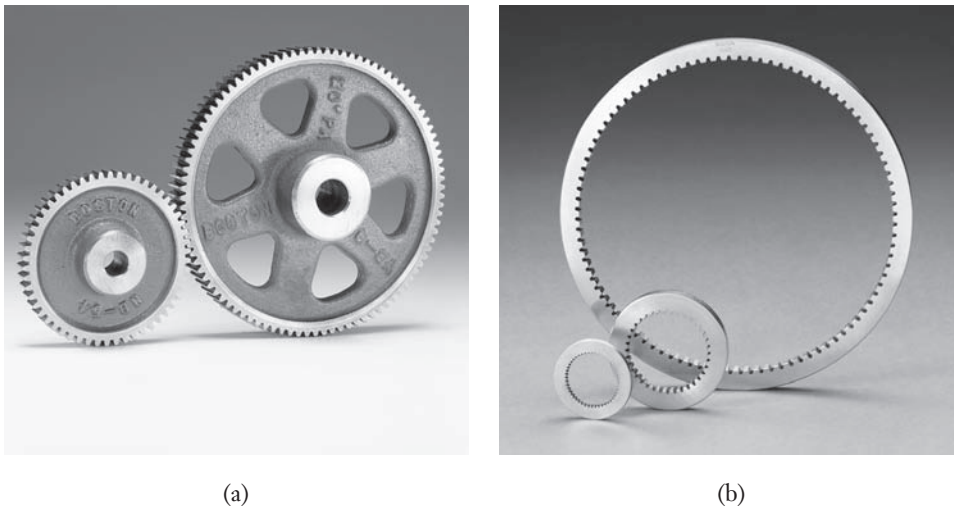


**Figure 8.9**

(a) Two external spur gears in mesh. (b) Internal or ring gears of several sizes.

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Gearset

engage and motion is transmitted from one shaft to another, the two gears are said to form a *gearset*. Figure 8.10 depicts a spur gearset and some of the terminology used to describe the geometry of its teeth. By convention, the smaller (driving) gear is called the *pinion*, and the other (driven) one is

Pinion**Figure 8.10**

Terminology for a spur gearset and the geometry of its teeth.

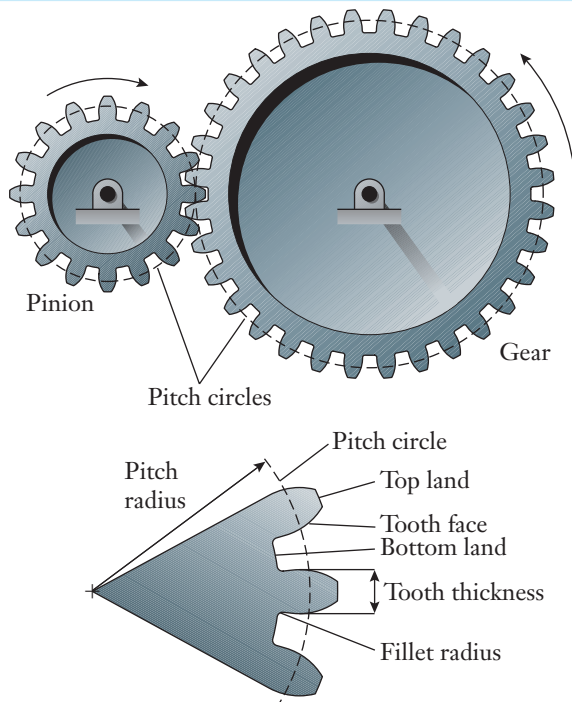
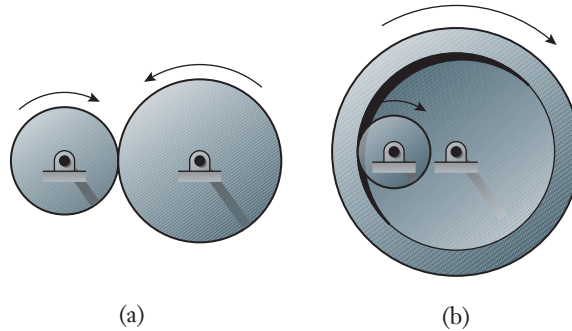


Figure 8.11

Configurations of gearsets having (a) two external gears and (b) one external and one internal gear. In each case, the rotations are analogous to two cylinders that roll on one another.



simply called the gear. The pinion and gear mesh at the point where the teeth approach, contact one another, and then separate.

Although a collection of many individual teeth are continuously contacting, engaging, and disengaging in a gearset, the pinion and gear can be regarded conceptually as two cylinders that are pressed against one another and that smoothly roll together. As illustrated in Figure 8.11, those cylinders roll on the outside of one another for two external gears, or one can roll within the other if the gearset comprises external and internal gears. Referring to the nomenclature of Figure 8.10, the effective radius r of a spur gear (which is also the radius of its conceptual rolling cylinder) is called the *pitch radius*. Continuous contact between the pinion and gear is imagined to take place at the intersection of the two *pitch circles*. The pitch radius is not the distance from the gear's center to either the top or bottom lands of a tooth. Instead, r is simply the radius that an equivalent cylinder would have if it rotated at the same speed as the pinion or gear.

The thickness of a tooth and the spacing between adjacent teeth are measured along the gear's pitch circle. The tooth-to-tooth spacing must be slightly larger than the tooth's thickness itself to prevent the teeth from binding against one another as the pinion and gear rotate. On the other hand, if the space between the teeth is too large, the clearance and free play could cause undesirable rattle, vibration, speed fluctuations, fatigue wear, and eventual gear failure. In the USCS, the proximity of teeth to one another is measured by a quantity called the *diametral pitch*

$$p = \frac{N}{2r} \quad (8.6)$$

where N is the number of teeth on the gear, and r is the pitch radius. The diametral pitch has the units of teeth per inch of the gear's diameter. For a pinion and gear to mesh, they must have the same diametral pitch or the gearset will bind as the shafts begin to rotate. The product catalogs supplied by gear manufacturers will classify compatible gears according to their diametral pitch. By convention, values of $p < 20$ teeth/in. are regarded as being so-called coarse gears, and gears with $p \geq 20$ teeth/in. are said to

be fine pitched. Because r and N are proportional in Equation (8.6), if the number of teeth on a gear is doubled, its radius will likewise double. In the SI, the spacing between teeth is measured not by the diametral pitch, but by a quantity called the *module*

$$m = \frac{2r}{N} \quad (8.7)$$

with the units of millimeters.

In principle, pinions and gears with teeth having any complementary shape are capable of transmitting rotation between their respective shafts. However, arbitrarily shaped gear teeth may not be able to transmit consistent motion because the pinion teeth may not efficiently engage and disengage with the gear teeth. Therefore, the tooth shape on modern spur gears has been mathematically optimized so that motion is smoothly transferred between the pinion and gear. Figure 8.12 depicts a small portion of a gearset during three stages of its rotation. As the teeth on the pinion and gear begin to mesh, the teeth roll on the surface of one another, and their point of contact moves from one side of the pitch circles to the other. The cross-sectional shape of the tooth is called the *involute profile*, and it compensates for the fact that the tooth-to-tooth contact point moves during meshing. The term “involute” is synonymous with “intricate,” and this special shape for spur gear teeth ensures that, if the pinion turns at a constant speed, the gear will also. This

Figure 8.12

A pair of teeth in contact.

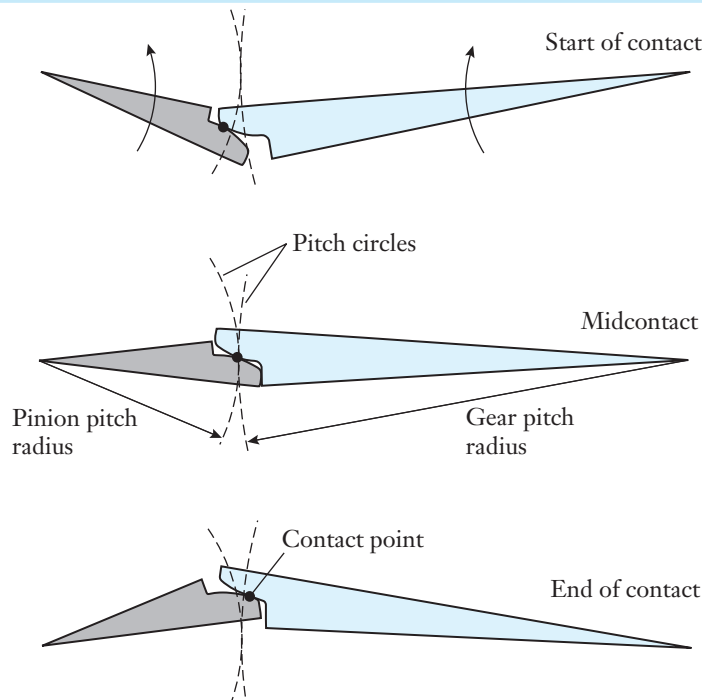
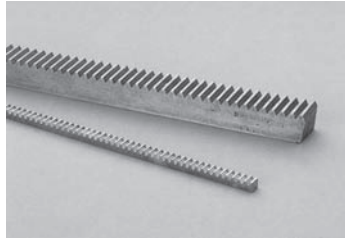


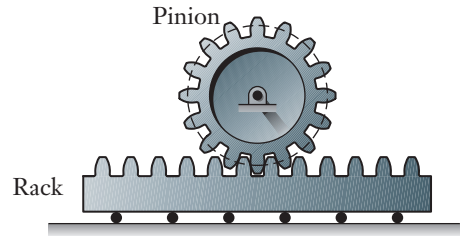
Figure 8.13

- (a) Gear racks.
(b) Rack-and-pinion mechanism.

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by Boston Gear Company.



(a)



(b)

Fundamental property of gearsets

mathematical characteristic of the involute profile establishes what is known as the *fundamental property of gearsets*, enabling engineers to view gearsets as two cylinders rolling on one another:

For spur gears with involute-shaped teeth, if one gear turns at a constant speed, so will the other.

We will use this property throughout the remainder of this chapter to analyze gearsets and geartrains.

Rack and Pinion

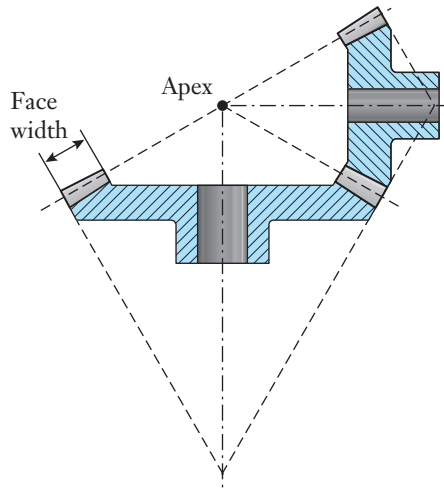
Gears are sometimes used to convert the rotational motion of a shaft into the straight-line, or translational, motion of a slider (and vice versa). The rack-and-pinion mechanism is the limiting case of a gearset in which the gear has an infinite radius and tends toward a straight line. This configuration of a rack and pinion is shown in Figure 8.13. With the center point of the pinion fixed, the rack will move horizontally as the pinion rotates—leftward in Figure 8.13(b) as the pinion turns clockwise. The rack itself can be supported by rollers, or it can slide on a lubricated smooth surface. Racks and pinions are often used in the mechanism for steering an automobile's front wheels, and for positioning machine tables in a milling or grinding machine where the cutting head is stationary.

Bevel Gears

Whereas the teeth of spur gears are arranged on a cylinder, a bevel gear is produced by alternatively forming teeth on a blank that is shaped like a truncated cone. Figure 8.14 depicts a photograph and cross-sectional drawing of a bevel gearset. You can see how its design enables a shaft's rotation to be redirected by 90° . Bevel gears (Figure 8.15) are appropriate for applications in which two shafts must be connected at a right angle and where extensions to the shaft centerlines would intersect one another.

Helical Gears

Because the teeth on spur gears are straight with faces parallel to their shafts, as two teeth approach one another, they make contact suddenly along the full width of each tooth. Likewise, in the meshing sequence of Figure 8.12,

**Figure 8.14**

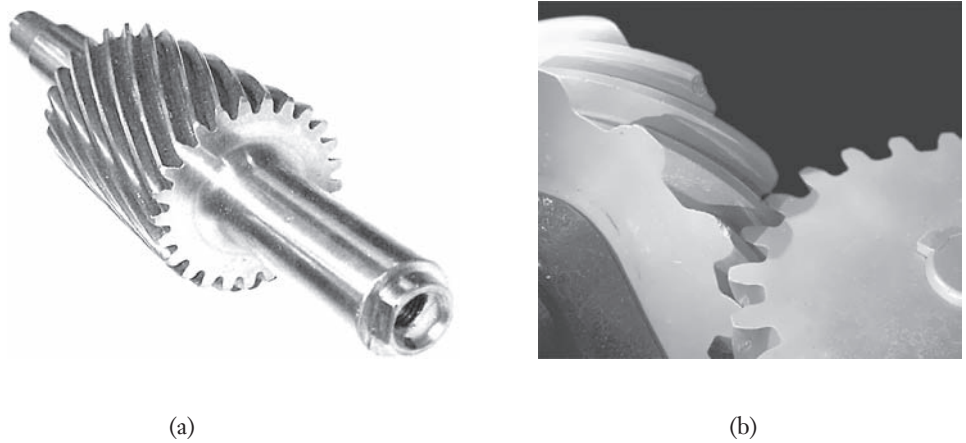
Two bevel gears in mesh.

Reprinted with permission by Boston Gear Company.

**Figure 8.15**

A collection of bevel gears, some having straight and others having spiral-shaped teeth.

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**Figure 8.16**

(a) A helical gear. (b) A Pair of crossed helical gears in mesh.

Image courtesy of the authors.

the teeth separate and lose contact along the tooth's entire width at once. Those relatively abrupt engagements and disengagements result in spur gears producing more noise and vibration than other types of gears.

Helical gears are an alternative to spur gears, and they offer the advantage of smoother and quieter meshing. Helical gears are similar to their spur counterparts in the sense that the teeth are still formed on a cylinder but not parallel to the gear's shaft. As their name implies, the teeth on a helical gear are instead inclined at an *angle* so that each tooth wraps around the gear in the shape of a shallow helix (Figure 8.16). With the same objective of having teeth mesh gradually, some of the *bevel gears* (shown in Figure 8.15) also have *spiral*, instead of straight, teeth.

