

Second Edition

Project Economics and Decision Analysis

Volume 1: Deterministic Models

M.A. Mian

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This book is dedicated to my brother, the late M. Rashid Mian, who passed away in a tragic car accident, and the memories that we shared together for more than 40 years. It was his idea to write this book with me. I have incorporated some of his handwritten notes on production sharing economics in chapter 5.

He was a warm, dynamic, vivacious, and fun-loving person. He was truly one of the most beloved and admired members of our family. We loved him deeply, and his loss leaves an enormous void in all our lives.

May God bless his soul in peace.

Contents

Preface	xi
Acknowledgments	xvii
1. Introduction	1
Profit Planning	2
Typical oil company objectives	3
The role of management in planning	4
Planning of capital expenditures	4
Some Basic Principles of Economics	6
Demand, supply, and equilibrium price	7
Crude oil price volatility	15
Energy independence	20
Inflation	22
Consumer price index	26
Uncertainty and Risk	30
Questions and Problems	32
2. The Time Value of Money	35
The Concept of Interest	37
Simple and compound interest	39
Nominal and effective interest rate	42
Cash Flow Diagram	45
The Time Value of Money	46
Present value of future sum	49
Future value of present sum	54
Present value of an annuity	57
Future value of an annuity	59
Present value of deferred annuity	62
Annuity from present value	64
Annuity from future value	67
Present value of uniform gradient series	68
Present value of shifted gradient series	70
Calculation of unknown interest rate	72
Loan Amortization Schedule	73
Funds Flow and Compounding/Discounting	81
Spreadsheet Applications	83
The Time Zero Dilemma	95
Questions and Problems	96

3. Before-Tax Cash Flow Models	111
Cash Flow Model	113
Data Required for Project Evaluation	116
Forecasting Product Stream	117
Volumetric calculations	119
Decline curve analysis	121
Gas reservoir material balance	142
Production forecast by analogy	146
Product Pricing	149
Capital Expenditures (CAPEX)	154
Geological and geophysical (G&G) costs	155
Estimating drilling costs	155
Facility costs	158
Operating Expenditure (OPEX)	162
Abandonment costs and sunk costs	166
Opportunity costs	167
Types of Cost Estimates	168
Order of magnitude estimate	168
Optimization study estimate	169
Budget estimate	169
Control estimate	170
Variations in Actual Project Costs	170
Cost overrun	170
The accuracy of the estimates	171
Contingencies and allowances	172
Transfer Pricing	174
Transfer price at cost	176
General formula for computing transfer price	177
Transfer at market price	177
Aspects of Leasing	178
System of land ownership identification	179
Ownership interest computation	181
Severance and Ad Valorem Taxes	185
Cash Flow Estimation Bias	190
Cash Flow Analysis of Foreign Investments	191
Cash Flow Analysis and Inflation	192
The Cost of Capital	193
The Cash Flow Spreadsheet	195
Questions and Problems	198

4. After-Tax Cash Flow Model	207
Depreciation	213
Tangible property	213
Intangible property	213
Modified accelerated cost recovery system	214
Accelerated cost recovery system	218
Straight-line depreciation	219
Declining balance depreciation	220
Sum-of-years'-digits depreciation	221
Units of production depreciation	223
Amortization and Depletion	223
Amortization	224
Depletion	224
Intangible Drilling Costs (IDCs)	228
Spreadsheet Application	235
Questions and Problems	243
5. International Petroleum Economics	245
Types of Contract Arrangements	247
The concessionary system	249
Sample concessionary system cash flow spreadsheet	251
The production sharing system	254
Sample PSC cash flow spreadsheet	259
Basic elements of production sharing and royalty tax systems	263
Contract initiation and award	270
Government and contractor take	271
Risk service contracts	273
Rate of return (ROR) contracts	274
Effects of Various Fiscal Terms	274
Efficient fiscal regime	275
The effect of cost recovery ceiling	277
Designing fiscal systems	280
Questions and Problems	294
6. Capital Budgeting Techniques	301
Accounting Approaches	306
Payback period	306
Average return on investment (AROI)	310
Discounted Cash Flow Approaches	311
Discounted payback period	312

Net present value (NPV)	313
Internal rate of return (IRR)	316
Profitability index (PI)	328
Unit technical cost (UTC) or long-run marginal cost (LRMC)	331
Netback value (NBV) and indexed pricing	337
Growth rate of return (GRR)	350
Adjusted net present value (ANPV)	353
Effect of Discounting Frequency	355
Treatment of Financing Mix	360
Pattern of project debt repayment schedule	362
After-tax weighted average cost of capital method	366
Equity residual method— NPV_{ER}	369
Arditti-Levy method— NPV_{AL}	371
Adjusted net present value— NPV_A	372
Comparison of the four methods	373
Questions and Problems	375
Further Reading	385
7. Investment Selection Decision Making	387
Service-Producing Investments	389
Equivalent cost analysis	390
Rate of return analysis	392
Present value ratio or PI analysis	394
Service life breakeven analysis	394
Unequal life service-producing alternatives	395
Lease versus Buy Decision Analysis	398
Income-Producing Investments	403
Project screening economics	403
Mutually exclusive investment analysis	406
Non-mutually exclusive investment analysis	415
Questions and Problems	420
Appendix A. Interest Factors for Annual Interest Compounding	433
Appendix B. Overview of Spreadsheets on CD-ROM that Accompany Volume 2	441
Answers to Selected End-of-Chapter Problems	445
Index	453

Preface

Welcome to the second edition of *Project Economics and Decision Analysis*. As mentioned in the previous edition, the international oil and gas industry remains one of the most important, highly capital-intensive, and risky industries at global, regional, and local levels. The Global E&P expenditure reached \$324 billion in 2007 (*World Oil* magazine, February 2008, Vol. 229, No. 2). Our profit margins are under real pressure from many factors, including the higher costs of developing new reserves, less oil found per foot drilled, rising inflationary costs of doing business, oversupply of crude, crude oil price volatility, competition for oil company investments, competition for acreage/concessions, competition for funds, and overall business risk and uncertainty.

Therefore, it is crucial to carry out prudent economic evaluations of any capital investments before resources are committed. This, of course, requires a thorough understanding of the techniques available and their application by all those involved in decision-making. To assist in achieving this goal, the industry deserves a comprehensive guide to provide all the necessary concepts of capital investment evaluation, capital budgeting, and decision analysis. This book, *Project Economics and Decision Analysis*, in its second edition, will hopefully meet this requirement.

Objective

One goal in writing this book has been to provide students, practicing engineers, geologists, economists, planners, and managers with a solid foundation in the dynamic and growing field of capital investment evaluation with emphasis on the uncertainty aspect. It describes how investment decisions are currently made under different stages of uncertainty and prescribes techniques for making rational decisions.

This two-volume set describes the philosophy, process, and methods of capital investment evaluation and decision analysis. In summary, the main objectives of the book are to:

- Explain the ever-expanding role of economics in prudent capital investment decision-making.
- Assist readers in developing a knowledgeable vocabulary of the terms associated with economic analysis.
- Review the procedures used in preparing capital investment evaluations and decision analysis.
- Relate the new vocabulary and knowledge to some specific problems.

- Present ways of interpreting estimates that include uncertainty (i.e., converting probabilistic description into a measure of profitability).
- Provide solid hands-on experience with capital investment evaluation and decision analysis.

The subject of project economics and decisions analysis is not difficult. It does not require you to remember your differential equations, nuclear physics, or atomic theory. However, it does require the use of a good deal of logic and common sense, and the rest is just number crunching. The analyst doing economic evaluations needs to think imaginatively to identify a problem, to weigh the various factors that will affect these variables, to evolve alternatives, to take a position, and finally to be prepared to defend his or her position.

I urge students and practitioners to have a helicopter view of the entire process and develop an eye for details in order to be able to resolve problems as opposed to completing a series of cookbook computations. This is what *Project Economics and Decision Analysis* is all about. While I have retained most of the features that have made these books so popular, I have further enhanced the coverage to make it more relevant and more accessible to students of varying backgrounds.

This edition is further improved in order to address audiences who are not sophisticated in the subject. An attempt has been made to present the subject matter clearly and simply so that the reader will be in a position, whether in the classroom or in practice, to move immediately to higher levels of sophistication in understanding and application.

Changes in the Second Edition

This edition continues the basic philosophy of the first edition, which is to provide users with coverage of all-important areas of project economics and decision analysis, while providing flexibility in the use of materials. This objective of flexibility has warranted the addition of some new material.

Those who will be using these books either as a college text, a learning guide, or as a reference in day-to-day economic decision making may be interested to know what is different about the second edition. Apart from polishing the material and making the concepts easier to understand, I have made the following changes:

1. All materials that have a time aspect have been updated.
2. Throughout the book I have added new, real-world examples to round out the coverage of concepts and to provide appropriate emphasis on areas of central importance.

3. Although most youngsters these days are becoming increasingly proficient in their MS Excel skills, many still need to be aware of the power and limitations of this valuable software—especially when it comes to economic evaluation of projects. In my five-day courses on the subject, I note that the participants really struggle with using Excel's full potential. Therefore, I have expanded the spreadsheet applications sections to elaborate on the capabilities of Excel and highlight the areas where analysts have misused it.
4. The section on the weighted average cost of capital (WACC) concept is expanded to make the concept easy to comprehend so that the analysts can easily recognize the weakness of this concept in practice.
5. A new section on netback pricing and indexed netback pricing is included.
6. The concept of unit technical cost, also referred to as long-run marginal cost (LRMC) is expanded to make it easier to calculate and apply.
7. The section on international economics (fiscal systems) is rewritten and expanded.
8. A new chapter on exploration economics is added to volume 2.
9. Many problems in volume 2 were solved using the previous versions of the Palisade DecisionTools suite. These problems are updated using the latest version of the DecisionTools suite.
10. After using the books for six years, with significant feedback from the participants in my five-day courses on the subject and some of the professors who are using these books in their courses, I found that there was room for improvement in adding clarity to some of the topics. Therefore, I have provided additional examples or reworked the existing examples.
11. For professors who are using these books, a test bank of approximately 300 multiple-choice and true-false questions was developed. In addition, a comprehensive set of some 300 MS PowerPoint slides is made available.

Emphasis and Style

The book presents a balanced blend of theoretical concepts and their practical utility. I prefer to focus less on extensive theoretical discussions than

might be found in other books. Theory, I feel, distracts the reader from the most important concepts and their practical application. Moreover, theory can seem sterile and pointless unless its usefulness is made clear. Therefore, I have focused more on practical application. The underlying concepts are stressed and made concrete with liberal use of illustrations, many of them taken from actual real-life capital investment evaluations. Algebraic formulations and spreadsheets are used side-by-side to help readers develop conceptual thinking skills. Emphasis is placed on model formulation and interpretation rather than algorithms.

The technical materials have been developed with considerable patience—assuming very little foreknowledge and proceeding in a step-by-step manner to minimize the possibility of the reader getting lost along the way. Moreover, I have resorted to a greater degree of informality in the presentation in the belief that readability should be an overriding consideration. Toward the same goal, intuitive and economic explanations are frequently provided to clarify the why of a particular concept/technique.

This book is primarily intended for use by economists, earth scientists, engineers, and students. It is also intended to serve as a refresher and perhaps as a self-study textbook. The problem-solving approach is instructive in nature, but the foundational principles show the practical application of the material. Its chief purpose is two-fold: (1) to render a systematic exposition of certain basic deterministic investment evaluation methods, and (2) to relate these to the decision analysis in such a way that the mutual relevance of the two is clearly brought out.

Therefore, the book is divided into two separate, yet complementary, volumes. This book, volume 1, is essentially introductory and deals with the deterministic evaluation tools used for capital investment evaluations. These concepts are seldom covered as broadly or from the same viewpoints in economics and other courses, yet they are fundamental to the proper understanding of all evaluation work.

Volume 2 in this series deals with the concepts of decision analysis (i.e., incorporating risk and uncertainty as applied to capital investments). Generally, each topic is introduced by a brief practical or conceptual overview, followed by a brief discussion related to its application in practice and a solved example. For optimum benefit, it is recommended that readers explore both volumes and benefit from their integrated instruction.

Examples and Assignment Problems

Being gratified by the success of the first edition, I have maintained the example-driven approach to the subject matter. I believe that the best way to learn any subject is by working through examples and completing plenty

of problems—the problems however have to represent reality, not textbook examples or problems. Although this active learning approach is not new, I believe these books have more fully developed this approach than any other book on the subject. This example-driven approach is further reinforced by imparting valuable modeling skills that students can appreciate and take with them into their careers.

Included in this second edition are additional solved real-life examples (100+) and end-of-chapter assignment material (300+ questions and problems). Examples help reinforce the learning process. Each solved example is straightforward, fully explained, and avoids sudden leaps to conclusions that might confuse the reader. The assignment material is divided into questions and problems. The questions primarily address key concepts and terms in the chapter. The problems either consolidate a number of chapter topics or focus on a comprehensive analysis of a single topic.

The wide variety of assignment material offers practical knowledge since the assignments include various combinations of breadth and depth, theory and procedures, simplicity and complexity. For maximum benefit, the reader should work out as many of these problems as possible, if indeed not all.

Spreadsheet Applications

During my five-day courses that I teach on this subject, I notice that most of the participants struggle a lot with the use of financial functions provided in MS Excel. Today, most of us are using spreadsheet programs to build models of the decision problems we face as a regular part of our daily work activities. Spreadsheets also capture users' interest and add a new relevance to investment evaluation and decision analysis. Since we extensively use Excel for our economic modeling, it is essential that those who are interested in working in this area be fully conversant with Excel.

In this second edition, I have repeatedly highlighted the mistakes committed by very senior analysts in modeling problems using MS Excel; such mistakes are unnecessary and avoidable. I stress again and again that the use of software is to organize and expedite our calculations rather than solve the problems for us. We should use software only if we know, given sufficient time, how to solve a given problem by hand.

A unique feature of this book is embedded application of computers in solving investment evaluation and decision analysis problems. I have tried to give both aspects of the required calculations—problem solving by hand and by using Excel. For those who build economic models, they also need to debug these models. So, how would one find bugs in any spreadsheet or economic model if one does not know how to solve the problems by hand?

Examples are provided to show how computers can be used to help make better evaluations and, hence, better decisions. The Microsoft Excel spreadsheet is making it increasingly easy and practical to do sensitivity and scenario analysis. Its use has gained acceptance in the industry and makes it feasible to do a variety of analyses with a multitude of problems. The latest version of DecisionTools Suite (an Excel add-on) by Palisade Corp. includes the award-winning @RISK and PrecisionTree applications, which are used where applicable. Screen captures of the various menus of DecisionTools Suite are used.

Reviewers' Comments

Many useful comments were received from PennWell's technical reviewers. These are incorporated wherever possible. Some reviewers noted that perhaps I may overkill some of the basic concepts, such as the Time Value of Money, etc. That may be true, but this is how I learned these concepts more than 30 years ago when I took a course from Dr. Franklin J. Stermole at Colorado School of Mines. It is very ironic that it is these simple concepts where very experienced analysts make mistakes or misinterpret the limitation of the concepts applied. I have specifically brought these mistakes to your attention so that they can be avoided. These basic concepts have been of considerable help to me over the years, enabling me to visualize the investment problems and their time horizons. Therefore, I have decided to leave them as they are. Similarly, the use of interest tables (in this modern age of computers) might be considered orthodox. The tables may not be used in practice, but they definitely add to understanding the concepts. I feel strongly that readers will benefit, so I have included them here.

Request for Suggestions

It should be noted that one cannot always add new material and never delete some of the existing material. Some examples and materials become dated and become natural candidates for deletion. In addition, some material needs to be enhanced by rewriting or restructuring in order to improve readability and understanding.

A considerable amount of dedication and investment (time and capital) goes into writing and publishing such a book. I have made every effort to introduce this two-volume set as a comprehensive desk reference. I sincerely welcome your thoughts as an end-user to help us further improve the contents, presentation, and utility of this book so as to make it a standard for the new generation of petroleum industry personnel. I will always be very grateful for your comments, suggestions, or corrections sent to me directly (asifmian55@yahoo.com) or through PennWell Publishing.

Acknowledgments

Many people are involved in the successful publication of a book. I wish to thank the following for making significant contributions to this book. Without their assistance, this project could not have been possible.

- All the people at PennWell who contributed their efforts to the production of this book, especially managing editor Stephen Hill, production manager Sheila Brock, production editor Tony Quinn, and book and cover designer Susan E. Ormston.
- The Palisade Corporation for sending me a copy of the latest version of their DecisionTools Suite . The use of this software to solve various examples in the book has significantly enhanced the utility of this book.
- I express my gratitude to the educators, students, and businessmen who have provided numerous suggestions for improving the materials in this second edition. I also thank PennWell's editorial team for editorial and substantive suggestions for the manuscript.
- I would like to thank all my students who have attended my short courses on the subject for making me think and improve my material to assist in their understanding of the subject. Their comments were very valuable.
- Finally, I thank my wife who was unaware when she married me that I was already married to *Project Economics and Decision Analysis*. I believe that I have been fair to both wives, or else one of them would have divorced me by now! Last but not least, knowing my weakness, I thank my four children for their work to obtain excellent educations—it is the best investment I have ever made. Special thanks to my son M. T. Mian at the Colorado School of Mines for being in the driver's seat; this has contributed a lot to my enthusiasm. I certainly hope that my grandchildren, Alishba Gul, Zaland, Zain, and others to come, will dedicate some of their precious time to transferring their knowledge to others.

God bless you all.

1

Introduction

This introductory chapter sets the stage for this book by presenting basic concepts and definitions for capital investment evaluation (also referred to as investment appraisal or economic evaluation) techniques presented in the following chapters. The chapter is divided into three sections:

1. Profit planning
2. Basic concepts of economics related to capital investments
3. Defining uncertainty and risk

Many terms, such as capital budgeting, inflation, supply and demand, and profit planning, presented in this chapter are referred to in day-to-day decision-making processes. However, many of us may not be fully aware of the theory behind them. These terms are often accepted as routine business jargon. The concepts presented in this chapter will assist in clarifying these terms and enabling us to relate them to the evaluation of capital investment projects.

The worldwide oil and gas industry applies advanced techniques of investment analysis. Because of the dynamics of high-risk, highly capital-intensive investments, complexity of operations, and profit potential, companies are compelled to seek the most sophisticated investment methods.

In virtually every segment of the oil and gas industry, oil and gas reserves estimates and complete economic evaluations are vital requirements for sales, mergers, acquisitions, estate settlements, litigations, and financing of exploration, field development, and major facilities projects.

For the protection of the public, every public company in the United States is required to submit an annual 10-K report with the Securities and Exchange Commission (SEC). For the 10-K filing, every public oil and gas company must update its oil and gas reserves estimates and accompanying economics. Petroleum engineering techniques are used to estimate oil and gas reserves and to make these forecasts. The economics constitute the oil and gas revenues, capital expenditures, operating expenses, taxes, cash flow, and various investment profitability measures.

Corporate or investor success or failure is based on the correct evaluation of investment opportunities and the results of decisions. The economic evaluations also serve as a good communication tool between the technical staff and the management team of the corporation. Most of the investment decisions are based on the overall financial impact of an investment opportunity rather than on any specific technical considerations.

It is not necessary for every investor or every person in the corporate orbit to be a petroleum engineer, geologist, or a financial analyst. Most of the time, specialists do investment evaluations with input from other specialists. Personnel involved in evaluating the data, recommending a course of action, and making the final decision must exchange sufficient and appropriate information/data. This total awareness considerably increases the quality of the decision-making process.

Profit Planning

The term *profit planning* is broadly defined as a systematic and formalized approach for accomplishing the planning, coordination, and control responsibilities of management. By making a continuing stream of well-conceived decisions, management can plan and control the long-range destiny of the enterprise. The concept speaks to planned prosperity as opposed to unplanned happenstance. Thus, the essence of the profit-planning concept is management's decision-making process. For the long-term success of an enterprise, a stream of these decisions must generate plans and actions. These will then provide the essential cash inflows required to support the planned cash outflows of the enterprise. This is necessary to ensure the organization earns realistic profits and return on investments, which are long-term objectives.¹

Long-term planning projects specify objectives and goals of an enterprise beyond the short term. Today's actions are influenced by increased consciousness of tomorrow's expected events. Therefore, management concentrates on those events that can produce the best and largest economic results. This is an important key to the success of the business. Peter Drucker defines planning as:

*The continuous process of making present entrepreneurial (risk-taking) decisions systematically and with the greatest knowledge of their futurity; organizing systematically the efforts needed to carry out these decisions; and measuring the results of these decisions against the expectations through organized, systematic feedback.*²

The management decisions must be *purposive* (i.e., primarily concerned with the purpose of developing enterprise objectives and devising realistic strategies to attain these objectives). Decisions must also be *futuristic* (i.e., concerned with the long term). This is called *strategic planning*. Objectives should be more than statements of good intentions—they should be specific strategic goals.

Typical oil company objectives

The typical primary objectives of any oil company are:

1. Add to the reserves inventory through exploration efforts or purchase of reserves in the most cost efficient manner.
2. Maximize ultimate oil and gas recovery from the reserves inventory through research efforts, studies, and the use of sound reservoir development and production practices.
3. Maintain sustained optimum production levels in line with sound reservoir management principles.
4. Ensure reliability of fuel and feedstock supplies to industry (where applicable).
5. Sustain the highest degree of reliability and availability of production facilities.
6. Conduct all operations in the most efficient, cost effective, safe, and environmentally acceptable manner.
7. Maximize return on investment and total worth of the company.

Once the objectives are set, they are then converted into actual doing. The decision-making process is a work program and will result in implementing many projects. These projects have to be coupled with specific and concrete work assignments with defined goals, deadlines, milestones, and clear accountability. Unless converted into action, objectives are nothing more than dreams.

The achievement of these objectives requires diverse activities within each sector of the enterprise. Attaining some of these objectives entails high risks. It requires effort, and that means capital. Businesses have to make a profit in order to survive.

The role of management in planning

The planning function is classified into long-term *strategic* plans and short-term *tactical* plans. Strategic plans provide a broader view of activities focusing on objectives and goals over a longer time period, say, from 5 to 10 years. The tactical plans provide a detailed view of the activities focusing on the means to attain goals.

After objectives are established, management must set goals and choose tactics to attain these goals. It will provide (a) timelines for attainment of the goals, (b) quantitative measurements, and (c) subdivision of responsibilities. The management team then develops strategies to specify the operational plan of attack to be used in pursuing the goals.

Finally, management operationalizes the objectives, goals, and strategies into one or more detailed profit plans (or budget). The profit plan explicitly states the goals in terms of time expectations and expected financial result (present worth, return on investment, profit, and costs).

Profit planning is a continuous and iterative process, as opposed to a periodic endeavor. The plans must be revised as conditions change and new information becomes available.

Planning of capital expenditures

In line with the attainment of the typical oil company objectives, the long-term and short-term profit plans of an oil company will include activities such as:

- Oil and gas exploration
- Infill drilling, including optimizing development of existing fields/reservoirs
- Installing artificial lift systems, which may be required from the inception of the field development or later during the life of the field
- Pressure maintenance (water or gas injection into the reservoir), which may be implemented from the inception of the field development or later during the life of the field
- Secondary and enhanced oil recovery
- Gas processing requirements
- Expansion and contraction of plants, buildings, and equipment
- Maintenance of production facilities and infrastructure

- Health, safety, and environment (HSE) projects
- Research and development (R&D) projects
- Major renovations, upgrading, and replacements of equipment/production facilities

These and other similar investment decisions comprise the capital expenditure plans, also referred to as the *capital expenditure budget*. In the planning process, each capital budget item is an identifiable project with its unique timeline and plan of implementation. Capital projects are normally capital intensive (requiring a large commitment of funds), and their impact on the enterprise extends far into the future.

The addition of each capital project to the profit plan has to be fully justified in terms of its derived financial, technical, and strategic benefits. Considerable planning and control are necessary to prevent idle operating capacity of plant and equipment, over-investment in operating capacity of plant and equipment, and investment in projects that will produce a low or marginal return on the committed funds. The required HSE projects are expenses that in most cases will not result in direct monetary benefits. However, there will always be various options available to achieve the same safety standards with different costs. These options have to be evaluated in order to select the most economic option.

The capital budget plans are normally comprised of a series of unique capital projects. Therefore, the overall plans include projects spread over three time periods:

- Projects that extend the farthest into the future (more than five years)
- Projects that extend from three to five years
- Projects that are part of the short-term profit plan (e.g., a one-year period)

The implementation of projects is scheduled according to operational priority, technical maturity, and financial rewards.

The capital expenditure plans normally include two principal types of projects: major projects, each involving considerable funds, and minor capital additions. The major capital projects normally cover more than one year. These projects will usually be considered and planned over a number of years before a final decision is reached. The minor capital additions comprise implementation of relatively low-cost machines and tools, computers, minor renovations to buildings, and other miscellaneous items. Most of the minor capital projects will be *service-producing* projects, while the major capital

projects will be *revenue-producing* projects. Some projects will require much more detailed analysis than others. The techniques used for the economic analysis of both service-producing and revenue-producing capital investments are presented in the following chapters.

Some Basic Principles of Economics

Economics is a social science that studies the allocation of scarce resources among competing ends. The principles, theories, or models of economics are used to explain and predict economic events. Policies are developed to correct economic problems. Economics studies/models are subdivided into *macroeconomics* and *microeconomics*.

Macroeconomics is the branch of economics dealing with the aggregate level of output and employment, national income, and the general price levels and the factors influencing it. Microeconomics, on the other hand, deals with the economic behavior of individuals such as consumers, resource owners, and business firms in a system where the economic activity is not directly controlled by the government. For example, it deals with the questions of:

1. How the purchasing power of an individual is influenced by the price of a commodity.
2. How a firm optimally combines various resources for the production of goods and services.
3. How the goods and services produced by the economy should be distributed.³

The primary purpose of this book is to provide tools for capital investment evaluations and decision analysis. In order to correctly use the relevant concepts presented here, it is important to understand some basic concepts related to macroeconomics, microeconomics, and their impact on variables used in capital investment evaluation and decision analysis.

For example, inflation and recession in an economy will significantly affect project cost estimates and returns from an investment. But most people are probably not aware of what causes inflation or recession, how they affect returns from investments, and how governments counteract these unfavorable economic trends. Similarly, many oil and gas companies invest in foreign countries where their investments are exposed to the risk of radical currency devaluation.

A brief exposure to the selected concepts and terms developed in economics (i.e., a description of tools and concepts most useful in investment analysis) is therefore considered appropriate.

Demand, supply, and equilibrium price

Supply and demand are terms used indiscriminately in casual discussions of economic events concerning services, commodities, and product prices. A comment such as “there are ample reserves of crude oil” may appropriately reflect the stock of oil in the ground. However, it does not say anything regarding the conditions under which producers will be willing or able to produce this stock of crude oil. Therefore,

- Supply is defined in terms of the quantity of a specific commodity offered at a given price.
- Demand is referred to as the consumption of that supplied quantity at that price.

Besides the crude oil prices, the cost of all other oil industry-related services are subject to the law of supply and demand. In 2007–08, the global demand for onshore and offshore rigs went up, resulting in an increase in the cost of rig rentals for onshore rigs from \$20,000 per day to \$50,000 per day and for offshore rigs from \$45,000 per day to \$150,000 per day. The cost of some offshore rigs for deep sea drilling went up to \$450,000 per day. This rise in rig demand was due to increased crude oil prices to more than \$70/barrel, which triggered increased exploration and field development activities.

The imbalance of supply and demand for any commodity and/or service affects project economics. Therefore, it is essential to be fully aware of the mechanism of how *equilibrium market price* for commodities is achieved in a free-enterprise economy.

Demand. Demand refers to the willingness as well as the ability of consumers to buy a commodity at a certain price. The existence of a need creates demand for a given commodity over a given time period. However, the need by itself is not a sufficient condition. The consumer must have the financial ability, i.e., money, to transform this need into effective demand for the commodity. Therefore, if the price of a commodity is increased, its demand will go down. This is because some of the consumers who could afford the commodity at the lower price may not want it at the higher price. This will also depend on whether or not the commodity has a substitute or if consumers can easily afford to forgo the consumption. At the lower product price, more of the product will be purchased because: (a) consumers can now purchase more with the same income, (b) the number of consumers who will be able to purchase the product at lower prices increases, and (c) consumers substitute this (now relatively cheaper) commodity for others in consumption.

Demand for a particular product or service is a function of the following factors:

$$\text{Demand} = f(P_n, P_n, \dots, P_{n-1}, Y, T, P, A, E)$$

where

- P_n = Price of product
- $P_n, \dots, P_{n-1}$ = Prices of other goods (substitutes and complements)
- Y = Income level and distribution
- T = Tastes and fashions
- P = Level and structure of the population
- A = Advertising
- E = Expectations of consumers

Supply. On the other hand, supply refers to the ability and willingness of a *producer* to supply a commodity over a given period at various alternative prices. The unit price a producer receives for its product is independent from the volume of the producer's output. The producer's objective is to determine what output level will maximize the short-term profit. This output level for any one producer is based on how production costs vary with output for that producer. This in turn determines the market price.

Supply for a particular product or service is a function of the following factors:

$$\text{Supply} = f(P_n, P_n, \dots, P_{n-1}, H, N, F_1, \dots, F_m, E, S_p)$$

where

- P_n = Price of product
- $P_n, \dots, P_{n-1}$ = Profitability of other goods in production and prices of goods in joint supply
- H = Technology
- N = Natural shocks
- $F_1, \dots, F_m$ = Costs of production
- E = Expectations of producers
- S_p = Social factors

Supply and demand curves. In models of free-enterprise economy (i.e., perfect competition) economists postulate the existence of two mathematical curves: the market demand curve and the industry supply curve. The industry supply curve is the sum of all the individual producers' supply curves for a

commodity. The market demand curve is the sum of the demand curves of *all* individuals in the market for a commodity.

As shown in figure 1–1, the demand curve is a downward sloping (negative slope) curve. This means that when the product price increases, the demand for the product (to a certain extent) will decrease, and vice versa. This is called the *law of demand*. The extent of change, of course, will depend on the elasticity of demand for the product (discussed later in the section). The demand curve in figure 1–1 shows that at price P1 (\$30/unit) the demand Q1 is only 10 units. As the price is reduced to P2 (\$15/unit), the demand Q2 for the product increases to about 17 units.

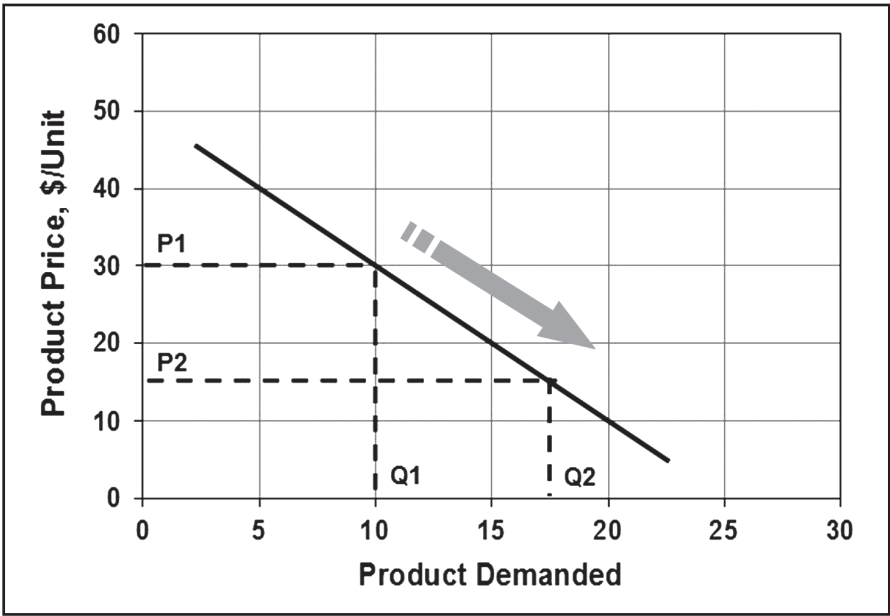


Fig. 1–1 The demand curve

As shown above, demand is a function of many factors. If any of these factors (other than the price) change, the entire demand curve for the product may shift upward or downward depending on the particular situation and the factor. If the income level of consumers increases and/or the population increases, there will be an upward shift in the demand curve as shown in figure 1–2. Because of the upward shift in the demand curve, the demand at P1 is now 20 units as compared to the 10 units with the original demand curve. Similarly, if there is a downward shift in the demand curve, then the units demanded at P1 will be 5 rather than 10 or 20 units.

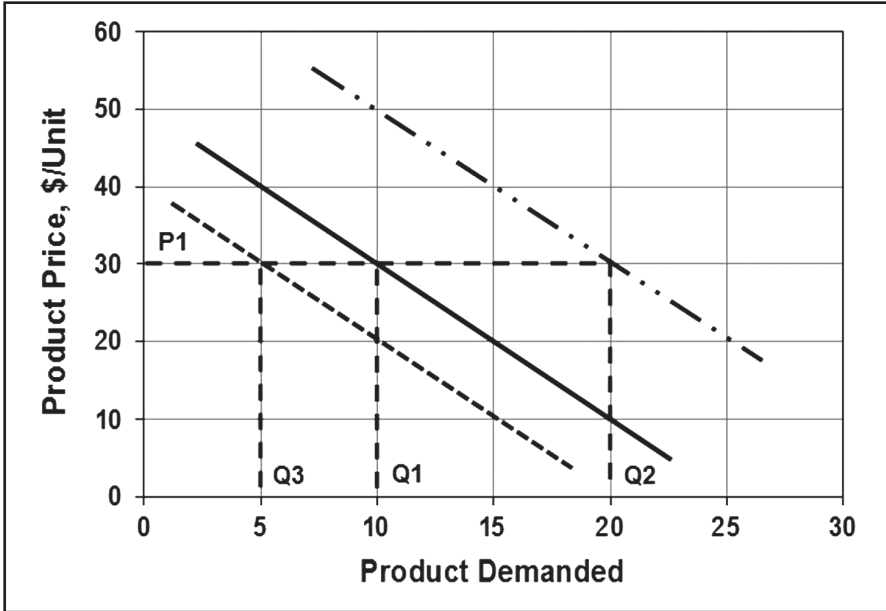


Fig. 1-2. The demand curve—shifts in demand curves

On the other hand, the supply curve as shown in figure 1-3 is an upward sloping (positive slope) curve. This means that when the product price increases, the supplier is willing to supply more of a product (to a certain extent), and vice versa. Alternatively, higher prices must be paid to induce the producer to supply more of the commodity. This is called the *law of supply*. The extent of change, of course, will depend on the elasticity of supply for the product (discussed later in the section). The supply curve in figure 1-3 shows that at price P1 (\$40/unit), the supplier is willing to supply Q1—20 units. As the price is reduced to P2 (\$20/unit), the supply for the product reduces to Q2—about 10 units.

As described above, supply is also a function of many factors. If any of these factors (other than the price) change, the entire supply curve for the product may shift upward or downward depending on the particular situation and the factor. Figure 1-4 shows the shift in supply curves and how the shift will impact the supplier's decision on supplying the market with a certain number of units.

It is worth mentioning that the supply and demand curves will not always be linear as shown in the figures in this book. The supply and demand curves come in many different forms and some will even have very complex correlations with the price.

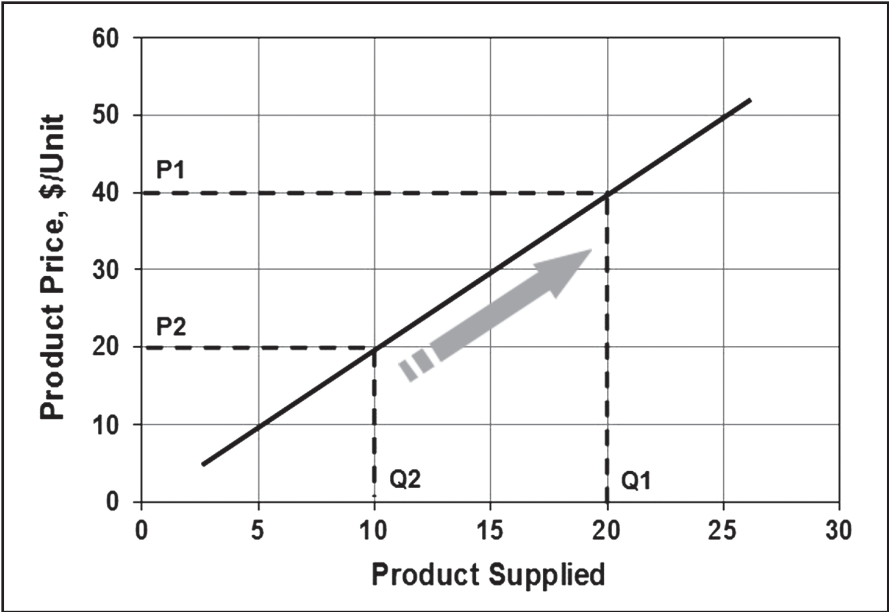


Fig. 1–3. The supply curve

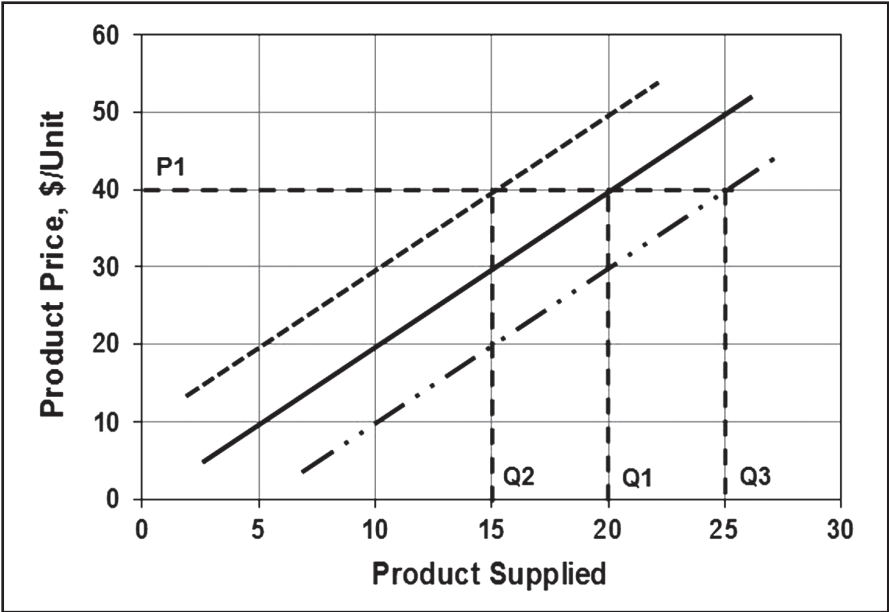


Fig. 1–4. The supply curve—shifts in supply curves

Equilibrium price. The intersection of the supply and demand curves determines the *equilibrium price* and the *equilibrium quantity* of a commodity. The equilibrium price is the price consumers are willing to pay over a given period that exactly equals the quantity producers are willing to supply. At a price higher than the equilibrium price, the demand for the commodity drops, resulting in *surplus* supply of the commodity. The producers then have to push the price down toward its equilibrium level. To get rid of the surplus stock, producers will have to accept lower prices in order to generate demand for their product. The lower prices will attract some consumers to the product. This process continues until the surplus disappears and a new equilibrium is reached. On the other hand, at a price lower than the equilibrium price, the quantity demanded exceeds the quantity supplied, resulting in *shortage* of the commodity. The producers therefore have to push the price up toward its equilibrium level. The shortage of a product in the market will drive its price upward, as some consumers will be ready to pay the higher price. The price will continue to rise until the shortage disappears and a new price equilibrium is established. Figure 1–5 shows the supply/demand price equilibrium.

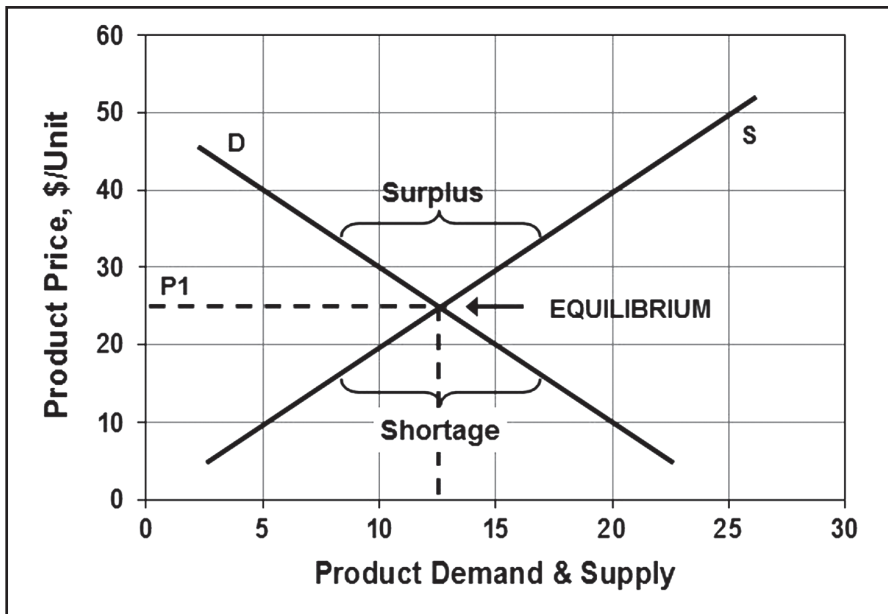


Fig. 1–5. Demand and supply price equilibrium

Shifts in equilibrium price. The market demand curve for a commodity is based on these assumptions:

- The number of consumers in the market stays constant.
- The consumers' tastes, income, and need stay the same.
- The prices of other related commodities remain unchanged.

Similarly, the market supply curve for a commodity is based on these assumptions:

- The number and size of producers of the commodity remain the same.
- The technology used to produce the commodity is unchanged.
- The factor price (the cost involved in producing a commodity) is unchanged.
- The prices of other related commodities remain unchanged.

However, changes in one or more of these assumptions will shift the curves, resulting in a new equilibrium price and equilibrium quantity for the commodity. For example, the market demand curve will *shift up* (the commodity's equilibrium price and quantity will both rise) if one or more of the factors *rises* (such as number of consumers, consumers' income, consumers' taste for the commodity, or the prices of related substitute commodities). Opposite changes will cause a downward shift in demand.

The market supply curve will *shift down* and to the right (increase) if one or more factors changes. Factors include increase in the number and/or size of the producers, improvement in technology, fall in factor price, or decrease in the prices of other commodities used in production. This will result in lower equilibrium price and a *higher* equilibrium quantity.

Elasticity of supply and demand. The elasticity of supply curve (ϵ_s) is the measurement of the average percentage change in the quantity of a commodity supplied because of an average percentage change in its price, expressed in absolute terms. The supply curve of a commodity is said to be *elastic* if $\epsilon_s > 1$, *unitary elastic* if $\epsilon_s = 1$, or *inelastic* if $\epsilon_s < 1$.

Similarly, the elasticity of demand curve (ϵ_d) is the measurement of the average percentage change in the quantity of a commodity demanded because of an average percentage change in its price, expressed in absolute terms. The demand curve of a commodity is said to be *elastic* if $\epsilon_d > 1$, *unitary elastic* if $\epsilon_d = 1$, or *inelastic* if $\epsilon_d < 1$.

Mathematically,

$$\varepsilon_s = \frac{\text{Change in Quantity Supplied}}{\text{Sum of Quantities Supplied}/2} \div \frac{\text{Change in Price}}{\text{Sum of Prices}/2} \quad (1.1)$$

$$\varepsilon_d = \frac{\text{Change in Quantity Demanded}}{\text{Sum of Quantities Demanded}/2} \div \frac{\text{Change in Price}}{\text{Sum of Prices}/2} \quad (1.2)$$

The concept of elasticity shows whether an increase or decrease in the price of a commodity will increase/decrease total revenue generated by the commodity. Therefore, if the price of a commodity falls, the total revenue of producers from the commodity will (a) increase if the demand curve is elastic, (b) remain constant for a unitary demand curve, or (c) decrease if the demand curve is inelastic.

The numerical value of ε_d also shows how expenditures for the commodity will change with price changes if no shift takes place in the demand curve. $\varepsilon_d > 1$ indicates an increase in price will result in decreased expenditure on the product, while $\varepsilon_d < 1$ indicates if the price is increased, total expenditure on the product will increase. The value of ε_d may be greater than 1 for some price ranges and less than 1 for other ranges.

Example 1–1

Assume that the demand curve for oil is represented by an equation $P_o = -2 \times D_o + 242$, where P_o is the price of oil and D_o is the demand for oil. Calculate the elasticity of demand (in million stock tank barrels) if the oil price drops from \$80/barrel to \$72/barrel.

Solution. Based on the given demand equation, the demand at \$80/barrel and \$72/barrel would be:

at \$80/barrel demand = 81 MMStb/day

at \$72/barrel demand = 85 MMStb/day

Using equation (1.2)

$$\varepsilon_d = \frac{85 - 81}{(85 + 81)/2} \div \frac{80 - 72}{(80 + 72)/2} = 0.0482 \div 0.1053 \approx 0.46$$

Since $\varepsilon_d < 1$, the demand curve in the \$72/barrel to \$80/barrel range is inelastic. Therefore, the total revenue of the producer will be decreased. This is because the percentage increase in quantity exceeds the percentage decline in price.

Crude oil price volatility

Figure 1–6 shows the historical crude oil prices and the events that coincide with the periodic fluctuations in these prices. Figure 1–7 shows inflation adjusted crude oil prices in 2008 dollars and that the inflation adjusted crude price in 1980 was an average of about \$90/barrel. This means that the average crude oil price in 2008 was still less than what it was in 1980.

Careful analysis of history can explain reasons for the price fluctuations. For example, when Iraq invaded Kuwait in August 1990 (the Gulf War), crude oil prices jumped to a five-year high (from an average of \$17 to \$33/barrel). This was due to the fear of oil supply disruption from the Gulf. However, the price fell back to an average of \$18/barrel in March 1991 when the Allied forces freed Kuwait from Iraq, and an embargo was imposed on Iraq.

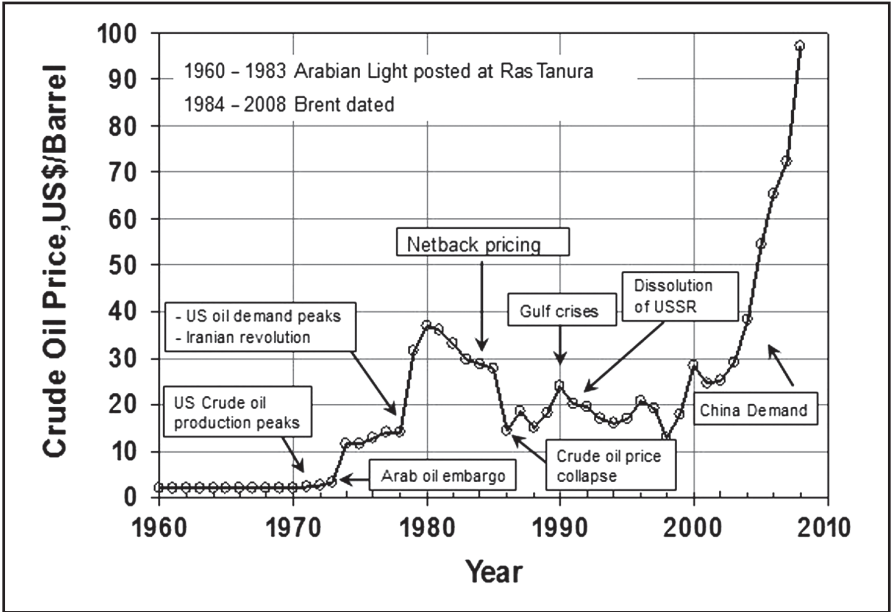


Fig. 1–6. Historical crude oil price (Source: *BP Statistical Review of World Energy*, June 2009)

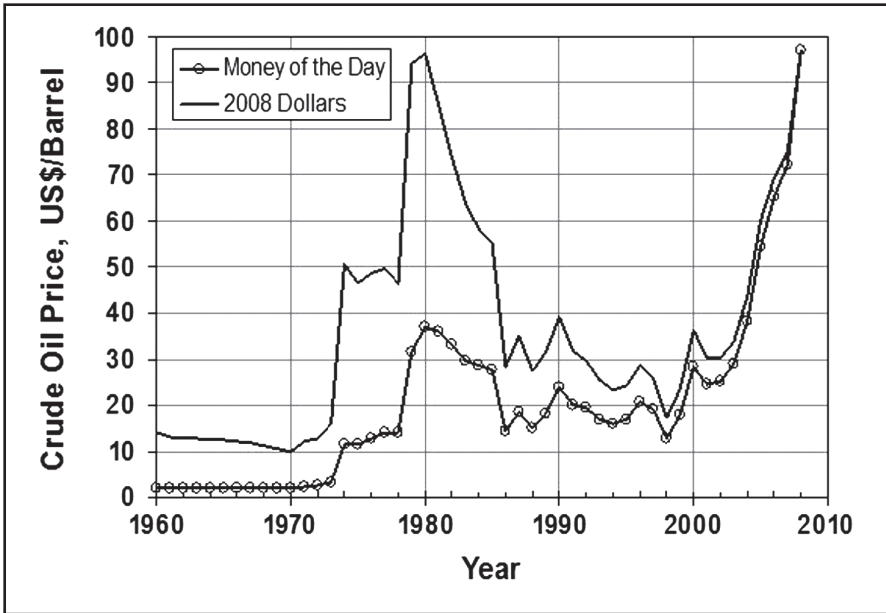


Fig. 1–7. Historical crude oil price in 2008 dollars (Source: *BP Statistical Review of World Energy*, June 2009)

Similarly, on May 20, 1996, an agreement was signed by the United Nations and Baghdad (Iraq) allowing Iraq to export oil for the first time since Iraqi troops invaded Kuwait in August 1990. This event led to the following news brief in the *Gulf Times*:

Iraq sends shivers through oil market and Brent crude prices fell by a dollar in London on the week. Dealers and analysts agreed that this extra oil would cripple the market already creaking under the strain of burgeoning supplies.

Such reports or comments do not always portray the reality or consequences of the event taking place. Many times speculators use such anecdotes to manipulate the market.

Similar analysis appeared in the Associated Press:

Oil lingers around \$60 on demand worries. (Associated Press, July 13, 2009)

Oil climbs to near \$60 in Asia as falling US crude inventories suggest demand pickup. (Associated Press, May 13, 2009)

Oil prices climbed more than 5 percent, surpassing \$70 a barrel Wednesday after a government report said the nation's gasoline supply dropped unexpectedly and demand increased from last year.
(Associated Press, September 30, 2009)

As noted in the news events above, the words *supply* and *demand* frequently coincide with crude oil price fluctuations. The increase in crude oil price since 2000 is mainly due to the oil consumption in fast-developing countries across Asia and the Middle East. The Organization of Petroleum Exporting Countries (OPEC) accounts for 40% of world production and two-thirds of world reserves. As shown in figure 1–8, the domestic consumption of these OPEC countries is constantly increasing over time. This means that these countries will have less and less oil available for export.

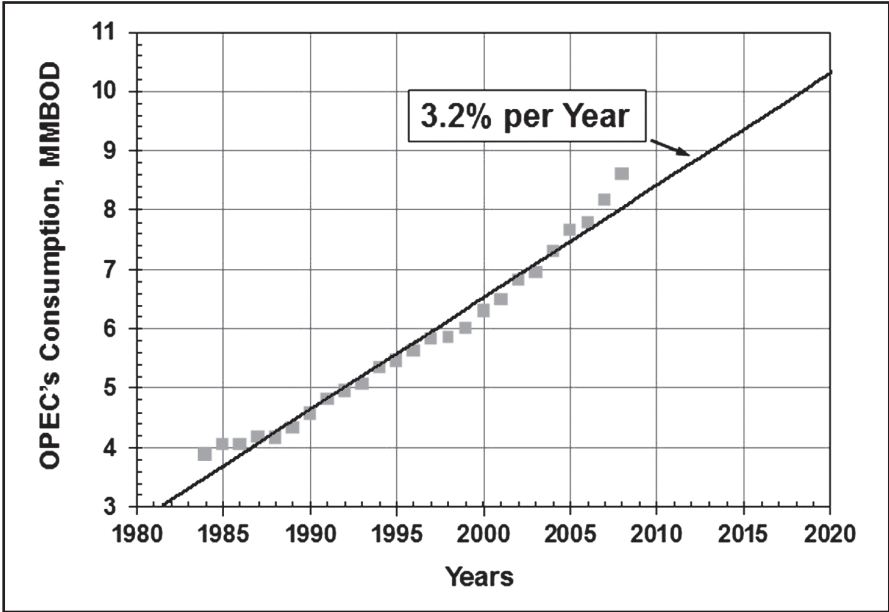


Fig. 1–8. OPEC's annual oil consumption (Source: *BP Statistical Review of World Energy*, June 2009)

Besides the crude oil supply and demand, market speculation (creating fear of inadequate supply or increasing demand), geopolitical tensions, and natural disasters play dominant roles in the crude oil price volatility. These events include (a) tensions between Iraq, Iran, and the United States, (b) tanker hijackings (Somali pirates demanding ransoms in the millions), and (c) disruption of supply due to tsunamis and hurricanes.

The enormous market volatility we have seen in 2008 is very problematic for our long-term business planning and economics. In 2008 the price of crude went up by \$11/bbl in one day. The peak crude oil price touched about \$147/bbl and the following week it was down to \$70/bbl and then further down to about \$36/bbl. Such price volatility is not merely due to the supply/demand imbalance but also due to market speculation. The supply/demand imbalance cannot just appear one day and then disappear the next day. The crude oil trading mechanisms such as futures trading, options trading, etc., active in the market are the main culprits for such volatility. The price of crude oil will be much more predictable if the trading of actual quantities of crude takes place rather than paper trading. The paper trading generates temporary imbalances in the market. The influx of speculative money into the oil futures market has played a dominant role in boosting the oil price. Such volatility will obviously be very detrimental to good business and operational planning.

The erratic price movements, as discussed above, prove that there are a wide variety of factors beyond the supply/demand fundamentals that are influencing the price. Such complexity requires us to develop a deep understanding of the underlying root causes of this volatility.

The crude oil price volatility is going to exist for as long as there is surplus oil supply capacity available from the crude oil exporting countries. The pure economic fundamentals of supply/demand and price equilibrium will come into play only when this surplus capacity disappears. Once the surplus capacity disappears, then the market price, like that for any other commodity, will adjust based on the supply/demand situation.

In this author's opinion, the time is not too far when there will not be enough supply available to offset the demand. At that time, the price itself will be the dominating factor in adjusting the demand. Figure 1–9 shows the historical crude oil imports and the net production availability by the OPEC countries. The net production is the total production of the OPEC countries less their domestic consumption. It is shown that about 26.5 million barrels of oil per day (MMBOD) of imports are currently fulfilled by non-OPEC countries such as Canada, Russia, Brazil, and Mexico. Although it is not a very likely scenario, let's assume that these non-OPEC countries will continue to supply the 26.5 MMBOD up to at least 2020. If the oil imports maintain the same trend as shown in figure 1–9, then the OPEC countries will have to increase their export availability from the 2008 rate of about 28 MMBOD (production of about 37 MMBOD) to more than 36 MMBOD (production of about 46 MMBOD) by 2020. The OPEC's domestic consumption by 2020 is estimated to be about 10 MMBOD.

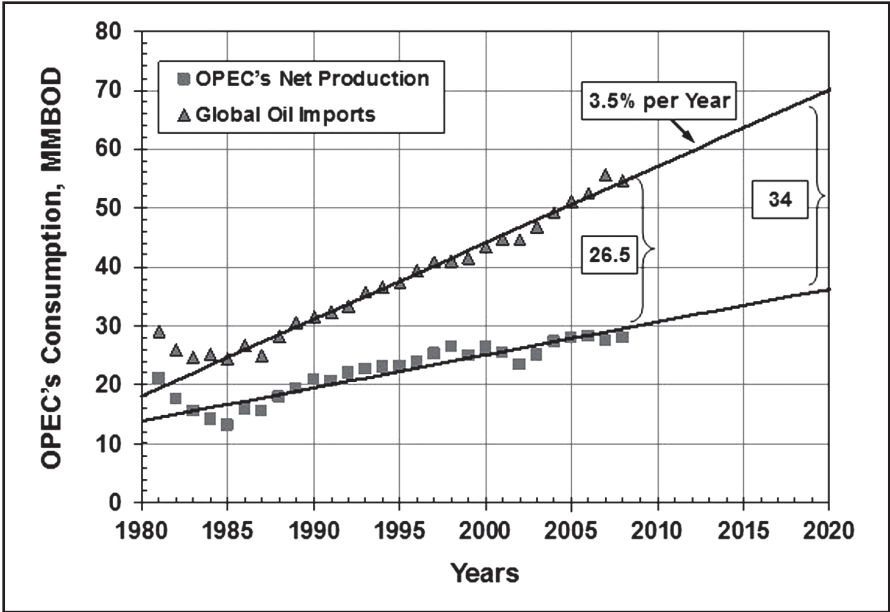


Fig. 1–9. Global crude oil imports and OPEC's net supply (Source: *BP Statistical Review of World Energy*, June 2009)

In this author's opinion, both these scenarios (the non-OPEC countries maintaining the same exports and OPEC's ability to produce 46 MMBOD, maintaining it at this level or even increasing it further with time) are very unlikely. Figure 1–10 shows a historic trend of cumulative reserves replacement ratio. The reserves replacement ratio is a key performance indicator (KPI) for the exploration and production (E&P) companies. The ratio shows whether the company is replacing whatever hydrocarbons it has produced during the year with new reserves. The new reserves may be added through (a) exploration, (b) purchasing reserves, (c) studies, (d) infill drilling, and (e) technology. Figure 1–10 shows a very interesting picture—we are not replacing the reserves we are producing every year. This situation is mainly applicable to the oil importing countries. This means the demand on imports will keep on increasing due to both increasing consumption of the oil importing countries and their declining reserves replacement ratio.

By looking at the trends in figures 1–8 to 1–10, it is inevitable that the crude price will go up as soon as the global economy improves. If not in the short term, it will definitely go up in the long term. In the short term, OPEC will be able to play an important role in stabilizing the oil price but only until they have surplus capacity. With the availability of surplus capacity, OPEC can keep the crude oil price stable at a certain level by imposing production

quotas on each member country. The production quotas will be adjusted to meet the demand.

Other critical information that figures 1–9 and 1–10 reveal is that the reserves life index, based on the 2008 crude oil production of about 81.82 MMBOD and reserves of 1,258 billion barrels (*BP Statistical Review of World Energy*, June 2009), is about 42 years. Increasing the rate further will further shorten the reserves life index. So, the energy independence (explained in the following section), to some extent, will be globally imminent.

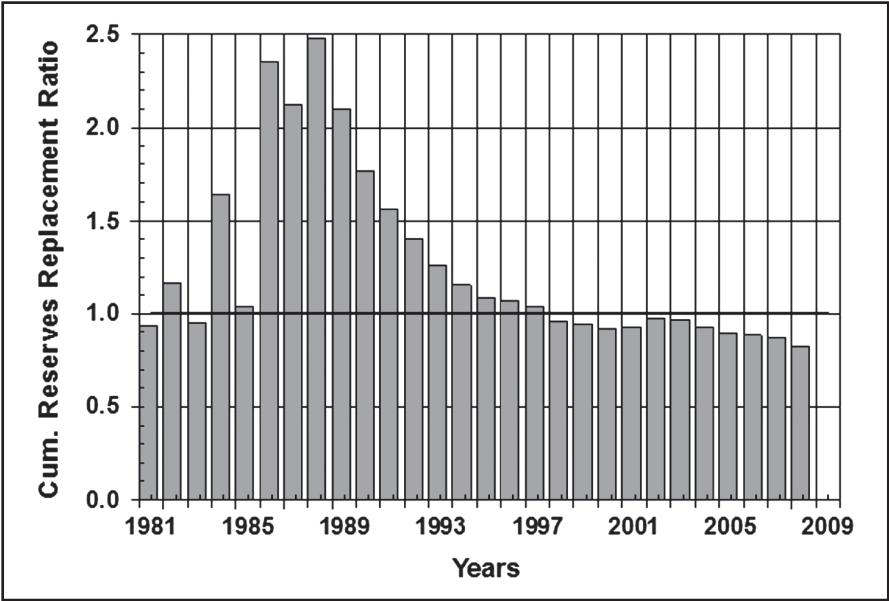


Fig. 1–10. Cumulative global reserves replacement ratio (Source: *BP Statistical Review of World Energy*, June 2009)

Energy independence

There are many buzzwords used by executives and politicians in order to impress laymen and solidify their positions of power. In the past decade, one such buzzword or phrase related to the oil and gas industry is “energy independence,” which has echoed in our ears from many different angles. Although there is no problem in coming up with such phrases, there has to be some reality behind them. The irony is that most of those who talk about energy independence do not even know about the full value chain of petroleum products.

Such buzzwords may be detrimental to good business, operational planning, and interdependent cross-border socioeconomic and geopolitical relationships. The term “energy conservation” probably more aptly addresses the issue of energy demand and the associated supply, and it is more appropriate to replace the phrase “energy independence” with “reduced energy dependence.” The reduction in dependence can be achieved through conservation and judicious use of exhaustible energy resources, but trying to achieve independence could be a painful effort.

It is very important that individuals, markets, oil and gas producers, and governments commit themselves to pursuing mutually beneficial collaborative efforts all along the petroleum value chain. They should all work together on the vast range of issues that are detrimental in shaping the future of energy rather than pursuing the “do it alone” scenario.

In this author’s opinion, the misguided policy statements about achieving “energy independence” will create more uncertainty and panic rather than do any good. This is because energy independence is a global issue and should not be used as a political issue by one or two countries. The issue is not solely related to the United States or the European Union or other major importers; it is a global issue. Therefore, calls for an abrupt shift to unproven alternative energy sources could be very detrimental to the long-term research and development investments applicable to the oil and gas industry operations.

The best way to handle the problem on global playing field is for everyone to pool their resources together to come up with alternative sources of energy. The alternative sources would not necessarily replace the natural exhaustible resources, but complement them so that these exhaustible resources may last longer than currently anticipated. It is important to acknowledge the fact that petroleum will continue to play a major role in satisfying the world’s demand for energy for many, many decades to come.

The International Energy Agency (IEA) forecasts that oil will remain the world’s largest single energy source in 2030, even as total energy demand continues to grow. Of course, with rising energy demand there will be a need to draw upon all available energy resources.

Renewable sources of energy currently account for roughly 30% of energy needs. Much of these renewable sources come in the form of traditional biomass such as wood and animal waste used for heating and cooking among the world’s poorest populations. This energy source is neither efficient nor environmentally acceptable. Geothermal, wind, and similar renewable energies currently account for less than 1% of today’s total energy supply. This means that significant breakthroughs in their efficiency and economic performance will be required before they will have a major impact on our energy roadmap. While we have heard a great deal about alternatives

such as ethanol and other biofuels, an objective assessment of these sources demonstrates that they still face considerable technical, economic, infrastructure, and even environmental obstacles.

It is worth highlighting that 60% of all produced oil is utilized in the transportation industry. This utilization will continue to increase due to the continued prosperity of developed countries and the rising levels of living standards in the developing countries. Could anyone, a decade ago, have imagined that such large numbers of Chinese would go from riding bicycles to driving cars? And don't forget that China has the largest population in the world, so even a small percentage of economic shift makes a huge difference. China's oil consumption has almost doubled over the last decade, from 4.2 MMBOD in 1998 to 8 MMBOD in 2008. The growth from 2007 to 2008 was 3%. This is evidence that the world's growing population will be increasingly mobile, and the cars, trucks, buses, planes, and ships they will use to move themselves and their goods and products from place to place will run almost entirely upon petroleum.

To claim 100% energy independence (energy here refers to oil and natural gas), the above means of transportation should be able to run on solar, wind, etc. In addition to the transportation industry, oil and gas are also critical inputs for the chemical and petrochemical industries, for agricultural and manufacturing concerns, and for many other economic sectors. Therefore, it is fully acknowledged that energy, and in particular petroleum, is an indispensable input for today's economies and growth. This means that enhanced energy security and the steps and strategies needed to achieve it must be at the top of virtually every nation's political and economic agenda today.

Inflation

Capital investment decision making is affected by inflation. It will make investments with costs incurred earlier in their economic life look more attractive than investment costs for the same purpose incurred later. Inflation occurs when there is an increase in general price levels (i.e., when the price of a given basket of goods and services increases).

But not all prices rise. Even during acute inflationary periods, some specific prices may be relatively constant while others actually fall. Nor does inflation mean prices rise evenly or proportionately. If some prices rise and others fall, the price level may not change. Inflation reduces the purchasing power of a unit of money over time. It is typically reported as a percentage increase in prices from one time to another. The percentage increase is referred to as the rate of inflation.³⁻⁴

Inflation affects real income and wealth. Real income is the amount of goods and services that can be purchased with a given money income. For example, a worker's salary increases by 10%. If prices remain stable, the worker can purchase 10% more goods and services, meaning real income also increased by 10%. If prices increased more than 10%, the worker cannot buy as much as he or she previously could, meaning real income is less than money income. If the price level doubles while the money income is constant, the real income is cut in half, meaning only half the basket of goods can now be bought.

The so-called "rule of 70" provides a convenient formula for gaining a quantitative appreciation of inflation. The rule allows a quick calculation of the number of years required to double the price level, by dividing the number 70 by the annual rate of inflation. For example, 5% annual rate of inflation doubles the price level in about 14 ($70 \div 5$) years.

There are three major causes of inflation: demand-pull inflation, cost-push inflation, and structural inflation. These three causes of inflation are related and not mutually exclusive (i.e., all three can be in action simultaneously).

Demand-pull inflation. The demand-pull inflation develops when economic resources are fully employed and one or more of the following occur:

1. There is an increase in aggregate demand
2. Aggregate demand grows at a faster rate than aggregate supply
3. Bottlenecks develop prior to full employment. (For example, some industries are fully utilizing their productive capacity before others and are, therefore, unable to respond to further increases in demand for their products by increasing supply, so their prices rise.)

Demand-pull inflation occurs because prices need to be increased in order to ration goods and services as the desire to purchase exceeds the ability of the producers to supply. The essence of demand-pull inflation is crudely expressed in the phrase, "too much money chasing too few goods."

Cost-push inflation. This type of inflation stems from the supply or cost side of the market. It may be due to the market power of labor unions, business firms or suppliers of raw materials, or to government regulation or taxation of business. In order to increase their profits, the firms with sufficient market control increase prices of goods and services they produce. Similarly, when government raises taxes on businesses, labor's wages increase, or suppliers of raw materials increase their prices, the result is a higher cost of producing goods and services and businesses passing those costs on to the consumer.

Therefore, cost-push inflation results when monopolistic power permits supply prices to be influenced by the actions of firms, labor unions, raw material suppliers, or government.

The economy will also suffer from cost-push inflation if there is a simultaneous increase in inflation and unemployment. This inflation/unemployment is supply driven in the sense that, initially, producers will produce goods that are not purchased. In order to reduce their financial losses, producers increase prices and reduce their workforce, the result of which is a simultaneous increase in inflation and unemployment. This is called *stagflation*.

A typical example of the cost-push inflation is the rapid increase in the price of steel since 2004. Steel is a material used extensively in the oil industry (drilling rigs, tubulars, pipelines, casings, and other facilities). Any increase in the price of steel will increase the cost of field development and production, which must be passed on as increases in crude oil prices. Similarly, an increase in crude oil prices will increase the price of gasoline and other products for which crude oil is used as a feedstock or fuel. Since gasoline is the major fuel used in transportation (every business has to rely on transportation to get its products to the consumers), the cost of the products will go up, pushing the price higher.

Structural inflation. This is also referred to as demand-shift inflation. In this case, the total demand is not excessive, but a sharp change in the structure or composition of total demand takes place. This results in increased prices and wages in those segments of the businesses where expanding demand occurred. However, the prices and wages do not fall in those businesses where demand declined. This results in net increase in price and wage levels, thus inflation occurs.

Impact of inflation. Let's shift from the causes of inflation to the impact/effects of inflation. How does inflation redistribute income? What is the possible impact of inflation on financial management of capital investments? What are possible ways of accounting for the effect of inflation in capital investment decision making? These will be more thoroughly addressed in chapter 3.

1. *Fixed income groups.* Inflation arbitrarily penalizes people with relatively fixed incomes. The households whose income is not keeping up with the pace of inflation will find their real incomes deteriorating because of inflation. The purchasing power of each dollar they receive in their pay envelopes will fall as prices rise, resulting in a decline of their standard of living.

2. *Savers.* As prices rise, the real value or purchasing power of savings will deteriorate if inflation is higher than the interest earned on the savings. For example, a person deposits \$1,000 in a saving account earning 6% interest per year. After one year, he will have \$1,060 in savings. However, if inflation is 8% during that year, the real value of the \$1,060 is then only \$981.48 ($\$1,060/1.08$). To put it another way, the basket of goods that could be bought for \$1,000 a year ago is now costing \$1,080 because of 8% inflation, while only \$1,060 is available now.
3. *Debtors and creditors.* Inflation tends to benefit debtors (borrowers) at the expense of the creditors (lenders). As the prices go up, the value of the dollar comes down. As a result, the debtor gets dear dollars but pays back cheap dollars. However, this will happen only if the inflation is unexpected. If the rate of inflation is known or it can be anticipated, lenders will add the percentage rate of inflation to the rate of interest charged on the loan. This analysis also implies that, rather than being a cause of inflation, high interest rates are a consequence of inflation.
4. *Planning difficulties.* Businesses operate based on long-term plans. For example, a firm commits capital investment in a plant in anticipation of receiving certain revenue over the economic life of the plant. However, if the anticipated costs are not in line with the anticipated inflationary increase in costs, the investment may become marginal or uneconomical. Therefore, it is important to use improved forecasting techniques in order to include more flexibility in reflecting the increased level of uncertainty in the economy.
5. *Demand for capital.* Inflation increases the amount of capital required to embark on a desired volume of business. The costs of expanding or replacing plants increase, while workers demand higher wages. All these require raising additional capital. On the other hand, the Federal Reserve System tends to restrict the supply of loanable funds in order to curb inflation. The ensuing scramble for limited funds drives interest rates higher (i.e., increase in cost of capital). High interest rates and shortage of capital cause firms to be especially careful in planning long-term capital outlays.
6. *Depreciation.* In financial statements and cash flows, depreciation charges are based on yesterday's currency value. With the shrinkage in an inflationary economic environment, this depreciation will rapidly become inadequate. Eventually, the asset has to be replaced,

at which time the depreciated currency will be insufficient to cover the replacement cost.

Expectations of long-term oil and gas prices, currency exchange rates, inflation rates, and financing and discount rates are, of course, critical in evaluating the potential rewards from a capital investment project.

Consumer price index

The consumer price index (CPI), also referred to as the cost of living index, provides a quantitative indication of inflation over a period of time. The CPI is compiled monthly by the U.S. Bureau of Labor Statistics (BLS). Two forms of CPI, CPI for Urban Wage Earners and Clerical Workers (CPI-W) and CPI for All Urban Consumers (CPI-U), are in common use in most analysis. The CPI-W includes about 32% of the population, while the CPI-U includes about 87% of the total population. The CPI-U also includes groups such as professionals, managerial, self-employed, short-term workers, retirees, unemployed, and technical workers.

The prices of the items in a basket of goods and services each month are checked and recorded. The basket is composed of thousands of day-to-day consumable goods and services. As mentioned above, the basket typically includes food, clothing, fuels, housing, shelter, transportation fares, medical charges, drugs, and other routinely consumable items. The CPI in each period is referenced to a pre-specified index period. The index starts with 100 in 1982–84; the CPIs of all other periods are referenced to this particular point. An increase of 10% from the base will mean a CPI of 110.

Table 1–1 shows the CPI-U for the U.S. over the last 25 years. The CPI that is normally referred to is the annual average of the monthly CPIs. In addition, a certain CPI is not a dollar value but a normalized index that gives indication of change in the index over time, relative to the base value. In table 1–1, the CPI in 1984 is 103.9, and in 2008, it is 215.3. This means that the basket of goods and services that we could purchase with \$103.90 in 1984 will cost \$215.30 in 2008. Alternatively, if your annual household income in 1984 was \$103,900 and your annual income in 2008 is \$215,300, then you will be able to maintain the same living standard. If your income in 2008 is \$175,000, then your living standard has to be significantly adjusted downward because your current income is not enough to get you the same goods and services that you used to get in 1984. This means that although your *money income* (nominal salary) has gone up from \$103,900 to \$175,000, your *real income* (real salary) has actually gone down.

Therefore, the CPI can be used to show how the purchasing power of money has changed over time. The purchasing power of a dollar in 2008 is about 96.3% of the purchasing power of the dollar in 2007.

$$\frac{CPI_{2007}}{CPI_{2008}} = \frac{207.34}{215.30} \times 100 = 96.30 \text{ percent}$$

Table 1–1 U.S. Consumer Price Index, All Urban Consumers (CPI-U)

Year	Average CPI per Year	Percent Change
1984	103.9	4.3
1985	107.6	3.6
1986	109.6	1.9
1987	113.6	3.6
1988	118.3	4.1
1989	124.0	4.8
1990	130.7	5.4
1991	136.2	4.2
1992	140.3	3.0
1993	144.5	3.0
1994	148.2	2.6
1995	152.4	2.8
1996	156.9	3.0
1997	160.5	2.3
1998	163.0	1.6
1999	166.6	2.2
2000	172.2	3.4
2001	177.1	2.8
2002	179.9	1.6
2003	184.0	2.3
2004	188.9	2.7
2005	195.3	3.4
2006	201.6	3.2
2007	207.3	2.8
2008	215.3	3.8

Source: U.S. Department of Labor, Bureau of Labor Statistics, Washington, D.C.

The inflation rate is calculated from the movement of the CPI from one period to another and is usually expressed as a percent change rather than as a change in index. The periods can be monthly, quarterly, semi-annually, and

annually. As shown in figure 1–11, the CPI usually shows a very smooth and consistent trend over the years, but the inflation rate shows wild fluctuations over time. The following calculations show the inflation rate for 2008.

Average CPI for 2008	215.30
Average CPI for 2007	207.34
Index Point Change	7.96
Divide Index Point Change by 2007 CPI.....	0.0384 or 3.84%

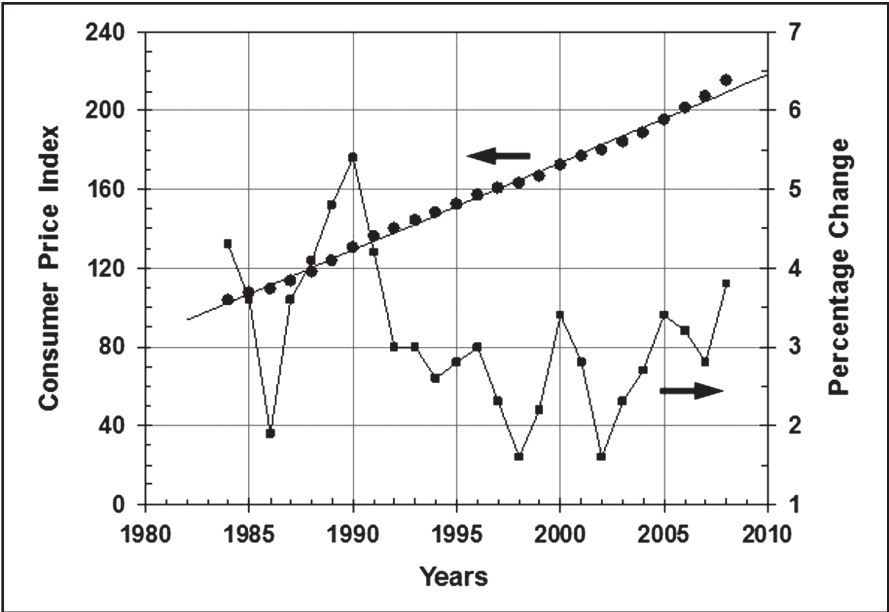


Fig. 1–11. U.S. Consumer Price Index, All Urban Consumers (CPI-U)

As shown above, to arrive at a percentage change over an entire year it is recommended to use CPIs based on 12-month averages. This should not be confused with over-the-year change in the CPI, such as December 2007 to December 2008. This may not always give the right inflation figure because there will be wide fluctuations in the monthly CPIs as shown in figure 1–12. The calculations below show the difference in the inflation rate calculated by using the December 2007 and December 2008 CPIs, versus the annual averages as shown in the calculations above. The difference could be significant; the over-the-year percent change may be misleading and not very useful.

CPI for December 2008.....	210.23
CPI for December 2007.....	210.04
Index Point Change	.0.19
Divide Index Point Change by Dec. 2007 CPI ...	0.00091 or 0.091%

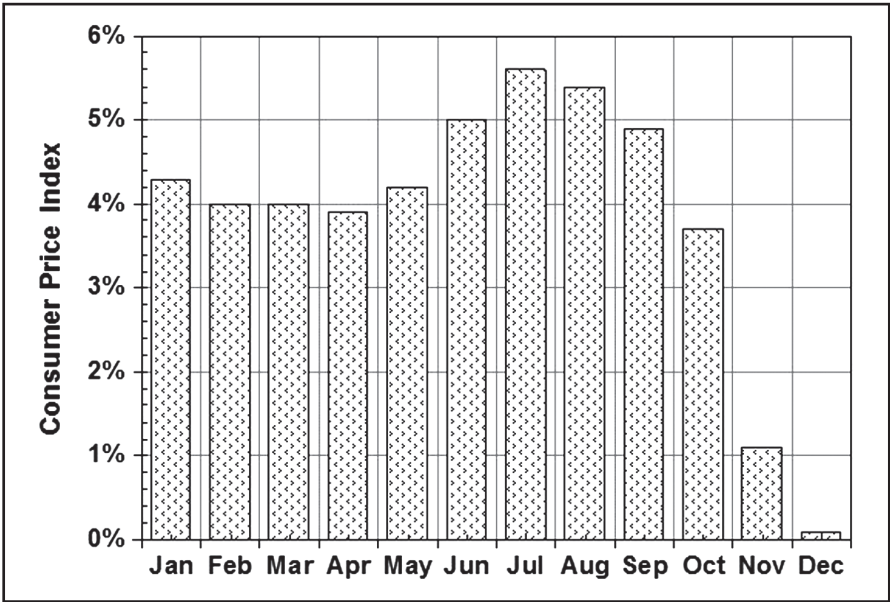


Fig. 1–12. U.S. inflation, month-to-month over 2007 and 2008

The *current* or *nominal* dollars can be converted into *constant* or *real* dollars by multiplying each dollar amount by a ratio of price indexes. Suppose the average crude oil prices per barrel in 1990, 2000, and 2008 were \$23.73, \$28.50, and \$97.26, respectively. These prices are referred to as the prices in the *money-of-the-day* (MOD). The CPI-U was 130.70 in 1990, 172.27 in 2000, and 215.30 in 2008. The conversion of the 1990 and 2000 crude oil prices to the *constant 2008 dollars* would be as follows:

1990: $(215.30/130.70) \times \$23.73 = \39.09 per barrel
2000: $(215.30/172.27) \times \$28.50 = \35.62 per barrel
2008: $(215.30/215.30) \times \$97.26 = \97.26 per barrel

The above calculations show that the MOD crude oil price in 2000 was about \$4.77 per barrel higher than the price in 1990. However, real crude oil price in 2000 was actually \$3.47 per barrel lower than the real price in 1990.

As shown in the preceding paragraphs, the average historical inflation over the years is calculated using the following equation:

$$\text{Average Inflation} = \left(\frac{CPI_{\text{Current Year}}}{CPI_{\text{Reference Year}}} \right)^{1/(\text{Current Year} - \text{Reference Year})} - 1 \quad (1.3)$$

Therefore, inflation over the years from 1984 to 2008 can be calculated using Equation (1.3) as:

$$\begin{aligned} \text{Average Inflation} &= \left(\frac{CPI_{2008}}{CPI_{1984}} \right)^{1/(2008-1984)} - 1 \\ &= \left(\frac{215.30}{103.90} \right)^{1/24} - 1 = 0.031 \text{ or } 3.08\% \end{aligned}$$

Uncertainty and Risk

In this book, it is implicitly assumed that the results and implications of investment decisions are known at the time of the decision (i.e., the future is known, and it is known with certainty). That is, the probability of success for each investment under evaluation is considered to be 100%. In reality, such conditions simply do not exist. In practice, different investment options will inherit different degrees of associated uncertainty and consequently different degrees of risk. Risk and uncertainty are complementary concepts.

Risk refers to situations where a project has a number of possible alternative outcomes, but the probability of each outcome is known or can be estimated. Uncertainty refers to a situation where these probabilities are not known or cannot be estimated.

When faced with decision choices under uncertain conditions, we can use informal analysis methods and quantitative methods to ascertain risk and uncertainty associated with the available investment options. Informal analysis involves the decision maker's experience, intuition, judgment, and gut feeling. The quantitative analyses use a logical and consistent decision strategy by incorporating the effects of risk and uncertainty into the analysis results. Several different approaches are available that can be used to quantitatively incorporate risk and uncertainty into investment analysis.⁵

Most methods of dealing with risk and uncertainty require assessments of probability of the uncertain outcomes. Therefore, the success of each method depends upon the accuracy of these probabilities. Thus, the

importance of developing accurate probability estimates is the essential part of decision analysis.

Volume 2 of this series is devoted to outlining a number of approaches for analyzing degrees of risk and allowing for them when making investment decisions. The following topics are included in volume 2:

1. *Introduction*
2. *Statistics and probability concepts.* This chapter is divided into two parts. The first part deals with the descriptive statistics (i.e., calculating measures of central tendency and measures of variability from grouped and ungrouped data). The second part deals with the concepts of probability and theoretical probability distributions.
3. *Expected value and decision trees.* In this chapter, the probability concepts and the deterministic concepts presented in this volume are combined to arrive at the expected value of investment alternatives under consideration.
4. *Incorporating attitudes toward risk.* In this chapter, the decision maker's particular risk attitudes (risk preferences) are incorporated into decision analysis.
5. *Determining venture participation.* The concept of Gambler's Ruin is presented in this chapter, and it is used to determine the optimum working interest in a certain investment. In addition, the concepts of risk tolerance and risk aversion are used to arrive at a balanced portfolio of investments while optimizing the working interest carried in each investment of the portfolio.
6. *Simulation in decision analysis.* The concepts related to Monte Carlo simulation are presented in this chapter.
7. *Simulation exercises.* In this chapter some topics of exploration economics are presented. In addition, several oil and gas industry related simulation problems are discussed.

Volume 2 also includes a companion CD-ROM. The CD-ROM includes spreadsheet solutions of all the examples presented in volume 1 and volume 2. The CD-ROM also includes three comprehensive economics analysis spreadsheets, the decision trees solved in the book, Monte Carlo simulation models, normal probability graph paper, log-normal probability graph paper, and interest tables. A description of the economic analysis spreadsheets is given in appendix B of this book.

Questions and Problems

- 1.1 Distinguish between strategic and tactical planning.
- 1.2 Briefly differentiate between objectives, goals, and strategies as applied to planning.
- 1.3 What is a profit plan? Why is planning a continuous and iterative process?
- 1.4 Contrast major and minor capital investments. Give examples of each.
- 1.5 What is meant by the term *capital budgeting*?
- 1.6 What is the significance of a demand curve and a supply curve? How are these curves determined?
- 1.7 What is a free-enterprise system? How is the market price of goods or services determined in a free-enterprise system?
- 1.8 Define the terms *elasticity of demand* and *elasticity of supply*. What are the conditions for a demand curve to be elastic, unitary elastic, or inelastic?
- 1.9 If the total revenue of the producer rises when the price falls, the demand curve is
 - a. Elastic
 - b. Unitary elastic
 - c. Inelastic
 - d. Any of the above
- 1.10 If the total revenue of the producer rises when the price of his commodity rises, the demand curve is
 - a. Elastic
 - b. Unitary elastic
 - c. Inelastic
 - d. Any of the above
- 1.11 Explain the basic causes of demand-pull and cost-push inflation.

- 1.12 What is the effect of inflation on (a) fixed income groups, (b) debtors, and (c) creditors? How can one be protected against the redistribution effect of inflation?
- 1.13 Explain the difference between money income and real income.
- 1.14 Your annual salary in 2005 was \$48,000, and in 2008, it was \$60,000. Based on the CPI listed in table 1–1, was your salary in 2008 more or less than what you were drawing in 2005?
- 1.15 What is the difference between risk and uncertainty?

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2

The Time Value of Money

This chapter introduces simple concepts concerning the role of *interest* (cost of money) in investment evaluation (capital budgeting). The concepts presented in this chapter are essential in day-to-day personal finances and important for the understanding of many topics presented throughout this book, whether dealing with investment analysis, involving risk and uncertainty, or otherwise. Although the concepts are simple, they are frequently misinterpreted. Most of the problems involving the cost of money can be solved with only a few simple formulas.

1. **Inflation.** Figure 2–1 shows a simple case in which a sum of money is expected to be received or disbursed at two different times, seven years apart. Although the two amounts (\$15,000) in option A and option B look equal, in reality they are not. The difference in the values is due to the time value of money, which is due to inflation (as discussed in chapter 1). The value of \$15,000 in option A, if left unused (not invested—kept under a mattress for seven years), deteriorates over time (its purchasing power decreases every year).
2. **Interest.** The \$15,000 in option A, if invested at a certain interest rate, will grow over time. After seven years its value will be different because it would have earned interest over this period.
3. **Risk.** Are we 100% certain that we will get the money in option B seven years down the road?
4. **Opportunity cost.** The \$15,000 in option A has opportunity cost; the dollars will not be kept under a mattress. The money may be kept in an interest-bearing account or placed in some other investment vehicle. When this money is taken out of the interest-bearing investment account, it forgoes the opportunity of earning interest. The forgone opportunity is its opportunity cost. The alternative investment must generate the minimum of what has been forgone. In addition, if the opportunity in option B has more risk than option A, then there will be an expectation of risk premium.

Therefore, the two numbers cannot be compared as is, added, or subtracted unless they are converted to equivalent bases. The equivalent bases can be to (a) convert the value in option A into future value at the end of year seven, (b) bring the value in option B to the present at the beginning of year one, or (c) convert each value into its annual equivalent values.

Based on the short discussion above, it becomes obvious that decision making in investment analysis requires anticipated receipts (*revenues*) and disbursements (*costs*) of alternative investment proposals be placed on *equivalent* basis. Only then we will be able to have an apple-to-apple comparison. The economic meaning of equivalence, methods used to arrive at the equivalent basis, and the consequences of not using the equivalent basis are discussed in this chapter.

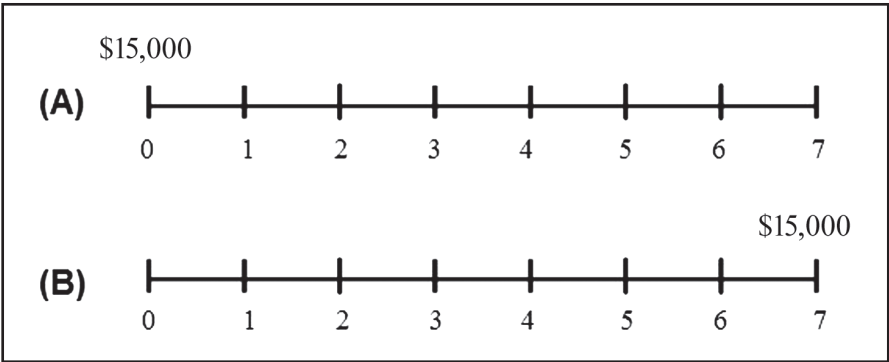


Fig. 2–1. Value of money over time

It will be clear from the discussion in this chapter that interest is the *time value of money*. Interest is further composed of risk-free rate, market risk premium (expected market rate of return and the risk-free rate of return), systematic risk (risk associated with overall aggregate market returns), and administrative costs. It must be taken into consideration in all investment analysis problems in order to arrive at valid decisions.

Implementation of projects, besides technical know-how and manpower resources, requires financial resources. Two major forms of financial resources are used by business firms: equity financing through common stock, and various forms of debt. Financial markets include commercial banks, savings and loan associations, finance companies, investment bankers, and insurance companies. Regardless of the source of financing, the financial resources cost money.

The Concept of Interest

Interest is the price/rent paid for the use of money or loanable funds, expressed as a percentage of loaned or borrowed funds. The amount of loaned or borrowed funds is referred to as the *principal*. The rate of interest, or rate of capital growth, is customarily stated on an annual basis. Thus, a 10% interest rate per year on \$100 borrowed or invested for one year will yield \$110. The \$100 is referred to as *principal*, and the \$10 is the interest charged or received. The gain of \$10 is called the rate of *return* of 10% on the investment of \$100. Due to this growth ability of capital over time, it is said, “money makes money.”

The interest charged becomes cost to the borrower and revenue for the lender. Since it is a cost to the borrower, interest is considered an important element in investment decisions involving the acquisition of financial resources. In this transaction between the borrower and the lender, both the borrower and the lender try their best to achieve their own profitability goals.

Interest rates in the market are linked to the *prime lending rate*, which is the interest rate charged by banks to their most creditworthy customers (usually the most prominent and stable business customers). The prime rate stems from the *discount rate* and the *federal funds rate*. These rates are fixed by the country’s central banks. For example, the Federal Reserve (the Fed) of the United States, European Central Bank (ECB) of the European Union, Bank of Japan, and so on. The *discount rate* is the interest rate the Fed charges banks to borrow reserves from the Fed. Banks borrow reserves from the Fed when it is impractical to borrow from other banks. The *federal funds rate* is the target interest rate for banks borrowing reserves among themselves.

Many consumer loans (including credit cards, home equity loans, and home equity lines of credit) are tied in some way to the prime rate. When it changes, so do the interest rates consumers must pay. When the Fed raises or lowers the federal funds rate and federal discount rate, banks typically raise or lower their prime rates by the same amount. Historically, the discount rate is lower than the Fed funds rate by about 50 basis points (0.5%). Similarly, the prime rate is higher than the Fed funds rate by about 3 percentage points. For example, in 2008 the Fed cut the discount rate from 1.25% on October 29, 2008, to 0.5% on December 16, 2008. The Fed funds rate was correspondingly cut from 1.00% to 0.00%–0.25%. Accordingly, the banks adjusted their prime rate to 3.25%. The rates were cut by 325 basis points (3.25%) from January to December of 2008. On June 23, 2010 the Federal Open Market Committee (FOMC) voted to keep the target range for the fed funds rate at 0%–0.25%. Therefore, the U.S. prime rate remains at 3.25% as of August 2010.

Consequently, most interest rates in the market had to be cut. Such change by the Federal Reserve authorities implies a change in economic conditions has occurred. The new conditions call for tightening or easing of monetary conditions. Lower rates are needed to keep the economy from slipping into recession, and the rates are raised to curb inflation.

The observed prevailing interest rates in the financial markets usually vary; at any given time there is an array of different rates. This depends on the risk, maturity of loan, administrative costs of the loan incurred by the financial institution, and competition for loanable funds. In general, the interest rate on loans will be higher due to:

- Lender's risk—which reflects the possibility that, for various reasons, the principal and the accrued interest may not be repaid by the borrower
- Longer period of loan—risk exposure for a longer period of time
- Smaller loan amounts—higher administrative costs
- A less competitive financial system
- The nature of the investment—rates are higher for riskier investments

Therefore, the interest rate facing a saver is always less than the rate facing a borrower. In the current market, the quoted interest rates may range from 3% to 21% or even higher. For example, interest charged on home mortgage loans may be 6.5%, credit unions may charge 10%, retailers and credit card companies typically charge 18% to 21%, and banks pay only around 0.5% to 2% on savings deposits.

The interest rate is also influenced by the demand and supply of *loanable funds*. A loanable fund is the actual amount of money available for lending to prospective borrowers. The *demand for loanable funds* depends on borrowing by:

- Firms in order to invest in various productive investments available to them
- Consumers as home mortgage loans, automobile loans, and loans required for other day-to-day requirements
- Governments in order to finance their budgets

On the other hand, *supply of loanable funds* stems from the past and current savings of individuals and firms. The central banks, commercial banks, and the public are the major sources of money supply.

Simple and compound interest

Simple interest is calculated using principal only, which earns interest over a fixed period (loan period). At the end of the period, both the principal and the accrued interest are returned. A simple interest loan may be made for any length of time. The total interest can be computed using the following equation:

$$\begin{aligned} \text{Interest} &= (\text{Principal}) \times (\text{Number of Periods}) \\ &\quad \times (\text{Interest Rate per Period, fraction}) \end{aligned} \quad (2.1)$$

When simple interest for a partial year is calculated, the year is commonly considered to be of 12 months with 30 days in each month, or 360 days.

Example 2–1

Suppose \$2,000 is borrowed at a simple interest rate of 8% per year. Calculate (a) the principal plus interest at the end of one year, and (b) principal and interest if the loan has to be repaid after 90 days.

Solution: (a) Using equation (2.1),

$$\begin{aligned} \text{Interest} &= (2,000) \times (1) \times (0.08) = \$160 \\ [\text{Principal} + \text{Interest}] &= \$2,000 + \$160 = \$2,160 \\ &= (\$2,000) \times (1 + 0.08) = \$2,160 \end{aligned}$$

(b) Principal plus interest at the end of 90 days will be

$$\text{Principal} + \text{Interest} = \$2,000 \times \left(1 + \frac{90}{360} \times 0.08\right) = \$2,040$$

Therefore, the interest in part (a) is \$160 while that in part (b) is \$40, which is simply $(160 \times 90/360)$.

Compound interest is the interest accrued on both the principal and the unpaid interest earned in the previous periods. At the end of one interest period, the principal and earned interest are left to accrue interest for another period. The principal and accrued interest essentially become the principal sum for interest computation in the following period. The accumulated interest depends on the number of compounding periods per year and the number of years interest is earned on the money. This is the type of interest widely used in home mortgage loans, automobile loans, credit card installments, and other installment loans. Interest compounding determines

the future sum (at some specified time and interest rate) of a sum of money invested today. Compound interest is calculated using the following equation:

$$Interest = \left[\left(1 + \frac{i_n}{m} \right)^{tm} - 1 \right] \times Principal \quad (2.2)$$

Or

$$[Principal + Interest] = \left(1 + \frac{i_n}{m} \right)^{tm} \times Principal \quad (2.3)$$

where

- i_n = the nominal interest rate, fraction per year
- m = compounding or interest periods per year (Where
 $m = 1$ for yearly compounding, $m = 2$ for semiannual
 compounding, $m = 4$ for quarterly compounding,
 $m = 12$ for monthly compounding, and so on.)
- t = the loan period or investment period, years

If the interest is compounded once a year, the nominal interest rate (i_n) equals the effective interest rate (i_e). The relationship between the nominal and effective interest rate is discussed in the next section. In this case, equation (2.2) reduces to

$$Interest = [(1 + i_e)^t - 1] \times Principal \quad (2.2a)$$

Example 2–2

Suppose that \$2,000 is borrowed for one year at an interest rate of 8%. Calculate the interest at the end of the year if the interest is compounded (a) yearly and (b) quarterly. (c) What will be the interest at the end of three years if the interest is compounded semi-annually?

Solution: (a) If the interest is compounded yearly, using equation (2.2), the interest is

$$Interest = \left[\left(1 + \frac{0.08}{1} \right)^{1 \times 1} - 1 \right] \times \$2,000 = \$160$$

The resulting interest is the same as that obtained in example 2–1.

- (b) If the interest is compounded quarterly, using equation (2.2), the interest is

$$\text{Interest} = \left[\left(1 + \frac{0.08}{4} \right)^{1 \times 4} - 1 \right] \times \$2,000 = \$164.86$$

The same interest can be calculated without using equation (2.2) as follows:

$$\text{First quarter's interest} = \frac{0.08}{4} \times \$2,000 = \$40$$

Principal at the end of first quarter is, therefore, $\$2,000 + \$40 = \$2,040$.

$$\text{Second quarter's interest} = \frac{0.08}{4} \times \$2,040 = \$40.80$$

Principal at the end of second quarter amounts to $\$2,000 + \$40 + \$40.80 = \$2,080.80$.

$$\text{Third quarter's interest} = \frac{0.08}{4} \times \$2,080.80 = \$41.62$$

Similarly, the interest for the fourth quarter amounts to \$42.45. Thus, the total interest at the end of the year (for four quarters) sums to

$$\$164.87 = \$40.00 + \$40.80 + \$41.62 + \$42.45$$

This is the same, within rounding, as \$164.86 calculated by using equation (2.2).

Using equation (2.2), the interest after three years (with semiannual compounding) will be

$$\text{Interest} = \left[\left(1 + \frac{0.08}{2} \right)^{3 \times 2} - 1 \right] \times \$2,000 = \$530.64$$

Example 2-3

Suppose an employee's monthly salary is currently \$4,000, and he receives a merit increase of 5% every year. What will be his salary after five years?

Solution: Using equation (2.3), salary after five years will be

$$\text{Salary} = (1 + 0.05)^5 \times \$4,000 \approx \$5,105$$

The effect of compounding at various interest rates over a period of 15 years is shown in figure 2–2. The figure shows how a sum of \$10 today grows with time and interest. Of course, at higher interest rates, the growth is faster relative to that at lower interest rates.

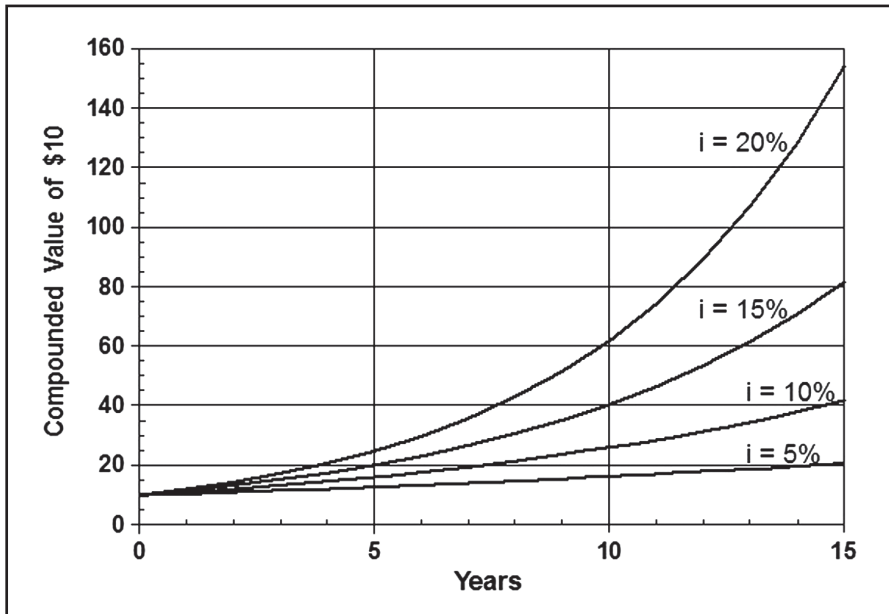


Fig. 2–2. Relationship between interest rates, time, and compound value

Nominal and effective interest rate

Compound interest rate is further classified into *nominal* rate (i_n) and the *effective* annual interest rate (i_e). Compound interest is normally reported as nominal interest rate per year, compounded periodically (i.e., daily, monthly, quarterly, or semiannually) or continuously (infinite number of times per year, approximately equal to daily compounding).

Effective interest is the rate that when applied to a sum of money once per year will give the same amount of interest as will be given by a nominal interest rate compounded m periods per year. Effective and nominal interest rates are equal ($i_e = i_n$) if interest is compounded annually; they are related to each other by the following equations.

For periodic compounding,

$$i_e = \left(1 + \frac{i_n}{m}\right)^m - 1 \quad (2.4)$$

Equation (2.4) is used for nominal annual interest rates compounded any number of times up to and including continuous compounding. However, as the frequency of compounding increases, m approaches infinity and the equation becomes equivalent to continuous compounding, as shown by equation (2.5).

For continuous compounding,

$$i_e = e^r - 1 \quad (2.5)$$

To differentiate between the periodic and continuous interest rates, the nominal continuous interest rate will be designated by r . The use of effective interest rate reduces equations (2.2) and (2.3) to

$$Interest = [(1 + i_e)^t - 1] \times Principal \quad (2.6)$$

$$[Principal + Interest] = (1 + i_e)^t \times Principal \quad (2.7)$$

Example 2–4

Suppose a sum of \$2,000 is borrowed for one year at an interest rate of 8%. Calculate the total amount to be paid back to the bank after one year if the interest is compounded quarterly.

Solution: First calculate i_e , using equation (2.4)

$$\begin{aligned} i_e &= \left(1 + \frac{0.08}{4}\right)^4 - 1 \\ &= 0.08243 \text{ or } 8.243\% \end{aligned}$$

Now using equation (2.7), we get

$$\begin{aligned} [Principal + Interest] &= (1 + 0.08243)^1 \times \$2,000 \\ &= \$2,164.86 \end{aligned}$$

The same answer, as calculated in example 2–2b, is obtained.

Example 2–5

A U.S. dollar currency account is maintained at a bank. The account is fixed for one month, starting 05/20/2010. After the expiration of one month, the account is either automatically renewed for another month or the money is withdrawn. The deposit receipt received from the bank gives the following details.

Current principal = \$50,000	Date of deposit = 05/20/2010
Maturity = 06/20/2010	Interest rate = 5.875%
Interest on maturity = \$253.60	

Determine the basis used by the bank to calculate interest on maturity. What effective interest rate is the bank paying?

Solution: The interest is either compounded monthly or daily. On a monthly basis, using equation (2.3), the principal plus interest is

$$\begin{aligned}
 [Principal + Interest] &= \left(1 + \frac{0.05875}{12}\right)^{1/12 \times 12} \times \$50,000 \\
 &= \$50,250.79
 \end{aligned}$$

On a daily basis for 31 days (from 05/20/2002 to 06/20/2002) it is

$$\begin{aligned}
 [Principal + Interest] &= \left(1 + \frac{0.05875}{360}\right)^{31} \times \$50,000 \\
 &= \$50,253.60
 \end{aligned}$$

This presumes an interest rate of 5.875%, compounded daily. The effective interest rate equivalent to a nominal rate of 5.875%, compounded daily, is as follows.

Using equation (2.4):

$$\begin{aligned}
 [Principal + Interest] &= \left[\left(1 + \frac{0.05875}{360}\right)^{360} - 1\right] \times 100 \\
 &= 6.051\%
 \end{aligned}$$

or using equation (2.5):

$$i_e = (e^{0.05875} - 1) \times 100 = 6.051\%$$

The preceding example shows one has to be very careful with the quoted interest rates.

Cash Flow Diagram

Every capital investment project will typically have cash receipts (inward flow of cash) and cash disbursements (outward flow of cash) into or out of the treasury, respectively. Typical examples of cash flows are seismic acquisition, processing, and interpretation costs; drilling and completion costs; production facility costs; lease operating costs; corporate overhead costs; revenues from the sale of crude and natural gas; etc. Subtracting the cash disbursements from the cash receipts will generate net negative or positive cash flow. Written mathematically,

$$\text{Net Cash Flow} = \text{Receipts} - \text{Disbursements} \quad (2.8)$$

Cash flow diagrams graph net cash flows and cumulative net cash flows over time. The diagram typically shows (1) cash receipts at the end of each year generated by the investment, (2) cash disbursements of all costs (initial and subsequent costs) per year required for the operations, and (3) total time span of the investment in years. Therefore, the major point of the cash flow diagram is to show that a capital investment is an amount paid to receive expected net cash inflows over the economic life of the investment.

For ease of recognizing the various elements of cash flow, cash receipts (positive) are shown as upward-pointing arrows while the cash disbursements (negative) are shown as downward-pointing arrows on the diagram.

On the diagram, the beginning of year one is considered *time zero*, at which the equivalent (present value) of all the future cash flows is calculated. Time zero shows the beginning of the first year or period, time one shows the end of the first year (year one) or period and the beginning of the second year (year two) or period, and so on. Figure 2–3 shows a typical cash flow diagram.

A clearly drawn cash flow diagram serves as a valuable tool in problem solving because it provides a visual representation of an investment problem. It helps in simplifying complicated descriptive problems. The cash flow diagrams used throughout this book assist in solving investment alternatives.

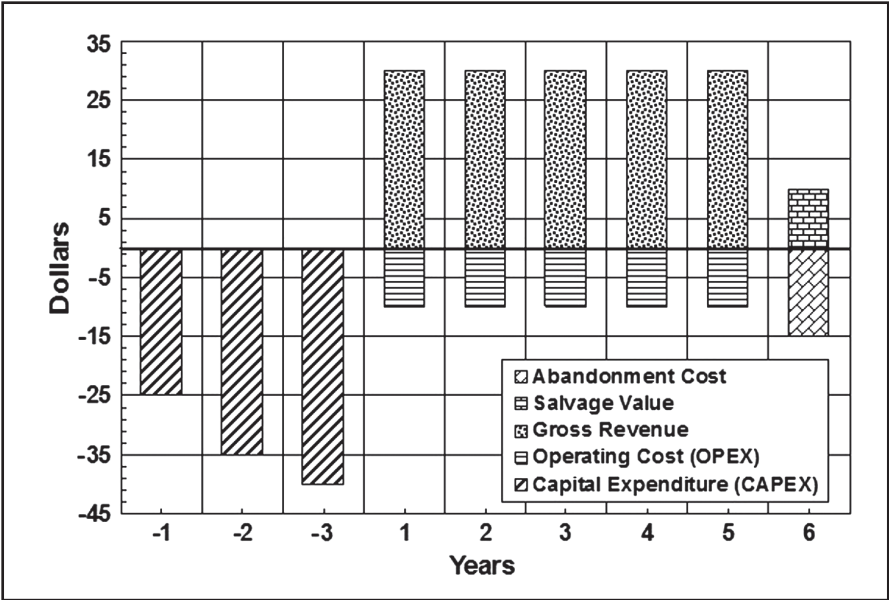


Fig. 2–3. A typical cash flow diagram

The Time Value of Money

If you were asked, “Would you rather have \$100 today or \$100 one year from today?” your obvious answer would most likely be that you would rather have \$100 today. On the other hand, if the choice was between \$100 today or \$110 one year from today, you would probably be indifferent.

Your replies intuitively involve the consideration of the *time value of money*. The difference in the worth between the two amounts, one year apart, is *interest* (i.e., the growth potential of money over one year). You know in the back of your mind that if \$100 is received today, you can invest it at 10% interest per year and have \$110 ($\$100 \times 1.1 = \110) one year from today. The trade-off involves length of time and the available earning power of the money. The concept of interest is key to accounting for time value of money.

You are indifferent to the second offer because you know both amounts are the same. However, if you know of an alternative investment that will make you \$120, you will obviously again settle for the \$100 today. On the other hand, if the timing of benefits with *comparable risk* and *uncertainty* vary widely, more immediate benefits may be preferable to those benefits that will be obtained further out in time.

In the same way, if you were asked, “Would you rather have \$100 today or \$130 one year from today?” you will most likely settle for the \$130. This is because you know you cannot find an investment that will yield 30% annual return if \$100 is invested today. This is why it is important to consider time value of money in all investment evaluations and decision analysis.

By looking at the above example from another perspective, assume the \$130 to be received one year from today has a higher associated risk. In this case, you will also assess the risk factor. Based on the risk involved, you may decide to take the immediately available funds (\$100) and invest them at a profit commensurate with your *risk preference*. Your decision obviously will reflect your attitude to risk. Are you a person who prefers to select the least risky alternatives in a decision (a risk averter), or are you prepared to tolerate some level of risk (a risk seeker or indifferent to risk)?

Capital investments begin with an objective to generate a stream of positive net cash flows in the years ahead. Investing in oil and gas projects are typical examples—a well is drilled, completed (capital expenditure), and then put on production to generate revenue. The well may produce for 5, 10, or 20 years or even longer.

Besides the receipt of revenues in future periods, the pattern of receipts also varies from property to property, i.e., the pattern of how cash is generated. Some wells or fields produce at higher rates, decline severely in the early years, and then stabilize at a steady decline. Other wells or fields start at lower rates and keep a steady decline for a longer time period. Although both wells/fields may give the same ultimate recovery, the former is of more value due to the effect of the time value of money (higher revenue generated in the early years).

Therefore, to account for the time value of money, all future expenditures and revenues need to be converted to a *common denominator*, i.e., a common equivalent value of all the future cash flows calculated at a common point in time. This common point in time may be the present, future, or even annual. Customarily, analysts choose the *present*, which is also referred to as *time zero*. When comparing capital investments, time zero is the same for all alternatives subject to evaluation for decision analysis. This is achieved by *discounting* future cash flow streams. Discounting is the mirror to the *compounding interest* calculations. The distinction is whether we are paying or receiving interest. Compounding converts a present sum of money into its equivalent *future* sum. Discounting converts a future sum of money into its equivalent *present* sum.

The rate used for discounting is typically referred to as the *discount rate*, i_d . The effect of discounting at various interest rates over a period of 15 years is shown in figure 2–4. The figure shows how a sum of money received in the future deteriorates in value over time. Figure 2–4 shows how a sum of \$10

received after 15 years is only worth \$0.65 today, at an interest rate of 20%. The present value of a dollar available at some date in the future declines as the interest rate increases; if the interest rate is fixed, a dollar received further out in the future is worth less today.

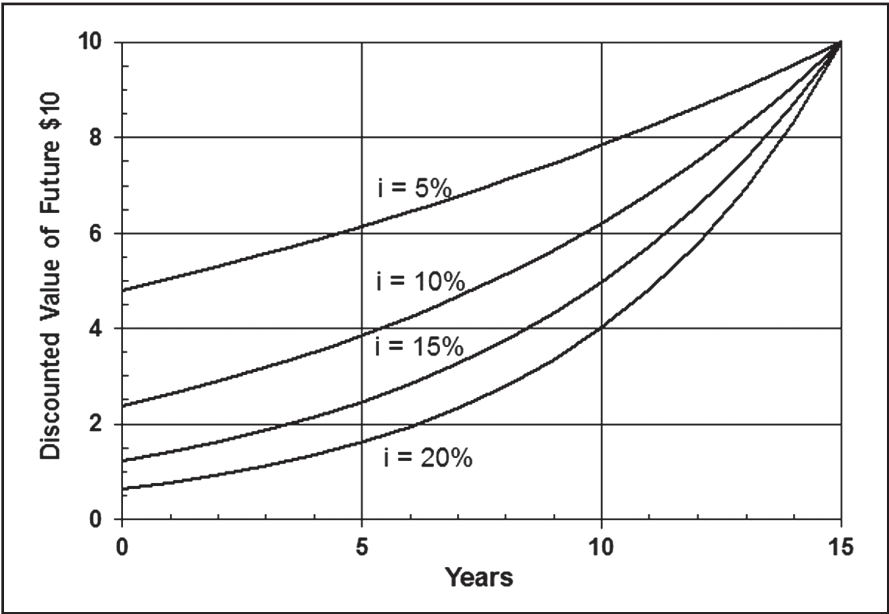


Fig. 2-4. Relationship between interest rate, time, and discounted value

As a result, the discounting of future cash flows favors earlier receipts from investments over longer-term alternatives. This increases the chances investors will be biased toward the short-term investment alternatives versus the long-term alternatives.

The “time value of money calculation” is achieved by using (1) interest tables, (2) mathematical equations, (3) financial calculators, or (4) computerized spreadsheet applications (such as Microsoft Excel™). Excel has built-in financial functions to facilitate such calculations. Equations and interest tables may not be used in practice because of limited availability. However, understanding the basic concepts presented here helps knowing which equations are in use and why they are used.

