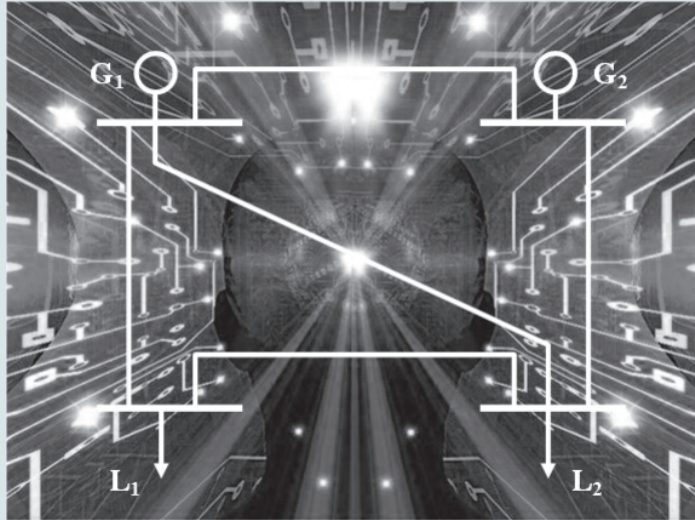


Artificial Intelligence in Power System Optimization

Weerakorn Ongsakul
Dieu Ngoc Vo



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PREFACE

In recent years, there have been many books published on power system optimization. Most of these books do not cover applications of artificial intelligence based methods. Moreover, with the recent increase of artificial intelligence applications in various fields, it is becoming a new trend in solving optimization problems in engineering in general due to its advantages of being simple and efficient in tackling complex problems. For this reason, the application of artificial intelligence in power systems has attracted the interest of many researchers around the world during the last two decades. This book is a result of our effort to provide information on the latest applications of artificial intelligence to optimization problems in power systems before and after deregulation.

The book is intended as a reference for graduate students in electric power system management, and will also be useful for researchers who are interested in this field. The contents of the book are based on recent studies specializing in the application of artificial intelligence such as particle swarm optimization, evolutionary programming, fuzzy logic, and augmented Lagrange Hopfield network to power system optimization problems. Issues widely considered in power system operation comprise economic dispatch, unit commitment, hydrothermal scheduling, optimal power flow, reactive power dispatch, and available transfer capacity. Moreover, subjects of the electricity market, such as day-ahead generating scheduling and transmission pricing, are also included.

The purpose of this book is to provide students of electric power system management the basic knowledge of different optimization problems in the power system operation of utilities as well as the electricity market. The presentation is simple and easy to understand, and also up to date, especially in methods adopting artificial intelligence. At the end of each chapter, practical problems have been provided. The book has been written with the help of several experts. We hope to get the readers interested in finding state-of-the-art solutions based on artificial intelligence to power system optimization.

You are most welcome to send your comments or queries to us.

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CONTENTS

<i>Preface</i>	v
1. Introduction	1
1.1 Importance of Power System Optimization	1
1.2 Artificial Intelligence as a New Trend in Optimization Problems	3
1.3 Artificial Intelligence Applications in Power Systems	6
1.4 Overview of the Book	7
1.5 References	9
2. Economic Dispatch	11
2.1 Introduction	11
2.2 Generator Incremental Cost Curve	12
2.3 Economic Dispatch Problem Formulation without Regarding Loss	13
2.4 Economic Dispatch Considering Transmission Losses	20
2.5 Economic Dispatch with Ramp Rate Constraint	29
2.6 Fuel Constrained Economic Dispatch	31
2.7 Economic Dispatch Considering Emissions	42
2.8 Economic Dispatch with Transmission Constraint	49
2.9 Economic Dispatch with Non-smooth Cost Functions	59
2.10 Combined Heat and Power Economic Dispatch	77
2.11 Hydrothermal Economic Dispatch	86
2.12 Optimal Power Dispatch in a Competitive Electricity Supply Industry	96
2.13 Summary	105
2.14 Problems for Exercise	106
2.15 References	113
3. Unit Commitment	118
3.1 Introduction	118
3.2 Unit Commitment Problem Formulation	127
3.3 Unit Commitment Solution Methods	130
3.4 Constrained Unit Commitment	191
3.5 Security Constrained Unit Commitment	210
3.6 Price-based Unit Commitment	219

3.7 Summary	227
3.8 Problems	227
3.9 References	230
4. Hydrothermal Scheduling	232
4.1 Introduction	232
4.2 Hydroelectric Plant Model	233
4.3 Hydrothermal Scheduling Formulation	233
4.4 Hydrothermal Scheduling Methods	235
4.5 Hydroelectric Units in Series	244
4.6 Pumped Storage Hydroelectric Plants	245
4.7 Problem Formulation for Hydrothermal Scheduling for Both Hydroelectric and Pumped Storage Hydroelectric Plants	246
4.8 Solution Methods for Hydrothermal Scheduling Including Pumped Storage Hydroelectric Plants	249
4.9 Summary	288
4.10 Problems	289
4.11 References	290
5. Optimal Power Flow	292
5.1 Introduction	292
5.2 Optimal Power Flow Problem Formulation	293
5.3 Optimal Real Power Dispatch with Network Limit Constraints	295
5.4 Neural Network Application to Optimal Power Flow	304
5.5 Particle Swarm Optimization for Optimal Power Flow	307
5.6 Summary	312
5.7 Problems	312
5.8 References	314
6. Optimal Reactive Power Dispatch	317
6.1 Introduction	317
6.2 Reactive Power in Power Systems	318
6.3 Conventional Optimal Reactive Power Dispatch	333
6.4 Optimal Reactive Power Dispatch in Deregulated Electricity Markets	339
6.5 TVAC-PSO based Optimal Reactive Power Dispatch under Deregulated Electricity Market Conditions	348
6.6 Summary	352
6.7 Problems	352
6.8 References	353

7. Available Transfer Capability	355
7.1 Introduction	355
7.2 Transmission Transfer Capability Concepts	356
7.3 Available Transfer Capability Principles	359
7.4 Available Transfer Capability Definition and Determination	360
7.5 Methodologies to Calculate ATC	367
7.6 Available Transfer Capability Calculation	370
7.7 Calculation of Total Transfer Capability by Evolutionary Programming	376
7.8 Total Transfer Capability Enhancement by Hybrid Evolutionary Algorithm	384
7.9 Optimal Placement of Multi-type Facts Devices for ATC Enhancement Using HEA	393
7.10 Summary	405
7.11 Problems	406
7.12 References	408
Appendix A: Mathematical Model Derivations	415
Appendix B: Data of Example Systems	440
Appendix C: Results of Examples	448
Appendix D: Tips for Programming in Matlab	470
<i>Index</i>	493
<i>Color Plate Section</i>	497

INTRODUCTION

1.1 IMPORTANCE OF POWER SYSTEM OPTIMIZATION

Power system engineering has the longest history of development among the various areas of electrical engineering. Ever since practical numerical optimization methods have been applied to power system engineering and operation, they have played a very important role. The value contributed by system optimization is considerable in economical terms with hundreds of millions of dollars saved annually in large utilities in terms of fuel cost, improved operational reliability and system security [1, 2]. As power systems are getting larger and more complicated due to the increase of load demand, the fossil fuel demand of thermal power plants increases which causes rising costs and rising emissions into the environment. Therefore, optimization has become essential for the operation of power system utilities in terms of fuel cost savings and environmental preservation [3].

The aim can be to minimize the cost of power generation in regulated power systems or to maximize social welfare in deregulated power systems while satisfying various operating constraints. In general, optimization problems are nonlinear, including nonlinear objective functions and nonlinear equality and inequality constraints. Moreover, with dwindling fossil fuel resources such as oil and coal, the limitations to large scale renewable energy development and controversial nuclear energy as well as concerns about unsustainable levels of environmental emissions, optimization has become even more important in power system operation for economical and environmental reasons [4-6].

There have been numerous methods including conventional and artificial intelligence techniques applied to solve power system optimization problems [7-18]. These methods are being constantly improved and developed to deal with large size systems and ever more interconnected systems. Optimization problems are complex due to a large number of constraints. Hence, finding better solutions with shorter computation times is the goal of these methods, and power system researchers have proposed several new, improved methods to this end [6].

Several optimization issues have been considered in power system operation such as economic dispatch, unit commitment, hydrothermal scheduling, optimal power flow, maintenance scheduling, etc.

Economic dispatch [1, 4] determines the optimal real power outputs for the generating units online so that fuel cost of generating units is minimized while all unit and system operating constraints are satisfied. Emissions may also be added to the objective function of this problem. In the economic dispatch approach, fuel cost functions of generating units are nonlinear curves and the optimal economical solution is found at that point where the total power output of online generating units meets the total load demand in an optimal manner. The fuel cost function of generating units in the economic dispatch approach can be approximated by a quadratic function. However, other real fuel cost functions such as higher order, multiple fuel types, and valve point effect can be included. In addition, generating units facing the practical constraint of prohibited operating zones are taken into consideration in this approach. All these issues make this a large-scale and nonlinearly constrained optimization problem. Therefore, it is of great interest to find a practical, i.e., economical and fast solution to this problem.

In the economic dispatch approach, it is assumed that all generating units are committed online to satisfy load demand. For off-line schedule planning, generating units are hourly scheduled on/off, based on load forecast for a planned time horizon. This is called *unit commitment* [1, 2]. The unit commitment problem is to schedule generators such that the total production cost of the system is minimized while the required spinning reserve is maintained and all other generator and system constraints are satisfied over a scheduled time horizon, ranging from one day to one week. Unit commitment scheduling is commonly a large-scale combinatorial problem. The optimal solution of this can be obtained by exhaustive enumeration of all sorts of feasible combinations of generating units, which is impractical for implementation. Moreover, in a deregulated environment, power system operation is price or profit based rather than driven by cost based centralized optimization. Hence, the aim of price-based unit commitment planning in deregulated power systems is to maximize the total profit of utilities without regard to any obligation to satisfy load demand and spinning reserve. Hence, a more inclusive solution for price-based unit commitment needs to be found.

In most power systems, there are not only thermal power plants but also hydro power plants. The optimal generation scheduling of a power system with both thermal and hydro power plants, called *hydrothermal scheduling* [1, 4], includes additional hydraulic constraints and others regarding systems with pumped-storage or cascaded hydro units. The hydrothermal scheduling method is generally used to minimize the total fuel cost of thermal units neglecting the marginal cost of hydro units subject to generator, system and hydraulic constraints in a given time horizon. In a short term hydrothermal scheduling approach, the schedule time horizon usually ranges from one day to one week.

Optimal power flow [1, 6] is also an important issue in power system optimization. It is naturally a nonlinear programming approach to optimize a certain objective function such as generation cost or transmission power losses while satisfying a set of equality and inequality of generator and system operational constraints which are imposed by capacity limitations and security requirements. The optimal power flow function was mathematically formulated as an extension of the economic dispatch problem. In the optimal power flow approach, several factors are considered such as bus voltages, bus angles and transformer taps. Other devices can also be integrated in the considerations such as FACTS devices and phase shifters. Solving this problem requires optimization and power flow analysis. Optimal power flow has become a powerful tool for off-line planning and online control.

Another important optimization approach in power systems is the *optimal reactive power dispatch* (ORPD) [19]. The objective of the ORPD is to determine the control variables such as generator voltage magnitudes, switchable VAR compensators and transformer tap setting so as to minimize the objective functions while satisfying the unit and system constraints. In ORPD, the objective can be the prevention of total power loss or voltage deviation at load buses for voltage profile improvement, or the voltage stability index for voltage stability enhancement. ORPD is a complex and large-scale optimization problem with a nonlinear objective and related operating constraints. In power system operation, the major role of ORPD is to maintain the load bus voltages within their limits to provide high quality of service to consumers.

In competitive electric power markets, electric utilities have to operate closer to their limits, which may lead to line overloading and voltage stability problems. *Available transfer capability* (ATC) [20] is a measure of the transfer capability remaining in a physical transmission network for further commercial activity over and above already committed use. ATC is calculated for each control area and posted on a public communication system for open access by a transmission network delivering electricity. Determination of ATC is a complicated task in which the total transfer capability (TTC) and two transmission margins are calculated: the transmission reliability margin (TRM) and the capacity benefit margin (CBM). An accurate determination of ATC is essential to maximize the utilization of the existing transmission network while maintaining system security. An underestimated ATC may lead to under-utilization of the transmission system, while an overestimated ATC can lower the system reliability.

1.2 ARTIFICIAL INTELLIGENCE AS A NEW TREND IN OPTIMIZATION PROBLEMS

Recently, methods based on artificial intelligence have been widely used for solving optimization problems. These methods have the advantage that they can deal with complex problems that cannot be solved by conventional methods. Moreover, these

methods are easy to apply due to their simple mathematical structure and easy to combine with other methods to hybrid systems adding the strengths of each single method. Artificial intelligence based methods generally simulate natural phenomena or the social behavior of humans or animals.

An *expert system* [21], also known as a knowledge based system, is a computer program that incorporates knowledge derived from experts in a specific subject to provide problem analysis to users. The common form of an expert system is a computer program containing the rules for analysis and recommendations for users who have less experience in solving a specific problem. Expert systems were developed during the 1960s and 1970s and commercially applied throughout the 1980s. The methodologies of expert systems can be classified into the categories of rule-based systems, knowledge-based systems, neural networks, object-oriented methodology, case-based reasoning, system architecture, intelligent agent systems, database methodology, modeling, and ontology. Expert systems are also combined with fuzzy systems to fuzzy-expert systems or combined with neural networks to neuron-expert systems. Recently, with the development of computer techniques, expert systems are applicable to online applications.

Fuzzy systems [12, 14] are considered a mathematical means of describing vagueness in linguistic terms instead of an exact mathematical description. They are appropriate for dealing with uncertainties and approximate reasoning. In a fuzzy system, the membership functions are vaguely defined to represent the degree of truth of some events or conditions. The values of membership functions range from 0 to 1 in their linguistic form associated with imprecise concepts. Fuzzy systems were developed in 1965 and have become popular in technical problem solving.

Artificial neural networks [6] are mathematical models simulating the human biological neural network for processing information. A neural network consists of some layers of artificial neurons linked by weight connections. There are several types of neural networks defined by their structure such as feed forward, back propagation, radial basis function, recurrent networks, etc. Each type of neural network is capable of some specific work after being trained. Neural networks are able to infer a function from observations which is particularly useful for applications with the complex tasks faced in real life like function approximation, classification, data processing, etc. The primary advantage of neural networks is the capability to learn algorithms, the online adaption of dynamic systems, quick parallel computation, and intelligent interpolation of data.

Simulated annealing [6] is a meta-heuristic search algorithm for solving optimization problems by locating a good approximation at the global optimum point of a given function in a search space. This method simulates the annealing in metallurgy used for heating and controlled cooling of a metal for its crystal resizing and effect reduction. Simulated annealing was developed in the 1980s for solving optimization problems in a discrete searching space and proved more efficient than the method of exhaustive enumeration of the search space.

Taboo search [6] is also a meta-heuristic search for solving combinatorial optimization problems in management science, industrial engineering, economics, and computer science. This method belongs to the local search techniques but it enhances the performance of local search methods using memory structures to match them with local minima at the beginning. Once a potential solution has been obtained, it is marked as taboo, thus the algorithm does not visit that possibility again and again during the search process. Taboo search was developed in the 1970s and recently has been widely used for its powerful search capabilities.

Ant colony optimization algorithm [22] is a probabilistic technique to solve optimization problems. It can be reduced to the problem of finding the shortest paths through graphs based on the behavior of ants in finding food for their colony by marking their trails with pheromones. The shortest path is the trail with the most pheromone marks which the ants will use to carry their food back home. The first algorithm was developed in 1991 and since then, many variants of this principle have been developed.

Genetic algorithm [2, 4] is a search technique used to find the exact or approximately best solution for optimization problems. The genetic algorithm belongs to evolutionary computation using the techniques inspired by evolutionary biology such as inheritance, mutation, selection and crossover. The genetic algorithm was developed part by part from the 1950s onward and is one of the most popular methods applied to various optimization problems in bioinformatics, phylogenetics, computer science, engineering, economics, chemistry, manufacturing, mathematics, physics and other fields. This method can take long computational times to find the optimal solution.

Evolutionary programming [8] also follows the evolutionary computation paradigms to find the globally optimal solution for an optimization problem. Evolutionary programming was developed in 1960 placing emphasis on the behavior of the linkage between parents and their offspring rather than trying to emulate the specific genetic operators as observed in nature. The main operators of evolutionary programming consist of mutation, evaluation and selection. This method is also widely used in different optimization techniques due to its powerful search capabilities.

Particle swarm optimization [6] is one of the most heuristic algorithms developed under emulation of the simplified social behavior of animals in swarms, e.g., in fish schools and bird flocks. It is a population based evolutionary algorithm found to be efficient in solving continuous non-linear optimization problems. Particle swarm optimization provides a population-based search procedure, in which individuals (particles) change their positions (states) over time. It uses a velocity vector based on the social behavior of the individuals of the population to update the current position of each particle in the swarm flying in a multidimensional search space of a problem. During the flight, each particle with a certain velocity is dynamically adjusted according to its flight experience and that of its neighboring particles to find the best position for itself among its neighbors. The particle swarm

optimization technique can deliver a high-quality solution within shorter calculation time with more stable convergence characteristics than other stochastic methods. Developed since 1995, particle swarm optimization has been successfully applied in many researches and application areas such as engineering, finance and management systems and is now one of the most widely used methods in optimizations.

Differential evolution [23], belonging to the class of evolution strategy optimizers, is a method of mathematical optimization of multidimensional functions to find the global minimum of a multidimensional and multimodal function fairly fast and reasonably robust. Developed in the mid 1990s, the differential evolution method is a simple population based and stochastic function minimizer and has become one of the most popular methods used by researchers. The central idea of this method is a scheme to generate trial parameter vectors by adding the weight difference between two population vectors to a third one that makes the scheme completely self-organizing. The trial vector is used for the next generation if it yields a reduction in the value of an objective function.

In general, the methods based on artificial intelligence are continuously developed further for other application in different power system optimization problems. Recently, hybrid systems combining the strengths of each single method have been favored by researchers due to various advantages over the single methods as presented above.

1.3 ARTIFICIAL INTELLIGENCE APPLICATIONS IN POWER SYSTEMS

For the two recent decades, artificial intelligence based methods have become popular for solving different problems in power systems such as control, planning, forecast, scheduling, etc. These methods can deal with the complex tasks faced by applications in modern large power systems with ever more interconnections installed to meet the increasing load demand. The application of these methods has been successful in many areas of power system engineering.

Some major problems of power systems that artificial intelligence methods have been applied to, include planning, operation and modeling analysis.

- *Power system expansion planning* [24]: The structure of a typical electrical power system is very large and complex including basic components as generators, AC and DC transmission systems, a distribution system, load, and FACTS devices. The main objective of least cost system expansion planning is to optimize the components necessary to provide an adequate energy supply at minimum cost. In power system expansion planning, many factors have to be taken into consideration such as demand, line and FACTS placement, environmental effects, etc. This includes the planning of any expansion of generation, transmission and distribution. Further issues also considered in this task are reactive power planning, reliability analysis and network design among others.

- *Power system operation* [6]: In real-time power system operation, the online power generation should meet the total load requirement plus transmission losses in a reliable and secure manner. In this approach, operating power plants and the transmission system are considered. The problems involved include generation scheduling, economic dispatch, optimal power flow, unit commitment, hydrothermal scheduling, reactive power dispatch, voltage control, load frequency control, static and dynamic security assessment, maintenance scheduling, fuel scheduling, contract management, equipment monitoring and dynamic generator rescheduling. For supervisory purposes, data acquisition and the energy management system (SCADA/EMS), load forecasting, load management, alarm processing, fault analysis and diagnosis, service restoration, network/substation switching, contingency analysis, optimal power flow and state estimation functions are included in the design.
- *Power system modeling/analysis* [11]: The issues of power system modeling/analysis include power flow analysis, transient stability, harmonics, dynamic stability, control design, simulation, protection and bad data detection.

Most recently, artificial intelligence based methods have been applied to problems in deregulated environments aiding congestion management, optimization of bidding strategies and generation scheduling [25].

However, there are still many challenges ahead. Therefore, these methods are continuously developed and improved to deal with ever larger complex power systems with an increasing number of constraints in both the traditional electric power supply industry and the competitive electricity market environment.

1.4 OVERVIEW OF THE BOOK

This book includes seven chapters solving optimization problems in power systems before and after deregulation addressed by both conventional and artificial intelligence methods. In each chapter, the problem formulation and the related solution methods are given. At the end of every chapter, practical problems are included to enable readers to practice further with the insight gained. The main contents of the chapters are briefly given as follows:

Chapter 1 gives an introduction to the significance of optimization problems in power systems and reviews the newest trends in applying artificial intelligence. These applications are also mentioned in this chapter.

The economic dispatch problem with different objective functions and constraints is comprehensively covered in *Chapter 2*. Economic dispatch is one of the attempts in power system operation to optimize the online units' schedule in the least expensive manner. The problems with economic dispatch and its various constraints such as ramp rate, transmission and emissions are considered first. Economic dispatch problems like non-smooth cost functions including prohibited

operating zones and multiple fuels are introduced. Other economic dispatch problems of multi-objective, combined heat and power and hydrothermal systems are also presented. An optimal dispatch solution for a competitive electricity market is also covered. Complementary, in this chapter, solution methods based on artificial intelligence including particle swarm optimization and augmented Hopfield Lagrange network are described. The particle swarm optimization method developed through simulation of simplified social models is one of the most modern heuristic algorithms. It can deal with non-convex optimization problems similar to other evolutionary algorithms. The Augmented Lagrange Hopfield network is a continuous Hopfield neural network having its energy function based on an augmented Lagrange function. This network is simpler than a Hopfield network and more efficient in finding the optimal solution within a shorter computational time. In addition, fuzzy linear programming, a method based on the vagueness in linguistics, is also used for solving economic dispatch problems in traditional and competitive markets. The mathematical models and explanations for these methods are also provided so that the reader can easily understand them.

In *Chapter 3*, unit commitment is used as the criteria for operation planning for one day up to one week ahead based on load forecast considering generating unit statuses as discrete variables and time dependent constraints. The solution methods have to handle both continuous and discrete variables. In this problem, many constraints are considered such as ramp rate, transmission line, environmental emissions, fuel limitations etc. Additionally, the unit commitment approach in competitive markets based on the maximization of total revenue is also considered. The methods suggested in this chapter are a new adaptive Lagrangian relaxation and hybrid systems based on generating a unit merit order, Hopfield network and Lagrangian relaxation. In the adaptive Lagrangian relaxation method, the Lagrangian multipliers are adaptively adjusted to enhance the convergence rate. Moreover, introducing a new on/off criterion also improves the convergence in this method. The augmented Lagrange-augmented Hopfield network is also an efficient method to solve the unit commitment problem. This method is a combination of both continuous and discrete Hopfield networks with its energy function based on the augmented Lagrange function. Another method based on a merit order of generating units is also efficient in resolving unit commitment issues. The merit order of generating units based on their average production cost is effective in determining the thermal unit scheduling. In the methods based on this, heuristic search is needed to enhance the results or repair constraint violations. Besides this, the sub-procedures of these methods for certain tasks are illustrated.

One of the most complex problems in generation scheduling optimization, hydrothermal scheduling, is presented in chapter 4. The role of hydro power plants has become more important in power systems due to their comparably low environmental impact and—over lifetime—marginal cost. However, the capacity of hydro power plants depends on reservoir capacity and weather. Therefore, the optimal scheduling of hydro-thermal systems is of utmost importance, especially

in systems with dominant hydro power plants. This problem includes hydraulic constraints in addition to the constraints of thermal systems in unit commitment as described in chapter 3. In the methods including adaptive Lagrangian relaxation and hybrid systems based on generating merit order, Hopfield network and Lagrangian relaxation are applied. The methods in this chapter are similar to the ones applied to the unit commitment problem.

Optimal power flow is addressed in chapter 5. The optimal power flow approach has been researched for a long time and has drawn more attention recently due to advancing numerical optimization techniques and computer and communication technology. With this approach, many constraints can be handled such as transmission capacity, real and reactive power limits of generators, bus voltages, security margins, transformer taps etc. The solution methods for this problem include conventional and artificial intelligence methods such as linear programming, quadratic programming and augmented Lagrange Hopfield network. The conventional methods are applicable only to convex power flow optimization problems while artificial intelligence based methods can easily resolve a non-convex non-differentiable OPF by both augmented Lagrange and sigmoid function of the Hopfield network.

Chapter 6 introduces the optimal reactive power dispatch in a power system. The reactive power performs no real work but, in the contrary, consumes resources. In fact, it is consumed throughout network elements and loads and has to be supplied by reactive power sources in the system. This chapter provides various models for optimal reactive power dispatch for passive and active elements consuming reactive power. In addition, the optimal reactive power dispatch before and after deregulation in power systems is also covered.

The problem of available transfer capability (ATC) is presented in chapter 7. With the movement towards a competitive market, open access to the transmission system plays an important role in interconnected transmission networks. Electric power transfers will increase and the reliable operation of the transmission networks becomes a difficult task. In this chapter, the concept, definition, principles and calculation methodologies of available transfer capacity are introduced. Besides, the conventional methods applied to calculate the available transfer capability such as linear approximation, continuation power flow and repetitive power flow, stability-constrained ATC, and optimal power flow based methods, meta-heuristic based methods including evolutionary programming and hybrid evolutionary algorithms are also proposed for ATC calculation.

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ECONOMIC DISPATCH*

2.1 INTRODUCTION

The economic dispatch problem plays an important role in power system operation. The principal objective of ED is to obtain the minimum operating cost needed to satisfy power balance, generator and network operating limit constraints. The optimal operating point of a power generation system is where the operating level of each generating unit is adjusted such that the total cost of delivered power is at a minimum. In an energy management system (EMS), Economic Dispatch (ED) is used to determine each generating level in the system in order to minimize the total generator fuel cost or total generator cost and emission of thermal units while still covering load demand plus transmission losses [1].

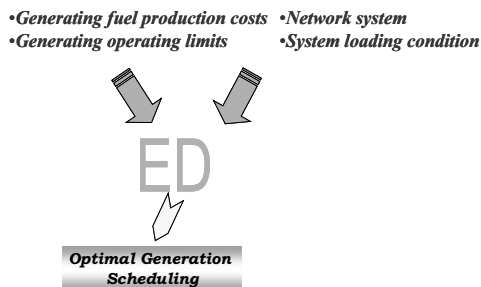


Fig. 2.1 The process of economic dispatch.

The cost function for each generator can be approximately represented by a quadratic function for mathematical convenience. Mathematical programming including gradient method, linear programming or quadratic programming (QP) can be used to determine the ED [1-2].

*This chapter has been written with assistance from Keerati Chayakulkheeree

2.2 GENERATOR INCREMENTAL COST CURVE

The analysis of the problems associated with the controlled operation of power systems contains many parameters of interest. Fundamental to the economic operating problem is the set of input-output characteristics of a thermal power generation unit. That is, gross input to the plant represents the total generator fuel cost whether measured in terms of dollars per hour or tons of coal per hour or millions of cubic feet or meters of gas per hour or any other unit. The net output of the plant is the electrical power output (P_G) in MW supplied to the electric utility system.

These data may be obtained from design calculations or from heat rate tests. When heat rate test data are used, it will usually be found that the data points do not form a smooth curve. Steam turbine generating units have several critical operating constraints. Generally, the minimum load at which a unit can operate is influenced more by the steam generator and the regenerative cycle than by the turbine itself. The only critical parameters for the turbine are shell and rotor metal differential temperatures, exhaust hood temperature, and rotor and shell expansion. A limitation to the minimum load is required to maintain fuel combustion stability and by inherent steam generator design constraints. For example, most supercritical units cannot operate below 30% of design capacity. A minimum flow of 30% is required to cool the tubes in the furnace of the steam generator adequately. Turbines do not have any inherent overload capability so the data shown on these curves normally do not extend much beyond 5% of the manufacturer's stated valve-wide-open capacity [1].

In practice, two approximation models are widely used: quadratic approximation as shown in (2.1) and piecewise linear approximation as shown in (2.2).

$$F(P_G) = a + bP_G + cP_G^2 \quad (2.1)$$

$$F(P_G) = \begin{cases} r_1 + q_1 P_G, & P_1 \leq P_G < P_2 \\ r_2 + q_2 P_G, & P_2 \leq P_G < P_3 \\ r_3 + q_3 P_G, & P_3 \leq P_G \leq P_4 \end{cases} \quad (2.2)$$

where

$F(P_G)$ generating unit operating cost,
 P_G real power generation.

and a, b, c, r_i and q_i with $i = 1, \dots, 3$ are cost coefficients.

The incremental cost of a unit is the slope (the derivative) of the unit cost curve. The data relates \$/MWh to net power output of the unit in MW. This relation is widely used in the economic dispatching of units. Figures 2.2 and 2.3 show the operating cost and incremental costs of quadratic and piecewise linear approximations, respectively.

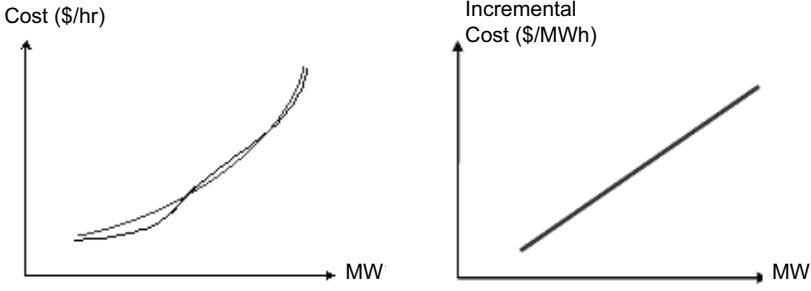


Fig. 2.2 Quadratic approximation cost function.

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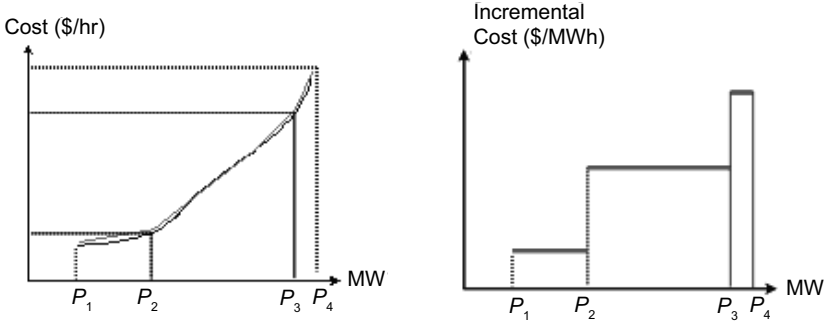


Fig. 2.3 Piecewise linear approximation cost function.

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2.3 ECONOMIC DISPATCH PROBLEM FORMULATION WITHOUT REGARDING LOSS

Consider a lossless system consisting of NG thermal generating units connected to a single bus-bar serving electrical load as shown in Fig. 2.4 [1] with the aim to minimize total power generation cost:

$$F_T = F_1(P_{G1}) + F_2(P_{G2}) + \dots + F_{NG}(P_{G,NG}) = \sum_{i=1}^{NG} F_i(P_{Gi}) \quad (2.3)$$

Subject to the simplified power balance constraint,

$$P_{G1} + P_{G2} + \dots + P_{G,NG} = P_L \text{ or } w(P_G) = P_D - \sum_{i=1}^{NG} P_{Gi} = 0 \quad (2.4)$$

where P_D is load demand and NG is the number of generating units. Thus, the Lagrange function can be given as

$$L = F_T(P_G) + \lambda \cdot w(P_G) \quad (2.5)$$

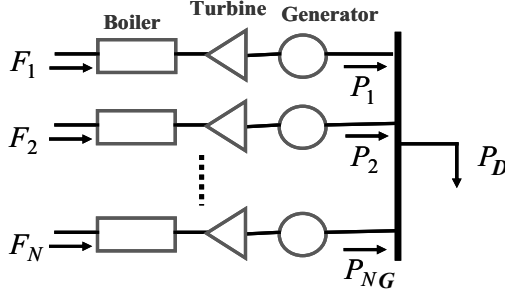


Fig. 2.4 Dispatch of generating units without network loss.

$$L = \sum_{i=1}^{NG} F_i(P_{Gi}) + \lambda \cdot (P_D - \sum_{i=1}^{NG} P_{Gi}) \quad (2.6)$$

The optimality condition is

$$\frac{\partial L}{\partial P_{Gi}} = \frac{\partial F_i(P_{Gi})}{\partial P_{Gi}} - \lambda = 0, \quad \text{for } i = 1, 2, \dots, NG \quad (2.7)$$

$$\frac{\partial F_i(P_{Gi})}{\partial P_{Gi}} = \lambda, \quad \text{for } i = 1, 2, \dots, NG \quad (2.8)$$

The necessary condition for the existence of a minimum cost operating point of a thermal power system is that the incremental cost of all units are equal to some undetermined value, λ , known as Lagrangian multiplier.

In addition, the power output of each unit must not exceed its operating limits. That is

$$P_{Gi}^{\min} \leq P_{Gi} \leq P_{Gi}^{\max}, \quad \text{for } i = 1, 2, \dots, NG \quad (2.9)$$

whereby $P_{Gi}^{\min}$ and $P_{Gi}^{\max}$ are generator minimum and maximum operating limits. With respect to the generators' operating limits, the necessary condition is slightly expanded to:

$$\frac{\partial F_i(P_{Gi})}{\partial P_{Gi}} = \lambda, \quad \text{for } P_{Gi}^{\min} < P_{Gi} < P_{Gi}^{\max} \quad (2.10)$$

$$\frac{\partial F_i(P_{Gi})}{\partial P_{Gi}} \leq \lambda, \quad \text{for } P_{Gi} = P_{Gi}^{\max} \quad (2.11)$$

$$\frac{\partial F_i(P_{Gi})}{\partial P_{Gi}} \geq \lambda, \quad \text{for } P_{Gi} = P_{Gi}^{\min} \quad (2.12)$$

Example 2.1

The economic operating point of the following three units as shown in Fig. 2.5 when delivering a total load of 320 MW is determined as follows:

Unit 1: Max output = 200 MW

Min output = 20 MW

Cost curve (\$/hr): $F_1(P_{G1}) = 300 + 10P_{G1} + 0.15P_{G1}^2$

Unit 2: Max output = 200 MW

Min output = 20 MW

Cost curve (\$/hr): $F_2(P_{G2}) = 200 + 20P_{G2} + 0.2P_{G2}^2$

Unit 3: Max output = 200 MW

Min output = 20 MW

Cost curve (\$/hr): $F_3(P_{G3}) = 100 + 10P_{G3} + 0.3P_{G3}^2$

The incremental costs of three units can be expressed in \$/MWh, as:

$$\frac{\partial F_1(P_{G1})}{\partial P_{G1}} = 10 + 0.3 \cdot P_{G1}$$

$$\frac{\partial F_2(P_{G2})}{\partial P_{G2}} = 20 + 0.4 \cdot P_{G2}$$

$$\text{and } \frac{\partial F_3(P_{G3})}{\partial P_{G3}} = 10 + 0.6 \cdot P_{G3}$$

Hence, the optimality condition is

$$10 + 0.3 \cdot P_{G1} = \lambda$$

$$F_1(P_{G1}) = 300 + 10P_{G1} + 0.15P_{G1}^2 \quad F_2(P_{G2}) = 200 + 20P_{G2} + 0.2P_{G2}^2 \quad F_3(P_{G3}) = 100 + 10P_{G3} + 0.3P_{G3}^2$$

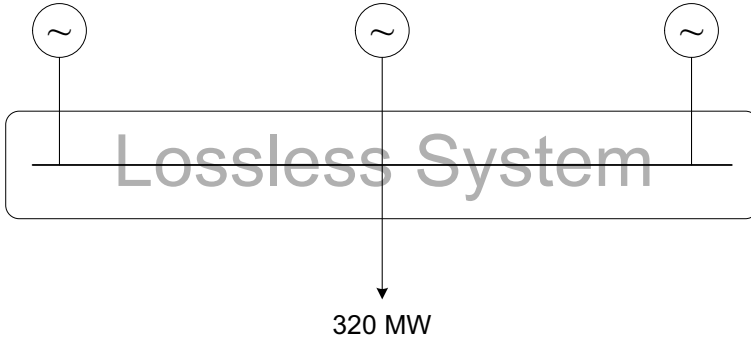


Fig. 2.5 The three unit system in example 2.1.

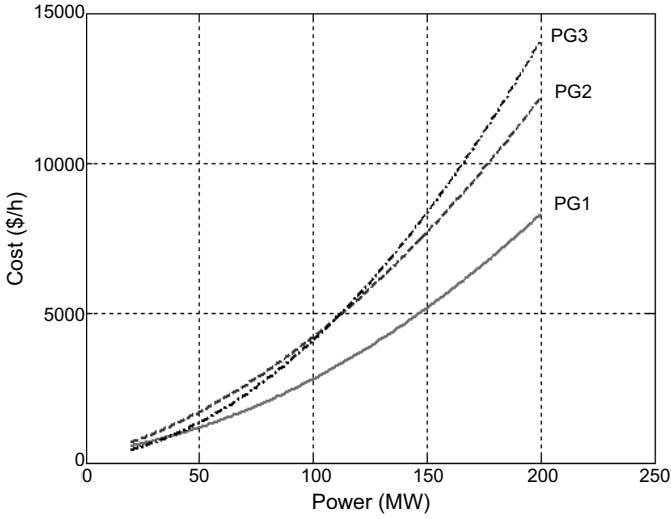


Fig. 2.6 Generation costs of three units in example 2.1.

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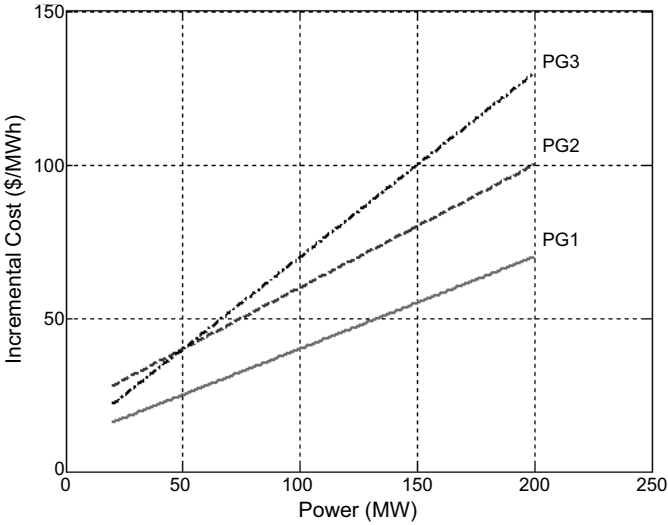


Fig. 2.7 Incremental costs of the three units in example 2.1.

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$$20 + 0.4 \cdot P_{G2} = \lambda$$

$$10 + 0.6 \cdot P_{G3} = \lambda$$

$$\text{and } P_{G1} + P_{G2} + P_{G3} = 320$$

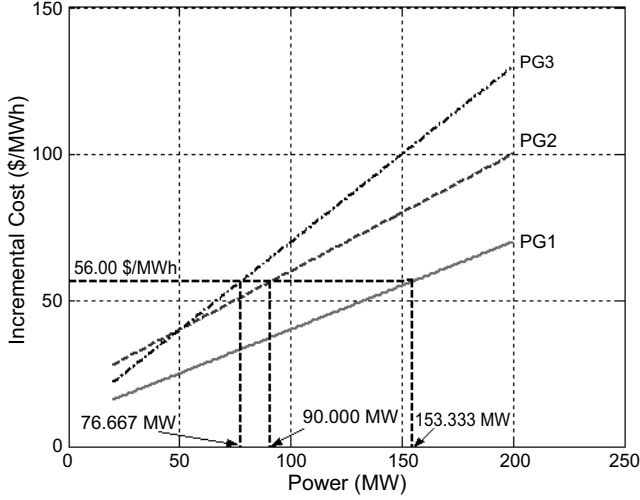


Fig. 2.8 Graphical representation of the dispatch result in the example 2.1.

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The equations can be re-arranged as:

$$\begin{bmatrix} 0.3 & 0 & 0 & -1 \\ 0 & 0.4 & 0 & -1 \\ 0 & 0 & 0.6 & -1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} P_{G1} \\ P_{G2} \\ P_{G3} \\ \lambda \end{bmatrix} = \begin{bmatrix} -10 \\ -20 \\ -10 \\ 320 \end{bmatrix}$$

The solution is

$$P_{G1} = 153.333 \text{ MW}$$

$$P_{G2} = 90.000 \text{ MW}$$

$$P_{G3} = 76.667 \text{ MW}$$

and $\lambda = 56.000 \text{ $/MWh}$

Note that each unit is within its minimum and maximum limits and the total output of all three units meets the required 320 MW load. The total generation cost is \$ 11,610 for 320 MWh.

Example 2.2

In the system in example 2.1, let's suppose that unit 1 cannot operate at its rated power and delivers only 100 MW.

Unit 1: Max output = 100 MW

Min output = 20 MW

Cost curve (\$/hr): $F_1(P_{G1}) = 300 + 10P_{G1} + 0.15P_{G1}^2$

Unit 2: Max output = 200 MW

Min output = 20 MW

Cost curve (\$/hr): $F_2(P_{G2}) = 200 + 20P_{G2} + 0.2P_{G2}^2$

Unit 3: Max output = 200 MW

Min output = 20 MW

Cost curve (\$/hr): $F_3(P_{G3}) = 100 + 10P_{G3} + 0.3P_{G3}^2$

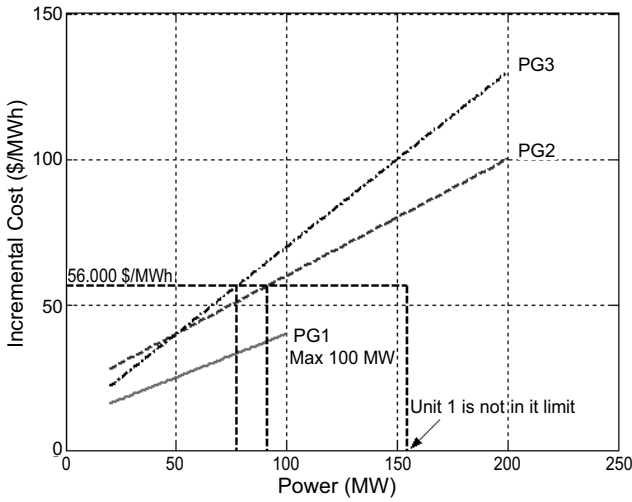


Fig. 2.9 Graphical representation of the dispatch result in example 2.2 when unit 1 is exceeding its limit.

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Thus, the optimality condition is

$$10 + 0.3 \cdot P_{G1} = \lambda$$

$$20 + 0.4 \cdot P_{G2} = \lambda$$

$$10 + 0.6 \cdot P_{G3} = \lambda$$

$$\text{and } P_{G1} + P_{G2} + P_{G3} = 320$$

Thus, the optimal dispatch solution is

$$P_{G1} = 153.333 \text{ MW}$$

$$P_{G2} = 90.000 \text{ MW}$$

$$P_{G3} = 76.667 \text{ MW}$$

$$\text{and } \lambda = 56.000 \text{ \$ / MWh}$$

Here, unit 1 exceeds its limit of 100 MW. Therefore, unit 1 is set to its maximum output (100 MW) and unit 2 and 3 are to be dispatched for the remaining load (320–100 = 220 MW).

Then, the constrained optimality condition becomes

$$P_1 = 100$$

$$20 + 0.4 \cdot P_{G2} = \lambda$$

$$10 + 0.6 \cdot P_{G3} = \lambda$$

$$\text{and } 100 + P_{G2} + P_{G3} = 320$$

The solution is

$$P_{G1} = 100.000 \text{ MW}$$

$$P_{G2} = 122.000 \text{ MW}$$

$$P_{G3} = 98.000 \text{ MW}$$

$$\lambda = 68.800 \text{ \$ / MWh}$$

Each unit is within its minimum and maximum limits and the total output of all three units meets the required 320 MW load.

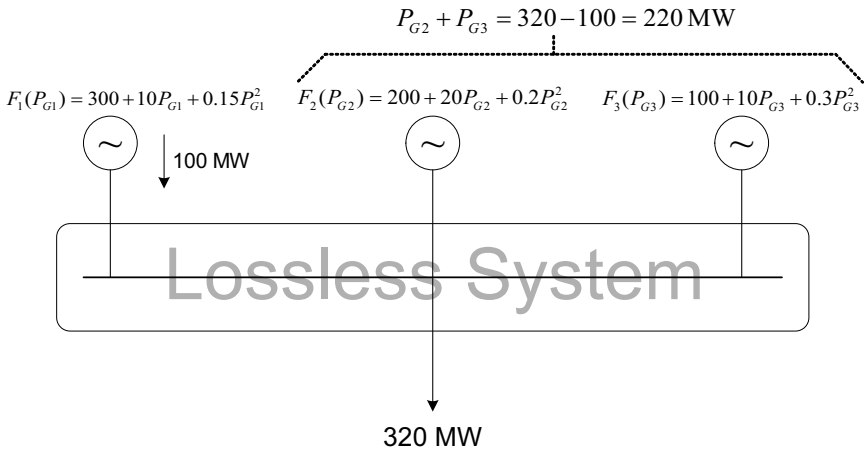


Fig. 2.10 Condition for optimal dispatch of example 2.2.

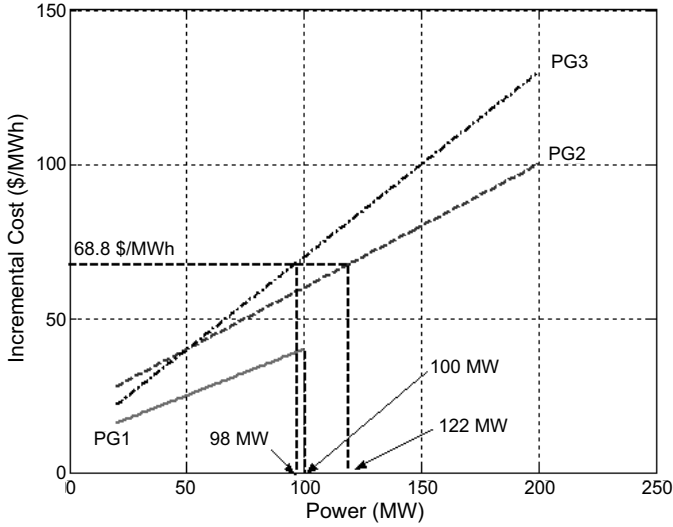


Fig. 2.11 Graphical representation of the dispatch result in example 2.2 with all units within their limits.

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2.4 ECONOMIC DISPATCH CONSIDERING TRANSMISSION LOSSES

Considering that all thermal power generation systems are connected to an equivalent load bus through a transmission network, the economic dispatching problem becomes slightly more complicated than discussed in the calculation neglecting the losses. With respect to the network losses, the objective function is the same with the following load balance constraint equation:

$$w(P_G) = P_D + P_{loss} - \sum_{i=1}^{NG} P_{Gi} = 0 \quad (2.13)$$

where P_{loss} is the total real power loss.

The Lagrange function can be set up

$$L = \sum_{i=1}^{NG} F_i(P_{Gi}) + \lambda \cdot (P_D + P_{loss} - \sum_{i=1}^{NG} P_{Gi}) \quad (2.14)$$

The optimality condition is

$$\frac{\partial L}{\partial P_i} = \frac{\partial F_i(P_i)}{\partial P_i} - \lambda \cdot (1 - \frac{\partial P_{loss}}{\partial P_i}) = 0, \text{ for } i = 1, 2, \dots, NG \quad (2.15)$$

$$\frac{\partial F_i(P_i)}{\partial P_i} \cdot \frac{1}{(1 - \frac{\partial P_{loss}}{\partial P_i})} = \lambda, \quad \text{for } i=1,2,\dots, NG \quad (2.16)$$

It is much more difficult to solve this set of equations than that of the lossless case since this second set consists of nonlinear equations which are difficult to resolve by analytical methods. The calculation of an incremental transmission loss is illustrated in appendix A.

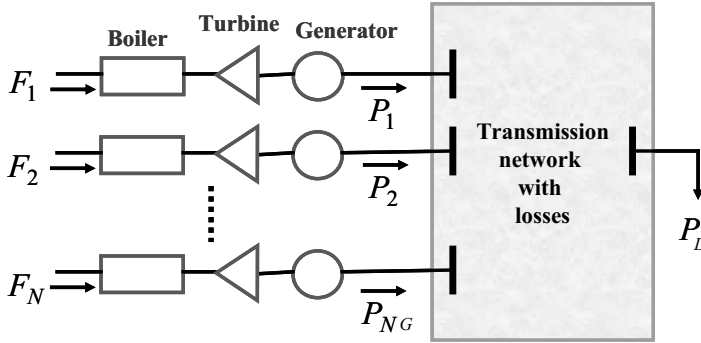


Fig. 2.12 Dispatch of generating units with network losses.

For exact loss calculation, the power flow solution can be incorporated in the ED problem formulation [1-2] as in (2.17) and (2.18).

$$P_{Gi} - P_{Di} = \sum_{j=1}^{NB} |V_i| |V_j| |y_{ij}| \cos(\theta_{ij} - \delta_{ij}), \quad i = 1, \dots, NB \quad (2.17)$$

$$Q_{Gi} - Q_{Di} = - \sum_{j=1}^{NB} |V_i| |V_j| |y_{ij}| \sin(\theta_{ij} - \delta_{ij}), \quad i = 1, \dots, NB \quad (2.18)$$

where

- P_{Di} total real power demand at bus i (MW),
- P_{Gi} real power generation at bus i (MW),
- Q_{Di} total reactive power demand at bus i (MVar),
- Q_{Gi} reactive power generation at bus i (MVar),
- θ_{ij} angle of the y_{ij} element of Y_{bus} (radian),
- δ_{ij} voltage angle difference between bus i and j (radian),
- $|y_{ij}|$ magnitude of the y_{ij} element of Y_{bus} (mho),
- $|V_i|$ voltage magnitude at bus i (kV),
- NB total number of buses.

To approximate transmission losses, several other methods can be used, involving, e.g., B-coefficients, Y_{bus} , sensitivity factors and transmission loss coefficients.

One widely used approximation method using B-coefficients is also known as Kron's formula which is expressed as follows:

$$P_{loss} = \sum_{i=1}^{NG} \sum_{j=1}^{NG} P_{Gi} B_{ij} P_{Gj} + \sum_{i=1}^{NG} B_{i0} P_{Gi} + B_{00} \quad (2.19)$$

where B_{ij} , B_{0i} and B_{00} are constant loss coefficients under certain system conditions. The detailed evaluation of these coefficients by the classical method via power flow is given in appendix A.

For simplicity, the power loss in (2.19) can be reduced to:

$$P_{loss} = \sum_{i=1}^{NG} \sum_{j=1}^{NG} P_{Gi} B_{ij} P_{Gj} \quad (2.20)$$

In (2.20), the loss coefficients B_{ij} can be evaluated by generalized as generation distribution factor (GGDF) given in appendix A.

The power loss in the transmission system is also represented as a function of real and reactive power:

$$P_{loss} = \sum_{i=1}^{NB} \sum_{j=1}^{NB} \left[a_{ij} (P_i P_j + Q_i Q_j) + b_{ij} (Q_i P_j - P_i Q_j) \right] \quad (2.21)$$

where

$$a_{ij} = \frac{R_{ij}}{|V_i| |V_j|} \cos(\delta_i - \delta_j) \quad (2.22)$$

$$b_{ij} = \frac{R_{ij}}{|V_i| |V_j|} \sin(\delta_i - \delta_j) \quad (2.23)$$

P_i real power injection at bus i , $P_i = P_{Gi} - P_{Di}$

Q_i reactive power injection at bus i , $Q_i = Q_{Gi} - Q_{Di}$

$R_{ij} + jX_{ij}$ element ij of impedance matrix.

The detailed evaluation of power loss in (2.21) is given in appendix A.

Augmented Lagrange Hopfield network (ALHN) is used for solving the ED problem including power loss as expressed in Kron's formula. ALHN is a continuous Hopfield network with its energy function based on an augmented Lagrange function. In ALHN, the energy function is augmented by Hopfield terms from a Hopfield neural network and penalty factors from the augmented Lagrangian function to

damp out oscillation of the Hopfield network during the convergence process [3]. Thus, ALHN can overcome the drawbacks of the conventional Hopfield network due to its simplicity while getting closer to the optimal solution and featuring faster convergence. The detailed model of ALHN is given in appendix A.

The augmented Lagrangian function of the problem is formulated as follows:

$$L = \sum_{i=1}^{NG} (a_i + b_i P_{Gi} + c_i P_{Gi}^2) + \lambda \left(P_{loss} + P_D - \sum_{i=1}^{NG} P_{Gi} \right) + \frac{1}{2} \beta \left(P_{loss} + P_D - \sum_{i=1}^{NG} P_{Gi} \right)^2 \quad (2.24)$$

where

- L augmented Lagrange function,
- a_i, b_i, c_i cost coefficients of generating unit i ,
- P_D total power load demand,
- P_{loss} total power loss,
- λ Lagrangian multiplier,
- β penalty factor.

To represent power outputs in ALHN, NG continuous neurons and one multiplier neuron are required. The energy function of the problem is formulated based on the augmented Lagrangian function as follows:

$$E = \sum_{i=1}^{NG} (a_i + b_i V_i + c_i V_i^2) + V_\lambda \left(P_{loss} + P_D - \sum_{i=1}^{NG} V_i \right) + \frac{1}{2} \beta \left(P_{loss} + P_D - \sum_{i=1}^{NG} V_i \right)^2 \quad (2.25)$$

$$+ \sum_{i=1}^{NG} \left(\int_0^{V_i} g^{-1}(V) dV + \int_0^{V_\lambda} g^{-1}(V) dV \right)$$

where

- E energy function of ALHN,
- V_i output of continuous neuron i representing output power P_{Gi} ,
- V_λ output of the multiplier neuron representing Lagrangian multiplier.

The sums of integral terms are Hopfield terms where their global effect is a displacement of solutions towards the interior of the state space.

The dynamics of neuron inputs are defined as the derivatives of the energy function with respect to outputs of neurons which are derived as follows:

$$\frac{dU_i}{dt} = -\frac{\partial E}{\partial V_i} = - \left\{ \begin{aligned} & (b_i + 2c_i V_i) \\ & + \left[V_\lambda + \beta \left(P_{loss} + P_D - \sum_{i=1}^{NG} V_i \right) \right] \left(\frac{\partial P_{loss}}{\partial V_i} - 1 \right) + U_i \end{aligned} \right\} \quad (2.26)$$

$$\frac{dU_\lambda}{dt} = + \frac{\partial E}{\partial V_\lambda} = P_{loss} + P_D - \sum_{i=1}^{NG} V_i \quad (2.27)$$

where

$$\frac{\partial P_{loss}}{\partial V_i} = 2 \sum_{j=1}^{NG} B_{ij} V_j + B_{i0} \quad (2.28)$$

U_i input of continuous neuron i ,

U_λ input of the multiplier neuron.

The inputs of neurons at iteration n are updated from the iteration $n-1$ as follows:

$$U_i^{(n)} = U_i^{(n-1)} + \alpha_i \frac{dU_i}{dt} = U_i^{(n-1)} - \alpha_i \frac{\partial E}{\partial V_i} \quad (2.29)$$

$$U_\lambda^{(n)} = U_\lambda^{(n-1)} + \alpha_\lambda \frac{dU_\lambda}{dt} = U_\lambda^{(n-1)} + \alpha_\lambda \frac{\partial E}{\partial V_\lambda} \quad (2.30)$$

where α_i and α_λ are updating step sizes for neurons.

The outputs of continuous neurons representing the unit power outputs are calculated via a sigmoid function:

$$V_i = g(U_i) = \left(\frac{P_{Gi}^{\max} - P_{Gi}^{\min}}{2} \right) \left[1 + \tanh(\sigma U_i) \right] + P_{Gi}^{\min} \quad (2.31)$$

where σ is the slope of the sigmoid function determining the shape of this function.

The outputs of multiplier neurons are defined by a transfer function as follows:

$$V_\lambda = g_\lambda(U_\lambda) = U_\lambda \quad (2.32)$$

For the selection of parameters in the neural network, the slope of the sigmoid function σ and the penalty factor β are fixed at 100 and 0.001, respectively. The updating step sizes for neurons including α_i and α_λ , which are smaller than one, will be tuned depending on the problem.

The algorithm requires initial conditions for all neurons. For the continuous neurons representing the unit power outputs, their outputs are initiated by “mean distribution”. That is, the initial output of a generating unit is given proportional to its maximum power output as follows:

$$V_i^{(0)} = P_D \cdot \frac{P_{Gi}^{\max}}{\sum_{j=1}^{NG} P_{Gj}^{\max}} \quad (2.33)$$

where $V_i^{(0)}$ is the initial value of the output of continuous neuron i .

The inputs of continuous neurons are calculated via the inverse function of the sigmoid function:

$$U_i^{(0)} = \ln \left[\frac{1}{2\sigma} \frac{V_i^{(0)} - P_{Gi}^{\min}}{P_{Gi}^{\max} - V_i^{(0)}} \right] \quad (2.34)$$

where $U_i^{(0)}$ is the initial value of the input of continuous neuron i .

For the multiplier neuron associated with the power balance constraint, the output is initialized by the mean value which is obtained by solving $\partial E / \partial V_i = 0$ in (2.26), neglecting penalty factor and inputs of neurons:

$$V_\lambda^{(0)} = \frac{1}{NG} \sum_{i=1}^{NG} \frac{b_i + 2c_i V_i^{(0)}}{1 - \frac{\partial P_{loss}}{\partial V_i}} \quad (2.35)$$

where $V_\lambda^{(0)}$ is the initial value of the output of the multiplier neuron which. The initial value of the input of the multiplier neuron is initialized by its output value.

In the ALHN model, the errors are calculated from the constraint errors and neural iterative errors. The power balance constraint error at iteration n is determined as:

$$\Delta P^{(n)} = \left| P_{loss} + P_D - \sum_{i=1}^{NG} V_i^{(n)} \right| \quad (2.36)$$

where $\Delta P^{(n)}$ is the power balance constraint error at iteration n . The iterative errors of neurons at iteration n are defined as:

$$\Delta V_i^{(n)} = \left| V_i^{(n)} - V_i^{(n-1)} \right| \quad (2.37)$$

$$\Delta V_\lambda^{(n)} = \left| V_\lambda^{(n)} - V_\lambda^{(n-1)} \right| \quad (2.38)$$

where $\Delta V_i^{(0)}$ and $\Delta V_\lambda^{(0)}$ are iterative errors of continuous and multiplier neurons at iteration n , respectively. The maximum error of the model at iteration n is determined by the combination of power balance and iterative error:

$$Err_{\max}^{(n)} = \max \left\{ \Delta P^{(n)}, \Delta V_i^{(n)}, \Delta V_\lambda^{(n)} \right\} \quad (2.39)$$

where $Err_{\max}^{(n)}$ is the maximum error of ALHN at iteration n .

The algorithm will be terminated when either the maximum error is lower than a pre-specified tolerance or maximum number of iterations is reached. The algorithm of ALHN for solving the ED problem is as follows:

- Step 1: Read parameters for the problem including cost coefficients, maximum power outputs, load demand, and loss coefficients.
- Step 2: Select parameters for ALHN including slope of sigmoid function, penalty factor, and updating step sizes.

- Step 3: Set the maximum number of iterations and the threshold for maximum error of ALHN.
- Step 4: Initiate outputs of all neurons and calculate their corresponding inputs from (2.33) to (2.35).
- Step 5: Set the iteration $n = 1$.
- Step 6: Calculate dynamics of neurons by (2.26) and (2.27).
- Step 7: Update inputs of neurons by (2.29) and (2.30).
- Step 8: Calculate outputs of neurons by (2.31) and (2.32).
- Step 9: Calculate maximum error by (2.39).
- Step 10: If $n < N_{max}$ or $Err_{max}^{(n)} > \varepsilon$, $n = n + 1$ and return to Step 6.
- Step 11: Calculate total cost using the objective function.

where N_{max} is the maximum number of iterations and ε is the threshold of the maximum error of ALHN.

Example 2.3

Suppose the fuel cost characteristics and generation limits of a three-unit system are the following:

$$F_1(P_{G1}) = 213.1 + 11.669P_{G1} + 0.00533P_{G1}^2 \text{ \$ /h}$$

$$50 \leq P_{G1} \leq 200 \text{ MW}$$

$$F_2(P_{G2}) = 200.0 + 10.333P_{G2} + 0.00889P_{G2}^2 \text{ \$ /h}$$

$$37.5 \leq P_{G2} \leq 150 \text{ MW}$$

$$F_3(P_{G3}) = 240.0 + 10.833P_{G3} + 0.00741P_{G3}^2 \text{ \$ /h}$$

$$45 \leq P_{G3} \leq 180 \text{ MW}$$

These units supply a load demand of 210 MW. The loss coefficients for transmission loss calculation are as follows:

$$B_{ij} = 10^{-3} \times \begin{bmatrix} 0.6760 & 0.0953 & -0.0507 \\ 0.0953 & 0.5210 & 0.0901 \\ -0.0507 & 0.0901 & 0.2940 \end{bmatrix}$$

$$B_{i0} = [-0.0766 \quad -0.00342 \quad 0.0189]^T$$

$$B_{00} = 4.0357$$

The parameters for ALHN are selected as shown below:

$$\sigma = 100; \beta = 0.001; \alpha_i = 0.011; \alpha_\lambda = 0.005$$

$$N_{max} = 2500; \varepsilon = 10^{-4}.$$

The initial values of neurons are determined:

$$V_i = \begin{bmatrix} 79.2453 \\ 59.4340 \\ 71.3208 \end{bmatrix}; U_i = \begin{bmatrix} -0.0071 \\ -0.0071 \\ -0.0071 \end{bmatrix}$$

$$V_\lambda = 12.7089; U_\lambda = 12.7089.$$

1st iteration

Dynamics of all neurons are calculated:

$$\frac{\partial E}{\partial V_i} = \begin{bmatrix} 0.2304 \\ -0.2345 \\ -0.0262 \end{bmatrix}; \frac{\partial E}{\partial V_\lambda} = 7.7797$$

The inputs of neurons are updated:

$$U_i = \begin{bmatrix} -0.0094 \\ -0.0047 \\ -0.0068 \end{bmatrix}; U_\lambda = 12.7401$$

The outputs of neurons are calculated by the sigmoid function:

$$V_i = \begin{bmatrix} 69.8785 \\ 68.8967 \\ 72.4484 \end{bmatrix}; V_\lambda = 12.7401$$

Power loss $P_{loss} = 8.4375$ MW and error are determined:

$$\Delta P^{(1)} = 7.2138; \Delta V_i^{(1)} = \begin{bmatrix} -9.3667 \\ 9.4628 \\ 1.1276 \end{bmatrix}; \Delta V_\lambda^{(1)} = 0.0311$$

$$Err_{max}^{(1)} = 9.4628.$$

2nd iteration

Dynamics of neurons are recalculated:

$$\frac{\partial E}{\partial V_i} = \begin{bmatrix} -0.0412 \\ 0.0137 \\ 0.0045 \end{bmatrix}; \frac{\partial E}{\partial V_\lambda} = 7.2138$$

The inputs of neurons are re-updated:

$$U_i = \begin{bmatrix} -0.0090 \\ -0.0049 \\ -0.0069 \end{bmatrix}; U_\lambda = 12.7689$$

The outputs of neurons are calculated:

$$V_i = \begin{bmatrix} 71.3412 \\ 68.2806 \\ 72.2535 \end{bmatrix}; V_\lambda = 12.7689$$

The power loss P_{loss} is 8.4023 MW and the error is re-calculated:

$$\Delta P^{(2)} = 6.5269; \Delta V_i^{(2)} = \begin{bmatrix} 1.4627 \\ -0.6161 \\ -0.1948 \end{bmatrix}; \Delta V_\lambda^{(2)} = 0.0289$$

$$Err_{max}^{(2)} = 6.5269.$$

After 15 iterations, the final result with a maximum error of 6.8517×10^{-5} is obtained as follows:

$$P_{G1} = 73.7211 \text{ MW}; P_{G2} = 69.9227 \text{ MW}; P_{G3} = 75.1771 \text{ MW}$$

$$P_{loss} = 8.8209 \text{ MW}; \lambda = 12.8153 \text{ \$/MWh}; \text{Total cost: \$ 3164.57/h}$$

The convergence characteristic of ALHN for the problem based on the maximum error is shown in Fig. 2.13.

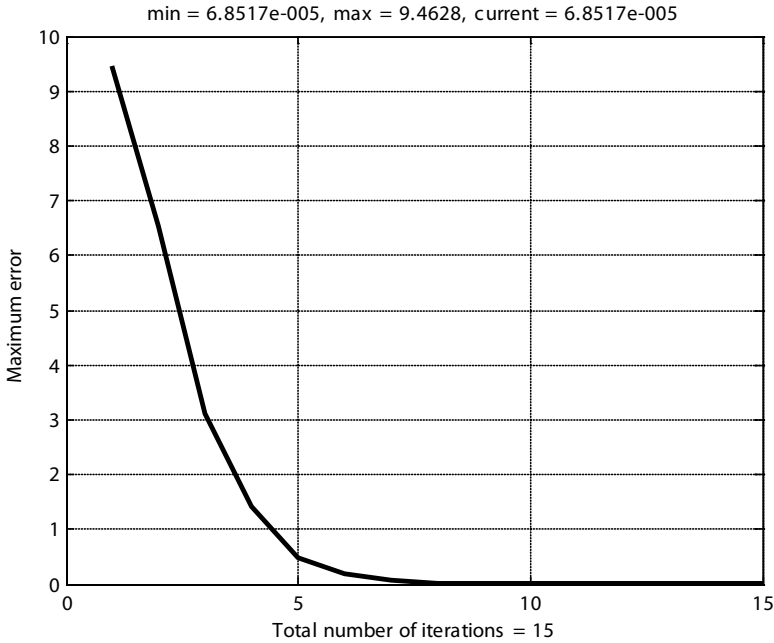


Fig. 2.13 Convergence characteristics of ALHN in the ED problem of example 2.3.

2.5 ECONOMIC DISPATCH WITH RAMP RATE CONSTRAINT

In an ED design without ramp rate constraint, it is assumed that generating units have the capability to increase stepwise from zero to the rated capacity and vice versa. However, in practice, because of the generating units' physical limitations, their capability to increase output is constrained by a ramp function, called ramp-up. Likewise, when a generating unit has to reduce its generation, the decrease is constrained by a ramp function called ramp-down. An ED with ramp rate constraint is also referred to as dynamic economic dispatch [4].

In the ED calculation, the extra constraints for the ramp rate will be added:

$$P_{Gi} - P_{Gi}^0 \leq UR_i, \text{ if generation increases} \quad (2.40)$$

$$P_{Gi}^0 - P_{Gi} \leq DR_i, \text{ if generation decreases} \quad (2.41)$$

where

P_{Gi}^0 initial power from the previous operating hour of generating unit i ,

UR_i ramp up limit of unit i ,

DR_i ramp down limit of unit i .

An ED with ramp rate constraint may have higher operating costs as in finding the optimal operating point of the generating units, their ramp rate limits must not be exceeded.

ALHN can handle these constraints properly by replacing the maximum and minimum power outputs of units for calculating outputs of continuous neurons in (2.31) by the new values:

$$V_i = g(U_i) = \left(\frac{P_{Gi}^{high} - P_{Gi}^{low}}{2} \right) \cdot [1 + \tanh(\sigma U_i)] + P_{Gi}^{low} \quad (2.42)$$

where

$$P_{Gi}^{high} = \min \{ P_{Gi}^{\max}, P_{Gi}^0 + UR_i \} \quad (2.43)$$

$$P_{Gi}^{low} = \max \{ P_{Gi}^{\min}, P_{Gi}^0 - DR_i \} \quad (2.44)$$

P_{Gi}^{high} new upper limit of unit i ,

P_{Gi}^{low} new lower limit of unit i .

The procedure for solving the ED problem with ramp rate constraints using ALHN is similar to that without ramp rate.

Example 2.4

The system in example 2.3 is used with the additional data including initial power outputs and ramp rates as follows:

$$P_{Gi}^0 = \begin{bmatrix} 107.5 \\ 72.0 \\ 122.5 \end{bmatrix}; UR_i = \begin{bmatrix} 27.5 \\ 27.5 \\ 22.5 \end{bmatrix}; DR_i = \begin{bmatrix} 47.5 \\ 39.0 \\ 32.0 \end{bmatrix}$$

Due to the change of generating limits, the updating step sizes of neurons are also changed by tuning other values. The values of updating step sizes for neurons in this example are $\alpha_i = 0.018$ and $\alpha_\lambda = 0.007$. The other parameters and initial values of neurons for ALHN are similar to the ones in example 2.3.

The new generation limits of units are determined as follows:

$$P_{Gi}^{high} = \begin{bmatrix} 135.0 \\ 99.5 \\ 145.0 \end{bmatrix}; P_{Gi}^{low} = \begin{bmatrix} 60.0 \\ 37.5 \\ 90.5 \end{bmatrix}$$

Applying the same procedure of example 2.3 to solve this example leads to the final result with a maximum error of 9.0598×10^{-5} as follows:

$$P_{G1} = 66.5117 \text{ MW}; P_{G2} = 62.0171 \text{ MW}; P_{G3} = 90.5000 \text{ MW}$$

$$P_{loss} = 9.0287 \text{ MW}; \lambda = 12.5669 \text{ \$/MWh}; \text{total cost: \$ 3168.89/h}$$

The total cost in this example is slightly higher than in example 2.3 due to the effect of the ramp rate constraint. The convergence characteristics of ALHN in the problem based on the maximum error are shown in Fig. 2.14.

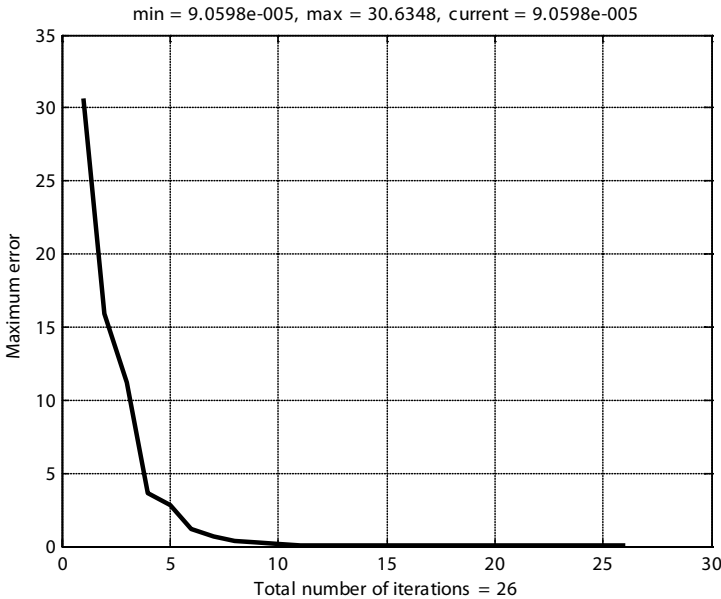


Fig. 2.14 Convergence characteristics of ALHN in the ED problem of example 2.4.

2.6 FUEL CONSTRAINED ECONOMIC DISPATCH

Fuel constrained economic dispatch (FCED) is increasingly important for utility operation and planning since it involves an even more extensive problem including a wide range of time periods of operation and planning and a larger set of constraints and variables. The fuel used in a generating unit may be obtained through different contracts at different prices. Fuel contracts are generally a take-or-pay agreement and include both maximum and minimum limits on delivery of fuel to the generating units over the life time of the contract. The fuel storage kept to prepare for inaccurate load forecasts and untimely delivery supplies is usually within a specified limit [5].

The objective of FCED is to minimize simultaneously the total fuel cost and emission level of thermal generators over a schedule time horizon while satisfying power balance, fuel delivery, and fuel storage constraints and generator operating limits. FCED can be divided into long-term (weeks up to a year), short-term (days up to a week), daily (hours up to one day), and real-time (minutes up to one hour). In long-term FCED which is subject of this study, the schedule time is divided into sub-periods (months or weeks) to obtain an optimal fuel use strategy [6-7].

Assume that the entire long-term schedule time horizon is divided into M subintervals each having a constant load demand and that all generating units are available and remain on-line for M subintervals. The objective is to simultaneously minimize generation cost and emission levels of generating units over the M subintervals such that the constraints of power balance, fuel delivery and fuel storage for any given subinterval, and maximum-minimum fuel delivery, fuel storage, and generator operating constraints for each generating unit are satisfied.

The problem formulation for a system having NG thermal generating units scheduled in M subintervals is as follows:

$$\text{Min} \sum_{k=1}^M \sum_{i=1}^{NG} t_k \cdot [F_i(P_{Gi}^k) + E_i(P_{Gi}^k)] \quad (2.45)$$

where

$$F_i(P_{Gi}^k) = a_i + b_i P_{Gi}^k + c_i (P_{Gi}^k)^2 \quad (2.46)$$

$$E_i(P_{Gi}^k) = d_i + e_i P_{Gi}^k + f_i (P_{Gi}^k)^2 \quad (2.47)$$

subject to

Power balance constraint

$$\sum_{i=1}^{NG} P_{Gi}^k - P_{loss}^k - P_D^k = 0 \quad (2.48)$$

$$P_{loss}^k = \sum_{i=1}^{NG} \sum_{j=1}^{NG} P_{Gi}^k B_{ij} P_{Gj}^k + \sum_{i=1}^{NG} B_{i0} P_{Gi}^k + B_{00} \quad (2.49)$$

Fuel delivery constraint

$$\sum_{i=1}^{NG} F_i^k - F_D^k = 0 \quad (2.50)$$

Fuel storage constraint

$$X_i^k = X_i^{k-1} + F_i^k - t_k Q_i^k \quad (2.51)$$

$$Q_i^k = q_{0i} + q_{1i} P_{Gi}^k + q_{2i} (P_{Gi}^k)^2 \quad (2.52)$$

Generator operating limits

$$P_{Gi}^{\min} \leq P_{Gi}^k \leq P_{Gi}^{\max} \quad (2.53)$$

Fuel delivery limits

$$F_i^{\min} \leq F_i^k \leq F_i^{\max} \quad (2.54)$$

Fuel storage limits

$$X_i^{\min} \leq X_i^k \leq X_i^{\max} \quad (2.55)$$

where

$F_i(P_{Gi}^k)$	fuel cost function of unit i (\$/h),
$E_i(P_{Gi}^k)$	emission function of unit i (kg/h),
M	number of subintervals during schedule time horizon,
t_k	number of hours in subinterval k ,
P_{Gi}^k	power output of unit i in subinterval k (MW),
a_i, b_i, c_i	fuel cost coefficients of unit i (\$/h, \$/MWh, \$/MW ² h),
d_i, e_i, f_i	emission coefficients of unit i (kg/h, kg/MWh, kg/MW ² h),
P_D^k	total power load demand during subinterval k (MW),
P_{loss}^k	system power loss during subinterval k (MW),
F_D^k	total fuel delivered during subinterval k (tons),
F_i^k	fuel delivery for thermal unit i during subinterval k (tons),
X_{ik}	fuel storage for unit i during subinterval k (tons),
Q_i^k	fuel consumption of unit i during subinterval k (tons/h),

- q_{0i}, q_{1i}, q_{2i} fuel consumption coefficients of unit i during subinterval k (tons/h, tons/MWh, tons/MW²h),
- $P_{Gi}^{max}, P_{Gi}^{min}$ maximum and minimum power outputs of unit i (MW),
- F_i^{max}, F_i^{min} maximum and minimum fuel delivery to unit i (tons),
- X_i^{max}, X_i^{min} maximum and minimum fuel storage of unit i (tons).

The fuel storage at subinterval k in (2.51) can be rewritten in terms of initial fuel storage as follows:

$$X_i^k = X_i^0 + \sum_{l=1}^k (F_i^l - t_l Q_i^l) \quad (2.56)$$

where X_i^0 is the initial storage for unit i .

ALHN is efficient in solving this problem with the equality constraints properly handled by an augmented Lagrangian function and sigmoid function. The continuous variables will be handled by continuous neurons including power outputs of units P_{Gi}^k , fuel deliveries to units F_i^k and fuel storages for units X_i^k while the constraints will be handled by an augmented Lagrange function consisting of power balance, fuel delivery and fuel storage constraints.

The augmented Lagrange function L of the problem is formulated as follows:

$$\begin{aligned} L = & \sum_{k=1}^M \sum_{i=1}^{NG} t_k \left[F_i(P_{Gi}^k) + E_i(P_{Gi}^k) \right] \\ & + \sum_{k=1}^M \left[\lambda^k \left(P_{loss}^k + P_D^k - \sum_{i=1}^{NG} P_{Gi}^k \right) + \frac{1}{2} \beta_\lambda^k \left(P_{loss}^k + P_D^k - \sum_{i=1}^{NG} P_{Gi}^k \right)^2 \right] \\ & + \sum_{k=1}^M \left[\gamma^k \left(\sum_{i=1}^{NG} F_i^k - F_D^k \right) + \frac{1}{2} \beta_\gamma^k \left(\sum_{i=1}^{NG} F_i^k - F_D^k \right)^2 \right] \\ & + \sum_{k=1}^M \sum_{i=1}^{NG} \left[\eta_i^k \left(X_i^k - X_i^0 - \sum_{l=1}^k (F_i^l + t_l Q_i^l) \right) + \frac{1}{2} \beta_{\eta,i}^k \left(X_i^k - X_i^0 - \sum_{l=1}^k (F_i^l + t_l Q_i^l) \right)^2 \right] \end{aligned} \quad (2.57)$$

where

- $\lambda^k, \gamma^k, \eta_i^k$ Lagrangian multiplier associated with power balance, fuel delivery and fuel storage, respectively.
- $\beta_\lambda^k, \beta_\gamma^k, \beta_{\eta,i}^k$ penalty factor associated with power balance, fuel delivery and fuel storage, respectively.

To represent power outputs in ALHN, $3NG \times M$ continuous neurons and $(NG+2) \times M$ multiplier neurons are required. The energy function of ALHN is formulated:

$$\begin{aligned}
 E = & \sum_{k=1}^M \sum_{i=1}^{NG} t_k \left[F_i(V_{i,p}^k) + E_i(V_{i,p}^k) \right] \\
 & + \sum_{k=1}^M \left[V_{\lambda}^k \left(P_{loss}^k + P_D^k - \sum_{i=1}^{NG} V_{i,p}^k \right) + \frac{1}{2} \beta_{\lambda}^k \left(P_{loss}^k + P_D^k - \sum_{i=1}^{NG} V_{i,p}^k \right)^2 \right] \\
 & + \sum_{k=1}^M \left[V_{\gamma}^k \left(\sum_{i=1}^{NG} V_{i,f}^k - F_D^k \right) + \frac{1}{2} \beta_{\gamma}^k \left(\sum_{i=1}^{NG} V_{i,f}^k - F_D^k \right)^2 \right] \\
 & + \sum_{k=1}^M \sum_{i=1}^{NG} \left[V_{i,\eta}^k \left(V_{i,x}^k - X_i^0 - \sum_{l=1}^k (V_{i,f}^l + t_l Q_i^l) \right) \right. \\
 & \quad \left. + \frac{1}{2} \beta_{i,\eta}^k \left(V_{i,x}^k - X_i^0 - \sum_{l=1}^k (V_{i,f}^l + t_l Q_i^l) \right)^2 \right] \\
 & + \sum_{k=1}^M \sum_{i=1}^{NG} \left(\int_0^{V_{i,p}^k} g_c^{-1}(V) dV + \int_0^{V_{i,f}^k} g_c^{-1}(V) dV + \int_0^{V_{i,x}^k} g_c^{-1}(V) dV \right)
 \end{aligned} \tag{2.58}$$

where

- $V_{i,p}^k$ output of continuous neuron $p(i,k)$ representing P_{Gi}^k ,
- $V_{i,f}^k$ output of continuous neuron $f(i,k)$ representing F_i^k ,
- $V_{i,x}^k$ output of continuous neuron $x(i,k)$ representing X_i^k ,
- V_{λ}^k output of multiplier neuron $\lambda(k)$ representing λ^k ,
- V_{γ}^k output of multiplier neuron $\gamma(k)$ representing γ^k ,
- $V_{i,\eta}^k$ output of multiplier neuron $\eta(i,k)$ representing η_i^k ,
- g_c^{-1} inverse function of sigmoid function.

The dynamics of ALHN for updating inputs of neurons are defined as:

$$\begin{aligned} \frac{dU_{i,p}^k}{dt} = -\frac{\partial E}{\partial V_{i,p}^k} = & \left\{ \begin{aligned} & t_k \left(\frac{dF_i(V_{i,p}^k)}{dV_{i,p}^k} + \frac{dE_i(V_{i,p}^k)}{dV_{i,p}^k} \right) \\ & + \left[V_{\lambda}^k + \beta_{\lambda}^k \left(P_{loss}^k + P_D^k - \sum_{i=1}^{NG} V_{i,p}^k \right) \right] \left(\frac{\partial P_{loss}^k}{\partial V_{i,p}^k} - 1 \right) \\ & + \left[V_{i,\eta}^k + \beta_{i,\eta}^k \left(V_{i,x}^k - X_i^0 - \sum_{l=1}^k (V_{i,f}^l + t_l Q_i^l) \right) \right] \times t_k \frac{dQ_i^k}{dV_{i,p}^k} + U_{i,p}^k \end{aligned} \right\} \quad (2.59) \end{aligned}$$

$$\begin{aligned} \frac{dU_{i,f}^k}{dt} = -\frac{\partial E}{\partial V_{i,f}^k} = & \left\{ \begin{aligned} & \left[V_{\gamma}^k + \beta_{\gamma}^k \left(\sum_{s=1}^{NG} V_{i,f}^k - F_D^k \right) \right] \\ & - \left[V_{i,\eta}^k + \beta_{i,\eta}^k \left(V_{i,x}^k - X_i^0 - \sum_{l=1}^k (V_{i,f}^l + t_l Q_i^l) \right) \right] + U_{i,f}^k \end{aligned} \right\} \quad (2.60) \end{aligned}$$

$$\frac{dU_{i,x}^k}{dt} = -\frac{\partial E}{\partial V_{i,x}^k} = -\left\{ V_{i,\eta}^k + \beta_{i,\eta}^k \left(V_{i,x}^k - X_i^0 - \sum_{l=1}^k (V_{i,f}^l + t_l Q_i^l) \right) + U_{i,x}^k \right\} \quad (2.61)$$

$$\frac{dU_{\lambda}^k}{dt} = +\frac{\partial E}{\partial V_{\lambda}^k} = P_{loss}^k + P_D^k - \sum_{i=1}^{NG} V_{i,p}^k \quad (2.62)$$

$$\frac{dU_{\gamma}^k}{dt} = +\frac{\partial E}{\partial V_{\gamma}^k} = \sum_{i=1}^{NG} V_{i,f}^k - F_D^k \quad (2.63)$$

$$\frac{dU_{i,\eta}^k}{dt} = +\frac{\partial E}{\partial V_{i,\eta}^k} = V_{i,x}^k - X_i^0 - \sum_{l=1}^k (V_{i,f}^l + t_l Q_i^l) \quad (2.64)$$

where

$$\frac{dF_i(V_{i,p}^k)}{dV_{i,p}^k} = b_i + 2c_i V_{i,p}^k \quad (2.65)$$

$$\frac{dE_i(V_{i,p}^k)}{dV_{i,p}^k} = e_i + 2f_i V_{i,p}^k \quad (2.66)$$

$$\frac{\partial P_{loss}^k}{\partial V_{i,p}^k} = 2 \sum_{j=1}^{NG} B_{ij} V_{j,p}^k + B_{i0} \quad (2.67)$$

$$\frac{\partial Q_i^k}{\partial V_{i,p}^k} = q_{1i} + 2q_{2i} V_{i,p}^k \quad (2.68)$$

$U_{i,p}^k$ input of continuous neuron $p(i,k)$,

$U_{i,f}^k$ input of continuous neuron $f(i,k)$,

$U_{i,x}^k$ input of continuous neuron $x(i,k)$,

U_{λ}^k input of multiplier neuron $\lambda(k)$,

U_{γ}^k input of multiplier neuron $\gamma(k)$,

$U_{i,\eta}^k$ input of multiplier neuron $\eta(i,k)$.

Inputs of neurons at iteration n are updated from iteration $n-1$:

$$U_{i,p}^{k(n)} = U_{i,p}^{k(n-1)} + \alpha_p \frac{dU_{i,p}^k}{dt} = U_{i,p}^{k(n-1)} - \alpha_p \frac{\partial E}{\partial V_{i,p}^k} \quad (2.69)$$

$$U_{i,f}^{k(n)} = U_{i,f}^{k(n-1)} + \alpha_f \frac{dU_{i,f}^k}{dt} = U_{i,f}^{k(n-1)} - \alpha_f \frac{\partial E}{\partial V_{i,f}^k} \quad (2.70)$$

$$U_{i,x}^{k(n)} = U_{i,x}^{k(n-1)} + \alpha_x \frac{dU_{i,x}^k}{dt} = U_{i,x}^{k(n-1)} - \alpha_x \frac{\partial E}{\partial V_{i,x}^k} \quad (2.71)$$

$$U_{\lambda}^{k(n)} = U_{\lambda}^{k(n-1)} + \alpha_{\lambda} \frac{dU_{\lambda}^k}{dt} = U_{\lambda}^{k(n-1)} + \alpha_{\lambda} \frac{\partial E}{\partial V_{\lambda}^k} \quad (2.72)$$

$$U_{\gamma}^{k(n)} = U_{\gamma}^{k(n-1)} + \alpha_{\gamma} \frac{dU_{\gamma}^k}{dt} = U_{\gamma}^{k(n-1)} + \alpha_{\gamma} \frac{\partial E}{\partial V_{\gamma}^k} \quad (2.73)$$

$$U_{i,\eta}^{k(n)} = U_{i,\eta}^{k(n-1)} + \alpha_{\eta} \frac{dU_{i,\eta}^k}{dt} = U_{i,\eta}^{k(n-1)} + \alpha_{\eta} \frac{\partial E}{\partial V_{i,\eta}^k} \quad (2.74)$$

where $\alpha_p, \alpha_f, \alpha_x$ are updating step sizes for continuous neurons, and $\alpha_\lambda, \alpha_\gamma, \alpha_\eta$ are updating step sizes for multiplier neurons. For simplicity, the updating step sizes for continuous neurons can be equally chosen.

The outputs of continuous neurons are determined by:

$$V_{i,p}^k = g_c(U_{i,p}^k) = \left(\frac{P_{Gi}^{\max} - P_{Gi}^{\min}}{2} \right) \left[1 + \tanh(\sigma U_{i,p}^k) \right] + P_{Gi}^{\min} \quad (2.75)$$

$$V_{i,f}^k = g_c(U_{i,f}^k) = (F_i^{\max} - F_i^{\min}) \left(\frac{1 + \tanh(\sigma U_{i,f}^k)}{2} \right) + F_i^{\min} \quad (2.76)$$

$$V_{i,x}^k = g_c(U_{i,x}^k) = (X_i^{\max} - X_i^{\min}) \left(\frac{1 + \tanh(\sigma U_{i,x}^k)}{2} \right) + X_i^{\min} \quad (2.77)$$

The outputs of multiplier neurons are defined by transfer function:

$$V_\lambda^k = g_m(U_\lambda^k) = U_\lambda^k \quad (2.78)$$

$$V_\gamma^k = g_m(U_\gamma^k) = U_\gamma^k \quad (2.79)$$

$$V_{i,\eta}^k = g_m(U_{i,\eta}^k) = U_{i,\eta}^k \quad (2.80)$$

Outputs of continuous neurons representing power output of and fuel delivery to units are initiated by “mean distribution” as follows:

$$V_{i,p}^{k(0)} = P_D^k \cdot \frac{P_{Gi}^{\max}}{\sum_{j=1}^{NG} P_{Gj}^{\max}} \quad (2.81)$$

$$V_{i,f}^{k(0)} = F_D^k \cdot \frac{F_i^{\max}}{\sum_{j=1}^{NG} F_j^{\max}} \quad (2.82)$$

where $V_{i,p}^{k(0)}$ and $V_{i,f}^{k(0)}$ are initial values of outputs of continuous neurons representing power generation of unit i and fuel delivery to this unit.

Outputs of neurons representing fuel storage levels of units are initiated at the middle point between their maximum and minimum values of fuel storage:

$$V_{i,x}^{k(0)} = (X_i^{\max} + X_i^{\min}) / 2 \quad (2.83)$$

where $V_{i,x}^{k(0)}$ is the initial value of output of a continuous neuron representing the fuel storage level for unit i .

Outputs of multiplier neurons associated with the power balance constraint are initialized by mean values as follows:

$$V_{\lambda}^{k(0)} = \frac{1}{NG} \sum_{i=1}^{NG} \frac{t_k (b_i + 2c_i V_{i,p}^{k(0)}) + t_k (e_i + 2f_i V_{i,p}^{k(0)})}{1 - \frac{\partial P_{loss}^k}{\partial V_{i,p}^k}} \quad (2.84)$$

The initial outputs of other multiplier neurons are zeros. The inputs of neurons are calculated based on the outputs via inversed sigmoid and transfer functions.

The maximum error of ALHN at iteration n is determined as:

$$Err_{\max}^{(n)} = \max \left\{ \Delta P^{k(n)}, \Delta F_i^{k(n)}, \Delta X_i^{k(n)}, \Delta V_{i,p}^{k(n)}, \Delta V_{i,f}^{k(n)}, \Delta V_{i,x}^{k(n)} \right\} \quad (2.85)$$

where

$$\Delta P^{k(n)} = \left| P_{loss}^{k(n)} + P_D^k - \sum_{i=1}^{NG} V_{i,p}^{k(n)} \right| \quad (2.86)$$

$$\Delta F_i^{k(n)} = \left| \sum_{j=1}^{NG} V_{j,f}^{k(n)} - F_D^k \right| \quad (2.87)$$

$$\Delta X_i^{k(n)} = \left| V_{i,x}^{k(n)} - X_i^0 - \sum_{l=1}^k (V_{i,f}^{l(n)} + t_l Q_i^{l(n)}) \right| \quad (2.88)$$

$$\Delta V_{i,p}^{k(n)} = \left| V_{i,p}^{k(n)} - V_{i,p}^{k(n-1)} \right| \quad (2.89)$$

$$\Delta V_{i,f}^{k(n)} = \left| V_{i,f}^{k(n)} - V_{i,f}^{k(n-1)} \right| \quad (2.90)$$

$$\Delta V_{i,x}^{k(n)} = \left| V_{i,x}^{k(n)} - V_{i,x}^{k(n-1)} \right| \quad (2.91)$$

The overall procedure of ALHN for determining the FCED is similar to that in Section 2.4. In the multi-objective problem solution by ALHN, the slope of the sigmoid function of continuous neurons is adjusted to obtain results compromising between the objectives.

Example 2.5

A system with five coal-burning generating units remains online for a 3-week period. In this example, the load demand is assumed to be constant over a week. The load demand for each week is 700, 800, and 650 MW, respectively. The coal supplier is to deliver a total of 7,000 tons of coal at the beginning of each week to

the five generating units, and the initial fuel storage levels are assumed to be 2000, 5000, 5000, 8000, and 8000 tons at the beginning of the first week. The other data of this system are given in Table 2.1.

In this example, the number of subintervals is 3 with a 168 hour duration each. The sub-cases will be considered here including minimization of fuel cost only, minimization of emissions only and minimization of two or multi objectives to find a trade-off solution. In the single objective minimization problems, the slope of the sigmoid function of continuous neurons is fixed, whereas in the simultaneous multi-objective minimization, the slope is adjusted so that the best compromise solution can be obtained.

In a single objective case, the problem is that of a normal ED with fuel constraint. Therefore, the algorithm of ALHN for solving these problems is similar to a simple ED problem. In these cases, the parameters of ALHN differ from case to case except the slope of the sigmoid function and the penalty factors. On the other hand, in a multi-objective problem, the slope of the sigmoid function will change to obtain a trade-off, e.g., between cost and emission curves. As the slope of the sigmoid function changes, the other parameters may also change to achieve a faster convergence, except for the penalty factors.

Table 2.1 Unit data, fuel delivery and storage capacity for the system in example 2.5.

Unit	1	2	3	4	5
a_i (\$/h)	25.0	60.0	100.0	120.0	40.0
b_i (\$/MWh)	2.0	1.8	2.1	2.2	1.8
c_i (\$/MW ² h)	0.008	0.003	0.0012	0.004	0.0015
d_i (kg/h)	80	50	70	45	30
e_i (kg/MWh)	-0.805	-0.555	-0.955	-0.600	-0.555
f_i (kg/MW ² h)	0.018	0.015	0.0115	0.008	0.012
q_{0i} (tons/h)	0.83612	2.00669	3.34448	4.01338	1.33779
q_{1i} (tons/MWh)	0.066889	0.060200	0.070234	0.073578	0.060200
q_{2i} (tons/MW ² h)	0.00026756	0.00010033	0.00004013	0.00013378	0.00005017
P_i^{max} (MW)	20	20	30	40	50
P_i^{min} (MW)	75	125	175	250	300
F_i^{min} (tons)	0.0	0.0	0.0	0.0	0.0
F_i^{max} (tons)	1,000	1,000	2,000	3,000	3,000
X_i^{min} (tons)	0.0	0.0	0.0	0.0	0.0
X_i^{max} (tons)	10,000	10,000	20,000	30,000	30,000

Single fuel cost objective

The problem here is to minimize the fuel cost of generating units, neglecting the emission objective. Steps of ALHN for solving this problem are similar to that for the normal ED. The values of the slope of the sigmoid function and penalty factors are fixed at 100 and 10^{-5} , respectively. The other parameters are selected as follows: $\alpha_p = 0.00015$, $\alpha_f = \alpha_x = 0.04$, $\alpha_\lambda = 0.2$, $\alpha_\gamma = \alpha_\eta = 8 \times 10^{-7}$.

The algorithm converges after 434 iterations with a maximum error of 9.9667×10^{-5} for the pre-specified threshold of 10^{-4} . The obtained total cost and emissions are \$ 1,002,551.42/h and 756,288.65 kg/h, respectively. The optimal solution for power generation (MW), fuel delivery (tons) and fuel storage (tons) is:

$$P_{Gi}^k = \begin{bmatrix} 41.6570 & 75.0000 & 38.4301 \\ 125.0000 & 125.0000 & 125.0000 \\ 175.0000 & 175.0000 & 175.0000 \\ 58.3430 & 125.0000 & 40.0000 \\ 300.0000 & 300.0000 & 271.5699 \end{bmatrix}; F_i^k = 10^3 \times \begin{bmatrix} 0.7455 & 0.7353 & 0.6931 \\ 0.5496 & 0.6066 & 0.6889 \\ 1.5326 & 1.5882 & 1.6837 \\ 1.9934 & 1.8876 & 1.7278 \\ 2.1790 & 2.1823 & 2.2064 \end{bmatrix}$$

$$X_i^k = 10^3 \times \begin{bmatrix} 2.1351 & 1.8838 & 2.0028 \\ 3.9462 & 2.9493 & 2.0347 \\ 3.9046 & 2.8649 & 1.9207 \\ 8.5966 & 8.2620 & 8.8202 \\ 6.9176 & 5.8385 & 5.0714 \end{bmatrix}$$

Single emission objective

In this case, only the emission objective is observed neglecting the fuel cost objective. The values corresponding to the slope of the sigmoid function and penalty factors are fixed similarly to the case of the single fuel cost objective. The other parameters are reselected as: $\alpha_p = 2.5 \times 10^{-6}$, $\alpha_f = \alpha_x = 0.0015$, $\alpha_\lambda = 0.015$, $\alpha_\gamma = \alpha_\eta = 2 \times 10^{-8}$.

The obtained total cost and emissions in this case are \$1,072,673.57/h and 521,793.48 kg/h, respectively. Compared to the single fuel objective case, the total cost in this case is somewhat higher while the emission level is considerably lower. Therefore, depending on the selected objective of minimization, a solution will be obtained accordingly. The obtained values of power outputs (MW), fuel delivery (tons) and fuel storage (tons) are:

$$P_{Gi}^k = \begin{bmatrix} 75.0000 & 75.0000 & 75.0000 \\ 111.4054 & 125.0000 & 102.1930 \\ 162.6840 & 175.0000 & 150.6701 \\ 211.6589 & 250.0000 & 194.3871 \\ 139.2517 & 175.0000 & 127.7498 \end{bmatrix}; F_i^k = 10^3 \times \begin{bmatrix} 0.7809 & 0.7679 & 0.7710 \\ 0.5321 & 0.5853 & 0.6377 \\ 1.5110 & 1.5614 & 1.6159 \\ 2.1715 & 2.2237 & 2.2506 \\ 2.0046 & 1.8617 & 1.7249 \end{bmatrix}$$

$$X_i^k = 10^3 \times \begin{bmatrix} 1.7943 & 1.5755 & 1.3599 \\ 4.0664 & 3.0482 & 2.3136 \\ 4.0284 & 2.9619 & 2.2371 \\ 6.8761 & 5.3297 & 4.4988 \\ 8.3703 & 8.2359 & 8.4429 \end{bmatrix}$$

Multi-objectives

If more or all objectives are to be considered, a trade-off between the objectives has to be reached. Generally, the best compromise between objectives is based on the trade-off curve obtained by changing the weight contribution of each single objective to the general objective. In practical operation, depending on the operating policy, the appropriate objectives will be selected. In ALHN implementation, the best trade-off between two objectives can be found by adjusting the slope of the sigmoid function. At different points in the slope, total cost obtained and emission level will vary inversely. The point where the best compromise between the objectives is reached is the best trade-off between cost and emission.

To observe the trade-off between two objectives, the slope of the sigmoid function is changed from 10^{-5} to 10^5 while the penalty factors are fixed at 10^{-5} and other parameters change corresponding to the slope. The obtained generation cost and emission level curves with different slopes are shown in Fig. 2.15, illustrating

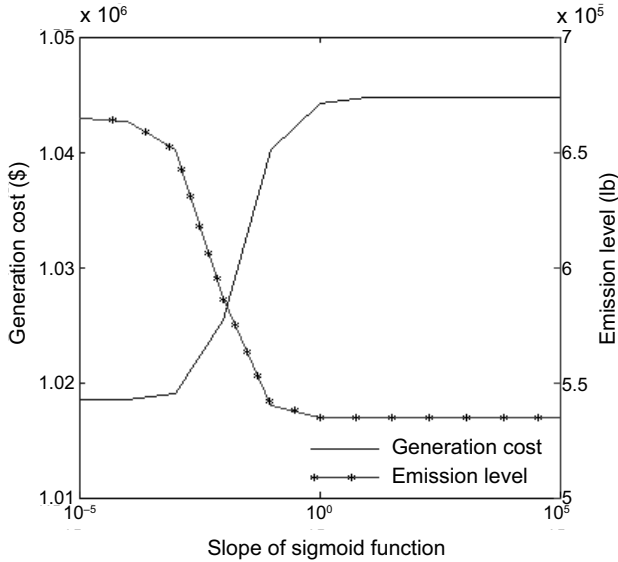


Fig. 2.15 Generation cost and emission level curves with different values of slope.

the trade-off between the two objectives. When $\sigma < 0.01$ the objective of generation cost is more important than the one of the emission level. By contrast, when $\sigma > 0.01$ the objective of the emission level is more important. Therefore, the value of the slope has to be adjusted in accordance to the operation policy. The point of a balanced trade-off between generation cost and emission level curves appears around $\sigma = 0.01$. At this slope of the sigmoid function, the other parameters of ALHN are $\alpha_p = 0.04$, $\alpha_f = \alpha_x = 0.12$, $\alpha_\lambda = 1.4$, $\alpha_\gamma = \alpha_\eta = 0.01$. The corresponding generation cost and emission level are \$1,025,326.22 and 586,769.44 kg, respectively, after 398 iterations with a maximum error of 8.8817×10^{-5} . The power outputs (MW), fuel delivery (tons) and fuel storage (tons) of generating units are:

$$P_{Gi}^k = \begin{bmatrix} 58.2520 & 70.8170 & 57.7157 \\ 124.9997 & 124.9999 & 120.2325 \\ 157.5702 & 170.4563 & 125.2352 \\ 149.4267 & 179.8063 & 138.5289 \\ 209.7514 & 253.9205 & 208.2878 \end{bmatrix}; F_i^k = 10^3 \times \begin{bmatrix} 0.7630 & 0.7468 & 0.7299 \\ 0.5474 & 0.6037 & 0.6812 \\ 1.5083 & 1.5563 & 1.5768 \\ 2.0995 & 2.0611 & 2.0229 \\ 2.0818 & 2.0321 & 1.9891 \end{bmatrix}$$

$$X_i^k = 10^3 \times \begin{bmatrix} 1.9653 & 1.7727 & 1.7110 \\ 3.9439 & 2.9442 & 2.0703 \\ 4.0861 & 3.0681 & 2.6045 \\ 7.5748 & 6.7351 & 6.3683 \\ 7.7340 & 6.9711 & 6.6272 \end{bmatrix}$$

In multi-objective problems, each objective can also be assigned a weight factor according to its contribution to the general objective. The weight factors can be adjusted depending on their importance.

2.7 ECONOMIC DISPATCH CONSIDERING EMISSIONS

The main purpose of optimal power dispatch is to minimize the operating cost of the power system while still satisfying power balance constraints. However, this objective may not necessarily be the best in terms of the environment. Several options for achieving emission reductions include the installation of a gas cleaner, switching from regular fuel to low sulfur fuel and adopting a new power dispatch. Among these options, modifications in power dispatch are the most effective and require the least additional costs. Therefore, the unit dispatch considering emissions besides cost minimization has received widespread attention as an effective short-term option with less investment requirement. The major environmental concerns in power dispatch include SO_2 , NO_x and CO_2 emissions [8-19].

Techniques for handling emissions in power dispatch optimization focus on two distinct directions. In the one approach, emissions are treated as constraints. In the other, due to the conflicting and non-commensurable nature of operating

cost and emissions, the objective function integrates emission minimization into the total operating cost minimization problem.

2.7.1 Economic Dispatch with Emission Constraints

SO₂, NO_x and CO₂ emissions can be included in the ED problem. The problem formulation can be expressed as

$$\text{Minimize } F_T = \sum_{i \in BG} F(P_{Gi}) \quad (2.92)$$

subject to the power balance constraints in equations (2.17) and (2.18) and the generators minimum and maximum limit constraints

$$P_{Gi}^{\min} \leq P_{Gi} \leq P_{Gi}^{\max} \quad (2.93)$$

and emission constraints

$$\sum_{i \in BG} E_{NO_x}(P_{Gi}) \leq EC_{NO_x} \quad (2.94)$$

$$\sum_{i \in BG} E_{SO_x}(P_{Gi}) \leq EC_{SO_x} \quad (2.95)$$

$$\sum_{i \in BG} E_{CO_2}(P_{Gi}) \leq EC_{CO_2} \quad (2.96)$$

where

EC_{NO_x} , EC_{SO_2} , EC_{CO_2} are total system NO_x, SO₂ and CO₂ emissions, respectively (tons/h),

$EC_{NO_x}(P_{Gi})$, $EC_{SO_2}(P_{Gi})$, $EC_{CO_2}(P_{Gi})$ are NO_x, SO₂, and CO₂ emissions of the generator connected to bus i , respectively (tons/h),

F_T total system operating cost (\$/h),

$F(P_{Gi})$ operating cost of the generator connected to bus i (\$/h),

BG set of buses connected with generators,

$P_{Gi}^{\max}$ maximum real power generation at bus i (MW),

$P_{Gi}^{\min}$ minimum real power generation at bus i (MW).

2.7.2 ED with Emission Objective

The emission objective is combined with the total cost objective using a weight factor as shown in equation (2.97).

$$F_T = \sum_{i \in BG} F(P_{Gi}) + W \cdot \sum_{i \in BG} E(P_{Gi}) \quad (2.97)$$

where W is the weight factor and $E(P_{Gi})$ is the emission function of unit i .

Note that the weight factors are in fact the emission factors. With the conflicting nature of these objectives, the multi-objective optimization technique can help handle the power dispatch optimization problem under the requirement of emission consideration. The multi-objective function can be expressed as follows:

Minimizing the generator fuel cost:

$$F_T = \sum_{i \in BG} F(P_{Gi}) \quad (2.98)$$

and minimizing total NO_x emissions:

$$EC_{\text{NO}_x} = \sum_{i \in BG} E_{\text{NO}_x}(P_{Gi}) \quad (2.99)$$

and minimizing total SO_x emissions:

$$EC_{\text{SO}_x} = \sum_{i \in BG} E_{\text{SO}_x}(P_{Gi}) \quad (2.100)$$

and minimizing total CO_2 emissions:

$$EC_{\text{CO}_2} = \sum_{i \in BG} E_{\text{CO}_2}(P_{Gi}) \quad (2.101)$$

subject to the power balance constraints in equations (2.17) and (2.18), and the generator minimum and maximum limit constraints:

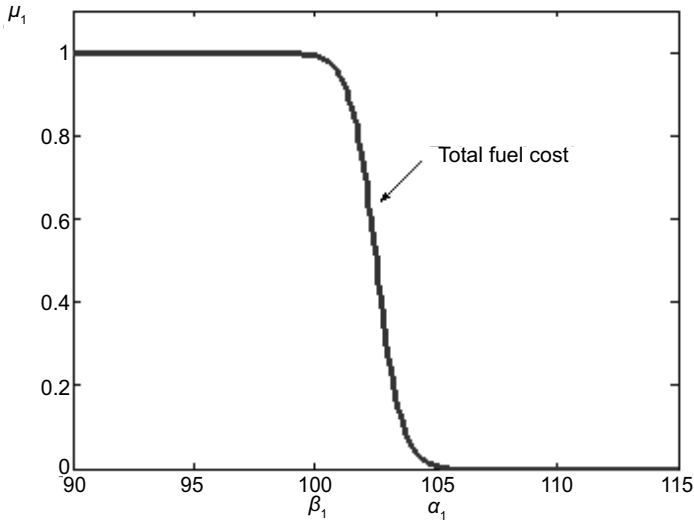


Fig. 2.16 Membership function of total operating cost.

$$P_{Gi}^{\min} \leq P_{Gi} \leq P_{Gi}^{\max} \quad (2.102)$$

In this section, the solution technique used is fuzzy linear programming (FLP). In FLP [20], the goal of decision-maker can be expressed as a fuzzy set and the solution space is defined by constraints that can be modeled by a fuzzy set. The multi-objective fuzzy minimization problem can be formulated as

$$\text{Maximize } \{\mu_1(x), \mu_2(x), \mu_3(x), \mu_4(x)\} \quad (2.103)$$

$$\text{subject to } \mathbf{B} \cdot \mathbf{P}_{Gi} \lesssim \mathbf{d} \quad (2.104)$$

and power balance constraints in (2.17) and (2.18), and low and high limits of P_{Gi} in (2.93).

$\mathbf{P}_{Gi}$ is the column matrix representing the set of real power generation of the generator connected to bus i ; $\mathbf{d}$ is the vector representing a fuzzy objective function. Each row of $\mathbf{B}$ in (2.99) is represented by a fuzzy set with the membership functions of $\mu_i(x)$. $\mu_i(x)$ that can be interpreted as the degree to which $\mathbf{P}_{Gi}$ satisfies the fuzzy objective function. Here, $\mu_1(x)$ is the degree of satisfaction of $\mathbf{P}_{Gi}$ for the objective function, whereas $\mu_2(x)$ to $\mu_4(x)$ are the degrees of satisfaction of $\mathbf{P}_{Gi}$ for the total system's NO_x , SO_x and CO_2 emissions, respectively. In this chapter, the hyperbolic function is used to represent the nonlinear, S-shaped membership function. The function can be expressed as:

$$\mu_i(x) = \frac{1}{2} \cdot \tanh \left(\left(\mathbf{B}_i \cdot \mathbf{P}_{Gij} - \frac{\alpha_i + \beta_i}{2} \right) \cdot \gamma_i \right) + \frac{1}{2} \quad (2.105)$$

where α_i , β_i and γ_i are the parameters representing the shape of $\mu_i(x)$ depending on the decision maker and $\mathbf{B}_i$ is the row i of $\mathbf{B}$.

To obtain the membership function of the objective function, α_i is defined as the minimum total generator fuel cost as solved by the linear programming (LP) ignoring emissions. On the other hand, β_i is the maximum total fuel cost found among the minimization solutions of other fuzzy objective functions. γ_i is obtained by α_i/β_i . Similarly, α_2 , α_3 , and α_4 are the minimum SO_2 , NO_x and CO_2 emissions determined by LP without considering total generator fuel cost and any other fuzzy objective functions; β_2 , β_3 , and β_4 are the maximum NO_x , SO_x and CO_2 emissions found among the total fuel cost minimization solutions and fuzzy objective functions.

With the defined membership functions of objective functions and fuzzy constraints, the fuzzy optimization problem can be reformulated as

$$\text{Maximize } \mu' \quad (2.106)$$

$$\text{subject to } \mu' \leq \mu_i(x), \text{ for } i = 1, \dots, 4 \quad (2.107)$$

$$\text{and } 0 \leq \mu' \leq 1 \quad (2.108)$$

and power balance constraints as well as low and high limits of P_{Gi} in (2.17), (2.18) and (2.93).

The FLP computational procedure is as described below:

- Step 1: Solve the linear programming for individual objective functions.
- Step 2: Compute the individual objective value of each case.
- Step 3: Obtain α_i and β_i from minimum and maximum of all objective values computed in Step 2.
- Step 4: Solve the fuzzy linear programming of the multi-objective problem using α_i and β_i from Step 3.

Example 2.6

IEEE 30-bus system data are used as test data. The diagram of the network is depicted in Fig. 2.17. Generator fuel cost and SO_2 , NO_x and CO_2 emission functions are given in Tables 2.2 and 2.3.

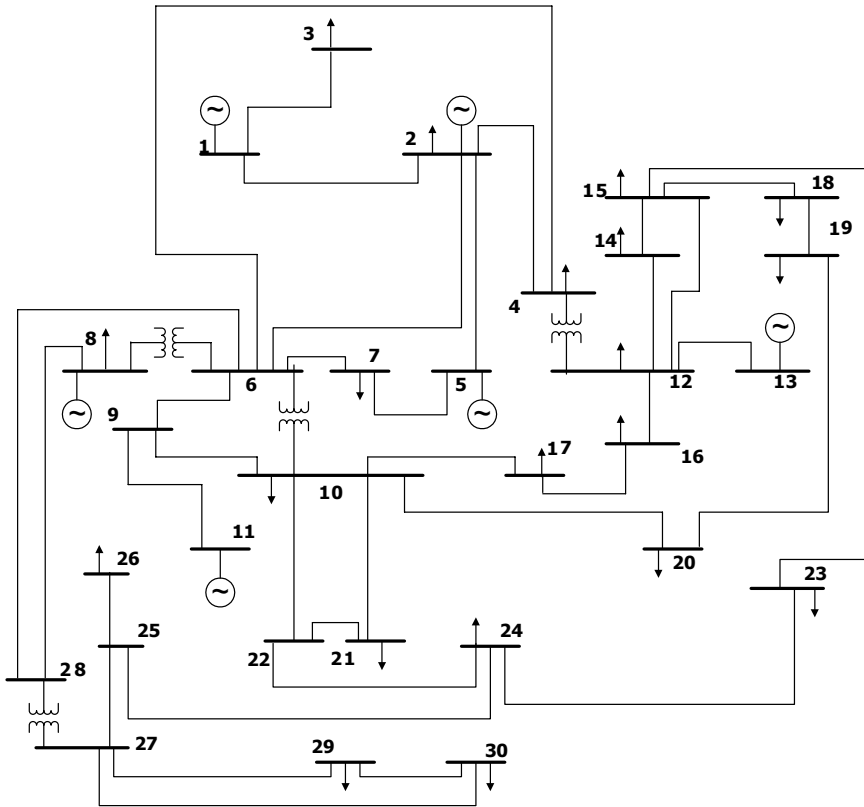


Fig. 2.17 IEEE 30 bus test system.

Table 2.2 Generator cost function of the IEEE 30 bus system.

Gen bus	Min (MW)	Max (MW)	$F(P_{Gi}) = d_i \cdot P_{Gi}^3 + c_i \cdot P_{Gi}^2 + b_i \cdot P_{Gi} + a_i$			
			d_i	c_i	b_i	a_i
1	50	200	0.0010	0.092	14.5	-136
2	20	80	0.0004	0.025	22	-3.5
5	15	50	0.0006	0.075	23	-81
8	10	50	0.0002	0.1	13.5	-14.5
11	10	50	0.0013	0.12	11.5	-9.75
13	12	40	0.0004	0.084	12.5	75.6

Table 2.3 Emissions of the IEEE 30 bus system.

$E_{SO_2}(P_{Gi}) = d_{SO_2i} \cdot P_{Gi}^3 + c_{SO_2i} \cdot P_{Gi}^2 + b_{SO_2i} \cdot P_{Gi} + a_{SO_2i}$ $E_{NO_x}(P_{Gi}) = d_{NO_xi} \cdot P_{Gi}^3 + c_{NO_xi} \cdot P_{Gi}^2 + b_{NO_xi} \cdot P_{Gi} + a_{NO_xi}$ $E_{CO_2}(P_{Gi}) = d_{CO_2i} \cdot P_{Gi}^3 + c_{CO_2i} \cdot P_{Gi}^2 + b_{CO_2i} \cdot P_{Gi} + a_{CO_2i}$												
Gen bus	SO ₂				NO _x				CO ₂			
	d_{SO_2i}	c_{SO_2i}	b_{SO_2i}	a_{SO_2i}	d_{NO_xi}	c_{NO_xi}	b_{NO_xi}	a_{NO_xi}	d_{CO_2i}	c_{CO_2i}	b_{CO_2i}	a_{CO_2i}
1	0.0005	0.150	17.0	-90.0	0.0012	0.052	18.5	-26.0	0.0015	0.092	14.0	-16.0
2	0.0014	0.055	12.0	-30.5	0.0004	0.045	12.0	-35.0	0.0014	0.025	12.5	-93.5
5	0.0010	0.035	10.0	-80.0	0.0016	0.050	13.0	-15.0	0.0016	0.055	13.5	-85.0
8	0.0020	0.070	23.5	-34.5	0.0012	0.070	17.5	-74.0	0.0012	0.010	13.5	-24.5
11	0.0013	0.120	21.5	-19.75	0.0003	0.040	8.5	-89.0	0.0023	0.040	21.0	-59.0
13	0.0021	0.080	22.5	25.6	0.0014	0.024	15.5	-75.0	0.0014	0.080	22.0	-70.0

For the implementation of the linear fuzzy programming method, these functions are linearized into 5 piece-wise linear functions.

Tables 2.4–2.7 address the dispatch results of minimum operating cost, SO₂, NO_x and CO₂ emissions, respectively. Table 2.8 shows the dispatch results of the multi-objective solution. The results show that the single objective approaches result in inferior results in the other objectives and a lower degree of satisfaction. For example, the total cost minimization solution carries high SO₂, NO_x, and CO₂ emission values.

In this test case, the minimum operating cost solution results in the highest SO₂ emission of 7035.94 kg/h. On the other hand, the minimum NO_x emission results in the highest total operating cost and CO₂ emissions, of 6161.4 \$/h and 6104.2 kg/h, respectively, while the minimum CO₂ solution results in the highest NO_x emission of 5187.67 kg/h.

By contrast, the proposed FMOPD effectively trades off between the objectives of total system operating cost, SO₂, NO_x and CO₂ emissions in a fuzzy reasoning sense leading to the best compromise solution while satisfying transmission line limits and transformer loading constraints. Note that the FMOPD results in a satisfaction degree of 0.881.

Table 2.4 Dispatch results for the minimum total operating cost condition.

*** Generation Cost ***			
BUS	P_GEN (MW)	Cost (\$/h)	Inc-Cost (\$/MWh)
1	50.00	944.00015	21.60000
2	68.00	1733.87260	25.54960
5	36.00	872.19306	26.47760
8	50.00	935.49803	18.99999
11	43.07	812.15973	19.08103
13	40.00	735.59962	16.50000
Total Cost	=	6033.32319 \$/h	
Total SO2	=	7035.94301 kg/h	
Total NOX	=	5060.62788 kg/h	
Total CO2	=	5917.43863 kg/h	

Table 2.5 Dispatch results for the minimum SO2 emission condition.

*** Generation Cost ***			
BUS	P_GEN (MW)	Cost (\$/h)	Inc-Cost (\$/MWh)
1	50.00	944.00013	21.60000
2	68.00	1733.87267	25.54960
5	50.00	1331.49953	28.25000
8	36.68	625.17418	17.43752
11	42.00	781.24482	18.83320
13	40.00	735.59969	16.50000
Total Cost	=	6151.39102 \$/h	
Total SO2	=	6709.76760 kg/h	
Total NOX	=	5009.04237 kg/h	
Total CO2	=	5976.29335 kg/h	

Table 2.6 Dispatch results for the minimum NOx emission condition.

*** Generation Cost ***			
BUS	P_GEN (MW)	Cost (\$/h)	Inc-Cost (\$/MWh)
1	50.00	944.00000	21.60000
2	69.86	1791.70996	25.69841
5	43.00	1094.37920	27.33440
8	34.00	567.96080	17.13120
11	50.00	1027.75000	20.75000
13	40.00	735.60000	16.50000
Total Cost	=	6161.39996 \$/h	
Total SO2	=	6874.94078 kg/h	
Total NOX	=	4897.48710 kg/h	
Total CO2	=	6104.20084 kg/h	

Table 2.7 Dispatch results of minimum CO2 emission condition.

*** Generation Cost ***			
BUS	P_GEN (MW)	Cost (\$/h)	Inc-Cost (\$/MWh)
1	50.00	943.99989	21.60000
2	68.00	1733.87299	25.54960
5	50.00	1331.50050	28.25000
8	50.00	935.50167	19.00001
11	34.00	571.06491	17.08280
13	34.68	626.79931	15.89414
Total Cost	= 6142.73928 \$/h		
Total SO2	= 6766.27879 kg/h		
Total NOX	= 5187.67201 kg/h		
Total CO2	= 5805.99644 kg/h		

Table 2.8 Dispatch results of the proposed FMOPD.

*** Generation Cost ***			
BUS	P_GEN (MW)	Cost (\$/h)	Inc-Cost (\$/MWh)
1	9.98	943.48013	21.59680
2	63.29	1590.34897	25.18430
5	44.74	1151.97467	27.55687
8	45.35	822.01699	18.44621
11	43.82	833.92697	19.25416
13	39.56	726.39639	16.44944
Total Cost	= 6068.14412 \$/h		
Total SO2	= 6822.92570 kg/h		
Total NOX	= 5018.07405 kg/h		
Total CO2	= 5889.96324 kg/h.		

Thus, FMOPD potentially can be applied to overcome the shortcomings of the method using emission factors in the combined objective.

2.8 ECONOMIC DISPATCH WITH TRANSMISSION CONSTRAINT

In the transmission constrained ED, the power flow limits of transmission lines are taken into consideration [21-22]. The power flow on transmission lines can be obtained by either AC or DC load flow [2]. In this section, the power flow is calculated via DC load flow analysis. The problem is formulated as follows:

$$\text{Min } F = \sum_{k=1}^M \sum_{i=1}^{NG} (a_i + b_i P_{Gi}^k + c_i (P_{Gi}^k)^2) \quad (2.109)$$

subject to the

power balance constraint:

$$\sum_{i=1}^{NG} P_{Gi}^k = P_D^k + P_{loss}^k \quad (2.110)$$

$$P_{loss}^k = \sum_{i=1}^{NG} \sum_{j=1}^{NG} P_{Gi}^k B_{ij} P_{Gj}^k \quad (2.111)$$

Power generation limits

$$P_{Gi}^{\min} \leq P_{Gi}^k \leq P_{Gi}^{\max} \quad (2.112)$$

Ramp rate constraints

$$P_{Gi}^k - P_{Gi}^{k-1} \leq UR_i \text{ as generating unit increases power} \quad (2.113)$$

$$P_{Gi}^{k-1} - P_{Gi}^k \leq DR_i \text{ as generating unit decreases power} \quad (2.114)$$

Emission constraint

$$\sum_{i=1}^{NG} \frac{ER_i}{FC_i} (a_i + b_i P_{Gi}^k + c_i (P_{Gi}^k)^2) \leq EM^k \quad (2.115)$$

Transmission constraint

$$-P_l^{\max} \leq P_l^k \leq P_l^{\max}; l = 1, \dots, NL \quad (2.116)$$

$$P_l^k = \sum_{i=1}^{NG} D_{li} P_{Gi}^k \quad (2.117)$$

where

P_{loss}^k system power loss in subinterval k (MW),

ER_i emission rate of unit i (kg/MWh),

FC_i cost of the fuel consumed by unit i (\$/MWh),

EM^k maximum emission allowance for subinterval k (kg/h),

P_l^k power flow on transmission line l (MW),

$P_l^{\max}$ maximum power flow on transmission line l (MW),

D_{li} generalized generation distribution factor for calculating power flow on transmission line l due to generator i ,

NL number of transmission lines.

Other parameters are similar to those in sections 2.4, 2.5 and 2.6.

The calculation of D_{li} and the loss coefficients B_{ij} derived from D_{li} are given in appendix A.

ALHN is applied to solve this problem where variables are considered such as power generation P_{Gi}^k and power flow on transmission lines P_l^k for each subinterval k with respect to their limits, power balance equality constraint, as well as ramp rate and emission inequality constraints. For implementation in ALHN, $(NG+NL) \times M$ continuous neurons and $(2+M) \times NL$ multiplier neurons are used.

An augmented Lagrange function is formulated:

$$\begin{aligned}
 L = & \sum_{k=1}^M \sum_{i=1}^{NG} \left(a_i + b_i P_{Gi}^k + c_i (P_{Gi}^k)^2 \right) \\
 & + \sum_{k=1}^M \left[\lambda^k (P_D^k + P_{loss}^k - \sum_{i=1}^{NG} P_{Gi}^k) + \frac{1}{2} \beta_\lambda (P_D^k + P_{loss}^k - \sum_{i=1}^{NG} P_{Gi}^k)^2 \right] \\
 & + \sum_{k=1}^M \frac{1+s_k}{2} \left\{ \gamma^k \left[\sum_{i=1}^{NG} \frac{ER_i}{FC_i} (a_i + b_i P_{Gi}^k + c_i (P_{Gi}^k)^2) - EM^k \right] \right. \\
 & \quad \left. + \frac{1}{2} \beta_\gamma \left[\sum_{i=1}^{NG} \frac{ER_i}{FC_i} (a_i + b_i P_{Gi}^k + c_i (P_{Gi}^k)^2) - EM^k \right]^2 \right\} \quad (2.118) \\
 & + \sum_{k=1}^M \frac{1-s_k}{2} \left(-\frac{(\gamma^k)^2}{2\beta_\gamma} \right) \\
 & + \sum_{k=1}^M \sum_{l=1}^{NL} \left[\eta_l^k \left(P_l^k - \sum_{i=1}^{NG} D_{li} P_{Gi}^k \right) + \frac{1}{2} \beta_\eta \left(P_l^k - \sum_{i=1}^{NG} D_{li} P_{Gi}^k \right)^2 \right]
 \end{aligned}$$

where,

$$s_k = \begin{cases} 1 & \text{if } \sum_{i=1}^{NG} \frac{ER_i}{FC_i} (a_i + b_i P_{Gi}^k + c_i (P_{Gi}^k)^2) - EM^k + \frac{\gamma^k}{\beta_\gamma} \geq 0 \\ -1 & \text{otherwise} \end{cases} \quad (2.119)$$

λ^k Lagrangian multiplier for power balance constraint at subinterval k ,

γ^k Lagrangian multiplier for emission constraint at subinterval k ,

η_l^k Lagrangian multiplier for transmission constraint at subinterval k ,

s_k sign function for emission constraint in Lagrangian relaxation handling inequality constraint derived from appendix A.

The augmented Lagrange function (2.118) is converted to the energy function of ALHN:

$$\begin{aligned}
E = & \sum_{k=1}^M \sum_{i=1}^{NG} \left(a_i + b_i V_{i,p}^k + c_i (V_{i,p}^k)^2 \right) \\
& + \sum_{k=1}^M \left[V_{\lambda}^k (P_D^k + P_{loss}^k - \sum_{i=1}^{NG} V_{i,p}^k) + \frac{1}{2} \beta_{\lambda} (P_D^k + P_{loss}^k - \sum_{i=1}^{NG} V_{i,p}^k)^2 \right] \\
& + \sum_{k=1}^M \frac{1+s_k}{2} \left\{ V_{\gamma}^k \left[\sum_{i=1}^{NG} \frac{ER_i}{FC_i} (a_i + b_i V_{i,p}^k + c_i (V_{i,p}^k)^2) - EM^k \right] \right. \\
& \quad \left. + \frac{1}{2} \beta_{\gamma} \left[\sum_{i=1}^{NG} \frac{ER_i}{FC_i} (a_i + b_i V_{i,p}^k + c_i (V_{i,p}^k)^2) - EM^k \right]^2 \right\} \quad (2.120) \\
& + \sum_{k=1}^M \frac{1-s_k}{2} \left(-\frac{(V_{\gamma}^k)^2}{2\beta_{\gamma}} \right) \\
& + \sum_{k=1}^M \sum_{l=1}^{NL} \left[V_{l,\eta}^k \left(V_{l,p}^k - \sum_{i=1}^{NG} D_{li} V_{i,p}^k \right) + \frac{1}{2} \beta_{\eta} \left(V_{l,p}^k - \sum_{i=1}^{NG} D_{li} V_{i,p}^k \right)^2 \right] \\
& + \sum_{k=1}^M \left(\sum_{i=1}^{NG} \int_0^{V_{i,p}^k} g^{-1}(V) dV + \sum_{l=1}^{NL} \int_0^{V_{l,p}^k} g^{-1}(V) dV \right)
\end{aligned}$$

where

- $V_{i,p}^k$ output of continuous neuron $p(i,k)$ representing power output of unit i in subinterval k , P_{Gi}^k ,
- $V_{l,p}^k$ output of continuous neuron $p(l,k)$ representing power flow on line l in subinterval k , $P_{l,p}^k$,
- V_{λ}^k output of multiplier neuron $\lambda(k)$ representing Lagrangian multiplier associated with power balance constraint λ^k ,
- V_{γ}^k output of multiplier neuron $\gamma(k)$ representing Lagrangian multiplier for emission constraint γ^k ,
- $V_{l,\eta}^k$ output of multiplier neuron $\eta(l,k)$ representing Lagrangian multiplier for transmission constraint η_l^k .

The dynamics of neurons based on the derivative of the energy function with respect to outputs of neurons are determined:

$$\begin{aligned}
\frac{dU_{i,p}^k}{dt} &= -\frac{\partial E}{\partial V_{i,p}^k} \\
&= -\left\{ \begin{aligned} &(b_i + 2c_i V_{i,p}^k) \\ &+ \left[V_{\lambda}^k + \beta_{\lambda} (P_D^k + P_{loss}^k - \sum_{i=1}^{NG} V_{i,p}^k) \right] \times \left(\frac{\partial P_{loss}^k}{\partial V_{i,p}^k} - 1 \right) \\ &+ \frac{1+s_k}{2} \left\{ V_{\gamma}^k + \beta_{\gamma} \left[\sum_{i=1}^{NG} \frac{ER_i}{FC_i} (a_i + b_i V_{i,p}^k + c_i (V_{i,p}^k)^2) - EM^k \right] \right\} \\ &\times \frac{ER_i}{FC_i} (b_i + 2c_i V_{i,p}^k) \\ &+ \sum_{l=1}^{NL} \left[V_{l,\eta}^k + \beta_{\eta} \left(V_{l,p}^k - \sum_{i=1}^{NG} D_{li} V_{i,p}^k \right) \right] \times (-D_{li}) + U_{i,p}^k \end{aligned} \right\} \quad (2.121)
\end{aligned}$$

$$\frac{dU_{l,p}^k}{dt} = -\frac{\partial E}{\partial V_{l,p}^k} = -\left\{ V_{l,\eta}^k + \beta_{\eta} \left(V_{l,p}^k - \sum_{i=1}^{NG} D_{li} V_{i,p}^k \right) + U_{l,p}^k \right\} \quad (2.122)$$

$$\frac{dU_{\lambda}^k}{dt} = \frac{\partial E}{\partial V_{\lambda}^k} = P_L^k + P_{loss}^k - \sum_{i=1}^{NG} V_{i,p}^k \quad (2.123)$$

$$\frac{dU_{\gamma}^k}{dt} = \frac{\partial E}{\partial V_{\gamma}^k} \quad (2.124)$$

$$= \frac{1+s_k}{2} \left[\sum_{i=1}^{NG} \frac{ER_i}{FC_i} (a_i + b_i V_{i,p}^k + c_i (V_{i,p}^k)^2) - EM^k \right] + \frac{1-s_k}{2} \left(-\frac{V_{\gamma}^k}{\beta_{\gamma}} \right)$$

$$\frac{dU_{l,\eta}^k}{dt} = \frac{\partial E}{\partial V_{l,\eta}^k} = V_l^k - \sum_{i=1}^{NG} D_{li} V_i^k \quad (2.125)$$

where,

$$\frac{\partial P_{loss}^k}{\partial V_{i,p}^k} = 2 \sum_{j=1}^{NG} B_{ij} V_{j,p}^k \quad (2.126)$$

$U_{i,p}^k$ input of continuous neuron $p(i,k)$ corresponding to the output $V_{i,p}^k$,
 $U_{l,p}^k$ input of continuous neuron $p(l,k)$ corresponding to the output $V_{l,p}^k$,
 U_{λ}^k input of multiplier neuron $\lambda(k)$ corresponding to the output V_{λ}^k ,
 U_{γ}^k input of multiplier neuron $\gamma(k)$ corresponding to the output V_{γ}^k ,
 $U_{l,\eta}^k$ input of multiplier neuron $\eta(l,k)$ corresponding to the output $V_{l,\eta}^k$.

The inputs of neurons are updated as follows:

$$U_{i,p}^{k(n)} = U_{i,p}^{k(n-1)} - \alpha_i \frac{\partial E}{\partial V_{i,p}^k} \quad (2.127)$$

$$U_{l,p}^{k(n)} = U_{l,p}^{k(n-1)} - \alpha_l \frac{\partial E}{\partial V_{l,p}^k} \quad (2.128)$$

$$U_{\lambda}^{k(n)} = U_{\lambda}^{k(n-1)} + \alpha_{\lambda} \frac{\partial E}{\partial V_{\lambda}^k} \quad (2.129)$$

$$U_{\gamma}^{k(n)} = U_{\gamma}^{k(n-1)} + \alpha_{\gamma} \frac{\partial E}{\partial V_{\gamma}^k} \quad (2.130)$$

$$U_{l,\eta}^{k(n)} = U_{l,\eta}^{k(n-1)} + \alpha_{\eta} \frac{\partial E}{\partial V_{l,\eta}^k} \quad (2.131)$$

where α_i , α_p , α_{λ} , α_{γ} and α_{η} are updating step sizes for neurons.

The outputs of continuous neurons are determined:

$$V_{i,p}^k = g_c(U_{i,p}^k) = \left(\frac{P_{Gi}^{high} - P_{Gi}^{low}}{2} \right) \left[1 + \tanh(\sigma U_{i,p}^k) \right] + P_{Gi}^{low} \quad (2.132)$$

$$V_{l,p}^k = g_c(U_{l,p}^k) = \tanh(\sigma U_{l,p}^k) \quad (2.133)$$

where the new generation limits are determined by:

$$P_{Gi}^{high} = \min \left\{ V_{i,p}^{(k-1)} + UR_i, P_{Gi}^{\max} \right\} \quad (2.134)$$

$$P_{Gi}^{low} = \max \left\{ V_{i,p}^{(k-1)} - DR_i, P_{Gi}^{\min} \right\} \quad (2.135)$$

The outputs of multiplier neurons are equal to their inputs by the linear transfer function.

The output of a continuous neuron representing output power is initialized by “mean distribution” as in (2.81). The initial output of a continuous neuron representing power flow in transmission lines is zero. The initial inputs of these continuous neurons are determined by the inverse sigmoid function.

The output of the multiplier neuron representing the Lagrangian multiplier associated with power balance constraint is initialized:

$$V_{\lambda}^{k(0)} = \frac{1}{NG} \sum_{i=1}^{NG} \frac{(b_i + 2c_i V_{i,p}^{k(0)})}{1 - \frac{\partial P_{loss}^k}{\partial V_{i,p}^k}} \quad (2.136)$$

where $V_{\lambda}^{k(0)}$ is the initial output of the multiplier neuron associated with power balance constraint.

The initial outputs of the other multiplier neurons are zero. The corresponding inputs of these neurons are equal to their outputs.

The maximum error of ALHN in this problem is determined from the maximum value of constraint errors of power balance, emissions and transmission and the iterative errors of neurons.

The algorithm of ALHN for solving this problem is similar to the one for solving the ED problem with transmission losses in section 2.5.

Example 2.7

The New England test system having 10 generating units, 39 buses and 46 transmission lines is viewed in this example. The schedule time horizon is divided into 12 subintervals with a duration of 1h each. The characteristics, emission rates and fuel cost of units and the load demand are given in Tables 2.9, 2.10 and 2.11. The transmission data and the load bus data are given in Tables 2.12 and 2.13. The diagram of the test system is given in Fig. 2.18. The maximum emission allowance of SO_2 for each interval is 5.4 tons.

Table 2.9 Characteristics of power units in example 2.7.

Unit	a_i (\$/h)	b_i (\$/MWh)	c_i (\$/MW ² h)	P_i^{max} (MW)	P_i^{min} (MW)	UR_i (MW/h)	DR_i (MW/h)	P_i^0 (MW)
1	180	26.4408	0.03720	360	155	20	25	239
2	275	21.0771	0.03256	680	320	20	25	355
3	352	18.6626	0.03102	718	323	50	50	412
4	792	16.8894	0.02871	680	275	50	50	476
5	440	17.3998	0.03223	600	230	50	50	416
6	348	21.6180	0.02064	748	350	50	50	547
7	588	15.1716	0.02268	620	220	100	100	620
8	984	14.5632	0.01776	643	225	100	150	643
9	1260	14.3448	0.01644	920	350	100	150	907
10	1200	13.5420	0.01620	1050	450	100	150	946

Table 2.10 SO₂ emission rate and fuel cost of units in example 2.7.

Unit	1	2	3	4	5	6	7	8	9	10
FC_i (\$/MWh)	12	11	11	11	11	12	12	12	12	12
ER_i (kg/MWh)	0.11	0.13	0.14	0.14	0.15	0.14	0.12	0.52	0.73	0.63

Table 2.11 Load demand in example 2.7.

Hour	1	2	3	4	5	6
Demand P_k (MW)	5560	5620	5800	5810	5990	6040
Hour	7	8	9	10	11	12
Demand P_k (MW)	6000	5790	5680	5540	5690	5750

Table 2.12 Transmission data of the New England test system in example 2.7.

Line no.	From bus	To bus	Resistance R (pu)	Reactance X (pu)	Susceptance B (pu)	Line limit P_l^{max} (MW)
1	1	2	0.0035	0.0411	0.6987	300
2	1	39	0.0010	0.0250	0.7500	300
3	2	3	0.0013	0.0151	0.2572	500
4	2	25	0.0070	0.0086	0.1460	300
5	3	4	0.0013	0.0213	0.2214	300
6	3	18	0.0011	0.0133	0.2138	300
7	4	5	0.0008	0.0128	0.1342	300
8	4	14	0.0008	0.0129	0.1382	300
9	5	6	0.0002	0.0026	0.0434	400
10	5	8	0.0008	0.0112	0.1476	300
11	6	7	0.0006	0.0092	0.1130	400
12	6	11	0.0007	0.0082	0.1389	400
13	7	8	0.0004	0.0046	0.0780	300
14	8	9	0.0023	0.0363	0.3804	300
15	9	39	0.0010	0.0250	1.2000	300
16	1	11	0.0004	0.0043	0.0729	400
17	10	13	0.0004	0.0043	0.0729	300
18	13	14	0.0009	0.0101	0.1723	300
19	14	15	0.0018	0.0217	0.3660	300
20	15	16	0.0009	0.0094	0.1710	400
21	16	17	0.0007	0.0089	0.1342	300
22	16	19	0.0016	0.0195	0.3040	300
23	16	21	0.0008	0.0135	0.2548	400
24	16	24	0.0003	0.0059	0.0680	300
25	17	18	0.0007	0.0082	0.1319	300
26	17	27	0.0013	0.0173	0.3216	300
27	21	22	0.0008	0.0140	0.2565	650
28	22	23	0.0006	0.0096	0.1846	300

Table 2.12 contd....

Table 2.12 contd....

Line no.	From bus	To bus	Resistance R (pu)	Reactance X (pu)	Susceptance B (pu)	Line limit P_l^{max} (MW)
29	23	24	0.0022	0.0350	0.3610	400
30	25	26	0.0032	0.0323	0.5130	300
31	26	27	0.0014	0.0147	0.2396	400
32	26	28	0.0043	0.0474	0.7802	300
33	26	29	0.0057	0.0625	1.0290	300
34	28	29	0.0014	0.0151	0.2490	400
35	12	11	0.0016	0.0435	0	400
36	12	13	0.0016	0.0435	0	400
37	6	31	0	0.0250	0	1200
38	10	32	0	0.0200	0	750
39	19	33	0.0007	0.0142	0	750
40	20	34	0.0009	0.0180	0	750
41	22	35	0	0.0143	0	750
42	23	36	0.0005	0.0272	0	750
43	25	37	0.0006	0.0232	0	750
44	2	30	0	0.0181	0	400
45	29	38	0.0008	0.0156	0	1000
46	19	20	0.0007	0.0138	0	400
38	10	32	0	0.0200	0	750
39	19	33	0.0007	0.0142	0	750
40	20	34	0.0009	0.0180	0	750
41	22	35	0	0.0143	0	750
42	23	36	0.0005	0.0272	0	750
43	25	37	0.0006	0.0232	0	750

Table 2.13 Load bus data of the New England test system in example 2.7.

Bus no.	Gen. no.	P_D (MW)	Bus no.	Gen. no.	P_L (MW)	Bus no.	Gen. no.	P_D (MW)
1	-	0	14	-	0	27	-	281.00
2	-	0	15	-	320.00	28	-	206.00
3	-	322.00	16	-	329.40	29	-	283.50
4	-	500.00	17	-	0	30	1	0
5	-	0	18	-	158.00	31	2	9.20
6	-	0	19	-	0	32	3	0
7	-	233.80	20	-	680.00	33	4	0
8	-	522.00	21	-	274.00	34	5	0
9	-	0	22	-	0	35	6	0
10	-	0	23	-	247.50	36	7	0
11	-	0	24	-	308.60	37	8	0
12	-	8.50	25	-	224.00	38	9	0
13	-	0	26	-	139.00	39	10	1104.00

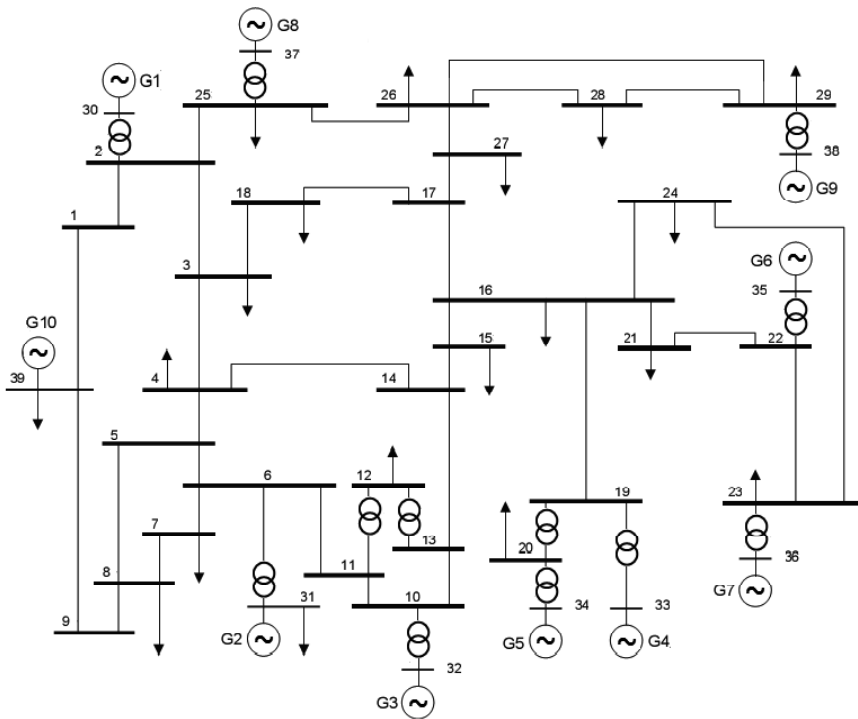


Fig. 2.18 Diagram of the New England test system in example 2.7.