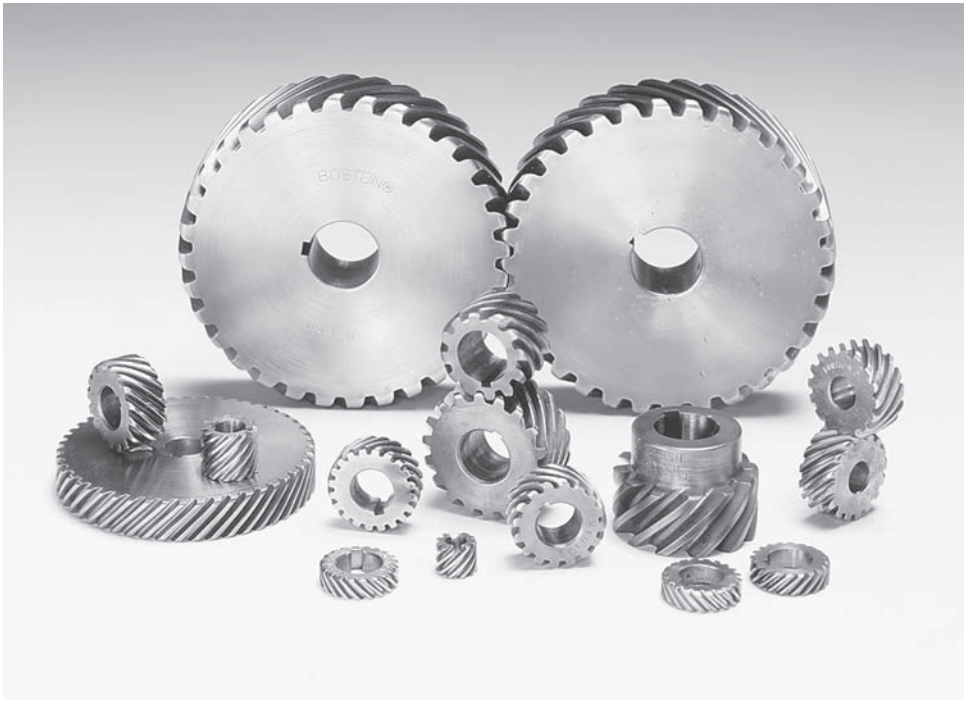
Helical gears are more complex and expensive to manufacture than spur gears. On the other hand, helical gearsets offer the advantage that they produce less noise and vibration when used in high-speed machinery. Automobile automatic transmissions, for instance, are typically constructed using both external and internal helical gears for precisely that reason. In a helical gearset, tooth-to-tooth contact starts at one edge of a tooth and proceeds gradually across its width, thus smoothing out the engagement and disengagement of teeth. Another attribute of helical gears is that they are capable of carrying greater torque and power when compared to similarly sized spur gears, because the tooth-to-tooth forces are spread over more surface area and the contact stresses are reduced.

We have seen how helix-shaped teeth can be formed on gears mounted on parallel shafts, but gears that are attached to perpendicular shafts can also incorporate helical teeth. The *crossed helical gears* of Figure 8.16(b) connect two perpendicular shafts, but, unlike the bevel gearset application, the shafts here are offset from one another, and extensions to their centerlines do not intersect. A sampler of helical and crossed helical gears is shown in Figure 8.17.

Helix angle

Spiral bevel gear

Crossed helical gears

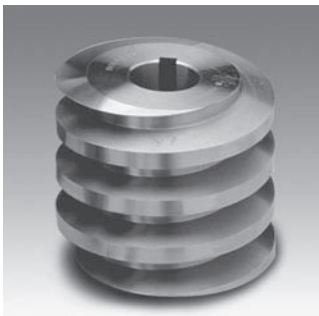
**Figure 8.17**

A collection of helical and crossed helical gears.

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Worm Gearsets

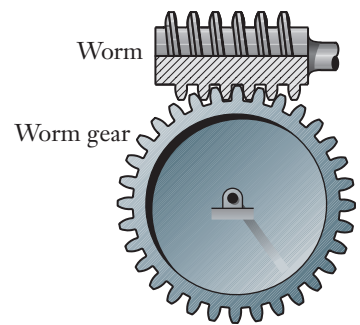
If the helix angle on a pair of crossed helical gears is made large enough, the resulting pair is called a worm and worm gear. Figure 8.18 illustrates this type of gearset, in which the worm itself has only one tooth that wraps several times



(a)



(b)



(c)

Figure 8.18

(a) Worm having one continuous tooth. (b) Worm gears. (c) Worm gearset.

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around a cylindrical body, similar to a thread of a screw. For each revolution of the worm, the worm gear advances by just one tooth in its rotation.

Worm gearsets are capable of large speed-reduction ratios. For instance, if the worm has only one tooth and the worm gear has 50 teeth, the speed reduction for a gearset [Figure 8.18(c)] would be 50-fold. The ability to package a geartrain with large speed reduction into a small physical space is an attractive feature of worm gearsets. However, the tooth profiles in worm gearsets are not involutes, and significant sliding occurs between the teeth during meshing. That friction is a major source of power loss, heating, and inefficiency when compared to other types of gears.

Self-locking

Another feature of worm gearsets is that they can be designed so that they are capable of being driven in only one direction, namely, from the worm to the worm gear. For such *self-locking* gearsets, the power flow cannot be reversed by having the worm gear drive the worm. This capability for motion transmission in only one direction can be exploited in such applications as hoists or jacks, where it is desirable for safety reasons to prevent the system mechanically from being back driven. Not all worm gearsets are self-locking, however, and this characteristic depends on such factors as the helix angle, the amount of friction between the worm and worm gear, and the presence of vibration.

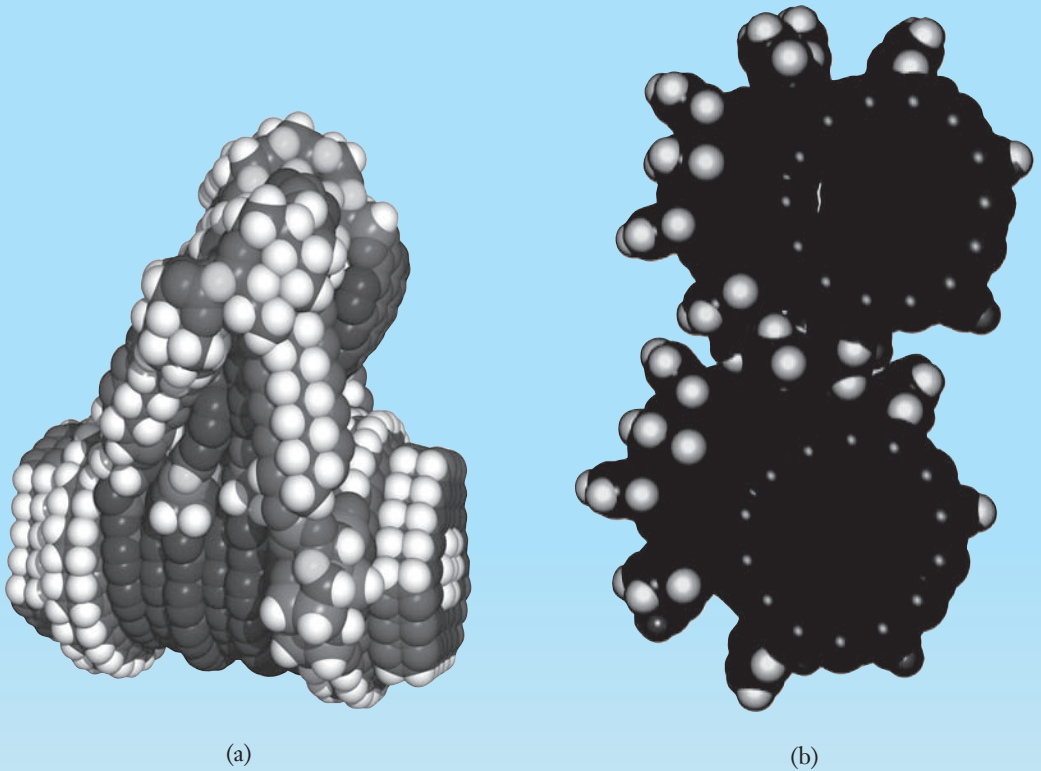
Focus On NANOMACHINES

Although the machines discussed in this chapter are typically quite large (e.g., industrial robotics, transmissions), a new breed of machines is being developed at the nano-scale and manufactured out of individual atoms. These nanomachines could perform tasks never before possible with larger machines. For example, nanomachines could someday be used to disassemble cancer cells, repair damaged bones and tissues, eliminate landfills, detect and remove impurities in drinking water, deliver drugs internally with unmatched precision, build new ozone molecules, and clean up toxins or oil spills.

In Figure 8.19(a), a conceptual design for a nanomachine that would provide fine-motion control for molecular assembly is shown. This proposed device includes cam mechanisms that use levers to drive rotating rings. Simpler machines such as nanogears have been developed that may pave the way for more complex and

functional nanomachines. Researchers developed these stable nanogears by attaching benzene molecules to the outside of carbon nanotubes [Figure 8.19(b)]. Rack and pinion versions of these nanogears, developed by research teams in Germany and France, could be used to move a scalpel back and forth or to push the plunger of a syringe.

Creating these gears, racks, and pinions at the nano or even micro level has typically been an expensive and time-consuming process. However, a research team at Columbia University has developed self-assembling microgears that are created as two materials shrink at different rates when cooled. A thin metal film is deposited on top of a heat-expanded polymer. When the polymer is cooled, the metal buckles, creating regularly spaced “teeth” in the polymer. Variations in metal stiffness and cooling rates allow for differently sized gears and teeth to form.

**Figure 8.19**

(a) A mechanism made up of approximately 2500 atoms to move and assemble individual molecules. (b) Nanogears made up of hundreds of individual atoms.

(a) © Institute for Molecular Manufacturing. www.imm.org; (b) Courtesy of NASA.

Powering the nanomachines of the future will be an additional challenge for engineers to overcome. Currently, the power for these nanomachines must come from chemical sources such as light from a laser, imitating the photosynthetic process that plants use to create energy from sunlight. However, scientists and

engineers are currently developing other sources of power including nanogenerators powered by ultrasonic waves, magnetic fields, or electric currents. Nanomachines designed, developed, and tested by engineers could soon be having widespread impact on medical, environmental, and energy issues around the globe.

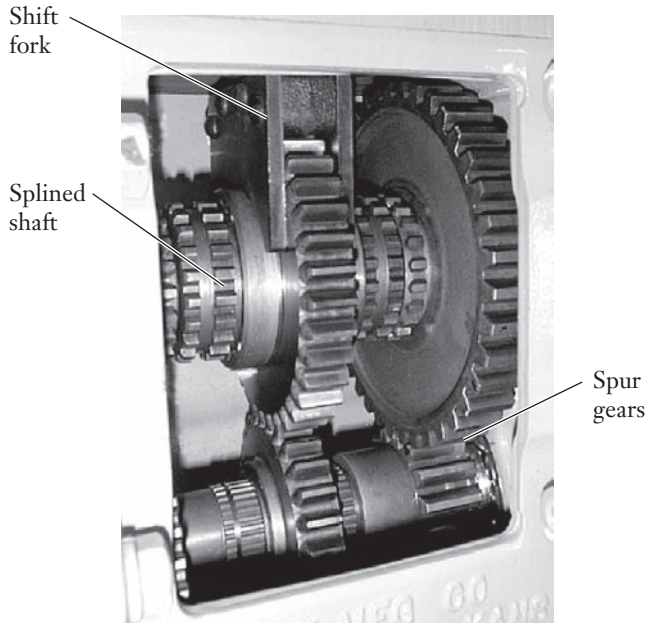
© 8.4 SPEED, TORQUE, AND POWER IN GEARSETS

A gearset is a pair of gears that mesh with one another, and it forms the basic building block of larger-scale systems, such as transmissions, that transmit rotation, torque, and power between shafts (Figure 8.20, see on page 370). In this section, we examine the speed, torque, and power characteristics of two meshing gears. In later sections, we will extend those results to simple, compound, and planetary geartrains.

Figure 8.20

The shift fork in the transmission is used to slide a gear along the upper shaft so that the speed of the output shaft changes.

Image courtesy of the authors.



Speed

In Figure 8.21(a), the smaller of the two gears is called the pinion (denoted by p), and the larger is called the gear (denoted by g). The pitch radii of the pinion and gear are denoted by r_p and r_g , respectively.

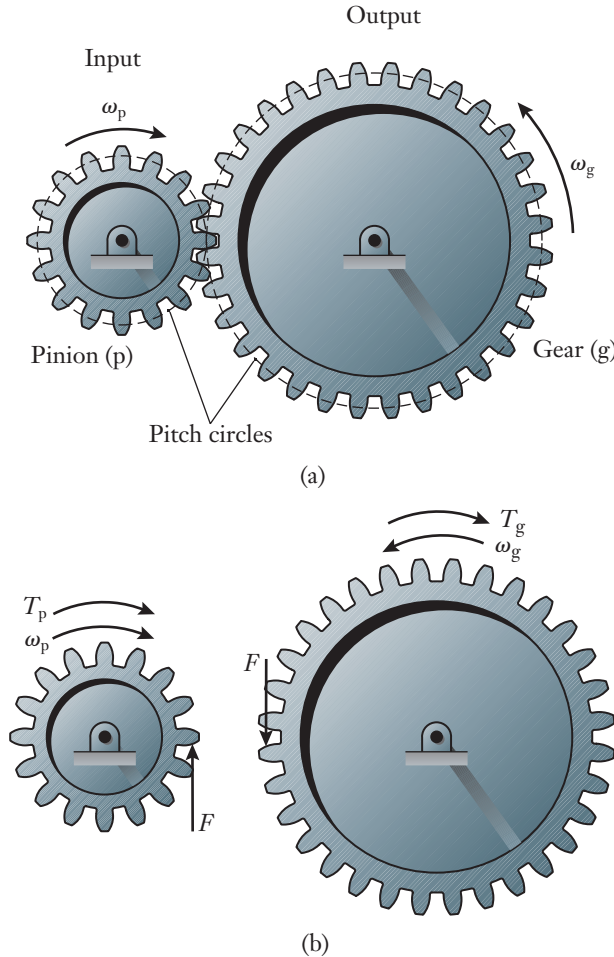
As the pinion rotates with angular velocity ω_p , the speed of a point on its pitch circle is $v_p = r_p \omega_p$, following Equation (8.2). Likewise, the speed of a point on the pitch circle of the gear is $v_g = r_g \omega_g$. Because the teeth on the pinion and gear do not slide past one another, the velocity of points in contact on the pitch circles are the same, and $r_g \omega_g = r_p \omega_p$. With the angular velocity of the pinion's shaft being known as the input to the gearset, the speed of the output shaft in Figure 8.21(a) is

$$\omega_g = \left(\frac{r_p}{r_g} \right) \omega_p = \left(\frac{N_p}{N_g} \right) \omega_p \quad (8.8)$$

Rather than perform calculations in terms of the pitch radii r_p and r_g , it is simpler to work with the numbers of teeth N_p and N_g . Although it is a simple matter to count the number of teeth on a gear, measuring the pitch radius is not so straightforward. So that the pinion and gear mesh smoothly, they must be manufactured with the same distance between teeth, as measured by either the diametral pitch or module. In both Equations (8.6) and (8.7), the number of teeth on a gear is proportional to its pitch radius. Since the diametral pitch (or module) of the pinion and gear are the same, the ratio of radii in Equation (8.8) also equals the ratio of their teeth numbers.

Figure 8.21

- (a) A pinion and gear form a gearset.
 (b) The pinion and gear are conceptually separated to expose the driving component of the tooth force and the input and output torques.