Present value of future sum

In the preceding section, it was mentioned that a present point in time is customarily chosen at which the equivalent value of all the future cash flows is calculated. All the future values are converted to their equivalent present value. This equivalent value is typically referred to as the *present value* or *present worth* of future cash flows.

The present value of a single sum of money, received at some future point in time, is calculated using the following equation:

$$P_v = F_v \left[\frac{1}{(1 + i_e)^t} \right] \quad (2.9)$$

where

P_v = present value (at time zero) of future sum

F_v = future sum received at time t

i_e = effective interest rate (discount rate, fraction)

The present value of periodic cash flows is calculated with equation (2.10).

$$P_v = (F_v)_1 \frac{1}{(1 + i_e)^1} + (F_v)_2 \frac{1}{(1 + i_e)^2} + (F_v)_3 \frac{1}{(1 + i_e)^3} + \cdots + (F_v)_t \frac{1}{(1 + i_e)^t} \quad (2.10)$$

OR

$$P_v = \sum_{t=1}^n (F_v)_t \left[\frac{1}{(1 + i_e)^t} \right]$$

In equations (2.9) and (2.10), the term $[1/(1 + i_e)^t]$ is referred to as the *present worth factor*, P/F . Tables are available in every finance, project economics, and accounting book to provide these factors for a value of \$1 at several i_e and t values. Interest factors (for $i = 1\%$ to $i = 35\%$) for year-end payments and annual compounding/discounting are given in appendix A of this book. Additional tables are provided on the CD-ROM that accompanies volume 2 of this series. Table 2-1 is an excerpt from table A-8 (appendix A), showing interest factors for $i_e = 8\%$.

Although the interest tables are provided for annual compounding/discounting, they can be used in situations where the interest is compounded/discounted more than once a year. For example, when the interest rate is 8% compounded semiannually, the present value factor will be read as $P/F = 0.9246$ from table A-4 for $i = 4\%$ at $t = 2$. Similarly, the compound factor F/P will be 1.0816.

Table 2–1. Excerpt from table A–8 (appendix A) – 8% interest factors for annual compounding

<i>t</i>	Single Payment		Equal Payment Series				Uniform Gradient Series A/G
	Compound Amount Factor F/P	Present Worth Factor P/F	Compound Amount Factor F/A	Sinking Fund Factor A/F	Present Worth Factor P/A	Capital Recovery Factor A/P	
1	1.0800	0.9259	1.0000	1.0000	0.9259	1.0800	0.0000
2	1.1664	0.8573	2.0800	0.4808	1.7833	0.5608	0.4808
3	1.2597	0.7938	3.2464	0.3080	2.5771	0.3880	0.9487
4	1.3605	0.7350	4.5061	0.2219	3.3121	0.3019	1.4040
5	1.4693	0.6806	5.8666	0.1705	3.9927	0.2505	1.8456
6	1.5869	0.6302	7.3359	0.1363	4.6229	0.2163	2.2763
7	1.7138	0.5835	8.9228	0.1121	5.2064	0.1921	2.6937
8	1.8509	0.5403	10.6366	0.0940	5.7466	0.1740	3.0985
9	1.9990	0.5002	12.4876	0.0801	6.2469	0.1601	3.4910
10	2.1589	0.4632	14.4866	0.0690	6.7101	0.1490	3.8713
11	2.3316	0.4289	16.6455	0.0601	7.1390	0.1401	4.2395
12	2.5182	0.3971	18.9771	0.0527	7.5361	0.1327	4.5957
13	2.7196	0.3677	21.4953	0.0465	7.9038	0.1265	4.9402
14	2.9372	0.3405	24.2149	0.0413	8.2442	0.1213	5.2731
15	3.1722	0.3152	27.1521	0.0368	8.5595	0.1168	5.5945
16	3.4259	0.2919	30.3243	0.0330	8.8514	0.1130	5.9046
17	3.7000	0.2703	33.7502	0.0296	9.1216	0.1096	6.2037
18	3.9960	0.2502	37.4502	0.0267	9.3719	0.1067	6.4920
19	4.3157	0.2317	41.4463	0.0241	9.6036	0.1041	6.7697
20	4.6610	0.2145	45.7620	0.0219	9.8181	0.1019	7.0369

The interest factor tables in appendix A are provided for whole interest rates (such as 8% or 9%). The interest factors for intermediate interest rates (such as 8.25%) between two consecutive interest factor tables can be estimated using *linear interpolation*. The following relationship is used for interpolation:

$$(P/F) = \left[\frac{i_e - (i_e)_L}{(i_e)_H - (i_e)_L} \right] \left[(P/F)_H - (P/F)_L \right] + (P/F)_L \tag{2.11}$$

where

(P/F) = the present worth factor

i_e = the desirable interest rate

$(i_e)_L$ = the lower than desired interest rate

$(i_e)_H$ = the higher than desired interest rate

$(P/F)_L$ = interest factor at lower interest rate

$(P/F)_H$ = interest factor at higher interest rate

Similarly, if the time period t or the number of interest periods tm are beyond the values listed in the available interest factor tables, the desired factor can be obtained from a combination of various tables. For example, if a value of $(P/F)_{i=10\%, t=50}$ is desired that is not in table A-10 (interest factors table for $i = 10\%$), then the following relationship can be used to obtain the desired factor:

$$\begin{aligned}(P/F)_{i=10\%, t=50} &= (P/F)_{i=10\%, t=40} \times (P/F)_{i=10\%, t=10} \\ &= (0.0221) \times (0.3856) \\ &= 0.00852\end{aligned}$$

Example 2-6

In six years, it is estimated \$5,000 (future value) will be required to purchase a piece of equipment. What amount should be deposited in a bank today, at an effective interest rate of 8%, to accumulate \$5,000 at the end of the sixth year?

Solution: The cash flow diagram for this example is shown in figure 2-5.

Using equation (2.9), the present value is

$$\begin{aligned}P_v &= \$5,000 \times \left[\frac{1}{(1 + i_e)^6} \right] \\ &= \$3,150.85\end{aligned}$$

Using interest factor (P/F) , the value at the intersection of column P/F and row $t = 6$ from table 2-1:

$$\begin{aligned}(P/F)_{i_e=8\%, t=6} &= 0.6302 \\ P_v &= \$5,000(0.6302) \cong \$3,151\end{aligned}$$

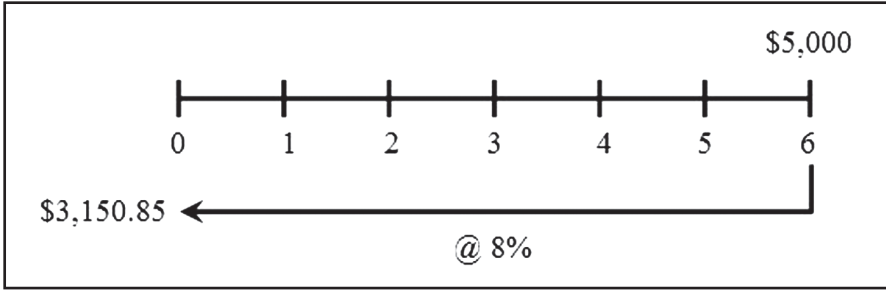


Fig. 2-5. Cash flow diagram showing present value equivalent of \$5,000 received after six years (example 2-6)

Example 2-7

Suppose a landowner receives annual royalty payments of \$2,000, \$2,200, \$1,900, \$2,500, and \$1,500 over the next five years. Calculate the present worth of these payments at an interest (discount) rate of 8%.

Solution: The cash flow diagram for the problem is shown in figure 2-6.

Using equation (2.10)

$$\begin{aligned}
 P_v &= \$2,000 \left[\frac{1}{(1+0.08)^1} \right] + \$2,200 \left[\frac{1}{(1+0.08)^2} \right] + \$1,900 \left[\frac{1}{(1+0.08)^3} \right] \\
 &\quad + \$2,500 \left[\frac{1}{(1+0.08)^4} \right] + \$1,500 \left[\frac{1}{(1+0.08)^5} \right] \\
 &= \$1,851.85 + \$1,886.15 + \$1,508.28 + \$1,837.57 + \$1,020.87 \\
 &= \$8,104.72
 \end{aligned}$$

The same problem is solved by using the (P/F) factor at $i_e = 8\%$, $t = 1$ to 5 from Table 2-1, as shown in figure 2-6.

Example 2-8

Rework example 2-6 for an effective interest rate of 8.5%. Use interest tables from appendix A.

Solution: Since an interest factor table for 8.5% is not provided, the factor is obtained by linear interpolation between interest factors from table A-8 and table A-9.

From table A-8: $(P/F) i = 8\%$, $t = 6 = 0.6302$

From table A-9: $(P/F) i = 9\%$, $t = 6 = 0.5963$

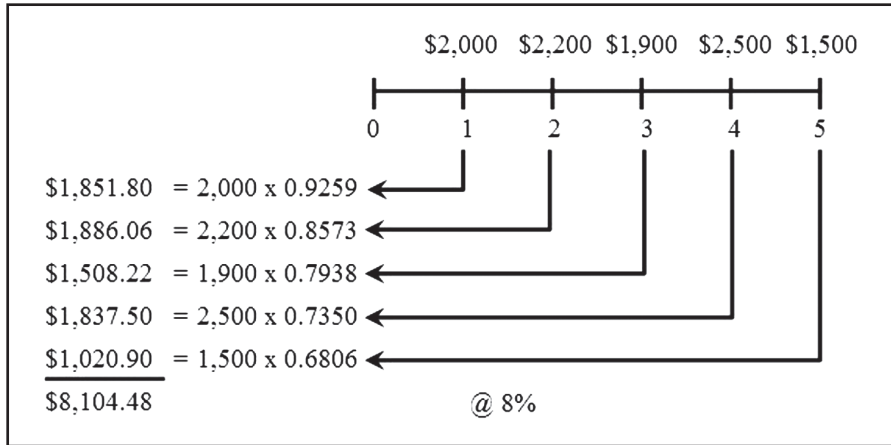


Fig. 2-6. Cash flow diagram showing present value of five annual receipts (example 2-7)

Using equation (2.11)

$$\begin{aligned}
 (P/F)_{i=8\%, t=6} &= \left(\frac{0.085 - 0.08}{0.09 - 0.08} \right) (0.5963 - 0.6302) + 0.6302 \\
 &= (0.50)(-0.0339) + (0.6302) \\
 &= 0.61325
 \end{aligned}$$

Therefore

$$\begin{aligned}
 P_v &= F_v(P/F)_{i=8.5\%, t=6} \\
 &= \$5,000(0.61325) \\
 &= \$3,066.25
 \end{aligned}$$

Using equation (2.9), the present value is

$$\begin{aligned}
 P_v &= \$5,000 \left[\frac{1}{(1 + 0.085)^6} \right] \\
 &= \$3,064.73
 \end{aligned}$$

Note the small difference between the P_v calculated by using equation (2.9) and the P_v calculated by using the interpolated interest factor.

The difference is due to the nonlinear relationship between interest compounding and time as shown in figures 2-1 and 2-2. Therefore, the linear interpolation gives an approximate interest factor of 0.61325, versus 0.61295 from the equation.

Example 2–9

Calculate the present value of \$5,000 received at the end of the sixth year (example 2–6) if the interest rate is 8% per year compounded quarterly (i.e., $m = 4$).

Solution: In this example, $t = 6$, $i_n = 0.08$, and $m = 4$.

Therefore

$$i = \frac{i_n}{m} = \frac{0.08}{4} = 0.02$$

$$t = tm = 6 \times 4 = 24$$

The interest factor (P/F) at $i = 2\%$ and $t = 24$ is obtained from table A–2 as 0.6217. Thus

$$\begin{aligned} P_v &= \$5,000 \times 0.6217 \\ &= \$3,108.50 \end{aligned}$$

Using equation (2.3) in terms of discounting, the present value is

$$\begin{aligned} P_v &= F_v \frac{1}{\left(1 + \frac{i_n}{m}\right)^{tm}} \\ &= \$5,000 \times \frac{1}{\left(1 + \frac{0.08}{4}\right)^{6 \times 4}} = \$3,108.61 \end{aligned}$$

Future value of present sum

As previously mentioned, the future value of a present sum is a reciprocal of the present value of future sum. Here, interest is added to the cash flows (i.e., compounding versus discounting). Thus equation (2.9) becomes

$$F_v = P_v(1 + i_e)^t \quad (2.12)$$

The factor $(1 + i_e)^t$ is referred to as the *single payment compound amount factor*, represented by the factor F/P in interest tables. The problem solving methodology is the same as discussed in the previous section.

The future value of a single sum received/invested (at a compound interest) at present is the original sum plus the compound interest thereon, stated as of a specific future reference date. The future value of several periodic receipts or disbursements between the present and future value reference dates can be calculated by using equation (2.12). This is done by calculating

the future value of each year's cash flow and then adding them together. The receipt/disbursement at each interval is treated as a single sum at present. The following equation is used to achieve this:

$$F_v = (P_v)_1(1+i_e)^{t-1} + (P_v)_2(1+i_e)^{t-2} + (P_v)_3(1+i_e)^{t-3} + \dots + (P_v)_n(1+i_e)^{t-n} \quad (2.13)$$

OR

$$F_v = \sum_{t=1}^{n-1} (P_v)_t (1+i_e)^t$$

where

n = number of periods, years

$(P_v)_t$ = present value at time t , $t = 1$ to n

Example 2-10

Calculate the equivalent future value of \$3,150.85 invested today at 8% interest for a period of six years.

Solution: The cash flow diagram for example 2-10 is shown in figure 2-7.

Using equation (2.12), the future value is

$$\begin{aligned} (F_v)_{i=8\%, t=6} &= \$3,150.85(1 + 0.08)^6 \\ &= \$5,000 \end{aligned}$$

Using the interest factors, the single payment compound amount factor $F/P = 1.5869$ is obtained from table 2-1 at the intersection of column F/P and row $t = 6$.

Thus:

$$(F_v)_{i=8\%, t=6} = \$3,150.85(1.5869) \cong \$5,000$$

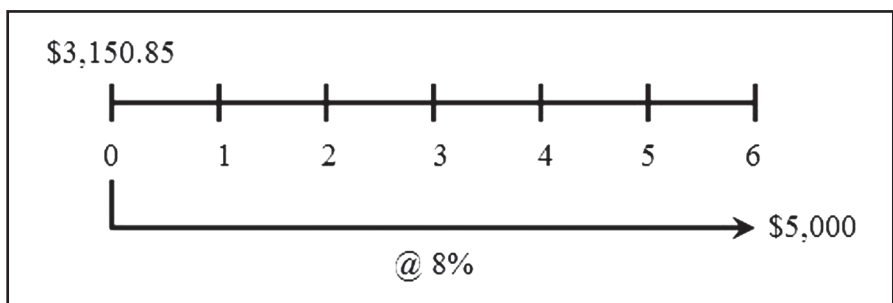


Fig. 2-7. Cash flow diagram for example 2-10, showing future value of a present sum of money

Example 2–11

Using the royalty payments in example 2–7, calculate the future value of these payments at 8% effective interest.

Solution: The cash flow diagram for example 2–11 is shown in figure 2–8. Using equation (2.13),

$$\begin{aligned}
 F_v &= \$2,000(1 + 0.08)^4 + \$2,200(1 + 0.08)^3 + \$1,900(1 + 0.08)^2 \\
 &\quad + \$2,500(1 + 0.08)^1 + \$1,500(1 + 0.08)^0 \\
 &= \$2,000(1.3605) + \$2,200(1.2597) + \$1,900(1.1664) \\
 &\quad + \$2,500(1.08) + \$1,500 \\
 &= \$11,908.50
 \end{aligned}$$

The present value of this amount is calculated by treating it as a single future sum,

$$\begin{aligned}
 P_v &= \$11,908.50 \frac{1}{(1 + 0.08)^5} \\
 &= \$8,104.73
 \end{aligned}$$

which is the same value as calculated in example 2–7. Similarly, the future value of \$8,104.73 (obtained in example 2–7) could have been calculated by treating it as a single payment present sum.

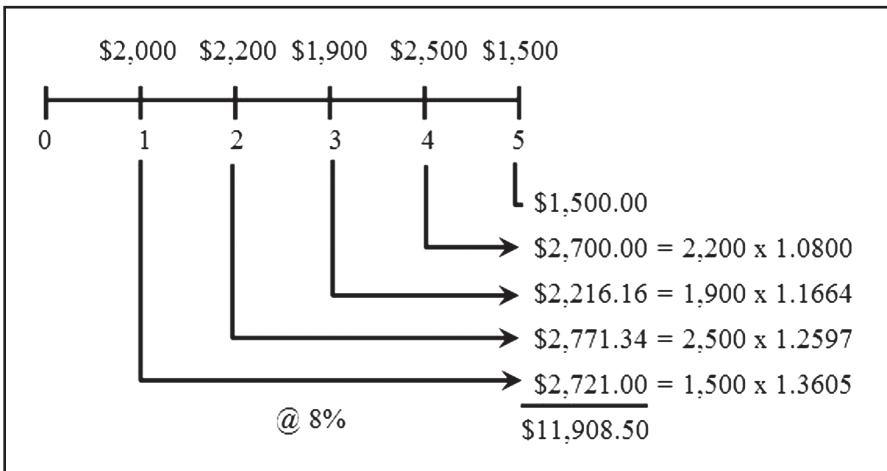


Fig. 2–8. Cash flow diagram for example 2–11, showing future value of periodic receipts

Assuming the payments are received at the end of each year, the payment received at the end of the fifth year is already at future value. The payment received at the end of the first year earns interest for four periods, receipt at the end of the second year earns interest for three years, and so on, except for the fifth payment, which does not earn any interest.

The future value of the present sum at a nominal interest rate may be calculated by using equations (2.12) and (2.13) while replacing the i_e by i_n/m and t by tm , where m is the number of compounding periods per year.

Present value of an annuity

A series of equal periodic payments or disbursements at regular intervals (normally at the end of a period, over a period of time), while the interest is compounded at a certain rate, is called *annuity*. Typical examples of annuities are (1) home-mortgage payments, (2) automobile loans, (3) salaries and social security receipts, (4) insurance premiums, (5) payment into pension funds, and (6) other installment loans. The concept is helpful in calculating the equivalent present value of alternative annuities for comparison.

The present value of a series of cash flows (annuity) determined *one period before* the receipt or payment of the first sum of money is referred to as an *ordinary annuity*. If the present value of an annuity is determined on the date when the first cash flow occurred, the present value is referred to as the present value of *annuity due*.

Thus, the present value of an annuity is the single sum of money invested today at a certain interest rate in order to withdraw equal amounts each year. At the end of the annuity period, with the withdrawal of the last amount, the balance is equal to zero. To qualify cash flows for annuity, it is required that:

1. The periodic payments or disbursements are of equal amount.
2. The interest rate is constant throughout the annuity period under consideration.
3. The time intervals between each payment must be of the same length.
4. The interest rate is compounded at the end of each time period.

In example 2-7, the present value of unequal annual royalty receipts was calculated. The present value of an annuity is determined in a similar way. However, the condition of equal periodic cash flows simplifies the solution that can be achieved by using a single equation. The following equation is used to calculate the present value of an annuity:

$$P_v = A_v \left[\frac{(1+i_e)^t - 1}{i_e(1+i_e)^t} \right] \quad (2.14)$$

The resulting factor, $[(1 + i_e)^t - 1/i_e(1 + i_e)^t]$, is referred to as the equal payment series present worth factor for ordinary annuity. The factor is represented by the P/A factor in the interest tables.

Example 2–12

An oil company has to pay \$10,000 per year, starting one year from today, on a loan obtained for five years at an effective interest rate of 8%. Calculate the equivalent present value of these five yearly payments. The payments include both principal and interest.

Solution: The cash flow diagram for example 2–12 is shown in figure 2–9. Using equation (2.14), the present value of annuity is

$$\begin{aligned} P_v &= \$10,000 \left[\frac{(1 + 0.08)^5 - 1}{0.08(1 + 0.08)^5} \right] \\ &= \$10,000(3.9927) \\ &= \$39,927 \end{aligned}$$

The *equal payment series present worth factor* (P/A) in table 2–1 is found at the intersection of column P/A and row $t = 5$ as 3.9927.

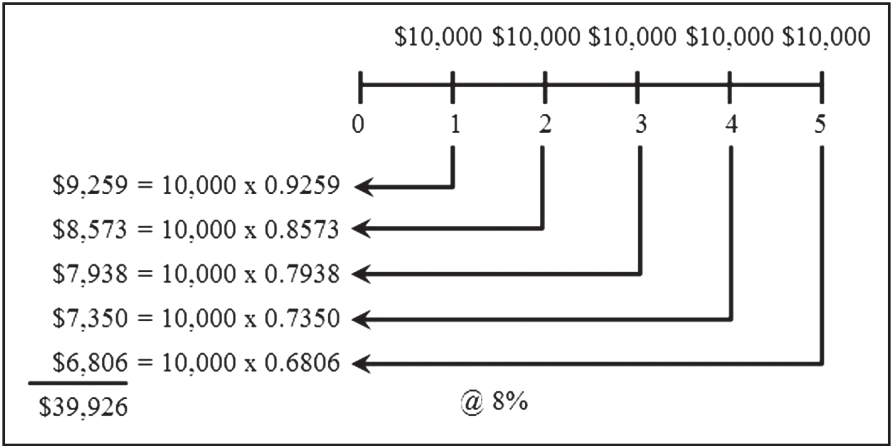


Fig. 2–9. Cash flow diagram for example 2–12, showing present value of ordinary annuity

To calculate the present value of an *annuity due* while only the *ordinary annuity* interest tables are available (such as the tables in appendix A), use the following steps:

1. Obtain the P/A factor from the ordinary annuity interest tables for the desired interest rate and for $t = n - 1$, where n = number of payments.
2. Add 1 to the factor obtained in step 1.
3. Calculate the present value by multiplying the periodic cash flow by the factor calculated in step 2.

Example 2–13

An oil producer acquired production equipment by paying \$10,000 as down payment and agreed to pay the balance in four year-end installments of \$10,000. Calculate the present value of the annuity due at an interest rate of 8%. Note that the total number of payments (including the down payment) is five.

Solution: The cash flow diagram for example 2–13 is shown in figure 2–10.

1. The P/A factor from table 2–1 for $n-1$ and $i = 8\%$ (i.e., four payments) is 3.3121.
2. Add 1 to the P/A factor in step 1 (i.e., $3.3121 + 1 = 4.3121$).
3. Therefore, $P_v = \$10,000 \times (4.3121) = \$43,121$

Future value of an annuity

To calculate the equivalent single future value that will result from an annuity, a series of equal periodic cash flows occurring at the end of succeeding periods, the following equation is used:

$$F_v = A_v \left[\frac{(1 + i_e)^t - 1}{i_e} \right] \quad (2.15)$$

The resulting factor, $[(1 + i_e)^t - 1] / i_e$, designated by F/A factor in the interest tables, is referred to as the *equal payment compound amount factor*. If the future value of an annuity is determined at the point when last payment of an annuity is made, such a series of cash flows is called *ordinary annuity*. If the future value of an annuity is calculated at the end of a period following the last cash flow in the series, this is referred to as the future value of an *annuity due*. The following modification of equation (2.15) is used to calculate the future value of annuity due.

$$(F_v)_{t+1} = A_v \left[\frac{(1+i_e)^{t+1} - 1}{i_e} - 1 \right]$$

(2.15a)

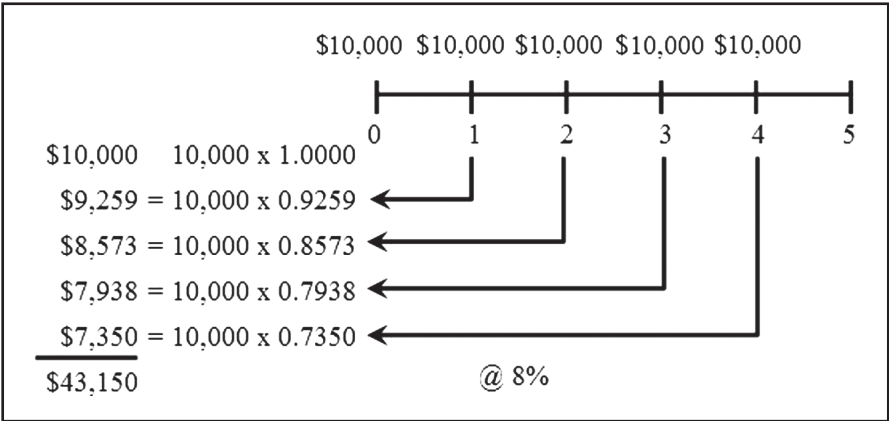


Fig. 2-10. Cash flow diagram for example 2-13, showing present value of annuity due

Example 2-14

Calculate the future value of the annuity given in example 2-12 by treating it as (a) an ordinary annuity and (b) an annuity due.

Solution: The cash flow diagram for the ordinary annuity is shown in figure 2-11.

(a) Using equation (2.15), the future value of ordinary annuity is

$$F_v = \$10,000 \left[\frac{(1 + 0.08)^5 - 1}{0.08} \right] = \$58,666$$

The same answer can be obtained by using the *F/A* factor from table 2-1. Similarly, the *F/P* factor or equation (2.12) can be applied to the *P_v* calculated in example 2-12 to get *F_v*.

(b) The cash flow diagram for the annuity due is shown in figure 2-12.

Using equation (2.15a):

$$\begin{aligned} F_v &= \$10,000 \left[\frac{(1 + 0.08)^{5+1} - 1}{0.08} - 1 \right] \\ &= \$10,000 (6.3359) = \$63,359 \end{aligned}$$

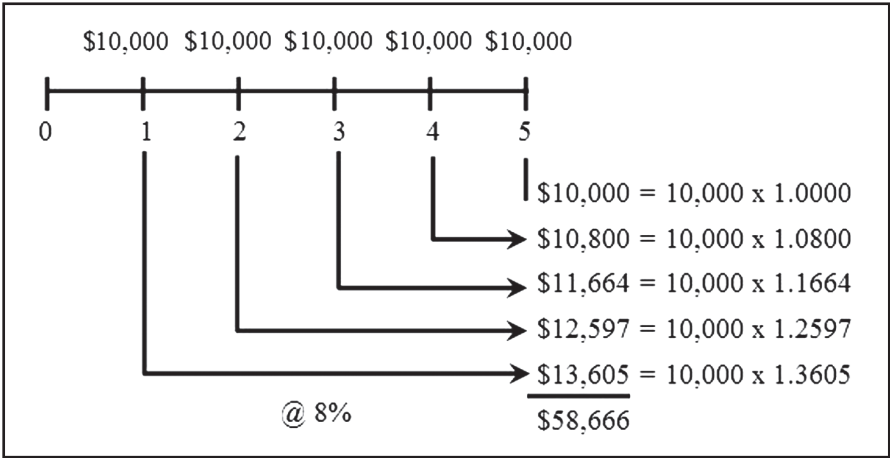


Fig. 2-11. Cash flow diagram for example 2-14, showing future value of ordinary annuity

In order to use the ordinary annuity interest factors for determining future value of annuity due, these steps should be followed:

1. Look up the value of $t + 1$ payments at an interest rate i_e in the interest tables.

$$(F/A)_{i_e=8\%,t=6} = 7.3359$$

2. Subtract 1 from the factor obtained in step 1.

$$7.3359 - 1 = 6.3359$$

3. The factor 6.3359 is now the equal payment compound amount factor for an annuity of five periodic payments at an interest rate of 8%. Thus, the future value is

$$(F_v)_{5+1} = \$10,000 \times 6.3359 = \$63,359$$

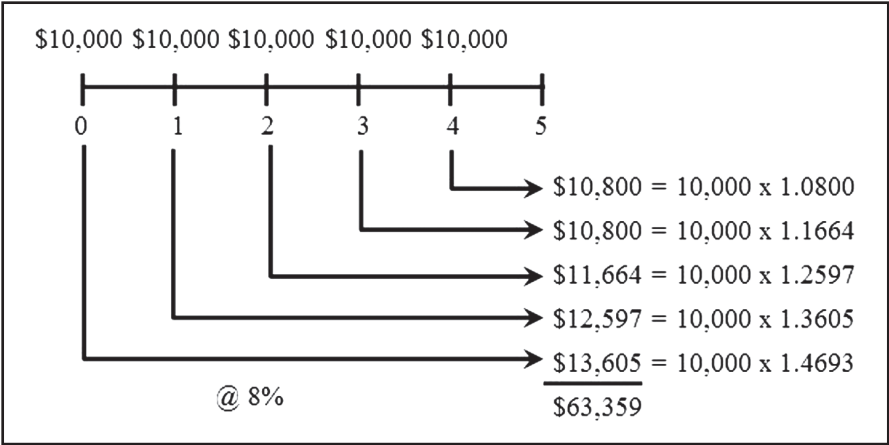


Fig. 2-12. Cash flow diagram for example 2-14, showing future value of annuity due

Present value of deferred annuity

In some cases, the series of equal periodic cash flows (i.e., annuity) will not start until some time in the future, meaning the annuity will be postponed by a number of periods from the present time (time zero). There is no exclusive equation or interest factor for calculating the present value of deferred annuity. The concepts presented in the preceding sections are used in combination to arrive at the present value. An example of a deferred annuity is shown in figure 2-13.

The present value of deferred annuity may be calculated in any one of the following three ways. Refer to the annuity presented in figure 2-13.

1. Calculate the present value of year three, $(P_v)_3$, treating the cash flows as ordinary annuity by using equation (2.14) or interest factor (P/A) . Then calculate the present value by treating the $(P_v)_3$ as the future value at end of year three and using equation (2.9) or (P/F) factor.
2. Calculate the future value by treating the cash flow as ordinary annuity and using equation (2.15) or factor (F/A) . Then calculate the present value of this future value by using equation (2.9) or factor P/F .
3. Calculate the present value of the entire series by treating the problem as if the annuity started from year one. Then subtract the present value of the annuity of the deferred period (i.e., the first

three years). Note that by calculating the present value of deferred annuity in this way, it is understood the same cash flow occurs over the entire period including the deferred period. Thus, the present value calculation using this method involves a combination of two ordinary annuities.

The practical utility of such equivalence calculations is applicable to deferred loan amortization. For example, money is borrowed for a period of five years; however, the loan is paid back in three equal annual installments with the first installment due at the end of the third year.

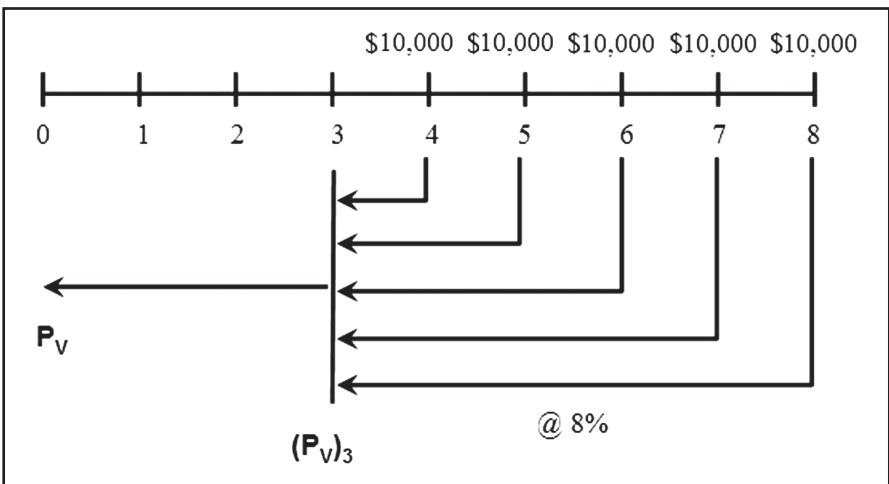


Fig. 2–13. Graphical presentation of deferred annuity

Example 2–15

Calculate the present value of the deferred annuity presented in figure 2–13. Use the three methods presented for calculating the present value of deferred annuity. Presume an interest rate of 8%.

Solution: Using equations (2.14) and (2.9):

$$P_v = \$10,000 \left[\frac{(1 + 0.08)^5 - 1}{0.08(1 + 0.08)^5} \right] \times \left[\frac{1}{(1 + 0.08)^3} \right]$$

OR

$$\begin{aligned}
 &= \$10,000(P/A)_{i=8\%,t=5} (P/F)_{i=8\%,t=3} \\
 &= \$10,000(3.9927)(0.7938) \\
 &= \$31,694.05
 \end{aligned}$$

Using equations (2.15) and (2.9):

$$P_v = \$10,000 \left[\frac{(1 + 0.08)^5 - 1}{0.08} \right] \times \left[\frac{1}{(1 + 0.08)^8} \right]$$

OR

$$\begin{aligned}
 &= \$10,000(F/A)_{i=8\%,t=5} (P/F)_{i=8\%,t=8} \\
 &= \$10,000(5.8666)(0.54027) \\
 &= \$31,695.48
 \end{aligned}$$

Using the third method, with equation (2.14):

$$P_v = \$10,000 \left[\frac{(1 + 0.08)^8 - 1}{0.08(1 + 0.08)^8} - \frac{(1 + 0.08)^3 - 1}{0.08(1 + 0.08)^3} \right]$$

OR

$$\begin{aligned}
 &= \$10,000[(P/A)_{i=8\%,t=8} - (P/F)_{i=8\%,t=3}] \\
 &= \$10,000(5.7466 - 2.5771) \\
 &= \$31,695
 \end{aligned}$$

Annuity from present value

This is a useful concept of equivalence and can be applied in our day-to-day activities. The periodic installments on home mortgage loans, automobile loans, or other fixed installment loans are typical examples of this type of calculation. The concept is also applicable in situations where a certain amount is deposited at an annual interest rate, and you wish to withdraw the principal plus earned interest in a series of equal year-end amounts over the following t years. Thus, in such cases the present value, interest rate, and number of periods are known, while the annuity needs to be calculated. The following equation is used for such calculations.

$$A_v = P_v \left[\frac{i_e(1 + i_e)^t}{(1 + i_e)^t - 1} \right] \quad (2.16)$$

The term, $[i_e (1 + i_e)^t / (1 + i_e)^t - 1]$, is referred to as the *equal payments series capital recovery factor*, designated by the *A/P* factors in appendix A. The calculated annuity (series of equal periodic cash flows) includes both the principal and the accrued interest thereon. In a debt payment situation, as each periodic payment is made on the due date, a portion of it goes to the accrued interest on the outstanding principal balance and a portion reduces the principal. Therefore, on home mortgage loans only a small amount of the actual borrowed principal reduces in the first few years of the mortgage period. A considerable portion of the monthly mortgage payment in the beginning pays for the accrued interest on the outstanding principal.

Example 2–16

Suppose \$20,000 is borrowed from a bank at an interest rate of 8% per year in exchange for a promise the loan will be paid off through a series of five equal year-end installments. Calculate the yearly installment if the first payment is due one year from the day money is borrowed.

Solution: The cash flow diagram for example 2–16 is shown in figure 2–14.

Using equation (2.16), the annual installment is

$$\begin{aligned} A_v &= \$20,000 \left[\frac{0.08(1 + 0.08)^5}{(1 + 0.08)^5 - 1} \right] \\ &= \$20,000(0.2505) \\ &= \$5,010 \text{ per year} \end{aligned}$$

Note that over the five-year period \$25,050 will be paid on the \$20,000 borrowed. The \$5,050 is the interest charged by the bank.

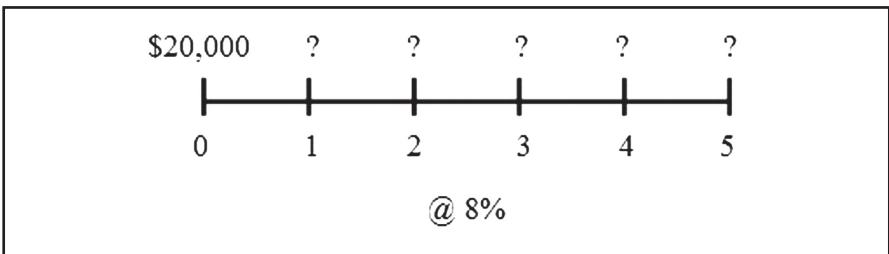


Fig. 2–14. Cash flow diagram for calculating annuity from present value (example 2–16)

Example 2–17

A piece of equipment is acquired for \$50,000. The anticipated useful economic life of the equipment is 10 years. Calculate the equivalent annual cost of the equipment if the market interest rate is 8%.

Solution: Using the $(A/P)_{i=8\%, t=10}$ factor from table 2–1:

$$\begin{aligned} A_v &= \$50,000(A/P)_{i=8\%, t=10} \\ &= \$50,000(0.14903) \\ &= \$7,451.50 \text{ per year} \end{aligned}$$

Example 2–18

On January 1, 2002, a sum of \$20,000 is deposited in a bank account paying 8% interest per year. The money stays in the account for three years. Starting at the end of year four, the money will be withdrawn in five equal year-end installments. How much money should be withdrawn each year in order to have five equal year-end withdrawals?

Solution: The problem is similar to the case of deferred annuity. First, the future value of the deposit is calculated at the end of year three. Then, treating this future value as a present value, the annuity for the remaining five years is calculated. The cash flow diagram for this problem is shown in figure 2–15.

Using equations (2.12) and (2.16) in combination:

$$A_v = \$20,000 \left\{ (1 + 0.08)^3 \left[\frac{0.08(1 + 0.08)^5}{(1 + 0.08)^5 - 1} \right] \right\}$$

OR

$$\begin{aligned} A_v &= \$20,000[(F/P)_{i=8\%, t=3} \times (A/P)_{i=8\%, t=5}] \\ &= \$20,000(1.2597)(0.2505) \\ &= \$6,311.10 \text{ per year starting end of year 4} \end{aligned}$$

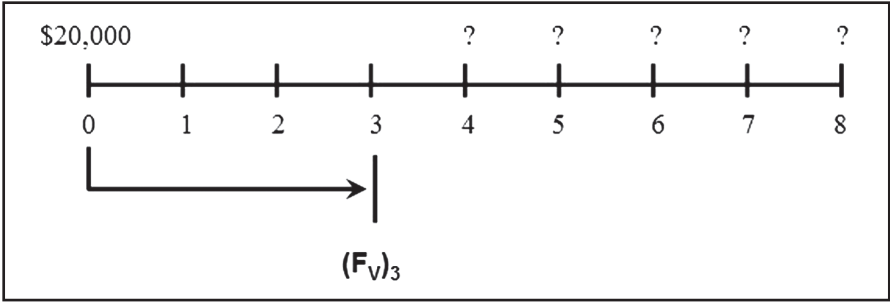


Fig. 2-15. Cash flow diagram for example 2-18

Annuity from future value

An annuity to generate a desired future value is determined using the following equation.

$$A_v = F_v \left[\frac{i_e}{(1 + i_e)^t - 1} \right] \quad (2.17)$$

The factor, $i_e / [(1 + i_e)^t - 1]$, in equation (2.17) is referred to as the *equal payment series sinking fund factor*, designated by the interest factor (A/F) in the interest factor tables. Equation (2.17) or the factor A/F may be used to find the required year-end payments for the accumulation of a desired future amount.

Example 2-19

An oil producer plans to replace certain equipment at a cost of \$100,000 five years from today. If the interest rate is 8%, how much money must the oil producer put aside per year in order to generate \$100,000?

Solution: The cash flow diagram for example 2-19 is shown in figure 2-16.

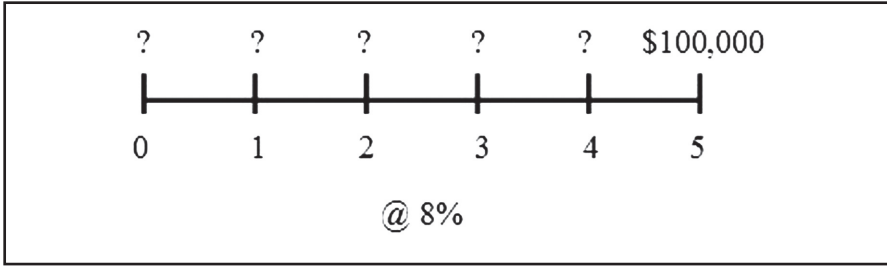


Fig. 2-16. Cash flow diagram for example 2-19, to calculate annuity from given future sum

Using equation (2.17), annual value is

$$A_v = \$100,000 \left[\frac{0.08}{(1 + 0.08)^5 - 1} \right]$$

OR

$$\begin{aligned} A_v &= \$100,000(A/F)_{i=8\%,t=5} \\ &= \$100,000(0.1705) = \$17,050 \text{ per year} \end{aligned}$$

Present value of uniform gradient series

In the preceding sections, annuities and how to calculate the present value or future value of an annuity were discussed. An annuity, however, requires the series of periodic cash flows are equal in value. In many cases, the series of periodic cash flows may not be equal but may increase or decrease at a certain uniform rate per period.

In general, a uniformly increasing series of cash flows may be expressed as $A_1, A_1 + G, A_1 + 2G, \dots, A_1 + (t - 1)G$. Similarly, a uniformly decreasing series of cash flows may be expressed as $A_1, A_1 - G, A_1 - 2G, \dots, A_1 - (t - 1)G$.

As previously shown, first calculating the present value of each payment and then adding them all together will give the present value of a uniform gradient series. However, this method of calculation is tedious and time consuming. Another recommended method is to first convert the uniform gradient series into its equivalent annuity. This annuity is then used to calculate its present or future value using equation (2.14) or equation (2.5), respectively. The following equation is used to convert the uniform gradient series into its equivalent annuity.

$$A_v = A_1 \pm G \left[\frac{1}{i_e} - \frac{t}{(1 + i_e)^t - 1} \right] \tag{2.18}$$

where

G = annual change, positive or negative
 A_1 = cash flow at the end of the first year

The factor, $[1 / i_e - t / (1 + i_e)^t - 1]$, in equation (2.18) is referred to as the uniform gradient series factor, designated by the A/G factor in the interest factors tables. The A_v calculated by using equation (2.18) is then used to calculate the present or future value.

Example 2–20

Suppose an oil well produces 2,000 barrels per year in the first year of production. The production declines by 200 barrels per year for each of the following five years. If the market rate of interest is 8%, calculate the equivalent uniform annual production rate. If the price of oil is \$75 per barrel, calculate the present value of the production.

Solution: The cash flow diagram for the problem is shown in figure 2–17.

Using equation (2.18)

$$A_v = 2,000 \pm 200 \left[\frac{1}{0.08} - \frac{6}{(1 + 0.08)^6 - 1} \right]$$

OR

$$\begin{aligned} A_v &= 2,000 - 200(A/G)_{i=8\%,t=6} \\ &= 2,000 - 200(2.2763) \\ &= 1,544.74 \text{ barrels/year} \end{aligned}$$

and

$$A_v = 1,544.74 \times \$75 = \$115,855.50$$

Using equation (2.14) or interest factor $(P/A)_{i=8\%,t=6}$

$$\begin{aligned} P_v &= \$115,855.50(P/A)_{i=8\%,t=6} \\ &= \$115,855.50(4.6229) \\ &= \$535,588.39 \end{aligned}$$

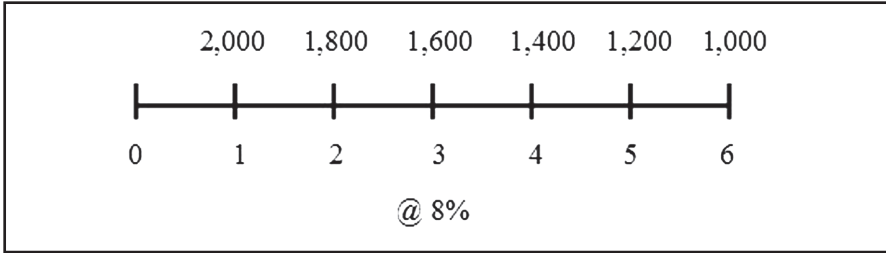


Fig. 2-17. Cash flow diagram for example 2-20, showing negative gradient series

Present value of shifted gradient series

In the previous section, the uniform gradient series and how to calculate the present value of such a series was discussed. There, it was presumed the gradient occurs in the second year of the series of payments and continues throughout the entire period under evaluation. In some cases, the gradient series may start to occur at any time during the series of payments and may not continue for the entire period under evaluation. Such a series of cash flows is referred to as the *shifted gradient*.

In order to calculate the present value of shifted gradients, a combination of various interest factors will be required. No single interest factor or equation is available to solve such gradients. The following example provides the steps required to calculate the present value of shifted gradients.

Example 2-21

Calculate the present value and annuity of the cash flows illustrated in the cash flow diagram in figure 2-18.

1. Calculate the P_v of the three uniform cash flows of \$100.

Using $(P/A)_{i_c=8\%,t=3}$

$$\begin{aligned} P_v &= \$100(2.5771) \\ &= \$257.71 \end{aligned}$$

2. Calculate the equivalent annuity A_v of the gradient series at year four using $(A/G)_{i_c=8\%,t=5}$.

$$(A_v)_{t=4} = \$100 + 50(1.8465) = \$192.33$$

3. Calculate the P_v at year three of the annuity in step 2 using $(P/A)_{i_e=8\%,t=5}$.

$$(P_v)_{t=3} = \$192.33(3.9927) = \$767.92$$

4. Calculate the $(P_v)_0$ of the $(P_v)_{t=3}$ in step 3 using $(P/F)_{i_e=8\%,t=3}$.

$$P_v = \$767.92(0.7938) = \$609.57$$

5. Now add the present value calculated in step 1 and step 4.

$$P_v = \$257.71 + \$609.57 = \$867.28$$

To calculate the equivalent annuity, use $(A/P)_{i_e=8\%,t=8}$, and the P_v calculated in step 5.

$$\begin{aligned} A_v &= \$867.28(0.1740) \\ &= \$150.91 \text{ per year} \end{aligned}$$

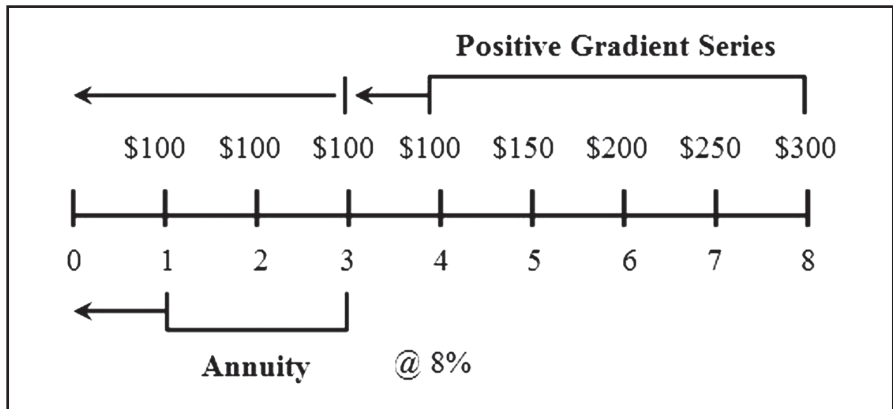


Fig. 2-18. Cash flow diagram for example 2-21

Note very carefully how the various conversions took place in example 2-21. The most critical part of the problem is the proper placement of P_v , A_v , or F_v . The same problem can be solved in several other ways. Therefore, a clear understanding of the various equivalence relationships presented in this chapter is important.

Calculation of unknown interest rate

In all examples presented in this chapter, a certain interest rate is presumed. In some cases, however, the series of cash flows and the timing of the occurrence of these cash flows will be given, but the interest rate, which has impacted these cash flows, may be unknown.

For example, a person has borrowed \$20,000 today with a promise to pay back to the lender \$30,037.13 five years from today. In this example, the P_v , F_v , and t are known, and we need to calculate the interest rate charged by the lender. The unknown interest rate may be calculated using one of the following methods.

1. Using the interest tables: setting up the P/F or F/P relationship and solving for the factor value can find the interest. For example:

$$(F/P)_{i=?, t=5} = \frac{\$30,037.13}{\$20,000} = 1.5037$$

From the interest tables, the F/P factor of 1.5037 for $t = 5$ lies between the 8% and 9% tables. The F/P value from the 8% table is 1.4693 and from the 9% table is 1.5386. Interpolating between these values:

$$\begin{aligned} i &= 8 + \left(\frac{1.5386 - 1.5037}{1.5386 - 1.4693} \right) \times (9 - 8) \\ &= 8.5\% \end{aligned}$$

2. The following equation may be used to directly calculate the interest rate.

$$\begin{aligned} i &= \exp \left[\frac{\ln(F/P)}{t} \right] - 1 \\ &= \exp \left[\frac{\ln(30,073.13/20,000)}{5} \right] - 1 \\ &= \exp(0.08158) - 1 \\ &= 0.085 \quad \text{or} \quad 8.5\% \end{aligned} \tag{2.19}$$

Similarly, if P_v , F_v , and the interest rate are known, the unknown number of years may be determined by interpolation of the interest tables.

Loan Amortization Schedule

Loan amortization means paying off a debt, normally a gradual retirement of debt with a series of equal periodic payments. Depending on the type of loan repayment arrangements, the series of payments will normally include interest accrued on the outstanding loan balance and some portion of the principal. The loan amortization calculations shown in this section will prove useful when investment projects are financed. In practice, the percent of total project cost being financed will depend on the type of project, project risk, availability of funds, and the investors' policy of maintaining a certain debt-equity ratio or debt-to-value ratio. This information will also be used in the after-tax cash flow generation discussed in chapter 4. Interest is tax deductible and therefore it is much more critical to add in the after-tax economics.

In granting the loan, the lender is buying an opportunity to receive a certain periodic cash flow over a certain period. In negotiating the loan, the lender establishes the following information in consultation with the borrower:

1. The amount of the loan requested
2. The number of years and times per year payments of interest and principal are made
3. The rate of interest charged

Based on this, the periodic payment and amortization schedule is calculated.

The borrowed money (debt) is paid back over a period of time following agreed payment arrangements. The loan can be repaid in many different ways. The following four methods are commonly used to amortize or pay back debt.

1. **Equal periodic payments.** Make equal end-of-period payments, which will consist of accrued interest and a portion of the outstanding loan balance. This is also the most commonly used loan repayment method in our day-to-day financing arrangements. With each payment the interest portion of the total payment for the following period decreases and the principal portion increases. The loan payment is calculated using the *equal payment series capital recovery factor*, A/P , or equation (2.16). As each periodic payment is made, the payment is split between interest on the outstanding balance and principal amount by using the concept of simple

interest, equation (2.1). A majority of our personal loans, car loans, and home mortgage loans are structured using this method.

2. **Interest-only loan payment.** In this type of loan repayment, the accrued interest on the loan is paid periodically and the lump sum loan balance is paid at the end of the agreed period. The lump sum principal payment is also referred to as a “balloon” payment. Therefore, a balloon loan payment is one in which it is planned to pay off the loan in a big chunk after a number of small regular periodic payments. The balloon payment can also be applied in the case of equal periodic payments loan amortization. The interest to be paid each period is calculated using the concept of simple interest, equation (2.1).
3. **Equal principal payments.** Make equal principal payments and interest payments on the unpaid balance at the end of each interest period.
4. **Single lump sum payment.** Make a single payment of principal and interest at the end of the agreed loan period. This method is not usually used in long-term debts and larger loans but can be used for short-term debts and smaller loans.

Note: It is very important and strongly recommended to always incorporate some checks for the loan calculations in the cash flow model to make sure that the calculations are done correctly. The checks include (a) the sum of the principal payments must equal the total loan amount, (b) the outstanding loan balance at the end of the loan period must equal zero, and (c) the sum of the P_v of interest payments and the P_v of the principal payments must equal the total loan amount.

Example 2–22

An oil producer borrows \$100,000 at an interest rate of 8% per year for a period of three years. The loan has to be paid back in equal quarterly payments over the three-year period, with the first payment due exactly three months from the date money is borrowed. Calculate the quarterly payment and show the loan amortization schedule.

Solution: Using equation (2.16), the quarterly payment is

$$\begin{aligned}
 A_v &= \$100,000 \left[\frac{\frac{i_n}{4} \left(1 + \frac{i_n}{4} \right)^{t \times 4}}{\left(1 + \frac{i_n}{4} \right)^{t \times 4} - 1} \right] \\
 &= \$100,000 \left[\frac{0.02(1 + 0.02)^{12}}{(1 + 0.02)^{12} - 1} \right] \\
 &= \$100,000(0.09456) \\
 &= \$9,456 \text{ per quarter}
 \end{aligned}$$

The amortization schedule is given in table 2–2. The \$92,544 outstanding loan balance at the end of the first quarter (after the first payment has been made) becomes the principal owed during the second quarter, and so on. *Simple interest* calculations are used at the end of each payment to calculate the interest and principal for that payment. The first quarter's calculations are shown below.

Cell C6 = quarterly payment = −\$9,456

Cell E8 = outstanding loan = +\$100,000

Cell A9 = end of quarter or period one

Cell B9 = cell E8 × − cell C6/cell C3 = \$100,000 × −0.08/4 = −\$2,000

Cell C9 = cell C6 − cell B9 = −\$9,456 − (−\$2,000) = −\$7,456

Cell D9 = cell B9 + cell C9 = −\$2,000 − \$7,456 = −\$9,456

Cell E9 = cell E8 + cell C9 = \$100,000 − \$7,456 = \$92,544

The rest of the table is completed by copying cells B9 to E9 and pasting them into cells B10 to E20. Note that the sign could be positive or negative. A negative sign is preferred to indicate that the interest and principal payments are cash outflows.

Table 2–2. Loan amortization schedule for example 2–22

	A	B	C	D	E
1	Money Borrowed		\$100,000		
2	Payment Periods, Years		3		
3	Payments per Year		4		
4	Nominal Interest Rate		8%		
5					
6	Quarterly Payments		(\$9,455.96)		
7	End of Quarter	Interest Payment \$	Principal Payment \$	Total Quarterly Payment, \$	Outstanding Balance \$
8	0	0.00	0.00	0.00	100,000.00
9	1	(2,000.00)	(7,455.96)	(9,455.96)	92,544.04
10	2	(1,850.88)	(7,605.08)	(9,455.96)	84,938.96
11	3	(1,698.78)	(7,757.18)	(9,455.96)	77,181.78
12	4	(1,543.64)	(7,912.32)	(9,455.96)	69,269.46
13	5	(1,385.39)	(8,070.57)	(9,455.96)	61,198.89
14	6	(1,223.98)	(8,231.98)	(9,455.96)	52,966.90
15	7	(1,059.34)	(8,396.62)	(9,455.96)	44,570.28
16	8	(891.41)	(8,564.55)	(9,455.96)	36,005.73
17	9	(720.11)	(8,735.85)	(9,455.96)	27,269.88
18	10	(545.40)	(8,910.56)	(9,455.96)	18,359.32
19	11	(367.19)	(9,088.77)	(9,455.96)	9,270.55
20	12	(185.41)	(9,270.55)	(9,455.96)	(0.00)
21	TOTAL	(13,471.52)	(100,000.00)	(113,471.52)	

Example 2–23

An oil producer borrows \$100,000 at an interest rate of 8% per year for a period of three years. The total loan amount will be paid back at the end of the agreed three-year period, while interest on the loan has to be paid back quarterly over the three-year period, with the first payment due exactly three months from the date money is borrowed. Calculate the quarterly interest payment and show the last installment at the end of year three. Briefly explain the difference between the amount of interest paid in example 2–22 and this example.

Solution: Using equation (2.1), the quarterly interest payment is

$$\begin{aligned} \text{Interest} &= \text{Principal} \times \text{Quarterly Interest Rate} \\ &= \$100,000 \times -0.08/4 = -\$2,000 \text{ per quarter} \end{aligned}$$

The calculations are shown in table 2–3.

Table 2–3. Loan repayment schedule for example 2–23, assuming interest-only loan payment and balloon payment at end of year three

	A	B	C	D	E
1	Money Borrowed		\$100,000		
2	Payment Periods, Years		3		
3	Payments per Year		4		
4	Nominal Interest Rate		8%		
5					
6	Quarterly Interest Payment		(\$2,000.00)		
7	End of Quarter	Interest Payment \$	Principal Payment \$	Total Quarterly Payment, \$	Outstanding Balance \$
8	0	0	0	0	100,000
9	1	(2,000)	0	(2,000)	100,000
10	2	(2,000)	0	(2,000)	100,000
11	3	(2,000)	0	(2,000)	100,000
12	4	(2,000)	0	(2,000)	100,000
13	5	(2,000)	0	(2,000)	100,000
14	6	(2,000)	0	(2,000)	100,000
15	7	(2,000)	0	(2,000)	100,000
16	8	(2,000)	0	(2,000)	100,000
17	9	(2,000)	0	(2,000)	100,000
18	10	(2,000)	0	(2,000)	100,000
19	11	(2,000)	0	(2,000)	100,000
20	12	(2,000)	(100,000)	(102,000)	0
21	TOTAL	(24,000)	(100,000)	(124,000)	

The total interest over the three years will amount to \$24,000 ($\$2,000 \times 4$ quarters/year $\times 3$ years = \$24,000). The last payment at the end of year three will be \$102,000, which will consist of the interest for the last quarter and

the total borrowed amount. Note the difference in the total interest (\$13,470) paid in example 2–22 as compared to the \$24,000 paid in this example. The difference is because the outstanding loan balance in example 2–22 keeps on reducing while the loan balance in the interest-only payment option stays the same.

Example 2–24

An oil producer borrows \$100,000 at an interest rate of 8% per year for a period of three years. The total loan amount will be paid back over the three-year period by (a) 12 equal (constant) quarterly payments of principal amount and (b) quarterly interest payments on the outstanding loan balance. The first payment is due exactly three months from the date money is borrowed. Calculate the quarterly principal and interest payment and show the last installment at the end of year three.

Solution: The following steps are followed for loan repayment calculations using the constant principal payment. The calculations are shown in table 2–4.

Cell C6: $= -C1/(C3 \times C2) = -\$100,000 / (4 \times 3) = -\$8,333.33/\text{quarter}$

Cell E8: outstanding loan balance = \$100,000

Cell B9: interest payment $= -C4/C3 \times E8$

Cell C9: principal payment = C\$6

Cell D9: total quarterly payment $= B9 + C9$

Cell E9: outstanding balance $= E8 + C9$

The rest of the table is completed by copying cells B9 to E9 and pasting them into cells B10 to E20.

Example 2–25

An oil producer borrows \$100,000 at an interest rate of 8% per year for a period of three years. The total loan amount with accrued interest will be paid back in one lump sum payment at the end of year three. It is assumed that the interest on the loan will accrue quarterly. Calculate the total amount (principal and interest) to be paid back at the end of year three.

Solution: The following steps are used for loan repayment calculations using the lump sum interest and principal payment.

$$\begin{aligned} F_v &= P_v (F/P_{i=8\%, t=12}) \\ &= \$100,000(1.26824) = \$126,824 \end{aligned}$$

The quarterly accrued interest and the outstanding loan balance are shown in table 2-5. The accrued interest is compounded each quarter. Since there are no quarterly payments, zeros are entered in cells C7 to D18.

$$\text{Cell B8} = \text{cell E7} \times \text{cell C\$4} / \text{cell C\$3}$$

$$\text{Cell E8} = \text{E7} + \text{B8} - \text{C8}$$

$$\text{Cell E19} = \text{E18} - \text{C19} - \text{SUM(B8:B18)}$$

Enter the principal payment of \$100,000 in cell C19 (treated as payment at the end of year three). Cells B8 to E8 are then copied into cells B9 to E18. The calculations are shown in table 2-5.

Table 2-4. Loan repayment schedule for example 2-24, assuming constant quarterly principal payment and quarterly interest payment on outstanding loan balance.

	A	B	C	D	E
1	Money Borrowed		\$100,000		
2	Payment Periods, Years		3		
3	Payments per Year		4		
4	Nominal Interest Rate		8%		
5					
6	Quarterly Principal Payment		(\$8,333.33)		
7	End of Quarter	Interest Payment \$	Principal Payment \$	Total Quarterly Payment, \$	Outstanding Balance \$
8	0	0	0	0	100,000
9	1	(2,000)	(8,333)	(10,333)	91,667
10	2	(1,833)	(8,333)	(10,167)	83,333
11	3	(1,667)	(8,333)	(10,000)	75,000
12	4	(1,500)	(8,333)	(9,833)	66,667
13	5	(1,333)	(8,333)	(9,667)	58,333
14	6	(1,167)	(8,333)	(9,500)	50,000
15	7	(1,000)	(8,333)	(9,333)	41,667
16	8	(833)	(8,333)	(9,167)	33,333
17	9	(667)	(8,333)	(9,000)	25,000
18	10	(500)	(8,333)	(8,833)	16,667
19	11	(333)	(8,333)	(8,667)	8,333
20	12	(167)	(8,333)	(8,500)	0
21	TOTAL	(13,000)	(100,000)	(113,000)	

Table 2–5. Loan repayment schedule for example 2–25, assuming single payment (total interest and loan amount) at the end of year three.

	A	B	C	D	E
1	Money Borrowed		\$100,000		
2	Payment Periods, Years		3		
3	Payments per Year		4		
4	Nominal Interest Rate		8%		
5					
6	End of Quarter	Interest Accrued \$	Principal Payment \$	Total Quarterly Payment, \$	Outstanding Balance \$
7	0	0	0	0	100,000
8	1	2,000	0	0	102,000
9	2	2,040	0	0	104,040
10	3	2,081	0	0	106,121
11	4	2,122	0	0	108,243
12	5	2,165	0	0	110,408
13	6	2,208	0	0	112,616
14	7	2,252	0	0	114,869
15	8	2,297	0	0	117,166
16	9	2,343	0	0	119,509
17	10	2,390	0	0	121,899
18	11	2,438	0	0	124,337
19	12	2,487	100,000	0	0
20	TOTAL	26,824	100,000	0	

The four loan payback methods result in different interest payments over the loan payback period. The interest amounts paid in the four methods (as shown in tables 2–2 to 2–5) are:

1. \$13,472 for the equal periodic payments
2. \$24,000 for the interest-only payment
3. \$13,000 for the equal principal payments
4. \$26,824 for the single lump sum payment

The more conventional types of loan paybacks are the (a) equal periodic payments and (b) constant principal payment. Both these methods give a reasonable interest over the loan payback period. The other two methods charge the highest interest over the loan payback period. The single lump sum loan payment and the interest-only payment methods are much more risky for the lender in comparison to the remaining two methods. The reason for the risk is because the lender is not getting his money back until the end of the loan period and therefore has longer risk exposure.

Funds Flow and Compounding/Discounting

Throughout this chapter, it was assumed the cash flows, regardless of their actual frequency of occurrence, occurred at the end of the year or interest period in which they occurred. In addition, it was assumed that the interest is compounded/discounted at the end of the year or interest period. This is referred to as *discrete period* interest compounding/discounting and *end of period* cash flows.

It should be kept in mind that all the equivalence factors presented in the preceding sections are applicable only in case of the above assumptions. Variations to the above assumptions that have practical applications and are frequently used in the industry are:

1. Continuous *interest compounding/discounting* and *discrete period cash flows*. The continuous interest compounding/discounting (i.e., compounding/discounting interest an infinite number of times per year) closely approximates daily compounding/discounting. To differentiate between the periodic interest rate, i_e , and the continuous interest rate, the latter is designated by r . Equations for the continuous compounding/discounting and discrete period cash flows are given in table 2–6.

The continuous interest rate r may be converted to effective interest rate by using equation (2.5). This effective interest rate can then be used in the equations for discrete period interest compounding/discounting and end-of-year cash flows.

2. Continuous *interest compounding/discounting* and *continuous flow of funds*: In many instances, it may be reasonable to presume cash flows occur on a relatively continuous basis throughout the year or interest period. Such situations involve a *funds-flow* process. In this case, an infinite number of cash flows (received or disbursed continuously) per year to match the interest rate with an infinite number of compounding/discounting periods. Equations for the

continuous interest compounding/discounting and continuous flow of funds are given in table 2–6.

3. Discrete *period interest compounding/discounting* and *mid-period cash flows*. The use of this assumption in handling time value of money problems has gained considerable popularity in the oil industry. According to this assumption, all cash flows are presumed to occur at the middle of the interest period instead of at the end. Consequently, all cash flows after time zero occur half a period sooner than the end of period convention.

The equivalent values obtained by using the end-of-year cash flow convention may be converted to conform to the mid-period convention. This is done by multiplying the former value by the factor $(1 + i)^{0.5}$ or by replacing the t in equations (2.9) through (2.17) by $t - 0.5$.

For example, recalculating the present value of the \$5,000 in example 2–6 using the mid-year convention as

$$\begin{aligned} P_v &= \$5,000 \left[\frac{1}{(1 + 0.08)^{5.5}} \right] \\ &= \$5,000(0.6549) \\ &= \$3,274.46 \end{aligned}$$

OR, from example 2–6:

$$\begin{aligned} P_v &= \$3,150.85(1 + 0.08)^{0.5} \\ &= \$3,150.85 \times 1.0392 = \$3,274.46 \end{aligned}$$

The selection of the type of compounding/discounting (e.g., annual, monthly, daily, continuous, etc.) and the assumption of the pattern of cash flows (e.g., year end, mid-year, monthly, uniform, etc.) used in cash flow analysis should be representative of the fiscal practices of the firm's treasury and the projects being considered.

In general, the year-end cash flows and the mid-year discounting are widely used in the industry. The year-end cash flows and year-end discounting at a certain discount rate will give conservative present value, followed by mid-year discounting and continuous discounting. Some analysts argue that continuous cash flows and discounting more closely reflect reality. However, (1) the cash flow reinvestment may not be as efficient as implied by the calculations, and (2) the cash flow projections are not accurate enough to warrant the extra accuracy.

Table 2–6. Equations for two different types of funds flow and continuous interest compounding/discounting

	Factor	Continuous Interest Discrete Cash Flow	Continuous Interest Continuous Cash Flow
Single payment compound amount factor	F/P	$F_v = P_v(e^{rt})$	$F_v = P_v\left(\frac{re^{rt}}{e^r - 1}\right)$
Single payment present worth factor	P/F	$P_v = F_v\left(\frac{1}{e^{rt}}\right)$	$P_v = F_v\left(\frac{e^r - 1}{re^{rt}}\right)$
Uniform series compound amount factor	F/A	$F_v = A_v\left(\frac{e^{rt} - 1}{e^r - 1}\right)$	$F_v = \overline{A}_v\left(\frac{e^{rt} - 1}{r}\right)$
Sinking fund factor	A/F	$A_v = F_v\left(\frac{e^r - 1}{e^{rt} - 1}\right)$	$\overline{A}_v = F_v\left(\frac{r}{e^{rt} - 1}\right)$
Uniform series present worth factor	P/A	$P_v = A_v\left[\frac{e^{rt} - 1}{(e^r - 1)e^{rt}}\right]$	$P_v = \overline{A}_v\left(\frac{e^{rt} - 1}{re^{rt}}\right)$
Capital recovery factor	A/P	$A_v = P_v\left[\frac{(e^r - 1)e^{rt}}{e^{rt} - 1}\right]$	$\overline{A}_v = P_v\left(\frac{re^{rt}}{e^{rt} - 1}\right)$
Uniform gradient series factor	A/G	$A_v = A_1 + G\left[\frac{1}{e^r - 1} - \frac{1}{e^{rt} - 1}\right]$	

$\overline{A}_v$ = the single total amount of funds flowing continuously during a period, in a uniform series of equal payments

The analysts are also advised that before using any canned economic analysis models or calculators with built-in financial functions, they should check the assumptions to make sure they are in accordance with your requirement and standards used.

Spreadsheet Applications

The use of computers, specifically spreadsheet applications such as Microsoft Excel, has revolutionized the working environment of engineers, geologists, economists, financial/investment analysts, accountants, stockbrokers, and others. The use of spreadsheets greatly simplifies the calculations covered in the preceding pages.

The Microsoft Excel financial functions are invaluable tools for answering important financial questions. There are some financial calculations that the spreadsheets do not handle directly. However, with a little effort, the scope of these spreadsheets can be expanded to handle such calculations by combining

functions or building straightforward formulas. This section explains how, with a little finesse, the spreadsheets can be used to calculate

1. The term of a present value, or how long it will take to pay off a loan at a given payment schedule and interest rate
2. The future value of an initial lump-sum payment
3. The present value of a future lump-sum payment
4. Balloon loan payments
5. The interest and principal parts of a payment
6. The present value of constantly changing annual receipts/payments
7. The future value of constantly changing annual receipts/payments
8. Periodic interest rates

The data given in examples 2–1 through 2–22 show how to calculate these problems manually. In the following pages, many tips not normally shown in the user manuals of the spreadsheets are introduced. These assist in solving a variety of real-life problems. Table 2–7 presents the Microsoft Excel functions needed to solve examples 2–2 to 2–19.

The interest rate in all relationships is used as a fraction. If a nominal interest rate is given and the compounding periods per year is more than one (1), then the nominal interest rate is divided by the number of compounding periods per year and the years are multiplied by the compounding periods per year. For example, to make monthly payments on a four-year car loan at 12% nominal interest rate, use 12%/12 for the interest rate and 4×12 for the *periods (time)*.

1. *=EFFECT(nominal interest rate, compounding periods per year)*: Returns the effective annual interest rate when the nominal annual interest rate and the compounding frequency per year are given.
2. *=FV(interest rate, periods, payment, present value, type)*: Returns the future value of an annuity or a lump-sum present value. If the future value of an annuity is required, the present value is input as zero (0) and vice versa. Type is the number zero (0) or one (1) and indicates when payments are due (i.e., zero is used for end-of-period payments like in ordinary annuity, and one is used when payments are made at the beginning of the periods like in annuity due).

3. `=INTRATE(settlement, maturity, investment, redemption, basis)`: Returns the interest rate when the present value, future value, date of present value, and date of future value are given.
4. `=RATE(nper, pmt, pv, fv, type, guess)`: Returns the interest rate charged when the number of periodic payments, periodic payment, and the present value of future value are given. Type is the number zero (0) or one (1) and indicates when payments are due (i.e., zero is used for end-of-period payments like in ordinary annuity, and one is used when payments are made at the beginning of the periods like in annuity due). RATE is calculated by iteration. The guess is normally not required as Excel will internally *guess* the rate start iterations.
5. `=NPER(rate, pmt, pv, fv, type)`: Returns the number of periods required to achieve a certain sum of money in the future given the periodic rate, present value, and periodic payments. Type is the number zero (0) or one (1) and indicates when payments are due (i.e., zero is used for end-of-period payments like in ordinary annuity, and one is used when payments are made at the beginning of the periods like in annuity due).
6. `=NPV(interest rate, value 1, value 2, .... value n)`: Returns the net present value of a series of equally spaced periodic cash flows at a certain discount rate. The cash flows do not have to be the same each period, as required for annuity. Excel, by default, assumes each value in the cell to be at the end of the period. To treat the values at the beginning of the period, the first value should be placed outside the NPV function. The two versions of NPV are shown in table 2–8.

Another thing to note is that no cells should be left blank when using the NPV function. Excel normally treats blank cells as zeros, but when using the NPV function, the default is not applicable. If zeros are omitted in the cells that are supposed to have the zeros, then the NPV will not be correct. This is shown in table 2–9.

Table 2-7. Excel spreadsheet representing the calculations in examples 2-2 to 2-19

	A	B	C
1	Example 2-2 (a)		
2	Borrowed Money (Pv)	\$2,000	
3	Interest Rate	8.0%	
4	Payback Period, Years	1	
5	Periodic Payments / Year	1	
6	Interest at End of Year	\$160.00	=FV(B3/B5,B4*B5,-B2)-B2
7	Example 2-2 (b)		
8	Borrowed Money (Pv)	\$2,000.00	
9	Interest Rate	8.0%	
10	Payback Period, Years	1	
11	Periodic Payments / Year	4	
12	Interest at End of Year	\$164.86	=FV(B9/B11,B10*B11,-B8)-B8
13	Example 2-2 (c)		
14	Borrowed Money (Pv)	\$2,000.00	
15	Interest Rate	8.0%	
16	Payback Period, Years	3	
17	Periodic Payments / Year	2	
18	Interest at End of Year	\$530.64	=FV(B15/B17,B16*B17,-B14)-B14
19	Example 2-3		
20	Current Monthly Salary	\$4,000	
21	Merit Increase / Year	5.0%	
22	Total Time Elapsed, Years	5	
23	Salary after Five Years	\$5,105	=FV(B21,B22,-B20)
24	Example 2-4		
25	Borrowed (Pv)	\$2,000	
26	Interest Rate	8.0%	
27	Payback Period, Years	1	
28	Periodic Payments / Year	4	
29	Amount to be Paid Back (Fv)	\$2,165	=FV(B3/B5,B4*B5,-B2)
30		OR	
31	Effective Interest Rate	8.24%	=EFFECT(B3,B5)
32	Amount to be Paid Back (Fv)	\$2,165	=FV(B9,B4,-B2)
33		OR	
34	Amount to be Paid Back (Fv)	\$2,165	=FV(EFFECT(B3,B5),B4,-B2)

Table 2-7. (Continued)

	A	B	C
35	Example 2-6		
36	Future Value (Fv)	\$5,000	
37	Interest Rate	8.0%	
38	Number of Years	6	
39	Number of Periods / Year	1	
40	Present Value (Pv)	\$3,151	=PV(B37/B39,B38*B39,-B36)
41	Example 2-7		
42	Royalty Payment 1	\$2,000	
43	Royalty Payment 2	\$2,200	
44	Royalty Payment 3	\$1,900	
45	Royalty Payment 4	\$2,500	
46	Royalty Payment 5	\$1,500	
47	Interest Rate	8.0%	
48	Present Value (Pv)	\$8,105	=B42*(1+B47)^-1+B43*(1+B47)^-2+B44*(1+B47)^-3+B45*(1+B47)^-4+B46*(1+B47)^-5
49		OR	
50	Present Value (Pv)	\$8,105	=NPV(B47,B42:B46)
51	Example 2-8		
52	Future Value (Fv)	\$5,000.00	
53	Interest Rate	8.5%	
54	Number of Years	6	
55	Number of Periods / Year	1	
56	Present Value (Pv)	\$3,064.73	=PV(B53/B55,B54*B55,-B52)
57	Example 2-9		
58	Future Value (Fv)	\$5,000	
59	Interest Rate	8.0%	
60	Number of Years	6	
61	Number of Periods / Year	4	
62	Present Value (Pv)	\$3,109	=PV(B59/B61,B60*B61,-B58)
63	Example 2-10		
64	Present Value (Pv)	\$3,150.85	
65	Interest Rate	8.0%	
66	Number of Years	6	
67	Number of Periods / Year	1	
68	Future Value (Fv)	(\$5,000)	=FV(B65/B67,B66*B67,-B64)

Table 2-7. (Continued)

	A	B	C
69	Example 2-11		
70	Royalty Payment 1	\$2,000	
71	Royalty Payment 2	\$2,200	
72	Royalty Payment 3	\$1,900	
73	Royalty Payment 4	\$2,500	
74	Royalty Payment 5	\$1,500	
75	Interest Rate	8.0%	
76	Number of Years	5	
77	Present Value (Pv)	\$8,105	=NPV(B75,B70:B74)
78	Future Value (Fv)	\$11,909	=FV(B75,B76,,-B77)
79	Example 2-12		
80	Annual Loan Payment	\$10,000	
81	Interest Rate	8.0%	
82	Number of Years	5	
83	Number of Periods / Year	1	
84	Present Value (Pv)	\$39,927	=PV(B81/B83,B82*B83,-B80)
85	Example 2-13		
86	Down Payment	\$10,000	
87	Year-End Installments	\$10,000	
88	Interest Rate	8.0%	
89	Number of Years	4	
90	Present Value (Pv)	\$43,121	=PV(B88,B89,-B87)+B86
91		OR	
92	Present Value (Pv)	\$43,121	=PV(B88,B89+1,B87,,1)
93	Example 2-14		
94	Each Installment	\$10,000	
95	Interest Rate	8.0%	
96	Number of Installments	5	
97	Fv of Ordinary Annuity	\$58,666	=FV(B95,B96,-B94,,0)
98	Fv of Annuity Due	\$63,359	=FV(B95,B96,-B94,,1)
99	Example 2-15		
100	Deferred Annuity	\$10,000	
101	Interest Rate	8.0%	
102	Number of Installments	5	
103	Deferred Periods, Years	3	
104	Present Value (Pv)	(\$31,695)	=PV(B101,B103,,PV(B101,B102,-B100))

Table 2-7. (Continued)

	A	B	C
105	Example 2-16		
106	Borrowed from Bank (Pv)	\$20,000	
107	Interest Rate	8.0%	
108	Payback Years	5	
109	Payments / Year	1	
110	Amount of Each Payment	\$5,009	=PMT(B107/B109,B108*B109,-B106)
111	Example 2-17		
112	Cost of Equipment (Pv)	\$50,000	
113	Anticipated Useful Life, Years	10	
114	Interest Rate	8.0%	
115	Equivalent Annual Cost	\$7,451.5	=PMT(B114,B113,-B112)
116	Example 2-18		
117	Money Deposited in Bank	\$20,000	
118	Interest Rate	8.0%	
119	Money Kept in Account, Yrs	3	
120	Number of Equal Withdrawals	5	
121	Amount of Each Withdrawal	(\$6,310.06)	=PMT(B118,B120,FV(B118,B119,-B117))
122	Example 2-19		
123	Cost of Equipment (Fv)	\$100,000	
124	Interest Rate	8.0%	
125	Years to Replacement	5	
126	Money to be kept aside	\$17,045.65	=PMT(B124,B125,-B123)
127	Example 2-21		
128	Payment 1	\$150	
129	Payment 2	\$200	
130	Payment 3	\$250	
131	Payment 4	\$300	
132	Annuity Ammount	\$100	
133	Number of Annuity Payments	4	
134	Interest Rate	8.0%	
135	Pv of Payments 1 to 4	\$536	=PV(B134,B133,-NPV(B134,B128:B131))
136	Pv of Annuity (4 Payments)	\$331	=PV(B134,B133,-B132)
137	Total Present Value (Pv)	\$867	=SUM(B135:B136)
138		OR	
139	Total Present Value (Pv)	\$867	=PV(B134,B133,-B132)+NPV(B134,0,0,0,0,B128:B131)

Table 2–7. (Continued)

	A	B	C
140	Calculation of Unknown Interest Rate		
141	Borrowed Today (Pv)	\$20,000	
142	Payback Amount	\$30,073	
143	Payback Period, Years	5	
144	Number of Periods / Year	1	
145	Interest Rate Charged	8.5%	=RATE(B143*B144,,B141,-B142)*B144

Table 2–8. Two versions of the NPV function

	A	B	C	D	E	F
1	Interest Rate	8.0%				
2	Years	1	2	3	4	5
3	Net Cash-Flow	(1,000)	500	350	250	175
4						
5	NPV @ 8% - Year-End Cash-Flow	\$83.44	=NPV(B1,B3:F3)			
6	NPV @ 8% - Beginning-of-Year Cash-Flow	\$90.12	=NPV(B1,C3:F3)+B3			

Table 2–9. Effect of blank cells on NPV

	A	B	C	D	E	F	G	H
1	Interest Rate	8.0%						
2	Years	1	2	3	4	5	6	7
3	Net Cash-Flow			(1,000)	500	350	250	175
4								
5		0	0	(1,000)	500	350	250	175
6								
7	NPV @ 8% - Year-End Cash-Flow	\$83.44	=NPV(B1,B3:H3)					
8	NPV @ 8% - Year-End Cash-Flow	\$71.54	=NPV(B1,B5:H5)					

1. =PMT(*interest rate*, *nper*, *pv*, *fv*, *type*): Calculates the payment for a loan based on a constant payment and a constant interest rate when the present value of a loan is given. The payment calculated by PMT includes principal and interest payment but no taxes. The PMT can also be used to calculate a sinking fund (annuity or equivalent annual values) that will accumulate to some required future value. In the former case, FV=0 and in the latter case PV=0.
2. =IPMT(*rate*, *per*, *nper*, *pv*, *fv*, *type*): Returns the interest payment for the specified period, given the rate, period, number of periods, the present or future value, and the type. Similarly, the =CUMIPMT(*rate*, *nper*, *pv*, *start_period*, *end_period*, *type*) function

can be used to calculate the cumulative interest amount paid on a loan between two time periods. Type is the number zero (0) or one (1) and indicates when payments are due (i.e., zero is used for end-of-period payments like in ordinary annuity, and one is used when payments are made at the beginning of the periods like in annuity due).

3. `=PPMT(rate, per, nper, pv, fv, type)`: Returns the principal payment for the specified period, given the rate, period, number of periods, the present or future value, and the type. Similarly, the `=CUMPRINC(rate, nper, pv, start_period, end_period, type)` function can be used to calculate the cumulative principal amount paid on a loan between two time periods. Type is the number zero (0) or one (1) and indicates when payments are due (i.e., zero is used for end-of-period payments like in ordinary annuity, and one is used when payments are made at the beginning of the periods like in annuity due).
4. `=PV(interest rate, periods, payment, future value, type)`: Returns the present value of an annuity or a lump-sum future value. If the future value of an annuity is required, the future value is input as zero (0) and vice versa. Type is the number zero (0) or one (1) and indicates when payments are due (i.e., zero is used for end-of-period payments like in ordinary annuity, and one is used when payments are made at the beginning of the periods like in annuity due).
5. `=XNPV(rate, values, dates)`: Returns the net present value for a schedule of cash flows that is not necessarily periodic and is specified by dates. The XNPV calculates the PV assuming the beginning-of-the-year cash flow assumption regardless of the date entered for the first value. To calculate the PV at time zero using the XNPV function and the year-end cash flow assumption, the first value must be zero. The use of the XNPV function is shown in table 2–10.

Table 2–10. Use of the XNPV function

	A	B	C	D	E	F	G
1	Interest Rate	8.0%					
2	Years		12/31/2010	12/31/2011	12/31/2012	12/31/2013	12/31/2014
3	Net Cash-Flow		(1,000)	500	350	250	175
4	NPV @ 8% - Beginning-of-Year Cash-Flow	\$89.99	=XNPV(B1,C3:G3,C2:G2)				
5							
6	Years		1/1/2010	12/31/2010	12/31/2011	12/31/2012	12/31/2013
7	Net Cash-Flow		(1,000)	500	350	250	175
8	NPV @ 8% - Beginning-of-Year Cash-Flow	\$90.28	=XNPV(B1,C7:G7,C6:G6)				
9							
10	Years	1/1/2010	12/31/2010	12/31/2011	12/31/2012	12/31/2013	12/31/2014
11	Net Cash-Flow	0	(1,000)	500	350	250	175
12	NPV @ 8% - Year-End Cash-Flow	\$83.34	=XNPV(B1,B11:G11,B10:G10)				

It is very critical to pay attention to the units required by these financial functions. If monthly payments are made on a four-year loan at 12% annual interest, use 12%/12 for guess (if guess is required), 12%/12 for rate in the PV, FV, IPMT, PPMT, and PMT functions, and 4*12 for NPER. If annual payments are made on the same loan, then there will be no need to divide the rate by 12, multiply the number of periods (NPER) by 12, and so on.

It is also very critical to pay attention to the signs of the various variables. Excel, by default, gives negative value for the PV, FV, IPMT, PPMT, and PMT functions. These negative values, if required, can be changed into positive values by either entering a minus before the function [such as =-PV(rate, nper, pmt, fv, type)] or before the given variable (such as [PV(rate, nper, -pmt, -fv, type)]). Similarly, the variables used inside the RATE, INTRATE, CUMPRINC, CUMIPMT, and NPER should have the appropriate sign. It even becomes more critical to pay attention to the sign when *embedded* functions (two or three functions used together) are used in Excel.

Table 2–11 duplicates the amortization schedule of example 2–22, similar to the one shown in table 2–2. In this table, three spreadsheet commands (=PMT, =IPMT, and =PPMT) are used. The IPMT function calculates the interest payment for a given period on an investment based on periodic, constant payments and a constant periodic interest rate. PPMT calculates the principal payment corresponding to the IPMT. The following steps are used to set up the amortization schedule of table 2–11:

1. Input the loan parameters in cells C1 to C4.
2. In cell C6, input the formula =PMT(C4/C3,C3*C2, -C1).
3. Enter the original loan amount (principal) in cell E8.
4. In cells A9 to A20, input the quarters 1 to 12.
5. In cell B9, input the formula =IPMT(C4/C3,A9,C3*C\$2,C\$1).

6. In cell C9, input the formula =PPMT(C\$4/C\$3,A9,C\$3*C\$2,C\$1).
7. In cell D9, input =sum(B9:C9).
8. In cell E9, input =E8+C9.
9. Copy cell B9 to E9 and paste the contents all the way from cell B10 to E20.

Note that the outstanding balance at the end of 12th quarter (cell E20) is zero, the total principal payment in cell C21 amounts to \$100,000, and the sum of the IPMT (column B) and PPMT (column C) equals the total payment (column D).

Table 2–11. Amortization schedule of example 2–22

	A	B	C	D	E
1	Money Borrowed		\$100,000		
2	Payment Periods, Years		3		
3	Payments per Year		4		
4	Nominal Interest Rate		8%		
5					
6	Quarterly Payment		9,455.96		
7	End of Quarter	Interest Payment \$	Principal Payment \$	Total Quarterly Payment, \$	Outstanding Balance \$
8	0	0	0	0	100,000
9	1	(2,000)	(7,456)	(9,456)	92,544
10	2	(1,851)	(7,605)	(9,456)	84,939
11	3	(1,699)	(7,757)	(9,456)	77,182
12	4	(1,544)	(7,912)	(9,456)	69,269
13	5	(1,385)	(8,071)	(9,456)	61,199
14	6	(1,224)	(8,232)	(9,456)	52,967
15	7	(1,059)	(8,397)	(9,456)	44,570
16	8	(891)	(8,565)	(9,456)	36,006
17	9	(720)	(8,736)	(9,456)	27,270
18	10	(545)	(8,911)	(9,456)	18,359
19	11	(367)	(9,089)	(9,456)	9,271
20	12	(185)	(9,271)	(9,456)	(0)
21	TOTAL	(13,472)	(100,000)	(113,472)	

Example 2–26

Use the appropriate Excel financial functions to solve the following problems.

1. Calculate the cumulative principal payment for the loan in table 2–11.
2. Calculate the cumulative interest payment for the loan in table 2–11.
3. In table 2–11, calculate the interest rate (assuming that it is not known) given the present value (loan amount) and the number of periods over which the loan has to be paid back. The periodic payment amount should be calculated using the =PTM function *embedded* in the =RATE function.
4. In table 2–11, calculate the loan period (assuming it is not known) given the interest rate, periodic payment, and loan amount.
4. Use the data of figure 2–7 to calculate the interest rate required to make \$5,000 six years from today by investing \$3,150.85 today.

Solution: The problems are solved using Excel as shown in table 2–12.

Table 2–12. Solution of the problems in example 2–26

	A	B	C	D
1	Example 2-26 (Loan Amortization of Table 2-10)			
2		Money Borrowed	\$100,000	
3		Payment Periods, Years	3	
4		Payments per Year	4	
5		Nominal Interest Rate	8%	
6				
7	1.	Cumulative Principal Payment	(100,000)	=CUMPRINC(C5/C4,C3*C4,C2,1,12,0)
8	2.	Cumulative Interest Payment	(13,472)	=CUMIPMT(C5/C4,C4*C3,C2,1,12,0)
9	3.	Interest Rate for the Loan	8.0%	=RATE(C4*C3,PMT(C5/C4,C4*C3,C2),C2)*C4
10	4.	Loan Period, Quarters	12	=NPER(C5/C4,PMT(C5/C4,C4*C3,C2),C2)
11	Example 2-26 (Problem of Figure 2-27)			
12	5.	Present Value	\$3,150.85	
13		Future Value	\$5,000.00	
14		Interest Rate	8%	
15		Number of Periods per Year	1	
16		Number of Years Required	6	=NPER(C14/C15,,-C12,C13,0)

The Time Zero Dilemma

Many analysts have confused the issue of time zero and often the timing assumptions are jumbled up to an extent that they start violating the logic and common sense of the events taking place. We need to very clearly define how to treat the cash flows. The main purpose of the time zero is to identify the inception of the investment or project implementation. This will normally be associated with the business plan or operating plan of the company (e.g., implementations to start 01/01/2010).

The question now is whether all the cash flows taking place in the first year of the project should be treated at time zero (e.g., 01/01/2010) or at the end of the year (e.g., 12/31/2010) or even be treated as occurring at mid-year. Many analysts treat whatever has happened during the first year as if it occurred at time zero. In reality, time zero is actually applicable only to instantaneous investments or investments that do not take longer than a couple of months, such as well workover, equipment replacement, pump installation, drilling a single well, sidetracking an existing well, depositing funds in bank account, and so on.

Although it may look like a very simple issue, it really is not that simple when it comes to dealing with multi-billion dollar projects. The difference in the NPV using the first year's cash flow at time zero versus treating it as if it occurred at the year end will make a significant difference depending on the discount rate used. With a \$5 billion project and a desired internal rate of return (IRR) of 10%, the difference in the two NPVs will be 10%. Two hypothetical alternatives are shown in figure 2–19. The NPV of each alternative at IRR = 10% is:

$$\begin{aligned}
 (P_v)_A &= -\$1,000 - \$2,000(1 + 0.10)^{-1} - \$2,000(1 + 0.10)^{-2} \\
 &= -\$1,000 - \$1,818.18 - \$1,652.89 \\
 &\cong -\$4,471 \\
 (P_v)_B &= -\$1,000(1 + 0.10)^{-1} - \$2,000(1 + 0.10)^{-2} - \$2,000(1 + 0.10)^{-3} \\
 &= -\$909.09 - \$1,652.89 - \$1,502.63 \\
 &\cong -\$4,065
 \end{aligned}$$

The difference in the net present value (NPV) of the two alternatives shown in figure 2–19 is \$406 million, or 10%. Later in the book it will be shown that shifting certain cash flow by one period will alter the final decision whether the investment should be accepted or rejected. Besides this major difference, does the shift in cash flow reflect the reality in the following scenarios?

1. Alternative A in figure 2–19 shows that \$3 billion is spent in year one (\$1 billion at the beginning of year one and \$2 billion at the end of year one) and then \$2 billion is spent at the end of year two. Does the year one cash-outflow represent reality?

2. Is the project implementation taking two years or three years to be on stream? Alternative A shows that the cash-inflow starts in year three.
3. If the project is financed, does the first loan payment start end of year three or end of year four?
4. When is the facility actually put in use? Does it qualify for depreciation at end of year three or at end of year four?
5. If any rental equipment is used, is the rent paid at end of year or beginning of year?
6. What is the first tax year (assume that the investment generates positive taxable income when on-stream) tax would be paid: end of year three or end of year four?

All of the above and other similar questions must be answered in order to treat the cash flows properly.¹

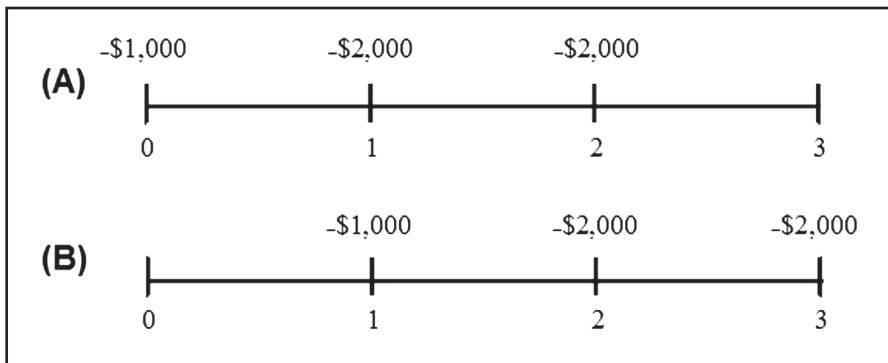


Fig 2-19 The time zero dilemma

Questions and Problems

- 2.1 What is meant by the term *time value of money*? Explain the concept of *equivalence*.
- 2.2 What is meant by the terms *discounting* and *compounding*? Why are these important in investment analysis?
- 2.3 What is the difference between *simple* and *compound interest*, *nominal* and *effective interest rates*?

- 2.4 What are the necessary conditions for an ordinary annuity?
- 2.5 What are the cash flow assumptions with respect to the following?
- a. Periodic discounting and year-end cash flows
 - b. Periodic discounting and mid-year cash flow
 - c. Continuous discounting and periodic cash flows
 - d. Continuous discounting and continuous cash flows
- 2.6 The time value of money is due to
- a. Inflation
 - b. Interest rate
 - c. Opportunity cost
 - d. Risk
- I. All of the above
 - II. None of the above
 - III. B and D
 - IV. A and C
- 2.7 Your current salary is \$60,000/year. What will be your salary after five years if you receive a 5% per year merit increase?
- a. \$85,245
 - b. \$76,577
 - c. \$66,245
 - d. \$70,650
- 2.8 If inflation over the next five years is 3% per year, then the answer in problem 2.7 is your
- a. Money income
 - b. Real income
 - c. Both (a) and (b)
 - d. None of the above

2.9 Your current salary is \$60,000/year, your salary is increased every year by 5% per year, and the inflation rate is 3% per year. Your inflation adjusted income after five years will be

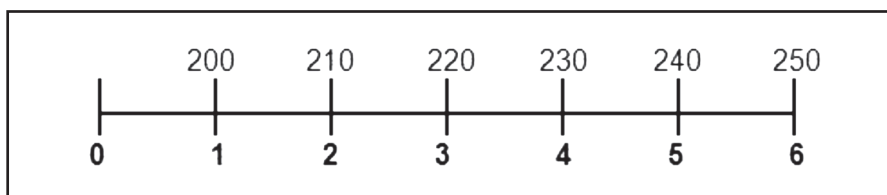
- a. \$76,577
- b. \$85,245
- c. \$66,245
- d. \$70,650

2.10 Conditions for ordinary annuity are:

- a. The periodic payments or disbursements are of equal amount.
 - b. The interest rate is constant throughout the annuity period under consideration.
 - c. The time intervals between each payment are of same length.
 - d. The interest rate is compounded at the end of each time period.
- I. Only A and C are true.
 - II. Only B and D are true.
 - III. All four conditions are true.
 - IV. None of the above conditions are true.

2.11 The following time diagram shows

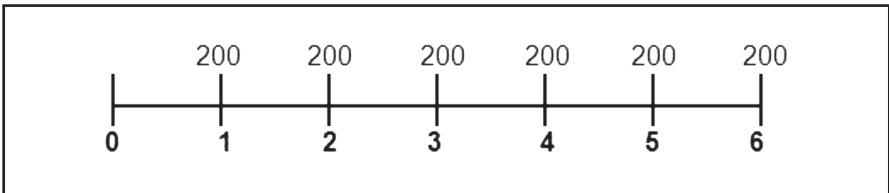
- a. Ordinary annuity
- b. Annuity due
- c. Deferred annuity
- d. None of the above



Problem 2.11 diagram

2.12 The following time diagram shows

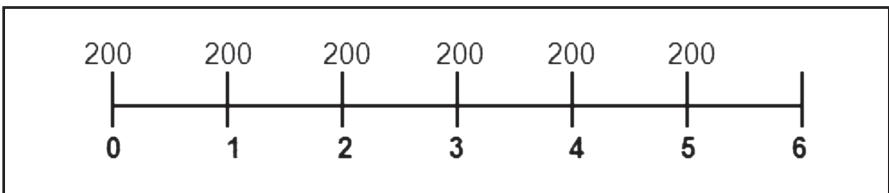
- a. Ordinary annuity
- b. Annuity due
- c. Deferred annuity
- d. None of the above



Problem 2.12 diagram

2.13 The following time diagram shows

- a. Ordinary annuity
- b. Annuity due
- c. Deferred annuity
- d. None of the above



Problem 2.13 diagram

2.14 Assuming an interest rate of 8% per year, the present value of the annuity in problem 2.12 is

- a. \$925
- b. \$999
- c. \$825
- d. \$888

- 2.15 Assuming an interest rate of 8% per year, the present value of the annuity in problem 2.13 is
- \$925
 - \$999
 - \$825
 - \$888
- 2.16 Assuming an interest rate of 8% per year, the future value of the annuity in problem 2.12 is
- \$925
 - \$999
 - \$1,467
 - \$1,585
- 2.17 Assuming an interest rate of 8% per year, the future value of the annuity in problem 2.13 is
- \$925
 - \$999
 - \$1,467
 - \$1,585
- 2.18 The following equation represents $PV = FV \left(1 + \frac{i}{12}\right)^{-t \times 12}$
- Annual compounding
 - Semi-annual discounting
 - Daily compounding
 - Monthly discounting
- 2.19 The following equation represents $FV = PV \left(1 + \frac{i}{12}\right)^{t \times 12}$
- Annual discounting
 - Semi-annual discounting
 - Daily compounding
 - Monthly compounding
- 2.20 You have been quoted a 10% per year interest rate for a car loan from a local bank. If you borrow money from this bank at this

rate, you have to pay it back in 12 equal consecutive monthly installments. The quoted interest rate is

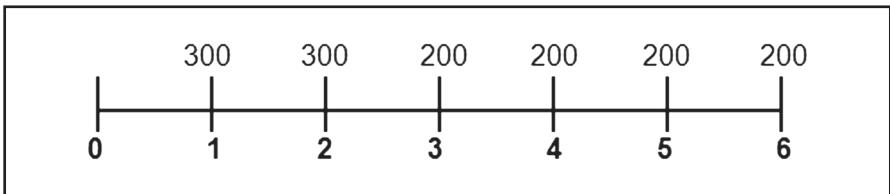
- a. Effective interest rate
- b. Simple interest rate
- c. Nominal interest rate
- d. None of the above

2.21 If the interest rate in problem 2.20 is nominal interest rate, then the effective interest rate is

- a. 10%
- b. 10.6%
- c. 10.47%
- d. None of the above

2.22 Assuming an interest rate of 8% per year, the present value of the cash flow in the following time diagram is

- a. \$1,467
- b. \$1,585
- c. \$1,060
- d. \$1,103



Problem 2.22 diagram

2.23 What is the future value of \$100 after five years at an interest rate of 10% per year compounded monthly?

- a. \$161.05
- b. \$164.53
- c. \$151.05
- d. \$174.53

- 2.24 What will be the present value of \$164.53 received five years from today, using the effective interest rate 10.47% per year?
- a. \$161.05
 - b. \$164.53
 - c. \$100.00
 - d. \$110.00
- 2.25 You have borrowed \$10,000 at an interest rate of 8% per year. The loan has to be paid back in five year-end installments. Each installment will be equal to
- a. \$2,000
 - b. \$2,505
 - c. \$2,550
 - d. \$2,600
- 2.26 In problem 2.25, the interest and principal payments in year one will be
- a. (\$664) (\$1,841)
 - b. (\$800) (\$1,705)
 - c. (\$357) (\$2,147)
 - d. (\$516) (\$1,988)
- 2.27 In problem 2.25, the interest and principal payments in year two will be
- a. (\$664) (\$1,841)
 - b. (\$800) (\$1,705)
 - c. (\$357) (\$2,147)
 - d. (\$516) (\$1,988)
- 2.28 The following diagram shows that 60% of the total CAPEX required for a project is borrowed from a bank at an interest rate of 8% per year. What will be the total loan amount at the end of year three if the loan proceeds take place at the end of each year and interest accrues from the day actual amount is withdrawn?
- a. \$1,500
 - b. \$1,598
 - c. \$1,661
 - d. \$1,726

	A	B	C	D
1	Interest on Loan	8%		
2		1	2	3
3	Total Capex Required, MM\$	500	1,000	1,000
4	Loan Proceeds, MM\$	-300	-600	-600
5	Total Equity, MM\$	200	400	400

Problem 2.28 diagram

- 2.29 In problem 2.28, what will be the total loan amount at the end of year three if the loan proceeds take place at the beginning of each year?
- \$1,500
 - \$1,598
 - \$1,661
 - \$1,726
- 2.30 In problem 2.28, what will be the total loan amount at the end of year three if the loan proceeds take place in the middle of each year?
- \$1,500
 - \$1,598
 - \$1,661
 - \$1,726
- 2.31 If the loan calculated in problem 2.30 has to be paid back in five equal end-of-year payments, what should be the amount of each payment?
- \$376
 - \$400
 - \$416
 - \$432
- 2.32 In problem 2.31, what will be the interest in year one and year two? What will be the remaining loan balance at the end of year two?
- \$120, \$100, \$968
 - \$128, \$106, \$1,031
 - \$133, \$110, \$1,072
 - \$138, \$115, \$1,114

2.33 Which of the following Excel financial functions can be used to calculate the total loan amount in problem 2.28?

- I. $=FV(B1,3,,NPV(B1,B4:D4))$
- II. $=B4*(1+B1)^2+C4*(1+B1)^1+D4$
- III. $=FV(B1,2,D4)+FV(B1,2,,B4)$
 - a. I only
 - b. I and II
 - c. None of the above
 - d. All of the above

2.34 In the following diagram, assuming the payments are at the end of each year, which of the following Excel financial functions can be used to calculate the present value?

- a. $=NPV(C1,A3:F3)$ OR $=-PV(C1,F2,F3,,1)$
- b. $=NPV(C1,A3:F3)$ OR $=-PV(C1,F2,F3)$
- c. $=NPV(C1,A3:F3)$ OR $=-PV(C1,F2,,F3)$
- d. $=NPV(C1,A3:F3)$ OR $=-PV(C1,F2,,F3,1)$

	A	B	C	D	E	F
1	Interest Rate		8%			
2	1	2	3	4	5	6
3	200	200	200	200	200	200
4						

Problem 2.34 diagram

2.35 In problem 2.34, assuming the payments are at the beginning of each year, which of the following Excel financial functions can be used to calculate the present value?

- a. $=NPV(C1,A3:F3)$ OR $=-PV(C1,F2,F3,,1)$
- b. $=NPV(C1,A3:F3)$ OR $=-PV(C1,F2,F3)$
- c. $=NPV(C1,B3:F3)+A3$ OR $=-PV(C1,F2,,F3)$
- d. $=NPV(C1,B3:F3)+A3$ OR $=-PV(C1,F2,F3,,1)$

- 2.36 In problem 2.34, assuming the payments are at the end of each year, which of the following Excel financial functions can be used to calculate the future value?
- a. $=-FV(C1,F2,,NPV(C1,A3:F3))$ OR $=-FV(C1,F2,F3,,1)$
 - b. $=-FV(C1,F2,,NPV(C1,A3:F3))$ OR $=-FV(C1,F2,F3)$
 - c. $=-FV(C1,F2,,NPV(C1,B3:F3)+A3)$ OR $=-FV(C1,F2,F3,,1)$
 - d. $=-FV(C1,F2,,NPV(C1,B3:F3)+A3)$ OR $=-FV(C1,F2,F3)$
- 2.37 In problem 2.34, assuming the payments are at the beginning of each year, which of the following Excel financial functions can be used to calculate the future value?
- a. $=-FV(C1,F2,,NPV(C1,A3:F3))$ OR $=-FV(C1,F2,F3,,1)$
 - b. $=-FV(C1,F2,,NPV(C1,A3:F3))$ OR $=-FV(C1,F2,F3)$
 - c. $=-FV(C1,F2,,NPV(C1,B3:F3)+A3)$ OR $=-FV(C1,F2,F3,,1)$
 - d. $=-FV(C1,F2,,NPV(C1,B3:F3)+A3)$ OR $=-FV(C1,F2,F3)$
- 2.38 Assuming the payments in the following diagram are at the end of each year, which of the following Excel financial functions can be used to calculate the present value?
- a. $=NPV(C1,B3:F3)+A3$ OR $=-(PV(C1,B2,B3)-PV(C1,B2,,PV(C1,F2-B2,F3)))$
 - b. $=NPV(C1,A3:F3)$ OR $=-(PV(C1,B2,B3)-PV(C1,B2,,PV(C1,F2-B2,F3)))$
 - c. $=NPV(C1,A3:F3)$ OR $=-(PV(C1,B2,,B3)-PV(C1,B2,,PV(C1,F2-B2,F3)))$
 - d. $=NPV(C1,A3:F3)$ OR $=-(PV(C1,B2,,B3)-PV(C1,B2,,PV(C1,F2-B2,,F3)))$

	A	B	C	D	E	F
1	Interest Rate		8%			
2	1	2	3	4	5	6
3	300	300	200	200	200	200
4						

Problem 2.38 diagram

- 2.39 Calculate the effective interest rate for the following nominal interest rates:
- Interest rate of 12% per year, compounded annually
 - Interest rate of 12% per year, compounded semi-annually
 - Interest rate of 12% per year, compounded quarterly
 - Interest rate of 12% per year, compounded daily
 - Interest rate of 12% per year, compounded continuously
- 2.40 Calculate the interest earned on \$1,500 over:
- A period of 10 years at an interest rate of 8% per year, compounded yearly
 - A period of 10 years at an interest rate of 8% per year, compounded quarterly
- 2.41 Calculate the future value of the following:
- An investment of \$1,000 in 10 years at an interest rate of 8% per year, compounded annually
 - An investment of \$15,000 in eight years at an interest rate of 8% per year, compounded monthly
- 2.42 Calculate the present value of \$15,000 to be received 10 years from today. Assume an interest rate of 10% per year, compounded annually.
- 2.43 The annual amount of a series of payments to be made at the end of each of the next 10 years is \$500. What is the present worth of the payments at 7% interest per year, compounded annually?
- 2.44 Calculate the future value of the following investments at an effective interest rate of 12% per year.
- \$800 invested at the end of each of the next eight years
 - \$800 invested every six months for the next four years
 - \$800 received at the end of each of the following four years and \$1,000 per year for the four years thereafter
 - \$1,000 invested monthly for the next four years
- 2.45 Calculate the present value of the cash flows in problem 2.44.
- 2.46 Calculate the equal series of annual payments that will accumulate to \$20,000 in five years at an effective interest rate of 8% per year.

- 2.47 A piece of machinery costs \$20,000 and has an estimated life of eight years and a scrap value of \$2,000. What uniform annual amount must be set aside at the end of each of the eight years for replacement if the interest rate is 6% per year, compounded annually?
- 2.48 If \$15,000 is put in a bank account today at an interest rate of 8% per year, what annual amount of money can be withdrawn every year over the next seven years?
- 2.49 What equal series of payments must be paid into a sinking fund to accumulate the following?
- a. \$5,000 in seven years at an effective interest rate of 8% per year when payments are made annually
 - b. \$20,000 in five years at an interest rate of 12% per year when payments are made quarterly
- 2.50 An oil company employs a petroleum engineer on his 45th birthday at a salary of \$60,000 per year, which is expected to increase by an average of \$1,500 at the end of each year until his retirement age of 65. The company's retirement package pays one-half the average salary over the last three years of the employment. Since vesting is required after 10 years, he may leave the company at that time and begin drawing retirement on his 65th birthday. The company invests its employees' pension fund at 8% per year, and assumes a life expectancy of 75 years. It pays half the cost of the retirement system. What fraction of this employee's pay (on an annual basis) must be withheld in order to ensure an adequate fund for his retirement at any time after 10 years?
- 2.51 A property costs \$12,000 today. If it is appreciating at a rate of 5% per year, calculate its price in 50 years.
- 2.52 The operating cost of a new machine is \$500 for the first year. Starting the second year, the operating cost increases by \$200 per year for the next 10 years. Calculate the equivalent annual operating cost of the machine. What will be the present and future value of the operating costs over the 11-year period? Assume the market interest rate of 8.0% per year.
- 2.53 Confirm your answer in question 2.52 by using the NPV function.

- 2.54 Calculate the rate of interest compounded annually if:
- An investment of \$8,000 today accumulates to \$20,750 in 10 years
 - A series of five year-end payments of \$5,000 results in an accumulated amount of \$30,525
- 2.55 A person wins \$10,000 in a state lottery. He plans to deposit this money in a savings account to earn 8% annual interest for six years. If he wants to withdraw equal annual amounts from the account for six years, starting with the first withdrawal one year from the date of deposit, what will be the amount of each withdrawal?
- 2.56 Calculate the equal quarterly series equivalent to the decreasing gradient series given below. Assume the interest rate is 8% per year.
- \$10,000 at the end of the 1st quarter
 - \$9,500 at the end of the 2nd quarter
 - \$9,000 at the end of the 3rd quarter
 - \$8,500 at the end of the 4th quarter
 - \$8,000 at the end of the 5th quarter
 - \$7,500 at the end of the 6th quarter
 - \$7,000 at the end of the 7th quarter
 - \$6,500 at the end of the 8th quarter
 - \$5,000 at the end of the 9th quarter
 - \$4,000 at the end of the 10th quarter
 - \$3,000 at the end of the 11th quarter
 - \$2,000 at the end of the 12th quarter
- 2.57 How many years will it take \$5,000 to double at an interest rate of 8% per year, compounded annually?
- 2.58 An oil producer has borrowed \$100,000 at an interest rate of 12% per year for a period of three years. Calculate (a) the quarterly payment, and (b) the monthly payment. If the loan has to be paid back using quarterly payments, show the amortization schedule for the first two years.

- 2.59 If \$100,000 is borrowed by an oil producer right now at 6% interest per annum (compounded annually) to be paid back in eight yearly installments, how much of the principal will remain to be paid after a \$30,000 payment is made four years from now?
- 2.60 An oil producer has purchased a pumping unit for \$30,000, and the loan is to be paid back in 24 equal monthly installments of \$1,771.43 per month. Calculate the nominal interest rate charged for this financing arrangement. Calculate the effective interest rate.
- 2.61 An engineer has generated an oil production forecast for a group of wells. According to this forecast, the wells produce 30,000 barrels in the first year. Starting the second year, production declines by 2,000 barrels per year for four years. Starting the sixth year, production declines by 3,000 barrels per year for another four years. Calculate the present value of the revenues if the oil price is \$75 per barrel for the first five years and \$80 per barrel thereafter. Also, calculate the equivalent annual value of these revenues. Assume an interest rate of 8% per year.
- 2.62 An oil company has installed an offshore production facility for \$10 million in year 1. The annual maintenance cost of the facility is \$60,000 per year for the 2nd year, increasing by \$10,000 per year for the next 9 years. In the 12th year, a major overhaul is conducted at a cost of \$500,000. The overhaul has helped in keeping the maintenance costs fixed at \$150,000 per year for the remaining 10 years. At the end of 22 years, the facility is sold for a sum of \$2 million. If the market interest rate is 8% per year, calculate the present value of all the costs over the 22-year period. Also, calculate the equivalent annual cost of the facility.

Reference

1. Mian, M.A., "Unnecessary & Avoidable Mistakes in Financial Calculations," *Oil & Gas Financial Journal*, January 2006, pp. 46–53.

3

Before-Tax Cash Flow Models

Capital investment analysis is the analytic process of reaching a decision between alternative investment projects. The decision process, in general, is based on the following three steps:

1. Gathering data about each investment alternative through estimation and forecasting of relevant variables
2. Combining the quantitative information gathered in step 1 into profitability measures (also referred to as decision criteria or profitability yardsticks)
3. Making the final decision based on the criteria calculated in step 2 and judgment on the non-quantified information

The variables required to arrive at the before federal income tax (BFIT or BTAX) project cash flow (step 1) are discussed in this chapter. The discussions of step 2 and step 3 are presented in chapters 6 and 7, respectively. Therefore, this chapter is intended to provide groundwork for the decision analysis methods presented in the following chapters. Before embarking on the sophisticated decision analysis methods, awareness of the process of converting the available investment project description into a series of realistic estimates of future cash flows is essential. These cash flows incorporate initial costs of project implementation and the resulting long-term net benefits.

Forecasting cash flow is the foundation of almost all economic analysis carried out for investment decision making. Some cash flow forecasts are simple (merely involving estimates of future cost and timing of a single well or lease equipment). Others may be complicated (estimating cash flows of full field development over a longer period of time, together with detailed fiscal calculations for each year).

