Circuits
    1. Chapter Contents
    2. Chapter Objectives
    3. 11.1 Balanced Three-Phase Voltages
    4. 11.2 Three-Phase Voltage Sources
    5. 11.3 Analysis of the Wye-Wye Circuit
        1. Terminology
        2. Constructing a Single-Phase Equivalent Circuit
    6. 11.4 Analysis of the Wye-Delta Circuit
    7. 11.5 Power Calculations in Balanced Three-Phase Circuits
        1. Average Power in a Balanced Wye Load
        2. Reactive and Complex Power in a Balanced Wye Load
        3. Power Calculations in a Balanced Delta Load
        4. Instantaneous Power in Three-Phase Circuits
    8. 11.6 Measuring Average Power in Three-Phase Circuits
        1. The Two-Wattmeter Method
    9. Summary
    10. Problems
        1. Section 11.1
        2. Section 11.2
        3. Section 11.3
        4. Section 11.4
        5. Section 11.5
        6. Section 11.6
        7. Sections 11.1–11.6
24. Chapter 12 Introduction to the Laplace Transform
    1. Chapter Contents
    2. Chapter Objectives
    3. 12.1 Definition of the Laplace Transform
    4. 12.2 The Step Function
    5. 12.3 The Impulse Function
        1. Describing the Impulse Function
        2. Defining the Impulse Function and Its Sifting Property
        3. Laplace Transform and Derivatives of the Impulse Function
    6. 12.4 Functional Transforms
    7. 12.5 Operational Transforms
        1. Multiplication by a Constant

2. Addition and Subtraction
3. Differentiation
4. Integration
5. Translation in the Time Domain
6. Translation in the Frequency Domain
7. Scale Changing

8. 12.6 Applying the Laplace Transform
9. 12.7 Inverse Transforms
    1. Partial Fraction Expansion: Proper Rational Functions
    2. Partial Fraction Expansion: Distinct Real Roots of D(s)
    3. Partial Fraction Expansion: Distinct Complex Roots of D(s)
    4. Partial Fraction Expansion: Repeated Real Roots of D(s)
    5. Partial Fraction Expansion: Repeated Complex Roots of D(s)
    6. Partial Fraction Expansion: Improper Rational Functions
10. 12.8 Poles and Zeros of F(s)
11. 12.9 Initial- and Final-Value Theorems
12. Summary
13. Problems
    1. Section 12.2
    2. Section 12.3
    3. Sections 12.4–12.5
    4. Section 12.6
    5. Section 12.7
    6. Sections 12.8–12.9
    7. Sections 12.1–12.9

25. Chapter 13 The Laplace Transform in Circuit Analysis
    1. Chapter Contents
    2. Chapter Objectives
    3. 13.1 Circuit Elements in the s Domain
        1. A Resistor in the s Domain
        2. An Inductor in the s Domain
        3. A Capacitor in the s Domain
    4. 13.2 Circuit Analysis in the s Domain
    5. 13.3 Applications
    6. 13.4 The Transfer Function
        1. The Location of Poles and Zeros of H(s)
    7. 13.5 The Transfer Function in Partial Fraction Expansions

1. Observations on the Use of H(s) in Circuit Analysis
8. 13.6 The Transfer Function and the Convolution Integral
    1. The Concepts of Memory and the Weighting Function
9. 13.7 The Transfer Function and the Steady-State Sinusoidal Response
10. 13.8 The Impulse Function in Circuit Analysis
    1. Switching Operations
        1. Capacitor Circuit
    2. Impulsive Sources
11. Summary
12. Problems
    1. Section 13.1
    2. Section 13.2
    3. Section 13.3
    4. Sections 13.4–13.5
    5. Section 13.6
    6. Section 13.7
    7. Section 13.8
    8. Sections 13.1–13.8

26. Chapter 14 Introduction to Frequency Selective Circuits
    1. Chapter Contents
    2. Chapter Objectives
    3. 14.1 Some Preliminaries
    4. 14.2 Low-Pass Filters
        1. The Series RL Circuit—Qualitative Analysis
        2. Defining the Cutoff Frequency
        3. The Series RL Circuit—Quantitative Analysis
        4. A Series RC Circuit
        5. Relating the Frequency Domain to the Time Domain
    5. 14.3 High-Pass Filters
        1. The Series RC Circuit—Qualitative Analysis
        2. The Series RC Circuit—Quantitative Analysis
    6. 14.4 Bandpass Filters
        1. Center Frequency, Bandwidth, and Quality Factor
        2. The Series RLC Circuit—Qualitative Analysis
        3. The Series RLC Circuit—Quantitative Analysis
        4. Relating the Frequency Domain to the Time Domain

7. 14.5 Bandreject Filters
    1. The Series RLC Circuit—Qualitative Analysis
    2. The Series RLC Circuit—Quantitative Analysis
8. Summary
9. Problems
    1. Section 14.2
    2. Section 14.3
    3. Section 14.4
    4. Section 14.5
    5. Sections 14.1–14.5
27. Chapter 15 Active Filter Circuits
    1. Chapter Contents
    2. Chapter Objectives
    3. 15.1 First-Order Low-Pass and High-Pass Filters
        1. A Note About Frequency Response Plots
    4. 15.2 Scaling
        1. The Use of Scaling in the Design of Filters
    5. 15.3 Op Amp Bandpass and Bandreject Filters
    6. 15.4 Higher-Order Op Amp Filters
        1. Cascading Identical Filters
        2. Butterworth Filters
        3. Butterworth Filter Circuits
        4. The Order of a Butterworth Filter
        5. Butterworth High-Pass, Bandpass, and Bandreject Filters
    7. 15.5 Narrowband Bandpass and Bandreject Filters
    8. Summary
    9. Problems
        1. Section 15.1
        2. Section 15.2
        3. Section 15.3
        4. Section 15.4
        5. Section 15.5
        6. Sections 15.1–15.5
28. Chapter 16 Fourier Series
    1. Chapter Contents
    2. Chapter Objectives
    3. 16.1 Fourier Series Analysis: An Overview

4. 16.2 The Fourier Coefficients
5. 16.3 The Effect of Symmetry on the Fourier Coefficients
    1. Even-Function Symmetry
    2. Odd-Function Symmetry
    3. Half-Wave Symmetry
    4. Quarter-Wave Symmetry
6. 16.4 An Alternative Trigonometric Form of the Fourier Series
7. 16.5 An Application
    1. An Application of the Direct Approach to the Steady-State Response
8. 16.6 Average-Power Calculations with Periodic Functions
9. 16.7 The rms Value of a Periodic Function
10. 16.8 The Exponential Form of the Fourier Series
11. 16.9 Amplitude and Phase Spectra
12. Summary
13. Problems
    1. Sections 16.1–16.2
    2. Section 16.3
    3. Section 16.4
    4. Section 16.5
    5. Section 16.6
    6. Section 16.7
    7. Section 16.8
    8. Section 16.9
    9. Sections 16.1–16.9

29. Chapter 17 The Fourier Transform
    1. Chapter Contents
    2. Chapter Objectives
    3. 17.1 The Derivation of the Fourier Transform
    4. 17.2 The Convergence of the Fourier Integral
    5. 17.3 Using Laplace Transforms to Find Fourier Transforms
    6. 17.4 Fourier Transforms in the Limit
        1. The Fourier Transform of a Signum Function
        2. The Fourier Transform of a Unit Step Function
        3. The Fourier Transform of a Cosine Function
    7. 17.5 Some Mathematical Properties
    8. 17.6 Operational Transforms

1. Multiplication by a Constant
2. Addition (Subtraction)
3. Differentiation
4. Integration
5. Scale Change
6. Translation in the Time Domain
7. Translation in the Frequency Domain
8. Modulation
9. Convolution in the Time Domain
10. Convolution in the Frequency Domain

9. 17.7 Circuit Applications
10. 17.8 Parseval''s Theorem
    1. Demonstrating Parseval''s Theorem
    2. Interpreting Parseval''s Theorem
11. Summary
12. Problems
    1. Sections 17.1 17.2
    2. Sections 17.3—17.5
    3. Section 17.6
    4. Section 17.7
    5. Section 17.8

30. Chapter 18 Two-Port Circuits
    1. Chapter Contents
    2. Chapter Objectives
    3. 18.1 The Terminal Equations
    4. 18.2 The Two-Port Parameters
       1. Relationships Among the Two-Port Parameters
       2. Reciprocal Two-Port Circuits
    5. 18.3 Analysis of the Terminated Two-Port Circuit
       1. The Six Characteristics in Terms of the z Parameters
    6. 18.4 Interconnected Two-Port Circuits
    7. Summary
    8. Problems
       1. Sections 18.1–18.2
       2. Section 18.3
       3. Section 18.4
       4. Sections 18.1–18.4

31. Appendix A The Solution of Linear Simultaneous Equations
    1. A.1 Preliminary Steps
    2. A.2 Calculator and Computer Methods
        1. Using Excel
        2. Using MATLAB
    3. A.3 Paper-and-Pencil Methods
        1. Back-Substitution
        2. Cramer's Method
    4. A.4 Applications
32. Appendix B Complex Numbers
    1. B.1 Notation
    2. B.2 The Graphical Representation of a Complex Number
    3. B.3 Arithmetic Operations
        1. Addition (Subtraction)
        2. Multiplication (Division)
    4. B.4 Useful Identities
    5. B.5 The Integer Power of a Complex Number
    6. B.6 The Roots of a Complex Number
33. Appendix C More on Magnetically Coupled Coils and Ideal Transformers
    1. C.1 Equivalent Circuits for Magnetically Coupled Coils
        1. The T-Equivalent Circuit
        2. The π-Equivalent Circuit
    2. C.2 The Need for Ideal Transformers in the Equivalent Circuits
34. Appendix D The Decibel
35. Appendix E Bode Diagrams
    1. E.1 Real, First-Order Poles and Zeros
    2. E.2 Straight-Line Amplitude Plots
    3. E.3 More Accurate Amplitude Plots
    4. E.4 Straight-Line Phase Angle Plots
    5. E.5 Bode Diagrams: Complex Poles and Zeros
    6. E.6 Straight-Line Amplitude Plots for Complex Poles
    7. E.7 Correcting Straight-Line Amplitude Plots for Complex Poles
    8. E.8 Phase Angle Plots for Complex Poles
36. Appendix F An Abbreviated Table of Trigonometric Identities
37. Appendix G An Abbreviated Table of Integrals
38. Appendix H Common Standard Component Values

39. Answers to Selected Problems
40. Index
    1. A
    2. B
    3. C
    4. D
    5. E
    6. F
    7. G
    8. H
    9. I
    10. K
    11. L
    12. M
    13. N
    14. O
    15. P
    16. Q
    17. R
    18. S
    19. T
    20. U
    21. V
    22. W
    23. Z

List of Illustrations

Landmarks

1. Brief Contents
2. Frontmatter
3. Start of Content
4. backmatter
5. List of Illustrations

1. i
2. ii
3. iii
4. iv
5. v
6. vi
7. vii
8. viii
9. ix
10. x
11. xi
12. xii
13. xiii
14. xiv
15. xv
16. xvi
17. xvii
18. xviii
19. xix
20. xx
21. xxi
22. xxii
23. xxiii
24. xxiv
25. xxv
26. xxvi
27. 1
28. 2
29. 3
30. 4
31. 5
32. 6
33. 7
34. 8
35. 9
36. 10
37. 11

38. 12
39. 13
40. 14
41. 15
42. 16
43. 17
44. 18
45. 19
46. 20
47. 21
48. 22
49. 23
50. 24
51. 25
52. 26
53. 27
54. 28
55. 29
56. 30
57. 31
58. 32
59. 33
60. 34
61. 35
62. 36
63. 37
64. 38
65. 39
66. 40
67. 41
68. 42
69. 43
70. 44
71. 45
72. 46
73. 47
74. 48

75. 49
76. 50
77. 51
78. 52
79. 53
80. 54
81. 55
82. 56
83. 57
84. 58
85. 59
86. 60
87. 61
88. 62
89. 63
90. 64
91. 65
92. 66
93. 67
94. 68
95. 69
96. 70
97. 71
98. 72
99. 73
100. 74
101. 75
102. 76
103. 77
104. 78
105. 79
106. 80
107. 81
108. 82
109. 83
110. 84
111. 85

12. 86
13. 87
14. 88
15. 89
16. 90
17. 91
18. 92
19. 93
20. 94
21. 95
22. 96
23. 97
24. 98
25. 99
26. 100
27. 101
28. 102
29. 103
30. 104
31. 105
32. 106
33. 107
34. 108
35. 109
36. 110
37. 111
38. 112
39. 113
40. 114
41. 115
42. 116
43. 117
44. 118
45. 119
46. 120
47. 121
48. 122

49. 123
50. 124
51. 125
52. 126
53. 127
54. 128
55. 129
56. 130
57. 131
58. 132
59. 133
60. 134
61. 135
62. 136
63. 137
64. 138
65. 139
66. 140
67. 141
68. 142
69. 143
70. 144
71. 145
72. 146
73. 147
74. 148
75. 149
76. 150
77. 151
78. 152
79. 153
80. 154
81. 155
82. 156
83. 157
84. 158
85. 159

186. 160
187. 161
188. 162
189. 163
190. 164
191. 165
192. 166
193. 167
194. 168
195. 169
196. 170
197. 171
198. 172
199. 173
200. 174
201. 175
202. 176
203. 177
204. 178
205. 179
206. 180
207. 181
208. 182
209. 183
210. 184
211. 185
212. 186
213. 187
214. 188
215. 189
216. 190
217. 191
218. 192
219. 193
220. 194
221. 195
222. 196

223. 197
224. 198
225. 199
226. 200
227. 201
228. 202
229. 203
230. 204
231. 205
232. 206
233. 207
234. 208
235. 209
236. 210
237. 211
238. 212
239. 213
240. 214
241. 215
242. 216
243. 217
244. 218
245. 219
246. 220
247. 221
248. 222
249. 223
250. 224
251. 225
252. 226
253. 227
254. 228
255. 229
256. 230
257. 231
258. 232
259. 233

260. 234
261. 235
262. 236
263. 237
264. 238
265. 239
266. 240
267. 241
268. 242
269. 243
270. 244
271. 245
272. 246
273. 247
274. 248
275. 249
276. 250
277. 251
278. 252
279. 253
280. 254
281. 255
282. 256
283. 257
284. 258
285. 259
286. 260
287. 261
288. 262
289. 263
290. 264
291. 265
292. 266
293. 267
294. 268
295. 269
296. 270

297. 271
298. 272
299. 273
300. 274
301. 275
302. 276
303. 277
304. 278
305. 279
306. 280
307. 281
308. 282
309. 283
310. 284
311. 285
312. 286
313. 287
314. 288
315. 289
316. 290
317. 291
318. 292
319. 293
320. 294
321. 295
322. 296
323. 297
324. 298
325. 299
326. 300
327. 301
328. 302
329. 303
330. 304
331. 305
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The given model shows a square circuit which has a vertical wire connecting its lateral sides, and a horizontal wire connecting the middle of the vertical wire with the left side. Eight components are marked on various limbs of the circuit, as follows:

* a and b on the left side, on either side of the horizontal wire

* c and e on the two lateral sides

* d on the horizontal wire

* f and g on the vertical wire, on either side of the horizontal wire

* h on the right side

The given drawing shows the various components of a telephone system, which are connected by wires and cables. A private telephone is connected to a telephone box, which connects to three telephone towers. The towers communicate with the local exchange, which is further connected to a main exchange. Several main exchanges communicate with each other through fiber optic cables.

A cellphone connects to a cell tower. Several cell towers, each having its own cells, are connected to a mobile exchange, which further communicates with the main exchange through optic cables.

The main exchanges of a region are connected to an international exchange, which communicates with another international exchange through an undersea cable, and also through a communication satellite.

The scan shows the Femur on top and the Tibia at the bottom, with the joint being cushioned by the Meniscus. The Patella or Knee cap is identified at the knee joint, covered by the Patellar tendon.

The drawing shows a commercial aircraft on which the different electrical and communication systems are marked, which are connected, as follows:

Electrical distribution (behind the wings):

- Starter generator (left engine)
- Ice protection (left wing)
- Primary controls (right wing)
    - - Secondary controls (right wing)
- Primary controls (tail)
- Environmental control (behind cockpit)
- Engine systems (right engine)
    - - Landing gear (underneath)
    - - Air traffic communications (cockpit)
        - Flight control (cockpit)
            - * Navigation (cockpit)

A table of the International System of Units (SI) lists the seven quantities along with the basic unit and symbol for each.

The table lists Quantity, Basic unit, and Symbol as follows:

Length; meter; m

Mass; kilogram; kg

Time; second; s

Electric current; ampere; A

Thermodynamic temperature; degree kelvin; K

Amount of substance; mole; mol

Luminous intensity; candela; cd

A table of the derived units in SI lists several quantities along with their unit name (symbol) and formula.

The table lists Quantity, Unit name (Symbol in bracket), and Formula, as follows:

Frequency; hertz (Hz); s raised to the power of minus 1

Force; newton (N); kg multiplied by m over s squared

Energy or work; joule (J); N multiplied by m

Power; watt (W); J over s

Electric charge; coulomb (C); A multiplied by s

Electric potential; volt (V); J over C

Electric resistance; ohm (ohm); V over A

Electric conductance; Siemens (S); A over V

Electric capacitance; farad (F); C over V

Magnetic flux; weber (Wb); V multiplied by s

Inductance; henry (H); W multiplied by b over A

A table lists several standardized prefixes to signify powers of ten.

A table lists Prefix, Symbol, and the Powers of ten as follows:

atto; a; 10 raised to minus 18

femto; f; 10 raised to minus 15

pico; p; 10 raised to minus 12

nano; n; 10 raised to minus 9

micro; mu; 10 raised to minus 6

milli; m; 10 raised to minus 3

centi; c; 10 raised to minus 2

deci; d; 10 raised to minus 1

deka; da; 10

hecto; h; 10 raised to 2

kilo; k; 10 raised to 3

mega; M; 10 raised to 6

giga; G; 10 raised to 9

tera; T; 10 raised to 12

The given model shows six steps with arrows pointing from one step to next, as follows:

1. Need
2. Design specifications (points to steps 3, 4, and 6 next)
3. Concept
    - - with inputs from Physical insight
4. Circuit model
    - - Refinement based on analysis
    - - Inputs from step 5 (Circuit analysis)
    - - Iterative inputs from step 6 (Physical prototype)
5. Circuit analysis
6. Physical prototype
    - - inputs from Laboratory measurements
    - - Refinements based on measurements

Physical prototype leads to a Circuit which meets design specifications.

Diagram shows a rectangle with two terminals marked as 1 and 2 at its ends. Voltage is indicated as v, with polarities at 1 and 2 marked as plus and minus, respectively. A right facing arrow pointing at terminal 1, and labeled as i, represents current in circuit.

A table lists interpretations of positive and negative values for voltage polarity and current direction.

The given table lists positive interpretations on the left and negative interpretations on the right, as follows:

Positive value:

v: voltage drop from terminal 1 to terminal 2, or voltage rise from terminal 2 to terminal 1.

i: positive charge flowing from terminal 1 to terminal 2, or negative charge flowing from terminal 2 to terminal 1.

Negative value:

v: voltage rise from terminal 1 to terminal 2, or voltage drop from terminal 2 to terminal 1.

i: positive charge flowing from terminal 2 to terminal 1, or negative charge flowing from terminal 1 to terminal 2.

A diagram shows a rectangle with two terminals marked as 1 and 2 at its ends. Positive voltage is indicated as v, with polarities at 1 and 2 marked as plus and minus, respectively. A right-facing arrow pointing at terminal 1, and labeled as i, represents positive current in the circuit. Power in the circuit is represented as "p = vi".

A diagram shows a rectangle with two terminals marked as 1 and 2 at its ends. Positive voltage is indicated as v, with polarities at 1 and 2 marked as plus and minus, respectively. A left facing arrow pointing at terminal 2, and labeled as i, represents negative current in circuit. Power in the circuit is represented as "p = minus vi".

A diagram shows a rectangle with two terminals marked as 1 and 2 at its ends. Negative voltage is indicated as v, with polarities at 1 and 2 marked as minus and plus, respectively. A right-facing arrow pointing at terminal 1, and labeled as i, represents positive current in the circuit. Power in the circuit is represented as "p = minus vi".

A diagram shows a rectangle with two terminals marked as 1 and 2 at its ends. Negative voltage is indicated as v, with polarities at 1 and 2 marked as minus and plus, respectively. A left-facing arrow pointing at terminal 2, and labeled as i, represents negative current in the circuit. Power in the circuit is represented as "p = vi".

The given model shows a square circuit which has a vertical wire connecting its lateral sides, and a horizontal wire connecting the middle of the vertical wire with the left side. Eight components are marked on various limbs of the circuit, along with voltage and current polarities, as follows:

* Component a, on top left wire. Voltage - va; Current - ia, flowing from positive to negative.

* Component b, on bottom left wire. Voltage - vb; Current - ib, flowing from negative to positive.

* Component d on horizontal wire. Voltage - vd; Current - id, flowing from negative to positive.

* Component c on top wire. Voltage - vc; Current - ic, flowing from positive to negative.

* Component e on bottom wire. Voltage - ve; Current - ie, flowing from positive to negative.

* Component f on top part of vertical cross wire. Voltage - vf; Current - if, flowing from negative to positive.

* Component g on bottom part of vertical cross wire. Voltage - vg; Current - ig, flowing from positive to negative.

* Component h, on right wire. Voltage - vh; Current - ih, flowing from positive to negative.

A table shows voltage and current values for a home circuit which has eight components (shown in previous figure).

Components, with their voltage in volts and current in amps, are listed as follows:

1. 120V; minus 10A
2. 120V; 9A
3. 10V; 10A
4. 10V; 1A
5. minus 10V; minus 9A
6. minus 100V; 5A
7. 120V; 4A
8. minus 220V; minus 5A

The drawing shows the cars A and B facing each other with their hoods open, and two wires connecting the positive and the negative terminals of their batteries. Both the cars have 12V batteries. A current i is shown flowing from A to B between the positive terminals.

The X-axis plots time in kilo seconds, ranging from 4 to 20 in increments of 4, while the y-axis plots voltage v in volts, ranging from 4 to 12 in increments of 4. The graph shows a line which starts at y equals 8, moves with an upward slope to a point with coordinates (16, 12), and then moves horizontally. From the peak value of the line, dotted lines are drawn to both the axes, marking time as 16 ks and voltage as 12V.

The X-axis plots time in kilo seconds, ranging from 4 to 20 in increments of 4, while the y-axis plots current i in Amps, ranging from 8 to 24 in increment of 8. The graph shows a line which starts at y equals 24, moves in a downward slope to a point with coordinates (12, 16), and then moves with a steeper slope, ending on the x - axis at 16. Dotted lines are drawn to both the axes from the changeover point, marking time as 12 ks and current as 16A.

The X-axis plots time in kilo seconds, ranging from 4 to 56 in increments of 4, while the y-axis plots voltage in volts, ranging from minus 10 to 10 in increments of 2. The graph shows a line which starts at the origin, rises with an upward slope to (4, 10), moves horizontally to (8, 10), and then moves with a downward slope, intersecting the x-axis at (12, 0). The line then moves in a downward slope in the fourth quadrant up to (16, minus 10), moves horizontally to (20, minus 10), and then moves with an upward slope, intersecting the x-axis at (30, 0). In the last cycle, the line moves further in an upward slope to (36, 6), moves horizontally to (46, 6), and then moves with a downward slope, ending at 50 on the x-axis.

Dotted lines are drawn from the line to both the axes at the following points, where voltage is constant: (4, 10) (8, 10), (16, minus 10), (20, minus 10), (36, 6), and (46, 6).

The X-axis plots time in kilo seconds, ranging from 4 to 56 in increments of 4, while the y-axis plots current i in micro Amps, and marks minus 1.0, minus 0.6, 0, 0.4, and 1.0. The graph shows a line which starts on the y axis at (0, 1.0), moves horizontally to (4, 1.0), and then moves vertically down to reach the x-axis at (4,0). The line then moves to (8,0), and moves vertically down in the fourth quadrant to (minus 1.0, 8), moves horizontally to (minus 1.0, 16), before moving up vertically to reach the x-axis at (16, 0). In the next cycle, the line moves through (20, 0), (20, 0.4), (36, 0.4), and (36, 0). In the last cycle, the line moves through (44, 0), (44, minus 0.6), and (50, minus 0.6).

Dotted lines are drawn from the line to the y axis at the following points, where current is constant: (20, 0.4) and (46, minus 0.6).

A diagram shows a square circuit which has a vertical wire connecting its lateral sides, and a horizontal wire connecting the middle of the vertical wire with the left side. Six components are marked on various limbs of the circuit, along with their voltage and current polarities, as follows:

* Component a, on top left wire. Voltage - va; Current - ia, flowing from negative to positive.

* Component b, on bottom left wire. Voltage - vb; Current - ib, flowing from positive to negative.

* Component c on horizontal wire. Voltage - vc; Current - ic, flowing from positive to negative.

* Component d on top part of vertical cross wire. Voltage - vd; Current - id, flowing from negative to positive.

* Component e on bottom part of vertical cross wire. Voltage - ve; Current - ie, flowing from negative to positive.

* Component f, on right wire. Voltage - vf; Current - if, flowing from positive to negative.

A table shows the voltage and current values for a circuit which has six elements.

The elements, with their voltage in volts and current in amps, are listed as follows:

1. minus 18V; minus 51A
2. minus 18V; 45A
3. 2V; minus 6A
4. 20V; minus 20A
5. 16V; minus 14A
6. 36V; 31A

A diagram shows a square circuit with an adjacent circuit on top, and a smaller adjacent circuit on its right. Seven components are marked on various limbs of the three circuits, along with their voltage and current polarities, as follows:

* Component a, on left wire of main circuit. Voltage - va; Current - ia, flowing from negative to positive.

* Component b, on top wire of second circuit on top. Voltage - vb; Current - ib, flowing from positive to negative.

* Component c on left side of horizontal wire, common to main circuit and second circuit. Voltage - vc; Current - ic, flowing from positive to negative.

* Component d on right side of horizontal wire, common to main circuit and second circuit. Voltage - vd; Current - id, flowing from positive to negative.

* Component e on right wire of second circuit. Voltage - ve; Current - ie, flowing from positive to negative.

* Component f, on vertical wire common to main circuit and third circuit on right. Voltage - vf; Current - if, flowing from negative to positive.

* Component g, on right wire of third circuit. Voltage - vg; Current - ig, flowing from positive to negative.

A table shows the power and voltage values for a circuit which has seven elements.

The elements, with their power in kilowatts and voltage in Volts, are listed as follows:

1. 0.6 supplied; 400V
2. 0.05 supplied; minus 100V
3. 0.4 absorbed; 200V
4. 0.6 supplied; 300V
5. 0.1 absorbed; minus 200V
6. 2.0 absorbed; 500V
7. 1.25 supplied; minus 500V

A diagram shows a square circuit which has two vertical wires connecting its lateral sides at points which are equidistant from each other and from both ends. Six components are marked on various limbs of the circuit, along with their voltage and current polarities, as follows:

* Component a, on left wire. Voltage - va; Current - ia, flowing from negative to positive.

* Component b, on top of left vertical wire. Voltage - vb; Current - ib, flowing from negative to positive.

* Component c at bottom of left vertical wire. Voltage - vc; Current - ic, flowing from negative to positive.

* Component d on top wire, between the two vertical wires. Voltage - vd; Current - id, flowing from positive to negative.

* Component e on right vertical wire. Voltage - ve; Current - ie, flowing from positive to negative.

* Component f, on right wire. Voltage - vf; Current - if, flowing from positive to negative.

A table shows the voltage and current values for a circuit which has six elements.

The elements, with their voltage in kilovolts and current in milliamps, are listed as follows:

1. minus 3 kV; minus 250 mA
2. 4 kV; 200 mA
3. 1 kV; 400 mA
4. 1 kV; 150 mA
5. minus 4 kV; 200 mA
6. 4 kV; 50 mA

A diagram shows a square circuit with two vertical wires connecting its lateral sides, and an adjacent circuit on top. Seven components are marked on various limbs of the circuits, along with their voltage and current polarities, as follows:

* Component a, on top wire of second circuit. Voltage - va; Current - ia, flowing from positive to negative.

* Component b, on left wire of main circuit. Voltage - vb; Current - ib, flowing from positive to negative.

* Component c on top left wire of the main circuit. Voltage - vc; Current - ic, flowing from negative to positive.

* Component d on left vertical wire. Voltage - vd; Current - id, flowing from positive to negative.

* Component e on right vertical wire. Voltage - ve; Current - ie, flowing from negative to positive.

* Component f, on top right wire of the main circuit. Voltage - vf; Current - if, flowing from positive to negative.

* Component g, on right wire of the main circuit. Voltage - vg; Current - ig, flowing from positive to negative.

A table shows the power and current values for a circuit which has seven elements.

The elements, with their power in milliWatts and current in milliamps, are listed as follows:

1. 175 mW; 25 mA
2. 375 mW; 75 mA
3. 150 mW; minus 50 mA
4. minus 320 mW; 40 mA
5. 160 mW; 20 mA
6. 120 mW; minus 30 mA
7. minus 660 mW; 55 mA

A diagram shows a square circuit which has a horizontal wire connecting its left and right sides, and an adjacent circuit on its left. Eight components are marked on various limbs of the two circuits, along with their voltage and current polarities, as follows:

* Component a, on left wire of second circuit. Voltage - va; Current - ia, flowing from negative to positive.

* Component b, on top wire of main circuit. Voltage - vb; Current - ib, flowing from negative to positive.

* Component c, on top left wire of the main circuit. Voltage - vc; Current - ic, flowing from negative to positive.

* Component d, on top right wire of the main circuit. Voltage - vd; Current - id, flowing from positive to negative.

* Component e, on horizontal cross wire of the main circuit. Voltage - ve; Current - ie, flowing from negative to positive.

* Component f, on bottom left wire of the main circuit. Voltage - vf; Current - if, flowing from positive to negative.

* Component g, on bottom right wire of the main circuit. Voltage - vg; Current - ig, flowing from negative to positive.

* Component h, on bottom wire of the main circuit. Voltage - vh; Current - ih, flowing from negative to positive.

A table shows the voltage and current values for a circuit which has eight elements.

The elements, with their voltage in volts and current in amps, are listed as follows:

1. 900 V; minus 22.5 A
2. 105 V; minus 52.5 A
3. minus 600 V; minus 30.0 A
4. 585 V; minus 52.5 A
5. minus 120 V; 30.0 A
6. 300 V; 60.0 A
7. 585 V; 82.5 A
8. minus 165 V; 82.5 A

The first two diagrams show circuit symbols for voltage sources, while the last two diagrams depict circuit symbols for current sources. The first symbol for Ideal, dependent, voltage controlled voltage source shows a diamond with a plus and a minus symbol written one below the other inside it. It has two vertical lines drawn out from the top and the bottom. The symbol is represented by formula v sub s equals mu multiplied by v sub x, where supplied voltage (v sub s) depends on controlling voltage (v sub x).

The second symbol for Ideal, dependent, current controlled voltage source also shows a diamond with a plus and a minus symbol written one below the other inside it. It also has two vertical lines drawn out from the top and the bottom. The symbol is represented by formula v sub s equals pi multiplied by i sub x, where supplied voltage (v sub s) depends on controlling current (i sub x).

Third symbol for Ideal, dependent, voltage controlled current source shows a diamond with an upward arrow drawn inside it. It has two vertical lines drawn out from the top and the bottom. The symbol is represented by formula i sub s equals alpha multiplied by v sub x, where supplied current (i sub s) depends on controlling voltage (v sub x).

The last symbol for Ideal, dependent, current controlled current source also shows a diamond with an upward arrow drawn inside it. It also has two vertical lines drawn out from the top and the bottom. The symbol is represented by formula i sub s equals beta multiplied by i sub x, where supplied current (i sub s) depends on controlling current (i sub x).

The circuit shows the following:

The sources supply voltage across the same pair of terminals, marked a and b on the top and bottom wires. The independent source provides a voltage of v sub x equals 5V, while the dependent source provides a voltage of v sub s equals 3v sub x.

The circuit shows the following:

The sources supply a voltage of v sub x equals 5V across two terminals, marked a and b on the top and bottom wires. The dependent current source supplies a current of i sub s equals 3v sub x through the same terminals.

The circuit shows the following:

The independent source supplies a current of i sub x equals 2A across two terminals, marked a and b on the top and bottom wires. The dependent voltage source supplies a voltage of v sub s equals 4i sub x through the same terminals.

The circuit shows the following:

The independent source supplies a current of i sub x equals 2A across two terminals, marked a and b on the top and bottom wires. The dependent current source supplies a current of i sub s equals 3i sub x through the same terminals.

A rectangular circuit has an independent current source on the right wire, supplying an 8A current in a clockwise direction, while a dependent voltage source on the left wire supplies a voltage of i sub b over 4 in the same direction. I sub b is the value of current flowing anti-clockwise in the top wire. The circuit also shows an independent voltage source on a cross wire connecting two terminals on the top and bottom wires, whose voltage supply is marked as v sub g.

A rectangular circuit has an independent current source on the left wire, supplying a 15A current in an anti-clockwise direction, while an independent voltage source on the right wire provides a 25V voltage in the opposite direction. A dependent current source on the top wire supplies current in a clockwise direction, whose value is marked as alpha multiplied by v sub x, where v sub x is voltage at the independent voltage source.

The resistance of shown resistor is R, with a voltage v across the positive and negative terminals on the top and the bottom, respectively, and a current i flowing from the top to the bottom. The equation for this resistor is v = i multiplied by r.

The resistance of shown resistor is R, with a voltage v across the positive and negative terminals on the top and the bottom, respectively, and a current i flowing from the bottom to the top. The equation for this resistor is v equals minus i multiplied by r.

The interior arrangement of a flashlight's components shows a lamp at the front, which is held over a filament terminal, and a connector spring at the bottom, with a broken metal strip joining these two. In between the filament terminal and the connector spring, two dry cells are placed, one behind the other, with their opposite terminals connected. The components are arranged inside a case with a sliding switch in the middle, which completes this circuit by joining the broken pieces of the metal strip to switch on the flashlight.

The given circuit shows an ideal independent voltage source, v sub s, on the left wire, and a resistor, R sub l, on the right wire. Two resistors, R1 and R sub c, and a switch in an off state are shown on the bottom wire of the circuit.

In the table, Voltage, v sub t Volts, and current, i sub t Amps, are tabulated as follows:

minus 40; minus 10

minus 20; minus 5

0; 0

20; 5

40; 10

The X-axis plots current, i sub t in Amps, while the y-axis plots voltage, v sub t in Volts. Five points are plotted on this graph at the following coordinate values: (minus 10, minus 40), (minus 5, minus 20), (0, 0), (5, 20), and (10, 40). Dotted lines are drawn from these points to both the axes. A diagonal line is drawn passing through these points.

The given circuit shows an ideal independent voltage source, v sub s, on the left wire, and a resistor, R sub l, on the right wire. Two resistors, R1 and R sub c, and a switch in an ON state are shown on the bottom wire of the circuit.

The diagram shows four nodes, labeled from a to d, and also assigns voltage and current passing through these nodes. Node a connects the battery to R1. A current, i sub s, flows clockwise through the battery. Node b connects R1 and R sub c. The current and voltage through R1 are i sub 1 and v sub 1, respectively, with flow shown from b to a. Node c is the switch. The current and voltage through R sub c are i sub c and v sub c, respectively, with the flow shown from b to c. Node d connects R sub l with the battery. The current and voltage through R sub l are i sub l and v sub l, respectively, with the flow shown from d to c.

Node c is the switch, while node d connects the battery with the lamp resistor, R sub l.

The shown circuit is diamond shaped, with four nodes at its four corners, labeled as a, b, c, and d. Cross wires connect the opposite nodes, a and c, and b and d. Two additional wires connect the adjacent nodes, a and b, and b and c.

The circuit shows three ideal independent current sources between the nodes, as follows:

* First source is between b and d, with a current i sub a flowing from d to b.

* Second source is between d and c, with a current i sub c flowing from d to c.

* Third source is on the additional wire between b and c, with a current i sub b flowing from c to b.

Five resistors are shown in the circuit at the following locations:

* A 1 ohm resistor is shown on the additional wire between a and b, with a current i sub 1 flowing from a to b.

* A 2 ohm resistor is shown between a and b, with a current i sub 2 flowing from b to a.

* A 3 ohm resistor is shown between b and c, with a current i sub 3 flowing from b to c.

* A 4 ohm resistor is shown between a and c, with a current i sub 4 flowing from a to c.

* A 5 ohm resistor is shown between a and d, with a current i sub 5 flowing from d to a.

The shown circuit is square shaped, with two cross wires connecting the left and the right wires at equidistant nodes. A vertical wire connects the two cross wires at their midpoint.

Four ideal independent voltage sources are located on the circuit, as follows:

* A voltage source, v sub a, is shown on the left wire between the two cross wires.

* The second source, v sub b, is shown on the upper cross wire, towards the right wire.

* The third source, v sub c, is shown on the right wire, between the two cross wires, with its polarities reversed.

* The fourth source, v sub d, is shown on the bottom wire of the circuit.

The circuit shows seven resistors with the following voltages:

* A 1 ohm resistor on the top wire, to the left, shows a voltage drop of v sub 1.

* A 2 ohm resistor on the top wire, to the right, shows a voltage rise of v sub 2.

* A 3 ohm resistor on the left portion of the top cross wire shows a voltage drop of v sub 3.

* A 4 ohm resistor on the right portion of the top cross wire shows a voltage rise of v sub 4.

* A 5 ohm resistor on the vertical wire between the cross wires shows a voltage rise of v sub 5.

* A 6 ohm resistor on the right portion of the bottom cross wire shows a voltage drop of v sub 6.

* A 7 ohm resistor on the bottom wire shows a voltage rise of v sub 7.

The circuit also shows four designated paths marked clockwise for the voltages. Path a is marked between the top wire and the top cross wire. Path b is marked between the two cross wires on the left portion, while path c is marked on the right portion. Path d is marked across the outer circumference of the entire circuit.

The shown rectangular circuit has a 120V independent voltage source on the left wire, and a 6A independent current source on the right wire. It also shows a cross wire connecting the top and bottom wires, carrying a 5 ohm resistor. A 10 ohm resistor on the left of the top wire shows a current, i sub o, flowing from the left to the right.

The shown rectangular circuit has a 120V independent voltage source on the left wire and a 6A independent current source on the right wire. It also shows a cross wire connecting the top and bottom wires at nodes b and c, respectively, carrying a 5 ohm resistor. The voltage drop across the nodes is v sub 1. A current, i sub 1, moves through the resistor from b to c.

A 10 ohm resistor on the left of the top wire connects nodes a and b. It shows a current, i sub o, flowing from a to b. The voltage drop across the resistor is v sub 0.

In the table, voltage, v sub t Volts, and current, i sub t Amps, are tabulated as follows:

30; 0

15; 3

0; 6

The X-axis plots current, i sub t, in Amps, while the y-axis plots voltage, v sub t, in Volts. Three points are plotted on this graph at the following coordinate values: (0, 30), (3, 15), and (6, 0). A downward-sloping line is drawn passing through these points, with dotted lines drawn from (3,15) on the line to both the axes.

The shown device is represented as a dotted square which contains a small circuit within it, connected to two terminals outside. The circuit has a 30V ideal independent voltage source on the left wire, and a 5 ohm resistor on the top wire, carrying a current i in a clockwise direction.

Two terminals, a and b, have a positive and a negative polarity, respectively, and carry a voltage v sub t. They are joined by a 10 ohm resistor on the right wire, which completes the circuit.

A rectangular circuit shows a 24V ideal independent voltage source on the left wire, a 3 ohm resistor on the top wire with a positive voltage v2, a 2 ohm resistor on the bottom wire with a negative voltage v1, and a 7 ohm resistor on the right wire with a positive voltage v5, along with a current i5 moving clockwise.

A rectangular circuit shows a 200V ideal independent voltage source on the left wire, a resistor R on the top left wire, and an 8 ohm resistor on the right wire. A cross wire joins two terminals on the top and bottom wires, carrying a 24 ohm resistor with a positive voltage of 120V.

In the table, voltage, v sub t Volts, and current, i sub t Amps, are tabulated as follows:

25; 0

15; 0.1

5; 0.2

0; 0.25

A rectangular circuit shows a 500V ideal independent voltage source on the left wire, and an ideal dependent current source on the right wire with a current 5i sub delta moving anticlockwise. A cross wire joins two terminals, b and c, on the top and bottom wires, carrying a 20 ohm resistor with a positive voltage v0 and a clockwise current i0. A terminal a is marked on the left end of the top wire. A 5 ohm resistor is located on the top wire between terminals a and b, carrying a clockwise current of i sub delta.

The circuit on left has a 10V independent voltage source and a 6 ohm resistor, with a clockwise current, i sub s, flowing through it. The circuit on the right has a dependent current source generating a current of 3i sub s, a 2 ohm resistor, and a 3 ohm resistor. The clockwise current in the 2 ohm resistor is marked as i sub 0, while the positive voltage in the 3 ohm resistor is marked as v sub 0. A wire connects the right end of the first circuit with the left end of the second circuit.

A rectangular circuit has a vertical cross wire connecting the top and the bottom wires at terminals a and d, and a horizontal cross wire connecting the left wire and the vertical cross wire at terminals b and c. Three more terminals are marked on the cross wires, with terminal 1 between c and a, terminal 2 between c and b, and terminal 3 between c and d.

Three power sources are located as follows. An independent voltage source, V sub CC, is on the right wire, supplying a current i sub CC in an anticlockwise direction. Another independent voltage source, V0, is on the horizontal cross wire between terminals c and 2, with a current i sub B flowing through it from left to right. Plus, a dependent current source, Beta i sub B, is on the vertical cross wire between terminals c and 1, with the current moving from the top to the bottom.

The circuit shows four resistors. R1 is between terminals a and b on the top left wire with the current i sub 1, while R2 is between terminals b and d on the bottom left wire, with the current i sub 2. R sub C is on the top vertical wire, between terminals a and 1, with the current i sub C, while R sub E is on the bottom vertical wire, between terminals d and 3, with the current, i sub E. All the currents flow from the top to the bottom.

A rectangular circuit has a cross wire in the middle between the top and the bottom wires, carrying a 6 ohm resistor. A 5V independent voltage source is on the left wire, and a 8V independent voltage source is on the right wire. A 54k ohm resistor, carrying a i sub 1 current, and a 1V independent voltage source with negative voltage are located on the top left wire. A 1.8k ohm resistor and a dependent current source, carrying 30i sub 1 current from right to left, are located on the top right wire.

A rectangular circuit has a 5A independent current source on the left wire and an independent voltage source with voltage v sub s on the right wire. A 10 ohm resistor is located on the top wire to the right, while a 30 ohm resistor is located on the cross wire between the top and bottom wires. A current, i sub theta, flows from the top to the bottom through the 30 ohm resistor.

An additional wire is drawn over the 10 ohm resistor. This wire carries a dependent current source with current 2i sub theta flowing from the left to the right.

A rectangular circuit shows a 240V independent voltage source on the left wire, a resistor R with a positive voltage v2 on the right wire, and another resistor R with a positive voltage v1 on the cross wire which connects the top and bottom wires. A current i sub s flows clockwise on the top wire, while a current i sub 1 flows from the top to the bottom along the cross wire.

A rectangular circuit shows a 240V independent voltage source on the left wire, a 48 ohm resistor with a positive voltage v sub x on the top wire, and another 48 ohm resistor with a positive voltage v sub y on the right wire. A current i sub x flows clockwise toward terminal a located between the two resistors. Another current i sub y moves down from a toward 48 ohm on the right wire.

A rectangular circuit shows a 10A independent current source on the left wire and a 5A independent current source on the right wire, with both sources having the current flow from the bottom to the top. A cross wire connects the top and bottom wires of the circuit. A 100V independent voltage source is located on this cross wire, while a 40V independent voltage source with opposite polarity is located on the top right wire.

The circuit shows a 50V independent voltage source on the left wire, a 40V independent voltage source with same polarity on the right wire, a 10V independent voltage source with opposite polarity on the top wire, and a 5A independent current source on the bottom wire, with current flowing from the left to the right.

A rectangular circuit has a cross wire in the middle, between the top and bottom wires, which carries a 15V independent voltage source. A 25V independent voltage source of opposite polarity is located on the right wire. Two independent current sources are located on the top right wire and the bottom right wire, with a 3A current and 4A current flowing through them, respectively, from the left to the right. A third independent current source of 8A capacity is located on the left wire, with the current flowing from the top to the bottom.

The shown rectangular circuit has a cross wire in the middle between the top and bottom wires, which carries a 15mA independent current source, with current flowing from the bottom to the top. The left wire carries a 5mA current source, where the current flows from the bottom to the top. Voltage v2, across terminals, has the same polarity as the current flow. A third independent current source is located on the bottom right wire, carrying a 20mA current from the right to the left. Voltage v1, across terminals, has an opposite polarity to the current flow.

The circuit shows two independent voltage sources. A 60V voltage source is located on the right wire, while a 20V voltage source with opposite polarity is located on the top right wire.

In the circuit, the right wire has a 15mA independent current source, with current flowing from the top to the bottom, while the left wire has a 6V independent voltage source with opposite polarity. A current, i sub delta, flows from the right to the left in the top wire. A cross wire in the middle connects two terminals between the top and the bottom wires. It carries a dependent voltage source with opposite polarity, whose voltage is marked as alpha multiplied by i sub delta.

In the circuit, the left wire has a 400mA independent current source, with current flowing from the bottom to the top, while the right wire has a dependent current source, with current flowing from the top to the bottom, marked as v1 over 50. An independent voltage source with voltage v1 is located on the top wire.

In the circuit, a cross wire connects two terminals across the top and the bottom wires of a rectangular circuit. Three independent voltage sources are located on the left wire, the cross wire, and the right wire, with respective voltages of 30V, 40V, and 60V, and with same polarity. A current, i sub x, flows from the top to the bottom through the 40V voltage source on the cross wire.

A 50 mA independent current source is located below the voltage source on the cross wire, with current flowing from the top to the bottom through it. Plus, a dependent voltage source of voltage 1800i sub x is located above the independent voltage source on the right wire, with the same polarity.

In the circuit, a cross wire connects two terminals across the top and the bottom wires of a rectangular circuit. Two independent voltage sources are located on the left wire and cross wire, with respective voltages of 20V and v sub g, and with opposite polarities. A 9A independent current source is located on the left wire, above the voltage source, where current flows from the top to the bottom. Plus, a 6A independent current source is located on the cross wire, below the voltage source, with the current flowing from the top to the bottom. The voltage across this current source is marked as v sub a, with a polarity that's opposite to the current flow. A dependent voltage source of voltage 10v sub a is located on the right wire. The voltage across the terminals is marked as v0, with the same polarity as in the voltage source.

In the circuit, a cross wire connects two terminals across the top and the bottom wires of a rectangular circuit. Two independent voltage sources are located on the left wire and the cross wire, with respective voltages of 20V and 100V, and having the same polarities. An 8A independent current source is located on the left wire, below the voltage source, where current flows from the bottom to the top. Plus, a 16A independent current source is located on the cross wire, above the voltage source, with current flowing from the bottom to the top. A current, i sub delta, flows from the right to the left on the left part of the top wire.

A dependent current source is located on the right wire where a current, 3i sub delta, flows from the bottom to the top.

Lamp A and Lamp B are located on parallel wires. A common wire from both the lamps connects to one terminal of a switch. A wire from the other terminal of the switch connects to the positive terminal of a 12V battery. A wire from the negative terminal of the battery connects to the metal frame of the car.

The values of current i in milliAmps and voltage v in Volts are tabulated as follows:

minus 10; minus 120

minus 5; minus 60

5; 60

10; 120

15; 180

Values of current i in milliAmps and power p in milliWatts are tabulated as follows:

0.5; 8.25

1.0; 33.0

1.5; 74.25

2.0; 132.00

2.5; 206.25

3.0; 297.00

Values of voltage v in Volts and power p in milliWatts are tabulated as follows:

minus 8; 640

minus 4; 160

4; 160

8; 640

12; 1440

16; 2560

In the circuit, a cross wire connects two terminals across the top and the bottom wires of a rectangular circuit. A 100 ohm resistor is located on the cross wire, with current i1 flowing through it from the top to the bottom. A 150 ohm resistor is located on the top right wire, with current i2 flowing through it from the left to the right. The third resistor of 250 ohm capacity is located on the right wire. Plus, a 1.5A independent current source is located on the left wire, with current flowing from the bottom to the top. Voltage v sub 0, across the current source, has the same polarity as the current flow.

In the circuit, a cross wire connects two terminals on the top and the bottom wires of a rectangular circuit. A 300 ohm resistor is located on the cross wire, with current i sub a flowing through it from the top to the bottom. A 40 ohm resistor is located on the left of the top wire. The third resistor of 75 ohm capacity is located on the right wire, with voltage v0 marked across it. Plus, a 200V independent voltage source is located on the left wire with the same polarity as the voltage in the resistor on the right wire. Current i sub b flows on the right of the top wire, from the left to the right.

In the circuit, a cross wire connects two terminals on the top and the bottom wires of a rectangular circuit. A 10 ohm resistor is located on the cross wire, with current i0 flowing through it from the top to the bottom. An 8 ohm resistor is located on the right of the top wire, with current i sub a flowing through it from the left to the right. The third resistor of 14 ohm capacity is located on the right wire. Plus, a fourth resistor of 18 ohm capacity is located on the right of the bottom wire. An independent current source is located on the left wire, with current i sub g flowing from the bottom to the top.

In the circuit, a cross wire connects two terminals across the top and bottom wires of a rectangular circuit. A 5k ohm resistor is located on the cross wire, with voltage v0 marked across it. A 2k ohm resistor is located on the right of the top wire. An independent current source is located on the left wire, with 20mA current flowing through it from the bottom to the top. Plus, a 5V independent voltage source is located on the right wire, with a polarity that's opposite to the current flow of the current source.

In the circuit, a cross wire connects two terminals across the top and bottom wires of a rectangular circuit. A 250 ohm resistor is located on the cross wire, with voltage v1 marked across it. A 50 ohm resistor is on the left of the top wire, with current i sub x flowing through it from the left to the right, while a 200 ohm resistor is on the right of the top wire, with current i sub 1 flowing through it from the left to the right. Plus, a 175 ohm resistor is located on the right wire, with voltage v sub x marked across it.

An independent voltage source with voltage v sub g is located on the left wire of the circuit.

In the circuit, a horizontal cross wire connects two terminals on the left and right wires of a rectangular circuit. A vertical cross wire connects the middle of the horizontal cross wire to the bottom wire. A 4 ohm resistor is located on the vertical cross wire, with current i sub 1 flowing though it from the top to the bottom. Plus, a 13 ohm resistor and a 4 ohm resistor are located on the left and right arms of the horizontal cross wire, respectively.

A 20 ohm resistor is located on the right wire, below the horizontal cross wire, with current i sub 0 flowing through it from the top to the bottom. An 8 ohm resistor is shown on the top wire. Power is supplied by an 80V independent voltage source, which is located on the left wire, below the horizontal cross wire.

The small circuit is diamond shaped, with a horizontal cross wire connecting its left and right terminals and carrying a 5 ohm resistor. Four resistors are located on four arms of this circuit, with resistance values 40, 22.5, 15, and 20 ohms, starting from the top left and moving clockwise. Voltage across the 22.5 ohm resistor is marked as 90V.

Voltage in the bigger circuit is supplied by a 240V independent voltage source, located on the left wire.

In the circuit, a vertical cross wire connects two terminals across the top and bottom wires of a rectangular circuit. A horizontal cross wire connects the middle of the vertical cross wire with the left wire. A 4 ohm resistor is located on the horizontal cross wire. An 8 ohm resistor and a 16 ohm resistor are located on the top and bottom arms of the vertical cross wire, respectively, with currents i sub 1 and i sub 2 flowing through the two resistors from the top to the bottom.

Two resistors with resistance of 2 ohms each are located on the top and bottom wires, on the left of the vertical cross wire, while the last resistor of 80 ohm resistance is located on the right wire. The circuit has two independent voltage sources on the left wire. The one on top has a voltage supply of 230V while the one at the bottom has a voltage supply of 260V.

In the circuit, a horizontal cross wire connects two terminals on the left and right wires of a rectangular circuit. A vertical cross wire connects the middle of the horizontal cross wire to the bottom wire. An independent current source, with current i sub g flowing from the top to the bottom, is located on this vertical wire. Voltage across its terminals is v sub g, with an opposite polarity to the flow of current. The circuit also has an independent voltage source of 100V voltage, located on the lower part of the left wire, with opposite polarity to the current source.

Eight resistors in the circuit are located at the following locations:

* A 5 ohm resistor and a 30 ohm resistor are found on the left and right arms of the horizontal cross wire.

* A 15 ohm resistor and a 16 ohm resistor are found on the left and right arms of the bottom wire.

* A 10 ohm resistor and a 4 ohm resistor are located on the top and bottom arms of the right wire.

* A 9 ohm resistor is found on the upper part of the left wire, above the voltage source.

* An 11 ohm resistor is located on the top wire.

Current i sub a flows from the left to the right through the 15 ohm resistor on the left of the bottom wire, while another current, i sub b, flows from the right to the left through the 11 ohm resistor on the top wire.

In the circuit, a vertical cross wire connects two terminals across the top and bottom wires of a rectangular circuit. A horizontal cross wire connects the middle of the vertical cross wire with the right wire. A 4 ohm resistor is located on the horizontal cross wire.

Six more resistors are found at the following locations in the circuit. A 5 ohm resistor and a 6 ohm resistor are located on the right arms of the top and bottom wires, respectively. A 10 ohm resistor and a 14 ohm resistor are located on the upper and lower arms of the right wire. Plus, a 10 ohm resistor and another resistor with resistance R are located on the bottom and top arms of the vertical cross wire, respectively.

A 4A current flows from the left to the right in the 5 ohm resistor on the top wire.

In the circuit, a horizontal cross wire connects two terminals on the left and right wires of a rectangular circuit. A vertical cross wire connects the middle of the horizontal cross wire to the bottom wire. A 5k ohm resistor is located on the vertical cross wire, while a 1.5k ohm resistor and a 3k ohm resistor are located on the left and right arms of the horizontal cross wire, respectively.

A 500 ohm resistor is found on the lower arm of the right wire. Plus, a variable resistor with resistance R is found on the top wire. Voltage in the circuit is provided by an 80V independent voltage source, located on the bottom arm of the left wire.

The values of voltage, v sub t in Volts, and current, i sub t in Amps, are tabulated as follows:

minus 30; 0

10; 2

50; 4

90; 6

130; 8

The values of voltage, v sub t in Volts, and current, i sub t in Amps, are tabulated as follows:

50; 0

65; 3

80; 6

95; 9

110; 12

The values of voltage, v sub s in Volts, and current, i sub s in milliAmps, are tabulated as follows:

24; 0

22; 8

20; 16

18; 24

15; 32

10; 40

0; 48

The values of current, i sub s in milliAmps, and voltage, v sub s in Volts, are tabulated as follows:

40; 0

35; 10

30; 20

25; 30

18; 40

8; 50

0; 55

The circuit on the left has a 10mA independent current source on the left wire and a 4k ohm resistor on the right wire, with voltage v1 across the terminals. The circuit on the right has a dependent voltage source on the left wire, with voltage of v1 over 2 and a 2k ohm resistor on the top wire, and a 6k ohm resistor on the right wire. Clockwise current in the 6k ohm resistor is marked as i sub 0. A wire connects the right end of the first circuit with the left end of the second circuit.

The circuit on the left has a 20V independent voltage source on the left wire, a 450 ohm resistor on the top wire, and a 150 ohm resistor on the right wire, with voltage v sub x marked across it. Circuit on the right has a dependent current source on the left wire, with a current of v sub x over 100 flowing from the bottom to the top, and a 300 ohm resistor on the right wire, with voltage v sub 0. A wire connects the right end of the first circuit with the left end of the second circuit.

In the circuit, a vertical cross wire connects two terminals across the top and bottom wires of a rectangular circuit. A 6 ohm resistor is located on this cross wire, with current i sub x flowing through it from the top to the bottom. A second resistor, with resistance of 2 ohm, is found on the right arm of the top wire, with voltage v sub 0 marked across it. The left wire has a 45A independent current source, with current flowing from the bottom to the top, while the right wire has a dependent voltage source with the same polarity, whose voltage is marked as 2i sub x.

source on the left wire, a 8k ohm resistor on the top wire, and a 12k ohm resistor with voltage v sub delta on the right wire. In the circuit on the right, a vertical cross wire connects the terminals across the top and bottom wires. A 9k ohm resistor is located on this cross wire, carrying current i sub 1 from the top to the bottom, while another resistor, with a 3k ohm resistance, is found on the right wire, carrying current i sub 2 from the top to the bottom.

A wire connects the right end of the first circuit with the left end of the second circuit. Current i sub 0 flows through this connecting wire from the right to the left.

In the circuit, two vertical cross wires connect the terminals across the top and bottom wires of a rectangular circuit at equal distances from each other and from the ends. A 5 ohm resistor is located on the left vertical cross wire, with current i sub sigma flowing through it from the top to the bottom. A 2 ohm resistor is located on the right vertical cross wire, with current i sub sigma flowing through it from the top to the bottom. The voltage across the second resistor is marked as v sub 0.

Two independent voltage sources, with opposite polarities, are located on the left and right wires, with respective voltages of 12V and 8V. Two dependent voltage sources, with same polarities, are located on the left and middle arms of the top wire, with respective voltages marked as 2i sub sigma and 8i sub sigma. A dependent current source is located on the right arm of the top wire, with current 8i sub sigma flowing through it from the left to the right.

The circuit on the left has an independent voltage source, v sub g, located on the left wire, a 60 ohm resistor on the top wire, carrying current i sub 1 from the left to the right, and a 260 ohm resistor on its right wire.

In the middle circuit, a vertical cross wire connects two terminals across the top and bottom wires. It carries a 20 ohm resistor, with voltage v sub 1 marked across it. An 80 ohm resistor is located on the right wire, with current i sub 2 flowing from the left to the right between the two resistors. A dependent current source is located on the left wire, with a current of 25 i sub 1 flowing through it from the top to the bottom.

In the right circuit, a vertical cross wire connects two terminals across the top and bottom wires. It carries a 40 ohm resistor, with voltage v sub 0 marked across it. A 10 ohm resistor is located on the right wire. A dependent current source is located on the left wire, with a current of 40 i sub 2 flowing through it from the top to the bottom.

All the three circuits are connected at their bottom edges to their adjacent circuit.

The shown switch has terminal 1 on top and terminals 2 and 3 at the bottom. In position 1, terminal 1 is connected to terminal 2. In position 2, terminal 1 is connected to terminal 3.

The shown switch has terminals 1 and 2 on top and terminals 3 and 4 at the bottom. Position 1 shows straight connections between terminals 1 and 3, and 2 and 4. Position 2 shows cross connections between terminals 1 and 4, and 2 and 3.

In the circuit, two vertical cross wires connect the terminals across the top and bottom wires of a rectangular circuit at equal distances from each other and from the ends. Two radiators are located on these cross wires, while a third radiator is found on the right wire. An independent voltage source on the left wire supplies a voltage of 240V.

In the circuit, a vertical cross wire connects the terminals across the top and bottom wires of a rectangular circuit. A radiator is located on this cross wire, while another radiator is located on the right wire. The third radiator is found on the right arm of the top wire. An independent voltage source on the left wire supplies a voltage of 240V.

In the circuit, a vertical cross wire connects the terminals across the top and bottom wires of a rectangular circuit. A radiator is located on this cross wire, while another radiator is located on the right wire. The third radiator is found on the left arm of the top wire. An independent voltage source on the left wire supplies a voltage of 240V.

A drawing shows a grid of resistors in the x-direction, x1-x2, and a grid of resistors in the y-direction, y1-y2, which model a touch screen. Since the screen is not touched, the two grids overlap but do not connect.

A diagram shows a top layer and a bottom layer of a touch screen, covered with a resistive coating towards each other. When a finger touches the top layer, a part of it moves down, connecting to the bottom layer through the spacer in between.

A drawing shows a grid of resistors in the x-direction, x1-x2, and a grid of resistors in the y-direction, y1-y2, which model a touch screen. Since the screen is touched, the two grids connect to each other at the origin.

The rectangular circuit has an independent voltage source, v sub s, located on the left wire, with a current, i sub s, flowing through the wire from the bottom to the top. Eight nodes are marked on the circuit, with nodes a, b, c, and d located on the top wire from the left to the right, and nodes e, f, g, and h located on the bottom wire from the right to the left. The circuit shows seven resistors between the nodes and also marks the currents flowing through each of them, as follows:

Top wire:

R1 between a and b, with a current i sub 1 flowing from left to right.

R2 between b and c, with a current i sub 2 flowing from right to left.

R3 between c and d, with a current i sub 3 flowing from left to right.

Right wire:

R4 between d and e, with a current i sub 4 flowing from top to bottom.

Bottom wire:

R5 between e and f, with a current i sub 5 flowing from left to right.

R6 between f and g, with a current i sub 6 flowing from left to right

R7 between g and h, with a current i sub 7 flowing from right to left

The rectangular circuit has an independent voltage source, v sub s, located on the left wire, with a current, i sub s, flowing through the wire from the bottom to the top. Eight nodes are marked on the circuit, with nodes a, b, c, and d located on the top wire from the left to the right, and nodes e, f, g, and h located on the bottom wire from the right to the left. The circuit shows seven resistors between the nodes as follows:

Top wire:

R1 between a and b.

R2 between b and c.

R3 between c and d.

Right wire:

R4 between d and e.

Bottom wire:

R5 between e and f.

R6 between f and g.

R7 between g and h.

The simplified circuit has an independent voltage source, v sub s, on the left wire, with a current, i sub s, flowing through the wire from the bottom to the top. Nodes a and h are marked at the top and the bottom of the left wire. A resistor, R sub eq, is located on the right wire.

The drawing shows two boxes representing two circuits, separated by a bidirectional arrow. The box on the left encloses a rectangular circuit with seven resistors, whose left wire is removed at terminal a above and terminal h below. Resistors R1, R2, and R3 are located on the top wire, resistor R4 is on the right wire, and resistors R5, R6, and R7 are on the bottom wire. Two wires project outside the box on the left from the two terminals. The voltage across the wires is marked as v sub s, while a current, i sub s, flows rightward on the top wire.

The box on the right represents a simplified version of the box on the left. It encloses a similar circuit where the seven resistors are replaced by a single resistor, R sub eq, on the right wire. The voltage and current at the wires are the same.

The drawing shows a rectangular circuit with four resistors on parallel wires, powered by an independent voltage source, v sub s, on the left wire. The right part of the circuit is enclosed in a box, with the voltage source depicted outside. Two terminals a and s are located on the top wire and the bottom wire on the left, inside the box. The circuit has three nodes on the top wire and three nodes on the bottom wire which are connected by parallel wires. Resistors R1, R2, and R3 are located on these parallel wires from left to right while resistor R4 is on the right wire. A current, i sub s, flows from the right to the left at terminal a on the top wire.

The square circuit has two resistors, R2 and R3, on the top and right wires, and a parallel wire to the left connecting two nodes on the top and bottom wires. The parallel wire carries a resistor R1.

The drawing shows a rectangular circuit with an independent voltage source, v sub s, on the left wire. The right part of the circuit is enclosed in a box, with the voltage source depicted outside. Two terminals a and s are located on the top wire and the bottom wire on the left, inside the box. A resistor, R sub eq, is located on the right wire of the circuit, while a current, i sub s flows in the top wire from left to right.

The circuit shows a parallel wire connecting two nodes on the top and bottom wires. A resistor R1 is located on the parallel wire while another resistor R2 is located on the right wire. The left wire is opened, and two terminals, a and b, are marked at the ends of the top and bottom wires.

The rectangular circuit shows three nodes on the top wire, and three nodes on the bottom wire, joined by three parallel cross wires. Three resistors with resistances of 6k ohm, 24k ohm, and 20k ohm are located on these cross wires from left to right. A 3k ohm resistor is found on the right wire. And two series resistors with resistances of 4k ohm and 2k ohm are located on the third and fourth arms of the top wire, respectively.

The circuit is powered by a 50mA independent current source on the left wire, with an upward bound current.

The rectangular circuit has a 50mA independent current source on the left wire with an upward bound current. Three parallel cross wires are drawn connecting three nodes on the top wire and three nodes on the bottom wire. Three resistors with resistances of 6k ohm, 24k ohm, and 20k ohm are located on these cross wires from left to right. A 4k ohm resistor is located on the third arm of the upper wire.

Two series resistors with resistances of 2k ohm and 3k ohm at the right end of the circuit, from figure 3.9, are replaced in this circuit with a single 5k ohm resistor, located on the right wire.

The rectangular circuit has a 50mA independent current source on the left wire with an upward bound current. Two parallel cross wires are drawn connecting two nodes on the top wire and two nodes on the bottom wire. Two resistors with resistances of 6k ohm and 24k ohm are located on these cross wires from left to right. A 4k ohm resistor is located on the third arm of the upper wire.

From figure 3.10a, the two parallel resistors at the right end with resistances of 20k ohm and 5k ohm are replaced in this circuit with a single 4k ohm resistor, located on the right wire.

The rectangular circuit has a 50mA independent current source on the left wire with an upward bound current. Two parallel cross wires are drawn connecting two nodes on the top wire and two nodes on the bottom wire. Two resistors with resistances of 6k ohm and 24k ohm are located on these cross wires from left to right.

From 3.10b, the two series resistors at the right end with resistances of 4k ohm each are replaced in this circuit with a single 8k ohm resistor, located on the right wire.

The rectangular circuit has a 50mA independent current source on the left wire with an upward bound current. From 3.10c, the three parallel resistors with resistances of 6k ohm, 24k ohm, and 8k ohm are replaced in this circuit with a single 3k ohm resistor, located on the right wire.

The rectangular circuit is powered by a 120V independent voltage source on the left wire. Two nodes are located on the top wire, equidistant from each other and from either end, while two similar nodes are marked on the bottom wire. A parallel wire connects one pair of nodes to the right. The left pair of nodes remain unconnected, and are labeled as x and y, on the top and bottom wires, respectively.

A 4 ohm series resistor is located on the first arm of the top wire to the left of node x, with a rightward current i sub s flowing through it. An 18 ohm resistor is found on the cross wire, with a current, i sub 1, flowing through it from the top to the bottom. A 9 ohm parallel resistor is located on the right wire, with a current i sub 2 flowing through it from the top to the bottom.

The rectangular circuit is powered by a 120V independent voltage source on the left wire. Two nodes are located on the top wire, equidistant from each other and from either end, while two similar nodes are marked on the bottom wire. A parallel wire connects one pair of nodes to the right. The left pair of nodes remain unconnected, and are labeled as x and y, on the top and bottom wires, respectively.

A 4 ohm series resistor is located on the first arm of the top wire to the left of node x, with a rightward current i sub s flowing through it. An 18 ohm resistor is found on the cross wire, with a current, i sub 1, flowing through it from the top to the bottom. Two series resistors with resistances of 3 ohm and 6 ohm at the right end of the circuit, from figure 3.11, are replaced in this circuit with a single 9 ohm resistor, located on the right wire, with a current i sub 2 flowing through it from the top to the bottom.

The rectangular circuit is powered by a 120V independent voltage source on the left wire. Two nodes are marked on the top and bottom wires, and are labeled as x and y, respectively. A 4 ohm series resistor is located on the first arm of the top wire to the left of node x, with a rightward current i sub s flowing through it.

From figure 3.12a, the two parallel resistors of 18 and 9 ohm resistance are replaced with a single resistor on the right wire, with a resistance of 6 ohm.

The rectangular circuit has two nodes, x and y, marked on the top and bottom wires. A 4 ohm resistor is located on the top wire to the left of x, and a 6 ohm resistor is located on the right wire. The circuit is powered by a 120V independent voltage source on the left wire. A 12A current flows through the top wire from left to right.

The rectangular circuit shows two nodes on the top wire and two equivalent nodes on the bottom wire which are connected by two parallel cross wires. A 30 ohm resistor and a 64 ohm resistor are located on the left and right cross wires, respectively. A 10 ohm resistor is found on the right wire. Two series resistors with resistances of 7.2 ohm and 6 ohm are located on the second and third arms of the top wire.

The circuit is powered by a 5A independent current source on the left wire, with an upward-bound current. A voltage v is marked across the top and bottom wires across the current source.

The rectangular circuit shows an independent voltage source, v sub s, on the left wire, and two series resistors, R1 and R2, on the right wire. The voltages across R1 and R2 are marked as v1 and v2, respectively.

The rectangular circuit shows an independent voltage source, v sub s, on the left wire, and two series resistors, R1 and R2, on the right wire. The voltages across R1 and R2 are marked as v1 and v2, respectively. A current i flows from left to right on the top wire.

The rectangular circuit shows a 20V independent voltage source on the left wire, and two series resistors, R1 equals 30k ohm and R2 equals 10k ohm on the right wire. An additional wire is drawn across R2, and a resistive load of R sub L equals 10k ohm is connected in parallel to R2. The voltage across the resistive load is marked as v sub 0.

The rectangular circuit shows a 20V independent voltage source on the left wire, and two series resistors with resistances of 1200 ohm and 400 ohm on the right wire. An additional wire is drawn across the 400 ohm resistor, and a resistive load of 10k ohm is connected in parallel to the 400 ohm resistor. The voltage across the resistive load is marked as v sub 0.

The rectangular circuit shows an independent voltage source, v sub s, on the left wire, and two series resistors, R1 and R2, on the right wire. An additional wire is drawn across R2, and a resistive load of R sub L is connected in parallel to R2. Two nodes are marked on the top and bottom wires of the resistive load loop to the left of R sub L. The voltage across these nodes is marked as v sub 0.

The rectangular circuit shows a 100V independent voltage source on the left wire, and two series resistors, R1 equals 25k ohm and R2 equals 100k ohm on the right wire. Two wires are drawn across the 100k ohm resistor connecting them to two nodes. The voltage across these nodes is marked as v sub 0.

The rectangular circuit shows two nodes on the top wire and two equivalent nodes on the bottom wire. The right pair of nodes are joined by a cross wire. A resistor R1 is located on the cross wire, and a parallel resistor, R2, is located on the right wire.

The circuit is powered by an independent current source, i sub s, on the left wire. The downward currents in R1 and R2 are marked as i sub 1 and i sub 2, respectively. A voltage v is marked across the top and bottom wires between the two parallel resistors.

The rectangular circuit shows a 200V independent voltage source on the left wire, and two series resistors with resistances of 25k ohm and 75k ohm on the right wire. An additional wire is drawn across the 75k ohm resistor, carrying a parallel resistive load of R sub L. Two nodes are marked on the top and bottom wires of the resistive load loop to the left of R sub L. The voltage across these nodes is marked as v sub 0.

The rectangular circuit has a cross wire connecting a node on the top wire with another node on the bottom wire. Two resistors with resistances of 40 ohm and 80 ohm are located on this cross wire. A 60 ohm resistor is located on the top wire to the left of the cross wire, and a parallel resistor R is located on the right wire. The circuit is powered by a 20A independent current source, located on the left wire.

The drawing shows a box on the left with two internal terminals, representing a circuit. The terminals lead out to an external closed loop which contains n resistors in series, labeled from R1, R2, etc. to R sub n-1 and R sub n. The current in the loop is represented as i. One of the resistors on the right wire, R sub j, has a voltage drop of v sub j marked across it.

The drawing shows a box on the left with two internal terminals, representing a circuit. The terminals lead out to an external closed loop which contains n resistors in parallel, located on cross wires. The resistors are labeled from R1, R2, etc. to R sub n-1 and R sub n. The current in the loop is represented as i. One of the resistors R sub j has a current, i sub j, moving through it from top to bottom. The voltage across the loop is marked as v.

The rectangular circuit has three nodes on the top wire and three nodes on the bottom wire which are joined by three cross wires. An 8A independent current source is located on the left wire with an upward bound current. The left cross wire has two resistors with resistances of 36 ohm and 44 ohm. The middle cross wire carries a 10 ohm resistor. The right cross wire carries three resistors with resistances of 40 ohm, 10 ohm, and 30 ohm. And the right wire carries a 24 ohm resistor, with a voltage v marked across it.

An unknown voltage, v sub 0, is marked across the 30 ohm resistor in the third cross wire. And an unknown current, i sub 0, flows from the top to the bottom through the right wire.

The rectangular circuit has two nodes on the top wire and two nodes on the bottom wire which are joined by two cross wires. A 60V independent voltage source is located on the left wire. A 20 ohm resistor and a 30 ohm resistor are located on the left and right cross wires, respectively. The top wire has a 40 ohm resistor on its left arm and a 50 ohm resistor on its right arm. The bottom wire has a 70 ohm resistor on its left arm. And the right wire carries a 10 ohm resistor.

The voltage across the 40 ohm resistor on the left arm of the top wire is marked as v sub 0.

The rectangular circuit has a cross wire connecting two nodes on the top and bottom wires. An independent voltages source, v sub s, is located on the left wire. A parallel resistor, R2, is located on the cross wire, while a series resistor, R1, is located on the top wire. An ammeter is located in series with resistor R1 to its right on the top wire. And a voltmeter is located on the right wire, parallel to resistor R2.

The rectangular circuit has a cross wire connecting two nodes on the top and bottom wires. An independent voltages source, v sub s, is located on the left wire. A parallel resistor, R2, is located on the cross wire, while a series resistor, R1, is located on the top wire. An ideal ammeter is located in series with resistor R1 to its right on the top wire. The right wire of the circuit is opened, and a voltmeter is connected to its open terminals, parallel to resistor R2.

The d'Arsonval meter movement consists a hemispherical shaped permanent magnet with a circular magnetic steel core at its base. A semi circular scale is located on top of the permanent magnet. A pointer moves on this scale. The base of the pointer is fastened to a movable coil with a restoring spring. And the movable coil is wrapped around the circular magnetic steel core, at the base of the permanent magnet.

The analog ammeter is represented as a box with a circuit inside, projecting two Ammeter terminals outside. The circuit consists of a d'Arsonval movement, in parallel with a single resistor, R sub A.

The circuit shows four resistors located on the four arms of a diamond shaped circuit, which is powered by a dc voltage source, v. A galvanometer is located on a crosswire which connects the two opposite nodes of the diamond shaped circuit. The resistors in the circuit are labeled as R1, R2, R sub x, and R3, starting from the top left and moving clockwise. R3 is a variable resistor.

The circuit shows four resistors located on the four arms of a diamond shaped circuit, which is powered by a dc voltage source, v. A galvanometer is located on a crosswire which connects the two opposite nodes, a and b, on the diamond shaped circuit. The resistors in the circuit are labeled as R1, R2, R sub x, and R3, starting from the top left and moving clockwise. R3 is a variable resistor.

The currents across the five arms of the Wheatstone bridge are labeled as follows:

i sub 1: moves through R1, towards node a.

i sub 2: moves through R2, towards node b.

i sub 3: moves through R3, away from node a.

i sub x: moves through R sub x, away from node b.

i sub g: moves from a to b.

The circuit shows four resistors located on the four arms of a diamond shaped circuit, which is powered by a dc voltage source, v. A galvanometer is located on a crosswire which connects the two opposite nodes of the diamond shaped circuit. Two resistors of 1k ohm and 4k ohm resistance are located on the top arms, while R sub x and R3 are on the bottom arms of the diamond shaped circuit. R3 is a variable resistor.

The circuit shows four resistors located on the four arms of a diamond shaped circuit, which is powered by a dc voltage source, v. A galvanometer is located on a crosswire which connects the two opposite nodes of the diamond shaped circuit. The resistors in the circuit are labeled as R1, R2, R sub x, and R3, starting from the top left and moving clockwise. R3 is a variable resistor.

The circuit shows four resistors located on the four arms of a diamond shaped circuit, which is powered by a dc voltage source, v. A resistor, R sub m is locate,d in place of the galvanometer on a crosswire which connects the two opposite nodes of the diamond shaped circuit. The resistors in the circuit are labeled as R1, R2, R sub x, and R3, starting from the top left and moving clockwise. R3 is a variable resistor.

The left circuit shows a delta configuration where three nodes--a, b, c--are placed in a triangular shape, connected by three resistors as follows: R sub c between a and b on top, R sub a between b and c on the right, and R sub b between a and c on the left.

The circuit on the right shows a pi configuration, with two nodes a and b on the top wire, and node c on the bottom wire. Resistor R sub c is located between a and b. Two cross wires are drawn between the top and bottom wires on either side of R sub c on the top wire and node c on the bottom wire. Resistor R sub b is located on the left cross wire while resistor R sub a is located on the right cross wire.

The left circuit shows a y shaped structure, with three nodes--a, b, c--at three ends of the Y shape, with an unnamed node at the center. Resistors R1, R2, and R3 are located on the three arms of the y shape, connected to nodes a, b, and c, respectively.

The right circuit shows the same y shaped structure in the form of alphabet T. Nodes a and b are at the two ends of the horizontal arm while node c is at the end of the vertical arm. Resistors R1 and R2 are on the horizontal arms, closer to a and b, respectively, while resistor R3 is on the vertical arm.

The left circuit shows a delta configuration where three nodes--a, b, c--are placed in a triangular shape, connected by three resistors as follows: R sub c between a and b on top, R sub a between b and c on the right, and R sub b between a and c on the left.

The right circuit shows a y shaped structure, with three nodes--a, b, c--at three ends of the Y shape, with an unnamed node at the center. Resistors R1, R2, and R3 are located on the three arms of the y shape, connected to nodes a, b, and c, respectively.

A bidirectional arrow is located between the two circuits.

The circuit shows a 40V independent dc voltage source on the left wire and a 5 ohm series resistor on the top wire. A Wheatstone bridge circuit is shown on the right with four resistors of 100, 125, 37.5, and 40 ohm resistances from the top left, moving clockwise, and a 25 ohm resistor on the parallel wire across the nodes.

The drawing shows the upper triangle of the Wheatstone bridge circuit, with resistors of resistance 100 ohm, 125 ohm, and 25 ohm on the left, right, and bottom wires, respectively. Three wires are drawn from the three nodes of the triangle which meet at a central node to form an inverted Y shaped structure. Three resistors are located on these wires with R1 on the wire from the top node, R2 on the wire from the right node, and R3 on the wire from the left node.

The circuit shows a 40V independent dc voltage source on the left wire and a 5 ohm series resistor on top wire. The Wheatstone bridge circuit on the right is now replaced by a y structure on top and the two resistors which make the lower delta structure. The y-structure has a 50 ohm resistor towards the top, and two resistors of 10 ohm and 12.5 ohm resistance which make the left and right arms of the structure. The lower delta structure has a 40 ohm resistor and a 37.5 ohm resistor on its left and right arms.

The rectangular circuit has a y structure between its sides formed by a horizontal cross wire and a vertical cross wire. The horizontal cross wire connects nodes on the left and right wires, while the vertical cross wire connects nodes on the horizontal cross wire and bottom wire. A 20 ohm resistor and a 10 ohm resistor are located on the left and right arms of the horizontal cross wire. A 5 ohm resistor is located on the vertical cross wire.

A 28 ohm resistor is found on the top wire, and a 105 ohm resistor is found on the lower arm of the right wire. The circuit is powered by a 2A independent current source, located on the lower arm of the left wire.

The circuit shows four resistors in series represented horizontally, collectively labeled as R sub x, and an independent voltage source, V sub s. A downward arrow pointing to the R sub x resistors indicates the x-location where the screen is touched. The total resistance is divided into two parts, with the right part of the resistance labeled as "alpha R sub x", and the left part labeled as (1 minus alpha)R sub x.

The voltage drop across alpha R sub x is marked as V sub x.

The drawing shows a rectangle which represents a mobile screen with the coordinates marked at the four corners. The x-y coordinates of the four corners are as follows:

Upper left: (0, 0)

Upper right: (p sub x minus 1, 0)

Bottom right: (p sub x minus 1, p sub y minus 1)

Bottom left: (0, p sub y minus 1)

The circuit shows four resistors in series represented vertically, collectively labeled as R sub y, and an independent voltage source, V sub s. A leftward arrow pointing to the R sub y resistors indicates the y-location where the screen is touched. The total resistance is divided into two parts, with the lower part of the resistance labeled as "Beta R sub y", and the upper part labeled as (1 minus Beta)R sub y

The voltage drop across Beta R sub y is marked as V sub y.

The circuit shows an independent voltage source, v sub s on the left wire, and two series resistors, R1 and R2 on the right wire. The voltage across R1 and R2 is marked as v1 and v2, respectively.

The rectangular circuit shows a cross wire across the top and bottom wires. A resistor R1 is located on the cross wire, and a parallel resistor, R2 is located on the right wire. An independent current source, i sub s is located on the left wire. A current i1 flows through R1, and a current i2 flows through R2, both from top to bottom.

The rectangular circuit has a 10V independent voltage source on its left wire. A vertical cross wire connects nodes on the top and bottom wires. A 4 ohm resistor is located on the cross wire, while a 7 ohm parallel resistor is located on the right wire. Three series resistors are located on the top wire, with a 6 ohm resistor and 12 ohm resistor on the left arm, and a 9 ohm resistor found on the right arm.

The rectangular circuit has two resistors on the right wire with resistances of 5k ohm and 7k ohm, and two resistors on the left wire with resistances of 3k ohm and 10k ohm. Two nodes are marked across the 10k ohm resistor on the left wire. An additional rectangular wire is drawn across these two nodes, with a 4k ohm resistor on the top wire, and a 3mA independent current source on the left wire.

The rectangular circuit has a vertical cross wire connecting nodes on the top and bottom wires. A 600 ohm resistor is found on this cross wire, while a 1.2k ohm parallel resistor is found on the right wire. A 300 ohm resistor and a 400 ohm resistor are located on the left arms of the top and bottom wires, while a 500 ohm resistor is located on the left wire. The circuit is powered by a 200mV independent voltage source in reverse polarity, located on the left arm of the top wire, to the left of the 300 ohm resistor.

The rectangular circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. A 300 ohm resistor and a 100 ohm resistor are located on the left and right cross wires, respectively, while a 70 ohm resistor is located on the right wire. The top wire shows a 50 ohm and 90 ohm resistor on the middle arm, and a 80 ohm resistor on the right arm. A 30mA independent current source is located on the left wire of the circuit.

The rectangular circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. A 100 ohm resistor and a 25 ohm resistor are located on the left and right cross wires, respectively, while a 22 ohm resistor is located on the right wire. The top wire shows an 18 ohm and 40 ohm resistor on the left arm and right arm, respectively. An additional wire drawn across the nodes of the 40 ohm resistor on the top wire carries a 10 ohm resistor. A 5V independent voltage source is located on the left wire of the circuit.

The rectangular circuit shows three parallel vertical cross wires connecting three pairs of nodes on the top and bottom wires. Three resistors with resistances of 750, 500, and 1.5k ohm respectively are located on the left, middle, and right cross wires, while a 3k ohm resistor is located on the right wire. The top wire shows a 600 ohm and a 2k ohm resistor on the first and third arms, respectively. A 900 ohm resistor is located on the left wire. A 65V independent voltage source with reverse polarity is located on the third arm of the top wire, to the left of the 2k ohm resistor.

The rectangular circuit has a vertical cross wire connecting nodes on the top and bottom wires. A 10k ohm resistor is found on this cross wire, while two resistors with resistances of 2k ohm and 9k ohm are located on the right wire. A 5k ohm resistor is found on the right arm of the top wire. The circuit is powered by a 50mA independent current source with an upward current, located on the left wire.

An additional wire is drawn from the nodes across the 9k ohm resistor on the right wire, forming a small rectangular circuit to the right. A cross wire is drawn across the top and bottom wires of this smaller circuit, carrying an 18k ohm resistor, while a 6k ohm resistor is located on the right arm of the top wire.

The rectangular circuit shows three parallel vertical cross wires connecting three pairs of nodes on the top and bottom wires. Two resistors with resistances of 200 and 300 ohm respectively are located on the left and right cross wires, while a 150 ohm resistor is located on the right wire. The top wire shows a 250 ohm resistor on its fourth arm to the right, while a 600 ohm

resistor is located on the left wire. A 300 ohm independent current source with upward current is found on the middle cross wire.

The rectangular circuit shows a cross wire drawn across the top and bottom wires. A 24 ohm resistor is located on the cross wire while an 18 ohm resistor is found on the right wire. A 12 ohm and 30 ohm resistor are located on the left and right arms on the top wire, while a 10 ohm resistor is found on the left arm of the bottom wire.

The left wire of the circuit is removed, and the top and bottom wires are connected to two terminals, a and b.

The rectangular circuit shows four parallel vertical cross wires connecting four pairs of nodes on the top and bottom wires. Four resistors with resistances of 4k ohm, 30k ohm, 60k ohm, and 7.2k ohm are located on the four cross wires, from left to right respectively. A 2.4k ohm resistor is found on the right wire. And a 1.2k ohm resistor and 2k ohm resistor are located on the fourth arms of the top and bottom wires, respectively.

The left wire of the circuit is removed, and the top and bottom wires are connected to two terminals, a and b.

The rectangular circuit shows a cross wire drawn across the top and bottom wires. A 8k ohm resistor is located on the cross wire while a 6k ohm resistor is found on the left wire. A 4k ohm and 5.2k ohm resistor are located on the left and right arms on the top wire, while a 2k ohm resistor is found on the left arm of the bottom wire.

The right wire of the circuit is removed, and the top and bottom wires are connected to two terminals, a and b.

The rectangular circuit shows two pairs of nodes on the top and bottom wires. A cross wire is drawn across one pair of nodes on the left, while the second pair of nodes are connected to two open nodes, marked as a and b. A 720 ohm resistor is located on the cross wire while a 480 ohm resistor is found on the right wire. A 1200 ohm and 320 ohm resistor are located on the left and right arms on the top wire.

The rectangular circuit shows a cross wire drawn across a pair of nodes on the top and bottom wires. A 36 ohm resistor is located on the cross wire while a 10 ohm resistor is found on the right wire. A 6 ohm resistor is found on the right arm of the bottom wire, while an unmarked resistor is located on the right arm of the top wire. A diagonal cross wire is drawn from the top node of the vertical cross wire to the bottom node of the right wire. An 18 ohm resistor is located on this diagonal cross wire.

The left wire of the circuit is removed, and the top and bottom wires are connected to two terminals, a and b.

The rectangular circuit shows two parallel cross wires drawn across two pairs of nodes on the top and bottom wires. A 30 ohm resistor and a 15 ohm resistor are located on the left and right cross wires. Three resistors with resistances of 4 ohm, 16 ohm, and 12 ohm are located on the left, middle, and right arms of the top wire. A 14 ohm resistor is located on the left arm of the bottom wire while a 18 ohm resistor is located on the right wire. The circuit also shows two diagonal wires, the first of which is drawn between the lower node of the left cross wire and upper node of the right cross wire, carrying a 20 ohm resistor. The second diagonal wire is drawn from the upper node of the right cross wire and lower node of the right wire, and carries a 10 ohm resistor.

The left wire of the circuit is removed, and the top and bottom wires are connected to two terminals, a and b.

The rectangular circuit shows two parallel cross wires drawn across two pairs of nodes on the top and bottom wires. A 50 ohm resistor and a 15 ohm resistor are located on the left and right cross wires. A 25 ohm resistor and 30 ohm resistor are located on the left and middle arms of the top wire, while a 10 ohm resistor and 20 ohm resistor are located on the left and middle arms of the bottom wire. A 60 ohm resistor is found on the right wire. An additional wire is drawn across the terminals of the 30 ohm resistor on the top wire. A 45 ohm resistor is located on this additional wire.

The left wire of the circuit is removed, and the top and bottom wires are connected to two terminals, a and b.

The rectangular circuit shows two parallel cross wires drawn across two pairs of nodes on the top and bottom wires. A 40 ohm resistor and a 75 ohm resistor are located on the left and right cross wires. A 60 ohm resistor and 18 ohm resistor are located on the middle and right arms of the top wire, while a 20 ohm resistor and 30 ohm resistor are located on the middle and right arms of the bottom wire. A 12 ohm resistor is found on the right wire. An additional wire is drawn across the terminals of the 60 ohm resistor on the top wire. A 20 ohm resistor is located on this additional wire.

The circuit also shows two diagonal wires, the first of which is drawn between the lower node of the left cross wire and upper node of the right cross wire, carrying a 50 ohm resistor. The second diagonal wire is drawn from the upper node of the right cross wire and lower node of the right wire, and carries a 60 ohm resistor.

The left wire of the circuit is removed, and the top and bottom wires are connected to two terminals, a and b.

The rectangular circuit shows a cross wire drawn across a pair of nodes on the top and bottom wires. A 360 ohm resistor is located on the cross wire while a 160 ohm resistor is found on the right wire. A 90 ohm resistor is found on the right arm of the top wire, while a 200 ohm resistor is located on the right arm of the bottom wire. A diagonal cross wire is drawn from the bottom node of the cross wire to the top node of the right wire. A 120 ohm resistor is located on this diagonal cross wire.

The circuit is powered by a 30mA independent current source located on the left wire, with an upward flowing current.

The circuit also shows two diagonal wires, the first of which is drawn between the lower node of the left cross wire and upper node of the middle cross wire, carrying a 12 ohm resistor. The second diagonal wire is drawn from the upper node of the right cross wire and lower node of the right wire, and carries a 15 ohm resistor.

The circuit is powered by a 144V independent voltage source, located on the left wire.

The rectangular circuit shows a cross wire drawn across a pair of nodes on the top and bottom wires. A 50mA independent current source is located on the cross wire, with an upward flowing current. The left wire carries a 750 ohm resistor while the right wire carries a 600 ohm resistor. Two resistors with resistances of 100 ohm and 150 ohm are found on the left and right arms of the top wire, while two resistors with resistances of 250 and 300 ohm are located on the two arms of the bottom wire.

The circuit also shows two diagonal wires, the first of which is drawn between the upper node of the left wire and lower node of the cross wire, carrying a 1k ohm resistor. The second diagonal wire is drawn from the upper node of the cross wire and lower node of the right wire, and carries a 500 ohm resistor.

The rectangular circuit shows four parallel cross wires drawn across four pairs of nodes on the top and bottom wires. The first cross wire on the left carries two resistors with resistance of 40 ohm and 20 ohm, while the next

three cross wires carry resistors with respective resistances of 80 ohm, 30 ohm, and 45 ohm. The top wire carries three resistors with resistances of 24 ohm, 10 ohm, and 15 ohm on the first, third and fourth arms respectively, while the bottom wire carries a 10 ohm resistor on its first arm, and a 20 ohm resistor on its third arm.

The circuit also shows a diagonal wire, drawn between the lower node of the second cross wire and upper node of the third cross wire, carrying a 60 ohm resistor. A 80mV independent voltage source is located on the left wire.

The rectangular circuit shows a cross wire drawn across a pair of nodes on the top and bottom wires, carrying a resistor, R2. A 40 ohm resistor is located on the left arm of the top wire. The left wire carries a 20V independent voltage source while the right wire is removed, and the top and bottom wires are connected to two open terminals, a and b. The voltage across the two terminals is marked as v sub 0.

A parallel wire carrying a load resistor, R sub L is attached across the terminals, a and b.

The rectangular circuit has a 75V independent voltage source on the left wire and two series resistors, R1=2k ohm and R2=500 ohm on its right wire. Two wires are drawn from the nodes on either side of R2, and are connected to two open terminals. The voltage across the open terminals is marked as v sub 0.

The rectangular circuit has a cross wire drawn across a pair of nodes on the top and bottom wires. A resistor, R2 is located on the cross wire, while another resistor R sub L is located on the right wire. A resistor R1 is located on the left arm of the top wire. The circuit is powered by a 40V independent voltage source located on the left wire. The voltage across R sub L is marked as v sub 0.

The rectangular circuit has a cross wire drawn across a pair of nodes on the top and bottom wires. A 75k ohm resistor is located on the cross wire, while a 25k ohm resistor is located on the top wire. The circuit is powered by a 240V independent voltage source located on the left wire. The right wire is removed, and the top and bottom wires are connected to two open terminals, a and b.

A current, i, flows from left to right through the 25k ohm series resistor.

The rectangular circuit has a cross wire drawn across a pair of nodes on the top and bottom wires. A 120k ohm resistor is located on the cross wire, while a 30k ohm resistor is located on the top wire. The right wire is removed, and the top and bottom wires are connected to two open terminals, with a voltage, v sub 0 marked across them. Similarly, the left wire is removed, and the top and bottom wires are connected to two open terminals, marked as a-dash and b-dash.

The drawing shows two circuits which are joined by a wire at their lower nodes. The circuit on the left has a 240V independent voltage source on the left wire, a 25k ohm resistor on the top wire and, a 75k ohm resistor on the right wire. A current, i, flows from left to right through the 25k ohm resistor.

The rectangular circuit on the right has a cross wire drawn across a pair of nodes on the top and bottom wires. A 120k ohm resistor is located on the cross wire, while a 30k ohm resistor is located on the top wire. The right wire is removed, and the top and bottom wires are connected to two open terminals, with a voltage, v sub 0 marked across them. A dependent voltage source is located on the left wire, with the voltage marked as 75,000i.

The rectangular circuit has a 24V independent voltage source on the left wire, and three series resistors on the right wire with resistances marked as R1, R2, and R3. Four nodes are marked on the right wire on either side of the three resistors, and wires are drawn from them, connecting them to four open terminals. The third terminal between R2 and R3 is marked as a common terminal. The voltages across the first, second, and fourth terminals are marked as v1, v2, and v3, respectively.

The rectangular circuit shows two parallel cross wires drawn across two pairs of nodes on the top and bottom wires. A 2.4A independent current source is located on the left cross wire, with an upward flowing current, while a 90 ohm resistor is located on the right cross wire. A 20 ohm resistor is located on the left wire while a 10 ohm resistor is located on the right wire. The top wire carries a 6 ohm resistor on the middle arm while the bottom wire carries a 10 ohm resistor on the left arm. The voltage across the left wire is marked as v sub 0. A downward flowing current through the right cross wire is marked as i sub 0.

The rectangular circuit shows two parallel cross wires drawn across two pairs of nodes on the top and bottom wires. A 6 ohm resistor and a 30 ohm resistor are located on the left and right cross wires, while a 20 ohm resistor is located on the right cross wire. The circuit is powered by a 30A independent current source on the left wire, with an upward flowing current.

The rectangular circuit shows three parallel cross wires drawn across three pairs of nodes on the top and bottom wires. Three resistors with resistances of R1, R2, and R3 are located on the left, middle, and right cross wires, respectively, with a resistor R4 is found on the right cross wire. The circuit is powered by an independent current source, i sub g, located on the left wire, with an upward flowing current.

The voltage across the current source is marked as v sub g. Four downward flowing currents are marked across R1, R2, R3, and R4, and are marked as i1, i2, i3, and i4, respectively.