**Velocity ratio**

The *velocity ratio* of the gearset is defined as

$$VR = \frac{\text{output speed}}{\text{input speed}} = \frac{\omega_g}{\omega_p} = \frac{N_p}{N_g} \quad (8.9)$$

which is a constant that relates the output and input speeds, just as the mechanical advantage is a constant that relates the input and output forces acting on a mechanism.

Torque

We next consider how torque transfers from the shaft of the pinion to the shaft of the gear. For example, imagine that the pinion in Figure 8.21(a) is driven by a motor and that the shaft of the gear is connected to a mechanical load such as a crane or pump. In the diagrams of Figure 8.21(b), the motor applies

torque T_p to the pinion, and torque T_g is applied to the gear's shaft by the load being driven. The tooth force F that is exposed in this diagram is the physical means by which torque is transferred between the pinion and gear. The full meshing force is the resultant of F and another component that acts along the line of action passing through the centers of the two shafts. That second force component, which would be directed horizontally in Figure 8.21(b), tends to separate the pinion and gear from one another, but it does not contribute to the transfer of torque, and so it is omitted from the diagram for clarity. As the pinion and gear are imagined to be isolated from one another, F and the separating force act in equal and opposite directions on the teeth in mesh.

When the gearset runs at a constant speed, the sum of torques applied to the pinion and gear about their centers are each zero; therefore, $T_p = r_p F$ and $T_g = r_g F$. By eliminating the unknown force F , we obtain an expression for the gearset's output torque

$$T_g = \left(\frac{r_g}{r_p} \right) T_p = \left(\frac{N_g}{N_p} \right) T_p \quad (8.10)$$

in terms of either the pitch radii or the numbers of teeth. Analogous to Equation (8.9), the *torque ratio* of the gearset is defined as

$$TR = \frac{\text{output torque}}{\text{input torque}} = \frac{T_g}{T_p} = \frac{N_g}{N_p} \quad (8.11)$$

As you can see from Equations (8.9) and (8.11), the torque ratio for a gearset is exactly the reciprocal of its velocity ratio. If a gearset is designed to increase the speed of its output shaft relative to the input shaft ($VR > 1$), then the amount of torque transferred will be reduced by an equal factor ($TR < 1$). A gearset exchanges speed for torque, and it is not possible to increase both simultaneously. As a common example of this principle, when the transmission of an automobile or truck is set into low gear, the rotation speed of the engine's crankshaft is reduced by the transmission to increase the torque applied to the drive wheels.

Power

Following Equation (8.5), the power supplied to the pinion by its motor is $P_p = T_p \omega_p$. On the other hand, the power transmitted to the mechanical load by the gear is $P_g = T_g \omega_g$. By combining Equations (8.9) and (8.11), we find that

$$P_g = \left(\frac{N_g}{N_p} T_p \right) \left(\frac{N_p}{N_g} \omega_p \right) = T_p \omega_p = P_p \quad (8.12)$$

which shows that the input and output power levels are exactly the same. The power supplied to the gearset is identical to the power that it transfers to the load. From a practical standpoint, any real gearset incurs frictional losses, but Equation (8.12) is a good approximation for gearsets made of quality gears and bearings where friction is small relative to the overall power level. In short, any reduction in power between the input and output of a gearset will be associated with frictional losses, not with the intrinsic changes of speed and torque.

8.5 SIMPLE AND COMPOUND GEARTRAINS

Simple Geartrain

For most combinations of a single pinion and gear, a reasonable limit on the velocity ratio lies in the range of 5 to 10. Larger velocity ratios often become impractical either because of size constraints or because the pinion would need to have too few teeth for it to smoothly engage with the gear. One might therefore consider building a geartrain that is formed as a serial chain of more than two gears. Such a mechanism is called a *simple geartrain*, and it has the characteristic that each shaft carries a single gear.

Figure 8.22 shows an example of a simple geartrain having four gears. To distinguish the various gears and shafts, we set the convention that the input gear is labeled as gear 1, and the other gears are numbered sequentially. The numbers of teeth and rotational speeds of each gear are represented by the symbols N_i and ω_i . We are interested in determining the speed ω_4 of the output shaft for a given speed ω_1 of the input shaft. The direction that each gear rotates can be determined by recognizing that, for external gearsets, the direction reverses at each mesh point.

We can view the simple geartrain as a sequence of connected gearsets. In that sense, Equation (8.9) can be applied recursively to each pair of gears. Beginning at the first mesh point,

$$\omega_2 = \left(\frac{N_1}{N_2}\right)\omega_1$$

(8.13)

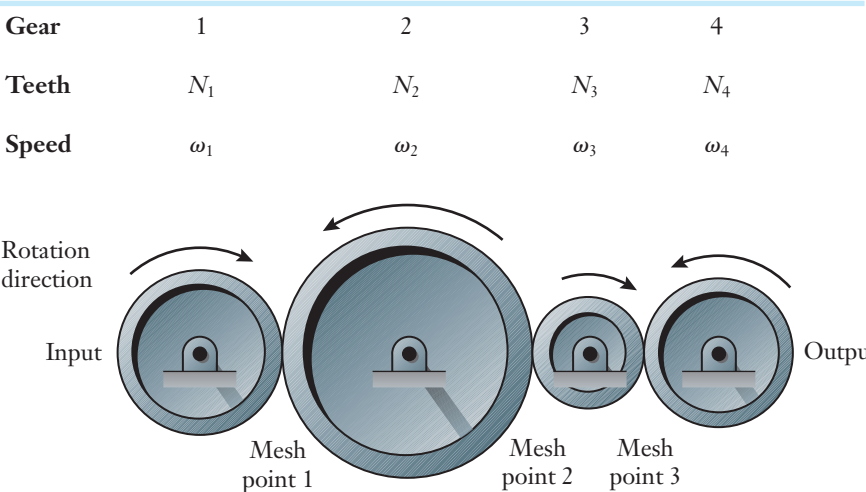
in the counterclockwise direction. At the second mesh point,

$$\omega_3 = \left(\frac{N_2}{N_3}\right)\omega_2$$

(8.14)

Figure 8.22

A simple geartrain formed as a serial connection of four gears on separate shafts. The gears' teeth are omitted for clarity.



In designing a geartrain, we are more interested in the relationship between the input and output shaft speeds, than in the speeds ω_2 and ω_3 of the intermediate shafts. By combining Equations (8.13) and (8.14), we can eliminate the intermediate variable ω_2 :

$$\omega_3 = \left(\frac{N_2}{N_3}\right) \left(\frac{N_1}{N_2}\right) \omega_1 = \left(\frac{N_1}{N_3}\right) \omega_1 \quad (8.15)$$

In other words, the effect of the second gear (and in particular, its size and number of teeth) cancel as far as the third gear is concerned. Proceeding to the final mesh point, the velocity of the output gear becomes

$$\omega_4 = \left(\frac{N_3}{N_4}\right) \omega_3 = \left(\frac{N_3}{N_4}\right) \left(\frac{N_1}{N_3}\right) \omega_1 = \left(\frac{N_1}{N_4}\right) \omega_1 \quad (8.16)$$

For this simple geartrain, the overall velocity ratio between the output and input shafts is

$$VR = \frac{\text{output speed}}{\text{input speed}} = \frac{\omega_4}{\omega_1} = \frac{N_1}{N_4} = \frac{N_{\text{input}}}{N_{\text{output}}} \quad (8.17)$$

The sizes of the intermediate gears 2 and 3 have no effect on the geartrain's velocity ratio. This result can be extended to simple geartrains having any number of gears, and, in general, the velocity ratio will depend only on the sizes of the input and output gears. In a similar manner, by applying Equation (8.12) for power conservation with pairs of ideal gears, the power supplied to the first gear must balance the power transmitted by the final gear. Since $VR \times TR = 1$ for an ideal gearset, the torque ratio is likewise unaffected by the numbers of teeth on gears 2 and 3, and

$$TR = \frac{\text{output torque}}{\text{input torque}} = \frac{T_4}{T_1} = \frac{N_4}{N_1} = \frac{N_{\text{output}}}{N_{\text{input}}} \quad (8.18)$$

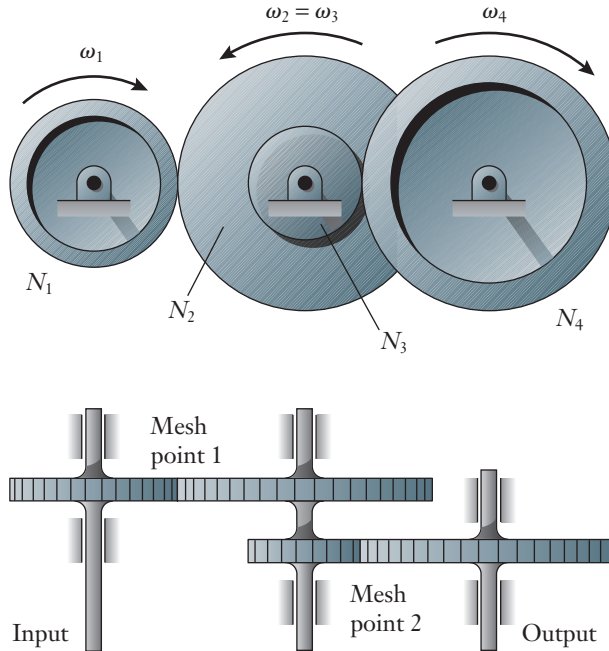
Idler gear Because the intermediate gears of a simple geartrain provide no speed or torque modifications as a whole, they are sometimes called *idler gears*. Although they may have no direct effect on VR and TR , the idler gears contribute indirectly to the extent that a designer can insert them to gradually increase or decrease the dimensions of adjacent gears. Additional idler gears also enable the input and output shafts to be separated farther from one another, but the power transmission chains and belts discussed later in this chapter could also be used for that purpose.

Compound Geartrain

As an alternative to simple geartrains, compound geartrains can be used in transmissions when larger velocity or torque ratios are needed or when the gearbox must be made physically compact. A *compound geartrain* is based on the principle of having more than one gear on each intermediate shaft. In the geartrain of Figure 8.23, the intermediate shaft carries two gears having different numbers of teeth. To determine the overall velocity ratio ω_4/ω_1 between the input and output shafts, we apply Equation (8.9) to each pair of

Figure 8.23

A compound geartrain in which the second and third gears are mounted on the same shaft and rotate together. The gears' teeth are omitted for clarity.



meshing gears. Beginning with the first pair in mesh,

$$\omega_2 = \left(\frac{N_1}{N_2}\right) \omega_1 \quad (8.19)$$

Because gears 2 and 3 are mounted on the same shaft, $\omega_3 = \omega_2$. Proceeding to the next mesh point between gears 3 and 4, we have

$$\omega_4 = \left(\frac{N_3}{N_4}\right) \omega_3 \quad (8.20)$$

Combining Equations (8.19) and (8.20), the speed of the output shaft becomes

$$\omega_4 = \left(\frac{N_3}{N_4}\right) \left(\frac{N_1}{N_2}\right) \omega_1 \quad (8.21)$$

and the velocity ratio is

$$VR = \frac{\text{output speed}}{\text{input speed}} = \left(\frac{\omega_4}{\omega_1}\right) = \left(\frac{N_1}{N_2}\right) \left(\frac{N_3}{N_4}\right) \quad (8.22)$$

A compound geartrain's velocity ratio is the product of the velocity ratios between the pairs of gears at each mesh point, a result that can be generalized to compound geartrains having any number of additional stages. Unlike the gears in a simple geartrain, the sizes of the intermediate gears here influence the velocity ratio. The clockwise or counterclockwise rotation of any gear is determined by

the rule of thumb: With an even number of mesh points, the output shaft rotates in the same direction as the input shaft. Conversely, if the number of mesh points is odd, the input and output shafts rotate opposite to one another.

Focus On GEARTRAIN DESIGN

Segway Inc. faced a difficult design problem to develop a new personal clean-energy transportation system that could be used for both recreation and professional purposes. The company developed a two-wheeled conceptual vehicle but had to make sure that the vehicle's detailed design was technically sound. For the drivetrain, engineers needed to ensure that the noise produced by the gears meshing in the transmission would not disturb the rider or pedestrians. What resulted was the Segway® Personal Transporter, an electrically powered personal vehicle that uses gyroscopes to maintain balance [Figure 8.24(a)].

A two-stage geartrain was designed to connect an electric motor to a wheel, and helical gears were used to reduce noise. The spiral shape of the teeth on the gears enables the teeth to engage gradually, therefore reducing noise and increasing the transmission's load-carrying

capability. Engineers also selected the sizes of the gears in the transmission so that the two primary sounds that the geartrain makes are exactly two musical octaves apart. In that manner, the transmission produces sounds that are in harmony, pleasing to the ear, and not discordant or annoying. Focus groups were conducted with potential customers in which they listened to the sounds produced by the transmission and rated their quality. The engineers then improved and optimized the transmission's design based on the evaluations.

Segway Inc. has now, in partnership with General Motors and the Shanghai Automotive Industry Corporation, designed a two-person Electric Networked Vehicle (EN-V) based on the drivetrain in the Segway® Personal Transporter. The EN-V, shown in Figure 8.24(b) is designed to be an urban transportation system for personal, professional, and recreational use.

Figure 8.24

(a) The single-rider Segway® Personal Transporter. (b) The two-person EN-V designed to have more power, range, speed, payload, and the capability to operate autonomously and connect to other EN-Vs.

(a) Reprinted with permission of Segway LLC;
(b) testing/Shutterstock.



(a)

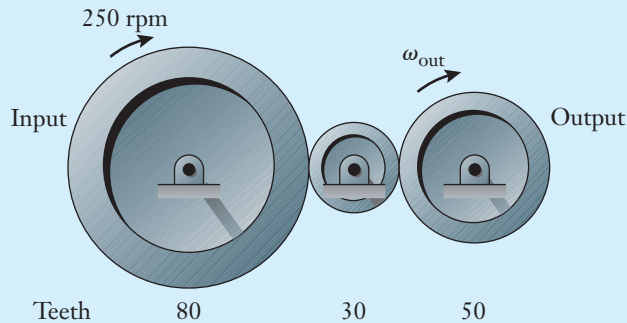


(b)

Example 8.4 Speed, Torque, and Power in a Simple Geartrain

The input shaft to the simple geartrain is driven by a motor that supplies 1 kW of power at the operating speed of 250 rpm. (See Figure 8.25.) (a) Determine the speed and rotation direction of the output shaft. (b) What magnitude of torque does the output shaft transfer to its mechanical load?

Figure 8.25



Approach

To find the rotation direction of the output shaft, we recognize that the input shaft rotates clockwise in the figure, and, at each mesh point, the direction of rotation reverses. Thus, the 30-tooth gear rotates counterclockwise, and the 50-tooth output gear rotates clockwise. To determine the speed of the output shaft, we will apply Equation (8.17). In part (b), for an ideal geartrain where friction can be neglected, the input and output power levels are identical. We can apply Equation (8.5) to relate speed, torque, and power.