The most important and the most difficult step in capital investment analysis is estimating its cash flows—both the investment outlays required and the annual net cash inflows after the project is implemented. Cash

flow forecasts consist of many variables, and many individual departments participate in the process. For example:

- Reservoir engineers, together with geologists and geophysicists, generate production forecasts and the wells required to achieve this forecast.
- The drilling department will suggest various well configurations (vertical, deviated, horizontal, long radius, short radius, or multilateral) that can be achieved in the field to be developed. Once the well configuration is determined, the drilling department will then design the well and generate drilling cost forecast of the wells required for the project.
- The production department identifies subsurface facilities and well completion requirements and estimates the costs.
- The engineering department determines the surface facilities (flow lines, headers, pipeline, oil/gas/water separation, storage tanks, compression facilities for gas, gas processing facilities, and pumps) required and estimates the cost of these.
- Economists, together with marketing staff's knowledge of price elasticity, advertising effects, and the state of the economy, forecast product prices.

The project evaluators' or economists' role in this process includes:

- Coordinating the efforts of other departments
- Ensuring all involved in generating the forecasts use a consistent set of economic assumptions
- Noting and resolving any anomalies in the information received from other departments
- Building and debugging the economics model, if one is not already available
- Making sure no biases are inherent in the forecasts

The last point is important, because division managers often become emotionally involved with pet projects. This leads to cash flow forecasting biases in order to make bad projects look good on paper. If estimation bias occurs, the project cash flows and the resulting project profitability could be either too optimistic or too pessimistic.

Some people argue that the decision analysis (risk analysis) tools take care of any inherent biases. This is not true. The unbiased point estimates of the key variables are insufficient, therefore data on appropriate probability distributions or other indications of the likely range of errors are also essential.

There are several basic principles of cash flow analysis vital to the correct analysis of available investment alternatives. These include basic definitions, the treatment of depreciation and depletion, capital costs, intangible and tangible drilling costs, incorporating inflation, concepts of nominal and real cash flow, the treatment of interest on loans and loan payments (financing mix), and the tax treatment of various costs. Each of these principles is discussed in this and the following chapter in order to arrive at a clear understanding. The after federal income tax (AFIT or ATAX) cash flow model is discussed in chapter 4. Chapter 5 deals with the cash flow model required for international agreements such as the production sharing agreements and the concessionary (royalty/tax) system agreements. The focus of this chapter is on

- Information required for arriving at project cash flows
- Practical tips on how to generate estimates of various variables required
- Measures used in determining an investment's profitability, etc.

In this chapter, it is temporarily assumed that the generated cash flows are known with certainty—an unrealistic presumption to be certain, but a useful one for expository purposes.

The difficulties one may encounter in generating cash flow forecasts and the importance of these forecasts are duly acknowledged. However, forecast errors can be minimized if the principles discussed and the practical tips presented in the following sections are observed.

A prerequisite for generating the inputs in a cash flow is to know the past and, based on this knowledge, to forecast the future. A wise man once said, “Those who do not study the past are destined to repeat it.” Those who study the past are more likely to know what to expect in the future than those who do not. Therefore, using statistical forecasting (studying the past) helps in making better plans for the future.

Cash Flow Model

As stated in chapter 2, in its basic form, *cash flow* of an investment is simply the cash expended over a defined period. *Net cash flow* (as shown in equation (2.8)) is simply *cash received* less *cash spent* during a period. Each

period is usually considered as one year, and the net cash flow projections are made over the economic project life or up to 28 years. The 28 years will usually include a 3-year project construction (project implementation) period followed by 25 years of production. If the economic limit is reached before the 25 years of production, then the project will be abandoned. The cash flow projection beyond the 25-year production life, even of it is economical, usually does not add much value to the decision making. This is because the present value of the net cash flow occurring after 25 years is very small. In addition, the uncertainty of the forecast increases beyond this period. In its simplest form, the cash flow projection would be as shown in table 3–1. In this table, year 1 is the first year of projected cash flow.

Table 3–1. Simple net cash flow projection

		Year 1	Year 2	Year 3	Year 4	Year 5	Year n
	Cash received	\$	\$	\$	\$	\$	\$
Less	Cash expended	\$	\$	\$	\$	\$	\$
Equals	Net cash flow	\$	\$	\$	\$	\$	\$

The cash received, also referred to as gross revenue, is the product stream times the projected price of the product. The product streams could be crude oil, natural gas, liquefied petroleum gas (LPG—propane and butane), natural gas liquids [NGL—propane, butane, and pentane (C5+)], liquefied natural gas (LNG), heating oil, petrochemicals, sulfur, etc. The crude oil is reported in stock tank barrels (Stb) or standard cubic meters; gas in million standard cubic feet (MMScf), thousand standard cubic feet (MScf), cubic meters, or million British thermal units (MMBtu); heating oil in barrels; and the other products in metric tons or barrels.

The cash expended is further divided into three main categories as capital expenditure (CAPEX); operating expenditure (OPEX); and abandonment costs, production taxes, and sunk costs. The CAPEX is further classified into geological and geophysical (G&G) costs, drilling, and facility costs. The estimate of the costs (capital as well as operating) is not only of prime importance for the required economic analysis, but it also plays an important role in the preparation of the company’s budgets. The operating cost during the project implementation phase is also considered as capital cost (CAPEX).

Underestimating these costs may lead to project cost overruns that result in disappointing profitability, not to mention the painful justification of budget increases. On the other hand, if the estimate is set too high, it may result in either prematurely killing a project or unnecessary freezing (or even

spending) of funds that could have been usefully employed elsewhere in the company's program.

The estimation of costs depends on the cost of a commodity or service that fluctuates with time and varies from location to location. Costs are affected by factors such as the following:

1. General inflation
2. Market conditions (e.g., worldwide or regional peak of oilfield construction activity) that temporarily drive up costs of equipment and services due to the law of supply and demand
3. Escalation, which is the combined effect of items 1 and 2. Sometimes this term is also used to indicate the overall cost increases of a project as compared to its original estimates.
4. Local wage levels and productivity
5. Cost of transportation from source
6. Taxes and import duties
7. Climactic and terrain conditions (e.g., the cost of an onshore project will be much less than the cost of the same project offshore)

The combined effect of items 4 to 7 on project costs is usually expressed in estimates as a *location factor*, referenced to a standard location such as the Gulf of Mexico, Middle East, or North Sea.

In addition to capital and operating costs, major items of cash flow for most projects are royalties, profit sharing (international contracts), severance and ad valorem taxes, income taxes, etc. These costs are collectively referred to as *state* or *government take*.

The economic evaluation of each project as a whole consists of multiple sub-evaluations. For example, the type of services and materials to be used, such as (a) drilling contract, (b) drilling fluid, (c) drilling bits, and (d) equipment (such as leased equipment or purchased equipment). In the following pages, each of the revenue and expense items and how they are estimated is further explained. Putting the forecasts of all these variables together and calculating profitability yardsticks from these is called a cash flow model.

Data Required for Project Evaluation

To make sound economic evaluation of a given investment project, certain basic data is required. The following checklist serves as a guide for collecting appropriate data for projects (wells, fields, upstream and downstream facilities, etc.) under evaluation.¹

1. Ownership maps, geological structure maps, isopach maps, geological cross sections, etc.
2. Lease location data
3. Complete suite of well logs
4. Core analysis data for the zones cored and analyzed
5. Reservoir fluid sample analysis data
6. Chronological history of all well operations including original drilling and completion
7. Tabulation of monthly oil, water, and gas production by lease, by wells, and by pay zones since original completion along with tabulation of any bottomhole pressure surveys, etc.
8. Summary of allowable formula and current daily allowable rates for each well
9. Gross oil and gas prices accompanied by a summary of the oil and gas sales contract agreement with the respective purchasers and the term of agreement
10. Severance and local taxes actually paid to the local authorities accompanied by randomly selected payment vouchers for confirmation
11. Tabulation of actual historical gross operating expenses per well per month for each property. Identify the routine OPEX versus the non-routine expenditure, such as equipment replacement, workover, stimulation, etc.
12. In case of undeveloped or nonproducing reserves, an estimate of completed well costs or recompletion costs for reserves behind the casing
13. Tabulation of the ownership interests including reversionary interests and the latest payout amounts or status

14. A copy of all previously prepared geological or engineering reports/studies containing data pertinent to the current evaluation of the property
15. A summary of lease and assignment provisions, lease facilities, operating agreements, unitization agreements, etc.

Forecasting Product Stream

Petroleum engineers, together with geologists and geophysicists, use several methods to forecast future oil and/or gas production resulting from the development of a field. Process engineers and chemical engineers forecast the products generated by petrochemical plants by using the oil and/or gas forecasts generated by the reservoir engineers.

The forecast of product stream for downstream projects such as refinery, methanol plant, ethylene cracker, NGL recovery plant, LNG plant, and power plants is much easier as compared to the forecast for field development. This is because the forecast for the downstream projects is usually based on the design capacity of the plant with an estimated on-stream factor. It is also assumed that the required feed to the plant is available when needed and that the design capacity is based on market demand for the products.

An oil and/or gas property goes through several development stages. Most likely, at each stage, a different reserve estimate is developed with a different degree of confidence. The first stage in the development of any oil and/or gas property is the exploratory stage when a wildcat well is drilled. The reserve estimates at this stage are based on analogy with other similar prospects and geologic settings. If the estimates are favorable, the drilling of a wildcat is approved.

Once the first well is drilled and the productive horizon identified, more physical information about the rock and fluid properties is obtained with the help of well logs, core analysis, PVT (pressure-volume-temperature) analysis, drill stem test, transient well test (pressure test) analysis, etc. Based on this information, the reserve estimates are revised appropriately.

After the first well starts producing oil and/or gas, reservoir pressure and production performance become available. This information may further warrant revision of the reserve estimates. With the passage of time, the data on the property accumulates and the estimates are revised accordingly. At different stages, more appropriate techniques may be used to obtain better estimates.

The drilling of every new well into the same productive horizon (reservoir) provides added information (such as the areal extent of the reservoir, the

type of oil and/or gas accumulation, the reservoir-producing mechanism, etc.). The reserves numbers remain an estimate until the abandonment of the well/lease. Unfortunately, the reserve estimates are no longer estimates when the property is abandoned; they are exact amounts.

Several techniques, depending on the type of data available (stage of field/reservoir development), are in common use for estimating oil and/or gas reserves. These techniques include:

- Volumetric calculations
- Historical production and reservoir pressure performance analysis (decline curve analysis)
- Analogy to similar reservoirs in the close vicinity of the area under evaluation
- Material balance calculations
- Reservoir simulation and process simulation

In this chapter, reserves estimated by volumetric calculations, analogy, and from historical production performance of the reservoir are discussed. An in-depth study of material balance calculations and reservoir simulation is beyond the scope of this book; these topics are covered in detail in many other technical books.²

In dealing with oil and/or gas reserve estimates, the following terminology is commonly used:

Original oil in place (OOIP) and/or original gas in place (OGIP), initial oil in place (IOIP) and/or initial gas in place (IGIP), gas initially in place (GIIP), and stock tank oil initially in place (STOIIP). These terms refer to the total estimated volume of hydrocarbon accumulation.

Ultimate oil and/or gas recovery. This refers to the total volume of hydrocarbons eventually produced/recovered over the economic productive life of the reservoir. Having determined the original oil and/or gas in place, some empirical correlations and experience are applied in order to predict the percentage of the total volume of hydrocarbons economically and technically producible. This producible volume is referred to as the ultimate oil and/or gas recovery.

Cumulative oil and/or gas production. When a property starts producing, the production of oil, gas, and/or water is accumulated on a daily, monthly, and yearly basis. These accumulated amounts are referred to as cumulative production of oil, gas, and/or water at a specific point in time.

Oil and/or gas reserves. The reserves produced after a certain point in time are referred to as the remaining oil and/or gas recovery or simply oil and/or gas reserves as of that time. These estimates assume existing economic conditions, established operation practices, and current government regulations.

The sum of the *cumulative* production and the reserves at a point in time is again referred to as the *ultimate* oil and/or gas recovery.

Volumetric calculations

Volumetric estimates of oil and gas reserves are generally used early in the life of a field. These estimates are considered preliminary as compared to the estimates obtained from using historical performance of wells/fields. Reservoir heterogeneities are a commonly overlooked factor, which often makes the volumetric estimates different from those obtained by evaluating performance.

The data required for estimating OOIP are (a) formation thickness (h , feet), (b) drainage area (A , acres), (c) porosity (ϕ , fraction), (d) formation oil saturation (S_o , fraction), and (e) oil formation volume factor (β_o , RB/Stb). The following equation is used to estimate OOIP (N) in stock tank barrels.

$$N = \frac{7,758\phi(1 - S_w)hA}{\beta_o} \quad (3.1)$$

The porosity and water saturation (S_w , fraction) are obtained from well logs or core analysis or both. The formation thickness is estimated from resistivity logs or from geologic maps if the well is in a developed reservoir. The drainage area is estimated based on experience, type of reservoir, producing mechanism, analogy to wells producing from similar horizons in the other areas, and from geologic maps. The oil formation volume factor is either determined in the laboratory from fluid analysis or it is estimated from empirical correlations. The constant 7,758 in equation (3.1) is used to convert acre-feet into reservoir barrels. The B_o (Stb/RB) is used to convert reservoir barrels into their equivalent stock tank (surface) barrels (Stb).

The OOIP is then multiplied by a recovery factor (E_R , fraction) to estimate the recoverable oil. The recovery factor is selected based on experience, reservoir drive mechanism, analogy, and rock and fluid properties. The primary recovery factor, depending upon the type of reservoir drive, ranges from 12% to 30%.

The volumetric calculation of gas reserves requires about the same information as what is used for oil volumetric reserve calculations. The following equation calculates OGIP (G).

$$G = 43,560 \phi(1 - S_w)hA\beta_g \quad (3.2)$$

The β_g (Scf/Cf) in equation (3.2) is called the gas formation volume factor; it is used to convert reservoir gas volume into its equivalent volume at the standard surface conditions. It should be noted that the standard convention for reporting B_o is in RB/Stb while for B_g it is Scf/CF. Thus, using these standard conventions, the terms have units that are the inverse of each other. Therefore, B_o appears in the numerator of the volumetric equation [Equation (3.1)] and B_g in the denominator of the volumetric equation [Equation (3.2)]. The constant 43,560 converts acre feet into cubic feet. The recovery factor for gas wells may be based on experience or it may be calculated from an estimated abandonment pressure. Since gas flows more readily than oil, usually a higher percentage (70–90%) of the OGIP is recoverable. At the same time, a larger area could be drained by each well. For this reason, the spacing of oil wells is usually denser than the spacing of gas wells. The β_g can also be calculated in terms of reservoir cubic feet per surface (standard) cubic feet (CF/Scf), in which case equation (3.2) is divided by the β_g instead of multiplying it by β_g .

Example 3–1

An oil well has been drilled and completed. The productive zone has been encountered at a depth of 7,815–7,830 feet. The log analysis showed an average porosity of 15% and an average water saturation of 35%. The oil formation volume factor is determined in the laboratory to be 1.215 RB/Stb. Experience shows other reservoirs of about the same properties drain 80 acres with a recovery factor of 12%. Compute the OOIP and the ultimate oil recovery.

Solution: Using equation (3.1), the OOIP is

$$\begin{aligned} N &= \frac{7,758 \phi(1 - S_w)hA}{\beta_o} = \frac{7,758 \times 0.15(1 - 0.35) \times 15 \times 80}{1.215} \\ &= 747,000 \text{ Stb} \end{aligned}$$

The ultimate recovery is given by

$$\begin{aligned} N_{ul} &= N \times E_R \\ &= 747,000 \times 0.12 = 89,600 \text{ Stb} \end{aligned}$$

Decline curve analysis

The purpose of decline curve analysis is to determine future production and therefore ultimate recovery for wells/fields with some production history. Since decline curve analysis depends on a curve-fit of past performance, the accuracy is expected to be higher for a well/field with several months or years of production history than for a well with only a limited history. Decline curve analyses are based on the following assumptions.

1. Sufficient past production performance is available in order to make a reasonable match of this performance and extrapolating its future performance.
2. The past production history is based on capacity (unrestricted) production with no changes in operational policy such as artificial lift, stimulation, etc. It is assumed the property will continue to be operated in the same manner in the future.

In its simple form, decline curve analysis can be performed by finding a curve that approximates the past production history and extrapolating this curve into the future. A more rigorous procedure is to fit the past performance with a mathematical curve (represented by a mathematical equation). Once the characteristics (parameters) of this curve are known, they can be used to predict future performance.

Three rate-time decline curves are discussed in this section: (1) exponential decline, (2) hyperbolic decline, and (3) harmonic decline (special case of hyperbolic decline). In addition to these rate-time curves, several other decline curves used in the industry, such as rate-cumulative production plot, cumulative oil–cumulative gas curve, water cut versus cumulative production curve, and gas-oil ratio (GOR) versus cumulative oil production curve are also discussed. The rate-time curves are commonly plotted on semi-logarithmic graph paper as $\log q$ vs. t .

To work with rate-time curves, monthly or daily production is first plotted on semi-logarithmic graph paper (monthly is most common). After production is plotted, a declining pattern is observed and the future trend is forecasted. This future trend is extrapolated to the *economic limit* (EL) of the property. Production is abandoned before it would cease due to natural causes (depletion) because it declines to a rate where it costs more to produce the hydrocarbons than those hydrocarbons are worth.³

Therefore, the EL is the production rate below which the net cash flow is negative. The EL is computed as follows.

$$EL_{oil} = \frac{WI \times LOE}{NRI \left[P_o(1 - T_o) + P_g(1 - T_g) \left(\frac{GOR}{1,000} \right) \right] (1 - T)} \quad (3.3)$$

$$EL_{gas} = \frac{WI \times LOE}{NRI [P_g(1-T_g) + P_o(Y)(1-T_o)](1-T)} \quad (3.4)$$

where

EL_{oil} = economic limit for oil well, Stb/month

EL_{gas} = economic limit for gas well, MScf/month

P_o, P_g = oil and gas prices, \$/Stb and \$/MScf

LOE = lease operating expenses, \$/well/month

WI = working interest, fraction

NRI = net revenue interest, fraction

GOR = gas-oil ratio, Scf/Stb

Y = condensate yield, Stb/MScf

T_o, T_g = oil and gas severance/production taxes, fraction

T = Ad valorem tax, fraction

Example 3–2

Calculate the economic limit for the following data:

$$\begin{array}{llll} P_o = \$60/\text{Stb} & P_g = \$5.5/\text{MScf} & T_g = 5\% & T_o = 7.5\% \\ LOE = \$5,000/\text{month} & T = 12\% & NRI = 87.5\% & GOR = 500 \text{ Scf/Stb} \end{array}$$

Solution: Calculate the economic limit using equation (3.3).

$$\begin{aligned} EL_{oil} &= \frac{1 \times 5,000}{0.875 \left[60(1 - 0.075) + 5.5(1 - 0.05) \left(\frac{500}{1,000} \right) \right] (1 - 0.12)} \\ &= \frac{\$5,000}{0.875(55.50 + 2.613)(1 - 0.12)} \\ &= \frac{\$5,000}{44.75} = 111.73 \text{ Stb/month} \end{aligned}$$

The calculated economic limit of 111.73 barrels/month means that if the production drops below this limit, it will not generate enough revenue to offset the operating costs of \$5,000 per month. This can be checked by calculating the revenue generated by the 111.73 barrels of production as shown below. The revenue should be about the same as the operating cost, i.e., breakeven point.

$$\begin{aligned}
\text{Oil Revenue} &= EL_{Oil} \times P_o (1 - T_o) \times NRI \times (1 - T) \\
&= 111.73 \times 60(1 - 0.075) \times 0.875 \times (1 - 0.12) \\
&= \$4,774.78 \\
\text{Gas Production, } Q_g &= 111.73 \times 500 = 55,865 \text{ Scf or } 55.865 \text{ MScf} \\
\text{Gas Revenue} &= Q_g \times P_g (1 - T_g) \times NRI \times (1 - T) \\
&= 55.865 \times 5.5 \times (1 - 0.05) \times 0.875 \times (1 - 0.12) \\
&= \$224.76 \\
\text{Total Revenue} &= \$4,774.78 + 224.76 = \$4,999.54 \cong \$5,000
\end{aligned}$$

Since the calculated revenue is equal to the operating cost of \$5,000, it shows that the economic limit is calculated correctly.

The above calculations of the economic limit are based on the assumption that all the variables (oil price, gas price, OPEX, taxes, and ownership) remain constant over the economic life of the asset. The above equations cannot be used if any of the variables will change with time. Alternatively, a logic statement can be entered in the net cash flow column of the economics model, which will check the net cash flow before it adds it to the model. If the net cash flow is negative in any particular year, the model will stop the cash flow at this time. This may be a little tricky if there are larger investments during the life of the asset that will make the net cash flow temporarily negative. Nevertheless, the cash flows must always be checked to make sure that none are negative. These cash flows will usually be toward the end of the asset life. The long-run marginal cost (LRMC), discussed in chapter 6, can also be used in the same way.

Exponential decline. Exponential decline is also referred to as *constant percentage decline*. The mathematical expression that defines this type of decline is an exponential equation of the form $y = ae^{bx}$. The exponentially declining production plots as a straight line on semi-logarithmic graph paper (production on log scale and time on linear scale). The following equations are used to interpret the exponential decline curve.

Oil produced during interval $t = 0$ and t_o

$$\Delta N_p = \frac{(q_i - q_o) \times 12}{a} \quad (3.5)$$

Time required to produce ΔN_p

$$t = \frac{\ln\left(\frac{q_i}{q_o}\right)}{a} \quad (3.6)$$

Rate at time t_o

$$\begin{aligned}
 q_{ot} &= q_i e^{-at} \\
 d &= \frac{(q_i - q_o)}{q_i} = 1 - \frac{q_o}{q_i} = 1 - e^{-at} \\
 a &= -\ln(1 - d) = \frac{\ln(q_i) - \ln(q_o)}{t}
 \end{aligned} \tag{3.7}$$

where

- q_i = rate at the beginning of time period, Stb/month
- q_o = rate at end of time period or EL, Stb/month
- t = time period between q_i and q_o , years
- ΔN_p = cumulative production during time period, Stb
- a = nominal decline rate, fraction
- d = effective decline rate, fraction

Changing effective monthly decline rate to effective yearly decline rate, and vice versa:

$$d_y = 1 - (1 - d_m)^{12} \tag{3.8}$$

OR

$$d_m = 1 - (1 - d_y)^{1/12}$$

Example 3–3

In figure 3–1, there is a rate-time plot of actual oil production for years 1997 through 2001 and the forecasted oil production thereafter. The cumulative oil production (N_p) on 12/31/2001 is 78,044 Stb. Calculate:

- (a) Effective and nominal decline rate— $q_{1/1/2002}$ and $q_{1/1/2003}$ are read from the graph as 843 Stb/month and 717 Stb/month, respectively
- (b) Remaining oil reserves from 1/1/2002 to EL of 200 barrels per month
- (c) Time required to produce the oil calculated in (b)
- (d) Production rate at the end of year 2004
- (e) Ultimate oil recovery (N_{ul})

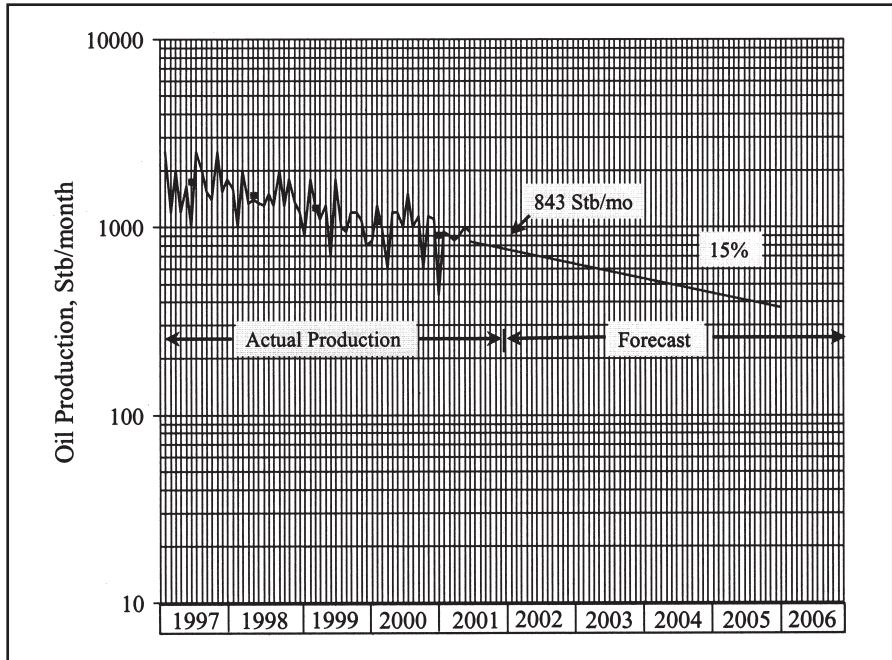


Fig. 3-1. Rate-time plot showing exponential decline for example 3-3

Solution:

(a) Effective decline rate/year: $d = \frac{843 - 717}{843} \times 100 = 14.94\% \approx 15\%$

$$a = -\ln(1 - 0.15) = 0.1625$$

(b) Remaining oil reserves $N_r = \frac{(q_i - q_{EL})}{a}$

$$= \frac{(843 - 200) \times 12}{0.1625} = 47,483 \text{ Stb}$$

(c) Time required $t = \frac{\ln\left(\frac{843}{200}\right)}{0.1625} = 8.85 \text{ years}$

(d) Production rate $q_{ot} = q_i e^{-at_o}$

$$= 843 e^{(-0.1625 \times 3)} = 518 \text{ Stb/month}$$

(e) Ultimate recovery, $N_{ul} = N_p + N_r$

$$= 78,044 + 47,483 = 125,527 \text{ Stb}$$

It is normal for the raw production data plot to exhibit wild fluctuations. To make the data easier to extrapolate into the future, it can be smoothed by calculating averages for periods of time and plotting the averages at the middle of the time period used for averaging (yearly averages shown in figure 3–1). For steeper decline rates, averages over a shorter period, say three or six months, should be used because the average rate for the period does not represent a true logarithmic average rate. This is particularly true for steeply declining production.

Hyperbolic decline. The hyperbolic decline curve is a concave upward curve when plotted on semi-logarithmic graph paper. As a consequence, the decline characteristic, a , is not a constant value but rather the slope of the tangent to the rate-time curve at any point. The slope of this tangent to the curve may be evaluated graphically in the same manner as the exponential decline. The decline characteristic a , however, changes with producing time. The shape of the curve is defined by the hyperbolic exponent, b (also denoted by n in some books). The hyperbolic exponent is constant with time. A typical hyperbolically declining production curve is shown in figure 3–2.

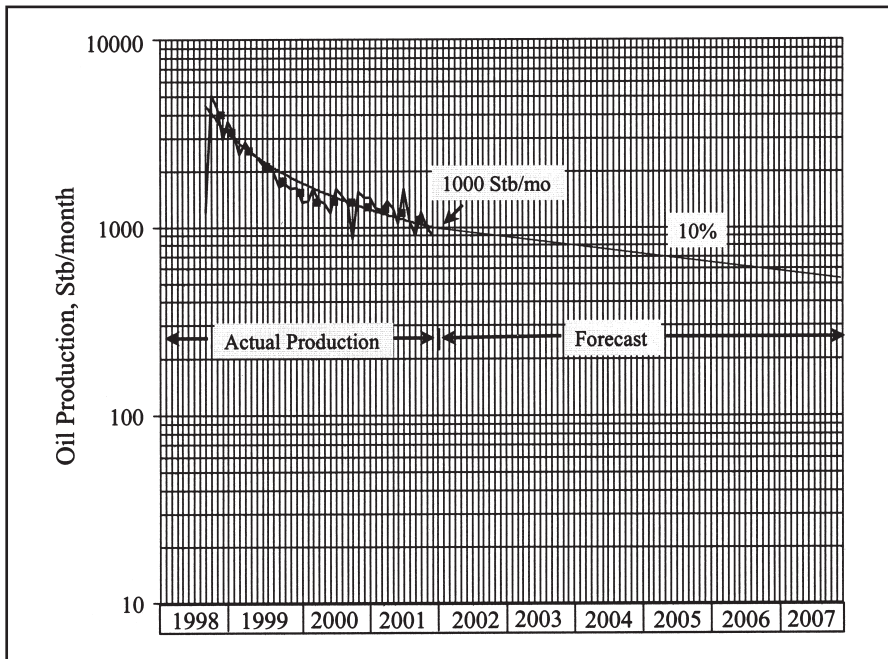


Fig. 3–2. A typical hyperbolically declining production curve

In normal petroleum operations, the value of b ranges between 0 and 1.0, with $b = 0$ being an exponential decline and $b = 1.0$ being a harmonic decline (special case of hyperbolic decline). However, it is found that in fractured tight (low-permeability) formations, exponents in excess of 1.0 may be calculated. Care should be exercised in these cases, as a high value for b will result in an unrealistically low decline rate late in the well life. The following equations are used to interpret hyperbolic decline curves.

Several methods are available to evaluate the hyperbolic exponent b . Three of the most commonly used methods are (1) French curve method, (2) shifting the curve on log-log graph paper, and (3) type-curve fitting method. Only the first two methods are presented here. Other methods are described in technical journals such as the *Journal of Petroleum Technology* and the *Oil & Gas Journal*.⁴⁻⁸

French curve method: A French curve is a template made in the shape of a hyperbolic curve that is used to draw smooth curve shapes. This is a preferred method because it is easy to use, gives comparable results, and is not time consuming. The following steps should be followed:

1. Plot the monthly production data on a semi-logarithmic graph paper.
2. With the help of a French curve, draw a best-fit curve through the plotted data.
3. Select a point q_i on this curve and draw a line tangent to the curve at this point.
4. Compute the effective decline rate, d , per year and compute $a_i = -\ln(1 - d)$.
5. Select another point q_t at some point t from the initial rate q_i , and try different values of b in equation (3.11) until a value of b is found that gives a q_t value close to the one selected. This trial-and-error calculation can be easily done with the help of a programmable calculator.

$$\Delta N_p = \frac{(q_i)^b}{(1-b)a_i[q_i^{(1-b)} - q_o^{(1-b)}]} \quad (3.9)$$

$$t = \frac{\left(\frac{q_i}{q_o}\right)^b - 1}{ba_i} \quad (3.10)$$

$$q_{ot} = q_i(1 + ba_it_o)^{-1/b} \quad (3.11)$$

$$a_t = a_i \left(\frac{q_t}{q_i} \right)^b = \frac{1}{b} \left[(1 - d_t)^{-b} - 1 \right]$$

$$d = \frac{q_i - q_o}{q_i}$$

$$d_t = 1 - (1 + ba_t)^{-1/b}$$

These values of b , a_i , and q_i are then used to forecast production performance. If the q_i is different from the q_i at the effective date of evaluation, a new value of a_i must be calculated by using the following equation.

$$(a_i)_{new} = \frac{(a_i)_{old}}{[1 + n(a_i)_{old} \Delta t]} \quad (3.12)$$

Curve shift on log-log paper. This method involves a log-log trial-and-error plot of the rate versus time data until a straight line is obtained. The procedure involves plotting the log of the rate versus the log of the time plus some constant, $(t + A)$. The constant A is varied until a straight line is obtained. Over-shifting the curve to the right shows upward curvature. The hyperbolic exponent b is then given by the reciprocal of the slope ($b = -1/\text{slope}$) of the straight line. The method is illustrated in figure 3-3 for the data in table 3-2.

Trying to find the constant A in the term $(t + A)$ by hand is tedious and time consuming. Therefore, a Newton-Raphson iteration technique is utilized to simplify the computation of the constant A . The following relationship is used.

The iteration is carried out by guessing the initial value of $A_i = 0$ and calculating $f(A)$ and $f'(A)$. If $f(A_i)$ is significantly different from zero, A_{i+1} is calculated using equation (3.15) and the process is repeated until the function converges to zero (or close to zero).

Table 3-2. Hyperbolically declining oil production data

No.	Month	Q_o , Stb	No.	Month	Q_o , Stb	No.	Month	Q_o , Stb
1	1	28,200	4	19	6,635	7	37	2,850
2	7	15,680	5	25	4,775	8	43	2,300
3	13	9,700	6	31	3,628	9	49	1,905

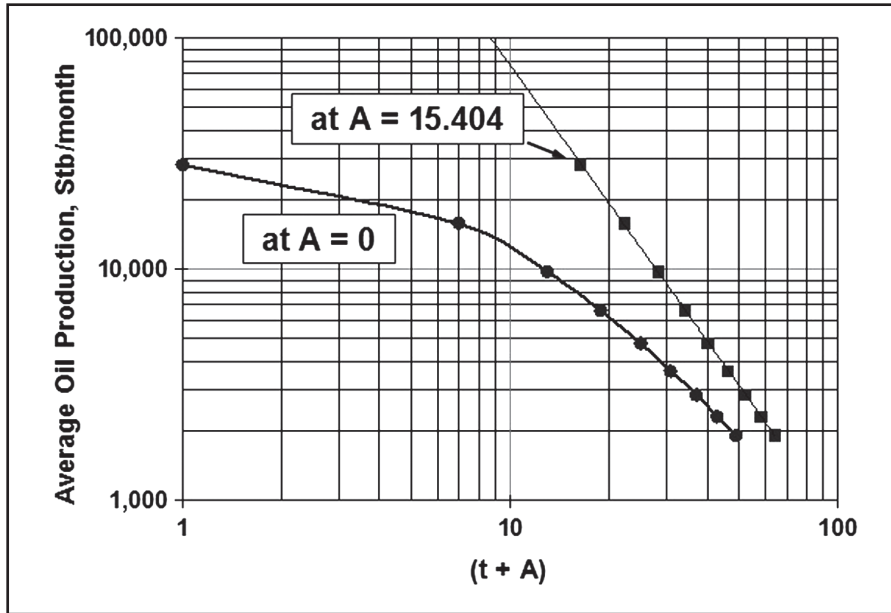


Fig. 3-3. Log-log extrapolation of hyperbolically declining production of table 3-2

$$f(A) = \frac{\log Q_2 - \log Q_1}{\log(t_2 + A) - \log(t_1 + A)} - \frac{\log Q_3 - \log Q_1}{\log(t_3 + A) - \log(t_1 + A)} \quad (3.13)$$

$$f'(A) = \frac{(\log Q_1 - \log Q_2) \left[\frac{1}{t_2 + A} - \frac{1}{t_1 + A} \right]}{[\log(t_2 + A) - \log(t_1 + A)]^2} - \frac{(\log Q_1 - \log Q_3) \left[\frac{1}{t_3 + A} - \frac{1}{t_1 + A} \right]}{[\log(t_3 + A) - \log(t_1 + A)]^2} \quad (3.14)$$

$$A_{i+1} = A_i - \frac{f'(A)}{f(A)} \quad (3.15)$$

The above equations require only three data points on the curve. The two end points and the median or central point from the given set of data points give sufficient data to compute the constant A , provided the data are correct and smooth. If the data are not smooth, a best-fit curve is first drawn through the data points. The two end points are placed on the curve (x_1 and x_3 , and the corresponding y_1 and y_3) and the median point is chosen at $x_2 = (x_1 + x_3)/2$ with the corresponding y_2 read from the curve.

An Excel spreadsheet can be used to perform the above iterative process in order to compute the variable A . The following systematic instructions help in developing the model in Excel as shown in table 3–3. The same information is included in table 3–3 of spreadsheet Chapter3.xlsx on the CD-ROM that accompanies volume 2 in this series.⁹

1. Select three data points from table 3–2 at $t = 1, 25$, and 49 months and the corresponding production rates as $Q_3 = 28,200$, $Q_2 = 4,775$, and $Q_1 = 1,905$ Stb/month.
2. Enter the t in cells B3 to B5 and the corresponding oil production rate (Q_o) in cells C3 to C5.
3. Enter $A=0$ (initial guess) in cell A8.
4. Enter the following formula in cell B8:

$$=(\text{LOG}(C4)-\text{LOG}(C5))/(\text{LOG}(B4+A8)-\text{LOG}(B5+A8))-(\text{LOG}(C3)-\text{LOG}(C5))/(\text{LOG}(B3+A8)-\text{LOG}(B5+A8))$$
5. To calculate the hyperbolic exponent b , enter the following formula in cell C10:

$$=-1/((\text{LOG}(C4)-\text{LOG}(C5))/(\text{LOG}(B4+A8)-\text{LOG}(B5+A8)))$$
6. In cells E3 through E11, input the $t(s)$ from table 3–2 and input the corresponding production rate in cells F3 through F11. Enter the formula $=D3+A\$8$ in cell F3 and then copy it all the way to cell F11. The data thus calculated has shifted the curve (plot of E3:F11) by the variable A , as shown in figure 3–3.
7. Now click on **Data** $\Rightarrow$ **What_If Analysis** $\Rightarrow$ **Goal Seek** in the Excel toolbar menu. In the Goal Seek menu select the following:
 - a. Set target cell: B8
 - b. By changing cells: A8
 - c. Equal to: click on **Value of: 0**
8. Now click **OK**, and the Goal Seek will perform the iterations. The final value of A will appear in cell A8 by setting cell B8 = 0.

Table 3–3. Spreadsheet to calculate hyperbolic exponent b and constant $(t+A)$ for curve shifting

	A	B	C	D	E	F
2	Point	t, month	Q_o	t	Q_o	(t + A)
3	3	1	28,200	1	28,200	16.419
4	2	25	4,775	7	15,680	22.419
5	1	49	1,905	13	9,700	28.419
6				19	6,635	34.419
7	Estimated A	f(A)	b	25	4,775	40.419
8	15.419	0.00	0.507	31	3,628	46.419
9				37	2,850	52.419
10				43	2,300	58.419
11				49	1,905	64.419

Harmonic decline. Harmonic decline is a special case of hyperbolic decline with hyperbolic exponent $b = 1.0$. The rate-time relationship, in this case, can also be straightened out on log-log graph paper after shifting and assumes a slope of 45° (unit slope line). In this specific case, a plot of the inverse of the production rate versus time on a linear scale should also yield a straight line. The following equations are used in the same way as with hyperbolic decline.

$$\Delta N_p = \frac{q_i}{a_i} \ln \frac{q_i}{q_o} \quad (3.16)$$

$$t = \frac{\left(\frac{q_i}{q_o}\right) - 1}{a} \quad (3.17)$$

$$q_{ot} = \frac{q_i}{1 + a_i t} \quad (3.18)$$

Low-permeability (tight) gas reservoirs. The Federal Energy Regulatory Commission (FERC) classifies low-permeability gas reservoirs (tight gas sands) as those reservoirs with average in-situ permeability to gas of 0.1 millidarcy (md) or less. These tight reservoirs must be hydraulically fractured to achieve economical production rates. Massive hydraulic fracturing (MHF) treatments have made it possible to economically produce these reservoirs. The MHF treatments improve production by exposing larger surface areas of these low-permeability formations to flow into the wellbore through the creation of vertical fractures.

Gas production from the tight sands in the early life of the well experiences a sharp initial decline (linear flow characteristics) followed by a long transition (pseudosteady state) into exponential decline. If the well's production history indicates it has passed the linear flow regime, then the evaluation is relatively simple (i.e., the conventional exponential decline forecast can be applied).

Most of the performance prediction methods discussed in the previous sections do not give good results when applied to tight sand gas production. The use of hyperbolic decline curves, discussed earlier, is not recommended because these curves do not account for the transformation of fracture production into matrix production. Further, most wells show a hyperbolic exponent between 0 and 1, while a majority of the tight sands production shows a value of b greater than 1.0. In case of high-capacity wells showing longer productive lives, the use of higher exponents (greater than 1.0) makes the forecasted lives even longer and less realistic. The hyperbolic decline curves, in general, have a tendency to flatten with time, making the forecasts optimistic later in the life of the well.

There are several techniques for evaluating reserves and predicting future performance of tight sand gas wells. Some techniques yield excellent results but are time consuming or require data that may not be available. The method presented here is easy to use, requires only the routinely recorded production data, and yields good results. This author has used this method, and in a majority of the cases it has provided excellent results. However, exceptions are always there, and engineers are advised to test the method in each particular case before using it. The best way to check the accuracy is to determine whether the method gives a reasonable match with the actual production used.

Production from these low-permeability fractured reservoirs when plotted on semi-logarithmic graph paper exhibits hyperbolic decline characteristics with a hyperbolic exponent greater than 1.0. The same production data when plotted on log-log paper (rate vs. time) generally shows a decline slope of -0.50 . Formation damage reduces the early flush production rates but has diminishing effect. So the slope will be less than -0.50 initially and then will approach -0.50 after three to six months. Sometimes, the early time slope is greater than -0.50 , which indicates the depletion of thin lenses of high permeability near the wellbore. This typical behavior is observed in many low-permeability reservoirs.¹⁰⁻¹²

The following equations can be used to forecast the past production performance of these reservoirs:

$$\log q_g = -\frac{1}{2} \log t + \log C \quad (3.19)$$

which gives

$$q_g = \frac{C}{\sqrt{t}} \quad (3.19a)$$

and

$$\frac{1}{q_g} = m\sqrt{t} + C_1 \quad (3.20)$$

Equation (3.19) is used with the production plotted on log-log paper (rate q_g vs. elapsed time t) and a half-unit slope line is drawn through the data points to give intercept C at $t = 1$ month. This intercept is then used in equation (3.19a) to forecast future performance. This method can be used on wells with at least 12 months of production or when sufficient data fall on a half-unit slope.

The second equation, equation (3.20), can be used with as little as four months of initial production. The reciprocal production rate ($1/q_g$) is plotted versus the square root of time ($\sqrt{t}$) on coordinate graph paper. The method of linear regression is usually used to determine the values of C_1 and m . Feeding these values into equation (3.20) generates production rates at any given time, t .

In most low-permeability gas reservoirs, the linear flow characteristics can last for up to three or four years. When using these methods, the production should be scheduled for up to this period and then switched to an average exponential decline (draw a line tangent to this forecast) to EL. This incorporates the boundary effects into the long-term production. Failing to do this will result in unrealistically long productive life and very high gas recovery. The exponential declines in tight sands are usually in the range of 6% to 10% per year.

An example from a tight sand formation is presented to clarify the use of the technique and the limitations of using it. Remember, if the forecast generated by the given method does not match (say within 5%) the actual production, do not use it. Always plot the forecast back on the rate-time plot showing the actual production and make sure the forecast is reasonable based on the actual production and experience with the formation.

Example 3–4

Use the technique presented in this section to forecast the future production of a well producing from the Pictured Cliffs formation in San Juan County, New Mexico. The monthly production for the last 20 months is reported in table 3–4.

Table 3–4. Production data for example 3–4

Month	Production MScf	Month	Production MScf	Month	Production MScf
1	10,106	8	4,667	15	3,693
2	9,240	9	4,396	16	2,641
3	6,738	10	3,512	17	3,528
4	6,138	11	4,230	18	3,250
5	5,608	12	3,918	19	3,209
6	3,957	13	3,834	20	2,038
7	3,943	14	3,712		

Solution:

1. Plot the production data of table 3–4 on log-log graph paper as shown in figure 3–4.
2. Draw a half-unit slope line through the data points.
3. Read the value of C at $t = 1$ month as $C = 14,000$.
4. Forecast the future production rate at 12, 24, 36, and 48 months as follows:

$$Q_{12} = \frac{14,000}{12^{0.50}} = 4,041 \text{ MScf}$$

$$Q_{24} = \frac{14,000}{24^{0.5}} = 2,858 \text{ MScf}$$

$$Q_{36} = \frac{14,000}{36^{0.5}} = 2,333 \text{ MScf}$$

$$Q_{48} = \frac{14,000}{48^{0.50}} = 2,021 \text{ MScf}$$

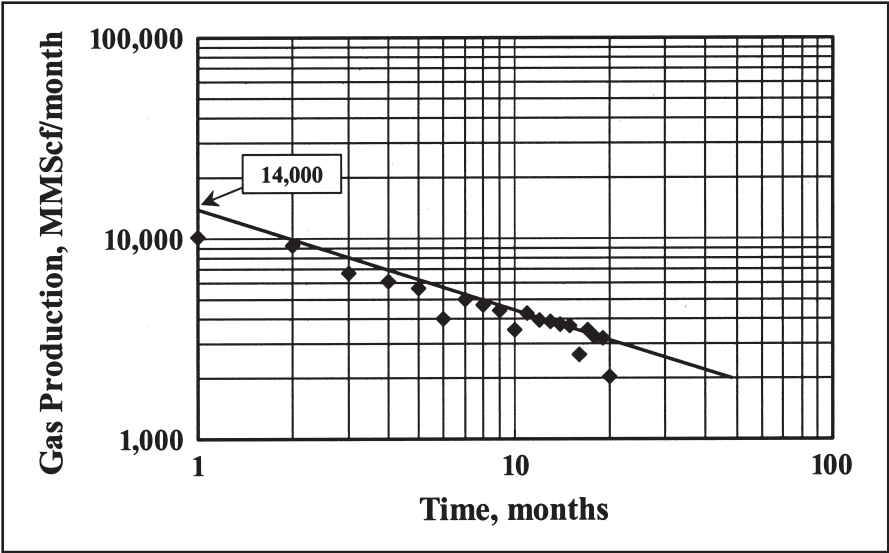


Fig. 3–4. Half-unit slope log-log plot for example 3–4

Example 3–5

Rework the data in the previous example in order to compare the forecast with the actual production. The actual yearly production from the same well is given in table 3–5. Plot the actual production and forecast on a rate-time plot. Compute the cumulative production for the first 4 years (48 months) and compare it with the actual 48 months of production.

Table 3–5. Production data for example 3–5

Year	Production MScf	Cumulative MScf	Year	Production MScf	Cumulative MScf
1	67,453	67,453	7	20,947	233,422
2	38,831	106,284	8	21,215	254,637
3	30,744	137,028	9	17,751	272,388
4	28,396	165,424	10	15,633	288,021
5	24,512	189,936	11	15,014	303,035
6	22,539	212,475			

Solution: The rate-time curve for the actual production and the forecast is shown in figure 3–5.

$$\begin{aligned}\text{Cumulative, 4 years} &= 2 \times 14,000(12 \times 4)^{0.50} \\ &= 193,990 \text{ MScf (Actual} = 165,424 \text{ MScf)}\end{aligned}$$

This method usually gives higher cumulative because the first few months of production do not fall on the half-unit slope line while the forecast is accounting for it as it is on the half-unit slope line. However, this method gives a good forecast as shown in figure 3–5.

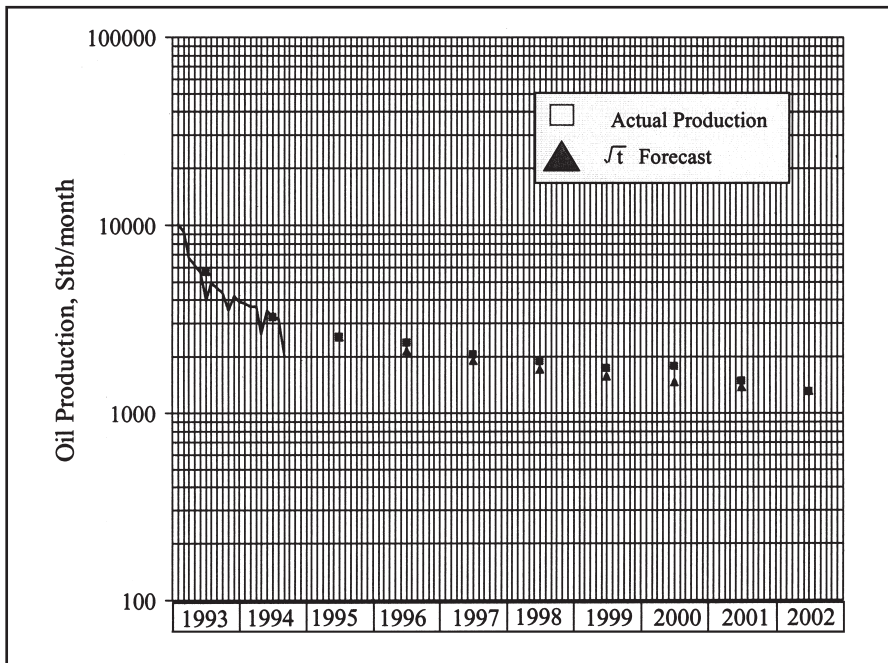


Fig. 3–5. Rate-time plot for the data in table 3–5

Rate-cumulative production plot. The rate-cumulative plots can be used for estimating the ultimate recovery of oil from a well or a producing lease. These types of plots have an advantage over *rate-time* plots because any interruptions due to mechanical or other failures or downtime do not affect the plotting coordinates. The evaluation of workovers and remedial treatments tends to be directly evident on these plots, and any change in recovery can be read easily by extrapolating the plotted data. These plots have the following characteristics:

1. The rate-cumulative data forms a straight line on *Cartesian coordinate* graph paper if the rate-time plot of the same data shows a straight line on semi-logarithmic graph paper, i.e., exponential decline with $b = 0$. Theoretically, the slope of the rate-cumulative curve should be the same as the nominal decline rate calculated from the rate-time plot. However, because of the difference in “eyeballing” the curves and data smoothing, some differences may exist. A typical rate-cumulative plot corresponding to the rate-time plot in figure 3–1 is shown in figure 3–6. Note the slope of the rate-cumulative curve is 15%, which is the same as the 15% decline of the rate-time curve. Also, the ultimate recovery from the rate-cumulative plot at $EL = 200$ Stb/month is 125,000 barrels, versus 125,527 barrels from the rate-time curve.

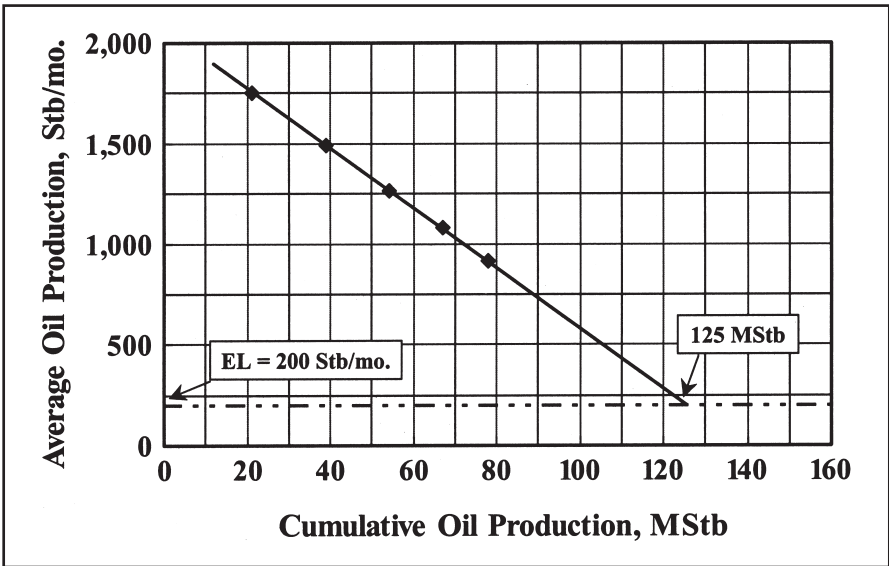


Fig. 3–6. Rate-cumulative plot for the data in figure 3–1

2. The rate-cumulative data forms a straight line when plotted on *semi-logarithmic* graph paper (with rate plotted on log scale) if the rate-time plot of the same data shows a harmonic decline with $b = 1.0$.

3. If the rate-time plot of the data exhibit hyperbolic decline with $0 < b < 1.0$, then the rate-cumulative data forms a straight line after being shifted (in the X direction) on log-log graph paper, i.e., to plot $\log q$ versus $\log(A - N_p)$. The value of A can be found by trial-and-error, using different values until one is found that gives a straight line plot on log-log graph paper with a $slope = [1/(1 - b)]$. Also for a hyperbolically declining production, the rate-cumulative plot is a straight line on a special graph paper with the rate scale marked off so it is linear in $q - b$ and the cumulative scale is linear.

Cumulative gas (G_p)–cumulative oil (N_p) plot. The projection of associated gas in oil production or condensate in gas production is not as straightforward as the projection of the primary product. There are several methods of projecting associated gas production from solution gas drive reservoirs. The material balance techniques (Tarner, Tracy, etc.) can be used to predict gas-oil ratio versus cumulative oil production. However, these methods require data about the produced fluid's properties and relative permeability that are often not available.

Another frequently used method of projecting associated gas is to assume constant GOR throughout the productive economic life of the property. This assumption is correct as long as the reservoir is producing at above the bubble-point pressure. However, the GOR increases dramatically as the reservoir pressure drops below the bubble point. In this case, a pessimistic gas rate is predicted. On the other hand, near abandonment the GOR trends downward, so a constant GOR assumption at this stage may be too optimistic.

A log-log plot of cumulative gas production (G_p) versus the corresponding cumulative oil production (N_p) is most often utilized to project the associated gas production. On this plot, G_p and N_p plot as a straight line as shown in figure 3–7, with unit slope until the reservoir pressure is above the bubble-point pressure. Below the bubble-point pressure, the GOR increases and the slope on the log-log plot increases as shown in figure 3–7.

Another plot of G_p versus N_p , which according to this author's experience is a better forecasting tool, is to plot $\log G_p$ versus N_p (on semi-logarithmic graph paper) as shown in figure 3–8. On this plot, the data plots as a straight line after shifting it by a constant A , ($G_p + A$) in the direction of the y-axis. Knowing the cumulative oil production at a particular point in time, its associated cumulative gas production can be projected using this plot.

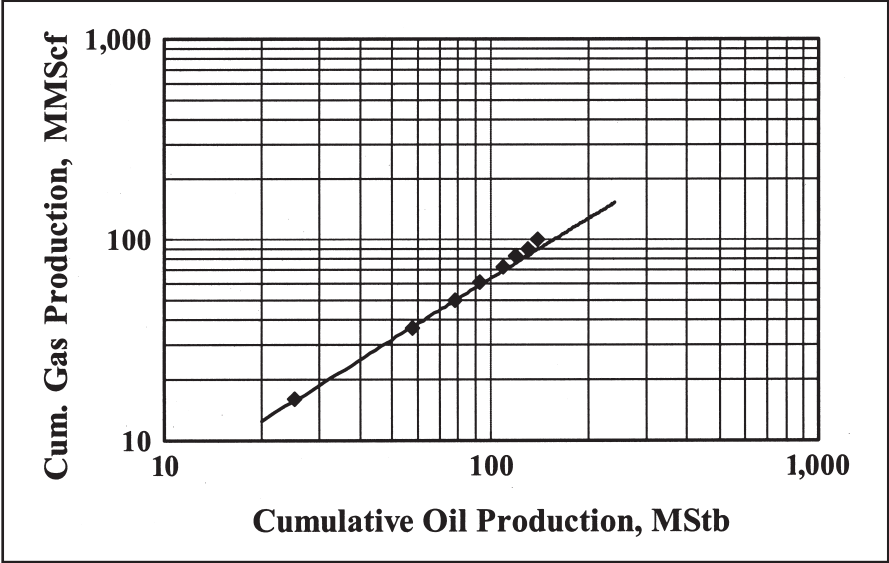


Fig. 3–7. Cumulative gas (G_p) versus cumulative oil (N_p) plot

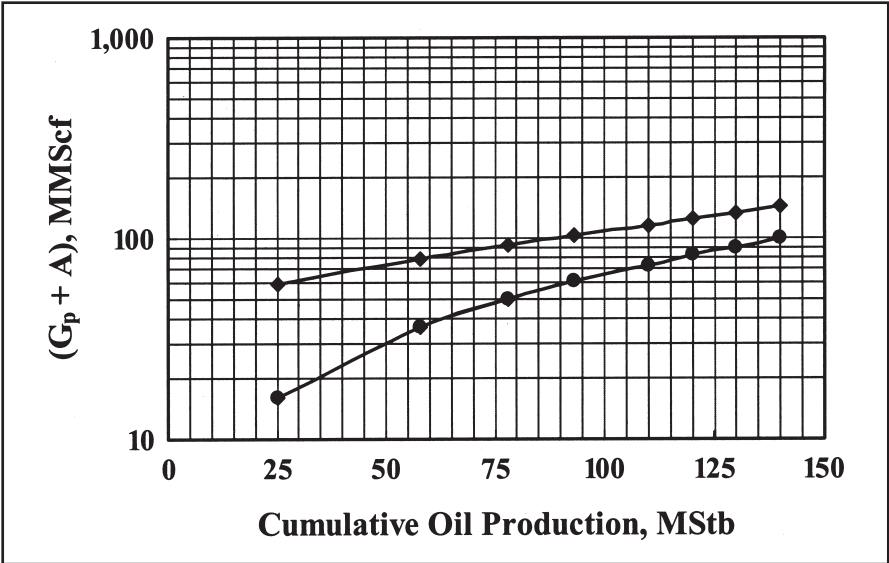


Fig. 3–8. Semi-logarithmic plot of cumulative gas production versus cumulative oil production

Water cut versus cumulative oil plot. This plot is most often used in water-drive reservoirs and reservoirs producing under waterflood. However, its use is not limited only to these two types. The method can be applied to any reservoir showing increasing water production. In the case of waterdrive or waterflood, the oil production is consistent and maintains a shallow decline while the water production increases relatively faster.

Since the disposal of produced water costs money, the variable monthly operating expenses increase, hence raising the EL, which in turn limits the oil production. The wells showing increasing water production, if forecasted with the regular rate-time curves or rate-cumulative curves without accounting for water production and the associated cost, may not provide realistic results.

To determine the ultimate oil recovery, the water cut that is the percentage of water produced per total fluid produced $[q_w/(q_o + q_w)]$, is plotted versus N_p on semi-logarithmic graph paper. The plot usually provides a straight line or shows some relationship between the two variables. After the trend is defined, the cumulative oil production limit is obtained from the trend at about 98% water cut.

But arbitrarily choosing the 98% water cut may not always provide a true EL of cumulative oil production. At this percentage, one does not know the EL. Therefore, it is suggested to use a companion plot of total fluid versus water cut, plotted on semi-logarithmic graph paper. This plot gives an idea of the specific ratio of water and oil at certain water cut. Now it can be determined whether the percentage of oil in the total fluid is above or below the EL. These curves may also be used to forecast the water production and hence the associated variable cost.

A water cut–cumulative oil production plot and a water cut–total fluid production plot for a well producing from Mission Canyon formation in North Dakota are shown in figures 3–9 and 3–10, respectively. The cumulative oil production at 98% water cut is read from figure 3–9 as 47,250 barrels. At 98% water cut, the total monthly fluid production is read from figure 3–10 as 3,610.6 barrels/month. This gives oil production of approximately 72 barrels $[3,610.6 \times (1 - 0.98) = 72.21]$, which meets the requirement of the estimated EL.

If the EL was more than 72 barrels/month, then one would have to determine the cumulative at a percentage lower than 98%, and so on. In fact, one would first determine the water cut percentage giving the forecasted EL, and then determine the ultimate oil recovery at this percentage.

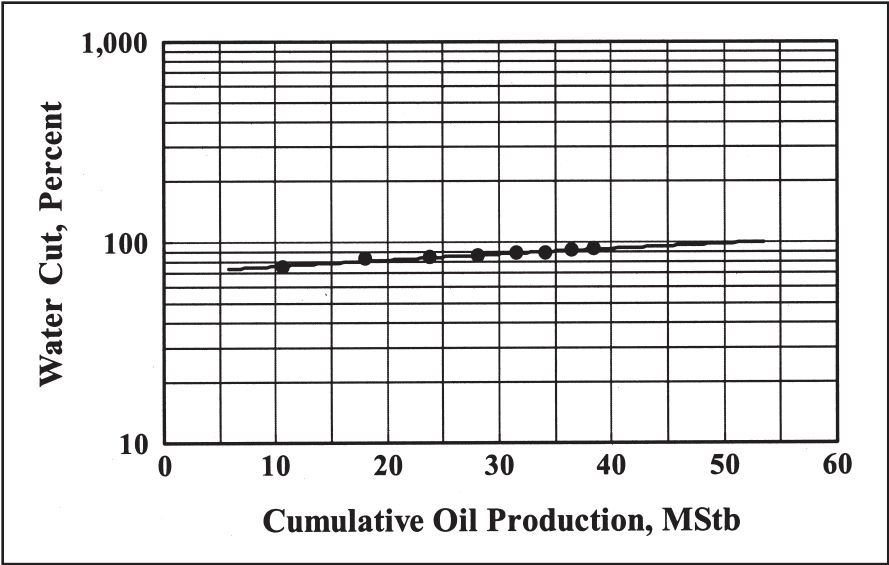


Fig. 3–9. Water cut versus cumulative oil production plot

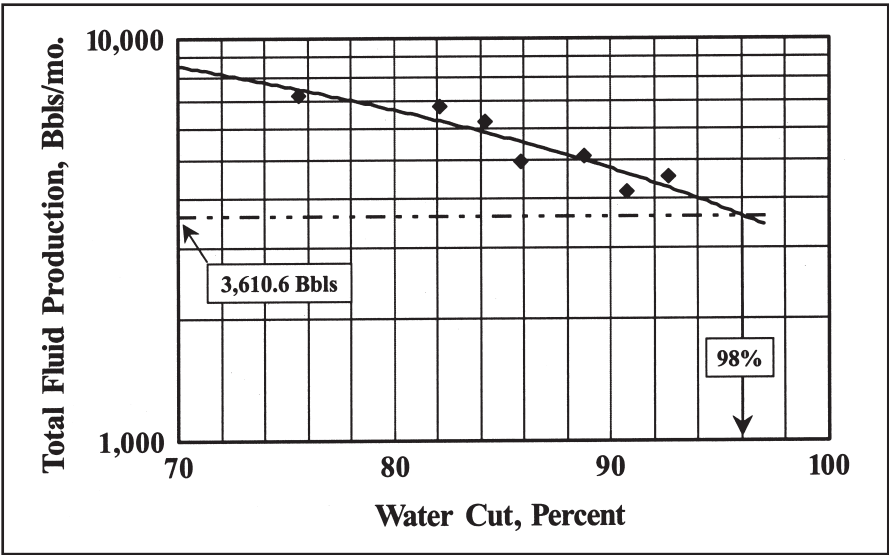


Fig. 3–10. Water cut versus total fluid production plot for the data in figure 3–9

Gas reservoir material balance

The gas material balance calculations are relatively simple as compared to oil material balance and therefore they are used routinely. The relative simplicity is because unlike saturated crude oil and condensate, natural gas does not undergo phase changes upon reduction in reservoir pressure (i.e., below bubble point, at bubble point, and above bubble point pressure).

The gas material balance is of interest especially in situations when gas production is severely curtailed due to market demand. The production performance (rate-time performance) in this case cannot be relied upon for calculating gas reserves. Using material balance, gas in place and reserves may be estimated from pressure performance history. The gas material balance calculations are based on the following assumptions:

1. No large pressure gradients exist across the reservoir at any given time (i.e., pressure equilibrium exists throughout the reservoir).
2. Reliable production, injection, and periodic reservoir pressure measurements are available.
3. Laboratory PVT data apply to the reservoir gas at the average pressure used.
4. The change in volume of interstitial water and the change in porosity with pressure are negligible.

In practice, the most common way to perform gas material balance calculations is to plot (p/z) versus G_p on Cartesian coordinate graph paper (i.e., p/z on the y-axis and G_p on the x-axis). In this case, p is the periodic measurement of reservoir pressure, z is the corresponding gas deviation factor, and G_p is the corresponding cumulative gas production. Theoretically, the plot should be a straight line. A best-fit line drawn through the points and extended to the $p/z = 0$ gives gas in place. The ultimate recovery can also be read from the curve at (p_a/z_a) i.e., the reservoir pressure at abandonment and its corresponding z -factor.

$$\frac{p}{z} = a + b(G_p) \quad (3.21)$$

where

$$a = \frac{p_i V_i T_{sc}}{z_i p_{sc} T_R}$$

and

$$b = \frac{V_i T_{sc}}{p_{sc} T_R}$$

In equation (3.21), p/z is the dependent variable, G_p is the cumulative at p_i/z_i , and a and b are the coefficients of the equation, corresponding to the intercept and slope, respectively. The coefficients a and b may be found by using the method of *least squares (regression)*. The following equations are utilized.

$$b = \frac{n \sum_{i=1}^n (p/z)_i (G_p)_i - \sum_{i=1}^n (p/z)_i \sum_{i=1}^n (G_p)_i}{n \sum_{i=1}^n (G_p)_i^2 - \left(\sum_{i=1}^n G_p \right)^2} \quad (3.22)$$

$$a = \overline{p/z} - b \overline{G_p} \quad (3.23)$$

$$G_p = \frac{a}{-b} \quad (3.24)$$

$$\text{Recoverable gas at } p_x/z_x = \frac{a - p_x/z_x}{-b} \quad (3.24a)$$

where

$$\begin{aligned} n &= \text{the number of } p/z \text{ vs. } G_p \text{ pairs} \\ \overline{p/z} &= \text{average } p/z \text{ value, } (\sum p/z)/n \\ \overline{G_p} &= \text{average } G_p \text{ value, } (\sum G_p)/n \end{aligned}$$

In addition, a plot of $G_p B_g$ versus $B_g - B_{gi}$ on a Cartesian coordinate graph paper should result in a straight line going through the origin, with G being the slope of this straight line.

Example 3–6

Following is the gas well test information on a Morrow formation gas well in Clark County, Kansas. Compute (a) the OGIP, (b) the recoverable gas (G_{ul}) at an abandonment $p_a = 200$ psi, and (c) the remaining gas reserves at $p/z = 800$. Other pertinent reservoir gas PVT data are given below.

$$\begin{aligned} T_{pc} &= 370.88^\circ \text{ R} & p_{pc} &= 671.24 \text{ psi} \\ \gamma_g &= 0.64 \text{ (air} = 1.0) & T_R &= 123^\circ \text{ F} \end{aligned}$$

Date	p_{ts} , psi	G_p , MMScf
01/01/2006	1,422	166.406
01/01/2007	1,208	293.967
01/01/2008	980	406.317
01/01/2009	847	490.430

Solution: The following steps are required:

1. Convert wellhead shut-in pressure to bottomhole shut-in pressure.
2. Determine gas deviation factor z at each bottomhole shut-in pressure.
3. Compute p/z value for each pressure.
4. Plot p/z versus G_p .

The calculations are tabulated in table 3–6 and plotted in figure 3–11. The p_i/z_i from figure 3–11 is approximately equal to 2,460. The OGIP G is also calculated individually at each pressure and presented in table 3–6. The calculated G_s are in good agreement.

Table 3–6. p/z data for example 3–6

Date	p_{ts} (psi)	p_{ws} (psi)	z	p/z	G_p (MMScf)	$\frac{G_p \times p_i/z_i}{p_i/z_i - p/z}$
01/01/2006	1,422	1,661	0.8287	2,004	166.406	897.717
01/01/2007	1,208	1,406	0.8456	1,663	293.967	907.351
01/01/2008	980	1,135	0.8687	1,307	406.317	866.904
01/01/2009	847	978	0.8841	1,106	490.430	891.032

- (a) The OGIP at $p/z = 0$ in figure 3–11, $G \approx 881$ MMScf.
- (b) To find recoverable gas at $p_a = 200$ psi, first find z at 200 psi.

$$p_{pr} = \frac{p_a}{p_{pc}} = \frac{200}{671.24} = 0.298 \text{ psi}$$

$$T_{pr} = \frac{T_R + 460}{T_{pc}} = \frac{123 + 460}{370.88} = 1.572^\circ R$$

(c) Therefore, to calculate remaining gas reserves,

$$z = 0.975 \text{ and } p_a/z_a = 205.13$$

$$G_p \text{ @ } p_a = 200 \text{ psi} \approx 807 \text{ MMScf (from figure 3-11)}$$

$$G_p \text{ @ } p/z = 800 \approx 600 \text{ MMScf (from figure 3-11)}$$

$$\begin{aligned} G_{ul} \text{ @ } p/z = 800 &= G_{p205} - G_{p800} \\ &= 807 - 600 = 207 \text{ MMScf} \end{aligned}$$

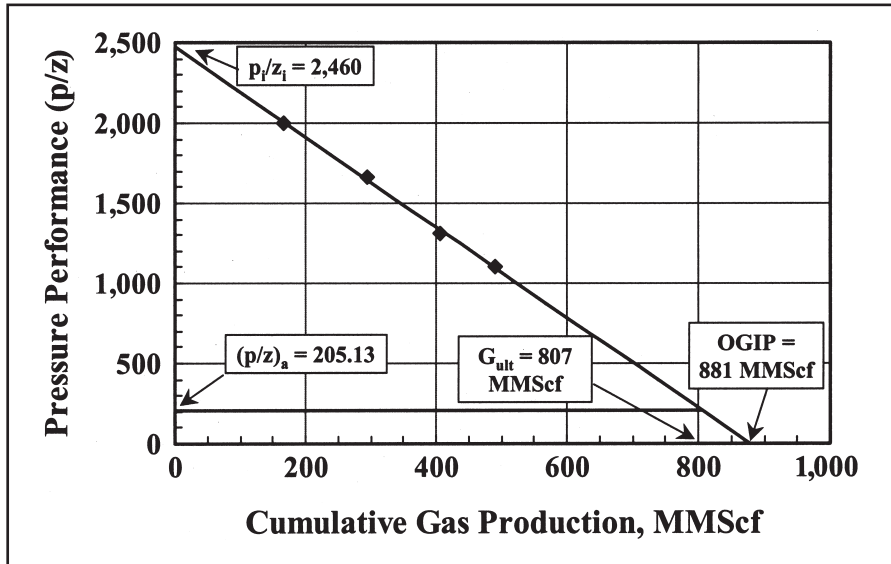


Fig. 3-11. Pressure-production graph for gas reservoir using equation (3.24)

Example 3-7

Rework the data in example 3-6 to calculate OGIP and recoverable gas at $p/z = 800$, using regression analysis.

Solution: The data for regression analysis are tabulated in table 3-7.

Using equation (3.22)

$$b = \frac{4(1,895,816.644) - (6,080)(1,357.120)}{4(519,722.644) - (1,357.120)^2} = -2.817$$

$$\overline{p/z} = \frac{6,080}{4} = 1,520$$

$$\overline{G_p} = \frac{1,357,120}{4} = 339.280$$

Using equation (3.23)

$$\begin{aligned} a &= 1,520 - (-2.817)(339.280) \\ &= 2,475.752 \end{aligned}$$

Using equation (3.24)

$$G = \frac{2,475.752}{-(-2.817)} \cong 879 \text{ MMScf}$$

$$G_p @ p/z = 800 = \frac{2,475.752 - 800}{-(-2.817)} \cong 595 \text{ MMScf}$$

Table 3–7. Regression analysis of p/z data

p/z	G_p (MMScf)	$(p/z)(G_p)$	$(G_p)^2$
2,004	166.406	333,477.624	27,690.957
1,663	293.967	488,867.121	86,416.597
1,307	406.317	531,056.319	165,093.505
1,106	490.430	542,415.580	240,521.585
6,080	1,357.120	1,895,816.644	519,722.644

Production forecast by analogy

This is a useful way of determining reserves and the associated production forecast, specifically for the fields/wells with insufficient information. The method is widely used on new wells drilled in a developed field and/or exploratory wells (wildcats). The analogy used for wells drilled in a developed field is expected to constitute the reservoir characteristics in the wells producing in the adjacent sections. For wildcats, the analogy can be used from other fields producing from the same type of expected hydrocarbon accumulations. The closer the analogous properties are to the properties to be evaluated, the better and more reliable the estimates will be.

Analogous information can be used to determine average recovery factor, ultimate oil and/or gas reserves, and the most likely production behavior. The data used are the production, completion reports, well logs, fluid properties, structure maps, isopach maps, and isovolume maps. The more information is available, the better the analogy will be.

The recovery factor is determined by determining reserves using the performance prediction methods, and these reserves are then compared with volumetric calculations. The ultimate oil and/or gas recovery is estimated from the average performance trends of the analogous reservoirs. In new wells, a major problem is finding out how they will perform. This is important because it assists in forecasting the pattern of revenue generation. The forecast is based on the average performance of the wells producing from the same type of reservoirs.

Note that analogy must be from the same type of reservoirs with approximately the same geological age, reservoir drive mechanism, and petrophysical properties. For example, one should not apply the properties of J-Sand gas wells in Colorado to the Austin Chalk wells in Texas, and vice versa. The following example clarifies the use of production forecasting by analogy.

Example 3–8

In figure 3–12, there is an acreage plat with four producing wells in sections 11, 13, and 23. All four wells are producing from the Pictured Cliffs formation in San Juan County, New Mexico. A company proposes to drill another infill well in section 14 as shown on the plat. What reserves and production behavior would be expected if the well is successfully drilled into the same formation? The yearly production data of the four producing wells is given in table 3–8.

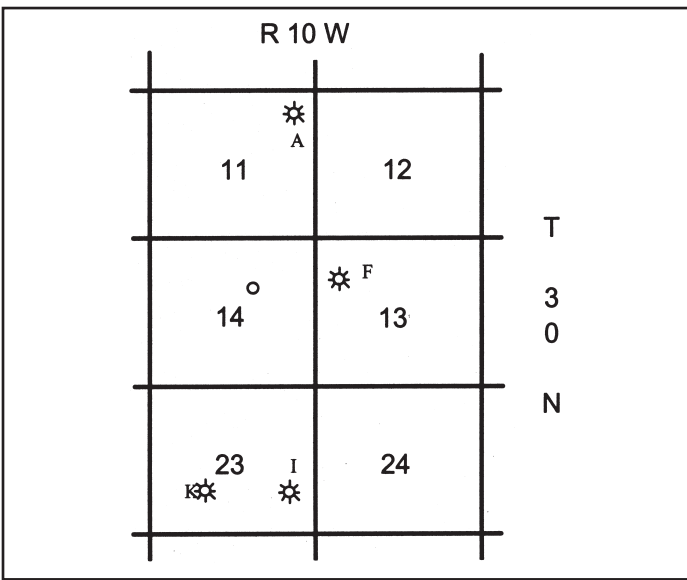


Fig. 3–12. Acreage plat for example 3–8

Table 3–8. Production (MScf) data for example 3–8

Year	Sec. 11A	Sec. 13F	Sec. 23K	Sec. 23I	Average
1	48,612	54,678	73,236	111,992	72,130
2	30,365	27,487	44,601	65,050	41,876
3	26,420	23,987	36,847	55,711	35,741
4	21,889	20,783	34,973	51,513	32,290
5	21,639	19,908	30,798	46,378	29,681
6	23,158	16,795	28,550	41,901	27,601
7	9,451	15,238	26,435	38,030	22,289

The average production per well per month is then plotted on the rate-time plot as shown in figure 3–13. The cumulative for the seven years of production is an average of 261,608 MScf/well. The initial rate is 1,900 MScf/month and the expected EL is 800 MScf/month. The production is forecast to decline exponentially at 10% per year.

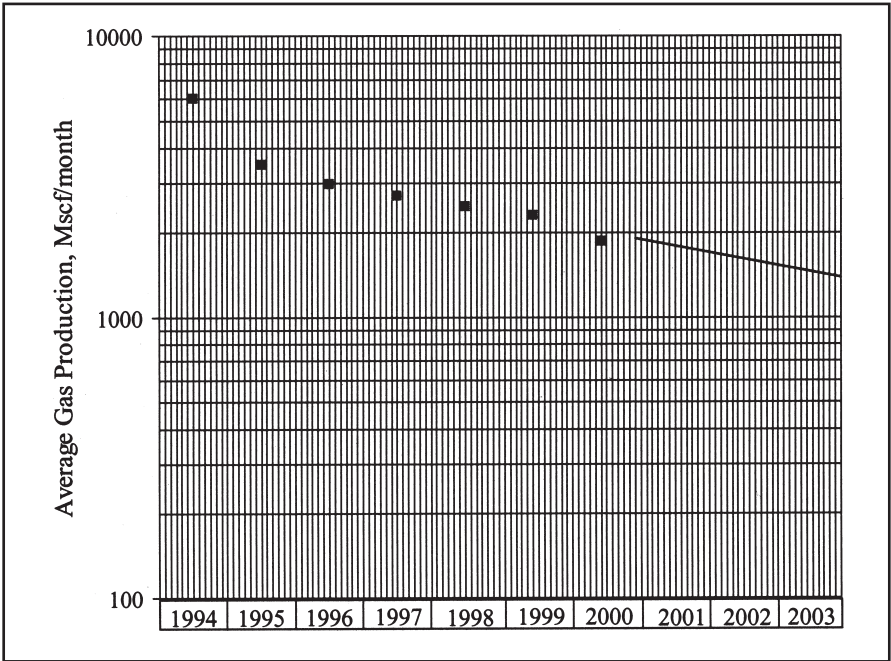


Fig. 3–13. Rate-time plot for the data in table 3–8

$$\begin{aligned}
 \text{Remaining Reserves} &= \frac{(1,900 - 800) \times 12}{-\ln(1 - 0.10)} \\
 &= \frac{13,200}{0.10536} = 125,284 \text{ MScf}
 \end{aligned}$$

$$\begin{aligned}
 \text{Ultimate Recovery} &= 261,608 + 125,284 \\
 &= 386,892 \text{ MScf}
 \end{aligned}$$

The undeveloped location, therefore, will be assigned an ultimate gas recovery of 386,892 MScf and the forecast as presented in figure 3–13, with an expected initial rate of 10,000 MScf/month.

In reality, some companies may assign some risk factor to the undeveloped reserves. The above analysis could have been further refined if isopach and isovolume maps were available. Since the isovolume map shows the $h\phi(1 - S_w)$ of each well, it can be used to determine the approximate value of volumetric reserves for the undeveloped location. In practice, the engineers/geologists prorate the reserves according to this value and any other known parameters from the analogous wells. For example, if an analogous well has $h\phi(1 - S_w) = 50$ and its reserves are estimated at 500 MMScf, then the well to be drilled at $h\phi(1 - S_w) = 25$ contour will be assigned 250 MMScf. Doing this is valid only if such correlation has been demonstrated by other producing wells. Averaging always works well; however, there may be other factors that will provide a better analogy. Remember, $h\phi(1 - S_w)$ is not the only parameter controlling the recoverable reserves, other factors such as kh and kh/μ , etc. should also be considered.

If there is a wide variation in the reserves of the analogous properties, then the simple averaging of reserves or production forecast will not be valid (i.e., it will give a higher mean). In this case, the median (not the mean) of the reserves or production forecast should be determined. Alternatively, the concepts of risk analysis presented in volume 2 of this series can be used to arrive at expected reserves and production forecast.

Product Pricing

Prices of crude oil in international trade are universally quoted in U.S. dollars per American Petroleum Institute (API) barrel of 42 U.S. gallons at 60° F and atmospheric pressure, also known as stock tank barrel (Stb). Crude oils differ in quality. Consequently, the products extracted and manufactured from crude oil also differ significantly from one crude type to another.

Crude oils from different sources are categorized primarily according to their API gravity (a measure of oil gravity in comparison to the gravity of water, expressed in degrees), and then the sulfur content (percent by weight) in the crude. The API gravity of crude oils varies from 5° to 55° API. Average crude oils have a 25° to 35° API range. Light oils are 35° to 45° API and heavy oils are below 25° API. The price of crude oil is dependent on these two factors at any particular market conditions. The lighter crudes (higher API gravity) receive a higher price as compared to the heavier crude (lower API gravity). This is because the lighter crudes tend to have more gasoline by volume than the heavier crudes, while the heavier crudes have proportionately more gas-oil (diesel) and residue cracking stock. Typical refinery yields from a simple distillation refinery, based on the crude type (API gravity), are shown in figure 3–14.

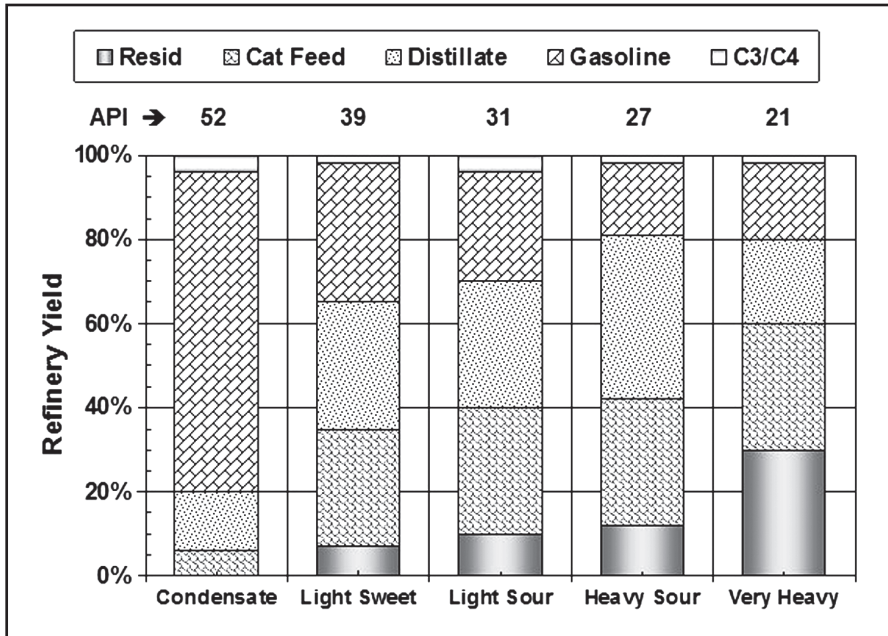


Fig. 3–14. Refinery yields, based on simple distillation refinery, from various crudes

As shown in figure 3–14, there are five main product groups that come out of the simple distillate refinery. These are:

1. Gasoline
2. Propane and butane

3. Cat feed—partially processed product, requires further refining
4. Distillate—includes diesel oil and furnace oil
5. Residual fuel—the heaviest and lowest-valued part of product output, used to make heavy fuel oil and asphalt

In addition to the refining yield of the crude, its sulfur content is an important factor in reducing its value. Sulfur content tells the refiner the amount of basic impurity in the crude oil. Even though the API gravity figure might be attractively high, this might reflect high sulfur content. The sulfur content for most crude oils falls between 1% and 2.5%, where 1% sulfur content is considered *sweet* crude and 2.5% sulfur content is considered *sour* crude. Besides discounting the price due to sulfur, some refiners simply will not purchase sour crude (high sulfur content) at any price because they may not be equipped to process such crude.

For example, a benchmark price is applied to crudes with gravity between 40° and 45° API, having sulfur content of 0.5% or less. A typical quality deduction by a refiner is then \$0.15 per degree API for crudes between 30° and 40° API gravity. For crudes below 30° API, the deduction is \$3.00 plus \$0.10 per degree API less than 30° API. For crudes with gravity greater than 45° API, the price is increased by \$0.075 per degree API. The sulfur deduction is \$0.05 per 0.1% sulfur over 0.5% sulfur, with no sulfur penalty for crudes of less than 30° API.

The amount of entrained water is an additional factor for discounting the price. The salt-water contamination is generally expressed in pounds of salt per barrel. All of these factors impact the actual price the refiner is willing to pay for each barrel of crude oil, regardless of where in the world it is produced.

There is no single benchmark pricing source for crude oil. The high volume of trading in crude oils has resulted in the following major price references/benchmarks. All the trading in the world is done in reference to these price benchmarks (i.e., adjusted for quality from one of these several marker crudes).

1. *West Texas intermediate* (WTI) is based on 38° to 40° API and 0.3% sulfur. West Texas sour, a secondary benchmark, has 33° API and 1.6% sulfur. All the crudes produced in the United States are traded in reference to these benchmarks.
2. *Saudi Arabian* (Arab light) is based on 33.4° API and 1.8% sulfur.
3. *Brent crude from the North Sea* is based on 38.3° API, while Ekofisk crude is based on 42.8° API.

4. *Urals-Mediterranean* is used for the Russian production entering the western markets.
5. *Singapore* quotations are increasingly employed as a reference for crude oil pricing in the Far East.
6. *OPEC reference basket* (ORB), introduced on 16 June 2005, is currently made up of the following crudes: *Saharan Blend* (Algeria), *Girassol* (Angola), *Oriente* (Ecuador), *Iran Heavy* (Islamic Republic of Iran), *Basra Light* (Iraq), *Kuwait Export* (Kuwait), *Es Sider* (Libya), *Bonny Light* (Nigeria), *Qatar Marine* (Qatar), *Arab Light* (Saudi Arabia), *Murban* (UAE), and *Merey* (Venezuela).

WTI in the United States, OPEC reference basket price from OPEC, and the highly important Brent from the United Kingdom have become the principal marker, or reference, crudes. Their prices are considered the most indicative of the world's market conditions. Since WTI is traded on the New York Mercantile Exchange (NYMEX), it has gained higher visibility, resulting in its becoming the industry's principal reference crude.

Historical FOB crude oil spot selling prices of various crudes are shown in figure 3–15 for reference. The OPEC countries spot price FOB is weighted by estimated export volume, and the United States spot price FOB is weighted by estimated import volume. Data is taken from the U.S. Energy Information Administration's (EIA) website. The figure shows that the 2000–2009 average Brent blend spot price FOB is about \$2.43/barrel higher than the OPEC countries' spot price FOB. The 2000–2009 average Brent blend spot price FOB is about \$4.05/barrel higher than the U.S. spot price FOB. Similarly, the 2000–2009 average OPEC countries spot price FOB is about \$1.62/barrel higher than the U.S. spot price FOB.

Oil prices are influenced by many other independent factors. These include items such as conservation incentives/practices, the overall state of the global economy (recession versus growth), exploration efforts and the success of exploration activities, production in non-OPEC countries, political stability of OPEC members, alternate fuels, and so on.

Figure 3–16 shows how the crude oil price varies with its API gravity. The 2005–2009 average FOB spot price for Saudi Arabian light (34° API), Saudi Arabian medium (31° API), and Saudi Arabian heavy (27° API) are shown. The average difference between the Arab light and Arab medium is about \$2.22/barrel, the average difference between the Arab medium and Arab heavy is about \$2.01/barrel, and the average difference between the Arab light and Arab heavy is about \$4.23/barral. The differences vary daily depending on the demand for certain crude, especially the demand for heavy crude.

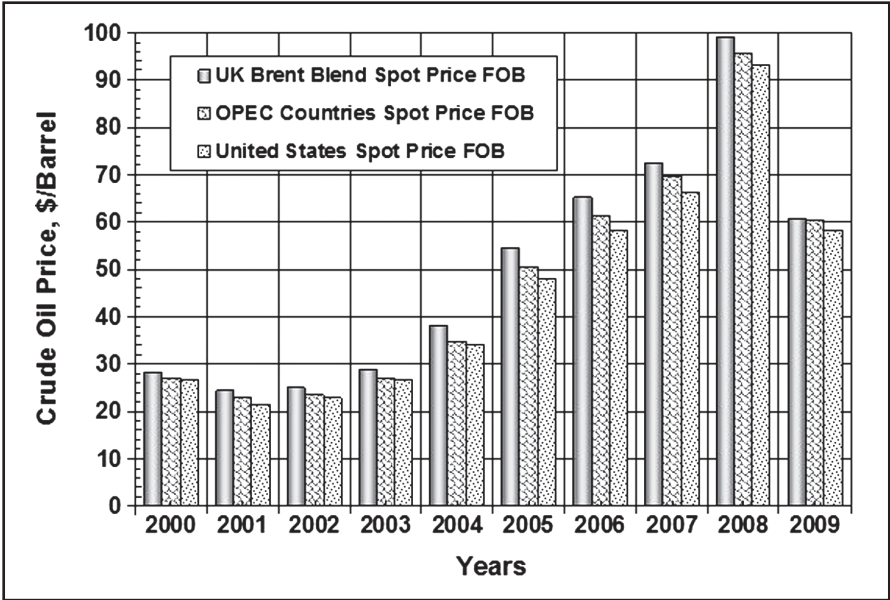


Fig. 3–15. Historical FOB crude oil spot selling prices of various crudes (Source: EIA)

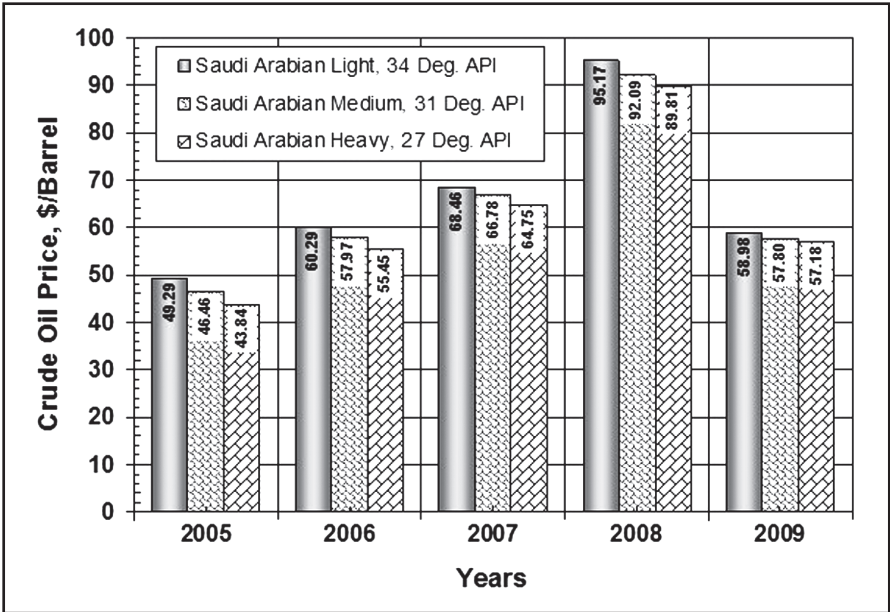


Fig. 3–16. Historical FOB crude oil spot selling prices for Arab light, medium, and heavy crudes (Source: EIA)

Therefore, at present the uncertainties of crude oil price forecasting are so high, it is imprudent to base one's cash flow projections on a single price scenario. Sensitivity analyses have to be conducted at a range of pricing scenarios. The use of decision analysis techniques to account for the crude oil price volatility will be discussed later in this book.

The price of natural gas is based on dollars per million British thermal units (\$/MMBtu); one million Btus are equivalent to 1,000 Scf (1 Mscf) of 1,000 Btu/Scf gas. If the price of a certain gas is \$2/MMBtu and the gas is 1,200 Btu/Scf, then the price translates into $\$2 \times 1.2 = \2.4 per MScf. Similarly, for 950 Btu/Scf gas the price will be \$1.80/MScf ($\2×0.95). Refined product prices for a specific location are developed in relation to projected prices for these products in the major price-setting markets, which are the United States, Western Europe (Rotterdam), and Asia (Singapore). Refined product prices for these primary locations are developed based on each market's expected crude and product slate, operating costs, and profit margins for typical refinery configurations.

Capital Expenditures (CAPEX)

The main characteristic of CAPEX is they are one-off costs, usually incurred at the beginning of a project (also referred to as front-end costs). These usually large expenditures often must be incurred several years before any revenue is obtained. CAPEX typically consists of G&G costs; drilling costs; tankers; offshore platform construction and installation, process facilities; wellheads, flow lines, and trunk lines to transport oil and gas; supply bases; camps and accommodation; storage tanks or vessels; etc. The capital costs may also occur during the economic life of a project, for example:

- Recompletion of wells into another formation
- Sidetracking an existing well with a horizontal well
- Installing artificial lift facilities if the wells were initially on natural flow
- Major upgrading/replacement of existing facilities
- Installing facilities for secondary recovery (waterflood or gas injection) or enhanced oil recovery (EOR)

Geological and geophysical (G&G) costs

G&G costs are pre-drilling exploration costs. These costs include topographical, geological, and geophysical studies. The geophysical costs include rights of access to properties, acquisition of seismic lines (seismic surveys), processing of seismic surveys, and interpretation of these surveys. These costs may be incurred either before or after acquisition of working interest in the property (lease). G&G costs are considered inherently capital in nature and must be added to costs of property acquired or retained. *For tax purposes, these costs are expensed in the year the costs were incurred.*

Seismic acquisition, in most cases, is done by outside contractors who specialize in providing this service to the industry. The company's geophysical staff can either do the processing and interpretation of the acquired data in-house or it may be contracted out to the same contractor who acquired the data.

The cost estimate is made based on the type of acquisition, for example, onshore versus offshore, 2D seismic versus 3D seismic surveys, and many other parameters involved in acquisition and processing of data. The best way to arrive at a cost estimate is to either approach various contractors for quotation, or compare your current requirements with some of the analogous surveys conducted in the past. Remember, the costs of seismic acquisition, processing, and interpretation depend on supply and demand for these services in the industry. Therefore, obtaining fresh quotations from contractors is recommended.

Estimating drilling costs

Preparation of drilling cost estimates may require as much engineering work as the actual well design. The cost estimate is heavily dependent on the technical aspects of the well to be drilled. Therefore, these aspects have to be established. These aspects include the following:

1. The type of wells to be drilled (i.e., exploration, appraisal, delineation, or development wells). Exploration and appraisal wells generally cost more because of extensive testing and coring, etc. In addition, since exploration wells are drilled in wildcat or rank wildcat areas, optimization of drilling parameters is difficult.
2. The configuration of wells to be drilled (i.e., vertical, deviated, horizontal, multilateral, new well or sidetrack, water disposal well, or injection well)

3. The type of drilling contract to be used and the rig type. Rig cost constitutes a major percentage of the total drilling cost. The onshore rig costs range from \$10,000 to \$45,000 per day, while the offshore rig costs range from \$35,000 to \$150,000 per day. The rig costs in a particular area depend on the supply and demand of the rigs available in that area, contract type, and size of rig. The deep offshore rigs cost \$450,000 to \$1 million per day.
4. The depth of the well and the complexity of the formations the well is drilled through. For example, the cost of drilling increases in high pressured, sloughing shale, unconsolidated formations, and formations with thief zones.
5. The casing scheme to be used and the type of casing required
6. The drilling mud's rheology and the type of bits to be used
7. Testing and coring requirements
8. Hydraulic fracturing and acidization and the size of these treatments
9. Completion equipment

Once the technical aspects are established, the expected time required to drill the well is determined. The actual well cost is then obtained by integrating the anticipated drilling and completion times with the well design.

As is obvious from the daily rig costs given above, the time required to drill a well has a significant impact on many items in the well cost estimate. These items may include the following:

- Type of drilling contract
- Drilling rig
- Drilling fluid
- Offshore transportation (helicopter and marine)
- Rental tools
- Time-dependent support services

The drilling time can be estimated based on the data available on analogous wells in the same or similar area. Numerous sources are available to enable estimating of drilling time of a well. These include bit records, mud records, and operator's well histories on offset or analogous wells. Several factors affect the amount of time spent in drilling a well, as listed below. Each factor may vary with drilling geology, geographical location, and operator philosophy and efficiency.

- *Drilling rate*: dependent on rock type, bit selection, mud type and properties, weight on bit, and rotary speeds
- *Trip time*: dependent on well depth, amount of mud trip margin, hole problems, rig capacity, and crew efficiency. As a rule of thumb, trip time is estimated as 1 hr/1,000 ft of well depth.
- *Hole problems*: hole sloughing, lost circulation, and slower drilling rates are considered standard hole problems
- *Running casing*: dependent on casing size, depth of well, hole conditions, efficiency of crew, and use of special equipment
- *Logging, coring, and pressure (DST, drawdown or buildup) surveys*
- *Directional/horizontal drilling*: directional control of a well requires increases in the drilling time
- *Well completion*: dependent on the complexity of well completion (i.e., single completion, dual completion, commingled completion, gravel pack, acidizing, fracturing, and other forms of well treatments). The use of completion rig versus drilling rig, for completion operations, should be considered where technically and economically feasible. A completion rig is a smaller workover rig that costs significantly less than a large drilling rig.

The well cost estimate is usually divided into several cost categories for engineering and accounting purposes, such as dry hole versus completed well costs and tangible versus intangible costs. General summaries of typical well costs are given in the *Authority for Expenditure* (AFE), as shown in table 3–9.

Table 3–9. Authority for expenditure (AFE)

Well Name:	Date:
County/State:	AFE No.
Location:	Operator

Items and Description	Qty.	Unit Price	Total Cost	
			Dry Hole	Producer
Intangible Drilling Costs				
Labor, company, and contract			×	
Fuel, water, power, and lubricants			×	
Hauling			×	
Permit, survey, etc.			×	
Roads, location, and contract			×	
Contract drilling			×	
Day work			×	
Drilling bits			×	
Compressor and tool rentals			×	
Pulling unit or cable tools			×	
Mud and chemicals			×	
Mud logger			×	
Cement and cementing			×	
Coring			×	
Well surveys (deviation)			×	
Well surveys (logging)			×	
Drill stem test			×	
Perforating			×	
Acidizing, fracturing, and gravel Pack			×	
Supplies and miscellaneous			×	
Total Intangible Drilling Costs				

Facility costs

A typical lease service installation (onshore) is shown in figure 3–17. Many different elements of surface facilities are required to produce crude oil and/or gas. Some of these facilities are required from day one of the production; others may be required later during the economic life of a well. For example, the free-water-knockout tanks may not be required in the beginning. Similarly, artificial lift facilities (sucker rod pumping unit, electrical submersible pumps, or hydraulic pumps) may not be required from the beginning.

Table 3–9. *(Continued)*

Tangible Drilling Costs	
Casing (conductor)	×
Casing (surface)	×
Casing (intermediate)	×
Casing (production)	×
Tubing	×
Packer(s)	×
Rods, sucker	×
Pumping unit	×
Electric motor	×
Engine	×
Subsurface pump	×
Christmas tree	×
Miscellaneous completion equipment	×
Total Tangible Drilling Costs	
Lease Equipment and Installation	
Tanks, storage	×
Treater	×
Heater	×
Meter run	×
Separator	×
Line pipe, flow line	×
Header	×
Miscellaneous connections	×
Trucking and labor	×
Total Lease Equipment, etc.	
Total AFE Cost	

The facilities requirement in offshore fields is even more complex and expensive. The offshore production facilities require (1) various platforms (process, flare, utilities, etc.), (2) wellhead jackets with topsides and flow lines, and (3) crude oil and/or gas lines from offshore production stations to the storage facilities onshore. Facilities such as heat exchangers, compressors, desalination units, seawater pumps, and power generation units are required on offshore platforms.

The facilities have to be planned and designed in such a way that benefit is achieved due to economies of scale. For example, installation of nine

single-slot wellhead jackets with flow lines to drill nine wells in a field may cost \$12 million. In this case, the possibility of drilling the nine wells from three three-slot wellhead jackets should be considered because installing these jackets with flow lines will cost \$7 million. This option, if technically feasible, will save \$5 million due to economies of scale. However, since longer reach wells have to be drilled from each of these three jackets, the incremental drilling cost may increase. The net benefit should therefore be considered. The same applies to compressors, separation facilities, flow lines, etc. Therefore, a long-term assessment of the requirements rather than a short-term assessment is necessary.

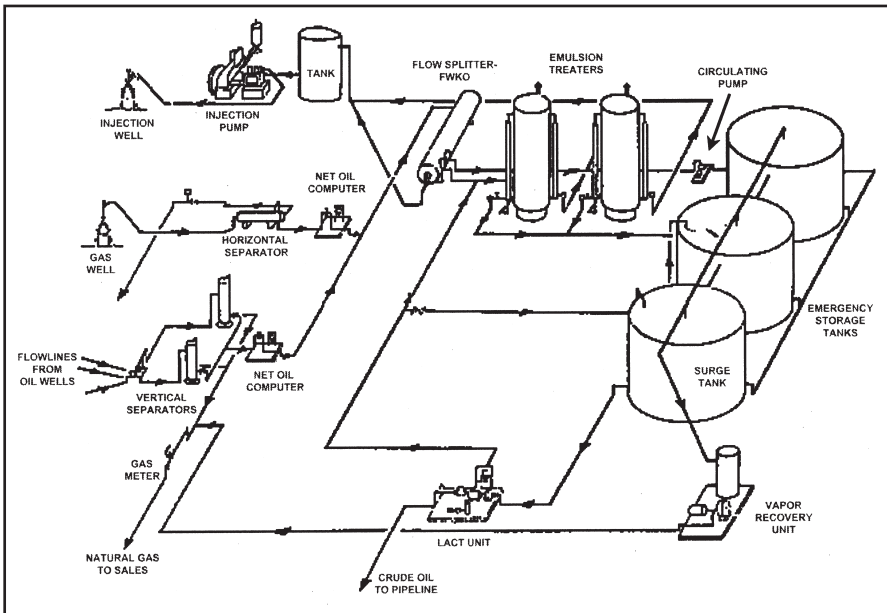


Fig. 3-17. A typical lease service installation (onshore)

Using the power law and sizing model can often accomplish equipment cost estimating. The power law model requires that the equipment must be similar in type and vary only in size. The economies of scale are in terms of size expressed in the relationship, which is given as:

$$C_d = C_r \left(\frac{Q_d}{Q_r} \right)^m \quad (3.25)$$

where

C_d = cost of design size Q_d
 C_r = known cost of reference size Q_r
 Q_d = design size
 Q_r = reference design size
 m = correlation exponent, $0 < m < 1$

If $m = 1$, a linear relationship exists and the economies of scale do not apply. For most equipment, m is approximately 0.5, and for chemical processing equipment it is approximately 0.6. The determination of the correct exponent is essential to the estimation process.

$$m = \frac{\log(C_L) - \log(C_S)}{\log(Q_L) - \log(Q_S)} \quad (3.25a)$$

where

C_L, Q_L = cost and capacity of larger unit
 C_S, Q_S = cost and capacity of smaller unit

Example 3–9

Your company has completed two gas processing plants in 2009. The production capacity of the larger plant is 500 MMScf/day and that of the smaller plant is 300 MMScf/day. The actual cost (CAPEX) of the larger plant was \$1,800 million and that of the smaller plant was \$1,305 million. Based on this information,

- What will be the scaling factor (power exponent) for economies of scale?
- What will be the cost of a new gas processing plant with a capacity of 750 MMScf/day?
- What will be the cost of the new gas processing plant to be built in 2011 if the 2009 costs have increased by about 5% per year?
- The previous gas processing plants did not require a nitrogen rejection unit, but the new plant will require one. What will be the total cost of the gas processing plant if the unit costs about \$120 million?

Solution:

- Using equation (3.25a)

$$m = \frac{\log(1,800) - \log(1,305)}{\log(500) - \log(300)} = \frac{0.1397}{0.2218} = 0.63$$

- (b) Using equation (3.25), the cost of the 750 MMScf/day gas processing plant would be

$$\begin{aligned}
 C_{\text{newGP}} &= C_{500 \text{ MMScf/day}} \left(\frac{Q_{750 \text{ MMScf/day}}}{Q_{500 \text{ MMScf/day}}} \right)^m \\
 &= 1,800 \left(\frac{750}{500} \right)^{0.63} = \$2,323.86 \approx \$2,324 \text{ million}
 \end{aligned}$$

- (c) If the costs have escalated by about 5% per year, then the price of the new gas processing plant to be built in 2011 would be approximately

$$C_{\text{newGP}} = 2,324 \times (1 + 0.05)^2 = \$2,562 \text{ million}$$

- (d) The cost of the new gas processing plant along with the nitrogen rejection unit will cost $\$2,562 + \$120 = \$2,682$ million.

Operating Expenditure (OPEX)

The main characteristic of OPEX, also referred to as lease operating expenditure (LOE), is that these expenditures occur periodically and are necessary for the day-to-day operations of the field. In cash flow analysis, operating costs are usually expressed in terms of expenditure per year, per barrel, or per MScf. The operating costs typically consist of five elements: (1) fixed cost, (2) variable cost per unit of production (determined as a function of production rate), (3) maintenance of facilities, (4) maintenance (workover) of wells, and (5) overheads. The breakdown of various cost categories is as follows:

1. *Feedstocks*: In economic analysis, required feedstocks have to be costed at their fair market value. In the absence of a fair market value, costs should fairly reflect the economic cost of manpower, materials, and capital, and the values of co-product credits should be used. For downstream project evaluations, intermediate products are transferred from upstream to downstream plants at the cash cost of production. The value of this transfer cost has no effect on the net cash flow for an integrated complex (i.e., internal transfer revenue for one plant is a production cost for another). Using cash cost as a basis of transfer is, however, convenient for the calculation of inventory values.

2. *Utilities:* Utilities are estimated based on the economic cost of energy (gas and electricity), manpower, materials, and capital.
3. *Maintenance:* Maintenance comprises material costs and manpower (skilled and unskilled) costs. These costs are associated with keeping the oilfield equipment in good working order. The total maintenance costs are related to the cost of the equipment to be maintained. The maintenance costs can be subdivided into
 - a. Inspection costs
 - b. Preventative maintenance costs (e.g., lubricants, periodic replacement of small parts, painting, and minor repairs)
 - c. Remedial costs (e.g., replacement and major parts)
4. *Administrative and general overhead:* General overhead is comprised of (a) labor (administration, laboratory, stores, medical, security, etc.), (b) materials and supplies (electricity, stationery, food, lodging, miscellaneous consumables, etc.), and (c) services (communications, insurance, travel, training, etc.). The corporate overheads cover company costs not related directly to the production site. These include corporate R&D, financial administration, and head office and policy functions. Corporate overheads are normally estimated from previous projects at 2% of gross revenue.

Some companies prefer to subdivide the employee benefits category. Items such as employee health plans, life insurance, etc. are separated from the expense of providing offices, motor pool, berths aboard ship, offshore catering, and other per employee support costs.

5. *Production costs:* These costs can be subdivided into
 - a. Lifting costs—related to bringing the oil and/or gas to the surface
 - b. Treatment costs of dehydration and oil/gas separation
 - c. Workover costs of well stimulation and downhole repairs
 - d. Secondary recovery costs related to the injection of water, gas, steam, or chemicals
 - e. Water disposal costs
6. *Evacuation costs:* These costs are related to the transport of oil and/or gas from the field to a refinery or gas processing facility, an export terminal, or any other point of sale. The costs depend strongly on the throughput, the distance to be covered, and the means of transport. Typical evacuation cost items are

- a. Pump and compressor fuel and losses
 - b. Tanker rentals according to world scale rate, which depends on the voyage and are indexed monthly
 - c. Pipeline tariffs
 - d. Cost of operating terminals and jetties
7. *Insurance costs*: Particularly in the early phases of the lifetime of the field, the oilfield equipment is insured. The rates vary from 0.5% to 4% of the replacement cost, in accordance with the vulnerability of the insured unit.

Although the economic effect of the last years of the project is rather small due to the discounting procedure, an overestimate of the late operating expenses will result in an underestimate of the economic lifetime of the project. This underestimated lifetime may have engineering consequences in the design of the oilfield equipment. The percentage of each of the above elements in the total OPEX varies from company to company and location to location. However, the following breakdown should be kept in mind.

- Production costs—35% of total OPEX
- Evacuation costs—23% of total OPEX
- Insurance premium—21% of total OPEX
- Maintenance costs—17% of total OPEX
- Overhead—4% of total OPEX

Another rule of thumb this author has used in the Middle East is given below. The total OPEX arrived at through this combination is then checked with the actual historical unit of OPEX in \$/barrel.

- 5% of engineering CAPEX
- \$1/barrel of oil production for offshore and \$0.60/barrel for onshore field
- Workover 10% of the wells every year, each workover on the average costs \$130,000 for offshore and \$22,000 for onshore wells

Both fixed and variable cost items are important in order to arrive at a realistic projection of operating expenses for a given project. Some of the expenses may be on a monthly per barrel or per well basis. Individual well workover maintenance or production stimulation treatments occur less frequently. These are projected on a statistical basis over the economic life of the project. For example, it may be assumed that each well will be worked

over on the average once every five years. This, however, does not mean expenses are assigned on five-year intervals (five-year bumps); rather they should be spread on a yearly basis throughout the economic life of the wells. The spread may be estimated based on the fact that a certain percentage of the total wells may require an annual workover.

Operating cost data generally must be developed from historical records for the project or from analogous operations. Some elements of direct operating expense (onshore and offshore) and allocated operating expense are summarized in tables 3–10 and 3–11, respectively.

Table 3–10. Elements of direct operating expense

Onshore Operations	Offshore Operations
Pumper wages, benefits	Supply boats, marine
Pumper transportation	Helicopters
Purchased power, fuel, and water	Standby vessels
Field power, fuel, water	Docking charges
Treating chemicals	Shore base expense
Small tools, supplies	Underwater inspectors
Teaming, trucking, heavy equipment	• Platform
Gas gathering, compression	• Pipelines
Salt water disposal	Communications and data transmission
Roustabout gang wages, benefits	Personnel
Roads, bridges, docks	• Process maintenance
Flow lines, tank batteries	• Process operators
Wells	Warehouse
• pulling, clean-out	Pipeline tariffs
• Tubing and casing repair	Fuel
• Plug back in zone	Equipment standby
• Stimulation zone	• Cementing pumps
• Gas shut-off	• Wireline service
Outside labor and equipment	Food and lodging for personnel
• Service company personnel equipment	
• Service company deliveries	
• Contractor services	
Crop damage	
Pipeline gauging/pigging	

Table 3–11. Elements of allocated operating expense

Office expenses, including rent and utilities
Lease supervision wages, benefits
Engineering salaries, benefits
Clerical, accounting wages, benefits
Tool room, warehouse, shop wages, benefits
Motor pool expense, not recovered as direct charge on mileage basis
Legal, financial, fire and safety, human resources wages, benefits
Training of personnel
Research and development (R&D)
Health insurance, etc.

Abandonment costs and sunk costs

Abandonment costs are a special category of CAPEX associated with the environmentally safe abandonment of the wells and facilities at the end of the economic life of the project. These costs may constitute a significant component of cash flow, particularly in offshore operations. Abandonment costs may be as high as the original development costs in real terms.

There are companies that specialize in abandonment of wells and facilities. In some cases, these companies take care of the abandonment in an environmentally safe manner in exchange for the salvage value of the facilities to be abandoned and some additional charge.

Sunk costs are costs incurred before the first period of a cash flow projection. They are usually historic costs—for instance, previous exploration costs incurred before a development being analyzed gets underway. It is important for prior costs or sunk costs not to appear directly in a projection of future cash flow. Investment decisions are based on future costs and revenues because we can choose whether or not to incur them. We cannot do anything about sunk costs. By definition, they have already been spent and therefore cannot directly affect the future decisions in a financial sense.

For example, consider an oil company deciding whether to drill new oil wells on a lease in 2002. The company conducted a seismic survey on the lease in 1996 at a cost of \$10 million, which showed a small oil accumulation.

Given the results of this, the firm estimates capital investment of \$55 million. This results in net cash flow (after OPEX and taxes) of \$10 million each year for the next 10 years. If the firm’s cost of capital is 10%, should the company develop the lease? Based on this information, the net present

value (NPV) of the lease is \$6.45 million (i.e., the investment is profitable), thus drilling should take place. However, if the cost of the seismic test office is taken into consideration, the NPV of this project would be negative \$3.55 million ($-\3.55), thus the project should be aborted. Since the firm has already spent the \$10 million whether it drills or not, this sum represents a sunk cost and the investment decision should be independent of it. In other words, the firm would be better off taking the investment (drilling) since, on an incremental basis, the NPV is positive. Another way of looking at it is that taking the investment will reduce the loss from \$10 to \$3.55 million.

As discussed in the previous paragraphs, the sunk costs should be ignored when evaluating a subsequent investment opportunity. However, frequently there are tax credits generated by these costs and they should not be ignored. For example, tax deductions due to depreciation, depletion, or investment tax credits yet to be accounted for should be included where appropriate. Such credits affect the after-tax economics and may have a significant impact on decision alternatives. For example, the tax credit available on the surrender of a nonproducing lease and the write-off of its cost (sunk) may be greater than the value of developing a marginal producing property. Similarly, the depreciation tax benefits due to the seismic sunk costs (presented in the previous paragraph) may actually result in positive after-tax net cash flow.