The rectangular circuit shows (N minus 1) number of parallel cross wires drawn across nodes on the top and bottom wires. Parallel resistors are located on these cross wires, and are marked as R1, R2, R3, etc. The right wire carries a resistor marked as R sub N. One of the parallel resistors is marked as R sub k, and the downward current flowing through it is marked as i sub k. The circuit is powered by an independent current source, i sub g, located on the left wire.

The rectangular circuit shows five parallel cross wires drawn across five pairs of nodes on the top and bottom wires. Five parallel resistors are located on these cross wires, with resistances marked from left to right as 0.5 ohm, 5 ohm, 8 ohm, 10 ohm, and 20 ohm respectively. A 40 ohm resistor is found on the right wire. The circuit is powered by a 40A independent current source on the left wire, with an upward flowing current.

The rectangular circuit shows a vertical cross wire connecting two nodes on the top and bottom wires. A diagonal cross wire connects a node at the midpoint of the vertical cross wire with the bottom node on the right wire. A 20 ohm resistor is located on the diagonal cross wire. A 6 ohm resistor and a 5 ohm resistor are located on the top and bottom arms of the vertical cross wire, while two resistors with resistances of 12 ohm and 13 ohm are located on the right wire. The top wire carries a 2 ohm resistor on the left arm, and a 15 ohm resistor on the right arm. The circuit is powered by a 125V independent voltage source, located on the left wire.

A current, i sub g flows from left to right through the 2 ohm resistor on the top wire. Another current, i sub 0 flows from top to bottom through the 20 ohm resistor on the diagonal cross wire.

The rectangular circuit shows a vertical cross wire connecting two nodes on the top and bottom wires. A 30 ohm resistor is located on this cross wire. Two diagonal cross wires are drawn from the bottom node of the vertical cross wire to the top nodes of the left and right wires. A 60 ohm resistor is located on the left diagonal cross wire, while a 80 ohm resistor is located on the right diagonal cross wire.

The top wire carries an 8 ohm resistor on the left arm, and a 4 ohm resistor on the right arm. A 20 ohm resistor is located on the right wire. The circuit is powered by a 250mA independent current source, located on the left wire, with an upward flowing current.

A current, i1 flows top to bottom through the 30 ohm resistor on the vertical cross wire. Another current, i2 flows top to bottom through the 20 ohm resistor on the right wire.

The rectangular circuit shows three nodes on the top wire, and a wire drawn from the middle of the bottom wire, ending in a node. A 40 ohm resistor is located on this wire drawn from the bottom wire. The node at the end of this wire is connected to the left node on the top wire, carrying a 150 ohm resistor, and is also connected to the right node on the top wire, carrying a 75 ohm resistor. The voltage across this node and the middle node of the top wire is marked as v1.

The top wire carries a 90 ohm resistor on its left arm and a 60 ohm resistor on its right arm. A 30 ohm resistor is found on the right wire, with a voltage, v2 marked across it. The circuit is powered by a 3V independent voltage source.

The rectangular circuit shows a vertical cross wire connecting two nodes on the top and bottom wires. A horizontal cross wire is drawn connecting the vertical cross wire with the right wire at their midpoints. On this horizontal cross wire, two nodes are marked in the middle, and the wire between them is removed. The voltage across these open nodes is marked as v sub 0.

A 2k ohm resistor is located on the left arm of the horizontal cross wire, while a 4k ohm resistor is located on its right arm. The vertical cross wire carries a 10k ohm resistor and a 15k ohm resistor on its top and bottom arms. And the right wire carries a 3k ohm resistor and a 12k ohm resistor on its top and bottom arms.

The circuit is powered by a 18mA independent current source on the left wire, with a downward flowing current.

The rectangular circuit shows a vertical cross wire connecting two nodes on the top and bottom wires. Two nodes are marked on the midpoints of the vertical cross wire and the right wire. The voltage across these open nodes is marked as v sub x.

The vertical cross wire carries a 5k ohm resistor and a 20k ohm resistor on its top and bottom arms. And the right wire carries a 60k ohm resistor and a 90k ohm resistor on its top and bottom arms.

The circuit is powered by a 45V independent voltage source on the left wire.

The drawing shows an Ammeter drawn across a parallel resistor whose resistance is (10 over 99) ohm. The ammeter has a voltage of 100 mu V, and a current of 10 mu A. The current flowing into the resistor is marked as i sub meas, while the current flowing through the ammeter is marked as i sub m.

The drawing shows an Ammeter drawn across a shunt resistor whose resistance is marked as R sub A. The ammeter has a voltage of 150mV, and a current of 3mA. The circuit is enclosed in a box, and is connected to two open terminals outside.

The drawing shows a Voltmeter connected to a series resistor, R sub v. The voltmeter has a voltage of 20 mV and a current of 1 mA. The circuit is enclosed in a box, and is connected to two open terminals outside.

The rectangular circuit shows a vertical cross wire connecting a pair of nodes on the top and bottom wires. A 15 ohm resistor is located on the cross wire while a 45 ohm resistor is located on the right wire. The left wire carries a 50mA independent current source with an upward flowing current. A rightward flowing current, i sub 0 is marked on the right arm of the top wire.

The rectangular circuit shows two nodes on the top wire which hold another rectangular Ammeter circuit between them. A 10 ohm series resistor is located to the left of this Ammeter circuit, while a 50V independent voltage source is located on the left arm.

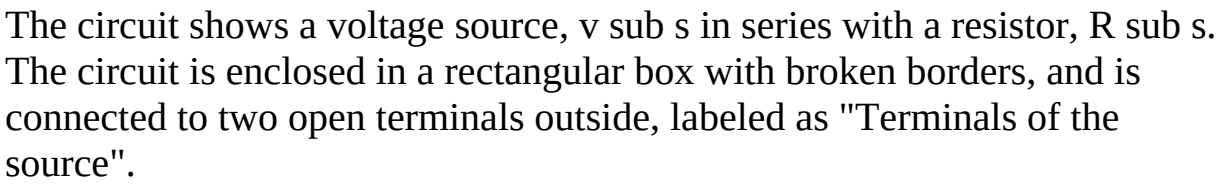
The Ammeter circuit shows an Ammeter and a 20 ohm resistor on the bottom wire, and a 60 ohm resistor on the top wire.

The circuit is represented as a box with broken borders, with a voltmeter inside in series with a resistor, R sub m. The meter movement of the voltmeter is shown as 100mV and 0.5mA. Two terminals outside the box connect to the voltmeter circuit, with one circuit showing a reading of 500V while the other terminal is marked as common.

The rectangular circuit shows two parallel crosswire connecting two pairs of nodes on the top and bottom wires. A 50 ohm resistor and a 250 ohm resistor are located on the left and right cross wires. A 10mA independent current source is located on the left wire, with an upward moving current. The right wire is removed, and the top and bottom wires are connected to two open terminals with a voltage v marked across them.

The circuit shows three parallel wires which carry resistors, R1, R2, and R3. A voltmeter connects between a common terminal and the terminals across these three resistors. The meter movement of the voltmeter is shown as 50 mV and 2 mA. The voltage across each of the resistances is marked as follows: R1 - 100V; R2 - 10V; R3 - 1V.

The rectangular circuit has a 180V independent dc voltage source on the left wire and two series resistors measuring 20k ohm and 70k ohm on its right wire. Two wires are drawn from the nodes on either side of the 70k ohm resistor, and are connected to two open terminals. The voltage across the open terminals is marked as v sub 0.


The circuit shows a voltage source, v sub s in series with a resistor, R sub s. The circuit is enclosed in a rectangular box with broken borders, and is connected to two open terminals outside, labeled as "Terminals of the source".

The circuit shows two resistors in series connected to a voltmeter which has a meter movement of 100 mV and 2mA. Each resistor is connected to an open terminal, while the voltmeter is connected to a common terminal. The resistances and corresponding voltage readings across the two resistors is marked as follows: 49.95k ohms - 100V; 360k ohms - 820V. The circuit is shown inside a box with the terminals depicted outside.

A wire connects the common terminal and the 100V terminal outside the box, running parallel to the 49.95k ohm resistor. A 200k ohm resistor is located on this wire.

The rectangular circuit shows a vertical cross wire drawn across a pair of nodes on the top and bottom wires. A 300V independent voltage source and a 60k ohm series resistor are located on the left wire, while a 600k ohm resistor is located on the cross wire. The right wire is removed, and the top and bottom wires are connected to two open terminals. A rectangular box representing a modified voltmeter is connected across these two open terminals. The lower terminal is marked as common while the upper terminal shows a voltage reading of 820V.

The circuit shows a 75V independent voltage source on the left wire and a Wheatstone bridge circuit on the right wire with four resistors of 6k ohm, 30k ohm, 20k ohm, and 6k ohm marked from the top left, moving clockwise. A voltmeter is located on the bridge, with a detector current, i sub d flowing from left to right.

The circuit shows a 300V independent voltage source on the left wire, a 3 ohm series resistor on top wire and a Wheatstone bridge circuit on the right wire with four resistors of 10 ohm, 30 ohm, 18 ohm, and 6 ohm marked from the top left, moving clockwise. A 50 ohm resistor is located on the bridge.

The rectangular circuit shows a horizontal cross wire connecting two nodes on the left and right wires, and a vertical cross wire connecting two nodes on the horizontal cross wire and the bottom wire. A 40 ohm resistor is located on the vertical cross wire, while a 1 ohm resistor and 10 ohm resistor are located on the left and right arms of the horizontal cross wire. A 15 ohm resistor is located on the top wire while a 50 ohm resistor is located on the lower arm of the right wire.

The circuit is powered by a 24V independent voltage source located on the lower arm of the left wire. The voltage across the vertical cross wire is marked as v1 while the voltage across the lower arm of the right wire is marked as v2.

The smaller rectangular circuit shows a vertical cross wire connecting a pair of nodes on the top and bottom wires. A 60 ohm resistor is located on the vertical cross wire. The top wire shows a 22 ohm resistor and a 30 ohm resistor on its left and right arms while the bottom wire shows a 4 ohm resistor and a 10 ohm resistor on its left and right arms. The voltage across the cross wire is marked as v. Two currents, i1 and i2 flow from left to right through the left and right arms of the bottom wire, respectively.

The smaller circuit is located on the top wire of a bigger circuit, which has a 1 ohm resistor on its right wire, and a 5A independent current source on its left wire.

The rectangular circuit shows a vertical cross wire connecting two nodes on the top and bottom wires. A 140 ohm resistor is located on this cross wire. Two diagonal cross wires are drawn from the bottom node of the vertical cross wire to the top nodes of the left and right wires. A 75 ohm resistor is located on the left diagonal cross wire, while a 60 ohm resistor is located on the right diagonal cross wire.

The top wire carries an 22 ohm resistor on the left arm, and a 20 ohm resistor on the right arm. The bottom wire carries a 10 ohm resistor on the left arm, and a 12 ohm resistor on the right arm.

The circuit is powered by a 240V independent voltage source, located on the left wire. A current, i sub 0 flows downwards on the left diagonal wire.

The rectangular circuit shows a node marked at the midpoint of the right wire. Three wires are drawn from the top, middle, and bottom nodes of the right wire to a common external node, located to the right of the circuit. Three resistors are located on these three wires, with resistances of R2 = 30 ohm, R3 = 60 ohm, and R5 = 18 ohm, from top to bottom, respectively. Two resistors are located on the top and bottom arms of the right wire, with resistances of R1 = 10 ohm and R4 = 30 ohm.

The top wire of the circuit carries a 20 ohm resistor while the bottom wire carries a 9 ohm resistor. The left wire is removed, and the top and bottom wires are connected to two open terminals marked as a and b, respectively.

The rectangular circuit shows a cross wire connecting a pair of nodes on the top and bottom wires. A 40 ohm resistor carrying a downward bound current, i2 is located on this cross wire. Two resistors with resistances of 44 ohm and 80 ohm are located on the left arm of the top wire, while a 20 ohm resistor is located on the right arm. A 210 ohm resistor is located on the right wire, carrying a downward bound current, i sub 0.

An additional wire is drawn across two nodes on the top wire, encompassing the 80 ohm resistor and the 20 ohm resistor. A 56 ohm resistor is located on this additional wire, carrying a rightward bound current, i1.

The circuit is powered by a 5V independent voltage source.

The rectangular circuit shows a vertical cross wire connecting a pair of nodes on the top and bottom wires. Two horizontal cross wires are drawn parallel to each other across two pairs of nodes on the vertical cross wire and the right wire. A 100 ohm resistor and an 80 ohm resistor are located on the top and bottom horizontal cross wires. And a 20 ohm resistor and a 40 ohm resistor are located on the right arms of the top and bottom wires.

Three resistors are located on the three arms of the vertical cross wire with resistances of 50 ohm, 71 ohm, and 60 ohm from top to bottom. Similarly, three resistors are located on the three arms of the right wire with resistances of 30 ohm, 31 ohm, and 20 ohm from top to bottom. A 1.5 ohm resistor is located on the left arm of the top wire. Two nodes, a and b are located on the top and bottom wires, towards the left wire.

The circuit is powered by an independent voltage source, v sub ab located on the left wire.

The rectangular circuit shows a vertical cross wire connecting a pair of nodes on the top and bottom wires. A resistor, R2 is located on this cross wire. Four resistors with a resistance R1 are located on the left and right arms of the top wire, and the left and right arms of the bottom wire. These five resistors are enclosed in a rectangle with broken edges, labeled as "Attenuator".

The right wire carries a load resistor, R sub L. Two nodes, c and d are marked on the top and bottom wires between the Attenuator and the load resistor. The voltage across these nodes is marked as v sub 0.

The left wire of the circuit is removed, and the top and bottom wires are connected to two terminals, marked as a and b. A voltage, v sub i is marked across these two terminals.

The rectangular circuit shows a vertical cross wire connecting a pair of nodes on the top and bottom wires. A resistor, R is located on this cross wire. Two more resistors with a resistance R are located on the left and right arms of the top wire. An additional wire is drawn across the top wire, encompassing the two resistors. A resistor R is located on the additional wire. These four resistors are enclosed in a rectangle with broken edges, labeled as "Fixed-Attenuator pad".

The right wire carries a load resistor, R sub L. Two nodes, c and d are marked on the top and bottom wires between the Attenuator and the load resistor. The voltage across these nodes is marked as v sub 0.

The left wire of the circuit is removed, and the top and bottom wires are connected to two terminals, marked as a and b. A voltage, v sub i is marked across these two terminals.

The rectangular circuit shows a vertical cross wire connecting a pair of nodes on the top and bottom wires. A resistor, R is located on this cross wire. Two more resistors with a resistance R are located on the left and right arms of the top wire. An additional wire is drawn across the top wire, encompassing the two resistors. A resistor R2 is located on the additional wire. These four resistors are enclosed in a rectangle with broken edges, representing a bridged tee.

The right wire carries a load resistor, R sub L. Two nodes, c and d are marked on the top and bottom wires between the bridged tee and the load resistor. The voltage across these nodes is marked as v sub 0.

The left wire of the circuit is removed, and the top and bottom wires are connected to two terminals, marked as a and b. A voltage, v sub i is marked across these two terminals.

The circuit shows an independent voltage source, "v sub in" on the left wire, and a Wheatstone bridge circuit on the right wire. Four resistors of the bridge circuit are marked from top left, moving clockwise as "R sub 0 plus delta R", R2, R3, and R4. The voltage across the bridge is marked as v sub 0.

The left column displays Band color while the right column shows the corresponding Value, as follows:

Black: 0

Brown: 1

Red: 2

Orange: 3

Yellow: 4

Green: 5

Blue: 6

Violet: 7

Grey: 8

White: 9

Gold: 0.1

Silver: 0.01

The left column displays Band color while the right column shows plus or minus percentage, as follows:

Brown: 1

Red: 2

Gold: 5

Silver: 10

None: 20

An illustration of a Resistor shows four bands drawn on it from top to bottom, with the bands being closer to the top end. The four bands are labeled and explained as follows:

Band 1: First figure of value

Band 2: Second figure of value

Band 3: Number of zeros/multiplier

Band 4: Tolerance (plus or minus percentage)

A note explains that the bands are closer to one end than the other.

Four bands of the shown resistor are identified as follows:

Band 1: Brown - 1

Band 2: Green - 5

Band 3: Orange - 000

Band 4: Gold - 5 percent

Resistor is 15000 ohm or 15k ohm, plus or minus 5 percent

Four bands of the shown resistor are identified as follows:

Band 1: Yellow - 4

Band 2: Violet - 7

Band 3: Silver - multiplied by 0.01

Band 4: Red - 2 percent

Resistor is 47 multiplied by 0.01 ohm or 0.47 ohm, plus or minus 2 percent

Four bands of the shown resistor are identified as follows:

Band 1: Red - 2

Band 2: Red - 2

Band 3: Green - 00000

Band 4: No color - 20 percent

Resistor is 2200000 ohm or 2.2M ohm, plus or minus 20 percent

Four bands of the shown resistor are identified as follows:

Band 1: Brown - 1

Band 2: Green - 5

Band 3: Red - 00

Band 4: Gold - 5 percent

Resistor is 1500 ohm or 1.5k ohm, plus or minus 5 percent

A rectangular circuit with eight resistors has an independent voltage source, v sub s, on its left wire. Two nodes are located at midpoints of the top and bottom wires. The top wire has resistors R1 and R2 on its left and right arms, while the bottom wire has resistors R5 and R4 on its left and right arms, respectively. Resistor R3 is located on the right wire between two nodes located on the top and bottom.

A vertical cross wire connects the two nodes at midpoints of the top and bottom wires. This wire carries resistor R6. Two diagonal cross wires are drawn between the two midpoint nodes and end points of the right wire. The diagonal wire from the top midpoint to bottom right node carries resistor R7, while the diagonal wire from the bottom midpoint to top right node carries resistor R8.

A rectangular circuit with eight resistors has an independent voltage source, v sub s, on its left wire. Two nodes are located at midpoints of the top and bottom wires. The top wire has resistors R1 and R2 on its left and right arms, while the bottom wire has resistors R5 and R4 on its left and right arms, respectively. Resistor R3 is located on the right wire, between two nodes on the top and bottom.

A vertical cross wire connects the two nodes at the midpoints of the top and bottom wires. This wire carries resistor R6. A diagonal wire from the bottom midpoint to the top right node carries resistor R8. An additional wire between the top midpoint and bottom right node carries resistor R7.

A rectangular circuit with eleven resistors has an independent voltage source, v sub s, on its left wire. Two nodes are located on the top wire, equidistant from each other and from both ends. Similarly, two nodes are located on the bottom wire. A single node is located at the midpoint of the right wire.

Three resistors, R1, R2, and R3, are located on the left, middle, and right arms of the top wire. Two resistors, R4 and R5, are located on the right and middle arms of the bottom wire.

Two vertical cross wires connect the nodes in the top and bottom wires. The left vertical cross wire carries resistor R6, while the right vertical cross wire carries resistor R11. A diagonal cross wire from the top left node to bottom right node carries resistor R7, while the second diagonal cross wire from the top right node to bottom left node carries resistor R9.

Two more diagonal cross wires are drawn from the midpoint node on the right wire to the top left node and bottom left node, carrying resistors R8 and R10, respectively.

The Names, definitions, and examples from figure 4.3 are listed in the given table, as follows:

Node: A point where two or more circuit elements join; a

Essential node: A node where three or more circuit elements join; b

Path: A trace of adjoining basic elements with no elements included more than once; v1 - R1 - R5 - R6

Branch: A path that connects two nodes; R1

Essential branch: A path that connects two essential nodes without passing through an essential node; v1 - R1

Loop: A path whose last node is the same as the starting node; v1 - R1 - R5 - R6 - R4 - v2

Mesh: A loop that does not enclose any other loops; v1 - R1 - R5 - R3 - R2

Planar circuit: A circuit that can be drawn on a plane with no crossing branches; Fig. 4.3 is a planar circuit. Fig. 4.2 is a non-planar circuit.

A rectangular circuit has three nodes on the left wire, marked as "a" on top, c at midpoint, and f at bottom. An independent voltage source, v1, is located between "a" and c, while a second independent voltage source with same polarity, v2, is located between c and f. The circuit also has an independent current source, I, located on the right wire, with current flowing from bottom to top.

Two nodes, b and g, are located on the top and bottom wires, close to the right wire. A vertical cross wire connects these two nodes. At midpoint of this cross wire, a node e is marked. A horizontal cross wire joins node e on the vertical cross wire and node c at the midpoint of the left wire. Midpoint of this horizontal cross wire is marked as node d.

Seven resistors are located in the circuit. Resistor R1 is on the left arm of the top wire, while resistors R2 and R3 are respectively on the left and right arms of the horizontal cross wire between c and e. Resistor R4 is on the left arm of the bottom wire. Resistors R5 and R6 are respectively on the top and bottom arms of the vertical cross wire between nodes b and g. Seventh resistor, R7, is located on an additional vertical cross wire, drawn between unnamed nodes, in-between the vertical cross wire between nodes b and g and the right wire.

A rectangular circuit has three nodes on its left wire, marked as (a) on top, c at midpoint, and f at bottom. An independent voltage source, v1, is located between (a) and c, while a second independent voltage source with the same polarity, v2, is located between c and f. The circuit also has an independent current source, I, located on its right wire, with current flowing from bottom to top.

Two nodes, b and g, are located on the top and bottom wires, close to the right wire. A vertical cross wire connects these two nodes. At the midpoint of this cross wire, a node e is marked. A horizontal cross wire joins node e on the vertical cross wire and node c at the midpoint of the left wire. The midpoint of this horizontal cross wire is marked as node d.

Seven resistors are located in the circuit, carrying six unknown branch currents. Resistor R1 is on the left arm of the top wire, with current i1 flowing from left to right. Resistors R2 and R3 are respectively on the left and right arms of the horizontal cross wire between c and e, with current i3 flowing from right to left. Resistor R4 is on the left arm of the bottom wire, carrying current i5 from right to left. Resistors R5 and R6 are respectively on the top and bottom arms of the vertical cross wire between nodes b and g. Current i2 flows through R5, while current i4 flows through R6; both from top to bottom. Seventh resistor, R7, is located on an additional vertical cross wire, which is drawn closer to the right wire, between unnamed nodes. Current i6 flows through this resistor from top to bottom.

A rectangular circuit has a 10V independent voltage source on the left wire and a 2A independent current source on the right wire, with current flowing from bottom to top. Two nodes are located on the top wire, equidistant from each other and from either end. Similarly, two nodes are located on the bottom wire. Two vertical cross wires are drawn between these two pairs of nodes.

A 5 ohm resistor is located on the left cross wire, while a 10 ohm resistor is located on the right cross wire. A 1 ohm resistor and a 2 ohm resistor are respectively located on the left and middle arms of the top wire.

A rectangular circuit has a 10V independent voltage source on the left wire and a 2A independent current source on the right wire, with current flowing from bottom to top. Two nodes are located on the top wire, equidistant from each other and from either end. Similarly, two nodes are located on the bottom wire. Two vertical cross wires are drawn between these two pairs of nodes.

A 5 ohm resistor is located on the the left cross wire, while a 10 ohm resistor is located on the right cross wire. A 1 ohm resistor and a 2 ohm resistor are respectively located on the left and middle arms of the top wire.

The two nodes on the top wire are encircled separately and marked as 1 and 2. The two nodes on the bottom wire are encircled together, with a downward arrow drawn from the left node, marking it as a reference node. Voltage across the 5 ohm resistor is marked as v1, with node 1 marked positive and the reference node below marked negative. Voltage across the 10 ohm resistor is marked as v2, with node 2 marked positive and the reference node below marked negative.

A rectangular circuit has a 10V independent voltage source on the left wire and a 1 ohm resistor on the top wire. Two nodes are marked on the top and bottom wires, with the bottom node marked as a reference node. Voltage between the two nodes is marked as v1, with positive and negative polarities marked on top and bottom, respectively.

Current i flows from right to left through the 1 ohm resistor. Voltage across the resistor is marked as iR, with its polarity marked opposite to flow of current i.

A rectangular circuit has a 50V independent voltage source on the left wire and a 3A independent current source on the right wire, with current flowing from bottom to top. Two nodes are located on the top wire, equidistant from each other and from either end. Similarly, two nodes are located on the bottom wire. Two vertical cross wires are drawn between these two pairs of nodes.

A 5 ohm resistor is located on the left arm of the top wire, with current i sub a flowing through it from left to right. A 10 ohm resistor is located on the left vertical cross wire, with current i sub b flowing through it from top to bottom. Plus, a 40 ohm resistor is located on the right vertical cross wire, with current i sub c flowing through it from top to bottom.

A rectangular circuit has a 50V independent voltage source on the left wire and a 3A independent current source on the right wire, with current flowing from bottom to top. Two nodes are located on the top wire, equidistant from each other and from either end. Similarly, two nodes are located on the bottom wire. Two vertical cross wires are drawn between these two pairs of nodes.

A 5 ohm resistor is located on the left arm of the top wire. A 10 ohm resistor is located on the left vertical cross wire, while a 40 ohm resistor is located on the right vertical cross wire. The two top nodes are encircled together, with the left node marked as 1. The two bottom nodes are similarly encircled together, with the left node marked as a reference node. The unknown node voltage between node 1 and the reference node is marked as v1, with node 1 marked positive and the reference node marked negative.

A rectangular circuit has a 15A independent current source on the left wire with current flowing from bottom to top, and a 5A independent current source on the right wire with current flowing from top to bottom. Three nodes are located on the top wire, equidistant from each other and from either end. Similarly, three nodes are located on the bottom wire. Three vertical cross wires are drawn between these three pairs of nodes.

Three resistors with resistances of 60, 15, and 2 ohms are located on the left, middle, and right cross wires respectively. A 5 ohm resistor is located between the second and third nodes on the top wire. An unknown current, i1, flows through this resistor from left to right.

An unknown voltage, v1, is marked across the 60 ohm resistor on the left vertical cross wire. A second unknown voltage, v2, is marked across the 5A current source on the right wire.

A rectangular circuit has a 4.5A independent current source on the left wire with current flowing from bottom to top, and a 30V independent voltage source on the right wire with polarity matching the current source. Two nodes are located on the top wire at approximately one-fourth the total distance from either end. Similarly, two nodes are located on the bottom wire. Two vertical cross wires are drawn between these two pairs of nodes.

The left and right cross wires carry 1 ohm and 2 ohm resistors respectively. Two more resistors are located in the long middle arm of the top wire, with resistances of 6 ohms and 2 ohms. A fifth resistor of 4 ohm resistance is located on the right arm of the top wire.

An unknown voltage, v, is marked at the midpoints of the top and bottom wires.

A rectangular circuit has a 20V independent voltage source on the left wire, and a dependent voltage source on the right wire, whose same polarity voltage is marked as 8i sub phi. Two nodes are located on the top wire, equidistant from each other and from either end. Similarly, two nodes are located on the bottom wire. Two vertical cross wires are drawn between these two pairs of nodes.

Three resistors with resistances of 2, 5, and 2 ohms are located respectively on the left, middle, and right arms of the top wire. A 20 ohm resistor and a 10 ohm resistor are located on the left and right cross wires. An unknown current, i sub phi, flows through the 5 ohm resistor in the middle arm of the top wire.

A rectangular circuit has a 20V independent voltage source on the left wire, and a dependent voltage source on the right wire, whose same polarity voltage is marked as 8i sub phi. Two nodes are located on the top wire, equidistant from each other and from either end. They are encircled separately and labeled as 1 and 2. Similarly, two nodes are located on the bottom wire. These two nodes are encircled together, with a downward arrow marked on left node, indicating it as a reference node. Opposite pairs of nodes on the top and bottom wires are joined by two vertical cross wires.

Three resistors with resistances of 2, 5, and 2 ohms are located respectively on the left, middle, and right arms of the top wire. A 20 ohm resistor and a 10 ohm resistor are located on the left and right cross wires respectively. A node voltage, v1, is marked across the 20 ohm resistor, while a second node voltage, v2, is marked across the 10 ohm resistor. An unknown current, i sub phi, flows through the 5 ohm resistor in the middle arm of the top wire.

A rectangular circuit has a 50V independent voltage source on the left wire, and a 5A independent current source on the right wire, with current flowing from bottom to top. Two nodes are located on the top wire, equidistant from each other and from either end. Similarly, two nodes are located on the bottom wire. Two vertical cross wires are drawn between these two pairs of nodes.

An 8 ohm resistor and a 4 ohm resistor are located on the left and right cross wires respectively. A 6 ohm resistor and a 2 ohm resistor are located on the left and middle arms of the top wire. Current i1 flows from left to right through the 6 ohm resistor.

A parallel line connects the two nodes of the top wire over the 2 ohm resistor. This line carries a dependent current source in which a 3i1 current flows from right to left.

A rectangular circuit has a 100V independent voltage source on the left wire, and a 5A independent current source on the right wire with current flowing from bottom to top. Two nodes are located on the top wire, equidistant from each other and from either end. They are encircled separately and labeled as 1 and 2. Similarly, two nodes are located on the bottom wire. These two nodes are encircled together, with a downward arrow marked on the left node, indicating it as a reference node. Opposite pairs of nodes on the top and bottom wires are joined by two vertical cross wires.

A 25 ohm resistor is located on the left cross wire, between node 1 and the reference node, while a 50 ohm resistor is located on the right cross wire, between node 2 and the reference node. A 10 ohm resistor is located on the middle arm of the top wire, between nodes 1 and 2. Voltage across node 1 and the reference node is marked as v1, and voltage across node 2 and the reference node is marked as v2.

A rectangular circuit has a 100V independent voltage source on the left wire, and a 5A independent current source on the right wire with current flowing from bottom to top. Two nodes are located on the top wire, equidistant from each other and from either end. They are encircled separately and labeled as 1 and 2. Similarly, two nodes are located on the bottom wire. These two nodes are encircled together, with a downward arrow marked on the left node, indicating it as a reference node. Opposite pairs of nodes on the top and bottom wires are joined by two vertical cross wires.

A 25 ohm resistor is located on the left cross wire, between node 1 and the reference node, while a 50 ohm resistor is located on the right cross wire, between node 2 and the reference node. A 10 ohm resistor is located on the middle arm of the top wire, between nodes 1 and 2. Voltage across node 1 and the reference node is marked as 100V, and voltage across node 2 and the reference node is marked as v2.

A rectangular circuit has a 50V independent voltage source on the left wire, and a 4A independent current source on the right wire with current flowing from bottom to top. Three nodes are located on the top wire, equidistant from each other and from either end. Similarly, three nodes are located on the bottom wire. Three vertical cross wires are drawn between these three pairs of nodes.

Three resistors are located on the three cross wires from left to right, with resistances of 40, 50, and 100 ohms respectively. A 5 ohm resistor is located on the top wire between the first and second nodes. An unknown current, i sub phi, flows through this branch from right to left. A dependent voltage source with an opposite polarity voltage of 10i sub phi is located between the second and third nodes on the top wire.

A rectangular circuit has a 50V independent voltage source on the left wire, and a 4A independent current source on the right wire with current flowing from bottom to top. Three nodes are located on the top wire, equidistant from each other and from either end. They are encircled separately and labeled as 1, 2, and 3 from left to right. Similarly, three nodes are located on the bottom wire. These three nodes are encircled together, with the left node marked as a reference node. Three vertical cross wires are drawn between these opposite pairs of nodes.

Three resistors are located on the three cross wires from left to right, with resistances of 40, 50, and 100 ohms respectively. Voltages across these resistors are marked respectively as 50V, v2, and v3. A 5 ohm resistor is located on the top wire between the first and second nodes. An unknown current, i sub phi, flows through this branch from right to left. A dependent voltage source with an opposite polarity voltage of 10i sub phi is located between the second and third nodes on the top wire. A current i flows to the right from the second to third node.

A rectangular circuit has a 50V independent voltage source on the left wire, and a 4A independent current source on the right wire with current flowing from bottom to top. Three nodes are located on the top wire, equidistant from each other and from either end. They are encircled separately and labeled as 1, 2, and 3 from left to right. Similarly, three nodes are located on the bottom wire. These three nodes are encircled together, with the left node marked as a reference node. Three vertical cross wires are drawn between these opposite pairs of nodes.

Three resistors are located on the three cross wires from left to right, with resistances of 40, 50, and 100 ohms respectively. Voltages across these resistors are marked respectively as 50V, v2, and v3. A 5 ohm resistor is located on the top wire, between the first and second nodes. An unknown current, i sub phi, flows through this branch from right to left. A dependent voltage source with an opposite polarity voltage of 10i sub phi is located between the second and third nodes on the top wire.

Node 2 and node 3 in the top wire are encircled together to indicate that these two form a super node.

A rectangular circuit has two nodes, a and d, at midpoints of its top and bottom wires respectively, which are joined by a vertical cross wire. A horizontal cross wire joins node c on the vertical cross wire with node b on the left wire, with both the nodes located closer to the bottom wire. An independent voltage source, V sub 0, is located on the horizontal cross wire. An unknown current, i sub B, flows through this branch from left to right.

A second independent voltage source, V sub CC, is located on the right wire. A dependent current source, with current Beta i sub B flowing from top to bottom, is located on the top arm of the vertical cross wire.

The circuit has four resistors between the following nodes: R1 between a and b on the top arm of the left wire, R2 between b and d on the bottom arm of the left wire, R sub C between a and c on the top arm of the vertical cross wire, and R sub E between c and d on the bottom arm of the vertical cross wire.

A rectangular circuit has two nodes, a and d, at midpoints of its top and bottom wires respectively, which are joined by a vertical cross wire. A horizontal cross wire joins node c on the vertical cross wire with node b on the left wire, with both the nodes located closer to the bottom wire. An independent voltage source, V sub 0, is located on the horizontal cross wire. An unknown current, i sub B, flows through this branch from left to right.

A second independent voltage source, V sub CC, is located on the right wire. A dependent current source, with current Beta i sub B flowing from top to bottom, is located on the top arm of the vertical cross wire.

The circuit has four resistors between the following nodes: R1 between a and b on the top arm of left wire, R2 between b and d on the bottom arm of the left wire, R sub C between a and c on the top arm of the vertical cross wire, and R sub E between c and d on the bottom arm of the vertical cross wire.

The four nodes, a, b, c, and d, are marked as essential nodes, with node d identified as a reference node. A bigger circle is drawn around b and c, combining them into a super node. Voltage across resistor R2 is marked as v sub b, while voltage across resistor R sub E is marked as v sub c. Voltage across nodes a and d is marked as V sub CC.

A rectangular circuit has two nodes on its left and right wires, which are joined by a horizontal cross wire. A vertical cross wire joins a node on the horizontal cross wire with a node on the bottom wire. Two resistors of 10 ohm and 20 ohm resistance are located on the left and right arms of the horizontal cross wire, respectively. Another resistor of 40 ohm resistance is located on the vertical cross wire. The fourth resistor of 30 ohm resistance is found on the top wire.

A 10V independent voltage source is located on the bottom arm of the left wire. Current i sub delta flows through it from bottom to top. A dependent voltage source, with a voltage of 20i sub delta, is located on the bottom arm of the right wire. Voltage across the nodes of the vertical cross wire is marked as v sub 0, and voltage across the 20 ohm resistor on the right arm of the horizontal cross wire is marked as v sub 1.

A rectangular circuit has a 4.8A independent current source on the left wire, with current flowing from bottom to top, and a 12V voltage source on the right wire. Three nodes are located on the top wire, equidistant from each other and from either end. Similarly, three nodes are located on the bottom wire. Three vertical cross wires are drawn between these three pairs of nodes.

Three resistors are located on the three cross wires from left to right, with resistances of 7.5, 10, and 2.5 ohms respectively. A 2.5 ohm resistor is located between the first and second nodes on the top wire, while a 1 ohm resistor is located on the right arm of the top wire, after the third node. A dependent voltage source with opposite polarity to independent voltage source is found between the second and third nodes on the top wire.

Current in the dependent voltage source is marked as i sub x. The same current flows from top to bottom through the 7.5 ohm resistor on the left cross wire. Voltage v is marked across the 10 ohm resistor on the middle cross wire.

A rectangular circuit has one node on its left wire and another node on its right wire, which are joined by a horizontal cross wire. A vertical cross wire joins a node on the middle of the horizontal cross wire with a node on the middle of the bottom wire. Two resistors of 2 ohm and 3 ohm resistance are located on the left and right arms of the horizontal cross wire, respectively. Another resistor of 24 ohm resistance is located on the vertical cross wire. The fourth resistor of 3 ohm resistance is found on the bottom arm of the right wire.

A 60V independent voltage source is located on the bottom arm of the left wire. A dependent voltage source, with opposite polarity voltage of 6i sub phi, is located on the top wire. Voltage across the node of the vertical cross wire is marked as v sub 1. Current i sub phi flows from right to left through the 3 ohm resistor on the right arm of the horizontal cross wire.

A rectangular circuit has two nodes on its top and bottom wires, which are joined by a vertical cross wire. A 4 ohm resistor, carrying current i sub 1 flowing from left to right, is located on the left arm of the top wire, while a 5 ohm resistor, carrying current i sub 2 flowing from left to right, is located on the right arm of the top wire. The third resistor, on the vertical cross wire, has a resistance of 10 ohms, and it carries current i sub 3, which flows from top to bottom.

The circuit has a 100V independent voltage source on the left wire and a 40V independent voltage source on the right wire.

A rectangular circuit has two nodes on its top and bottom wires, which are joined by a vertical cross wire. A 4 ohm resistor, carrying current i sub 1 flowing from left to right, is located on the left arm of the top wire, while a 5 ohm resistor, carrying current i sub 2 flowing from left to right, is located on the right arm of the top wire. The third resistor, on the vertical cross wire, has a resistance of 10 ohms, and it carries current i sub 3, which flows from top to bottom.

The circuit has a 100V independent voltage source on the left wire and a 40V independent voltage source on the right wire. Mesh current in the left loop is indicated by a clockwise curved arrow labeled as i sub a, and mesh current in the right loop is indicated by another clockwise curved arrow labeled as i sub b.

A rectangular circuit has a 40V independent voltage source on the left wire and a 20V independent voltage source on the right wire. Two nodes are located on the top wire, equidistant from each other and from either end. Similarly, two nodes are located on the bottom wire. Two vertical cross wires are drawn between these two pairs of nodes.

An 8 ohm resistor and a 6 ohm resistor are located on the left and right cross wires, respectively. A 2 ohm resistor, 6 ohm resistor, and 4 ohm resistor are located on the left, middle, and right arms of the top wire. Voltage across the nodes of the left vertical cross wire is marked as v sub 0.

A rectangular circuit has a 40V independent voltage source on the left wire and a 20V independent voltage source on the right wire. Two nodes are located on the top wire, equidistant from each other and from either end. Similarly, two nodes are located on the bottom wire. Two vertical cross wires are drawn between these two pairs of nodes.

An 8 ohm resistor and a 6 ohm resistor are located on the left and right cross wires respectively. A 2 ohm resistor, a 6 ohm resistor, and a 4 ohm resistor are located on the left, middle, and right arms of the top wire.

Mesh currents in the left, middle, and right loops of the circuit are indicated by clockwise curved arrows, which are respectively labeled as i sub a, i sub b, and i sub c.

A rectangular circuit has one node on its left wire and another node on its right wire, which are joined by a horizontal cross wire. A vertical cross wire joins a node on the middle of the horizontal cross wire with a node on the middle of bottom wire. Two resistors of 5 ohm and 90 ohm resistance are located on the left and right arms of the horizontal cross wire, respectively. Another resistor of 26 ohm resistance is located on the vertical cross wire. The fourth resistor of 8 ohm resistance is found on the bottom arm of the right wire, while the fifth resistor of 30 ohm resistance is located on the top wire.

An 80V independent voltage source is located on the bottom arm of the left wire.

A rectangular circuit has one node on its left wire and another node on its right wire, which are joined by a horizontal cross wire. A vertical cross wire joins a node on the middle of the horizontal cross wire with a node on the middle of bottom wire. Two resistors of 5 ohm and 4 ohm resistance are located on the left and right arms of the horizontal cross wire, respectively. Another resistor of 20 ohm resistance is located on the vertical cross wire. The fourth resistor of 1 ohm resistance is located on the top wire.

A 50V independent voltage source is located on the bottom arm of the left wire. A dependent voltage source of voltage 15i sub phi is located on the bottom arm of the right wire. Current i sub phi flows from top to bottom across the 20 ohm resistor on the vertical cross wire.

A rectangular circuit has one node on its left wire and another node on its right wire, which are joined by a horizontal cross wire. A vertical cross wire joins a node on the middle of the horizontal cross wire with a node on the middle of bottom wire. Two resistors of 5 ohm and 4 ohm resistance are located on the left and right arms of the horizontal cross wire, respectively. Another resistor of 20 ohm resistance is located on the vertical cross wire. The fourth resistor of 1 ohm resistance is located on the top wire.

A 50V independent voltage source is located on the bottom arm of the left wire. A dependent voltage source of voltage 15i sub phi is located on the bottom arm of the right wire. Current i sub phi flows from top to bottom across the 20 ohm resistor on the vertical cross wire.

Three mesh currents are represented by clockwise curved arrows in the top loop, bottom left loop, and bottom right loop, and are respectively labeled as i sub 2, i sub 1, and i sub 3.

A rectangular circuit has one node on its left wire and another node on its right wire, which are joined by a horizontal cross wire. A vertical cross wire joins a node on the middle of the horizontal cross wire with a node on the middle of bottom wire. Two resistors of 2 ohm and 3 ohm resistance are located on the left and right arms of the horizontal cross wire, respectively. Another resistor of 5 ohm resistance is located on the vertical cross wire. The fourth resistor of 1 ohm resistance is located on the bottom arm of right wire, while a fifth resistor of 14 ohm resistance is located on the top wire.

A 25V independent voltage source is located on the bottom arm of the left wire. Another independent voltage source of 10V voltage is located on the vertical cross wire, below the 5 ohm resistor. A dependent voltage source of voltage minus 3v sub phi is located on the top wire, to the left of the 14 ohm resistor. Voltage across the 3 ohm resistor on the right arm of the vertical cross wire is marked as v sub phi.

A rectangular circuit has one node on its left wire and another node on its right wire, which are joined by a horizontal cross wire. A vertical cross wire joins a node on the middle of the horizontal cross wire with a node on the middle of bottom wire. Two resistors of 6 ohm and 8 ohm resistance are located on the left and right arms of the horizontal cross wire, respectively. Another resistor of 8 ohm resistance is located on the vertical cross wire. The fourth resistor of 2 ohm resistance is located on the top wire.

A 25V independent voltage source is located on the bottom arm of the left wire. A dependent voltage source of voltage 5i sub phi is located on the bottom arm of the right wire. Current i sub phi flows from bottom to top across the 25V voltage source on the left wire. Voltage across the 8 ohm resistor on the vertical cross wire is marked as v sub 0.

A rectangular circuit has a 50V independent voltage source on the left wire, and a 3A independent current source on the right wire, with a current flowing from bottom to top. Two nodes are located on the top wire, equidistant from each other and from either end. Similarly, two nodes are located on the bottom wire. Two vertical cross wires are drawn between these two pairs of nodes.

A 10 ohm resistor and a 40 ohm resistor are located on the left and right cross wires, respectively. A 5 ohm resistor is located on the left arm of the top wire. An unknown current, i sub a, flows left to right through the 5 ohm resistor, and two unknown currents, i sub b and i sub c, flow from top to bottom through the 10 ohm and 40 ohm resistors, respectively.

A rectangular circuit has a 50V independent voltage source on the left wire, and a 3A independent current source on the right wire, with a current flowing from bottom to top. Two nodes are located on the top wire, equidistant from each other and from either end. Similarly, two nodes are located on the bottom wire. Two vertical cross wires are drawn between these two pairs of nodes.

A 10 ohm resistor and a 40 ohm resistor are located on the left and right cross wires, respectively. A 5 ohm resistor is located on the left arm of the top wire. An unknown current, i sub a, flows left to right through the 5 ohm resistor, and two unknown currents, i sub b and i sub c, flow from top to bottom through the 10 ohm and 40 ohm resistors, respectively.

Three mesh currents are identified by clockwise curved arrows in the left, middle and right loops of the circuit, labeled respectively as i sub 1, i sub 2, and minus 3A.

A rectangular circuit has one node on its left wire and another node on its right wire, which are joined by a horizontal cross wire. A vertical cross wire joins a node on the middle of the horizontal cross wire with a node on the middle of bottom wire. Two resistors of 3 ohm and 2 ohm resistance are located on the left and right arms of the horizontal cross wire, respectively. Two more resistors of 6 ohm and 4 ohm resistance are located on the left and right arms of the bottom wire. A 10 ohm resistor is found on the top wire.

A 100V independent voltage source is located on the bottom arm of the left wire. A 50V independent voltage source is located on the bottom arm of the right wire. A 5A independent current source is located on the vertical cross wire, with current flowing from bottom to top. Voltage across the vertical cross wire is marked as v.

Three mesh currents are represented by clockwise curved arrows in the top loop, bottom left loop, and bottom right loop, and are respectively labeled as i sub b, i sub a, and i sub c.

A rectangular circuit has one node on its left wire and another node on its right wire, which are joined by a horizontal cross wire. A vertical cross wire joins a node on the middle of the horizontal cross wire with a node on the middle of bottom wire. Two resistors of 3 ohm and 2 ohm resistance are located on the left and right arms of the horizontal cross wire, respectively. Two more resistors of 6 ohm and 4 ohm resistance are located on the left and right arms of the bottom wire. A 10 ohm resistor is found on the top wire.

A 100V independent voltage source is located on the bottom arm of the left wire. A 50V independent voltage source is located on the bottom arm of the right wire. A 5A independent current source is located on the vertical cross wire, with current flowing from bottom to top.

Three mesh currents are represented by clockwise curved arrows in the top loop, bottom left loop, and bottom right loop, and are respectively labeled as i sub b, i sub a, and i sub c. A dotted rectangle is drawn around meshes i sub a and i sub c, and is labeled as Supermesh.

A rectangular circuit has two nodes at midpoints of its top and bottom wires, which are joined by a vertical cross wire. A horizontal cross wire joins a node on the vertical cross wire with another node on the left wire, with both the nodes located closer to the bottom wire. An independent voltage source, V sub 0, is located on the horizontal cross wire. An unknown current, i sub B, flows through this branch from left to right.

A second independent voltage source, V sub CC, is located on the right wire. A dependent current source with current Beta i sub B, flowing from top to bottom, is located on the top arm of the vertical cross wire.

The circuit has four resistors. R1 and R2 are located respectively on the top and bottom arms of the left wire. Resistor R sub c is found on the top arm of the vertical cross wire, above the dependent current source, and resistor R sub E is found on the bottom arm of the vertical cross wire.

A rectangular circuit has two nodes at midpoints of its top and bottom wires, which are joined by a vertical cross wire. A horizontal cross wire joins a node on the vertical cross wire with another node on the left wire, with both the nodes located closer to the bottom wire. An independent voltage source, V sub 0, is located on the horizontal cross wire. An unknown current, i sub B, flows through this branch from left to right.

A second independent voltage source, V sub CC, is located on the right wire. A dependent current source with current Beta i sub B, flowing from top to bottom, is located on the top arm of the vertical cross wire.

The circuit has four resistors. R1 and R2 are located respectively on the top and bottom arms of the left wire. Resistor R sub c is found on the top arm of the vertical cross wire, above the dependent current source, and resistor R sub E is found on the bottom arm of the vertical cross wire.

Three meshes are indicated through clockwise curved arrows in the top left, bottom left, and right loops, and are labeled respectively as i sub a, i sub b, and i sub c. Loops i sub a and i sub c are joined by a dotted closed line to form a Superloop.

A rectangular circuit has a 30V independent voltage source on the left wire, and a 16A independent current source on the right wire, with a current flowing from bottom to top. Two nodes are located on the top wire, equidistant from each other and from either end. Similarly, two nodes are located on the bottom wire. Two vertical cross wires are drawn between these two pairs of nodes.

A 2 ohm resistor and a 5 ohm resistor are located on the left and right cross wires, respectively. A 3 ohm resistor and an 8 ohm resistor are located on the left and middle arms of the top wire, while a 6 ohm resistor and a 4 ohm resistor are located on the left and middle arms of the bottom wire, respectively.

A rectangular circuit has two nodes at midpoints of the left and right wires, which are joined by a horizontal cross wire. A vertical cross wire joins two nodes at midpoints of the horizontal cross wire and bottom wire. A 5 ohm resistor is located on the vertical cross wire, with voltage v sub phi marked across the nodes. Two resistors of 2 ohm and 1 ohm resistance are located on the left and right arms of the horizontal cross wire, respectively.

A 75V independent voltage source is located on the bottom arm of the left wire. A 10A independent current source is located on the top wire, with current flowing from right to left. An independent current source is located on the bottom arm of the right wire, with current 2v sub phi over 5 flowing from top to bottom.

A mesh current, i sub a, is represented by a clockwise curved arrow in the bottom left loop.

A rectangular circuit has two nodes at midpoints of the left and right wires, which are joined by a horizontal cross wire. A vertical cross wire joins two nodes at midpoints of the horizontal cross wire and bottom wire. A 2 ohm resistor is located on this vertical cross wire. Another 2 ohm resistor is on the top wire, while a third 2 ohm resistor is located on the right arm of the horizontal cross wire. A 1 ohm resistor is located on the right arm of the bottom wire.

A 10V independent voltage source is located on the bottom arm of the left wire. A 6V independent voltage source with opposite polarity is located on the bottom arm of the right wire. A 2A independent current source is located on the left arm of the horizontal cross wire, with current flowing from left to right.

A rectangular circuit has a 256V independent voltage source on the left wire, and a 128V independent voltage source with opposite polarity on the right wire. Three nodes are located on the top wire, equidistant from each other and from either end. Similarly, three nodes are located on the bottom wire. Three vertical cross wires are drawn between these three pairs of nodes.

A 200 ohm resistor and a 400 ohm resistor are located on the left and right cross wires, respectively. A dependent voltage source, with a voltage of 50i sub delta, is located on the middle cross wire. Four resistors are located on the four arms of the top wire, between the nodes, with resistances of 150, 100, 250, and 500 ohms respectively from left to right.

An additional wire is drawn between the first node and the third node of the top wire. A 300 ohm resistor is located on this additional wire, with current i sub delta flowing through it from right to left.

A rectangular circuit has a 256V independent voltage source on the left wire, and a 128V independent voltage source with opposite polarity on the right wire. Three nodes are located on the top wire, equidistant from each other and from either end. Similarly, three nodes are located on the bottom wire. Three vertical cross wires are drawn between these three pairs of nodes.

A 200 ohm resistor and a 400 ohm resistor are located on the left and right cross wires, respectively. A dependent voltage source, with a voltage of 50i sub delta, is located on the middle cross wire. Four resistors are located on the four arms of the top wire, between the nodes, with resistances of 150, 100, 250, and 500 ohms respectively from left to right.

An additional wire is drawn between the first node and the third node of the top wire. A 300 ohm resistor is located on this additional wire, with current i sub delta flowing through it from right to left.

Five mesh currents are depicted in the circuit as clockwise curved arrows. Four mesh currents in four loops of the rectangular circuit are labeled as i sub a, i sub c, i sub d, and i sub e respectively from left to right. The fifth current in the additional wire on top of the upper wire is labeled as i sub b.

A rectangular circuit has a 256V independent voltage source on the left wire, and a 128V independent voltage source with opposite polarity on the right wire. Three nodes are located on the top wire, equidistant from each other and from either end. Similarly, three nodes are located on the bottom wire. Three vertical cross wires are drawn between these three pairs of nodes.

A 200 ohm resistor and a 400 ohm resistor are located on the left and right cross wires, respectively. A dependent voltage source, with a voltage of 50i sub delta, is located on the middle cross wire. Four resistors are located on the four arms of the top wire, between the nodes, with resistances of 150, 100, 250, and 500 ohms respectively from left to right.

An additional wire is drawn between the first node and the third node of the top wire. A 300 ohm resistor is located on this additional wire, with current i sub delta flowing through it from right to left.

The left node on the top wire is marked as a reference node with a downward arrow. The middle node is marked as node 1, with voltage v1, while the right node is marked as node2, with voltage v2. The middle node of the bottom wire is marked as node 3, with voltage v3. Node 1 and node 3 are encircled together, along with the dependent voltage source on the vertical cross wire between them, to indicate that these two form a super node.

A rectangular circuit has a 256V independent voltage source on the left wire, and a 128V independent voltage source with opposite polarity on the right wire. Three nodes are located on the top wire, equidistant from each other and from either end. Similarly, three nodes are located on the bottom wire. Three vertical cross wires are drawn between these three pairs of nodes.

A 200 ohm resistor and a 400 ohm resistor are located on the left and right cross wires, respectively. A dependent voltage source, with a voltage of 50i sub delta, is located on the middle cross wire. Four resistors are located on the four arms of the top wire, between the nodes, with resistances of 150, 100, 250, and 500 ohms respectively from left to right.

An additional wire is drawn between the first node and the third node of the top wire. A 300 ohm resistor is located on this additional wire, with current i sub delta flowing through it from right to left.

The three nodes on the top wire are labeled as a, b, and c, with respective voltages of v sub a, v sub b, and v sub c, from left to right. The middle node of the bottom wire is marked as a reference node, with a downward pointing arrow drawn from it.

A rectangular circuit has a 193V independent voltage source on the left wire, and a dependent voltage source with the same polarity on the right wire, with voltage marked as 0.8v sub theta. Two nodes are located on the top wire, equidistant from each other and from either end. Similarly, two nodes are located on the bottom wire. Two vertical cross wires are drawn between these two pairs of nodes.

A dependent current source is located on the left cross wire, with current 0.4v sub delta flowing from bottom to top. Voltage across the nodes on this cross wire is v sub o. A 0.5A independent current source is located on the right cross wire, with current flowing from bottom to top.

Three resistors with resistances of 4, 2.5, and 2 ohms are located on three arms of the top wire, from left to right. Voltage across the 2 ohm resistor is marked as v sub delta. Similarly, three resistors with resistances of 6, 7.5, and 8 ohms are located on three arms of the bottom wire, from left to right. Voltage across the 7.5 ohm resistor is marked as v sub theta.

A rectangular circuit has a 193V independent voltage source on the left wire, and a dependent voltage source with the same polarity on the right wire, with voltage marked as 0.8v sub theta. Two nodes are located on the top wire, equidistant from each other and from either end. Similarly, two nodes are located on the bottom wire. Two vertical cross wires are drawn between these two pairs of nodes.

A dependent current source is located on the left cross wire, with current 0.4v sub delta flowing from bottom to top. Voltage across the nodes on this cross wire is v sub o. A 0.5A independent current source is located on the right cross wire, with current flowing from bottom to top.

Three resistors with resistances of 4, 2.5, and 2 ohms are located on three arms of the top wire, from left to right. Voltage across the 2 ohm resistor is marked as v sub delta. Similarly, three resistors with resistances of 6, 7.5, and 8 ohms are located on three arms of the bottom wire, from left to right. Voltage across the 7.5 ohm resistor is marked as v sub theta.

Three mesh currents are marked with clockwise curving arrows in three loops of the circuit, as i sub a, i sub b, and i sub c, from left to right. A dotted rectangle is drawn combining the three meshes into a single super-mesh.

A rectangular circuit has a 193V independent voltage source on the left wire, and a dependent voltage source with the same polarity on the right wire, with voltage marked as 0.8v sub theta. Two nodes are located on the top wire, equidistant from each other and from either end. Similarly, two nodes are located on the bottom wire. Two vertical cross wires are drawn between these two pairs of nodes.

A dependent current source is located on the left cross wire, with current 0.4v sub delta flowing from bottom to top. Voltage across the nodes on this cross wire is v sub o. A 0.5A independent current source is located on the right cross wire, with current flowing from bottom to top.

Three resistors with resistances of 4, 2.5, and 2 ohms are located on three arms of the top wire, from left to right. Voltage across the 2 ohm resistor is marked as v sub delta. Similarly, three resistors with resistances of 6, 7.5, and 8 ohms are located on three arms of the bottom wire, from left to right. Voltage across the 7.5 ohm resistor is marked as v sub theta.

Node voltage on the right node of the top wire is marked as v sub a, and node voltage on the right node of the bottom wire is marked as v sub b.

A rectangular circuit has two nodes at midpoints of the top and bottom wires, which are connected by a vertical cross wire. A 2A independent current source is located on this cross wire with current flowing from bottom to top. A 20V independent voltage source is found on the left wire, while a 25V independent voltage source is found on the right wire. The circuit has two resistors on the left and right arms of the top wire, with respective resistances marked as 15 ohms and 10 ohms.

A rectangular circuit has a 128V independent voltage source on the left wire, and a 30i sub x dependent voltage source with the same polarity on the right wire. A 4A independent current source is located on the top wire, with current flowing from left to right.

Two nodes are located at midpoints of the left and right wires, which are joined by a horizontal cross wire. A vertical cross wire joins two nodes at midpoints of the horizontal cross wire and bottom wire. A 6 ohm resistor is located on the vertical cross wire. Two resistors with resistances of 4 ohms and 3 ohms are located on the left and right arms of the horizontal cross wire, and two resistors with resistances of 2 ohms and 5 ohms are located on the left and right arms of the bottom wire.

Current i sub x flows from left to right through the 4 ohm resistor on the left arm of the horizontal cross wire.

A circuit on top shows an independent voltage source, v sub s, with a resistor R in series, between nodes a and b. The circuit below shows an independent current source, i sub s, with a resistor R running parallel to it between nodes a and b. A bi-directional arrow between the two circuits indicates that these two sources can be interchanged.

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More information

VtSource™

A rectangular circuit has a 6V voltage source on the left wire. Two resistors of 30 ohms and 20 ohms are located parallel to the voltage source. Two serial resistors of 4 ohms and 6 ohms are located on the top wire, while a 10 ohm serial resistor is found on the bottom wire.

The power source and resistor on the right part of the circuit are highlighted in a shaded box. The earlier voltage source and serial resistor are replaced by an 8A independent current source on the right wire and a 5 ohm parallel resistor.

A rectangular circuit has a 6V voltage source on the left wire, along with a 30 ohm parallel resistor. Two serial resistors of 4 ohms and 6 ohms are located on the top wire, while a 10 ohm serial resistor is found on the bottom wire.

The power source and resistor on the right part of the circuit are highlighted in a shaded box. The earlier current source and parallel resistor from step 1 are replaced by a 32V independent voltage source on the right wire and a 4 ohm serial resistor on the top wire.

A rectangular circuit has a 6V voltage source on the left wire, along with a 30 ohm parallel resistor and a 4 ohm serial resistor on the top wire.

The power source and resistor on the right part of the circuit are highlighted in a shaded box. The earlier voltage source and serial resistor from step 2 are replaced by a 1.6A independent current source on the right wire and a 20 ohm parallel resistor.

A rectangular circuit has a 6V voltage source on the left wire, along with a 4 ohm serial resistor on the top wire.

The power source and resistor on the right part of the circuit are highlighted in a shaded box. The earlier current source and parallel resistor from step 3 are replaced by a 19.2V independent voltage source on the right wire and a 12 ohm serial resistor on the top wire.

A circuit on the left shows a voltage source, v sub s, with a serial resistor R and a parallel resistor R sub p between terminals a and b. The circuit on the right shows the same configuration, but without the parallel resistor R sub p. Both the circuits are equivalent, as indicated by the arrow in between.

The circuit on the left shows a current source, i sub s, with a serial resistor R sub s and a parallel resistor R between terminals a and b. The circuit on the right shows the same configuration but without the serial resistor R sub s. Both the circuits are equivalent, as indicated by the arrow in between.

A rectangular circuit has a 250V independent voltage source on the left wire. Three nodes are located on the top wire, equidistant from each other and from either end. Similarly, three nodes are located on the bottom wire. Three vertical cross wires are drawn between these three pairs of nodes.

The circuit has four parallel resistors with resistances of 125 ohms, 10 ohms, 100 ohms, and 15 ohms on the three cross wires and the right wire, respectively. Two serial resistors of 25 ohms and 5 ohms resistance are located on the middle and right arms of the top wire, respectively. An 8A independent current source is located on the middle cross wire, with current flowing from top to bottom. Voltage across the right cross wire is marked as v sub o.

A rectangular circuit has a 250V independent voltage source on the left wire. Two nodes are located on the top wire, equidistant from each other and from either end. Similarly, two nodes are located on the bottom wire. Two vertical cross wires are drawn between these two pairs of nodes.

The circuit has a 25 ohm serial resistor on the left arm of the top wire, and two parallel resistors with 100 ohm and 20 ohm resistances on the right vertical cross wire and right wire, respectively. An 8A independent current source is located on the left cross wire, with current flowing from top to bottom. Voltage across the right cross wire is marked as v sub 0.

A rectangular circuit has a 10A independent current source on the left wire, with current flowing from bottom to top. Three nodes are located on the top wire, equidistant from each other and from either end. Similarly, three nodes are located on the bottom wire. Three vertical cross wires are drawn between these three pairs of nodes.

The circuit has three parallel resistors with resistances of 25 ohms, 100 ohms, and 20 ohms on the first cross wire, third cross wire, and right wire respectively. An 8A independent current source is located on the middle cross wire, with current flowing from top to bottom. Voltage across the right cross wire is marked as v sub o.

The circuit has a 2A independent current source on the left wire, with current flowing from bottom to top, and a 10 ohm resistor on the right wire, with voltage marked as v sub o.

A rectangular circuit has a 120V independent voltage source with a 20 ohm resistor on the left wire, and a 8 ohm resistor on the right wire. Three nodes are located on the top wire, equidistant from each other and from either end. Similarly, three nodes are located on the bottom wire. Three vertical cross wires are drawn between these three pairs of nodes.

The left cross wire has a 60V independent voltage source with opposite polarity, and a 5 ohm resistor. The middle cross wire has a 36A independent current source, with current flowing from bottom to top. The right cross wire has a 6 ohm resistor. A serial resistor of 1.6 ohm resistance is located on the right arm of the top wire.

A fourth pair of nodes is located on the top and bottom wires, close to the right wire. Voltage across these two nodes is marked as v.

The circuit inside the box has two cross wires connecting the nodes on the top and bottom wires. The left cross wire has a 20 ohm resistor, while the right cross wire has a 3A independent current source, with current flowing from bottom to top. Voltage across the right cross wire is marked as v1. Two serial resistors of 5 ohm and 4 ohm resistance are located on the left arm and right arm of the top wire, respectively. A 25V independent voltage source is located on the left wire.

The circuit has two open terminals outside the box, marked as a and b, leading out of the top and bottom wires respectively. Voltage across the terminals is marked as v sub ab.

The circuit inside the box has two cross wires connecting nodes on the top and bottom wires. The left cross wire has a 20 ohm resistor, while the right cross wire has a 3A independent current source, with current flowing from bottom to top. Voltage across the right cross wire is marked as v1. Two serial resistors of 5 ohm and 4 ohm resistance are located on the left arm and right arm of the top wire, respectively. A 25V independent voltage source is located on the left wire.

The circuit has two open terminals outside the box, marked as a and b, leading out of the top and bottom wires, respectively. The terminals are short circuited with a wire. A current, marked as i sub sc flows through the short circuit from a to b.

The circuit has two open terminals, a and b, on the top and bottom wires respectively. The left wire has a 32V independent voltage source, while the top wire has an 8 ohm serial resistor.

The first diagram shows a circuit inside a box, which has two cross wires connecting the nodes on the top and bottom wires. The left cross wire has a 20 ohm resistor, while the right cross wire has a 3A independent current source, with current flowing from bottom to top. Two series resistors of 5 ohm and 4 ohm resistance are located on the left and right arms of the top wire, respectively. A 25V independent voltage source is located on the left wire. The circuit has two open terminals outside the box, marked as a and b, leading out of the top and bottom wires respectively.

The second diagram shows step 1, where power sources are transformed. Voltage source and series resistor on the left part of the circuit are replaced by a 5A current source on the left wire with current flowing from bottom to top, and a parallel resistor with 5 ohm resistance. Rest of the circuit on the right remains as it is.

In step 2, shown next, parallel sources and parallel resistors are combined. The two parallel current sources of 5A and 3A capacity are replaced by a single current source, measuring an 8A current on the left wire. The two parallel resistors of 20 ohm and 5 ohm resistances are replaced by a single parallel resistor of 4 ohm resistance. The series resistor of 4 ohm resistance remains on the top wire.

Source transformation is achieved in step 3, shown next, where series resistors are combined to produce a Thevenin equivalent circuit. The two resistors of 4 ohms resistance each are replaced by a single series resistor of 8 ohm resistance on the top wire. The current source is replaced by an independent voltage source, with 32 V voltage.

In the final step 4, another source transformation is achieved, producing a Norton equivalent circuit. The voltage source and series resistor are replaced by a 4A independent current source and an 8 ohm parallel resistor, with the terminals a and b remaining open ended outside the box.

The circuit shows two closed small circuits whose bottom wires are connected, with current i sub x flowing through the connecting wire from left to right. The circuit on the left has a 5V independent voltage source on the left wire, and a 3v dependent voltage source on the right wire, along with a 2k ohm series resistor on the top wire with current i flowing from left to right. The circuit on the right has a 20i dependent current source on the left wire, with current flowing from top to bottom, and a 25 ohm parallel resistor with a voltage v across its terminals.

Both the circuits are enclosed in a box, with two external open terminals, a and b, leading out of the top and bottom wires of the right circuit. Voltage between a and b is marked as v sub ab.

The circuit shows two closed small circuits with connected bottom wires inside a box, having two external terminals, a and b, which are short circuited. Current i sub sc flows in the short circuit from a to b.

The left circuit inside the box has a 5V independent voltage source on the left wire, and a 2k ohm series resistor on the top wire, with current i flowing from left to right. The circuit on the right has a 20i dependent current source on the left wire, with current flowing from top to bottom, and a 25 ohm parallel resistor.

A drawing shows a circuit inside a box with a 5V independent voltage source on the left wire, and a 100 ohm series resistor on the top wire, with two external open terminals, a and b, leading out of the top and bottom wires.

The circuit has two nodes on the left and right wires which are joined by a horizontal cross wire. A vertical cross wire joins two nodes at midpoints of the horizontal cross wire and bottom wire. The bottom arm of the left wire has a 72V independent voltage source, while the bottom arm of the right wire is open, leading to two terminals, a and b.

Two resistors with 5 ohm and 8 ohm resistances are located respectively on the left and right arms of the horizontal cross wire. A 20 ohm resistor is found on the vertical cross wire, and a 12 ohm resistor is located on the top wire.

A rectangular circuit has two nodes located on the top wire, equidistant from each other and from either end, and two similar nodes on the bottom wire. These pairs of nodes are joined by two vertical cross wires. An 8 ohm resistor and a 12 ohm resistor are located on the left and right cross wires, respectively, while a 2 ohm resistor and a 10 ohm resistor are located on the middle arms of the top and bottom wires, respectively.

The left wire of the circuit carries a 15A current source, with current flowing from bottom to top. The right wire is open, with two terminals, a and b, leading out of the top and bottom wires.

A rectangular circuit has two nodes located on the top wire, equidistant from each other and from either end, and two similar nodes on the bottom wire. These pairs of nodes are joined by two vertical cross wires. Two resistors of 12k ohm and 15k ohm resistance are located on the left and right arms of the top wire, respectively. The right cross wire carries a 60k ohm resistor.

The left wire carries a 36V independent voltage source with opposite polarities, while the left cross wire carries an 18mA current source, with current flowing from bottom to top. The right wire is open, with two terminals, A and B, leading out of the top and bottom wires. Voltage across these open terminals is marked as v sub AB.

A rectangular circuit has two nodes located on the top wire, equidistant from each other and from either end, and two similar nodes on the bottom wire. These pairs of nodes are joined by two vertical cross wires. Two resistors of 5 ohm and 4 ohm resistance are located on the left and right arms of the top wire, respectively. The left cross wire carries a 20 ohm resistor.

The left wire carries a 25V independent voltage source, while the right cross wire carries a 3A current source, with current flowing from bottom to top. The right wire is open, with two terminals, a and b, leading out of the top and bottom wires. Voltage across these open terminals is marked as v sub ab.

A rectangular circuit has a vertical cross wire connecting two nodes on the top and bottom wires. The cross wire carries a 20 ohm resistor, while the left and right arms of the top wire carry 5 ohm and 4 ohm resistors, respectively. The right wire is open, with two terminals, a and b, leading out of the top and bottom wires. Terminal resistance is marked as R sub ab.

A circuit shows two closed small circuits, whose bottom wires are connected. The circuit on the left has a 3v sub T dependent voltage source on the right wire, and a 2k ohm series resistor on the top wire, with current i flowing from left to right. The circuit on the right has a 20i dependent current source on the left wire, with current flowing from top to bottom, and a 25 ohm parallel resistor.

Both the circuits are enclosed in a box, with two external terminals leading out of the top and bottom wires of the right circuit. The two terminals are connected to an independent voltage source, v sub T. Current i sub T flows from right to left through the terminal on top.

The circuit has a vertical cross wire connecting nodes at the midpoints of the top and bottom wires. A dependent current source, v sub x over 6, is located on this cross wire, with current flowing from bottom to top. The circuit has a 30 ohm resistor on its right wire, a 20 ohm resistor on the left arm of the top wire, and a 10 ohm resistor on the left wire, with voltage v sub x marked across its terminals.

Two terminals, a and b, lead out of the top and bottom wires on right.

The circuit has a vertical cross wire connecting the nodes at the midpoints of the top and bottom wires. A dependent current source, v sub x over 6, is located on this cross wire, with current flowing from bottom to top. The circuit has a 30 ohm resistor on its right wire, a 20 ohm resistor on the left arm of the top wire, and a 10 ohm resistor on the left wire, with voltage v sub x marked across its terminals.

Two terminals, a and b, lead out of the top and bottom wires on right. These two terminals are short circuited and connected to an independent current source, i sub T, on right. Voltage across this source is marked as v sub T.

A rectangular circuit has two nodes, a and d, at the midpoints of its top and bottom wires, respectively. They are joined by a vertical cross wire. A horizontal cross wire joins node c on the vertical cross wire with node b on the left wire, with both the nodes located closer to the bottom wire. An independent voltage source, V sub 0, is located on the horizontal cross wire. An unknown current, i sub B, flows through this branch from left to right.

A second independent voltage source, V sub CC, is located on the right wire. Plus, a dependent current source with current Beta i sub B, flowing from top to bottom, is located on the top arm of the vertical cross wire.

The circuit has four resistors between the following nodes: R1 between a and b on the top arm of the left wire, R2 between b and d on the bottom arm of the left wire, R sub C between a and c on the top arm of the vertical cross wire, and R sub E between c and d on the bottom arm of the vertical cross wire. Currents i sub 1, i sub 2, and i sub E flow through R1, R2, and R sub E respectively, from top to bottom.

The circuit shows two small closed circuits, which are joined by two wires. The circuit on the left has an independent voltage source, V sub CC, on the left wire, and two resistors, R1 and R2, on the top and bottom arms of the right wire, separated by node b. Currents i sub 1 and i sub 2 flow through R1 and R2 respectively from top to bottom. A node a is marked at the top of the right wire, while another unnamed node is marked at the bottom.

The circuit on the right has an independent voltage source, V sub CC, on the right wire, two resistors R sub C and R sub E on the top and bottom arms of the left wire, separated by node c, and a dependent current source, Beta i sub B, with current flowing downwards, on the top arm of the left wire. A node a-dash is marked at the top of the left wire, while a node d is marked at the bottom. Current i sub E flows downwards through resistor R sub E.

Nodes b and c on the left and right circuits are joined by a wire, which carries an independent voltage source V sub 0, and current i sub B, flowing from left to right. Bottom nodes of both the circuits are also joined.

The circuit shows two small closed circuits, which are joined by two wires. The circuit on the left is modified and depicted inside a box. It has an independent voltage source, V sub Th, a series resistor, R sub Th, and a terminal b. The top and bottom wires, leading out of the left circuit, connect to terminals c and d respectively on the right circuit. The top wire connecting the two circuits carries an independent voltage source, V sub 0, and a current i sub B, flowing from left to right.

The circuit on the right has an independent voltage source, V sub CC, on the right wire, two resistors R sub C and R sub E on the top and bottom arms of the left wire, separated by node c, and a dependent current source, Beta i sub B, with current flowing downwards, on the top arm of the left wire. Node a is marked at the top of the left wire, while node d is marked at the bottom. Current i sub E flows downwards through resistor R sub E.

Nodes b and c on the left and right circuits are joined by a wire, which carries an independent voltage source V sub 0, and a current i sub B, flowing from left to right. Bottom nodes of both the circuits are also joined.

A rectangular circuit has a vertical cross wire connecting two nodes on the top and bottom wires. A 24V independent voltage source is located on the left wire, while a 4A independent current source is located on the cross wire, with current flowing from top to bottom. A 2 ohm resistor is located on the left arm of the top wire. An additional wire is drawn over the nodes on either side of this resistor. This wire carries a dependent current source, 3i sub x, which has current flowing from right to left.

An 8 ohm resistor is located on the right wire, with current i sub x flowing through it from top to bottom. Two open terminals leading out of the top and bottom wires on the right are labeled as a and b.

A rectangular circuit has two vertical cross wires connecting two pairs of nodes on the top and bottom wires. A 4A independent current source is located on the left cross wire, with current flowing from bottom to top, while a 160i sub delta dependent voltage source is located on the middle arm of the top wire.

The circuit shows a series resistor of 20 ohm resistance on the middle arm of the top wire, and three parallel resistors of 60 ohm, 80 ohm, and 40 ohm resistances on the left wire, right cross wire, and right wire, respectively. Current i sub delta runs through the 40 ohm resistor on the right wire, from top to bottom. Two nodes, a and b, lead out of the top and bottom wires on the right.

The diagram shows a box with two internal terminals, a and b, connected to a resistive network containing independent and dependent sources. The two terminals are short circuited outside, connecting to a resistor, R sub L.

The diagram shows a box with two internal terminals, a and b, connected to a network containing an independent voltage source, V sub Th, and a series resistor, R sub Th. The two terminals are short circuited outside, connecting to a resistor, R sub L, through which current i flows from top to bottom.

The diagram shows a box with two external terminals, a and b, connected to a network containing a 360V independent voltage source, a 30 ohm series resistor, and a 150 ohm parallel resistor. The two terminals are short circuited, connecting to a resistor, R sub L.

The diagram shows a box with two external terminals, a and b, connected to a network containing a 360V independent voltage source and a 25 ohm series resistor. The two terminals are short circuited, connecting to a resistor, R sub L.

A rectangular circuit has a vertical cross wire connecting two nodes on the top and bottom wires. This cross wire carries a 20V independent voltage source at the bottom and a 4 ohm resistor on the top, with the voltage across it marked as v sub phi. Two resistors of 4 ohm resistance each are located on the left and right arms of the top wire. A 100V independent voltage source is located on the left wire of the circuit.

An additional wire is drawn across two nodes on the top wire, on either side of the two 4 ohm resistors. This additional wire carries a dependent voltage source, v sub phi, with opposite polarity and a 4 ohm resistor to its right.

Two terminals, a and b, are marked on the top and bottom wires, close to the right wire. A resistor R is located on the right wire.

A rectangular circuit has two nodes on the top wire, equidistant from each other and from either end, and two similar nodes on the bottom wire. The two pairs of nodes are connected by two vertical cross wires. A 3 ohm resistor, carrying a downward flowing current i sub 2, and a 4 ohm resistor, carrying a downward flowing current i sub 4, are located on the left and right cross wires, respectively. A 6 ohm resistor, carrying a rightward flowing current i sub 1, and a 2 ohm resistor, carrying a rightward flowing current i sub 3, are located on the left and middle arms of the top wire, respectively.

The circuit shows a 120V independent voltage source on the left wire, and a 12A independent current source on the right wire, with current flowing from top to bottom.

A rectangular circuit has two nodes on the top wire, equidistant from each other and from either end, and two similar nodes on the bottom wire. The two pairs of nodes are connected by two vertical cross wires. A 3 ohm resistor, carrying a downward flowing current i-dash sub 2, and a 4 ohm resistor, carrying a downward flowing current i-dash sub 4, are located on the left and right cross wires, respectively. A 6 ohm resistor, carrying a rightward flowing current i-dash sub 1, and a 2 ohm resistor, carrying a rightward flowing current i-dash sub 3, are located on the left and middle arms of the top wire, respectively.

The circuit shows a 120V independent voltage source on the left wire. The top node of the left cross wire has a voltage marked as v1, while the bottom node is marked as a reference node. The independent current source on the right wire is removed and terminals are opened.

A rectangular circuit has two nodes on the top wire, equidistant from each other and from either end, and two similar nodes on the bottom wire. The two pairs of nodes are connected by two vertical cross wires. A 3 ohm resistor, carrying a downward flowing current i-double dash sub 2, and a 4 ohm resistor, carrying a downward flowing current i-double dash sub 4, are located on the left and right cross wires, respectively. A 6 ohm resistor, carrying a rightward flowing current i-double dash sub 1, and a 2 ohm resistor, carrying a rightward flowing current i-double dash sub 3, are located on the left and middle arms of the top wire, respectively.

The circuit shows a 12A independent current source on the right wire, with current flowing from top to bottom. Voltage source on the left wire is removed.

A rectangular circuit has two nodes on the top wire, equidistant from each other and from either end, and two similar nodes on the bottom wire. The two pairs of nodes are connected by two vertical cross wires. A 3 ohm and a 4 ohm resistor are located on the left and right cross wires, respectively. A 6 ohm resistor and a 2 ohm resistor are located on the left and middle arms of the top wire, respectively.

The circuit shows a 12A independent current source on the right wire, with current flowing from top to bottom. Voltage source on the left wire is removed. Voltage v sub 3 is marked across the nodes of the left cross wire, and voltage v sub 4 is marked across the nodes of the right cross wire.

A rectangular circuit has two nodes on the top wire, equidistant from each other and from either end, and two similar nodes on the bottom wire. The two pairs of nodes are connected by two vertical cross wires. A 20 ohm resistor with voltage v sub 0, and a 10 ohm resistor with voltage v sub delta, are located on the left and right cross wires, respectively. A 5 ohm resistor, carrying a rightward current i sub delta, is located on the left arm of the top wire.

The circuit shows a 10V independent voltage source on the left wire, and a 5A independent current source on the right wire, with current flowing from bottom to top. A dependent current source, 0.4 v sub delta, is located on the middle arm of the top wire, with current flowing from right to left. A dependent voltage source, 2i sub delta, with opposite polarity, is located on the middle arm of the bottom wire.

A rectangular circuit has two nodes on the top wire, equidistant from each other and from either end, and two similar nodes on the bottom wire. The two pairs of nodes are connected by two vertical cross wires. A 20 ohm resistor with voltage v sub 0-dash, and a 10 ohm resistor with voltage v sub delta-dash, are located on the left and right cross wires, respectively. A 5 ohm resistor, carrying a rightward current i sub delta-dash, is located on the left arm of the top wire.

The circuit shows a 10V independent voltage source on the left wire. A dependent current source, 0.4 v sub delta-dash, is located on the middle arm of the top wire, with current flowing from right to left. A dependent voltage source, 2i sub delta-dash, with opposite polarity, is located on the middle arm of the bottom wire.

A 5A current source on the right wire is removed, and the two terminals on the top and bottom wires are left open.

A rectangular circuit has two nodes on the top wire, equidistant from each other and from either end, and two similar nodes on the bottom wire. The two pairs of nodes are connected by two vertical cross wires. A 20 ohm resistor with voltage v sub 0-double dash, and a 10 ohm resistor with voltage v sub delta-double dash, are located on the left and right cross wires, respectively. A 5 ohm resistor, carrying a rightward current i sub delta-double dash, is located on the left arm of the top wire.

The circuit shows a 5A independent current source on the right wire, with current flowing from bottom to top. A dependent current source, 0.4 v sub delta-double dash, is located on the middle arm of the top wire, with current flowing from right to left. A dependent voltage source, 2i sub delta-double dash, with opposite polarity, is located on the middle arm of the bottom wire. Voltage source on the left wire is removed.

A rectangular circuit has three vertical cross wires connecting three pairs of nodes between the top and bottom wires. Three resistors with resistances of R1, R3, and R4 are located on these cross wires, from left to right. Voltage across R1 is marked as v1, while voltage across R3 is marked as v2. A fourth resistor with resistance R2 is located between the first and second nodes on the top wire.

An independent current source, I sub g1, is located on the left wire, with current flowing from top to bottom. A second independent current source, I sub g2, is located on the right wire, with current flowing from bottom to top.

A table displays data for Element name, Element value, Element sensitivity (in Volts per unit), and Normalized sensitivity (Volts per percent). First part of the table displays DC sensitivities of Node voltage V1, as follows:

R1: 25; 0.5833; 0.1458

R2: 5; minus 5.417; minus 0.2708

R3: 50; 0.45; 0.225

R4: 75; 0.2; 0.15

IG1: 12; minus 14.58; minus 1.75

IG2: 16; 12.5; 2

Second part displays sensitivities of output V2, as follows:

R1: 25; 0.5; 0.125

R2: 5; 6.5; 0.325

R3: 50; 0.54; 0.27

R4: 75; 0.24; 0.18

IG1: 12; minus 12.5; minus 1.5

IG2: 16; 15; 2.4

Step heading, Node-voltage method step, and Mesh-Current method step are outlined as follows:

Step 1 - Identify nodes/meshes:

(node-voltage) Identify the essential nodes by circling them on the circuit diagram

(mesh current) Identify the meshes by drawing directed arrows inside each mesh

Step 2 - Label node voltages/mesh currents. Recognize special cases:

(node-voltage) Pick and label a reference node; then label the remaining essential node voltages

- If a voltage source is the only component in a branch connecting the reference node and another essential node, label the essential node with the value of the voltage source

- If a voltage source is the only component in a branch connecting two nonreference essential nodes, create a supernode that includes the voltage source and the two nodes on either side

(mesh current) Label each mesh current

- If a current source is in a single mesh, label the mesh current with the value of the current source

- If a current source is shared by two adjacent meshes, create a supermesh by combining the two adjacent meshes and temporarily eliminating the branch that contains the current source.

Step 3 – Write the equation:

(node-voltage) Write the following equations:

- A KCL equation for any supernodes

- A KCL equation for any remaining essential nodes where the voltage is unknown
- A constraint equation for each dependent source that defines the controlling variable for the dependent source in terms of the node voltages
- A constraint equation for each supernode that equates the difference between the two node voltages in the supernode to the voltage source in the supernode

(mesh current) Write the following equations:

- A KVL equation for any supermeshes
- A KVL equation for any remaining meshes where the current is unknown
- A constraint equation for each dependent source that defines the controlling variable for the dependent source in terms of the mesh currents
- A constraint equation for each supermesh that equates the difference between the two mesh currents in the supermesh to the current source eliminated to form the supermesh

Step 4 – Solve the equations:

(node-voltage) Solve the equations to find the node voltages

(mesh current) Solve the equations to find the mesh currents

Step 5 - Solve for other unknowns

(node-voltage) Use the node voltage values to find any unknown voltages, currents, or powers

(mesh current) Use the mesh current values to find any unknown voltages, currents, or powers

A rectangular circuit shows three nodes on its top wire, which are equidistant from each other and from either end, and three similar nodes on its bottom wire. Three vertical cross wires connect these three pairs of nodes across the top and bottom wires. Resistors R1, R2, and R3 are located on the left, middle, and right cross wires, while resistors R6, R7, and R8 are located on the second, third, and fourth arms of the top wire, respectively.

An additional wire connects the first and third nodes on the top wire, forming another rectangular circuit on top. In this new circuit, a horizontal cross wire connects the midpoints of the left and right wires. Two resistors, R4 and R5, are located on this horizontal cross wire, separated by a node in between. An dependent current source, 4i sub x, is located on the top wire of this additional circuit, with current flowing from left to right.

The circuit is powered by a 10V independent voltage source on the right wire and an 8A independent current source on the left wire, with current flowing through it from bottom to top. An unknown current, i sub x, flows from top to bottom through resistor R2 on the middle cross wire.

A rectangular circuit has a vertical cross wire connecting two nodes on the top and bottom wires. A horizontal cross wire connects two nodes at the midpoints of the vertical cross wire and right wire. Resistors R2 and R3 are located on the top and bottom arms of the vertical cross wire, respectively. Resistors R6 and R7 are located on the top and bottom arms of the right wire, respectively. R1 and R4 are found on the left and right arms of the top wire, while R5 is located on the right arm of the bottom wire.

The circuit is powered by an independent voltage source, v sub s, located on the left wire.

A vertical cross wire connects two nodes on the top and bottom wires of a rectangular circuit. A 25 ohm parallel resistor is located on this cross wire, with voltage v sub 0 marked across its nodes. A 5 ohm resistor and a 120 ohm resistor are located on the left wire and left arm of the top wire, respectively. The circuit is powered by a 25V independent voltage source on the left wire and a 40mA independent current source on the right wire, with current flowing from top to bottom.

A vertical cross wire connects two nodes on the top and bottom wires of a rectangular circuit. A 50 ohm parallel resistor is located on this cross wire, while a 1 ohm resistor and a 4 ohm resistor are located on the right arms of the top and bottom wires, respectively. The circuit is powered by a 45V independent voltage source on the right wire and 2A independent current source on the left wire, with current flowing from bottom to top.

A rectangular circuit shows three nodes on its top wire, which are equidistant from each other and from either end, and three similar nodes on its bottom wire. Three vertical cross wires connect these three pairs of nodes across the top and bottom wires. A 40 ohm resistor, 80 ohm resistor, and 120 ohm resistor are located on the left, middle, and right cross wires, respectively. Plus, an 8 ohm resistor is located between the first and second nodes on the top wire.

The circuit is powered by a 6A independent current source on the left wire, with current flowing from bottom to top, and a 1A independent current source on the right wire, with current flowing through it from top to bottom.

Two voltages, v1 and v2, are marked across the left and right cross wires, respectively, with the same polarity as the independent voltage source.

A rectangular circuit shows two nodes on its top wire, which are equidistant from each other and from either end, and two similar nodes on its bottom wire. Two vertical cross wires connect these two pairs of nodes across the top and bottom wires. A 10 ohm resistor is located on the left cross wire. A 4 ohm resistor and an 80 ohm resistor are located on the left and middle arms of the top wire, respectively. Plus, a 5 ohm resistor is located on the right wire.

The circuit is powered by a 144V independent voltage source on the left wire and a 3A independent current source on the right cross wire, with current flowing through it from bottom to top.

Two voltages, v1 and v2, are marked across the left and right cross wires, respectively, with the same polarity as the independent voltage source.

A rectangular circuit shows two nodes on its top wire, which are equidistant from each other and from either end, and two similar nodes on its bottom wire. Two vertical cross wires connect these two pairs of nodes across the top and bottom wires. A 6 ohm resistor and a 3 ohm resistor are located on the left cross wire and right wire, respectively. Plus, 4 ohm, 1 ohm, and 2 ohm resistors are located on the left, middle, and right arms of the top wire, respectively.

The circuit is powered by a 44V independent voltage source on the left wire and a 2V independent voltage source on the right wire, with an opposite polarity.

Three unknown currents, i sub a, i sub c, and i sub e, flow from left to right through the left, middle, and right arms of the top wire, respectively. Plus, two currents, i sub b and i sub d, flow from top to bottom through the left and right cross wires, respectively.

A rectangular circuit shows two nodes on its top wire, which are equidistant from each other and from either end, and two similar nodes on its bottom wire. Two vertical cross wires connect these two pairs of nodes across the top and bottom wires. A 40 ohm resistor and a 4 ohm resistor are located on the left cross wire and right wire, respectively. A 3 ohm resistor and a 2 ohm resistor are located on the left and middle arms of the top wire, respectively. Plus, a 1 ohm resistor and a 2 ohm resistor are located on the left and middle arms of the bottom wire, respectively.

The circuit is powered by a 40V independent voltage source on the left wire and a 28A independent current source on the right wire, with current flowing through it from bottom to top.

Two voltages, v1 and v2, are marked across the left and right cross wires, respectively, with the same polarity as the independent voltage source. A third voltage, v3, is marked across the middle arm of the bottom wire, with an opposite polarity.

A rectangular circuit has a vertical cross wire connecting two nodes on its top and bottom wires. A horizontal cross wire connects two nodes at midpoints of the vertical cross wire and left wire. A 2 ohm resistor is located on the horizontal cross wire, while a 6 ohm resistor and a 12 ohm resistor are located on the top and bottom arms of the vertical cross wire, respectively. A 1 ohm resistor is located on the left arm of the top wire, while another 1 ohm resistor is located on the left arm of the bottom wire. A 24 ohm resistor is located on the right wire.

The circuit is powered by two independent voltage sources, with the same polarity voltage of 125V, located on the top and bottom arms of the left wire.

Two currents, i4 and i5, flow from top to bottom through the top and bottom arms of the vertical cross wire, respectively. Two more currents, i2 and i3, flow from right to left through the horizontal cross wire and left arm of bottom wire, respectively. Current i1 flows from left to right on the left arm of the top wire, while current i6 flows from top to bottom on the right wire.

A rectangular circuit shows two nodes on its top wire, which are equidistant from each other and from either end, and two similar nodes on its bottom wire. Two vertical cross wires connect these two pairs of nodes across the top and bottom wires. A 25 ohm resistor and 40 ohm resistor are located on the left and right cross wires, respectively. A 12 ohm resistor and a 20 ohm resistor are located on the left and middle arms of the top wire, respectively. Plus, a 40 ohm resistor is found on the middle arm of the bottom wire.

The circuit is powered by a 40V independent voltage source on the left wire with opposite polarity, and a 7.5A independent current source on the right wire, with current flowing through it from bottom to top. An additional wire connects the two nodes on the top wire. A 5A independent current source is located on this wire, with current flowing from left to right.

A rectangular circuit shows three nodes on its top wire, which are equidistant from each other and from either end, and three similar nodes on its bottom wire. Three vertical cross wires connect these three pairs of nodes across the top and bottom wires. Four resistors with resistance R are located on these three cross wires and on the left wire.

The circuit is powered by four independent voltage sources, v1, v2, v3, and v sub n, which are located on the left wire, left cross wire, middle cross wire, and right cross wire, respectively. All four voltage sources have similar polarities.

The right wire of the circuit is open and connected to two terminals. The same polarity voltage across these terminals is marked as v sub 0.

A vertical cross wire connects two nodes on the top and bottom wires of a rectangular circuit. A 100 ohm parallel resistor is located on the cross wire, a 20 ohm resistor is located on the right arm of the bottom wire, and two resistors of 10 ohm and 30 ohm resistance are located on the left and right arms of the top wire.

The circuit is powered by a 160V independent voltage source on the left wire and a dependent voltage source with the same polarity on the right wire, whose voltage is marked as 150i sub sigma.

A rectangular circuit shows two nodes on its top wire, which are equidistant from each other and from either end, and two similar nodes on its bottom wire. Two vertical cross wires connect these two pairs of nodes across the top and bottom wires. A 1k ohm resistor and a 200 ohm resistor are located on the left cross wire and right wire, respectively. A 500 ohm resistor and a 2k ohm resistor are located on the left and middle arms of the top wire, respectively.

The circuit is powered by a 50V independent voltage source on the left wire, and a dependent current source on the right cross wire, with a current v sub delta over 750 flowing through it from bottom to top.

Voltage across the 1k ohm resistor on the left cross wire is marked as v sub delta, and voltage across the 200 ohm resistor on the right wire is marked as v sub 0.

A rectangular circuit shows three nodes on its top wire, which are equidistant from each other and from either end, and three similar nodes on its bottom wire. Three vertical cross wires connect these three pairs of nodes across the top and bottom wires. A 1k ohm resistor, a 4k ohm resistor, and a 2k ohm resistor are located on the left, middle, and right cross wires, respectively. Plus, a 1250 ohm resistor is located between the first and second nodes on the top wire, while a 200 ohm resistor is found on the right arm of the top wire.

The circuit is powered by a 20mA independent current source on the left wire, with current flowing from top to bottom, and a dependent voltage source, 2500i sub delta, on the right wire.

Current i sub delta flows from right to left through the 1250 ohm resistor on the top wire.

A rectangular circuit shows two nodes on its top wire, which are equidistant from each other and from either end, and two similar nodes on its bottom wire. Two vertical cross wires connect these two pairs of nodes across the top and bottom wires. A 100 ohm resistor and a 5 ohm resistor are located on the left and right cross wires, respectively. Plus, a 25 ohm resistor is located on the right arm of the top wire.

The circuit is powered by a 40V independent voltage source on the right wire, a 450mA independent current source on the left wire, with current flowing from top to bottom, and a 6.25i sub delta dependent voltage source on the right cross wire.

Voltage across the 100 ohm resistor on the left cross wire is marked as v sub 0. Also, Current i sub delta flows from right to left through the 25 ohm resistor on the right arm of the top wire.

A rectangular circuit shows three nodes on its top wire which are equidistant from each other and from either end, and three similar nodes on its bottom wire. Three vertical cross wires connect these three pairs of nodes across the top and bottom wires. A 20 ohm resistor, a 40 ohm resistor, and a 5 ohm resistor are located on the left, middle, and right cross wires, respectively. Plus, a 5 ohm resistor, a 10 ohm resistor, and a 4 ohm resistor are located on the second, third, and fourth arms of the top wire.

The circuit is powered by a 96V independent voltage source on the right wire and a 5i sub 0 dependent current source on the left wire, with current flowing from bottom to top.

Current i sub 0 flows from top to bottom through the 40 ohm resistor on the middle cross wire of the circuit. Voltages v1, v2, and v3 are marked across the left, middle, and right cross wires, respectively.

A rectangular circuit shows two nodes on its top wire, which are equidistant from each other and from either end, and two similar nodes on its bottom wire. Two vertical cross wires connect these two pairs of nodes across the top and bottom wires. A 50 ohm resistor is located on the left cross wire. A 5 ohm resistor and a 10 ohm resistor are located on the left and middle arms of the top wire, respectively. An additional wire connects the right node and left extreme node on the top wire. An 8 ohm resistor is located on this wire.

The circuit is powered by a 40V independent voltage source on the left wire and a 10A independent current source on the right cross wire, with current flowing through it from bottom to top.

Voltage across the 5 ohm resistor on the left arm of the top wire is marked as v sub 0, with a polarity that's opposite to that of the independent voltage source.

A rectangular circuit shows two nodes on its top wire, which are equidistant from each other and from either end, and two similar nodes on its bottom wire. Two vertical cross wires connect these two pairs of nodes across the top and bottom wires. A 500 ohm resistor and 4k ohm resistor are located on the left and right cross wires, respectively. Plus, a 5k ohm resistor is located on the left arm of the top wire. An additional wire connects the left node and right extreme node on the top wire. A 1k ohm resistor is located on this wire.

The circuit is powered by a 80V independent voltage source on the right wire, a 30V independent voltage source with opposite polarity on the left wire, and a 10 mA independent current source on the middle arm of the top wire, with current flowing from left to right.

An unknown current, i1, flows from left to right through the 5k ohm resistor on the left arm of the top wire. Two currents, i2 and i3, flow from top to bottom through the left cross wire and the right wire, respectively.