Solution

- (a) The speed of the output shaft is

$$\begin{aligned}\omega_{out} &= \left(\frac{80}{50}\right)(250 \text{ rpm}) \quad \leftarrow \left[VR = \frac{N_{input}}{N_{output}}\right] \\ &= 400 \text{ rpm}\end{aligned}$$

- (b) The instantaneous power is the product of torque and rotation speed. For the calculation to be dimensionally consistent, the speed of the output shaft first must be converted to the units of radians per second by using the conversion factor listed in Table 8.1:

$$\begin{aligned}\omega_{out} &= 400 \text{ rpm} \left(0.1047 \frac{\text{rad/s}}{\text{rpm}}\right) \\ &= 41.88 \left(\frac{\text{rpm} \cdot \text{rad/s}}{\text{rpm}}\right) \\ &= 41.88 \frac{\text{rad}}{\text{s}}\end{aligned}$$

Example 8.4 *continued*

The output torque becomes

$$\begin{aligned}
 T_{\text{out}} &= \frac{1000 \text{ W}}{41.88 \text{ rad/s}} && \leftarrow \left[T = \frac{P}{\omega} \right] \\
 &= 23.88 \frac{\text{W} \cdot \text{s}}{\text{rad}} \\
 &= 23.88 \left(\frac{\text{N} \cdot \text{m}}{\text{s}} \right) \left(\frac{1}{\text{rad}} \right) \\
 &= 23.88 \text{ N} \cdot \text{m}
 \end{aligned}$$

Here we used the definition that $1 \text{ W} = 1 (\text{N} \cdot \text{m})/\text{s}$ and the fact that $1 \text{ kW} = 1000 \text{ W}$.

Discussion

Because Equation (8.17) involves the ratio of the output and input angular velocities, the shaft speeds can be expressed in any of the dimensions that are appropriate for angular velocity, including rpm. Also, the speed of the 30-tooth gear will be larger than the input or output gears because of its smaller size. When calculating the torque from the expression $P = T\omega$, the radian unit in the angular velocity can be directly canceled because it is a dimensionless measure of an angle.

$$\omega_{\text{out}} = 400 \text{ rpm}$$

$$T_{\text{out}} = 23.88 \text{ N} \cdot \text{m}$$

Example 8.5 Money Changer Geartrain

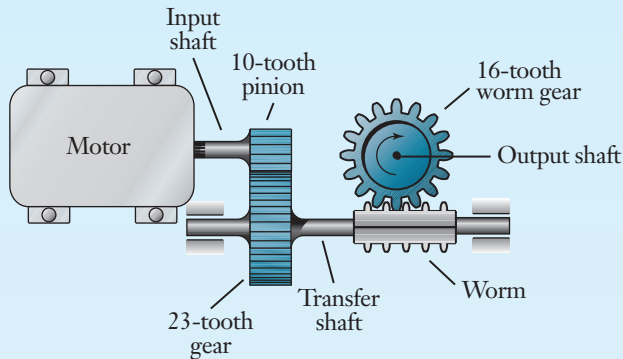
A gear reduction mechanism is used in a machine that accepts bill currency, inspects it, and returns coins in change. The numbers of teeth on the pinion and gear in the first speed-reduction stage are shown in Figure 8.26. The worm has a single tooth that meshes with the 16-tooth worm gear. Determine the velocity ratio between the output shaft's speed and the speed of the direct-current motor.

Approach

To find the velocity ratio between the output shaft and the input speed, we recognize that the motor's speed is reduced in two stages: first, in the gearset comprising the 10-tooth pinion and 23-tooth gear, and second, at the mesh between the worm and worm gear. We denote the angular velocity of the motor's shaft as ω_{in} . To determine the speed of the transfer shaft, we will apply Equation (8.9). In the second speed-reduction stage, the worm gear will advance by one tooth for each rotation of the transfer shaft.

Example 8.5 *continued*

Figure 8.26

**Solution**

The speed of the transfer shaft is

$$\begin{aligned}\omega_{\text{transfer}} &= \frac{10}{23} \omega_{\text{in}} && \leftarrow \left[\omega_g = \left(\frac{N_p}{N_g} \right) \omega_p \right] \\ &= 0.4348 \omega_{\text{in}}\end{aligned}$$

or $\omega_{\text{in}} = 2.3 \omega_{\text{transfer}}$. Because the worm gear has 16 teeth and the worm has only one, the speeds of the transfer shaft and output shaft are related by

$$\begin{aligned}\omega_{\text{out}} &= \left(\frac{1}{16} \right) \omega_{\text{transfer}} \\ &= 0.0625 \omega_{\text{transfer}}\end{aligned}$$

The geartrain's overall velocity ratio is

$$\begin{aligned}VR &= \frac{0.0625 \omega_{\text{transfer}}}{2.3 \omega_{\text{transfer}}} && \leftarrow \left[VR = \frac{\omega_{\text{out}}}{\omega_{\text{in}}} \right] \\ &= 2.717 \times 10^{-2}\end{aligned}$$

Discussion

The speed of the output shaft is reduced to 2.7% of the input shaft's speed, for a speed reduction factor of 36.8-fold. This is consistent with the use of worm gears in design applications where large speed reduction ratios are required.

$$VR = 2.717 \times 10^{-2}$$

◎ 8.6 DESIGN APPLICATION: BELT AND CHAIN DRIVES

Similar to geartrains, belt and chain drives can also be used to transfer rotation, torque, and power between shafts. Some applications include compressors, appliances, machine tools, sheet-metal rolling mills, and automotive engines (such as the one shown in Figure 7.13). Belt and chain drives have the abilities to isolate elements of a drivetrain from shock, to have relatively long working distances between the centers of shafts, and to tolerate some degree of misalignment between shafts. Those favorable characteristics stem largely from the belt's or chain's flexibility.

The common type of power transmission belt shown in Figures 8.27 and 8.28 is called a *V-belt*, and it is named appropriately after the wedge-shaped appearance of its cross section. The grooved pulleys on which the V-belt rides are called *sheaves*. To have efficient transfer of power between the two shafts, the belt must be tensioned and have good frictional contact with its sheaves. In fact, the belt's cross section is designed to wedge tightly into the sheave's groove. The capability of V-belts to transfer load between shafts is determined by the belt's wedge angle and by the amount of friction between the belt and the surface of the sheaves. The exterior of a V-belt is made from a synthetic rubber to increase the amount of friction. Because elastomer materials have low elastic modulus and stretch easily, V-belts are usually reinforced internally with fiber or wire cords that carry most of the belt's tension.

Although V-belts are well suited for transmitting power, some amount of slippage invariably occurs between the belt and sheaves because the only contact between the two is friction. Slippage between gears, on the other hand, cannot occur because of the direct mechanical engagement between

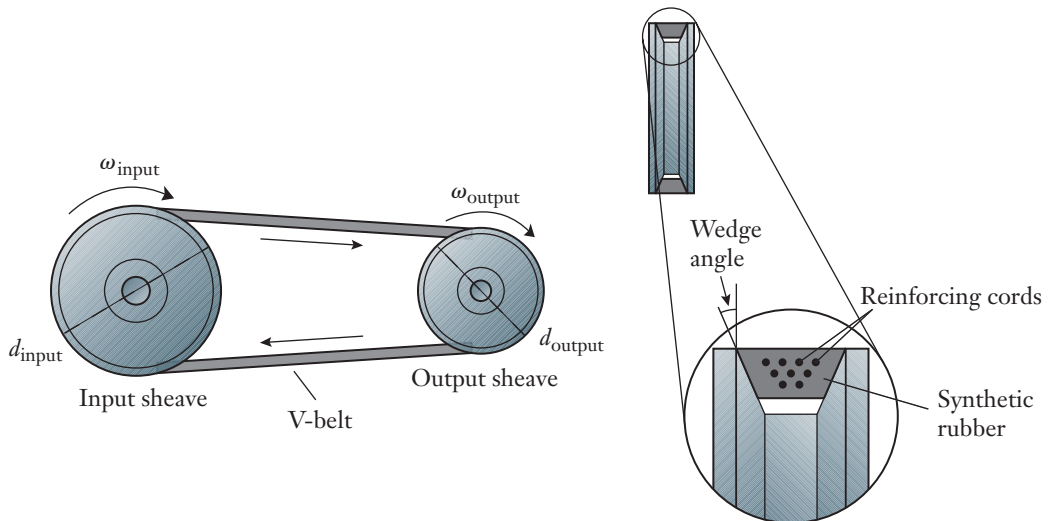


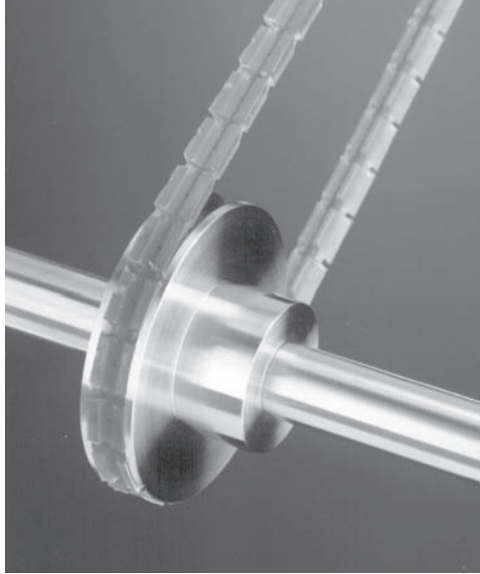
Figure 8.27

A V-belt and its sheaves.

Figure 8.28

A segmented V-belt
in contact with its
sheave.

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Synchronous rotation

Timing belt

Chain drives

their teeth. Geartrains are a *synchronous* method of rotation; that is, the input and output shafts are synchronized and they rotate together exactly. Belt slippage is not a concern if the engineer is interested only in transmitting power, as in a gasoline engine that drives a compressor or generator. On the other hand, for such precision applications as robotic manipulators and valve timing in automotive engines, the rotation of the shafts must remain perfectly synchronized. Timing belts address that need by having molded teeth that mesh in matching grooves on their sheaves. *Timing belts*, such as the one shown in Figure 8.29, combine some of the best features of belts—mechanical isolation and long working distances between shaft centers—with a gearset's ability to provide synchronous motion.

Chain drives (Figures 8.30 and 8.31, see on page 382) are another design option when synchronous motion is required, particularly when high torque or power must be transmitted. Because of their metallic link construction,

Figure 8.29

A magnified view of
the teeth on a timing
belt. The reinforcing
cords are exposed at
the belt's cross
section.

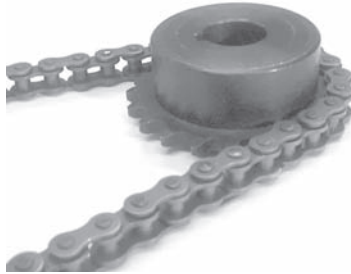
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Figure 8.30

A chain and sprocket used in a power-transmission drive.

Image courtesy of the authors.



chain and sprocket mechanisms are capable of carrying greater forces than belts, and they are also able to withstand high-temperature environments.

For timing belts and chains (and for V-belts when slippage on the sheaves can be neglected), the angular velocities of the two shafts are proportional to one another in a manner similar to a gearset. In Figure 8.27, for instance, the speed of the belt when it wraps around the first sheave $[(d_{\text{input}}/2) \omega_{\text{input}}]$ must be the same as when it wraps around the second sheave $[(d_{\text{output}}/2) \omega_{\text{output}}]$. In much the same manner as Equation (8.9), the velocity ratio between the output and input shafts is given by

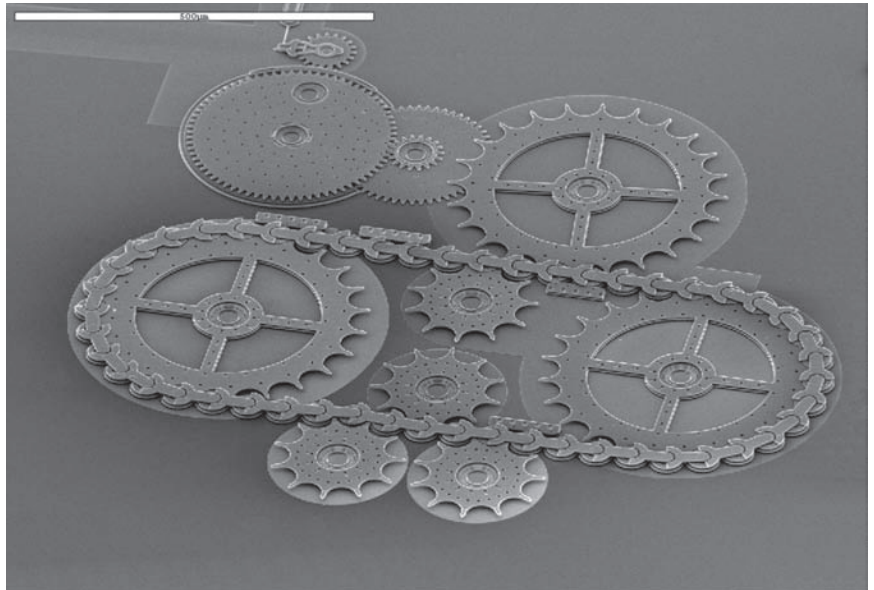
$$VR = \frac{\text{output speed}}{\text{input speed}} = \frac{\omega_{\text{output}}}{\omega_{\text{input}}} = \frac{d_{\text{input}}}{d_{\text{output}}} \quad (8.23)$$

where the pitch diameters of the sheaves are denoted by d_{input} and d_{output} .

Figure 8.31

Engineers and scientists have manufactured micro mechanical chain drives by adapting techniques that are used in the production of integrated electronic circuits. The distance between each link in this chain is 50 microns, smaller than the diameter of a human hair.

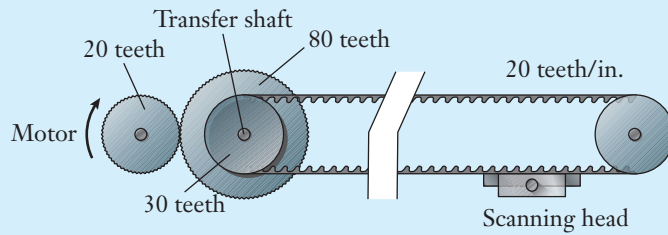
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Example 8.6 Computer Scanner

The mechanism in a desktop computer scanner converts the rotation of a motor's shaft into the side-to-side motion of the scanning head (Figure 8.32). During a portion of a scan operation, the drive motor turns at 180 rpm. The gear attached to the motor's shaft has 20 teeth, and the timing belt has 20 teeth/in. The two other gears are sized as indicated. In the units of in./s, calculate the speed at which the scanning head moves.

Figure 8.32



Approach

To find the speed of the scanning head, we must first determine the speed of the transfer shaft by applying Equation (8.9). Equation (8.2) can then be used to relate the rotational speed of the transfer shaft to the belt's speed.