Opportunity costs

Opportunity cost is defined as the potential benefit lost or sacrificed when the choice of one course of action requires giving up an alternative course of action. When estimating the cash flow of a proposed capital investment project, the opportunity or alternative costs, and not just the direct outlay costs, must be taken into consideration.

Take as an example a lease under evaluation, that among other things, requires five sucker rod-pumping units at a total installed cost of \$125,000. The company already has five idle units in the yard that can be used. Under the circumstances, what should be the cost of the pumps (if any) assigned to the proposed project? Remember in this case, it is a sunk cost. If one adopts the incremental cash flow principle, it would appear that, on the surface at least, the cost of pumps should not be charged to the project. However, suppose the company can sell the pumps in the open market for a total price of \$75,000. In such a case the opportunity cost of using the pumps, which is \$75,000, should be accounted for in the cash flow. In the absence of an alternative use (i.e., the opportunity cost is zero), no cost should be charged to the new project.

Similarly, the opportunity cost of using limited services of key personnel may have to be estimated, especially if their involvement in the new project

creates a need to hire additional people elsewhere in the company. Despite the difficulties involved, a careful analysis of the opportunity costs of transferring existing assets to the new project must be made to ensure that the overall return to the firm as a whole is reflected in the calculations.

Types of Cost Estimates

During the life of a project, four types of estimates are typically made. Each type is linked to a certain stage in the development of the project and is based on the available design and planning information at the respective stage. The different types of these estimates, their purpose, and accuracy are discussed in this section.

Order of magnitude estimate

This estimate is based on general information, for example, the information available immediately after the discovery well is drilled. The design base, on which the cost-engineering study is then based, could for example cover the following data:

- Location
- Prevailing weather conditions
- Water depth in case of offshore discovery
- Terrain conditions in case of onshore discovery
- Distance to export terminal or refinery
- Recoverable reserves estimate
- Well potential, number, and type of wells required
- Most probable reservoir mechanism
- Crude oil or gas properties (gravity, gas-oil ratio, sulfur or H₂ content, etc.)

For this type of estimate, analysts use known costs from existing similar projects and apply the appropriate scaling rules to downsize or upsize the facilities as required by the project on hand. These include the following:

- Drilling and completion
- Flow lines and trunk lines

- Facilities for separation, compression, gas treatment, water injection, etc.
- Platform structure(s) in case of offshore development
- Storage facilities
- Evacuation system (pipeline, tanker, rail, jetty, trucking, etc.)

The *order of magnitude* estimate can be generated in a short time period with the resulting accuracy in the range of 25%–40%. These estimates are used for the earliest economic appraisals and ranking of prospects/alternatives. A promising subsequent economic analysis may result in an appraisal campaign.

Because of the appraisal campaign, well production data and detailed maps are made available. Based on this information, specialists are able to decide on the most likely production mechanism; this results in one or more production scenarios. These plans with detailed information allow the engineers to create a *conceptual* development plan or plans. These development plans are then used for the *optimization study estimates*, one for each of the alternatives put forward by the engineers.

Optimization study estimate

The costing method used to generate these estimates mostly depends on scaling rules. However, this time they are applied to individual parts (as these are now better defined) of the various development units mentioned in the preceding paragraph. The costing study is far more detailed than in the order of magnitude case. It therefore takes more time with the resulting accuracy in the range of 15% to 25%.

The results of this *optimization study* estimate are used in the economic analysis in order to select the most economically attractive development option. If that is not possible, another appraisal campaign may be necessary and the process is repeated.

Budget estimate

Once the development plan has been selected and approved by the management of the company, the *design base* is considered frozen. This decision has important consequences for the engineering department. A team of engineers create the *basis of design* (BOD), using preliminary surveys, process and instrument diagrams, weight breakdowns, etc. System and equipment specifications are agreed on, and vendors and contractors are

invited to bid on the required specifications. By freezing the BOD and the timing of the project, the largest potential for uncertainty is removed.

At this stage, the material and manpower requirements become clear; the resulting *budget estimate* is based on the price of equipment, steel, man-hour rates, barge-day rates, contractor bids, etc.

This cost estimate requires considerable amount of time to create with the resulting accuracy in the range of 10% to 15%. This phase is terminated with either the approval or abandonment of the project. In the first case, the budget is firmed up and formally presented to the company's executive management for approval.

Control estimate

Once the management approval is granted, the project execution phase commences, and contracts can be awarded. During this phase, the actual expenditure is continuously monitored versus the budget estimate. In this comparison, use is made of the so-called *S-curve*, a graph of the cumulative expenditure versus time. By using this method, one obtains forewarning of any possible expenditure overruns.

When new information becomes available during this phase (e.g., from new well data), the development plan is updated accordingly and new control estimates prepared. In case of large discrepancies (more than 10%) between the original budget estimate and the control estimate, the budget is revised upward and downward as warranted.

Variations in Actual Project Costs

The post project implementation audits show that historically there are variations in the actual project cost versus the estimated costs before the project implementation. There are various factors that contribute to these variations.

Cost overrun

Costs may exceed the original budget for several reasons:

1. In order to induce contractors to deliver on time, they are often paid by *milestone payments* as soon as a certain agreed milestone is reached. If progress is still slow, the subsequent development steps are delayed and possibly more expensive according to inflation clauses in the contracts. This type of cost overrun need not necessarily be sanctioned by a separate management approval.

2. Contractors may encounter unforeseen difficulties such as extreme weather conditions or worker strikes. This often leads to increased costs, which if over a certain threshold, may be subject to management approval.
3. New information may induce the petroleum engineers to change the BOD or scope of the project. Such scope changes can lead to delays and large cost increases, which are subject to management approval. To prevent this embarrassing situation, one is often tempted to leave the options open and build some flexibility into the project. This flexibility can be costly. For an offshore development, it can result in an increase of the topside facilities, leading to a disproportionate increase in the overall weight and thus the price of the substructure. Cost overruns are often combined with an extended development period. Both are harmful to sound economics. Therefore, a CAPEX-overrun sensitivity without a corresponding extended development period yields optimistic results.

The accuracy of the estimates

Due to the nature of the business (i.e., too many unknowns), even using perfect estimating skills, methods, and databases, an estimate can only be as good as the available definition of the project. Figure 3–18 shows qualitatively how the accuracy of the estimates generally improves with time. An estimate prepared at the earlier stage of the project often is less accurate, but as time passes, the project definition improves and so does the corresponding accuracy of cost estimates. A major improvement occurs when the BOD is frozen. This freezing reduces the amount of uncertainty, which simplifies the project planning, reduces the costs for uncertain elements (contingency), and thus improves the accuracy of the estimate.

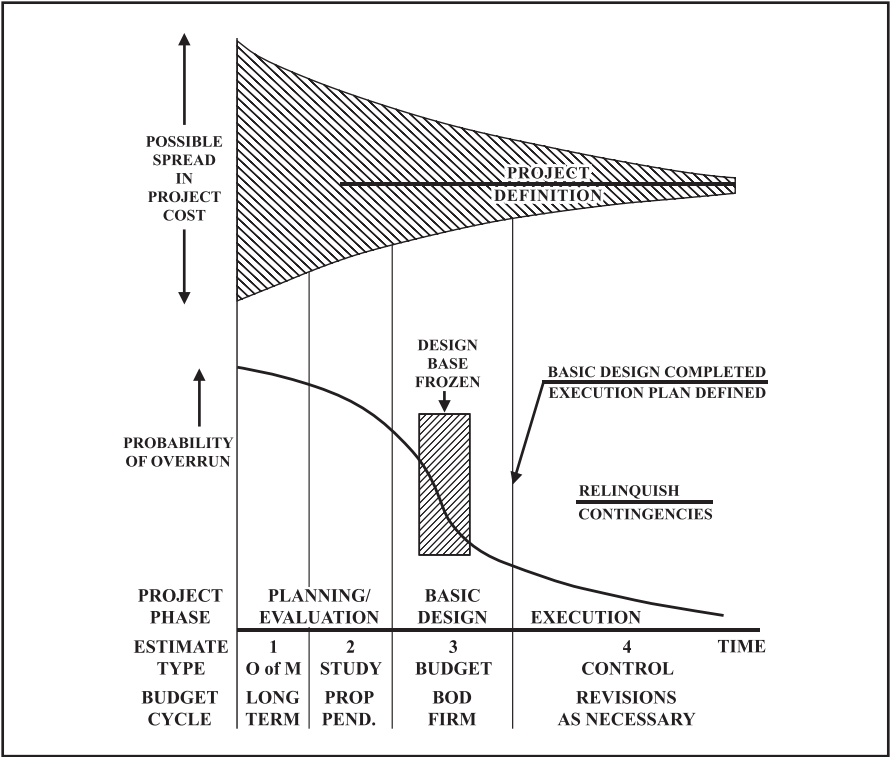


Fig. 3–18. Accuracy of cost estimate improves with time

Contingencies and allowances

Probably no other single element in an estimate is subject to as much conflict and confusion as contingency. Despite its importance, contingencies are rarely well planned for or analyzed in detail. The purpose of contingency is to provide a budget for uncertainties (i.e., expected but undefined elements in an estimate). It is linked to the estimate’s accuracy and should thus cover:

- Estimate omissions due to incomplete project definition
- Inadequacies in the estimating method
- Design and construction changes within the agreed project scope
- Variances in inflation and escalation rates

Whereas contingencies cover the *unknown unknowns* relating to the total project or a large portion of it, allowances are supposed to cover the *known unknowns* relating to individual aspects of the project. Allowances can be defined as the probable extra costs (probability of occurrence more than

50%) related to specific activities, or cost elements, and can be identified and quantified (usually from statistical data). Examples of allowances are:

- Material take-off allowances (e.g., 3% extra line pipe for a pipeline scaled from a map)
- Identified risks (e.g., buckle allowance, trenching allowance for an offshore pipeline)
- Foreseeable market conditions
- Foreseeable weather downtime
- New technology (in certain parts of the project)
- Growth (normal extra work)

The value of the estimating contingency should be such that the probability of cost overrun of the *median estimate* equals 50% (by definition). The *base estimate* equals the modal cost of the project. The relationship between these two statistical quantities is shown in figure 3–19. The contingency is supposed to cover the difference between the mode and the median estimate. The contingency factor therefore depends on the skewness of the distribution curve in figure 3–19. In practice, the contingency factor is about two-thirds of the accuracy shown in the section on types of cost estimates.

The resulting median estimate is used for the *base case* economic analysis. For the CAPEX *overrun* sensitivity, the budget proposal estimate should also include an overrun *allowance*, which should be selected in such a way that the probability of cost overrun of the budget estimate is only some 10% for a large project or some 20% for a small project.

This procedure implies that more funds are reserved or frozen for the project than is probably necessary. It is, therefore, important to unfreeze these extra funds as soon as it becomes clear that certain expected cost overruns are not going to be realized and the funds will not be required.

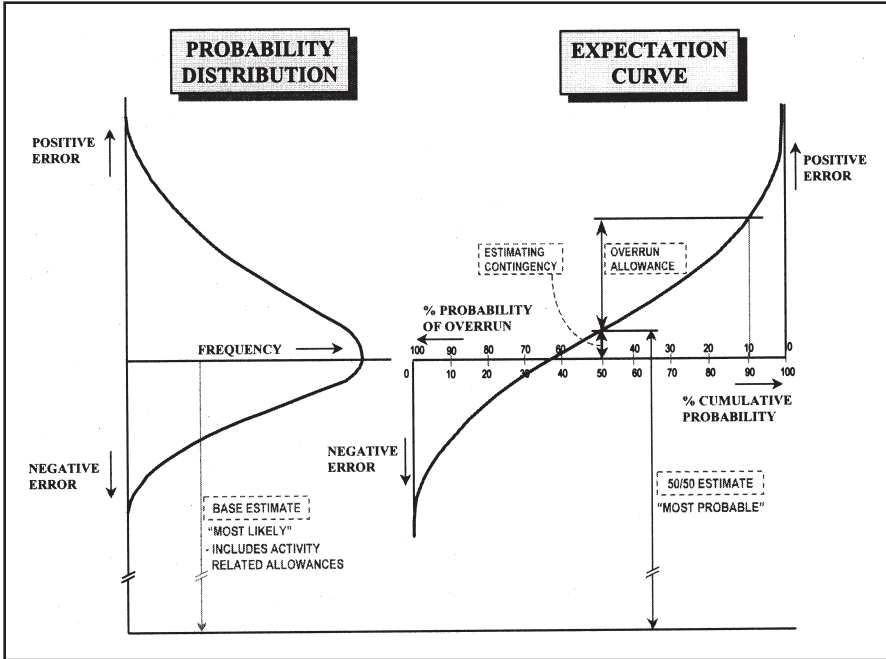


Fig. 3-19. A statistical view of contingencies and allowances

Transfer Pricing

Special problems can arise when evaluating projects in integrated companies—when one division or profit center of a company supplies goods and services to another division or profit center. The problems revolve around the question of what *transfer price* to charge for the transfer of goods or for the exchange of services.

Suppose a major, vertically integrated, oil company has three divisions:

- Upstream, oil, and gas production (division A)
- Downstream, gas processing (NGL) facility (division B)
- Downstream, petrochemical facility (division C)

Division A develops a gas field and strips liquids (condensate and raw NGL) from the gas in a stripping plant located in the field. The condensate is spiked into the company's crude oil line passing through the field, the stripped gas is injected back into the reservoir for pressure maintenance, and the raw NGL is sent to the company's division B, where the C_2 , C_3 , C_4 , and C_5+ are separated from the raw NGL. The C_3 and C_4 are sold; the C_5+ is

combined with the condensate from division A and spiked into the crude oil line. Division C takes the C_2 for producing petrochemicals and related products. Ethane (C_2) is used to produce ethylene, low-density polyethylene (LDPE), high-density polyethylene (HDPE), propylene, ethylene glycol, etc. The above activities are schematically shown in figure 3–20.

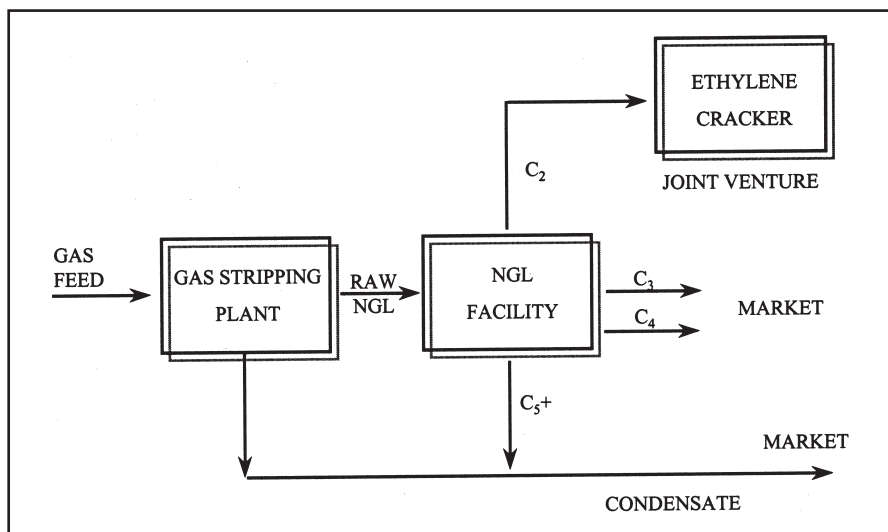


Fig. 3–20. Schematic of integrated divisions

In this example, there are two product transfers between divisions within the same company. The question is, what price should control these transfers? Should the price be set to include some *profit* element for the transferring division, or should it be set to include only the accumulated costs to that point? Alternatively, should it be set at some other figure still? The choice of a transfer price can be complicated by the fact that each division may be supplying portions of its output to outside customers, as well as to sister divisions. A further complication is if the selling price of one division becomes a cost expense to the other division. For example, an ethylene cracker takes 1,000 tons of ethane from division A at a price of \$20/ton, so the \$20,000 becomes revenue for division A and an expense (part of OPEX) for the ethylene cracker. The higher this cost, the lower the purchasing division's profitability. Thus, the purchasing division would like the transfer price to be low, whereas the selling division would like it to be high. Perhaps the transfer could be at *market* price, so the same price is charged internally as charged to outside customers.

All three projects are set up from scratch and presented for evaluation. The problem is further compounded if the company owning divisions A and B does not solely own division C (i.e., it is a joint venture with another company). In practice, the following three general approaches are used in setting transfer prices for project evaluation and then later for the performance evaluation of each division or profit center.

1. Set transfer prices at cost using:
 - a. Variable cost
 - b. Full (absorption) cost
2. Set transfer prices at market price
3. Set transfer prices at a negotiated market price

Transfer price at cost

Many companies make transfers between divisions based on accumulated cost of the goods being transferred, thus ignoring any profit element to the selling division. A transfer price computed in this way might be based only on the variable costs involved, or fixed costs might also be considered and the transfer price thus based on full (absorption) costs accumulated to the point of transfer. In our example above, the transfer price for raw NGL is \$/ton of NGL transferred to division B and \$/ton of C_2 transferred to division C.

The cost approach to setting transfer prices, however, has the following disadvantages:

1. This approach has no built-in mechanism for telling when transfer should or should not be made between divisions (i.e., if the product can be sold outside the company for a higher price than the cost).
2. The only division to show any profits is the one making the final sales to an outside party. The transferring division will not show any profit, thus evaluation of setting up this division by itself will be difficult.
3. The mechanism does not provide any incentive for the transferring division to control costs. If the costs of one division are simply passed on to the next, then there is little incentive for anyone to control costs. Therefore, the final selling division is simply burdened with the accumulated waste and inefficiency of intermediate processors, and the division will be penalized with a rate of return that is deficient in comparison to its competitors.

General formula for computing transfer price

A general formula exists and is commonly used by the engineers/economists as a starting point in computing the appropriate transfer price between projects, divisions, or segments of a multidivisional company. The formula is: *the transfer price should be equal to the unit variable costs of the good/services being transferred, plus the contribution margin per unit lost to the transferring division because of giving up outside sales.* The formula can be mathematically expressed as:

$$\begin{aligned}\text{Transfer Price} &= \text{Variable cost per unit} \\ &+ \text{Lost contribution margin per unit on outside sales}\end{aligned}$$

A transfer price computed by the above formula is based on competitive market conditions.

Transfer at market price

This transfer price mechanism is based on some form of competitive *market price* and is considered the most appropriate approach to the transfer pricing problem. The mechanism dovetails well with the profit center concept and enables the engineers to make profit-based performance evaluations feasible at many levels of an organization. By using this mechanism for transfer pricing, all projects/divisions/segments of a company are adequately evaluated for their respective profitability (economic viability).

The idea behind using this type of transfer price mechanism is to create competitive market conditions that would exist if the various divisions were indeed separate firms and engaged in arm's length, open-market bargaining. The market price depends on (1) if a well-defined intermediate market exists for the product or (2) the price changes in the intermediate market.

1. *A well-defined intermediate market exists.* The market conditions may differ from division to division, project to project, and company to company. If no intermediate market exists for the product, then the only customer a division has for its output is a sister division. If an intermediate market exists for part or all of the division's product, then it has a choice between selling to outside customers or selling to other divisions within the same company. The existence of intermediate market price represents an upper limit on the charge that can be made on transfers between divisions. In many situations, a lesser price can also be justified due to reduction in administrative and marketing expenses when inter-company sales are involved.

2. *The price changes in the intermediate market.* Here it becomes a problem of opportunity cost, as discussed in the preceding sections. For example, if a division is already selling all it can produce to outside customers, then the division's opportunity cost is the purchase price paid by the outside customers. Therefore, if the inside business is accepted, it will be at a cost of giving up the outside business. Thus, when the selling division is at full capacity, the transfer price inside should never be less than this opportunity cost, or the selling division suffers. However, the situation will change if the selling division is not at full capacity. Under this condition, the opportunity costs may be zero (depending on what alternative uses the seller has for the excess capacity). Under such situations, so long as the selling division can receive a price greater than its variable costs, all parties will benefit by keeping business inside rather than having the buying division go outside. Some type of *negotiated market price* can best serve such situations.

Similarly, one of the divisions may require a product unavailable from outside sources and must be produced internally by a sister division. In such situations, the buying division negotiates with the selling division to arrive at a mutually agreed transfer price sufficiently attractive for the economic viability of both divisions.

Aspects of Leasing

Note that nowhere in the world do the companies involved in exploration and oil and gas production activities own the land or the underneath minerals. In some areas of the world both the land (surface) and the underneath minerals belong to the state/government. In other places the surface rights may belong to one party (individual) while the mineral rights belong to another (state or individual). In the United States and Canada, individuals may own rights to both the land and the underneath minerals. If the surface owner also owns the minerals (ownership in fee), he will usually be just as interested as the producer in bringing about the oil and gas production. An owner in fee may sell his surface ownership of the land while retaining the mineral rights. A legal instrument called a *mineral deed* accomplishes transfers of mineral ownership.

Before an oil and gas company can start drilling activities on a prospective piece of land, permissions to conduct such operations must be obtained from both the surface and mineral owners. Usually a *landman*, whose job is to seek

out the owners of both estates and negotiate leases with them on behalf of the intended oil and gas company, takes care of the required legal formalities.

In the United States, there are also vast stretches of federal land. Here the government owns both the land and the minerals. If an oil and gas lease on federal land is successful and produces petroleum in commercial quantities, the U.S. Minerals Management Service collects the royalties and divides them evenly with the state where the tract is situated. Joint federal/state mineral income sharing may also occur from offshore leases.¹³

In Canada, the surface and mineral rights may be owned separately. Owners may be individuals, the federal government (Crown land), or the provincial government. For example, in Alberta, 80% of the mineral rights belong to the provincial government. In eastern Canada, individuals or private enterprises own almost all mineral rights. In southern Ontario, minerals are mostly privately owned, while in northern Ontario, almost all minerals belong to the provincial government. In Manitoba, about 80% of the mineral rights are owned by freeholders. The other provinces also have a higher percentage of freeholder mineral owners than Alberta.

The land department provides appropriate ownership interest rates for each property for economic evaluation. However, the mechanism of how the interests of each partner in an oil deal are derived must be understood.

System of land ownership identification

Just as every person has to have a name for identification, lands are also assigned some identification so ownership rights and privileges can be easily identified. Most of the land in the United States has been surveyed by the General Land Office of the U.S. government and each state is divided into small squares based on the survey. The system of survey is referred to as the USGS (United States Geological Survey) system or rectangular survey system.

According to this survey system, *principal meridians* and *base lines* were established. The principal meridians run in a true north-south direction, and the base lines run east and west at right angles to the meridians.

The General Land Office surveyed the states with a system of north-south and east-west coordinates, which divided the states into squares called *townships*. The squares are consecutively arranged into *north-to-south* townships and *east-to-west* ranges, as shown in figure 3–21(a). Each township is further divided into 36 sections, each of which contains 640 acres (1 acre = 43,560 sq. ft) and is one mile square. The numbering of the sections start as section 1 (one) in the NE corner of the township, consecutively numbered with section 6 (six) in the NW corner. Section 7 then starts with the section below section 6 and consecutively numbered toward the east, and so on. The 36 sections per township are shown in figure 3–21(b).

In Canada, the same system is used, but there the sections in a township are numbered in a different way. Section 1 starts in the SE corner of the township, as shown in figure 3–21(c). Each section is then further divided into 40-acre tracts (16 subsections), as shown in figure 3–21(d). Similarly, in New Mexico, each section is further divided into 40-acre tracts and each tract is identified by a letter, as shown in figure 3–21(e).

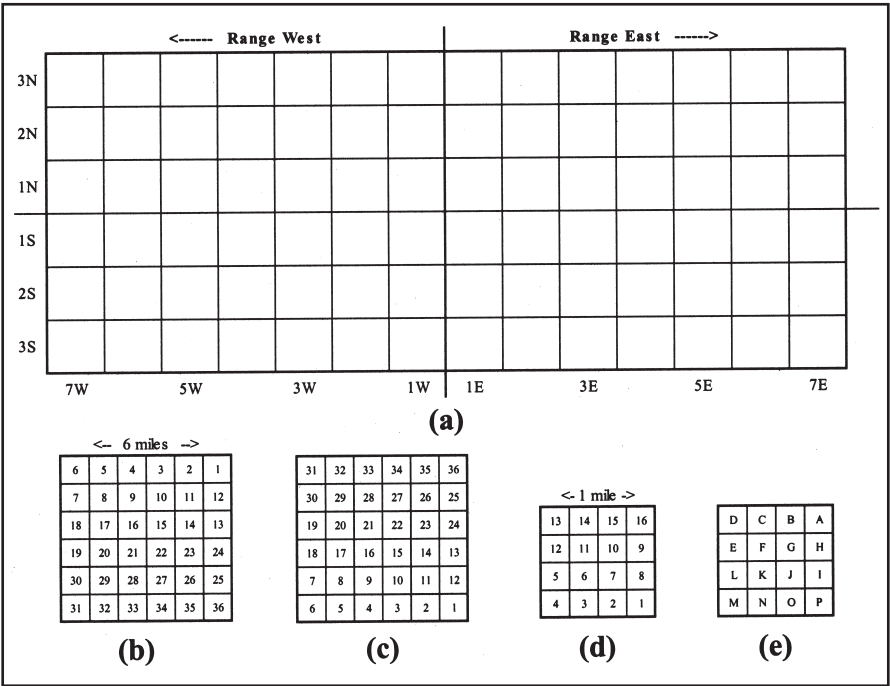


Fig. 3–21. The system of land ownership identification

Another land survey method that is used in some states and offshore is referred to as the *metes and bounds* system. In this system, the ownership is irregular and unsystematic and the land is divided into blocks. This system is mostly applied in the state of Texas and most of the offshore.

It is extremely important to identify the exact legal location of the oil and gas wells in oil and gas property evaluation. Although the wells are assigned names, it is recommended to use both the name of the well and the legal location. The wells are also assigned API numbers. By including the legal location, it is also easier to locate the well on the acreage map, which is then used for constructing the structure map, isovolume map, isopach map, etc.

The wells are identified either by a corner-corner location or by specific distances from the boundaries of the section. For example, in figure 3–22(a), the well is located in the southwest quarter (SW–¼) of the southeast quarter (SE–¼) of Section 15, Township 1 North Range 65 West; this description is abbreviated as SWSE–15–T1N–R65W. Similarly, in figure 3–22(b), the well is located 2,640 feet from the east line and 1,760 feet from the north line of Section 15, Township 1 North Range 65 West. As mentioned above, each section is a one-mile square (5,280 ft × 5,280 ft). If the section is drawn with a scale of three inches to a mile, then the well is located one and a half inches ($2,640/5,280 \times 3 \text{ in} = 1.5 \text{ in}$) from the east line and one inch ($1,760/5,280 \times 3 \text{ in} = 1.0 \text{ in}$) from the north line of Section 15, T1N–R65W. This is abbreviated as 2640 FEL and 1760 FNL–15–T1N–R65W.

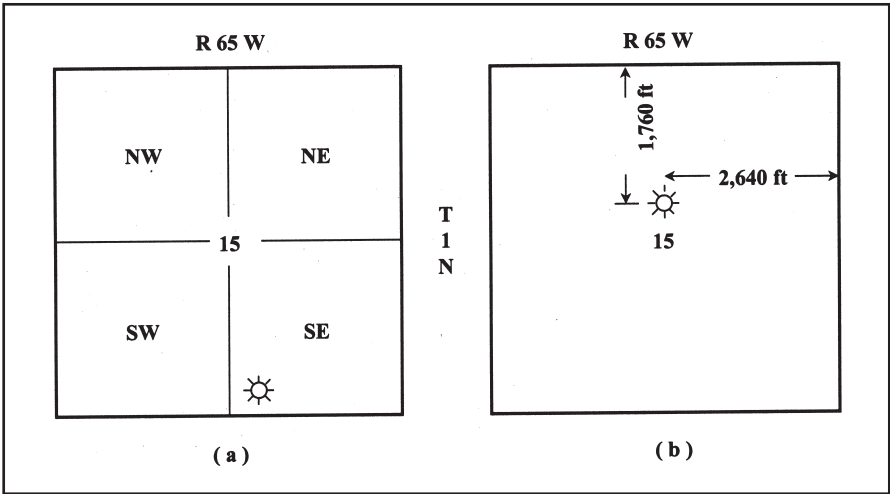


Fig. 3–22. Legal identification of well locations

Ownership interest computation

The mineral interest in a property is divided into *working* (operating) interests and *non-operating interests*. A working interest (WI) can be classified as a basic, joint, pooled, or unitized interest. A non-operating interest can be classified as a royalty, overriding royalty, production payment, or net profits interest. Associated with each working interest there is a *net revenue interest* (NRI) reduced proportionately by *non-operating working interest*. For an investor in an oil and gas deal, NRI is the key economic figure, which

measures the percent of profits earned by investor. This NRI is earned by a specific working interest.

When a lease is obtained, a royalty, commonly *one-eighth* (12.5%) is retained by the land owner/mineral interest owner (*lessor*). The working interest owner at this stage earns a maximum of only 87.5% of any oil and gas revenues from the lease while paying 100% of the costs. On the other hand, the lessor receives 12.5% of the oil and gas free of any costs except for the severance, ad valorem, and any other surcharge taxes levied by the state. The various types of interest arrangements and oil deals are defined below:

1. *Leasehold interest*: The interest owner (lessor) conveys leasehold interest to the lessee. The leasehold interest owner has the right to explore for and produce minerals.
2. *Primary term*: Leases are usually in effect for a period of 1 to 10 years and as long thereafter as hydrocarbons are produced. When hydrocarbons are produced on a lease, the lease is referred to as *held by production*.
3. *Lease bonus*: Upon acquiring mineral rights, the lessee pays a sign-up bonus to the lessor. The amount ranges from \$1 to \$3,000 per acre, depending on the exploration activity within or nearby the leased property.
4. *Delay rental*: Unless operations are commenced on or before one year from the effective date of the lease, the lease terminates regardless of the length of the primary term of the lease, unless the lessee pays a previously agreed delay rental, which enables the lessee to defer operations for one more year.
5. *Overriding royalty interests (ORRI)*: This type of non-operating interest is similar to a landowner's royalty in that the ORRI owner receives a portion of the production free and clear of all costs. An override is often said to be *carved out* of the WI since it is created from the leasehold's share of the revenue.
6. *Joint working interest*: When more than one working interest owner is involved, each owns an individual fraction of the WI in a single lease. A joint WI may result from leases, purchases or exchanges, or sharing arrangements. One party known as the operator (usually the party with the largest interest) in the property has the responsibility for developing and operating the property.
7. *Pooled working interest*: Two or more properties combined that may have the same or different WI owner. The NRIs are

combined separately at the same time. Pooling may be voluntary or mandatory; it will usually result in operations that are more efficient and a greater recovery of hydrocarbons.

8. *Unitized working interest*: When two or more WIs in different properties are combined, the resulting WI is referred to as unitized WI. Unitizations usually occur after production starts because the WIs and the associated NRIs are prorated based on the ultimate recovery of hydrocarbons dedicated to each lease. On the other hand, pooling usually occurs before development is started.
9. *Production payment interest (PPI)*: Non-operating interest normally created out of the WI and usually expressed as a certain amount of money, a certain period, or a certain quantity of oil or gas. The PPI reverts to the WI from which it was created after the specified amount of money, period of time, or oil or gas are recovered. It is often used to pay down debt since the amount of money going for debt services is proportional to the producing rate.
10. *Carried interest*: A form of deal in which an interest owner does not receive any revenue and does not pay any costs until a certain condition is met. Carried interests are more often carried to the *casing point* or through the tanks of the first well. A farmout could be expressed as a carried interest. If a company has an $x\%$ WI to the casing point, then the carrying party pays all of the dry hole costs. At the casing point, the carrying party may decide to put up $x\%$ of the completion costs to gain the $x\%$ of WI with its associated NRI. A third for a quarter ($\frac{1}{3}$ for $\frac{1}{4}$) is a typical farmout agreement in which the farmee pays one third of the costs to some predetermined point in order to receive one quarter WI.

The computation of ownership interest in oil and gas deals ranges from a simple two-owner arrangement (landowner and investor) to complex multiple owner arrangements. The best way to determine these interests is to always flowchart the investors and their associated interests in a systematic manner. The following example clarifies the computations.

Example 3–10

Suppose a lease is obtained by company X for 12.5% of a landowner's royalty. To reduce the risk exposure in development, company X further sells 30% of the deal to company Y. Compute the NRI of each company.

Solution: Ownership interests of each party are given in table 3–12.

Table 3–12. Working and net revenue interest computation for example 3–10

	Working Interest Amount Spent (%)	Revenue Interest Amount Earned (%)
Lessor	0	12.50
Company X	70	61.25
Company Y	30	26.25
Total	100	100.00
Net Lease = $1.0 - 0.125 = 0.875$		
Company X = $0.70 \times 0.875 = 0.6125$		
Company Y = $0.30 \times 0.875 = 0.2625$		

Example 3–11

A lease broker, Z, obtains a lease for a 12.5% landowner’s royalty. He sells 100% of the deal to A retaining a 2.5% ORRI. A sells 100% of the deal to B, retaining 2.5% ORRI. B sells 50% of the deal to C and 25% to D. Compute the ownership interests of each party in the deal.

Solution: Ownership interest computations are shown in table 3–13.

Table 3–13. WI and NRI computation for example 3–11

	WI (fraction)	NRI (fraction)
Landowner	0.00	0.125
Broker Z	0.00	0.025
Broker A	0.00	0.025
Company B	0.25	0.206
Company C	0.50	0.413
Company D	0.25	0.206
Total	1.00	1.000

Example 3–12

A lease is obtained for a 12.5% landowner royalty. A geologist is retained to study the lease for 2% ORRI. A promoter who sells the deal to investors keeps 5.5% ORRI with 25% back-in working interest after payout (after the

WI participants have recovered all their costs from a successful producing well). Compute the before and after payout ownership interests for each party.

Solution: The various interests are shown in table 3–14.

Table 3–14. WI and NRI comparison for the deal in example 3–12

	Before Payout (%)		After Payout (%)	
	WI	NRI	WI	NRI
Lessor	0	12.50	0	12.50
Geologist	0	2.00	0	2.00
Promoter	0	5.50	25.00	21.375
Investor	100	80.00	75.00	64.125
Total	100	100.00	100.00	100.000

Severance and Ad Valorem Taxes

In addition to CAPEX and OPEX, major items of cash outflow for most petroleum projects are royalties and taxes deducted from the pre-tax (federal income tax) cash flow. In the oil and gas industry, besides the federal and state income taxes, each state has its own surcharge on oil and gas production. This surcharge is referred to as ad valorem and severance taxes. These taxes, collectively referred to as production taxes, are usually applied on a percentage basis or on a dollar per unit basis of the gross value at the point of production.

Production taxes are levied upon the owner of oil and gas interests by a state or local government. The tax is determined from the volume (or value) of production (or sales). State oil and gas taxes are quite simple in some states and extremely complicated in others.

Many states have tax incentive programs; however, the programs are constantly being revised, extended, or have expired. In addition, many states allow tax exemptions to qualified wells and projects but stipulate unique criteria for “qualification” or “certification.” It would be impossible to describe all of the various qualifications or, for that matter, to include all special tax exemptions, especially those exceptions so specific that they require more knowledge than the date of first production and the producing rate of the property being evaluated.

Generally, though, these taxes range anywhere from zero in some states to as high as 15% of the gross value in other states. Therefore, it is extremely

important to allow for the proper tax expenses in oil and gas property evaluations. The states with higher taxes will have a significant financial impact on the profitability of the property.

If two field development alternatives are available that are comparable in reserves, investment lives, and initial and ongoing costs, then the tax environment applicable to each alternative will have a significant influence on which alternative is chosen for further development. Obviously, the alternative with lower production taxes yields better economics.

As previously mentioned, many states have complicated exemptions, limits, or sliding scales. In order to provide a quick reference, a summary of the severance and ad valorem taxes is presented in table 3–15 and table 3–16, respectively. The American Petroleum Institute's publication *TC-10: State and Local Oil and Gas Severance and Production Taxes* provides a 26-page summary of these taxes. The production taxes, by state, are also listed on the website of the Society of Petroleum Evaluation Engineers (SPEE) at <http://www.spee.org>. The taxes presented here are given as a sample. Because the tax laws are constantly changing, individual state departments of revenue should be contacted for further and more accurate details.

The following descriptions, additional limitations, or concessions levied by each state relate to the notes in table 3–15.¹⁴

1. For wells producing less than 25 Stb/d or 200 MScf/d in unitized fields and all wells commencing production after September 1, 1979. Baldwin County has an additional 1% severance tax.
2. For wells producing more than 25 Stb/d or 200 MScf/d commencing production on or before September 1, 1979. Baldwin County has an additional 1% severance tax.
3. For oil wells commencing production after June 30, 1981. For first five years only and subject to oil gravity and EL.
4. For oil wells commencing production on or before June 30, 1981. Rate is affected by oil gravity and EL.
5. Oil production is 10 Stb/d or less during any calendar month.
6. Production is greater than 10 Stb/d during any calendar month.
7. Varies, depending on needs of the oil and gas division. Usually small when compared to the estimated ad valorem tax rate. 1983 rate = \$0.0144/Stb.
8. For gross income under \$25,000. Wells averaging less than 10 Stb/d for taxable year are exempt.

Table 3–15. Severance and ad valorem taxes by state

Note	State	Oil	Gas	Note	State	Oil	Gas
1	Alabama	0.06	0.06	17	Louisiana	0.03125	\$0.07
2		0.10		18		0.0625	
3	Alaska	0.1225	0.10	19		0.125	
4		0.15		20		Note	
	Arizona	0.025	0.025	21	Michigan	0.066	0.05
5	Arkansas	0.04	\$0.003	22		0.04	
6		0.05			Mississippi	0.06	0.06
		5 mil/Stb		23	Montana	0.05	0.0265
7	California	Varies	Varies	24		0.06	
8	Colorado	0.02	0.02	25		0.0002	
9		0.03	0.03	26	Nebraska	0.02	0.02
10		0.04	0.04	27		0.03	0.03
11		0.05	0.05	28		0.0004	0.0005
12		0.0006		29	Nevada	\$0.005	\$0.0001
13		0.0009		30	New Mexico	0.0375	\$0.152
14	Florida	0.05	0.05	31		0.0019	
15		0.08	0.05	32		0.0255	
	Georgia	\$0.005	\$0.0005		New York	None	None
	Idaho	0.02	0.02	33	N. Carolina	\$0.005	\$0.0005
	Indiana	0.01	0.01	34	N. Dakota	0.05	0.05
16	Kansas	0.08	0.08	35		0.115	
	Kentucky	0.045	0.045	36		0.001875	
	Oklahoma	0.07085	0.07085		Ohio	\$0.10	\$0.03
	Oregon	0.06	0.06	37	Utah	0.04	0.04
	S. Dakota	0.045	0.045		Virginia	None	None
	Tennessee	0.03	0.03	38	W. Virginia	0.0434	0.0863
	Texas	0.046	0.075	39	Wyoming	0.06	0.06
				40		0.04	
				41		0.0001	

\$ taxes are based on \$/Stb and \$/MScf

Table 3–16. Approximate ad valorem tax rates (fraction of gross value)

State	Oil	Gas	State	Oil	Gas
Alabama	None	None	Nebraska	0.06	0.06
Alaska	0.002	0.002	Nevada	0.06	0.06
Arizona	0.05	0.05	New Mexico	0.05	0.05
Arkansas	0.04	0.04	New York	0.10	0.10
California	0.09	0.09	North Carolina	0.05	0.05
Colorado	0.03	0.03	North Dakota	None	None
Florida	0.02	0.02	Ohio	0.07	0.07
Georgia	0.05	0.05	Oklahoma	None	None
Idaho	0.05	0.05	Oregon	None	None
Illinois		0.08	Pennsylvania	0.08	0.08
Indiana	0.07	0.07	S. Carolina	0.05	0.05
Kansas	0.08	0.08	S. Dakota	0.02	0.02
Kentucky	0.06	0.06	Tennessee	None	None
Louisiana	None	None	Texas	0.04	0.04
Maryland	0.07	0.07	Utah	0.05	0.05
Michigan	0.02	0.02	Virginia	None	None
Minnesota	0.05	0.05	W. Virginia	None	None
Mississippi	None	None	Wyoming	0.07	0.07
Montana	0.09	0.09			

Ad valorem taxes are often based on assessed value of equipment and property and are administered on county/parish/township basis.

9. For gross income greater than \$25,000 and less than \$100,000.
Wells averaging less than 10 Stb/d for taxable year are exempt.
10. For gross income greater than \$100,000 but less than \$300,000.
Wells averaging less than 10 Stb/d for taxable year are exempt.
11. For gross income \$300,000 and over. Stripper production is exempt.
12. A conservation tax, for production on or after January 1, 1982.
13. A conservation tax, for production after January 1, 1979 and before July 1, 1981.
14. For wells producing less than 100 Stb/d and by tertiary methods.
15. For wells producing more than 100 Stb/d and not utilizing tertiary methods.

16. Any ad valorem taxes paid may be used as credit against the severance tax up to 3.67% of the gross value. Thus, the effective severance tax rate may only be 4.33% if the ad valorem taxes paid are at least 3.67% of the gross value.
17. For 10 Stb/d or less.
18. For 25 Stb/d or less and at least 50% salt water.
19. For wells producing more than 25 Stb/d and any condensate production.
20. A working interest in newly discovered fields is exempt from 50% of the severance tax for 24 months on the first 100 Stb/d. The 24-month period starts when regular production begins.
21. All production except stripper production.
22. Stripper production as per definition of the State of Michigan.
23. After April 1, 1985.
24. Until March 31, 1985.
25. Conservation tax. Montana also has a Resource Indemnity Trust tax of \$25 + 0.5% of gross value more than \$5,000.
26. Wells producing 10 Stb/d or less.
27. Wells producing more than 10 Stb/d.
28. Conservation tax.
29. Conservation tax.
30. Severance tax, gas rate changes yearly.
31. Conservation tax.
32. Emergency school tax.
33. Conservation tax.
34. Production tax on all crude oil and gas.
35. Extraction tax of 6.5% on all crude oil except stripper production and royalty interests on first 100 Stb/d in addition to 5% production tax.
36. Oil conservation tax.

37. Occupation tax effective 1/1/84. Wells producing less than 20 Stb/d are exempt. Production for the first six months is also exempt. The state has a conservation tax of 0.002.
38. The 0.0434 tax is the business and occupation tax. The gas tax is on the value more than \$5,000 per year.
39. For production where average daily rate is in excess of 10 Stb/d for preceding calendar year.
40. For production where average daily rate is less than or equal to 10 Stb/d for preceding calendar year.
41. Oil and gas conservation tax. Rate varies, with maximum 0.0004.

Cash Flow Estimation Bias

As stated at the beginning of this chapter, cash flow estimation bias is the most critical—and the most difficult—part of the capital investment (project evaluation) process. Cash flow components must be forecast many years into the future, and estimation errors are bound to occur.¹⁵

Input into project economics is contributed by many departments. It is likely that, in the process, some departments will provide conservative estimates while some others will provide optimistic estimates. In totality, the end result should balance out. Similarly, since large companies evaluate and accept many projects every year, and as long as the cash flow estimates are unbiased and the errors are random, the estimation errors may tend to cancel each other out.

Several studies indicate capital budgeting cash flow forecasts are biased. Many departments tend to be overly optimistic in their forecasts, and as a result, revenues tend to be overstated and costs understated. The result is an upward bias in net operating cash flows and thus an upward bias in estimating the profitability of the proposed project. This practice could be dangerous in evaluating projects with borderline profitability.¹⁶

Many times, division managers/engineers become emotionally attached to their proposed projects, and hence unable to objectively assess their project's potential negative factors. Realizing biases may exist, economists or project evaluators should develop historical data on the forecast accuracy of the projects already completed. This information should then be considered in the project evaluation process. The data highlights the departments consistently providing either optimistic or pessimistic data. The economists can then adjust this situation, or at least it will provide the highs and lows.

This information can be incorporated into the decision analysis process (to be discussed in volume 2 of this series).

A first step in recognizing cash flow estimation bias, especially for projects estimated to be either extraordinarily profitable or below expectation profitability, is to ask this question: What is the underlying cause of the project's unexpected profitability (high or low)? If the company has some inherent advantage, such as patent protection, unique project execution methods, unique manufacturing advantage, or marketing expertise, then the projects utilizing such an advantage may truly be extraordinarily profitable. At the end of the day, the project evaluators need to have experience in identifying the right information and/or anomalies in the data provided to them for evaluation.

Cash Flow Analysis of Foreign Investments

Cash flow analysis is much more complex when capital investments are being made in a foreign country. The relevant cash flows from a foreign investment are the dollar equivalent cash flows to be returned, or repatriated, to the parent company. Since in most cases, the cash flows are earned in the currency of the foreign country, they must be converted to dollars, and thus they are subject to future currency exchange rate fluctuations.

Fortunately, in the oil and gas industry, most transactions are in U.S. dollars. However, many cash flow items are either paid or received in local currency. For example, most of the operating expenses are paid in the local currency and some of the locally produced capital equipment to be used in the project is billed in local currency. The services provided locally, like helicopter and marine service for offshore operations, are usually billed in local currency. Similarly, if the product produced by the project is predominantly absorbed in the local market, the demand for this product is affected if the local currency is weakened/devalued. Since the company has to maintain its return in dollars, the product price in local currency will be increased to compensate for the devaluation. Thus, the consumers have to pay more for the same product. The revenue generated from the sale of the product in the local market when converted into the U.S. dollars at the weaker local currency results in fewer dollars than forecasted, even if the demand for the product stays the same. Therefore, the dollar cash flows and hence the value of the investment would decline.

The currency exchange rates change daily, and long-term trends associated with inflation and other fundamental factors occur, though not in a predictable manner. The currency crises of Malaysia, Indonesia, Turkey, Korea, and Taiwan are typical examples. Therefore, over the economic life of

a project, substantial changes could occur, but it is exceedingly difficult to predict what they will be. These exchange rate changes, beside other factors, make international capital investment quite complex, thus increasing the riskiness of foreign investments.

Following is an example of a *true/real* investment situation. An American investor made a \$12,500,000 real estate investment in a foreign country on 1/1/1990. At the time, the local currency equivalent amounted to LC 250,000,000. At the end of 1996, the property was sold for LC 575,000,000, at which time \$1 was equal to LC 40. Do you think it was a sound investment? Not at all! By analyzing the investment, we find the investor made some 12.63% return [$250,000,000 \text{ cv } (1 + 0.1263)^7 = 574,802,432.65 \approx 575,000,000$] in the local currency. However, when the funds were repatriated, he ended up with \$14,375,000 ($575,000,000 \div 40 = 14,375,000$), which is a return of only 2%. On the other hand, if the \$12,500,000 were invested in a bank at an average interest rate of 5% (net of income tax), his net worth at 12/31/1996 would have been \$17,588,755 (i.e., a net opportunity loss of \$3,213,755).

Similarly, different countries have different local tax regimes to be incorporated into the project cash flow. On the other hand, in the United States, the taxes paid in the foreign country are deductible against the taxes to be paid in the United States.

Cash Flow Analysis and Inflation

It is a usual practice to incorporate estimates of future inflation in cash flow projections of revenues and costs. This reflects an expectation that the different elements discussed in the preceding section, which form the basis of the project cash flow, will be larger in the future years than they are now because of inflation. Note that any reserves evaluation reports submitted to the Securities and Exchange Commission (SEC) are required to be in constant (unescalated or uninflated) terms.

In cash flow projections, simple assumptions about the general anticipated level of inflation are normally acceptable. For example, it may be assumed that the capital cost, OPEX, and product prices will all inflate (escalate) at 3% per year or so. Each stream may be subject to a different escalation factor (i.e., product prices escalating at 2% per year, capital cost escalating at 5% per year, and OPEX escalating at 3% per year). Depending upon the assumptions made, the escalations may be applied from start of the project to end. They may be kept constant for some years and then escalated onward, and they may be subject to a ceiling. For example, oil price may be kept constant at \$75/barrel until year 2010; from 2011, it is escalated at a rate of 3% to a

maximum of \$100/barrel (i.e., the price will be \$77.25 in 2011, \$79.56 in 2012, \$81.95 in 2013, and so on).

Therefore, the cash flow analysis starts by making estimates of capital, OPEX, and product prices in today's terms (that is, as if they were all incurred in the current year) and then inflating or escalating them year by year based on the inflation projection.

The escalated costs, prices, and cash flow discussed above are referred to as *inflated*, *nominal*, or *money-of-the-day* costs, prices, and cash flow. These are distinguished from today's *real*, *unescalated*, *constant*, or *deflated* costs, prices, and cash flow. Real or deflated quantities represent a measure of the purchasing power of a future amount of money in today's terms.

If the inflationary expectations are embedded in the cash flow model, the resulting net cash flow is called *nominal net cash flow*. On the other hand, if the inflationary expectations are not embedded in the cash flow model, the resulting net cash flow is called *real net cash flow*. The nominal net cash flow can be converted to real net cash flow by deflating it at the projected inflation rate, provided each element has been escalated at the same rate.

It is important that each element of the cash flow is escalated separately, rather than escalating the net cash flow. This is because the bottom line economics are dependent on the sensitivity of each element to the escalation rate. For example, a high-capacity oil producer will respond differently to a 2% escalation than a low-capacity producer at the same rate. Similarly, a project with a high unit operating cost will be more sensitive to escalation compared to a low unit operating cost project.

The escalation rates might correspond to the general levels of inflation as measured, for instance, by the Consumer Price Index or some similar indicator.

The Cost of Capital

As mentioned in the previous chapters, most companies today use discounted cash flow (DCF) techniques for project evaluation. The method involves forecasting the future cash flows, choosing the appropriate cost of capital (also referred to as the discount rate), and finding the present value of the forecasted net cash flows. The NPV is defined as the present value of receipts less the present value of disbursements. If the NPV at the project's cost of capital is positive, then accepting the project adds value to the firm and vice versa.

Assume that estimates on the future cash flows of a proposed project are sound. The success of the discounted cash flow technique then depends on how well the analysts choose the discount rate. If a picked rate is too high, projects that add value to the firm will be unnecessarily rejected. On the other hand, if

the discount rate is too low, projects that do not add value to the firm will be accepted. Therefore, choosing the appropriate discount rate is as important as the estimation of appropriate future cash flows. For detailed discussion, the reader is referred to an excellent book by Michael C. Ehrhardt.¹⁷

There are several different types of discounted cash flow approaches, and each has a different specification of cash flows and a different appropriate discount rate. The *weighted average cost of capital* approach is highly recommended by some experts. To implement this approach, one has to first estimate the costs of the various sources of financing (including equity). In general, these costs are a function of the riskiness of the project being financed and the amount of debt used to finance the project. The technique involves

- *Estimating the current cost of each source of funds.* This is not the historical cost of the capital raised, but the marginal cost of the incremental funds required for the project.
- *Finding the weighted average of the costs.* These separate sources of capital are based on the market values of the sources of financing. The result is the weighted average cost of capital.

The different sources of financing include the cost of (1) common equity (i.e., the return the shareholders expect the project to earn on their money), (2) preferred stock, (3) debt, and (4) other elements in the capital structure. The mixture of these elements is known as *capital structure*. The ratio of borrowed funds to either total funds or shareholders' equity is known as the *gearing* ratio. A firm is *highly geared* when it has a larger proportion of funds borrowed from intermediaries in relation to its share capital. Alternatively, *low gearing* occurs when a large bulk of the borrowed funds is in the form of share capital. The higher the level of capital gearing, the greater the risk associated with the project.

It is important to differentiate between two types of risks: project risk and financial risk. Project risk is the inherent uncertainty surrounding the level of pre-tax profits that will be generated by the project. It arises because it is usually impossible to accurately forecast several variables that form the project cash flow. Financial risk is the additional risk introduced because of gearing in the capital structure. The level of fixed interest financing in the capital structure determines the financial risk. The probability of cash insolvency increases with the level of gearing used by the firm, as does the variability of the earnings available for ordinary shareholders. The fixed interest loans have to be paid regardless of whether the company is making profit, but paying dividends to the providers of share capital can be avoided in poor years.

Determining the appropriate cost of each source is beyond the scope of this book, and analysts will not normally be required to do so. The finance department or the corporate planning department of the firm normally specifies the appropriate discount rate. In the oil and gas industry, the general trend is to use a discount rate of 10%–12%.

The Cash Flow Spreadsheet

Once all the data is collected, it is put together in a certain interpretive form to enable the analysts and management to easily comprehend the profitability of the project. Many commercial software packages are available to take care of this task, such as:

- eXpress and CONTINUUM— Oil and gas reserve evaluation applications (OGRE Systems)
- EUREKA—Oil and gas economic evaluation software (Richard J. Miller and Associates, Inc.)
- Petrocalc3: Reservoir Economics and Evaluation (disc and manual) (Gulf Publishers)
- Merak Peep—Petroleum economic evaluation and decline analysis software (Schlumberger)

In addition to these commercially available economic evaluation applications, many major oil companies have their own software tools. Nevertheless, there are situations when the analysts have to generate their own tools for analysis. One basic approach used by many analysts is to develop economic evaluation spreadsheets (or worksheets as they are sometimes called) in Excel. Table 3–17 shows a cash flow for a typical North American producing lease, covering a simulated span of 16 years.

The output can be formatted in many different ways to adapt the spreadsheet to particular needs.

The cash flows typically show project identification information (such as well/lease name, API number, location, field name, county, state, operator, reserve category, etc.). A description of the various columns of the spreadsheet in table 3–17 follows.

1. Columns B and C list the gross (wellhead) oil and gas or other products (such as propane, butane, ethane, sulfur, etc.) production forecast to EL. Using the methods discussed in this chapter helps to generate the forecast.

Table 3–17. Typical (before federal income tax) cash flow

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O																		
1	RESERVES AND ECONOMICS																																
2	AS OF JANUARY 1, 2011																																
3	Proved Undeveloped																																
4	Sun Oil Company																																
5	Ref: Well #2-25 (Sec. 25-T5N-R65W)																																
6	Weld County, Colorado																																
7	Gross Production			Net Production			Product Prices			Expenditure			Operating Revenue			Cum. NCF																	
8	Oil (MStb)			Gas (MMScf)			Oil (\$/Stb)			Gas (\$/MMScf)			Capital (M\$)				Taxes (M\$)			TOTAL (M\$)			Gross (M\$)			Net (M\$)			Cum. NCF (M\$)				
9	Year			(MStb)			(MMScf)			(\$/Stb)			(\$/MMScf)			(M\$)			(M\$)			(M\$)			(M\$)			(M\$)			(M\$)		
10	2011	-	-	-	-	-	-	-	-	60.00	5.50	5.50	0.00	0.00	750.00	0.00	0.00	750.00	0.00	480.65	417.85	(750.00)	(750.00)	(715.10)									
11	2012	5,174	43,432	4,527	38,003	60.00	5.50	5.50	24.35	38.45	62.80	480.65	417.85	(332.15)	(352.91)																		
12	2013	2,393	22,860	2,094	20,003	60.00	5.50	5.50	24.35	18.85	43.20	235.65	192.45	(139.70)	(201.26)																		
13	2014	1,820	16,407	1,593	14,356	60.00	5.50	5.50	24.35	13.96	38.31	174.51	136.20	(3.50)	(103.70)																		
14	2015	1,207	10,801	1,056	9,451	60.00	5.50	5.50	24.35	9.23	33.58	115.35	81.77	78.27	(50.45)																		
15	2016	0,971	8,625	0,850	7,547	60.00	5.50	5.50	24.35	7.40	31.75	92.49	60.74	139.01	(14.49)																		
16	2017	0,812	7,181	0,711	6,283	60.00	5.50	5.50	24.35	6.18	30.52	77.19	46.67	185.67	10.63																		
17	2018	0,698	6,152	0,611	5,383	60.00	5.50	5.50	24.35	5.30	29.65	66.25	36.60	222.28	28.54																		
18	2019	0,650	5,381	0,569	4,708	60.00	5.50	5.50	24.35	4.80	29.15	60.02	30.87	253.15	42.27																		
19	2020	0,631	5,048	0,552	4,417	60.00	5.50	5.50	24.35	4.59	28.94	57.42	28.48	281.63	53.78																		
20	2021	0,591	4,728	0,517	4,137	60.00	5.50	5.50	24.35	4.30	28.65	53.78	25.13	306.76	63.02																		
21	2022	0,572	4,576	0,501	4,004	60.00	5.50	5.50	24.35	4.16	28.51	52.05	23.54	330.30	70.89																		
22	2023	0,569	4,552	0,498	3,983	60.00	5.50	5.50	24.35	4.14	28.49	51.78	23.29	353.59	77.96																		
23	2024	0,567	4,536	0,496	3,969	60.00	5.50	5.50	24.35	4.13	28.48	51.60	23.12	376.71	84.35																		
24	2025	0,560	4,480	0,490	3,920	60.00	5.50	5.50	24.35	4.08	28.42	50.96	22.54	399.24	90.01																		
25	2026																																
26	Sub	17,215	148,759	15,063	130,164				750.00	340.87	129.58	1,619.69	399.24																				
27	Rem	0.000	0.000	0.000	0.000				0.00	0.00	0.00	0.00	0.00																				
28	Total	17,215	148,759	15,063	130,164				750.00	340.87	129.58	1,619.69	399.24																				
29	Initial Wt Fraction						Lease Operating Exp. (\$/Month)						Net Present Value @10% (M\$)																				
30	Final Wt Fraction						Production Start Date						Rate of Return (ROR)																				
31							01/01/12						15.00%																				
32							Oil Severance Tax (Percent)						Profitability Index																				
33	Initial Net Oil Fraction						8.00						1.126																				
34	Initial Net Gas Fraction						8.00						6.58																				
35	Final Net Oil Fraction						0.00						20%																				
36	Final Net Gas Fraction						0.00						25%																				
37							0						41.79																				
38							9,778						97.96																				
39													51.81																				
H:\Spreadsheet Applications\Volume 1\Chapter 3.xlsx\TABLE 3-17																January 11, 2011																	

2. Columns D and E list the net product forecast, net to the interest owner whose interest is evaluated or net to the total lease (net revenue interest).
3. Columns F and G list the oil and gas prices.

4. Columns H through J list the CAPEX, LOE, and taxes (severance and ad valorem), respectively. The CAPEX and OPEX correspond to the working interest (WI) under evaluation. The CAPEX column may be further subdivided into drilling CAPEX, facility CAPEX, and so on. Similarly, the LOE column may be further subdivided into fixed cost, variable cost, workovers cost, and so on.

State and local taxes, which are directly related to the property under evaluation, are itemized in column J. In North America, these are normally severance taxes (also referred to as production taxes). This is a form of sales tax applied to oil and gas sales (revenues) levied by the state and ad valorem taxes assessed by other local jurisdictions. In western Canada, where the government is both the royalty owner and the taxing authority, the royalty (referred to as Crown royalty, normally a sliding scale) and taxes are normally lumped together under the term government take.

The practice of including the state and local taxes as part of the direct operating expenditure as is done in column J is quite common. Income and other general taxes pertaining to the corporation as a whole are generally not recognized (in the before income tax cash flows) specifically as part of the operating costs at the well/lease level.

5. Column K lists the total cash outflow, which is the sum of columns H, J, and I.
6. Column L lists the gross revenue (i.e., before any expenses are deducted) corresponding to the net production in columns D and E (i.e., 100% WI and 87.5% NRI in this particular case). This is the sum of column D times column F and column E times column G.
7. Column M lists the net revenue (also referred to as the net cash flow), which is column L less column K. The net cash flow (in this case, before income tax) is an important indicator of how the operation is progressing year-by-year from a business standpoint.
8. Column N lists cumulative cash flow, the cumulative of column M by year.
9. Column O lists the cumulative discounted net cash flow. As discussed in chapter 2, the net cash flow generated in each year has to be brought to its equivalent of present value. This is obtained by multiplying the entries in column M by $[1/(1 + 0.1)^{(t - 0.5)}]$, where 0.1 (10%) is the discount rate as discussed in the preceding section.

This factor assumes that cash flows are received and disbursed in the middle of a year. Therefore the discounting in the first year is over half a year. Some companies use a year-end discounting assumption, in that case the entries in Column M will be multiplied by $(1 + 0.1)^{-t}$.

The cash flow shown in table 3–17 is included on the CD-ROM included with volume 2 in this series. In addition, a more detailed cash flow model is also included as a part of ECONPAC on the CD-ROM.

Questions and Problems

- 3.1 Your company has recently completed the following projects. What is the scaling factor (economy of scale exponent) based on this information?
- 300 MMScf/d project for 1,500 MM\$
 - 500 MMScf/d project for 1,997 MM\$
- a. 0.50
 - b. 0.60
 - c. 0.56
 - d. 0.65
- 3.2 Based on the scaling factor calculated in problem 3.1, what will be the approximate cost of a 750 MMScf/d project?
- a. 2,446 MM\$
 - b. 2,547 MM\$
 - c. 2,506 MM\$
 - d. 2,599 MM\$
- 3.3 If the projects in problem 3.1 were completed three years ago, what will be the cost of the project in problem 3.2 if the costs have escalated by 3% per year?
- a. 2,673 MM\$
 - b. 2,783 MM\$
 - c. 2,738 MM\$
 - d. 2,840 MM\$

- 3.4 If the project in problem 3.3 requires an additional nitrogen separation unit that costs 50 MM\$, what will be the cost of this new project?
- a. 2,723 MM\$
 - b. 2,833 MM\$
 - c. 2,788 MM\$
 - d. 2,890 MM\$
- 3.5 What will be the gross annual oil revenue based on 10 MBOD at a price of \$60/barrel?
- a. 200 MM\$
 - b. 219 MM\$
 - c. 229 MM\$
 - d. 239 MM\$
- 3.6 What will be the annual gross revenue if the oil produced in problem 3.5 has an associated gas production of 10 MScf/Stb with 1,000 Btu/Scf and the gas price is \$1.25/MScf?
- a. 45.63 MM\$
 - b. 49.28 MM\$
 - c. 55.73 MM\$
 - d. 65.83 MM\$
- 3.7 What will be the annual gross revenue if the gas in problem 3.6 is 1,080 Btu/Scf?
- a. 45.63 MM\$
 - b. 49.28 MM\$
 - c. 55.73 MM\$
 - d. 65.83 MM\$
- 3.8 A field is producing 750 MMScf/d of wellhead gas of 1,175 Btu/Scf and a sales gas of 600 MMScf/d of 1,080 Btu/Scf. If the price of sales gas is \$1.5/MMBtu, what will be the gross annual gas revenue?
- a. 354.78 MM\$
 - b. 328.50 MM\$
 - c. 443.48 MM\$
 - d. 410.63 MM\$

- 3.9 What will be the variable operating cost for the project in problem 3.8 if the variable cost is \$0.25/MMBtu of wellhead gas?
- a. 73.91 MM\$
 - b. 59.13 MM\$
 - c. 64.33 MM\$
 - d. 80.41 MM\$
- 3.10 What will be the annual fixed operating cost if the total facilities CAPEX of a project is 2,506 MM\$ and the fixed cost is 4% of this CAPEX?
- a. 110.24 MM\$
 - b. 100.24 MM\$
 - c. 105.25 MM\$
 - d. 95.25 MM\$
- 3.11 What will be the annual workover cost per well (based on total number of wells) if a field is producing from 50 wells, on the average 10% of the wells require workover every year, and each workover costs 1.25 MM\$?
- a. \$135,000 per well
 - b. \$145,000 per well
 - c. \$125,000 per well
 - d. \$105,000 per well
- 3.12 What will be the net annual revenue based on the answers you have generated in problems 3.8 to 3.11? The CAPEX is spent in the years before the start of production.
- a. 137.60 MM\$
 - b. 167.88 MM\$
 - c. 271.85 MM\$
 - d. 245.80 MM\$
- 3.13 Calculate the EL (economic limit) in barrels per month for an oil lease with $WI = 100\%$, $NRI = 87.5\%$, lease operating expense = \$5,000/month, price of oil = \$60/Stb, price of associated gas = \$4.25/MScf, gas-oil ratio = 500 Scf/Stb, and the severance tax for oil and gas is 7.085% each.

- 3.14 Calculate the EL in MScf per month for a gas lease with
WI = 100%, NRI = 87.5%, lease operating expense = \$5,000/month,
condensate price = \$60/Stb, gas price = \$4.25/MScf, condensate
yield = 5 Stb/MScf, and the severance tax for gas and condensate is
4.5% each.
- 3.15 An oil well has been drilled and completed. The productive zone
has been encountered at a depth of 7,815–7,830 feet. The log
analysis showed an average porosity of 15% and an average water
saturation of 35%. The oil formation volume factor was determined
from empirical equations to be 1.215 RB/Stb. Experience shows
that other reservoirs of about the same properties drained 80
acres with a recovery factor of 12%. Compute the STOIP and the
ultimate recovery.
- 3.16 The initial rate and EL of the well in problem 3.15 is 1,680 Stb/
month and 200 Stb/month, respectively. At what exponential
decline rate will it recover the reserves calculated in problem 3.15?
How long it will take the well to reach the EL?
- 3.17 A gas well is completed at a depth of 8,550 feet. The log analysis
showed total formation thickness of 12 feet of 16% porosity and
30% water saturation. On potential test, the well produced dry
gas with a specific gravity of 0.75. The reservoir pressure was
determined from a drill stem test (DST) to be 3,850 psi and the log
heading showed a reservoir temperature of 155° F. The gas will be
produced at the surface where the standard pressure is 14.65 psi
and the standard temperature is 60° F. The study of the offset wells
producing from the same formation has shown that the wells are
capable of draining 160 acres at a recovery factor of 85%. Compute
the GIIP and the recoverable gas reserves. The gas formation
volume factor is 259.89 Scf/CF.
- 3.18 In problem 3.17, the abandonment pressure has been determined
to be 800 psi below which no gas could be economically produced.
Based on this information, compute the ultimate gas recovery and
the recovery factor. The formation volume factor at abandonment
pressure is 51.88 Scf/CF.

- 3.19 An oil well has produced consistently for the last five years. The monthly oil production for each year is given in the table below. It costs \$2,000 per month to operate the well, the current oil price is \$60/Stb, and the severance tax is 7.085% of the gross value. The well is not producing any associated gas. Compute the following:
- The EL based on 100% WI and 87.5% NRI
 - The exponential decline based on a graphed plot of production versus time
 - The remaining oil reserves as of 1/1/2011
 - Production rate at the end of 2011
 - The remaining life of the well
 - Ultimate oil recovery
 - Production for 2011, 2012, and 2013

Month	Oil Production, Stb				
	2006	2007	2008	2009	2010
January	2,399	1,882	1,637	1,334	1,151
February	2,059	1,621	1,265	1,177	1,028
March	1,808	1,536	1,386	1,208	1,151
April	1,627	1,551	1,310	1,191	1,075
May	1,699	1,526	1,300	1,370	1,081
June	1,575	1,553	1,241	1,315	983
July	1,618	1,514	1,289	1,268	1,037
August	1,660	1,428	1,299	1,312	959
September	1,575	1,320	1,240	1,224	989
October	1,538	1,534	1,290	1,229	897
November	1,541	1,309	1,240	1,202	1,044
December	1,702	1,474	1,348	1,229	996

- 3.20 Confirm the reserves obtained in problem 3.19 by using the rate-cumulative oil production plot.
- 3.21 In problem 3.19, generate the cash flow to the EL of the lease.

- 3.22 A gas well is producing from the Morrow formation at a depth of 6,462 feet in the Camrick gas area, Texas County, Oklahoma. The productive interval is perforated from 6,462 to 6,593 feet. The reservoir temperature is 162° F and the gas gravity is 0.70. Based on the following pressure test data, compute the following:
- Initial gas in place (IGIP)
 - Ultimate gas recovery, assuming an abandonment p/z of 500 psi
 - Recovery factor
 - Remaining gas reserves as of 06/04/2009 to an abandonment pressure of 243 psi ($p/z = 250$ psi)

Test Date	Cumulative to Test (BScf)	BHP/z, psi
07/15/1997	1.774	1,546
07/15/2000	2.305	1,366
07/15/2003	3.151	1,102
07/19/2006	4.071	862
06/04/2009	4.808	612

- 3.23 A lease owned by company XYZ has to be evaluated. The lease is currently producing at a rate of 4,000 Stb/month. It is estimated that production will be constant for one year, and then decline exponentially at a rate of 30% per year. Current operating expenses are \$15,000 per month. Oil price is \$60/Stb, and severance tax is 12% of gross revenue. The company has a 50% working interest and a corresponding net revenue interest (NRI) of 40%. Determine:
- Economic Limit (EL)
 - Remaining 8/8ths reserves
 - Remaining net reserves (your company's share)
 - The economic productive life of the lease
 - The cash flow to the EL

3.24 A 320-acre lease has to be evaluated. Data on the lease are as follows:

- Average porosity = 15%
- Net pay = 15 ft
- Water saturation = 30%
- Reservoir oil formation volume factor = 1.2
- Initial producing rate = 9,026 Stb/month
- Economic limit = 200 Stb/month
- Recovery factor = 20%

Determine by calculations the following:

- a. Stb initially in place
- b. Recoverable reserves
- c. Exponential decline rate required to recover the reserves
- d. Life of field, years
- e. Average producing rate after five years, Stb/month
- 3.25 Calculate the cost of a nine-slot wellhead jacket if a three-slot wellhead jacket for the same location costs \$6 million. The correlation exponent in the area is 0.50.
- 3.26 The XYZ Oil Company acquires an oil and gas lease from a landowner Mr. A. The mineral deed shows Mr. A will be entitled to landowner’s royalty interest (LORI) of 1/8th. The geologist who found the area retains an ORRI of 1.5%. Calculate the NRI of the XYZ Oil Company assuming the company has a 100% working interest.
- 3.27 If the XYZ Oil Company in problem 3.26 shares the working interest with three other investors as follows, compute the NRI of each investor assuming the same LORI and ORRI are applicable.

Investors	WI Share, %
A—XYZ Oil Company	50%
B—ABC Oil Company	25%
C—Gas Producers, Inc.	15%
D—Dome Oil and Gas, Inc.	10%

- 3.28 The XYZ Oil Company has successfully completed an oil well for a total cost of \$2,400,000. The well will produce 30,000 barrels of oil in the first year, 18,000 barrels in the second year, and 10,800 barrels in the third year. The selling price of oil is \$65 per barrel, and the gross operating expenses for the first year are \$50,000, \$46,000 for the second year, and \$40,000 for the third year. The XYZ Oil Company has a 50% WI with a net lease of 80%. The ABC Oil Company has a 25% WI in the lease that reverts to 30% WI after the XYZ Oil Company recovers its share of the drilling and completion costs. What is the payout amount at the beginning of the first year? After how long will the interest of ABC Oil Company revert? Assume the operating costs include the applicable production taxes.
- 3.29 Generate a cash flow for the information given below..
- 3.30 Assume (a) initial production rate is 375 Stb/month and declining exponentially at 20% per year, (b) the price of oil is \$60/Stb, (c) severance tax is 7%, (d) working interest is 100%, (e) net revenue interest is 87.5%, and (f) monthly lease operating cost is \$3,700. The total cost of drilling and completing the well is \$350,000. Also calculate the productive life of the field and the economic limit.

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4

After-Tax Cash Flow Model

Up to this point it has been emphasized that investment decisions should be based on cash flows. In chapter 3, a cash flow model without any reference to corporate taxation was presented. Federal income tax is a disbursement that must be included in the economic evaluations. Although its inclusion in investment analysis increases their complexity, performing economic evaluations without accounting for tax effects is misleading. In some cases, incorporating taxes in economic evaluation may reverse the decisions based on the before-tax cash flows.

The interest payments on debt and the depreciation, depletion, and amortization (DD&A) of assets do influence the value and timing of the taxable income and the resulting tax payment (actual cash *outflows*). These items are known as tax-deductible items. Corporate taxes may not affect all investments to the same degree. Therefore, for meaningful comparisons, project cash flows must always be expressed on an after-tax basis. The variables required in addition to the ones discussed in chapter 3 to arrive at after federal income tax (AFIT or ATAX) cash flow are discussed in this chapter.

The tax laws vary considerably from country to country due to small business allowances, investment tax credits, adjustments for double taxation, and so on. Tax rates also vary with time. It is worth noting, however, that the corporate tax structures of many countries are similar, and for most countries range between 30% and 40%.

The net after-tax cash flow equals the pre-tax cash flow minus the applicable taxes (a specified percent of the firm's taxable income), mathematically expressed as:

$$\begin{aligned} NCF_{AFIT} &= NCF_{BFIT} - T_c(NCF_{BFIT} - DD\&A - I) \\ &= (1 - T_c)NCF_{BFIT} + T_c(DD\&A + I) \end{aligned} \quad (4.1)$$

where

T_c = the corporate tax rate, fraction

NCF_{AFIT} = the after corporate income tax net cash flow

NCF_{BFIT} = the before corporate income tax net cash flow

$DD\&A$ = depreciation, depletion, and amortization

I = interest payment on debt (loan)

The after-tax cash flow is central to all financial decision problems and can be derived as follows:

1. Calculate the project's net income by subtracting the operating costs, severance and ad valorem taxes, depreciation, intangible drilling costs (IDCs), and interest on debt to arrive at the taxable income before depletion. The concept of interest deduction is further discussed in the following pages. There are different ways of handling interest payments in cash flows; these will be discussed in chapter 6.
2. Subtract the allowable depletion (cost depletion or percent depletion) allowance (discussed in the following sections) to arrive at the taxable income after allowable depletion.
3. If the taxable income is negative (i.e., there is no tax payment), the loss is carried forward to the following years. This will depend on the size of the project and/or the company. If the company/investor has other operations from which the cash flow is positive, then negative tax on the project should be accounted for. This is because the negative cash flow of the project will reduce the total taxable income of the investor and hence the tax liability.
4. Calculate the tax liability by multiplying the positive taxable income (or negative taxable income as explained in step 3 above) by the applicable corporate tax rate.
5. Calculate the after-tax income by subtracting the tax payable from the taxable income.
6. Add the DD&A, less carried forward, and then subtract the CAPEX, loan proceeds, and principal payment on loan to the after-tax income in step 5 to arrive at the net after-tax cash flow.

The topics that form the basis for tax liability calculations and the after-tax cash flow are discussed in this chapter. The discussion is intended to be brief and not an exhaustive analysis of the more complex tax laws. Specific tax questions should be addressed to tax professionals. In its simplest form, the after-tax cash flow projection would be as shown in tables 4-1 and 4-1a. A framework for the development of an AFIT (after federal income tax) cash flow model is illustrated in figures 4-1 through 4-3.

Table 4–1. Simple after-tax net cash flow projection

		Year 1	Year 2	Year 3	Year <i>n</i>
	Gross revenue (8/8ths)	\$	\$	\$	\$
Less	Royalty	\$	\$	\$	\$
	Operating expenditure (OPEX)	\$	\$	\$	\$
	Depreciation	\$	\$	\$	\$
	Intangible drilling costs (IDCs)	\$	\$	\$	\$
	Interest on loan	\$	\$	\$	\$
Equals	Taxable before depletion	\$	\$	\$	\$
Less	Depletion allowance	\$	\$	\$	\$
Equals	Income after DD&A and interest	\$	\$	\$	\$
	Loss carried forward	\$	\$	\$	\$
Equals	Taxable income	\$	\$	\$	\$
Less	Corporate tax @ 30%	\$	\$	\$	\$
Equals	Net after-tax income	\$	\$	\$	\$
Less	Capital expenditure (CAPEX)	\$	\$	\$	\$
	Loan proceeds	\$	\$	\$	\$
	Principal payment on loan	\$	\$	\$	\$
Plus	Depreciation	\$	\$	\$	\$
	Allowable depletion allowance	\$	\$	\$	\$
	Loss carried forward	\$	\$	\$	\$
Equals	After-tax net cash flow	\$	\$	\$	\$

Table 4–1a. Simple after-tax net cash flow projection

		Year 1	Year 2	Year 3	Year <i>n</i>
	Gross revenue (8/8ths)	\$	\$	\$	\$
Less	Royalty	\$	\$	\$	\$
	Operating cost (OPEX)	\$	\$	\$	\$
	Intangible drilling costs (IDCs)	\$	\$	\$	\$
	Capital expenditure (CAPEX)	\$	\$	\$	\$
	Interest on loan	\$	\$	\$	\$
	Principal payment on loan	\$	\$	\$	\$
	Corporate tax	\$	\$	\$	\$
Plus	Loan proceeds	\$	\$	\$	\$
Equals	After-tax net cash flow	\$	\$	\$	\$

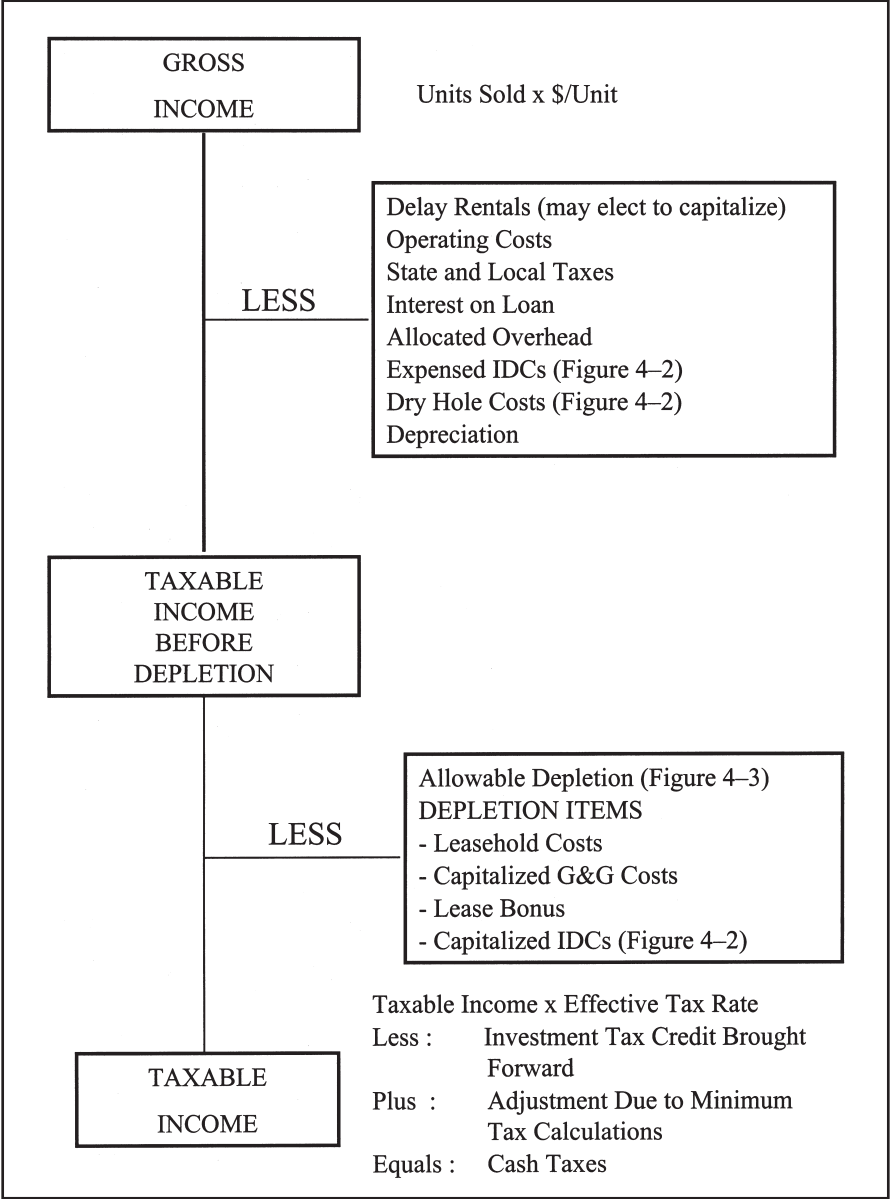


Fig. 4-1. Federal income tax model for oil and gas transactions (after Thompson and Wright)¹

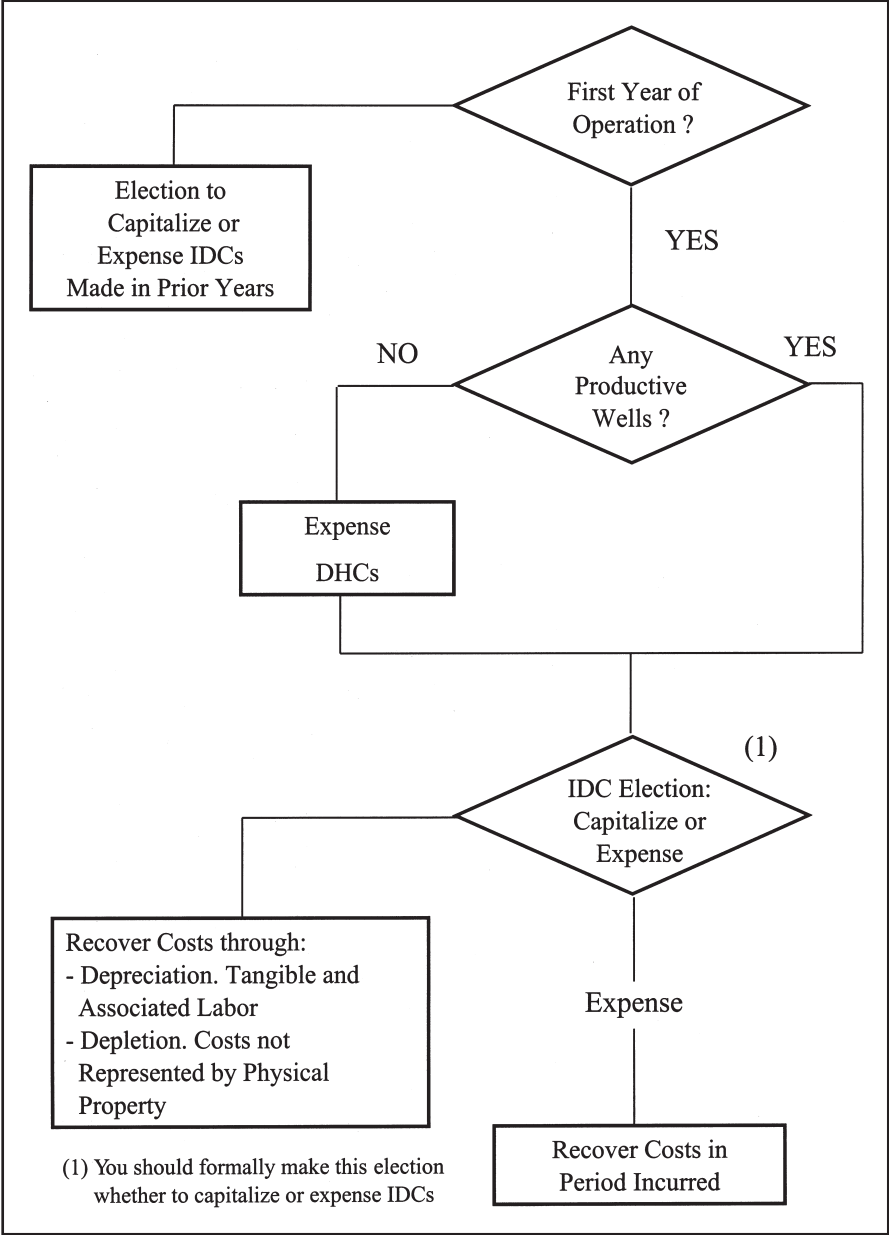


Fig. 4–2. Recovery of drilling and completion costs (after Thompson and Wright)¹

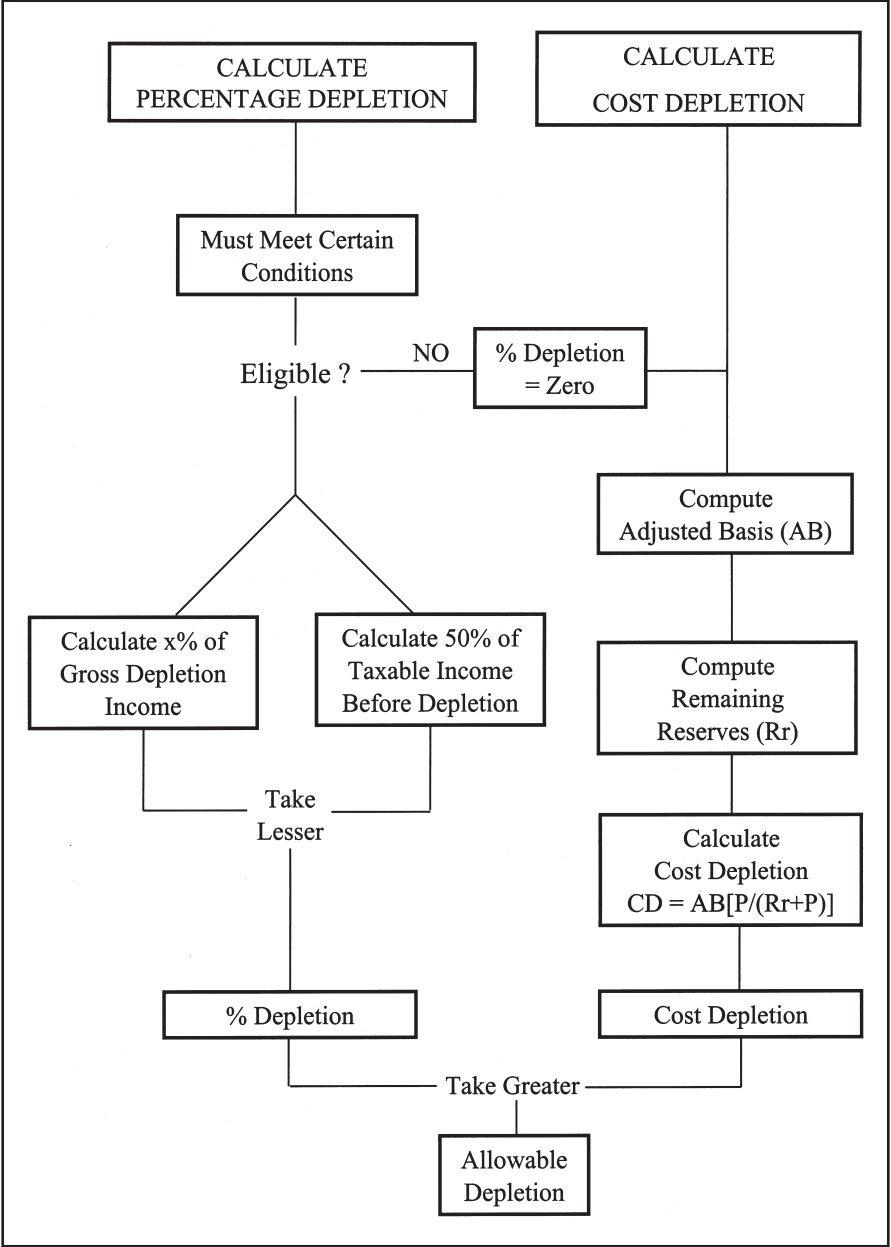


Fig. 4–3. Depletion allowance calculation (after Thompson and Wright)¹

Depreciation

Depreciation is a deductible *non-cash* expense for income tax purposes. The higher the depreciation allowance being deducted in any given year, the lower the taxable income and the cash disbursements (cash outflow) in the form of income tax. As noted earlier, *depreciation is not an actual cash outflow and therefore is not subtracted from the annual cash flow as an expense*. However, the tax deductibility of depreciation decreases the firm's tax liability. Therefore, its inclusion in calculating the project's after-tax cash flow is essential.

Depreciation is defined as a loss in the value of an asset over the time it is being used. Events that can cause property to depreciate include wear and tear, age, deterioration, and obsolescence. The asset subject to depreciation can be either *tangible property* or *intangible property*.

Depreciation of a property begins at the time the property is placed in service for use in trade or business or for the production of income. Depreciation of a property ends when the cost or other basis of the property is recovered or when it is retired (i.e., permanently withdrawn from use in trade or business) from service. The property is retired from service by selling or exchanging it, abandoning it, or destroying it. The cost of a property is fully recovered when its depreciation deductions are equal to the cost or investment in the project.

Tangible property

Tangible property is that which can be seen or touched. Tangible property includes two main types: real property and personal property. Real property is land and buildings and, generally, anything built or constructed on land or anything growing on or attached to the land. Land by itself can never be depreciated because it does not wear out or become obsolete and it cannot be used up. Personal property includes cars, trucks, machinery, furniture, equipment, and anything that is tangible except real property.

Intangible property

Intangible property is generally any personal property that has a value but cannot be seen or touched. It includes items such as copyright, franchises, patents, trademarks, and trade names. Intangible property must either be amortized or depreciated using the straight-line method.

Unless the costs of patents or copyrights are amortized, the costs can be recovered through depreciation. The useful life of a copyright or patent is the life granted to it by the government. If it becomes valueless in any year

before its useful life expires, the remaining cost can be deducted in full for that year. If patents or copyrights were acquired as part of the acquisition of a substantial portion of a business after August 10, 1993 (or after July 25, 1991, if elected), the cost must be amortized over 15 years. If these are not acquired as part of an acquisition of a substantial portion of a business, the costs must be depreciated as mentioned above.

A franchise can be depreciated only if it has a determinable useful life and cannot be amortized. If a franchise was acquired after August 10, 1993 (after July 10, 1991, if elected), the cost must be amortized over 15 years.

For property to be depreciable, it must first meet all of the following basic requirements:

1. The property must be used in business or held to produce income.
2. The property must have a determinable useful life longer than one year.
3. The property must be something that is subject to wearing out, decaying, being used up, becoming obsolete, or losing its value from natural causes.
4. If a repair or replacement increases the value of a property, makes it more useful, or lengthens its life, the repairs or replacement cost must be capitalized and depreciated. If the repair or replacement does not increase the value of a property, make it more useful, or lengthen its life, the cost of the repair or replacement is deductible in the same way as any other business expense.

For additional details on depreciation and how to depreciate property, refer to Publication 946 of the U.S. Internal Revenue Service.

Modified accelerated cost recovery system

The modified accelerated cost recovery system (MACRS) is the name given to tax rules for getting back (recovering) through depreciation deductions the cost of a depreciable property. These rules generally apply to tangible property placed in service after 1986. MACRS consists of two main systems that determine how a property can be depreciated. The main system is called the *general depreciation system* (GDS), while the second system is called the *alternative depreciation system* (ADS). Unless the law specifically requires ADS or you elect it, GDS is generally used. The main difference between the two systems is that ADS generally provides for a longer recovery period and uses a straight-line method of depreciation.

If a property is depreciated under a method not based on a term of years, such as the *unit-of-production* method, you can elect to exclude that property from MACRS. Once it is determined that a property can be depreciated under MACRS and whether it falls under GDS or ADS, the following information is required to figure out depreciation:

1. Its basis (i.e., the cost paid for the property).
2. Its property class and recovery period. Under MACRS, property is assigned to one of several property classes. These property classes establish the recovery periods (number of years) over which the basis of the property can be recovered. The list of class lives and recovery periods for various oil and gas related properties is given in table 4-2.

Under GDS, most tangible property is assigned to one of the following eight main property classes:

- a. *3-year property*. This class includes tractor units for over-the-road use and any racehorse more than two years old when placed in service.
- b. *5-year property*. This class includes automobiles, taxis, buses, trucks, computers and peripheral equipment, office machinery (such as typewriters, calculators, copiers, etc.), and any property used in research and experimentation. It also includes breeding cattle and dairy cattle.
- c. *7-year property*. This class includes office furniture and fixtures such as desks, files, safes, etc. Any property that does not have a class life and that has not been designated by law as being in any other class is also a 7-year property.
- d. *10-year property*. This class includes vessels, barges, tugs, similar water transportation equipment, any single-purpose agricultural or horticultural structure, and any tree or vine bearing fruits or nuts.
- e. *15-year property*. This class includes certain depreciable improvements made directly to land or added to it, such as shrubbery, fences, roads, and bridges.
- f. *20-year property*. This class includes farm buildings (other than agricultural or horticultural structures).
- g. *Residential rental property*. This class includes real property such as rental home or structure (including a mobile home) if 80% or more of its gross rental income for the tax year is for dwelling units. The recovery period for this property is 27.5 years.

- h. *Nonresidential real property.* The recovery period for nonresidential real property is 39 years if it is placed in service after May 12, 1993; otherwise, it is 31.5 years.

Table 4–2. Class lives and recovery periods for various oil and gas properties

Description of Assets Includes	Class Life (in years)	Recovery Periods (in years)	
		GDS MACRS	ADS
1. Office Furniture, Fixtures, and Equipment Includes furniture and fixtures that are not a structural component of a building. Includes such assets as desks, files, safes, and communication equipment. Does not include communication equipment that is included in other classes.	10	7	10
2. Information Systems Includes computers and peripheral equipment used in administering normal business transactions and the maintenance of business records, their retrieval and analysis. Information systems are defined as: a) Computers: A computer is a programmable electrically activated device capable of accepting information, applying prescribed processes to the information, and supplying the results of these processes with or without human intervention. It usually consists of a central processing unit containing extensive storage, logic, arithmetic, and control capabilities. Excluded from this category are adding machines, electronic desk calculators, etc., and other equipment described in Class 3 below. b) Peripheral equipment consists of the auxiliary machines designed to be placed under control of the central processing unit. Non-limiting examples are: card readers, card punches, magnetic tape feeds, high speed printers, optical character readers, tape cassettes, mass storage units, paper tape equipment, keypunches, data entry devices, teleprinters, terminals, tape devices, disc drives, disc files, disc packs, visual image projector tubes, card sorters, and collators. Peripheral equipment may be used on-line or off-line. Does not include equipment that is an integral part of other capital equipment that is included in other classes of economic activity.	6	5	5
3. Data Handling Equipment; Except Computers Includes only typewriters, calculators, adding and accounting machines, copiers, and duplicating equipment	6	5	6
4. Light General Purpose Trucks Includes trucks for use over the road (actual weight less than 13,000 pounds).	4	5	5
5. Heavy General Purpose Trucks Includes heavy general-purpose trucks, concrete ready mix-trucks, and ore trucks over the road (actual unloaded weight 13,000 pounds or more).	6	5	6
6. Offshore Drilling Includes assets used in offshore drilling for oil and gas such as floating, self-propelled and other drilling vessels, barges, platforms, and drilling equipment and support vessels such as tenders, barges, towboats and crew boats. Excludes oil and gas production assets.	7.5	5	7.5
7. Drilling of Oil and Gas Wells Includes assets used in the drilling of onshore oil and gas wells and the provision of geophysical and other exploration services; and the provision of such oil and gas field services as chemical treatment, plugging and abandoning of wells and cementing or perforating well casings. Does not include assets used in the performance of any of these activities and services by integrated petroleum and natural gas producers for their own account.	6	5	6
8. Exploration for and Production of Petroleum and Natural Gas Deposits Includes assets used by petroleum and natural gas producers for drilling of wells and production of petroleum and natural gas, including gathering pipeline and related storage facilities. Also includes petroleum and natural gas offshore transportation facilities used by producers and others consisting of platforms (other than drilling platforms classified in Class 6 above), compression or pumping equipment and gathering and transmission lines to the first onshore transshipment facility. The assets used in the first onshore transshipment facility are also included and consists of separation equipment (used for separation of natural gas, liquids), and liquid holding or storage facilities (other than those classified in Class 12). Does not include support vessels.	14	7	7
9. Petroleum Refining Includes assets used for the distillation, fractionation, and catalytic cracking of crude petroleum into gasoline and its other components.	16	10	16

Table 4–2. (Continued)

	Description of Assets Includes	Class Life (in years)	Recovery Periods (in years)	
			GDS MACRS	ADS
10. Pipeline Transportation	Includes assets used in the private, commercial, and contract carrying petroleum, gas, and other products by means of pipes and conveyers. The trunk lines and related storage facilities of integrated petroleum and natural gas producers are included in this class. Excludes initial clearing and grading land improvements but includes all other related land improvements.	22	15	22
11. Natural Gas Production Plant		14	7	14
12. Liquefied Natural Gas Plant	Includes assets used in the liquefaction, storage, and re-gasification of natural gas including loading and unloading connections, instrumentation equipment and controls, pumps, vaporizers and odorizers, tanks, and related land improvements. Also includes pipeline interconnections with gas transmission lines and distribution systems and marine terminal facilities.	22	15	22

3. Its placed-in-service date.
4. Its convention period. To figure out the depreciation deduction under GDS or ADS, one of the following three conventions is used:
 - a. *The half-year convention.* This convention is generally used for property other than nonresidential real and residential rental property. Under this convention, all depreciable property placed in service or disposed of during a tax year is treated as placed in service or disposed of at the midpoint of the year, regardless of when during the year. First, figure out the depreciation for a full tax year using the method you select. Then apply the half-year convention by taking only half of the total amount calculated for the first year.
 - b. *The mid-month convention.* This convention is used for both nonresidential real property and residential real property. Under this convention, all depreciable property placed in service or disposed of during a month is treated as placed in service or disposed of at the midpoint of the month, regardless of when during the month. First, figure out the depreciation for a full tax year using the straight-line method for residential rental or nonresidential real property. Then multiply this amount by a fraction. The numerator of the fraction is the number of full months in the tax year that the property is in service plus 1/2 (or 0.5). The denominator is 12.
 - c. *The mid-quarter convention.* This convention can apply to any depreciable property (other than nonresidential real property and residential rental property) in certain circumstances. These circumstances occur during any tax year when the total depreciable bases of your MACRS property placed in service

during the last three months of that tax year is more than 40% of the total depreciable basis of all MACRS property placed in service during the entire tax year. When that happens, you must use this convention. To figure the MACRS deduction using this convention, first figure your depreciation for the full tax year. Then multiply by the percentages 87.5%, 62.5%, 37.5%, and 12.5% for the first, second, third, and fourth quarters, respectively, of the tax year the property is placed in service.

5. Its depreciation method. Under MACRS, five methods can be used to depreciate a property:
 - a. The 200% declining-balance method over the GDS recovery period (for non-farm property in the 3-, 5-, 7-, and 10-year property classes), which switches to the straight-line method when that method provides a greater deduction.
 - b. The 150% declining balance method over the GDS recovery period (for property in the 15- and 20-year property classes and property used in farming businesses), which switches to the straight-line method when that method provides a greater deduction.
 - c. The straight-line method over the GDS recovery period used for nonresidential real property and residential rental property, and, if you elect it, property in the 3-, 5-, 7-, 10-, 15- and 20-year classes.
 - d. The 150% declining balance method over fixed ADS recovery periods, which switches to the straight-line method when that method provides a greater deduction.
 - e. The straight-line method over fixed ADS recovery periods.

Accelerated cost recovery system

The accelerated cost recovery system (ACRS) applies to property placed in service after 1980 and before 1987. For property placed in service after 1986, the MACRS method is used. Property depreciated under ACRS was called *recovery property*. The recovery class of property determined the recovery period. Generally, the class life of property placed it in a 3-, 5-, 10-, 15-, 18-, or 19-year recovery class.

Under the ACRS, prescribed percentages were used to recover the unadjusted basis of recovery property. The depreciation deduction was calculated by multiplying the prescribed percentage for the recovery class by the unadjusted basis of the recovery property.

ACRS also provided an alternative ACRS method that could be elected. This alternate ACRS method used a recovery percentage based on a modified straight-line method. Under the ACRS method, only the half-year of depreciation was deducted for the year a property was placed in service. This applied regardless of when in the tax year the property was placed in service. For each of the remaining years in the recovery period, full deduction was taken. If the property was held for the entire recovery period, a half-year of depreciation was allowable for the year following the end of the recovery period.

Straight-line depreciation

Before 1981, one could use a reasonable method for every kind of depreciable property. One of these methods was the straight-line method.

In this method of depreciation, the depreciable cost or cost basis of the property is equally distributed over the useful life of the asset. The following equation is used.

$$D_n = \frac{C - S_v}{n} \quad (4.2)$$

where D_n is the depreciation in n th year, C is the total cost of the depreciable asset, S_v is the salvage value, and n is the useful life of the asset. Salvage value, also referred to as residual value, terminal value, or scrap value, is the estimated value of property at the end of its useful life. It is what one would expect to get for the property if it is sold after it can no longer be used productively. The difference between the total acquisition cost of the asset and its predicted salvage value is sometimes called *depreciable value*.

The salvage value (S_v) is not considered for U.S. federal income tax calculations but it may be applicable for financial accounting calculations. It should be remembered that if a depreciable asset is expected to have any salvage value at the end of the project's economic life and this value exceeds the equipment's book value (original price less accumulated depreciation at that time), a corporate tax liability would be incurred on the profit (net of abandonment costs) from the sale of such equipment. Starting with the 1986 tax year, corporate long-term capital gains are to be taxed as ordinary income and are subject to a 34% tax rate. The tax and sales proceeds appear in the last year of the project's cash flow. Similarly, if equipment is replaced at the beginning or in the middle of a planned project, the transaction should be reflected in the cash flow in the respective period. Tax payments generated by such transactions can be crucial to the investment decision.

Example 4–1

Compute the straight-line depreciation for a pumping unit acquired at a cost of \$35,000 with a salvage value of \$5,000 at the end of its useful life of five years.

$$\begin{aligned} D_n &= \frac{\$35,000 - \$5,000}{5} \\ &= \$6,000 \text{ per year} \end{aligned}$$

Declining balance depreciation

This method is also referred to as the accelerated depreciation method. The method is appropriate when it can be reasonably estimated that the benefits derived from an asset will decline with time. The declining balance depreciation is also used under the MACRS method of depreciation. A fixed value of percentage is applied to the book value (total value of asset less accumulated depreciation of previous years) of the asset each year.

The declining balance depreciation is further divided into different percentages, such as the 200% declining balance method (also referred to as the *double declining balance depreciation*), 175% declining balance, 150% declining balance, and 125% declining balance. The fixed percentage is calculated by dividing the above-mentioned percentages by the recovery period assigned to the asset. The following example clarifies the computations.

Under this method, the salvage value is not subtracted when figuring yearly deductions. However, the property may not be depreciated below its reasonable salvage value. In addition, it is allowed to switch to the straight-line depreciation in the year when this method provides greater deduction over the declining balance method.

Example 4–2

For the pumping unit in example 4–1, calculate 200% declining balance depreciation and switch to straight-line when feasible. Assume zero salvage value at the end of five years.

Solution: Fixed percentage = $\frac{200\%}{5} = 40\% \text{ or } 0.40$

The depreciation schedule is shown in table 4–3. The book value at the beginning of the second year (adjusted basis) is \$21,000 (i.e., \$35,000 – \$14,000 = \$21,000), and so on. The straight-line depreciation is calculated by dividing the book value at the beginning of the year by the remaining life of the asset.

Table 4–3. 200% declining balance with switch to straight-line depreciation

Year	Book Value of Asset at Beginning of Year	Rate	D_n Declining Balance	D_n Straight Line	Deduction Taken
1	\$35,000	0.40	\$14,000	\$7,000	\$14,000
2	\$21,000	0.40	\$8,400	\$5,250	\$8,400
3	\$12,600	0.40	\$5,040	\$4,200	\$5,040
4	\$7,560	0.40	\$3,024	\$3,780	\$3,780
5	\$3,780	0.40		\$3,780	\$3,780
TOTAL					\$35,000

Sum-of-years'-digits depreciation

This method produces a declining depreciation charge each year by applying a declining charge to the total cost of the asset (depreciation base). The declining charge is determined each year by dividing the remaining life of the asset by the sum of the years' digits. For an asset with a five-year life, the sum of years' digits is $5 + 4 + 3 + 2 + 1 = 15$, and the declining charge for the first year is $5/15$, and so on. The following equation is used to calculate the sum of years' digits.

$$SYD = \frac{n(n+1)}{2} \tag{4.3}$$

$$SYD = \frac{\text{Number of Remaining Years of Life}}{SYD} \times (C - S_v) \tag{4.4}$$

where SYD is the sum of years' digits, and n is the useful life of the asset to be depreciated.

In this method of depreciation, the depreciation rate is equal to the years remaining divided by the sum of the years' digits. Clearly, the annual depreciation is higher toward the beginning of the project because this is when the remaining life is longer.

Example 4–3

Rework example 4–2 using the half-year convention. In this convention, the deduction taken is half of the calculated depreciation. For the five-year property, the final year is the *sixth* year because of applying the half-year

convention in the first year. For the straight-line depreciation in the second and third year, the adjusted basis needs to be divided by 4.5 and 3.5 years, respectively, and so on. The rate for the final year need not be determined because the remaining recovery period is less than one year.

Solution: The calculations are shown in table 4–4.

Table 4–4. Depreciation calculation using half-year convention (example 4–3)

Year	Book Value of Asset at Beginning of Year	Rate	D_n Declining Balance	D_n Straight Line	Deduction Taken
1	\$35,000	0.40	\$14,000	\$3,500	\$7,000
2	\$28,000	0.40	\$11,200	\$6,222	\$11,200
3	\$16,800	0.40	\$6,720	\$4,800	\$6,720
4	\$10,080	0.40	\$4,032	\$4,032	\$4,032
5	\$6,048	0.40	\$2,419	\$4,032	\$4,032
6	\$2,016	0.40	\$806	\$2,016	\$2,016
TOTAL					\$35,000

Example 4–4

Compute the sum-of-years'-digits depreciation for the asset in example 4–2.

The depreciation schedule for example 4–4 is shown in table 4–5.

Solution: $\frac{5(5+1)}{2} = 15$

Table 4–5. Depreciation schedule for example 4–5

Year	Depreciation Base	Fraction	D_n Depreciation Charge	Book value at Year End
1	\$35,000	5/15	\$11,667	\$23,333
2	\$35,000	4/15	\$9,333	\$14,000
3	\$35,000	3/15	\$7,000	\$7,000
4	\$35,000	2/15	\$4,667	\$2,333
5	\$35,000	1/15	\$2,333	0
TOTAL			\$35,000	

Units of production depreciation

When physical wear and tear is the dominating factor in the useful life of the asset, depreciation may be based on units of service rather than on the units of time. Therefore, this method is used when it is determined that the life of the asset is dependent on how much the asset is used or it produces, rather than the passage of time. The activity may be measured in hours, days, months, years, the number of items produced, etc. The following equation is used to establish the units of production depreciation rate. The depreciation charge for each year is calculated by multiplying the activity per period by the depreciation rate.

$$\text{Depreciation Rate} = \frac{\text{Cost} - \text{Salvage Value}}{\text{Total Lifetime Activity}} \quad (4.5)$$

Example 4–5

A drilling rig is purchased for \$1,300,000. It is estimated that the rig will be able to drill a total of 10,000,000 feet. After drilling that many feet, the rig can be sold for \$150,000. In the first year of service, the rig drills 155,000 feet, and in the second year, it drills 100,000 feet. What is the depreciation deduction for the two years?

Solution:

$$\begin{aligned} \text{Depreciation Rate} &= \frac{\$1,300,000 - \$150,000}{10,000,000} \\ &= \$0.115 \text{ per foot} \\ \text{Depreciation for Year 1} &= \$155,000 \times 0.115 \\ &= \$17,825 \\ \text{Depreciation for Year 2} &= \$100,000 \times 0.115 \\ &= \$11,500 \end{aligned}$$

Amortization and Depletion

A part of a certain business capital expenditure may be amortized (deducted) each year. Amortization generally allows a write-off of costs that are not ordinarily deductible. On the other hand, if you have an exhaustible natural resource or timber, you are allowed a depletion allowance.

Amortization

All costs incurred to get a business started are treated as capital expenditure and are part of your basis in the business. If you elect to amortize, these costs are deducted in equal amounts over a period of 60 months or more.

Business startup costs and organizational expenses for a corporation or partnership are capital expenditures. To be amortizable, the costs must be paid or incurred before the day the active trade or business begins. However, startup costs do not include deductible interest, taxes, or research and experimental costs.

Startup costs include the cost of items such as survey of potential market; analysis of available facilities, labor supply, etc.; initial advertisements; salaries and wages for training employees and their instructors; and salaries and fees for executives, consultants, and other professional services.

Organizational costs include the cost of temporary directors; organizational meetings; incorporation fees; accounting services; and legal services for drafting the charter, bylaws, terms of the original stock certification, and minutes of organizational meetings. However, costs of issuing and selling stock or securities, such as commissions, professional fees, and printing costs, are not amortizable.

The amortization deduction each year is determined by dividing the total of startup and organizational costs by the number of months in the amortization period. The resulting amount can be deducted each month.

Depletion

Natural exhaustible resources such as minerals, oil, natural gas, and timber, etc. are sometimes called wasting assets. Depletion is the gradual exhaustion of the original amounts of the resources acquired. Depletion differs from depreciation and amortization because the former focuses mainly on a physical phenomenon and the latter focuses more broadly on any cause of reduction of the economic value of a fixed asset, including physical wear and tear plus obsolescence.

The depletion deduction is available to you as an owner and operator only if you have a legal interest in the oil and gas in place and you have the right to income from the extraction and sale of these. Typically in oil and gas deals, more than one party may have an economic interest in the reserves (or mineral deposit). Therefore, each interest owner accounts for his/her portion of the depletion deduction against his share of the costs paid for the acquisition of these reserves and the proportionate share of the reserves and production. Depletion is figured separately for each mineral property (i.e., each separate interest you own in each mineral deposit in each separate tract or parcel of land).

There are two ways of calculating depletion on mineral property: *cost depletion* and *percentage depletion*. Although percentage depletion generally is not allowed for oil or gas wells, an exemption from this rule applies to certain oil and gas producers known as independent producers and royalty owners (small producers).

The small producer exemption applies to independent producers and royalty owners whose average daily oil production is up to 1,000 barrels of oil equivalent (6,000 standard cubic feet of natural gas is the equivalent of a barrel of oil).

Depletion allowed or allowable each year is the greater of percentage depletion (if allowed) or cost depletion. However, the adjusted basis may never be less than zero.

Cost depletion. Cost depletion is determined by dividing the adjusted basis of the mineral property by the total recoverable units in the property's natural deposit, then multiplying the resulting rate per unit by one of the following:

1. The units sold for which you receive payment during the tax year, if you use the cash method of accounting.
2. The units sold if you use the accrual method of accounting.

The total recoverable units is the number of units of mineral remaining at the end of the year (including units recovered but not sold) plus the number of units sold during the tax year. The recoverable units in place in a natural deposit is an engineering problem to be determined by using generally accepted industry practice.

The adjusted basis of the mineral property is the original cost or other basis plus any capitalized costs, minus all the depletion allowed or allowable on the property. The basis for depletion would therefore be the same adjusted basis you would use in determining gain on the sale or other disposition of property but not including:

1. The amounts recoverable:
 - a. Through depreciation deductions
 - b. Through deferred expenses (including deferred exploration and development costs), and
 - c. Through deductions other than depletion
2. The residual value of land and improvements at the end of operations, and
3. The cost or value of land acquired for purposes other than mineral production.

In equation form, cost depletion can be represented by:

$$\text{Cost depletion} = AB \left(\frac{Q}{R_r + Q} \right) \quad (4.6)$$

where AB is the adjusted basis for the taxable year, Q is the number of units sold during the year, and R_r is the number of remaining reserves at the end of the taxable year. This means that each year the remaining reserves have to be determined. The remaining reserves may not necessarily be the previous year's remaining reserves less the production during the year (i.e., the reserves may have been revised upward or downward due to additional studies, etc). Items included in the depletable basis for cost depletion are geological and geophysical (G&G) costs, lease bonus paid by the lessee, and capitalized intangible costs not represented by physical property.

Percentage depletion. Percentage depletion is determined by multiplying a certain percentage, specified for each mineral, by the gross income from the property during the tax year. Gross income from an oil and gas property is the amount received from the sale of the oil or gas in the immediate vicinity of the well. The depletion deduction under this method cannot be more than 50% of the taxable income from the property figured without the depletion deduction.