A rectangular circuit has a horizontal cross wire connecting two nodes on the left and right wires. A vertical cross wire connects two nodes at the midpoints of the horizontal cross wire and bottom wire. A 3 ohm resistor is located on the vertical cross wire with voltage v sub 0 marked across its nodes, while a 1 ohm resistor is located on the right arm of the horizontal cross wire. A 2 ohm resistor is located on the top wire, with voltage v sub x marked across its nodes.

The circuit is powered by a 4V independent voltage source at the bottom arm of the left wire and a 7A independent current source on the left arm of the horizontal cross wire, with current flowing from right to left. A dependent current source is located on the bottom arm of right wire, with current 2v sub x flowing from bottom to top.

A rectangular circuit has a vertical cross wire connecting two nodes on the top and bottom wires. Two nodes are located on this cross wire, equidistant from each other and from either end. Two similar nodes are located on the right wire. Two horizontal cross wires connect these two pairs of nodes on the vertical cross wire and the right wire.

Three resistors with resistance of 1 ohm each are located on the top, middle, and bottom arms of the vertical cross wire. Four resistors with resistance of 1 ohm each are located on the two horizontal cross wires, right arm of the top wire, and right arm of the bottom wire. Plus, a 5 ohm resistor, a 2 ohm resistor, and a 5 ohm resistor are located on the top, middle, and bottom arms of the right wire.

The circuit is powered by a 230V independent voltage source located on the left wire.

A rectangular circuit has a 20V independent voltage source on its left wire, and a diamond shaped smaller circuit across two nodes on its right wire. The diamond shaped circuit has four resistors on its outer wires, starting from top left and moving clockwise, with resistances of 2k ohm, 5k ohm, 1k ohm, and 30k ohm, respectively. A horizontal cross wire connects the left and right nodes of the diamond circuit, carrying a 5k ohm resistor. A current i sub 0 flows from left to right on this horizontal cross wire.

A rectangular circuit shows two nodes on its top wire, which are equidistant from each other and from either end, and two similar nodes on its bottom wire. Two vertical cross wires connect these two pairs of nodes across the top and bottom wires. A 50 ohm resistor and a 150 ohm resistor are located on the left and right cross wires, respectively. Plus, a 20 ohm resistor and a 55 ohm resistor are located on the right arm of the top wire and right wire, respectively.

The circuit is powered by a 25V independent voltage source located on the middle arm of the top wire and a 2A independent current source on the left wire, with current flowing from top to bottom.

Nodes across the left cross wire are marked as b and a on the top and bottom wires, respectively. Voltage across these nodes is marked as v sub 0.

A rectangular circuit shows three nodes on its top wire, which are equidistant from each other and from either end, and three similar nodes on its bottom wire. Three vertical cross wires connect these three pairs of nodes across the top and bottom wires. An 80 ohm resistor, a 60 ohm resistor, and a 20 ohm resistor are located on the left, middle, and right cross wires, respectively. Plus, a 10 ohm resistor is found on the second arm of the top wire from left, while a 30 ohm resistor is found on the right wire.

The circuit is powered by a 100V independent voltage source on the left wire, and a 4v sub delta dependent voltage source on the third arm from left of the top wire, with the same polarity as the independent voltage source.

Voltages v sub 0 and v sub delta are marked on the 10 ohm resistor on the top wire and the 30 ohm resistor on the right wire, respectively.

A rectangular circuit shows three nodes on its top wire, which are equidistant from each other and from either end, and three similar nodes on its bottom wire. Three vertical cross wires connect these three pairs of nodes across the top and bottom wires. A 40 ohm resistor and an 80 ohm resistor are located on the middle and right cross wires, respectively. A 20 ohm resistor is found on the left wire. Plus, a 2 ohm resistor, a 1 ohm resistor, and a 4 ohm resistor are located on the first three arms of the top wire.

The circuit is powered by a 20V independent voltage source on the left cross wire. A dependent current source, with current 3.125v sub delta, flowing from top to bottom, is located on the right wire. An additional wire connects the third node and extreme left node on the top wire. This wire carries a dependent voltage source, with voltage of 35i sub phi.

An unknown voltage, v sub delta, is marked across the 1 ohm resistor on the second arm of the top wire, with the same polarity voltage as that of the independent voltage source. Current i sub phi flows from top to bottom through the 40 ohm resistor on the middle cross wire.

A rectangular circuit has a horizontal cross wire connecting two nodes on the left and right wires. Two nodes are located on the horizontal cross wire, equidistant from each other and from either end. Two similar nodes are located on the bottom wire. Two vertical cross wires connect these two pairs of nodes. The two nodes on the horizontal cross wire are marked as 1 and 2. The left node on the bottom wire is marked as a reference node.

A 4.25k ohm resistor is located on the right vertical cross wire, while a 2.5k ohm resistor is located on the bottom arm of the left wire. Three resistors of 1k ohm, 500 ohm, and 500 ohm resistance are located on the left, middle, and right arms of the horizontal cross wire, respectively.

The circuit is powered by a 75V independent voltage source on the bottom arm of the right wire. A dependent voltage source with voltage of 1250 i sub phi is located on the top wire. A dependent current source, with current of 12 multiplied by 10 raised to minus 3v sub delta, is located on the left cross wire, with current flowing from bottom to top.

An unknown current, i sub phi, flows from left to right through the middle arm of the horizontal cross wire. An unknown voltage, v sub delta, is marked across the right arm of the horizontal cross wire, with the same polarity as the independent voltage source.

A vertical cross wire connects two nodes on the top and bottom wires of a rectangular circuit. A 200 ohm parallel resistor is located on this cross wire. Two resistors of 75 ohm and 150 ohm resistance are located on the left and right arms of the top wire, while two resistors of 125 ohm and 250 ohm resistance are located on the left and right arms of the bottom wire.

The circuit is powered by an 80V independent voltage source on the left wire and a 140V independent voltage source on the right wire.

Three currents flow through three branches of the circuit as follows: i sub a flows from left to right on the left branch of the top wire, i sub c flows from right to left on the right branch of the top wire, and i sub b flows from top to bottom on the cross wire.

A rectangular circuit has a horizontal cross wire connecting two nodes on the left and right wires. A vertical cross wire connects the two nodes at the midpoints of the horizontal cross wire and bottom wire. A 2 ohm resistor is located on this vertical cross wire, while a 1 ohm resistor and a 3 ohm resistor are located on the left and right arms of the horizontal cross wire, respectively. The bottom wire shows a 4 ohm resistor and a 5 ohm resistor on its left and right arms.

The circuit is powered by three independent voltage sources with the same polarity, located on the bottom arm of the left wire, vertical cross wire, and bottom arm of right wire, with respective voltages marked as 230V, 115V, and 460V.

A vertical cross wire connects two nodes on the top and bottom wires of a rectangular circuit. A 5 ohm parallel resistor is located on this cross wire. Two resistors of 4 ohm and 8 ohm resistance are located on the left and right arms of the top wire, while two resistors of 6 ohm and 15 ohm resistance are located on the left and right arms of the bottom wire.

The circuit is powered by a 65V independent voltage source on the left wire and a dependent voltage source on the right wire, whose voltage is marked as 3v sub delta.

Voltage across the 4 ohm resistor on the left arm of the top wire is marked as v sub delta.

A rectangular circuit has a horizontal cross wire connecting two nodes on the left and right wires. A vertical cross wire connects two nodes at the midpoints of the horizontal cross wire and bottom wire. A 20 ohm resistor is located on this vertical cross wire, with current i sub delta flowing from top to bottom through it. A 3 ohm resistor and a 5 ohm resistor are located on the left and right arms of the horizontal cross wire, respectively. The bottom wire shows a 7 ohm resistor and a 2 ohm resistor on its left and right arms.

The circuit is powered by a 30V independent voltage source at the bottom arm of the left wire, and a 30V independent voltage source with opposite polarity at the bottom arm of the right wire. A dependent voltage source is located on the top wire, with voltage of 53i sub delta.

A rectangular circuit has a vertical cross wire connecting two nodes on the top and bottom wires. A horizontal cross wire connects two nodes at the midpoints of the vertical cross wire and left wire. A 10 ohm resistor is located on this horizontal cross wire, while a 15 ohm resistor and a 50 ohm resistor are located on the top and bottom arms of the vertical cross wire, respectively. A 5 ohm resistor is located on the left arm of the top wire, while a 25 ohm resistor is located on the right wire.

The circuit is powered by a 660V independent voltage source on the top arm of the left wire and a 20i sub phi dependent voltage source, with same polarity, on the bottom arm of the left wire.

An unknown current, i sub phi, flows from top to bottom through the 50 ohm resistor on the bottom arm of the vertical cross wire.

The circuit on left has a vertical cross wire connecting nodes at the midpoints of the top and bottom wires. A 16 ohm resistor is located on this cross wire, with an unknown voltage, v sub 0, marked across the nodes. Two resistors of 2 ohm and 12 ohm resistance are located on the left and right arms of the top wire.

The circuit is powered by a 10V independent voltage source on the left wire, and a dependent voltage source on the right wire, with the same polarity voltage of 4i sub delta.

The circuit on the right has a 5 ohm resistor on the top wire and a 3 ohm resistor on the right wire, carrying an unknown current of i sub delta, flowing from top to bottom. A 4V independent voltage source is located on the left wire of this circuit.

The two circuits are connected to each other at their bottom nodes.

A rectangular circuit has a horizontal cross wire connecting two nodes on the left and right wires. A vertical cross wire connects two nodes at the midpoints of the horizontal cross wire and bottom wire. A 30 ohm resistor is located on this vertical cross wire, while a 38 ohm resistor and a 6 ohm resistor are located on the left and right arms of the horizontal cross wire, respectively. The bottom wire shows a 12 ohm resistor and a 40 ohm resistor on its left and right arms.

The circuit is powered by a 5V independent voltage source, with opposite polarity, at the bottom arm of the left wire and a 67V independent voltage source at the bottom arm of the right wire. A 5A independent current source is located on the top wire, with the current moving from left to right.

A vertical cross wire connects two nodes on the top and bottom wires of a rectangular circuit. A 5.4k ohm parallel resistor is located on this cross wire. Two resistors of 10k ohm and 1k ohm resistance are located on the left and right arms of the top wire, while a 2.7k ohm resistor is located on the right arm of the bottom wire.

The circuit is powered by a 5mA independent current source on the left wire, with current flowing from bottom to top, and a dependent voltage source with opposite polarity on the right wire, whose voltage is marked as 150i sub delta.

Current i sub delta flows from top to bottom through the 5.4k ohm resistor on the cross wire.

A rectangular circuit shows two nodes on its top wire, which are equidistant from each other and from either end, and two similar nodes on its bottom wire. Two vertical cross wires connect these two pairs of nodes across the top and bottom wires. A 4 ohm resistor and a 20 ohm resistor are located on the left and right cross wires, respectively. Plus, a 2 ohm resistor and a 5 ohm resistor are located on the left and middle arms of the top wire, respectively.

The circuit is powered by a 50V independent voltage source on the left wire, a dependent voltage source, 9i sub delta with the same polarity, on the left cross wire, and a dependent current source, 1.7v sub delta, on right wire, with current flowing from bottom to top.

Current i sub delta flows from left to right through the 5 ohm resistor on the middle arm of the top wire. Voltage across the 2 ohm resistor on the left arm of the top wire is marked as v sub delta.

A rectangular circuit has a horizontal cross wire connecting two nodes on the left and right wires. A vertical cross wire connects two nodes at the midpoints of the horizontal cross wire and bottom wire. A 20 ohm resistor is located on this vertical cross wire, while a 5 ohm resistor and a 4 ohm resistor are located on the left and right arms of the horizontal cross wire, respectively. A 1 ohm resistor is found on the top wire, with a rightward current, i sub delta, flowing through it.

The circuit is powered by a 20A independent current source at the bottom arm of the left wire, with current flowing from bottom to top, and a dependent voltage source with voltage of 6.5 i sub delta at the bottom arm of the right wire.

A vertical cross wire connects two nodes on the top and bottom wires of a rectangular circuit. Two resistors of 4 ohm and 9 ohm resistance are located on the left and right arms of the top wire, while two more resistors of 1 ohm and 6 ohm resistance are located on the left and right arms of the bottom wire.

The circuit is powered by a 20V independent voltage source on the left wire, a 90V independent voltage source with opposite polarity on the right wire, and a 6A independent current source on the cross wire, with current flowing from top to bottom.

A rectangular circuit has a vertical cross wire connecting two nodes on the top and bottom wires. A horizontal cross wire connects two nodes at the midpoints of the vertical cross wire and left wire. A 5 ohm resistor is located on the horizontal cross wire, while a 10 ohm resistor is located on the bottom arm of the vertical cross wire. A 2 ohm resistor is located on the top arm of the left wire, a 20 ohm resistor is located on the right wire, and a 10 ohm resistor is found on the left arm of the bottom wire.

The circuit is powered by a 100V independent voltage source on the bottom arm of the left wire and a dependent current source on the top arm of the vertical cross wire, with a 1.2 i sub d current flowing from bottom to top.

Five unknown currents flow through different branches of the circuit as follows: i sub a and i sub b flow from bottom to top on the top arm and bottom arm of the left wire, respectively; i sub c flows from top to bottom on the right wire; i sub e flows from top to bottom on the bottom arm of the vertical cross wire; and i sub d flows from left to right on the horizontal cross wire.

A rectangular circuit has a horizontal cross wire connecting two nodes on the left and right wires. A vertical cross wire connects two nodes at the midpoints of the horizontal cross wire and bottom wire. A 5 ohm resistor and a 10 ohm resistor are located on the left and right arms of the horizontal cross wire, respectively. A 40 ohm resistor is located on the lower arm of the left wire.

An additional wire is drawn between the nodes across the 40 ohm resistor on the lower arm of the left wire. On this additional wire, a 19A independent current source is located, with current flowing through it from bottom to top. A 240V independent voltage source with reverse polarity is located on the lower arm of the right wire. Plus, a dependent voltage source with reverse polarity voltage of 4i sub d is located on the top wire.

Five unknown currents flow through different branches of the circuit as follows: i sub a and i sub d flow from top to bottom on the lower arms of the left and right wires, respectively; i sub b and i sub c flow from right to left on the left and right arms of the horizontal cross wire, respectively; and i sub e flows from left to right on the top wire.

A rectangular circuit has a horizontal cross wire connecting two nodes on the left and right wires. A vertical cross wire connects two nodes at the midpoints of the horizontal cross wire and bottom wire. A 10 ohm resistor is located on the vertical cross wire, while a 5 ohm resistor and a 15 ohm resistor are located on the left and right arms of the horizontal cross wire, respectively. A 30 ohm resistor is found on the top wire. Plus, a 20 ohm resistor and 25 ohm resistor are located on the left and right arms of the bottom wire, respectively.

The circuit is powered by two fixed independent voltage sources with same polarity voltages of 23V and 46V, located on the bottom arm of the left wire and the vertical cross wire, respectively. A variable independent voltage source with the same polarity, labeled as v sub dc, is located on the bottom arm of the right wire. Current i sub o flows from left to right on the left arm of the bottom wire through the 20 ohm resistor.

A rectangular circuit has two nodes on its top wire, equidistant from each other and from either end, and two similar nodes on its bottom wire. Two vertical cross wires connect these two pairs of nodes on the top and bottom wires. A 20 ohm resistor and a 50 ohm resistor are located on the left and right cross wires, respectively, while a 15 ohm resistor and a 40 ohm resistor are located on the middle arm of the top and bottom wires, respectively. A fifth resistor of 12 ohm resistance is found on the left arm of the top wire.

An additional wire connects the two nodes on the top wire, carrying a 4A independent current source with a leftward bound current. A 240V independent voltage source is located on the left wire. Plus, a variable independent current source, i sub dc, is located on the right wire, with an upward bound current.

A rectangular circuit has a 10mA independent current source on its left wire, with current flowing from bottom to top, and a diamond shaped smaller circuit across two nodes on its right wire. The diamond shaped circuit has four resistors on its outer wires, starting from the top left and moving clockwise, with resistances of 2.5k ohm, 2k ohm, 1k ohm, and 5k ohm. A horizontal cross wire connects the left and right nodes of the diamond circuit, carrying a 1k ohm resistor.

A rectangular circuit has a horizontal cross wire connecting two nodes on the left and right wires. Two nodes are located on the horizontal cross wire, equidistant from each other and from either end. Two similar nodes are located on the bottom wire. Two vertical cross wires connect these two pairs of nodes.

A 250 ohm resistor and a 500 ohm resistor are located on the left and right vertical cross wires, respectively. A 100 ohm resistor and a 200 ohm resistor are located on the bottom arms of the left and right wires, respectively.

Three power sources are located on three arms of the horizontal cross wire. A 20V independent voltage source with reverse polarity is on the left arm, while a 200mA independent current source with a leftward bound current is on the middle arm. A dependent voltage source with reverse polarity of 0.4v sub a is located on the right arm. Plus, a dependent current source with a rightward bound current of 0.003v sub delta is located on the top wire.

Two voltages, v sub a and v sub delta, are marked across the left and right vertical cross wires, respectively.

A rectangular circuit has two nodes located on the top wire, equidistant from each other and from either end. Two similar nodes are located on the bottom wire. Two vertical cross wires connect these two pairs of nodes.

A 330 ohm resistor and a 270 ohm resistor are located on the left and right vertical cross wires, respectively. A 150 ohm resistor is located on the middle arm of the top wire. An unknown current, i sub o, flows from right to left through the 150 ohm resistor on the middle arm of the top wire.

The circuit is powered by a 25mA independent current source on the left wire, with a downward bound current, and a 25mA independent current source on the right wire, with an upward bound current.

A rectangular circuit shows three nodes on its top wire, which are equidistant from each other and from either end, and three similar nodes on its bottom wire. Three vertical cross wires connect these three pairs of nodes across the top and bottom wires. A 60 ohm resistor and a 90k ohm resistor are located on the left and right cross wires, respectively. Three resistors with resistances of 40k ohm, 4k ohm, and 2.5k ohm are located on the first, second and fourth arms of the top wire, respectively. A 2k ohm resistor is found on the second arm of the bottom wire, while a 15k ohm resistor is located on the right wire.

The circuit is powered by a 120V independent voltage source on the left wire and an 8.4mA independent current source on the middle cross wire, with current flowing from top to bottom.

Two terminals are located on the top and bottom wires, close to the right wire. Current i sub o flows from top to bottom between these two terminals.

A rectangular circuit shows three nodes on its top wire, which are equidistant from each other and from either end, and three similar nodes on its bottom wire. Three vertical cross wires connect these three pairs of nodes across the top and bottom wires. A 30k ohm resistor and a 25k ohm resistor are located on the middle and right cross wires, respectively. Two resistors with resistances of 10k ohm and 15k ohm are located on the second and third arms of the top wire, respectively. Plus, a 5k ohm resistor is located on the left wire.

The circuit is powered by an 8mA independent current source on the left cross wire, with current flowing from bottom to top, and a 1mA independent current source on the right wire, with current flowing from top to bottom. An unknown voltage, v sub o, is marked across the 25k ohm resistor on the right cross wire.

A rectangular circuit has two vertical cross wires connecting two pairs of nodes on the top and bottom wires. A 6 ohm resistor and 17 ohm resistor are located on the left and right vertical cross wires, respectively. Two resistors with resistances of 6 ohm and 5 ohm are located on the middle arm of the top wire, separated by a node. Plus, a 1.5 ohm resistor is found on the right wire.

The circuit is powered by a 2A independent current source on the left wire, with an upward bound current, and a 34V independent voltage source, with reverse polarity, on the right cross wire. An additional wire is drawn between nodes on either side of the 5 ohm resistor on the top wire. A 1A independent current source, with a rightward bound current, is located on this wire.

An unknown current, i sub o, flows from left to right on the right arm of the top wire.

A rectangular circuit has three nodes located on the top wire, equidistant from each other and from either end. Two similar nodes are located on the bottom wire, equivalent to the left and right nodes of the top wire. A vertical cross wire connects the left pair of nodes on the top wire and the bottom wire.

A 40 ohm resistor is located on the vertical cross wire. Three resistors with resistances of 16 ohm, 260 ohm, and 4 ohm are located on the first, second, and third arms of the top wire. A 6 ohm resistor is located between the two nodes on the bottom wire, while the right wire carries a 250 ohm resistor.

An additional wire is drawn between the nodes on either side of the 260 ohm resistor on the top wire. A 520V independent voltage source with reverse polarity is located on this additional wire. The circuit also has a 1A independent current source on the left wire, with current flowing from top to bottom.

Voltage across the third node on the top wire and its equivalent node on the bottom wire is marked as v sub o.

A vertical cross wire connects two nodes on the top and bottom wires of a rectangular circuit. Two resistors of 10 ohm and 8 ohm resistance are located on the left and right arms of the top wire, while a 40 ohm resistor is located on the cross wire. The circuit is powered by a 60V independent voltage source on the left wire.

The right wire of the circuit is open, and two terminals on the top and bottom wires are labeled as a and b.

A rectangular circuit has two vertical cross wires connecting two pairs of nodes on the top and bottom wires. A 4 ohm resistor and a 3 ohm resistor are located on the left and right vertical cross wires, respectively. Plus, a 2k ohm resistor is located on the middle arm of the top wire. The circuit is powered by a 75mA independent current source on the left wire, with current flowing from bottom to top.

The right wire of the circuit is removed, and the top and bottom wires are connected to two open terminals, a and b.

A rectangular circuit has a horizontal cross wire connecting two nodes on the left and right wires. A vertical cross wire connects two nodes at the midpoints of the horizontal cross wire and bottom wire. A 6 ohm resistor is located on the vertical cross wire, while a 12 ohm resistor and 2 ohm resistor are located on the left and right arms of the horizontal cross wire, respectively.

The circuit is powered by a 12V independent voltage source located on the bottom arm of the left wire and an 8A independent current source on the top wire, with current flowing from left to right. The bottom arm of the right wire is open and connected to two open terminals, a and b.

A rectangular circuit has two nodes on the left wire, equidistant from each other and from either end. Two similar nodes are found on the right wire. Two horizontal cross wires are drawn connecting these two pairs of nodes between the left and right wires. A vertical cross wire connects the node which is located at the midpoint of the lower horizontal cross wire with another node at the midpoint of the bottom wire.

A 40 ohm resistor and a 10 ohm resistor are found on the left and right arms of the lower horizontal cross wire. A 150 ohm resistor is located on the upper horizontal cross wire. The vertical cross wire of the circuit carries an 8 ohm resistor. The circuit is powered by a 300V independent voltage source on the lower arm of the left wire and a 3A independent current source located on the top wire, with current flowing from right to left.

The lower arm of the right wire is removed, and the top and bottom ends of this arm are connected to two terminals, a and b.

A rectangular circuit has three nodes located on the top wire, equidistant from each other and from either end. Three similar nodes are found on the bottom wire. Three vertical cross wires are drawn connecting these three pairs of nodes between the top and bottom wires.

A 20 ohm resistor is located on the left cross wire. A 15k ohm resistor is located on the third arm of the top wire, while a 30k ohm resistor is found on the right wire.

The circuit is powered by a 30V independent voltage source on the middle cross wire, a 10mA independent current source on the right cross wire, with current flowing from top to bottom, and an 8mA independent current source located on the left wire, with current flowing from top to bottom.

The circuit is connected to two open terminals, a and b, from nodes at the right edge of the top and bottom wires.

A rectangular circuit has four nodes located on the top wire, equidistant from each other and from either end. Four similar nodes are found on the bottom wire. Three vertical cross wires are drawn connecting the first three pairs of nodes between the top and bottom wires. The fourth pair of nodes to the right are not connected directly. The unknown voltage across these two nodes is marked as v sub o.

A 400 ohm resistor and a 240 ohm resistor are located on the first and third cross wires. The top wire has two resistors with resistances of 120 ohm and 40 ohm on the second and third arms, respectively. A 100 ohm resistor is found on the left wire. Plus, a resistor, R0, is found on the right wire.

The circuit is powered by a 3V independent voltage source on the left wire, below the 100 ohm resistor, and an 18mA independent current source on the second cross wire, with current flowing from bottom to top.

A rectangular circuit shows three nodes on its top wire which are equidistant from each other and from either end, and three similar nodes on its bottom wire. Three vertical cross wires connect these three pairs of nodes across the top and bottom wires. A 20k ohm resistor and a 45k ohm resistor are located on the left and right cross wires, respectively. A 1k ohm resistor is found on the third arm of the top wire, while a 5k ohm resistor is located on the left wire.

The circuit is powered by a 50V independent voltage source with reverse polarity on the left wire, and a 25mA independent current source on the middle cross wire, with current flowing from bottom to top.

The right wire is open and the top and bottom wires are connected to two terminals, a and b, respectively.

A rectangular circuit has a vertical cross wire which connects two nodes on the top and bottom wires, closer to the left wire. A horizontal cross wire connects a node at the midpoint of the vertical cross wire to another node at the midpoint of the right wire. A Galvanometer is located on this horizontal cross wire. The Galvanometer is depicted as a rectangle with an internal current source and two connecting terminals.

Two resistors, R1 and R3, with respective resistances of 100 ohm and 500 ohm are located on the top and bottom arms of the vertical cross wire. Two more resistors, R2 and Rx, with respective resistances of 1000 ohm and 5000 ohm are located on the top and bottom arms of the right wire. The circuit is powered by a 5V independent voltage source located on the left wire.

The circuit on the left has a vertical cross wire connecting nodes at the midpoints of the top and bottom wires. A 625 ohm resistor is located on this cross wire. A 2.5k ohm resistor is located on the left arm of the top wire, with an unknown current, i sub 1, flowing through it from left to right.

The circuit is powered by a 100V independent voltage source on the left wire and a dependent current source on the right wire, with an upward flowing current of 10 raised to minus 3 times v sub 2.

The circuit on the right has a 4k ohm resistor on the top wire and a 6k ohm resistor on the right wire, with an unknown voltage, v sub 2, marked across its terminals. A dependent voltage source is located on the left wire of this circuit, with voltage of 5000i sub 1. The circuit is connected to two open nodes, a and b, from the nodes on either side of the right wire.

The left and right circuits are connected to each other at their bottom nodes.

A rectangular circuit has two nodes on its top wire, equidistant from each other and from either end, and two similar nodes on its bottom wire. Two vertical cross wires connect these two pairs of nodes on the top and bottom wires. A 2k ohm resistor and a 5.6k ohm resistor are located on the left and right cross wires, respectively, while two resistors with resistance of 2k ohm each are found on the first and second arms of the top wire.

An additional wire connects the two nodes on the top wire, carrying a dependent current source with a rightward bound current of 0.2i sub delta. A 280V independent voltage source is located on the left wire.

The right wire of the circuit is open, with the top and bottom wires connected to two open terminals, a and b.

A rectangular circuit has a horizontal cross wire connecting two nodes on the left and right wires. A vertical cross wire connects two nodes at the midpoints of the horizontal cross wire and bottom wire. A 50 ohm resistor is located on the vertical cross wire, while a 16 ohm resistor and a 96 ohm resistor are located on the left and right arms of the horizontal cross wire, respectively. Two diagonal wires are drawn from the midpoint of the horizontal cross wire to the left and right bottom nodes. An 80 ohm resistor and a 40 ohm resistor are located on these left and right diagonal wires. Plus, a 114 ohm resistor is located on the bottom arm of the right wire.

The circuit is powered by a 40V independent voltage source located on the bottom arm of the left wire, and a dependent current source on the top wire with a rightward current of 10i sub phi.

An unknown current, i sub phi, flows from top to bottom through the 50 ohm resistor on the vertical cross wire.

A rectangular circuit has a horizontal cross wire connecting two nodes on the left and right wires. A vertical cross wire connects two nodes at the midpoints of the horizontal cross wire and bottom wire. A 25 ohm resistor is located on the vertical cross wire, while a 5 ohm resistor and a 1.8A independent current source, with rightward current, are located on the left and right arms of the horizontal cross wire, respectively. The bottom arm of the right wire carries a 60 ohm resistor, and the right arm of the bottom wire carries a 10 ohm resistor. A 20 ohm resistor is located on the top wire.

A 9V independent voltage source is located on the bottom arm of the left wire. The bottom arm of the right wire is connected to two open terminals, a and b.

A rectangular circuit has a horizontal cross wire connecting two nodes on the left and right wires. A vertical cross wire connects two nodes at the midpoints of the horizontal cross wire and bottom wire. A 100 ohm resistor is located on the bottom arm of the left wire, while a 200 ohm resistor and a 50 ohm resistor are located on the left and right arms of the horizontal cross wire, respectively. A 150 ohm resistor is located on the top wire.

A dependent voltage source with voltage of 250i sub delta is located on the vertical cross wire of the circuit. Current i sub delta flows from right to left on the top wire. The bottom arm of the right wire is open and connected to two open terminals, a and b.

The circuit has a vertical cross wire connecting the nodes at the midpoints of the top and bottom wires. A 5 ohm resistor and a dependent voltage source, 10i sub x are located on this cross wire. The circuit also has a 20 ohm resistor on the right wire, a 10 ohm resistor on the right arm of the top wire, and a 40 ohm resistor on the left wire, carrying a downward current, i sub x.

The top and bottom nodes of the right wire are connected to two open terminals, a and b.

The circuit has a vertical cross wire connecting nodes at the midpoints of the top and bottom wires. A dependent current source with an upward bound current of 1.5i sub x is located on this cross wire. The circuit also has a dependent voltage source, 250i sub x, on the left wire, a 500 ohm resistor on the left arm of the top wire, and a 750 ohm resistor on the right wire, carrying a downward current i sub x.

The top and bottom nodes of the right wire are connected to two open terminals, a and b.

A rectangular circuit has three nodes located on the top wire, equidistant from each other and from either end. Three similar nodes are found on the bottom wire. Three vertical cross wires are drawn connecting these three pairs of nodes between the top and bottom wires. An additional pair of nodes is located between the third pair and the right wire.

A 4.8k ohm resistor and a 5k ohm resistor are located on the first and third cross wires. The top wire has two resistors, with resistances of 2.4k ohm and 1.6k ohm, on the first and third arms, respectively. A 1.8k ohm resistor is found on the third arm of the bottom wire. A variable resistor, R0, is found on the right wire.

The circuit is powered by a 60V independent voltage source on the left wire, and a 15mA independent current source on the second cross wire, with current flowing from top to bottom.

The circuit has a 30V independent voltage source on the left wire, a 6 ohm resistor on the right wire, and a variable resistor, R sub o, on the top wire, close to the left wire. Two terminals are located on the top and bottom wires, close to the right wire.

A rectangular circuit has two nodes on the top wire, equidistant from each other and from either end, and two similar nodes on the bottom wire. Two vertical cross wires connect these two pairs of nodes. A 100 ohm resistor is located on the left cross wire, while a 20 ohm resistor and a dependent voltage source, 30i sub x, are located on the right cross wire. The top wire has a 25 ohm resistor and a 10 ohm resistor on its first and second arms. A variable resistor, R sub o, is located on the right wire.

The circuit is powered by a 200V independent voltage source on the left wire.

A rectangular circuit has two nodes on the top wire, equidistant from each other and from either end, and two similar nodes on the bottom wire. Two vertical cross wires connect these two pairs of nodes. A dependent current source with a downward current of 2v sub delta is located on the left cross wire, while a dependent voltage source, 4i sub delta, and a 5 ohm resistor are located on the right cross wire. A variable resistor, R sub o, is located on the right wire. The circuit is powered by a 60V independent voltage source on the left wire.

The top wire has a 2 ohm resistor and a 4 ohm resistor on its first and second arms. An unknown voltage, v sub delta, is marked across the 2 ohm resistor. An unknown current, i sub delta, flows from left to right through the 4 ohm resistor.

A rectangular circuit has a horizontal cross wire connecting two nodes on the left and right wires. Two nodes are located on the horizontal cross wire, equidistant from each other and from either end. Two similar nodes are located on the bottom wire. Two vertical cross wires connect these two pairs of nodes.

A 30 ohm resistor is located on the top wire, while a 300 ohm resistor and a variable resistor, R sub L, are located on the left and right vertical cross wires, respectively. A 45 ohm resistor and a 60 ohm resistor are located on the left and right arms of the horizontal cross wire, respectively. Plus, a 30 ohm resistor and a 15 ohm resistor are located on the left and right arms of the bottom wire, respectively.

The circuit is powered by a 3.6k V independent voltage source on the bottom arm of the left wire, and a dependent voltage source on the bottom arm of the right wire, with reverse voltage polarity of 150i sub beta.

An unknown current, i sub beta, flows from left to right through the 45 ohm resistor on the left arm of the horizontal cross wire.

A rectangular circuit has a horizontal cross wire connecting two nodes on the left and right wires. Two nodes are located on the horizontal cross wire, equidistant from each other and from either end. Two similar nodes are located on the bottom wire. Two vertical cross wires connect these two pairs of nodes.

An 80 ohm resistor and a variable resistor, R sub o, are located on the left and right vertical cross wires, respectively. A 4 ohm resistor and an 8 ohm resistor are located on the left and right arms of the horizontal cross wire, respectively. Plus, a 16 ohm resistor and a 12 ohm resistor are located on the left and right arms of the bottom wire, respectively.

The circuit is powered by a 100V independent voltage source with reverse polarity on the bottom arm of the left wire, and a 50V independent voltage source on the bottom arm of the right wire. A dependent voltage source is located on the top wire, with voltage marked as 124i sub delta.

An unknown current, i sub delta, flows from bottom to top through the 80 ohm resistor on the left vertical cross wire.

A rectangular circuit has a horizontal cross wire connecting two nodes on the left and right wires. Two nodes are located on the horizontal cross wire, equidistant from each other and from either end. Two similar nodes are located on the bottom wire. Two vertical cross wires connect these two pairs of nodes.

A variable resistor, R sub o, is located on the right vertical cross wire. A 20 ohm resistor, a 2 ohm resistor, and an 8 ohm resistor are located on the left, middle and right arms of the horizontal cross wire, respectively. Plus, a 16 ohm resistor and a 10 ohm resistor are located on the bottom arms of the left and right wires, respectively.

The circuit is powered by a 180V independent voltage source located on the left vertical cross wire. A dependent voltage source is located on the top wire, with voltage marked as 184i sub phi. An additional wire is drawn connecting the nodes of the bottom arm of the right wire, with a dependent current source of 0.1v sub delta located on it. Current flow in this source is upward.

An unknown current, i sub phi, flows downward through the variable resistor on the right vertical cross wire. Voltage v sub delta is marked with reverse polarity across the 2 ohm resistor located on the middle arm of the horizontal cross wire.

A rectangular circuit has a horizontal cross wire connecting two nodes on the left and right wires, close to the top wire. A vertical cross wire connects two nodes at the midpoints of the horizontal cross wire and bottom wire. A 10 ohm resistor is located on the vertical cross wire, while a 5 ohm resistor and 2 ohm resistor are located on the left and right arms of the horizontal cross wire, respectively. The bottom arm of the right wire carries a 12 ohm resistor.

The circuit is powered by a 110V independent voltage source at the bottom arm of the left wire, and a 4A independent current source on the top wire, with current moving from right to left. An unknown voltage, v, is marked across the nodes of the vertical cross wire.

A rectangular circuit has a horizontal cross wire connecting two nodes on the left and right wires. Two nodes are located on the horizontal cross wire, equidistant from each other and from either end. Two similar nodes are located on the bottom wire. Two vertical cross wires connect these two pairs of nodes.

A 60 ohm resistor and an 80 ohm resistor are located on the left and right vertical cross wires, respectively. A 30 ohm resistor and a 20 ohm resistor are located on the left and right arms of the horizontal cross wire, respectively. Plus, a 40 ohm resistor is located on the top wire, while a 25 ohm resistor is located on the bottom arm of the right wire.

The circuit is powered by a 135V independent voltage source on the bottom arm of the left wire, and an 18A independent current source located on the middle arm of the horizontal cross wire, carrying a rightward bound current.

An unknown current, i sub o, flows from right to left on the top wire. Voltage across the 60 ohm resistor on the left vertical cross wire is marked as v sub o, with a reverse polarity.

A rectangular circuit has a horizontal cross wire connecting two nodes on the left and right wires. A vertical cross wire connects two nodes at the midpoints of the horizontal cross wire and top wire. A 40 ohm resistor is located on the vertical cross wire, while a 15 ohm resistor and 30 ohm resistor are located on the left and right arms of the horizontal cross wire, respectively. A 5 ohm resistor and a 10 ohm resistor are located on the left and right arms of the top wire, respectively.

The circuit is powered by a 10V independent voltage source located on the top arm of the left wire, a 20V independent voltage source located on the top arm of the right wire, and a 6A independent current source on the bottom wire, with a leftward current.

An unknown voltage, v sub 0, is marked across the 40 ohm resistor on the vertical cross wire.

A rectangular circuit has two nodes, a and b, on the top wire, equidistant from each other and from either end, and two similar nodes on the bottom wire. Two vertical cross wires connect these two pairs of nodes. A 10k ohm resistor and an 18k ohm resistor are located on the left and right cross wires, respectively. A 2k ohm resistor is found between nodes a and b on the top wire.

The circuit is powered by a 20V independent voltage source on the left wire and a 5mA current source on the right wire, with an upward bound current.

The two nodes, a and b, have an additional wire attached to them. This wire carries a 10mA independent current source with a rightward bound current.

A rectangular circuit has four nodes located on the top wire, equidistant from each other and from either end, and four similar nodes on the bottom wire. Four vertical cross wires are drawn connecting these four pairs of nodes.

Three resistors with resistances of 12 ohm, 14 ohm, and 10 ohm are located on the first, third, and fourth cross wires, respectively. A 15 ohm resistor is found on the right wire, while a 1.8 ohm resistor is found on the second arm of the top wire.

The circuit is powered by a 4.5A independent current source with a downward current on the left wire, a 20A independent current source with an upward current on the second cross wire, and a 50V independent voltage source with reverse polarity on the fourth cross wire.

An unknown current, i sub o, flows from top to bottom across the 12 ohm resistor on the first cross wire.

A rectangular circuit has a horizontal cross wire connecting two nodes on the left and right wires, close to the top wire. A vertical cross wire connects two nodes at the midpoints of the horizontal cross wire and bottom wire. A 2k ohm resistor and a 4k ohm resistor are located on the left and right arms of the horizontal cross wire, respectively. The right arm of the bottom wire carries a 2k ohm resistor, while the top wire carries a 1k ohm resistor.

The circuit is powered by a 90V independent voltage source at the bottom arm of the left wire, and a 40V independent voltage source at the bottom arm of the right wire. A dependent voltage source, 2.5v sub b, is located on the vertical cross wire, with current i flowing through the cross wire from top to bottom. An unknown voltage, v sub b, is marked across the 1k ohm resistor on the top wire.

A rectangular circuit has a horizontal cross wire connecting two nodes on the left and right wires. A vertical cross wire connects two nodes at the midpoints of the horizontal cross wire and bottom wire. A 5k ohm resistor is located on the left arm of the horizontal cross wire, while a 20k ohm resistor is located on the bottom arm of the right wire.

The circuit is powered by a 35V independent voltage source located on the bottom arm of the left wire, a 7mA independent current source located on the vertical cross wire with a downward current, and a dependent current source on the top wire, with a rightward current of 5i sub phi.

An unknown current, i sub phi, flows rightward through the 5k ohm resistor on the left arm of the horizontal cross wire. The bottom arm of the right wire is connected to two open terminals. Voltage across these two terminals is marked as v sub o.

A rectangular circuit has two nodes on its top wire, equidistant from each other and from either end, and two similar nodes on its bottom wire. Two vertical cross wires connect these two pairs of nodes between the top and bottom wires. A horizontal cross wire is drawn from the midpoint of the left wire, connecting to the midpoints of the two vertical cross wires.

A pair of 36 ohm resistors and 45 ohm resistors are located on the top and bottom arms of the left and right vertical cross wires, respectively. Four 0.1 ohm resistors are located on the first and second arms of the top wire, and the first and second arms of the bottom wire. Two 0.2 ohm resistors are located on the left and right arms of the horizontal wire. Plus, a 27 ohm resistor is located on the right wire.

The circuit is powered by two 120V independent voltage sources, located on the top and bottom arms of the left wire.

Voltages v1 and v2 are marked on the top and bottom arms of the right vertical cross wire, while voltage v3 is marked across the right wire.

A rectangular circuit has a vertical cross wire connecting two nodes at the midpoints of the top and bottom wires. Two nodes are located on the vertical cross wire, equidistant from each other and from either end. Two similar nodes are located on the right wire. Two horizontal cross wires connect these two pairs of nodes between the vertical cross wire and the right wire. The circuit also shows three diagonal cross wires between nodes on the vertical cross wire and the right wire.

Three 10 ohm resistors are located on the first, second, and third arms of the vertical cross wire. Three resistors with resistances of 10 ohm, 20 ohm, and 10 ohm are located on the first, second, and third arms of the right wire, respectively. Two 20 ohm resistors are located on the two horizontal cross wires between the vertical cross wire and the right wire. A 10 ohm resistor is found on the right arm of the top wire, while a 5 ohm resistor is located on the right arm of the bottom wire.

A 20 ohm resistor is located on the top diagonal wire between the left node on the vertical cross wire and the right node on the right wire. Another 20 ohm resistor is located on the middle diagonal wire between the right node on the right wire and the left node on the vertical cross wire. Plus, a 10 ohm resistor is located on the bottom diagonal wire between the left node on the vertical cross wire and the right node on the right wire.

A 120V independent voltage source is located on the left wire. An unknown current, i1, flows from the right to left on the top horizontal cross wire, while another unknown current, i2, flows from left to right on the bottom horizontal cross wire.

A rectangular circuit has a vertical cross wire connecting two nodes at the midpoints of the top and bottom wires. It also has a horizontal cross wire connecting two nodes at the midpoints of the left and right wires. The two cross wires intersect at a node in the middle. From this intersection point, a diagonal wire is drawn to the top left node of the circuit.

The left wire shows a 50 ohm resistor on its top arm, and a dependent voltage source, 1.25v sub x, on its bottom arm. The right wire shows a 50 ohm resistor on its top arm, and a 250 ohm resistor on its bottom arm. The top wire shows a 200 ohm resistor on the left arm and a 50V independent voltage source on the right arm. Plus, the bottom wire shows a 50 ohm resistor on its right arm.

The vertical cross wire shows a 100 ohm resistor on its top arm and a 0.9A independent current source with an upward bound current on its bottom arm. The horizontal cross wire shows a 100 ohm resistor on the left arm and a dependent current source with a leftward bound current, v sub y over 25, on the right arm.

An additional wire is drawn across the two nodes of the bottom arm of the right wire. A 100 ohm resistor and a dependent voltage source, 50i sub x, are located on this additional wire.

Two unknown voltages, v sub x and v sub y, are marked across the top arms of the vertical cross wire and the right wire, respectively. An unknown current, i sub x, flows downward on the bottom arm of the right wire.

The circuit has an independent dc voltage source v1 on the left wire and another independent dc voltage source v2 on the right wire. Two pairs of electrical conductors, leading out of the voltage sources, have a resistance of r ohm per meter. The horizontal distance between v1 and v2 is depicted as L.

A movable load with resistance R is placed on a parallel wire between v1 and v2, at a horizontal distance of x from v1.

A drawing shows a top view of an eight-lead DIP package.

The drawing shows a device in the shape of a rectangular block with four terminals on the right and four on the left, numbered and labeled as follows:

Left - top to bottom:

1. Offset null
2. Inverting input
3. Noninverting input
4. V minus

Right - bottom to top

5. Offset null
6. Output
7. V plus
8. NC

The symbol shows a triangle with the base to the left and the pointed edge to the right. Two terminals are located one below the other on the base, labeled as Noninverting input and Inverting input. A terminal at the pointed edge to the right is labeled as Output. Two perpendicular lines are drawn above and below from the middle of the triangle, leading to two terminals. The upper terminal represents Positive power supply while the lower terminal represents Negative power supply.

The symbol shows a triangle pointing to the right having five terminals. Two terminals at the base on the left are marked plus and minus, with wires pointing out while a terminal at the top on the right has a wire pointing out. Two wires point above and below from the center of the triangle, leading to two terminals, marked as V plus and V minus, respectively.

The drawing shows the op amp device connected to a circuit which has two supply voltages and a common node. The positive supply voltage (V sub CC) is connected at one end to the V plus node of the op amp and at the other end to the common node. The negative supply voltage (minus V sub CC) is connected at one end to the V minus node and at the other end to the common node.

Three voltages are marked for the remaining three terminals of the op amp against the common node as follows:

Inverting input terminal: v sub p

Noninverting input terminal: v sub n

Output terminal: v sub o

The drawing shows the op amp device connected to a circuit which has two supply voltages and a common node. The positive supply voltage (V sub CC) is connected at one end to V plus node of the op amp and at the other end to the common node. The negative supply voltage (minus V sub CC) is connected at one end to V minus node and at the other end to the common node.

Five inward currents are marked for the five terminals of the op amp as follows:

Inverting input terminal: i sub p

Noninverting input terminal: i sub n

Output terminal: i sub o

Positive power supply terminal: i sub c plus

The x axis plots (v sub p minus v sub n), and marks "minus V sub CC over A" and "V sub CC over A" on either side of the origin. The y axis plots v sub o, and marks V sub CC on either side of the origin. The graph shows an upward sloping line passing through the origin in the first and third quadrants. The sloping part of the line is labeled as "linear region". The line flattens out at V sub CC in the first quadrant and third quadrant at V sub CC, where it is labeled as Positive saturation and Negative saturation, respectively.

The op amp has a positive supply voltage of 10V and a negative supply voltage of minus 10V. An independent voltage source, v sub a is connected in series with a 25k ohm resistor between the inverting input terminal and the common node. Another independent voltage source, v sub b connects between the non-inverting input terminal and the common node. A 100k ohm resistor is connected between the output terminal and inverting input terminal. The voltage across the output terminal is marked as v sub o.

A current, i sub 25 flows through the 25 ohm series resistor into the inverting input terminal. Another current, i sub 100 flows through the 100k ohm resistor, out of the output terminal.

The op amp has a positive supply voltage of 10V and a negative supply voltage of minus 15V. An independent voltage source, v sub s is connected in series with a 16k ohm resistor between the inverting input terminal and the common node. The non-inverting input terminal is short circuited with the common node. A 80k ohm resistor is connected between the output terminal and inverting input terminal. The voltage across the output terminal is marked as v sub o.

The op amp has a positive supply voltage, V sub CC and a negative supply voltage, minus V sub CC. An independent voltage source, v sub s is connected in series with a resistor R sub s between the inverting input terminal and the common node. The non-inverting input terminal is short circuited with the common node. A resistor R sub f is connected between the output terminal and inverting input terminal. The voltages across the output terminal, non inverting input terminal and the inverting input terminal are marked as v sub o, v sub p, and v sub n, respectively.

A current, i sub s flows through resistor R sub s into the inverting input terminal. Another current, i sub f flows through resistor R sub f out of the output terminal.

The circuit contains an op amp connected in series with a resistor and an independent voltage source. The op amp has a positive supply voltage, V sub CC and a negative supply voltage, minus V sub CC. An independent voltage source, v sub s is connected in series with a resistor R sub s between the inverting input terminal and the common node. The non-inverting input terminal is short circuited with the common node. The voltages across output terminal, and inverting input terminal are marked as v sub o, and v sub n, respectively.

The op amp has a positive supply voltage of 15V and a negative supply voltage of minus 15V. An independent voltage source, v sub s is connected in series with a 1k ohm resistor between the inverting input terminal and the common node. The non-inverting input terminal is short circuited with the common node. A 12k ohm resistor is connected between the output terminal and inverting input terminal. The voltage across the output terminal and common node is marked as v sub o.

The op amp has a positive supply voltage, V sub CC and a negative supply voltage, minus V sub CC. A resistor R sub f is connected between the output terminal and inverting input terminal, and carries a current, i sub n into the output terminal. The non-inverting input terminal is short circuited with the common node, and shows a voltage, v sub n. The voltage across the output terminal and common node is marked as v sub o.

Three input voltages--v sub a, v sub b, and v sub c are connected along with series resistors--R sub a, R sub b, and R sub c, respectively between the inverting input terminal and the common node.

The op amp has a 12V positive supply voltage, and a minus 12V negative supply voltage. A 20k ohm resistor is connected between the output terminal and inverting input terminal, and carries a current, i sub n into the output terminal. The non-inverting input terminal is short circuited with the common node, and shows a voltage, v sub n. The voltage across the output terminal and common node is marked as v sub o.

Three input voltages--v sub a, v sub b, and v sub c are connected along with series resistors, with respective resistances of 5k ohm, 10k ohm, and 4k ohm between the inverting input terminal and the common node.

The op amp has a 15V positive supply voltage, and a minus 10V negative supply voltage. A 250k ohm resistor is connected between the output terminal and inverting input terminal. The non-inverting input terminal is short circuited with the common node. The voltage across the output terminal and common node is marked as v sub o.

Two input voltages, v sub a and v sub b, are connected along with series resistors with respective resistances of 5k ohm and 25k ohm between the inverting input terminal and the common node.

The op amp has a positive supply voltage, V sub CC, and a negative supply voltage, minus V sub CC. A resistor R sub f is connected between the output terminal and inverting input terminal. The inverting input terminal is connected to a series resistor, R sub s, and is short circuited with the common node. It shows a voltage v sub n. The noninverting input terminal has another series resistor, R sub s, and is connected to an independent voltage source, V sub g. The voltage across the output terminal and common node is marked as v sub o.

The op amp has a positive supply voltage, V sub CC, and a negative supply voltage, minus V sub CC. A 10k ohm resistor is connected between the output terminal and inverting input terminal. The inverting input terminal is connected to two series resistors with resistances of 1k ohm each, and is short circuited with the common node. The noninverting input terminal has a series resistor of 1k ohm resistance, and is connected to an independent voltage source, V sub g. The voltage across the output terminal and common node is marked as v sub o.

The op amp has a 5V positive supply voltage and a minus 5V negative supply voltage. A 63k ohm resistor is connected between the output terminal and inverting input terminal. The inverting input terminal is connected to a 4.5k ohm series resistor, and is short circuited with the common node. The noninverting input terminal is connected to a 400mV independent voltage source which is in series with a 15k ohm resistor and has a parallel connection with a variable resistor, R sub x. The voltage across the output terminal and common node is marked as v sub o.

The op amp has a positive supply voltage, V sub CC, and a negative supply voltage, minus V sub CC. A resistor R sub b is connected between the output terminal and inverting input terminal. The inverting input terminal is connected to an independent voltage source, v sub a in series with a resistor, R sub a. The noninverting input terminal is connected to an independent voltage source, V sub b in series with a resistor, R sub c, and in parallel with a resistor, R sub d. The voltages across the output terminal, non inverting input terminal and inverting input terminal are marked as v sub o, v sub p, and v sub n, respectively.

The op amp has a 8V positive supply voltage, and a minus 8V negative supply voltage. A 12k ohm resistor is connected between the output terminal and inverting input terminal. The inverting input terminal is connected to an independent voltage source, v sub a, in series with a 1.5k ohm resistor. The noninverting input terminal is connected to an independent voltage source, V sub b, in series with a 1.5k ohm resistor, and in parallel with a 12k ohm resistor. The voltage across the output terminal and common node is marked as v sub o.

The op amp has a 10V positive supply voltage, and a minus 10V negative supply voltage. A 50k ohm resistor is connected between the output terminal and inverting input terminal. The inverting input terminal is connected to an independent voltage source, v sub a, in series with a 10k ohm resistor. The noninverting input terminal is connected to an independent voltage source, V sub b, in series with a 4k ohm resistor, and in parallel with a 20k ohm resistor. The voltage across the output terminal and common node is marked as v sub o.

The op amp has a positive supply voltage, V sub CC, and a negative supply voltage, minus V sub CC. A resistor R sub b is connected between the output terminal and inverting input terminal. The inverting input terminal is connected to a differential input voltage, "v sub dm over 2", in series with a resistor, R sub a. The noninverting input terminal is connected to another differential input voltage, "v sub dm over 2", in series with a resistor R sub c, and in parallel with a resistor, R sub d. The two differential input voltages are connected to a common mode input voltage, "v sub cm". The voltage across the output terminal is marked as v sub o.

The drawing shows two circuits inside a triangle which points to the right. One circuit shows a resistor, R sub i, drawn between two terminals at the base. The positive terminal measures a voltage v sub p with the common node outside, and has an inward current i sub p, while the negative terminal measures a voltage v sub n with the common node, and has an inward current, i sub n.

The second circuit is connected between a terminal at the apex of the triangle and the common node outside. It has a dependent voltage source "A (v sub p minus v sub n)" in series with a resistor, R sub o. This terminal measures a voltage, v sub o, with the common node outside, and has an inward current, i sub o.

The drawing shows two circuits inside a triangle which points to the right. One circuit shows a resistor, R sub i, drawn between two terminals at the base. The positive terminal measures a voltage v sub p with the common node outside, while the negative terminal measures a voltage v sub n with the common node. The negative terminal is connected to an independent voltage source, v sub s, in series with a resistor, R sub s, both of which are shown outside the triangle. A node, a is marked on this circuit close to the negative terminal, outside the triangle.

The second circuit is connected between a terminal at the apex of the triangle and the common node outside. It has a dependent voltage source "A (v sub p minus v sub n)" in series with a resistor, R sub o. This terminal measures a voltage, v sub o, with the common node outside. A node, b, is marked on this circuit, close to the apex terminal.

A wire connects terminals a and b, and carries a resistor R sub f, outside the triangle.

The drawing shows two circuits inside a triangle which points to the right. One circuit shows a resistor, R sub i, drawn between two terminals at the base. The positive terminal measures a voltage v sub p with the common node outside, while the negative terminal measures a voltage v sub n with the common node. The positive terminal is connected to an independent voltage source, v sub g, in series with a resistor, R sub g, both of which are shown outside the triangle. The negative terminal is in series with a resistor, R sub s, shown outside the triangle. A node, a is marked on this circuit outside the triangle close to the negative terminal.

The second circuit is connected between a terminal at the apex of the triangle and the common node outside. It has a dependent voltage source "A (v sub p minus v sub n)" in series with a resistor, R sub o. This terminal measures a voltage, v sub o, with the common node outside, in series with a load resistor, R sub L. A node, b, is marked on this circuit, close to the apex terminal.

A wire connects terminals a and b, and carries a resistor R sub f, outside the triangle.

The drawing shows two circuits inside a triangle which points to the right. One circuit shows a resistor, R sub i, drawn between two terminals at the base. The positive terminal measures a voltage v sub p with the common node outside, while the negative terminal measures a voltage v sub n with the common node. The positive terminal is connected to an independent voltage source, v sub g, in series with a 1k ohm resistor both of which are shown outside the triangle. The negative terminal is in series with a 2k ohm resistor shown outside the triangle. A node is marked on this circuit outside the triangle close to the negative terminal.

The second circuit is connected between a terminal at the apex of the triangle and the common node outside. It has a dependent voltage source "A (v sub p minus v sub n)" in series with a resistor, R sub o. This terminal measures a voltage, v sub o, with the common node outside. A node is marked on this circuit, close to the apex terminal.

A wire connects terminals a and b, and carries a 10k ohm resistor outside the triangle.

The op amp has a positive supply voltage of 20V, and a negative supply voltage of minus 20V. An independent voltage source v sub g is connected in series with a 5k ohm resistor between the inverting input terminal and the common node. The non-inverting input terminal is short circuited with the common node. A 100k ohm resistor is connected between the output terminal and inverting input terminal. The voltage across the output terminal and common node is marked as v sub o.

The strain gage bridge is represented by a diamond shaped circuit with four resistors on its four arms. Two resistors on the top left and bottom right have resistances of "R minus delta R", while two resistors on the top right and bottom left have resistances of "R plus delta R". The top node of the diamond circuit is powered by an independent voltage source, v sub ref, while the bottom node is grounded with the common node. The node on the right is connected to the inverting terminal of an op amp while the node on the left is connected to the non-inverting terminal.

The op amp has a positive supply voltage, V sub CC, and a negative supply voltage, minus V sub CC. A resistor R sub f is connected between the output terminal and the inverting terminal. The non-inverting terminal is grounded with the common terminal in series with resistor, R sub f. The voltages across the inverting, non-inverting, and output terminals are marked as v sub n, v sub p, and v sub o, respectively.

The op amp has a positive supply voltage of 15V and a negative supply voltage of minus 15V. A 2.5V independent voltage source is connected in series with a 10k ohm resistor between the inverting input terminal and the common node. The non-inverting input terminal is short circuited with the common node. A 40k ohm resistor is connected between the output terminal and inverting input terminal. The output terminal is in series with a 3.5k ohm resistor, and is grounded with the common terminal.

The voltages across the inverting, non-inverting, and output terminals are marked as v sub n, v sub p, and v sub o, respectively. A current, i sub n flows between the inverting terminal and the output terminal.

The op amp has a positive supply voltage of 15V and a negative supply voltage of minus 15V. A 2.5V independent voltage source is connected in series with a 10k ohm resistor between the inverting input terminal and the common node. The non-inverting input terminal is short circuited with the common node. A 40k ohm resistor is connected between the output terminal and inverting input terminal. The output terminal is in series with a 3.5k ohm resistor, and is grounded with the common terminal.

The voltages across the inverting, non-inverting, and output terminals are marked as v sub n, v sub p, and v sub o, respectively. A current, i sub n, flows between the inverting terminal and the output terminal.

The op amp has a positive supply voltage of 16V and a negative supply voltage of minus 16V. An independent voltage source, v sub a, is connected in series with a 20k ohm resistor between the inverting input terminal and the common node. Another independent voltage source, v sub b, is connected in series with a 40k ohm resistor between the non-inverting input terminal and the common node. A 100k ohm resistor is connected between the output terminal and inverting input terminal. The output terminal is in series with a 50k ohm resistor, and is grounded with the common terminal. The voltage across the output terminal and the common terminal is marked as v sub o.

The op amp has a positive supply voltage of 15V and a negative supply voltage of minus 15V. A 240mV independent voltage source is connected in series with a 8k ohm resistor between the inverting input terminal and the common node. The non-inverting input terminal is grounded with the common node. The output terminal is in series with a 20k ohm resistor, and is grounded with the common terminal, with a voltage, v sub 0, marked across it.

Three resistors are located on the wire connecting the output terminal with the inverting input terminal. Two of them with resistances of 60k ohm and 30k ohm are in series while the third resistor with resistance of 40k ohm is on a parallel wire that connects the node between the two series resistors and the common node. The voltage across the parallel resistor is marked as v sub a. A current, i sub a, flows from the inverting input terminal through the series resistors, while a current, i sub o, flows into the output terminal.

The op amp has a positive supply voltage of 20V and a negative supply voltage of minus 20V. A 5V independent voltage source is connected in series with a 3k ohm resistor between the inverting input terminal and the common node. A 6V independent voltage source is connected in series with a 10k ohm resistor and in parallel with a 5k ohm resistor between the non-inverting input terminal and the common node. The output terminal is in series with a 8k ohm resistor, and is grounded with the common terminal, with a current, i sub L, flowing out of it.

A 6k ohm resistor is located on the wire connecting the output terminal with the inverting input terminal.

The op amp has a positive supply voltage of 9V and a negative supply voltage of minus 9V. A 3.3M ohm resistor connects the output terminal with the inverting terminal. The non-inverting terminal is grounded with the common terminal while the inverting terminal is connected to a 2.5 muA independent current source. A voltmeter, v sub m, is connected to the output terminal which is grounded with the common node. The voltage across the output terminal is marked as v sub 0.

The op amp has a positive supply voltage of 5V and a negative supply voltage of minus 5V. The non-inverting terminal is grounded with the common terminal. The inverting terminal is connected to a 250 mV independent voltage source along with a 1.6k ohm series resistor and a 6.4k ohm parallel resistor . The output terminal which is grounded with the common terminal is in series with a 10k ohm resistor, with a voltage v sub 0 marked across it, along with an inward current, i sub 0.

A 12k ohm series resistor and a 50k ohm adjustable resistor with a variable value of "alpha 50k" ohm connects the output terminal with the inverting terminal.

The op amp has a positive supply voltage of 7V and a negative supply voltage of minus 7V. The non-inverting terminal is grounded with the common terminal. The inverting terminal is connected to an independent voltage source, v sub g, along with a 2k ohm series resistor. The output terminal which is grounded with the common terminal is in series with a 10k ohm resistor, with a voltage v sub 0 marked across it.

A 50k ohm series resistor and a "R sub delta" adjustable resistor with a variable value of "alpha R sub delta" connects the output terminal with the inverting terminal.

The op amp has a 6V positive supply voltage, and a minus 6V negative supply voltage. A 220k ohm resistor is connected between the output terminal and inverting input terminal. The non-inverting input terminal is short circuited with the common node. The output terminal which is grounded with the common terminal is in series with a 3.3K ohm resistor, with a voltage v sub 0 marked across it.

Three input voltages--v sub a, v sub b, and v sub c--are marked across three parallel resistors, with respective resistances of 33k ohm, 22k ohm, and 80k ohm which are connected between the inverting input terminal and the common node.

The op amp has a 15V positive supply voltage, and a minus 15V negative supply voltage. A 220k ohm resistor is connected between the output terminal and inverting input terminal. The non-inverting input terminal is connected to an independent voltage source, v sub d before being grounded with the common node. The output terminal which is grounded with the common terminal is in series with a 10k ohm resistor, with a voltage v sub 0 marked across it.

Three input voltages--v sub a, v sub b, and v sub c are marked across three parallel resistors, with respective resistances of 40k ohm, 22k ohm, and 100 ohm which are connected along with a 352k ohm series resistor between the inverting input terminal and the common node.

The op amp has a 12V positive supply voltage, and a minus 15V negative supply voltage. A 56k ohm resistor is connected between the inverting input terminal and the output terminal. The inverting input terminal which is grounded with the common terminal is in series with a 7 ohm resistor. The output terminal which is grounded with the common terminal is in series with a 12k ohm resistor, with a voltage v sub 0 marked across it.

The non-inverting input terminal is connected to a small circuit before being grounded with the common node. The small circuit contains an independent voltage source, v sub s, a 32k ohm series resistor, and an 8k ohm parallel resistor.

The op amp has a 16V positive supply voltage, and a minus 16V negative supply voltage. A 150k ohm resistor is connected between the inverting input terminal and the output terminal. The inverting input terminal which is grounded with the common terminal is in series with a 25k ohm resistor. The output terminal is grounded with the common terminal, and has a voltage v sub 0 marked across it.

A 2V independent voltage source connects the non-inverting input terminal with the common terminal.

The op amp has a 10V positive supply voltage, and a minus 10V negative supply voltage. A 48k ohm resistor is connected between the inverting input terminal and the output terminal. The inverting input terminal which is grounded with the common terminal is in series with a 15k ohm resistor. The output terminal which is grounded with the common terminal is in series with a 30k ohm resistor, with a voltage v sub 0 marked across it.

The non-inverting input terminal is connected to an independent voltage source, v sub g, which is in series with a 30k ohm resistor, and in parallel with a 45k ohm resistor.

The op amp has a 16V positive supply voltage, and a minus 16V negative supply voltage. A 60k ohm resistor is connected between the inverting input terminal and the output terminal. The inverting input terminal which is grounded with the common terminal is in series with a 20k ohm resistor. The output terminal which is grounded with the common terminal is in series with a 2k ohm resistor, with a voltage v sub 0 marked across it.

The non-inverting input terminal is connected to a 6V independent voltage source in series with a 30k ohm resistor, and a second independent voltage source, v sub g, in series with a 15k ohm resistor.

The op amp has a 5V positive supply voltage, and a minus 5V negative supply voltage. A 100k ohm resistor is connected between the output terminal and inverting input terminal. The inverting input terminal which is grounded with the common terminal is in series with a 20k ohm resistor. The output terminal which is grounded with the common terminal is in series with a 4.7k ohm resistor, with a voltage v sub 0 marked across it.

Three parallel resistors with respective resistances of R sub a, R sub b = 15k ohm, and R sub c are connected between the common terminal and the non-inverting input terminal, with respective voltages marked as v sub a, v sub b, and v sub c, and respective currents, i sub a, i sub b, and i sub c flowing into the input terminal.

The op amp has a 10V positive supply voltage, and a minus 10V negative supply voltage. A resistor, R sub f = 20 ohm, is connected between the inverting input terminal and the output terminal. An independent voltage source, v sub a, connects the inverting input terminal with the common terminal. The output terminal which is grounded with the common terminal is in series with a 27k ohm resistor, with a voltage v sub 0 marked across it.

The non-inverting input terminal is connected to a small circuit before being grounded with the common node. The small circuit contains a 5V independent voltage source, a 8k ohm series resistor, and a 2k ohm parallel resistor.

The op amp has a 9V positive supply voltage, and a minus 9V negative supply voltage. A variable resistor, R sub f, is connected between the inverting input terminal and the output terminal. The output terminal which is grounded with the common terminal is in series with a 5.6k ohm resistor.

A 1.6k ohm resistor in series with the inverting input terminal, and a 7.5k ohm resistor in series with the non inverting input terminal are connected to an 18V independent voltage source. A 1.5k ohm resistor is also connected in parallel across the non inverting input terminal.

The op amp has a 15V positive supply voltage, and a minus 15V negative supply voltage. A resistor, R sub f, is connected between the output terminal and inverting input terminal. The output terminal which is grounded with the common terminal has a voltage v sub 0 marked across it.

An independent current source, i sub a, in parallel with a 2k ohm resistor connects the inverting input terminal with the common terminal. Another independent current source, i sub b, in parallel with a resistor, R sub b, connects the non inverting input terminal with the common terminal.

The op amp has a 20V positive supply voltage, and a minus 20V negative supply voltage. A 180k ohm resistor is connected between the output terminal and inverting input terminal. The output terminal which is grounded with the common terminal is in series with a 47k ohm resistor, with a voltage v sub 0 marked across it.

The inverting input terminal which is grounded with the common terminal is connected to two parallel resistors with respective resistances of 20k ohm and 18k ohm, and with respective voltages of v sub a and v sub b marked across them. The non-inverting input terminal which is grounded with the common terminal is connected to a 20k ohm series resistor, and two parallel resistors with respective resistances of 30k ohm and 20k ohm, and with respective voltages of v sub c and v sub d marked across them.

The op amp has a 10V positive supply voltage, and a minus 10V negative supply voltage. A 100k ohm resistor is connected between the inverting input terminal and the output terminal. The output terminal is grounded with the common terminal, and has a voltage, v sub o, marked across it.

An independent voltage source, v sub a, in series with a 5k ohm resistor connects the inverting terminal with the common terminal. Another independent voltage source, v sub b, which is in series with a 5k ohm resistor and in parallel with a 95k ohm resistor connects the non inverting terminal with the common terminal.

The op amp has a 10V positive supply voltage, and a minus 10V negative supply voltage. A 50k ohm resistor is connected between the inverting input terminal and the output terminal. The output terminal is grounded with the common terminal, and has a voltage, v sub o, marked across it.

An independent voltage source, v sub a, in series with a 20k ohm resistor connects the inverting terminal with the common terminal. Another independent voltage source, v sub b, which is in series with a resistor, R sub x, and in parallel with a 50k ohm resistor connects the non inverting terminal with the common terminal.

The op amp has a positive supply voltage of 10V and a negative supply voltage of minus 10V. A resistor, Rf connects the inverting terminal with the output terminal. The inverting terminal is connected to an independent voltage source, v sub g, along with a series resistor, R1. The output terminal which is grounded with the common terminal is in series with a resistor, R sub L, with a voltage v sub 0 marked across it.

An adjustable resistor, R sub g, with a variable value of "alpha R sub g" connects the inverting terminal with the independent voltage source, v sub g.

The op amp has a resistor, R2, connecting the output terminal with the inverting input terminal. The output terminal is grounded with the common terminal, with a voltage, v sub o, marked across it. The inverting input terminal is in series with a resistor, R1, and is grounded with the common terminal. An independent voltage source, v sub s, in series with a resistor, Rs, connects the non-inverting input terminal with the common terminal.

The X-axis shows time, t in seconds, marking 0.5 to 4 in increments of 0.5, while the y-axis shows voltage, v sub g, marking 0.5V and minus 0.5V on either side of the origin. The graph shows a zigzag curve starting at the origin, and moving from left to right on either side of the x-axis. The curve hits peak values of 0.5V in the first quadrant and minus 0.5V in the fourth quadrant alternatively for every minute, starting from 0.5 seconds.

The op amp has a positive supply voltage of 5V and a negative supply voltage of minus 5V. A 120k ohm resistor connects the inverting terminal with the output terminal. The inverting input terminal is connected to an independent voltage source, v sub g, along with a 7.5k series resistor. The output terminal which is grounded with the common terminal is in series with a 6.8k resistor, with a voltage v sub o marked across it. The non-inverting input terminal is grounded with the common resistor.

The op amp has a positive supply voltage of 10V and a negative supply voltage of minus 10V. A 60k ohm resistor connects the inverting terminal with the output terminal. The inverting input terminal which is grounded with common terminal is connected to a 20k series resistor. The output terminal which is grounded with the common terminal is in series with a 5k ohm resistor, with a voltage v sub o marked across it.

An independent voltage source, v sub g, which is in series with a 1.8k ohm resistor and in parallel with a 5.4k ohm resistor connects the non-inverting input terminal with the common node.

The op amp has a positive supply voltage, V sub CC, and a negative supply voltage, minus V sub CC. A resistor R connects the inverting terminal with the output terminal.

A common node connects three wires to the output terminal, inverting input terminal, and non-inverting input terminal. The output terminal is in series with a resistor R while the inverting input terminal is in series with another resistor R. The non-inverting input terminal is connected to an independent voltage source, v sub g. The common node which is grounded with the common terminal is in series with a resistor, R sub a, with a downward current i sub a flowing through it.

The op amp on the left has a positive supply voltage of 6V and a negative supply voltage of minus 6V. A 47k ohm resistor connects the inverting input terminal with the output terminal. A 1V independent voltage source in series with a 10k ohm resistor connects the inverting input terminal with the common terminal. The non-inverting input terminal is grounded with the common terminal.

The output terminals of the two op amps are connected by 1k ohm resistor which carries a current i sub a from right to left. The op amp on the right has a positive supply voltage of 6V and a negative supply voltage of minus 6V. A 220k ohm resistor connects the inverting input terminal with the output terminal. A 150mV independent voltage source with reverse polarity, in series with a 33k ohm resistor, connects the inverting input terminal with the common terminal. The non-inverting input terminal is grounded with the common terminal.

The op amp on top has a positive supply voltage of 15V and a negative supply voltage of minus 15V. A 500 ohm resistor connects the inverting input terminal with the output terminal. The non-inverting input terminal is connected to a 15V voltage source. The voltage across the output terminal is marked as v sub o1.

The op amp below has a positive supply voltage of 15V and a negative supply voltage of minus 15V. A 400 ohm resistor connects the inverting input terminal with the output terminal. The non-inverting input terminal is connected to a 10V voltage source. The voltage across the output terminal is marked as v sub o2.

The output terminal of the top op amp is connected to the output terminal of the bottom op amp by a 2k series resistor and a 5k ohm parallel resistor. The output terminal of the bottom op amp is grounded, in series with a 1k ohm resistor.

The circuit on the right shows an independent current source, i sub g, with an upward bound current on the left wire, and a load resistor, R sub L, on the right wire, with a current i sub L flowing from top to bottom. The left portion of the circuit including the current source is shaded in a box.

The circuit on left shows the expanded representation of the circuit on the right. The load resistor R sub L with a downward current i sub L is shown as earlier. A shaded box to the left replaces the current source with an op amp circuit. The op amp has a positive supply voltage of 20V and a negative supply voltage of minus 20V. An 8V independent voltage source in series with a 50k ohm resistor connects the non-inverting input terminal with the common node. The resistor R sub L connects the output terminal with the inverting input terminal. The inverting input terminal is grounded with the common node, in series with a 4 ohm resistor.

The op amp has a short circuit between the output terminal and the inverting input terminal. A 320 mV independent voltage source in series with a 48k ohm resistor connects the non inverting input terminal with a Source on the left. The output terminal in series with a 16k ohm resistor is connected to a Load on the right. The source and load circuits, excluding the op amp, are shown in two shaded boxes.

The op amp has a positive supply voltage of 15V and a negative supply voltage of minus 15V. A 24k ohm resistor connects the inverting input terminal with the output terminal. An independent voltage source, v sub s, in series with a 1.6k ohm resistor connects the inverting input terminal with the common node. The output terminal is grounded with common node, with a voltage v sub o marked across it. The non-inverting input terminal is grounded with the common node.

The op amp has a supply voltage of 6V and a negative supply voltage of minus 6V. A 135k ohm resistor connects the inverting input terminal with the output terminal. An independent voltage source, v sub g, in series with a 15k ohm resistor connects the inverting input terminal with the common node. The output terminal is grounded with common node, with a voltage v sub o marked across it. The non-inverting input terminal is grounded with the common node.

The op amp has a supply voltage of 18V and a negative supply voltage of minus 18V. A 80k ohm resistor connects the inverting input terminal with the output terminal. An independent voltage source, v sub g, in series with a 100k ohm resistor connects the non-inverting input terminal with the common node. The output terminal is grounded with common node in series with a 4k ohm resistor, with a voltage v sub o marked across it. The inverting input terminal is grounded with the common node, in series with a 40k ohm resistor.

The strain gage bridge is represented by a diamond shaped circuit with four resistors on its four arms. The resistor on top left has a resistance of "R plus delta R", while the other three resistors have resistances marked as R. The top node of the diamond circuit is powered by an independent voltage source, v sub in, while the bottom node is grounded with the common node. The node on the right is connected to the inverting terminal of an op amp while the node on the left is connected to the non-inverting terminal.

The op amp has a resistor R sub f connected between the output terminal and the inverting terminal. The non-inverting terminal is grounded with the common terminal in series with resistor, R sub f. The voltage across the output terminal is marked as v sub o.

The X-axis plots time, t in seconds, while the y-axis plots current, i in Amps. The graph shows a line starting from the origin, which rises with a steep upward curve to a peak value of 0.736A in 0.2 seconds, and then moves with a decreasing slope to reach close to x-axis, where it runs parallel. Dotted lines are drawn from the peak value of (0.2, 0.736), perpendicular to both the axes.

The X-axis plots time, t in seconds, while the y-axis plots voltage, v in Volts. The graph shows a line starting from y = 1.0, which moves with a decreasing slope to reach x = 0.2, and moves further down into the fourth quadrant, and rises up again to reach x-axis at x = 0.6

The X-axis plots time, t in seconds, ranging from 0 to 1 in increments of 0.1, while the y-axis plots voltage, v in Volts, ranging from 0 to 0.6 in increments of 0.2. The graph shows a line starting from the origin, which rises with a steep upward slope to reach peak value of 0.736 Volts in 0.1 seconds, and then moves down with a steep curve, moving closer to and uniting with the x axis at 1. Dotted lines are drawn from the peak value of (0.1, 0.736), perpendicular to both the axes.

The X-axis plots time, t in seconds, ranging from 0 to 1 in increments of 0.1, while the y-axis plots voltage, v in Volts, ranging from 0 to 2.0 in increments of 0.5. The graph shows a line starting from the origin, which rises with a steep upward curve to reach a plateau of y equal to 2, from where it runs parallel to the x-axis. A dotted line is drawn at y equal to 2.

The X-axis plots time, t in seconds, ranging from 0 to 1.0 in increments of 0.2, while the y-axis plots current, i in mA, ranging from 0 to 800 in increments of 400. The graph shows a line starting from the origin, which rises with a steep upward curve to reach peak value of approximately 700 mA in 0.2 seconds, and then moves in a decreasing slope, moving closer to and parallel to the x-axis at 1.0.

The X-axis plots time, t in seconds, ranging from 0 to 1.0 in increments of 0.2, while the y-axis plots voltage, v in Volts, ranging from 0 to 1.0 in increments of 0.5. The graph shows a line starting from y=1.0, which moves with a decreasing slope to reach x=0.2, moves further down into fourth quadrant, and rises up again, moving closer to and parallel to the x-axis at x=1.0

The X-axis plots time, t in seconds, ranging from 0 to 1.0 in increments of 0.2, while the y-axis plots power, p in mW, ranging from 0 to 200 in increments of 100. The graph shows a line which rises with an upward slope to reach a peak value of y=225 at x=0.1, then moves with a steep downward slope to reach x=0.2, moves further down into fourth quadrant, and finally again rises up, moving closer to and parallel to the x-axis at x=1.0.

The X-axis plots time, t in seconds, ranging from 0 to 1.0 in increments of 0.2, while the y-axis plots energy, w in mJ, ranging from 0 to 30 in increments of 15. The graph shows a line starting from origin, which rises with an upward slope to reach peak value of y=28 at x=0.2, and then moves with a downward slope, ending at x=1.0.

The X-axis plots time, t in seconds, ranging from 0 to 0.6 in increments of 0.1, while the y-axis plots voltage, v in Volts, ranging from 0 to 1.0 in increments of 0.5. The graph shows a line starting from origin, which moves with an upward slope to reach peak value of 0.7V in 0.1 seconds, and then moves with a downward slope to run close to and parallel to the x-axis at x=0.6.

The X-axis plots time, t in seconds, ranging from 0 to 0.6 in increments of 0.2, while the y-axis plots current, i in Amps, ranging from 0 to 2.0 in increments of 1.0. The graph shows a line starting from origin, which rises with a steep upward curve to reach a plateau at y=2.0 and x=0.5, from where it continues to run parallel to the x-axis.

The X-axis plots time, t in seconds, ranging from 0 to 0.6 in increments of 0.2, while the y-axis plots power, p in mW, ranging from 0 to 600 in increments of 300. The graph shows a line, which rises with an upward slope to reach peak value of y=600 at x=0.17, and then moves with a steep downward slope to reach x=0.6.

The X-axis plots time, t in seconds, ranging from 0 to 0.6 in increments of 0.2, while the y-axis plots energy, w in mJ, ranging from 0 to 200 in increments of 100. The graph shows a line starting from origin, which rises from x=0.05 with an upward slope to reach a plateau at y=180 and x=0.5, from where it continues to run parallel to the x-axis.

The X-axis plots time, t in seconds, ranging from 0 to 6 in increments of 1, while the y-axis plots voltage, v in Volts, ranging from 0 to 4 in increments of 2. The graph shows a line starting from the origin, which moves in a steep slope to peak value of 4 Volts in 1 second, and then moves with a downward curve to reach x=6.

The X-axis plots time, t in seconds, ranging from 0 to 6 in increments of 1, while the y-axis plots current, i in microAmps, ranging from minus 2 to 2 in increments of 1. The graph shows two lines which are joined by a vertical dotted line at x=1. The first line runs parallel to the x-axis at y=2, between x=0 and x=1. The second line starts at (1, minus 2) in the fourth quadrant, rises with an upward slope to reach x=3, and continues on the x-axis up to x =6.

The X-axis plots time, t in seconds, ranging from 0 to 6 in increments of 1, while the y-axis plots current, i in microAmps, ranging from minus 8 to 8 in increments of 4. The graph shows two lines which are joined by a vertical dotted line at x=1. The first line starts at the origin and moves straight in an upward slope to (1, 8). The second line starts at (1, minus 8) in the fourth quadrant and rises with an upward slope to reach x=3, and continues on the x axis up to x=6.

The X-axis plots time, t in seconds, ranging from 0 to 6 in increments of 1, while the y-axis plots voltage, v in Volts, ranging from 0 to 4 in increments of 2. The graph shows a line starting from the origin, which moves in a steep slope to a peak value of 4 volts in 1 second, and then moves with a downward curve to reach x=3, and continues on the x-axis up to x=6.

The X-axis plots time, t in microseconds, ranging from 0 to 60 in increments of 10, while the y-axis plots current, i in microAmps, ranging from 0 to 100 in increments of 50. The graph shows a line starting at the origin, which moves in a steep upward sloping line to reach a peak value at (20, 100), and then moves in a downward sloping line to reach x=40, and continues on the x-axis up to x=60.

The X-axis plots time, t in microseconds, ranging from 0 to 60 in increments of 10, while the y-axis plots voltage, v in volts, ranging from 0 to 10 in increments of 5. The graph shows a line starting at the origin, which moves in an upward slope to reach a plateau at (40, 10), and then continues to run parallel to the x-axis.

The X-axis plots time, t in microseconds, ranging from 0 to 60 in increments of 10, while the y-axis plots power, p in milliWatts, ranging from 0 to 500 in increments of 100. The graph shows a line starting at the origin, which moves in an upward slope to reach a peak at (25, 550), and then moves with a downward slope to reach the x-axis at x=40, and continues on the x-axis up to x=60.

The X-axis plots time, t in microseconds, ranging from 0 to 60 in increments of 10, while the y-axis plots energy, w in microJoule, ranging from 0 to 10 in increments of 2. The graph shows a line starting at the origin, which moves in an upward slope to reach a plateau at (40, 10), and continues to run parallel to the x-axis.

The circuit shows the following:

Three inductors are connected in series, with inductances marked from left to right as L1, L2, and L3, and their respective voltage drops marked as v1, v2, and v3.

Current i flows through the three inductors from left to right, while the voltage drop across the entire circuit is marked as v.

The circuit on top shows three inductors in series, with inductances marked from left to right as L1, L2, and L3. Current i flows through the three inductors from left to right, while the voltage drop across the entire circuit is marked as v. Initial current in the circuit is marked as i of t sub 0.

An equivalent circuit is shown below with a single inductor, with an inductance marked as L sub eq = L1 plus L2 plus L3. Current i flows from left to right, while the voltage drop is marked as v. The initial current is marked as i of t sub 0.

A rectangular circuit shows two vertical cross wires connecting two pairs of nodes on the top and bottom wires. Three inductors are located on the left cross wire, right cross wire, and right wire, with the respective inductances marked as L1, L2, and L3. The left wire is removed and the two open terminals marked on the top and bottom wires show a voltage drop of v.

Current i flows through the top wire from left to right. Current and initial current flowing from top to bottom through the three inductors are as follows:

L1: i1; i1 of t sub 0

L2: i2; i2 of t sub 0

L3: i3; i3 of t sub 0

A square circuit shows an inductor, L sub eq, on the right wire, with initial current i of t sub 0 flowing through it from top to bottom. The left wire is removed and voltage drop across the two terminals on the top and bottom wires is marked as v. Current i flows through the top wire from left to right.

Two equations are shown next to this diagram, as follows:

1 over L sub eq = 1 over L1 plus 1 over L2 plus 1 over L3

i open parenthesis t sub 0 close parenthesis = i1 open parenthesis t sub 0 close parenthesis plus i2 open parenthesis t sub 0 close parenthesis plus i3 open parenthesis t sub 0 close parenthesis

A circuit shows several capacitors in series on the right wire, marked from top to bottom as C1, C2 and so on to Cn, with respective initial voltages marked as v1 of t sub 0, v2 of t sub 0 and so on to vn of t sub 0. The left wire is removed and voltage drop across the two terminals on the top and bottom wires is marked as v. Current i flows through the top wire from left to right.

A square circuit shows a capacitor, C sub eq, on the right wire, with an initial voltage drop, v of t sub 0, marked across it. The left wire is removed and voltage drop across the two terminals on the top and bottom wires is marked as v. Current i flows through the top wire from left to right.

Two equations are shown next to this diagram as follows:

1 over C sub eq = 1 over C1 plus 1 over C2 plus and so on to plus 1 over Cn

v open parenthesis t sub 0 close parenthesis = v1 open parenthisis t sub 0 close parenthesis plus v2 open parenthesis t sub 0 close parenthesis plus and so on to plus vn open parenthesis t sub 0 close parenthesis

A rectangular circuit shows two vertical cross wires connecting two pairs of nodes on the top and bottom wires. Three capacitors are located on the left cross wire, right cross wire, and right wire, with respective capacitances marked as C1, C2, and C3. The left wire is removed and the two open terminals marked on the top and bottom wires show voltage drop of v. Current i flows through the top wire from left to right.

A square circuit shows a capacitor, C sub eq, on the right wire. The left wire is removed and voltage drop across the two open terminals on the top and bottom wires is marked as v. Current i flows through the top wire from left to right.

The following equation is shown next to this diagram:

C sub eq = C1 plus C2 plus and so on to plus Cn.

A circuit shows a 24mH inductor on the top wire, a 6mH inductor on the right wire, and a 10mH inductor on the bottom wire. The left wire is removed and the two open terminals at the left ends of the top and bottom wires are marked respectively as a and b.

An additional wire is drawn between the two nodes across the inductor on the top wire. A 12mH inductor is located on this additional wire, with a 10A current flowing through it from left to right. A 6A current flows through the 24mH inductor on the top wire from right to left.

The circuit shows an inductor, L sub eq, on the right wire, with current i flowing rightwards through the top wire. The left wire is removed and the two open terminals at the left ends of the top and bottom wires are marked respectively as a and b.

The circuit shows two nodes on the top and bottom wires joined by a parallel wire, which carries a 14 microF capacitor. The top wire carries two capacitors on its left and right arm, with respective capacitances of 20 microF and 10 microF, and respective voltages of 12V and minus 8V. A 15 microF capacitor is located on the right wire with voltage of 16V. The left wire is removed and the two open terminals at the left ends of the top and bottom wires are marked respectively as a and b.

The circuit shows a capacitor, C sub eq, located on the right wire. The left wire is removed and the two open terminals at the left ends of the top and bottom wires are marked respectively as a and b, with the voltage drop marked as v.

The circuit shows two nodes on the top and bottom wires joined by a parallel wire, which carries a 60 mH inductor and downward current i1 of t. A 240mH inductor is located in parallel on the right wire, with the downward current marked as i2 of t. Current i of t flows rightward through the top wire. The left wire is removed and the voltage drop across the two open terminals at the left ends of the top and bottom wires is marked as v.

The circuit shows a 2 microF capacitor on the top wire with a voltage drop of v1, and an 8 microF capacitor on the right wire with a voltage drop of v2. Current i flows rightward through the top wire. The left wire is removed and the two open terminals are located at the left ends of the top and bottom wires.

The table depicts the following relationships between Inductors and Capacitors, over nine parameters:

*Primary v-i equations:

Inductors: v of t equals L multiplied by numerator di of t over denominator dt

Capacitors: i of t equals C multiplied by numerator dv of t over denominator dt

*Alternate v-i equation:

Inductors: i of t equals 1 over L multiplied by Integral (t sub 0 to t) of v(tau) d-tau plus i of t0

Capacitors: v of t equals 1 over C multiplied by Integral (t sub 0 to t) of i(tau) d-tau plus v of t0

*Initial condition:

Inductors: i of t0

Capacitors: v of t0

*Behavior with a constant source:

Inductors: If i of t equals I, v of t is equal to 0 and the inductor behaves like a short circuit

Capacitors: If v of t equals V, i of t is equal to 0 and the capacitor behaves like an open circuit

* Continuity requirement:

Inductors: i of t is continuous for all time so v of t is finite

Capacitors: v of t is continuous for all time so i of t is finite

* Power equation:

Inductors: p of t equals v of t i of t is equal to L multiplied by i of t multiplied by numerator d i of t over denominator d of t

Capacitors: p of t equals v of t i of t is equal to C multiplied by v of t multiplied by numerator d v of t over denominator d of t

* Energy equation:

Inductors: w of t equals 1 over 2 multiplied by L multiplied by i of t squared

Capacitors: w of t equals 1 over 2 multiplied by C multiplied by v of t squared

* Series-connected equivalent:

Inductors: L sub eq equals sigma (from j = 1 to n) L sub j; i sub eq of t sub 0 equals i sub j of t sub 0 for all j

Capacitors: 1 over C sub eq equals sigma (from j=1 to n) 1 over C sub j; v sub eq of t sub 0 = sigma (from j=I851 to n) v sub j of t sub 0

* Parallel-connected equivalent:

Inductors: 1 over L sub eq equals sigma (from j=1 to n) 1 over L sub j; i sub eq of t sub 0 = sigma (from j=1 to n) i sub j of t sub 0

Capacitors: C sub eq equals sigma (from j = 1 to n) C sub j; v sub eq of t sub 0 equals v sub j of t sub 0 for all j

The circuit on the left shows an inductor L1 on the right wire, a resistor R1 on the top wire, and an independent voltage source, v sub g, on the left wire. The circuit on the right shows an inductor L2 on the left wire and a resistor R2 on the right wire. Mutual inductance associated with magnetic coupling of L1 and L2 is indicated by M, with a double sided arrow pointing to both inductors.

The circuit on the left shows an inductor L1 on the right wire, a resistor R1 on the top wire, and an independent voltage source, v sub g, on the left wire. The mesh current in this circuit is indicated by a clockwise pointing curved arrow, i1.

The circuit on the right shows an inductor L2 on the left wire and a resistor R2 on the right wire. The mesh current in this circuit is indicated by an anticlockwise pointing curved arrow, i2.

Mutual inductance of two coils is marked as M.

A dot is marked near the top right corner of the left circuit, while another dot is marked near the bottom left corner of the right circuit.

The circuit on the left shows an inductor L1 on the right wire, a resistor R1 on the top wire, and an independent voltage source, v sub g, on the left wire. The circuit on the right shows an inductor L2 on the left wire and a resistor R2 on the right wire. Mutual inductance of the two coils is marked as M.

A dot is marked near the top right corner of the left circuit. Current i1 flows rightward through the top wire and enters this terminal, resulting in a self-induced voltage in L1, and a mutually induced voltage in L2. The self induced voltage in L1 is marked as L1 multiplied by di1 over dt, with the dotted terminal marked positive and non-dotted terminal marked negative. The mutually induced voltage in L2 is marked as M multiplied by di1 over dt, with the dotted terminal marked positive and non-dotted terminal marked negative.

Similarly, another dot is marked near the bottom left corner of the right circuit. Current i2 flows leftward through the top wire and enters this terminal, resulting in a self-induced voltage in L2, and a mutually induced voltage in L1. The self induced voltage in L2 is marked as L2 multiplied by di2 over dt, with the dotted terminal marked negative and non-dotted terminal marked positive. The mutually induced voltage in L1 is marked as M multiplied by di2 over dt, with the dotted terminal marked negative and non-dotted terminal marked positive.

The drawing shows a circular magnetic core with a coil wrapped on the left, between terminals A and B, and another coil wrapped on the right, between terminals C and D. Five steps are marked in this diagram to indicate the method of determining dot markings, as follows:

Step 1: Arbitrarily dotted terminal (a dot is marked near terminal D).

Step 2: Refers to the current i sub D, which enters the circuit on the right at terminal D.

Step 3: Refers to the magnetic field generated by the coil on the right, phi sub D, pointing in a clockwise direction.

Step 4: Refers to the current i sub A, which enters the circuit on the left at terminal A (a dot is marked near terminal A).

Step 5: Refers to the magnetic field generated by the coil on the left, phi sub A, pointing in a clockwise direction.

The drawing shows two circuits with the inductor coils facing each other, connected by a dc voltmeter. The circuit on the left has a dc voltage source V sub BB on the left wire, and a resistor R and a Switch on the top wire. Two terminals are marked on the top and bottom wires close to the right wire.

The circuit on the right has an independent voltage source on the right wire. Two terminals are marked on the top and bottom wires close to the left wire.

A shaded box is drawn covering both the coils.

A dotted terminal is marked near the top right corner of the circuit on the left.

A rectangular circuit shows a horizontal cross wire connecting two nodes on the left and right wires, and a vertical cross wire connecting a node at the midpoint of the horizontal cross wire with the bottom wire. The horizontal cross wire carries a 5 ohm resistor and a 20 ohm resistor on its left and right arms respectively, while a 60 ohm resistor is located on the bottom arm of the right wire. An independent current source, i sub g is located on the bottom arm of the left wire, with an upward bound current. A 4H inductor is located on the top wire, with a dot terminal marked near its top left, while a 16H inductor is located on the vertical cross wire, with a dot terminal marked close to its top left. Mutual inductance between the two coils is marked as 8H.

Three mesh currents, i1, i2, and i sub g are respectively marked in the top, bottom right, and bottom left coils of the circuit, indicated by clockwise curving arrows.

A rectangular circuit shows a horizontal cross wire connecting two nodes on the left and right wires, and a vertical cross wire connecting a node at the midpoint of the horizontal cross wire with the bottom wire. The horizontal cross wire carries a 5 ohm resistor and a 20 ohm resistor on its left and right arms, while a 60 ohm resistor is located on the bottom arm of the right wire. A 16A independent current source is located on the bottom arm of the left wire, with an upward bound current. Current i1 flows from left to right on the top wire, while current i2 flows downward through the 60 ohm resistor on the right wire. The node at the midpoint of the right wire is marked as a.

The diagram shows a circuit where a coil with N turns is shown on the right wire. Voltage across the coil is v, while a rightward current, i, flows through the top wire into the coil. The magnetic field around the coil is represented by two rectangles labeled phi threading the coil on the left and right, with an upward arrow marked on their sides.

The diagram shows a rectangular magnetic frame with two coils wrapped on the left and right arms. The coil on the left arm has N1 turns, and forms a closed circuit with an independent current source, i sub s. Voltage across the coil is v1, while current i1 flows through the top wire from left to right. The coil on the right arm has N2 turns, and has a voltage v2 marked across its open ends on the right. Two dot terminals are located close to the top left and top right arms of this frame.

Current i1 creates a flux, phi sub 11, in the left coil, indicated by two rectangles threading the N1 coil on the left and right, with respective directions marked counter clockwise and clockwise. A flux, phi sub 21, is created in both the left and right coils, indicated by a rectangle threading through this square frame, with a clockwise direction marked on it.

The diagram shows a rectangular magnetic frame with two coils wrapped on the left and right arms. The coil on the left arm has N1 turns, and has voltage v1 marked across its open ends, toward the left. The coil on the right arm has N2 turns, and forms a closed circuit with an independent current source, i sub s. Voltage across the coil is v2, while current i2 flows through the top wire from right to left. Two dot terminals are located close to the top left and top right arms of this frame.

Current i2 creates a flux, phi sub 22, in the right coil, indicated by two rectangles threading the N2 coil on the left and right, with respective directions marked clockwise and counter clockwise. A flux, phi sub 12, is created in both the left and right coils, indicated by a rectangle threading through this square frame, with a clockwise direction marked on it.

The circuit on the left shows an inductor L1 on the right wire, with a dot terminal marked close to the top right corner. Voltage across the open wire on the left is v1, while current i1 flows rightward on the top wire.

The circuit on the left shows an inductor L1 on the left wire, with a dot terminal marked close to the top left corner. Voltage across the open wire on the right is v2, while current i2 flows leftward on the top wire.

Mutual inductance between the coils is indicated as M.

The drawing shows a rectangular screen with a grid formed by four vertical wires and four horizontal wires. The vertical wires are labeled as X0, X1, X2, and X3 from left to right, while the horizontal wires are labeled as Y0, Y1, Y2, and Y3 from bottom to top.

The drawing shows a rectangular screen with a small rectangle beneath it representing an electrode. Dotted lines are drawn from the electrode to indicate its charge. A wire connecting the electrode to the ground carries a capacitor, C sub p.