Solution

The angular velocity of the transfer shaft is

$$\begin{aligned}\omega &= \left(\frac{20}{80}\right) (180 \text{ rpm}) \quad \leftarrow \left[\omega_g = \left(\frac{N_p}{N_g}\right) \omega_p\right] \\ &= 45 \text{ rpm}\end{aligned}$$

Because the 30-tooth and 80-tooth gears are mounted on the same shaft, the 30-tooth gear also rotates at 45 rpm. With each rotation of the transfer shaft, 30 teeth of the timing belt mesh with the gear, and the timing belt advances by the distance

$$\begin{aligned}x &= \frac{30 \text{ teeth/rev}}{20 \text{ teeth/in.}} \\ &= 1.5 \left(\frac{\text{teeth}}{\text{rev}}\right) \left(\frac{\text{in.}}{\text{teeth}}\right) \\ &= 1.5 \frac{\text{in.}}{\text{rev}}\end{aligned}$$

Example 8.6 *continued*

The speed v of the timing belt is the product of the shaft's speed ω and the amount x by which the belt advances with each revolution. The scanning head's speed becomes

$$\begin{aligned} v &= \left(1.5 \frac{\text{in.}}{\text{rev}}\right)(45 \text{ rpm}) \\ &= 67.5 \left(\frac{\text{rev}}{\text{min}}\right) \left(\frac{\text{in.}}{\text{rev}}\right) \\ &= 67.5 \frac{\text{in.}}{\text{min}} \end{aligned}$$

In the units of inches per second, the velocity is

$$\begin{aligned} v &= \left(67.5 \frac{\text{in.}}{\text{min}}\right) \left(\frac{1}{60} \frac{\text{min}}{\text{s}}\right) \\ &= 1.125 \left(\frac{\text{in.}}{\text{min}}\right) \left(\frac{\text{min}}{\text{s}}\right) \\ &= 1.125 \frac{\text{in.}}{\text{s}} \end{aligned}$$

Discussion

This mechanism achieves a speed reduction and a conversion between rotational and straight-line motion in two stages. First, the (input) angular velocity of the motor's shaft is reduced to the speed of the transfer shaft. Second, the timing belt converts the transfer shaft's rotational motion to the (output) straight-line motion of the scanning head.

$$v = 1.125 \frac{\text{in.}}{\text{s}}$$

Example 8.7 Treadmill's Belt Drive

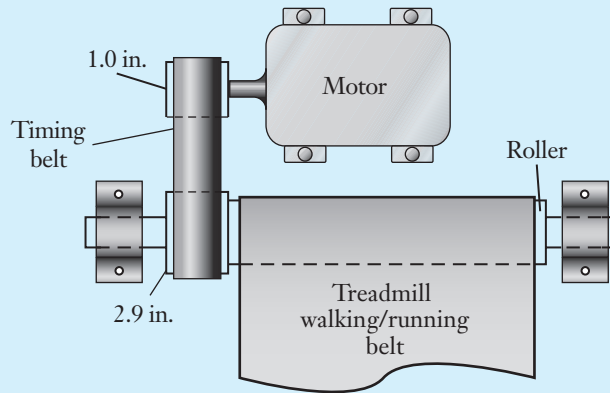
A timing belt is used to transfer power in the exercise treadmill from the motor to the roller that supports the walking/running belt (Figure 8.33). The pitch diameters of the sheaves on the two shafts are 1.0 in. and 2.9 in., and the roller has a diameter of 1.75 in. For a running pace of 7 min/mi, at what speed should the motor be set? Express your answer in the units of rpm.

Approach

To find the appropriate speed of the motor, we first find the roller's angular velocity and then relate this to the motor speed. The speed of the walking/running belt is related to the roller's angular velocity by Equation (8.2). Since

Example 8.7 *continued*

the timing belt's speed is the same whether it is in contact with the sheave on the roller or with the sheave on the motor, the angular velocities of those two shafts are related by Equation (8.23).

Figure 8.33**Solution**

In dimensionally consistent units, the speed of the walking/running belt is

$$\begin{aligned}
 v &= \left(\frac{1 \text{ mi}}{7 \text{ min}} \right) \left(5280 \frac{\text{ft}}{\text{mi}} \right) \left(12 \frac{\text{in.}}{\text{ft}} \right) \left(\frac{1}{60} \frac{\text{min}}{\text{s}} \right) \\
 &= 150.9 \left(\frac{\text{mi}}{\text{min}} \right) \left(\frac{\text{ft}}{\text{mi}} \right) \left(\frac{\text{in.}}{\text{ft}} \right) \left(\frac{\text{min}}{\text{s}} \right) \\
 &= 150.9 \frac{\text{in.}}{\text{s}}
 \end{aligned}$$

The angular velocity of the roller is

$$\begin{aligned}
 \omega_{\text{roller}} &= \frac{150.9 \text{ in./s}}{(1.75 \text{ in.})/2} \quad \leftarrow [v = r\omega] \\
 &= 172.4 \left(\frac{\text{in.}}{\text{s}} \right) \left(\frac{1}{\text{in.}} \right) \\
 &= 172.4 \frac{\text{rad}}{\text{s}}
 \end{aligned}$$

where the dimensionless radian unit has been used for the roller's angle. In the engineering units of rpm, the roller's speed is

$$\begin{aligned}
 \omega_{\text{roller}} &= \left(172.4 \frac{\text{rad}}{\text{s}} \right) \left(9.549 \frac{\text{rpm}}{\text{rad/s}} \right) \\
 &= 1646 \left(\frac{\text{rad}}{\text{s}} \right) \left(\frac{\text{rpm}}{\text{rad/s}} \right) \\
 &= 1646 \text{ rpm}
 \end{aligned}$$

Example 8.7 *continued*

Since the 2.9-in.-diameter sheave turns at the same speed as the roller, the speed of the motor's shaft is

$$\begin{aligned}\omega_{\text{motor}} &= \left(\frac{2.9 \text{ in.}}{1.0 \text{ in.}} \right) (1646 \text{ rpm}) \quad \leftarrow \left[VR = \frac{d_{\text{input}}}{d_{\text{output}}} \right] \\ &= 4774 \left(\frac{\text{in.}}{\text{in.}} \right) (\text{rpm}) \\ &= 4774 \text{ rpm}\end{aligned}$$

Discussion

This belt drive is similar to a compound geartrain in the sense that two belts (the timing belt and the walking/running belt) are in contact with sheaves on the same shaft. Because the roller and the 2.9-in. sheave have different diameters, the two belts move at different speeds v and are assumed not to slip. Many treadmill motors are rated for a maximum of 4000–5000 rpm. So our answer, though on the high end of the range, is certainly reasonable for the speed of the runner.

$$\omega_{\text{motor}} = 4774 \text{ rpm}$$

⊙ 8.7 PLANETARY GEARTRAINS

Up to this point, our geartrain shafts have been connected to the housing of a gearbox by bearings, and the centers of the shafts themselves did not move. The gearsets, simple geartrains, compound geartrains, and belt drives of the previous sections were all of this type. In some geartrains, however, the centers of certain gears may be allowed to move. Such mechanisms are called planetary geartrains because the motion of their gears is (in many ways) analogous to the orbit of a planet around a star.

Sun gear
Planet gear

Carrier

Simple and planetary gearsets are contrasted in Figure 8.34. Conceptually, we can view the center points of gears in the simple gearset as being connected by a link that is fixed to the ground. In the planetary system of Figure 8.34(b), on the other hand, although the center of the *sun gear* is stationary, the center of the *planet gear* can orbit around the sun gear. The planet gear rotates about its own center, meshes with the sun gear, and then orbits as a whole about the center of the sun gear. The link that connects the centers of the two gears is called the *carrier*. Planetary geartrains are often used as speed reducers, and one application is the geartrain of Figure 8.35 that is used in the transmission of a light-duty helicopter.

In Figure 8.34(b), it is straightforward enough to connect the shaft of the sun gear to a power source or to a mechanical load. However, because of the planet gear's orbital motion, connecting the planet gear to the shaft

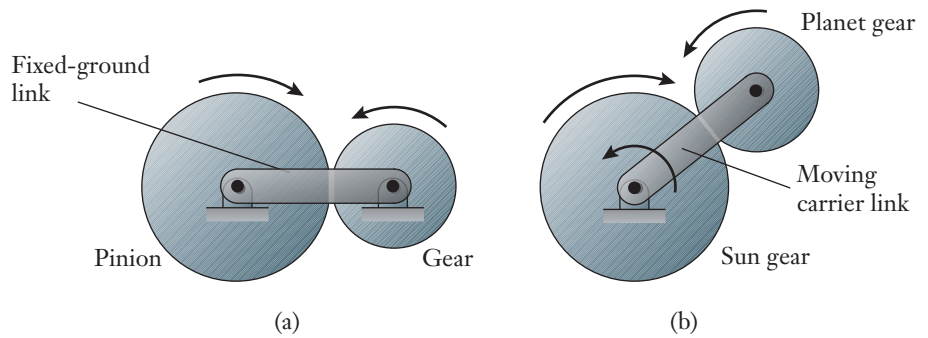


Figure 8.34

(a) A simple gearset in which the pinion and gear are connected by a fixed-ground link. (b) A planetary gearset where the planet and carrier link can rotate about the center of the sun gear.

Ring gear

of another machine directly is not feasible. To construct a more functional geartrain, the *ring gear* shown in Figure 8.36 (see on page 388) is used to convert the motion of the planet gear into rotation of the ring gear and its shaft. The ring gear is an internal gear, whereas the sun and planet gears are external gears.

In this configuration, a planetary geartrain has three input–output connection points, as indicated in Figure 8.37 (see on page 388): the shafts of the sun gear, carrier, and ring gear. Those connections can be configured to form a geartrain having two input shafts (for instance, the carrier and the sun gear) and one output shaft (the ring gear in this case), or a geartrain having one input and two output shafts. A planetary geartrain can therefore combine power from two sources into one output, or it can split the power from one source into two outputs. In a rear-wheel-drive automobile, for instance, power supplied by the engine is split between the two drive wheels by a special type of planetary geartrain called a differential (described later in this section). It is also possible to fix one of the planetary geartrain's connections to ground (say, the ring gear) so that there are only one input shaft and one output shaft (in this case, the carrier and sun gear). Alternatively, two of the shafts can be coupled, again reducing the number of connection points from three to two. Versatile

Figure 8.35

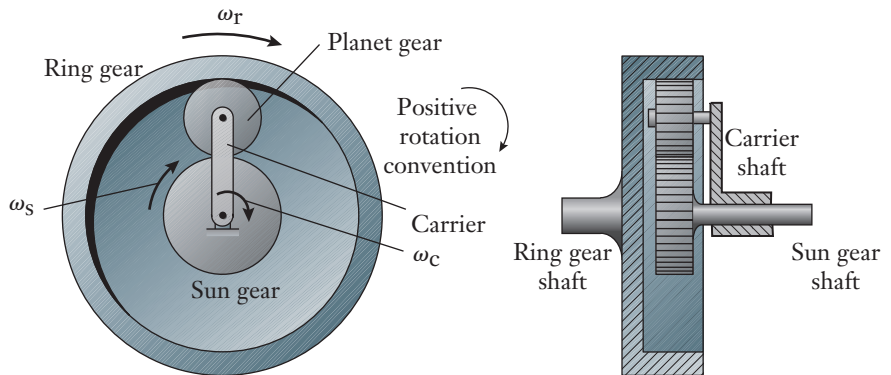
A planetary geartrain used in a helicopter's transmission. The diameter of the ring gear is approximately one foot.

Courtesy of NASA.



Figure 8.36

Front and cross-sectional views of a planetary geartrain.



configurations such as these are exploited in the operation of automotive automatic transmissions. By the proper sizing of the sun, planet, and ring gears and by selecting the input and output connections, engineers can obtain either very small or very large velocity ratios in a geartrain of compact size.

Balanced geartrain
Spider

Planetary geartrains are usually constructed with more than one planet gear to reduce noise, vibration, and the forces applied to the gear teeth. A *balanced planetary geartrain* is depicted in Figure 8.38. When multiple planet gears are present, the carrier is sometimes called the *spider*, because it has several (although perhaps not as many as eight) legs that evenly separate the planet gears around the circumference.

The rotations of shafts and the flow of power through simple and compound geartrains are usually straightforward enough to visualize. Planetary systems are more complicated to the extent that power can flow through the geartrain in multiple paths, and our intuition may not always be sufficient to determine the rotation directions. For instance, if the carrier and sun gears in Figure 8.36 are both driven clockwise, and the carrier's speed is greater than the sun's, then the ring gear will rotate clockwise. However, as the speed of the sun gear is gradually increased, the ring gear will slow down, stop, and then actually reverse its direction and rotate counterclockwise. With that in mind, instead of relying on our intuition, we can apply a design equation that relates the rotational velocities of the sun gear (ω_s), carrier (ω_c), and ring gear (ω_r):

$$\omega_s + n\omega_r - (1 + n)\omega_c = 0 \tag{8.24}$$

Figure 8.37

Three input-output connection points to a planetary geartrain.

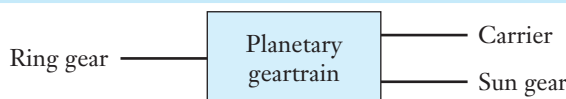
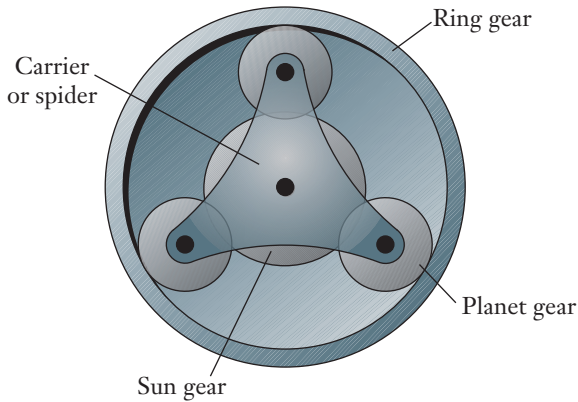


Figure 8.38

A balanced planetary geartrain having three planet gears.



Form factor

With the numbers of teeth on the sun and ring gears denoted by N_s and N_r , the geartrain's *form factor* n in Equation (8.24) is

$$n = \frac{N_r}{N_s} \quad (8.25)$$

This parameter is a size ratio that enters into the speed calculations for planetary geartrains and makes them more convenient; specifically, n is not the number of teeth on any gear. When n is a large number, then the planetary geartrain has a relatively small sun gear, and vice versa. The numbers of teeth on the sun, planet, and ring gears are related by

$$N_r = N_s + 2N_p \quad (8.26)$$

Sign convention

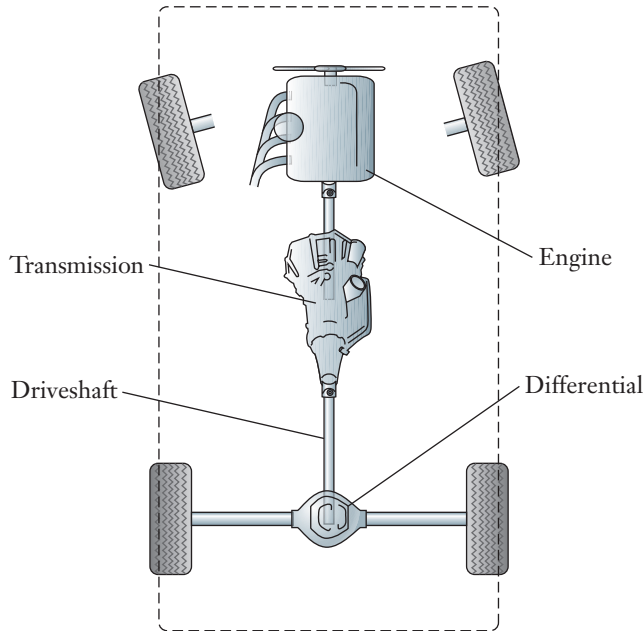
The direction in which each shaft rotates can be determined from the positive or negative signs of the angular velocity terms in Equation (8.24). As shown in Figure 8.36, we apply the convention that rotations of the shafts are positive when they are directed clockwise and negative when counterclockwise. By consistently applying the *sign convention*, we rely on the outcome of the calculation to indicate the directions that the shafts rotate. Of course, the clockwise convention is arbitrary, and we could just as well have chosen the counterclockwise direction as positive. In any event, once we define the convention for positive rotations, we use it consistently throughout a calculation.