When figuring the percentage depletion and the taxable income limit, an amount equal to any rents and royalties (which are depletable income to the payee) paid or incurred for the property is excluded from the gross income from the property. Also, reduce the gross income from the property by the allocable part of any bonus paid for a mineral lease or an oil and gas lease.

For a qualified independent producer or royalty owner of oil and gas, the deduction for percentage depletion cannot be more than the smaller of:

1. The taxable income from the property figured without the deduction for depletion
2. 65% of the taxable income from all sources figured without the depletion allowance, any net operating loss carryback, and any capital loss carryback. Any amount that cannot be deducted because of the 65% limit can be carried over and added to the depletion allowance (before applying any limits) for the following year.

As stated earlier, the use of percentage depletion for oil and gas is only allowed for certain domestic oil and gas production that qualifies for the small producer exemption. If applicable, the percentage depletion is 15% of the gross income from the *qualifying* production from oil and gas property.

Gross income from the property does not include lease bonuses, advance royalties, or other amounts payable without regard to production from the property received or incurred after August 16, 1986. Independent producers and royalty owners (small producers) qualify for the 15% depletion rate for a maximum daily average production of 1,000 barrels of oil equivalent (e.g., 6,000 cubic feet of natural gas).

Percentage depletion is allowed for regulated natural gas and for gas sold under a fixed contract or produced from geopressured brine, regardless of whether it qualifies for the small-producer exemption. *Income from natural gas sold under a fixed contract qualifies for percentage depletion at 22%.* Natural gas sold under a fixed contract is the domestic gas sold by the producer under a contract in effect on February 1, 1975, and at all times thereafter before the sale.

If you qualify for percentage depletion and it is less than the cost depletion for any year, you must use cost depletion for that year. To figure depletion allowance for mineral properties, calculate it under both depletion methods for each property, and claim a deduction for the greater of the two amounts if the percentage depletion is less than 50% of the taxable income before depletion.

Example 4–6

It is estimated that a taxpayer has remaining reserves (net to his interest) at the end of his tax year of 50,000 barrels of oil. His production share in the year was 6,000 barrels sold at \$18 per barrel. His net taxable income before depletion from the property was \$80,000 and the taxpayer's total taxable income was \$450,000. Compute the appropriate depletion charge for the year. The adjustable bases of the capitalized leasehold costs are \$45,000.

Solution:

Using equation (4.6):

$$\begin{aligned}\text{Cost Depletion} &= \$45,000 \left(\frac{6,000}{50,000 + 6,000} \right) \\ &= \$4,821\end{aligned}$$

Percentage depletion computation and limitation check:

- Percentage depletion = $0.15 (6,000 \times 18) = \$16,200$
- 50% net income limitation = \$40,000
- 65% taxable income limitation = \$292,500

Therefore, the allowable percentage depletion is \$16,200 since the percentage depletion is greater than the cost depletion of \$4,821. Note that the adjusted basis (*AB*) for the next year's cost depletion calculations is reduced by \$16,200 (i.e., *AB* for the next year is \$28,800 [\$45,000–\$16,200] and *R_t* is 44,000 [50,000–6,000] barrels).

Intangible Drilling Costs (IDCs)

The pre-1986 tax laws allow taxpayers to expense domestic IDCs. Under this law, the amount that the integrated producer can expense is 80% of its IDCs with the difference being amortized over 36 months, beginning in the month costs are incurred. The independent producer may elect to expense IDCs in the year incurred. The Tax Reform Act of 1986 changed the rules as summarized below.

1. The independent producer may elect to expense 100% of IDCs in the year incurred for domestic wells. For foreign IDCs, the cost may be recovered over a 10-year period using straight-line amortization or may be added to its depletable basis.
2. The integrated producer may elect to expense 70% of IDCs in the year they were incurred and amortize the remaining 30% over 60 months for domestic wells. The foreign IDCs must be capitalized over a 10-year period using straight-line amortization or may be added to its depletable basis.

Dry hole costs may be expensed in the year incurred, regardless of the way the IDCs are treated. This is applicable to both the integrated and independent producers. The breakdown of various expenditures involved in drilling a well and their respective tax treatment is shown in table 4–6.

There are a number of other tax items (e.g., alternative minimum tax) that affect the oil and gas business, but these are not included here because they are not directly applicable to project evaluations and a large number of assumptions are necessary for their calculations.

Table 4–6. Breakdown of various expenditures involved in drilling a well and their respective tax treatment¹

	Tax Treatment	
	Dry Hole Costs	Completion Costs
Administrative costs	E	IDC
Location—stake, permit, damages	E	IDC
Rig tariff	E	IDC
Rig transportation	E	IDC
Company labor—drilling	E	IDC
Cementing—surface casing	E	IDC
Cementing—production casing	E	IDC
Bits and reamers	E	IDC
Logging	E	IDC
Coring services	E	IDC
Mud logger	E	IDC
Engineering while drilling	E	IDC
Mud, water, and fuel	E	IDC
Drillstem tests (DST)	E	IDC
Casing crew—surface casing	E	IDC
Casing crew—production casing		IDC
Completion unit		IDC
Completion costs		IDC
Swabbing costs		IDC
Company labor—completion		IDC
Perforating		IDC
Surface casing		Tangible
Production casing		Tangible
Tubing		Tangible
Wellhead equipment		Tangible
Float equipment, centralizers		Tangible
Packers		Tangible
Line pipe		Tangible
Tanks and fittings		Tangible
Separators and fittings		Tangible
Pumping unit		Tangible

Example 4–7

An independent oil company acquired mineral rights for \$500,000 in 2009. The company drilled and completed a well on the lease for a total of \$3,000,000, of which \$1,000,000 was depreciable completion costs. The drilling cost of \$2,000,000 is expensed for tax purposes and deducted against income in 2010. Initial well tests show an estimated daily production of 750 barrels per day, declining at a rate of 50 barrels per day in each succeeding year. Lease operating costs are projected at \$400,000 per year, escalating by \$50,000 per year in each succeeding year. The crude oil will be sold at \$10 per barrel. Recoverable oil reserves are estimated to be 1,500,000 barrels. Assume that the company has 100% WI and 87.5% NRI in the lease, salvage value is zero, the income tax rate is 30%, and there is no production tax. Use double declining balance depreciation using 5-year asset life where applicable. Calculate the net after-tax cash flow for the first four years.

Solution: First calculate cost depletion for the four years as

$$CD_{2010} = \$500,000 \left[\frac{273,750}{(1,500,000 - 273,750) + 273,750} \right] = \$91,250$$

$$CD_{2011} = (\$500,000 - \$91,250) \left[\frac{255,500}{(1,500,000 - 529,250) + 255,500} \right] = \$85,167$$

$$CD_{2012} = (\$500,000 - \$176,417) \left[\frac{237,250}{(1,500,000 - 766,500) + 237,250} \right] = \$79,083$$

$$CD_{2013} = (\$500,000 - \$255,500) \left[\frac{219,000}{(1,500,000 - 985,500) + 219,000} \right] = \$73,000$$

The percent depletion is calculated as follows:

$$PD_{2010} = 273,750 \times \$10 \times 0.875 \times 0.15 = \$359,297$$

$$PD_{2011} = 255,500 \times \$10 \times 0.875 \times 0.15 = \$335,344$$

$$PD_{2012} = 237,250 \times \$10 \times 0.875 \times 0.15 = \$311,391$$

$$PD_{2013} = 219,000 \times \$10 \times 0.875 \times 0.15 = \$287,438$$

The cash flow for example 4–7 is shown in table 4–7. The before-tax cash flow for the same example is \$3,500,000; \$1,995,312; \$1,785,625; \$1,575,937; and \$1,366,250, with a total for the period from 2009 to 2013 of \$3,223,124 versus the after-tax cash flow of \$2,455,348. The difference between the two cash flows is the tax deduction of \$767,776.

Table 4–7. After-tax cash flow for example 4–7

	2009	2010	2011	2012	2013
Production, Stb	0	273,750	255,500	237,250	219,000
Gross (8/8ths) revenue	0	\$2,737,500	\$2,555,000	\$2,372,500	\$2,190,000
Royalty at 12.5%	0	–\$342,188	–\$319,375	–\$296,563	–\$273,750
Net operating revenue	0	\$2,395,312	\$2,235,625	\$2,075,937	\$1,916,250
Operating costs	0	–\$400,000	–\$450,000	–\$500,000	–\$550,000
Depreciation	0	–\$400,000	–\$240,000	–\$144,000	–\$86,400
IDCs	–\$2,000,000	0	0	0	0
Taxable before depletion	–\$2,000,000	\$1,595,312	\$1,545,625	\$1,431,937	\$1,279,850
Allowable depletion	0	–\$359,297	–\$335,344	–\$311,391	–\$287,438
Percent depletion (15%)	0	\$359,297	\$335,344	\$311,391	\$287,438
50% Limit	0	\$797,656	\$772,813	\$715,969	\$639,925
Cost depletion	0	\$91,250	\$85,167	\$79,083	\$73,000
Taxable after depletion	–\$2,000,000	\$1,236,015	\$1,210,281	\$1,120,546	\$992,412
Loss forward	0	–\$2,000,000	–\$763,985	0	0
Taxable income	–2,000,000	–\$763,985	\$446,296	\$1,120,546	\$992,412
Tax @ 30%	0	0	–\$133,889	–\$336,164	–\$297,724
Net income	–\$2,000,000	–\$763,985	\$312,407	\$784,382	\$694,688
+ Depreciation	0	\$400,000	\$240,000	\$144,000	\$86,400
+ Allowable depletion	0	\$359,297	\$335,344	\$311,391	\$287,438
+ Loss forward	0	\$2,000,000	\$763,984	0	0
Acquisition cost	–\$500,000	0	0	0	0
Tangible drilling costs	–\$1,000,000	0	0	0	0
After-tax net cash flow	–\$3,500,000	\$1,995,312	\$1,651,735	\$1,239,773	\$1,068,526

Table 4–7a. After-tax cash flow for example 4–7

	2005	2006	2007	2008	2009
Production, Stb	0	273,750	255,500	237,250	219,000
Gross (8/8ths) revenue	0	\$2,737,500	\$2,555,000	\$2,372,500	\$2,190,000
Royalty at 12.5%	0	–\$342,188	–\$319,375	–\$296,563	–\$273,750
Operating costs	0	–\$400,000	–\$450,000	–\$500,000	–\$550,000
IDCs	–\$2,000,000	0	0	0	0
Tax @ 30%	0	0	–\$133,889	–\$336,164	–\$297,724
Acquisition cost	–\$500,000	0	0	0	0
Tangible drilling costs	–\$1,000,000	0	0	0	0
After-tax net cash flow	–\$3,500,000	\$1,995,312	\$1,651,736	\$1,239,773	\$1,068,526

Example 4–8

A gas field development project is proposed for 2009. The total drilling cost of the wells is \$185 million, of which \$111 million is intangible drilling cost and \$74 million is tangible drilling cost. The tangible drilling costs are subject to depreciation with annual recovery of 14.28% in 2010, 24.49% in 2011, 17.49% in 2012, 12.49% in 2013, 8.93% in 2014 to 2016, and 4.46% in 2017. The interest of an integrated company is to be evaluated; therefore, 70% of the IDCs will be expensed and the remaining 30% will be amortized over 60 months. The oil and gas produced from the lease will be sold at \$14/Stb and \$2.25/MScf, respectively. Severance and ad valorem taxes are 8% for oil and gas. The NRI is 87.5% against a WI of 100%. The lease operating expenses are \$700 per month. Develop an after-tax cash flow for this example. The oil and gas production is given in table 4–8. Assume a corporate tax rate of 34%.

Solution: The cash flow for example 4–8 is shown in table 4–8.

Table 4-8. (Continued)

32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260	261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	289	290	291	292	293	294	295	296	297	298	299	300	301	302	303	304	305	306	307	308	309	310	311	312	313	314	315	316	317	318	319	320	321	322	323	324	325	326	327	328	329	330	331	332	333	334	335	336	337	338	339	340	341	342	343	344	345	346	347	348	349	350	351	352	353	354	355	356	357	358	359	360	361	362	363	364	365	366	367	368	369	370	371	372	373	374	375	376	377	378	379	380	381	382	383	384	385	386	387	388	389	390	391	392	393	394	395	396	397	398	399	400	401	402	403	404	405	406	407	408	409	410	411	412	413	414	415	416	417	418	419	420	421	422	423	424	425	426	427	428	429	430	431	432	433	434	435	436	437	438	439	440	441	442	443	444	445	446	447	448	449	450	451	452	453	454	455	456	457	458	459	460	461	462	463	464	465	466	467	468	469	470	471	472	473	474	475	476	477	478	479	480	481	482	483	484	485	486	487	488	489	490	491	492	493	494	495	496	497	498	499	500	501	502	503	504	505	506	507	508	509	510	511	512	513	514	515	516	517	518	519	520	521	522	523	524	525	526	527	528	529	530	531	532	533	534	535	536	537	538	539	540	541	542	543	544	545	546	547	548	549	550	551	552	553	554	555	556	557	558	559	560	561	562	563	564	565	566	567	568	569	570	571	572	573	574	575	576	577	578	579	580	581	582	583	584	585	586	587	588	589	590	591	592	593	594	595	596	597	598	599	600	601	602	603	604	605	606	607	608	609	610	611	612	613	614	615	616	617	618	619	620	621	622	623	624	625	626	627	628	629	630	631	632	633	634	635	636	637	638	639	640	641	642	643	644	645	646	647	648	649	650	651	652	653	654	655	656	657	658	659	660	661	662	663	664	665	666	667	668	669	670	671	672	673	674	675	676	677	678	679	680	681	682	683	684	685	686	687	688	689	690	691	692	693	694	695	696	697	698	699	700	701	702	703	704	705	706	707	708	709	710	711	712	713	714	715	716	717	718	719	720	721	722	723	724	725	726	727	728	729	730	731	732	733	734	735	736	737	738	739	740	741	742	743	744	745	746	747	748	749	750	751	752	753	754	755	756	757	758	759	760	761	762	763	764	765	766	767	768	769	770	771	772	773	774	775	776	777	778	779	780	781	782	783	784	785	786	787	788	789	790	791	792	793	794	795	796	797	798	799	800	801	802	803	804	805	806	807	808	809	810	811	812	813	814	815	816	817	818	819	820	821	822	823	824	825	826	827	828	829	830	831	832	833	834	835	836	837	838	839	840	841	842	843	844	845	846	847	848	849	850	851	852	853	854	855	856	857	858	859	860	861	862	863	864	865	866	867	868	869	870	871	872	873	874	875	876	877	878	879	880	881	882	883	884	885	886	887	888	889	890	891	892	893	894	895	896	897	898	899	900	901	902	903	904	905	906	907	908	909	910	911	912	913	914	915	916	917	918	919	920	921	922	923	924	925	926	927	928	929	930	931	932	933	934	935	936	937	938	939	940	941	942	943	944	945	946	947	948	949	950	951	952	953	954	955	956	957	958	959	960	961	962	963	964	965	966	967	968	969	970	971	972	973	974	975	976	977	978	979	980	981	982	983	984	985	986	987	988	989	990	991	992	993	994	995	996	997	998	999	1000	1001	1002	1003	1004	1005	1006	1007	1008	1009	1010	1011	1012	1013	1014	1015	1016	1017	1018	1019	1020	1021	1022	1023	1024	1025	1026	1027	1028	1029	1030	1031	1032	1033	1034	1035	1036	1037	1038	1039	1040	1041	1042	1043	1044	1045	1046	1047	1048	1049	1050	1051	1052	1053	1054	1055	1056	1057	1058	1059	1060	1061	1062	1063	1064	1065	1066	1067	1068	1069	1070	1071	1072	1073	1074	1075	1076	1077	1078	1079	1080	1081	1082	1083	1084	1085	1086	1087	1088	1089	1090	1091	1092	1093	1094	1095	1096	1097	1098	1099	1100	1101	1102	1103	1104	1105	1106	1107	1108	1109	1110	1111	1112	1113	1114	1115	1116	1117	1118	1119	1120	1121	1122	1123	1124	1125	1126	1127	1128	1129	1130	1131	1132	1133	1134	1135	1136	1137	1138	1139	1140	1141	1142	1143	1144	1145	1146	1147	1148	1149
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Example 4–9

A gas field is proposed to be acquired in 2010 for a total cost of \$12,000 million, including \$2,400 million worth of tangible costs. Most of the field is already developed except for one well that will be drilled in 2011 at a total cost of \$910 million, which consists of \$673 million as IDC and \$273 million as tangible costs. Calculate the after-tax net cash flow for the proposal at an oil price of \$70/Stb and gas price of \$4/MScf. Severance and ad valorem taxes equal 8% of gross income. The NRI against 100% WI is 87.5%. The production forecast and operating expenditures are listed in table 4–9. Assume a corporate tax rate of 34%.

Solution: The after-tax cash flow for example 4–9 is shown in table 4–9. Note that oil and gas property acquisition is different from other oil and gas deals in that there are almost no preferential tax treatments. The acquisition cost must be recovered through cost depletion except for that portion allocated to tangible equipment and recovered under ACRS. Percentage depletion on an acquired oil and gas property is not allowed.

Spreadsheet Application

The following Excel functions can be used to calculate the straight-line, declining balance, and sum-of-years'-digits depreciation.

1. $=SLN(cost, salvage, life)$. Returns the straight-line depreciation for a period. The application is shown in table 4–10.
 - a. Enter 0 in cell A7.
 - b. Enter $=C4$ in cell B7.
 - c. Enter $=C2$ in cell D7.
 - d. Enter $=A7+1$ in cell A8.
 - e. Enter $=B7-1$ in cell B8.
 - f. Enter the function $=SLN(D7, C\$3, B7)$ in cell C8.
 - g. Enter the formula $=D7-C8$ in cell D8.
 - h. Copy cells A8:D8 and paste to cells A9 to A12.
2. $=DB(cost, salvage, life, period, month)$. Returns the depreciation of an asset for a specified period using the fixed declining balance depreciation. In this function, *period* is the year for which a depreciation charge is to be calculated, and *month* is the number of months the asset had been in use in the first year. If the month is

Table 4–9. Typical AFIT cash flow for acquisition, integrated oil company

Year	Gross Production			Net Production		Product Prices		Expenditure				Operating Revenue, BFIT			Cum. NCF Disc. @10% (M\$)
	Oil (MStb)	Gas (MMScf)		Oil (MStb)	Gas (MMScf)	Oil \$/Stb	Gas \$/Mscf	Intangible (M\$)	Tangible (M\$)	LOE (M\$)	Taxes (M\$)	Total (M\$)	Gross (M\$)	Net (M\$)	
2010				-	-	70.00	4.00	-	7,200		-	36,000	-	(36,000)	(34,325)
2011	77	3,394		67.38	2,970	70.00	4.00	1,911	819	485	1,328	4,523	16,595	12,073	(23,860)
2012	81	4,205		70.88	3,679	70.00	4.00			918	1,574	2,492	19,679	17,186	(10,318)
2013	69	3,619		60.38	3,167	70.00	4.00			957	1,351	2,308	16,893	14,584	130
2014	56	2,880		49.00	2,520	70.00	4.00			996	1,081	2,077	13,510	11,433	7,575
2015	47	2,385		41.13	2,087	70.00	4.00			1,038	898	1,936	11,226	9,290	13,076
2016	40	2,002		36.00	1,752	70.00	4.00			1,083	757	1,840	9,457	7,617	17,175
2017	35	1,686		30.63	1,475	70.00	4.00			1,128	644	1,772	8,045	6,273	20,245
2018	30	1,445		26.25	1,264	70.00	4.00			1,176	552	1,728	6,895	5,167	22,543
2019	11	1,100		9.63	963	70.00	4.00			972	362	1,334	4,524	3,190	23,833
2020	6	673		5.25	589	70.00	4.00			564	218	782	2,723	1,941	24,546
2021	4	468		3.50	410	70.00	4.00			357	151	508	1,883	1,375	25,006
2022	2	280		1.75	245	70.00	4.00			204	88	292	1,103	810	25,252
2023	2	259		1.75	227	70.00	4.00			204	82	286	1,029	743	25,457
2024	2	240		1.75	210	70.00	4.00			204	77	281	963	682	25,628
2025	2	224		1.75	196	70.00	4.00			204	73	277	907	630	25,772
2026															25,772
2027															25,772
2028															25,772
2029															25,772
Sub Total	464	24,860		406.00	21,753			1,911	8,019	10,470	9,234	58,434	115,430	56,996	
Rem.	-	-		-	-			-	-	-	-	-	-	-	-
Total	464	24,860		406.00	21,753			1,911	8,019	10,470	9,234	58,434	115,430	56,996	25,772

- omitted, it will assume 12 months by default. Table 4–11 shows the application of this function.
- a. Enter 0 in cell A7.
 - b. Enter =C4 in cell B7.
 - c. Enter =C2 in cell D7.
 - d. Enter =A7+1 in cell A8.
 - e. Enter =B7–1 in cell B8.
 - f. Enter the function =DB(D\$7,C\$3,C\$4,A8) in cell C8. Cells D7, C3, and C4 have to be the same in all rows, only the period in cell A8 is changing when copied to the other cells.
 - g. Enter the formula =D7–C8 in cell D8.
 - h. Copy cells A8:D8 and paste to cells A9 to A12.

Table 4–10. Straight line depreciation calculation (using Excel functions) for example 4–1

	A	B	C	D
1	Example 4-1. Straight Line Depreciation			
2	Cost of Pumping Unit		\$35,000	
3	Salvage Value		\$5,000	
4	Useful Economic Life, Years		5	
5				
6	End of Year	Remaining Life	Depreciation Charge	Book value
7	0	5		\$35,000
8	1	4	\$6,000	\$29,000
9	2	3	\$6,000	\$23,000
10	3	2	\$6,000	\$17,000
11	4	1	\$6,000	\$11,000
12	5	0	\$6,000	\$5,000

Table 4–11. Declining balance depreciation calculation (using Excel functions)

	A	B	C	D
1	Declining-Balance Depreciation			
2	Cost of Pumping Unit		\$35,000	
3	Salvage Value		\$5,000	
4	Useful Economic Life, Years		5	
5				
6	End of Year	Remaining Life	Depreciation Charge	Book value
7	0	5		\$35,000
8	1	4	\$11,270	\$23,730
9	2	3	\$7,641	\$16,089
10	3	2	\$5,181	\$10,908
11	4	1	\$3,512	\$7,396
12	5	0	\$2,381	\$5,014

3. $=DDB(cost,salvage,life,period,factor)$. Returns the depreciation of an asset for a specified period using the double declining balance or some other method specified by the user. In the function, *period* is the year for which depreciation charge is to be calculated, and *factor* is the rate at which the balance declines. If the factor is omitted, it assumes double declining balance (200% declining balance). Table 4–12 shows the application of this function.

- a. Enter 0 in cell A7.
- b. Enter $=C4$ in cell B7.
- c. Enter $=C2$ in cell D7.
- d. Enter $=A7+1$ in cell A8.
- e. Enter $=B7-1$ in cell B8.
- f. Enter the function $=DDB(D\$7,C\$3,C\$4,A8,C\$5)$ in cell C8.
Cells D7, C3, C4, and C5 have to be the same in all rows; only

the period in cell A8 is changing when copied to the other cells. The cell C5 specifies the percent declining (i.e., use 2 for 200%, 1.75 for 175%, 1.25 for 125%, and so on).

- g. Enter the formula =D7-C8 in cell D8.
- h. Copy cells A8:D8 and paste to cells A9 to A12.

Table 4-12. Double declining balance depreciation calculation (using Excel functions) for example 4-2

	A	B	C	D
1	Example 4-2. Double-Declining Balance Depreciation			
2	Cost of Pumping Unit		\$35,000	
3	Salvage Value		\$0	
4	Useful Economic Life, Years		5	
5	Declining-Balance		2.00	2 = 200% 1.75 = 175% 1.25 = 125%
6	End of Year	Remaining Life	Depreciation Charge	Book value
7	0	5		\$35,000
8	1	4	\$14,000	\$21,000
9	2	3	\$8,400	\$12,600
10	3	2	\$5,040	\$7,560
11	4	1	\$3,024	\$4,536
12	5	0	\$1,814	\$2,722

4.=VDB(*cost,salvage,life,start_period,end_period,factor,no_switch*). Returns the depreciation of an asset for any specified period, including partial periods, using the declining balance method (200%, 175%, 150%, and so on). VDB stands for variable declining balance. The *start_period* is the starting period for which depreciation has to be calculated; it must use the same units as *life*. The *end_period* is the ending period for which depreciation has to be calculated; it must use the same units as *life*. The *no_switch* is a logical value specifying whether to switch to straight-line depreciation when depreciation is greater than the declining balance depreciation. If the *no_switch* is omitted, the depreciation charge will switch from DDB to SL when the SL depreciation is higher than the DDB. If switching is not required, then enter TRUE. Table 4–13 shows the application of this function.

- a. The switching will require one more cell where the option can be specified. Therefore, another row (row 6) is added. The switching option is specified in cell C6.
- b. Enter 0 in cell A8.
- c. Enter =C4 in cell B8.
- d. Enter =C2 in cell D8.
- e. Enter =A8+1 in cell A9.
- f. Enter = B8–1 in cell B9.
- g. Enter the function =VDB(D\$8,C\$3,C\$4,A8,A9,C\$5,C\$6) in cell C9. Note that the start period in Year 1 is 0, and end period is Year 1, and so on.
- h. Enter =D8–D7 in cell D9.
- i. Copy cells A9:D9 and paste to cells A10 to A13.

Table 4–13. Double-declining balance depreciation with switch to straight-line depreciation calculation (using Excel functions) for example 4–2

	A	B	C	D
1	Example 4-2. DDB with Switch to Straight Line			
2	Cost of Pumping Unit		\$35,000	
3	Salvage Value		\$0	
4	Useful Economic Life, Years		5	
5	Declining-Balance		2.00	2 = 200% 1.75 = 175% 1.25 = 125%
6	Switch to Straight Line		FALSE	True or False
7	End of Year	Remaining Life	Depreciation Charge	Book value
8	0	5		\$35,000
9	1	4	\$14,000	\$21,000
10	2	3	\$8,400	\$12,600
11	3	2	\$5,040	\$7,560
12	4	1	\$3,780	\$3,780
13	5	0	\$3,780	\$0

5. =SYD(cost,salvage,life,per). Returns the sum-of-years'-digits depreciation of an asset for a specified period. Table 4–14 shows the application of this function.
- a. Enter 0 in cell A7.
 - b. Enter =C4 in cell B7.
 - c. Enter =C2 in cell D7.
 - d. Enter =A7+1 in cell A8.
 - e. Enter =B7–1 in cell B8.
 - f. Enter the function =SYD(D\$7,C\$3,C\$4,A8) in cell C8. Cells D7, C3, and C4 have to be the same in all rows, only the period in cell A8 is changing when copied to the other cells.
 - g. Enter the formula =D7–C8 in cell D8.
 - h. Copy cells A8:D8 and paste to cells A9 to A12.

Table 4–14. Sum-of-years'-depreciation calculation (using Excel functions) for example 4–4

	A	B	C	D
1	Example 4-4. Sum-of-the-Year's Digits Depreciation			
2	Cost of Pumping Unit		\$35,000	
3	Salvage Value		\$0	
4	Useful Economic Life, Years		5	
5				
6	End of Year	Remaining Life	Depreciation Charge	Book value
7	0	5		\$35,000
8	1	4	\$11,667	\$23,333
9	2	3	\$9,333	\$14,000
10	3	2	\$7,000	\$7,000
11	4	1	\$4,667	\$2,333
12	5	0	\$2,333	\$0

Questions and Problems

- 4.1 A compressor is purchased for \$50 million. What will be the straight-line depreciation per year if the depreciable life of the compressor, by tax rules, is 10 years? Assume zero salvage value at the end of 10 years.
- a. 10.0 MM\$
 - b. 8.0 MM\$
 - c. 5.0 MM\$
 - d. 6.4 MM\$

- 4.2 Calculate the first year's depreciation for the compressor in question 4.1 using double-declining balance (DDB) depreciation.
- a. 10.0 MM\$
 - b. 8.0 MM\$
 - c. 5.0 MM\$
 - d. 6.4 MM\$
- 4.3 If you are allowed to switch from DDB depreciation to straight-line depreciation, in which year will it be advantageous to switch to straight-line in question 4.2?
- a. 4.0
 - b. 5.0
 - c. 6.0
 - d. 7.0
- 4.4 In question 4.3, what will be the depreciation charge in the year when the switch from DDB to straight-line takes place?
- a. 3.28 MM\$
 - b. 4.10 MM\$
 - c. 5.12 MM\$
 - d. 6.40 MM\$
- 4.5 Calculate the sum-of-years'-digits depreciation for year one and year two for the asset in question 4.1.
- 4.6 In question 4.5, what will be the book value of the asset at the end of year two?

Reference

1. Thompson, R. S., and Wright, J. D., *Oil Property Evaluation*, Thompson-Wright Associates, Golden, Colorado, 1984.

5

International Petroleum Economics

In chapters 3 and 4, cash flow models applicable to oil and gas exploration and production or to any other related investment projects were discussed. In the international oil and gas business, financial and contractual arrangements for the exploration and production of oil and gas are different. These arrangements are typically referred to as the *fiscal regime*. The fiscal regime or fiscal system includes all legislative, taxation, contractual, and fiscal elements. The major difference is that in the United States, the mineral rights belong to individuals or the state. Ownership of minerals in places outside the United States is vested almost entirely in a national government. Federal petroleum law is the basis for all petroleum operations. Such laws often vest important discretionary powers in federal administrative or legislative bodies.

The basic cash flow variables, such as the ones discussed in chapters 3 and 4, are still applicable to the economic evaluations of international oil and gas investments. For example, the production forecast, product price forecast, reserves, expenditure forecast (CAPEX and OPEX), depreciation, depletion, royalty, time value of money consideration, and income tax calculations are still applicable.

In the United States, an oil company (lessee) acquires mineral rights from the state government or private landowner (lessor). In international oil and gas exploration and production operations, a host government grants *license* or enters into a *contract* with a contractor for a given *contract area*. The terms *license*, *concession*, *acreage position*, *lease*, or *block* are used interchangeably. The *host government* is represented by either a national oil company, an oil ministry of the country, or both. The contractor or operator refers to an international oil company, contractor group, or consortium of these.

The prime objectives of the host government are sovereignty, economic growth, and environment (quality of life). The subobjectives are the optimal exploitation and use of mineral resources, satisfying domestic demand, fostering direct and indirect employment, accumulating expertise, and so forth. To meet these objectives, the host government may have the following constraints.¹

1. Lack of technological expertise of the national oil company (NOC)
2. Lack of manpower resources to execute a project
3. Availability of risked capital
4. Availability of financing
5. Sharing of risk
6. A need for expansion into new markets
7. A need for the use of patented technology
8. A need for diversification of operations

To maximize profits, national oil companies may be tempted to handle big projects on their own in times of high oil prices and to invite international companies to handle projects when prices are low.

Enhanced oil recovery (EOR) and improved oil recovery (IOR) projects require high technology in addition to intensive capital and risk. The international companies are able to provide risk capital and the necessary expertise in return for a reward if the exploration is successful. Therefore, a government as the owner of mineral resources engages a foreign oil company (international oil company or a consortium of oil companies) as a contractor to provide technical and financial resources for exploitation of these resources.

As shown in figure 5–1, these agreements are typically between a host government (HG) and: (a) an international oil company (IOC), (b) an international national oil company (INOC), or (c) a consortium of IOCs and INOCs. Host governments are usually represented by either a national oil company (NOC), an oil ministry, or both. They are collectively referred to as the state or the HG. The IOC, INOC, or their consortium is simply referred to as a contractor.

The HG may also be a joint interest partner with the contractor from the inception of the venture or it may retain an option (back-in option) to become a joint venture partner at some specified future time or milestone.

Details relevant to the economic evaluation of various fiscal systems are discussed in this chapter. Authors Kirsten Bindemann and Daniel Johnston provide a comprehensive overview of the international petroleum fiscal systems and production sharing contracts.^{1–2}

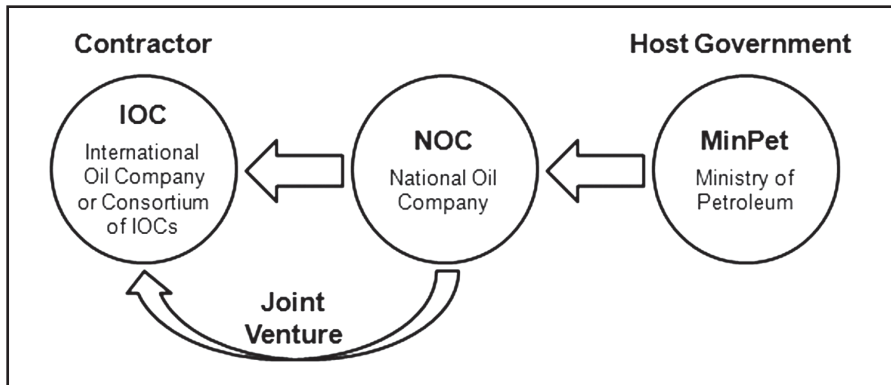


Fig. 5–1. Parties to petroleum agreements

Types of Contract Arrangements

The following different forms of fiscal arrangements are used in the industry:

1. Concessionary systems. Also referred to as the tax and royalty system. The agreement could be for exploration and development or only for development such as enhanced oil recovery.
2. Production sharing systems. The agreement could be for exploration, development and production (abbreviated as EPSA), or it could be for the development of a certain field (no exploration), which is known as a development and production sharing agreement (DPSA). The combination is also referred to as EPSA/DPSA. If the host government is also a joint venture partner with the contractor, then both partners will pay their proportionate share of the costs and receive a proportionate share of the revenue. In addition, the contractor will also share a percent of its share with the host government. The production sharing contract (PSC) is also referred to as the *production sharing agreement* (PSA).
3. Joint venture contracts
4. Service contracts. The service contracts are further classified into *pure service contracts* and *risk service contracts*. The difference is primarily based upon whether the fees are based upon a flat fee (pure) or profit (risk). Some countries offer concessionary arrangements as well as service or production sharing contracts.

The bottom line in either case is a financial issue (i.e., how costs are recovered, risks are shared, and profits are divided).

Under a concessionary system and production sharing agreement, the contractor bears all exploration costs (unless the host government is also a joint venture partner with the contractor), field development costs, operating costs, and risks in return for a stipulated share in the resulting production from the field. The risk and reward under each of the above contract types are summarized in table 5–1. In the joint venture contracts, all the partners are exposed to proportionate risk and reward in line with their respective partnership share.

Table 5–2 shows the summary of terms in the royalty and tax system and the production sharing system. The detailed mechanics of each will be presented in the following pages.

Table 5–1. Risk and reward of main contract types²

Contract Type	Contractor	Host Government
Concession	All risk/all reward	Share in reward
Production sharing agreement	Exploration risk/share in reward	Share in reward
Joint venture	Proportionate share in risk and reward	Proportionate share in risk and reward
Pure service contract	Some risk	All risk
Risk service contract	Some risk	Some risk

Table 5–2. Summary of the terms in production sharing and royalty tax systems

	Concessionary System	Production Sharing Agreement
Bonuses	Signature bonus	Signature bonus
	Discovery bonus	Discovery bonus
	Production bonus	Production bonus
	Cumulative production	Cumulative production
Profit sharing	Royalty	Royalty (rare)
	• Fixed rate	Profit share oil (PO—profit oil)
	• Sliding scale	Cost recovery oil (CR—cost oil)
	Taxes	Petroleum costs
	• Fixed rate	Excess cost recovery (ECR) oil
	• Sliding scale	Income tax (rare)
Sliding scale—tranches	Excess profit tax	
	Oil price	Oil price
	Production rate	Production rate
	Cumulative production	Cumulative production
	R-factor or IRR	R-factor or IRR

The concessionary system

The cash flow generated in the concessionary system is similar to the cash flows presented in tables 3–17, 4–8, and 4–9. The provisions of royalty/tax regimes are normally laid down in the country or state's legislation and regulations and are the same for all companies. In the concessionary system, the hydrocarbons extracted by the companies normally belong to them, and they pay royalties and taxes to the government of the country where they operate. This fiscal system may range from a simple single-royalty and single-tax system to more sophisticated formulas with various layers of royalties and taxation. The flow diagram in figure 5–2 shows the revenue distribution under a simple concessionary system.

Table 5–3 is a cash flow projection for a concessionary system. Sample calculations are also shown in example 5–1. Typical mathematical calculations in the concessionary system involve the following steps:

1. The royalty comes right off the top (i.e., deducted from the gross/wellhead revenue). This arrangement is similar to the landowner's royalty and/or overriding royalty of most lease arrangements in the United States.
2. The operating costs, applicable depreciation, depletion if allowed, amortization, and intangible drilling costs are deducted from the revenue left after the royalty deductions. The deductions are subject to respective tax rules as stipulated in the agreement. In many systems, the intangible capital costs are expensed in the year incurred. Bonuses are not always deductible for tax purposes.
3. The revenue remaining after royalty and deductions is called taxable income, which is subject to the tax deduction. In some cases, there are two or three layers of taxation. For example, provincial taxes are deducted from the taxable income before it is subjected to the federal income tax. In other cases, there are windfall profit taxes deducted after the provincial and federal tax.
4. Typically, model royalty and tax configurations are put forward by the host government as a basis for bidding and negotiations. Once a contract is signed, the fiscal terms remain fixed for the contract period of 25 to 30 years. These types of agreements are not limited to exploration only. In order to avail the technical expertise of international oil companies, development of existing oil fields is also done under the royalty and tax system. Some gas field development projects have a combination of agreements, where the upstream is developed under the royalty and tax system and the downstream is a joint venture.

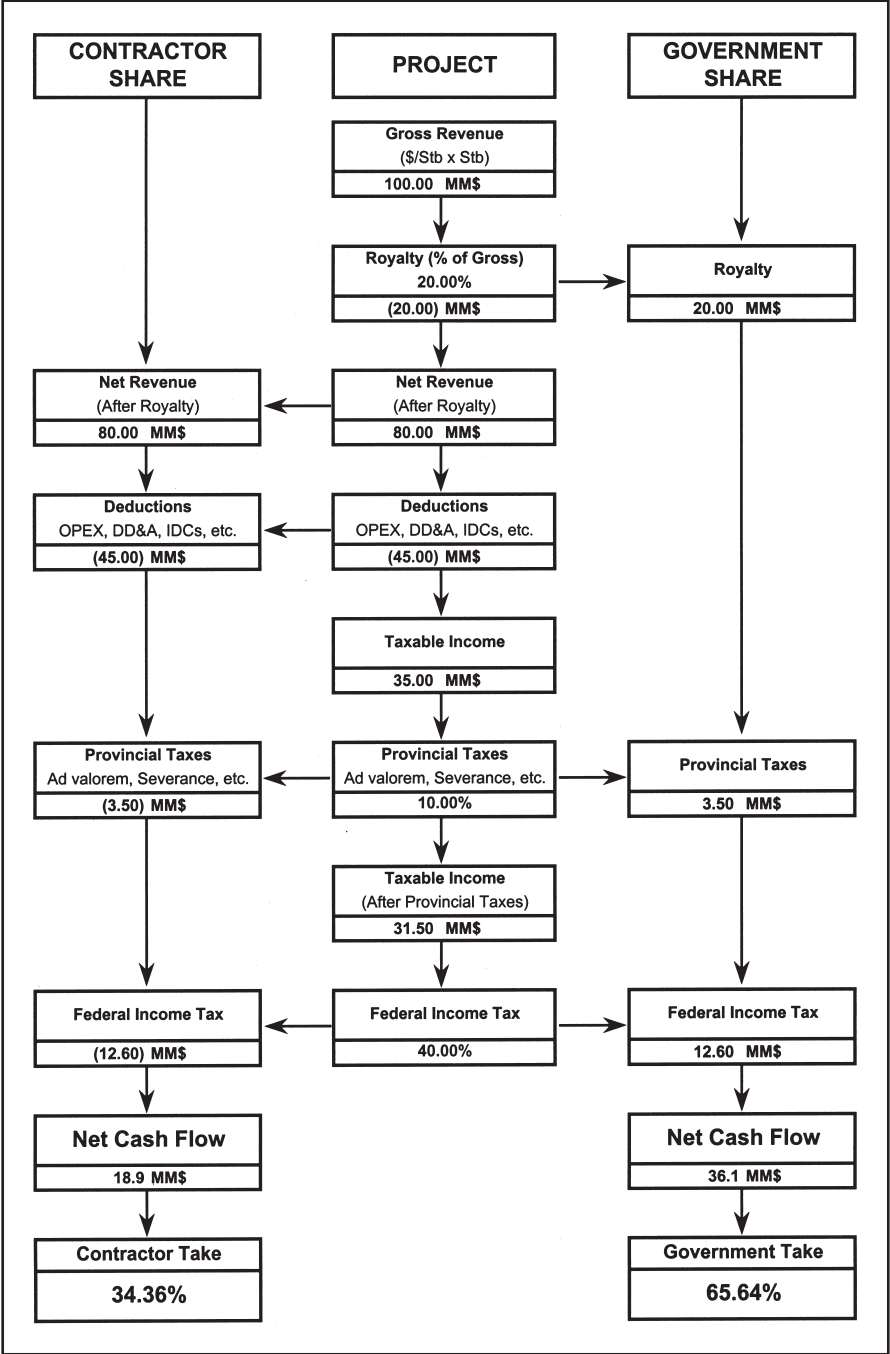


Fig. 5–2. Concessionary system flow diagram

Sample concessionary system cash flow spreadsheet

The spreadsheet has to be flexible enough to evaluate the effect of the fiscal terms for different field sizes, work commitments, pricing scenarios, etc. A description of the various columns of the spreadsheet in table 5–3 follows.

1. *Column A* shows the contract period from year 2010 to year 2023.
2. *Column B* is the expected production forecast in 1,000 Stb/d for a successful exploration. The column can also be programmed to utilize decline curve formulas.
3. *Column C* shows the oil price in \$/Stb.
4. *Column D* shows the gross revenue, which is the oil production times the price of oil.
5. *Column E* shows the royalty payment to the host government. This is 20% of the gross revenue in Column D.
6. *Column F* is the net revenue after the royalty payment.
7. *Column G* shows estimated intangible exploration geological and geophysical (G&G) costs.
8. *Column H* is the estimated drilling and facilities costs required for exploration and field development.
9. *Column I* shows the estimated operating expenditure to operate the field when it is developed.
10. *Column J* is the depletion, depreciation, and amortization (DD&A). In this column the tangible CAPEX of 436,500 M\$ is depreciated straight line over a five year period. Any other depreciation and asset life can be used as is specified by the government tax rules or as specified in the contract. The intangible CAPEX in this cash flow is expensed, depending on the tax rules and the contract.
11. *Column K* shows total deductions, which includes expensed CAPEX, OPEX, and DD&A.
12. *Column L* shows tax loss carry forward. The loss carry forward provision will again depend on the tax rules of the host government or they will be specific to the contract. The loss carry forward may be 100% of the loss available during a certain year or it may be limited to a certain percentage (say 25%) of the income before tax. If the loss carry forward has a limitation of 25%, this means that 75% of the income before tax will become taxable income, it will be taxed, and the loss carry will continue until it is fully recovered.

Table 5-3. Typical cash flow for a concessionary (royalty tax) system

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
Year		Oil Production (MStb)	Oil Price (\$/Stb)	Gross Revenue (M\$)	20.0% Royalty (M\$)	Net Revenue (M\$)	CAPEX		Operating Cost (M\$)	DD&A (M\$)	Total Deductions (M\$)	Tax Loss C/F (M\$)	Taxable Income (M\$)	35.0% Income Tax (M\$)	NET CASH FLOW	
							Intangible (M\$)	Tangible (M\$)							Contractor (M\$)	Government (M\$)
2010		-	70.00	-	-	-	67,500	54,000	-	-	67,500	-	(67,500)	-	(121,500)	-
2011		-	70.00	-	-	-	45,000	45,000	-	-	45,000	(67,500)	(112,500)	-	(90,000)	-
2012		-	70.00	-	-	-	31,500	202,500	-	-	31,500	(112,500)	(144,000)	-	(234,000)	-
2013		3,784	70.00	264,845	52,969	211,876	-	135,000	13,742	87,300	101,042	(144,000)	(33,166)	-	63,134	52,969
2014		3,216	70.00	225,118	45,024	180,095	-	-	11,756	87,300	99,056	-	81,039	28,364	139,975	73,387
2015		2,734	70.00	191,351	38,270	153,080	-	-	10,068	87,300	97,368	-	55,713	19,500	123,513	57,770
2016		2,324	70.00	162,648	32,530	130,118	-	-	8,632	87,300	95,932	-	34,186	11,965	109,521	44,495
2017		1,975	70.00	138,251	27,650	110,601	-	-	7,413	87,300	94,713	-	15,888	5,561	97,627	33,211
2018		1,679	70.00	117,513	23,503	94,011	-	-	6,376	-	6,376	-	87,635	30,672	56,963	54,175
2019		1,427	70.00	99,886	19,977	79,909	-	-	5,494	-	5,494	-	74,415	26,045	48,370	46,022
2020		1,213	70.00	84,903	16,981	67,923	-	-	4,745	-	4,745	-	63,177	22,112	41,065	39,093
2021		1,031	70.00	72,168	14,434	57,734	-	-	4,108	-	4,108	-	53,626	18,769	34,857	33,203
2022		876	70.00	61,343	12,269	49,074	-	-	3,567	-	3,567	-	45,507	15,927	29,580	28,196
2023		745	70.00	52,141	10,428	41,713	-	-	3,107	-	3,107	-	38,606	13,512	25,094	23,940
2024		21,002		1,470,167	294,033	1,176,133	144,000	436,500	79,008	436,500	659,508		192,625	192,427	324,198	486,460
2025																
2026																
2027																
2028																
2029																
2030																

Contractor				Government	
Net Present Value, M\$ @ 10.0%				24,340	232,153
Internal Rate of Return				11.5%	
Discounted Take				9.5%	90.5%
Un-Discounted Take				40.0%	60.0%

13. *Column M* shows the taxable income.
14. *Column N* shows the income tax of 35% on the taxable income.
15. *Column O* shows the contractor's net cash flow. The contractor's net cash flow is typically: Gross Revenue – Royalty – CAPEX – OPEX – Taxes.
16. *Column P* shows the host government's cash flow, which in this case is Royalty + Tax.
17. *Cell E26* shows the contractor's net present value (NPV) at 10% discount rate.
18. *Cell F26* shows the host government's net present value (NPV) at 10% discount rate.
19. *Cell E27* shows contractor's internal rate of return (IRR).
20. *Cells E28 to F29* show contractor's and host government's discounted and undiscounted takes.

To make sure that the calculations in the model are correct, it is recommended to always incorporate a check in the cash flow. The check in this case is that the sum of the two net cash flows (contractor and government) + CAPEX + OPEX must equal the gross revenue.

Example 5–1

A 30,000-barrel-per-year field is developed with a total cost (tangible and intangible) of \$250,000. The operating cost over the productive life of the field is estimated at \$120,000 per year. Calculate the contractor and government share if the royalty is 20% and the tax rate is 35%. Assume the crude oil price of \$60/Stb.

Solution: The calculations in table 5–4 are summarized from a detailed cash flow. In the cash flow, the tangible costs are depreciated (straight line) over a period of five years. The intangible costs are expensed in the year incurred. Calculations are shown for the first year only.

Table 5–4. Sample calculations for a concessionary (royalty tax) system (Example 5–1)

	Contractor Share	Host Government Share
Production, barrels	30,000	
Crude oil price, \$/Stb	60	
Gross revenue, \$	1,800,000	
Less royalty @ 20%, \$	–360,000	360,000
Gross revenue after royalty, \$	1,440,000	
Capital expenditure (CAPEX), \$	–250,000	
Operating expenditure (OPEX), \$	–120,000	
Taxable income, \$	1,070,000	
Income tax @ 35%, \$	–374,500	374,500
Net cash flow (NCF), \$	695,500	734,500
<i>Government Take</i> = $734,500 / (734,500 + 695,500) \times 100 = 51\%$		

The production sharing system

Production sharing systems or agreements are more common now as compared to the concessionary system. The first PSA was signed in 1966 between the Indonesian National Oil Company and IIAPCO. Many aspects of the government/contractor relationship under a PSA may be negotiated, but some are fixed. The basic structure is effectively predetermined by the legislation of the host government. Similar to the royalty and tax system, typically model PSAs are put forward by the host government as a basis for bidding and negotiations. Once a contract is signed, the fiscal terms remain fixed for the contract period of 25 to 30 years. The PSAs are not limited to exploration only. In order to avail the technical expertise of international oil companies, development of existing oil fields is also done under PSAs. Some gas field development projects have a combination of agreements, where the upstream is developed under a PSA and the downstream is a joint venture.

1. *Royalties.* Although rare, some PSAs include provision for royalty payments. Royalties are taken right off the top of gross (wellhead) revenues. Some systems allow a netback of transportation costs. This occurs when there is a difference between the point of valuation for royalty calculation purposes and the point of sale. If royalties are included, they are usually in the range of 8% to 15% of gross revenue. Royalty is paid directly to the host government as oil and gas are produced and sold, irrespective of the level of profit or loss from the contract area.

2. *Cost recovery (CR)*. The sequence of calculations following the bonus and royalty calculations leads to the recovery of costs. Under the concessionary system, these are called deductions; while in the contractual systems these are called cost recovery. The gross revenue, after royalty deduction if any, is split into two portions. One portion is called *cost recovery oil* (or simply *cost oil*, *cost hydrocarbons*, *cost petroleum*, etc.) and the second is called profit oil (PO). The contractor's costs (CAPEX and OPEX), also referred to as *petroleum costs*, are recovered from the cost recovery portion. It has to be very clearly specified in the agreements as to what is included in the petroleum costs. In some agreements, any unrecovered costs are carried forward and recovered in the succeeding years. Any *excess cost recovery* (ECR) left after all the costs have been recovered is again split between the contractor and the host government according to the fiscal terms in the contract. Cost recovery limits, also referred to as cost recovery ceilings, typically range from 30% to 60%. The recoverable costs, recovered from the cost recovery oil, normally are:
- Unrecovered costs carried over from the previous years. These may include (i) tax loss carry forward, (ii) unrecovered depreciation balance, (iii) unrecovered amortization balance, and (iv) cost recovery carry forward. There may be a ceiling on the loss carry forward, i.e. only a certain percentage can be deducted each year and the remaining carried forward.
 - Operating costs. These may include some of a contractor home office's general and administrative (G&A) costs. Some contracts allow a percentage of the total petroleum costs to be recovered as G&A costs.
 - Expensed capital costs. Some contracts allow the capital expenditure to be expensed in the year incurred and recovered from the cost recovery. In other cases, these have to be amortized, and then only the amortized amount is allowed for recovery, while the unrecovered balance is carried forward.
 - Current year DD&A. Sometimes depreciation of fixed capital assets is differentiated from the amortization of intangible capital costs. In some contracts, the intangible exploration and development costs are not amortized. They are expensed in the year incurred.
 - Interest on financing (usually with limitations).
 - Investment credits and uplift. These are incentives. Uplift allows the contractor to recover an additional percentage of

capital costs through cost recovery. For example, an uplift of 15% on \$70 million of capital expenditure allows the contractor to recover \$80.5 million, thereby reducing the contractor's tax. Uplifts can create incentives for the contractor during a low product price environment or marginal field development.

- g. Abandonment cost recovery fund. Anticipated cost of abandonment is accumulated through a sinking fund that matures at the anticipated time of abandonment of the block.
3. *Profit oil (PO) split and taxation.* The oil left after the royalty payment and cost recovery deduction is referred to as profit oil. This is immediately split between the contractor and the host government according to the fiscal terms of the contract. The split in most countries ranges from 10% to more than 55% for the contractor. It is negotiated before the contract is signed and is dependent on geopotential, costs, infrastructure, political stability, and other key factors that may influence the business decisions of the contractor. If the contractor has to pay taxes, these are deducted from the profit oil portion of the contractor. To attract investors, governments usually introduce royalty and tax holidays in the contract. These specify that for a given holiday period, say 15 years, royalty or taxes are not payable. The contractor's share of profit oil, excess cost recovery, and cost recovery are often referred to as the contractor entitlement. The taxes can be as high as 56%, so this is another cash flow to the host government. In some production sharing contracts, the national oil company pays the taxes for the contractor.

Sample mathematical calculations for a typical production sharing agreement are shown in example 5-2. A flowchart showing the sequence of the required mathematical calculations is shown in figure 5-3. A typical production sharing cash flow is shown in table 5-5.

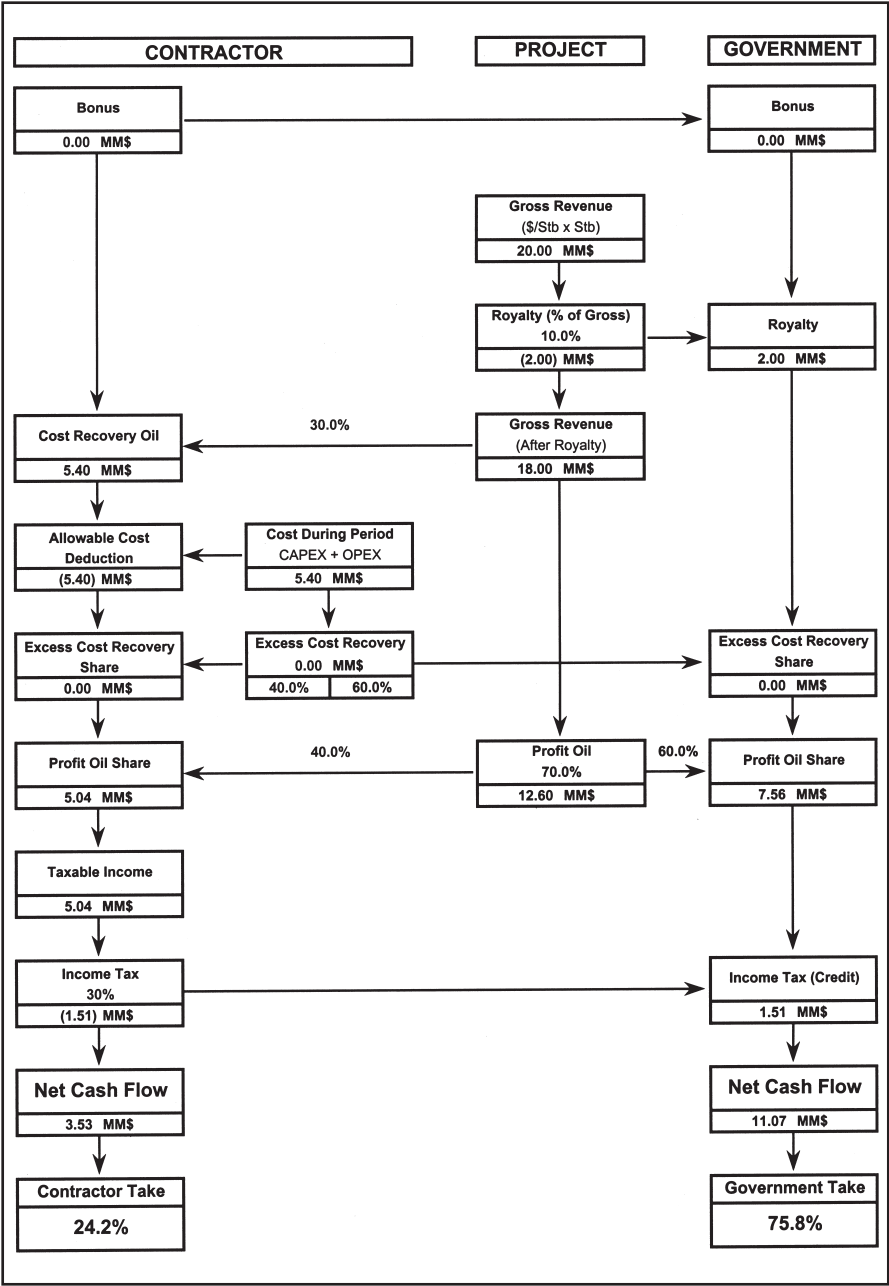


Fig. 5–3. Flow diagram showing disbursements under a typical production sharing contract (PSC)

Table 5–5. Typical cash flow for a production sharing contract (PSC)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	
	Year	Prod. (MStd/d)	Oil (MStd/d)	CAPEX (MM\$)	CAPEX (MM\$)	OPEX (MM\$)	OTHER COSTS (MM\$)	TOTAL COST (MM\$)	BONUS (MM\$)	PRICE (\$/bbl)	OIL REC. (%)	COST P. OIL (%)	HOST GOVT. SHARE (%)	COST TO RECOVER (MM\$)	CR (MM\$)	PO (MM\$)	ECR (MM\$)	CONT. (MM\$)	NET REVENUE (MM\$)	H. GOVT. (MM\$)	CUM. NET REVENUE (MM\$)	H. GOVT. (MM\$)	R. FACTOR
9	1	2010		24.72				24.72	5.00	70.00	35.00	58.00	58.00	24.72	-	-	-	(29.72)	5.00	(29.72)	5.00	-	
10	2	2011		25.48				25.48	-	70.00	35.00	58.00	58.00	50.20	-	-	-	(25.48)	-	(55.20)	5.00	-	
11	3	2012			93.96			93.96	-	72.10	35.00	58.00	58.00	144.16	-	-	-	(93.96)	-	(149.16)	5.00	-	
12	4	2013			149.24			149.24	-	74.26	35.00	58.00	58.00	293.40	-	-	-	(149.24)	-	(298.40)	5.00	-	
13	5	2014			241.12			241.12	-	76.49	35.00	58.00	58.00	534.52	-	-	-	(241.12)	-	(539.52)	5.00	-	
14	6	2015	6.986			24.20	1.91	26.11	5.00	78.79	35.00	58.00	58.00	490.32	70.31	130.58	-	94.04	80.74	(445.48)	85.74	0.22	
15	7	2016	13.973			24.92	3.83	28.75	5.00	81.15	35.00	58.00	58.00	374.21	144.86	269.02	-	224.10	161.03	(221.38)	246.77	0.63	
16	8	2017	13.973			25.68	3.83	29.51	5.00	83.58	35.00	58.00	58.00	254.51	149.20	277.09	-	231.07	165.71	9.70	412.48	1.02	
17	9	2018	13.973			26.44	3.83	30.27	-	86.09	30.00	61.00	61.00	153.05	131.72	307.35	-	221.33	187.49	231.02	599.96	1.35	
18	10	2019	13.973			27.24	3.83	31.07	-	88.67	25.00	70.00	70.00	71.06	113.06	339.19	-	183.75	237.43	414.78	837.40	1.59	
19	11	2020	12.301			28.04	3.37	31.41	-	91.33	20.00	72.00	72.00	20.45	82.02	328.06	-	142.47	236.20	557.24	1,073.60	1.76	
20	12	2021	10.822			28.88	2.96	31.84	10.00	94.07	20.00	72.00	72.00	-	74.32	297.28	22.03	99.85	239.90	657.10	1,313.50	1.85	
21	13	2022	9.534			29.76	2.61	32.37	-	96.90	20.00	72.00	72.00	-	67.44	269.75	35.07	85.35	219.47	742.44	1,532.97	1.92	
22	14	2023	8.384			30.64	2.30	32.94	-	99.80	20.00	72.00	72.00	-	61.08	244.33	28.15	76.29	196.18	818.74	1,729.15	1.98	
23	15	2024	7.397			31.56	2.02	33.58	-	102.80	20.00	72.00	72.00	-	55.51	222.03	21.92	68.31	175.65	887.05	1,904.80	2.02	
24	16	2025	6.493			32.52	1.78	34.30	-	105.88	20.00	75.00	75.00	-	50.19	200.75	15.89	54.16	162.48	941.21	2,067.28	2.04	
25	17	2026	5.726			33.48	1.57	35.05	-	109.06	20.00	75.00	75.00	-	45.59	182.34	10.54	48.22	144.66	989.43	2,211.94	2.05	
26	18	2027	5.041			34.52	1.38	35.90	-	112.33	20.00	75.00	75.00	-	41.34	165.35	5.44	42.70	128.09	1,032.12	2,340.03	2.06	
27	19	2028	4.438			35.52	1.21	36.73	-	115.70	20.00	75.00	75.00	-	37.48	149.93	0.75	37.67	113.01	1,069.79	2,463.04	2.05	
28	20	2029	3.918			36.60	1.07	37.67	-	119.17	20.00	75.00	75.00	3.59	34.08	136.34	-	30.50	102.25	1,100.29	2,555.29	2.05	
29	21	2030							-	122.75	20.00	75.00	75.00	3.59	-	-	-	-	-	1,100.29	2,555.29	2.05	
30	22	2031							-	126.43	20.00	75.00	75.00	3.59	-	-	-	-	-	1,100.29	2,555.29	2.05	
31	23	2032							-	130.22	20.00	75.00	75.00	3.59	-	-	-	-	-	1,100.29	2,555.29	2.05	
32	24	2033							-	134.13	20.00	75.00	75.00	3.59	-	-	-	-	-	1,100.29	2,555.29	2.05	
33	25	2034							-						-	-	-	-	-				
34																							
35																							
36																							
37																							
38																							
39																							
40																							
41																							
42	MSB/d	1.00	1.25	1.50	2.00	2.50	> 2.50		OIL PRICE \$/bbl	1.00	1.25	1.50	2.00		BONUS PAYMENTS				NPV				
43		58.0	61.0	70.0	72.0	75.0	80.0	-		50.0	40.0	40.0	35.0	Year	BONUS (MMB/D)	PROD. (MMB/D)	BONUS (MM\$)		10%	IRR (%)	TAKE %		
44	15.00	58.0	61.0	70.0	75.0	80.0	85.0	40.00	50.0	40.0	30.0	20.0	20.0	2010	5.00	5.00	5.00	CONTRACTOR	252.88	21.0%	23.61		
45	25.00	58.0	61.0	70.0	75.0	80.0	85.0	70.00	35.0	30.0	25.0	20.0	20.0	2015	5.00	10.00	5.00	HOST GOVERNMENT	818.32		76.39		
46	35.00	58.0	61.0	70.0	75.0	80.0	85.0	100.00	35.0	30.0	25.0	20.0	20.0	2015	5.00	10.00	5.00	TOTAL PROJECT	1,071.20		39.6%		

Sample PSC cash flow spreadsheet

The spreadsheet has to be flexible enough to evaluate the effect of the fiscal terms for different field sizes, work commitment, pricing scenarios, etc. Description of the various columns of the spreadsheet in table 5–5 follows.

1. *Column A* shows the contract period from year 1 to year 25.
2. *Column B* shows the calendar years of the contract period.
3. *Column C* is the expected production forecast in MStb/d (1,000 stock tank barrels per day) for a successful exploration. The column can also be programmed to utilize decline curve formulas.
4. *Column D* shows estimated exploration geological and geophysical (G&G) costs.
5. *Column E* is the estimated drilling and facilities costs required if the exploration effort is successful and the field has to be developed.
6. *Column F* shows the estimated operating expenditure to operate the field when it is developed.
7. *Column G* is other costs, such as tariffs or variable operating expenditures. For example, \$0.60/Stb is used in this spreadsheet.
8. *Column H* shows total recoverable cost or petroleum cost.
9. *Column I* shows bonus payments by the contractor. Note the bonuses are normally not considered petroleum costs and therefore cannot be recovered from cost recovery oil. The model accommodates up to three bonus payments tied to a particular year (e.g., 2010, 2015, and 2016 and so on). In addition, it accommodates up to three bonus payments tied to user specified cumulative production limits. When the respective cumulative production is reached, bonus is paid in the corresponding year.
10. *Column J* shows oil price. In the spreadsheet, the base price is \$70/Stb in 2010 and escalated at 3% per year from year 2012 onward.
11. *Column K* shows the cost recovery split based on the R-factor and the price of oil.
12. *Column L* shows the profit oil share of the host government based on the R-factor and the oil production during the year.

13. *Column M* shows the ECR (excess cost recovery) share of the host government. In this spreadsheet, it is assumed that the ECR share of the host government is the same as its profit oil share.
14. *Column N* shows the balance of recoverable cost each year (i.e., the balance in 2010 is to be recovered from the cost recovery petroleum in year 2011 and so on). Any costs that are not recovered in a particular year will be carried forward to the following years until fully recovered.
15. *Column O* shows the amount of cost recovery oil revenue allocated to the contractor, based on the balance in column N and the cost recovery split in column K.
16. *Column P* shows the profit oil portion of the gross revenue, which will be shared between the contractor and the host government according to their allocated shares.
17. *Column Q* shows the excess cost recovery amount (left after the contractor has recovered its costs from the cost recovery portion of the gross revenue), which will be shared between the contractor and the host government according to their allocated shares.
18. *Column R* shows the ultimate net revenue share or net cash flow of the contractor. This is its profit oil share + share of the ECR if any + the CR revenue – the bonuses and costs.
19. *Column S* shows the ultimate net revenue share or net cash flow of the host government. This is its profit oil share + share of the ECR if any + bonuses.
20. *Columns T and U* show the cumulative net revenue of the contractor and the host government, respectively.
21. *Column V* shows the contractor's R-factor for each year. The R-factor in this case is the cumulative gross revenue of the contractor divided by the cumulative cost of the contractor. These calculations are shown in *Columns Z and AA* (not shown in the figure).
22. *Cells T44–V44* show the profitability indicators of the contractor and the host government.

As discussed in the royalty/tax system, there has to be a check in the spreadsheet to make sure that the calculations are done correctly. In table 5–5, the check should be that the contractor's revenue (Cell R38) plus the

host government's revenue (Cell S38) plus the total costs (Cell H38) must equal the gross revenue (Cell O38 + Cell P38).

Example 5–2

Information for a typical EPSA is given in table 5–6. Calculate the yearly net revenue (net cash flow) for the contractor and the host government. Assume the price of oil to be %78.79/Stb in 2011 and \$81.15/Stb in 2012.

Table 5–6. Data for production sharing agreement (for example 5–2)

Year	Oil Production (Stb/d)	Bonus (MM\$)	CAPEX (MM\$)	OPEX (MM\$)	Cost Rec. (CR) (%)	Contractor's Share	
						ECR (%)	PO (%)
2010	0.0	5.0	25.0		35	42	42
2011	6,986.0		50.0	6.0	35	42	42
2012	13,973.0			10.0	25	30	30

Solution:

For 2010:

The subscripts c and g refer to the contractor and government, respectively.

Since production is zero, $CR = 0$, $PO = 0$, and $ECR = 0$

$$\begin{aligned}\text{Contractor's Net Rev.} &= (CR - ECR) + PO_c + ECR_c - \text{Bonus} - \text{CAPEX} - \text{OPEX} \\ &= (0 - 0) + 0 + 0 - 5 - 25.0 - 0 = -30.0 \text{ MM\$}\end{aligned}$$

$$\begin{aligned}\text{Government's Net Rev.} &= PO_g + ECR_g + \text{Bonus} \\ &= 0 + 0 + 5 = 5 \text{ MM\$}\end{aligned}$$

$$\begin{aligned}\text{Cumulative Contractor's Cost} &= \text{Bonus} + \text{CAPEX} + \text{OPEX} \\ &= 5 + 25.0 + 0 = 30.0 \text{ MM\$}\end{aligned}$$

$$\begin{aligned}\text{Cost to Recover by Contractor} &= \text{CAPEX} + \text{OPEX} \\ &= 25.0 + 0 = 25.0 \text{ MM\$ (note that the} \\ &\quad \text{bonus is not treated as petroleum cost)}\end{aligned}$$

$$\text{Cumulative Contractor's Rev.} = (CR - ECR) + PO_c + ECR_c = 0 \text{ MM\$}$$

For 2011:

$$\begin{aligned}\text{Oil Revenue} &= \text{Oil production} \times \text{Oil Price} \times 0.365 \text{ (days/year)} = 6.986 \times \\ &78.79 \times 0.365 = 200.89 \text{ MM\$}\end{aligned}$$

$$\text{Cost Recovery} = 200.89 \times 0.35 = 70.31 \text{ MM\$}$$

$$\text{Profit oil} = 200.89 \times (1 - 0.35) = 130.58 \text{ MM\$}$$

$$\begin{aligned}\text{Contractor's PO Share, POc} &= 130.58 \times 0.42 \\ &= 54.84 \text{ MM\$}\end{aligned}$$

$$\begin{aligned}\text{Government's PO Share, POg} &= 130.58 \times 0.58 \\ &= 75.74 \text{ MM\$}\end{aligned}$$

$$\begin{aligned}\text{Cumulative Contractor's Cost} &= (\text{Cumulative Cost})_{2010} + \text{CAPEX}_{2011} + \\ &\quad \text{OPEX}_{2011} \\ &= 25 + 50 + 6 = 81.0 \text{ MM\$ (note that the} \\ &\quad \text{bonuses in this case are not treated as} \\ &\quad \text{petroleum costs)}\end{aligned}$$

$$\begin{aligned}\text{Cost to Recover} &= (\text{Cumulative CAPEX \& OPEX})_{2010-2011} - \text{CR}_{2011} \\ &= 81.0 - 70.31 = 10.69 \text{ MM\$}\end{aligned}$$

$$\text{Excess Cost Recovery, ECR} = 0 \text{ (Since Cost to Recover is } > 0 \text{)}$$

$$\begin{aligned}\text{Contractor's Net Rev.} &= (\text{CR} - \text{ECR}) + \text{POc} + \text{ECRc} - \text{Bonus} - \text{CAPEX} \\ &\quad - \text{OPEX} \\ &= (70.31 - 0) + 54.84 + 0 - 0 - 0 - 50.0 - 6 = 69.15 \text{ MM\$}\end{aligned}$$

$$\begin{aligned}\text{Cum. Contractor's Gross Rev.} &= \text{POc} + \text{CRc} + \text{ECRc} \\ &= 54.84 + 70.31 + 0 = 125.15 \text{ MM\$}\end{aligned}$$

$$\begin{aligned}\text{R-factor} &= \text{Cum. Contractor's Gross Rev.} \div \text{Cumulative Cost} \\ &= 125.15 \div 81 = 1.55\end{aligned}$$

$$\begin{aligned}\text{Government's Net Rev.} &= \text{POg} + \text{ECRg} + \text{Bonus} \\ &= 75.74 + 0 + 0 = 75.74 \text{ MM\$}\end{aligned}$$

For 2012:

$$\begin{aligned}\text{Oil Revenue} &= \text{Oil production} \times \text{Oil Price} \times 0.365 \text{ (days/year)} = 13.973 \\ &\quad \times 81.15 \times 0.365 = 413.88 \text{ MM\$}\end{aligned}$$

$$\text{Cost Recovery} = 413.88 \times 0.25 = 103.47 \text{ MM\$}$$

$$\text{Profit Oil} = 413.88 \times (1 - 0.25) = 310.41 \text{ MM\$}$$

$$\begin{aligned}\text{Contractor's PO Share, POc} &= 310.41 \times 0.30 \\ &= 93.12 \text{ MM\$}\end{aligned}$$

$$\begin{aligned}\text{Government's PO Share, POg} &= 310.41 \times 0.7 \\ &= 217.29 \text{ MM\$}\end{aligned}$$

$$\begin{aligned}\text{Cumulative Contractor's Cost} &= [\text{Cumulative Cost}]_{2011} + \text{CAPEX}_{2012} \\ &\quad + \text{OPEX}_{2012} \\ &= 81.0 + 10 + 0 = 91.0 \text{ MM\$}\end{aligned}$$

$$\begin{aligned}\text{Cost to Recover} &= [\text{Cost to Recover}]_{2011} + \text{CAPEX}_{2012} + \text{OPEX}_{2012} - \text{CR}_{2012} \\ &= 10.69 + 0 + 10.0 - 103.47 = -82.78 \text{ MM\$}\end{aligned}$$

$$\text{Excess Cost Recovery} = 82.78 \text{ MM\$}$$

$$\begin{aligned}\text{Contractor's Net Rev.} &= (\text{CR} - \text{ECR}) + \text{POc} + \text{ECRc} - \text{Bonus} - \text{CAPEX} \\ &\quad - \text{OPEX} \\ &= (103.47 - 82.78) + 93.12 + (82.78 \times 0.30) - 0 - 0 - 10.0 \\ &= 128.64 \text{ MM\$}\end{aligned}$$

$$\begin{aligned}\text{Cum. Contractor's Gross Rev.} &= (\text{Cum. Contractor's Gross Rev.})_{2011} + \text{CR}_{2012} \\ &\quad + \text{PO}_{2012} + \text{ECR}_{2012} \\ &= 125.14 + 20.69 + 93.12 + (82.78 \times 0.30) \\ &= 263.78 \text{ MM\$}\end{aligned}$$

$$\text{R-factor} = 263.78 \div 91 = 2.90$$

$$\begin{aligned}\text{Government's Net Rev.} &= \text{POg} + \text{ECRg} + \text{Bonus} \\ &= 217.29 + (82.78 \times 0.70) + 0 \\ &= 275.24 \text{ MM\$}\end{aligned}$$

Basic elements of production sharing and royalty tax systems

Petroleum laws and requirements vary widely from country to country. The most common provisions and regulations have to do with the following:

1. *Title.* Title to the hydrocarbons remains with the host government.
2. *Management.* The host government maintains the management control, and the contractor is responsible to the host government for execution of petroleum operations in accordance with the fiscal terms of the contract. The contractor's operations are normally monitored by a management committee and a technical committee. These committees have representation on behalf of the contractor and the government.
3. *Type of agreement:* production sharing, concessionary system (tax and royalty), risk service contract, or joint venture. In some cases the upstream could be either on production sharing or tax royalty systems and the downstream will be on joint venture and so on.
4. Size, shape, and geographic limits of area to be explored and developed.
5. *Grant of rights (oil, gas, deepest zone, etc.).* Each agreement will grant rights to the contractor to certain depth and/or geologic age

(such as Paleozoic, Mesozoic, Cenozoic, etc.). If the agreement is for oil and the contractor finds gas, the contractor will have no right to develop the gas and vice versa.

6. *The duration of each agreement.* Typically, each agreement will be for 30 to 40 years. For example, 3 years of exploration period followed by 3 years of field development period (if commercial discovery) and 25 years of production period (total of 31 years). In some cases, depending on the size of the contracted area/block/concession and the expected number of prospects in the contracted area, there may be a first exploration period (FEP) of 5 years, followed by a second exploration period (SEP) of 3 years, and then a third exploration period (TEP) of 2 years (total of 10 years). In other cases the FEP, SEP, and TEP may be of equal duration. The relinquishment at the end of each exploration period is based on a formula specified in the agreement.
7. *Fees and bonuses.* Various amounts of fees and bonuses have to be paid by the contractor to the host government. These will be normally linked to various milestones, e.g., signing of the contract, upon commercial discovery, achieving a certain production level, or achieving a certain cumulative production.
8. *Relinquishment or surrender.* There will be a formula in the agreement that if the contractor is not successful in finding commercial hydrocarbons, it has to surrender the contract area. If the contractor has a discovery or discoveries, then it will keep those areas for appraisal and possible development and relinquish the rest of the area. A set formula will be given in the upstream project agreement, which will be used when it is time for relinquishment. Relinquishment, in almost all countries, is a requirement to relinquish a certain portion of the exploration area within a stated time limit. The area to be relinquished normally constitutes between 50% and 75% of the original area, and relinquishment is made typically in two or three steps (for example, 25% every two years).
9. Selection and convertibility of acreage.
10. Assignment or transfer of acreage, lease, or concession.
11. Royalty payments, sharing of profits, and cost recovery.

12. Investment credit and uplift. These are paper-based incentives. The investment credit and uplift help in reducing the tax burden of the contractor. The effect of uplift will be shown later in this chapter.
13. Tax obligations.
14. *Ringfencing*. This provision restricts a contractor's recovery of costs to those associated only with the successful prospect or block. For example, a contractor has interest in two blocks, A and B. He has spent \$100 million on block A, but there is no commercial discovery. Block B has a commercial discovery but it is ring fenced, so the contractor cannot recover the \$100 million of block A from the cost recovery of block B.
15. *Domestic obligation*. Obligation to supply domestic markets first and to build local refineries. Some host governments have domestic requirements for oil and gas produced from the contract area. These requirements are referred to as the domestic supply requirements or domestic market obligation (DMO). If the contract has such provision, a certain percentage of the contractor's pretax profit oil share is sold to the host government, usually at a price lower than the market price.
16. Employment and training of nationals.
17. Equity participation by government and repatriation of capital by the contractor. The equity participation could be from the inception or through a back-in option. The back-in option will be tied to a certain milestone at which the host government of the NOC will have an option to join as a joint venture partner. The participation percentage will also be outlined in the upstream project agreement.
18. *Financing and technology*. The contractor provides all the required financing and technology in accordance with the work commitment in the contract and bears all the risk.
19. *Annual work program*. The contractor is required to submit annual work programs and budgets for review and approval by the management and technical committees.
20. *Work commitment*. Work commitment in the exploration phase is generally measured in kilometers of seismic data to be acquired and the number of exploration wells to be drilled. In some cases, the work commitment may involve acquisition of seismic data with an option to drill. These are referred to as seismic options.

The duration of the right to explore for hydrocarbons normally ranges from three to six years for the initial exploration period. The exploration phase may be divided into a number of periods (two or three), each having the related work obligation. Linked to the exploration phase or any of its periods, an obligatory work program is stipulated, such as a number of seismic lines and a number of exploration wells within a specified time limit. This obligation may be limited to a certain minimum cost ceiling. However, due to inflation, the budgeted amount may not deliver a desired scope of work. Instead, if the scope of the work is specified, then no matter how many dollars are spent on the project it must be done. Therefore, it is useful to determine the work obligation in terms of scope of work to be performed instead of monetary value. The distinction of the wells to be drilled should be quite specific, indicating the depth or geologic horizons to be penetrated. If the contract covers several blocks, the work program should specify the work in each block, rather than in the area as a whole. On the expiry of the exploration period, if no commercial discovery is made, the contract automatically expires. The terms of work commitments outline penalties for nonperformance.

21. *Bonus payments.* Cash bonuses are sometimes paid upon finalization of negotiations and contract signing. This bonus is referred to as a signature bonus. The amount of the bonus depends on the prospectivity of the contract area and competition. Once the field is put on production, production bonuses are paid when production from a given contract area or block reaches a specified level. For example, the contractor may pay \$2 million when the production from the block reaches 10,000 Stb/d and another \$2 million when the production reaches 20,000 Stb/d, and so on. Production bonuses may also be tied to the cumulative production. For example, \$2 million may be paid when 5 million barrels are produced and another \$2 million may be paid when the cumulative production reaches 10 million barrels. The bonuses are normally not recoverable costs to the contractor. These bonuses become a cost to the contractor and revenue to the host government.
22. *Sliding scales.* In order to account for uncertainties in field size, oil price, average daily production, etc., many fiscal terms (royalties, taxes, cost recovery, excess cost recovery, and profit oil splits) are based on a sliding scale. These factors are either tied to the average barrels of oil produced, to the R-factor, crude oil price,

or a combination of these. Each increment in the sliding scale is called a tranche. For example, if average daily production is 10,000 Stb/d, the cost recovery is 40%. If the daily oil production is 20,000 Stb/d, the cost recovery changes to 35%, and so on. Alternatively, on the first 10,000 Stb/d, it is 40%, and on the second 10,000 Stb/d, it is 35%. The objective of the sliding scale fiscal terms is to create an environment to benefit contractors in lower production on marginal fields and/or in the low product price environment. As the production increases, the contractor's take flexes downward. This theoretically allows reasonable terms for the contractor to develop both large and small fields. Fixed fiscal terms for a contract area can become regressive when applied to the small and marginal discoveries, and they may even completely prevent their development. Similarly, if the terms are tied to the oil price, the contractor's take is automatically adjusted when the price variations take place. Such terms also restrict the contractor from making excessive profits in case a larger than anticipated field is encountered or the product price increases. Variables typically subject to sliding scale are:

- a. Bonuses
- b. Royalty
- c. Cost recovery oil
- d. Profit oil
- e. Excess cost recovery (ECR)
- f. Economic rent tax
- g. Excess/windfall profits tax (EPT or WPT)
- h. Corporate tax
- i. Uplifts (increases depreciated capital thus reducing taxes)

Some typical sliding scale factors are given in table 5–7.

In reality, the triggers normally take place on the results of the previous quarter, i.e. R-factor or IRR in the preceding quarter. In spreadsheet models, the triggers are activated on the results of the previous year. They can be incorporated in the same year but *circularity* results in the calculations, and this may unstabilize the model. In table 5–7, the PO split for production $\leq 10,000$ Stb/d is 25% for R-factor less than or equal to 1.0 and 23% for R-factor less than or equal to 1.25 but greater than 1.0 and so on.

Table 5–7. Typical sliding scale factors

Profit Oil Split (Contractor’s Share), %					Cost Recovery Limit, %				
Production Stb/d	R-Factor				Price \$/Stb				
	1.00	1.25	1.50	2.00		1.00	1.25	1.50	2.00
≤ 10,000	25	23	21	19	≤ 40	50	48	46	44
10,001–15,000	20	18	16	14	50	48	46	44	42
15,001–20,000	15	13	12	10	65	46	44	42	40
> 20,000	10	10	10	10	75	44	42	40	38
					85	42	40	38	36

23. *R-factor*. Most contracts use this factor for triggering sliding scale factors such as cost recovery, profit oil split, royalty, taxes, etc. The R-factor is the ratio of the cumulative contractor’s revenue after taxes and royalty, and the cumulative contractor’s cost from the day the contract is signed. The $R = 1$ implies a breakeven point for the contractor. The R-factor is calculated each year or quarterly and is used as the basis for adjusting the royalty fraction, cost recovery, profit oil splits, and taxes in accordance with a predetermined schedule. Thus, as the contractor makes more profit, a larger portion of the gross revenue is paid to the host government in the form of royalty, taxes, profit oil, etc. Mathematically, the R-factor is calculated by the following equations.
24. *Rate of return (ROR)*. As in item 23 above, the ROR (also referred to as internal rate of return, or IRR) can be used to trigger the sliding scale factors. In fact, experience shows that most of the other variables like production rate, cumulative production, or product price will most of the time fail to cover the boundary conditions appropriately. The main reason for this is because these variables are very uncertain. For example, an agreement was done at a time when the long-term price expectation was not to exceed \$50/Stb. At that time it was decided to link the royalty to the oil price (e.g., when the price is less than <\$20/Stb the royalty is 2.5%, when the price is between \$20/Stb and \$30/Stb the royalty is 5%, and so on), while the maximum royalty will be 10% when the oil price is equal to or above \$50/Stb. The market conditions change and the price over the contract period stays much higher than the \$50/Stb limit.

This will significantly improve the contractor’s profitability but the host government will not gain as much.

Therefore, linking the terms to the contractor’s profitability such as ROR will be much more responsive to the changing conditions than the other variables. This is because the ROR reflects changes in project cost, production column, reserves, product prices, operating costs, etc. Figure 5–4 shows some of the advantages and disadvantages of linking the fiscal terms to various outcomes.

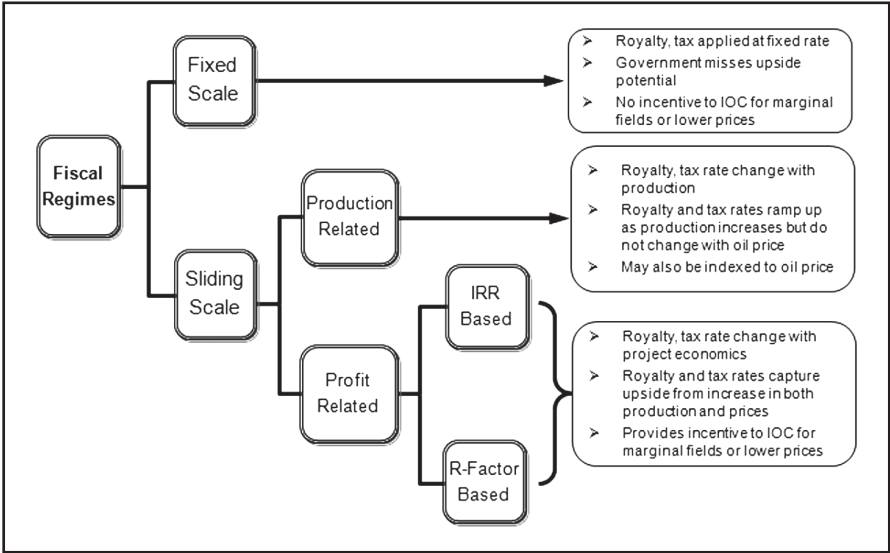


Fig. 5–4. Advantages and disadvantages of linking fiscal terms to various outcomes

$$R = \frac{X}{Y} \tag{5.1}$$

where

- X = Cumulative net revenue actually received by the contractor equals *turnover* (gross revenues) for all tax years less royalties and taxes
- Y = Total cumulative expenditure, exploration, and appraisal expenses and operating costs actually incurred by contractor from date contract is signed. A variation of the above equation, typically used by the Colombian government, is given below.

$$R = \frac{X}{[ID + A - B + (\alpha \times C) + GO]} \quad (5.2)$$

where

X = Accumulated earnings of the contractor

$[ID + A - B + (\alpha \times C) + GO]$ = Accumulated investment + Accumulated costs of the contractor

ID = 50% of cumulative gross development costs

A = Cumulative gross successful exploration costs

B = Cumulative successful exploration costs reimbursed by NOC (50% partner)

C = Cumulative gross unsuccessful exploration costs

α = Proportion of dry-hole costs reimbursed by NOC

GO = Contractor cumulative net operating costs including war tax payments and duties on imports

Contract initiation and award

To start with, the national oil company or the oil ministry of a host government divides all the prospective exploration areas into concession areas or blocks. Data packages are prepared for each block, and they are advertised so interested international oil companies can bid on them. The bids are normally received in two versions: a technical bid and a commercial bid. The technical bids are passed on to the exploration department for technical evaluation, and the legal/financial departments evaluate the commercial bids.

The technical bids are evaluated based on the following criteria:

1. Work commitment the contractor is proposing
2. The technical standing of the contractor
3. Experience in the area or similar areas
4. Financial position of the company
5. Previous experience and success in exploration, etc.

The economists build a production sharing cash flow model as shown in table 5–5 (an Excel spreadsheet is included on the CD that accompanies volume 2). This model is then used to test the fiscal terms proposed by each contractor to determine which contractor's terms are more favorable for the host government. Based on these evaluations, only the contractors whose terms are favorable to the host government are short-listed for further

detailed negotiations. Ultimately, a contract is signed with a contractor whose terms give the best return to the host government.

In some cases, the minimum work commitment is predefined in the bid packages and the legal terms are outlined. Economists evaluate the commercial bids for the fiscal terms proposed by the contractor.

Government and contractor take

The division of net cash flow (based on the agreed fiscal regime) between the contractor and the host government is called *contractor take* and *government take*, respectively. The contractor take provides a good yardstick for comparing the fiscal terms of one contractor to another while competing for the same concession area. It focuses exclusively on the division of profits and correlates directly with reserve values, field size thresholds, and other measures of relative economics.¹

The government and contractor takes are quantitative indicators that measure the overall effect of a particular fiscal system, thus allowing a meaningful comparison between them. The government take is defined as the amount of money received by the government over the life of a project. It is a percentage of the total net cash flow that the government would have received if the project was done by the government itself. The contractor take is a complement to the government take.³

A prerequisite for calculating the contractor or government takes is to first develop a detailed cash flow model that incorporates (a) intended fiscal terms, (b) exploration and facilities' capital expenditure (CAPEX), (c) estimated operating expenditure (OPEX), and (d) the anticipated production forecast. For exploration projects, the sensitivity of the fiscal terms is normally evaluated at various field sizes. For example, what will be the contractor take if a 20-million-barrel oil field is discovered versus a discovery of a 50-million-barrel field? Once the cash flow model has been built, the merit of any proposed fiscal terms can be evaluated. Based on these economic evaluations, technical capabilities of the contractors, and the contractor's work commitments, the fiscal terms are negotiated. The result of government efforts to arrive at feasible fiscal terms, which are considered appropriate under a variety of unknown circumstances, are sometimes referred to as *fiscal marksmanship*.

Mathematically, the takes are calculated using the following equations.

$$\text{Government Take} = \frac{\text{Government's NCF}}{\text{Government's NCF} + \text{Contractor's NCF}} \quad (5.3)$$

$$\text{Contractor's Take} = 1 - \text{Government Take} \quad (5.4)$$

Equations (5.3) and (5.4), used for calculating the government/contractor takes, do not account for the time value of money. The disbursements such as signature bonuses, discovery and commercial discovery bonuses, and indirect taxes (sales tax, value-added tax, and import duties) that are levied on the capital items greatly affect the net present value of the contractor's cash flow. A fiscal regime with such up-front disbursements is referred to as a front-end loaded fiscal regime. Therefore, the discounted takes for significantly front-end loaded fiscal regimes is different from the undiscounted takes.

Since the timing of payments to governments can vary widely depending on the structure of fiscal terms, it is important to include the effect of this timing in the calculation of government/contractor takes. Front-end loaded fiscal terms tend to increase government take on a discounted basis. Rapp et. al. introduced a concept called the *front-end loading index*.⁴

The front-end loading index (FLI) highlights the spread in the discounted and undiscounted takes. A value of FLI = 0 indicates no front-end loading at all (an ideal condition). The higher the FLI becomes, the more front-end loaded the fiscal regime becomes. The fiscal regime with excessive front-end loading becomes less attractive for the contractor. The FLI is given by the following equation.

$$FLI = \frac{\text{Discounted Government Take}}{\text{Undiscounted Government Take}} - 1 \quad (5.5)$$

The following example clarifies the calculations of government and contractor takes.

Example 5–3

Gross revenue from a field over its economic life is \$500 million. The capital and operating costs over the project life is \$200 million. The contractor pays \$129 million in royalties and taxes to the host government. Calculate the government and contractor takes for this concessionary arrangement.

Solution: The calculations are shown in table 5–8.

Applying equation (5.5) to the fiscal terms in tables 5–3 and 5–5 gives

$$FLI(\text{Table 5-3}) = \frac{52.7}{49.6} - 1 = 0.063$$

$$FLI(\text{Table 5-5}) = \frac{79.37}{65.49} - 1 = 0.212$$

Table 5–8. Calculation of government/contractor take (example 5–3)

	Contractor Share	Host Government Share
Gross revenue, MM\$	500.00	
CAPEX and OPEX, MM\$	–200.00	
Net operating revenue, MM\$	300.00	
Royalty and taxes, MM\$	–129.0	129.0
Net after-tax cash flow, MM\$	171.00	129.00
Total Project Cash Flow, MM\$	300.00	
Host government take	$129/300 \times 100 = 43\%$	
Contractor take	$171/300 \times 100 = 57\%$	

The above comparison shows that the fiscal terms in table 5–3 are less front-end loaded as compared to those of table 5–5. Although the above concept makes sense, the question is “do the contractor and the governments have the same discount rates?”

Risk service contracts

The countries importing crude oil normally use the risk service contract approach. According to this type of contract, the contractor provides all the capital required for the exploration and development of petroleum resources. If the exploration and development efforts are successful, the contractor is allowed to recover its costs from the revenues generated by the sale of oil and/or gas. The government keeps the oil. In addition, the contractor is paid a fee based on a percentage of the remaining revenues. The fee may or may not be subject to taxes. The term *risk* implies that if the contractor is not successful in finding oil and/or gas, all his costs of exploration are to his account with no liability to the host government.

On the other hand, some governments have a lot of money but lack the technical know-how. They may favor pure service contracts. According to such contracts, although rare, the host government bears the cost of exploration and development, and the contractor is paid a fee. This type of contract is also referred to as the *technical service agreement* (TSA).

There is little difference between the risk service contracts and the PSCs. The difference lies in how payments are made to the contractor. Other than that, the arithmetic and terminology are quite similar. Therefore, many service contracts are commonly referred to as PSCs.

Rate of return (ROR) contracts

A rate of return contract is just another version of the PSC or concessionary systems where the fiscal terms are on a sliding scale, tied to the ROR rather than the simple R-factor. Some governments have developed progressive taxes or sharing arrangements based on a project's rate of return. The effective government take in such agreements increases as the project ROR increases. The ROR system takes into account product price, field size, production capacity, operating cost, cost of capital, and timing of cash flows (inflows and outflows).

A modest royalty and tax in the beginning or early production phase characterize these types of contracts. The host government does not receive any other share until the contractor has recovered the initial investment plus a predetermined minimum (threshold/hurdle) rate of return. The contractor's share in this type of contract is calculated by accumulating the contractor's net cash flows and compounding them at the minimum (threshold) rate of return until the cumulative value becomes positive, i.e., payout time or when the contractor's IRR becomes equal to the pre-agreed IRR. At this point, additional taxes are levied. The contractor, in some cases, still receives some of the profits in excess of the minimum rate of return. In other cases, the contractor's share is limited only to his predetermined rate of return; any amounts in excess of this rate of return are made to the host government. However, sharing of the excess encourages the contractor to achieve cost effectiveness and improve hydrocarbon recovery. The additional taxes are typically referred to as the resource rent tax (RRT), excess profits tax (EPT), windfall profits tax (WPT), or additional profits tax (APT).

Effects of Various Fiscal Terms

In order for international oil companies to invest in oil-producing countries, their fiscal regime has to be sufficiently flexible and efficient to economically encourage the international oil companies. The fiscal regime has to be in balance with many other aspects of petroleum exploration and development in a particular country (e.g., the country's prospectivity, political stability, economic environment, etc.). For example, a country with an expected low prospectivity should set lenient fiscal terms as compared to the countries with high expected hydrocarbon prospectivity. Similarly, less politically stable countries should set lenient fiscal terms as compared to the more politically stable countries. The fiscal terms should also address broader boundary conditions such as field size, development costs, and product prices so that both the HG and the contractor are in a win-win situation.

The efficiency of a particular fiscal regime can be judged from the host government's take in a particular situation and the rate of return of the contractor. Because the fiscal severity of different oil producing countries varies widely, the effects of various fiscal terms on the host government's take and the contractor's profitability should be considered. The economic effects of petroleum fiscal regimes in more than 100 countries were examined by Petroconsultants Australasia Pty, Ltd. Their conclusions show how onerous or severe the regime is in a particular country in comparison with those of other countries.⁵

The discovery of giant or large oil/gas fields is becoming rare. Therefore, it is important that more attention is focused on developing marginal fields. The economic development of these marginal fields (borderline of being economic because of size, production rates, costs, etc.) requires flexible fiscal terms. The terms should be flexible enough to provide high levels of government take in case of a profitable development and a reasonable rate of return if there is a marginal discovery or the product prices are low.

Efficient fiscal regime

An efficient fiscal regime is one in which the contractor's net present value (NPV) before the government take honors the NPV after its take. Projects that will be profitable before the government take will also be profitable after the government take and vice versa. For example, some fiscal regimes tax very profitable projects heavily and marginal projects lightly or not at all. Similarly, royalty plays an important role in the contractor's profitability since this payment is made regardless of whether the contractor is going to make money out of the post royalty deduction revenue. This type of fiscal regime is referred to as a *progressive* fiscal regime. Similarly, there should be upward protection for the host government so that the contractor does not end up with a bonanza if there is a big discovery.

On the other hand, some fiscal regimes tax marginal projects relatively heavily as compared to the relatively profitable projects. This type of fiscal regime is referred to as a *regressive* or *inefficient* fiscal regime. When a fiscal regime is designed in such way that less profitable projects are taxed relatively lightly and the more profitable projects are taxed relatively heavily, the fiscal regime is referred to as a *flat*, *stable*, or *efficient* fiscal regime.

The progressive, regressive, or inefficient fiscal regimes may be liable to change in the future as the economic or exploration/field development conditions change. For example, severe fiscal terms may render a new marginal discovery uneconomical for the contractor. These terms may require revision if the host government is interested in the development of this marginal field. On the other hand, if the fiscal terms are very lenient (i.e.,

giving away too much to the contractor), the government may see a need to revise fiscal terms or renegotiate the contract.

One way to determine if a particular fiscal regime is efficient is to plot the NPV of net cash flow from an oil field development *after* government take (i.e., net contractor's share or contractor's NPV) versus NPV of the same *before* government take (i.e., total project NPV) as shown in figure 5–5. This figure is developed using two different fiscal systems on hypothetical 20 to 100 million barrels of oil (MMBO) oil fields. These particular runs are calculated using \$60/Stb crude oil price throughout the 25-year contract price.

In figure 5–5, fiscal system *B* represents an efficient fiscal regime. This line shows that if a project is economical (positive NPV as shown on the x-axis) before the host government take, it is also economical after the host government take, and vice versa. The line shows that the contractor will have positive NPV (at IRR = 10%) for a field size as small as 20 MMBO or even 10 MMBO at \$60/Stb constant price over the 25-year contract period. On the other hand, fiscal system *A* represents an inefficient or less efficient fiscal regime. In this type of fiscal regime, projects that are profitable for the contractor before host government take become unprofitable after host government take. Figure 5–5 shows that the contractor will have negative NPV (at IRR = 10%) even if he finds a field as large as 35 MMBO if the oil price stays at \$60/Stb over the contract period. The contractor's threshold field size for fiscal system *B* is about 10 MMBO, and for that of fiscal system *A* is about 35 MMBO.

Such a situation discourages the contractor from further involvement in the project. The contractor may walk away from the project, thus leaving petroleum in the ground that could be developed. Alternatively, he could ask for renegotiations of the fiscal terms. Such a regime prevents the development of marginally profitable fields.

The negative effect in fiscal system *A* is mainly due to some components of fiscal terms that are not tied to profitability of the project as a whole but are instead tied to the crude oil price and cumulative oil production. The royalty is one of the culprits in the fiscal regimes. This is because royalty is deducted from the gross revenue without any consideration for the project costs or profitability. Marginal developments are particularly affected by such terms. Payment of royalty reduces or even eliminates the positive difference between the revenues and the costs. Taxes, on the other hand, reduce the overall contractor's profitability from a marginal project since taxes are deducted only from the contractors' positive net cash flow before host government takes.

The ROR type of production sharing or concessionary contracts are considered efficient. These types of fiscal regimes are efficient because they

are based on the contractor's profitability. The more profit the contractor makes, the larger slice the government takes out of the pie. The contractor's ROR is calculated each year, and if it is positive, only then are the incremental benefits allocated to the host government. In other words, high ROR projects are taxed heavily as compared to low ROR projects.

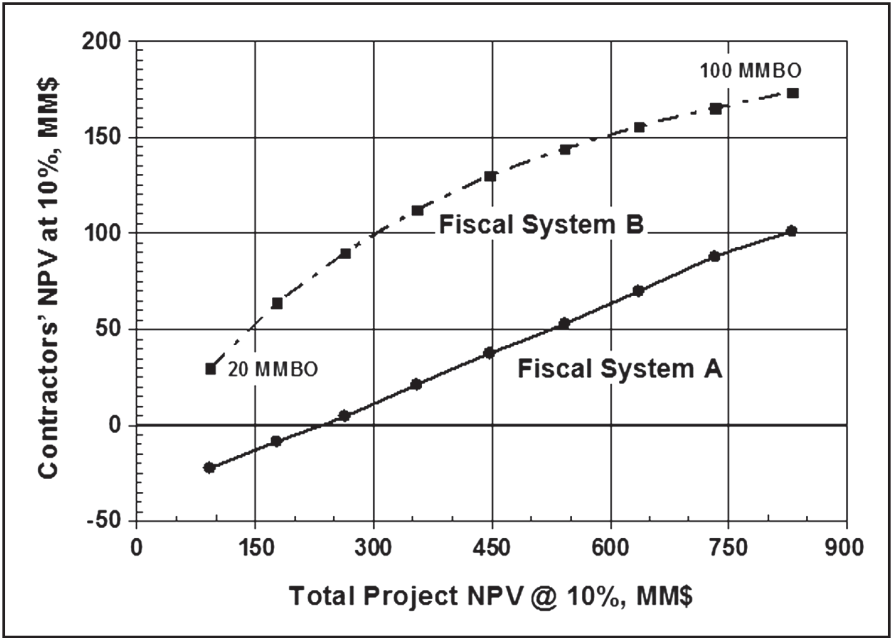


Fig. 5–5. Effect of host government's take on contractor's economics

The effect of cost recovery ceiling

The PSC provides for cost recovery. The effect of an inappropriate cost recovery ceiling lengthens the costs' recovery time. The efficiency of the PSC also depends on how the excess cost recovery is shared between the host government and the contractor. The excess cost recovery oil can be allocated in two ways: (1) it is entirely attributed to the host government or (2) it is treated as profit oil and shared in the same way between the host government and the contractor. Case 1 does not provide any economic incentive to the contractor to reduce costs by increasing efficiency, introducing new technologies, or by further investments in this direction.

Therefore, it is economically convenient for the host government to regard a portion of the excess cost oil as profit oil and shared with the contractor. In doing so, the contractor strives to reduce costs and by doing so ultimately benefits. The sharing of the excess cost recovery oil as profit oil neutralizes any further economic effect for the contractor as the cost recovery oil varies. The changes in the cost recovery percentage will have an insignificant effect on the economics of the contractor. Figure 5–6 shows that the NPV of the contractor after CR equals 40% is constant even if the CR is raised to 60%.

At the beginning of production, the cost recovery oil is usually insufficient to recover the exploration, development, and operating costs. Consequently, a portion of these costs (not fully recovered through the allocated cost recovery oil) must be carried forward to subsequent years. This mechanism penalizes the contractor since it delays his cost recovery. In principle, the value of the cost recovery oil must minimize, or even better *prevent*, this carrying forward. Some contracts acknowledge this effect in the contracts by either increasing the cost recovery by a certain percentage (uplift) or allowing interest to be charged on the unrecovered portion. Such mechanisms have a positive economic effect and, if well devised, they benefit both the contractor and the host government.

Another consideration is the effect of *ringfencing*. Ordinarily, all costs associated with a given block (concession area) must be recovered from the cost recovery oil generated within that block. This means the block is *ringfenced*. This element may have a significant impact on the recovery of costs of exploration and development. Some countries allow certain types of costs associated with a given field or license to be recovered from cost recovery oil from another field or license operated in the same country by the same contractor. Allowing the contractor's exploration costs to *cross the fence* can be a strong financial incentive, especially on marginal fields.

The production sharing cash flow in table 5–5 shows the effects of the cost recovery oil variation on the economic indicators. The excess cost recovery oil has been treated as profit oil, shared between the contractor and the host government in the same way as the profit oil. Figures 5–6 and 5–7 show how the economic indicators change with the change in cost recovery oil percentage.

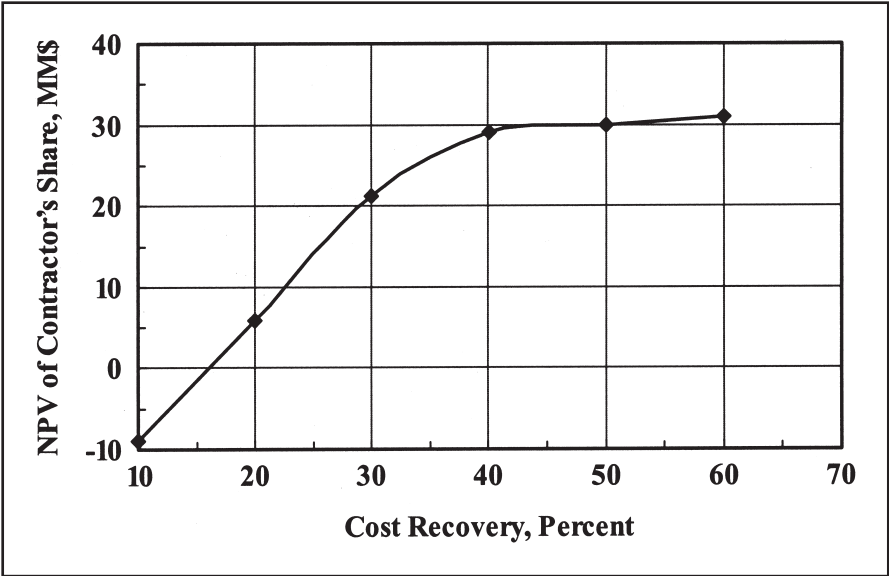


Fig. 5–6. Contractor's NPV versus cost recovery percent

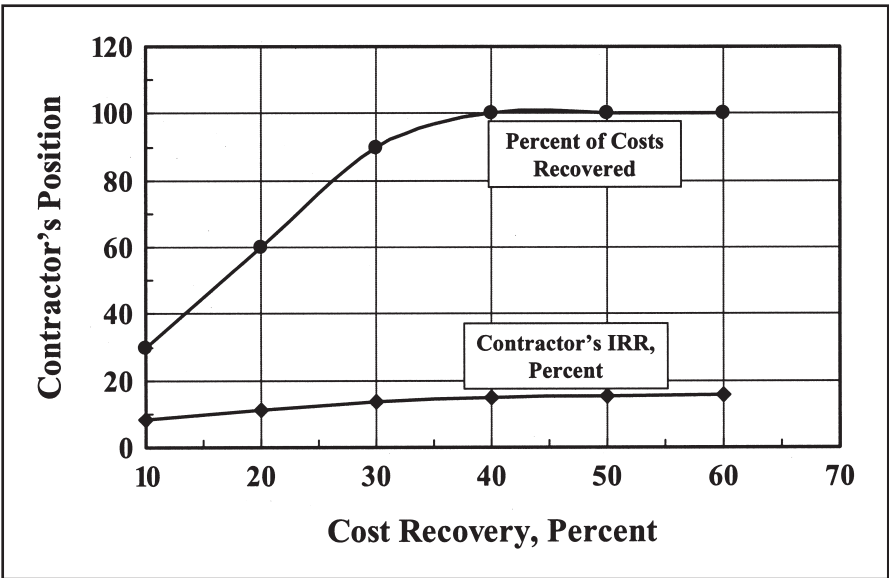


Fig. 5–7. Contractor's IRR and percent of costs recovered versus cost recovery percent

Figures 5–6 and 5–7 show that as the cost recovery percentage is increased, the contractor's IRR, NPV, and percentage of costs recovered increase up to a maximum cost recovery of approximately 40%. This occurs when all the costs are recovered according to the allocation allowed by the contract. Since the excess cost recovery oil is treated as profit oil and shared between the contractor and the host government, the economic indicators remain relatively constant after the 40% cost recovery percentage. If the excess cost recovery oil was entirely allocated to the host government, the economic indicators start to decline as the cost recovery percentage increases beyond the 40%. Therefore, any cost recovery percentage in excess of 40% is disproportionate to the need for the recovery of the costs.⁶

Designing fiscal systems

The fiscal systems for hydrocarbon exploration and development have come a long way during the last 75 years. Although the current fiscal systems are much more sophisticated, some very critical factors are still often overlooked. The ideal fiscal system has to be designed in such a way that it is simple to apply and provides the contractor with a fair rate of return (ROR) on investment commensurate with the project risks, and provides the host government (HG) with an adequate resource for rent, thereby resulting in a win-win situation. Some fiscal systems are unnecessarily cumbersome and do not achieve the main objective.⁷

The viability of any fiscal system is typically judged by two main economic parameters: (a) contractor take in terms of rate of return (ROR), and (b) government take (GT). The “take” is the portion of the total net cash flow (NCF) or total net present value (NPV) from the deal that each party is entitled to. These are useful profitability indicators that are used to rank alternatives and the economic viability of the fiscal system.

The information given here is applicable to the design of fiscal systems worldwide. Every year new exploration blocks and/or development fields are farmed out to either international oil companies (IOCs) or international national oil companies (INOCs). These farm-outs are based on fiscal and legal arrangements.

No matter how sophisticated a fiscal system you design, it is not a good system if it fails to address the expected boundary conditions of the project under consideration. A poor fiscal system will either give more to the contractor at the expense of the HG or vice versa. Developing a fiscal system without knowing the effect of various variables on the end result of the fiscal system is merely an academic exercise.

An important feature of any fiscal system is its simplicity. The perceived benefits intended by complicated fiscal rules and agreements usually

disintegrate when unforeseen events occur, or exotic legal loopholes are discovered. Simple rules are easier to manage, and more efficient to implement and audit. Their transparency will also make them less likely to be abused. Flexibility is another desirable feature; building in the ability for the government and contractor to easily negotiate adjustments in the future will allow the agreement to last longer. As far as the agreement's financial outcome is concerned, both the PSA and the concessionary systems can be designed in such a way that the financial outcome from both will be identical.

Some analysts try to compare the fiscal terms or GT of different countries. Such comparison may be misleading. The PSAs or the concessionary fiscal systems cannot be easily benchmarked because each contract is unique and is dependent upon the following boundary conditions:

1. Prospectivity of the area and associated risks
2. Competition for the prospects
3. Expected limits of resources and production levels
4. Expected development and production costs
5. Expected product prices over the contract duration
6. Proximity to the market and infrastructure
7. The profit expectation of the contractor (how much return the contractor is expecting) and the HG (how much the HG is willing to give), which will be linked to the factors mentioned above

Several hypothetical field development and product pricing scenarios are used to test various fiscal systems. The outcome of each is evaluated in terms of the end benefit to the contractor and the HG. It is not necessary to have a cumbersome fiscal system if the same objective (reasonable return for the contractor and a reasonable government take (GT)) can be achieved with a simple system.

In these scenarios, the key elements of a fiscal system are examined as well as desirable features that should be considered in the design of a fiscal policy with the objective of optimizing the HG's take while providing an acceptable contractor's ROR.

To show how GT and contractor's ROR varies with varying boundary conditions (field size and product prices), hypothetical fields carrying reserves of 20 to 100 MMBO are evaluated using two different hypothetical fiscal terms and five different crude oil price scenarios. The reserves are assumed to be recovered during the contract period of 25 years. The crude oil price scenarios used in the analysis are shown in figure 5–8. Fiscal system

A is a slightly modified version of the Kazakhstan fiscal terms as shown in table 5–9. Fiscal system B is an alternative to fiscal system A. Fiscal system A is used as a benchmark for comparison with the alternative fiscal terms. In each of these systems, the losses are carried forward until they are fully recovered. The terms of each of these agreements are shown below.

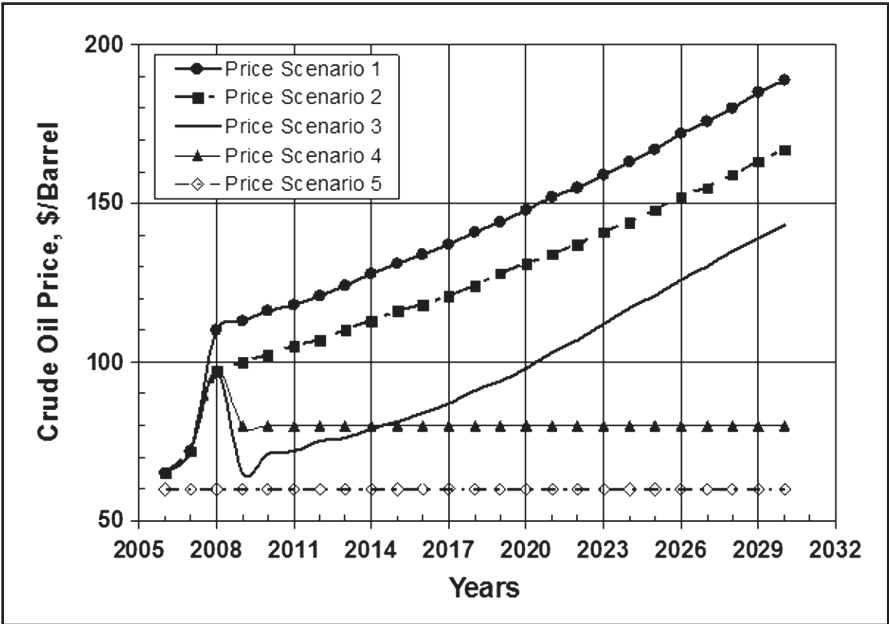


Fig. 5–8. Crude oil price scenarios used to test hypothetical fiscal systems

Fiscal system A (table 5–9)

1. Progressive royalty. The royalty rate is linked to the cumulative oil production. It increases as the cumulative production increases. If the cumulative production is between 3.8 MMBO and 7.6 MMBO, then the royalty rate is 2.5%, and so on. It is obvious from the royalty terms that the HG does not expect a discovery significantly higher than 38 MMBO (boundary condition on cumulative reserves over the contractor period).
2. Economic rent tax (ERT) or export tax (ET). The ERT is linked to the oil price. The ERT increases as the oil price increases. The ERT is zero if the oil price is below \$19/bbl. If the price is between \$19/bbl and \$20/bbl, then the ERT is 4%, and so on. The calculations involve taking out royalty and ERT from the gross revenue. The

income after royalty and ERT is subjected to the deduction of all costs to arrive at taxable income. Note the limit on the oil price of \$40/bbl. This shows that the HG is not expecting the oil price to go significantly higher than \$40/bbl (another boundary condition). Normally the cost of transportation is deducted from the export price for the calculation of ERT. The analysis in this scenario ignores the adjustment of export price for the cost of transportation.