When the transmission constraint is neglected, ALHN with the set of parameters of $\alpha_i = 0.017$, $\alpha_\lambda = 0.06$, $\alpha_\gamma = 0.0006$ gives a total cost of \$ 2,225,811 and total emissions of 63,522 kg SO₂ for 12 subintervals. Note that the slope of the sigmoid function and penalty factors are fixed at 100 and 0.001, respectively.

Remark that the transmission constraint is violated in this case where line no. 4 is overloaded for the entire schedule time horizon while lines no. 3, 22 and 27 are overloaded for some hours as shown in Table 2.14.

Table 2.14 Overloaded power on transmission lines without transmission constraint in example 2.7.

Hour	1	2	3	4	5	6
Line 3	0	0	9.7354	9.1049	0	0
Line 4	62.8862	61.2384	56.6717	55.3901	35.2739	27.0814
Line 22	0	0	0.4088	3.0047	76.5487	70.3894
Line 27	0	0	0	0	0	1.2962
Hour	7	8	9	10	11	12
Line 3	0	11.4442	2.2588	0	3.2935	6.9526
Line 4	30.9432	50.9988	59.4988	63.1396	59.5593	58.0830
Line 22	54.3007	0	0	0	0	0
Line 27	0	0	0	0	0	0

When the transmission constraints are introduced, the overloads of lines are eliminated. The parameters of ALHN in this case are $\alpha_i = \alpha_l = 0.003$, $\alpha_\lambda = 0.005$, $\alpha_\gamma = 0.0003$, $\alpha_\eta = 0.0007$, and σ and β are fixed as in the case without transmission constraint. In this case, ALHN takes longer for convergence than in the case without regard to transmission line limits with the transmission constraint error set to 0.1 and the others set to 10^{-3} , resulting in total cost of \$ 2,229,733 and total emissions of 62,280 kg SO₂. Compared to the case without transmission constraint, the total cost in this case is higher but the total emissions are lower because with the transmission constraint included, the power units with high operating cost are enforced to increase their power, leading to higher overall cost. The emission level for each interval which is proportional to the load demand is given in Fig. 2.19 whereby the emission levels at the peak load hours (which are considerably higher than the maximum allowance of 5.4 tons) will be limited to this maximum value.

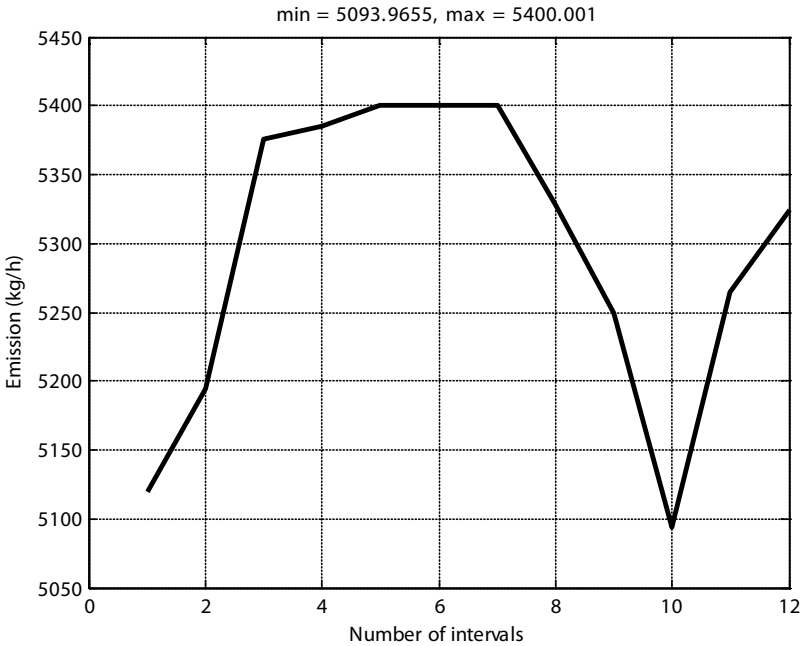


Fig. 2.19 Emission levels during the schedule time horizon in example 2.7.

2.9 ECONOMIC DISPATCH WITH NON-SMOOTH COST FUNCTIONS

The input-output characteristics of large units are essentially discontinuous cost curves due to valve-point and multi-fuel effects. The discontinuities of cost curves were neglected in the ED method leading to an inaccurate dispatch result. To obtain realistic dispatch results, approaches without restrictions on the shape of incremental fuel cost functions are needed [22-47].

Table 2.15 Power generation and power loss with transmission constraint in example 2.7.

P_1^k	P_2^k	P_3^k	P_4^k	P_5^k	P_6^k	P_7^k	P_8^k	P_9^k	P_{10}^k	P_{loss}^k
259.0	375.0	452.6	485.7	426.2	561.9	620	611.3	763.1	1046.0	40.7463
279.0	395.0	451.9	486.4	426.8	562.6	620	621.1	768.4	1050.0	41.1575
299.0	415.0	486.8	503.7	442.3	607.3	620	624.3	795.0	1050.0	43.3320
305.5	426.5	480.0	504.3	442.8	601.7	620	628.1	794.4	1050.0	43.2957
321.5	446.5	523.6	514.9	452.4	651.7	620	627.5	827.6	1050.0	45.6629
325.6	466.5	537.5	518.5	454.3	663.4	620	643.0	809.7	1047.0	45.4231
321.5	461.8	516.6	515.5	452.9	652.2	620	630.8	824.2	1050.0	45.5077
303.0	436.8	466.6	499.3	438.4	602.2	620	631.7	784.8	1050.0	42.7831
293.5	411.8	455.3	490.5	430.5	568.5	620	627.0	774.8	1050.0	41.7007
281.9	388.2	440.0	475.5	417.1	547.1	620	619.3	758.6	1032.7	40.3941
295.1	406.0	458.7	493.5	433.1	572.6	620	626.1	776.9	1050.0	41.9482
300.5	415.4	468.4	500.8	439.7	585.8	620	627.3	784.9	1050.0	42.6290

2.9.1 Economic Dispatch with Prohibited Operating Zones

A. Problem Formulation

In real power systems, a thermal generating unit may face certain prohibited zones within its overall operating range due to physical limitations of individual components such as vibrations in a shaft bearing amplified in certain operating regimes or faults in either the machines themselves or the associated auxiliaries such as boilers, feed pumps etc. These physical limitations or faults may lead to instabilities in the operation at certain levels of unit loading. Thus, the units may be unable to work at any load level within its entire operating range at any given time. To resolve this problem, the model of prohibited operating zones (POZ) is used. For a generating unit with prohibited zones, its whole operating region will be broken down into several isolated feasible sub-regions. These isolated feasible sub-regions will form multiple decision spaces, where each of the decision spaces can be either feasible or infeasible with respect to the system load demand. The optimal operation region will reside in one of the feasible decision spaces. Consequently, the associated economic dispatch problem is a non-convex optimization problem which is difficult to be solved by conventional methods [48-51].

The objective of the ED with POZ problem is to minimize the total cost of thermal generating units in a system over an appropriate period (typically one hour) while satisfying various constraints including power balance, generator power limits, prohibited zones, spinning reserve and ramp rate constraints. The ED problem with POZ is formulated as follows:

$$\text{Min } F = \sum_{i=1}^{NG} (a_i + b_i P_{Gi} + c_i P_{Gi}^2) \quad (2.137)$$

subject to

Power balance constraint

$$\sum_{i=1}^{NG} P_{Gi} - P_L - P_{loss} = 0 \quad (2.138)$$

$$P_{loss} = \sum_{i=1}^{NG} \sum_{j=1}^{NG} P_{Gi} B_{ij} P_{Gj} + \sum_{i=1}^{NG} B_{0i} P_{Gi} + B_{00} \quad (2.139)$$

Generator operating limits

$$P_{Gi}^{\min} \leq P_{Gi} \leq P_{Gi}^{\max}; \forall i \in (\Omega - \Theta) \quad (2.140)$$

Prohibited operating zone constraint

$$\begin{cases} P_{Gi}^{\min} \leq P_{Gi} \leq P_{Gi1}^l \\ \dots \\ P_{Gik-1}^u \leq P_{Gi} \leq P_{Gik}^l; \forall i \in \Theta; k = 2, \dots, n_i \\ \dots \\ P_{Gin_i}^u \leq P_{Gi} \leq P_{Gi}^{\max} \end{cases} \quad (2.141)$$

Spinning reserve constraint

$$\sum_{i=1}^{NG} S_{Gi} \geq S_R \quad (2.142)$$

$$S_{Gi} = \min \{P_{Gi}^{\max} - P_{Gi}, S_{Gi}^{\max}\}; \forall i \in (\Omega - \Theta) \quad (2.143)$$

$$S_{Gi} = 0; \forall i \in \Theta \quad (2.144)$$

Ramp rate constraint

$$P_{Gi} - P_{Gi}^0 \leq UR_i, \text{ if generation increases} \quad (2.145)$$

$$P_{Gi}^0 - P_{Gi} \leq DR_i, \text{ if generation decreases} \quad (2.146)$$

where

Ω set of generating units,

Θ set of generating units with POZ,

P_{Gik}^l lower bound for the prohibited zone k of unit i ,

P_{Gik}^u upper bound for the prohibited zone k of unit i ,

S_{Gi} spinning reserve contribution of unit i ,

S_R total spinning reserve requirement,

S_{Gi}^{max} maximum spinning reserve contribution of unit i .

Other parameters are similar to the ones used in the previous sections.

A typical input-output curve of a unit with POZ is given in Fig. 2.20.

It is obvious from equation (2.141) that if a unit has n_i prohibited zones, its entire operating region will be decomposed into (n_i+1) feasible sub-regions, leading to multiple search spaces for the ED with POZ problem. The total number of decision spaces can be enumerated as follows:

$$N = \prod_{i \in \Theta} (n_i + 1) \quad (2.147)$$

From equation (2.147), it is implied that, as the number of units with POZ increases, the total number of decision spaces increases sharply. To compute all those decision spaces classical methods are not viable. Therefore, new approaches that can overcome or better say bypass these difficulties are needed.

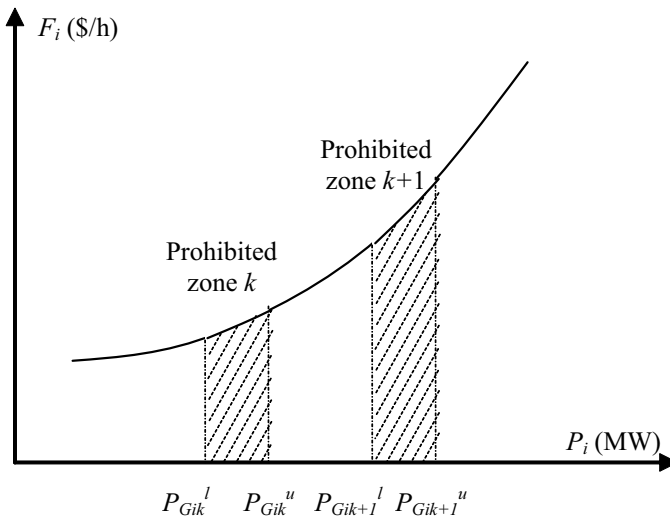


Fig. 2.20 Typical input-output of a unit with POZ.

B. Particle Swarm Optimization Implementation

The practical ED problem with valve-point and multi-fuel effects is represented as a non-smooth optimization problem with equality and inequality constraints, and this makes the problem of finding the global optimum difficult. To solve this problem, many salient methods have been proposed and genetic algorithm, evolutionary programming, Taboo search, and particle swarm optimization (PSO) are considered powerful methods to obtain usable solutions to power system optimization problems.

Among those methods, PSO is one of the most modern heuristic algorithms and has gained a lot of attention in various power system applications. PSO can be applied to nonlinear and non-continuous optimization problems with continuous variables. It has been developed through simulation of simplified social models. PSO is similar to other evolutionary algorithms in that the system is randomly initialized with a population of solutions [52].

The algorithm for obtaining the optimal power dispatch with a non-smooth cost function is shown in Fig. 2.21.

As an example, a hybrid particle swarm quadratic programming based economic dispatch (PSO-QP-ED) with a network and generator constrained algorithm is used to illustrate the non-smooth cost function ED.

In the PSO-QP-ED algorithm, the real power generation sets at the generator bus are used as particles in the PSO. The QP based ED with transmission line limit and transformer loading constraints is performed for every generation to obtain the best solution for each population search. PSO-QP-ED is compared to PSO-ED (without QP) on the IEEE 30 bus system under transmission line and transformer loading limit constraints and with generators' discontinuous fuel cost functions.

Generally, PSO is characterized as a simple heuristic of a well-balanced mechanism with flexibility to adapt and enhance both global and local exploration abilities. It is a stochastic search technique with reduced memory requirement, computationally effective and easier to implement than other artificial intelligence techniques. PSO also has a greater global searching ability at the beginning of the run while conducting a local search near the end of the run. Therefore, when solving problems with several local optimal solutions, there is a high possibility that PSO will explore more local optimal solutions with the potential of global optimal solution after convergence.

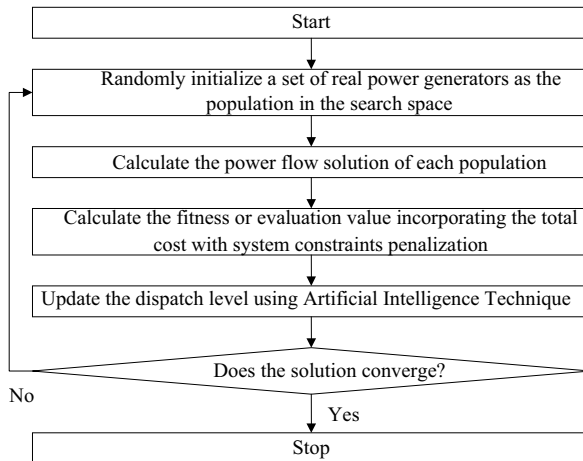


Fig. 2.21 Computational procedure of a non-smooth cost function ED.

In a PSO system, particles fly around in a multidimensional search space. During flight, each particle adjusts its position according to its own experience and the experience of neighboring particles, making use of the best position encountered by itself and its neighbors. The swarm direction of a particle is guided by the set of neighboring particles and its historical experience.

In the hybrid PSO-QP-ED algorithm, the set of particles is represented as follows:

$$\mathbf{P}_{Gi} = [\mathbf{P}_{Gi}^1, \mathbf{P}_{Gi}^2, \dots, \mathbf{P}_{Gi}^M] \quad (2.148)$$

$$\mathbf{P}_{Gi}^j = [P_{G1}^j, P_{G2}^j, \dots, P_{G,NG}^j]^T \quad (2.149)$$

where $\mathbf{P}_{Gi}$ is a matrix representing the set of individual searches. More specifically, in this case, it is a set of real power generators. The sub-matrix $\mathbf{P}_{Gi}^j$ is the set of the current position of particle j representing the real power generation of the generator connected to bus i (P_{Gi}^j). Each particle is used to solve the quadratic programming-optimal power flow (QPOPF) and the best previous position of the j th particle is recorded and represented as:

$$\mathbf{pbest} = [pbest_1, pbest_2, \dots, pbest_{NG}]^T \quad (2.150)$$

The index of the best particle among all the particles in the group is represented by the $gbest^j$. The velocity rate of particle j is represented as

$$\mathbf{u}^j = [u_1^j, u_2^j, \dots, u_{NG}^j]^T \quad (2.151)$$

The modified velocity and position of each particle can be calculated using the current velocity and the distance from $pbest_i$ to $gbest_i$ as shown in the following formulas:

$$\begin{aligned} u_i^{j(k+1)} &= w \cdot u_i^{j(k)} + c_1 \cdot rand_1(\bullet) \cdot (pbest_i - P_{Gi}^{j(k)}) \\ &\quad + c_2 \cdot rand_2(\bullet) \cdot (gbest_i - P_{Gi}^{j(k)}) \\ j &= 1, 2, \dots, M; \quad i = 1, 2, \dots, NG \end{aligned} \quad (2.152)$$

$$P_{Gi}^{j(k+1)} = P_{Gi}^{j(k)} + v_i^{j(k+1)}, \quad j = 1, 2, \dots, M; \quad i = 1, 2, \dots, NG \quad (2.153)$$

$$u_i^{\min} \leq u_i^j \leq u_i^{\max} \quad (2.154)$$

where

- M number of particles in a group,
- NG number of members in a particle,
- k pointer of iterations (generations),
- w inertia weight factor,
- c_1, c_2 acceleration coefficients,

$u_i^{j(k)}$ velocity of particle j corresponding to P_{Gi} at iteration k ,
 $P_{Gi}^{j(k)}$ current position of particle j corresponding to P_{Gi} at iteration k ,
 $rand_1(\bullet), rand_2(\bullet)$ uniform random values in the range $[0,1]$.

The parameter u_i^{max} determines the resolution, or fitness, which regions are to be searched with between present position and target position. If u_i^{max} is too high, particles might fly over good solutions. If u_i^{max} is too low, particles may get stuck to local solutions. From experience with PSO, u_i^{max} has often been set at 10–20% in each dimension of the dynamic range of a variable.

The constants c_1 and c_2 represent the weight of the stochastic acceleration terms that pull each particle towards the $pbest_i$ and $gbest_i$ positions. Low values may move particles past the target regions before being pulled back. On the other hand, high values may result in abrupt movement towards, or past, target regions. Hence, the acceleration constants c_1 and c_2 have often been set to 2.0 which works reasonably well for ED problem. A suitable selection of the inertia weight w provides a balance between global and local explorations, thus on average requiring less iterations to find an optimal solution. As originally developed, w often decreases linearly from about 0.9 to 0.4 during a run. In general, the inertia weight w is set according to the following equation:

$$w = w_{\max} - \frac{w_{\max} - w_{\min}}{M} \cdot k \quad (2.155)$$

where $w_{\max}$ and $w_{\min}$ are maximum and minimum inertia weight factors, respectively, and M is the maximum number of iterations.

The evaluation value is normalized into the range between 0 and 1 as:

$$EV = 1 / (F_{\text{cost}} + P_{pbc}) \quad (2.156)$$

where

$$F_{\text{cost}} = 1 + abs \left(\frac{\sum_{i \in BG} F(P_{Gi}) - F_{\min}}{F_{\max} - F_{\min}} \right) \quad (2.157)$$

$$P_{pbc} = 1 + \left(\sum_{i \in BG} P_{Gi} - \sum_{i=1}^{NB} P_{Di} - P_{\text{loss}} \right)^2 \quad (2.158)$$

$F_{\max}$ maximum generation cost among all individuals in the initial population,

$F_{\min}$ minimum generation cost among all individuals in the initial population,

BG number of buses with generation.

To limit the evaluation value of each individual of the population within a feasible range, before estimation, the generated power output must satisfy all constraints. If one individual satisfies all constraints, it is a feasible individual

and F_{cost} has a small value. Otherwise, the F_{cost} value of the individual is penalized with a very large positive constant. The computational procedure is shown in Fig. 2.22.

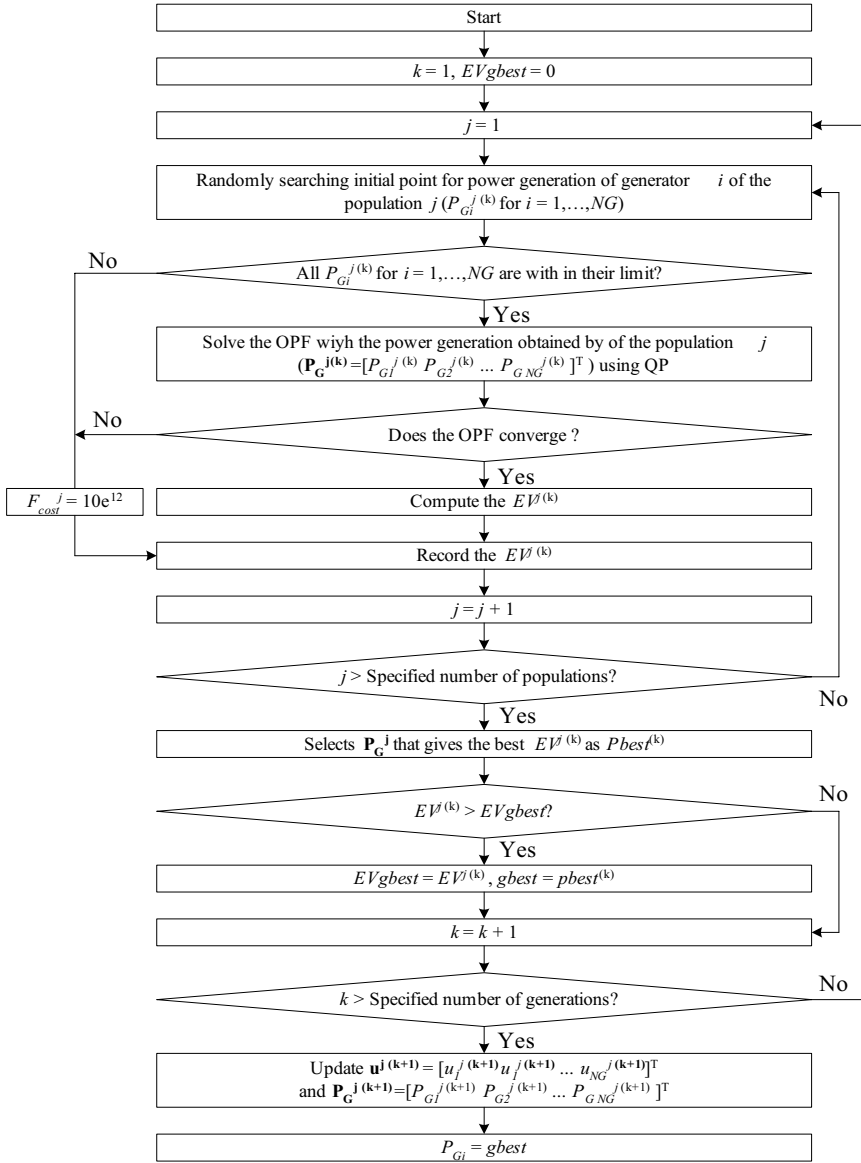


Fig. 2.22 PSO-QP-ED Computational Procedure.

Example 2.8

The IEEE 30 bus system in example 2.6 is used as test data. The generator fuel cost functions and prohibited operating zones are given in Table 2.16. The parameters of the proposed PSO-QP-ED and the PSO-ED are as follows:

Population size = 200

Generation (M) = 10

$w_{min} = 0.4$, $w_{max} = 0.9$

$u_i^{\max} = 0.5 \cdot P_{Gi}^{\max}$, $u_i^{\min} = -0.5 \cdot P_{Gi}^{\min}$

$c_1 = 2$, and $c_2 = 2$

Table 2.17 addresses the summarized results of PSO-ED and the proposed PSO-QP-ED from 50 trials. The convergence properties of the best solutions of PSO-QP-ED and PSO-ED are shown in Fig. 2.23. Figure 2.24 shows the total system operating cost from 50 trials of PSO-ED and PSO-QP-ED. The results show that the total generator fuel costs of the proposed PSO-QP-ED are lower than that of the PSO-ED. The computational time of PSO-QP-ED is longer than that of PSO-ED because PSO-QP-ED solves QPOPF for each individual search in the search space. The computational time of the proposed method could be decreased by parallel computation. However, the PSO-QP-ED offers a higher probability of obtaining the global minimum of total generator fuel cost.

Table 2.16 Generator operating cost functions and constraints.

Gen bus	$F(P_{Gi}) = a_i + b_i P_{Gi} + c_i P_{Gi}^2$			$P_{Gi}^{\min}$	$P_{Gi}^{\max}$	Generator Prohibited Operating Zone			
	a_i	b_i	c_i	MW	MW	From MW	To MW	From MW	To MW
1	0	2	0.00375	50	200	100	120	150	160
2	0	1.75	0.0175	20	80	25	30	40	60
5	0	1	0.0625	15	50	20	25	40	45
8	0	3.25	0.00834	10	35	15	20	25	30
11	0	3	0.025	10	30	15	18	22	25
13	0	3	0.025	12	40	20	25	30	35

Table 2.17 Results of the IEEE 30-bus test system.

	PSOED			PSO-QP-ED		
	Min	Aver.	Max	Min	Aver.	Max
Total system operating cost (\$/h)	806.10	818.13	834.05	805.14	809.46	816.47
Computation time of the best trial solution (sec)	52.67			69.39		

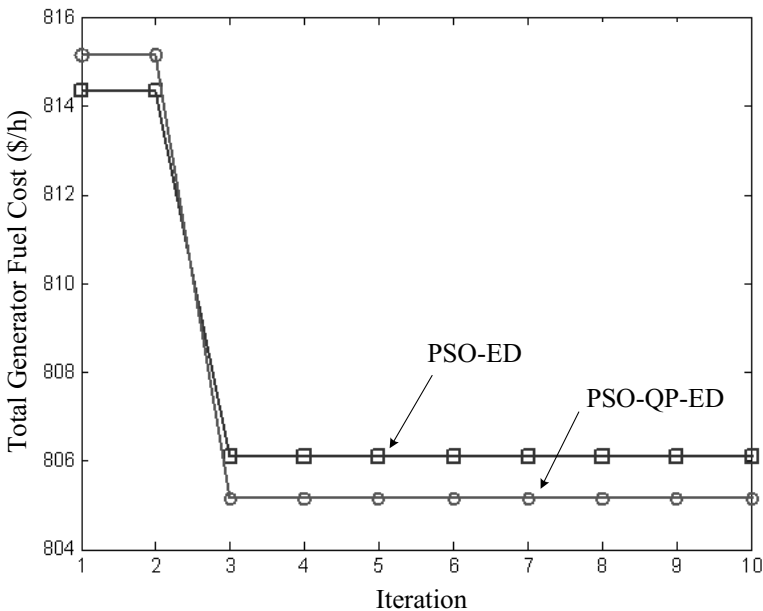


Fig. 2.23 Convergence properties of the IEEE 30 bus test system.

Color image of this figure appears in the color plate section at the end of the book.

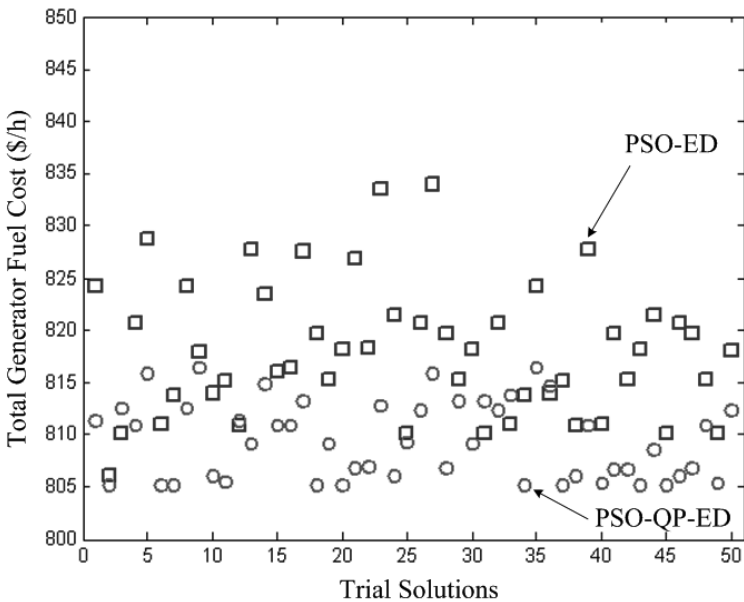


Fig. 2.24 The results from 50 trials with the IEEE 30 bus test system.

Color image of this figure appears in the color plate section at the end of the book.

A hybrid particle swarm optimization and quadratic programming for economic dispatch (PSO-QP-ED) is effectively minimizing the total generator fuel cost while satisfying transmission line limits and transformer loading constraints with generator prohibited operating zone constraints. PSO-QP-ED achieves lower minimum total generator fuel cost in the constrained ED than PSO-ED.

C. Enhanced ALHN Implementation

Enhanced ALHN is an ALHN method enhanced by a heuristic search handling the non-convex objective function in which ALHN is used for solving the ED problem and the heuristic search is used to find the most suitable operating zones of generating units to avoid the prohibited zones.

The procedure for ALHN implementation is the same as in the ramp rate constrained ED problem in section 2.5. The heuristic search algorithm is used here to analyze the operating points of generating units and to modify them such that any affected unit is moved out of a prohibited zone.

For the generation units currently at an operating point not violating any prohibited zone as determined by the suggested solution of the ED problem, new operating limits are fixed within feasible operating zones. For those units currently at an operating point violating a prohibited zone, the margins from the operating point to the lower and upper bounds of the prohibited zone must be recalculated and compared. If the former is smaller than the latter, the upper limit of the unit will be fixed at the lower bound of the prohibited zone. By contrast, if the latter is smaller than the former, the lower limit of the unit will be fixed at the upper bound of the prohibited zone. An explanation of generation limit modification of the prohibited zone k without ramp rates is given in Fig. 2.25.

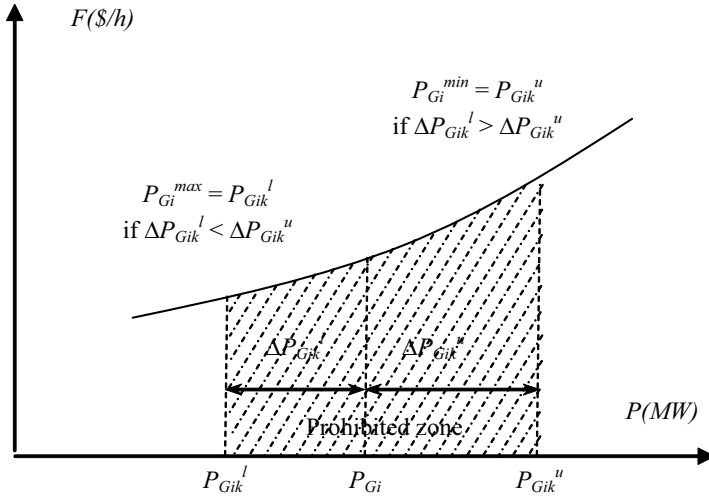


Fig. 2.25 Modification of generation limits of units.

The lower and upper margins from the operating point to the lower and upper bounds of the prohibited zone are calculated as obtained from the CED problem.

$$\Delta P_{Gik}^l = P_{Gi} - P_{Gik}^l; \forall i \in \Theta \quad (2.159)$$

$$\Delta P_{Gik}^u = P_{Gik}^u - P_{Gi} \quad (2.160)$$

where

ΔP_{Gik}^l margin from operating point to the lower bound of prohibited zone k of unit i ,

ΔP_{Gik}^u margin from operating point to upper bound of prohibited zone k of unit i .

The new generation limits with ramp rates are modified according to the following possibilities:

$$P_{Gi}^{\text{high(new)}} = P_{Gik}^l \quad \text{if } \Delta P_{Gik}^l \leq \Delta P_{Gik}^u \\ \text{and } P_{Gi}^{\text{low}} \leq P_{Gik}^l, \forall i \in \Theta \quad (2.161a)$$

$$\text{or } P_{Gi}^{\text{low(new)}} = P_{Gik}^u \quad \text{if } \Delta P_{Gik}^l \leq \Delta P_{Gik}^u \\ \text{and } P_{Gi}^{\text{low}} > P_{Gik}^l, \forall i \in \Theta \quad (2.161b)$$

$$P_{Gi}^{\text{low(new)}} = P_{Gik}^u \quad \text{if } \Delta P_{Gik}^l > \Delta P_{Gik}^u \\ \text{and } P_{Gi}^{\text{high}} \geq P_{Gik}^u, \forall i \in \Theta \quad (2.162a)$$

$$\text{or } P_{Gi}^{\text{high(new)}} = P_{Gik}^l \quad \text{if } \Delta P_{Gik}^l > \Delta P_{Gik}^u \\ \text{and } P_{Gi}^{\text{high}} < P_{Gik}^u, \forall i \in \Theta \quad (2.162b)$$

The procedure of modifying generation limits is as follows:

Step 1: Obtain a list of all units having prohibited operating zones.

Step 2: Choose the first unit from the list.

Step 3: Check the operating point against prohibited zones.

Step 4: If no prohibited zone is violated, fix the operating limits of the unit in this feasible operating zone and go to Step 7.

Step 5: Calculate lower and upper margins from (2.159) and (2.160).

Step 6: Fix new generation operating limits as (2.161) and (2.162).

Step 7: Eliminate the unit from the list. If the list is not empty, choose the next unit in the list and go to Step 3. Otherwise, stop.

After obtaining a solution for the ED problem by ALHN, the system spinning reserve contributed by units without prohibited operating zones has to be checked for violations.

If the contributed spinning reserve of units does not satisfy the system spinning reserve, the amount of insufficient spinning reserve is calculated.

$$\Delta S = S_R - \sum_{i \in \Omega - \Theta} S_{Gi} \quad (2.163)$$

where ΔS is insufficient power for spinning reserve.

To determine new generation limits for units satisfying the spinning reserve, a merit order is used:

$$M_i = \frac{F_i(P_{Gi})}{P_{Gi}} \Big|_{x_i P_{Gi}^{\max}}; \forall i \in (\Omega - \Theta) \quad (2.164)$$

where M_i is the priority index for the unit i based on its average fuel cost. The value of x_i , fixed fraction of the maximum output of unit i , is chosen as follows:

$$x_i = \frac{1}{2} \left(1 + \frac{P_{Gi}^{\min}}{P_{Gi}^{\max}} \right) \quad (2.165)$$

The units without prohibited zones are classified in two categories.

Category 1: the units whose contributions are at their maximum satisfying level:

$$P_{Gi}^{\max} - P_{Gi} \geq S_{Gi}^{\max} \quad (2.166)$$

These units cannot contribute additional power to add to spinning reserve requirements; hence their new generation limits are redefined as:

$$P_{Gi}^{\max(\text{new})} = P_{Gi}^{\max} - S_{Gi}^{\max} \quad (2.167)$$

$$P_{Gi}^{\min(\text{new})} = P_{Gi} \quad (2.168)$$

Category 2: the units whose contributions are lower than their maximum satisfying level:

$$P_{Gi}^{\max} - P_{Gi} < S_{Gi}^{\max} \quad (2.169)$$

These units can contribute additional power to the system spinning reserve. Therefore, the units with highest M_i will be selected to increase their contribution to the system spinning reserve by fixing their new maximum output power as (2.166). The power contributions of these units are determined:

$$\Delta P_{Gi} = P_{Gi} - (P_{Gi}^{\max} - S_{Gi}^{\max}) \quad (2.170)$$

and the satisfying level:

$$\sum_{i \in L} \Delta P_{Gi} \geq \Delta S \quad (2.171)$$

where

ΔP_{Gi} power contribution of unit i to spinning reserve by reducing its generation,

L set of selected units to increase spinning reserve contribution.

The procedure for reducing generation limits satisfying spinning reserve is as follows:

Step 1: Calculate the amount of insufficient power for spinning reserve requirement as (2.163).

Step 2: If the spinning reserve requirement is satisfied, go to step 8.

Step 3: Calculate the merit order of each unit without POZ as given in (3.164).

Step 4: Find the units whose contributions to spinning reserve are at their maximum level, fix their new generation limits as in (3.167)–(3.168).

Step 5: Obtain a list of units whose contributions to spinning reserve can be increased.

Step 6: Choose the unit with the highest M_i , fix its new maximum limit as in (3.167) and calculate the extra contribution ΔP_{Gi} as in (3.170).

Step 7: If $\Delta P_{Gi} < \Delta S$, $\Delta S = \Delta S - \Delta P_{Gi}$ and eliminate the unit from the list and return to step 5.

Step 8: Stop the procedure.

The overall procedure for enhanced ALHN for solving ED with POZ is as follows:

Step 1: Solve the ED by ALHN without prohibited zones and spinning reserve.

Step 2: If no prohibited zone violation is found, go to step 7.

Step 3: Modify new maximum and minimum power outputs of units with violated prohibited zones using heuristic.

Step 4: Solve the ED problem again by ALHN with new generator operating limits.

Step 5: If the system spinning reserve is satisfied, go to step 7.

Step 6: Modify the generation limits of units to the new values using heuristic search, and solve the final ED problem by ALHN.

Step 7: Compute the total cost and stop.

Example 2.9

A test system consisting of six generating units is considered in this example. Each unit has two prohibited zones and supplies a total load demand of 1,230 MW. Ramp rate constraint and power losses are included while spinning reserve is neglected. The characteristics of generating units and their initial prohibited zones are given in Table 2.18. The B-matrix power loss coefficients for the system are given as follows:

Table 2.18 Unit data for example 2.9.

Unit	1	2	3	4	5	6
a_i (\$/h)	240	200	220	200	220	190
b_i (\$/MWh)	7.0	10.0	8.5	11.0	10.5	12.0
c_i (\$/MW ² h)	0.0070	0.0095	0.0090	0.0090	0.0080	0.0075
$P_{i,min}$ (MW)	100	50	80	50	50	50
$P_{i,max}$ (MW)	500	200	300	150	200	120
UR_i (MW/h)	80	50	65	50	50	50
DR_i (MW/h)	120	90	100	90	90	90
P_i^0 (MW)	440	170	200	150	190	110
POZ (MW)	[210,240] [350,380]	[90,110] [140,160]	[150,170] [210,240]	[80,90] [110,120]	[90,110] [140,150]	[75,85] [100,105]

$$B_{ij} = 10^{-4} \times \begin{bmatrix} 17 & 12 & 7 & -1 & -5 & -2 \\ 12 & 14 & 9 & 1 & -6 & -1 \\ 7 & 9 & 31 & 0 & -10 & -6 \\ -1 & 1 & 0 & 24 & -6 & -8 \\ -5 & -6 & -10 & -6 & 129 & -2 \\ -2 & -1 & -6 & -8 & -2 & 150 \end{bmatrix}$$

$$B_{i0} = 10^{-3} \times [-0.3098 \quad -0.1297 \quad 0.7047 \quad 0.0591 \quad 0.2161 \quad -0.6635]^T$$

$$B_{00} = 0.056$$

In the first step, ALHN is used for solving the ramp rate constrained ED similar to section 2.5 neglecting POZ with the parameters of $\sigma = 100$, $\beta = 0.001$, $\alpha_i = 0.015$ and $\alpha_\lambda = 0.00125$. The operating point of generating units obtained in the first solution by ALHN is:

$$P_{Gi} = [441.1915 \quad 168.5799 \quad 255.9113 \quad 133.6728 \quad 160.5604 \quad 82.2937]^T$$

with a power loss of 12.2095 MW and total cost of 15,002.99 \$/h.

However, this solution is infeasible as unit 6 violates its prohibited zones. Therefore, rescheduling is needed to move the operating point of unit 6 out of its prohibited zone without causing other units to violate their POZ. Based on the obtained solution, the units are classified in two categories, one for the units not violating their POZ including units 1–5 and one for the units violating their POZ including unit 6. Unit 6 violates its first prohibited zone, thus the margins from its current operating point to the bound of that prohibited zone are calculated:

$$\Delta P_{G61}^l = P_{G6} - P_{G61}^l = 82.2937 - 75 = 7.2937 \text{ MW}$$

$$\Delta P_{G61}'' = P_{G61}'' - P_{G6} = 85 - 82.2937 = 2.7063 \text{ MW}$$

It is obvious that the upper margin is smaller than the lower. Moreover, the maximum and minimum possible operating range of this unit are $P_6^{high} = P_6^0 + UR_6 = 110 + 50 = 160$ MW and $P_6^{low} = P_6^0 - DR_6 = 110 - 90 = 20$ MW which cover the upper bound of the first prohibited zone of 85 MW. Therefore, the new low generation limit of unit 6 will be fixed at the upper bound of the first prohibited zone and its new high generation limit fixed at the lower bound of the second prohibited zone as follows:

$$P_{G6}^{low(new)} = P_{G61}^u = 85 \text{ MW}; P_{G6}^{high(new)} = P_{G62}^l = 100 \text{ MW}$$

The generation limits of the others will be fixed at their current feasible operating zone:

$$P_{G1}^{low(new)} = P_{G12}^u = 380 \text{ MW}; P_{G1}^{high(new)} = P_{G1}^{\max} = 500 \text{ MW}$$

$$P_{G2}^{low(new)} = P_{G22}^u = 160 \text{ MW}; P_{G2}^{high(new)} = P_{G2}^{\max} = 200 \text{ MW}$$

$$P_{G3}^{low(new)} = P_{G32}^u = 240 \text{ MW}; P_{G3}^{high(new)} = P_{G3}^{\max} = 265 \text{ MW}$$

$$P_{G4}^{low(new)} = P_{G42}^u = 120 \text{ MW}; P_{G4}^{high(new)} = P_{G4}^{\max} = 150 \text{ MW}$$

$$P_{G5}^{low(new)} = P_{G52}^u = 160 \text{ MW}; P_{G5}^{high(new)} = P_{G5}^{\max} = 200 \text{ MW}$$

In the last step, ALHN is used again to find the final solution based on the new generation limits with the same parameters as in the first step. The obtained solution is as follows:

$$P_{Gi} = [440.1625 \ 168.2497 \ 255.5809 \ 133.1487 \ 160.0609 \ 85.0006]^T$$

with a power loss of 12.2035 MW and total cost of \$15003.08.

In the final solution, there is no more prohibited zone violated, thus this is a feasible solution. As the power generation of unit 6 increases due to repair, power generation of the other units slightly decreases as compared to the original solution without prohibited zones. The total cost in the final solution is slightly higher than without considering prohibited zones due to the increase of power generation by units with higher fuel cost and the decrease of power generation by units using less expensive fuel.

2.9.2 Economic Dispatch with Piecewise Quadratic Cost Function

A. Problem Formulation

Under practical power system operation conditions, many thermal generating units, especially those with multiple fuel sources like natural gas and oil, require that their fuel cost functions are segmented as piecewise quadratic cost functions for different fuel types. The aim of ED with piecewise quadratic fuel cost functions is

to minimize the fuel cost by choosing among the available fuels of each unit with respect to load demand and generation limits. This is a non-convex optimization problem. Since this representation contains discontinuous values at each boundary and many local optima, classical solution methods are difficult to apply to this problem [3, 53].

Mathematically, the problem is formulated as:

$$\text{Min } F = \sum_{i=1}^{NG} F_i(P_{Gi}) \quad (2.172)$$

$$F_i(P_{Gi}) = \begin{cases} a_{i1} + b_{i1}P_i + c_{i1}P_{Gi}^2, \text{fuel1}, P_{Gi}^{\min} = P_{Gi1}^{\min} \leq P_{Gi} \leq P_{Gi1}^{\max} \\ a_{i2} + b_{i2}P_i + c_{i2}P_{Gi}^2, \text{fuel2}, P_{Gi2}^{\min} \leq P_{Gi} \leq P_{Gi2}^{\max} \\ \dots \\ a_{ik} + b_{ik}P_{Gi} + c_{ik}P_{Gi}^2, \text{fuel}k, P_{Gik}^{\min} \leq P_{Gi} \leq P_{Gik}^{\max} = P_{Gi}^{\max} \end{cases} \quad (2.173)$$

subject to

Power balance constraint

$$\sum_{i=1}^{NG} P_{Gi} - P_D - P_{loss} = 0 \quad (2.174)$$

$$P_{loss} = \sum_{i=1}^{NG} \sum_{j=1}^{NG} P_{Gi} B_{ij} P_{Gj} + \sum_{i=1}^{NG} B_{i0} P_{Gi} + B_{00} \quad (2.175)$$

Generator operating limits

$$P_{Gi}^{\min} \leq P_{Gi} \leq P_{Gi}^{\max} \quad (2.176)$$

where $P_{Gik}^{\min}$ and $P_{Gik}^{\max}$ are the respective lower and upper generation limits of unit i using fuel k .

A typical unit with multiple fuel types is represented in Fig. 2.26.

B. Enhanced ALHN Implementation

Each unit has multiple fuel types represented by a quadratic cost function. The enhanced ALHN method used here is similar to its application to the ED problem with prohibited zones in using heuristic search to find the most appropriate fuel type for the respective units to satisfy the load demand before the ED problem is finally solved. The heuristic search is based on a combined priority index indicating which fuel type will be selected preferentially.

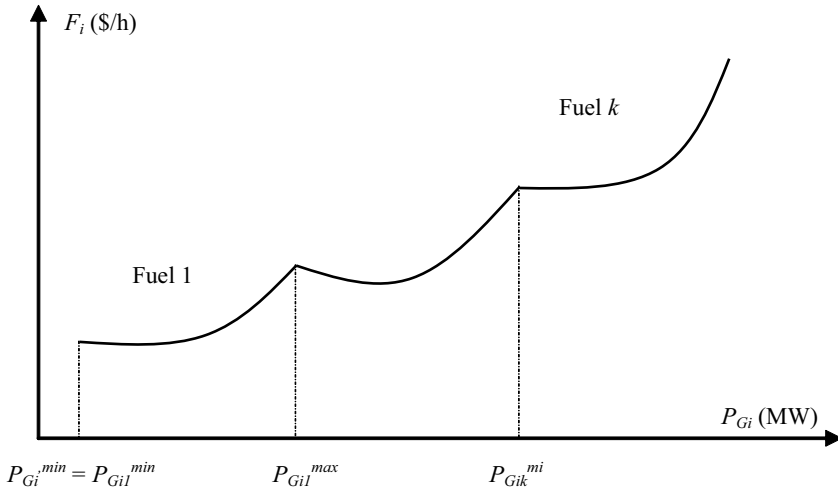


Fig. 2.26 Typical unit with multiple fuel types.

The average production cost for each fuel type is defined as follows:

$$M_{ik} = \frac{F_i(P_{Gik}^{avg})}{P_{Gik}^{avg}} \quad (2.177)$$

where M_{ik} is the priority index for the i with fuel k , and P_{Gik}^{avg} is the average power output of unit i with fuel k as calculated by:

$$P_{Gik}^{avg} = w_1 \cdot P_{Gik}^{max} + (1 - w_1) \cdot P_{Gik}^{min}; 0 \leq w_1 \leq 1 \quad (2.178)$$

where w_1 is the weight factor of P_{Gik}^{max} in the priority index M_{ik} .

The average incremental cost for each fuel type is defined as follows:

$$\lambda_{ik}^{avg} = \frac{F_i(P_{Gik}^{max}) - F_i(P_{Gik}^{min})}{P_{Gik}^{max} - P_{Gik}^{min}} \quad (2.179)$$

where λ_{ik}^{avg} is the average incremental cost of fuel k of unit i .

A combined priority index x_{ik} based on the priority index M_{ik} and average incremental cost λ_{ik}^{avg} is defined as:

$$x_{ik} = w_2 \cdot M_{ik} + (1 - w_2) \cdot \lambda_{ik}^{avg}; 0 \leq w_2 \leq 1 \quad (2.180)$$

where w_2 is the weighting factor of M_{ik} in the combined average priority index x_{ik} .

In each unit, the segment with a lower x_{ik} will have a higher priority to be selected for operation.

The procedure for fuel type determination for each unit to satisfy load demand is given as follows:

- Step 1: Determine the combined priority index x_{ik} of each fuel type of each unit as in (2.180).
- Step 2: Create a priority list of fuels of all units.
- Step 3: Start with the first fuel type of each unit to check the total power supply against load demand. Delete these fuels from the list.
- Step 4: If the total power supply is sufficient to meet the load demand, go to Step 7.
- Step 5: Sort the list by ascending x_{ik} associated with fuel types.
- Step 6: Select the next fuel in the list. Delete this fuel from the list and check the total power supply against load demand.
- Step 7: Repeat Step 5 until the total power supply is sufficient to meet the load demand.
- Step 8: Stop the procedure.

After obtaining the preferable fuel type for each generating unit, ALHN is used to finally solve the ED problem similar to section 2.4.

Example 2.10

The test system consists of 10 generating units, each with two or three piecewise quadratic cost functions representing different fuel types as given in Table 2.19. Total demand gradually changes from 2,400 MW to 2,700 MW neglecting power losses.

In the first step, a heuristic search is used to find the most economic fuel type among the available fuel types for each unit. The values of w_1 and w_2 are equally set to 0.5 for the load demand of 2,400 MW, 0.75 for 2,500 MW and 2,600 MW, and 0.8 for 2,700 MW. The values of M_{ik} , λ_{ik}^{avg} and x_{ik} for the different fuels used in the units and the fuel type for each unit selected to supply the load demand of 2,400 MW are shown in Table 2.20. The priority indices for the other cases are calculated similarly but with different weight factors.

In the last step, ALHN is applied to solve the ED problem with the fuel type determined for each unit with parameters of $\sigma = 10^6$, $\alpha_p = 3 \cdot 10^{-6}$, $\alpha_\lambda = 5 \cdot 10^{-4}$, $\beta = 0.001$ for each load demand. The results obtained for the system with different load demands including power generation and fuel type of each unit and total costs are given in Table 2.21.

2.10 COMBINED HEAT AND POWER ECONOMIC DISPATCH

Combined heat and power generation (co-generation) is a promising and widely proven technology providing not only power but also heat to customers. For most co-generation units, there is a mutual dependency between heat and power, i.e.,

Table 2.19 Characteristics of the 10-unit system in example 2.10.

Unit	$P_{i,min}$ P_{i1} P_{i2} $P_{i,max}$ F_1 F_2 F_3	Fuel type (F)	a_i (\$/h)	b_i (\$/MWh)	c_i (\$/MW ² h)
1	100 196 250	1	0.2697E2	-0.3975E0	0.2176E-2
	1 2	2	0.2113E2	-0.3059E0	0.1861E-2
2	50 114 157 230	1	0.1184E3	-0.1269E1	0.4194E-2
	2 3 1	2	0.1865E1	-0.3988E-1	0.1138E-2
		3	0.1365E2	-0.1980E0	0.1620E-2
3	200 332 388 500	1	0.3979E2	-0.3116E0	0.1457E-2
	1 3 2	2	-0.5914E2	0.4864E0	0.1176E-4
		3	-0.2876E1	0.3389E-1	0.8035E-3
4	190 338 407 490	1	0.1983E1	-0.3114E-1	0.1049E-2
	1 2 3	2	0.5285E2	-0.6348E0	0.2758E-2
		3	0.2668E3	-0.2338E1	0.5935E-2
5	190 338 407 490	1	0.1392E2	-0.8733E-1	0.1066E-2
	1 2 3	2	0.9976E2	-0.5206E0	0.1597E-2
		3	-0.5399E2	0.4462E0	0.1498E-3
6	85 138 200 265	1	0.5285E2	-0.6348E0	0.2758E-2
	2 1 3	2	0.1983E1	-0.3114E-1	0.1049E-2
		3	0.2668E3	-0.2338E1	0.5935E-2
7	200 331 391 500	1	0.1893E2	-0.1325E0	0.1107E-2
	1 2 3	2	0.4377E2	-0.2267E0	0.1165E-2
		3	-0.4335E2	0.3559E0	0.2454E-3
8	99 138 200 265	1	0.1983E1	-0.3114E-1	0.1049E-2
	1 2 3	2	0.5285E2	-0.6348E0	0.2758E-2
		3	0.2678E3	-0.2338E1	0.5935E-2
9	130 213 370 440	1	0.8853E2	-0.5675E0	0.1554E-2
	3 1 3	2	0.1530E2	-0.4514E-1	0.7033E-2
		3	0.1423E2	-0.1817E-1	0.6121E-3
10	200 362 407 490	1	0.1397E2	-0.9938E-1	0.1102E-2
	1 3 2	2	-0.6113E2	0.5084E0	0.4164E-4
		3	0.4671E2	-0.2024E0	0.1137E-2

the heat production capacity depends on power generation and vice versa. Thus, the combined heat and power economic dispatch (CHPED) problem implies new complexities in the integration of co-generation units into the general power system economic dispatch since both power and heat demand must be satisfied. Although the combined heat and power systems are well known, only few research works have been reported in the literature in the area of CHPED problems [54-55].

The objective of the CHPED problem is to minimize the total operation cost of power and heat production while satisfying both power and heat load demands and unit power and heat output limits.

The system has three types of units including pure power, combined power and heat, and pure heat units. The feasible heat-power operation region of a combined power and heat unit is shown in Fig. 2.27, where the boundary curve ABCDEF determines the feasible region. Along the boundary, there is a trade-off between power and heat production. It can be seen that along the curve AB the unit reaches maximum power output. By contrast, the unit reaches maximum heat production along the curves BC and DC.

Table 2.20 Priority index of different fuels in the 10-unit system of example 2.10.

Unit	Fuel (F)	M_{ik} (\$/MWh)	λ_{ik} (\$/MWh)	x_{ik} (\$/MWh)	Selected fuel
1	1	0.1068	0.2466	0.1767	1
	2	0.2039	0.5241	0.3640	
2	1	0.0762	0.1468	0.1115	1
	2	0.1222	0.2410	0.1816	
	3	0.1544	0.3541	0.2543	
3	1	0.2255	0.4635	0.3445	1
	2	0.3152	0.6124	0.4638	
	3	0.3584	0.4968	0.4276	
4	1	0.1099	0.2175	0.1637	3
	2	0.1440	0.2974	0.2207	
	3	0.1894	0.4218	0.3056	
5	1	0.2468	0.4755	0.3612	1
	2	0.3421	0.6692	0.5056	
	3	0.3930	0.5806	0.4868	
6	1	0.1036	0.2028	0.1532	3
	2	0.1440	0.2974	0.2207	
	3	0.1894	0.4218	0.3056	
7	1	0.2327	0.4553	0.3440	1
	2	0.3151	0.6144	0.4648	
	3	0.3679	0.5746	0.4712	
8	1	0.1099	0.2175	0.1637	3
	2	0.1440	0.2974	0.2207	
	3	0.1894	0.4218	0.3056	
9	1	0.1698	0.1918	0.1808	1
	2	0.1892	0.3385	0.2638	
	3	0.2649	0.4776	0.3712	
10	1	0.2600	0.5199	0.3900	1
	2	0.3563	0.6720	0.5141	
	3	0.3908	0.5458	0.4683	

Table 2.21 Solutions for the 10-unit system with different load demands in example 2.10.

Unit no.	Load of 2,400 MW		Load of 2,500 MW		Load of 2,600 MW		Load of 2,700 MW	
	P_{Gi} (MW)	F	P_{Gi} (MW)	F	P_{Gi} (MW)	F	P_{Gi} (MW)	Fuel
1	189.7397	1	206.5188	2	216.5441	2	218.2502	2
2	202.3427	1	206.4573	1	210.9057	1	211.6627	1
3	253.8954	1	265.7392	1	278.5441	1	280.7230	1
4	233.0456	3	235.9531	3	239.0967	3	239.6316	3
5	241.8299	1	258.0178	1	275.5195	1	278.4975	1
6	233.0456	3	235.9531	3	239.0967	3	239.6316	3
7	253.2752	1	268.8636	1	285.7170	1	288.5847	1
8	233.0456	3	235.9531	3	239.0967	3	239.6316	3
9	320.3831	1	331.4876	1	343.4932	1	428.5203	3
10	239.3973	1	255.0564	1	271.9863	1	274.8671	1
Total cost (\$)	481.273		536.239		574.381		623.809	

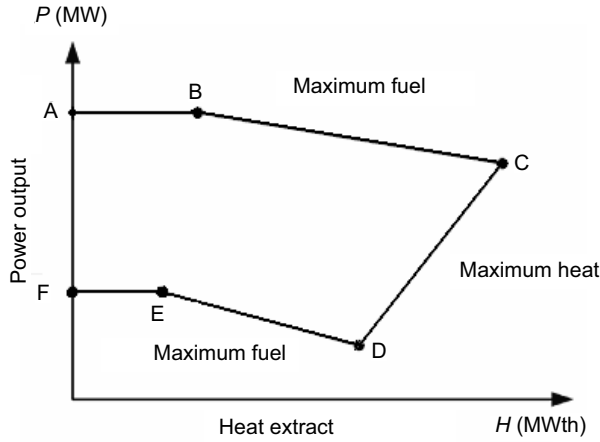


Fig. 2.27 Typical heat-power feasible region for co-generation units.

Mathematically, the problem is formulated as

$$\text{Min} \left\{ \sum_{i=1}^{NP} F_i(P_{Gi}) + \sum_{j=1}^{NC} F_j(P_{Gj}, H_{Gj}) + \sum_{k=1}^{NH} F_k(H_{Gk}) \right\} \quad (2.181)$$

subject to

Power balance constraint

$$P_D - \sum_{i=1}^{NP} P_{Gi} - \sum_{j=1}^{NC} P_{Gj} = 0 \quad (2.182)$$

Heat balance constraint

$$H_L - \sum_{j=1}^{NC} H_{Gj} - \sum_{k=1}^{NH} H_{Gk} = 0 \quad (2.183)$$

Generation limit constraints

$$P_{Gi}^{\min} \leq P_{Gi} \leq P_{Gi}^{\max} \quad (2.184)$$

$$P_{Gj}^{\min}(H_{Gj}) \leq P_{Gj} \leq P_{Gi}^{\max}(H_{Gj}) \quad (2.185)$$

$$H_{Gj}^{\min}(P_{Gj}) \leq H_{Gj} \leq H_{Gj}^{\max}(P_{Gj}) \quad (2.186)$$

$$H_{Gk}^{\min} \leq H_{Gk} \leq H_{Gk}^{\max} \quad (2.187)$$

where $F_i(P_{Gi})$, $F_j(P_{Gj}, H_{Gj})$ and $F_k(H_{Gk})$ are convex functions and $F_i(P_{Gi})$ cost function of pure power generating unit i ,

- $F_j(P_{Gj}, H_{Gj})$ cost function of co-generating unit j ,
 $F_k(H_{Gk})$ cost function of pure heat producing unit k ,
 H_L system heat demand,
 H_{Gj} heat production of co-generation unit j ,
 H_{Gk} heat production of pure heat production unit k ,
 NC number of co-generation units,
 NH number of pure heat production units,
 NP number of pure power generating units,
 P_L system power demand, in MW,
 P_{Gi} output power of pure power generating unit i ,
 P_{Gj} output power of co-generation unit j ,
 $P_{Gi}^{max}, P_{Gi}^{min}$ maximum and minimum output of pure power unit i ,
 $H_{Gk}^{max}, H_{Gk}^{min}$ maximum and minimum output of pure heat production unit k ,
 $H_{Gj}^{max}(P_{Gj}), H_{Gj}^{min}(P_{Gj})$ maximum and minimum heat output of co-generation unit j , a function of output power,
 $P_{Gj}^{max}(H_{Gj}), P_{Gj}^{min}(H_{Gj})$ maximum and minimum output power of co-generation unit j , a function of heat production.

For implementation in ALHN, the augmented Lagrangian function is formulated first:

$$\begin{aligned}
 L = & \sum_{i=1}^{NP} F_i(P_{Gi}) + \sum_{j=1}^{NC} F_j(P_{Gj}, H_{Gj}) + \sum_{k=1}^{NH} F_k(H_{Gk}) \\
 & + \lambda_p \left(P_D - \sum_{i=1}^{NP} P_{Gi} - \sum_{j=1}^{NC} P_{Gj} \right) + \frac{1}{2} \beta_p \left(P_D - \sum_{i=1}^{NP} P_{Gi} - \sum_{j=1}^{NC} P_{Gj} \right)^2 \\
 & + \lambda_h \left(H_L - \sum_{j=1}^{NC} H_{Gj} - \sum_{k=1}^{NH} H_{Gk} \right) + \frac{1}{2} \beta_h \left(H_L - \sum_{j=1}^{NC} H_{Gj} - \sum_{k=1}^{NH} H_{Gk} \right)^2
 \end{aligned} \tag{2.188}$$

where

- β_p, β_h penalty factors for power and heat demand balances, respectively,
 λ_p, λ_h Lagrange multipliers for power and heat demand balances, respectively.

There will be $NP+2*NC+NH$ continuous neurons and two multiplier neurons required in ALHN. The energy function of ALHN is defined as follows:

$$\begin{aligned}
E = & \sum_{i=1}^{NP} F_i(V_{i,p}) + \sum_{j=1}^{NC} F_j(V_{j,p}, V_{j,h}) + \sum_{k=1}^{NH} F_k(V_{k,h}) \\
& + V_{\lambda p} \left(P_D - \sum_{i=1}^{NP} V_{i,p} - \sum_{j=1}^{NC} V_{j,p} \right) + \frac{1}{2} \beta_p \left(P_D - \sum_{i=1}^{NP} V_{i,p} - \sum_{j=1}^{NC} V_{j,p} \right)^2 \\
& + V_{\lambda h} \left(H_L - \sum_{j=1}^{NC} V_{j,h} - \sum_{k=1}^{NH} V_{k,h} \right) + \frac{1}{2} \beta_h \left(H_L - \sum_{j=1}^{NC} V_{j,h} - \sum_{k=1}^{NH} V_{k,h} \right)^2 \\
& + \sum_{i=1}^{NP} \int_0^{V_{i,p}} g^{-1}(V) dV + \sum_{j=1}^{NC} \int_0^{V_{j,p}} g^{-1}(V) dV + \sum_{j=1}^{NC} \int_0^{V_{j,h}} g^{-1}(V) dV + \sum_{k=1}^{NH} \int_0^{V_{k,h}} g^{-1}(V) dV
\end{aligned} \tag{2.189}$$

where

- $V_{\lambda h}$ output of multiplier neuron representing λ_h ,
- $V_{\lambda p}$ output of multiplier neuron representing λ_p ,
- $V_{j,h}$ output of continuous neuron $h(j)$ representing H_{Gj} ,
- $V_{k,h}$ output of continuous neuron $h(k)$ representing H_{Gk} ,
- $V_{i,p}$ output of continuous neuron $p(i)$ representing P_{Gi} ,
- $V_{j,p}$ output of continuous neuron $p(j)$ representing P_{Gj} .

The dynamics for updating neuron inputs are derived as follows:

$$\begin{aligned}
\frac{dU_{i,p}}{dt} &= -\frac{\partial E}{\partial V_{i,p}} \\
&= -\left\{ \frac{\partial F_i(V_{i,p})}{\partial V_{i,p}} - \left[V_{\lambda p} + \beta_p \left(P_D - \sum_{i=1}^{NP} V_{i,p} - \sum_{j=1}^{NC} V_{j,p} \right) \right] + U_{i,p} \right\}
\end{aligned} \tag{2.190}$$

$$\begin{aligned}
\frac{dU_{j,p}}{dt} &= -\frac{\partial E}{\partial V_{j,p}} \\
&= -\left\{ \frac{\partial F_j(V_{j,p}, V_{j,h})}{\partial V_{j,p}} - \left[V_{\lambda p} + \beta_p \left(P_D - \sum_{i=1}^{NP} V_{i,p} - \sum_{j=1}^{NC} V_{j,p} \right) \right] + U_{j,p} \right\}
\end{aligned} \tag{2.191}$$

$$\begin{aligned}
\frac{dU_{j,h}}{dt} &= -\frac{\partial E}{\partial V_{j,h}} \\
&= -\left\{ \frac{\partial F_j(V_{j,p}, V_{j,h})}{\partial V_{j,h}} - \left[V_{\lambda h} + \beta_h \left(H_L - \sum_{j=1}^{NC} V_{j,h} - \sum_{k=1}^{NH} V_{k,h} \right) \right] + U_{j,h} \right\}
\end{aligned} \tag{2.192}$$

$$\frac{dU_{k,h}}{dt} = -\frac{\partial E}{\partial V_{k,h}} \quad (2.193)$$

$$= -\left\{ \frac{\partial F_k(V_{k,h})}{\partial V_{k,h}} - \left[V_{\lambda h} + \beta_h \left(H_L - \sum_{j=1}^{NC} V_{j,h} - \sum_{k=1}^{NH} V_{k,h} \right) \right] + U_{k,h} \right\}$$

$$\frac{dU_{\lambda p}}{dt} = +\frac{\partial E}{\partial V_{\lambda p}} = P_L - \sum_{i=1}^{NP} V_{i,p} - \sum_{j=1}^{NC} V_{j,p} \quad (2.194)$$

$$\frac{dU_{\lambda h}}{dt} = +\frac{\partial E}{\partial V_{\lambda h}} = H_L - \sum_{j=1}^{NC} V_{j,h} - \sum_{k=1}^{NH} V_{k,h} \quad (2.195)$$

where

$U_{\lambda,h}$ input of multiplier neuron with corresponding output $U_{\lambda,h}$,

$V_{\lambda,p}$ input of multiplier neuron with corresponding output $V_{\lambda,p}$,

$V_{j,h}$ input of continuous neuron $h(j)$ with corresponding output $V_{j,h}$,

$V_{k,h}$ input of continuous neuron $h(k)$ with corresponding output $V_{k,h}$,

$V_{i,p}$ input of continuous neuron $p(i)$ with corresponding output $V_{i,p}$,

$V_{j,p}$ input of continuous neuron $p(j)$ with corresponding output $V_{j,p}$.

Inputs of neurons are updated similarly to ALHN as applied to other ED problems. Outputs of neurons are calculated by:

$$V_{i,p} = g_c(U_{i,p}) = (P_{Gi}^{\max} - P_{Gi}^{\min}) \left(\frac{1 + \tanh(\sigma U_{i,p})}{2} \right) + P_{Gi}^{\min} \quad (2.196)$$

$$\begin{aligned} V_{j,p} &= g_c(U_{j,p}) \\ &= [P_{Gj}^{\max}(H_{Gj}) - P_{Gj}^{\min}(H_{Gj})] \left(\frac{1 + \tanh(\sigma U_{j,p})}{2} \right) + P_{Gj}^{\min}(H_{Gj}) \end{aligned} \quad (2.197)$$

$$\begin{aligned} V_{j,h} &= g_c(U_{j,h}) \\ &= [H_{Gj}^{\max}(P_{Gj}) - H_{Gj}^{\min}(P_{Gj})] \left(\frac{1 + \tanh(\sigma U_{j,h})}{2} \right) + H_{Gj}^{\min}(P_{Gj}) \end{aligned} \quad (2.198)$$

$$V_{k,h} = g_c(U_{k,h}) = (H_{Gk}^{\max} - H_{Gk}^{\min}) \left(\frac{1 + \tanh(\sigma U_{k,h})}{2} \right) + H_{Gk}^{\min} \quad (2.199)$$

As shown in Fig. 2.27, maximum and minimum output power and maximum and minimum heat production of co-generation units are determined as follows:

$$P_{Gj}^{\max}(H_{Gj}) = \min \left\{ P_{Gj}(H_{Gj})|_{AB}, P_{Gj}(H_{Gj})|_{BC} \right\} \quad (2.200)$$

$$P_{Gj}^{\min}(H_{Gj}) = \max \left\{ P_{Gj}(H_{Gj})|_{CD}, P_{Gj}(H_{Gj})|_{DE}, P_{Gj}(H_{Gj})|_{EF} \right\} \quad (2.201)$$

$$H_{Gj}^{\max}(P_{Gj}) = \min \left\{ H_{Gj}(P_{Gj})|_{BC}, H_{Gj}(P_{Gj})|_{CD} \right\} \quad (2.202)$$

$$H_{Gj}^{\min}(P_{Gj}) = 0 \quad (2.203)$$

The initialization of ALHN in this problem is similar to that in the normal ED problem in Section 2.4.

Example 2.11

A system having one pure power generation unit, two co-generation units and one pure heat production unit have to cover a load demand of 200 MW power and 115 MWth heat. The data of system are described below:

$$F_1(P_{G1}) = 50P_{G1} \quad (\$/h)$$

$$F_2(P_{G2}, H_{G2}) = 2650 + 14.5P_{G2} + 0.0345P_{G2}^2 + 4.2H_{G2} + 0.03H_{G2}^2 + 0.031P_{G2}H_{G2} \quad (\$/h)$$

$$F_3(P_{G3}, H_{G3}) = 1250 + 36P_{G3} + 0.0435P_{G3}^2 + 0.6H_{G3} + 0.027H_{G3}^2 + 0.011P_{G3}H_{G3} \quad (\$/h)$$

$$F_4(H_{G4}) = 23.4H_{G4} \quad (\$/h)$$

$$0 \leq P_{G1} \leq 150 \text{ MW}$$

$$0 \leq H_{G4} \leq 2695.2 \text{ MWth}$$

Output limits of heat production and power generation for co-generation units 2 and 3 are given in Figs. 2.28 and 2.29.

The key factor in this problem is how to handle the power generation and heat production of co-generation units within their feasible operating zone enclosed by the non-convex functions.

For co-generation unit 2, the feasible operating region is enclosed by four lines intersecting at four points A, B, C and D. Therefore, the maximum and minimum power generation and heat production will combine these lines. The maximum and minimum limits of power and heat production are determined as follows:

$$P_{G2}^{\max}(H_{G2}) = P_{G2}(H_{G2})|_{AB} = \frac{1}{45}(1115 - 8H_{G2})$$

$$\begin{aligned} P_{G2}^{\min}(H_{G2}) &= \max \left\{ P_{G2}(H_{G2})|_{BC}, P_{G2}(H_{G2})|_{CD} \right\} \\ &= \max \left\{ \frac{1}{75.2}(-2886120 + 134H_{G2}), \frac{1}{104.8}(10354.24 - 17.8H_{G2}) \right\} \end{aligned}$$

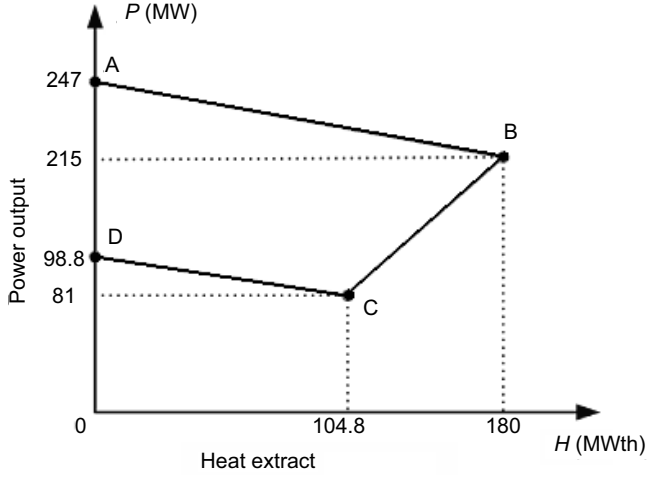


Fig. 2.28 Heat-power feasible operation region for co-generation unit 2 in example 2.11.

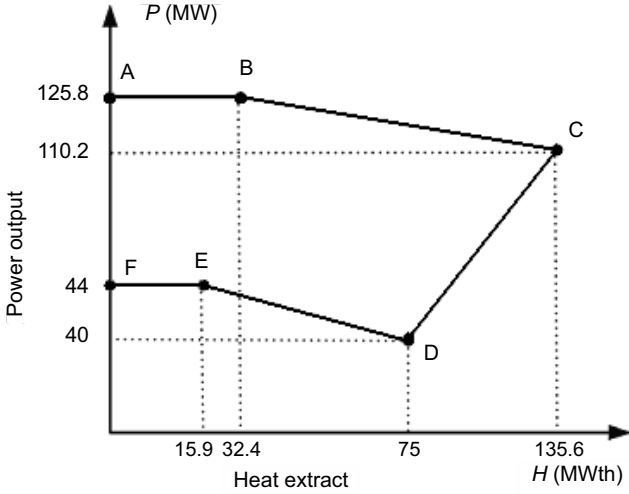


Fig. 2.29 Heat-power feasible operation region for co-generation unit 3 in example 2.11.

$$\begin{aligned}
 H_{G2}^{\max}(P_{G2}) &= \min \left\{ H_{G2}(P_{G2})|_{AB}, H_{G2}(P_{G2})|_{BC} \right\} \\
 &= \min \left\{ \frac{1}{8}(11115 - 45P_{G2}), \frac{1}{134}(7952 - 75.2P_{G2}) \right\}
 \end{aligned}$$

$$H_{G2}^{\min}(P_{G2}) = 0$$

The maximum and minimum power and heat generation limits are calculated similarly:

$$P_{G3}^{\max}(H_{G3}) = \min \left\{ P_{G3}(H_{G3})|_{FA}, P_{G3}(H_{G3})|_{AB} \right\}$$

$$= \min \left\{ 125.8, \frac{1}{103.2} (13488 - 15.6H_{G3}) \right\}$$

$$P_{G3}^{\min}(H_{G3}) = \max \left\{ P_{G3}(H_{G3})|_{BC}, P_{G3}(H_{G3})|_{CD}, P_{G3}(H_{G3})|_{DE} \right\}$$

$$= \max \left\{ \frac{1}{60.6} (-2841 + 70.2H_{G3}), \frac{1}{59.1} (2664 - 4H_{G3}), 44 \right\}$$

$$H_{G3}^{\max}(P_{G3}) = \min \left\{ H_{G3}(P_{G3})|_{AB}, H_{G3}(P_{G3})|_{BC} \right\}$$

$$= \min \left\{ \frac{1}{15.6} (13488 - 103.2P_{G3}), \frac{1}{70.2} (2841 - 60.6P_{G2}) \right\}$$

$$H_{G3}^{\min}(P_{G3}) = 0$$

The parameters of ALHN are selected as follows: $\sigma = 100$, $\beta_p = 0.1$, $\beta_h = 0.0125$, updating step sizes for neurons associated with power generation and heat production are 0.00015, that for multiplier neurons associated with power balance is 0.01, and for heat balance 0.0015.

The obtained solution is: $P_1 = 0$ MW, $P_2 = 160$ MW, $H_2 = 40$ MWth, $P_3 = 40$ MW, $H_3 = 75$ MW, and $H_4 = 0$ MWth with the total cost of \$ 9,257.10. This solution is feasible since it satisfies the constraints and the operating points of co-generation units 2 and 3 are within their feasible operating zones.

2.11 HYDROTHERMAL ECONOMIC DISPATCH

Power systems usually include thermal and hydro units. Therefore, the ED problem with thermal and hydro units is usually more complicated than the ED problem with only thermal units. Hydrothermal ED is the allocation of all online thermal and hydro units so as to minimize the total generation cost of thermal units while satisfying power balance, generator operating and water availability constraints over a given period of time [56-58].

The hydrothermal ED problem with NG thermal units and NH hydro units scheduled in M sub-intervals is formulated as follows:

$$\text{Min } F = \sum_{k=1}^M \sum_{i=1}^{NG} t_k \left[a_i + b_i P_{Gi}^k + c_i (P_{Gi}^k)^2 \right] \quad (2.204)$$

subject to

Power balance constraint

$$\sum_{i=1}^{NG} P_{Gi}^k + \sum_{h=1}^{NH} P_{Gh}^k - P_{loss}^k - P_D^k = 0 \quad (2.205)$$

$$P_{loss}^k = \sum_{i=1}^{NG+NH} \sum_{j=1}^{NG+NH} P_{Gi}^k B_{ij} P_{Gj}^k + \sum_{i=1}^{NG+NH} B_{i0} P_{Gi}^k + B_{00} \quad (2.206)$$

Continuity of reservoir head constraints

$$d_h^k = d_h^{k-1} + \frac{t_k}{f_h} (r_h^k - q_h^k) \quad (2.207)$$

$$q_h^k = \Phi_h(P_{Gh}^k) \times \Psi_h(d_h^k) \quad (2.208)$$

$$\Phi_h(P_{Gh}^k) = \phi_{0h} + \phi_{1h} P_{Gh}^k + \phi_{2h} (P_{Gh}^k)^2 \quad (2.209)$$

$$\Psi_h(d_h^k) = \psi_{0h} + \psi_{1h} d_h^k + \psi_{2h} (d_h^k)^2 \quad (2.210)$$

Generator operating limits

$$P_{Gi}^{\min} \leq P_{Gi}^k \leq P_{Gi}^{\max} \quad (2.211)$$

$$P_{Gh}^{\min} \leq P_{Gh}^k \leq P_{Gh}^{\max} \quad (2.212)$$

where

P_{Gh}^k generation output of hydro unit h during subinterval k ,

NH number of hydro units,

d_h^k the height of the reservoir head of hydro unit h in interval k ,

q_h^k rate of water flow from hydro unit h in interval k ,

r_h^k reservoir inflow for hydro unit h in interval k ,

f_h the surface of the vertical sided tank of hydro unit h ,

$\Phi_h(P_{Gh}^k)$ water flow function of hydro unit h ,

$\Psi_h(d_h^k)$ head variation function of hydro unit h ,

$\phi_{0h}, \phi_{1h}, \phi_{2h}$ coefficients of water rate flow,

$\psi_{0h}, \psi_{1h}, \psi_{2h}$ coefficients of reservoir head variation function,

$P_{Gh}^{\max}, P_{Gh}^{\min}$ maximum and minimum generation output of hydro unit h .

The other symbols are similar to the ones in section 2.6.

In terms of water availability, the equation (2.207) can be rewritten as follows:

$$\sum_{k=1}^M t_k (q_h^k - r_h^k) = (d_h^0 - d_h^M) f_h = W_h \quad (2.213)$$

where W_h is the volume of water available for generation by hydro unit h during the scheduling period, and d_h^0 and d_h^M at the beginning and final schedule are given.

Neglecting the reservoir head variation of a hydro unit leads to the fixed-head hydrothermal ED problem. In this case, the water discharge of a hydro unit in (2.208) is purely a function of its generation. The Lagrange function L is formulated as follows:

$$\begin{aligned}
 L = & \sum_{k=1}^M \sum_{i=1}^{NG} t_k \left[a_i + b_i P_{Gi}^k + c_i (P_{Gi}^k)^2 \right] \\
 & + \sum_{k=1}^M \left[\lambda^k \left(P_D^k + P_{loss}^k - \sum_{i=1}^{NG} P_{Gi}^k - \sum_{h=1}^{NH} P_{Gh}^k \right) \right. \\
 & \left. + \frac{1}{2} \beta_{\lambda}^k \left(P_D^k + P_{loss}^k - \sum_{i=1}^{NG} P_{Gi}^k - \sum_{h=1}^{NH} P_{Gh}^k \right)^2 \right] \\
 & + \sum_{h=1}^{NH} \left\{ \gamma_h \left[\sum_{k=1}^M t_k (q_h^k - r_h^k) - W_h \right] + \frac{1}{2} \beta_{h,\gamma} \left[\sum_{k=1}^M t_k (q_h^k - r_h^k) - W_h \right]^2 \right\}
 \end{aligned} \tag{2.214}$$

where

λ_k, γ_h Lagrangian multipliers associated with power balance and water discharge, respectively,

$\beta_{\lambda}^k, \beta_{h,\gamma}$ penalty factors associated with power balance and water discharge, respectively.

To represent the problem in a Hopfield Lagrange model, $(NG+NH) \times M$ continuous neurons and $NH+M$ multiplier neurons are required. The energy function E for the neural network is formulated as follows:

$$E = \left\{ \begin{aligned} & \sum_{k=1}^M \sum_{i=1}^{NG} t_k \left[a_i + b_i V_{i,p}^k + c_i (V_{i,p}^k)^2 \right] \\ & + \sum_{k=1}^M \left[V_{\lambda}^k \left(P_D^k + P_{loss}^k - \sum_{i=1}^{NG} V_{i,p}^k - \sum_{h=1}^{NH} V_{h,p}^k \right) \right. \\ & \left. + \frac{1}{2} \beta_{\lambda}^k \left(P_D^k + P_{loss}^k - \sum_{i=1}^{NG} V_{i,p}^k - \sum_{h=1}^{NH} V_{h,p}^k \right)^2 \right] \\ & + \sum_{h=1}^{NH} \left\{ V_{h,\gamma} \left[\sum_{k=1}^M t_k (q_h^k - r_h^k) - W_h \right] + \frac{1}{2} \beta_{h,\gamma} \left[\sum_{k=1}^M t_k (q_h^k - r_h^k) - W_h \right]^2 \right\} \\ & + \sum_{k=1}^M \left(\sum_{i=1}^{NG} \int_0^{V_{i,p}^k} g^{-1}(V) dV + \sum_{h=1}^{NH} \int_0^{V_{h,p}^k} g^{-1}(V) dV \right) \end{aligned} \right\} \tag{2.215}$$

where

$V_{i,p}^k$ output of continuous neuron $p(i,k)$ representing P_{Gi}^k ,
 $V_{h,p}^k$ output of continuous neuron $p(h,k)$ representing P_{Gh}^k ,
 $V_{\lambda}^k, V_{h,\gamma}$ outputs of the multiplier neurons associated with power balance and water constraint, respectively.

The dynamics of neurons are derived as:

$$\begin{aligned} \frac{dU_{i,p}^k}{dt} &= -\frac{\partial E}{\partial V_{i,p}^k} \\ &= -\left\{ t_k \left(b_i + 2c_i V_{i,p}^k \right) \right. \\ &\quad \left. + \left[V_{\lambda}^k + \beta_{\lambda}^k \left(P_D^k + P_{loss}^k - \sum_{i=1}^{NG} V_{i,p}^k - \sum_{h=1}^{NH} V_{h,p}^k \right) \right] \left(\frac{\partial P_{loss}^k}{\partial V_{i,p}^k} - 1 \right) + U_{i,p} \right\} \end{aligned} \quad (2.216)$$

$$\begin{aligned} \frac{dU_{h,p}^k}{dt} &= -\frac{\partial E}{\partial V_{h,p}^k} \\ &= -\left\{ \left[V_{\lambda}^k + \beta_{\lambda}^k \left(P_D^k + P_{loss}^k - \sum_{i=1}^{NG} V_{i,p}^k - \sum_{h=1}^{NH} V_{h,p}^k \right) \right] \left(\frac{\partial P_{loss}^k}{\partial V_{h,p}^k} - 1 \right) \right. \\ &\quad \left. + V_{h,\gamma} t_k \frac{\partial q_h^k}{\partial V_{h,p}^k} + U_{h,p} \right\} \end{aligned} \quad (2.217)$$

$$\frac{dU_{\lambda}^k}{dt} = +\frac{\partial E}{\partial V_{\lambda}^k} = P_D^k + P_{loss}^k - \sum_{i=1}^{NG} V_{i,p}^k - \sum_{h=1}^{NH} V_{h,p}^k \quad (2.218)$$

$$\frac{dU_{h,\gamma}}{dt} = +\frac{\partial E}{\partial V_{h,\gamma}} = \sum_{k=1}^M t_k (q_h^k - r_h^k) - W_h \quad (2.219)$$

where

$$\frac{\partial P_{loss}^k}{\partial V_{i,p}^k} = 2 \sum_{i=1}^{NG} BTT_{ij} V_{j,p}^k + 2 \sum_{h=1}^{NH} BTH_{ih} V_{h,p}^k + BTT_{i0} \quad (2.220)$$

$$\frac{\partial P_{loss}^k}{\partial V_{h,p}^k} = 2 \sum_{i=1}^{NG} BHT_{hi} V_{i,p}^k + 2 \sum_{j=1}^{NH} BHH_{hj} V_{j,p}^k + BHH_{h0} \quad (2.221)$$

$$\frac{\partial q_h^k}{\partial V_{h,p}^k} = \Psi(d_h^k) \frac{d\Phi(V_{h,p}^k)}{dV_{h,p}^k} = \Psi(d_h^k) (\phi_{1h} + 2\phi_{2h} V_{h,p}^k) \quad (2.222)$$

$$B_{ij} = \begin{vmatrix} BTT_{ij} & BHT_{ih} \\ BHT_{hi} & BHH_{hj} \end{vmatrix} \quad (2.223)$$

$$B_{0i} = |BTT_{0i} \ BHH_{0h}| \quad (2.224)$$

where

$$\begin{aligned} U_{i,p}^k, U_{h,p}^k & \text{ inputs of the neurons } p(i,k) \text{ and } p(h,k) \text{ respectively,} \\ U_{\lambda}^k, U_{h,\gamma} & \text{ inputs of the multiplier neurons,} \\ BTT_{ij} & \text{ loss coefficients related to thermal plants,} \\ BHH_{hj} & \text{ loss coefficients related to hydro plants,} \\ BTH_{ih}, BHT_{hi} & \text{ loss coefficients between thermal and hydro plants, with } BTH_{ih} \\ & = BHT_{hi}^T. \end{aligned}$$

The algorithm for updating inputs of neurons at step n is as follows:

$$U_{i,p}^{k(n)} = U_{i,p}^{k(n-1)} - \alpha_i \frac{\partial E}{\partial V_{i,p}^k} \quad (2.225)$$

$$U_{h,p}^{k(n)} = U_{h,p}^{k(n-1)} - \alpha_h \frac{\partial E}{\partial V_{h,p}^k} \quad (2.226)$$

$$U_{\lambda}^{k(n)} = U_{\lambda}^{k(n-1)} - \alpha_{\lambda} \frac{\partial E}{\partial V_{\lambda}^k} \quad (2.227)$$

$$U_{h,\gamma}^{(n)} = U_{h,\gamma}^{(n-1)} - \alpha_{\gamma} \frac{\partial E}{\partial V_{h,\gamma}} \quad (2.228)$$

where α_i , α_h , α_{λ} and α_{γ} are updating step sizes for neurons.

The outputs of neurons are calculated as follows:

$$V_{i,p}^k = (P_{Gi}^{\max} - P_{Gi}^{\min}) \left(\frac{1 + \tanh(\sigma U_{i,p}^k)}{2} \right) + P_{Gi}^{\min} \quad (2.229)$$

$$V_{h,p}^k = (P_{Gh}^{\max} - P_{Gh}^{\min}) \left(\frac{1 + \tanh(\sigma U_{h,p}^k)}{2} \right) + P_{Gh}^{\min} \quad (2.230)$$

$$V_{\lambda}^k = U_{\lambda}^k \quad (2.231)$$

$$V_{h,\gamma} = U_{h,\gamma} \quad (2.232)$$

The inputs of continuous neurons are initialized by:

$$V_{i,p}^{k(0)} = (P_{Gi}^{\max} + P_{Gi}^{\min})/2 \quad (2.233)$$

$$V_{h,p}^{k(0)} = (P_{Gh}^{\max} + P_{Gh}^{\min})/2 \quad (2.234)$$

and the initial multiplier neurons are:

$$V_{\lambda}^{k(0)} = \frac{1}{NG} \sum_{i=1}^{NG} \frac{t_k (b_i + 2c_i V_{i,p}^k)}{1 - \frac{\partial P_{loss}^k}{\partial V_{i,p}^k}} \quad (2.235)$$

$$V_{h,\gamma}^{(0)} = \frac{1}{NH} \sum_{h=1}^{NH} V_{\lambda}^{k(0)} \frac{1 - \partial P_{loss}^k / \partial V_{h,p}^k}{t_k \partial q_h^k / \partial V_{h,p}^k} \quad (2.236)$$

where $V_{i,p}^{k(0)}$, $V_{h,p}^{k(0)}$, $V_{\lambda}^{k(0)}$ and $V_{h,\gamma}^{(0)}$ are initial values of $V_{i,p}^k$, $V_{h,p}^k$, V_{λ}^k and $V_{h,\gamma}$, respectively. Note that the initial value of the height of the reservoir head depending on water discharge at each subinterval is calculated as follows:

$$d_h^{k(0)} = d_h^0 - k \frac{d_h^0 - d_h^M}{M} \quad (2.237)$$

The stopping criteria of ALHN in this problem are similar to other ED problems including constraint errors and iterative errors of neurons.

Example 2.12

In this example, fixed-head hydro power plants are considered. This means the effect of reservoir head variation is neglected. As mentioned above, the water flow rate in this case becomes a mere function of power generation. Suppose that a system includes two thermal plants and two hydro plants scheduled over a period of 48 hours divided into four subintervals with 12 hours each. The load demand for the four subintervals is 1200, 1500, 1400 and 1700 MW. The unit characteristics for units 1 and 2 defined as thermal and units 3 and 4 as hydro are given as follows:

$$F_1(P_{G1}^k) = 380 + 6.75P_{G1}^k + 0.00225(P_{G1}^k)^2 \quad \$/h$$

$$47.5 \text{ MW} \leq P_{G1}^k \leq 450 \text{ MW}$$

$$F_2(P_{G2}^k) = 600 + 5.28P_{G2}^k + 0.0055(P_{G2}^k)^2 \quad \$/h$$

$$100 \text{ MW} \leq P_{G2}^k \leq 1000 \text{ MW}$$

$$q_3^k(P_{G3}^k) = 260 + 8.5P_{G3}^k + 0.00986(P_{G3}^k)^2 \quad 10^3 \text{ m}^3/h$$

$$0 \text{ MW} \leq P_{G3}^k \leq 250 \text{ MW}$$

$$q_4^k(P_{G4}^k) = 250 + 9.8P_{G4}^k + 0.0114(P_{G4}^k)^2 \quad 10^3 \text{ m}^3/h$$

$$0 \text{ MW} \leq P_{G4}^k \leq 500 \text{ MW}$$

The allowable volumes of water for units 3 and 4 operating over the whole period are $W_3 = 125,000,000 \text{ m}^3$; $W_4 = 286,000,000 \text{ m}^3$.

The water inflows of reservoirs during the schedule period are supposed to be zeros. The transmission loss coefficient matrix is shown below, per MW:

$$B_{ij} = 10^{-5} \times \begin{bmatrix} 4.0 & 1.0 & 1.5 & 1.5 \\ 1.0 & 3.5 & 1.0 & 1.2 \\ 1.5 & 1.0 & 3.9 & 2.0 \\ 1.5 & 1.2 & 2.0 & 4.9 \end{bmatrix}$$

The parameters of ALHN tuned in this case are $\sigma = 100$, $\beta = 10^{-6}$, $\alpha_i = \alpha_h = 0.0003$, $\alpha_k = 0.0225$ and $\alpha_v = 1.75 \times 10^{-6}$. ALHN found the optimal solution with a total cost of \$ 375,933.644. The obtained solution for the problem is detailed in Table 2.22.

Table 2.22 Solution of the fixed-head problem in example 2.12.

Duration t_k (h)	Load P_L^k (MW)	Loss P_{loss}^k (MW)	Thermal 1 P_1^k (MW)	Thermal 2 P_2^k (MW)	Hydro 1 P_3^k (MW)	Hydro 2 P_4^k (MW)
12	1200	31.4146	441.6994	326.4081	160.7018	302.6053
12	1500	48.9164	450.0000	445.2333	247.0123	406.6708
12	1400	42.5663	450.0000	404.2848	217.3759	370.9057
12	1700	63.6895	450.0000	563.6895	250.0000	500.0000

Example 2.13

The ED problem of a hydrothermal system including reservoir head variation is considered in this example. The hydrothermal system includes two thermal and two hydro plants scheduled over a period of 24 hours, in which units 1 and 2 are thermal and units 3 and 4 are hydro. The unit data are given as follows:

$$F_1(P_{G1}^k) = 25 + 3.2P_{G1}^k + 0.0025(P_{G1}^k)^2 \quad \$/h$$

$$50 \leq P_{G1}^k \leq 500 \text{ MW}$$

$$F_2(P_{G2}^k) = 30 + 3.4P_{G2}^k + 0.0008(P_{G2}^k)^2 \quad \$/h$$

$$150 \leq P_{G2}^k \leq 800 \text{ MW}$$

$$\Phi_3(P_{G3}^k) = 0.1980 + 0.306P_{G3}^k + 0.000216(P_{G3}^k)^2 \quad 10^3 \text{ m}^3/h$$

$$\Psi_3(d_3^k) = 0.90 - 0.0030d_3^k + 0.00001(d_3^k)^2 \quad \text{m}$$

$$0 \leq P_{G3}^k \leq 500 \text{ MW}$$

$$\Phi_4(P_{G4}^k) = 0.9360 + 0.612P_{G4}^k + 0.000360(P_{G4}^k)^2 \quad 10^3 \text{ m}^3/h$$

$$\Psi_4(d_4^k) = 0.95 - 0.0025d_4^k + 0.00002(d_4^k)^2 \quad \text{m}$$

$$0 \leq P_{G4}^k \leq 250 \text{ MW}$$

The surface area of the reservoir for the two hydro plants are 10^7 m^2 and $4 \times 10^6 \text{ m}^2$ with water availability of $2.850 \times 10^6 \text{ m}^3$ and $2.450 \times 10^6 \text{ m}^3$, respectively. The beginning heads of the reservoirs are 30 m and 25 m. Water inflow for the hydro

plants is zero during the schedule period. The load demand during the 24-hour schedule time is given in Table 2.23.

The B-matrix coefficients for the variable-head case are:

$$B_{ij} = 10^{-4} \times \begin{bmatrix} 1.40 & 0.10 & 0.15 & 0.15 \\ 0.15 & 0.60 & 0.10 & 0.13 \\ 0.15 & 0.10 & 0.68 & 0.65 \\ 0.15 & 0.13 & 0.65 & 0.70 \end{bmatrix}$$

ALHN is implemented with parameters of $\sigma = 100$, $\beta = 0.0001$, $\alpha_i = \alpha_h = 0.017$, $\alpha_\lambda = 0.0008$ and $\alpha_\gamma = 0.00017$.

Table 2.23 Load demand in example 2.13.

Hour	1	2	3	4	5	6	7	8
Load demand P_L (MW)	800	700	600	600	600	650	800	1000
Hour	9	10	11	12	13	14	15	16
Load demand P_L (MW)	1330	1350	1450	1500	1300	1350	1350	1370
Hour	17	18	19	20	21	22	23	24
Load demand P_L (MW)	1450	1570	1430	1350	1270	1150	1000	900

Initialization

The outputs of continuous neurons are initialized by $V_{G1}^{k(0)} = 175$ MW, $V_{G2}^{k(0)} = 475$ MW, $V_{G3}^{k(0)} = 250$ MW, and $V_{G4}^{k(0)} = 125$ MW for the entire schedule time horizon. Their corresponding inputs are zero. The power loss is $P_{loss}^k = 34.7813$ MW for the entire schedule time horizon. Therefore, the initial error of power balance constraint will be calculated accordingly.

The initial values of reservoir heads and the corresponding initial values of water flow rates are given in Table 2.24. The corresponding error of water discharge constraint is $\Delta W_h = [-911.3008, 642.5408]^T 10^3 \text{ m}^3$.

The initial value of the multiplier neuron associated with power balance constraint is $V_\lambda^{k(0)} = 4.4238$ \$/MWh and valid for the entire schedule time horizon. The initial value of the multiplier neuron associated with water discharge is $V_{h,\gamma}^{(0)} = [11.1561, 3.7878]^T$.

1st iteration

The dynamics of neurons are calculated in the first iteration given in Table 2.25 and $\partial E / \partial V_{h,\gamma} = \Delta W_h = [-911.3008, 642.5408]^T 10^3 \text{ m}^3$.

The new inputs and outputs of neurons are given in Table 2.26, and the input and output of multiplier neurons associated with water discharge are $V_{h,\gamma} = U_{h,\gamma} = [11.0011, 3.8970]^T$.

The power balance constraint error is given in Table 2.27. The water discharge error is $\Delta W_h = [-733.2243, 322.2474]^T 10^3 \text{ m}^3$. The maximum error in this iteration is 733.2243. Therefore, the values of reservoir heads and water flow rates are updated for the next iteration.

Table 2.24 Initial values of reservoir heads and flow rates in example 2.13.

Hour	1	2	3	4	5	6	7	8
$d_3^k (10^{-1} \text{m})$	299.88	299.76	299.64	299.52	299.41	299.29	299.17	299.05
$d_4^k (10^{-1} \text{m})$	249.74	249.49	249.23	248.98	248.72	248.47	248.21	247.96
$q_3^k (10^3 \text{ m}^3/\text{h})$	81.146	81.114	81.082	81.050	81.018	80.986	80.954	80.922
$q_4^k (10^3 \text{ m}^3/\text{h})$	130.66	130.50	130.35	130.19	130.03	129.87	129.71	129.56
Hour	9	10	11	12	13	14	15	16
$d_3^k (10^{-1} \text{m})$	298.93	298.81	298.69	298.57	298.46	298.34	298.22	298.10
$d_4^k (10^{-1} \text{m})$	247.70	247.45	247.19	246.94	246.68	246.43	246.17	245.92
$q_3^k (10^3 \text{ m}^3/\text{h})$	80.890	80.858	80.826	80.794	80.763	80.731	80.699	80.667
$q_4^k (10^3 \text{ m}^3/\text{h})$	129.40	129.24	129.09	128.93	128.77	128.62	128.46	128.31
Hour	17	18	19	20	21	22	23	24
$d_3^k (10^{-1} \text{m})$	297.98	297.86	297.74	297.63	297.51	297.39	297.27	297.15
$d_4^k (10^{-1} \text{m})$	245.66	245.41	245.15	244.90	244.64	244.39	244.13	243.88
$q_3^k (10^3 \text{ m}^3/\text{h})$	80.636	80.604	80.572	80.541	80.509	80.477	80.446	80.414
$q_4^k (10^3 \text{ m}^3/\text{h})$	128.15	127.99	127.84	127.68	127.53	127.38	127.22	127.07

Table 2.25 Dynamics of neurons for 1st iteration in example 2.13.

Hour	1	2	3	4	5	6	7	8
$\partial E / \partial V_{1,p}^k$	-0.023	-0.013	-0.004	-0.004	-0.004	-0.009	-0.023	-0.041
$\partial E / \partial V_{2,p}^k$	0.058	0.067	0.077	0.077	0.077	0.072	0.058	0.039
$\partial E / \partial V_{3,p}^k$	0.003	0.010	0.018	0.016	0.015	0.009	-0.007	-0.027
$\partial E / \partial V_{4,p}^k$	0.147	0.151	0.155	0.150	0.145	0.135	0.116	0.092
$\partial E / \partial V_{\lambda}^k$	-190.2	-190.2	-390.2	-390.2	-390.2	-340.2	-190.2	9.8
Hour	9	10	11	12	13	14	15	16
$\partial E / \partial V_{1,p}^k$	-0.072	-0.074	-0.083	-0.088	-0.069	-0.074	-0.074	-0.076
$\partial E / \partial V_{2,p}^k$	0.009	0.007	-0.003	-0.007	0.011	0.007	0.007	0.005
$\partial E / \partial V_{3,p}^k$	-0.060	-0.063	-0.074	-0.081	-0.064	-0.070	-0.071	-0.075
$\partial E / \partial V_{4,p}^k$	0.056	0.049	0.035	0.025	0.039	0.029	0.024	0.017
$\partial E / \partial V_{\lambda}^k$	339.8	359.8	459.8	509.8	309.8	359.8	359.8	379.8
Hour	17	18	19	20	21	22	23	24
$\partial E / \partial V_{1,p}^k$	-0.083	-0.094	-0.081	-0.074	-0.066	-0.055	-0.041	-0.032
$\partial E / \partial V_{2,p}^k$	-0.003	-0.014	-0.001	0.007	0.014	0.025	0.039	0.049
$\partial E / \partial V_{3,p}^k$	-0.084	-0.097	-0.085	-0.080	-0.074	-0.064	-0.052	-0.044
$\partial E / \partial V_{4,p}^k$	0.004	-0.012	-0.004	-0.001	0.001	0.007	0.016	0.020
$\partial E / \partial V_{\lambda}^k$	459.8	579.8	439.8	359.8	279.8	159.8	9.8	-90.2

Table 2.26 Inputs and outputs of neurons after the 1st iteration in example 2.13.

Hour	1	2	3	4	5	6	7	8
$U_{1,p}^k$	0.0004	0.0002	0.0001	0.0001	0.0001	0.0001	0.0004	0.0007
$U_{2,p}^k$	-0.0010	-0.0011	-0.0013	-0.0013	-0.0013	-0.0012	-0.0010	-0.0007
$U_{3,p}^k$	-0.0000	-0.0002	-0.0003	-0.0003	-0.0003	-0.0001	0.0001	0.0005
$U_{4,p}^k$	-0.0025	-0.0026	-0.0026	-0.0025	-0.0025	-0.0023	-0.0020	-0.0016
U_{λ}^k	4.2717	4.1917	4.1117	4.1117	4.1117	4.1517	4.2717	4.4317
$V_{1,p}^k$	179.79	177.82	175.84	175.84	175.84	176.83	179.79	183.74
$V_{2,p}^k$	443.05	437.96	432.89	432.89	432.89	435.42	443.05	453.26
$V_{3,p}^k$	248.87	245.59	242.31	243.00	243.69	246.37	253.02	261.64
$V_{4,p}^k$	94.44	93.61	92.78	93.80	94.83	96.80	100.68	105.58
V_{λ}^k	4.2717	4.1917	4.1117	4.1117	4.1117	4.1517	4.2717	4.4317
Hour	9	10	11	12	13	14	15	16
$U_{1,p}^k$	0.0012	0.0013	0.0014	0.0015	0.0012	0.0013	0.0013	0.0013
$U_{2,p}^k$	-0.0001	-0.0001	0.0000	0.0001	-0.0002	-0.0001	-0.0001	-0.0001
$U_{3,p}^k$	0.0010	0.0011	0.0013	0.0014	0.0011	0.0012	0.0012	0.0013
$U_{4,p}^k$	-0.0010	-0.0008	-0.0006	-0.0004	-0.0007	-0.0005	-0.0004	-0.0003
U_{λ}^k	4.6957	4.7117	4.7917	4.8317	4.6717	4.7117	4.7117	4.7277
$V_{1,p}^k$	190.20	190.59	192.53	193.50	189.61	190.59	190.59	190.98
$V_{2,p}^k$	470.21	471.24	476.39	478.96	468.67	471.24	471.24	472.27
$V_{3,p}^k$	275.37	276.83	281.43	284.06	276.91	279.55	280.22	281.68
$V_{4,p}^k$	113.08	114.55	117.60	119.67	116.79	118.86	119.93	121.41
V_{λ}^k	4.6957	4.7117	4.7917	4.8317	4.6717	4.7117	4.7117	4.7277
Hour	17	18	19	20	21	22	23	24
$U_{1,p}^k$	0.0014	0.0016	0.0014	0.0013	0.0011	0.0009	0.0007	0.0005
$U_{2,p}^k$	0.0000	0.0002	0.0000	-0.0001	-0.0002	-0.0004	-0.0007	-0.0008
$U_{3,p}^k$	0.0014	0.0016	0.0015	0.0014	0.0013	0.0011	0.0009	0.0007
$U_{4,p}^k$	-0.0001	0.0002	0.0001	0.0000	-0.0000	-0.0001	-0.0003	-0.0003
U_{λ}^k	4.7917	4.8877	4.7757	4.7117	4.6477	4.5517	4.4317	4.3517
$V_{1,p}^k$	192.53	194.85	192.14	190.59	189.03	186.68	183.74	181.77
$V_{2,p}^k$	476.39	482.56	475.36	471.24	467.13	460.96	453.26	448.15
$V_{3,p}^k$	285.47	290.80	286.04	283.59	281.13	277.10	271.87	268.60
$V_{4,p}^k$	124.06	127.52	125.81	125.30	124.78	123.47	121.57	120.65
V_{λ}^k	4.7917	4.8877	4.7757	4.7117	4.6477	4.5517	4.4317	4.3517

Table 2.27 Power balance error of the 1st iteration in example 2.13.

Hour	1	2	3	4	5	6	7	8
ΔP^k (MW)	30.932	30.222	29.522	29.626	29.730	30.238	31.580	33.385
Hour	9	10	11	12	13	14	15	16
ΔP^k (MW)	36.407	36.708	37.740	38.323	36.621	37.197	37.320	37.626
Hour	17	18	19	20	21	22	23	24
ΔP^k (MW)	38.486	39.728	38.551	37.938	37.330	36.363	35.142	35.142

This process is continued until the maximum error is less than a pre-specified threshold.

Final solution

The final solution is obtained after 160 iterations with an accuracy of 10^{-5} . The obtained total cost is \$ 67,952.42 with a maximum error of 9.78×10^{-6} . The solution of the problem is given in Table 2.28.

Table 2.28 Solution for the hydrothermal ED problem in example 2.13.

Hour	Thermal 1 P_1 (MW)	Thermal 2 P_2 (MW)	Hydro 1 P_3 (MW)	Hydro 2 P_4 (MW)	P_{loss}^k (MW)	Hydro head 1 d_3 (10^{-1} m)	Hydro head 2 d_4 (10^{-1} m)
1	151.2964	363.3024	277.0064	30.7073	22.3124	299.9086	249.9210
2	134.8577	312.7834	256.6154	12.7150	16.9715	299.8250	249.8865
3	117.6827	259.9843	234.7355	0	12.4024	299.7496	249.8828
4	117.6423	259.8645	234.8957	0	12.4026	299.6741	249.8791
5	117.6019	259.7448	235.0560	0	12.4027	299.5985	249.8754
6	126.5721	287.3155	247.1017	3.6013	14.5906	299.5185	249.8631
7	151.0125	362.4633	278.0364	30.7993	22.3115	299.4269	249.7839
8	184.0215	463.7738	319.7904	67.7842	35.3699	299.3190	249.6107
9	239.6645	634.0666	390.1161	130.3426	64.1897	299.1822	249.2700
10	242.8271	643.7473	394.3943	135.2714	66.2401	299.0436	248.9162
11	259.7863	695.5360	416.0642	155.6460	77.0325	298.8958	248.5061
12	268.1385	721.0342	426.8927	166.7058	82.7712	298.7432	248.0653
13	233.3410	614.8344	383.3267	129.6804	61.1825	298.6095	247.7290
14	241.6345	640.1937	394.0605	140.3542	66.2429	298.4713	247.3638
15	241.3506	639.3479	393.9869	141.5589	66.2442	298.3333	246.9958
16	244.4960	648.9767	398.2337	146.6256	68.3320	298.1934	246.6145
17	257.9807	690.1636	415.5150	163.3805	77.0398	298.0462	246.1871
18	278.4961	752.7475	441.6714	188.2893	91.2043	297.8876	245.6898
19	253.8200	677.5182	410.8402	162.6365	74.8149	297.7425	245.2663
20	239.7501	634.5763	393.3821	148.5485	66.2569	297.6050	244.8828
21	225.8092	591.9868	376.0849	134.3901	58.2710	297.4749	244.5387
22	205.2531	529.1044	350.4520	112.5297	47.3392	297.3554	244.2538
23	179.9571	451.5992	318.8485	84.9905	35.3954	297.2486	244.0412
24	163.2444	400.3254	298.0476	66.8487	28.4661	297.1500	243.8750

2.12 OPTIMAL POWER DISPATCH IN A COMPETITIVE ELECTRICITY SUPPLY INDUSTRY

Deregulation of the electric power system is intended to improve the efficiency of the electricity supplier industry. It was pioneered in Latin America since the early 1990s. Earlier, communication and information systems may not have been fast enough to capture the full potential benefits of deregulation. Nowadays, the

technology reaches the point where electricity deregulation of smart grids through automatic meter infrastructure (AMI) can offer better value added services such as demand response, direct load control, integration of renewables etc. Deregulation restructures the electricity supplier industry but may not lead the system towards a higher efficiency in real power systems [59-90].

In deregulated power system, one of the main challenges for the independent system operator (ISO) is to match supply and demand in an optimal and secure manner. This means, the ISO must hold an available aggregate generating supply that is sufficient to meet the required aggregate consumer demand under the given generating and system constraints and be capable of responding quickly to changes in load in real-time.

A large interconnected system with a competitive generation environment is referred to as a power pool, where each generator is a participant in an auction. In the Poolco model, each participant sends the offers (electricity offered prices) to the ISO who has the right to control the interconnected power system. The ISO can be either the transmission company (TransCo) or a separate entity.

The auction method could be either a

- **single sided auction**, which has only one participant on either the seller's side or the buyer's side, or
- **double sided auction**, which has more than one participants on both the seller's side and the buyer's side.

Two types of bid (offers) protocols that are widely used are

- **block bid protocol** which means that the bid consists of the offered MW and its price in each block and
- **linear bid protocol** where the bid is a linear function of price and offered MW.

For generation settlements, there are currently two types including

- **uniform price rule** where each generator participant is paid the same market clearing price (MCP) and
- **discriminatory price rule** where each generator participant is paid according to their own offer.

Block bid protocol and linear bid protocol of the generator offered prices are shown in Figs. 2.30a and 2.30b.

Figure 2.31 shows a deregulated model where the market allocation rule used is the uniform price rule in an hour-ahead market. The auction method is the block bid protocol. In this model, the ISO sorts the offered price in the ascending order to obtain the aggregate supply curve. The equilibrium point or MCP is the intersection of the aggregate supply curve and the required gross demand (total system load plus loss).

Figure 2.32 shows the deregulated model where the auction uses the block bid protocol with the market allocation rule of uniform price rule in an hour-ahead

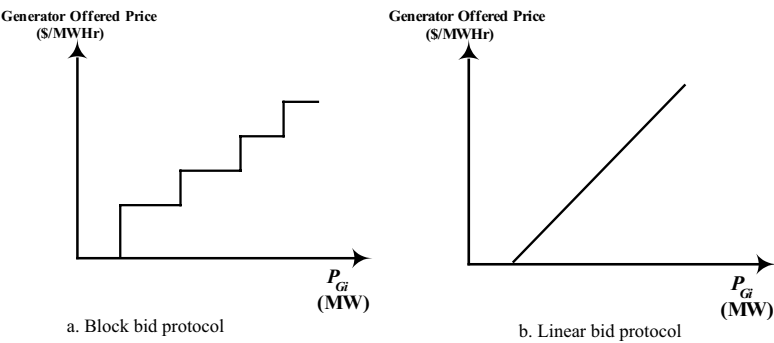


Fig. 2.30 Block bid protocol and linear bid protocol.

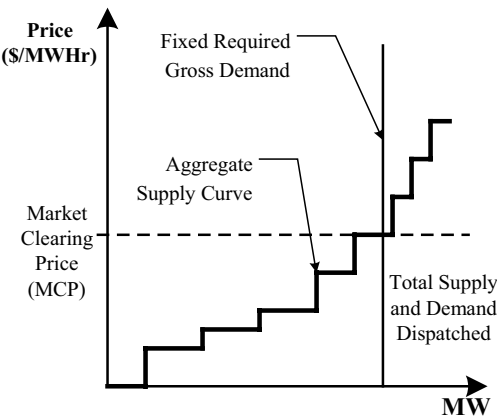


Fig. 2.31 Uniform price rule dispatch model with single side bidding.

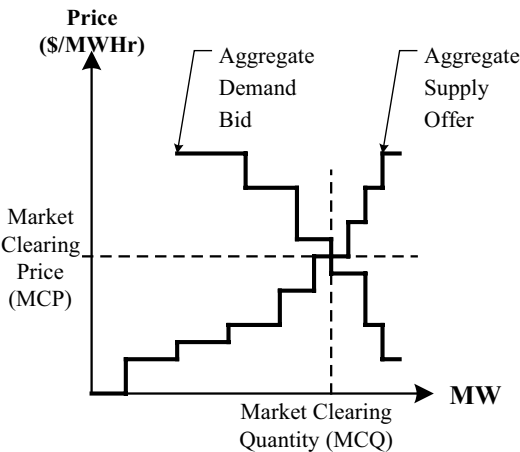


Fig. 2.32 Uniform price rule dispatch model with double side bidding.

market. In this model, the ISO sorts the offered price in the ascending order to obtain the aggregate supply curve and sorts the demand bid in descending order to obtain the aggregate demand curve. The equilibrium point or MCP is the intersection of the aggregate supply curve and the aggregate demand curve.

2.12.1 The Objective Function without Demand Side Bidding

In single side bidding with block bid protocol, the generator offered prices are represented by an increasing staircase function. Thus, the supply cost is a piecewise linear function as shown in Fig. 2.33. The auction based dispatch problem is formulated as:

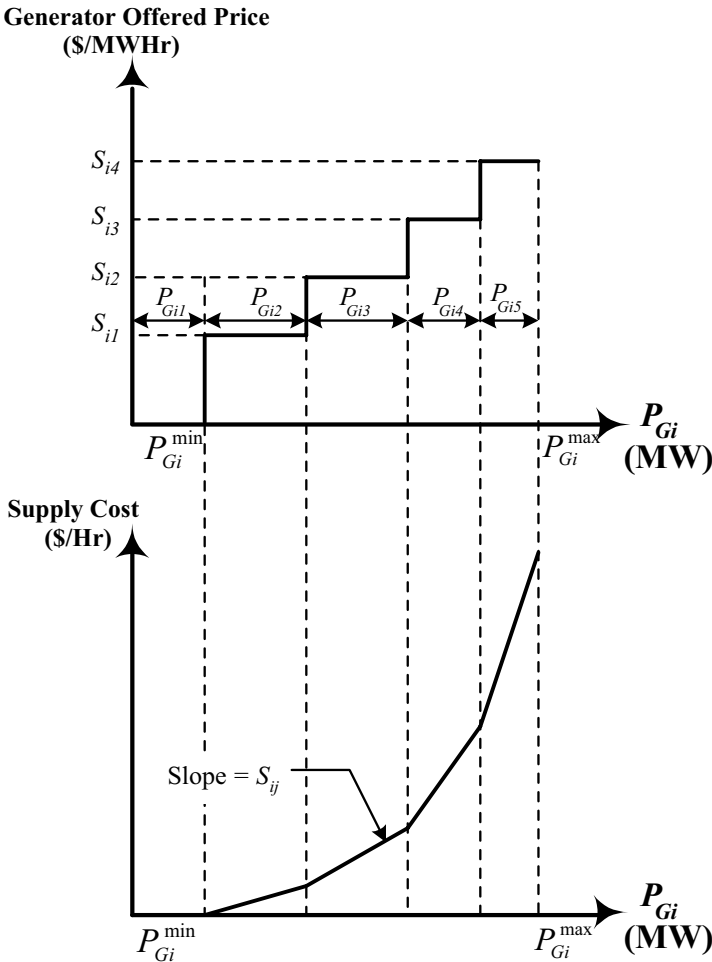


Fig. 2.33 Offered prices of a generator participant.

$$\text{Minimize } S(P_{Gij}) = \sum_{i=1}^{NG} \sum_{j=1}^{NS_i} S_{ij} P_{Gij} \quad (2.238)$$

subject to

the power balance constraint

$$\sum_{i=1}^{NG} \sum_{j=1}^{NS_i} P_{Gij} + \sum_{i=1}^{NG} P_{Gi}^{\min} = P_D + P_{loss} \quad (2.239)$$

and generator operating limit constraints

$$P_{Gi}^{\min} \leq P_{Gi} \leq P_{Gi}^{\max}, i=1, \dots, NG \quad (2.240)$$

and linearized line flow limit constraints

$$f_l = f_l^o + \sum_{i=1}^{NG} a_{li} \left(\sum_{j=1}^{NS_i} P_{Gij} + P_{Gi}^{\min} - P_{Gi}^o \right) \leq f_l^{\max}, \text{ for } l = 1, 2, \dots, NC \quad (2.241)$$

$$\text{where } P_{Gi} = P_{Gi}^{\min} + \sum_{j=1}^{NS_i} P_{Gij} \quad (2.242)$$

$$0 \leq P_{Gij} \leq P_{Gij}^{\max} \text{ for } j = 1, 2, 3, \dots, NS_i$$

$S(P_{Gij})$ supply cost curve (\$/hr),

P_{Gij} offered block j of generator at bus i (MW),

$P_{Gi}^{\max}$ maximum power output at bus i (MW),

$P_{Gi}^{\min}$ minimum power output at bus i (MW),

S_{ij} slope of segment j at bus i ,

P_L total system real power load (MW),

P_{loss} total power system losses (MW),

f_l real power flow at line or transformer l (MW),

$f_l^{\max}$ real power flow limit at line or transformer l (MW),

f_l^o real power flow at line or transformer l at previous iteration,

NS_i number of segments of supply cost of generator at bus i ,

NG total number of generator participants,

NC total number line flow constraints,

a_{li} line l sensitivity factor to the real injection power at bus $i = \Delta f_l / \Delta P_i$,

P_{Gi}^o real power generation at bus i at previous iteration.

2.12.2 The Objective Function with Demand Side Bidding

In a deregulated power system, electricity is considered a tradable commodity. The demand side curve can be shown as in Fig. 2.34. When the market price is “ p ” the surplus to the electricity consumer can be defined as

$$\text{Surplus to electricity consumer} = \int_0^q (D(q) - p) dq \quad (2.243)$$

where

p = product market price,

q = amount of the product that has been consumed,

$D(q)$ = demand cost curve in the function of q (per unit cost).

Similarly, the supply cost curve that presents the producer willingness to sell is shown as in Fig. 2.35 and the surplus to the electricity producer is:

$$\text{Surplus to electricity producer} = \int_0^q (p - S(q)) dq \quad (2.244)$$

$S(q)$ = supply price curve in the function of q (per unit price).

The integration of $D(q)$ and $S(q)$ with respect to q is the total cost and total price. The surplus to the total welfare is the summation of the surplus to electricity consumer and the surplus to the electricity producer and can be expressed as:

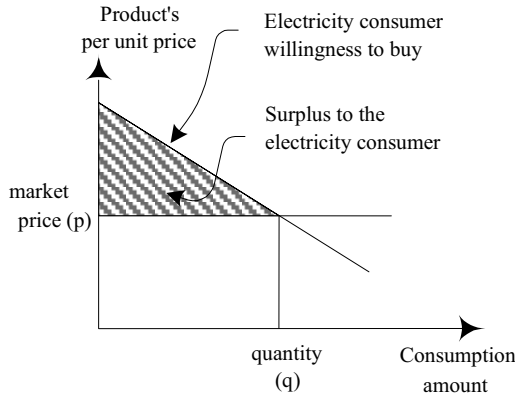


Fig. 2.34 Demand cost curve and the surplus to the consumer.

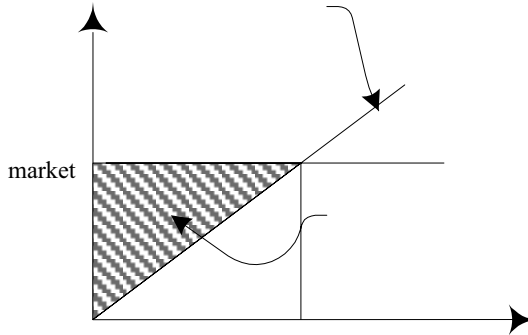


Fig. 2.35 Supply cost curve and the surplus to the producer.

Total Social Welfare

$$\begin{aligned}
 &= \text{Surplus to electricity consumer} + \text{Surplus to electricity producer} \\
 &= \int_0^q (D(q) - pq) dq + \int_0^q (pq - S(q)) dq \\
 &= \int_0^q (D(q) - S(q)) dq
 \end{aligned} \tag{2.245}$$

In the perfect competitive market, the market moves itself towards the maximum social benefit. Figure 2.36 shows the market equilibrium point where supply and demand are equal. The price that reflects this condition is the price that “clears the market”. Therefore, in a deregulated power market, the Independent System Operator can use the objective of maximization of social welfare in the dispatch strategy. The objective function for this purpose is

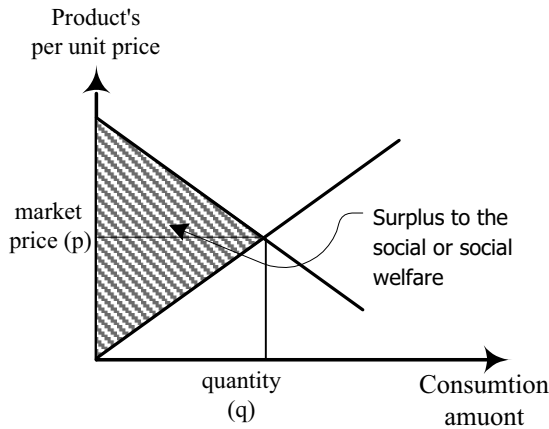


Fig. 2.36 Maximize social welfare condition.

$$\text{Maximize } \int_0^P (D(P_d) - S(P_s)) dP \quad (2.246)$$

$$\text{or Maximize } B(P_d) - C(P_s) \quad (2.247)$$

where

$D(P_d)$ demand per unit cost function (\$/MWhr),

$S(P_s)$ incremental price of power (\$/MWhr),

$B(P_d)$ customer benefit in electricity consumption (\$/hr),

$C(P_s)$ total price of electricity (\$/hr).

On the other hand

$$D(P_d) = \frac{dB(P_d)}{dP_d} \text{ and } S(P_s) = \frac{dC(P_s)}{dP_s} \quad (2.248)$$

Figure 2.37 clarifies the objective of maximizing social welfare when the customer benefit is the function of $B(P_d) = -0.01P_d^2 + 80P_d$ and $C(P_s) = 0.05P_s^2 + 18P_s$. It can be seen that the quantity of MW that corresponds to the maximum value of $B(P_d) - C(P_s)$ is the intersection of $D(P_d)$ and $S(P_s)$.

The supply and demand curves in deregulated power system are in the form of bid and offer of energy. When the offer to sell is the same as in Fig. 2.31 and the bid to buy is conducted by block bid protocol as in Fig. 2.38, the objective function can be expressed as

$$\text{Maximize } \sum_{i=1}^{NP} \sum_{j=1}^{ND_i} D_{ij} P_{Gij} - \sum_{i=1}^{NG} \sum_{j=1}^{NS_i} S_{ij} P_{Gij} \quad (2.249)$$

$$\text{or Minimize } \sum_{i=1}^{NG} \sum_{j=1}^{NS_i} S_{ij} P_{Gij} - \sum_{i=1}^{NP} \sum_{j=1}^{ND_i} D_{ij} P_{Dij} \quad (2.250)$$

subject to the power balance constraint

$$\sum_{i=1}^{NG} \sum_{j=1}^{NS_i} P_{Gij} = \sum_{i=1}^{NP} \sum_{j=1}^{ND_i} P_{Dij} + P_{loss} \quad (2.251)$$

and the line flow limit constraint in equation (2.241).

where

$$P_{Gi} = \sum_{j=1}^{NS_i} P_{Gij} \quad (2.252)$$

$$0 \leq P_{Gij} \leq P_{Gij}^{\max} \text{ for } j = 1, 2, 3, \dots, NS_i \quad (2.253)$$

and

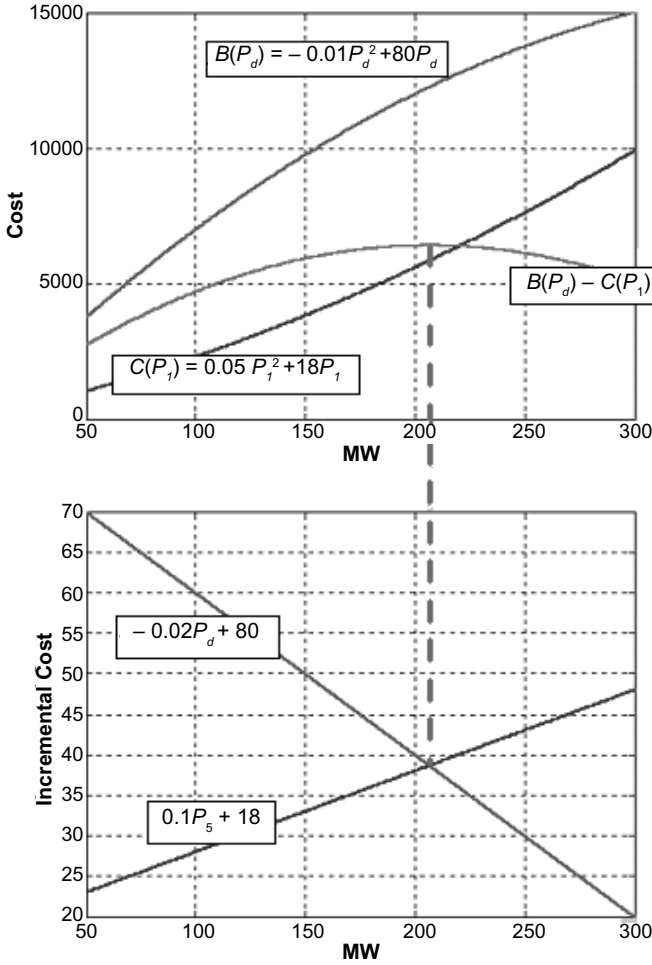


Fig. 2.37 The maximum social welfare objective.

Color image of this figure appears in the color plate section at the end of the book.

$$P_{Di} = \sum_{j=1}^{ND_i} P_{Dij} \quad (2.254)$$

$$0 \leq P_{Dij} \leq P_{Dij}^{\max} \text{ for } j = 1, 2, 3, \dots, NP_i \quad (2.555)$$

ND_i number of segments of demand bid of purchaser at bus i ,

NP number of purchasers in the system,

D_{ij} slope of segment j of demand (purchaser) i ,

P_{Di} real power consumption at bus i (MW),

P_{Dij} power bid to buy block j of the purchaser at bus i (MW).

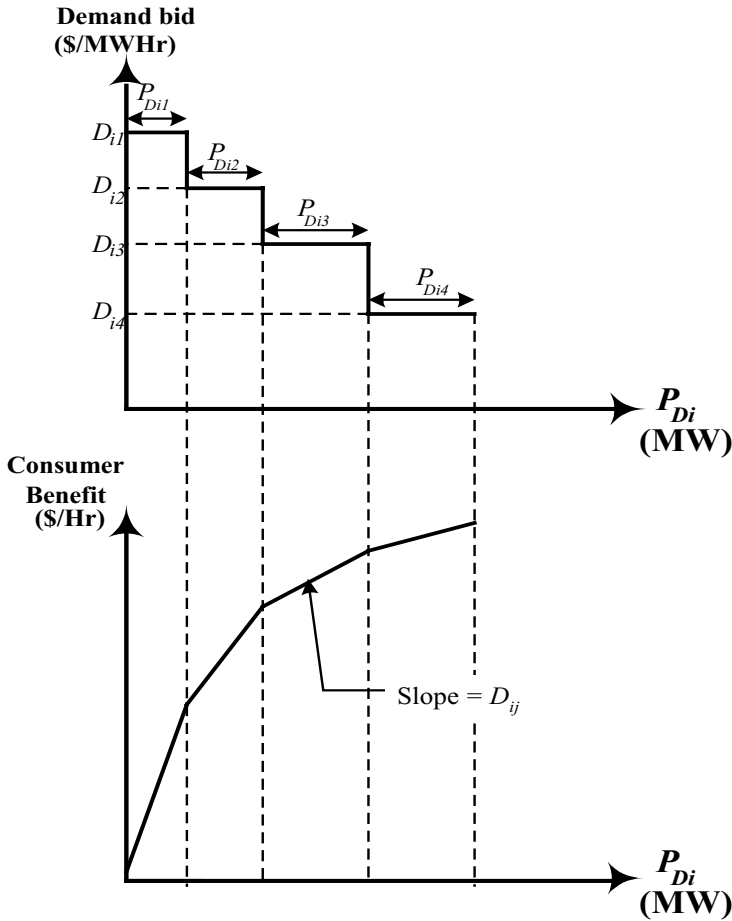


Fig. 2.38 Block bid protocol in double side bidding.

2.13 SUMMARY

The optimal results for the scheduling of generation power are selected from an infinite number of power solutions, leading to the optimal power flow solution. The main aim in the economic dispatch problem is to minimize the total cost of generating real power at various stations while satisfying the loads and losses in transmission links. The optimal system operation, in general, involves the consideration of economy of operation, system security, emissions of certain fossil-fuel plants, optimal release levels of water at hydro power plants etc. All these aspects may lead to conflicting requirements which makes a compromise solution necessary for optimal system operation. Economy of operation is naturally

predominant in determining the best choice, though there are other increasingly important factors such as emissions, transmission loss, voltage stability and network reliability that should be given consideration.

2.14 PROBLEMS FOR EXERCISE

- 2.14.1 Determine the economic operating point and total system operating cost for the following two units, when delivering a total of 500 MW.

Unit 1: Max output: $P_{G1}^{max} = 500$ MW
Min output: $P_{G1}^{min} = 20$ MW

$$\text{Cost curve (\$/hr): } F_1(P_{G1}) = 300 + 10P_{G1} + 0.15P_{G1}^2$$

Unit 2: Max output: $P_{G2}^{max} = 500$ MW
Min output: $P_{G2}^{min} = 20$ MW

$$\text{Cost curve (\$/hr): } F_2(P_{G2}) = 200 + 20P_{G2} + 0.2P_{G2}^2$$

- 2.14.2 Determine the economic operating point and total system operating cost for the following three units, when delivering a total of 1000 MW.

Unit 1: Max output: $P_{G1}^{max} = 500$ MW
Min output: $P_{G1}^{min} = 50$ MW

$$\text{Cost curve (\$/hr): } F_1(P_{G1}) = 300 + 10P_{G1} + 0.15P_{G1}^2$$

Unit 2: Max output: $P_{G2}^{max} = 500$ MW
Min output: $P_{G2}^{min} = 20$ MW

$$\text{Cost curve (\$/hr): } F_2(P_{G2}) = 200 + 20P_{G2} + 0.2P_{G2}^2$$

Unit 3: Max output: $P_{G3}^{max} = 500$ MW
Min output: $P_{G3}^{min} = 20$ MW

$$\text{Cost curve (\$/hr): } F_3(P_{G3}) = 100 + 10P_{G3} + 0.3P_{G3}^2$$

- 2.14.3 From problem 2, if the generators have prohibited zones as below, determine the economic operating point and operating cost for the three units, when delivering a total load of 1000 MW.

Unit 1: Max output: $P_{G1}^{max} = 500$ MW
Min output: $P_{G1}^{min} = 50$ MW

$$\text{Cost curve (\$/hr): } F_1(P_{G1}) = 300 + 10P_{G1} + 0.15P_{G1}^2$$

Prohibited Zone: 220–250 MW

Unit 2: Max output: $P_{G2}^{max} = 500$ MW

Min output: $P_{G2}^{min} = 50$ MW

Cost curve (\$/hr): $F_2(P_{G2}) = 200 + 20P_{G2} + 0.2P_{G2}^2$

Prohibited Zone: 220–250 MW

Unit 3: Max output: $P_{G3}^{max} = 500$ MW

Min output: $P_{G3}^{min} = 50$ MW

Cost curve (\$/hr): $F_3(P_{G3}) = 100 + 10P_{G3} + 0.3P_{G3}^2$

Prohibited Zone: 220–250 MW

2.14.4 A system includes five units with the data given as follows:

Unit 1: Max output: $P_{G1}^{max} = 455$ MW

Min output: $P_{G1}^{min} = 50$ MW

Cost curve (\$/hr):

$$F_1(P'_{G1}) = 516.175 + 7.748P'_{G1} + 0.00023(P'_{G1})^2$$

Ramp rate: $UR_1 = DR_1 = 60$ MW

Initial power: $P_{G1}^{(0)} = 300$ MW

Unit 2: Max output: $P_{G2}^{max} = 455$ MW

Min output: $P_{G2}^{min} = 50$ MW

Cost curve (\$/hr):

$$F_2(P'_{G2}) = 459.629 + 8.178P'_{G2} + 0.000147(P'_{G2})^2$$

Ramp rate: $UR_2 = DR_2 = 24$ MW

Initial power: $P_{G2}^{(0)} = 50$ MW

Unit 3: Max output: $P_{G3}^{max} = 130$ MW

Min output: $P_{G2}^{min} = 40$ MW

Cost curve (\$/hr):

$$F_3(P'_{G3}) = 258.335 + 6.070P'_{G3} + 0.000776(P'_{G3})^2$$

Ramp rate: $UR_3 = DR_3 = 130$ MW

Initial power: $P_{G3}^{(0)} = 40$ MW

Unit 4: Max output: $P_{G4}^{max} = 300$ MW

Min output: $P_{G4}^{min} = 40$ MW

Cost curve (\$/hr):

$$F_4(P'_{G4}) = 174.686 + 8.850P'_{G4} + 0.00026(P'_{G4})^2$$

Ramp rate: $UR_4 = DR_4 = 300$ MW

Initial power: $P_{G4}^{(0)} = 40$ MW

Unit 5: Max output: $P_{G5}^{max} = 162$ MW

Min output: $P_{G4}^{min} = 25$ MW

Cost curve (\$/hr):

$$F_5(P_{G5}^t) = 133.628 + 8.620P_{G5}^t + 0.00062(P_{G5}^t)^2$$

Ramp rate: $UR_5 = DR_5 = 160$ MW

Initial power: $P_{G5}^{(0)} = 25$ MW

The system is scheduled over a 24-hour period supplying to a load demand as in Table 2.29 below.

- 1) Find the optimal operating point of the system neglecting ramp rate constraints.
- 2) Find the optimal operating point of the system with ramp rate constraints.
- 3) When the ramp-up and down rates of units are reduced to a half, can a feasible solution be found? Explain why?

Table 2.29 Load demand for the five-unit system.

Hour	1	2	3	4	5	6	7	8
P_L (MW)	360	420	455	455	455	405	345	405
Hour	9	10	11	12	13	14	15	16
P_L (MW)	355	415	455	455	455	455	455	455
Hour	17	18	19	20	21	22	23	24
P_L (MW)	455	405	345	405	355	415	455	455

2.14.5 Solve the problem in example 2.7 without transmission constraint with the maximum emission allowance of 5.2 tons per interval.

2.14.6 Solve the problem in example 2.7 without emission constraint.

2.14.7 Two coal-burning generating units must both remain online for a three-week period supplying to a load demand of 1200, 1500 and 800 MW during the first, second and third week respectively, with the assumption that the load is constant for every week. The two units are supplied 40,000 tons of coal per week. Both units have the same initial inventory of 70,000 tons and the maximum storage capacity of each is 200,000 tons. The fuel cost and fuel consumption of the two units each week is:

$$F_1(P_{G1}) = 83269.2 + 1811P_{G1} + 2.2(P_{G1})^2 \quad \$/wk$$

$$F_2(P_{G2}) = 127814.4 + 1789P_{G2} + 13.9(P_{G2})^2 \quad \$/wk$$

$$q_3(P_{G3}) = 2775.4 + 60.4P_{G3} + 0.21(P_{G3})^2 \quad \text{tons/wk}$$

$$q_4(P_{G4}) = 4260.5 + 59.7P_{G4} + 0.15(P_{G4})^2 \quad \text{tons/wk}$$

- 1) Solve the problem for optimal solution.
- 2) Solve the problem when the initial inventory of unit 2 is 50,000 tons.

2.14.8 A system includes 15 online units to supply a system load demand of 2,650 MW with a system spinning reserve requirement of 200 MW. Among these

units, units 2, 5, 6 and 12 have prohibited zones. The unit data and prohibited zones are given in Tables 2.30 and 2.31.

The loss coefficients are given as follows:

$$B_{ij} = 10^{-4} \times$$

14	12	7	-1	-3	-1	-1	-1	-3	5	-3	-2	4	3	-1
12	15	13	0	-5	-2	0	1	-2	-4	-4	0	4	10	-2
7	13	76	-1	-13	-9	-1	0	-8	-12	-17	0	-26	111	-28
-1	0	-1	34	-7	-4	11	50	29	32	-11	0	1	1	-26
-3	-5	-13	-7	90	14	-3	-12	-10	-13	7	-2	-2	-24	-3
-1	-2	-9	-4	14	16	0	-6	-5	-8	11	-1	-2	-17	3
-1	0	-1	11	-3	0	15	17	15	9	-5	7	0	-2	-8
-1	1	0	50	-12	-6	17	168	82	79	-23	-36	1	5	-78
-3	-2	-8	29	-10	-5	15	82	129	116	-21	25	7	-12	-72
5	-4	-12	32	-13	-8	9	79	116	200	-27	-34	9	-11	-88
-3	-4	-17	-11	7	11	-5	-23	-21	-27	140	1	4	-38	168
-2	0	0	0	-2	-1	7	-36	25	-34	1	54	-1	-4	28
4	4	-26	1	-2	-2	0	1	7	9	4	-1	103	-101	28
3	10	111	1	-24	-17	-2	5	-12	-11	-38	-4	-101	578	-94
-1	-2	-28	-26	-3	3	-8	-78	-72	-88	168	28	28	-94	1283

$$B_{0i} = 10^{-4} \times \begin{bmatrix} -1 & -2 & 28 & -1 & 1 & -3 & -2 & -2 & 6 & 39 & -17 & 0 & -32 & 67 & -64 \end{bmatrix}^T$$

$$B_{00} = 0.0055$$

Table 2.30 Unit data of the 15-unit system.

Unit	a_i (\$/h)	b_i (\$/MWh)	c_i (\$/MW ² h)	P_{Gi}^{min} (MW)	P_{Gi}^{max} (MW)	P_{Gi}^0 (MW)	UR_i (MW/h)	DR_i (MW/h)
1	671.03	10.07	0.000299	150	455		400	80
2	574.54	10.22	0.000183	150	455	400	300	80
3	374.59	8.80	0.001126	20	130	300	105	130
4	374.59	8.80	0.001126	20	130	105	100	130
5	461.37	10.40	0.000205	150	470	100	90	80
6	630.14	10.10	0.000301	135	460	90	400	80
7	548.20	9.87	0.000364	135	465	400	350	80
8	227.09	11.50	0.000338	60	300	350	95	65
9	173.72	11.21	0.000807	25	162	95	105	60
10	175.95	10.72	0.001203	20	160	105	110	60
11	186.86	11.21	0.003586	20	80	110	60	80
12	230.27	9.90	0.005513	20	80	60	40	80
13	225.28	13.12	0.000371	25	85	40	30	80
14	309.03	12.12	0.001929	15	55	30	20	55
15	323.79	12.41	0.004447	15	55	20	20	55

Table 2.31 Prohibited zones of the 15-unit system.

Unit	Zone 1	Zone 2	Zone 3
2	[185,225]	[305,335]	[420,450]
5	[180,200]	[260,335]	[390,420]
6	[230,255]	[365,395]	[430,455]
12	[30,55]	[65,75]	-

- 1) Find the optimal operating point of the system neglecting ramp rate constraints and power losses.
- 1) Solve the problem with all constraints.

2.14.9 A system consists of one pure power unit, three co-generation units and one pure heat unit. The cost functions of the units are:

$$F_1(P_{G1}) = 254.8863 + 7.6997P_{G1} + 0.00172P_{G1}^2 + 0.000115P_{G1}^3 \text{ ($/h)}$$

$$F_2(P_2, H_2) = 1250 + 36P_{G2} + 0.0435P_{G2}^2 + 0.6H_{G2} + 0.027H_{G2}^2 + 0.011P_{G2}H_{G2} \text{ ($/h)}$$

$$F_3(P_{G3}, H_{G3}) = 2650 + 34.5P_{G3} + 0.1035P_{G3}^2 + 2.203H_{G3} + 0.025H_{G3}^2 + 0.051P_{G3}H_{G3} \text{ ($/h)}$$

$$F_4(P_{G4}, H_{G4}) = 1565 + 20P_{G4} + 0.072P_{G4}^2 + 2.3H_{G4} + 0.02H_{G4}^2 + 0.04P_{G4}H_{G4} \text{ ($/h)}$$

$$F_5(H_{G5}) = 950 + 2.0109H_{G5} + 0.038H_{G5}^2 \text{ ($/h)}$$

$$35 \leq P_{G1} \leq 135 \text{ MW}$$

$$0 \leq H_{G5} \leq 60$$

The feasible operation zones of co-generation units 2–4 are given in Figs. 2.39–2.41.

- 1) Find the optimal operating point of the units for a load demand of 300 MW and 150 MWth.
- 2) Find the optimal operating point of the units for a load demand of 250 MW and 175 MWth.
- 3) Find the optimal operating point of the units for a load demand of 160 MW and 220 MWth.

2.14.10 A system with one thermal plant and one hydro plant has the following characteristics:

$$F_1(P_{G1}) = 575 + 9.2 P_{G1} + 0.00184P_{G1}^2 \quad \text{\$/h}$$

$$q_2(P_{G2}) = 330 + 4.97P_{G2} \quad 10^3 \text{ m}^3/\text{h}$$

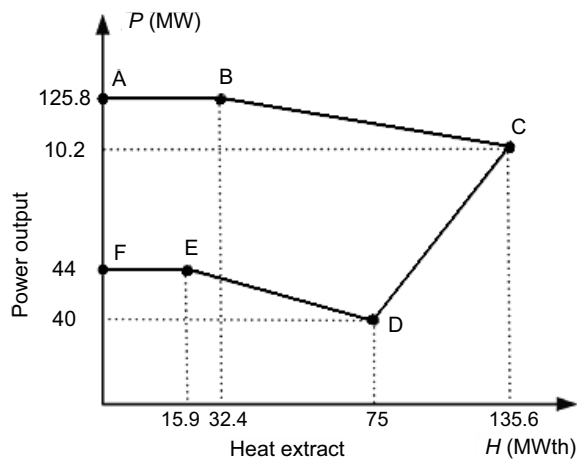


Fig. 2.39 Heat-power feasible operation region of co-generation unit 2.

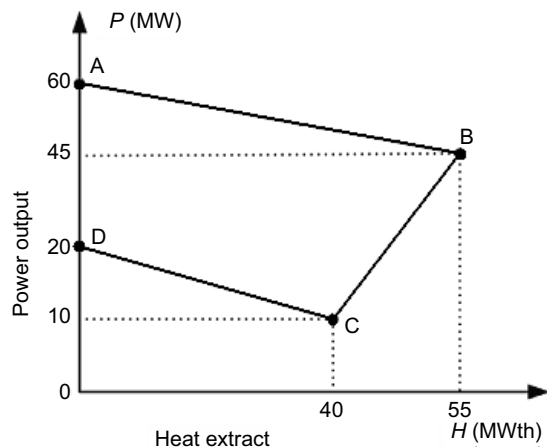


Fig. 2.40 Heat-power feasible operation region of co-generation unit 3.

The hydro plant is located a good distance from the load. The loss formula coefficients are:

$$B_{11} = B_{12} = B_{21} = 0 \text{ and } B_{22} = 8 \times 10^{-5} \text{ per MW}$$

The hydro unit’s reservoir is limited to a drawdown of 10^8 m^3 over a dispatch period of one day divided into two intervals with 12 hours each and the load demand for the two intervals is 1200 and 1500 MW. Find the optimal solution for the problem.