Differential

A *differential* is a special type of planetary geartrain used in automobiles. The layout of the drivetrain for a rear-wheel-drive vehicle is shown in Figure 8.39 (see on page 390). The engine is located at the front of the automobile, and the crankshaft feeds into the transmission. The speed of the engine's crankshaft is reduced by the transmission, and the driveshaft extends down the length of the vehicle to the rear wheels. The transmission adjusts the velocity ratio between the rotation speeds of the engine's crankshaft and the driveshaft. In turn, the differential transfers torque from the driveshaft

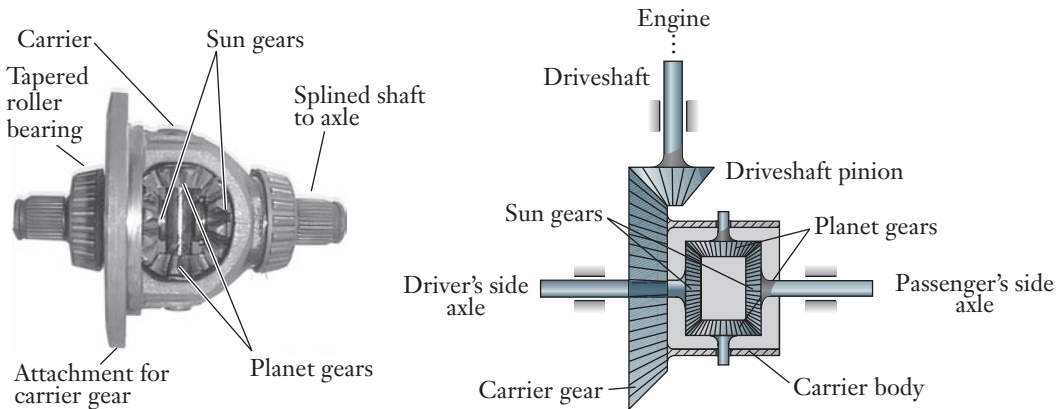
Figure 8.39

Drivetrain in a rear-wheel-drive automobile.



and splits it between the wheels on the driver's and passenger's sides. The differential therefore has one input (the driveshaft) and two outputs (the wheel axes). Look underneath a rear-wheel-drive automobile, and see if you can identify the transmission, driveshaft, and differential.

The differential enables the rear wheels to rotate at different speeds when the vehicle turns a corner, all the while being powered by the engine. As an automobile turns, the drive wheel on the outside of the curve rolls through a greater distance than does the wheel on the inside. If the drive wheels were connected to the same shaft and constrained to rotate at the same speed, they would need to slide and skid during the turn because of the velocity difference between the inside and outside of the corner. To solve that problem, mechanical engineers developed the differential for placement between the driveshaft and the axles of the rear wheels, allowing the wheels to rotate at different speeds as the vehicle follows curves along a road. Figure 8.40 shows the construction of an automobile's differential. The carrier is a structural shell that is attached to an external ring gear, which, in turn, meshes with a pinion on the driveshaft. As the carrier rotates, the two planet gears orbit in and out of the plane of the figure. Because of the bevel-type gear construction, the ring and sun gears of an equivalent planetary geartrain would have the same pitch radius. For that reason, the ring and sun gears in a differential are interchangeable, and both are conventionally called sun gears.

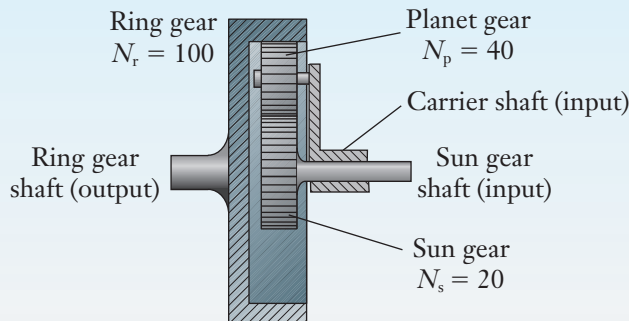
**Figure 8.40**

A differential for a small sedan automobile, which can be viewed as a planetary geartrain constructed from bevel-type gears.

Image courtesy of the authors.

Example 8.8 Planetary Geartrain Speeds

The planetary geartrain has two inputs (the sun gear and carrier) and one output (the ring gear). (See Figure 8.41.) When viewed from the right-hand side, the hollow carrier shaft is driven at 3600 rpm clockwise, and the shaft for the sun gear turns at 2400 rpm counterclockwise. (a) Determine the speed and direction of the ring gear. (b) The carrier's shaft is driven by an electric motor capable of reversing its direction. Repeat the calculation for the case in which it is instead driven at 3600 rpm counterclockwise. (c) For what speed and direction of the carrier's shaft will the ring gear not rotate?

Figure 8.41

Example 8.8 *continued***Approach**

To find the speed and direction of the appropriate gear in each operating condition, we apply Equations (8.24) and (8.25), together with the sign convention shown in Figure 8.36. With clockwise rotations being positive, the known speeds in part (a) are $\omega_c = 3600$ rpm and $\omega_s = -2400$ rpm. In part (b), when the carrier's shaft reverses direction, $\omega_c = -3600$ rpm.

Solution

- (a) Based on the numbers of teeth shown in the diagram, the form factor is

$$\begin{aligned} n &= \frac{100 \text{ teeth}}{20 \text{ teeth}} && \leftarrow \left[n = \frac{N_r}{N_s} \right] \\ &= 5 \frac{\text{teeth}}{\text{teeth}} \\ &= 5 \end{aligned}$$

Substituting the given quantities into the planetary geartrain design equation, $[\omega_s + n\omega_r - (1 + n)\omega_c = 0]$

$$(-2400 \text{ rpm}) + 5\omega_r - 6(3600 \text{ rpm}) = 0$$

and $\omega_r = 4800$ rpm. Because ω_r is positive, the ring gear rotates clockwise.

- (b) When the carrier's shaft reverses, $\omega_c = -3600$ rpm. The speed of the ring gear using the planetary geartrain design equation $[\omega_s + n\omega_r - (1 + n)\omega_c = 0]$ becomes

$$(-2400 \text{ rpm}) + 5\omega_r - 6(-3600 \text{ rpm}) = 0$$

and $\omega_r = -3840$ rpm. The ring gear rotates in the direction opposite that in part (a) and at a lower speed.

- (c) The speeds of the sun gear and carrier are related by the planetary geartrain design equation $[\omega_s + n\omega_r - (1 + n)\omega_c = 0]$, and at that special condition

$$(-2400 \text{ rpm}) - 6\omega_c = 0$$

and $\omega_c = -400$ rpm. The carrier's shaft should be driven counterclockwise at 400 rpm for the ring gear to be stationary.

Discussion

The form factor is a dimensionless number because it is the ratio of the numbers of teeth on the ring and sun gears. Because this calculation involves only rotational speeds, not the velocity of a point as in Equation (8.2), it is acceptable to use the dimensions of rpm rather than convert the units for

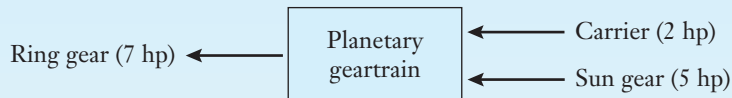
Example 8.8 *continued*

angular velocity into radians per unit time. These results demonstrate the complicated yet flexible nature of planetary geartrains to produce a wide range of output motions.

Clockwise carrier: $\omega_r = 4800$ rpm (clockwise)
 Counterclockwise carrier: $\omega_r = 3840$ rpm (counterclockwise)
 Stationary ring: $\omega_c = 400$ rpm (counterclockwise)

Example 8.9 Torque in a Planetary Geartrain

In part (a) of the previous example, the carrier and sun gear are driven by engines producing 2 hp and 5 hp, respectively (Figure 8.42). Determine the torque applied to the shaft of the output ring gear.

Figure 8.42**Approach**

To calculate the torque applied to the output shaft, we recognize that, to balance the power supplied to the geartrain, the output shaft must transfer a total of 7 hp to a mechanical load. To apply Equation (8.5) in dimensionally consistent units, first convert the units for power from horsepower to $(\text{ft} \cdot \text{lb})/\text{s}$ with the conversion factor in Table 7.2.

Solution

In dimensionally consistent units, the speed of the ring gear's output shaft is

$$\begin{aligned}\omega_r &= (4800 \text{ rpm}) \left(0.1047 \frac{\text{rad/s}}{\text{rpm}} \right) \\ &= 502.6 \text{ (rpm)} \left(\frac{\text{rad/s}}{\text{rpm}} \right) \\ &= 502.6 \frac{\text{rad}}{\text{s}}\end{aligned}$$

The total power supplied the geartrain is $P = 7$ hp, and, in dimensionally consistent units, this becomes

$$\begin{aligned}P &= (7 \text{ hp}) \left(550 \frac{(\text{ft} \cdot \text{lb})/\text{s}}{\text{hp}} \right) \\ &= 3850 \text{ (hp)} \left(\frac{(\text{ft} \cdot \text{lb})/\text{s}}{\text{hp}} \right) \\ &= 3850 \frac{\text{ft} \cdot \text{lb}}{\text{s}}\end{aligned}$$

Example 8.9 *continued*

The output torque is therefore

$$\begin{aligned}
 T &= \frac{3850 \text{ (ft} \cdot \text{lb)/s}}{502.6 \text{ rad/s}} && \leftarrow [P = T\omega] \\
 &= 7.659 \left(\frac{\text{ft} \cdot \text{lb}}{\cancel{\text{s}}} \right) \left(\frac{\cancel{\text{s}}}{\text{rad}} \right) \\
 &= 7.659 \text{ ft} \cdot \text{lb}
 \end{aligned}$$

Discussion

As before, we can directly cancel the radian unit when calculating the torque because the radian is a dimensionless measure of an angle. For this geartrain as a whole, the input power is equivalent to the power transferred to the mechanical load to the extent that friction within the geartrain can be neglected.

$$T = 7.659 \text{ ft} \cdot \text{lb}$$

SUMMARY

In this chapter, we discussed the topics of motion and power transmission in machinery in the context of geartrains and belt drives. The important quantities introduced in this chapter, common symbols representing them, and their units are summarized in Table 8.2, and Table 8.3 reviews the key equations used. The motion of geartrains and belt or chain drives encompasses mechanical components, forces and torques, and energy and power.

Gearsets, simple geartrains, compound geartrains, planetary geartrains, and belt and chain drives are used to transmit power, to change the rotation speed of a shaft, and to modify the torque applied to a shaft. More broadly, geartrains and belt and chain drives are examples of mechanisms commonly encountered in mechanical engineering. Mechanisms are combinations of gears, sheaves, belts, chains, links, shafts, bearings, springs, cams, lead screws, and other building-block components that can be assembled to convert one type of motion into another. Thousands of recipes for various mechanisms are available in printed and online mechanical engineering resources with applications including robotics, engines, automatic-feed mechanisms, medical devices, conveyor systems, safety latches, ratchets, and self-deploying aerospace structures.

Table 8.2

Quantities, Symbols,
and Units that Arise
when Analyzing
Motion and Power-
Transmission
Machinery

Quantity	Conventional Symbols	Conventional Units	
		USCS	SI
Velocity	v	in./s, ft/s	mm/s, m/s
Angle	θ	deg, rad	
Angular velocity	ω	rpm, rps, deg/s, rad/s	
Torque	T	in · lb, ft · lb	N · m
Work	W	ft · lb	J
Instantaneous power	P	hp, (ft · lb)/s	W
Diametral pitch	p	teeth/in.	—
Module	m	—	mm
Number of teeth	N	—	—
Velocity ratio	VR	—	—
Torque ratio	TR	—	—
Form factor	n	—	—

Table 8.3

Key Equations that
Arise when Analyzing
Motion and Power-
Transmission
Machinery

Velocity	$v = r\omega$
Work of a torque	$W = T\Delta\theta$
Instantaneous power	
Force	$P = Fv$
Torque	$P = T\omega$
Gear design	
Module	$m = \frac{2r}{N}$
Diametral pitch	$p = \frac{N}{2r}$
Velocity ratio	
Gearset	$VR = \frac{N_p}{N_g}$
Simple geartrain	$VR = \frac{N_{\text{input}}}{N_{\text{output}}}$
Compound geartrain	$VR = \left(\frac{N_1}{N_2}\right)\left(\frac{N_3}{N_4}\right)\cdots$
Belt/chain drive	$VR = \frac{d_{\text{input}}}{d_{\text{output}}}$
Torque ratio	$TR = \frac{1}{VR}$
Planetary geartrain	
Form factor	$n = \frac{N_r}{N_s}$
Velocity	$\omega_s + n\omega_r - (1 + n)\omega_c = 0$
Teeth	$N_r = N_s + 2N_p$

Self-Study and Review

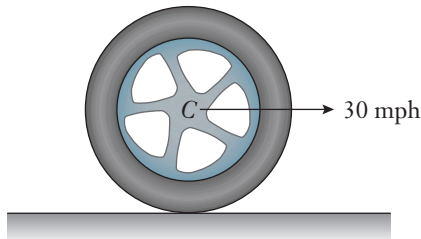
- 8.1. List several of the units that engineers use for angular velocity.
- 8.2. In what types of calculations must the unit of rad/s be used for angular velocity?
- 8.3. What is the difference between average and instantaneous power?
- 8.4. Sketch the shape of a spur gear's teeth.
- 8.5. What is the fundamental property of gearsets?
- 8.6. Define the terms “diametral pitch” and “module.”
- 8.7. What are a rack and pinion?
- 8.8. How do helical gears differ from spur gears?
- 8.9. Make a sketch to show the difference in shaft orientations when bevel gears and crossed helical gears are used.
- 8.10. How do simple and compound geartrains differ?
- 8.11. How are the velocity and torque ratios of a geartrain defined?
- 8.12. What relationship exists between the velocity and torque ratios for an ideal geartrain?
- 8.13. What are some of the differences between a V-belt and a timing belt?
- 8.14. Sketch a planetary geartrain, label its main components, and explain how it operates.
- 8.15. Describe the main components of an automobile's drivetrain.
- 8.16. What function does the differential in an automobile serve?

PROBLEMS

Problem P8.1

An automobile travels at 30 mph, which is also the speed of the center C of the tire (Figure P8.1). If the outer radius of the tire is 15 in., determine the rotational velocity of the tire in the units of rad/s, deg/s, rps, and rpm.