Table 5–9. Fiscal terms of fiscal system A⁸

Accumulated Production (MMBO)	Royalty Rate (%)	Oil Basket Price (\$/Stb)	Economic Rent Tax (ERT) (%)	R-Factor	Excess Profits Tax (EPT) (%)
< 3.80	2.0	< 19	1.0	< 1.2	0
3.8 – 7.6	2.5	19 – 20	4.0	1.2 – 1.3	10
7.6 – 11.4	3.0	20 – 21	7.0	1.3 – 1.4	20
11.4 – 15.2	3.5	21 – 22	10.0	1.4 – 1.5	30
15.2 – 19.0	4.0	22 – 23	12.0	1.5 – 1.6	40
19.0 – 26.6	4.5	23 – 25	14.0	1.6 – 1.7	50
26.6 – 34.2	5.0	25 – 26	16.0	> 1.7	60
34.2 – 38.0	5.5	26 – 27	17.0		
> 38.0	6.0	27 – 28	19.0		
		28 – 29	21.0		
		29 – 30	22.0		
		30 – 31	23.0		
		31 – 32	25.0		
		32 – 34	26.0		
		34 – 36	28.0		
		36 – 37	29.0		
		37 – 38	30.0		
		38 – 40	31.0		
		> 40	33.0		

- 3. Corporate tax. The taxable income is subject to a fixed tax rate of 30%.
- 4. Excess profit tax (EPT). Finally, the EPT (same as windfall profits tax) is linked to the contractor’s R-factor. The EPT increases as the R-factor increases. The EPT is 10% if the R-factor is between 1.2 and 1.3, and so on. The after-tax income (contractor’s pre-EPT net

cash flow) is subject to EPT if the net after-tax income is in excess of 20% of tax deductions.

Fiscal system B. The cash flow for this is shown in table 5–10.

5. Progressive royalty. The royalty rate is linked to the contractor's pre-EPT ROR. The royalty is 5% if the ROR $\leq$ 5% and 20% if the ROR $>$ 12%. The royalty rate between the ROR of 5% and 12% is a sliding scale and calculated by: $\text{Royalty} = 2.1429 \times \text{ROR} - 0.0571$.
6. Economic rent tax (ERT). There is no ERT, therefore one less calculation.
7. Corporate tax. The taxable income is subject to progressive corporate tax. The tax rate is 30% if ROR $\leq$ 10% and 85% if the ROR $>$ 20%. The tax rate between the ROR of 10% and 20% is a sliding scale and calculated by: $\text{Tax} = 5.50 \times \text{ROR} - 0.25$.
8. Excess profit tax (EPT). The EPT is also linked to the contractor's pre-EPT ROR. The EPT rate is 0% if the contractor's pre-EPT ROR $\leq$ 15% and 70% if the ROR $>$ 20%. The EPT rate between the ROR of 15% and 20% is a sliding scale and calculated by: $\text{EPT} = 14.0 \times \text{ROR} - 2.10$.

In the following steps, the outcome from the two systems is analyzed to highlight the areas where changes in some of the terms in each fiscal system could have improved the outcome to both the HG and the contractor.

1. As mentioned above, if the terms of the agreement do not cover appropriate expected boundary conditions of the prospect, then it may lead to a situation where the contractor will lose money. Such a situation will either lead to renegotiation of the agreement or the contractor will walk away from the deal. As shown in figure 5–5, the project's NPV at 10% (x-axis) is positive (economical project) at all levels of reserves, and the crude oil price is consistent with scenario 5. The contractor's NPV (under fiscal system A) is negative up until reserves of about 35 MMBO. In comparison, using fiscal system B, the contractor generates positive NPV even if the reserves are 20 MMBO. Fiscal system B could be further improved by reducing the royalty from 5% to 2.5% and so on. In this way even if the crude oil price goes down further, the contractor will still end up with positive economics. In fiscal system A, the contractor's ROR for 40 MMBO reserves is 10.3% while in fiscal system B it is 15.6%. Even at 20 MMBO reserves, the contractor's ROR in fiscal system B is 12.5% while in fiscal system A it is only 7.7%.

2. Analysts typically use undiscounted NCF to calculate GT. As shown in figure 5–9 (based on undiscounted NCF, the four crude oil price scenarios, and fiscal system A), the GT increases as the reserves increase. This is a logical situation as the HG should recover a higher percentage of the incremental benefits when the project profitability increases. In comparison, figure 5–10 shows that the GT (as a percentage of the project's NPV at 10%) reduces as reserves increase.
3. As shown in figure 5–11, another observation is that the GT as a percent of project NCF (under crude price scenario 5 and fiscal system A) increases with increased reserves while the GT as a percent of project NPV decreases with increased reserves. In fiscal system B, the GT as a percent of project NCF increases with increased reserves while the GT as a percent of project NPV first decreases (up to 30 MMBO reserves) and then it starts to increase as the reserves increase. A positive aspect of fiscal system B is that the GT is automatically adjusted when the contractor is losing money.
4. As shown in figure 5–12 (based on NPV at 10%, the four crude oil price scenarios, and fiscal system A), the GT (as a percentage of the project's NPV at 10%) constantly decreases as the project NPV increases while the contractor take is constantly increasing. The fiscal system B has attempted to correct this problem. As shown in figure 5–13, the GT (as a percentage of the project's NPV at 10%) increases as the project NPV increases. This does not mean that the contractor is not allowed to benefit from the increased project NPV, but only up to a certain limit.
5. Figure 5–14 shows the effect of item 4 above in terms of contractor's ROR. This figure shows the spread of contractor's ROR for the two fiscal systems for reserves of 20 MMBO and 100 MMBO. As shown (under fiscal system A), the contractor's ROR ranges from 10.1% to 22.5%, while the spread is an average of about eight percentage points. The other message figure 5–14 gives is that the contractor's ROR in most of the price scenarios is less than 15% for the 20 MMBO reserves. On the other hand, the minimum contractor's ROR in fiscal system B is 15.1% and the high is 21.1%. The most important is the upward spread from the lowest ROR, which is an average of about 3.5%. This means that the contractor will not end up with a bonanza at the expense of reduced GT.

Table 5–10. Cash flow model for fiscal system A

Year	Oil Production		Seismic	Drilling CAPEX		Maintenance	1% of CAPEX		Total Cost	Oil Price	Gross Revenue	Total Project Cash Flow	Royalty Rate	Royalty	Profitability Summary	
	Production BOPD	Cumulative MMB	MM\$	Exploration	Development	CAPEX	Production Facilities	3% of CAPEX	MM\$	\$/Bbl	MM\$	MM\$	%	MM\$		
2010	0	0	0	5		0.00			5.00	78	0.0	(6)	0.0%	0.00	Total Project IRR	38.4%
2011	0	0	0	5	10	0.00			15.00	80	0.0	(15)	0.0%	0.00	Total Project Net Cash Flow, MM\$	11.39
2012	0	0	0	20		0.00			20.00	82	0.0	(20)	0.0%	0.00	Total Project NPV @ 10%, MM\$	1,891.4
2013	0	0	0			0.00	68		68.29	84	0.0	(68)	0.0%	0.00	LRMC of Oil @ 10%, \$/Bbl	19.0
2014	0	0	0			0.00	137		136.58	86	0.0	(137)	0.0%	0.00		
2015	0	0	0	15		0.00			151.58	88	0.0	(152)	0.0%	0.00	Total Net Cash Flow of Contractor, MM\$	943
2016	1,667	0.6				3.41			24.63	90	55.0	30	0.0%	0.00	Total Net Cash Flow of Government, MM\$	10,195
2017	3,333	1.8				3.41			20.96	93	112.8	92	0.0%	0.00	Government's Take	92%
2018	5,000	3.7				3.41			57.75	95	173.4	116	0.0%	0.00		
2019	10,000	7.3				3.41			97.9	97	355.6	287	0.0%	0.00	Contractor's NPV @ 10%, MM\$	183.4
2020	15,625	13.0				3.41			18.82	100	563.4	551	0.0%	0.00	Government's NPV @ 10%, MM\$	1,708.0
2021	15,625	18.7				3.41			18.92	102	583.7	565	20.0%	116.73	Government's Take @ 10%	90%
2022	15,625	24.4				3.41			19.02	105	598.3	579	20.0%	119.65	Contractor's IRR	20.7%
2023	15,625	30.1				3.41			19.13	108	613.2	584	20.0%	125.71	Cumulative Production (1 = Yr, 2 = Tot)	2
2024	15,625	35.8				3.41			19.23	110	628.6	609	20.0%	128.85	Reserves, MMB	100
2025	15,625	41.5				3.41			19.33	113	644.3	625	20.0%	132.07		
2026	15,625	47.2				3.41			16.02	116	660.4	641	20.0%	135.38		
2027	15,625	52.9				3.41			16.23	119	676.9	657	20.0%	138.76		
2028	15,625	58.6				3.41			16.33	122	693.8	674	20.0%	142.23		
2029	15,625	64.3				3.41			16.43	125	711.1	691	20.0%	145.79		
2030	15,625	70.0				3.41			16.53	128	728.9	709	20.0%	149.43		
2031	15,625	75.7				3.41			16.63	131	747.2	727	20.0%	153.17		
2032	15,625	81.4				3.41			20.05	134	765.8	746	20.0%	156.91		
2033	12,500	86.0				3.41			16.80	138	628.0	611	20.0%	125.60		
2034	10,000	89.7				3.41			16.80	141	514.9	498	20.0%	102.39		
2035	8,000	92.6				3.41			16.80	145	422.3	405	20.0%	69.25		
2036	6,400	94.9				3.41			16.80	148	346.2	329	20.0%	46.56		
2037	5,120	96.8				3.41			16.80	152	263.9	267	20.0%	28.18		
2038	4,096	98.3				3.41			16.80	156	232.8	216	20.0%	13.91		
2039	3,277	99.5				3.41			16.80	160	190.9	174	20.0%	38.18		
2040	2,621	100.4				3.41			16.80	164	156.5	140	20.0%	31.31		

Table 5-10. (Continued)

Year	Contractor's cash-flows										IRR Before WPT	Windfall Profit Tax Rate	Windfall Profit Tax MM\$	R-Factor	Post-WPT Net Cash-Flow MM\$	IRR after EPT	Government Take MM\$
	Revenue Before-Tax MM\$	Loss Carried Forward MM\$	Taxable Income MM\$	Income Tax MM\$	Pre-EPT Net Cash-Flow MM\$												
2010	(5.00)	(5.00)	0.00	0.00	(5.00)	-	0.00	0.00	0.00	0.00	(5.00)	-	0.00	0.00	0.00	0.00	
2011	(15.00)	(20.00)	0.00	0.00	(15.00)	-	0.00	0.00	0.00	0.00	(15.00)	-	0.00	0.00	0.00	0.00	
2012	(20.00)	(40.00)	0.00	0.00	(20.00)	-	0.00	0.00	0.00	0.00	(20.00)	-	0.00	0.00	0.00	0.00	
2013	(68.29)	(108.29)	0.00	0.00	(68.29)	-	0.00	0.00	0.00	0.00	(68.29)	-	0.00	0.00	0.00	0.00	
2014	(136.58)	(244.87)	0.00	0.00	(136.58)	-	0.00	0.00	0.00	0.00	(136.58)	-	0.00	0.00	0.00	0.00	
2015	(151.58)	(396.45)	0.00	0.00	(151.58)	-	0.00	0.00	0.00	0.00	(151.58)	-	0.00	0.00	0.00	0.00	
2016	30.40	(366.05)	0.00	0.00	30.40	-	0.00	0.00	0.13	0.30	30.40	-	0.00	0.00	0.00	0.00	
2017	91.84	(274.21)	0.00	0.00	91.84	-	0.00	0.00	0.38	0.98	91.84	-	0.00	0.00	0.00	0.00	
2018	115.69	(158.52)	0.00	0.00	115.69	-	0.00	0.00	0.68	1.58	115.69	-	0.00	0.00	0.00	0.00	
2019	287.34	128.82	128.82	0.00	287.34	6.7%	0.00	0.00	1.23	3.67	287.34	6.7%	17.5%	165.19	165.19	0.00	
2020	550.62	679.44	550.62	165.19	385.43	17.9%	0.00	0.00	1.88	5.19	385.43	17.9%	19.9%	587.42	587.42	0.00	
2021	448.02	1,127.46	448.02	328.67	119.15	19.9%	40.4%	48.10	1.97	71.24	119.15	19.9%	20.9%	602.47	602.47	0.00	
2022	469.59	1,587.04	469.59	387.72	71.66	20.8%	68.4%	49.00	1.97	22.86	71.66	20.8%	20.5%	617.90	617.90	0.00	
2023	471.45	2,058.49	471.45	400.73	70.52	21.5%	70.0%	49.36	1.98	21.36	70.52	21.5%	19.8%	633.72	633.72	0.00	
2024	483.61	2,542.11	483.61	411.07	72.34	22.0%	70.0%	50.64	1.98	21.90	72.34	22.0%	19.9%	649.93	649.93	0.00	
2025	496.08	3,038.19	496.08	421.67	74.21	22.4%	70.0%	51.95	1.99	22.46	74.21	22.4%	20.0%	666.55	666.55	0.00	
2026	508.86	3,547.05	508.86	432.53	76.13	22.8%	70.0%	53.29	1.99	23.04	76.13	22.8%	20.2%	677.90	677.90	0.00	
2027	521.97	4,069.02	521.97	443.67	78.10	23.0%	70.0%	54.67	2.00	23.63	78.10	23.0%	20.3%	693.33	693.33	0.00	
2028	535.40	4,604.42	535.40	455.09	80.11	23.3%	70.0%	56.08	2.00	24.23	80.11	23.3%	20.4%	709.93	709.93	0.00	
2029	549.18	5,153.60	549.18	466.80	82.18	23.4%	70.0%	57.52	2.01	24.85	82.18	23.4%	20.5%	726.93	726.93	0.00	
2030	563.30	5,716.90	563.30	478.80	84.29	23.6%	70.0%	59.01	2.02	25.49	84.29	23.6%	20.5%	744.93	744.93	0.00	
2031	577.77	6,294.67	577.77	491.11	86.47	23.7%	70.0%	60.53	2.02	26.14	86.47	23.7%	20.6%	762.93	762.93	0.00	
2032	592.61	6,887.29	592.61	503.72	88.69	23.8%	70.0%	62.08	2.03	26.81	88.69	23.8%	20.6%	780.93	780.93	0.00	
2033	485.58	7,372.87	485.58	412.74	72.64	23.8%	70.0%	50.85	2.04	21.99	72.64	23.8%	20.7%	798.93	798.93	0.00	
2034	395.15	7,768.02	395.15	335.88	59.07	23.9%	70.0%	41.35	2.04	17.92	59.07	23.9%	20.7%	816.93	816.93	0.00	
2035	321.00	8,089.02	321.00	272.85	47.95	23.9%	70.0%	33.56	2.03	14.58	47.95	23.9%	20.7%	834.93	834.93	0.00	
2036	260.19	8,349.21	260.19	221.17	38.83	23.9%	70.0%	27.18	2.03	11.85	38.83	23.9%	20.7%	852.93	852.93	0.00	
2037	210.33	8,559.55	210.33	178.78	31.35	23.9%	70.0%	21.95	2.02	9.61	31.35	23.9%	20.7%	870.93	870.93	0.00	
2038	169.45	8,729.00	169.45	144.03	25.22	23.9%	70.0%	17.65	2.01	7.77	25.22	23.9%	20.7%	888.93	888.93	0.00	
2039	135.92	8,864.92	135.92	115.54	20.19	23.9%	70.0%	14.13	2.00	6.26	20.19	23.9%	20.7%	906.93	906.93	0.00	
2040	108.43	8,973.36	108.43	92.17	16.06	24.0%	70.0%	11.25	1.98	5.02	16.06	24.0%	20.7%	924.93	924.93	0.00	
	8,973	112,527	8,973	7,160	1,803.41			870.15		943.26	1,803.41			10,196	10,196		

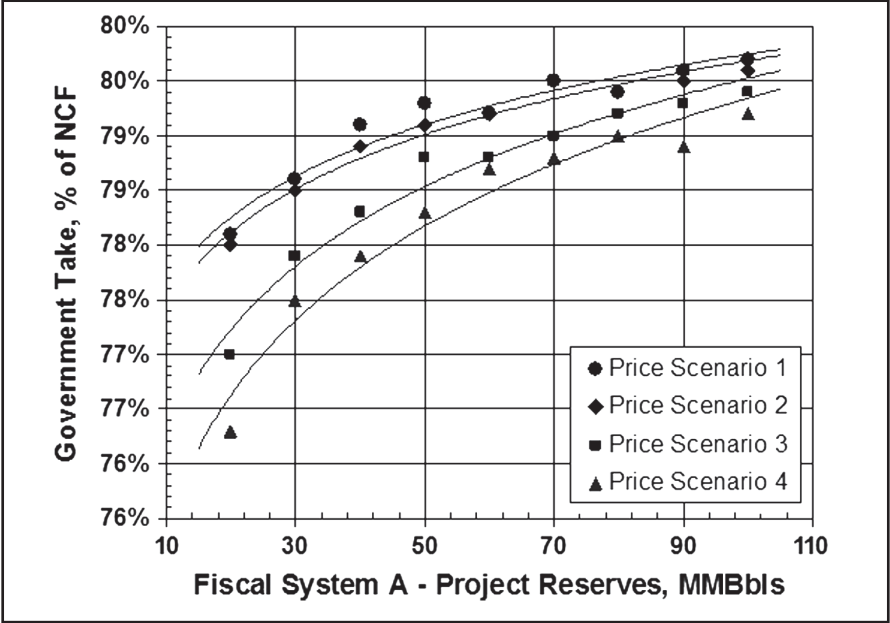


Fig. 5–9. Fiscal system A. Effect of higher reserves and crude oil price on GT as percent of project NCF

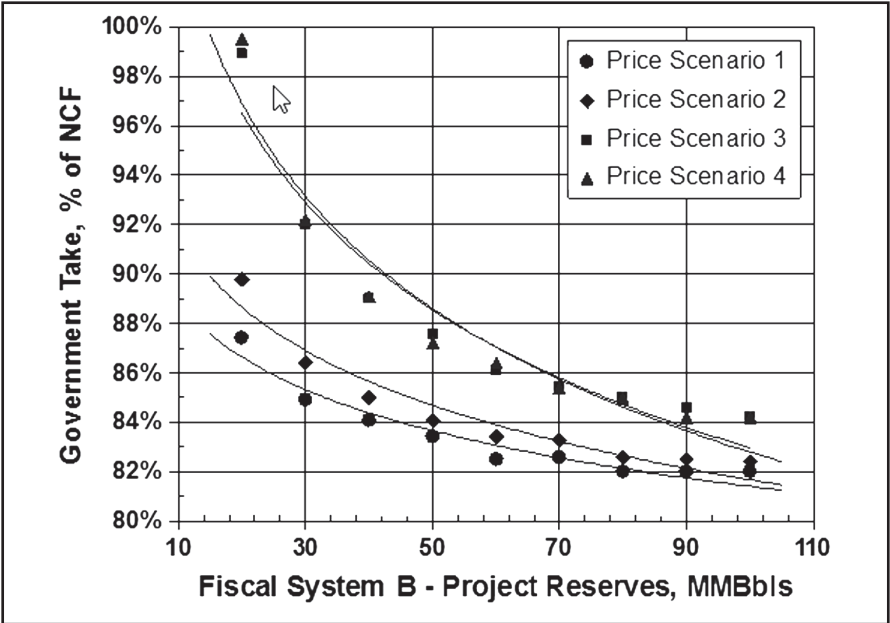


Fig. 5–10. Fiscal system A. Effect of higher reserves and crude oil price on GT as percent of project NPV at 10%

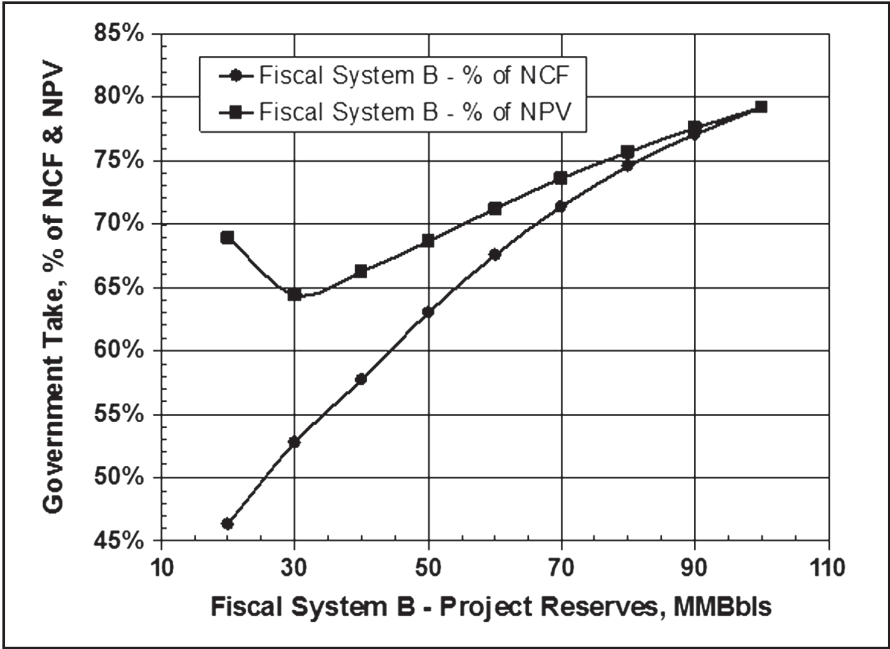


Fig. 5–11. Fiscal system B. GT as percent of project NCF and NPV at 10% at price scenario 5

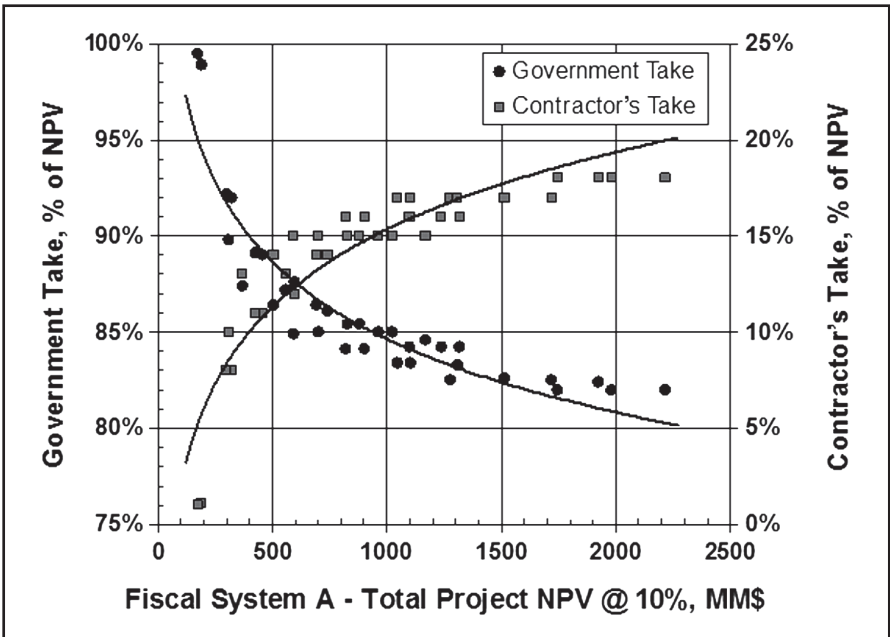


Fig. 5–12. Fiscal system A. Variation of government and contractor takes with increased project NPV

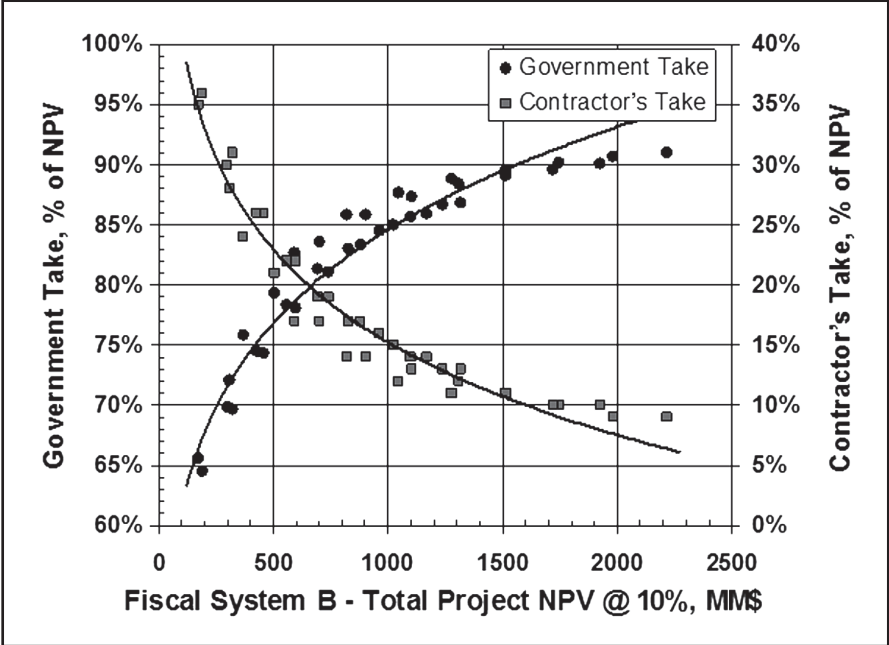


Fig. 5–13. Fiscal system B. Variation of government and contractor takes with increased project NPV

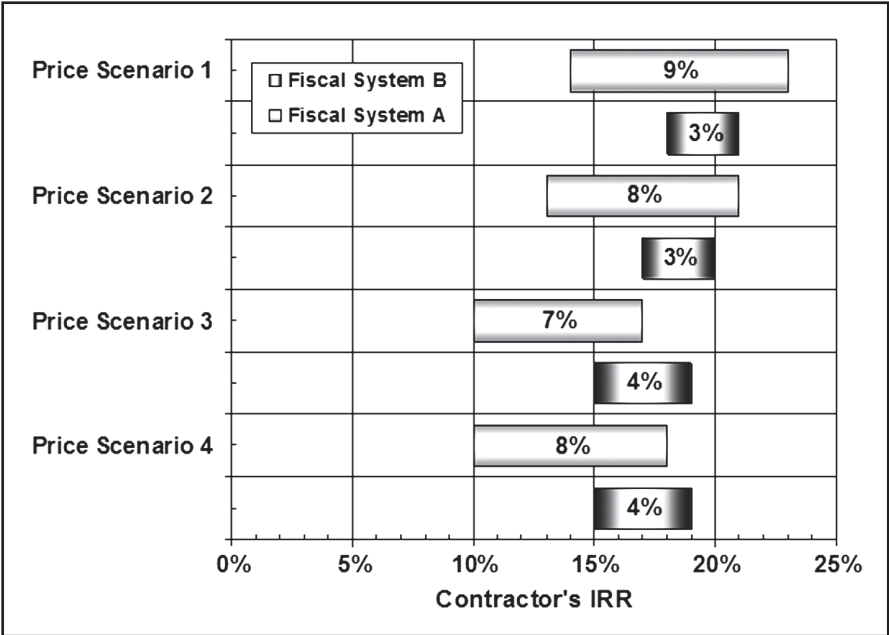


Fig. 5–14. Fiscal systems A and B. Effect of crude oil price and reserves on contractor's ROR

6. Figure 5–15 shows that if the fiscal system is optimally designed, then the GT as a percent of NCF and NPV should show the same trend, i.e., GT increasing with increased reserves at any price scenarios. As mentioned above, GT as a percent of NPV is a better indicator in comparison to the percent of NCF.

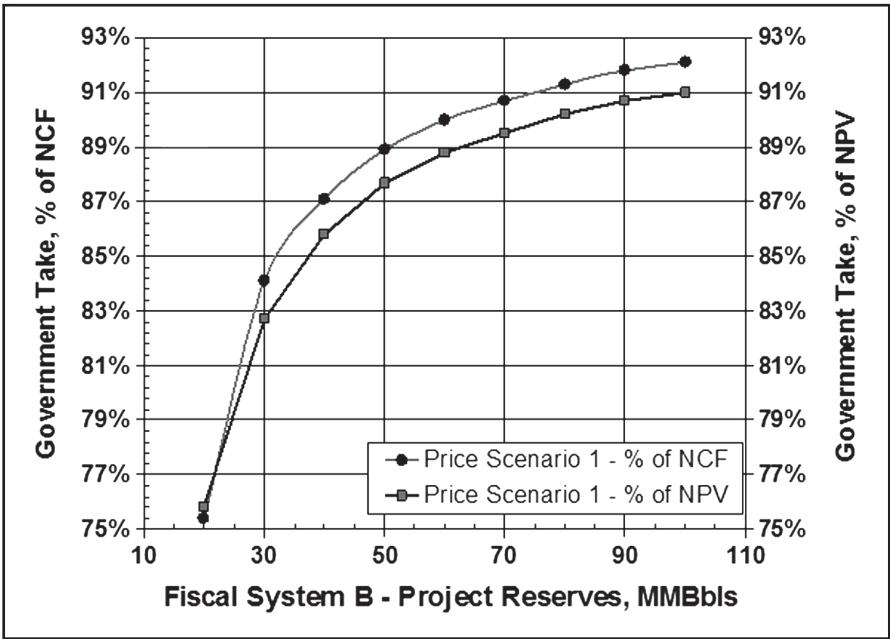


Fig. 5–15. Fiscal system B. Comparison of GT as a percent of project NCF and project NPV at crude oil price scenario 1

7. The problem with fiscal system A is also highlighted in figure 5–16. Since the contractor has spent a significant amount of capital up front, the fiscal system should give a higher share of the cash flow to the contractor during this period. This allows the contractor to break even in a shorter period of time. Due to the front-end costs, the contractor's ROR and NPV are also significantly affected during the first five or so years following the production. Fiscal system A fails to respond to this issue while fiscal system B gives more to the contractor in the years when the contractor needs higher revenue to recover its costs.

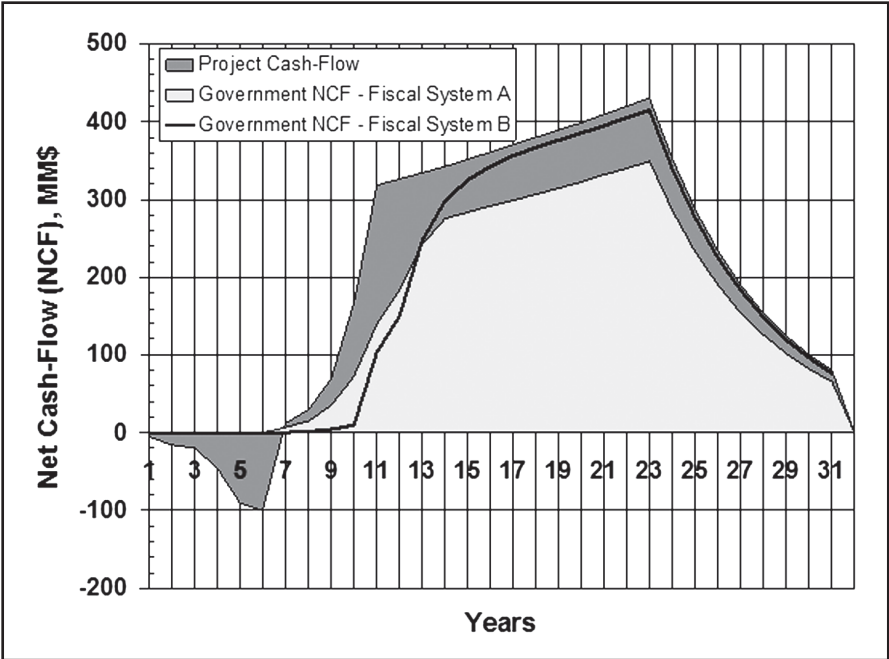


Fig. 5-16. Total project and government net cash flow at price scenario 1 and 50 MMBO reserves

- 8. As mentioned above, many analysts compare the contractor's ROR or undiscounted GT resulting from fiscal terms of different countries applied to a hypothetical prospect and using a hypothetical pricing scenario. This could be very misleading. As shown in figure 5-11, the undiscounted GT for 100 MMBO reserves and pricing scenario 5 is 78.8% (contractor's ROR = 14.3%) for country A (using fiscal system A) and 79.2% (contractor's ROR = 18.3%) for country B (using fiscal system B). This comparison shows that both countries have almost the same GT. If country B's reserves were 20 MMBO, then the GT would be 46.3% (contractor's ROR = 12.5%). This will mean that the two fiscal systems are significantly different.
- 9. Fear of gold plating. There is an unsubstantiated concern that contractors will spend an excessive amount of money (gold plating) on the facilities or that they will not optimize the costs when uplift or investment credit is given. This is merely an academic concern. No contractor will try to spend an excessive amount of money in

an effort to reduce GT in a properly designed fiscal system for the following reasons:

- a. The contract is for a limited period of time and at the end of this period the facility is left to the HG.
- b. In the past this could have been the situation when the HG did not have the capability to monitor the operations. The contracts today call for maintaining strict corporate governance policies. Most major decisions have to be approved by technical committees, audit committees, contract committees, and management committees. These committees are formed with representatives from the HG or its representatives and the contractor.
- c. Even if there is a slight chance of misalignment, this could be controlled by not leaving the uplift or investment credit open ended. It should also be linked to the profitability of the contractor. So, the uplift automatically kicks in when the contractor is not going to achieve its minimum desired ROR (such as in cases of marginal field, low product price environment, or higher than expected cost environment). On the other hand, the uplift should automatically cease if the contractor is expected to obtain its desired profitability, especially after contractor's R-factor of 1.0.

The above analyses show that regardless of how exotic a fiscal system is, it will not be a good system if it fails to address wider boundary conditions. However, the individual boundary conditions such as product prices, costs, field size, and production rates are not easy to predict with any degree of certainty. Therefore, a realistic progressive fiscal system in the true sense will be the one that will capture the profitability. The contractor's IRR is one parameter that truly reflects the profitability of the contractor. At the same time, this parameter is calculated by incorporating all other variables involved.

As mentioned in the beginning of this chapter, the industry has made a lot of progress in efficiently designing these contracts. However, as the fiscal system A (discussed in this section and it is a very recent one) shows, many analysts still fail to realize the end result of their designed fiscal system.

Questions and Problems

5.1 The cash flow shown below is for a royalty tax system. The contractor's net cash flow in this is:

- a. \$25,000
- b. \$35,000
- c. \$45,000
- d. \$55,000

	A	B	C
1		Contractor's Share	Gouvernement Share
2	Crude Oil Production, STB	20,000	
3	Crude Oil Price, \$/Stb	50	
4	Gross Revenue, \$		
5	Less Royalty @ 20%		
6	Gross Revenue after Royalty, \$		
7	Capital Expenditure, \$	(300,000)	
8	Operating Expenditure, \$	(200,000)	
9	Taxable Income, \$		
10	Income Tax at @ 85%		
11	Net Cash Flow (NCF), \$		

- 5.2 The government's net cash flow for the cash flow in question 5.1 is:
- a. \$455,000
 - b. \$475,000
 - c. \$500,000
 - d. \$525,000
- 5.3 The government take as a percent of the total project's net cash flow for the cash flow in question 5.1 is:
- a. 85%
 - b. 75%
 - c. 91%
 - d. 95%
- 5.4 The royalty and tax in a concessionary system is based on the R-factor of the contractor. What is the R-factor of the contractor if his cumulative revenue and cumulative cost at a certain point in time is \$500,000 and \$350,000, respectively?
- a. 1.0
 - b. 1.23
 - c. 1.43
 - d. 1.53
- 5.5 In question 5.1, what will be the government's net cash flow if the capital expenditure (normally it will be the depreciation on the CAPEX that will be subjected to the uplift) is subject to 50% uplift?
- a. \$455,000
 - b. \$127,500
 - c. \$327,500
 - d. \$22,500
- 5.6 If the capital expenditure in question 5.1 is subject to 50% uplift, what will be the contractor's net cash flow?
- a. \$22,500
 - b. \$45,000
 - c. \$127,500
 - d. \$255,000

- 5.7 Gross revenue of \$1,000,000 is generated by a contractor under a production sharing agreement. If the cost recovery percent is 40%, what will be the contractor's cost recovery share?
- a. \$600,000
 - b. \$500,000
 - c. \$400,000
 - d. \$300,000
- 5.8 In question 5.7, if the government's share of the profit oil is 70%, what will be the government's profit share out of the \$1,000,000 gross revenue?
- a. \$700,000
 - b. \$420,000
 - c. \$280,000
 - d. \$180,000
- 5.9 In question 5.7, if the government's share of the profit oil is 70%, what will be the contractor's profit share out of the \$1,000,000 gross revenue?
- a. \$700,000
 - b. \$420,000
 - c. \$280,000
 - d. \$180,000
- 5.10 In question 5.7, if the contractor has only \$200,000 of cost remaining to be recovered, what will be the excess cost recovery share of the government if the sharing is the same as the profit share?
- a. \$280,000
 - b. \$210,000
 - c. \$140,000
 - d. \$70,000

- 5.11 In question 5.7, if the contractor has only \$200,000 of cost remaining to be recovered, what will be the excess cost recovery share of the contractor if the sharing is the same as the profit share?
- a. \$120,000
 - b. \$90,000
 - c. \$60,000
 - d. \$30,000
- 5.12 In questions 5.7 to 5.11, what is the total NCF of the contractor?
- a. \$240,000
 - b. \$440,000
 - c. \$560,000
 - d. \$660,000
- 5.13 In questions 5.7 to 5.11, what is the total NCF of the government?
- a. \$240,000
 - b. \$440,000
 - c. \$560,000
 - d. \$660,000
- 5.14 In questions 5.7 to 5.11, what is the government take as a percent of the total NCF from the project?
- a. 50%
 - b. 60%
 - c. 70%
 - d. 80%
- 5.15 What will be the gross revenue if the government's NCF is \$560,000, contractor's NCF is \$240,000 and contractor's cost is \$200,000?
- a. \$440,000
 - b. \$760,000
 - c. \$800,000
 - d. \$1,000,000

- 5.16 An international contractor signs an EPSA on 1/1/2010 for a block in Nigeria. On signing the contract, the contractor pays \$5 million in bonus. During the first two years, the contractor conducts exploration activities at a cost of \$8 million each year. If a commercial oil field is discovered, it will take another two years to develop the field at a cost of \$35 million in year one and \$28 million in year two. Production from the field starts at a rate of 15,000 Stb/day in 2014, declining exponentially at a rate of 15% each year. The yearly operating expenditure is 5% of the total development cost. If the oil price is \$60 per barrel, calculate the net cash flow of the contractor and the host government for the next five years. The fiscal terms of the contract are CR = 30%, PO share of the contractor is 25%, and the excess cost recovery oil is shared between the contractor and the host government in the same way as the profit oil.
- 5.17 Evaluate problem 5.16 if the contractor has to pay another \$6 million when the cumulative production from the field reaches 10 million barrels and the contractor has to pay 5% royalty to the host government out of the gross production.
- 5.18 Data on a prospective exploration concession is given in table 5–11. Build a production sharing economics model and then design fiscal terms in such a way that the contractor will receive a rate of return of approximately 15% no matter what field size he discovers. Assume the oil price of \$15 per barrel, escalating at 3% per year. The costs are in 2010 dollars and assume the costs will be escalating at 3% per year. The operating cost in each case is 5% per year of the total development cost in each case. The contractor has to pay a tariff of \$0.45 per barrel to the host government for using their oil storage and handling facility. The contractor pays a signature bonus of \$6 million on 01/01/2011 and pays production bonuses of \$5 million and \$4 million when the cumulative production reaches 10 million and 25 million barrels, respectively.
- 5.19 Repeat problem 5.18 if the contractor has to pay 5% royalty and 35% tax to the host government. Assume a tax holiday of 10 years from the first production and ignore depreciation and depletion.

Table 5–11. Estimated production and cost data for a prospective exploration concession (problem 5–18)

Year	Production Forecast, Stb/day				CAPEX for each Case, MM\$			
	A	B	C	D	A	B	C	D
2009					6.00	6.00	6.00	6.00
2010					6.00	6.00	6.00	6.00
2011					19.40	20.80	27.60	23.60
2012					14.50	32.20	43.30	40.40
2013	3,493	6,986	13,973	20,959	22.60	50.30	79.10	82.30
2014	6,986	13,973	13,973	20,959			30.90	44.20
2015	6,986	13,973	27,945	45,000				42.90
2016	6,986	13,973	27,945	43,000				
2017	6,986	13,973	27,945	41,918				
2018	6,151	12,301	27,945	41,918				
2019	5,411	10,822	27,945	41,918				
2020	4,767	9,543	27,945	41,918				
2021	4,192	8,384	16,986	41,918				
2022	2,822	7,397	14,411	34,608				
2023		6,493	12,192	31,147				
2024		5,724	10,329	21,600				
2025		5,041	9,590	16,240				
2026		4,438	7,425	14,616				
2027		3,973	7,425	13,154				
2028				11,839				
2029				10,655				
2030				9,589				
2031				8,630				
2032				7,767				
2033				6,990				
2034				6,292				
2035				5,662				
2036				5,096				
2037				4,587				

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6

Capital Budgeting Techniques

The term *capital* refers to expenditure for the purchase or expansion of physical assets, such as property, plants, and equipment. The term *budget* refers to a plan that details projected inflows resulting from projected outflows. Thus, the *capital budget* is an outline of planned capital investments, and *capital budgeting* is the whole process of analyzing the projects and deciding whether they should be included in the capital budget. Capital budgeting is the deliberate allocation of present resources with the expectation of future returns. Through the capital budgeting process, an investor is able to answer three main questions:

1. How large should the budget be?
2. Which projects/investments should be included in the portfolio?
3. How should the required budget be financed?

Although we are referring to property, plants, and equipment, the parameters presented here may also be used in other investment situations.

Prudent capital investment decision making is an important and critical managerial responsibility. The long-term survival, profitability, value, and growth of a business are dependent on these decisions. Capital budgeting decisions also play an important role in providing a mechanism for the implementation of business strategies.

Capital projects normally entail large commitments of funds, and their impact on the financial well-being of the organization extends over relatively long time spans. The investments once made are difficult to amend or retrieve. An ill-advised decision concerning capital investments frequently cannot be reversed before it seriously affects the financial health of the organization. Because of their importance, systematic procedures are essential for effective planning of these investments. Each investment proposal should be viewed as an identifiable incremental project having its own unique time dimension and reward structure. The term *incremental* refers to all those costs or investments that otherwise would not be incurred in the absence of the project/investment.

An important part of the planning process is to define and lay down rules and techniques to form what is called *profitability analysis*. It is essential for management to define what is meant by a *profitable project* or *investment*. In order to achieve this objective, management must provide a way to choose between alternative schemes of carrying out a given investment or project. Unless these guidelines are provided clearly, it is difficult to delegate the responsibility of evaluating investment projects to the lower levels of management.

The overall capital expenditure plan of an oil company consists of a series of unique capital projects/investments. These may include:

1. Exploration for oil and gas
2. New oil and gas fields development
3. Infill drilling
4. Facilities replacements/upgrades
5. Installation of new facilities such as artificial lift systems and pressure maintenance
6. Secondary and enhanced oil recovery projects
7. A variety of downstream projects

Major capital additions should be accorded special analysis, management evaluation, and judgment. Approaches to determine their investment worth should loom large in the decision-making process.

To judge the attractiveness of any investment, three main elements are considered: (a) *investment amount*, (b) the *operating benefits*, and (c) the *economic life*. The economic life of an investment is the time period where positive net benefits are expected from the investment alternative. For example, it is technically possible to produce a certain reservoir down to 10 Stb/day at the end of 15 years. However, at the end of year 10, costs incurred to produce this reservoir exceed the revenues generated from 50 Stb/day. Therefore, the economic life is only 10 years and not 15. Similarly, the economic life of equipment may be shorter than its physical life. Therefore, all investment analysis must combine these three elements in a logical manner in order to provide a clue to whether or not the investment is worthy of consideration. These basic conditions are true of all investment proposals under consideration. On the other hand, the cash flows are normally restricted to accommodate up to a 4-year project implementation period (construction period) followed by a period of about 25 years of production unless the economic limit occurs before the 25-year period. It is not recommended to run cash flows for longer than the 28- to 29-year time

span because the present value of the variables occurring after 25 years is very small and will not significantly change the economics and because the accuracy of the variables' forecast decreases with time.

Capital budgeting, in practice, involves understanding an integrated use of various techniques presented in

1. Managerial and corporate finance
2. Industrial economics
3. Strategic planning (i.e., developing a sustainable competitive edge over other investors in the industry)
4. Decision theory

A considerable amount of literature on the subject of capital budgeting is available in the finance, economics, accounting, and oil industry trade journals. In addition to the articles and books directly referenced within this chapter, some additional references are given at the end of the chapter in the section titled "Further Reading."

Briefly stated, capital budgeting is an exercise that requires estimating the net value of future cash flows and other benefits as compared to the initial investment required. Many financial analyses entail the search for an answer to the question: "Is the investment worthwhile considering potential future returns?"

For the capital budgeting, management needs objective measures of the economic worth of investment proposals. They should be able to rank, accept, reject, compare, or choose among them and select those most beneficial to the company's long-term prosperity. The parameters used to facilitate the above decisions are frequently called *measures of profitability*, *profit indicators*, *profitability yardsticks*, *economic yardsticks*, *decision criteria*, or *measures of investment worth*. The investment evaluation techniques should have the capability of estimating whether the cash returns from the investment will be large enough to make the initial investment worthwhile.

One of the main difficulties in capital budgeting is the large number of economic evaluation yardsticks used. These criteria have different properties and are used in various ways. Studying the mathematical properties of these criteria, as well as working through examples, is an effective way to put them into focus.

This chapter considers the meaning, mathematics, and properties (advantages and disadvantages) of the most commonly used measures of profitability. In the course of our discussion, the methods are compared in order to highlight their relative strengths and weaknesses, as well as evaluate performances in terms of identifying investments that will maximize the

corporate worth. At this point, no attempt is made to rank the desirability of these measures. The measures of profitability presented in this chapter do not include any explicit statements of risks and uncertainty. *Both the accounting measures and the time-adjusted measures of profitability are presented. However, common time-adjusted measures are emphasized and recommended for use.*

No unique profitability measure accounts for all possible decision factors or dimensions of a capital investment. Economists should select profitability measures and assumptions related to each particular financial environment. Ideally, profitability measures should be backed up by experience and sound economic judgment. The ideal measures of profitability should have the following characteristics:

1. It must be suitable for comparing and ranking the profitability of available investment alternatives.
2. It should account for the time value of money.
3. It should provide a means for telling whether profitability exceeds some minimum *cut-off* (threshold) requirement of the organization.
4. It should be able to include probability estimates to generate expected values.
5. If possible it should reflect other factors such as corporate goals, decision makers' risk preferences, and corporate asset position.
6. It should reflect the size of the initial investment; otherwise, there might be an inappropriate bias toward small investments. Such a situation may strain the company resources essential for the implementation of the available projects.

The following capital budgeting techniques/approaches are discussed in this chapter:

- Net present value (NPV)
- Payback period
- Internal rate of return (IRR)
- Profitability index
- Unit technical cost or long-run marginal cost
- Cost benefit ratio

In chapter 7, these parameters are used in arriving at economic decisions in a variety of investment situations. Note that all subsequent techniques for

Accounting Approaches

This section deals with often-used simple capital budgeting techniques, their key elements, and their advantages and disadvantages. The next section addresses more advanced but increasingly popular methods that employ the discounted cash flow concepts, including adjustment for the timing of cash flows and the recognition of economic opportunity costs.

Payback period

The *payback period*, also referred to as the *breakeven point*, is defined as the expected number of years required for recovering the original investment. At this point, the cash receipts exactly equal the cash disbursements. This yardstick is used along with at least one other measure of profitability, since it does not provide a meaningful decision criterion by itself.

The only question the calculated payback period answers is “How long will it take to get my money back?” From an economic viewpoint this is not enough, of course, since one would also hope to earn a profit on the investment. When related to the useful economic life of an investment, the payback figure is used as an indication of whether the investment is repaid within the economic life.

The payback period is calculated from the net cash flow by two different methods. The first method requires accumulating the negative net cash flow (NCF) each year until it turns positive, as shown in column N of table 6–1 (between cells N13 and N14). The payback period is then the *cumulative time* (each year with negative cumulative net cash flow from time zero) to a point between the negative and positive cumulative net cash flow. The partial payback period in the year when the NCF turns positive is calculated by interpolating between the two values using the following equation.

$$\begin{aligned}
 \text{Payback Period} &= \text{Cum. - ve NCF Years} + \frac{1}{+ \text{ve NCF} - (- \text{ve NCF})} \times -(- \text{ve NCF}) \\
 &= 4 + \frac{1}{78.27 - (-3.50)} \times -(-3.50) \\
 &= 4.04 \text{ years}
 \end{aligned}
 \tag{6.1}$$

The second method involves plotting the cumulative net cash flow versus time, as shown in figure 6–1. The payback period is read at the intersection of the time line at zero cumulative net cash flow, the point at which the cumulative net cash flow moves from the negative quadrant into the positive quadrant.

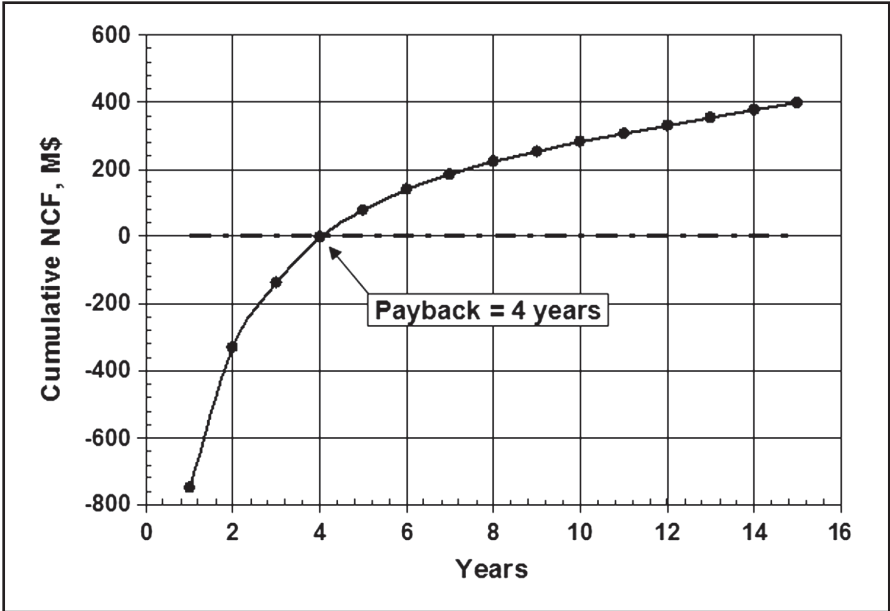


Fig. 6–1. Graphical calculation of payback period

The payback period does not provide any idea of the benefits to follow. The method is not concerned with profitability. For an investment to provide profit, the useful economic life beyond the payback period is important. For example, consider two investments of \$150,000 each and both have equal payback periods of three years (the net cash flow is \$50,000 each year). Furthermore, project A does not generate any cash flow after the payback period, while project B generates \$60,000 in three years after the payback period. Therefore, project B is the preferred choice. The payback period fails to highlight this fact.

In fact, if economic life and payback period are equal, then an opportunity loss occurs, since the same capital invested elsewhere would presumably have earned some return. For example, in table 6–1 the 750 M\$ if invested in a bank at an interest rate of 10% per year accumulates to approximately 1,267 M\$ by the end of 2025 as shown in figure 6–2. On the other hand, the *positive net revenue* generated each year if reinvested at 10% accumulates to approximately 1,414 M\$ by the end of 2025 as shown in figure 6–3. By plotting the yearly cumulative position from figures 6–2 and 6–3 as shown in figure 6–4, the breakeven point or the payback period is approximately 6.5 years. Thus the 4.04 years previously calculated is not correct.

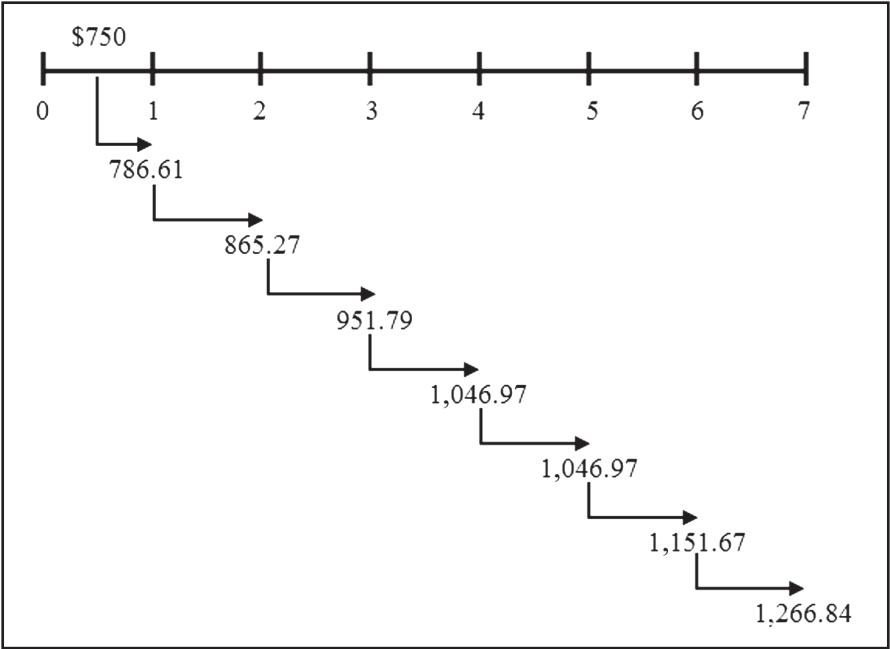


Fig. 6–2. Graphical view of the growth of initial investment of 750 M\$ at 10%

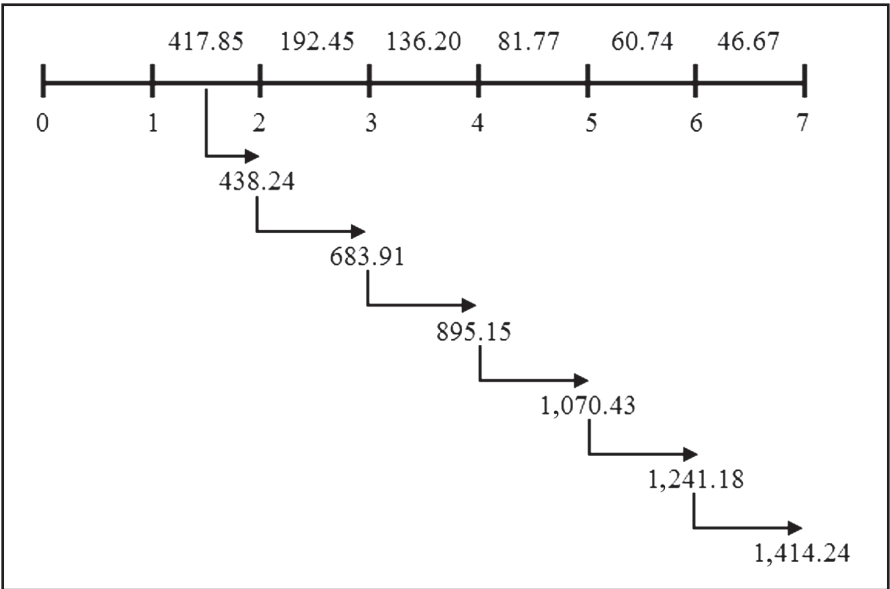


Fig. 6–3. Graphical view of the growth of reinvestment of net revenue at 10%

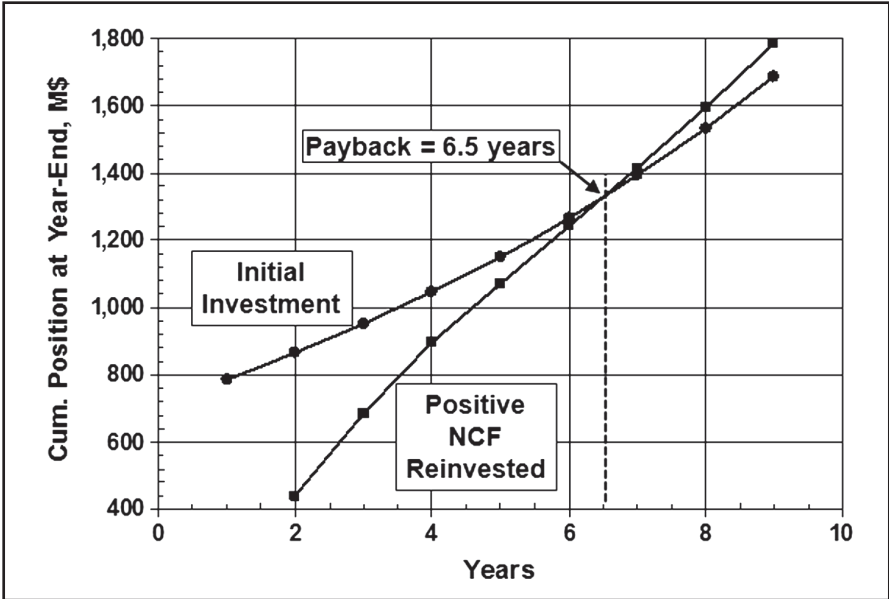


Fig. 6-4. Plot of the cumulative growth positions from figures 6-2 and 6-3

In figure 6-2, it is assumed the 750 M\$ earns interest over a partial year of 2011 (Year 1). Therefore, $786.61 \text{ M\$} = [750 \times (1 + 0.1)^{0.5}]$ is taken in 2011, $865.27 \text{ M\$} = (786.61 \times 1.1)$ in 2012, and so on. On the other hand, in figure 6-3, it is assumed the revenues received each year are reinvested at 10%.

The mid-year compounding assumption is used for the first year of reinvestment since the revenues are received monthly. Therefore, it does not earn interest for the whole year. The calculations are shown below.

$$\text{Cumulative at end of 2012} = 417.85 \times (1 + 0.1)^{0.5} = 438.24 \text{ M\$}$$

$$\text{Cumulative at end of 2013} = 192.45 \times (1 + 0.1)^{0.5} + 438.24$$

$$\times (1 + 0.1) = 683.91 \text{ M\$}$$

$$\text{Cumulative at end of 2013} = 136.20 \times (1 + 0.1)^{0.5} + 683.90$$

$$\times (1 + 0.1) = 895.15 \text{ M\$}$$

and so on.

The method as outlined in this section does not account for the time value of money. The payback period is useful because it provides information on how long funds are tied up, i.e., the shorter the payback period (other things held constant), the greater the liquidity. Also, the future expected cash flows are generally believed to be riskier than near-term cash flows. Thus payback may indicate aspects of a project's riskiness (i.e., the longer the

payback period, the higher the risk exposure). This is particularly true if the investment is taking place in one of the following:

- An industry where the rate of technological change is high (i.e., technological advances quickly make state-of-the-art products obsolete)
- An environment where the produced product's price is highly volatile, such as the oil price
- A politically and/or economically unstable country

Average return on investment (AROI)

This method is also referred to as the return on investment (ROI) or accounting rate of return (ARR). The method simply relates the average annual cash inflow to the average investment. The formula is:

$$\text{Average Return on Investment} = \frac{\text{Annual Average Cash Inflow or Savings}}{\text{Net Investment Outlay}} \quad (6.2)$$

Substituting the data from table 6–1,

$$\begin{aligned} \text{Average Return on Investment} &= \frac{\frac{(\text{Gross Revenue} - \text{Taxes} - \text{LOE})}{\text{Economic Life}}}{\text{Investment}} \\ \text{Average Return on Investment} &= \frac{\frac{1,619.69 - 129.58 - 340.87}{15}}{750} \\ &= 0.1022 \text{ or } 10.22\% \end{aligned}$$

The ROI method shares most of the technical problems associated with the payback method discussed in the previous section. Following are the properties of the ROI method.

- The technique does not account for the time value of money.
- The averaging process ignores the length of the life of the alternatives being evaluated.
- It does not give any indication about the size of the initial investment.

- It does not give any indication of the value of the project (i.e., no contribution to wealth maximization effort).
- The technique is not additive (i.e., if one project has an ROI of 10% and another project has an ROI of 15%, the total ROI does not become simply 25%). The average annual cash inflows from the two projects have to be first added together then divided by the total investment of the two projects to arrive at the ROI of the total investment in the two projects.
- It does not provide any indication of capital's exposure to risk and uncertainty.
- The ROI is not comparable with the cost of capital of the investment (i.e., opportunity cost or interest rate).

Discounted Cash Flow Approaches

The discounted cash flow methods are widely accepted and used in the industry for all types of capital investment evaluations because they recognize the time value of money. This is critical when assessing the profitability of long-term investments. Future and past values of money can be converted into their present value equivalent by using the time value of money concepts presented in chapter 2.

In this section, the mathematics of the various discounted cash flow methods and their utility in practical investment evaluations are discussed. The various methods presented are (1) discounted payback period, (2) net present value, (3) internal rate of return, (4) profitability index, (5) unit technical cost or long-run marginal cost, (6) growth rate of return or modified internal rate of return, and (7) adjusted net present value. At the end of this chapter the effect of the frequency of discounting/compounding and flow of funds (continuous or discrete) on these measures of profitability is presented.

Discounted payback period

The discounted payback period is calculated in exactly the same way as presented in the previous section. The exception here is accounting for the time value of money. Referring to table 6-1, the cumulative discounted NCF of column O instead of the cumulative NCF of column N is used for computing the discounted payback period. A weighted cost of capital of $i_d = 10\%$, the mid-year assumption to calculate the discounted NCF of column M, and the following equation is used.

$$\begin{aligned}
 \text{Cell O12} &= \text{Cell M12} \times \frac{1}{(1+i_d)^{n-0.50}} \\
 &= -750 \times \frac{1}{(1+0.10)^{1-0.50}} = -750 \times 0.9535 = -715.13
 \end{aligned}$$

The n for the first year is 1, for the second year is 2, and so on. If the year-end assumption is used, then the 0.5 in the power is omitted. Each year the present value of the net revenue is added to the discounted NCF of the previous year to compute the cumulative. This cumulative discounted NCF is then used to calculate the payback period in the same way as in the previous case. Figure 6–5 illustrates the plot of the cumulative discounted NCF of column O and the discounted payback period.

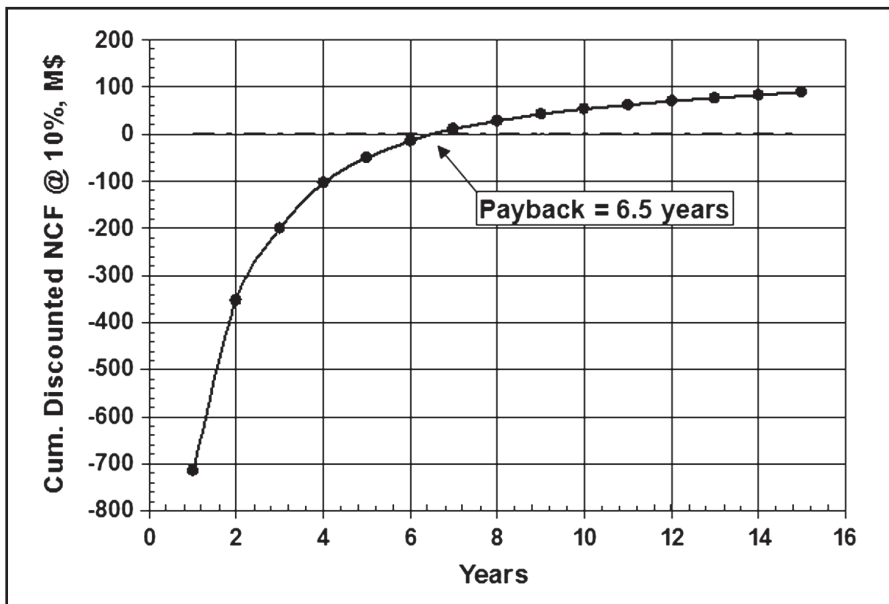


Fig. 6–5. Graphical calculation of the discounted payback period

Figures 6–3 and 6–5 show that it takes longer to break even in discounted terms as compared to the previous case (6.5 years versus 4.04 years). This is due to the recovery of the 10% interest on the initial investment. Therefore, this method removes the time value of money limitation from the previously calculated payback period. The rest of the properties are the same in both cases. The payback period is a subjective measure and depends on project

type and the preference of the investor. Some investors may reject projects if their payback period is more than 5 years even if they are otherwise profitable. Others may limit their choice to only 3-year payback projects, while some may not mind even longer payback projects. In most cases it may be a strategic decision more than a financial decision.

Net present value (NPV)

The net present value (NPV), also referred to as the present value of *cash surplus or present worth*, is obtained by subtracting the present value of periodic cash outflows from the present value of periodic cash inflows. The present value is calculated using the *weighted average cost of capital* of the investor, also referred to as the *discount rate* or *minimum acceptable rate of return*.

The discount rate should reflect the value of the alternative use of funds. What is an alternative use of funds? Money has value regardless of how it is utilized. If it is kept in a bank savings account, it earns interest; if it is kept in a mattress, it loses *real* value with time because of the missed opportunity of earning interest. For example, if \$100,000 invested in a bank earning 5% interest is withdrawn for investing it in another project, the 5% becomes the cost of capital or the opportunity cost of the \$100,000 that is forgone by investing it in another project. The bank account is the alternative use of funds. Therefore, when evaluating the project it must generate at least 5% to break even with the returns from the bank account. The risk factor also comes into play (if return from the project is riskier than the return on the bank account, then some risk premium more than the 5% return is desired).

When the NPV of an investment at a certain discount rate is positive, it pays for the cost of financing the investment or the cost of the alternative use of funds. The *investment generates revenue that is equal to the positive present value*. It also implies the rate of return on the investment is at least equal to the discount rate. Conversely, a negative NPV indicates the investment is not generating earnings equivalent to those expected from the alternative use of funds, thus causing opportunity loss. The net present value method of evaluating the desirability of investments is mathematically represented by the following equation.

$$NPV = \frac{S_1}{(1+i_d)^1} + \frac{S_2}{(1+i_d)^2} + \frac{S_3}{(1+i_d)^3} + \cdots + \frac{S_n}{(1+i_d)^n} - I_o \quad (6.3)$$

OR

$$NPV = \sum_{t=1}^n \frac{S_t}{(1+i_d)^t} - I_o \quad (6.3a)$$

where

S_t = the expected net cash flow (gross revenue – LOE – taxes)
at the end of year t

I_o = the initial investment outlay at time zero, i.e., 1/1/2011

i_d = the discount rate, i.e., the required minimum annual rate
of return on new investment

n = the project's economic life in years

Since the capital outlay may stretch longer than one period, a more general formulation would be to define I_o as the present value of the capital outlays. On the other hand, to further simplify the calculations, a net cash flow (NCF) is calculated as shown in table 6–1. The NPV of this NCF stream is then calculated. Therefore, equation (6.3) is modified as:

$$NPV = \sum_{t=1}^n \frac{NCF_t}{(1+i_d)^t} \quad (6.4)$$

The above equations assume cash flows are received or disbursed at the end of the year. In reality this is not a valid assumption because cash flow transactions take place continuously or at the least monthly. To account for this, it is assumed all cash flow transactions occur in mid-year (average over the year). To do this the t in equations (6.3) and (6.4) is changed to $(t - 0.5)$.

Example 6–1

Calculate the NPV of the NCF stream (column M) in table 6–1 using a 10% discount rate and mid-year cash flow assumption.

Solution: The calculations for example 6–1 are shown in table 6–2.

The NPV of 90.01 M\$, at 10% discount rate and mid-year cash flow assumption, is calculated in table 6–2. Combining the analysis with the payback period calculation, the initial capital investment is recovered in 6.5 years and 10% is earned on the declining outstanding NCFs during the investment's economic life. Moreover, a cushion of 90.01 M\$ of extra dollars in present value can be counted on if the project lives out its economic life and the cash flow estimates are correct.

Spreadsheets have a built-in function for NPV calculation. In MS Excel, the =NPV(rate, NCF₁:NCF_n) function calculates the NPV of an investment by using a discount rate and a series of future net cash flows. The =NPV function assumes year-end cash flows. The NPV calculated by Excel is adjusted to the mid-year cash flow assumption by multiplying it by the factor $(1 + i_d)^{0.5}$. For example, the NPV in cell L30 of table 6–1 is calculated by using the function =NPV(0.1,M10:M25)*(1+0.1)^0.5.

Table 6–2. Calculations of NPV for example 6–1

Year	NCF M\$	<i>t</i>	$\frac{1}{(1 + 0.10)^{t-0.5}}$	Cum. NPV @ 10% M\$
2011	(750.00)	0.50	0.9535	(715.13)
2012	417.85	1.50	0.8668	362.19
2013	192.45	2.50	0.7880	151.65
2014	136.20	3.50	0.7164	97.57
2015	81.77	4.50	0.6512	53.25
2016	60.74	5.50	0.5920	35.96
2017	46.67	6.50	0.5382	25.12
2018	36.60	7.50	0.4893	17.91
2019	30.87	8.50	0.4448	13.73
2020	28.48	9.50	0.4044	11.52
2021	25.13	10.50	0.3676	9.24
2022	23.54	11.50	0.3342	7.87
2023	23.29	12.50	0.3038	7.08
2024	23.12	13.50	0.2762	6.39
2025	22.54	14.50	0.2511	5.66
	399.24			90.01

Once the NPV of the investment alternative is calculated, the following decision rules apply:

- If the NPV is positive, accept the proposal.
- If the NPV is negative, reject the proposal.

If the NPV is zero, the analyst will be indifferent because the proposal is generating the same return as the alternative use of funds will generate (assuming both alternatives have the same risk). The NPV decision criterion follows directly from the assumption that the analyst is required to maximize the worth of the firm. This criterion results in *optimal* choice of projects.

The size of a project's NPV depends, among other things, on the discount factor. A plot showing this is called the NPV profile. Figure 6–6 illustrates the NPV *profile* for the NCF generated in table 6–1. The NPV profile shows the sensitivity of the NCF to the discount rate and gives a range of the discount rates below which the project generates a positive NPV. Therefore, if one is not sure of the firm's cost of capital, this profile gives a useful range. When the

discount rate is equal to zero (at the intercept with the vertical axis), the NPV equals the algebraic sum of the periodic *undiscounted* NCFs.

The main features of this method are summarized below.

1. The technique accounts for the time value of money.
2. It considers cash flows over the economic life of the investment.
3. It does not give any indication about the size of the initial investment (i.e., the NPV of a \$100 investment may be \$100, and the NPV of a \$10,000 investment may also be \$100).
4. It gives direct indication of the value of the investment (i.e., contributes to the wealth maximization effort—when a firm takes on an investment with a positive NPV, its value increases by the amount of the NPV).
5. The NPVs are additive (i.e., if there are 10 projects each with an NPV of \$10, the total NPV will be \$100).
6. The NPV method implicitly assumes all cash flows are reinvested at the weighted average or opportunity cost of capital.
7. The same discount rate does not have to be used for the entire period of cash flows.
8. NPV is suitable for use with probabilities, i.e., for calculating the expected monetary value (see chapter 3 of volume 2 in this series).

Internal rate of return (IRR)

Internal rate of return (IRR) is another important and widely reported measure of profitability. IRR is reported as a percentage rather than a dollar figure such as NPV. The internal rate of return (IRR) is also referred to as the discounted cash flow rate of return (DCFROR), rate of return (ROR), internal yield, marginal efficiency of capital, and the investor's method. IRR is the discount rate at which the net present value is exactly equal to zero, or the present value of cash inflows is equal to the present value of cash outflows. Another definition of IRR is the interest rate received for an investment consisting of payments (negative values) and income (positive values) that occur at regular periods. The equation for calculating IRR is

$$\sum_{t=1}^n \frac{NCF_t}{(1 + IRR)^t} = 0 \quad (6.5)$$

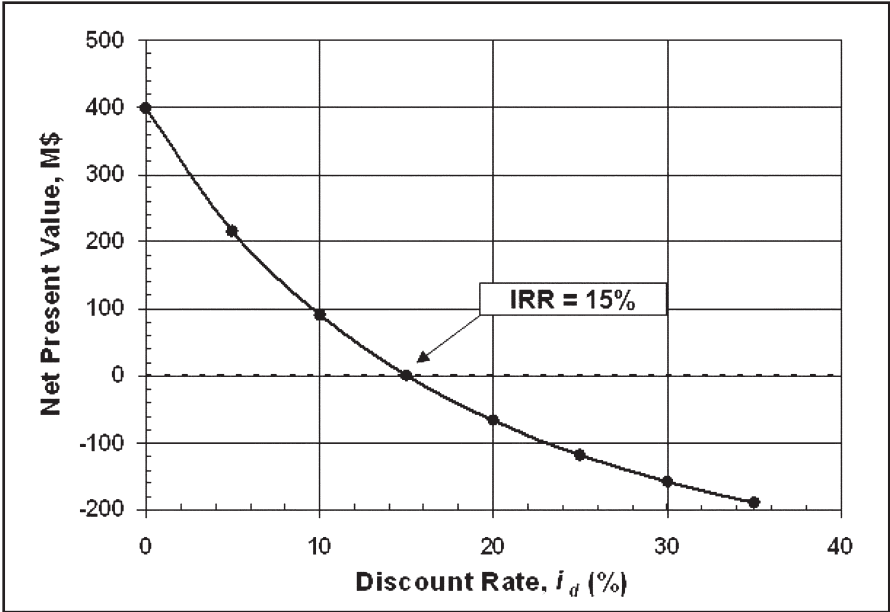


Fig. 6–6. The NPV profile of the NCFs of table 6–1

In equation (6.5) the net cash flows at time t are known, but the IRR (fraction) is unknown. Some value of IRR causes the discounted NCFs of the project to equal zero; this value of IRR is then called the internal rate of return. IRR resembles the interest rate paid on financial investments or debts.

The IRR formula, equation (6.5), is simply the NPV formula, equation (6.4), solved for the value of i_d that causes the NPV to equal zero. The only difference is that in the NPV method i_d is known and NPV is calculated; in the IRR method NPV is known (equals zero) and the value of IRR that forces the NPV to equal zero is calculated. Therefore, the calculation of IRR requires an iterative solution.

IRR can be calculated manually in two ways: by trial-and-error and graphically. The IRR calculation by trial-and-error requires at least two interest rates that when substituted for IRR in equation (6.5) give a positive and a negative NPV. The first step is to make a best guess of the IRR for the investment. If a positive NPV is calculated, the IRR should be increased and NPV calculated again. The best guess is repeated until a negative NPV is calculated. The IRR is then between the two adjacent interest rates that gave positive and negative NPVs, which is determined by linear interpolation using the following equation:

$$IRR = \frac{NPV_A}{NPV_A - NPV_B} (IRR_B - IRR_A) + IRR_A$$

(6.6)

where

NPV_A

= positive net present value

NPV_B

= negative net present value

IRR_A

= interest rate associated with NPV_A

IRR_B

= interest rate associated with NPV_B

As shown in figure 6–6, the relationship between interest rate and NPV is not linear; therefore, linear interpolation is not recommended over more than 5 or at the most 10 percentage points.

To determine the IRR graphically, three to five NPVs are calculated at different interest rates. These NPVs are then plotted versus their respective interest rates on Cartesian coordinate graph paper. A line is drawn through these points. The intersection of this line at NPV of zero gives the IRR.

Example 6–2

A net present value profile for the NCFs in table 6–1 follows. Using the methods discussed previously, calculate the internal rate of return for the project. Note that the NPV at $i_d = 0$ is equal to the cumulative undiscounted net cash flow.

NPV Profile								
Percent	0	5	10	15	20	25	30	35
NPV, M\$	399.25	215.60	90.01	0.06	(66.81)	(118.00)	(158.11)	(190.13)

Solution: From the NPV profile above, it is obvious the IRR is between 15% (+ve NPV) and 20% (–ve NPV). Using equation (6.6), the IRR is calculated as

$$IRR = \frac{0.06}{0.06 - (-66.81)} (20 - 15) + 15 \approx 15.00\%$$

The “approximately equal to” (symbol, $\approx$), is used rather than the equality (=) sign because interpolation provides an approximate solution, especially when the relationship is not linear. To determine IRR graphically, the NPV profile is plotted as shown in figure 6–6 and the IRR read at the point where the NPV profile intersects the NPV at zero.

Microsoft Excel has built-in functions for the IRR calculation. In Excel the =IRR(values, guess) function returns the internal rate of return for a

series of cash flows represented by the numbers in *values*. *Values* is an array or a reference to cells that contain desirable numbers for the internal rate of return.. For example, the IRR in cell L31 (table 6–1) is calculated using the NCFs in column M by entering the formula =IRR(M10:M25). Excel uses an iterative technique for calculating IRR. Starting with *guess*, IRR cycles through the calculation until the result is accurate within 0.00001%. If *guess* is omitted, it is assumed to be 0.1 (10%).

The rule for making the investment decision when using IRR is:

- *Accept* the investment if its calculated IRR is greater than the return on the alternative use of funds or cost of capital.
- *Reject* the investment if its calculated IRR is less than the return on the alternative use of funds or cost of capital. If the investment is financed 100% by borrowed capital, then the rate of return should at least exceed the interest rate being paid on the loan. A company could also set a minimum required rate of return or hurdle rate.

If the calculated IRR is (1) greater than the required rate of return, the NPV is positive; (2) less than the required rate of return, the NPV is negative; and (3) equal to the required rate of return, the NPV is zero.

Dual or multiple rates of return. It is mathematically possible to have multiple rates of return in a complex project, which adds to the problem of interpreting IRR. Multiple rates of return occur when there are multiple sign changes (i.e., multiple payback periods or negative, positive, negative and positive cash flows) in the net cash flow. The second or third sign reversal, however, is not sufficient to result in multiple rates of return. It also depends on when the second or third reversals take place and the magnitude of the negative cash flow causing the reversal.

Such situations may occur whenever a proposal calls for advanced payment or high terminal costs. Also, rate acceleration projects (which attempt to increase production in early years at the cost of decreased production in the later years) and projects requiring a large expenditure at a later point in the project life (such as abandonment costs related to the removal of production facilities) generate multiple rates of return.

When analyzing alternative measures of profitability, it is important to keep in mind the type of cash flow. The forecasted cash flows of a project are classified as either *conventional* or *nonconventional*. A conventional cash flow is one in which the initial capital investment outlay is followed by a stream of positive net revenues of the form: $- + + + + \dots$; or if the initial outlay takes place over a number of years, the cash flow has the form: $- - - + + + +$. The cumulative net cash flow of such conventional cash flows shows only one sign

change from negative to positive. Nonconventional cash flows are those that have more than one sign change, such as $+-++$ or $-+++$. Many oil field investments may follow nonconventional cash flows. They have initial negative capital outlays, followed by positive cash flows, and finally negative cash flow for the abandonment of the facilities preceding the terminal year.

Stermole has presented some useful observations regarding the interpretation of IRR resulting from nonconventional cash flows, such as:¹

- Income or savings precedes costs
- Investment—income—investment situation
- Income—investment—income situation

In the dual IRR situation, the calculated IRR has a combination rate of return and rate of reinvestment meaning. In such situations, other profitability yardsticks should be used. The following rules of thumb can be used to determine if dual rates of return exist.

When the undiscounted net cash flow (i.e., NPV at $i_d = 0\%$) is negative for an investment followed by income followed by an investment situation, dual positive IRRs exist if any real interest rate solutions exist for the NPV equation. On the other hand, if the undiscounted net cash flow (i.e., NPV at $i_d = 0\%$) is positive, the dual IRR values exist with one being negative and the other positive. Further, if significant income follows the second investment, such as investment—income—investment—income, a dual rate of return situation may not exist depending on the magnitude and timing of the income following the second investment.

In an income—investment—income situation, dual rates of return occur for the conditions opposite to the investment—income—investment situation. In this situation, dual IRRs exist if the undiscounted net cash flow is positive, and positive dual IRRs do not exist if it is negative. When the dual IRR does not exist, an investment is acceptable for i_d greater than the investment's IRR because in this region the NPV is positive. When dual IRRs exist, an investment is acceptable only for i_d values outside the region between the dual rates because in this region the NPV is positive.

Example 6–3

A strip mining company invests \$70,000 at time zero to generate net revenue of \$40,000 per year for the following five years. During the sixth year, the company has to spend \$140,000 in order to restore the land to its original condition. Calculate the IRR of this investment. Assume year-end discounting.

Solution: The total undiscounted net cash flow (i.e., NPV at 0%) is $-\$10,000$. The NPV at 5% discount factor is calculated as follows. Since the net revenue of $\$40,000$ is an annuity, equation (2.14) is used to calculate its present value. The present value of $\$140,000$ is calculated using equation (2.10).

$$P_v = A_v \left[\frac{(1+i_e)^t - 1}{i_e (1+i_e)^t} \right] \tag{2.14}$$

$$P_v = (F_v)_1 \frac{1}{(1+i_e)^1} + (F_v)_2 \frac{1}{(1+i_e)^2} + (F_v)_3 \frac{1}{(1+i_e)^3} + \cdots + (F_v)_t \frac{1}{(1+i_e)^t} \tag{2.10}$$

$$\begin{aligned} NPV @ 5\% &= -\$70,000 + \$40,000 \frac{(1+i_e)^t - 1}{i_e (1+i_e)^t} - \$140,000 \left(\frac{1}{(1+i_e)^t} \right) \\ &= -\$70,000 + \$40,000 \frac{(1+0.05)^5 - 1}{0.05(1+0.05)^5} - \$140,000(1+0.05)^{-6} \\ &= -\$70,000 + \$40,000(4.3295) - \$140,000(0.7462) \\ &= -\$1,288 \end{aligned}$$

Similarly, the NPV at other discount rates is calculated as shown below and in figure 6-7.

Rate, %	NPV	Rate, %	NPV
5	$-\$1,288$	25	$\$871$
10	$\$2,605$	30	$-\$1,582$
15	$\$3,560$	35	$-\$4,329$
20	$\$2,739$	40	$-\$7,187$

Figure 6-7 shows the NPV profile crosses the NPV = 0 line at approximately 6.2% and 26.78%, or dual rates of return of 6.2% and 26.78%. These rates cannot be used directly for decision-making purposes. However, they do provide useful information by providing a range of i_e when the investment's NPV is positive. Thus the investment is accepted if i_e is between 6.2% and 26.78%. Whenever a dual IRR situation exists, it is recommended to use NPV for decision making. If the =IRR(values, guess) function of MS Excel is used to calculate IRR of the cash flows, it calculates an IRR of only 6.2%. It does not show if a dual rate of return has occurred unless the NPV profile of the cash flow is examined. The Excel function will give the second or third IRR (if more than one IRR exist) if a guess is entered close to where the

second or third IRR could be. However, it is not a very definite way of finding the second or third IRR. The best way, as mentioned above, is to review the NPV profile.

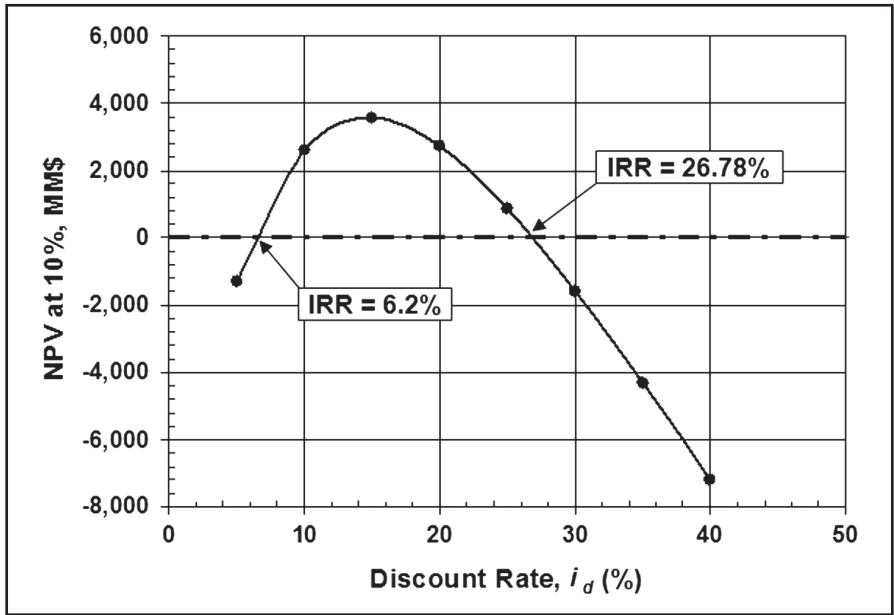


Fig. 6-7. NPV profile for the data of example 6-3

Another example is of a heavy oil field. The field has been in production for the last 10 years. However, since the oil is heavy the recovery is very small and the rates are very low. It has been now decided to implement steam injection in order to improve the oil recovery. The project was economically evaluated; the NPV profile of the project is as shown in table 6-3. The NPV profile is plotted as shown in figure 6-8.

In this particular case, since the curve is continually upward, the IRR may be between 17.7% and 37.2%. It may be closer to 37.2% rather than 17.7%. In such situations, it is recommended to evaluate the incremental investment and the resulting benefit, i.e., there is no need to lump the incremental costs and benefits with the existing cash flow.

Reinvestment in the IRR calculation. To achieve the calculated IRR, the IRR method implicitly assumes the generated cash flows are reinvested at the calculated IRR. However, in the views of most authors and evaluators, the question of whether the IRR calculation involves the reinvestment of

income still remains (see the section titled “Further Reading” at the end of this chapter).

Table 6–3. Net cash flow and NPV profile of a steam injection project

Year	NCF (MM\$)	Discount Rate	NPV (MM\$)
1	94.40	0%	6,813.70
2	167.30	5%	2,081.75
3	163.80	10%	583.08
4	(82.10)	15%	98.77
5	(14.40)	20%	(42.22)
6	(387.90)	25%	(63.16)
7	(991.60)	30%	(43.78)
8	(1,153.10)	35%	(13.72)
9	155.60	40%	16.09
10	116.40	45%	41.98
11	383.90	50%	63.15
12	338.80	55%	79.85
13	598.40	60%	92.71
14	503.60	65%	102.41
15	427.70	70%	109.55
16	507.90		
17	435.50		
18	429.90		
19	404.10		
20	396.90		
21	396.40		
22	394.70		
23	386.00		
24	378.20		
25	381.00		
26	375.20		
27	372.50		
28	467.30		
29	447.30		
30	388.80		
31	331.20		
			6,813.70

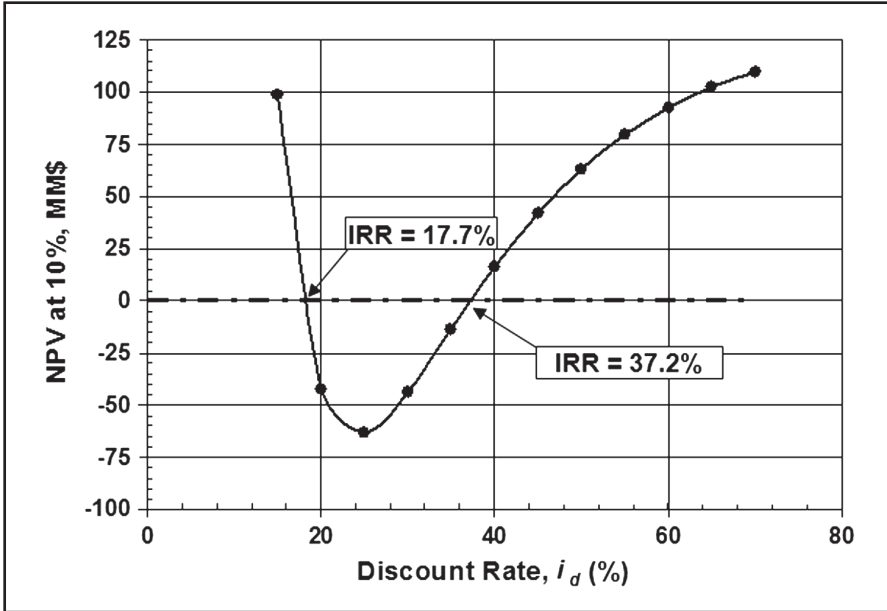


Fig. 6–8. NPV profile for the data of table 6–3

Some feel that inherent in the calculation procedure is a reinvestment of income at the computed IRR. Others argue there is no reinvestment implied and it does not matter what is done with the interim revenue generated by the investment. Many authors state categorically that reinvestment at the IRR is implied in the calculation procedure. Others state just as emphatically that no reinvestment is implied. Collarini states, “the discounted cash flow (or internal) ROR is simply a measure of project efficiency that is more useful in the banking industry than in the petroleum business.”²

Stermole states *reinvestment is not implied or required to achieve the calculated project rate of return when investment precedes income or savings*. The statement is further mathematically proven by Dougherty^{3,4} and Aguilera.⁵

The authors arguing against the reinvestment assumption requirement state the calculated IRR gives the rate of return on the unamortized investment (unpaid principal) at the end of each year and is not the rate of return on the initial investment for the life of the project. The confusion is caused due to the lack of understanding of the difference between *project rate of return* and *overall growth rate*.

Weston and Brigham state the reinvestment requirement in the following way, which does not explicitly say that without the reinvestment the calculated IRR or NPV values are not valid.

Suppose the cash flows from a project are not reinvested but are used for current consumption. No reinvestment is involved; yet an IRR for the project could still be calculated. Does this show that the reinvestment assumption is not always implied in the IRR calculation? The answer is no; reinvestment itself is not necessarily assumed, but the opportunity of reinvestment is assumed. Because that assumption is made in the very construction of the NPV equations and because calculation of both NPV and IRR is based on those equations, we simply could not define or interpret the concepts of NPV and IRR without assuming reinvestment.⁶

To show how the internal rate of return is unrelated to reinvestment of incomes derived from the particular investment, consider a person deposits \$100,000 in a bank saving account at an annual interest of 8%. At the end of each year, he withdraws the earned interest of \$8,000 to meet his expenses, i.e., the money is not reinvested. At the end of year five he withdraws all his money (the \$8,000 for the fifth year and his initial deposit of \$100,000). This generates an IRR of 8% over the five-year period while the interim cash flows are not reinvested at all (option A in the following table).

From the above example it is evident the 8% return received on this investment (bank account) is not related to reinvestment of the \$8,000 annual interest dividends or the \$100,000 principal received at the end of year five. Whether the earned interest is spent or reinvested at the same or larger rate of return does not change the fact that this is an 8% IRR investment. However, what is done with the earned interest does affect the overall individual or corporate growth rate. For example, assume the \$8,000 interest earned each year is left in the account. Its accrued interest would give \$146,933 at the end of five years, which also results in an IRR of 8% as shown below (option B in the following table).

Year	Option A	Option B
0	-\$100,000	-\$100,000
1	\$8,000	0
2	\$8,800	0
3	\$8,800	0
4	\$8,800	0
5	\$108,000	\$146,933
	8.00%	8.00%

The above example is valid for bank accounts. Is it really applicable to other capital investments where the initial investment is not recovered at the end of the project, but rather recovered from the revenues generated by the project every year? Assume \$100,000 is invested in a capital project that will generate a net yearly cash flow of \$33,400 for the next five years. The IRR for this will be about 20%. On the other hand, instead of receiving \$33,400 per year for five years, \$167,000 ($\$33,400 \times 5 = \$167,000$) is received at the end of year five. The IRR for this option is 10.8%. The difference is obviously due to the time value of money, hence the reinvestment. If the \$33,400 is received each year and reinvested at 20%, it accumulates to \$248,549 in five years to yield a rate of return of 20% as shown below.

Year	Option A	Option B	Option C
0	−\$100,000	−\$100,000	−\$100,000
1	\$33,400	0	0
2	\$33,400	0	0
3	\$33,400	0	0
4	\$33,400	0	0
5	\$33,400	\$167,000	\$248,549
	20.00%	10.80%	20.00%

Terry states:

*Those that argue that the rate of return calculation does not imply reinvestment say that the incomes can be removed without affecting the calculation. While this may appear to be true on the surface, the IRR calculation does not allow the removal of incomes without assigning a value to them. Since all incomes in the IRR calculation are brought back to the present at the IRR, the calculation is placing a worth on them. By removing the incomes without stating that they are worth the IRR, then the problem has been changed.*⁷

The above discussion is provided for academic interest. For the purpose of economic evaluation, due to the unresolved controversy of reinvestment assumption, IRR is losing its credibility as a decision yardstick. However, it is suggested not to worry too much about the issue of reinvestment. If properly used, IRR can still provide meaningful results. The investment situations where IRR can be successfully used are discussed in chapter 7.

Specific characteristics of IRR. Following are the salient features of the internal rate of return (IRR).

1. It accounts for the time value of money.
2. It considers cash flows over the economic life of the investment (i.e., it has a built-in economic earnings requirement beyond the recovery of the original investment). Thus an IRR higher than the minimum desired rate of return provides an incremental economic gain beyond satisfying a normal earnings standard.
3. Since it is a percentage measure, it does not give any indication about the size of the initial investment. Under a constrained budget situation, it is desirable to invest in a lower-yielding, larger-size project rather than a higher-yielding, smaller-size project. Generation of the largest addition to total portfolio remains the ultimate objective.
4. It does not give direct indication of the value of the investment.
5. The IRRs are not additive (i.e., if there are three projects with an IRR of 10%, 12%, and 18%, the total IRR will not be 40%). The only valid procedure is to combine the cash flows of the projects under consideration and then determine the IRR for the combined cash flow. In joint venture partnerships, the IRR for each partner is the same as the IRR of the total investment. On the other hand, the NPV is proportional to the share of the individual partners.
6. IRR cannot be calculated for the following situations:
 - a. When cash flows are all negative
 - b. When cash flows are all positive, i.e., investment is paid out of the future revenues
 - c. Total undiscounted revenues are less than the investment (e.g., a marginal producing well or field depleted before reaching payback)
 - d. When the cumulative cash flow stream goes negative more than once
7. To achieve the calculated IRR, the IRR method implicitly assumes the generated cash flows are *reinvested* at the calculated IRR, which may not be possible for the higher IRR investments. For example, if a project's IRR is calculated at 35%, it achieves this rate of return only if all monies received from this project are reinvested at 35%. If at the time, a reinvestment opportunity at a lower than 35% rate is available, then the project IRR is between this lower rate and the 35%. This argument is frequently used against the use of this concept.

8. The timing of cash flows is especially significant to accuracy of the IRR, as IRR is usually more sensitive to the assumptions of timing than other profitability measures.
9. IRR is not affected by delays in project start dates. If a project starts several years in the future, then the net present value of the project is reduced because its annual net cash flows must be discounted more. For example, if NPV of a project is \$100 million, a two-year delay in the project start date would mean a reduction in the NPV to $\$100(1 + \text{IRR})^{-2}$. However, the IRR does not change because of the delay. This is then a disadvantage of the IRR, as it cannot distinguish between projects that start late as compared to the ones that start early. This limitation of IRR is applicable only when the cash flows of the delayed project are identical to the cash flows of the original project. If the cash flows change due to inflation or price escalation, then the IRR also changes.
10. The method gives conflicting ranking when used for ranking mutually exclusive investments or selecting non-mutually exclusive investments. This problem is generally attributed to the reinvestment assumption. This author feels it is due to the pattern of particular cash flows and that discounting favors the early cash flows over the cash flows received later in the economic life of the investment.
11. Risk (probability) cannot be mathematically incorporated in the IRR equation.
12. IRR can be computed for after-tax and before-tax cash flows.

Profitability index (PI)

The discussion of NPV and IRR shows how these two methods fail to reflect the size of the initial investment. These yardsticks pose problems when an analyst has to make a choice among several alternative investments of different sizes. For example, the NPV and IRR for two investments may be equal or close, but the amount of investment required by the two alternatives may vary widely. Measures like the NPV tend to emphasize the adoption of larger capital investments since such investments tend to have higher NPVs. Therefore, NPV as an investment criterion frequently comes under attack because it does not measure the efficiency of an investment.

To overcome this efficiency problem, *profitability index* (PI) is introduced as a profitability criterion. PI is a dimensionless ratio obtained by dividing

the present value of future operating cash flows by the present value of the investment. Mathematically, the PI is given by the following equation.^{8,9}

$$PI = 1 + \frac{NPV}{PV \text{ of Capital Investment}} \tag{6.7}$$

The PI simply answers the question: How much in present value benefits is created per dollar of investment? The PI shows the relative profitability of an investment, or the present value of benefits per the present worth of every dollar invested (i.e., higher net return for each dollar invested). Consider the following two investments being mutually exclusive (i.e., only one of the investments can be selected).

	Option A	Option B
PV of investment, M\$	\$100	\$1,000
Net present value, M\$	\$100	\$100

The two options cited above, therefore, give PIs as follows.

$$PI_A = 1 + \frac{100}{100} = 2.0$$
$$PI_B = 1 + \frac{100}{1,000} = 1.10$$

Note that both the numerator and denominator are discounted by the same discount rate. Investment A is generating more PV dollars for every dollar invested as compared to investment B. The $PI = 2.00$ means total return on the investment dollar is \$2, where \$1 is the cost and \$1 is the net gain in PV terms. The definition of PI implies that projects with the largest PI values generate the largest cumulative NPV that can be accumulated from the available present worth investment dollars (i.e., the higher the PI, the better the investment). A variation to the definition of the profitability index is the *present value ratio* (PVR), *present value index* (PVI), *discounted profit-to-investment ratio* (DPIR), or *investment efficiency*. The PVR is the ratio of the NPV to the present value of capital investment rather than the ratio of the PV of future operating cash flows to the PV of capital investment. Mathematically, this relationship is represented by the following equation.

$$PVR = \frac{NPV}{PV \text{ of Capital Investment}} \tag{6.8}$$

According to equation (6.8), the above calculations become

$$PVR_A = \frac{100}{100} = 1.0$$

$$PVR_B = \frac{100}{1,000} = 0.10$$

As shown in the above calculations, the PI is equal to $PVR+1$. The PVR shows net PV dollars generated per the PV of every capital investment dollar (i.e., the capital investment is already recovered and the value of the PVR is the net gain over every dollar invested). *Sometimes the two definitions are reversed. Therefore, the analyst needs to be careful when interpreting these values.* The PI for the data in table 6-1 is calculated as follows:

$$PI = 1 + \frac{90.01}{715.10} = 1.126$$

The PI generates a number greater than one (1.0) for investments with a positive NPV and a number less than one for investments with a negative NPV. The decision rules for use of the PI are:

1. *Accept* all independent investment proposals with PI greater than one.
2. *Reject* all independent investment proposals with PI less than one.

A PI equal to one (1.0) means the project breaks even (i.e., it neither makes money nor loses money). Similarly, accept all investments with PVR greater than zero (0.0) and reject those with PVR less than zero. Following are properties of the profitability index.

- It shares all the advantages of NPV (such as realistic reinvestment rate, no multiple rates, not a trial-and-error solution, etc.).
- It provides a measure of profitability per dollar invested. This is a particularly important consideration when faced with the selection of investments from a list containing more opportunities than the available funds can cover.¹
- It is the most representative measure of true earning potential of an investment, thus contributing to the portfolio maximization effort.
- As a ranking criterion, it can be used in capital allocation.

Unit technical cost (UTC) or long-run marginal cost (LRMC)

The unit technical cost (UTC), also referred to as the finding cost or long-run marginal cost (LRMC), is another useful indicator for screening capital investment projects. The unit technical cost is defined as the ratio of the total cost (CAPEX and OPEX) over the economic life of a project to the total expected reserves from the project. The indicator is thus reported in \$/Stb, \$/MMBtu, \$/MScf, \$/ton, etc. If other products such as gas, natural gas liquids (NGL), or condensate are also recovered along with oil, then the indicator is reported in dollars per barrel of oil equivalent (\$/BOE) or dollars per million British thermal units (\$/MMBtu).

The conversion from any other units to BOE can be done through British thermal units (Btu) content, physical constants, or revenue basis. For example, when using physical constants, 5.615 cubic feet of gas is equivalent to one barrel of oil equivalent, one metric ton of NGLs is equivalent to 0.1342 barrels of oil equivalent, and one barrel of crude oil is approximately equal to six million Btu's (depending on the API gravity of the oil). The revenue conversion involves product prices.

Alternatively, the LRMC of the major phase could be calculated. This is done by subtracting the present value of the revenue generated by the by-products (such as sulfur, ethane, and condensate extracted from gas production) from the present value of all the costs, and then dividing this numerator by the present value of the sales gas volume in MMBtu. Similarly, if NGL recovery is also part of the gas field development project, then the present value of the NGL revenues are subtracted from the present value of all the costs.

For example, if the oil price is \$70/Stb and the gas price is \$4.25/MScf, then 16.47 MScf is equal to one barrel of oil equivalent. Similarly, if the price of NGL is \$600/ton, then one ton of NGL is 8.57 barrels of oil equivalent. The revenue conversion gives a more realistic and meaningful cash flow evaluation as compared to the other conversions.

The unit technical cost (UTC) may be undiscounted and/or discounted. The calculation of discounted UTC requires both yearly costs and production be discounted to the present value by using the same discount rate used for the calculation of NPV.

The positive point about UTC is that it is independent of the price of the product involved. It gives an indication of what the product is costing to develop and produce. It provides a measure of the cushion available when compared to the actual estimated product prices. For example, the profit margin is \$45/Stb if the discounted (at 10%) UTC for an oil project is \$25/Stb and the anticipated oil price over the life of the project is \$70/Stb. The project breaks even if the price of oil, over the life of the project, remains at \$25/Stb.

Unit technical cost for the data in table 3–17 and table 6–1 are calculated as shown below. The total cost (CAPEX and LOE) over the productive life of the lease (table 3–17) is 1,090.87 MM\$. The net oil and gas reserves (net of severance tax) are 13.858 MStb and 119.751 MMScf, respectively. The product prices are \$60/Stb and \$5.50/MScf. The gas production is converted to its barrel of oil equivalent by revenue basis, i.e., 10.91 MScf (\$60/\$5.50 = 10.91) is equivalent to one barrel of oil equivalent. The following steps are followed to calculate the UTCs.

1. Convert the 119.751 MMScf (net of taxes) to its BOE as:

$$\text{Total Reserves} = 13,858 + \frac{119,751}{10.91} = 24,834.26 \text{ BOE}$$

2. Calculate the undiscounted unit technical cost as:

$$\text{Unit Total Cost in BOE} = \frac{\$750,000 + \$340,870}{24,834.26} = \$43.93 \text{ per BOE}$$

Table 6–1 shows UTC of \$41.79/BOE because the gas was converted to BOE using a factor of 9.778 MScf/Stb as shown in Cell H37 of table 3–17.

3. If it was only an oil producing well, then the \$/Stb and \$/BOE are equal. If it is predominantly a gas well, then the technical cost should be calculated in \$/MScf. The technical cost in \$/Stb for the same cash flow is:

$$\text{Unit Technical Cost in Stb} = \frac{\$750,000 + \$340,870}{13,858} = \$78.72 \text{ per Stb}$$

4. To calculate *discounted* unit technical cost, the product stream and the total cost are discounted using the 10% discount factor (the same discount factor used for discounting cash flow is used for technical cost calculation). The rest of the calculations are the same as shown above.
5. Discounted oil production (net of severance tax) = 8.62 MStb
Discounted gas production (net of severance tax) = 75.13 MMScf
Discounted cost (LOE + CAPEX) = \$163.06 + \$681.82 = \$844.88 M\$

$$\text{Discounted Unit Technical Cost in BOE} = \frac{\$844.88}{8.62 + \frac{75.13}{9.778}} = \frac{\$844.88}{16.304} = \$51.82 \text{ per BOE}$$

$$\text{Discounted Unit Technical Cost in Stb} = \frac{\$844.88}{8.62} = \$98.0 \text{ per Stb}$$

The technical cost in Stb, in this particular cash flow, is high (higher than the oil price of \$60/Stb) because it is predominantly a gas well with high oil yield. The decision rule for the use of technical cost is:

- *Accept* the project if its discounted technical cost in \$/BOE (or \$/MScf or \$/MMBtu for gas lease) is less than the expected oil price (or gas price for gas lease) over the economic life of the project.
- *Reject* the project if its discounted technical cost in \$/BOE (or \$/MScf or \$/MMBtu for gas lease) is higher than the expected oil price (or gas price for gas lease) over the economic life of the project.

If the barrel of oil equivalent replaces the oil production in table 3–17 and the oil price is replaced by the discounted technical cost of \$51.82/BOE, the measures of profitability are (a) NPV = 0, (b) PI = 1.0, (c) IRR = 10%, and (d) UTC = \$51.82/BOE. Remember this is the breakeven case. The NPV = 0 means the present value of net revenues equals the present value of CAPEX. The PI = 1.0 means all the costs are recovered, and the IRR = 10% means the project's rate of return offsets the cost of capital.

If the *undiscounted* UTC (\$43.93/BOE) replaces oil price, the cash flow results in negative NPV and the IRR is less than the discount rate. This means the project is losing in PV terms. The UTC can be obtained graphically from a graph of NPV (calculated at various prices) versus the respective prices. The product price at the intersection of NPV = 0 is the desired technical cost.

Some analysts have a problem reconciling the idea of discounting production when calculating the discounted UTC. An engineer once said, "I don't believe in discounting production." Therefore, the mathematical derivation of the UTC is given here. As per our definition, UTC is the product price (oil, gas, or BOE) at which the present value of costs equals the present value of gross revenues. Therefore

$$PV \text{ of Costs} = PV \text{ of Gross Revenue}$$

$$\begin{aligned} \sum_{t=1}^n \frac{(\text{Costs})_t}{(1+i_d)^t} &= \sum_{t=1}^n \frac{(\text{Gross Revenue})_t}{(1+i_d)^t} \\ &= \sum_{t=1}^n \frac{(\text{Production} \times \text{Price})_t}{(1+i_d)^t} \end{aligned}$$

OR

By assuming constant price over the economic life of the project

$$= Price \sum_{t=1}^n \frac{(Production)_t}{(1+i_d)^t}$$

(6.9)

Therefore,

$$Price (UTC) = \frac{\sum_{t=1}^n \frac{(Costs)_t}{(1+i_d)^t}}{\sum_{t=1}^n \frac{(Production)_t}{(1+i_d)^t}}$$

Example 6–4

Using the cash flow in table 6–4, calculate the LRMC of the gas production in \$/MMBtu. What will be the net present value (show your calculations) and IRR of this investment if the gas is sold at the calculated LRMC? Assume the minimum acceptable IRR is 10%.

Table 6–4. Gas project’s cash flow, used for calculating LRMC and NPV

	A	B	C	D	E	F	G	H	I	J	K	L
1	Initial Gas Price	\$/MMBTU	0.85									
2	Initial Condensate Price	\$/Bbl	65.00									
3		Units	Escalation	TOTAL	-1	-2	-3	1	2	3	4	5
4	Year				2009	2010	2011	2012	2013	2014	2015	2016
5	Gross Gas Production, 1150 Btu/Scf	MMSCFD		1,369	0	0	0	750	750	750	750	750
6	Sales Gas Production, 1080 Btu/Scf	MMSCFD		1,232	0	0	0	675	675	675	675	675
7	Condensate Production	MBD		48	0	0	0	26	26	26	26	26
8	Gas Price	\$/MMBTU	0.0%		0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85
9	Condensate Price	\$/Bbl	2.5%		65.00	66.63	68.29	70.00	71.75	73.54	75.38	77.26
10	Gas Revenue	MM\$		1,131	0	0	0	226	226	226	226	226
11	Condensate Revenue	MM\$		3,525	0	0	0	671	687	705	722	740
12	Total Gross Revenue	MM\$		4,656	0	0	0	897	914	931	948	966
13	Drilling Cost (CAPEX)	MM\$		(180)	0	(90)	(90)					
14	Facilities Cost (CAPEX)	MM\$		(2,506)	(501)	(1,002)	(1,002)					
15	Variable OPEX	MM\$		(236)	0	0	0	(47)	(47)	(47)	(47)	(47)
16	Fixed OPEX	MM\$		(671)	0	0	0	(134)	(134)	(134)	(134)	(134)
17	Total Cost	MM\$		(3,593)	(501)	(1,092)	(1,092)	(182)	(182)	(182)	(182)	(182)
18	Net Cash-Flow	MM\$		1,063	(501)	(1,092)	(1,092)	715	732	749	767	785

Solution: Calculate the present value of all the costs (CAPEX and OPEX).

$$\begin{aligned}(P_v)_{Costs} &= 182 (P/A)_{i=10\%,5} (P/F)_{i=10\%,3} + 1,092(1.1)^{-3} + 1,092(1.1)^{-2} + 501(1.1)^{-1} \\ &= 182 (3.7908)(0.7513) + 1,092 \times 0.7513 + 1,092 \times 0.8264 + 501 \times 0.9091 \\ &= 518.34 + 820.42 + 902.43 + 455.46 \\ &= 2,696.65\end{aligned}$$

The present value of condensate is calculated as shown below

$$\begin{aligned}
 (P_v)_{\text{Cond. Rev.}} &= 670.67 \times 1.1^{-4} + 687.43 \times 1.1^{-5} + 704.62 \times 1.1^{-6} + 722.24 \times 1.1^{-7} \\
 &\quad + 740.29 \times 1.1^{-8} \\
 &= 458.07 \times 426.82 \times 397.76 \times 370.65 \times 345.35 \\
 &= 1,998.65
 \end{aligned}$$

Calculate the present value of the gas sales in MMBtu. The units are very critical; a common error is not accounting for the proper units.

$$\begin{aligned}
 (P_v)_{\text{Gas Sales}} &= 675(P/A)_{i=10\%,5} (P/F)_{i=10\%,3} \times 365 \times 1,080 \times 10^{-6} \\
 &= 675 \times 3.7908 \times 0.7513 \times 365 \times 1,080 \times 10^{-6} \\
 &= 757.82
 \end{aligned}$$

The sales gas is 1,080 Btu/Scf, therefore, the total sales gas production has to be converted to Btu by multiplying it by 1,080. The LRMC is now calculated as shown below.

$$\begin{aligned}
 LRMC &= \frac{(P_v)_{\text{Costs}} - (P_v)_{\text{Cond. Rev.}}}{(P_v)_{\text{Gas Sales}}} = \frac{2,696.65 - 1,998.65}{757.82} = \frac{698}{757.82} \\
 &= \$0.92/\text{MMBtu}
 \end{aligned}$$

This means that if the gas is sold for \$0.92/MMBtu constantly over the project life (five years in this example), then the IRR of the project will be 10% and the NPV = 0. One should always check to see if the LRMC/UTC is calculated correctly. This can be done by replacing the sales gas price in the model by the calculated LRMC. The result should be NPV = 0 and IRR = 10%.