The drawing shows a rectangular screen with a small rectangle beneath it representing an electrode. Dotted lines are drawn from the electrode to indicate its charge. A wire connecting the electrode to the ground carries a capacitor, C sub p. A finger is shown touching the screen and forming a circuit with the ground. This new circuit carries a capacitor, C sub t.

The drawing shows a rectangular screen with two small rectangles beneath it representing an x-grid electrode and a y-grid electrode. Dotted lines are drawn connecting the two electrodes to indicate their charge. A wire connecting the two electrodes carries a capacitor, C sub mxy.

The drawing shows a rectangular screen with two small rectangles beneath it representing an x-grid electrode and a y-grid electrode. Dotted lines are drawn connecting the two electrodes to indicate their charge. A wire connecting the two electrodes carries a capacitor, C sub mxy.

A finger is shown touching the screen and forming a circuit with the ground. This new circuit carries a capacitor, C sub t.

The X-axis plots time t in milliseconds, while the y-axis plots voltage v sub s in microVolts. The graph shows a rectangular pulse moving between x=0 and x=2, peaking at y=5.

The X-axis plots time t in milliseconds, while the y-axis plots current i in millAmps. The graph shows a triangular pulse which starts from the origin, moves with a steep upward slope to (5, 250), and then moves with a steep downward slope to (10, 0).

The X-axis plots time t in seconds, ranging from 0 to 6 in increments of 1, while the y-axis plots voltage v in Volts. The graph shows a line starting from the origin, which moves with a steep downward slope in the fourth quadrant to (1, minus 100), then moves with a steep upward slope, crossing the x-axis at x=2, and moving further to reach a plateau at (3, 100). The line then moves parallel to the x-axis up to (5, 100), before moving with a downward slope to end at x=6.

The circuit shows a switch on the top wire with a connecting wire that moves between its two open parts, labeled as a and b, and located on the right and left, respectively. A 5H inductor connects the switch connector with the bottom wire. The circuit is powered by a 3mV independent voltage source on the left wire and a d'Arsonval Voltmeter on the right wire.

The X-axis plots time t in seconds, ranging from 0 to 8 in increments of 2, while the y-axis plots voltage v in Volts. The graph shows a triangular pulse which moves between x=0 and x=4, peaking at x=2. The pulse then moves in the fourth quadrant, moving between x=4 and x=8, peaking at x=6.

The X-axis plots time t in microseconds, ranging from 0 to 500 in increments of 100, while the y-axis plots current i in mA, ranging from 0 to 50 in increments of 25. The graph shows a downward sloping line drawn from y=50. The line is labeled as 50e raised to the power of minus 2000t mA, t greater than or equal to 0.

The X-axis plots time t in microseconds, ranging from 0 to 50 in increments of 10, while the y-axis plots current i in mA. The graph shows a line starting from the origin, which moves with an upward slope to (5, 400), then moves horizontally to (20, 400) before dropping vertically to (20, minus 300) in the fourth quadrant. The line then moves with an upward slope to reach the x axis at x=50.

The X-axis plots time t in microseconds, ranging from 0 to 35 in increments of 5, while the y-axis plots current i in mA, ranging from minus 4 to 8 in increments of 2. The graph shows a rectangular shaped pulse moving between x=0 and x=5, peaking at y=4. The pulse then moves in the fourth quadrant, between x=5 and x=20, peaking at y=minus 2. In the third cycle, the pulse moves between x=20 and x=25, peaking at y=6 and dropping down to y=4, and then moves between x=25 and x=35, with peak value of y=4.

A rectangular circuit shows a vertical crosswire connecting two nodes on the top and bottom wires. A 20mH inductor is located on the cross wire, while a 15mH inductor is located on the right wire. The top wire shows two inductors with inductances of 24mH and 10mH on its left and right arms, while the bottom wire shows an 8mH inductor on its left arm. An additional wire is drawn between two terminals across the 24mH inductor on the top wire, carrying a 12mH inductor.

Two diagonal wires are drawn from the bottom node of the crosswire to the top left and top right nodes, carrying a 9mH inductor and a 30mH inductor, respectively.

The left wire of the circuit is removed and the two terminals to the left of the top and bottom wires are respectively marked as a and b.

A rectangular circuit shows two parallel vertical crosswires connecting two pairs of nodes onthe top and bottom wires. A 30microH inductor is located on the left cross wire while a 75microH inductor is located on the right cross wire. The top wire shows two inductors with inductances of 25 microH and 18 microH on its left and right arms, while the bottom wire shows two inductors with inductances of 15 microH and 38 microH on its left and right arms. A 12 microH inductor is located on the right wire.

A diagonal wire is drawn from the top node of the left cross wire to the bottom left wire, carrying a 60 microH inductor. A second diagonal wire is drawn from the top node of the right cross wire to the bottom right wire, carrying a 20 microH inductor.

The left wire of the circuit is removed and two terminals to the left of the top and bottom wires are respectively marked as a and b.

A rectangular circuit shows a vertical crosswire connecting two nodes on the top and bottom wires. A switch, t=0, is located on the crosswire. A portion of the left wire is removed between the two terminals and a small rectangular circuit is drawn in this removed portion. A 1H inductor and a 4H inductor are located on the left and right wires of the small rectangular circuit, carrying two respective upward currents, i1 and i2.

A 3.2H inductor is located on the left arm of the bottom wire, carrying a rightward current i sub o. The right wire is removed and two terminals are marked on the top and bottom wires to the right, with reverse voltage, v sub o, marked across them. A rectangular Black box is drawn around these two terminals.

A rectangular circuit shows two vertical crosswires connecting two pairs of nodes on the top and bottom wires. A switch, t=0, is located on the right crosswire. A 3H inductor with an upward current i1 of t is located on the left wire, while a 6H inductor with an upward current i2 of t is located on the left cross wire.

The right wire is removed, and two terminals are marked on the top and bottom wires to the right, with reverse voltage v marked across them. A rectangular Black box is drawn around the two terminals.

A rectangular circuit shows a vertical crosswire connecting two nodes on the top and bottom wires, carrying an 8nF capacitor. A diagonal wire is drawn from the top node of the cross wire to the bottom right node, carrying a 4nF capacitor. A 24nF capacitor with 10V voltage is located on the right wire.

The top wire carries two capacitors on the left and right arms, with respective capacitances of 20nF and 48nF, and respective voltages of minus 40V and 30V. The bottom wire carries two capacitors on the left and right arms, with respective capacitances of 10nF and 30nF, and respective voltages of 5V and minus 20V.

The left wire is removed and two terminals on the top and bottom wires to the left are respectively marked as a and b.

A rectangular circuit shows a vertical crosswire connecting two nodes on the top and bottom wires, carrying a 24microF capacitor. An 18 microF capacitor with minus 2V voltage is located on the right wire.

The top wire carries two capacitors on the left and right arms, with respective capacitances of 5 microF and 36 microF. Voltage across the 36 microF capacitor is marked as 5V. An additional wire is drawn between two nodes across the 5 microF capacitor. This wire carries a 25 microF capacitor, with voltage marked as 10V. The bottom wire carries two capacitors on the left and right arms, with respective capacitances of 30 microF and 12 microF, and respective voltages of minus 20V and 8V.

The left wire is removed and two terminals on the top and bottom wires to the left are respectively marked as a and b.

A rectangular circuit shows a switch, t=0, located on the top wire. Two capacitors are located in series on the left wire, with capacitances marked as 3 microF and 6 microF, and respective voltages marked as 4V and 6V. A rightward current, i of t, flows in the top wire.

The right wire is removed and two terminals are marked on the top and bottom wires to the right, with voltage v marked across them. A rectangular Black box is drawn around the two terminals.

A rectangular circuit shows a switch, t=0, located on the top wire, with a 5 microF capacitor located to its left, having voltage v sub a. A rightward current, i sub b, flows in the top wire to the right of the switch. The bottom wire carries a 1.25 microF capacitor with reverse voltage v sub c. A portion of the left wire is removed between the two terminals, and a small rectangular circuit is drawn in this removed portion. Reverse voltage across these terminals is marked as v sub d. A 200 microF capacitor and an 800 microF capacitor are located on the left and right wires of the small rectangular circuit, carrying two respective upward currents, i1 and i2.

The right wire of the circuit is removed and two terminals are marked on the top and bottom wires to the right, with voltage v sub b marked across them. A rectangular Black box is drawn around the two terminals.

A rectangular circuit shows a switch, t=0, located on the top wire, with a rightward current i sub o flowing to its right. A 25mH inductor is located to the right of the switch on the top wire. A 625 nF capacitor with voltage v sub c is located on the left wire.

The right wire of the circuit is removed and two terminals are marked on the top and bottom wires to the right, with voltage v sub o marked across them. A rectangular Black box is drawn around the two terminals.

A rectangular circuit has a 5 microF capacitor on the left wire with voltage v1, a 10 mH inductor on the right wire with voltage v2, and a 40 ohm resistor on the top wire. The mesh current in the middle of the circuit is marked as i sub o.

A rectangular circuit shows a vertical cross wire connecting two terminals on the top and bottom wires. An inductor L1 is located on this cross wire, with a dot terminal marked to its top left. A second inductor L2 is located on the right wire, with a dot terminals marked at its top left. Mutual inductance between these two coils is marked as M. The left wire is removed and two terminals at the top and bottom wires to the left are respectively marked as a and b.

The circuit on the left shows an inductor L1 on the right wire, with a dot terminal marked close to the top right corner. An independent current source, i sub g, with an upward current flow, is located on the left wire, with voltage marked as v1. A cross wire is drawn between the top and bottom wires, carrying a switch, t=0.

The circuit on the right shows an inductor L2 on the left wire, with a dot terminal marked close to the bottom left corner. Resistor R sub o is located on the right wire. A mesh current, i2, is marked at the middle of this circuit.

Mutual inductance between the two coils is marked as M.

The drawing shows two circuits with inductor coils facing each other. A shaded box is drawn covering both the coils. The circuit on the left has a dc voltage source V sub BB on the left wire, and a resistor R and a Switch t=0 on the top wire.

The circuit on the right has a dc voltmeter on the right wire. Two terminals are marked at the midpoints of the top and bottom wires.

A dotted terminal is marked near the top right corner of the circuit on the left.

The drawing shows a rectangular frame with a coil located on its left arm having end terminals marked as 1 and 2 near the top and bottom. A second coil is located on the bottom arm of the frame, with end terminals marked as 3 and 4 on the left and right.

The drawing shows a rectangular frame with a cross arm connecting the top and bottom arms. Two coils are located on this cross arm, one below the other. The coil on top has its end terminals marked as 1 and 2, while the coil at the bottom has its end terminals marked as 3 and 4.

The circuit shows inductor L on the right wire, with voltage v marked across it, and resistor R sub Th on the top wire, with a rightward current i flowing through it. The circuit is powered by an independent voltage source, V sub Th, which is located on the left wire. Two nodes are marked on the top and bottom wires close to the right wire.

The circuit shows inductor L on the right wire, with voltage v marked across it, and resistor R sub Th located on a vertical crosswire connecting two nodes on the top and bottom wires. A rightward current, i, flows through the top wire. Two nodes are marked on the top and bottom wires close to the right wire. The circuit is powered by an independent voltage source, V sub Th over R Sub Th, which is located on the left wire.

The circuit shows capacitor C on the right wire, with voltage v marked across it, and resistor R sub Th on the top wire, with a rightward current i flowing through it. The circuit is powered by an independent voltage source, V sub Th, which is located on the left wire. Two nodes are marked on the top and bottom wires close to the right wire.

The circuit shows a capacitor C on the right wire, with voltage v marked across it, and resistor R sub Th located on a vertical crosswire connecting two nodes on the top and bottom wires. A rightward current, i, flows through the top wire. Two nodes are marked on the top and bottom wires close to the right wire. The circuit is powered by an independent voltage source, V sub Th over R Sub Th, which is located on the left wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes located on the top and bottom wires. Resistor R sub 0 is located on the left cross wire, while inductor L is located on the right cross wire. A switch, t=0, is located on the middle arm of the top wire, in between the two cross wires.

Resistor R with voltage v is located on the right wire. A rightward current, i, flows through the right arm of the top wire. The circuit is powered by an independent current source, I sub s, with a downward current, located on the left wire.

The circuit shows inductor L on the left wire, with an upward current, i(0) = I sub s, flowing through it. Resistor R with voltage v is located on the right wire. A rightward current, i, flows through the top wire.

The X-axis plots time t, while the y-axis plots current i of t. The graph shows a line which starts at y=I sub 0, and moves with a downward slope to reach the x-axis at the bottom right corner.

The circuit shows three parallel vertical cross wires connecting three pairs of nodes on the top and bottom wires. Two resistors with resistances of 0.1 ohm and 10 ohm are located on the left and right cross wires, while a 2H inductor with a downward current i sub L flowing through it, is located on the middle cross wire. A 2 ohm resistor is located on the third arm of the top wire, and a 40 ohm resistor with voltage v sub o is located on the right wire.

A switch, t=0, is found on the second arm of the top wire. The circuit is powered by a 20A independent current source with an upward bound current, located on the left wire.

The circuit shows three parallel vertical cross wires connecting three pairs of nodes on the top and bottom wires. Two resistors with resistances of 0.1 ohm and 10 ohm are located on the left and right cross wires, while a downward current, I sub 0, flows through the middle cross wire. A 2 ohm resistor is located on the third arm of the top wire, and a 40 ohm resistor is located on the right wire. The circuit is powered by a 20A independent current source with an upward bound current, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 10 ohm resistor. A 40 ohm resistor with voltage v sub o is located on the right wire, while a 2 ohm resistor is found on the left arm of the top wire. A rightward current, i sub o, is marked on the right arm of the top wire. A 2H inductor with a downward current, i sub L, is located on the left wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes located on the top and bottom wires. A 30 ohm resistor is located on the left cross wire, while an 8mH inductor with an upward current i is located on the right cross wire. Two resistors with resistances of 3 ohm and 6 ohm are found respectively on the left and middle arms of the top wire, while a 2 ohm resistor is located on the right wire. A switch, t=0, is located on the middle arm of the top wire, to the right of the 6 ohm resistor. The circuit is powered by a 120V independent voltage source, located on the left wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes located on the top and bottom wires. A 10 ohm resistor with voltage v sub o is located on the left vertical cross wire, while a 0.32H inductor is located on the right cross wire. A 6 ohm resistor is located on the middle arm of the top wire, and a 4 ohm resistor is located on the right wire. The circuit is powered by a 6A independent current source with an upward current, located on the left wire.

Two unconnected short wires, labeled as a and b, are drawn from the left arms of the top and bottom wires respectively. A switch, t=0, connects these short wires to a node on top of the left wire.

The table displays the following data:

t: e raised to the power of minus t over tau

Tau: 3.6788 multiplied by 10 raised to minus 1

2Tau: 1.3534 multiplied by 10 raised to minus 1

3Tau: 4.9787 multiplied by 10 raised to minus 2

4Tau:1.8316 multiplied by 10 raised to minus 2

5Tau: 6.7379 multiplied by 10 raised to minus 3

6Tau: 2.4788 multiplied by 10 raised to minus 3

7Tau: 9.1188 multiplied by 10 raised to minus 4

8Tau: 3.3546 multiplied by 10 raised to minus 4

9Tau: 1.2341 multiplied by 10 raised to minus 4

10Tau: 4.5400 multiplied by 10 raised to minus 5

The X-axis plots time t, while the y-axis plots current i. The graph shows a line which starts at y=I sub 0, and moves with a downward slope to reach close to the x-axis at the bottom right corner. This sloping line is labeled as i = I sub 0 multiplied by e raised to the power of minus t over tau.

There is another straight line that is drawn with a downward slope, tangential to the sloping line, between y=I sub 0 and x=tau. This line is labeled as i = I sub 0 minus multiplication of (I sub 0 over tau) by t.

The circuit shows four parallel vertical cross wires drawn between four pairs of nodes on the top wire and bottom wire. Inductor L1=5H, with initial current 8A moving upward, is located on the left wire. A second inductor, L2=20H, with initial current 4A moving upward, is located on the first cross wire from the left. A switch, t=0, is located on the second cross wire.

Two resistors with resistances of 40 ohm and 15 ohm are located on the third and fourth cross wires, respectively. A 4 ohm resistor is located on the top wire, between the third and fourth cross wires, while a 10 ohm resistor is located on the right wire.

Two unconnected nodes are located at the midpoints of the top and bottom wires. Voltage across these nodes is marked as v of t. Three downward currents, i1, i2, and i3, are marked in the left wire, first cross wire from left, and right wire, respectively.

The circuit shows a pair of nodes on the top and bottom wires, with voltage across them marked as v of t. A 4H inductor, with initial current of 12A moving upward, is located on the left wire, while an 8 ohm resistor is located on the right wire. A current i flows rightward on the top wire.

The circuit shows two nodes, a and b, located on the top wire, with connection between them removed. A vertical cross wire is drawn from a node on the bottom wire, with a switch, t=0, located on its top end. The switch connects to both the nodes a and b.

A capacitor, C, is located on the vertical cross wire below the switch. Resistor R1 is found on the left arm of the top wire, while a second resistor R is found on the right wire. The circuit is powered by an independent voltage source, V sub g, on the left wire.

The circuit shows capacitor C with voltage V sub g on the left wire, and resistor R with a downward current i on the right wire. Voltage across the top and bottom wires is marked as v.

The X-axis plots time t, while the y-axis plots voltage v of t. The graph shows a line which starts at y=V sub 0, and moves with a downward slope to reach close to the x-axis at the bottom right corner. This sloping line is labeled as v of t = V sub 0 multiplied by e raised to the power of minus t over tau.

A straight line is drawn with a downward slope, tangential to the sloping line, intersecting the x-axis at x=tau. This line is labeled as v of t = V sub 0 minus multiplication of (V sub 0 over tau) by t.

The circuit shows two nodes, x and y, located on the top wire, with connection between them removed. A vertical cross wire is drawn from a node on the bottom wire, with a switch, t=0, located on its top end. The switch connects to both the nodes x and y. A 0.5 microF capacitor, with voltage v sub C, is located on the vertical cross wire, below the switch.

A second vertical cross wire is located close to the right wire, connecting two nodes on the top and bottom wires. This cross wire carries a 240k ohm resistor, with voltage v sub o marked across it. The right wire carries a 60k ohm resistor. Two resistors are located on the top wire, on either side of the x and y nodes, with respective resistances marked as 10k ohm and 32k ohm. A rightward current, i sub o, flows in the right arm of the top wire.

The circuit is powered by a 100V independent voltage source located on the left wire.

The circuit shows a 100V independent voltage source on the left wire and a 10k ohm resistor on the top wire. Two open terminals are located on the right wire, with voltage between them marked as V sub 0.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 240 ohm resistor, with voltage v sub o marked across it. A 32k ohm resistor is located on the left arm of the top wire, while a 60k ohm resistor is located on the right wire. A rightward current, i sub o, flows in the right arm of the top wire. A 0.5 microF capacitor, with initial voltage of v sub C, is located on the left wire.

The circuit shows a capacitor C2 on the lower part of the left wire, with capacitance of 20 microF and initial voltage of 24 V. Another resistor, C1, is located on the upper part of the left wire, with capacitance of 5 microF and reverse voltage of 4V. A switch, t=0, is located on the top wire, while a 250k ohm resistor is found on the right wire. Current i of t flows downwards on the right wire.

Voltages across the top capacitor, bottom capacitor, and resistor are marked respectively as v1 of t, v2 of t, and v of t.

The circuit shows a 4 microF capacitor on the left wire, with 20V voltage; a 250k ohm resistor on the right wire, with voltage marked as v of t and a downward current i of t flowing through it; and a switch t=0 located on the top wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries an 80k ohm resistor, while the right cross wire carries a 0.4 microF capacitor. A 20k ohm resistor and a switch, t=0, are located in the middle arm of the top wire, between the two cross wires.

A 50k ohm resistor, with voltage v of t marked across it, is located on the right wire. The circuit is powered by a 7.5mA independent current source with an upward current, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, close to the right wire. A 1 microF capacitor is located on this cross wire, while a 40k ohm resistor is located on the right wire. The top wire carries two resistors on its left arm, with resistances of 15k ohm and 20k ohm. A switch, t=0, is located between the two resistors.

An additional wire is drawn between the two nodes on either side of the 20k ohm resistor on the top wire. This wire carries a 5 microF capacitor. The circuit is powered by a 15V independent voltage source. Voltage between the top and bottom wires is marked as vo of t.

The circuit shows resistor R and switch t=0 on the top wire, and inductor L with voltage v of t on the right wire. A DC voltage source, V sub s, is located on the left wire. A mesh current, "i" is indicated in the middle of the circuit by a curved clockwise arrow.

The circuit shows two points, b and a, located on the top wire, with connection between them removed, and two semicircular wires drawn from them toward each other. A vertical cross wire is drawn from a node on the bottom wire, with switch t=0 located on its top end. The switch connects to both semicircular wires from a and b.

A 200mH inductor is located on the vertical cross wire, below the switch. Voltage across the inductor is marked as v, while current i flows through it from top to bottom. Another cross wire is drawn across two nodes on the top and bottom wires, close to the right wire. A 10 ohm resistor is located on this cross wire, while a 2 ohm resistor is located on the left arm of the top wire. The circuit is powered by a 24V DC voltage source located on the left wire, and an 8A independent current source with a downward current located on the right wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 10 ohm resistor. An 8A independent voltage source, with a downward current, is located on the right wire. Two nodes are located on the left wire, with a downward current between them marked as I sub 0.

The circuit shows a 200mH inductor on the right wire, a 2 ohm resistor on the top wire, and a 24V DC voltage source on the left wire.

The circuit shows a 2 ohm resistor on the top wire, a 24V DC voltage source on the left wire, and two nodes on the right wire, with a downward current I sub f flowing between them.

The X-axis plots time t in ms, ranging from 100 to 500 in increments of 100. The Y-axis plots current i in Amps, ranging from minus 8 to 12 in increments of 4, and voltage v in Volts on another scale, ranging from 8 to 40 in increments of 8. This graph shows two curved lines, representing current and voltage waveforms.

A line starts from y=minus 8 in fourth quadrant and rises with an upward slope, moves into the first quadrant, reaches a plateau at y=12, and runs parallel to the x-axis. This line is labeled as i. A second line starts from y=40 and moves with a downward slope to reach the x-axis at x=400. This line is labeled as v.

A dotted line is drawn between the two curves from the point where the current waveform intersects the x-axis.

X-axis plots time t, ranging from 1 tau to 5 tau in increments of 1 tau, while y-axis plots current i. Graph shows a line which starts at origin and rises with a steep upward slope to reach a plateau, from where it runs parallel to the x-axis. This line is represented by equation i = V sub s over R minus multiplication of V sub s over R and e raised to the power of minus t over tau."

An upward sloping straight line is drawn tangential to the curve close to the origin. This line is represented by the equation, "i = V sub s over L multiplied by t. A dotted vertical line is drawn from 1Tau, intersecting the curved line and straight line at two points, whose respective y-coordinates are marked by dotted lines as 0.632 V sub s over R and V sub s over R.

The X-axis plots time t, ranging from 1 tau to 5 tau in increments of 1 tau, while the y-axis plots voltage v. The graph shows a line which starts at y=V sub s and moves down with a steep downward slope to reach the x-axis at x=5 tau. This line is represented by equation v = V sub s multiplied by e raised to the power of minus R over L multiplied by t.

A downward sloping straight line is drawn tangential to the curve at y=V sub s. This line is represented by equation v = V sub s minus multiplication of R over L and V sub s and t.

A dotted vertical line is drawn from 1Tau, intersecting the curved line at a point whose y-coordinate is marked by a dotted horizontal line as 0.368 multiplied by V sub s.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying resistor R. A switch, t=0, is located on the right arm of the top wire, while capacitor C, with voltage v, is located on the right wire. A leftward current, i, flows on the right arm of the bottom wire. The circuit is powered by an independent current source, I sub s, with an upward current, located on the left wire.

The circuit shows two nodes, 1 and 2, located on the top wire, with connection between them removed. A vertical cross wire is drawn from a node on the bottom wire, with a switch, t=0, located on its top end. The switch connects to both the nodes 1 and 2. A 0.25 microF capacitor, with voltage v sub o, is located on the vertical cross wire, below the switch.

The circuit shows two more cross wires on either side of the switch-carrying cross wire, connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a 60k ohm resistor, while the right cross wire carries a 160k ohm resistor. The top wire carries three resistors on its left, middle, and right arms, with respective resistances of 20k ohm, 8k ohm, and 40k ohm. A leftward current, i sub o, flows through the 8k ohm resistor on the top wire.

The circuit is powered by a 40V independent voltage source located on the left wire, and a 75V independent voltage source with reverse voltage located on the right wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 60k ohm resistor. A 20k ohm resistor is located on the left arm of the top wire. A 40 V independent voltage source is located on the left wire. Two open terminals are located on the right wire, with voltage V sub 0 marked between them.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 160k ohm resistor. The top wire carries an 8k ohm resistor and a 40k ohm resistor on its left and right arms, respectively. A 0.25 microF capacitor is located on the left wire. The circuit is powered by a 75V independent voltage source with reverse polarity, located on the right wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 40k ohm resistor. A 0.25 microF capacitor is located on the left wire, and a 1.5mA independent current source, with dow nward current, is located on the right wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 40k ohm resistor. Two open terminals are located on the left wire, with voltage between them marked as V sub f. A 1.5mA independent current source, with a downward current, is located on the right wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 100 ohm resistor, with voltage v sub o marked across it. A switch, t=0, is located on the left arm of the top wire, while a 25 ohm resistor, with a rightward current i sub o, is located on the right arm of the top wire. A 50 mH inductor is located on the right wire. The circuit is powered by a 75mA independent current source, with an upward current, located on the left wire.

The circuit shows two nodes, b and a, located on the top wire, with connection between them removed. A vertical cross wire is drawn from a node on the bottom wire, with a switch, t=0, located on its top end. The switch connects to both nodes b and a.

A 0.5microF capacitor is located on the vertical cross wire, below the switch. Voltage across the capacitor is marked as v sub C, while current i flows through it from top to bottom. Another cross wire is drawn across two nodes on the top and bottom wires, close to the right wire. A 60 ohm resistor is located on this cross wire, while a 400k ohm resistor and a 20 ohm resistor are located on the left and right arms of the top wire. The circuit is powered by a 90V DC voltage source on the left wire, and a 40V DC voltage source with reverse polarity on the right wire.

The X-axis plots time t in ms, ranging from 0 to 800 in increments of 200. The Y-axis plots current i in microAmps, ranging from minus 50 to 300 in increments of 50, and voltage v sub C in Volts on another scale, ranging from minus 20 to 120 in increments of 20. This graph shows two curved lines, representing current and voltage waveforms.

A line starts from y=minus 30 in the fourth quadrant, rises with an upward slope, moves into the first quadrant, reaches a plateau at y=90, and runs parallel to the x-axis. This line is labeled as v sub C. A second line starts from y=300 and moves with a downward slope to reach the x-axis at x=800. This line is labeled as i.

A dotted line is drawn between the two curves from the point where the voltage waveform intersects the x-axis.

The Circuit shows a 1 ohm resistor and a 3 ohm resistor on the top wire, an 80mH inductor on the right wire, and a 20V independent voltage source on the left wire. An additional wire is drawn between two nodes on either side of the 3 ohm resistor on the top wire. A switch, t=0, is located on this additional wire. Two unconnected nodes are located on the top and bottom wires, close to the right wire. Voltage between these nodes is marked as v of t. A rightward current, i of t, flows through the right arm of the top wire.

The circuit shows a vertical cross wire drawn between two nodes on the top and bottom wires, carrying a 5nF capacitor with voltage v sub o. A 75k ohm resistor and a switch, t=0, are located on the left arm of the top wire, while another resistor with resistance of 25k ohm is located on the right wire. The circuit is powered by a 60V independent voltage source, located on the left wire.

The circuit shows a vertical cross wire drawn between two nodes on the top and bottom wires, carrying a 20k ohm resistor with voltage v of t. A switch, t=0, and a 0.1microF capacitor are located on the right arm of the top wire, while another resistor with resistance of 30k ohm, is located on the right wire. A rightward current, i of t, flows through the capacitor. The circuit is powered by a 7.5mA independent current source, located on the left wire.

The circuit shows a vertical cross wire drawn between two nodes on the top and bottom wires, carrying a 3H inductor with voltage v sub o and a downward current i1. A dot terminal is marked near the top left corner of this inductor. A second inductor with an inductance of 15H is located on the right wire, with a downward current i2 flowing through it. Another dot terminal is marked near the top left corner of this inductor. Mutual inductance between the two inductors is marked as 6H.

A 7.5 ohm resistor and a switch, t=0, are located on the left arm of the top wire, with current i sub 0 flowing rightward from the switch. The circuit is powered by a 120V independent voltage source located on the left wire.

The circuit shows a 1.5H inductor on the right wire, with voltage v sub o marked across it. A rightward current, i sub o, is marked in the top wire. A 7.5 ohm resistor and a switch, t=0 are located on the top wire as well. The circuit is powered by a 120V independent voltage source, located on the left wire.

The circuit shows three parallel vertical cross wires connecting three pairs of nodes on the top and bottom wires. Switch 1, where t=0, is located between the first two cross wires on the left, while switch 2, with t=35 ms, is located between the third cross wire and the right wire.

A 12 ohm resistor and a 6 ohm resistor are located respectively on the left and middle cross wires, while a 150mH inductor is located on the right cross wire, with voltage marked as v sub L and a downward current marked as i sub L. An 18 ohm resistor is located on the right wire, while the top wire carries a 4 ohm resistor on its left arm and a 3 ohm resistor on its middle right arm.

The circuit is powered by a 60V independent voltage source located on the left wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. A 12 ohm resistor and a 6 ohm resistor are located on the left and right cross wires, respectively. The top wire carries a 4 ohm resistor and a 3 ohm resistor on its left and right arms. The circuit is powered by a 60V independent voltage source on the left wire. A downward current, i sub L(0 minus), is marked on the right wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. A 6 ohm resistor and a 150 mH inductor are located on the left and right cross wires, respectively. The top wire carries a 3 ohm resistor on its middle arm, between the two cross wires. An open switch, labeled as 1, is located on the left arm of the top wire, while a closed switch, labeled as 2, is located on the right arm. An 18 ohm resistor is located on the right wire.

The left wire of the circuit is removed. Voltage across the inductor is marked as v sub L, while initial current, which flows downward through the inductor, is marked as i sub L. The inductor current is marked as i sub L(0 plus) = 6A.

The circuit shows a 6 ohm resistor on the left wire and a 3 ohm resistor on the top wire. A 150 mH inductor is located on the right wire with voltage marked across it as v sub L. The initial current, which flows downward through the inductor, is marked as i sub L. The inductor current is marked as i sub L(0.035) = 1.48A.

The circuit shows a switch in the middle of the top wire, which moves between three nodes marked as a, b, and c. Node a is an independent node positioned above the top wire. Node b is on the left arm of the top wire. Node c is on top of a vertical cross wire, which connects to a node on the bottom wire.

The cross wire carries a 50k ohm resistor, while the left arm of the top wire carries a 100k ohm resistor. A 0.1 microF capacitor is located on the right wire, with voltage across it marked as v of t. The circuit is powered by a 400V independent voltage source located on the left wire.

The X-axis plots time t in ms, ranging from 0 to 25 in increments of 5, while the y-axis plots voltage v in Volts, ranging from 0 to 300 in increments of 100. The graph shows a line starting from the origin, which moves with an upward slope to a peak value of (15, 300), and then moves with a downward slope to reach close to x = 25. A dotted line is drawn to the x-axis from the peak point. The upward slope is marked as v = 400 minus multiplication of 400 and e raised to the power of minus 100t. The downward slope is marked as v = 310.75 multiplied by e raised to the power of minus 200 times t minus 0.015.

The circuit shows three parallel vertical cross wires connecting three pairs of nodes on the top and bottom wires. A 40k ohm resistor, a 24k ohm resistor, and a 1 microF capacitor are located on the left, middle, and right cross wires, respectively. Voltage across the capacitor is marked as v sub c of t.

A 60k ohm resistor is located on the second arm of the top wire, while a 100k ohm resistor is located on the right wire. The top wire carries two switches on its second and fourth arms, labeled as 1 and 2. Switch 1, which is located to the left of the 60k ohm resistor, has a t value of 0, while switch 2 has a t value of 10ms. The circuit is powered by a 10mA independent current source with an upward current, located on the left wire.

The circuit shows three parallel vertical cross wires connecting three pairs of nodes on the top and bottom wires. A 3 ohm resistor, a 2H inductor, and a 3 ohm resistor are located on the left, middle, and right cross wires, respectively. A downward current, i, flows through the inductor.

A 10V independent voltage source is located on the left wire, while a 6 ohm resistor is located on the right wire. The top wire carries a 2 ohm resistor, a 0.8 ohm resistor, and a 9 ohm resistor on its first, second, and fourth arms, respectively. A switch labeled as a is located on the second arm of the top wire, to the right of the 0.8 ohm resistor. Time gap of the switch is indicated as t = 0 and t = 1s.

An additional wire connects two nodes of the fourth arm of the top wire on either side of the 9 ohm resistor. A horizontal cross wire, carrying an 8A independent current source, connects the midpoints of the left and right arms of the additional wire. A second switch, b, is located on the top arm of the additional wire. Time duration of this switch is marked as t = 1s.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. A 10k ohm resistor and a 7i sub delta dependent current source with an upward current are located on the left and right cross wires, respectively. The left wire carries a 5 microF capacitor with 10V voltage, while the right wire carries a 20k ohm resistor with a downward current i sub delta. A switch, t = 0, is located on the left arm of the top wire. Voltage between the top and bottom wires is marked as v sub o.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. A 10k ohm resistor and a 7i sub delta dependent current source with an upward current are located on the left and right cross wires, respectively. The left wire is removed and two nodes are marked on the top and bottom wires to the left, with voltage marked across them as v sub T. A rightward current, i sub T, flows through the top wire. The right wire carries a 20k ohm resistor with a downward current i sub delta.

The circuit shows a minus 5k ohm resistor on the right wire and a 5 microF capacitor with 10V voltage on the left wire. A switch, t = 0, is located on the top wire. Voltage between the top and bottom wires is marked as v sub o.

The shown op amp has a positive supply voltage, V sub CC, and a negative supply voltage, minus V sub CC. A capacitor, C sub f, is connected between the output terminal and inverting input terminal, with the leftward current i sub f flowing through it. The output terminal, which is grounded with the common node, has voltage v sub 0 marked across it.

An independent voltage source, v sub s, which is in series with resistor R sub s, connects the inverting terminal with the common node. A rightward current, i sub s, flows through the resistor. The non-inverting terminal is grounded with the common node. Voltages across the inverting and non-inverting terminals are marked respectively as v sub p and v sub n.

The X-axis plots time t and marks t1 and 2t1, while the y-axis plots voltage v sub s and marks minus V sub m and V sub m on either side of the origin. The graph shows an input voltage signal represented as a rectangular pulse between x = 0 and x = t1, peaking at y = V sub m. The pulse continues in the fourth quadrant, between x = t1 and x = 2t1, peaking at y = minus V sub m.

The X-axis plots time t and marks t1 and 2t1, while the y-axis plots voltage v sub o. The graph shows a triangular pulse in the fourth quadrant, between x = 0 and x = 2t1, peaking at y = minus numerator V sub m multiplied by t1 over denominator R sub s multiplied by C sub f. The x value for peak value of the pulse is t1. Dotted lines are drawn from the peak to both the axes.

The X-axis plots time t and marks 1 and 2, while the y-axis plots voltage v sub o. The graph shows a triangular pulse in the fourth quadrant, between x = 0 and x = 2, peaking at y = minus 5 for x = 1. Dotted lines are drawn from this peak to both the axes.

The shown op amp has a 6V positive supply voltage and a minus 6V negative supply voltage. A 0.1 microF capacitor with 5V voltage is connected between the output terminal and the inverting input terminal. The output terminal, which is grounded with the common node, has voltage v sub 0 marked across it. The non-inverting terminal is grounded with the common node.

The inverting terminal is connected to a 100k ohm resistor and a switch, t = 9ms, which alternates between two nodes, a and b. Node a connects to the common node through a 10V independent voltage source with reverse polarity. Node b connects to the common node through an 8V independent voltage source.

The shown op amp has a 10V positive supply voltage and a minus 15V negative supply voltage. A 0.2 microF capacitor is connected between the output terminal and the inverting input terminal. The output terminal, which is grounded with the common node, has voltage v sub 0 marked across it. The non-inverting terminal is grounded with the common node.

The inverting terminal is connected to a 160k ohm resistor and a switch, t = 32ms, which alternates between two nodes, a and b. Node a connects to the common node through a 40k ohm series resistor and a 10V independent voltage source with reverse polarity. Node b connects to the common node through a 90k ohm series resistor and a 5V independent voltage source.

The shown op amp has a 5V positive supply voltage and a minus 5V negative supply voltage. A 40k ohm resistor is connected between the output terminal and the inverting input terminal. The output terminal is in series with a 6.8k ohm resistor, before being grounded with the common node, and has voltage v sub 0 marked across it. The inverting terminal is similarly in series with a 10k ohm resistor before being grounded with the common node.

The non-inverting terminal is connected to a 2V independent voltage source with reverse polarity through a switch, t = 0. Voltage source is in series with a 160k ohm resistor, located to the left of the switch, and is in parallel with a 0.01microF capacitor, located to the right of the switch. The voltage source and the capacitor are grounded with the common node.

The circuit shows a vertical crosswire connecting two nodes on the top and bottom wires. Capacitor C is located on the cross wire, while resistor R is located on the left arm of the top wire. A DC voltage source, V sub s, is located on the left wire. Voltage across the top and bottom wires is marked as v sub C.

A box labeled as container is located on the right wire, with two internal terminals connected to the circuit.

The X-axis plots time t, while the y-axis plots voltage v sub C of t. The graph shows continuous near-triangular pulses between x=0 and x=t sub c, peaking at y=V sub max. Each pulse moves with an upward slope to reach maximum value and then drops abruptly to the x-axis.

The circuit shows capacitor C on the right wire with voltage v sub C marked across it, and resistor R on the top wire. A DC voltage source, V sub s, is located on the left wire.

The circuit shows a vertical crosswire connecting two nodes on the top and bottom wires. A 2k ohm resistor with a downward current i2 is located on this cross wire, while a 400mH inductor is located on the right wire. The top wire carries a 500 ohm resistor and a 6k ohm resistor on its left and right arms. A switch, t = 0, is located on the left arm of the top wire, to the right of the resistor. Current i1 flows rightward through the 6k ohm resistor on the top wire.

The circuit is powered by a 40V independent voltage source located on the left wire.

The circuit shows a switch, t = 0, in the middle of the top wire, which moves between two wires marked as a and b. The wire a extends into the left arm of the top wire. Wire b is at the top end of a vertical cross wire, which connects to a node on the bottom wire.

A 70 ohm resistor with voltage v2 is located on the cross wire. A 30 ohm resistor and a 90 ohm resistor are located on the left and right arms of the top wire, respectively. A rightward current i flows through the 90 ohm resistor on the top wire. The right wire shows a 0.32H inductor with voltage v1. The circuit is powered by a 60V independent voltage source located on the left wire.

The circuit shows a vertical crosswire connecting two nodes on the top and bottom wires. A switch, t = 0, is located on the cross wire, with a downward current i sub o flowing through it. The top wire carries a 2 ohm resistor and a 4 ohm resistor on its left and right arms, respectively. A 1H inductor is located on the right wire.

The circuit is powered by a 12V independent voltage source located on the left wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. A switch, t = 0, is located on the left cross wire, with a downward current i sub o flowing through it. A 50mH inductor with a downward current i sub L is located on the right cross wire. The top wire carries a 50 ohm resistor and a 200 ohm resistor on its left and middle arms. The right wire is removed and voltage across the two open terminals at the top and bottom is marked as v sub L.

The circuit is powered by a 25V independent voltage source located on the left wire.

The circuit shows a switch, t = 0, toward the left of the top wire, which moves between two nodes marked as 1 and 2. Node 1 extends into the left arm of the top wire. Node 2 is at the top end of a vertical cross wire, which connects to a node on the bottom wire. A second cross wire to the right connects two nodes on the top and bottom wires.

A 6 ohm resistor is located on the left cross wire, while a 40 ohm resistor with reverse polarity voltage of v sub o is located on the right cross wire. A 10 ohm resistor is located on the right wire. The top wire carries a 12 ohm resistor on its left arm, a 4 ohm resistor on its middle arm, and a 72mH inductor on its right arm. The circuit is powered by a 240V independent voltage source located on the left wire.

The circuit shows three parallel vertical cross wires connecting three pairs of nodes on the top and bottom wires. A 75 ohm resistor, a 0.02H inductor, and a 60 ohm resistor are located on the left, middle, and right cross wires, respectively. A downward current, i sub o, flows through the middle cross wire, while a voltage v sub o is marked across the right cross wire.

The top wire carries a 20 ohm resistor, a 50 ohm resistor, and a 3 ohm resistor on its first, second, and third arms. A switch, t = 0, is located on the second arm of the top wire, to the right of the resistor. A 15 ohm resistor is located on the right wire.

The circuit is powered by a 50V independent voltage source located on the left wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. A 1k ohm resistor and a 10 ohm resistor are located on the left and right cross wires, respectively. The middle arm of the top wire carries a switch, t = 0, and a 9k ohm resistor to its right. An inductor L, with voltage v sub o, is located on the right wire, while a 30mA independent current source with an upward current is located on the left wire.

The circuit shows a switch, t = 0, located on one end of a vertical cross wire, which is drawn from a node on the bottom wire. The switch connects to two unconnected wires on the top wire, which are marked as 1 and 2. Wire 1 connects to a node on the top wire towards the left, while wire 2 connects to the right wire. A 30mH inductor is located on this cross wire.

A second vertical cross wire is located to the left, connecting two nodes on the top and bottom wires. This cross wire carries a 4k ohm resistor. Another resistor, R, is located on the right wire. The circuit is powered by a 3A independent current source with an upward current, located on the left wire.

The circuit shows three parallel vertical cross wires connecting three pairs of nodes on the top and bottom wires. Three resistors with resistances of 1k ohm, 4k ohm, and 20k ohm are located on the left, middle, and right cross wires, respectively. An 80k ohm resistor is located on the right wire, while a 6H inductor is located on the third arm of the top wire. Two switches with t value of 0 are located on the second and fourth arms of the top wire. The circuit is powered by a 105mA independent current source with an upward current, located on the left wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. A 2 ohm resistor is located on the left wire while a 4v sub 0 dependent current source with a downward current is located on the right wire. The top wire carries a switch, t = 0, on its middle arm.

A 5mH inductor is located on the right wire with voltage v sub o marked across it. The circuit is powered by a 4A independent current source with an upward current located on the left wire.

The circuit shows three parallel vertical cross wires connecting three pairs of nodes on the top and bottom wires. A 70 ohm resistor, a 20mH inductor and a 90 ohm resistor are located on the left, middle, and right cross wires, respectively. The top wire carries a 30 ohm resistor on its first arm, a 4 ohm resistor and switch t = 0 on its second arm, and a 30i sub delta dependent voltage source on its third arm. A 70 ohm resistor is located on the right wire. The circuit is powered by a 400V independent voltage source, located on the left wire.

Voltage across the middle cross wire is marked as v sub L, while a downward current i sub L flows through it. Another downward current, i sub delta, flows through the right cross wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires. A 5 ohm resistor and a 12.5mH inductor are located on this cross wire. A 3 ohm resistor and a 3.75mH inductor are located on the right wire.

Two nodes are marked on the top and bottom wires, to the left of the cross wire, and are labeled as a and b, respectively. A 5 ohm resistor is located on the left arm of the top wire, to the left of node a. The circuit is powered by a 165V independent voltage source, located on the left wire.

The circuit shows a switch, t = 0, located on one end of a vertical cross wire, which is drawn from a node on the bottom wire. The switch connects to two unconnected wires on the top wire, which are marked as a and b. Wire a connects to a 30 ohm resistor on the top wire towards the left, while wire b connects to another node on the top wire toward the right. A 8mH inductor is located on this cross wire.

Two vertical cross wires are located on either side of the central cross wire with the switch, connecting two pairs of nodes on the top and bottom wires. A 150 ohm resistor is located on the left cross wire while an 8 ohm resistor with a downward current, i sub o, is located on the right cross wire. A 2mH inductor is located on the right wire.

The circuit is powered by a 12A independent current source with an upward current, located on the left wire.

The drawing shows a small rectangular circuit with its base located between two nodes of the bottom wire of a bigger rectangular circuit. A 10H inductor and a 6H inductor are located on the left and right wires of the small circuit, while a switch, t = 0, is located on its top wire. Voltage across the top and bottom wires is marked as v sub o, while an upward current, i sub o, flows through its right wire.

A second switch, t = 0, is located on one end of a vertical cross wire, which is drawn from a node on the top wire of the smaller circuit. The switch connects to two unconnected wires on the top wire of the bigger circuit. The left wire of the bigger circuit carries a 10 ohm resistor, while its right wire carries a 7.5k ohm resistor.

An additional wire is drawn across two nodes on either side of the 10 ohm resistor on the left wire of the bigger circuit. A 2A independent current source with a downward current is located on this additional wire.

The circuit shows a switch, t = 0, located on one end of a vertical cross wire, which is drawn from a node on the bottom wire. The switch connects to two unconnected parts of the top wire. A 10k ohm resistor is located on the top wire, toward the left of the switch, while a 5k ohm resistor is located toward the right. A 40nF capacitor with voltage v is located below the switch on this cross wire.

A second cross wire is located to the right, connecting two nodes on the top and bottom wires, and carrying a 30k ohm resistor. A 60k ohm resistor is located on the right wire. The circuit is powered by a 150V independent voltage source located on the left wire.

The circuit shows a switch, t = 0, located on one end of a vertical cross wire, which is drawn from a node on the bottom wire. The switch connects to two unconnected parts of the top wire. A 0.5 microF capacitor with voltage v is located below the switch on this cross wire.

Two cross wires are located to the left of the switch-carrying cross wire, carrying a 2.7k resistor and a 3.3k ohm resistor. Another cross wire is located towards the right, carrying a 3k ohm resistor. A 2.4k ohm resistor is located on the right arm of the top wire, with a rightward current i flowing through it. A 3.6k ohm resistor is located on the right wire. The circuit is powered by a 40mA independent current source located on the left wire.

The circuit shows three parallel vertical cross wires connecting three pairs of nodes on the top and bottom wires. A switch, t = 0, is located on the left cross wire, while a 20k ohm resistor and a 60k ohm resistor are located on the middle and right cross wires. A 10k ohm resistor is located on the right arm of the top wire. The right wire carries a 40nF capacitor with voltage v marked across it. The circuit is powered by a 20mA independent current source with an upward current, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, close to the left wire. A switch, t = 0, is located on this cross wire. The right wire carries a 3 ohm resistor, while the left wire carries a 1V independent voltage source.

The top wire carries a 20 ohm resistor on its left arm. Two resistors are located on the right arm of the top wire, with resistances of 5 ohm and 2 ohm, carrying rightward currents, i2 and i1, respectively.

An additional wire is drawn across two nodes on either side of the 20 ohm resistor, located on the left arm of the top wire. A 100 mA independent current source with a rightward current is located on top of this cross wire. A second additional wire is drawn across two nodes on either side of the 2 ohm resistor, located on the right arm of the top wire. A 2 microF capacitor is located on top of this cross wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. A 10 over 3 microF capacitor is located on the left cross wire, while a 12k ohm resistor is located on the right cross wire. The right wire carries a 68k ohm resistor while the left arm of the top wire carries a 1.8k ohm resistor. Two switches with t value as 0 are located on the left and right arm of the top wire.

The circuit is powered by a 120V independent voltage source with reverse polarity, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 15 ohm resistor. A switch, t = 0, is located on the top node of this cross wire, connecting to two nodes which are labeled as 1 and 2. Node 1 is located on the left arm of the top wire, while node 2 is on the cross wire.

The top wire carries a 4.7k ohm resistor on its left arm, and a 5i sub o dependent voltage source on its right arm. A 2 microF capacitor is located on the right wire with a downward current, i sub o. The circuit is powered by a 15V independent voltage source located on the left wire.

The circuit shows a switch, t = 0, located on one end of a vertical cross wire, which is drawn from a node on the bottom wire. The switch connects to two unconnected nodes on the top wire, marked as x and y. A 0.8 microF capacitor is located below the switch on this cross wire.

A second cross wire is located to the left, connecting two nodes on the top and bottom wires, and carrying a 3.6k ohm resistor. A 5k ohm resistor is located on the right wire, with voltage v sub delta marked across it. The circuit is powered by a 5mA independent current source with a downward current, located on the left wire.

A dependent current source, alpha v sub delta, is located on the right arm of the top wire. An additional wire is drawn from two nodes on either side of this current source. A 20k ohm resistor is located on top of this additional wire.

Circuit shows a vertical cross wire connecting two nodes on top and bottom wires, and carrying a 24k ohm resistor. Left arm of the top wire carries a 250nF capacitor with voltage of 40V, a switch, t = 0, and a 400 ohm resistor. Right wire carries a 16k ohm resistor.

A small square circuit is drawn on lower part of left wire, between two nodes on the left wire and the bottom wire. Voltage between these two nodes is marked as 50V. A 200nF capacitor is located on left wire of this small circuit, while a 800nF capacitor is located on its right wire.

The circuit shows a switch, t = 0, located on one end of a vertical cross wire, which is drawn from a node on the bottom wire. The switch connects to two unconnected nodes on the top wire, marked as a and b. A 1 microF capacitor is located below the switch on this cross wire, with voltage across it marked as v1. Another capacitor with capacitance of 4 microF and voltage v2 is located on the right wire.

The top wire carries a 3.3k ohm resistor on its left arm, and a 25k ohm resistor on its right arm. The circuit is powered by a 40V independent voltage source located on its left wire.

Circuit shows two parallel vertical cross wires connecting two pairs of nodes on top and bottom wires. A 30 ohm resistor is located on left cross wire while a 0.2 ohm resistor and a 1 microF capacitor are located on right cross wire. Right wire carries a 100 microF capacitor and a 0.5 ohm resistor. Two nodes, a and b, are located on the top and bottom wires, respectively, between the two cross wires. The circuit is powered by a 5A independent current source with an upward current, located on left wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. A switch, t = 0, is located on the left cross wire while a 30k ohm resistor is located on the right cross wire. The top wire carries a 25k ohm resistor on its left arm, and a 200mH inductor on its middle arm. The circuit is powered by a 250V independent voltage source, located on the left wire, and a 1mA independent current source with an upward current, located on the right wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. A 8 ohm resistor is located on the left cross wire, while a 40 ohm resistor with voltage v sub o marked across it is located on the right cross wire. The left wire carries a 50A independent current source with an upward current. The right wire carries a 40mH inductor. A 10 ohm resistor is locate on the right arm of the top wire, with arightward current i sub o flowing through it.

A portion of the top wire, between the two cross wires, is removed. Two unconnected nodes, b and a, are located on the top wire. Node b connects to the left arm of the top wire. Node a is positioned on top of a third cross wire, which is drawn from a node on the bottom wire, and is located between the two cross wires of the circuit. This cross wire carries a 120 ohm resistor and a 800V independent voltage source. The two nodes, b and a, are connected to a switch, t = 0, which is located on an extended wire that connects to the right arm of the top wire.

The drawing shows a switch, t = 0, located on top of a vertical cross wire drawn from a node on the bottom wire. The switch alternates between two small wires on the top wire, labeled as a and b. Wire a connects to the right wire while wire b connects to another node to the left. The cross wire carries a 6A independent current source.

A second cross wire is located to the left, drawn between two nodes on the top and bottom wires. This cross wire carries an 8 ohm resistor, and has voltage v sub o marked across it. The left arm of the top wire carries a 12 ohm resistor and a 5mh inductor with voltage v sub L. A rightward current, i sub L, flows to the right of the inductor.

The left wire carries a 32V independent voltage source.

The drawing shows a vertical cross wire close to the left wire, drawn between two nodes on the top and bottom wires. A resistor R1 is located on this cross wire, while another resistor, R2, is located on the right arm of the top wire, with voltage v sub sw marked across it. Inductor L is located on the right wire, with voltage vo of t marked across it. The circuit is powered by an independent current source, I sub g, located on the left wire, with an upward flowing current.

An additional wire is drawn across two nodes on either side of the resistor R2 on the top wire. A switch, t = 0, is located on top of this additional wire.

The drawing shows a vertical cross wire drawn between two nodes on the top and bottom wires. A d'Arsonval voltmeter is located on this cross wire. Resistor R and switch t = 0 are located on the left arm of the top wire. The right wire carries inductor L. The left wire carries an independent voltage source, V sub bb.

The drawing shows four parallel vertical cross wires connecting four pairs of nodes on the top and bottom wires. A 15 ohm resistor, a 4 mH inductor with voltage v sub o, an 8 ohm resistor, and a dependent current source 9i sub delta with an upward current are located on each of these cross wires from left to right.

The top wire shows a 10 ohm resistor and a 5 ohm resistor on its first and second arms, and a switch t = 0 on its third arm. A rightward current, i sub delta, flows through the third arm of the top wire. The circuit is powered by a 20mA independent current source with an upward current on its left wire, and a 50mA independent current source with a downward current on its right wire.

The drawing shows three parallel vertical cross wires connecting three pairs of nodes on the top and bottom wires. A dependent current source with an upward current, 0.8v sub phi, is located on the left cross wire. Switch t = 0, a 5 ohm resistor, and a 280V independent voltage source are located on the middle cross wire. A 15 ohm resistor is located on the right cross wire.

The top wire carries an 80mH inductor with a leftward current io of t on its first arm, a 1 ohm resistor with voltage v sub phi on its second arm, and a 20 ohm resistor on its third arm. A 15 ohm resistor is located on the left wire. The circuit is powered by a 480V independent voltage source located on the right wire.

The drawing shows a switch, t = 0, located on top of a vertical cross wire drawn from a node on the bottom wire. The switch alternates between two small wires on the top wire, labeled as 1 and 2. Wire 1 connects to a 10 ohm resistor located on the left arm of the top wire. Wire 2 connects to a node on the right arm of the top wire. A 3H inductor is located below the switch on this cross wire.

A second vertical cross wire is located to the right, connecting two nodes on the top and bottom wires, and carrying a 1.5H inductor. A 40 ohm resistor, with voltage v sub o, is located on the right wire. The circuit is powered by a 50V independent voltage source with reverse polarity, located on the left wire.

The drawing shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. A 10 ohm resistor and a 50V independent voltage source are located on the left cross wire, while a 40 ohm resistor is located on the right cross wire. A switch, t = 0, is located on the middle arm of the top wire, between the two cross wires.

The left arm carries a 20mH inductor and a 80mH inductor with voltage v sub o. The right arm carries a 15A independent current source with a downward current.

The drawing shows a small rectangular circuit with its base located between two nodes of the bottom wire of a bigger rectangular circuit. A 60mH inductor with a downward flowing current i1, and a 40mH inductor with a downward flowing current i2 are located respectively on the left and right wires of the small circuit. A switch, t = 0, is located on its top wire. The switch connects to two unconnected wires, a and b, on the top wire of the bigger circuit. Wire a connects to a node to the left, while wire b connects to a node to the right.

Two parallel vertical cross wires connect two pairs of nodes on the top and bottom wires of the bigger circuit. These cross wires are located on either side of the smaller circuit. A 15 ohm resistor is located on the left cross wire, while a 120 ohm resistor with a voltage, v sub o is located on the right cross wire. The left wire of the bigger circuit carries a 25mA independent current source with an upward current, while the right wire carries a 50mA independent current source with a downward current.

The circuit shows three parallel vertical cross wires connecting three pairs of nodes on the top and bottom wires. Resistor R sub g is located on the left cross wire. The right cross wire carries an inductor L1, while the right wire carries another inductor L2. Two downward currents, i1 of t and i2 of t, flow through the right cross wire and the right wire, respectively.

A switch, t = 0, is located on the middle cross wire. The circuit is powered by an independent current source, I sub g, located on the left wire.

The circuit shows a switch, t = 0, located on top of a vertical cross wire which is drawn from a node on the bottom wire. The switch connects to two nodes, a and b, on the top wire. Node a connects to a 400 ohm resistor on the left arm of the top wire, while node b connects to another node on the top wire to the right. A 25mF capacitor is located on the cross wire below the switch. Its voltage and downward current are marked as v sub C and i, respectively.

A second cross wire is located to the right, joining two nodes on the top and bottom wires. A 20 ohm resistor is located on this cross wire, while a 5 ohm resistor is located on the right arm of the top wire. The circuit is powered by a 50V independent voltage source with reverse polarity on the left wire, and a 30V independent voltage source with reverse polarity on the right wire.

The circuit shows a switch, t = 0, located on top of a vertical cross wire, which is drawn from a node on the bottom wire. The switch connects to two nodes, a and b, on the top wire to the left and right. A 40nF capacitor is located on the cross wire below the switch, with voltage marked as v sub o of t.

A second cross wire is located to the right, joining two nodes on the top and bottom wires, and carrying a 40k ohm resistor. A third cross wire is located to the left, joining two nodes on the top and bottom wires, and carrying a 10k ohm resistor. The top wire carries a 5k ohm resistor on its left arm and a 10k ohm resistor on its right arm. A current, io of t, flows rightward on the right arm of the top wire.

The circuit is powered by a 75V independent voltage source, located on the left wire, and a 100V independent voltage source with reverse polarity, located on the right wire.

The circuit shows a switch, t = 0 located on top of a vertical cross wire which is drawn from a node on the bottom wire. The switch connects to two nodes, a and b, on the top wire. Node a connects to a 40 ohm resistor on the middle arm of the top wire to the right, while node b connects to a 25 ohm resistor on the left arm of the top wire. A 100 microF capacitor is located on the cross wire below the switch, with voltage marked as v sub C.

A second cross wire is located to the right, joining two nodes on the top and bottom wires. A 10 ohm resistor is located on this cross wire. The circuit is powered by a 40V independent voltage source on the left wire, and a 2A independent current source with an upward current, located on the right wire.

The circuit shows three parallel vertical cross wires connecting three pairs of nodes on the top and bottom wires. A 40k ohm resistor and a 24k ohm resistor are located respectively on the left and right cross wires, while a 10k ohm resistor and a 4k ohm resistor are located respectively on the left and right arms of the top wire. A 0.05 microF capacitor is located on the right wire, with its voltage marked as v sub o.

The circuit is powered by a 80V independent voltage source on the left wire, and a 3mA independent current source with a downward current located on the middle cross wire.

The circuit shows three vertical parallel cross wires between the top and bottom wires. The left cross wire carries a 3k ohm resistor and a 9k ohm resistor, separated by a node in between. The right cross wire carries a 40k ohm resistor. The middle cross wire has a switch, t = 0, on its top node. This switch connects to two nodes, labeled as a and b. Node a connects to the node located at the midpoint of the left cross wire. Node b connects to a 10k ohm resistor located on the middle arm of the top wire, towards the right. A 20nF capacitor is located below the switch on the middle cross wire, with voltage v sub C marked across it.

The circuit is powered by a 120V independent voltage source on the left wire, and a 1.5mA independent current source with downward current, located on the right wire.

The circuit shows three vertical parallel cross wires connecting three pairs of nodes on the top and bottom wires. The left cross wire carries a 30k ohm resistor and a 50k ohm resistor, separated by a node in between. An additional rectangular wire connects this node to another node located to the right on the bottom wire, between the left and middle cross wires. A 16nF capacitor with voltage vo of t is located on this additional wire.

A 20k ohm resistor is located on the middle cross wire. The right cross wire carries a switch, t = 0, on its bottom node. The switch connects to two unconnected small wires, a and b, on the bottom wire, towards the right and left, respectively.

The circuit is powered by a 10mA independent current source with an upward current on the left wire, and a 15mA independent current source with a downward current, located on the right cross wire.

The circuit shows four vertical parallel cross wires connecting four pairs of nodes on the top and bottom wires. The first cross wire on the left carries a 60k ohm resistor. The second cross wire has switch 1, with t = 0 on its top node. This switch connects to two nodes, labeled as a and b, toward left and right on the top wire, respectively. A 250nF capacitor is located below the switch on the second cross wire. The third cross wire carries a 20k ohm resistor with voltage marked as v sub o. And the last cross wire to the right carries a 5mA independent current source with a downward current.

A switch 2 with t = 0 is located on the right wire. The left arm of the top wire carries a 40k ohm resistor. A 50V independent voltage source is located on the left wire.

The circuit shows three vertical parallel cross wires connecting three pairs of nodes on the top and bottom wires. The left cross wire carries a 15k ohm resistor. The middle cross wire has a switch, t = 0, on its top node. This switch connects to two nodes, labeled as OFF and ON, towards the left and the right on the top wire, respectively. A 62.5nF capacitor is located below the switch on the middle cross wire, with voltage v sub o marked across it. The right cross wire carries a 10k ohm resistor.

The top wire carries a 5k ohm resistor on its second arm, to the left of the OFF button, and a dependent voltage source with reverse polarity on the third arm, to the right of the ON button. Voltage of the voltage source is marked as 20 multiplied by 10 cube i sub delta. A 40k ohm resistor is located on the right arm of the top wire, with a rightward current i sub delta flowing through it.

The circuit is powered by a 3mA independent current source with a downward current, located on the left wire, and a 75V independent voltage source on the right wire.

The drawing shows two circuits which are connected by nodes on their bottom wires. The circuit on the left shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. A 33k ohm resistor is located on the left cross wire, while a switch, t = 0, is located on the right cross wire. A 47k ohm resistor is located on the right wire, and a 120 microA independent current source with an upward current is located on the left wire. A rightward current, i sub b, flows through the right arm of the top wire.

The circuit on the right shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 16k ohm resistor. A 0.25nF capacitor is located on the right wire, while a dependent current source with a downward current, 25i sub b, is located on the left wire.

A circuit shows a switch, t = 0, positioned on top of a vertical cross wire that is drawn from a node on the bottom wire. The switch connects to two nodes, a and b, located to the left and right, respectively on the top wire. Two capacitors are located below the switch on the cross wire, with capacitances of 0.2 microF and 0.8 microF, and respective voltages of v1 and v2. Voltage across the cross wire is marked as v sub o.

The top wire carries two resistors on its left and right arms, with respective resistances of 2.2k ohm and 6.25k ohm. A rightward current, i sub o, flows through the right arm of the top wire. The circuit is powered by a 40V independent voltage source on the left wire, and an 80V independent voltage source on the right wire.

The circuit shows a switch, t = 0, positioned on top of a vertical cross wire, that is drawn from a node on the bottom wire. The switch connects to two nodes, x and y, located to the left and right respectively on the top wire. A 30nF capacitor is located below the switch on the cross wire.

The circuit shows two more parallel vertical cross wires drawn on either side of the switch-carrying cross wire, connecting two pairs of nodes on the top and bottom wires. A 30k ohm resistor is located on the left cross wire, while a 60k ohm resistor with voltage v sub o is located on the right cross wire. The top wire carries a 15k ohm resistor on its second arm, to the left of node x; a 60nF capacitor with voltage v1 on its third arm, to the right of node y; and a 120k ohm resistor on its fourth arm. The circuit is powered by a 4mA independent current source with an upward current on the left wire, and a 90V independent voltage source on the right wire.

The circuit shows two capacitors on the right wire with capacitances marked as C1 and C2, and respective voltages marked as v1 of t and v2 of t. Resistor R sub g and a switch, t = 0, are located on the top wire. An independent voltage source, V sub g, is located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires. A 5H inductor with a dot terminal to its top left is located on this cross wire. Voltage across the inductor is v sub o, while a downward current i1 flows through it. The right wire carries a 10H inductor with a dot terminal located to its bottom left. Mutual inductance between the inductors is marked as 5H.

Current i2 flows rightward on the right arm of the top wire. A 20 ohm resistor, with a rightward current i sub o, and a switch, t = 0, are located on the left arm of the top wire. The circuit is powered by an 80V independent voltage source on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires. A 2H inductor, with a dot terminal to its top left, is located on this cross wire. Voltage across the inductor is v sub o, while a downward current, i1, flows through it. The right wire carries an 18H inductor with a dot terminal located to its top right. A downward current, i2, flows through it. Mutual inductance between the inductors is marked as 4H.

A 50 over 3 ohm resistor, with rightward current i sub o, and a switch, t = 0, are located on the left arm of the top wire. The circuit is powered by a 100V independent voltage source on the left wire.

The circuit shows a 5H inductor on the top wire close to the right wire, with a voltage marked as v1 of t, and a dot terminal located to its bottom left. The right wire carries a 10H inductor, with a voltage marked as v2 of t, and a dot terminal located to its bottom right. Mutual inductance between the two inductors is marked as 2.5H.

A 40 ohm resistor, with a rightward current i of t, and a switch, t = 0, are located on the top wire, to the left of the inductor. The circuit is powered by an 80V independent voltage source located on the left wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. A switch A, with t = 0, is located on the left cross wire, while a second switch B, with t = 5s, is located on the middle arm of the top wire. The right cross wire carries a 1 ohm resistor while the left arm of the top wire carries a 5 ohm resistor. A rightward current, i sub L of t, flows through the right arm of the top wire. A 5H inductor is located on the right wire. The circuit is powered by a 10V independent voltage source, located on the left wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a 10 ohm resistor while the right cross wire carries a 50mH inductor with voltage v sub o. A 5 ohm resistor is located on the middle arm of the top wire, while a 20 ohm resistor is located on the right wire.

Two nodes are located on the top wire and bottom wire in between the two cross wires. A switch 1 with t = 0 is located on the top node. This switch connects to two more nodes, a and b, where a is located on the top wire toward the left, and b is located on a cross wire drawn from the node on the bottom wire. A second switch 2, labeled as 0 plus 10ms, is located on the right arm of the top wire.

The circuit is powered by a 15A independent current source with an upward current located on the left wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a 40 ohm resistor while the right cross wire carries a 60 ohm resistor. Two more shorter cross wires are located in between the left and right cross wires. These two short cross wires are drawn from nodes on the bottom wire, while the top nodes are unconnected with the top wire. The left short wire carries a 120 ohm resistor, with its top node marked as b. The right wire has its top node marked as c. A closed switch with node a is located above b and c on the top wire.

A 40 ohm resistor with a rightward current i is located on the right arm of the top wire. An 80 mH inductor with voltage v is located on the right wire. The circuit is powered by a 20A independent current source with an upward current, located on the left wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. A switch 1, with t = 0, is located on the middle arm of the top wire, while switch 2, with t = 10 micro seconds, is located on the right arm of the top wire. The left cross wire carries a 4 ohm resistor, while the right cross wire carries a 20nF capacitor with voltage v sub o. A 16k ohm resistor is located on the right arm of the top wire. The circuit is powered by a 30V independent voltage source, located on the right wire, and a 5mA independent current source with a downward current, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires. A horizontal cross wire connects a node at the midpoint of the vertical cross wire with another node at the midpoint of the right wire. A 30k ohm resistor and a 120k ohm resistor are located on the top and bottom arms of the vertical cross wire. A 60k ohm resistor and a 40k ohm resistor are located on the top and bottom arms of the right wire.

Switch 1, with t = 0, is located on the left arm of the top wire. Switch 2, with t = 0 plus 200 micro seconds, is located on the horizontal cross wire. A 10 over 3 nF capacitor with voltage of 300V is located on the left wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a 500 ohm resistor. The middle arm of the top wire carries a 4.7k ohm resistor, and the right wire carries a 6.25k ohm resistor.

The right cross wire carries a 160nF capacitor, and a switch t = 0 on its top node, which connects to node a, located to the left on the top wire. Voltage across the capacitor is marked as v sub o. The second switch, with t = 50ms, is located to the right of the first switch, with two nodes, b and c, where node b is located above the top wire and node c is located on the top wire. The circuit is powered by a 20mA independent current source with an upward current, located on the left wire.

The circuit shows three parallel vertical cross wires connecting three pairs of nodes on the top and bottom wires. The left cross wire carries a 10k ohm resistor while the right cross wire carries a 500 nF capacitor. The middle cross wire carries a 4k ohm resistor and switch 1 on its top node. The switch, with t = 0, connects to two nodes, a and b, where a is located to the left on the top wire and b is located on the cross wire above the resistor. Voltage across the middle cross wire is marked as v sub o.

A 2k ohm resistor is located on the third arm of the top wire, while a 3k ohm resistor is located on the right wire. Switch 2 is located on the right arm of the top wire. Two time intervals for this switch are given as 0 plus 800 micro seconds and 0 plus 1.1 ms.

The circuit is powered by a 7.5mA independent current source with a downward current located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 2k ohm resistor, with voltage v sub o. A 250 mH inductor is located on the right wire, while a rightward current, i sub o, flows through the right arm of the top wire. The circuit is powered by an independent current source, i sub s with an upward current, located on the left wire.

The X-axis plots time t in micro seconds, while the y-axis plots current i sub s in mA. The graph shows a rectangular pulse between x = 0 and x = 75, with a constant value of y = 25.

The X-axis plots time t in ms, while the y-axis plots voltage v sub s in Volts. The graph shows a rectangular pulse between x = 0 and x = 2.5, with a constant value of y = 80.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 40mH inductor with a downward flowing current, i sub 0. A 20 ohm resistor is located on the left arm of the top wire. The left and right wires are open and two pairs of open terminals are shown at the top and bottom ends. Voltage at the left terminals is marked as v sub s, while voltage at the right terminals is marked as v sub o.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 25 nF capacitor. Resistor R = 200k ohm is located on the left arm of the top wire. An independent voltage source, v sub s, is located on the left wire. The right wire is removed and voltage across the open terminals is marked as v sub 0.

The X-axis plots time t in ms, while the y-axis plots voltage v sub s in Volts. The graph shows a rectangular pulse that moves from x = 0 to x = 4 with a constant value of y = 100, and continues in the fourth quadrant from x = 4 to x = 8 with a constant value of y = minus 100

The X-axis plots time t in ms, while the y-axis plots voltage v sub s in Volts. The graph shows a rectangular pulse between x = 0 and x = 1 with a constant value of y = 50.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 400k ohm resistor. A 10nF capacitor is located on the left arm of the top wire. The left and right wires are open and two pairs of open terminals are shown at the top and bottom ends. Voltage at the left terminals is marked as v sub s, while voltage at the right terminals is marked as v sub o.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. A switch, t = 0, is located on the left cross wire. The right cross wire shows a dependent current source with a downward current, marked as 2 multiplied by 10 raised to minus 3 times v sub phi.

A 2k ohm resistor is located on the middle arm of the top wire, while a 4k ohm resistor is located on the right wire. The left wire carries a 10H inductor with an upward current of 25mA flowing through it.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 20k ohm resistor. The left arm of the top wire shows a switch t = 0, and a dependent voltage source, marked as 12 multiplied by 10 raised to 4 times i sub delta.

A 80k ohm resistor with a leftward current, i sub delta, is located on the right arm of the top wire. The left wire carries a 2.5 microF capacitor with a 20V voltage.

The circuit shows four vertical parallel cross wires connecting four pairs of nodes on the top and bottom wires. A 2k ohm resistor, a 1.6 microF capacitor, a 4k ohm resistor with a downward current i sub delta, and a switch t = 0 are located on these cross wires, from left to right. The top wire carries a 1k ohm resistor on its middle arm between the second and third cross wires. The circuit is powered by a 5A independent current source with an upward current, located on the right wire, and a dependent current source with an upward current, 4 i sub delta, located on the left wire.

The circuit shows three vertical parallel cross wires connecting three pairs of nodes on the top and bottom wires. A 20k ohm resistor, a dependent current source with an upward current, marked as Beta i sub sigma, and a 200 mH inductor are located respectively on the left, middle, and right cross wires. A gap is indicated on the right wire.

The top wire carries a switch t = 0 and a 5k ohm resistor with a leftward current, i sub sigma. The circuit is powered by a 40V independent voltage source located on the left wire.

The circuit shows a 2 microF capacitor on its left wire, and a 4k ohm resistor and 80V DC voltage source on the right wire. A 25k ohm electric relay is located in the middle of the circuit, with its lower end connected to a node on the bottom wire, and its upper end barely connecting to the top wire. A leftward current, i, flows on the left arm of the top wire.

A T shaped switch is shown with its lower arm connected to the top wire, above the electric relay, and its upper arm connected to a horizontal wire, which connects to a node on its left, labeled as a. Another horizontal arm is shown to the right of the switch, leading to a node labeled as b. A push button is provided to the left of the switch, pushing which the circuit is completed between nodes a and b.

The op amp has a positive supply voltage of 10V, and a negative supply voltage of minus 10V. A 500 nF capacitor connects the output terminal with the inverting input terminal. The output terminal, which is grounded with the common node, is in series with a 5.1k ohm resistor with voltage v sub o. The non-inverting input terminal is grounded with the common node. A 4V independent voltage source, in series with switch t = 0 and resistor R, connects the inverting input terminal with the common node.

The X-axis plots time t in ms, while the y-axis plots voltage v sub g in mV. The graph shows a rectangular pulse that moves from x = 0 to x = 250 with a constant value of y = 200, and continues in the fourth quadrant from x = 250 to x = 500 with a constant value of y = minus 200.

The op amp has a positive supply voltage of 6V, and a negative supply voltage of minus 6V. A 400 nF capacitor connects the output terminal with the inverting input terminal. The output terminal which is grounded with the common node has a voltage marked as, v sub o. The non-inverting input terminal is grounded with the common node. An independent voltage source, v sub g, in series with a switch t = 0 and a 25k ohm resistor, connects the inverting input terminal with the common node.

The op amp has a positive supply voltage, V sub CC, and a negative supply voltage, minus V sub CC. Capacitor C connects the output terminal with the inverting input terminal. The output terminal, which is grounded with the common node, has voltage marked as v sub o. An independent voltage source, v sub a, in series with switch t = 0 and resistor R, connects the inverting input terminal with the common node. Another independent voltage source, v sub b, in series with a switch t = 0 and resistor R, and in parallel with a capacitor C, connects the non-inverting input terminal with the common node.

The op amp has a positive supply voltage of 25V, and a negative supply voltage of minus 25V. A 2.5 microF capacitor with a reverse polarity voltage of 56V connects the output terminal with the inverting input terminal. The output terminal, which is grounded with the common node, has voltage marked as v sub o.

A 14V independent voltage source, in series with switch t = 0 and two resistors on either side of the switch, with resistances of 33k ohm and 47k ohm, connects the inverting input terminal with the common node. Another independent voltage source with reverse polarity voltage of 45V, in series with a 20k ohm resistor, and in parallel with an 80 ohm resistor, connects the non-inverting input terminal with the common node.

The op amp has a positive supply voltage of 15V and a negative supply voltage of minus 15V. An 800 pF capacitor connects the output terminal with the inverting input terminal. The output terminal, which is grounded with the common node, has voltage marked as v sub o.

An independent voltage source, v sub g, in series with a switch t = 0 and a 1k ohm resistor, connects the inverting input terminal with the common node. The non-inverting input terminal is grounded with the common node.

The X-axis plots time t in microseconds, while the y-axis plots voltage v sub g in Volts. The graph shows a triangular pulse that moves from x = 0 to x = 2, peaking at y = 2, and continues in the fourth quadrant from x = 2 to x = 4, peaking at y = minus 2.

A rectangular circuit shows two square shaped electronic switches, T1 and T2, on its lower left wire and lower right wire, respectively. Switch T1 has three internal nodes, out of which c1 is connected to the left wire, e1 is connected to the bottom wire, and b1 is connected to a diagonal wire which links to a node on the top portion of right wire, in series with resistor R1. Similarly, switch T2 has three nodes, out of which c2 is connected to the right wire, e2 is connected to the bottom wire, and b2 is connected to a diagonal wire which links to a small square circuit at the top left corner. The small square circuit has resistor R sub L on its left wire, another resistor R on its right wire, and a capacitor C on its bottom wire.

Resistor R sub L is located on the top portion of the right wire. An additional wire is drawn between node e2 from switch T2 and a node at the midpoint of the top wire. This additional wire carries an independent voltage source, V sub CC. Similarly, another additional wire is drawn between node e1 from switch T1 and a node at the midpoint of the left wire. This additional wire carries an open switch, S.

Voltage across node b1 is marked as v sub be1, while voltage across node b2 is marked as v sub be2. The voltage across node c2 is marked as v sub ce2. Two downward currents, i sub b1 and i sub b2, flow into the nodes b1 and b2, respectively.

A rectangular circuit shows two square shaped electronic switches, T1 and T2, on its lower left wire and lower right wire, respectively. Switch T1 has three internal nodes, out of which c1 is connected to the left wire, e1 is connected to the bottom wire, and b1 is connected to a diagonal wire which links to a small square circuit at the top right corner. The small square circuit has a resistor R sub 1 on its left wire, another resistor R sub L on its right wire, and a capacitor C sub 1 on its bottom wire.

Similarly, switch T2 has three nodes, out of which c2 is connected to the right wire, e2 is connected to the bottom wire, and b2 is connected to a diagonal wire which links to a small square circuit at the top left corner. The small square circuit has a resistor R sub L on its left wire, another resistor R sub 2 on its right wire, and a capacitor C sub 2 on its bottom wire.

An additional wire is drawn between node e2 from switch T2 and a node at the midpoint of the top wire. This additional wire carries an independent voltage source, V sub CC.

The voltage across node b1 is marked as v sub be1, while the voltage across node b2 is marked as v sub be2. The voltage across node c1 is marked as v sub ce1, while the voltage across node c2 is marked as v sub ce2. Two downward currents, i sub b1 and i sub b2, flow into the nodes b1 and b2, respectively.

The drawing shows a rectangular relay box consisting of four terminals at its four corners. The top left terminal connects to a 30V DC voltage source. The top right and bottom right terminals together connect to DC loads through a DC BUS.

Two compressed springs, with a relay coil (R, L) in between them, are suspended on a wire that connects the bottom left and bottom right terminals. The relay coil and the bottom left terminal are connected to a 30V dc generator.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries an inductor L with initial current I sub 0, while the right cross wire carries resistor R. Capacitor C with voltage V sub 0 is located on the left wire. The right wire is removed and voltage v is marked across the open terminals on the top and bottom wires. Three downward currents, i sub C, i sub L, and i sub R, flow through the capacitor, inductor, and resistor, respectively.

The circuit shows four parallel vertical cross wires connecting four pairs of nodes on the top and bottom wires. A switch, t = 0, is located on the first cross wire to the left. Capacitor C, inductor L, and resistor R are located on the second, third, and fourth cross wires, respectively. The right wire is removed and voltage v is marked across the open terminals on the top and bottom wires. The circuit is powered by an independent current source I with an upward current, located on the left wire.

The top wire of the circuit carries resistor R and inductor L with a rightward current I sub 0. The right wire carries capacitor C with voltage V sub 0. A clockwise arrow, labeled as i, is depicted in the middle of the circuit.

The top wire of the circuit carries resistor R and inductor L. Switch t = 0 separates the resistor and inductor. The right wire carries capacitor C, while the left wire carries an independent voltage source V. A clockwise arrow, labeled as i, is depicted in the middle of the circuit.

The table shows three parameters along with their respective terminologies and values in natural response, as follows:

s1, s2; Characteristic roots; s1 = minus alpha plus square root of (alpha squared minus omega sub 0 sq+I18uared), s2 = minus alpha minus square root of (alpha squared minus omega sub 0 square)

alpha; Neper frequency; alpha equals 1 over 2RC

omega sub 0; Resonant radian frequency; omega sub 0 equals 1 over square root of LC.

The Circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries inductor L with an initial current I sub 0, while the right cross wire carries resistor R. Capacitor C with voltage V sub 0 is located on the left wire. The right wire is removed and voltage v is marked across the open terminals on the top and bottom wires. Three downward currents, i sub C, i sub L, and i sub R, flow through the capacitor, inductor, and resistor, respectively.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a 50mH inductor with initial current I sub 0, while the right cross wire carries a 200 ohm resistor. A 0.2 microF capacitor with voltage V sub 0 is located on the left wire. The right wire is removed and voltage v is marked across the open terminals on the top and bottom wires. Three downward currents, i sub C, i sub L, and i sub R flow through the capacitor, inductor, and resistor, respectively.

The X-axis plots time t in micro seconds, ranging from 0 to 250 in increments of 50, while the y-axis plots voltage v of t in Volts, ranging from minus 6 to 12 in increments of 2. The graph shows a line which starts on the y-axis at y = 12 and moves with a steep downward slope, crossing over the x-axis into the fourth quadrant at approximately x = 45. This curve reaches a negative peak of y = minus 5 at x = 145, and then turns upward, moving up to y = minus 3 at approximately x = 250.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries inductor L with initial current I sub 0, while the right cross wire carries resistor R. Capacitor C with voltage V sub 0 is located on the left wire. The right wire is removed and voltage v is marked across the open terminals on the top and bottom wires. Three downward currents, i sub C, i sub L, and i sub R flow through the capacitor, inductor, and resistor, respectively.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries an 8H inductor with initial current I sub 0, while the right cross wire carries a 20k ohm resistor. A 125nF capacitor with voltage V sub 0 is located on the left wire. The right wire is removed and voltage v is marked across the open terminals on the top and bottom wires. Three downward currents, i sub C, i sub L, and i sub R flow through the capacitor, inductor, and resistor, respectively.

The X-axis plots time t in ms, ranging from 0 to 11 in increments of 1, while the y-axis plots voltage v in Volts, ranging from minus 40 to 80 in increments of 20. The graph shows a sinusoidal wave moving between x = 0 and x = 3.1, peaking approximately at y = 78, and then moving in the fourth quadrant between x = 3.1 and x = 6.5, peaking at minus 40, and then moving in the first quadrant between x = 6.5 and x = 9.6, peaking at 20, and so on. Peak values reduce with every pulse of the wave.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries inductor L with initial current I sub 0, while the right cross wire carries resistor R. Capacitor C with voltage V sub 0 is located on the left wire. The right wire is removed and voltage v is marked across the open terminals on the top and bottom wires. Three downward currents, i sub C, i sub L, and i sub R flow through the capacitor, inductor, and resistor, respectively.

Equations are as follows:

Characteristic equation: s square plus multiplication of 1 over RC by s plus 1 over LC = zero

Neper, resonant, and damped frequencies: alpha = 1 over 2RC; omega sub 0 = square root of 1 over LC; omega sub d = square root of (omega sub 0 square minus alpha square)

Roots of the characteristic equation: s1 = minus alpha plus square root of (alpha square minus omega sub 0 square); s2 = minus alpha minus square root of (alpha square minus omega sub 0 square)

alpha square greater than omega sub 0 square: overdamped:

- v of t = A1 multiplied by e raised to s1t plus A2 multiplied by e raised to s2t, t is greater than or equal to zero
- v(0 plus) = A1 plus A2 = V sub 0
- dv(0 plus) over dt = s1A1 plus s2A2 = 1 over C (minus V sub 0 over R minus I sub 0)

alpha square lesser than omega sub 0 square: underdamped:

- v of t = (B1 multiplied by e raised to alpha t multiplied by cos omega sub d multiplied by t) plus (B2 multiplied by e raised to minus alpha t multiplied by sine omega sub d multiplied by t), t is greater than or equal to zero
- v(0 plus) = B1 = V sub 0
- dv(0 plus) over dt = minus alpha B1 plus omega sub d multiplied by B2 = 1 over C (minus V sub 0 over R minus I sub 0)

alpha square equal to omega sub 0 square: critically damped:

- v of t = (D1 multiplied by t multiplied by e raised to minus alpha t) plus

(D2 multiplied by e raised to minus alpha t), t is greater than or equal to zero

- v(0 plus) = D2 = V sub 0
- dv(0 plus) over dt = D1 minus (alpha multiplied by D2)= 1 over C (minus V sub 0 over R minus I sub 0)

The X-axis plots time t in ms, ranging from 0 to 7 in increments of 1, while the y-axis plots voltage v in Volts, ranging from 0 to 40 in increments of 8. The graph shows a line which starts at the origin and rises with a steep upward slope to peak at (1.5, 36). The line then moves with a relaxed downward slope, ending at x = 7 on the x-axis.

The circuit shows four parallel vertical cross wires connecting four pairs of nodes on the top and bottom wires. A switch, t = 0, is located on the first cross wire to the left. Capacitor C, inductor L, and resistor R are located on the second, third, and fourth cross wires, respectively. The right wire is removed and voltage v is marked across the open terminals on the top and bottom wires. The circuit is powered by an independent current source I with an upward current, located on the left wire. Three downward currents labeled as i sub C, i sub L, and i sub R flow through the capacitor, inductor, and resistor, respectively.

The circuit shows three parallel vertical cross wires connecting three pairs of nodes on the top and bottom wires. The left cross wire has two open nodes, while the middle cross wire has two nodes which are short circuited, with a downward current I sub f = I flowing through it. Resistor R is located on the right wire.

The right wire is removed and voltage V sub f = 0 is marked across the open terminals on the top and bottom wires. The circuit is powered by an independent current source I with an upward current, located on the left wire.

Equations are as follows:

Characteristic equation: s square plus multiplication of 1 over RC by s plus 1 over LC = 1 over LC

Neper, resonant, and damped frequencies: alpha = 1 over 2RC; omega sub 0 = square root of 1 over LC; omega sub d = square root of (omega sub 0 square minus alpha square)

Roots of the characteristic equation: s1 = minus alpha plus square root of (alpha square minus omega sub 0 square); s2 = minus alpha minus square root of (alpha square minus omega sub 0 square)

alpha square greater than omega sub 0 square: overdamped:

- i sub L of t = I sub f plus A1-dash multiplied by e raised to s1t plus A2-dash multiplied by e raised to s2t, t is greater than or equal to zero
- i sub L(0 plus) = I sub f plus A1-dash plus A2-dash = I sub 0
- di sub L(0 plus) over dt = s1A1-dash plus s2A2-dash = V sub 0 over L

alpha square lesser than omega sub 0 square: underdamped:

- i sub L of t = I sub f plus (B1-dash multiplied by e raised to minus alpha t multiplied by cos omega sub d multiplied by t) plus (B2-dash multiplied by e raised to minus alpha t multiplied by sine omega sub d multiplied by t), t is greater than or equal to zero
- i sub L(0 plus) = I sub f plus B1-dash = I sub 0
- d i sub L(0 plus) over dt = minus alpha B1-dash plus omega sub d multiplied by B2-dash = V sub 0 over L

alpha square equal to omega sub 0 square: critically damped:

- i sub L of t = I sub f plus (D1-dash multiplied by t multiplied by e raised to minus alpha t) plus (D2-dash multiplied by e raised to minus alpha t),

t is greater than or equal to zero

- i sub L(0 plus) = I sub f plus D2-dash = I sub 0
- d i sub L(0 plus) over dt = D1-dash minus (alpha multiplied by D2-dash)=

The circuit shows four parallel vertical cross wires connecting four pairs of nodes on the top and bottom wires. A switch, t = 0, is located on the first cross wire to the left. A 25nF capacitor, a 25mH inductor, and resistor R are located on the second, third, and fourth cross wires, respectively. The right wire is removed and voltage v is marked across the open terminals on the top and bottom wires. The circuit is powered by an independent current source I with an upward current, located on the left wire. Three downward currents labeled as i sub C, i sub L, and i sub R flow through the capacitor, inductor, and resistor, respectively.

The X-axis plots time t in micro seconds, ranging from 0 to 180 in increments of 40, starting from 20. The Y-axis plots current i sub L in mA, ranging from 0 to 26 in increments of 4, starting from 2. The graph shows three lines which start together just above the origin on the y-axis, and rise up separately with steep upward slopes to reach a plateau from where they run parallel to the x-axis.

A dotted line is drawn horizontal to the x-axis at y = 22, which intersects three curves at three points, from where dotted vertical lines are drawn to the x-axis. Points of intersection for each line represent 90 percent of the final value. The line to left is labethe led as Underdamped (R = 625 ohm), with the point of intersection marked approximately at (75, 22). The line in the middle is labeled as Critically damped (R = 500 ohm) with the point of intersection marked approximately at (98, 22). The line to the right is labeled as Overdamped (R = 400 ohm), with its point of intersection marked at (128, 22).

The circuit shows four parallel vertical cross wires connecting four pairs of nodes on the top and bottom wires. A switch, t = 0, is located on the first cross wire to the left. Capacitor C, inductor L, and resistor R are located on the second, third, and fourth cross wires, respectively. The right wire is removed and voltage v is marked across the open terminals on the top and bottom wires. The circuit is powered by an independent current source I with an upward current, located on the left wire. Three downward currents labeled as i sub C, i sub L, and i sub R flow through the capacitor, inductor, and resistor, respectively.

The top wire of the circuit carries resistor R and inductor L with a rightward current I sub 0. The right wire carries capacitor C, with voltage marked as V sub 0. A clockwise arrow, labeled as I, is depicted in the middle of the circuit.

Equations are as follows:

Characteristic equation: s square plus multiplication of R over L by s plus 1 over LC = zero

Neper, resonant, and damped frequencies: alpha = R over 2L; omega sub 0 = square root of 1 over LC; omega sub d = square root of (omega sub 0 square minus alpha square)

Roots of the characteristic equation: s1 = minus alpha plus square root of (alpha square minus omega sub 0 square); s2 = minus alpha minus square root of (alpha square minus omega sub 0 square)

alpha square greater than omega sub 0 square: overdamped:

* i of t = A1 multiplied by e raised to s1t plus A2 multiplied by e raised to s2t, t is greater than or equal to zero

* i(0 plus) = A1 plus A2 = I sub 0

* di(0 plus) over dt = s1A1 plus s2A2 = 1 over L(minus multiplication of R by I sub 0 minus V sub 0)

alpha square lesser than omega sub 0 square: underdamped:

* i of t = (B1 multiplied by e raised to alpha t multiplied by cos omega sub d multiplied by t) plus (B2 multiplied by e raised to minus alpha t multiplied by sine omega sub d multiplied by t), t is greater than or equal to zero

* i(0 plus) = B1 = i sub 0

* di(0 plus) over dt = minus alpha B1 plus omega sub d multiplied by B2 = 1 over L (minus multiplication of R by I sub 0 minus V sub 0)

alpha square equal to omega sub 0 square: critically damped:

* i of t = (D1 multiplied by t multiplied by e raised to minus alpha t) plus (D2 multiplied by e raised to minus alpha t), t is greater than or equal to zero

* i(0 plus) = D2 = I sub 0

* di(0 plus) over dt = D1 minus (alpha multiplied by D2)= 1 over L (minus multiplication of R by I sub 0 minus V sub 0)

The top wire of the circuit carries switch t = 0, resistor R with voltage v sub R, and inductor L with voltage v sub L. The right wire carries capacitor C with voltage v sub C, while the left wire carries an independent voltage source, V. A clockwise arrow, labeled as i, is depicted in the middle of the circuit.

Equations are as follows:

Characteristic equation: s square plus multiplication of R over L by s plus 1 over LC = V over LC

Neper, resonant, and damped frequencies: alpha = R over 2L; omega sub 0 = square root of 1 over LC; omega sub d = square root of (omega sub 0 square minus alpha square)

Roots of the characteristic equation: s1 = minus alpha plus square root of (alpha square minus omega sub 0 square); s2 = minus alpha minus square root of (alpha square minus omega sub 0 square)

alpha square greater than omega sub 0 square: overdamped:

* v sub C of t = V sub f plus A1-dash multiplied by e raised to s1t plus A2-dash multiplied by e raised to s2t, t is greater than or equal to zero

* v sub C(0 plus) = V sub f plus A1-dash plus A2-dash = V sub 0

* dv sub C(0 plus) over dt = s1A1-dash plus s2A2-dash = I sub 0 over C

alpha square lesser than omega sub 0 square: underdamped:

* v sub C of t = V sub f plus (B1-dash multiplied by e raised to minus alpha t multiplied by cos omega sub d multiplied by t) plus (B2-dash multiplied by e raised to minus alpha t multiplied by sine omega sub d multiplied by t), t is greater than or equal to zero

* v sub C(0 plus) = V sub f plus B1-dash = V sub 0

* d v sub C(0 plus) over dt = minus alpha B1-dash plus omega sub d multiplied by B2-dash = I sub 0 over L

alpha square equal to omega sub 0 square: critically damped:

* v sub C of t = V sub f plus (D1-dash multiplied by t multiplied by e raised to minus alpha t) plus (D2-dash multiplied by e raised to minus alpha t), t is

greater than or equal to zero

* v sub C(0 plus) = V sub f plus D2-dash = V sub 0

* d v sub C(0 plus) over dt = D1-dash minus (alpha multiplied by D2-dash)= I sub 0 over C

The circuit shows switch t = 0 and a 100mH inductor on the top wire, a 560 ohm resistor on the right wire, and a 0.1 microF capacitor with 100V voltage on the left wire. Two nodes are located at the midpoints of the top and bottom wires. Voltage between these nodes is marked as v sub C. An anticlockwise curved arrow, marked as i, is depicted at the center of the circuit.

The circuit shows a switch t = 0, a 0.1H inductor, and a 1250 ohm resistor on the top wire, and a 0.4 microF capacitor with voltage v sub C on the right wire. A 48V independent voltage source is located on the left wire.

The circuit shows a switch, t = 0, located on top of a vertical cross wire which is drawn from a node at the midpoint of the bottom wire. The switch connects to node a and node b, which are respectively located to the left and right on the top wire. A 2 microF capacitor with voltage v sub c is located below the switch on the cross wire.

Another parallel vertical cross wire is located to the left of the switch-carrying crosswire. A 15k ohm resistor is located on this cross wire. The top wire carries a 9k ohm resistor on its left arm. The right arm of the top wire carries an 80 ohm resistor and a 5mH inductor, along with a leftward current i. The circuit is powered by two independent voltage sources located on the left and right wires, with respective voltages of 80V and 100V.

The circuit shows two op amps where output terminal of the left op amp is connected to an inverting input terminal of the right op amp, with resistor R2 located in series between them. Both op amps have positive supply voltage, V sub CC, and negative supply voltage, minus V sub CC. The non-inverting input terminals of both the op amps are grounded with the common node.

In the left op amp, capacitor C1 connects the output terminal with the inverting input terminal, which is in turn grounded with the common node, in series with resistor R1. Voltage across the inverting input terminal is v sub g, while voltage across the output terminal is v1 sub o. In the right op amp, capacitor C2 connects the output terminal with the inverting input terminal. The output terminal is grounded with the common node and shows voltage V sub o.

The circuit shows two op amps where the output terminal of the left op amp is connected to the inverting input terminal of the right op amp, with 500k ohm resistor located in series between them. The non-inverting input terminals of both the op amps are grounded with the common node.

In the left op amp, a 0.1 microF capacitor connects the output terminal with the inverting input terminal, which is in turn grounded with the common node, in series with a 250k ohm resistor. Voltage across the inverting input terminal is v sub g, while voltage across the output terminal is v sub o 1. The op amp has a 5V positive voltage supply and a minus 5V negative voltage supply.