Figure P8.1

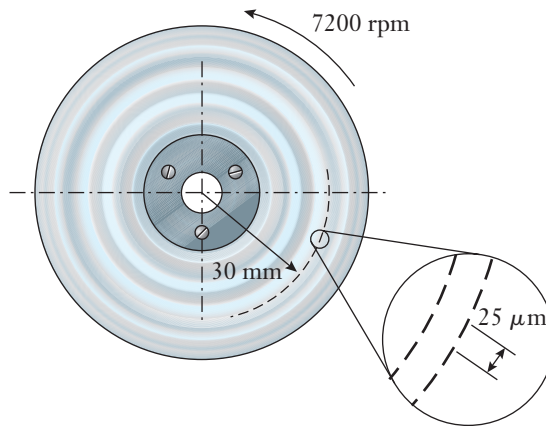


Problem P8.2

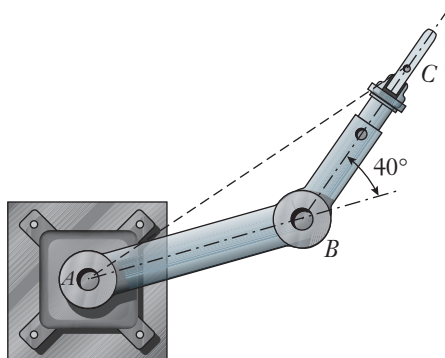
Assume that the tire in Problem P8.1 is now sitting on ice spinning with the same rotational velocity, but not translating forward. Determine the velocities of points at the top of the tire and at the bottom of the tire where it contacts the ice. Also, identify the direction of these velocities.

Problem P8.3

The disk in a computer hard drive spins at 7200 rpm (Figure P8.3). At the radius of 30 mm, a stream of data is magnetically written on the disk, and the spacing between data bits is $25\ \mu\text{m}$. Determine the number of bits per second that pass by the read/write head.

Figure P8.3**Problem P8.4**

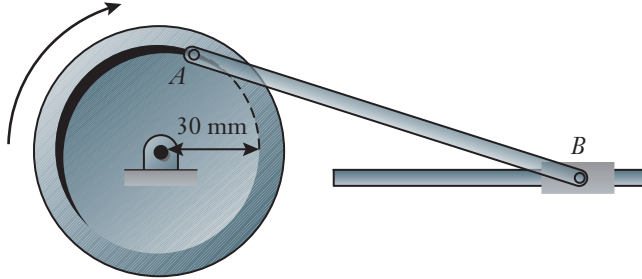
The lengths of the two links in the industrial robot are $AB = 22\ \text{in.}$ and $BC = 18\ \text{in.}$ (Figure P8.4). The angle between the two links is held constant at 40° as the robot's arm rotates about base A at $300^\circ/\text{s}$. Calculate the velocity of the center C of the robot's gripper.

Figure P8.4**Problem P8.5**

For the industrial robot in Problem P8.4, assume that the robot arm stops rotating about base A . However, the link BC begins to rotate about joint B . If link BC rotates 90° in $0.2\ \text{s}$, calculate the velocity of point C .

Problem P8.6

The driving disk spins at a constant 280 rpm clockwise (Figure P8.6). Determine the velocity of the connecting pin at A . Also, does the collar at B move with constant velocity? Explain your answer.

Figure P8.6**Problem P8.7**

A gasoline-powered engine produces 15 hp as it drives a water pump at a construction site. If the engine's speed is 450 rpm, determine the torque T that is transmitted from the output shaft of the engine to the pump. Express your answer in the units of $\text{ft} \cdot \text{lb}$ and $\text{N} \cdot \text{m}$.

Problem P8.8

A small automobile engine produces $260 \text{ N} \cdot \text{m}$ of torque at 2100 rpm. Determine the engine's power output in the units of kW and hp.

Problem P8.9

A diesel engine for marine propulsion applications produces a maximum power of 900 hp and torque of $5300 \text{ N} \cdot \text{m}$. Determine the engine speed necessary for this production in rpm.

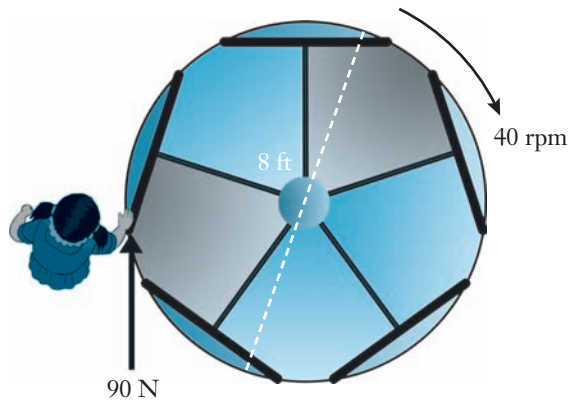
Problem P8.10

A diesel engine for on-highway construction applications produces 350 hp at 1800 rpm. Determine the torque produced by the engine at this speed in $\text{N} \cdot \text{m}$ and $\text{lb} \cdot \text{ft}$.

Problem P8.11

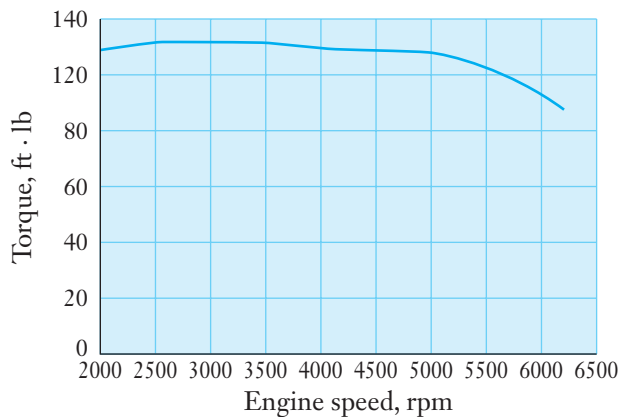
A child pushes a merry-go-round by applying a force tangential to the platform (Figure P8.11). To maintain a constant rotational speed of 40 rpm, the child must exert a constant force of 90 N to overcome the slowing effects of friction in the bearings and platform. Calculate the power exerted by the child in horsepower to operate the merry-go-round. The diameter of the platform is 8 ft.

Figure P8.11

**Problem P8.12**

The torque produced by a 2.5-L automobile engine operating at full throttle was measured over a range of speeds (Figure P8.12). By using this graph of torque as a function of the engine's speed, prepare a second graph to show how the power output of the engine (in the units of hp) changes with speed (in rpm).

Figure P8.12

**Problem P8.13**

A spur gearset has been designed with the following specifications:

Pinion gear: number of teeth = 32, diameter = 3.2 in.

Output gear: number of teeth = 96, diameter = 8.0 in.

Determine whether this gearset will operate smoothly.

Problem P8.14

The radius of an input pinion is 3.8 cm, and the radius of an output gear is 11.4 cm. Calculate the velocity and torque ratios of the gearset.

Problem P8.15

The torque ratio of a gearset is 0.75. The pinion gear has 36 teeth and a diametral pitch of 8. Determine the number of teeth on the output gear and the radii of both gears.

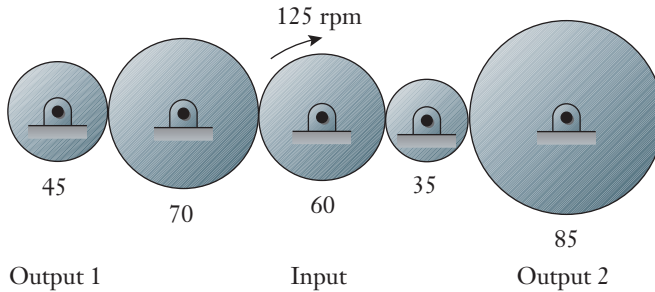
Problem P8.16

You are designing a geartrain with three spur gears: one input gear, one idler gear, and one output gear. The diametral pitch for the geartrain is 16. The diameter of the input gear needs to be twice the diameter of the idler gear and three times the diameter of the output gear. The entire geartrain needs to fit into a rectangular footprint of no larger than 16 in. high and 24 in. long. Determine an appropriate number of teeth and diameter of each gear.

Problem P8.17

The helical gears in the simple geartrain have teeth numbers as labeled (Figure P8.17). The central gear rotates at 125 rpm and drives the two output shafts. Determine the speeds and rotation directions of each shaft.

Figure P8.17

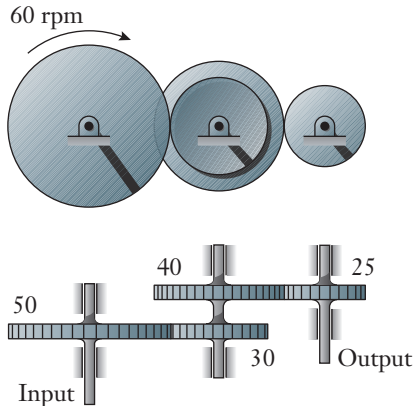


Problem P8.18

The spur gears in the compound geartrain have teeth numbers as labeled (Figure P8.18).

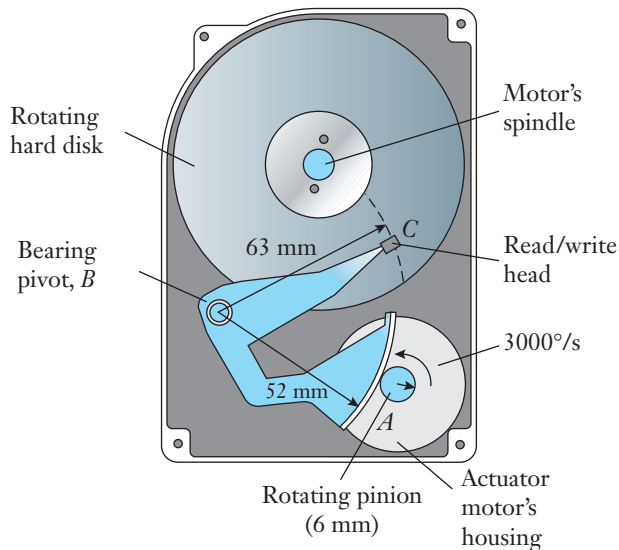
- Determine the speed and rotation direction of the output shaft.
- If the geartrain transfers 4 hp of power, calculate the torques applied to the input and output shafts.

Figure P8.18

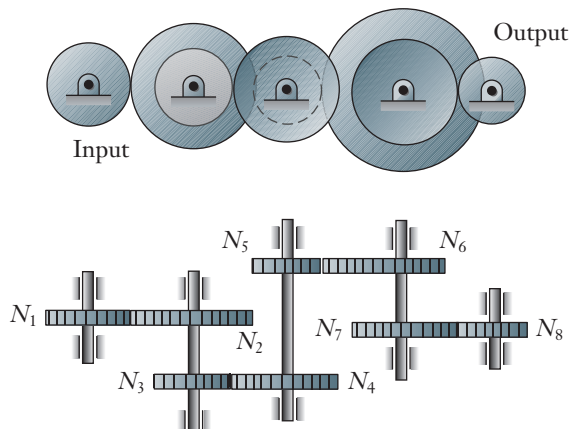


Problem P8.19

The disk in a computer's magnetic hard drive spins about its center spindle while the recording head C reads and writes data (Figure P8.19). The head is positioned above a specific data track by the arm, which pivots about its bearing at B . As the actuator motor A turns through a limited angle of rotation, its 6-mm-radius pinion rotates along the arc-shaped segment, which has a radius of 52 mm about B . If the actuator motor turns at 3000 deg/s over a small range of motion during a track-to-track seek operation, calculate the speed of the read/write head.

Figure P8.19**Problem P8.20**

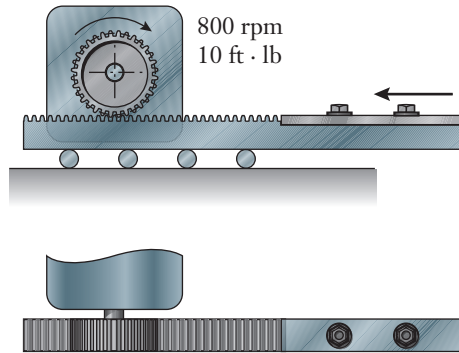
For the compound geartrain, obtain an equation for the velocity and torque ratios in terms of the numbers of teeth labeled in Figure P8.20.

Figure P8.20

Problem P8.21

In the motorized rack-and-pinion system in a milling machine, the pinion has 50 teeth and a pitch radius of 0.75 in. (Figure P8.21). Over a certain range of its motion, the motor turns at 800 rpm.

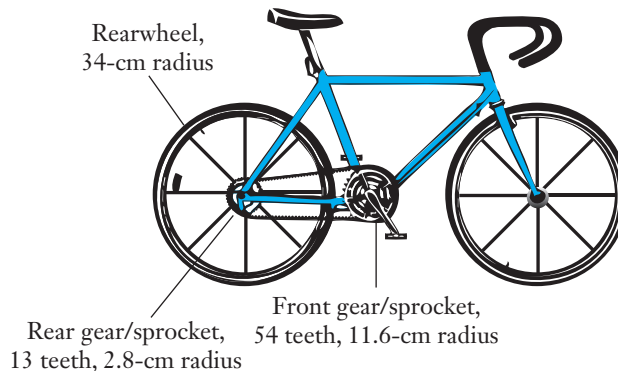
- Determine the horizontal velocity of the rack.
- If a peak torque of $10 \text{ ft} \cdot \text{lb}$ is supplied by the motor, determine the pulling force that is produced in the rack and transferred to the milling machine.

Figure P8.21**Problem P8.22**

Two gears, whose centers are 1.5 m apart, are connected by a belt. The smaller 0.40-m diameter gear is rotating with an angular velocity of 50 rad/s. Calculate the angular velocity of the larger 1.1-m diameter gear.

Problem P8.23

The world distance record for riding a standard bicycle in 1 h is 49.70 km, set in 2005 in Russia (Figure P8.23). A fixed-gear bike was used, which means that the bike effectively had only one gear. Assuming that the rider pedaled at a constant speed, how fast did the rider pedal the front gear/sprocket in rpm to achieve this record?

Figure P8.23

Problem P8.24

A motor running at 3000 rpm is connected to an output sheave through a V-belt. The output sheave drives a separate conveyer belt at 1000 rpm. The distance between the centers of the motor and output sheaves should be at least 12 in. but no more than 24 in. There should be at least 4 in. of clearance between the sheaves as well. Determine appropriate diameters for the motor and output sheaves.

Problem P8.25

- (a) By directly examining sprockets and counting their numbers of teeth, determine the velocity ratio between the (input) pedal crank and the (output) rear wheel for your multiple-speed bicycle, a friend's, or a family member's. Make a table to show how the velocity ratio changes depending on which sprocket is selected in the front and back of the chain drive. Tabulate the velocity ratio for each speed setting.
- (b) For a bicycle speed of 15 mph, determine the rotational speeds of the sprockets and the speed of the chain for one of the chain drive's speed settings. Show in your calculations how the units of the various terms are converted to obtain a dimensionally consistent result.