In Microsoft Excel, the LRMC will be calculated as shown by the following equation.

$$LRMC = \frac{= -NPV(0.10, E17 : L17) - = NPV(0.10, E11 : L11)}{= NPV(0.1, E6 : L6) * 365 * MID(A6, 23, 4) * 10^{-6}}$$

Note the use of Excel's MID(text, start_num, num_chars) function. Such functions are convenient and make the models flexible to use. The function reads the text in cell A6 and returns four digits starting at digit 23. Thus the function reads the value of 1080 Btu from the text. Therefore, any time the Btu content of the gas changes, the LRMC can be easily recalculated by just changing the LRMC in the text in cell A6.

The above formula can be applied when the before-tax LRMC/UTC is calculated and the gas price over the project life is assumed to be constant. The formula will not work with after-tax LRMC/UTC calculations and/or when the sales gas price is not constant over time. If the gas price is not

constant (escalating at a certain fixed percent every year) and there are tax implications, then Excel's Goal Seek feature must be used.

Example 6–5

Using the cash flow in table 6–5, calculate the LRMC of the gas production if the gas price has to be escalated by 3% per year starting from the inception of the project.

Table 6–5. Gas project's cash flow, calculating LRMC using the Goal Seek feature

	A	B	C	D	E	F	G	H	I	J	K	L
1	Initial Gas Price	\$/MMBTU	0.798		Net Present Value (0) MMS							
2	Initial Condensate Price	\$/Bbl	65.00		Rate of Return, IRR 10%							
3					-1	-2	-3	1	2	3	4	5
4	Year	Units	Escalation	TOTAL	2009	2010	2011	2012	2013	2014	2015	2016
5	Gross Gas Production, 1150 Btu/Scf	MMSCFD		1,369	0	0	0	750	750	750	750	750
6	Sales Gas Production, 1080 Btu/Scf	MMSCFD		1,232	0	0	0	675	675	675	675	675
7	Condensate Production	MBD		48	0	0	0	26	26	26	26	26
8	Gas Price	\$/MMBTU	3.0%		0.80	0.82	0.85	0.87	0.90	0.92	0.95	0.98
9	Condensate Price	\$/Bbl	2.5%		65.00	66.63	68.29	70.00	71.75	73.54	75.38	77.26
10	Gas Revenue	MMS		1,231	0.00	0.00	0.00	231.89	238.84	246.01	253.39	260.99
11	Condensate Revenue	MMS		3,525	0.00	0.00	0.00	670.67	687.43	704.62	722.24	740.29
12	Total Gross Revenue	MMS		4,756	0	0	0	903	926	951	976	1,001
13	Drilling Cost (CAPEX)	MMS		(180)	0	(90)	(90)					
14	Facilities Cost (CAPEX)	MMS		(2,506)	(501)	(1,002)	(1,002)					
15	Variable OPEX	MMS		(236)	0	0	0	(47)	(47)	(47)	(47)	(47)
16	Fixed OPEX	MMS		(671)	0	0	0	(134)	(134)	(134)	(134)	(134)
17	Total Cost	MMS		(3,593)	(501)	(1,092)	(1,092)	(182)	(182)	(182)	(182)	(182)
18	Net Cash-Flow	MMS		1,163	(501)	(1,092)	(1,092)	721	745	769	794	820

Solution: Follow the following steps to calculate the base gas price at the inception of the project.

1. Enter 3.0% in *cell C8*.
2. Enter the initial gas price (as a guess) in *cell 1* and link this to *cell E8* by entering =C1 in *cell E8*.
3. Enter $c=E8*(1+\$C8)$ in *cell F8* and copy this to *cells G8 to L8*. This will escalate the price at 3% per year. The model can be run instantly at a different escalation rate if desired.
4. Enter =NPV(0.1,E18:L18) in *cell G1* to calculate the net present value of the net cash flow.
5. Keep the cursor in *cell G1*, click on **Data**, click on **What-If Analysis**, and then click on **Goal Seek**. The Goal Seek window appears, with *cell G1* already shown as the *Set cell*.

6. Enter 0 (zero) for *To value* and \$C\$1 for *By changing cell*, then click on OK. Figure 6–9 shows the Goal Seek window with the final input.
7. Excel will perform iterations and give the initial gas price as \$0.798/MMBtu in cell C1.
8. Calculate the NPV and IRR in cell G1 [by entering =NPV(0.1,E18:L18)] and cell G2 [by entering =IRR(E18:L18)], respectively.

This means that the gas can be sold for \$0.92/MMBtu (constant over the life of the project) or the initial price can start at \$0.798/MMBtu and be escalated by 3% per year over the project life. Both will give the same NPV and the same IRR.

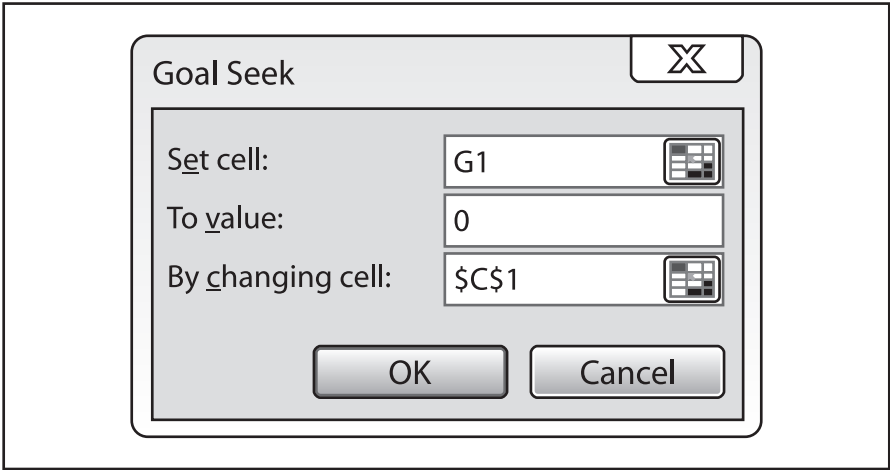


Fig. 6–9. The Goal Seek window with data entry for example 6–5

Netback value (NBV) and indexed pricing

There will be many situations when long-term contracts are required, such as in sales and purchase agreements for resources. Since the contracts, once signed, will be fixed for a longer period of time, it has to be assured that we have addressed the expected price fluctuations over the contract period. In every contract there will be at least two parties: the seller and the buyer. The seller has to know (a) what it costs to produce a resource and (b) what can the buyer afford to pay for the resource.

Therefore, you will need to know the LRMC of your resource (how much it costs you to produce the resource) and what is the netback of the buyer (how

much the buyer can pay you for the resource and still make money). Knowing these two parameters will enable you to negotiate a mutually beneficial price for the resource so that both parties are in a win-win situation.

The purpose of this section is to numerically show three different methods that can be used to calculate netback value (NBV) for pricing a feedstock or fuel. Netback pricing is a contractual arrangement in which the price of the feedstock or fuel is based upon the processed products.

Using netback, the supplier-processor starts with the sales price of the processed products, and then subtracts certain costs (such as capital, operating, processing, taxes, and transportation) to determine the value of the feedstock when production is complete. In essence, it is a backward calculation of price starting from the consumer to the wellhead. The concept is applicable to many cases, such as pricing feed to refinery, ethane to ethylene cracker, methane to a methanol plant, gas to an NGL (natural gas liquids) recovery plant, and methane to an LNG plant.

Technically, the NBV is defined as the present value of the net benefits of a project (excluding the cost of gas/feedstock used), divided by the present value of the volume of gas consumed. It represents the gas price that would cause the project to break even at the investor's expected rate of return (ROR).¹⁰

When gas is used as fuel for a power plant that can also use liquid fuel (combine cycle), then the economic NBV of natural gas is the value the consumer saves on the alternative fuel—what the consumer would have purchased had gas not been available. Therefore, NBV is the maximum price that the consumer is willing to pay for the fuel.¹¹

The netback approach used for calculating the value of natural gas is only a starting point. The magnitude of the NBV is highly dependent on the assumptions used in calculating it. Its value is also dependent on the discount rate used and the duration over which it is desired.

Three commonly used methods of calculating the NBV and the resulting gas prices are shown here. The objective is to show which of the three methods closely represents the reality, limitations of each method in practice, and how to interpret the derived NBV in terms of gas price. The method of NBV calculation is very simple and straightforward, but failure to base it on adequate methodologies could be detrimental.

The data used in demonstrating the different methods of NBV calculation are presented in table 6–6. The calculations require pricing gas feed for a hypothetical 300 metric tons per day (MT/day) methanol plant. It is assumed that about 35 MMBtu of methane-rich sales gas (feedstock) produces one metric ton of methanol. The plant's capital cost (CAPEX) is estimated to be \$80 million (80 MM\$) and its annual operating cost (OPEX) is estimated to be about 4% of the CAPEX (i.e., 3.2 MM\$). The construction period of

the plant is three years. The CAPEX will be spent 25% in year one, 40% in year two, and 35% in year three. The investor is expecting 15% ROR. The contract duration for all calculations is assumed to be five years. The owner of the methanol plant is referred to as “investor” and the sales gas provider is referred to as “gas supplier.”

Table 6–6. Data used in demonstrating the NBV calculations

Year	-1	-2	-3	1	2	3	4	5
Sales gas volume, MMBtu/day	0	0	0	10,500	10,500	10,500	10,500	10,500
Methanol price, \$/MT				320	320	320	320	320
Methanol production, MT/day				300	300	300	300	300
Operating cost (OPEX), MM\$	0	0	0	(3.20)	(3.20)	(3.20)	(3.20)	(3.20)
Capital expenditure (CAPEX), MM\$	(20.00)	(32.00)	(28.00)					

Base year and ROI approach. This method involves calculating the base NBV using an engineering/economic assessment of costs and product revenues in a base year. The base year is referred to as the first year of production. The costs will include CAPEX and OPEX (inclusive of transportation costs).

The return on investment (ROI), also referred to as accounting rate of return (ARR), is calculated by dividing a proposal’s annual net income by either the total or the average initial investment outlay. Because there are many variations of these methods, just referring to the return as ROI or ARR could be misleading and open to interpretation.

Mathematically, the NBV for this method (in the base year) is given by the following equation (all variables are in MM\$ in the base year).

$$\text{NBV} = \text{Methanol Revenue} - \text{CAPEX} - \text{OPEX} - \text{Target ROI} \quad (6.10)$$

The target ROI for this case (based on investor’s desired ROI of 15% and investment of 80 MM\$) is 12 MM\$ ($80 \times 0.15 = 12$) per year. The CAPEX is also amortized over the duration for which the NBV is desired (five years in this example). The detailed calculations are shown in table 6–7 and schematically illustrated in figure 6–10. The y-axis of figure 6–10 is represented in terms of methanol’s price. The variables of equation (6.10), from table 6–7, are converted to \$/MT by dividing each by the base year methanol production in MM-MT and plotted in figure 6–10. The NBV for this method calculates to 3.84 MM\$ or \$35.07/MT [$= 3.84 \times 106 / (300 \times 365)$] of methanol.

The netback value in terms of methanol, \$35.07/MT or 3.84 MM\$, is now converted into \$/MMBtu to calculate the maximum gas purchase price to the methanol plant as shown below.

$$NBV_{Gas} @ 15\% ROI = \frac{3.84 \times 10^6}{10,500 \times 365} = \$1.002/MMBtu$$

Table 6–7. Netback value—base year and ROI approach

Sales gas volume, MMBtu/day	10,500
Methanol price, \$/MT	320
Methanol production, MT/day	300
Annual methanol revenue, MM\$	35.04 = 320 × 300 × 365 × 10 ^{−6}
Annual operating cost (OPEX), MM\$	(3.20) = 4% of 80 MM\$ CAPEX
Amortized CAPEX (5-year amortization), MM\$	(16.00) = 80 MM\$ ÷ 5 years
Desired return on investment, MM\$	(12.00) = 15% of 80 MM\$ CAPEX
Netback value, MM\$	3.84 = 35.04 − 3.2 − 16.0 − 12.0
Netback value, \$/MT	35.07 = (3.84 × 10 ⁶)/(300 × 365)

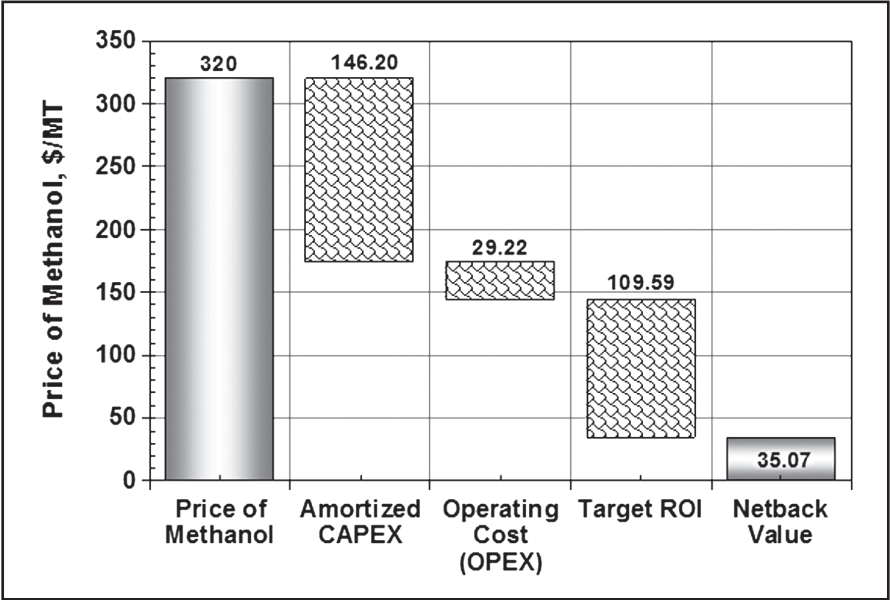


Fig. 6–10. Netback value—schematic of the base year and ROI approach

Base year and ROR approach. This method is also based on the base year concept, i.e., basing calculations on the information of the first year only. However, it uses the time value of money concept to arrive at the NBV.

It is worth mentioning here that there is a difference between ROI, ROR, and internal rate of return (IRR). The ROI or ARR are undiscounted methods and the IRR (also referred to as ROR) are typically discounted methods. As discussed in the preceding sections, the IRR/ROR is defined as the discount rate that makes the net present value (NPV), the net future value (NFV), or the net annual value (NAV) of the investment's net cash flow equal to zero. It is recommended to always use IRR/ROR as these terms are widely used and understood in the industry.

The calculations for this method are shown in table 6–8. In table 6–8, the CAPEX over the three-year construction period is rolled into its annual equivalent value (AEV or annuity) based on the investor's expected ROR of 15%.

Table 6–8. Netback value—base year and ROI approach

Sales gas volume, MMBtu/day	10,500
Methanol price, \$/MT	320
Methanol production, MT/day	300
.....	
Annual methanol revenue, MM\$	$35.04 = 320 \times 300 \times 365 \times 10^{-6}$
Annual operating cost (OPEX), MM\$	$(3.20) = 4\% \text{ of } 80 \text{ MM\$ CAPEX}$
Annual equivalent CAPEX (5-year annuity), MM\$ (27.22)	
.....	
Netback value, MM\$	(4.62) Seek the value that will make the net annual value below equal to zero
.....	
Net annual value, MM\$	0.00
Netback value, \$/MT	$42.19 = (4.62 \times 10^6)/(300 \times 365)$

CAPEX value at the beginning of year three (base year—first year of production), or the future value (FV) at the end of year three of the CAPEX in the construction period, is first calculated as shown below.

$$CAPEX_{Year1} @15\% = 20(1 + 0.15)^2 + 32(1 + 0.15)^1 + 28 = 91.25 \text{ MM\$}$$

The above CAPEX is then converted into its annual equivalent value or annuity (AEV or A_v), using the investor's expected ROR of 15%, for the period over which the netback value is desired (five years in this example).

$$AEV @ 15\% = 91.25 \left[\frac{0.15(1 + 0.15)^5}{(1 + 0.15)^5 - 1} \right] = 91.25 \times 0.2983 = 27.22 \text{ MM\$}$$

Or using the interest tables in appendix A (table A-11)

$$\begin{aligned} A_v &= \$91.25(A/P)_{i=15\%, t=5} \\ &= \$91.25(0.2983) \\ &= \$27.22 \text{ MM\$/Year} \end{aligned}$$

Using the following Excel financial function will give the same annuity.

$$A_v = pmt(0.15, 5, -91.25)$$

The NBV is now calculated as shown below. The NBV so calculated is 4.62 MM\$ or \$42.18/MT of methanol, and it is schematically shown in figure 6-11. The calculations are shown in table 6-8.

$$\begin{aligned} NBV &= \text{Annual Revenue} - \text{Annual OPEX} - \text{Annual Equivalent of CAPEX} \\ NBV &= \$35.04 - \$3.20 - \$27.22 = 4.62 \text{ MM\$} \end{aligned}$$

The NBV of 4.62 MM\$ is now converted to \$/MT of methanol by dividing it by the total annual methanol production.

$$NBV, \$/\text{MT} = \frac{4.62 \times 10^6}{300 \times 365} = \$42.19/\text{MT of Methanol}$$

Accordingly, the maximum gas purchase price is:

$$NBV_{\text{Gas}} @ 15\% \text{ ROR} = \frac{4.62 \times 10^6}{10,500 \times 365} = \$1.21/\text{MMBtu}$$

Note that the ROI approach gave a maximum gas price of \$1.002/MMBtu while the ROR approach gives \$1.21/MMBtu. The difference is basically because the ROI approach is not accounting for the time value of money in its calculations. The difference of \$0.208/MMBtu is significant when it comes to multi-billion dollars worth of gas purchases. This means that the methanol plant owner can afford to pay a maximum of \$1.21/MMBtu for methane as feedstock and make an ROR of 15% on his investment.

Long-run marginal cost approach. The long-run marginal cost (LRMC) is defined as the total cost, per unit of incremental production, required by an investor to produce and deliver a unit of product into the market. In this section it is referred to the cost per unit of production regardless of incremental or existing units. If the LRMC of a product is below the prevailing

market price for the product, then it is economical to produce this product and vice versa.

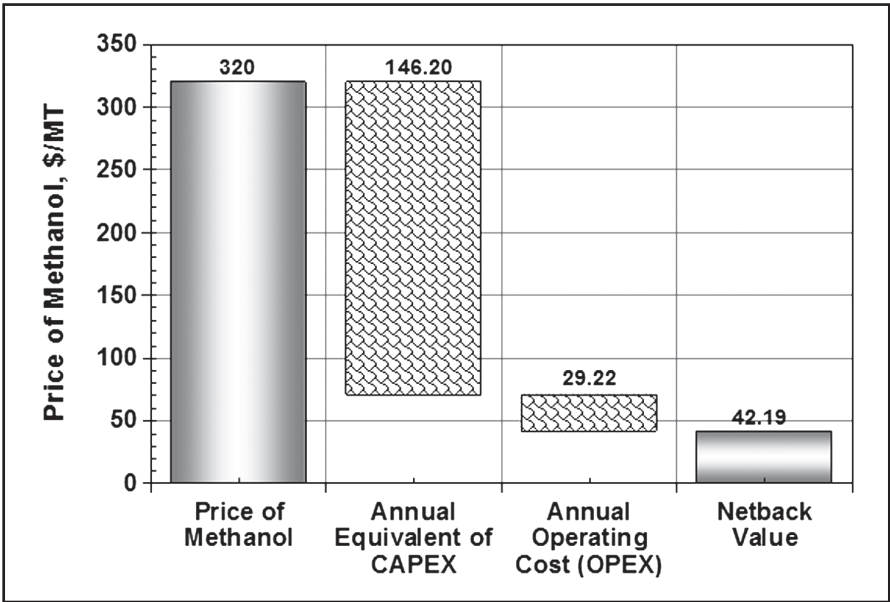


Fig. 6–11. Netback value—schematic of the base year and ROR approach

This implies that the minimum methanol price, at the start of the construction of the methanol plant, is required to provide the investor’s minimum desired ROR (i.e., discount rate at which the project’s net present value becomes zero). On the other hand, a gas supplier will not sell his gas as a fuel or feedstock if the price offered to him is below the LRMC of his gas production.

The LRMC calculations are based on a detailed cash flow of a project, a cash flow that is typically required for assessing the economic viability of a project. Therefore, it may be after-tax or before-tax; it may include adjustments to CAPEX, OPEX, price of product to reflect cost increases over time; and so on. The cash flow may also incorporate financing if the project is to be financed. If after-tax LRMC is desired, then any type of allowable depreciation scenario can be included in the cash flow.

To determine the NBV of methanol using this method, the LRMC of the methanol’s production is calculated without the cost of the sales gas (feedstock). The difference between the market price of methanol and the LRMC is then the NBV. The detailed calculations for this case are shown in table 6–9 and schematically in figure 6–12.

Table 6–9. Netback value—base year and ROI approach

	A	B	C	D	E	F	G	H	I
1		-1	-2	-3	1	2	3	4	5
2	CAPEX, MMS	(20.00)	(32.00)	(28.00)					
3	OPEX, MMS	0.00	0.00	0.00	(3.20)	(3.20)	(3.20)	(3.20)	(3.20)
4	Methanol Production, MT/day	0	0	0	300	300	300	300	300
5	Investor's Desired ROR	15%							
6	Present Value of CAPEX, MMS	(60.00)	=NPV(B5,B2:D2)						
7	Present Value of OPEX, MMS	(7.05)	=NPV(B5,B3:I3)						
8	Present Value of Methanol Production, MM-MT	0.2413	=NPV(B5,B4:I4)*365/1000000						
9									
10	Price of Methanol, \$/MT	320							
11	LRMC of Methanol Production, \$/MT	277.82	=(B6+B7)/B8						
12	Netback Value of Methanol Production, \$/MT	42.18	=B10-B11						

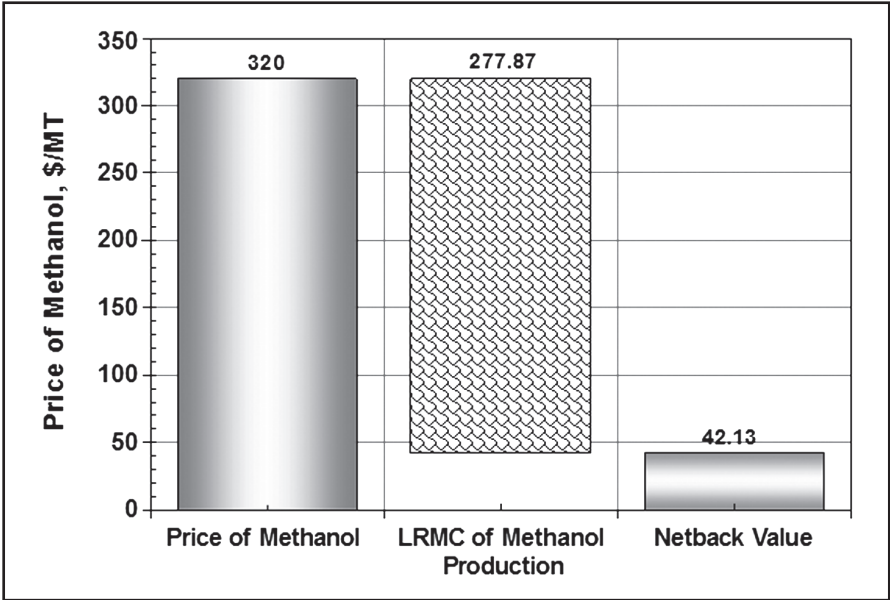


Fig. 6–12. Netback value—schematic of the LRMC approach

$$P_v \text{ of CAPEX} = 20(1 + 0.15)^{-1} + 32(1 + 0.15)^{-2} + 28(1 + 0.15)^{-3}$$
$$= 20(0.8696) + 32(0.7561) + 28(0.6575) = 59.997 \cong 60 \text{ MM\$}$$

$$P_v \text{ of OPEX} = 3.2(P/A)_{i=15\%,t=5} (P/F)_{i=15\%,t=3} = 3.2(3.3522)(0.6575)$$
$$= 7.053 \cong 7.05 \text{ MM\$}$$

$$P_v \text{ of Methanol Production} = 300(P/A)_{i=15\%,t=5} (P/F)_{i=15\%,t=3} \times 365 \times 10^{-6}$$
$$= 300(3.3522)(0.6575) \times 365 \times 10^{-6} = 0.2413 \text{ MM} - \text{MT}$$

The LRMC of the methanol's production is now calculated as shown below.

$$\begin{aligned} \text{LRMC, \$/MT} &= \frac{P_v \text{ of CAPEX} + P_v \text{ of OPEX}}{P_v \text{ of Methanol Production}} \\ &= \frac{60 + 7.05}{0.2413} = \$277.87/\text{MT of Methanol} \end{aligned}$$

The netback value is then simply the difference between the price of methanol and the LRMC of the methanol production as shown below.

$$\begin{aligned} \text{NBV} &= \text{Price of Methanol} - \text{LRMC of Methanol Production} \\ &= 320 - 277.87 = \$42.13/\text{MT of Methanol} \end{aligned}$$

The NBV calculated using the LRMC approach (table 6–9) is exactly the same as that calculated using the AEV approach (table 6–8). This is because the costs, methanol price and methanol volume, etc. are all constant in both cases. The results derived from the AEV approach will be different from the LRMC approach if any of the variables varied over time (e.g., costs escalated), unless all numbers are converted to their equivalent annual values.

The NBV can also be calculated using equation (6.11); the equation also accommodates an increase in the price of methanol over time.¹⁰

$$\text{NBV, \$/MT} = \frac{\sum_{t=1}^T \frac{P_t \times Q_t}{(1 + \text{IRR})^t} - \sum_{t=1}^T \frac{(\text{CAPEX}_t + \text{OPEX}_t)}{(1 + \text{IRR})^t}}{\sum_{t=1}^T \frac{Q_t}{(1 + \text{IRR})^t}} \quad (6.11)$$

where

$P_t Q_t$ = annual gross revenue of methanol (product) in year t , MM\$

Q_t = annual volume of methanol (product) in year t , MM-MT

IRR = discount rate (investor's desired rate of return)

T = time horizon for development of the project as well as production life

It has been noticed that equation (6.11) works well only for before-tax NBV calculations. If the project is debt-financed, the PV of $(\text{CAPEX}_t + \text{OPEX}_t)$ term is changed to PV of $(\{\text{OPEX}_t + \text{EQUITY}_t + \text{INTEREST}_t + \text{PRINCIPAL}_t\})$ divided by PV of product volumes to calculate the LRMC or NBV. For after-tax calculations it is recommended to use Excel's Goal Seek feature rather than the equation.

The detailed cash flow for the sample methanol project is shown in table 6–10. Perform the following steps to use the Goal Seek feature to determine the gas purchase price that will make the investor's IRR = 15%.

1. Connect the sales gas price (*cell E3*) in year one with the respective cells from year two to the end of project evaluation life, i.e. enter =E3 in cell F3 and so on.
2. Go to *cell B12* and enter =NPV(0.15,B10:I10) and stay in the cell.
3. From the *Data* menu choose *What-If Analysis* and click on *Goal Seek*.
4. Enter 0 (zero) for *To value*:.
5. Enter E3 for *By changing cell*.
6. Click on OK. The program will calculate the gas purchase price.

Table 6–10. Goal Seek function—detailed cash flow of methanol plant

	A	B	C	D	E	F	G	H	I
		-1	-2	-3	1	2	3	4	5
1									
2	Sales Gas Volume, MMBtu/day				10,500	10,500	10,500	10,500	10,500
3	Purchase Gas Price, \$/MMBtu				1.21	1.21	1.21	1.21	1.21
4	Methanol Production, MT/day				300	300	300	300	300
5	Sales Methanol Price, \$/MT				320	320	320	320	320
6	Gross Revenue, MMS	0.00	0.00	0.00	35.04	35.04	35.04	35.04	35.04
7	CAPEX, MMS	(20.00)	(32.00)	(28.00)					
8	OPEX, MMS	0.00	0.00	0.00	(3.20)	(3.20)	(3.20)	(3.20)	(3.20)
9	Cost of Feedstock, MMS	0.00	0.00	0.00	(4.62)	(4.62)	(4.62)	(4.62)	(4.62)
10	Net Cash-Flow (NCF), MMS	(20.00)	(32.00)	(28.00)	27.22	27.22	27.22	27.22	27.22
11									
12	Net Present Value (NPV) @ 15%, MMS	\$0.00	=NPV(0.15,B10:I10)						
13	Investor's Rate of Return (ROR)	15%	=IRR(B10:I10)						

If the feedstock yields more than one product, then the weighted average price and weighted average quantity of these products is used in the NBV and LRMC calculations, respectively.

The gas contract will include a “floor” to ensure that the price of sales gas will never drop below this pre-agreed level. The floor will be determined by the cost of producing the gas; typically the LRMC of the gas supplier. This LRMC should be inclusive of all costs and any expected increases in these costs over the contract period.

The LRMC calculation of the gas supplier requires special attention when costing associated gas or unassociated gas with significant condensate and NGL production. It has to be decided whether the gas cost should be credited with the revenue from the associated products. Alternatively, the development and production costs should be split to adequately reflect the costs associated with each product produced.

The contract duration, financing, taxation, etc. have significant effects on the NBV, hence the feedstock price. Therefore, it is necessary that assumptions used are as close to the real-life situation as possible. Otherwise, the calculations will be merely an academic exercise.

Indexed netback pricing. The above calculations assume that the price of the feedstock is constant over time. This may not be a very realistic case, especially if the price of the end product (methanol in the case discussed in this section) increases over time. Therefore, it will be fair that if the methanol owner makes more money because of higher methanol price, the gas seller should also benefit from this increase. This leads us into another contract term called *indexed pricing*. Indexed pricing is when the price of the feedstock/fuel is linked to the price of the end product.

In this way both the gas supplier and investor share the incremental benefit due to increased product price. The incremental increase may be over a pre-agreed base price of methanol or over a previous period. The sharing is negotiated at the contract stage, say 30/70 split between the sales gas supplier and the investor of the methanol plant. The following equation is used for indexed NBV calculations.

$$NBV_{New} = NBV_{Base} + f(P_{New} - P_{Base})_{End\ Product} \quad (6.12)$$

where

$$f = \text{share factor (30\% to the gas supplier in this example)}$$

$$NBV_{Base} = \text{NBV calculated at the base price } (P_{Base}) \text{ (\$320/MT in this example)}$$

If the price of methanol increases to \$380/MT, then the new NBV will be calculated using equation (6.12).

$$\begin{aligned} NBV_{New} &= 42.18 + 0.30(380 - 320)_{Methanol} \\ &= 42.18 + 18.00 = \$60.18/\text{MT} \end{aligned}$$

In terms of MM\$

$$\begin{aligned} &= 60.18 \times 300 \text{ MT/day} \times 365 \text{ days/year} \times 10^{-6} \\ &= 6.59 \text{ MM\$} \end{aligned}$$

$$\text{Net Sales Gas Price} = \frac{6.59 \times 10^6}{10,500 \times 365} = \$1.72 / \text{MMBtu}$$

As shown above, when the methanol price increases from the base price of \$320/MT to the new price of \$380/MT, the gas seller will charge \$1.72/MMBtu for the methane supplied to the methanol plant.

Alternatively, the new sales gas price can be calculated using the following version of equation (6.13).

$$\begin{aligned}
 \text{Net Sales Gas Price} &= (P_{\text{Base}})_{\text{Gas}} + f(P_{\text{New}} - P_{\text{Base}})_{\text{Methanol}} \times \frac{Q_{\text{Methanol}}}{Q_{\text{Gas}}} \\
 &= 1.21 + 0.30 (380 - 320) \times \frac{300}{10,500} \\
 &= 1.21 + 18 \times 0.029 = \$1.72 / \text{MMBtu}
 \end{aligned}$$

The f -factor could be linked to the investor's IRR. The investor initially gets a higher share of the incremental price until a certain pre-agreed ROR is reached. The percent of sharing (f -factor) keeps on changing as the IRR of the investor changes until it reaches a certain limit, after which it is kept constant. Figure 6–13 shows a hypothetical relationship of the investor's IRR with the percent of incremental gain in the product price shared with the supplier. The curve shows that the supplier gets only 30% of the incremental product price until the investor's IRR is 8%. Between the investor's IRR of 8% and 12%, the f -factor = $10 \times \text{IRR} - 0.5$. The f -factor stays constant once the investor achieves the IRR of 12%.

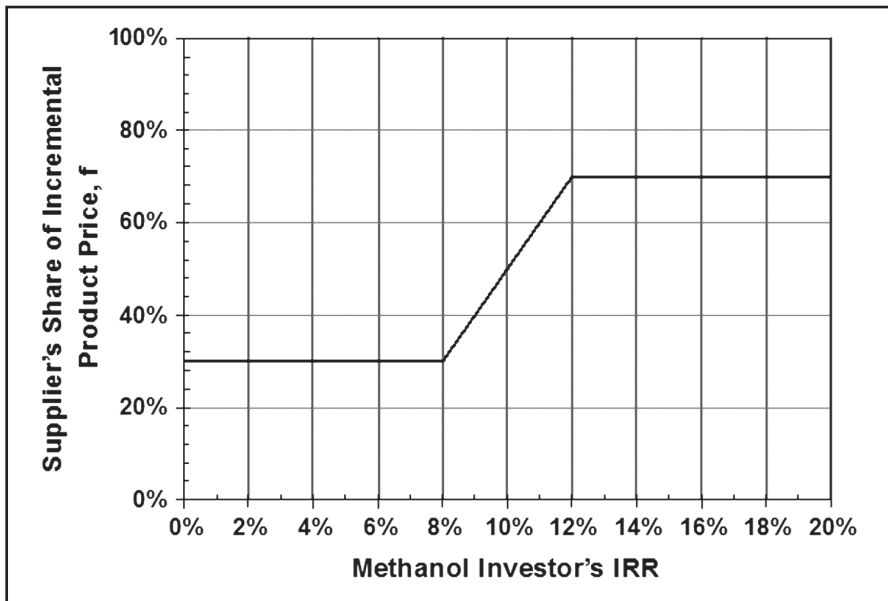


Fig. 6–13. Incremental benefit split in indexed pricing

Advantages and disadvantages of the three approaches. Some advantages and disadvantages of the three methods are presented below.

1. Base year and ROI approach

- a. The method does not account for the time value of money.
- b. It assumes constant cash receipts and disbursements over time.
- c. The ROI is not comparable with the cost of capital of the investor.
- d. The method loses accuracy if after-tax NBV is required, unless straight line depreciation and fixed tax rate is used for the duration over which the NBV is desired.
- e. The method cannot be used to calculate NBV if the investment is debt-financed.
- f. It will not be able to account for any escalations in product prices or costs. The costs should be escalated in order to reflect the increasing trend over time.
- g. The method will not reflect major equipment replacements or upgrades during the time when the calculations are performed.
- h. Detailed cash flow must be developed when incorporating the above variations. The average of each of the variables, presented above, may be used in the base year but it will further reduce accuracy. The main attraction of the base year estimates is to avoid detailed cash flow. However, once the detailed cash flow is developed, the LRMC approach can be used, which gives a more realistic NBV.

2. Base year and ROR approach

- a. The method accounts for time value of money.
- b. A detailed cash flow is required when the variables are expected to change, as discussed above. All variables have to be converted to their annual equivalent values at the investor's desired ROR. Simple averaging will affect accuracy.

3. Long-run marginal cost approach

- a. This is the most recommended approach.
- b. The method is flexible, so any scenario of cash flow can be easily accommodated.
- c. The after-tax LRMC can be accurately calculated using the Microsoft Excel Goal Seek function.

- d. Since detailed cash flows are normally available for a new project, the NBV calculation can be integrated into the same worksheet.

Growth rate of return (GRR)

The PI does not have much meaning to most managers who are accustomed to using IRR or some other rate-of-return measure. On the other hand, the IRR is not a reliable profitability indicator in all cases. This leads to introducing a new term called *growth rate of return* (GRR), also referred to as the *equity rate of return* or *modified internal rate of return* (MIRR). The GRR resolves the shortcomings of the IRR (i.e., multiple rates of return, reinvestment rate assumption, and trial-and-error calculation).

The GRR is calculated by first compounding all the positive net cash flows forward to some time horizon t years in the future. For the GRR comparison, it is recommended that the same time horizon be used for all projects under consideration. Any cash flow beyond that time is discounted back to this point. Secondly, all the negative cash flows (capital investments) are discounted to time zero to get the present value of the investment, I . The rate at which the positive cash flows are compounded and the negative investments are discounted is the reinvestment rate or opportunity rate of the organization. The GRR implies total growth from the investment at time t , counting the revenue from the investment plus the interest earned by reinvesting in future projects. We identify the total growth as B (FV of positive net cash flows).

The investment promises a return of B dollars at time t if the equivalent of one dollar is invested now. If these investment dollars were put in a bank instead, what interest rate would they have to earn to do as well? That interest rate is the project's GRR. It shows the investor is just as well off investing in this project and reinvesting the proceeds at his/her opportunity rate as he/she would be if the money was put in a bank and earned the GRR.¹²

If the available reinvestment rate is the same as the project's IRR, then the GRR is equal to IRR. If the available reinvestment rate is less than the project's IRR, then the GRR is somewhere between the project's IRR and the reinvestment rate. In this case, the GRR is a weighted average of a project's IRR and the reinvestment rate. Thus in GRR, the worth of a project plus the reinvestment opportunities are calculated (i.e., total growth) rather than the worth of a project alone. Mathematically, the GRR is represented by the following equations.

For annual compounding:

$$GRR = \left(\frac{B = \text{FV of Positive Cash Flow}}{I = \text{PV of CAPEX}} \right)^{1/t} - 1 \quad (6.13)$$

For continuous compounding:

$$GRR = \frac{1}{t} \ln \left(\frac{B = \text{FV of Positive Cash Flow}}{I = \text{PV of CAPEX}} \right) \quad (6.14)$$

In terms of the PVR and PI, for annual compounding:

$$GRR = (PVR + 1)^{1/t} (1 + i_d) - 1 \quad (6.15)$$

$$GRR = (PI)^{1/t} (1 + i_d) - 1 \quad (6.16)$$

For continuous compounding:

$$GRR = \frac{1}{t} \ln(PVR + 1) + i_d \quad (6.17)$$

$$GRR = \frac{1}{t} \ln(PI) + i_d \quad (6.18)$$

where

i_d = the reinvestment rate, fraction

The GRR yields the same accept/reject decisions as the net present value (NPV), it yields the same ranking of projects as the profitability index (PI) or present value ratio (PVR), and it is expressed as a rate of return. In addition, as previously mentioned, the calculation of GRR is straightforward (i.e., it does not require trial-and-error calculations as does the IRR) and it does not result in multiple solutions. Accept the project if its GRR is greater than the minimum acceptable rate of return and reject the project if its GRR is less than the minimum acceptable rate of return.

If NPV is negative, the $GRR < i_d$. If NPV is positive, the $GRR > i_d$. Thus, GRR is equivalent to NPV as an accept/reject criterion. If one project has a higher PI or NPV than another, it also has a higher GRR provided i_d and t are the same for both projects.

Example 6–6

The cash flows for two projects are given in table 6–11. Calculate the IRR and GRR for each project assuming the reinvestment rate is 10%. Comment on the difference between the IRR and GRR of the two projects.

Solution: The calculations in table 6–11 were performed using Excel. The detailed calculations are shown below.

The future value of positive net cash flow, each year, is calculated as

$$FV_A \text{ in year one} = \$210 (1 + 0.10)^9 = \$495.17$$

$$FV_A \text{ in year two} = \$210 (1 + 0.10)^8 = \$450.15, \text{ and so on.}$$

Similarly, for option B the future value of positive net cash flow, each year, is

$$FV_B \text{ in year one} = \$750 (1 + 0.10)^9 = \$1,768.46$$

$$FV_B \text{ in year two} = \$750 (1 + 0.10)^8 = \$1,607.69$$

Using equation (6.13), the GRR is

$$GRR_A = \left[\left(\frac{3,346.86}{1,000} \right)^{1/10} - 1 \right] \times 100 = 12.84\%$$

$$GRR_B = \left[\left(\frac{3,376.15}{1,000} \right)^{1/10} - 1 \right] \times 100 = 12.94\%$$

Using equation (6.16) and the PVR in table 6–3, the GRR is

$$GRR_A = [(1 + 0.29)^{1/10} (1 + 0.10) - 1] \times 100 = 12.84\%$$

$$GRR_B = [(1 + 0.30)^{1/10} (1 + 0.10) - 1] \times 100 = 12.92\%$$

From the above calculations, both projects have GRR greater than 10%. Therefore, both are accepted. If a choice has to be made between the two projects, project B will be selected over project A since $GRR_B > GRR_A$. According to IRR, project B seems to be much better than project A. On the other hand, GRR ranks project B only slightly higher than project A. To judge which criterion gives a more realistic comparison between the two projects, compare the total worth (assuming reinvestment opportunity of 10% per year) of each project at the end of 10 years. The positive cash flows reinvested from project A gives the future worth of \$3,346.86 as compared to \$3,376.15 for project B. These values suggest there is hardly any difference (less than 1%) between the two. Yet IRR suggests project B is almost “twice as good” as project A. Note that a project that looks far superior to another in terms of IRR may have only a slightly higher GRR, as shown in table 6–11. In such cases, as shown above, the IRR can be misleading.

The GRR calculated for a project depends on the time horizon chosen. As the time horizon is moved further beyond the end of the project, the reinvestment rate becomes more heavily weighted in calculating GRR.

However, the relative ranking of the projects do not change as the time horizon changes.

Table 6–11. Project cash flows for example 6–6

	A	B	C	D	E	F
2	Year	Reinvestment Years	PROJECT A		PROJECT B	
3			NCF, \$	FV @ 10%	NCF, \$	FV @ 10%
4	0		-1,000	0.00	-1,000	
5	1	9	210	495.17	750	1,768.46
6	2	8	210	450.15	750	1,607.69
7	3	7	210	409.23		0.00
8	4	6	210	372.03		0.00
9	5	5	210	338.21		0.00
10	6	4	210	307.46		0.00
11	7	3	210	279.51		0.00
12	8	2	210	254.10		0.00
13	9	1	210	231.00		0.00
14	10	0	210	210.00		0.00
15			1,100	3,346.86	500	3,376.15
16	IRR		16.40%		31.87%	
17	NPV @ 10%		290.36		301.65	
18	GRR @ 10%		12.84%		12.94%	
19	PI		1.29		1.30	

Adjusted net present value (ANPV)

This method can be used to evaluate capital investment projects when the investment and financing decisions cannot be separated. The method is particularly suitable for evaluating projects in a multinational setting. The traditional NPV, which is calculated by using the weighted average cost of capital of the corporation, may be inadequate in a multinational setting in view of the following.¹³

1. International variations exist in capital structure type and the widespread use of subsidized project-specific financing, loan guarantees, and insurance against political risks.
2. Political and currency risks may be unsystematic in nature for a capital budgeting proposal in a multinational firm.
3. Project systematic risk may not reflect the systematic risk of the parent company.

Thus the adjusted net present value approach has been widely suggested as being more appropriate for the capital budgeting process in a multinational corporation. In this method, each element of the investment is explicitly accounted for. It uses an all-equity discount rate that reflects local inflation and interest rates and a systematic part of the business risk of a particular project. In addition, (1) it uses the value-additivity approach and (2) adds to the present value of the operating cash flows the present value of after-tax amounts of any subsidies inherent in project-specific financing, as well as the present value of debt-related tax shields reflecting the capital structure appropriate for the particular project being evaluated.

As an example, consider the following for the adjusted net present value of a project being considered by a foreign affiliate of a multinational company.¹⁴

$$ANPV = -I_o + \sum_{t=1}^n \frac{CF_t}{(1+k_e)^t} + \sum_{t=1}^n \frac{T_t}{(1+k_d)^t} + \sum_{t=1}^n \frac{S_t}{(1+k_d)^t} + \sum_{t=1}^n \frac{O_t}{(1+k_e)^t} + \sum_{t=1}^n \frac{TV_t}{(1+k_e)^t} \quad (6.19)$$

where

- I_o = Initial investment
- k_e = all-equity cost or discount rate reflecting the riskiness and diversification benefits of the project
- k_d = cost of debt
- n = number of periods in the investment horizon
- CF_t = after-tax net cash inflows for period t
- T_t = tax shield on debt service payments for period t reflecting the capital structure of the affiliate undertaking the project
- S_t = after-tax value of special financial or other subsidies associated with the project for the period t
- O_t = estimated value in period t of any options, such as the ability to enter a new business created by the project
- TV_t = estimated terminal value in period n at the end of the investment horizon, which can also be the estimated present value of compensation received for an expected government takeover

In equation (6.19), the first term covers the initial investment at time zero. The second term reflects the present value of the net after-tax cash inflows the project is expected to generate, which are discounted at the all-equity cost of capital. The third term reflects the present value of the tax savings with the use of debt in the capital structure. By explicitly accounting for the tax shields, it is possible to account for the unique capital structure being used by the affiliate undertaking the project. The fourth term reflects the present value of

any financial or other subsidies often received by international projects from home, host, or other governments. The fifth term reflects the value of any options, such as the ability to enter a new business, whether exercised or not, generated by the project. These values may be zero or very small, at least for the first few years, and may often be difficult to estimate. The last term reflects the estimated terminal value at the end of the investment horizon. Although there are many ways to estimate the terminal value, one approach commonly used is to set it equal to the present value of all future cash flows, that is, equal to $CF/(k - g)$, where CF is the annual cash flow after the investment horizon, k is the discount rate, and g is the expected growth rate for these cash flows.¹³

Effect of Discounting Frequency

The objective of this section is to show how the discounting frequency and the funds flow assumption, as presented in chapter 2, affect the profitability indicators discussed in the preceding sections and how to account for these differences. In chapter 2, various assumptions were presented that can be used for discounting future cash flows.

As discussed in the preceding sections, profitability indicators such as NPV, IRR, PI, UTC, and GRR are often used when evaluating investment opportunities. The magnitude of the profitability indicators is highly dependent on the assumptions used to calculate them. Blindly dealing with the profitability indicators without knowing these assumptions can lead to costly and hazardous decisions. For an investor, it is essential to not only use an appropriate discount rate but also to make sure consistent funds flow and discounting frequency assumptions are used for the alternative investments under consideration.

Table 6–12 provides five cash flow and discounting method assumptions and their corresponding formulas used to calculate present value factors. These five cases (cases A and B being the most common) are frequently encountered in the oil industry. Any one of these assumptions, if consistently used, yields consistent investment choices. However, if at any time different investment opportunities are evaluated using different assumptions, the decisions will be hazardous. The differences also result in considerable confusion when two negotiating companies use different methods and assumptions.

Table 6–12. Funds flow and discounting frequency assumptions¹⁵

Case	Funds Flow Assumptions	Discounting Frequency	Present Value Discount Factor
A	Year-end	Periodic	$(1 + i_d)^{-t}$
B	Mid-year	Periodic	$(1 + i_d)^{-(t-0.5)}$
C	Continuously	Periodic	$[i/\ln(1 + i)](1 + i)^{-t}$
D	Year-end	Continuous	$e^{-r t}$
E	Continuously	Continuous	$(e^r - 1)/(re^r t)$

Note the =NPV and =IRR functions of Microsoft Excel use the case A assumptions. Two hypothetical investment profiles, as shown in table 6–13, are used to highlight the significance of the various assumptions used. All cash flows are assumed to be lump sum payments/receipts at the end of each year.

Table 6–13 shows the three profitability indicators calculated for two investments (A & B) using the five cash flow assumptions and discounting methods. Note that the NPV of investment A varies from a low of \$112,378 to a high of \$123,075, and investment B varies from a low of \$117,445 to a high of \$131,212. Similar differences exist between the other two profitability indicators. The differences in NPV are even higher at higher discount rates.

If 10% is the minimum acceptable rate of return, both investments are acceptable yielding higher IRR regardless of the assumptions used. However, if the investments are mutually exclusive, the selection will be different. For example, if investment A has used case C assumptions and investment B has used case D assumptions, investment A will be selected since the NPV of case C for investment A is higher than the NPV of case D for investment B. However, under most assumptions, investment B is the preferred investment of the two alternatives.

To avoid such misleading decisions, the profitability indicators (if not calculated using consistent assumptions) need to be adjusted to a common cash flow assumption and discounting method for each investment, and then compared. This can be achieved using the following relationships.

The adjustment of IRR is based on the following equation (this equation was labeled (2.5) and fully explained in chapter 2) because cases A through C give *effective* IRR and cases D through E give *nominal* IRR. The IRRs of cases D and E can be converted to effective IRR by using the same equation. For example,

$$i_e = e^r - 1$$
$$IRR_A = IRR_B = IRR_C = i_e$$
$$IRR_A = e^{IRR_D} - 1 = e^{0.2582} - 1$$
$$= 0.2946 \text{ or } 29.46\%$$

Table 6–13. Hypothetical investment profiles and their profitability indicators

	A	B	C	D	E	F	G	H	I	J	K	L	M	
	Net Present Value @ 10 %													
Discount Rate	Investmen		10%		Case A		Case B		Case C		Case D		Case E	
	Year	A	B			A	B	A	B	A	B	A	B	
1														
2														
3														
4														
5														
6														
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CASE	NPV @ 10%		IRR		PI	
	A	B	A	B	A	B
A	117,303	125,059	29.46%	22.15%	1.480	1.399
B	123,028	131,163	29.46%	22.15%	1.480	1.399
C	123,075	131,212	29.46%	22.15%	1.480	1.399
D	112,378	117,445	25.82%	20.01%	1.462	1.376
E	118,189	123,518	25.82%	20.01%	1.462	1.376

The net present values are adjusted using the following relationships.

$$NPV_A = \frac{NPV_B}{(1 + i_d)^{0.50}} \quad (6.20)$$

$$NPV_A = NPV_C \left[\frac{\ln(1 + i_d)}{i_d} \right] \quad (6.21)$$

$$NPV_E = NPV_D \left(\frac{e^{i_d} - 1}{i_d} \right) \quad (6.22)$$

$$NPV_B \cong NPV_C \quad (6.23)$$

Another assumption commonly used in many economic evaluations, textbooks, and technical articles is the *time zero* investment assumption, which introduces further significant variation in most profitability indicators calculated using different discounting methods.

The complication arises due to the mixed cash flow assumption in the first year, i.e., for year one, investment is treated as it occurred at the beginning of the year (instantaneous cash outflow), and the cash inflow in this year is treated as year-end cash flow. Also, all other cash flows after the first year are assumed as being received at the end of each year. So the cash flow assumption in year one is not consistent with the future years.

For example, rearranging investments A and B as shown in table 6–14, the profitability indicators calculated using these profiles changes comparing the net present values (NPV) for the investments given in table 6–14 are significantly different from those in table 6–13. The IRR and PI for cases A, B, and C are similar, but there is a significant difference in IRR for case A versus cases B and C (in table 6–13 versus 6–14). The difference is dependent on the magnitude of the time zero investment as shown in figure 6–14. The data used to generate the graph in figure 6–14 is given in table 6–15.

Then using the time zero assumption for cash flows, the IRR and NPV cannot be easily converted from one method to another as previously shown. The time zero investment has to be known in order to achieve the NPV conversions. For IRR conversion, the NPV profile at various discount rates has to be known. For example, converting NPV_B to NPV_A in table 6–14 using equation (6.20) is

$$\begin{aligned} NPV_A &= \frac{NPV_B + I_o}{(1 + i_d)^{0.50}} - I_o \\ &= \frac{148,441 + 268,600}{(1 + 0.10)^{0.50}} - 268,600 \\ &= \$129,033 \end{aligned}$$

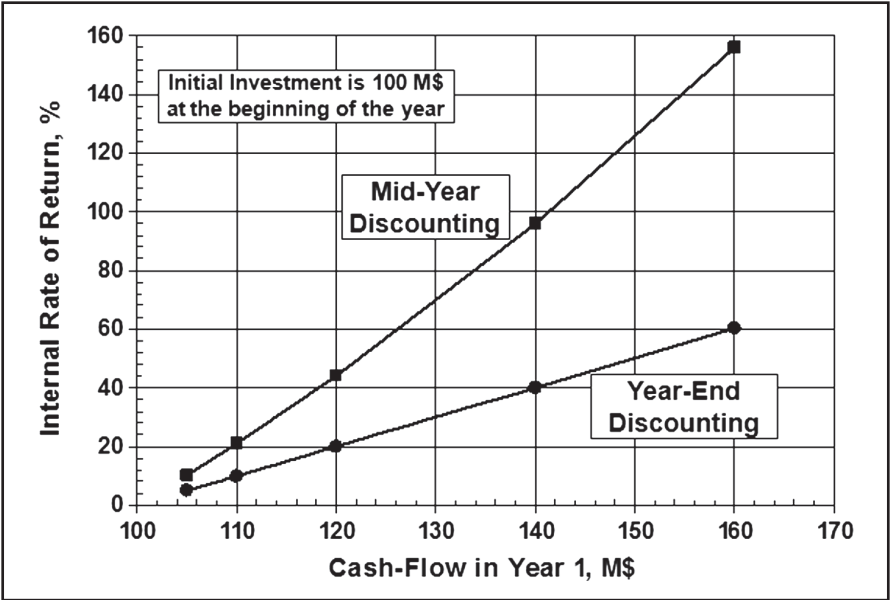


Fig. 6–14. Rates of return comparison with lump sum cash flow¹⁶

Table 6–15. Data used to generate the graph in figure 6–14

	A	B	C	D	E	F	G
1		Date	A	B	C	D	E
2		1/1/2009	(\$100,000)	(\$100,000)	(\$100,000)	(\$100,000)	(\$100,000)
3		12/31/2009	\$105,000	\$110,000	\$120,000	\$140,000	\$160,000
4	IRR, Year-End CF		5.0%	10.0%	20.1%	40.1%	60.2%
5	IRR, Mid-Year CF		10.3%	21.0%	44.0%	96.0%	156.0%
6	NPV @ 10%, Year-End CF		(\$4,521)	\$26	\$9,119	\$27,306	\$45,493
7	NPV @ 10%, Mid-Year CF		\$114	\$4,881	\$14,416	\$33,485	\$52,554

Treatment of Financing Mix

The approach used for the analysis of the interactions of corporate financing and investment decisions is reviewed in this section. In the preceding sections, it was assumed that all investment decisions are tied to the *weighted average cost of capital* (WACC). This is the classical approach

presented in most finance textbooks. However, the literature of finance does not give full analysis of the use of the weighted average cost of capital as a standard for capital budgeting. Most of us are probably not aware of the limitations/assumptions inherent in the calculations when using the WACC. The common understanding regarding the use of WACC is:

1. Use *total cash flows* and *pre-tax* cost of capital (pre-tax weighted average cost of capital).

OR

2. Use *operating cash flows* and *after-tax* cost of capital (after-tax weighted average cost of capital).

Both of these approaches should theoretically result in the same answer. Although this statement is true, it is true only under special circumstances. If the special circumstances are not met, the two approaches will result in numerically different net present values (NPVs). These may occasionally result in conflicting signals to decision makers. If the special circumstances are met, the NPVs calculated through each method will be same, but the rest of the profitability indicators and the cash available (as depicted from the cash flows) for reinvestment will still be different.

The total cash flow approach explicitly includes in its cash flow estimates the reduction in corporate income taxes resulting from the deductibility of interest payments generated by debt used in financing the investment. The discount rate (pre-tax cost of capital) used with this type of cash flow recognizes the total cash returns expected from the organization by various groups of capital providers, i.e., banks, stockholders, etc.

On the other hand, the classic NPV uses operating cash flows that do not require the estimates of tax deduction on the interest payments. The discount rate (after-tax cost of capital) used with this type of cash flow recognizes the cash flows that the firm must actually generate from its operations (after-tax) in order to service its capital claims. Thus the cash flows have to exclude nonoperational subsidies, such as the tax deduction on interest.¹⁷⁻¹⁸

In the following pages, examples are given to show if the two approaches are equivalent and under what conditions. Four standard methods that incorporate the interaction of investment and financing decisions in evaluating investment opportunities are presented. Various features of these four methods are compared to explore the implications of using them. The four methods reported are:¹⁹

- The after-tax weighted average cost of capital method
- The equity residual method

- The Arditti-Levy weighted average cost of capital method
- The adjusted net present value method

Pattern of project debt repayment schedule

Most standard approaches to the analysis of prospective capital investment projects attempt to take into account the method of financing the investment. Theoretically, the investment and financing decisions are interdependent. Firms obtain financing in relatively large blocks. Although a given financing decision may be made to support a particular investment, the decision regarding what percentage of the investment should be financed by debt and what percentage by equity varies from company to company and project to project within the company. Each company determines its own capital structure to fit its investment environment.

For the capital investment evaluation, it is important to know how the debts that relate to a particular investment will be paid back once the projects are implemented. Debt repayment schedule is the key issue in the discussion of the proceeding four methods. In practice, the standard term loans are serviced in a variety of ways, such as:

- Equal periodic payments as in mortgage or car loan payments (principal and interest)
- Equal principal payments and the resulting interest on outstanding loan balance
- Level debt schedule that assumes all issued debt remains outstanding until the end of the project's life and thus implies more average project debt than the first two
- Constant debt ratio schedule, which implies the debt weight is constant over the life of the project

In the constant debt ratio schedule, project debt at time t is the present value of all net operating cash flows in periods beyond t discounted at the weighted average cost of capital i_w , and multiplied by the debt weight W_d . Consequently, project debt remains a constant percentage of the market value of the project. The principal payment each year is the difference between the present values of two adjacent periods.

The constant debt schedule implies more debt is initially issued as compared to the above-mentioned debt schedules. On the other hand, the constant ratio schedule has project debt declining as cash flows from the project are realized (i.e., the debt value decreases in constant proportion to

total value across time). As debt value decreases, so do the tax savings due to interest deduction. This means the i_w discount rate and the classic NPV calculation adjust for these decreased tax savings due to interest across time, because i_w is calculated in constant value ratio of debt to total capital.

The following example illustrates how the various debt schedules are calculated.

Example 6–7

A capital investment project involves an initial investment, at time zero, of \$1,000 financed \$400 by equity and \$600 by debt, i.e., the debt to investment ratio is 0.6. Interest on debt is 12% with a repayment schedule of five years. The corporate tax rate is 46%. The project generates after-tax operating cash flows $[(R_t - C_t - D_t)(1 - T_c) + D_t]$ of \$300 per year for the next five years. Calculate (a) equal periodic debt payment schedule, (b) equal principal payment debt schedule, (c) level debt schedule, and (d) constant ratio debt schedule.

Solution: The calculations are performed using an Excel spreadsheet. The respective spreadsheets are given on the CD-ROM accompanying volume 2 of this series.

(a) The equal periodic debt schedule is presented in table 6–16.

(b) The equal principal payment debt schedule is presented in table 6–17.

Out of the \$1,057.41, \$600 is the principal payment, and \$457.41 is the interest.

In tables 6–16 and 6–17, interest is the previous year's outstanding loan balance multiplied by the interest rate per period. Note the present value of the year-end payments at an interest rate of 12% is equivalent to the total debt of \$600. In these schedules, initial debt issued is equal to the debt ratio W_d times the investment outlay (i.e., W_d times the total investment) on the grounds that this is likely what the analysts would do in practice.

(c) The level debt schedule is calculated as follows:

$$\text{Level Debt Schedule} = \$600 (1 + 0.12)^5 = \$1,057.41$$

Table 6–16. Calculation of equal periodic debt payment schedule

	A	B	C	D	E
1	Loan Amount (Present Value)				\$600
2	Loan Repayment Schedule, Years				5
3	Interest Rate Per Period				12%
4					
5	Period	Interest	Principal	Year-End Payment	Outstanding Balance
6	0				\$600.00
7	1	(\$72.00)	(\$94.45)	(\$166.45)	\$505.55
8	2	(\$60.67)	(\$105.78)	(\$166.45)	\$399.77
9	3	(\$47.97)	(\$118.47)	(\$166.45)	\$281.30
10	4	(\$33.76)	(\$132.69)	(\$166.45)	\$148.61
11	5	(\$17.83)	(\$148.61)	(\$166.45)	\$0.00

Table 6–17. Calculation of equal principal payment debt schedule

	A	B	C	D	E
2	Loan Amount (Present Value)				\$600
3	Loan Repayment Schedule, Years				5
4	Interest Rate Per Period				12%
5					
6	Period	Interest	Principal	Year-End Payment	Outstanding Balance
7	0				\$600.00
8	1	(\$72.00)	(\$120.00)	(\$192.00)	\$480.00
9	2	(\$57.60)	(\$120.00)	(\$177.60)	\$360.00
10	3	(\$43.20)	(\$120.00)	(\$163.20)	\$240.00
11	4	(\$28.80)	(\$120.00)	(\$148.80)	\$120.00
12	5	(\$14.40)	(\$120.00)	(\$134.40)	\$0.00

The calculation of equal periodic payment debt schedule is presented in chapter 2 in the section titled “Loan Amortization Schedule.”

- (d) The constant ratio debt schedule calculations are shown below and in table 6–18.

Table 6–18. Calculation of constant ratio debt payment schedule

	A	B	C	D	E	F	G	H	I	J
1	Loan Repayment Schedule, Years			5	=E5*E2*(1-E3)+(1-E5)*E4					
2	Interest Rate Per Period (Cost of Debt)			12%						
3	Tax Rate			46%						
4	Return on Equity			7%						
5	Weight of Debt (Percent of Total Capex)			60%						
6	Weighted Average Cost of Capital (WACC)			6.69%						
7										
8					0	1	2	3	4	5
9	Net Operating Cash-Flow (NCF)					300.00	300.00	300.00	300.00	300.00
10	Present Value of NCF @ WACC				1,240.40	1,023.36	791.80	544.76	281.19	
11										
12	Outstanding Loan				744.24	614.02	475.08	326.86	168.72	
13										
14	Interest on Outstanding Loan					89.31	73.68	57.01	39.22	20.25
15	Principal Payment					130.23	138.93	148.23	158.14	168.72
16	Total Payment					219.53	212.62	205.24	197.36	188.96

1. The weighted average cost of capital (WACC) is calculated as

$$\begin{aligned}
 i_{AT-WACC} &= W_d i_d (1 - T_c) + (1 - W_d) i_e \\
 &= 0.60(0.12)(1 - 0.46) + (1 - 0.60)(0.070) \\
 &= 0.0669 \text{ or } 6.688\%
 \end{aligned}$$

2. Using the after-tax weighted average cost of capital (discussed in the following section), calculate the present value of the periodic net cash flows.

$$\begin{aligned}
 PV_0 &= 300(1 + 0.06688)^{-1} + 200(1 + 0.06688)^{-2} + 300(1 + 0.06688)^{-3} \\
 &\quad + 300(1 + 0.06688)^{-4} + 300(1 + 0.06688)^{-5} \\
 &= 281.19 + 263.57 + 247.04 + 231.56 + 217.04 \\
 &= 1,240.40
 \end{aligned}$$

$$\begin{aligned}
 PV_1 &= 300(1 + 0.06688)^{-1} + 300(1 + 0.06688)^{-2} + 300(1 + 0.06688)^{-3} \\
 &\quad + 300(1 + 0.06688)^{-4} \\
 &= 281.19 + 263.57 + 247.04 + 231.56 \\
 &= 1,023.36, \text{ and so on}
 \end{aligned}$$

3. The debt each year is then calculated by multiplying the debt weight by each year's present value of the cash flows.

$$\text{Debt}_0 = 1,240.40 \times 0.6 = \$744.24$$

$$\text{Debt}_1 = 1,023.36 \times 0.6 = \$614.02, \text{ and so on}$$

4. Interest each year is the previous year's outstanding debt times the interest rate (cost of debt) of debt, i.e., interest at end of year one is $\$744.24 \times 0.12 = \89.31 , and so on.
5. The principal payment each year is the difference between the current year's outstanding debt and the previous year's outstanding debt, i.e., principal payment in year one is $\$744.24 - \$614.02 = \$130.22$, and so on.

It would be possible for an analyst, in considering investment alternatives, to assume debt schedules consistent with the theoretical requirements of the models. Practitioners are unlikely to do this because most firms do not manage their financial structures in this way. The constant ratio debt schedule is not in common use. In practice, due to the way corporate funds are raised and the way equity values rise and fall (for a variety of reasons), maintaining constant debt ratios for individual projects would be next to impossible. Once the pattern of project debt is determined, it has to be evaluated in line with the other costs and benefits of the project. The basic question is, should net loan proceeds, principal payments, and interest rate or tax savings due to the interest be included in the cash flows? In practice it is typically assumed the repayment of interest and principal exactly offsets the market value of the net proceeds.

After-tax weighted average cost of capital method

The after-tax weighted average cost of capital (WACC_{AT}) method is discussed in most major textbooks of corporate finance. It is well known and widely used in the industry. However, the method does not give a true picture of the available cash flows generated by an investment. This results in calculation of varying profitability indicators, depending on which method is used.

In this method, the NPV is calculated by discounting the after-tax operating cash flows of an investment alternative at a rate i_w . The i_w reflects both the equity and debt required rates of return. The cash flow combination is represented by the following equation.

$$NPV_{WACC-AT} = \sum_{t=1}^n \frac{(R_t - C_t - D_t)(1 - T_c) + D_t}{(1 + i_w)^t} - I_o \quad (6.24)$$

$$WACC_{AT} = i_w = W_d i_d (1 - T_c) + (1 - W_d) i_e \quad (6.25)$$

where

R_t = pre-tax operating cash revenues generated by the investment during period t

C_t = pre-tax operating cash expenses due to the investment during time period t

D_t = additional depreciation due to the investment during period t

T_c = corporate tax rate, fraction

I_o = initial after-tax cash outlay to undertake the investment

i_w = weighted average cost of capital, fraction

i_e = required rate of return on equity over the investment life, fraction

i_d = cost of debt, fraction

W_d = weight of debt to total capital investment I_o , fraction

The key point to note in equation (6.24) is that the *financial flows* (*interest and principal payments on debt*) do not appear in the cash flows. The effects of financing flows, on the other hand, rather than being treated explicitly in the cash flows, are handled implicitly via the discount rate i_w . Thus i_w takes into account (1) the debt/equity mix used to finance the project, and (2) the tax deductibility of interest. In theory, this weight is to be expressed in market value terms with the assumption that the market value debt-equity mix of the firm remains constant over time. In practice, it is assumed i_w is constant over the life of the investment.

Some authors overextend the assumptions of discounting and claim that depreciation is also accounted for in the discounting process. Therefore, they say, it should not be included in the cash flows: "Fundamentally, the analysis of present value must exclude depreciation as a cost because the present value concept *automatically* provides for the return of the initial investment over its useful life."²⁰ This statement is not correct; depreciation is not added because it is a non-cash item. However, since depreciation is deducted for tax purposes, the tax outflow savings achieved due to depreciation shall be accounted for in the cash flows. This is shown in equation (6.24) and in chapter 4. The following example clarifies the use of equations (6.24) and (6.25).

Example 6–8

A capital investment project involves an initial investment of \$1,000, financed 75% by equity and 25% by debt. The corporate tax rate is 35%, interest on the debt is 6.15%, and the minimum acceptable rate of return on equity is 10% (assuming the equity can be invested elsewhere at a rate of return of 10%). The project will generate after-tax operating cash flows as shown in table 6–19. Calculate the NPV, IRR, and PI for the investment.

Solution: The cash flow for example 6–8 is shown in table 6–19.

Table 6–19. Cash flow for the data in example 6–8

	A	B	C	D	E	F	G	H
1			0	1	2	3	4	5
2		Gross Revenue		548	471	394	317	286
3		Operating Cost		(40)	(40)	(40)	(40)	(40)
4	Less	Interest Payment						
5		Depreciation		(200)	(200)	(200)	(200)	(200)
6		Taxable Income	0	308	231	154	77	46
7		Tax @ 35%	0	108	81	54	27	16
8		Income After-Tax	0	200	150	100	50	30
9	Less	Capital Expenditure	(1,000)					
10	Plus	Loan Proceeds						
11	Plus	Interest Tax Savings, $T_c \times I$						
12	Less	Principal Payment						
13	Plus	Depreciation	0	200	200	200	200	200
14		Capital Cash-Flow (ECF)	(1,000)	400	350	300	250	230

Using equation (6.25), the after-tax weighted average cost of capital is

$$\begin{aligned}
 WACC_{AT} &= i_w = 0.25 \times 0.0615(1 - 0.35) + (1 - 0.25) \times 0.10 \\
 &= 0.25 \times 0.0615 + (1 - 0.25) \times 0.10 \\
 &= 0.0850 \text{ or } 8.5\%
 \end{aligned}$$

The NPV is calculated as

$$\begin{aligned}
 NPV_{WACC-AT} &= -\$1,000 + \frac{\$400}{(1 + 0.085)^1} + \frac{\$350}{(1 + 0.085)^2} + \frac{\$300}{(1 + 0.085)^3} \\
 &\quad + \frac{\$250}{(1 + 0.085)^4} + \frac{\$230}{(1 + 0.085)^5} \\
 &= -\$1,000 + \$368.66 + \$297.31 + \$234.87 + \$180.39 + \$152.96 \\
 &= \$234.19 \approx \$234
 \end{aligned}$$

Similarly, the IRR and PI are

$$\begin{aligned}\text{IRR} &= 18.13\% \text{ (from Excel)} \\ \text{PI} &= 1 + 234.19/1,000 = 1.23\end{aligned}$$

Note the cash flow in table 6–19 does not include any loan proceeds from the bank, tax savings due to interest (interest tax shield), and principal payments on debt in the cash flow. The cash flow represents total asset cash flow. The cash flow gives an impression that it is done 100% with equity, i.e. no financing. These are implicitly incorporated in the calculation of the i_w . This means that the cash flow is subject to debt and tax payment liabilities. The other three methods, presented in the following pages, do account for the tax savings on interest payments and/or the principal payments in the cash flows. The objective here is to formulate all the methods in such a way that they all calculate the same NPV.

Equity residual method— NPV_{ER}

The objective of pursuing any investment project is to maximize the shareholder's interest in the firm. According to the equity residual method, the value of these shareholder interests, NPV_{ER} , can be expressed by the following equation.

$$\text{NPV}_{\text{ER}} = \sum_{t=1}^n \frac{[(R_t - C_t - D_t - i_d B_t)(1 - T_c) + D_t] - (B_t - B_{t+1})}{(1 + i_e)^t} - (I_o - NP) \quad (6.26)$$

where

B_t = the project debt outstanding from time $t-1$ to time t
 NP = the net proceeds (time zero) of issuing B_t of project debt,
 which is outstanding from time 0 to time t

The first term in equation (6.26) is the present value of the after-tax cash income less repayment from equity sources. Therefore, equation (6.26) is the present value of the residual cash flows to equity owners matched against their initial equity outlay. A positive NPV_{ER} means a desirable investment from the viewpoint of the equity holders. In applications where the project-specific financing scheme is known, the shareholder interests in the project could be calculated directly by equation (6.26). The cash flows generated and calculations under the equity residual method are shown in table 6–20. The constant ratio debt schedule of table 6–20 is calculated in the following table, using the weighted average cost of capital of 8.50% and the cash flow of table 6–19.

Table 6–20. Cash flow for the equity residual method (data of example 6–8)

	A	B	C	D	E	F	G	H
1			0	1	2	3	4	5
2		Gross Revenue		548	471	394	317	286
3		Operating Cost		(40)	(40)	(40)	(40)	(40)
4	Less	Interest Payment		(19)	(14)	(10)	(7)	(3)
5		Depreciation		(200)	(200)	(200)	(200)	(200)
6		Taxable Income	0	289	217	144	70	43
7		Tax @ 35%	0	101	76	50	25	15
8		Income After-Tax	0	188	141	94	46	28
9	Less	Capital Expenditure	(1,000)					
10	Plus	Loan Proceeds	309					
11	Plus	Interest Tax Savings, $T_c \times I$	0					
12	Less	Principal Payment	0	(74)	(68)	(61)	(53)	(53)
13	Plus	Depreciation	0	200	200	200	200	200
14		Equity Cash-Flow (ECF)	(691)	314	273	233	193	175

Year	0	1	2	3	4	5
NPV @ 8.5%, \$	1,234.56	939.30	668.99	425.75	211.89	0
Debt Weight	25%	25%	25%	25%	25%	
Outstanding Debt, \$	308.64	234.83	167.25	106.44	52.97	0
Principal Payments, \$		73.81	67.58	60.81	53.47	52.97
Interest Payment @ 6.5%		18.98	14.44	10.29	6.55	3.26

The NPV is calculated by using the rate of equity of 10% (as specified in Example 6–8) as

$$\begin{aligned}
 NPV_{ER} &= -\$691 + \frac{\$314}{(1+0.10)^1} + \frac{\$273.19}{(1+0.10)^2} + \frac{\$232.61}{(1+0.10)^3} + \frac{\$192.33}{(1+0.10)^4} + \frac{\$174.81}{(1+0.10)^5} \\
 &= -\$691.36 + \$285.45 + \$225.78 + \$174.76 + \$131.36 + \$108.56 \\
 &= \$234.55 \approx \$234
 \end{aligned}$$

Similarly, the IRR and PI are

$$\begin{aligned}
 IRR &= 24.20\% \\
 PI &= 1 + 234.55/691.36 = 1.34
 \end{aligned}$$

The NPVs for the weighted average cost of capital method and the residual equity method are the same. However, the IRR and PIs are different. The IRR in the case of the equity residual cash flow is the return on equity while in the

weighted average cash flow the return is on total asset. This differentiation is very critical to keep in mind.

Arditti-Levy method— NPV_{AL}

Because interest payments are tax deductible, the cash flow estimates used in the weighted average cost of capital (WACC) model overstates (by the amount $T_c i_d B_t$) the corporate taxes actually paid by the corporation. The model proposed by Arditti and Levy accounts for the cash flows actually received by both the debt and equity claimants. Their model is represented by the following equation.

$$NPV_{AL} = \sum_{t=1}^n \frac{[(R_t - C_t - D_t - i_d B_t)(1 - T_c) + D_t] + i_d B_t}{(1 + i_{AL})^t} - I_o \quad (6.27)$$

where

$$i_{AL} = W_d i_d + (1 - W_d) i_e \quad (6.28)$$

The Arditti-Levy approach presented in equations (6.27) and (6.28) differ from the $WACC_{AT}$ approach represented by equations (6.24) and (6.25) in two fundamental aspects. The cash flows in the numerator of equation (6.27) are the sum of flows to both the equity and debt owners. Comparison of equations (6.27) and (6.24) show the cash flows in the numerator of equation (6.27) exceed those in equation (6.24) by the amount of the tax savings due to interest on debt ($i_d B_t T_c$). Thus, equation (6.27) shows lower taxes paid to the government than those paid by the $WACC_{AT}$ method. The comparison of equations (6.25) and (6.28) shows the i_{EL} has no adjustment for the tax savings due to interest payments; instead, this arrangement has been accounted for in the cash flows. The cash flows generated and calculations under the equity residual method are shown in table 6–21 (the constant ratio debt schedule of table 6–20 is assumed).

The i_{AL} is actually referred to as the before-tax weighted average cost of capital ($WACC_{BT}$). This is because the term $(1 - T_c)$ in the $WACC_{AT}$ equation is removed, and the resulting interest tax savings are included in the cash flow.

The cost of capital for the Arditti-Levy method is calculated using equation (6.28) as

$$\begin{aligned} i_{AL} &= 0.25 \times 0.0615 + (1 - 0.25) \times 0.10 \\ &= (0.25 \times 0.0615 + 0.75 \times 0.10) \times 100 = 9.04\% \end{aligned}$$

The NPV is calculated using the Arditti-Levy before-tax weighted average cost of capital as

$$\begin{aligned} NPV_{AL} &= -\$1,000 + \frac{\$406.84}{(1 + 0.0904)^1} + \frac{\$355.20}{(1 + 0.0904)^2} + \frac{\$303.70}{(1 + 0.0904)^3} \\ &\quad + \frac{\$252.34}{(1 + 0.0904)^4} + \frac{\$231.04}{(1 + 0.0904)^5} \\ &= -\$1,000 + \$373.11 + \$298.74 + \$234.25 + \$178.50 + \$149.88 \\ &= \$234.48 \cong \$234 \end{aligned}$$

Similarly, the IRR and PI are

$$\begin{aligned} \text{IRR} &= 18.78\% \\ \text{PI} &= 1 + 34.48/1,000 = 1.23 \end{aligned}$$

Table 6–21. Cash flow for the Arditti-Levy method (data of example 6–8)

	A	B	C	D	E	F	G	H
1			0	1	2	3	4	5
2		Gross Revenue		548	471	394	317	286
3		Operating Cost		(40)	(40)	(40)	(40)	(40)
4	Less	Interest Payment						
5		Depreciation		(200)	(200)	(200)	(200)	(200)
6		Taxable Income	0	308	231	154	77	46
7		Tax @ 35%	0	108	81	54	27	16
8		Income After-Tax	0	200	150	100	50	30
9	Less	Capital Expenditure	(1,000)					
10	Plus	Loan Proceeds						
11	Plus	Interest Tax Savings, $T_c \times I$		7	5	4	2	1
12	Less	Principal Payment						
13	Plus	Depreciation	0	200	200	200	200	200
14		Total Cash-Flow (ECF)	(1,000)	407	355	304	253	231

Adjusted net present value—NPV_A

Unlike the other three methods, this method treats operating cash flows and financing in two separate steps as shown in the following equation. The first term in the equation (6.29) discounts the after-tax operating (without any effects due to interest payments on debt) cash flows at the discount rate, ρ , that would be appropriate if the project was entirely financed by equity. The last term of equation (6.29) accounts for the value of the projects’ financing, which is the discounted value of the tax savings due to the interest payments on the debt.

$$NPV_A = \sum_{t=1}^n \frac{[(R_t - C_t - D_t)(1 - T_c) + D_t]}{(1 + \rho)^t} - I_o + \sum_{t=1}^n \frac{T_c i_d B_t}{(1 + i_d)^t} \quad (6.29)$$

where

$$\rho = \frac{i_w}{(1 - T_c W_d)} \quad (6.30)$$

The cash flow streams generated in table 6–21 for the Arditti-Levy method are used to calculate the NPV_A as follows.

Using equation (6.30), the rate used to discount the after-tax operating cash flow is

$$\rho = \frac{0.085}{(1 - 0.35 \times 0.25)} = 0.0932 \text{ or } 9.32\%$$

The NPV_A is calculated as given below

$$\begin{aligned} NPV_A &= -\$1,000 + \frac{\$400.20}{(1 + 0.0932)^1} + \frac{\$350.15}{(1 + 0.0932)^2} + \frac{\$300.10}{(1 + 0.0932)^3} + \frac{\$250.05}{(1 + 0.0932)^4} \\ &\quad + \frac{\$229.90}{(1 + 0.0932)^5} + \frac{\$6.64}{(1 + 0.0615)^1} + \frac{\$5.05}{(1 + 0.0615)^2} + \frac{3.60}{(1 + 0.0615)^3} \\ &\quad + \frac{\$2.29}{(1 + 0.0615)^4} + \frac{1.14}{(1 + 0.0615)^5} \\ &= -\$1,000 + \$366.08 + \$292.99 + \$229.70 + \$175.08 + \$147.25 \\ &\quad + \$6.26 + \$4.48 + \$3.01 + \$1.80 + \$0.85 \\ &= \$227.50 \end{aligned}$$

Similarly, the IRR and PI are

$$\begin{aligned} \text{IRR} &= 19.19\% \\ \text{PI} &= 1 + 227.50/1,000 = 1.23 \end{aligned}$$

Comparison of the four methods

The application of the four methods to a simple numerical example were illustrated in the preceding pages. In these analyses, the *constant ratio debt schedule* and a tax rate of 35% were used. Further analyses were done based on equal periodic payments on \$250 (25% of capital expenditure) debt and a tax rate of 35%. The results of these two scenarios are presented in table 6–22 for discussion.

When the four methods are applied to projects with cash flow patterns and debt repayment schedules typically encountered in practice, they yield different results. The four methods yield similar results only under special conditions. The conclusions are:

1. Under the constant ratio debt schedule and $T_c > 0\%$ assumption, $NPV_{WACC-AT}$, NPV_{ER} , and NPV_{AL} are equal. The IRR of the equity residual method is much higher than the other methods. The reason for the higher IRR is that it is the return on equity and the others are returns on total asset. The same difference is shown in the PI as well. The PI in the equity residual method is PI on equity and not the total asset.
2. Under the conventional debt schedules (constant periodic payments, constant principal payments, or level debt) and $T_c > 0\%$, the profitability indicators in most of the methods are different. Thus, if the project debt ratio is not kept constant, $NPV_{WACC-AT}$, NPV_{ER} , and NPV_{AL} are not equivalent.
3. If the tax rate is zero or no consideration is given to the tax savings due to interest payments on debt, any debt payment schedule yields the same profitability indicators in most of the methods.
4. For conventional projects and higher debt weighting, the differences in the NPVs calculated by the four methods are even larger because:
 - a. There is a 3-year construction period followed by long revenue generation period.
 - b. The debt cash flow is over only a limited number of years as compared to the total project life.
 - c. The four methods assume constant debt/equity ratio. The debt/equity ratio keeps on changing for the real project as the debt is being paid.
 - d. The four methods assume that there will be tax payment every year. This may not be the case in real projects because of loss carried forward or tax holidays, etc.

The analyst has to be careful when dealing with the profitability indicators generated by the different methods. The reason why these methods yield different profitability indicators is due to the implicit assumption each makes as to the treatment of tax savings due to interest payments.

There is a reasonable intuitive appeal to the use of the equity residual method because it accounts for the total dollars generated to service capital

at each point in time. From the viewpoint of the net cash flow, the equity residual method gives a more realistic picture of the cash flows available for distribution as dividends or reinvestment. The cash flows in the other methods are overstated (i.e., they are subject to the payment of the principal and interest on the debt). As long as the equity residual method is consistently used from project to project, the selection of any practical (close to reality) debt payment schedule may be adopted. NOTE: This author recommends using the equity residual method over other methods, the only enhancement it requires is to vary the discount rate as the debt-equity ratio of the project changes. It can be easily handled in Excel. The adjusted NPV method is also strongly recommended but only for NPV calculation, the IRR should be reported for both the equity case and the total asset case.

Table 6–22. Comparison of profitability indicators from the four methods

	A	B	C	D	E
		WACC _{AT}	Equity Residual	Arditti Levy	Ajusted NPV
2					
3	Constant Debt Ratio, Tc = 35%				
4	Discount Rate	8.50%	10.00%	9.04%	9.31%
5	Net Present Value	235	235	235	227
6	Internal Rate of Return (IRR)	18.13%	24.20%	18.78%	18.78%
7	Profitability Index (PI)	1.23	1.34	1.23	1.23
8	Equal Periodic Payment, Tc = 35%				
9	Discount Rate	8.50%	10.00%	9.04%	9.31%
10	Net Present Value	235	230	233	225
11	Internal Rate of Return (IRR)	18.13%	23.18%	18.70%	18.70%
12	Profitability Index (PI)	1.23	1.31	1.23	1.22

Questions and Problems

- 6.1 Define the profitability measures payback period, NPV, IRR, PI, PVR, TC, and GRR. What is the logic behind each method?
- 6.2 Which payback period (discounted or undiscounted) is appropriate to be used in investment analysis and why?
- 6.3 What are the three most important indications provided by the payback period?

- 6.4 Briefly list the advantages and disadvantages of the following:
- payback period
 - net present value
 - internal rate of return
 - profitability index
 - growth rate of return
- 6.5 What is the technical cost of development? How would you interpret the technical cost of \$12/Stb calculated at 10% discount factor?
- 6.6 For the calculation of discounted technical cost, the production stream and the total costs have to be discounted at the desired rate. An engineer once said, "I don't believe in discounting the production." Comment on this statement.
- 6.7 List three different ways of converting gas production into its barrels of oil equivalent (BOE). Which method is the most appropriate for calculating technical cost of development for an oil and gas project?
- 6.8 List the accept/reject criteria for the following:
- payback period
 - net present value
 - internal rate of return
 - profitability index
 - present value ratio
 - technical cost of development
 - growth rate of return.
- 6.9 For independent projects, is it true that if $PI > 1$, then NPV will also be greater than zero, and $IRR > i_d$?
- 6.10 The IRR of a project with a constant cash flow S for n years is positive. Hence the NPV of the project at a zero discount rate must be positive. Do you agree? Prove your answer.
- 6.11 In what sense is a reinvestment rate assumption embodied in NPV and IRR calculations? What is the assumed reinvestment rate of each method? Which assumption is correct and why?

- 6.12 What is the significance of the GRR and how does it differ from the IRR?
- 6.13 For a project with five years of useful economic life, the PI is 1.5, and the available reinvestment rate is 7%. Calculate its growth rate of return.
- 6.14 The NPV of a stream of net cash flow (NCF), using year-end discounting and a discount rate of 8%, is 250 MM\$. What will be the NPV of the same NCF using a mid-year discounting assumption?
- a. 250 MM\$
 - b. 260 MM\$
 - c. 270 MM\$
 - d. 280 MM\$
- 6.15 The net present value (NPV), at 8%, of the NCF shown above is:
- a. 1,470 MM\$
 - b. 1,461 MM\$
 - c. 1,361 MM\$
 - d. 1,261 MM\$

	A	B	C	D	E
1	Discount Rate	8%			
2	1	2	3	4	5
3	-1,800	-4,200	3,000	3,000	3,000

Problem 6.15 diagram

- 6.16 The NPV of the cash-flow in question 6.15 is (a) 438.32 MM\$ at 15% and (b) -28.16 MM\$ at 20%. What is the rate of return of this cash flow?
- a. 21.65%
 - b. 19.70%
 - c. 18.65%
 - d. 22.65%

- 6.17 What is the profitability index (PI) of the cash flow in question 6.15?
- a. 1.56
 - b. 1.46
 - c. 1.36
 - d. 1.26
- 6.18 The discounted payback period for the cash flow in question 6.15 is:
- a. ~One year
 - b. ~Two years
 - c. ~Three years
 - d. ~Four years
- 6.19 Which of the following Excel financial functions is used to calculate the correct NPV in question 6.15?
- a. =NPV(B1,B3:E3)+A3
 - b. =NPV(B1,A3:E3)
 - c. =XNPV(B1,A3:E3,A2:E2)
 - d. =NPV(B1,C3:E3)+A3+B3
- 6.20 Which of the following two Excel financial functions will give the correct NPV for the data in the table below?
- I. =NPV(B1,A3:E3)
 - II. =NPV(B1,A6:E6)
- a. I and II
 - b. I only
 - c. II only
 - d. None of the above

	A	B	C	D	E	F
1	Discount Rate	8%				
2	1	2	3	4	5	(I)
3	0	0	3,000	3,000	3,000	
4						
5	1	2	3	4	5	(II)
6			3,000	3,000	3,000	

Problem 6.20 diagram

6.21 In question 6.20, what will be the NPV in each case (calculated by using =NPV(B1, A3:E3) and =NPV(B1,A6:E6)?

- 6,628 and 7,731
- 6,658 and 7,761
- 6,678 and 7,781
- 6,698 and 7,801

6.22 The Excel financial function =NPV(B1,B3:E3)+A3 is used on the cash flow below to calculate the NPV of 1,470. What will be its equivalent NPV if mid-year discounting is used?

- 1,470
- 1,598
- 1,698
- 1,670

	A	B	C	D	E
1	Discount Rate	8%			
2	0	1	2	3	4
3	-1,800	-4,200	3,000	3,000	3,000

Problem 6.22 diagram

- 6.23 What will be the LRMC, in \$/MMBtu, for the cash flow shown below?
- a. 9.76
 - b. 11.09
 - c. 10.09
 - d. 9.09

	A	B	C	D	E	F
1	Discount Rate	8%				
2	Sales Gas Heating Value	1,080	Btu/Scf			
3		1	2	3	4	5
4	Sales Gas Volume, MMSCFD	0	0	600	600	600
5	CAPEX, MM\$	-1,800	-4,200			
6	OPEX, MM\$	0	0	-240	-240	-240

Problem 6.23 diagram

- 6.24 What will be the LRMC, in \$/MMBtu, for the cash flow shown in question 6.23 if the gas is accompanied by condensate production? The present value of the condensate revenue is 700 MM\$.
- a. 9.76
 - b. 11.09
 - c. 10.09
 - d. 9.09
- 6.25 In which of the following situations will you get correct IRR?
- I. + + + + - - - + + + +
 - II. + + + + + + + + + +
 - III. - - - + + + + + + +
 - IV. - - - + + + + - - - -
- a. I only
 - b. I, II, and III
 - c. III only
 - d. None of the above

6.26 Given the following cash flow:

0	1	2	3	4	5
−\$3,352	\$1,000	\$1,000	\$1,000	\$1,000	\$1,000

- a. Calculate the NPV of the project using the following discount rates: 5%, 10%, 15%, 20%, and 25%.
- b. Graph the NPVs of the project as a function of the discount rates.
- c. Find the IRR, mathematically and graphically.
- d. Calculate the NPV and IRR of the project using an Excel spreadsheet.

6.27 For each of the following investment projects, calculate:

Project	0	1	2	3	4	5	6	7
A	−4,564	1,000	1,000	1,000	1,000	1,000	1,000	1,000
B	−2,000	524.7	524.7	524.7	524.7	524.7	524.7	524.7
C	−21,000	3,000	3,000	3,000	3,000	3,000	3,000	3,000

- a. The net present value (NPV) using a 15% discount rate.
 - b. The internal rate of return.
 - c. Based on the two profitability measures calculated, which project(s) should be accepted?
- 6.28 Two projects are considered for evaluation. Project A has a cost of \$10,000 and is expected to produce benefits of \$3,000 per year for five years. Project B costs \$25,000 and is expected to produce cash flows of \$7,500 per year for five years. Calculate the NPV, IRR, PI, and PVR for the two projects. Assume the minimum acceptable rate of return and the available reinvestment rate of 8%. Should both projects be accepted if they are independent?
- 6.29 Calculate the NPV and IRR for the cash flows given in the following table. Assume the minimum acceptable rate of return of 8%. Which projects should be accepted if they are independent projects? Would the selection of the projects change if the cost of capital were 12%?

Expected Net Cash Flow		
Year	Project A	Project B
0	−\$10,000	−\$10,000
1	\$6,500	\$3,500
2	\$3,000	\$3,500
3	\$3,000	\$3,500
4	\$1,000	\$3,500

- 6.30 A company is deciding whether or not it should develop a strip mine at an initial cost of 4.5 MM\$. Net cash inflows are expected to be 9 MM\$ for each of the first three years of operations. In the fourth year, the land must be returned to its natural state at a cost of 25 MM\$. Develop and plot the NPV profile for the project. Should the project be accepted at a rate of return of 8%? Should it be accepted at a rate of return of 15%? What are the project GRRs if the available reinvestment rate is (a) 8% and (b) 15%?
- 6.31 Certain oil wells are expected to produce consistently for the following five years. The annual oil production forecast is given in the table below. It costs \$20,000 per month to operate the wells, the current oil price is \$60/Stb, and the severance tax is 7.085% of the gross value. The wells are not producing any associated gas. Calculate the NPV, PI, LRMC, and IRR, as of 01/01/2011, of this production stream assuming the minimum acceptable rate of return of 8%, 100% WI, 87.5% NRI, and investment of \$2,000,000 in 2011.

Oil Production, Stb				
2012	2013	2014	2015	2016
20,801	18,248	15,845	15,059	12,391

- 6.32 How would you confirm that the LRMC calculated in question 6.31 is correct? Show your work and the steps required to prove your answer.
- 6.33 Assume (a) initial production rate is 375 Stb/month and declining exponentially at 20% per year, (b) the price of oil to be \$60/Stb, (c) severance tax is 7%, (d) working interest is 100%, (e) net revenue interest is 87.5%, and (f) monthly lease operating cost is \$3,700.

The total cost of drilling and completing the well is \$350,000. Assume the after-tax weighted average cost of capital of 8%. Forecast the production until the economic limit and calculate its NPV, IRR, PI, and LRMC.

- 6.34 Develop a simple spreadsheet to enable calculation of payback period (discounted), NPV, PI, PVR, IRR, and UTC for the data in problem 6.32. Assume the cost of capital and the reinvestment rate is 8%, and it is constant throughout the economic life of the investment.
- 6.35 Suppose there is an investment proposal requiring \$16,000 outlay now (time zero) and returning a constant cash flow of \$7,000 per period before tax savings due to interest payments for the next three years. The proposal is to have a market debt proportion of 50% (i.e., 0.50). The capital market requires per period rate of return on equity of 27% and on debt of 9%. The corporate tax rate is 40%, and interest is deductible for the calculation of income tax. Calculate the NPV and IRR based on (a) weighted average cost of capital method, (b) Arditty-Levy method, (c) equity residual method, and (d) adjusted net present value method.
- 6.36 Rework problem 6.35, assuming the corporate tax rate is not applicable.
- 6.37 Rework problem 6.35, assuming the loan is paid in three equal periodic year-end payments. Comment on the outcomes as compared to the outcomes in problem 6.35.
- 6.38 Two investment alternatives are to be economically evaluated. Both projects require initial capital outlay of \$1,000 (in time zero). Project A generates a net before-tax cash flow of \$300 per year for the next five years. Project B generates a net before-tax cash flow of \$200 per year for the next four years and \$792.58 in year five. The rate of equity is 7%, the interest rate on debt is 11%, the corporate tax rate is 46%, and the debt weight is 30% (i.e., equity = \$700 and debt = \$300). Using the methods of (a) weighted average cost of capital, (b) Arditti-Levy, and (c) equity residual, calculate the NPV, IRR, and GRR (assuming reinvestment rate of 7%). Assume the debt repayment follows a constant ratio debt schedule, and the loan is paid back in five years.
- 6.39 Rework problem 6.37, assuming the loan is serviced in (a) five year-end equal principal payments of \$60, and (b) five year-end

equal payments. Comment on the profitability measures in comparison to the ones calculated in problem 6.37.

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7

Investment Selection Decision Making

The data requirement for each investment alternative under evaluation and the methods used to forecast relevant factors were presented in chapters 3 and 4. In chapter 6, the data organized in chapters 3 and 4 were combined and translated into meaningful profitability/decision measures. In this chapter, the information gathered/calculated in chapters 3 to 6 is used to make a decision whether or not to select the proposed investment and to rank the selected investment if a number of investment alternatives are available.

Capital expenditure plans must be consistent with the cash position and financing considerations created. Both the timing of the capital expenditures and the amount of funds to invest in projects involve serious policy decisions. Capital additions should be analyzed in terms of their probable effect on return on investment. Prudent management should not undertake a capital addition unless its analysis indicates it will yield a return equal to, or greater than, the long-range company objective for return on investment.

A basic assumption of microeconomics is that the primary objective of a firm is to maximize profits throughout some planning horizon. To achieve this objective, what particular capital investments out of a number of available alternatives should the firm acquire? The simple answer is that the firm should invest only in those capital projects that will maximize its worth.

The investment evaluation process is comprised of two components: screening and ranking investment opportunities. Screening of all known investment opportunities identifies those projects that meet or exceed a predetermined standard (i.e., opportunities of value to the investor warranting further detailed evaluation). Screening criteria are based upon the investors' long-term goals and objectives. The criteria set the minimum acceptable profitability standard for potential investment opportunities. If the profitability of a potential investment meets or exceeds the screening criteria, it may be accepted. A screening criterion is necessary for every economic evaluation. The screening ensures only investments projected to achieve the desired goals of the investing organization are considered.

Once an array of investment opportunities have fulfilled the screening criteria, the second step is to rank them in some order for a systematic

evaluation. Ranking places these screened projects in order of most desirable to least desirable. Ranking criteria are only needed to allocate resources when they are in short supply. Selection of projects from the investment opportunities being considered, in accordance with proper ranking criterion, will yield the maximum return possible within the limited resources (capital requirement) available. If the investor has unlimited resources, all projects (even those that exceed the screening criteria) may be approved.

The investments are normally grouped according to their nature and requirement. Although practices vary from firm to firm, investment proposals are frequently grouped according to the following categories:

1. Routine replacements and/or upgrades of service-producing assets
2. Expansions providing additional capacity in existing facilities or introduction of new product lines
3. Acquisitions of producing properties
4. Infill/development drilling—primary recovery
5. Infill/development drilling—secondary and enhanced recovery
6. Exploration activities

While evaluating capital investments, it is recommended they be classified into groups. This will help in developing standardized estimation and administration procedures for handling particular classes of proposals. A few of the many possible ways of classifying investment proposals are:

1. *By project size* (i.e., the amount of cash resources required for implementing the project). Three general classes of project sizes are major projects, regular capital expenditures, and small proposals.
2. *By type of benefit* (e.g., benefits can arise from cost reduction, expansion of existing product lines, risk reduction, or social overhead investment)
3. *By degree of dependence or interdependence* between two or more investment proposals. These can arise for several reasons such as:
 - a. *Proposals are mutually exclusive*: The acceptance of one precludes the acceptance of the other. The early identification of these investments is crucial for a logical screening of investments.
 - b. *Proposals are complements*: The decision to execute one investment increases the expected benefit from another investment.

- c. *Proposals are substitutes*: The acceptance of one investment decreases the profitability of another investment.
- d. *Proposals are independent or dependent*: Two investments are considered statistically dependent when an increase (decrease) in the benefits from one investment is accompanied by an increase (decrease) in the benefits of the second, or vice versa.

In using the various measures of profitability discussed in chapter 6 and applied in this chapter, the role of *sound management judgment and experience* must not be overlooked. Most evaluations are based on estimates of their potential. Therefore, the results cannot be better than the estimates. The estimates themselves may be subjective, and there may be other numerous, unquantifiable factors related to a proposal. Therefore, a keen appreciation of the assumptions underlying the computations in each technique is essential.

Service-Producing Investments

In this section, the profitability yardsticks discussed in chapter 6 are used to select the economic service-producing investment alternative. The service-producing investments are mainly the facilities required for oil and gas production. These include separators, pumps, tanks, engines, turbines, flow lines, etc. These capital additions, replacements, or upgrades are assumed to provide the same revenue contribution, but at different costs. For example, a decision has to be made to install one of four artificial lift systems: (1) gas lift, (2) beam pumping unit, (3) hydraulic pump, or (4) electrical submersible pump. The beam pumping unit may have three additional alternatives of whether it should be primed by electric motor, gas engine, or diesel engine. Each of these pumps will produce the same amount of oil, but at different initial and annual maintenance/replacement costs.

The selection of service-producing investments is mutually exclusive (i.e., the selection of a beam pumping unit precludes the selection of the other three artificial lift systems). If the revenue contribution differs from one alternative to the other, then it should be considered as a revenue-producing investment (i.e., accounting for the revenue contribution along with the costs). A decision whether to install any type of artificial lift system, by itself, is a revenue-producing investment proposal.

Ordinarily, replacement decisions are the simplest to make. Facilities wear out or become obsolete, and they must be replaced if production efficiency is to be maintained. The firm has a good idea of the cost savings obtained by replacing old equipment/facilities, and it knows the consequences if

replacements are not made. Overall, the outcomes of most replacement decisions can be predicted with a high degree of confidence.

Equivalent cost analysis

In this section, it is assumed the alternatives considered will be of equal economic life, technically comparable, and provide the same service. The selection will be based on the equivalent cost basis: (1) equivalent present value of costs, (2) equivalent annual value of costs, or (3) equivalent future value of costs. Comparison of either of the equivalent costs will assist in selection of the most economical and cost effective alternative.

The equivalent costs will be calculated at a discount rate equivalent to the opportunity cost of investing the funds elsewhere. The following example will clarify the computation. Once the equivalent costs of each of the alternatives are calculated, one of the following decision yardsticks will be applied:

- 1. Select the alternative with the lowest present value (PV) of cost.
- 2. Select the alternative with the lowest annual value (AV) of cost.
- 3. Select the alternative with the lowest future value (FV) of cost.

Example 7–1

A choice has to be made between a diesel engine and a secondhand gasoline engine for a service expected to last seven years. The minimum attractive rate of return is 8%. Select the most economical alternative using (a) PV of costs, (b) AV of costs, and (c) FV of costs. The two alternatives are diagrammatically presented in figure 7–1.

	A Diesel	B Gasoline
Initial cost	\$15,000	\$5,000
Salvage value after seven years	-5,000	0
Annual fuel cost	2,500	6,000
Annual maintenance cost	500	1,000

Close attention should be paid to the units. All the costs should be either reported as negative or positive. Revenues are always treated as positive. Salvage value in this table is negative because it is offsetting the costs and costs are listed as positive. If the costs were listed as negative then the salvage value will be positive.

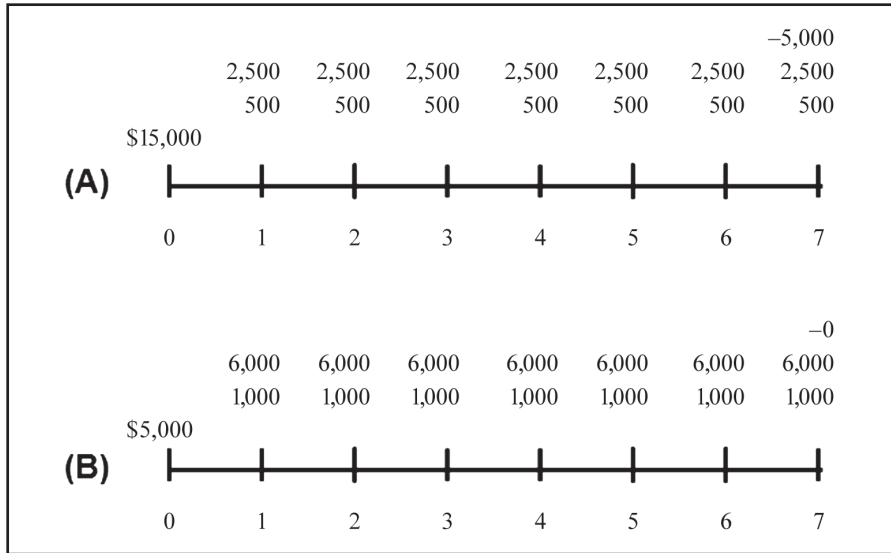


Fig. 7-1. Cash flow diagram for the data in example 7-1

Solution: Present, annual, and future values of costs for each alternative are:

$$\begin{aligned}
 \text{(a) } PV_A &= \$15,000 + \$2,500(P/A_{8\%,7}) + \$500(P/A_{8\%,7}) - \$5,000(P/F_{8\%,7}) \\
 &= \$15,000 + \$2,500(5.2064) + \$500(5.2064) - \$5,000(0.5835) \\
 &= \$27,701.70
 \end{aligned}$$

$$\begin{aligned}
 PV_B &= \$5,000 + \$6,000(P/A_{8\%,7}) + \$1,000(P/A_{8\%,7}) - \$0(P/F_{8\%,7}) \\
 &= \$5,000 + \$6,000(5.2064) + \$1,000(5.2064) - \$0(0.5835) \\
 &= \$41,444.80
 \end{aligned}$$

Select alternative A: the diesel engine with the lowest PV cost of \$27,701.70.

$$\begin{aligned}
 \text{(b) } AV_A &= \$15,000(A/P_{8\%,7}) + \$2,500 + \$500 - \$5,000(A/F_{8\%,7}) \\
 &= \$15,000(0.19207) + \$2,500 + \$500 - \$5,000(0.11207) \\
 &= \$5,320.70
 \end{aligned}$$

$$\begin{aligned}
 AV_B &= \$5,000(A/P_{8\%,7}) + \$6,000 + \$1,000 - \$0(A/F_{8\%,7}) \\
 &= \$5,000(0.19207) + \$6,000 + \$1,000 - \$0(0.11207) \\
 &= \$7,960.35
 \end{aligned}$$

Select alternative A: the diesel engine with the lowest AV cost of \$5,320.70.

$$\begin{aligned}
 \text{(c) } FV_A &= \$15,000(F/P_{8\%,7}) + \$2,500(F/A_{8\%,7}) + \$500(F/A_{8\%,7}) - \$5,000 \\
 &= \$15,000(1.7138) + \$2,500(8.9228) + \$500(8.9228) - \$5,000 \\
 &= \$47,475.40
 \end{aligned}$$

$$\begin{aligned}FV_B &= \$5,000(F/P_{8\%,7}) + \$6,000(F/A_{8\%,7}) + \$1,000(F/A_{8\%,7}) - \$0 \\&= \$5,000(1.7138) + \$6,000(8.9228) + \$1,000(8.9228) - \$0 \\&= \$71,028.60\end{aligned}$$

Select alternative A: the diesel engine with the lowest FV cost of \$47,475.40.

The preceding example shows that regardless of which method is used, an appropriate alternative will be selected.

Rate of return analysis

Using this profitability measure for the selection of economical service-producing investments will not give the internal rate of return (IRR). This is because of the negative cash flow (costs) and lack of positive cash flow to offset these costs. Remember, one of the limitations of IRR is it cannot be calculated when cash flows are either all positive or all negative.

This limitation of IRR can be avoided by performing incremental IRR analysis for selecting among service-producing investment alternatives. This is done by subtracting the costs of one alternative from the costs of the other alternative. The difference between the two costs is the savings (positive and negative) one alternative generates over the other. The IRR analyses are then performed on these incremental costs.

If the *incremental* IRR is greater than the minimum acceptable IRR, select the high initial cost alternative, and vice versa. If there are more than two alternatives, the survivor of the two is then compared with the next alternative. This is repeated until all the available alternatives are exhausted. Remember the negative incremental costs are treated as positive savings, and vice versa.

Example 7-2

Using the IRR analysis on the investments presented in example 7-1, which alternative will be the best economic choice? Is the selection consistent with the selection made in example 7-1? The incremental cash flow diagram for the investment is shown in figure 7-2. Note that all costs are treated as negative costs, and the salvage value is treated as positive benefit.

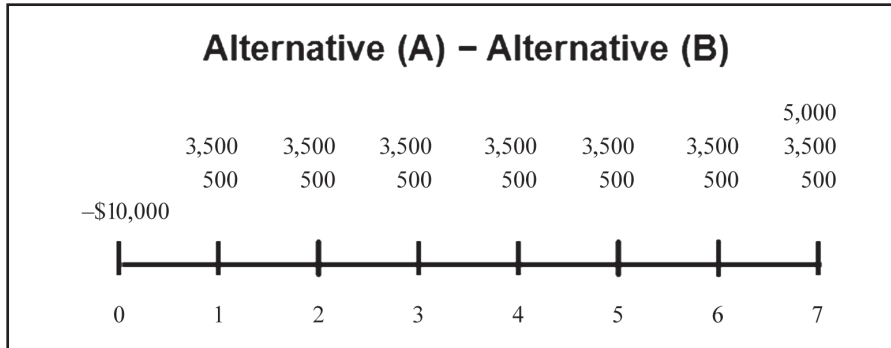


Fig. 7–2. Incremental cash flow diagram for the data in example 7–1

Solution: Calculate the PV cost at $i = 8\%$.

$$\begin{aligned}
 PV_{(A-B)} &= -\$10,000 + \$3,500(P/A_{8\%,7}) + \$500(P/A_{8\%,7}) + \$5,000(P/F_{8\%,7}) \\
 &= -\$10,000 + \$3,500(5.2064) + \$500(5.2064) + \$5,000(0.5835) \\
 &= \$13,743
 \end{aligned}$$

Since the present value of the incremental costs is positive at $i = 8\%$ (minimum acceptable rate of return), the incremental IRR will be greater than 8%. Therefore, select alternative A. The selection is consistent with the selection made in example 7–1. The incremental IRR is calculated as follows.

Try $i = 35\%$:

$$\begin{aligned}
 PV_{(A-B)} @ i = 0.35 &= -\$10,000 + \$3,500(P/A_{35\%,7}) + \$500(P/A_{35\%,7}) \\
 &\quad + \$5,000(P/F_{35\%,7}) \\
 &= -\$10,000 + \$3,500(2.5075) + \$500(2.5075) + \$5,000(0.1224) \\
 &= \$642.00
 \end{aligned}$$

Similarly, for $i = 40\%$, the $PV_{(A-B)} @ i = 0.40 = -\474.30

Therefore, IRR by interpolation is

$$\begin{aligned}
 IRR_{(A-B)} &= \frac{642.00}{642.00 + 474.30} (40 - 35) + 35 \\
 &= 37.88\%
 \end{aligned}$$

Since the incremental IRR is greater than the minimum acceptable rate of return of 8%, the alternative with the higher initial cost (alternative A) is selected.

Present value ratio or PI analysis

The present value ratio (PVR) and profitability index (PI) can be used in the same way as IRR is used. If the PVR of the incremental cash flow is greater than zero, the alternative with the higher initial cost is selected and vice versa. On the other hand, if the PI of the incremental cash flow is greater than one ($PI > 1$), the alternative with the higher initial cost is selected and vice versa. The calculations of the PVR and PI for the data in example 7–2 follow.

From example 7–2, the present value of the incremental costs, $PV_{(A-B)}$, at the minimum acceptable rate of return of 8%, is \$7,908.10, and the incremental investment over alternative B is \$10,000. Therefore, the PVR and PI are

$$PVR_{(A-B)} = \frac{13,743.00}{10,000.00} = 1.374$$

$$PI_{(A-B)} = 1 + \frac{13,743.00}{10,000.00} = 2.374$$

Both the PVR and PI confirm the selection of alternative A.

Service life breakeven analysis

The use of this method gives the same selection obtained by using the methods (PV, AV, FV, IRR, PVR, or PI) discussed in the preceding sections. The following steps should be followed for service life breakeven analysis.

1. Select one alternative and calculate the AV of its costs.
2. For the second alternative, compute n (service life) that gives the AV equal to the AV calculated in step 1.
3. If n in step 2 is less than the service life of the first alternative, select the alternative in step 2. If n in step 2 is greater than the service life calculated in step 1, select the alternative in step 1. The following example will clarify the calculations.

Example 7–3

Use service life breakeven analysis to select the least costly investment alternative out of the alternatives given in example 7–1.

Solution: $AV_A = \$5,320.70$ (from example 7-1)

$$n = 7 \text{ years}$$

Compute n for alternative B (by trial and error), which will make the $AV_A = AV_B$. For $n = 10$ years,

$$\begin{aligned} AV_B &= \$5,000(A/P_{8\%,10}) + \$6,000(P/A_{8\%,7})(A/P_{8\%,10}) + \$1,000(P/A_{8\%,7})(A/P_{8\%,10}) \\ &= \$5,000(0.14903) + \$6,000(5.2064)(0.14903) + \$1,000(5.2064)(0.14903) \\ &= \$6,176.52 \end{aligned}$$

Since the $AV_B = \$6,176.52$ for $n = 10$ years is greater than the $AV_A = \$5,320.70$ for $n = 7$ years, another trial is needed.

For $n = 15$ years,

$$\begin{aligned} AV_B &= \$5,000(A/P_{8\%,15}) + \$6,000(P/A_{8\%,7})(A/P_{8\%,15}) + \$1,000(P/A_{8\%,7})(A/P_{8\%,15}) \\ &= \$5,000(0.11683) + \$6,000(5.2064)(0.11683) + \$1,000(5.2064)(0.11683) \\ &= \$4,842.00 \end{aligned}$$

For $n = 15$ years, the $AV_B = \$4,842.00$ is less than the $AV_A = \$5,320.70$. Therefore, the required n is calculated by linear interpolation as shown below.

$$\begin{aligned} n &= \frac{\$5,320.70 - \$4,842.00}{\$6,176.52 - \$5,320.70} (15 - 10) + 10 \\ &= \frac{\$478.70}{855.82} \times 5 + 10 = 12.8 \text{ or } 13 \text{ years} \end{aligned}$$

The service life of alternative B (13 years) is greater than the service life of alternative A for the same annual costs. Therefore, alternative A is selected. This selection is again consistent with the selection made using the other methods discussed above.