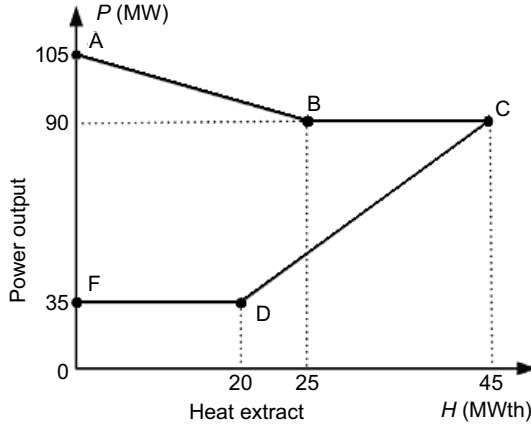


Fig. 2.41 Heat-power feasible operation region of co-generation unit 4.

2.14.11 A hydrothermal system has two thermal and two hydro units with the following characteristics:

$$\begin{aligned}
 F_1(P_{G1}) &= 25 + 3.2P_{G1} + 0.0025P_{G1}^2 & \$/\text{h} \\
 F_2(P_{G2}) &= 30 + 3.4P_{G2} + 0.0008P_{G2}^2 & \$/\text{h} \\
 q_3(P_{G3}) &= 550 + 85P_{G3} + 0.06P_{G3}^2 & 10^3 \text{ m}^3/\text{s} \\
 q_4(P_{G4}) &= 260 + 170P_{G4} + 0.1P_{G4}^2 & 10^3 \text{ m}^3/\text{s}
 \end{aligned}$$

The transmission loss formula coefficients are:

$$B_{00} = 0$$

$$B_{0i} = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B_{ij} = 10^{-4} \times \begin{bmatrix} 1.40 & 0.10 & 0.15 & 0.15 \\ 0.10 & 0.60 & 0.10 & 0.13 \\ 0.15 & 0.10 & 0.68 & 0.65 \\ 0.15 & 0.13 & 0.65 & 0.70 \end{bmatrix} \text{ per MW}$$

The allowable volumes of water for a dispatch period of one day are $W_3 = 70.7925 \times 10^6 \text{ m}^3$ and $W_4 = 59.4657 \times 10^6 \text{ m}^3$. The load demand of the system is given in Table 2.32. Find the optimal operating point of the system.

2.14.12 From the above offer prices and quantities of the three generation companies in Table 2.33:

- 1) Construct the aggregate supply curve of the system.
- 2) Determine dispatch results and total system cost for a total load of 500 MW.

Table 2.32 Load demand for problem 2.14.11.

Hour	1	2	3	4	5	6	7	8
P_L (MW)	400	300	250	250	250	300	450	900
Hour	9	10	11	12	13	14	15	16
P_L (MW)	1230	1250	1350	1400	1200	1250	1250	1270
Hour	17	18	19	20	21	22	23	24
P_L (MW)	1350	1470	1330	1250	1170	1050	900	600

Table 2.33 Offer price and quantities of three generation companies.

GEN No.	From (MW)	To (MW)	Offer Cost (\$/MWhr)
1	20	56	17.6
	56	92	24.8
	92	128	32
	128	164	39.2
	164	200	46.4
2	20	56	35.2
	56	92	49.6
	92	128	64
	128	164	78.4
	164	200	92.8
3	20	56	32.8
	56	92	54.4
	92	128	76
	128	164	97.6
	164	200	119.2

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UNIT COMMITMENT*

3.1 INTRODUCTION

Since electrical load varies over a daily period and a weekly period in a cyclic manner, electric utilities plan in advance an economic schedule to decide which generating units to switch on/off and connect to/disconnect from the network system at a certain time. In general, load profiles aren't uniform. Total loads on the electric power system are generally higher during day time and early evening when industries are (still) active while household demand picks up towards the evening, and lower during late evening and early morning when most people sleep. Due to less economic activity, the overall load is lower over the weekends. Sufficient reliable power generation to meet the peak load demand must therefore be synchronized prior to the actual load occurrence. Simply committing a sufficient number of units to cover the maximum system load and leaving them running online for the entire duration scheduled may not be economic because the supply is probably excessive. Note that to 'commit' a generating unit is to 'turn it on', i.e., to bring the unit up to speed, synchronize it to the system and connect it so that it can deliver power to the network. Since an appropriate schedule can save costs significantly, unit commitment (UC) is essential to provide an economic on/off schedule regarding unit characteristics and system restrictions such as reserve and network constraints etc. It determines start-up, shut down, production levels of all units in each period considering unit operation constraints and cost.

In general, each generating unit has its own characteristics such as minimum up and down time, minimum and maximum generation level, and up/down ramping limit. Furthermore, there are some system requirements such as power balance, spinning reserve, transmission line limit, fuel supply limit, etc. UC is a scheduling problem usually covering a discrete time range from 24 hours (1 day) to 168 hours (1 week) ahead and is handled by the operator in the pre-dispatch stage. An electric utility system operator, who has the knowledge of system components and operating

*This chapter has been written with assistance from Nit Petcharakas

costs of generating units, is the one who makes decisions on UC to minimize the utility's generation cost. The input for the operator is the forecasted demand for the next week or next day aggregated for the whole system, unit availability and network status. In the UC approach, the operator seeks to minimize system costs while meeting the forecasted demand and satisfying unit and system constraints over the planning horizon to decide upon the units' on/off status. The presence of binary decision variables of the unit status (on/off) makes the UC problem complex. Hence, the UC problem is a large scale nonlinear mixed integer combinatorial problem. Consequently, optimal solutions are very difficult to obtain, especially for a large system. The global optimal solution can be obtained by complete enumeration, which is not applicable to large power systems due to its excessive computational time requirements. According to the size and complexity of UC problems and the large economic benefits that could result from improved solutions, considerable attention has been devoted to algorithm development. An efficient algorithm for UC planning could lead to substantial fuel cost savings even with only a few percent of improvement.

Recently, the importance of UC has been realized as in addition to basic rules of system economy, growth in system size, a variety of practical constraints and types of power sources have to be considered. The generation resource mix includes fossil-fuel units, peak load units such as combustion turbines, stored and run-off-river hydro-power, pumped storage hydro-power, nuclear units, purchase and sale over tie lines (interregional power exchange). Furthermore, UC is used to simulate the effects of unit selection methods on the choice of new generation.

Many constraints should be included in the UC approach. The list presented here does not cover all practical constraints since each individual power system has its own characteristics and constraints. However, the common constraints are presented as follows.

3.1.1 Unit Constraints

Thermal units require some time to establish working temperature and pressure and to bring it online. Therefore, each generating unit has its own generation limit as described below. This also means that a certain amount of energy must be expended to bring the unit online. This energy does not result in any MW of power generated by the unit and shows up in the UC problem as a startup cost. Conventional unit constraints are as follows:

- a. Minimum and maximum power output constraints: Each unit should generate not less than its minimum power and not more than its maximum.
- b. Minimum up and down time constraints: Once the unit is running, it should not be turned off immediately and vice versa, once the unit is de-committed, there is a minimum time before it can be recommitted.

3.1.2 Spinning Reserve Constraint

Spinning reserve is the term used to describe the total amount of generation available from all units synchronized (spinning) on the system, minus the present load and losses. The amount of spinning reserve is usually designed to equal either a certain percentage of the system load demand, the maximum capacity of the committed units or the maximum tie line capacities. The impact of the spinning reserve on the UC problem can be illustrated by numerical examples in series as follows:

System I: Suppose load is 900 MW. There are six generator units

$$F_1(P_1) = 300 + 10P_1 \text{ \$}/\text{h}, \quad 50 \leq P_1 \leq 500 \text{ MW}$$

$$F_2(P_2) = 200 + 12P_2 \text{ \$}/\text{h}, \quad 20 \leq P_2 \leq 300 \text{ MW}$$

$$F_3(P_3) = 200 + 13P_3 \text{ \$}/\text{h}, \quad 20 \leq P_3 \leq 300 \text{ MW}$$

$$F_4(P_4) = 200 + 14P_4 \text{ \$}/\text{h}, \quad 20 \leq P_4 \leq 300 \text{ MW}$$

$$F_5(P_5) = 200 + 15P_5 \text{ \$}/\text{h}, \quad 20 \leq P_5 \leq 300 \text{ MW}$$

$$F_6(P_6) = 200 + 16P_6 \text{ \$}/\text{h}, \quad 20 \leq P_6 \leq 300 \text{ MW}$$

The spinning reserve requirement in this illustration is first set as a percentage of forecasted demand so that the reserve is capable of coping with any expected increase/decrease of demand. Suppose that the units are committed according to their merit order neglecting startup cost. In plan A, the spinning reserve requirement is set as 90 MW computed as 10% of the load. This results in committing U#1, U#2 and U#3 as shown in Table 3.1. The power is dispatched economically with 90% of unit maximum capacity. The remaining 10% of unit capacity is kept as reserve. However, a spinning reserve must be secured so that the loss of one or more units does not cause too much a drop in the system frequency. If one unit is lost, there must be ample reserve on the other units to make up for the loss in a specific time

Table 3.1 Plan A with the corresponding total cost of \$10,780, spinning reserve requirement is set as 10% of the load.

Unit	Unit status	Generation output (MW)	Spinning reserve (MW)
U#1	1	450	50
U#2	1	270	30
U#3	1	180	120
U#4	0	0	-
U#5	0	0	-
U#6	0	0	-
Total		900	200

period. Even though the total cost of plan A is the lowest one at \$10,780, this schedule may face a problem in the contingency of losing the largest unit. This problem can be solved by setting the spinning reserve requirement at the capacity level of the largest generating unit so that the reserve is capable of covering the loss of the most heavily loaded unit in a given period of time. Thus, the spinning reserve requirement rises to 500 MW and one more unit is needed which results in committing U#4. This schedule is designated as plan B shown in Table 3.2 with a higher total cost of \$11,000. Note: To obtain a feasible solution, the sum of all maximum capacities of the committed units must be larger than the forecasted load plus spinning reserve, and the sum of all minimum levels of the committed units must be greater than the load.

Table 3.2 Plan B with the corresponding total cost of \$11,000, the spinning reserve requirement is set at the capacity of the largest generating unit.

Unit	Unit status	Generation output (MW)	Spinning reserve (MW)
U#1	1	450	50
U#2	1	270	30
U#3	1	160	140
U#4	1	20	280
U#5	0	0	-
U#6	0	0	-
Total		900	500

3.1.3 Transmission Line Constraint

One common approach including the transmission line constraint is known as indirect method, considering the UC problem into two steps. This approach ignores transmission constraints in UC, then account for these constraints in an ED process by re-dispatching. A direct method for security constrained unit commitment was proposed in 1995. Transmission line constraints present a challenge to researchers of the UC problem since the early 1990s. Generally, a linear dc power flow was considered in the UC problem formulation for security reasons. However, optimal power flow was incorporated in the UC formulation in 1999.

Transmission line constraints are important factors influencing the UC solution. Suppose there are two areas: Area A containing U#1, U#2 and U#3 with a load demand of 350 MW and area B containing U#4, U#5 and U#6 with a load demand of 550 MW. These two areas are linked with two transmission lines, 100 MW thermal limit each as shown in Fig. 3.1. For plan B, the power flowing in the tie lines violates the line limits shown in Table 3.3. To solve this transmission line problem, one more unit in area B must be committed. Thus in plan C, unit U#5 in area B is committed as shown in Table 3.4. This schedule results in higher cost of \$11,800 but satisfies spinning reserve and tie line constraints. However, it may commit to many generating units in area A, therefore unit U#3 is de-committed

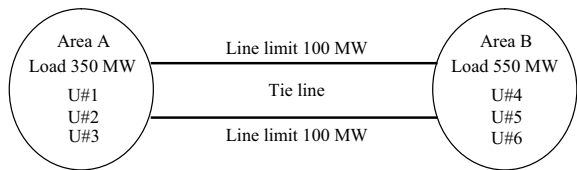


Fig. 3.1 System I, including transmission line limit.

Table 3.3 Plan B, the power flow in tie lines is higher than the line limit.

	Unit	Unit status	Generation output (MW)	Spinning reserve (MW)	Load (MW)	Each tie line flow (MW)
Area A	U#1	1	450	50		
	U#2	1	130	170		
	U#3	1	20	280		
	Subtotal		600	500	350	125
Area B	U#4	1	300	0		
	U#5	0	0	-		
	U#6	0	0	-		
	Subtotal		30	0	550	−125
Total			900	500	900	

Table 3.4 Plan C with the corresponding total cost of \$11,800, unit U#5 is committed to satisfy the transmission line constraints.

	Unit	Unit status	Generation output (MW)	Spinning reserve (MW)	Load (MW)	Tie line flow (MW)
Area A	U#1	1	450	50		
	U#2	1	80	220		
	U#3	1	20	280		
	Subtotal		550	550	350	100
Area B	U#4	1	270	30		
	U#5	1	80	220		
	U#6	0	0	-		
	Subtotal		350	250	550	−100
Total			900	800	900	

as shown in plan D, Table 3.5, resulting in lower cost of \$11,580. This illustration shows that, in addition to providing a sufficient reserve to make up for a generation unit failure, the reserve must be spread around the system to avoid transmission system limitations. Furthermore, the reserve must be allocated among the fast-responding units and slow-responding units. This allows the automatic generation control system to restore frequency and interchange quickly in the event of a generating unit outage.

Table 3.5 In Plan D with the corresponding total cost of \$11,580, unit U#3 is de-committed to reduce total cost.

	Unit	Unit status	Generation output (MW)	Spinning reserve (MW)	Load (MW)	Tie line flow (MW)
Area A	U#1	1	450	50		
	U#2	1	100	200		
	U#3	0	0	-		
	Subtotal		550	250	350	100
Area B	U#4	1	270	30		
	U#5	1	80	220		
	U#6	0	0	-		
	Subtotal		350	250	550	−100
Total			900	500	900	

3.1.4 Ramp Constraints

Ramp limits should be taken into account. On a unit basis, a change in a unit's generation level between any two successive periods must not exceed its ramp rate limitation. The sum of ramp limits of committed units must still allow coping with the change in the system load from one period to the next. On the system basis, the spinning reserve amount contributed by each unit must be calculated by considering these ramp rate constraints. The spinning reserve calculated from the difference between maximum committed power and actual load can be large enough to meet the reserve requirement, but ramping limitations may make the actual available spinning reserve insufficient.

The sum of ramp limits of committed units must be at least sufficient to meet a change in system load from one period to the next. In other words, the spinning reserve calculated from the difference between maximum committed power and actual load can be large enough to meet the reserve requirement, but ramping limitations may offset a significant part of the available spinning difference. Consequently, the ramp rate constraints link the generation variables of the previous period to that of the present, and hence introduce a dynamic characteristic in the UC model resulting in more difficulties.

Ramp rate constrained unit commitment (RUC) was solved by mixed integer linear programming (MILP) [1], enhanced dynamic programming, artificial neural network (ANN), Lagrangian relaxation (LR), augmented Lagrangian relaxation (ALR) and dynamic priority list. Recently, the objective function was augmented with an additional ramping cost, which was related to the depreciation in shaft life. Ramping constraints in UC cause an additional computation burden by slowing the convergence rate. Furthermore, backward economic dispatch is required to reduce the generation output from its constrained maximum to zero under the ramp

down limit within one period. Ongsakul and Petcharaks [2] and [3] proposed an enhanced adaptive Lagrangian relaxation (ELR) for the ramp rate constrained unit commitment problem.

The generation scheduling with relaxed ramp rate constraints is generally used since it can greatly simplify the problem. However, it does not reflect the actual operating process of generating units. The use of ramp rate constraints to simulate the unit state and generation changes, however, has a strong effect on optimal scheduling.

Three types of ramping constraints for each unit are considered.

- A. Startup Ramp Constraints:** when an offline unit is turned on, it takes $T_{i,SR}$ minutes to increase its generation capability from zero to its minimum level as shown in Fig. 3.2, left side. During this period, this unit is offline but consuming fuel.
- B. Shutdown Ramp Constraints:** when an online unit is turned off, it takes some time to decrease its generation capability. However, de-committing the online unit with generation output higher than minimum level is not allowed unless in a forced shutdown (generation shedding). This is because before the unit status is changed from '1' at period t to '0' at period $t+1$, this unit is operated at the equilibrium point where mechanical input power equals the electrical output power. At period $t+1$ when this unit is disconnected from the system, the electrical power of this unit is changed from its current generation to zero. This results in accelerating power on the rotor and causes transient instability, which may further affect the generator's life time and its performance. Hence, the generation output of this unit should be at the minimum level at the last committed period. It will take $T_{i,SDR}$ minutes to decrease its generation capability from its minimum level to zero as shown in Fig. 3.2, right side.
- C. Operating Ramp Constraints:** The generation output of the current period confined by the up ramp limit and the down ramp limit, i.e., the generation output of the current period must be less than the generation output of the previous period plus up ramp rate (MW/h) and it must be higher than the generation output of the previous period minus down ramp rate (MW/h).

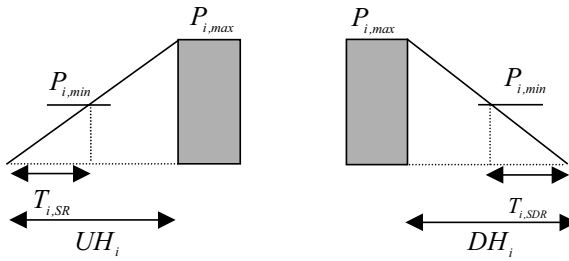


Fig. 3.2 Startup and shutdown ramp constraints.

- 1) *Generation ramp limit*: To satisfy the generation operating limit constraint, generation ramp limit, $P_{i,high}^t$ and $P_{i,low}^t$ are introduced as shown in Fig. 3.3.
- 2) *Minimum down time*: The minimum down time includes the time for the shutdown process (from minimum generation level to zero) and the time for the startup process (from zero to minimum generation level) as shown in Fig. 3.4.
- 3) *Unit reserve contribution*: The up spinning reserve is required to ensure that spinning reserve is sufficient to make up for any unexpected load increase or loss of generation. Whereas, the down spinning reserve is required to ensure that a sudden outage of load won't cause the forced shutdown of thermal units. Due to the unit ramp up and ramp down limits, the up (down) spinning reserve contributed by each unit are determined by two terms: the difference between the maximum (minimum) capacity and the current generation output, and the up/down ramp capability within the specific time. The smaller of these two terms is selected as up/down spinning reserve of that unit.
- 4) *On/offline minimum level*: To reduce the disturbance due to an output change (between zero and 'on' line generation output) and to satisfy the minimum level constraints, the generation output of the shutdown period and the startup period is limited to its minimum level.

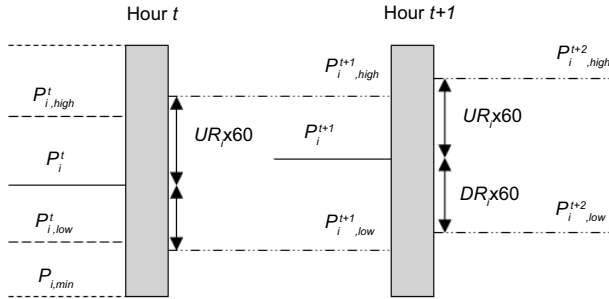
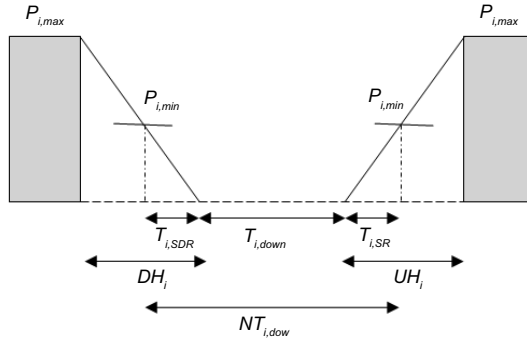
Fig. 3.3 Generation ramp limit for unit i .

Fig. 3.4 New minimum down time.

Table 3.6 Plan C provides an optimal feasible schedule when including ramping constraints.

	Unit	Unit status	Generation output (MW)	Spinning reserve (MW)	Load (MW)	Tie line flow (MW)	Ramp limit (MW/min.)	Sp. reserve in 10 min.
Area A	U#1	1	450	50			2.5	25
	U#2	1	130	170			2.0	20
	U#3	1	20	280			2.0	20
	Subtotal		600	500	400	200	-	65
Area B	U#4	1	270	30			2.0	20
	U#5	1	30	270			2.0	20
	U#6	0	0	-			-	-
	Subtotal		300	300	500	-200	-	40
Total			900	300	900		-	105

Considering the UC problem of system I described in Section 3.1.2 and 3.1.3, it is supposed that the ramp rate limit of unit #1 is 2.5 MW/min whereas that of other units is 2 MW/min. The spinning reserve requirement within 10 minutes is assumed to be 90 MW. Even though the total cost of the schedule in plan D in Section 3.1.3 is lower than that of plan C, the plan D schedule cannot provide sufficient spinning reserve within 10 minutes as required (90 MW) due to ramp rate limits whereas the schedule of plan C can provide a spinning reserve of 105 MW in the specified time. The illustration shows that a spinning reserve solution including ramp rate limits may render a unit schedule infeasible. Therefore, it is suggested to consider a unit ramp rate limit in this UC problem.

3.1.5 Fuel Constraints

Some units have limited fuel supplies or have constraints that require them to burn a specified amount of fuel in a specific time. Thus, fuel constraints complicate the short term thermal unit commitment. Generally, electric utilities have the problem that they either have too much take-or-pay fuels and/or insufficient low cost fuel.

When the energy resource available to a particular plant is limited, the unit commitment and economic dispatch must be done differently. Some fuel resources are under ‘take or pay’ agreement. The utility agrees to use a minimum of fuel during a period (the ‘take’) or failing to use this amount, it agrees to pay the minimum charge (the ‘pay’). Gas supply constraints were included in recent work [4]. The fuel mixture ratio has been considered as well.

3.1.6 Environmental Constraints

Environmental constraints receive great attention nowadays. The Clean Air Act Amendment was passed by the United States congress in 1990. This requires utilities to limit emissions to a specific number of tons per year. The Amendments cover a myriad of air quality concerns including the control of the constituents

of “acid rain”, specifically sulphur dioxide (SO_2) and oxides of nitrogen (NO_x) in power plant emissions. To reduce power plant emissions, the use of natural gas is preferred to coal and oil-fired generating units. In addition, for existing coal power plants, either the installation of scrubbers to remove SO_2 or switching to coal with a lower sulfur content are required. These and other environmental constraints were recently included in many research works on generation scheduling.

3.1.7 Must Run Constraints

Some units are scheduled with a must-run status during certain times due to different reasons such as voltage support for the transmission line network or steam supply requirements.

3.1.8 Derating Constraint

The capacity limit of a thermal unit may change frequently according to maintenance or unscheduled outages of various equipment in the plant.

3.2 UNIT COMMITMENT PROBLEM FORMULATION

The objective of UC problem is to minimize the production cost over the scheduled time horizon (e.g., 24 hours) under the conditions of power balance, spinning reserve, 15 minute spinning reserve, generation limit, minimum up and down time, generation ramp limit, transmission line constraints, environmental constraints, limited fuel, fuel mixing ratio, gas delivery, mixing ratio of different gas sources and alternative fuel. The objective function to be minimized is:

$$\text{Min } F(P_i^t, U_i^t) = \sum_{t=1}^T \sum_{i=1}^N [F_i(P_i^t) + ST_{i,t} (1 - U_i^{t-1})] U_i^t \quad (3.1)$$

subject to:

Power balance constraint

$$P_{load}^t - \sum_{i=1}^N P_i^t U_i^t = 0 \quad (3.2)$$

Spinning reserve constraint

$$P_{load}^t + R^t - \sum_{i=1}^N P_{i,\max} U_i^t \leq 0 \quad (3.3)$$

Generation limit constraints

$$P_{i,\min} U_i^t \leq P_i^t \leq P_{i,\max} U_i^t, i = 1, \dots, N \quad (3.4)$$

Minimum up and new down time constraints

$$U_i^t = \begin{cases} 1 & \text{if } T_{i,on}^{t-1} < T_{i,up} \\ 0 & \text{if } T_{i,off}^{t-1} < T_{i,down} \\ 0 \text{ or } 1 & \text{otherwise} \end{cases} \quad (3.5)$$

Startup cost

$$ST_{i,t} = [\chi_i + \delta_i (1 - \exp(-\frac{T_{i,off}^{t-1}}{\kappa_i}))], i = 1, \dots, N \quad (3.6)$$

On/offline minimum level constraints

$$P_i^t = P_{i,min}, \text{ if } U_i^{t-1} = 0 \text{ and } U_i^t = 1, \\ \text{or } U_i^t = 1 \text{ and } U_i^{t+1} = 0 \quad (3.7)$$

Generation ramp limit constraints

$$P_{i,low}^t U_i^t \leq P_i^t \leq P_{i,high}^t U_i^t, i = 1, \dots, N \quad (3.8)$$

where

$$P_{i,high}^t = \min[P_{i,max}, P_i^{t-1} + UR_i \cdot 60], \text{ if } U_i^t = U_i^{t-1} = 1 \quad (3.9)$$

$$P_{i,low}^t = \max[P_{i,min}, P_i^{t-1} - DR_i \cdot 60], \text{ if } U_i^t = U_i^{t-1} = 1 \quad (3.10)$$

Transmission line constraints

$$-f_l \leq \sum_{\substack{j=1 \\ j \neq \text{slack bus}}}^{NB} s_{l,j} (P_{bus,j}^t - P_{load,j}^t) \leq f_l, l = 1, \dots, NL \quad (3.11)$$

Environmental constraints

$$\sum_{t=1}^T \sum_{i \in \Omega_e} ER_i H_i(P_i^t) U_i^t \leq EM \quad (3.12)$$

Fuel limitation constraints

$$\sum_{t=1}^T q_i(P_i^t) \leq q_{i,TOT}, i \in \Omega_f \quad (3.13)$$

Unit hourly fuel mixing ratio constraints

$$\frac{P_{i,gas}^t}{P_{i,oil}^t} \geq \frac{1}{k_r}, i \in \Omega_g \quad (3.14)$$

where a constrained unit is separated into two units using gas either—or oil as fuel with an additional constraint:

$$P_{i,\min} U_i^t \leq P_{i,gas}^t + P_{i,oil}^t \leq P_{i,\max} U_i^t \quad (3.15)$$

Gas delivery constraints

$$\sum_{i \in \Omega_g} \sum_{t=1}^T q_g(P_i^t) \leq Q_g \quad (3.16)$$

Different gas source mixing ratio

$$\frac{P_{i, source1}^t}{P_{i, source2}^t} \neq k_g, i \in \Omega_{dgs} \quad (3.17)$$

where a constrained unit is separated into two units using east and west gas as fuel with an additional constraint

$$P_{i,\min} U_i^t \leq P_{i,source1}^t + P_{i,source2}^t \leq P_{i,\max} U_i^t \quad (3.18)$$

Alternative fuel constraints

$$P_{i,\min} U_i^t \leq P_{i,gas}^t + P_{i,diesel}^t \leq P_{i,\max} U_i^t \quad (3.19)$$

where units are separated into two units using gas either or diesel as fuel.

$F_i(P_i^t)$ fuel cost function of generating unit i in quadratic form or linear form:

$$F_i(P_i^t) = a_i + b_i P_i^t + c_i (P_i^t)^2 \text{ or } F_i(P_i^t) = a_i + b_i P_i^t$$

P_i^t power generation of unit i at hour t

U_i^t status of unit i at hour t (on = 1, off = 0)

$ST_{i,t}$ start up cost of unit i at hour t

N number of generating units

T schedule time horizon

P_{load}^t load demand of system at hour t

R^t system spinning reserve requirement at hour t

$P_{i,\max}$ maximum generation limit of unit i

$P_{i,\min}$ minimum generation limit of unit i

$T_{i,up}$ minimum up time of unit i

$T_{i,down}$ minimum down time of unit i

$T_{i,on}^{t-1}$ continuous on time of unit i up to hour $t-1$

$T_{i,off}^{t-1}$ continuous off time of unit i up to hour $t-1$

$\chi_i, \delta_i, \kappa_i$ startup cost parameters for thermal unit i

$P_{i,high}^t$	the highest possible power generation of unit i at hour t
$P_{i,low}^t$	the lowest possible power generation of unit i at hour t
UR_i	ramp up rate of unit i
DR_i	ramp down rate of unit i
f_l	power flow on transmission line l
s_{lj}	sensitivity factor for calculation of power flow on line due to power generation at bus j
$P_{load,j}^t$	load demand at hour t at bus j
$P_{bus,j}^t$	power generation at bus j at hour t
NB	total number of buses
NL	total number of transmission lines
ER_i	emission rate of unit i including nitrate oxide (NO_x) and sulfur oxide (SO_2)
$H_i(P_i^t)$	heat rate function of unit i equivalent to $F_i(P_i^t)/FC_i$
FC_i	fuel cost of unit i
EM	maximum emission allowance of system
Ω_e	set of units having emission constraints
$q_i(P_i^t)$	fuel consumption function of unit i
$q_{i,TOT}$	total allowed fuel consumption
Ω_f	set of units having fuel constraints
$P_{i,gas}^t$	power generation by gas turbine unit i
$P_{i,oil}^t$	power generation by oil-fired unit i
k_r	fuel mixing ratio in constant number
Ω_g	set of generating units with a fuel mix from different source
$q_g(P_i^t)$	gas consumption function of unit i
Q_g	total allowed gas consumption
$P_{i,source1}^t$	generation by the first gas source of unit i at hour t
$P_{i,source2}^t$	generation by the second gas source of unit i at hour t
k_g	mixing ratio of gas from different sources
Ω_{dgs}	set of generating units with different gas sources

3.3 UNIT COMMITMENT SOLUTION METHODS

UC is a large scale nonlinear mixed integer combinatorial problem. Consequently, optimal solutions are very difficult to obtain especially for a large system. Various

techniques used to solve the UC problem are described in subsequent sections. Assuming there are N units to be scheduled discretely in time, e.g., in one hour intervals, for a period of T hours with a constant forecasted load over each time interval, the unit status U_i^t is defined as

$U_i^t = 0$, unit i is offline during period t ,

$U_i^t = 1$, unit i is online during period t .

For a power system consisting of N generating units, there are $2^N - 1$ candidate combinations or states when each unit is considered either ‘on’ of ‘off’.

3.3.1 Priority List Method

Generating unit priorities are determined according to their average production costs. For each hour, units are committed one by one according to their priorities until power balance and security constraints are satisfied. The minimum up and down time constraints are checked during this process. Thereafter, a shut down searching procedure is carried out to de-commit some inefficient units for additional cost savings. However, the start up cost is not included in the optimization process. Moreover, only small subsets of possible schedules are examined in these methods. Consequently, the resulting solution is usually far from the optimal. In addition, this algorithm cannot be applied to systems consisting of fuel constrained and/or hydro units. The simplest priority list (PL) solution method based on startup heuristic ordering by operating cost combined with transition cost was presented in 1980. The pre-determined order is then used to commit units such that the system load is satisfied. Variation dynamically ranks the units sequentially in this technique. The ranking process is based on specific guidelines. The Commitment Utilization Factor (CUF) and the classical economic index Average Full Load Cost (AFLC) can be combined to determine the priority commitment order. Recently, the Extended Priority List has been employed to solve the problem of thermal unit commitment.

The priority list is obtained by computing the full load average production cost of each unit and sorting them in the ascending order to obtain a sorted set of units according to their full load average production cost.

Example 3.1

Consider System II comprising three units with their fuel cost functions given below:

$$F_1(P_1) = 500 + 10P_1 + 0.002P_1^2 \text{ \$ /h, } 100 \leq P_1 \leq 700 \text{ MW}$$

$$F_2(P_2) = 400 + 9P_2 + 0.003P_2^2 \text{ \$ /h, } 100 \leq P_2 \leq 400 \text{ MW}$$

$$F_3(P_3) = 300 + 8P_3 + 0.005P_3^2 \text{ \$/h, } 50 \leq P_3 \leq 200 \text{ MW}$$

These three units are to be scheduled to supply a five hour load pattern. Unit data and load profile are shown in Table 3.7.

Note: If the shutdown time is less than the minimum down time plus cold start hour, the startup cost will be that of a hot startup, otherwise, that of a cold startup.

The priority order is unit U#3–U#2–U#1 as shown in Table 3.9. Priority list solution I neglecting spinning reserve and minimum up and down time is shown in Table 3.10, whereas solution II as shown in Table 3.11, including minimum up and down times neglecting spinning reserve constraints, result in a production cost increase of \$402. When both minimum up and down time and spinning reserve constraints are included, solution III (Table 3.12) leads to a rise in production cost of \$682 as compared to solution I.

Table 3.7 Unit characteristics and initial status in system II.

		Unit		
		U#1	U#2	U#3
Maximum capacity (MW)		700	400	200
Minimum capacity (MW)		100	100	50
Full load average cost (\$/MWh)		12.0333	11.2000	10.5000
Minimum	Up time (h)	2	2	2
	Down time(h)	2	2	2
Initial status (-): hours offline (+): hours online		-2	-2	2
Startup cost	Hot (\$)	300	200	100
	Cold (\$)	500	300	200
	Cold start (h)	1	1	1

Table 3.8 Load profile of System II.

	Hour				
	H 1	H 2	H 3	H 4	H 5
Load (MW)	200	500	1100	600	300

Table 3.9 Priority order of System II.

State	Priority order
001	Committed unit#3
011	Committed unit#3, 2
111	Committed unit#3, 2, 1

Table 3.10 Priority list solution I for System II relaxing minimum up and down time and spinning reserve constraints.

	Load (MW)	Generating power (MW)			Reserve (MW)	Operating cost (\$)	Transition cost (startup cost) (\$)	Sum cost (\$)
		P ₁	P ₂	P ₃				
Hour 1	200	0	0	200	0	2,100	-	2,100
Hour 2	500	0	300	200	100	5,470	300	5,770
Hour 3	1100	500	400	200	200	12,580	500	13,080
Hour 4	600	0	400	200	0	6,580	-	6,580
Hour 5	300	0	125	175	300	3,425	-	3,425
Total Cost (\$)								30,955

Table 3.11 Priority list solution II for System II respecting minimum up and down time but relaxed spinning reserve constraints.

	Load (MW)	Generating power (MW)			Reserve (MW)	Operating cost (\$)	Transition cost (startup cost) (\$)	Sum cost (\$)
		P ₁	P ₂	P ₃				
Hour 1	200	0	0	200	0	2,100	-	2,100
Hour 2	500	0	300	200	100	5,470	300	5,770
Hour 3	1100	500	400	200	200	12,580	500	13,080
Hour 4	600	140	260	200	700	6,982	-	6,580
Hour 5	300	0	125	175	300	3,425	-	3,425
Total Cost (\$)								31,357

Table 3.12 Priority list solution III for System II respecting minimum up and down time and spinning reserve constraints.

	Load (MW)	Generating power (MW)			Reserve (MW)		Operat. cost (\$)	Trans. cost (Startup cost) (\$)	Sum cost (\$)
		P ₁	P ₂	P ₃	Required	Available			
Hour 1	200	0	100	200	20	400	2,480	200	2,680
Hour 2	500	0	300	200	50	100	5,470	-	5,770
Hour 3	1100	500	400	200	110	200	12,580	500	13,080
Hour 4	600	140	260	200	60	700	6,982	-	6,580
Hour 5	300	0	125	175	30	300	3,425	-	3,425
Total Cost (\$)									31,637

3.3.2 Enumeration Method

Earlier, the UC problem has been solved by enumerating all possible combinations of generating units and then the combinations that yielded the least operation cost were chosen as the optimal solutions [5], [6]. This method was not applicable to large power systems due to its excessive computational time requirements [7]. The total number of combinations in a UC problem comprising of N units and T periods is $(2^N - 1)^T$. The numerical results for a variety of units and periods are shown in Table 3.13.

Table 3.13 Total number of combinations in a UC problem.

<i>N</i> units	<i>T</i> periods	Number of solutions
5	24	6.20×10^{35}
	168	3.54×10^{250}
10	24	1.73×10^{72}
	168	1.02×10^{504}
20	24	3.12×10^{144}
	168	1.05×10^{1008}
40	24	9.75×10^{288}
	168	1.1×10^{2016}

For System II comprising 3 units and 5 periods, the total number of combinations is 16807. However, not all 16807 solutions are feasible.

3.3.3 Dynamic Programming

Dynamic programming (DP) was a major approach proposed in the 1960s. DP was the earliest optimization based method to be applied to the UC problem. In DP process, an optimization problem is first divided into several stages, i.e., operation periods within the study horizon. The combinations of units in a certain period are known as states. DP searches the solution space that consists of the units for an optimal solution. Forward DP finds the most economical schedule by starting at the initial stage, accumulating total costs, followed by backtracking from the combination of least accumulated cost starting at the last stage and ending with the initial stage. DP builds and evaluates the complete decision tree to optimize the solution for the problem at hand. Thus, DP suffers from the curse of dimensionality because the problem grows rapidly with the number of generator units to be considered. To reduce the search space and hence the dimension of the DP problem, a PL technique is adopted.

An application of the DP approach in the unit commitment problem has been proposed in 1976. The solution procedure considered the entire time span from the first to the last period of a sequence, so that all the transitional information would be carried over. In this regard, the time dependent startup cost as well as minimum up and down time could be incorporated in this procedure. However, it had not been considered efficient enough to be applied to large scale problems. Hence, additional techniques were proposed to specify a subset of combinations for every period. According to the fuel cost of the generating units and load demand in that period, certain units were selected into a window for searching the optimal policy, and commitment states of remaining units were set either to ON (must-run units) or OFF (excess units). This was known as the truncated DP approach. Thus, a small search range could reduce the computation time but might not include the optimal policy. Additional features for limiting the search range were in another method called sequential DP. The subset of combinations being examined was

obtained by committing each unit in a PL sequence. To reduce the dimension of the DP problem, the generating units were organized into classes within which the units were prioritized. A threshold and a window were defined in each class to determine which units should be automatically committed (threshold), which units were evaluated for commitment (window), and which units were not considered at all. Various combinations were presented such as dynamic programming truncated combination (DP-TC), dynamic programming sequential combination (DP-SC), a combination of DP-SC and DP-TC (DP-STC) called variable window truncated dynamic programming (DP-VW).

An enhanced dynamic programming including ramp rate constraints by enlarging state spaces was proposed in 1988. The DP approach was applied to multi area unit commitment including tie line limitation and power system dynamic stability. Conflicts between the computational time and the solution accuracy may occur. Consequently, this DP approach is not suitable for systems consisting of more than 100 units. The limitations of the DP method are more apparent when hydro units are involved. For hydrothermal scheduling, an additional iteration loop is required that executes the thermal unit commitment several times. Therefore, computational efficiency suffers. To improve it, the application of vectorization and parallelization techniques has been proposed to solve the unit commitment problem on a super computer. A DP algorithm was applied to fuel constrained unit commitment. A practical hydro dynamic UC and loading model was developed for the British Columbia Hydro Power Authority.

DP decomposes the main problem into a series of single stage (period or hour) decision problems and optimization is performed at each stage. For the start period, the transition cost from the given initial state plus operating cost is computed for each state as accumulated cost. Then, some of the states with the lowest accumulated cost are saved and the DP process moves to the next period. For each state, the cost of the transition from each saved strategy at the previous period to that state is determined to find which strategy will provide the lowest accumulated cost. In this manner, strategies with the lowest cost from the initial state up to the considered period are saved until the final period is reached. Ultimately, the path with the lowest production cost is obtained and is tracked back.

DP can be performed in either direction, forward or backward. Forward DP begins at the initial stage and is more appropriate for UC than backward DP due to two advantages: 1. Forward DP is able to consider unit startup cost depending on offline hours; 2. Initial conditions can be specified easily.

In general, the forth DP has some disadvantages. While it is able to satisfy some constraints such as power balance, spinning reserve and minimum up and down time, additional constraints increase the complexity exponentially. Furthermore, as mentioned above, the DP approach suffers from a major drawback known as the curse of dimensionality, i.e., the number of states to be handled quickly gets out of control with the number of generating units in larger systems. In addition,

only a few of the potential strategies are saved, thus the optimal solution may not be found as illustrated in the following examples. Therefore, a strict priority was imposed to reduce the number of considered states in the problem dramatically. But this still does not guarantee that the obtained solution will be optimal.

At period t , state k , the minimum cost is computed by the following equation:

$$F_a(t, k) = P_{\text{cost}}(t, k) + \min[S_{\text{cost}}(t-1, L : t, k) + F_a(t-1, L)]$$

where

- $F_a(t, k)$ total accumulated fuel cost to arrive at state k of at period t ,
- $P_{\text{cost}}(t, k)$ production cost of state k at period t (excluding startup and shutdown cost),
- $S_{\text{cost}}(t-1, L : t, k)$ transition cost from state L at period $t-1$ to state k at period t ,
- $F_a(t-1, L)$ total accumulated fuel cost to arrive at state L of period $t-1$, $L \in \Omega_s$, and
- Ω_s the set of saved strategies.

For a system comprising N generating units considered over T periods, the number of states to search at each period is 2^N-1 for completed enumeration or a lower number denoted as X when a strict priority is imposed. Hence, X is less than 2^N-1 . The selection procedure is illustrated in Fig. 3.5, limiting the number of states to be searched $X = 5$ and strategies saved $S = 3$. At period $t-1$, 5 states (a, b, c, d, e) are searched for the lowest cost. The shadow circle indicates the saved states due to their lowest cost: c, d and e. At period t , for each state (a, b, c, d, e) is considered to determine which path from the three saved strategies (c, d, e) from period $t-1$ will provide the lowest accumulated cost. Saved strategies are states b, c and d. Then, the same calculation is performed for period $t+1$.

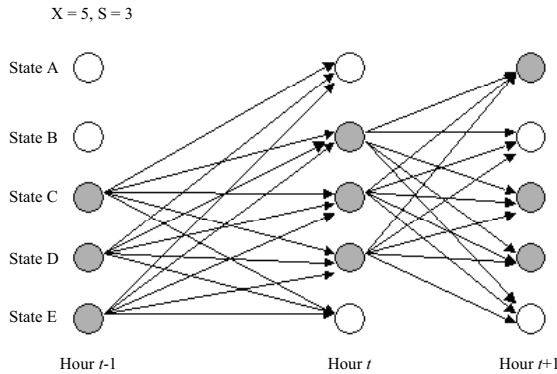


Fig. 3.5 State X and saved strategies S ($X = 5, S = 3$).

The DP procedure for UC is shown below.

- Step 1 Set $t = 1$.
- Step 2 Specify all feasible states satisfying spinning reserve (the state regarding spinning reserve constraint logically should satisfy power balance).
- Step 3 Perform economic dispatch using load at period t for all feasible states and calculate production cost, $P_{cost}(t, k)$.
- Step 4 If $t < T$, set $t = t + 1$. Go to Step 2.
- Step 5 Set $t = 1$.
- Step 6 Set $a = 1$
- Step 7 Set K be the a^{th} member in set Ω_f^t , $K \in \Omega_f^t$, where Ω_f^t is a set of feasible states at period t .
- Step 8 Examine transition paths from each saved previous path to state K at period t which satisfies minimum up and down time constraints. Compute transition cost $S_{cost}(t-1, L:t, k)$ and accumulated cost $F_a(t, k)$. Specify the lowest value strategy that arrives at State K .
- Step 9 $a = a + 1$, if $a < f$, go to Step 7, where f is the number of members in set Ω_f^t .
- Step 10 Sort the accumulated cost in the ascending order to obtain a sorted set. Save S strategies from the top of this set as set of Ω_s .
- Step 11 $t = t + 1$. If $t < T$, return to Step 6.
- Step 12 Select the lowest accumulated cost and trace back to obtain the optimal solution.

If a strict priority list is imposed, the full load average production cost of each state is calculated. Then, generating units are sorted according to their full load average production cost $FLAC_i$ in descending order to obtain a sorted set, Ω_{SFLAC} . The first state is obtained by committing the first unit in set Ω_{SFLAC} , and then the subsequent states are obtained by committing the generating units one by one until all units are committed. Thus, there will be only N states to be considered instead of all states. Only feasible states in set Ω_{SFLAC} will be saved in Ω_f^t .

Example 3.2

For system II, there are only three states to be searched when priority order is employed in DP as shown in Tables 3.14–3.17. For case 1, neglecting minimum up and down time, and spinning reserve constraints, at stage 1, there are only two feasible states, 1 and 3, since the summation of minimum capacities of committed units in state 7 is higher than the load demand. Whereas, at stages 2–5, state 1 is infeasible due to insufficient capacity to supply the load demand. At peak load period, there is only one feasible state, state 7, whereas states 1 and 3 could not provide sufficient capacity. At stage 1, the transition cost from initial state 1 at hour

Table 3.14 DP (based on priority list) solution I of system II of 3 generator units including minimum up and down time but neglecting spinning reserve.

State	$U_1U_2U_3$	$\sum_{i=1}^3 P_{i,min}$ (MW)	$\sum_{i=1}^3 P_{i,max}$ (MW)	Hour 0	Hour 1	Hour 2	Hour 3	Hour 4	Hour 5
7	111	250	1300		Infeasible due to $\sum_{i=1}^3 P_{i,min} > Load$	$[P_1, P_2, P_3] = [100 \ 200 \ 200]$ <1> from Path 1 Operating cost \$5,940	$[P_1, P_2, P_3] = [500 \ 400 \ 200]$ <3> from Path 3 Operating cost \$12,580	$[P_1, P_2, P_3] = [140 \ 260 \ 200]$ <7> from Path 7 Operating cost \$6,982	$[P_1, P_2, P_3] = [100 \ 100 \ 100]$ <3> from Path 3 Operating cost \$4,000
						Path 7 Infeasible Path 3 2,680+500 = 3,180 Path 1 2,100+800 = 2,900 Accumulated Cost \$8,840	Path 7 8,840+0 = 8,840 Path 3 7,870+500 = 8,370 Path 1 Infeasible Accumulated Cost \$20,950	Path 7 20,950+0 = 20,950 Path 3 Infeasible Path 1 Infeasible Accumulated Cost \$27,932	Path 7 27,932+0 = 27,932 Path 3 27,530+0 = 27,530 Path 1 Infeasible Accumulated Cost \$31,530
						$[P_1, P_2, P_3] = [0 \ 300 \ 200]$ <1> from Path 1 Operating Cost \$5,470	Infeasible due to insufficient capacity	$[P_1, P_2, P_3] = [0 \ 400 \ 200]$ <7> from Path 7 Operating Cost \$6,580	$[P_1, P_2, P_3] = [0 \ 125 \ 175]$ <3> from Path 3 Operating Cost \$3,425
3	011	250	1300		Accumulated Cost \$2,680 $[P_1, P_2, P_3] = [0 \ 0 \ 200]$ <1> from Path 1 Operating cost (\$) Path 1 0+0 = 0 Accumulated Cost \$2,100	Path 3 2,680+0 = 2,680 Path 1 2,100+300 = 2,400 Accumulated Cost \$7,870	Infeasible due to insufficient capacity	Path 7 20,950+0 = 20,950 Path 3 Infeasible Path 1 Infeasible Accumulated Cost \$27,530	Path 7 27,932+0 = 27,932 Path 3 27,530+0 = 27,530 Path 1 Infeasible Accumulated Cost \$30,955
						Infeasible due to insufficient capacity	Infeasible due to insufficient capacity	Infeasible due to insufficient spinning reserve	Infeasible due to insufficient capacity
1	001	50	200	Initial Status					
Load (MW)					200	500	1100	600	300

Table 3.15 DP (based on priority list) solution II of system II of 3 generator units including minimum up and down time but neglecting spinning reserve.

State	$U_1 U_2 U_3$	$\sum_{i=1}^3 P_{i,min}$ (MW)	$\sum_{i=1}^3 P_{i,max}$ (MW)	Hour 0	Hour 1	Hour 2	Hour 3	Hour 4	Hour 5
7	111	250	1300		Infeasible due to $\sum_{i=1}^3 P_{i,min} > Load$	[P, P, P] ₃ = [100 200 200] <1> from Path 1 Operating Cost \$5,940 Path 7 Infeasible Path 3 2,680+500 = 3,180 Path 1 2,100+800 = <u>2,900</u> Accumulated Cost \$8,840	[P, P, P] ₃ = [500 400 200] <3> from Path 3 Operating Cost \$12,580 Path 7 8,840+0 = 8,840 Path 3 7,870+500 = <u>8,370</u> Path 1 Infeasible Accumulated Cost \$20,950	[P, P, P] ₃ = [140 260 200] <7> from Path 7 Operating Cost \$6,982 Path 7 20,950+0 = <u>20,950</u> Path 3 Infeasible Path 1 Infeasible Accumulated Cost \$27,932	[P, P, P] ₃ = [100 100 100] <3> from Path 3 Operating Cost \$4,000 Path 7 27,932+0 = 27,932 Path 3 27,530+0 = <u>27,530</u> Path 1 Infeasible Accumulated Cost \$31,530
3	011	250	1300		[P, P, P] ₃ = [0 100 100] <1> from Path 1 Operating Cost \$2,480 Path 1 0+200 = <u>200</u> Accumulated Cost \$2,680	[P, P, P] ₃ = [0 300 200] <1> from Path 1 Operating Cost \$5,470 Path 7 Infeasible Path 3 2,680+0 = 2,680 Path 1 2,100+300 = <u>2,400</u> Accumulated Cost \$7,870	Infeasible due to insufficient capacity	Infeasible due to minimum up and down time constraints	[P, P, P] ₃ = [0 125 175] <7> from Path 7 Operating Cost \$3,425 Path 7 27,932+0 = <u>27,932</u> Path 3 Infeasible Path 1 Infeasible Accumulated Cost \$31,357
1	001	50	200	Initial Status	[P, P, P] ₃ = [0 0 200] <1> from Path 1 Operating cost \$2,100 Path 1 0+0 = 0 Accumulated Cost \$2,100	Infeasible due to insufficient capacity	Infeasible due to insufficient capacity	Infeasible due to insufficient spinning reserve	Infeasible due to insufficient capacity
Load (MW)					200	500	1100	600	300

Table 3.16 DP (based on priority list) solution III of system II of 3 generator units including minimum up and down time but neglecting spinning reserve.

State	$U_1 \cup U_2 \cup U_3$	$\sum_{i=1}^3 P_{i,min}$ (MW)	$\sum_{i=1}^3 P_{i,max}$ (MW)	Hour 0	Hour 1	Hour 2	Hour 3	Hour 4	Hour 5
7	111	250	1300		Infeasible due to $\sum_{i=1}^3 P_{i,min} > Load$	[P, P ₂ , P ₃] = [100 200 200] <1> from Path 1 Operating Cost \$5,940 Path 7 Infeasible Path 3 2,680+500 = <u>3,180</u> Path 1 Infeasible Accumulated Cost \$9,120	[P, P ₂ , P ₃] = [500 400 200] <3> from Path 3 Operating Cost \$12,580 Path 7 9,120+0 = 8,840 Path 3 8,150+500 = <u>8,650</u> Path 1 Infeasible Accumulated Cost \$21,230	[P, P ₂ , P ₃] = [140 260 200] <7> from Path 7 Operating Cost \$6,982 Path 7 21,230+0=21,230 Path 3 Infeasible Path 1 Infeasible Accumulated Cost \$28,212	[P, P ₂ , P ₃] = [100 100 100] <7> from Path 7 Operating Cost \$4,000 Path 7 27,932+0 = <u>28,212</u> Path 3 Infeasible Path 1 Infeasible Accumulated Cost \$32,212
						[P, P ₂ , P ₃] = [0 300 200] <1> from Path 1 Operating Cost \$5,470 Path 7 Infeasible Path 3 2,680+0= <u>2,680</u> Path 1 Infeasible Accumulated Cost \$8,150	Infeasible due to insufficient capacity Infeasible due to insufficient capacity Infeasible due to insufficient spinning reserve	Infeasible due to insufficient spinning reserve Infeasible due to insufficient spinning reserve Infeasible due to insufficient spinning reserve	[P, P ₂ , P ₃] = [0 125 175] <7> from Path 7 Operating Cost \$3,425 Path 7 28,212+0= <u>28,212</u> Path 3 Infeasible Path 1 Infeasible Accumulated Cost \$31,637
									Infeasible due to insufficient capacity
3	011	250	1300		[P, P ₂ , P ₃] = [0 100 100] <1> from Path 1 Operating Cost \$2,480 Path 1 0+200 = <u>200</u> Accumulated Cost \$2,680				
1	010	50	200	Initial Status	Infeasible due to spinning reserve				
Load (MW)				200	500	1100	600	300	

Table 3.17 DP solution IV of system II of 3 generator units including spinning reserve and, minimum up and down time, with 7 saving lowest cost strategy.

State	$U_1 U_2 U_3$	$\sum_{i=1}^3 P_{i,min}$ (MW)	$\sum_{i=1}^3 P_{i,max}$ (MW)	Hour 0	Hour 1	Hour 2	Hour 3	Hour 4	Hour 5
7	111	250	1300	Infeasible due to $\sum_{i=1}^3 P_{i,min} > Load$	Infeasible due to $<3>$ from Path 3 Path 7 Infeasible Path 6 Infeasible Path 5 2,970+300 = 3,270 Path 4 Infeasible Path 3 2,680+500 = <u>3,180</u> Path 2 Infeasible Accumulated Cost \$9,120 [P, P, P] ₂ = [200, 300, 0]	Infeasible due to $<3>$ from Path 3 Path 7 Infeasible Path 6 Infeasible Path 5 2,970+300 = 3,270 Path 4 Infeasible Path 3 2,680+500 = <u>3,180</u> Path 2 Infeasible Accumulated Cost \$9,120 [P, P, P] ₂ = [200, 300, 0]	Infeasible due to $<6>$ from Path 6 Operating Cost \$5,940 Path 7 9,120+0 = 9,120 Path 6 8,070+100 = <u>8,170</u> Path 5 8,750+300 = 9,050 Path 4 8,880+400 = 9,280 Path 3 8,150+500 = 8,650 Path 2 Infeasible Accumulated Cost \$20,750 [P, P, P] ₂ = [500, 400, 200]	Infeasible due to $<7>$ from Path 7 Operating Cost \$6,982 Path 7 20,750+0 = <u>20,750</u> Path 6 Infeasible Path 5 Infeasible Path 4 Infeasible Path 3 Infeasible Path 2 Infeasible Accumulated Cost \$ 27,732 [P, P, P] ₂ = [260, 340, 0]	Infeasible due to $<7>$ from Path 7 Operating Cost \$4,000 Path 7 27,732+0 = <u>27,732</u> Path 6 Infeasible Path 5 Infeasible Path 4 Infeasible Path 3 Infeasible Path 2 Infeasible Accumulated Cost \$31,732 [P, P, P] ₂ = [100, 200, 0]
6	110	200	1100	Infeasible due to $<4>$ from Path 1 Operating Cost \$2,850 Path 1 0+500 = <u>500</u>	Infeasible due to $<4>$ from Path 1 Operating Cost \$2,850 Path 1 0+500 = <u>500</u>	Infeasible due to $<4>$ from Path 1 Operating Cost \$5,950 Path 6 3,350+0 = 3,350 Path 5 2,970+300 = 3,270 Path 4 2,880+300 = 3,180 Path 3 2,680+500 = 3,180 Path 2 2,520+500 = <u>3,020</u> Accumulated Cost \$8,070 [P, P, P] ₂ = [300, 0, 200]	Infeasible due to insufficient spinning reserve Path 7 Infeasible Path 6 Infeasible Path 5 Infeasible Path 4 Infeasible Path 3 Infeasible Path 2 Infeasible Accumulated Cost \$7,042 [P, P, P] ₂ = [400, 0, 200]	Infeasible due to $<5>$ from Path 5 Operating Cost \$6,920 Path 7 20,750+0+0 = 20,750 Path 6 Infeasible Path 5 Infeasible Path 4 Infeasible Path 3 Infeasible Path 2 Infeasible Accumulated Cost \$ 27,670 [P, P, P] ₂ = [100, 0, 200]	Infeasible due to $<5>$ from Path 5 Operating Cost \$3,620 Path 7 27,732+0 = 27,732 Path 6 Infeasible Path 5 27,670+0 = <u>27,670</u> Path 4 Infeasible Path 3 Infeasible Path 2 Infeasible Accumulated Cost \$31,290 [P, P, P] ₂ = [100, 0, 200]
5	101	250	1300	Infeasible due to $<4>$ from Path 1 Operating Cost \$2,670 Path 1 0+300 = <u>300</u>	Infeasible due to $<4>$ from Path 1 Operating Cost \$2,670 Path 1 0+300 = <u>300</u>	Infeasible due to insufficient capacity Path 7 Infeasible Path 6 Infeasible Path 5 2,970+0=2,970 Path 4 Infeasible Path 3 Infeasible Path 2 Infeasible Accumulated Cost \$8,750 [P, P, P] ₂ = [300, 0, 200]	Infeasible due to insufficient capacity Path 7 Infeasible Path 6 Infeasible Path 5 2,970+0=2,970 Path 4 Infeasible Path 3 Infeasible Path 2 Infeasible Accumulated Cost \$ 27,670 [P, P, P] ₂ = [100, 0, 200]	Infeasible due to $<5>$ from Path 5 Operating Cost \$3,620 Path 7 27,732+0 = 27,732 Path 6 Infeasible Path 5 27,670+0 = <u>27,670</u> Path 4 Infeasible Path 3 Infeasible Path 2 Infeasible Accumulated Cost \$31,290 [P, P, P] ₂ = [100, 0, 200]	Infeasible due to $<5>$ from Path 5 Operating Cost \$3,620 Path 7 27,732+0 = 27,732 Path 6 Infeasible Path 5 27,670+0 = <u>27,670</u> Path 4 Infeasible Path 3 Infeasible Path 2 Infeasible Accumulated Cost \$31,290 [P, P, P] ₂ = [100, 0, 200]

Table 3.17 contd....

Table 3.17 contd....

State	$U_1 U_2 U_3$	$\sum_{i=1}^3 P_{i,min}$ (MW)	$\sum_{i=1}^3 P_{i,max}$ (MW)	Hour 0	Hour 1	Hour 2	Hour 3	Hour 4	Hour 5
4	100	200	1100		$[P_1, P_2, P_3] = [200 \ 0 \ 0]$ <1> from Path 1 Operating Cost \$2,580 Path 1 0+300=300 Path 7 Infeasible Path 6 Infeasible Path 5 2970+0 = 2970 Path 4 2880+0 = 2880 Path 3 Infeasible Path 2 Infeasible Accumulated Cost \$8,880	$[P_1, P_2, P_3] = [500 \ 0 \ 0]$ <4> from Path 4 Operating Cost \$6,000 Path 7 Infeasible Path 6 Infeasible Path 5 2970+0 = 2970 Path 4 2880+0 = 2880 Path 3 Infeasible Path 2 Infeasible Accumulated Cost \$8,880	Infeasible due to insufficient capacity	$[P_1, P_2, P_3] = 600 \ 0 \ 0$ Infeasible Operating Cost \$7,220 Path 7 Infeasible Path 6 Infeasible Path 5 Infeasible Path 4 Infeasible Path 3 Infeasible Path 2 Infeasible Accumulated Cost \$31,350	$[P_1, P_2, P_3] = 300 \ 0 \ 0$ <5> from Path 5 Operating Cost \$3,680 Path 7 27,732+0 = 27,732 Path 6 Infeasible Path 5 27,670+0 = 27,670 Path 4 Infeasible Path 3 Infeasible Path 2 Infeasible Accumulated Cost \$31,350
3	011	250	1300		$[P_1, P_2, P_3] = [0 \ 100 \ 100]$ <1> from Path 1 Operating Cost \$2,480 Path 1 0+200 = 200 Path 7 Infeasible Path 6 Infeasible Path 5 Infeasible Path 4 Infeasible Path 3 2,680+0 = <u>2,680</u> Path 2 Infeasible Accumulated Cost \$8,150	$[P_1, P_2, P_3] = [0 \ 300 \ 200]$ <3> from Path 3 Operating Cost \$5,470 Path 7 Infeasible Path 6 Infeasible Path 5 Infeasible Path 4 Infeasible Path 3 2,680+0 = <u>2,680</u> Path 2 Infeasible Accumulated Cost \$8,150	Infeasible due to insufficient capacity	Infeasible due to insufficient spinning reserve Operating Cost \$3,425 Path 7 27,732+0 = <u>27,732</u> Path 6 Infeasible Path 5 Infeasible Path 4 Infeasible Path 3 Infeasible Path 2 Infeasible Accumulated Cost \$31,157	$[P_1, P_2, P_3] = [0 \ 125 \ 175]$ <7> from Path 7 Operating Cost \$3,425 Path 7 27,732+0 = <u>27,732</u> Path 6 Infeasible Path 5 Infeasible Path 4 Infeasible Path 3 Infeasible Path 2 Infeasible Accumulated Cost \$31,157
2					$[P_1, P_2, P_3] = [100 \ 100 \ 0]$ <1> from Path 1 Operating Cost \$2,850 Path 1 0+500=500 Accumulated Cost \$3,350	$[P_1, P_2, P_3] = [100 \ 100 \ 0]$ <1> from Path 1 Operating Cost \$2,850 Path 1 0+500=500 Accumulated Cost \$3,350	Infeasible due to insufficient capacity	Infeasible Operating Cost \$3,370 Path 7 27,732+0 = <u>27,732</u> Path 6 Infeasible Path 5 Infeasible Path 4 Infeasible Path 3 Infeasible Path 2 Infeasible Accumulated Cost \$31,102	$[P_1, P_2, P_3] = [0 \ 300 \ 0]$ <7> from Path 7 Operating Cost \$3,370 Path 7 27,732+0 = <u>27,732</u> Path 6 Infeasible Path 5 Infeasible Path 4 Infeasible Path 3 Infeasible Path 2 Infeasible Accumulated Cost \$31,102
1	001	50	200	Initial Status	Infeasible due to spinning reserve Accumulated Cost \$3,350	Infeasible due to insufficient capacity	Infeasible due to insufficient capacity	Infeasible due to insufficient capacity	Infeasible due to insufficient capacity
Load (MW)					200	500	1100	600	300

0 to be afforded to arrive at state 1 at hour 1 and state 3 at hour 1 is computed from initial state 1 at hour 0 and added to the operation cost to form the accumulated cost for each state. The path and accumulated cost for each state (states 1 and 3) are saved as strategies. At stage 2, considering state 7, the transition cost from saved strategies 1 and 3 to arrive at state 7 are compared to find the lowest accumulated cost which are then added to the operation cost of state 7 (\$5,940) to form the accumulated cost of strategy 7 at stage 2. In this case, strategy 1 is selected since it provides lower transition cost (\$2,900). Thus, accumulated cost of strategy 7 at stage 2 is \$8,840. Similarly, strategy 3 at stage 2 is calculated. Its resulting accumulated cost is \$7,870. Strategies 3 and 7 at stage 2 are saved. The subsequent stages 3, 4 and 5, are calculated the same way, leading, ultimately, to the total production cost of each feasible path. Strategy 1, offering the lowest cost of \$30,955, is chosen. Subsequently, the path is tracked backward to the initial stage to obtain the final solution consisting of states 1, 3, 7, 3 and 3 for stages 1–5, respectively.

For case 2, minimum up and down time constraints are included but spinning reserve constraints are neglected. The details of the computation are shown in Table 3.15. A difference from case 1 occurs at stage 4 where state 3 is infeasible due to minimum down time constraints. This results in a different solution consisting of states 1, 3, 7, 7 and 3 for stages 1–5, respectively, and results in production cost of \$31,357.

For case 3, one more constraint, the spinning reserve constraint, is added to the constraints of case 2. The details of the computation are shown in Table 3.16. Case 3 differs from case 2 already at stage 1 where state 1 is infeasible due to insufficient spinning reserve. This results in a different strategic solution with states 3, 3, 7, 7 and 3 for the stages 1–5, respectively, that entails \$31,637 of production cost.

For system II, when there is no priority order employed in DP, 7 states are to be considered as shown in Table 3.17 including minimum up and down time and spinning reserve. This increases the number of states to be searched dramatically which consequently leads to a longer computation time. However, it could provide the solution with the lowest production cost of \$31,102 as shown in Fig. 3.6. The numbers in a three-level square are operation cost of that state, transition cost and accumulated cost. In the forward DP approach, the calculation proceeds from stage 1 to stage 5, and accumulated cost and its corresponding path are saved. At the final stage, the least accumulated cost is \$31,120 in state 2. Then, the path is tracked backward to the initial stage, leading to the final solution consisting of states 2, 6, 7, 7 and 2 for stages 1–5, respectively.

For system II, DP solutions based on priority order are similar to those from priority list as shown in Table 3.17 whereby the DP solution without imposing a priority order, including minimum up and down time and spinning reserve constraints results in \$535 lower production cost than that of the priority list solution. However, the DP solution depends on the number of saved strategies which leads to different production cost as shown in Table 3.17 and Fig. 3.6. If only one strategy is saved the solution will consist of states 2, 6, 7, 5 and 5 for stages 1–5, respectively, with \$31,290 production cost.

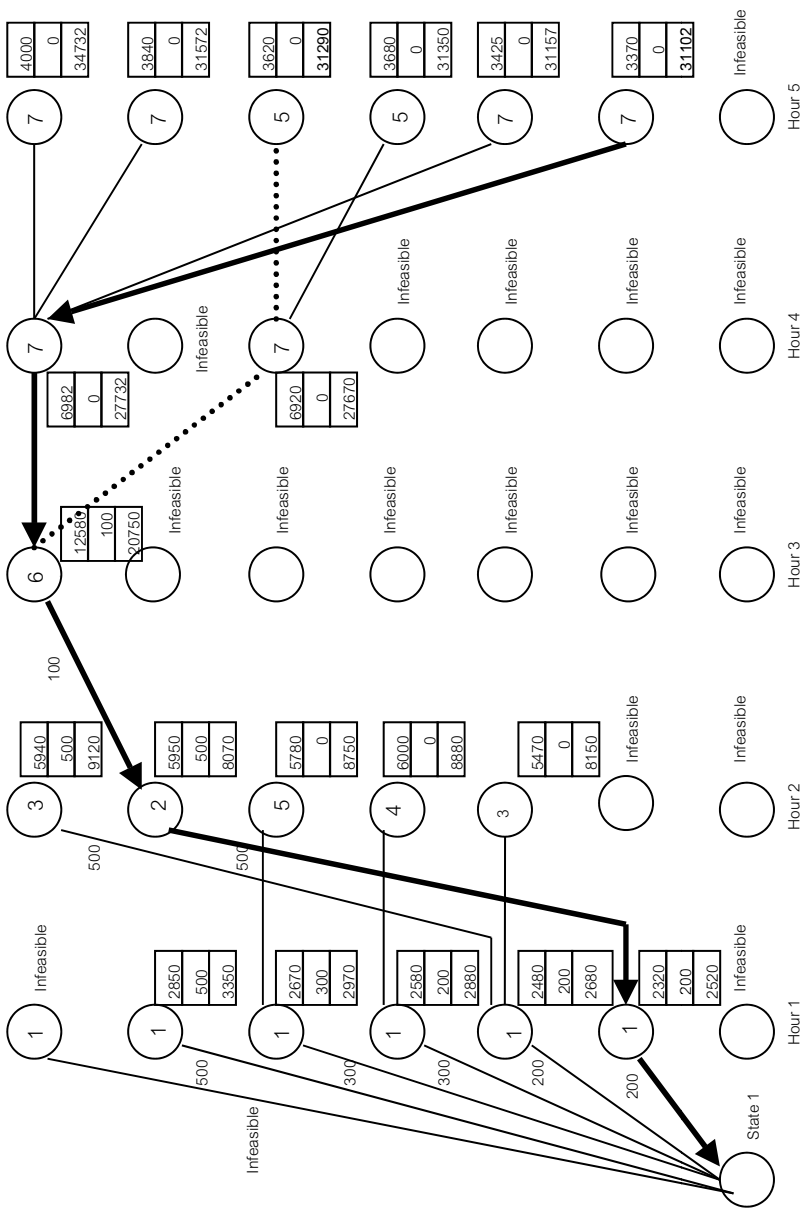


Table 3.18 Comparison of results of priority list and dynamic programming methods under different conditions.

Method		Saving lowest cost strategies	Constraints		Cost (\$)
			Min. up and Min. down Time (h)	Spinning reserve requirement (% of Load)	
Priority List		-	-	-	30,955
		-	2	-	31,357
		-	2	10	31,637
Dynamic Programming	Priority	3	-	-	30,955
		3	2	-	31,357
		3	2	10	31,637
	Non-priority	1	2	10	31,290
		2	2	10	31,102
		3	2	10	31,102
		7	2	10	31,102

3.3.4 Lagrangian Relaxation

The LR technique is a mathematical tool for mixed integer programming problems. The LR optimization technique decomposes the master problem into subproblems which are solved independently. Each subproblem is defined as a single unit commitment. The problems are coupled by Lagrangian multipliers added to the master problem to yield a dual problem. The primal function is always greater than or equal to the dual function. The difference between the primal and dual functions yields the duality gap for which the primal function is an upper bound. The duality gap provides a measure of the near optimality of the solution. The Lagrangian multipliers are updated at the master problem level by maximizing the dual function. Then, the Lagrangian multipliers are passed to the subproblems which are solved by forward DP. Thereafter, the subproblem solutions are fed back to the master problem, and the Lagrangian multipliers are updated. These processes are performed iteratively until the solutions converge and a near optimal solution is obtained. To reduce computation time for feasible search, only spinning reserve constraints were used for examining the feasibility of a solution. For short term UC problems, the multipliers are generally updated by a subgradient method with scaling factors and tuning constants that are determined heuristically [7].

The principle is to use Lagrange multipliers to relax system wide demand and reserve requirements, and decompose the problem into unit sub-problems. Then these multipliers are updated, most commonly by subgradient (SG) methods. LR-based UC approaches have used subgradient methods as dual maximization tool because of its simplicity and low per-iteration computing time. However, LR with subgradient optimization is considered an unstable technique for UC models since it depends on parameter settings and initial values of Lagrange multipliers. Different parameter settings or different initial values of Lagrange multipliers

may lead to different and multiple solutions and consequently lead to conflicts of interest when LR-based methods are used in a competitive environment [8]. Therefore, other methods are proposed to update Lagrangian multipliers. Reduced Complexity Bundle Method (RCBM), Bundle Trust Region Method (BTRM) and Dynamically Constrained Cutting Plane (DC-CP) have been proposed for the update of Lagrange multipliers. Nevertheless, all proposed cutting plane methods still have the disadvantage that parameters need to be carefully tuned whereas updating Lagrange multipliers by an interior point/cutting plane (IP/CP) method is free of parameter tuning requirements. The subgradient method generally needs a large number of iterations to converge to near the dual optimum, but per iteration multiplier update is very fast. However, the adaptive subgradient method was used in co-operation with the new Lagrangian multiplier initialization in [9]. This produced high quality initial feasible multipliers requiring much lower numbers of iterations to converge, leading to short computation times. In addition, each sub-problem was solved by new on/off decision criteria instead of dynamic programming. For long term UC problems, the multipliers are updated by variable metric method to prevent the solution from oscillating.

The quality of the final LR solution depends on the sensitivity of the commitment to Lagrangian multipliers. Augmented Lagrangian relaxation algorithms were originally developed to overcome the oscillation in LR solutions caused by linearized cost functions. An augmented Lagrangian algorithm was formed by incorporating a quadratic penalty function only with power balance equality constraints into the Lagrangian function. ALR has been applied in some recent researches, using penalty functions with spinning reserve and emission inequality constraints. Nevertheless, the convergence rate was related to penalty coefficients, and it was still difficult to select those effective penalty coefficients that were suitable for different types of sub-problems at the same time [10].

A mathematical method for dealing with ramp rate limits in UC and the rotor fatigue effect in economic scheduling is presented. The ramp rate constraint is an important factor of the feasibility of the UC solution. However, ramping constraints in UC problems require dramatically enlarged state spaces for dynamic programming to solve each unit sub-problem in the LR algorithm. The total number of states is the sum of the numbers of down states, ramp up states, up states and ramp down states. Moreover, additional Lagrangian multipliers corresponding to ramping constraints at each stage and each unit were proposed recently. These extra Lagrangian multipliers cause additional computational burden, leading to a slower convergence rate. In some research work, optimal power flow is incorporated in the UC formulation. Lagrangian multipliers are interpreted as the prices that the system is willing to pay to preserve the power balance and fulfill the spinning reserve requirement during each period neglecting ramp rate constraints.

As mentioned earlier, the LR method solves the UC problem by decomposing the main problem into sub-problems which are coupled by the Lagrangian

multipliers λ^t and μ^t . LR solves the UC problem by relaxing or temporarily ignoring the coupling constraints and solving the problem as if they did not exist. This is done through a dual optimization procedure attempting to reach the constrained optimum by maximizing the Lagrangian function:

$$L(P, U, \lambda, \mu, \gamma) = F(P_i^t, U_i^t) + \sum_{t=1}^T \lambda^t (P_{load}^t - \sum_{i=1}^N P_i^t U_i^t) \\ + \sum_{t=1}^T \mu^t (R^t + P_{load}^t - \sum_{i=1}^N P_{i,max} U_i^t)$$

with respect to nonnegative Lagrangian multipliers λ^t and μ^t . The Lagrangian function is rewritten as follows:

$$L = \sum_{i=1}^N \sum_{t=1}^T \{ [F_i(P_i^t) + ST_{i,t}(1 - U_i^{t-1})] U_i^t - \lambda^t P_i^t U_i^t - \mu^t P_{i,max} U_i^t \} \\ + \sum_{t=1}^T (\lambda^t P_{load}^t + \mu^t (P_{load}^t + R^t)) \quad (3.20)$$

The term $\sum_{t=1}^T \{ [F_i(P_i^t) + ST_{i,t}(1 - U_i^{t-1})] U_i^t - \lambda^t P_i^t U_i^t - \mu^t P_{i,max} U_i^t \}$ can be minimized separately for each generating unit, when the coupling constraints are temporarily ignored. Then the minimum of the Lagrangian function is solved for each generating unit, that is

$$\underset{P_i^t, U_i^t}{Min} L(P, U, \lambda, \mu, \gamma) \\ = \sum_{i=1}^N \min \sum_{t=1}^T \{ [F_i(P_i^t) + ST_{i,t}(1 - U_i^{t-1})] U_i^t - \lambda^t P_i^t U_i^t - \mu^t P_{i,max} U_i^t \} \quad (3.21)$$

subject to the constraints (3.5), (3.6) and (3.7). This could be accomplished by performing dynamic programming as described in Section A.1.

A. Dual Problem Solution

In the conventional Lagrangian relaxation method, dynamic programming (DP) is used to solve the dual optimization. Recently, new on/off decision criteria were proposed in [9] to obtain a dual solution instead of DP. This results in shorter computation time.

1) Dynamic Programming

In the Lagrangian relaxation method, a dual solution is obtained by using DP for each unit separately. This can be visualized in Fig.3.7 showing the only two possible states for unit i (i.e., $U_i^t = 0$ or 1):

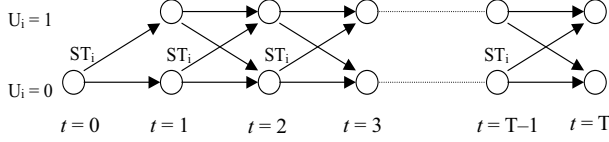


Fig. 3.7 Search paths of each unit in dynamic programming.

In the $U_i^t = 0$ state, the value of the function to be minimized is trivial (i.e., it equals zero), at the state where $U_i^t = 1$, the function to be minimized is (the start-up cost and $\mu' P_{i,max}$ are dropped here since the minimization is with respect to P_i^t) $\min[F_i(P_i^t) - \lambda^t P_i^t]$. To find the dual power of each generating unit, the optimality condition and actual dual power are

$$\frac{d}{dP_i^t} [F_i(P_i^t) - \lambda^t P_i^t] = 0 \quad (3.22)$$

$$\frac{dF_i(P_i^{t,opt})}{dP_i^t} = \lambda^t \quad (3.23)$$

The dual power output is obtained

$$P_i^{t,opt} = \frac{\lambda^t - b_i}{2c_i} \quad (3.24)$$

There are three cases to check optimal power output $P_i^{t,opt}$ at period t against its limits:

1. If $P_i^{t,opt} < P_{i,low}^t$, $P_i^t = P_{i,low}^t$
2. If $P_{i,low}^t \leq P_i^{t,opt} \leq P_{i,high}^t$, $P_i^t = P_i^{t,opt}$
3. If $P_i^{t,opt} > P_{i,high}^t$, $P_i^t = P_{i,high}^t$

Dynamic programming is used to determine the optimal schedule of each unit over the scheduled time period. More specifically, for each state and each stage, on/off decision making is needed to select the lowest cost by comparing the combination of the start up cost and the accumulated costs of two historical routes. At period t , the dual power as calculated by (3.24) and within the limits in (3.4), will be substituted in the negative fictitious revenue (NFR),

$[F_i(P_i') + ST_{i,t}(1 - U_i^{t-1})]U_i^t - \lambda^t P_i' U_i^t - \mu^t P_{i,\max} U_i^t$, for each generating unit. Then, dynamic programming searches for the optimal scheduling for each unit to obtain the lowest value of the NFR term subject to minimum up and down time constraints. DP considers the unit status from the first to the last period ($t = T$), one unit at a time, each generating unit, one unit at a time, from the first unit to the last, and searches for the route that minimizes the NFR term. After all units are determined by DP at each stage, the spinning reserve constraints are calculated by evaluating all unit statuses as obtained by the dynamic programming process. If constraints are violated, the value of Lagrangian multipliers, λ^t and μ^t , at the corresponding stage will be adjusted as described in the subsequent Section D. In the next iteration, DP considers each generating unit with the new values of λ^t and μ^t ; some more generators will be committed at those stages that need whereas some excess units may be de-committed at other stages. The unit sub-problem minimization by DP is illustrated in the following examples.

Example 3.3

Assume the Lagrangian multipliers shown in Table 3.19 are used in the test case of system II, spinning reserve requirement is zero.

For unit #1, the details of computation are shown in Table 3.20 whereas the selected path including minimum up and down time constraint is shown in Fig. 3.8. The numbers in a three-level rectangular are NFR term, $a_i + b_i P_i' + c_i (P_i')^2 - \lambda^t P_i'$, transition cost and accumulated cost shown ordered from the highest level to the lowest level at each stage. Eventually, the total accumulated cost is -\$120. Similarly, the computation for unit #2 and unit #3 is shown in Tables 3.21 and 3.22, whereas the selected paths are shown in Figs. 3.9 and 3.10, respectively, the corresponding accumulated cost are \$860 and \$750 respectively.

Table 3.19 The value of Lagrangian multipliers.

	Hour 1	Hour 2	Hour 3	Hour 4	Hour 5
λ^t (\$/MWh)	9	11.5	13	12	10

Table 3.20 Subproblem of unit #1 solved by DP approach.

Hour	λ^t (\$/MWh)	P_i (MW)	$a_i + b_i P_i' + c_i (P_i')^2$ (\$)	$\lambda^t P_i'$ (\$)	U_i^t
Hour 1	9	100	1520	900	0
Hour 2	11.5	375	4531.25	4312.5	0
Hour 3	13	700	8480	9100	1
Hour 4	12	500	6000	6000	1
Hour 5	10	100	1520	1000	0

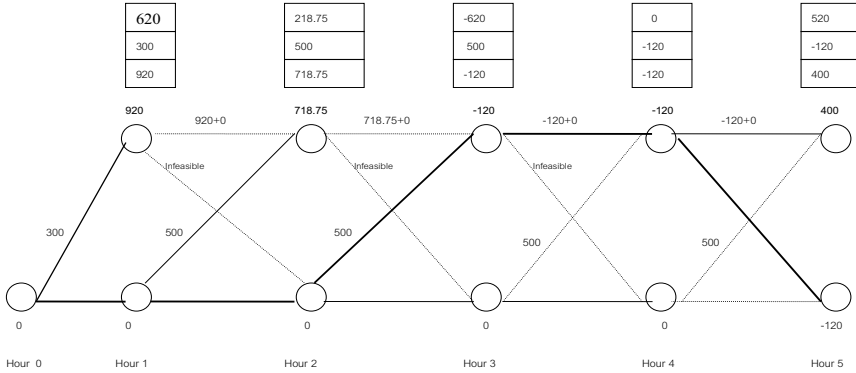


Fig. 3.8 The selected path for unit #1 solved by DP approach.

Table 3.21 Subproblem of unit#2 solved by DP approach.

Hour	λ' (\$/MWh)	P_i (MW)	$a_i + b_i P_i' + c_i (P_i')^2$ (\$)	$\lambda' P_i'$ (\$)	U_i'
Hour 1	9	100	1330	900	0
Hour 2	11.5	400	4480	4600	1
Hour 3	13	400	4480	5200	1
Hour 4	12	400	4480	4800	1
Hour 5	10	166.7	1983.7	1667	0

Table 3.22 Subproblem of unit#3 solved by DP approach.

Hour	λ' (\$/MWh)	P_i (MW)	$a_i + b_i P_i' + c_i (P_i')^2$ (\$)	$\lambda' P_i'$ (\$)	U_i'
Hour 1	9	100	1150	900	0
Hour 2	11.5	200	2100	2300	0
Hour 3	13	200	2100	2600	1
Hour 4	12	200	2100	2400	1
Hour 5	10	200	2100	2000	0

2) On/Off Decision Criteria

Using minimal startup cost as on/off decision criterion to decide the schedule of each unit was proposed in [9]. The scaled down startup cost is calculated by dividing the startup cost by the minimum up time. The dual power, calculated by (3.24) and within the limits, is substituted in the on/off decision criterion (ODC),

$$[F_i(P_i') + \frac{ST_{i,t}(1 - U_i^{t-1})}{T_{i,up}} - \lambda' P_i' - \mu' P_{i,max}].$$

If the full startup cost is used in the above term, ODC, the units with high startup cost and low operation cost will be committed only when λ' and μ' are large enough. This would therefore result in higher production cost due to the over-

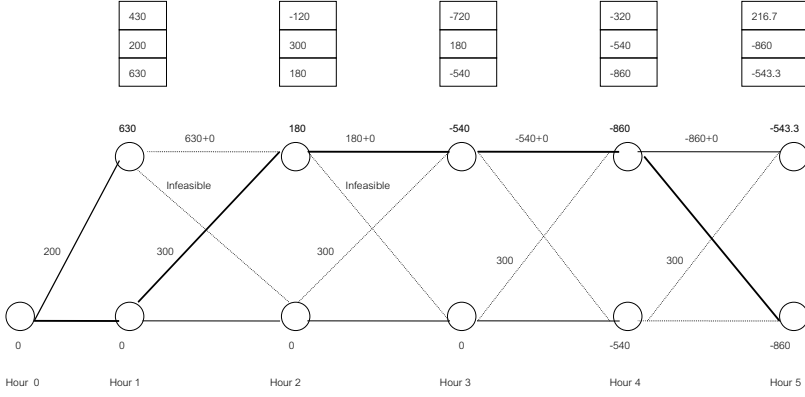


Fig. 3.9 The selected path for unit #2 solved by DP approach.

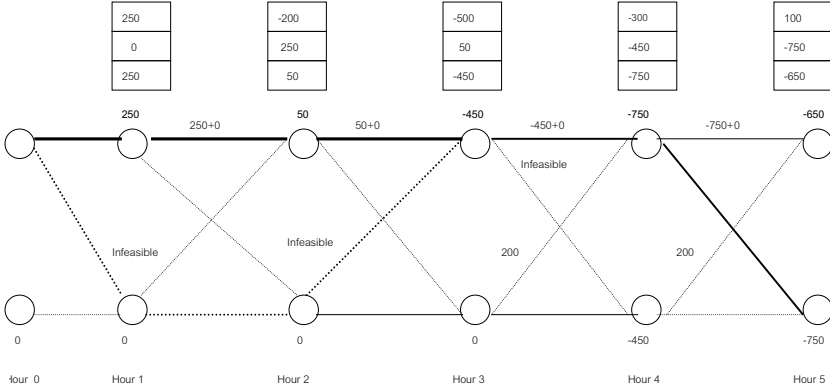


Fig. 3.10 The selected path for unit #3 solved by DP approach.

commitment. With high λ^t and μ^t values, it is more difficult for LR to converge to the optimal solution. The Lagrangian multipliers λ^t and μ^t at period t have an economic interpretation: λ^t is the marginal price of the next MWh of energy and μ^t is the marginal price of the next MW of spinning reserve at period t . $\lambda^t P_i^t + \mu^t P_{i,\max}$,

is the fictitious total cost of unit i , whereas $F_i(P_i^t) + \frac{ST_{i,t}(1-U_i^{t-1})}{T_{i,up}}$, is the unit i generator fuel cost with reduced startup cost.

To minimize the above term (ODC) at each stage, if

$$[F_i(P_i^t) + \frac{ST_{i,t}(1-U_i^{t-1})}{T_{i,up}} - \lambda^t P_i^t - \mu^t P_{i,\max}] \leq 0$$

or

$$F_i(P_i^t) + \frac{ST_{i,t}(1-U_i^{t-1})}{T_{i,up}} \leq \lambda^t P_i^t + \mu^t P_{i,\max}$$

this unit will be committed if it does not violate the minimum down time constraint ($U_i^t = 1$). Otherwise, this unit will not be committed if it does not violate the minimum up time constraint ($U_i^t = 0$).

Example 3.4

The same test case, system II, as in DP in Section 3.3.4.A.1 is used. The details of the computation for solving the unit sub-problems for units #1–3 is shown in Tables 3.23–3.25.

Table 3.23 Subproblem of unit #1 solved by on/off decision criteria.

Hour	λ^t (\$/MWh)	P_i (MW)	$F_i(P_i^t) + \frac{ST_{i,t}(1-U_i^{t-1})}{T_{i,up}}$ (\$)	$\lambda^t P_i^t$ (\$)	ODC (\$)	u_i^t
Hour 1	9	100	1820	900	920	0
Hour 2	11.5	375	5031.2	4312.5	718.7	0
Hour 3	13	700	8980	9100	-120	1
Hour 4	12	500	6000	6000	0	1
Hour 5	10	100	1520	1000	520	0

Table 3.24 Subproblem of unit #2 solved by on/off decision criteria.

Hour	λ^t (\$/MWh)	P_i (MW)	$F_i(P_i^t) + \frac{ST_{i,t}(1-U_i^{t-1})}{T_{i,up}}$ (\$)	$\lambda^t P_i^t$ (\$)	ODC (\$)	u_i^t
Hour 1	9	100	1430	900	530	0
Hour 2	11.5	400	4630	4600	30	0
Hour 3	13	400	4630	5200	-570	1
Hour 4	12	400	4480	4800	-320	1
Hour 5	10	166.7	1983.7	1667	316.7	0

Table 3.25 Subproblem of unit #3 solved by on/off decision criteria.

Hour	λ^t (\$/MWh)	P_i (MW)	$F_i(P_i^t) + \frac{ST_{i,t}(1-U_i^{t-1})}{T_{i,up}}$ (\$)	$\lambda^t P_i^t$ (\$)	u_i^t
Hour 1	9	100	1150	900	0
Hour 2	11.5	200	2200	2300	0
Hour 3	13	200	2200	2600	1
Hour 4	12	200	2100	2400	1
Hour 5	10	200	2100	2000	0

The comparison of the unit schedules as from DP and ODC is shown in Table 3.26. The unit status of unit #1 is similar to that of DP in Section A.1. However, the unit schedule of unit #2 and unit #3 is different from that of DP at stage 2. U_2^2 determined by ODC is '0' whereas U_2^2 determined by DP is '1'. The corresponding

value of the term $\sum_{t=1}^T \{[F_i(P_i^t) + ST_{i,t}(1 - U_i^{t-1})]U_i^t - \lambda^t P_i^t U_i^t - \mu^t P_{i,\max} U_i^t\}$

(−\$860) of the DP solution is lower than that of ODC (−\$740). Thus, DP could provide a lower value of the objective term of the unit sub-problem. More details of the comparison are discussed in the section 3.3.4.F.

Table 3.26 Comparison of the unit status of each unit sub-problem as solved by DP and ODC.

	DP solution						ODC solution					
	Unit status					NFR	Unit status					NFR
	H 1	H 2	H 3	H 4	H 5		H 1	H 2	H 3	H 4	H 5	
U #1	0	0	1	1	0	−120	0	0	1	1	0	−120
U #2	0	1	1	1	0	−860	0	0	1	1	0	−740
U #3	1	1	1	1	0	−750	0	0	1	1	0	−550

B. Initialization

The initial values of Lagrangian multipliers are very critical to the LR solution since they may cause an elongated computation time to reach a solution or even prevent LR from reaching the optimal solution [8]. Different initial values may also lead LR to different solutions. In some research works, the initial multiplier λ' was set to the hourly system marginal cost of the schedule to satisfy the power balance constraint and the initial multiplier μ' was set to zero, leading to an infeasible initial solution. On the other hand, the initial multiplier λ' was set to the hourly system marginal cost of the schedule satisfying both power balance and spinning reserve constraints, whereas the initial multiplier μ' was set to zero which was generally lower than the optimal value μ' .

The initialization procedure in [9] intends to create a high quality feasible schedule in the first few iterations. The generating units are sorted in an ascending order of full load average production cost, $FLAC_i$. For each of the 24 hours (stages), the groups of identical units with the least $FLAC_i$ will be committed one by one until the power balance constraint is satisfied. Subsequently, economic dispatch is carried out for each hour to obtain the hourly lambda value which was initially set as Lagrangian multiplier $\lambda^{(0)}$. For the hours with insufficient spinning reserves, more units (here, in groups of identical units with the least $FLAC_i$) need to be committed to produce an initial feasible solution satisfying the spinning reserve requirement. Thus, the initial unit schedule is achieved. For each of the 24 hours of the schedule, each nonnegative $\mu_i^{(0)}$ is determined by the upper bound of zero on/off decision criteria of the committed unit i as follows:

$$\mu_i^{(0)} = \max\left[\frac{1}{P_{i,\max}}[F_i(P_i^t) + \frac{ST_i}{T_{i,up}} - \lambda^{(0)} P_i^t], 0\right] \quad (3.25)$$

where ST_i is the tartup cost of unit i and $\lambda^{(0)}$ is the initial value of the Lagrangian multiplier associated with power balance.

The initial Lagrangian multiplier associated with spinning reserve $\mu^{(0)}$ is determined by the highest $\mu_i^{(0)}$ among the committed units as

$$\mu^{(0)} = \max[\mu_1^{(0)}, \dots, \mu_m^{(0)}] \quad (3.26)$$

where m is the marginal unit with the highest $FLAC_i$ providing sufficient spinning reserve at hour t .

In case the spinning reserve constraint is neglected, the initial multiplier λ' should be modified as follows:

$$\lambda_i^{t(0)} = 2c_i P_{i,ed}^{t(0)} + b_i + \frac{a_i}{P_{i,ed}^{t(0)}} + \frac{ST_i(1 - U_i^{t-1})U_i^t}{P_{i,ed}^{t(0)}} \quad (3.27)$$

The initial $\lambda^{(0)}$ is determined by the highest $\lambda^{t(0)}$ among the committed units as

$$\lambda^{(0)} = \max[\lambda_1^{t(0)}, \dots, \lambda_m^{t(0)}] + 0.1 \quad (3.28)$$

where m is the marginal unit with the highest $FLAC_i$ satisfying the power balance constraint at hour t . The number 0.1 in (3.28) is added to assure that it is high enough to produce a negative value of the NFR and ODC terms resulting in units committed as in the initial unit schedule.

C. Updating of Lagrangian Multipliers

In general, adjusting Lagrangian multipliers by a subgradient method is not efficient in the presence of the spinning reserve constraint. The LR performance is heavily dependent on the method used to update the multipliers. One rule for updating Lagrangian multipliers is to design large size steps at the beginning of an iteration and smaller steps as the iteration proceeds as proposed in [8], [9]. The values of α and β are determined heuristically [11]. Each nonnegative λ' and μ' are adaptively updated by

$$\lambda^{t(k)} = \max\left[\lambda^{t(k-1)} + \frac{pdif^t}{(\alpha + \beta \times k) \times \text{norm}(pdif)}, 0\right] \quad (3.29)$$

$$\text{where } pdif^t = P_{load}^t - \sum_{i=1}^N P_i^t U_i^t \quad (3.30)$$

$$\text{norm}(pdif) = \sqrt{(pdif^1)^2 + (pdif^2)^2 + \dots + (pdif^T)^2} \quad (3.31)$$

$$\mu^{t(k)} = \max\left[\mu^{t(k-1)} + \frac{rdif^t}{(\alpha + \beta \times k) \times \text{norm}(rdif)}, 0\right] \quad (3.32)$$

$$rdif^t = \max \left[\begin{array}{l} P_{load}^t + R^t - \sum_{i=1}^N P_{i,max} U_i^t, \\ R^t - \sum_{i=1}^N \tau \cdot UR_i U_i^t, R^t - \sum_{i=1}^N \tau \cdot DR_i \cdot U_i^t \end{array} \right] \quad (3.33)$$

$$RampDif^t = \begin{cases} Dif_{up}^t & \text{as load increases} \\ Dif_{down}^t & \text{as load decreases} \end{cases} \quad (3.34)$$

$$Dif_{up}^t = \left| P_{load}^{t+1} - P_{load}^t \right| - \sum_{i=1}^N \left[UR_i \cdot 60 \cdot U_i^t \cdot U_i^{t+1} + P_{i,min} \cdot U_i^{t+1} \cdot (1 - U_i^t) \right] \quad (3.35)$$

$$Dif_{down}^t = \left| P_{load}^{t+1} - P_{load}^t \right| - \sum_{i=1}^N \left[DR_i \cdot 60 \cdot U_i^t \cdot U_i^{t+1} + P_{i,min} \cdot U_i^t \cdot (1 - U_i^{t+1}) \right] \quad (3.36)$$

$$norm(rdif) = \sqrt{(rdif^1)^2 + (rdif^2)^2 + ... + (rdif^T)^2} \quad (3.37)$$

α and β are divided into three cases depending on the signs of $pdif^t$ and $rdif^t$ as follows:

Case 1: $pdif^t \geq 0$ and $rdif^t \geq 0$: updating both λ^t and μ^t by using $\alpha = 0.02$ and $\beta = 0.05$,

Case 2: $pdif^t < 0$ and $rdif^t < 0$: updating both λ^t and μ^t by using $\alpha = 0.6$, $\beta = 0.25$,

Case 3: $pdif^t < 0$ and $rdif^t > 0$: updating only μ^t by using $\alpha = 0.02$, $\beta = 0.05$.

In fact, updating these two multipliers, λ^t and μ^t , in hour t must move them into the same direction. In period t , if $pdif^t$ and $rdif^t$ have the same signs, either positive or negative, λ^t and μ^t will be updated (increased or decreased) by (3.29) and (3.32) respectively. When the total dual generation output is larger than the load in that period ($pdif^t < 0$) but the spinning reserve is insufficient ($rdif^t > 0$), more committed unit(s) are required to satisfy the spinning reserve constraints. However, updating λ^t by (3.29) will decrease its value, resulting in committing fewer units. Therefore, when $pdif^t < 0$ and $rdif^t > 0$, only μ^t will be updated.

Note that the subgradient method generally needs a large number of iterations to converge to near the dual optimum, but per iteration multiplier update is very fast. The proposed adaptive subgradient method using high quality initial feasible multipliers requires a much lower number of iterations to converge, leading to shorter computation times.

D. Economic Dispatch

If the 24 hour schedule does not violate power balance (3.2) and spinning reserve constraints (3.3) an hourly constrained economic dispatch (CED) by linear/quadratic programming is used to minimize the total production cost subject to power balance constraint and unit operating limit. Otherwise, current Lagrangian multipliers are not suitable to obtain a feasible schedule. The objective function of constrained economic dispatch is

$$\text{Minimize } \sum_{i=1}^N F_i(P_{i,ed}^t) \quad (3.38)$$

subject to power balance constraint

$$P_{load}^t - \sum_{i=1}^N P_{i,ed}^t U_i^t = 0 \quad (3.39)$$

where $P_{i,ed}^t$ is economic dispatch generation output of thermal unit i at hour t .

E. Duality Gap

The relative duality gap is defined as follows

$$G^{(k)} = \frac{J([U_{i,t}^{(k)}]) - L(P^{(k)}, U^{(k)}, \lambda^{(k)}, \mu^{(k)}, \gamma^{(k)})}{L(P^{(k)}, U^{(k)}, \lambda^{(k)}, \mu^{(k)}, \gamma^{(k)})} \quad (3.40)$$

where $G^{(k)}$ is the relative duality gap at iteration k and the values of

$$J([U_{i,t}^{(k)}]) = \sum_{t=1}^T \sum_{i=1}^N [F_i(P_{i,ed}^t) + ST_{i,t}(1 - U_{i,t-1})] U_{i,t} \text{ and } L(P^{(k)}, U^{(k)}, \lambda^{(k)}, \mu^{(k)}, \gamma^{(k)})$$

are calculated from (3.20).

The relative duality gap is used to measure the solution quality by checking against the stopping criteria. The iteration process stops when either the relative duality gap is less than the specified tolerance or the iteration counter exceeds the maximum allowable number of iterations.

F. Overall Procedure

- Step 1: Initialize λ^t and μ^t as described in Section 3.3.4.B. The constraints (3.2) and (3.3) are satisfied by appropriate initial λ^t and μ^t values.
- Step 2: Initialize the ELR iteration counter, $k = 1$ and $J_B = \$10^7$.
- Step 3: Solve the unit sub-problems considering constraints (3.5), (3.7) and (3.8) as described in Section 3.3.4.A.
- Step 4: If the dual solution does not satisfy the constraints (3.3), go to Step 9.

- Step 5: Carry out transmission and ramp rate constrained economic dispatch considering constraints (3.2), (3.4), (3.5), (3.7), and (3.8) by linear/quadratic programming as described in Section 3.3.4.D.
- Step 6: Calculate the primal cost $J([U_{i,t}^k])$, the dual cost $L(P^{(k)}, U^{(k)}, \lambda^{(k)}, \mu^{(k)}, \gamma^{(k)})$ and the relative dual gap $G^{(k)}$ as described in Section 3.3.4.E.
- Step 7: If $J([U_{i,t}^k]) < J_B$, $J_B = J([U_{i,t}^k])$ and $[U_{i,t}^B] = [U_{i,t}^k]$.
- Step 8: If the relative dual gap $G^{(k)} < \varepsilon$, go to Step 10.
- Step 9: If $k < K_{max}$, $k = k + 1$, update Lagrangian multipliers adaptively to satisfy constraints (3.2) and (3.3) as described in Section 3.3.4.C and go to Step 3.
- Step 10: Terminate.

where

- J_B best total economic dispatch production cost reached (\$)
- $J([U_{i,t}^{(k)}])$ total economic dispatch production cost at iteration k (\$)
- $[U_{i,t}^B]$ best feasible solution reached
- $[U_{i,t}^k]$ feasible solution at iteration k
- K_{max} maximum allowable number of iterations

Example 3.5A

This problem is illustrated by the data from system II and solved by the Lagrangian relaxation technique. The spinning reserve is neglected. Sub-problems are solved by DP. Lagrangian multipliers are updated by the adaptive subgradient method.

The initialization starts by sorting the generating units in the ascending order of $FLAC_i$. At each stage, the units with the least $FLAC_i$ are committed one by one until the power balanced constraint (3.2) is satisfied. The corresponding unit schedule and the dispatch power during the initialization process are shown below. The starting Lagrangian multipliers $\lambda^{(0)}$, shown below, are set as described in Section 3.3.4.B.

Initialization

Hour	λ^t	$u_1^{t(0)}$	$u_2^{t(0)}$	$u_3^{t(0)}$	$P_{ed,1}^{t(0)}$	$P_{ed,2}^{t(0)}$	$P_{ed,3}^{t(0)}$	Load
0	-	0	0	1	-	-	-	-
1	11.600	0	0	1	0	0	200	200
2	13.233	0	1	1	0	300	200	500
3	14.100	1	1	1	500	400	200	1100
4	12.500	0	1	1	0	400	200	600
5	13.050	0	1	1	0	125	175	300

Iteration 1

Hour	λ^t	u_1^t	u_2^t	u_3^t	P_1^t	P_2^t	P_3^t	$pdif^t$	$P_{ed,1}^t$	$P_{ed,2}^t$	$P_{ed,3}^t$
1	11.600	1	1	1	400	400	200	-800	-	-	-
2	13.233	1	1	1	700	400	200	-800	-	-	-
3	14.100	1	1	1	700	400	200	-200	-	-	-
4	12.500	1	1	1	625	400	200	-625	-	-	-
5	13.050	1	1	1	700	400	200	-1000	-	-	-

The corresponding unit schedule is infeasible. The $norm(pdif)$ in (3.31) for this iteration is 1646.397. The Lambda Lagrangian multipliers are updated each hour by (3.29) as shown below.

	Hour 1	Hour 2	Hour 3	Hour 4	Hour 5
$\lambda^{\pi(0)}$	11.600	13.233	14.100	12.500	13.050
$pdif^t$	-800	-800	-200	-624.5	-1000
$\frac{pdif^t}{(\alpha + \beta k) \cdot norm(pdif)}$	-0.572	-0.571	-0.164	-0.143	-0.715
$\lambda^{\pi(1)}$	11.028	12.662	13.957	12.053	12.335

Iteration 2

Hour	λ^t	u_1^t	u_2^t	u_3^t	P_1^t	P_2^t	P_3^t	$pdif^t$	$P_{ed,1}^t$	$P_{ed,2}^t$	$P_{ed,3}^t$
1	11.028	1	1	1	257.1	338.1	200	-595.2	-	-	-
2	12.662	1	1	1	665.4	400	200	-765.4	-	-	-
3	13.957	1	1	1	700	400	200	-200	-	-	-
4	12.053	1	1	1	513.3	400	200	-513.3	-	-	-
5	12.335	1	1	1	583.8	400	200	-883.8	-	-	-

The corresponding unit schedule is infeasible. The $norm(pdif)$ in (3.31) for this iteration is 1422.951. The Lambda Lagrangian multipliers are updated each hour by (3.29) as shown below.

	Hour 1	Hour 2	Hour 3	Hour 4	Hour 5
$\lambda^{\pi(1)}$	11.028	12.662	13.957	12.053	12.335
$pdif^t$	-595.2	-765.4	-200	-513.3	-838.8
$\frac{pdif^t}{(\alpha + \beta k) \cdot norm(pdif)}$	-0.380	-0.490	-0.128	-0.328	-0.564
$\lambda^{\pi(2)}$	10.648	12.172	13.829	11.725	11.771

Iteration 3

Hour	λ'	u'_1	u'_2	u'_3	P'_1	P'_2	P'_3	$pdif'$	$P'_{ed,1}$	$P'_{ed,2}$	$P'_{ed,3}$
1	10.648	1	1	1	162	274.7	200	-436.7	-	-	-
2	12.172	1	1	1	543	400	200	-643	-	-	-
3	13.829	1	1	1	700	400	200	-200	-	-	-
4	11.725	0	1	1	0	400	200	0	-	-	-
5	11.771	0	1	1	0	400	200	-300	-	-	-

The corresponding unit schedule is infeasible. The $norm(pdif')$ in (3.31) for this iteration is 856.962. The Lambda Lagrangian multipliers are updated each hour by (3.29) as shown below.

	Hour 1	Hour 2	Hour 3	Hour 4	Hour 5
$\lambda^{(2)}$	10.648	12.172	13.829	11.725	11.771
$pdif'$	-436.7	-643	-200	0	-300
$\frac{pdif'}{(\alpha + \beta k) \cdot norm(pdif')}$	-0.377	-0.555	-0.173	0	-0.260
$\lambda^{(3)}$	10.271	11.617	13.656	11.725	11.511

Iteration 4

Hour	λ'	u'_1	u'_2	u'_3	P'_1	P'_2	P'_3	$pdif'$	$P'_{ed,1}$	$P'_{ed,2}$	$P'_{ed,3}$
1	10.271	0	1	1	0	211.8	200	-211.8	0	100	100
2	11.617	0	1	1	0	400	200	-100	0	300	200
3	13.656	1	1	1	700	400	200	-200	500	400	200
4	11.725	1	1	1	431.3	400	200	-431.3	140	260	200
5	11.511	0	1	1	0	377.9	200	-300	0	125	175

The total cost calculated from power dispatch is \$31,637. The relative duality gap calculated by (3.40) is 0.0323. The $norm(pdif')$ in (3.31) for this iteration is 609.028. The Lambda Lagrangian multipliers are updated each hour by (3.29) as shown below.

	Hour 1	Hour 2	Hour 3	Hour 4	Hour 5
$\lambda^{(3)}$	10.271	11.617	13.656	11.725	11.511
$pdif'$	-211.8	-100	-200	-431.3	-300
$\frac{pdif'}{(\alpha + \beta k) \cdot norm(pdif')}$	-0.218	-0.103	-0.205	-0.442	-0.308
$\lambda^{(4)}$	10.053	11.514	13.451	11.283	11.203

Iteration 5

Hour	λ'	u'_1	u'_2	u'_3	P'_1	P'_2	P'_3	$pdif'$	$P'_{ed,1}$	$P'_{ed,2}$	$P'_{ed,3}$
1	10.053	0	0	1	0	0	200	0	0	0	200
2	11.514	0	1	1	0	400	200	-100	0	300	200
3	13.451	1	1	1	700	400	200	-200	500	400	200
4	11.283	1	1	1	320.7	380.5	200	-301.2	140	260	200
5	11.203	0	1	1	0	367.3	200	-267.3	0	125	175

The total cost calculated from power dispatch is \$31,357. The relative duality gap calculated by (3.40) is 0.0185. The $norm(pdif')$ in (3.31) for this iteration is 460.564. The Lambda Lagrangian multipliers are updated each hour by (3.29) as shown below.

	Hour 1	Hour 2	Hour 3	Hour 4	Hour 5
$\lambda^{(4)}$	10.053	11.514	13.451	11.283	11.203
$pdif'$	0	-100	-200	-301.2	-267.3
$\frac{pdif'}{(\alpha + \beta k) \cdot norm(pdif')}$	0	-0.117	-0.235	-0.354	-0.313
$\lambda^{(5)}$	10.053	11.397	13.216	10.929	10.890

Example 3.5B

The data is similar to that of Example 3.5A but sub-problems are solved by ODC.

Iteration 1

Hour	λ'	u'_1	u'_2	u'_3	P'_1	P'_2	P'_3	$pdif'$	$P'_{ed,1}$	$P'_{ed,2}$	$P'_{ed,3}$
1	11.600	0	1	1	0	400	200	-400	0	100	100
2	13.233	1	1	1	700	400	200	-800	100	200	200
3	14.100	1	1	1	700	400	200	-200	500	400	200
4	12.500	1	1	1	625	400	200	-625	140	260	200
5	13.050	1	1	1	700	400	200	-1000	100	100	100

The total cost calculated from power dispatch is \$32,682. The relative duality gap calculated by (3.40) is 0.1815. The $norm(pdif')$ in (3.31) for this iteration is 1493.52. The Lambda Lagrangian multipliers are updated each hour by (3.29) as shown below.

	Hour 1	Hour 2	Hour 3	Hour 4	Hour 5
$\lambda^{(0)}$	11.600	13.233	14.100	12.500	13.050
$pdif^t$	-400	-800	-200	-625	-1000
$\frac{pdif^t}{(\alpha + \beta k) \cdot norm(pdif)}$	-0.315	-0.627	-0.16	-0.492	-0.788
$\lambda^{(1)}$	11.285	12.603	13.94	12.008	12.262

Iteration 2

Hour	λ^t	u'_1	u'_2	u'_3	P'_1	P'_2	P'_3	$pdif^t$	$P'_{ed,1}$	$P'_{ed,2}$	$P'_{ed,3}$
1	11.285	0	1	1	0	381	200	-381	0	100	100
2	12.603	1	1	1	650.8	400	200	-750.8	100	200	200
3	13.942	1	1	1	700	400	200	-200	500	400	200
4	12.007	1	1	1	0	400	200	0	140	100	200
5	12.262	1	1	1	0	400	200	-300	100	100	100

The total cost calculated from power dispatch is \$32,682. The relative duality gap calculated by (3.40) is 0.113. The $norm(pdif)$ in (3.31) for this iteration is 1322.818. The Lambda Lagrangian multipliers are updated each hour by (3.29) as shown below.

	Hour 1	Hour 2	Hour 3	Hour 4	Hour 5
$\lambda^{(0)}$	11.285	12.603	13.94	12.008	12.262
$pdif^t$	-381	-750.8	-200	0	-300
$\frac{pdif^t}{(\alpha + \beta k) \cdot norm(pdif)}$	-0.378	-0.745	-0.198	0	-0.298
$\lambda^{(1)}$	10.907	11.858	13.744	12.007	11.964

Iteration 3

Hour	λ^t	u'_1	u'_2	u'_3	P'_1	P'_2	P'_3	$pdif^t$	$P'_{ed,1}$	$P'_{ed,2}$	$P'_{ed,3}$
1	10.907	0	0	1	0	0	200	0	0	0	200
2	11.858	0	1	1	0	400	200	-100	0	300	200
3	13.744	1	1	1	700	400	200	-200	500	400	200
4	12.007	1	1	1	415.7	400	200	-415.7	140	2600	200
5	11.964	0	1	1	0	400	200	-300	0	125	175

The total cost calculated from power dispatch is \$31,357. The relative duality gap calculated by (3.40) is 0.032. The $norm(pdif)$ in (3.31) for this iteration is 559.28. The Lambda Lagrangian multipliers are updated each hour by (3.29) as shown below.

	Hour 1	Hour 2	Hour 3	Hour 4	Hour 5
$\lambda^{(0)}$	11.023	12.017	13.805	11.663	11.667
$pdif^t$	0	-100	-200	-502	-300
$\frac{pdif^t}{(\alpha + \beta k) \cdot norm(pdif)}$	0	-0.119	-0.237	-0.593	-0.355
$\lambda^{(1)}$	11.023	11.955	13.540	11.112	11.270

Example 3.6

The data is similar to that of Example 3.5A and 3.5B but the spinning reserve requirement is set at 10%. The results are shown as follows.

	Hour 1	Hour 2	Hour 3	Hour 4	Hour 5
$\lambda^{(0)}$	10.00	10.80	12.00	13.40	9.75
$\mu^{(0)}$	1.075	0.575	0.214	0.704	1.133

	No. of iterations	CPU time (s)	k^{th} iteration solution found.	Total cost of best solution
Unit sub-problems solved by DP	100	1.625	33	31,637
Unit sub-problems solved by ODC	100	0.547	32	32,212

3.3.5 Enhanced Augmented Lagrange Augmented Hopfield Method

The UC problem to be solved by the enhanced augmented Lagrange augmented Hopfield network (ALAHN) method includes objective functions (3.1) subject to the constraints of power balance (3.2), spinning reserve (3.3), generation limit (3.4), and minimum up/down times (3.5). The enhanced ALAHN method for solving UC problems explained in this section consists of three stages. In the first stage, ALAHN is used to commit units to satisfy load demand and spinning reserve neglecting minimum up and down time constraints. In this step, the spinning reserve is handled by constraint neurons. Mathematically, the minimum up/down times of all units are set to one hour. In the second stage, a heuristic search based algorithm is applied to repair the minimum up and down time constraint violations as well as de-commit excessive units based on the unit schedule of the first stage. In the last stage, ALHN is used to solve ED and obtain the final UC schedule. A model of ALAHN is given in Appendix A.

A. ALAHN for Unit Scheduling

ALAHN is a combination of discrete and continuous Hopfield networks with its energy function based on augmented Lagrangian relaxation. The augmented

Lagrangian function of the problem for equality constraints is formulated as follows:

$$L = \left\{ \sum_{t=1}^T \sum_{i=1}^N \left[a_i + b_i P_i^t + c_i (P_i^t)^2 + ST_{i,t} (1 - U_i^{t-1}) \right] U_i^t + \sum_{t=1}^T \left[\lambda^t \left(P_{load}^t - \sum_{i=1}^N P_i^t U_i^t \right) + \frac{1}{2} \beta_\lambda^t \left(P_{load}^t - \sum_{i=1}^N P_i^t U_i^t \right)^2 \right] \right\} \quad (3.41)$$

where λ^t and β_λ^t are a Lagrangian multiplier and a penalty factor, respectively, associated with power balance constraint.

To represent the problem in form of ALAHN, $N \times T$ discrete neurons, $N \times T$ continuous neurons and T multiplier neurons are required. The energy function for a Hopfield network augmented by Hopfield terms is formulated based on the augmented Lagrangian function as follows:

$$E = \left\{ \sum_{t=1}^T \sum_{i=1}^N \left[a_i + b_i V_{i,p}^t + c_i (V_{i,p}^t)^2 + ST_{i,t} (1 - V_{i,u}^{t-1}) \right] V_{i,u}^t + \sum_{t=1}^T \left[V_\lambda^t \left(P_{load}^t - \sum_{i=1}^N V_{i,p}^t V_{i,u}^t \right) + \frac{1}{2} \beta_\lambda^t \left(P_{load}^t - \sum_{i=1}^N V_{i,p}^t V_{i,u}^t \right)^2 \right] + \sum_{t=1}^T \sum_{i=1}^N \int_0^{V_{i,p}^t} g_c^{-1}(V) dV \right\} \quad (3.42)$$

where

$V_{i,p}^t$: output of continuous neuron $p(i,t)$ represents output power P_i^t

$V_{i,u}^t$: output of discrete neuron $u(i,t)$ represents unit status U_i^t

V_λ^t : output of multiplier neuron $\lambda(t)$ represents Lagrangian multiplier λ^t

The dynamics of ALAHN for updating the neuron inputs are derived as follows:

$$\begin{aligned} \frac{dU_{i,p}^t}{dt} &= - \frac{\partial E}{\partial V_{i,p}^t} \\ &= - \left\{ (b_i + 2c_i V_{i,p}^t) V_{i,u}^t - \left[V_\lambda^t + \beta_\lambda^t \left(P_{load}^t - \sum_{i=1}^N V_{i,p}^t V_{i,u}^t \right) \right] V_{i,u}^t + U_{i,p}^t \right\} \end{aligned} \quad (3.43)$$

$$\begin{aligned} \frac{dU_{i,u}^t}{dt} &= -\frac{\partial E}{\partial V_{i,u}^t} \\ &= -\left\{ a_i + b_i V_{i,p}^t + c_i (V_{i,p}^t)^2 + ST_{i,t} (1 - V_{i,u}^{t-1}) - ST_i^{t+1} V_{i,u}^{t+1} \right. \\ &\quad \left. - \left[V_{\lambda}^t + \beta_{\lambda}^t \left(P_{load}^t - \sum_{i=1}^N V_{i,p}^t V_{i,u}^t \right) \right] V_{i,p}^t \right\} \end{aligned} \quad (3.44)$$

$$\frac{dU_{\lambda}^t}{dt} = +\frac{\partial E}{\partial V_{\lambda}^t} = P_{load}^t - \sum_{i=1}^N V_{i,p}^t V_{i,u}^t \quad (3.45)$$

where

$U_{i,p}^t$: input of continuous neuron $p(i,t)$ corresponds to output $V_{i,p}^t$

$U_{i,u}^t$: input of discrete neuron $u(i,t)$ corresponds to output $V_{i,u}^t$

U_{λ}^t : input of multiplier neuron $\lambda(t)$ corresponds to output V_{λ}^t

To handle spinning reserve constraints, constraint neurons are used as inputs of discrete Hopfield neurons. The input of constraint neurons is defined as follows:

$$U_{i,s}^t = P_{load}^t + R^t - \sum_{\substack{j=1 \\ j \neq i}}^N V_{j,u}^t P_{j,\max} \quad (3.46)$$

where $U_{i,s}^t$ is the input of constraint neuron $s(i,t)$ associated with spinning reserve constraints.

The output of the inequality constraint neurons is determined using:

$$V_{i,s}^t = g_s(U_{i,s}^t) = \max \{0, \gamma U_{i,s}^t\} \quad (3.47)$$

where $V_{i,s}^t$ is the output of constraint neuron $s(i,t)$ and always positive, and γ is the penalty factor.

The output $V_{i,s}^t$ of the inequality constraint neuron is then added to the input of the discrete neuron $U_{i,u}^t$. This way, the generator i is encouraged to go online at hour t if the total maximum power from generators already online is not sufficient for load demand and spinning reserve requirements.

To enhance the capability of the constraint neurons, a dynamic penalty factor γ at iteration n is defined as follows:

$$\gamma(n) = \gamma(n-1) + \exp(n/K) \quad (3.48)$$

where $\gamma(n)$ is the value of γ at the iteration n , and K is a positive constant. This dynamic penalty factor γ ensures that sufficient spinning reserve is maintained as the number of iterations increases.

The inputs of neurons at iteration n are updated as:

$$U_{i,p}^{t(n)} = U_{i,p}^{t(n-1)} - \alpha_p \frac{\partial E}{\partial V_{i,p}^t} \quad (3.49)$$

$$U_{i,u}^{t(n)} = U_{i,u}^{t(n-1)} - \alpha_u \frac{\partial E}{\partial V_{i,u}^t} + V_{i,s}^{t(n-1)} \quad (3.50)$$

$$U_{\lambda}^{t(n)} = U_{\lambda}^{t(n-1)} + \alpha_{\lambda} \frac{\partial E}{\partial V_{\lambda}^t} \quad (3.51)$$

where α_p , α_u and α_{λ} are updating step sizes of neurons.

The outputs of continuous neurons are determined by a sigmoid function:

$$V_{i,p}^t = g_c(U_{i,p}^t) = (P_{i,\max} - P_{i,\min}) \left(\frac{1 + \tanh(\sigma U_{i,p}^t)}{2} \right) + P_{i,\min} \quad (3.52)$$

The outputs of discrete neurons are determined by a sigmoid function:

$$V_{i,u}^t = g_d(U_{i,u}^t) = \begin{cases} 1 & \text{if } U_{i,u}^t > 0 \\ 0 & \text{if } U_{i,u}^t < 0 \\ \text{no change} & \text{if } U_{i,u}^t = 0 \end{cases} \quad (3.53)$$

The outputs of multiplier neurons are defined by a linear transfer function

$$V_{\lambda}^t = g_m(U_{\lambda}^t) = U_{\lambda}^t \quad (3.54)$$

The selection of parameters for ALAHN is similar to that for ALHN.

The initial input values of discrete neurons $U_{i,u}^{t(0)}$ are random and their initial output values $V_{i,u}^{t(0)}$ are all set to one. The initial inputs of continuous neurons are set by mean distribution. The initial value of each unit is, e.g., proportional to its maximum contribution to the total load demand:

$$V_{i,p}^{t(0)} = P_{load}^t \frac{P_{i,\max}^{(0)}}{\sum_{i=1}^N P_{i,\max}^{(0)}} \quad (3.55)$$

where $P_{i,\max}^{(0)}$ and $P_{i,\min}^{(0)}$ are the maximum and minimum power of generator i corresponding to the initial value $V_{i,u}^{t(0)}$ determined as follows:

$$P_{i,\max}^{(0)} = P_{i,\max} V_{i,u}^{t(0)} \quad (3.56)$$

$$P_{i,\min}^{(0)} = P_{i,\min} V_{i,u}^{t(0)} \quad (3.57)$$

The initial outputs of multiplier neurons are set as:

$$V_{\lambda}^{t(0)} = \frac{1}{N} \sum_{i=1}^N (b_i + 2c_i V_{i,p}^{t(0)}) \quad (3.58)$$

Stopping criteria are applied to errors in power balance, spinning reserve, and iterative process of neurons except discrete neurons since their values always change between 0 and 1 during the iterative process. Therefore, the convergence characteristics of ALAHN are not applied to these neurons.

The overall procedure of ALAHN is similar to ALHN and its diagram for primary unit scheduling is given in Fig. 3.11.

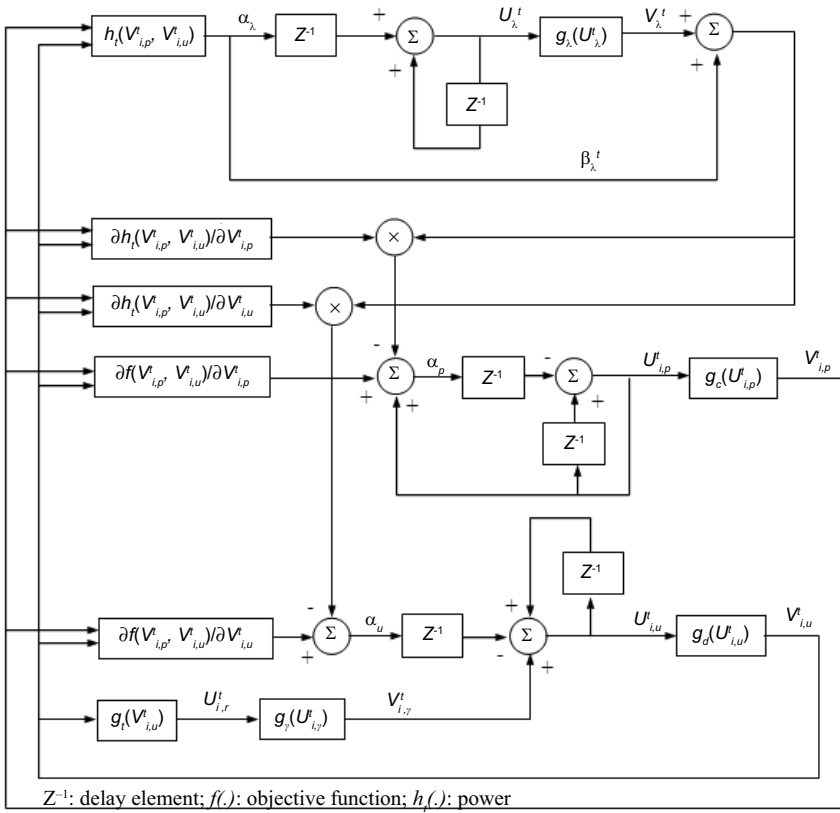


Fig. 3.11 Diagram of ALAHN for primary unit scheduling.

B. Heuristic Search Applications

1) Primary unit de-commitment

The primary unit schedule obtained by ALAHN may not be optimal in terms of spinning reserve in some cases due to the random initial values. That means some units may be redundant in the obtained unit schedule. Therefore, these units are de-committed to reduce excessive spinning reserve based on their merit order defined in (2.164) of Chapter 2.

The algorithm for primary de-commitment of excessive units is as follows:

Step 1: Set $t = 1$.

Step 2: Calculate excessive spinning reserve at period t .

Step 3: Find a list of committed units with the maximum capacity of each smaller than the excessive spinning reserve.

Step 4: If the list is not empty, choose a unit having the highest value M_i as defined in (2.164). Otherwise, go to Step 7.

Step 5: If the excessive spinning reserve is larger than the maximum capacity of the chosen unit, de-commit this unit and reduce the excessive spinning reserve by the maximum capacity of this unit.

Step 6: Delete the unit from the list and return to Step 4.

Step 7: If $t < T$, $t = t + 1$ and return to Step 2. Otherwise, stop.

2) Minimum up and down time repairing

Since the obtained primary UC schedule may not satisfy the minimum up and down time constraints, a heuristic search based algorithm is required to repair the violations.

To check for violations, on and off times of units are determined in advance. The continuous on/off times of unit i up to period t are calculated as follows:

$$T_{i,on}^t = \begin{cases} T_{i,on}^{t-1} + 1, & \text{if } U_i^t = 1 \\ 0, & \text{if } U_i^t = 0 \end{cases} \quad (3.59)$$

$$T_{i,off}^t = \begin{cases} T_{i,off}^{t-1} + 1, & \text{if } U_i^t = 0 \\ 0, & \text{if } U_i^t = 1 \end{cases} \quad (3.60)$$

Minimum up and down time constraints may be violated by the obtained unit schedule since they are neglected in the previous procedures. Therefore, they will be checked and the schedule repaired in case of violations.

Minimum up time of units is usually violated at peak load where the peak load hours are shorter than minimum up time, “hills” exist. The algorithm will check for banking these “hills”. An example is shown in Fig. 3.12. Based on the obtained unit scheduling, unit i is on for only 2 consecutive hours but let’s suppose that this unit has a minimum up time of 3 hours. Therefore, the minimum up time of this unit is violated. To repair this violation, this unit will be further committed in the appending hours so that its total committed hours are not less than its minimum up time. At hour t , this unit is committed and the total committed time of this unit is 3 hours. Consequently, the minimum up time is satisfied. The violation can be also repaired by committing the unit in advance the first committed hour, e.g., at hour $t-3$. However, this way is not convenient for the repairing due to the reverse counting of committed hours.

Minimum down time of units is usually violated at low load levels when low load level hours are shorter than minimum down times of units, and “valleys” exist. The algorithm will check to fill the “valleys”. An example of minimum down time repairing is shown in Fig. 3.13. Suppose that unit i has a minimum down time of 3 hours. Based on the obtained unit scheduling, the total off time of this unit is only 2 hours. Therefore, the minimum down time of this unit is violated. To repair this violation, this unit remains committed during the off hours $t-1$ and t . Consequently, the violation is relieved. This procedure does not check the possibility to prolong the number of de-committed hours of the violated units to satisfy their minimum down time constraint since new units with higher average production costs will be committed to satisfy spinning reserve constraint, leading to a higher operation cost. Moreover, this procedure does not cause further violations of minimum down time constraints by committing units at the violated hours since the units to be committed are the existing committed units violating minimum down time. Therefore, filling the “valleys” is repairing the infeasible solution.

Unit/Time		$t-3$	$t-2$	$t-1$	t	$t+1$			
i	0	0	1	1	0	0	0	Min. up time unsatisfied	
									
i	0	0	1	1	1	0	0	Min. up time satisfied	

Fig. 3.12 Repairing of minimum up time.

Unit/Time			$t-1$	t	$t+1$				
i	1	1	0	0	1	1	1	Min. down time unsatisfied	
									
i	1	1	1	1	1	1	1	Min. down time violation repaired.	

Fig. 3.13 Repairing of minimum down time.

The procedure for minimum up/down time repairing is as follows:

Step 1: Calculate on and off times of all units for the whole schedule time horizon by using (3.56) and (3.57).

Step 2: Set $t = 1$.

Step 3: Set $i = 1$.

Step 4: If $U_i^t = 0$ and $U_i^{t-1} = 1$ and $T_{i,on}^{t-1} < T_{i,up}$, then $U_i^t = 1$.

Step 5: If $U_i^t = 0$ and $U_i^{t-1} = 1$ and $t + T_{i,down} - 1 \leq T$ and $T_{i,off}^{t+T_{i,down}-1} < T_{i,down}$, then $U_i^t = 1$.

Step 6: If $U_i^t = 0$ and $U_i^{t-1} = 1$ and $t + T_{i,down} - 1 > T$ and $\sum_{j=t}^T U_i^j > 0$, then $U_i^t = 1$.

Step 7: Update the on/off status for unit i by (3.56) and (3.57).

Step 8: If $i < N$, $i = i + 1$ and return to Step 4.

Step 9: If $t < T$, $t = t + 1$ and return to Step 3. Otherwise, stop.

3) De-commitment of excessive units

The further excessive spinning reserve is the consequence of repairing minimum up and down time constraints. To de-commit the excessive units, the algorithm is based on a priority list of units.

Starting from the units with the highest values of M_p , the algorithm determines the units that can be de-committed without violating minimum up and down times and spinning reserve constraints until no more unit can be de-committed. An example is shown in Fig. 3.14. The minimum up time violation of unit i is repaired by committing this unit at hour t which leads to excessive spinning reserve at this hour. Therefore, unit j is checked for de-committing to reduce excessive spinning reserve at this hour.

Unit/Time				$t-1$	t	$t+1$		
i	1	1	1	1	1	0	0	
j	0	0	1	1	0	0	0	Min. up time violated
								
i	1	1	1	1	1	0	0	
j	0	0	1	1	1	0	0	The violated repaired.
								
i	1	1	1	1	0	0	0	Excessive unit decommitted
j	0	0	1	1	1	0	0	

Fig. 3.14 De-commitment of excessive units due to minimum up time repairing.

In this procedure, spinning reserve and minimum up and down time constraints must be checked before de-committing a unit. Therefore, the de-commitment process of an excessive unit may not result in a feasible schedule. In other words, this procedure is to de-commit the redundant units due to the repair of minimum up and down time, thereby reducing the operating cost.

The procedure for de-commitment of excessive units is as follows:

Step 1: Set $t = 1$.

Step 2: Set $i = 1$.

Step 3: If unit i can be de-committed without violating minimum up/down time and spinning reserve constraints, put the unit into an excessive list.

Step 4: If $i < N$, $i = i + 1$ return to Step 3.

Step 5: If the excessive list is empty go to Step 9.

Step 6: De-commit the unit with the highest M_i in the excessive list and eliminate it from the list.

Step 7: If the list is not empty and the unit with the highest M_i in the list can be de-committed without violating spinning reserve, return to Step 6.

Step 8: If the list is not empty and the unit with the highest M_i in the list can not be de-committed, delete the unit from the list and return to Step 7.

Step 9: If $t < T$, $t = t + 1$ return to Step 2. Otherwise, stop.

C. ALHN for ED Problems

Having a feasible UC schedule, ALHN can be applied to solve ED. The ED problem formulation consists of an objective function (3.1) neglecting startup cost subject to power balance (3.2) and generation limits (3.4), in which the status of units U_i^t has been determined from ALAHN. ALHN applied to ED is described in Chapter 2. Note that the total production cost considered in the UC problem includes operating cost of the ED solution and startup cost of the UC schedule.

D. Overall procedure

Step 1: Read in all data of the system.

Step 2: Apply ALAHN to determine unit scheduling.

Step 3: Perform primary unit de-commitment for excessive units.

Step 4: Repair minimum up/down time violations by heuristic search.

Step 5: De-commit the excessive units due to the minimum up/down time violation repairing.

Step 6: Apply ALHN to solve the final ED problem.

Step 7: Calculate total cost including operating cost and startup cost and stop.

Example 3.7

A system comprising 10 units scheduled over a time horizon of 12 hours is considered in this example. The system reserve is set to 50 MW. The unit characteristics and load demand are given in Tables 3.27 and 3.28.

Table 3.27 Unit characteristics of the 10-unit system in Example 3.7.

Unit	a_i (\$/h)	b_i (\$/MWh)	c_i (\$/MW ² h)	$T_{i,up}$ (h)	$T_{i,down}$ (h)	ST_i (\$)	$P_{i,min}$ (MW)	$P_{i,max}$ (MW)	Init. stat.
1	200	9.8	0.000903	8	6	5552	100	330	8
2	157	10.7	0.000903	8	5	4987	130	298	8
3	800	13.6	0.000903	8	2	2453	120	154	8
4	547	14.8	0.001457	6	4	989	95	123	6
5	532	15.2	0.001323	4	4	2675	37	234	4
6	532	16.1	0.005040	2	4	2985	37	246	-4
7	590	16.1	0.005040	5	2	3334	17	91	-5
8	612	16.4	0.032397	2	2	3789	25	95	-2
9	580	17.1	0.032397	4	6	2976	54	274	-4
10	317	17.1	0.004701	2	6	2543	58	276	-6

Table 3.28 Load demand for the 10-unit system in Example 3.7.

Hour	1	2	3	4	5	6	7	8	9	10	11	12
Load (MW)	1031	912	856	816	809	808	849	1034	1168	1252	1250	1227

Since ALAHN is randomly initialized and different initialization will lead to different results, it is run several times for obtaining the best solution. In this example, neurons initialized for one run are as follows:

$$V_{i,u}^{t(0)} = 1 \text{ and } U_{i,u}^{t(0)} = 1, i = 1 \dots, N; t = 1, \dots, T$$

$$V_{i,p}^{t(0)} = (P_{i,max} + P_{i,min})V_{i,u}^{t(0)} / 2 \text{ and } U_{i,u}^{t(0)} = 0$$

$$V_{\lambda}^{t(0)} = \sum_{i=1}^N (b_i + 2c_i V_{i,p}^{t(0)}) / \sum_{i=1}^N V_{i,u}^{t(0)}$$

The parameters of ALAHN are selected as follows: $\sigma = 100$; $\alpha_p = 0.0225$; $\alpha_u = 0.005$; $\alpha_{\lambda} = 0.00075$; $\beta = 0.001$, $\gamma = 100$ and $\Gamma = 250$.

In the first step, ALAHN is used to find a primary unit scheduling. The obtained power generation of units is given in Table 3.29.

Based on this unit scheduling, the excessive spinning reserve is calculated at each stage/hour as follows:

$$\begin{aligned}
 ESR^t &= \sum_{i=1}^N P_{i,max} V_{i,u}^t - P_{load}^t - R^t \\
 &= [58 \quad 177 \quad 233 \quad 273 \quad 280 \quad 713 \quad 948 \quad 1037 \quad 903 \quad 819 \quad 821 \quad 844]
 \end{aligned}$$

A heuristic search is used to de-commit the units with a maximum smaller capacity than ESR' and the lowest priority index. The priority index of units based on their average production cost is given in Table 3.30.

After performing primary de-commitment, some further units will be de-committed. For example, at stage 1, no further unit is de-committed because no unit has a smaller maximum capacity than the excessive spinning reserve. At stage 2, the excessive spinning reserve is 177 MW. Based on the obtained unit scheduling and the priority index, the committed units 3 and 4 have maximum capacities of 154 MW and 123 MW respectively, which are smaller than the excessive spinning reserve at this stage. Therefore, units 3 or 4 can be further de-committed to reduce excessive spinning reserve. As unit 4 has a lower priority index, it is de-committed first. The remaining excessive spinning reserve after de-committing unit 4 is $177 - 123 = 54$ MW. This remaining excessive spinning reserve is smaller than the maximum capacity of any committed unit. Therefore, no further unit can be de-committed at this stage. The process is repeated for the other stages.

The new unit scheduling after performing primary unit de-commitment is given in Table 3.31.

The new excessive spinning reserve is calculated as follows:

$$ESR' = [58 \ 54 \ 110 \ 150 \ 3 \ 4 \ 117 \ 55 \ 74 \ 113 \ 115 \ 15]$$

In the next step, the minimum up/down time constraint violations are considered for repairing. In the new unit scheduling, only minimum up/down times of unit 4 are violated from stage 8 onward. Since this unit has a minimum up time of 6

Table 3.29 Primary unit scheduling by ALAHN in Example 3.7

UT	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1	1	1
6	0	0	0	0	0	1	1	1	1	1	1	1
7	0	0	0	0	0	1	1	1	1	1	1	1
8	0	0	0	0	0	1	1	1	1	1	1	1
9	0	0	0	0	0	0	0	1	1	1	1	1
10	0	0	0	0	0	0	1	1	1	1	1	1

Table 3.30 Priority index of 10-unit system in Example 3.7.

Unit	1	2	3	4	5	6	7	8	9	10
M_i (\$/MWh)	10.9	11.6	19.6	20.0	19.3	20.6	27.4	28.5	26.0	20.1
Priority	1	2	4	5	3	7	9	10	8	6

hours, it is further committed at hours 9 and 12 to satisfy the constraint. The unit scheduling after repairing minimum up/down time constraint violations are given in Table 3.32.

Table 3.31 Unit scheduling after primary unit de-commitment in Example 3.7.

UNT	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1	1
3	1	1	1	1	0	0	1	1	1	1	1	1
4	1	0	0	0	0	0	0	1	0	1	1	0
5	1	1	1	1	1	1	1	1	1	1	1	1
6	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	1	1	1	1

Table 3.32 Unit scheduling after repairing minimum up/down time constraint violations in Example 3.7.

UNT	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1	1
3	1	1	1	1	0	0	1	1	1	1	1	1
4	1	0	0	0	0	0	0	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1	1	1
6	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	1	1	1	1

The spinning reserve after repairing minimum up/down time constraint violations is recalculated:

$$ESR' = [58 \ 54 \ 110 \ 150 \ 3 \ 4 \ 117 \ 55 \ 197 \ 113 \ 115 \ 138]$$

After repairing the minimum up/down time constraint violations, no further unit can be de-committed. So far, the spinning reserve and minimum up/down time constraints are satisfied.

In the last step, ALHN is used to solve the final ED problem. The total cost of the power generation in this schedule is \$ 175,596 with the following values:

$$\lambda' = [15.5 \ 15.5 \ 15.4 \ 13.9 \ 15.7 \ 15.7 \ 15.3 \ 15.5 \ 15.7 \ 18.1 \ 18.1 \ 17.9]$$

The power outputs of units are given in Table 3.33.

Table 3.33 Final power outputs (MW) of units in Example 3.7.

UNT	1	2	3	4	5	6	7	8	9	10	11	12
1	330	330	330	330	330	330	330	330	330	330	330	330
2	298	298	298	298	298	298	298	298	298	298	298	298
3	154	154	154	151	0	0	154	154	154	154	154	154
4	123	0	0	0	0	0	0	123	123	123	123	123
5	126	130	74	37	181	180	67	129	205	234	234	234
6	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	58	113	111	88

ALAHN can be also used for solving the UC problem without using constraint neurons. In this case, the spinning reserve is included into the energy function of ALAHN as an inequality constraint. The augmented Lagrangian function including spinning reserve constraint is formulated as follows:

$$\begin{aligned}
L = & \sum_{t=1}^T \sum_{i=1}^N \left[a_i + b_i P_i^t + c_i (P_i^t)^2 + S_{T_{i,t}} (1 - U_i^{t-1}) \right] U_i^t \\
& + \sum_{t=1}^T \left[\lambda^t \left(P_{load}^t - \sum_{i=1}^N P_i^t U_i^t \right) + \frac{1}{2} \beta_\lambda^t \left(P_{load}^t - \sum_{i=1}^N P_i^t U_i^t \right)^2 \right] \\
& + \sum_{t=1}^T \left\{ \frac{1 + S_t}{2} \left[\mu^t \left(P_{load}^t + R^t - \sum_{i=1}^N P_{i,\max} U_i^t \right) \right. \right. \\
& \left. \left. + \frac{1}{2} \beta_\mu^t \left(P_{load}^t + R^t - \sum_{i=1}^N P_{i,\max} U_i^t \right)^2 \right] - \frac{1 - S_t}{2} \left(\frac{(\mu^t)^2}{2\beta_\mu} \right) \right\} \quad (3.61)
\end{aligned}$$

where

$$S_t = \text{sign} \left\{ \frac{\mu^t}{\beta_\mu^t} + P_{load}^t + R^t - \sum_{i=1}^N P_{i,\max} U_i^t \right\} \quad (3.62)$$

in which $S_t = 1$ if $\frac{\mu^t}{\beta_\mu^t} + P_{load}^t + R^t - \sum_{i=1}^N P_{i,\max} U_i^t \geq 0$ and $S_t = -1$ otherwise.

The energy function of ALAHN is formulated as:

$$\begin{aligned}
 E = & \sum_{t=1}^T \sum_{i=1}^N \left[a_i + b_i V_{i,p}^t + c_i (V_{i,p}^t)^2 + ST_{i,t} (1 - V_{i,u}^{t-1}) \right] V_{i,u}^t \\
 & + \sum_{t=1}^T \left[V_{\lambda}^t \left(P_{load}^t - \sum_{i=1}^N V_{i,p}^t V_{i,u}^t \right) + \frac{1}{2} \beta_{\lambda}^t \left(P_{load}^t - \sum_{i=1}^N V_{i,p}^t V_{i,u}^t \right)^2 \right] \\
 & + \sum_{t=1}^T \left\{ \frac{1+S_t}{2} \left[\frac{V_{\mu}^t \left(P_{load}^t + R^t - \sum_{i=1}^N P_{i,max} V_{i,u}^t \right)}{+ \frac{1}{2} \beta_{\mu} \left(P_{load}^t + R^t - \sum_{i=1}^N P_{i,max} V_{i,u}^t \right)^2} \right] - \frac{1-S_t}{2} \left(\frac{(V_{\mu}^t)^2}{2\beta_{\mu}} \right) \right\} \\
 & + \sum_{t=1}^T \sum_{i=1}^N \int_0^{V_{i,p}^t} g_c^{-1}(V) dV
 \end{aligned} \tag{3.63}$$

where V_{μ}^t is the output of multiplier neuron $\mu(t)$ representing Lagrangian multiplier μ^t .