In the right op amp, a 1 microF capacitor connects the output terminal with the inverting input terminal. The op amp has a 9V positive voltage supply and a minus 9V negative voltage supply. The output terminal is grounded with the common node and shows voltage v sub o.

The circuit shows two op amps where the output terminal of the left op amp is connected to the inverting input terminal of the right op amp, with resistor R sub b located in series between them. The non-inverting input terminals of both the op amps are grounded with the common node.

In the left op amp, capacitor C1 connects the output terminal with the inverting input terminal, which is in turn grounded with the common node, in series with resistor R sub a. A resistor R1 is connected in parallel across the capacitor C1. The op amp has a positive voltage supply of V sub CC1 and a negative voltage supply of minus V sub CC1. Voltage across the inverting input terminal is v sub g, while voltage across the output terminal is v sub o 1.

In the right op amp, capacitor C2 connects the output terminal with the inverting input terminal. Resistor R2 is connected in parallel across the capacitor C2. The op amp has a positive voltage supply of V sub CC2 and a negative voltage supply of minus V sub CC2. The output terminal is grounded with the common node and shows voltage V sub o.

The circuit shows a switch, t = 0, on top of a vertical cross wire that is drawn from a node on the bottom wire. The switch connects to wires a and b, which are located to the left and right respectively on the top wire. Capacitor C with reverse polarity voltage of v sub 0 of t is located below the switch on the cross wire.

Resistor R sub s is located on the left arm of the top wire, while inductor L is located on the right wire. A clockwise mesh current, i, is shown in the right mesh. The circuit is powered by an independent voltage source, V sub s, located on the left wire.

The circuit shows four parallel vertical cross wires. The first and third cross wires from the left are drawn between nodes on the top and bottom wires, and carry 500 ohm resistor and 100 ohm resistor with voltage v sub 0 of t, respectively. The second and fourth cross wires are drawn from nodes on the bottom wire, and carry switches, t = 0, on their top.

The switch on the second cross wire connects to two wires, a and b, located to the left and right on the top wire, with wire a being the initial connection. A 62.5 mH inductor is located on this cross wire. The switch on the fourth cross wire connects to two wires, c and d, located to the left and right on the top wire, with wire d being the initial connection. A 1 microF capacitor is located on this cross wire.

A 50 ohm resistor is located on the right arm of the top wire. The circuit is powered by a 60mA independent current source with a downward current located on the left wire, and a 15V independent voltage source located on the right wire.

The circuit shows four parallel vertical cross wires. The second cross wire from the left is drawn from a node on the bottom wire, and carries switch t = 0 on its top. The switch connects to two nodes, a and b, located to the left and right on the top wire, with node a being the initial connection. An 8 microF capacitor is located on this cross wire.

The other three cross wires are drawn between nodes on the top and bottom wires. The first and fourth cross wires carry resistors with respective resistances of 4k ohm and 150 ohm. The third cross wire carries a 51.2 mH inductor with voltage v sub o.

A 6k ohm resistor is located on the left arm of the top wire, while a 60 ohm resistor is located on the right wire. The circuit is powered by a 50V independent voltage source on the left wire, and a dependent voltage source with reverse polarity of 10i sub phi, located on the fourth arm of the top wire. A leftward current, i sub phi, flows through the fifth arm of the top wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 1.25 microF capacitor. The top wire carries a 400 ohm resistor on the left arm and switch t = 0 on the right arm, while the right wire carries an 800 mH inductor with voltage v sub o. A rightward current, i sub o, flows to the right of the switch on the top wire. The circuit is powered by a 12V independent voltage source, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 6.25H inductor. The top wire carries a 200 ohm resistor on the left arm and switch t = 0 on the right arm, while the right wire carries a 25 microF capacitor with voltage v sub o. The circuit is powered by a 7.5V independent voltage source, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 6.25 microF capacitor. The top wire carries a 125 ohm resistor and switch t = 0 on the left arm, while the right wire carries a 250 mH inductor with voltage v sub o. A rightward current, i sub o, flows through the right arm of the top wire. The circuit is powered by a 25V independent voltage source, located on the left wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a 25mH inductor with a downward current, i sub L of t, while the right cross wire carries a 62.5 microF capacitor. Resistor R is located on the right wire. The circuit is powered by a 2A independent current source with an upward current, located on the left wire.

The circuit shows four parallel vertical cross wires connecting four pairs of nodes on the top and bottom pairs. A 3750 ohm resistor, a 250nF capacitor, a 4H inductor with a downward flowing current i sub L, and a 500 ohm resistor are located on the four cross wires, from left to right. Two switches with t = 0 and with opposite action, labeled as Switch 1 and Switch 2, are located on the third and fourth arms of the top wire. A 7.5k ohm resistor is located on the left arm of the top wire.

The circuit is powered by a 150V independent voltage source on the left wire, and a 100mA independent current source with an upward current on the right wire.

The circuit shows four parallel vertical cross wires connecting four pairs of nodes on the top and bottom pairs. A 3k ohm resistor, a 25 microF capacitor with voltage v sub C of t, a 250mH inductor with a downward flowing current i sub L of t, and a 40 ohm resistor are located on the four cross wires, from left to right. A switch, t = 0, is located on top of the second cross wire, connecting to two wires on the top wire. A 1k ohm resistor is located on the left arm of the top wire.

The circuit is powered by a 100V independent voltage source on the left wire, and a 100mA independent current source with an upward current on the right wire.

The circuit shows three parallel vertical cross wires connecting three pairs of nodes on the top and bottom pairs. A 20 mH inductor with a downward flowing current i sub L, a 500 nF capacitor, and a 150 ohm resistor are located on the left, middle, and right cross wires, respectively. A switch, t = 0 is located on the third arm of top wire. A 300 ohm resistor is located on the left arm of top wire.

The circuit is powered by a 36V independent voltage source on the left wire, and a 20mA independent current source with a downward current on the right wire.

The circuit shows a 500nF capacitor with voltage v sub C on the left wire, a switch t = 0 and resistor R with a rightward current i on the top wire, and a 80mH inductor on the right wire.

The circuit shows a 50nF capacitor on the left wire, a switch t = 0 and resistor R with a rightward current i of t on the top wire, and an inductor L with voltage v sub L of t on the right wire.

The circuit shows a switch t = 0 positioned on top of a vertical cross wire drawn from a node on the bottom wire. The switch connects to two wires, a and b, located on the left and right of the top wire. An 80mH inductor is located on the cross wire below the switch.

Another vertical cross wire is located to the left of the switch-carrying cross wire, drawn between two nodes on the top and bottom wires. A 50 ohm resistor is located on this cross wire, while a 40 ohm resistor with a rightward current i is located on the right arm of the top wire. The right wire carries a 200 microF capacitor. A 75 mA independent current source with an upward current is located on the left wire.

The circuit shows switch t = 0 positioned on top of a vertical cross wire drawn from a node on the bottom wire. The switch connects to two nodes, a and b, located on the left and right of the top wire. A 100nF capacitor is located on the cross wire below the switch.

Two nodes are located on the top and bottom wire, toward the right of the cross wire, with voltage between them marked as v sub a. The top wire carries a 50 ohm resistor on the left arm, and two resistors on the right arm with resistances of 5k ohm and 7.5k ohm on either side of the node. The right wire carries a 2.5 H inductor. A 72V independent voltage source with reverse polarity is located on the left wire.

The circuit shows three parallel vertical cross wires connecting three pairs of nodes on the top and bottom pairs. A 100 ohm resistor, a 10 microF capacitor with reverse polarity voltage of v sub o of t, and a 400mH inductor with a downward current i sub o of t are located on the left, middle, and right cross wires, respectively. A switch t= 0 is located on the top of the middle cross wire, connecting to two wires on the top wire. A 500 ohm resistor and a 50 ohm resistor are located on the third and fourth arms of the top wire.

The circuit is powered by a 100V independent voltage source on the right wire, and a 4A independent current source with an upward current on the right wire.

The circuit shows a rectangular shaped small circuit drawn between two nodes on the top wire. A 10 ohm resistor is located on the top wire of this small circuit, while a 70 ohm resistor and a 20 ohm resistor are located on its bottom wire. An 11 ohm resistor is located on the top wire of the bigger circuit, to the left of the smaller circuit.

A cross wire is drawn connecting the bottom wire of the smaller circuit, in between the two resistors, and a node at the midpoint of the bottom wire of the bigger circuit. An 8 ohm resistor, a 240V independent voltage source, and a switch t = 0 are located on this cross wire.

The left wire carries a 1H inductor while the right wire carries a 5 mF capacitor with voltage v sub o.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 10 nF capacitor. The left arm of the top wire carries an 8k ohm resistor and a 160mH inductor, while the left wire carries a 50V independent voltage source. The right wire is removed and voltage between the open terminals on the top and bottom wires is marked as v sub o of t.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 200 ohm resistor. The left arm of the top wire carries switch t = 0, while the right arm carries a 250mH inductor. A 16 microF capacitor with voltage v sub o of t is located on the right wire. The circuit is powered by an 80mA independent current source with an upward current, located on the left wire.

The circuit shows two parallel vertical cross wires. The cross wire on the right is drawn between two nodes on the top and bottom wires, and carries a 2mF capacitor with voltage v sub C of t. The cross wire on the left is drawn between a node on the bottom wire and a small wire b, which connects to a switch on the top wire. The switch, labeled as 1 with t = 0, alternates between wire b and wire a, which is located on the left arm of the top wire. An 8 ohm resistor and a 100 mh inductor are located to the right of the switch, on the middle arm of the top wire. A 4 ohm resistor is located to the left of the wire a on the left arm of the top wire, while a 2 ohm resistor is located on the cross wire to the left of wire b. A second switch, labeled as 2 with t = 0, is located on the right arm of the top wire. This switch has an opposite action to switch 1. The right wire carries an 18 ohm resistor.

The circuit is powered by a 150V independent voltage source with reverse polarity, located on the left wire, and a 60V independent voltage source located on the left cross wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 12.5 nF capacitor with voltage v sub o of t. The top wire carries a 160 ohm resistor on the left arm, and a 480 ohm resistor and 0.5mH inductor on the right arm. An additional wire is drawn from two terminals on either side of the 480 ohm resistor on the top wire. This additional wire carries a 960 ohm resistor on its top wire.

Switch t = 0 is located on the top terminal of the cross wire. The switch alternates between node a, located to left on the top wire, and node b, which is located on the additional wire. The circuit is powered by a 28V independent voltage source with reverse polarity, located on the left wire, and a 20V independent voltage source, located on the right wire.

The circuit shows a voltage source, V sub g, on the left wire, a capacitor C and resistor R on the top wire, and an inductor L with voltage v sub o of t on the right wire. An additional wire is drawn across two nodes on either side of the capacitor. A switch t = 0 is located on this additional wire.

The circuit shows an op amp which has a positive supply voltage, V sub CC, and a negative supply voltage, minus V sub CC. Two capacitors in series, labeled as C, and a parallel resistor in between them, labeled as R over 2, connect output terminal of the op amp with inverting input terminal. Non-inverting input terminal and the output terminal are grounded with common node. The inverting input terminal is grounded with the common node, while connected in series with two resistors labeled as R, and in parallel with capacitor 2C, located between the two resistors. Voltage across the input inverting terminal is v sub g while the voltage across the output terminal is v sub o.

Thr X-axis plots time t in seconds, while the y-axis plots voltage v sub g in mV. The graph shows a rectangular pulse drawn between x = 0 and x = 0.5, with a steady value of y = 80. The pulse continues in the fourth quadrant from x = 0.5 onwards, with a steady value of y = minus 40.

The circuit shows two op amps where output terminal of the left op amp is connected to the inverting input terminal of the right op amp, with a 400k ohm resistor located in series between them. The non-inverting input terminals of both the op amps are grounded with a common node.

In the left op amp, a 500nF capacitor connects the output terminal with the inverting input terminal, which is in turn grounded with the common node, in series with a 100k ohm resistor and an independent voltage source v sub g. Voltage across the output terminal is v sub o (1). The op amp has a 5V positive voltage supply and a minus 5V negative voltage supply.

In the right op amp, a 200nF capacitor connects the output terminal with the inverting input terminal. The op amp has a 12.5 positive voltage supply and a minus 12.5 negative voltage supply. The output terminal is grounded with the common node and shows voltage v sub o.

The drawing shows a rectangular mass M connected by a spring K to the vertical arm of an L shaped support. A force f of t moves the mass horizontally to right. Small gap between the mass and horizontal arm of the support is marked as D. Displacement of the mass, when the force is applied, is indicated by x of t.

Shown op amps are labeled from 1 to 6. Op amp 1 to 4 are depicted in a series, with output terminal of first connecting to inverting input terminal of second, and so on. Non-inverting input terminals of these four op amps are grounded with common node. Op amp 5 and 6 are shown below the series.

Op amp 1 shows resistor R8 connecting its output terminal with the inverting input terminal, which is connected to terminal f of t in series with resistor R7.

Op amp 2 shows resistor R connecting its output terminal with the inverting input terminal, which in turn is connected to three parallel resistors labeled as R, and connected to three nodes labeled as F, E, and D. Node F connects to the output terminal of op amp 1, node E connects to the output terminal of op amp 5, and Node D connects to the output terminal of op amp 6.

Op amp 3 shows a capacitor C1 connecting its output terminal with the inverting input terminal, which is connected to node A in series with resistor R1. The output terminal is also connected to the inverting input terminals of op amp 4 and op amp 5.

Op amp 4 shows a capacitor C2 connecting its output terminal with inverting input terminal, which is connected to node B in series with resistor R2. The output terminal is also connected to node C and to the non-inverting input terminal of op amp 6.

Op amp 5, shown below the series, has resistor R3 connecting its output terminal with the inverting input terminal, which in turn is connected to output terminal of op amp 3, in series with resistor R4. Non-inverting input terminal is grounded with the common node.

Op amp 6, at the bottom, has resistor R5 connecting its output terminal with inverting input terminal, which in turn is grounded with the common node in

series with resistor R6.

The X-axis plots time t, while the y-axis plots voltage v, marking minus V sub m, 0, and V sub m. The graph shows a sinusoidal wave moving on either side of the x-axis with an amplitude marked as V sub m and frequency marked as T.

The X-axis plots time t, while the y-axis plots voltage v, marking minus V sub m, 0, and V sub m. The graph shows a sinusoidal wave moving on either side of the x-axis, hitting peak values at y = V sub m and y = minus V sub m. A dotted sinusoidal wave is depicted along the same time axis, shifted to the right by a distance of phi over omega, but moving with the same frequency and amplitude.

The X-axis plots time t, marking minus T over 2 and minus T over 4 to the left of the origin, and T over 4, T over 2, 3T over 4, and T to the right of the origin. The Y-axis plots current i, marking minus I sub p and I sub p on either side of the origin. The graph shows a triangular pulse with a frequency of T over 2 passing through the origin and moving on either side of the x-axis, peaking at y = I sub p and y = minus I sub p.

The X-axis plots time t, marking minus T over 2 and minus T over 4 to the left of the origin, and T over 4, T over 2, 3T over 4, and T to the right of the origin. The Y-axis plots current i square, and marks I sub p square. The graph shows a triangular pulse, with curved edges at the bottom, passing through the origin and moving above the x-axis. The curve moves with a frequency of T over 2, and peaks at y = I sub p square.

The Circuit shows an independent voltage source V sub s on the left wire, switch t = 0 and resistor R with a rightward current i of t on the top wire, and inductor L on the right wire.

A drawing shows a right angled triangle used in the solution for y, with the following dimensions:

Hypotenuse: 44.72

Base: 37.32

Lateral side: 24.64

Base angle: 33.43 degrees

The drawing shows resistor R with voltage V between its terminals, and current I flowing rightward.

The X-axis plots time t, marking minus T over 4 at the left of the origin, and T over 2, T, 3T over 2, and 2T to the right of the origin. The Y-axis plots current i and voltage v. The graph shows a sinusoidal wave hitting a peak value of v on either side of the time-axis, along with another sinusoidal wave with the same frequency but a lesser amplitude, overlapping the first wave and hitting a peak value of i on either side of the time-axis.

The drawing shows inductor j omega L, with voltage V between its terminals and current I flowing rightward.

The X-axis plots time t, marking minus T over 4 at left of origin, and T over 2, T, and 3T over 2 to the right of the origin. The Y-axis plots current i and voltage v. The graph shows a sinusoidal wave hitting a peak value of v on either side of the time-axis, along with another sinusoidal wave with the same frequency but a lesser amplitude, separated from the first wave by 90 degrees and hitting a peak value of i on either side of the time-axis.

The drawing shows capacitor 1 over j omega C, with voltage V between its terminals and current I flowing rightward.

The X-axis plots time t, marking minus T over 4 at the left of the origin, and T over 2, T, 3T over 2, and 2T to the right of the origin. The Y-axis plots current i and voltage v. The graph shows a sinusoidal wave hitting a peak value of v on either side of the time-axis, along with another sinusoidal wave with the same frequency but a lesser amplitude, separated from the first wave by 90 degrees, and hitting a peak value of i on either side of the time-axis.

Impedance and reactance values are shown for the three circuit elements, respectively, as follows.

Resistor: R; blank

Inductor: j omega L; omega L

Capacitor: j (minus 1 over omega C); minus 1 over omega C

The drawing shows a 100 ohm resistor in series with a 50 mH inductor, with current i flowing rightward through the node between them. The respective voltages in the resistor and inductor are v sub R and v sub L.

The drawing shows resistor Z sub R in series with inductor Z sub L, with current I flowing rightward through the node between them. The respective voltages in the resistor and inductor are V sub R and V sub L.

The circuit shows several impedances in series, between terminals a and b. The impedances are labeled as Z1, Z2, etc., up to Z sub n. A phasor current I flows rightward through all of them. The Phasor voltage between the terminals a and b is marked as V sub ab.

The circuit shows a 90 ohm resistor and a 32mH inductor on the top wire, a 5 microF capacitor with voltage v on the right wire, and an independent voltage source, v sub s, on the left wire.

The circuit shows a 90 ohm resistor and a j160 ohm inductor on the top wire, a minus j40 ohm capacitor with voltage V on the right wire, and an independent voltage source with voltage of 750-angled 30 degree V on the left wire. Two nodes, a and b, are marked on the top and bottom wires, close to the left wire.

The circuit shows several impedances in parallel between terminals a and b. The impedances are labeled as Z1, Z2, etc., up to Z sub n, with respective phasor currents of I1, I2, etc., up to I sub n. The Phasor voltage between the terminals a and b is marked as V, while current I flows rightward from a.

Admittance(Y) and Susceptance values are shown respectively as follows:

Resistor: G (conductance); blank

Inductor: j of minus 1 over omega L; minus 1 over omega L

Capacitor: j omega C; omega C

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a 10 ohm resistor while the right cross wire carries a 6 ohm resistor and a 40 microH inductor, along with a downward current i. The right wire carries a 1 microF capacitor. The circuit is powered by an independent current source, i sub s with an upward current, located on the left wire. Voltage across the left wire is marked as v.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a 10 ohm resistor while the right cross wire carries a 6 ohm resistor and a j8 ohm inductor, along with a downward current I. The right wire carries a minus j5 ohm capacitor. The circuit is powered by an independent current source located on the left wire, with an upward current 8 angled 0 degree A. Voltage across the left wire is marked as V.

The drawing shows a triangular circuit with three nodes, a, b, and c, at three corners. Three delta connected impendences are located on the wires connecting the nodes, as follows:

Z sub c: between a and b

Z sub a: between b and c

Z sub b: between c and a

A node n is depicted at the middle of the triangle and is connected to a, b, and c. Three wye connected impendences are located on wires connecting a, b, and c to n, as follows:

Z1: between a and n

Z2: between b and n

Z3: between c and n

Circuit shows an independent voltage source on left wire, with voltage of 120 angled 0 degree V. A rightward current, I sub 0, flows through top wire. A diamond shaped circuit is shown in place of right wire, between nodes a and d on top and bottom, and nodes b and c at sides. This network is made of an upper delta (abc) and lower delta (bcd), with common bridge wire bc between them.

The upper delta has a minus j4 ohm capacitor on wire ab with downward current I1, a 63.2 ohm resistor and a j2.4 ohm inductor on wire ac with downward current I2, and a 10 ohm resistor with rightward current I3 on wire bc .

The lower delta has a 20 ohm resistor and j60 ohm inductor on wire bd with downward current I4, and a minus j20 ohm capacitor on wire cd with downward current I5.

Voltages at node b and node c are marked as V1 and V2, respectively.

The circuit shows an independent voltage source on the left wire, with voltage of 120 angled 0 degree V. A rightward current, I sub 0, flows through the top wire. A diamond shaped circuit is shown in place of the right wire, between nodes a and d on the top and bottom, and nodes b and c at the sides. Upper delta, abc, remains as in the earlier circuit from figure 9.23. The lower delta of bcd is replaced by a wye network, consisting of three nodes, b, c, and d, forming three end points, and node n, forming the midpoint of the wye-network.

The upper delta has a minus j4 ohm capacitor on wire ab, and a 63.2 ohm resistor and a j2.4 ohm inductor on wire ac.

The wye network has a 12 ohm resistor and a j4 ohm inductor on wire bn, a minus j2.4 capacitor and a minus 3.2 ohm resistor on wire cn, and an 8 ohm resistor and minus j2.4 ohm capacitor on wire dn.

The circuit shows an independent voltage source on the left wire, with voltage of 120 angled 0 degree V. A rightward current, I sub 0, flows through the top wire. The right wire, which has node a on top, carries an 18 ohm resistor and a minus j24 ohm capacitor.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and a horizontal cross wire connecting a node at the midpoint of the vertical cross wire and another node at the midpoint of the right wire. The horizontal cross wire carries a 50 ohm resistor. The vertical cross wire carries a j40 ohm inductor on the top arm and a 40 ohm resistor on the bottom arm. The right wire carries a minus j15 ohm capacitor on the top arm and a 10 ohm resistor on the bottom arm. A 14 ohm resistor with a rightward current i is located on the left arm of the top wire. The circuit is powered by a voltage source with voltage marked as 136 angled 0 degree V.

The drawing shows two circuits with a voltage source and a current source. The left circuit shows an independent voltage source V sub s on the left wire, an impedance Z sub s on the top wire, and two open terminals, a and b, in place of the right wire. The value of V sub s is shown as equal to Z sub s multiplied by I sub s.

The circuit on the right shows an independent current source I sub s on the left wire, an impedance Z sub s on a parallel cross wire, and two open terminals, a and b, in place of the right wire. Value of I sub s is shown as equal to V sub s over Z sub s.

The drawing shows two boxes where the left box contains two internal terminals, a and b, along with the following text: Frequency-domain linear circuit; may contain both independent and dependent sources. An arrow points from the box on the left to the box on the right, which contains a circuit with an independent voltage source, V sub Th on the left wire, an impedance Z sub Th on the top wire, and two open terminals, a and b, in place of the right wire.

The drawing shows two boxes where the left box contains two internal terminals, a and b, along with the following text: Frequency-domain linear circuit; may contain both independent and dependent sources. An arrow points from the box on the left to the box on the right, which contains a circuit with an independent current source I sub N on the left wire, an impedance Z sub N on a parallel cross wire, and two open terminals, a and b, in place of the right wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a 9 ohm resistor and a minus j3 ohm capacitor. The right cross wire carries a 10 ohm resistor and a minus j19 ohm capacitor. The top arm carries a 1 ohm resistor and j3 ohm inductor on the left arm, and a 0.2 ohm resistor and a j0.6 ohm inductor on the middle arm. The right wire is removed and the two open terminals on the top and bottom wires have voltage across them marked as V sub 0. An independent voltage source with voltage of 40 angled 0 degree V is located on the left wire.

The circuit shows three parallel vertical cross wires connecting three pairs of nodes on the top and bottom wires. The middle cross wire carries a 9 ohm resistor and a minus j3 ohm capacitor. The right cross wire carries a 10 ohm resistor and a minus j19 ohm capacitor. The top arm carries a 0.2 ohm resistor and a j0.6 ohm inductor on its third arm. The right wire is removed and two open terminals on the top and bottom wire have voltage across them marked as V sub 0.

An independent current source with an upward current 4 minus j12 A is located on the left wire. The left cross wire shows a 1 ohm resistor and a j3 ohm inductor.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires. A 10 ohm resistor and a minus j19 ohm capacitor are located on this cross wire. The right wire is removed and the two open terminals on the top and bottom wires have voltage across them marked as V sub 0.

The left arm of the top wire carries a 1.8 ohm resistor, a j2.4 ohm inductor with a rightward current I sub 0, a 0.2 ohm resistor, and a j0.6 ohm inductor. The circuit is powered by an independent voltage source on the left wire, with voltage of 36 minus j12 V.

The drawing shows two small circuits inside a box connected to each other. The circuit on the left shows an independent voltage source 120 angled 0 degree V on the left wire, a 12 ohm resistor on the top wire, and a 60 ohm resistor on the right wire, with voltage between nodes marked as V sub x. The circuit on the right shows a dependent voltage source, 10 V sub x, on the left wire, and a 120 ohm resistor on the top wire. The right wire is removed and the two open terminals on the top and bottom wires are shown outside the box.

A wire connects the bottom nodes of the two circuits. Another additional wire on top connects the top right node of the left circuit with a node on the right circuit, located to the right of the 120 ohm resistor on the top wire. This additional wire carries a minus j40 ohm capacitor.

The drawing shows two small circuits inside a box connected to each other. The circuit on the left shows an independent voltage source 100 angled 0 degree V on the left wire and a 10 ohm resistor on the top wire. The right wire from figure 9.32 is removed and the voltage in its place is marked as V sub x.

The circuit on the right shows a dependent voltage source 10 V sub x on the left wire and a 120 ohm resistor on the top wire. The right wire is removed and the two open terminals, a and b, on the top and bottom wires are shown outside the box. Voltage between a and b is marked as V sub Th.

The two circuits are connected at the bottom. Another additional wire on top connects the top right end of the left circuit with a node on the right circuit, located to the right of the 120 ohm resistor on the top wire. This additional wire carries a minus j40 ohm capacitor on top, with a rightward current I.

The drawing shows two small circuits inside a box connected to each other. The circuit on the left shows a 12 ohm resistor on the top wire and a 60 ohm resistor on the right wire, with voltage V sub x marked across its nodes.

The circuit on the right a shows dependent voltage source 10 V sub x on the left wire and a 120 ohm resistor on the top wire, with a leftward current I sub b. The right wire is removed and the two terminals, a and b, on the top and bottom wires are shown outside the box. The two terminals are connected to an independent voltage source, V sub Th, on the right. A leftward current, I sub Th, flows through node a.

The two circuits are connected at the bottom. Another additional wire on top connects the node at the top right end of the left circuit with a node on the right circuit, located to the right of the 120 ohm resistor on the top wire. This additional wire carries a minus j40 ohm capacitor on top, with a leftward current I sub a.

The circuit inside a box shows an independent voltage source on the left wire with voltage 784 minus j288 V. The top wire carries a 91.2 ohm resistor and a minus j38.4 ohm capacitor. The right wire is removed and the two terminals, a and b, on the top and bottom wire are shown outside the box.

Circuit shows two parallel vertical cross wires connecting two pairs of nodes on top and bottom wires. A 30 ohm resistor is located on left cross wire while a 25 over 6 microF capacitor is located on right cross wire. Voltage between top and bottom wires is marked as v sub o of t. Top wire carries a 15 mH inductor on left arm and a 20 ohm resistor on right arm. The circuit is powered by an independent voltage source, v1, on left wire and another independent voltage source with reverse polarity, v2, on right wire.

The circuit shows three parallel vertical cross wires connecting three pairs of nodes on the top and bottom wires. A 20 ohm resistor with a downward current I sub x is located on the left cross wire, while a minus j10 ohm capacitor is located on the right cross wire. A 10 I sub x dependent voltage source is located on the middle cross wire.

The top wire carries a j10 ohm inductor on the second arm and a 10 ohm resistor on the third arm. The right wire is removed and two nodes, a and b, are marked on the top and bottom wires, respectively. The circuit is powered by an independent current source on the left wire, with an upward current 2 angled 45 degree A.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. A 10 ohm resistor with a downward current i sub a is located on the left cross wire, while a 400 microF capacitor with a downward current i sub c is located on the right cross wire. The top wire carries a 1 ohm resistor, with a rightward current i sub x, and a 4 mH inductor on the middle arm, and a 5 ohm resistor with a rightward current I sub b on the right arm. The circuit is powered by an independent current source with an upward current i sub s on the left wire, and a dependent voltage source, 20 i sub x, on the right wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. A 10 ohm resistor with a downward current I sub a is located on the left cross wire, while a minus j5 ohm capacitor with a downward current I sub c is located on the right cross wire. The top wire carries a 1 ohm resistor with a rightward current I sub x and a j2 ohm inductor on the middle arm, and a 5 ohm resistor with a rightward current I sub b on the right arm. The circuit is powered by an independent current source with an upward current 10.6 angled 0 degree A on the left wire and a dependent voltage source, 20 I sub x, on the right wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The two nodes on the top wire are labeled as 1 and 2, while the left node on the bottom wire is marked as the reference node. A 10 ohm resistor with voltage V sub 1 is located on the left cross wire, while a minus j5 ohm capacitor with voltage V sub 2 is located on the right cross wire. The top wire carries a 1 ohm resistor and a j2 ohm inductor on the middle arm, with a rightward current I sub x, and a 5 ohm resistor on the right arm. The circuit is powered by an independent current source with an upward current, 10.6 angled 0 degree A, on the left wire and a dependent voltage source, 20 I sub x, on the right wire.

The circuit shows three parallel vertical cross wires connecting three pairs of nodes on the top and bottom wires. A 5 ohm resistor, a 9 microF capacitor, and a 100 microH inductor are located on the left, middle, and right cross wires, respectively. Voltage across the top and bottom wires is marked as v of t. A 20 ohm resistor is located on the right arm of the top wire. The circuit is powered by an independent current source with an upward current, i sub s, on the left wire, and an independent voltage source, v sub s, on the right wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires. A 12 ohm resistor and a minus j16 ohm capacitor are located on this cross wire. Voltage across the capacitor is marked as V2 while a downward current, I sub x, flows through it.

The left arm of the top wire carries a 1 ohm resistor and a j2 ohm inductor, while the right arm carries a 1 ohm resistor and a j3 ohm inductor. The respective voltages across the left and right arms of the top wire are marked as V1 and V3. The circuit is powered by an independent voltage source, 150 angled 0 degree V, located on the left wire, and a dependent voltage source, 39I sub x, located on the right wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires. A 12 ohm resistor and a minus j16 ohm capacitor are located on this cross wire. A downward current, I sub x, flows through the resistor.

The left arm of the top wire carries a 1 ohm resistor and a j2 ohm inductor, while the right arm carries a 1 ohm resistor and a j3 ohm inductor. The circuit is powered by an independent voltage source, 150 angled 0 degree V, located on the left wire, and a dependent voltage source, 39 I sub x, located on the right wire. Two currents, I1 and I2, are shown in the left and right mesh respectively, as two clockwise curving arrows.

The circuit shows two vertical cross wires connecting two pairs of nodes on the top and bottom wires. A 3 ohm resistor and a minus j5 ohm capacitor are located on the left cross wire, while a 2 ohm resistor is located on the right cross wire. Voltage across the capacitor on the left cross wire is marked as V sub x.

The left arm of the top wire carries a 1 ohm resistor and a j2 ohm inductor, with a rightward current I. The circuit is powered by an independent voltage source, 33.8 angled 0 degree V, located on the left wire, and a dependent voltage source, 0.75 V sub x, located on the right wire.

The drawing shows two circuits where two inductor coils on adjacent wires at the center represent a transformer. The inductor j omega L1 on the left has a dot terminal on the top left, while the inductor j omega L2 on the right has a dot terminal on the top right. Mutual inductance between the two coils is marked as j omega M.

The circuit on the left shows an independent voltage source, V sub s, on the 0left wire, marked as Source. The left arm of the top wire shows an internal impedance, Z sub S, with a rightward current I sub s, while the right arm shows resistance R1. These two elements are separated by node a, while another node b is located on the bottom wire.

The circuit on the right shows impendance Z sub L on the right wire, marked as Load. A resistor R2, with a rightward current I2, is located on the left arm of the top wire. Two nodes, c and d, are located on the top and bottom wires, respectively.

The drawing shows two circuits where two inductor coils on adjacent wires at the center represent a linear transformer, with mutual inductance marked as j1200. A j3600 ohm inductor on the left has a dot terminal on the top left, while a j1600 ohm inductor on the right has a dot terminal on the top right. The left circuit shows two nodes, a and b, on the top and bottom wires, with voltage marked as V1. A 200 ohm resistor is located on the right arm of the top wire. The right circuit shows two nodes, c and d, on the top and bottom wires, with voltage marked as V2. A 100 ohm resistor is located on the left arm of the top wire. A shaded box is drawn encompassing the nodes a, b, c, and d on the two circuits.

The circuit on the left shows an independent voltage source, 300 angled 0 degree V, on the left wire. The left arm of the top wire shows a 500 ohm resistor and a j100 ohm inductor, with a rightward current I1.

The circuit on the right shows a 800 ohm resistor on the right arm of the top wire and a minus j2500 ohm capacitor on the right wire with a downward current I2.

The circuit shows a 171.09 ohm resistor and a j1224.26 ohm inductor on the top wire. The left wire carries an independent voltage source 95.60 angled 10.71 degree V. The right wire is removed and the two open terminals, c and d, are marked on the top and bottom wires, respectively.

The drawing shows two circuits where two inductor coils on adjacent wires at the center represent a linear transformer, with mutual inductance marked as j omega M. Inductor j omega L1 on the left has a dot terminal on the top left, while j omega L2 inductor on the right has a dot terminal on the top right. The left circuit shows two nodes, a and b, on the top and bottom wires. Resistor R1 is located on the right arm of the top wire. The right circuit shows two nodes, c and d, on the top and bottom wires. Resistor R2 is located on the left arm of the top wire. A shaded box is drawn encompassing the nodes a, b, c, and d on the two circuits.

The circuit on the left shows an independent voltage source, V sub s, on the left wire, marked as Source. The left arm of the top wire shows an internal impedance, Z sub S, with a rightward current I sub s.

The circuit on the right shows impedance Z sub L on the right wire, marked as Load.

The drawing shows two circuits where two inductor coils on adjacent wires at the center represent an ideal transformer, with mutual inductance marked as j omega M. Inductor j omega L1 on the left, with N1 turns, has a dot terminal on the top left, while j omega L2 inductor on the right, with N2 turns, has a dot terminal on the top right.

The circuit on the left shows an independent voltage source, V1, on the left wire, and a rightward current, I1, on the top wire. Two nodes are located at the midpoints of the top and bottom wires. The circuit on the right has its right wire removed, with two nodes located on the top and bottom wires to the right, with voltage V2 marked between them.

The drawing shows two circuits where two inductor coils on adjacent wires at the center represent an ideal transformer, with mutual inductance marked as j omega M. Inductor j omega L1 on the left, with N1 turns, has a dot terminal on the top left, while j omega L2 inductor on the right, with N2 turns, has a dot terminal on the top right.

The circuit on the left shows an independent voltage source, V1, on the left wire, and a rightward current, I1, on the top wire. Two nodes are located at the midpoints of the top and bottom wires. The circuit on the right has two nodes located on the top and bottom wires, with a rightward current I2 flowing through the top wire.

The figure depicts two circuits where two inductor coils on adjacent wires are shown at the center, separated by two vertical lines. The inductor on the left has N1 turns and a dot terminal on the top left, while the inductor on the right has N2 turns and a dot terminal on the top right.

The drawing shows an ideal transformer, with the left coil having N1 turns and the right coil having N2 turns. The left coil has voltage V1, and current I1 that is directed away from the dot terminal on the top left. The right coil has voltage V2, and current I2 that is directed away from the dot terminal on the top right. Voltage and current equations are given as follows:

V1 over N1 = V2 over N2

N1 multiplied by I1 = minus N2 multiplied by I2.

The drawing shows an ideal transformer, with the left coil having N1 turns and the right coil having N2 turns. The left coil has voltage V1, and current I1 that is directed away from the dot terminal on the top left. The right coil has voltage V2, and current I2 that is directed into the dot terminal on the bottom right. Voltage and current equations are given as follows:

V1 over N1 = minus V2 over N2

N1 multiplied by I1 = N2 multiplied by I2.

The drawing shows an ideal transformer, with the left coil having N1 turns and the right coil having N2 turns. The left coil has voltage V1, and current I1 that is directed away from the dot terminal on the top left. The right coil has voltage V2, and current I2 that is directed into the dot terminal on the top right. Voltage and current equations are given as follows:

V1 over N1 = V2 over N2

N1 multiplied by I1 = N2 multiplied by I2.

The drawing shows an ideal transformer, with the left coil having N1 turns and the right coil having N2 turns. The left coil has voltage V1, and current I1 that is directed away from the dot terminal on the top left. The right coil has voltage V2, and current I2 that is directed away from the dot terminal on the bottom right. Voltage and current equations are given as follows:

V1 over N1 = minus V2 over N2

N1 multiplied by I1 = minus N2 multiplied by I2.

The drawing shows an ideal transformer, with the left coil having voltage V1 and a dot terminal on the top left, and the right coil having voltage V2 and a dot terminal on the top right. The number of turns of the left and right coils are shown respectively as N1 = 500 and N2 = 2500.

The drawing shows an ideal transformer, with the left coil having voltage V1 and a dot terminal on the top left, and the right coil having voltage V2 and a dot terminal on the top right. Turns ratio between the left coil and the right coil is mentioned as 1 is to 5.

The drawing shows an ideal transformer, with the left coil having voltage V1 and a dot terminal on the top left, and the right coil having voltage V2 and a dot terminal on the top right. Turns ratio between the left coil and the right coil is mentioned as 1 over 5 is to 1.

The circuit shows an ideal transformer with voltage v1 on the left inductor and voltage v2 on the right inductor. Turns ratio is indicated as 10 is to 1. Dot terminals are shown at the top left of the left inductor and the top right of the right inductor. The top wire of the left circuit shows a 0.25 ohm resistor with a rightward current i1, along with a 5mH inductor, while the left wire carries an independent voltage source, v sub g. Two nodes are displayed on the top and bottom wires of the right circuit. The right arm of the top wire of the right circuit shows a 237.5m ohm resistor with a rightward current i2, while the right wire carries a 125 microH inductor.

The circuit shows an ideal transformer with voltage V1 on the left inductor and voltage V2 on the right inductor. Turns ratio is indicated as 10 is to 1. Dot terminals are shown at the top left of the left inductor and the top right of the right inductor. The top wire of the left circuit shows a 0.25 ohm resistor with a rightward current I1, and a j2 ohm inductor, while the left wire carries an independent voltage source with voltage 2500 angled 0 degree V. The top wire of the right circuit shows a 0.2375 ohm resistor with a rightward current I2, while the right wire carries a j0.05 ohm inductor.

The circuit shows an ideal transformer with voltage V1 on the left inductor and voltage V2 at the right inductor. Turns ratio is indicated as 25 is to 1. Dot terminals are shown at the top left of the left inductor and bottom right of the right inductor. The top wire of the left circuit shows a 1.5 ohm resistor with a rightward current I1, and a j6k ohm inductor, while the left wire carries an independent voltage source, v sub s. The top wire of the right circuit shows a 4 ohm resistor, while the right wire carries a minus j14.4 ohm capacitor with a downward current I2.

The circuit shows an ideal transformer with turns ratio of 1 is to a. Dot terminals are shown at the top left of the left inductor and the top right of the right inductor. The left circuit shows an independent voltage source, V sub s, on the left wire, and an impedance Z sub s on the left arm of the top wire, with a rightward current I1. Two nodes are shown on the top and bottom wires, with voltage V1 marked between them.

The right circuit shows load impedance Z sub L on the right wire. The top wire carries a rightward current I2. Two nodes are shown on the top and bottom wires, with voltage V2 marked between them.

The drawing shows four phasor quantities represented as arrows drawn from the origin of a plane. The positive horizontal axis is labeled as Real while the positive vertical axis is labeled as Imag. The magnitude and angle of the four lines, represented in the four quadrants in a counter-clockwise direction are as follows:

First quadrant: Magnitude-10 angled 30 degree; Angle-30 degree

Second quadrant: Magnitude-12 angled 150 degree; Angle-150 degree

Third quadrant: Magnitude-8 angled minus 170 degree; Angle-Minus 170 degree

Fourth quadrant: Magnitude-5 angled minus 45 degree; Angle-minus 45 degree

The drawing shows a plane where the positive horizontal axis is labeled as Real and the positive vertical axis is labeled as Imag. A phasor is shown in the third quadrant with magnitude 7 marked on the negative horizontal axis and magnitude j3 marked on the negative vertical axis. A dotted line is drawn from the origin below the phasor, with a positive angle of 225 degrees and a negative angle of minus 135 degrees.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a 0.2mH inductor with voltage v sub m, while the right cross wire carries a 50 microF capacitor. The right wire carries resistor R, while the left wire carries an independent current source, i sub s, with an upward current. Three downward currents, i sub L, i sub C, and i sub R flow through the inductor, capacitor, and resistor respectively.

The drawing shows a plane where the positive horizontal axis is labeled as Real and the positive vertical axis is labeled as Imag. Three phasors are depicted in this graph, as follows:

I sub R = V sub m over R (shown along positive horizontal axis)

I sub C = j4V sub m (shown along positive vertical axis)

I sub L = minus j1V sub m (shown along negative vertical axis)

A dotted arrow is drawn from the origin at a 45 degree angle in the first quadrant and marked as I sub s.

The circuit shows two pairs of nodes on the top and bottom wires. The nodes to the right are connected by a vertical cross wire, which carries resistor R2. The nodes to the left have voltage v sub L marked across them. The top wire carries resistor R1 and inductor L1 to the left of the nodes. Inductor L2 is located on the right wire. An independent voltage source, v sub s, is located on the left wire.

The circuit shows two pairs of nodes on the top and bottom wires. The nodes to the right are connected by a vertical cross wire, which carries resistor R2 with a downward current I sub a. The nodes to the left have voltage V sub L marked across them. The top wire carries resistor R1 and inductor j omega L1 to the left of the nodes, along with a rightward current I. Inductor j omega L2 is located on the right wire, with a downward current I sub b. An independent voltage source, V sub s, is located on the left wire.

A phasor, V sub L, is placed on the positive real axis. Another phasor, I sub a, is placed in phase with V sub L. A third phasor, I sub b, is placed perpendicular to I sub a, pointing downward.

A phasor, V sub L, is placed on the positive real axis. Another phasor, I sub a, is placed in phase with V sub L. A third phasor, I sub b, is placed perpendicular to I sub a, pointing downward. A phasor for current I is placed at 45 degrees between I sub a and I sub b.

A phasor, V sub L, is placed on the positive real axis. Another phasor, I sub a, is placed in phase with V sub L. A third phasor, I sub b, is placed perpendicular to I sub a, pointing downward. A phasor for current I is placed at 45 degrees between I sub a and I sub b.

A phasor for R1I is placed in phase with I. A sixth phasor for j omega L1 I is placed at 90 degrees above I, at 45 degrees above the horizontal axis.

A phasor, V sub L, is placed on the positive real axis. Another phasor, I sub a, is placed in phase with V sub L. A third phasor, I sub b, is placed perpendicular to I sub a, pointing downward. A phasor for current I is placed at 45 degrees between I sub a and I sub b.

A phasor for R1I is placed in phase with I. A sixth phasor for j omega L1 I is placed at 90 degrees above I, at 45 degrees above the horizontal axis.

A phasor, V sub s, is placed between V sub L and j omega L1 I. Another phasor, R1I, is placed at the right end of V sub L, at an angle of 45 degrees in the fourth quadrant. A third phasor, j omega L1 I, is placed between the tips of R1I and V sub s.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries resistor R2 with a downward current I sub a, and the right cross wire carries inductor j omega L2 with a downward current I sub b.

Two nodes are shown on the top and bottom wires to the left of the left cross wire. Voltage V sub L is marked across them. The top wire carries resistor R1 and inductor j omega L1 to the left of the nodes, along with a rightward current I. Capacitor 1 over j omega C is located on the right wire, with a downward current I sub c. An independent voltage source, V sub s, is located on the left wire.

A phasor, V sub L, is placed on the positive real axis. Another phasor, I sub a, is placed in phase with V sub L. A third phasor, I sub b, is placed perpendicular to I sub a, pointing downward. Another phasor, I sub c, is placed perpendicular to I sub a, pointing upward. A phasor for current I is placed at 45 degrees between I sub a and I sub b.

A phasor, V sub L, is placed on the positive real axis. Another phasor, I sub a, is placed in phase with V sub L. A third phasor, I sub b, is placed perpendicular to I sub a, pointing downward. Another phasor, I sub c, is placed perpendicular to I sub a, pointing upward. A phasor for current I is placed at 45 degrees between I sub a and I sub b.

Three phasors are shown in dotted lines, representing currents and voltage before adding capacitor. A phasor, V sub s, is placed between V sub L and I sub c. Another phasor, R1I, is placed at the right end of V sub L, at an angle of 45 degrees in the fourth quadrant. A third phasor, j omega L1 I, is placed between the tips of R1I and V sub s.

Three phasors are shown in solid lines for V sub s, R1I, and j omega L1 I, representing currents and voltages after adding the capacitor. These phasors are shown at a slight angle above their dotted counterparts.

The drawing shows a rectangular circuit which represents a household circuit. A vertical cross wire connects two nodes on the top and bottom wires, while a horizontal cross wire connects a node at the midpoint of the vertical cross wire with a node at the midpoint of the left wire.

The left arm of the top wire carries a 100A fuse labeled as Fuse A, while the left arm of the bottom wire carries another 100A fuse labeled as Fuse B. A 30A fuse is located on the right arm of the top wire. The top arm of the vertical cross wire carries a 24 ohm resistor and a 15A fuse, while the bottom arm carries a 12 ohm resistor and a 15A fuse. The right wire carries a rectangular Fan motor which contains an 8.4 ohm resistor and a j6.30 ohm inductor.

The circuit is powered by two independent voltage sources located on the top and bottom arms of the left wire, with voltages marked as 120 angled 0 degree V. A vertical dotted line with an x is drawn to the right of fuse A, indicating a momentary short circuit which interrupts the fuse.

Six currents are shown in the circuits, at the following locations:

I1: Rightward on the left arm of the top wire

I2: Rightward on the horizontal cross wire

I3: Leftward on the left arm of the bottom wire

I4: Downward on top arm of the vertical cross wire

I5: Downward on bottom arm of the vertical cross wire

I6: Rightward on the right arm of the top wire

The drawing shows a rectangular circuit which represents the household circuit. A vertical cross wire connects two nodes on the top and bottom wires, while a horizontal cross wire connects a node at the midpoint of the vertical cross wire with a node at the midpoint of the left wire.

The left arm of the top wire carries two open nodes to indicate the interrupted Fuse A, while the left arm of the bottom wire carries a 100A fuse labeled as Fuse B. A 30A fuse is located on the right arm of the top wire. The top arm of the vertical cross wire carries a 24 ohm resistor and a 15A fuse, while the bottom arm carries a 12 ohm resistor and a 15A fuse. The right wire carries no elements, indicating a stalled fan motor.

The circuit is powered by two independent voltage sources, located on the top and bottom arms of the left wire, with voltage marked as 120 angled 0 degree V. The mesh current in the bottom left mesh is marked as I sub a, while the mesh current in the right mesh is marked as I sub b.

The table displays Impedance (Z), Reactance, Admittance (Y), and Susceptance, respectively for the following elements:

Resistor: R (resistance); no value; G (conductance); no value

Capacitor: j (minus 1 over omega C); minus 1 over omega C; j omega C; omega C

Inductor: j omega L; omega L; j (minus 1 over omega L); minus 1 over omega L

The X-axis plots time t, ranging from 0 to 2T in increments of T over 2, while the y-axis plots voltage v. The graph shows a sinusoidal half-wave, which moves from x = 0 to x = T over 2, peaking at y = V sub m. It then moves from x = T over 2 to x = T on the x-axis, and again moves as a half wave, and so on. This line is represented by equation v = V sub m multiplied by sine 2 pi over T multiplied by t, where 0 is less than or equal to t which is less than or equal to T over 2.

The drawing shows a rectangular box on the right, labeled as Circuit, with two internal terminals. The terminals are connected to an independent voltage source v sub g to the left, with a rightward current i sub g flowing on the top wire.

The circuit shows three parallel vertical cross wires connecting three pairs of nodes on the top and bottom wires. The left cross wire carries a 6 ohm resistor and a minus j2 ohm capacitor. The middle cross wire carries a j12 ohm inductor and a 4 ohm resistor. The right cross wire carries a 5 ohm resistor.

A j10 ohm inductor is located on the right wire. The left arms of the top and bottom wires carry a minus j12.8 ohm capacitor and a 13.6 ohm resistor, respectively. The left wire is removed and the two open terminals are marked as a and b on the top and bottom wires to the left. Admittance between the two terminals is marked as Y sub ab.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and a horizontal cross wire connecting two nodes on the vertical cross wire and right wire. The top arm of the vertical cross wire carries a j4 ohm inductor and a 2 ohm capacitor, while the bottom arm carries a 40 ohm resistor. The top arm of the right wire carries a 10 ohm resistor and a minus j20 ohm capacitor, while the bottom arm carries a j20 ohm inductor. The left arm of the top wire carries a 1 ohm resistor and a minus j8 ohm capacitor. The left wire is removed and the two open terminals are marked as a and b on the top and bottom wires to the left. Impedance between the two terminals is marked as Z sub ab.

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The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 30 ohm resistor. The left arm of the top wire carries a 1 microF capacitor, while the right wire carries a 1.2 mH inductor with voltage v sub o marked across it. The circuit is powered by an independent voltage source, v sub g, located on the left wire.

The circuit shows an independent voltage source, v sub s, on the left wire, a 4 ohm resistor with a rightward current i sub o of t and a 240 mH inductor on the top wire, and a 2.5 mF capacitor on the right wire.

The circuit shows two vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a 2.5 mH inductor while the right cross wire carries a 20 ohm resistor. The right wire carries a 12.5 microF capacitor. The top wire carries a 5 ohm resistor with a rightward current i sub o on its left arm, and a 10 ohm resistor on its middle arm. The circuit is powered by an independent voltage source, v sub s, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 12.5 microF capacitor and a 120 ohm resistor. The right wire carries a 40 ohm resistor and a 60 mH inductor with voltage v sub o marked across it. The circuit is powered by an independent current source, i sub g, with an upward current, located on the left wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a j250 ohm inductor and a 500 ohm resistor, with a downward current I sub a. The right cross wire carries a minus j1000 ohm capacitor and a 2000 ohm resistor, with a downward current I sub b. The right arm of the top wire carries a 1000 ohm resistor, with a rightward current I sub c. The circuit is powered by an independent current source, I sub g, with an upward current, located on the left wire, and a j50V independent voltage source with reverse polarity, located on the right wire.

The circuit shows three parallel vertical cross wires connecting three pairs of nodes on the top and bottom wires. A 4 mH inductor, an 80nF capacitor, and a 500 ohm resistor are located on the left, middle, and right cross wires, respectively. A 100 ohm resistor is located on the second arm of the top wire. The circuit is powered by an independent current source, I sub g, with an upward current, located on the left wire. The right wire is removed and the voltage across two nodes on the top and bottom wires to the right is marked as v sub o.

The circuit shows a horizontal cross wire connecting two nodes on the left and right wires. Two parallel vertical cross wires connect two pairs of nodes on the horizontal cross wire and bottom wire. The left cross wire carries a j5 ohm inductor, while the right cross wire carries a minus j10 ohm capacitor with voltage V1.

The horizontal wire carries a 20 ohm resistor on its left arm, and a 12 ohm resistor in series with a j16 ohm inductor on its middle arm. An independent voltage source, V sub g, is located on the bottom arm of the left wire, while an independent current source with an upward current, I sub g, is located on the bottom arm of the right wire. An impedance Z is located on the top wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and a horizontal cross wire connecting two nodes on the vertical cross wire and right wire. The top arm of the vertical cross wire carries a j3 ohm inductor and a 1 ohm capacitor, while the bottom arm carries an impedance Z. The top arm of the right wire carries a minus j2 ohm capacitor with a downward current I sub a, while the bottom arm carries a minus j3 ohm capacitor. The horizontal cross wire carries a minus j5 ohm inductor with a rightward current I sub b. A 4 ohm resistor is located on the right arm of the bottom wire.

The circuit is powered by an independent voltage source, V sub g, located on the left wire.

The circuit shows an independent voltage source, v sub g, on the left wire, a 3k ohm resistor and a 1 H inductor with a rightward current i sub o on the top wire, and a 31.25 microF capacitor on the right wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 62.5 nF capacitor. The left arm of the top wire carries a 1k ohm resistor, with a rightward current i sub g. The right wire carries a 240 ohm resistor and a 10 mH inductor. The circuit is powered by an independent voltage source, v sub g, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 200 mH inductor. The left arm of the top wire carries a 500 ohm resistor with a rightward current i sub o, while the right arm carries a 200 ohm resistor. The right wire carries a 1 microF capacitor. The circuit is powered by an independent voltage source, v sub g, located on the left wire.

The circuit shows two vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a 480 ohm resistor with a voltage, v sub g of t marked across it. The right cross wire carries a 3.125 microF capacitor. The right wire carries a 200 ohm resistor. The circuit is powered by an independent current source, i sub g, with an upward current located on the left wire.

The circuit shows a variable capacitor C and a 10k ohm resistor connected in parallel between two nodes on the top wire. A 800 mH inductor is located on the left arm of the top wire, while a rightward current i sub g flows through its right arm. The circuit is powered by an independent voltage source, v sub g, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 2k ohm resistor. The left arm of the top wire carries a 400 ohm resistor with a rightward current i sub g, and a 2.5 microF capacitor. The right wire carries inductor L. The circuit is powered by an independent voltage source, v sub g, located on the left wire.

The shown triangular circuit connects to node a and node b on its base to the left and right, respectively. It shows nodes marked at the midpoints of each of its sides. Three cross wires are drawn connecting these nodes to form a small triangle inside this bigger triangular circuit.

The following elements are located on the sides of the small triangle:

Top cross wire: 1 ohm resistor; minus j1 ohm capacitor

Left cross wire: j1 ohm inductor

Right cross wire: minus j1 ohm capacitor

The following elements are located on the sides of the bigger triangle:

Left wire: 1 ohm resistor on the lower arm; 1 ohm resistor on the upper arm

Right wire: 1 ohm resistor on the lower arm; j1 ohm inductor on the upper arm

Base: minus j1 ohm capacitor on the left arm; j1 ohm inductor on the right arm.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 36 ohm resistor. The left arm of the top wire carries a j60 ohm inductor, while the left arm of the bottom wire carries a minus j48 ohm capacitor. An independent voltage source is located on the left wire, with voltage marked as 240 angled 0 degree V. The right wire is removed and two terminals on the top and bottom wires to the right are marked as a and b.

The circuit shows two vertical cross wires connecting two pairs nodes on the top and bottom wires. A 50 ohm resistor and a minus j 100 ohm capacitor are located on the left and right cross wires, respectively. A j60 ohm inductor and a 30 ohm resistor are located on the middle arms of the top and bottom wires, respectively. An independent current source is located on the left wire with an upward current, 4 angled 0 degree A. The right wire is removed and two terminals on the top and bottom wires to the right are marked as a and b.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. A 31.25 microF capacitor is located on the left cross wire, while a 400mH inductor is located on the right cross wire. The left arm of the top wire carries a 320 ohm resistor, while the left wire carries an independent voltage source, v sub g. The right wire is removed and two open terminals on the top and bottom wires to the right are marked as a and b.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and a horizontal cross wire connecting two nodes on the left wire and the vertical cross wire. The vertical cross wire carries a 40 ohm resistor on its top arm and a j16 ohm inductor on its bottom arm. A 20 ohm resistor is located on the horizontal cross wire. The top arm of the left wire carries a minus j20 ohm capacitor, while the bottom arm carries an independent current source with an upward current, marked as 0.4 plus j0.2 A.

The right wire is removed and two open terminals on the top and bottom wires to the right are marked as a and b.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a minus j10 ohm capacitor with resistance marked as 1 ohm. The left arm of the top wire carries a 10 ohm resistor, with voltage marked as V sub x, while the right arm carries a dependent voltage source, 5V sub x, with reverse polarity. The left wire carries an independent voltage source with voltage marked as (minus 40 plus j40)V. The right wire is removed and two open terminals on the top and bottom wire to the right are marked as a and b.

The circuit shows three parallel vertical cross wires connecting three pairs of nodes on the top and bottom wires. The left cross wire carries a V2 over 5 dependent voltage source, the middle cross wire carries a 88I sub phi dependent current source with a downward current, and the right cross wire carries a 10 ohm resistor with voltage V2. A 200 ohm resistor with a rightward current I sub phi is located on the left arm of the top wire, while a minus j50 ohm capacitor is located on its second arm. The left wire carries an independent voltage source, V sub s. The right wire is removed and two nodes on the top and bottom wires to the right are marked as a and b.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. A 0.03V sub o independent current source with an upward current is located on the left cross wire, while a minus j100 ohm capacitor with voltage V sub o is located on the right cross wire. The left arm of the top wire carries a 20 ohm resistor and a j10 ohm inductor, while the middle arm carries a 50 ohm resistor. The circuit is powered by an independent voltage source on the left wire, marked as 250 angled 0 degree V. The right wire is removed and two nodes on the top and bottom wires to the right are marked as a and b.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 100 ohm resistor. The top node on the cross wire connects to two L shaped wires, which lead to the left and right arms of the top wire. The left arm of the top wire carries a 1 microF capacitor and a 10k ohm resistor with a rightward current i sub delta, while the right arm carries a 199 i sub delta dependent current source and a 1 microF capacitor. A 4.7k ohm resistor is located on the right wire. The left wire is removed and two nodes on the top and bottom wires to the left are marked as a and b.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 600 microH inductor. The left arm of the top wire carries a 400 nF capacitor and a 5i sub delta dependent voltage source. A 30 ohm resistor with an upward current i sub delta is located on the right wire. The left wire is removed and two nodes on the top and bottom wires to the left are marked as a and b.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 1k ohm resistor with voltage v sub delta. The right arm of the top wire carries a 100 microF capacitor, while the right wire carries a dependent voltage source, alpha v sub delta. The left wire is removed and two nodes on the top and bottom wires to the left are marked as a and b.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 150 ohm resistor with voltage v sub o. The top wire carries a 50 microF capacitor on the left arm and a 50mH inductor on the right arm. The circuit is powered by two independent voltage sources, v sub g1 and v sub g2, located on the left wire and right wire, respectively.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 50 ohm resistor. The top wire carries two j10 ohm inductors on its left and right arms. Two nodes are located on the top and bottom wires, close to the right wire, with voltage V sub o marked between them. A 30 ohm resistor is located on the right wire. The circuit is powered by an independent voltage source on the left wire, with voltage marked as 240 angled 0 degree V.

The circuit shows a horizontal cross wire connecting two nodes on the left and right wires, and a vertical cross wire connecting two nodes on the horizontal cross wire and bottom wire. The horizontal cross wire carries a 50 microF capacitor on the left arm and a 12 ohm resistor on the right arm, while the vertical cross wire carries a 1.6mH inductor with a downward current i sub o.

A 100 microF capacitor is located on the top wire. The circuit is powered by an independent current source, i sub g with an upward current, located on the lower arm of the left wire, and an independent voltage source, v sub g, located on the lower arm of the right wire.

The circuit on the left shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 40 ohm resistor. The right arm of the top wire carries a j20 ohm inductor with a rightward current I sub 0. The circuit is powered by an independent current source on the left wire with an upward current, 25 angled 0 degree mA, and a dependent voltage source on the right wire, V sub 0 over 8.

The circuit on the right shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 50 ohm resistor. A minus j25 ohm capacitor with voltage V sub o is located on the right wire. A dependent current source with a downward current 16I sub 0 is located on the left wire. The two circuits are linked at their bottom wires.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a j4 ohm inductor and a 2.4I sub delta dependent voltage source, while the right cross wire carries a 5 ohm resistor with voltage V sub 0. The left wire carries a minus j8 ohm capacitor, with a downward current, I sub delta flowing through it. An independent current source with an upward current, 10 plus j10, is located on the right wire.

The circuit shows a horizontal cross wire connecting two nodes on the left and right wires, and a vertical cross wire connecting two nodes on the horizontal cross wire and bottom wire. The horizontal cross wire carries a 100 nF capacitor on the left arm and a 10 ohm resistor on the right arm, while the vertical cross wire carries a 10 ohm resistor.

A 10 microH inductor is located on the top wire. The circuit is powered by an independent voltage source, v sub a, located on the lower arm of the left wire, and an independent voltage source, v sub b, with reverse polarity, located on the lower arm of the right wire. A downward current, i sub a, flows through the lower arm of the left wire, while an upward current, i sub b, flows through the lower arm of the right wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 100 microH inductor with a downward current i sub o. The top wire carries a 10 ohm resistor on the left arm, and a 2 microF capacitor on the right arm. The circuit is powered by an independent voltage source, v sub a, located on the left wire, and another independent voltage source with reverse polarity, v sub b, located on the right wire.

A circuit shows two horizontal cross wires connecting two pairs of nodes on the left and right wires. A vertical cross wire connects a pair of nodes located on the bottom wire and lower horizontal cross wire. A 1 ohm resistor with a downward current I sub d is located on the vertical cross wire. The lower horizontal cross wire carries a minus j1 ohm capacitor with a rightward current I sub b on its left arm, and a j1 ohm inductor with a rightward current I sub c on its right arm. The upper horizontal cross wire carries a 5 ohm resistor with a rightward current I sub a.

The circuit is powered by two independent voltage sources and an independent current source. Voltage source on the lower arm of the left wire is marked as 10 angled 0 degree V, while on the lower arm of the right wire, it is marked as 5 angled 0 degree V. Current source on the top wire is marked as 1 angled 0 degree A, with a rightward current.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires. A 50 ohm resistor with a downward current i sub delta, and a dependent voltage source, 150i sub delta, with reverse polarity, are located on this cross wire. The top wire carries a 60mH inductor on its left arm and a 2 microF capacitor on its right arm, while the right wire carries a 100 ohm resistor with voltage v sub o. The circuit is powered by an independent voltage source, v sub g, located on the left wire.

The circuit shows two unconnected nodes on the top and bottom wires, with voltage v sub o marked across them. The top wire carries a 300 ohm resistor and a 200 mH inductor to the left of the node, and a 500 ohm resistor to the right of the node. A 125 nF capacitor is located on the right wire, while an independent voltage source, v sub g, is located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 200 ohm resistor with a downward current i sub o, and a 125 nF capacitor. The right wire carries a 60 ohm resistor and a 60 mH inductor. The circuit is powered by an independent current source, i sub g, with an upward current, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 10 ohm resistor with voltage v sub o. The top wire carries a 1 mH inductor on the left arm and a 100 microF capacitor on the right arm. The circuit is powered by two independent voltage sources, v1 and v2, located respectively on the left wire and the right wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 25 mH inductor with voltage v sub o. The top wire carries a 625 nF capacitor on the left arm and a 400 ohm resistor on the right arm. The circuit is powered by two independent voltage sources, v sub a and v sub b, located respectively on the left wire and right wire.

The op amp has a 10V positive voltage supply and a minus 10V negative voltage supply. A wire connecting the output terminal with the inverting input terminal carries a 250 pF capacitor and a 40k ohm resistor connected in parallel. The inverting input terminal is grounded with the common node, in series with an 80k ohm resistor. The output terminal is grounded with the common node, with voltage v sub o marked across it. An independent voltage source, v sub g, which is in series with an 80k ohm resistor and in parallel with a 20k ohm resistor, connects the non-inverting input terminal with the common node. Value of v sub g is indicated as v sub g = 25 cos 50,000t V.

The op amp has a 5V positive voltage supply and a minus 5V negative voltage supply. A 10pF capacitor connects the output terminal with the inverting input terminal. The non-inverting input terminal is grounded with the common node. The output terminal is grounded with the common node, in series with a 40k ohm resistor with voltage v sub o marked across it.

An independent voltage source, v sub g, which is in series with a 5k ohm resistor and a 20k ohm resistor, and in parallel with a 100 pF capacitor, connects the inverting input terminal with the common node. A node is marked between two series resistors connected to the inverting input terminal. A wire connects this node to the output terminal, carrying a 100k ohm resistor.

The op amp has a 12V positive voltage supply and a minus 12V negative voltage supply. A 10k ohm resistor connects the output terminal with the inverting input terminal. The non-inverting input terminal is grounded with the common node. The output terminal is grounded with the common node, in series with a 33k ohm resistor with voltage v sub o marked across it.

An independent voltage source, v sub g, which is in series with two 20k ohm resistors and in parallel with a 250 nF capacitor, connects the inverting input terminal with the common node.

The op amp has a 6V positive voltage supply and a minus 6V negative voltage supply. A 500 nF capacitor connects the output terminal with the inverting input terminal. The output terminal is grounded with the common node, in series with a 100 ohm resistor with voltage v sub o marked across it. The inverting input terminal is grounded with the common node in series with a 20 ohm resistor.

An independent voltage source, v sub g, which is in series with a 5 ohm resistor and in parallel with variable capacitor C sub o, connects the non-inverting input terminal with the common node.

The drawing shows a 2 mH coil on the left with a dot terminal on the top left and a 8 mH coil on the right with a dot terminal on the bottom right. Mutual inductance between the coils is marked as 2mH. A 10 ohm resistor with a rightward current i sub g is located on the top wire of the left circuit, while its left wire carries an independent voltage source, v sub g. A rightward current, i sub L, is marked on the top wire of the right circuit, while its right wire carries a 30 ohm resistor.

The drawing shows a j5 ohm coil on the left with a dot terminal on the top left and a j125 ohm coil on the right with a dot terminal on the bottom right. Mutual inductance between the coils is marked as j20 ohm. A 5 ohm resistor is located on the top wire of the left circuit, while its left wire carries an independent voltage source, 425 angled 0 degree V(rms). A 45 ohm resistor is located on the top wire of the right circuit. The right wire of the right circuit is removed and two open terminals, c and d, are located on the top and bottom wires to the right.

The drawing shows a 125 mH coil on the left with a dot terminal on the top left and a 8 mH coil on the right with a dot terminal on the top right. Mutual inductance between the coils is marked as k. A 5 ohm resistor is located on the top wire of the right circuit, while its right wire carries a 12.5 microF capacitor. A 20 ohm resistor is located on the top wire of the left circuit. The left wire of the left circuit is removed and two open terminals, a and b, are located on the top and bottom wires to the left.

The drawing shows a 1 mH coil on the left with a dot terminal on the top left and a 4 mH coil on the right with a dot terminal on the top right. Mutual inductance between the coils is marked as k.

A pair of nodes are located on the top and bottom wires of the left circuit. The top wire carries a 150 ohm resistor and a 50 ohm resistor on either side of the node, with a rightward current i1 flowing through it. The left wire carries an independent voltage source, v sub g.

Similarly, a pair of nodes are located on the top and bottom wires of the right circuit. The top wire carries a 100 ohm resistor and a 200 ohm resistor on either side of the node. The right wire carries a 12.5 nF capacitor.

The drawing shows an ideal transformer between the left and middle circuit, with a turns ratio of 50 is to 1. Dot terminals are located at the top left of the left coil and the top right of the right coil. Two nodes are marked on the top and bottom wires of the middle circuit. The left wire of the left circuit is removed and two open terminals, a and b, are marked on the top and bottom wires to the left. Impendance at the left circuit is marked as Z sub ab.

A second ideal transformer is located between the middle and right circuits, with a turns ratio of 1 is to 20. Dot terminals are located at the top left of the left coil and the bottom right of the right coil. An impendance, Z sub L, is located on the right wire of the right circuit.

The circuit shows two coils, N1 and N2, located on the ight wire, separated by an ideal transformer. Both the coils have dot terminals located on their top right. An additional wire is drawn to the right, across two nodes on either side of N2, carrying an internal impendance, Z sub L. The left wire of the circuit is removed and two nodes, a and b, are located on the top and bottom wires to the left. Impendance at a and b is marked as Z sub ab.

The circuit shows two coils, N1 and N2, located on the left wire, separated by an ideal transformer. Both the coils have dot terminals located on their top right. An additional wire is drawn to the left, across two nodes on either side of N2, connected to two open terminals, a and b. Impendance at a and b is marked as Z sub ab. A load impendance, Z sub L, is located on the right wire of the circuit.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 7.5 ohm resistor. The left wire of the circuit is removed and two open terminals are marked at the top and bottom wires to the left, with voltage between them marked as V sub s.

Two nodes are located on the top and bottom wires, to the left of the cross wire, with the voltage across them marked as 120 angled 0 degree V. The voltage between the node on the top wire and the open terminal on the left wire is marked as V sub l.

The left arm of the top wire carries a 0.15 ohm resistor and a j6 ohm inductor. The right wire carries a j12 ohm inductor. A minus j2 ohm capacitor is shown to be connected in parallel with the right wire, and is depicted in broken lines.

The left wire and the right wire of the shown circuit are removed and open terminals are marked on the top and bottom wires on both sides. Voltage across the terminals on the left is marked as v sub s = V sub m multiplied by cos omega t. Voltage across the terminals on the right is marked as v sub o.

Two pairs of nodes are marked on the top and bottom wires. Two cross wires are drawn between these nodes in an X shape, with the top left node connecting to the bottom right node, and the top right node connecting to the bottom left node. A capacitor C and a resistor R1 are located on the top arms of this X formation, to the left and right respectively. A resistor R1 is located on the top wire between the nodes, while a variable resistor, R sub x, is located on the bottom wire between the nodes.

The drawing shows an ideal transformer between two coils of two circuits, with dot terminals located on the top left and top right of the left and right coils, respectively. Voltage across the top and bottom wires of the left circuit is marked as 13.2 angled 0 degree kV. A rightward current, I sub p, flows through the top wire of the left circuit.

The right circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and a horizontal cross wire connecting a node on the left wire with a node on the vertical cross wire. The horizontal cross wire divides the coil on the left wire into two parts, each of which has voltage marked as 120 angled 0 degree V. Each part of the coil shows a dot terminal on its top right.

The left arm of the top wire carries a 1 ohm resistor with a rightward current I1, the left arm of the horizontal cross wire carries a 2 ohm resistor with a rightward current I2, and the left arm of the bottom wire carries a 1 ohm resistor with a rightward current I3. The top arm of the vertical cross wire carries a 20 ohm resistor with a downward current I5, while the bottom arm carries a 40 ohm resistor with a downward current I6. The right wire carries a 10 ohm resistor with a downward current I4.

The drawing shows an ideal transformer between two coils of two circuits, with dot terminals located on the top left and top right of the left and right coils, respectively. Voltage across the top and bottom wires of the left circuit is marked as 14 angled 0 degree kV. A rightward current, I sub p, flows through the top wire of the left circuit.

The right circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and a horizontal cross wire connecting a node on the left wire with a node on the vertical cross wire. The horizontal cross wire divides the coil on the left wire into two parts, each of which has voltage marked as 125 angled 0 degree V. Each part of the coil shows a dot terminal on its top right.

The left arm of the top wire carries a 0.02 ohm resistor and a j0.02 ohm inductor with a rightward current I1, the left arm of the horizontal cross wire carries a 0.03 ohm resistor and a j0.03 ohm inductor with a leftward current I sub n, and the left arm of the bottom wire carries a 0.02 ohm resistor and a j0.02 ohm inductor with a leftward current I2. The top arm of the vertical cross wire carries resistor R1 with voltage V1, while the bottom arm carries resistor R2 with voltage V2. The right wire carries resistor R3 with voltage V3 marked across it.

The diagram shows a rectangular box with two terminals that have wires pointing outside to the left. Voltage between these two wires is marked as v, while a rightward current, i, flows inward through the top wire.

The X-axis plots omega t in radians, ranging from 0 to 4 pi in increments of pi. The Y-axis plots voltage, current, and power, marked as v, i, and p. This graph shows three sinusoidal waves of different frequencies and amplitudes, drawn on either side of the x-axis.

The wave for the current, which has lowest amplitude, starts at I sub m on the y-axis and completes two cycles, peaking at minus I sub m below the x-axis and at I sub m above the x-axis. The wave representing voltage is shown in a broken line and has a slightly higher amplitude and frequency than the current wave. This line starts below I sub m on the y-axis and completes two cycles in the graph, peaking at V sub m above the x-axis and at minus V sub m below the x-axis. The wave for power has the highest frequency and amplitude, as compared to the other two waves. It starts at (V sub m multiplied by I sub m) over 2 on the y-axis and completes four cycles for the two cycles of voltage and power. The wave peaks at 3 multiplied by V sub m multiplied by I sub m over 4 above the x-axis and at 1 multiplied by V sub m multiplied by I sub m over 4 below the y-axis.

The X-axis plots Time in seconds, ranging from 0 to 0.025 in increments of 0.005, while the y-axis plots Instantaneous and average power (W), ranging from 0 to 2.0 in increments of 0.5. This graph shows a horizontal line drawn at y=1.0, and a sinusoidal wave that moves on either side of the horizontal line above the x-axis. The horizontal line is labeled as P (upper case), while the sinusoidal line is labeled as p (lower case). The sinusoidal line starts at 2.0 on the y-axis, which is the upper peak, and hits a lower peak on the x-axis.

The X-axis plots Time in seconds, ranging from 0 to 0.025 in increments of 0.005, while the y-axis plots Instantaneous, average, and reactive power, ranging from minus 1.0 to 1.0 in increments of 0.5. This graph shows a horizontal line drawn at y=0, above the x-axis, and a sinusoidal wave that moves on either side of this horizontal line. The horizontal line is labeled as P (W) (upper case), while the sinusoidal wave is labeled as p (w) (lower case). The sinusoidal wave starts at y=0 and peaks at y=1.0 and y=minus 1.0, completing three cycles between 0 and 0.025 seconds. A second horizontal line, labeled as Q (WAR), is drawn at y=1.0, passing through the upper peaks of the sinusoidal wave.

The X-axis plots Time in seconds, ranging from 0 to 0.025 in increments of 0.005, while the y-axis plots Instantaneous, average, and reactive power, ranging from minus 1.5 to 1.0 in increments of 0.5. This graph shows a horizontal line drawn at y=0 above the x-axis, and a sinusoidal wave that moves on either side of this horizontal line. The horizontal line is labeled as P of W (upper case), while the sinusoidal wave is labeled as p of w (lower case). The sinusoidal wave starts at y=0 and peaks at y=1.0 and y=minus 1.0, completing three cycles between 0 and 0.025 seconds. A second horizontal line, labeled as Q (WAR) is drawn at y=minus 1.0, passing through the lower peaks of the sinusoidal wave.

The diagram shows a rectangular box with two terminals that have wires pointing outside to left. Voltage between these two wires is marked as v, while a rightward current, i, flows inward through the top wire.

Appliances with their respective Hours in use per month, number of months used, Annual kWh, and Annual cost are displayed as follows:

A/C—central: 120; 3; 1080; 173 dollars

Clothes dryer—electric: 24; 12; 901; 144 dollars

Clothes washer (does not include cost of hot water): 28; 12; 108; 17 dollars

Coffee maker (residential): 30; 12; 60; 10 dollars

Computer—desktop with monitor: asterix; asterix; 127; 20 dollars

Computer—laptop: asterix; asterix; 23; 4 dollars

Dishwasher—heat dry (does not include hot water): 30; 12; 293; 47 dollars

DVD player: 60; 12; 18; 3 dollars

Fan—ceiling (does not include lights): 150; 6; 72; 12 dollars

Fan—table/box/floor: 60; 3; 28; 4 dollars

Game console (includes standby/phantom load): asterix; asterix; 65; 10 dollars

Heating system—electric heat—baseboard, 10 ft: 240; 5; 1500; 240 dollars

Humidifier: 240; 12; 360; 58 dollars

Lighting—incandescent, 75 Watt: 60; 12; 54; 9 dollars

Lighting—CFL, 20 Watt (75W incandescent equivalent): 90; 12; 22; 3 dollars

Lighting—LED, 10 Watt (75W incandescent equivalent): 90; 12; 12; 2 dollars

Microwave: 9; 12; 101; 16 dollars

Oven—electric: 9; 12; 284; 45 dollars

Refrigerator—19–21.4 cu ft —2001–2008: 720; 12; 533; 85 dollars

Refrigerator—19–21.4 cu ft (new ENERGY STAR): 720; 12; 336; 54 dollars

Set-top box, cable/satellite receiver: 720; 12; 249; 40 dollars

Television—50”+ non-ENERGY STAR TV: asterix; asterix; 215; 34 dollars

Water heater—electric (newer base model .95 energy factor): n/a; 12; 4559; 729 dollars

Asterix: stands for Draws power in standby mode.

Following notes are displayed below this table:

- Hours in Use per Month is based on a typical four-person household in a northern US state.
- Annual kWh may vary considerably depending on model, age, and use.
- Annual Cost is based on 16 cents per kilowatt hour (kWH).
- Data used with permission from https://www.efficiencyvermont.com/tips-tools/tools/electric-usage-chart-tool.

The circuit on top shows an independent voltage source on the left wire and resistor R on the right wire. Voltage delivered to the resistor is marked as v sub s equals 100 V(rms).

An equivalent circuit below this one shows an independent voltage source on the left wire and resistor R on the right wire. Voltage delivered to the resistor is marked as v sub s equals 100 V(dc).

The X-axis plots time t, ranging from minus T over 2 in the second quadrant to T in the first quadrant, in increments of T over 4. The Y-axis plots current I, and marks I sub p and minus I sub p on either side of the origin. This graph shows a periodic triangular current moving from left to right on either side of the x-axis, passing through the origin. The triangular wave peaks at I sub p in the first quadrant and at minus I sub p in the fourth quadrant.

The shown right angled triangle has angle theta between its hypotenuse and an adjacent side. The hypotenuse represents |S| = apparent power, while the adjacent side represents P = average power. The opposite side represents Q = reactive power.

The shown right angled triangle has angle theta between its hypotenuse and an adjacent side. The hypotenuse represents |S| while the adjacent side represents P. The opposite side represents Q.

The diagram shows a rectangular box labeled Circuit, with two terminals that have wires pointing outside to the left. Voltage between these two wires is marked as V sub rms, while a rightward current, I sub rms, flows inward through the top wire.

The diagram shows a rectangular box which carries an impedance with two terminals that have wires pointing outside to the left. Voltage between these two wires is marked as V sub rms, while a rightward current, I sub rms, flows inward through the top wire.

The circuit shows two pairs of nodes on the top and bottom wires. A 1 ohm resistor and a j4 ohm inductor, located between the nodes on the top wire, represent Line impedance, while a 39 ohm resistor and a j26 ohm inductor on the right wire represent Load impedance. A current, I sub L, flows downward through the right wire, while voltage across the two nodes on the top and bottom wire close to the right wire is marked as V sub L. The circuit is powered by an independent voltage source on the left wire, marked as 250 angled 0 degree V(rms).

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires. A load impedance L1 is located on the cross wire, with a downward current I1 flowing through it, and a voltage marked across it as 250 angled 0 degree V(rms). A second impedance, L2, with a downward current I2 is located on the right wire. The left arm of the top wire carries a 0.05 ohm resistor and a j0.50 ohm inductor in series, with a rightward current I sub s flowing through it. The left wire is open and two open terminals are located on the top and bottom wires to the left, with voltage marked across them as V sub s.

The hypotenuse makes an angle of minus 36.87 degrees with the adjacent side. Apparent power at the hypotenuse is 10kVA, while the average power at the adjacent side is 8kW, and reactive power at the opposite side is minus 6kVAR.

The hypotenuse makes an angle of 53.13 degrees with the adjacent side. The apparent power at the hypotenuse is 20kVA, while the average power at the adjacent side is 12kW, and reactive power at the opposite side is 16kVAR.

The hypotenuse makes an angle of 26.565 degrees with the adjacent side. The apparent power at the hypotenuse is 22.36 kVA, while average power at the adjacent side is 20kW, and reactive power at the opposite side is 10kVAR.

The hypotenuse makes an angle of 26.565 degrees with the adjacent side. The apparent power at the hypotenuse is 22.36 kVA, while average power at the adjacent side is 20kW, and reactive power at the opposite side is 10kVAR.

The diagram shows a vertical line with reactive power marked as minus 10 kVAR.

The diagram shows a horizontal line with average power marked as 20kW.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires. A 12 ohm resistor and a minus j16 ohm capacitor are located on the cross wire, with voltage marked as V2 and a downward current I2 flowing it. The left and right arms on the top wire carry two impedances, with respective voltages marked as V1 and V3, and respective rightward currents through them marked as I1 and I3. A 1 ohm resistor and a j2 ohm inductor are located on the left arm, while a 1 ohm resistor and a j3 ohm inductor are located on the right arm. The circuit is powered by an independent voltage source, V sub s, on the left wire, and a dependent voltage source, 39 I2, on the right wire.

The circuit shows two pairs of nodes on the top and bottom wires. A 1 ohm resistor and a j4 ohm inductor located between the nodes on the top wire represent Line impedance, while a 39 ohm resistor and a j26 ohm inductor on right wire represent Load impedance. A current, I sub L, flows downward through the right wire, while voltage across the two nodes on the top and bottom wire, close to the right wire, is marked as V sub L. The circuit is powered by an independent voltage source on the left wire, marked as 250 angled 0 degree V(rms).

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a load impedance L1. A parallel load, L2, is located on the right wire, while a j1 ohm inductor is located on the left arm of the top wire. Voltage between the top and bottom wires to the left of L1 is marked as 200 angled 0 degree V (rms). The circuit is powered by an independent voltage source, V sub s, located on the left wire.

The circuit carries a load, Z sub L, on the right wire. The left wire is removed and two open terminals, labeled a and b, are located on the top and bottom wires to the left. The terminals connect to a Generalized linear network operating in the sinusoidal steady state.

The circuit carries a load, Z sub L, on the right wire. Two nodes, a and b, are marked on the top and bottom wires, respectively. A load, Z sub Th, is located on the top wire, to left of a, with current I flowing through it from left to right. An independent voltage source, V sub Th, is located on the left wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a 20 ohm resistor while the right cross wire carries a minus j6 ohm capacitor. The top wire carries a 5 ohm resistor on the left arm and a j3 ohm inductor on the middle arm. A load impedance, Z sub L, is located on the right wire. Two nodes, a and b, are marked on the right arms of the top and bottom wires, respectively. The circuit is powered by a 20 angled 0 degree V independent voltage source, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a minus j6 ohm capacitor. The left arm of the top wire carries a 4 ohm resistor and a j3 ohm inductor. The right wire is removed and two open terminals, labeled as a and b, are located on the top and bottom ends. Voltage between the two terminals is marked as V sub Th. The circuit is powered by a 16 angled 0 degree V independent voltage source, located on the left wire.

The circuit shows two nodes, a and b, located on the top and bottom wires respectively. On the top wire, to the left of node a, a 5.76 ohm resistor with a rightward current I, and a minus j1.68 ohm capacitor are located. The right wire carries a 5.76 ohm resistor and a plus j1.68 ohm inductor. The circuit is powered by a 19.2 angled minus 53.13 degree V independent voltage source, located on the left wire.

The circuit shows two nodes, a and b, located on the top and bottom wires, respectively. On the top wire, to the left of node a, a 3000 ohm resistor and a j4000 ohm inductor are located. The right wire carries a resistor, R sub L, and a capacitor, jX sub L, which are shown inside a box labeled as Z sub L. The circuit is powered by a 10 angled 0 degree V(rms) independent voltage source, located on the left wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires.The left cross wire carries a 20 ohm resistor while the right cross wire carries a 5 microF capacitor. A 3.6 mH inductor is located on the right arm of the top wire, while a 4 ohm resistor is located on the right arm of the bottom wire.The right wire is removed and two open terminals, labeled as a and b, are located on the top and bottom wires to the right. The circuit is powered by an independent current source, i sub g, located on the left wire.

The diagram shows a circuit where the middle of the top wire is bent down in a U shape. Two inductors are located on two arms of the U, and an ideal transformer, with a turns ratio 4 is to 1, is located in between. A dot terminal is located on the top left of the left inductor while another dot terminal is located on the bottom right of the right inductor.

A vertical cross wire connects a node at the bottom of the U shape with another node on the bottom wire. A 20 ohm resistor is located on this cross wire, while a 60 ohm resistor is located on the left arm of the top wire. A variable resistor, R sub L, is located on the right wire. Two nodes, a and b, are located on the top and bottom wires respectively, close to the right wire. The circuit is powered by an 840 angled 0 degree V(rms) independent voltage source, located on the left wire.

The diagram shows a circuit where the middle of the top wire is bent down in a U shape. Two inductors are located on two arms of the U, and an ideal transformer, with a turns ratio 4 is to 1, is located in between. A dot terminal is located on the top left of the left inductor while another dot terminal is located on the bottom right of the right inductor. Two voltages, V1 and V2, are marked across the left and right inductors, respectively. Plus, two rightward currents, I1 and I2, are marked on the left and right arms of the top wire, respectively.

A vertical cross wire connects a node at the bottom of the U shape with another node on the bottom wire. A 20 ohm resistor is located on this cross wire, while a 60 ohm resistor is located on the left arm of the top wire. The right wire is removed and two open terminals, a and b, are located on the top and bottom wires respectively, to the right. Voltage between the two terminals is marked as V sub Th. The circuit is powered by an 840 angled 0 degree V(rms) independent voltage source, located on the left wire.

The diagram shows a circuit where the middle of the top wire is bent down in a U shape. Two inductors are located on two arms of the U, and an ideal transformer, with a turns ratio 4 is to 1, is located in between. A dot terminal is located on the top left of the left inductor while another dot terminal is located on the bottom right of the right inductor. Two voltages, V1 and V2, are marked across the left and right inductors, respectively. Plus, two rightward currents, I1 and I2, are marked on the left and right arms of the top wire, respectively.

A vertical cross wire connects a node at the bottom of the U shape with another node on the bottom wire. A 20 ohm resistor is located on this cross wire, while a 60 ohm resistor is located on the left arm of the top wire. Two nodes, marked as a and b, are located at either end of the right wire. The circuit is powered by an 840 angled 0 degree V(rms) independent voltage source, located on the left wire.

The circuit shows two nodes, marked as a and b, located respectively on the top and bottom wires. A 35 ohm resistor is located on the top wire, to the left of node a, while another 30 ohm resistor is located on the right wire. The circuit is powered by a 210 angled 0 degree V(rms) independent voltage source, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires. A 20 mH inductor, with a dot terminal at the bottom left, is located on this cross wire. A 10 mH inductor, with a dot terminal on the top right, is located on the left arm of the top wire, close to the cross wire. Mutual inductance between the two inductors is marked as 8 mH. A 34 ohm resistor is located on the top wire, to the left of the inductor, while a 100 ohm resistor is located on the right wire. The circuit is powered by an independent voltage source, v sub g, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 375 ohm resistor. The right wire carries a 400 ohm resistor. A 40 m H inductor, with a dot terminal on the bottom right, is located on the left arm of the top wire. A 100 mH inductor, with a dot terminal on the bottom left, is located on the right arm of the top wire. Mutual inductance between the two is marked as 50 mH. Th e-circuit is powered by an independent voltage source, v sub g, located on the left wire.

Power in watts is listed as follows for these electrical devices:

Mobile phone charger

- Attached to phone, phone charging: 3.68
- Plugged into wall outlet but not into phone: 0.26

Notebook computer AC adapter

- Attached to computer, computer charging: 44.28
- Attached to computer, computer sleeping: 15.77
- Attached to computer, computer off: 8.9
- Plugged into wall outlet but not into computer: 4.42

DVD player

- On and playing: 9.91
- On and not playing: 7.54
- Off: 1.55

Microwave oven

- Ready with door closed: 3.08
- Ready with door open: 25.79
- Cooking: 1433.0

Inkjet multifunction printer

- On: 9.16

- Off: 5.26

The drawing shows two adjacent circuits. An inductor L1, with a dot terminal on the top left, is located on the right wire of the left circuit, while another inductor L2, with a dot terminal on the top right, is located on the left wire of the right circuit. Mutual inductance between these two inductors is marked as M. The top wire of the left circuit carries two resistors in series, marked as R sub s and R1. A 120 V(rms) independent voltage source is located on the left wire of the left circuit. The top wire of the right circuit carries resistor R2, while a phone is located on the right wire of the right circuit.

The drawing shows two adjacent circuits. An inductor j omega L1, with a dot terminal on the top left, is located on the right wire of the left circuit, while another inductor j omega L2, with a dot terminal on the top right, is located on the left wire of the right circuit. Mutual inductance between these two inductors is marked as j omega M. The top wire of the left circuit carries two resistors in series, marked as R sub s and R1. A 120 V(rms) independent voltage source is located on the left wire of the left circuit. The top wire of the right circuit carries resistor R2. The phone on the right wire is removed in this circuit and two open nodes are located in its place.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying an 8H inductor. A 5k ohm resistor is located on the right arm of the top wire, while an 80nF capacitor is located on the right wire. The circuit is powered by an independent current source, i sub g, located on the left wire.

The circuit shows an op amp which has a 5V positive supply voltage and a minus 5V negative supply voltage. A 20k ohm resistor, in parallel with a 100 nF capacitor, connects the output terminal with an inverting input terminal. The output terminal is grounded with the common node, in series with a 1 k ohm resistor. The non-inverting input terminal is likewise grounded with the common node. An independent voltage source, v sub g, connects the inverting input terminal with the common node, in series with a 500 nF capacitor and a 2k ohm resistor.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 0.5 mH inductor with a downward current i sub delta. The right arm of the top wire carries a 50 i sub delta dependent voltage source and a 1 over 3 micro F capacitor. A 40 ohm resistor is located on the right wire. The circuit is powered by an independent current source, i sub g, located on the left wire.

The circuit shows two pairs of nodes on the top and bottom wires. Part of the circuit between the two nodes is marked as Line, while the circuit to the right of the nodes is marked as Load, and the circuit to the left of the nodes is marked as Source. A 20 ohm resistor and a jX ohm inductor are located on the top wire, between these two nodes. The right wire carries a load impedance, marked as Z sub L. A 500 angled 0 degree V(rms) independent voltage source is located on the left wire.

The X axis plots time t in ms, ranging from 25 to 100 in increments of 25, while the y-axis plots current i in amps. The graph plots a line which starts from the origin and moves with a diagonal upward slope, peaking at (15, 75), before moving with a downward slope to reach x=100. The line continues in the second cycle from x=100.

The X-axis plots time t in seconds, ranging from 0 to 40 in increments of 5, while the y-axis plots voltage v sub g in Volts. The graph shows a rectangular pulse which moves between 0 and 5, with a uniform peak value of y=40, and then moves along the x-axis between x=5 and x=10. The pulse then moves in the fourth quadrant, between x=10 and x=15, with a uniform peak value of y=minus 40, before moving along the x-axis between x=15 and x=20. This pattern repeats for the next cycle between x = 20 and x=40, and continues beyond.

The X-axis plots time t in microseconds, and marks 30, 67.5, 100, 130, 167.5, and 200, while the y-axis plots voltage v sub g in Volts, and marks minus 5, minus 2, 2, and 5. The graph shows a rectangular pulse which moves between 0 and 30, with a uniform peak value of y=5, and then steps down to move between 30 and 67.5 on the x-axis, at a uniform peak value of y=2. The line then moves along the x-axis between x=67.5 and x=100. The pulse then moves in the fourth quadrant, between x=100 and x=130, at a uniform peak value of y=minus 5, before stepping down to move between x=130 and x=167.5 at a uniform peak value of y=minus 2. The line then moves along the x-axis between x=167.5 and x=200. This pattern repeats for subsequent cycles.

The circuit shows a 1 ohm resistor and j2 ohm inductor on the top wire, and a load on the right wire, with voltage marked across as V sub L angled 0 degree. The left wire is removed and two open terminals are located on the top and bottom wires, toward the left, with voltage across them marked as 250 angled theta degree V(rms).

The circuit shows a 25 nF capacitor and an 80 microH inductor on the top wire, and a 60 ohm resistor on the right wire. A 40 ohm resistor is drawn parallel across two nodes on either side of the capacitor. The circuit is powered by an independent voltage source, v sub g, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on top and bottom wires, carrying a minus j20 ohm capacitor. A 10 ohm resistor and a 12 ohm resistor are located on the left and right arms of the top wire, while a j16 ohm inductor is located on the right wire. The circuit is powered by an independent voltage source, V sub g, located on the left wire.

The circuit shows a 50 ohm resistor on the top wire and a 100 mH inductor on the right wire, which are depicted inside a box that represents Load. An 80 micro F capacitor is located on the top wire, to the left of the resistor. The circuit is powered by an independent voltage source, v sub g, located on the left wire.

The circuit shows two pairs of nodes on the top wire and bottom wire. The pair to the right are connected by a vertical cross wire that carries a load impedance L1, while the right wire carries another load, L2. The pair of nodes to the left have voltage V sub L marked across them. A 0.05 ohm resistor and a j0.5 ohm inductor are located in series on the top wire, to the left of the nodes, while a similar resistor and inductor are located on the bottom wire, to the left of the nodes. The circuit is powered by an independent voltage source, V sub g, located on the left wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. Two loads, labeled as L1 and L2, are located on the left and right cross wires respectively, while a third load, L3, is found on the right wire. Voltage across the top and bottom wires is marked as V sub o. A j 0.1 ohm inductor is located on the left arm of the top wire. The circuit is powered by an independent voltage source, V sub g, located on the left wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. Two loads, labeled as 1 and 2, are located on the left and right cross wires respectively, while a third load, labeled as 3, is found on the right wire. The left wire is removed and voltage across the open terminals on the top and bottom wires is marked as 250V(rms).

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. Two loads, labeled as L1 and L2, are located on the left and right cross wires respectively, while a third load, labeled as L3, is found on the right wire. The left wire is removed and voltage across the open terminals on the top and bottom wires is marked as V sub s. A 0.01 ohm resistor and a j0.08 ohm inductor are located on the left arm of the top wire. Voltage across L3 on the right wire is marked as 250 angled 0 degree V(rms).

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. Two loads, labeled as Load1 and Load2, are located on the left and right cross wires respectively, while a third load, labeled as Load3, is found on the right wire. An independent voltage source, V sub g, is located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and a horizontal cross wire connecting a node on the vertical cross wire with another node on the left wire. Loads L1 and L2 are located on the top and bottom arms of the vertical cross wire, while load L3 is located on the right wire. Two independent voltage sources, V sub g1 and V sub g2, are located on the top and bottom arms of the left wire, respectively.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and a horizontal cross wire connecting a node on the vertical cross wire with another node on the left wire. Loads S1 and S2, with individual voltages of 125 angled 0 degree V(rms), are located on the top and bottom arms of the vertical cross wire, while load S3 is located on the right wire.

A 0.05 ohm resistor, a 0.15 ohm resistor, and a 0.05 ohm resistor are located on the left arms of the top wire, the horizontal cross wire, and the bottom wire, respectively. Two independent voltage sources, V sub g1 and V sub g2, are located on the top and bottom arms of the left wire, respectively.