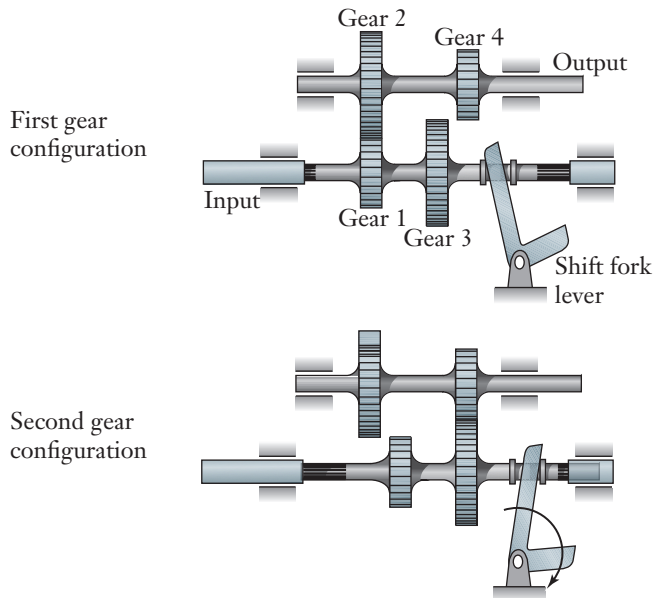
Problem P8.26

Estimate the velocity ratio between the (input) engine and the (output) drive wheels for your automobile (or a friend's or a family member's) in several different transmission settings. You will need to know the engine's speed (as read on the tachometer), the vehicle's speed (on the speedometer), and the outer diameter of the wheels. Show in your calculations how the units of the various terms are converted to obtain a dimensionally consistent result.

Problem P8.27

In a two-speed gearbox (Figure P8.27, see on page 404), the lower shaft has splines, and it can slide horizontally as an operator moves the shift fork. Design the transmission and choose the number of teeth on each gear to produce velocity ratios of approximately 0.8 (in first gear) and 1.2 (in second gear). Select gears having only even numbers of teeth between 40 and 80. Note that a constraint exists on the sizes of the gears so that the locations of the shaft's centers are the same in the first and second gears.

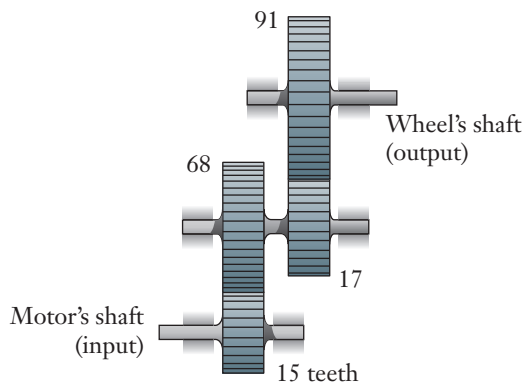
Figure P8.27

**Problem P8.28**

The geartrain in the transmission for the Segway® Personal Transporter uses helical gears to reduce noise and vibration (Figure P8.28). The vehicle's wheels have a diameter of 48 cm, and it has a top speed of 12.5 mph.

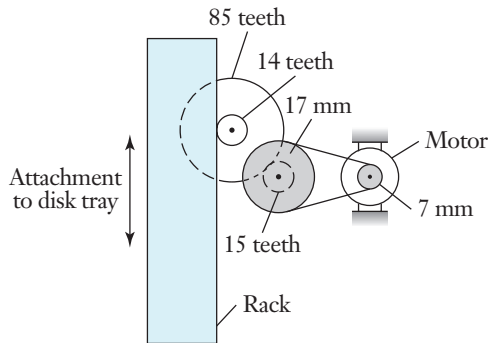
- Each gearset was designed to have a noninteger gear ratio so that the teeth mesh at different points from revolution to revolution, therefore reducing wear and extending the transmission's life. What is the velocity ratio at each mesh point?
- What is the velocity ratio for the entire geartrain?
- In the units of rpm, how fast does the motor's shaft turn at top speed?

Figure P8.28



Problem P8.29

The mechanism that operates the load/unload tray for holding the disk in a tabletop Blu-ray™ player uses nylon spur gears, a rack, and a belt drive (Figure P8.29). The gear that meshes with the rack has a module of 2.5 mm. The gears have the numbers of teeth indicated, and the two sheaves have diameters of 7 mm and 17 mm. The rack is connected to the tray that holds the disk. For the tray to move at 0.1 m/s, how fast must the motor turn?

Figure P8.29**Problem P8.30**

Explain why the number of teeth on the ring gear in a planetary geartrain is related to the sizes of the sun and planet gears by the equation $N_r = N_s + 2N_p$.

Problem P8.31

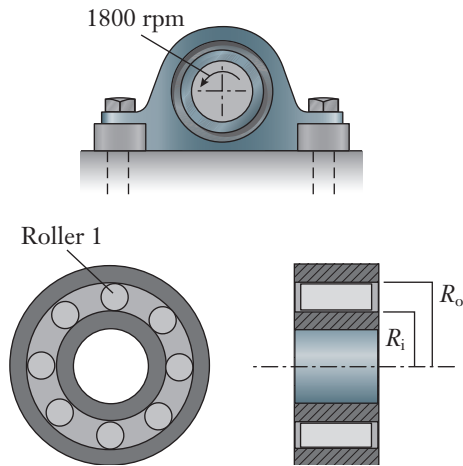
A planetary geartrain with $N_s = 48$ and $N_p = 30$ uses the carrier and ring gear as inputs and the sun gear as output. When viewed from the right-hand side in Figure 8.36, the hollow carrier shaft is driven at 1200 rpm clockwise, and the shaft for the ring gear is driven at 1000 rpm counterclockwise.

- Determine the speed and rotation direction of the sun gear.
- Repeat the calculation for the case in which the carrier is instead driven counterclockwise at 2400 rpm.

Problem P8.32

Rolling element bearings (Section 4.6) are analogous to the layout of a balanced planetary geartrain (Figure P8.32, see on page 406). The rotations of the rollers, separator, inner race, and outer race are similar to those of the planets, carrier, sun gear, and ring gear, respectively, in a planetary geartrain. The outer race of the straight roller bearing shown is held by a pillow-block mount. The inner race supports a shaft that rotates at 1800 rpm. The radii of the inner and outer races are $R_i = 0.625$ in. and $R_o = 0.875$ in. In the units of ms, how long does it take for roller 1 to orbit the shaft and return to the topmost position in the bearing? Does the roller orbit in a clockwise or counterclockwise direction?

Figure P8.32

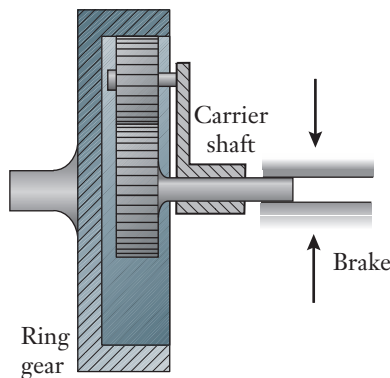
**Problem P8.33**

The ring gear on a planetary geartrain similar to Figure 8.38 has 60 teeth and is rotating clockwise at 120 rpm. The planet and sun gears are identically sized. The sun gear is rotating clockwise at 150 rpm. Calculate the speed and rotation direction of the carrier.

Problem P8.34

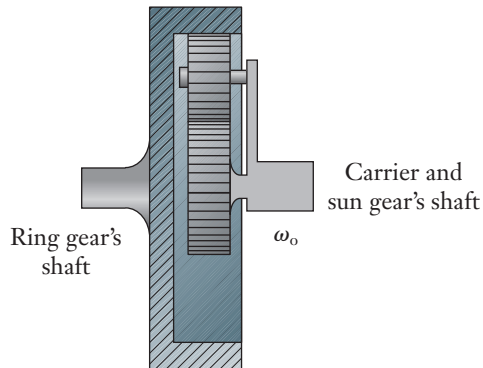
- The shaft of the sun gear in the planetary geartrain is held stationary by a brake (Figure P8.34). Determine the relationship between the rotational speeds of the shafts for the ring gear and carrier. Do those shafts rotate in the same direction or opposite directions?
- Repeat the exercise for the case in which the ring gear shaft is instead held stationary.
- Repeat the exercise for the case in which the carrier shaft is instead held stationary.

Figure P8.34



Problem P8.35

In one of the planetary geartrain configurations used in automotive automatic transmissions, the shafts for the sun gear and carrier are connected and turn at the same speed ω_o (Figure P8.35). In terms of the geartrain's form factor n , determine the speed of the ring gear's shaft for this configuration.

Figure P8.35**References**

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Greek Alphabet

Table A.1

Name	Uppercase	Lowercase
Alpha	A	α
Beta	B	β
Gamma	Γ	γ
Delta	Δ	δ
Epsilon	E	ϵ
Zeta	Z	ζ
Eta	H	η
Theta	Θ	θ
Iota	I	ι
Kappa	K	κ
Lambda	Λ	λ
Mu	M	μ
Nu	N	ν
Xi	Ξ	ξ
Omicron	O	$\omicron$
Pi	Π	π
Rho	P	ρ
Sigma	Σ	σ
Tau	T	τ
Upsilon	Y	υ
Phi	Φ	ϕ
Chi	X	χ
Psi	Ψ	ψ
Omega	Ω	ω

Trigonometry Review

○ B.1 DEGREES AND RADIANs

The magnitude of an angle can be measured by using either degrees or radians. One full revolution around a circle corresponds to 360° or 2π radians. The radian unit is abbreviated as rad. Likewise, half of a circle corresponds to an angle of 180° or π rad. You can convert between radians and degrees by using the following factors:

$$1 \text{ deg} = 1.7453 \times 10^{-2} \text{ rad} \quad (\text{B.1})$$

$$1 \text{ rad} = 57.296 \text{ deg} \quad (\text{B.2})$$

○ B.2 RIGHT TRIANGLES

A right angle has a measure of 90° , or equivalently, $\pi/2$ rad. As shown in Figure B.1, a right triangle is composed of one right angle and two acute angles. An acute angle is one that is smaller than 90° . In this case, the acute angles of the triangle are denoted by the lowercase Greek characters phi (ϕ) and theta (θ), as listed in Appendix A. Since the magnitudes of the triangle's three angles sum to 180° , the two acute angles in the triangle are related by

$$\phi + \theta = 90^\circ \quad (\text{B.3})$$

The lengths of the two sides that meet and form the right angle are denoted by x and y . The remaining longer side is called the hypotenuse, and it has length z . An acute angle is formed between the hypotenuse and either side adjacent to it. The three side lengths are related to one another by the Pythagorean theorem

$$x^2 + y^2 = z^2 \quad (\text{B.4})$$

Figure B.1

Right triangle with side lengths x , y , z and acute angles ϕ and θ .

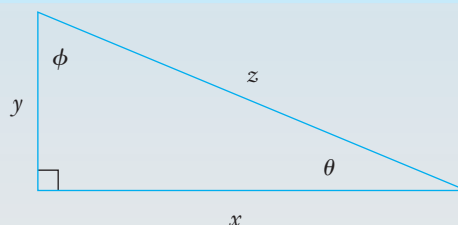


Table B.1

Some Values of the
Sine, Cosine, and
Tangent Functions

	0°	30°	45°	60°	90°
sin	0	0.5	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	0.5	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞

If the lengths of two sides in a right triangle are known, the third length can be determined from this expression.

The lengths and angles in a right triangle are also related to one another by properties of the trigonometric functions called sine, cosine, and tangent. Each of these functions is defined as the ratio of one side's length to another. Referring to the angle θ in Figure B.1, the sine, cosine and tangent of θ involve the following ratios of the adjacent side's length (x), the opposite side's length (y), and the hypotenuse's length (z):

$$\sin \theta = \frac{y}{z} \quad \left(\frac{\text{opposite side's length}}{\text{hypotenuse's length}} \right) \quad (\text{B.5})$$

$$\cos \theta = \frac{x}{z} \quad \left(\frac{\text{adjacent side's length}}{\text{hypotenuse's length}} \right) \quad (\text{B.6})$$

$$\tan \theta = \frac{y}{x} \quad \left(\frac{\text{opposite side's length}}{\text{adjacent side's length}} \right) \quad (\text{B.7})$$

Similarly, for the other acute angle (ϕ) in the triangle,

$$\sin \phi = \frac{x}{z} \quad (\text{B.8})$$

$$\cos \phi = \frac{y}{z} \quad (\text{B.9})$$

$$\tan \phi = \frac{x}{y} \quad (\text{B.10})$$

From these definitions, you can see such characteristics of these functions as $\sin(45^\circ) = \cos(45^\circ) = \frac{\sqrt{2}}{2}$ and $\tan(45^\circ) = 1$. Other numerical values of the sine, cosine, and tangent functions are listed in Table B.1.

◎ B.3 IDENTITIES

The sine and cosine of an angle are related to one another by

$$\sin^2 \theta + \cos^2 \theta = 1 \quad (\text{B.11})$$

This expression can be deduced by applying the Pythagorean theorem and the definitions in Equations (B.5) and (B.6) to a right triangle. When two angles

θ_1 and θ_2 are combined, the sines and cosines of their sums and differences can be determined from

$$\sin(\theta_1 \pm \theta_2) = \sin \theta_1 \cos \theta_2 \pm \cos \theta_1 \sin \theta_2 \quad (\text{B.12})$$

$$\cos(\theta_1 \pm \theta_2) = \cos \theta_1 \cos \theta_2 \pm \sin \theta_1 \sin \theta_2 \quad (\text{B.13})$$

In particular, when $\theta_1 = \theta_2$, these expressions are used to deduce the double angle formulas

$$\sin 2\theta = 2 \sin \theta \cos \theta \quad (\text{B.14})$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta \quad (\text{B.15})$$

◎ B.4 OBLIQUE TRIANGLES

Simply put, an oblique triangle is any triangle that is not a right triangle. Therefore, an oblique triangle does not contain a right angle. There are two types of oblique triangles. In an acute triangle, all three angles have magnitudes of less than 90° . In an obtuse triangle, one of the angles is greater than 90° , and the other two are each less than 90° . In all cases, the sum of the angles in a triangle is 180° .

Two theorems of trigonometry can be applied to determine an unknown side length or angle in an oblique triangle. These theorems are called the laws of sines and cosines. The law of sines is based on the ratio of a triangle's side length to the sine of the opposing angle. That ratio is the same for the three pairs of lengths and opposite angles in the triangle. Referring to the side lengths and angles labeled in Figure B.2, the law of sines is

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad (\text{B.16})$$

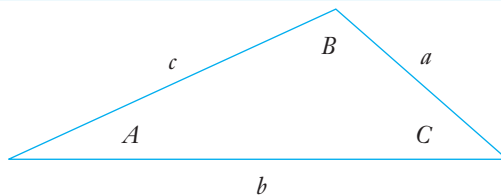
When we happen to know the lengths of two sides in a triangle and their included angle, the law of cosines can be used to find the length of the triangle's third side:

$$c^2 = a^2 + b^2 - 2ab \cos C \quad (\text{B.17})$$

When $C = 90^\circ$, this expression reduces to $c^2 = a^2 + b^2$, which is a restatement of the Pythagorean theorem (B.4).

Figure B.2

An oblique triangle with side lengths a , b , c and opposing angles A , B , and C .



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