The dynamics of ALAHN for updating neuron inputs are defined:

$$\frac{dU_{i,p}^t}{dt} = - \frac{\partial E}{\partial V_{i,p}^t} \tag{3.64}$$

$$\begin{aligned}
 = & - \left\{ (b_i + 2c_i V_{i,p}^t) V_{i,u}^t - \left[V_{\lambda}^t + \beta_{\lambda}^t \left(P_{load}^t - \sum_{i=1}^N V_{i,p}^t V_{i,u}^t \right) \right] V_{i,u}^t + U_{i,p}^t \right\} \\
 \frac{dU_{i,u}^t}{dt} = & - \frac{\partial E}{\partial V_{i,u}^t} = - \left\{ \begin{aligned} & a_i + b_i V_{i,p}^t + c_i (V_{i,p}^t)^2 \\ & + ST_{i,t} (1 - V_{i,u}^{t-1}) - ST_i^{t+1} V_{i,u}^{t+1} \\ & - \left[V_{\lambda}^t + \beta_{\lambda}^t \left(P_{load}^t - \sum_{i=1}^N V_{i,p}^t V_{i,u}^t \right) \right] V_{i,p}^t \\ & - \frac{1+S_t}{2} \left[V_{\mu}^t + \beta_{\mu} \left(P_{load}^t + R^t - \sum_{i=1}^N P_{i,max} V_{i,u}^t \right) \right] P_{i,max} \end{aligned} \right\}
 \end{aligned} \tag{3.65}$$

$$\frac{dU_{\lambda}^t}{dt} = + \frac{\partial E}{\partial V_{\lambda}^t} = P_{load}^t - \sum_{i=1}^N V_{i,p}^t V_{i,u}^t \tag{3.66}$$

$$\frac{dU_{\mu}^t}{dt} = + \frac{\partial E}{\partial V_{\mu}^t} = \frac{1+S_t}{2} \left[\left(P_{load}^t + R^t - \sum_{i=1}^N P_{i,max} V_{i,u}^t \right) \right] - \frac{1-S_t}{2} \left(\frac{V_{\mu}^t}{\beta_{\mu}} \right) \quad (3.67)$$

where U_{μ}^t is the input of multiplier neuron $\mu(t)$ corresponding to output V_{μ}^t .

Steps of this model for solving the UC problem are similar to those of the model with constraint neurons.

3.3.6 Enhanced Merit Order and Augmented Lagrange Hopfield Method

In the enhanced merit order and augmented Lagrange Hopfield network (EMO-ALHN), EMO, a merit order based on the average-load production cost of generating units improved by heuristic search, and ALHN are used to solve ED problems. The method works in three stages. In the first stage, EMO is applied to find the unit scheduling. It consists of two sub-stages. In the first, a heuristic search algorithm is used to commit units based on their merit order while satisfying load demand and spinning reserve requirements but neglecting the minimum up and down time constraints. In the second sub-stage, a heuristic search is applied to repair minimum up and down time constraints and de-commit excessive units after minimum up and down time repairing. In the second stage, ALHN is used to solve the ramp rate constraint ED problem based on the obtained unit schedule. A strategy for repairing ramp rate constraint violations is applied if no feasible solution is found.

The EMO-ALHN method is based on the new merit order of generating units enhanced by heuristic search. The purpose of the heuristic search is to refine the primary unit scheduling and repair the constraint violations. In fact, the heuristic search does not change the main unit scheduling obtained from the merit order since it only affects some aspects of the main unit scheduling. Moreover, the heuristic search used in this method is simple and efficient in different systems. Thus, solutions are always found in a low-cost and fast manner.

The priority method uses the priority index based on either capacity of generating units or full-load average production cost (here in \$/MWh) of generating units. These priority indices are efficient with the base load units but maybe not with intermediate or peak load units since these units can operate some hours a day with low unit loading. In contrast, the proposed method uses a priority index based on the average load (50%) production cost. The advantage of the new priority index over the conventional priority indices is that the new priority index can utilize the intermediate and peak load units better while the order of base load units remains the same. Moreover, the proposed method is enhanced by heuristic search to refine the obtained unit scheduling, leading to a lower total cost than the conventional priority list method that can handle only minimum up/down time constraints. In large scale problems with complicated constraints, the normal priority list method

is not favorable as it leads to a higher operation cost and is ineffective in handling ramp rate and transmission constraints. Due to these restrictions, the proposed method is much more efficient than the conventional priority list in terms of the practicality of the derived list and the efficiency of the applied heuristic search. If the usual priority list is used with the same heuristic search in the same way as the proposed method, the results will still be different due to its different priority list. Despite its simplicity, the proposed priority index and heuristic search are new and efficient in solving UC problems. Although the proposed method does not always find the best solution for every problem due to the huge number of combinations, it still always finds better solutions than many other mature methods available in the literature on the same benchmark test systems. It is very difficult to recognize whether a feasible solution is really optimal or not, but it is acceptable if it is better than those produced by many other mature methods for the same problem.

The UC problem solved by the EMO-ALHN method includes the objective function (3.1) subject to power balance (3.2), spinning reserve (3.3), generation limit (3.4), minimum up/down time constraints (3.5), startup cost (3.7), and ramp rate constraints described as follows:

$$P_i^t - P_i^{t-1} \leq SUR_i, \text{ as unit } i \text{ starts up} \quad (3.68)$$

$$P_i^{t-1} - P_i^t \leq SDR_i, \text{ as unit } i \text{ shuts down} \quad (3.69)$$

$$P_i^t - P_i^{t-1} \leq UR_i, \text{ if } U_i^t = 1 \text{ and } U_i^{t-1} = 1 \quad (3.70)$$

$$P_i^{t-1} - P_i^t \leq DR_i, \text{ if } U_i^t = 1 \text{ and } U_i^{t-1} = 1 \quad (3.71)$$

where

SUR_i : startup ramp constraint of unit i

SDR_i : shutdown ramp constraint of unit i

UR_i : ramp up rate limit of unit i

DR_i : ramp down rate limit of unit i

A. EMO for Unit Scheduling

The EMO method finds a feasible solution of unit scheduling according to least total cost based on the merit order of generating units that commits the units with the lowest average production cost first whereas the heuristic procedures are used to satisfy the constraints. Multiple heuristic procedures are intended to improve the solution quality while satisfying the unit and system constraints since each procedure is to either perform a single task which is independent from the others or to improve the result of the previous ones. In other words, a heuristic procedure is not harmful to the result of the previous ones since these procedures have horizontal effects on the primary unit schedule. Therefore, the final result is not affected by the superposition of sequential heuristic procedures. Even though a huge number

of combinations in a UC problem may arise due to constraints determined by unit and system characteristics, finally this is not too difficult to deal with since most of the combinations are not feasible. Even though there is not a rigorous heuristic procedure that can properly handle all combinations for every UC problem, the proposed heuristic procedures always guarantee satisfying the constraints so that a feasible solution can always be obtained.

1) Merit order for primary unit scheduling

The merit order M_i of units based on the average fuel cost of each unit operating at a certain fixed fraction of maximum output is defined in (2.164) of Chapter 2.

The procedure for committing units satisfying power balance and spinning reserve constraints based on merit order is as follows:

Step 1: Set $U_i^t = 0$ for all units and $P_{cap}^t = 0$ for all hours.

Step 2: Construct a list based on values of M_i .

Step 3: Set $t = 1$.

Step 4: Choose unit i with the lowest priority index M_i .

Step 5: Set $U_i^t = 1$ and calculate $P_{cap}^t = P_{cap}^t + P_{i,max}$.

Step 6: If $P_{cap}^t < P_{load}^t + R^t$, delete unit i from the list and return to Step 4.

Step 7: If $t < T$, $t = t + 1$ and return to Step 4. Otherwise, stop.

where P_{cap}^t is the capacity of all committed units at hour t .

For unit de-commitment or substitution, the excessive spinning reserve at each hour is calculated as follows

$$ESR^t = \sum_{i=1}^N P_{i,max} U_i^t - P_{load}^t - R^t \quad (3.72)$$

where ESR^t is the excessive spinning reserve at hour t .

2) Minimum up and down time repairing

Since the obtained unit schedule may not satisfy minimum up and down time constraints, heuristic search is required to relieve these violations. To check for violations, continuous on and off times of units must be determined in advance. The continuous on/off times of units up to hour t are calculated as in (3.59) and (3.60).

The minimum up/down time repair of the obtained primary unit schedule is similar to that of the enhanced ALAHN method for UC in Section 3.3.5.B.2.

3) De-commitment of excessive units

The procedure for de-commitment of excessive units after minimum up/down time repair is also similar to that of the enhanced ALHN method for UC in Section 3.3.5.B.3.

B. ALHN for Solving the ED Problem

1) ALHN for ED

With the unit schedule obtained by EMO, ALHN is used to solve the ED problem. ALHN can efficiently handle more variables and constraints as well as more complex load demand profiles than Hopfield networks due to its sigmoid and energy functions based on augmented Lagrange relaxation. Moreover, ALHN is a recurrent neural network with parallel processing, which is fast enough to solve time critical ED problems.

The ED problem is formulated to minimize operation cost neglecting startup cost subject to power balance, ramp rate, and generation limit constraints.

To handle the ramp rate constraints in ALHN, an up frame based on forward ramp rate starting from the first committed hour and backward ramp rate stating from the last committed hour is needed. For application in ALHN, the startup, shutdown and operating ramp rates will be combined into the upper and lower bounds of generating units handled by a sigmoid function of continuous neurons.

The highest forward possible power output $P_{i,fwd}^t$ of unit i is calculated based on a continuous on time starting from the first committed hour as follows:

$$P_{i,fwd}^t = \begin{cases} \min[P_{i,max}, SUR_i \times SUH_i + T_{i,fwd}^{t-1} UR_i] & , \text{ if } U_i^t = U_i^{t-1} = 1 \\ \min[P_{i,max}, SUR_i \times SUH_i] & , \text{ if } U_i^t = 1, U_i^{t-1} = 0 \end{cases} \quad (3.73)$$

where SUH_i is the startup time of unit i to increase its output power from zero to $P_{i,min}$ or above and $t = 1, \dots, T$.

The highest backward possible power output $P_{i,bwd}^t$ of unit i is determined based on a continuous on time starting from the last committed hour as follows:

$$P_{i,bwd}^t = \begin{cases} \min[P_{i,max}, SDR_i \times SDH_i + T_{i,bwd}^{t-1} DR_i] & , \text{ if } U_i^t = U_i^{t-1} = 1 \\ \min[P_{i,max}, SDR_i \times SDH_i] & , \text{ if } U_i^t = 1, U_i^{t+1} = 0 \end{cases} \quad (3.74)$$

where SDH_i is the shutdown time of unit i to decrease its output power from $P_{i,min}$ or above to zero and $t = T-1, \dots, 1$.

The up frame limit $F_{i,high}^t$ for unit i at hour t is defined as:

$$F_{i,high}^t = \begin{cases} \min[P_{i,fwd}^t, P_{i,bwd}^t], & \text{if } U_i^t = U_i^{t-1} = 1 \\ P_{i,fwd}^t, & \text{if } U_i^t = 1, U_i^{t-1} = 0 \\ P_{i,bwd}^t, & \text{if } U_i^t = 1, U_i^{t+1} = 0 \end{cases} \quad (3.75)$$

The actual highest and lowest possible outputs of unit i at hour t are determined as follows:

$$P_{i,high}^t = \begin{cases} \min[F_{i,high}^t, P_i^{t-1} + UR_i], & \text{if } U_i^t = U_i^{t-1} = 1 \\ F_{i,high}^t, & \text{if } U_i^t = 1, U_i^{t-1} = 0 \text{ or } U_i^t = 1, U_i^{t+1} = 0 \end{cases} \quad (3.76)$$

$$P_{i,low}^t = \begin{cases} \max[P_{i,min}, P_i^{t-1} - DR_i], & \text{if } U_i^t = U_i^{t-1} = 1 \\ P_{i,min}, & \text{if } U_i^t = 0, U_i^{t-1} = 1 \text{ or } U_i^t = 1, U_i^{t+1} = 0 \end{cases} \quad (3.77)$$

where $P_{i,high}^t$ and $P_{i,low}^t$ are the highest and lowest possible power outputs of unit i at hour t .

Thus, the new generation limits of units are defined:

$$P_{i,low}^t \leq P_i^t \leq P_{i,high}^t \quad (3.78)$$

The highest and lowest possible outputs will be used as new generation limits in the ED problem solved by ALHN. The highest forward possible power output, the highest backward possible power output, and the highest possible output of a unit with startup and shutdown times of one hour are shown in Figs. 3.15, 3.16 and 3.17.

ALHN applied to the ED problem is detailed in Chapter 2 with the new generation limits of units as in (3.78).

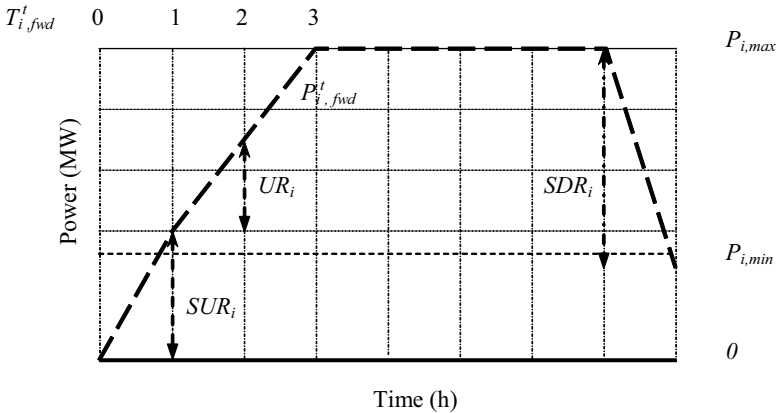


Fig. 3.15 The highest forward power output of unit i starting from the first committed hour.

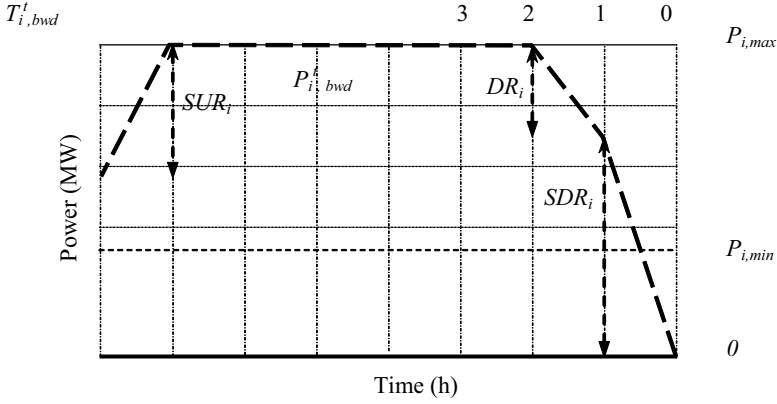


Fig. 3.16 The highest backward power output of unit i starting from the last committed hour.

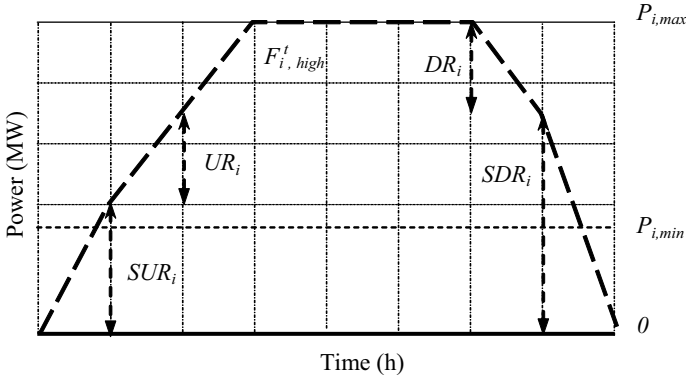


Fig. 3.17 The highest power output of unit i .

2) Repair strategy for ramp rate constraint violations

ALHN in ED may obtain an infeasible solution due to ramp rate constraints leading to power shortages at certain times. There are two scenarios to be considered.

Scenario 1: A power shortage may occur at the hours where the units just committed suffer from the online minimum level constraint. Similarly, power shortages may also occur at those times when the committed units due to be de-committed in the next hour suffer from the off line minimum level constraint. To increase power generation at those times with power shortage to satisfy power balance, the units just committed have to be committed earlier and the units to be de-committed will be de-committed later. Figures 3.18 and 3.19 show the advanced commitment of units to increase power generation during the power shortage periods.

Unit\Time	t-2	t-1	t	t+1		
i	0	0	1	1		
i	0	1	1	1		

Fig. 3.18 Extended commitment of unit i at hour $t-1$ to increase power at hour t .

Unit\Time	t-2	t-1	t	t+1		
i	0	0	1	1		
j	0	0	0	1		
i	0	0	1	1		
j	0	0	1	1		

Fig. 3.19 Extended commitment of unit j at hour t to increase power at this hour.

Scenario 2: Power shortage may occur at those hours where the load demand increase from one hour to another exceeds the ramp rate response capacity of the committed units. That means:

$$P_{load}^t - P_{load}^{t-1} > \sum_{i=1}^N UR_i U_i^t \quad (3.79)$$

To relieve the power shortage at these times, the power generation at hour $t-1$ should be increased to satisfy the condition:

$$\sum_{i=1}^N P_i^{t-1} U_i^{t-1} \geq P_{load}^t - \sum_{i=1}^N UR_i U_i^{t-1} \quad (3.80)$$

Note that the units' status at hour t and hour $t-1$ is the same, i.e., $U_i^t = U_i^{t-1}$. To satisfy (3.80), the lowest possible output power of committed units at hour $t-1$ is updated as follows:

$$P_{i,low}^{t-1} = \max \{P_{i,min}^t, P_{i,ed}^t - UR_i\} \quad (3.81)$$

where $P_{i,ed}^t$ is the solution of ED without ramp rate constraints.

When a power shortage occurs, a heuristic search algorithm will be used to determine which scenario it belongs to and perform the repair accordingly. In the heuristic search algorithm, the selection of units corresponding to the scenarios and used to relieve the power shortage, is based on their priority. If all units corresponding to the two scenarios do not increase enough power to relieve the power shortage, other uncommitted units will be chosen to commit based on their priority to relieve the power shortage. Finally, the minimum up/down time repair procedure in Section 3.3.5.B.2 of Chapter 2 is needed to satisfy the minimum up/down time constraints before solving the final ED problem.

Example 3.8

The system of Example 3.7 is considered in this example. Startup and shutdown rates of generating units are equal to their maximum power outputs while both ramp up and down rates of all units are set to 45 MW/h and 60 MW/h, respectively. The initial power outputs of units are:

$$P_i^{(0)} = \{285 \quad 253 \quad 120 \quad 95 \quad 89 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\}^T$$

Applying EMO to find a unit schedule obtained the result given in Table 3.34.

Table 3.34 Primary unit scheduling by EMO in Example 3.8.

UNT	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1	1
3	1	1	1	1	0	0	1	1	1	1	1	1
4	1	0	0	0	0	0	0	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1	1	1
6	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	1	1	1	1

The new excessive spinning reserve at each hour is calculated as:

$$ESR' = [58 \quad 54 \quad 110 \quad 150 \quad 3 \quad 4 \quad 117 \quad 55 \quad 197 \quad 113 \quad 115 \quad 138]$$

As the minimum up/down time constraints have already been satisfied by the primary unit scheduling, there is no need to repair violations for these constraints. Consequently, there is no necessity for further unit de-commitment due to minimum up/down time violation repair.

In the next step, ALHN is used to solve the ED with ramp rate constraints with parameters selected as follows: $\sigma = 100$, $\beta = 0.001$, and updating step sizes for continuous and multiplier neurons are 0.0375 and 0.022, respectively. The obtained solution for the power generation of each unit is given in Table 3.35.

The obtained solution is infeasible due to unsatisfying power balance at hours 5 and 6. The power mismatch is calculated:

$$\Delta P' = [0 \quad 0 \quad 0 \quad 0 \quad 96 \quad 50 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$$

The power shortage is due to the ramp rate constraints that do not allow enough power increase from hour 4 to meet the power requirement in hour 5. This type of violation belongs to category 2 which can be relieved by increasing power outputs

Table 3.35 Primary power outputs (MW) of units in Example 3.8.

Unit	1	2	3	4	5	6	7	8	9	10	11	12
1	330	330	330	330	330	330	330	330	330	330	330	330
2	298	298	298	298	298	298	298	298	298	298	298	298
3	154	154	143	148	0	0	136	154	154	154	154	154
4	123	0	0	0	0	0	0	123	123	123	123	123
5	126	130	85	40	85	130	85	129	174	219	234	234
6	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	89	128	111	88

of units operating with lower than maximum power output at hour 4. In this case, the power shortage at hour 5 is 96 MW which can be compensated by increasing power output of unit 5 to $40 + 96 = 136$ MW. This is done by setting the minimum power output of unit 5 to this value. Power outputs of units 1–4 cannot be adjusted since units 1 and 2 are already operating at their maximum while units 3 and 4 are off at hour 5. To prevent a further power shortage at hour 5 due to ramp rate constraint after repairing the violation, the minimum power generation of units committed at hour 5 are set to their current power output that means $P_{1,min}^5 = 330$ MW, and $P_{2,min}^5 = 298$ MW. This guarantees that the power generation of unit 5 at hour 5 will be increased to $85 + 96 = 181$ MW to satisfy the power balance constraint. It is not necessary to repair the power shortage of 50 MW at hour 6 since power balance is automatically satisfied via the above repair. In fact, as the power output of unit 5 is increased to 181 MW, the power output of this unit of $130 + 50 = 180$ MW to satisfy the power balance constraint at hour 6 is easily reached. Therefore, by increasing power output of unit 5 at hour 4, the power shortage at hours 5 and 6 will be alleviated while the unit schedule remains unchanged.

After repairing the violations, ALHN is used again to solve the ED problem with ramp rate constraints. The obtained total cost is \$ 176,269 which is slightly higher than the case without ramp rate constraint in Example 3.7. The final optimal solution for the problem of defining power outputs of generating units is given in Table 3.36.

3.3.7 ALHN Based Lagrangian Relaxation Method

ALHN based Lagrangian relaxation (ALHN-LR) is a combination of improved Lagrangian relaxation (ILR) and ALHN enhanced by heuristic search based algorithms. The proposed method solves ramp rate constrained UC problems in three stages. In the first stage, improved Lagrange relaxation (ILR) is used to solve the unit scheduling, satisfying load demand and spinning reserve constraints but neglecting minimum up and down time constraints. In the second stage, heuristic

Table 3.36 Final power outputs (MW) of units in Example 3.8.

UNT	1	2	3	4	5	6	7	8	9	10	11	12
1	330	330	330	307	330	330	330	330	330	330	330	330
2	298	298	298	253	298	298	264	298	298	298	298	298
3	154	154	143	120	0	0	120	154	154	154	154	154
4	123	0	0	0	0	0	0	123	123	123	123	123
5	126	130	85	136	181	180	135	129	174	219	234	234
6	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	89	128	111	88

search based algorithms are applied to refine the obtained unit schedule including primary unit de-commitment, unit substitution, minimum up and down time repair, de-commitment of excessive units, and ramp rate constraint violation repair. In the last stage, ALHN is applied to solve the ED problem, and a repair strategy for ramp rate constraint violations is used if no feasible solution is found.

The UC problem solved by the ALHN-LR method is formulated similar to the one in section 3.3.6.

A. ILR for Primary Unit Scheduling

The ILR applied in this research is only used to determine a unit schedule satisfying load demand and spinning reserve while neglecting minimum up and down time constraints.

The Lagrangian function is formulated similar to the one in the enhanced ALAHN method in Section 3.3.5 or in 3.3.6 as follows:

$$\begin{aligned}
 L(P, U, \lambda, \pi) = & \sum_{t=1}^T \sum_{i=1}^N [F_i(P_i^t) + ST_{i,t}(1 - U_i^{t-1})] U_i^t \\
 & + \sum_{t=1}^T \lambda^t (P_{load}^t - \sum_{i=1}^N P_i^t U_i^t) + \sum_{t=1}^T \mu^t (P_{load}^t + R^t - \sum_{i=1}^N P_{i,max} U_i^t)
 \end{aligned} \quad (3.82)$$

where λ^t and μ^t are Lagrangian multipliers associated with power balance and spinning reserve constraints, respectively, and

$$F_i(P_i^t) = a_i + b_i P_i^t + c_i (P_i^t)^2 \quad (3.83)$$

The dual optimization procedure attempts to reach the constrained optimum by maximizing the Lagrangian with respect to nonnegative Lagrangian multipliers whereas minimizing it with respect the other variables:

$$q^*(\lambda, \mu) = \underset{\lambda^t, \pi^t}{Max} q(\lambda, \mu) \quad (3.84)$$

where $q(\lambda, \mu)$ represents the dual problem of the Lagrangian relaxation with

$$q(\lambda, \mu) = \min_{P'_i, U'_i} L(P, U, \lambda, \mu) \quad (3.85)$$

The Lagrange function (3.82) is rewritten as

$$L(P, U, \lambda, \mu) = \sum_{t=1}^T \sum_{i=1}^N \left\{ [F_i(P'_i) + ST_{i,t}(1 - U_i^{t-1})] U_i^t - \lambda^t P'_i U_i^t - \mu^t P_{i,\max} U_i^t \right\} \quad (3.86)$$

$$+ \sum_{t=1}^T [\lambda^t P'_{load} + \pi^t (P'_{load} + R^t)]$$

The first summation of (3.86) can be separately minimized for each generating unit as the coupling constraints are temporarily ignored. Therefore, the Lagrange function can be minimized by solving each generating unit over the schedule time horizon, that is

$$\min_{P'_i, U'_i} L(P, U, \lambda, \mu) \quad (3.87)$$

$$= \sum_{i=1}^N \min \sum_{t=1}^T \left\{ [F_i(P'_i) + ST_{i,t}(1 - U_i^{t-1})] U_i^t - \lambda^t P'_i U_i^t - \mu^t P_{i,\max} U_i^t \right\}$$

subject to generation limits as (3.4).

For solving unit scheduling, the on/off decision criterion C_i^t based on reduced startup and shutdown costs is used as follows:

$$C_i^t = F_i(P'_i) + \frac{ST_{i,t}(1 - U_i^{t-1})}{T_{i,up}} - \lambda^t P'_i - \mu^t P_{i,\max} \quad (3.88)$$

where P'_i is the solution of the optimality condition:

$$\frac{d}{dP'_i} [F_i(P'_i) - \lambda^t P'_i] = 0 \quad (3.89)$$

and satisfies generation limits (3.4).

From (3.88), if $C_i^t \leq 0$, unit i is committed at hour t ($U_i^t = 1$). Otherwise, this unit will remain/be de-committed ($U_i^t = 0$). The Lagrangian multipliers at iteration n are adjusted as follows:

$$\lambda^{t(n)} = \lambda^{t(n-1)} + \frac{\partial q(\lambda, \mu)}{\partial \lambda} \psi \quad (3.90)$$

$$\mu^{t(n)} = \mu^{t(n-1)} + \frac{\partial q(\lambda, \mu)}{\partial \mu} \phi \quad (3.91)$$

where

$$\psi = \begin{cases} \frac{0.00125}{\Psi} & \text{if } \frac{\partial q(\lambda, \mu)}{\partial \lambda} \geq 0 \\ \frac{0.00025}{\Psi} & \text{if } \frac{\partial q(\lambda, \mu)}{\partial \lambda} < 0 \end{cases} \quad (3.92)$$

$$\phi = \begin{cases} \frac{0.00125}{\Phi} & \text{if } \frac{\partial q(\lambda, \mu)}{\partial \mu} \geq 0 \\ \frac{0.00025}{\Phi} & \text{if } \frac{\partial q(\lambda, \mu)}{\partial \mu} < 0 \end{cases} \quad (3.93)$$

in which, positive Ψ and Φ can be tuned depending on load demand and spinning reserve to obtain a better solution.

The algorithm is terminated whenever the spinning reserve constraint is satisfied or the maximum number of allowed iterations N_{max} is reached. Since the purpose of ILR is to obtain a unit schedule, the dual gap is not considered.

The algorithm of ILR for primary unit scheduling is as follows:

Step 1: Set Lagrangian multipliers and unit status to zero.

Step 2: Set iteration counter $n = 1$.

Step 3: Calculate dual power P_i^t as from (3.89).

Step 4: Calculate the new on/off decision criterion C_i^t for unit scheduling as (3.88).

Step 5: If $C_i^t \leq 0$, $U_i^t = 1$. Otherwise, $U_i^t = 0$.

Step 6: Update Lagrangian multipliers as (3.90) and (3.91).

Step 7: If $n < N_{max}$ and spinning reserve constraint is not satisfied, $n = n + 1$ and return to Step 3. Otherwise, stop.

B. Heuristic Search for Refining and Repairing

After obtaining the primary unit schedule from the ILR, heuristic search based algorithms are applied for primary unit de-commitment, unit substitution, minimum up and down time repair and de-commitment of excessive units. Moreover, the heuristic search is also applied to repairing ramp rate constraint violations.

1) Primary unit de-commitment

The primary unit schedule obtained by ILR may not be optimal regarding spinning reserve. This means that the spinning reserve may be excessive at some hours. Therefore, the primary unit de-commit procedure is to de-commit units at these

hours based on their merit order in (2.164) in Chapter 2, neglecting minimum up and down time constraints for further reduction of excessive spinning reserve if possible.

The procedure for primary unit de-commitment is given in Section 3.3.5.B.1.

2) Minimum up and down time repair

The minimum up and down time constraints may be violated by the obtained unit commitment schedule since they were neglected in the previous procedures. Therefore, they will be checked and repaired if any violation occurs.

The procedure of heuristic search for repairing the minimum up/down time violations is given in Section 3.3.5.B.2.

3) Excessive unit de-commitment

The minimum up and down time violation repair may lead to excessive spinning reserve at certain times. Therefore, some committed units except the ones have been committed by the minimum up and down time repair in section 3.3.7.B.2 may be de-committed during these hours based on their merit order, without violating spinning reserve and minimum up and down time constraints to reduce excessive spinning reserve if possible.

The procedure for de-committing units without violating minimum up/down time and spinning reserve constraints is given in Section 3.3.5.B.3.

C. ALHN Applied to the ED Problem

1) ALHN for ED

Based on the obtained unit schedule, ALHN is applied to solve the ED problem with ramp rate constraints. ALHN for ED problems with ramp rate constraints is detailed in Section 3.3.6.B.1.

2) Ramp rate constraint repair

Due to ramp rate constraints, the solution for the RED problem obtained by ALHN may be infeasible. Therefore, a strategy for repairing ramp rate constraint violations is executed if no feasible solution is found. The procedure for repairing ramp rate violations is given in Section 3.3.6.B.2.

D. Overall Procedure

The overall procedure of the proposed ALHN-LR for solving UC problems with ramp rate constraints is as follows:

- Step 1: Solve the primary unit scheduling by ILR.
- Step 2: Perform the primary unit de-commitment on the obtained primary unit schedule.
- Step 3: Repair minimum up and down time constraint violations.
- Step 4: De-commit excessive units to refine the obtained unit schedule.
- Step 5: Solve the ED problem with ramp rate constraints using ALHN.
- Step 6: If no feasible solution is found, repair ramp rate constraint violations and return to Step 5. Otherwise, stop.

Example 3.9

The system in Example 3.8 is used in this example with ramp up and down rates of all units set to 60 MW/h.

Firstly, ILR is used to find a unit schedule neglecting minimum up/down time constraints. The parameters of ILR are selected as follows: $\Psi = \Phi = 5$. The obtained unit scheduling by ILR is given in Table 3.37.

In unit scheduling, unit 9 is committed instead of unit 3 as a result obtained from the merit order method. The excessive spinning reserve for the primary unit schedule is calculated:

$$ESR' = [55 \quad 174 \quad 230 \quad 270 \quad 277 \quad 278 \quad 237 \quad 52 \quad 194 \quad 233 \quad 235 \quad 258]$$

Based on the primary unit scheduling by ILR, the committed units have high capacity. Therefore, the primary unit de-commitment does not have effect to the obtained unit scheduling. Moreover, this unit scheduling does not violate the minimum up/down time constraints of any unit. Consequently, there are no further units committed or de-committed as of the obtained unit schedule.

In the next step, ALHN is used to solve the ED with ramp rate constraints based on the obtained unit scheduling with parameters selected as follows: $\sigma = 100$, $\beta = 0.001$ and updating step sizes of continuous and multiplier neurons of 0.005 and 0.001, respectively. The obtained primary solution is shown in Table 3.38.

Table 3.37 Primary unit scheduling by ILR in Example 3.9.

UNT	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1	1
3	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	1	1	1
5	1	1	1	1	1	1	1	1	1	1	1	1
6	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0
9	1	1	1	1	1	1	1	1	1	1	1	1
10	0	0	0	0	0	0	0	0	1	1	1	1

Table 3.38 Primary power outputs (MW) of units in Example 3.9.

UNT	1	2	3	4	5	6	7	8	9	10	11	12
1	330	330	330	330	330	330	330	330	330	330	330	330
2	298	298	298	298	298	298	298	298	298	298	298	298
3	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0
5	234	175	174	134	127	126	167	227	210	234	234	234
6	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0
9	169	109	54	54	54	54	54	114	54	114	112	89
10	0	0	0	0	0	0	0	0	276	276	276	276

However, this is an infeasible solution due to the power balance constraint that is not satisfied at hour 8 with a power shortage of 65 MW. The power mismatch during the schedule time horizon is determined:

$$\Delta P' = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 65 \ 0 \ 0 \ 0 \ 0]$$

A heuristic search is employed to repair the ramp rate constraint violation and relieve the power shortage. This power shortage caused by ramp rate constraints of the previous hour can be alleviated by increasing the minimum power generation of committed units at the previous hours to release more power generation in the next hours. Based on the obtained solution of ED, the power shortage at hour 8 can be compensated by units 5 and 9. Of the two units, unit 5 has higher priority than unit 9, thus it will have priority to increase its power output first. The power generation increase at unit 5 is $234 - 227 = 7$ MW. Therefore, the new minimum generation limit of unit 5 at hour 7 is set to $167 + 7 = 174$ MW. The remaining power shortage of $65 - 7 = 58$ MW will be compensated by unit 9. Therefore, the new minimum generation limit of unit 9 at hour 7 is set to $54 + 58 = 112$ MW. The new minimum generation limits of other committed units (1 and 2) at hour 8 are set to the current values of the ED solution (330 and 298 MW) to prevent a power shortage at this hour.

Finally, ALHN is used again to solve the ED with ramp rate constraints with new generation limits of units. The obtained total cost in this case is \$ 180,232. In the ALHN-LR method, the units with high capacity but low priority index are committed, leading to higher total cost than the schedule derived by the EMO-AHLN method. The final solution of the problem by ALHN-LR is detailed in Table 3.39.

Table 3.39 Final power outputs (MW) of units in Example 3.9.

UNT	1	2	3	4	5	6	7	8	9	10	11	12
1	330	330	330	330	330	330	325	330	330	330	330	330
2	298	298	298	298	298	298	238	298	276	298	298	298
3	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0
5	234	175	174	134	127	126	174	234	174	234	234	234
6	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0
9	169	109	54	54	54	54	112	172	112	114	112	89
10	0	0	0	0	0	0	0	0	276	276	276	276

3.4 CONSTRAINED UNIT COMMITMENT

In the constrained unit commitment problem (CUC), 15 minute spinning reserve, generation ramp limit and transmission line constraints are considered in addition to conventional constraints. The up and down 15 minute spinning reserve response time constraints are:

$$R^t - \sum_{i=1}^N r_{up,i}^t U_i^t \leq 0 \quad (3.94)$$

$$R^t - \sum_{i=1}^N r_{dn,i}^t U_i^t \leq 0 \quad (3.95)$$

where

$r_{up,i}^t$ unit up reserve contribution at hour t , $\min[P_{i,\max} - P_i^t, \tau \cdot UR_i]$ (MW)

$r_{dn,i}^t$ unit down reserve contribution at hour t , calculated by $\min[P_i^t - P_{i,\min}, \tau \cdot DR_i]$ (MW)

The constrained unit commitment in [4] is solved by the Enhanced Lagrangian Relaxation Program (ELRP). The ELRP comprises two sub procedures: enhanced Lagrangian relaxation (ELR) and heuristic search using linear/quadratic programming (HSLQP). The ELRP applied to CUC (including only thermal power plants) is described and tested using two cases.

3.4.1 Enhanced Lagrangian Relaxation

LR in Section 3.3.4 is enhanced to ELR by adding more sub-procedures such as identical marginal unit de-commitment, congestion management, HSLQP. Furthermore, transmission and ramp constrained economic dispatch is used instead of the economic dispatch in Section 3.3.4.D.

A. Dual Optimization Procedure

This sub-procedure is similar to Section 3.3.4.A.

B. Initialization

This sub-procedure is similar to Section 3.3.4.B.

C. Identical Marginal Unit De-commitment

LR can find only sub-optimal solutions when there are identical or similar units [8] with identical cost parameters a_p , b_p , c_i and startup cost which causes them to be simultaneously turned on/off as a result of the same on/off decision criterion. This cannot lead to the optimal solution because committing one unit at a time will be less expensive than committing a whole group of units, which may lead to over-commitment. As an example, the shadow blocks illustrate the excessive reserve due to the identical units no. 25, 26, 27, and 28 as shown in Fig. 3.20. Thus, after committing a whole group of identical marginal units, we shall de-commit one identical marginal unit at a time if it does not violate the minimum up time constraint until either the spinning reserve requirement is not satisfied or there is only one marginal unit left. In general, generating units are classified into three types based on responsiveness, startup time, fuel and other constraints: base load units, intermediate load units, and peak load units corresponding to their low, medium, and high full load operating cost respectively [9].

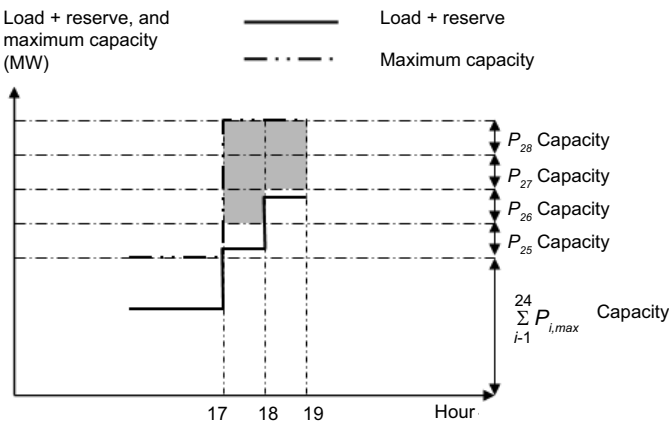


Fig. 3.20 Excessive spinning reserve due to identical marginal units.

The identical marginal unit de-commitment procedure is as follows.

Step 1: Set $t = 1$.

Step 2: Sort the committed unit(s) excluding the base load units in the descending order of negative value of on/off decision criteria to obtain $SS(DC_{i,t}(U_i^t))$. Let the first group of marginal identical units from $SS(DC_{i,t}(U_i^t))$ be $S(MU^t)$.

Step 3: If $S(MU^t)$ has more than one unit, let the first unit in $S(MU^t)$ be CU^t . Otherwise, go to Step 8.

Step 4: Calculate the excess spinning reserve of hour t .

Step 5: If the excess spinning reserve is less than maximum generation of CU^t , go to Step 8.

Step 6: If de-committing CU^t violates its minimum up and down time constraint, go to Step 8.

Step 7: De-commit CU^t and delete it from the set $S(MU^t)$. Update $[U_{i,t}^k]$ and return to Step 3.

Step 8: If $t < T$, $t = t + 1$ return to Step 2. Otherwise, terminate this procedure.

where

$SS(DC_{i,t}(U_i^t))$ set of committed units at hour t sorted in the descending order of negative value of $DC_{i,t}(U_i^t)$

$S(MU^t)$ set of identical marginal units at hour t

CU^t current unit at hour t

$U_{i,t}^k$ feasible solution at iteration k

D. Adaptive Updating of Lagrangian Multipliers

This sub-procedure is similar to Section 3.3.4.C.

E. Transmission and Ramp Rate Constrained Economic Dispatch

1) New limit frame

Since forward economic dispatch proceeds from the first hour to the last hour, the resulting generation level at the last committed hour may be higher than the minimum level required for de-committing this unit (shutdown generation level constraint). Similarly, backward economic dispatch, proceeding in the opposite direction, may result in a higher than the minimum generation level at the first committed hour (startup generation level constraint). This problem is solved by combining forward and backward procedure in [12]. If the dispatch procedure could not find a feasible solution due to ramp rate limitation, it returned to the previous

hour, $t-1$ and re-dispatched the power at that hour so that it could obtain a feasible power dispatch satisfying the ramping constraints at hour t . However, if load changes dramatically, this dispatch procedure may not be able to follow the load changes, resulting in infeasible solutions. On the other hand, if an economic dispatch is initially performed at the maximum demand hour as in [13], performing forward economic dispatch for the subsequent hours and backward economic dispatch for the previous hours does not guarantee that the startup and shutdown generation level constraints will be satisfied. Therefore, a new limit frame consisting of down and up frame is used in transmission constrained dynamic economic dispatch [14], [2] and [3]. The down frame limit is created to guarantee that the generation output at the last committed hour of a unit will be at its minimum power output before de-committing. On the other hand, the up frame limit is used to guarantee that the generation output at the first committed hour will be at its minimum power output.

At the last committed hour, the generation output is set to the minimum level. Thus, at the hour prior to this hour, the upper operating limit is set by a down frame limit, $DF_{i,high}^{t_l-t_b}$.

$$DF_{i,high}^{t_l-t_b} = \begin{cases} \min[P_{i,max}, P_{i,min} + DR_i \cdot 60 \cdot t_b] & \text{if } t_l = T-1, \dots, 2 \text{ and } U_i^{t_l-t_b} = 1 \\ P_{i,max} & \text{if } U_i^t = 1 \text{ and } t = 1, \dots, T \\ 0 & \text{if } U_i^{t_l-t_b} = 0 \end{cases} \quad (3.96)$$

as shown in Fig. 3.21a.

Afterward, a limit frame is constructed starting from the first to the last scheduled hour. At the first committed hour, the generation output is set to the minimum level. At the subsequent hours, the upper operating limit is set by an up frame limit, $F_{i,high}^{t_f+t_a}$.

$$F_{i,high}^{t_f+t_a} = \begin{cases} \min[DF_{i,high}^{t_f+t_a}, P_{i,min} + UR_i \cdot 60 \cdot t_a], & \text{if } U_i^{t_f+t_a} = 1, t_f = 1, \dots, T, t_a \geq 0 \\ P_{i,max} & \text{if } U_i^t = 1 \text{ and } t = 1, \dots, T \end{cases} \quad (3.97)$$

as shown in Fig. 3.21b.

where

- $DF_{i,high}^{t_l-t_b}$ down frame of unit i at t_b hours prior to the last committed hour t_l (MW)
- DR_i ramp down rate limit of thermal unit i (MW/min)
- t_b number of hours prior to the last committed hour
- t_l last committed hour
- $F_{i,high}^{t_f+t_a}$ limit frame of unit i at t_a hours after the first committed hour t_f (MW)
- UR_i ramp up rate limit of thermal unit i (MW/min)
- t_a number of hours after the first committed hour
- t_f first committed hour

A whole frame for unit i is shown in Fig. 3.22, where the dotted block represents the unconstrained capacity neglecting ramp rate limit, whereas the solid line represents the ramp rate limit frame.

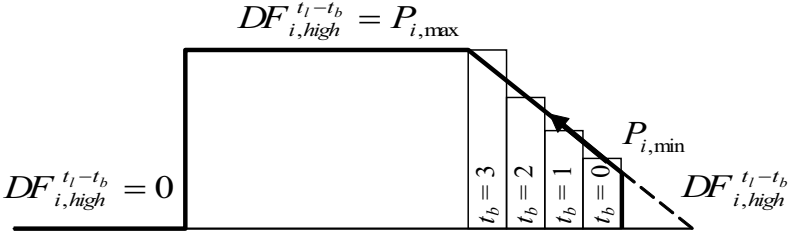


Fig. 3.21a A down frame for unit i .

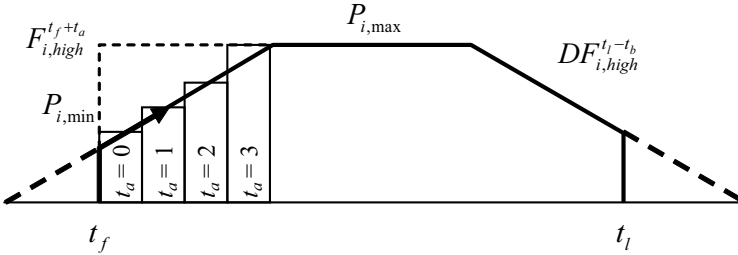


Fig. 3.21b An up frame for unit i .

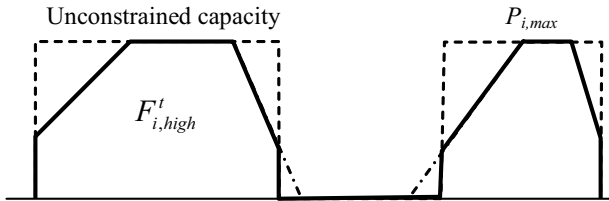


Fig. 3.22 A whole frame for unit i .

After the complete frame for each unit is obtained, the economic dispatch is performed initially at the hour that corresponds to the maximum system demand. Then, the dispatching process proceeds forward and backward in the other time intervals.

The unit commitment schedule obtained from Section 3.4.1. A is checked for its feasibility at each point of time by considering the maximum capacity constraint (3.3), up and down reserve ramping capacity constraint

$$R^t - \sum_{i=1}^N \tau \cdot UR_i \cdot U_i^t \leq 0 \quad (3.98)$$

$$R^t - \sum_{i=1}^N \tau \cdot DR_i \cdot U_i^t \leq 0 \quad (3.99)$$

and committed unit constraint:

$$\sum_{i=1}^N P_{i,\min} U_i^t \leq P_{load}^t \quad (3.100)$$

where τ is reserve response time frame (i.e., 10–15 min).

To satisfy the up and down 15 minute spinning reserve response time constraints (3.94) and (3.95), the sum of the reserve contributions of every unit determined by both the difference between its capacity and current generation, and ramping capability must be at least sufficient to meet the spinning reserve requirement within 15 minutes. The maximum capacity constraint (3.3) forces commitment of sufficient generation to meet the demand and reserve requirement. Whereas the up and down reserve ramping capacity constraints (3.98) and (3.99) are necessary to assure that the committed generators have sufficient reserve ramping capacity to satisfy the reserve requirements. To satisfy the up and down 15 minute spinning reserve response time constraints (3.94) and (3.95), all maximum capacity (3.3) and reserve ramping capacity constraints (3.98) and (3.99) must be satisfied. Because if only the maximum capacity constraint (3.3) is satisfied, the spinning reserve may be limited by the ramping capability whereas with satisfying only reserve ramping capacity constraints (3.98) and (3.99), the spinning reserve may be limited by the generation capacity. The committed unit constraint (3.100) assures that the demand can be met when all dispatched generators are loaded above their respective lower limits.

2) *Fictitious 15 minute ahead demand*

In general, economic dispatch, following either backward or forward procedure, cannot guarantee satisfying the 15 minute response time spinning reserve constraint. Recently, improved Lagrangian relaxation (ILR) [14], [2] and [3] has solved this problem by utilizing a repair strategy after performing a transmission constrained

dynamic economic dispatch. However, ILR could not provide the required 15 minute spinning reserve directly, if up and down spinning reserve constraints were violated simultaneously. A repair strategy was not be able to re-dispatch power to satisfy both constraints at the same time. A fictitious 15 minute ahead demand is proposed in this dissertation [4] by assuming a fictitious load in the next 15 minutes. The fictitious demand is computed from load at hour t plus spinning reserve at hour t . This causes the economic dispatch procedure to dispatch power of thermal units for hour t and hour $t + 15$ minutes simultaneously. This fictitious demand will force the economic procedure to satisfy the up and down 15 minute spinning reserve directly. Thus, no repair strategy is needed. Since a fictitious demand is not realistic, the production cost of this fictitious demand is set to be trivial.

3) Transmission and ramp rate constrained economic dispatch

If the 24 hour schedule does not violate constraints (3.3), (3.98), (3.99) and (3.100), hourly transmission and ramp rate constrained economic dispatch (TRED) by linear/quadratic programming is used to minimize the total production cost subject to power balance constraints and unit operating limits. Otherwise, current Lagrangian multipliers are not suitable to obtain a feasible schedule. The objective function of the constrained economic dispatch is:

$$\text{Minimize } \sum_{i=1}^N F_i(P_{i,ed}^t) + \sum_{i=1}^N F_i(P_{i,ed}^{t+15M}) \quad (3.101)$$

subject to

i) power balance constraint

$$P_{load}^t - \sum_{i=1}^N P_{i,ed}^t U_i^t = 0 \quad (3.102)$$

ii) fictitious demand constraint

$$P_{load}^t + R^t - \sum_{i=1}^N P_{i,ed}^{t+15M} \cdot U_i^t = 0 \quad (3.103)$$

iii) new generation ramp operating frame limit

$$P_{i,ed-low}^t U_i^t \leq P_{i,ed}^t \leq P_{i,ed-high}^t U_i^t, i = 1, \dots, N \quad (3.104)$$

where

$$P_{i,ed-high}^t = \begin{cases} \min[F_{i,high}^t, P_{i,ed}^{t-1} + UR_i \cdot 60] & \text{if } U_i^t = U_i^{t-1} = 1 \\ P_{i,min} & \text{if } U_i^{t-1} = 0, U_i^t = 1 \text{ or } U_i^t = 1, U_i^{t+1} = 0 \end{cases} \quad (3.105)$$

$$P_{i,ed-low}^t = \begin{cases} \max[P_{i,min}, P_{i,ed}^{t-1} - DR_i \cdot 60] & \text{if } U_i^t = U_i^{t-1} = 1 \\ P_{i,min} & \text{if } U_i^{t-1} = 0, U_i^t = 1 \text{ or } U_i^t = 1, U_i^{t+1} = 0 \end{cases} \quad (3.106)$$

iv) transmission line constraints in (3.11).

where

$P_{i,ed}^t$ economic dispatch generation output of thermal unit i at hour t (MW), for generator supply cost function in a piecewise linear form

$$P_{i,ed}^t = P_{i,min} + \sum_{s=1}^{NS_i} P_{i,ed}^{s,t} \quad (\text{MW})$$

NS_i Number of segments of unit i

$P_{i,ed}^{s,t}$ economic dispatch generation output at segment s of thermal unit i at hour t (MW)

$P_{i,ed}^{t+15M}$ economic dispatch generation output of unit i at hour $t + 15$ minutes (MW)

$P_{i,ed-high}^t$ highest possible dispatched power output of unit i at hour t (MW)

$P_{i,ed-low}^t$ lowest possible dispatched power output of unit i at hour t (MW)

$F_{i,high}^t$ limit frame of unit i at hour t (MW)

$P_{i,high}^t$ highest possible power output of unit i at hour t (MW)

$P_{i,low}^t$ lowest possible power output of unit i at hour t (MW)

$P_{i,ed-high}^t$ in (3.105) and $P_{i,ed-low}^t$ in (3.106) are different from $P_{i,high}^t$ in (3.9) and $P_{i,low}^t$ in (3.10). $P_{i,ed-high}^t$ uses the limit frame as the upper limit instead of the maximum real power generation. The economic dispatch using linear/quadratic programming is performed initially at the hour that corresponds to the maximum system demand. Then, the dispatching process proceeds forward and backward to the other time intervals.

F. Congestion Management

If TRED in Section 3.4.1.E cannot find a feasible solution, rescheduling is needed. The uncommitted units at this congested hour are considered to be committed by evaluating their capability to relieve all congested lines in the least cost manner. If there are C' congested lines at hour t , the congestion index (ACI') of uncommitted units is calculated as:

$$ACI_i^t = \left\{ \sum_{l=1}^{C'} s_{l,i} P_{i,max} \Delta P_l^t \right\} / FLAC_i \quad (3.107)$$

where

$$\Delta P_l^t = \begin{cases} f_l - P_l^t < 0 & \text{if } P_l^t > f_l \geq 0 \\ -f_l - P_l^t > 0 & \text{if } P_l^t < -f_l < 0 \end{cases} \quad (3.108)$$

$$P_l^t = \sum_{j=1}^{NB} (s_{l,j} P_{bus,j}^t - s_{l,j} P_{load,j}^t) \quad (3.109)$$

Where

ACI_i^t congestion index of unit i at hour t

C^t total number of congested lines at hour t

$s_{l,i}$ element in the matrix relating DC line flow l to generation output of unit i

ΔP_l^t excess line flow over f_l at hour t (MW)

$FLAC_i$ full load average production cost of thermal unit k , $F(P_{i,max}) / P_{i,max}$ (\$/MWh)

P_l^t total real power flow in line l ($j-k$) at hour t (MW)

f_l upper operating limit of line l (MW)

NB total number of buses

$s_{l,j}$ element in the matrix relating DC line flow l to a net injection at bus j

$P_{bus,j}^t$ sum of generation outputs at bus j at hour t (MW)

$P_{load,j}^t$ load demand at hour t at bus j (MW)

A negative $s_{l,i}$ implies that the committing of unit i will decrease the power flow in line l , whereas committing unit i with a positive $s_{l,i}$ will increase the power flow in line l . To relieve congestion, a negative ΔP_l^t implies that the power flow in line l ($j-k$) must be decreased whereas a positive ΔP_l^t requires the power flow in line l ($j-k$) be increased or overload of line l ($k-j$) be decreased. Thus, a positive value of $s_{l,i} \Delta P_l^t$ means that committing unit i may help alleviating the congestion in line l , while a negative value means it will increase the overload. Note that even if unit i may help relieve one congested line it may increase an overload in another

congested line. Hence, the term $\sum_{l=1}^{C^t} s_{l,i} P_{i,max} \Delta P_l^t$ represents the capability of unit

i to relieve the total of congested lines in arithmetic summation. The higher the

value of $\sum_{l=1}^{C^t} s_{l,i} P_{i,max} \Delta P_l^t$, the better the effects on relieving the overload. On

the other hand, the lower the full load average cost $FLAC_i$, the less increase in cost. Therefore, the uncommitted unit with the highest ACI_i^t is the first unit required to alleviate the congestion. After committing one more unit, CED is carried out. If there is no feasible solution, more units will be committed based on the next highest

ACI_i^t , one at a time until all line constraints as well as minimum up and down time constraints are satisfied.

G. Stopping Criteria

Stopping criteria are handled similar to the sub-procedure of the duality gap in Section 3.3.4.E.

3.4.2. Heuristic Search Using Linear/Quadratic Programming (HSLQP)

HSLQP comprises the whole dispatch period constrained economic dispatch by linear/quadratic programming (EDLQP), thermal unit substitution and thermal unit de-commitment.

A. Whole Dispatch Period Constrained EDLQP

ILR [2], [3] and ELR in Section 3.4.1 solve the unit commitment problem by dispatching power for each hour t separately and using constrained economic dispatch linking between hour t and hour $t+1$. This may not be able to dispatch power if an unforeseen load change causes a ramp rate limit violation. In contrast, EDLQP, using linear/quadratic programming to dispatch power for overall periods simultaneously, may see the load change in advance. In addition, the 15 minute up and down spinning reserve constraints are enforced directly by applying a 15 minute ahead fictitious load. The on/offline minimum level constraints (3.7) and generation ramp constraints (3.8) are implemented directly. Hence, limit frame and repair strategy procedures are not necessary in EDLQP. The objective function of EDLQP considering the fictitious demand is:

$$\text{Minimize}_{P_{i,ed}^t} \sum_{t=1}^T \left\{ \sum_{i=1}^N F_i(P_{i,ed}^t) + \sum_{i=1}^N F_i(P_{i,ed}^{t+15M}) \right\} \quad (3.110)$$

subject to power balance (3.012), fictitious demand (3.103), on/offline minimum level (3.7), generation ramp limit (3.8) and transmission line constraints (3.11).

B. Thermal Unit Substitution

In peak load periods, some committed units cannot be shutdown due to their minimum uptime constraints. This may result in an excessive spinning reserve as the load is going to decrease over the next few hours. In the example, the shadow blocks show the excessive spinning reserve due to the minimum up time constraints of units 6 and 7 as shown in Fig. 3.23. Therefore, those intermediate load units could be de-committed for their entire minimum up time hours and replaced by one

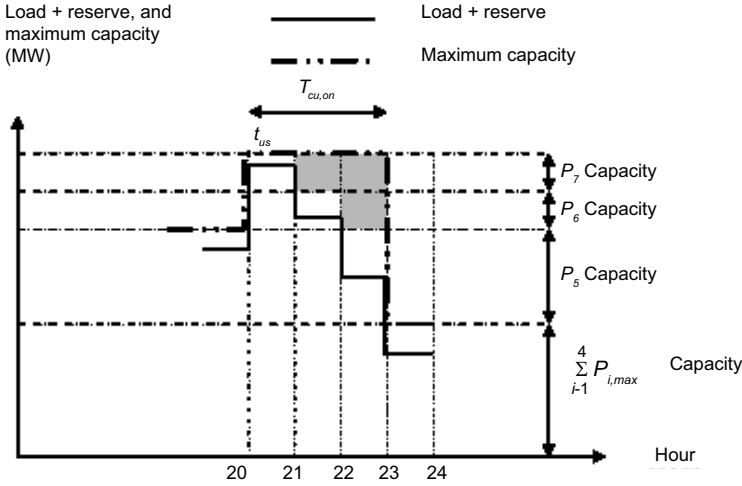


Fig. 3.23 Excessive spinning reserve due to minimum up time constraints.

uncommitted peak load unit based on the least full load average production cost index at a certain time until the spinning reserve requirement is satisfied.

For each peak period, if there is some excessive spinning reserve, a unit substitution will be carried out, starting from two hours after peak hour. This procedure continues de-committing the next highest average production cost intermediate load unit until one of the following three conditions are not satisfied: i) de-committing the unit does not violate its minimum uptime constraints (unit substitution is needed), ii) the total production cost of the new schedule is lower, iii) there are sufficient uncommitted units to replace the de-committed unit. Subsequently, the unit substitution procedure will be repeated at the next peak period until the last peak period. The solution of this procedure will be the best solution reached and used as an initial solution for the unit de-commitment. The unit substitution procedure is as follows:

Step 1: Obtain $[U_{i,t}^B]$ from ELR.

Step 2: Determine the number of major load peaks over the scheduled time horizon (NP). Let $j = 1$.

Step 3: Let $[U_{i,t}^B]$ be an initial solution. Set $t = HR_{peak,j} + 2$.

Step 4: Calculate $F_{avg}(P_{i,ed}^t)$ for each committed intermediate load unit at hour t and sort them in a descending order of $F_{avg}(P_{i,ed}^t)$ to obtain $SS(F_{avg}(P_{i,ed}^t))$. Let the first unit in $SS(F_{avg}(P_{i,ed}^t))$ be CU^t . If the excess spinning reserve at hour t is less than the maximum generation of CU^t , go to Step 14.

Step 5: If de-committing CU^t violates its minimum uptime constraint, de-commit CU^t for all previous $T_{cu,on}$ hours. Otherwise, go to Step 14.

Step 6: Starting from the first hour of $T_{cu,on}$, referred to as t_{us} .

- Step 7: Check the spinning reserve at hour t_{us} . Calculate the full load average production cost $FLAC_i$ of the uncommitted peak load units with one hour minimum up time. Sort them in the ascending order of $FLAC_i$ to obtain $SS(FLAC_j)$.
- Step 8: If the spinning reserve at hour t_{us} is sufficient, go to Step 11.
- Step 9: If the total capacity of uncommitted peak load units at hour t_{us} is insufficient to replace the de-committed CU^t , go to Step 14.
- Step 10: Commit an uncommitted unit in $SS(FLAC_j)$, one unit at a time, until the spinning reserve constraint is satisfied.
- Step 11: If t_{us} is not the last hour of $T_{cu,on}$, $t_{us} = t_{us} + 1$, and return to Step 7.
- Step 12: Calculate the total production cost of the modified schedule. If the total production cost is smaller than J_B , update $[U_{i,t}^B]$ and J_B , delete CU^t from $SS(F_{avg}(P_{i,ed}^t))$. Otherwise, go to Step 14.
- Step 13: Let the first unit in $SS(F_{avg}(P_{i,ed}^t))$ be CU^t and return to Step 5.
- Step 14: If $j < NP$, $j = j+1$ —return to Step 3. Otherwise, the unit substitution procedure is terminated, $[U_{i,t}^B]$ is the solution of unit substitution.

where

- NP number of major load peaks
- $HR_{peak,j}$ j^{th} local peak load hour
- $F_{avg}(P_{i,ed}^t)$ average production cost of unit i at hour t $F(P_{i,ed}^t) / P_{i,ed}^t$
- $SS(F_{avg}(P_{i,ed}^t))$ set of committed units at hour t sorted in descending order of $F_{avg}(P_{i,ed}^t)$
- $T_{cu,on}$ continuous ontime of CU^t (h)
- $SS(FLAC_j)$ set of committed units sorted in descending order of $FLAC_j$

C. Thermal Unit De-commitment

In general, over-commitment may occur in a Lagrangian relaxation solution, hence, it is checked by setting a minimum limit ($P_{i,min}$) of all committed intermediate and peak load units at zero and performing EDLQP as described in Section 3.4.2.A. Thermal units with a lower economic dispatch power output than their minimum limits are considered for de-commitment based on their de-commitment index, (DCI_i) calculated by:

$$DCI_i = \sum_{t=1}^T dci_i^t \quad (3.111)$$

where

$$dci_i^t = \begin{cases} \frac{P_{i,ed}^t}{MAC_i} \cdot P_{i,max} & \text{if } P_{i,ed}^t \leq P_{i,min} \\ \frac{P_{i,ed}^t}{DAC_i} \cdot P_{i,max} & \text{otherwise} \end{cases} \quad (3.112)$$

where

DCI_i decommitment index

MAC_i minimum load average production cost of thermal unit i ,
 $F(P_{i,min}) / P_{i,min}$ (\$/MWh)

DAC_i dispatch load average production cost of thermal unit i , $F(P_{i,ed}^t) / P_{i,ed}^t$ (\$/MWh)

A lower $P_{i,ed}^t$ implies less power needed from unit i . Whereas, the denominator in (3.112), MAC_i and DAC_i will lower dci_i^t . MAC_i is used in (3.111) when $P_{i,ed}^t \leq P_{i,min}$ because the dispatched power is lower than $P_{i,min}$. If it is committed, power must be dispatched at $P_{i,min}$. Note that using $FLAC_i$ instead of MAC_i in (3.111) may lead to less cost savings. Furthermore, $P_{i,max}$ is weighted in (3.111) as larger units can provide a higher spinning reserve. In addition, startup cost of a larger unit is generally higher than that of a smaller unit. However, the dispatch power from unit i may be much needed in certain hours but it may be less needed in other hours. Hence, $\sum_{t=1}^T dci_i^t$ represents the index of unit i for the total periods. The units

with the least DCI_i will be de-committed first.

If economic dispatch power outputs are below their minimum limits, the unit de-commitment procedure will determine whether de-committing the unit with the lowest de-commitment index at a time will leave sufficient spinning reserve, satisfying the minimum up and down time constraints. The thermal unit de-commitment procedure is carried out until the unit status of all thermal intermediate and peak load units are considered.

The unit de-commitment procedure is as follows:

Create a sorted set

Step 1: Calculate the de-commitment index of the thermal intermediate and peak load units. Sort the units in ascending order of DCI_i to obtain $SS(DCI_i)$.

De-commitment

Step 2: Set $t = 1$.

Step 3: If the dispatched thermal power output is not lower than the minimum limit, go to Step 5.

Step 4: If de-committing CU does not violate the spinning reserve constraints in (3.3), (3.98) and (3.99) de-commit CU .

Step 5: If $t < T$, $t = t+1$ —return to Step 3.

Adjust individual unit schedules to satisfy minimum up and down time constraints

Step 6: Starting ch with the first hour, $T_{cu,on}^{ch}$ and $T_{cu,off}^{ch}$ are checked. If $T_{cu,on}^{ch} < T_{cu,up}$, the unit status of CU at hour ch is changed to ‘on’. If $T_{cu,off}^{ch} < T_{cu,down}$, the unit status of CU at hour $ch-1$ is changed to ‘on’. Restore the minimum limit constraint of CU .

Perform EDLQP

Step 7: Restore the minimum limit of CU . Delete CU from $SS(DCI_i)$.

Step 8: Update $[U_{i,t}]$ and run EDLQP as described in Section 3.4.2.A.

Step 9: If $SS(DCI_i)$ is not empty, let CU be the first unit in $SS(DCI_i)$ and go to Step 2. Otherwise, stop, and $[U_{i,t}]$ is the thermal unit de-commitment solution.

where

$SS(DCI_i)$ sorted set of committed units sorted in descending order of DCI_i

CU current unit

$T_{cu,on}^{ch}$ continuous on time of thermal current unit CU up to hour ch at which its unit status is changed to off

$T_{cu,off}^{ch}$ continuous off time of thermal current unit CU up to hour ch at which its unit status is changed to on

$T_{cu,down}$ minimum down time of thermal current unit (h)

$T_{cu,up}$ minimum up time of thermal current unit (h)

3.4.3 ELRP Overall Procedures**ELR procedure**

Step 1: Initialize λ' and μ' as described in Section 3.4.1.B.

Step 2: Initialize the ELR iteration counter, $k = 1$ and $J_B = \$10^7$.

Step 3: Solve the unit sub-problems as described in Section 3.4.1.A.

- Step 4: If the dual solution does not satisfy the constraints described in Section 3.4.1.E.3, go to Step 10. Otherwise, perform the identical marginal unit decommitment as explained in Section 3.4.1.C.
- Step 5: Carry out transmission and ramp rate constrained economic dispatch by linear/quadratic programming as described in Section 3.4.1.E.
- Step 6: If there is no feasible solution, congestion management as detailed in Section 3.4.1.F is performed.
- Step 7: Calculate the primal cost $J([U_{i,t}^k])$, the dual cost $L(P^{(k)}, U^{(k)}, \lambda^{(k)}, \mu^{(k)})$ and the relative dual gap $G^{(k)}$ as described in Section 3.4.1.G.
- Step 8: If $J([U_{i,t}^k]) < J_B$, $J_B = J([U_{i,t}^k])$ and $[U_{i,t}^B] = [U_{i,t}^k]$.
- Step 9: If the relative dual gap $G^{(k)} < \varepsilon$, go to Step 11.
- Step 10: If $k < K_{max}$, $k = k + 1$, update Lagrangian multipliers adaptively as described in Section 3.4.1.D and go to Step 3. Otherwise, go to Step 11.

HSLQP procedure

- Step 11: Perform EDLQP (see Section 3.4.2.A) based on the ELR solution, $[U_{i,t}^B]$ neglecting minimum limit constraints (setting minimum limits to zero) of committed intermediate and peak load thermal units of the ELR schedule.
- Step 12: Perform the unit thermal substitution as described in Section 3.4.2.B.
- Step 13: Perform the unit thermal de-commitment described in Section 3.4.2.C.
- Step 14: Perform a full constrained 24 hour economic dispatch, EDLQP (see Section 3.4.2.A). Calculate the total production cost.

where

$J([U_{i,t}^k])$ total economic dispatch production cost at iteration k (\$)

ε relative duality gap tolerance

K_{max} maximum allowable number of iterations

3.4.4 Numerical Results

ELRP was tested on two cases: (3.4.4.A) 10–100 unit systems and (3.4.4.B) an IEEE 24 bus Reliability Test System without hydro power plants, shown in Appendix B.1 and B.2 and Table 3.38. ELRP was tested on a Pentium IV, 1.6 GHz personal computer. In dual optimization procedures, DP consumed larger CPU times but it could find better solutions in some specific cases. Moreover, DP could deal with unit sub-problems regarding ramp rate constraints better than the new on/off decision criteria. Therefore, the new on/off decision criteria is only used in Case 3.4.4.1.

Table 3.40 Test system for cases 3.4.4.A and 3.4.4.B.

Constraints	Case 3.4.4.A	Case 3.4.4.B	
		Case 1	Case 1
Spinning reserve	$0.1 * P'_{load}$ (3.3)	400 MW (3.3)	$0.04 * P'_{load}$ (3.3)
15 minutes up and down spinning reserve response time	Not included	Not included	(3.38), (3.39)
Generation limit	(3.4)	(3.4)	(3.4)
Minimum up and down time	(3.5)	(3.5)	(3.5)
On/off minimum level	Not included	(3.7)	(3.7)
Generation ramp limit	Not included	(3.8)	(3.8)
Transmission line	Not included	Not included	(3.11)

A. 10–100 Unit Systems

The ELRP algorithm is tested on a 10 unit system given in [15]. For unit data and load demand refer to Appendix B.1. The 20, 40, 60, 80, and 100 units' data are obtained by duplicating the base case (10 unit system), whereas the load demands are adjusted in proportion to the system size. In the simulation, the required reserve is 10% of the load demand. The startup cost in this test system depends on shutdown times expressed as:

$$ST_{i,t} = \begin{cases} HST_i & \text{if } T_{i,down} \leq T_{i,off} \leq T_{i,cold} + T_{i,down} \\ CST_i & \text{if } T_{i,off} > T_{i,cold} + T_{i,down} \end{cases} \quad (113)$$

where

CST_i cold startup cost of unit i (\$)

HST_i hot startup cost of unit i (\$)

The dynamic programming based LR (DPLR) uses single unit dynamic programming to determine the optimal path whereas ELR uses new on/off decision criteria (in this case only). The identical marginal unit de-commitment is not applicable to DPLR. In addition, HSLQP is not used to fine tune the DPLR solution. The duality gap tolerances for ELR and DPLR are both set to 0.001. Since transmission and ramp rate constraints are not included in this test system, the constraints in (3.38)–(3.39) and (3.7)–(3.11) are neglected. Moreover, new limit frame and fictitious 15 minute ahead demand as in Section 3.4.1.E.1 and 3.4.1.E.2, congestion management as in Section 3.4.1.F and EDLQP as in Section 3.4.2.A are not active. In addition, $rdif^t$ of (3.33) is reduced as follows:

$$rdif^t = P'_{load} + R^t - \sum_{i=1}^N P_{i,max} U_i^t \quad (3.114)$$

In other words, only power balance (3.2), spinning reserve (3.3), generation limit (3.4) and minimum up and down time constraints (3.5) are included. In this case, ELRP as applied for this test system comprises ELR, thermal unit substitution and thermal unit de-commitment as in Sections 3.4.1, 3.4.2.B and 3.4.2.C, respectively. However, for this case, the thermal unit de-commitment sub procedure as described in Section 3.4.2.C is modified. The committed thermal units are considered de-committed based on full load average cost ($FLAC$) instead of the de-commitment index (DCI). In addition, this procedure determines whether de-committing the highest full load average cost unit, one unit at a time, will leave sufficient spinning reserve and satisfy the minimum up and down time constraints as in (3.5). Thus, adjusting the individual unit schedules to satisfy these constraints (3.5) is not required since it is already addressed in the de-commitment procedure. Note that the number of major load peaks (NP) is two.

As shown in Table 3.40, the total production costs of ELRP appear to be lower than those of LR [15], GA [15], EP [16], GAUC (genetic algorithm based on characteristic classification) in [17], DPLR, and ELR on all generating unit systems. For the 60, 80 and 100 generating unit systems, ELRP total production cost is lower than that of LRGA [18]. Obviously, ELR alone vastly improves DPLR in terms of both solution quality and CPU time especially withfor large systems with many generation units. ELR converges to the solution at a faster rate than DPLR. If the full startup cost is used as on/off decision criteria (ODC) as explained in section 3.3.4.A.2 instead, μ' of the ELR with full startup cost (ELR-FST) solution

Table 3.41 Comparison of total production cost (\$).

No. of units	10	20	40	60	80	100
LR [15]	565,825	1,130,660	2,258,503	3,394,066	4,526,022	5,657,277
GA [15]	565,825	1,126,243	2,251,911	3,376,625	4,504,933	5,627,437
EP [16]	564,551	1,125,494	2,249,093	3,371,611	4,498,479	5,623,885
LRGA [18]	564,800	1,122,622	2,242,178	3,371,079	4,501,844	5,613,127
GAUC [17]	563,977	1,125,516	2,249,715	3,375,065	4,505,614	5,626,514
DPLR	564,049	1,128,098	2,256,195	3,384,293	4,512,391	5,640,488
ELR	565,508	1,126,720	2,249,790	3,371,188	4,494,487	5,615,893
ELRP	563,977	1,123,297	2,244,237	3,363,491	4,485,633	5,605,678

Table 3.42 Comparison of CPU time (s).

No. of units	10	20	40	60	80	100
GA [15]	221	733	2697	5840	10036	15733
EP [16]	100	340	1176	2267	3584	6120
LRGA [18]	518	1147	2165	2414	3383	4045
GAUC [17]	85	225	614	1085	1975	3547
DPLR	108	299	1200	3199	8447	12437
ELR	3.2	12	34	67	111	167
ELRP	4.0	16	52	113	209	345

in the 10 unit system is higher than that of DPLR and ELR as shown in Fig. 3.24, leading to a higher production cost of \$581,694. This clearly demonstrates the advantage of using the reduced startup cost over the full startup cost as new on-off decision criteria.

The CPU times shown in Table 3.40 may not be directly comparable due to different computers used. CPU times of GA [15] and EP [15], EP [16] were obtained from an HP Apollo 720 workstation and an HP C160 workstation respectively, whereas CPU times of DPLR, ELR and ELRP were obtained from a Pentium IV, 1.6 GHz personal computer. LRGA [18] and GAUC [17] did not report the computers used. However, the CPU times of ELRP are much smaller than those of the other methods. Moreover, the ELRP CPU time increases linearly with the system size which is favorable for large scale implementation.

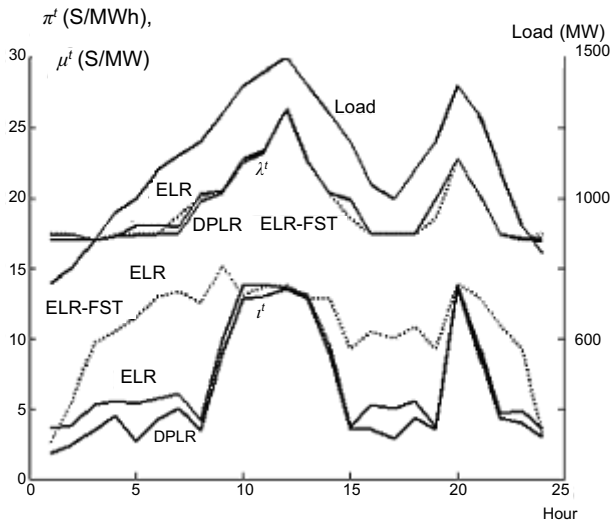


Fig. 3.24 Lagrangian multipliers of ELR, ELR-FST and DPLR solutions for the 10 unit system.

B. IEEE 24-Bus Reliability Test System without Hydro Power Plants

The ELRP algorithm is tested on a IEEE reliability system consisting of 26 generating units [19]-[25]. Transmission line and ramp rate constraints are considered. Unit data, network data and load demand are shown in Appendix B.2. There are two sub-cases:

Case 1: To compare the results with those of ANN in [19], the spinning reserve is calculated from the difference between the maximum capacity and the current generation output of each unit. The spinning reserve requirement is set to the largest committed unit with respect to the constraint in (3.3), neglecting

transmission line constraints in (3.11) and the 15 minute up and down spinning reserve response time constraint in (3.38) and (3.39). There are two load levels, levels 1 and 2, in Appendix B.2. Load level 2 is smaller than load level 1, thus there are more medium size units to start up and shut down.

Case 2: Transmission line constraints as in (3.11) and 15 minute up and down spinning reserve response time constraints as in (3.38) and (3.39) are included. The spinning reserve is calculated from unit reserve contribution available within 15 minutes. The spinning reserve is set to 4% of the total load demand. There are two load levels: load level 4 and load level 5 as shown in Appendix B.2. Load level 4 is used in [23]–[25] whereas load level 5 is similar to load level 4 but the load demand at hours 15–17 has been changed to simulate a dramatically changed load during peak hours while the spinning reserve is maintained at 4%.

Since identical units do not exist in this case, the identical marginal unit decommitment as detailed in Section 3.4.1.C is not used. However, EDLQP as in Section 3.4.2.A is necessary because of the 15 minute up and down spinning reserve response time constraints in (3.38) and (3.39). Consequently, EDLQP in co-operation with the thermal unit decommitment as in Section 3.4.2.C renders the thermal unit substitution in Section 3.4.2.B redundant. In other words, the identical marginal unit decommitment as discussed in Section 3.4.1.C and the thermal unit substitution as in Section 3.4.2.B are not used.

For case 1, only the ELR procedure without congestion management is used whereas HSLQP is not required since the 15 minute up and down spinning reserve response time constraints as in (3.38) and (3.39) are neglected. The results are shown in Table 3.42. ELR production costs are lower than that of ANN for both load levels.

For case 3.3.2.B, the results of load levels 4 and 5 are shown in Tables 3.43 and 3.44, respectively. Dynamic priority list (DPL) was proposed in [23], whereas ILR [2], [3] used a repair strategy. Alternatively, ELR used the fictitious 15 minute ahead demand in the hourly economic dispatch described in Section 3.4.1.E.2. On the other hand, ELRP cooperates in ELR with HSLQP in which a whole schedule economic dispatch regarding the 15 minute fictitious load demand is performed. For load level 4, ELRP results in the lowest cost in both cases (15 minute and 60 minute spinning reserve) as shown in Table 3.43. However, the cost may not allow a direct comparison since the total cost in [23] includes network losses. At load level 5, ELRP also delivers the lowest cost as shown in Table 3.44. This is because

Table 3.43 Production cost in Case 3.4.2.A.

Load	Method	CPU time (s)	Cost (\$)
Load level 1	ANN [19]	12.0	729,326.5
	ELR	76.9	725,996.9
Load level 2	ANN [19]	14.0	613,653.6
	ELR	119.6	594,116.5

Table 3.44 Production cost in Case 3.4.2.B, load level 4.

Method	For 15 minute spinning reserve			For 60 minute spinning reserve		
	CPU time (s)	Cost (\$)	Cost difference (%)	CPU time(s)	Cost (\$)	Cost difference (%)
DPL [23]	-	-	-	8	775,807	2.75
ILR [2]	341	764,101	0.0012	518	760,552	0.72
ELR	902	764,100	0.0010	257	755,094	0.01
ELRP	85,227	764,092	0	10,988	754,980	0

Table 3.45 Production cost in Case 3.4.2.B, load level 5.

Method	For 60 minute spinning reserve		
	CPU time (s)	Cost (\$)	Cost difference (%)
ILR [2]	551	768,571	1.2
ELR	318	765,406	0.8
ELRP	21,109	759,444	0

conventional ED, dispatching power for each hour separately, could not foresee the load change. With ramp rate constraints, conventional ED could not dispatch enough power at hour t and increase power to simultaneously meet the load change at hour $t+1$ and satisfy the 15 minute ramp rate constraints. With dramatic load changes or frequent steep changes such as in load level 5, ELRP could find the better solution.

The computation time was still large due to the whole dispatch schedule constrained economic dispatch and time coupling constraints. However, this can be greatly cut down by using a state of the art high performance computer with sufficient random access memory (RAM).

3.5 SECURITY CONSTRAINED UNIT COMMITMENT

Security constrained unit commitment (SCUC) intends to provide a schedule minimizing cost while satisfying power balance constraints and maintaining security. Satisfying power balance constraints is an obligation for SCUC, thus it is a hard constraint. In addition, SCUC maintains security by providing sufficient spinning reserve and satisfying practical constraints such as ramp rate limits, fuel limit constraints, environmental constraints, transmission constraints etc. In other words, in SCUC, fuel limit constraints and environmental constraints are added to the constraints of CUC as discussed in Section 3.4.

In SCUC, ELR is performed as shown in Section 3.5.1 to produce a basic thermal schedule to be used in Section 3.5.2. In this case, the transmission and environmental constraints are relaxed in the ELR procedure and directly implemented in Section 3.5.2.B. Consequently, there is no CED solution infeasible

due to transmission line constraints, hence, the congestion management of Section 3.4.1.F and the identical marginal unit de-commitment of Section 3.4.1.C are not required. The thermal unit substitution sub procedure in Section 3.4.2.B is redundant and the related sub procedures mentioned above are not required.

The objective function (3.1) is used to minimize the production cost over the schedule time horizon (e.g., 24 hours) under constraints in power balance, 15 minute spinning reserve, generation limit, minimum up and down time, generation ramp limit, transmission line, environmental, limited fuel, fuel mixing ratio, gas delivery, different source gas mixing ratio and alternative fuel.

As mentioned above, the enhanced Lagrangian relaxation program (ELRP) comprises two coordination procedures, enhanced Lagrangian relaxation (ELR) and a heuristic search using linear/quadratic programming (HSLQP). In SCUC, transmission line (3.11) and environmental constraints (3.12) are neglected during the ELR procedure, but they are implemented directly in EDLQP as in Section 3.5.2.B. Thus, there is no need for the congestion management procedure in SCUC. Regarding units in maintenance, their capacities are set to zero at the corresponding hours/days, whereas the unit status of must-run units are set to one.

3.5.1 Enhanced Lagrangian Relaxation

The first step of the solution is an enhanced Lagrangian relaxation to provide an LR based thermal solution. The ELR procedure approaches the unit commitment problem by decomposing the main problem into sub-problems which are coupled by the Lagrangian multipliers and by relaxing or temporarily ignoring the coupling constraints and solves the problem as if they did not exist. This is done through a dual optimization procedure attempting to reach the constrained optimum by maximizing the Lagrangian:

$$\begin{aligned}
 L(P, U, \lambda, \mu, \gamma) = & F(P'_i, U'_i) + \sum_{t=1}^T \lambda^t (P'_{load} - \sum_{i=1}^N P'_i U'_i) \\
 & + \sum_{t=1}^T \mu^t (R^t + P'_{load} - \sum_{i=1}^N P_{i,max} U'_i) + \sum_{i \in \Omega_f} \gamma_i (\sum_{t=1}^T q_i(P'_i) U'_i - q_{i,TOT})
 \end{aligned} \tag{3.115}$$

with respect to nonnegative Lagrangian multipliers λ^t , μ^t , and γ_i , and

$q_{i,TOT}$ total fuel supply (MLitre)

$q_i(P'_i)$ consumed fuel function (Mcf/h)

Ω_f set of thermal units with limited fuel supply

Note: The limited fuel constraint in (3.13) is appended in the Lagrangian function (3.114), by multiplying with γ_i , which is a Lagrangian multiplier or pseudo price related with the consumed fuel. The Lagrangian function is rewritten as:

$$L = \sum_{i=1}^N \sum_{t=1}^T \left\{ \left[F_i(P_i^t) + ST_{i,t} (1 - U_i^{t-1}) \right] U_i^t - \lambda^t P_i^t U_i^t - \mu^t P_{i,\max} U_i^t + \gamma_i q_i(P_i^t) U_i^t \right\} \quad (3.116)$$

$$+ \sum_{t=1}^T (\lambda^t P_{load}^t + \mu^t (P_{load}^t + R^t)) - \sum_{i \in \Omega_f} \gamma_i \cdot q_{i,TOT}$$

γ_i is zero if $i \notin \Omega_f$. The term

$$\sum_{t=1}^T \left\{ \left[F_i(P_i^t) + ST_{i,t} (1 - U_i^{t-1}) \right] U_i^t - \lambda^t P_i^t U_i^t - \mu^t P_{i,\max} U_i^t + \gamma_i q_i(P_i^t) U_i^t \right\} \quad (3.117)$$

can be minimized separately for each generating unit, when the coupling constraints are temporarily ignored. Then, the minimum of the Lagrangian function is solved for each generating unit, that is:

$$\min_{P_i^t, U_i^t} L(P, U, \lambda, \mu, \gamma) = \sum_{i=1}^N \min \sum_{t=1}^T \left\{ \left[F_i(P_i^t) + ST_{i,t} (1 - U_i^{t-1}) \right] U_i^t - \lambda^t P_i^t U_i^t - \mu^t P_{i,\max} U_i^t + \gamma_i q_i(P_i^t) U_i^t \right\} \quad (3.118)$$

subject to constraints (3.5), (3.7) and (3.8).

A. Dynamic Programming

This procedure is similar to Section 3.4.1.A.1 but modified. At the $U_i^t = 0$ state, the value of the function to be minimized is trivial (i.e., it equals zero), at the state where $U_i^t = 1$, the function to be minimized is (startup cost and $\mu^t P_{i,\max}$ are dropped here since the minimization is with respect to P_i^t) $\min[F_i(P_i^t) - \lambda^t P_i^t + \gamma_i q_i(P_i^t)]$.

The dual power of each generating unit is determined based on the optimality condition given as follows:

$$\frac{d}{dP_i^t} [F_i(P_i^t) - \lambda^t P_i^t] + \frac{d}{dP_i^t} [\gamma_i q_i(P_i^t)] = 0 \quad (3.119)$$

$$\frac{dF_i(P_i^{t,opt})}{dP_i^t} - \lambda^t + \gamma_i \frac{dq_i(P_i^{t,opt})}{dP_i^t} = 0 \quad (3.120)$$

$$P_i^{t,opt} = \frac{\lambda^t - \gamma_i \frac{dq_i(P_i^t)}{dP_i^t} - b_i}{2c_i} \quad (3.121)$$

Using the relationship

$$q_i(P_i^t) = r_i P_i^t \quad (3.122)$$

equation (3.68) can be rewritten as:

$$P_i^{t,opt} = \frac{\lambda^t - r_i \cdot \gamma_i - b_i}{2c_i} \quad (3.123)$$

Dynamic programming is used to determine the optimal schedule of each unit over the schedule time period. More specifically, for each state in each hour, on/off decision making is needed to achieve the lowest cost by comparing the combination of start up cost and accumulated costs of two historical routes. At hour t , the dual power calculated by (3.123) and within the limits described in (3.8) will be substituted by

$$[F_i(P_i^t) + ST_{i,t}(1 - U_i^{t-1})]U_i^t - \lambda^t P_i^t U_i^t - \mu^t P_{i,max} U_i^t + \gamma_i q_i(P_i^t) U_i^t \quad (3.124)$$

for each generating unit. Then, dynamic programming searches for the optimal scheduling for each unit to obtain the lowest value of the term in (3.117) subject to minimum up and down time constraints.

For thermal units with limited fuel supply, DP must be performed repeatedly to satisfy fuel limitation constraints in (3.13). Therefore, after the unit schedule is obtained by DP, sub-iterations are needed to adjust γ_i to these. In the first sub-iteration, pseudo price γ_i is initially set to zero. The dual power is computed for each hour by (3.123). Then, dynamic programming is used to search for an optimal

24 hour schedule. Thereafter, the total consumed fuel is calculated and $\gamma_i^{(k_f+1)}$ is adjusted by

$$\gamma_i^{(k_f+1)} = \gamma_i^{(k_f)} + \alpha_f \left(\sum_{t=1}^T q_i(P_i^t) - q_{i,TOT} \right) \quad (3.125)$$

where

k_f sub-iteration counter in dynamic programming

α_f step size coefficient

If the total consumed fuel exceeds the limited fuel supply $\gamma_i^{(k_f+1)}$ will be higher and dual power obtained in (3.123) lower, resulting in less consumed fuel.

Otherwise, $\gamma_i^{(k_f+1)}$ will be lower, dual power in (3.123) will be higher leading to more fuel consumed. The sub-iteration procedure is as follows:

Step 1: Set pseudo price to zero, $\gamma_i^{(0)} = 0$, $k_f = 1$ and $MM_{least} = 10^6$.

Step 2: Calculate dual power for each hour in (3.123).

Step 3: Perform dynamic programming to search for an optimal 24 hour schedule for unit i .

Step 4: Compute the total consumed fuel, $\sum_{t=1}^T q_i(P_i^t)$.

Step 5: If the consumed fuel mismatch MM is less than MM_{least} and the fuel limitation constraint (3.13) is satisfied, set $MM_{least} = MM$ and $[U_{i,t}^{best}]_i = [U_{i,t}^{kf}]_i$.

Step 6: If MM is less than ϵ_f , go to Step 9.

Step 7: If $k_f < K_{f,max}$, update $\gamma_i^{(k_f+1)}$ by (3.125) and go to Step 2.

Step 8: If $[U_{i,t}^{best}]_i$ is empty, set a $k_f = 1$ and go to Step 2.

Step 9: Terminate sub-iteration. Set $[U_{i,t}]_i = [U_{i,t}^{best}]_i$.

where

MM consumed fuel mismatch, $\left| \sum_{t=1}^T q_i(P_i^t) - q_{i,TOT} \right|$ (Mcf)

MM_{least} least consumed fuel mismatch

$[U_{i,t}^{best}]_i$ best individual generator schedule of unit i

$[U_{i,t}^{kf}]_i$ individual generator schedule of unit i at iteration k_f ;

$[U_{i,t}]_i$ individual generator schedule of unit i ;

$K_{f,max}$ maximum allowable number of sub-iterations in dynamic programming

ϵ_f limited fuel mismatch tolerance.

The sub-iteration procedure schedules only the thermal units with limited fuel supply. Without the sub-iteration procedure, these units will be economically dispatched at their maximum capacity, leading to exhaustion of fuel in the first few committed periods and inability to generate power during the remaining committed hours, resulting in an infeasible solution. The schedule obtained from the ELR procedure will be further modified in the second procedure, HSLQP, in which the fuel limitation constraint (3.13) will be implemented directly.

B. Initialization

This procedure is similar to the procedure in Section 3.3.4.B.

C. Adaptive Updating of Lagrangian Multipliers

This procedure is similar to Section 3.3.4.C.

D. Ramp Rate Constrained Economic Dispatch

This procedure is similar to the TRED procedure in Section 3.4.1.E but transmission line constraints are neglected.

E. Stopping Criteria

This procedure is similar to the procedure in Section 3.3.4.E.

3.5.2 Heuristic Search Using Linear/Quadratic Programming (HSLQP)

In the ELR procedure, the sub-iteration in dynamic programming indirectly solves the unit sub-problem with fuel limitation constraints (3.13) which may not be satisfied since they are not considered in the hourly economic dispatch. This may result in too much fuel consumed (violating fuel limitation constraints) or too little

fuel consumed for the total dispatch hours $(\sum_{t=1}^T q_i (P_i^t) < q_{i,TOT})$ leading to higher

total production cost, e.g., due to take-or-pay contracts or marginal power dispatch with higher incremental cost. Furthermore, the generation ramp limit constraints may prevent ELR from finding the optimal feasible solution since the load change is unforeseen if economic dispatch is carried out hourly. By contrast, these problems are solved directly by HSLQP.

HSLQP comprises an initial HSLQP schedule, whole dispatch period constrained economic dispatch by linear/quadratic programming (EDLQP), thermal unit de-commitment, and hydro unit commitment.

A. Initial HSLQP Schedule

The initial HSLQP schedule is constructed by adding 24-hour committed limited fuel supply units to the base schedule obtained from ELR. As discussed earlier, generating units are classified into three types: base load units, intermediate and peak load units according to their low, medium, or high full load operating cost, respectively [9]. To satisfy time coupling constraints (3.12), (3.13), (3.16) and use limited resources efficiently (3.13), (3.16), economic dispatch must be performed over the scheduled period simultaneously while neglecting minimum limits of (3.4) (setting minimum limit to zero) of committed intermediate and peak thermal power plants in the ELR schedule, and of all 24 hour limited fuel supply thermal power plants and hydro power plants.

B. Whole Dispatch Period Constrained Economic Dispatch by Linear/Quadratic Programming (EDLQP)

EDLQP in this Section is similar to EDLQP in Section 3.4.2.A but environmental constraints (3.12) and fuel supply constraints (3.13)–(3.19) are added.

C. Thermal Unit De-commitment

Adding 24-hour committed limited fuel supply units to the ELR base solution causes over-commitment. Hence, thermal unit de-commitment is required considering economic power dispatch derived from EDLQP. The thermal unit de-commitment procedure is similar to that in Section 3.4.2.C but some additional procedure is needed to deal with the limited fuel constraints (3.13). Thus, the following constraint (3.126) is used to ensure that the committed hours are sufficient for full fuel consumption if this is economically relevant:

$$\sum_{t=1}^T q_i (P_{i,max}) \cdot U_i^t \geq q_{i,TOT}, \quad i \in \Omega_f \quad (3.126)$$

The obtained schedule for units with limited fuel supply is feasible if the following constraint is satisfied:

$$\sum_{t=1}^T q_i (P_{i,min}) \cdot U_i^t \leq q_{i,TOT}, \quad i \in \Omega_f \quad (3.127)$$

Consequently, the unit de-commitment procedure is modified as follows:

Create a sorted set

Step 1: Calculate the de-commitment index of the thermal intermediate and peak load units without limited fuel supply. Sort the units in ascending order of DCI_i to obtain $SSI(DCI_i)$.

Step 2: Calculate the de-commitment index of the thermal units with limited fuel supply. Sort the units in ascending order of DCI_i , $i \in \Omega_f$ to obtain $SS2(DCI_i)$.

Step 3: Create $SS(DCI_i)$ by adding $SSI(DCI_i)$ on the top of $SS2(DCI_i)$. Let the first unit in $SS(DCI_i)$ be CU .

De-commitment

Step 4: If $i \notin \Omega_f$, go to Step 7. Otherwise, set $t = 1$.

Step 5: If the dispatched thermal power output is equal to zero and de-committing CU does not violate the spinning reserve constraints in (3.3), de-commit CU .

Step 6: If $t < T$, $t = t+1$ —return to Step 5.

Step 7: Set $t = 1$.

Step 8: If the dispatched thermal power output is not lower than the minimum limit, go to Step 10.

Step 9: If de-committing CU does not violate the spinning reserve constraints in (3.3) and the constraint in (3.126), de-commit CU .

Step 10: If $t < T$, $t = t+1$ —return to Step 8.

Adjust the individual unit schedule to satisfy minimum up and down time constraints

Step 11: Starting ch from the first hour, $T_{cu,on}^{ch}$ and $T_{cu,off}^{ch}$ are checked. If $T_{cu,on}^{ch} < T_{cu,up}$, the unit status of CU at hour ch is changed to ‘on’. If $T_{cu,off}^{ch} < T_{cu,down}$, the unit status of CU at hour $ch-1$ is changed to ‘on’. Restore the minimum limit constraint of CU .

Step 12: If $CU \notin \Omega_f$ or the constraint in (3.127) is satisfied, go to Step 14.

Adjust the individual unit schedule to satisfy a fuel limitation fuel constraint

Step 13: De-commit CU at the first or last committed hour in a sequence of the lowest dispatch power if it does not violate minimum up and down time constraints in (3.5) and spinning reserve constraints in (3.3). Perform this de-committing procedure until the constraint in (3.126) is satisfied.

Perform EDLQP

Step 14: Restore the minimum limit of CU . Delete CU from $SS(DCI_i)$.

Step 15: Update $[U_{i,t}]$ and run EDLQP as described in Section 3.5.2.B.

Step 16: If $SS(DCI_i)$ is not empty, let CU be the first unit in $SS(DCI_i)$ and go to Step 4. Otherwise, stop, and $[U_{i,t}]$ is the thermal unit de-commitment solution.

where

$SS(DCI_i)$ sorted set of committed units sorted in descending order of DCI_i

$SS1(DCI_i)$ sorted set of committed units without limited fuel supply, sorted in descending order of DCI_i

$SS2(DCI_i)$ sorted set of committed units with limited fuel supply, sorted in descending order of DCI_i

$T_{cu,on}^{ch}$ continuous on-time of thermal current unit CU up to hour ch at which its unit status is changed to off

$T_{cu,off}^{ch}$ continuous off-time of thermal current unit CU up to hour ch at which its unit status is changed to on

$T_{cu,down}$ minimum down time of thermal current unit (h)

$T_{cu,up}$ minimum up time of thermal current unit (h)

3.5.3 ELRP Overall Procedures

ELR procedure

- Step 1: Initialize λ' and μ' as described in Section 3.5.1.B.
- Step 2: Initialize the ELR iteration counter, $k = 1$ and $J_B = \$10^7$.
- Step 3: Solve the unit sub-problems as described in Section 3.5.1.A.
- Step 4: If the dual solution does not satisfy the constraints described in Section 3.5.1.D, go to Step 9.
- Step 5: Carry out a constrained economic dispatch by linear/quadratic programming as described in Section 3.5.1.D.
- Step 6: Calculate the primal cost $J([U_{i,t}^k])$, the dual cost $L(P^{(k)}, U^{(k)}, \lambda^{(k)}, \mu^{(k)}, \gamma^{(k)})$ and the relative dual gap $G^{(k)}$ as described in Section 3.5.1.E.
- Step 7: If $J([U_{i,t}^k]) < J_B$, $J_B = J([U_{i,t}^k])$ and $[U_{i,t}^B] = [U_{i,t}^k]$.
- Step 8: If the relative dual gap $G^{(k)} < \varepsilon$, go to Step 10.
- Step 9: If $k < K_{max}$, $k = k + 1$, update Lagrangian multipliers adaptively as described in Section 3.5.1.C and go to Step 3. Otherwise, go to Step 10.

HSLQP procedure

- Step 10: Construct an initial HSLQP schedule as described in Section 3.5.2.A.
- Step 11: Perform EDLQP as described in Section 3.5.2.B based on the initial HSLQP schedule neglecting minimum limit constraints (setting minimum limit to zero) of committed intermediate and peak load thermal units in the ELR schedule, and the minimum limit constraints of all units with 24 hour limited fuel supply.
- Step 12: Perform the thermal unit de-commitment as described in Section 3.5.2.C.
- Step 13: Perform a 24 hour economic dispatch, EDLQP as described in Section 3.5.2.B based on the updated schedule. Calculate the total production cost.

where

- $G^{(k)}$ relative duality gap at iteration k
- $J([U_{i,t}^k])$ total economic dispatch production cost at iteration k (\$)
- K_{max} maximum allowable number of iterations
- ε relative duality gap tolerance

3.6 PRICE-BASED UNIT COMMITMENT

In a competitive environment, different entities may be responsible for executing unit commitment. In California and New England, generation companies (GENCOs) execute a price-based unit commitment whereas in Pennsylvania, Jersey, Maryland power pool (PJM) and New York, an independent system operator (ISO) follows a security constrained unit commitment [26].

The conventional UC is referred to as cost-based unit commitment since its objective is to minimize operation cost while satisfying prevailing constraints whereas in a competitive environment, the UC used by individual GENCOs is referred to as price-based unit commitment (PBUC) since its objective is to maximize the GENCO's profits depending on the electricity price and generation cost. Signals concerning energy sales price, fuel purchase prices and ancillary service sales prices are important since these prices affect the GENCOs' profits directly. Profits depend on both revenue and cost since profit is defined as the revenue minus cost.

In PBUC, satisfying load is no longer an obligation and security is unbundled from energy and priced as ancillary service. Load-balance is not a hard constraint but load forecasting is needed to forecast the market price. In addition, security is not considered in formulating PBUC.

Suppose a GENCO has many generation units and some bilateral contracts binding it to supply energy at an agreed price, amount and time. Thus, the GENCO must consider whether to purchase or sell energy at any given time. In addition, a GENCO could sell its reserves, either spinning or operating reserves. Spinning reserve is the unloaded synchronized generation that can ramp up in a specific time, e.g., 10 minutes, whereas the operating reserve is the quick start capacity from 'off' status, and non-spinning reserve is the unsynchronized generating capacity that can ramp up within a specific time.

The objective is to maximize the total profit

$$\text{Max } PF = RV - TC \quad (3.128)$$

$$\text{or Min } \{TC - RV\} \quad (3.129)$$

where

$$RV = \sum_{t=1}^T \sum_{i=1}^N (P_i^t SP^t) U_i^t + \rho \sum_{t=1}^T \sum_{i=1}^N (R_i^t RP^t) U_i^t \quad (3.130)$$

$$\begin{aligned} TC = & (1 - \rho) \sum_{t=1}^T \sum_{i=1}^N F_i(P_i^t) U_i^t + \rho \sum_{t=1}^T \sum_{i=1}^N F_i(P_i^t + R_i^t) U_i^t \\ & + \sum_{t=1}^T \sum_{i=1}^N ST_{i,t} (1 - U_i^{t-1}) U_i^t \end{aligned} \quad (3.131)$$

subject to

(a) Power demand

$$\sum_{i=1}^N P_i^t U_i^t \leq P_{load}^t \quad (3.132)$$

(b) Reserve constraint

$$\sum_{i=1}^N R_i^t U_i^t \leq P_{res}^t \quad (3.133)$$

(c) Generation and reserve limits

$$P_{i,min} \leq P_i^t \leq P_{i,max} \quad (3.134)$$

$$0 \leq R_i^t \leq P_{i,max} - P_{i,min} \quad (3.135)$$

$$P_i^t + R_i^t \leq P_{i,max} \quad (3.136)$$

(d) Minimum up/down time constraints

$$U_i^t = \begin{cases} 1 & \text{if } T_{i,on}^t < T_{i,up} \\ 0 & \text{if } T_{i,off}^t < T_{i,down} \\ 0 \text{ or } 1 & \text{otherwise} \end{cases} \quad (3.137)$$

where

PF total profit of GENCO

RV total revenues of GENCO

TC total cost of generation of GENCO

P_{res}^t system spinning requirement at hour t

P_i^t power output of unit i at hour t

R_i^t reserve generation of unit i at hour t

SP^t forecasted spot price at hour t

RP^t forecasted reserve price at hour t

ρ probability that the reserve is called and generated

A. ALAHN for Unit Scheduling

The augmented Lagrange function is formulated for the problem:

$$\begin{aligned}
E = & \sum_{t=1}^T \sum_{i=1}^N \left[(1-\rho)F_i(V_{i,p}^t) + \rho F_i(V_{i,p}^t + V_{i,r}^t) \right] V_{i,u}^t - \sum_{t=1}^T \sum_{i=1}^N \left[V_{i,p}^t S P^t + \rho V_{i,r}^t R P^t \right] V_{i,u}^t \\
& + \sum_{t=1}^T \left\{ \frac{1+S_p^t}{2} \left[V_{\lambda}^t \left(\sum_{i=1}^N V_{i,p}^t V_{i,u}^t - P_{load}^t \right) + \frac{1}{2} \beta_{\lambda}^t \left(\sum_{i=1}^N V_{i,p}^t V_{i,u}^t - P_{load}^t \right)^2 \right] \right. \\
& + \sum_{t=1}^T \left\{ \frac{1+S_p^t}{2} \left[V_{\lambda}^t \left(\sum_{i=1}^N V_{i,p}^t V_{i,u}^t - P_{load}^t \right) + \frac{1}{2} \beta_{\lambda}^t \left(\sum_{i=1}^N V_{i,p}^t V_{i,u}^t - P_{load}^t \right)^2 \right] \right. \\
& + \frac{1-S_p^t}{2} \left(-\frac{(V_{\lambda}^t)^2}{2\beta_{\lambda}^t} \right) \left. \right\} + \sum_{t=1}^T \left\{ \frac{1+S_r^t}{2} \left[V_{\mu}^t \left(\sum_{i=1}^N V_{i,r}^t V_{i,u}^t - P_{res}^t \right) + \frac{1}{2} \beta_{\mu}^t \left(\sum_{i=1}^N V_{i,r}^t V_{i,u}^t - P_{res}^t \right)^2 \right] \right. \\
& + \frac{1-S_r^t}{2} \left(-\frac{(V_{\mu}^t)^2}{2\beta_{\mu}^t} \right) \left. \right\} + \sum_{t=1}^T \sum_{i=1}^N \left\{ \frac{1+S_{i,pr}^t}{2} \left[V_{i,\gamma}^t \left(V_{i,p}^t V_{i,u}^t + V_{i,r}^t V_{i,u}^t - P_{i,max} V_{i,u}^t \right) \right. \right. \\
& + \frac{1}{2} \beta_{i,\gamma}^t \left(V_{i,p}^t V_{i,u}^t + V_{i,r}^t V_{i,u}^t - P_{i,max} V_{i,u}^t \right)^2 \left. \right\} + \frac{1-S_{i,pr}^t}{2} \left(-\frac{(V_{i,\gamma}^t)^2}{2\beta_{i,\gamma}^t} \right) \left. \right\} \\
& + \sum_{t=1}^T \sum_{i=1}^N \left[\int_0^{V_{i,p}^t} g_c^{-1}(V) dV + \int_0^{V_{i,r}^t} g_c^{-1}(V) dV \right]
\end{aligned} \tag{3.138}$$

where

$$S_p^t = \text{sign} \left\{ \sum_{i=1}^N P_i^t U_i^t - P_{load}^t + \frac{\lambda^t}{\beta_{\lambda}^t} \right\} \tag{3.139}$$

$$S_r^t = \text{sign} \left\{ \sum_{i=1}^N R_i^t U_i^t - P_{reserve}^t + \frac{\mu^t}{\beta_{\mu}^t} \right\} \tag{3.140}$$

$$S_{i,pr}^t = \text{sign} \left\{ P_i^t U_i^t + R_i^t U_i^t - P_{i,max} U_i^t + \frac{\gamma^t}{\beta_{\gamma}^t} \right\} \tag{3.141}$$

in which, $\text{sign}(x) = 1$ if $x \geq 0$ and $\text{sign}(x) = -1$ otherwise, and λ^t , μ^t and γ_i^t are Lagrangian multiplier associated constraints.

To represent the problem in a Hopfield network, $2 \times N \times T$ continuous neurons and $(N_i + 2) \times T$ multiplier neurons are needed. The energy function of the Hopfield network is defined based on the augmented Lagrange function as follows:

$$\begin{aligned}
E = & \sum_{t=1}^T \sum_{i=1}^N \left[(1-\rho)F_i(V_{i,p}^t) + \rho F_i(V_{i,p}^t + V_{i,r}^t) \right] V_{i,u}^t - \sum_{t=1}^T \sum_{i=1}^N \left[V_{i,p}^t S P^t + \rho V_{i,r}^t R P^t \right] V_{i,u}^t \\
& + \sum_{t=1}^T \left\{ \frac{1+S_p^t}{2} \left[V_{\lambda}^t \left(\sum_{i=1}^N V_{i,p}^t V_{i,u}^t - P_{load}^t \right) + \frac{1}{2} \beta_{\lambda}^t \left(\sum_{i=1}^N V_{i,p}^t V_{i,u}^t - P_{load}^t \right)^2 \right] \right. \\
& + \sum_{t=1}^T \left\{ \frac{1+S_p^t}{2} \left[V_{\lambda}^t \left(\sum_{i=1}^N V_{i,p}^t V_{i,u}^t - P_{load}^t \right) + \frac{1}{2} \beta_{\lambda}^t \left(\sum_{i=1}^N V_{i,p}^t V_{i,u}^t - P_{load}^t \right)^2 \right] \right. \\
& + \frac{1-S_p^t}{2} \left(-\frac{(V_{\lambda}^t)^2}{2\beta_{\lambda}^t} \right) \left. \right\} + \sum_{t=1}^T \left\{ \frac{1+S_r^t}{2} \left[V_{\mu}^t \left(\sum_{i=1}^N V_{i,r}^t V_{i,u}^t - P_{res}^t \right) \right. \right. \\
& + \frac{1}{2} \beta_{\mu}^t \left(\sum_{i=1}^N V_{i,r}^t V_{i,u}^t - P_{res}^t \right)^2 \left. \right] + \frac{1-S_r^t}{2} \left(-\frac{(V_{\mu}^t)^2}{2\beta_{\mu}^t} \right) \left. \right\} \\
& + \sum_{t=1}^T \sum_{i=1}^N \left\{ \frac{1+S_{i,pr}^t}{2} \left[V_{i,\gamma}^t \left(V_{i,p}^t V_{i,u}^t + V_{i,r}^t V_{i,u}^t - P_{i,max} V_{i,u}^t \right) \right. \right. \\
& + \frac{1}{2} \beta_{i,\gamma}^t \left(V_{i,p}^t V_{i,u}^t + V_{i,r}^t V_{i,u}^t - P_{i,max} V_{i,u}^t \right)^2 \left. \right] + \frac{1-S_{i,pr}^t}{2} \left(-\frac{(V_{i,\gamma}^t)^2}{2\beta_{i,\gamma}^t} \right) \left. \right\} \\
& + \sum_{t=1}^T \sum_{i=1}^N \left[\int_0^{V_{i,p}^t} g_c^{-1}(V) dV + \int_0^{V_{i,r}^t} g_c^{-1}(V) dV \right]
\end{aligned} \tag{3.142}$$

where

$V_{i,p}^t$ continuous neuron $p(i,t)$ representing power output P_i^t

$V_{i,r}^t$ continuous neuron $r(i,t)$ representing power output R_i^t

$V_{i,u}^t$ discrete neuron $u(i,t)$ representing unit status U_i^t

V_{λ}^t multiplier neuron $\lambda(t)$ representing Lagrangian multiplier λ'

V_{μ}^t multiplier neuron $\mu(t)$ representing Lagrangian multiplier μ'

$V_{i,\gamma}^t$ multiplier neuron $\gamma(i,t)$ representing Lagrangian multiplier γ_i^t

Dynamics of neurons are derived as follows:

$$\frac{dU_{i,p}^t}{dt} = -\frac{\partial E}{\partial V_{i,p}^t} = \quad (3.143)$$

$$-\left\{ \begin{aligned} & \left[(1-\rho) \frac{dF_i(V_{i,p}^t)}{dV_{i,p}^t} + \rho \frac{dF_i(V_{i,p}^t + V_{i,r}^t)}{dV_{i,p}^t} \right] V_{i,u}^t - SP^t V_{i,u}^t \\ & + \frac{1+S_p^t}{2} \left[V_{\lambda}^t + \beta_{\lambda}^t \left(\sum_{i=1}^N V_{i,p}^t V_{i,u}^t - P_{load}^t \right) \right] V_{i,u}^t \\ & \frac{1+S_{i,pr}^t}{2} \left[V_{i,\gamma}^t + \beta_{i,\gamma}^t \left(V_{i,p}^t V_{i,u}^t + V_{i,r}^t V_{i,u}^t - P_{i,max} V_{i,u}^t \right) \right] V_{i,u}^t + U_{i,p}^t \end{aligned} \right\}$$

$$\frac{dU_{i,r}^t}{dt} = -\frac{\partial E}{\partial V_{i,r}^t} = \quad (3.144)$$

$$-\left\{ \begin{aligned} & \rho \frac{dF_i(V_{i,p}^t + V_{i,r}^t)}{dV_{i,r}^t} V_{i,u}^t - \rho RP^t V_{i,u}^t \\ & + \frac{1+S_r^t}{2} \left[V_{\mu}^t + \beta_{\mu}^t \left(\sum_{i=1}^N V_{i,r}^t V_{i,u}^t - P_{res}^t \right) \right] V_{i,u}^t \\ & \frac{1+S_{i,pr}^t}{2} \left[V_{i,\gamma}^t + \beta_{i,\gamma}^t \left(V_{i,p}^t V_{i,u}^t + V_{i,r}^t V_{i,u}^t - P_{i,max} V_{i,u}^t \right) \right] V_{i,u}^t + U_{i,r}^t \end{aligned} \right\}$$

$$\frac{dU_{i,u}^t}{dt} = -\frac{\partial E}{\partial V_{i,u}^t} = \quad (3.145)$$

$$-\left\{ \begin{aligned} & \left[(1-\rho) F_i(V_{i,p}^t) + \rho F_i(V_{i,p}^t + V_{i,r}^t) \right] - \left[V_{i,p}^t SP^t + \rho V_{i,r}^t RP^t \right] \\ & + \frac{1+S_p^t}{2} \left[V_{\lambda}^t + \beta_{\lambda}^t \left(\sum_{i=1}^N V_{i,p}^t V_{i,u}^t - P_{load}^t \right) \right] V_{i,p}^t \\ & + \frac{1+S_r^t}{2} \left[V_{\mu}^t + \beta_{\mu}^t \left(\sum_{i=1}^N V_{i,r}^t V_{i,u}^t - P_{res}^t \right) \right] V_{i,r}^t \\ & \frac{1+S_{i,pr}^t}{2} \left[V_{i,\gamma}^t + \beta_{i,\gamma}^t \left(V_{i,p}^t V_{i,u}^t + V_{i,r}^t V_{i,u}^t - P_{i,max} V_{i,u}^t \right) \right] \\ & \times \left(V_{i,p}^t + V_{i,r}^t - P_{i,max} \right) \end{aligned} \right\}$$

$$\frac{dU_{\lambda}^t}{dt} = \frac{\partial E}{\partial V_{\lambda}^t} = \frac{1+S_p^t}{2} \left(\sum_{i=1}^N V_{i,p}^t V_{i,u}^t - P_{load}^t \right) + \frac{1-S_p^t}{2} \left(-\frac{V_{\lambda}^t}{\beta_{\lambda}^t} \right) \quad (3.146)$$

$$\frac{dU_{\mu}^t}{dt} = \frac{\partial E}{\partial V_{\mu}^t} = \frac{1+S_r^t}{2} \left(\sum_{i=1}^N V_{i,r}^t V_{i,u}^t - P_{res}^t \right) + \frac{1-S_r^t}{2} \left(-\frac{V_{\mu}^t}{\beta_{\mu}^t} \right) \quad (3.147)$$

$$\begin{aligned} \frac{dU_{i,\gamma}^t}{dt} &= \frac{\partial E}{\partial V_{i,\gamma}^t} \\ &= \frac{1+S_{i,pr}^t}{2} \left(V_{i,p}^t V_{i,u}^t + V_{i,r}^t U_{i,u}^t - P_{i,max} U_{i,u}^t \right) + \frac{1-S_{i,pr}^t}{2} \left(-\frac{V_{i,\gamma}^t}{\beta_{i,\gamma}^t} \right) \end{aligned} \quad (3.148)$$

where

$$\frac{dF_i(V_{i,p}^t)}{dV_{i,p}^t} = b_i + 2c_i V_{i,p}^t \quad (3.149)$$

$$\frac{dF_i(V_{i,p}^t + V_{i,r}^t)}{dV_{i,p}^t} = \frac{dF_i(V_{i,p}^t + V_{i,r}^t)}{dV_{i,r}^t} = b_i + 2c_i (V_{i,p}^t + V_{i,r}^t) \quad (3.150)$$

$U_{i,p}^t$ input of continuous neuron $p(i,t)$ with corresponding output $V_{i,p}^t$

$U_{i,r}^t$ input of continuous neuron $r(i,t)$ with corresponding output $V_{i,r}^t$

$U_{i,u}^t$ input of discrete neuron $u(i,t)$ with corresponding output $V_{i,u}^t$

U_{λ}^t input of multiplier neuron $\lambda(t)$ with corresponding output V_{λ}^t

U_{μ}^t input of multiplier neuron $\mu(t)$ with corresponding output V_{μ}^t

$U_{i,\gamma}^t$ input of multiplier neuron $\gamma(i,t)$ with corresponding output $V_{i,\gamma}^t$

The inputs of neurons are updated based on the dynamics of neurons are calculated via sigmoid and transfer functions. The selection of parameters for ALAHN is similar to that described under ALAHN for conventional UC problems.

The neurons are initialized as follows:

$$U_{i,u}^{t(0)} = \text{rand}\{-1,1\} \quad (3.151)$$

$$V_{i,u}^{t(0)} = \begin{cases} 1 & \text{if } U_{i,u}^{t(0)} \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (3.152)$$

$$V_{i,p}^{t(0)} = \frac{P_{i,\max} V_{i,u}^{t(0)}}{\sum_{j=1}^N P_{j,\max} V_{j,u}^{t(0)}} P_{load}^t \quad (3.153)$$

$$U_{i,p}^{t(0)} = g^{-1}(V_{i,p}^{t(0)}) \quad (3.154)$$

$$V_{i,r}^{t(0)} = \frac{R_{i,\max} V_{i,u}^{t(0)}}{\sum_{j=1}^N P_{j,\max} V_{j,u}^{t(0)}} P_{res}^t \quad (3.155)$$

$$U_{i,r}^{t(0)} = g^{-1}(V_{i,r}^{t(0)}) \quad (3.156)$$

$$U_{\lambda}^{t(0)} = \frac{1}{\sum_{j=1}^N V_{i,u}^{t(0)}} \sum_{i=1}^N \left[(1-\rho) \frac{dF_i(V_{i,p}^t)}{dV_{i,p}^t} + \rho \frac{dF_i(V_{i,p}^t + V_{i,r}^t)}{dV_{i,p}^t} - SP^t \right] \quad (3.157)$$

$$V_{\lambda}^{t(0)} = U_{\lambda}^{t(0)} \quad (3.158)$$

$$U_{\mu}^{t(0)} = \frac{1}{\sum_{j=1}^N V_{i,u}^{t(0)}} \sum_{i=1}^N \left[\rho \frac{dF_i(V_{i,p}^t + V_{i,r}^t)}{dV_{i,r}^t} - \rho RP^t \right] \quad (3.159)$$

$$V_{\mu}^{t(0)} = U_{\mu}^{t(0)} \quad (3.160)$$

$$V_{i,\gamma}^{t(0)} = U_{i,\gamma}^{t(0)} = 0 \quad (3.161)$$

The algorithm of ALAHN for unit scheduling in this problem is similar to the application of ALAHN to conventional UC problems.

B. Heuristic Search for Unit De-commitment and Violation Repair

1) Unit de-commitment

In cost-based UC, the intermediate units are usually committed at load peaks to maintain sufficient spinning reserve. However, in PBUC, the intermediate units are also committed at minor load peaks which may cause profit losses for GENCOs due to minimum up/down time constraints leading to negative profits for these

units during low load hours. The primary unit de-commitment procedure is to de-commit the intermediate units committed at minor load peaks with minimum up time constraint violations to reduce operation cost.

The procedure includes the work of finding the minor load peaks, finding the units with the least minimum up time constraint unsatisfied at the load peaks and de-committing the units found.

2) Minimum up/down time violation repair

The procedure for repairing minimum up/down time violations is given in Section 3.3.5.B.2.

C. ALHN for Optimal Power Dispatch

1) ALHN for optimal power dispatch

ALHN applied to optimal power dispatch is similar to ALAHN for unit scheduling with defined U_i^t .

2) Solution refining

The final unit de-commitment procedure de-commits further units with power outputs at their minimum generation limits, based on the solution of optimal power dispatch by ALHN to reduce operation cost since units operating at their minimum generation limits cannot contribute to the total profit. In other words, the operation costs of these units are higher than the revenues they offer. The procedure for the final unit de-commitment is as follows:

Step 1: Set $t = 1$.

Step 2: Find a list of units with power outputs at their minimum generation limits.

Step 3: If the list is not empty, de-commit all units in the list.

Step 4: Update the continuous on/off times as in (6.27) and (6.28).

Step 5: If $t < T$, $t = t + 1$ —return to Step 2. Otherwise, stop.

Finally, the minimum up/down time repair procedure is applied again to satisfy the constraints if violated.

D. Overall Procedure

The overall procedure of the enhanced ALAHN method for solving the PBUC problem is as follows:

Step 1: Find unit scheduling by ALAHN neglecting minimum up/down time constraints.

- Step 2: De-commit the units committed at the minor load peak having their minimum up time constraints violated found by heuristic search.
- Step 3: Repair minimum up/down time constraint violations according to heuristic search.
- Step 4: Apply ALHN to obtain the optimal schedule for the economic dispatch problem.
- Step 5: De-commit the units operating at their minimum power level based on the solution of optimal power dispatch without violating their minimum down time constraint as of heuristic search.
- Step 6: Establish the final optimal power dispatch by ALHN.

3.7 SUMMARY

This chapter presented the formulation and approaches for solving the conventional unit commitment including many practical constraints such as environmental, limited fuel, unit hourly fuel mixing ratio constraints etc. This could help utilities to implement the UC approach in real-life systems effectively and efficiently.

The formulation of the PBUC problem in a competitive environment can be used by GENCOs in some power markets where GENCOs are responsible for unit commitment. Under these circumstances, GENCOs could commit and schedule their units for selling or purchasing power or selling spinning and operating reserves in order to maximize their profits.

3.8 PROBLEMS

- 3.8.1 A system includes 3 generating units scheduled over a period of six hours. Unit characteristics and load demand are given in Tables 3.45 and 3.46. The required system spinning reserve is set to 10% of the total load demand.

Table 3.46 Unit characteristics of units in Problem 3.8.1.

Unit	$P_{i,max}$	$P_{i,min}$	a_i	b_i	c_i	$T_{i,up}$	$T_{i,down}$	Init.	UR_i	DR_i
1	600	100	78	7.97	0.00482	4	2	4	50	50
2	400	100	561	7.92	0.001562	2	2	2	30	30
3	200	50	310	7.85	0.00194	1	1	-1	10	10

Table 3.47 Load demand in Problem 3.8.1.

Hour	1	2	3	4	5	6
P'_{load}	850	800	650	700	950	1000

- 1) Solve the UC problem without ramp rate constraints.
- 2) Solve the UC problem with ramp rate constraints.

3.8.2 A system with 10 generating units is scheduled over a period of 8 hours. The system load demand and spinning reserve requirement are given in Table 3.47 and the unit characteristics in Table 3.48. Find the optimal scheduling for the generating units. Solve the problem again with ramp rate constraints supposed that both ramp up and down rates of all units are 100 MW/h.

3.8.3 A system has 10 units with the unit characteristics, load demand and spinning reserve given in Tables 3.49 and 3.50.

- 1) Find the optimal dispatch for the system.
- 2) Solve the problem when units 1, 7–9 are initially off within 6 hours prior to the beginning of the schedule time horizon.
- 3) Solve the problem again when units 1, 7–9 are initially off within 4 hours prior to the beginning of the schedule time horizon.

3.8.4 A 15-unit system is scheduled over a period of 8 hours. Load demand, spinning reserve and unit characteristics are given in Tables 3.51 and 3.52.

- 1) Find the optimal dispatch for the system.
- 2) Solve the problem with ramp rate constraints of 80 MW/h for units 1–6, 50 MW/h for units 7–10 and 30 MW/h for the rest.

Table 3.48 Load demand and spinning reserve in Problem 3.8.2.

Hour	1	2	3	4	5	6	7	8
Load	1025	1000	900	850	1025	1400	1970	2400
Reserve	85	85	65	55	85	110	165	190

Table 3.49 Unit characteristics in Problem 3.8.2.

Unit	$P_{i,max}$	$P_{i,min}$	a_i	b_i	c_i	$T_{i,up}$	$T_{i,down}$	χ_i	δ_i	κ_i	Init.
1	200	50	175	7.054	0.00515	2	2	1360	750	2	-1
2	400	130	400	7.654	0.00160	3	2	1460	650	3	5
3	375	110	400	7.762	0.00171	1	3	1370	550	3	-1
4	420	130	420	8.431	0.00150	1	3	1480	650	4	-2
5	250	75	200	8.149	0.00452	2	1	1180	625	2	7
6	850	275	725	8.162	0.00128	4	3	2200	950	4	2
7	600	165	600	8.752	0.00147	2	4	2100	950	4	1
8	750	250	600	9.121	0.00131	3	4	2300	950	4	6
9	1000	300	820	9.023	0.00113	5	4	2050	825	4	-4
10	700	540	540	9.223	0.00234	4	5	2100	900	3	-8

Table 3.50 Unit characteristics in Problem 3.8.3.

Unit	$P_{i,max}$	$P_{i,min}$	a_i	b_i	c_i	$T_{i,up}$	$T_{i,down}$	χ_i	δ_i	κ_i	Init.
1	200	75	82	1.2136	0.00148	5	2	227	0.641	0.11	6
2	60	15	15	1.4000	0.00510	5	2	85	0.588	0.20	6
3	150	50	29	1.5400	0.00212	5	2	113	0.639	0.18	6
4	120	25	32	1.4000	0.00382	5	2	94	0.650	0.18	6
5	80	20	25	1.5000	0.00396	5	2	101	0.594	0.20	6
6	100	30	40	1.3500	0.00393	5	2	114	0.570	0.20	6
7	150	50	100	1.3285	0.00135	5	2	282	0.749	0.09	6
8	520	250	105	1.3954	0.00127	5	2	267	0.749	0.09	6
9	280	75	72	1.3500	0.00261	5	2	176	0.568	0.15	6
10	320	120	49	1.2643	0.00289	5	2	187	0.617	0.13	6

Table 3.51 Load demand and spinning reserve in Problem 3.8.3.

Hour	1	2	3	4	5	6	7	8
Load	1459	1372	1299	1285	1271	1314	1372	1314
Reserve	146	137	130	129	127	131	137	131

Table 3.52 Load demand and spinning reserve in Problem 3.8.4.

Hour	1	2	3	4	5	6	7	8
Load	1200	1250	1350	1450	1500	1600	1650	1700
Reserve	120	125	135	145	150	160	165	170

Table 3.53 Unit characteristics in Problem 3.8.4.

Unit	$P_{i,max}$	$P_{i,min}$	a_i	b_i	c_i	$T_{i,up}$	$T_{i,down}$	SH_i	SC_i	$T_{i,c}$	Init.
1	455	150	671.03	10.07	0.000299	8	8	4500	9000	5	8
2	455	150	574.54	10.22	0.000183	8	8	5000	10000	5	8
3	470	105	461.37	10.40	0.000205	7	7	3200	6400	4	7
4	460	135	630.14	10.10	0.000301	7	7	3700	7400	4	7
5	465	135	548.20	9.87	0.000364	7	7	3500	7000	4	-7
6	300	60	227.09	11.21	0.000338	7	7	4000	8000	4	-7
7	162	25	173.72	11.21	0.000807	6	6	900	1800	4	-6
8	160	20	175.95	10.72	0.001203	6	6	950	1900	4	-6
9	130	20	374.59	8.80	0.001126	5	5	550	1100	4	-5
10	130	20	374.59	8.80	0.001126	5	5	560	1120	4	-5
11	80	20	186.86	10.21	0.003586	3	3	170	340	2	3
12	85	25	230.27	9.90	0.005513	3	3	260	520	2	3
13	55	10	225.28	13.12	0.000371	1	1	30	60	0	-1
14	55	10	309.03	12.12	0.001929	1	1	30	60	0	-1
15	55	10	323.79	12.41	0.00447	1	1	30	60	0	-1

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HYDROTHERMAL SCHEDULING*

4.1 INTRODUCTION

A modern power system comprises a large number of thermal and hydro power plants of various types. The integrated operation of hydro and thermal generation in a hydrothermal system to minimize the cost of generation has been referred to as hydrothermal coordination scheduling. The hydrothermal scheduling problem is even more complex than a purely thermal system optimization problem since hydroelectric plants are coupled both electrically and hydraulically. In addition, all energy resources should be fully utilized in the most economic manner. The operating cost of thermal plants is very high, though their capacity cost is low. In general, the startup of thermal plants is slow and their response speed is also low. Hence, once they are started, they have to be operated continuously for many hours/days. On the contrary, the operating cost of hydroelectric plants is low but their capital cost is high. Hydroelectric plants can be started quickly and their response is so fast that they can easily cope with fluctuating loads. Therefore, it is economical and convenient to have both thermal and hydro power plant in a generation system. Thermal power plants are generally scheduled as base load plants whereas hydroelectric plants are used during peak load periods. Hydrothermal optimal scheduling offers a substantial savings on fuel cost and enables the utilization of the full potential of limited water resources.

Hydroelectric plants are constrained by limited storage reservoirs and discharge rates. Available water resources are limited by the capacities of reservoirs and inflows, pre-specified amounts of water withdrawals from the reservoirs for meeting agricultural or flood control demands, or amounts to be preserved for navigational purposes or ecological requirements (e.g., for fish ladders).

Due to the usually cyclic nature of reservoir water inflows, hydro scheduling is classified into long range and short range problems. The long range problem is normally confined to a year, a season or a month, the short range problem may cover a day or a week.

*This chapter has been written with assistance from Nit Petcharakas

Long range problems

Long range scheduling (1 week to 1 year or several years) involves the inflow into the reservoirs in an annual cycle. The solution of long range scheduling determines water quantities to be drawn from the reservoirs for generation in each sub-interval and utilizes the water available for power generation during the total interval. It involves meteorological and statistical analyses. For optimization, it is necessary to predict the load demand, inflows, evaporation from the reservoirs, etc. for the entire year. Long range load demand, unit availabilities and hydraulic inflows are probabilistic in nature and are projected by statistical prediction methods.

Short range problems

Short range problems usually have an optimization horizon of a day or a week involving the hour-by-hour scheduling of all generation units in a hydrothermal system to achieve minimum production cost over the planning schedule. Concerning hydro power, the solution for the long range problem determines the water available each day or week. As the scheduling intervals of a short range problem are not longer than an hour, the head which is the height between reservoir surface and penstock of power plant can be considered fairly constant. The amount of water to be utilized in a short range schedule is known from the solution of the long range scheduling. Thus, the assumed loads, hydraulic inflows and unit availabilities are known. A set of starting conditions (e.g., reservoir levels, initial status of thermal power plants) is given, and the optimal hourly schedule must satisfy hydraulic stream and electric system constraints. At the end of the scheduling horizon, the “end point” conditions must be satisfied to conform to the long range water-releasing schedule established previously. In addition, at any time, the storage must remain within minimum and maximum levels regarding inflow, spillage and water discharge per hour.

4.2 HYDROELECTRIC PLANT MODEL

Hydro electric plants can be classified into three types, high head, medium head and low head plants. High head plants (typically over 300 m height) use impulse or Pelton turbines in which the water is directed into spoon shaped buckets on the wheel by means of one or more water jets located around the outside of the wheel. Medium head plants (range from 18–300 m height) use Francis turbines exclusively. Low head plants (3 to 18 m height) use propeller turbines. A hydroelectric plant model is shown in Fig. 4.1.

4.3 HYDROTHERMAL SCHEDULING FORMULATION

The objective of HTS is to minimize the production cost over the scheduled time horizon (e.g., 24 hours) under power balance, generation limit and water discharge. Neglecting hydro power operational cost, the objective function to be minimized is:

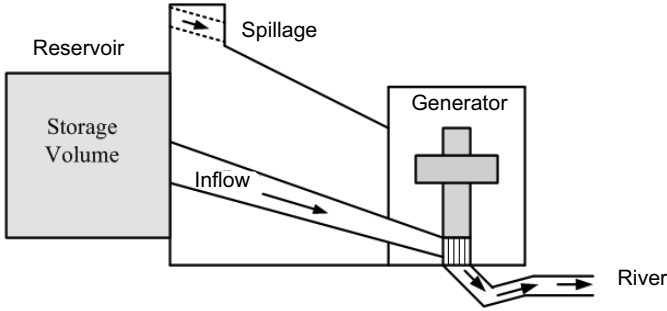


Fig. 4.1 Hydro electric plant.

$$\text{Min } F(P'_i, U_{i,t}) = \sum_{t=1}^T \sum_{i=1}^N [F_i(P'_i) + ST_{i,t}(1 - U_i^{t-1})] U_i^t \quad (4.1)$$

subject to:

(a) power balance constraint

$$P'_{load} - \sum_{i=1}^N P'_i U_i^t - \sum_{h=1}^{NH} P'_h U_h^t = 0 \quad (4.2)$$

(b) generation limit constraints

$$P_{i,\min} U_i^t \leq P'_i \leq P_{i,\max} U_i^t, i = 1, \dots, N \quad (4.3)$$

$$P_{h,\min} U_h^t \leq P'_h \leq P_{h,\max} U_h^t, h = 1, \dots, NH \quad (4.4)$$

(c) water discharge constraints

$$\sum_{t=1}^T q_h(P'_h) = q_{h,TOT}, h = 1, \dots, NH \quad (4.5)$$

$$\text{where } q_h^T = q_h^{end} \quad (4.6)$$

$$q_h(P'_h) \leq q_h^{(0)} - q_h^{end} + \sum_{t_c=1}^t \inf_{t_c}^{t_c} - \sum_{t_c=1}^t sp_h^{t_c} - \sum_{t_c=1}^{t-1} q_h(P_h^{t_c}) \quad (4.7)$$

(d) reservoir constraints

$$V_{\min} \leq V_h^t \leq V_{\max} \quad (4.8)$$

$$\text{where } V_h^t = q_h^{(0)} + \sum_{t_c=1}^t \inf_{t_c}^{t_c} - \sum_{t_c=1}^{t-1} q_h(P_h^{t_c}) - \sum_{t_c=1}^t sp_h^{t_c} \quad (4.9)$$

$F_i(P_i^t)$ generator fuel cost function in a quadratic or linear form:

$$F_i(P_i^t) = a_i + b_i P_i^t + c_i (P_i^t)^2 \text{ or } F_i(P_i^t) = a_i + b_i P_i^t$$

P_i^t output power of thermal unit i at hour t (MW)

N total number of thermal generator units

T total number of hours

$ST_{i,t}$ startup cost of unit i at hour t (\$)

U_i^t status of thermal unit i at hour t (on = 1, off = 0)

P_{load}^t load demand at hour t (MW)

NH total number of hydro generator units

P_h^t output power of hydro unit h at hour t (MW)

U_h^t status of hydro unit h at hour t (on = 1, off = 0)

$P_{i,min}$ minimum real power generation of thermal unit i (MW)

$P_{i,max}$ maximum real power generation of thermal unit i (MW)

$P_{h,min}$ minimum real power generation of hydro unit h (MW)

$P_{h,max}$ maximum real power generation of hydro unit h (MW)

$q_h(P_h^t)$ consumed water function ($10^6 \text{ m}^3/\text{h}$)

$q_{h,TOT}$ total water discharge (10^6 m^3)

q_h^T water volume for hydro power plant h at the end of hour T ($10^6 \text{ m}^3/\text{h}$)

q_h^{end} final water volume for hydro power plant h after T hours ($10^6 \text{ m}^3/\text{h}$)

$q_h^{(0)}$ initial water volume for hydro power plant h (10^6 m^3)

inf_h^t inflow water to hydro power plant h during hour t

sp_h^t spillage discharge from hydro power plant h during hour t ($10^6 \text{ m}^3/\text{h}$)

V_h^t storage volume for hydro power plant h at hour t (10^3 m^3)

V_{max} maximum storage volume (10^3 m^3)

V_{min} minimum storage volume (10^3 m^3)

4.4 HYDROTHERMAL SCHEDULING METHODS

The hydrothermal scheduling problem was solved long time ago by λ - γ iteration or hydrothermal coordination or coordination equation method (CEM) as described in [1], [2], where λ is the thermal marginal cost and γ is a fictitious or shadow price of water (currency/water volume). The concept assigns a fictitious price to water to encourage an efficient use of limited water resources by coordination with thermal marginal cost management.

In another approach, hydrothermal scheduling is solved by decomposing the problem into hydro and thermal sub-problems, and subsequently applying

Lagrangian relaxation [3] or augmented Lagrangian relaxation [4], [5] whereby in some research work, hydropower generation may be adjusted by heuristics either manually or by peak shaving [6], [7].

Alternatively, an augmented Hopfield network can be employed to provide a unit schedule and dispatch power simultaneously [8], [9].

In a recent approach, hydrothermal scheduling has been accomplished by an enhanced Lagrangian relaxation program (ELRP) [10].

4.4.1 Coordination Equation Method

The coordination equation method (CEM) or λ - γ coordination minimizes the total production cost

$$\text{Min} \sum_{t=1}^T \sum_{i=1}^N [F_i(P'_i) + ST_{i,t}(1 - U_i^{t-1})] U_i^t + \sum_{t=1}^T \sum_{h=1}^{NH} [F_h(P'_h)] U_h^t$$

where $F_h(P'_h) = a_h \gamma + b_h \gamma(P'_h) + 2c_h \gamma(P'_h)^2$ and a_h , b_h and c_h are coefficients for fictitious water discharge cost in hydro units.

Suppose that γ is the fictitious or shadow price of water (currency/water volume) and c_h is the fictitious cost which is insignificant to assist the calculation P'_h

The Lagrangian function is:

$$L = \sum_{t=1}^T \sum_{i=1}^N [F_i(P'_i) + ST_{i,t}(1 - U_i^{t-1})] U_i^t + \sum_{t=1}^T \sum_{h=1}^{NH} [F_h(P'_h)] U_h^t + \lambda(P'_{load} - P'_i - P'_h) \quad (4.10)$$

The optimum conditions are:

$$\frac{\partial L}{\partial P'_i} = \frac{\partial F_i(P'_i)}{\partial P'_i} - \lambda = 0 \quad (4.11)$$

$$P'_i = \frac{\lambda - b_i}{2c_i} \quad (4.12)$$

$$\frac{\partial L}{\partial P'_h} = \frac{\partial F_h(P'_h)}{\partial P'_h} - \lambda = 0 \quad (4.13)$$

$$P'_h = \frac{\lambda - b_h \gamma}{2c_h \gamma} \quad (4.14)$$

The solution procedure is an iteration process. The λ - γ iteration tries to adjust λ and γ to satisfy the constraints. The solution procedure is shown in Fig. 4.2. Thereby, the total volume of available water must be pre-specified.

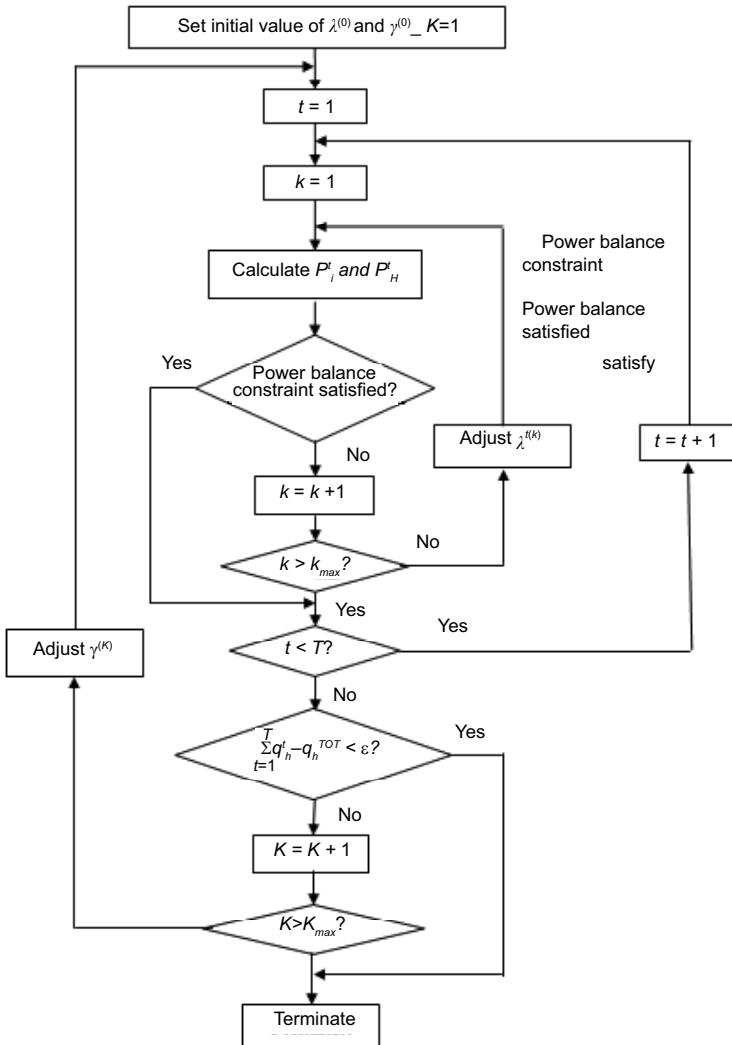


Fig. 4.2 Iteration procedure.

4.4.2 Peak Shaving Method

Suppose there are two sources of electrical energy to supply a load, one hydro and another thermal. The total amount of energy from hydroelectric plants should be used up to minimize the cost of running the thermal plants. The required energy from thermal plants is

$$\sum_{t=1}^T P'_{load} - \sum_{t=1}^T P'_h = E \quad (4.15)$$

The objective is to minimize the thermal production cost

$$F_T = \sum_{t=1}^T F(P'_i) \quad (4.16)$$

subject to

$$\sum_{t=1}^T P'_i = E \quad (4.17)$$

where E is the total energy to be supplied.

The Lagrangian function is

$$L(\alpha, P'_i) = \sum_{t=1}^T F(P'_i) + \alpha(E - \sum_{t=1}^T P'_i) \quad (4.18)$$

The optimum

$$\frac{dL}{dP'_i} = \frac{dF(P'_i)}{dP'_i} - \alpha = 0 \quad (4.19)$$

$$\frac{dF(P'_i)}{dP'_i} = \alpha \quad (4.20)$$

This means that the thermal power plants should be run at constant incremental cost for the entire period they are on.

$$F_T = \sum_{t=1}^{T_i} F(P'_i) = P_i^* T_i \quad (4.21)$$

Suppose, the thermal cost function is

$$F(P'_i) = a_i + b_i P'_i + c_i (P'_i)^2.$$

$$F_T = [a_i + b_i P'_i + c_i (P'_i)^2] T_i \quad (4.22)$$

$$\sum_{t=1}^{T_i} P'_i = P_i^* T_i = E \quad (4.23)$$

$$T_i = \frac{E}{P_i^*} \quad (4.24)$$

$$F_T = \left[a_i + b_i P_i^* + c_i (P_i^*)^2 \right] \frac{E}{P_i^*} \quad (4.25)$$

$$\frac{dF_T}{dP_i^*} = -\frac{a_i E}{(P_i^*)^2} + c_i E = 0 \quad (4.26)$$

$$P_i^* = \sqrt{\frac{a_i}{c_i}} \quad (4.27)$$

Note: The heat rate is

$$\frac{H(P_i)}{P_i} = \frac{1}{f_c} \left(\frac{a_i}{P_i} + b_i + c_i P_i \right) \quad (4.28)$$

$$\frac{d}{dP_i} \left(\frac{H(P_i)}{P_i} \right) = 0 \quad (4.29)$$

$$-\left(\frac{a_i}{P_i} \right)^2 + c_i = 0 \quad (4.30)$$

$$P_i = \sqrt{\frac{a_i}{c_i}} = P_i^* \quad (4.31)$$

where

P_i^* optimum real power output of thermal unit i (MW)

$H_i(P_i^t)$ heat function of unit i which is equivalent to $\frac{F_i(P_i^t)}{FC_i} (10^9 \text{ J/h})$

FC_i fuel cost of unit i (10^{-9} \$/J)

f_c fuel cost (10^{-9} currency/J)

The output power at best efficiency P_i^* as obtained from (4.27) or (4.31) is illustrated in Fig. 4.3.