The circuit shows two parallel dotted lines drawn vertically near the left and right wires. Part of the circuit between these two lines is marked as Line, while the circuit to the right of these lines is marked as Load, and the circuit to the left of these lines is marked as Source. A 4 ohm resistor and a j3 ohm inductor are located on the top wire, between these two nodes. The right wire carries a 120 ohm resistor in series with a j90 ohm inductor. A 465 angled 0 degree V(rms) independent voltage source is located on the left wire.

The circuit shows a pair of nodes on the top and bottom wires, with voltage across them marked as 4800 angled 0 degrees V(rms). A 2 ohm resistor and a j10 ohm inductor are located on the top wire, to the left of the nodes. The right wire carries a load marked as 192 kVA 0.8 lag. The left wire is removed and two open terminals on the top and bottom wires to the right have voltage across them marked as V sub s.

The circuit shows a horizontal cross wire connecting two nodes on the left and right wires, and a vertical cross wire connecting a node on the horizontal cross wire with another node on the bottom wire. The left arm of the horizontal cross wire carries a j1 ohm inductor, with a dot terminal near its bottom left while the right arm carries a j2 ohm inductor with two dot terminals below it on the left and right. Mutual inductance between the two inductors is marked as j1 ohm. Two rightward currents, I sub b and I sub d, flow through these left and right inductors, respectively.

A capacitor with capacitance of minus j1 ohm is located on the vertical cross wire, carrying a downward current, I sub e. The lower arm of the right wire carries a 1 ohm resistor, along with a downward current I sub f, while the top wire carries another 1 ohm resistor, with a rightward current I sub c. The circuit is powered by an independent voltage source, 10 angled 0 degree V (rms), located on the lower arm of the left wire, with an upward current, I sub a, flowing through it.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires. A j20 ohm inductor is located on this cross wire with a dot terminal marked to its top left. A j10 ohm inductor is located on the left arm of the top wire, with a dot terminal to its top left, while a j4 ohm inductor is located on the right arm of the top wire, with a dot terminal to its top right.

Mutual inductance between the two inductors on the top wire is marked as j6 ohm. The mutual inductance between the inductor on the top left wire and the vertical cross wire is marked as j14 ohm. And the mutual inductance between the inductor on the top right wire and the vertical cross wire is marked as j8 ohm.

Two nodes, a and b, are located on the top and bottom wires respectively, to the left of the cross wire. A rightward power, Z sub ab, is marked between the two nodes. A 2 ohm resistor is located on the top wire, to the left of node a, while an 8 ohm resistor is located on the right wire. The circuit is powered by a 272 angled 0 degree V(rms) independent voltage source, located on the left wire.

The diagram shows a circuit where the middle of the top wire is bent down in a U shape. Two inductors are located on two arms of the U, and an ideal transformer, with a turns ratio 4 is to 1, is located in between. A dot terminal is located on the top left of the left inductor while another dot terminal is located on the bottom right of the right inductor.

A vertical cross wire connects a node at the bottom of the U shape with another node on the bottom wire. A 20 ohm resistor is located on this cross wire, while a 40 ohm resistor is located on the right wire. Another vertical cross wire is drawn to the left of the first cross wire, connecting two nodes on the top and bottom wires, and carrying a 60 ohm resistor. The circuit is powered by a 5 angled 0 degree A(rms) independent current source, located on the left wire.

The circuit shows an ideal transformer made of two inductors on the right wire, with dot terminals located to their top left. The inductor on top has 400 turns while the one below has 100 turns. A 15 ohm resistor is located on the top wire, while an independent voltage source with voltage of 100 angled 0 degree V(rms) is located on the left wire.

An additional wire is drawn across two nodes located on either side of the lower inductor, forming a small circuit. A minus j2 ohm capacitor is located on the right wire of this small circuit, while a 1 ohm resistor is located on its top wire.

The drawing shows three circuits where two are placed one above the other on the right, and the third circuit is shown on the left. The circuit on the left shows two inductors on its right wire, with dot terminals marked on their top left. The circuit on the top right has a 25 ohm resistor on its right wire and an inductor on its left wire, with a dot terminal to its top right. An ideal transformer, with a turns ratio of a1 is to 1, is marked between this inductor and the top inductor on the left circuit. Similarly, the circuit on the bottom right has a 4 ohm resistor on its right wire and an inductor on its left wire, with a dot terminal to its top right. An ideal transformer, with a turns ratio of a2 is to 1, is marked between this inductor and the bottom inductor on the left circuit.

The left circuit carries a 500 ohm resistor and an independent voltage source, 2000 angled 0 degree V(rms), on its left wire.

The drawing shows a box with two nodes, a and b, that are connected to two open terminals outside to the right. Voltage between these two open terminals is marked as V sub ab. The box represents A circuit operating in the sinusoidal steady state.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 40 ohm resistor. A 4 mH inductor and a 2.5 microF capacitor are located on the left and right arms of the top wire. The right wire carries a load impedance, marked as Z sub L. The circuit is powered by an independent voltage source, v sub g, located on the left wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a 5I sub phi dependent voltage source, while the right cross wire carries a j3 ohm inductor. The top wire carries a 25 ohm resistor, with a rightward current I sub phi, and a j10 ohm inductor on its left arm, and a 1 ohm resistor on its middle arm. The right wire carries a variable load impedance, marked as Z sub L. Two nodes are marked on the top and bottom wires, close to the right wire. The circuit is powered by a 100 angled 0 degree V(rms) independent voltage source, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a minus j8 ohm capacitor. The left arm of the top wire carries a 3 ohm resistor and a j4 ohm inductor, while the right arm carries a 7.32 ohm resistor and a minus j17.24 ohm capacitor. A variable resistor, R, is located on the right wire. Two nodes are marked on the top and bottom wires, close to the right wire. The circuit is powered by a 112.5 angled 0 degree V(rms) independent voltage source, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a j5 ohm capacitor, with voltage V sub phi marked across it. The left arm of the top wire carries a 5 ohm resistor, while the right arm carries a minus j5 ohm capacitor. A variable resistor, R sub o, is located on the right wire. A node is marked on the right arm of the top wire. An additional wire is drawn between this node and the node on top of the left wire. This wire carries a 5 ohm resistor and a dependent current source, V sub phi over 10, with a rightward current. The circuit is powered by a 100 angled 0 degree V(rms) independent voltage source, located on the left wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a 300 ohm resistor, with voltage across it marked as 13,800 angled 0 degree V(rms). The right cross wire carries a j100 ohm inductor. A 1.5 ohm resistor and a j12 ohm inductor are located in series on the left arm of the top wire. The right wire carries a variable capacitor, marked as minus jX sub C. The left wire of the circuit is removed and two open terminals are marked on the top and bottom wires to the left, with voltage between them marked as V sub s.

The circuit shows two pairs of nodes located on the top and bottom wires. A vertical cross wire is drawn between the pair of nodes on the left, carrying a 12k ohm resistor. A 6k ohm resistor is located on the left arm of the top wire, while a 0.6H inductor is located on the right arm of the top wire, to the left of the node, and a variable resistor, R sub o, is located to the right of the node on the top wire. A variable capacitor, C sub o, is located on the right wire. The circuit is powered by an independent voltage source, v sub g, located on the left wire.

The circuit on left shows a 625mH inductor on the right wire, with a dot terminal to its top left, while the circuit on the right shows another 625mH inductor on its left wire, with a dot terminal to its top right. Mutual inductance between these two inductors is marked as 312.5mH. A rightward current, i sub L, flows through the top wire of the right circuit, while its right wire carries a 375 ohm resistor. The top wire of the left circuit carries a 125 ohm resistor, with a rightward current, i sub g. Both the circuits are powered by an independent voltage source, v sub g, located on the left wire of the left circuit.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying an inductor L2 with a dot terminal to its top left. Another inductor, L1, is located on the right arm of the top wire, with a dot terminal to its top left. Mutual inductance between these two inductors is marked as k. The left arm of the top wire carries resistor R sub g, while the right wire carries resistor R sub L, with voltage v sub o marked across it. The circuit is powered by an independent voltage source, v sub g, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a j30 ohm inductor with a dot terminal to its bottom right. A j15 ohm inductor is located on the right arm of the top wire, with a dot terminal to its top right. Mutual inductance between these two inductors is marked as j18 ohm. The left arm of the top wire carries a 15 ohm resistor, while the right wire carries a variable impedance, Z sub o. The circuit is powered by a 40 angled 0 degree V(rms) independent voltage source, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a j9 ohm inductor with a dot terminal to its top right. A j4 ohm inductor is located on the left arm of the top wire, with a dot terminal to its bottom left. Mutual inductance between these two inductors is marked as j3 ohm. The left arm of the top wire also carries a 3 ohm resistor, while the right wire carries a 9 ohm resistor, with voltage V sub o marked across it. The circuit is powered by a 180 angled 0 degree V(rms) independent voltage source, located on the left wire.

The drawing shows two circuits placed side by side. The circuit on the left carries a j5 ohm inductor on its right wire, with a dot terminal to its top left, and a 12 ohm resistor on its top wire, close to the right wire. The circuit on the right carries a j23 ohm inductor on its left wire, with a dot terminal to its top right, and an 11 ohm resistor on its top wire, close to the left wire. Mutual inductance between these inductors is marked as j10 ohm. A dotted square is drawn encompassing these two resistors and inductors of the two circuits. This square is labeled as Transformer.

The right circuit, to the right of the transformer, represents Load. A variable impedance, Z sub L, is located on the right wire, while two nodes are marked on the top and bottom wires, close to the right wire. The left circuit, to the left of the transformer, represents Source. Two nodes are marked on the top and bottom wires, to the immediate left of the transformer. A 4 ohm resistor and a j7 ohm inductor are located in series on the left arm of the top wire, to the left of the node. An independent voltage source, 120 angled 0 degree V(rms), is located on the left wire.

The circuit shows an ideal transformer between the left and middle circuits, with a turns ratio of 25 is to 1. The iInductor on the left circuit has a dot terminal on the top left, while the inductor on the middle circuit has a dot terminal on the top right. Another ideal transformer is shown between the middle and right circuits, with a turns ratio of 1 is to 50. The inductor on the middle circuit has a dot terminal on the bottom left, while the inductor on the right circuit has a dot terminal on the top right.

A 5k ohm resistor is located on the right wire of the right circuit. A 200 ohm resistor is located on the top wire of the left circuit. The three circuits are powered by a 145 angled 0 degree V(rms) independent voltage source, located on the left wire of the left circuit.

The circuit shows an ideal transformer made of two inductors on the right wire. The inductor on top has N1 turns and a dot terminal to its bottom right, while the one below has N2 turns and a dot terminal to its top right. A 50 ohm resistor and a j400 ohm inductor are located on the top wire, while an independent voltage source, with voltage of 24 angled 0 degree V(rms), is located on the left wire.

An additional wire is drawn across two nodes located on either side of the lower inductor, forming a small circuit. A load impedance, Z sub L, is located on the right wire of this small circuit. Voltage across its top and bottom wires is marked as V sub L, while a rightward current, I sub L, flows through its top wire.

The circuit shows three pairs of nodes on the top and bottom wires. A vertical cross wire connects the left pair of nodes, carrying an 80k ohm resistor, while a 10k ohm resistor is located on the top wire, to the right of this cross wire. Another vertical cross wire connects the right pair of nodes, carrying an inductor with N2 turns and a dot terminal to its bottom right. The middle arm of the top wire carries a second inductor, with N1 turns, adjacent to the right cross wire, and having a dot terminal to its top left. An ideal transformer is marked between these two inductors.

Voltage between the middle node and the right node on the top wire is marked as V1. A 400 ohm resistor is located on the right wire. The circuit is powered by a 2.25 angled 0 degree mA(rms) independent current source, located on the left wire.

The drawing shows two circuits connected by an ideal transformer, with a turns ratio of 1 is to 5. The inductor on right wire of the left circuit has a dot terminal to its top left, while the inductor on the left wire of the right circuit has a dot terminal to its top right. The right circuit carries a 100 ohm resistor on the top wire and a variable capacitor, C, on its right wire.

The left circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires, with the left and right cross wires carrying a 20 ohm resistor and a j10 ohm inductor, respectively. A 2 ohm resistor is located on the right arm of the top wire. The circuit is powered by a 15 angled 0 degree A(rms) independent current source, located on the left wire.

The drawing shows two circuits connected by an ideal transformer, with a turns ratio of 1 is to 4. The inductor on the right wire of the left circuit has a dot terminal to its top left, while the inductor on the left wire of the right circuit has a dot terminal to its top right. The right circuit carries a 16 ohm resistor on the top wire, close to the left wire, and a variable resistor, R sub L, on its right wire. The bottom wires of the two circuits are connected.

The left circuit shows a 4 ohm resistor and a 1 ohm resistor, in series on the top wire. A node is located in between these two resistors. An additional wire is drawn connecting this node with another node located to the right of the 16 ohm resistor located on the top wire of the right circuit. This additional wire carries a 12 ohm resistor.

The circuit is powered by a 40 angled 0 degree V(rms) independent voltage source, located on the left wire.

The drawing shows two circuits connected by an ideal transformer, with a turns ratio of 1 is to 2. The inductor on the right wire of the left circuit has a dot terminal to its top left, while the inductor on the left wire of the right circuit has a dot terminal to its top right. The right circuit carries a 40 ohm resistor on the top wire, close to the left wire, and a 360 ohm resistor on its right wire. The bottom wires of the two circuits are connected.

The left circuit shows an 80 ohm resistor and a 20 ohm resistor, in series on the top wire. A node is located in between these two resistors. An additional wire is drawn connecting this node with another node located on top of the right wire of the right circuit. This additional wire carries a variable resistor, R sub L.

The circuit is powered by a 500 angled 0 degree V(rms) independent voltage source, located on the left wire.

The drawing shows two circuits. The circuit on the left shows a j30 ohm inductor on its right wire, with a dot terminal to its top left, while the circuit on the right shows a j1500 ohm inductor on its left wire, with a dot terminal to its top right. Mutual inductance between these two inductors is marked as j200 ohm. The circuit on the left also carries a 40 ohm resistor on its top wire, and a 255 angled 0 degree V(rms) independent voltage source on its left wire.

The circuit on the right shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying an inductor with N2 turns, with a dot terminal located on its top left. The right arm of the top wire carries an inductor with N1 turns, with a dot terminal to its bottom right. An ideal transformer is marked between these two inductors. The left arm of the top wire carries a 720 ohm resistor, while the right wire carries a 6800 ohm resistor.

The drawing shows two boxes containing three terminals each, which are connected by three parallel wires. The box on the left represents a three-phase voltage source, while the box on the right represents a three-phase load. The connecting lines are labeled as three-phase line.

The phasor diagram shows three phasors drawn from the origin. The horizontal phasor is labeled V sub a. A phasor which leads the a-phase voltage by 120 degrees is shown above it, and is labeled as V sub c. The phasor which lags the a-phase voltage by 120 degrees is shown below it, and is labeled as V sub b.

The phasor diagram shows three phasors drawn from the origin. The horizontal phasor is labeled V sub a. A phasor which leads the a-phase voltage by 120 degrees is shown above it, and is labeled as V sub b. The phasor which lags the a-phase voltage by 120 degrees is shown below it, and is labeled as V sub c.

The sketch shows a generator consisting of a rotor surrounded by a circular stator. The rotor in the middle has a clockwise rotating electromagnet, padded with field winding on two sides. The surrounding stator consists of two sets of three separate windings distributed on its periphery, with these sets labeled as a-phase winding, c-phase winding, and b-phase winding. Three dotted lines are drawn from the middle of the rotor to the periphery of the stator, separated by an angle of 120 degrees. These lines represent axes of a-phase winding, b-phase winding, and c-phase winding.

The drawing shows an inverted y-shaped circuit with three wires labeled as a, b, and c, joined at a common node n, which is a neutral conductor. Three independent voltage sources are located on these three wires. The voltage source on the top wire a is labeled as V sub a, while two sources in the two arms below are labeled as V sub c on wire c and V sub b on wire b.

The drawing shows a triangular circuit with three nodes at three vertices. Three wires are drawn from the top, right, and left vertices and are respectively labeled as a, b, and c. Three independent voltage sources are located on the three wires of the triangle, labeled as V sub b on base, V sub a on the right wire, and V sub c on the left wire.

The drawing shows a y-connection, with three independent voltage sources connected to wires a, b, and c, labeled respectively as V sub a, V sub b, and V sub c. Each wire also contains an inductor, jX sub w, and a resistor, R sub w, which are connected in series with each voltage source.

The drawing shows a delta-connection, with three independent voltage sources located on the sides of a triangle and connected to wires a, b, and c, and labeled respectively as V sub a, V sub b, and V sub c. Each side also contains an inductor, jX sub w, and a resistor, R sub w, which are connected in series with each voltage source.

The drawing shows two y-connections enclosed in two boxes. The y-connection in the left box has a voltage source and an internal impedance in each of its three wires, which connect to three nodes outside, labeled as a, b and c. The neutral conductor connects to node n outside this box. The voltage sources connected to the nodes a, b, and c are labeled as V sub a-dash n, V sub b-dash n, and V sub c-dash n, respectively. The internal impedances are labeled as Z sub ga, Z sub gb, and Z sub gc.

The y-connection in the right box has three impedances on its three wires, labeled as Z sub A, Z sub B, and Z sub C, which are connected to three nodes outside, labeled as A, B, and C. The neutral conductor of this circuit connects to node N outside this box.

Four parallel wires are drawn connecting nodes a, n, b, c from the left box with nodes A, N, B, C from the right box, respectively. Each of these wires carry impedances, labeled as Z sub la, Z sub 0, Z sub lb, and Z sub lc. Three rightward currents move from nodes a, b, c to nodes A, B, C, and are labeled respectively as I sub aA, I sub bB, and I sub cC. A leftward current moves from node N to node n, and is labeled as I sub 0.

The circuit shows a voltage source, V sub a-dash n, on the left wire between node a-dash on the top and node n at the bottom. A load impedance, Z sub A, is located on the right wire, with node N below it, at the bottom. Two nodes, a and A, are located on the top wire, with load Z sub la located between them and carrying a rightward current, I sub aA. Another load, Z sub ga, is located on the top wire to the left, between the nodes a-dash and a.

The circuit shows a y-circuit with three nodes A, B, and C connected to a neutral conductor N. Three impedances, labeled as Z sub A, Z sub B, and Z sub C, are located on wires leading from node N to the nodes A, B, and C, respectively. Voltages between AB, AC, and BC are marked as V sub AB, V sub CA, and V sub BC, respectively. Voltages between AN, BN, and CN are marked as V sub AN, V sub BN, and V sub CN, respectively.

The phasor diagram for abc sequence shows three line-to-line phasors drawn as follows: V sub AB pointing diagonally right upward, V sub BC pointing downward, and V sub CA pointing diagonally left upward. Three line-to-neutral phasors are drawn at 30 degree angles to each of the line-to-line phasors, as follows: V sub AN below V sub AB, V sub BN to left of V sub BC, and V sub CN to right of V sub CA.

Phasor diagram for acb sequence shows three line-to-line phasors drawn as follows: V sub AB pointing diagonally right downward, V sub BC pointing upward, and V sub CA pointing diagonally left downward. Three line-to-neutral phasors are drawn at 30 degree angles to each of the line-to-line phasors, as follows: V sub AN above V sub AB, V sub BN to left of V sub BC, and V sub CN to right of V sub CA.

The circuit shows a 120 angled 0 degree V independent voltage source on the left wire, between node a-dash on the top and node n at the bottom. The right wire carries a 39 ohm resistor and a j28 ohm inductor, with node N at the bottom. Two nodes, a and A, are located on the top wire, with a 0.8 ohm resistor and a j1.5 ohm inductor located between them and carrying a rightward current, I sub aA. A 0.2 ohm resistor and a j0.5 ohm inductor are located between the nodes a-dash and a. The voltage between nodes a and n is marked as V sub an, while the voltage between nodes A and N is marked as V sub AN.

The drawing shows a y-connection and a delta-connection enclosed in two boxes. The y-connection in the left box has a voltage source and an internal impedance in each of its three wires, which connect to three nodes outside, labeled as a, b, and c. The voltage sources connected to nodes a, b, and c are labeled as V sub a-dash n, V sub b-dash n, and V sub c-dash n, respectively. The internal impedances are labeled as Z sub g on each wire.

The delta-connection in the right box has two parallel wires. The wire on the right carries impedance Z sub delta, while the wire on the left carries two impedances, labeled as Z sub delta each, and separated by a neutral conductor N in between. The left wire connects to three nodes outside the box, labeled as A, B, and C, which are connected to the top, node N, and bottom of the left wire. Two downward currents flow in the top arm and bottom arm of the left wire, labeled respectively as I sub AB and I sub BC, while an upward current, I sub CA, flows through the right wire.

Three parallel wires are drawn connecting nodes a, b, and c from the left box with nodes A, B, C from the right box, respectively. Each of these wires carry impedances labeled as Z sub l. Three rightward currents move from nodes a, b, and c to nodes A, B, and C, and are labeled respectively as I sub aA, I sub bB, and I sub cC.

The circuit shows a voltage source, V sub a-dash n, on the left wire, between node a-dash on top and node n at the bottom. A load impedance, Z sub A, is located on the right wire, with node N below it, at the bottom. Two nodes, a and A, are located on the top wire, with load Z sub la located between them and carrying a rightward current, I sub aA. Another load, Z sub ga, is located on the top wire to the left, between the nodes a-dash and a.

The phasor diagram for the abc sequence shows three phasors representing line currents, drawn as follows: I sub aA pointing diagonally left downward, I sub bB pointing diagonally right downward, and I sub cC pointing upward. Three phasors representing phase currents are drawn at 30 degree angles to each of the line current phasors, as follows: I sub AB above I sub aA, I sub CA to the left of I sub cC, and I sub BC to the right of I sub bB.

The phasor diagram for the acb sequence shows three phasors representing line currents, drawn as follows: I sub aA pointing diagonally right upward, I sub cC pointing downward, and I sub bB pointing diagonally left upward. Three phasors representing phase currents are drawn at 30 degree angles to each of the line current phasors, as follows: I sub AB below I sub aA, I sub CA to the left of I sub cC, and I sub BC to the right of I sub bB.

The circuit shows a 120 angled 0 degree V independent voltage source on the left wire, between node a-dash on the top and node n at the bottom. The right wire carries a 39.5 ohm resistor and a j28.6 ohm inductor, with node N at the bottom. Two nodes, a and A, are located on the top wire, with a 0.3 ohm resistor and a j0.9 ohm inductor located between them and carrying a rightward current, I sub aA. A 0.2 ohm resistor and a j0.5 ohm inductor are located between the nodes a-dash and a.

The drawing shows a y-circuit with a neutral conductor N connected to three nodes labeled as A, B, and C. Three loads are located on wires connecting N to A, B, and C, labeled respectively as Z sub A, Z sub B, and Z sub C. Respective voltages across them are marked as V sub AN, V sub BN, and V sub CN. Three inward currents are marked flowing into the nodes A, B, and C from outside, and labeled respectively as I sub aA, I sub bB, and I sub cC.

The given delta-connection shows two parallel wires. Wire on the right carries impedance Z sub delta, while the wire on the left carries two impedances, labeled as Z sub delta each, and separated by a node B in between. The left wire also has two nodes on the top and bottom, labeled as A and C. Two downward currents flow in the top arm and bottom arm of the left wire, labeled respectively as I sub AB and I sub BC. An upward current, I sub CA, flows through the right wire. Voltages in the top and bottom arms of the left wire are labeled respectively as V sub AB and V sub BC, while reverse-polarity voltage in the right wire is labeled as V sub CA.

The circuit shows a 0.005 ohm resistor, with a rightward current I sub aA, and a j0.025 ohm inductor on the top wire. Node A is marked on top of the right wire while neutral conductor N is marked at its bottom. Voltage between A and N is marked as 600 over square root 3 angled 0 degree V. The right wire carries a 160kW power source at 0.8 lag. The left wire is removed and two open terminals are marked on top and bottom to the left, marked as a and b, with voltage V sub an marked between them.

The drawing shows a box shaped device with two terminals on the left towards the base, labeled as Potential-coil terminals, and two terminals on the right towards the base, labeled as Current-coil terminals. A Watt scale is displayed in front, with a pointer that moves over a semi-circular reading, from 0 to 500 and above.

The drawing shows a box with terminals depicted one below the other, and labeled sequentially from 1 to n. Each terminal has a wire drawn from it to outside towards the left. Inward currents flow through each wire, labeled sequentially as i1, i2, i3, and so on, for terminals 1, 2, 3 etc. Voltage across each wire against the nth wire is marked sequentially as v1, v2, v3, etc.

The drawing shows a y circuit on the right, with a neutral conductor N connected to three nodes labeled as A, B, and C. Three loads are located on wires connecting N to A, B, and C, and are labeled as Z sub phi, which is equivalent to |Z| angled theta. Voltage between A and N is marked as V sub AN, while voltage between C and N is marked as V sub CN. A rightward current, I sub aA, flows through the load next to node A, while another rightward current, I sub cC, flows through the load next to node C.

Three parallel wires connect nodes A, B, and C on the right with nodes a, b, and c on the left. Two inductors, labeled as cc, are located in series with nodes a and c. Two more inductors, labeled as pc, are located in parallel with nodes a and c, connecting the top-middle and middle-bottom parallel wires. A box labeled W1 is drawn around the a series inductor and a parallel inductor around node a, while another box labeled W2 is drawn around another series inductor and a parallel inductor around node c.

The drawing shows two boxes containing three terminals each which are connected by three parallel wires. The box on the left represents a Generator plant while the box on the right represents Sub station. Connecting lines are labeled as 3-phi line.

The circuit shows two nodes on the top wire and two nodes on the bottom wire. The node to the left on the top wire is marked as A, while the node on the bottom wire is marked as N. Voltage between A and N is marked as 13,800 over square root 3 angled 0 degree V. A vertical cross wire connects nodes on the right, carrying a 1.2 MW power source, while the right wire carries a 1.2 MVAR source.

A 0.6 ohm resistor and a j4.8 ohm inductor are located on the left arm of the top wire. The left wire is removed and two terminals on the top and bottom wires to the left are marked as a and n, respectively. Voltage between them is marked as V sub an.

Drawing shows two y-connections enclosed in two boxes. The y-connection in left box has a voltage source and a 1 ohm resistor in each of its three wires, which connect to three nodes outside, labeled as a, b, and c. Voltages of the voltage sources connected to a, b, and c are marked as 200 angled 0 degree V, 200 angled minus 120 degree V, and 200 angled 120 degree V, respectively.

The y-connection in right box has three resistors on its three wires, which are connected to three nodes outside, labeled as A, B, and C, with respective resistances marked as 20 ohm, 25 ohm, and 75 ohm. Neutral conductor N of this circuit connects to neutral conductor n of left circuit, with leftward current I sub 0 flowing through it.

Three parallel wires are drawn connecting nodes a, b, c in the left box with nodes A, B, C in the right box, respectively. Each of these wires carries a 4 ohm resistor.

The drawing shows two y-connections connected to each other. The y-connection on the left has a voltage source in series with a resistor and inductor in each of its three wires, which connect to three nodes on the right, labeled as a, b, and c. Voltage, resistance, and inductance in each wire is marked respectively as follows:

From node a: 277 angled 0 degree V voltage, 0.4 ohm resistance, j2 ohm inductance

From node b: 277 angled minus 120 degree V voltage, 0.4 ohm resistance, j1.6 ohm inductance

From node c: 277 angled 120 degree V voltage, 0.2 ohm resistance, j1.2 ohm inductance

The y-connection on the right has three resistors and three inductors on its three wires, which are connected to three nodes to the left, labeled as A, B, and C. Respective resistances on these wires from A, B, and C are marked as 78 ohm, 77 ohm, and 79 ohm, while the respective inductances are marked as j54 ohm, j56 ohm, and j55 ohm. Neutral conductor N of this circuit connects to neutral conductor n of left circuit, with leftward current I sub 0 flowing through it.

Three parallel wires are drawn connecting nodes a, b, c in the left box with nodes A, B, C in the right box, respectively. Each wire carries a resistor and inductor, as follows:

Top wire between a and A: 1.6 ohm resistor, j4 ohm inductor

Middle wire between b and B: 2.6 ohm resistor, j2.4 ohm inductor

Bottom wire between c and C: 0.8 ohm resistor, j3.8 ohm inductor

Drawing shows two y-connections connected to each other. The y-connection on left has a voltage source in series with a resistor and inductor in each of its three wires, which connect to three nodes on right, labeled as a, b and c. Voltage, resistance, and inductance in each wire is marked respectively as follows:

From node a: 240 angled 0 degree V voltage, 0.1 ohm resistance, j0.8 ohm inductance

From node b: 240 angled 120 degree V voltage, 0.1 ohm resistance, j0.8 ohm inductance

From node c: 277 angled minus 120 degree V voltage, 0.1 ohm resistance, j0.8 ohm inductance

The y-connection on right has three resistors and three inductors on its three wires, which are connected to three nodes to left, labeled as A, B, and C. Respective resistances on these wires from A, B, and C are marked as 59.5 ohm, 39.5 ohm, and 19.5 ohm, while respective inductances are marked as j76 ohm, j26 ohm, and j11 ohm. Neutral conductor N of this circuit connects to neutral conductor n of left circuit, with leftward current I sub 0 flowing through a 10 ohm resistor located on connecting wire.

Three parallel wires are drawn connecting nodes a, b, c in the left with nodes A, B, C on the right, respectively. Each wire carries a 0.4 ohm resistor and a j3.2 inductor.

A drawing shows a delta-connected triangular network with three nodes marked as a, b, and c, connected by three independent voltage sources with the following voltages: 4.16 angled 0 degree kV between a and b, 4.16 angled 120 degree kV between b and c, and 4.16 angled minus 120 degree kV between c and a.

Source network is connected to a delta-connected triangular network, with three nodes marked as A, B, and C, which are connected by three impedances marked as Z.

Nodes a, b, c are respectively connected to nodes A, B, C, with rightward currents I sub aA, I sub bB, and I sub cC flowing through them respectively.

The drawing shows a delta-connected triangular network with three nodes marked as a, b, and c, connected by three independent voltage sources with following voltages: 480 angled 0 degree V between a and b, 480 angled 120 degree V between b and c, and 480 angled minus 120 degree V between c and a.

The source network is connected to a delta-connected triangular network, with three nodes marked as A, B, and C, which are connected by three impedances, as follows: Z1 between A and B, Z2 between B and C, and Z3 between C and A.

Nodes a, b, and c are connected to nodes A, B, and C respectively.

The drawing shows a delta-connected source network where three nodes, a, b, and c, are connected by an independent voltage source, a resistor, and an inductor each, as follows:

Between a and b: 4156 angled 0 degree V voltage source, 2.7 ohm resistor, j13.5 ohm inductor

Between b and c: 4156 angled 120 degree V voltage source, 2.7 ohm resistor, j13.5 ohm inductor

Between c and a: 4156 angled 120 degree V voltage source, 2.7 ohm resistor, j13.5 ohm inductor

Nodes a, b, and c are connected on the right to respective load nodes, marked as A, B, and C.

The drawing shows a y-connected source network where three nodes, a, b, and c, are connected to a neutral conductor by an independent voltage source and a j2 ohm inductor each, as follows:

Between a and neutral conductor: 1365 angled 0 degree V voltage source

Between b and neutral conductor: 1365 angled minus 120 degree V voltage source

Between c and neutral conductor: 1365 angled 120 degree V voltage source

Nodes a, b, and c are connected on the right to respective load nodes, marked as A, B, and C, with a 1.5 ohm series resistor located in between each connection.

Load is made of a delta connected network where three nodes, A, B, and C, are connected to each other by a 85.5 ohm resistor and a j114 ohm inductor on each connection.

The drawing shows a y-connected source network where three nodes, a, b, and c, are connected to a neutral conductor by an independent voltage source, as follows:

Between a and neutral conductor: V sub an

Between b and neutral conductor: V sub bn

Between c and neutral conductor: V sub cn

A rightward current, I sub na, flows into node a from its voltage source.

Nodes a, b, and c are connected on the right to respective load nodes, marked as A, B, and C, with a 0.18 ohm resistor and a j1.44 ohm inductor located in series on each connection. Voltage between A and B is marked as 500 kVA while voltage between B and C marks a lag of 0.96 pf.

Two capacitors are connected in parallel to the right, between a and b, while a single capacitor and a short circuit line are connected in parallel to the right between b and c. These three capacitors have capacitance marked as minus j 180 ohm each.

The drawing shows two boxes containing three terminals each, which are connected by three parallel wires. The box on the left represents Balanced three-phase source while box on the right represents Balanced three-phase load. Each parallel wire carries a 0.04 ohm resistor and a j0.03 ohm inductor, connected in series.

The drawing shows three parallel wires connecting nodes a, b, and c on the left to nodes A, B, and C on right, respectively. Line aA carries a current coil, cc, while a potential coil, pc, is drawn across the wires connecting nodes b and c. A dotted-line rectangle is drawn encompassing two wattmeters.

Nodes A, B, C are connected to the neutral conductor N, in series with load impedances, each marked individually as Z sub phi.

The drawing shows two boxes containing three terminals each, which are connected by three parallel wires. The box on the left with terminals a, b, and c is labeled as Source, while the box on the right has terminals marked as A, B, and C. A 432k VA reading is shown between terminals A and B, and a 0.96 pf lead is shown between terminals B and C.

A current coil is located on line bB while a potential coil is drawn across wires connecting nodes b and c. A dotted rectangle is drawn encompassing these two coils, and is labeled as W sub m.

A drawing shows a y-connected source network, where three nodes, a, b, and c, are connected to neutral conductor by an independent voltage source as follows:

Between a and neutral conductor: 240 angled 0 degree V

Between b and neutral conductor: 240 angled 120 degree V

Between c and neutral conductor: 240 angled minus 120 degree V

Nodes a, b, and c are connected on the right to respective load nodes, marked as A, B, and C, which are in turn connected to each other in a delta-network, with three load impedances marked as Z sub phi located between them.

A wattmeter W1 is located between the aA and bB lines, with a current coil drawn on bB, and a potential coil drawn across lines connecting a and b. A second wattmeter, W2, is located between the cC and bB lines, with a current coil drawn on cC, and a potential coil drawn across lines connecting C and B.

The drawing shows a y-connected source network where three nodes are connected to a neutral conductor by an independent voltage source, as follows:

Between the top node and neutral conductor: 600 angled 0 degree V

Between the middle node and neutral conductor: 600 angled minus 120 degree V

Between bottom node and neutral conductor: 600 angled plus 120 degree V

Nodes from source are connected on the right to respective load nodes, which are in turn connected to each other in a delta-network, with three load impedances marked as Z located between them.

A wattmeter W1 is located between the top and the middle connecting lines, with a current coil drawn on top line and a potential coil drawn across top and middle lines. A second wattmeter, W2, is located between the middle and bottom connecting lines, with a current coil drawn on the middle line, and a potential coil drawn across the middle and bottom lines.

The drawing shows a y-connected source network, where three nodes a, b, and c, are connected to a neutral conductor by an independent voltage source, as follows:

Between a and neutral conductor: 240 angled 0 degree V

Between b and neutral conductor: 240 angled 120 degree V

Between c and neutral conductor: 240 angled minus 120 degree V

Nodes a, b, and c are connected on the right to respective load nodes, marked as A, B, and C, which are in turn connected to each other in a delta-network, with three load impedances marked as Z sub A, Z sub B, and Z sub C, respectively between AB, BC, and CA.

A wattmeter W sub m1 is located between aA and bB lines, with a current coil drawn on aA, and a potential coil drawn across lines connecting a and b. A second wattmeter, W sub m2, is located between cC and bB lines, with a current coil drawn on cC, and a potential coil drawn across lines connecting C and B.

The X-axis of the given graph plots time t, while the y-axis plots f of t. The graph shows a downward sloping line representing e raised to minus at. The line is drawn from the top left corner of the graph, in the second quadrant, to the bottom right corner, in the first quadrant, cutting through the y-axis at y = 1.0.

The X-axis of the given graph plots time t, while the y-axis plots f of t. The graph shows a straight line drawn along the x-axis in the second quadrant, up to the origin and labeled as 0, t less than 0. The second part of the line moves in a downward slope in the first quadrant, starting from y = 1.0, and is labeled as e raised to minus at, t is greater than 0.

The X-axis of the given graph plots time t, while the y-axis plots f of t. The graph shows a straight line drawn along the x-axis in the second quadrant, up to the origin. The line steps up in the first quadrant and moves horizontal to the x-axis, starting from y = K.

The X-axis of the given graph plots time t, while the y-axis plots f of t. The graph shows a straight line drawn along the x-axis in the second quadrant, up to 0 minus, which is just to the left of the origin. The line steps up in the first quadrant and moves horizontal to the x-axis, starting from (0 plus, K), where 0 plus is just to the right of the origin. An upward slanting line is drawn between the end of the first line and the beginning of the second line, cutting the y-axis at y = 0.5K.

The X-axis of the given graph plots time t, while the y-axis plots f of t. The graph shows a straight line drawn along the x-axis in the second quadrant, up to point a, which is on the x-axis in the first quadrant. The line then steps up to y = K and moves horizontal to the x-axis.

The X-axis of the given graph plots time t, while the y-axis plots f of t. The graph shows a straight line drawn parallel to the x-axis in the second quadrant, up to point a, which is on the x-axis in the first quadrant. The line then steps down to reach the x-axis, and runs along the x-axis.

The X-axis of the given graph plots time t, ranging from 0 to 4 in increments of 1, while the y-axis plots f of t, ranging between minus 2 and 2. The graph shows a triangular pulse between x = 0 and x = 2, which peaks at (1, 2). The pulse moves into the fourth quadrant, running between x = 2 and x = 4, peaking at (3, minus 2).

The X-axis of the given graph plots time t, ranging from 0 to 4 in increments of 1, while the y-axis plots f of t, ranging between minus 2 and 2. The graph shows a triangular pulse formed by two intersecting straight line segments. One line, which moves in an upward slope from the origin, represents 2t, while the second line, which moves in a downward slope from y = 4, represents minus 2t plus 4. The triangular pulse is formed between these two lines from x = 0 and x = 2, peaking at (1, 2).

The downward sloping line, minus 2t plus 4, moves into the fourth quadrant, intersecting with an upward sloping line, which is drawn from x = 4 in the fourth quadrant and is labeled as 2t minus 8. Another triangular pulse is formed between these two intersecting lines, from x = 2 and x = 4, peaking at (3, minus 2).

The X-axis of the given graph plots time t, while the y-axis plots f of t, ranging from 0 to 1.0 in increments of 0.5. The graph shows a straight line drawn along the x-axis in the second quadrant, up to minus epsilon, which is just to the left of the origin. The line steps up in the first quadrant and moves with a downward slope, starting from (epsilon, 1.0), where epsilon is just to the right of the origin. An upward slanting line is drawn between the end of the first line and the beginning of the second line, cutting the y-axis at y = 0.5. This line is represented by the equation t over 2 epsilon plus 0.5.

The X-axis of the given graph plots time t, while the y-axis plots f-dash of t. The graph shows a straight line drawn along the x-axis in the second quadrant, up to minus epsilon, which is just to the left of the origin. The line steps up to 1 over 2 epsilon in the second quadrant itself, and moves parallel to the x-axis, up to epsilon in the first quadrant, where epsilon is just to the right of the origin. The line then steps down to minus a in the fourth quadrant and then moves in an upward slope, reaching close to the x-axis from below. This slanting line is represented by the equation minus a multiplied by e raised to minus a(t minus epsilon).

The X-axis of the given graph plots time t, while the y-axis plots f of t. The graph shows a line which starts on the x-axis in the second quadrant and moves with an upward slope to reach K over 2 epsilon2 on the y-axis. The line then moves with a downward slope in the first quadrant to end on the x-axis. This line is represented by equation K over 2 epsilon 2 multiplied by e raised to mod t over epsilon 2.

A second line starts close to the x-axis in the second quadrant and moves with an upward slope to reach K over 2 epsilon1 on the y-axis, where epsilon 1 is lesser in value than epsilon 2. The line then moves with a downward slope in the first quadrant to move close to the x-axis. This line is represented by equation K over 2 epsilon 1 multiplied by e raised to mod t over epsilon 1.

The X-axis of the given graph plots time t, while the y-axis plots f of t. The graph shows an upward arrow drawn from the origin to (K) on the y-axis, labeled as K delta of t. A second upward arrow is drawn at x = a, from y = 0 to y = (K), and labeled as K delta I25(t minus a).

The X-axis of the given graph plots time t, while the y-axis plots f of t. The graph shows a line drawn along the x-axis in the second quadrant, up to minus epsilon, which is to the left of the origin. The line then moves with an upward slope in the second quadrant to reach 1 over epsilon on the y-axis. It then moves with a downward slope in the first quadrant, from 1 over epsilon to epsilon on the x-axis, which is to the right of the origin. The line then continues to move along the x-axis.

The X-axis of the given graph plots time t, while the y-axis plots f of t. The graph shows a line drawn along the x-axis in the second quadrant, up to minus epsilon, which is to the left of the origin. The line then steps up to y = 1 over epsilon square and moves parallel to the x-axis, up to the origin. From this point, the line steps down to y = minus 1 over epsilon square in the fourth quadrant, and moves parallel to the x-axis up to x = epsilon, which is to right of the origin. The line then steps up to the x-axis and continues to move along the x-axis.

The X-axis of the given graph plots time t, while the y-axis plots f of t. The graph shows a line drawn along the x-axis in the second quadrant, up to minus epsilon, which is to the left of the origin. The line then moves with an upward slope in the the second quadrant to reach (epsilon, 1) in the first quadrant, cutting across the y-axis. From this point, the line moves parallel to the x-axis.

The X-axis of the given graph plots time t, while the y-axis plots f of t. The graph shows a line drawn along the x-axis in the second quadrant, up to minus epsilon, which is to the left of the origin. The line then steps up to y = 1 over 2 epsilon and moves parallel to the x-axis up to x = epsilon in the first quadrant. From this point, the line steps down to the x-axis and continues to move along the x-axis.

The X-axis of the given graph plots time t, while the y-axis plots f of t. The graph shows a straight line drawn along the x-axis in the second quadrant, up to the origin and labeled as 0, t less than 0. The second part of the line moves in a downward slope in the first quadrant, starting from y = 1.0 and labeled as e raised to minus at, t is greater than 0.

The X-axis of the given graph plots time t, while the y-axis plots f of t. The graph shows a sinusoidal wave moving on either side of the x-axis, peaking at y = 1.0 and y = minus 1.0 in the first and fourth quadrants, respectively.

The table shows the following data in three columns:

Type: f of t (t greater than 0 minus); F of s

(impulse): delta of t; 1

(step): u of t; 1 over s

(ramp): t; 1 over s squared

(exponential): e raised to minus at; 1 over (s plus a)

(sine): sin omega t; omega over (s squared plus omega squared)

(cosine): cos omega t; s over (s squared plus omega squared)

(damped ramp): t multiplied by e raised to minus at; 1 over (s plus a) squared

(damped sine): e raised to minus at multiplied by sin omega t; omega over (s plus a) squared plus omega squared

(damped cosine): e raised to minus at multiplied by cos omega t; (s plus a) over (s plus a) squared plus omega squared

The table shows the following data in three columns:

Operation: f of t; F of s

* Multiplication by a constant: Kf of t; KF of s

* Addition/Subtraction: f1 of t plus f2 of t minus f3 of t plus…; F1 of s plus F2 of s minus F3 of s plus…

* First derivative (time): df of t over dt; sF of s minus f(0 minus)

* Second derivative (time): d squared f of t over dt-squared; s-squared F of s minus sf(0 minus) minus df(0 minus) over dt

* nth derivative (time): d raised to n f of t over dt-raised to n; s-raised to n F of s minus s raised to (n minus 1) f(0 minus) minus s raised to (n minus 2) df(0 minus) over dt minus s raised to (n minus 3) d f-squared (0 minus) over dt-squared minus…..minus d raised to (n minus 1) f(0 minus) over d t-raised to (n minus 1)

* Time integral: Integral from 0 to 1 of f(x) dx; F of s over s

* Translation in time: f (t minus a) u (t minus a), a greater than 0; e raised to (minus as) multiplied by F of s

* Translation in frequency: e raised to (minus at) multiplied by f of t; F (s plus a)

* Scale changing: f(at), a greater than 0; 1 over a multiplied by F(s over a)

* First derivative of s: tf of t; minus d F of s over ds

* nth derivative of s: t raised to n multiplied by f of t; (minus 1) raised to n multiplied by d raised to n multiplied by F of s over d s-raised to n

* sintegral: f of t over t; integral from s to infinity of F(u) multiplied by du

The circuit shows four parallel vertical cross wires connecting pairs of nodes on the top and bottom wires. A switch, t = 0, a resistor R, an inductor L, and a capacitor C are located on these cross wires from left to right, respectively. A dc current source, I sub dc, with an upward current, is located on the left wire. The right wire is removed and the voltage across two open terminals on the top and bottom wires is marked as v of t.

The table shows the following data in four columns:

Pair number: Nature of roots; F of s; f of t

1. Distinct real; K over (s plus a); K e raised to minus at multiplied by u of t

2. Repeated real; K over (s plus a)squared; K t e raised to minus at multiplied by u of t

3. Distinct complex: K over (s plus alpha minus j beta) plus K-star over (s plus alpha plus j beta); 2 mod K e raised to minus alpha t cos (beta t plus omega) multiplied by u of t

4. Repeated complex: K over (s plus alpha minus j beta)squared plus K-star over (s plus alpha plus j beta)squared; 2t mod K e raised to minus alpha t cos (beta t plus omega) multiplied by u of t

The graph shows the s plane where R of s is plotted on the horizontal axis and I of s is plotted on the vertical axis. Poles are plotted as Xs at 0, minus 10, (minus 6 plus j8), and (minus 6 minus j8), of which the last two are joined by dotted lines. Zeros are plotted as Os at minus 5, (minus 3 plus j4), and (minus 6 minus j8), of which the last two are joined by dotted lines.

The graph shows the s plane where R of s is plotted on the horizontal axis and I of s is plotted on the vertical axis. Poles are plotted as Xs at minus 4, minus 3, minus 2, and minus 1. Zeros are plotted as Os at minus 10 and minus 5 on the horizontal axis.

The circuit shows a 10mH inductor, with a rightward current i sub L, and a 100 microF capacitor on the top wire, and a 15 ohm resistor on the right wire. An independent voltage source, cos 120 pie tV, is located on the left wire.

The X-axis of the given graph plots time t in milliseconds, ranging from 0 to 50 in increments of 10, while the y-axis plots current i sub L of t in milliAmps, ranging from minus 50 to 50 in increments of 10. The graph plots a sinusoidal wave that moves from the left to the right on either side of the x-axis, peaking close to plus 40 and minus 40, completing 3 and a half cycles between 0 and 50 milliseconds.

The X-axis of the given graph plots time t, while the y-axis plots f of t. The graph shows a line which runs along the x-axis up to minus 10 in the second quadrant, and then moves with an upward slope to reach y = 10. The line then moves with a downward slope in the first quadrant to reach the x-axis at x = 10, and then runs along the x-axis.

The X-axis of the given graph plots time t, while the y-axis plots f of t. The graph shows a trapezoidal pulse between minus 3 and the origin, which peaks at a constant value of y = minus 8 between x = minus 2 and x = minus 1 in the third quadrant. The pulse continues in the first quadrant between 0 and 3, peaking at a constant value of y = 8 between x= 1 and x=2.

The X-axis plots time t in seconds, ranging between 2 and 8 in increments of 2, while the y-axis plots f of t. The graph shows a line which runs along the x-axis in the second quadrant up to the origin. The line then moves with an upward slope to reach (2, 10), then runs parallel to the x-axis up to (6, 10), and then moves with a downward slope to reach (8, 0), from where it runs along the x-axis.

The X-axis plots time t in seconds, ranging between 0 and 2 in increments of 0.5, while the y-axis plots f of t. The graph shows a line which runs along the x-axis in the second quadrant up to the origin. The line then runs as a sinusoidal wave, marked as 10 sin pie t between 0 and 2.0, peaking at y = 10 in the first quadrant for x = 0.5, and at y = minus 10 in the fourth quadrant for x = 1.5. The line then continues to run along the x-axis.

The X-axis plots time t in seconds, while the y-axis plots f of t. The graph shows a line which runs along the x-axis in the second quadrant, up to the origin. The line then runs with an upward slope to reach a peak value of (5, 20) in the first quadrant, from where it drops to the x-axis and continues to run along the x-axis.

The X-axis of the graph plots time t, while the y-axis plots f of t. The graph shows a line drawn along the x-axis in the second quadrant, up to minus epsilon, which is to the left of the origin. The line then moves in a triangular pulse between minus epsilon and the origin in the second quadrant, peaking at (minus epsilon over 2, 2 over epsilon square). It then moves in another triangular pulse in the fourth quadrant, between the origin and x = epsilon, peaking at (minus epsilon over 2, minus 2 over epsilon square). The line continues to move along the x-axis in the first quadrant.

The X-axis plots time t in seconds, ranging from 0 to 8 in increments of 2, while the y-axis plots f of t. The graph shows a line drawn along the x-axis in the second quadrant, up to the origin. The line then moves in a triangular pulse between 0 and 4, peaking at (2, 10) in the first quadrant. It then moves in another triangular pulse in the fourth quadrant, between 4 and 8, peaking at (6, minus 10). The line continues to move along the x-axis in the first quadrant.

The circuit shows three parallel vertical cross wires connecting three pairs of nodes on the top and bottom wires. The cross wires carry a switch t = 0, an inductor L with voltage v sub o, and a resistor R, from left to right, respectively. The right wire carries capacitor C with a downward current, i sub o. The circuit is powered by an independent current source, I sub dc, with an upward current, located on the left wire.

The circuit shows three parallel vertical cross wires connecting three pairs of nodes on the top and bottom wires. Switch t = 0 is located on the top of the left wire, connecting to nodes a and b on the top wire and left cross wire, respectively. The second and third cross wires from the left carry resistor R and capacitor C, respectively, while the right wire carries an inductor L with a downward current i sub o. The circuit is powered by an independent current source, I sub dc, with an upward current, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires. Switch t = 0 is located on the top of this cross wire, connecting nodes a and b. A resistor R and an inductor L are located on the top wire, to the right of the cross wire, while a capacitor C with voltage v sub o is located on the right wire. The circuit is powered by an independent voltage source, V sub dc, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires. An inductor L, with a downward current i sub o, is located on this cross wire. Switch t = 0, and resistor R are located on the left arm of the top wire, while capacitor C, with voltage v sub o, is located on the right wire. The circuit is powered by an independent voltage source, V sub dc, located on the left wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries switch t = 0, while the right cross wire carries capacitor C with voltage v1. The right wire carries inductor L, with voltage v2, while the right arm of the top wire carries resistor R. The circuit is powered by an independent current source, i sub g, with an upward current, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires. A 5H inductor, with a dot terminal on the bottom right, is located on this cross wire. A 25H inductor, with a dot terminal on the bottom left, is located on the top wire, to the left of the cross wire. Mutual inductance between these two inductors is marked as 10H.

A 60 ohm resistor is located on the top wire, close to the left wire, while a 40 ohm resistor is located on the right wire. Mesh currents in the left and right meshes on either side of the cross wire are marked as i1 and i2. The circuit is powered by a 300 u of tV independent voltage source, located on the left wire.

The circuit shows an inductor sL located on a vertical wire between two terminals, a and b, in series with an independent voltage source, LI sub 0, with a reverse polarity. Voltage across a and b is marked as V, while a downward current, I, flows through the inductor.

The circuit shows an inductor sL located on a vertical wire between two terminals, a and b, in parallel with an independent current source with a downward current, I sub 0 over s. Voltage across this parallel connection is marked as V, while a downward current, I, flows down from terminal a.

The circuit shows capacitor 1 over sC located on a vertical wire between two terminals, a and b, in parallel with an independent current source with an upward current, CV sub 0. Voltage across this parallel connection is marked as V, while a downward current, I, flows down from terminal a.

The circuit shows capacitor 1 over sC located on a vertical wire between two terminals, a and b, in series with an independent voltage source V sub 0 over s. Voltage across the terminals is marked as V, while a downward current, I, flows through the capacitor.

The table shows two columns for time domain and frequency domain circuits, depicting resistor, inductor, and capacitor one below the other in separate rows. The circuits are shown as follows:

First row:

A time-domain circuit shows resistor R on a vertical wire between two terminals, a and b, with voltage v marked across the terminals, and a downward current i flowing through the wire. An equation, v = Ri, is displayed below the circuit.

A frequency-domain circuit shows resistor R on a vertical wire between two terminals, a and b, with voltage V marked across the terminals, and a downward current I flowing through the wire. An equation, V = RI, is displayed below the circuit.

Second row:

A time-domain circuit shows an inductor L on a vertical wire between two terminals, a and b, with voltage v marked across the terminals. An initial current, I sub 0, is marked downwards. A downward current in the inductor is marked as i.

The following equations are shown below the circuit:

v = L multiplied by di over dt

i = 1 over L multiplied by integral between 0 to t of minus vdx plus I sub 0

A series-equivalent circuit shows an inductor sL located on a vertical wire between two terminals, a and b, in series with an independent voltage source, LI sub 0, with a reverse polarity. Voltage across a and b is marked as V, while a downward current I flows through the inductor.

The following equation is shown below the circuit:

V = sLI minus LI sub 0

A parallel-equivalent circuit shows an inductor sL located on a vertical wire between two terminals, a and b, in parallel with an independent current source with a downward current, I sub 0 over s. Voltage across this parallel connection is marked as V, while a downward current I flows down from terminal a.

The following equation is shown below the circuit:

I = V over sL plus I sub 0 over s

Third row:

A time-domain circuit shows capacitor C on a vertical wire between two terminals, a and b. Voltage between the terminals is marked as v, while a downward current i flows through the wire. The initial voltage in the capacitor is marked as V sub 0.

The following equations are shown below the circuit:

i = C multiplied by dv over dt

v = 1 over C multiplied by integral 0 to t of minus idx plus V sub 0

A series-equivalent circuit shows capacitor 1 over sC located on a vertical wire between two terminals, a and b, in series with an independent voltage source, V sub 0 over s. Voltage across the terminals is marked as V, while a downward current I flows through the capacitor.

The following equation is shown below the circuit:

V = I over sC plus V sub 0 over s

A parallel-equivalent circuit shows capacitor 1 over sC located on a vertical wire between two terminals, a and b, in parallel with an independent current source with an upward current, CV sub 0. Voltage across this parallel connection is marked as V, while a downward current I flows down from terminal a.

The following equation is shown below the circuit:

I = sCV minus CV sub 0

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. A 0.016s ohm inductor is located on the left cross wire, while a 500 ohm resistor is located on the right cross wire. A capacitor located on the left wire has a capacitance of numerator 40 multiplied by 10 raised to 6 over denominator s ohms. The right wire is removed and two open terminals are located on the top and bottom wires to the right, with an equivalent admittance marked as Y sub eq.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. A 0.5 microF capacitor is located on the left cross wire, with voltage marked across as v sub C, while a 240k ohm resistor with voltage v sub o is located on the right cross wire. A switch, t = 0, is located on the top of the left cross wire, connecting to nodes x and y on the top wire. Node x leads to a 10k ohm resistor on the left arm of the top wire, while node y leads to a 32k ohm resistor on the middle arm of the top wire. A 60k ohm resistor is located on the right wire. The circuit is powered by a 100V independent voltage source, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 240k ohm resistor with voltage V sub o. A 32k ohm resistor is located on the left arm of the top wire, while a 60k ohm resistor is located on the right wire. The left wire carries an independent voltage source, 100 over s V-s, and a capacitor with capacitance marked as numerator 2 multiplied by 10 raised to 6 over denominator s ohms.

The circuit shows four parallel vertical cross wires connecting four pairs of nodes on the top and bottom wires. A switch, t = 0, is located on the left cross wire. A 25 nF capacitor is located on the second cross wire from the left. The third cross wire carries a 25mH inductor with a downward current i sub L, while the fourth cross wire carries a 500 ohm resistor. The right wire is open and two open terminals on the top and bottom wires to the right show voltage v between them. The circuit is powered by a 24 mA independent current source with an upward current, located on the left wire.

The circuit shows five parallel vertical cross wires connecting five pairs of nodes on the top and bottom wires. A capacitor, inductor, and resistor are located on the first, third, and fifth cross wires, respectively. Capacitance is marked as numerator 40 multiplied by 10 raised to 6 over denominator s ohms. Inductance is marked as s over 40 ohms, while resistance is 500 ohms. the Right wire is removed and two open terminals on the top and bottom wires to the right show voltage V between them.

The circuit is powered by three independent current sources, located on the left wire, second cross wire, and fourth cross wire. Current source on the left wire has an upward current, 24 over s mA-s, while the current source on the second cross wire has an upward current marked as 1.25 microV-s. The current source on the fourth cross wire has a downward current, 29 over s mA-s.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, with switch t = 0 located on top, connecting to nodes a and b on the top wire. Node a leads to a 10k ohm resistor on the left arm of the top wire. A 5k ohm resistor, with a downward current i, is located on the right wire. The cross wire carries a 0.2 microF capacitor with voltage v1, and a 0.8 microF capacitor with voltage v2, one below the other. The circuit is powered by a 100V independent voltage source, which is located on the left wire.

The circuit shows a switch, t = 0, a 4.8 ohm resistor, and a 4H inductor with voltage v, on the top wire. A 0.25F capacitor is located on the right wire, while a 160V independent voltage source is located on the left wire. A mesh current, i, is represented by a clockwise curved arrow inside the circuit.

The circuit shows a switch, t = 0, and a 10nF capacitor on the top wire, a 4k ohm resistor on the right wire, and a 62.5 mH inductor on the bottom wire. An independent voltage source, v of t, is located on the left wire. A mesh current, i of t, is represented by a clockwise curved arrow inside the circuit.

The circuit shows a switch, t = 0, and a numerator 10 raised to 8 over denominator s capacitor on the top wire, a 4k ohm resistor on the right wire, and a s over 16 ohm inductor on the bottom wire. An independent voltage source is located on the left wire, with voltage marked as numerator 15s over denominator s-square plus 40,000-square V-s. A mesh current, I, is represented by a clockwise curved arrow inside the circuit.

The circuit shows three parallel vertical cross wires connecting three pairs of nodes on the top and bottom wires. A switch, t = 0, is located on the left cross wire, while a 25 nF capacitor and a 625 ohm resistor are located on the middle and right cross wires, respectively. The right wire carries a 25 mH inductor with a downward current i. The circuit is powered by an independent current source with an upward current, i of t, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires. A 42 ohm resistor is located on this cross wire, while a 48 ohm resistor is located on the right wire. The left arm of the top wire carries an 8.4 H inductor with a rightward current i1, while the right arm carries a 10H inductor with a rightward current i2. A switch, t = 0, is located on the top wire, close to the left wire. The circuit is powered by a 336V independent voltage source, which is located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires. A 42 ohm resistor is located on this cross wire, while a 48 ohm resistor is located on the right wire. The left arm of the top wire carries an 8.4s ohm inductor, while the right arm carries a 10s ohm inductor. The circuit is powered by a 336 over s V independent voltage source, which is located on the left wire. Two mesh currents, I1 and I2, are indicated by a clockwise curved arrows located in the left and right meshes, respectively.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires. A 42 ohm resistor is located on this cross wire, while a 48 ohm resistor is located on the right wire. A rightward current, i sub 1f, flows through the left arm of the top wire, while another rightward current, i sub 2f, flows through the right arm. The circuit is powered by a 336V independent voltage source, located on the left wire.

The circuit shows three parallel vertical cross wires connecting three pairs of nodes on the top and bottom wires. A switch, t = 0, is located on the left cross wire, while a 1 F capacitor with voltage v1 and a 3 ohm resistor with voltage v2 are located on the middle and right cross wires, respectively. A 1H inductor is located on the third arm of the top wire, while a 15 ohm resistor and a second switch, t = 0, are located on the fourth arm. The circuit is powered by a 15V independent voltage source, located on the right wire, and a 5A independent current source, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 2 mH inductor. A 20 ohm resistor and a switch, t = 0, are located on the left arm of the top wire, while a 60 ohm resistor is located on the right arm. A 5 microF capacitor with voltage v sub C and a downward current i sub C is located on the right wire. The circuit is powered by a 480V independent voltage source, located on the left wire.

Two nodes, a and b, are located on the top and bottom wires, close to the right wire. The circuit to left of the nodes is enclosed in a shaded box.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 0.002s ohm inductor. A 20 ohm resistor is located on the left arm of the top wire, while a 60 ohm resistor is located on the right arm. A capacitor with capacitance of numerator 2 multiplied by 10 raised to 5 over denominator s ohm and downward current I sub C, is located on the right wire. The circuit is powered by a 480 over s V-s independent voltage source, located on the left wire.

Two nodes, a and b, are located on the top and bottom wires, close to the right wire, with voltage between them marked as V sub C. The circuit to the left of the nodes is enclosed in a shaded box.

The circuit shows a resistor on the top wire, with resistance marked as numerator 80 multiplied by sum (s plus 7500) over denominator s plus 10 raised to 4 ohm. A capacitor is located on the right wire, with capacitance marked as numerator 2 multiplied by 10 raised to 5 over denominator s ohm. A downward current, I sub C, flows through the capacitor. The circuit is powered by an independent voltage source on the left wire, with voltage marked as numerator 480 over denominator s plus 10 raised to 4 V-s.

Two nodes, a and b, are located on the top and bottom wires, close to the right wire, with voltage between them marked as V sub C. The circuit to the left of the nodes is enclosed in a shaded box.

The circuit shows a 5 ohm resistor, a 0.2 v sub x dependent voltage source with reverse polarity, a 1 ohm resistor, and a 2 ohm resistor on the top wire. A 1H inductor is located on the right wire. An additional wire is drawn across two terminals located on either side of the 1 ohm resistor and the dependent voltage source on the top wire. A 0.5microF capacitor is located on this additional wire. The voltage between the top and bottom wires to the left of the dependent circuit is marked as "v sub x". The circuit is powered by a 20 u of t independent voltage source, located on the left wire.

Two nodes, a and b, are located on the top and bottom wires, close to the right wire, with node a located to the left of the 2 ohm resistor on the top wire. The circuit to the left of the nodes is enclosed in a shaded box.

The diagram shows two circuits with magnetically coupled coils facing each other. A 2H inductor, with a dot terminal on the top left, is located on the right wire of left the circuit, while an 8H inductor, with a dot terminal on the top right, is located on the left wire of the right circuit. Mutual inductance between the two is marked as the 2H.

The right circuit shows a 2 ohm resistor on top wire, a 10 ohm resistor on the right wire, and downward current i2 on the left wire. The left circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, with a switch, t = 0, located on top of the cross wire, connecting wire a on the top wire and wire b on the cross wire. Wire a leads to a 9 ohm resistor on the left arm of the top wire. A 3 ohm resistor with rightward current i1 is located on the right arm of the top wire. The circuit is powered by a 60V independent voltage source, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 2H inductor with mutual inductance marked as (M). The left arm of the top wire carries a 3 ohm resistor and a 0H inductor, whose mutual inductance is (L1 minus H), while the right arm carries a 2 ohm resistor and a 6H inductor, whose mutual inductance is (L2 minus M). A rightward current, i1, flows through the left arm while a leftward current, i2, flows through the right arm of the top wire. A 10 ohm resistor is located on the right wire. The left wire of the circuit represents the vertical cross wire of the left circuit in figure 13.23. Node b of the switch is displayed on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 2s ohm inductor and a 10 V-s independent voltage source with reverse polarity. The left arm of the top wire carries a 3 ohm resistor with a rightward current I1, while the right arm carries a 2 ohm resistor and a 6s ohm inductor with a leftward current I2. A 10 ohm resistor is located on the right wire. Two nodes are displayed on the top and bottom wires, to the right of the cross wire. The left wire of the circuit represents the vertical cross wire of the left circuit in figure 13.23. Node b of the switch is displayed on the left wire.

The X-axis plots time t in ms, while the y-axis plots current i2 in mA. The graph shows a line which runs along the x-axis in the second quadrant, up to the origin. The line then moves with an upward slope, reaching a peak value of (549.31, 481.13), before descending with a downward slope and moving close and parallel to the x-axis.

The circuit shows three parallel vertical cross wires connecting three pairs of nodes on the top and bottom wires. The left cross wire carries an independent current source with an upward current, 1.25 micro V-s. The middle cross wire carries a s over 40 ohm inductor. The right cross wire carries a 500 ohm resistor. The right wire is removed and two open terminals are located on the top and bottom wires to the right, with voltage V-dash marked across them. A capacitor is located on the left wire, with its capacitance marked as numerator 40 multiplied by 10 raised to 6 over denominator s ohm.

The circuit shows three parallel vertical cross wires connecting three pairs of nodes on the top and bottom wires. The left cross wire carries a s over 40 ohm inductor. The middle cross wire carries an independent current source with a downward current 29 over s mA-s. The right cross wire carries a 500 ohm resistor. The right wire is removed and two open terminals are located on the top and bottom wires to the right, with voltage V-double dash marked across them. A capacitor is located on the left wire, with its capacitance marked as numerator 40 multiplied by 10 raised to 6 over denominator s ohm.

The circuit shows three parallel vertical cross wires connecting three pairs of nodes on the top and bottom wires. The left cross wire carries a capacitor whose capacitance is marked as numerator 40 multiplied by 10 raised to 6 over denominator s ohm. The middle cross wire carries a s over 40 ohm inductor. The right cross wire carries a 500 ohm resistor. The right wire is removed and two open terminals are located on the top and bottom wires to the right, with voltage V- triple dash marked across them. An independent current source with an upward current, 24 over s mA-s, is located on the left wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. A 1.25H inductor is located on the left cross wire, while a 50mF capacitor is located on the right cross wire. Voltage between the top and bottom wires, between the two cross wires, is marked as v. A 2 ohm resistor and is located on the left arm of the top wire.

The circuit is powered by a 20u of tV independent voltage source, located on the left wire, and a 5u of tA independent current source with an upward current, located on the right wire.

The circuit shows resistor R, with a rightward current I, and inductor sL on the top wire. The right wire carries capacitor 1 over sC, with voltage V marked across it. An independent voltage source, V sub g, is located on the left wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a 250 ohm resistor and a 50 mH inductor, while the right cross wire carries a 1 microF capacitor. A 1000 ohm resistor is located on the left arm of the top wire. The right wire is removed and two open terminals are located on the top and bottom wires to the right, with voltage v sub o marked between them. The circuit is powered by an independent voltage source, v sub g, located on the left wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a 250 ohm resistor and a 0.05s ohm inductor, while the right cross wire carries a numerator 10 raised to 6 over denominator s ohm capacitor. A 1000 ohm resistor is located on the left arm of the top wire. The right wire is removed and two open terminals are located on the top and bottom wires to the right, with voltage V sub o marked between them. The circuit is powered by an independent voltage source, V sub g, located on the left wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a 2 ohm resistor and a 1H inductor, while the right cross wire carries a 0.1F capacitor. The right wire is removed and two open terminals are located on the top and bottom wires to the right, with voltage v sub o marked between them. The circuit is powered by an independent current source with an upward current, i sub g, located on the left wire.

The X-axis plots time t in ms, ranging from 0 to 1.4 in increments of 0.2, the while y-axis plots voltage v sub o in mV, ranging from 0 to 16 in increments of 2. The graph shows a line representing v sub 0, which starts at the origin and moves with a steep upward slope toward the y-axis, reaching (0.9, 8). The line then moves diagonally upward.

A second diagonal line is drawn starting from the origin and moving close to the line representing v sub o. This line is labeled as (10,000t minus 0.4) mV.

The drawing shows a square block labeled as h of t. An inward arrow labeled x of t is drawn toward the block from the left. An outward arrow labeled y of t is drawn from the block toward the right.

The X-axis plots time t, while the y-axis plots input signal x of t. The graph shows a straight line which runs along the x-axis in the second quadrant, up to the origin. The line discontinues at the origin and starts again from a point on the y-axis, from where it moves with an upward curve for a short distance, and then moves down with a downward slope, tapering off toward the end to move close to the x-axis.

The X-axis plots time t, while the y-axis plots input signal x of t. The graph shows a straight line which runs along the x-axis in the second quadrant, up to the origin, which is depicted as lambda 0. The line discontinues at the origin and starts again from a point on the y-axis, from where it moves with an upward curve for a short distance, and then moves down with a downward slope, tapering off toward the end to move close to the x-axis.

A series of rectangular pulses are drawn between the curve and horizontal axis, for values on the x-axis marked as lambda1, lambda2, lambda3...lambda i,...etc. Value of x of t on the curve, where individual pulses intersect, are marked as x(lambda 0), x(lambda1), x(lambda2) and so on to x(lambda i) etcetra.

The X-axis plots time t while the y-axis plots input signal x of t. The graph shows a series of upward arrows drawn from the horizontal axis, at values marked as lambda0, lambda1, lambda2, lambda3 and so on to lambda i. Corresponding x of t values for these impulses are marked as [x(lambda0) delta lambda], [x(lambda1) delta lambda], [x(lambda2) delta lambda], [x(lambda3) delta lambda] and so on to [x(lambda i) delta lambda].

The X-axis plots time t, while the y-axis plots impulse response h of t. The graph shows a line which moves along the x-axis in the second quadrant, up to the origin. The line then steps up to a point on the y-axis, from where it moves in a downward slope, moving close to and running parallel to the x-axis.

The X-axis plots time t, while y-the axis plots output signal y of t. The graph shows a series of impulse responses which run along the x-axis, up to points marked on the x-axis as lambda 0, lambda 1, lambda 2, and so on. These impulses then step up vertically to reach a peak point and then move with a downward slope, hovering close to and running parallel to the x-axis.

The impulse responses are summed up and a line is drawn through them, which moves in an upward slope above the impulses, reaches a peak value, and then moves down. This line represents Approximation of y of t.

The X-axis plots lambda, while the y-axis plots h of lambda. The graph shows a line which moves along the x-axis in the second quadrant, up to the origin. The line then steps up to point A on the y-axis, from where it moves in a downward slope, moving close to and running parallel to the x-axis.

The X-axis plots lambda, while the y-axis plots x of lambda. The graph shows a line which moves along the x-axis in the second quadrant, up to the origin. The line then moves in a straight upward slope to reach a peak value of (tau 1, M), runs parallel to the x-axis up to (tau 2, M), and then drops vertically to the x-axis and continues to run along it.

The X-axis plots lambda, while the y-axis plots x of minus lambda. The graph shows a line which moves along the x-axis in the second quadrant, up to minus tau 2. The line then steps up to (minus tau 2, M), moves parallel to the x-axis up to (minus tau 1, M), and then slopes down to the origin, from where it continues to run along the x-axis in the first quadrant.

The X-axis plots lambda, while the y-axis plots x of t minus lambda. The graph shows a line which moves along the x-axis in the second quadrant, up to t minus tau 2. The line then steps up to (t minus tau 2, M), moves parallel to the x-axis up to (t minus tau 1, M) in the first quadrant, and then slopes down to t on the x-axis, from where it continues to run along it.

The X-axis plots lambda, while the y-axis plots h of lambda x of t minus lambda. The graph shows a line which moves along the x-axis, up to the origin in the second quadrant. The line then steps up to M on the y-axis, from where it curves down in the first quadrant to a point whose x-coordinate value is (t minus tau 1). From this point, the curve moves down with a steeper slope to reach the x-axis at t, from where it continues to move along the x-axis.

Region between the curve and the x-axis in the first quadrant is marked as y of t = Area.

The X-axis plots lambda, while the y-axis plots h of lambda. The graph shows a line which moves along the x-axis in the second quadrant, up to the origin. The line then steps up to point A on the y-axis, from where it moves in a downward slope, moving close to and running parallel to the x-axis.

The X-axis plots lambda, while the y-axis plots x of lambda. The graph shows a line which moves along the x-axis in the second quadrant, up to the origin. The line then moves in a straight upward slope to reach a peak value of (tau 1, M), runs parallel to the x-axis up to (tau 2, M), and then drops vertically to the x-axis and continues to run along it.

The X-axis plots lambda, while the y-axis plots h(minus lambda). The graph shows a line which starts on the x-axis in the second quadrant, moves with an upward slope to reach A on the y-axis, and then drops down to the origin, from where it continues to run along the x-axis in the first quadrant.

The X-axis plots lambda, while the y-axis plots h(t minus lambda). The graph shows a line which starts on the x-axis in the second quadrant, moves with an upward slope to reach (t, A) in the first quadrant, and then drops down to the x-axis, from where it continues to run along it.

The X-axis plots lambda, while the y-axis plots h of lambda x(t minus lambda). The graph shows a line which moves along the x-axis, up to the origin in the second quadrant. The line then curves up in the first quadrant, to a point whose x-coordinate value is (tau 1). From this point, the line moves up with a steeper slope to reach (t, MA). The line then drops vertically down to the x-axis and continues to run along it. Region between the curve and the x-axis in the first quadrant is marked as y of t = Area.

The circuit shows a 1H inductor on the top wire, a 1 ohm resistor with voltage v sub o on the right wire, and an independent voltage source, v sub i, on the left wire.

The X-axis plots time t in seconds, while the y-axis plots v sub i. The graph shows a line which runs along the x-axis, up to the origin in the second quadrant. The line then moves with an upward slope to reach (5, 20V), runs parallel to the x-axis up to (10, 20V), and then drops vertically down to the x-axis, from where it runs along it.

A graph depicts folded input function for the circuit in example 13.11.

The X-axis plots lambda, while the y-axis plots h of lambda. The graph shows a line which moves along the x-axis in the second quadrant, up to the origin. The line then steps up to y = 1.0 on the y-axis, from where it moves in a downward slope, moving close to and running parallel to the x-axis. This line representing e raised to minus lambda is marked as Impulse response.

The X-axis plots lambda, while the y-axis plots v sub i of minus lambda. The graph shows a line which moves along the x-axis in the second quadrant, up to x = minus 10. The line then steps up to (minus 10, 20V), runs parallel to the x-axis up to (minus 5, 20V), and then moves with a downward slope up to the origin, from where it runs along the x-axis in the first quadrant. The line is marked as Folded excitation.

A graph depicts displacement curve v sub i of t minus lambda, when t is greater than zero and less than 5.

A graph depicts displacement curve v sub i of t minus lambda, when t is greater than 5 and less than 10.

A graph depicts displacement curve v sub i of t minus lambda, when t is between 10 and infinity.

The X-axis plots lambda, while the y-axis plots h of lambda. The graph shows a line which moves along the x-axis in the second quadrant, up to the origin. The line then steps up to y = 1.0 on the y-axis, from where it moves in a downward slope, moving close to and running parallel to the x-axis.

The X-axis plots lambda, while the y-axis plots v sub i of t minus lambda. The graph shows a line which moves along the x-axis in the second quadrant, up to x = t minus 10. The line then steps up to (t minus 10, 20), runs parallel to the x-axis up to (t minus 5, 20), and then moves with a downward slope to reach t on the x-axis in the first quadrant, from where it runs along the x-axis. Value of t is depicted as greater than 0 and less than 5.

The X-axis plots lambda, while the y-axis plots v sub i of t minus lambda. The graph shows a line which moves along the x-axis in the second quadrant, up to x = t minus 10. The line then steps up to (t minus 10, 20), runs parallel to the x-axis up to (t minus 5, 20) in the first quadrant, and then moves with a downward slope to reach t on the x-axis, from where it runs along it. Value of t is depicted as greater than 5 and less than 10.

The X-axis plots lambda, while the y-axis plots v sub i of t minus lambda. The graph shows a line which moves along the x-axis in the second quadrant, up to x = t minus 10 in the first quadrant. The line then steps up to (t minus 10, 20), runs parallel to the x-axis up to (t minus 5, 20), and then moves with a downward slope to reach t on the x-axis, from where it runs along it. Value of t is depicted as greater than 10 and less than infinity.

The X-axis plots time t in seconds, ranging from 0 to 14 in increments of 2, while the y-axis plots voltage v sub o in volts, ranging from 0 to 20 in increments of 2. The graph shows a line which starts at the origin and moves with a steep upward slope up to (6, 19). The line then moves almost parallel to the x-axis, but with a slight upward slant up to (10, 20), from where it drops almost vertically down in a steep downward slope up to (2, 12), and then moves with a lesser steep to end on the x-axis beyond 14.

The X-axis plots lambda, while the y-axis plots v sub i of t minus lambda. The graph shows a line which moves along the x-axis in the second quadrant, up to x = t minus 10. The line then steps upward, runs parallel to the x-axis up to x value of t minus 5 in the the second quadrant, and then moves with a downward slope to reach t on the x-axis in the first quadrant, from where it runs along the x-axis. The horizontal line in the first quadrant is labeled as Past (has happened), the slope between the second and first quadrants is labeled as Present, while the curve in the second quadrant is labeled as Future (will happen).

The X-axis plots time t, while the y-axis plots h of t. The graph shows a line which runs along the x-axis in the second quadrant up to the origin, then steps up to y = 1.0 on the y-axis, and then runs parallel to the x-axis.

The X-axis plots time t, while the y-axis plots h of t. The graph shows a vertical arrow drawn along the y-axis, from the origin to y = 1.0.

The X-axis plots time t in seconds, ranging from 0 to 14 in increments of 2, while the y-axis plots voltage v sub o, v sub i in volts, ranging from 0 to 20 in increments of 2. The graph shows a curve representing Response and a dotted line representing Excitation. The Response curve starts at the origin and moves with a steep upward slope, up to (6, 19). The line then moves almost parallel to the x-axis, but with a slight upward slant, up to (10, 20), from where it drops almost vertically down in a steep downward slope to (2, 12), and then moves with a lesser steep to end on the x-axis beyond 14.

The Excitation line moves with an upward slope from the origin to (5, 20) in a straight line, then runs parallel to the x-axis up to (10, 20), where it intersects the response curve. The line then falls vertically down to reach the x-axis at x = 10.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a 250 ohm resistor and a 50mH inductor, while the right cross wire carries a 1 microF capacitor. The left arm of the top wire carries a 1000 ohm resistor. The right wire is removed and two open terminals are located on the top and bottom wires to the right, with voltage v sub o marked between them. The circuit is powered by an independent voltage source, v sub g, located on the left wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a 2 ohm resistor and a 1H inductor, while the right cross wire carries a 0.1F capacitor. The right wire is removed and two open terminals are located on the top and bottom wires to the right, with voltage v sub o marked between them. The circuit is powered by an independent current source, i sub g, with an upward current, located on the left wire.

The circuit shows an op amp with a positive supply voltage of 15V and a negative supply voltage of minus 15V. A 10k ohm resistor connects the output terminal with the inverting input terminal. The output terminal is grounded with the common node, with voltage v sub o marked between them. The non-inverting input terminal is grounded with the common node in series with a 50k ohm resistor. An independent voltage source, v sub g, connects the common node with the inverting input terminal in series with a 10k ohm resistor, and also connects to the non-inverting input terminal in series with a 400pF capacitor. The resistor and capacitor at the input terminals are parallel to each other.

The circuit shows a capacitor C1 with voltage V sub 0 on the left wire, and another capacitor C2 on the right wire. A switch, t = 0, and a resistor R, with a rightward current i of t, are located on the top wire.

The circuit shows a capacitor, 1 over sC1, and an independent voltage source, V sub 0 over s, on the left wire, and another capacitor, 1 over sC2, on the right wire. A resistor R, with a rightward current I, is located on the top wire.

The X-axis plots time t, while the y-axis plots current i. The graph shows two curves, representing R1 and R2, where R2 is lesser than R1. The curve for R1 starts at V sub 0 over R1 on the y-axis, and moves with a steep downward slope to move close to the x-axis. The curve for R2 moves along the x-axis in the second quadrant, up to the origin. It then discontinues and starts at V sub 0 over R2 on the y-axis, which is above the starting point of the R1 curve. It then moves with a downward slope in the first quadrant, intersects R1, and ends up on the x-axis, from where it runs along it.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries switch t = 0, while the right cross wire carries a 15 ohm resistor with a downward current i2 and an inductor L2 with inductance of 2H. The left arm of the top wire carries a 10 ohm resistor with a rightward current i1, and an inductor L1 with inductance of 3H. The right wire is removed and two open terminals are located on the top and bottom wires to the right, with voltage between them marked as v sub o. The circuit is powered by a 100V independent voltage source, located on the left wire.

The circuit shows a vertical cross wire connecting a pair of nodes on the top and bottom wires, carrying a 15 ohm resistor and a 2s ohm inductor. The left arm of the top wire carries a 10 ohm resistor with a rightward current I1, a 3s ohm inductor, and a 30V-s independent voltage source with reverse polarity. The right wire is removed and two open terminals are located on the top and bottom wires to the right, with voltage between them marked as V sub o. The circuit is powered by a 100 over s V-s independent voltage source, located on the left wire.

The X-axis plots time t, while the y-axis plots currents i1, i2 in Amps, ranging from 0 to 10 in increments of 2. The graph shows a line representing current i1, which runs parallel to the x-axis in the second quadrant at y = 10. A second line, representing current i2, runs along the x-axis in the second quadrant, up to the origin. It then discontinues and starts again at y = 6 on the y-axis, and then moves with a downward slope in the first quadrant to reach y = 4, from where it runs parallel to the x-axis. Value of i1 is the same as i2 at y = 4.

The circuit shows a resistor R on the top wire, an inductor L on the right wire, and an independent voltage source, V0 delta of t, on the left wire.

The circuit shows a resistor R with a rightward current I on the top wire, an inductor sL on the right wire, and an independent voltage source, V sub 0, on the left wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries switch t = 0, while the right cross wire carries a 15 ohm resistor with a downward current i2 and a 2H inductor. The left arm of the top wire carries a 10 ohm resistor with a rightward current i1, and a 3H inductor. The right wire is removed and two open terminals are located on the top and bottom wires to the right, with voltage between them marked as v sub o. The circuit is powered by a 100V independent voltage source and a 50 delta of tV independent voltage source, both of which are located on the left wire.

The circuit shows a vertical cross wire connecting a pair of nodes on the top and bottom wire, carrying a 15 ohm resistor and a 2s ohm inductor. The left arm of the top wire carries a 10 ohm resistor with a rightward current I1, a 3s ohm inductor, and a 30V-s independent voltage source with reverse polarity. The right wire is removed and two open terminals are located on the top and bottom wires to the right, with voltage between them marked as V sub o. The circuit is powered by a 100 over s V-s independent voltage source and a 50 V-s independent voltage source, both of which are located on the left wire.

The X-axis plots time t, while the y-axis plots currents i1, i2 in Amps, ranging from 0 to 16 in increments of 2. The graph shows a line, representing current i1, which runs parallel to the x-axis in the second quadrant at y = 10. A second line, representing current i2, runs along the x-axis in the the second quadrant, up to the origin. It then discontinues and starts again at y = 16 on the y-axis, and then moves with a downward slope in the first quadrant to reach y = 4, from where it runs parallel to the x-axis. Value of i1 is same as i2 at y = 4.

The X-axis plots time t, while the y-axis plots di1 over dt. The graph shows a line which runs along the x-axis in the second quadrant up to the origin, and then moves along the y-axis up to y = 6. The second part of the line is a curve that runs in an upward slope in the fourth quadrant, from y = minus 60 on the y-axis to a point on the x-axis.

The X-axis plots time t, while the y-axis plots di2 over dt. The graph shows a line which runs along the x-axis in the second quadrant up to the origin, and then moves along the y-axis up to y = 16. The second part of the line is a curve that runs in an upward slope in the fourth quadrant, from y = minus 60 on the y-axis to a point on the x-axis.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries resistor R sub a with a downward current I1, while the right cross wire carries inductor j omega L sub a with a downward current I2. Voltage between the top and bottom wires between the cross wires is marked as V sub o. A second inductor, j omega L sub iota, is located on the left arm of the top wire, with a rightward current I sub 0 marked to its right. A switch, t = 0, is located on the right arm of the top wire. Plus, resistor R sub b, with a downward current I3, is located on the right wire. The circuit is powered by an independent voltage source, V sub g, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying resistor R sub a with voltage V sub o marked across it. The left arm of the top wire carries an inductor, sL sub iota, and an independent voltage source with reverse polarity, L sub iota multiplied by I sub 0. A second inductor, sL sub a, is located on the right wire. The circuit is powered by an independent voltage source, V sub g, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and a horizontal cross wire connecting a node on the vertical cross wire with another node on the right wire. A 1 F capacitor is located on the horizontal cross wire, while two 1 H inductors are located on the top and bottom arms of the vertical cross wire. The right wire carries two 1 ohm resistors on its top and bottom arms. The left wire is removed and two open terminals, a and b, are located on the top and bottom wires to the left.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a 4 ohm resistor, while the right cross wire carries a 1 H inductor. A 62.5 microF capacitor is located on the middle arm of the top wire, while a 4 ohm resistor is located on the right wire. The left wire is removed and two open terminals, a and b, are located on the top and bottom wires to the left. An impedance looking rightward into the open terminals is marked as Z sub ab.

Circuit shows two parallel vertical cross wires connecting two pairs of nodes on top and bottom wires. Left cross wire carries a 50 ohm resistor, while right cross wire carries a 625 nF capacitor and switch t = 0, which connects to nodes x and y on the top wire. Node y leads to a 5k ohm resistor on right arm of the top wire. A 2.5 H inductor with voltage v sub o is located on right wire. The circuit is powered by a 3A independent current source, with a downward current, located on left wire.

The circuit shows a 125 nF capacitor and a 4k ohm resistor, with a rightward current i sub o, on the top wire, and a 0.5 H inductor with voltage v sub o on the right wire. An additional wire, carrying switch t = 0, is drawn across two nodes located on either side of the capacitor. The circuit is powered by a 20V independent voltage source, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 200 nF capacitor. The left arm of the top wire carries switch t = 0, a 1120 ohm resistor, and a 0.8 H inductor. The right wire is removed and two open terminals are located on the top and bottom wires to the right, with voltage v sub o marked across them. The circuit is powered by a 240V independent voltage source, located on the left wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a 1 H inductor, while the right cross wire carries a 0.1 F capacitor. A 7 ohm resistor is located on the middle arm of the top wire. The right wire is removed and two open terminals are located on the top and bottom wires to the right, with voltage v sub o marked across them. The circuit is powered by a 15u of tA independent current source with an upward current, located on the left wire.

The circuit shows four parallel vertical cross wires connecting four pairs of nodes on the top and bottom wires. The left cross wire carries an 8 ohm resistor. The second cross wire from the left carries switch t = 0 on top, which moves between wire a on the top wire and wire b on the cross wire. The third cross wire from the left carries an 80 ohm resistor, while the last cross wire on the right carries a 6.4 mH inductor with a downward current i sub L. A 20 ohm resistor is located on the fourth arm of the top wire, between the third and fourth cross wires. An additional wire is drawn over this resistor and a 6.25 microF capacitor is located on this additional wire. The ight wire is removed and two open terminals are located on the top and bottom wires to the right, with voltage v sub o marked between them. The circuit is powered by a 9A independent current source with an upward current, located on the left wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. A 240 ohm resistor is located on the left cross wire, while an 80 microF capacitor with voltage v sub o is located on the right cross wire. The top wire carries an 80 ohm resistor on its left arm and a 100 ohm resistor on its right arm, while the bottom wire carries a 16 mH inductor on its right arm. A 100V independent voltage source is located on the left wire and a 275V independent voltage source is located on the right wire.

An additional wire is drawn over the right arm of the top wire. A 25 ohm resistor is located on top of this additional wire. A switch, t = 0, is located at the left junction of the additional wire and the top wire, connecting to node a on the top wire and node b on the additional wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a 25 H inductor and a 25 ohm resistor, while the right cross wire carries a 100 ohm resistor. A 5 ohm resistor is located on the left arm of the top wire, while a 10 mF capacitor is located on the right wire. A rightward current, i sub o, flows through the right arm of the top wire. A switch, t = 0, is located on top of the left cross wire, connecting to wires a and b on the left and right of the top wire respectively. The circuit is powered by a 500V independent voltage source, located on the left wire.

The circuit shows a small rectangular circuit located in the middle of the top wire. This smaller circuit carries a 240 ohm resistor on the top wire and two 80 ohm resistors on its bottom wire. A vertical cross wire connects a node located in between the resistors on the bottom wire of the small circuit, with another node located on bottom wire of the bigger circuit. The cross wire carries a 32 ohm resistor, a 40V independent voltage source, and a switch t = 0.

The bigger circuit shows a 25 microF capacitor on its left wire, and a 64 ohm resistor and a 200 mH inductor on its right wire, with a downward current i sub o.

The circuit shows a horizontal cross wire connecting two nodes on the left and right wires, and a vertical cross wire connecting two nodes on the horizontal cross wire and bottom wire. A 50 ohm resistor is located on the vertical cross wire, while a 500 mH inductor and a 400 microF capacitor are located on the left and right arms of the horizontal cross wire, respectively. The top wire carries a 100 ohm resistor with a rightward current i sub 1. The circuit is powered by a 30u of tA independent current source with an upward current, located on the bottom arm of the left wire, and a 375u of tV independent voltage source with a downward current i sub 2, located on the bottom arm of the right wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 400 mH inductor with a downward current i sub o and voltage v sub o. The top wire carries two 200 ohm resistors on its left and right arms, while the right wire carries a 10 microF capacitor. The circuit is powered by a 200u of tV independent voltage source, located on the left wire.

The circuit shows a horizontal cross wire connecting two nodes on the left and right wires, and a vertical cross wire connecting two nodes on the horizontal cross wire and bottom wire. A 10 ohm resistor is located on the vertical cross wire, while a 40 mF capacitor and a 4 H inductor are located on the left and right arms of the horizontal cross wire, respectively. The top wire carries a 30 ohm resistor. The circuit is powered by a 50u of tV independent voltage source, located on the bottom arm of the left wire, and a 5u of tA independent current source with an upward current and voltage v sub o, located on the bottom arm of the right wire.

The circuit shows a horizontal cross wire connecting two nodes on the left and right wires, and a vertical cross wire connecting two nodes on the horizontal cross wire and bottom wire. A 500 ohm resistor is located on the vertical cross wire, while two 250 ohm resistors are located on the left and right arms of the horizontal cross wire. The top wire carries a 800 nF capacitor with voltage v sub o, while the bottom arm of the right wire carries a 625 mH inductor with a downward current i sub o. The circuit is powered by an independent voltage source, v sub g, located on the bottom arm of the left wire.

The circuit shows switch t = 0, a 4 ohm resistor, and a 240 mH inductor with a rightward current i sub o of t, located on the top wire. A 2.5 mF capacitor is located on the right wire. The circuit is powered by a 0.1sin 50tV independent voltage source, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 30 ohm resistor. The left arm of the top wire carries switch t = 0 and a 1 microF capacitor, while the right wire carries a 1.2 mH inductor with voltage v sub o. The circuit is powered by a 40 sin 50,000tV independent voltage source, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires. A dependent current source with a downward current, 0.4v sub delta, with voltage v sub o, is located on this cross wire. The right wire carries a dependent voltage source 8 i sub phi. The left arm of the top wire carries a 2 ohm resistor, with a rightward current i sub phi, and switch t = 0. The right arm carries a 1 H inductor, with voltage v sub delta, and a 4 mF capacitor. A 35V independent voltage source is located on the left wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a 200 ohm resistor in series with a 250 nF capacitor with voltage v sub phi. The right cross wire carries a dependent current source with a downward current, 3.75 multiplied by 10 raised to minus 3 v sub phi. Voltage across the current source is marked as v sub o. The right wire carries a 40 mH inductor. The circuit is powered by an independent current source, i sub g, located on the left wire.

The circuit shows a horizontal cross wire connecting two nodes on the left and right wires, and a vertical cross wire connecting two nodes on the horizontal cross wire and bottom wire. A 4 mF capacitor with a reverse voltage of v sub phi is located on the vertical cross wire. The horizontal cross wire carries a 50 ohm resistor on its left arm and a 20 H inductor with a rightward current i sub o on its right arm. A 700 ohm resistor is located on the bottom arm of the right wire.

The top wire carries a dependent current source with a rightward current, v sub phi over 20. An independent voltage source, v sub g, is located on the bottom arm of the left wire.

The circuit shows four parallel vertical cross wires connecting four pairs of nodes on the top and bottom wires. The left cross wire carries a 1K ohm resistor, while the second cross wire carries switch t = 0. The third cross wire carries a 25 ohm resistor, while the last cross wire to the right carries a 100 nF capacitor. A dependent current source with an upward current, 0.3i sub beta, is located on the right wire, with voltage v sub o marked across it. A 50 mA independent current source with an upward current is located on the left wire.

The circuit shows three parallel vertical cross wires connecting three pairs of nodes on the top and bottom wires. The left cross wire carries a 5.625 H inductor and a switch t = 0 on top, which connects to wires a and b to the left and right on the top wire. The middle cross wire carries a 5 ohm resistor with voltage v sub o marked across it. The right cross wire carries a 0.1 F capacitor. A 20 ohm resistor is located on the right wire while a 3 ohm resistor is located on the left arm of the top wire. A dependent voltage source, 25i sub phi, is located on the third arm of the top wire, while a rightward current, i sub phi, flows through the fourth arm of the top wire. The circuit is powered by a 24V independent voltage source with reverse polarity, located on the left wire.

The circuit shows a horizontal cross wire connecting two nodes on the left and right wires, and a vertical cross wire connecting two nodes on the horizontal cross wire and bottom wire. An independent current source with an upward current, i sub g, is located on the vertical cross wire. The horizontal cross wire carries a 25 ohm resistor on its left arm and a 25 H inductor on its right arm. A 5 ohm resistor is located on the bottom arm of the left wire, while a 20 mF capacitor with a downward current i sub phi is located on the bottom arm of the right wire. The top wire carries a dependent voltage source, 20i sub phi, with a rightward current i sub o.

The circuit shows a small rectangular circuit located at the bottom right corner, between two nodes on the right wire and bottom wire. Voltage between these nodes is marked as v2. A 25 microF capacitor and a 15 microF capacitor are located on the left and right wires of this small circuit, respectively. A 10 microF capacitor with voltage v1 is located on the right wire of the bigger circuit, above the smaller circuit. The top wire carries a 2.5k ohm resistor, with a rightward current i1, and a switch, t = 0. The circuit is powered by a 25V independent voltage source, located on the left wire.

The circuit shows a small rectangular circuit located on the bottom wire, between two nodes on the bottom wire and one node on a vertical cross wire which connects to a switch on the top wire. Voltage between the top and bottom wires of the small circuit is marked as v2. A 16 microF capacitor and a 24 microF capacitor are located on the left and right wires of this small circuit, respectively. The cross wire carries a 10 microF capacitor and switch t = 0, which connects to nodes a and b on the top wire. A 50k ohm resistor is located on the left arm of the top wire. The right wire carries a 1.25 mH inductor and a 25 ohm resistor with voltage v1. A 450V independent voltage source is located on the left wire.