Based on the concept of constant output power from thermal power plants, the peak shaving method (PSM) tries to shave out peak loads by using hydropower at the far right side of the load duration curve as shown in Fig. 4.4.

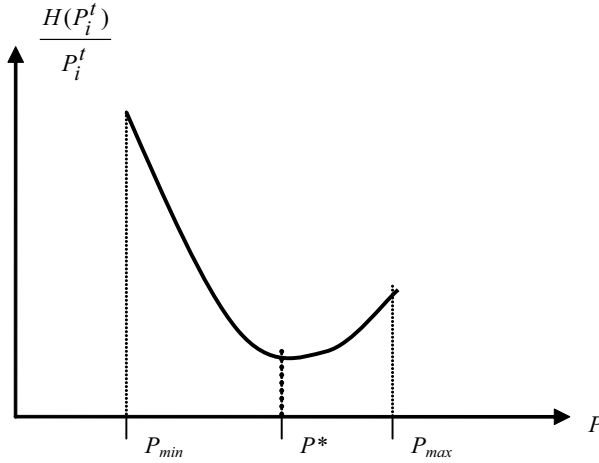


Fig. 4.3 Heat rate of a thermal power plant.

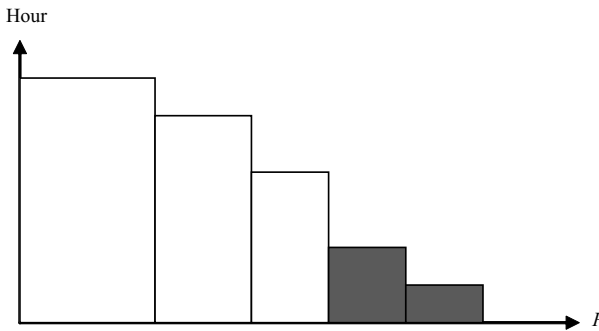


Fig. 4.4 Load duration curve.

4.4.3 Enhanced Lagrangian Relaxation Program

The Enhanced Lagrangian Relaxation Program (ELRP) resolves the hydrothermal scheduling problem with two coordination procedures: enhanced Lagrangian relaxation (ELR) and heuristic search by linear/quadratic programming (HSLQP).

The first step is an ELR excluding hydro power plants and related water discharge (4.5) to provide an LR basic thermal solution. Then, a basic schedule is created by adding hydro power plants to the LR basic solution with continuous 'on' status and relaxed minimum levels but the water discharge in (4.5) are still implemented. Subsequently, quadratic programming is used to dispatch the output power of each unit for each period over the entire period, and to adjust the unit status

accordingly. Then, all constraints are implemented. Finally, the fully constrained quadratic programming is performed on the updated schedule. The overall procedure is shown in Fig. 4.4.

Example 4.1

A system consists of 1 hydroelectric plant unit and 1 thermal power plant unit. The details of load, hydroelectric and thermal power plants are shown below.

Load: 1000 MW during 0.00–8.00 h
 1500 MW during 8.00–16.00 h
 1200 MW during 16.00–24.00 h

Thermal power plant:

$$F(P'_i) = 500 + 9P'_i + 0.002(P'_i)^2 \text{ \$/h}$$

$$150 \leq P'_i \leq 1500 \text{ MW}$$

Hydro power plant:

$$q(P'_H) = 300 + 5P'_H \cdot 10^3 \text{ m}^3/\text{h}$$

$$0 \leq P'_H \leq 1200 \text{ MW}$$

The reservoir is limited to a drawdown of 10^8 m^3 over the entire 24-h period. Inflow to the reservoir is to be neglected.

Method I: Coordination equation method

The start λ' for each hour is 10.775 \\$/MWh whereas the start fictitious price of water is $2.2 \times 10^{-3} \text{ \$/m}^3$. The solution of the coordination equation method is shown below. The corresponding total production cost is \\$ 121,685.7. The fictitious cost of water is $2.168 \times 10^{-3} \text{ \$/m}^3$.

Thermal output power:	460.4 MW during	0.00–8.00 h
	460.9 MW during	8.00–16.00 h
	460.0 MW during	16.00–24.00 h

Hydro output power:	540.4 MW during	0.00–8.00 h
	1039.4 MW during	8.00–16.00 h
	740.2 MW during	16.00–24.00 h

The solution of this method depends on the updating of $\lambda^{(i)}$ and the fictitious price of water. The total production cost is higher than that of other methods because of the problems with the power balance constraint unsatisfied.

Method II: Peak shaving method

Assume that the hydropower plant is 'on' for 24 hours, the available hydro energy is $E_H = \frac{100,000 - 300 \cdot 24}{5} = 18,560 \text{ MWh}$. The load requires a total energy of 29,600

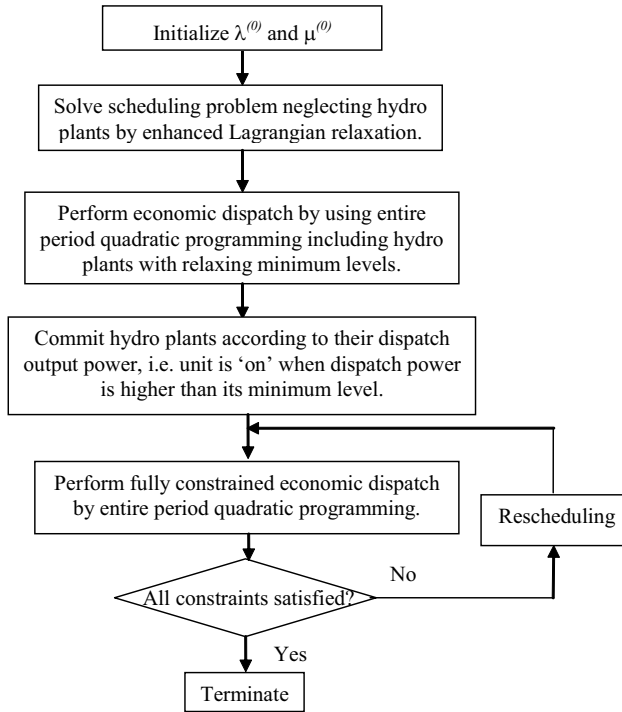


Fig. 4.5 ELRP procedure.

MWh $(1,000 \cdot 8 + 1,500 \cdot 8 + 1,200 \cdot 8)$. Hence, the energy required from the thermal plant is 11,040 MWh. The output power of the thermal plant is 460 MW $(\frac{11,040}{24})$.

Note: The power at best efficiency is 500 MW $(\sqrt{\frac{500}{.002}})$. The total production cost is 121,526.8 \$. The solution is:

Thermal output power:	460 MW during	0.00–8.00 h
	460 MW during	8.00–16.00 h
	460 MW during	16.00–24.00 h
Hydro output power:	540 MW during	0.00–8.00 h
	1040 MW during	8.00–16.00 h
	740 MW during	16.00–24.00 h

Method III: Enhanced Lagrangian relaxation method

The solution and the related total production cost is exactly the same as that of PSM.

Example 4.2

The problem is similar to that of Example 4.1 except the drawdown is limited to $2 \times 10^7 \text{ m}^3$.

Method I: Coordination equation method

The start cost of the thermal unit is set at 10.775 \$/MWh whereas the fictitious start price of water is 2.2×10^{-3} \$/m³. The solution of the coordination equation method is shown below. The total production cost is 316,765.4 \$. The fictitious final cost of water is 2.75×10^{-3} \$/m³.

Thermal output power:	1001.0 MW during	0.00–8.00 h
	1190.2 MW during	8.00–16.00 h
	1189.7 MW during	16.00–24.00 h
Hydro output power:	0 MW during	0.00–8.00 h
	310.1 MW during	8.00–16.00 h
	9.7 MW during	16.00–24.00 h

The total production cost of this method is higher than that of other methods which are shown later because power balance is mismatched and the available water cannot be used up.

Method II: Peak shaving method

The hydro plant is 'on' for 16 hours due to the load curve requirements as shown in Fig. 4.6. The available hydro energy is used during 8.00–24.00 h. At the far right of load duration curve, hydro energy supplies load of 300 MW for 8 hours. The corresponding total water discharge is $14,400 \times 10^3 \text{ m}^3$ ($q(P_H) = (300 + 5.300) \cdot 8$). The residual hydro energy supplies the second block of load partially,

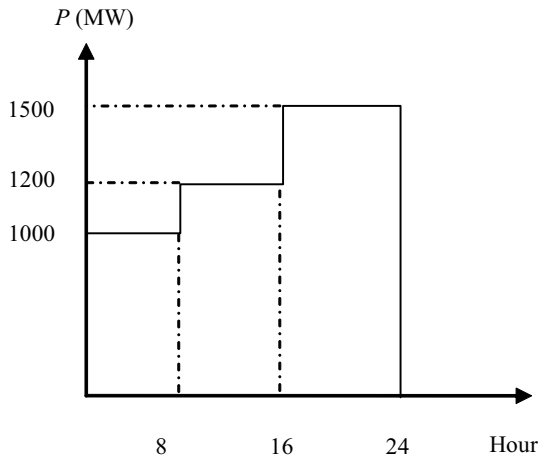


Fig. 4.6 Load duration curve of Example 4.2.

with 40 MW ($P_H = \frac{(20,000 - 14,400) - 300 \cdot 8}{5 \cdot 16}$). Thus, the hydro power output

is 340 MW during 8.00–16.00 h and 40 MW during 16.00–24.00 h, respectively. Thermal power is needed to supply the remaining load.

The total production cost is 310,099.2 \$. The solution is:

Thermal output power:	1000 MW during	0.00–8.00 h
	1160 MW during	8.00–16.00 h
	1160 MW during	16.00–24.00 h
Hydro output power:	0 MW during	0.00–8.00 h
	340 MW during	8.00–16.00 h
	40 MW during	16.00–24.00 h

Method III: Enhanced Lagrangian relaxation method

The solution and the total production cost of ELRP are exactly the same as those of PSM.

Comparing the solutions of Example 1 and Example 2, the total production cost of Example 2 is higher than that of Example 1 due to the lower amount of available water for hydro power in Example 2. In the calculation, this results in higher fictitious cost of water than in Example 1. The solutions of PSM and ELRP are similar because the problem is simple. With more constraints implemented, PSM has difficulties in finding the optimum solution since it cannot cope with hard constraints such as hydraulic or thermal constraints like minimum up and down time, ramp rate, line, fuel constraints, etc. In addition, PSM does not determine the startup cost of thermal power plants. In contrast, ELRP can be applied to systems with various constraints [10].

4.5 HYDROELECTRIC UNITS IN SERIES

Hydroelectric plants may be on the same stream building a hydraulically coupled system as shown in Fig. 4.7. A discharge from any upstream reservoir is assumed to flow directly to the succeeding downstream plant without any time lag. The hydraulic constraints are:

$$V_{h-1}^t = V_{h-1}^{t-1} + \inf_{h-1}^t - sp_{h-1}^t - q_{h-1}^t \quad (4.32)$$

$$V_{h-2}^t = V_{h-2}^{t-1} + sp_{h-1}^t - sp_{h-2}^t + q_{h-1}^t - q_{h-2}^t \quad (4.33)$$

$$V_{h-3}^t = V_{h-3}^{t-1} + sp_{h-1}^t + sp_{h-2}^t - sp_{h-3}^t + q_{h-1}^t + q_{h-2}^t - q_{h-3}^t \quad (4.34)$$

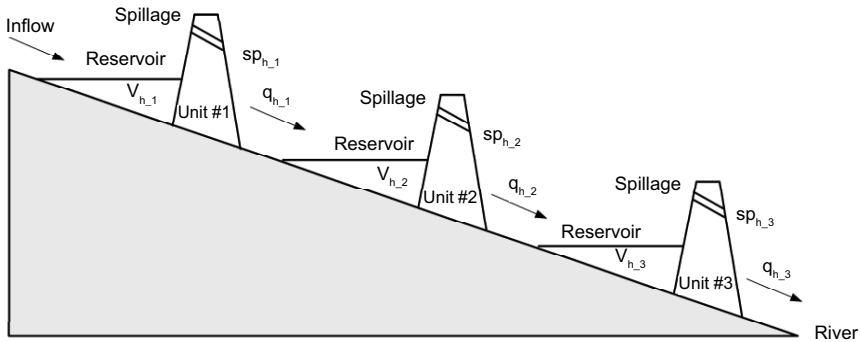


Fig. 4.7 Hydroelectric units in series.

4.6 PUMPED STORAGE HYDROELECTRIC PLANTS

Pumped storage hydroelectric plants are designed to save fuel costs by serving peak loads by hydro energy and then pumping water back up into the reservoir during light load periods (using low-cost power). Their operation is illustrated by the graphs in Fig. 4.8. To formulate a pumped storage hydroelectric plant in a hydrothermal system, it is separated into two units: the generator and the pump. To ensure that water is chronologically available for generation, an additional constraint is:

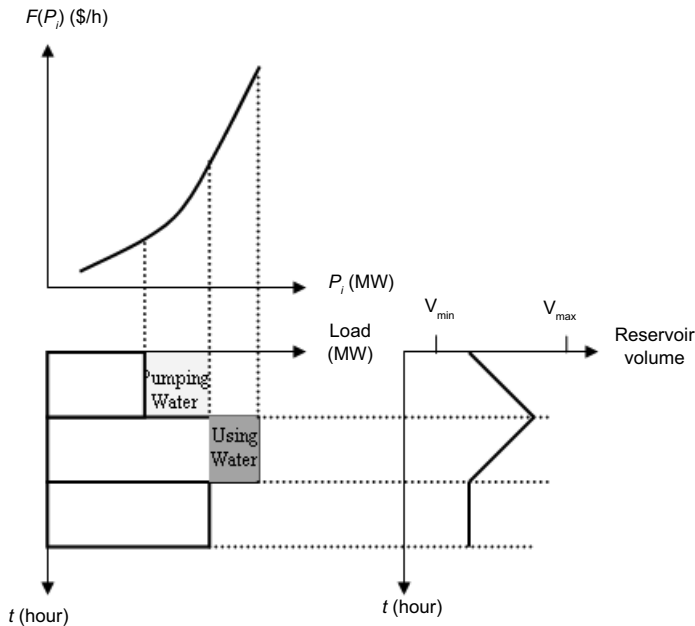


Fig. 4.8 Operation of a pumped storage hydro plant.

$$q_{ps_g}(P_{ps_g}^t) \leq q_{ps}^{(0)} + \eta \sum_{t_c=1}^{t-1} q_{ps_p}(P_{ps_p}^{t_c}) - \sum_{t_c=1}^{t-1} q_{ps_g}(P_{ps_g}^{t_c}) - q_{ps}^{end} \quad (4.35)$$

where

$q_{ps_g}(P_{ps_g}^t)$ consumed water function when a pumped storage hydroelectric plant is operated as a generator ($10^6 \text{ m}^3/\text{h}$)

$q_{ps_p}(P_{ps_p}^t)$ pumped water function when a pumped storage hydroelectric plant is operated as a pump ($10^6 \text{ m}^3/\text{h}$)

$q_{ps}^{(0)}$ initial water volume for pumped storage hydroelectric plant ps (10^6 m^3)

q_{ps}^{end} water volume for pumped storage hydroelectric plant ps at the end of hour T (10^6 m^3)

$P_{ps_p}^t$ consumed power of pumped storage hydro unit ps operated as a pump at hour t (MW)

$P_{ps_g}^t$ generation output power of pumped storage hydro unit ps , operated as a generator at hour t (MW)

η pumped storage plant efficiency, typically about 0.67.

$$\eta = \frac{c_p}{c_g} \quad (4.36)$$

based on the assumption that

$$q_{ps_g}(P_{ps_g}^t) = c_g P_{ps_g}^t \quad (4.37)$$

$$q_{ps_p}(P_{ps_p}^t) = c_p P_{ps_p}^t \quad (4.38)$$

4.7 PROBLEM FORMULATION FOR HYDROTHERMAL SCHEDULING FOR BOTH HYDROELECTRIC AND PUMPED STORAGE HYDROELECTRIC PLANTS

The objective of HTS is to minimize the production cost over the scheduled time horizon (e.g., 24 hours) under power balance, 15 minute spinning reserve, generation limit, minimum up and down time, generation ramp limit, transmission line, environmental, limited fuel, fuel mixing ratio, gas delivery, different source gas mixing ratio, alternative fuel, water discharge and water balance constraints. Neglecting hydro production cost, the objective function to be minimized is:

$$\text{Min } F(P_i^t, U_i^t) = \sum_{t=1}^T \sum_{i=1}^N [F_i(P_i^t) + ST_{i,t}(I - U_i^{t-1})] U_i^t \quad (4.39)$$

subject to:

(a) power balance constraint

$$P_{load}^t - \sum_{i=1}^N P_i^t U_i^t - \sum_{h=1}^{NH} P_h^t U_h^t - \sum_{ps=1}^{PS} P_{ps_g}^t U_{ps_g}^t + \sum_{ps=1}^{PS} P_{ps_p}^t U_{ps_p}^t = 0 \quad (4.40)$$

(b) spinning reserve constraint

$$P_{load}^t + R^t + \sum_{ps=1}^{PS} P_{ps_p}^t U_{ps_p}^t - \sum_{i=1}^N P_{i,max} U_i^t - \sum_{h=1}^{NH} P_{h,max} U_h^t - \sum_{ps=1}^{PS} P_{ps_g}^t U_{ps_g}^t \leq 0 \quad (4.41)$$

(d) generation limit constraints

$$P_{i,min} U_i^t \leq P_i^t \leq P_{i,max} U_i^t, i = 1, \dots, N \quad (4.42)$$

$$P_{h,min} U_h^t \leq P_h^t \leq P_{h,max} U_h^t, h = 1, \dots, NH \quad (4.43)$$

$$P_{ps_g}^{\min} U_{ps_g}^t \leq P_{ps_g}^t \leq P_{ps_g}^{\max} U_{ps_g}^t, ps = 1, \dots, PS \quad (4.44)$$

$$P_{ps_p}^t = P_{ps_p}^{rated} U_{ps_p}^t, ps = 1, \dots, PS \quad (4.45)$$

(e) minimum up and new down time constraints

$$U_i^t = \begin{cases} 1 & \text{if } T_{i,on}^{t-1} \leq T_{i,up} \\ 0 & \text{if } T_{i,off}^{t-1} \leq NT_{i,down} \\ 0 \text{ or } 1 & \text{otherwise} \end{cases} \quad (4.46)$$

(f) startup cost

$$ST_{i,t} = [\chi_i + \delta_i (1 - \exp(\frac{-T_{i,off}^{t-1}}{\kappa_i}))], i = 1, \dots, N \quad (4.47)$$

(g) on/offline minimum level constraints

$$P_i^t = P_{i,min}, \text{ if } U_i^{t-1} = 0 \text{ and } U_i^t = 1, \text{ or } U_i^t = 1 \text{ and } U_i^{t+1} = 0 \quad (4.48)$$

(h) generation ramp limit constraints

$$P_{i,low}^t U_i^t \leq P_i^t \leq P_{i,high}^t U_i^t, i = 1, \dots, N \quad (4.49)$$

where

$$P_{i,high}^t = \min[P_{i,max}, P_i^{t-1} + UR_i \cdot 60], \text{ if } U_i^t = U_i^{t-1} = 1 \quad (4.50)$$

$$P_{i,low}^t = \max[P_{i,min}, P_i^{t-1} - DR_i \cdot 60], \text{ if } U_i^t = U_i^{t-1} = 1 \quad (4.51)$$

(i) transmission line constraints

$$-f_l \leq \sum_{\substack{j=1 \\ j \neq \text{slack bus}}}^{NB} s_{l,j} (P'_{bus,j} - P'_{load,j}) \leq f_l, l = 1, \dots, NL \quad (4.52)$$

(j) environmental constraints

$$\sum_{t=1}^T \sum_{i \in \Omega_e} ER_i H_i (P'_i) U_i^t \leq EM \quad (4.53)$$

(k) fuel limitation constraints

$$\sum_{t=1}^T q_i (P'_i) \leq q_{i,TOT}, i \in \Omega_f \quad (4.54)$$

(l) water discharge constraints

$$\sum_{t=1}^T q_h (P'_h) = q_{h,TOT}, h = 1, \dots, NH \quad (4.55)$$

(m) water balance of pumped storage hydroelectric plant constraints

$$\sum_{t_c=1}^t q_{ps_g} (P'^{t_c}_{ps_g}) \cdot U^t_{ps_g} \leq \sum_{t_c=1}^{t-1} q_{ps_p} (P'^{t_c}_{ps_p}) \cdot U^{t_c}_{ps_p} + q_{ps,reserve} \quad (4.56)$$

The constraint in (4.56) is used to ensure that water is chronologically available for generation whereas the reserved water volume $q_{ps,reserve}$ at the beginning hour and the final hour must remain the same. To satisfy water balance constraints in (4.55), each pumped storage hydroelectric plant is treated as two separate machines, generator and pump, as explained in Section 4.6.

(n) committed unit constraints

$$\sum_{i=1}^N P_{i,min} U_i^t \leq P'_{load} \quad (4.57)$$

where

PS	total number of pumped storage hydroelectric plants
$U^t_{ps_g}$	status of pumped storage unit ps , operated as a generator at hour t (on = 1, off = 0)
$U^t_{ps_p}$	status of pumped storage unit ps , operated as a pump at hour t (on = 1, off = 0)
R^t	spinning reserve at hour t (MW)
$P'^{rated}_{ps_p}$	rated power of pumped storage hydro unit ps , operated as a pump (MW)

$P_{psg}^{\min}, P_{psg}^{\max}$	minimum and maximum output power of pumped storage hydro unit ps , operated as a generator (MW)
$T_{i,down}$	minimum down time of thermal unit i (h)
$NT_{i,down}$	new minimum down time of thermal unit i (h)
$T_{i,up}$	minimum up time of thermal unit i (h)
$T_{i,on}^{t-1}$	continuous on time of thermal unit i up to hour $t-1$
$T_{i,off}^{t-1}$	continuous off time of thermal unit i up to hour $t-1$
$\chi_i, \delta_i, \kappa_i$	startup cost parameters for thermal units
$P_{i,high}^t, P_{i,low}^t$	highest and lowest possible power output of unit i at hour t (MW)
UR_i	ramp up rate limit of thermal unit i (MW/min)
DR_i	ramp down rate limit of thermal unit i (MW/min)
f_l	upper operating limit of line l (MW)
NB	total number of buses
NL	total number of lines
$P_{bus,j}^t$	sum of generation output at bus j at hour t (MW)
$P_{load,j}^t$	load demand at hour t at bus j (MW)
Ω_e	set of thermal units releasing emissions
$q_i(P_i^t)$	consumed fuel function (10^6 m ³ /h)
$q_{i,TOT}$	total fuel supply (10^3 m ³ /h)
Ω_f	set of thermal units with limited fuel supply
$q_{ps,reserve}$	reserved water volume for pumped storage hydro plant unit ps (10^3 m ³) or (10^6 m ³)

4.8 SOLUTION METHODS FOR HYDROTHERMAL SCHEDULING INCLUDING PUMPED STORAGE HYDROELECTRIC PLANTS

Enhanced Lagrangian Relaxation Program

ELRP as described in Section 3.5 of Chapter 3 is used as a method for hydrothermal scheduling but the HSLQP procedure is modified as follows. In addition, transmission line and environmental constraints are neglected in ELR and implemented directly in Section 4.8.2. Thus, congestion management procedures aren't needed in ELR.

4.8.1 Enhanced Lagrangian Relaxation

The ELR procedure is similar to the procedure in Section 3.4.1. First, the problem is solved by enhanced Lagrangian relaxation excluding hydro power plants to provide a LR based thermal solution.

A. Dynamic Programming

This procedure is similar to the procedure in Section 3.4.1.A.

B. Initialization

This procedure is similar to the procedure in Section 3.3.4.B.

C. Adaptive updating of Lagrangian Multipliers

This procedure is similar to the procedure in Section 3.3.4.C.

D. Constrained Economic Dispatch

This procedure is similar to the procedure in Section 3.4.1.D.

E. Stopping Criteria

This procedure is similar to the procedure in Section 3.3.4.E.

4.8.2 Heuristic Search Using Linear/Quadratic Programming (HSLQP)

HSLQP in this section is similar to HSLQP in Section 3.4.2. HSLQP comprises initial HSLQP schedule, whole dispatch period constrained economic dispatch by linear/quadratic programming (EDLQP), thermal unit de-commitment and hydro unit commitment. The initial HSLQP schedule of Section 3.4.2.A is slightly changed due to hydraulic constraints whereas the thermal unit decommitment does not differ from that of Section 3.4.2.C. However, the hydro unit commitment procedure is added to commit hydro and pumped storage hydro power plants appropriately.

A. Initial HSLQP Schedule

The initial HSLQP schedule is constructed by adding 24-hour committed limited fuel supply units and hydro power units to the base schedule obtained from ELR. However, in case of insufficient thermal capacity to meet the system load, the base

schedule is created by committing all thermal units for all 24 hours. In general, generating units are classified into three types: base load units, intermediate load units, and peak load units corresponding to their low, medium, and high full load operating cost respectively [11], [12]. To satisfy time coupling constraints (4.53)–(4.55) and to use limited resources efficiently (4.54)–(4.55), economic dispatch must be performed simultaneously over the scheduled period when neglecting minimum limits in (4.42)–(4.45) (setting minimum limit to zero) of committed intermediate and peak thermal power plants of the ELR schedule, and all thermal power plants with 24 hour limited fuel supply and hydro power plants.

B. Whole Dispatch Period Constrained Economic Dispatch by Linear/Quadratic Programming (EDLQP)

EDLQP is similar to EDLQP in Section 3.4.2.B but water discharge constraints (4.55) and water balance constraints (4.56) are added.

C. Thermal Unit De-commitment

This procedure is similar to the procedure in Section 3.4.2.C.

D. Hydroelectric Unit Commitment

1. Hydro power plants

Since minimum up and down times are less than 1 hour, the constraint in (4.46) is not needed to be checked. Furthermore, with the limited amount of water available, the commitment of hydro power plants will be performed hour by hour following the ranking of dispatch power $P_{h,ed}^t$ value from hour t with the highest benefit from hydro power. The commitment procedure starts committing hydro power plants at the hour with the highest $P_{h,ed}^t$. If there is remaining water determined with regard to the following constraint (4.58), the hydro power plant concerned will be committed at the hour with the next highest $P_{h,ed}^t$.

$$q_{h,TOT} - q_{h,used} > q_h(P_{h,min}) \quad (4.58)$$

where

$$q_{h,used} = \max \left[\sum_{t=1}^T q_h(P_{h,ed}^t) \cdot U_h^t, \sum_{t=1}^T q_h(P_{h,min}) \cdot U_h^t \right] \quad (4.59)$$

The commitment procedure is as follows:

Step 1: Set $h = 1$.

Step 2: Set $U_h^t = '0'$, for $t = 1, \dots, T$.

Step 3: Sort the hours of hydro unit h in the descending order of $P_{h,ed}^t$ to obtain $SS(P_{h,ed}^t)$. Let CT_c be the first hour in $SS(P_{h,ed}^t)$.

Step 4: If the constraint in (4.58) is satisfied, set $U_h^{CT_c} = '1'$. Otherwise go to Step 6.

Step 5: Delete CT_c from $SS(P_{h,ed}^t)$ and let CT_c be the first hour in $SS(P_{h,ed}^t)$, go to Step 4.

Step 6: If $h < NH$, $h = h + 1$, go to Step 2.

Step 7: Restore minimum limit of remaining committed hydro power plants.

where

$P_{h,ed}^t$ economic dispatch generation output of hydro unit h at hour t (MW)
 $SS(P_{h,ed}^t)$ sorted set of hours sorted in descending order of $P_{h,ed}^t$ for committing hydro unit h
 CT_c current hour when a unit is committed

2. Pumped storage hydro power plants

Similar to hydro power plants, minimum up and down times are less than 1 hour, hence, the constraint in (4.46) is not active. But water balance constraints (4.55) and (4.56) must be satisfied. Hence, the pumped storage hydro power plant operated as a pump will be committed hour by hour depending on the value of $P_{ps_p}^t$. Consequently, the commitment of the pumped storage hydro power plant as a generator will be carried out similar to the procedure in Section 4.8.2.D.1 but with the following water utilization constraints:

$$q_{ps_p}^{TOT} - q_{ps_g}^{TOT} > q_{ps_g}(P_{ps_g}^{\min}) \quad (4.60)$$

where

$$q_{ps_p}^{TOT} = \sum_{t=1}^T q_{ps_p}(P_{ps_p}^{rated}) \cdot U_{ps_p}^t \quad (4.61)$$

$$q_{ps_g}^{TOT} = \max \left[\sum_{t=1}^T q_{ps_g}(P_{ps_g}^t) \cdot U_{ps_g}^t, \sum_{t=1}^T q_{ps_g}(P_{ps_g}^{\min}) \cdot U_{ps_g}^t \right] \quad (4.62)$$

where

$q_{ps_g}^{TOT}$ total water consumed by a pumped storage hydro plant operated as a generator
 (10^6 m^3)

$q_{ps_p}^{TOT}$ total water pumped by a pumped storage hydro plant operated as a pump
 (10^6 m^3)

If the remaining water is sufficient to operate a pumped storage power plant as a generator, it should be committed at the hour with the next highest $P_{ps_g}^t$. The commitment procedure is as follows:

- Step 1: Set $ps = 1$.
- Step 2: Set $U_{ps_p}^t = '0'$, for $t = 1, \dots, T$.
- Step 3: Set $t_c = 1$.
- Step 4: If $P_{ps_p}^{tc} > 0.8P_{ps_p}^{rated}$ and the spinning reserve constraint (4.41) including this pump is satisfied, or the committed unit constraint (4.57) is violated, set $U_{ps_p}^{t_c} = '1'$.
- Step 5: If $t_c < T$, $t_c = t_c + 1$, go to Step 4.
- Step 6: Set $U_{ps_g}^t = '0'$, for $t = 1, \dots, T$.
- Step 7: Sort the hours of pumped storage unit ps used as generator in descending order of $P_{ps_g}^t$ to obtain $SS(P_{ps_g}^t)$. Let CT_c be the first hour in $SS(P_{ps_g}^t)$.
- Step 8: If the constraints in (4.56) and (4.60) are satisfied, set $U_{ps_g}^{CT_c} = '1'$. Otherwise go to Step 10.
- Step 9: Delete CT_c from $SS(P_{ps_g}^t)$ and let CT_c be the first hour in $SS(P_{ps_g}^t)$, go to Step 8.
- Step 10: If $ps < PS$, $ps = ps + 1$, go to Step 2.
- Step 11: Restore minimum limits of remaining committed pumped storage hydro power plants.

where $SS(P_{ps_g}^t)$ is a sorted set of hours sorted in descending order of $P_{ps_g}^t$ for committing pumped storage hydro unit ps .

4.8.3 ELRP Overall Procedures

ELR procedure

- Step 1: Initialize λ^t and μ^t as described in Section 4.8.1.B.
- Step 2: Initialize the ELR iteration counter, $k = 1$ and $J_B = \$10^7$.
- Step 3: Solve the unit sub-problems as described in Section 4.8.1.A.
- Step 4: If the dual solution does not satisfy the constraints described in Section 4.8.1.D, go to Step 9.
- Step 5: Carry out constrained economic dispatch by linear/quadratic programming as described in Section 4.8.1.D.
- Step 6: Calculate primal cost $J([U_{i,t}^k])$, dual cost $L(P^{(k)}, U^{(k)}, \lambda^{(k)}, \mu^{(k)}, \gamma^{(k)})$ and relative dual gap $G^{(k)}$ as described in Section 4.8.1.E.
- Step 7: If $J([U_{i,t}^k]) < J_{B'}$, $J_B = J([U_{i,t}^k])$ and $[U_{i,t}^B] = [U_{i,t}^k]$.

Step 8: If the relative dual gap $G^{(k)} < \varepsilon$, go to Step 10.

Step 9: If $k < K_{max}$, $k = k + 1$, update Lagrangian multipliers adaptively as described in Section 4.8.1.C and go to Step 3. Otherwise, go to Step 10.

HSLQP procedure

Step 10: Construct an initial HSLQP schedule as described in Section 4.8.2.A.

Step 11: Perform EDLQP as described in Section 4.8.2.B based on the initial HSLQP schedule neglecting minimum limit constraints (setting minimum limit to zero) of committed intermediate and peak load thermal units of the ELR schedule, and the minimum limit constraints of all 24 hour fuel supply limited and hydroelectric units.

Step 12: Perform thermal unit decommitment as described in Section 4.8.2.C.

Step 13: Commit hydro power plants as described in Section 4.8.2.D.1.

Step 14: Perform a 24 hour economic dispatch and EDLQP as described in Section 4.8.2.B based on the updated schedule.

Step 15: Commit pumped storage hydro power plants as described in Section 4.8.2.D.2.

Step 16: Perform a full constrained 24 hour economic dispatch and EDLQP as described in Section 4.8.2.B. Calculate the total production cost.

where

J_B best total economic dispatch production cost reached (\$)

$J([U_{i,t}^{(k)}])$ total economic dispatch production cost at iteration k (\$)

$[U_{i,t}^B]$ best feasible solution reached

$[U_{i,t}^k]$ feasible solution at iteration k

$G^{(k)}$ relative duality gap at iteration k

K_{max} maximum allowable number of iterations

ε relative duality gap tolerance

Example 4.3 Hydrothermal System

This system is obtained from [8] and [9]. Since environmental constraint (4.53) is neglected, but limited fuel constraints (4.54) are considered. Unit data and load demand are shown in Appendix B.3. There are two cases as shown in Table 4.1.

Case A: the hydrothermal system comprises 17 thermal units, 2 hydroelectric plants and 2 pumped storage units [8]. The spinning reserve is 100 MW. Water discharge is sufficient for the two hydro power plants to generate electricity of 230 MWh and 100 MWh respectively. The pumped storage units have an efficiency of 75% ($\eta = 0.75$). The reserved water is assumed to be sufficient

Table 4.1 Test system for Example 4.3, Cases A and B.

	Case A	Case B
No. of all thermal units	17	17
No. of limited fuel thermal units	0	1 (unit #10)
No. of all hydro units	2	2
No. of pumped storage units	2	0
Spinning reserve (MW)	100	100
M_{up} (h)	6	3
M_{down} (h)	5	3
Transmission line constraints	Not included	$\sum_{l=1}^3 P'_{l,ed} \leq 700$
Ramp rate (MW/h)	Not included	75
Emission constraints	Not included	Not included
Limited Fuel constraint (limited energy for unit #10 in MWh)	Not included	(800 MWh)
Water discharge constraint (water discharge converted into MWh)	(230 and 100)	(235 and 110)
Water balance of pumped storage hydro plant (reserved water converted into MWh)	(4.55), (4.56) 210	Not included

for both pumped storage units for the generation of 210 MWh. Minimum up and down times of each generating unit are 6 hours and 5 hours, respectively. Transmission line, ramp rate and fuel constraints of thermal units are neglected. The load demand is load level 1 in Appendix B.3.

Case B: the hydrothermal system comprises 17 thermal units and 2 hydro plants [9]. The spinning reserve is 100 MW. Water discharge is sufficient for two hydro power plants to produce electricity of 235 MWh and 110 MWh respectively. Transmission line, ramp rate and fuel constraints of thermal units are included. All units have minimum up and down times of 3 hours. Because of transmission line constraints, the total output of the first three generators can not be higher than 700 MW. Thermal unit #10 is a fuel constrained unit with 800 MWh of energy available. The ramp rate limit is 75 MW/h. The load demand is load level 2 in Appendix B.3.

The numerical results for case A are shown in Table 4.2. If hydro power plants are not included, ELR produces a solution with a total cost of \$959,014. When hydro power plants are included, the achieved ELRP solution results in cost savings of \$28,706 as compared to the system without hydroelectric plants due to “free” hydro power. As expected, hydro power plants are scheduled to generate their output during peak load periods as shown in Fig. 4.9. However, only one pumped storage power plant is assigned to run as a pump during hours 21 and 22,

Table 4.2 Comparison of total cost in Example 3, Case A.

Problem	Method	Total cost (\$)
Thermal system	ELR	959,014
Hydrothermal system	ELRP	930,308

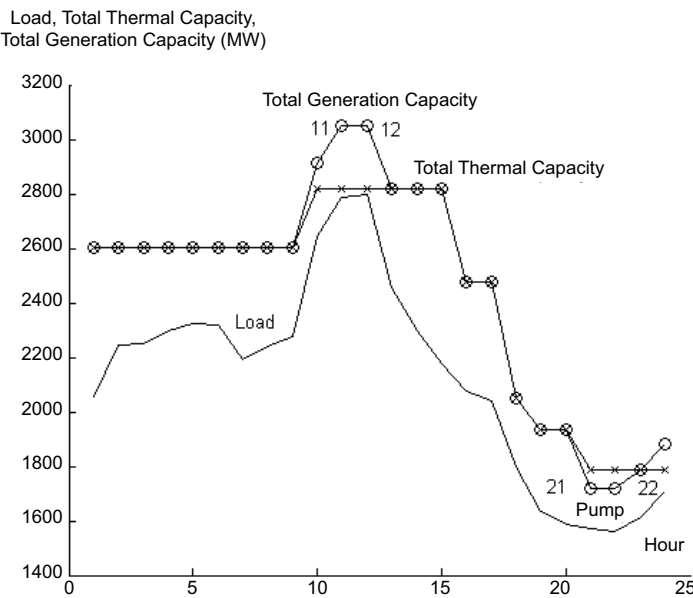


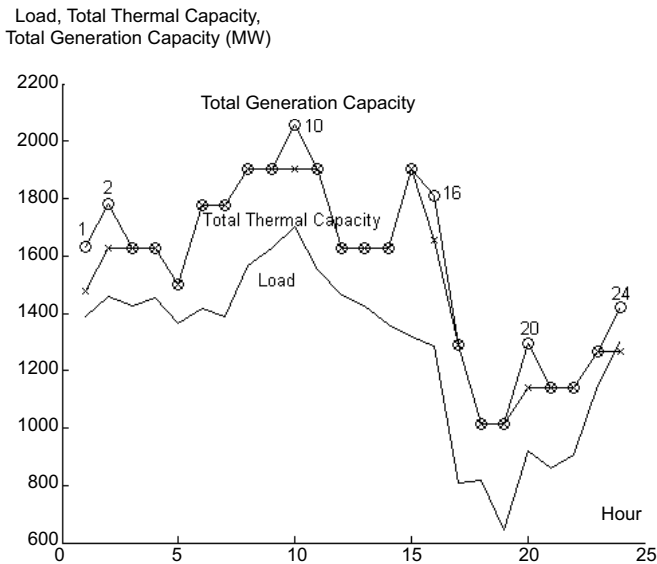
Fig. 4.9 Load curve and generation capacity in Example 4.3 Case A.

the lowest load hours, and run as a generator during hours 11 and 12, the peak load hours. The other pumped storage power plant is not used as no cost savings could be obtained. The corresponding ELRP solution resulting in production cost of \$930,308 is shown in Appendix B.3. Note: PSM & ELRP methods solve the problem by first allocating water by PSM and then performing ELRP. In other words, PSM & ELRP commits hydro power plants in the first sub-procedure, and then thermal units are committed.

In case B, the load is lower and there is no pumped storage power plant. There are two subcases considered: a fully constrained problem and a partially constrained problem neglecting limited fuel (4.54), ramp limit (4.49), on/offline minimum level (4.48) and transmission line constraints (4.52). The production costs of the partially constrained problem calculated by ELRP are lower than those of the fully constrained problem as shown in Table 4.3. This is because the more constraints implemented in a hydrothermal scheduling problem, the more the output power of each power plant will be limited. Load curve and generation capacity are shown in Fig. 4.10. The ELRP production costs are lower than those of PSM & ELRP in both cases as shown in Table 4.3. Using ELRP, the hydroelectric generation could

Table 4.3 Comparison of total cost of Case B in Example 4.3.

Hydrothermal system	Method	Total cost (\$)
Partially constrained problem	PSM & ELRP	467,935
	ELRP	467,720
Fully constrained problem	PSM & ELRP	514,848
	ELRP	487,888

**Fig. 4.10** Load curve and generation capacity in Example 4.3 Case B.

reduce the cost of thermal units during the peak load period at hour 10. In addition, the hydro power plants are used at hours 1 and 16 to shut down thermal units #7 and #6 respectively, and the remaining water discharge is used at hours 20 and 24 to avoid starting a new unit, leading to more cost savings than pre-scheduled by peak shaving method (PSM) and LRQP by 0.05% and 5.2% for partial and fully constrained problem, respectively. Based on peak shaving method, hydro power plants are used during on-peak load hours 8, 9, 10, and 11 whereas scheduled by ELRP, they are used at hours 1, 2, 10, 16, 20 and 24 which are not always at the peak of the daily load demand. Production costs of \$467,720 and \$487,888 are derived from ELRP and solutions are shown in Appendix C.

Example 4.4: IEEE 24 bus RTS with hydro power plants

The data used in this section is obtained from [12], [13] and shown in Appendix B.2. Six hydro plants with a capacity of 50 MW each are located at bus 22. Fuel limit constraints are not included, thus constraints (4.54) are neglected. There are two major cases as shown in Table 4.4.

Case A: The IEEE 24 bus reliability test system comprises 26 thermal units with CO₂ emission allowances of 125 ton/day and 95 ton/day and 6 hydroelectric plants [4]. The daily load is shown in Appendix B.2. The spinning reserve is set to 10% of the load. Water discharge is sufficient for six hydro power plants to produce electricity of 900, 900, 650, 800, 900 and 900 MWh, respectively. Ramp rates and environmental constraints are included.

Case B: This test system is similar to Case A with CO₂ emission allowances of 125 ton/day, and transmission line constraints. There are two subcases, B-1 and B-2. In case B-1, units #27–32 are all hydroelectric units whereas in case B-2, units #27–30 are hydro power units but units #31 and 32 are pumped storage hydro power plants with a maximum generation/pumping power of 50 MW [5]. The reserved water is set to 10⁶ m³ [5]. The values of the water-power conversion parameters are set as in the example described in [2].

For case A, the ELRP calculated production cost for the emission allowance of 125 ton/day is \$458,544 whereas that for the emission allowance of 95 ton per day is \$459,240.0, resulting in higher cost of \$696.5. This is because the higher production cost units with low pollution record must be dispatched to produce more power while the less expensive units causing more pollution generate less power to reduce CO₂ emission, leading to higher operating cost. The ELRP solution provides lower total cost than that of the conventional augmented Lagrangian relaxation LR, as shown in Table 4.4.

Regarding cases B-1 and B-2, the ELRP calculated production cost in case B-1 is lower than that of case B-2 because two hydro power plants in case B-1 are replaced by pumped storage hydro power plants in case B-2 as detailed in Table 4.5. This results in higher production cost due to water pumping efforts. Due to transmission line constraints, the power generation from some generating units is limited. All hydro power plants located at bus 22, shown in Fig. B.1, Appendix B.2, are dispatched to generate as much power output as possible during peak load periods resulting in decreasing power generated from nearby thermal power

Table 4.4 Comparison total cost for Example 4.4, Case A.

Hydrothermal system	Method	Total cost (\$)
Emission allowance of 125 ton/day	LR	498,385.6
	ELRP	458,544.4
Emission allowance of 95 ton/day	LR	Infeasible solution
	ELRP	459,240.9

Table 4.5 Comparison total cost for Example 4.4, Case B-1 and Case B-2.

Hydrothermal system	Method	Total cost (\$)
Case B-1	ELRP	471,997.6
Case B-2	ELRP	490,922.5

plants, units #19, 20 and 24 which are also located in the northern region at bus 23. This renders these units marginal. Note that units #19 and 20 are also marginal units during light load periods since the intermediate and peak load thermal units are decommitted during these hours. Thus, incremental costs during light load and peak load periods are very close. Principally, a pumped storage hydro plant is designed to operate whenever it can save fuel costs due to the cost difference between light load and peak load periods, limited by the machine efficiency, $\lambda^{peak} > \lambda^{light}/\eta$. Therefore, in the ELRP solution, the pumped storage hydroelectric plants are not operated.

4.8.4 Improved Merit Order and ALHN

In this section, Improved Merit Order (IMO) with ALHN is applied to the HTS problem with the objective (4.39) subject to the constraints (4.40)-(3.57).

In the IMO-ALHN method, IMO is a merit order based on the average production cost of generating units improved by heuristic search used to determine unit scheduling and repair constraint violations while ALHN is applied to solve the CED problem.

A. IMO for Thermal Unit Scheduling

The IMO method uses a heuristic search based on a merit order of generating units to commit thermal units. The merit order based average production cost of generating units which is defined as cost per MWh at their average output power between minimum and maximum operating limits is given in Chapter 3.

1) Fuel constrained unit scheduling

The fuel constrained units usually have low average production cost but can only operate on a limited number of hours due to limited fuel supply. Therefore, the proposed IMO will commit these units based on their allowable energy generation if the following condition is satisfied

$$\frac{E_i}{P_{i,\min}} < T \quad (4.63)$$

where E_i is the target energy for fuel constrained unit i corresponding to $Q_{i,TOT}$ in MWh.

That means, as the allowable energy of fuel constrained unit i does not allow it to operate during the schedule time horizon at its minimum output power, it needs to be committed first satisfying the fuel constraint. Otherwise, it will be committed similar to other thermal units and the constraint will be satisfied by solving ED.

These units are committed at peak load hours to prevent starting more expensive units which would lead to a higher operation cost. To commit the fuel constrained units, their number of committed hours corresponding to the allowable fuel supply considering ramp rate constraints is calculated so that they are neither over committed violating the fuel constraint nor under committed leading to higher operation cost.

The duration for committing the fuel constrained units before and after each peak load hour is calculated:

$$T_i = \text{round} \left(\frac{T_{i,opt} / NP - 1}{2} \right) \quad (4.64)$$

where

$$T_{i,opt} = NP(T_{i,rup} + T_{i,rdn}) + \text{ceil} \left(\frac{E_i - NP \left[\sum_{t_1=1}^{T_{i,rup}} \left(P_{i,\min} + \sum_{t_1=0}^{t_1-1} t_1 UR_i \right) + \sum_{t_2=1}^{T_{i,rdn}} \left(P_{i,\min} + \sum_{t_2=0}^{t_2-1} t_2 DR_i \right) \right]}{P_{i,\max}} \right) \quad (4.65)$$

in which

$$T_{i,rup} = \text{ceil} \left(\frac{P_{i,\max} - P_{i,\min}}{UR_i} \right) \quad (4.66)$$

$$T_{i,rdn} = \text{ceil} \left(\frac{P_{i,\max} - P_{i,\min}}{DR_i} \right) \quad (4.67)$$

T_i Duration before and after each peak load hour for commitment of fuel constrained unit i , in hours

$T_{i,opt}$ Optimal number of committed hours of fuel constrained unit i corresponding to its available fuel supply, in hours

NP Number of load peaks

$T_{i,rup}$ Number of hours for fuel constrained unit i to increase its power from $P_{i,\min}$ to $P_{i,\max}$, in hours

$T_{i,rdn}$ Number of hours for fuel constrained unit i to decrease its power from $P_{i,\max}$ to $P_{i,\min}$, in hours

If T_i of unit i is smaller than its minimum up time $T_{i,up}$, the commitment is carried out only at the highest load peak.

The procedure for fuel constrained unit scheduling is as follows:

- Step 1: Determine the number of major load peaks NP over the scheduling time horizon.
- Step 2: Set $i = 1$.
- Step 3: Determine the optimal number of committed hours $T_{j,opt}$ for fuel constrained unit i as (4.65).
- Step 4: Calculate the duration T_i before and after each load peak as (4.64).
- Step 5: If $T_i < T_{i,up}$, go to Step 10.
- Step 6: Set load peak index $p = 1$.
- Step 7: Determine the hour corresponding to the load peak p .
- Step 8: Commit this unit at this hour, and before and after the peak load hour by T_i hours.
- Step 9: If $p < NP$, $p = p + 1$ and return to Step 7. Otherwise, go to Step 11.
- Step 10: Find the highest load peak of the load demand and commit this unit around this peak with a total of $T_{i,opt}$ hours.
- Step 11: If $i < N_f$, $i = i + 1$, return to Step 3. Otherwise, stop.

where N_f is the number of fuel constrained units.

To commit other thermal units, the new spinning reserve NR' of the system excluding the fuel constrained units is recalculated as follows:

$$NR' = R' - \sum_{i \in \Omega_f} P_{i,max} U_i' \quad (4.68)$$

where U_i' has been determined from the procedure for fuel constrained unit scheduling.

2) Thermal unit scheduling

The proposed IMO for unit scheduling of other thermal units satisfying power balance and new spinning reserve in (4.68) neglecting transmission power loss is similar to IMO for unit scheduling in Chapter 3.

B. Combined ALHN and Heuristic Search for Hydroelectric Unit Scheduling

Because of the negligible operational cost of hydroelectric power, hydro units are committed based on the available water for power generation. In this case, the hydro units are committed using a combined ALHN and heuristic search algorithm if the following condition is satisfied

$$\frac{E_h}{P_{h,\min}} < T \quad (4.69)$$

where E_h is the target energy for hydro unit h corresponding to $q_{h,TOT}$ in MWh.

Similar to the fuel constrained units, it means that if the allowable energy of hydro unit h does not allow it to operate throughout the schedule time horizon at its minimum output power it needs to be committed after the thermal units satisfy the minimum up and down time and spinning constraints. Otherwise, it will be committed similar to thermal units by assigning a small fictitious cost for this unit. As committed together with thermal units, the hydro units which always have the highest merit order will be committed first. However, the hydro units are lately de-committed to reduce the excessive spinning reserve caused by the minimum up time constraint of thermal units.

The combined ALHN and heuristic search algorithms are applied for committing hydro units in three steps. In the first step, all hydro units are committed for the entire scheduling time horizon, adding to the obtained unit schedule of thermal units. In the second step, ALHN is used to solve the ED problem and satisfy power balance, limited fuel, and water balance constraints based on the obtained unit schedule whereby the minimum power outputs of the hydro units are temporarily set to zero. In the last step, hydro power units with lower power outputs than their minimum operating limits are de-committed.

However, the additional commitment of hydro units results in excessive spinning reserve at some hours. Therefore, some thermal units have to be de-committed to reduce excessive spinning reserve and thus, operation cost if feasible. Moreover, hydro units are also used to substitute thermal units to reduce operation cost.

For de-committing and substituting units, the excessive spinning reserve for each hour is calculated as follows:

$$ESR^t = \sum_{i=1}^N P_{i,\max} U_i^t + \sum_{\substack{h=1 \\ h \notin \Omega_g}}^{NH} P_{h,\max} U_h^t - P_{load}^t - R^t \quad (4.70)$$

where ESR^t is the excessive spinning reserve at hour t and Ω_g is the set of pumped-storage units operated as generators.

The procedure for de-committing thermal units is as follows:

Step 1: Set $t = 1$.

Step 2: Find a list of committed thermal units which were off at the earlier hour or will be off in the next hour.

- Step 3: If the list is not empty, choose unit i with the lowest merit order from the list. Otherwise, go to Step 6.
- Step 4: If de-commit unit i satisfying $ESR^t > P_{i,max}$ or $ESR^t < P_{i,max}$ with a spinning reserve shortage that can be compensated by the off hydro units, de-commit unit i and reduce the excessive spinning reserve ESR^t by $P_{i,max}$.
- Step 5: Delete the unit from the list and return to Step 3.
- Step 6: If $t < T$, $t = t + 1$, return to Step 2.
- Step 7: Update the continuous on/off times of thermal units as in (3.59) and (3.60).

After completing the unit de-commitment procedure, minimum up and down time constraints may be violated. Therefore, the procedure for repairing these violations in Section 3.3.5.B.2 of Chapter 3 is performed to satisfy the constraints if necessary.

The procedure for committing hydro units to repair a spinning reserve shortage is as follows:

- Step 1: Set $t = 1$.
- Step 2: If $ESR^t < 0$, find a list of uncommitted hydro units. Otherwise, go to Step 5.
- Step 3: Choose a hydro unit h having the least total number of committed hours up to this step, commit this hydro unit and increase ESR^t by $P_{h,max}$.
- Step 4: If $ESR^t < 0$, delete the unit from the list and return to Step 3.
- Step 5: If $t < T$, $t = t + 1$, return to Step 2. Otherwise, stop.

C. Pumped-Storage Unit Scheduling

Pumped-storage units are usually scheduled to save fuel cost by generating power at the peak load (otherwise at high fuel cost) and pumping water into a reservoir at low load (and low cost) hours. Based on the gradient method in [2] pumped-storage units operate as pumps during times with low operational cost λ'_{low} and operate as generators during times with high operational cost λ'_{high} , satisfying

$$\frac{\lambda'_{low}}{\xi} < \lambda'_{high} \quad (4.71)$$

Based on the obtained unit schedule and the ED solution with power balance, fuel limitation, and water discharge constraints, the pumped-storage units are scheduled to pump water at low load periods without committing new thermal units to make use of the excessive spinning reserve from thermal units and generate power at peak load periods so that equation (4.71) is satisfied.

The procedure for pumped-storage unit scheduling is as follows:

- Step 1: Determine the hour t_p where the positive λ' is the lowest to pump water with the excessive spinning reserve ESR' which is greater than the capacity of each single unit in Ω_p .
- Step 2: Determine the hour t_g where the positive λ' is the highest to generate power.
- Step 3: If equation (4.71) is not satisfied, set all the values of λ' to zero and go to Step 7.
- Step 4: Choose an uncommitted unit p in Ω_p and a corresponding unit g in Ω_g .
- Step 5: If $ESR' > P_{ps_p}^{rated}$, commit unit p at hour t_p and unit g at hour t_g and reduce ESR' by $P_{ps_p}^{rated}$ at hour t_p , and increase ESR' by $P_{ps_g}^{max}$ at hour t_g . Otherwise, set the value of λ' at t_p to zero and go to Step 7.
- Step 6: If the number of committed units in Ω_p at hour t_p is smaller than PS , return to Step 4. Otherwise, set the values of λ' at t_p and t_g to zero.
- Step 7: If there are at least two hours where $\lambda' > 0$, return to Step 1. Otherwise, stop.

where Ω_p is the set of pumped-storage units operated as pumps.

D. ALHN for Constrained Economic Dispatch

ALHN is applied to solve the CED based on the unit schedule obtained from IMO. The objective of the CED problem is to minimize the operational cost of thermal units in (4.39) neglecting startup costs subject to power balance (4.40), generator operating limits for hydro units (4.43) and pumped-storage (4.44)–(4.45), on/offline minimum level (4.48), generation ramp limit (4.49), transmission line (4.52), environmental (4.53), limited fuel (4.54), water discharge (4.55) and water balance with pumped storage hydro plant (4.56) constraints.

The power flow on transmission lines are calculated via generalized generation distribution factor (GGDF) given in Appendix A.

To handle the on/offline minimum level and operation ramp rate constraints in power generation limits, the forward and backward ramp rates are combined. The method is similar to the one in Section 3.3.6.B.1 but startup and shutdown ramp rates are replaced by the minimum generation limit.

The highest forward possible power output of unit i is calculated based on continuous on time starting from the first committed hour as follows:

$$P_{i,fwd}^t = \begin{cases} \min \left[P_{i,max}, P_{i,min} + T_{i,fwd}^{t-1} UR_i \right], & \text{if } U_i^t = U_i^{t-1} = 1 \\ P_{i,min}, & \text{if } U_i^{t-1} = 0, U_i^t = 1 \end{cases} \quad (4.72)$$

where $t = 1, \dots, T$ and $T_{i,fwd}^{t-1}$ is the continuous on time starting from the first committed hour up to hour $t-1$. The highest backward possible power output of unit i is determined based on the continuous on time starting from the last committed hour as follows:

$$P_{i,bwd}^t = \begin{cases} \min[P_{i,max}, P_{i,min} + T_{i,bwd}^{t-1} DR_i], & \text{if } U_i^t = U_i^{t-1} = 1 \\ P_{i,min}, & \text{if } U_i^t = 1, U_i^{t+1} = 0 \end{cases} \quad (4.73)$$

where $t = T-1, \dots, 1$ and $T_{i,bwd}^{t-1}$ is the continuous on time starting from committed hour $t-1$ back to the first committed hour. The up frame limit $F_{i,high}^t$ for unit i at hour t is defined as:

$$F_{i,high}^t = \begin{cases} \min[P_{i,fwd}^t, P_{i,bwd}^t], & \text{if } U_i^t = U_i^{t-1} = 1 \\ P_{i,min}, & \text{if } U_i^{t-1} = 0, U_i^t = 1 \text{ or } U_i^t = 1, U_i^{t+1} = 0 \end{cases} \quad (4.74)$$

The highest and lowest possible outputs of unit i at hour t are determined as follows:

$$P_{i,high}^t = \begin{cases} \min[F_{i,high}^t, P_i^{t-1} + UR_i], & \text{if } U_i^t = U_i^{t-1} = 1 \\ F_{i,high}^t, & \text{if } U_i^{t-1} = 0, U_i^t = 1 \text{ or } U_i^t = 1, U_i^{t+1} = 0 \end{cases} \quad (4.75)$$

$$P_{i,low}^t = \begin{cases} \max[P_{i,min}, P_i^{t-1} - DR_i], & \text{if } U_i^t = U_i^{t-1} = 1 \\ P_{i,min}, & \text{if } U_i^{t-1} = 0, U_i^t = 1 \text{ or } U_i^t = 1, U_i^{t+1} = 0 \end{cases} \quad (4.76)$$

The new generation limits of thermal units are redefined as:

$$P_{i,low}^t \leq P_i^t \leq P_{i,high}^t \quad (4.77)$$

The highest and lowest possible outputs will be used as new generation limits in ALHN for solving the RED. The highest forward possible power output, the highest backward possible power output, and the highest possible output of a unit are shown in Figs. 4.11, 4.12 and 4.13, respectively.

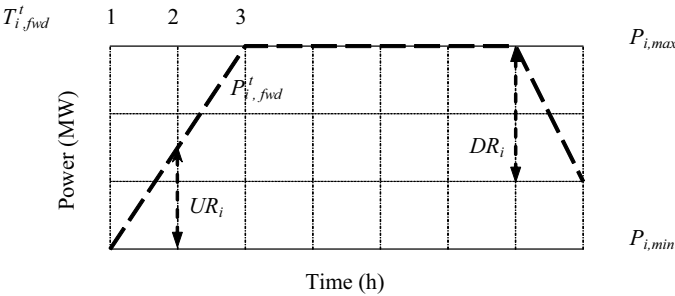


Fig. 4.11 The highest forward power output of unit i starting from the first committed hour.

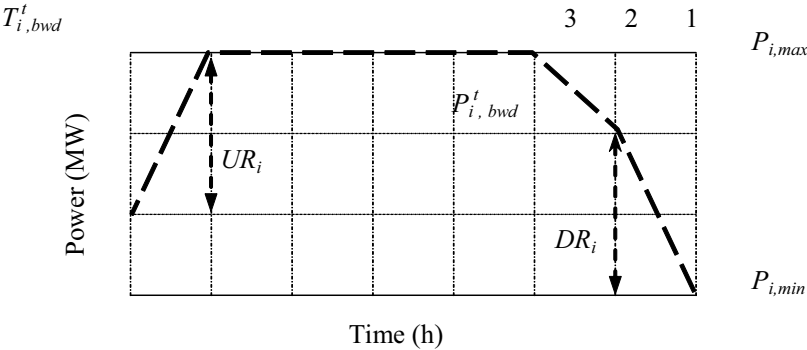


Fig. 4.12 The highest backward power output of unit i starting from the last committed hour.

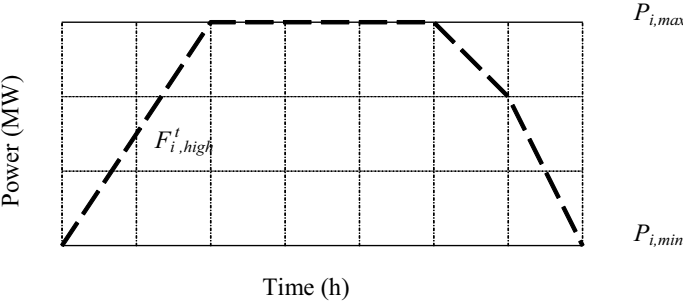


Fig. 4.13 The highest power output of unit i .

The Augmented Lagrange function of the ED problem is formulated:

$$\begin{aligned}
L = & \sum_{t=1}^T \sum_{i=1}^N \left[a_i + b_i P_i^t + c_i (P_i^t)^2 \right] U_i^t \\
& + \sum_{t=1}^T \left[\lambda^t \left(P_{load}^t + \sum_{ps=1}^{PS} P_{ps_p}^t U_{ps_p}^t - \sum_{i=1}^N P_i^t U_i^t - \sum_{h=1}^{NH} P_h^t U_h^t - \sum_{ps=1}^{PS} P_{ps_g}^t U_{ps_g}^t \right) \right. \\
& \left. + \frac{1}{2} \beta_\lambda^t \left(P_{load}^t + \sum_{ps=1}^{PS} P_{ps_p}^t U_{ps_p}^t - \sum_{i=1}^N P_i^t U_i^t - \sum_{h=1}^{NH} P_h^t U_h^t - \sum_{ps=1}^{PS} P_{ps_g}^t U_{ps_g}^t \right)^2 \right] \\
& + \sum_{i \in \Omega_f} \left[\gamma_i \left(\sum_{j=1}^T q_i(P_j^t) U_j^t - q_{i,TOT} \right) + \frac{1}{2} \beta_{i,\gamma} \left(\sum_{j=1}^T q_i(P_j^t) U_j^t - q_{i,TOT} \right)^2 \right] \\
& + \sum_{h=1}^{NH} \left[\eta_h \left(\sum_{t=1}^T q_h(P_h^t) U_h^t - q_{h,TOT} \right) + \frac{1}{2} \beta_{h,\eta} \left(\sum_{t=1}^T q_h(P_h^t) U_h^t - q_{h,TOT} \right)^2 \right] \\
& + \left(\frac{1+S_e}{2} \right) \left[v \left(\sum_{t=1}^T \sum_{i=1}^N ER_i H_i(P_i^t) U_i^t - EM \right) \right. \\
& \left. + \frac{1}{2} \beta_v \left(\sum_{t=1}^T \sum_{i=1}^N ER_i H_i(P_i^t) U_i^t - EM \right)^2 \right] + \left(\frac{1-S_e}{2} \right) \left(-\frac{v^2}{2\beta_v} \right) \quad (4.78) \\
& + \sum_{t=1}^T \sum_{i=1}^{NL} \left[\mu_i^t \left(P_i^t - \sum_{i=1}^{NL} D_{l,i} P_i^t U_i^t - \sum_{h=1}^{NH} D_{l,h} P_h^t U_h^t - \sum_{ps=1}^{PS} D_{l,ps} P_{ps_g}^t U_{ps_g}^t \right) \right. \\
& \left. + \frac{1}{2} \beta_{l,\mu}^t \left(P_i^t - \sum_{i=1}^{NL} D_{l,i} P_i^t U_i^t - \sum_{h=1}^{NH} D_{l,h} P_h^t U_h^t - \sum_{ps=1}^{PS} D_{l,ps} P_{ps_g}^t U_{ps_g}^t \right)^2 \right] \\
& + \varphi \left(\sum_{t=1}^T \sum_{ps=1}^{PS} P_{ps_g}^t U_{ps_g}^t - \xi \sum_{t=1}^T \sum_{p=1}^{PS} P_{ps_p}^t U_{ps_p}^t \right) \\
& + \frac{1}{2} \beta_\varphi \left(\sum_{t=1}^T \sum_{ps=1}^{PS} P_{ps_g}^t U_{ps_g}^t - \xi \sum_{t=1}^T \sum_{p=1}^{PS} P_{ps_p}^t U_{ps_p}^t \right)^2
\end{aligned}$$

where

$$S_e = \text{sign} \left\{ \sum_{t=1}^T \sum_{i=1}^N ER_i H_i(P_i^t) U_i^t - EM + \frac{v}{\beta_v} \right\} \quad (4.79)$$

in which $S_e = 1$ if $\sum_{t=1}^T \sum_{i=1}^N ER_i H_i(P_i^t) U_i^t - EM + \frac{v}{\beta_v} \geq 0$ and $S_e = -1$ otherwise.

$\lambda^t, \beta_\lambda^t$ Lagrangian multiplier and penalty factor associated with power balance constraint

- γ_p, β_i Lagrangian multiplier and penalty factor associated with fuel limitation constraint
- η_h, β_η Lagrangian multiplier and penalty factor associated with hydro water discharge constraint
- μ_l^t, β_l^t Lagrangian multiplier and penalty factor associated with transmission constraint
- ϕ, β_ϕ Lagrangian multiplier and penalty factor associated with emission constraint
- v, β_v Lagrangian multiplier and penalty factor associated with water balance constraint for pumped-storage units
- S_e sign function of emission constraint
- P_l^t power flow on line l at hour t
- $D_{l,i}$ sensitivity coefficient for power flow in line l with respect to power output of thermal unit i
- $D_{l,h}$ sensitivity coefficient for power flow in line l with respect to power output of hydro unit h
- $D_{l,ps}$ sensitivity coefficient for power flow in line l with respect to power output of pumped-storage unit operated as generator ps
- ξ efficiency of pumped-storage units

The energy function for continuous Hopfield network is defined:

$$\begin{aligned}
 E = & \sum_{t=1}^T \sum_{i=1}^{N_t} \left[a_i + b_i V_{i,p}^t + c_i (V_{i,p}^t)^2 \right] U_i^t \\
 & + \sum_{t=1}^T \left[V_\lambda^t \left(P_{load}^t + \sum_{ps=1}^{PS} P_{ps_p}^t U_{ps_p}^t - \sum_{i=1}^N V_{i,p}^t U_i^t - \sum_{h=1}^{NH} V_{h,p}^t U_h^t - \sum_{ps=1}^{PS} V_{ps,g}^t U_{ps_g}^t \right) \right. \\
 & \left. + \frac{1}{2} \beta_\lambda^t \left(P_{load}^t + \sum_{ps=1}^{PS} P_{ps_p}^t U_{ps_p}^t - \sum_{i=1}^N V_{i,p}^t U_i^t - \sum_{h=1}^{NH} V_{h,p}^t U_h^t - \sum_{ps=1}^{PS} V_{ps,g}^t U_{ps_g}^t \right)^2 \right] \\
 & + \sum_{i \in \Omega_f} \left[V_{i,\gamma} \left(\sum_{t=1}^T q_i(V_{i,p}^t) U_i^t - q_{i,TOT} \right) + \frac{1}{2} \beta_{i,\gamma} \left(\sum_{t=1}^T q_i(V_{i,p}^t) U_i^t - q_{i,TOT} \right)^2 \right] \\
 & + \sum_{h=1}^{NH} \left[V_{h,\eta} \left(\sum_{t=1}^T q_h(V_{h,p}^t) U_h^t - q_{h,TOT} \right) + \frac{1}{2} \beta_{h,\eta} \left(\sum_{t=1}^T q_h(V_{h,p}^t) U_h^t - q_{h,TOT} \right)^2 \right]
 \end{aligned}$$

$$\begin{aligned}
& + \left(\frac{1+S_e}{2} \right) \left[V_v \left(\sum_{t=1}^T \sum_{i=1}^N ER_i H_i(V_{i,p}^t) U_i^t - EM \right) \right] + \left(\frac{1-S_e}{2} \right) \left(-\frac{v^2}{2\beta_v} \right) \\
& + \frac{1}{2} \beta_v \left(\sum_{t=1}^T \sum_{i=1}^N ER_i H_i(V_{i,p}^t) U_i^t - EM \right)^2 \Bigg] + \left(\frac{1-S_e}{2} \right) \left(-\frac{v^2}{2\beta_v} \right) \\
& + \sum_{t=1}^T \sum_{l=1}^{NL} \left[V_{l,\mu}^t \left(V_{l,p}^t - \sum_{i=1}^N D_{l,i} V_{i,p}^t U_i^t - \sum_{h=1}^{NH} D_{l,h} V_{h,p}^t U_h^t - \sum_{ps=1}^{PS} D_{l,ps} P_{ps,g}^t U_{ps,g}^t \right) \right. \\
& \left. + \frac{1}{2} \beta_{l,\mu}^t \left(V_{l,p}^t - \sum_{i=1}^N D_{l,i} V_{i,p}^t U_i^t - \sum_{h=1}^{NH} D_{l,h} V_{h,p}^t U_h^t - \sum_{ps=1}^{PS} D_{l,ps} P_{ps,g}^t U_{ps,g}^t \right)^2 \right] \\
& + V_\phi \left(\sum_{t=1}^T \sum_{ps=1}^{PS} V_{ps,g}^t U_{ps,g}^t - \xi \sum_{t=1}^T \sum_{ps=1}^{PS} P_{ps,p}^t U_{ps,p}^t \right) + \frac{1}{2} \beta_\phi \left(\sum_{t=1}^T \sum_{ps=1}^{PS} V_{ps,g}^t U_{ps,g}^t - \xi \sum_{t=1}^T \sum_{ps=1}^{PS} P_{ps,p}^t U_{ps,p}^t \right)^2 \\
& + \sum_{t=1}^T \left(\sum_{i=1}^N \int_0^{V_{i,p}^t} g_c^{-1}(V) dV + \sum_{h=1}^{NH} \int_0^{V_{h,p}^t} g_c^{-1}(V) dV + \sum_{ps=1}^{PS} \int_0^{V_{ps,g}^t} g_c^{-1}(V) dV + \sum_{l=1}^{NL} \int_0^{V_{l,p}^t} g_c^{-1}(V) dV \right)
\end{aligned} \quad (4.80)$$

where

$V_{i,p}^t$ output of continuous neuron $p(i,t)$ representing P_i^t

$V_{h,p}^t$ output of continuous neuron $h(j)$ representing P_h^t

$V_{ps,g}^t$ output of continuous neuron $ps(g,t)$ representing $P_{ps,g}^t$

$V_{l,p}^t$ output of continuous neuron $p(l,k)$ representing P_l^k

V_λ^t output of multiplier neuron $\lambda(t)$ representing λ^t

$V_{i,\gamma}$ output of multiplier neuron $\gamma(i)$ representing γ_i

$V_{l,\mu}^t$ output of multiplier neuron $\mu(l,t)$ representing μ_l^t

V_v output of the multiplier neuron representing v

V_ϕ output of the multiplier neuron for water balance representing ϕ

Dynamics of neurons are derived as follows:

$$\begin{aligned}
\frac{dU_{i,p}^t}{dt} &= -\frac{\partial E}{\partial V_{i,p}^t} = -\left[\left[b_i + 2c_i (V_{i,p}^t)^2 \right] U_i^t \right. \\
&+ \left[V_\lambda^t + \beta_\lambda^t \left(P_{load}^t + \sum_{ps=1}^{PS} P_{ps,p}^t U_{ps,p}^t - \sum_{i=1}^N V_{i,p}^t U_i^t - \sum_{h=1}^{NH} V_{h,p}^t U_h^t - \sum_{ps=1}^{PS} V_{ps,g}^t U_{ps,g}^t \right) \right] \\
&+ \left(\frac{1+S_e}{2} \right) \left[V_v + \beta_v \left(\sum_{t=1}^T \sum_{i=1}^N ER_i H_i(V_{i,p}^t) U_i^t - EM \right) \right] \times \left(ER_i \frac{\partial H_i(V_{i,p}^t)}{\partial V_{i,p}^t} U_i^t \right) \\
&+ \left[V_{l,\mu}^t + \beta_{l,\mu}^t \left(V_{l,p}^t - \sum_{i=1}^N D_{l,i} V_{i,p}^t U_i^t - \sum_{h=1}^{NH} D_{l,h} V_{h,p}^t U_h^t - \sum_{ps=1}^{PS} D_{l,ps} V_{ps,g}^t U_{ps,g}^t \right) \right] \\
&\times \left(-D_{l,i} U_i^t \right) + U_{i,p}^t \Bigg] \quad i \notin \Omega_f
\end{aligned} \quad (4.81)$$

$$\begin{aligned}
\frac{dU_{i,p}^t}{dt} = & -\frac{\partial E}{\partial V_{i,p}^t} - \left\{ [b_i + 2c_i(V_{i,p}^t)^2] U_i^t \right. \\
& + \left[V_{\lambda}^t + \beta_{\lambda}^t \left(P_{load}^t + \sum_{ps=1}^{PS} P_{ps_p}^t U_{ps_p}^t - \sum_{i=1}^N V_{i,p}^t U_i^t - \sum_{h=1}^{NH} V_{h,p}^t U_h^t - \sum_{ps=1}^{PS} V_{ps,g}^t U_{ps_g}^t \right) \right] \\
& + \left[V_{i,\gamma}^t + \beta_{i,\gamma}^t \left(\sum_{t=1}^T q_i(V_{i,p}^t) U_i^t - q_{i,TOT} \right) \right] \times \left(\frac{dq_i(V_{i,p}^t)}{dV_{i,p}^t} U_i^t \right) \\
& + \left(\frac{1+S_e}{2} \right) \left[V_v + \beta_v \left(\sum_{t=1}^T \sum_{i=1}^N ER_i H_i(V_{i,p}^t) U_i^t - EM \right) \right] \times \left(ER_i \frac{\partial H_i(V_{i,p}^t)}{\partial V_{i,p}^t} U_i^t \right) \\
& + \left[V_{l,\mu}^t + \beta_{l,\mu}^t \left(V_{l,p}^t - \sum_{i=1}^N D_{l,i} V_{i,p}^t U_i^t - \sum_{h=1}^{NH} D_{l,h} V_{h,p}^t U_h^t - \sum_{ps=1}^{PS} D_{l,ps} V_{ps,g}^t U_{ps_g}^t \right) \right] \\
& \times (-D_{l,i} U_i^t) + U_{i,p}^t \Big\}, \quad i \in \Omega_f
\end{aligned} \tag{4.82}$$

$$\begin{aligned}
\frac{dU_{h,p}^t}{dt} = & -\frac{\partial E}{\partial V_{h,p}^t} \\
= & - \left\{ \left[V_{\lambda}^t + \beta_{\lambda}^t \left(P_{load}^t + \sum_{ps=1}^{PS} V_{ps_p}^t U_{ps_p}^t - \sum_{i=1}^N V_{i,p}^t U_i^t - \sum_{h=1}^{NH} V_{h,p}^t U_h^t - \sum_{ps=1}^{PS} V_{ps,g}^t U_{ps_g}^t \right) \right] \right. \\
& + \left[V_{h,\eta}^t + \beta_{h,\eta}^t \left(\sum_{t=1}^T q_h(V_{h,p}^t) U_h^t - q_{h,TOT} \right) \right] \times \left(\frac{\partial q_h(V_{h,p}^t)}{\partial V_{h,p}^t} U_h^t \right) \\
& + \left[V_{l,\mu}^t + \beta_{l,\mu}^t \left(V_{l,p}^t - \sum_{i=1}^N D_{l,i} V_{i,p}^t U_i^t - \sum_{h=1}^{NH} D_{l,h} V_{h,p}^t U_h^t - \sum_{ps=1}^{PS} D_{l,ps} V_{ps,g}^t U_{ps_g}^t \right) \right] \\
& \times (-D_{l,h} U_h^t) + U_{h,p}^t \Big\}
\end{aligned} \tag{4.83}$$

$$\begin{aligned}
\frac{dU_{ps,g}^t}{dt} = & -\frac{\partial E}{\partial V_{ps,g}^t} \\
= & - \left\{ \left[V_{\lambda}^t + \beta_{\lambda}^t \left(P_{load}^t + \sum_{ps=1}^{PS} P_{ps_p}^t U_{ps_p}^t - \sum_{i=1}^N V_{i,p}^t U_i^t - \sum_{h=1}^{NH} V_{h,p}^t U_h^t - \sum_{ps=1}^{PS} V_{ps,g}^t U_{ps_g}^t \right) \right] \right. \\
& + \left[V_{l,\mu}^t + \beta_{l,\mu}^t \left(V_{l,p}^t - \sum_{i=1}^N D_{l,i} V_{i,p}^t U_i^t - \sum_{h=1}^{NH} D_{l,h} V_{h,p}^t U_h^t - \sum_{ps=1}^{PS} D_{l,ps} V_{ps,g}^t U_{ps_g}^t \right) \right] \\
& \times (-D_{l,ps} V_{ps,g}^t U_{ps_g}^t) \\
& + \left[V_{\varphi} + \beta_{\varphi} \left(\sum_{t=1}^T \sum_{ps=1}^{PS} V_{ps,g}^t U_{ps_g}^t - \xi \sum_{t=1}^T \sum_{p=1}^{N_{ps}} P_{ps_p}^t U_{ps_p}^t \right) \right] \times (U_{ps_g}^t) + U_{ps,g}^t
\end{aligned} \tag{4.84}$$

$$\frac{dU_{l,p}^t}{dt} = -\frac{\partial E}{\partial U_{l,p}^t} = -\left\{ \left[V_{l,\mu}^t + \beta_{l,\mu}^t \left(V_{l,p}^t - \sum_{i=1}^N D_{l,i} V_{i,p}^t U_i^t - \sum_{h=1}^{NH} D_{l,h} V_{h,p}^t U_h^t - \sum_{ps=1}^{PS} D_{l,ps} V_{ps,g}^t U_{ps_g}^t \right) \right] + U_{l,p}^t \right\} \quad (4.85)$$

$$\frac{dU_{\lambda}^t}{dt} = +\frac{\partial E}{\partial V_{\lambda}^t} = P_{load}^t + \sum_{ps=1}^{PS} P_{ps_p}^t U_{ps_p}^t - \sum_{i=1}^N V_{i,p}^t U_i^t - \sum_{h=1}^{NH} V_{h,p}^t U_h^t - \sum_{ps=1}^{PS} V_{ps,g}^t U_{ps_g}^t \quad (4.86)$$

$$\frac{dU_{i,\gamma}}{dt} = +\frac{\partial E}{\partial U_{i,\gamma}} = \sum_{t=1}^T q_i(V_{i,p}^t) U_i^t - q_{j,TOT}, \quad i \in \Omega_f \quad (4.87)$$

$$\frac{dU_{h,\eta}}{dt} = +\frac{\partial E}{\partial V_{h,\eta}} = \sum_{t=1}^T q_h(V_{h,p}^t) U_h^t - q_{h,TOT} \quad (4.88)$$

$$\frac{dU_v}{dt} = +\frac{\partial E}{\partial V_v} = \left(\frac{1+S_e}{2} \right) \left(\sum_{t=1}^T \sum_{i=1}^N ER_i H_i(V_{i,p}^t) U_i^t - EM \right) + \left(\frac{1-S_e}{2} \right) \left(-\frac{v}{\beta_v} \right) \quad (4.89)$$

$$\frac{dU_{l,\mu}^t}{dt} = +\frac{\partial E}{\partial V_{l,\mu}^t} = V_{l,p}^t - \sum_{i=1}^N D_{l,i} V_{i,p}^t U_i^t - \sum_{h=1}^{NH} D_{l,h} V_{h,p}^t U_h^t - \sum_{ps=1}^{PS} D_{l,ps} V_{ps,g}^t U_{ps_g}^t \quad (4.90)$$

$$\frac{dU_{\varphi}}{dt} = +\frac{\partial E}{\partial V_{\varphi}} = \sum_{t=1}^T \sum_{ps=1}^{PS} V_{ps,g}^t U_{ps_g}^t - \xi \sum_{t=1}^T \sum_{ps=1}^{PS} P_{ps_p}^t U_{ps_p}^t \quad (4.91)$$

Steps of ALHN for solving the CED are similar to ALHN for ED problems in Chapter 2.

E. Heuristic Search Algorithms for Repairing Constraint Violations

1) Ramp rate constraint violation repairing

ALHN for the CED problem with all constraints except environmental emission and transmission limit constraints may obtain an infeasible solution due to ramp rate and on/offline minimum level constraints leading to power shortages during some hours. To satisfy these constraints, these power shortages will be repaired similar to IMO-ALHN for UC problems in Section 3.3.6.B.2 of Chapter 3.

i) Emission constraint violation repairing

The emission constraint may also lead to unfeasible solutions of the CED including all constraints except the transmission constraint by ALHN since to the thermal units with low production cost but high emission rates are operated on full power. Therefore, a heuristic search is applied to commit more units with low average production cost, low emission rates and high capacity to relieve this violation.

The procedure for repairing emission constraint violations is as below:

Step 1: Solve the CED with all constraints except transmission constraint by ALHN.

Step 2: If no feasible solution is found, choose an uncommitted unit with the lowest $M_i * ER_i / P_{i,max}$ ratio for committing at peak load hours while satisfying its minimum up time constraint and return to Step 1. Otherwise, stop.

ii) Transmission constraint violation repairing

Rescheduling is needed if the solution of the CED with full constraints is not feasible, committing units from the congestion index based on their capacity to relieve all congested lines during congested hours.

Based on the obtained unit scheduling, only DC power flow in transmission lines is calculated using GGDF. Consequently, some transmission lines may be overloaded by the committed units. The congestion index is used to relieve the overload on transmission lines by committing uncommitted units based on their ability to alleviate the overload in a merit order manner. The uncommitted unit with the largest congestion index gets the highest priority. The calculation of the congestion index is only based on DC power flow and GGDF without any other assumptions except network assumptions for DC power flow.

If there are CL' congested lines at hour t , the congestion index CI'_i of uncommitted unit i is calculated as follows:

$$CI'_i = \frac{P_{i,max}(1 - U'_i)}{M_i} \sum_{l=1}^{CL'} D_{l,i} \Delta P'_l \quad (4.92)$$

where

$$\Delta P'_l = \begin{cases} f_l - P'_l & \text{if } P'_l > f_l \\ -f_l - P'_l & \text{if } P'_l < -f_l \\ 0 & \text{otherwise} \end{cases} \quad (4.93)$$

CL' Number of congested lines at hour t

CI'_i Congestion index for unit i at hour t

$\Delta P'_l$ difference between power flow on line l and its maximum limit

M_i priority index of unit i defined as (2.164)

Negative D_{il} and positive P_i^t imply that committing unit i will help to decrease the power flow on line l , or vice versa, increase it. A nonzero ΔP_l^t means that line l overloaded at hour t must be relieved. Therefore, a positive term $D_{il}\Delta P_l^t$ implies that committing unit i may help alleviating overload on line l at hour t . The term is enhanced by $P_{i,max}/M_i$ to represent the capacity of unit i to relieve congested line l in the least cost manner. Thus, equation (4.92) shows the capacity of unit i to alleviate all congested lines at hour t , whereby the higher the value of CI_i^t , the better the effects alleviating the congestion.

At each congested hour, after committing the unit with the highest CI_i^t to relieve overloaded lines, a transmission constrained economic dispatch is carried out by QP. If no feasible solution is found, the next unit in CI_i^t is committed until all congested lines are relieved. After relieving all congestions at all hours, the minimum up and down time repairing procedure as in Section 3.3.5.B.2 of Chapter 3 is applied to satisfy these constraints.

F. Overall Procedure of IMO-ALHN for HTS Problems

- Step 1: Apply IMO for thermal unit scheduling.
- Step 2: Solve the ED problem by ALHN for thermal units including power balance and fuel limitation constraints.
- Step 3: Apply a heuristic search for hydro unit scheduling and unit decommitment.
- Step 4: Solve the ED by ALHN for thermal and hydro units including power balance, fuel limitation, and water discharge constraints.
- Step 5: Apply a heuristic search for pumped-storage unit scheduling.
- Step 6: Solve the CED by ALHN with all constraints except emission and transmission constraints.
- Step 7: If no feasible solution is found, repair the violations of ramp rate constraints by heuristic search and return to Step 6.
- Step 8: Solve the CED by ALHN with all constraints except transmission constraint.
- Step 9: If no feasible solution is found, repair any violation of the emission constraint by heuristic search and return to Step 8.
- Step 10: If the transmission constraint is satisfied, go to Step 13.
- Step 11: Solve the CED by ALHN with all constraints.
- Step 12: If no feasible solution is found, repair the transmission violation by heuristic search and return to Step 11.
- Step 13: Stop.

For partial problems, the unrelated steps in the overall procedure are neglected.

Example 4.5

The system includes the 17 thermal, 2 hydroelectric and 2 pumped-storage plants from Example 4.3 scheduled over a 24-hour horizon. The two cases considered in this example have the same data as the two cases in Example 4.3.

Case A: As the fuel constrained units are not included in this case, the unit scheduling is conducted for the thermal, hydro and pumped-storage units. Thermal units are committed first, based on their merit order defined in Table 4.6.

Table 4.6 Merit order of 17 thermal units in Example 4.5, Case A.

Unit	1	2	3	4	5	6	7	8	9
Merit order	1	2	4	5	3	7	11	14	10
Unit	10	11	12	12	14	15	16	17	
Merit order	6	16	15	8	9	17	13	12	

Since the total capacity of thermal units is not sufficient to satisfy load demand and spinning reserve at peak load hours, the power shortages at these peak load hours will be compensated by the hydro units. The unit scheduling of thermal units by IMO is given in Table 4.7. The excessive spinning reserve at each hour is calculated:

$$ESR' = \begin{bmatrix} 109 & 39 & 36 & 50 & 24 & 25 & 152 & 106 & 70 & 39 & -6 & -16 \\ 56 & 213 & 335 & 97 & 35 & 1 & 41 & 88 & 106 & 113 & 60 & 95 \end{bmatrix}$$

Based on the obtained unit scheduling, hydro units are all added with $U_h' = 1$. ALHN is applied to solve the ED problem with power balance and water discharge constraints for hydro units with the temporary setting $P_{h,min} = 0$ for all hydro units. Then the hydro units with a smaller power output than their actual $P_{h,min}$ are decommitted. The schedule of thermal and hydro units is obtained as shown in Table 4.8. The excessive units are not necessarily de-committed since this does not lead to total cost savings.

The excessive spinning reserve including committed thermal and hydro units is determined:

$$ESR' = \begin{bmatrix} 109 & 194 & 191 & 205 & 179 & 180 & 152 & 106 & 70 & 194 & 149 & 139 \\ 56 & 213 & 335 & 97 & 127 & 1 & 41 & 88 & 106 & 113 & 60 & 95 \end{bmatrix}$$

Table 4.7 Thermal unit scheduling by IMO in Example 4.5, Case A.

TU	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1	1	1	1	1	1	1	0	1	1	0	0	1	1	0	0	0
2	1	1	1	1	1	1	1	0	1	1	0	0	1	1	0	1	0
3	1	1	1	1	1	1	1	0	1	1	0	0	1	1	0	1	0
4	1	1	1	1	1	1	1	0	1	1	0	0	1	1	0	1	1
5	1	1	1	1	1	1	1	0	1	1	0	0	1	1	0	1	1
6	1	1	1	1	1	1	1	0	1	1	0	0	1	1	0	1	1
7	1	1	1	1	1	1	1	0	1	1	0	0	1	1	0	1	1
8	1	1	1	1	1	1	1	0	1	1	0	0	1	1	0	1	1
9	1	1	1	1	1	1	1	0	1	1	0	0	1	1	0	1	1
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1
11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
13	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	0	0
14	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	0	0
15	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	0	0
16	1	1	1	1	1	1	0	0	1	1	0	0	1	1	1	0	0
17	1	1	1	1	1	1	0	0	1	1	0	0	1	1	0	0	0
18	1	1	1	1	1	1	0	0	0	1	0	0	1	1	0	0	0
19	1	1	1	1	1	1	0	0	0	1	0	0	1	0	0	0	0
20	1	1	1	1	1	1	0	0	0	1	0	0	1	0	0	0	0
21	1	1	1	1	1	1	0	0	0	1	0	0	1	0	0	0	0
22	1	1	1	1	1	1	0	0	0	1	0	0	1	0	0	0	0
23	1	1	1	1	1	1	0	0	0	1	0	0	1	0	0	0	0
24	1	1	1	1	1	1	0	0	0	1	0	0	1	1	0	0	0

ALHN is applied again to solve the ED problem with power balance and water discharge constraints, and the new generation limits of hydro units. The obtained energy costs are as follows:

$$\lambda' = \begin{bmatrix} 21.30 & 25.31 & 25.31 & 25.30 & 25.31 & 25.31 & 25.00 & 25.24 \\ 25.29 & 25.31 & 25.31 & 25.31 & 24.93 & 22.20 & 20.75 & 22.29 \\ 25.30 & 20.14 & 18.93 & 18.54 & 18.45 & 18.41 & 18.75 & 19.00 \end{bmatrix}$$

Heuristic search is used to commit pumped-storage units by a gradient method where the units are used for pumping water at times with low energy cost without committing new thermal units and generate power at times with high energy cost. Based on the obtained energy cost, the pumped-storage units are used for pumping water at hours 20–22 and generating power at hours 5 and 11–12. The obtained unit scheduling for all units is given in Table 4.9, where G_i and P_i ($i = 1, 2$) represent pumped-storage unit i separately, operated as generator or pump, respectively. After committing pumped-storage units, it is not necessary to de-commit other thermal units since it does not lead to cost savings.

Table 4.8 Unit scheduling with thermal and hydro units in Example 4.5, Case A.

TU	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	H1	H2
1	1	1	1	1	1	1	1	0	1	1	0	0	1	1	0	0	0	0	0
2	1	1	1	1	1	1	1	0	1	1	0	0	1	1	0	1	0	1	1
3	1	1	1	1	1	1	1	0	1	1	0	0	1	1	0	1	0	1	1
4	1	1	1	1	1	1	1	0	1	1	0	0	1	1	0	1	1	1	1
5	1	1	1	1	1	1	1	0	1	1	0	0	1	1	0	1	1	1	1
6	1	1	1	1	1	1	1	0	1	1	0	0	1	1	0	1	1	1	1
7	1	1	1	1	1	1	1	0	1	1	0	0	1	1	0	1	1	0	0
8	1	1	1	1	1	1	1	0	1	1	0	0	1	1	0	1	1	0	0
9	1	1	1	1	1	1	1	0	1	1	0	0	1	1	0	1	1	0	0
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1
11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
13	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	0	0	0	0
14	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	0	0	0	0
15	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	0	0	0	0
16	1	1	1	1	1	1	0	0	1	1	0	0	1	1	1	0	0	0	0
17	1	1	1	1	1	1	0	0	1	1	0	0	1	1	0	0	0	1	0
18	1	1	1	1	1	1	0	0	0	1	0	0	1	1	0	0	0	0	0
19	1	1	1	1	1	1	0	0	0	1	0	0	1	0	0	0	0	0	0
20	1	1	1	1	1	1	0	0	0	1	0	0	1	0	0	0	0	0	0
21	1	1	1	1	1	1	0	0	0	1	0	0	1	0	0	0	0	0	0
22	1	1	1	1	1	1	0	0	0	1	0	0	1	0	0	0	0	0	0
23	1	1	1	1	1	1	0	0	0	1	0	0	1	0	0	0	0	0	0
24	1	1	1	1	1	1	0	0	0	1	0	0	1	1	0	0	0	0	0

ALHN is used to solve the final ED problem resulting in total cost of \$928,498. The capacity curves of the committed units are given in Fig. 4.14 and the final solution of ED is given in Appendix D.

Case B: In this case, the fuel constrained units are committed first, followed by thermal and hydro units. Fuel constrained unit 10 is committed first due to the heuristic search as shown below:

$$U'_{10} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

The thermal units are committed by IMO based on the merit order in Table 4.6. The obtained unit scheduling is given in Table 4.10.

The excessive spinning reserve is calculated:

$$ESR' = \begin{bmatrix} 9 & 65 & 99 & 73 & 158 & 108 & 11 & 108 & 50 & 100 & 249 & 337 \\ 101 & 41 & 79 & 0 & 110 & 100 & 272 & 120 & 180 & 407 & 164 & 11 \end{bmatrix}$$

Table 4.9 Unit scheduling for all units in Example 4.5, Case A.

T\U	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	H1	H2	G1	G2	P1	P2
1	1	1	1	1	1	1	1	0	1	1	0	0	1	1	0	0	0	0	0	0	0	0	0
2	1	1	1	1	1	1	1	0	1	1	0	0	1	1	0	1	0	1	1	0	0	0	0
3	1	1	1	1	1	1	1	0	1	1	0	0	1	1	0	1	0	1	1	0	0	0	0
4	1	1	1	1	1	1	1	0	1	1	0	0	1	1	0	1	1	1	1	0	0	0	0
5	1	1	1	1	1	1	1	0	1	1	0	0	1	1	0	1	1	1	1	1	0	0	0
6	1	1	1	1	1	1	1	0	1	1	0	0	1	1	0	1	1	1	1	0	0	0	0
7	1	1	1	1	1	1	1	0	1	1	0	0	1	1	0	1	1	0	0	0	0	0	0
8	1	1	1	1	1	1	1	0	1	1	0	0	1	1	0	1	1	0	0	0	0	0	0
9	1	1	1	1	1	1	1	0	1	1	0	0	1	1	0	1	1	0	0	0	0	0	0
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	0	0	0	0
11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
13	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
14	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
15	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
16	1	1	1	1	1	1	0	0	1	1	0	0	1	1	1	0	0	0	0	0	0	0	0
17	1	1	1	1	1	1	0	0	1	1	0	0	1	1	0	0	0	1	0	0	0	0	0
18	1	1	1	1	1	1	0	0	0	1	0	0	1	1	0	0	0	0	0	0	0	0	0
19	1	1	1	1	1	1	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0
20	1	1	1	1	1	1	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	1	0
21	1	1	1	1	1	1	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	1	0
22	1	1	1	1	1	1	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	1	0
23	1	1	1	1	1	1	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0
24	1	1	1	1	1	1	0	0	0	1	0	0	1	1	0	0	0	0	0	0	0	0	0

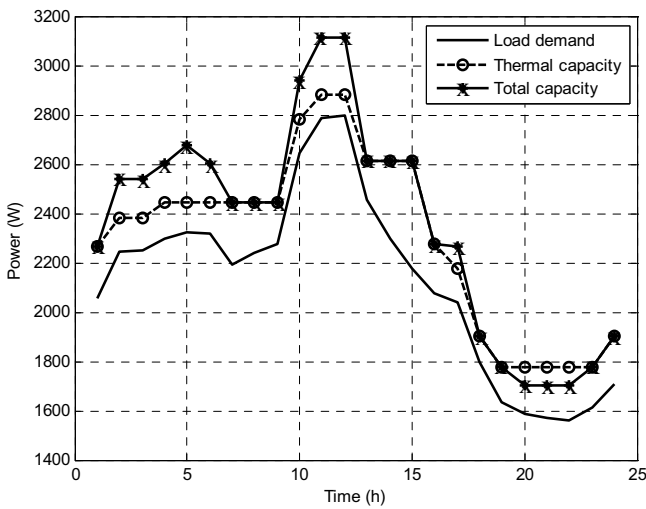


Fig. 4.14 Capacity curves for the hydrothermal system in Example 4.5, Case A.

Table 4.10 Unit scheduling for thermal units in Example 4.5, Case B.

T\U	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1	1	1	1	1	1	1	0	0	0	0	0	1	0	0	0	0
2	1	1	1	1	1	1	0	0	0	0	0	0	1	1	0	0	0
3	1	1	1	1	1	1	0	0	0	0	0	0	1	1	0	0	0
4	1	1	1	1	1	1	0	0	0	0	0	0	1	1	0	0	0
5	1	1	1	1	1	1	0	0	0	0	0	0	1	1	0	0	0
6	1	1	1	1	1	1	0	0	0	0	0	0	1	1	0	0	0
7	1	1	1	1	1	1	0	0	0	0	0	0	1	0	0	0	0
8	1	1	1	1	1	1	0	0	0	1	0	0	1	0	0	0	0
9	1	1	1	1	1	1	0	0	0	1	0	0	1	0	0	0	0
10	1	1	1	1	1	1	0	0	0	1	0	0	1	1	0	0	0
11	1	1	1	1	1	1	0	0	0	1	0	0	1	1	0	0	0
12	1	1	1	1	1	1	0	0	0	1	0	0	1	1	0	0	0
13	1	1	1	1	1	1	0	0	0	0	0	0	1	1	0	0	0
14	1	1	1	1	1	1	0	0	0	0	0	0	1	0	0	0	0
15	1	1	1	1	1	1	0	0	0	0	0	0	1	0	0	0	0
16	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
17	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0
18	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0
19	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0
20	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
21	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
22	1	1	1	1	1	0	0	0	0	1	0	0	0	0	0	0	0
23	1	1	1	1	1	0	0	0	0	1	0	0	0	0	0	0	0
24	1	1	1	1	1	0	0	0	0	1	0	0	0	0	0	0	0

To commit hydro units, the unit status of all hydro units are set to 1. Then ALHN is used to solve the ED problem with energy constraints of hydro units and their minimum generation limit is temporarily set to zero. In the obtained solution, only hydro units with greater power output than zero are actually committed. The hydro units can be committed by this way because they are able to contribute more power during power shortages as they do not suffer from complicated constraints such as ramp rate and transmission constraints. Moreover, it is not necessary to de-commit the thermal units since the transmission imposes a power generation limit on generating units 1–3 causing an increased power shortage. The hydro units are committed as shown in Table 4.11.

Table 4.11 Unit scheduling for hydro units in Example 4.5, Case B.

Unit/Time	1	2	3	4	5	6	7	8	9	10	11	12
H1	0	0	0	0	0	0	0	1	1	1	0	0
H2	0	0	0	0	0	0	0	1	1	1	0	0
Unit/Time	13	14	15	16	17	18	19	20	21	22	23	24
H1	0	0	1	1	0	0	0	0	0	0	1	1
H2	0	0	1	1	0	0	0	0	0	0	1	1

Based on the obtained unit scheduling of all units, ALHN is used to solve the ED problem with ramp rate, fuel limitation and transmission constraints. The solution is infeasible due to power shortages due to power balance constraints as follows:

$$\Delta P' = \begin{bmatrix} 15.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 58.82 \\ 58.82 & 58.82 & 0.00 & 0.00 & 0.00 & 0.00 & 58.82 & 64.00 \\ 0.00 & 0.00 & 0.00 & 8.00 & 0.00 & 0.00 & 58.82 & 58.82 \end{bmatrix}$$

Moreover, the energy constraints of fuel limited and hydro units are also not satisfied as they are scheduled to generate more power than the limits. However, these violations are automatically relieved when the power balance constraint is satisfied.

The power balance constraint is violated at hours 1, 8–10, 15–16, 20 and 23–24. Heuristic search will be used to increase power generation during the hours of power shortage. For the power shortages at hours 1 and 8, a hydro unit is committed to compensate due to the small amount needed. In fact, hydro units are very flexible in repairing power shortages at single hours as they have no minimum up/down time constraints. For the power shortage during hours 8–10, unit 13 is committed during hours 8–9 since this unit has the highest merit order among the uncommitted units at this time. The power shortage at hour 10 is automatically relieved when unit 13 is committed during hours 8–9 since the power output of this unit is no further limited to its minimum generation limit. However, hydro units active during hours 8–10 should be de-committed due to redundant power generation. On the other hand, the hydro units will be used to relieve power shortages at other times such as hours 1 and 20. For the power shortage during hours 15–16, unit 13 is further committed during hours 17–18 since it has higher merit order than other uncommitted units. If unit 13 is only committed at hour 17, the power shortage at hour 15 is still relieved but the power shortage at hour 16 is not alleviated since unit 13 will be forced to operate at its minimum generation limit due to ramp rate constraints. For the power shortage during hours 23–24, unit 13 is only committed at hour 24 to alleviate the violation as hydro power units are still able to generate power at hour 23 to satisfy the constraint due to their de-commitment during hours 8–10.

Finally, ALHN is used again to solve the ED problem with all constraints. The obtained total cost without any further constraint violations is \$488,085. The capacity curves of committed units are given in Fig. 4.15 and the final solution for ED is given in Appendix D.

4.8.5 ALHN Based Lagrangian Relaxation Method

The ALHN-LR method consists of ILR and ALHN guided by heuristic search based algorithms. For solving the HTS problem, the method applies ILR combined with heuristic search algorithms to find thermal unit scheduling, heuristic search

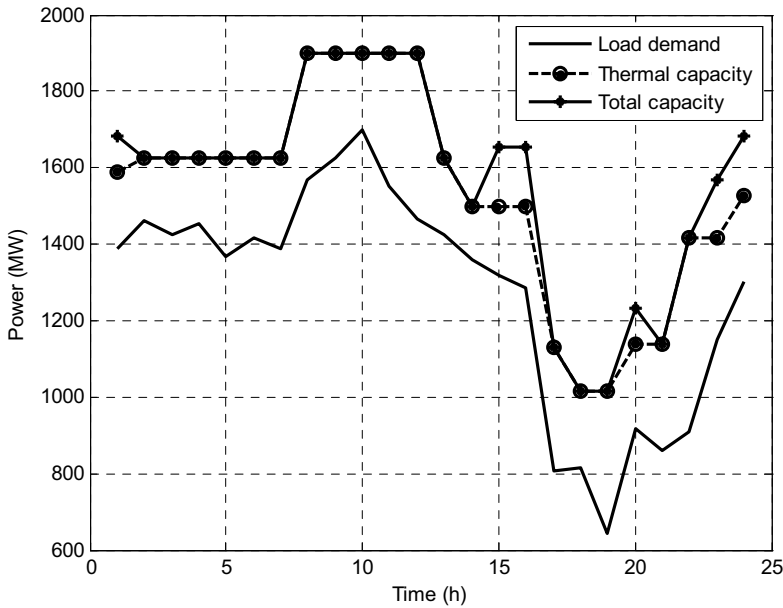


Fig. 4.15 Capacity curves for the hydrothermal system in Example 4.5, Case B.

based algorithms for committing hydro and pumped-storage units, and to repair violations of constraints such as ramp rate, emissions, and transmission, and finally ALHN for solving the CED.

This method employs some heuristic search algorithms previously used in ALHN-LR applied to UC and IMO-ALHN applied to HTS.

A. Thermal Unit Scheduling

In this method, the thermal units are committed prior to hydro and pumped-storage units satisfying system load demand and spinning reserve. The fuel-constrained units are committed first at peak load hours satisfying energy and ramp rate constraints using heuristic search whereas the other thermal units are committed by ILR.

1) Fuel-constrained unit scheduling

The unit scheduling of fuel constrained units solved by heuristic search is similar to that of IMO for fuel constrained unit scheduling given in Section 4.8.4.A.1.

2) Thermal unit scheduling

The unit scheduling for thermal units excluding fuel constrained units solved by ILR and heuristic search is similar to ALHN-LR in UC in Section 3.3.7.A of Chapter 3.

B. Hydro Unit Scheduling

The hydro unit scheduling solved by ALHN and heuristic search is similar to that of IMO for unit scheduling in Section 4.8.4.B.

C. Pumped-Storage Unit Scheduling

The unit scheduling for pumped-storage units solved by heuristic search is similar to that of IMO for unit scheduling in Section 4.8.4.C.

D. ALHN for Constrained Economic Dispatch

The CED problem formulation with new generation limits and GGDF calculation for transmission power flow is similar to the IMO-ALHN method given in Section 4.8.4.D.

E. Heuristic Search for Repairing Constraint Violations

1) Ramp rate constraint violation repairing

The heuristic search used for ramp rate violation repairing is similar to the one in Section 4.8.4.E.1.

2) Emission constraint violation repairing

The heuristic search for repairing emission constraint violations is given in Section 4.8.4.E.2.

3) Transmission constraint violation repairing

The transmission constraint violation is repaired by heuristic search as can be found in Section 4.8.4.E.3.

F. Overall Procedure

The overall procedure of ALHN-LR applied to an HTS problem is as follows:

- Step 1: Apply a heuristic search for fuel-constrained unit scheduling.
- Step 2: Apply ILR and heuristic search for thermal unit scheduling except for fuel-constrained units.
- Step 3: Solve the ED problem by ALHN with power balance and fuel constraints.
- Step 4: Use heuristic search for hydro unit scheduling and unit decommitment.

- Step 5: Solve the ED problem with power balance, fuel and water discharge constraints by ALHN.
- Step 6: Apply a gradient based heuristic search for pumped-storage unit scheduling.
- Step 7: Solve the CED problem with all constraints except emission and transmission constraints by ALHN.
- Step 8: If no feasible solution is found, repair violations and return to Step 7.
- Step 9: If the emission constraint is satisfied, go to Step 12.
- Step 10: Solve the CED problem with all constraints except transmission constraint by ALHN.
- Step 11: If no feasible solution is found, repair violations and return to Step 10.
- Step 12: If the transmission constraint is satisfied, go to Step 15.
- Step 13: Solve the CED problem with all constraints by ALHN.
- Step 14: If no feasible solution is found, repair violations and return to Step 13.
- Step 15: Stop.

For partial problems, the unrelated sub-procedures are neglected.

Example 4.6

In this example, the considered system is that of Example 4.3 with 17 thermal, 2 hydro and 2 pumped-storage units scheduled over a period of 24 hours. Two cases will be considered in this example similar to the two cases in Example 4.5.

Case A: First of all, ILR is used to find the unit scheduling for thermal units. After repairing minimum up/down time constraint violations, the unit scheduling of thermal units is given in Table 4.12. The excessive spinning reserve is determined:

$$ESR' = \begin{bmatrix} 204 & 196 & 193 & 145 & 119 & 120 & 247 & 201 & 165 & 39 & -6 & -16 \\ 209 & 366 & 331 & 188 & 126 & 1 & 41 & 88 & 106 & 113 & 60 & 95 \end{bmatrix}$$

The excessive spinning reserve is negative during hours 11–12 since the total committed capacity of thermal units is not enough to satisfy power demand and spinning reserve requirements during these hours. This amount of power shortage has to be compensated by hydro units.

In the next step, hydro units are committed by heuristic search. The hydro units are initially committed all time but their minimum generation limit is temporarily set to zero. Based on the solution of the ED problem with power balance and energy limit constraints of hydro units by ALHN, hydro units with a smaller power output than their actual minimum generation limit are de-committed. Note that the hydro units are not de-committed during the hours with negative excessive

Table 4.12 Thermal unit scheduling by ILR in Example 4.6, Case A.

TU	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	0
2	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1	1
3	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1	1
4	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1	1
5	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1	1
6	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1	1
7	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1	1
8	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1	1
9	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1	1
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1
11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1
14	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1
15	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	0	0
16	1	1	1	1	1	1	1	0	1	1	0	0	1	1	1	0	0
17	1	1	1	1	1	1	1	0	1	1	0	0	1	1	0	0	0
18	1	1	1	1	1	1	0	0	0	1	0	0	1	1	0	0	0
19	1	1	1	1	1	1	0	0	0	1	0	0	1	0	0	0	0
20	1	1	1	1	1	1	0	0	0	1	0	0	1	0	0	0	0
21	1	1	1	1	1	1	0	0	0	1	0	0	1	0	0	0	0
22	1	1	1	1	1	1	0	0	0	1	0	0	1	0	0	0	0
23	1	1	1	1	1	1	0	0	0	1	0	0	1	0	0	0	0
24	1	1	1	1	1	1	0	0	0	1	0	0	1	1	0	0	0

spinning reserve as they are used to satisfy load demand and spinning reserve. The thermal units becoming excessive due to the commitment of hydro units are de-committed to reduce excessive spinning reserve. The schedule of thermal and hydro units after de-committing excessive thermal units and repairing minimum up/down time constraint violations is given in Table 4.13. The excessive spinning reserve is now determined accordingly as follows:

$$ESR' = \begin{bmatrix} 90 & 16 & 13 & 27 & 1 & 2 & 129 & 83 & 47 & 112 & 49 & 39 \\ 227 & 227 & 349 & 170 & 35 & 1 & 41 & 88 & 106 & 113 & 60 & 95 \end{bmatrix}$$

The pumped-storage units are committed based on the energy cost obtained by the ED solution including power balance and hydro energy limit constraints. The energy cost is obtained by ALHN as follows:

$$\lambda^t = \begin{bmatrix} 22.56 & 27.34 & 27.53 & 26.63 & 28.31 & 28.25 & 22.39 & 25.20 \\ 25.33 & 26.96 & 26.97 & 26.98 & 23.10 & 22.23 & 20.27 & 20.27 \\ 26.11 & 20.14 & 19.40 & 18.96 & 18.78 & 18.72 & 19.22 & 19.48 \end{bmatrix}$$

Table 4.13 Unit schedule of thermal and hydro units in Example 4.6, Case A.

T\U	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	H1	H2
1	1	1	1	1	1	1	1	1	1	1	0	0	0	1	0	0	0	0	0
2	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	0	0	0
3	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	0	0	0
4	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	1	0	0
5	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	1	0	0
6	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	1	0	0
7	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	1	0	0
8	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	1	0	0
9	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	1	0	0
10	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	1
11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1
12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1
13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0	0
14	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0	1	0	0	0
15	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0	1	0	0	0
16	1	1	1	1	1	1	1	0	1	1	1	0	1	1	0	0	0	0	0
17	1	1	1	1	1	1	0	0	1	1	0	0	1	1	0	0	0	0	0
18	1	1	1	1	1	1	0	0	0	1	0	0	1	1	0	0	0	0	0
19	1	1	1	1	1	1	0	0	0	1	0	0	1	0	0	0	0	0	0
20	1	1	1	1	1	1	0	0	0	1	0	0	1	0	0	0	0	0	0
21	1	1	1	1	1	1	0	0	0	1	0	0	1	0	0	0	0	0	0
22	1	1	1	1	1	1	0	0	0	1	0	0	1	0	0	0	0	0	0
23	1	1	1	1	1	1	0	0	0	1	0	0	1	0	0	0	0	0	0
24	1	1	1	1	1	1	0	0	0	1	0	0	1	1	0	0	0	0	0

Based on the obtained energy cost, the pumped-storage units are committed during hours 20–22 and 24 for pumping water and committed during hours 3–4 and 5–6 for generating power. The final unit schedule for all units is given in Table 4.14. The de-commitment of excessive thermal units is not necessary since it does not lead to cost savings.

ALHN is applied to solve the final ED problem with all constraints. The obtained total cost is \$918,686. The capacity curves of committed units are given in Fig. 4.16 and the solution of the ED problem is given in Appendix D.

Case B: In this case, ILR is used to determine the thermal unit scheduling. The scheduling of fuel constrained, hydro and pumped-storage units as well as constraint violation repairing procedures are similar to those of the IMO-ALHN method in Case B of Example 4.5.

The obtained schedule for fuel constrained unit 10 is similar to Case B of Example 4.5. The obtained schedule for thermal units after using ILR and de-committing excessive units and repairing minimum up/down time constraint violations is given in Table 4.15.

Table 4.14 Final unit schedule for all units in Example 4.6, Case A.

TU	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	H1	H2	G1	G2	P1	P2
1	1	1	1	1	1	1	1	1	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0
2	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	0	0	0	1	0	0	0
3	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	0	0	0	1	0	0	0
4	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	1	0	0	0	0	0	0
5	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	1	0	0	1	0	0	0
6	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	1	0	0	1	0	0	0
7	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	1	0	0	0	0	0	0
8	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	1	0	0	0	0	0	0
9	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	1	0	0	0	0	0	0
10	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	1	0	0	0	0
11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	0	0	0	0
12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	0	0	0	0
13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0	0	0	0	0	0
14	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0	1	0	0	0	0	0	0	0
15	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0	1	0	0	0	0	0	0	0
16	1	1	1	1	1	1	1	0	1	1	1	0	1	1	0	0	0	0	0	0	0	0	0
17	1	1	1	1	1	1	0	0	1	1	0	0	1	1	0	0	0	0	0	0	0	0	0
18	1	1	1	1	1	1	0	0	0	1	0	0	1	1	0	0	0	0	0	0	0	0	0
19	1	1	1	1	1	1	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0
20	1	1	1	1	1	1	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	1	0
21	1	1	1	1	1	1	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	1	0
22	1	1	1	1	1	1	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	1	0
23	1	1	1	1	1	1	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0
24	1	1	1	1	1	1	0	0	0	1	0	0	1	1	0	0	0	0	0	0	0	1	0

The excessive spinning reserve is determined:

$$ESR' = \begin{bmatrix} 100 & 65 & 99 & 73 & 158 & 108 & 137 & 234 & 176 & 100 & 249 & 337 \\ 221 & 287 & 325 & 86 & 202 & 192 & 364 & 89 & 149 & 376 & 133 & 194 \end{bmatrix}$$

The hydro units are committed similar to the IMO-ALHN method in Example 4.5 by solving the ED problem by ALHN including power balance, ramp rate and hydro water discharge constraints with all hydro units temporarily committed and their minimum generation output temporarily set to zero. Based on the ED solution, the hydro units with an output close to zero are de-committed. In this case, the thermal units excessive due to the committing of hydro units are de-committed to reduce excessive spinning reserve. The obtained schedule for thermal and hydro units after committing hydro units, de-committing thermal units and repairing minimum up and down time constraint violations is shown in Table 4.16.

The excessive spinning reserve after committing hydro units and de-committing thermal units is determined as follows:

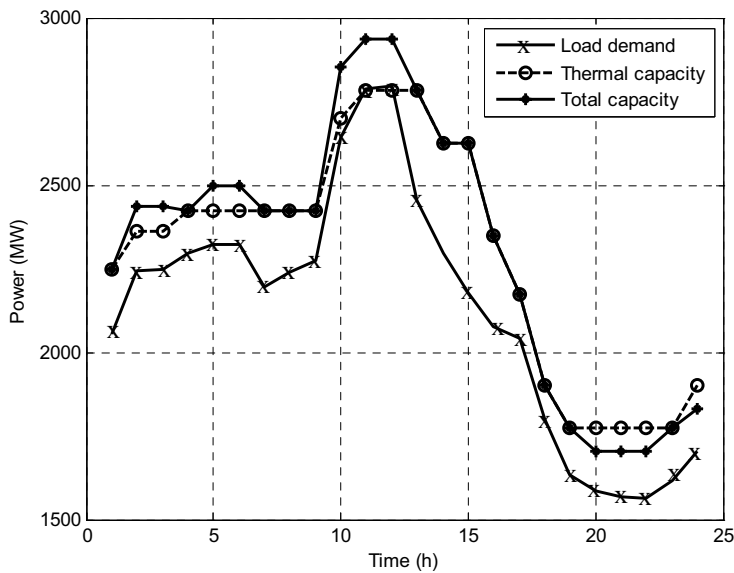


Fig. 4.16 Capacity curves for the hydrothermal system in Example 4.6, Case A.

Table 4.15 Unit scheduling for thermal units in Example 4.6, Case B.

T\U	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1	1	1	1	1	1	1	0	0	0	0	0	1	0	0	0	0
2	1	1	1	1	1	1	0	0	0	0	0	0	1	1	0	0	0
3	1	1	1	1	1	1	0	0	0	0	0	0	1	1	0	0	0
4	1	1	1	1	1	1	0	0	0	0	0	0	1	1	0	0	0
5	1	1	1	1	1	1	0	0	0	0	0	0	1	1	0	0	0
6	1	1	1	1	1	1	0	0	0	0	0	0	1	1	0	0	0
7	1	1	1	1	1	1	0	0	0	0	0	0	1	1	0	0	0
8	1	1	1	1	1	1	0	0	0	1	0	0	1	1	0	0	0
9	1	1	1	1	1	1	0	0	0	1	0	0	1	1	0	0	0
10	1	1	1	1	1	1	0	0	0	1	0	0	1	1	0	0	0
11	1	1	1	1	1	1	0	0	0	1	0	0	1	1	0	0	0
12	1	1	1	1	1	1	0	0	0	1	0	0	1	1	0	0	0
13	1	1	0	1	1	1	0	0	1	0	0	0	1	1	0	0	0
14	1	1	0	1	1	1	0	0	1	0	0	0	1	1	0	0	0
15	1	1	0	1	1	1	0	0	1	0	0	0	1	1	0	0	0
16	1	1	0	1	1	1	0	0	0	0	0	0	1	1	0	0	0
17	1	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0
18	1	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0
19	1	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0
20	1	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0
21	1	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0
22	1	1	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0
23	1	1	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0
24	1	1	0	0	1	1	0	0	0	1	0	0	1	0	0	0	0

Table 4.16 Unit scheduling for all units in Example 4.6, Case B.

T\U	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	H1	H2
1	1	1	1	1	1	1	1	0	0	0	0	0	1	0	0	0	0	0	0
2	1	1	1	1	1	1	0	0	0	0	0	0	1	1	0	0	0	0	0
3	1	1	1	1	1	1	0	0	0	0	0	0	1	1	0	0	0	0	0
4	1	1	1	1	1	1	0	0	0	0	0	0	1	1	0	0	0	0	0
5	1	1	1	1	1	1	0	0	0	0	0	0	1	1	0	0	0	0	0
6	1	1	1	1	1	1	0	0	0	0	0	0	1	1	0	0	0	0	0
7	1	1	1	1	1	1	0	0	0	0	0	0	1	1	0	0	0	0	0
8	1	1	1	1	1	1	0	0	0	1	0	0	1	1	0	0	0	1	1
9	1	1	1	1	1	1	0	0	0	1	0	0	1	1	0	0	0	1	1
10	1	1	1	1	1	1	0	0	0	1	0	0	1	1	0	0	0	1	1
11	1	1	1	1	1	1	0	0	0	1	0	0	1	1	0	0	0	0	0
12	1	1	1	1	1	1	0	0	0	1	0	0	1	1	0	0	0	0	0
13	1	1	0	1	1	1	0	0	0	0	0	0	1	1	0	0	0	1	1
14	1	1	0	1	1	1	0	0	0	0	0	0	1	1	0	0	0	0	0
15	1	1	0	1	1	1	0	0	0	0	0	0	1	1	0	0	0	0	0
16	1	1	0	1	1	1	0	0	0	0	0	0	1	1	0	0	0	0	0
17	1	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
18	1	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
19	1	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
20	1	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	1	1
21	1	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
22	1	1	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0
23	1	1	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	1	1
24	1	1	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	1	1

$$ESR' = \begin{bmatrix} 100 & 65 & 99 & 73 & 158 & 108 & 137 & 389 & 331 & 225 & 249 & 337 \\ 102 & 13 & 51 & 86 & 202 & 192 & 364 & 244 & 149 & 376 & 288 & 135 \end{bmatrix}$$

After obtaining the schedule for all units, ALHN is used to solve the ED problem with all constraints. However, the solution in this case is infeasible but only due to a power shortage violating the power balance constraint at hour 1 with the amount of 15 MW. A repair strategy by heuristic search is applied to satisfy the power balance constraint. As the amount of power shortage is very small it can be compensated by hydro units. Based on the ED solution, the power outputs of hydro units at hour 13 are at their minimum generation limits. Therefore, hydro unit H1 is de-committed at hour 13 to reduce excessive spinning reserve and committed at hour 1 to satisfy the power shortage. Note that if hydro unit H2 is de-committed at hour 13, the spinning reserve at this hour is not satisfied.

ALHN is used again to solve the ED problem including all constraints after repairing the violation. The final solution is feasible and indicates a total cost of \$479926. The capacity curves of committed units are given in Fig. 4.17 and the final ED solution by ALHN is given in Appendix C.

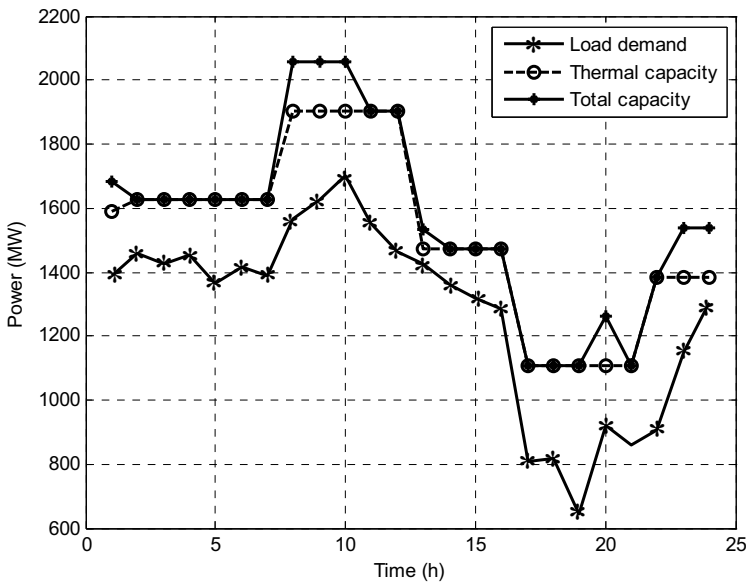


Fig. 4.17 Capacity curves for the hydrothermal system in Example 4.6, Case B.

4.9 SUMMARY

Hydrothermal scheduling is much more complex than the scheduling of a pure thermal system due to hydraulic constraints which are time interval coupled. However, ELRP can be efficiently and effectively implemented to solve the HTS problem. ELRP minimizes the total cost by simultaneously scheduling thermal units and hydro power plants. Using 24 hour EDLQP, the solution can satisfy the time coupling constraints such as limited fuel supply and water discharge constraints. In addition, when time coupling constraints such as limited fuel supply, environmental and water discharge constraints are considered, ELRP is the most appropriate method. However, ELRP requires a large dimension matrix corresponding to equivalent and inequivalent constraints, i.e., at least in the form of (No. of inequivalent constraints) $\times$ NT size.

On the other hand, PSM is a simple method. Its' effectiveness depends on the hydro fraction in a system. If hydro plants are a small fraction and PSM is implemented strategically, it can lead to a good solution within a short time appropriate for a large hydrothermal scheduling system. In addition, IMO-ALHN and ALHN-LR methods have also successfully been applied to solving the HTS problem. These methods are applicable to large scale systems whereby ALHN-LR is more efficient than the IMO-ALHN method when applied to problems with complex constraints.

4.10 PROBLEMS

4.10.1 The incremental cost of a steam plant is:

$$IC_i = 2 + 0.002P'_i, \text{ \$/MWh, } 100 \leq P'_i \leq 800 \text{ MW.}$$

The constant value in the thermal cost function (no load cost) is 160 \\$/h. The incremental water/power rate of hydro power plants is:

$$IW_h = 5 + 0.02P'_h, 10^6 \text{ m}^3/\text{MWh, } 0 \leq P'_h \leq 500 \text{ MW}$$

The available water in a reservoir is $40,000 \times 10^6 \text{ m}^3$. The water input for $P_h = 0$, may be assumed zero, that is $q'_h(P'_h = 0) = 0$.

Table 4.17 Load for 3 periods (8 hours/period) for Problem 4.10.1.

Time Period	1	2	3
Load (MW)	400	700	500

The losses are neglected and the thermal power plant remains on-line for the whole 24-h period. Compare the optimum hydro-thermal schedules over the 24-h period obtained by three methods: the method with a constant thermal power dispatch, peak shaving method and coordination equation method.

4.10.2 Review Problem 4.1, remark that the hydroelectric unit's water consumption function which is $q'_h(P'_h) = 5P'_h$ and the steam unit is not necessarily 'on' all the time.

4.10.3 Review Problem 4.1 considering that the available water in the reservoir is 10,000 MCM.

4.10.4 The composite fuel cost characteristics of the thermal generation system are assumed as:

$$F_i(P'_i) = 160 + 2P'_i + 0.001(P'_i)^2 \text{ \$/h, } 200 \leq P'_i \leq 1500 \text{ MW}$$

The water discharge function of a pumped storage hydro power plant when operated as a generator is $q'_h(P'_h) = 5P'_h$ MCM/h, $0 \leq P'_h \leq 150$ MW. Its pumping load when operated as a pump is either 100 or 200 MW. The cycle efficiency is 70% (i.e., pumping energy of 100MWh can generate 70MWh of electrical energy). The water stored in the reservoir at the start is $12,000 \times 10^6 \text{ m}^3$. The water stored in the reservoir at the end of the schedule is required to be the same amount as at its beginning. The loads during 6 periods (4 hours/period) are given below. Find the optimal pumped storage power plant schedule to minimize the thermal system cost.

Table 4.18 Load demand in Problem 4.10.4.

Time Period	1	2	3	4	5	6
Load (MW)	400	1100	1300	800	600	300

- 4.10.5 Repeat Problem 4.10.4 assuming that the water storage volume of the reservoir is unknown and it should be at the same level at the end of the day.
- 4.10.6 Repeat Problem 4.10.4 assuming that the storage capacity of the reservoir must be within the limits of $5,000 \leq V \leq 30,000$ MCM. The start reservoir volume is 20,000 MCM and it should be at the same level at the end of the day.
- 4.10.7 Solve the HTS problem in Case A of Example 4.3 with the additional pumped-storage units from Case B.
- 4.10.8 Solve the HTS problem in Example 4.4 by IMO-ALHN.
- 4.10.9 Solve the HTS problem in Example 4.4 by ALHN-LR.
- 4.10.10 Solve the HTS problem in Example 4.4 with additional fuel limit for units 10–13 by imposing an energy generation limit for each unit of 250 MWh throughout the schedule time horizon.

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OPTIMAL POWER FLOW*

5.1 INTRODUCTION

The optimal power flow or OPF has a long history since it was first discussed by Carpentier in 1962 as an extension of conventional economic dispatch to determine the optimal settings for control variables while respecting various constraints. OPF development took a long time to turn out as a successful algorithm capable of solving everyday problems. The development of OPF during the last two decades has tracked progress closely in numerical optimization technique and advanced computer technology. Current commercial OPF programs are able to solve very large and complex power system optimization problems in a relatively short time. Many different approaches have been proposed to solve OPF problems. Current interest in OPF centers on its ability to find the optimal solution taking into account the security of the system.

If power flow is solved simultaneously with generation cost minimization, the power balance constraint is included in the formulation and the representation of incremental losses is exact. It also guarantees a feasible dispatch solution. In addition, many more of the power system limits can be included to guarantee that the dispatch solution is within the limits of:

- transmission system capacity,
- generators' real and reactive power,
- bus voltages,
- security margins, etc.

Many more adjustable variables can be included in the optimization effort to achieve the optimum conditions of:

- transformer tap,
- static VAR compensator,
- generator voltage,
- FACTS devices, etc.

*This chapter has been written with assistance from Keerati Chayakulkheeree

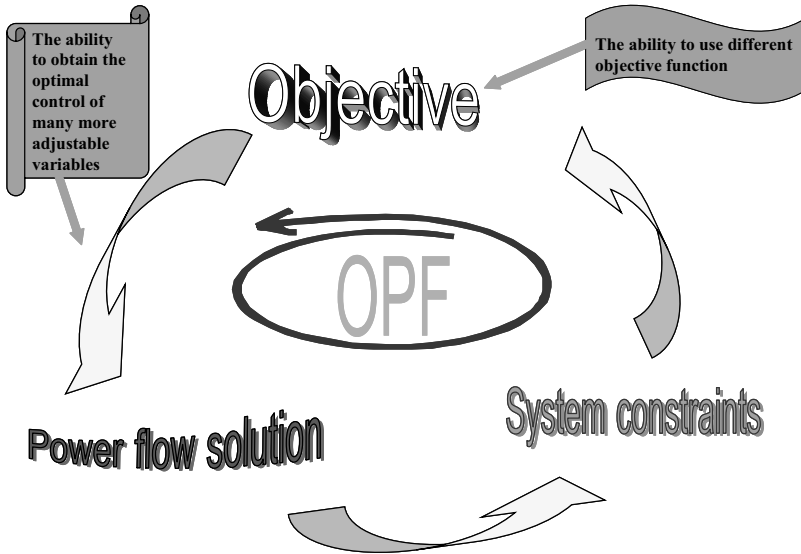


Fig. 5.1 Function of the OPF.

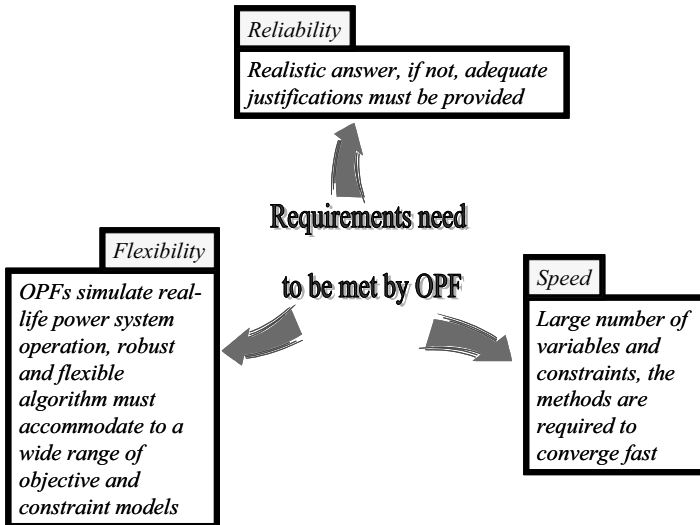


Fig. 5.2 Requirements to be met by OPF.

5.2 OPTIMAL POWER FLOW PROBLEM FORMULATION

The objectives commonly found in OPF problems are:

- Minimum generation cost,
- Minimum transmission loss,

Minimum deviation from target schedule,
Minimum control shift to alleviate violations,
Minimum emissions.

Minimum generation cost is found to be the most common objective in OPF. Fuel cost minimization requires knowledge of the fuel cost curve for each of the generating units. An accurate representation of the cost curves may require a piecewise polynomial form or can be approximated in several ways.

The piecewise linear form is used in many production-grade linear programming applications, a quadratic approximation in most nonlinear programming applications.

If the objective function is to minimize total generation cost, it is expressed as follows:

$$TF = \text{Min} \sum_{i \in BG} F_i(P_{Gi}) \quad (5.1)$$

subject to

$$\sum_{i \in BG} P_{Gi} = P_{load} + P_{loss} \quad (5.2)$$

$$P_i = P_{Gi} - P_{loadi} = \sum_{j=1}^{NB} |V_i| |V_j| |Y_{ij}| \cos(\theta_{ij} - \delta_{ij}) \quad (5.3)$$

$$Q_i = Q_{Gi} - Q_{loadi} = - \sum_{j=1}^{NB} |V_i| |V_j| |Y_{ij}| \sin(-\theta_{ij} - \delta_{ij}) \quad (5.4)$$

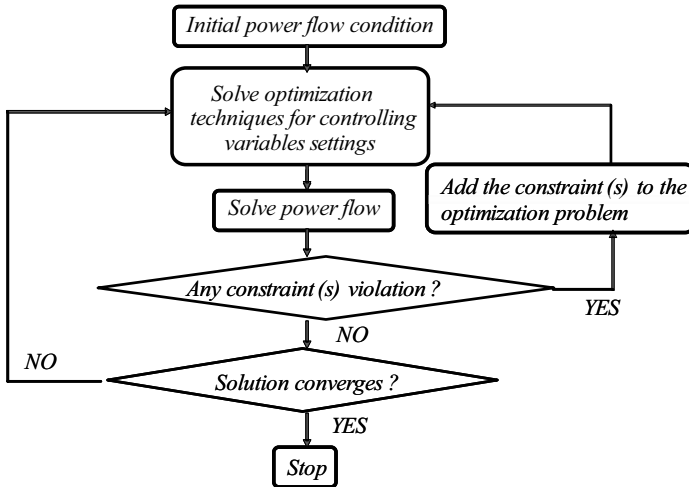


Fig. 5.3 Typical computation procedure in OPF.

$$|V_i^{\min}| \leq |V_i| \leq |V_i^{\max}|, \quad i = 1, \dots, NB \quad (5.5)$$

$$f_l \leq f_l^{\max}, \quad l = 1, \dots, NL \quad (5.6)$$

where

$F_i(P_{Gi})$ generating unit operation cost of generator connected to bus i (\$/h)

P_{Gi} real power generation of generator connected to bus i (MW)

P_{loadi} total real power load at bus i (MW)

$P_{Gi}^{\max}$ maximum real power generation at bus i (MW)

$P_{Gi}^{\min}$ minimum real power generation at bus i (MW)

$|V_i|$ voltage magnitude at bus i (kV)

$|V_i^{\min}|$ minimum voltage magnitude at bus i (kV)

$|V_i^{\max}|$ maximum voltage magnitude at bus i (kV)

θ_{ij} angle of the Y_{ij}^{th} element of Y_{bus} (radian)

δ_{ij} voltage angle difference between bus i and j (radian)

$|y_{ij}|$ magnitude of the Y_{ij}^{th} element of Y_{bus} (mho)

NB total number of buses

BG set of buses connected with generators

NL total number of line flow violations

$f_l^{\max}$ maximum line MVA flow in line l

f_l line MVA flow in line l .

5.3 OPTIMAL REAL POWER DISPATCH WITH NETWORK LIMIT CONSTRAINTS

5.3.1 Linear Programming of Optimal Power Flow (LPOPF)

For implementation of linear programming to this problem, we can approximate the nonlinear function (5.1) as a series of strength-line segments as shown in Fig. 5.4. Considering the LP formulation with real power variable only, the entire model used in the LP is based on dispatching real power only.

$$P_{Gi} = \sum_{j=1}^{NS} P_{Gij}, \quad i \in BG \quad (5.7)$$

The problem formulation can be expressed as

$$\text{Minimize } TF = \sum_{i \in BG} F_i(P_{i,\min}) + \sum_{i \in BG} \sum_{j=1}^{NSi} S_{ij} P_{ij} \quad (5.8)$$

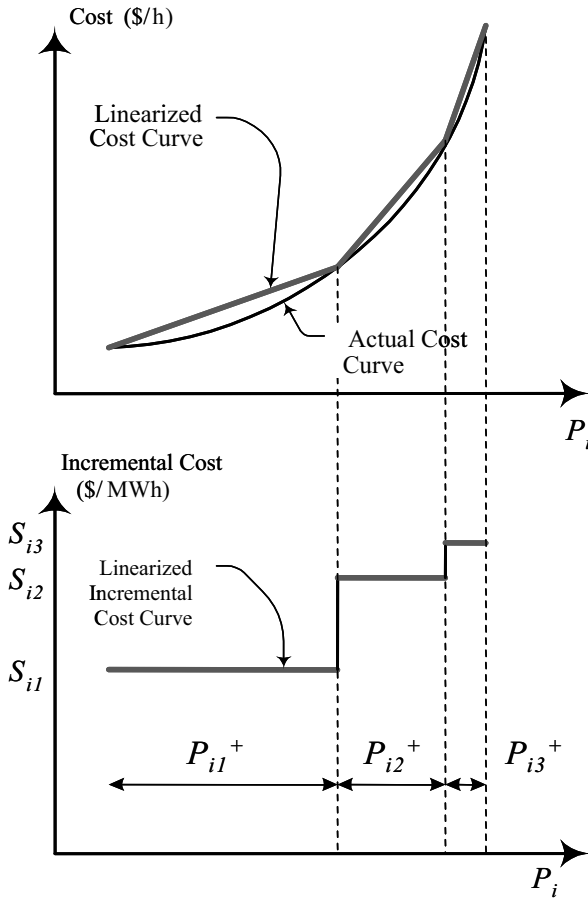


Fig. 5.4 The linearized cost function.

Color image of this figure appears in the color plate section at the end of the book.

subject to the constraint in Eqs.(5.2)–(5.6).

Linear sensitivity coefficients give an indication of the change in one system quantity (e.g., MW flow, MVA flow, bus voltage, etc. as another quantity changes (e.g., generator power output, transformer tap position, etc.)

$$\text{Linear sensitivity coefficient} = \frac{\partial (\text{Line } l \text{ MVA flow})}{\partial (\text{Real power generation at bus } i)}$$

Sensitivity to changes such as the variations of power flow with respect to changes in generator output is rather linear across a wide range of adjustments, whereas voltage and MVAR flows have a nonlinear relationship.

The linearized line flow constraints can be represented as:

$$f_l = f_l^o + \sum_{i \in BG} \frac{\partial f_l}{\partial P_i} dP_i \leq f_l^{\max}, l = 1, 2, 3, \dots, NL \quad (5.8)$$

where

f_l^o the original power flow in line l

$\frac{\partial f_l}{\partial P_{Gi}} dP_{Gi}$ the change in power flow through line l due to the change in real power injected at bus i

After running the power flow to obtain the line flow in each branch, the inequality can be written as:

$$f_l = f_l^o + \sum_{i=1}^{NG} \frac{\partial f_l}{\partial P_i} (P_{Gi} - P_{Gi}^o) \leq f_l^{\max} \quad (5.9)$$

where P_{Gi}^o is the original real power generation at bus i ,

The computation is done iteratively until the total generation calculated by LP is equal to the solution from power flow as shown in Fig. 5.5.

Example 5.1

LPOPF on a 9 Bus System

The 9 bus system network diagram and the generators operating cost functions are shown in Fig. 5.6.

The network data are shown in Tables 5.1–5.4. The cost functions are linearized as shown in Table 5.5.

The base case power flow solution is shown in Fig. 5.7. In the base case, the total real and reactive power generations are 319.64 MW and 22.84 MVAR, respectively. The total real power loss is 4.64 MW and the total system cost is 13677.53 \$/h.

The result with a single run of LPOPF is as shown in Fig. 5.8. The system total cost is decreased to 11487.19 \$/h from 13677.53447 \$/h in the base case. However, the flow in line 4–5 is 91.202 MVA and violates its limit of 85 MVA.

Then the line flow constraint is included in the LPOPF problem and the LPOPF is run iteratively until no significant changes in results occur anymore and no further line limits are violated.

In this case, the computation converges in 12 iterations. The result is shown in Fig. 5.9. Including the line flow limit in the LP optimization problem, the flow in line 4–5 is still within its limit. However, the total system cost is 11711.24 \$/h which is around 1.95% higher than that of the case without line flow limit consideration.

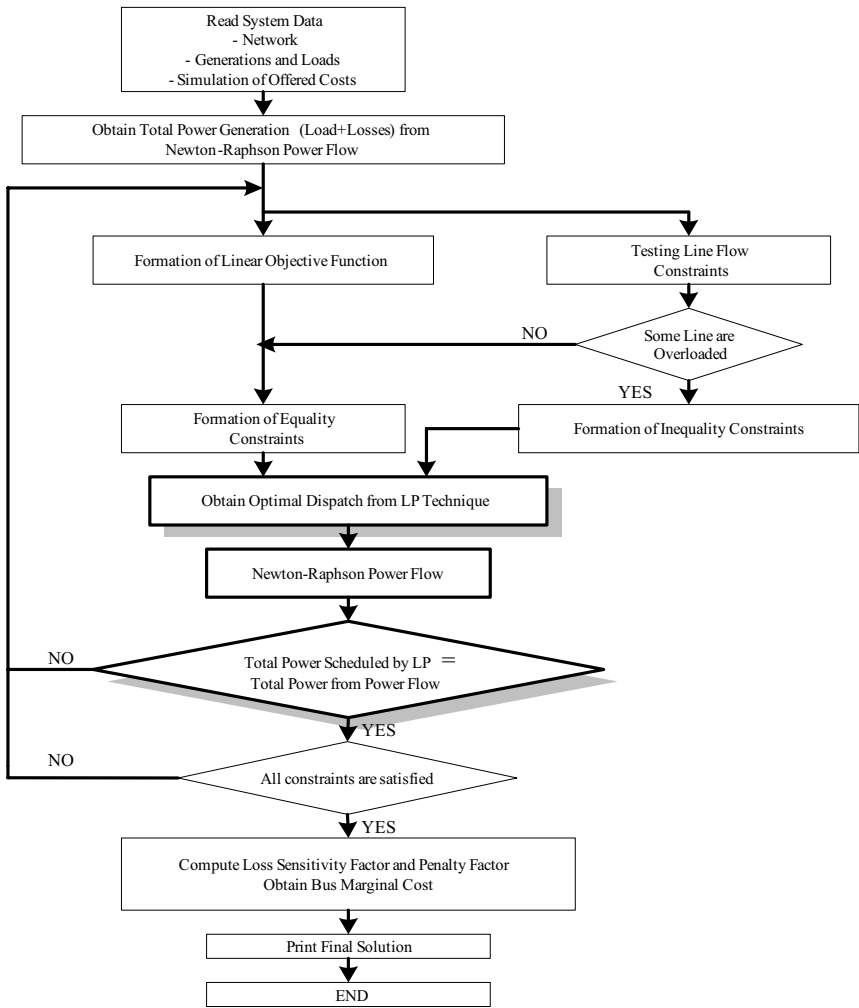


Fig. 5.5 Computational procedure of LPOPF.