A circuit shows a horizontal cross wire connecting two nodes on the left and right wires, and a vertical cross wire connecting two nodes on the horizontal cross wire and bottom wire. A 100 mF capacitor with voltage v sub c is located on the vertical cross wire. The horizontal cross wire carries a 10 ohm resistor with a rightward current i sub b on its left arm, and a 100 mF capacitor with voltage v sub b on its right arm. A 10 ohm resistor is located on the bottom arm of the right wire, while a 100 mF capacitor with voltage v sub a, and a rightward current i sub a, is located on the top wire. The circuit is powered by a 9u of tA independent current source with an upward current, located on the bottom arm of the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 5 H inductor with a downward current i sub L and voltage v sub o. A 100 ohm resistor with a downward current i sub o is located on the right wire. The circuit is powered by an independent current source with an upward current, 50t e raised to minus 50t u of t A, located on the left wire.

The circuit shows a vertical cross wire connecting a pair of nodes on the top and bottom wires, and carrying a 10 H inductor. The top wire carries switch t = 0 and a 1k ohm resistor on its left arm, and another switch t = 0 and a 1k ohm resistor on its right arm. Node a is located in between the switch and the resistor on the right arm of the top wire, while node b is located on the right arm of the bottom wire. A 2 microF capacitor with a downward current i of t is located on the right wire. The circuit is powered by a 40V independent voltage source, located on the left wire.

The drawing shows two circuits with two inductors facing each other. The right wire on the left circuit carries a 15 H inductor with a dot terminal on its top left, while the left wire on the right circuit carries a 20 H inductor with a dot terminal on its bottom right. Mutual inductance between these two coils is marked as 10 H. The right circuit carries a 40 ohm resistor on its top wire and a 160 ohm resistor with voltage v sub o on its right wire. The left circuit carries a switch t = 0 and a 100 ohm resistor on its top wire, and a 180V independent voltage source on its left wire.

The drawing shows two circuits with two inductors facing each other. The right wire on the left circuit carries an 8 H inductor with a 15A downward current and a dot terminal located on its top left, while the left wire on the right circuit carries an 18 H inductor with a 10A downward current and a dot terminal located on its top right. Mutual inductance between these two coils is marked as 6 H. The right circuit carries a 270 ohm resistor on its right wire, while the left circuit carries a 120 ohm resistor on its left wire. Mesh currents in the left and right circuits are marked as i1 and i2, respectively.

The drawing shows two circuits with two inductors facing each other. The right wire on the left circuit carries a 3 H inductor with a dot terminal on its top left, while the left wire on the right circuit carries a 15 H inductor with a dot terminal on its bottom right. Mutual inductance between these two coils is marked as 3 H.

The left circuit carries a 120 ohm resistor with a rightward current i1 and switch 1 with t = 0 on its top wire, and a 24V independent voltage source on its left wire. The right circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 360 ohm resistor. Switch 2 with t = 0 is located to the left of the cross wire, connecting to wire a and wire b on the cross wire. A 10 ohm resistor is located on the right arm of the top wire, while a 20V independent voltage source is located on the right wire.

The circuit shows a vertical cross wire connecting a pair of nodes on the top and bottom wires, and carrying a 20 ohm resistor and a switch t = 0 at its top. The switch connects to wire a on the top wire and wire b on the cross wire. A 5 ohm resistor is located on the left arm of the top wire, while a rightward current i sub o flows through its right arm. A 90V independent voltage source is located on the left wire.

The right wire carries a 3 H inductor on the top and a 2 H inductor at its bottom, with dot terminals located to the top right of these coils. Mutual inductance between the coils is marked as 1 H. An additional wire is drawn across the 2 H inductor at the bottom of the right wire. A 10 ohm resistor is located on this additional wire.

The drawing shows two circuits with two inductors facing each other. The right wire on the left circuit carries a 0.8 H inductor with a dot terminal on its top left, while the left wire on the right circuit carries a 1.6 H inductor with a dot terminal on its top right. Mutual inductance between these two coils is marked as 0.8 H.

The left circuit carries a 10 mF capacitor and a 4 ohm resistor on its top wire, and a 48V independent voltage source on its left wire. An additional wire is drawn across nodes on either side of the capacitor on the top wire, and switch t = 0 is located on this additional wire. The right circuit carries a 20 ohm resistor with voltage v sub o on its right wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a 12.5 mF capacitor with voltage v sub o marked across it, while the right cross wire carries a 1.5u of tA independent current source with an upward current. The top wire carries a 10 ohm resistor on its left arm and a 10 H inductor on its middle arm. A 20 ohm resistor is located on the right wire, while a 60u of tV independent voltage source is located on the left wire.

The circuit shows an op amp with a positive supply voltage of 4V and a negative supply voltage of minus 4V. Two series capacitors with capacitance of 250 nF each, and a 100k ohm resistor in parallel, connect output terminal with inverting input terminal. The output terminal is grounded with the common node, with voltage between them marked as v sub o. The non-inverting input terminal is similarly grounded with the common node. The inverting input terminal is grounded with the common node in series with two 200k ohm resistors and in parallel with a 500nF capacitor. Voltage between the common node and the inverting input terminal is marked as 0.5u of tV.

The circuit shows an op amp with a positive supply voltage of 5V and a negative supply voltage of minus 5V. A 100 nF capacitor C2 and a 1k ohm resistor R2 are connected in parallel between the output terminal and the inverting input terminal. The output terminal is grounded with the common node, with voltage between them marked as v sub o. The non-inverting input terminal is similarly grounded with the common node. The inverting input terminal is grounded with the common node in series with a 400 ohm resistor R1 and a 500 nF capacitor C1. Voltage between the common node and the inverting input terminal is marked as v sub g.

The circuit shows an op amp with a positive supply voltage of 5V and a negative supply voltage of minus 5V. A 10 mF capacitor and a 5 ohm resistor are connected in parallel between the output terminal and the inverting input terminal. The output terminal is grounded with the common node, in series with a 10 ohm resistor, with voltage between them marked as v sub o. An independent voltage source, v sub g1, connects the inverting input terminal with the common node, in series with a 20 ohm resistor. Another independent voltage source, v sub g2, connects the non-inverting input terminal with the common node, in series with a 5 ohm resistor and in parallel with a 20 mF capacitor.

The circuit shows an op am in which two parallel wires connect the output terminal with the inverting input terminal. An outer wire carries two 1 F capacitors, while the inner wire carries a 1.25 ohm resistor. The output terminal is grounded with the common node, with voltage between them marked as v sub o of t. The non-inverting input terminal is also grounded with the common node. An independent voltage source, v sub g, connects the common node with the inverting input terminal, in series with a 0.8 ohm resistor.

The circuit on the left shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a resistor R. A capacitor C is located on the left arm of the top wire. The left and right wires are removed and are replaced by pairs of open terminals on the top and bottom wires on either side. The voltage across the terminals on the left is marked as v sub i, while the voltage across the terminals on the right is marked as v sub o.

The circuit on the right shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying an inductor L. A resistor R is located on the left arm of the top wire. The left and right wires are removed and are replaced by pairs of open terminals on the top and bottom wires on either side. Voltage across the terminals on the left is marked as v sub i, while the voltage across the terminals on the right is marked as v sub o.

The circuit on the left shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a capacitor C. A resistor R is located on the left arm of the top wire. The left and right wires are removed and are replaced by pairs of open terminals on the top and bottom wires on either side. Voltage across the terminals on the left is marked as v sub i, while the voltage across the terminals on the right is marked as v sub o.

The circuit on the right shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a resistor R. An inductor L is located on the left arm of the top wire. The left and right wires are removed and are replaced by pairs of open terminals on the top and bottom wires on either side. Voltage across the terminals on the left is marked as v sub i, while the voltage across the terminals on the right is marked as v sub o.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 100 nF capacitor. A 50k ohm resistor is located on the left arm of the top wire. The left and right wires are removed and are replaced by pairs of open terminals on the top and bottom wires on either side. Voltage across the terminals on the left is marked as v sub i, while the voltage across the terminals on right is marked as v sub o.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 50k ohm resistor. A 100 nF capacitor is located on the left arm of the top wire. The left and right wires are removed and are replaced by pairs of open terminals on the top and bottom wires on either side. Voltage across the terminals on the left is marked as v sub i, while the voltage across the terminals on the right is marked as v sub o.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 250 mH inductor. A 2k ohm resistor is located on the left arm of the top wire. The lLeft and right wires are removed and are replaced by pairs of open terminals on the top and bottom wires on either side. Voltage across the terminals the n left is marked as v sub i, while the oltage across the terminals on the ight is marked as v sub o.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 2k ohm resistor. A 250mH inductor is located on the left arm of the top wire. The left and right wires are removed and are replaced by pairs of open terminals on the top and bottom wires on either side. Voltage across the terminals on the left is marked as v sub i, while the voltage across the terminals on the right is marked as v sub o.

The circuit shows two vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a 10k ohm resistor, while the right cross wire carries a 250 nF capacitor. A 40k ohm resistor is located on the left arm of the top wire. The left and right wires are removed and are replaced by pairs of open terminals on the top and bottom wires on either side. Voltage across the terminals on the left is marked as v sub i, while the voltage across the terminals on the right is marked as v sub o.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a resistor R. The left arm of the top wire carries a capacitor C and an inductor L. The left and right wires are removed and two pairs of open terminals are located on the top and bottom wires on either side. Voltage at the terminals on the left is marked as v sub i, while the voltage at the terminals on the right is marked as v sub o.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a capacitor C while the right cross wire carries a resistor R. The left arm of the top wire carries an inductor L. The left and right wires are removed and two pairs of open terminals are located on the top and bottom wires on either side. Voltage at the terminals on the left is marked as v sub i, while the voltage at the terminals on the right is marked as v sub o.

The circuit shows two diagonal cross wires drawn across each other in an X shape between two pairs of nodes on the top and bottom wires. The cross wire drawn from the left node on the top wire to the right node on the bottom wire carries a 50nF capacitor on its top arm. The other cross wire carries a 20k ohm resistor on its bottom arm. The top and bottom wires between the two cross wires carry an 80k ohm resistor and a 500k ohm resistor, respectively. The left and right wires are removed and two pairs of open terminals are located on the top and bottom wires on either side. Voltage at the terminals on the left is marked as v sub i, while the voltage at the terminals on the right is marked as v sub o.

The circuit shows an op amp in which two parallel wires connect an output terminal with an inverting input terminal. The outer wire carries two 200nF capacitors while the inner wire carries a 5k ohm resistor. The output terminal is grounded with the common node, with voltage between them marked as v sub o. The non-inverting input terminal is also grounded with the common node. An independent voltage source, v sub g, connects the common node with the inverting input terminal, in series with a 1k ohm resistor. The op amp has a positive supply voltage of V sub CC and a negative supply voltage of minus V sub CC.

The circuit shows an op amp with a positive supply voltage of V sub CC and a negative supply voltage of minus V sub CC. A 125k ohm resistor, which is in series with a 1.6nF capacitor and in parallel with a 400pF capacitor, connects the output terminal with the inverting input terminal. The output terminal is grounded with the common node, with voltage between them marked as v sub o. The non-inverting input terminal is also grounded with the common node. The inverting terminal is grounded with the common node in series with a 250k ohm resistor, with voltage between the terminals marked as v sub g.

The circuit shows an op amp with a positive supply voltage of 5V and a negative supply voltage of minus 5V. A resistor, R2 = 40k ohm, which is in parallel with capacitor C2 = 25nF, connects the output terminal with the inverting input terminal. The output terminal is grounded with the common node, with voltage between them marked as v sub o. The non-inverting input terminal is also grounded with the common node. The inverting terminal is grounded with the common node in series with a 1k ohm resistor R1 and a 200nF capacitor C1, with voltage between the terminals marked as v sub g.

The circuit shows three parallel vertical cross wires connecting three pairs of nodes on the top and bottom wires. The left cross wire carries switch t = 0, the middle cross wire carries a 5k ohm resistor, and the right cross wire carries a 200mH inductor. A 100nF capacitor, with a downward current i sub o, is located on the right wire. The circuit is powered by an independent current source with an upward current, i sub g, located on the left wire.

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The circuit shows a 5H inductor and a 25H inductor on the left wire, with dot terminals located to their top left, and mutual inductance between them marked as 10H. An additional wire is drawn across two nodes located on either side of the 25H inductor at the bottom of the left wire. This additional wire carries an independent voltage source, v sub g. The right wire carries a 10k ohm resistor with a downward current i sub o, and a 62.5nF capacitor.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 1 ohm resistor. The left arm of the top wire carries a 1H inductor. The left and right wires are removed and two pairs of open terminals are located on the top and bottom wires on either side. Voltage at the terminals on the left is marked as v sub i, while the voltage at the terminals on the right is marked as v sub o.

The graph on the left plots time t on the x-axis and h of t on the y-axis. The graph shows an unbroken rectangular pulse moving in from the second quadrant and scaling to y = 1 between x = 0 and x = 40.

The graph on the right plots time t on x-axis and x of t on y-axis. The graph shows a rectangular pulse moving in from second quadrant and scaling to y = 10 between x = 0 and x = 40. The pulse is disrupted on the y-axis.

The graph plots time t on the x-axis and h of t on the y-axis. The graph shows an unbroken rectangular pulse moving in from the second quadrant and scaling to y = 4 between x = 0 and x = 10.

The graph plots time t on the x-axis and h of t on the y-axis. The graph shows a rectangular pulse moving in from the second quadrant and scaling to y = 40 between x = 0 and x = 1. The pulse is disrupted on the y-axis.

The graph plots time t in seconds on the x-axis and h of t in Volts on the y-axis. The graph shows a triangular pulse moving in from the second quadrant and peaking at (5, 2) while running between x = 0 and x = 10. Dotted perpendicular lines are drawn from the peak of the pulse to both axes.

The graph on the left plots time t on the x-axis and h of t on the y-axis. The graph shows a rectangular pulse moving in from the second quadrant and scaling to y = 25 between x = 0 and x = 10.

The graph on the right plots time t on the x-axis and x of t on the y-axis. The graph shows a rectangular pulse moving in from the second quadrant and scaling to y = 25 between x = 0 and x = 10.

The X-axis plots time t while the y-axis plots x of t. The graph shows a rectangular pulse moving in from the second quadrant and scaling to y = 12.5 between x = 0 and x = 20.

The X-axis plots time t while y-the axis plots h of t. The graph shows a rectangular pulse moving in from the second quadrant and scaling to y = 25 between x = 0 and x = 1.

The X-axis plots time t in ms while the y-axis plots voltage v sub i in Volts. The graph shows a rectangular pulse moving in from the second quadrant and scaling to y = 20 between x = 0 and x = 5.

The X-axis plots time t in seconds while y-the axis plots h of t in Volts. The graph shows a trapezoidal pulse moving in from the second quadrant and scaling to a peak value of y = 10 between x = 10 and x = 20, as the pulse moves from the origin to x = 30.

The X-axis plots lambda in seconds while the y-axis plots voltage v sub i of lambda in Volts. The graph shows a rectangular pulse between the second and first quadrants, scaling to y = 10 between x = minus 1 and x = 4.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 16H inductor. The left arm of the top wire carries a 40 ohm resistor and a 4H inductor. The left and right wires are removed and two pairs of open terminals are located on the top and bottom wires on either side. Voltage at the terminals on the left is marked as v sub i, while the voltage at the terminals on the right is marked as v sub o.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 100mF capacitor. The left arm of the top wire carries a 2 ohm resistor and a 100mH inductor. The left and right wires are removed and two pairs of open terminals are located on the top and bottom wires on either side. Voltage at the terminals on the left is marked as v sub i, while the voltage at the terminals on the right is marked as v sub o.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying an 80 ohm resistor. The left arm of the top wire carries a 4 ohm resistor, while the right wire carries a 20 ohm resistor with the voltage marked across as v sub o. The circuit is powered by an independent voltage source, v sub g, located on the left wire.

The X-axis plots time t in seconds while the y-axis plots voltage v sub g in Volts. The graph shows a rectangular pulse moving in from the second quadrant and scaling to y = 75 between x = 0 and x = 0.5. The pulse next moves in the fourth quadrant, scaling to y = minus 75 between x = 0.5 and x = 1.0.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 250mF capacitor. The left arm of the top wire carries a 5 ohm resistor, while the right wire carries a 20 ohm resistor with the voltage marked across as v sub o. The circuit is powered by an independent voltage source, v sub g, located on the left wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a 100k ohm resistor, while the right cross wire carries a 0.2 microF capacitor. The right wire is removed and two open terminals are located on the top and bottom wires to the right, with voltage between them marked as v sub o. A rightward current, i sub o, flows through the middle arm of the top wire. The circuit is powered by an independent current source, i sub g, with an upward current, located on the left wire.

The X-axis plots time t in ms while y-the axis plots current i sub g in microAmps. The graph shows a rectangular pulse moving in from the second quadrant and scaling to y = 50 between x = 0 and x = 100. The pulse next moves in the fourth quadrant, scaling to y = minus 50 between x = 100 and x = 200.

The X-axis plots time t in seconds while the y-axis plots voltage v sub i of t. The graph shows a sinusoidal pulse moving in from the second quadrant and scaling to a peak value of y = 20 at x = pie over 4, when the pulse moves between x = 0 and x = pie over 2.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 5 ohm resistor. The left arm of the top wire carries a 2.5H inductor. The left and right wires are removed and two pairs of open terminals are located on the top and bottom wires on either side. Voltage at the terminals on the left is marked as v sub i, while the voltage at the terminals on the right is marked as v sub o.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a 5k ohm resistor, while the right cross wire carries a 20k ohm resistor. A 0.4 microF capacitor is located on the middle arm of the top wire. The right wire is removed and two open terminals are marked on the top and bottom wires to the right, with voltage between them marked as v sub o. The circuit is powered by an independent current source, i sub g, with an upward current, located on the left wire.

The X-axis plots time t in ms, ranging from 0 to 6 in increments of 1, while the y-axis plots current i sub g in microAmps. The graph shows a rectangular pulse moving in from the second quadrant and scaling to y = 10 between x = 0 and x = 4. The pulse next moves in the fourth quadrant, scaling to y = minus 20 between x = 4 and x = 6

The circuit shows an op amp with a positive supply voltage of 6V and a negative supply voltage of minus 6V. A 25k ohm resistor, which is in parallel with an 8nF capacitor, connects the output terminal with the inverting input terminal. The output terminal is grounded with the common node, in series with a 15k ohm resistor, with voltage between them marked as v sub o. The inverting input terminal is also grounded with the common node, in series with a 5k ohm resistor. An independent voltage source, v sub g, connects the common node with the non-inverting input terminal, in series with an 80 ohm resistor and in parallel with a 10mH inductor.

The circuit shows an op amp with a positive supply voltage of 5V and a negative supply voltage of minus 5V. A 100k ohm resistor, which is in parallel with a 10nF capacitor, connects the output terminal with the inverting input terminal. The output terminal is grounded with the common node, in series with a 20k ohm resistor, with voltage between them marked as v sub o. The non-inverting input terminal is grounded with the common node. An independent voltage source, v sub g, connects the common node with the inverting input terminal, in series with a 10k ohm resistor and a 250nF capacitor.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 250 ohm resistor and a 50mH inductor. Voltage across the cross wire is marked as v sub o. The left arm of the top wire carries a 1000 ohm resistor, while the right arm carries switch t = 0. The right wire carries a 1 microF capacitor, with 1000V voltage marked across it.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a 10k ohm resistor, while the right cross wire carries a 20 microF capacitor. The middle arm of the top wire carries a 5 microF capacitor. The right wire is removed and two open terminals are marked on the top and bottom wires to the right, with voltage v sub o marked across them. The circuit is powered by a 10 delta of t microA independent current source with an upward current, located on the left wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a resistor R, while the right cross wire carries switch t = 0, with voltage v marked across it. The left wire carries an inductor L1 with two upward currents, i1 and rho, while the right wire carries another inductor, L2, with a downward current i2.

The drawing shows a circuit with a 1H inductor on its right wire, having a dot terminal to its top left, and another circuit with a 1H inductor on its left wire, having a dot terminal to its top right. Mutual inductance between these coils is marked as 0.5H. The left circuit carries a 20 delta of t V independent voltage source on its left wire, with a rightward current i1 marked on its top wire. The right circuit carries a 1 over 3F capacitor on its right wire, with voltage marked as v sub o, and carries a rightward current i2 on its top wire.

A circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a 50k ohm resistor, while the right cross wire carries a 1 microF capacitor. The middle arm of the top wire carries a 250 nF capacitor. The right wire is removed and two open terminals are marked on the top and bottom wires to the right, with voltage v sub o marked across them. The circuit is powered by a 10 delta of t microA independent current source, with an upward current, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 0.5 microF capacitor with voltage v1, and a 2.0 microF capacitor with voltage v2. Switch t = 0 is located on the top of the cross wire, connected to nodes a and b to the left and the right on the top wire. Node a leads to a 20k ohm resistor located on the left arm of the top wire. A rightward current, i of t, flows through the right arm of the top wire. The right wire carries a 1.6 microF capacitor, with voltage marked as v3. The circuit is powered by a 100V independent voltage source, located on the left wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a 0.8k ohm resistor, while the right cross wire carries an 8mH inductor and a 4k ohm resistor, with a downward current i1. Switch t = 0 is located on the middle arm of the top wire, while voltage between the top and bottom wires, toward the right wire, is marked as v of t. The right wire carries a 2mH inductor and a 16k ohm resistor, with a downward current i2 flowing through them. The circuit is powered by a 5A independent current source, with an upward current, located on the left wire.

The circuit shows a rectangular box in the middle representing a circuit. Two input wires drawn from the left of the box connect to an independent voltage source, V sub i of s. Two output wires drawn from the right of the box connect to two open terminals, with voltage V sub o of s marked between them.

The X-axis plots frequency omega while the y-axis plots mod H of j omega. The graph shows a rectangular pulse drawn on the x-axis, and a downward sloping line drawn in the fourth quadrant, starting from the y-axis. The pulse scales up to y = 1 between origin theta of j omega and x = omega sub c. The area enclosed inside the pulse is labeled as Passband, while the area to the right of the pulse is labeled as Stopband. The downward sloping line in the fourth quadrant is drawn starting from y = 0 degrees.

The X-axis plots frequency omega while the y-axis plots mod H of j omega. The y-axis marks 0 degree and theta of j omega sub c below origin theta of j omega. The graph shows a rectangular pulse drawn on the x-axis, and a downward sloping line drawn in the fourth quadrant. The pulse scales up to y = 1 between x = omega sub c and x = omega. The area enclosed inside the pulse is labeled as Passband, while the area to the left of the pulse, up to the y-axis, is labeled as Stopband. The downward sloping line in the fourth quadrant is drawn below the Passband.

The X-axis plots frequency omega while the y-axis plots mod H of j omega. The y-axis marks theta of j omega sub c2, 0 degree, and theta of j omega sub c1 below origin theta of j omega. The graph shows a rectangular pulse drawn on the x-axis, and a downward sloping line drawn in the fourth quadrant. The pulse scales up to y = 1 between x = omega sub c1 and x = omega sub c2. The area enclosed inside the pulse is labeled as Passband, while the areas to the left and right of the Passband are labeled as Stopband. The downward sloping line in the fourth quadrant is drawn below the Passband, starting approximately from omega sub c1, minus theta of j omega sub c1.

The X-axis plots frequency omega while the y-axis plots mod H of j omega. The y-axis marks theta of j omega sub c1, 0 degree, and theta of j omega sub c2 below origin theta of j omega. The graph shows two rectangular pulses drawn on the x-axis, and two downward sloping lines drawn in the fourth quadrant. The first pulse scales up to y = 1 between the origin and x = omega sub c1, while the second pulse scales to y = 1 between x = omega sub c2 and x = omega. The areas enclosed inside the two pulses are labeled as Passband, while the area between them is labeled as Stopband. In the fourth quadrant, one sloping line is drawn below the left Passband, starting from y = 0 degree on the y-axis, going up to (omega sub c1, minus theta of j omega sub c1). The second sloping line is drawn below the right passband, approximately between omega sub c2 and minus theta of j omega sub c2.

The circuit shows an inductor L on the top wire, a resistor R with voltage v sub o on the right wire, and an independent voltage source v sub i on the left wire.

The drawing shows a circuit where the top wire is short circuited because of very low inductance, L. A resistor R with voltage v sub o is shown on the right wire, while an independent voltage source, v sub i, is shown on the left wire.

A drawing shows a circuit where the top wire is open circuited because of very high inductance, L. A resistor R with voltage v sub o is shown on the right wire, while an independent voltage source, v sub i, is shown on the left wire.

The X-axis plots frequency omega while the y-axis plots mod H of j omega. The y-axis marks minus 90 degree and 0 degree below origin theta of j omega. The graph shows a line starting from y = 1.0, which moves in a downward slope, running close to and parallel to the x-axis. From point (omega sub c, 1.0), two dotted lines are drawn perpendicular to both the axes.

A second line is shown in the fourth quadrant, starting from y = 0 degree on the y-axis, moving in a downward slope, and running parallel to the x-axis toward the end.

The circuit shows an inductor sL on the top wire, a resistor R with voltage V sub o of s on the right wire, and an independent voltage source, V sub i of s, on the left wire.

The circuit shows a resistor R on the top wire, a capacitor C with voltage v sub o on the right wire, and an independent voltage source, v sub i, on the left wire.

The circuit shows a resistor R on the top wire, a capacitor 1 over sC with voltage V sub o of s on the right wire, and an independent voltage source, V sub i of s, on the left wire.

A drawing shows a circuit behaving as a series RC low-pass filter; and it also displays the transfer function and cutoff frequencies.

A circuit shows an inductor sL on the top wire, a resistor R with voltage V sub o on the right wire, and an independent voltage source, V sub i, on the left wire. The transfer function and cutoff frequency are shown in the following equation next to the circuit:

H of s = numerator R over L over denominator s plus R over L

omega sub c = R over L

The circuit shows a resistor R on the top wire, a capacitor 1 over sC with voltage V sub o on the right wire, and an independent voltage source, V sub i, on the left wire. The transfer function and cutoff frequency are shown in the following equation next to the circuit:

H of s = numerator 1 over RC over denominator s plus 1 over RC

omega sub c = 1 over RC

The circuit shows a capacitor C on the top wire, a resistor R with voltage v sub o on the right wire, and an independent voltage source, v sub i, on the left wire.

The drawing shows a circuit where the top wire is open circuited, with capacitance marked as C. A resistor R with voltage v sub o is shown on the right wire, while an independent voltage source, v sub i, is shown on the left wire.

The drawing shows a circuit where the top wire is short circuited, with capacitance marked as C. A resistor R with voltage v sub o is shown on the right wire, while an independent voltage source, v sub i, is shown on the left wire.

The X-axis plots frequency omega while the y-axis plots mod H of j omega. The y-axis marks 0 degree and minus 90 degree below origin theta of j omega. The graph shows a line starting from the origin and moving with an upward slope to reach a point where y = 1.0. The line then runs parallel to the x-axis. From point (omega sub c, 1), a dotted vertical line is drawn intersecting the x-axis at x = omega sub c, while another perpendicular dotted line is drawn from same point, tangential to the upper part of the curve.

A second line is shown in the fourth quadrant, starting from y = 90 degree on the y-axis, moving in a downward slope, and running parallel to the x-axis toward the end.

The circuit shows a capacitor 1 over sC on the top wire, a resistor R with voltage V sub o of s on the right wire, and an independent voltage source, V sub i of s, on the left wire.

The circuit shows a resistor R on the top wire, an inductor L with voltage v sub o on the right wire, and an independent voltage source, v sub i, on the left wire.

The circuit shows a resistor R on the top wire, an inductor sL with voltage V sub o of s on the right wire, and an independent voltage source, V sub i of s, on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying an inductor L. A resistor R is located on the left arm of the top wire, while another resistor, R sub L with voltage v sub o, is located on the right wire. The left wire carries an independent voltage source, v sub i.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying an inductor sL. A resistor R is located on the left arm of the top wire, while another resistor, R sub L with voltage V sub o of s, is located on the right wire. The left wire carries an independent voltage source, V sub i of s.

The X-axis plots frequency in kHz, ranging from 0 to 50 in increments of 10. The Y-axis plots voltage V, ranging from 0 to 1.0 in increments of 0.2. The graph shows two lines which start from the origin and move with an upward slope to reach a plateau, from where they run parallel to the x-axis. The line on the left, representing unloaded RL high-pass filter, reaches up to approximately 0.9 volts, from where it runs parallel to the x-axis. A point is marked near the top of this line, from where a perpendicular dotted line is drawn to the x-axis, marking frequency as f sub c, and another perpendicular line is drawn to the y-axis, marking voltage as 1 over square root 2.

The line on the right, representing loaded RL high-pass filter, reaches up to approximately 0.5 volts, from where it runs parallel to the x-axis. A point is marked near the top of this line, from where a perpendicular dotted line is drawn to the x-axis, marking frequency as f sub c-dash, and another perpendicular line is drawn to the y-axis, marking voltage as 1 over 2 multiplied by square root 2.

A drawing shows a circuit behaving as a series RL high-pass filter, and it also displays the transfer function and cutoff frequencies.

The circuit shows a capacitor 1 over sC on the top wire, a resistor R with voltage V sub o on the right wire, and an independent voltage source, V sub i, on the left wire. The transfer function and cutoff frequencies are shown in the following equation next to the circuit:

H of s = numerator s over denominator s plus 1 over RC

omega sub c = 1 over RC

The circuit shows a resistor R on the top wire, an inductor sL with voltage V sub o on the right wire, and an independent voltage source, V sub i, on the left wire. The transfer function and cutoff frequencies are shown in following the equation next to the circuit:

H of s = numerator s over denominator s plus R over L

omega sub c = R over L

The circuit shows an inductor L and a capacitor C on the top wire, a resistor R with voltage v sub o on the right wire, and an independent voltage source, v sub i, on the left wire.

The drawing shows a circuit where the left part of the top wire is short circuited, with inductance marked as L, while the right part is open circuited, with capacitance marked as C. A resistor R with voltage v sub o is shown on the right wire, while an independent voltage source, v sub i, is shown on the left wire.

The drawing shows a circuit where the left part of the top wire is open circuited, with inductance marked as L, while the right part is short circuited, with capacitance marked as C. A resistor R with voltage v sub o is shown on the right wire, while an independent voltage source, v sub i, is shown on the left wire.

The X-axis plots frequency omega while the y-axis plots mod H of j omega. The y-axis marks minus 90 degree, 0 degree, and 90 degree below origin theta of j omega. Another horizontal line, representing frequency omega, is drawn below the x-axis, where y = 90 degree. The graph shows a line starting from the origin and moving with an upward slope to reach a peak point where y = 1.0. The line then slopes down to move close to the x-axis, from where it runs parallel to the x-axis. From the peak-point of the curve, a dotted line is drawn perpendicular to the x-axis, marking coordinate as omega sub o. A horizontal dotted line is drawn from y = 1 over square root 2, intersecting the curve at its ascent and descent. From these two intersection points, dotted lines are drawn perpendicular to the x-axis, marking the coordinates as omega sub c1 and omega sub c2. The horizontal distance between these two dotted perpendicular lines is marked as Beta.

A second line is shown in the fourth quadrant, starting from y = 90 degree on the y-axis, moving in a downward slope, and running parallel to the x-axis toward the end. A point is marked on this line, from where a horizontal dotted line is drawn to the y-axis marking the coordinate as 0 degree, while a vertical dotted line is drawn to the horizontal line drawn at y = 90 degrees.

The circuit shows an inductor sL and a capacitor 1 over sC on the top wire, a resistor R with voltage V sub o of s on the right wire, and an independent voltage source, V sub i of s, on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a capacitor C. The left arm of the top wire carries a resistor R, while the right wire carries an inductor L with voltage v sub o. An independent voltage source, v sub i, is located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a capacitor 1 over sC. The left arm of the top wire carries a resistor R, while the right wire carries an inductor sL with voltage V sub o of s. An independent voltage source, V sub i of s, is located on the left wire. The impedance of the right arm of the top wire is marked as Z sub eq of s.

The circuit shows a resistor R sub i, an inductor L, and a capacitor C on the top wire, and another resistor, R with voltage v sub o, on the right wire. An independent voltage source, v sub i, is located on the left wire.

The circuit shows a resistor R sub i, an inductor sL, and a capacitor 1 over sC on the top wire, and another resistor, R with a voltage V sub o of s, on the right wire. An independent voltage source, V sub i of s, is located on the left wire.

The X-axis plots frequency in kHz, ranging from 0 to 20 in increments of 5. The Y-axis plots H|(jf)|, ranging from 0 to 1.0 in increments of 0.2. The graph shows two lines which start from the origin and move with an upward slope to a peak value, and then move down to reach a plateau, from where they run parallel to the x-axis. The line on the left, representing R sub i = 0, reaches a peak value of y = 1.0 and then moves down to approximately y = 0.5, from where it runs parallel to the x-axis. A point is marked near the top of this line, from where a perpendicular dotted line is drawn to the x-axis, marking frequency as f sub o, and another perpendicular line is drawn to the y-axis at y = 1.0. A horizontal dotted line is drawn from y = 1 over square root 2, intersecting the curve at its ascent and descent. From these two intersection points, dotted lines are drawn perpendicular to the x-axis, marking coordinates as 1 (approximately) and 10, respectively. The horizontal distance between these two dotted perpendicular lines is marked as Beta(R sub i = 0).

The line on the right, representing R sub i = R, reaches a peak value of y = 0.5 and then moves down to approximately y = 0.4, from where it runs parallel to the x-axis. A point is marked near the top of this line, from where a perpendicular dotted line is drawn to the x-axis, marking frequency as f sub o, and another perpendicular line is drawn to the y-axis at y = 0.5. A horizontal dotted line is drawn from y = 0.5 over square root 2, intersecting the curve at its ascent and descent. From these two intersection points, dotted lines are drawn perpendicular to the x-axis, marking approximate x-coordinates as 0.5 and 18, respectively. The horizontal distance between these two dotted perpendicular lines is marked as Beta(R sub i = R).

A drawing shows a parallel RLC circuit behaving as a bandpass filter, and also displays its transfer function, center frequency, and bandwidth.

The circuit shows an inductor sL and a capacitor 1 over sC on the top wire, a resistor R with voltage V sub o on the right wire, and an independent voltage source, V sub i, on the left wire. The transfer function, center frequency, and bandwidth are shown in the following equations below the circuit:

H of s = numerator(R over L)s over denominator s-squared (R over L)s plus 1 over LC

Omega sub o = square root of 1 over LC

Beta = R over L

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying capacitor 1 over sC. The left arm of the top wire carries a resistor R, while the right wire carries an inductor sL with voltage V sub o. The left wire carries an independent voltage source, V sub i. The transfer function, center frequency, and bandwidth are shown in the following equations below the circuit:

H of s = numerator s over RC over denominator s-squared plus s over RC plus 1 over LC

Omega sub o = square root of 1 over LC

Beta = 1 over RC

The circuit shows a resistor R on the top wire and an inductor L in series with capacitor C on the right wire, with voltage across the right wire marked as v sub o. The left wire carries an independent voltage source, v sub i.

The drawing shows a circuit where the bottom part of the right wire is open circuited, with capacitance marked as C, while the top part is short circuited, with inductance marked as L. Voltage across the right wire is marked as v sub o. A resistor R is shown on the top wire, while an independent voltage source, v sub i, is shown on the left wire.

The drawing shows a circuit where the bottom part of the right wire is short circuited, with capacitance marked as C, while the top part is open circuited, with inductance marked as L. Voltage across the right wire is marked as v sub o. A resistor R is shown on the top wire, while an independent voltage source, v sub i, is shown on the left wire.

The X-axis plots frequency omega while the y-axis plots mod H of j omega. The y-axis marks minus 90 degree, 0 degree, and 90 degree below origin theta of j omega, while the x-axis marks omega sub c1, omega sub o, and omega sub c2. The graph shows a line which starts from y = 1.0 and curves down to reach the x-axis at omega sub o. The line then moves with an upward slope to reach a plateau close enough to y = 1.0, and then runs parallel to the x-axis. A horizontal dotted line is drawn from y = 1 over square root 2, intersecting these two curves. From these two intersection points, dotted lines are drawn perpendicular to the x-axis, marking x-coordinates as omega sub c1 and omega sub c2. A rectangular pulse is drawn for the left curve, between x = 0 and x = omega sub c1, scaling up to y = 1.0, while a second rectangular pulse is drawn for the curve on the right, starting from x = omega sub c2, scaling up to y = 1.0.

Two downward sloping short lines are drawn in the fourth quadrant, corresponding to the two curves above the x-axis. The line on the left is drawn from the y-axis at y = 0 degree. The line on the right is drawn approximately from (omega sub o, 90 degree).

The circuit shows a resistor R on the top wire, and an inductor sL in series with a capacitor 1 over sC on the right wire, with voltage across the right wire marked as V sub o of s. The left wire carries an independent voltage source, V sub i of s.

A drawing shows a parallel RLC circuit behaving as a bandreject filter; and also displays its transfer function, center frequency, and bandwidth.

The circuit shows an inductor sL and a capacitor 1 over sC on the right wire, with voltage marked as V sub o. It shows a resistor R on the top wire and an independent voltage source, V sub i, on the left wire. The transfer function, center frequency, and bandwidth are shown in the following equations below the circuit:

H of s = numerator s-squared plus 1 over LC over denominator s-squared plus (R over L)s plus 1 over LC

Omega sub o = square root of 1 over LC

Beta = R over L

The circuit shows an independent voltage source V sub i on the left wire and a resistor R on the right wire, with voltage marked as V sub o. A small rectangular circuit is located between two nodes on the top wire, carrying an inductor sL on its top wire and a capacitor 1 over sC on its bottom wire. The transfer function, center frequency, and bandwidth are shown in the following equations below the circuit:

H of s = numerator s-squared plus 1 over LC over denominator s-squared plus s over RC plus 1 over LC

Omega sub o = square root of 1 over LC

Beta = 1 over RC

The frequencies for the rows are labeled as Low-frequency group, while the frequencies for the columns are labeled as High-frequency group. Four rows have the following frequencies marked, from top to bottom.

Row 1: 697 Hz

Row 2: 770 Hz

Row 3: 852 Hz

Row 4: 941 Hz

Three columns have the following frequencies marked, from left to right:

Column 1: 1209 Hz

Column 2: 1336 Hz

Column 3: 1477 Hz

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 127 ohm resistor. The left arm of the top wire carries a 10mH inductor. The left and right wires are removed, and are replaced by two pairs of open terminals on the top and bottom wires. Voltage between the terminals on the left is marked as v sub i, while the voltage between the terminals on the right is marked as v sub o.

The circuit shows an inductor L on the top wire, a resistor R with voltage v sub o on the right wire, and an independent voltage source, v sub i, on the left wire. A load resistor, R sub L, is located on an additional wire drawn across the right wire. The additional wire is shown in dotted lines. Output voltage at the load resistor is shown as H of s = V sub o of s over V sub i of s.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a resistor R. The left arm of the the top wire carries a resistor R sub l, in series with an inductor L. The left and right wires are removed, and are replaced by two pairs of open terminals on the top and bottom wires. Voltage between the terminals on the left is marked as v sub i, while the voltage between the terminals on the right is marked as v sub o.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 100nF capacitor. The left arm of the top wire carries a 1k ohm resistor. The left and right wires are removed, and are replaced by two pairs of open terminals on the top and bottom wires. Voltage between the terminals on the left is marked as v sub i, while voltage between the terminals on the right is marked as v sub o.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries capacitor C, while the right cross wire carries resistor R sub L. The left arm of the top wire carries resistor R. The left and right wires are removed, and are replaced by two pairs of open terminals on the top and bottom wires. Voltage between the terminals on the left is marked as v sub i, while the voltage between the terminals on the right is marked as v sub o.

The circuit shows a 150 ohm resistor on the top wire, a 10 mH inductor with voltage v sub o on the right wire, and an independent voltage source, v sub i, on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 50k ohm resistor. The left arm of the top wire carries a 5nF capacitor. The left and right wires are removed, and are replaced by two pairs of open terminals on the top and bottom wires. Voltage between the terminals on the left is marked as v sub i, while the voltage between the terminals on the right is marked as v sub o.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a resistor R. The left arm of the top wire carries a resistor R sub c, in series with a capacitor C. The left and right wires are removed, and are replaced by two pairs of open terminals on the top and bottom wires. Voltage between the terminals on the left is marked as v sub i, while the voltage between the terminals on the right is marked as v sub o.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 180 ohm resistor. The left arm of the top wire carries a 20 ohm resistor, a 40mH inductor, and a 40nF capacitor. The left and right wires are removed, and are replaced by two pairs of open terminals on the top and bottom wires. Voltage between the terminals on the left is marked as v sub i, while the voltage between the terminals on the right is marked as v sub o.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a 10mH inductor, while the right cross wire carries a 10nF capacitor. The left arm of the top wire carries an 8k ohm resistor. The left and right wires are removed, and are replaced by two pairs of open terminals on the top and bottom wires. Voltage between the terminals on the left is marked as v sub i, while the voltage between the terminals on the right is marked as v sub o.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a 200pF capacitor, while the right cross wire carries a 5mH inductor. The left arm of the top wire carries a 100k ohm resistor, while the right wire carries a 400k ohm resistor. The left wire is removed and is replaced by a pair of open terminals on the top and bottom wires. Voltage between the terminals on the left is marked as v sub i. Two nodes are located on the top and bottom wires, close to the right wire. Voltage between these nodes is marked as v sub o.

The diagram shows three blocks placed horizontally next to each other, and connected by rightward pointing arrows. From left to right, the blocks are labeled as Source, Filter, and Load, respectively.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a capacitor C, while the right cross wire carries an inductor L. The left arm of the top wire carries a resistor R, while the right wire carries a load resistor R sub L. The left wire is removed and is replaced by a pair of open terminals on the top and bottom wires. Voltage between the terminals on the left is marked as v sub i. Two nodes are located on the top and bottom wires, close to the right wire. Voltage between these nodes is marked as v sub o.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 50mH inductor in series with a 20microF capacitor. The left arm of the top wire carries a 25 ohm resistor. The left and right wires are removed and are replaced by two pairs of open terminals on the top and bottom wires. Voltage between the terminals on the left is marked as v sub i, while the voltage between the terminals on the right is marked as v sub o.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a resistor R. A small rectangular circuit is located between two nodes on the left arm of the top wire, carrying an inductor L and a capacitor C on its top and bottom wires, respectively. The left and right wires of the bigger circuit are removed, and are replaced by two pairs of open terminals on the top and bottom wires. Voltage between the terminals on the left is marked as v sub i, while the voltage between the terminals on the right is marked as v sub o.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 750 ohm resistor. A small rectangular circuit is located between two nodes on the left arm of the top wire, carrying a 50 microF inductor and a 20nF capacitor on its top and bottom wires, respectively. The left and right wires of the bigger circuit are removed, and are replaced by two pairs of open terminals on the top and bottom wires. Voltage between the terminals on the left is marked as v sub i, while the voltage between the terminals on the right is marked as v sub o.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a resistor R. A small rectangular circuit is located between two nodes on the left arm of the top wire, carrying a 500nF capacitor and an inductor L on its top and bottom wires, respectively. The left and right wires of the bigger circuit are removed and are replaced by two pairs of open terminals on the top and bottom wires. Voltage between the terminals on the left is marked as v sub i, while the voltage between the terminals on the right is marked as v sub o.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying an inductor L in series with capacitor C. The left arm of the top wire carries a resistor R, while the right wire carries a load resistor, R sub L. The left wire is removed, and is replaced by a pair of open terminals on the top and bottom wires. Voltage between the terminals on the left is marked as v sub i. Two nodes are located on the top and bottom wires, close to the right wire. Voltage between these nodes is marked as v sub o.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying an inductor L. The left arm of the top wire carries resistor R, in series with capacitor C. The left and right wires are removed, and are replaced by two pairs of open terminals on the top and bottom wires. Voltage between the terminals on the left is marked as v sub i, while the voltage between the terminals on the right is marked as v sub o.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a capacitor C. The left arm of the top wire carries a resistor R, in series with an inductor L. The left and right wires are removed, and are replaced by two pairs of open terminals on the top and bottom wires. Voltage between the terminals on the left is marked as v sub i, while the voltage between the terminals on the right is marked as v sub o.

The circuit shows an op amp where a resistor R2 in parallel with a capacitor C connects the output terminal with the inverting input terminal. The output terminal is grounded with the common node, and the output voltage between them is marked as v sub o. The non-inverting input terminal is similarly grounded with the common node. An independent voltage source, v sub i, in series with a resistor R1, connects the inverting input terminal with the common node.

The circuit shows an op amp where an impedance Z sub f connects the output terminal with the inverting input terminal. The output terminal is grounded with the common node, with output voltage between them marked as V sub o. The non-inverting input terminal is similarly grounded with the common node. An independent voltage source, V sub i, in series with an impedance Z sub i, connects the inverting input terminal with the common node.

The X-axis plots omega in rad per second, and marks 0.1, 0.5, 1.0, 5.0, and 10, while the y-axis plots mod H of j omega in dB, ranging from minus 20 to 10 in increments of 5. A grid of vertical and horizontal lines spaced unevenly is drawn all over this graph. The graph shows a line which starts at the origin and runs along the x-axis up to the first vertical grid line at x = 0.2. The line then curves down in the fourth quadrant and moves in a downward slope, ending at (10, minus 20) at the bottom left of the graph.

The circuit shows an op amp where a resistor R2 connects the output terminal with the inverting input terminal. The output terminal is grounded with the common node, with the output voltage between them marked as V sub o. The non-inverting input terminal is similarly grounded with the common node. An independent voltage source, V sub i, in series with a resistor R1 and a capacitor 1 over sC, connects the inverting input terminal with the common node.

The X-axis plots omega in rad per second, and marks 1, 5, 10, 50, 100, 500, 1000, 5000, and 10,000, while the y-axis plots mod H of j omega in dB, ranging from minus 40 to 30 in increments of 10. A grid of vertical and horizontal lines spaced unevenly is drawn all over this graph. The graph shows a line which starts on the y-axis between minus 30 and minus 40 in the fourth quadrant. The line moves with an upward slope to reach (2000, 20) in the first quadrant, and then runs parallel to the x-axis. A point is marked on the curve close to the peak, at approximately (600, 17). A dotted line is drawn perpendicular to the y-axis from this point.

The circuit shows an op amp where a 200k ohm resistor connects the output terminal with the inverting input terminal. The output terminal is grounded with the common node, with the output voltage between them marked as v sub o. The non-inverting input terminal is grounded with the common node. An independent voltage source, v sub i, in series with a 20k ohm resistor and a 0.1 microF capacitor, connects the inverting input terminal with the common node.

The circuit shows a 1H inductor in series with a 1F capacitor on the top wire, a 1 ohm resistor with voltage v sub s on the right wire, and an independent voltage source, v sub i, on the left wire.

The X-axis plots f in Hertz and marks 10, 50, 100, 500, 1000, 5000, and 10,000, while the y-axis plots mod H of jf in dB, ranging from minus 20 to 20 in increments of 5. A grid of vertical and horizontal lines spaced unevenly is drawn all over this graph. The graph shows a line which starts on the y-axis, close to y = 15, and moves parallel to the x-axis up to (200, 14). The line then curves downward and moves with a downward slope, ending at (10,000, minus 6).

The X-axis plots omega in rad per second, and marks 1, 5, 10, 50, 100, 500, 1000, 5000, and 10,000, while the y-axis plots mod H of j omega in dB, ranging from minus 40 to 30 in increments of 10. A grid of vertical and horizontal lines spaced unevenly is drawn all over this graph. The graph shows two curves, resembling inverted U shapes. The curve in the fourth quadrant starts at y = minus 20 on the y-axis, rises up with an upward slope to reach the x-axis at x = 100, and then moves with a downward slope to end at (minus 20, 10,000). The upward ascent of the curve on the left is labeled as High pass, while the descent of the curve on the right is labeled as Low pass. A dotted horizontal line is drawn at approximately y = 2, intersecting the curve at omega sub c1 on its ascent, and at omega sub c2 on its descent.

The second curve, representing Cascaded bandpass, starts at the origin and moves with an upward slope to reach (100, 20), and then moves with a downward slope to end on the x-axis at x = 10,000. A horizontal line is drawn tangential to this curve at y = 20 and is labeled as Gain.

The drawing shows three blocks placed side by side horizontally, with rightward arrows pointing from one block to another. The blocks represent a Low-pass filter, a High-pass filter, and an Inverting amplifier, from left to right respectively. Input voltage into the low-pass filter at the left is marked as v sub i, while the output voltage from the Inverting amplifier on the right is marked as v sub o.

The drawing shows three op amp circuits placed side by side horizontally and connected to each other. The non-inverting input terminals of the op amps in all three circuits are grounded with the common node.

The circuit on the left shows an op amp where a resistor R sub L, in parallel with a capacitor C sub L, connects the output terminal with the inverting input terminal. An independent voltage source, v sub i, connects the common node with the inverting input terminal, in series with a resistor R sub L. The output terminal connects to the inverting input terminal of the second op amp in the middle circuit, in series with a resistor R sub H and a capacitor C sub H.

The middle circuit shows an op amp where a resistor R sub H connects the output terminal with the inverting input terminal. The output terminal connects to the inverting input terminal of the third op amp in the right circuit, in series with a resistor R sub i.

The circuit on the right shows an op amp where a resistor R sub f connects the output terminal with the inverting input terminal. The output terminal is grounded with the common node, with voltage v sub o marked between them.

The drawing shows three op amp circuits placed side by side horizontally and connected to each other. The non-inverting input terminals of the op amps in all three circuits are grounded with the common node.

The circuit on the left shows an op amp where an 80 ohm resistor, in parallel with a 0.2 microF capacitor, connects the output terminal with the inverting input terminal. An independent voltage source, v sub i, connects the common node with the inverting input terminal, in series with an 80 ohm resistor. The output terminal connects to the inverting input terminal of the second op amp in the middle circuit, in series with a 7958 ohm resistor and a 0.2 microF capacitor.

The middle circuit shows an op amp where a 7958 ohm resistor connects the output terminal with the inverting input terminal. The output terminal connects to the inverting input terminal of the third op amp in the right circuit, in series with a 1k ohm resistor.

The circuit on the right shows an op amp where a 2k ohm resistor connects the output terminal with the inverting input terminal. The output terminal is grounded with the common node, with voltage v sub o marked between them.

The X-axis plots omega in rad per second, and marks 1, 5, 10, 50, 100, 500, 1000, 5000, and 10,000, while the y-axis plots mod H of j omega in dB, ranging from minus 40 to 30 in increments of 10. A grid of vertical and horizontal lines spaced unevenly is drawn all over this graph. The graph shows two curves in the fourth quadrant, that represent a High pass filter and a Low pass filter, and also shows a curve in the first quadrant, that represents Parallel band reject filter.

The line for the low-pass filter starts from the origin and curves down with a downward slope, moving straight to end at (1000, minus 40). The line for high-pass filter starts from (10, minus 40) and moves with an upward slope, curving horizontally at the top to end at x = 10,000 on the x-axis. A horizontal dotted line is drawn from approximately y = minus 3, marking the cut off frequency omega sub c1 on the low-pass filter curve at (10, minus 3), and omega sub c2 on the high-pass filter curve at (900, minus 3).

A horizontal line is drawn at y = 20 and labeled as Gain. The line for Parallel band reject filter starts on the y-axis at y = 20 and curves down, moving vertically down to end at (90, minus 20). The second part of the curve starts from (100, minus 20) and moves up vertically before curving away to the right with an upward slope and turning horizontal when it reaches (5,000, 20).

The drawing shows a block representing a Low-pass filter placed above another block representing a High-pass filter. The input voltage into both the blocks on the left is marked as v sub i. The blocks point to a third block on their right, labeled as Summing amplifier, which has an output voltage marked as v sub o.

The drawing shows three connected op amp circuits, where two are placed one above another, while the third is placed to their right. The non-inverting input terminals of the op amps in all three circuits are grounded with the common node.

The circuit on top shows an op amp where the resistor R sub L, in parallel with capacitor C sub L, connects the output terminal with the inverting input terminal. The circuit below shows an op amp where resistor R sub H connects the output terminal with the inverting input terminal. An independent voltage source, v sub i, connects the common node with the inverting input terminals of both the op amps, with resistor R sub L placed in series with the top op amp, and resistor R sub H and capacitor C sub H placed in series with the lower op amp. The output terminals of both the op amps, in series with separate resistors, both marked as R sub i, connect to the inverting input terminal of the third op amp on the right.

The circuit on the right shows an op amp where resistor R sub f connects the output terminal with the inverting input terminal. The output terminal is grounded with the common node, with voltage v sub o marked between them.

The X-axis plots omega in rad per second, and marks 10, 50, 100, 500, 1000, 5000, and 10,000, while the y-axis plots mod H of j omega in dB, ranging from minus 30 to 20 in increments of 5. A grid of vertical and horizontal lines spaced unevenly is drawn all over this graph. The graph shows a line which starts on the y-axis, approximately at y = 9, and curves down to touch a negative peak at (500, minus 10.5) in the fourth quadrant. The line then moves with an upward slope, curving away to the right and turning horizontal once it reaches (10,000, 9). A dotted horizontal line is drawn at y = 6.54, which marks the cut-off frequencies omega sub c1 for the descending line at (100, 6.54) and omega sub c2 for the ascending line at (2500, 6.54), approximately.

The drawing shows three connected op amp circuits, where two are placed one above another, while the third is placed to their right. The non-inverting input terminals of the op amps in all three circuits are grounded with the common node.

The circuit on top shows an op amp where a 20k ohm resistor, in parallel with a 0.5 microF capacitor, connects the output terminal with the inverting input terminal. The circuit below shows an op amp where a 1k ohm resistor connects the output terminal with the inverting input terminal. An independent voltage source, v sub i, connects the common node with the inverting input terminals of both the op amps, with a 20k ohm resistor placed in series with the top op amp, and a 1k ohm resistor and 0.5 microF capacitor placed in series with the lower op amp. The output terminals of both the op amps, in series with separate 1k ohm resistors, connect to the inverting input terminal of the third op amp on the right.

The circuit on the right shows an op amp where a 3k ohm resistor connects the output terminal with the inverting input terminal. The output terminal is grounded with the common node, with voltage v sub o marked between them.

The X-axis plots omega in rad per second and marks 0.1, 0.5, 1, 5, and 10, while the y-axis plots mod H of j omega in dB, ranging from minus 80 to 20 in increments of 10. A grid of vertical and horizontal lines spaced unevenly is drawn all over this graph. The graph shows four lines which start at the origin and curve down into the fourth quadrant. The line on top with least curvature represents the First order filter, which ends at (10, minus 20). The line below it represents the Second order filter and ends at (10, minus 40). The third line represents the Third order filter and ends at (10, minus 60). The line with greatest curvature represents the Fourth-order filter and ends at (10, minus 80).

A dotted line is drawn at y = minus 3, which intersects the four curves to mark their respective cut off frequencies.

The drawing shows three blocks placed side by side horizontally, with rightward arrows pointing from one block to another. The blocks represent multiple Low-pass filters from left to right, with dots between the second and third blocks representing several filters. The input voltage into the Low-pass filter at the left is marked as v sub i, while the output voltage from the Low-pass filter on the right is marked as v sub o.

The drawing shows three op amp circuits placed side by side horizontally and connected to each other. The non-inverting input terminals of the op amps in all three circuits are grounded with the common node.

The circuit on the left shows an op amp where resistor R2, in parallel with capacitor C, connects the output terminal with the inverting input terminal. An independent voltage source, v sub i, connects the common node with the inverting input terminal, in series with resistor R1. The output terminal connects to the inverting input terminal of the second op amp in the middle circuit, in series with resistor R1.

The second circuit shows another op amp, where resistor R2, in parallel with capacitor C, connects the output terminal with the inverting input terminal. Dots are displayed after the second circuit to indicate several op amp circuits.

The circuit on the right represents the last op am circuit, where resistor R1 is connected to the inverting input terminal from the output terminal of the previous circuit. Resistor R2, in parallel with capacitor C, connects the output terminal with the inverting input terminal. The output terminal is grounded with the common node, with voltage v sub o marked between them.

The drawing shows five op amp circuits arranged horizontally, such that the output terminal of each op amp connects to the inverting input terminal of the next op amp. The non-inverting input terminals of all the op amps are grounded with the common node.

For the first four op amp circuits, a 138.6 ohm resistor, in parallel with a 1 microF capacitor, connects the output terminal with the inverting input terminal. Plus, each output terminal is connected to the next op amp's inverting input terminal in series with a 138.46 ohm resistor.

The first circuit on the left shows an independent voltage source, v sub i, connecting the common node with the inverting input terminal of the op amp, in series with a 138.46 ohm resistor. The last circuit on the right shows a 138.46 ohm resistor connecting the output terminal with the inverting input terminal. The output terminal is grounded with the common node, with output voltage marked as v sub o.

The X-axis plots frequency in hertz, and marks 10, 50, 100, 500, 1000, 5000, and 10,000, while the y-axis plots mod H of jf in dB, ranging from minus 30 to 30 in increments of 10. A grid of vertical and horizontal lines spaced unevenly is drawn all over this graph. The graph shows a line which starts at y = 20 on the y-axis and runs horizontal up to (70, 20). The line then curves down in a steep downward slope, running almost vertical towards the end, and ends at (5,000, minus 30) in the fourth quadrant.

The diagram shows three blocks connected by rightward arrows. The input voltage at the first block on the left is V sub i, while the output voltage from the third block on the right is V sub o. The following values are depicted in each of the blocks from left to right:

1 over (s plus 1); 1 over (s-square plus 0.618s plus 1); 1 over (s-square plus 1.618s plus 1)

The circuit shows an op amp where the output terminal is short circuited with the inverting input terminal. An independent voltage source, V sub i, connects the common node with the non-inverting input terminal, in series with two resistors marked as R and in parallel with a 1 over sC2 capacitor. A node with voltage V sub a is located between the two resistors. An additional line connects this node with the short-circuited wire between the output and inverting input terminals, in series with a 1 over sC1 capacitor. The output terminal is grounded with the common node, with voltage between them marked as V sub o.

The drawing shows three op amp circuits placed horizontally and connected to each other. The circuit on the left shows an op amp where the output terminal is short circuited with the inverting input terminal. An independent voltage source, v sub i, connects the common node with the non-inverting input terminal, in series with two resistors marked as R, and in parallel with capacitor C sub 2a. A node is located between the two resistors. An additional line connects this node with the short-circuited wire between the output and inverting input terminals, in series with capacitor C sub 1a. The output terminal is connected to the non-inverting input terminal of the second op amp to its right, in series with two resistors marked as R, and in parallel with capacitor C sub 2b.

The second circuit in the middle shows an op amp where the output terminal is short circuited with the inverting input terminal. An additional wire connects this short circuited wire with the non-inverting input terminal in between two R resistors, in series with capacitor C sub 1b. The output terminal is connected to the inverting input terminal of the third op amp to its right, in series with resistor R1.

The third circuit on the right shows an op amp where resistor R sub f connects the output terminal with the inverting input terminal. The non-inverting input terminal is grounded with the common node. The output terminal is similarly grounded with the common node, with output voltage marked as v sub o.

The X-axis plots frequency in hertz, and marks 10, 50, 100, 500, 1000, 5000, and 10,000, while the y-axis plots mod H of jf in dB, ranging from minus 30 to 30 in increments of 10. A grid of vertical and horizontal lines spaced unevenly is drawn all over this graph. The graph shows a line representing Identical cascade, which starts at y = 20 on the y-axis and runs horizontal up to (100, 20). The line then curves down in a steep downward slope, running almost vertical toward the end, and ends at (5,000, minus 30) in the fourth quadrant.

A second curve, representing Butterworth, also starts at y = 20 on the y-axis and runs horizontally up to (200, 20). The line then curves down, intersecting the Identical cascade curve at a point marked as f sub c, and moves almost vertically downward, ending at (2,000, minus 30) in the fourth quadrant.

The X-axis plots log 10 omega while the y-axis plots mod H of j omega in dB. The graph is divided into three zones by two vertical dotted lines, labeled as Pass band, Transition band, and Stop band, from left to right. A line representing a low-pass filter is depicted in the graph, starting from the origin and moving along the x-axis in the pass-band zone. Once the line enters the transition-band zone, it moves diagonally down into the fourth quadrant and enters the stop band zone.

Two points are marked on the line in the transition band zone. Dotted perpendicular lines are drawn from these two points to the axes, marking x-coordinates as omega sub p and omega sub s, and y-coordinates as A sub p and A sub s.

The circuit shows an op amp where the output terminal is short circuited with the inverting input terminal. An independent voltage source, v sub i, connects the common node with the non-inverting input terminal, in series with two capacitors marked as C, and in parallel with resistor R2. A node is located between the two capacitors. An additional line connects this node with the short-circuited wire between the output and inverting input terminals, in series with resistor R1. The output terminal is grounded with the common node, with voltage between them marked as v sub o.

The drawing shows five op amp circuits placed horizontally and connected to each other. The first four circuits show op amps where the output terminal is short circuited with the inverting input terminal. The last circuit on the right has resistor R sub f connecting the output terminal with the inverting input terminal

The first circuit on the left shows an independent voltage source, v sub i, connecting the common node with the non-inverting input terminal, in series with two resistors marked as R, and in parallel with capacitor C sub 2a. A node is located between the two resistors. An additional line connects this node with the short-circuited wire between the output and the inverting input terminals, in series with capacitor C sub 1a. The output terminal is connected to the non-inverting input terminal of the second op amp to its right, in series with two resistors marked as R, and in parallel with capacitor C sub 2b.

The second circuit shows an op amp where an additional wire connects the short circuited output wire with the non-inverting input terminal in between two R resistors, in series with capacitor C sub 1b. The output terminal is connected to the non-inverting input terminal of the third op amp to its right, in series with two capacitors labeled as C, and in parallel with resistor R sub 2a.

The third circuit shows an op amp where an additional wire connects the short circuited output wire with the non-inverting input terminal in between two C capacitors, in series with resistor R sub 1a. The output terminal is connected to the non-inverting input terminal of the fourth op amp to its right, in series with two capacitors labeled as C, and in parallel with resistor R sub 2b.

The fourth circuit shows an op amp where an additional wire connects the short circuited output wire with the non-inverting input terminal in between two C capacitors, in series with resistor R sub 1b. The output terminal is connected to the inverting input terminal of the last op amp to its right, in series with resistor R.

The fifth circuit at the right end shows an op amp where resistor R sub f connects the output terminal with the inverting input terminal. The non-

inverting input terminal is grounded with the common node. The output terminal is similarly grounded with the common node, with output voltage marked as v sub o.

The circuit shows an op amp where the output terminal is connected with the inverting input terminal in parallel with resistor R3 and capacitor 1 over sC. An independent voltage source, V sub i, connects the common node with the inverting input terminal, in series with resistor R1 and capacitor 1 over sC, and in parallel with resistor R2. A node with voltage V sub a is marked at the juncture of resistors R1 and R2 and the capacitor 1 over sC. The non-inverting input terminal is grounded with the common node. The output terminal is also grounded with the common node, with voltage between them marked as V sub o.

The circuit shows an op amp where the output terminal is connected with the inverting input terminal in parallel with a 106.1k ohm resistor and a 0.01 microF capacitor. An independent voltage source, v sub i, connects the common node with the inverting input terminal, in series with a 26.5k ohm resistor and a 0.01 microF capacitor, and in parallel with a 268 ohm resistor. The non-inverting input terminal is grounded with the common node. The output terminal is also grounded with the common node, with voltage between them marked as v sub o.

The X-axis plots frequency in hertz, and marks 100, 500, 1000, 5000, 10,000, 50,000, and 100,000, while the y-axis plots mod H of jf in dB, ranging from minus 40 to 10 in increments of 5. A grid of vertical and horizontal lines spaced unevenly is drawn all over this graph. The graph shows a line which starts approximately at x = 150 on the x-axis and rises up with an upward slope to reach (6, 3000). The line then moves down in a steep downward slope and ends at (70,000, minus 40) in the fourth quadrant. The peak point of the curve is labeled as 6 dB (gain of 2).

The circuit shows two op amps connected to each other and to a voltage source through an RC circuit. In both op amps, the output terminal is short circuited with the inverting input terminal. The output terminal of the top op amp is connected to the non-inverting input terminal of the bottom op amp, in series with a resistor with resistance marked as (1 minus sigma)R. The non-inverting input terminal of the bottom op amp is grounded with the common node, in series with resistor sigma R. Voltage across sigma R resistor is marked as sigma V sub o. The voltage at the output terminal of the top op amp is marked as V sub o.

The RC circuit in the middle of this drawing shows two capacitors on its top wire, with capacitances of 1 over sC, and two resistors on its bottom wire, with resistances R. A node a is marked on the top wire between the capacitors, with its voltage marked as V sub a. Another point, b, is marked on the bottom wire between the resistors, with its voltage marked as V sub b. An additional wire connects points a and b, with resistor R over 2 and capacitor 1 over 2sC located in parallel on this wire.

The RC circuit has a node on its left wire, which connects to an independent voltage source, V sub i, which is grounded with the common node. Another node is located on its right wire, which connects to the non-inverting input terminal of the top op amp. A third node is located on its additional wire connecting nodes a and b. This node connects to the output terminal of the bottom op amp, with voltage marked as sigma V sub o.

The circuit shows two op amps connected to each other, and to a voltage source through an RC circuit. In both the op amps, the output terminal is short circuited with the inverting input terminal. The output terminal of the top op amp is connected to the non-inverting input terminal of the bottom op amp, in series with a 10 ohm resistor. The non-inverting input terminal of the bottom op amp is grounded with the common node, in series with a 190 ohm resistor. Voltage across the 190 ohm resistor is marked as sigma v sub o. The voltage at the output terminal of the top op amp is marked as v sub o.

The RC circuit in the middle of this drawing shows two 1 microF capacitors on its top wire, and two 200 ohm resistors on its bottom wire. A node is located between the capacitors on the top wire, while another node is located between the resistors on the bottom wire. An additional wire connects these nodes, carrying a 100 ohm resistor located in parallel with a 2 microF capacitor.

The RC circuit has a node on its left wire, which connects to an independent voltage source, v sub i, which is grounded with the common node. Another node is located on its right wire, which connects to the non-inverting input terminal of the top op amp. A third node is located on its additional wire, which connects to the output terminal of the bottom op amp, with voltage marked across as sigma v sub o.

The X-axis plots omega in rad per seconds, and marks 1000, 5000, 10,000, 50,000, and 100,000, while the y-axis plots mod H of j omega in dB, ranging from minus 20 to 10 in increments of 5. A grid of vertical and horizontal lines spaced unevenly is drawn all over this graph. The graph shows a line which starts at the origin and moves along the x-axis up to approximately x = 1500. The line then curves down and drops vertically to reach (5000, minus 20). From the same point, the line moves up vertically and curves away towards the right, reaching the x-axis at approximately x = 3000, from where it moves along the x-axis.

The circuit shows an op amp in which two resistors, marked as R1, in series with a potentiometer with variable resistor R2, connect the output terminal with the inverting input terminal. Capacitor C1 is connected in parallel across the nodes of the potentiometer. The input voltage into the op amp is marked as v sub s, while its output voltage at the output terminal is marked as v sub o. The non-inverting input terminal is grounded with the common node.

The X-axis plots omega while the y-axis plots mod V sub o over V sub s in dB. The graph shows three pairs of curves marked on either side of the x-axis, representing three different frequencies. Each pair, representing a particular frequency, shows two curves which are drawn from two points on the y-axis on either side of the x-axis, to a common point on the x-axis. The outermost pair shows a downward sloping curve drawn from y = dB1, and an upward sloping curve drawn from y = minus dB1. These two curves end at a common point on the x-axis.

The middle pair shows a downward sloping curve drawn from y = dB2, and an upward sloping curve drawn from y = minus dB2. Both the curves end at a common point on the x-axis, which is closer to the origin than the common point of the outermost curves.

The innermost pair shows a downward sloping curve drawn from y = dB1, and an upward sloping curve drawn from y = minus dB1, which end at a common point that is closest to the origin.

The circuit shows an op amp in which two resistors marked as R1, in series with a potentiometer with variable resistor R2, connect the output terminal with the inverting input terminal. Capacitor 1 over sC1 is connected in parallel across the nodes of the potentiometer. The variable resistor R2 is divided into two components, marked as (1 minus alpha) R2 and alpha R2.

The voltages at nodes a and b on either side of the potentiometer are marked respectively as V sub a and V sub b. Input voltage into the op amp is marked as V sub s while its output voltage at the output terminal is marked as V sub o. The non-inverting input terminal is grounded with the common node.

The circuit shows an op amp where an impedance, Z sub f, connects the output terminal with the inverting input terminal. The non-inverting input terminal is grounded with the common node. The inverting input terminal is grounded with the common node in series with impedance Z sub i, with an input voltage marked as V sub i. The output terminal is grounded with the common node, with voltage marked as V sub o.

The circuit shows an op amp where resistor R2, in parallel with capacitor C2, connects the output terminal with the inverting input terminal. The non-inverting input terminal is grounded with the common node. The inverting input terminal is grounded with the common node in series with a parallel RC connection consisting of resistor R1 and capacitor C1. Input voltage is marked as v sub i. The output terminal is grounded with the common node, with voltage marked as v sub o.

The circuit shows an op amp where resistor R2, in parallel with capacitor C2, connects the output terminal with the inverting input terminal. The non-inverting input terminal is grounded with the common node. The inverting input terminal is grounded with the common node in series with resistor R1 and capacitor C1. The input voltage is marked as v sub i. The output terminal is grounded with the common node, with voltage marked as v sub o.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying resistor R with resistance of 1 ohm. The left arm of the top wire carries capacitor C with capacitance of 1F.The lLeft and right wires are removed and are replaced by open terminals on the top and bottom wires. Input voltage at the left terminals is marked as v sub i, while the output voltage at the right terminals is marked as v sub o.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying inductor L with inductance of 1 H. The left arm of the top wire carries resistor R with resistance of 1 ohm. The left and right wires are removed and are replaced by open terminals on the top and bottom wires. Input voltage at the left terminals is marked as v sub i, while the output voltage at the right terminals is marked as v sub o.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying capacitor C with capacitance of 1 F. The left arm of the top wire carries resistor R with resistance of 1 ohm. The left and right wires are removed and are replaced by open terminals on the top and bottom wires. Input voltage at the left terminals is marked as v sub i, while the output voltage at the right terminals is marked as v sub o.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying resistor R with resistance of 1 ohm. The left arm of the top wire carries inductor L with inductance of 1 H. The left and right wires are removed and are replaced by open terminals on the top and bottom wires. Input voltage at the left terminals is marked as v sub i, while the output voltage at the right terminals is marked as v sub o.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying resistor R with resistance of 1 over Q ohm. The left arm of the top wire carries inductor L with inductance of 1 H, in series with a capacitor with capacitance of 1 F. The left and right wires are removed and are replaced by open terminals on the top and bottom wires. Input voltage at the left terminals is marked as v sub i, while the output voltage at the right terminals is marked as v sub o.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 1 H inductor in series with a 1 F capacitor. The left arm of the top wire carries a resistor with resistance of (1 over Q) ohm. The left and right wires are removed and are replaced by open terminals on the top and bottom wires. Input voltage at the left terminals is marked as v sub i, while the output voltage at the right terminals is marked as v sub o.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying an inductor with an inductance of Q H, in series with a capacitor with capacitance of (1 over Q) F. The left arm of the top wire carries a 1 ohm resistor. The left and right wires are removed and are replaced by open terminals on the top and bottom wires. Input voltage at the left terminals is marked as v sub i, while the output voltage at the right terminals is marked as v sub o.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a resistor with resistance of Q ohms. The left arm of the top wire carries a 1 H inductor connected in parallel with a 1 F capacitor. The left and right wires are removed and are replaced by open terminals on the top and bottom wires. Input voltage at the left terminals is marked as v sub i, while the output voltage at the right terminals is marked as v sub o.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 1 ohm resistor. The left arm of the top wire carries a (1 over Q) H inductor connected in parallel with a Q F capacitor. The left and right wires are removed and are replaced by open terminals on the top and bottom wires. Input voltage at the left terminals is marked as v sub i, while the output voltage at the right terminals is marked as v sub o.

The circuit shows an op amp where a 5 ohm resistor, in parallel with a 10 microF capacitor, connects the output terminal with the inverting input terminal. The non-inverting input terminal is grounded with the common node. The inverting terminal is grounded with the common node, in series with a 400 ohm resistor and a 50 microF capacitor. Input voltage is marked as v sub i. The output terminal is grounded with the common node, with output voltage marked as v sub o.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a capacitor with capacitance of 1 over sC. The left arm of the top wire carries resistor R. The left and right wires are removed and are replaced by open terminals on the top and bottom wires. Input voltage at the left terminals is marked as V sub i, while the output voltage at the right terminals is marked as V sub o.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a capacitor with capacitance of 1 over sC, while the right cross wire carries an inductor with inductance of sL. The left arm of the top wire carries resistor R1, while the middle arm carries resistor R2. The left and right wires are removed and are replaced by open terminals on the top and bottom wires. The Input voltage at the left terminals is marked as V sub i, while the output voltage at the right terminals is marked as V sub o.

The circuit shows an op amp where resistor R3, in series with capacitor 1 over sC and a parallel short circuit wire, connects the output terminal with the inverting input terminal. The non-inverting input terminal is grounded with the common node. The inverting input terminal is grounded with the common node, in series with resistor R1 and capacitor 1 over sC, and in parallel with resistor R2. Input voltage is marked as V sub i. Output voltage at the output terminal is marked as V sub o.

The circuit shows an op amp where resistor R2, in parallel with capacitor C1, connects the output terminal with the inverting input terminal. The non-inverting input terminal is grounded with the common node. The inverting input terminal is grounded with the common node, in series with two resistors, R1 and R3, and in parallel with capacitor C2. Input voltage is marked as v sub i. Output voltage at the output terminal is marked as v sub o. A downward arrow is marked at the common node to indicate that the op amp circuit is part of a bigger circuit.

The circuit shows two op amps connected to each other, with their respective output terminals short circuited with their inverting input terminals. The output terminal of the left op amp is connected to the non-inverting input terminal of the right op amp, in series with resistor R2 and in parallel with capacitor C2. Capacitor C1 connects the output terminal of the right op amp with the non-inverting input terminal of the left op amp. The output terminal of the right op amp is grounded with common node, with output voltage marked as v sub o. The non-inverting input terminal of the left op amp is also grounded with the common node, with input voltage marked as v sub i.

The circuit shows a horizontal wire drawn between two terminals. A small rectangular circuit is located between two nodes on this wire. The circuit shows capacitor 1 over sC1 on its top wire and two resistors on its bottom wire, with resistances of (1 minus alpha)R2 and alpha R2.

The circuit shows a horizontal wire drawn between two terminals. A small rectangular circuit is located between two nodes on this wire. The top wire of the circuit shows two capacitors separated by node x, with capacitances marked as 1 minus alpha over sC1 and alpha over sC1. The bottom wire shows two resistors separated by node y, with resistances marked as 1 minus alphaR2 and alpha R2.

The circuit shows an op amp where the output terminal is connected to the inverting input terminal by a resistor which is in parallel with two series RC circuits. The resistor has resistance marked as R4 plus 2R3. The RC circuit on the left has capacitor 1 minus alpha over sC1 on its top wire, and resistor 1 minus alphaR2 on its bottom wire. Resistor R1 is located to the left of this circuit. The RC circuit on the right has capacitor alpha over sC1 on its top wire, and resistor alpha R2 on its bottom wire. Resistor R1 is located to the right of this circuit. A node marked as (x, y) is located in between these two RC circuits, and connects to the inverting input terminal of the op amp.

Input voltage to the left of the left RC circuit is marked as Vs. The output voltage at the output terminal is marked as V sub o. The non-inverting input terminal is grounded with the common node.

The circuit shows a vertical cross wire which connects a node on the top wire with variable resistor R4 on the bottom wire. The resistor on the bottom wire is divided into two parts, marked as (1 minus Beta R4 and Beta R4). The left and right arms of the top wire carry two resistors with resistances marked as R1, while the left and right arms of the bottom wire carry two resistors with resistances marked as R3. The vertical cross wire carries resistor R2 on top and capacitor C2 at the bottom.

An op amp is located between the vertical cross wire and the right wire. The output node of the op amp connects to a node on the right wire, marking output voltage as v sub o. The inverting input terminal of the op amp connects to a node on the vertical cross wire, located between the resistor and capacitor. The non-inverting input terminal is grounded with the common node. Input voltage of the entire circuit is marked on a node at the midpoint of the left wire, as v sub s.

X-axis plots time t and marks minus T, T, and 2T, while y-axis plots f of t. Graph plots a non-sinusoidal waveform between minus T and 0, which shows a triangular pulse in first half, peaking at y = A above the x-axis, and also shows another triangular pulse in second half, peaking at y = B below the x-axis. This is followed by another similar waveform between 0 and T, between T and 2T, and so on.

A horizontal dotted line is drawn above the x-axis, intersecting the triangular waveform just below its peak value. In each waveform, the horizontal dotted line intersects with upward slanting line at a point from where a vertical dotted line is drawn to the x-axis, marking x-coordinate value. These values are marked at (t0 minus T), t0, (t0 plus T), (t0 plus 2T), and so on.

The X-axis plots time t and marks 0, T, and 2T, while the y-axis plots v of t and marks V sub m. The graph shows a non-sinusoidal waveform between 0 and T, in the form of an inverted U which peaks at V sub m. This is followed by another waveform between T and 2T, and so on.

The X-axis plots time t and marks 0, T over 2, and T, while the y-axis plots v of t and marks V sub m. The graph shows a non-sinusoidal waveform between 0 and T over 2, in the form of an inverted U which peaks at V sub m. This line then runs along the x-axis between T over 2 and T. It is followed by another similar waveform after T.

The X-axis plots time t and marks T, 2T, and 3T, while the y-axis plots voltage v of t. The graph shows a triangular pulse between 0 and T, which peaks at voltage V sub m, followed by another pulse between T and 2T with the same peak, and another pulse between 2T and 3T, and so on. Each pulse shows an upward sloping line which reaches the peak and then drops down in a very steep slope.

The X-axis plots time t and marks T and 2T, while the y-axis plots voltage v of t and marks V sub m and minus V sub m. The graph shows a square waveform between 0 and T, which contains a positive rectangular pulse peaking at voltage V sub m, followed by a negative rectangular pulse peaking at voltage minus V sub m. This is followed by another similar pulse between T and 2T, and so on.

The X-axis plots time t and marks T, while the y-axis plots voltage v of t and marks V sub m and minus V sub m. The graph shows a triangular waveform between 0 and T, which contains a positive triangular pulse peaking at voltage V sub m, followed by a negative triangular pulse peaking at voltage minus V sub m. This is followed by another similar pulse after T.

The X-axis plots time t and marks T and 2T, while the y-axis plots voltage v of t and marks V sub m. The graph shows a rectangular waveform between 0 and T, which contains a positive rectangular pulse peaking at voltage V sub m, followed by a line which runs along the x-axis up to x = T. This is followed by another similar pulse between T and 2T, and so on.

The first drawing on the top shows multiple sinusoidal waves with different frequencies summed up. A text next to the drawing reads: The sum of sinusoids whose frequencies are integer multiples of the fundamental frequency.

The second drawing shows a square wave which the sum of the sinusoidal waves shown. A text next to it reads: The sum is a square wave at the fundamental frequency.

The square wave points to a High-Q bandpass filter below, with an accompanying text which reads: The center frequency of the filter is the fundamental frequency.

The bandpass filter points to a single sinusoidal wave depicted below it, along with a text that reads: The filter extracts the sinusoid at the fundamental frequency from the square wave.

The X-axis plots time t and marks minus T, T, and 2T, while the y-axis plots voltage v of t. The graph shows a triangular pulse between minus T and 0 which peaks at voltage V sub m, followed by another pulse between 0 and T with same peak, followed by another pulse between T and 2T with the same peak, and so on. Each pulse shows an upward sloping line which reaches a peak and then drops down almost vertically.

The X-axis plots time t and marks T over 3, 2T over 3, T, 4T over 3, 5T over 3, and 2T. The Y-axis plots voltage v of t and marks V sub m over 3, and V sub m. The graph shows a waveform between 0 and T, which consists of a rectangular pulse followed by a horizontal line. The rectangular pulse is plotted between x-coordinates 0 and 2T over 3, and y-coordinates V sub m over 3 and V sub m. The horizontal line is drawn at V sub m over 3, between x-coordinates 2T over 3 and T. A similar waveform is drawn between T and 2T, between 2T and 3T, and so on.

The X-axis plots time t and marks minus T, 0, and T. The Y-axis plots voltage f of t. The graph shows an M-shaped waveform drawn with x = minus T as its central axis. The waveform which extends slightly below the x-axis shows a v shape on top, exactly above the x-coordinate value of minus T. This is followed by a similar waveform, which is drawn around x = 0, and another waveform drawn around x = T, and so on.

The X-axis plots time t and marks minus T, minus T over 2, 0, T over 2, and T. The Y-axis plots f of t and marks A and minus A. The graph shows a near-triangular waveform between minus T and 0, which contains a positive pulse peaking at A and a negative pulse peaking at minus A. The positive pulse runs from minus T to minus T over 2, while the negative pulse runs from minus T over 2 to 0. At peak value, the waveform runs parallel to the x-axis for a short distance. A similar waveform is drawn from 0 to T, and so on.

The X-axis plots time t and marks minus T, minus T over 2, 0, T over 2, and T. The Y-axis plots f of t and marks A and minus A. The graph shows a triangular waveform between minus T and 0, which contains a positive pulse which peaks at y = A, and a negative pulse which peaks at y = minus A. The x-coordinate of the positive pulse is located just to the right of minus T, while the x-coordinate of the negative pulse is located just to the right of minus T over 2. A similar waveform is drawn from 0 to T, and so on.

The X-axis plots time t and marks minus T, minus T over 2, 0, T over 2, and T. The Y-axis plots f of t and marks A and minus A. The graph shows a triangular waveform between minus T and 0, which contains a positive pulse which peaks at y = A, and a negative pulse which peaks at y = minus A. The x-coordinate of the positive pulse is located exactly at minus T while the x-coordinate of the negative pulse is located at minus T over 2. A similar waveform is drawn from 0 to T, and so on.

The X-axis plots time t and marks minus T, minus T over 2, 0, T over 2, and T. The Y-axis plots f of t and marks A and minus A. The graph shows a triangular waveform between minus T and 0, which contains a positive pulse which peaks at y = A and a negative pulse which peaks at y = minus A. The x-coordinate of the positive pulse is located at the midpoint between minus T and minus T over 2, while the x-coordinate of the negative pulse is located at the midpoint of minus T over 2 and 0. A similar waveform is drawn from 0 to T, and so on.

The X-axis plots time t and marks 0, T over 4, T over 2, 3T over 4, and T. The Y-axis plots f of t and marks minus A and A. The graph shows a waveform between 0 and T, which has a near-square shaped pulse between 0 and T over 2 and a symmetrical pulse in the negative half-cycle between T over 2 and T. The pulse reaches a maximum value of A in the positive half-cycle and displays a U shaped depression at its midpoint, where x value is T over 4. Similarly, the pulse reaches a maximum value of minus A in the negative half-cycle and displays a U shaped depression at its midpoint, where x value is 3T over 4. The waveform shows a similar pattern in the second quadrant to the left of the origin, and also continues in a similar pattern after T in the first quadrant.

The X-axis plots time t and marks 0, T over 4, T over 2, 3T over 4, and T. The Y-axis plots f of t and marks minus A and A. The graph shows a waveform between 0 and T, which has a stepping-up pulse between 0 and T over 2 and a symmetrical pulse in the negative half-cycle between T over 2 and T. The pulse in the positive half-cycle shows a small rectangle from 0 to a little over its midpoint at T over 4, and then steps up to a bigger rectangle up to T over 2, reaching a peak value of A. A symmetrical pattern is displayed in the negative half-cycle, between T over 2 and T, with a peak value of minus A. The waveform shows a similar pattern in the second quadrant to the left of the origin, and also continues in a similar pattern after T in the first quadrant.

The X-axis plots time t and marks minus T over 2, 0, T over 2, T, 3T over 2, 2T, 5T over 2, and 3T. The Y-axis plots i of t and marks minus I sub m and I sub m. The graph shows a waveform between minus T over 2 and T over 2, with two triangular pulses on either sides of the x-axis. The first triangular pulse is shown between minus T over 2 and 0, which peaks at minus I sub m below the x-axis. It is followed by another triangular pulse between 0 and T over 2, which peaks at I sub m in the first quadrant. Similar waveforms are drawn between T over 2 and 3T over 2, and so on.

The X-axis plots time t and marks 0, T over 6, T over 3, T over 2, 2T over 3, 5T over 6, and T. The Y-axis plots v sub g of t and marks minus V sub m and V sub m. The graph plots a waveform between 0 and T, which consists of a trapezoidal pulse above the x-axis between 0 and T over 2, and another trapezoidal pulse below the x-axis between T over 2 and T. The positive trapezoidal pulse peaks at y = V sub m between x values of T over 6 and T over 3. The negative trapezoidal pulse peaks at y = minus V sub m between x values of 2T over 3 and 5T over 6. The waveform continues from x = T onwards.

The X-axis plots time t, ranging from 0 to 2T in increments of T over 4. The Y-axis plots v of t and marks V sub m. The graph shows a waveform between 0 and T, consisting of a rectangular pulse drawn from 0 to T over 4, peaking at V sub m, followed by a horizontal line drawn along the x-axis from T over 4 to T.

A similar waveform is drawn between T and 2T, and so on.

The X-axis plots time t and marks T over 3, 2T over 3, T, 4T over 3, 5T over 3, and 2T. The Y-axis plots voltage and marks V sub m over 3, and V sub m. The graph shows a waveform between 0 and T, which consists of a rectangular pulse followed by a horizontal line. The rectangular pulse is plotted between x-coordinates 0 and 2T over 3, and y-coordinates V sub m over 3 and V sub m. The horizontal line is drawn at V sub m over 3, between x-coordinates 2T over 3 and T. A similar waveform is drawn between T and 2T, and so on.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying capacitor C. The left arm of the top wire carries resistor R. The left wire carries an independent voltage source, v sub g. The right wire is removed and is replaced by two open terminals located on the top and bottom wires. Output voltage between the two terminals is marked as v sub o.

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The X-axis plots time t and marks 0, T over 2, T, 3T over 2, and 2T, while the y-axis plots v sub o and marks V1 in the fourth quadrant and V2 in the first quadrant. The graph shows a near-triangular waveform that starts on the y-axis at y = V1 and moves with an upward slope, crossing the x-axis and reaching a peak value of (T over 2, V2). The line then moves in a downward slope to reach a negative peak value of (T, minus V1). This waveform continues in same pattern. Dotted lines are drawn extending the upward rise of the pulse at positive peaks, and are marked as Towards plus V sub m. Similar dotted lines are drawn extending the downward movement of the pulse at negative peaks, and are marked as Towards minus V sub m.

The X-axis plots time t and marks 0, T over 2, T, 3T over 2, and 2T, while the y-axis plots v sub o and marks minus V sub m and V1 in the fourth quadrant, and V2 and V sub m in the first quadrant. The graph shows a rectangular waveform in dotted lines, peaking at V sub m and minus V sub m between 0 and T. This waveform continues in a similar pattern beyond T.

Two near-triangular waveforms are depicted, representing output voltage for two different capacitor sizes. A solid black line is drawn starting on the y-axis at y = V1, which moves with an upward slope, crosses the x-axis, and reaches a peak value of (T over 2, V2). The line then moves in a downward slope to reach a negative peak value of (T, minus V1). This waveform continues in the same pattern and is marked as Large C. The second waveform shows a solid colored line which moves similarly but reaches a higher peak on both sides at V sub m and minus V sub m. This line is marked as Small C.

The circuit shows a 20 ohm resistor on the right wire with voltage marked as v sub o, and an independent voltage source, v sub g, on the left wire. A small rectangular circuit is marked between two nodes on the top wire. The top wire of this circuit carries an 8 mH inductor, while its bottom wire carries a 500 microF capacitor.

The X-axis plots time t and marks 0, T over 2, and T, while the y-axis plots v sub i and marks minus V sub m and V sub m. The graph shows a triangular waveform with a positive pulse which peaks at V sub m at the origin. It is followed by a symmetrical negative pulse, which peaks at minus V sub m at x = T over 2. The triangular waveform continues onwards from T over 2.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 100 nF capacitor. The left arm of the top wire carries a 100 k ohm resistor. The left and right wires are removed and are replaced by two pairs of open terminals on the top and bottom wires. The input voltage at the left terminals is marked as v sub i, while the output voltage at the right terminals is marked as v sub o.

The X-axis plots time t and marks 0, T over 2, and T, while the y-axis plots v sub g and marks minus V sub m and V sub m. The graph shows a square waveform with a positive pulse which has its midpoint at the origin and peaks at V sub m. It is followed by a symmetrical negative pulse, which has its midpoint at T over 2 and peaks at minus V sub m. The square waveform continues onwards from T over 2.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a 20 nF capacitor, while the right cross wire carries a 20 mH inductor. The left arm of the top wire carries a 10 k ohm resistor. The left and right wires are removed and are replaced by two pairs of open terminals on the top and bottom wires. Input voltage at the left terminals is marked as v sub g, while the output voltage at the right terminals is marked as v sub o.

The circuit shows a 1 H resistor in series with a 1 F capacitor on its top wire, a 2 ohm resistor with voltage v sub R on its right wire, and an independent voltage source, v sub g, on its left wire.

The X-axis plots time t and marks minus tau over 2, 0, tau over 2, T minus tau over 2, T, and T plus tau over 2. The Y-axis plots voltage v of t and marks V sub m. The graph shows a waveform which consists of a horizontal line along the x-axis that rises up in a rectangular pulse between minus tau over 2 and tau over 2 on either side of the origin, peaking at V sub m. The waveform continues along the x-axis, rising up again between T minus tau over 2 and T plus tau over 2 on either side of x = T.

The X-axis plots time t in ms, ranging from 0 to 44 in increments of 4, while the y-axis plots current i in Amps and marks minus 8, minus 2, 2, and 8. The graph shows a waveform between 0 and 16, consisting of a short square pulse between x values of 0 and 4 that peaks at y = 2. The pulse then steps up to y = 8 between x values of 4 and 12, and then steps down back to y = 2 between x-values of 12 and 16. This pattern repeats with a negative pulse between x-values of 16 and 32, and continues further.

The X-axis plots x, ranging from minus 2 phi to 2 phi in increments of 0.5 phi. The Y-axis plots mod of sin x over x, ranging from 0 to 1.0 in increments of 0.2. The graph shows an inverted U shaped curve between x values of minus 2 phi and minus phi, that peaks at y = 0.2. The waveform then shows a bigger inverted U shaped curve between x values of minus phi and phi, that peaks at y = 1.0, followed by a smaller curve between x values of phi and 2 phi, that peaks at x = 0.2.

In the first graph, the x-axis plots frequency f in hertz, ranging from minus 1000 to 1000 in increments of 200. The Y-axis plots mod of C sub n, ranging from 0 to 1.0 in increments of 0.2. The graph shows an inverted U shaped formation of coordinate points between x values of minus 1000 and minus 500, that peaks at y = 0.2. The waveform then shows a bigger inverted U shaped formation of coordinate points between x values of minus 500 and 500, that peaks at y = 1.0, followed by a smaller formation between x values of 500 and 1000, that peaks at x = 0.2. Vertical lines are drawn from each of these points to the x-axis.

In the second graph shown below the first graph, the x-axis plots frequency f in hertz, ranging from minus 1000 to 1000 in increments of 200. The Y-axis plots mod theta sub n, ranging from 0 to 180 degrees in increments of 90 degrees. Graph shows a waveform which consists of coordinate points moving in a horizontal line along the x-axis, that rises up in a rectangular pulse between minus 900 and minus 600, peaking at y = 180 degree. The waveform continues along the x-axis, rising up again between x-values of 600 and 900 in the first quadrant, peaking at y = 180 degree.

Each of the rectangular pulses consists of four coordinate points, separated by x values of 100. Vertical lines are drawn from these points to the x-axis.

The X-axis plots frequency f in hertz, ranging from minus 1000 to 1000 in increments of 200, while the y-axis plots theta sub n, ranging from minus 180 degrees to 180 degrees in increments of 45 degrees. The graph shows a waveform in the second quadrant, consisting of two triangular formations of coordinate points. The first pulse on the left is between minus 1000 and minus 600, consisting of five coordinate points at approximate values of (minus 1000, 0), (900, 150), (800, 100), (700, 75), and (600, 40). The second pulse is between minus 500 and 0, consisting of five coordinate points at approximate values of (minus 500, 180), (400, 150), (300, 100), (200, 75), and (100, 40).

The waveform then shifts to the fourth quadrant once it crosses the origin, depicting two triangular formations of coordinate points. The first pulse is between 0 and 500, consisting of five coordinate points at approximate values of (100, 40), (200, 75), (300, 100), (400, 150) and (500, 180). The second pulse on the right is between 600 and 1000, consisting of five coordinate points at approximate values of (600, 40), (700, 75), (800, 100), (900, 150) and (1000, 0).

Vertical lines are drawn from these points to the x-axis.

The circuit shows an op amp where a 10k ohm resistor R3, parallel with 100 nF capacitor C1, connects the output terminal with the inverting input terminal. The output terminal is grounded with the common node and the output voltage is marked as v sub o. The non-inverting input terminal is also grounded with the common node. The inverting input terminal is grounded with the common node in series with 3912.50 ohm resistor R1 and 100 nF capacitor C2, and parallel with 6.26 ohm resistor R2. The input voltage is marked as v sub g.

The X-axis plots time t in micro seconds, ranging from minus 50 phi to 50 phi in increments of 12.5 phi, while the y-axis plots voltage v sub g in Volts and marks minus 15.65 phi and 15.65 phi. The graph shows a square waveform drawn in the second quadrant, which consists of a positive pulse that peaks at y value of 15.65 phi at its x-coordinate midpoint of minus 50 phi, and a negative pulse that peaks at y value of minus 15.65 phi at its x-coordinate midpoint of minus 25 phi. The waveform continues in a similar pattern between minus 12.5 phi and 37.5 phi, and so on.

The X-axis plots time t in microseconds and marks 100, while the y-axis plots voltage v sub g in Volts, ranging from minus 80 to 80 in increments of 20. The graph shows a waveform consisting of a line that starts on the y-axis at y = 55 (approximately). The line moves with a downward slope to rise up again to reach y = 60. It then moves down with a steep downward slope, crosses the x-axis at approximately x = 40, and reaches a negative peak at (50, minus 60). The line fluctuates between y-values of minus 60 and minus 40 up to x-value of 100. It then rises up in an upward slope and crosses the x-axis at x = 120 to reach a positive peak value at (140, 60).