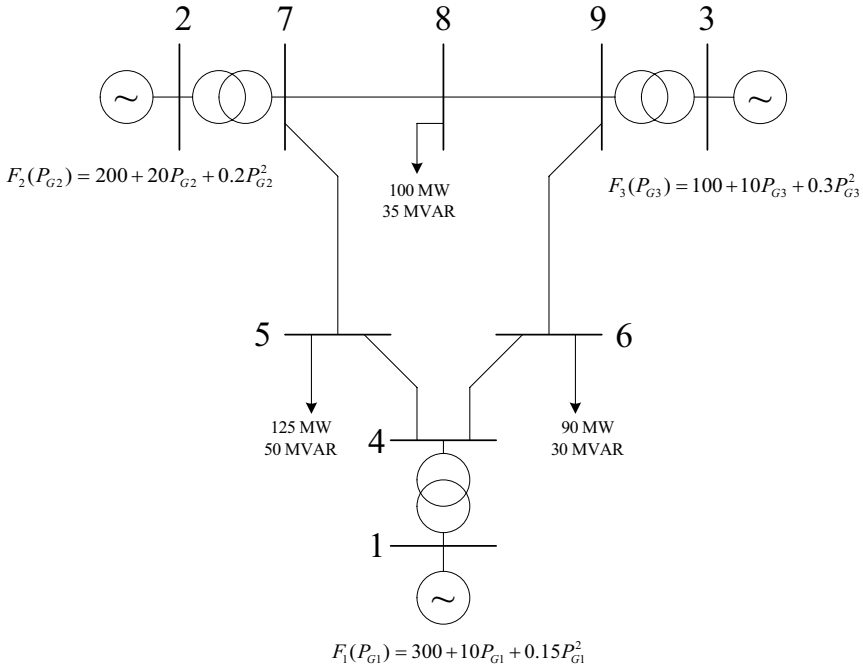


Fig. 5.6 The 9 bus system.

Table 5.1 Transformer data of the 9 bus system.

Form Bus	To Bus	R	X	Tap	Limit (MVA)
4	1	0	.05760	1.00	300
7	2	0	.06250	1.00	300
9	3	0	.05860	1.00	300

Table 5.2 Transmission line of the 9 bus system.

From Bus	To Bus	R	X	B	Limit (MVA)
7	8	0.00850	0.07200	0.07450	85.0
9	8	0.01190	0.10080	0.10450	85.0
7	5	0.03200	0.16100	0.15300	85.0
9	6	0.03900	0.17000	0.17900	85.0
5	4	0.01000	0.08500	0.08800	85.0
6	4	0.01700	0.09200	0.07900	85.0

Table 5.3 Load data of the 9 bus system.

At Bus	P (MW)	Q (MVAR)
5	125	50
6	90	30
8	100	35

Table 5.4 Generator Data of the 9 Bus System.

At Bus	$P_{Gi}^{\min}$ (MW)	$P_{Gi}^{\max}$ (MVAR)	Set up voltage (p.u.)
1	20	200	1.040
2	20	200	1.025
3	20	200	1.025

Table 5.5 Linearized generator cost functions of example 5.1.

Generator 1			Generator 2			Generator 3		
From MW	To MW	COST \$/MWh	From MW	To MW	COST \$/MWh	From MW	To MW	COST \$/MWh
20	29	17.35	20	29	29.8	20	29	24.7
29	38	20.05	29	38	33.4	29	38	30.1
38	47	22.75	38	47	37	38	47	35.5
47	56	25.45	47	56	40.6	47	56	40.9
56	65	28.15	56	65	44.2	56	65	46.3
65	74	30.85	65	74	47.8	65	74	51.7
74	83	33.55	74	83	51.4	74	83	57.1
83	92	36.25	83	92	55	83	92	62.5
92	101	38.95	92	101	58.6	92	101	67.9
101	110	41.65	101	110	62.2	101	110	73.3
110	119	44.35	110	119	65.8	110	119	78.7
119	128	47.05	119	128	69.4	119	128	84.1
128	137	49.75	128	137	73	128	137	89.5
137	146	52.45	137	146	76.6	137	146	94.9
146	155	55.15	146	155	80.2	146	155	100.3
155	164	57.85	155	164	83.8	155	164	105.7
164	173	60.55	164	173	87.4	164	173	111.1
173	182	63.25	173	182	91	173	182	116.5
182	191	65.95	182	191	94.6	182	191	121.9
191	200	68.65	191	200	98.2	191	200	127.3

5.3.2 Quadratic Programming Optimal Power Flow (QPOPF)

We can approximate the nonlinear function as a series of strength-line segments as shown in Fig. 5.4. The problem formulation can be expressed as:

$$\text{Minimize } TF = \sum_{i \in BG} F_i(P_{Gi}) = \sum_{i \in BG} a_i + b_i P_{Gi} + c_i P_{Gi}^2 \quad (5.10)$$

subject to the constraints in Eqs. (5.2)–(5.5) and the linearized line flow constraints in Eq.(5.8). The computational procedure is similar to LPOPF.

BUS	NAME	V	DEL	Pgen	Qgen	Pload	Qload	ShtVAR
===	=====	=====	=====	=====	=====	=====	=====	=====
1	GEN***1	1.040	0.0	71.6	27.0	0.0	0.0	0.0
2	GEN***2	1.025	9.3	163.0	6.7	0.0	0.0	0.0
3	GEN***3	1.025	4.7	85.0	-10.9	0.0	0.0	0.0
4	LOAD***4	1.026	-2.2	0.0	0.0	-0.0	-0.0	0.0
5	LOAD***5	0.996	-4.0	0.0	0.0	125.0	50.0	0.0
6	LOAD***6	1.013	-3.7	0.0	0.0	90.0	30.0	0.0
7	LOAD***7	1.026	3.7	0.0	0.0	0.0	0.0	0.0
8	LOAD***8	1.016	0.7	0.0	0.0	100.0	35.0	0.0
9	LOAD***9	1.032	2.0	0.0	0.0	-0.0	-0.0	0.0

** ** System-Grid Total ** **								

Total Generation		=	319.64	MW	22.84	MVAR		
Total P-Q Load		=	315.00	MW	115.00	MVAR		
SHUNT VAR		=			0.00	MVAR		
Total Power Losses		=	4.64	MW	-92.16	MVAR		
Overall P Mismatch		=	-0.0000	MW				

** ** Generation Cost ** **								

BUS	P_GEN	Cost						
	(MW)	(\$/h)						
1	71.64	1786.26642						
2	163.00	8773.78080						
3	85.00	3117.48725						
Total Cost = 13677.53447 \$/h								

Fig 5.7 Base case power flow of the 9 bus system.

```

** Optimal Active Power Schedule From LP**
*****
BUS 1      153.64 MW
BUS 2      92.00 MW
BUS 3      74.00 MW

-----
** ** Generation Cost ** **
-----
BUS      P_GEN      Cost      Inc-Cost      Cost
          (MW)      (UM/HR) *      (UM/MWHR)      (UM/HR)
1      151.75      8368.89980      55.15000      5271.59591
2      92.00      5073.79999      55.00000      3732.79999
3      74.00      4081.10000      51.70000      2482.80000

Total Cost = 11487.19589 $/h

** OVERLOADED LINE **-----
*****
** Line From      Line To      MW flow      MVAR flow      MVA flow      MVA limit
-----
4      5      89.046      19.714      91.202      85.000

```

Fig 5.8 Solution of the first run of LPOPF for the 9 bus system.

Example 5.2 QPOPF in a 9 Bus System

The 9 bus system used in the last section is now tested with QPOPF. The original system is the same as in Fig. 5.7. The generator operating cost functions are given in Table 5.6.

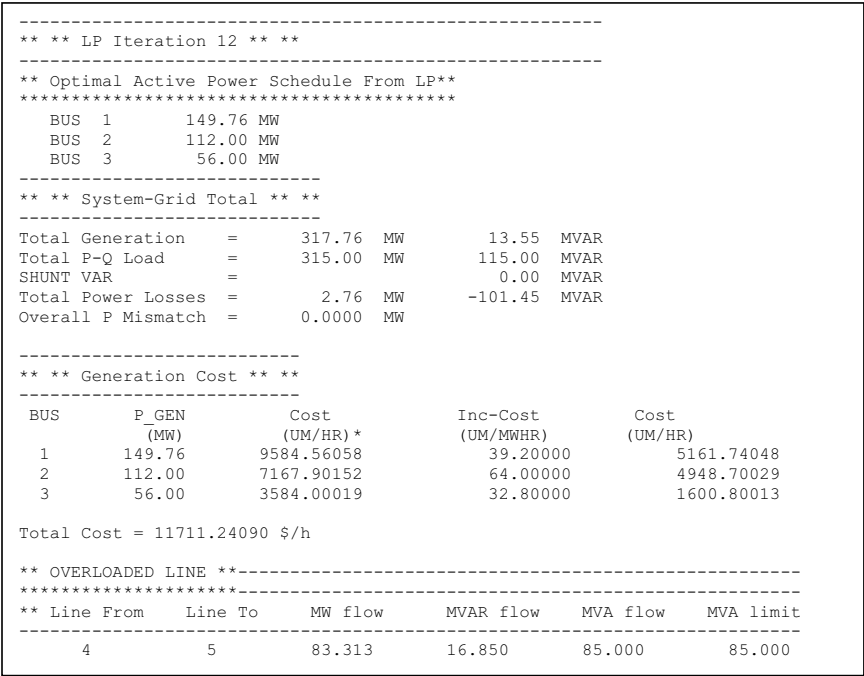


Fig. 5.9 Solution of LPOPF for the 9 bus system after 12 iterations.

Table 5.6 Generator operating cost functions of the 9 bus system.

Gen Bus	$P_{Gi}^{\min}$	$P_{Gi}^{\max}$	$F_i = a_i + b_i P_{Gi} + c_i P_{Gi}^2$		
			a_i	b_i	c_i
1	20	200	300	10	0.15
2	20	200	200	20	0.2
3	20	200	100	10	0.3

Figure 5.10 shows the solution obtained from the first run of QPOPF. The total system cost is decreased to 11,484.601 \$/h from 13677.53447 \$/h for the base case. However, this solution results in a violation of line 4–5 limits. Therefore, the line 4–5 flow constraint is added to the QP formulation. The computation converges within 15 iterations to a result quantifying the total system cost at 11,521.103 \$/h, without any line overload, as shown in Fig. 5.11.

```

** ** QP Iteration 1 ** **
-----
** Optimal Active Power Schedule From LP**
*****
BUS 1      153.17 MW
BUS 2      89.88 MW
BUS 3      76.59 MW
-----
** ** System-Grid Total ** **
-----
Total Generation   =      317.75 MW      21.75 MVAR
Total P-Q Load    =      315.00 MW      122.93 MVAR
SHUNT VAR         =              0.00 MVAR
Total Power Losses =        2.75 MW      -101.18 MVAR
Overall P Mismatch =        0.0000 MW
-----
** ** Generation Cost ** **
-----
BUS      P_GEN      Cost
          (MW)      ($/h)
1        151.28      5245.78742
2         89.88      3613.28837
3         76.59      2625.52559
Total Cost = 11484.60138 $/h
** OVERLOADED LINE **-----
*****
** Line From   Line To   MW flow   MVAR flow   MVA flow   MVA limit
-----
4             5         89.391     19.699     91.536     85.000

```

Fig. 5.10 Solution after the first run of QPOPF on the 9 bus system.

```

** ** QP Iteration 15 ** **
-----
** Optimal Active Power Schedule From LP**
*****
BUS 1      141.41 MW
BUS 2      98.97 MW
BUS 3      77.36 MW
-----
** ** System-Grid Total ** **
-----
Total Generation   =      317.74 MW      11.82 MVAR
Total P-Q Load    =      315.00 MW      115.00 MVAR
SHUNT VAR         =              0.00 MVAR
Total Power Losses =        2.74 MW      -103.18 MVAR
Overall P Mismatch =        0.0000 MW
-----
** ** Generation Cost ** **
-----
BUS      P_GEN      Cost
          (MW)      ($/h)
1        141.41      4713.52482
2         98.97      4138.52602
3         77.36      2669.05166
Total Cost = 11521.10249 $/h
** OVERLOADED LINE **-----
*****
** Line From   Line To   MW flow   MVAR flow   MVA flow   MVA limit
-----
4             5         83.331     16.760     85.000     85.000

```

Fig. 5.11 Solution after 15 iterations of QPOPF on the 9 bus system.

5.4 NEURAL NETWORK APPLICATION TO OPTIMAL POWER FLOW

A method based on ALHN is used to solve the OPF problem formulated in Section 5.2, in which ALHN is used to minimize objective (5.1) satisfying power balance (5.2) and transmission (5.6) constraints while the Newton-Raphson method is used to solve the power flow problem (5.3)–(5.5). In the considered system, a slack bus is numbered 1 and the other generation buses are numbered from 2 to N . The slack bus 1 is used in this case since it is needed for the power flow solution by Newton-Raphson method.

The augmented Lagrangian function is formulated as follows:

$$\begin{aligned}
 L = & \sum_{i=1}^N (a_i + b_i P_{Gi} + c_i P_{Gi}^2) \\
 & + \left[\lambda \left(P_{load} + P_{loss} - \sum_{i=1}^N P_{Gi} \right) + \frac{1}{2} \beta \left(P_{load} + P_{loss} - \sum_{i=1}^N P_{Gi} \right)^2 \right] \\
 & + \sum_{l=1}^{NL} \left[\gamma_l (f_l - S_l) + \frac{1}{2} \beta_l (f_l - S_l)^2 \right]
 \end{aligned} \quad (5.11)$$

where

$$P_{load} = \sum_{i=1}^{NB} P_{loadi} \quad (5.12)$$

$$P_{loss} = \sum_{i=1}^N \sum_{j=1}^N (P_{Gi} - P_{loadi}) B_{ij} (P_{Gj} - P_{loadj}) \quad (5.13)$$

$$B_{ij} = \frac{r_{ij} \cos(\psi_i - \psi_j)}{|V_i| |V_j| \cos \phi_i \cos \phi_j} \quad (5.14)$$

$$\psi_i = \delta_i - \phi_i \quad (5.15)$$

$$\phi_i = \tan^{-1}(Q_i / P_i) \quad (5.16)$$

$$S_l = V_i \left[(V_i^* - V_j^*) Y_{ij}^* + V_i^* \frac{(Y'_{ij})^*}{2} \right] \quad (5.17)$$

and

λ, γ_l Lagrangian multipliers associated with power balance and transmission constraints

β, β_l penalty factors associated with power balance and transmission constraints
 S_l power flow in line l connecting bus i to bus j .

The energy function of ALHN is formulated based on the augmented Lagrangian function:

$$\begin{aligned}
 E = & \sum_{i=1}^N (a_i + b_i V_{i,p} + c_i V_{i,p}^2) \\
 & + \left[V_\lambda \left(P_{load} + P_{loss} - \sum_{i=1}^N V_{i,p} \right) + \frac{1}{2} \beta \left(P_{load} + P_{loss} - \sum_{i=1}^N V_{i,p} \right)^2 \right] \\
 & + \sum_{l=1}^{NL} \left[V_{l,\gamma} (V_{l,f} - S_l) + \frac{1}{2} \beta_l (V_{l,f} - S_l)^2 \right] \\
 & + \sum_{i=1}^N \int_0^{V_{i,p}} g^{-1}(V) dV + \sum_{l=1}^{NL} \int_0^{V_{l,f}} g^{-1}(V) dV
 \end{aligned} \tag{5.18}$$

where

$V_{i,p}$ output of continuous neuron $p(i)$ representing generator output P_{Gi}

$V_{l,f}$ output of continuous neuron $f(l)$ representing power flow f_l

V_λ output of multiplier neuron representing λ

$V_{l,\gamma}$ output of multiplier neuron $\gamma(l)$ representing λ_l .

The dynamics of neurons are defined as follows:

$$\begin{aligned}
 \frac{dU_{i,p}}{dt} &= - \frac{\partial E}{\partial V_{i,p}} \\
 &= - \left\{ b_i + 2c_i V_{i,p} + \left[V_\lambda + \beta \left(P_{load} + P_{loss} - \sum_{i=1}^N V_{i,p} \right) \right] \left(\frac{\partial P_{loss}}{\partial V_{i,p}} - 1 \right) + U_{i,p} \right\}
 \end{aligned} \tag{5.19}$$

$$\frac{dU_{l,f}}{dt} = - \frac{\partial E}{\partial V_{l,f}} = - \left\{ \left[V_{l,\gamma} + \beta_l (V_{l,f} - S_l) \right] + U_{l,f} \right\} \tag{5.20}$$

$$\frac{dU_\lambda}{dt} = + \frac{\partial E}{\partial V_\lambda} = P_{load} + P_{loss} - \sum_{i=1}^N V_{i,p} \tag{5.21}$$

$$\frac{dU_{l,\gamma}}{dt} = + \frac{\partial E}{\partial V_{l,\gamma}} = V_{l,f} - S_l \tag{5.22}$$

where

$$\frac{\partial P_{loss}}{\partial V_{i,p}} = \sum_{j=1}^N B_{ij} (P_{Gj} - P_{loadj}) \quad (5.23)$$

and

$U_{i,p}$ input of the continuous neuron with corresponding output of $V_{i,p}$

$U_{l,f}$ input of the continuous neuron with corresponding output of $V_{l,f}$

U_{λ} input of the multiplier neuron with corresponding output of V_{λ}

$U_{l,\gamma}$ input of the multiplier neuron with corresponding output of $V_{l,\gamma}$

The inputs of neurons at iteration n are updated as follows:

$$U_{i,p}^{(n)} = U_{i,p}^{(n-1)} - \alpha_i \frac{\partial E}{\partial V_{i,p}} \quad (5.24)$$

$$U_{l,f}^{(n)} = U_{l,f}^{(n-1)} - \alpha_{l,f} \frac{\partial E}{\partial V_{l,f}} \quad (5.25)$$

$$U_{\lambda}^{(n)} = U_{\lambda}^{(n-1)} - \alpha_{\lambda} \frac{\partial E}{\partial V_{\lambda}} \quad (5.26)$$

$$U_{l,\gamma}^{(n)} = U_{l,\gamma}^{(n-1)} - \alpha_{l,\gamma} \frac{\partial E}{\partial V_{l,\gamma}} \quad (5.27)$$

where α_p , $\alpha_{l,p}$, α_{λ} and $\alpha_{l,\gamma}$ are updating step sizes of neurons.

The outputs of neurons are calculated by:

$$V_{i,p} = g(U_{i,p}) = \frac{P_{Gi}^{\max} - P_{Gi}^{\min}}{2} \left[1 + \tanh(\sigma U_{i,p}) \right] + P_{Gi}^{\min} \quad (5.28)$$

$$V_{l,f} = g(U_{l,f}) = f_l^{\max} \tanh(\sigma U_{l,f}) \quad (5.29)$$

$$V_{\lambda} = h(U_{\lambda}) = U_{\lambda} \quad (5.30)$$

$$V_{l,\gamma} = h(U_{l,\gamma}) = U_{l,\gamma} \quad (5.31)$$

where σ is the slope of the sigmoid function of continuous neurons.

All neurons are initialized as follows:

$$V_{i,p}^{(0)} = P_{load} \frac{P_{Gi}^{\max}}{\sum_{j=1}^N P_{Gj}^{\max}} \quad (5.32)$$

$$V_{\lambda}^{(0)} = \frac{1}{N} (b_i + 2c_i P_{Gi}^{(0)}) \quad (5.33)$$

$$V_{l,y}^{(0)} = 0 \quad (5.34)$$

The algorithm of the ALHN based method for solving the OPF problem is as follows:

- Step 1: Read in data of the problem and parameters for ALHN.
- Step 2: Form Y_{bus} of system.
- Step 3: Initialize all neurons $V_{i,p}^{(0)}$ except slack bus, $V_{\lambda}^{(0)}$ and $V_{l,y}^{(0)}$.
- Step 4: Solve the power flow problem by the Newton-Raphson method from initial $V_{i,p}^{(0)}$.
- Step 5: Calculate P_i , Q_i , power flow S_i and power loss P_{loss} and obtain the initial power output $V_{l,p}^{(0)}$ for the slack bus.
- Step 6: Calculate the maximum error by combining power balance and transmission constraint errors.
- Step 7: If the maximum error is lower than a pre-specified threshold, go to Step 16.
- Step 8: Set the number of iterations $n = 1$.
- Step 9: Calculate the dynamics of all neurons.
- Step 10: Update inputs of all neurons.
- Step 11: Calculate outputs of all neurons.
- Step 12: Use the obtained $V_{i,p}^{(n)}$ to solve the power flow problem by the Newton-Raphson method.
- Step 13: Calculate P_i , Q_i , power flow S_i and power loss P_{loss} and obtain the new power output $V_{l,p}^{(n)}$ for the slack bus.
- Step 14: Calculate the maximum error from the constraints.
- Step 15: If $n < N_{max}$ and the maximum error is greater than the pre-specified threshold, $n = n + 1$ and return to Step 9.
- Step 16: Calculate total cost and stop.

5.5 PARTICLE SWARM OPTIMIZATION FOR OPTIMAL POWER FLOW

5.5.1 Particle Swarm Optimization

Particle swarm optimization (PSO) is a population based evolutionary computation technique inspired from the social behaviors of bird flocking or fish schooling. Since

its invention in 1995 by Kennedy and Eberhart, PSO has become one of the most popular methods applied to various optimization problems due to its simplicity and capability to find near optimal solutions. In conventional PSO, a population of particles moves in the search space of a problem to approach the global optimum. The movement of each particle in the population is determined via its location and velocity. During the movement, the velocity of each particle is changed over time and its position is updated accordingly.

Consider an n -dimensional optimization problem

$$\text{Min } f(x) \quad (3.35)$$

where $x = [x_1, x_2, \dots, x_n]$ is a vector of variables.

For implementation to the problem, the position and velocity vectors of particle d are represented by $x_d = [x_{1d}, x_{2d}, \dots, x_{nd}]$ and $v_d = [v_{1d}, v_{2d}, \dots, v_{nd}]$, respectively, where $d = 1, \dots, NP$ and NP is the number of particles. The best previous position of particle d is based on the valuation of the fitness function represented by $pbest_d = [p_{1d}, p_{2d}, \dots, p_{nd}]$ and the best particle among all particles represented by $gbest$. The velocity and position of each particle in the next iteration ($k+1$) for fitness function evaluation are calculated as follows:

$$\begin{aligned} v_{id}^{(k+1)} &= w^{(k+1)} \times v_{id}^{(k)} + c_1 \times rand_1 \times (pbest_{id}^{(k)} - x_{id}^{(k)}) \\ &\quad + c_2 \times rand_2 \times (gbest_i^{(k)} - x_{id}^{(k)}) \end{aligned} \quad (3.36)$$

$$x_{id}^{(k+1)} = x_{id}^{(k)} + v_{id}^{(k+1)} \quad (3.37)$$

where w is the inertia weight factor, c_1 and c_2 are cognitive and social parameters, respectively, and $rand_1$ and $rand_2$ are random values in $[0, 1]$.

In conventional PSO, the inertia weight factor and cognitive and social parameters are constants. Position and velocity of each particle have their own limits. Regrading position limits, the lower and upper bounds are defined by the limits of variables represented by the particle's position. However, the velocity limits for the particles can be defined by the user. Generally, the solution quality of PSO is sensitive to cognitive and social parameters and velocity limits for particles. Therefore, there have been several attempts to control, explore, and exploit the abilities of the PSO algorithm such as modifications in inertia weight factor and cognitive and social parameters. In this chapter, the PSO with time-varying inertia weight factor (PSO-TVIW) is implemented to solve the given OPF problem. In PSO-TVIW, the inertia weight factor linearly declines from its maximum to the minimum value as the number of iterations increases from 0 to IT_{max} . The inertia weight factor at iteration k is updated as follows:

$$w^{(k)} = w_{max} - (w_{max} - w_{min}) \frac{k}{IT_{max}} \quad (3.38)$$

where w_{max} and w_{min} are maximum and minimum weight factor, respectively, and IT_{max} is the maximum number of iterations.

PSO-TVIW is one of the suggested improvements of the PSO method. With the variation of the inertia weight factor, the PSO-TVIW method can obtain an even better solution quality than the conventional PSO method. Thus, the PSO-TVIW method can be implemented in complex practical optimization problems.

5.5.2 Implementation of PSO-TVIW

For the implementation of PSO-TVIW to the OPF problem, each particle position representing a control variable is defined as follows:

$$x_d = [P_{G2d}, \dots, P_{BGGd}, V_{G1d}, \dots, V_{BGGd}]^T, \quad d = 1, \dots, NP \quad (3.39)$$

where NP is the number of particles and V_{Gi} is the voltage magnitude at generation buses with $i = 1, \dots, BG$.

The vector of dependent variables includes voltage magnitude at load buses V_{li} and apparent power flow in transmission lines f_l which is determined via power flow solution based on the control variables.

The upper and lower limits for velocity of each particle are determined based on their lower and upper bounds of position defined by:

$$v_{d,\max} = R \times (x_{d,\max} - x_{d,\min}) \quad (3.40)$$

$$v_{d,\min} = -v_{d,\max} \quad (3.41)$$

where R is the limiting factor for particle velocity.

Both particle positions and velocities are initialized within their limits given by:

$$x_d^{(0)} = x_{d,\min} + rand_3 \times (x_{d,\max} - x_{d,\min}) \quad (3.42)$$

$$v_d^{(0)} = v_{d,\min} + rand_4 \times (v_{d,\max} - v_{d,\min}) \quad (3.43)$$

where $rand_3$ and $rand_4$ are random values in $[0, 1]$ and $x_{d,\max}$ and $x_{d,\min}$ are maximum and minimum limits for the position of particle d .

During the iterative process, position and velocity of particles are always adjusted within their limits after being recalculated after each iteration as follows:

$$v_d^{new} = \min \{v_{d,\max}, \max \{v_{d,\min}, v_d\}\} \quad (3.44)$$

$$x_d^{new} = \min \{x_{d,\max}, \max \{x_{d,\min}, x_d\}\} \quad (3.45)$$

The fitness function to be minimized is based on the problem's objective function and dependent variables including load bus voltages and power flows in transmission lines. The fitness function is defined as follows:

$$FT = \sum_{i=1}^{BG} F_i(P_{Gi}) + K_v * \sum_{i=1}^{ND} (V_{li} - V_{li}^{\lim})^2 + K_f * \sum_{l=1}^{NL} (f_l - f_l^{\max})^2 \quad (3.46)$$

where K_v and K_s are penalty factors for load bus voltages and power flows in transmission lines, respectively; ND and NL are the numbers of load buses and transmission lines, respectively; and V_{li} is the voltage magnitude at the load buses.

The upper and lower limits of the voltage magnitude at load buses in (3.46) are determined based on its calculated value as follows:

$$V_{li}^{\lim} = \begin{cases} V_{li}^{\max} & \text{if } V_{li}^{\lim} > V_{li}^{\max} \\ V_{li}^{\min} & \text{if } V_{li}^{\lim} < V_{li}^{\min} \\ V_{li} & \text{Otherwise} \end{cases} \quad (3.47)$$

where $V_{li}^{\max}$ and $V_{li}^{\min}$ are maximum and minimum voltage magnitudes at load buses.

The overall procedure of the proposed PSO-TVIW for solving the OPF problem is as follows:

Step 1: Choose the controlling parameters for PSO-TVIW including number of particles NP , maximum number of iterations $IT_{\max}$, maximum and minimum cognitive and social acceleration factors c_{1f} , c_{1r} , c_{2f} , and c_{2r} , limiting factor for maximum velocity R , and penalty factors for constraints K_v , K_f .

Step 2: Generate NP particles for control variables in their limits including initial particle position x_{id} representing vector of control variables in (3.39) and velocity v_{id} as in (3.42) and (3.43), where $i = 1, \dots, 2*N_g - 1$ and $d = 1, \dots, NP$.

Step 3: For each particle, calculate the value of dependent variables based on power flow solution by the Newton-Raphson method and evaluate their fitness function F_{pbestd} in (3.46). Determine the global best value of fitness function $F_{gbest} = \min(F_{pbestd})$.

Step 4: Set $pbest_{id}$ to x_{id} for each particle and $gbest_i$ to the position of the particle corresponding to F_{pbestd} . Set iteration counter $k = 1$.

Step 5: Calculate new velocity $v_{id}^{(k)}$ and update position $x_{id}^{(k)}$ for each particle using (3.36) and (3.37), respectively. Note that the obtained position and velocity of each particle should be limited in their lower and upper bounds given by (3.44) and (3.45).

Step 6: Solve power flow using the Newton-Raphson method based on the newly obtained position value of each particle.

Step 7: Evaluate fitness function FT_d in (3.46) ($d = 1, \dots, NP$) for each particle with the newly obtained position. Compare the calculated FT_d to $F_{pbestd}^{(k-1)}$ to obtain the best fitness function up to the current iteration $F_{pbestd}^{(k)}$.

Step 8: Pick up the position $pbest_{id}^{(k)}$ corresponding to $F_{pbestd}^{(k)}$ for each particle and determine the new global best fitness function $F_{pbestd}^{(k)}$ and the corresponding position $gbest_i^{(k)}$.

Step 9: If $k < IT_{max}$, $k = k + 1$ —return to Step 5. Otherwise, stop.

Example 5.3

The test system in this example is from Example 5.1. The upper and lower voltage limits at buses are set to 0.95 pu and 1.10 pu, respectively. The parameters for PSO-TVIW are selected as follows: $NP = 10$, $IT_{max} = 200$, $c_1 = c_2 = 2.0$, $w_{max} = 0.9$, $w_{min} = 0.4$, $R = 0.15$, $K_v = K_f = 10^6$. The PSO-TVIW method is run 20 times. The obtained results including minimum total cost, average total cost, maximum total cost, and standard deviation are given in Table 5.7. The ratio of standard deviation to the minimum total cost here is about 0.095%. The optimal solution by PSO-TVIW including power output at generation buses and voltage at buses are given in Table 5.8. In this case, the power flows in the transmission lines do not violate their thermal limits. The convergence characteristics of PSO-TVIW during the iterative process are shown in Fig. 5.12.

Table 5.7 Result obtained by PSO-TVIW for Example 5.3.

Minimum total cost (\$/h)	11,522.0227
Average total cost (\$/h)	11,524.6766
Maximum total cost (\$/h)	11,572.0615
Standard deviation (\$/h)	10.8724

Table 5.8 Optimal solution by PSO-TVIW for Example 5.3.

Variable	Value
P_{g1}	142.2097
P_{g2}	98.1559
P_{g3}	77.4907
V_1	1.0499
V_2	1.1000
V_3	1.0498
V_4	1.0369
V_5	1.0148
V_6	1.0220
V_7	1.0673
V_8	1.0609
V_9	1.0555
Power loss (MW)	2.8563
Total real power generation	317.8563

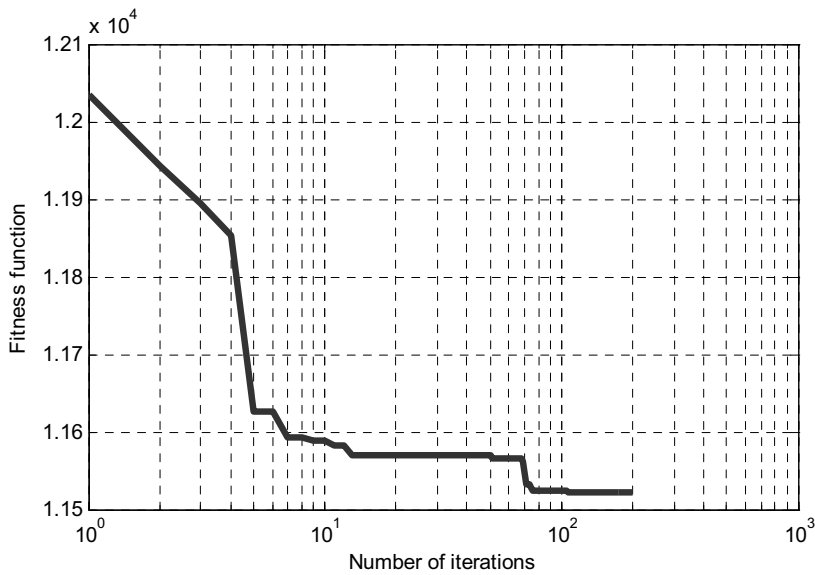


Fig. 5.12 Convergence characteristic of PSO-TVIW for Example 5.3.

5.6 SUMMARY

This chapter discussed the classical and modern methods to determine the optimal operation of power systems called optimal power flow (OPF). Several objective and system constraints can be considered in the OPF approach. In practice, the OPF problem is non-convex with a non-polynomial cost curve of generators. It also includes several discrete control actions and discontinuous variables. There are still many challenges in developing the OPF to reflect the optimal operation under real world conditions.

5.7 PROBLEMS

5.7.1 Determine the linearized fuel cost functions of the following generators.

Table 5.9 Unit characteristics of the system in Problem 5.7.1.

Gen Bus	$P_{Gi}^{\min}$	$P_{Gi}^{\max}$	$F_i = a_i + b_i P_{Gi} + c_i P_{Gi}^2$		
			a_i	b_i	c_i
1	10	200	100	10	0.1
2	20	200	200	20	0.2
3	30	200	300	30	0.3

5.7.2 From the IEEE 5 bus system below, formulate the problem of solving the optimal power flow using the QP method.

Table 5.10 Transformer data of the IEEE 5-bus system.

Form Bus	To Bus	R	X	Tap	Limit (MVA)
3	5	0	0.03	1.05	65
2	4	0	0.15	1.05	65

Table 5.11 Transmission line of the IEEE-5 bus system.

From Bus	To Bus	R	X	B	Limit (MVA)
1	2	0.04	0.25	0.25	130
1	3	0.10	0.35	0	130
2	3	0.08	0.30	0.25	65

Table 5.12 Load Data of the IEEE 5-bus system.

At Bus	P (MW)	Q (MVAR)
1	160	80
2	200	100
3	370	130

Table 5.13 Generator data of the 9-bus system.

At Bus	$P_{Gi}^{\min}$ (MW)	$P_{Gi}^{\max}$ (MVAR)	Set up Voltage (p.u.)	a_i	b_i	c_i
4	50	500	1.05	0	2.00	0.00375
5	50	500	1.05	0	1.75	0.0175

5.7.3 From the IEEE 5-bus system in Problem 2, determine the linearized fuel cost functions of the following generators and formulate the problem of solving the optimal power flow using the LP method.

5.7.4 The data of a six bus system is given in the tables below:

Table 5.14 Line characteristics of the 6-bus system.

From	To	R (pu)	X (pu)	B (pu)	Limit (MVA)
1	2	0.04	0.08	0.02	100
1	5	0.04	0.08	0.02	100
2	4	0.04	0.08	0.02	100
3	5	0.04	0.08	0.02	100
3	6	0.04	0.08	0.02	100
4	5	0.04	0.08	0.02	50
4	6	0.04	0.08	0.02	100

Table 5.15 Bus characteristics of the 6-bus system.

Bus	Load (MW)	Load (MVar)	P_{Gi}^{min} (MW)	P_{Gi}^{max} (MW)	a_i (\$)	b_i (\$/MW)	c_i (\$/MWh)
1	100	20	50	250	105	12.0	0.012
2	100	20	50	250	96	9.6	0.0096
3	100	20	50	250	105	13.0	0.0130
4	100	20	50	250	94	9.4	0.0094
5	100	50	0	0	105	12.0	0.012
6	100	10	0	0	96	9.6	0.0096

Find the optimal power flow solution for the given system.

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OPTIMAL REACTIVE POWER DISPATCH*

6.1 INTRODUCTION

Reactive power (or reactive volt-amperes) is a specific phenomenon occurring in an alternative current (AC) system. This kind of power does no real work for the electricity consumers but plays a very important role in power system engineering. Since the impedances of the network components are predominantly reactive, the transmission of real power requires a difference in voltage phase angle between the sending and receiving points, whereas the transmission of reactive power requires a difference in voltage magnitudes of these two adjacent buses, which are usually restricted to a 5-percent margin. Hence, to stay within the voltage limits, transmission of reactive power doesn't reach very far and is regarded as local [1].

Reactive power is consumed not only by most of the network elements which are passive but also by most of the consumer loads, predominantly caused by induction motors [2, 3]. So, reactive power must be supplied somehow, either by passive or active elements at power systems.

Overhead lines, depending on the load current, either absorb or supply reactive power. At loads transmitted along the line below the natural (surge impedance) load, lines produce net reactive power; at loads above the natural load, lines absorb reactive power (see 6.2.1.A). Underground cables, owing to their high capacitance, have high natural loads; they are always loaded below their natural loads, and hence generate reactive power under all operating conditions [2]. Transformers always absorb reactive power regardless of their loading; at zero load, the shunt magnetizing reactance effects predominate while at full load, the series leakage inductance effects predominate [2].

*This chapter has been written with assistance from Chira Achayuthakan

Compensating devices, which are active elements, are usually added to supply or absorb reactive power and thereby control the system voltage in the desired manner balancing reactive power. This chapter begins with the discussing the essence of reactive power in power systems and continues with optimal reactive power dispatch methodologies: in both conventional and deregulated electricity markets.

6.2 REACTIVE POWER IN POWER SYSTEMS

In the context of power systems, transferring power from sources to loads makes the passive elements of the network either consume reactive power from or inject it into the system which is also the case with parts of the load themselves. With the restriction of reactive power to travel from afar sources, compensation of reactive power to such passive elements or reactive loads highly depends on local reactive sources. As the reactive compensation requirements change over time, the reactive devices should be controllable—so called active elements.

6.2.1 Reactive Power with Passive Elements

For power system analysis, most of the passive elements in a network are modeled by admittance matrix. The simple network model of a two bus system may help understand the phenomena of reactive power flow in a passive network system. This section explains some key terms of surge impedance load (SIL) and steady state voltage stability.

A. Surge Impedance Load (SIL)

The fundamental concept of SIL or natural load can be found in many textbooks [2, 4, 5, 6]. SIL is a conventional analytical model explaining the reactive power-related system characteristics. The term SIL is usually reserved for the special case of lossless line [6]. SIL can be formulated as V^2/Z_0 , where V is the rated voltage, $Z_0 = \sqrt{X/B}$ (pure resistive), X is the line reactance and B is the line susceptance, see Fig. 6.1. Most approaches deriving SIL involve an ordinary differential equation. Put in simple terms, it means that SIL is critical dependent on the load side when production of reactive power equals consumption, $V^2B = I^2X$, and that $Z_0 = V/I = \sqrt{X/B}$ [4]. In this case, the load level constituting a balance of inductive and capacitive effects (netted out) is called SIL [7]. Firstly, SIL cancels out reactive power required by the network, secondly, voltage and current profiles become uniform (flat) along the line [4]. A uniform voltage profile is especially desirable as it allows the voltage to be held near the maximum value.

It is convenient to express the power transmitted by a line in terms of per unit of SIL [6]. For instance, the permissible loading of a transmission line may be expressed as units of its SIL, and SIL provides a comparison of load-carrying capabilities of lines. Typically, SIL is 40% of the line's thermal loading limit [7], or, vice versa, that the thermal loading limit is $2.5 \times \text{SIL}$.

An example of power flow analysis conducted on a non-lossless two-bus system is shown in Fig. 6.2, with real power load varying at the receiving end from $0.5 \times \text{SIL}$ to $1.5 \times \text{SIL}$, and the load's power factor varying from 0.7 leading to 0.8 lagging.

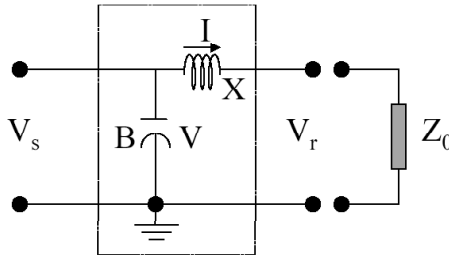


Fig. 6.1 Long power line model.

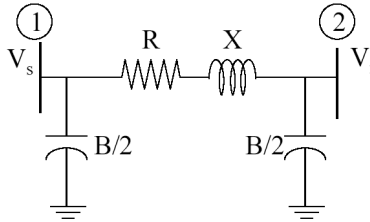


Fig. 6.2 Two-bus system.

The specifications of the example line here are: 500kV, 4×795 MCM, 211.438 km, thermal limit = 2,833.635 MVA, $R = 4.395796 \, \Omega$, $X = 57.15592 \, \Omega$, and $B = 2.75268 \, \mu\text{F}$. This leads to $R = 0.0017583$ p.u., $X = 0.02286$ p.u., and $B = 2.161949$ p.u. This line's SIL is reckoned to be 972.438 MW and the thermal limit is then $2.9139 \times \text{SIL}$. We can see the power line carrying characteristics as presented in Figs. 6.3–6.4.

From Fig. 6.3, we can see that with the high power factor loading (near to unity), the network will absorb the net reactive power at real power loads above SIL. It can also be seen that the higher the load with increasingly poor lagging power factor, the more reactive power (Q_{Loss}) is required by the network.

This analysis can be used to plot the P – V curve as in Fig. 6.4. The comparison of Figs. 6.3 and 6.4 reveals a high correlation indicating that the more reactive power is required by the network (Q_{Loss}), the lower the voltage magnitude at the receiving end because there is the only source of reactive power at the sending bus end. With voltage limits imposed in power system operation, e.g., 0.95–1.05,

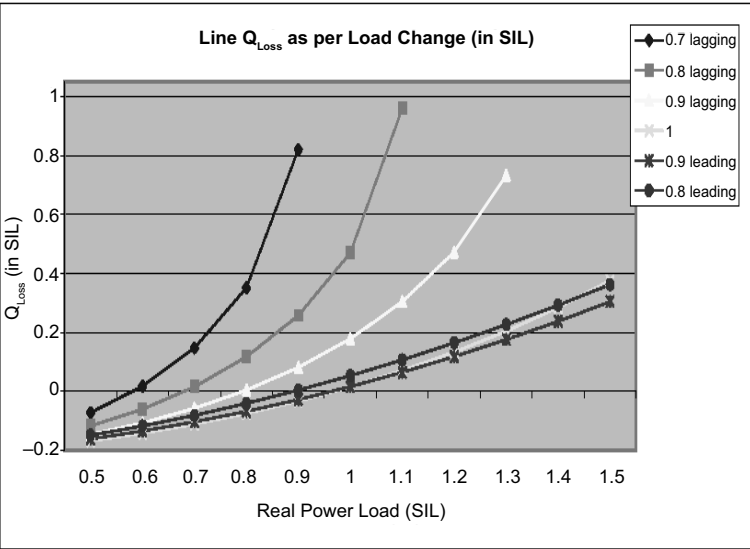


Fig. 6.3 Q_{Loss} as per power loading (211 km).

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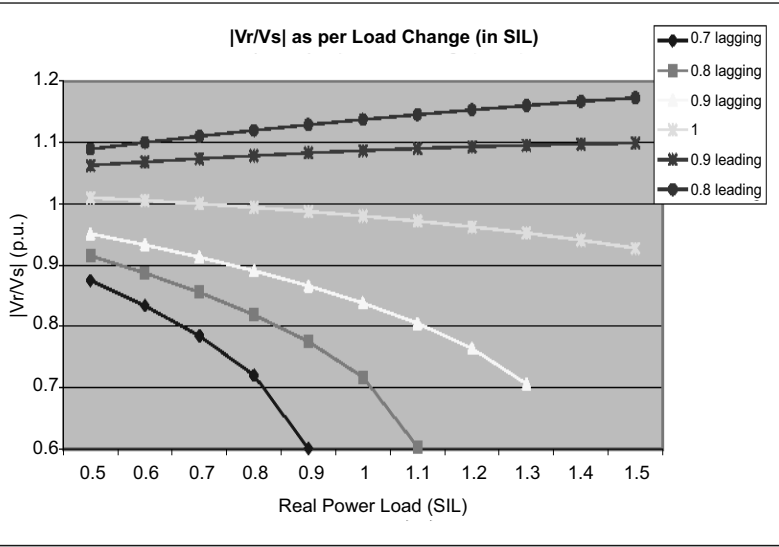


Fig. 6.4 Receiving end voltage as per real power loading (211 km).

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networks may not be able to transfer even a power of $0.6 \times \text{SIL}$ for the 0.9 lagging power factor load required at the receiving bus. This introduces the idea that the line limit can be reduced from its thermal limit of $2.9 \times \text{SIL}$ to only $0.6 \times \text{SIL}$ because of voltage limits.

Under abnormal conditions, when the load goes with the leading power factor of capacitive load, the receiving end bus voltages are even higher than at the sending end. Loading more power into the network constitutes no violation of voltage limits. Additionally, we can see that the basic potential of local reactive power sources can increase the transfer capability of networks.

B. Steady State Voltage Stability

Without appropriate reactive power compensation, network loading may be more restricted by the steady-state voltage stability limit. The maximum loadability of a system, which indicates how much the system can be further stressed before becoming unstable is commonly expressed as voltage stability margin [8]. The fundamental explanation of the critical point of the steady state voltage stability limit can be derived from a simple two-bus-one-line system with pure reactance as in Fig. 6.5.

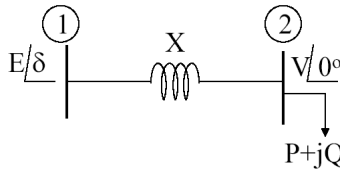


Fig. 6.5 Pure reactance two-bus-one-line system.

From system in Fig. 6.5, we get:

$$P = |E||V|/X \sin(\delta) \quad (6.1)$$

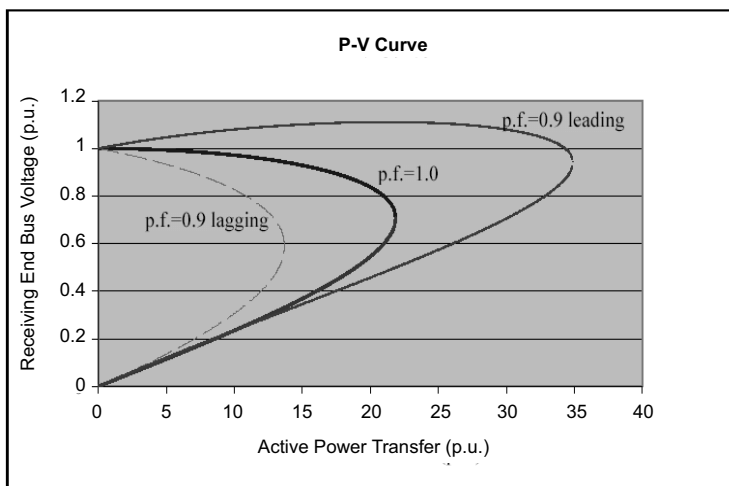
$$Q = |E||V|/X \cos(\delta) - |V|^2/X \quad (6.2)$$

where (6.1) is called the power-angle equation. Transforming (6.1) and (6.2), we get:

$$|V|^2 = [PX|E|]^2 + [(QX + |V|^2)/|E|]^2 \quad (6.3)$$

With (6.3), we can draw the P-V curve as shown in Fig. 6.6 ($|E| = 1.0$ p.u. and $X = 0.02286$ p.u.). Figure 6.6 shows that for a load with a power factor of 0.9 lagging, the voltage magnitude at the receiving end may collapse when line loading is above 14 p.u. (and 22 p.u. in the case of unit power factor attributed to the receiving end bus). At this critical point, the gradient between voltage and power loading becomes infinite ($\partial V/\partial P = \infty$). These terms of voltage instability and voltage collapse are used somewhat interchangeable by most engineers [9].

For more sophisticated networks with conventional power flow analysis, the determination of the critical point may face the problem of singularity as of the Jacobian matrix. However, this problem can be solved by the continuous power flow (CPF) technique [10], which shall not be included in this book.

**Fig. 6.6** P-V Curve.

Color image of this figure appears in the color plate section at the end of the book.

6.2.2 Reactive Power with Active Elements

As explained earlier, the reactive power required in a system changes subject to changes in reactive loads, and system loading and loads above SIL may cause a system to absorb the reactive power totally so that it has to be compensated by some other devices, here called active devices.

Not only such passive power systems require reactive power, but the loads, at consumer locations dominated by induction machinery also absorb reactive power. Multiplicatively, the real power loads induce loading into the lines in the vicinity of them, resulting in more reactive power required by those lines. With the knowledge that the amount of reactive power transferred along a line is highly correlated to the voltage difference between the line ends [4], serving one location reactive power demanded from the source located afar can deteriorate the voltage profile.

The primary active resources of reactive power in the system are lying with synchronous generators, which are prevalently considered as sources of real power. The placement of synchronous generators usually depends on fuel or other primary energy supply availability rather than locational requirements of reactive power in the system. Locations of synchronous generators and loads (also consuming reactive power) are practically far from one another. Voltage profile problems may not be avoided when the system depends solely on synchronous generators as reactive power sources. Hence, other reactive power compensating devices are needed. A concept including various kinds of compensators is described below.

A. Ideal Reactive Power Compensator

The ideal reactive power compensator device is able to inject or absorb reactive power regardless of its capability limits; hence it can perfectly regulate its terminal voltage to the desired level (see Fig. 6.7).

Another idea about the characteristics of an ideal reactive compensator was that it should be able to operate independently in the three phases [2].

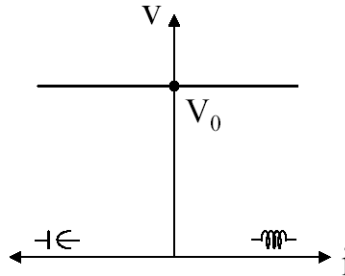


Fig. 6.7 Terminal (v-i) characteristics of ideal reactive compensator.

B. Synchronous Generator/Synchronous Condenser

Synchronous generators can generate or absorb reactive power depending on the excitation [2]. When overexcited they supply reactive power, and when under-excited they absorb reactive power. The capability to continuously supply and absorb reactive power is, however, limited by the field current, armature current, and end-region heating limits. This is known as the generator capability curve, see Fig. 6.8. Synchronous generators are normally equipped with automatic voltage regulators, which continually adjust the excitation so as to control the armature voltage.

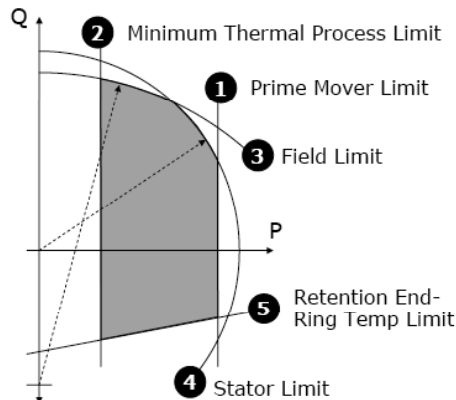


Fig. 6.8 Typical capability curve of a generator.

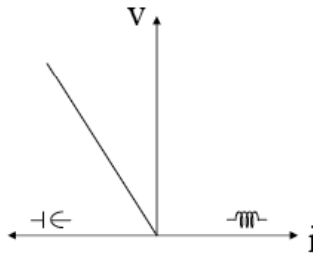


Fig. 6.9 Terminal (v-i) characteristics of a shunt capacitor.

C. Shunt Capacitor/Reactor

A shunt capacitor always supplies reactive power and boosts local voltage. Shunt capacitors are usually installed at the customer's load side for power factor correction. They are also installed in a transmission system as voltage support equipment. Shunt capacitors are treated as a source of reactive power that is relatively inexpensive to install and maintain [11]. Installing the shunt capacitors in the load area or at the location where reactive power is needed increases voltage stability.

The main advantages of shunt capacitors are their low cost and flexibility of installation and operation. The main disadvantages of shunt capacitors are that they supply reactive power in steps rather than smoothly, and that their reactive power output is proportional to the square of the voltage. Consequently, the reactive power output is reduced at low voltages when it is likely to be needed more urgently.

A shunt reactor works in a similar manner as shunt capacitor, but it usually absorbs reactive power from the system when installed.

D. FACTS Devices

Flexible AC transmission system (FACTS) technologies open up new opportunities for controlling power and enhancing the usable capacity of present, new, and upgraded lines. These opportunities arise through the ability of FACTS controllers to control the interrelated parameters that govern the operation of transmission systems including series impedance, shunt impedance, current, voltage, phase angle and the damping of oscillations at various frequencies. By providing added flexibility with rapid response of power electronics, FACTS controllers can also diminish burdens of reactive power to the system. As the FACTS technology has been proven to solve some problems in power systems so far, new types of FACTS controllers are being studied and developed. Some FACTS devices related to reactive power operation explained in this section are the static Var compensator (SVC), the static synchronous compensator (STATCOM) and the thyristor-controlled series capacitor (TCSC).

1) Static Var Compensator (SVC)

The SVC is a shunt connected static Var generator/absorber, the output of which is adjusted to exchange the capacitive or inductive current so as to maintain or control system variables [11, 12, 13]. An SVC is different to a synchronous compensator in that it is used to supply or absorb reactive power without rotating parts. It is also capable of acting as an equivalent of an automatic voltage regulator system to set and maintain a target voltage level. It is composed of a shunt reactor and a shunt capacitor as shown in Fig. 6.10. Typically, the control parameter is the terminal voltage. There are two popular configurations of the SVC. One is a fixed capacitor and thyristor-controlled reactor (TCR) configuration, the other is a thyristor-switched capacitor (TSC) and TCR configuration. An SVC control system sends a signal to the thyristor to control the amount of current flow through its total susceptance. In other words, the SVC represents the controller with variable impedance that changes with the firing angle of TCR. Figure 6.11 illustrates the terminal ($v-i$) characteristics of each component of an SVC, i.e., TCR and fixed capacitor respectively. The structure and terminal ($v-i$) characteristics of an SVC are presented in Figs. 6.10 and 6.12.

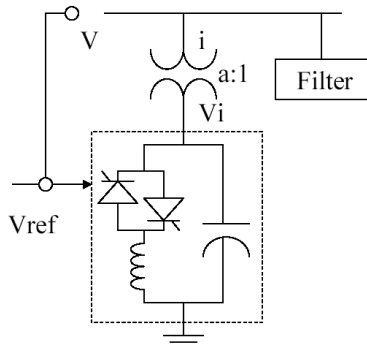


Fig. 6.10 Structure of SVC.

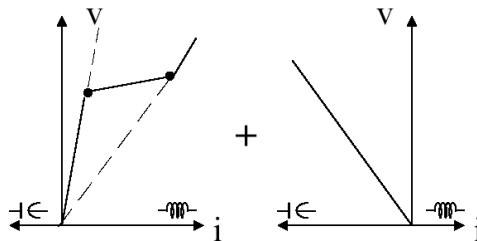


Fig. 6.11 SVC Terminal ($v-i$) characteristics by components (TCR and fixed capacitor).

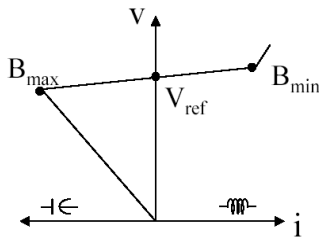


Fig. 6.12 Terminal (v - i) characteristics of SVC.

2) Static Synchronous Compensator (STATCOM)

STATCOM is a voltage-source converter that converts a DC input voltage into AC output voltage in order to compensate the real and reactive voltage needed by the system [11, 13]. In general, with STATCOM, a reduction in the physical size of the installation of more than 50% can be expected as compared with SVC. STATCOM also offers better characteristics than SVC in the case that when the system voltage drops enough to force the STATCOM output to its ceiling, its maximum reactive power output is still not affected by the voltage magnitude. Therefore, it exhibits constant current characteristics even when the voltage is dropping under the limit. The structure and terminal (v - i) characteristics of STATCOM are presented in Fig. 6.13.

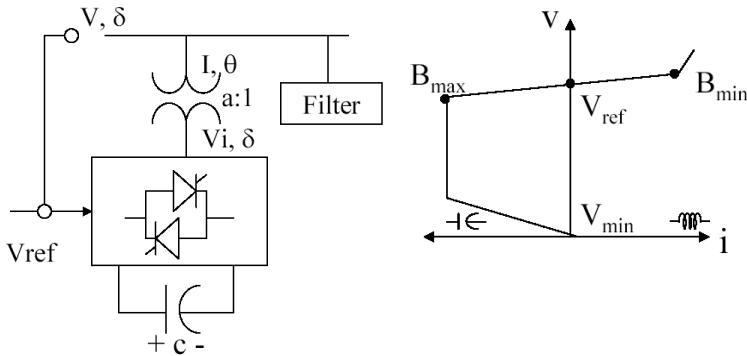


Fig. 6.13 STATCOM: (a) Structure and (b) Terminal (v - i) characteristics.

3) Thyristor-Controlled Series Capacitor (TCSC)

TCSC is a capacitive reactance compensator which consists of a series capacitor bank shunted by a thyristor-controlled reactor as shown in Fig. 6.14, which provides for a smoothly variable series capacitive reactance. It is a very important FACTS controller. The TCSC is based on a thyristor without gate turn-off capability. A variable reactor such as a Thyristor-Controlled Reactor (TCR) is connected across a

series capacitor. When the TCR firing angle is 180 degrees, the reactor becomes non-conducting and the series capacitor has its normal impedance. As the firing angle is advanced from 180 degrees (to less than 180 degrees), the capacitive impedance increases. At the other end, when the TCR firing angle is 90 degrees, the reactor becomes fully conductive and the total impedance becomes inductive because the reactor impedance is designed to be much lower than the series capacitor impedance. With a 90 degree firing angle, the TCSC helps in limiting the fault current. The TCSC may be a single large unit consist of several equal or different-sized smaller capacitors in order to achieve a superior, flexible performance.

Series compensation reduces the series impedance of the line (see Fig. 6.15) which is the main cause of voltage drop and the most important factor in determining the maximum power transmission capability. Under heavy load condition, i.e., the line loading is beyond the surge impedance load (SIL), the transmission line absorbs reactive power; installing a series compensator can then provide reactive power.

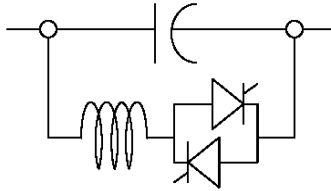


Fig. 6.14 TCSC Structure.

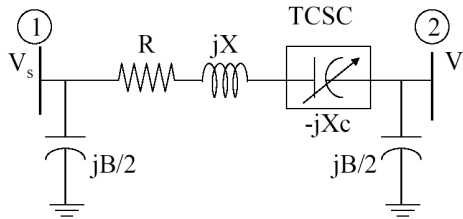


Fig. 6.15 Model of TCSC.

E. Induction Generator

Brushless and more rugged, particularly due to its lower cost than a synchronous machine, the induction generator has been considered for applications in wind and micro-hydro turbines etc. In either motor or generator mode of operation, an induction machine requires reactive power through its winding terminal for magnetization. If an induction generator is isolated from the grid and needs no external reactive power compensation, it is called self-excitation induction generator (SEIG). If an induction generator is switched parallel to the grid as grid-connected induction generator (GCIG) this causes an excessive inrush current and voltage drop at connection. Compared with a synchronous machine, an induction generator

does not contribute more fault current to the system [14]. The drop of voltage that accompanies a short circuit current automatically reduces the excitation and limits the short circuit current. The disadvantages are its poor voltage and frequency regulation and low power factor [15].

Various kinds of reactive power compensation for induction machines have been proposed. One study [16] suggested the use of a synchronous condenser that provides reactive power to the induction generator through slip rings, while the induction supplies real power to the synchronous condenser. Additional series capacitors in its short shunt or long shunt configurations also can be used to control the voltage in SEIG, but the value of capacitors have to be optimized for the entire loading range. The large reactive power drain from GCIG can be compensated by shunt capacitors but they cause excessive over voltage during disconnection. Wind generators may pose significant voltage problems that will require installation of synchronous condensers, STATCOMs, and SVCs for the purpose of managing voltage levels in the vicinity of wind farms [17].

For modeling the induction generation for power flow analysis, the PV model does no longer suit such as asynchronous generator features [18]. It can be stated that the PX model in (6.4), proposed by [19] is more accurate than the conventional PQ model in (6.5), proposed by [20]. Another adopted model is the RX model that parameterizes the rotor slip to find the value of slip corresponding to electrical power, steady state model of induction machine, and mechanical power associated with wind speed [21].

$$Q = \frac{-V_i^2}{X_i} \quad (6.4)$$

$$Q \approx V^2 \frac{X_c - X_m}{X_c X_m} + \frac{X}{V^2} P^2 \quad (6.5)$$

6.2.3 Passive Network with Active Reactive Compensator

The analysis from section 6.2.1.A can be applied to the case of a line length reduced by a half, say 105 km, with the same configuration. The SIL remains the same regardless of the line length; all network parameters are reduced to a half. The results are presented similar to those shown in Figs. 6.16–6.17. We can see some changes in network power carrying characteristics: reactive power required by the network is reduced and the receiving end voltage profile is much better.

Some interpretations from section 6.2.1.A can also be extended: Without reactive power compensation, an increase in line length causes more operational constraints on the line, and the line transfer capability limit is reduced [7] as shown in Fig. 6.18. This depends on a few factors: Explicitly, the line's thermal limit, sometimes known as MVA limit depending on its conductor sizes, appears to be the

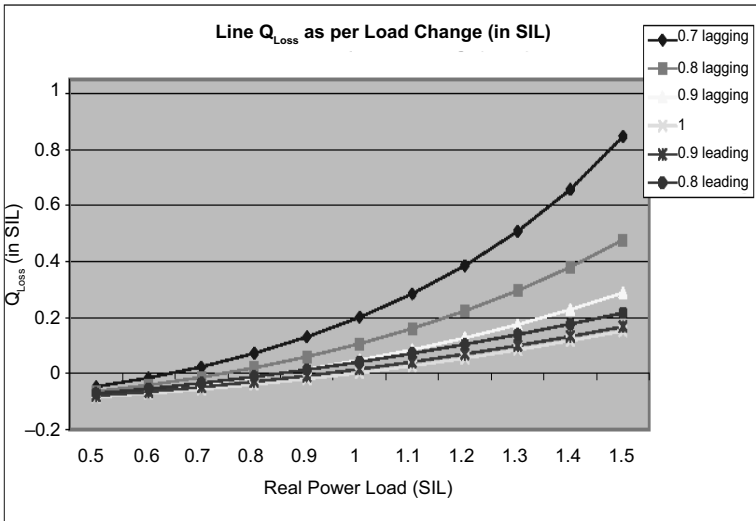


Fig. 6.16 Q_{Loss} as per power loading (105 km).

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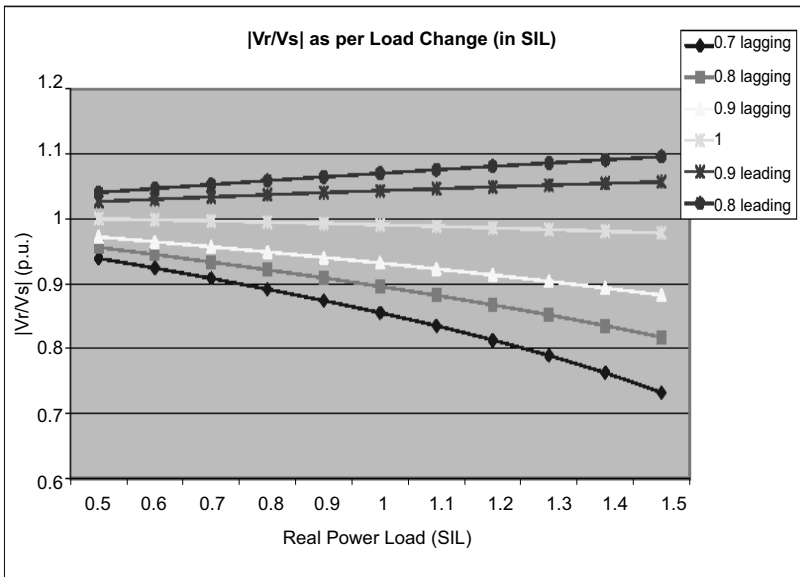


Fig. 6.17 Receiving end voltage as per real power loading (105 km).

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most important factor. The second factor is voltage limits. The third factor limiting line transfer capacity is the stability limit, which covers various aspects, e.g., rotor angle stability, voltage stability limits etc.

The P - V Curve as shown in Fig. 6.18 visualizes the line capability limitations by three causes: line current flow (or MVA) limits, receiving end minimum voltage

limit, and voltage stability limit. Figure 6.19 shows that the different causes limit the line capability with different line lengths. The limit in a short line is defined by its thermal limit. The limit in a medium length line is usually the voltage limit at the receiving end. And lastly, power transmitted along a long line may not guarantee its receiving end voltage stability. In a preferable design, line power transfer capability should be defined by its intrinsic capacity constrained by the thermal limit.

Transmission networks are mainly designed to transfer real power [22]. It may be expected that decreasing the impedance can increase the line transfer capability. This question is applied to the line in Fig. 6.2, that is divided in half as shown in Fig. 6.21 (turning to a 3-bus 2-line system). How the line transfer capability from bus 1 to bus 2 is observed. At first view, the impedance between buses 1 and 2 is

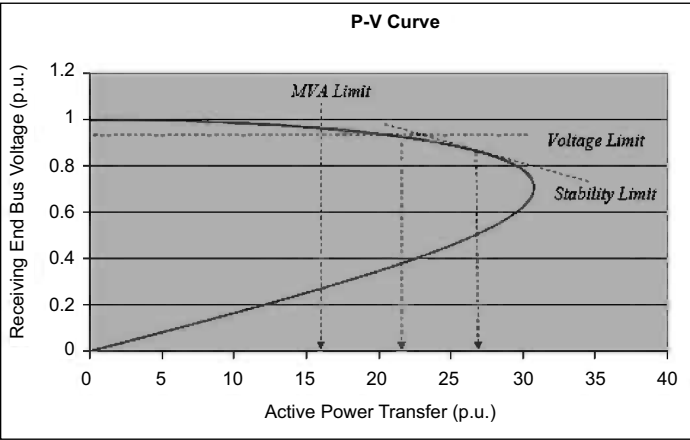


Fig. 6.18 Limits in transmission line capability.

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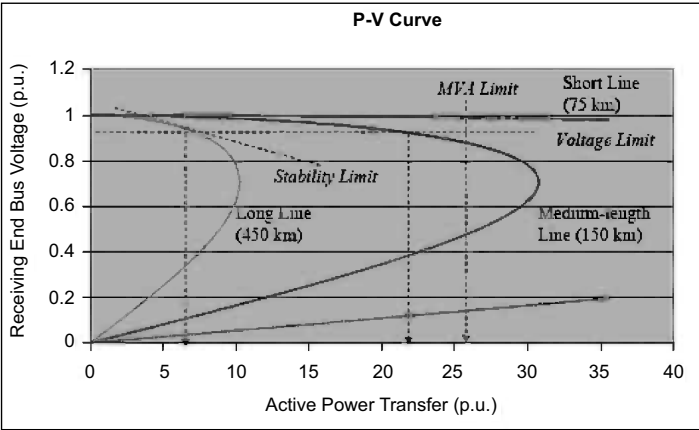


Fig. 6.19 Line capability limits of three different line lengths.

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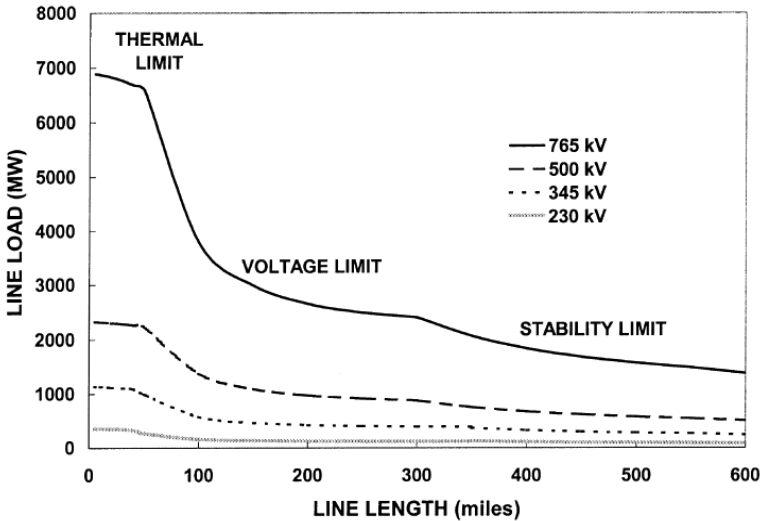


Fig. 6.20 Transmission line capability without reactive compensation to maintain voltage (source from [7]).

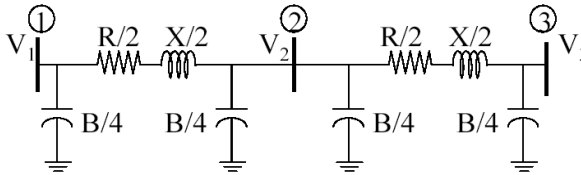


Fig. 6.21 The 3-bus 2-line power system.

reduced so the line capability is expected to increase. This is physically not really true, since the line is still the same as in Fig. 6.2 and the voltage at V_3 still restricts its transfer capability.

Without any reactive power compensation at the new intermediate bus (bus 2), the transfer capability is limited by the terminal voltage (V_3). The results are as shown in Figs. 6.22 and 6.23. Actually V_3 as shown in Fig. 6.23 is comparable to V_r in Fig. 6.4 since the system is the same. The new bus (bus 2) has no significant effect on the system.

For the system in Fig. 6.21 instated with a reactive power compensator, here is an analysis considering an ideal compensator and that bus 2 becomes a PV bus with $P=0$ MW as demonstrated in Fig. 6.24. The results of the analysis are shown in Fig. 6.25. We may also note that V_2 will always be 1.0 because its reactive power is fully compensated.

When we compare the results shown in Fig. 6.23 and Fig. 6.25, we can see that the voltage profile is improved and transfer capability increased when the line impedance is decreased; V_3 in Fig. 6.25 is comparable to V_r in Fig. 6.17.

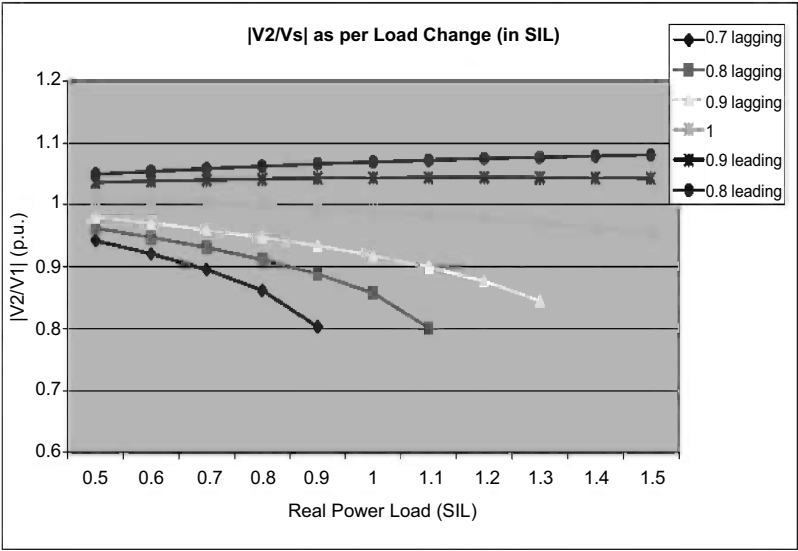


Fig. 6.22 V_2 voltage as per power loading.

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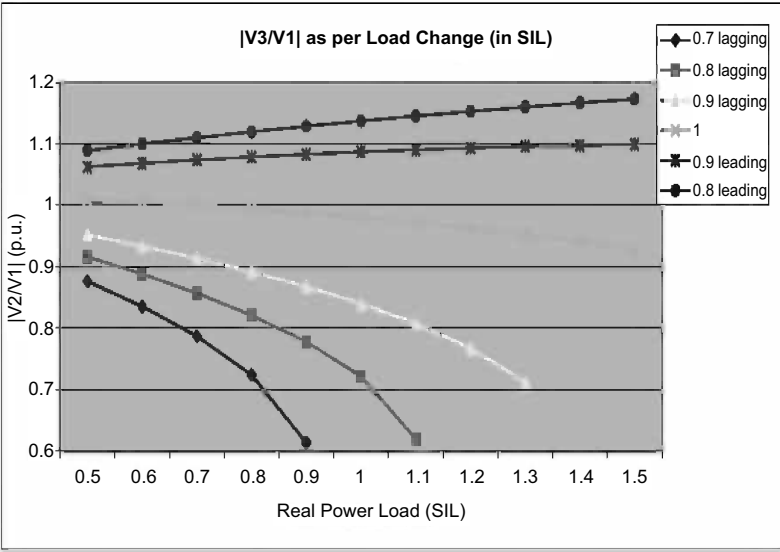


Fig. 6.23 V_3 Voltage as per power loading.

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The important role of reactive power supply becomes evident: It supplies power for direct loads' consumption and is essential for power transfer across the network.

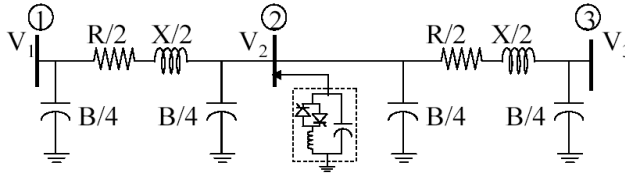


Fig. 6.24 The 3-bus 2-line power system with full compensation of reactive power.

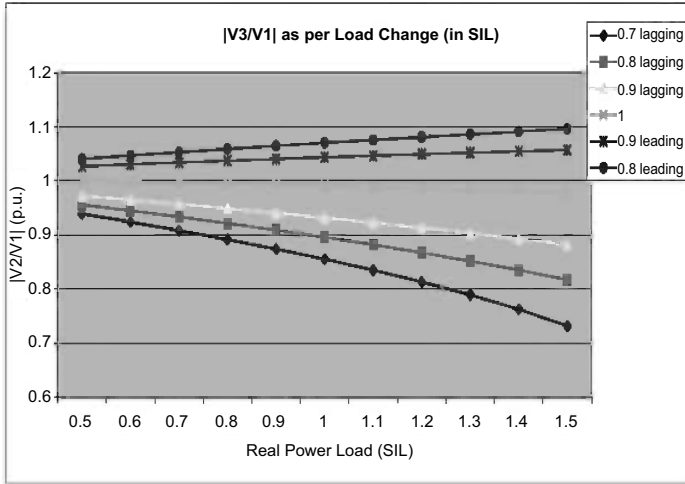


Fig. 6.25 V_3 voltage as per real power loading with full compensation of reactive power.

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6.3 CONVENTIONAL OPTIMAL REACTIVE POWER DISPATCH

Optimal reactive power dispatch (ORPD) is considered as part of the optimal power flow (OPF) problem; the economic dispatch (ED) or real power dispatch problem can be expanded to the OPF problem by adding more control variables of reactive control devices. Generally, the OPF problem is decoupled into two sub-problems: ED and ORPD. They can be alternately solved until their convergence is reached as illustrated in Fig. 6.26. The generation of real power is usually determined by the ED sub-problems and considered constant in the ORPD problem.

Prior to the emergence of the era of electricity market deregulation, most concepts of the ORPD problem were confined to conventional operation regimes based on cost aspects. ORPD solutions using OPF are focused on real power loss minimization. This section presents the formulation of the structure of conventional ORPD using an optimization framework consisting of objective function, constraints and control or decision variables.

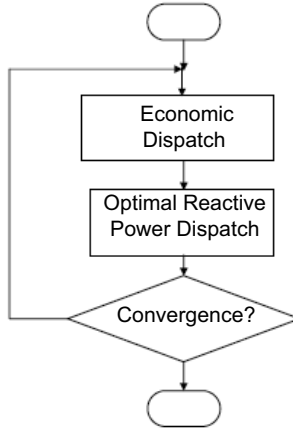


Fig. 6.26 Two sub-problem model of OPF.

6.3.1 Objective Function

Most objective functions modelled for solving ORPD problems are designed to minimize real power loss. Anyhow, other alternatives can also be formulated. Typical objective functions are mentioned below.

A. Real Power Loss Minimization

The formula of the objective function for real power loss (P_{Loss}) minimization is:

$$P_{Loss} = \sum_{k=1}^{nline} g_k [V_i^2 + V_j^2 - 2V_i V_j \cos(\delta_i - \delta_j)] \quad (6.6)$$

where

V_i, δ_i voltage magnitude and angle of bus i , respectively.

g_k conductance of line k .

$nline$ number of lines.

In most ORPD power system analysis models, the generator at swing bus ($P_{G,swing}$) is finally adjusted to balance the system power including system loss while all other real power generators (P_G) are unchanged; in some ORPD cases, the minimization of $P_{G,swing}$ can be used as objective function in lieu of P_{Loss} .

B. Voltage Deviation Minimization

Usually, the objective function of voltage deviation (VD) minimization is defined as:

$$VD = \sum_{k=1}^{nbus} |V_k - V_{dk}| \quad (6.7)$$

where

V_k voltage magnitude of bus k .

V_{dk} desired voltage magnitude of bus k , usually equals 1.0 p.u.

$nbus$ number of buses.

C. Cost Minimization

The cost to be minimized by ORPD is mainly that of system losses. However, ORPD cost minimization can also be extended to other cost components, e.g., cost of reactive power production, and cost of adjusting of control devices. The below description expresses more details of each cost component:

1) Cost of energy loss due to system losses

The objective function of real power loss as stated in section 6.3.1.A can be directly converted to this objective function of cost caused by system losses by multiplication with the energy price (*Price*).

$$Cost_{Loss} = Price \times P_{Loss} \quad (6.8)$$

2) Cost of reactive power production

To detail the cost of providing reactive power, reactive devices are divided into generation and transmission devices. The former type is constituted by synchronous generators, the latter one of shunt capacitors/reactors, transformer tap changers, FACTS etc. Both types of devices mainly account for fixed cost.

ORPD is short term optimization problem by nature with the variable cost being the relevant cost component of reactive power production. However, particularly the amount attributed to transmission devices is usually so small that its inclusion in the objective function is sometimes negligible whereas regarding generation devices, variable cost is still determinable. Two main components can represent the variable cost of reactive power production: loss of opportunity and energy loss.

Various proposed models for reactive power production cost are based on loss of opportunity, (6.9) or (6.10):

$$C_{qgi}(Q_{gi}) = k \times [C_{pgi}(S_{gi,\max}) - C_{pgi}(\sqrt{S_{gi,\max}^2 - Q_{gi}^2})] \quad (6.9)$$

where

$C_{qgi}(Q_{gi})$ = Reactive power production cost of generator i

k = Profit rate of real power generation usually between 5%–10%

$C_{pgi}(P_{gi})$ = real power production cost of generator i

$S_{gi,\max}$ = Nominal apparent power of generator i

Q_{Gi} = Operating level of reactive power of generator i

P_{Gi} = Operating level of real power of generator i

$$C_{qgi}(Q_{gi}) = C_{pgi}(\sqrt{P_{gi}^2 + Q_{gi}^2}) - C_{pgi}(P_{gi}) \quad (6.10)$$

where

$C_{qgi}(Q_{gi})$ = Reactive power production cost of generator i

$C_{pgi}(P_{gi})$ = Real power quadratic production cost of generator i

Q_{gi} = Operating level of reactive power of generator i

P_{gi} = Operating level of real power of generator i

3) Cost of adjustment of control devices

As indicated above, the fuel-related cost of transmission related reactive power devices is very small and eventually negligible. Anyhow, it is more sensible that the variable cost of reactive power production appears in the form of cost per device operation rather than cost per reactive power itself because the more frequent the operation of a device, e.g., the number of switching, the shorter the maintenance/overhaul period and life expectancy of the equipment (requiring earlier re-investment for replacement).

The cost of the adjustment per control device unit for transformer on-load tap changers (OLTC) can be formulated:

$$C_i = \frac{1}{N_{OLTC,i}} (FC_{OLTC,i} + \frac{\alpha_{Ti} - \alpha'_{Ti}}{\alpha_{Ti}} FC_{trx,i} + \frac{FC_{OH,OLTC,i} \times \alpha'_{Ti}}{T_{OH,OLTC,i}}) + C_M \quad (6.11)$$

where

$N_{OLTC,i}$ allowable adjustment times of the OLTC of transformer i

$FC_{OLTC,i}$ investment cost of OLTC of transformer i (\$)

α_{Ti} life expectancy of the transformer i without OLTC operation (years)

α'_{Ti} life expectancy of the transformer i after N_{OLTC} times of operation (years)

- $FC_{tx,i}$ investment cost of transformer i (\$)
 $FC_{OH,OLTC,i}$ unit overhaul cost of OLTC of transformer i (\$)
 $T_{OH,OLTC,i}$ overhaul period cost of OLTC of transformer i (year)
 C_M unit cost of manpower (\$)

The cost of adjustment per control device unit for any switched reactive device can be formulated as:

$$C_j = \frac{1}{N_{Op,Qj}} (FC_{QSWj} + FC_{Qj} + \frac{FC_{OH,Qj} \times \alpha'_{Qj}}{T_{OH,Qj}}) + FC_M \quad (6.12)$$

where

- $N_{Op,Qj}$ allowable adjustment times of reactive compensator j
 FC_{QSWj} investment cost of switching device of reactive compensator j (\$)
 FC_{Qj} investment cost of reactive compensator j (\$)
 $FC_{OH,Qj}$ unit overhaul cost of reactive compensator j (\$)
 α'_{Qj} life expectancy of reactive compensator j after $N_{Op,Qj}$ times of operation (years)
 $T_{OH,Qj}$ overhaul period of reactive compensator j (\$)
 C_M unit cost of manpower (\$)

With the modelling in this style of variable cost (cost per device operation), the static load model is no longer sufficient. A dynamic load over a specified interval of time, e.g., 24 hours, is required to solve this ORPD problem.

D. Combined or Multi-Objective Functions

The objective functions to be minimized can be generally combined in a single objective function using weight factors (W_i). The below expression is a template for combined or multi objective functions formulation.

$$Obj = \sum_{i=1}^{nobj} W_i \times Obj_i \quad (6.13)$$

where

- W_i weight factor of objective function i
 Obj_i objective function i
 $nobj$ number of objective functions considered in problem

6.3.2 Constraints

The constraints affecting ORPD here are equality and inequality. The crucial factors are real and reactive power flow balances. They can be represented in the power flow equations:

$$P_{G,i} - P_{L,i} = V_i \sum_{j=1}^{nbus} V_j [G_{ij} \cos(\delta_i - \delta_j) + B_{ij} \sin(\delta_i - \delta_j)] \quad (6.14)$$

$$Q_{G,i} - Q_{L,i} = V_i \sum_{j=1}^{nbus} V_j [G_{ij} \sin(\delta_i - \delta_j) - B_{ij} \cos(\delta_i - \delta_j)] \quad (6.15)$$

where

$P_{G,i}$ real power generation of bus i .

$P_{L,i}$ real power load of bus i .

V_i, δ_i voltage magnitude and angle of bus i .

G_{ij} conductance of line i - j .

B_{ij} susceptance line i - j .

$Q_{G,i}$ reactive power generation of bus i .

$Q_{L,i}$ reactive power load of bus i .

Other constraints are various kinds of system limits representing inequality such as

Bus voltage limits:

$$V_{i,\min} \leq V_i \leq V_{i,\max} \quad ; \text{ for all } i=1 \text{ to } nbus \quad (6.16)$$

Line flow (MVA) limits:

$$S_{i,\min} \leq S_i \leq S_{i,\max} \quad ; \text{ for all } i = 1 \text{ to } nline \quad (6.17)$$

Generator reactive power limits:

$$Q_{Gi,\min} \leq Q_{Gi} \leq Q_{Gi,\max}; \text{ for all } i=1 \text{ to } ngen \quad (6.18)$$

or other control variable limits, e.g., the tap of transformer having OLTC etc.

6.3.3 Control Variables

Since ORPD is usually a OPF sub-problem, the real power dispatch in an ORPD problem is pre-determined (except for swing bus). The conventional controllable parameters are related to reactive devices, e.g., generators' reactive power generation noted as Q_G (or desired operating voltage level) noted as V_G , transformer tap, FACTS device control parameters etc.

6.3.4 Example of a Conventional ORPD Problem

One of the ways to model a conventional ORPD problem is the structure given below.

Objective:	Loss minimization
Subject to:	Power flow equations and limits
Control variables:	Generator's reactive power production

As an example, given the system, adapting Fig. 6.5 by including resistance and reactance of 0.0017583 and 0.02286 p.u. in the line impedance, respectively, with $|E| = 1.0$ p.u., load $P = 10.0$ p.u., and $Q = 0.0$ p.u., we get the below conditions:

$$|E| = 1.000, \delta = 17.85 \text{ degrees.}$$

$$|V| = 0.933. P_{loss} = 20.20 \text{ MW.}$$

Supposed we have an ideal reactive power source connected to bus 2 (Q_2), the conventional ORPD problem can be formulated as:

$$\text{Objective:} \quad \text{Min. } P_{Loss} = \frac{R}{R^2 + X^2} [1 + |V|^2 - 2|V|\cos(\delta)]$$

$$\text{Subject to:} \quad \text{Power flow equations, and } 0.9 \leq V \leq 1.1$$

$$\text{Control Variable: } V$$

$$\text{The results obtained are } |V| = 1.024, P_{loss} = 18.2299 \text{ MW, } Q_2 = 295.224 \text{ MVar.}$$

6.4 OPTIMAL REACTIVE POWER DISPATCH IN DEREGULATED ELECTRICITY MARKETS

In the new environment of deregulated electricity markets, a price-based approach is of essence. Compensating reactive power delivered to a system requires some resources to be deployed. The mechanism to settle reactive power acquisition is market-oriented. The matured electricity market usually is distinguished into the main real power market and other ancillary service (AS) markets. Reactive power is usually regarded as a component of AS.

6.4.1 Reactive Power as an Ancillary Service

As the electric supply industry moves towards full competition, various services previously provided by electric utilities are unbundled. Much attention paid to the developments around the independent system operator (ISO) has focused on the structure of markets for energy and power transmission, and for AS, which is becoming more crucial. AS is generally referred to as services other than those providing energy that are essential for ensuring the reliable operation of the electrical

grid [24]. The reliable operation of a power system requires generation reserves, of both real and reactive power, to be available to cover system contingencies. As restructuring evolves, determining the cost of AS supply and finding out how these costs would change with respect to operating decisions is becoming a major issue [25].

The US Federal Energy Regulatory Commission (FERC) has recognized the importance of voltage control by including it as an AS in order 888, *Reactive Supply and Voltage Support from Generation Sources* [26, 27]. FERC and the industry differentiate generation-related activities from transmission-related activities, with the latter usually addressed under the basic transmission tariff. Reactive power requirements and compensation for generation-related resources are subject to FERC's voltage control service [7]. This means that only reactive power provided by generators can be marketed in the AS market. The range of physical requirements such as speed of response, contingency reserve etc., still exists. This also means that the real-power production of generators that are required to supply excessive amounts of reactive power must be curtailed. This is accounted for under opportunity cost whereby it is difficult to unambiguously separate reactive from real power costs.

As with most AS, the need for resources capable of voltage control stems from an overall system demand, and there must be also a central function directly controlling those resources to meet the requirements in real time [7]. Suppliers of reactive power resources are not capable of independently determining the system's voltage-control needs.

To ensure reliability by supplying sufficient reactive power, two mechanisms are employed. One is AS-market-driven. The other mandates that grid-connected generators supply reactive power, e.g., in California who has no generation resources of its own. The California ISO approach is to establish long-term contracts for the refund of the embedded costs of these units and ISO shall compensate generators who operate outside the pre-specified range of power factors [7].

Generators, synchronous condensers, SVCs, and STATCOMs all provide fast, continuously controllable voltage control. Load-tap changer transformers provide continuous voltage control. However, as the transformer alters reactive power from one bus to another, the control gained at one bus is at the expense of the other. In other words, though load-tap changer transformers are common tools of voltage-control, they do not really supply reactive power. Hence, the scheme of remuneration for transformer tap changer providers should be different from that for other providers who supply real reactive power to the system. An unfortunate characteristic of capacitors and capacitor-based SVCs is that output drops dramatically when voltage is low and support is needed most. STATCOMs provide more support under under-voltage conditions than capacitors or SVCs. Voltage control characteristics favor the use of generators and synchronous condensers. Cost, on the other hand, favors capacitors.

Generators are sometimes turned on to provide voltage support even though their energy costs are higher than the current spot price. Appropriately deciding how to compensate these “must-run” units (to provide reactive power) can be difficult.

The introduction of advanced-technology devices, such as STATCOMs and SVCs, further complicates the split between transmission- and generation-based voltage controls. The fast response of these devices often allows them to substitute the generation-based voltage control. But their high capital costs limit their use. A competitive voltage-control market would encourage effective investment [7].

In areas with high concentrations of generation, the operation of a competitive market requires sufficient interaction among generators. In other locations, the installation of a small amount of controllable reactive devices in the transmission system might enable to provide the bulk of the reactive support market. Unfortunately, in these areas, particularly lowly meshed areas, existing generation facilities would be able to exercise market power and thus still require economic regulations for this service. Longer-term contracts may help to mitigate generator market power by encouraging investment in lower-cost transmission-based alternatives. In general, regulators should encourage market provision of reactive power rather than regulate the provision of the service. To achieve this end, regulators should encourage utilities and ISOs to analyze the areas within each region where competitive vs. regulated voltage-control situations exist [7].

Here confined to the domain of reactive power, any participants' devices would either absorb or inject reactive power to the grid. Some devices are passive, some are active. We cannot conclude that the devices absorbing reactive power should be always charged, like in the case of the owner of a shunt reactor. The concept of remuneration and cost charging should incline to that the active device owners get remunerated for their operation for reactive power compensation (either absorbing or injecting reactive power) to the grid and the passive element owners must be charged as reactive consumers (again, either absorbing or injecting reactive power to the grid). Following this concept, the split of generation-related and transmission-related devices becomes blurred. The controllable FACTS devices can regulate the system voltage, as generators can, and should be allowed to enter the reactive power market.

AS costs, like for maintaining frequency and voltage, are not straightforward to quantify, only market conditions will tell [19]. So generating companies must know the quality of their products to compete in the markets.

The next section introduces the production cost of reactive power.

6.4.2 General Reactive Power Production Cost

Again, the two components, fixed and variable costs, are considered in the calculation of reactive power production cost.

A. Fixed Cost of Generation-Based Reactive Power Production

The determination of the fixed cost component of reactive power production is ambiguous since it is difficult to divide the cost of real and reactive power produced by the facilities, and it is less straightforward to quantify which part of the generating equipment is producing reactive power. A review of cost-based [3] or fixed cost methods [28] for quantifying the portion of generation capital cost ($CC_{kVA-Gen}$) indicates that the costs which should be attributed to reactive power are:

(1) Incremental cost

$$CC_{kVA-Gen} = CC_{kVA-Gen} - CC_{kW-Gen} \quad (6.19)$$

where

$CC_{kVA-Gen}$ = Generator's capital cost for providing reactive power

$CC_{kVA-Gen}$ = Generator's capital cost

CC_{kW-Gen} = Generator's capital cost for providing real power

(2) Costs of synchronous condensers

$$CC_{kVA-Gen} = CC_{kVA-Sync} \times CAP_{kVA-Gen} / CAP_{kVA-Sync} \quad (6.20)$$

where

$CC_{kVA-Gen}$ = Generator's capital cost for providing reactive power

$CC_{kVA-Sync}$ = Synchronous condenser's capital cost

$CAP_{kVA-Gen}$ = Generator's reactive power capability

$CAP_{kVA-Sync}$ = Synchronous condenser's reactive power capability

(3) Embedded costs

$$CC_{kVA-Gen} = CC_{kVA-Gen} \times CAP_{kVA-Gen} / CAP_{kVA-Gen} \quad (6.21)$$

where

$CC_{kVA-Gen}$ = Generator's capital cost for providing reactive power

$CC_{kVA-Gen}$ = Generator's capital cost

$CAP_{kVA-Gen}$ = Generator's reactive power capability

$CAP_{kVA-Gen}$ = Generator's apparent power capability

(4) Costs based on the ratio of kW to kVA

$$CC_{kVA-Gen} = CC_{kVA-Gen} \times (1 - CAP_{kW-Gen} / CAP_{kVA-Gen}) \quad (6.22)$$

where

$CC_{kVA-Gen}$ = Generator's capital cost for providing reactive power

$CC_{kVA-Gen}$ = Generator's capital cost

CAP_{kW-Gen} = Generator's real power capability

$CAP_{kVA-Gen}$ = Generator's apparent power capability

- (5) Cooperative game theory “allocates to each product at least the separable portion of the total cost for producing each product and at most the alternative cost for producing the individual product alone”

$$CC_{kVA-Gen} - CC_{kW-Gen} \leq CC_{kVAr-Gen} \leq \frac{CC_{kVAr-Sync} \times CAP_{kVAr-Gen}}{CAP_{kVAr-Sync}} \quad (6.23)$$

where

$CC_{kVAr-Gen}$ = Generator's capital cost for providing reactive power

$CC_{kVA-Gen}$ = Generator's capital cost

CC_{kW-Gen} = Generator's capital cost for providing real power

$CC_{kVAr-Sync}$ = Synchronous condenser's capital cost

$CAP_{kVAr-Gen}$ = Generator's reactive power capability

$CAP_{kVAr-Sync}$ = Synchronous condenser's reactive power capability

- (6) Triangle method

$$CC_{kVAr-Gen} = CC_{kVA-Gen} \times \sqrt{CAP_{kVA-Gen}^2 - CAP_{kW-Gen}^2} / CAP_{kVA-Gen} \quad (6.24)$$

where

$CC_{kVAr-Gen}$ = Generator's capital cost for providing reactive power

$CC_{kVA-Gen}$ = Generator's capital cost

$CAP_{kVA-Gen}$ = Generator's apparent power capability

CAP_{kW-Gen} = Generator's real power capability

B. Variable Cost of Generation-Based Reactive Power Production

Two determinable components of a generator's variable cost of providing reactive power are cost of energy loss and cost of loss of opportunity to generate real power. The models of loss of opportunity expressed in 6.3.1.C are rectified in the model below as shown in Fig. 6.27 and (6.25).

$$RL = \mu(P_A - P_B) - [C(P_A) - C(P_B)] \quad (6.25)$$

where

RL = Revenue loss due to production of reactive power

μ = Real power price.

$C(P_A)$ = Real power production cost, at P_A .

The equation (6.25) expresses that the cost of loss of opportunity will take effect only when the generator is forced to reduce real power production in order to generate more reactive power, along (P_A, Q_A) to (P_B, Q_B) . The cost of loss of opportunity shall be considered a reduction in profit (revenue minus cost) of real power production referring to its sales value (price) and its cost of real power production.

It is worth considering that the cost of fuel spent on real power production is also a kind of loss of opportunity: the opportunity to sell the fuel in the fuel market.

The variable cost of energy loss by producing reactive power is incremental corresponding to the higher current flow in stator windings as well as more magnetization in field windings. These losses need to be compensated by a slightly increased power from the prime mover.

C. Generators' Expected Payment Function

One of the generators' reactive power bidding frameworks is based on the expected payment function (EPF). The EPF derived from the generator capability curve as shown in Figs. 6.8 and 6.27 is applied as in Fig. 6.28. The obtained result is shown in Fig. 6.29. Q_{base} is the reactive power required by the generator for its auxiliary equipment as well as any obligatory requirement by the ISO. If the operating point lies inside the limiting curves, let's say at (P_A, Q_{base}) , then the unit can increase its reactive generation from Q_{base} up to Q_A without requiring a readjustment of P_A . This will, however, result in increased losses in the windings and, hence, increase the *cost of loss*.

If the generator operates at the limiting curve, any increase in reactive power (Q) will require a decrease in real power (P) to adhere to the winding heating limits. Consider the operating point "A" on the curve defined by (P_A, Q_A) . If more reactive power is required from the unit, say Q_B , the operating point requires shifting back along the curve to point B (P_B, Q_B) , where $P_B < P_A$. This signifies

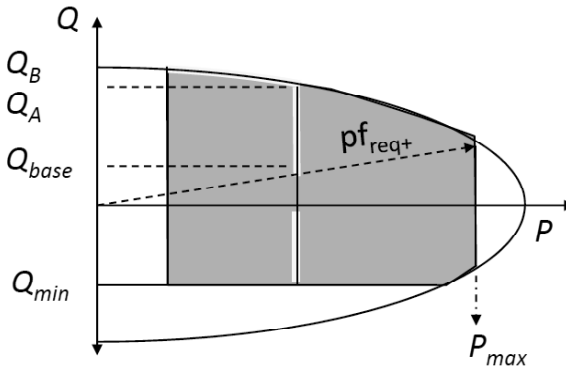


Fig. 6.27 Synchronous generator capability curve.

Color image of this figure appears in the color plate section at the end of the book.

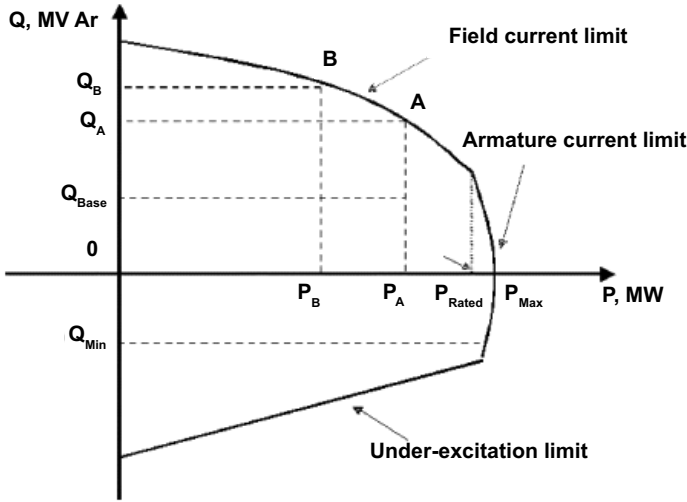


Fig. 6.28 Synchronous generator capability curve (source from [22]).

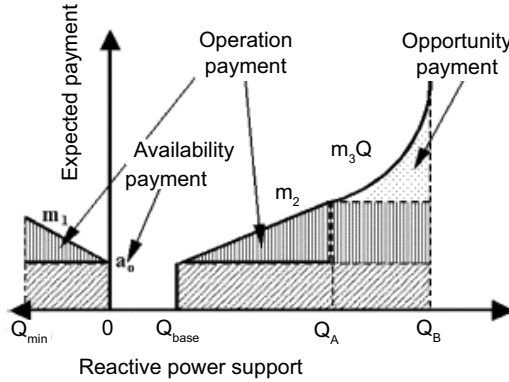


Fig. 6.29 Expected payment function for reactive power generation (source from [23]).

that the unit has to reduce its real power production to adhere to operating limits when higher reactive power is demanded. The loss in revenue from the generator due to reduced production of real power is termed *lost opportunity cost* and is a significant issue.

The full EPF model can be described by four components of a price offer, a_0 , m_1 , m_2 , and m_3Q , constituting a generalized expected payment function (EPF) as shown in (6.26).

$$EPF = a_0 + \int_{Q_{\min}}^0 m_1 \times dQ + \int_{Q_{\text{base}}}^{Q_A} m_2 \times dQ + \int_{Q_A}^{Q_B} (m_3 \times Q) dQ \quad (6.26)$$

where

a_0 , Availability price offer, in \$/hr

m_1 Operating price offer for operating in under-excitation mode, $Q_{min} \leq Q \leq 0$, in \$/MVarh

m_2 , Operating price offer for operating in the region of $Q_{base} \leq Q \leq Q_A$, in \$/MVarh

$m_3 Q$, Opportunity price offer for operating in the region of $Q_A \leq Q \leq Q_B$, in \$/MVarh.

Note that the opportunity offer is a function of reactive power output. Hence, the corresponding EPF component will be a quadratic function of Q .

The EPF model is a comprehensive model reflecting various considered costs incurred as stated in earlier sections. The a_0 represents the embedded cost of capital of providing reactive power (see section 6.4.2.A), m_1 and m_2 represent the costs of stator winding losses, and m_3 represents the loss of opportunity (see section 6.4.2.B).

D. Transmission-based Reactive Power Production Cost

A possible determination of short term variable cost of transmission-based reactive power production devices is by cost of energy losses due to current flow through the equipment although this component is very small and mostly negligible.

With the EPF representation as stated in section 6.4.2.C, the cost of reactive power production by transmission-based devices may heavily tend toward the availability offer (a_0) leaving other components diminished. But in a deregulated electricity market environment, transmission-based reactive devices owned by the private sector may manipulate the EPF parameters for competition in the reactive power market subject to the sector's own business risks.

6.4.3 Reactive Power Market Settlement and Pricing

ORPD in the deregulated electricity market can be derived from the optimization problem of cost minimization. In some studies, ORPD is carried out in a separate reactive power market [22, 23]. Here, the cost function refers to the total cost of procuring reactive power, hence their objective function includes only the cost of reactive power acquisition ($Cost_Q$). Experience shows that in conventional ORPD, reactive power dispatch considerably affects system losses and cost of losses. Hence, for the completion of the optimization function handling the reactive power market, it is appropriate to include the cost of real power loss ($Cost_{PLoss}$) together with the reactive power acquisition from the reactive power market ($Cost_Q$) as shown in (6.27).

$$\text{Obj. Min. Payment} = Cost_{PLoss} + Cost_Q \quad (6.27)$$

where

$Cost_{PLoss}$ = Cost of real power loss.

$$= Price_{pool} \times P_{Loss}$$

$Cost_Q$ = Cost of reactive power acquisition derived from the market. The cost structure is subject to market design. One worth to be mentioned here was proposed together with the EPF representation, stated in 6.4.2.C, where the $Cost_Q$ is defined as shown in (6.28).

$$Cost_Q = \sum_{gen} (\rho_0 W_{0,gen} - \rho_1 W_{1,gen} \times Q_{1,gen}) + \sum_{gen} (\rho_2 W_{2,gen} \times Q_{2,gen} - \rho_2 W_{3,gen} \times Q_{A,gen} + \frac{1}{2} \rho_3 \times Q_{3,gen}^2) \quad (6.28)$$

ρ_0 uniform market price of availability offer, a_0 .

ρ_1 uniform market price of operating offer, leading, m_1 .

ρ_2 uniform market zonal price of operating offer, lagging, m_2 .

ρ_3 uniform market zonal price of opportunity offer, m_3 .

$W_{1,gen}$ binary index, equal to 1 when $Q_{min} < Q_{gen} < 0$.

$W_{2,gen}$ binary index, equal to 1 when $Q_{base} < Q_{gen} < Q_A$.

$W_{3,gen}$ binary index, equal to 1 when $Q_A < Q_{gen} < Q_B$.

$$W_{0,gen} = W_{1,gen} + W_{2,gen} + W_{3,gen}.$$

In general, the problem formulation can be expressed as:

Objective:	Loss and reactive production cost minimization
Subject to:	Power flow equations and limits
Control variables:	Generator's reactive power production

The composition of loss and reactive power cost components in an objective function creates a trade-off of resources of reactive power (supply) and real power (loss), which can be illustrated as below.

As a concrete example, again the system given in section 6.3.4, adapting Fig. 6.5 with regard to the line impedance which here also includes resistance and reactance of 0.0017583 and 0.02286 p.u. respectively, with $|E| = 1.0$ p.u., load $P=10.0$ p.u., and load $Q = 0.0$ p.u., and especially with regard to the reactive power source which is connected to bus 2 (Q_2). Furthermore the prices of real and reactive power (of only Q_2) are 1 \$/h/MW p.u. and 0.01 \$/h/MVAr p.u. respectively.

Before we go to the problem formulation, it is worth noting that in the optimal dispatch derived from section 6.3.4, total cost of real power loss and reactive power production on bus 2 is 21.1822 \$/hr.

The problem of ORPD under deregulated electricity market conditions can be formulated as:

$$\text{Objective:} \quad \text{Min } 1 \times P_{\text{Loss}} + 0.01 \times Q_2 \quad (6.29)$$

$$\text{Subject to:} \quad \text{Power flow equations, and } 0.9 \leq V \leq 1.1$$

$$\text{Control Variable: } V$$

$$\text{where} \quad P_{\text{Loss}} = \frac{R}{R^2 + X^2} [1 + |V|^2 - 2|V| \cos(\delta)] \quad (6.30)$$

The results obtained are $|V| = 0.959$, $P_{\text{loss}} = 19.2352$ MW, Q_2 is 78.0893 MVar and total cost is 20.0161 \$/hr. We here can see the differences between the optimal results in a conventional market as detailed in section 6.3.4, and those in a deregulated market as explained in this section. Real power loss may not be minimized under market-based ORPD, since the conventional ORPD pay no cost for any resources in producing reactive power. As long as that cost of reactive power production is relevant, the minimization of total cost including both system real power loss and reactive power production could be the better form of optimization problem modelling.

6.5 TVAC-PSO based Optimal Reactive Power Dispatch under Deregulated Electricity Market Conditions

The ORPD under the conditions of a deregulated electricity market can be exemplarily carried out using the time-varying acceleration coefficients—particle swarm optimization (TVAC-PSO) [29] as shown in this section. The prescript TVAC of this PSO method indicates its particle updating technique.

6.5.1 Problem Formulation

The ORPD problem of minimizing reactive power supply cost subject to power balance constraints, generator operating constraints and network constraints is formulated as shown in section 6.4.3.

6.5.2 Particle Swam Optimization

Particle swam optimization (PSO) is a search algorithm that can solve nonlinear optimization problems without any restriction on the shape of the objective function. PSO was first introduced by Kennedy and Eberhard [30] in 1995. It is widely used in various power system optimization problems. It has been successfully implemented for economic load dispatch (ELD) [31], strategic bidding [32, 33], trading strategy [34], and congestion management [35].

The PSO algorithm is an iterative multiple-search optimization. It includes representation, initialization, fitness function, particle updating, and best particle updating. Each component are described in the following sub-sections A–E.

The procedure of PSO applied to ORPD is shown in Fig. 6.30.

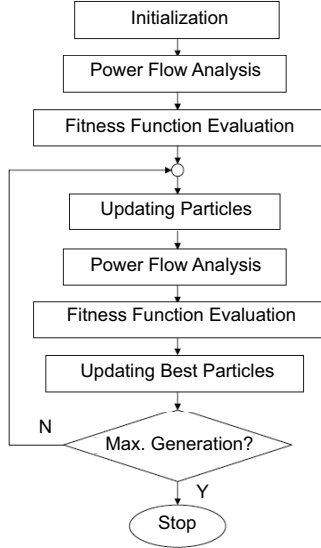


Fig. 6.30 Procedure of PSO for optimal power dispatch (source from [29]).

A. Representation

In a problem with N decision variables, a particle X is defined as a set of decision variables, $X = (x_1, x_2, x_3, \dots, x_N)$. With a searching size of d , searching particles are denoted $X_1, X_2, X_3, \dots, X_d$. A particle i in the previous iteration or generation k , denoted X_i^k shall be updated by a leap of velocity V_i^{k+1} resulting in an updated particle X_i^{k+1} . The updating process is detailed below.

B. Initialization

Unless there is a particular strategy for initializing a particle, each variable in a particle can be initialized with any value within the variable's range of limit. The initial particle i is denoted as X_i^0 . The initial velocity V_i^0 can be pre-defined as zero.

C. Fitness Function

The fitness function is usually defined as the objective function of a problem plus a penalty function representing constraints of the problem.

D. Updating Best Particles

For convenient reading, the explanation of the process of updating of best particles shall be preceded by that of updating particles in general. At any current iteration or generation k , two types of best particles are determined: the local best particle i ($P_{best,i}^k$), defined as:

$$P_{best,i}^k = best(X_i^1, X_i^2, X_i^3, \dots, X_i^k) \quad (6.31)$$

and the global best particle (G_{best}^k), defined as:

$$G_{best,i}^k = best(P_{best,1}^k, P_{best,2}^k, P_{best,3}^k, \dots, P_{best,d}^k) \quad (6.32)$$

E. Updating Particles

The updating equations of TVAC-PSO are expressed as (6.33).

$$X_i^{k+1} = X_i^k + V_i^{k+1} \quad (6.33)$$

where

$$V_i^{k+1} = \omega^k \times V_i^k + a_1^k \times rand_1(P_{best,i}^k - X_i^k) + a_2^k \times rand_2(G_{best}^k - X_i^k) \quad (6.34)$$

$$\omega^k = \omega_{\max} - \frac{\omega_{\max} - \omega_{\min}}{k_{\max}} \times k \quad (6.35)$$

$$a_i^k = a_{i,\max} - \frac{a_{i,\max} - a_{i,\min}}{k_{\max}} \times k \quad ; i = 1, 2 \quad (6.36)$$

and $\omega_{\max}$, $\omega_{\min}$, $a_{1,\max}$, $a_{1,\min}$, $a_{2,\max}$ and $a_{2,\min}$ are PSO parameters; $k_{\max}$ is the pre-defined maximum number of iterations or generations, and $rand_1$ and $rand_2$ are random numbers.

The values of these PSO parameters are very much depending on the nature of the problem and PSO may need several iterations to converge if these parameters are not appropriately chosen. According to [29], those parameters are best set as shown in Table 6.1.

Table 6.1 PSO Parameters from [29].

Velocity Vector	$k = 1$	$k = k_{\max}$
Inertia weight	$\omega_{\min} = 0.9$	$\omega_{\max} = 0.4$
Toward local best	$a_{1,\min} = 2$	$a_{1,\max} = 0.4$
Toward global best	$a_{2,\min} = 0.4$	$a_{2,\max} = 2$

Equations (6.33)–(6.34) state that particles (solution candidates) shall be updated by velocity, as a result of three vectors, i.e., previous iteration velocity, vector toward local best particle, and vector toward global best particle. The vectors are named as presented in the first column of Table 6.1. Equations (6.34)–(6.36) identify the particle updating technique for TVAC. With the PSO parameters shown in Table 6.1 and iterations or generations progressing along with k progressing from 1 to k_{max} , inertia weight and vector toward local best particles will decline whereas the vector toward global best particle will increase. The PSO using TVAC shall perform a wide-area search in the beginning of the process and do a fine search around the global best particle.

6.5.3 Numerical Performance of PSO

The ORPD setup is tested on a modified IEEE 30-bus system with 41 lines and 6 generators. System load is 189.20 MW and 107.20 MVar. Since it employs a heuristic search, the PSO solution obtained is inconsistent. Observing its performance may lead to characteristics like solution statistics and convergence. Additionally, it is appropriate to compare its performance with that of some other heuristic methods, e.g., evolutionary programming (EP) which is also explained in this section. To this end, forty runs are conducted to compare TVAC-PSO and EP performance in this problem. Table 6.2 presents statistics of the final solutions obtained from both methods. As evident from Table 6.2, TVAC-PSO could provide better solutions in all aspects, i.e., lower minimum cost, lower average cost and less deviation. Figure 6.31 shows convergence characteristics of both methods. It is clear that TVAC-PSO converges faster. In some research studies, computation time is also used as a criteria of performance.

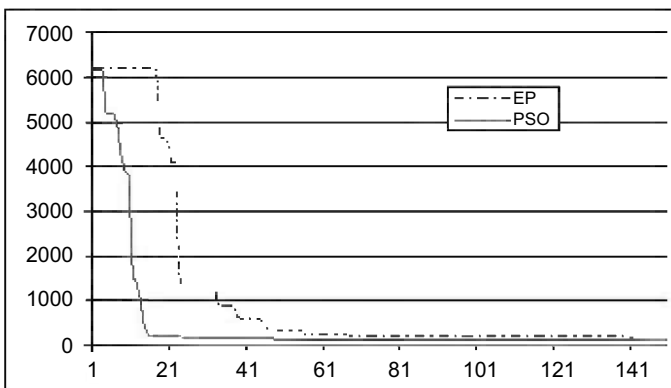


Fig. 6.31 Convergence characteristics (best solutions): TVAC-PSO vs. EP (Source from [29]).

Color image of this figure appears in the color plate section at the end of the book.

Table 6.2 Final $Cost_Q$ Solutions after 40 Runs.

	Min. $QCost$	Max. $QCost$	Avg. $QCost$	S.D.
TVAC-PSO	125.6251	167.1084	138.0197	13.2220
EP	130.3999	188.3945	151.1722	19.2345

6.6 SUMMARY

This chapter reviewed the concept and characteristics of reactive power and presented methods to formulate the optimal reactive power dispatch (ORPD) problem. Reactive power is essential for direct reactive load as well as for power transaction across the network. In the domain of reactive power, elements in the power system are considered either passive or active, regardless if they inject or absorb reactive power. Passive reactive power elements are regarded as users. Active elements providing reactive power, controllable by the system operator, are regarded as reactive resource that can be optimally utilized for ORPD. Conventional ORPD is usually employed for real power loss minimization. Such conventional models may overlook the cost of reactive power production that becomes relevant in deregulated electricity markets. ORPD under this new paradigm can be extended considering the cost of reactive power production. This includes a trade-off between costs of reactive power production and real power loss. Artificial intelligence based optimization, e.g., particle swarm optimization (PSO) using time-varying acceleration coefficients (TVAC), is exemplarily employed to solve the ORPD problem under the conditions of a deregulated electricity market.

6.7 PROBLEMS

- 6.7.1 Given a single transmission line with a line reactance of $0.02 \Omega/\text{km}$ and a line charging of $0.15 \Omega/\text{km}$: Find its surge impedance load.
- 6.7.2 A given transmission line is represented only by the reactance of 0.07 p.u. (Fig. 6.5). Assume that the load has a unity power factor and a voltage magnitude at the sending bus end of 1.0 p.u. 1) Find the real power flow limit corresponding to the constraint of voltage at the receiving bus end of not less than 0.9 p.u. 2) Find the real power flow limit corresponding to the constraint of steady state voltage stability limit with $\partial P/\partial V = 0$.
- 6.7.3 Carrying the result from 6.7.2, find the amount of reactive power compensated at bus 2 to maintain a receiving end voltage of 0.95 .
- 6.7.4 Given a synchronous generator with ratings of the prime mover limit = 230 MW and the stator winding limit = 270 MVA : 1) Find the power factor values when it is operating at 215 MW and 100 MVA . 2) From 1), how much can the generator increase its reactive power production without hitting the

limit? 3) From 2), if the generator is needed to operate at 200 MVar, how much does it need to reduce real power generation?

- 6.7.5 Given a shunt capacitor with a susceptance of 0.5 p.u.: How much reactive power can it produce when its terminal voltage is 1.0, Given that the base power is 100 MVA?

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AVAILABLE TRANSFER CAPABILITY*

7.1 INTRODUCTION

In 1992, the United States Congress developed the Energy Policy Act (EPAAct), which was interpreted by the Federal Energy Regulatory Commission (FERC) as a mandate to introduce open-access of transmission resources and generation competition to the electric power industry [1]. In 1995, the development of technical guidelines for transmission transfer capability definitions and determination was deferred from FERC to the North American Electric Reliability Council (NERC), an industry group that develops reliability standards and guides for the planning and operation of electric power systems.

NERC revised the earlier reference documents on transfer capability published in 1980 to provide additional clarifications and examples. The revised document [2] addresses transmission transfer capability from the perspective of the transmission systems physical characteristics and limitations. It provides the technical basis for discussions about transfer capability. Background information on industry practices related to transfer capability is also presented including definitions, concepts, technical issues, and simulation techniques used to calculate and report transmission transfer capability. The report also recommends two transfer capability measures: First Contingency Incremental Transfer Capability (FCITC) and First Contingency Total Transfer Capability (FCTTC). The FCITC is defined as the amount of electric power, incremental above normal base power transfers that can be transferred over the interconnected transmission systems in a reliable manner. The FCTTC is defined as the total amount of electric power (normal base power transfers plus FCITC) that can be transferred between two areas of the interconnected transmission systems.

In April 1996, FERC issued Order No. 888 [3] and Order No. 889 [4] to promote electric supply competition and accommodate customer demand for a choice in

*This chapter has been written with assistance from Peerapol Jirapong

the electric power industry. The Final Rule 888 was intended to promote utility competition through open-access of transmission resources. This rule required public utilities to functionally unbundle wholesale generation and transmission services. The Final Rule 889 mandated the calculation of available transfer capability (ATC) for each control area and the posting of this value on a communications system called the open access same-time information system (OASIS). The purpose of this rule was to enhance the open-access of bulk transmission systems by providing a market signal of the capability of a transmission system to deliver energy. The associated rulings of FERC have added considerable emphasis to the interest in quantifying transmission transfer capability. This interest has led to new definitions and recommended methods for transfer capability determination. In June 1996, NERC published a technical report regarding ATC definitions to provide a uniform framework for determining ATC and related terms. The report also provided ATC principles under which ATC values would have to be calculated.

7.2 TRANSMISSION TRANSFER CAPABILITY CONCEPTS

This section provides the key basic concepts of transmission transfer capability. Other terms related to transfer capability can be explored in detail in [2] and [5].

7.2.1 Transfer Capability versus Transmission Capacity

Electric power systems have used a common terminology to calculate and report transmission transfer limits to maintain the reliability of the interconnected transmission networks. These transfer values are called “capabilities” because they are highly dependent on the generation, customer demand, and transmission system conditions assumed during the time period analyzed. The electric industry generally uses the term “capacity” as a specific limit or rating of power system equipment. In transmission, capacity usually refers to the thermal limit or rating of a particular transmission element or component. The ability of a single transmission line to transfer electric power, when operated as part of the interconnected network, is a function of the physical relationship of that line to the other elements of the transmission network.

7.2.2 Transmission Transfer Capability

Transfer capability is the measure of the ability of interconnected electric systems to reliably move or transfer electric power from one area to another by way of all transmission lines or paths between those areas under specified system conditions. The unit of transfer capability is, in terms of electric power, generally expressed in megawatts (MW). In this context, area may refer to an individual electric system, power pool, control area, region, or a portion of any of these. Transfer capability

is also directional in nature. That is, the transfer capability from area A to area B is not generally equal to the transfer capability from area B to area A.

In both the planning and operation of electric systems, transfer capability is one of several performance measures used to assess the reliability of the interconnected transmission systems. System planners use transfer capability as a measure or indicator of transmission strength in assessing interconnected transmission system performance. It is often used to compare and evaluate alternative transmission system configurations. System operators use transfer capability to evaluate the real-time ability of the interconnected transmission systems to transfer electric power from one portion of the network to another or between control areas. In the operation of interconnected systems, transfer is synonymous with interchange.

The concept of transmission transfer capability may be explained in terms of a simplified interconnected systems network comprised of three areas or systems, A, B, and C, interconnected by transmission paths A-B, A-C, and B-C, as shown in Fig. 7.1. Each area represents a configuration of generating stations, substations, and internally connected transmission lines. The transmission paths or interconnections from area A to area B, area A to area C, and area B to area C may each represent one or more transmission lines. In this example, two transmission lines comprise each transmission path.

The determination of transfer capability from area A to area B is achieved using computer simulations of the interconnected systems network of Fig. 7.1. To simulate an electric power transfer, area A and area B generation (and/or electrical demand) is adjusted so as to create a generation excess in area A and a generation deficiency in area B, thereby automatically resulting in an electric power transfer from area A to area B. These differential adjustments in each area's generation

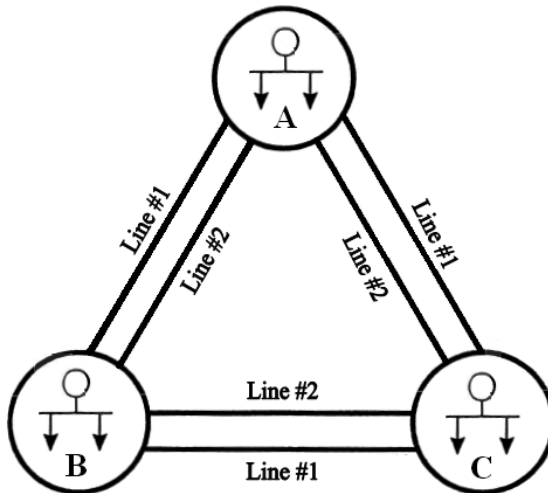


Fig. 7.1 Simplified interconnected systems network.

level are increased until an equipment or system limit is reached, or a transfer test level is achieved, taking into account the most critical single contingency (e.g., generating unit, transformer, transmission line, etc.) outage condition. In those cases where an equipment or system limit is reached with all facilities in service at a transfer level below that of the single contingency outage condition, then that lower transfer level defines the transfer capability limit.

To determine the transfer capability in the opposite direction, from area B to area A, the generation excess is created in area B and the generation deficiency in area A. As customer demands and transmission and generation facilities in areas A and B will rarely be symmetrical, and as the critical facility outage condition will likely be different, the transfer capability in each direction, area A to area B or area B to area A, will also generally be different, and must be determined separately.

As the generation levels in areas A and B are modified to increase the electric power transfer from area A to area B, the loading level on transmission path A-B, as well as on all other interconnection and internal transmission facilities, will change but at different rates. These different rates, called power transfer distribution factors, are determined according to the physical laws of electrical networks. Thus, all transmission paths will not simultaneously reach their capability limits at the same transfer level. However, the area A to area B transfer level at which a transmission path, system voltage, or system stability limit is reached for a single facility outage becomes the limiting transfer capability level for transfers from area A to area B. In the interconnected systems, it is possible that the critical single contingency facility outage and the associated limiting facility may not be in areas A or B, or at the interface (transmission paths) between areas A or B altogether, but in another area (or areas), such as area C.

The capabilities or ratings of the interconnecting transmission lines, lines A-B#1 and A-B#2, between areas A and B cannot be added to derive the transfer capability from area A to area B or from area B to area A. In addition, the sum of the non-simultaneous transfer capabilities from area A to area B and from area C to area B does not equal the total transfer capability to area B. Simultaneous transfer capability calculations from areas A and C to area B are required to determine that value.

When transfer capabilities between areas or systems are determined, it must be understood that these capabilities correspond to a specific set of system conditions for the interconnected systems network. The transfer capabilities can be significantly different for any other set of system conditions, such as a different customer demand level, a different network configuration, or a different generation dispatch. Also influencing the level of transfer capability between areas A and B are any electric power transfers under way between other neighboring systems, such as transfers from area A to area C or from area B to area C.

7.2.3 Limits to Transfer Capability

The ability of interconnected transmission networks to reliably transfer electric power may be limited by the physical and electrical characteristics of the systems including any one or more of the following:

1. **Thermal Limits:** Thermal limits establish the maximum amount of electrical current that a transmission line or electrical facility can conduct over a specified time period before it sustains permanent damage by overheating or before it violates public safety requirements.
2. **Voltage Limits:** System voltages and changes to them must be maintained within the range of acceptable minimum and maximum limits. For example, minimum voltage limits can establish the maximum amount of electric power that can be transferred without causing damage to the electric system or customer facilities. A widespread collapse of system voltage can result in a blackout of portions or the entire interconnected network.
3. **Stability Limits:** The transmission network must be capable of surviving disturbances through the transient and dynamic time periods (from milliseconds to several minutes, respectively) following a disturbance. All generators connected to AC interconnected transmission systems operate in synchronism with each other at the same frequency. Immediately following a system disturbance, generators begin to oscillate relative to each other, causing fluctuations in system frequency, line loadings and system voltages. For the system to be stable, these oscillations must diminish as the electric systems attain a new, stable operating point. If a new, stable operating point is not quickly established, the generators will likely lose synchronism with one another, and all or a portion of the interconnected electric systems may become unstable.

The limiting condition on some portions of the transmission network can shift among thermal, voltage, and stability limits as the network operating conditions change over time. Such variations further complicate the determination of transfer capability limits.

7.3 AVAILABLE TRANSFER CAPABILITY PRINCIPLES

ATC is used as a market signal of the capability of the transmission network to deliver electric energy in deregulated power systems. It is required to be calculated for each control area and posted on a public communication system called OASIS to enhance the open access transmission network and make competition reasonable and effective [3], [4].

As a measure bridging the technical characteristics of how interconnected transmission networks perform to the commercial requirements associated with transmission service requests, ATC must satisfy certain principles balancing both

technical and commercial issues. ATC must accurately reflect the physical realities of the transmission network, while not being so complicated that it unduly constrains commerce. The following principles identify the requirements for the calculation and application of ATC [5]:

1. ATC calculations must give a reasonable and dependable indication of transfer capabilities available to the electric power market to produce commercially viable results. The frequency and detail of individual ATC calculations must be consistent with the level of commercial activity and congestion.
2. ATC calculations must recognize time-variant power flow conditions on the entire interconnected transmission network. In addition, the effects of simultaneous transfers and parallel path flows throughout the network must be addressed from a reliability viewpoint. Regardless of the desire for commercial simplification, the laws of physics govern how the transmission network will react to customer demand and generation supply. Electrical demand and supply cannot, in general, be treated independently of one another. All system conditions, uses and limits must be considered to accurately assess the capabilities of the transmission network.
3. ATC calculations must recognize the dependency of ATC on the points of electric power injection, the directions of transfers across the interconnected transmission network, and the points of power extraction. All entities must provide sufficient information necessary for the calculation of ATC.
4. Regional or wide-area coordination is necessary to develop and post information that reasonably reflects the ATC of the interconnected transmission network so that ATC calculations capture the interactions of electric power flows among individual, subregional, regional, and multiregional systems.
5. ATC calculations must conform to NERC, regional, subregional, power pool, and individual system reliability planning and operating policies, criteria or guides. Appropriate system contingencies must be considered.
6. The determination of ATC must accommodate reasonable uncertainties in system conditions and provide operating flexibility to ensure the secure operation of the interconnected network.

7.4 AVAILABLE TRANSFER CAPABILITY DEFINITION AND DETERMINATION

7.4.1 Available Transfer Capability

A. Definition of Available Transfer Capability

ATC is a measure of the transfer capability remaining in the physical transmission network for further commercial activity over and above already committed uses. Mathematically, ATC is defined as the Total Transfer Capability (TTC) less the Transmission Reliability Margin (TRM), less the sum of existing transmission

commitments (which includes retail customer service) and the Capacity Benefit Margin (CBM). ATC can be expressed as:

$$\text{ATC} = \text{TTC} - \text{TRM} - \text{CBM} - \text{Existing Transmission Commitments}$$

The ATC between two areas provides an indication of the amount of additional electric power that can be transferred from one area to another for a specific time frame for a specific set of conditions. ATC can be a dynamic quantity because it is a function of variable and interdependent parameters. These parameters are highly dependent upon the conditions of the network. Consequently, ATC calculations may need to be periodically updated. Because of the influence of conditions throughout the network, the accuracy of the ATC calculation is highly dependent on the completeness and accuracy of available network data.

B. Determination of Available Transfer Capability

The calculation of ATC is generally based on computer simulations of the operation of the interconnected transmission network under a specific set of assumed operating conditions. These simulations are typically performed “offline,” well before the systems approach that operational state. Each simulation represents a single “snapshot” of the operation of the interconnected network based on the projections of many factors. As such, they are viewed as reasonable indicators of network performance.

The conditions on the interconnected network continuously vary in real time. Therefore, the transfer capability of the network will also vary from one instant to the next. For this reason, transfer capability calculations may need to be updated periodically for application in the operation of the network. In addition, depending on actual network conditions, transfer capabilities can often be higher or lower than those determined in the offline studies. The farther into the future those simulations are projected, the greater is the uncertainty in assumed conditions. However, transfer capabilities determined from simulation studies are generally viewed as reasonable indicators of actual network capability.

To more fully define ATC, specific commercial aspects of transmission service must be considered. To create reasonably consistent expectations regarding the transmission services that are being offered, the concepts of curtailability and recallability are introduced.

Curtailability is defined as the right of a transmission provider to interrupt all or part of a transmission service due to constraints that reduce the capability of the transmission network to provide that transmission service. Transmission service is to be curtailed only in cases where system reliability is threatened or emergency conditions exist.

Recallability is defined as the right of a transmission provider to interrupt all or part of a transmission service for any reason, including economic. Based on the recallability concept, two commercial applications of ATC are defined below and depicted graphically in Fig. 7.2. They are as follows:

Non-recallable Available Transfer Capability: Non-recallable ATC (NATC) is defined as TTC less TRM, less non-recallable reserved transmission service (including CBM). NATC has the highest priority use of the transmission network. The maximum amount of non-recallable service that can be reserved is determined based on what the network can reliably handle under normal operating conditions and during appropriate contingencies. Any lower priority service can be displaced by non-recallable service that is either new non-recallable service or non-recallable service that had been reserved but not scheduled.

Recallable Available Transfer Capability: Recallable ATC (RATC) is defined as TTC less TRM, less recallable transmission service, less non-recallable transmission service (including CBM). Portions of the TRM may be made available by the transmission provider for recallable use, depending on the time frame under consideration for granting additional transmission service. To the extent load serving entities reserve transmission transfer capability for CBM, portions of CBM may be made available for recallable use, depending on the time frame under consideration for granting additional transmission service.

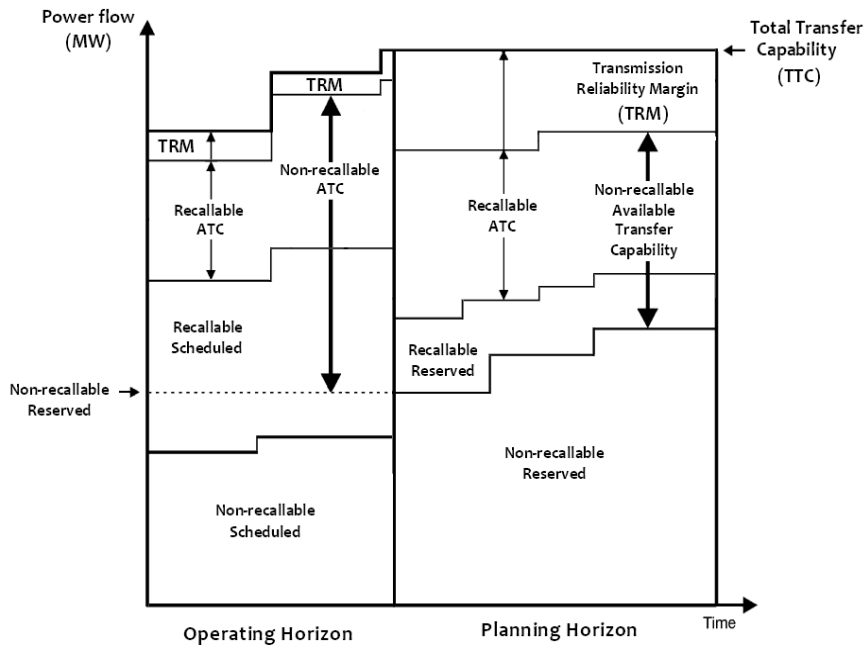


Fig. 7.2 ATC and related terms in the transmission reservation system.

7.4.2 Total Transfer Capability

A. Definition of Total Transfer Capability

TTC is the amount of electric power that can be transferred over the interconnected transmission network in a reliable manner based on all of the following conditions:

1. For the existing or planned system configuration and with normal (pre-contingency) operating procedures in effect, all facility loadings are within normal ratings and all voltages are within normal limits.
2. The electric systems are capable of absorbing the dynamic power swings, and remaining stable, following a disturbance that results in the loss of any single electric system element, such as a transmission line, transformer, or generating unit.
3. After the dynamic power swings subside following a disturbance that results in the loss of any single electric system element as described in 2 above, and after the operation of any automatic operating systems, but before any post-contingency operator-initiated system adjustments are implemented, all transmission facility loadings are within emergency ratings and all voltages are within emergency limits.
4. With reference to condition 1 above, in the case where pre-contingency facility loadings reach normal thermal ratings at a transfer level below that at which any first contingency transfer limits are reached, the transfer capability is defined as that transfer level at which such normal ratings are reached.
5. In some cases, individual system, power pool, subregional, or regional planning criteria or guides may require consideration of specified multiple contingencies, such as the outage of transmission circuits using common towers or rights-of-way, in the determination of transfer capability limits. If the resulting transfer limits for these multiple contingencies are more restrictive than the single contingency considerations described above, the more restrictive reliability criteria or guides must be observed.

B. Determination of Total Transfer Capability

The concepts for determining TTC are briefly outlined below.

System Conditions: Base system conditions are identified and modeled for the period being analyzed, including projected customer demands, generation dispatch, system configuration, and base scheduled transfers. As system conditions change, the base system conditions under which TTC is calculated may also need to be modified.

Critical Contingencies: During transfer capability studies, many generation and transmission system contingencies throughout the network are evaluated to

determine which facility outages are most restrictive to the transfer being analyzed. The types of contingencies evaluated are consistent with individual system, power pool, subregional, and Regional planning criteria or guides. The evaluation process should include a variety of system operating conditions because as those conditions vary, the most critical system contingencies and their resulting limiting system elements could also vary.

System Limits: As discussed earlier, the transfer capability of the transmission network may be limited by the physical and electrical characteristics of the systems including thermal, voltage, and stability considerations. Once the critical contingencies are identified, their impact on the network must be evaluated to determine the most restrictive of those limitations. As system operating conditions vary, the most restrictive limit on TTC may move from one facility or system limit to another as illustrated in Fig. 7.3. Therefore, the TTC becomes:

$$\text{TTC} = \text{Minimum of } \{ \text{Thermal Limit, Voltage Limit, Stability Limit} \}$$

Parallel Path Flows: When electric power is transferred across the network, parallel path flows occur. This complex electric transmission network phenomenon can affect all systems of an interconnected network, especially those systems electrically near the transacting systems. As a result, transfer capability determinations must be sufficient in scope to ensure that limits throughout the interconnected network are addressed. In some cases, the parallel path flows may result in transmission limitations in systems other than the transacting systems, which can limit the transfer capability between the two contracting areas.

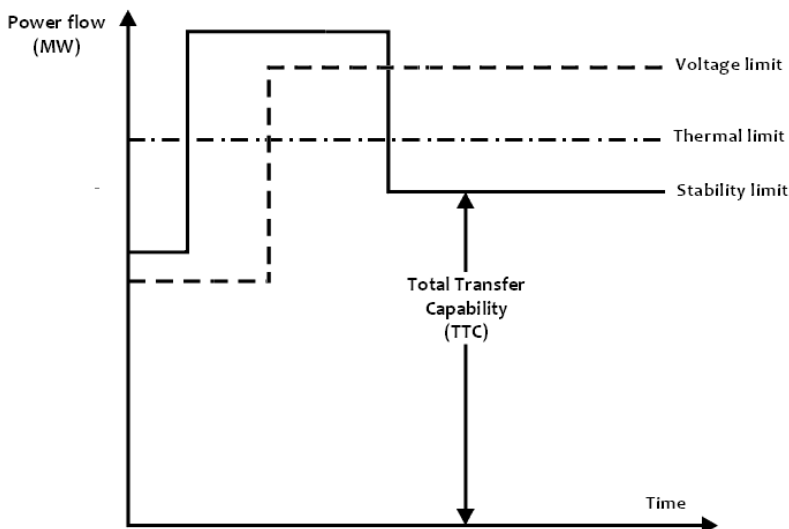


Fig. 7.3 Limits to total transfer capability.

Non-Simultaneous and Simultaneous Transfers: Transfer capability can be determined by simulating transfers from one area to another independently and non-concurrently with other area transfers. These capabilities are referred to as “non-simultaneous” transfers. Another type of transfer capability reflects simultaneous or multiple transfers concurrently. These capabilities are developed in a manner similar to that used for non-simultaneous capability, except that the interdependency of transfers among the other areas is taken into account. These interdependent capabilities are referred to as “simultaneous” transfers. No simple relationship exists between non-simultaneous and simultaneous transfer capabilities. The simultaneous transfer capability may be lower than the sum of the individual non-simultaneous transfer capabilities.

7.4.3 Transmission Reliability Margin

A. Definition of Transmission Reliability Margin

TRM is defined as the amount of transmission transfer capability necessary to provide a reasonable level of assurance that the interconnected transmission network will be secure. TRM accounts for the inherent uncertainty in system conditions and its associated effects on ATC calculations, and the need for operating flexibility to ensure reliable system operation as system conditions change. All transmission system users benefit from the preservation of TRM by transmission providers.

B. Determination of Transmission Reliability Margin

Typically, TRM is either calculated via a simple facility rating reduction (in percent of ratings) or a transfer capability quantity applied (in MW) at specific interfaces.

TRM applied by Rating Reduction: For systems in which the distribution of uncertainty among all of its facilities is relatively uniform, a TRM applied to all the transmission provider’s system facilities may be appropriate. In this case, the TRM is applied against the facility ratings themselves and is measured as a percentage reduction of facility ratings. The rating reduction is typically 2–5% and may increase over an extended time horizon. This determination is typically accomplished by a two-step method:

1. Determining TTC and ATC values using the full “customary” (normal or emergency ratings as appropriate) ratings (i.e., assuming that TRM is zero).
2. Determining ATC using facility ratings that are reduced from the “customary” ratings. The TRM (in terms of MW of transfer capability) is simply the algebraic difference between the ATC values determined using the “customary” ratings and the ATC values determined using reduced ratings.

TRM applied by Interface: In systems where uncertain contributions can be associated with specific interfaces or flow gates, a TRM applied to specific critical interfaces or flow gates may be appropriate. Systems that apply TRM in this manner typically would be able to quantify the uncertainty associated with TRM components through the use of historical transmission loading analysis. In this case, the TRM is applied against a particular facility or set of facilities and is measured as a megawatt reduction in transfer capability. The TRM applied in this manner is relatively constant but may change based on the actual experience.

Although the general methods to apply TRM differ in application and approach, they both serve to quantify a reasonable amount of transfer capability margin to provide the operating flexibility to ensure reliable system operation as system conditions change. However, the applications of TRM are related in that the amount of TRM is a factor of the limiting facility's response for the particular transfer.

7.4.4 Capacity Benefit Margin

A. Definition of Capacity Benefit Margin

CBM is defined as the amount of firm transmission transfer capability preserved for Load Serving Entities (LSEs) on the host transmission system where their load is located, to enable access to generation from interconnected systems to meet generation reliability requirements. Preservation of CBM for an LSE allows that entity to reduce its installed generating capacity below what may otherwise have been necessary without interconnections to meet its generation reliability requirements. The transmission capacity preserved as CBM is intended to be used by the LSE only in times of emergency generation deficiencies.

Unlike TRM, the direct beneficiaries of CBM can be identified. These beneficiaries are the LSEs that are network customers (including native load) of a host transmission provider. The benefit that LSEs receive from CBM is the sharing of installed capacity reserves elsewhere in the interconnection, which translates into a reduced need for installed generating capacity and ultimately, lower rates for customers.

B. Determination of Capacity Benefit Margin

The methodology used to derive CBM must be documented and consistent with published planning criteria. This is the case if the same components that comprise the CBM are also addressed in the planning criteria. The methodology used to determine and apply CBM does not have to involve the same mechanisms as the planning process, but the same uncertainties must be considered and any simplifying assumptions explained. It is recognized that ATC determinations are often constrained and thus will not permit the use of the same mechanisms employed

in a more rigorous planning process. The Generation Reserve Requirement can be determined via either deterministic or probabilistic methods.

Probabilistic Methodologies: Probabilistic calculation methods, such as loss of load probability, have inputs such as unit forced outages, maintenance outages, minimum downtimes, load forecasts etc. A typical benchmark is a generation reserve level to achieve a probabilistic loss of load expectation of 0.1 day per year.

Deterministic Methodologies: Deterministic methods typically are centered on maintaining a specified reserve or capacity margin, or may be based upon surviving the loss of the largest generating unit. Typical benchmarks for the determination methodology would be a multiple of the largest generation unit within the transmission provider's system.

Whether probabilistic or deterministic methods are used to determine the generation reserve requirement, the criteria applied must be consistently applied by the transmission provider to all LSEs. The determination of CBM for an LSE is a three-step process:

1. The amount of additional external generating capacity necessary to achieve a target reliability level (e.g., 0.1 day/year loss of load expectation) must be determined.
2. The total amount of transmission transfer capability necessary to import the external generating reserve requirement must be determined from the amount of required external generating capacity (less the TRM component for operating reserves).
3. This total amount of transmission transfer capability must be allocated to the specific transmission system interfaces or paths over which the imported power may flow.

These three steps can be accomplished either sequentially or simultaneously. Sequential determination often relies on deterministic rules. For example, the needed external generating capacity might be set at the capacity of the largest internal plant, the total CBM might be set at two times that amount, and the allocation among three interfaces might be set as 60/20/20%, based upon historical experience. Simultaneous determination can be accomplished with a probabilistic model, which includes both generation and transmission representation.

7.5 METHODOLOGIES TO CALCULATE ATC

NERC has described two methods for ATC calculation [5]. One is called the Network Response method, and the other is the Rated System Path (RSP) method. The Network Response method is used to calculate ATC in highly dense, meshed transmission networks where customer demand, generation sources, and transmission systems are tightly interconnected. In such networks, transmission paths that are critical to a certain power transfer cannot generally be identified in

advance because of the high dependency on system conditions that would exist at the time that power transfer is scheduled. This method is popular and applicable to many transmission networks. The Rated System Path method is used for transmission systems which are described as sparse networks with customer demand and generation centers distant from one another.

7.5.1 The Network Response Method

To calculate ATC using the Network Response method, for a particular transaction area pair at a single instance of time and under specific system conditions, computer simulations are needed to find the impact (response) of a power transaction, from one area to another, on the entire transmission network flows. The network responses are obtained based on load-flow studies of the interconnected transmission network. An initial load-flow analysis is used to establish the base-case power flow in the network. Then the generation in the power-selling area and the load in the power-buying area are increased by some specified amount to simulate a transfer of power, and then another load-flow study is carried out. The incremental flow on each transmission network component is calculated based on the two load-flow cases. The difference is then normalized according to the incremental generation (load) in the second case. This quantity is called the network response of a transmission line or network facility and denoted by R_{ij}^ℓ as in (7.1).

$$R_{ij}^\ell = \frac{(f_m^\ell - f_b^\ell)}{P_{ij}} \quad (7.1)$$

where

ij buying and selling area pair (from area i to area j),

ℓ transmission component,

f_m^ℓ power flow in ℓ for the modified case,

f_b^ℓ power flow in ℓ for the base case, and

P_{ij} amount of simulated power transfer

The Available Loading Capacity: ALC_{ij}^ℓ (i.e., unused capacity of each transmission component) is found by subtracting the base-case power flows from the rated value of the transmission component. The ALC_{ij}^ℓ is given in (7.2).

$$ALF_{ij}^\ell = C^\ell - f_b^\ell \quad (7.2)$$

where C^ℓ is the thermal limit of the transmission component ℓ .