The X-axis plots time t in microseconds and marks 100, while the y-axis plots voltage v sub g in Volts, ranging from minus 100 to 100 in increments of 20. The graph shows a waveform consisting of a solid line that starts on the y-axis at y = minus 80. It rises with an upward slope, crosses the x-axis at approximately x= 40, and reaches a peak value at approximately (80, 80). It then moves down to cross the x-axis and reach a negative peak in the fourth quadrant. A second dotted line is also shown, which moves closely with the solid line, rising slightly above and below the line.

The x-axis plots time t in microseconds, ranging from minus 150 to 250 in increments of 100, while the y-axis plots voltage v of t in Volts, ranging from minus 80 to 80 in increments of 40. The graph shows a waveform between x values of 0 and 200, that consists of symmetrical positive and negative pulses. The positive pulse shows a rectangular movement between the x values of 0 and 50 at the y value of 40. It then steps up to y = 80 between the x values of 50 and 100. The negative pulse shows a rectangular movement between the x values of 100 and 150 at the y value of minus 40, and then steps down to y = minus 80 between the x values of 150 and 200. The waveform shows a similar pattern to the right, and also on the left side before the origin.

The X-axis plots time t in microseconds and marks minus 45, minus 35, minus 5, 5, 35, and 45, while the y-axis plots voltage v of t in Volts. The graph shows a waveform consisting of a horizontal line moving along the x-axis, that shows rectangular pulses which peak at y = 100 at regular intervals. The first pulse on the left is shown between minus 45 and minus 35, the second pulse is between minus 5 and 5, and the third pulse on the right is between 35 and 45 on the x-axis.

The X-axis plots time t and marks T over 4, T over 2, 3T over 4, and T, while the y-axis plots voltage v of t in Volts, ranging from 20 to 100 in increments of 20. The graph shows a waveform consisting of the half-sinusoidal pulse between x-values of 0 and T over 2, that peaks at 100, followed by another half-sinusoidal pulse between x-values of T over 2 and T, that peaks at 60.

The X-axis plots time t and marks minus T, 0, T, 2T, and 3T, while the y-axis plots v of t and marks V sub m and minus V sub m. The graph shows a square waveform between minus T and 0, which shows a rectangular positive pulse that peaks at V sub m, followed by another rectangular negative pulse that peaks at minus V sub m. This square wave pattern continues between 0 and T, and so on.

The X-axis plots time t and marks minus T, 0, T, 2T, and 3T, while the y-axis plots v of t and marks V sub m. The graph shows a non-sinusoidal waveform between minus T and 0, in form of an inverted U which peaks at V sub m. This is followed by another waveform between 0 and T, and so on.

The X-axis plots time t and marks 0, T over 2, T, and 3T over 2, while the y-axis plots v of t and marks V sub m. The graph shows a non-sinusoidal waveform between 0 and T over 2, in form of an inverted U which peaks at V sub m. The line then runs along the x-axis between T over 2 and T. This is followed by another similar waveform between T and 3T over 2, and so on.

The X-axis plots time t and marks T over 4, T over 2, 3T over 4, T, and 5T over 4. The Y-axis plots voltage and marks V sub m over 2 and V sub m. The graph shows a waveform between 0 and T, which consists of a rectangular pulse followed by a horizontal line. The rectangular pulse is plotted between the x-coordinates 0 and T over 4, and the y-coordinates V sub m over 2 and V sub m. The horizontal line is drawn at V sub m over 2, between the x-coordinates T over 4 and T. The waveform continues in a similar pattern after T.

The X-axis plots time t and marks minus T, minus T over 2, 0, T over 2, and T, while the y-axis plots voltage v of t and marks minus V sub m and V sub m. The graph shows a square waveform in the second quadrant, which has a positive-half pulse peaking at V sub m with its midpoint at x = minus T, followed by a negative-half pulse peaking at minus V sub m with its midpoint at x = minus T over 2. The waveform continues in a similar pattern extending into the first quadrant.

The X-axis plots time t and marks minus T, minus T over 2, 0, T over 2, T, and 3T over 2, while the y-axis plots voltage v of t and marks minus V sub p and V sub p. The graph shows a triangular waveform in the second quadrant, which has a negative-half pulse peaking at minus V sub p with its peak point at x = minus T, followed by a positive-half pulse peaking at V sub p with its peak point at x = minus T over 2. The waveform continues in a similar pattern extending into the first quadrant.

The X-axis plots time t and marks T over 8 and T over 4, while the y-axis plots f of t, ranging from 0 to 20 in increments of 5. The graph shows a line which starts at the origin and moves in a diagonal upward slope up to (T over 8, 15). The line then moves upward in a less steeper slope up to (T over 4, 20).

The X-axis plots time t, ranging from minus 5T over 4 to 5T over 4 in increments of T over 4. The Y-axis plots current i of t and marks I sub m. The graph shows a waveform that moves along the x-axis in the second quadrant, and then steps up in a triangular pulse to (minus T, I sub m) between the x-values of minus 5T over 4 and minus 3T over 4. The waveform continues along the x-axis from minus 3T over 4 to minus T over 4, and again steps up in a triangular pulse to the y-value of I sub m on the y-axis. This pattern of waveform continues into the first quadrant.

The X-axis plots time t and marks minus T over 4, 0, T over 4, T over 2, 3T over 4, T, and 5T over 4. The Y-axis plots voltage v1 of t and marks V sub m over 2. The graph shows a horizontal line which is drawn at V sub m over 2.

The X-axis plots time t and marks minus T over 4, 0, T over 4, T over 2, 3T over 4, T, and 5T over 4. The Y-axis plots voltage v2 of t and marks V sub m over 2. The graph shows a waveform that moves along the x-axis in the second quadrant up to the origin, and then steps up in a rectangular pulse to the y-value of V sub m over 2 between the x-values of 0 and T over 4. The waveform continues along the x-axis from T over 4 to T, and again steps up to in a rectangular pulse to the y-value of V sub m over 2 between the x-values of T and 5T over 4. This pattern of waveform continues after 5T over 4.

The X-axis plots time t and marks minus T over 2, minus T over 4, 0, T over 4, T over 2, 3T over 4, T, and 5T over 4. The Y-axis plots voltage v2 of t plus T over 8 and marks V sub m over 2. The graph shows a waveform that moves in a horizontal line along the x-axis in the second quadrant, up to the mid-point between minus T over 4 and the origin. The line then steps up in a rectangular pulse to reach the y-value of V sub m over 2, up to the mid-point between the origin and T over 4 on x-axis in the first quadrant. The waveform continues along the x-axis up to the mid-point between 3T over 4 and T, and again steps up in a rectangular pulse to the y-value of V sub m over 2 on the x-axis. This pattern of waveform continues after 5T over 4.

The X-axis plots time t and marks 0, T over 2, and T, while the y-axis plots i of t and marks minus I sub m and I sub m. The graph shows a right-angled triangular waveform, which has a positive pulse between the x values of 0 and T over 2, peaking at I sub m on the y-axis and moving in a downward slope from y = I sub m to x = T over 2. The waveform then shows a negative pulse between the x values of T over 2 and T, peaking at y = minus I sub m and moving in an upward slope to reach x = T on the x-axis. The waveform pattern continues after x = T.

The X-axis plots time t and marks T over 2, T, and 3T over 2, while the y-axis plots voltage v sub g and marks V sub m. The graph shows a waveform that moves along the x-axis in the second quadrant, up to the origin, and then steps up in a rectangular pulse to y-value of V sub m between the x-values of 0 and T over 2. The waveform continues along the x-axis from T over 2 to T, and then again steps up in a rectangular pulse to the y-value of V sub m between the x-values of T and 3T over 2. This pattern of waveform continues after 3T over 2.

The circuit shows resistor R on the top wire with a rightward current i, the inductor L on the right wire, and an independent voltage source v sub g on the left wire.

The X-axis plots time t and marks 0, T over 2, and T, while the y-axis plots voltage v sub i and marks minus V sub m and V sub m. The graph shows a square waveform starting close to the origin in the second quadrant, which has a positive-half pulse peaking at V sub m with its midpoint at x = 0, followed by a negative-half pulse peaking at minus V sub m with its midpoint at x = T over 2 in the first quadrant. The waveform continues in a similar pattern moving further into the first quadrant.

The circuit shows a vertical cross wire connecting a pair of nodes on the top and bottom wires, and carrying a 10 mH inductor. The left arm of the top wire carries a 10 ohm resistor. The left and right wires are removed and are replaced by two pairs of open terminals on the top and bottom wires. The input voltage at the left terminals is marked as v sub i, while the output voltage at the right terminals is marked as v sub o.

The circuit shows a vertical cross wire connecting a pair of nodes on the top and bottom wires, and carrying a 20k ohm resistor. A small rectangular circuit is located on the left arm of the top wire between two nodes, carrying a 100 mH inductor on its top wire and a 100 nF capacitor on its bottom wire. The left and right wires are removed and are replaced by two pairs of open terminals on the top and bottom wires. Input voltage at the left terminals is marked as v sub g, while the output voltage at the right terminals is marked as v sub o.

The X-axis plots time t in seconds and marks 0, 1 over 120, 1 over 60, and 1 over 40, while the y-axis plots v sub g in Volts and marks 340. The graph shows a half-sinusoidal waveform between 0 and 1 over 120, in the form of an inverted U which peaks at y = 340. This is followed by another waveform between 1 over 120 and 1 over 60, and so on.

The circuit shows a vertical cross wire connecting a pair of nodes on the top and bottom wires, and carrying a 12.5 microF capacitor. The left arm of the top wire carries a 16 H inductor and the right wire carries a 1k ohm resistor with a downward current i sub o. The left wire carries an independent voltage source, v sub g.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a 400 ohm resistor, while the right cross wire carries a 3200 ohm resistor in series with a 10 mH inductor. The right wire carries a 312.5 nF capacitor, with output voltage marked across it as v sub o. The circuit is powered by an independent voltage source with an upward current, i sub g, located on the left wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires, and carrying a 10 mH inductor and a 4 pF capacitor, respectively from the left to the right. The left arm of the top wire carries a 250k ohm resistor, while the left wire carries an independent voltage source, v sub g. The right wire is removed and two open terminals are located on the top and bottom wires to the right, with output voltage marked across them as v sub o.

The X-axis plots time t and marks minus T over 2, 0, T over 2, and T, while the y-axis plots current in mA and marks 240. The graph shows a trapezoidal waveform in the second quadrant, that peaks at y = 240 between the x values of minus T over 2 and 0. The waveform continues in the first quadrant between 0 and T, peaking at y = 240 between T over 2 and T.

The X-axis plots time t, ranging from minus T over 4 to T in increments of T over 4. The Y-axis plots voltage v in Volts and marks 60. The graph shows a waveform that contains a triangular pulse between the x-values of minus T over 4 and T over 4, peaking at y = 60 on the y-axis. The waveform continues along the x-axis from minus T over 4 to minus 3T over 4, and again steps up in a triangular pulse to the y-value of 60.

The circuit shows a vertical cross wire connecting a pair of nodes on the top and bottom wires, and carrying a 10 microF capacitor. The left arm of the top wire carries a 100 mH inductor and the right wire carries a 50 square root 2 ohm resistor. The left wire carries an independent voltage source, v sub g.

The X-axis plots time t in ms, ranging from 0 to 8 pie in increments of 2 pie. The Y-axis plots voltage v sub g in Volts and marks 20. The graph shows a triangular waveform which starts from the second quadrant close to the origin. The waveform shows a pulse between 0 and 4 pie which peaks at y = 20 for the x-value of 2 pie. The next pulse is drawn between 4 pie and 8 pie, peaking at y = 20 for x-the value of 6 pie. The waveform continues beyond 8 pie in a similar pattern.

The X-axis plots time t, ranging from 0 to T in increments of T over 6. The Y-axis plots voltage v sub g of t and marks minus V sub m and V sub m. The graph shows a periodic waveform which has a trapezoidal positive-half pulse between 0 and T over 2, peaking at y = V sub m between the x-values of T over 6 and T over 3. A negative-half pulse is shown next, between T over 2 and T, peaking at y = minus V sub m between the x-values of 2T over 3 and 5T over 6. The waveform continues beyond T in a similar pattern.

The X-axis plots time t and marks T over 4, T over 2, and 3T over 4, while the y-axis plots voltage v in Volts and marks minus 10 pie, minus 2.5 pie, 2.5 pie, and 10 pie. The graph shows a waveform between 0 and T over 2, consisting of a short square pulse between the x values of 0 and half-way up to T over 4, that peaks at y = 2.5 pie. The pulse then steps up to y = 10 pie, with its midpoint at x = T over 4, and then steps down back to y = 2.5 pie up to T over 2. This pattern repeats with a negative pulse, and continues further.

The X-axis plots time t in ms, ranging from 0 to 10 in increments of 5, while the y-axis plots voltage v in Volts and marks minus 120 and 120. The graph shows a square waveform between 0 and 10 on the x-axis, consisting of a positive rectangular pulse between x = 0 and x = 5 peaking at y = 120, followed by a negative pulse between x = 5 and x = 10 peaking at y = 120. This pattern continues after x = 10.

The X-axis plots time t in ms, ranging from 0 to 10 in increments of 5, while the y-axis plots voltage v in Volts and marks minus 120 and 120. The graph shows a triangular waveform between 0 and 10 on the x-axis, consisting of a positive triangular pulse between x = 0 and x = 5 peaking at y = 120, followed by a negative pulse between x = 5 and x = 10 peaking at y = 120. This pattern continues after x = 10.

The X-axis plots time t in seconds and marks 0, 20, and 40, while the y-axis plots v in Volts and marks 170. The graph shows a half-sinusoidal waveform between 0 and 20, in the form of an inverted U which peaks at y = 170. This is followed by another waveform between 20 and 40, and so on.

The X-axis plots time t in seconds and marks 0, 20, and 40, while the y-axis plots v in Volts and marks 170. The graph shows a half-sinusoidal waveform between 0 and 20, in the form of an inverted U which peaks at y = 170. This is followed by a straight line that runs along the x-axis from 20 to 40 on the x-axis. This pattern continues further.

The X-axis plots time t, ranging from 0 to T in increments of T over 4, while the y-axis plots current i sub g and marks minus I sub m and I sub m. The graph plots a periodic waveform in which the positive pulse consists of a triangular wave between 0 and T over 4 on the x-axis, which peaks at y = I sub m and comes down perpendicular to the x-axis, followed by a straight line along the x-axis from T over 4 to T over 2. The negative pulse between T over 2 and T follows a similar pattern, peaking at y = minus I sub m.

The X-axis plots time t in seconds and marks t sub a, t sub b, and T. The Y-axis plots voltage v in Volts and marks minus V sub p and V sub p. The graph shows a periodic waveform between 0 and T, consisting of a small positive triangular pulse that peaks at y value of V sub p at x value of t sub a. It is followed by a larger negative triangular pulse that peaks at y value of minus V sub p at x value of t sub b. This pattern continues after x = T.

The X-axis plots time t in seconds, ranging from 0 to 1.00 in increments of 0.2. The Y-axis plots voltage v in Volts and marks minus 10 and 10. The graph shows a periodic waveform between 0 and 1.00, consisting of a small positive triangular pulse that peaks at y = 10 for x = 0.2, followed by a larger negative triangular pulse that peaks at y = minus 10 for x = 0.6. This pattern continues after x = 1.00.

The X-axis plots time t in seconds and marks 0.4 and 1.0. The Y-axis plots voltage v in Volts and marks minus 10 and 10. The graph shows a periodic waveform between 0 and 1.00, consisting of a small positive right-angled triangular pulse that peaks at y = 10 for x = 0.4, followed by a larger negative right-angled triangular pulse that peaks at y = minus 10 for the same value of x = 0.4. This pattern continues after x = 1.00.

The X-axis plots time t in seconds and marks 0 and 1, while the y-axis plots voltage v in Volts and marks minus 10 and 10. The graph plots a triangular waveform between x = 0 and x = 1, which peaks at y = 10 for x = 1. This pattern continues after x = 1.

The X-axis plots time t and marks minus T, T, and 2T, while the y-axis plots voltage v in Volts and marks V sub m. The graph plots a triangular waveform between x = minus T and x = 0, which peaks at y = V sub m for x = 0. It is followed by a second pulse between the x values of 0 and T, which peaks at y = V sub m for x = T. This pattern continues after x = T.

The X-axis plots time t and marks minus T over 4, 0, T over 4, T over 2, 3T over 4, T, and 5T over 4. The Y-axis plots voltage v of t and marks V sub m. The graph shows a waveform that moves along the x-axis in the second quadrant, up to the origin, and then steps up in a rectangular pulse to the y-value of V sub m between the x-values of 0 and T over 4. The waveform continues along the x-axis from T over 4 to T, and again steps up in a rectangular pulse to the y-value of V sub m between the x-values of T and 5T over 4. This pattern of waveform continues after 5T over 4.

The circuit shows three parallel vertical cross wires connecting three pairs of nodes on the top and bottom wires, and carrying a 25 microH inductor, a 1 microF capacitor, and a 250 ohm resistor, respectively from the left to the right. The left arm of the top wire carries a 62.5 ohm resistor, while the left wire carries an independent voltage source, v sub g. The right wire is removed and two open terminals are located on the top and bottom wires to the right, with output voltage marked across them as v sub o.

The X-axis plots time t and marks minus T, T over 2, and T, while the y-axis plots voltage v sub g in Volts and marks minus V sub m and V sub m. The graph plots a half-triangular waveform between x = minus T and x = 0, which consists of a positive triangular pulse for half the distance on the x-axis, followed by a straight line for the other half. The triangular pulse peaks at y = V sub m. A similar half-triangular waveform is plotted between x =0 and x = T, and so on.

The X-axis plots n omega sub o and marks 0, omega sub o, 3 omega sub o, 5 omega sub o, and 7 omega sub o. The Y-axis plots amplitude A sub n. The graph plots four vertical lines of decreasing heights, marking y = A1 at x = omega sub 0, y = A3 at x = 3 omega sub o, y = A5 at x = 5 omega sub o, and y = A7 at x = 7 omega sub o.

The X-axis plots n omega sub o and marks 0, omega sub o, 3 omega sub o, 5 omega sub o, and 7 omega sub o. The Y-axis plots phase spectra theta sub n and marks minus 90 degrees and 90 degrees. The graph plots four vertical lines of equal heights, which alternate on either side of the x-axis. The graph marks y = minus 90 degrees at x = omega sub 0, y = 90 degrees at x = 3 omega sub o, y = minus 90 degrees at x = 5 omega sub o, and y = 90 degrees at x = 7 omega sub o.

The X-axis plots krad per second and marks from 10 to 70 in increments of 20. The Y-axis plots amplitude A sub n in microA. The graph plots four vertical lines of decreasing heights, marking y = 11,025 at x = 10, y = 1225 at x = 30, y = 441 at x = 50, and y = 225 at x = 70. The line at x = 10 is broken.

The X-axis plots krad per second and marks from 10 to 70 in increments of 20. The Y-axis plots phase spectra theta sub n and marks 180 degrees. The graph plots a dot and a vertical line alternatively, marking y = 0 at x = 10, y = 180 degrees at x = 30, y = 0 at x = 50, and y = 180 degrees at x = 70.

The X-axis plots time t in ms, ranging from minus 0.6 pie to 0.6 pie in increments of 0.2 pie. The Y-axis plots voltage v sub g in Volts and marks 200. The graph shows a triangular waveform, consisting of a pulse between minus 0.6 pie and minus 0.2 pie, which peaks at y = 200 for the x-value of minus 0.4 pie. The next pulse is drawn between minus 0.2 pie and plus 0.2 pie, peaking at y = 200 for the x-value of 0. This pattern continues into the first quadrant.

The circuit shows an op amp where a 10k ohm resistor R3, in parallel with a 100 nF capacitor C1, connects the output terminal with the inverting input terminal. The output terminal is grounded with the common node and the output voltage is marked as v sub o. The non-inverting input terminal is also grounded with the common node. The inverting input terminal is grounded with the common node in series with 3912.5 ohm resistor R1 and 100 nF capacitor C2, and in parallel with 6.26 ohm resistor R2. The input voltage is marked as v sub g.

The X-axis plots time t in ms, ranging from minus 50 pie to 50 pie in increments of 12.5 pie. The Y-axis plots voltage v sub g in Volts and marks minus 15.65 pie and 15.65 pie. The graph shows a square waveform in the second quadrant, which consists of a positive pulse that peaks at y = 15.65 pie with its midpoint at x = minus 50 pie. It is followed by a negative pulse that peaks at y = minus 15.65 pie with its midpoint at x = minus 25 pie. This pattern continues into the first quadrant.

The circuit shows an op amp where a 50k ohm resistor R3, in parallel with a 100 nF capacitor C1, connects the output terminal with the inverting input terminal. The output terminal is grounded with the common node and the output voltage is marked as v sub o. The non-inverting input terminal is also grounded with the common node. The inverting input terminal is grounded with the common node in series with 25k ohm resistor R1 and 100 nF capacitor C2, and in parallel with 20.016 ohm resistor R2. Input voltage is marked as v sub g.

The X-axis plots time t in ms, ranging from minus 0.1 pie to 0.2 pie in increments of 0.1 pie. The Y-axis plots voltage v sub g in mV and marks minus 2.25 pie-square and 2.25 pie-square. The graph shows a triangular waveform in the second quadrant, which consists of a negative pulse that peaks at y = minus 2.25 pie-square for x = minus 0.1 pie. It is followed by a positive pulse that peaks at y = 2.25 pie-square for x = 0. This pattern continues into the first quadrant.

The illustration shows a drawing of a desktop computer on top. The information it transmits is depicted as a voltage waveform, along with the digits 0111010010.

A photo of telephone poles dotting a landscape is shown in the middle, with an arrow pointing to it on the left.

The drawing at the bottom shows another desktop computer, with received information depicted as a voltage waveform, along with the digits 0111010010. An arrow points to this drawing from the left.

The X-axis plots time t and the y-axis plots voltage v of t. The graph shows a line which moves along the x-axis, from the second quadrant to the first quadrant, stepping up to y = V sub m in a rectangular positive pulse between the x-values of minus tau over 2 and tau over 2.

The X-axis plots n omega sub 0, ranging from minus 4 pie over tau to 4 pie over tau, in increments of 2 pie over tau. The Y-axis plots C sub n and marks 0.2 V sub m. The graph shows a waveform that depicts a small sinusoidal negative pulse between the x values of minus 4 pie over tau and minus 2 pie over tau, followed by a large sinusoidal positive pulse between the x values of minus 2 pie over tau and plus 2 pie over tau. The pulse touches the peak value of y = 0.2 V sub m on the y-axis. The positive pulse is followed by a small sinusoidal negative pulse between the x values of 2 pie over tau and 4 pie over tau. Vertical lines are drawn from the wave to the x-axis.

The X-axis plots n omega sub 0, ranging from minus 4 pie over tau to 4 pie over tau, in increments of 2 pie over tau. The Y-axis plots C sub n and marks 0.1 V sub m. The graph shows a waveform that depicts a small sinusoidal negative pulse between the x values of minus 4 pie over tau and minus 2 pie over tau, followed by a large sinusoidal positive pulse between the x values of minus 2 pie over tau and plus 2 pie over tau. The pulse touches the peak value of y = 0.1 V sub m on the y-axis. The positive pulse is followed by a small sinusoidal negative pulse between the x values of 2 pie over tau and 4 pie over tau. The wave is depicted by vertical lines, which are drawn from the wave surface to the x-axis.

The X-axis plots omega, ranging from minus 4 pie over tau to 4 pie over tau, in increments of 2 pie over tau. The Y-axis plots V of omega and marks V sub m multiplied by tau. The graph shows a waveform that depicts a small sinusoidal negative pulse between the x values of minus 4 pie over tau and minus 2 pie over tau, followed by a large sinusoidal positive pulse between the x values of minus 2 pie over tau and plus 2 pie over tau. The pulse touches the peak value of y = V sub m multiplied by tau on the y-axis. The positive pulse is followed by a small sinusoidal negative pulse between the x values of 2 pie over tau and 4 pie over tau.

The X-axis plots time t and y-axis plots f of t. The graph shows a line which runs along the x-axis in the second quadrant, up to the origin, and then steps up to y = K along the y-axis. The line then slides down with a downward slope to reach the x-axis at the bottom right corner of the quadrant. This line represents K multiplied by e raised to minus at.

The X-axis plots time t and the y-axis plots f of t. The graph shows two upward sloping curves in the second quadrant and two downward sloping curves in the first quadrant, which meet at a common point on the y-axis. A horizontal line labeled A is drawn passing through this common intersection point.

The two curves in the second quadrant are labeled as A multiplied by e raised to epsilon2 t and A multiplied by e raised to epsilon1 t, from the top to the bottom. The two curves in the first quadrant are marked as A multiplied by e raised to minus epsilon2 t and A multiplied by e raised to minus epsilon1 t, from the top to the bottom. The value of epsilon 2 is shown as lesser than that of epsilon 1.

The X-axis plots time t and the y-axis plots f of t. The graph shows a small sinusoidal wave in the second quadrant, followed by one half of a large positive-pulse which intersects the y-axis and moves in a sharp downward slope to end at x = 0 plus on the x-axis. This waveform represents f of t. A symmetrical waveform is depicted in the first quadrant, with a slanting line drawn from x = 0 minus to the y-axis, which then continues as one half of a large positive pulse, followed by a small sinusoidal wave. This line is depicted as a broken line and represents f of minus t.

The X-axis plots time t and the y-axis plots sgn of t. The graph shows a horizontal line drawn in the third quadrant, which reaches the y-axis at y = minus 1.0, steps up to y = 1.0, and then continues as a horizontal line in the first quadrant.

The X-axis plots time t and the y-axis plots f of t. The graph shows a downward sloping line in the third quadrant, which reaches the y-axis at y = minus 1.0, then steps up to y = 1.0, and then moves with a downward slope in the first quadrant. The curve in the third quadrant represents e raised to epsilon t, multiplied by u of minus t, while the curve in the first quadrant represents e raised to minus epsilon t, multiplied by u of t.

The X-axis plots omega and the y-axis plots A of omega. The graph shows a line which moves along the x-axis from the second quadrant to the first quadrant, stepping up to y = M in a rectangular positive pulse between the x-values of minus omega sub 0 over 2 and omega sub 0 over 2.

The X-axis plots t and the y-axis plots v of t. The graph shows a line which moves along the x-axis from the second quadrant to the first quadrant, stepping up to y = V sub m in a rectangular positive pulse between the x-values of minus tau over 2 and tau over 2.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 1 ohm resistor. The right wire carries a 3 ohm resistor and a 1 H inductor, with a downward current i sub o of t flowing through it. The circuit is powered by an independent current source with an upward current, i sub g of t, located on the left wire.

The X-axis plots t and the y-axis plots i sub o of t. The graph shows a horizontal line in the third quadrant, which is drawn up to y = minus 5 on the y-axis. The line then moves with an upward slope, crosses the x-axis, and moves parallel to the x-axis. The curving portion of this wave is labeled as i sub o, while the straight line represents 5 sgn of t.

A horizontal straight line is drawn above this curve at y = 5, representing 5 sgn of t. Another curving line is drawn below the i sub o curve, which starts from y = minus 10 in the fourth quadrant, then moves with an upward slope, and then runs parallel to the x-axis below it. This curve is labeled as minus 10 e raised to minus 4t.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 4 ohm resistor with a downward current i1. The right arm of the top wire carries a 1 ohm resistor with a leftward current i2, while the right wire carries a 1H inductor with the voltage marked across it as v sub o. The circuit is powered by an independent current source with an upward current, i sub g, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 0.5 ohm resistor with voltage marked across it as v sub a. The left arm of the top wire carries a 1 ohm resistor, while the right wire carries a 1 F capacitor. The circuit is powered by an independent voltage source, v sub g, located on the left wire.

The X-axis plots omega and marks minus omega 2, minus omega 1, omega 1, and omega 2, while the y-axis plots mod F of omega squared. The graph shows a line which starts just above the x-axis in the second quadrant and rises with an upward slope to reach a peak value on the y-axis. The line then moves with a downward slope and reaches a point close to the x-axis. A shaded area is marked between this curve and the x-axis, between the x values of minus omega 2 and minus omega 1. Another shaded area is marked between the curve and the x-axis between the x values of omega 1 and omega 2.

The X-axis plots omega in rad per second, ranging from minus 60 to 60 in increments of 20, while the y-axis plots mod V of omega squared, ranging from 5 to 25 in increments of 5. The graph shows a line which starts just above the x-axis in the second quadrant at x = minus 60 and rises with an upward slope to reach a peak value on the y-axis at y = 25. The line then moves with a downward slope and reaches a point close to the x-axis at x = 60.

The X-axis plots omega in rad per second, ranging from minus 60 to 60 in increments of 20, while the y-axis plots mod V of omega squared, ranging from 5 to 25 in increments of 5. The graph shows a line which runs along the x-axis, up to a point between minus 60 and minus 40, and then moves vertically up until y = 5. The line then moves with an upward slope to reach peak at approximately (minus 22, 14), from where it drops vertically down to the the x-axis and moves up to the origin along the x-axis.

The line then moves from the origin to approximately x = 22 along the x-axis, and then rises vertically upwards to a peak value of (22, 14). The line then curves down to (5, 50), drops down to the x-axis, and continues along the x-axis in the first quadrant.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 10 microF capacitor. The left arm of the top wire carries a 10k ohm resistor. The left and right wires are removed. The input voltage at the left end of the circuit is marked as v sub i, while the output voltage at the right end is marked as v sub o.

The X-axis plots time t and marks minus tau over 2 and tau over 2, while the y-axis plots voltage v of t and marks V sub m. The graph shows a line which moves along the x-axis from the second quadrant to the first quadrant, stepping up to y = V sub m in a rectangular positive pulse between the x-values of minus tau over 2 and tau over 2.

The X-axis plots omega, ranging from minus 4 pie over tau to 4 pie over tau, in increments of 2 pie over tau. The Y-axis plots V of omega and marks V sub m multiplied by tau. The graph shows a waveform that depicts a small sinusoidal negative pulse between the x values of minus 4 pie over tau and minus 2 pie over tau, followed by a large sinusoidal positive pulse between the x values of minus 2 pie over tau and plus 2 pie over tau. The positive pulse touches a peak value of y = V sub m multiplied by tau on the y-axis. The positive pulse is followed by a small sinusoidal negative pulse between the x values of 2 pie over tau and 4 pie over tau.

The X-axis plots omega in rad per second and marks minus 2000 pie and 2000 pie, while the y-axis plots voltage mod V of j omega and marks 0 and 6. The graph shows a line which moves along the x-axis from the second quadrant to the first quadrant, stepping up to y = 6 in a triangular positive pulse between the x-values of minus 2000 pie and 2000 pie.

The X-axis plots time t and marks minus tau over 2 and tau over 2, while the y-axis plots voltage v of t and marks V sub m. The graph shows a line which rises up in a sinusoidal positive pulse between the x values of minus tau over 2 and tau over 2, peaking at y = V sub m on the y-axis.

The X-axis plots time t and marks minus tau over 2 and tau over 2, while the y-axis plots voltage f of t and marks 0 and A. The graph shows a right-angled negative triangular pulse between minus tau over 2 and the origin, followed by a right-angled positive triangular pulse between 0 and tau over 2.

The X-axis plots omega and marks minus omega sub 0 over 2 and omega sub 0 over 2, while the y-axis plots F of omega and marks minus jA and A. The graph shows a line which moves along the x-axis from the second quadrant to the first quadrant, stepping up to y = A in a triangular positive pulse between the x-values of minus omega sub 0 over 2 and omega sub 0 over 2.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 60k ohm resistor with the voltage marked across it as v sub o. The left arm of the top wire carries a 12k ohm resistor, while the right wire carries a 2 microF capacitor. A rightward current, i sub o, is depicted on the right arm of the top wire. The circuit is powered by an independent voltage source, v sub g, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 10 ohm resistor. The left arm of the top wire carries a 5H inductor. The right wire is removed and is replaced by two open terminals on the top and bottom wires, with output voltage marked across them as v sub o. The circuit is powered by an independent voltage source, v sub g, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 50k ohm resistor. The right wire carries a 500 nF capacitor with voltage across it marked as v sub o of t. A rightward current, i sub o of t, is depicted on the right arm of the top wire. The circuit is powered by an independent current source with an upward current, i sub g, located on the left wire.

The circuit shows two nodes on the top and bottom wires, close to the right wire, with voltage across them marked as v sub o. The top wire carries a 0.5 ohm resistor and a 10 mH inductor to the left of the node, with a rightward current i sub o flowing through it. The right wire carries a 250 mF capacitor. The circuit is powered by an independent voltage source, v sub g, located on the left wire.

The circuit shows two nodes on the top and bottom wires, close to the right wire, with output voltage across them marked as v sub o. A vertical cross wire connects two more nodes on the top and bottom wires, to the left of the output voltage nodes. A 10 ohm resistor is located on this cross wire, while a 100 mF capacitor with a downward current i sub o is located on the right wire. The circuit is powered by an independent current source with an upward current, i sub g, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 10mH inductor. The top wire carries a 25 ohm resistor on its left arm and a 800nF capacitor with a rightward current i sub o on its right arm. A 100 ohm resistor is located on the right wire. The circuit is powered by an independent voltage source, v sub g, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 20mH inductor with a downward current i sub o. The left arm of the top wire carries a 5 mH inductor, while the right wire carries a 120 ohm resistor. The circuit is powered by an independent voltage source, v sub g, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 4H inductor with a downward current i sub L. The left arm of the top wire carries a 1 microF capacitor with voltage v sub C, while the right wire carries an 800 ohm resistor with voltage marked as v sub o. The circuit is powered by an independent voltage source, v sub g, located on the left wire.

The circuit shows a 5 ohm resistor with a rightward current i sub o on the top wire, a 100 mF capacitor with voltage v sub o on the right wire, and an independent voltage source v sub g on the left wire.

The circuit shows a 10 ohm resistor in series with a 1 H inductor on the top wire, a 62.5 mF capacitor with voltage v sub o on the right wire, and an independent voltage source v sub g on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 25 ohm resistor. The right arm of the top wire carries a 10 mF capacitor, while the right wire carries a 1 H inductor with the voltage marked across it as v sub o. The circuit is powered by an independent current source with an upward current, i sub g, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 25 ohm resistor with a downward current i sub o. The right wire carries a 500 mH inductor. The circuit is powered by an independent current source with an upward current, i sub g, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 100k ohm resistor. The right wire carries a 1.25 microF capacitor with a downward current i sub o. The circuit is powered by an independent current source with an upward current, i sub g, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying an 80 ohm resistor. The left arm of the top wire carries a 20 ohm resistor, while the right wire carries a 125 microF capacitor with the voltage marked across it as v sub o. The circuit is powered by an independent voltage source, v sub g, located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 20k ohm resistor. The left arm of the top wire carries a 0.5 microF capacitor. The left and right wires are removed and replaced by two pairs of open terminals on the top and bottom wires. The input voltage at the left terminals is marked as v sub i, while the output voltage at the right terminals is marked as v sub o.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a resistor R. The left arm of the top wire carries a capacitor C. The left and right wires are removed and replaced by two pairs of open terminals on the top and bottom wires. The input voltage at the left terminals is marked as v sub i, while the output voltage at the right terminals is marked as v sub o.

The drawing shows a box representing a circuit, with four internal terminals labeled from a to d. Two terminals on the left, a and b, have wires connected to them and extending out. The wires represent the input port and have voltage v1 marked between them. A rightward current, i1, flows into the upper terminal, while a leftward current, i-dash 1, flows out of the bottom terminal.

Two terminals on the right, c and d, have wires connected to them and extending out. The wires represent the output port and have voltage v2 marked between them. A leftward current, i2, flows into the upper terminal, while a rightward current, i-dash 2, flows out of the bottom terminal.

The drawing shows a box with four internal terminals, representing an s-domain circuit. Two terminals on the left have wires connected to them and extending out. The wires have voltage V1 marked between them, with a rightward current I1 shown flowing into the upper terminal.

Two terminals on the right have wires connected to them and extending out. The wires have voltage V2 marked between them, with a leftward current I2 shown flowing into the upper terminal.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a 20 ohm resistor, while the right cross wire carries a 15 ohm resistor. The middle arm of the top wire, between the cross wires, carries a 5 ohm resistor. A rightward current I1 flows in the left arm of the top wire, while a leftward current I2 flows in the right arm. The middle part of the circuit, containing the cross wires, is enclosed in a shaded box.

The left and right wires are removed and are replaced by two pairs of open terminals on the top and bottom wires. Voltage between the terminals on the left is marked as V1, while the voltage between the terminals on the right is marked as V2.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a 20 ohm resistor, with voltage marked as V1, while the right cross wire carries a 15 ohm resistor, with voltage marked as V2. The middle arm of the top wire, between the cross wires, carries a 5 ohm resistor. A leftward current, I2=0, flows in the right arm of the top wire.

A 1A independent current source, with an upward current, is located on the left wire. The right wire is removed and is replaced by two open terminals located on the top and bottom wires.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a 20 ohm resistor, with voltage marked as V1, while the right cross wire carries a 15 ohm resistor, with voltage marked as V2. The middle arm of the top wire, between the cross wires, carries a 5 ohm resistor. A rightward current, I1=0, flows in the left arm of the top wire.

A 1A independent current source, with an upward current, is located on the right wire. The left wire is removed and is replaced by two open terminals located on the top and bottom wires.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying impedance Z sub b. Two impedances, marked as Z sub a, are located on the left and right arms of the top wire. A rightward current, I1, flows in the left arm of the top wire, while a leftward current, I2, flows in the right arm. The middle part of the circuit, containing the cross wires, is enclosed in a shaded box.

The left and right wires are removed and are replaced by two pairs of open terminals on the top and bottom wires. Voltage between the terminals on the left is marked as V1, while the voltage between the terminals on the right is marked as V2.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires, carrying impedances Z sub b each. The middle arm of the top wire, between the cross wires, carries impedance Z sub a. A rightward current, I1, flows in the left arm of the top wire, while a leftward current, I2, flows in the right arm. The middle part of the circuit, containing the cross wires, is enclosed in a shaded box.

The left and right wires are removed and are replaced by two pairs of open terminals on the top and bottom wires. Voltage between the terminals on the left is marked as V1, while voltage between the terminals on the right is marked as V2.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying impedance Z sub b. Two impedances marked as Z sub a are located on the left and right arms of the top wire. An additional wire is drawn on the top wire, carrying impedance Z sub c. A rightward current, I1, flows in the left arm of the top wire, while a leftward current, I2, flows in the right arm. The middle part of the circuit, containing the cross wires and additional wire, is enclosed in a shaded box.

The left and right wires are removed and are replaced by two pairs of open terminals on the top and bottom wires. Voltage between the terminals on the left is marked as V1, while voltage between the terminals on the right is marked as V2.

The circuit shows two crossed vertical cross wires connecting two pairs of nodes on the top and bottom wires, each carrying impedance Z sub b. The middle arms of both the top wire and bottom wire between the cross wires carry impedance Z sub a. A rightward current, I1, flows in the left arm of the top wire, while a leftward current, I2, flows in the right arm. The middle part of the circuit, containing the cross wires, is enclosed in a shaded box.

The left and right wires are removed and are replaced by two pairs of open terminals on the top and bottom wires. Voltage between the terminals on the left is marked as V1, while voltage between the terminals on the right is marked as V2.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 60 ohm resistor. A 30 ohm resistor and a 20 ohm resistor are located on the left and right arms of the top wire, respectively. An additional wire is drawn on the top wire, carrying a 10 ohm resistor. A rightward current, I1, flows in the left arm of the top wire, while a leftward current, I2, flows in the right arm.

The left and right wires are removed and are replaced by two pairs of open terminals on the top and bottom wires, labeled as a and d on the left, and c and d on the right. The top node of the cross wire is marked as b. Voltage between the terminals on the left is marked as V1, the while voltage between the terminals on the right is marked as V2.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 60 ohm resistor. A 30 ohm resistor and a 20 ohm resistor are located on the left and right arms of the top wire, respectively. An additional wire is drawn on the top wire, carrying a 10 ohm resistor. Two nodes, labeled as a and d, are located close to the left wire on the top and bottom wires, while two nodes, labeled as c and d, are located similarly close to the right wire. The top node of the cross wire is marked as b. The middle part of the circuit, containing the cross wires and the additional wire, is enclosed inside a shaded box.

A 15V independent voltage source is located on the left wire, while an ammeter with a downward current I is located on the right wire.

Circuit shows a vertical cross wire connecting two nodes on top and bottom wires, carrying a 60 ohm resistor. A 30 ohm resistor and a 20 ohm resistor are located on left and right arms of the top wire, respectively. An additional wire is drawn on the top wire, carrying a 10 ohm resistor. Two nodes, labeled as a and d, are located close to left wire on the top and bottom wires, while two nodes, labeled as c and d, are located similarly close to right wire. Top node of the cross wire is marked as b. Middle part of the circuit, containing the cross wires and the additional wire, is enclosed inside a shaded box.

A 15V independent voltage source is located on right wire, while an ammeter with downward current I is located on left wire.

The drawing shows a box with four internal terminals, representing a Two-port model of a network. Two terminals on the left have wires connected to them and extending out to form a closed circuit, with impedance Z sub g on the top wire and independent voltage source V sub g on the left wire. The terminals have voltage V1 marked between them, with a rightward current I1 shown flowing into the upper terminal.

The two terminals on the right also have wires connected to them and extending out to form a closed circuit, with impedance Z sub L on the right wire. The terminals have voltage V2 marked between them, with a leftward current I2 shown flowing into the upper terminal.

The drawing shows a box labeled as [b] and having four internal terminals. Two terminals on the left have wires connected to them and extending out to form a closed circuit, with a 500 ohm resistor on the top wire and an independent voltage source 500 angled 0 degree V on the left wire. The terminals have voltage V1 marked between them, with a rightward current I1 shown flowing into the upper terminal.

The two terminals on the right also have wires connected to them and extending out to form a closed circuit, with 5k ohm resistor located on the right wire. The terminals have voltage V2 marked between them, with a leftward current I2 shown flowing into the upper terminal.

The drawing shows a box with four internal terminals, representing a Two-port model of a network. Two terminals on the left have wires connected to them and extending out to form a closed circuit, with impedance Z sub g on the top wire and independent voltage source V sub g on the left wire. The terminals have voltage V1 marked between them, with a rightward current I1 shown flowing into the upper terminal.

The two terminals on the right also have wires connected to them and extending out to form a closed circuit, with impedance Z sub L on the right wire. The terminals have voltage V2 marked between them, with a leftward current I2 shown flowing into the upper terminal.

A drawing shows a two-port circuit on the left labeled as 1, connected to another two-port circuit on the right labeled as 2. The output terminals on the right of circuit 1 connect to the input terminals on the left of circuit 2.

The drawing shows a two-port circuit on top labeled as 1, connected to another two-port circuit below it, labeled as 2. The bottom left terminal of circuit 1 connects to the top left terminal of circuit 2, and the bottom right terminal of circuit 1 connects to the top right terminal of circuit 2.

The drawing shows a two-port circuit on top labeled as 1, connected to another two-port circuit below it, labeled as 2. The top left terminals of circuit 1 and 2 connect to each other, and their bottom left terminals also are interconnected. Similarly, the top right terminals and bottom right terminals of circuits 1 and 2 are also interconnected.

The drawing shows a two-port circuit on top labeled as 1, connected to another two-port circuit below it, labeled as 2. The bottom left terminal of circuit 1 connects to the top left terminal of circuit 2, while the top right terminals and bottom right terminals of both the circuits are interconnected.

The drawing shows a two-port circuit on top labeled as 1, connected to another two-port circuit below it, labeled as 2. The bottom right terminal of circuit 1 connects to the top right terminal of circuit 2, while the top left terminals and bottom left terminals of both the circuits are interconnected.

the The drawing shows a two-port circuit on left labeled as Circuit 1, connected to another two-port circuit on the right, labeled as Circuit 2. The output terminals on the right of Circuit 1 connect to the input terminals on the left of Circuit 2.

The input terminals of Circuit 1 are labeled as a-dash 11 and a-dash 21 on the top and bottom, respectively. The input voltage between these terminals is marked as V1, while a rightward current, I1, flows into the top left terminal. The output terminals are labeled as a-dash 12 and a-dash 22 on the top and bottom, respectively. The output voltage between these terminals is marked as V-dash 2, while a leftward current, I-dash 2, flows into the top right terminal.

The input terminals of Circuit 2 are labeled as a-double dash 11 and a-double dash 21 on the top and bottom, respectively. The input voltage between these terminals is marked as V-dash 1, while a rightward current, I-dash 1, flows into the top left terminal. The output terminals are labeled as a-double dash 12 and a-double dash 22 on the top and bottom, respectively. The output voltage between these terminals is marked as V2, while a leftward current, I2, flows into the top right terminal.

The drawing shows a two-port circuit on the left labeled as A1, connected to another two-port circuit on the right, labeled as A2. The output terminals on the right of A1 connect to the input terminals on the left of A2.

The input terminals of A1 are connected to an independent voltage source, V sub g, in series with a 500 ohm resistor. The output terminals of A2, which have voltage V2 marked across them, are connected to a 10k ohm resistor.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying impedance Z sub b. Two impedances, marked as Z sub a, are located on the left and right arms of the top wire. An additional wire is drawn on the top wire, carrying impedance Z sub c. A rightward current, I1, flows in the left arm of the top wire, while a leftward current, I2, flows in the right arm. The middle part of the circuit, containing the cross wires and the additional wire, is enclosed in a shaded box.

The left and right wires are removed and are replaced by two pairs of open terminals on the top and bottom wires. Voltage between the terminals on the left is marked as V1, while the voltage between the terminals on the right is marked as V2.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 12 ohm resistor. The left arm of the top wire carries a 1 ohm resistor, while the right arm carries a 4 ohm resistor. A rightward current, I1, flows in the left arm of the top wire, while a leftward current, I2, flows in the right arm.

The left and right wires are removed and are replaced by two pairs of open terminals on the top and bottom wires. Voltage between the terminals on the left is marked as V1, while voltage between the terminals on the right is marked as V2.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 50 ohm resistor. The left arm of the top wire carries a 100 ohm resistor, while the right arm carries a 20 ohm resistor. An additional wire is drawn over the top wire, carrying a 40 ohm resistor. A rightward current, I1, flows in the left arm of the top wire, while a leftward current, I2, flows in the right arm.

The left and right wires are removed and are replaced by two pairs of open terminals on the top and bottom wires. Voltage between the terminals on the left is marked as V1, while the voltage between the terminals on the right is marked as V2.

The circuit shows two crossed vertical cross wires connecting two pairs of nodes on the top and bottom wires, carrying a 10 ohm resistor on the left cross wire and a 15 ohm resistor on the right cross wire. A 5 ohm resistor is located on the middle arm of the top wire, while a 20 ohm resistor is located on the middle arm of the bottom wire. A rightward current, I1, flows in the left arm of the top wire, while a leftward current, I2, flows in the right arm.

The left and right wires are removed and are replaced by two pairs of open terminals on the top and bottom wires. Voltage between the terminals on the left is marked as V1, while the voltage between the terminals on the right is marked as V2.

The circuit shows three parallel vertical cross wires connecting three pairs of nodes on the top and bottom wires, carrying an 80 ohm resistor, a 20 ohm resistor, and a 60 ohm resistor from the left to the right. The second arm from the left on the top wire carries a 4 ohm resistor, while the third arm carries a 20 ohm resistor. A rightward current, I1, flows in the left arm of the top wire, while a leftward current, I2, flows in the right arm.

The left and right wires are removed and are replaced by two pairs of open terminals on the top and bottom wires. Voltage between the terminals on the left is marked as V1, while the voltage between the terminals on the right is marked as V2..

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires, carrying resistors R2 and R3 on the left and right wires, respectively. The middle arm of the top wire, between the cross wires, carries resistor R1. A rightward current, I1, flows in the left arm of the top wire, while a leftward current, I2, flows in the right arm. The middle part of the circuit, containing the cross wires, is enclosed in a box drawn with dotted lines.

The left and right wires are removed and are replaced by two pairs of open terminals on the top and bottom wires. Voltage between the terminals on the left is marked as V1, while the voltage between the terminals on the right is marked as V2.

The circuit shows an op amp with positive supply voltage V sub CC and negative supply voltage minus V sub CC. A 1000 ohm resistor, in parallel with a 500 ohm resistor, connects the output terminal with the inverting input terminal. The non-inverting input terminal is grounded with the common node, in series with a 400 ohm resistor carrying a rightward current I1 and in parallel with a 1200 ohm resistor. The input voltage between the non-inverting terminal and the output terminal is marked as V1. The output terminal is grounded with the common node, in series with a 200 ohm resistor with a leftward current I2. The output voltage between these two nodes is marked as V2.

The drawing shows two circuits which are joined at their bottom wires. The circuit on the left shows a 10 ohm resistor on the top wire, with a rightward current I1 flowing through it, and a 40 ohm resistor on the right wire. Voltage between the top and bottom wires is marked as V sub sigma. The left wire is removed and is replaced by two open terminals having voltage V1.

The circuit on the right shows a 3 ohm resistor on the top wire, with a leftward current I2 flowing through it, and a 3 V sub sigma dependent voltage source with reverse polarity on its left wire. The right wire is removed and is replaced by two open terminals having voltage V2.

An additional wire carrying a 5 ohm resistor connects a node on the top arm of the left circuit, close to the right wire, and another node on the top arm of the right circuit, to the right of the 3 ohm resistor.

The drawing shows two circuits placed side by side. The circuit on the left shows a 1k ohm resistor on the top wire, with a rightward current I1 flowing through it, and a 10 raised to minus 4 V2 dependent voltage source on the right wire. The left wire is removed and is replaced by two open terminals having voltage V1.

The circuit on the right shows a 40k ohm resistor on a cross wire, a leftward current I2 flowing through the top wire, and a 50 I1 dependent current source with a downward current on its left wire. The right wire is removed and is replaced by two open terminals having voltage V2.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries a 50 I2 dependent voltage source, while the right cross wire carries a minus j 100 ohm capacitor. The middle arm of the top wire, between the cross wires, carries a 200 ohm resistor, while the left arm carries a 10 ohm resistor in series with a j20 ohm inductor. The rightward current I1 flows in the left arm of the top wire, while a leftward current I2 flows in the right arm.

The left and right wires are removed and are replaced by two pairs of open terminals on the top and bottom wires. Voltage between the terminals on the left is marked as V1, while the voltage between the terminals on the right is marked as V2.

The drawing shows a box labeled as g, with four internal terminals. Two terminals on the left have wires connected to them and extending out. The wires have input voltage V1 marked between them, with a rightward current I1 shown flowing into the upper terminal.

Two terminals on the right also have wires connected to them and extending out. The wires have output voltage V2 marked between them, with a leftward current I2 shown flowing into the upper terminal.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 4 ohm resistor. The left arm of the top wire carries a 1 F capacitor, while the right arm carries a 1 H inductor. A rightward current i1 flows in the left arm of the top wire, while a leftward current i2 flows in the right arm.

The left and right wires are removed and are replaced by two pairs of open terminals on the top and bottom wires. Voltage between the terminals on the left is marked as v1, while the voltage between the terminals on the right is marked as v2.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying a 1 H inductor. The left and right arms of the top wire carry a 1 ohm resistor each. An additional wire is drawn over the top wire, carrying a 1 F capacitor. A rightward current, i1, flows in the left arm of the top wire, while a leftward current, i2, flows in the right arm.

The left and right wires are removed and are replaced by two pairs of open terminals on the top and bottom wires. Voltage between the terminals on the left is marked as v1, while the voltage between the terminals on the right is marked as v2.

The drawing shows two circuits placed side by side. The circuit on the left shows resistor R on the top wire and inductor L on the right wire, with a dot terminal to its top left. The left wire is removed and is replaced by two open terminals, with input voltage marked as v1. A rightward current, i1, flows through the top wire.

The circuit on the right shows resistor R on the top wire and inductor L on the left wire, with a dot terminal to its bottom right. The right wire is removed and is replaced by two open terminals, with output voltage marked as v2. A leftward current, i2, flows through the top wire.

Mutual inductance between the inductors is marked as M.

The circuit shows four parallel vertical cross wires connecting four pairs of nodes on the top and bottom wires. The left cross wire carries a minus j 50 ohm capacitor. The second cross wire from the left carries a 100 ohm resistor, in series with a 5V2 dependent voltage source. The third cross wire carries a V1 over 40 dependent current source with a downward current. The right cross wire carries a 40 ohm resistor. A 20 ohm resistor is located on the third arm of the top wire. A rightward current, I1, flows in the left arm of the top wire, while a leftward current, I2, flows in the right arm.

The left and right wires are removed and are replaced by two pairs of open terminals on the top and bottom wires. Voltage between the terminals on the left is marked as V1, while the voltage between the terminals on the right is marked as V2.

The circuit shows two crossed vertical cross wires connecting two pairs of nodes on the top and bottom wires, each carrying impedance Z sub b. The middle arm of the top wire carries impedance Z sub a, while the middle arm of the bottom wire carries another impedance, Z sub c. A rightward current, I1, flows in the left arm of the top wire, while a leftward current, I2, flows in the right arm. The middle part of the circuit, containing the cross wires, is enclosed in a shaded box.

The left and right wires are removed and are replaced by two pairs of open terminals on the top and bottom wires. Voltage between the terminals on the left is marked as V1, while the voltage between the terminals on the right is marked as V2.

The drawing shows a box representing a Resistive network, having two input and two output terminals. The input terminals have voltage V1 marked between them, and connect outside to a 5.25 mV independent voltage source, in series with a 250 ohm resistor on the top wire that carries a rightward current I1 into the upper terminal. The output terminals have voltage V2 marked between them, and connect outside to a variable resistor R sub o located on the right wire. A leftward current, I2, flows into the upper terminal on the right.

The drawing shows a box with broken borders, having two input terminals marked as b11 and b21 and two output terminals marked as b12 and b22. The input terminals have voltage V1 marked between them and they connect outside to an independent voltage source V sub g, in series with a 6 ohm resistor on the top wire. The output terminals have voltage V2 marked between them and they connect outside to impedance Z sub L.

The drawing shows a box with broken borders, having two input terminals marked as h11 and h21 and two output terminals marked as h12 and h22. The input terminals connect outside to an independent voltage source V sub g, in series with impedance Z sub g located on the top wire. The output terminals have voltage V2 marked between them and they connect outside to impedance Z sub L.

The drawing shows a box representing an Amplifier, having two input and two output terminals. The input terminals have voltage V1 marked between them and they connect outside to a 120 angled 0 degree V(rms) independent voltage source, in series with a 20 ohm resistor on the top wire that carries a rightward current I1 into the upper terminal. The output terminals have voltage V2 marked between them and they connect outside to a 100 ohm resistor located on the right wire. A leftward current, I2, flows into the upper terminal on the right.

The drawing shows two circuits placed side by side. The circuit on the left shows a 25 ohm resistor and a 50 ohm resistor on the top wire, separated by a node, and a 12.5 mH inductor on the right wire, with a dot terminal to its top left. The left wire carries an independent voltage source, v sub g. Two nodes are located at the midpoints of the top and bottom wires, with voltage v1 marked between them.

The circuit on the right shows a 400 ohm resistor on the top wire and a 200 mH inductor on the left wire, with a dot terminal to its bottom right. The right wire carries a 1k ohm resistor. Two nodes are located at the midpoints of the top and bottom wires, with voltage v2 marked between them.

Mutual inductance between the inductors is marked as 0.75 k.

A box with broken borders is drawn in the middle, enclosing the right part of the left circuit and the left part of the right circuit.

The circuit shows three pairs of nodes on the top and bottom wires, with a vertical cross wire connecting the middle pair and carrying a 1 over s ohm capacitor. The voltage between the left pair of nodes is marked as V1, while the voltage between the right pair is marked as V2. Two inductors with individual inductances of s ohms are located on the top wire, between the first-second and second-third nodes. A portion of the circuit between the three nodes is enclosed in a box with broken borders.

The left arm of the top wire, outside the box, carries a 1 ohm resistor, with a rightward current I1 marked on it. The left wire of the circuit carries an independent voltage source, V sub g. The right arm of the top wire carries a leftward current I2. A 1 ohm resistor is located on the right wire.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires, carrying two capacitors with individual capacitances of 1 over sC. An inductor, sL, is located on the top wire, between the two cross wires. A rightward current, I1, flows in the left arm of the top wire, while a leftward current, I2, flows in the right arm. The middle part of the circuit, containing the cross wires, is enclosed in a shaded box.

The left and right wires are removed and are replaced by two pairs of open terminals on the top and bottom wires. Voltage between the terminals on the left is marked as V1, while voltage between the terminals on the right is marked as V2.

The circuit shows a two-port circuit labeled as g placed to the left of another two-port circuit labeled as h. The output terminals of circuit g are connected to the input terminals of circuit h. The input terminals of circuit g connect to an independent voltage source v sub g, in series with a 10 ohm resistor on the top wire. The output terminals of circuit h connect to a 15k ohm resistor on the right wire, with output voltage marked as v sub 0.

The circuit on top shows a two-port circuit labeled as 1 [h], placed to the left of another two-port circuit labeled as 2. The output terminals of circuit 1, labeled as c and d, are connected to the input terminals of circuit 2, also labeled as c and d. The input terminals of circuit 1, labeled as a and b, connect to an independent voltage source v sub g, in series with an 800 ohm resistor on the top wire. The output terminals of circuit 2, labeled as e and f, connect to a 72k ohm resistor on the right wire, with output voltage marked as v sub 0.

The circuit below shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying resistor R. The left and right arms of the top wire carry two resistors with individual resistances marked as R. An additional wire carrying resistor R is drawn across two nodes on the top wire. Value of R is marked as 72k ohm.

The left and right wires are removed and are replaced by two pairs of open terminals, labeled as c and d on the left and e and f on the right.

The circuit shows a two-port network A on the left, labeled as [a-dash], and a symmetric tee network B on the right. The input terminals of network A have voltage V1 marked between them and they connect outside to an independent voltage source V sub g, in series with impedance Z sub g on the top wire. The output terminals connect to the top and bottom wires of network B.

Network B shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a minus j6 ohm capacitor. The left arm of the top wire carries a 1 ohm resistor, in series with a j6 ohm inductor with a dot terminal located to its bottom right. The right arm of the top wire carries a 1 ohm resistor, in series with another j6 ohm inductor with a dot terminal located to its bottom right. Mutual inductance between the two inductors is marked as j5 ohm.

The output terminals of network B have voltage V2 marked between them and they connect outside to impedance Z sub L located on the right wire.

The drawing shows two circuits which are joined by their bottom wires. The circuit on the left shows an h11 ohm resistor on the top wire, with a rightward current I1 flowing through it, and an h12 V2 dependent voltage source on the right wire. The left wire is removed and is replaced by two open terminals having voltage V1.

The circuit on the right shows a 1 over h22 S resistor located on a parallel wire connecting the top and bottom wires, and a h21 I1 dependent current source with a downward current located on the left wire. The right wire is removed and is replaced by two open terminals having voltage V2.

The two circuits, excluding the open terminals at their extremes, are enclosed inside a shaded box.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying z21 impedance. The left arm of the top wire carries z11 minus z21 impedance, in series with a dependent voltage source I2 (z12 minus z21), while the right arm carries z22 minus z21 impedance.

The rightward current I1 flows in the left arm of the top wire, while the leftward current I2 flows in the right arm. The left and right wires are removed and are replaced by two pairs of open terminals on the top and bottom wires. Voltage between the terminals on the left is marked as V1, while the voltage between the terminals on the right is marked as V2.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, and carrying z12 impedance. The left arm of the top wire carries z11 minus z12 impedance, while the right arm carries z22 minus z12 impedance, in series with a dependent voltage source with reverse polarity, I1 (z21 minus z12).

The rightward current I1 flows in the left arm of the top wire, while the leftward current I2 flows in the right arm. The left and right wires are removed and are replaced by two pairs of open terminals on the top and bottom wires. Voltage between the terminals on the left is marked as V1, while voltage between the terminals on the right is marked as V2.

The Excel sheet shows A matrix entered in a square collection of cells, as follows:

[21, minus 9, 12]

[minus 3, 6, minus 2]

[minus 8, minus 4, 22]

Vector B is entered in a column of cells, as follows:

minus 33

3

50

Vector X is entered in a column of cells, as follows:

1

2

3

Function box shows the following function entered:

{equals MMULT(MINVERSE(E2:G4),J2:J4)}

The illustration defines three unknown variables and three simultaneous equations. A matrix and B vector are constructed, and function linsolve(A, B) is used to calculate values of unknowns.

MatLab interface is shown as follows:

>> syms i1 i2 i3

>> eq1 equals (21 multiplied by i1) minus (9 multiplied by i2) minus (12 multiplied by i3) equals minus 33;

>> eq2 equals (minus 3 multiplied by i1) plus (6 multiplied by i2) minus (2 multiplied by i3) equals3;

>> eq3 equals (minus 8 multiplied by i1) minus (4 multiplied by i2) plus (22 multiplied by i3) equals 50;

>> [A,B] = equationsToMatrix([eq1, eq2, eq3], [i1, i2, i3])

A equals:

[21, minus 9, minus 12]

[minus 3, 6, minus 2]

[minus 8, minus 4, minus 22]

B equals:

minus 33

3

50

>> X = linsolve(A,B)

X equals:

1

2

3

The Excel sheet shows A matrix entered in a square collection of cells, as follows:

[25, minus 5, minus 20, 0]

[minus 5, 10, minus 4, 0]

[minus 20, minus 4, 24, 15]

[1, 0, minus 1, minus 1]

Vector B is entered in a column of cells, as follows:

50

0

0

0

Vector X is entered in a column of cells, as follows:

29.6

26

28

1.6

Function box shows following function entered:

{equals MMULT(MINVERSE(E2:H5),K2:K5)}

Matlab interface shows the following equations and calculations:

>> syms i1 i2 ix

>> eq1 equals complex(13, minus14) multiplied by i1 plus complex(minus12,16) multiplied by i2 equals 150;

>> eq2 equals complex(minus12,16) multiplied by i1 plus complex(13, minus13) multiplied by i2 plus 39 multiplied by ix equals 0;

>> eq3 equals i1 minus i2 minus ix equals 0,

>> [A,B] equals equationsToMatrix([eq1, eq2, eq3], [i1, i2, ix])

A equals:

[13 minus 14i, minus 12 plus 16i, 0]

[minus12 plus 16i, 13 minus 13i, 39]

[1, minus1, minus1]

B equals:

150

0

0

>> X equals linsolve(A,B)

X equals:

minus 26 minus 52i

minus 24 minus 58i

minus 2 plus 6i

The circuit shows a horizontal cross wire connecting two nodes on the left and right wire, and a vertical cross wire connecting a node on the horizontal cross wire and the bottom wire. The node on the top of the vertical cross wire is marked as 1, while the node at the right end of the horizontal cross wire is marked as 2.

The vertical cross wire carries capacitor 1 over sC, with voltage V1 marked across it. The horizontal cross wire carries resistor R on its left arm, and capacitor 1 over sC on its right arm. A third capacitor with capacitance of 1 over sC is located on the top wire, while resistor R with voltage marked across it as V2 is located on the bottom arm of the right wire. The circuit is powered by an independent voltage source, V sub g, located on the bottom arm of the left wire.

The X-axis plots the real component Re, while the y-axis plots the imaginary component Im. The graph shows a diagonal straight line drawn from the origin, which moves with an upward slope at an angle theta with the x-axis. The tip of this line is marked as c. Dotted lines are drawn perpendicular from point c to both the axes, marking a on the x-axis and b on the y-axis.

The X-axis plots the real component Re, while the y-axis plots the imaginary component Im. The graph shows a diagonal straight line drawn from the origin, which moves with an upward slope at an angle theta with the x-axis. The line represents the complex number 5 angled 36.87 degree. Values of a and b are marked respectively as 4 and 3.

The following is written below the graph:

4 plus j3 equals 5 angled 36.87 degree.

The X-axis plots the real component Re, while the y-axis plots the imaginary component Im. The graph shows a diagonal straight line drawn from the origin in the second quadrant, which moves with an upward slope at angle theta, drawn with the positive x-axis. The line represents the complex number 5 angled 143.13 degree. Values of a and b are marked respectively as minus 4 and 3.

The following is written below the graph:

minus 4 plus j3 equals 5 angled 143.13 degree.

The X-axis plots the real component Re, while the y-axis plots the imaginary component Im. The graph shows a diagonal straight line drawn from the origin in the third quadrant, which moves with a downward slope at an angle theta, drawn with the positive x-axis. The line represents the complex number 5 angled 216.87 degree. Values of a and b are marked respectively as minus 4 and minus 3.

The following is written below the graph:

minus 4 minus j3 equals 5 angled 216.87 degree.

The X-axis plots the real component Re, while the y-axis plots the imaginary component Im. The graph shows a diagonal straight line drawn from the origin in the fourth quadrant, which moves with a downward slope at an angle theta, drawn with the positive x-axis. The line represents the complex number 5 angled 323.13 degree. Values of a and b are marked respectively as 4 and minus 3.

The following is written below the graph:

4 minus j3 equals 5 angled 323.13 degree.

The x-axis plots the real component Re, while the y-axis plots the imaginary component Im. The graph shows four lines in the four quadrants, which represent the following complex numbers:

n1 = a plus jb = c angled theta 1:

A diagonal line is drawn in the first quadrant, at an angle theta 1 with the positive x-axis, marking coordinates a on the x-axis and b on the y-axis.

n2 = minus a plus jb = c angled theta 2:

A diagonal line is drawn in the second quadrant, at an angle theta 2 with the positive x-axis, marking coordinates minus a on the x-axis and b on the y-axis.

n-asterix 2 = minus a minus jb = c angled minus theta 2:

A diagonal line is drawn in the third quadrant, at an angle minus theta 2 with the positive x-axis, marking coordinates minus a on the x-axis and minus b on the y-axis.

n-asterix 1 = a minus jb = c angled minus theta 1:

A diagonal line is drawn in the fourth quadrant, at an angle minus theta 1 with the positive x-axis, marking coordinates a on the x-axis and minus b on the y-axis.

The X-axis plots the real component Re, while the y-axis plots the imaginary component Im. The graph shows four coordinate points located in the four quadrants, with a circle passing through them, with the origin as its center. The four points and values they represent are as follows:

- First quadrant: (3, 1) - 3 angled 15 degree
- Second quadrant: (minus 1, 3) - 3 angled 105 degree
- Third quadrant: (minus 3, minus 1) - 3 angled 195 degree
- Fourth quadrant: (1, minus 3) - 3 angled 285 degree

The drawing shows two circuits with a pair of magnetically coupled coils placed close to each other.

The circuit on the left shows inductor L1 on the right wire, with a dot terminal to its top left; two nodes, a and b, located on the top and bottom wires close to the right wire; and resistor R1 on the top wire, carrying a rightward current i1. Voltage between nodes a and b is marked as v1.

The circuit on the right shows inductor L2 on the left wire, with a dot terminal to its top right; two nodes, c and d, located on the top and bottom wires close to the left wire; and resistor R1 on the top wire, carrying a leftward current i2. Voltage between nodes c and d is marked as v2.

Mutual inductance between L1 and L2 is marked as M. The central part of the drawing, including nodes a, b, c, and d, and excluding the resistors, is enclosed inside a shaded box.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying an inductor M. Two nodes, a and b, are marked the on top and bottom wires, to the left of the cross wire, while two more nodes, c and d, are marked on the top and bottom wires to the right. Voltage between a and b is v1 and voltage between c and d is v2. An inductor, L1 minus M, carrying a rightward current i1, is located between node a and the cross wire. Another inductor, L2 minus M, carrying a leftward current i2, is located between node c and the cross wire. The middle part of the circuit, including the four nodes, is enclosed inside a shaded box.

Resistor R1 is located on the top wire, to left of the shaded box, while another resistor R2 is located on the top wire, towards the right.

The circuit shows four parallel vertical cross wires connecting four pairs of nodes on the top and bottom wires. The cross wires on the left and right extreme carry two independent current sources with downward flowing currents, i1(0) and i2(0), respectively. The cross wires in the middle carry inductors marked as L sub A and L sub C, respectively, from the left to the right. A third inductor, L sub B, is located on the top wire, between the middle two cross wires.

The left wire is removed and is replaced by two open terminals on the top and bottom wires, marked as a and b, with voltage v1 marked between them. The rightward current i1 flows through the left arm of the top wire. Similarly, the right wire is removed and is replaced by two open terminals on the top and bottom wires, marked as c and d, with voltage v2 marked between them. The leftward current i2 flows through the right arm of the top wire.

The circuit shows four parallel vertical cross wires connecting four pairs of nodes on the top and bottom wires. The cross wires on the left and right extreme carry two independent current sources with downward flowing currents, i1(0) and i2(0), respectively. The left cross wire in the middle carries an inductor with inductance marked as (L1L2 minus M-square) over (L2 minus M). The right cross wire in the middle has an inductor with inductance marked as (L1L2 minus M-square) over (L1 minus M). A third inductor, (L1L2 minus M-square) over M, is located on the top wire, between these two cross wires.

Two nodes, a and b, are located on the top and bottom wires to the left of the first cross wire on the left. Voltage between these two nodes is marked as v1. Similarly, two nodes, c and d, with voltage v2 marked between them, are located on the top and bottom wires to the right of the last cross wire on the right. The middle part of the circuit, including these four nodes and the four cross wires, is enclosed in a shaded rectangular box.

The left arm of the top wire carries resistor R1 with a rightward current i1, while the right arm of the top wire carries resistor R2 with a leftward current i2. The left and right wires of the circuit are removed and are replaced by open terminals on the top and bottom wires.

The circuit shows two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries an inductor with inductance marked as (L1L2 minus M-square) over (L2 minus M). The right cross wire has an inductor with inductance marked as (L1L2 minus M-square) over (L1 minus M). A third inductor, (L1L2 minus M-square) over M, is located on the top wire between these two cross wires.

Two nodes, a and b, are located on the top and bottom wires to the left of the left cross wire. Voltage between these two nodes is marked as v1. Similarly, two nodes, c and d, with voltage v2 marked between them, are located on the top and bottom wires to the right of the right cross wire. The middle part of the circuit, including these four nodes and two cross wires, is enclosed in a shaded rectangular box.

The left arm of the top wire carries resistor R1 with a rightward current i1, while the right arm of the top wire carries resistor R2 with a leftward current i2. The left and right wires of the circuit are removed and are replaced by open terminals on the top and bottom wires.

The drawing shows two circuits with a pair of magnetically coupled coils placed close to each other.

The circuit on the left shows a j3600 ohm inductor on the right wire, with a dot terminal to its top left; two nodes, a and b, located on the top and bottom wires, close to the right wire; and a 200 ohm resistor on the top wire, close to the right wire. Voltage between nodes a and b is marked as V1.

The circuit on the right shows a j1600 ohm inductor on the left wire, with a dot terminal to its top right; two nodes located on the top and bottom wires, close to the left wire; and a 100 ohm resistor on the top wire, close to the left wire. Voltage between the nodes is marked as V2. Mutual inductance between the two inductors in the middle is marked as j1200 ohm. The central part of the drawing, including the four nodes, is enclosed in a shaded rectangular box.

The left circuit is powered by a 300 angled 0 degree V independent voltage source on the left wire, and carries a 500 ohm resistor, in series with a j100 ohm inductor, on the top wire with a rightward current marked as I1, all outside the shaded box. The right circuit carries a minus j2500 ohm capacitor with a downward current I2 on its right wire, and an 800 ohm resistor on its top wire, located outside the shaded box.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a 3 H inductor. The top wire carries a 6 H inductor on the left arm and a 1 H inductor on the right arm. The left and right wires are removed and are replaced by two pairs of open terminals on the top and bottom wires.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a j1200 inductor. The top wire carries a j2400 inductor on the left arm and a j400 inductor on the right arm. The left and right wires are removed and are replaced by two pairs of open terminals on the top and bottom wires.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a j1200 ohm inductor, with voltage marked across it as V. The left arm of the top wire carries a 500 ohm resistor, a j100 ohm inductor, a 200 ohm resistor, and a j2400 ohm inductor from the left to the right, with a rightward current marked as I1. The right arm of the top wire carries a j400 ohm inductor and a 100 ohm resistor, with a rightward current marked as I2. An 800 ohm resistor, in series with minus j2500 ohm capacitor, is located on the right wire. The circuit is powered by a 300 angled 0 degree V independent voltage source located on the left wire.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a minus j1200 ohm capacitor. The top wire carries a j4800 ohm inductor on the left arm and a j2800 ohm inductor on the right arm. The left and right wires are removed and are replaced by two pairs of open terminals on the top and bottom wires.

The circuit shows a vertical cross wire connecting two nodes on the top and bottom wires, carrying a minus j1200 ohm inductor, with the voltage marked across it as V. The left arm of the top wire carries a 500 ohm resistor, a j100 ohm inductor, a 200 ohm resistor, and a j4800 ohm inductor from the left to the right, with a rightward current marked as I1. The right arm of the top wire carries a j2800 ohm inductor and a 100 ohm resistor, with a rightward current marked as I2. An 800 ohm resistor, in series with a minus j2500 ohm capacitor, is located on the right wire. The circuit is powered by a 300 angled 0 degree V independent voltage source located on the left wire. An additional node is located on the bottom wire, close to the right wire.

The drawing shows two circuits with an ideal transformer between them, with a turns ratio of 1 is to a. The inductor on the left has a dot terminal to its top left, while the inductor on the right has a dot terminal to its top right.

The circuit on the left has a vertical cross wire connecting two nodes on the top and bottom wires, carrying inductor M over a. The left arm of the top wire carries L1 minus M over a inductor with a rightward current i1, while the right arm carries L2 by a-square minus M over a inductor. Two nodes are located on the top and bottom wires, close to the right wire. The left wire of the circuit is removed and is replaced by two open terminals, with voltage v1 marked across them.

The circuit on the right shows a leftward current i2 on its top wire. Its right wire is removed and is replaced by two open terminals, with voltage v2 marked across them.

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More information

VtSource™

The drawing shows two circuits with an ideal transformer between them, with a turns ratio of 1 is to a. The inductor on the left has a dot terminal to its top left, while the inductor on the right has a dot terminal to its top right.

The circuit on the left has two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries inductor (L2L1 minus M-square) over (L2 minus Ma). The right cross wire carries inductor (L2L1 minus M-square) over (a-square L1 minus Ma). The middle arm of the top wire carries (L1L2 minus M-square) over Ma inductor. The left arm of the top wire shows a rightward current i1. Two nodes are located on the top and bottom wires, close to the right wire. The left wire of the circuit is removed and is replaced by two open terminals, with voltage v1 marked across them.

The circuit on the right shows a leftward current i2 on its top wire. Its right wire is removed and is replaced by two open terminals, with voltage v2 marked across them.

The drawing shows two circuits with an ideal transformer between them, with a turns ratio of 1 is to a. The inductor on the left has a dot terminal to its top left, while the inductor on the right has a dot terminal to its top right.

The circuit on the right has two parallel vertical cross wires connecting two pairs of nodes on the top and bottom wires. The left cross wire carries an inductor a-square (L1L2 minus M-square) over (L2 minus Ma). The right cross wire carries an inductor a-square (L1L2 minus M-square) over (a-square L1 minus Ma). The middle arm of the top wire carries a(L1L2 minus M-square) over M inductor. The right arm of the top wire shows a leftward current i2. Two nodes are located on the top and bottom wires, close to the left wire. The right wire of the circuit is removed and is replaced by two open terminals, with voltage v2 marked across them.

The circuit on the left shows a rightward current i1 on the top wire. Its left wire is removed and is replaced by two open terminals, with voltage v1 marked across them.

The drawing shows two circuits with an ideal transformer between them, with a turns ratio of N1N2. The inductor on the left has a dot terminal to its top left, while the inductor on the right has a dot terminal to its top right.

The circuit on the left has a vertical cross wire connecting two nodes on the top and bottom wires, carrying inductor M over a. The left arm of the top wire carries L1 minus M by a inductor, with a rightward current i1, while the right arm carries L2 by a-square minus M by a inductor, with a leftward current i sub 0. Two nodes are located on the top and bottom wires, close to right wire, with voltage v sub 0 marked across them. The left wire of the circuit is removed and is replaced by two open terminals, with voltage v1 marked across them.

The circuit on the right shows a leftward current i2 on its top wire. Its right wire is removed and is replaced by two open terminals, with voltage v2 marked across them.

The drawing shows two circuits with an ideal transformer between them, with a turns ratio of 1 is to a. The inductor on the left has a dot terminal to its top left, while the inductor on the right has a dot terminal to its top right.

The circuit on the left has a vertical cross wire connecting two nodes on the top and bottom wires, carrying inductor L1. The left arm of the top wire carries a rightward current i1, while the right arm carries the L1(1 over k-square minus 1) inductor. Two nodes are located on the top and bottom wires, close to the right wire. The left wire of the circuit is removed and is replaced by two open terminals, with voltage v1 marked across them.

The circuit on the right shows a leftward current i2 on its top wire. Its right wire is removed and is replaced by two open terminals, with voltage v2 marked across them.

The drawing shows two circuits with an ideal transformer between them, with a turns ratio of 1 is to a. The inductor on the left has a dot terminal to its top left, while the inductor on the right has a dot terminal to its top right.

The circuit on the right has a vertical cross wire connecting two nodes on the top and bottom wires, carrying inductor k-square multiplied by L2. The right arm of the top wire carries (1 minus k-square)L2 inductor, with a leftward current i2. Two nodes are located on the top and bottom wires, close to the left wire. The right wire of the circuit is removed and is replaced by two open terminals, with voltage v2 marked across them.

The circuit on the left shows a rightward current i1 on its top wire. Its left wire is removed and is replaced by two open terminals, with voltage v1 marked across them.

The drawing shows two circuits with an ideal transformer between them, with a turns ratio of 1 is to a. The inductor on the left has a dot terminal to its top left, while the inductor on the right has a dot terminal to its top right.

The circuit on the left has a vertical cross wire connecting two nodes on the top and bottom wires, carrying inductor k-square multiplied by L1. The left arm of the top wire carries the L1(1 minus k-square) inductor, with a rightward current i1. Two nodes are located on the top and bottom wires, close to the right wire. The left wire of the circuit is removed and is replaced by two open terminals, with voltage v1 marked across them.

The circuit on the right shows a leftward current i2 on its top wire. Its right wire is removed and is replaced by two open terminals, with voltage v2 marked across them.

The drawing shows two circuits with an ideal transformer between them, with a turns ratio of 1 is to a. The inductor on the left has a dot terminal to its top left, while the inductor on the right has a dot terminal to its top right.

The circuit on the right has a vertical cross wire connecting two nodes on the top and bottom wires, carrying inductor L2. The left arm of the top wire carries L2(1 by k-square minus 1) inductor, while the right arm shows a leftward current i2. Two nodes are located on the top and bottom wires, close to the left wire. The right wire of the circuit is removed and is replaced by two open terminals, with voltage v2 marked across them.

The circuit on the left shows a rightward current i1 on its top wire. Its left wire is removed and is replaced by two open terminals, with voltage v1 marked across them.

The drawing shows two circuits with magnetic coupled coils between them. The coil on the left has inductance of jwL1 and a turns ratio of N1, and shows a dot terminal to its top left. The coil on the right has inductance of jwL2 and a turns ratio of N2, and shows a dot terminal to its top right.

The circuit on the left shows an independent voltage source V1 on the left wire and current I1 on the top wire. The circuit on the right has its right wire removed and replaced by two open terminals, with voltage V2 marked across them.

The drawing shows two circuits with an ideal transformer between them. The ideal transformer has a turns ratio of 1 is to minus 1 by 3. It has an inductor to its left, having a dot terminal to its top left and voltage V sub a, and another inductor to its right, having a dot terminal to its top right and voltage V sub b.

The circuit on the left shows a vertical cross wire connecting a pair of nodes on the top and bottom wires, carrying j3600 ohm inductor. The left arm of the top wire carries a 500 ohm resistor, in series with a j100 ohm inductor and a 200 ohm resistor. The rightward current I1 flows through the left arm. The right arm of the top wire carries a j10,800 ohm inductor, with a rightward current I sub a. The circuit is powered by a 300 angled 0 degree V independent voltage source, located on the left wire.

The circuit on the right shows a 100 ohm resistor on its top wire, with a leftward current I sub b marked to the left of the resistor and a rightward current I2 marked to its right. The right wire carries an 800 ohm resistor, in series with a minus j2500 ohm capacitor.

The drawing shows three boxes placed side by side, representing circuits A, B, and C from the left to the right. The output terminals of circuit A are connected to the input terminals of circuit B. Similarly, the output terminals of circuit B are connected to the input terminals of circuit A.

The power input and output at circuit A are marked as p sub i and p1, at circuit B they are marked as p1 and p2, and for circuit C they are marked as p2 and p sub o.

The X-axis plots frequency omega in rad per second, and marks from 1 to 10 in increments of 1, and further from 10 to 50 in increments of 10. The Y-axis plots Amplitude A in dB, ranging from minus 5 to 25 in increments of 5. A grid of horizontal and vertical lines spaced unevenly is drawn all over this graph. The graph shows a horizontal line intersecting a diagonal line at a point marked as z1, whose coordinates are (0, 4). The horizontal line is drawn from the origin to z1. The diagonal line is drawn with an upward slope from z1 to approximately (50, 22). The X-coordinate of the tip of the diagonal line is marked on the horizontal axis as 10 z1. The horizontal distance between z1 and 10z1 represents a Decade.

The diagonal line represents 20 log 10 (omega over z1). The slope of the line is marked as 20 dB per decade.

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The diagonal line represents 20 log 10 (omega over z1). The slope of the line is marked as 20 dB per decade.

The X-axis plots frequency omega in rad per second, and marks from 1 to 10 in increments of 1, and further from 10 to 50 in increments of 10. The Y-axis plots Amplitude A in dB, ranging from minus 20 to 5 in increments of 5. A grid of horizontal and vertical lines spaced unevenly is drawn all over this graph. The graph shows a horizontal line intersecting a diagonal line at a point marked as p1, whose coordinates are (0, 4). The horizontal line is drawn from the origin to p1. The diagonal line is drawn with a downward slope from p1 to approximately (40, minus 20). The X-coordinate of the tip of the diagonal line is marked on the horizontal axis as 10 p1.

The diagonal line represents minus 20 log 10 (omega over p1). The slope of the line is marked as minus 20 dB per decade.

The X-axis plots frequency omega in rad per second, and marks 1, 5, 10, 50, 100, 500, and 1000. The Y-axis plots Amplitude A in dB, ranging from minus 60 to 40 in increments of 10. A grid of horizontal and vertical lines spaced unevenly is drawn all over this graph. The graph shows four broken lines that represent straight line approximations of amplitude plots of zeroes and poles as follows:

- A line representing 20 log 10 mod j omega is drawn with an upward slope from (0, 0) to (40, 90).
- A horizontal line representing 20 log 10 of 0.11 is drawn from y = minus 20.
- A line representing minus 20 log 10 mod 1 plus j multiplied by numerator omega over denominator 10 is drawn starting from the origin to (10, 0), from where it moves with a downward slope to end at (6000, minus 60).
- A line representing minus 20 log 10 mod 1 plus j multiplied by numerator omega over denominator 100 is drawn starting from the origin to (100, 0), from where it moves with a downward slope to end at (10,000, minus 40).
- Three points are marked on this line, with respective y-coordinates of (minus 0.311), (minus 12.5), and (minus 19.22).
- The sum of these four lines is represented by a colored line, labeled as 20 log 10 mod H of j omega. This line starts at y = minus 20 and moves with an upward slope to (1, 10), then moves horizontally to (1, 10), and then moves with a downward slope to end at (10,000, minus 40).

The circuit shows a 100 mH inductor in series with a 10 mF capacitor on the top wire, an 11 ohm resistor on the right wire, where voltage is marked across as v sub o, and an independent voltage source, v sub i, located on the left wire.

The X-axis plots frequency omega in rad per second, and marks 1, 5, 10, 50, 100, 500, and 1000. The Y-axis plots Amplitude A in dB, ranging from minus 60 to 40 in increments of 10. A grid of horizontal and vertical lines spaced unevenly is drawn all over this graph. The graph shows four broken lines that represent straight line approximations of amplitude plots of zeroes and poles as follows:

- A line representing 20 log 10 mod j omega is drawn with an upward slope from (0, 0) to (40, 90).
- A horizontal line representing 20 log 10 of 0.11 is drawn from y = minus 20.
- A line representing minus 20 log 10 mod 1 plus j multiplied by numerator omega over denominator 10 is drawn starting from the origin to (10, 0), from where it moves with a downward slope to end at (6000, minus 60).
- A line representing minus 20 log 10 mod 1 plus j multiplied by numerator omega over denominator 100 is drawn starting from the origin to (100, 0), from where it moves with a downward slope to end at (10,000, minus 40).
- Three points are marked on this line, with respective y-coordinates of (minus 0.311), (minus 12.5), and (minus 19.22).
- The sum of these four lines is represented by a colored line, labeled as 20 log 10 mod H of j omega. This line starts at y = minus 20 and moves with an upward slope to (1, 10), then moves horizontally to (1, 10), and then moves with a downward slope to end at (10,000, minus 40).

The X-axis plots frequency omega, and marks omega sub c over 2, omega sub c, and 2 omega sub c. The Y-axis plots amplitude A in dB, ranging from minus 25 to 25 in increments of 5. A grid of horizontal and vertical lines spaced unevenly is drawn all over this graph. The graph shows a broken-line curve, representing a first-order zero, that starts from the origin and moves with an upward slope up to amplitude value of 20. Another broken-line curve, representing a first-order pole, starts from the origin and curves down in a downward slope to an amplitude value of minus 20.

A straight line approximation of the amplitude plot is drawn as a horizontal line between the origin and the corner frequency, omega sub c. The line splits at (0, omega sub c) into three lines. One line moves as an upward sloping straight line, tangential to the first-order zero plot, while another line moves as a downward sloping straight line, tangential to the first-order pole plot. Another line moves along the horizontal axis.

Corrections of the two first-order plots from the amplitude plot are marked as plus 1 dB and minus 1 dB at one-half corner frequency (omega sub c, over 2), as plus 3 dB and minus 3 dB at the corner frequency (omega sub c), and as plus 1 db and minus 1 dB at twice the corner frequency (2 omega sub c).

The X-axis plots frequency omega in rad per second, and marks z1 over 10, p1 over 10, z1, p1, 10z1, and 10p1. The Y-axis plots theta (omega), ranging from minus 90 degrees to 90 degrees, in increments of 30 degrees. A grid of horizontal and vertical lines spaced unevenly is drawn all over this graph. The graph shows a straight-line approximation for a first-order zero, which moves as a horizontal line from the origin to (0, z1 over 10), then moves up with an upward slope to reach (10z1, 90), and then moves as a horizontal line. A dashed-curve is drawn almost in line with the straight-line approximation, representing the actual plot of the phase angle versus frequency. It is labeled as phi1= tan raised to minus 1 (omega over z1).

A straight-line approximation for a first-order pole is drawn as a horizontal line from the origin to (0, p1 over 10), which then moves with a downward slope to reach (10p1, minus 90), and then moves as a horizontal line. A dashed-curve is drawn almost in line with the straight-line approximation, representing the actual plot of phase angle versus frequency. It is labeled as minus Beta 1= minus tan raised to minus 1 (omega over p1).

The X-axis plots omega in rad per second, and marks 1, 5, 10, 50, 100, 500, and 1000. The Y-axis plots theta (omega), ranging from minus 120 degrees to 90 degrees in increments of 30 degrees. A grid of horizontal and vertical lines spaced unevenly is drawn all over this graph. Three straight-line approximations are drawn for three phase angles in broken lines, as follows:

Psi 1 = 90 degrees: A straight horizontal line drawn from y = 90 degrees.

Minus Beta 1 = minus tan raised to minus 1 (omega over 10): A straight horizontal line drawn along the x-axis from the origin to (0, 1), which moves with a downward slope from (0, 1) to (100, minus 90), and then runs horizontally up to (1000, minus 90).

Minus Beta 2 = minus tan raised to minus 1 (omega over 100): A straight horizontal line drawn along the x-axis from the origin to (0, 10), which then moves with a downward slope to (1000, minus 90). A point is marked on this line with a y-coordinate marked as (minus 77.54).

A straight-line approximation of theta (omega) is drawn as a colored line that starts at y = 90 degrees and runs along the x-axis up to (1, 90), and then moves with a downward slope to approximately (10, 45). The line continues with a steeper slope to (100, minus 45), and further with a less steeper slope to (1000, minus 90). It then runs horizontally. This line is marked as theta (omega) = Psi 1 minus Beta 1 minus Beta 2. Two points are marked on this line with the respective y-coordinates as (minus 15.25) and (minus 83.72).

The X-axis plots omega in rad per second, and marks omega sub n and 10 omega sub n. The Y-axis plots amplitude A in dB, ranging from minus 50 to 20 in increments of 10. A grid of horizontal and vertical lines spaced unevenly is drawn all over this graph. The graph shows an approximate amplitude plot consisting of two straight lines. One line runs along the x-axis, between the origin and omega = omega sub n. The second line runs with a downward slope for omega greater than omega sub n. The slope of this line is marked as minus 40 dB per decade. These two lines meet at omega = omega sub n.

The X-axis plots omega in rad per second, and marks omega sub n. The Y-axis plots amplitude A in dB, ranging from minus 50 to 20 in increments of 10. A grid of horizontal and vertical lines spaced unevenly is drawn all over this graph. The graph shows a straight-line amplitude plot as a horizontal line drawn from the origin to (0, omega sub n), which then moves with a downward slope for omega value exceeding omega sub n. Three variations of amplitude plot are depicted for different values of damping coefficient, zeta. When zeta = 0.707, the amplitude plot is shown as a broken line which curves below the straight-line plot, to meet it at a point where y-coordinate value is (minus 12). When zeta = 0.3, the amplitude plot is shown as a broken line which rises above the straight-line plot to reach a peak at (omega sub n, 5), and then curves down to meet the straight-line plot at y-coordinate value of (minus 11). Finally, when zeta = 0.1, the amplitude plot rises above the straight-line plot to the highest peak of (omega sub n, 15), and then curves down to meet the straight-line plot at y-coordinate value of (minus 11).

The X-axis plots omega in rad per second, and marks omega sub n over 2, omega sub p, omega sub n, and omega sub 0. The Y-axis plots amplitude A in dB, ranging from minus 7 to 3 in increments of 1. A grid of horizontal and vertical lines spaced unevenly is drawn all over this graph. The graph shows a straight-line amplitude plot as a horizontal line drawn from the origin to (0, omega sub n), which then moves with a downward slope for omega value exceeding omega sub n.

Four points are marked to locate the corrected amplitude plot at the following coordinate points:

1: (omega sub n over 2, 1.4)

2: (omega sub p, 2.8)

3: (omega sub n, 2)

4: (omega sub 0, 0)

A corrected amplitude plot is drawn starting just above the origin on the y-axis and rising up on a short upward slope, passing through point 1, peaking at point 2, curving down to move through point 3, moving almost vertically down to point 4, and then running parallel to the straight-line amplitude plot.

The circuit shows a 50 mH inductor in series with a 1 ohm resistor on the top wire, an 8 mF capacitor on the right wire, where voltage is marked across as v sub o, and an independent voltage source, v sub i, located on the left wire.

The X-axis plots omega in rad per second, and marks 10, 50, 100, and 500. The Y-axis plots amplitude A in dB, ranging from minus 45 to 15 in increments of 5. A grid of horizontal and vertical lines spaced unevenly is drawn all over this graph. The graph shows a straight-line amplitude plot as a horizontal line drawn from the origin to (0, 50), which then moves with a downward slope for omega value exceeding 50.

Four points are marked to locate corrected amplitude plot at the following coordinate points:

1: (27, 2)

2: (48, 8)

3: (51, 7)

4: (70, 0)

A corrected amplitude plot is drawn starting just above the origin on the y-axis and rising up on a short upward slope, passing through point 1, peaking at point 2, curving down to move through point 3, moving down to point 4, and then running parallel to the straight-line amplitude plot.

The X-axis plots u, and marks 0.2, 0.4, 1.0, 2, 4, and 8. The Y-axis plots theta (omega), ranging from minus 180 degrees to 15 degrees in increments of 15 degrees. A grid of horizontal and vertical lines spaced unevenly is drawn all over this graph.

Three variations of phase angle plot are depicted for different values of damping coefficient, zeta. When zeta = 0.707, the phase angle plot starts from (0.2, minus 15) and moves with a downward slope, passing through (1.0, minus 90). From this point, the line curves slightly to the right, passing through (2, minus 135), and ends at (6, minus 165).

When zeta = 0.3, the phase angle plot starts from (0.2, minus 7) above the previous line and curves down to reach (1.0, minus 90). From this point, the line moves below the previous curve, moving with a steeper slope and curving to the right at its bottom end, ending at (6, minus 175).

Finally, when zeta = 0.1, the phase angle plot starts from (0.2, 0) above the two previous lines, and curves down to reach (1.0, minus 90). The line then moves almost vertically down in a straight line below the two previous lines, curving to the right at its bottom end and finishing at (6, minus 178).

The X-axis plots u, and marks 1.0 and 2.0. The Y-axis plots theta (omega), ranging from minus 180 degrees to 0 degrees in increments of 30 degrees. A grid of horizontal and vertical lines spaced unevenly is drawn all over this graph.

A straight-line approximation is drawn by plotting three segments. A horizontal line is drawn at the top for omega value less than (0.62 = 4.81 raised to minus zeta). Another horizontal line is drawn at the bottom for omega value greater than (1.6 = 4.81 raised to zeta). A downward sloping line is drawn connecting these two lines, passing through (1.0, minus 90), with the slope marked as 440 degrees per decade (7.67 rad per decade).

The actual curve is depicted moving closely with the straight-line approximation, curving below the straight line at the top, and curving away toward the right at the bottom.

The circuit shows a 250mH inductor on the top wire, a 1 ohm resistor in series with a 40mF capacitor on the right wire, where voltage is marked across as v sub o, and an independent voltage source, v sub i, located on the left wire.

The X-axis plots frequency omega in rad per second, and marks 1, 5, 10, 50, 100, 500, and 1000. The Y-axis plots theta of omega, ranging from minus 180 degrees to 135 degrees in increments of 15 degrees. A grid of horizontal and vertical lines spaced unevenly is drawn all over this graph.

The graph shows the plot of psi of omega as a broken line that runs horizontally from (0, 1) to (0, 2.5), and then moves with an upward slope to reach (200, 90), from where it continues to run horizontally.

A second plot of minus Beta 1 of omega is shown as a broken line that moves with a downward slope from (2.5, minus 40) to (100, minus 180), from where it continues to run horizontally.

The phase angle plot is shown as a colored line, theta of omega, that runs horizontally from the origin to (0, 1), then moves with a downward slope to intersect minus B1 angled plot at (2.5, minus 40), and then continues downward with a slightly less steeper slope up to (100, minus 105) (approximately). The line then rises up slightly to reach (200, minus 90) from where it continues to run horizontally.