The ALC_{ij}^ℓ for each component ℓ in the transmission network, based on the assumption that the transmission network is linear, is calculated in (7.3).

$$ALC_{ij}^{\ell} = \frac{ALC_{ij}^{\ell}}{R_{ij}^{\ell}} \quad (7.3)$$

The ATC value for the area pair, ATC_{ij} , in (7.4) is established as the ALC_{ij}^{ℓ} of the most constraining transmission component.

$$ATC_{ij} = \min\{ATC_{ij}^{\ell}\} \quad (7.4)$$

This is the value used in the calculation of ATC. To evaluate the ATC for a contingency case (single or multiple) or when the system configuration is modified, the procedure is repeated for the altered system to find the ATC.

7.5.2 The Rated System Path Method

This method has been suggested by NERC as an option to allocate transmission services. In this method, paths between areas of the network are identified, appropriate system constraints are calculated, and the ATC is computed for these identified paths and interconnections between transmission providers. The RSP method involves three steps:

1. Determining the path's TTC,
2. Allocating the TTC among owners in a multi-owned path to calculate the owner's rights, and
3. Calculating the ATC for each right-holder by subtracting each of their uses from each of their individual TTC rights.

The RSP method includes a procedure to allocate TTC and ATC among owners of the transmission path(s), where the TTC development follows a regional review process in order to achieve wide-area coordination. The RSP method uses the modeling of realistic customer demands and generation patterns in advance of real-time operations to take into consideration the impacts of parallel path flows (or unscheduled flows) on interconnected systems. The method also uses a maximum power flow test to guarantee that the transfer path is capable of carrying power flows up to its rated transfer capability or TTC.

The procedure for calculating ATC in the RSP method is as follows:

1. Paths for which ATCs are to be calculated are identified, then a reliability-based TTC (reliability rating) is calculated for each path, and TTC is allocated among the owners based on the negotiated agreement. To calculate a reliability-based TTC of each path, based on consistent path rating methods agreed upon, and procedures within the Interconnection, the network is tested under a wide range of generation, customer demand, and outage conditions. The TTC rating

usually remains fairly constant, but when the system configuration changes, the TTC rating will be recalculated.

2. If necessary, based on prearranged agreements or tariffs, deratings for outages, maintenance schedules or unscheduled flows are allocated to the right-holders.
3. To calculate the ATC value, right-holders will take their respective allocated shares of TTC for a path and subtract the existing commitments.

When new commitments affect their ATC, right-holders update and re-post their ATC calculations. A transfer from one area to another involving several transmission owners requires locating and reserving capacities across multiple paths and potentially multiple right-holders.

7.6 AVAILABLE TRANSFER CAPABILITY CALCULATION

ATC calculations are generally based on computer simulations of the operation of interconnected transmission networks under a specific set of assumed operating conditions. Wide varieties of mathematical methods have been developed for calculating ATC, which can be summarized into four types as follows:

1. Linear approximation method based on a DC power flow model considering only thermal limits.
2. Continuation power flow method based on an AC power flow model considering thermal, voltage, and voltage stability limits.
3. Stability-constrained ATC method based on time domain simulations with a dynamic model considering stability limits.
4. Optimal power flow method based on an AC power flow model considering thermal and voltage limits.

7.6.1 Linear Approximation Method

Linear methods are based on linear incremental power flow approximation whereby network sensitivity indices are calculated to determine the transfer capabilities of power systems. Initial concepts applying a sensitivity index to transfer capability calculations are presented in [6]. Christie et al. [7] propose a power transfer distribution factor (PTDF) based on DC load flow for ATC determination. In addition, [8] also propose a linear ATC (LATC) method for calculating ATC values. In LATC, the method is regarded lossless and its limits are only the branch thermal ratings. It is assumed that the power transfer is non-simultaneous because it only considers the transfer of power from a single source to a single sink. This method utilizes three sets of network linear sensitivity factors: Power Transfer Distribution Factors (PTDFs), Line Outage Distribution Factors (LODFs), and Generator Outage Distribution Factors (GODFs). PTDF is the sensitivity of the power flow in a monitored branch to a one unit increase in power transfer with no outages in

the network. LODF is the sensitivity of the power flow in a branch to an injection of one unit of power flow across the terminals of another branch with no outages in the network. GODF is the sensitivity of the power flow in a monitored branch to the outage of a generator and the replacement of its power by injections from other generators.

PTDF is the sensitivity of line MW in monitored branch ij to a one unit increase in MW transfer with no outage in the network. Using the Chain Rule, the PTDF is simply a function of voltage and angle sensitivities as defined in (7.5). Then, the estimated maximum transfer without causing a line overload is defined in (7.6) and (7.7).

$$PTDF_M = \Delta P_{km} = \left[\frac{\partial P_{km}}{\partial V_k} \right] \Delta V_k + \left[\frac{\partial P_{km}}{\partial V_m} \right] \Delta V_m + \left[\frac{\partial P_{km}}{\partial \delta_k} \right] \Delta \delta_k + \left[\frac{\partial P_{km}}{\partial \delta_m} \right] \Delta \delta_m \quad (7.5)$$

$$T_M = \begin{cases} \frac{Limit_M - MW_M}{PTDF_M} & ; \quad PTDF_M > 0 \\ \infty & ; \quad PTDF_M = 0 \\ \frac{-Limit_M - MW_M}{PTDF_M} & ; \quad PTDF_M < 0 \end{cases} \quad (7.6)$$

$$ATC = Min\{T_M\} \quad (7.7)$$

where

P_{km} flow from bus k to bus m ,

T_M maximum power transfer of the monitored branch,

$Limit_M$ normal rating or emergency of the monitored branch, and

MW_M power flow in the monitored branch from the base case.

For example, consider a transmission line with a limit of 10 and present loading of 5 and a PTDF of 10%. The estimated maximum transfer without causing on overload on line is: Transfer Limitation = (Line Limit–Present Loading)/PTDF = (10–5)/0.1 = 50 MW.

This method is attractive because the network sensitivity indices are easy to calculate and they quickly provide estimated ATC values. However, these methods are based on DC load flow, considering only thermal limits. Ignoring voltage and reactive power effects may lead to unacceptable errors especially in a stressed system with insufficient reactive power support and voltage control.

7.6.2 Continuous Power Flow and Repetitive Power Flow Methods

Continuous power flow (CPF) is a general method for finding the maximum value of a scalar parameter in a linear function of changes in real power at a set of buses in a power flow problem [9]. The CPF method traces the power flow solution curve, starting at base load, leading to the steady state voltage stability limit or the critical maximum loading point of power systems. The main advantage of CPF is that it can overcome the singularity of the Jacobian matrix near the saddle-node bifurcation point, or the critical point [11]. Originally introduced for determining maximum loadability, CPF is adaptable without change in principle for ATC computation. In [12], a CPF method with adaptive localization is proposed to calculate ATC considering reactive power and voltage stability effects. The ATC problem is formulated in terms of a single unknown, the scalar parameter λ , whose value is to be maximized for each ATC case. The participations of the injections at source and sink buses included in the ATC problem are given as linear functions of λ . The mathematical formulation of TTC using CPF can be expressed as in (7.8)–(7.16).

$$\text{Maximize } \lambda \quad (7.8)$$

subject to

$$P_{Gi} - P_{Di} - \sum_{j=1}^N V_i V_j Y_{ij} \cos(\theta_{ij} - \delta_i + \delta_j) = 0 \quad (7.9)$$

$$Q_{Gi} - Q_{Di} + \sum_{j=1}^N V_i V_j Y_{ij} \sin(\theta_{ij} - \delta_i + \delta_j) = 0 \quad (7.10)$$

$$V_i^{\min} \leq V_i \leq V_i^{\max}, \quad \forall i \in N \quad (7.11)$$

$$|S_{Li}| \leq S_{Li}^{\max}, \quad \forall i \in NL \quad (7.12)$$

where

λ scalar parameter representing the increase in bus load or generation,

$\lambda = 0$ corresponds to no transfer (base case) and

$\lambda = \lambda_{\max}$ corresponds to the maximum transfer,

P_{Gi}, Q_{Gi} real and reactive power generations at bus i ,

P_{Di}, Q_{Di} real and reactive loads at bus i ,

V_i, V_j voltage magnitudes at bus i and j ,

δ_i, δ_j voltage angles of bus i and j ,

Y_{ij}, θ_{ij} magnitude and angle of the ij th element in the bus admittance matrix,

$V_i^{\min}, V_i^{\max}$	lower and upper limits of voltage magnitude at bus i ,
$S_{Li}^{\max}$	i th line or transformer loading limit,
$ S_{Li} $	i th line or transformer loading,
N	number of buses, and
NL	number of branches.

Real power generations in the source area, and real and reactive loads in the sink area are changed in (7.13)–(7.15).

$$P_{Gi} = P_{Gi}^0 \cdot (1 + \lambda \cdot k_{Gi}) \quad (7.13)$$

$$P_{Di} = P_{Di}^0 \cdot (1 + \lambda \cdot k_{Di}) \quad (7.14)$$

$$Q_{Di} = Q_{Di}^0 \cdot (1 + \lambda \cdot k_{Di}) \quad (7.15)$$

where

P_{Gi}^0 base case real power generation at bus i in the source area,

P_{Di}^0, Q_{Di}^0 base case real and reactive loads at bus i in the sink area, and

k_{Gi}, k_{Di} constant values used to specify the change rate in generation and load.

The TTC value can be calculated as shown in (7.16).

$$TTC = \sum_{i \in Sink} P_{Di}(\lambda_{\max}) - \sum_{i \in Sink} P_{Di}^0 \quad (7.16)$$

where

$\sum_{i \in Sink} P_{Di}(\lambda_{\max})$ sum of loads in the sink area, when $\lambda = \lambda_{\max}$, and

$\sum_{i \in Sink} P_{Di}^0$ sum of loads in the sink area, when $\lambda = 0$.

The CPF algorithm effectively increases the controlling parameter in discrete steps and solves the resulting power flow problem at each step. The procedure is continued until a given condition or physical limit preventing further increase is reached. CPF enables transfers by increasing the complex load with a uniform power factor at every load bus in the sink area and increasing the injected real power at generator buses in the source area in incremental steps until limits are reached. The advantage of CPF is that it will not encounter any numerical difficulty so that it can master the complete P-V and V-Q curves to calculate voltage stability margins. Its disadvantage is that due to the large number of contingencies taken into account in the determination, the computation is large, complicated, and time-consuming.

In [12], a repetitive power flow (RPF) technique is proposed to calculate ATC. The RPF method, based on a generalized search method, repeatedly solves conventional power flow equations conducted to establish the maximum transfer capability. The RPF method enables transfers by increasing the complex load with a uniform power factor at every load bus in the sink area, and by increasing the injected real power at generator buses in the source area in incremental steps until limits are reached. A generalized search algorithm is implemented to first find the thermal limit of non-simultaneous transfers between two control areas. Once the thermal limit of the transfers is determined, an AC contingency check for the voltage limit is performed.

The method can be utilized to determine not only the simultaneous real power transfer capability but also the non-simultaneous transfer between any source-sink pair. This method repeatedly solves conventional power flow equations at a succession of points along the specified transfer directions. Although it can be time-consuming, the implementation reduces the time to converge on the maximum transfer capability. It avoids the computationally expensive calculation of transfer step sizes as found in continuous power flow techniques. RPF possesses several advantages: Compared to any Optimum Power Flow (OPF) Method, RPF can provide P-V and P-Q curves for voltage stability study, and the method to adjust control variables is relatively easier in RPF. Compared to CPF, the method is much easier and the convergence time is reduced.

In both CPF and RPF methods, the total load in the sink area is taken as TTC value of the power transaction. However, the common load factor for a specific cluster of generators and loads used by both CPF and RPF methods to increase a certain power transfer may lead to a conservative ATC value because these methods do not result in the optimal generation, loading, and generator bus voltages.

7.6.3 Stability-Constrained ATC Method

Dynamic ATC is concerned with calculating the maximum increase in power transfers maintaining a stable and viable transient response. There are a number of constraints on transient behavior considered in dynamic ATC such as saddle node bifurcation, loss of stability mechanisms associated with phase angle behavior, and electromechanical swing modes [13].

Hiskens et al. [14] proposed an iterative approach to computing dynamic ATC. This method uses trajectory sensitivities and a set of differential-algebraic-discrete equations to model the power system. The application of this method is limited to the evaluation of a single free parameter that can be used to yield marginally stable trajectories and limit computational complexity for application to large systems. In [15], a dynamic ATC problem is formulated as an OPF-based optimization problem by integrating transient stability constraints into a conventional steady-state ATC determination. Then, an interior point algorithm is used to solve the optimization problem. The application of this method to the evaluation of a large scale nonlinear

programming problem is limited as this would result in a problem formulation of huge dimensions. An application of bifurcation criteria is developed by [16] for ATC calculation of bilateral and multilateral power transactions. The Hopf bifurcation limit is used for determining dynamic ATC while the saddle node bifurcation and bus voltage limits are used for static ATC determination.

7.6.4 Optimal Power Flow-based Methods

In this category, ATC determination is formulated as a OPF problem which can be implemented by many optimization techniques such as conventional OPF calculations and artificial intelligence techniques.

A. Conventional OPF Calculations

Conventional security constrained OPF (SCOPF) methods are commonly used to solve the OPF-based ATC problem to maximize power transfer capability between two control areas, assuming all OPF optimized parameters can be centrally dispatched [17], [18]. To overcome the deficiency of the conventional SCOPF, a transfer-based SCOPF (TSCOPF) was developed in [19]. It is assumed that only OPF optimized parameters involving the selected source and sink areas can be dispatched, which can be satisfied in deregulated power systems.

A sequential quadratic programming (SQP) algorithm [20], [21] and a Bender decomposition method [22] were proposed to determine TTC values considering reactive power and voltage limits. The objective function is to maximize power transfers between specific generators and loads subject to constraints of load flow equations and system operation limits. Linear programming (LP) [23] and non-linear programming [24] methods have also been used to solve the OPF-based ATC problem. Even though the LP method is fast and reliable, the power flow equations are largely simplified, which may result in unacceptable results. The nonlinear programming approach has some disadvantages associated with its insecure convergence properties and algorithmic complexity.

Dai et al. [25] proposed a direct interior point algorithm to calculate maximum loadability and minimum load curtailment. The algorithm can only be used to compute ATC values from one generation company to a customer. In [26] a hybrid stochastic technique was proposed to calculate ATC of prescribed interfaces in transmission networks.

These methods require convexity of the objective function to obtain the optimal solution. However, OPF problems are generally nonlinear and non-convex optimization problems and, as a result, many local solutions may exist. Therefore, conventional optimization methods may converge to local optimal solutions or diverge altogether. Moreover, these methods consider only thermal and voltage limits. Ignoring power system stability limits may lead to insecure system operation.

B. Artificial Intelligence Techniques

A multi-layer feed-forward neural network approach was used to calculate ATC values by Luo et al. [27]. The inputs for the neural network are generator status, line status and load status whereas the output is the transfer capability. The Quickprop algorithm is used to train the neural network. Test results show that the proposed method can determine power transfer capability between system areas with variations in load levels and the status of generator and transmission lines.

In the advent of EC techniques, novel search algorithms such as GA and EP are implemented to solve the OPF-based ATC problem. Shaaban et al. [28] propose a GA approach to determine TTC values of prescribed point-to-point power transactions. In [29], an EP approach combined with the Newton-Raphson (NR) power flow method was used for calculating TTC values of power transactions between different control areas. To improve the robustness of the existing EC techniques, an improved EP (IEP) approach was proposed in [30] to calculate TTC values.

7.7 CALCULATION OF TOTAL TRANSFER CAPABILITY BY EVOLUTIONARY PROGRAMMING

TTC is the main component in determination of ATC. It is the amount of electric power that can be transferred over the interconnected transmission network in a reliable manner while meeting all of a specific set of defined pre-and post-contingency system conditions [5]. Determination of TTC has been an area of active research in recent years, and a wide variety of mathematical methods and algorithms have been developed which can be divided into four types as follows: 1) linear ATC (LATC) [8], 2) continuous power flow (CPF) [11], 3) repetitive power flow (RPF) [12], and 4) optimal power flow (OPF) based methods, which can be implemented by many optimization techniques such as interior point algorithm [25], neural networks [27], and sequential quadratic programming (SQP) [31].

The LATC method is based on linear incremental power flow approximation which calculates power transfer distribution factors (PTDFs) to determine the transfer capabilities of power systems [8]. This method is attractive because the factors are easy to calculate and can quickly deliver rough figures of TTC. However the method is based on the DC load flow ignoring voltage and reactive power effects, therefore, it might lead to unacceptable errors, especially in a stressed system with insufficient reactive power support and voltage control. The CPF method traces the power flow solution curve, starting at base load, leading to the steady state voltage stability limit or the critical maximum loading point of a power system [11]. This method can overcome the singularity of the Jacobian matrix near the saddle-node bifurcation point, or the critical point.

The RPF method, based on a generalized search method, repeatedly solves conventional power flow equations to establish the maximum transfer capability [12]. To increase a certain power transfer, CPF and RPF use a common loading factor

for a specific cluster of generators and loads, which might lead to a conservative TTC value since the optimal distribution of generation and loads is ignored.

OPF has been a powerful tool under very active development for over 30 years. A wide variety of optimization techniques have been applied for solving OPF problems [32], [33]. These methods rely on convexity to obtain the optimal solution. However, the OPF problem is in general non-convex and, as a result, many local minima may exist especially in a highly nonlinear system [34], [35]. Therefore, conventional optimization methods might converge to a local optimal point due to its directed search using local information.

Evolutionary programming (EP) is a computational optimization method in the area of evolutionary computation (EC), which uses the mechanics of evolution to find the global optimal solution of a given problem [36]. It works by evolving a population of candidates toward the global solutions through the use of a mutation operator and selection scheme. This algorithm can move over hills and across valleys to discover a global optimal point. Because of this, EP is more robust than the existing directed search methods. EP has been quite successfully applied to various areas of power systems such as economic dispatch, reactive power planning, and power flow control in Flexible AC Transmission Systems (FACTS) [37]. Therefore in this paper, the EP algorithm is proposed to solve the OPF-TTC problem for the global optimal solution.

7.7.1 Problem Formulation

In this section, the TTC problem is formulated as an OPF problem. The objective function is to maximize the power that can be transferred from a specific set of generators in a source area to loads in a sink area, subject to the constraints of load flow equations and system operation limits. The sum of real power loads in the sink area at the maximum power transaction is defined as the TTC value.

The objective function for the OPF-based TTC calculation is expressed mathematically as (7.17).

$$\text{Maximize } F = \sum_{i=1}^{ND_{SNK}} P_{Di} \quad (7.17)$$

subject to

$$P_{Gi} - P_{Di} - \sum_{j=1}^N V_i V_j Y_{ij} \cos(\theta_{ij} - \delta_i + \delta_j) = 0 \quad (7.18)$$

$$Q_{Gi} - Q_{Di} + \sum_{j=1}^N V_i V_j Y_{ij} \sin(\theta_{ij} - \delta_i + \delta_j) = 0 \quad (7.19)$$

$$P_{Gi}^{\min} \leq P_{Gi} \leq P_{Gi}^{\max}, \forall i \in NG \quad (7.20)$$

$$Q_{Gi}^{\min} \leq Q_{Gi} \leq Q_{Gi}^{\max}, \forall i \in NG \quad (7.21)$$

$$V_i^{\min} \leq V_i \leq V_i^{\max}, \forall i \in N \quad (7.22)$$

$$|S_{Li}| \leq S_{Li}^{\max}, \forall i \in NL \quad (7.23)$$

where

F	function to be minimized,
P_{Gi}, Q_{Gi}	real and reactive power generation at bus i ,
P_{Di}, Q_{Di}	real and reactive loads at bus i ,
$P_{Gi}^{\min}, P_{Gi}^{\max}$	lower and upper limits of real power generation at bus i ,
$Q_{Gi}^{\min}, Q_{Gi}^{\max}$	lower and upper limits of reactive power generation at bus i ,
V_i, V_j	voltage magnitudes at bus i and j ,
δ_i, δ_j	voltage angles of bus i and j ,
Y_{ij}, θ_{ij}	magnitude and angle of the ij th element in the bus admittance matrix,
$V_i^{\min}, V_i^{\max}$	lower and upper limits of voltage magnitude at bus i ,
$S_{Li}^{\max}$	i th line or transformer loading limit,
$ S_{Li} $	i th line or transformer loading,
N	number of buses,
NG	number of generators,
NL	number of branches, and
ND_SNK	number of load buses in the sink area.

For the optimization problem, only generators in the source area are allowed to change their power output while others are not. Only loads in the sink area will be incremented while others remain constant. The load increment will be done at a constant power factor rate. During the optimization, all of the OPF problem's constraints have to be enforced. All equality constraints are satisfied by verifying the convergence of power flow, whereas inequality constraints are forced by using a penalty function as in (7.24).

$$h(x_i) = \begin{cases} (x_i - x_i^{\max})^2 & \text{if } x_i > x_i^{\max} \\ (x_i^{\min} - x_i)^2 & \text{if } x_i < x_i^{\min} \\ 0 & \text{if } x_i^{\min} \leq x_i \leq x_i^{\max} \end{cases} \quad (7.24)$$

where

$h(x_i)$ penalty function of variable x_i

x_i^{min}, x_i^{max} lower and upper limits of x_i

It is assumed that in each area, generators belong to the same owner and loads belong to the same load serving entity. The utility in a certain area wants to import power from another area. Therefore, TTC is evaluated between areas and the sum of the loads at the sink area is taken as the TTC value, which can be calculated by (7.25) [31].

$$TTC = \sum_{i=1}^{ND} \sum_{j=1}^{SNK} P_{Di} \quad (7.25)$$

7.7.2 Evolutionary Programming

The proposed EP algorithm for solving the OPF-TTC problem can be expressed as shown in Fig. 7.4. The main components of the algorithm are explained as follows:

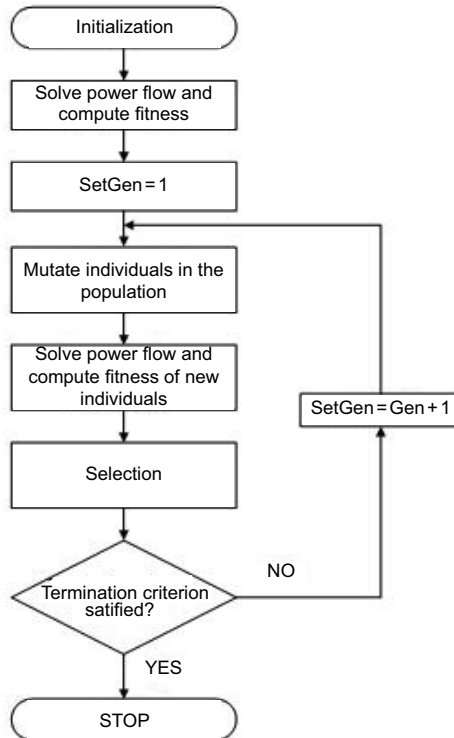


Fig. 7.4 EP algorithm for the OPF-TTC problem.

Representation of Solution: Each individual in a population is composed of OPF control variables coded by real numbers. The k th individual in a population is represented by a trial solution vector defined as in (7.26).

$$V_k^T = [P_{Gi}, Q_{Gi}, P_{Di}] \quad (7.26)$$

where P_{Gi} , $i = 2, \dots, NG_SCE$, is the real power generation at bus i , excluding slack bus; NG_SCE is the number of generator buses in the source area; V_{Gi} , $i = 1, \dots, NG$, is the voltage magnitude of generator at bus i including the slack bus; NG is the number of generator buses; P_{Di} , $i = 1, \dots, ND_SNK$, is the real power load at bus i ; and ND_SNK is the number of load buses in the sink area.

Initialization: The initial population is initialized randomly using sets of uniform random number distribution ranging over the feasible limits of each control variable as shown in (7.27).

$$x_i = x_i^{\min} + u * (x_i^{\max} - x_i^{\min}) \quad (7.27)$$

where x_i is the i th element of an individual in the population; $x_i^{\min}$ and $x_i^{\max}$ are the lower and upper limits of the i th element of the individual respectively; u is a uniform random number in the interval $[0, 1]$.

Power Flow Solution: During iterations, a power flow is calculated for each individual to compute its state variables. The Newton-Raphson (NR) method is used to solve the power flow computation.

Fitness Function: The fitness function is used to measure the optimality of each candidate solution with respect to the objective being optimized. The fitness of the k th individual can be calculated by using (7.28).

$$f_k = K_f * F \quad (7.28)$$

where f_k is the fitness of the k th individual; K_f is an arbitrary constant; and F is the objective function.

Mutation: A new population is generated by using the Gaussian mutation operator. Each element of the k th new trial solution vector, $V'_{k,i}$, is computed using (7.29) and (7.30).

$$x'_{k,i} = x_{k,i} + N(0, \sigma_{k,i}^2) \quad (7.29)$$

$$\sigma_{k,i} = (x_i^{\max} - x_i^{\min}) \left(\frac{f_{\max} - f_k}{f_{\max}} + a^g \right) \quad (7.30)$$

where $x'_{k,i}$ is the value of the i th element of the k th offspring individual; $x_{k,i}$ is the value of the i th element of the k th parent individual; $N(0, \sigma_{k,i}^2)$ is a Gaussian random number with a mean of zero and standard deviation of $\sigma_{k,i}$; $x_i^{\min}$ and $x_i^{\max}$ are the lower

and upper limits of the i th element of the k th parent individual; f_k is the fitness value of the k th individual; f_{max} is the maximum fitness of the parent population; a is a positive number constant slightly less than one; and g is the iteration counter.

Selection: The selection technique utilized is a tournament scheme, which can be expressed as in (7.31) and (7.32).

$$w_i = \begin{cases} 1 & \text{if } f_k > f_r \\ 0 & \text{otherwise} \end{cases} \quad (7.31)$$

$$s_k = \sum_{i=1}^{N_f} w_i \quad (7.32)$$

where f_k is the fitness of the k th individual in the combined population; f_r is the fitness of the r th opponent randomly selected from the combined population based on $r = \lfloor 2 * P * u + 1 \rfloor$; $\lfloor x \rfloor$ is the greatest integer less than or equal to x ; u is a uniform random number in the interval $[0,1]$; and P is the population size.

Termination Criterion: If the maximum generation number is reached, the iteration process is terminated. Otherwise, the mutation and selection process will be reiterated until the criterion is satisfied.

Example 7.1

The modified IEEE 30-bus system [8] shown in Fig. 7.5 is used as a test system. The system has three areas with two generators in each area. The base load flow of the system is given in Table 7.1.

Transactions between different control areas are investigated. Several cases are studied, four case results are presented below. To verify the validity of the results from the proposed algorithm, they are compared with the results from the CPF method using UWPFLOW [15]. For the transaction from area 2 to area 1, the results of the proposed EP-based method indicate that the generation of area 2 increases from 56.2 MW to 70.0 MW and the load of area 1 increases from 84.5 MW to 98.77 MW. The real power load of area 1 in the bus number sequence is [0.0, 30.14, 2.86, 7.94, 0.0, 0.0, 24.11, 30.74, 0.0, 0.0, 0.0 MW]. The TTC is found at 98.77 MW and the limiting condition is the generation upper limit in area 2. Figure 7.6 shows the convergence characteristics of the TTC value in the proposed EP method. The TTC calculated by CPF equals to 88.5 MW, due to the overload on line 6-8. The results show that the TTC from CPF is more conservative than that from the proposed method. This is because CPF does not optimize the generation and the loading together with the generator bus voltages while the EP-based method does. Therefore CPF can only present more conservative transfer capability results.

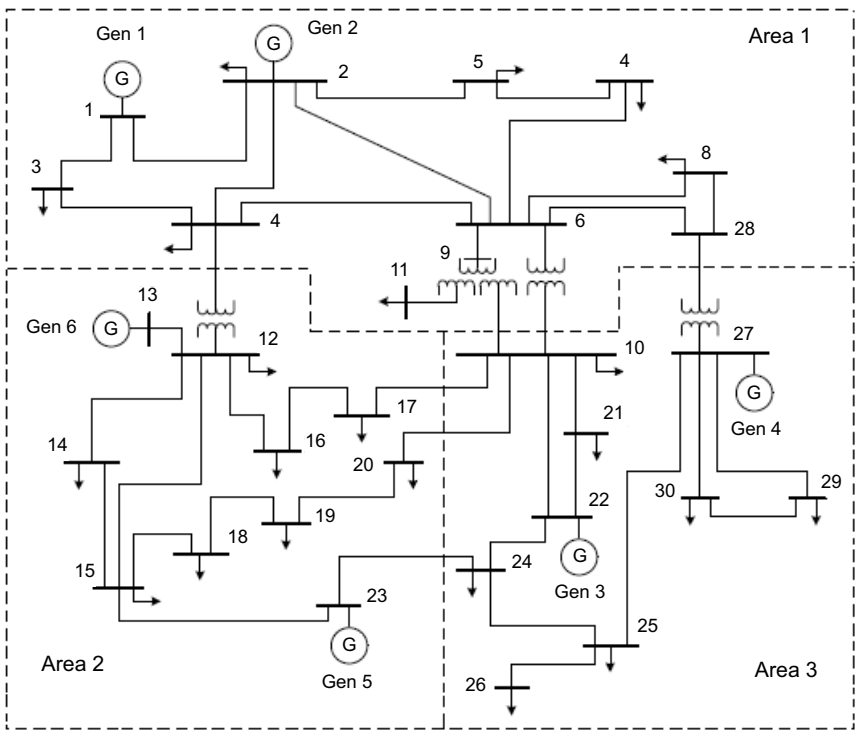


Fig. 7.5 Diagram of the IEEE 30-bus system.

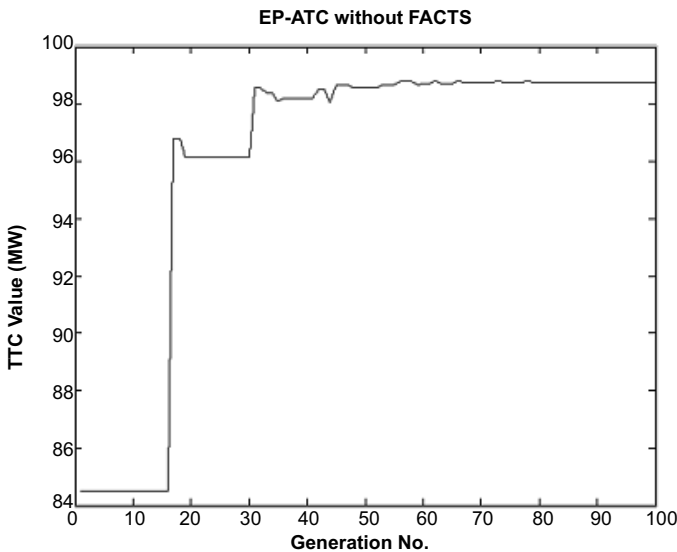


Fig. 7.6 TTC values from area 2 to area 1.

Other cases are investigated between different control areas to calculate the TTC. The results are given in Table 7.2 showing that the proposed EP approach works very well in determining TTC between different areas.

Table 7.1 The operating point of the 30-bus system (P/Q in MVA).

Bus no.	Area no.	V (p.u.)	Angle (°)	Generation		Load	
				P	Q	P	Q
1	1	1.000	0.0	26.0	0.90	0.0	0.0
2	1	1.001	-0.42	60.9	38.57	21.7	12.7
3	1	0.981	-1.50	-	-	2.4	1.2
4	1	0.978	-1.77	-	-	7.6	1.6
5	1	0.979	-1.82	-	-	0.0	0.0
6	1	0.971	-2.24	-	-	0.0	0.0
7	1	0.965	-2.61	-	-	22.8	10.9
8	1	0.959	-2.70	-	-	30.0	30.0
9	1	0.980	-2.97	-	-	0.0	0.0
10	3	0.984	-3.35	-	-	5.8	2.0
11	1	0.980	-2.97	-	-	0.0	0.0
12	2	0.985	-1.51	-	-	11.2	7.5
13	2	1.000	1.50	37.0	11.69	0.0	0.0
14	2	0.976	-2.28	-	-	6.2	1.6
15	2	0.980	-2.29	-	-	8.2	2.5
16	2	0.977	-2.62	-	-	3.5	1.8
17	2	0.976	-3.37	-	-	9.0	5.8
18	2	0.968	-3.46	-	-	3.2	0.9
19	2	0.965	-3.93	-	-	9.5	3.4
20	2	0.979	-3.85	-	-	2.2	0.7
21	3	0.993	-3.47	-	-	17.5	11.2
22	3	1.000	-3.38	21.5	40.34	0.0	0.0
23	2	1.000	-1.58	19.2	8.13	3.2	1.6
24	3	0.989	-2.62	-	-	8.7	6.7
25	3	0.990	-1.67	-	-	0.0	0.0
26	3	0.972	-2.12	-	-	3.5	2.3
27	3	1.000	-0.81	26.9	10.97	0.0	0.0
28	1	0.973	-2.25	-	-	0.0	0.0
29	3	0.980	-2.11	-	-	2.4	0.9
30	3	0.968	-3.02	-	-	10.6	1.9

Table 7.2 TTC results from the IEEE 30-bus system.

From area	To area	TTC (MW)		Limit condition	
		CPF	EP	CPF	EP
2	1	88.50	98.77	Line 6	Gen Area 2
2	3	51.50	62.65	Line 6	Gen Area 2
3	1	89.50	118.85	Line 6	Line 6-8
3	2	61.00	101.39	Line 21	Line 21-22

7.8 TOTAL TRANSFER CAPABILITY ENHANCEMENT BY HYBRID EVOLUTIONARY ALGORITHM

7.8.1 Problem Formulation

The Multi-objective OPF including TTC, transmission system real power loss and penalty functions in (7.33) is used to determine a feasible TTC value for the transfer from a specific set of generators in a source area to loads in a sink area within real and reactive power generation limits, thermal limits, voltage limits and steady-state stability limits.

$$\text{Maximize } F = \sum_{i=1}^{ND} P_{Di} - \sum_{i=1}^N (P_{Gi} - P_{Di}) - PF \quad (7.33)$$

subject to

$$P_{Gi} - P_{Di} - \sum_{j=1}^N V_i V_j Y_{ij} \cos(\theta_{ij} - \delta_i + \delta_j) = 0 \quad (7.34)$$

$$Q_{Gi} - Q_{Di} + \sum_{j=1}^N V_i V_j Y_{ij} \sin(\theta_{ij} - \delta_i + \delta_j) = 0 \quad (7.35)$$

$$P_{Gi}^{\min} \leq P_{Gi} \leq P_{Gi}^{\max}, \forall i \in NG \quad (7.36)$$

$$Q_{Gi}^{\min} \leq Q_{Gi} \leq Q_{Gi}^{\max}, \forall i \in NG \quad (7.37)$$

$$V_i^{\min} \leq V_i \leq V_i^{\max}, \forall i \in N \quad (7.38)$$

$$|S_{Li}| \leq S_{Li}^{\max}, \forall i \in NL \quad (7.39)$$

$$VCPI_i \leq 1, \forall i \in N \quad (7.40)$$

$$|\delta_{ij}| \leq \delta_{ij}^{crit}, \forall i \in NL \quad (7.41)$$

where

F multi-objective function,

PF penalty function

δ_{ij}^{crit} critical angle difference between bus i and j ,

ND number of load buses,

NG_SCE number of generator buses in a source area,

$VCPI_i$ voltage collapse proximity indicator at bus i ,

$|\delta_{ij}|$ angle difference between bus i and j ,

The other symbols are the same as the ones in Section 7.7.1.

The voltage collapse proximity indicator (VCPI) is used to directly determine voltage collapse conditions within voltage stability limits. The procedure for calculating VCPI can be found in [38]. Angle stability constraints considered can be either static ([39]) or dynamic [40]. This paper considers only static angle stability constraints. Critical angle displacement is used as a criterion to determine the steady-state angle stability limit. For a reasonable level of a typical heavy line loading situation, it is assumed that angle difference between bus i and j across a transmission line is kept within a critical angle difference, which is set to 44° as recommended in [41]. Voltage and angle stability limits are treated as OPF variables in (7.40) and (7.41), respectively. During the optimization, inequality constraints of state variables including bus voltage magnitudes and real power generation at slack bus, and output variables including reactive power generation and line or transformer loading are enforced, using the penalty function in (7.42).

$$PF = k_p * h(P_{G1}) + k_q * \sum_{i=1}^{NG} h(Q_{Gi}) + k_v * \sum_{i=1}^N h(V_i) + k_s * \sum_{i=1}^{NL} h(|S_{Li}|) \quad (7.42)$$

$$h(x) = \begin{cases} (x - x^{\max})^2 & \text{if } x > x^{\max} \\ (x^{\min} - x)^2 & \text{if } x < x^{\min} \\ 0 & \text{if } x^{\min} \leq x \leq x^{\max} \end{cases} \quad (7.43)$$

where

P_{G1} real and reactive power generation at slack bus,

k_p, k_q, k_v, k_s penalty coefficients for real power generation at slack bus, reactive power generation of all PV buses and slack bus, bus voltage magnitude and line loading, respectively,

$x^{\min}, x^{\max}$ lower and upper limits of variable x .

A multilateral transaction trading between source and sink areas is considered. Mathematically, a multilateral transaction involving several sellers and buyers can be expressed as:

$$\sum_{i \in S} P_{Gi} - \sum_{j \in B} P_{Dj} = 0 \quad (7.44)$$

where

P_{Gi} real power generation at bus i in a source area excluding slack bus,

P_{Dj} real power load at bus j in a sink area,

S set of sellers who sell the power to buyers,

B set of buyers who buy the power from sellers

The sum of real power loads in the sink area at the point of maximum power transfer is defined as the *TTC* value. Contingency analysis is also considered in the proposed model. Considering base case configuration, let TTC_0 be the maximum amount of power transfer without contingency constraints. Similarly, let TTC_k be the maximum amount of power transfer under the contingency k . Therefore, a feasible contingency *TTC* value is given in (7.45).

$$TTC = \min_k \{TTC_0, TTC_k\} \quad (7.45)$$

7.8.2 Hybrid Evolutionary Algorithm

To improve the robustness of EC techniques, a new hybrid evolutionary algorithm (HEA) integrating EP, TS and SA methods has been proposed. The HEA algorithm has special features and merits described as follows:

- i) Multiple population searches with various mutation operators are designed to enhance search diversity and improve population updates, providing a higher quality of solutions than those from single population search.
- ii) A reassignment strategy is carried out to fuse and exchange the search information of all subpopulations so that premature convergence caused by consistency of individuals in a single population will be alleviated.
- iii) Selection with a probabilistic updating strategy based on a TS algorithm and annealing schedule of SA is applied to avoid the dependency on the fitness function and to avoid being trapped in local optimal solutions.
- iv) The algorithm can easily facilitate parallel implementation on parallel computers to reduce the elapsed time without sacrificing the quality of the solution.

The proposed HEA algorithm is used to simultaneously search for real power generation in a source area excluding slack bus, generation bus voltages and real power loads in a sink area for determining a feasible *TTC* value. A flowchart of the HEA algorithm is shown in Fig. 7.7, which can be explained as follows.

Representation of the Solution: Each individual consists of OPF control variables coded by real numbers. The whole population P is divided into M subpopulations according to the number of mutation operators used. The p th individual in a population is represented by a trial vector in (7.46).

$$S_p = [P_{Gi}, V_{Gi}, P_{Dj}] \quad (7.46)$$

where

S_p trial solution vector of the p th individual in a population,

V_{Gi} voltage magnitude of generator at bus i including slack bus.

Initialization: Each element of the trial vector is initialized randomly within its search space by uniform random number distribution as formulated in (7.47).

$$x_i = x_i^{\min} + u * (x_i^{\max} - x_i^{\min}) \quad (7.47)$$

where x_i is the i th element of the individual in a population; $x_i^{\min}$ and $x_i^{\max}$ are the lower and upper limits of the i th element of the individual respectively; u is a uniform random number in the interval $[0,1]$.

Power Flow Solution: During iterations, a full AC Newton-Raphson (NR) power flow analysis is used to check the feasibility of each individual solution.

Fitness Function: The objective function in (7.33) is taken as the fitness function of the HEA algorithm.

Cooling Schedule Procedure: The initial temperature of each subpopulation is expressed in (7.48). The temperature is decreased by the temperature annealing function or cooling schedule in (7.49).

$$T_{0,m} = -(F_{\max,n} - F_{\min,n}) / \ln p_r \quad (7.48)$$

$$T_{r,m} = \lambda^{(r-1)} * T_{0,m} \quad (7.49)$$

where

- $T_{0,m}$ initial temperature of the m th subpopulation,
- $F_{\min,m}$ objective value of the worst individual in the m th subpopulation,
- $F_{\max,m}$ objective value of the best individual in the m th subpopulation,
- p_r probability of accepting the worst individual with respect to the best individual,
- $T_{r,m}$ annealing temperature of the m th subpopulation after the r th reassignment,
- λ rate of cooling,
- r iteration counter of reassignment.

Mutation: In different subpopulations, different mutation operators are used for the creation of a new offspring subpopulation so finally, many hybrid operators are applied to enhance the search diversity. Two mutation operators including Gaussian and Cauchy combined with the cooling schedule of SA are applied. Each element of the offspring individual is calculated according to (7.50).

$$x'_{k,i} = x_{k,i} + \sigma_{k,i} * \xi_m \quad (7.50)$$

$$\sigma_{k,i} = T_{r,m} * a^{(r-1)} * (x_i^{\max} - x_i^{\min}) \quad (7.51)$$

where

- $x'_{k,i}$ i th element of the k th offspring individual,
- $x_{k,i}$ i th element of the k th parent individual,

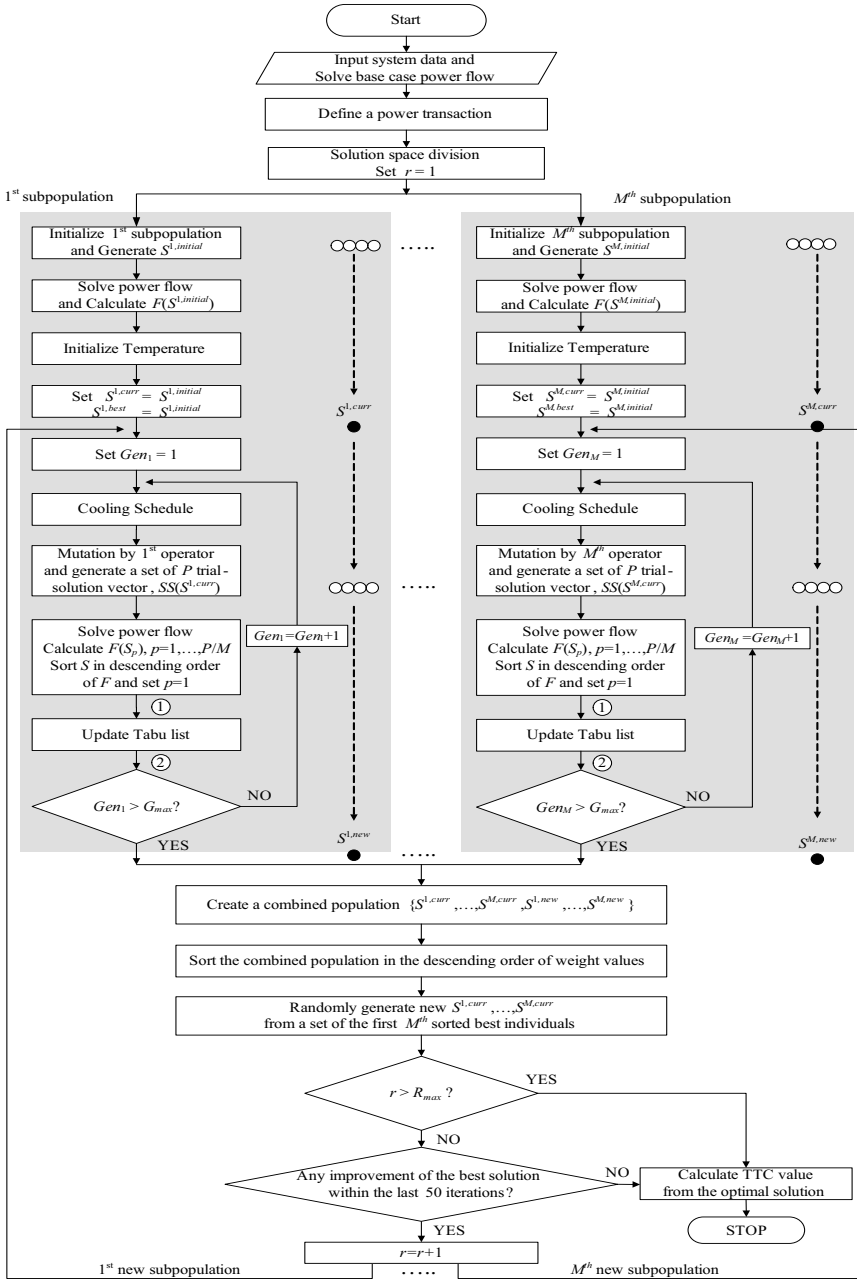


Fig. 7.7 Flowchart of the HEA algorithm.

- $\sigma_{k,i}$ mutation step size for the i th element of the k th individual,
 ξ_m mutation operator of the m th subpopulation, e.g., $N(0,1)$, $C(0,1)$, etc.,
 $N(0,1)$ Gaussian random number with mean 0 and standard deviation 1,
 $C(0,1)$ Cauchy random number with parameter $t=1$,
 a positive number slightly less than one.

Tabu List: A tabu list is a finite length one-in one-out first-in first-out structure, which records a set of current best solutions visited. A new trial vector is placed on top of the list and the oldest trial vector is taken out of the list.

Aspiration Criterion: The aspiration criterion adopts a probabilistic acceptance criterion of SA in (7.52). The tabu restriction is overridden if the aspiration criterion is satisfied. When the probabilistic acceptance criterion is higher than a uniform randomly generated variable in the interval $[0,1]$, the tabu restriction is overruled.

$$p_{k,m} = 1 / (1 + \exp(-\Delta / T_{r,m})) \quad (7.52)$$

where

- $P_{k,m}$ probabilistic acceptance criterion of the k th offspring individual within the m th subpopulation,
 Δ difference in objective value between the k th offspring individual and its corresponding parent individual

Reassignment Strategy: A tournament scheme is used to select a new current parent population from the combined population of current parents ($S^{l,curr}, \dots, S^{M,curr}$) and new offspring ($S^{l,new}, \dots, S^{M,new}$) individuals of all subpopulations [42]. Each individual in the combined population is assigned a weight value according to the competition in (7.53).

$$w_k = \sum_{t=1}^{N_t} \begin{cases} 1 & \text{if } F_k > F_r \\ 0 & \text{otherwise} \end{cases} \quad (7.53)$$

where

- w_k weight value of the k th individual in the combined population,
 F_k fitness value of the k th individual in the combined population,
 F_r fitness value of the r th opponent randomly selected from the combined population based on,
 N_t number of competitors.

After sorting the combined population of $2M$ individuals in the descending order of weight values, each new current parent solution individual of all subpopulations will be randomly selected from a set of the first M th sorted best solution individuals.

Termination Criteria: There are three termination criteria in the proposed HEA algorithm. The first termination criterion is set as the maximum number of generations of each subpopulation, the second is the number of reassignments required. The third termination criterion for a stop of the algorithm is a maximum of 50 generations without any improvement of the best fitness.

Example 7.2

The modified IEEE 24-bus RTS partitioned into 3 areas as shown in Fig. 7.8 is used to demonstrate the TTC calculation using the proposed HEA method. The modified system data are given in Tables 7.3–7.5 [19]. A multilateral transaction from area 1 to 2 with contingency constraints is considered. Only the outage of the largest generator in each area and the outage of tie lines are included in the contingency list. Parameter settings of the HEA algorithm suggested in [43] and [44] are utilized.

The normal case TTC computed by the HEA method is 716.82 MW. Considering the pre-specified contingency constraints as shown in Table 7.6, the contingency TTC value given by HEA is 635.44 MW without violating system constraints, which

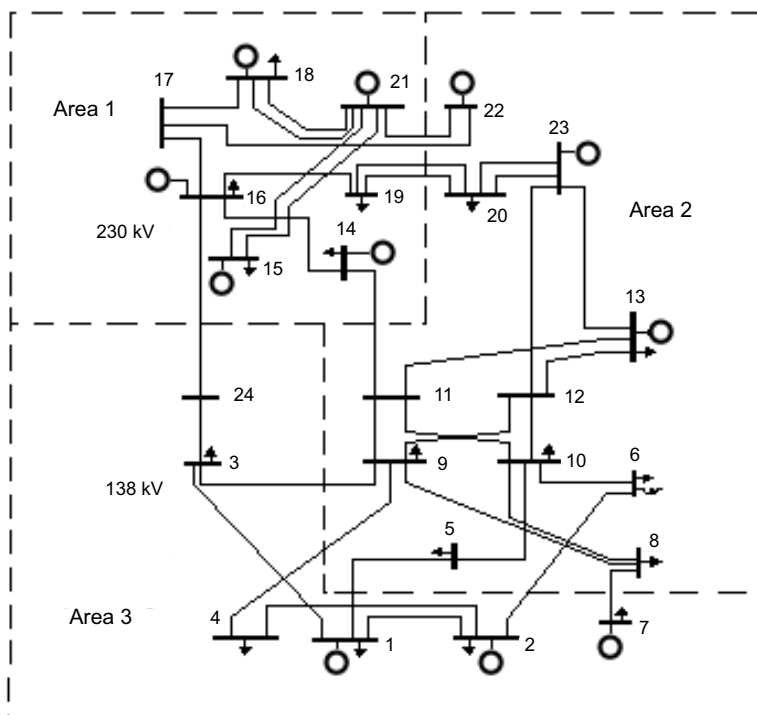


Fig. 7.8 Diagram of the modified IEEE 24-bus RTS.

Table 7.3 The three areas in the RTS.

Area	Bus	Gen. Cap (MW)	Load (MW)	Margin (MW)
1	14 ... 19, 21	1170	1125	45
2	5 ... 13, 20, 22–23	1551	1141	410
3	1 ... 4, 7, 24	648	584	100

Table 7.4 Tie-lines between areas.

Area	Tie lines
1 to 2	21–22, 17–22, 19–20 (2), 14–11
1 to 3	15–24
2 to 3	3–9, 4–9, 1–5, 2–6, 7–8

Table 7.5 Modification of the RTS system.

Area	Bus*	Number of gen #	Capacity/gen. (MW)
1	18	1	400
	16	2	155
	15	1	155
2	13	1	197
3	7	1	100

Table 7.6 TTC level and contingency TTC value of a multilateral transaction on the modified IEEE 24-bus RTS.

Case	TTC Level (MW)				
	EP	TS	TS/SA	IEP	HEA
Normal	713.99	714.90	716.01	714.80	716.82
Largest gen. in area 1 outage	717.19	714.12	715.00	715.02	716.26
Largest gen. in area 2 outage	731.23	727.36	729.04	743.25	743.77
Line 21–22 outage	711.75	699.90	710.35	713.15	715.68
Line 17–22 outage	714.68	707.01	716.65	716.98	718.24
Line 19–20 outage	700.56	701.32	707.64	705.99	717.52
Line 14–11 outage	630.73	629.30	630.83	633.11	635.44
Contingency TTC Value (MW)	630.73	629.30	630.83	633.11	635.44

is 0.75%, 0.97%, 0.73% and 0.37% higher than those derived from EP, TS, TS/SA and IEP methods, respectively. The TTC value decreases by 11.35% compared to the normal case without contingency constraints. The critical contingency case is the interconnected line 14–11 between those two outage areas. Even though test results indicate only a marginal improvement of HEA over the other methods, the

higher TTC of power value given by HEA than those of the other methods could lead to substantial cost savings in daily energy trading between different control areas under deregulated market conditions.

The comparisons of TTC results and CPU runtimes evaluated by EP, TS, TS/SA, IEP and HEA methods after 30 runs are shown in Table 7.7. Test results indicate that single-population searches of EP, TS and TS/SA are less effective than multi-population searches of IEP and HEA methods. The proposed HEA method can obtain better results regarding the best, average and the worst TTC values than those from the other optimization methods because the HEA algorithm uses a selection mechanism with a probabilistic updating strategy based on TS and SA algorithms to avoid the dependency on the fitness function and to escape from the entrapment in local optimal solutions. Furthermore, the variation of the HEA best solutions is smaller as evidenced by a smaller standard deviation than that of the other methods, leading to a more stable HEA algorithm.

CPU runtimes of IEP and HEA methods are longer than those of EP, TS and TS/SA because the best solutions of IEP and HEA are obtained based on acceptance probability, which depends on the improvement of the offspring as objective value and the annealing procedure of the SA algorithm. Moreover, the reassignment strategy requires additional computing effort. However, both IEP and HEA methods can easily be facilitated by parallel implementation reducing elapsed time without sacrificing the quality of the solution.

To compare the convergence characteristics, IEP and HEA algorithms utilize a probabilistic updating strategy based on an annealing schedule of SA, resulting in slower convergence requiring more generations than EP, TS and TS/SA methods as shown in Fig. 7.9. However, in contrast to the IEP algorithm, the convergence speed of HEA can be improved by introducing a flexible memory of TS search history to prevent cycling and to avoid entrapment in local optima.

Table 7.7 Optimal solutions for the multilateral transaction on the modified IEEE 24-bus RTS.

TTC Value (MW)	EP	TS	TS/SA	IEP	HEA
Best	630.73	629.3	630.83	633.11	635.44
Average	619.74	593.36	612.02	622.89	624.86
Worst	570.5	524.53	552.84	606.91	607.91
Standard Deviation	14.84	31.51	19.98	7.14	6.83
CPU Time (minute)	0.38	0.43	0.35	0.56	0.45

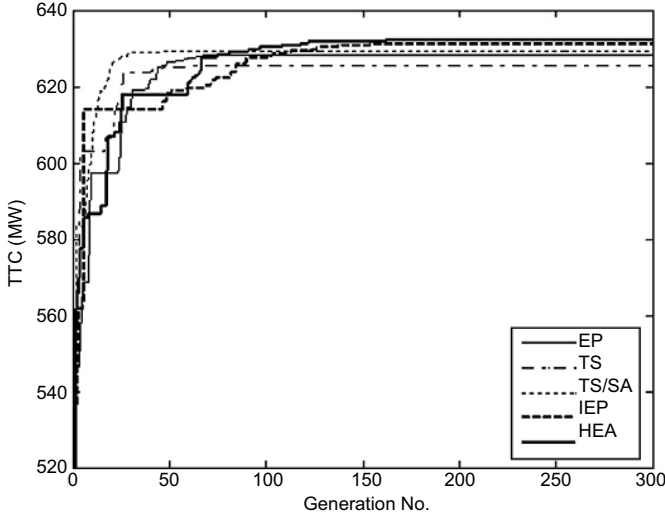


Fig. 7.9 Convergence characteristic of solutions.

7.9 OPTIMAL PLACEMENT OF MULTI-TYPE FACTS DEVICES FOR ATC ENHANCEMENT USING HEA

7.9.1 OPF with FACTS Problem Formulation

The OPF with optimal placement of FACTS is formulated as an MINLP problem. Real power generation, generator bus voltages, real power loads, and FACTS parameters are continuous variables. Types, locations, and numbers of FACTS devices are discrete variables of the MINLP problem. A combined objective function including TTC, TRM, CBM and system real power loss in (7.54) is used to determine a feasible ATC sufficient for the transfer of power from a specific set of generators in a source area to loads in a sink area within real and reactive power generation limits, line thermal limits, voltage limits, voltage and angle stability limits, and FACTS devices operating limits. Four types of FACTS devices are included: TCSC, TCPS, UPFC and SVC [45]. TCSC is modeled by the adjustable series reactance. TCPS and UPFC are modeled using the injected power model. SVC is modeled as shunt-connected static VAr generator or absorber. The problem is formulated as follows:

- The objective is

$$\text{Maximize } F = \sum_{i=1}^{ND} \sum_{j=1}^{SNK} P_{Di} - \sum_{i=1}^N (P_{Gi} - P_{Di}) - TRM - CBM \quad (7.54)$$

subject to the constraints

- Real and reactive power balance

$$P_{Gi} - P_{Di} + \sum_{k=1}^{m(i)} P_{Pi}(\alpha_{Pk}) + \sum_{k=1}^{n(i)} P_{Ui}(V_{Uk}, \alpha_{Uk}) - \sum_{j=1}^N V_i V_j Y_{ij}(X_S) \cos(\theta_{ij}(X_S) - \delta_i + \delta_j) = 0 \quad (7.55)$$

$$Q_{Gi} - Q_{Di} + \sum_{k=1}^{m(i)} Q_{Pi}(\alpha_{Pk}) + \sum_{k=1}^{n(i)} Q_{Ui}(V_{Uk}, \alpha_{Uk}) + Q_{Vi} + \sum_{j=1}^N V_i V_j Y_{ij}(X_S) \sin(\theta_{ij}(X_S) - \delta_i + \delta_j) = 0 \quad (7.56)$$

Real and reactive power limits

$$P_{Gi}^{\min} \leq P_{Gi} \leq P_{Gi}^{\max}, \quad \forall i \in NG \quad (7.57)$$

$$Q_{Gi}^{\min} \leq Q_{Gi} \leq Q_{Gi}^{\max}, \quad \forall i \in NG \quad (7.58)$$

- Voltage limit

$$V_i^{\min} \leq V_i \leq V_i^{\max}, \quad \forall i \in N \quad (7.59)$$

- Limits to random load variation

$$P_{Di}^{base} \leq P_{Di}^{rand} \leq 1.2 \cdot P_{Di}^{base}, \quad \forall i \notin ND_SNK \quad (7.60)$$

- Power flow limit

$$|S_{Li}| \leq S_{Li}^{\max}, \quad \forall i \in NL \quad (7.61)$$

- Limit to the voltage collapse proximity indicator

$$VCPI_i \leq 1, \quad \forall i \in N \quad (7.62)$$

- Transmission angle limit

$$|\delta_{ij}| \leq \delta_{ij}^{\text{crit}}, \quad \forall i \in NL \quad (7.63)$$

- Limits to TCSC

$$X_{Si}^{\min} \leq X_{Si} \leq X_{Si}^{\max} \quad (7.64)$$

- Limits to TCPS

$$\alpha_{Pi}^{\min} \leq \alpha_{Pi} \leq \alpha_{Pi}^{\max} \quad (7.65)$$

- Voltage and angle limits of UPFC

$$V_{Ui}^{\min} \leq V_{Ui} \leq V_{Ui}^{\max} \quad (7.66)$$

$$\alpha_{Ui}^{\min} \leq \alpha_{Ui} \leq \alpha_{Ui}^{\max} \quad (7.67)$$

- Limits of SVC

$$Q_{Vi}^{\min} \leq Q_{Vi} \leq Q_{Vi}^{\max} \quad (7.68)$$

- Limits to the number of FACTS components

$$0 \leq n_{CFk} \leq n_{CFk}^{\max} \quad (7.69)$$

- Limits to line or bus locations of FACTS

$$0 < location_k \leq N \text{ or } NL \quad (7.70)$$

where

F	combined objective function,
$P_{Di}^{base}, P_{Di}^{rand}$	base case real power load and random load variation at bus i ,
δ_{ij}^{crit}	critical angle difference between bus i and j ,
$X_{Si}^{\min}, X_{Si}^{\max}$	lower and upper limits of TCSC at line i ,
$\alpha_{Pi}^{\min}, \alpha_{Pi}^{\max}$	lower and upper limits of TCPS at line i ,
$V_{Ui}^{\min}, V_{Ui}^{\max}$	lower and upper voltage limits of UPFC at line i ,
$\alpha_{Ui}^{\min}, \alpha_{Ui}^{\max}$	lower and upper angle limits of UPFC at line i ,
$Q_{Vi}^{\min}, Q_{Vi}^{\max}$	lower and upper limits of SVC at bus i ,
$n_{CFk}^{\max}$	maximum allowable number of components of FACTS type k ,
P_{GI}, Q_{GI}	real and reactive power generation at slack bus,
$P_{Pi}(\alpha_{Pk})$	injected real power of TCPS at bus i ,
$Q_{Pi}(\alpha_{Pk})$	injected reactive power of TCPS at bus i ,
$P_{Ui}(V_{Uk}, \alpha_{Uk})$	injected real power of UPFC at bus i ,

$Q_{Ui}(V_{Uk}, \alpha_{Uk})$	injected reactive power of UPFC at bus i ,
$Y_{ij}(X_S), \theta_{ij}(X_S)$	magnitude and angle of the ij th element in the bus admittance matrix with TCSC included,
$m(i)$	number of injected power from TCPS at bus i ,
$n(i)$	number of injected power from UPFC at bus i ,
$ S_{Li} $	i th line or transformer loading,
$VCPI_i$	voltage collapse proximity indicator at bus i ,
$ \delta_{ij} $	angle difference between bus i and j ,
X_{Si}	reactance of TCSC at line i ,
α_{Pi}	phase shift angle of TCPS at line i ,
V_{Ui}, α_{Ui}	voltage magnitude and angle of UPFC at line i ,
Q_{Vi}	injected reactive power of SVC at bus i ,
n_{CFk}	integer value of the number of FACTS components of type k , and
$location_k$	integer value of line or bus locations of FACTS components of type k .

The other symbols are the same as in Section 7.7.1.

Voltage stability limits can be determined either by static or dynamic approaches. This paper considers only a static voltage collapse condition which is directly determined by a voltage collapse proximity indicator (VCPI). The VCPI is a general constraint which could be either static or dynamic. It represents accurate voltage collapse conditions without solving dynamic equations, which is computationally expensive. The VCPI at a load bus i is defined as the ratio of the Thevenin's equivalent impedance, Z_{ii} , looking into the port between bus i and ground, and the impedance of the load at bus i , Z_i [38]. The load at bus i can be increased to the maximum value at the voltage collapse point or critical point when Z_{ii} is equal to Z_i . Therefore, for maximum power transfer to a load at bus i without violating voltage stability limit, the VCPI at the load bus i must be equal to or less than one. VCPI is used as a constraint to ensure that the voltage will not violate the voltage lower stability limit. On the other hand, $|S_{Li}| \leq S_{Li}^{\max}$ in (7.61) concerns only the thermal line flow limit. Therefore, using both line thermal and voltage stability limits will not over-constrain the capacities. The limiting condition of some portions of the systems can shift among thermal, voltage, and angle stability limits as the network operating conditions change over time, which is illustrated in Fig. 7.4. TTC must be identified by the most restrictive of these limitations.

Even though dynamic models including various FACTS devices have been developed in voltage and angle stability studies [46]-[47], these models are based on the assumption that voltages and currents are sinusoidal, balanced, and operated near the fundamental frequency. However, they have several limitations, especially when studying large system changes due to these FACTS devices [46]. Therefore, dynamic characteristics of FACTS devices are not considered here.

Angle stability constraints of a system can be represented in either static or dynamic form [15]. In this paper, only the static form is considered. The angle stability limits will be determined by using critical angle displacement. For maintaining a reasonable level in typical situations of heavy line loading, it is assumed that the angle difference between two buses of a transmission line is kept within a critical angle difference of 44° as recommended in [41].

Voltage and angle stability limits are treated as OPF constraints in (7.62) and (7.63), respectively. During the optimization, inequality constraints are enforced using the penalty function in (7.71).

$$PF = k_p h(P_{G1}) + k_q \sum_{i=1}^{NG} h(Q_{Gi}) + k_v \sum_{i=1}^N h(V_i) + k_s \sum_{i=1}^{NL} h(|S_{Li}|) + k_d \sum_{p=1}^{NL} h(|\delta_{ij,p}|) + k_{vi} \sum_{i=1}^N h(VCPI_i) \quad (7.71)$$

$$h(x) = \begin{cases} (x - x^{\max})^2 & \text{if } x > x^{\max} \\ (x^{\min} - x)^2 & \text{if } x < x^{\min} \\ 0 & \text{if } x^{\min} \leq x \leq x^{\max} \end{cases} \quad (7.72)$$

where

PF penalty function,

$x^{\min}, x^{\max}$ lower and upper limits of variable x , and

k_p, k_q, k_v penalty coefficients for real power generation at slack bus, reactive power generation of all PV buses and slack bus, and bus voltage magnitude, respectively, and

k_s, k_d, k_{vi} penalty coefficients for line loading, angle difference, and voltage stability index, respectively.

By experiments, the penalty coefficients are set to 10^6 in all terms since lower coefficient values result in an oscillation of the HEA solution. The suggested range of the penalty coefficient is between 10^3 and 10^6 [48].

Two types of transactions [16] are considered:

Bilateral transaction: A bilateral transaction is made directly between a seller and a buyer. The seller injects a certain amount of power at a generator bus and the buyer receives this power at a load bus. Mathematically, each transaction satisfies the power balance relationship in (7.73) with the assumption that there is no power loss due to the transaction. In fact, the net real power generation should be equal to the sum of its active demand and associated transmission loss to form a transaction balance. However, the power loss allocation for each transaction is a very difficult task. Therefore, this paper considers only ideal transaction; that means all the

amount of power injected by the seller at a generation bus will be received by the buyer at a load bus.

$$P_{Gi} - P_{Dj} = 0 \quad (7.73)$$

Multilateral Transaction: A multilateral transaction trading among several parties is arranged by a scheduling coordinator. Mathematically, a multilateral transaction involving several sellers and buyers can be expressed in (7.74) where the power loss due to the transaction is also neglected similar to the bilateral transaction case. To facilitate the multilateral transaction, a weighting factor or a relative participation factor of generators involved on the seller bus side and loads on the sink bus side is set to 1.0. The optimization methods simultaneously search for the maximum allowable individual generation and load in each area. Using pre-specified participation factors, HEA will converge faster due to the reduced search space, but it may lead to a lower ATC value.

$$\sum_{i \in S} P_{Gi} - \sum_{j \in B} P_{Dj} = 0 \quad (7.74)$$

where

P_{Gi} real power generation at bus i in a source area,

P_{Dj} real power load at bus j in a sink area,

S set of sellers who sell power to buyers, and

B set of buyers who buy power from sellers.

To determine TTC values for power transactions between different control areas, the proposed optimization method enables transfers by increasing the complex load with a uniform power factor at every load bus in a sink area, a group of load buses, and increasing the injected real power at generator buses in a source area until a system limit is reached. The total real power load in the sink area given in (7.75) is used in the objective function to determine the maximum feasible TTC values without violating system constraints, similar to the TTC defined by the CPF and RPF methods.

$$TTC = \sum_{i=1}^{ND_SNK} P_{Di} \quad (7.75)$$

TRM is the amount of transmission transfer capability necessary to ensure that the transmission network is secure under system conditions with a reasonable range of uncertainties [49]. Few methods have been proposed to determine TRM [50]. For TRM determination considering load uncertainty, loads are randomly increased within 2% of base case values in every load flow calculation. Contingency analysis is also considered in the proposed model. Only the outage of the largest generator in each area and the outage of tie lines are included in the contingency list. Considering the base case configuration, let TTC_0 be the maximum amount of

power transfer without contingency constraints. Similarly, let TTC_k be the maximum amount of power transfer under contingency k . Therefore, a feasible contingency TTC value considering TRM can be given as in (7.76).

$$TTC = \text{Min}\{TTC_0, TTC_1, \dots, TTC_k\} \quad (7.76)$$

CBM is the amount of transmission transfer capability reserved by load serving entities to ensure access to generation from interconnected systems to meet generation reliability requirements [49]. Allocation of CBM to individual areas can be determined either by deterministic or probabilistic methods [51]. This paper considers only CBM determination based on single area generation reliability evaluation using the probabilistic method proposed in [19]. For hourly load model, loss of load expectation (LOLE) < 2.4 hours per year is selected as a reliability criterion. To incorporate CBM into ATC, CBM is subtracted from TTC directly.

Note: Incorporating the cost function of FACTS devices into the proposed objective function to minimize the investment cost of FACTS devices can be easily solved by HEA. The formulation of the optimal FACTS placement problem considering ATC, power loss, and cost of FACTS devices can be expressed as in (7.77).

$$\text{Maximize } F = w_1 \left(ATC - \sum_{i=1}^N (P_{Gi} - P_{Di}) \right) - w_2 (C_{FACTS}) \quad (7.77)$$

where

$ATC = TTC - TRM - CBM$

C_{FACTS} cost of FACTS devices,

w_1, w_2 weighting factors of the main objective function and FACTS cost function, respectively.

The investment cost functions, C_{FACTS} , of FACTS devices are expressed in (7.78)–(7.81) [52].

$$\text{UPFC: } C_{UPFC} = 0.0003s_{UPFC}^2 - 0.2691s_{UPFC} + 188.22 \quad (7.78)$$

$$\text{TCSC: } C_{TCSC} = 0.0015s_{TCSC}^2 - 0.7130s_{TCSC} + 153.75 \quad (7.79)$$

$$\text{SVC: } C_{SVC} = 0.0003s_{SVC}^2 - 0.3051s_{SVC} + 127.38 \quad (7.80)$$

$$\text{TCPS: } C_{TCPS} = dxP_{max} + IC_{TCPS} \quad (7.81)$$

where

C_{UPFC} , C_{TCSC} , and C_{SVC} investment costs (US\$/KVAR) of UPFC, TCSC, and SVC,

C_{TCPS} investment costs (US\$) of TCPS, and

s_{UPFC} , s_{TCSC} , and s_{SVC}	operating range in MVar of UPFC, TCSC, and SVC. s_{UPFC} and s_{TCSC} can be calculated from the power flow equations of the applied power injection (PI) model. s_{SVC} is determined by the injected reactive power compensation of SVC.
d	a positive constant representing the capital cost of TCPS,
IC_{TCPS}	installation cost of TCPS, and
P_{max}	thermal limit of the transmission line where TCPS is to be installed.

There is no additional variable to be searched by HEA since the cost functions of FACTS devices are easily calculated from the solution of HEA during each iteration. Note that the FACTS investment costs are not considered here, thus $w_1=1$ and $w_2=0$.

7.9.2 Implementation of HEA

HEA is used to simultaneously search for real power generation in a source area excluding slack bus, generation bus voltages, real power loads in a sink area, and optimal placement of multi-type FACTS devices for determining a feasible ATC value. A flowchart of the HEA approach is shown in Fig. 7.7 which can be explained as follows.

Representation of the Solution: Each individual consists of OPF control variables coded by real numbers. The whole population P is divided into M subpopulations according to the number of mutation operators used. The p th individual in a population is represented by a trial vector as defined in (7.82). There are four types of FACTS devices with a maximum of allowable n_{CFk} components for each type, which are assigned as input data. The placement configuration is represented by three parameters: n_{CFk} , $location_k$ and $parameter_k$ given in (7.83). For FACTS of type $k \in \{1,2,3,4\}$ representing the placement configuration of TCSC, TCPS, UPFC and SVC, respectively, the number of FACTS components of type k is $n_{CFk} = \{0,1,...,n\}$. More specifically, there is either no FACTS type k if $n_{CFk}=0$ or a number of FACTS of type k if $n_{CFk} \neq 0$. Therefore, the number of FACTS components, locations, and parameters of each type of FACTS devices are simultaneously searched by HEA.

$$S_p = [P_{Gi}, V_{Gi}, P_{Dj}, Loc_k] \quad (7.82)$$

$$Loc_k = [n_{CFk}, location_k, parameter_k] \quad (7.83)$$

where

S_p	trial solution vector of the p th individual,
V_{Gi}	voltage magnitude of generator at bus i including slack bus,

Loc_k allocation vector of FACTS device type k ,
 n_{CFk} number of FACTS components, $n_{CFk} = \{0, 1, \dots, n\}$,
 $location_k$ line or bus location of FACTS type k , and
 $parameter_k$ parameter settings of FACTS type k .

Note that the searched locations and parameters of FACTS type k are valid only when $n_{CFk} \neq 0$.

Initialization: Each element of the trial vector is initialized randomly within its search space using a uniform random number as defined in (7.84).

$$x_i = x_i^{\min} + u \cdot (x_i^{\max} - x_i^{\min}) \quad (7.84)$$

where

x_i i th element of the individual in a population,
 $x_i^{\min}, x_i^{\max}$ lower and upper limits of the i th element of the individual, and
 u uniform random number in the interval $[0, 1]$.

Power Flow Solution: During iterations, a full AC Newton-Raphson (NR) power flow analysis is used to check the feasibility of each individual solution.

Fitness Function: The extended objective function in (7.54) is taken as the fitness function of the HEA approach.

Cooling Schedule Procedure: The initial temperature of each subpopulation is determined in (7.85). The temperature is decreased by the temperature annealing function or cooling schedule given in (7.86).

$$T_{0,m} = -(F_m^{\max} - F_m^{\min}) / \ln p_r \quad (8.85)$$

$$T_{r,m} = \lambda^{(r-1)} \cdot T_{0,m} \quad (7.86)$$

where

$T_{0,m}$ initial temperature of the m th subpopulation,
 $F_m^{\min}, F_m^{\max}$ objective values of the worst and the best individuals in the m th subpopulation,
 p_r probability of accepting the worst individual with respect to the best individual,
 $T_{r,m}$ annealing temperature of the m th subpopulation after the r th reassignment,
 λ, r rate of cooling, and iteration counter of the reassignment strategy.

Mutation: In different subpopulations, different mutation operators are used to create new offspring subpopulations so that various hybrid operators enhance

the search diversity. In this case, two kinds of mutation operators are applied in HEA: Gaussian and Cauchy. Each element of the offspring is calculated as given in (7.87).

$$x'_{k,i} = x_{k,i} + \sigma_{k,i} \cdot \xi_m \quad (7.87)$$

$$\sigma_{k,i} = T_{r,m} \cdot a^{(r-1)} \cdot (x_i^{\max} - x_i^{\min}) \quad (7.88)$$

where

- $x'_{k,i}$, $x_{k,i}$ i th element of the k th offspring and parent individual,
- $\sigma_{k,i}$ mutation step size for the i th element of the k th individual,
- ξ_m mutation operator of the m th subpopulation, e.g., $N(0,1)$, $C(0,1)$, etc.,
- $N(0,1)$ Gaussian random number with mean 0 and standard deviation 1,
- $C(0,1)$ Cauchy random number,
- a positive number slightly less than one, and
- $x_i^{\max}$, $x_i^{\min}$ subinterval's or interval's upper and lower limits of the i th element of the individual.

Tabu List: A tabu list may be viewed as a “meta-heuristic” superimposed on another heuristic method. It is designed to jump over local optima and prevent a cycling movement. It stores the movement of a solution and forbids backtracking to previous movements [45].

Aspiration Criterion: In the proposed method, to avoid the dependence on the fitness function used in classical EP and to escape the entrapment in a local optimum, HEA uses the probabilistic updating strategy of SA to select the parent population for the next generation. Each offspring individual is accepted as a new parent individual according to its acceptance probability, which depends on the improvement of the offspring's objective value and the annealing temperature. The aspiration criterion in (7.89) adopts a probabilistic acceptance criterion of SA for the k th offspring individual in the m th subpopulation. When the probabilistic acceptance criterion is higher than a uniform randomly generated variable in the interval $[0,1]$, the tabu restriction is overruled.

$$p_{k,m} = 1/(1 + \exp(-\Delta / T_{r,m})) \quad (7.89)$$

where

- $p_{k,m}$ probabilistic acceptance criterion of the k th offspring individual within the m th subpopulation,
- Δ difference of objective values between the k th offspring individual and its corresponding parent individual, i.e., the k th parent individual.

Reassignment Strategy: A tournament scheme is used to select a new parent population from the combined population of current parents ($S^{l,curr}$, ..., $S^{M,curr}$) and

new offspring ($S^{1,new}, \dots, S^{M,new}$) individuals of all subpopulations. Each individual in the combined population is assigned a weight value according to the competition as defined in (7.90).

$$w_k = \sum_{t=1}^{N_t} \begin{cases} 1 & \text{if } F_k > F_r \\ 0 & \text{otherwise} \end{cases} \quad (7.90)$$

where

w_k weight value of the k th individual in the combined population,

N_t number of competitors,

F_k fitness value of the k th individual in the combined population, and

F_r fitness value of the r th opponent randomly selected from the combined population based on $r = \lfloor 2 \cdot M \cdot u + 1 \rfloor$.

After sorting the combined population of $2M$ individuals in descending order of weight values, each individual of the new parent solution is randomly selected from a set of the first M th individuals of the sorted best solutions of all subpopulations.

Termination Criteria: There are three termination criteria used in the proposed HEA approach. It will stop whenever any one of three criteria is met. The first termination criterion is set as the maximum number of generations of each subpopulation, the second is the number of required reassignments. The third termination criterion for the algorithm is a maximum of 50 generations without improvement of the best fitness. For a fair comparison, these criteria are applied to all methods.

Example 7.3

The modified IEEE 24-bus RTS of Example 7.2 is used to this end. A multilateral transaction from area 1 to 2 with contingency constraints is considered. Only the outage of the largest generator in each area and the outage of tie lines are included in the contingency list. For the transaction with optimally placed FACTS devices using the HEA method, the normal case TTC value of 906.03 MW is increased by 26.70% as compared to that without FACTS devices as shown in Table 7.8.

The parameter settings of the proposed algorithm suggested in [44] are utilized. The test results from HEA are compared to those from EP [53], TS and hybrid TS/SA [45], and IEP [54]. The reactance limit of TCSC in p.u. is $0 \leq X_{Si} \leq 60\%$ of line reactance; the phase shifting angle limit of TCPS is $-\pi/4 \leq \alpha_{pi} \leq \pi/4$ radian; the voltage limit of UPFC is $0 \leq V_{Ui} \leq 0.1$ p.u.; the angle limit of UPFC is $-\pi \leq \alpha_{Ui} \leq \pi$ radian; and the reactive power injection limit of SVC is $-10 \leq Q_{vi} \leq 10$ MVar. Loads are modeled as constant power factor loads.

Considering the pre-specified contingency constraints, using HEA, the contingency TTC value without violating network constraints is 814.69 MW, which is 30.10% higher than that without FACTS devices. The critical contingency case is the interconnected line 19-20 outage between those two areas. The TTC value is

Table 7.8 ATC Values for the Multilateral Transaction in the Modified IEEE 24-Bus RTS.

Case	TTC level (MW) without FACTS devices					TTC level (MW) with FACTS devices				
	EP	TS	TS/SA	IEP	HEA	EP	TS	TS/SA	IEP	HEA
Normal	706.44	708.39	708.82	711.90	715.33	794.18	788.84	803.24	905.24	906.03
Largest gen. in area 1 outage	707.58	701.93	711.76	708.77	711.57	781.33	833.64	752.83	883.60	974.04
Largest gen. in area 2 outage	710.76	710.86	714.91	711.98	714.31	739.02	715.19	748.41	826.32	835.38
Line 21-22 outage	705.07	706.43	706.52	712.24	705.07	784.12	833.43	838.15	915.83	965.51
Line 17-22 outage	712.05	705.18	708.83	713.34	713.53	809.09	791.53	803.96	914.19	916.15
Line 19-20 outage	679.50	688.83	691.56	697.95	700.09	721.88	720.96	725.06	782.65	814.69
Line 14-11 outage	623.99	622.76	624.19	624.12	626.18	684.39	674.66	765.89	859.61	913.47
Contingency TTC Value (MW)	623.99	622.76	624.19	624.12	626.18	684.39	674.66	725.06	782.65	814.69
ATC Value (MW)	563.99	562.76	564.19	564.12	566.18	624.39	614.66	665.06	722.65	754.69

12.46% lower than that without contingency constraints. It is evident that neglecting the effects of contingency constraints on TTC determination will inevitably lead to insecure system operation.

To meet the specific reliability criterion of $\text{LOLE} < 2.4$ hour/year, area 1 needs to import 60 MW and 40 MW from area 2 and 3, respectively. CBM for the critical contingency case is 60 MW. The ATC value using the HEA method for this case is 754.69 MW, which is 20.87%, 22.78%, 13.48%, and 4.43% higher than those computed by EP, TS, TS/SA, and IEP, respectively. Test results indicate that HEA can effectively re-dispatch real power generation, except for slack bus, in the source area, increase real power loads in the sink area, and optimally set generation bus voltages. Table 7.9 shows the optimal placement of FACTS devices for the contingency TTC values.

Table 7.9 Optimal Placement of FACTS Devices for Multilateral Transaction on the Modified IEEE 24-Bus RTS.

Method	TCSC			TCPS			UPFC			SVC		
	n_{CF1}	$location_1$	X_s (pu)	n_{CF2}	$location_2$	α_p (rad)	n_{CF3}	$location_3$	α_u (rad), V_u (pu)	n_{CF4}	$location_4$	Q_v (MVar)
EP	1	Line 15-24	0.007	1	Line 14-16	0.075	1	Line 12-13	-0.619, 0.001	1	Bus 22	9.160
TS	1	Line 10-12	0.021	1	Line 15-21	0.021	1	Line 15-24	0.377, 0.028	1	Bus 15	3.873
TS/SA	1	Line 16-17	0.013	1	Line 8-10	0.025	1	Line 12-13	-1.339, 0.012	1	Bus 5	1.382
IEP	1	Line 18-21	0.005	1	Line 6-10	0.006	1	Line 16-17	1.900, 0.046	1	Bus 1	6.332
HEA	1	Line 7-8	0.021	1	Line 15-24	0.016	1	Line 15-16	0.610, 0.017	1	Bus 5	5.332

7.10 SUMMARY

This chapter has presented the details of available transfer capability. The concept and definition have been provided for a basic understanding of ATC followed by the principles and methodologies for calculation. Methods for solving the problem have been presented including conventional and meta-heuristic search methods. EP has been proposed to solve the TTC problem through global optimal search. The results show that the proposed method is very effective and has a good convergence characteristic in determining TTC among different areas. The method is able to effectively re-dispatch the real power output of generators in a source area, increments of real and reactive loads in a sink area, and set the optimal bus voltage of generators. Thus, it achieves the highest TTC, while the CPF technique

usually leads to a conservative estimation of TTC. On the other hand, the proposed HEA algorithm is effectively implemented to determine the TTC value for power transactions between different control areas. Test results have indicated that the HEA algorithm can effectively re-dispatch real power generation, except for slack bus, in a source area, increments of real power loads in a sink area and set optimal generation bus voltages, leading to an efficient utilization of the existing power systems. Moreover, the algorithm can include additional voltage and angle stability limits, allowing for a higher trading level of energy transactions in secured power systems.

7.11 PROBLEMS

7.11.1 A 9-bus system with 2 areas, 3 generators and 3 loads is given in Fig. 7.10.

Table 7.10 Generator data of the 9-bus system.

Bus	P_{gen} (MW)	Q_{gen} (MVAR)	Q_{max} (MVAR)	Q_{min} (MVAR)	P_{max} (MW)	P_{min} (MW)
1	0	0	300	−300	250	10
2	163	0	300	−300	300	10
3	85	0	300	−300	270	10

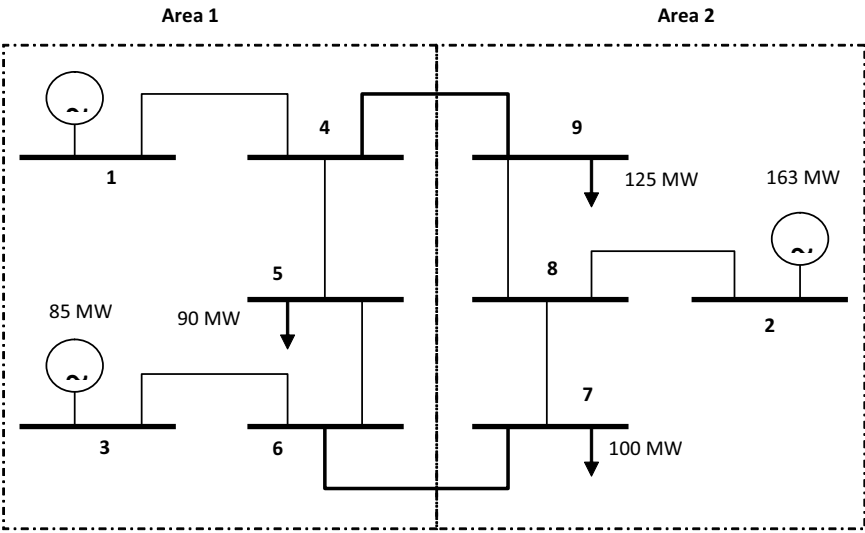


Fig. 7.10 9-bus system.

Table 7.11 Bus data of the 9-bus system.

From bus	To bus	R (pu)	X (pu)	$BCAP$ (pu)
1	4	0	.0576	0
4	5	0.017	0.092	0.158
5	6	0.039	0.17	0.358
3	6	0	.0586	0
6	7	0.0119	0.1008	0.209
7	8	0.0085	0.072	0.149
8	2	0	0.0625	0.250
8	9	0.032	0.161	0.306

Bus 1 is the slack bus. The generator and bus data are given in Tables 7.6 and 7.7, respectively. By increasing the loads on bus 7 and bus 9 to increase the power transfer from area 1 to area 2, calculate:

- Total transfer capability
- Available transfer capability.

7.11.2 A six-bus system with three generation buses and three load buses is given in Fig. 7.11. The bus data, transmission line data and flow limits on transmission lines are given in Tables 7.8–7.10.

- Calculate ATC based on the thermal limits.
- Calculate ATC under contingency with the line connected buses 3–5 opened.
- Calculate ATC under contingency with an outage of the generation at bus 3.

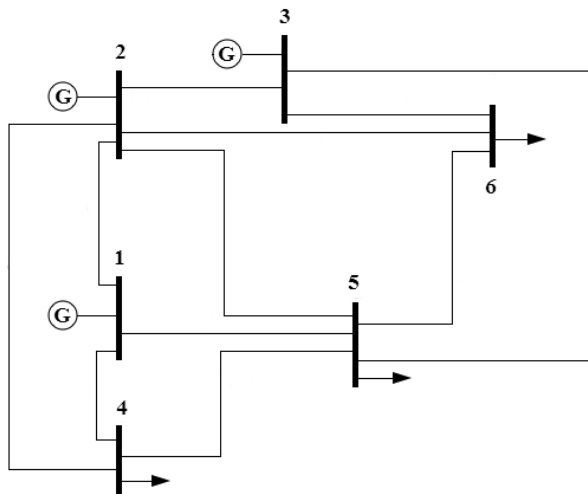
**Fig. 7.11** 6-bus system.

Table 7.12 Bus data of the 6-bus system.

Bus number	Bus type	Voltage schedule (pu V)	P_{gen} (pu MW)	P_{load} (pu MW)	Q_{load} (pu MVAR)
1	Swing	1.05			
2	Gen.	1.05	0.50	0.0	0.0
3	Gen.	1.07	0.60	0.0	0.0
4	Load		0.0	0.7	0.7
5	Load		0.0	0.7	0.7
6	Load		0.0	0.7	0.7

Table 7.13 Line data of the 6-bus system.

From bus	To bus	R (pu)	X (pu)	$BCAP$ (pu)
1	2	0.10	0.20	0.02
1	4	0.05	0.20	0.02
1	5	0.08	0.30	0.03
2	3	0.05	0.25	0.03
2	4	0.05	0.10	0.01
2	5	0.10	0.30	0.02
2	6	0.07	0.20	0.025
3	5	0.12	0.26	0.025
3	6	0.02	0.10	0.01
4	5	0.20	0.40	0.04
5	6	0.10	0.30	0.03

Table 7.14 Flow limits of the transmission lines.

Line	MW limit
1-2	30
1-4	50
1-5	40
2-3	20
2-4	40
2-5	20
2-6	30
3-5	20
3-6	60
4-5	20
5-6	20

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APPENDICES

APPENDIX A

MATHEMATICAL MODEL DERIVATIONS

A.1 COMPUTATION OF INCREMENTAL TRANSMISSION LOSS

Derived from the Newton-Raphson power flow method, the real and reactive power mismatch can be written as:

$$\begin{bmatrix} dP \\ dQ \end{bmatrix} = [J] \begin{bmatrix} d\delta \\ dV \end{bmatrix} \quad (\text{A.1})$$

where

$$\begin{bmatrix} dP \\ dQ \end{bmatrix}^T = [dP_2, \dots, dP_n, dQ_{g+1}, \dots, dQ_n] \quad (\text{A.2})$$

$$\begin{bmatrix} d\delta \\ dV \end{bmatrix}^T = [d\delta_2, \dots, d\delta_n, dV_{g+1}, \dots, dV_n] \quad (\text{A.3})$$

$$[J] = \begin{bmatrix} (\partial P / \partial \delta) & (\partial P / \partial V) \\ (\partial Q / \partial \delta) & (\partial Q / \partial V) \end{bmatrix}_{(2n-g-1) \times (2n-g-1)} \quad (\text{A.4})$$

n : total number of buses

g : number of generating buses.

The reference bus mismatch is determined:

$$dP_1 = \begin{bmatrix} \frac{\partial P_1}{\partial \delta} & \frac{\partial P_1}{\partial V} \end{bmatrix} \quad (\text{A.5})$$

where

$$\begin{bmatrix} \frac{\partial P_1}{\partial \delta} & \frac{\partial P_1}{\partial V} \end{bmatrix} = \begin{bmatrix} \frac{\partial P_1}{\partial \delta_2} & \dots & \frac{\partial P_1}{\partial \delta_n} & \frac{\partial P_1}{\partial V_{g+1}} & \dots & \frac{\partial P_1}{\partial V_n} \end{bmatrix} \quad (\text{A.6})$$

$$dP_1 = \begin{bmatrix} \frac{\partial P_1}{\partial \delta} & \frac{\partial P_1}{\partial V} \end{bmatrix} [JJ^{-1}] \begin{bmatrix} dP \\ dQ \end{bmatrix} \quad (\text{A.7})$$

$$dP_1 = [t] \begin{bmatrix} dP \\ dQ \end{bmatrix} \quad (\text{A.8})$$

where

$$[t]^T = [\gamma_2 \dots \gamma_n, \beta_{g+1} \dots \beta_n] \quad (\text{A.9})$$

$$[t] = \begin{bmatrix} \frac{\partial P_1}{\partial \delta} & \frac{\partial P_1}{\partial V} \end{bmatrix} [JJ]^{-1} \quad (\text{A.10})$$

The total transmission loss can be expressed as a function of bus power.

$$P_{loss} = P_{loss}(P_{G1}, \dots, P_{G,NG}) \quad (\text{A.11})$$

This can be expanded by Taylor series expansion around the initial bus power P_G^0 as follows:

$$P_{loss}(P_G) = P_{loss}(P_G^0 + dP) = P_{loss}(P_G^0) + P_{loss}(P_G) dP \quad (\text{A.12})$$

$$P_{loss}(P_G) = P_{loss}(P_G^0) + dP_{loss} \quad (\text{A.13})$$

The total transmission loss in a network is a function of bus loads and generation. From the power balance equation, the total transmission loss is

$$P_{loss} = \sum_{i=1}^{NB} P_i \quad (\text{A.14})$$

$$dP_{loss} = dP_1 + \sum_{i=1}^{NB} P_i \quad (\text{A.15})$$

Substituting for dP_i

$$dP_{loss} = \sum_{i=1}^{NB} (1 + \gamma_i) dP_i + \sum_{i=NG+1}^{NB} \beta_i dQ_i \quad (\text{A.16})$$

The incremental transmission loss (ITL_i) is defined as the change in transmission loss due to a change in generation at the i^{th} bus while generation at all other buses remains constant.

$$ITL_i = \frac{dP_{loss}}{dP_{Gi}} = \frac{dP_{loss}}{dP_i} = 1 + \gamma_i \quad (\text{A.17})$$

for slack bus $ITL_i = 0$.

A.2 AUGMENTED LAGRANGE HOPFIELD FOR OPTIMIZATION PROBLEMS

A.2.1 Background

Hopfield neural networks have been widely used for solving optimization problems in different fields. HNN is a recurrent network type that operates in an unsupervised manner. The action of a Hopfield network is based on the minimization of its energy function mapped from an optimization problem and the network will converge to a solution to the problem. One of the advantages of the Hopfield network is that it can

efficiently handle variable limits by its sigmoid function. However, the applications of Hopfield networks to optimization problems are limited to linear constraints and its convergence rate is very slow. Besides the resulting large number of iterations, oscillation is the major drawback HNN suffers from.

In this book, a newly improved continuous Hopfield neural network, called augmented Lagrange Hopfield network (ALHN), is presented to overcome the difficulties of Hopfield networks by introducing an augmented Lagrangian function as energy function of the Hopfield network. The advantages of the proposed neural network as compared to the conventional Hopfield network are as follows:

- In the proposed neural network, it is not necessary to predefine an energy function associated with penalty factors for mapping the problem into the Hopfield network in order to determine synaptic interconnections between neurons. Moreover, the penalty factors may lead to constraint mismatch and local optima if they are not carefully chosen.
- As the proposed neural network uses an augmented Lagrangian function as energy function for the Hopfield network, it can efficiently handle both equality and inequality constraints of a problem without causing a constraint mismatch. In addition, the proposed neural network is not limited to problems with linearized constraints, unlike the Hopfield network.
- The proposed neural network offers a very fast convergence compared to the conventional Hopfield network and can effectively find a global solution for a problem.
- The proposed neural network can easily deal with large-scale and complex optimization problems.
- The augmented term in LR can help to damp out oscillation during the convergence, especially in problems with complex constraints.

The proposed ALHN uses a sub-gradient technique with updating step sizes that can be easily tuned to each problem while the slope of the sigmoid function for continuous neurons is fixed.

The constrained optimization problem is formulated as follows:

$$\min f(x_k) \quad (\text{A.18})$$

subject to

$$g_i(x_k) = 0, i = 1, \dots, M \quad (\text{A.19})$$

$$h_j(x_k) \leq 0, j = 1, \dots, N \quad (\text{A.20})$$

$$x_{k,\min} \leq x_k \leq x_{k,\max}, k = 1, \dots, K \quad (\text{A.21})$$

where

$f(x_k)$: objective function to be minimized

$g_i(x_k)$: equality constraint

$h_j(x_k)$: inequality constraint

x_k : independent variable

$x_{k,min}$: lower bound of variable x

$x_{k,max}$: upper bound of variable x .

A.2.2 Algorithm

The augmented Lagrangian function of the problem is formulated as follows:

$$L = f(x_k) + \sum_{i=1}^M \left[\lambda_i g_i(x_k) + \frac{1}{2} \beta_i (g_i(x_k))^2 \right] + \sum_{j=1}^N \left\{ \frac{1+S_j}{2} \left[\gamma_j h_j(x_k) + \frac{1}{2} \beta_j (h_j(x_k))^2 \right] - \frac{1-S_j}{2} \left(\frac{\gamma_j^2}{2\beta_j} \right) \right\} \quad (\text{A.22})$$

where

λ_i : Lagrangian multiplier associated with equality constraint i

γ_j : Lagrangian multiplier associated with inequality constraint j

β_i, β_j : penalty factors of augmented Lagrangian function

S_j : sign function corresponding to inequality constraint j .

The inequality constraints are converted into equality constraints using the slack variables given in Section A.2.4.

To apply the augmented Lagrangian function in the Hopfield network, the continuous and multiplier neurons corresponding to independent variables and Lagrangian multipliers are respectively needed. The energy function of Hopfield network is formulated based on the augmented Lagrangian function as follows:

$$E = f(V_{k,x}) + \sum_{i=1}^M \left[V_{i,\lambda} g_i(V_{k,x}) + \frac{1}{2} \beta_i (g_i(V_{k,x}))^2 \right] + \sum_{j=1}^N \left\{ \frac{1+S_j}{2} \left[V_{j,\gamma} h_j(V_{k,x}) + \frac{1}{2} \beta_j (h_j(V_{k,x}))^2 \right] - \frac{1-S_j}{2} \left(\frac{V_{j,\gamma}^2}{2\beta_j} \right) \right\} + \sum_{k=1}^K \int_0^{V_{k,x}} g_c^{-1}(V) dV \quad (\text{A.23})$$

where

$V_{k,x}$: output of continuous neuron k corresponding to x_k

$V_{i,\lambda}$: output of multiplier neuron i corresponding to λ_i

$V_{j,\gamma}$: output of multiplier neuron j corresponding to γ_j

g_c^{-1} : inversed sigmoid function of continuous neurons.

The last term in (A.23) is the Hopfield term of continuous neurons whereby its global effect is the displacement of solutions towards the interior of the state space.

In ALHN, the neurons representing continuous variables are called continuous neurons, and the neurons representing Lagrange multipliers are called multiplier neurons.

The dynamics of this neural network are defined such that the energy function (A.23) should be minimized with respect to continuous neurons and maximized with respect to multiplier neurons as follows:

$$\frac{dU_{k,x}}{dt} = -\frac{\partial E}{\partial V_{k,x}} = -\left\{ \frac{\partial f(V_{k,x})}{\partial V_{k,x}} + [V_{i,\lambda} + \beta_i g_i(V_{k,x})] \frac{\partial g_i(V_{k,x})}{\partial V_{k,x}} + \frac{1+S_j}{2} [V_{j,\gamma} + \beta_j h_j(V_{k,x})] \frac{\partial h_j(V_{k,x})}{\partial V_{k,x}} + U_{k,x} \right\} \quad (\text{A.24})$$

$$\frac{dU_{i,\lambda}}{dt} = \frac{\partial E}{\partial V_{i,\lambda}} = g_i(V_{k,x}) \quad (\text{A.25})$$

$$\frac{dU_{j,\gamma}}{dt} = \frac{\partial E}{\partial V_{j,\gamma}} = \frac{1+S_j}{2} h_j(V_{k,x}) - \frac{1-S_j}{2} \left(\frac{V_{j,\gamma}}{\beta_j} \right) \quad (\text{A.26})$$

where

$U_{k,x}$: total inputs of continuous neuron k corresponding to the output $V_{k,x}$

$U_{i,\lambda}$: total inputs of multiplier neuron i corresponding to the output $V_{i,\lambda}$

$U_{j,\gamma}$: total inputs of multiplier neuron j corresponding to the output $V_{j,\gamma}$

The inputs of neurons at iteration n are updated based on the dynamics from (A.24) to (A.26) as follows:

$$U_{k,x}^{(n)} = U_{k,x}^{(n-1)} - \alpha_x \frac{\partial E}{\partial V_{k,x}} \quad (\text{A.27})$$

$$U_{i,\lambda}^{(n)} = U_{i,\lambda}^{(n-1)} + \alpha_\lambda \frac{\partial E}{\partial V_{i,\lambda}} \quad (\text{A.28})$$

$$U_{j,\gamma}^{(n)} = U_{j,\gamma}^{(n-1)} + \alpha_\gamma \frac{\partial E}{\partial V_{j,\gamma}} \quad (\text{A.29})$$

where α_x , α_λ , and α_γ are positive updating step sizes for inputs of continuous and multiplier neurons.

The sigmoid function of continuous neurons for determining the relationship between inputs and outputs of neurons is defined by a monotonically increasing function as follows:

$$V_{k,x} = g_c(U_{k,x}) = \frac{x_{k,\min} - x_{k,\max}}{2} \left[1 + \tanh(\sigma U_{k,x}) \right] + x_{k,\min} \quad (\text{A.30})$$

where positive σ is a scaling factor known as slope which determines the shape of the sigmoid function. The shape of the sigmoid function with various slopes is shown in Fig. A.1.

The transfer function to determine outputs of the multiplier neurons from their inputs is defined by a linear function as follows:

$$V_{i,\lambda} = g_m(U_{i,\lambda}) = U_{i,\lambda} \quad (\text{A.31})$$

$$V_{j,\gamma} = g_m(U_{j,\gamma}) = U_{j,\gamma} \quad (\text{A.32})$$

where g_m is a linear function for multiplier neurons.

The diagram for the augmented Lagrange Hopfield network is given in Fig. A.2.

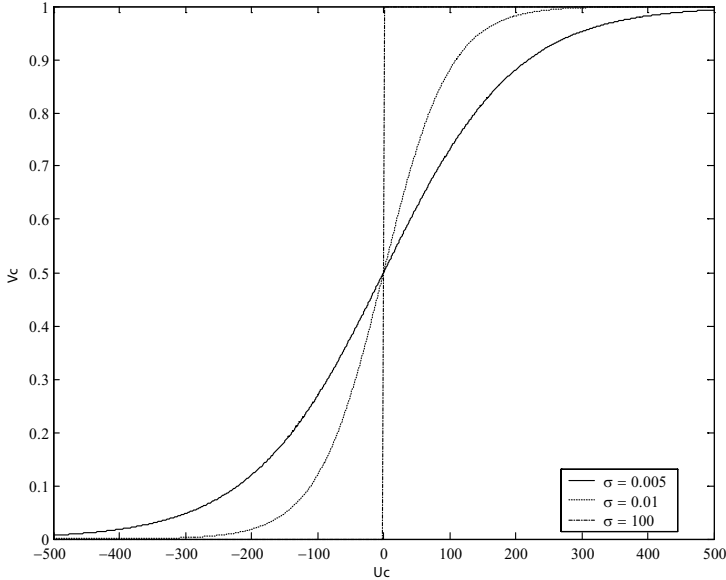
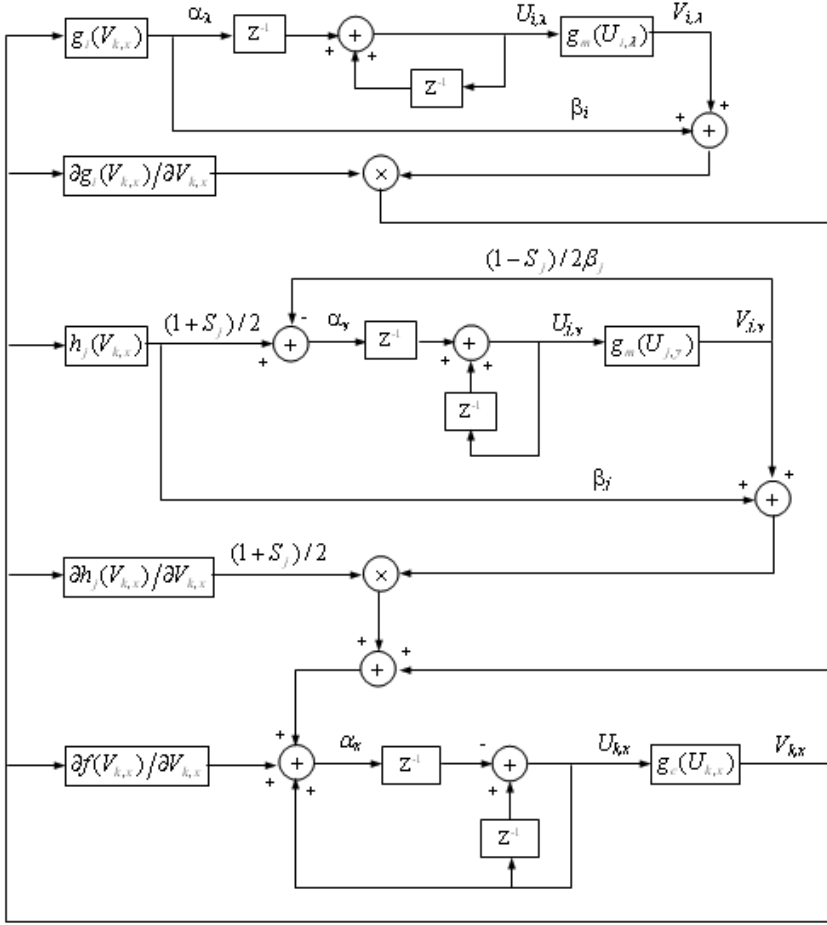


Fig. A.1 Sigmoid function of continuous neurons with different values of the slope where $x_{k,\max} = 1$ and $x_{k,\min} = 0$.



Z^{-1} : delay element

Fig. A.2 Discrete-time implementation of an augmented Lagrange Hopfield network.

1) Selection of Parameters

A proper parameter selection will guarantee rapid convergence for the neural network. So far, we have not found a method to find optimal parameters. Therefore, the parameters are tuned based on experiments which allow to draw some observations as follows.

If the slope $\sigma < 1$, the neural network converges very fast but the obtained solution may be a local optimum. In contrast, if the slope $\sigma > 1$, the solution from the neural network is the global optimum but obtained with a slightly slower convergence. The larger the value of σ , the better the obtained solution. Preferable values for σ range from 10 to 100.

The penalty factors β_i and β_j are introduced to damp the oscillation of the neural network during the convergence process, especially in complex problems. Based on the experiments, the values of the penalty factors are usually fixed in the range of 10^{-6} – 10^{-1} , with the most frequently used value of 0.001.

The neuron updating step sizes α_x , α_λ , and α_γ usually depend on the problem. It is observed that the larger the values of the updating step sizes, the closer to the behavior of the neural network to a discrete system, producing values closer to the upper and lower limits of each neuron. On the contrary, the smaller the values of updating step sizes, the slower the convergence of the neural network. To determine these parameters, we start with small values and gradually increase them until the network behaves like a discrete system. Proper values of the updating step sizes obtained usually lie in the range of 10^{-6} – 10^{-1} .

For simplicity, the penalty factors can be equally chosen. They are similar to the updating step sizes for continuous neurons.

2) Initialization

The neural network requires an initialization for each neuron. It is observed that a good initialization for multiplier neurons can speed up the convergence process.

In this research, the initialization for continuous neurons is based on “medium start”, e.g., the values of continuous neurons are initialized at the middle points between their limits as follows:

$$V_{k,x}^{(0)} = \frac{1}{2} (x_{k,\max} + x_{k,\min}) \quad (\text{A.33})$$

where $V_{k,x}^{(0)}$ is the initialization of $V_{k,x}$ of continuous neuron k .

In addition, a “mean distribution” initialization [446] is also applied, in which the initial values of continuous neurons are calculated based on their maximum capacity contribution to the equality constraints.

To speed up the convergence process of the neural network, multiplier neurons are initiated by the solution of the equation in (A.24) $\partial E / \partial V_{k,x} = 0$, in which penalty factors and total inputs of neurons are neglected and $V_{j,\gamma}^{(0)} = 0$. The obtained solution is as follows:

$$V_{i,\lambda}^{(0)} = - \frac{\partial f(V_{k,x})}{\partial V_{k,x}} \bigg/ \frac{\partial g_i(V_{k,x})}{\partial V_{k,x}} \quad (\text{A.34})$$

where

$V_{i,\lambda}^{(0)}$: initial value of $V_{i,\lambda}$ of multiplier neuron i

$V_{j,\gamma}^{(0)}$: initial value of $V_{j,\gamma}$ of multiplier neuron j .

The initial inputs of neurons are calculated based on their inversed sigmoid and transfer functions.

3) Stopping Criteria

The algorithm of the neural network will be terminated as soon as the maximum error $Err_{max}^{(n)}$ is lower than a pre-specified threshold ε or the maximum allowable number of iterations N_{max} is reached.

The maximum error at iteration n of the neural network is defined as follows:

$$Err_{max}^{(n)} = \max \left\{ \left| g_i(V_{k,x}^{(n)}) \right|, h_j(V_{k,x}^{(n)}), \right. \\ \left. V_{k,x}^{(n)} - V_{k,x}^{(n-1)}, V_{i,\lambda}^{(n)} - V_{i,\lambda}^{(n-1)}, V_{j,\gamma}^{(n)} - V_{j,\gamma}^{(n-1)} \right\} \quad (A.35)$$

4) Overall Procedure

The overall procedure of the proposed ALHN applied to an optimization problem is as follows:

- Step 1: Select parameters of the neural network as shown in Section A.2.2.1.
- Step 2: Initialize all neurons as shown in Section A.2.2.2.
- Step 3: Choose a threshold ε and the maximum number of iterations N_{max} .
Set $n = 1$.
- Step 4: Calculate dynamics of neurons as in (A.24) to (A.26).
- Step 5: Update total inputs of neurons as in (A.27) to (A.29).
- Step 6: Calculate outputs of neurons as in (A.30) to (A.32).
- Step 7: Calculate maximum error $Err_{max}^{(n)}$ as in Section A.2.2.3.
- Step 8: If $n < N_{max}$ and the maximum error $Err_{max}^{(n)} > \varepsilon$, $n = n + 1$ —return to Step 4. Otherwise, stop.

The algorithm is represented in the flow chart of Fig. A.3.

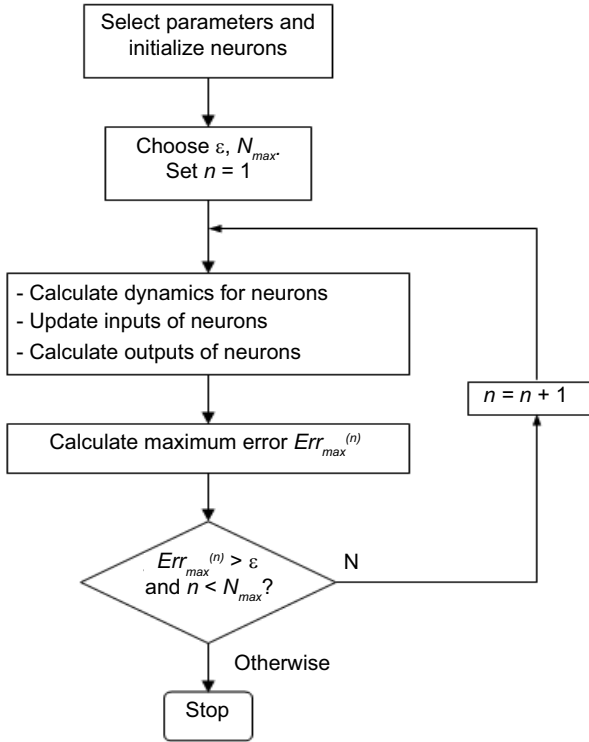
A.2.3 Proof of Convergence for ALHN

To illustrate how the dynamics of the neural network as detailed in (A.24) through (A.26) cause the minimization of the energy function in (A.23) with respect to continuous neurons and maximization with respect to multiplier neurons, the effects of changes in continuous neurons and multiplier neurons on the energy function are investigated.

Consider the effect of the change in continuous neurons on the energy function:

$$\frac{dE}{dt} = \frac{\partial E}{\partial V_{xk}} \frac{dV_{xk}}{dt} \quad (A.36)$$

Substituting V_{xk} in (A.30) into (A.36):

**Fig. A.3** Algorithm of ALHN.

$$\frac{dE}{dt} = \frac{\partial E}{\partial V_{xk}} \frac{dg_c(U_{xk})}{dU_{xk}} \frac{dU_{xk}}{dt} \quad (\text{A.37})$$

Substituting (A.24) into (A.37):

$$\frac{dE}{dt} = - \frac{dg_c(U_{xk})}{dU_i} \left(\frac{dU_{xk}}{dt} \right)^2 \quad (\text{A.38})$$

Since $g_c(U_{xk})$ is a monotonically increasing function, the value of the derivative $dg_c(U_{xk})/dU_{xk}$ is always positive. Consequently, the right hand side of equation (A.38) is always negative. Therefore, the energy function (A.23) is minimized when there is a change in the status of continuous neurons.

On the other hand, the effect of a change in the status of multiplier neurons associated with equality constraints on the energy function is considered as follows:

$$\frac{dE}{dt} = \frac{\partial E}{\partial V_{\lambda i}} \frac{dV_{\lambda i}}{dt} \quad (\text{A.39})$$

Substituting $V_{\lambda i}$ in (A.31) into (A.39):

$$\frac{dE}{dt} = \frac{\partial E}{\partial V_{\lambda i}} \frac{dg_m(U_{\lambda i})}{dU_{\lambda i}} \frac{dU_{\lambda i}}{dt} \quad (\text{A.40})$$

Substituting (A.25) into (A.40):

$$\frac{dE}{dt} = \left(\frac{dU_{\lambda i}}{dt} \right)^2 \quad (\text{A.41})$$

It is obvious that the right hand side of (A.41) is always positive. Therefore, the energy function always seeks for the maximum value when there is a change on multiplier neurons associated with equality constraints.

Similarly, a change in the multiplier neurons associated with inequality constraints also leads to the maximization of the energy function in (A.23).

A.2.4 LR with Inequality Constraint

The inequality constraint (A.20) is converted to an equality constraint using a slack variable as follows:

$$h_j(x_k) + z_j = 0 \quad (\text{A.42})$$

where $z_j > 0$ is the slack variable.

The augmented Lagrangian function is formulated:

$$\begin{aligned} L = & f(x_k) + \sum_{i=1}^M \left[\lambda_i g_i(x_k) + \frac{1}{2} \beta_i (g_i(x_k))^2 \right] \\ & + \sum_{j=1}^N \left\{ \gamma_j [h_j(x_k) + z_j] + \frac{1}{2} \beta_j [h_j(x_k) + z_j]^2 \right\} \end{aligned} \quad (\text{A.43})$$

To minimize the augmented Lagrangian function with respect to the non-negative z_j , the partial derivative of the function with respect to z_j is derived as follows:

$$\frac{\partial L}{\partial z_j} = \gamma_j + \beta_j [h_j(x_k) + z_j] = 0 \quad (\text{A.44})$$

$$z_j \geq 0 \quad (\text{A.45})$$

From (A.41) and (A.42), the obtained solution is:

$$Z_i = \max \left\{ 0, - \left(\frac{\gamma_j}{\beta_j} + h_j(x_k) \right) \right\} \quad (\text{A.46})$$

There are two possibilities:

$$1) \text{ If } -\left(\frac{\gamma_j}{\beta_j} + h_j(x)\right) \geq 0 \text{ or } -\frac{\gamma_j}{\beta_j} \geq h_j(x), \text{ from} \quad (\text{A.46})$$

$$z_j = -\left(\frac{\gamma_j}{\beta_j} + h_j(x_k)\right) \quad (\text{A.47})$$

Substituting z_j into (A.43), the augmented Lagrangian function becomes:

$$L = f(x_k) + \sum_{i=1}^M \left[\lambda_i g_i(x_k) + \frac{1}{2} \beta_i (g_i(x_k))^2 \right] - \sum_{j=1}^N \frac{1}{2} \left(\frac{\gamma_j}{\beta_j} \right)^2 \quad (\text{A.48})$$

$$2) \text{ If } -\left(\frac{\gamma_j}{\beta_j} + h_j(x_k)\right) < 0 \text{ or } -\frac{\gamma_j}{\beta_j} < h_j(x_k), \text{ from (A.46):}$$

$$z_j = 0 \quad (\text{A.49})$$

From (A.43), the augmented Lagrangian function can be rewritten:

$$L = f(x_k) + \sum_{i=1}^M \left[\lambda_i g_i(x_k) + \frac{1}{2} \beta_i (g_i(x_k))^2 \right] + \sum_{j=1}^N \left[\gamma_j h_j(x_k) + \frac{1}{2} \beta_j (h_j(x_k))^2 \right] \quad (\text{A.50})$$

By combining the two cases, the augmented Lagrangian function is determined:

$$L = f(x_k) + \sum_{i=1}^M \left[\lambda_i g_i(x_k) + \frac{1}{2} \beta_i (g_i(x_k))^2 \right] + \sum_{j=1}^N \left\{ \frac{1+S_j}{2} \left[\gamma_j h_j(x_k) + \frac{1}{2} \beta_j (h_j(x_k))^2 \right] - \frac{1-S_j}{2} \left(\frac{\gamma_j^2}{2\beta_j} \right) \right\} \quad (\text{A.51})$$

where

$$S_j = \text{sign} \left\{ \frac{\gamma_j}{\beta_j} + h_j(x_k) \right\} \quad (\text{A.52})$$

in which $S_j = 1$ if $\frac{\gamma_j}{\beta_j} + h_j(x_k) \geq 0$ and $S_j = -1$ otherwise.

The term of inequality constraint is redefined as follows:

$$A_j(x_k) = \frac{1}{2\beta_j} \left\{ \left[\max \left(0, \beta_j h_j(x_k) + \gamma_j \right) \right]^2 - \gamma_j^2 \right\} \quad (\text{A.53})$$

For a fixed value of $\gamma_j > 0$, $A_j(x_k)$ is a smooth function since its derivative with respect to $h_j(x_k)$ is equal to γ_j at $h_j(x_k) = 0$. The function is shown in Fig. A.4.

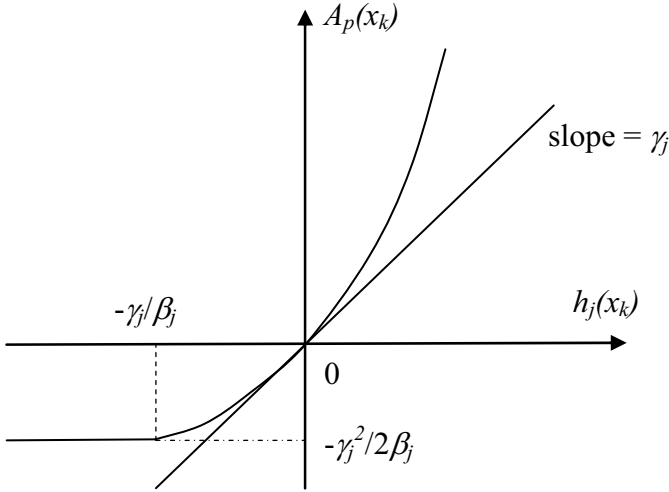


Fig. A.4 Smooth function of the inequality constraint.

A.3 DERIVATION OF A GENERALIZED GENERATION DISTRIBUTION FACTOR

A.3.1 Generation Shift Distribution Factor

The DC power flow relates to the net injection of real power P_G to buses with voltage angles δ_b neglecting line resistance with the assumption that the voltage magnitude at buses is 1 per-unit. The matrix equation is given by:

$$P_G = B' \cdot \delta_B \quad (\text{A.54})$$

$$\text{or } \delta_B = A' \cdot P_G \quad (\text{A.55})$$

where

B' matrix of bus susceptance

A' inversed matrix of the sub-matrix of B' with a zero row and column corresponding to the reference bus.

The elements of matrix B' are calculated as follows:

$$b'_{pq} = \begin{cases} -\frac{1}{x_{pq}} & \text{if } p \neq r \text{ and } q \neq r \\ 0 & \text{if } p = r \text{ or } q = r \end{cases} \quad (\text{A.56})$$

$$b'_{pp} = \begin{cases} \sum_{\substack{q=1 \\ q \neq i}}^{n_p} \frac{1}{x_{pq}} & \text{if } q \neq r \\ 0 & \text{if } q = r \end{cases} \quad (\text{A.57})$$

in which

x_{pq} reactance of line l connecting bus p and bus q

r reference bus

n_p number of buses connected to bus p .

Matrix B' is a singular matrix, and if a system has NB buses there will be $NB-1$ linearly independent equations. Therefore, there is only a $(NB-1) \times (NB-1)$ sub-matrix of B' which can be inverted. The power flow between two buses p and q is calculated:

$$P_{pq} = \frac{\delta_{Bp} - \delta_{Bq}}{x_{pq}} \quad (\text{A.58})$$

where P_{pq} is the power flow between bus p and bus q ; δ_{Bp} and δ_{Bq} are elements of vector δ_B .

Substituting (A.55) into (A.58)

$$P_l = P_{pq} = \frac{A'(\text{row } p - \text{row } q)}{x_{pq}} P_G(\text{col } m) = \sum_{m=1}^{NB} A_{lm} P_{Gm} \quad (\text{A.59})$$

So the generation shift distribution factor (GSDF) A_{lm} is derived out as follows:

$$A_{lm} = A_{p-q,m} = \frac{a_{pm} - a_{qm}}{x_{pq}} \quad (\text{A.60})$$

A.3.2 Generalized Generation Distribution Factor

Sensitivity factors have been used in system security and contingency analysis, especially GSDF and outage distribution factor (ODF) which are the two most important sensitivity factors in use. GSDF is based on the DC load flow and useful for shifting power generation between generators with unchanged system load demand. In other words, GSDF requires the initial power flow for each different

load demand. Therefore, it is not convenient to use GSDF for transmission calculation in CED problems. In this study, a generalized generation distribution factor (GGDF) is used for calculating the DC power flow based on GSDF. The advantage of GGDF over GSDF is that it is not necessary to recalculate it when the load demand changes.

The GGDF coefficient D_{lm} representing power on line l due to power generation at bus m is derived as follows:

$$D_{lm} = D_{lr} + A_{lm}, m \neq r \quad (\text{A.61})$$

where

$$D_{lr} = \frac{P_l^{(0)} - \sum_{\substack{m=1 \\ m \neq r}}^{NG} A_{lm} P_{Gm}^{(0)}}{\sum_{m=1}^{NG} P_{Gm}^{(0)}} \quad (\text{A.62})$$

r reference bus

A_{lm} GSDF representing power flow on line l due to power generation at bus m

D_{lm} GGDF representing power flow on line l due to power generation at bus m

$P_l^{(0)}$ power flow on line l by DC power flow

$P_{Gm}^{(0)}$ total generation at each bus m by DC power flow method.

The obtained D_{lm} is the matrix with the dimension of $NL \times NB$ where NL is the number of transmission lines and NB is the number of generation buses. However, the number of generators may be different from the number of generation buses since there may be more than one generator connected to a generation bus. Therefore, to calculate the power flow on a transmission line by generators, D_{lm} will be extended to D_{li} with the dimension of $NL \times NG$ where NG is the number of generators, in which the columns corresponding to generators connected to a bus are the same.

A.4 DETERMINATION OF TRANSMISSION LOSS COEFFICIENTS

A.4.1 B-Coefficients Derived from Power Flow

The total power system loss after determining the transmission power flow is calculated as follows:

$$P_{loss} + jQ_{loss} = \sum_{i=1}^{NB} V_i I_i^* \quad (\text{A.63})$$

where

P_{loss} : real power loss in the system

Q_{loss} : reactive power loss in the system

V_i : voltage at bus i

I_i : injected current at bus i

The voltage at buses can be represented in terms of Z_{bus} and injected currents as follows:

$$V_i = \sum_{j=1}^{NB} Z_{ij} I_j \quad (\text{A.64})$$

where $Z_{ij} = R_{ij} + jX_{ij}$ is an element of impedance matrix Z_{bus} .

Substituting (A.64) into (A.63), the obtained equation is:

$$\begin{aligned} P_{\text{loss}} + jQ_{\text{loss}} &= \sum_{i=1}^{NB} \sum_{j=1}^{NB} Z_{ij} I_j I_i^* \\ &= \sum_{i=1}^{NB} \sum_{j=1}^{NB} (R_{ij} + jX_{ij}) (|I_j| \angle \theta_j) (|I_i| \angle \theta_i)^* \\ &= \sum_{i=1}^{NB} \sum_{j=1}^{NB} (R_{ij} + jX_{ij}) |I_i| |I_j| \begin{bmatrix} (\cos \theta_i \cos \theta_j + \sin \theta_i \sin \theta_j) \\ +j(\cos \theta_i \sin \theta_j - \sin \theta_i \cos \theta_j) \end{bmatrix} \end{aligned} \quad (\text{A.65})$$

where

$$\theta_i = \delta_i - \phi_i$$

δ_i is voltage angle at bus i

$$\phi_i = \tan^{-1}(Q_i/P_i).$$

Sine Z_{bus} is symmetric, $Z_{ij} = Z_{ji}$, we obtain the following expression:

$$\begin{aligned} &\sum_{i=1}^{NB} |I_i| \cos(\theta_i) \left(\sum_{j=1}^{NB} (R_{ij} + jX_{ij}) |I_j| \sin(\theta_j) \right) \\ &= \sum_{j=1}^{NB} |I_j| \cos(\theta_j) \left(\sum_{i=1}^{NB} (R_{ji} + jX_{ji}) |I_i| \sin(\theta_i) \right) \end{aligned} \quad (\text{A.66})$$

From (A.65) and (A.66), we have:

$$P_{\text{loss}} + jQ_{\text{loss}} = \sum_{i=1}^{NB} \sum_{j=1}^{NB} (R_{ij} + jX_{ij}) |I_i| |I_j| \cos(\theta_i - \theta_j) \quad (\text{A.67})$$

The power losses on transmission lines are:

$$P_{\text{loss}} = \sum_{i=1}^{NB} \sum_{j=1}^{NB} R_{ij} |I_i| |I_j| \cos(\theta_i - \theta_j) \quad (\text{A.68})$$

$$Q_{loss} = \sum_{i=1}^{NB} \sum_{j=1}^{NB} X_{ij} |I_i| |I_j| \cos(\theta_i - \theta_j) \quad (\text{A.69})$$

Substituting the injected current in (A.68) by $|I_i| = P_i/|V_i|\cos(\phi_i)$, the obtained reactive power loss will be:

$$P_{loss} = \sum_{i=1}^{NB} \sum_{j=1}^{NB} R_{ij} \frac{P_i P_j}{|V_i| |V_j|} \frac{\cos(\theta_i - \theta_j)}{\cos \phi_i \cos \phi_j} \quad (\text{A.70})$$

where P_i is injected power at bus i .

Equation (A.70) can be rewritten in terms of B-coefficients as follows:

$$P_{loss} = \sum_{i=1}^{NB} \sum_{j=1}^{NB} P_i B_{ij} P_j \quad (\text{A.71})$$

where

$$B_{ij} = \frac{R_{ij}}{|V_i| |V_j|} \frac{\cos(\theta_i - \theta_j)}{\cos \phi_i \cos \phi_j} \quad (\text{A.72})$$

Substituting injected power $P_i = P_{Gi} - P_{Di}$ into (A.70)—note that B_{ij} is a symmetric matrix:

$$\begin{aligned} P_{loss} &= \sum_{i=1}^{NB} \sum_{j=1}^{NB} (P_{Gi} - P_{Di}) B_{ij} (P_{Gj} - P_{Dj}) \\ &= \sum_{i=1}^{NB} \sum_{j=1}^{NB} P_{Di} B_{ij} P_{Dj} - \sum_{i=1}^{NB} \left[\sum_{j=1}^{NB} (B_{ij} + B_{ji}) P_{Dj} \right] P_{Gi} + \sum_{i=1}^{NB} \sum_{j=1}^{NB} P_{Gi} B_{ij} P_{Gj} \end{aligned} \quad (\text{A.73})$$

Equation (A.72) can be rewritten in the form of Kron's formula:

$$P_{loss} = \sum_{i=1}^{NB} \sum_{j=1}^{NB} P_i B_{ij} P_j + \sum_{i=1}^{NB} B_{i0} P_i + B_{00} \quad (\text{A.74})$$

where

$$B_{i0} = - \sum_{j=1}^{NB} (B_{ij} + B_{ji}) P_{Dj} \quad (\text{A.75})$$

$$B_{00} = \sum_{i=1}^{NB} \sum_{j=1}^{NB} P_{Gi} B_{ij} P_{Gj} \quad (\text{A.76})$$

A.4.2 B-Coefficients from GGDF

The B-coefficients method is commonly used to calculate transmission losses in ED problems. The B-coefficients can be calculated in terms of GGDF under DC approximation as follows.

Total power loss expressed in terms of resistance and power flows in transmission lines is calculated as:

$$P_{loss} = \sum_{l=1}^{NL} r_l (P_l)^2 = \sum_{l=1}^{NL} r_l \left(\sum_{i=1}^{NG} D_{li} P_{Gi} \right)^2 \quad (A.77)$$

where

NL : number of transmission lines

r_l : resistance of transmission line l

D_{ln} : GGDF for line l due to generation n

The total power loss is also expressed in terms of B-coefficients:

$$P_{loss} = \sum_{i=1}^{NG} \sum_{j=1}^{NG} P_{Gi} B_{ij} P_{Gj} \quad (A.78)$$

From (A.78) and (A.77), the B-coefficients are derived out as follows:

$$B_{ij} = \frac{1}{2} \frac{\partial^2 P_{loss}}{\partial P_i \partial P_j} = \sum_{l=1}^{NL} r_l D_{li} D_{lj} \quad (A.79)$$

A.4.3 Power Loss as a Function of Real and Reactive Power

Substituting $\theta_i = \delta_i - \phi_i$ into (A.70), the obtained power loss will be:

$$\begin{aligned} P_{loss} &= \sum_{i=1}^{NB} \sum_{j=1}^{NB} R_{ij} \frac{P_i P_j}{|V_i| |V_j|} \frac{\cos[(\delta_i - \delta_j) - (\phi_i - \phi_j)]}{\cos \phi_i \cos \phi_j} \\ &= \sum_{i=1}^{NB} \sum_{j=1}^{NB} P_i P_j \left[a_{ij} \frac{\cos \phi_i \cos \phi_j + \sin \phi_i \sin \phi_j}{\cos \phi_i \cos \phi_j} + b_{ij} \frac{\sin \phi_i \cos \phi_j + \cos \phi_i \sin \phi_j}{\cos \phi_i \cos \phi_j} \right] \\ &= \sum_{i=1}^{NB} \sum_{j=1}^{NB} P_i P_j \left[a_{ij} (1 + \tan \phi_i \tan \phi_j) + b_{ij} (\tan \phi_i - \tan \phi_j) \right] \end{aligned} \quad (A.80)$$

where

$$a_{ij} = \frac{R_{ij}}{|V_i| |V_j|} \cos(\delta_i - \delta_j) \quad (A.81)$$

$$b_{ij} = \frac{R_{ij}}{|V_i| |V_j|} \sin(\delta_i - \delta_j) \quad (A.82)$$

Substituting $\phi_i = Q_i/P_i$ into (A.80), the obtained power loss will be:

$$P_{loss} = \sum_{i=1}^{NB} \sum_{j=1}^{NB} \left[a_{ij} (P_i P_j + Q_i Q_j) + b_{ij} (Q_i P_j - P_i Q_j) \right] \quad (\text{A.83})$$

A.5 AUGMENTED LAGRANGE AUGMENTED HOPFIELD NETWORK

An augmented Hopfield neural network (AHNN) which is a combination of continuous and discrete Hopfield neural networks is used for solving combinatorial optimization problems. However, due to the presence of both discrete and continuous neurons, the mapping of a combinatorial optimization problem into AHNN may be difficult due to the complicated mathematics evolving, and the constraints of the problem need to be linearized. Moreover, the predefined energy function associated with penalty factors for AHNN may lead to constraint mismatch. This research suggests an improvement of AHNN by using an augmented Lagrangian function as its energy function, called augmented Lagrange augmented Hopfield network (ALAHN) to overcome the mentioned difficulties with AHNN.

The combinatorial constrained optimization problem is formulated as follows:

$$\min f(x_k, u_l) \quad (\text{A.84})$$

subject to

$$g_i(x_k, u_l) = 0, i = 1, \dots, M \quad (\text{A.85})$$

$$h_j(x_k, u_l) \leq 0, j = 1, \dots, N \quad (\text{A.86})$$

$$x_k^{\min} \leq x_k \leq x_k^{\max}, k = 1, \dots, K \quad (\text{A.87})$$

$$u_l \in \{0,1\}, l = 1, \dots, L \quad (\text{A.88})$$

where

$f(x_k, u_l)$: objective function to be minimized

$g_i(x_k, u_l)$: equality constraint

$h_j(x_k, u_l)$: inequality constraint

x_k : continuous independent variable

u_l : discrete independent variable

The augmented Lagrangian function of the problem is formulated:

$$L = f(x_k, u_l) + \sum_{i=1}^M \left[\lambda_i g_i(x_k, u_l) + \frac{1}{2} \beta_i (g_i(x_k, u_l))^2 \right] + \sum_{j=1}^N \left\{ \frac{1+S_j}{2} \left[\gamma_j h_j(x_k, u_l) + \frac{1}{2} \beta_j (h_j(x_k, u_l))^2 \right] - \frac{1-S_j}{2} \left(\frac{\gamma_j^2}{2\beta_j} \right) \right\} \quad (\text{A.89})$$

where the used parameters are similar to those used in the ALHN model.

To implement the augmented Lagrangian function into AHNN, the continuous, discrete, and multiplier neurons corresponding to continuous independent variables, discrete independent variables and Lagrangian multipliers, respectively, are needed. The energy function of ALAHN based on the augmented Lagrangian function is formulated as follows:

$$E = f(V_{k,x}, V_{l,u}) + \sum_{i=1}^M \left[V_{i,\lambda} g_i(V_{k,x}, V_{l,u}) + \frac{1}{2} \beta_i (g_i(V_{k,x}, V_{l,u}))^2 \right] + \sum_{j=1}^N \left\{ \frac{1+S_j}{2} \left[V_{j,\gamma} h_j(V_{k,x}, V_{l,u}) + \frac{1}{2} \beta_j (h_j(V_{k,x}, V_{l,u}))^2 \right] - \frac{1-S_j}{2} \left(\frac{V_{j,\gamma}^2}{2\beta_j} \right) \right\} \quad (\text{A.90}) + \sum_{k=1}^K \int_0^{V_{k,x}} g_c^{-1}(V) dV$$

where $V_{l,u}$ is the output of discrete neuron k corresponding to u_l . The others are similar to those in the ALHN model.

In ALAHN, in addition to the continuous and multiplier neurons similar to ALHN, additional neurons representing discrete variables are called discrete neurons.

The dynamics of neurons are defined as follows:

$$\frac{dU_{k,x}}{dt} = -\frac{\partial E}{\partial V_{k,x}} = - \left\{ \frac{\partial f(V_{k,x}, V_{l,u})}{\partial V_{k,x}} + [V_{i,\lambda} + \beta_i g_i(V_{k,x}, V_{l,u})] \frac{\partial g_i(V_{k,x}, V_{l,u})}{\partial V_{k,x}} + \frac{1+S_j}{2} [V_{j,\gamma} + \beta_j h_j(V_{k,x}, V_{l,u})] \frac{\partial h_j(V_{k,x}, V_{l,u})}{\partial V_{k,x}} + U_{k,x} \right\} \quad (\text{A.91})$$

$$\frac{dU_{l,u}}{dt} = -\frac{\partial E}{\partial V_{l,u}} = - \left\{ \frac{\partial f(V_{k,x}, V_{l,u})}{\partial V_{l,u}} + [V_{i,\lambda} + \beta_i g_i(V_{k,x}, V_{l,u})] \frac{\partial g_i(V_{k,x}, V_{l,u})}{\partial V_{l,u}} + \frac{1+S_j}{2} [V_{j,\gamma} + \beta_j h_j(V_{k,x}, V_{l,u})] \frac{\partial h_j(V_{k,x}, V_{l,u})}{\partial V_{l,u}} \right\} \quad (\text{A.92})$$

$$\frac{dU_{i,\lambda}}{dt} = \frac{\partial E}{\partial V_{i,\lambda}} = g_i(V_{k,x}, V_{l,u}) \quad (\text{A.93})$$

$$\frac{dU_{j,\gamma}}{dt} = \frac{\partial E}{\partial V_{j,\gamma}} = \frac{1+S_j}{2} h_j(V_{k,x}, V_{l,u}) - \frac{1-S_j}{2} \left(\frac{V_{j,\gamma}}{\beta_j} \right) \quad (\text{A.94})$$

where $U_{l,u}$ is the total of inputs of discrete neuron l corresponding to output $V_{l,u}$ with the others similar to those in the ALHN model.

The input of discrete neurons at iteration n is updated based on the dynamic from (A.92) as follows:

$$U_{l,u}^{(n)} = U_{l,u}^{(n-1)} - \alpha_u \frac{\partial E}{\partial V_{l,u}} \quad (\text{A.95})$$

where α_u is a positive updating step size for the input of discrete neurons.

The inputs of other neurons are updated the same way as in ALHN. Outputs of the discrete neurons are determined by a sigmoid function as follows:

$$V_{l,u} = g_d(U_{l,u}) = \begin{cases} 1 & \text{if } U_{l,u} > 0 \\ 0 & \text{if } U_{l,u} < 0 \\ \text{no change} & \text{if } U_{l,u} = 0 \end{cases} \quad (\text{A.96})$$

where g_d is the sigmoid function of discrete neurons. Its shape is given in Fig. A.5.

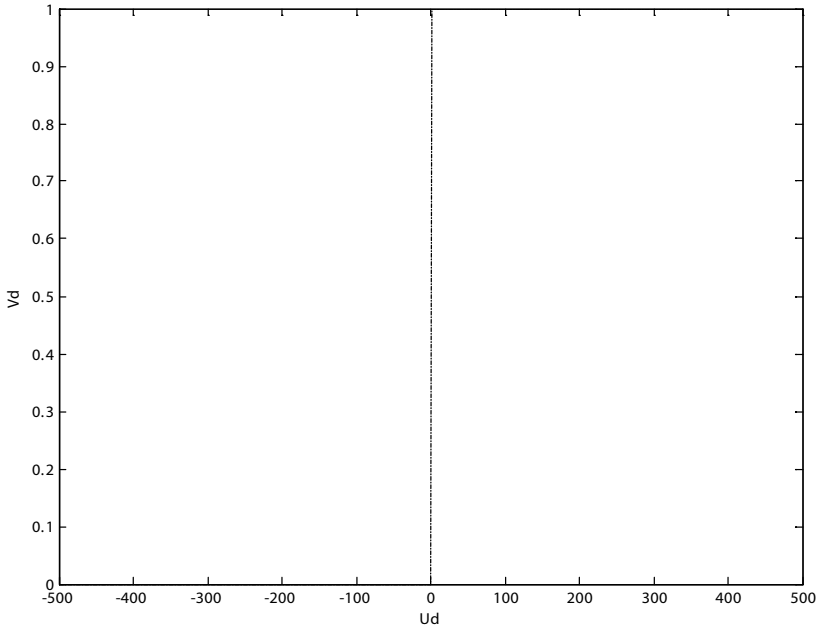


Fig. A.5 Sigmoid function of discrete neurons.

Outputs of continuous and multiplier neurons of the proposed neural network ALAHN are similar to those in ALHN.

In ALAHN, the inputs of discrete neurons can be randomly initialized and their initial outputs are calculated based on the sigmoid function in (A.96). The initialization of other neurons is similar to that of ALHN.

The updating step sizes for discrete neurons α_u as well as continuous and multiplier neurons are chosen the same way as in ALHN. The steps of the ALAHN algorithm are also similar to those of ALHN. A diagram of ALAHN is given in Fig. A.6.

Proof of convergence of ALAHN

Similar to ALHN, the energy function of ALAHN is minimized with respect to continuous neurons and maximized with respect to multiplier neurons with a change in these neurons. For the effect of discrete neurons on the energy function of ALAHN, the change in status of discrete neurons is considered as follows:

$$\frac{dE}{dt} = \frac{\partial E}{\partial V_{ul}} \frac{dV_{ul}}{dt} \quad (\text{A.97})$$

Substituting V_{ul} in (A.96) into (A.97):

$$\frac{dE}{dt} = \frac{\partial E}{\partial V_{ul}} \frac{dg_d(U_{ul})}{dU_{ul}} \frac{dU_{ul}}{dt} \quad (\text{A.98})$$

Substituting (A.92) into (A.98):

$$\frac{dE}{dt} = - \frac{dg_d(U_{ul})}{dU_{ul}} \left(\frac{dU_{ul}}{dt} \right)^2 \quad (\text{A.99})$$

Since $g_d(U_{ul})$ is also a monotonically increasing function as in Fig. A.5, the value of dE/dt is always negative. As a result, the energy function in (A.90) is minimized with respect to discrete neurons.

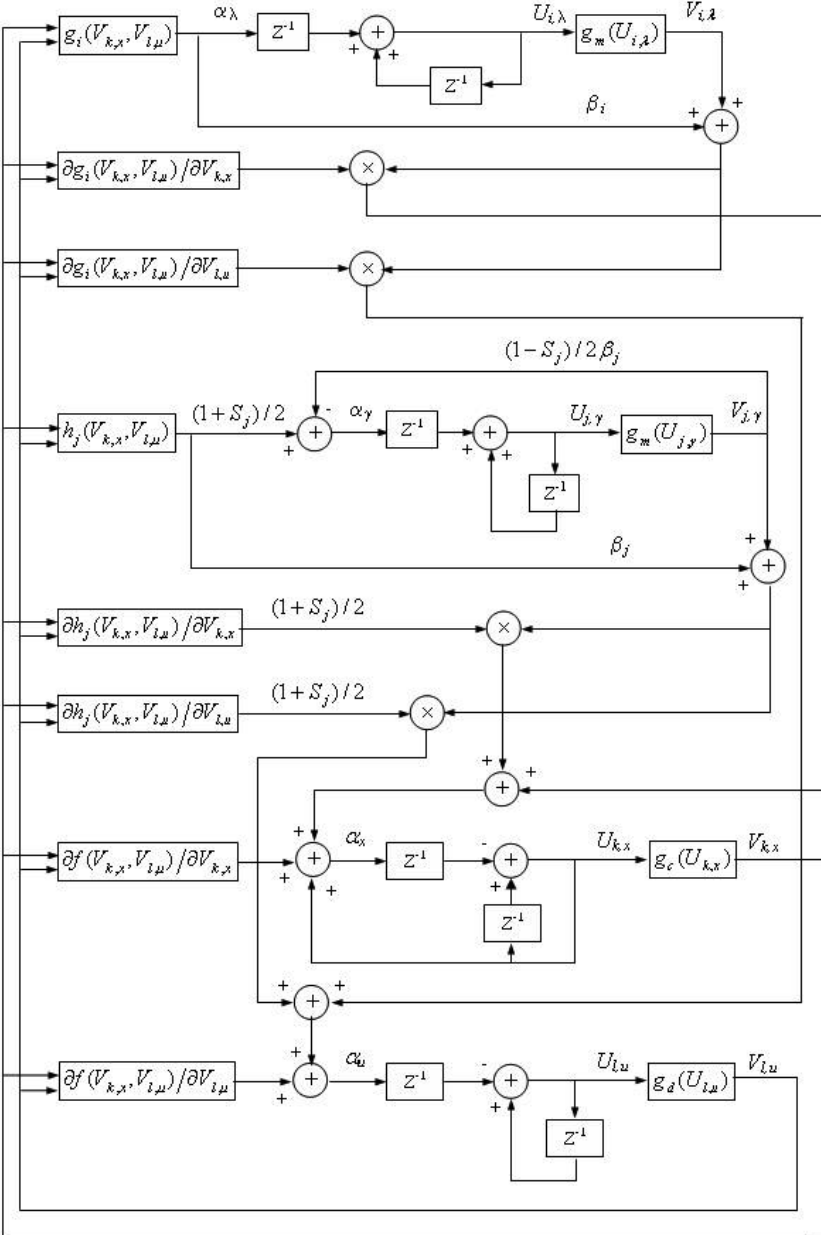


Fig. A.6 Discrete-time implementation of an augmented Lagrange augmented Hopfield network.

A.6 DC POWER FLOW

For the DC power flow, assuming that the voltage magnitude at each bus equals 1.0 per unit, the power flowing on each line is

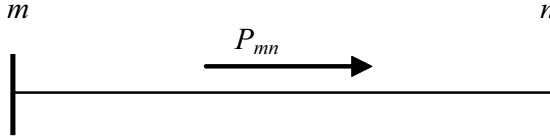


Fig. A.7 Power flow from bus m to bus n .

$$P_{mn} = \frac{1}{x_{mn}} (\theta_m - \theta_n) \quad (\text{A.100})$$

where P_{mn} is the power flow on line l from bus m to bus n , x_{mn} is the reactance of line l linking bus m and bus n , θ_m and θ_n are the phase angles at bus m and bus n , respectively. Assuming that bus number 1 is the slack bus and $\theta_1 = 0$, the relation of bus power injection and the phase angle at each bus can be expressed as:

$$\begin{bmatrix} P_2 \\ \cdot \\ \cdot \\ P_m \\ P_n \\ \cdot \\ \cdot \\ P_B \end{bmatrix} = \begin{bmatrix} B_{21} & B_{22} & \cdot & \cdot & B_{2B} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ B_{m1} & B_{m2} & \cdot & \cdot & B_{mB} \\ B_{n1} & B_{n2} & \cdot & \cdot & B_{nB} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ B_{B1} & B_{B2} & \cdot & \cdot & B_{BB} \end{bmatrix} \begin{bmatrix} \theta_2 \\ \cdot \\ \cdot \\ \theta_m \\ \theta_n \\ \cdot \\ \cdot \\ \theta_B \end{bmatrix} \quad (\text{A.101})$$

where $B_{mn} = -\frac{1}{x_{mn}}$, assuming a branch from m to n (zero otherwise),

$B_{mm} = \sum_{k=2}^B \frac{1}{x_{mk}}$, i.e., the summation of the reciprocal values of line reactance

connected to bus m , P_j is the injection power at bus j .

The equation (A.101) can be rewritten in matrix form as:

$$[P] = [B][\theta] \quad (\text{A.102})$$

$$\text{or } [\theta] = [B]^{-1} [P] \quad (\text{A.103})$$

The resulting equations are

$$\begin{bmatrix} \theta_2 \\ \cdot \\ \cdot \\ \theta_m \\ \theta_n \\ \cdot \\ \cdot \\ \theta_B \end{bmatrix} = \begin{bmatrix} X_{21} & X_{22} & \cdot & \cdot & X_{2B} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ X_{m1} & X_{m2} & \cdot & \cdot & X_{mB} \\ X_{n1} & X_{n2} & \cdot & \cdot & X_{nB} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ X_{B1} & X_{B2} & \cdot & \cdot & X_{BB} \end{bmatrix} \begin{bmatrix} P_2 \\ \cdot \\ \cdot \\ P_m \\ P_n \\ \cdot \\ \cdot \\ P_B \end{bmatrix} \quad (\text{A.104})$$

Representing θ_m and θ_n in (A.100), P_{mn} is expressed by

$$\begin{aligned} P_{mn} &= \frac{1}{x_{mn}} (\theta_m - \theta_n) \\ &= \frac{1}{x_{mn}} [(X_{m2}P_2 + \dots + X_{mB}P_B) - (X_{n2}P_2 + \dots + X_{nB}P_B)] \\ &= \frac{1}{x_{mn}} \sum_{j=2}^B (X_{mj} - X_{nj})P_j \\ &= \sum_{j=2}^B s_{l,j}P_j \end{aligned} \quad (\text{A.105})$$

where $s_{l,j}$ is the element corresponding to line l , linking bus m and bus n

$$s_{l,j} = \frac{X_{mj} - X_{nj}}{x_{mn}}, \text{ line } l, \text{ linking bus } m \text{ and bus } n \quad (\text{A.106})$$

$$s_{l,j} = \frac{X_{mj}}{x_{ms}}, \text{ line } l, \text{ linking bus } m \text{ and slack bus } s \quad (\text{A.107})$$

The resulting transmission line constraint for line l is

$$-f_l \leq \sum_{j=2}^B s_{l,j} (P_{bus,j}^t - P_{load,j}^t) \leq f_l, \text{ which appears in (3.11) and (5.52).}$$

APPENDIX B

DATA OF EXAMPLE SYSTEMS

B.1 10-UNIT BASIC SYSTEM

Table B.1 Unit data with quadratic cost function for the 10 unit system.

	Unit 1	Unit 2	Unit 3	Unit 4	Unit5
P_{max} (MW)	455	455	130	130	162
P_{min} (MW)	150	150	20	20	25
a (\$/h)	1000	970	700	680	450
b (\$/MWh)	16.19	17.26	16.60	16.50	19.70
c (\$/MW ² -h)	0.00048	0.00031	0.002	0.00211	0.00398
min up (h)	8	8	5	5	6
min down (h)	8	8	5	5	6
hot start cost (\$)	4500	5000	550	560	900
cold start cost (\$)	9000	10000	1100	1120	1800
cold start hours(h)	5	5	4	4	4
initial status (h)	8	8	-5	-5	-6
FLAC (\$/MWh)	18.576	19.533	22.245	22.005	23.122
	Unit 6	Unit 7	Unit 8	Unit 9	Unit 10
P_{max} (MW)	80	85	55	55	55
P_{min} (MW)	20	25	10	10	10
a (\$/h)	370	480	660	665	670
b (\$/MWh)	22.26	27.74	25.92	27.27	27.79
c (\$/MW ² -h)	0.00712	0.00079	0.00413	0.00222	0.00173
min up (h)	3	3	1	1	1
min down (h)	3	3	1	1	1
hot start cost (\$)	170	260	30	30	30
cold start cost (\$)	340	520	60	60	60
cold start hours(h)	2	2	0	0	0
initial status (h)	-3	-3	-1	-1	-1
FLAC (\$/MWh)	27.455	34.059	38.147	40.582	40.067

Table B.2 Load demand for the 10 unit system.

Hour	1	2	3	4	5	6	7	8
Load demand (MW)	700	750	850	950	1000	1100	1150	1200
Hour	9	10	11	12	13	14	15	16
Load demand (MW)	1300	1400	1450	1500	1400	1300	1200	1050
Hour	17	18	19	20	21	22	23	24
Load demand (MW)	1000	1100	1200	1400	1300	1100	900	800

B.2 24-BUS IEEE REILABILITY TEST SYSTEM**Table B.3** Unit data with quadratic cost function.

Unit	bus	$P_{i,min}$ (MW)	$P_{i,max}$ (MW)	c_i (\$/MW ² h)	b_i (\$/MW ² h)	a_i (\$/h)	$T_{i,up}$ (h)	$T_{i,down}$ (h)	Init. Stat. (h)	UH_i (h)	DH_i (h)
1	15	2.4	12.0	0.02533	25.5472	24.3891	0	0	-1	0	0
2	15	2.4	12.0	0.02649	25.6753	24.4110	0	0	-1	0	0
3	15	2.4	12.0	0.02801	25.8027	24.6382	0	0	-1	0	0
4	15	2.4	12.0	0.02842	25.9318	24.7605	0	0	-1	0	0
5	15	2.4	12.0	0.02855	26.0611	24.8882	0	0	-1	0	0
6	1	4.0	20.0	0.01199	37.5510	117.7511	0	0	-1	1	0
7	1	4.0	20.0	0.01261	37.6637	118.1083	0	0	-1	1	0
8	2	4.0	20.0	0.01359	37.7770	118.4576	0	0	-1	1	0
9	2	4.0	20.0	0.01433	37.8896	118.8206	0	0	-1	1	0
10	1	15.2	76.0	0.00876	13.3272	81.1364	3	2	3	2	1
11	1	15.2	76.0	0.00895	13.3538	81.2980	3	2	3	2	1
12	2	15.2	76.0	0.00910	13.3805	81.4641	3	2	3	2	1
13	2	15.2	76.0	0.00932	13.4073	81.6259	3	2	3	2	1
14	7	25.0	100.0	0.00623	18.0000	217.8952	4	2	-3	2	2
15	7	25.0	100.0	0.00612	18.1000	218.3350	4	2	-3	2	2
16	7	25.0	100.0	0.00598	18.2000	218.7752	4	2	-3	2	2
17	15	54.25	155.0	0.00463	10.6940	142.7348	5	3	5	3	2
18	16	54.25	155.0	0.00473	10.7154	143.0288	5	3	5	3	2
19	23	54.25	155.0	0.00481	10.7367	143.3179	5	3	5	3	2
20	23	54.25	155.0	0.00487	10.7583	142.5972	5	3	5	3	2
21	13	68.95	197.0	0.00259	23.0000	259.1310	5	4	-4	4	2
22	13	68.95	197.0	0.00260	23.1000	259.6490	5	4	-4	4	2
23	13	68.95	197.0	0.00263	23.2000	260.1760	5	4	-4	4	2
24	23	140.0	350.0	0.00153	10.8616	177.0575	8	5	10	5	3
25	18	100.0	400.0	0.00194	7.4921	310.0021	8	5	10	8	4
26	21	100.0	400.0	0.00195	7.5031	311.9102	8	5	10	8	4

Table B.3 contd....

Table B.3 Unit data with quadratic cost function (*contd.*)

Unit	DH_i (h)	UR_i (MW/min)	DR_i (MW/min)	$NT_{i,down}$ (h)	χ_i (\$)	δ_i (\$)	γ_i (h)	ER_i (kg/MBtu)	FC_i (\$/MBtu)
1	0	0.8	1.0	0	0	0	1	0.10	2.3
2	0	0.8	1.0	0	0	0	1	0.01	2.3
3	0	0.8	1.0	0	0	0	1	0.10	2.3
4	0	0.8	1.0	0	0	0	1	0.10	2.3
5	0	0.8	1.0	0	0	0	1	0.10	2.3
6	0	0.508	1.167	1	20	20	2	0.25	3.0
7	0	0.508	1.167	1	20	20	2	0.25	3.0
8	0	0.508	1.167	1	20	20	2	0.25	3.0
9	0	0.508	1.167	1	20	20	2	0.25	3.0
10	1	0.642	1.333	3	50	50	3	0.93	1.2
11	1	0.642	1.333	3	50	50	3	0.95	1.2
12	1	0.642	1.333	3	50	50	3	0.93	1.2
13	1	0.642	1.333	3	50	50	3	0.90	1.2
14	2	0.850	1.233	3	70	70	4	0.20	2.3
15	2	0.850	1.233	3	70	70	4	0.20	2.3
16	2	0.850	1.233	3	70	70	4	0.20	2.3
17	2	0.917	1.300	5	150	150	6	1.14	1.2
18	2	0.917	1.300	5	150	150	6	1.15	1.2
19	2	0.917	1.300	5	150	150	6	1.14	1.2
20	2	0.917	1.300	5	150	150	6	1.14	1.2
21	2	0.917	1.650	6	200	200	8	0.20	2.3
22	2	0.917	1.650	6	200	200	8	0.20	2.3
23	2	0.917	1.650	6	200	200	8	0.20	2.3
24	3	1.167	2.000	6	300	200	8	1.20	1.2
25	4	0.842	1.667	8	500	500	10	0	0.6
26	4	0.842	1.667	8	500	500	10	0	0.6

Table B.4 Bus load data.

Bus No.	1	2	3	4	5	6	7	8	9
Bus load (% of system load)	3.8	3.4	6.3	2.6	2.5	4.8	4.4	6.0	6.1
Bus No.	10	13	14	15	16	18	19	20	
Bus load (% of system load)	6.8	9.3	6.8	11.1	3.5	11.7	6.4	4.5	

Table B.5 Transmission line data.

Line No.	From Bus	To Bus	X (pu)	Continuous rating (MVA)
1	1	2	0.014	175
2	1	3	0.211	175
3	1	5	0.085	175
4	2	4	0.127	175
5	2	6	0.192	175
6	3	9	0.119	175
7	3	24	0.084	400
8	4	9	0.104	175
9	5	10	0.088	175
10	6	10	0.061	175
11	7	8	0.061	175
12-1	8	9	0.165	175
13-2	8	10	0.165	175
14	9	11	0.084	400
15	9	12	0.084	400
16	10	11	0.084	400
17	10	12	0.084	400
18	11	13	0.048	500
19	11	14	0.042	500
20	12	13	0.048	500
21	12	13	0.097	500
22	13	23	0.087	500
23	14	16	0.059	500
24	15	16	0.017	500
25-1	15	21	0.049	500
25-2	15	21	0.049	500
26	15	24	0.052	500
27	16	17	0.026	500
28	16	19	0.023	500
29	17	18	0.014	500
30	17	22	0.105	500
31-1	18	21	0.026	500
31-2	18	21	0.026	500
32-1	19	20	0.040	500
32-2	19	20	0.040	500
33-1	20	23	0.022	500
33-2	20	23	0.022	500
34	21	22	0.068	500

Table B.6 Daily load demand.

Hour	Load level 1 (Case 3.4.4.B.1)		Load level 2 (Case 3.4.4.B.1)		Load level 4 (Case 3.4.4.B.2)		Load level 5 (Case 3.4.4.B.2)	
	Load (MW)	Spinning reserve (MW)	Load (MW)	Spinning reserve (MW)	Load (MW)	Spinning reserve (MW)	Load (MW)	Spinning reserve (MW)
1	1700	400	1430	400	2223	400	2223	400
2	1730	400	1450	400	2052	400	2052	400
3	1690	400	1400	400	1938	400	1938	400
4	1700	400	1350	400	1881	400	1881	400
5	1750	400	1350	400	1824	400	1824	400
6	1850	400	1470	400	1825.5	400	1825.5	400
7	2000	400	1710	400	1881	400	1881	400
8	2430	400	2060	400	1995	400	1995	400
9	2540	400	2300	400	2280	400	2280	400
10	2600	400	2380	400	2508	400	2508	400
11	2670	400	2290	400	2565	400	2565	400
12	2590	400	2370	400	2593.5	400	2593	400
13	2590	400	2290	400	2565	400	2565	400
14	2550	400	2260	400	2508	400	2508	400
15	2620	400	2190	400	2479.5	400	2550	400
16	2650	400	2130	400	2479.5	400	2830	400
17	2550	400	2190	400	2593.5	400	2500	400
18	2530	400	2200	400	2850	255	2850	255
19	2500	400	2300	400	2821.5	283.5	2821.5	283.5
20	2550	400	2340	400	2764.5	340.5	2764.5	340.5
21	2600	400	2300	400	2679	400	2679	400
22	2480	400	2180	400	2622	400	2622	400
23	2200	400	1910	400	2479.5	400	2479.5	400
24	1840	400	1650	400	2308.5	400	2308.5	400

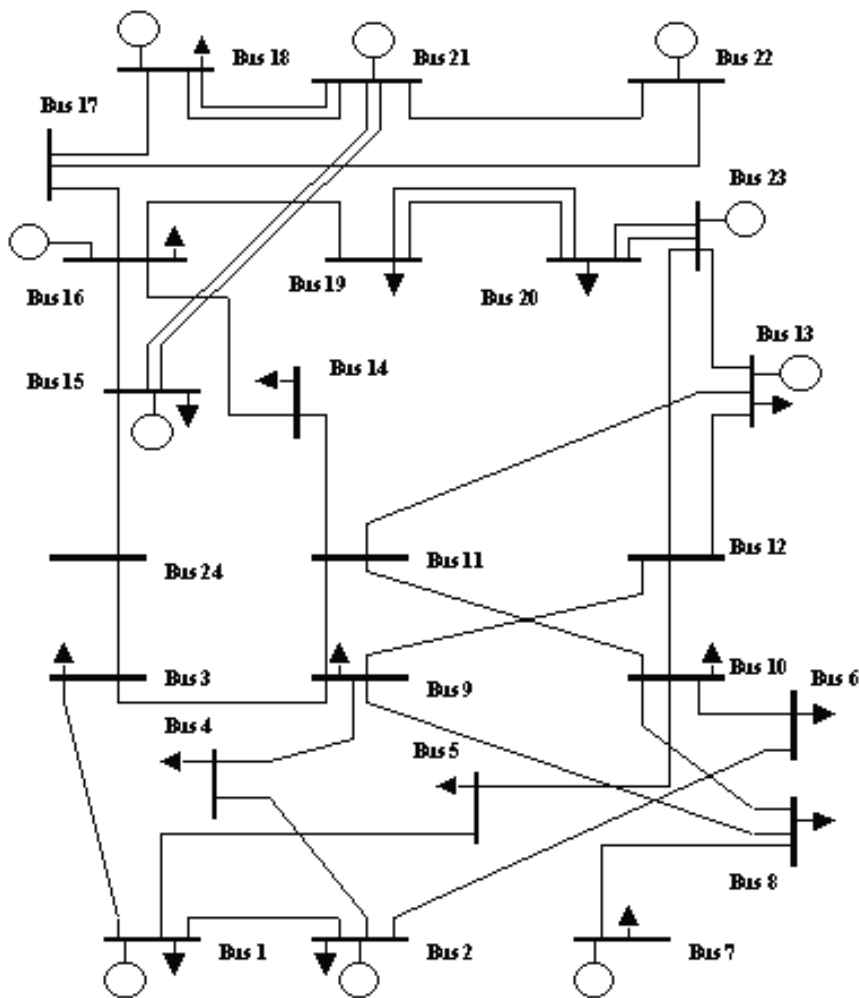


Fig. B.1 The IEEE 24-bus Reliability Test System.

Table B.7 Daily load demand.
(Example 4, Chapter 4, Section 4.8, Cases A, B-1, B-2)

Hour	1	2	3	4	5	6
Load (MW)	1462.2278	1370.8386	1325.1440	1279.4494	1279.4494	1325.1440
Hour	7	8	9	10	11	12
Load (MW)	1462.2278	1736.3956	1987.7160	2170.4944	2261.8837	2284.7310
Hour	13	14	15	16	17	18
Load (MW)	2261.8837	2284.7310	2284.7310	2216.1891	2193.3418	2193.3418
Hour	19	20	21	22	23	24
Load (MW)	2124.7998	2101.9525	2101.9525	2124.7998	1987.7160	1645.0063

**B.3 HYDROTHERMAL SYSTEMS WITH FUEL CONSTRAINED
PUMPED-STORAGE UNITS**

Table B.8 Unit data for Example 3, Case A, Section 4.8, Chapter 4.

Unit	c_i (\$/MW ² h)	b_i (\$/MWh)	a_i (\$/h)	Start cost	$P_{i,min}$ (MW)	$P_{i,max}$ (MW)	$M_{i,up}$ (h)	$M_{i,down}$ (h)	Ini. stat.	Remark
1	0.000903	9.8	220	5552	100	330	6	5	6	Thermal unit
2	0.00903	10.7	157	4987	130	298	6	5	6	Thermal unit
3	0.00903	13.6	800	2453	120	154	6	5	6	Thermal unit
4	0.001457	14.8	547	989	95	123	6	5	6	Thermal unit
5	0.001323	15.2	532	2675	37	234	6	5	6	Thermal unit
6	0.005040	16.1	532	2985	37	246	6	5	6	Thermal unit
7	0.005040	16.1	590	3334	17	91	6	5	6	Thermal unit
8	0.032397	16.4	612	3789	25	95	6	5	6	Thermal unit
9	0.032397	17.1	580	2976	54	274	6	5	-5	Thermal unit
10	0.004701	17.1	377	2543	58	276	6	5	-5	Thermal unit
11	0.003498	17.7	670	3245	14	82	6	5	6	Thermal unit
12	0.021183	18.3	910	2650	22	159	6	5	6	Thermal unit
13	0.005576	19.5	155	500	55	114	6	5	-5	Thermal unit
14	0.000875	20.0	170	500	64	126	6	5	6	Thermal unit
15	0.000938	22.1	658	2870	14	100	6	5	-5	Thermal unit
16	0.002441	24.8	297	701	52	118	6	5	-5	Thermal unit
17	0.000764	25.2	103	632	28	62	6	5	-5	Thermal unit
18	NA	NA	NA	NA	7	92	NA	NA	NA	Hydro, limited water 230 MWh
19	NA	NA	NA	NA	7	63	NA	NA	NA	Hydro, limited water 100 MWh
20	NA	NA	NA	NA	45	75	NA	NA	NA	PS as generator
	NA	NA	NA	NA	70	70	NA	NA	NA	PS as pump
21	NA	NA	NA	NA	45	75	NA	NA	NA	PS as generator
	NA	NA	NA	NA	70	70	NA	NA	NA	PS as pump

Table B.9 Unit data for data for Example 3, Case B, Section 4.8, Chapter 4.

Unit	c_i (\$/MW ² h)	b_i (\$/MWh)	a_i (\$/h)	Start cost	$P_{i,min}$ (MW)	$P_{i,max}$ (MW)	$M_{i,up}$ (h)	$M_{i,down}$ (h)	Ini. stat.	Remark
1	0.000903	9.8	220	5552	100	330	3	3	3	Thermal unit
2	0.00903	10.7	157	4987	130	298	3	3	3	Thermal unit
3	0.00903	13.6	800	2453	120	154	3	3	3	Thermal unit
4	0.001457	14.8	547	989	95	123	3	3	3	Thermal unit
5	0.001323	15.2	532	2675	37	234	3	3	3	Thermal unit
6	0.005040	16.1	532	2985	37	246	3	3	3	Thermal unit
7	0.005040	16.1	590	3334	17	91	3	3	3	Thermal unit
8	0.032397	16.4	612	3789	25	95	3	3	3	Thermal unit
9	0.032397	17.1	580	2976	54	274	3	3	-3	Thermal unit
10	0.004701	17.1	377	2543	58	276	3	3	-3	Thermal unit
11	0.003498	17.7	670	3245	14	82	3	3	-3	Thermal unit
12	0.021183	18.3	910	2650	22	159	3	3	-3	Thermal unit
13	0.005576	19.5	155	500	55	114	3	3	-3	Thermal unit
14	0.000875	20.0	170	500	64	126	3	3	-3	Thermal unit
15	0.000938	22.1	658	2870	14	100	3	3	-3	Thermal unit
16	0.002441	24.8	297	701	52	118	3	3	-3	Thermal unit
17	0.000764	25.2	103	632	28	62	3	3	-3	Thermal unit
18	NA	NA	NA	NA	7	92	NA	NA	NA	Hydro, limited water 235 MWh
19	NA	NA	NA	NA	7	63	NA	NA	NA	Hydro, limited water 110 MWh

Table B.10 Load demand for Example 3, Case A and B, Section 4.8, Chapter 4.

Hour	1	2	3	4	5	6	7	8
Load level 1(MW), Case A	2057	2245	2248	2296	2322	2321	2194	2240
Load level 2(MW), Case B	1390	1460	1426	1452	1367	1417	1388	1567
Hour	9	10	11	12	13	14	15	16
Load level 1(MW), Case A	2276	2643	2788	2798	2455	2298	2176	2078
Load level 2(MW), Case B	1625	1701	1552	1464	1424	1358	1320	1285
Hour	17	18	19	20	21	22	23	24
Load level 1(MW), Case A	2040	1800	1634	1587	1569	1562	1615	1706
Load level 2(MW), Case B	806	816	644	919	859	908	1151	1304

APPENDIX C

RESULTS OF EXAMPLES

Table C.1 Solution of Example 4.3, Case A: 17 thermal units and 2 hydro plants, 2 pumped storage hydro plants; ELRP total cost: \$930,308.

Unit	Hour												
	0	1	2	3	4	5	6	7	8	9	10	11	12
1		330	330	330	330	330	330	330	330	330	330	330	330
2		298	298	298	298	298	298	298	298	298	298	298	298
3		154	154	154	154	154	154	154	154	154	154	154	154
4		123	123	123	123	123	123	123	123	123	123	123	123
5		234	234	234	234	234	234	234	234	234	234	234	234
6		246	246	246	246	246	246	246	246	246	246	246	246
7		91	91	91	91	91	91	91	91	91	91	91	91
8		47.3	66.3	66.6	78.7	86.0	85.7	60.3	65.7	73.0	95	95	95
9		54	55.4	55.8	67.9	75.2	74.9	54	54.9	62.2	121.7	121.7	121.7
10		251.3	276	276	276	276	276	276	276	276	276	276	276
11		82	82	82	82	82	82	82	82	82	82	82	82
12		27.4	56.5	57.0	75.5	86.7	86.3	47.4	55.6	66.8	157.7	157.7	157.7
13		55	106.9	108.8	114	114	114	72.3	103.8	114	114	114	114
14		64	126	126	126	126	126	126	126	126	126	126	126
15		0	0	0	0	0	0	0	0	0	100	100	100
16		0	0	0	0	0	0	0	0	0	52	52	52
17		0	0	0	0	0	0	0	0	0	0	0	0
18		0	0	0	0	0	0	0	0	0	42.7	88.5	91.8
19		0	0	0	0	0	0	0	0	0	0	48.3	51.7
20		0	0	0	0	0	0	0	0	0	0	50.8	54.2
21		0	0	0	0	0	0	0	0	0	0	0	0
22		0	0	0	0	0	0	0	0	0	0	0	0
22		0	0	0	0	0	0	0	0	0	0	0	0
ΣP_{ed}		2057	2245	2248	2296	2322	2321	2194	2240	2276	2643	2788	2798
Load		2057	2245	2248	2296	2322	2321	2194	2240	2276	2643	2788	2798
ΣP_{max}		2502	2502	2502	2502	2502	2502	2502	2502	2502	2812	2950	2950
Rese.		445	257	254	206	180	181	308	262	226	169	162	152

Table C.1 Solution of Example 4.3, Case A: 17 thermal units and 2 hydro plants, 2 pumped storage hydro plants; ELRP total cost: \$930,308 (*contd.*).

Unit	Hour												Sum
	13	14	15	16	17	18	19	20	21	22	23	24	
1	330	330	330	330	330	330	330	330	330	330	330	330	
2	298	298	298	298	298	298	298	298	298	298	298	298	
3	154	154	154	154	154	154	154	154	154	154	154	154	
4	123	123	123	123	123	123	123	123	123	123	123	123	
5	234	234	234	234	234	234	234	234	234	234	234	234	
6	246	246	246	246	246	246	246	241.5	246	246	246	246	
7	91	91	91	91	91	0	0	0	0	0	0	0	
8	89.9	64.8	57.4	61.5	56.1	0	0	0	0	0	0	0	
9	79.1	54.0	54	54	54	54	54	54	0	0	0	0	
10	276	276	276	276	276	276	195	152.5	190.0	183.0	166	250	800
11	82	82	82	82	82	0	0	0	0	0	0	0	
12	92.7	54.3	42.9	49.2	40.92	0	0	0	0	0	0	0	
13	114	98.8	55.3	79.3	55	85	0	0	0	0	0	0	
14	126	126	66.5	0	0	0	0	0	64	64	64	64	
15	67.3	14	14	0	0	0	0	0	0	0	0	0	
16	52	52	52	0	0	0	0	0	0	0	0	0	
17	0	0	0	0	0	0	0	0	0	0	0	0	
18	0	0	0	0	0	0	0	0	0	0	0	7	235
19	0	0	0	0	0	0	0	0	0	0	0	0	110
20	0	0	0	0	0	0	0	0	0	0	0	0	
21	0	0	0	0	0	0	0	0	0	0	0	0	
22	0	0	0	0	0	0	0	0	70	70	0	0	
22	0	0	0	0	0	0	0	0	0	0	0	0	
ΣP_{ed}	2455	2298	2176	2078	2040	1800	1634	1587	1569	1562	1615	1706	
Load	2455	2298	2176	2078	2040	1800	1634	1587	1569	1562	1615	1706	
ΣP_{max}	2720	2720	2720	2376	2376	1949	1835	1835	1617	1617	1687	1779	
Rese.	265	422	544	298	336	149	201	248	47.9	54.9	72	73	

Table C.2 Partially constrained solution of Example 4.3, Case B: 17 thermal units and 2 hydro plants; ELRP total cost: \$467,720.

[illegible]

Table C.2 Partially constrained solution of Example 4.3, Case B: 17 thermal units and 2 hydro plants; ELRP total cost: \$ 467,720 (*contd.*).

[illegible]

Table C.3 Fully constrained solution of Example 4.3, Case B: 17 thermal units and 2 hydro plants; ELRP total cost: \$487,888.

Unit	Hour												
	0	1	2	3	4	5	6	7	8	9	10	11	12
1	320	330	330	330	330	330	330	330	330	330	330	330	330
2	200	250	250	250	250	250	250	250	250	250	250	250	250
3	154	120	120	120	120	120	120	120	120	120	120	120	120
4	123	123	123	123	123	123	123	123	123	123	123	123	123
5	234	234	234	234	234	234	234	234	234	234	234	234	234
6	245	246	246	246	246	246	246	218	246	246	246	246	246
7	50	17	0	0	0	0	0	0	0	0	0	0	0
8	25	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	58	58	123.1	120.9	133	58	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	55.0	55.4	85.0	64	56.0	55	76.9	75.1	114	65.0	60.6
14	0	0	64.0	67.6	64.0	0	0	0	64.00	126	126	126	100.4
15	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	46.4	30.4	0	0	0	0	0	0	0	18.0	0	0
19	0	23.6	7.6	0	0	0	0	0	0	0	7	0	0
ΣP_{ed}	1351	1390	1460	1426	1452	1367	1417	1388	1567	1625	1701	1552	1464
ΣP_{max}		1390	1460	1426	1452	1367	1417	1388	1567	1625	1701	1552	1464

Table C.3 Fully constrained solution of Example 4.3, Case B: 17 thermal units and 2 hydro plants; ELRP total cost: \$487,888 (*contd.*).

Unit	Hour												Sum
	13	14	15	16	17	18	19	20	21	22	23	24	
1	330	330	330	330	330	330	256	330	330	330	330	330	
2	250	250	250	250	175	250	175	250	250	239	250	250	
3	120	120	120	120	120	120	120	120	120	120	120	120	
4	123	123	123	95.0	0	0	0	0	0	0	95.0	123	
5	234	234	234	198	123	116	93.0	168.0	95	155	230	234	
6	187	112	37.0	0	0	0	0	0	0	0	0	0	
7	0	0	0	0	0	0	0	0	0	0	0	0	
8	0	0	0	0	0	0	0	0	0	0	0	0	
9	0	0	0	0	0	0	0	0	0	0	0	0	
10	0	0	58	133	58	0	0	0	0	0	0	0	800
11	0	0	0	0	0	0	0	0	0	0	0	0	
12	0	0	0	0	0	0	0	0	0	0	0	0	
13	63.2	64.4	61.5	55.0	0	0	0	0	0	0	0	0	
14	116.8	124.6	106.5	64.0	0	0	0	0	64	64	126	126	
15	0	0	0	0	0	0	0	0	0	0	0	0	
16	0	0	0	0	0	0	0	0	0	0	0	0	
17	0	0	0	0	0	0	0	0	0	0	0	0	
18	0	0	0	31.4	0	0	0	36.9	0	0	0	71.9	235
19	0	0	0	8.6	0	0	0	14.1	0	0	0	49.1	110
ΣP_{ed}	1424	1358	1320	1285	806	816	644	919	859	908	1151	1304	
ΣP_{max}	1424	1358	1320	1285	806	816	644	919	859	908	1151	1304	

Table C.4 Solution of Example 4.4, Case A: Environmental limit of 125 ton/day, ELRP cost: \$458,544.

Unit	Hour												
	0	1	2	3	4	5	6	7	8	9	10	11	12
1	−1	0	0	0	0	0	0	0	0	0	0	0	0
2	−1	0	0	0	0	0	0	0	0	0	0	0	0
3	−1	0	0	0	0	0	0	0	0	0	0	0	0
4	−1	0	0	0	0	0	0	0	0	0	0	0	0
5	−1	0	0	0	0	0	0	0	0	0	0	0	0
6	−1	0	0	0	0	0	0	0	0	0	0	0	0
7	−1	0	0	0	0	0	0	0	0	0	0	0	0
8	−1	0	0	0	0	0	0	0	0	0	0	0	0
9	−1	0	0	0	0	0	0	0	0	0	0	0	0
10	3	0	0	0	0	0	0	0	15.2	15.2	15.2	32.35	38.24
11	3	0	0	0	0	0	0	0	15.2	15.2	15.2	30.18	35.94
12	3	0	0	0	0	0	0	0	0	15.2	15.2	28.22	33.88
13	3	0	0	0	0	0	0	0	0	15.2	15.2	26.11	31.65
14	−4	0	0	0	0	0	0	0	0	25	25	25	25
15	3	0	0	0	0	0	0	0	0	25	25	25	25
16	3	0	0	0	0	0	0	0	0	25	25	25	25
17	5	91.52	91.51	85.74	79.14	79.14	85.74	91.52	104.6	125.7	153.6	155	155
18	5	87.32	87.32	81.67	75.20	75.20	81.67	87.32	100.1	120.8	148.1	155	155
19	5	83.66	83.65	78.09	71.74	71.74	78.09	83.65	96.30	116.6	143.4	155	155
20	5	80.41	80.40	74.91	68.63	68.63	74.91	80.41	92.90	112.9	139.4	155	155
21	−4	0	0	0	0	0	0	0	0	0	0	0	0
22	−4	0	0	0	0	0	0	0	0	0	0	0	0
23	−4	0	0	0	0	0	0	0	0	0	0	0	0
24	10	222.1	222.1	204.7	184.7	184.7	204.7	222.1	261.9	325.7	350	350	350
25	10	400	400	400	400	400	400	400	400	400	400	400	400
26	10	400	400	400	400	400	400	400	400	400	400	400	400
27	NA	24.28	1.44	0	0	0	0	24.28	50	50	50	50	50
28	NA	24.28	1.44	0	0	0	0	24.28	50	50	50	50	50
29	NA	0	0	0	0	0	0	0	0	0	50	50	50
30	NA	0	0	0	0	0	0	0	50	50	50	50	50
31	NA	24.28	1.440	0	0	0	0	24.28	50	50	50	50	50
32	NA	24.28	1.440	0	0	0	0	24.28	50	50	50	50	50
SP_{ed}		1462	1370	1325	1279	1279	1325	1462	1736	1987	2170	2261	2284
Load		1462	1370	1325	1279	1279	1325	1462	1736	1987	2170	2261	2284
SP_{max}													
Rese.		607.7	699.1	744.8	790.5	790.5	744.8	607.7	485.6	686.2	503.5	412.1	389.2
Res. Req.		146.2	137.0	132.5	127.9	127.9	132.5	146.2	173.6	198.7	217.0	226.1	228.4
Emis.		4116	4116	3847	3542	3542	3847	4116	5065	7017	7944	8645	8831

Table C.4 Solution of Example 4.4, Case A: Environmental limit of 125 ton/day, ELRP cost: \$458,544 (contd.).

Unit	Hour												Sum
	13	14	15	16	17	18	19	20	21	22	23	24	
1	0	0	0	0	0	0	0	0	0	0	0	0	
2	0	0	0	0	0	0	0	0	0	0	0	0	
3	0	0	0	0	0	0	0	0	0	0	0	0	
4	0	0	0	0	0	0	0	0	0	0	0	0	
5	0	0	0	0	0	0	0	0	0	0	0	0	
6	0	0	0	0	0	0	0	0	0	0	0	0	
7	0	0	0	0	0	0	0	0	0	0	0	0	
8	0	0	0	0	0	0	0	0	0	0	0	0	
9	0	0	0	0	0	0	0	0	0	0	0	0	
10	32.35	38.24	38.24	20.35	15.2	15.2	15.2	15.2	15.2	15.2	15.2	0	
11	30.18	35.94	35.94	18.61	15.2	15.2	15.2	15.2	15.2	15.2	15.2	0	
12	28.22	33.88	33.88	16.84	15.2	15.2	15.2	15.2	15.2	15.2	0	0	
13	26.11	31.65	31.65	15.2	15.2	15.2	15.2	15.2	15.2	15.2	0	0	
14	25	25	25	25	25	25	25	25	25	25	25	0	
15	25	25	25	25	25	25	25	25	25	25	25	0	
16	25	25	25	25	25	25	25	25	25	25	0	0	
17	155	155	155	155	155	155	141.9	136.0	136.0	141.9	129.3	98.67	
18	155	155	155	155	155	155	136.6	130.8	130.8	136.6	124.3	94.32	
19	155	155	155	155	150.8	150.8	132.1	126.5	126.5	132.1	120.0	90.54	
20	155	155	155	155	146.7	146.7	128.3	122.7	122.7	128.3	116.3	87.21	
21	0	0	0	0	0	0	0	0	0	0	0	0	
22	0	0	0	0	0	0	0	0	0	0	0	0	
23	0	0	0	0	0	0	0	0	0	0	0	0	
24	350	350	350	350	350	350	350	350	350	350	336.7	243.8	
25	400	400	400	400	400	400	400	400	400	400	400	400	
26	400	400	400	400	400	400	400	400	400	400	400	400	
27	50	50	50	50	50	50	50	50	50	50	50	50	900
28	50	50	50	50	50	50	50	50	50	50	50	50	900
29	50	50	50	50	50	50	50	50	50	50	0	0	650
30	50	50	50	50	50	50	50	50	50	50	50	0	800
31	50	50	50	50	50	50	50	50	50	50	50	50	900
32	50	50	50	50	50	50	50	50	50	50	50	50	900
SP_{ed}	2261	2284	2284	2216	2193	2293	2124	2102	2102	2124	1987	1645	
Load	2261	2284	2284	2216	2193	2293	2124	2102	2102	2124	1987	1645	
SP_{max}													
Rese.	412.1	389.2	389.2	457.8	480.6	480.6	549.2	572.0	572.0	549.2	586.2	576.9	
Res. Req.	226.1	228.4	228.4	221.6	219.3	219.3	212.4	210.2	210.2	212.4	198.7	164.5	
Emis.	8645	8831	8831	8277	8106	8106	7622	7463	7463	7622	6984	4784	

Emission allowance = $125 \times 2,200 / 24 = 11,458$ kg/h.

Table C.5 Solution of Example 4.4, Case A: Environmental limit of 95 ton/day, ELRP cost: \$459,241.

Unit	Hour												
	0	1	2	3	4	5	6	7	8	9	10	11	12
1	−1	0	0	0	0	0	0	0	0	0	0	0	0
2	−1	0	0	0	0	0	0	0	0	0	0	0	0
3	−1	0	0	0	0	0	0	0	0	0	0	0	0
4	−1	0	0	0	0	0	0	0	0	0	0	0	0
5	−1	0	0	0	0	0	0	0	0	0	0	0	0
6	−1	0	0	0	0	0	0	0	0	0	0	0	0
7	−1	0	0	0	0	0	0	0	0	0	0	0	0
8	−1	0	0	0	0	0	0	0	0	0	0	0	0
9	−1	0	0	0	0	0	0	0	0	0	0	0	0
10	3	0	0	0	0	0	0	0	15.2	15.2	15.2	32.35	29.4
11	3	0	0	0	0	0	0	0	15.2	15.2	15.2	30.18	24.5
12	3	0	0	0	0	0	0	0	0	15.2	15.2	28.2	20.1
13	3	0	0	0	0	0	0	0	0	15.2	15.2	26.11	15.33
14	−4	0	0	0	0	0	0	0	0	25	25	25	52.97
15	3	0	0	0	0	0	0	0	0	25	25	25	41.76
16	3	0	0	0	0	0	0	0	0	25	25	25	30.5
17	5	91.52	91.51	85.74	79.14	79.14	85.74	91.52	104.6	125.7	153.6	155	155
18	5	87.32	87.32	81.67	75.20	75.20	81.67	87.32	100.2	120.8	148.1	155	155
19	5	83.65	83.65	78.09	71.74	71.74	78.09	83.65	96.30	116.6	143.4	155	155
20	5	80.41	80.40	74.91	68.63	68.63	74.9	80.41	92.90	112.9	139.4	155	155
21	−4	0	0	0	0	0	0	0	0	0	0	0	0
22	−4	0	0	0	0	0	0	0	0	0	0	0	0
23	−4	0	0	0	0	0	0	0	0	0	0	0	0
24	10	222.1	222.1	204.7	184.7	184.7	204.7	222.1	261.9	325.7	350	350	350
25	10	400	400	400	400	400	400	400	400	400	400	400	400
26	10	400	400	400	400	400	400	400	400	400	400	400	400
27	NA	24.28	1.440	0	0	0	0	24.28	50	50	50	50	50
28	NA	24.28	1.440	0	0	0	0	24.28	50	50	50	50	50
29	NA	0	0	0	0	0	0	0	0	0	50	50	50
30	NA	0	0	0	0	0	0	0	50	50	50	50	50
31	NA	24.28	1.440	0	0	0	0	24.28	50	50	50	50	50
32	NA	24.28	1.440	0	0	0	0	24.28	50	50	50	50	50
SP_{ed}		1462	1370	1325	1279	1279	1325	1462	1736	1987	2170	2261	2284
Load		1462	1370	1325	1279	1279	1325	1462	1736	1987	2170	2262	2285
SP_{max}													
Rese.		607.7	699.1	744.8	790.5	790.5	744.8	607.7	485.6	686.2	503.5	412.1	389.2
Res. Req.		146.2	137.0	132.5	127.9	127.9	132.5	146.2	173.6	198.7	217.0	226.1	228.4
Emis.		4116	4116	3847	3542	3542	3847	4116	5065	7017	7944.	8645	8707

Table C.5 Solution of Example 4.4, Case A: Environmental limit of 95 ton/day, ELRP cost: \$459,241 (contd.).

Unit	Hour												Sum
	13	14	15	16	17	18	19	20	21	22	23	24	
1	0	0	0	0	0	0	0	0	0	0	0	0	
2	0	0	0	0	0	0	0	0	0	0	0	0	
3	0	0	0	0	0	0	0	0	0	0	0	0	
4	0	0	0	0	0	0	0	0	0	0	0	0	
5	0	0	0	0	0	0	0	0	0	0	0	0	
6	0	0	0	0	0	0	0	0	0	0	0	0	
7	0	0	0	0	0	0	0	0	0	0	0	0	
8	0	0	0	0	0	0	0	0	0	0	0	0	
9	0	0	0	0	0	0	0	0	0	0	0	0	
10	32.3	29.4	29.4	20.35	15.2	15.2	15.2	15.2	15.2	15.2	15.2	0	
11	30.1	24.5	24.5	18.61	15.2	15.2	15.2	15.2	15.2	15.2	15.2	0	
12	28.221	20.1	20.14	16.84	15.2	15.2	15.2	15.2	15.2	15.2	0	0	
13	26.11	15.33	15.33	15.2	15.2	15.2	15.2	15.2	15.2	15.2	0	0	
14	25	52.97	52.97	25	25	25	25	25	25	25	25	0	
15	25	41.76	41.76	25	25	25	25	25	25	25	25	0	
16	25	30.5	30.5	25	25	25	25	25	25	25	0	0	
17	155	155	155	155	155	155	141.9	136.0	136.0	141.9	129.37	98.677	
18	155	155	155	155	155	155	136.6	130.8	130.9	136.6	124.37	94.329	
19	155	155	155	155	150.8	150.8	132.1	126.5	126.5	132.2	120.09	90.546	
20	155	155	155	155	146.7	146.7	128.3	122.73	122.73	128.31	116.39	87.213	
21	0	0	0	0	0	0	0	0	0	0	0	0	
22	0	0	0	0	0	0	0	0	0	0	0	0	
23	0	0	0	0	0	0	0	0	0	0	0	0	
24	350	350	350	350	350	350	350	350	350	350	336.71	243.84	
25	400	400	400	400	400	400	400	400	400	400	400	400	
26	400	400	400	400	400	400	400	400	400	400	400	400	
27	50	50	50	50	50	50	50	50	50	50	50	50	900
28	50	50	50	50	50	50	50	50	50	50	50	50	900
29	50	50	50	50	50	50	50	50	50	50	0	0	650
30	50	50	50	50	50	50	50	50	50	50	50	0	800
31	50	50	50	50	50	50	50	50	50	50	50	50	900
32	50	50	50	50	50	50	50	50	50	50	50	50	900
SP_{ed}	2261	2284	2284	2216	2193	2293	2124	2102	2102	2125	1988	1645	
Load	2262	2284.	2284.	2216	2193	2293	2124	2102	2102	2125	1988	1645	
SP_{max}													
Rese.	412.1	389.3	389.3	457.8	480.6	480.6	549.2	572.0	572.0	549.2	586.3	577	
Res. Req.	226.1	228.4	228.4	221.6	219.3	219.3	212.4	210.2	210.2	212.4	198.7	164.5	
Emis.	8646	8708	8708	8278	8106	8106	7623	7463.	7463	7623	6984	4784.	

Emission allowance = $95 \times 2200 / 24 = 8708$ kg/h.

Table C.6 Solution of Example 4.4, Case B-1: Environmental limit of 95 ton/day, ELRP cost: \$471,977.60.

Unit	Hour												
	0	1	2	3	4	5	6	7	8	9	10	11	12
1		0	0	0	0	0	0	0	0	0	0	0	0
2		0	0	0	0	0	0	0	0	0	0	0	0
3		0	0	0	0	0	0	0	0	0	0	0	0
4		0	0	0	0	0	0	0	0	0	0	0	0
5		0	0	0	0	0	0	0	0	0	0	0	0
6		0	0	0	0	0	0	0	0	0	0	0	0
7		0	0	0	0	0	0	0	0	0	0	0	0
8		0	0	0	0	0	0	0	0	0	0	0	0
9		0	0	0	0	0	0	0	0	0	0	0	0
10		0	0	0	0	0	0	0	27.7	50.5	76	76	76
11		0	0	0	0	0	0	0	25.6	47.9	76	76	76
12		0	0	0	0	0	0	0	0	46.2	76	76	76
13		0	0	0	0	0	0	0	0	43.7	76	76	76
14		0	0	0	0	0	0	0	0	25	44.9	69.4	75.5
15		0	0	0	0	0	0	0	0	25	37.6	62.5	68.7
16		0	0	0	0	0	0	0	0	25	30.1	55.6	62.0
17		85.1	85.1	85.1	79.1	79.1	85.1	100	155	155	155	155	155
18		81.0	81.0	81.0	75.2	75.2	81.0	100	155	155	155	155	155
19		77.5	77.5	77.5	71.7	71.7	77.5	77.5	77.5	77.5	77.5	78.6	79.4
20		74.3	74.3	74.3	68.6	68.6	74.3	74.3	74.32	74.3	74.3	75.4	76.2
21		0	0	0	0	0	0	0	0	0	0	0	0
22		0	0	0	0	0	0	0	0	0	0	0	0
23		0	0	0	0	0	0	0	0	0	0	0	0
24		202.8	202.8	202.8	184.7	184.7	202.8	202.8	202.8	202.8	202.8	206.3	208.9
25		400	400	400	400	400	400	400	400	400	400	400	400
26		400	400	400	400	400	400	400	400	400	400	400	400
27		29.5	12.3	1.10	0	0	1.1	23.8	42.3	49.2	50	50	50
28		29.5	12.3	1.10	0	0	1.1	23.8	42.3	49.2	50	50	50
29		5.6	0	0	0	0	0	0	18.5	25.4	39.3	50	50
30		17.9	0.8	0	0	0	0	12.3	30.8	37.6	50	50	50
31		29.5	12.3	1.1	0	0	1.10	23.8	42.3	49.2	50	50	50
32		29.5	12.3	1.1	0	0	1.1	23.8	42.3	49.2	50	50	50
ΣP_{gd}		1462.2	1370.8	1325.1	1279.4	1279.4	1325.1	1462.2	1736.4	1987.8	2170.5	2261.8	2284.7
Rese.		307.8	399.2	444.9	642.6	642.6	596.9	459.8	337.6	386.3	400.5	369.1	286.3

Table C.6 Solution of Example 4.4, Case B-1: Environmental limit of 95 ton/day, ELRP cost: \$471,977.60 (contd.).

Unit	Hour													Sum
	13	14	15	16	17	18	19	20	21	22	23	24		
1	0	0	0	0	0	0	0	0	0	0	0	0		
2	0	0	0	0	0	0	0	0	0	0	0	0		
3	0	0	0	0	0	0	0	0	0	0	0	0		
4	0	0	0	0	0	0	0	0	0	0	0	0		
5	0	0	0	0	0	0	0	0	0	0	0	0		
6	0	0	0	0	0	0	0	0	0	0	0	0		
7	0	0	0	0	0	0	0	0	0	0	0	0		
8	0	0	0	0	0	0	0	0	0	0	0	0		
9	0	0	0	0	0	0	0	0	0	0	0	0		
10	76	76	76	76	76	76	76	75.3	75.3	76	57.1	15.2		
11	76	76	76	76	76	76	76	72.2	72.2	76	54.4	15.2		
12	76	76	76	76	76	76	76	70.2	70.2	76	52.6	0		
13	76	76	76	76	76	76	76	67.1	67.1	76	49.9	0		
14	69.4	75.6	75.6	57.2	51.0	51.0	25.1	25	25	25.1	25	0		
15	62.5	68.7	68.7	50.0	43.8	43.8	25	25	25	25	25	0		
16	55.6	62.0	62.0	42.8	36.5	36.5	25	25	25	25	0	0		
17	155	155	155	155	155	155	155	155	155	155	155	115.6		
18	155	155	155	155	155	155	155	155	155	155	155	130.7		
19	78.6	79.4	79.4	77.5	77.5	77.5	77.5	77.5	77.5	77.5	77.5	77.5		
20	75.4	76.2	76.2	74.3	74.3	74.3	74.3	74.3	74.3	74.3	74.3	74.3		
21	0	0	0	0	0	0	0	0	0	0	0	0		
22	0	0	0	0	0	0	0	0	0	0	0	0		
23	0	0	0	0	0	0	0	0	0	0	0	0		
24	206.3	208.9	208.9	202.8	202.8	202.8	202.8	202.8	202.8	202.8	202.8	202.8		
25	400	400	400	400	400	400	400	400	400	400	400	400		
26	400	400	400	400	400	400	400	400	400	400	400	400		
27	50	50	50	50	50	50	50	50	50	50	49.1	41.5	900	
28	50	50	50	50	50	50	50	50	50	50	49.1	41.5	900	
29	50	50	50	47.5	43.4	43.4	34.4	32.6	32.6	34.39	25.27	17.7	650	
30	50	50	50	50	50	50	46.7	44.9	44.9	46.7	37.5	29.9	800	
31	50	50	50	50	50	50	50	50	50	50	49.1	41.5	900	
32	50	50	50	50	50	50	50	50	50	50	49.1	41.5	900	
ΣP_{ed}	2261.8	2284.7	2284.7	2216.2	2193.3	2193.3	2124.8	2101.9	2101.9	2124.8	1987.7	1645.0		
Rese.	309.1	286.3	740.3	748.8	771.7	771.7	643.2	272.1	272.1	249.2	386.3	429.0		

Table C.7 Solution of Example 4.4, Case B-1: Environmental limit of 95 ton/day, ELRP cost: \$490,922.50.

Unit	Hour												
	0	1	2	3	4	5	6	7	8	9	10	11	12
1		0	0	0	0	0	0	0	0	0	0	0	0
2		0	0	0	0	0	0	0	0	0	0	0	0
3		0	0	0	0	0	0	0	0	0	0	0	0
4		0	0	0	0	0	0	0	0	0	0	0	0
5		0	0	0	0	0	0	0	0	0	0	0	0
6		0	0	0	0	0	0	0	0	0	0	0	0
7		0	0	0	0	0	0	0	0	0	0	0	0
8		0	0	0	0	0	0	0	0	0	0	0	0
9		0	0	0	0	0	0	0	0	0	0	0	0
10		0	0	0	0	0	0	0	27.9	66.4	76	76	76
11		0	0	0	0	0	0	0	25.4	63.9	76	76	76
12		0	0	0	0	0	0	0	0	69.2	76	76	76
13		0	0	0	0	0	0	0	0	66.1	76	76	76
14		0	0	0	0	0	0	0	0	0	44.9	69.4	75.5
15		0	0	0	0	0	0	0	0	0	37.6	62.5	68.7
16		0	0	0	0	0	0	0	0	0	30.1	55.6	62.0
17		97.7	92.4	85.7	79.1	79.1	85.7	100	155	155	155	155	155
18		93.4	88.1	81.7	75.2	75.2	81.7	100	155	155	155	155	155
19		89.6	84.5	78.1	71.7	71.7	78.1	89.6	94.8	94.8	94.9	98.1	98.9
20		86.3	81.2	74.9	68.6	68.6	74.9	86.3	91.4	91.4	91.5	94.7	95.5
21		0	0	0	0	0	0	0	0	0	0	0	0
22		0	0	0	0	0	0	0	0	0	0	0	0
23		0	0	0	0	0	0	0	0	0	0	0	0
24		240.8	224.7	204.7	184.7	184.7	204.8	240.9	257.2	257.2	257.5	267.6	270.1
25		400	400	400	400	400	400	400	400	400	400	400	400
26		400	400	400	400	400	400	400	400	400	400	400	400
27		27.2	0	0	0	0	0	22.8	50	50	50	50	50
28		27.2	0	0	0	0	0	22.8	50	50	50	50	50
29		0	0	0	0	0	0	0	0	19.5	50	50	50
30		0	0	0	0	0	0	0	29.8	49.3	50	50	50
31		0	0	0	0	0	0	0	0	0	0	0	0
32		0	0	0	0	0	0	0	0	0	0	0	0
33		0	0	0	0	0	0	0	0	0	0	0	0
34		0	0	0	0	0	0	0	0	0	0	0	0
ΣP_{ed}		1462.2	1370.8	1325.1	1279.4	1279.4	1325.1	1462.2	1736.4	1987.7	2170.5	2261.9	2284.7
Load		1462.3	1370.8	1325.1	1279.4	1279.4	1325.1	1462.2	1736.4	1987.7	2170.5	2261.9	2284.7
Rese.		307.8	399.2	444.9	642.6	642.6	596.9	459.8	337.6	386.3	400.5	369.1	286.3
Time resp. rese.		146.2	137.1	132.5	127.9	127.9	132.5	146.2	173.6	198.8	217.1	226.2	228.5

Table C.7 Solution of Example 4.4, Case B-1: Environmental limit of 95 ton/day, ELRP cost: \$490,922.50 (contd.).

Unit	Hour												Sum
	13	14	15	16	17	18	19	20	21	22	23	24	
1	0	0	0	0	0	0	0	0	0	0	0	0	
2	0	0	0	0	0	0	0	0	0	0	0	0	
3	0	0	0	0	0	0	0	0	0	0	0	0	
4	0	0	0	0	0	0	0	0	0	0	0	0	
5	0	0	0	0	0	0	0	0	0	0	0	0	
6	0	0	0	0	0	0	0	0	0	0	0	0	
7	0	0	0	0	0	0	0	0	0	0	0	0	
8	0	0	0	0	0	0	0	0	0	0	0	0	
9	0	0	0	0	0	0	0	0	0	0	0	0	
10	76	76	76	76	76	76	76	76	76	76	76	70.4	0
11	76	76	76	76	76	76	76	76	76	76	76	67.4	0
12	76	76	76	76	76	76	76	76	76	76	76	65.4	0
13	76	76	76	76	76	76	76	76	76	76	76	62.4	0
14	69.4	75.5	75.5	57.2	51.0	51.0	32.7	26.6	26.6	32.7	0	0	
15	62.5	68.7	68.8	50.0	43.8	43.8	25.1	18.9	18.9	25.1	0	0	
16	55.6	62.0	62.0	42.8	36.5	36.5	17.3	10.9	10.9	17.3	0	0	
17	155	155	155	155	155	155	155	155	155	155	155	155	132.8
18	155	155	155	155	155	155	155	155	155	155	155	155	147.1
19	98.1	98.9	98.9	96.5	95.7	95.7	94.8	94.8	94.8	94.8	94.8	94.8	
20	94.7	95.5	95.5	93.1	92.3	92.3	91.4	91.4	91.4	91.4	91.4	91.4	
21	0	0	0	0	0	0	0	0	0	0	0	0	
22	0	0	0	0	0	0	0	0	0	0	0	0	
23	0	0	0	0	0	0	0	0	0	0	0	0	
24	267.6	270.1	270.1	262.6	260.1	260.1	257.2	257.2	257.2	257.2	257.2	257.2	
25	400	400	400	400	400	400	400	400	400	400	400	400	
26	400	400	400	400	400	400	400	400	400	400	400	400	
27	50	50	50	50	50	50	50	50	50	50	50	50	900
28	50	50	50	50	50	50	50	50	50	50	50	50	900
29	50	50	50	50	50	50	42.3	38.2	38.2	42.3	19.51	0	650
30	50	50	50	50	50	50	50	50	50	50	49.2	21.7	800
31	0	0	0	0	0	0	0	0	0	0	0	0	
32	0	0	0	0	0	0	0	0	0	0	0	0	
33	0	0	0	0	0	0	0	0	0	0	0	0	
34	0	0	0	0	0	0	0	0	0	0	0	0	
ΣP_{ed}	2261.9	2284.7	2284.7	2216.2	2193.3	2193.3	2124.8	2101.9	2101.9	2124.8	1987.7	1645.0	
Load	2261.9	2284.7	2284.7	2216.2	2193.3	2193.3	2124.8	2101.9	2101.9	2124.8	1987.7	1645.0	
Rese.	309.1	286.3	740.3	748.8	771.7	771.7	643.2	272.1	272.1	249.2	386.3	429.0	
Time resp. rese.	226.2	228.5	228.5	221.6	219.3	219.3	212.5	210.2	210.2	212.5	198.8	164.5	

Table C.8 Solution for hydrothermal system with 17 thermal, 2 hydro and 2 pumped-storage units of Case A in Example 4.5 by EMO-ALHN method.

Unit	Hour												
	0	1	2	3	4	5	6	7	8	9	10	11	12
1	330	330	330	330	330	330	330	330	330	330	330	330	330
2	298	298	298	298	298	298	298	298	298	298	298	298	298
3	154	154	154	154	154	154	154	154	154	154	154	154	154
4	123	123	123	123	123	123	123	123	123	123	123	123	123
5	234	234	234	234	234	234	234	234	234	234	234	234	234
6	246	246	246	246	246	246	246	246	246	246	246	246	246
7	91	91	91	91	91	91	91	91	91	91	91	91	91
8	0	0	0	0	0	0	0	0	0	0	95	95	95
9	65	126.1	126.1	126.1	126.0	126.2	122.0	125.7	126.5	126.2	126.1	126.2	126.2
10	276	276	276	276	276	276	276	276	276	276.0	276.0	276.0	276
11	0	0	0	0	0	0	0	0	0	0	82	82	82
12	0	0	0	0	0	0	0	0	0	0	159	159	159
13	114	114	114.0	114.0	114	114	114	114	114	114.0	114	114.0	114
14	126	126	126.0	126.0	126	126	126	126	126	126	126	126.0	126
15	0	0	0	0	0	0	0	0	0	0	0	100	100
16	0	95.0	95.2	95.3	94.0	96.2	52.0	89.9	100.5	95.8	95.4	95.7	95.7
17	0	0	0	44.9	41.8	47.1	28	32.5	57.0	46.0	45.1	45.8	45.8
H1	0.0	21.3	23.5	25.7	12.3	41.9	0	0	0	32.9	27.1	32.3	32.3
H2	0	10.6	11.2	11.9	8.2	17.6	0	0	0	14.2	12.3	14.0	14.0
G1	0	0	0	0	47.5	0	0	0	0	0	53.9	56.0	56.0
G2	0	0	0	0	0	0	0	0	0	0	0	0	0
P1	0	0	0	0	0	0	0	0	0	0	0	0	0
P2	0	0	0	0	0	0	0	0	0	0	0	0	0
ΣP_{gd}	2057	2245	2248	2296	2322	2321	2194	2240	2276	2643	2788	2798	2798
Load	2057	2245	2248	2296	2322	2321	2194	2240	2276	2643	2788	2798	2798
ΣP_{max}	2266	2539	2539	2601	2676	2601	2446	2446	2446	2937	3112	3112	3112
Rese.	209	294	291	305	354	280	252	206	170	294	324	314	314

Table C.8 Solution for hydrothermal system with 17 thermal, 2 hydro and 2 pumped-storage units of Case A in Example 4.5 by EMO-ALHN method (*contd.*).

Unit	Hour												Sum
	13	14	15	16	17	18	19	20	21	22	23	24	
1	330	330	330	330	330	330	330	330	330	330	330	330	
2	298	298	298	298	298	298	298	298	298	298	298	298	
3	154	154	154	154	154	154	154	154	154	154	154	154	
4	123	123	123	123	123	123	123	123	123	123	123	123	
5	234	234	234	234	234	234	234	234	234	234	234	234	
6	246	246	246	246	246	246	246	246	246	246	246	246	
7	0	0	0	0	0	0	0	0	0	0	0	0	
8	95	89.4	67.2	0	0	0	0	0	0	0	0	0	
9	120.9	78.9	56.7	80.4	126.0	0	0	0	0	0	0	0	
10	276	276	276	276	276	276	194	217	199	192	175	202.0	
11	82	82	82	0	0	0	0	0	0	0	0	0	
12	156.1	92.1	58.0	0	0	0	0	0	0	0	0	0	
13	114.0	114	111.1	114.0	114	58.3	55	55	55	55	55	55	
14	126	126	126	126	126	80.7	0	0	0	0	0	64	
15	100	54.6	14	96.6	0	0	0	0	0	0	0	0	
16	0	0	0	0	0	0	0	0	0	0	0	0	
17	0	0	0	0	0	0	0	0	0	0	0	0	
H1	0	0	0.0	0.0	13.0	0	0	0	0	0	0.0	0.0	230
H2	0	0	0.0	0.0	0	0	0	0	0	0	0.0	0.0	110
G1	0	0	0	0	0	0	0	0	0	0	0	0	
G2	0	0	0	0	0	0	0	0	0	0	0	0	
P1	0	0	0	0	0	0	0	-70	-70	-70	0	0	
P2	0	0	0	0	0	0	0	0	0	0	0	0	
ΣP_{ed}	2455	2298	2176	2358	2040	1940	1634	1727	1709	1702	1615	1846	
Load	2455	2298	2176	2078	2040	1800	1634	1587	1569	1562	1615	1706	
ΣP_{max}	2611	2611	2611	2275	2267	1901	1775	1705	1705	1705	1775	1901	
Rese.	156	313	435	197	227	101	141	118	136	143	160	195	

Table C.9 Solution for hydrothermal system with 17 thermal and 2 hydro units of Case B in Example 4.5 by EMO-ALHN method.

Unit	Hour												
	0	1	2	3	4	5	6	7	8	9	10	11	12
1	320	330	330	330	330	330	330	330	330	330	330	330	330
2	200	250	250	250	250	250	250	250	250	250	250	250	250
3	154	120	120	120	120	120	120	120	120	120	120	120	120
4	123	123	123	123	123	123	123	123	123	123	123	123	123
5	234	234	234	234	234	234	234	234	234	234	234	234	234
6	245	246	246	246	246	191	241	212	246	246	246	246	230
7	50	17	0	0	0	0	0	0	0	0	0	0	0
8	25	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	58	108.9	158.0	83.0	58
11	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	55	93	55.9	59.8	55	55	55	80	87.1	114	62.3	55
14	0	0	64	67.1	89.2	64	64	64	126	126	126	103.7	64
15	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0
H1	0	15.0	0	0	0	0	0	0	0	0	0	0	0
H2	0	0	0	0	0	0	0	0	0	0	0	0	0
ΣP_{ed}	1351	1390	1460	1426	1452	1367	1417	1388	1567	1625	1701	1552	1464
ΣP_{max}		1600	1543	1543	1543	1543	1543	1543	1819	1819	1819	1819	1819

Table C.9 Solution for hydrothermal system with 17 thermal and 2 hydro units of Case B in Example 4.5 by EMO-ALHN method (*contd.*).

Unit	Hour												Sum
	13	14	15	16	17	18	19	20	21	22	23	24	
1	330	330	330	330	297	330	308	330	330	330	330	330	
2	250	250	250	250	175	250	175	250	250	250	250	250	
3	120	120	120	120	120	120	120	120	120	120	120	120	
4	123	123	123	95	0	0	0	95	118	113	123	123	
5	234	234	234	234	159	116	41	116	41	37	112	187	
6	246	187	112	37	0	0	0	0	0	0	0	0	
7	0	0	0	0	0	0	0	0	0	0	0	0	
8	0	0	0	0	0	0	0	0	0	0	0	0	
9	0	0	0	0	0	0	0	0	0	0	0	0	
10	0	0	0	0	0	0	0	0	0	58	133	143.1	800
11	0	0	0	0	0	0	0	0	0	0	0	0	
12	0	0	0	0	0	0	0	0	0	0	0	0	
13	57.0	114	113.1	113.7	55	0	0	0	0	0	0	55	
14	64	0	0	0	0	0	0	0	0	0	0	0	
15	0	0	0	0	0	0	0	0	0	0	0	0	
16	0	0	0	0	0	0	0	0	0	0	0	0	
17	0	0	0	0	0	0	0	0	0	0	0	0	
H1	0	0	25.1	68.4	0	0	0	8	0	0	55.4	63.1	235
H2	0	0	12.8	36.9	0	0	0	0	0	0	27.6	32.8	110
ΣP_{ed}	1424	1358	1320	1285	806	816	644	919	859	908	1151	1304	
ΣP_{max}	1543	1417	1572	1572	1048	934	934	1149	1057	1333	1488	1602	

Table C.10 Solution for hydrothermal system with 17 thermal, 2 hydro and 2 pumped-storage units of Case A in Example 4.6 by ALHN-LR method.

Unit	Hour												
	0	1	2	3	4	5	6	7	8	9	10	11	12
1		330	330	330	330	330	330	330	330	330	330	330	330
2		298	298	298	298	298	298	298	298	298	298	298	298
3		154	154	154	154	154	154	154	154	154	154	154	154
4		123	123	123	123	123	123	123	123	123	123	123	123
5		234	234	234	234	234	234	234	234	234	234	234	234
6		246	246	246	246	246	246	246	246	246	246	246	246
7		91	91	91	91	91	91	91	91	91	91	91	91
8		94.7	95	95	95	95	95	92.2	95	95	95	95	95
9		84.3	113	116	147	126.2	126.2	81.8	125	127	152.2	152.4	152.4
10		276	276	276	276	276	276	276	276	276	276	276	276
11		0	0	0	0	0	0	0	0	0	0	82	82
12		0	0	0	0	0	0	0	0	0	159	159	159
13		0	114	114	114	114	114	114	114	114	114	114	114
14		126	126	126	126	126	126	126	126	126	126	126	126
15		0	0	0	0	0	0	0	0	0	0	0	0
16		0	0	0	0	0	0	0	0	0	118	118	118
17		0	0	0	62	48.4	48.2	28	28	62	62	62	62
H1		0	0	0	0	0	0	0	0	0	52.9	87.5	89.6
H2		0	0	0	0	0	0	0	0	0	11.9	40.1	48.0
G1		0	45	45	0	60.4	59.6	0	0	0	0	0	0
G2		0	0	0	0	0	0	0	0	0	0	0	0
P1		0	0	0	0	0	0	0	0	0	0	0	0
P2		0	0	0	0	0	0	0	0	0	0	0	0
ΣP_{ed}		2057	2245	2248	2296	2322	2321	2194	2240	2276	2643	2788	2798
Load		2057	2245	2248	2296	2322	2321	2194	2240	2276	2643	2788	2798
ΣP_{max}		2247	2436	2436	2423	2498	2498	2423	2423	2423	2855	2937	2937
Rese.		190	191	188	127	176	177	229	183	147	212	149	139

Table C.10 Solution for hydrothermal system with 17 thermal, 2 hydro and 2 pumped-storage units of Case A in Example 4.5 by EMO-ALHN method (*contd.*).

Unit	Hour												Sum
	13	14	15	16	17	18	19	20	21	22	23	24	
1	330	330	330	330	330	330	330	330	330	330	330	330	
2	298	298	298	298	298	298	298	298	298	298	298	298	
3	154	154	154	154	154	154	154	154	154	154	154	154	
4	123	123	123	123	123	123	123	123	123	123	123	123	
5	234	234	234	234	234	234	234	234	234	234	234	234	
6	246	246	246	246	246	246	246	246	246	246	246	246	
7	91	91	91	91	0	0	0	0	0	0	0	0	
8	95	0	0	0	0	0	0	0	0	0	0	0	
9	92.7	79.2	49	49.1	139	0	0	0	0	0	0	0	
10	276	276	276	276	276	276	244	262.9	249	242	225	276	
11	82	82	82	82	0	0	0	0	0	0	0	0	
12	113.3	92.8	46.6	0	0	0	0	0	0	0	0	0	
13	114	114	68.5	69	114	57.3	5	9.1	5	5	5	51	
14	126	126	126.0	126.0	126	81.7	0	0	0	0	0	64	
15	0	0.0	0	0.0	0	0	0	0	0	0	0	0	
16	52	52	52	0	0	0	0	0	0	0	0	0	
17	28	0	0	0	0	0	0	0	0	0	0	0	
H1	0	0	0	0	0	0	0	0	0	0	0	0	230
H2	0	0	0	0	0	0	0	0	0	0	0	0	110
G1	0	0	0	0	0	0	0	0	0	0	0	0	
G2	0	0	0	0	0	0	0	0	0	0	0	0	
P1	0	0	0	0	0	0	0	70	70	70	0	70	
P2	0	0	0	0	0	0	0	0	0	0	0	0	
ΣP_{ed}	2455	2298	2176	2358	2040	1940	1634	1727	1709	1702	1615	1846	
Load	2455	2298	2176	2078	2040	1800	1634	1587	1569	1562	1615	1706	
ΣP_{max}	2782	2625	2625	2348	2175	1901	1775	1705	1705	1705	1775	1831	
Rese.	327	327	449	270	135	101	141	118	136	143	160	125	

Table C.11 Solution for hydrothermal system with 17 thermal and 2 hydro units of Case B in Example 4.6 by ALHN-LR method.

Unit	Hour												
	0	1	2	3	4	5	6	7	8	9	10	11	12
1	320	330	330	330	330	330	330	330	330	330	330	330	330
2	200	250	250	250	250	250	250	250	250	250	250	250	250
3	154	120	120	120	120	120	120	120	120	120	120	120	120
4	123	123	123	123	123	123	123	123	123	123	123	123	123
5	234	234	234	234	234	234	234	234	234	234	234	234	234
6	245	246	246	246	246	191	241	212	246	246	246	246	230
7	50	17	0	0	0	0	0	0	0	0	0	0	0
8	25	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	58	123.1	130.0	113.8	58
11	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	55	93	55.9	59.8	55	55	55	66.9	65.3	70.0	57.8	55
14	0	0	64	67.1	89.2	64	64	64	125.0	119.6	126.0	77.4	64
15	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0
H1	0	15	0	0	0	0	0	0	7	7	52.5	0	0
H2	0	0	0	0	0	0	0	0	7	7	19.5	0	0
ΣP_{avl}	1351	1390	1460	1426	1452	1367	1417	1388	1567	1625	1701	1552	1464
ΣP_{max}		1682	1625	1625	1625	1625	1625	1625	2056	2056	2056	1901	1901

Table C.11 Solution for hydrothermal system with 17 thermal and 2 hydro units of Case B in Example 4.6 by ALHN-LR method (*contd.*).

Unit	Hour												Sum
	13	14	15	16	17	18	19	20	21	22	23	24	
1	330	330	330	330	290	330	330	330	330	330	330	330	
2	298	298	298	298	223	298	223	298	298	298	298	298	
3	0	0	0	0	0	0	0	0	0	0	0	0	
4	123	123	123	95	0	0	0	0	0	0	0	0	
5	234	234	234	234	159	129	54	129	194	185	234	234	
6	246	246	216	209	134	59	37	112	37	37	112	187	
7	0	0	0	0	0	0	0	0	0	0	0	0	
8	0	0	0	0	0	0	0	0	0	0	0	0	
9	0	0	0	0	0	0	0	0	0	0	0	0	
10	0	0	0	0	0	0	0	0	0	58	128.1	131.0	800
11	0	0	0	0	0	0	0	0	0	0	0	0	
12	0	0	0	0	0	0	0	0	0	0	0	0	
13	65.5	56.5	55	55	0	0	0	0	0	0	0	0	
14	120.5	70.5	64	64	0	0	0	0	0	0	0	0	
15	0	0	0	0	0	0	0	0	0	0	0	0	
16	0	0	0	0	0	0	0	0	0	0	0	0	
17	0	0	0	0	0	0	0	0	0	0	0	0	
H1	0	0	0	0	0	0	0	36.5	0	0	35.6	81.4	235
H2	7	0	0	0	0	0	0	13.5	0	0	13.3	42.6	110
ΣP_{ed}	1424	1358	1320	1285	806	816	644	919	859	908	1151	1304	
ΣP_{max}	1534	1471	1471	1471	1108	1108	1108	1263	1108	1384	1539	1539	

APPENDIX D

TIPS FOR PROGRAMMING IN MATLAB

D.1 USEFUL TIPS FOR PROGRAMMING

Matlab is a technical programming language which is widely used by researchers due to convenience in programming and powerfulness in computation. The user interface of Matlab is simple and it is easy to learn for programming, especially for beginners. Matlab provides many powerful toolboxes developed for certain fields suitable for students and researchers to their researches. The basis of computation in Matlab is based on matrix operations. However, the performance of computation in Matlab is slower than that of some other programming languages such as C or Delphi. Therefore, speeding up the calculation in Matlab is also a very important work when coding an algorithm. The purpose of this appendix is not a tutorial on Matlab but to present the users some important tips to enhance the calculation and simplify the programs coded in Matlab. These tips are especially useful for large scale problems in terms of computational time and programming simplicity. By adhering to these tips, the user can save considerable computational time compared to the normal programming of the same algorithm. The difference in computational time can be clearly observed in large-scale problems with several variables. Moreover, with complex algorithms, these tips can help to considerably reduce the length of the program to ease its management. Moreover, readers can develop new ideas for faster calculation further based on these tips.

The essential idea to speed up to calculation in Matlab that we would like to introduce consists of the following points:

- *Try to use the built-in functions:* The built-in functions are the ones directly coded in Matlab; their source code cannot be viewed by the user as opposed to the functions developed in the toolboxes which are separately included in Matlab and whose source code can be viewed by the users by the *.m tag. The advantage of the built-in functions is that they perform a task faster than the functions in toolboxes do. Therefore, in programming the users should try to use the built-in functions or their combination first before choosing functions from toolboxes. To identify whether a function is a built-in one or not, use the command *type* <function name>. On the other hand, using the functions available in Matlab, either built-in functions or toolbox functions, would be better than functions developed by the user since the included functions are mature in terms of correctness and fast performance time.
- *Apply matrix operations in programming:* The basic calculation of Matlab is based on matrix. Therefore, matrix operations are most suitable for Matlab coded algorithms to provide for fast computation and simple programming.

Usually, people try to avoid using loops in programming since they are time consuming, especially loops in loops in large scale problems. When using a loop in Matlab, the program reserves a dynamic memory for dynamic variables and the memory is resized during performing the loop. This makes fix sized variables associated with fix memory more preferable as they allow for faster calculation. Consequently, matrix operations are more efficient than using loops. However, not all loops can be replaced by matrix operations. In such cases, matrix operations should be used wherever possible to reduce the computation time.

- *Simplify the program:* If a task is programmed using many commands, the performance of the program will be slower than of one using combined commands. Moreover, a compact and clear program is less likely to complicate programming management in terms of comprehensibility and performance time.

To learn how to use a certain command, use *help* <function name> for further information. Some tips may be useful for readers using Matlab version 7.1 or later as follows:

- *Functions and scripts:*

Scripts are the simplest kind of M-files in Matlab because they need no input or output arguments. They are useful for automating series of Matlab commands which are repeatedly performed from the command line. Functions are program routines usually run in M-files that accept input arguments and return output arguments. They operate on variables within their own workspace which is separate from the workspace you access at the MATLAB command prompt. When an M-file function is called, Matlab parses and stores it in the memory. Consequently, it runs faster on subsequent calls. The M-file functions are better than scripts as they make use of their own local variables and accept input arguments even though both M-file functions and scripts are similar.

- *Vector, matrix and cell:*

In Matlab, most operations are based on vectors and matrices. A vector has only one dimension while a matrix has two dimensions. A vector can be also considered as a specific case of a matrix with one row or one column. A cell can have more than two dimensions, in which each element can be a number, a vector or a matrix.

For example, a row vector can be created in two ways as follows:

```
V = [2 3 4];
V = ([2 3 4]);
```

A matrix can be also created in two ways:

```
M = [6 7 8; 10 11 12];
M = ([6 7 8; 10 11 12]);
```

A cell can be created by the combination of vectors and matrices:

```
C = {[1] [2 3 4]; [5; 9] [6 7 8; 10 11 12]};
```

To access an element in a vector or a matrix, the address of the element (i, j) to be accessed is put in parentheses, e.g., $V(i)$ and $M(i, j)$ for accessing a specific element in a vector or matrix, respectively. To access an element in a cell, the address of the element (i, j) to be accessed is put in braces, e.g., $C\{i, j\}$.

Moreover, many elements in a vector can be accessed by positioning their address via a vector of addresses. The obtained result is also a vector:

```
pos = [i, j, m:n]; % Positions of elements
V1 = V(pos);
```

In a similar way, rows or columns of elements of a matrix can be accessed, for example:

```
pos = [i, j, m:n]; % Pos. of row or columns
M1 = M(pos, :); % Access in rows
M2 = M(:, pos); % Access in columns
```

A cell array of matrices can be converted into a single matrix using the `cell2mat` command or vice versa using the `mat2cell` command.

A matrix or a row vector can be converted into a column vector using:

```
V = M(:); % Convert a matrix into a vector
```

- *Subscript and index:*

Subscript is the most common method used to refer to matrix elements, for example: $M(i, j)$ refers to row i , column j of matrix M while an index is usually used to refer to elements in a vector. There is no difference in the use of index between row and column vectors. Moreover, the index method is also an alternative method for referring to elements in a matrix. The conversion between the two methods can be performed using `sub2ind` or `ind2sub` command. However, these are M-file functions, hence, they are more efficient for direct computation of the conversions. For a two dimension matrix M having the size of $I \times J$, the conversion between subscript $M(i, j)$ and index $M(\text{index})$ gives:

$$M(i, j) \Leftrightarrow M(i + (j - 1) * I)$$

$$M(\text{index}) \Leftrightarrow A(\text{rem}(\text{index} - 1, M) + 1, \text{floor}((\text{index} - 1) / M) + 1)$$

The vector subscript can be also used for the subscript of a matrix, for example: The command $M([i, k], :)$ refers to row i and row k of matrix M .

- *Basic operators:*

The types of operators used in Matlab are necessarily differentiated for a proper use in programming such as arithmetic operators, relational operators, logical operators and set operators. Each type of operators is applied to certain objects.

- The arithmetic operators include:

Plus (plus or +): Used for adding the numerical and logic values with syntax `plus(a,b)` or `(a+b)`.

Minus (minus or -): Used for subtracting the numerical and logic values with syntax `minus(a,b)` or `(a-b)`.

Matrix multiply (`mtimes` or *): Used for multiplying between two matrices or numbers with the syntax `mtimes(A,B)` or `A*B`.

Array multiply (`times` or .*): Used for multiplying element by element between two matrices or two vectors of the same size with the syntax `times(A,B)` or `A.*B`.

Slash or right matrix divide (`mrdivide` or /): Used for dividing between two matrices or numbers with the syntax `mrdivide(A,B)` or `A/B`.

Right array divide (`rdivide` or ./): Used for dividing element by element between two matrices or two vectors of the same size with the syntax `rdivide(A,B)` or `A./B`.

Matrix power (`mpower` or ^): Used for raising a power of a square matrix or a number with the syntax `mpower(A,n)` or `A^n`.

Array power (`power` or .^): Used for raising a power of each element in a vector or a matrix with the syntax `power(A,n)` or `A.^n`.

- The relational operators include:

Equal (`eq` or ==): Used for comparing two values or two characters and returning a logical value of 1 if the comparison expression is true or 0 otherwise, with the syntax `eq(a,b)` or `(a==b)`.

Not equal (`ne` or ~=): This operator is the opposite of the operator *equal* with the syntax `ne(a,b)` or `(a~=b)`.

Less than (`lt` or <): Used for comparing two values or two characters and returning a logical value of 1 if the comparison expression is true or 0 otherwise, with the syntax `lt(a,b)` or `(a<b)`.

Greater than (`gt` or >): This operator is the opposite of the *less than* operator with the syntax `gt(a,b)` or `(a>b)`.

Less than or equal (`le` or <=): This operator is similar to the *less than* operator with the syntax `le(a,b)` or `(a<=b)`.

Greater than or equal (`ge` or >=): This operator is similar to the *greater than* operator with the syntax `ge(a,b)` or `(a>=b)`.

- The logical operators include:

Element-wise logical AND (`and` or &): Performs a logical AND of vector or scalar inputs and returns a vector containing elements set to either logical 1 (true) or logical 0 (false). An element of the output vector is set to 1 if all input

vectors contain a nonzero element at that same vector location. Otherwise, that element is set to 0. The syntax is `and (A, B)` or `A&B`.

Short-circuit logical AND (`&&`): Returns logical 1 (true) if both inputs evaluated are true, and logical 0 (false) if they are not. This operator evaluates its second operand only when the result from the first operand is fully determined. This is useful for reducing the time for processing the logical expression and avoiding divide-by-zero errors in computation.

Element-wise logical OR (`or` or `|`): Finds logical OR of vector or scalar inputs and returns a vector containing elements set to either logical 1 (true) or logical 0 (false). An element of the output vector is set to 1 if any input vectors contain a nonzero element at the same vector location. Otherwise, that element is set to 0. The syntax is `or (A, B)` or `(A|B)`.

Short-circuit logical OR (`||`): Returns logical 1 (true) if either input, or both, is true, and logical 0 (false) if they are not.

Logical NOT (`not` or `~`): Finds logical NOT of vector or scalar input and returns a vector containing elements set to either logical 1 (true) or logical 0 (false). An element of the output vector is set to 1 if the input vector contains a zero value element at the same vector location. Otherwise, that element is set to 0. The syntax is `not (A)` or `(~A)`.

- The set operators include:

Set union (`union`): Finds a set union of two vectors, matrix columns or matrix rows and returns the combined values of the inputs but without repetitions. The syntax is `union (A, B)`.

Set unique (`unique`): Finds unique elements of a vector, matrix column or matrix row and returns the same values as the input but without repetitions. The syntax is `unique (A)`.

Set intersection (`intersect`): Finds a set intersection of two vectors, matrix columns or matrix rows and returns the values common to the inputs. The syntax is `intersect (A, B)`.

Set difference (`setdiff`): Finds set difference of two vectors, matrix columns or matrix rows and returns the values in a vector that are not in the other. The syntax is `setdiff (A, B)`.

True for set member (`ismember`): Used to check whether vector elements are contained in a set or not and returns a vector of the same length as the input vector, containing logical 1 (true) where the elements of the input vector are in the set, and logical 0 (false) elsewhere. The syntax is `ismember (A, S)` where *A* is a vector and *S* is a set.

- Other operators

Matrix transpose ('): This is the linear algebraic transpose of a matrix. For complex matrices, this is the complex conjugate transpose. This operator can be also used for the vector transpose. The syntax is A' .

Colon (:): This operator is very popular in Matlab for indexing elements in a vector or matrix. For example, to index the first row of a matrix, the following statement is used $A(1, :)$. On the other hand, this operator is also used for creating a vector with a pre-specified rule. For instance, a vector with the dimension of N can be created with $V = 0:2:N$ or $V = N:-1:0$, etc.

Brackets ([]): Brackets are used to form vectors and matrices. Moreover, it is useful for recording the historical values with the number of elements not known in advance. For example, to record the historical values of a variable, the following statement is used $X = [X \text{ new_value}]$. A new matrix can be also formed based on a basic matrix. For instance, $M1 = [M \ M; \ M \ M]$ is to form a new matrix $M1$ with the dimension of double rows and columns of the basic matrix M .

Continuation (...): Used at the end of command line to show that the command is continued in the next line, applies to long command lines.

Semicolon (;): Used inside brackets to end rows or after an expression or statement to suppress printing or to separate statements.

- Find the maximum or minimum element in a vector or a matrix: In Matlab, finding elements in a vector or matrix is very simple and fast due to the developed built-in functions. To find the maximum value of an element regardless of its position, the command is as follows:

```
max_val = max(V);
max_val = max(M(:));
```

In these commands, the two-column sign (:) shows that the task will be performed on all elements in the matrix. Without this sign, Matlab will only find the maximum element for each column. This command is equivalent to $\max(\max(M))$. Therefore, it is not necessary to use this sign in finding the maximum element of a vector since a vector has only one dimension. If we need to find both value and position of an element in a vector or matrix, the following commands are used:

```
[max_val, max_pos] = max(V);
[max_val, max_pos] = max(M(:));
```

However, the returned position of the maximum element in matrix M is the absolute address. That means the value $\max_pos$ includes both numbers of row and column in form of $\max_pos = (\max_col-1)*N + \max_row$. To find the relative position of the found element in the matrix, a further computation is:

```
max_col = ceil(max_pos/N);
max_row = max_pos - (max_col-1)*N;
```

In the above command, `ceil(x)` is an operator to round x to the upper bound integer. The position of the maximum elements and their values are also found by the `find` command:

```
[max_col,max_row,max_vals] = find(M==max_val);
```

In this command, the relative position of the maximum element is determined by finding the element with a value not less than `max_val`.

To find the maximum element for each row or column in a matrix, the following control parameters are added:

```
max_val = max(M,[],1); % for each column
max_val = max(M,[],2); % for each row
```

By default, the command `max(M)` is to find the maximum elements in matrix M for each column. If we need to find the maximum element in a row or column of a matrix, the applied operation is similar to a vector operation:

```
max_val = max(M(i,:)); for row i
max_val = max(M(:,j)); for column j
```

In addition, the command `max` in Matlab can be used to replace elements in a matrix by a certain value where the values of the elements in the matrix are smaller than the value of the element to be replaced by the following command:

```
N = max(M,x);
```

In this command, replacements occur at those positions where the value of elements in matrix M are smaller than x , e.g., $M(i,j) < x$. In other words, the minimum value of all elements in the matrix M will be set to x by this command.

For the `min` operator, the performance is similar to the `max` operator but in the opposite meaning.

- *Calculate a sum or mean of all elements in a vector or matrix:*

The computation of a sum of all elements in a vector or matrix is easily performed using the command `sum` in Matlab. For a vector, its sum of elements is calculated as follows:

```
S = sum(V);
```

For a matrix M , the sum of all its elements is calculated by:

```
S = sum(M(:));
```

Moreover, the command `sum` can be flexibly used for matrix computations. The sum of elements in a matrix can be calculated by row or column as follows:

```
Sr = sum(M,1); % by row
Sc = sum(M,2); % by column
```

The command `sum` can also be used to make a sum of a group of elements in a vector or in a row or column of a matrix by specifying the range for computation. For example, the sum of elements from row/column `I` to `J` in a matrix is calculated:

```
Sij = sum(M(I:J,:)); % by row
Sij = sum(M(:,I:J)); % by column
```

The command `mean` in Matlab is used to calculate the average value of elements in a vector or matrix. The mean of a vector is equivalent to the sum of its elements divided by the number of elements in that vector. The method for the mean of a matrix is similar. The `mean` command can also be used to calculate the average value of elements in a matrix by row or column.

```
mean(V) ⇔ sum(V)/length(V);      % for vector
mean(M) ⇔ sum(M)/sum(size(M));   % for matrix
mean(M(1,:)) ⇔ sum(M(1,:))/size(M,1);
                                     % for 1st matrix row
mean(M(:,1)) ⇔ sum(M(:,1))/size(M,2);
                                     % for 1st matrix column
```

Note that the result from `size(M,1)` is equivalent to `length(M(1,:))` and `size(M,2)` equivalent to `length(M(:,1))`.

- *Create a matrix from a basic vector or matrix:*

Matlab usually applies vector and matrix operations in its computations. Operations between two matrices are only performed if these have the same size. Therefore, a basic vector or matrix sometimes needs to be converted to a matrix of a larger size for convenience in computation.

A basic vector can be converted into a matrix using `ones` command which is used to create a vector or matrix whose elements all have a value of 1. Suppose that we have to create a matrix `M` of the size of `[length(V) x J]` based on the basic column vector `V` having the size of `[length(V) x 1]`, the following command is used:

```
M = V*ones(1,J);
```

Similarly, if we create a matrix of the size of `[I x length(V)]` from the basic row vector `V` having the size of `[1 x length(V)]`, the computation is performed as follows:

```
M = ones(I,1)*V;
```

Note: If a square matrix `M` of the size of `[I x I]` is initialized with all elements being 1, one of the following commands can be used:


```
M = ones(I);
M = ones(I,I);
```

A matrix can be created based on a basic matrix using the `repmat` command. If the size of the new matrix is not much greater than the size of the basic matrix, operator `[]` can be used for a faster computation in which the basic matrix is considered an element in the new matrix.

On the other hand, a matrix can be converted to a vector using:

```
V = M(:);
```

- *Use matrix operations to replace loops:*

Suppose we calculate a product of a matrix `M[size(M,1) × size(M,2)]` and a row vector `V[1 × size(M,2)]` where each element of vector `V` is multiplied by a corresponding column of matrix `M`. To perform this computation, the following algorithm may be used:

```
W = zeros(size(M,1), size(M,2));
for j = 1:size(M,2)
    W(:,j) = M(:,j)*V(j);
end
```

However, to avoid the `for` loop, vector `V` is first converted into a matrix and then multiplied element by element with matrix `M` as follows:

```
W = M.*(ones(size(M,1),1)*V);
```

Similarly, when we need to calculate a product of a column vector `V[size(M,1) × 1]` with a matrix `M[size(M,1) × size(M,2)]` where each element of vector `V` is multiplied by a corresponding row of matrix `M`, the `for` loop can be avoided by converting vector `V` into a matrix and then multiplying the corresponding elements of the two matrices.

```
W = (V*ones(1,size(M,2)))*V;
```

When we need to multiply every column of matrix `M` of size `[M × N]` by another column vector `V` with the dimension `M`, the followed statement can be tried:

```
M1 = diag(sparse(V))*M;
```

where `diag(V)` is applied to create a square matrix with the main diagonal giving the elements of vector `V`.

Another example is finding the minimum distance from a set of points `(x, y, z)` in a space to the origin. The following algorithm can be applied:

```
dist = zeros(length(x),1); % Preallocation
for k = 1:length(x)
    dist(k) = sqrt(x(k)^2 + y(k)^2 + z(k)^2);
end
dist_min = min(dist);
```

However, the algorithm can be further improved by using the array operation to avoid the use of the loop. A combined command is used:

```
dist_min = min(sqrt(x.^2 + y.^2 + z.^2));
```

- *Eliminate an element out of a vector or a row or a column out of a matrix:*
An element or a vector can be easily eliminated out of a vector using the `setdiff` command. However, the `setdiff` command is not a built-in one. An element can be quickly eliminated out of a vector using the combined command with built-in functions:

```
V1 = V(find((a==V)<1));
```

where `a` is the element to be eliminated out of vector `V`.

In this command, the expression `(a==V)<1` returns the value 0 at the position where `a` is equal to an element in `V` and value 1 at the rest of the positions. Therefore, the expression `find((a==V)<1)` returns the position index of nonzero elements in vector `V`. Finally, `V1` contains the elements of vector `V` except the position where element `a` is equal to an element in vector `V`.

Element `a` is also eliminated from vector `V` by an alternative way:

```
V(find(a==V)) = [];
```

In this way, a row or a column is also easily deleted from a matrix, for example:

```
M(i,:) = []; % Delete row i
M(:,j) = []; % Delete column j
```

- *Use inequality operators instead of equality operators:*
Equality operators sometimes are used to replace equality operators to speed up computation since inequality operators are performed faster than equality operators. For example, elements of vector `V` have only two values 0 and 1. To speed up computation, we can use the greater operator (`>`) or less than operator (`<`) to replace the equal operator (`==`), where `V>0` implies that the elements of `V` have the value of 1 and `V<1` implies that the elements of `V` have the value of 0.
- *Seek and replace elements in a vector or matrix:*
Elements of a vector or matrix can be sought and replaced by other values using `max` and `min` commands. Other operators including `>`, `<` and `==` are also used for seeking and replacing values of elements in a vector or matrix. For example, the negative values of vector `V` are replaced by value 0 using `V(V<0) = 0`. A similar command is applied to a matrix. These operators can be used to replace an `if` statement. For example, there are statements like this:

```
for i = 1:length(V)
    if V(i) > Vmax(i)
```

```

        V(i) = Vmax(i);
    elseif V(i) < Vmin(i)
        V(i) = Vmin(i);
    end
end

```

These statements can be replaced by:

```

V(V>Vmax) = Vmax(V>Vmax) ;
V(V<Vmin) = Vmin(V<Vmin) ;

```

An alternative way can be tried as follows:

```

V = min(max(V,Vmin),Vmax);

```

- *Operation between a number and a logical result:*

In other programming languages, an operation between a number and a logical result may not be allowed. However, in Matlab this operation can be performed. Therefore, it is useful for counting in programming. For example, we need to count how many elements of vector V are greater than 0. The advantage of the combined operations in Matlab allows for the following command:

```

count = sum(V>0);

```

In this command, the expression $V>0$ returns the logical values of 1 for the elements of V greater than 1 and 0 otherwise, and the `sum` command makes a sum of all the returned values, resulting in the total number of elements in vector V greater than 0.

For another example, assume we need to calculate a sinc function defined as follows: $\text{sinc}(x) = \sin(x)/x$ if $x \neq 0$ and $\text{sinc}(x) = 1$ otherwise. The function can be calculated by:

```

sinc_x = (sin(x) + (x == 0))./(x + (x == 0));

```

An alternative way for computing this function is:

```

y = ones(size(x));
z = find(x ~= 0);
y(z) = sin(x(z))./x(z);

```

- *Other useful tips*

- Rescale the entries of a vector or matrix to a certain span [Min,Max]

```

min = min(V(:));
max = max(V(:));
V = (Max-Min) * (V-min)/(max-min) + Min;

```

- Pre-allocation of a vector or matrix

A vector or matrix should be pre-specified in advance by the `zeros` command to prevent Matlab from automatically resizing the vector and matrix, leading to extended time consumed. For example:

```
a = zeros(1,N); % Preallocation
b = zeros(1,N);
a(1) = 1;
b(1) = 0;
for k = 2:N
    a(k) = a(k-1) + b(k-1);
    b(k) = a(k-1) - b(k-1);
end
```

In case the final array size varies, use the upper bound on the array size and cut the excess after the loop. For example:

```
a = zeros(1,N); % Preallocate
count = 0;
for k = 1:N
    V = exp(rand(1) rand(1));
    if V > 0.5
        count = count + 1;
        a(count) = V;
    end
end
a = a(1:count); % Trim the result
```

- Eliminate coinciding elements from a vector

Coinciding elements can be deleted from a vector using the `unique(V)` command. Another inlined command can be used:

```
V1 = sort(V(:));
V1(find(V1((1:end-1)')==V1((2:end)')))=[];
```

where `end` is the last vector index.

- Repeat a column vector over the columns n times

A column vector V can be repeated over the columns n times by the `repmat(V,1,n)` command. Another way can also be applied to avoid calling an M-file function:

```
V1 = V(:,ones(1,n));
```

- Repeat a row vector over the rows m times

A row vector V can be repeated over the rows m times using the `repmat(V,m,1)` command. Again, another way can also be applied to avoid calling an M-file function:

```
V1 = V(ones(m,1),:);
```

- Repeat a scalar *a* into an *m*×*n* matrix

A scalar *a* can be repeated to form an *m*×*n* matrix by using the `repmat(a,m,n)` command. Other ways can also be applied to avoid calling an M-file function:

```
M = a(ones(m,n));
```

```
M = a + zeros(m,n);
```

```
M = a*ones(m,n);
```

- Swap rows or column of a matrix

The rows and columns in a matrix can be interchanged to each other. For example, the rows or columns of a matrix can be interchanged by using:

```
M([i,j],:) = M([j,i],:) % interchanges rows i and j
                        of matrix M
```

```
M(:, [i,j]) = M(:, [j,i]) % interchanges columns i and
                        j of matrix M
```

For a square matrix, the rows and columns of a matrix can be exchanged together using the `transpose` function.

- Operation on a subgroup of a vector

The vectors can be divided into subgroups and operations can be easily performed on these subgroups. For example, *V* is a vector of the size of 100 and we need to compute the sum of each group of 5 elements. To compute this sum, loops may be applied. However, the following inline is faster:

```
P = sum(reshape(V,[20 5]));
```

- Vectorization to replace loops

For example:

```
for i = 1:100
    for j = 1:100
        r(i,j) = sqrt(i^2+j^2);
    end
end
```

The computation time is shorter if the above statements are replaced by:

```
[i,j] = meshgrid(1:100,1:100);
r = sqrt(i.^2+j.^2);
```

- Reverse a vector

A vector can be reversed using the `flipdim` function. Another way is:

```
V = V(end:-1:1);
```

- *Some other useful commands in Matlab:*

Matlab has several powerful functions and utilities including built-in functions and M-file functions that make work a lot simpler and more convenient for users. Functions that can be used include:

sort: Sorts elements of an array in ascending or descending order. The syntax is `sort(A)`. If A is a vector, the function sorts the elements of A. If A is a matrix, the function sorts the columns of A.

sortrows: Sorts rows of a matrix in ascending order. The syntax is `sortrows(M, col)` sorting rows of matrix M in ascending order of elements in column col.

reshape: Reshapes an array to a vector or a matrix. The syntax is `reshape(A, m, n)` where A is an array and m and n define the size of the new array.

prod: Gives the product of elements in an array. The syntax is `prod(A)`. If A is a vector, the function returns the product of elements in the vector. If A is a matrix, the function returns the product of elements by column with `prod(A, 1)` or by row with `prod(A, 2)`.

norm: Computes the Euclidean length of a vector with the syntax `norm(V)`.

isvector: Determines whether the input is a vector, returning logical value 1 if the input is a 1xN or Nx1 vector and logical value 0 otherwise. The syntax is `isvector(V)`.

isempty: Determines whether the input is empty, returning logical value 1 if the input is an empty array and logical value 0 otherwise. The syntax is `isempty(A)`.

rand: Generates uniformly distributed pseudorandom numbers in the range [0,1]. The syntax is `rand(m, n)` where m and n define the size of the matrix to be created. To scale a randomly created vector with the size of n to a certain range [a, b], the computation is `a + (b-1)*rand(n)`.

inv: Inverses a square matrix with the syntax `inv(M)`.

eye: Creates an identity matrix with the syntax `eye(n)` where n is the size of the identity matrix.

num2str: Converts a number to a string with the syntax `num2str(A)` where A is an array. The opposite function is `str2num` which converts a string into a number.

plot: Creates a 2-D line plot. To plot 2-D lines in the same x-axis but different y-axis, the function `plotyy` is used. To plot the data at a logarithmic scale of the x-axis, the function `semilogx` is used.

disp: Displays text or array. When writing formatted data to file or displaying them on screen, the function `fprintf` is applied.

`clc`: Clears the Command Window. Similar to `clc`, `home` provides a clear screen but by moving the prompt down, but it does not clear the text so you can still scroll up to see it.

`clear`: Removes items from workspace and frees up system memory.

`who` and `whos`: `who` lists the current workspace variables and `whos` lists the variables plus information about size and class.

`tic` and `toc`: These stopwatch timer functions are used to measure how long a program takes to run or to compare the speed of different implementations of a program. The syntax is:

```
tic;
<Run the program section to be timed >
toc;
```

- *Symbols, subscript and superscript*

Matlab allows representing symbols, subscript and superscript in the title or axis labels of graphics.

Subscript and superscript characters: For subscript characters, use syntax “_”, for superscript characters syntax “^” directly followed by the characters or substring defined in braces.

Font style: The font style can be specified in the title of graphics using the syntax `\bf` for bold font and `\it` for italic font.

Symbols: Greek letters can also be used in titles of graphics in Matlab by using the syntax `\` directly followed by the letter name. The names of Greek letters can be found in the Matlab help.

Example, the command `title('\itAe^{\alpha}\itt {sin\beta}\itt, \alpha<<\beta')` will result in the display of Ae^{α} , $\alpha \ll \beta$ in the graphic title.

For a Matlab tutorial, readers can refer to the *User's Guide* available at the website <http://www.mathworks.com/access/helpdesk> and *How do I increase the speed or performance of MATLAB?* available at <http://www.mathworks.com/support/solutions/data/1-15NM7.html?solution=1-15NM7>.

D.2 SIMPLE EXAMPLES USING MATLAB

Example 1: ALHN for Solving an ED Problem

In this example, an ALHN algorithm coded in Matlab is applied to an ED problem including 3 generating units to supply a load demand of 210 MW with transmission loss coefficients provided. The purpose of this example is to provide the readers an overlook of how to apply the tips on Matlab use and the simple ALHN algorithm derived. Even for large scale ED problems scheduled over a 24-hour time horizon, a program can be easily developed based on this example.

```

% Clear the screen and variables
clear;
clc;

% Start counting time
tic;

% Generating unit data
N = 3;
ai = [213.1; 200; 240];
bi = [11.669; 10.333; 10.833];
ci = [0.00533; 0.00889; 0.00741];
Pimax = [200; 150; 180];
Pimin = [50; 37.5; 45];

% Load demand
PD = 210;

% Power loss coefficients
Bij = 1e-2*
    [0.06760 0.00953 -0.00507
     0.00953 0.05210  0.00901
    -0.00507 0.00901  0.02940];
B0i = [-0.0766; -0.00342; 0.0189];
B00 = 1e2*0.040357;

% Selection of parameters for ALHN
sigma = 100;
alpha_i = 0.01;
alpha_l = 0.004;
beta = 0.001;

% Parameters for iterations
maxiter = 2500;
curriter = 0;
maxerr = 10; % Choose high error at the beginning
accur = 1e-4;

% Initialization the records for errors
power_error = [];
model_error = [];
maxerror = [];
totalcost = [];
iter = [];

```



```

% =====
% Initialization for continuous neurons
Vi = ones(N,1)*PD.*Pimax./sum(Pimax);
Ui = real(log((Vi - Pimin)./(Pimax - Vi))/2/sigma);

% Initialization for Lagrange multiplier neurons
dPL_dVi = 2*Bij*Vi + B0i;
Ul = mean((bi + 2*ci.*Vi)./(1 - dPL_dVi));
Vl = Ul;

% =====
% Computation of initial values
PL = diag(Vi'*Bij*Vi + Vi'*B0i + B00);
dP = PD + PL - sum(Vi);
dPL_dVi = 2*B*Vi + B0i;
dE_dVi = (bi + 2*ci.*Vi).*(1 - dPL_dVi);

% =====
% Start the loop for whole algorithm
while (curriter <= maxiter) & (maxerr >= accur)
    curriter = curriter + 1;

    % Calculate dE/dVi at curriter-1 step
    dPL_dVi = 2*Bij*Vi + B0i;
    dE_dVi = (bi + 2*ci.*Vi) + ones(N,1)*(Vl +
        beta*dP).*(dPL_dVi - 1) + Ui;

    % Calculate dE/dVl at curriter-1 step
    dE_dVl = dP;

    % Calculate Ui and Ul at step curriter
    Ui = Ui - alpha_i*dE_dVi;
    Ul = Ul + alpha_l*dE_dVl;

    % Store the previous values
    Vi_1 = Vi;
    Vl_1 = Vl;

    % Calculate Vi and Vl at step curriter
    Vi = (Pimax - Pimin).*(1 + tanh(sigma.*Ui))/2 +
        Pimin;
    Vl = Ul;

```

```

% Calculate the power balance constraints errors
PL = diag(Vi'*B*Vi + Vi'*B0 + B00);
dP = PD + PL - sum(Vi);
max_dP = max(abs(dP(:)));
power_error = [power_error max_dP];

% Calculate the model errors
del_Vi = Vi - Vi_1;
del_Vl = Vl - Vl_1;

max_del = max([max(abs(del_Vi(:))) abs(del_Vl)]);
model_error = [model_error max_del];

% Calculate maximum error
maxerr = max([max_dP max_del]);
maxerror = [maxerror maxerr];

% Calculate cost
TC = sum(ai + bi.*Vi + ci.*Vi.^2);
totalcost = [totalcost TC];

iter = [iter curriter];
end % end of the loop

% Display the result
maxerr
curriter
fprintf('\n%6.2f\n\n',CT);

toc;      % Stop counting time

% Draw the convergence characteristics
figure(1); % 1st graph for power error
semilogx(iter,power_error);
xlabel(['Number of iterations = ',
num2str(curriter)]);
ylabel('Power balance constraints errors');
title(['min = ', num2str(min(power_error(:))), ',
max = ', num2str(max(power_error(:))), ', current
      = ', num2str(max(abs(dP(:))))]);
grid;

```

```

figure(2); % 2nd graph for model error
semilogx(iter,model_error);
xlabel(['Number of iterations = ',
num2str(curriter)]);
ylabel('Iteration errors');
title(['min = ', num2str(min(model_error(:))), ',
max = ', num2str(max(model_error(:))), ', current = ',
num2str(max_del)]);
grid;

figure(3); % 3rd graph for maximum error
plot(iter,maxerror);
xlabel(['Total number of iterations = ',
num2str(curriter)]);
ylabel('Maximum error');
title(['min = ', num2str(min(maxerror(:))), ', max
= ', num2str(max(maxerror(:))), ', current = ',
num2str(maxerr)]);
grid;

figure(4); % Last graph for total cost
plot(iter,totalcost);
xlabel(['Total number of iterations = ',
num2str(curriter)]);
ylabel('Total cost');
title(['min = ', num2str(min(totalcost(:))), ', max
= ', num2str(max(totalcost(:))), ', current = ',
num2str(CT)]);
grid;

```

Example 2: PSO for Solving ED Problems

In this example, a PSO algorithm coded in Matlab is applied to an ED problem with 3 generating units without power losses on transmission lines considered. Based on this example, readers may develop the program further themselves to solve larger scale ED problems scheduled over a certain period of time.

```

clear;
clc;

% Data for thermal units
D = 3;
ad = [459 310 78];

```

```

bd = [6.48 7.85 7.97];
cd = [0.00128 0.00194 0.00482];
Pdmax = [600 400 200];
Pdmin = [150 100 50];
PD = 850;

% Converting from vector to matrix
aid = ones(N,1)*ad;
bid = ones(N,1)*bd;
cid = ones(N,1)*cd;

% Initial values of PSO parameters
K = 100; % penalty factor
N = 100; % number of agents
c1 = 2;
c2 = 2;
wmax = 0.9;
wmin = 0.4;
itmax = 200; % Maximum iteration number

% Initialization for PSO
xidmax = ones(N,1)*Pdmax;
xidmin = ones(N,1)*Pdmin;
vidmax = 0.5*xidmax;
vidmin = -vidmax;
iter = 1:itmax;
w = wmax - (wmax - wmin)*iter/itmax;

% *****
% Initial positions of agents are randomly
% selected from the range (xmin,ximax)
x = xidmin + (xidmax-xidmin).*rand(N,D);

% Initial velocities of agents are randomly
% selected from the range (vmin,vimax)
v = vidmin + (vidmax - vidmin).*rand(N,D);

% Function to be minimized
f = sum(aid + bid.*x + cid.*x.^2,2) + K*(PD -
      sum(x,2)).^2;

```

```

% Initial best values of position and f
pbest = x;
fpbest = f;
[fgbest,ind] = min(fpbest);
gbest = pbest(ind,:);
% Store the historic values
Gbest = gbest;
Fgbest(1) = fgbest;

% Parameters for iterations
Errmax = 0.01;
Maxerr = 10;
k = 1;

% *****
% Iterative process
while k<itmax & Maxerr>Errmax
    k = k + 1;

    % velocity at iteration k cal. from v(k-1)
    v = w(k-1)*v + c1*rand(N,D).*(pbest - x) +
        c2*rand(N,D).*(ones(N,1)*gbest - x);
    v(v>vidmax) = vidmax(v>vidmax);
    v(v<vidmin) = vidmin(v<vidmin);

    % position at iteration k cal. from x(k-1)
    x = x + v;
    x(x>xidmax) = xidmax(x>xidmax);
    x(x<xidmin) = xidmin(x<xidmin);

    % Fitness function
    f = sum(aid + bid.*x + cid.*x.^2,2) + K*(PD
        - sum(x,2)).^2;

    % Find the best position each of agents
    fpbest_1 = fpbest;
    pbest = (f>fpbest_1)*ones(1,D).*pbest +
        (f<=fpbest_1)*ones(1,D).*x;
    fpbest = (f>fpbest_1).*fpbest_1 +
        (f<=fpbest_1).*f;

```

```
% Find global best position of all agents
fgbest_1 = fgbest;
[fgbest,ind] = min(fpbest);
gbest = pbest(ind,:);

% Calculate maximum error
Maxerr = max(abs(PD - sum(gbest)),abs(fgbest
        - fgbest_1));
% Store the historic values
Gbest = gbest;
Fgbest(k) = fgbest;

end

% Plot the convergence process of gbest
plot(1:k,Fgbest);
disp('RESULTS:');
k
Maxerr
Pd = gbest
TotalP = sum(Pd)
CT = sum(ad + bd.*Pd + cd.*Pd.^2);
fprintf(' %7.3f\n\n',CT);
```


Color Plate Section

Chapter 2

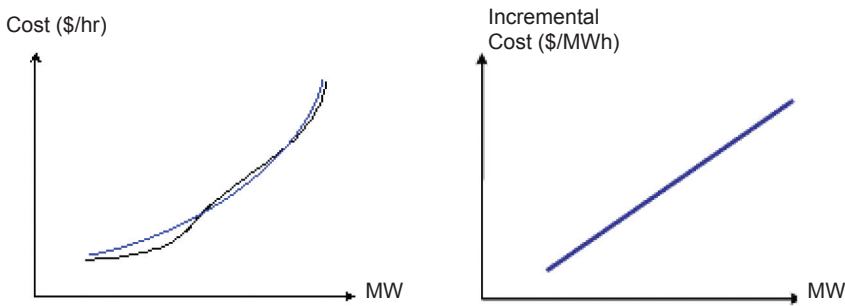


Fig. 2.2 Quadratic approximation cost function.

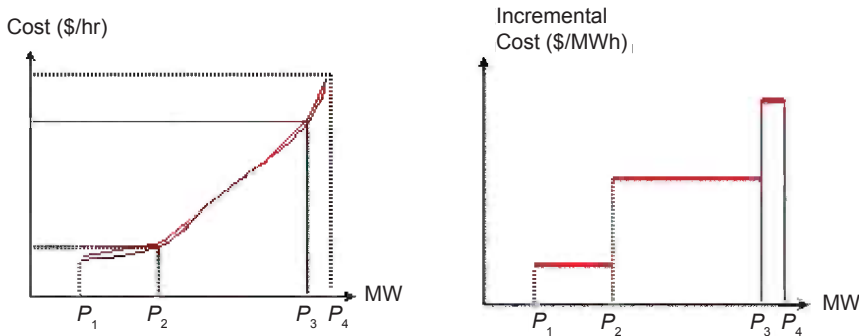


Fig. 2.3 Piecewise linear approximation cost function.

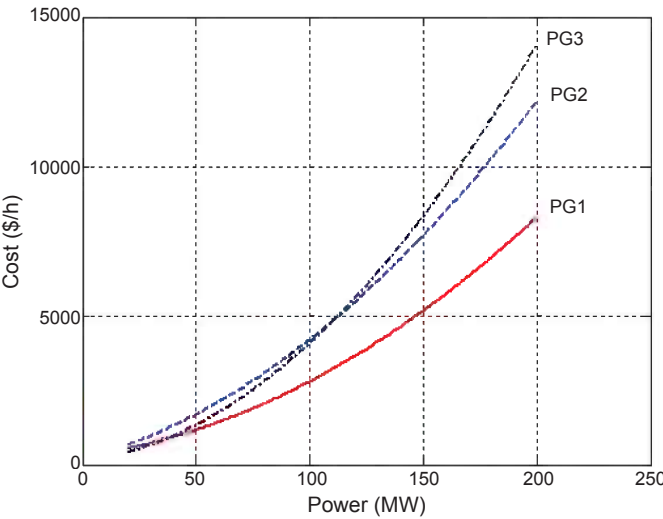


Fig. 2.6 Generation costs of three units in example 2.1.

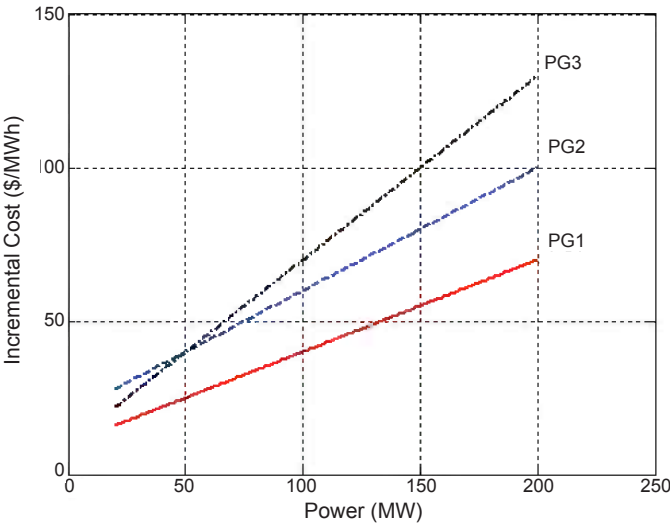


Fig. 2.7 Incremental costs of the three units in example 2.1.

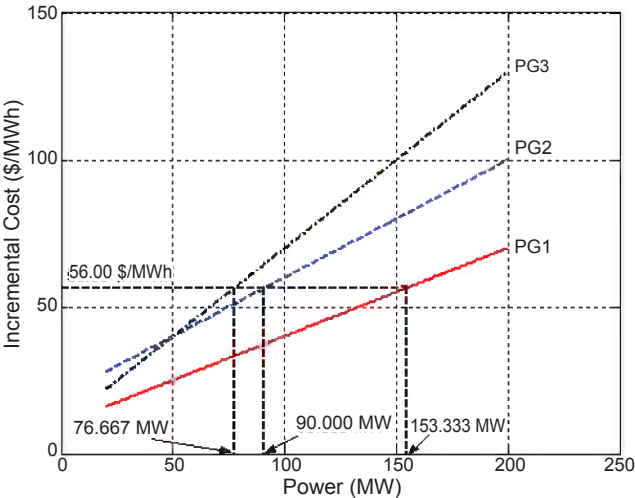


Fig. 2.8 Graphical representation of the dispatch result in the example 2.1.

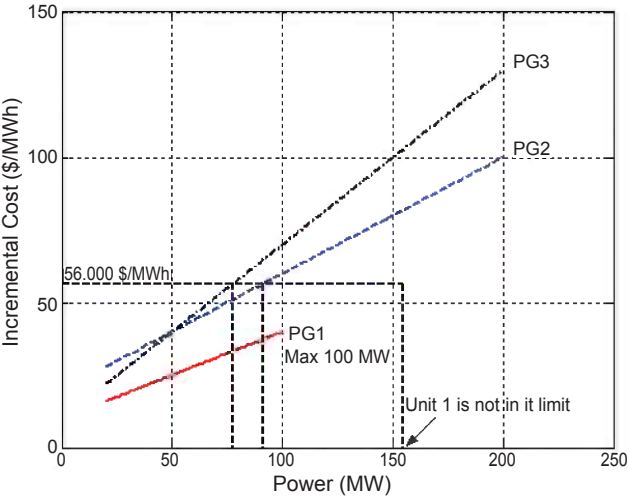


Fig. 2.9 Graphical representation of the dispatch result in example 2.2 when unit 1 is exceeding its limit.

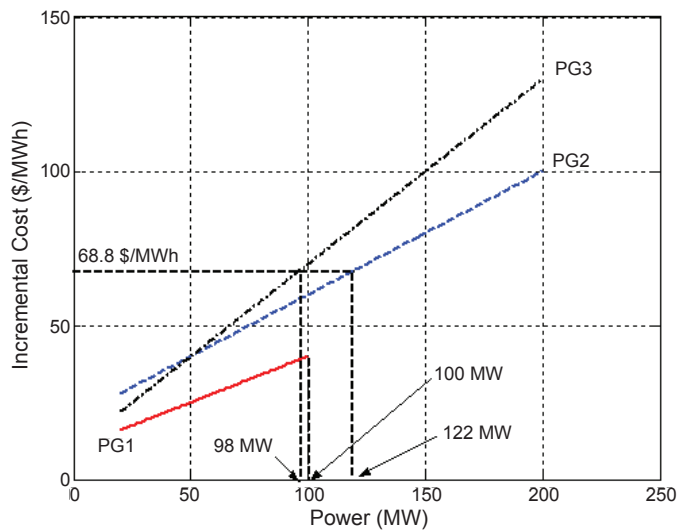


Fig. 2.11 Graphical representation of the dispatch result in example 2.2 with all units within their limits.

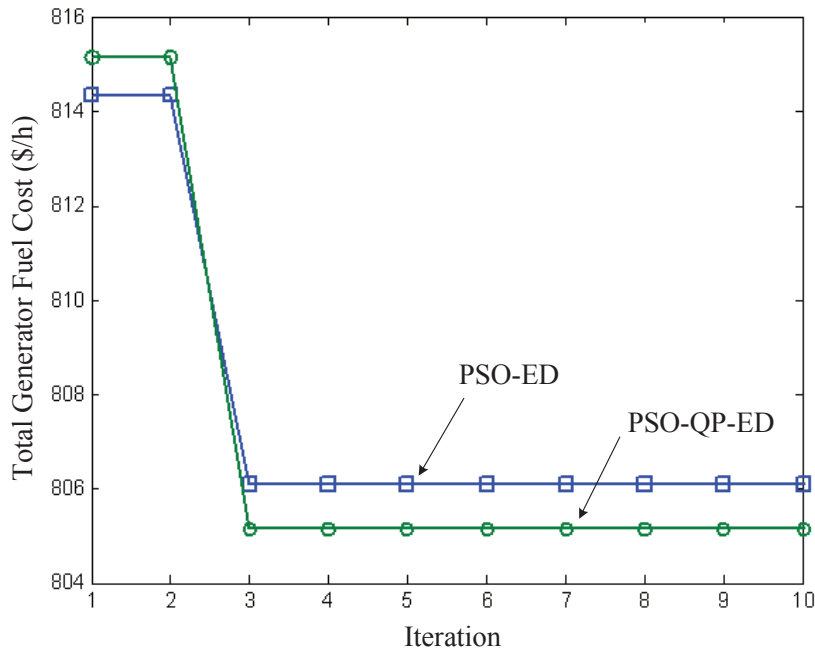


Fig. 2.23 Convergence properties of the IEEE 30 bus test system.

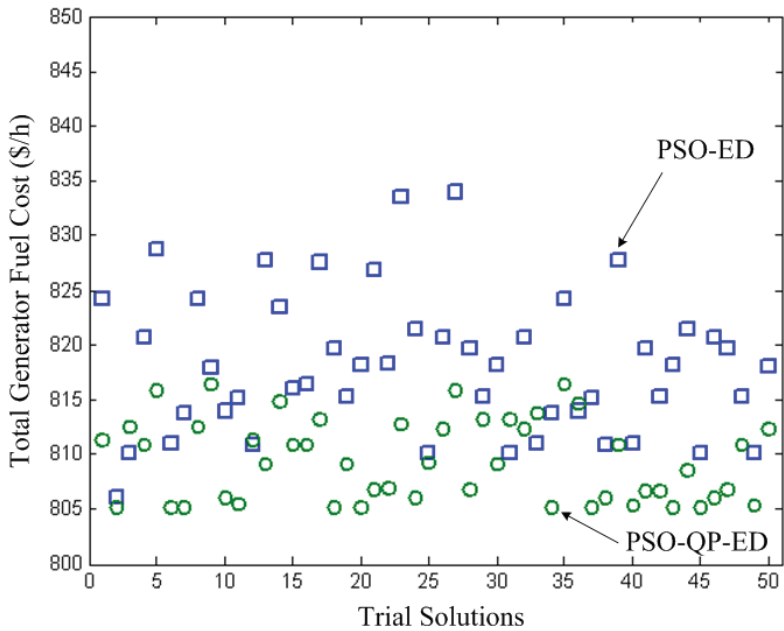


Fig. 2.24 The results from 50 trials with the IEEE 30 bus test system.

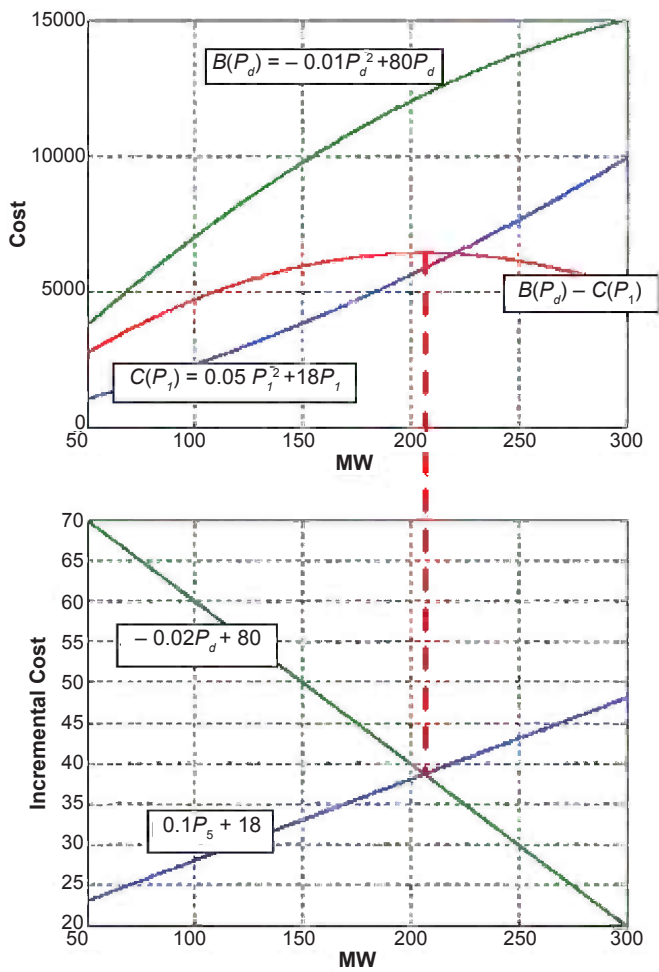


Fig. 2.37 The maximum social welfare objective.

Chapter 5

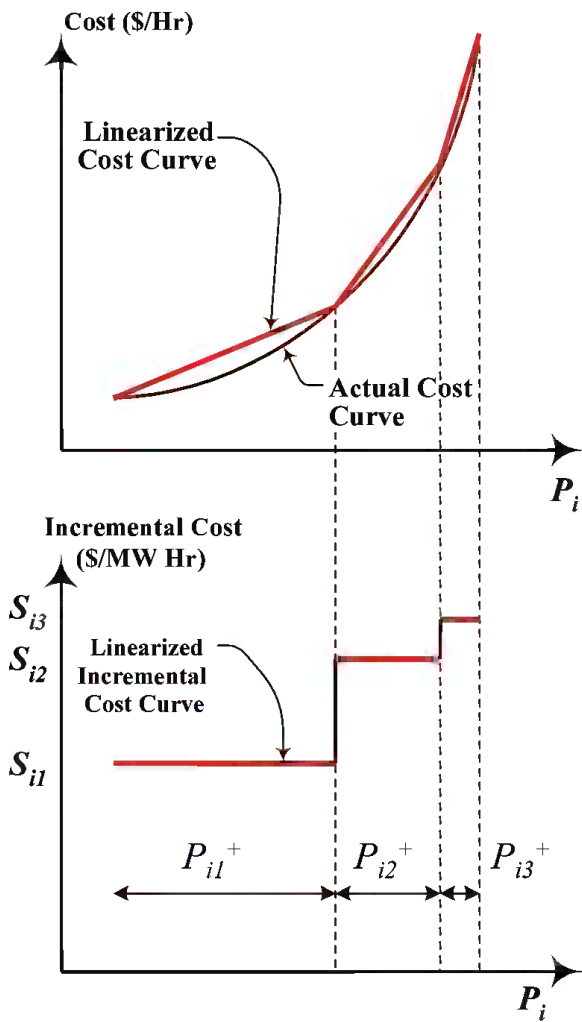


Fig. 5.4 The linearized cost function.

Chapter 6

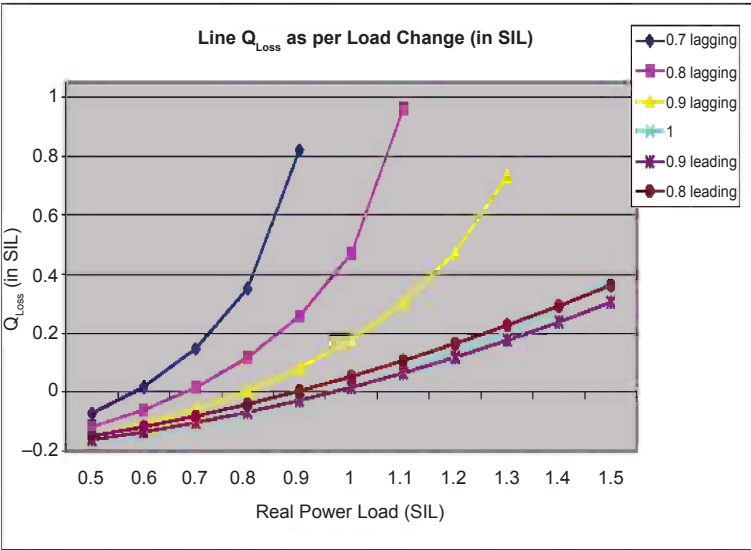


Fig. 6.3 Q_{Loss} as per power loading (211 km).

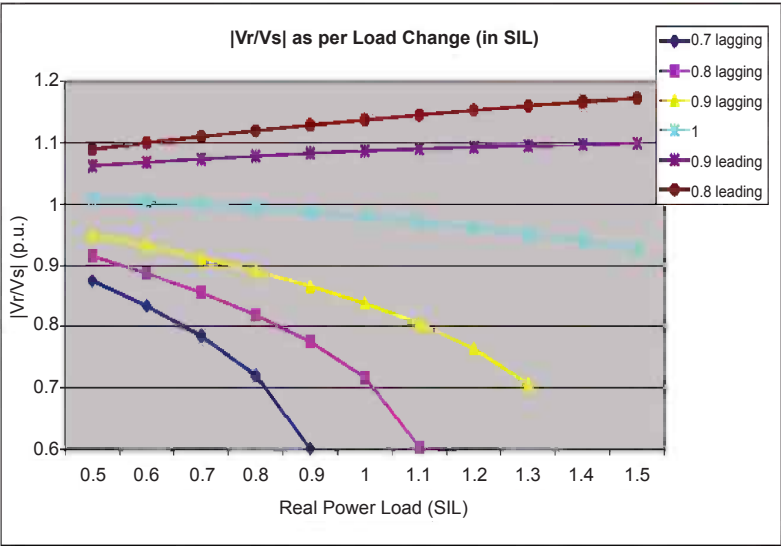


Fig. 6.4 Receiving end voltage as per real power loading (211 km).

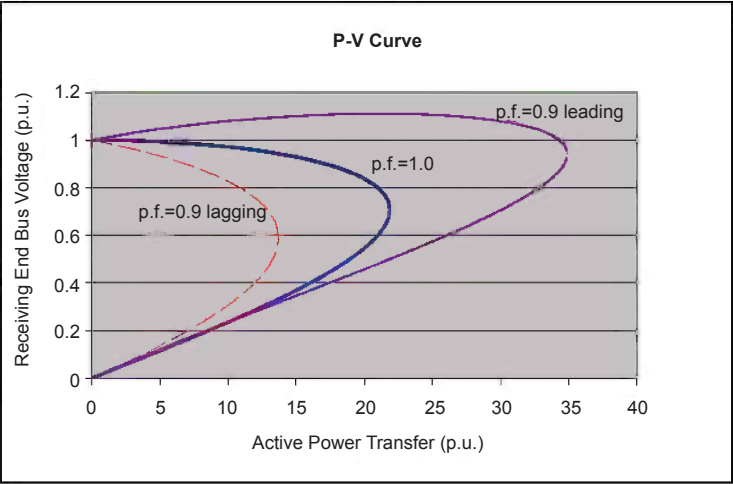


Fig. 6.6 P-V Curve.

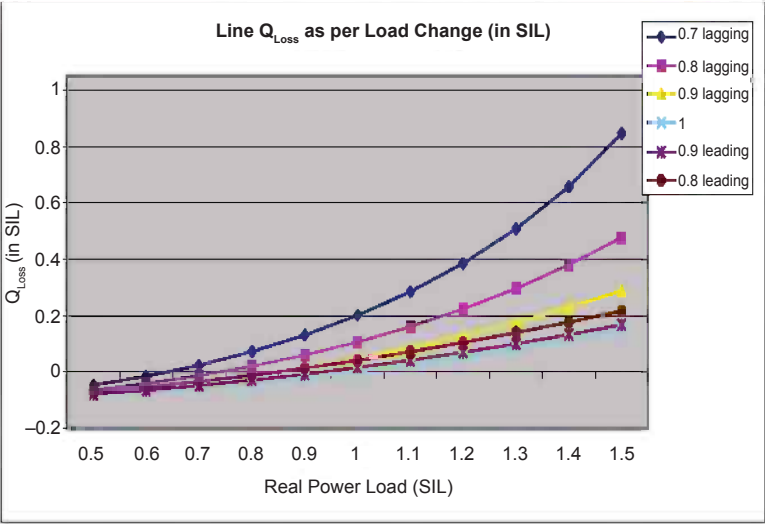


Fig. 6.16 Q_{Loss} as per power loading (105 km).

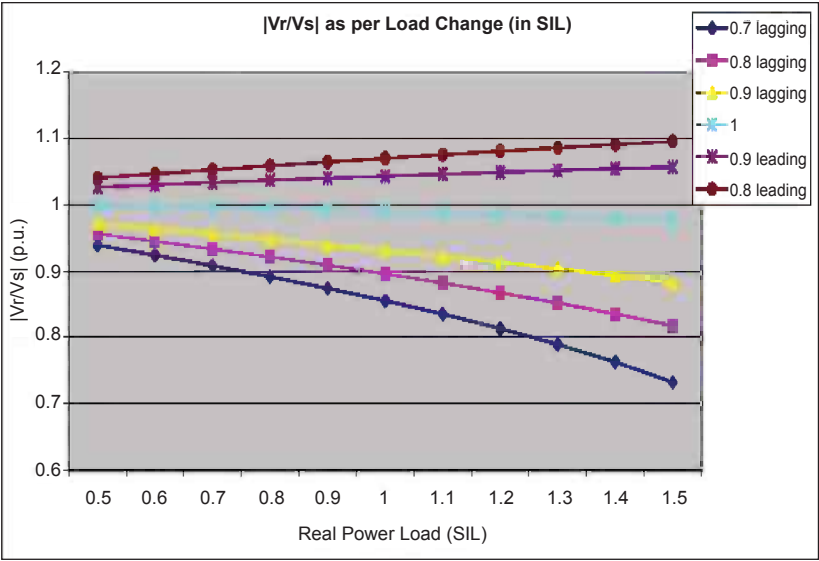


Fig. 6.17 Receiving end voltage as per real power loading (105 km).

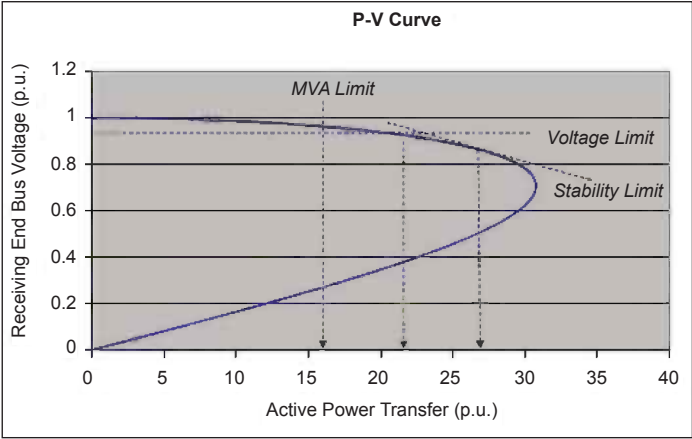


Fig. 6.18 Limits in transmission line capability.

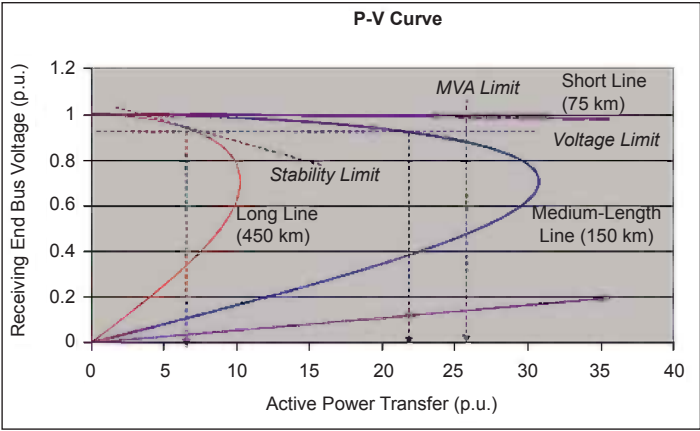


Fig. 6.19 Line capability limits of three different line lengths.

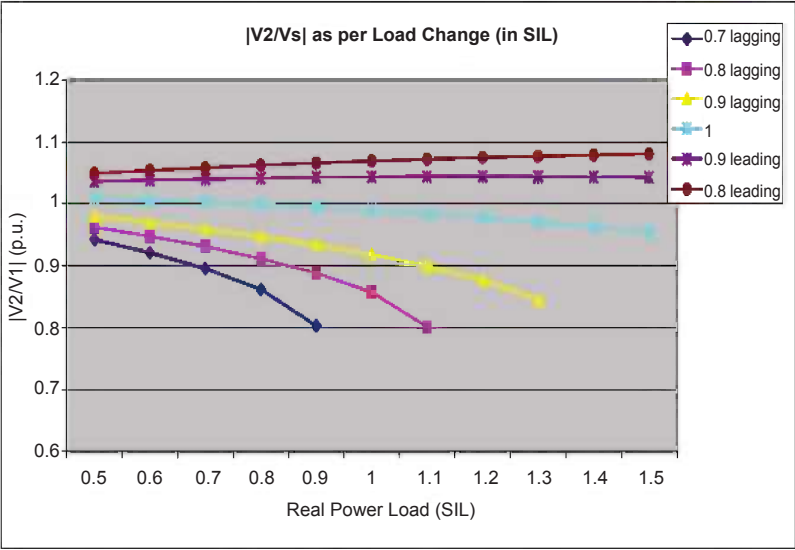


Fig. 6.22 V_2 voltage as per power loading.

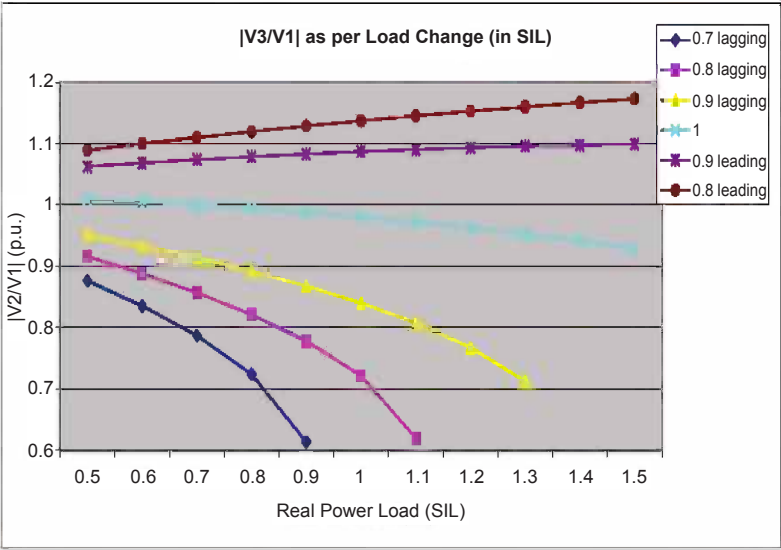


Fig. 6.23 V_3 Voltage as per power loading.

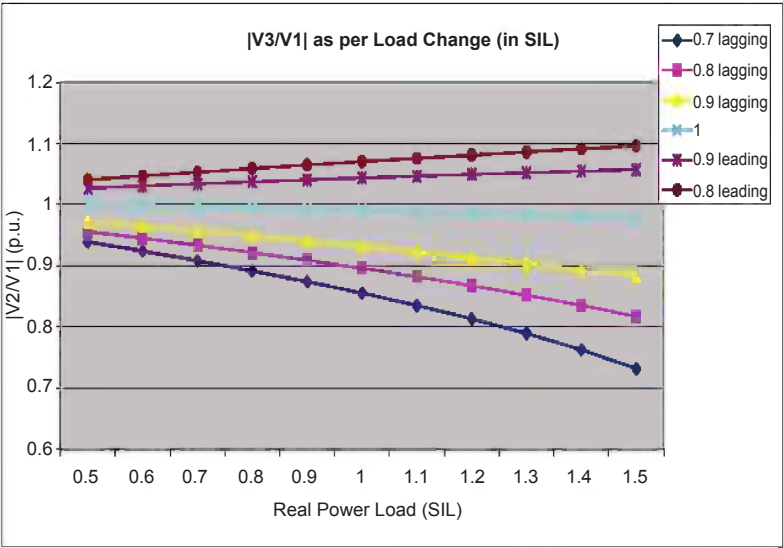


Fig. 6.25 V_3 voltage as per real power loading with full compensation of reactive power.

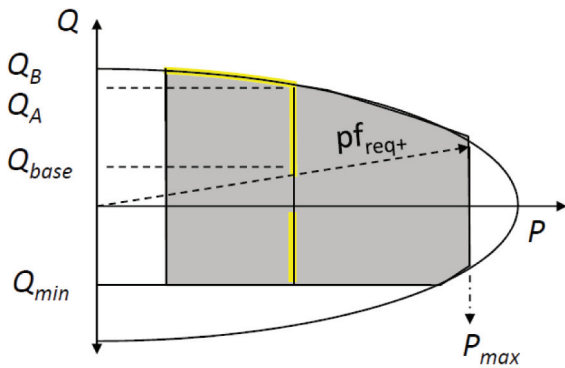


Fig. 6.27 Synchronous generator capability curve.

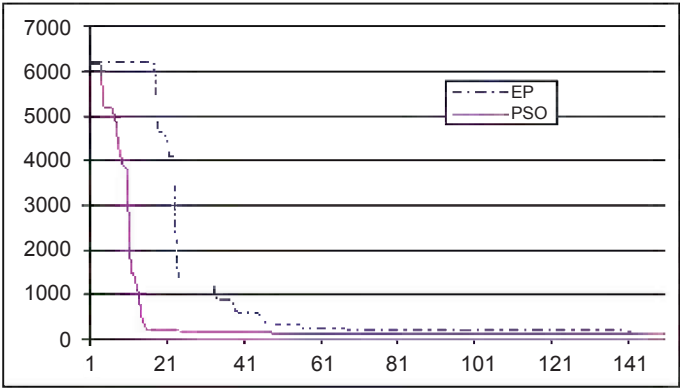


Fig. 6.31 Convergence characteristics (best solutions): TVAC-PSO vs. EP (Source from [29]).

In recent years, there have been many books published on power system optimization. However, in most books, the artificial intelligence (AI) based methods have been limited to the introduction rather than dealing with applications. With the considerable increase of AI applications AI is being increasingly used to solve optimization problems in engineering. Therefore in the past two decades, the applications of artificial intelligence in power systems have attracted much research. This book covers the current level of applications of artificial intelligence to the optimization problems in power systems.

This book will serve as a textbook for graduate students in electric power system management and will also be useful for those who are interested in using artificial intelligence in power system optimization.

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