Unequal life service-producing alternatives

In the examples above, the selection of service-producing investment alternatives with *equal service life* was assumed. The same techniques are used for selecting the economical service-producing investment alternatives with unequal lives, provided the lives of the alternatives are made equal for comparison purposes. In general, the methods presented here are not valid for comparing unequal life income or service-producing investments that do not result in the same service (these methods are presented in the next section).

To get a meaningful comparison of unequal life alternatives that provide the same service, assumptions or estimates must be made to permit comparison of the alternatives on an equal life (or equal study period) basis. Comparing project costs for unequal periods will mean comparing different total costs.¹

A common service life is obtained by first carefully defining the time span required for the service, for instance, 10 years. Assume there are two alternatives: one with 10-year life and the other with 5-year life. To make the lives common, it is assumed the 5-year alternative will be replaced by another (like-for-like) 5-year alternative and then compared. Alternatively, it may be assumed the life of the 5-year alternative will be extended for another 5 years with a major overhaul at the end of year 5.

The situation can be the other way around (i.e., the service is required for only 5 years). In this case, the extra 5 years of the 10-year life alternative are ignored and its salvage value at the end of year 5 is considered.

The alternative may be either sold at the end of year 5, or it will be used elsewhere. If it is used elsewhere, it will have opportunity cost. In either case, it will have an alternative value.

If capitalized cost analysis is used for comparison, the life of the permanent life (perpetual) alternative is used as a common life and the preceding assumptions are not required.

Example 7–4

Two conveyors are being considered to handle ore in a smelting operation. Conveyor A will cost \$15,000 with annual maintenance/operating costs of \$2,000 and an expected salvage value of \$10,000 at the end of its expected useful life of six years. Conveyor B will cost \$4,500 with annual operating/maintenance costs of \$4,000 and an expected salvage value of \$2,500 at the end of its expected useful service life of three years. Both conveyors are technically compatible. If the minimum acceptable rate of return before taxes for investments of this type is 8%, determine the most economical alternative by PV cost comparison if (a) service is required for six years and (b) service is required for only three years.

Solution: The useful life of conveyor A is six years and conveyor B is only three years. In the first case, it is assumed that at the end of three years, conveyor B can be replaced (like-for-like) with another conveyor so the total service life of six years is achieved. The cash flow diagram for this situation is as shown in figure 7–3.

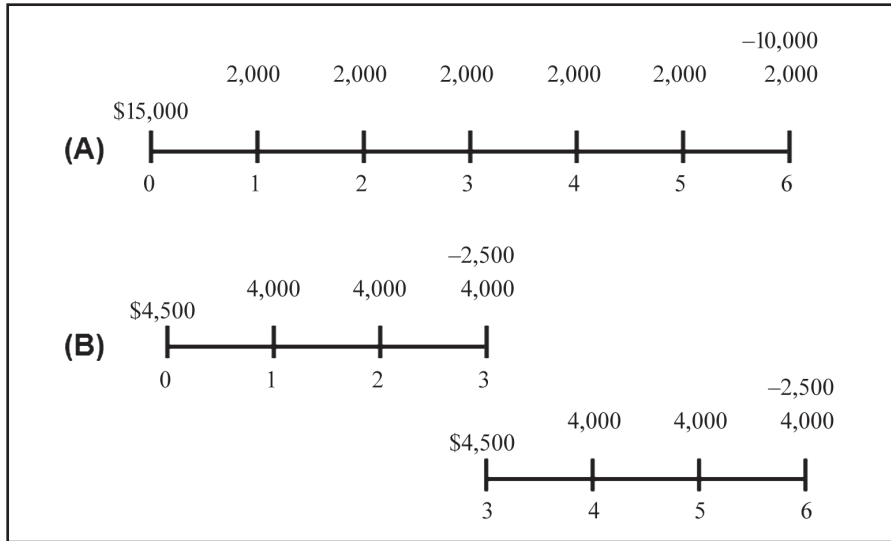


Fig. 7–3. Cash flow diagram for the data in example 7–4

$$\begin{aligned}
 \text{(a) } PV_A &= \$15,000 + \$2,000(P/A_{8\%,6}) - \$10,000(P/F_{8\%,6}) \\
 &= \$15,000 + \$2,000(4.623) - \$10,000(0.6302) \\
 &= \$17,944
 \end{aligned}$$

$$\begin{aligned}
 PV_B &= \$4,500 + \$4,000(P/A_{8\%,3}) - \$2,500(P/F_{8\%,3}) + \$4,500(P/F_{8\%,3}) \\
 &\quad + \$4,000(P/A_{8\%,3})(P/F_{8\%,3}) - \$2,500(P/F_{8\%,6}) \\
 &= \$4,500 + \$4,000(2.577) - \$2,500(0.7938) + \$4,500(0.7938) \\
 &\quad + \$4,000(2.577)(0.7938) - \$2,500(0.6302) \\
 &= \$23,002.60
 \end{aligned}$$

Therefore, alternative A with $PV_A < PV_B$ should be selected.

- (b) For this option, the life of the six-year conveyor is shortened to three years, and it is assumed its salvage value will be \$12,000 as compared to \$10,000 after six years.

$$\begin{aligned}
 PV_A &= \$15,000 + \$2,000(P/A_{8\%,3}) - \$12,000(P/F_{8\%,3}) \\
 &= \$15,000 + \$2,000(2.577) - \$12,000(0.7938) \\
 &= \$10,628.40
 \end{aligned}$$

$$\begin{aligned}
 PV_B &= \$4,500 + \$4,000(P/A_{8\%,3}) - \$2,500(P/F_{8\%,3}) \\
 &= \$4,500 + \$4,000(2.577) - \$2,500(0.7983) \\
 &= \$12,812.25
 \end{aligned}$$

For the three-year requirement, alternative A is the economical choice. Cost estimates (initial cost, annual operating/maintenance costs, and salvage

value) will increase due to inflation and tax consequences due to depreciation, if any, for the alternative to be replaced later. If replacement happens at the end of the initial three-year period, its cost three years from now may be higher due to inflation so this should be accounted for rather than assuming that even after three years, the machine can be replaced at the same cost as today.

Lease versus Buy Decision Analysis

For a business to use equipment for providing a product or service, it does not have to be owned. It can be leased. Equipment leasing provides customized financing with potentially unique tax features. J.S. Schallheim gives a detailed treatment of lease versus buy decisions in a very well-written book.²

In general, an asset is leased from a leasing company for a fixed duration of time at predetermined periodic lease payments during the lease period. Lease contracts can be written to provide the *lessee* with great flexibility in terms of the amount and timing of the lease payments. The ownership of the leased asset remains with the *lessor* in most leases (i.e., the lessee does not own the asset after completing all the contracted payments). However, the ownership does not affect the use of the asset by a lessee.

The economics of lease versus buy decisions is dependent on the tax rules applicable to a particular class of leases. The IRS specifies rules for tax-oriented leases. If a lease qualifies under the tax guidelines, it is called a *true lease*. Otherwise, the lease is a *conditional sales contract* under the IRS rules. Therefore, it is important to know the tax laws applicable for the leased asset under evaluation. Some of the major advantages of leasing are as follows.

1. Leasing offers tax savings, thus advantages for firms with:
 - a. Excess tax shields
 - b. Low earnings and/or low tax rates
2. Leasing offers financial cost savings.
 - a. Permits 100% financing (i.e., it does not tie up capital and may, in some cases, require lower initial capital outlay thus freeing working capital for more productive use)
 - b. Avoids restrictions inherent in debt contracts
 - c. Increases a firm's ability to acquire funds, thus leaving normal lines of bank credit undisturbed

3. Leasing offers transaction and information cost savings.
 - a. Avoids the purchase transaction and may be tailored to the lessee's needs
 - b. All costs related to the leased asset are known with certainty
 - c. Requires less record keeping
4. Leasing provides risk sharing.
 - a. Avoids the need to sell assets no longer required in the lessee's operations
 - b. Provides protection against asset obsolescence
 - c. Provides a hedge against inflation and business risk

In leasing, the lessee is saving the upfront cost of purchasing the asset and in return commits to a series of periodic lease payments that are generally tax deductible. On the other hand, the lessee gives up depreciation tax shields and any other tax credits associated with ownership. Since the lessee is saving the cost of purchasing the asset and is subject to tax swaps from tax shields, the net present value (NPV) or internal rate of return (IRR) can be used for the lease versus buy decision making. The NPV in this context is also referred to as the *net advantage to leasing* (NAL). The following equation is a general NPV or NAL formula used for lease versus buy analysis.

$$\begin{aligned}
 NPV = A_o - \sum_{t=0}^{n-1} \frac{L_t}{(1+r_1)^t} + \sum_{t=0}^{n-1} \frac{L_t(T)}{(1+r_2)^t} - \sum_{t=0}^{n-1} \frac{D_t(T)}{(1+r_3)^t} - \sum_{t=0}^{n-1} \frac{I_t(T)}{(1+r_4)^t} \\
 + \sum_{t=0}^{n-1} \frac{O_t(1-T)}{(1+r_5)^t} - \frac{S_n}{(1+r_6)^n} - TC
 \end{aligned} \tag{7.1}$$

where

A_o = price of the asset, if purchased, at present time ($t = 0$)

L_t = lease payment at time t

D_t = depreciation charge, if owned, at time t

I_t = interest charge on the equivalent loan, if purchased with debt, at time t

O_t = operating expenses that are higher if the asset is purchased but not if the asset is leased

S_n = expected after-tax salvage value of the asset, if purchased, at time n

r = appropriate discount rate for each of the cash flows above

T = marginal corporate tax rate of the lease

n = number of time periods covered by the lease

TC = tax credits applicable to the asset that are taken by the
lessor and not the lessee

As discussed in chapter 6, the asset cost A_o may be included in equation (7.1) and the after-tax weighted average cost of capital used as a discount rate. On the other hand, the principal and interest payments and the cost of debt used as the discount rate may replace the A_o term. The two approaches—asset cost versus loan payments—are equivalent if the total cost of the asset is borrowed (100% financing) and the discount rate is the after-tax cost of debt. If the borrowing is not 100%, then the weighted average cost of capital can still be used provided a constant ratio debt payment schedule is used.

Myers, Dill, and Bautista (MDB) present another model. The MDB model simultaneously solves the problem of the proper discount rate for the cash flows and the equivalent amount of borrowing. This approach treats the amount of borrowing equivalent to the lease as the present value of the after-tax lease payments and depreciation tax shields, all discounted at the after-tax cost of debt. The following equation is proposed by MDB.³

$$NPV = A_o - \sum_{t=1}^n \frac{L_t(1-T) + D_t(T)}{[1 + r_B(1-T)]^t} + \sum_{t=1}^n \frac{O_t(1-T)}{(1+r_1)^t} - \frac{S_n}{(1+r_2)^n} \quad (7.2)$$

where

r_B = the cost of debt
 r_1, r_2 = the discount rates appropriate for the operating expenses
and salvage value, respectively

If the before-tax cost of debt is used as the discount rate, equation (7.2) becomes

$$NPV = A_o - \sum_{t=1}^n \frac{L_t(1-T) + D_t(T) - I_t(T)}{(1+r_B)^t} + \sum_{t=1}^n \frac{O_t(1-T)}{(1+r_1)^t} - \frac{S_n}{(1+r_2)^n} \quad (7.3)$$

Example 7–5

An oil company must choose between leasing or purchasing a pumping unit for a well. The purchase price of the unit is \$40,000 with an estimated economic life of five years. The unit may be leased for annual year-end lease payments of \$10,000 for five years. Calculate the net value of leasing if (a) the salvage value of the unit is zero, (b) the operating costs for both options are the same, (c) depreciation is straight line over five years, (d) the cost of before-tax cost of capital is 10%, and (e) the corporate tax rate is 35%.

Solution: The cash flow for example 7–5 is shown in table 7–1.

Lease payment (net of taxes) = $\$10,000 \times (1 - 0.35) = \$6,500$
Depreciation = $40,000/5 = \$8,000$
Tax saving due to depreciation = $\$8,000 \times 0.35 = \$2,800$
After-tax cost of debt = $r_B(1 - T) = 10.0 \times (1 - 0.35) = 6.5\% \text{ or } 0.065$

Using equation (7.2), the net advantage of leasing (NAL) is,

$$\begin{aligned} \text{NAL} &= \$40,000 - 9,300(1 + 0.065)^{-1} - 9,300(1 + 0.065)^{-2} \\ &\quad - 9,300(1 + 0.065)^{-3} - 9,300(1 + 0.065)^{-4} - 9,300(1 + 0.065)^{-5} \\ &= \$40,000 - 8,732.39 - 8,199.43 - 7,699.00 - 7,229.10 - 6,787.89 \\ &= \$1,352.19 \end{aligned}$$

Since the NAL is positive (+ve), the leasing option is the preferred choice.

Table 7–1. Cash flow for example 7–5

		Year 1	Year 2	Year 3	Year 4	Year 5
Lease payment	$L_t(1-T)$	\$6,500	\$6,500	\$6,500	\$6,500	\$6,500
Depreciation tax shield	$D_t(T)$	\$2,800	\$2,800	\$2,800	\$2,800	\$2,800
Cost of leasing	$L_t(1-T)+D_t(T)$	\$9,300	\$9,300	\$9,300	\$9,300	\$9,300

Example 7–6

Rework example 7–5, assuming the lease payment includes maintenance by the lessor, which will otherwise cost \$1,000 per month. Also, assume the residual (salvage) value of the unit at the end of five years is 20% of the initial cost (i.e., $\$40,000 \times 0.2 = \$8,000$).

Solution: The cash flow for example 7–6 is shown in table 7–2. Note the new terms in the third line of the table show the after-tax operating cost savings (positive), and the fourth line captures the after-tax residual value lost (negative). The residual value *lost* is because the unit is fully depreciated (zero book value) at the end of year five. Therefore, the \$8,000 will be considered as a capital gain, which will be fully taxed.

Using equation (7.2), the net advantage of leasing (NAL) is

$$\begin{aligned} \text{NAL} &= \$40,000 - 8,090(1 + 0.065)^{-1} - 8,090(1 + 0.065)^{-2} \\ &\quad - 8,090(1 + 0.065)^{-3} - 8,090(1 + 0.065)^{-4} - 13,290(1 + 0.065)^{-5} \\ &= \$2,585.18 \end{aligned}$$

Since the NAL is positive (+ve), the leasing option is the preferred choice.

To calculate the internal rate of return (IRR) or yield for lease contracts, the before-tax cost of capital is calculated by trial and error that will result in NAL = 0. The following equation is used.

$$A_o - L_t \frac{(1 + IRR_{leasing})^t - 1}{IRR_{leasing}(1 + IRR_{leasing})^t} = 0$$

(7.4)

Using equation (7.4), the before-tax lease yield for example 7–5 is calculated as follows.

For $IRR_{leasing} = 10\%$, equation (7.4) becomes

$$\$40,000 - \$10,000 \frac{(1 + 0.10)^5 - 1}{0.10(1 + 0.10)^5} = \$2,092.13$$

For $IRR_{leasing} = 5\%$, equation (7.4) becomes

$$\$40,000 - \$10,000 \frac{(1 + 0.05)^5 - 1}{0.10(1 + 0.05)^5} = -\$3,294.77$$

Therefore, $IRR_{leasing}$ by linear interpolation is

$$IRR_{leasing} = \frac{\$3,294.77}{\$3,294.77 + \$2,092.13} (10 - 5) + 5 = 8.06\%$$

The before-tax yield is about 8.06%, compared to the before-tax cost of capital of 10%. According to this criterion, leasing is preferred. If the before-tax yield were higher than the before-tax cost of capital, the purchase option would be the preferred choice.

Table 7–2. Cash flow for example 7–6

		Year 1	Year 2	Year 3	Year 4	Year 5
Lease payment	$L_t(1-T)$	\$6,500	\$6,500	\$6,500	\$6,500	\$6,500
Depreciation tax shield	$D_t(T)$	\$2,240	\$2,240	\$2,240	\$2,240	\$2,240
Operating cost savings	$O_t(1-T)$	–\$650	–\$650	–\$650	–\$650	–\$650
Residual value	$S_N(1-T)$					\$5,200
Cost of leasing	$L_t(1-T) + D_t(T) + O_t(1-T) - S_N(1-T)$	\$8,090	\$8,090	\$8,090	\$8,090	\$13,290

Income-Producing Investments

In the preceding section, profitability measures were applied to the investment analysis of service-producing investments. In this section, the same profitability measures will be applied to the analysis of income-producing investment alternatives. The alternatives to be discussed include:

1. Project screening economics (single/independent investment analysis)
2. Mutually exclusive investment analysis
3. Non-mutually exclusive investment analysis
4. Economics of rate acceleration projects

Most of the profitability measures presented in the previous chapter, if properly used, will result in the same investment decision. The objective is to maximize the total worth of the investing organization.

Project screening economics

In this type of analysis, each investment alternative, whether it is a mutually exclusive alternative or a non-mutually exclusive, is screened. A decision is made whether the alternative is economical or not, whether to accept or reject. If the alternative is not economical, it is rejected and not considered for analysis as part of the mutually exclusive or non-mutually exclusive alternatives. All the economically accepted investments will then be further analyzed as part of either mutually exclusive alternatives or non-mutually exclusive alternatives.

As mentioned previously, if funds are unlimited, all investment alternatives that survive the screening criteria will be undertaken.

In this case, the first step is to carefully select screening criteria based on which alternatives will be screened (i.e., the minimum acceptable rate of return or cost of capital and the desired payback period). Benchmarks should be in place. For example, a company may not accept any project with payout greater than three years or with a rate of return less than 15%, and so on. Using this minimum acceptable rate of return, compute the various profitability measures discussed in chapter 6. As mentioned above, any one of the profitability measures, if properly used, will result in the same accept or reject decision. The decision rules are summarized in table 7–3.

The following example will clarify the use of profitability measures as applied to revenue-producing investment alternatives.

Table 7–3. Summary of profitability measures and decision rules

Profitability Measure	ACCEPT IF	REJECT IF
Payback period @ i_d	$\leq$ Desired	$>$ Desired
Net present value (NPV) @ i_d	> 0	< 0
Internal rate of return (IRR)	$> i_d$	$< i_d$
Profitability index (PI) @ i_d	> 1	< 1
Present value ratio (PVR) @ i_d	> 0	< 0
Unit technical cost @ i_d	$<$ Average product price	$>$ Average product price
Growth rate of return (GRR) @ i_d	$> i_d$	$< i_d$

Example 7–7

A company has an opportunity to invest money. Two investment alternatives are considered with estimated initial investment and expected cash inflows given below. If the company’s minimum acceptable rate of return is 8% and the company has enough funds to invest in both alternatives, should the investments be accepted? Base your decision on (a) NPV analysis, (b) IRR analysis, (c) PI analysis, (d) PVR analysis, and (e) GRR analysis.

	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8
A	–1,000	700	600	500	300	200	0	0	0
B	–1,500	750	650	550	450	300	250	150	100

Solution:

(a)
$$\begin{aligned} \text{NPV}_A &= -1,000 + 700(1 + 0.08)^{-1} + 600(1 + 0.08)^{-2} \\ &\quad + 500(1 + 0.08)^{-3} + 300(1 + 0.08)^{-4} + 200(1 + 0.08)^{-5} \\ &= \$916.10 \end{aligned}$$
$$\begin{aligned} \text{NPV}_B &= -1,500 + 750(1 + 0.08)^{-1} + 650(1 + 0.08)^{-2} \\ &\quad + 550(1 + 0.08)^{-3} + 450(1 + 0.08)^{-4} + 300(1 + 0.08)^{-5} \\ &\quad + 250(1 + 0.08)^{-6} + 150(1 + 0.08)^{-7} + 100(1 + 0.08)^{-8} \\ &= \$1,022.35 \end{aligned}$$

Both investment alternatives have positive NPV ($\text{NPV}_A > 0$ and $\text{NPV}_B > 0$). Therefore, both of them are acceptable provided there are enough funds to implement the two investments.

- (b) For the calculation of IRR, NPVs at various discount rates are calculated as shown below.

Discount Rate	NPV _A	NPV _B
0.05	\$1,046.32	\$1,245.08
0.10	\$836.98	\$890.61
0.15	\$662.10	\$608.91
0.20	\$514.40	\$381.10
0.25	\$388.42	\$193.99
0.30	\$279.98	\$38.19
0.35	\$185.88	–\$93.15
0.40	\$103.62	–\$205.09
0.45	\$31.21	–\$301.44
0.50	–\$32.92	–\$385.12

By linear interpolation, the IRR for each project is

$$IRR_A = \frac{\$31.21}{\$31.21 + \$32.92} (0.50 - 0.45) + 0.45 = 0.4743 \text{ or } 47.43\%$$

$$IRR_B = \frac{\$38.19}{\$38.19 + \$93.15} (0.35 - 0.30) + 0.30 = 0.3145 \text{ or } 31.45\%$$

Since the $IRR_A > i_d = 8\%$ and $IRR_B > i_d = 8\%$, both investments are acceptable.

- (c) The PI and PVR are calculated as follows:

$$PVR_A = \frac{NPV_A}{(I_o)_A} = \frac{\$916.10}{\$1,000} = 0.9161$$

$$PVR_B = \frac{NPV_B}{(I_o)_B} = \frac{\$1,022.35}{\$1,500} = 0.6816$$

Since $PVR_A > 0$ and $PVR_B > 0$, both investments are accepted.

- (d) For the PI analysis, $PI_A = 1 + PVR_A = 1.9161 > 1$ and $PI_B = 1 + PVR_B = 1.6816 > 1$, therefore both projects are accepted according to the PI decision criterion.
- (e) For the calculation of GRR, we assume the available reinvestment rate will be the same as the minimum acceptable rate of return of 8%. The GRR has to be calculated for the same project life (eight years) for both projects, even though project A's life is three years shorter than project B.

$$\begin{aligned} GRR_A &= (1 + PVR_A)^{1/8} \times (1 + i_d) - 1 \times 100 \\ &= (1 + 0.9161)^{1/8} \times (1 + 0.08) - 1 \times 100 \\ &= (1.0847 \times 1.08 - 1) \times 100 = 17.15\% \end{aligned}$$

$$\begin{aligned} GRR_B &= (1 + PVR_B)^{1/8} \times (1 + i_d) - 1 \times 100 \\ &= (1 + 0.6816)^{1/8} \times (1 + 0.08) - 1 \times 100 \\ &= (1.0671 \times 1.08 - 1) \times 100 = 15.25\% \end{aligned}$$

Based on the GRR, since $GRR_A > 8\%$ and $GRR_B > 8\%$, both projects are economical and therefore accepted.

Mutually exclusive investment analysis

In this type of investment alternative, several opportunities exist, but only one investment alternative is selected. The other alternatives are abandoned. The first step, as discussed previously, is to screen each alternative and consider which are economically and technically acceptable.

Therefore, the first step is always to apply “accept or reject” criteria. The accepted projects are then further analyzed against each other. Once again, any of the profitability yardsticks discussed in the previous chapter, if properly used, will provide selection of the most economical alternative out of the available alternatives. The following rules will result in the selection of the economic choice.

1. *Select the alternative with the highest NPV.* The NAV and NFV may be used in the same way except for unequal life investments, in which case the life of the longest life alternative should be used as a common life. The disadvantage in using NPV criterion is it does not provide the magnitude of the return per dollar invested.
2. *The use of PI, PVR, IRR, or GRR will give conflicting selections.* For example, if the investments given in example 7–7 are mutually exclusive, then the following selections will result.
 - a. NPV will select alternative B, since $NPV_B > NPV_A$.

- b. On the contrary, the IRR, PI, PVR, and GRR will select alternative A since $IRR_A > IRR_B$, $PI_A > PI_B$, $PVR_A > PVR_B$, and $GRR_A > GRR_B$. However, in order to maximize net worth, alternative B, with the higher NPV, will be the most economical choice.
3. *To avoid this conflicting selection situation, the IRR, PI, PVR, or GRR must be applied to the incremental cash flows of any two mutually exclusive alternatives.* For example, perform incremental IRR analysis on two alternatives at a time. If the incremental IRR is greater than the minimum acceptable rate of return, select the investment with the higher initial cost, or vice versa. If there are more than two mutually exclusive alternatives, the survivor of the first two sets of alternatives is then compared with the third alternative, and so on. Similarly, if the PI of the incremental cash flow is greater than zero, then select the investment with the higher initial cost, and vice versa.

Selection of the lower initial cost alternative will mean that it is not worth it to make the incremental investment in the higher initial cost investment alternative (i.e., it will be better to invest the incremental capital elsewhere at the minimum acceptable rate of return).

Example 7–8

Rework example 7–7, keeping in mind the alternatives are mutually exclusive (only one of the alternatives is selected). Based on the appropriate profitability analysis, (using NPV, IRR, PI, PVR, and GRR), which of the two alternatives will result in the most economical choice if only \$1,500 is available for investment?

Solution:

- (a) $NPV_A = \$916.10$ (from example 7–7)
 $NPV_B = \$1,022.35$ (from example 7–7)
 NPV_B is greater than NPV_A ; therefore, select investment B.
- (b) To use IRR, PI, PVR, and/or GRR, the incremental analyses are as follows. The incremental cash flows for the two investments follow.

	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8
A	-1,000	700	600	500	300	200	0	0	0
B	-1,500	750	650	550	450	300	250	150	100
B-A	-500	50	50	50	150	100	250	150	100

The NPV, IRR, PI, PVR, and GRR for the incremental cash flow (B - A) are shown below.

$NPV_{(B-A)} @ i_d = 8\%$ is

$$\begin{aligned}
 NPV_{(B-A)} &= -500 + 50(1 + 0.08)^{-1} + 50(1 + 0.08)^{-2} + 50(1 + 0.08)^{-3} \\
 &\quad + 150(1 + 0.08)^{-4} + 100(1 + 0.08)^{-5} + 250(1 + 0.08)^{-6} \\
 &\quad + 250(1 + 0.08)^{-7} + 100(1 + 0.08)^{-8} \\
 &= -500 + 46.30 + 42.87 + 39.69 + 110.25 + 68.06 + 157.54 \\
 &\quad + 145.87 + 54.03 \\
 &= \$164.61
 \end{aligned}$$

Since the $NPV_{(B-A)} > 0$, alternative B with the higher initial cost is selected. This selection is consistent with the selection made in (a) above. To calculate the IRR of the incremental cash flow, calculate the $NPV_{(B-A)}$ at another discount rate (e.g., $i_d = 15\%$).

$NPV_{(B-A)} @ i_d = 15\%$ is

$$\begin{aligned}
 NPV_{(B-A)} &= -500 + 50(1 + 0.15)^{-1} + 50(1 + 0.15)^{-2} + 50(1 + 0.15)^{-3} \\
 &\quad + 150(1 + 0.15)^{-4} + 100(1 + 0.15)^{-5} + 250(1 + 0.15)^{-6} \\
 &\quad + 250(1 + 0.15)^{-7} + 100(1 + 0.15)^{-8} \\
 &= -500 + 43.48 + 37.81 + 32.88 + 85.76 + 49.72 + 108.08 \\
 &\quad + 93.98 + 32.69 \\
 &= -\$15.60
 \end{aligned}$$

Therefore, $IRR_{(B-A)}$ by linear interpolation is

$$\begin{aligned}
 IRR_{(B-A)} &= \frac{\$164.61}{\$164.61 + \$15.60} (15 - 8) + 8 \\
 &= 0.9134 \times 7 + 8 = 14.39\%
 \end{aligned}$$

Since $IRR_{(B-A)} = 14.39\% > i_d = 8\%$, alternative B with the higher initial investment is accepted. Similarly, the PI, PVR, and GRR of the incremental cash flow at a discount rate of 8% are

$$PVR_{(B-A)} = \frac{NPV_{(B-A)} @ 8\%}{500} = \frac{164.61}{500} = 0.329$$

$$PI_{(B-A)} = 1 + PVR_{(B-A)} = 1.329$$

$$\begin{aligned} GRR_{(B-A)} &= [1 + PVR_{(B-A)}]^{1/t} \times (1 + i_d) - 1 \\ &= (1.329)^{1/8} \times 1.08 - 1 = 0.1191 \text{ or } 11.91\% \end{aligned}$$

Since $PVR_{(B-A)} = 0.329 > 0$, $PI_{(B-A)} = 1.329 > 1.0$, and $GRR_{(B-A)} = 11.91\% > i_d = 8\%$, select alternative B. If the $NPV_{(B-A)} < 0$, $IRR_{(B-A)} < i_d$, $PVR_{(B-A)} < 0$, $PI_{(B-A)} < 1.0$, and/or $GRR_{(B-A)} < i_d$, then alternative A will be selected and the remaining \$500 will be invested in the alternative providing a return of $i_d = 8\%$. This combination will maximize the return on total investment of \$1,500.

The NPV profile is used to find out if incremental analyses are required. The NPV profile of an investment under consideration is a graphic presentation of its NPVs at various discount rates. If several investment alternatives are under consideration, their NPVs are plotted versus their respective discount rate (i.e., NPV vs. i_d) on the same graph paper, as shown in figure 7-4. Figure 7-5 shows the NPV profiles for the investment alternatives in example 7-8. The NPV profile is a useful graphic illustration having the following advantages:

1. The profile shows sensitivity of each investment alternative to the timing and discount rate. In figure 7-5, investment B is more sensitive to discounting than investment A. In figure 7-4, the two investments have similar behavior.
2. The profile tells if a unique decision can be made by using any of the regular (*non-incremental*) investment analysis techniques. If the investments' NPV profiles are as shown in figure 7-4 (the profiles do not intersect), then using any of the regular investment analysis techniques will provide consistent selections. When the profiles intersect each other as shown in figure 7-5, then incremental analysis is required. The intersection is referred to as the Fisherian intersection.
3. The NPV profile shows the range of minimum acceptable rate of return in which each investment alternative is preferred. If there is no fixed minimum guideline (not sure of the cost of capital), the profile provides a range. In figure 7-5, if the cost of capital is less than 12.33% (the intersection of two profiles for investments A and B), then investment B is preferred. If the cost of capital is between

31.37% (IRR_B) and 47.36% (IRR_A), then investment A is preferred. At the Fisherian intersection (i.e., if the cost of capital is 12.33%), the investor will be indifferent (selection of either alternative will give the same result).

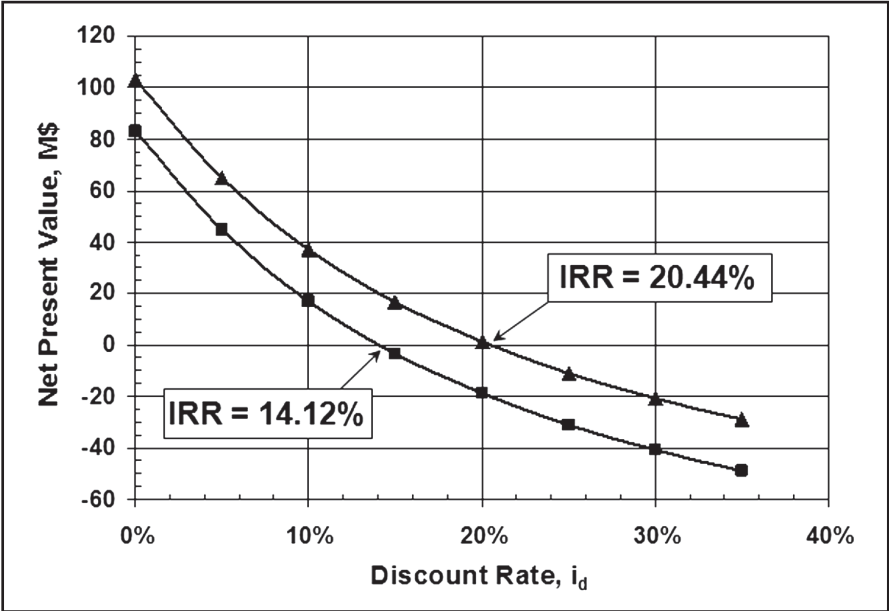


Fig. 7–4. NPV profiles of two investments, without Fisherian intersection

- 4. If a Fisherian intersection exists, one investment can have an IRR superior to the other, yet have an inferior NPV over one or more ranges of discount rate. Therefore, if an investment is selected according to the IRR only, the selection will be suboptimal if the actual discount rate falls below the Fisherian intersection. Investment selection using IRR is appropriate only if it can be proven the Fisherian intersection does not exist between zero and the IRR of the lower of the two investments' IRRs.

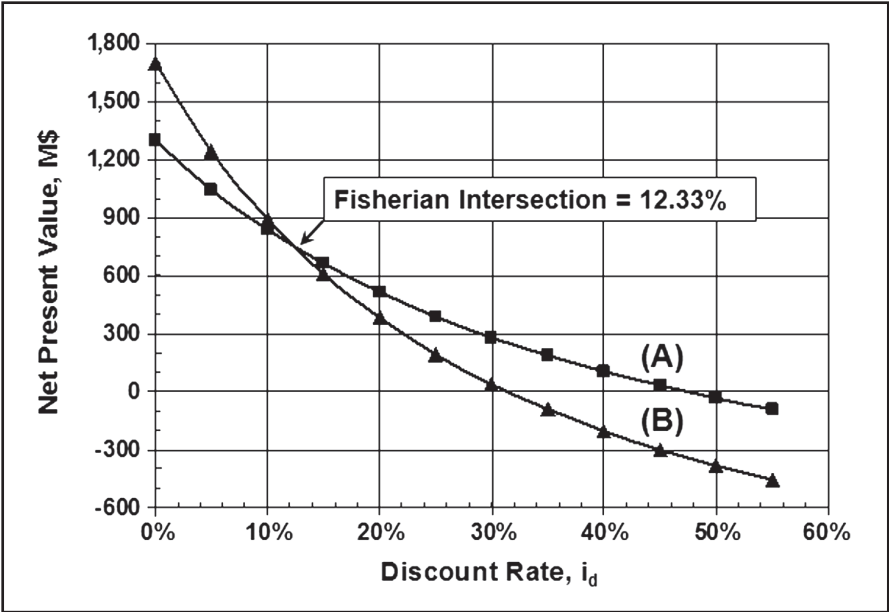


Fig. 7–5. NPV profiles (with Fisherian intersection) for the investments in example 7–8

Example 7–9

A company has an opportunity to invest \$1,500,000. Three mutually exclusive investment alternatives are considered with the investments and expected cash flows as given below. If the company’s cost of capital is 15%, which of the three investment alternatives should be selected in order to maximize the net worth of the company? Base your decision on (a) NPV analysis, (b) IRR analysis, and (c) PI analysis.

	Investment A	Investment B	Investment C
Initial investment	\$500,000	\$800,000	\$1,000,000
Annual revenues	100,000	150,000	200,000
Annual expenses	10,000	13,000	30,000
Investment life, years	20	20	20
Salvage value	50,000	75,000	100,000

Solution: First the NPV at $i = 15\%$ is calculated for each investment.

$$\begin{aligned} \text{(a) } NPV_A &= -\$500,000 + (100,000 - 10,000)(P/A_{15\%,20}) + 50,000(P/F_{15\%,20}) \\ &= -\$500,000 + 90,000(6.2593) + 50,000(.0611) \\ &= \$66,392.00 \end{aligned}$$

$$\begin{aligned} NPV_B &= -\$800,000 + (150,000 - 13,000)(P/A_{15\%,20}) + 75,000(P/F_{15\%,20}) \\ &= -\$800,000 + 857,524.10 + 4,582.50 \\ &= \$62,106.60 \end{aligned}$$

$$\begin{aligned} NPV_C &= -\$1,000,000 + (200,000 - 30,000)(P/A_{15\%,20}) + 100,000(P/F_{15\%,20}) \\ &= -\$1,000,000 + 1,064,081 + 6,110 \\ &= \$70,191 \end{aligned}$$

(b) For the calculation of IRR, NPV at $i = 20\%$ is calculated for each investment.

$$\begin{aligned} NPV_A &= -\$500,000 + 90,000(P/A_{20\%,20}) + 50,000(P/F_{20\%,20}) \\ &= -\$500,000 + 90,000(4.8696) + 50,000(0.0261) \\ &= -\$500,000 + 438,264 + 1,305 \\ &= -\$60,431.00 \end{aligned}$$

Similarly,

$$\begin{aligned} NPV_B &= -\$130,907.30 \\ NPV_C &= -\$169,558.00 \end{aligned}$$

Therefore, IRR for each investment alternative (by linear interpolation) is \$66,392.

$$IRR_A = \frac{66,392}{66,392 + 60,431} (20 - 15) + 15 = 17.62\%$$

$$IRR_B = \frac{66,106}{66,106 + 130,907.3} (20 - 15) + 15 = 16.61\%$$

and

$$IRR_C = 16.46\%$$

The PVR for each investment is as follows:

$$PVR_A = \frac{NPV_A @ 15\%}{I_A} = \frac{66,392}{500,000} = 0.133$$

$$PVR_B = \frac{NPV_B @ 15\%}{I_B} = \frac{62,106}{800,000} = 0.078$$

$$PVR_C = \frac{NPV_C @ 15\%}{I_C} = \frac{70,191}{1,000,000} = 0.07$$

The profitability indicators show all the investments are economical. Since the investments are mutually exclusive, only one is selected. Note the *NPV of each investment shows investment C should be selected while the IRR shows investment A should be selected*. The NPV profiles as shown in figure 7–6 intersect each other; therefore, incremental analyses are required.

First, incremental analysis of investments A and B are determined.

$$\begin{aligned} NPV_{(B-A)} &= -\$300,000 + 47,000(P/A_{15,20}) + 25,000(P/F_{15,20}) \\ &= -\$300,000 + 47,000(6.2593) + 25,000(0.0611) \\ &= -\$4,285.40 \end{aligned}$$

Since the $NPV_{(B-A)}$ is less than zero, investment A with the lower initial investment is selected. Similarly, since the $NPV_{(B-A)}$ at 15% is less than zero, the $IRR_{(B-A)}$ is less than $i_d = 15\%$. This also means investment A should be selected. Now the survivor (investment A) will be compared with investment C.

$$\begin{aligned} NPV_{(C-A)} &= -\$500,000 + 80,000(P/A_{15\%,20}) + 50,000(P/F_{15\%,20}) \\ &= -\$500,000 + 80,000(6.2593) + 50,000(0.0611) \\ &= \$3,799 \end{aligned}$$

Since $NPV_{(C-A)}$ is greater than zero, investment C with the higher initial cost is selected.

Example 7–10

A company is producing crude oil and associated gas from one of its offshore locations. The crude oil is transported to the tank farms through a dedicated flow line. The gas is stripped at the station into gas and raw NGL. The gas and raw NGL are transported to an onshore gas processing facility through 89-kilometer 24-inch and 12-inch pipelines, respectively. However,

the 12-inch NGL line is old and not reliable. The company is considering the following options.

1. Spike the stripped NGLs into the crude line and sell as crude oil.
2. Replace the 12-inch NGL pipeline with a new pipeline of the same diameter. This option will cost \$37 million. It will take approximately 30 months (starting 01/01/2003) to install the new line. The NGLs will be spiked into crude until the pipeline is replaced.

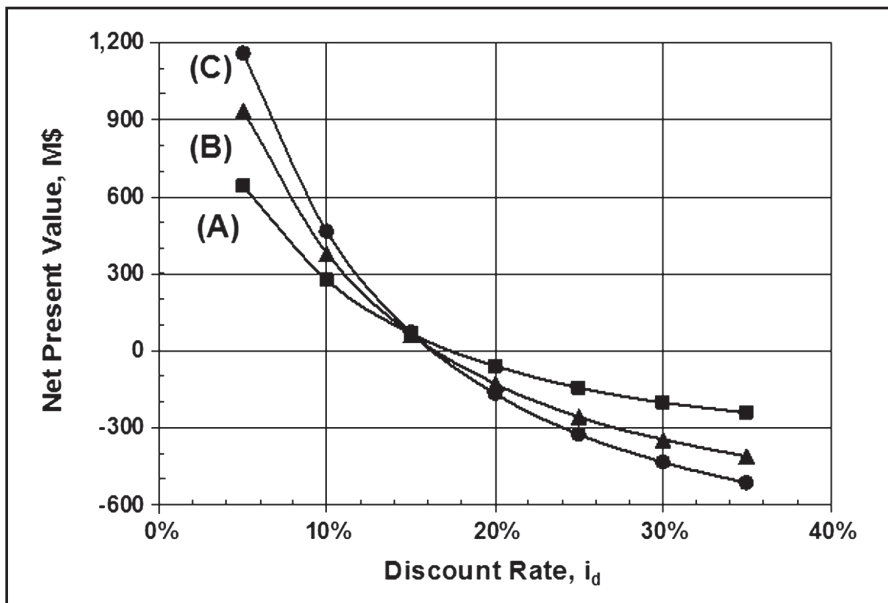


Fig. 7-6. NPV profiles for the investments in example 7-9

The resulting production streams for option 1 and option 2 are given in table 7-4. The raw NGL if transported to a gas processing facility is split into ethane (C_2) and LPG (C_{3+}). Assume the price of crude (net of operating expenses) to be \$17/Stb, the price of C_2 as \$22/ton, and the price of C_{3+} as \$175/ton. The capital expenditure is phased as 30% in 2002, 60% in 2003, and 10% in 2004. If the company's minimum acceptable rate of return is 10%, which option should be selected?

Solution: The cash flows for each option are shown in table 7-5. Since the NPV of option 1 is higher than the NPV of option 2, option 1 is recommended for implementation.

Table 7–4. Production forecast for the problem in example 7–10

	2010	2011	2012	2013	2014	2015	2016	2017	2018
Crude, Stb/d	1,807	1,580	2,174	2,045	1,988	1,889	1,785	1,669	1,540
Ethane, T/d	80	76	80	73	70	68	62	55	50
LPG, T/d	321	279	340	323	316	299	272	242	238

Table 7–5. Economics of pipeline replacement (example 7–10)

Year	OPTION 1			OPTION 2						
	Crude Prod. Stb/d	Crude Price \$/Stb	Net Rev. MM\$	Crude Prod. Stb/d	C ₂ Prod. T/d	C ₃ + Prod. T/d	C ₂ Price \$/T	C ₃ + Price \$/T	CAPEX MM\$	Net Rev. MM\$
2010	1,807	17.00	11.21	1,807			22	175	10.5	0.71
2011	1,580	17.00	9.80	1,580			22	175	21.00	(11.20)
2012	2,174	17.00	13.49	1,087	40	170	22	175	3.5	14.42
2013	2,045	17.00	12.69		73	323	22	175		21.22
2014	1,988	17.00	12.34		70	316	22	175		20.75
2015	1,889	17.00	11.72		68	299	22	175		19.64
2016	1,785	17.00	11.08		62	272	22	175		17.87
2017	1,669	17.00	10.36		55	242	22	175		15.90
2018	1,540	17.00	9.56		50	238	22	175		15.60
TOTAL			102.24						35.00	114.93
NPV			65.94							63.90

Non-mutually exclusive investment analysis

In most firms, there are more proposals for investments than the firm is able or willing to finance (i.e., budget constraint). Some proposals are good, others are bad, and methods must be developed for distinguishing between the good and the bad. Essentially, the result is ranking of the proposals and a cutoff point for determining how far down the ranked list to go. In this type of investment analysis, several investment alternatives will be available and more than one may be selected, depending on the availability of funds. The investments have to be selected in such a way that the total worth of the

investor is maximized within the budget constraint (i.e., available investment dollars).

The ranking criterion is only necessary when the investment funds are in short supply or are limited by management strategy/policy. The first step is the same as in the previous section: apply accept or reject criteria. Only the economically acceptable investments are then considered. The following rules apply.

1. Investments are selected in order of decreasing PI. The easiest way is to arrange all the investment opportunities in descending order. Accumulate the CAPEX of each investment opportunity until the budget constraint is met. Some minor fine-tuning at the end may be required to make sure that a majority of the available funds have been utilized.
2. IRR analysis, due to its limitations and conflicting results, cannot be applied to this type of decision analysis.
3. The other profitability indicators, such as NPV and GRR, will also fail to give the optimized ranking.

Remember, in the selection of non-mutually exclusive investments, not all of the available cash may be evenly invested in the available alternatives. The remaining cash will be invested elsewhere at the minimum acceptable rate of return, i.e. cost of capital. The following example shows the ranking of non-mutually exclusive investments.

Example 7–11

The cash flows of 10 different investment alternatives are given in table 7–6. Calculate the NPV, IRR, and PI for each investment. If the available funds are limited to \$1.2 million, select the investments that will maximize the net worth of the investor. The minimum acceptable rate of return and the reinvestment rate of the investor are 10%.

Solution: The ranking of the projects based on PI are shown in table 7–7. To maximize the net worth of the investor within the budget constraint, the PIs of the investments are arranged in descending order of PI and against the cumulative investments, as shown in table 7–8. The investments falling below the capital constraint of \$1.2 million are selected. The selected investments will yield a net worth of \$250.56 million for a total investment of \$1.075 million. The balance of \$0.125 million will be invested elsewhere at the investor's minimum acceptable rate of return.

Table 7–6. Cash flows for 10 different investments for example 7–11

Year	A	B	C	D	E	F	G	H	I	J
0	–100	–120	–100	–50	–80	–200	–250	–150	–300	–375
1	45	35	35	17	24	60	80	60	100	150
2	35	30	30	15	20	55	70	55	85	120
3	25	25	25	12	17	50	60	50	75	100
4	20	22	22	10	15	45	50	45	60	80
5	15	21	21	8	13	40	40	40	45	45
6		19	19	6	11	35	30	35	35	35
7		15	15	5	9	30	20	30	20	20
8		13	13	4	7		15		13	13
9		12	12	3	6		12		12	12
10		11	11	3	5		10		10	10
11				2	5		8		8	8
12					3		6		6	6
13					2					
14					2					
NPV, MM\$	11.6	17.3	37.3	9.6	11.8	28.3	32.5	78.3	36.2	68.0
IRR, %	15.48	14.12	20.44	16.1	14.12	14.75	14.20	26.73	14.08	16.75
GRR, %	10.87	11.06	12.52	11.39	11.09	11.04	10.96	13.35	10.9	11.32
PI	1.116	1.144	1.373	1.192	1.148	1.141	1.130	1.522	1.121	1.181

Table 7–7. Ranking investments of table 7–6 (example 7–11)

NPV	H	J	C	I	G	F	B	E	A	D
IRR	H	C	J	D	A	F	G	B	E	I
GRR	H	C	D	J	E	B	F	G	I	A
PI	H	C	D	J	E	B	F	G	I	A

Table 7–8. Ranking projects under budget constraint (using PI), table 7–6 (example 7–11)

Projects	PI	CAPEX MM\$	Cumulative CAPEX MM\$	NPV MM\$	Cumulative NPV MM\$
H	1.522	150	150	78.29	78.29
C	1.373	100	250	37.28	115.57
D	1.192	50	300	9.61	125.18
J	1.181	375	675	68.00	193.18
E	1.148	80	755	11.82	204.99
B	1.144	120	875	17.28	222.27
F	1.141	200	1,075	28.29	250.56
G	1.130	250	X	32.50	X
I	1.121	300	X	36.17	X
A	1.116	100	1,175	11.59	262.15

Many people ask me whether the NPV should also give the same result. The NPV will not always give the best selection and will not maximize the total NPV of the total portfolio. The investments are ranked in descending order of NPV and the associated CAPEX is accumulated in front of each alternative. The investments are selected until the accumulated budget is in line with the available budget. The ranking and selection of the investments based on the NPV approach is shown in table 7–9.

It is worth noting that both methods—NPV and PI—end up with spending the same CAPEX of \$1,075 million. However, the resulting total NPV from the portfolio is about \$9 million less in the case of the NPV selection versus the PI selection.

Another very useful way of ranking projects is to plot the PI versus the NPV, as shown in figure 7–7. The plot, with project numbers assigned to each data point, will visually show which of the investments are more promising as compared to the others.

Table 7–9. Ranking projects under budget constraint (using NPV), table 7–6 (example 7–11)

Projects	NPV MM\$	CAPEX MM\$	Cumulative CAPEX MM\$	Cumulative NPV MM\$
H	78.290	150	150	78.290
J	67.996	375	525	146.29
C	37.277	100	625	183.56
I	36.173	300	925	219.74
G	32.500	250	1,175	252.24
F	28.290	200	1,375	280.53
B	17.277	120	1,495	297.80
E	11.816	80	1,575	309.62
A	11.592	100	1,675	321.21
D	9.613	50	1,725	330.82

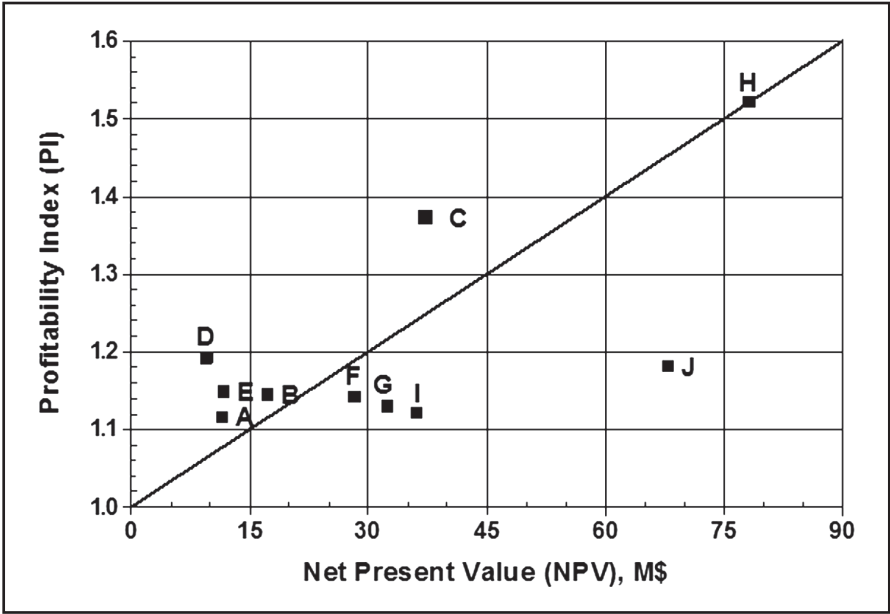


Fig. 7–7. Graphically ranking non-mutually exclusive investments

Questions and Problems

- 7.1 Give two examples of each of the following:
 - a. Mutually exclusive projects
 - b. Non-mutually exclusive projects
 - c. Complementary projects
 - d. Substitute projects
- 7.2 How would the NPV, IRR, GRR, and PI be used to select among mutually exclusive investment alternatives?
- 7.3 How would the NPV, IRR, GRR, and PI be used to rank non-mutually exclusive investment alternatives?
- 7.4 How would the NPV, IRR, GRR, and PI be used to screen investments?
- 7.5 Calculate the NPV of the cash flow shown in the following diagram if the minimum desired rate of return is 8%. Is the investment economically viable?

	A	B	C	D	E
1	1	2	3	4	5
2	(200)	(500)	300	300	300

Question 7.5 diagram

- 7.6 The IRR of the investment in question 7.5 is
 - a. > 8%
 - b. < 8%
 - c. = 8%
- 7.7 Calculate the IRR of the investment in question 7.5 if (1) the NPV at 5% is 97, (2) the NPV at 10% is 22, and (c) the NPV at 15% is -34.
 - a. 8.5%
 - b. 9.8%
 - c. 12.0%
 - d. 12.8%

7.8 What would be the PI of the investment in question 7.5?

- a. 0.08
- b. 1.25
- c. 1.15
- d. 1.08

7.9 If the investments in the following diagram are mutually exclusive, which project should be selected? Use NPV @ 8% for making the decision.

- a. A
- b. B
- c. C
- d. D

	A	B	C	D	E	F
1	Project	1	2	3	4	5
2	A	(200)	75	75	75	75
3	B	(450)	155	155	155	155
4	C	(700)	250	250	250	250
5	D	(950)	380	330	280	230

Question 7.9 diagram

7.10 If the investments in question 7.9 are mutually exclusive, which project should be selected? Use PI for making the decision.

- a. A
- b. B
- c. C
- d. D

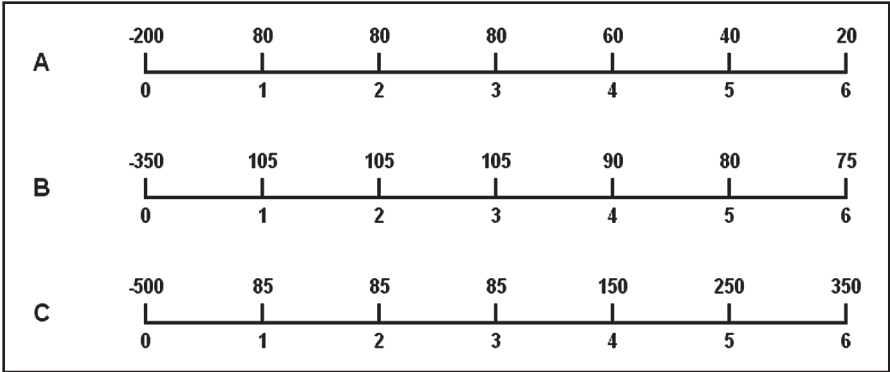
- 7.11 For the investments in question 7.9, perform incremental PI analysis to confirm your selection in question 7.9.
- a. A
 - b. B
 - c. C
 - d. D
- 7.12 Which of the investments in question 7.9 may be selected if they are non-mutually exclusive but we have a budget constraint of \$1,400 to optimize our portfolio?
- a. A, B, C, & D
 - b. A, B, & D
 - c. A, B, & C
 - d. A, C, & D
- 7.13 The cash flow in the following diagram shows the initial cost and the ongoing maintenance cost of two compressors. Which compressor should be selected if both give the same service? Use PV of cost analysis, discount rate is 8%.
- a. A
 - b. B

	A	B	C	D	E	F
1	Project	1	2	3	4	5
2	Compressor A	(200)	(135)	(135)	(135)	(135)
3	Compressor B	(350)	(100)	(100)	(100)	(100)

Question 7.13 diagram

- 7.14 Which of the two compressors in question 7.13 should be selected, using appropriate PI analysis?
- a. A
 - b. B

- 7.15 Calculate the NPV, using 8% discount rate, of each of the investments shown in the following diagram. Which investment should be selected if they are mutually exclusive?
- a. A
 - b. B
 - c. C



Question 7.15 diagram

- 7.16 Which project will be selected if $IRR_{(B-A)} = 8\%$, $IRR_{(C-B)} = 20\%$, and $IRR_{(C-A)} = 16\%$? The desired rate of return is 10%.
- a. Project A
 - b. Project B
 - c. Project C

- 7.17 After using machine A for five years, the company has an opportunity to purchase machine B, which would replace machine A. Machine B would have a five-year life, cost \$80,000, have a salvage value of \$20,000, generate gross revenue of \$150,000 per year, and reduce operating expenses by \$10,000 per year. If investing in machine B replaces machine A, the amount realized on the sale of machine A would be \$30,000.
- a. What change in cash flow does replacing machine A create?
 - b. Calculate the NPV, at discount rate of 10%, for the investment in machine B.
 - c. Compare the NPV of machine B to the NPV of machine A. Should machine A be replaced by machine B if the cost of capital is 12%?
 - d. Rework part C above using IRR as a decision criterion.
- 7.18 Two investment alternatives are under consideration and both require \$20,000 capital. Investment A will generate net cash flow of \$9,000 in each of the first two years and \$2,500 in the last three years. Investment B will generate \$3,500 in each of the first two years and \$7,000 in each of the last three years. Calculate the payback period and NPV for each investment. If the alternatives are mutually exclusive and the opportunity cost of the investor is 8%, which alternative should be selected based on (a) payback period and (b) net present value?
- 7.19 A small independent oil company is considering investing some money in a drilling prospect. Two prospects are under consideration out of which only one will be drilled this year. The initial drilling and completion costs and net revenues (revenues – expenses) are provided below.

	Prospect A	Prospect B
Drilling and completion cost	–\$800,000	–\$1,500,000
Net revenue, years 2 to 5	\$200,000	\$350,000
Net revenue, years 6 to 10	\$250,000	\$250,000

- a. Compute the payback period for each investment.
- b. Based on NPV analysis, which prospect should be selected if the company’s minimum acceptable rate of return is 12%?
- c. Compute the IRR for each investment. If the investment with the highest IRR is the economic choice, which prospect should be selected? Is the selection consistent with the selection made using NPV analysis? Why?

7.20 Assume an investor has to evaluate the following three projects:

	Cash flow					
	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
Project A	−\$1,200	\$358	\$340	\$322	\$306	\$290
Project B	−\$1,200	\$350	\$320	\$320	\$320	\$320
Project C	−\$1,200	0	0	0	0	\$2,000

- a. Calculate the NPV of each of the above projects, assuming a 10% discount rate, and rank the projects.
- b. Calculate the IRR for each of the above projects and rank the projects.
- c. Calculate the NPV of each of the above projects based on 6% discount rate and rank the projects.
- d. Calculate the NPV of each of the above projects based on 11% discount rate and rank the projects.
- e. Compare and explain the ranking obtained in parts (a) to (d) above, using a diagram that graphs the NPV as a function of discount rate.

7.21 Three mutually exclusive investment alternatives are under consideration. The initial capital outlays and the pattern of the net annual cash benefits (revenues – expenses) for each alternative are presented in the following table. Based on NPV analysis, if the company’s minimum acceptable rate of return is 10%, which alternative should be the best economic choice?

	Investment		
	A	B	C
Initial cost, M\$	–\$200	–\$350	–\$500
Net revenue, year 1 to 3	\$80	\$105	\$85
Net revenue, year 4	\$60	\$90	\$150
Net revenue, year 5	\$40	\$80	\$250
Net revenue, year 6	\$20	\$75	\$350

7.22 Consider the following two mutually exclusive investment opportunities:

	Cash flow					
	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
Project A	–\$48,675	\$17,000	\$17,000	\$17,000	\$17,000	\$17,000
Project B	–\$31,575	\$12,000	\$12,000	\$12,000	\$12,000	\$12,000

- a. Calculate each investment’s NPV and IRR (assume the cost of capital is 8%).

b. Which of the two investments would be chosen according to the IRR criterion? Which according to the NPV criterion?

c. Explain the differences in rankings given by the NPV and IRR methods in each case.
- 7.23 Calculate the PI for the three investments in question 7.22. Do all these indicators give the same selection? If the selection using these profitability yardsticks is not consistent with the selection in question 7.22, then use appropriate incremental analysis to defend your finding in question 7.22.

7.24 A small independent oil company has \$2,000,000 to invest in 2011. The company has four drilling prospects, all of which are considered of equal risk but their initial drilling and completion costs and the patterns of their generated benefits are different. The cash flow pattern for each prospect is given below. Using the various profitability indicators presented in this chapter, which one or more of the available prospects should be drilled to maximize the investment worth of the company? The money can be invested elsewhere at a rate of 8%.

Prospect	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
A	−\$450	\$150	\$150	\$150	\$150	\$150				
B	−\$550	\$300	\$210	\$147	\$103	\$72				
C	−\$1,000	\$400	\$350	\$300	\$250	\$200	\$150	\$100	\$50	
D	−\$1,500	\$500	\$475	\$450	\$425	\$400	\$350	\$300	\$250	\$200

- 7.25 Consider the investment of question 7.24. Use PVR analysis to select the best economic combination of prospects that will maximize the profit within the budget constraint of \$2,000,000.
- 7.26 The XYZ oil company has recently drilled an oil well. The company has tested two productive formations approximately 1,000 feet apart. According to state regulations, only one formation can be produced at a time. This means dual or commingle completions are not allowed. The lower formation will be put on production at a rate of 1,500 barrels/month declining exponentially at a rate of 30% per year to economic limit of 252 barrels/month. To produce the other formation, the company has two alternatives: (A) to complete the behind pipe formation after the lower formation is depleted, or (B) drill a twin well now to produce the upper formation. The recompletion cost of alternative A would be \$120,000. The drilling of twin wells will cost \$850,000 now. In either case, the formation will produce at a rate of 1,500 barrels/month declining exponentially at a rate of 25% per year to the economic limit of 252 barrels/month. In each case, the operating expenses are estimated to be \$14,277 per month, and the oil price would be \$70 per barrel. Using an appropriate technique, select the best economic choice (assume 100% WI, 87.5% NRI, 7.5% Severance Tax and the discount rate to be 15%). Assume that the cost escalation is offset by the increase in oil price and it takes one year to drill a new well or recomplete an existing well into another formation.

- 7.27 Repeat problem 7.27 assuming all costs are escalated at 3% per year and the oil price is escalating at 2% per year, starting from 2012.
- 7.28 Three mutually exclusive investment alternatives are available. Alternative A costs \$400,000 and will generate net revenue of \$200,000 per year for the next five years. Alternative B costs \$850,000 and will generate \$400,000 in year one and then decrease by a constant gradient of \$50,000 for each of the next four years. Alternative C will cost \$1,500,000 and will generate net revenue of \$500,000 in each of the first three years and \$300,000 per year in the remaining two years. The opportunity cost of capital is 15%. Basing your decision on NPV analysis, which alternative should be selected? Use appropriate IRR analysis to double-check your selection.
- 7.29 A gas well has just been completed at a cost of \$450,000. Due to the poor gas market, the pipeline companies will not purchase gas at deliverability. If they do purchase the gas at deliverability, they will pay only half the regular price of \$6/MScf for four years and then pay the regular price thereafter. Alternatively, they will pay the regular price but take half the deliverability for two years and starting the third year, they will purchase gas at the deliverability and pay \$7/MScf for the remaining life of the well. The well's deliverability is \$10,000 MScf/month declining exponentially at 30% per year to the economic limit of 575 MScf/month. If the well is curtailed, it will produce at 5,000 MScf/month for the next two years. Starting the third year, the rate will be back to normal at 6,400 MScf/month declining at 20% to the economic limit of 575 MScf/month. The operating expenses and taxes in each case amount to \$3,500/month. If the opportunity cost of capital of the company is 15%, which alternative should be selected?
- 7.30 A company is considering acquisition of proved reserves worth a total of \$3,000,000. Four properties are being considered. Alternative A will cost \$500,000 and will generate net revenue of \$200,000 for each of the next five years. Property B costs \$800,000 and will generate net revenue of \$500,000 in the first year and declining by \$50,000 from the second year for each of the next four years. Property C will cost \$1,700,000 and will generate net revenue of \$600,000 in the first year and then declining thereafter by \$25,000 per year for the next three years. In the fifth year it will generate \$300,000 and declining thereafter by \$50,000 per year for each of the next three years. Property D will cost \$2,500,000

and will generate net revenue of \$1,100,000 in the first year and declining thereafter by 25% per year until the economic limit of \$147,000 per year. If the company has the opportunity to invest all or part of the money elsewhere at 15% per year, which projects, if any, should be selected in order to maximize its investment worth?

- 7.31 Consider the initial costs and net revenue profiles of the following four investments. Use the appropriate techniques of investment analysis to select the most economic investment alternative. Carefully study the results obtained by each technique. Which techniques have given consistent selection of the investments? For all cases assume the opportunity cost of capital of 15%.

	A	B	C	D
Initial cost, M\$	−\$1,000	−\$2,500	−\$4,000	−\$7,000
Net revenue, M\$				
Year 1 through 4	\$350	\$800	\$1,300	\$2,300
Year 5 through 8	\$210	\$400	\$500	\$700
Year 9 through 12	\$60	\$160	\$250	\$420

- 7.32 Rework question 7.31, assuming the alternatives are non-mutually exclusive and we have a capital budget of \$6,500,000. All or any part of this money can be invested elsewhere at an interest rate of 15%. Which alternatives, if any, should be selected to optimize net worth?
- 7.33 Rework question 7.32, assuming the money can be invested elsewhere at an interest rate of 20%. Does this change your selection of the investments?
- 7.34 A company is examining the cost of some of its maintenance equipment. Its present equipment is costing \$20,000 per year to operate, and an additional \$5,000 expense is expected during the third year. The company estimates a salvage value of \$3,500 after six years of operation. The equipment can be upgraded at a cost of \$13,500. After upgrade, operating costs are expected to be \$17,000 per year, with no other costs. Salvage value after six years is expected to be \$8,500. Determine if it is feasible to upgrade the equipment, assuming the \$13,500 required for upgrade is earning interest at a rate of 8% per year.

7.35 A firm must decide between two models of a machine. The estimates shown in the following table apply. The firm evaluates all equipment purchases based on 8% rate of return. Which is the preferred model based on (a) equivalent annual cost, (b) present value of cost, (c) future value of cost, and (d) internal rate of return?

	Model A	Model B
Initial cost	\$90,000	\$125,000
Life, years	5	5
Salvage value	\$15,000	\$50,000
Operating/maintenance cost per year	\$44,000	\$40,000

7.36 Evaluate and compare the operating expenses of the 456D Mark II pumping units currently in use in a 6,000 ft waterflood and a planned hydraulic lift system for the same waterflood. The cost data for the two alternatives is given as follows.

456D API Beam Pumping Units	Hydraulic Pumping Units
Maintenance expenses for one-year show surface well equipment is \$20,403, pump repair costs \$15,319, and well service expense is \$32,723. Ninety percent of these costs are allocated to the producing wells. The waterflood has 26 × 228 API pumping units with 25 hp electric motors and 13 × 456D Mark II pumping units with 60 hp electric motors. Maintenance expenses are allocated to the producing wells based on electric motor horsepower. Electric power costs \$0.11/kwh.	The planned hydraulic system will utilize one 115 hp Ajax engine for every two wells. The Ajax engine’s maintenance will cost an average of \$22 per hp-year. The other surface hydraulic equipment per two well unit will cost \$1,880/year in maintenance. Hydraulic bottom pump repairs average \$600/repair. Three such pumps are repaired per year per well. Other service costs are \$600 per year per well. Gas costs \$0.14/MScf.

Calculate:

- a. The separate monthly maintenance cost for the Mark II Beam pumped wells and the hydraulic pumped wells (on a per-well basis).
- b. The separate monthly power costs per well for the two systems. Assume electric motors use $3/4$ kwh/hp-hr. Assume 65% utilization rate for the time clock controlled Mark II units. The Ajax engines use 13 cu ft of gas per hp-hr. The hydraulic surface equipment averages a utilization rate of 95% on any power system.
- c. The power cost of a 100 hp electric-powered hydraulic surface pump (\$/month/well).

7.37 Rework example 7–10 if a third, technically feasible, option of replacing the 12-inch NGL line with an 89-kilomer 6-inch pipeline is available. The 6-inch pipeline will cost \$31 million. Timing of the capital expenditure is the same as the 12-inch line.

References

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A

Appendix

Interest Factors for Annual Interest Compounding

Table A–1. Interest factors for annual interest compounding, $i = 1.0\%$

t	Single Payment		Equal Payment Series				Uniform Gradient Series, A/G
	Compound Amount	Present Worth	Compound Amount	Sinking Fund	Present Worth	Capital Recovery	
	Factor, F/P	Factor, P/F	Factor, F/A	Factor, A/F	Factor, P/A	Factor, A/P	
1	1.0100	0.9901	1.0000	1.0000	0.9901	1.0100	0.0000
2	1.0201	0.9803	2.0100	0.4975	1.9704	0.5075	0.4975
3	1.0303	0.9706	3.0301	0.3300	2.9410	0.3400	0.9934
4	1.0406	0.9610	4.0604	0.2463	3.9020	0.2563	1.4876
5	1.0510	0.9515	5.1010	0.1960	4.8534	0.2060	1.9801
6	1.0615	0.9420	6.1520	0.1625	5.7955	0.1725	2.4710
7	1.0721	0.9327	7.2135	0.1386	6.7282	0.1486	2.9602
8	1.0829	0.9235	8.2857	0.1207	7.6517	0.1307	3.4478
9	1.0937	0.9143	9.3685	0.1067	8.5660	0.1167	3.9337
10	1.1046	0.9053	10.4622	0.0956	9.4713	0.1056	4.4179
11	1.1157	0.8963	11.5668	0.0865	10.3676	0.0965	4.9005
12	1.1268	0.8874	12.6825	0.0788	11.2551	0.0888	5.3815
13	1.1381	0.8787	13.8093	0.0724	12.1337	0.0824	5.8607
14	1.1495	0.8700	14.9474	0.0669	13.0037	0.0769	6.3384
15	1.1610	0.8613	16.0969	0.0621	13.8651	0.0721	6.8143
16	1.1726	0.8528	17.2579	0.0579	14.7179	0.0679	7.2886
17	1.1843	0.8444	18.4304	0.0543	15.5623	0.0643	7.7613
18	1.1961	0.8360	19.6147	0.0510	16.3983	0.0610	8.2323
19	1.2081	0.8277	20.8109	0.0481	17.2260	0.0581	8.7017
20	1.2202	0.8195	22.0190	0.0454	18.0456	0.0554	9.1694

Table A–2. Interest factors for annual interest compounding, $i = 2.0\%$

t	Single Payment		Equal Payment Series				Uniform Gradient Series, A/G
	Compound Amount	Present Worth	Compound Amount	Sinking Fund	Present Worth	Capital Recovery	
	Factor, F/P	Factor, P/F	Factor, F/A	Factor, A/F	Factor, P/A	Factor, A/P	
1	1.0200	0.9804	1.0000	1.0000	0.9804	1.0200	0.0000
2	1.0404	0.9612	2.0200	0.4950	1.9416	0.5150	0.4950
3	1.0612	0.9423	3.0604	0.3268	2.8839	0.3468	0.9868
4	1.0824	0.9238	4.1216	0.2426	3.8077	0.2626	1.4752
5	1.1041	0.9057	5.2040	0.1922	4.7135	0.2122	1.9604
6	1.1262	0.8880	6.3081	0.1585	5.6014	0.1785	2.4423
7	1.1487	0.8706	7.4343	0.1345	6.4720	0.1545	2.9208
8	1.1717	0.8535	8.5830	0.1165	7.3255	0.1365	3.3961
9	1.1951	0.8368	9.7546	0.1025	8.1622	0.1225	3.8681
10	1.2190	0.8203	10.9497	0.0913	8.9826	0.1113	4.3367
11	1.2434	0.8043	12.1687	0.0822	9.7868	0.1022	4.8021
12	1.2682	0.7885	13.4121	0.0746	10.5753	0.0946	5.2642
13	1.2936	0.7730	14.6803	0.0681	11.3484	0.0881	5.7231
14	1.3195	0.7579	15.9739	0.0626	12.1062	0.0826	6.1786
15	1.3459	0.7430	17.2934	0.0578	12.8493	0.0778	6.6309
16	1.3728	0.7284	18.6393	0.0537	13.5777	0.0737	7.0799
17	1.4002	0.7142	20.0121	0.0500	14.2919	0.0700	7.5256
18	1.4282	0.7002	21.4123	0.0467	14.9920	0.0667	7.9681
19	1.4568	0.6864	22.8406	0.0438	15.6785	0.0638	8.4073
20	1.4859	0.6730	24.2974	0.0412	16.3514	0.0612	8.8433

Table A–3. Interest factors for annual interest compounding, $i = 3.0\%$

t	Single Payment		Equal Payment Series				Uniform Gradient Series, A/G
	Compound Amount	Present Worth	Compound Amount	Sinking Fund	Present Worth	Capital Recovery	
	Factor, F/P	Factor, P/F	Factor, F/A	Factor, A/F	Factor, P/A	Factor, A/P	
1	1.0300	0.9709	1.0000	1.0000	0.9709	1.0300	0.0000
2	1.0609	0.9426	2.0300	0.4926	1.9135	0.5226	0.4926
3	1.0927	0.9151	3.0909	0.3235	2.8286	0.3535	0.9803
4	1.1255	0.8885	4.1836	0.2390	3.7171	0.2690	1.4631
5	1.1593	0.8626	5.3091	0.1884	4.5797	0.2184	1.9409
6	1.1941	0.8375	6.4684	0.1546	5.4172	0.1846	2.4138
7	1.2299	0.8131	7.6625	0.1305	6.2303	0.1605	2.8819
8	1.2668	0.7894	8.8923	0.1125	7.0197	0.1425	3.3450
9	1.3048	0.7664	10.1591	0.0984	7.7861	0.1284	3.8032
10	1.3439	0.7441	11.4639	0.0872	8.5302	0.1172	4.2565
11	1.3842	0.7224	12.8078	0.0781	9.2526	0.1081	4.7049
12	1.4258	0.7014	14.1920	0.0705	9.9540	0.1005	5.1485
13	1.4685	0.6810	15.6178	0.0640	10.6350	0.0940	5.5872
14	1.5126	0.6611	17.0863	0.0585	11.2961	0.0885	6.0210
15	1.5580	0.6419	18.5989	0.0538	11.9379	0.0838	6.4500
16	1.6047	0.6232	20.1569	0.0496	12.5611	0.0796	6.8742
17	1.6528	0.6050	21.7616	0.0460	13.1661	0.0760	7.2936
18	1.7024	0.5874	23.4144	0.0427	13.7535	0.0727	7.7081
19	1.7535	0.5703	25.1169	0.0398	14.3238	0.0698	8.1179
20	1.8061	0.5537	26.8704	0.0372	14.8775	0.0672	8.5229

Table A–4. Interest factors for annual interest compounding, i = 4.0%

t	Single Payment		Equal Payment Series				Uniform Gradient Series, A/G
	Compound Amount Factor, F/P	Present Worth Factor, P/F	Compound Amount Factor, F/A	Sinking Fund Factor, A/F	Present Worth Factor, P/A	Capital Recovery Factor, A/P	
1	1.0400	0.9615	1.0000	1.0000	0.9615	1.0400	0.0000
2	1.0816	0.9246	2.0400	0.4902	1.8861	0.5302	0.4902
3	1.1249	0.8890	3.1216	0.3203	2.7751	0.3603	0.9739
4	1.1699	0.8548	4.2465	0.2355	3.6299	0.2755	1.4510
5	1.2167	0.8219	5.4163	0.1846	4.4518	0.2246	1.9216
6	1.2653	0.7903	6.6330	0.1508	5.2421	0.1908	2.3857
7	1.3159	0.7599	7.8983	0.1266	6.0021	0.1666	2.8433
8	1.3686	0.7307	9.2142	0.1085	6.7327	0.1485	3.2944
9	1.4233	0.7026	10.5828	0.0945	7.4353	0.1345	3.7391
10	1.4802	0.6756	12.0061	0.0833	8.1109	0.1233	4.1773
11	1.5395	0.6496	13.4864	0.0741	8.7605	0.1141	4.6090
12	1.6010	0.6246	15.0258	0.0666	9.3851	0.1066	5.0343
13	1.6651	0.6006	16.6268	0.0601	9.9856	0.1001	5.4533
14	1.7317	0.5775	18.2919	0.0547	10.5631	0.0947	5.8659
15	1.8009	0.5553	20.0236	0.0499	11.1184	0.0899	6.2721
16	1.8730	0.5339	21.8245	0.0458	11.6523	0.0858	6.6720
17	1.9479	0.5134	23.6975	0.0422	12.1657	0.0822	7.0656
18	2.0258	0.4936	25.6454	0.0390	12.6593	0.0790	7.4530
19	2.1068	0.4746	27.6712	0.0361	13.1339	0.0761	7.8342
20	2.1911	0.4564	29.7781	0.0336	13.5903	0.0736	8.2091

Table A–5. Interest factors for annual interest compounding, i = 5.0%

t	Single Payment		Equal Payment Series				Uniform Gradient Series, A/G
	Compound Amount Factor, F/P	Present Worth Factor, P/F	Compound Amount Factor, F/A	Sinking Fund Factor, A/F	Present Worth Factor, P/A	Capital Recovery Factor, A/P	
1	1.0500	0.9524	1.0000	1.0000	0.9524	1.0500	0.0000
2	1.1025	0.9070	2.0500	0.4878	1.8594	0.5378	0.4878
3	1.1576	0.8638	3.1525	0.3172	2.7232	0.3672	0.9675
4	1.2155	0.8227	4.3101	0.2320	3.5460	0.2820	1.4391
5	1.2763	0.7835	5.5256	0.1810	4.3295	0.2310	1.9025
6	1.3401	0.7462	6.8019	0.1470	5.0757	0.1970	2.3579
7	1.4071	0.7107	8.1420	0.1228	5.7864	0.1728	2.8052
8	1.4775	0.6768	9.5491	0.1047	6.4632	0.1547	3.2445
9	1.5513	0.6446	11.0266	0.0907	7.1078	0.1407	3.6758
10	1.6289	0.6139	12.5779	0.0795	7.7217	0.1295	4.0991
11	1.7103	0.5847	14.2068	0.0704	8.3064	0.1204	4.5144
12	1.7959	0.5568	15.9171	0.0628	8.8633	0.1128	4.9219
13	1.8856	0.5303	17.7130	0.0565	9.3936	0.1065	5.3215
14	1.9799	0.5051	19.5986	0.0510	9.8986	0.1010	5.7133
15	2.0789	0.4810	21.5786	0.0463	10.3797	0.0963	6.0973
16	2.1829	0.4581	23.6575	0.0423	10.8378	0.0923	6.4736
17	2.2920	0.4363	25.8404	0.0387	11.2741	0.0887	6.8423
18	2.4066	0.4155	28.1324	0.0355	11.6896	0.0855	7.2034
19	2.5270	0.3957	30.5390	0.0327	12.0853	0.0827	7.5569
20	2.6533	0.3769	33.0660	0.0302	12.4622	0.0802	7.9030

Table A–6. Interest factors for annual interest compounding, i = 6.0%

t	Single Payment		Equal Payment Series				Uniform Gradient Series, A/G
	Compound Amount	Present Worth	Compound Amount	Sinking Fund	Present Worth	Capital Recovery	
	Factor, F/P	Factor, P/F	Factor, F/A	Factor, A/F	Factor, P/A	Factor, A/P	
1	1.0600	0.9434	1.0000	1.0000	0.9434	1.0600	0.0000
2	1.1236	0.8900	2.0600	0.4854	1.8334	0.5454	0.4854
3	1.1910	0.8396	3.1836	0.3141	2.6730	0.3741	0.9612
4	1.2625	0.7921	4.3746	0.2286	3.4651	0.2886	1.4272
5	1.3382	0.7473	5.6371	0.1774	4.2124	0.2374	1.8836
6	1.4185	0.7050	6.9753	0.1434	4.9173	0.2034	2.3304
7	1.5036	0.6651	8.3938	0.1191	5.5824	0.1791	2.7676
8	1.5938	0.6274	9.8975	0.1010	6.2098	0.1610	3.1952
9	1.6895	0.5919	11.4913	0.0870	6.8017	0.1470	3.6133
10	1.7908	0.5584	13.1808	0.0759	7.3601	0.1359	4.0220
11	1.8983	0.5268	14.9716	0.0668	7.8869	0.1268	4.4213
12	2.0122	0.4970	16.8699	0.0593	8.3838	0.1193	4.8113
13	2.1329	0.4688	18.8821	0.0530	8.8527	0.1130	5.1920
14	2.2609	0.4423	21.0151	0.0476	9.2950	0.1076	5.5635
15	2.3966	0.4173	23.2760	0.0430	9.7122	0.1030	5.9260
16	2.5404	0.3936	25.6725	0.0390	10.1059	0.0990	6.2794
17	2.6928	0.3714	28.2129	0.0354	10.4773	0.0954	6.6240
18	2.8543	0.3503	30.9057	0.0324	10.8276	0.0924	6.9597
19	3.0256	0.3305	33.7600	0.0296	11.1581	0.0896	7.2867
20	3.2071	0.3118	36.7856	0.0272	11.4699	0.0872	7.6051

Table A–7. Interest factors for annual interest compounding, i = 7.0%

t	Single Payment		Equal Payment Series				Uniform Gradient Series, A/G
	Compound Amount	Present Worth	Compound Amount	Sinking Fund	Present Worth	Capital Recovery	
	Factor, F/P	Factor, P/F	Factor, F/A	Factor, A/F	Factor, P/A	Factor, A/P	
1	1.0700	0.9346	1.0000	1.0000	0.9346	1.0700	0.0000
2	1.1449	0.8734	2.0700	0.4831	1.8080	0.5531	0.4831
3	1.2250	0.8163	3.2149	0.3111	2.6243	0.3811	0.9549
4	1.3108	0.7629	4.4399	0.2252	3.3872	0.2952	1.4155
5	1.4026	0.7130	5.7507	0.1739	4.1002	0.2439	1.8650
6	1.5007	0.6663	7.1533	0.1398	4.7665	0.2098	2.3032
7	1.6058	0.6227	8.6540	0.1156	5.3893	0.1856	2.7304
8	1.7182	0.5820	10.2598	0.0975	5.9713	0.1675	3.1465
9	1.8385	0.5439	11.9780	0.0835	6.5152	0.1535	3.5517
10	1.9672	0.5083	13.8164	0.0724	7.0236	0.1424	3.9461
11	2.1049	0.4751	15.7836	0.0634	7.4987	0.1334	4.3296
12	2.2522	0.4440	17.8885	0.0559	7.9427	0.1259	4.7025
13	2.4098	0.4150	20.1406	0.0497	8.3577	0.1197	5.0648
14	2.5785	0.3878	22.5505	0.0443	8.7455	0.1143	5.4167
15	2.7590	0.3624	25.1290	0.0398	9.1079	0.1098	5.7583
16	2.9522	0.3387	27.8881	0.0359	9.4466	0.1059	6.0897
17	3.1588	0.3166	30.8402	0.0324	9.7632	0.1024	6.4110
18	3.3799	0.2959	33.9990	0.0294	10.0591	0.0994	6.7225
19	3.6165	0.2765	37.3790	0.0268	10.3356	0.0968	7.0242
20	3.8697	0.2584	40.9955	0.0244	10.5940	0.0944	7.3163

Table A–8. Interest factors for annual interest compounding, $i = 8.0\%$

t	Single Payment		Equal Payment Series				Uniform Gradient Series, A/G
	Compound Amount	Present Worth	Compound Amount	Sinking Fund	Present Worth	Capital Recovery	
	Factor, F/P	Factor, P/F	Factor, F/A	Factor, A/F	Factor, P/A	Factor, A/P	
1	1.0800	0.9259	1.0000	1.0000	0.9259	1.0800	0.0000
2	1.1664	0.8573	2.0800	0.4808	1.7833	0.5608	0.4808
3	1.2597	0.7938	3.2464	0.3080	2.5771	0.3880	0.9487
4	1.3605	0.7350	4.5061	0.2219	3.3121	0.3019	1.4040
5	1.4693	0.6806	5.8666	0.1705	3.9927	0.2505	1.8465
6	1.5869	0.6302	7.3359	0.1363	4.6229	0.2163	2.2763
7	1.7138	0.5835	8.9228	0.1121	5.2064	0.1921	2.6937
8	1.8509	0.5403	10.6366	0.0940	5.7466	0.1740	3.0985
9	1.9990	0.5002	12.4876	0.0801	6.2469	0.1601	3.4910
10	2.1589	0.4632	14.4866	0.0690	6.7101	0.1490	3.8713
11	2.3316	0.4289	16.6455	0.0601	7.1390	0.1401	4.2395
12	2.5182	0.3971	18.9771	0.0527	7.5361	0.1327	4.5957
13	2.7196	0.3677	21.4953	0.0465	7.9038	0.1265	4.9402
14	2.9372	0.3405	24.2149	0.0413	8.2442	0.1213	5.2731
15	3.1722	0.3152	27.1521	0.0368	8.5595	0.1168	5.5945
16	3.4259	0.2919	30.3243	0.0330	8.8514	0.1130	5.9046
17	3.7000	0.2703	33.7502	0.0296	9.1216	0.1096	6.2037
18	3.9960	0.2502	37.4502	0.0267	9.3719	0.1067	6.4920
19	4.3157	0.2317	41.4463	0.0241	9.6036	0.1041	6.7697
20	4.6610	0.2145	45.7620	0.0219	9.8181	0.1019	7.0369

Table A–9. Interest factors for annual interest compounding, $i = 9.0\%$

t	Single Payment		Equal Payment Series				Uniform Gradient Series, A/G
	Compound Amount	Present Worth	Compound Amount	Sinking Fund	Present Worth	Capital Recovery	
	Factor, F/P	Factor, P/F	Factor, F/A	Factor, A/F	Factor, P/A	Factor, A/P	
1	1.0900	0.9174	1.0000	1.0000	0.9174	1.0900	0.0000
2	1.1881	0.8417	2.0900	0.4785	1.7591	0.5685	0.4785
3	1.2950	0.7722	3.2781	0.3051	2.5313	0.3951	0.9426
4	1.4116	0.7084	4.5731	0.2187	3.2397	0.3087	1.3925
5	1.5386	0.6499	5.9847	0.1671	3.8897	0.2571	1.8282
6	1.6771	0.5963	7.5233	0.1329	4.4859	0.2229	2.2498
7	1.8280	0.5470	9.2004	0.1087	5.0330	0.1987	2.6574
8	1.9926	0.5019	11.0285	0.0907	5.5348	0.1807	3.0512
9	2.1719	0.4604	13.0210	0.0768	5.9952	0.1668	3.4312
10	2.3674	0.4224	15.1929	0.0658	6.4177	0.1558	3.7978
11	2.5804	0.3875	17.5603	0.0569	6.8052	0.1469	4.1510
12	2.8127	0.3555	20.1407	0.0497	7.1607	0.1397	4.4910
13	3.0658	0.3262	22.9534	0.0436	7.4869	0.1336	4.8182
14	3.3417	0.2992	26.0192	0.0384	7.7862	0.1284	5.1326
15	3.6425	0.2745	29.3609	0.0341	8.0607	0.1241	5.4346
16	3.9703	0.2519	33.0034	0.0303	8.3126	0.1203	5.7245
17	4.3276	0.2311	36.9737	0.0270	8.5436	0.1170	6.0024
18	4.7171	0.2120	41.3013	0.0242	8.7556	0.1142	6.2687
19	5.1417	0.1945	46.0185	0.0217	8.9501	0.1117	6.5236
20	5.6044	0.1784	51.1601	0.0195	9.1285	0.1095	6.7674

Table A–10. Interest factors for annual interest compounding, $i = 10.0\%$

t	Single Payment		Equal Payment Series				Uniform Gradient Series, A/G
	Compound Amount	Present Worth	Compound Amount	Sinking Fund	Present Worth	Capital Recovery	
	Factor, F/P	Factor, P/F	Factor, F/A	Factor, A/F	Factor, P/A	Factor, A/P	
1	1.1000	0.9091	1.0000	1.0000	0.9091	1.1000	0.0000
2	1.2100	0.8264	2.1000	0.4762	1.7355	0.5762	0.4762
3	1.3310	0.7513	3.3100	0.3021	2.4869	0.4021	0.9366
4	1.4641	0.6830	4.6410	0.2155	3.1699	0.3155	1.3812
5	1.6105	0.6209	6.1051	0.1638	3.7908	0.2638	1.8101
6	1.7716	0.5645	7.7156	0.1296	4.3553	0.2296	2.2236
7	1.9487	0.5132	9.4872	0.1054	4.8684	0.2054	2.6216
8	2.1436	0.4665	11.4359	0.0874	5.3349	0.1874	3.0045
9	2.3579	0.4241	13.5795	0.0736	5.7590	0.1736	3.3724
10	2.5937	0.3855	15.9374	0.0627	6.1446	0.1627	3.7255
11	2.8531	0.3505	18.5312	0.0540	6.4951	0.1540	4.0641
12	3.1384	0.3186	21.3843	0.0468	6.8137	0.1468	4.3884
13	3.4523	0.2897	24.5227	0.0408	7.1034	0.1408	4.6988
14	3.7975	0.2633	27.9750	0.0357	7.3667	0.1357	4.9955
15	4.1772	0.2394	31.7725	0.0315	7.6061	0.1315	5.2789
16	4.5950	0.2176	35.9497	0.0278	7.8237	0.1278	5.5493
17	5.0545	0.1978	40.5447	0.0247	8.0216	0.1247	5.8071
18	5.5599	0.1799	45.5992	0.0219	8.2014	0.1219	6.0526
19	6.1159	0.1635	51.1591	0.0195	8.3649	0.1195	6.2861
20	6.7275	0.1486	57.2750	0.0175	8.5136	0.1175	6.5081

Table A–11. Interest factors for annual interest compounding, $i = 12.0\%$

t	Single Payment		Equal Payment Series				Uniform Gradient Series, A/G
	Compound Amount	Present Worth	Compound Amount	Sinking Fund	Present Worth	Capital Recovery	
	Factor, F/P	Factor, P/F	Factor, F/A	Factor, A/F	Factor, P/A	Factor, A/P	
1	1.1200	0.8929	1.0000	1.0000	0.8929	1.1200	0.0000
2	1.2544	0.7972	2.1200	0.4717	1.6901	0.5917	0.4717
3	1.4049	0.7118	3.3744	0.2963	2.4018	0.4163	0.9246
4	1.5735	0.6355	4.7793	0.2092	3.0373	0.3292	1.3589
5	1.7623	0.5674	6.3528	0.1574	3.6048	0.2774	1.7746
6	1.9738	0.5066	8.1152	0.1232	4.1114	0.2432	2.1720
7	2.2107	0.4523	10.0890	0.0991	4.5638	0.2191	2.5515
8	2.4760	0.4039	12.2997	0.0813	4.9676	0.2013	2.9131
9	2.7731	0.3606	14.7757	0.0677	5.3282	0.1877	3.2574
10	3.1058	0.3220	17.5487	0.0570	5.6502	0.1770	3.5847
11	3.4785	0.2875	20.6546	0.0484	5.9377	0.1684	3.8953
12	3.8960	0.2567	24.1331	0.0414	6.1944	0.1614	4.1897
13	4.3635	0.2292	28.0291	0.0357	6.4235	0.1557	4.4683
14	4.8871	0.2046	32.3926	0.0309	6.6282	0.1509	4.7317
15	5.4736	0.1827	37.2797	0.0268	6.8109	0.1468	4.9803
16	6.1304	0.1631	42.7533	0.0234	6.9740	0.1434	5.2147
17	6.8660	0.1456	48.8837	0.0205	7.1196	0.1405	5.4353
18	7.6900	0.1300	55.7497	0.0179	7.2497	0.1379	5.6427
19	8.6128	0.1161	63.4397	0.0158	7.3658	0.1358	5.8375
20	9.6463	0.1037	72.0524	0.0139	7.4694	0.1339	6.0202

Table A–12. Interest factors for annual interest compounding, $i = 20.0\%$

t	Single Payment		Equal Payment Series				Uniform Gradient Series, A/G
	Compound Amount	Present Worth	Compound Amount	Sinking Fund	Present Worth	Capital Recovery	
	Factor, F/P	Factor, P/F	Factor, F/A	Factor, A/F	Factor, P/A	Factor, A/P	
1	1.2000	0.8333	1.0000	1.0000	0.8333	1.2000	0.0000
2	1.4400	0.6944	2.2000	0.4545	1.5278	0.6545	0.4545
3	1.7280	0.5787	3.6400	0.2747	2.1065	0.4747	0.8791
4	2.0736	0.4823	5.3680	0.1863	2.5887	0.3863	1.2742
5	2.4883	0.4019	7.4416	0.1344	2.9906	0.3344	1.6405
6	2.9860	0.3349	9.9299	0.1007	3.3255	0.3007	1.9788
7	3.5832	0.2791	12.9159	0.0774	3.6046	0.2774	2.2902
8	4.2998	0.2326	16.4991	0.0606	3.8372	0.2606	2.5756
9	5.1598	0.1938	20.7989	0.0481	4.0310	0.2481	2.8364
10	6.1917	0.1615	25.9587	0.0385	4.1925	0.2385	3.0739
11	7.4301	0.1346	32.1504	0.0311	4.3271	0.2311	3.2893
12	8.9161	0.1122	39.5805	0.0253	4.4392	0.2253	3.4841
13	10.6993	0.0935	48.4966	0.0206	4.5327	0.2206	3.6597
14	12.8392	0.0779	59.1959	0.0169	4.6106	0.2169	3.8175
15	15.4070	0.0649	72.0351	0.0139	4.6755	0.2139	3.9588
16	18.4884	0.0541	87.4421	0.0114	4.7296	0.2114	4.0851
17	22.1861	0.0451	105.9306	0.0094	4.7746	0.2094	4.1976
18	26.6233	0.0376	128.1167	0.0078	4.8122	0.2078	4.2975
19	31.9480	0.0313	154.7400	0.0065	4.8435	0.2065	4.3861
20	38.3376	0.0261	186.6880	0.0054	4.8696	0.2054	4.4643

Table A–13. Interest factors for annual interest compounding, $i = 25.0\%$

t	Single Payment		Equal Payment Series				Uniform Gradient Series, A/G
	Compound Amount	Present Worth	Compound Amount	Sinking Fund	Present Worth	Capital Recovery	
	Factor, F/P	Factor, P/F	Factor, F/A	Factor, A/F	Factor, P/A	Factor, A/P	
1	1.2500	0.8000	1.0000	1.0000	0.8000	1.2500	0.0000
2	1.5625	0.6400	2.2500	0.4444	1.4400	0.6944	0.4444
3	1.9531	0.5120	3.8125	0.2623	1.9520	0.5123	0.8525
4	2.4414	0.4096	5.7656	0.1734	2.3616	0.4234	1.2249
5	3.0518	0.3277	8.2070	0.1218	2.6893	0.3718	1.5631
6	3.8147	0.2621	11.2588	0.0888	2.9514	0.3388	1.8683
7	4.7684	0.2097	15.0735	0.0663	3.1611	0.3163	2.1424
8	5.9605	0.1678	19.8419	0.0504	3.3289	0.3004	2.3872
9	7.4506	0.1342	25.8023	0.0388	3.4631	0.2888	2.6048
10	9.3132	0.1074	33.2529	0.0301	3.5705	0.2801	2.7971
11	11.6415	0.0859	42.5661	0.0235	3.6564	0.2735	2.9663
12	14.5519	0.0687	54.2077	0.0184	3.7251	0.2684	3.1145
13	18.1899	0.0550	68.7596	0.0145	3.7801	0.2645	3.2437
14	22.7374	0.0440	86.9495	0.0115	3.8241	0.2615	3.3559
15	28.4217	0.0352	109.6868	0.0091	3.8593	0.2591	3.4530
16	35.5271	0.0281	138.1085	0.0072	3.8874	0.2572	3.5366
17	44.4089	0.0225	173.6357	0.0058	3.9099	0.2558	3.6084
18	55.5112	0.0180	218.0446	0.0046	3.9279	0.2546	3.6698
19	69.3889	0.0144	273.5558	0.0037	3.9424	0.2537	3.7222
20	86.7362	0.0115	342.9447	0.0029	3.9539	0.2529	3.7667

Table A–14. Interest factors for annual interest compounding, $i = 30.0\%$

t	Single Payment		Equal Payment Series				Uniform Gradient Series, A/G
	Compound Amount	Present Worth	Compound Amount	Sinking Fund	Present Worth	Capital Recovery	
	Factor, F/P	Factor, P/F	Factor, F/A	Factor, A/F	Factor, P/A	Factor, A/P	
1	1.3000	0.7692	1.0000	1.0000	0.7692	1.3000	0.0000
2	1.6900	0.5917	2.3000	0.4348	1.3609	0.7348	0.4348
3	2.1970	0.4552	3.9900	0.2506	1.8161	0.5506	0.8271
4	2.8561	0.3501	6.1870	0.1616	2.1662	0.4616	1.1783
5	3.7129	0.2693	9.0431	0.1106	2.4356	0.4106	1.4903
6	4.8268	0.2072	12.7560	0.0784	2.6427	0.3784	1.7654
7	6.2749	0.1594	17.5828	0.0569	2.8021	0.3569	2.0063
8	8.1573	0.1226	23.8577	0.0419	2.9247	0.3419	2.2156
9	10.6045	0.0943	32.0150	0.0312	3.0190	0.3312	2.3963
10	13.7858	0.0725	42.6195	0.0235	3.0915	0.3235	2.5512
11	17.9216	0.0558	56.4053	0.0177	3.1473	0.3177	2.6833
12	23.2981	0.0429	74.3270	0.0135	3.1903	0.3135	2.7952
13	30.2875	0.0330	97.6250	0.0102	3.2233	0.3102	2.8895
14	39.3738	0.0254	127.9125	0.0078	3.2487	0.3078	2.9685
15	51.1859	0.0195	167.2863	0.0060	3.2682	0.3060	3.0344
16	66.5417	0.0150	218.4722	0.0046	3.2832	0.3046	3.0892
17	86.5042	0.0116	285.0139	0.0035	3.2948	0.3035	3.1345
18	112.4554	0.0089	371.5180	0.0027	3.3037	0.3027	3.1718
19	146.1920	0.0068	483.9734	0.0021	3.3105	0.3021	3.2025
20	190.0496	0.0053	630.1655	0.0016	3.3158	0.3016	3.2275

Table A–15. Interest factors for annual interest compounding, $i = 35.0\%$

t	Single Payment		Equal Payment Series				Uniform Gradient Series, A/G
	Compound Amount	Present Worth	Compound Amount	Sinking Fund	Present Worth	Capital Recovery	
	Factor, F/P	Factor, P/F	Factor, F/A	Factor, A/F	Factor, P/A	Factor, A/P	
1	1.3500	0.7407	1.0000	1.0000	0.7407	1.3500	0.0000
2	1.8225	0.5487	2.3500	0.4255	1.2894	0.7755	0.4255
3	2.4604	0.4064	4.1725	0.2397	1.6959	0.5897	0.8029
4	3.3215	0.3011	6.6329	0.1508	1.9969	0.5008	1.1341
5	4.4840	0.2230	9.9544	0.1005	2.2200	0.4505	1.4220
6	6.0534	0.1652	14.4384	0.0693	2.3852	0.4193	1.6698
7	8.1722	0.1224	20.4919	0.0488	2.5075	0.3988	1.8811
8	11.0324	0.0906	28.6640	0.0349	2.5982	0.3849	2.0597
9	14.8937	0.0671	39.6964	0.0252	2.6653	0.3752	2.2094
10	20.1066	0.0497	54.5902	0.0183	2.7150	0.3683	2.3338
11	27.1439	0.0368	74.6967	0.0134	2.7519	0.3634	2.4364
12	36.6442	0.0273	101.8406	0.0098	2.7792	0.3598	2.5205
13	49.4697	0.0202	138.4848	0.0072	2.7994	0.3572	2.5889
14	66.7841	0.0150	187.9544	0.0053	2.8144	0.3553	2.6443
15	90.1585	0.0111	254.7385	0.0039	2.8255	0.3539	2.6889
16	121.7139	0.0082	344.8970	0.0029	2.8337	0.3529	2.7246
17	164.3138	0.0061	466.6109	0.0021	2.8398	0.3521	2.7530
18	221.8236	0.0045	630.9247	0.0016	2.8443	0.3516	2.7756
19	299.4619	0.0033	852.7483	0.0012	2.8476	0.3512	2.7935
20	404.2736	0.0025	1152.2103	0.0009	2.8501	0.3509	2.8075

B

Appendix

Overview of Spreadsheets on CD-ROM that Accompany Volume 2

A CD-ROM is included in volume 2 of this series. The CD contains Microsoft Excel spreadsheets with all the tables and example problems solved in the two volumes of this book. In addition to the spreadsheets used in the example problems and exercises in this volume, several comprehensive economics spreadsheets included on the CD are installed in a directory titled ECONPAC. These spreadsheets are designed for use by investors, analysts, engineers, and anyone interested in knowing more about the value and viability of oil and gas properties.

ECONOIL — This spreadsheet performs economic evaluations of oil and associated gas wells/lease for production life of up to 30 years.

ECONGAS — Same as ECONOIL, but used to evaluate gas and associated oil or condensate wells/leases.

ECONSUM — This is a utility spreadsheet used for creating summaries of individual cash flows that are created by ECONGAS and ECONOIL.

ECONPSA — This comprehensive spreadsheet can be used to evaluate economics of production-sharing deals. The spreadsheet is flexible in terms of handling various tranches linked to R-factor, production volumes, product price, etc.

ECONOIL and ECONGAS — Economic Evaluation Spreadsheets

The ECONOIL and ECONGAS spreadsheets are the same as far as functional capabilities are concerned. Following are the detailed functional capabilities of these two models.

1. Once the ECONOIL or ECONGAS file is opened, a main data window appears. This window is used for inputting all data.
2. The model uses an exponential decline rate to schedule oil or gas (major phase) production; gas–oil ratio; or oil or condensate yield, which calculates the associated gas, oil, or condensate production, respectively. The user specifies the initial monthly production rate of the major phase and the percent exponential decline rate per year.
3. The model is capable of calculating the economic limit of the production based on product prices, working interest, net revenue interest, monthly operating costs, severance and ad valorem taxes, and gas–oil ratio or yield. Production of the major phase will be scheduled to the economic limit or up to 30 years, whichever comes first.
4. Oil and gas severance taxes may be specified in \$/Stb, \$/MScf, or in percentages. In the former case, input the taxes with a minus (negative) sign.
5. In case of escalated economics or when evaluating overriding royalty interest only, the model will not calculate economic limit; therefore, the user must specify the ending rate (ERATE in the menu).
6. The models are capable of handling up to three escalation scenarios for oil price, gas price, and operating cost.
7. The presentation format of the cash flows is typical of many such cash flow formats encountered in the industry. The bottom line profitability indicators and economic parameters calculated/used by the models are listed at the bottom of the printout.

ECONSUM — Creating Summaries of Evaluations

Economic evaluations are normally performed for oil and gas properties classified into different reserve categories, locations (such as state), operators, etc. In order to get composite cash flows summarizing each group and the total of all groups (total portfolio), the individual cash flows generated by using ECONOIL and ECONGAS must be added together in the respective categories as required.

ECONSUM is a utility spreadsheet that can be used to perform this task. Once the cash flows for individual properties are generated, this spreadsheet can then be used to create their summaries. The spreadsheet requires that the ECONSUM and cash flows be added together on the same disk and in the same directory.

ECONSUM adds up to 30 files together at a time. If more than 30 files are to be added together in any one category, the summary file created for the first 30 wells is then used with file names of another 29 wells. This process is repeated until all the files in a specified category are added.

The following steps are used with this spreadsheet:

1. Open the file ECONSUM. When the following statement appears,
? This document contains links. Re-establish Link ?
click on NO.
2. In the window that appears next, enter the filenames to be added together in the empty cells (up to 30 files can be added at a time).
3. Click on **Tools** in the Excel toolbar. At the bottom of this dropdown menu the following commands appear.
SUMMARIZE
NAME CHECK
SAVE AS NEW FILE
MAIN MENU
4. Before creating the summary, determine whether you will need to add more than 30 files together. If 30 or fewer files must be added, click **NAME CHECK** and use the default filename **ECONSUM**. (Note, only ECONSUM can be printed. If more than 30 files must be added, click **NAME CHECK** and enter the filename for up to 29 additional files to be added to a summary file (e.g., PROVEPR for proved producing).
5. Click on **SUMMARIZE** to start adding the files.
6. Click **SAVE AS NEW FILE** to save the summary file and give it an appropriate name (e.g., PROVEPR).

Answers to Selected End-of-Chapter Problems

Chapter 1

1.9 a

1.10 c

Chapter 2

2.7 \$76,577

2.8 Real Income

2.9 \$66,245

2.10 III

2.11 d

2.12 a

2.13 b

2.14 a

2.15 b

2.16 c

2.17 d

2.18 d

2.19 d

2.20 c

2.21 c

2.22 d

2.23 b

2.24 b

2.25 b

2.26 b

2.27 a

2.28 b

2.29 d

2.30 c

2.31 c

2.32 c

2.33 d

2.34 b

2.35 d

2.36 b

2.37 c

2.38 b

2.39 –

a. 12%

b. 12.36%

c. 12.55%

d. 12.75%

e. 12.75%

2.40 1,738 & 1,812

2.41 2,159 & 28,387

2.42 6,948

2.43 3,512

2.44 9,840 & 7,918 & 10,796 & 61,223

- 2.45 3,974 & 4,968 & 4,582 & 37,974
- 2.46 3,409
- 2.47 1,819
- 2.48 2,881
- 2.49 560 & 744
- 2.50 –
- 2.51 137,609
- 2.52 1,348 & 9,623 & 22,436
- 2.53 9,623
- 2.54 10% and 10%
- 2.55 15,869 & 3,433
- 2.56 7,368
- 2.57 9 years
- 2.58 10,046 & 3,321
- 2.59 Remaining balance = \$25,801
- 2.60 Nominal rate = 36%
- 2.61 –
- a. PV of production = 138,372 barrels
 - b. Equivalent annual production = 22,150 barrels/year
 - c. PV = \$10,544,538
- 2.62 \$10,102,943 & \$990,412

Chapter 3

- 3.1 0.56
- 3.2 2,506 MM\$
- 3.3 2,739 MM\$
- 3.4 2,789 MM\$
- 3.5 219 MM\$

- 3.6 45.63 MM\$
- 3.7 49.28 MM\$
- 3.8 354.78 MM\$
- 3.9 80.41 MM\$
- 3.10 100.24 MM\$
- 3.11 \$125,000/well
- 3.12 167.88 MM\$
- 3.13 75.79 Stb/month
- 3.14 19.67 MScf/month
- 3.15 89,648 Stb
- 3.16 10.74 years
- 3.17 2,434 MMScf & 2,069 MMScf
- 3.18 –
- 3.19 –
 - a. 41 Stb/month
 - b. 18%
 - c. 55,323 Stb
 - d. 780 Stb/month
 - e. 16 years
 - f. 137,667 Stb
 - g. 10,347 & 8,495 & 6,975
- 3.20 –
- 3.21 –
- 3.22 –
- 3.23 –
 - a. 355 Stb/month
 - b. 170,629 Stb
 - c. 68,252 Stb
 - d. 7.79 years

3.24 –

- a. 3,258,360 Stb
- b. 651,672 Stb
- c. 15%
- d. 23.44 years
- e. 7,672 Stb/month

3.25 7.35 MM\$

3.26 86%

3.27 86%

3.28 –

3.29 75.78 Stb/month & 7.17 years & 13% IRR

Chapter 4

4.1 c

4.2 a

4.3 c

4.4 a

4.5 9.09 & 8.18

4.6 32.73

Chapter 5

5.1 \$45,000

5.2 \$455,000

5.3 91%

5.4 1.43

5.5 \$327,500

5.6 \$22,500

5.7 \$400,000

5.8 \$420,000

5.9 \$180,000

5.10 \$140,000

5.11 \$60,000

5.12 \$240,000

5.13 \$560,000

5.14 70%

5.15 \$1,000,000

Chapter 6

6.13 $GRR = 13.38\%$

6.14 b

6.15 c

6.16 b

6.17 d

6.18 d

6.19 b

6.20 b

6.21 a

6.22 b

6.23 b

6.24 a

6.25 c

6.28 Both are acceptable

6.29 Both are acceptable

6.30 $IRR = 6.84\%$

6.31 $NPV = \$261,486$; $IRR = 14\%$; $PI = 1.14$; $LRMC = \$54.77/Stb$

6.32 –

6.33 NPV = \$32,139; IRR = 12.4%; PI = 1.10; LRMC = \$56.74/Stb

Chapter 7

7.5 NPV = \$48.98

7.6 IRR = 11.76%

7.7 IRR = 12%

7.8 PI = 1.08

7.9 \$44.82; \$58.68; \$118.55; \$70.47

7.10 1.24; 1.14; 1.18; 1.08

7.11 Select C

7.12 A, B & C

7.13 Compressor A

7.14 Compressor A

7.15 Project C

7.16 Project C

7.17 $NPV_B @ 10\% = 0.33$; $NPV_B @ 12\% = (2.60)$; $IRR_B = 10\%$; Select A

7.18 $Payback_A = 4.08$; $Payback_B = 4.64$; $NPV_A = 1.57$; $NPV_B = 1.71$

7.19 –

a. $Payback_A = 8.99$; $Payback_B = 8.55$

b. $NPV_A = 32.64$; $NPV_B = 121.25$

c. $IRR_A = 13\%$; $IRR_B = 15\%$

7.20 –

a. $NPV_A = 37.44$; $NPV_B = 40.32$; $NPV_C = 41.84$ – Ranking is C, B, & A

b. $IRRA = 11.26\%$; $IRRB = 11.33\%$; $IRRC = 10.76\%$ - Select B, A, & C

c. $NPV_A = 169.78$; $NPV_B = 176.26$; $NPV_C = 294.52$ – Ranking is C, B, & A

d. $NPV_A = 7.59$; $NPV_B = 9.71$; $NPV_C = (13.10)$ – Ranking is B, A & C

7.21 $NPV_A = 76$; $NPV_B = 65$; $NPV_C = 167$

7.22 -

a. $NPV_A = 19,201$; $NPV_B = 16,338$

b. $IRR_A = 22\%$; $IRR_B = 26.1\%$

7.23 $NPV_{A-B} = 2,864$; $IRR_{A-B} = 14.2\%$; $PI_{A-B} = 1.18$ – Select A

7.24 Select Prospects A & D; Total NPV @ 8% = 916.28

7.25 -

7.26 Complete the behind pipe formation after the depletion of the lower formation

7.27 Complete the behind pipe formation after the depletion of the lower formation

7.28 Select A with NPV @ 15% = 235,157; $IRR_{B-A} = 5.5\%$; $IRR_{C-A} = (1,100,000)$

7.29 -

7.30 Invest in investments A, B & C – $PI_A = 1.34$; $PI_B = 1.73$; $PI_C = 1.18$; $PI_D = 1.06$.

7.31 Select D – $NPV_A = 398$; $NPV_B = 586$; $NPV_C = 761$; $NPV_D = 1,101$; $IRR_A = 27\%$; $IRR_B = 22\%$; $IRR_C = 21\%$; $IRR_D = 20\%$

7.32 Select A & B

7.33 Select A & B

7.34 $NPV_{As is} = (94,385)$; $NPV_{Upgrade} = (87,129)$

7.35 -

a. Equiv. Annual Cost_A = -53,263; Equiv. Annual Cost_B = -54,771

b. PV of Cost_A = -246,227; PV of Cost_B = -253,200

c. FV of Cost_A = -390,731; FV of Cost_B = -401,796

d. $IRR_{B-A} = 4.2\%$

Index

A

abandonment costs, 114, 166
accelerated cost recovery system, 218–219
accelerated depreciation method, 220
acceptable rate of return, 313
accounting approaches
 average rate of return (AROI), 310–311
 payback period, 306–310
 accounting measures, 304
accounting rate of return (ARR), 310, 339, 341
acreage position, 245
actual cash outflows, 207
additional profits tax (APT), 274
adjusted net present value (NPVA), 353–355, 362, 372–373
administration and general overhead, 163, 164
after federal income tax (AFT or ATAX), 113, 207
after-tax cash flow model
 about, 207–212
 amortization and depletion, 223–228
 depreciation, 213–223
 intangible drilling costs (IDCs), 228–235
 spreadsheet applications, 235–243
after-tax income, 208
after-tax weighted average cost of capital (WACCAT), 366–368, 371
after-tax weighted cost of capital method, 361, 366–369
agreement duration, 264
agreement types, 263
Aguilera, Roberto, 324
allowable depletion allowance, 208
allowances, 172
alternative ACS method, 219
alternative depreciation system (ADS), 214
American Petroleum Institute (API)
 barrel definition, 149
 TC-10: State and Local Oil and Gas

Severance and Production Taxes, 186

amortization and depletion
 about, 223
 amortization, 224
 depletion, 224–228
annual work program, 265
annuity, 57
annuity date, 58
annuity due, 57
annuity from future value, 67–68
annuity from present value, 64–67
API gravity, 150, 152
Arditti-Levy method (NPVAL), 362, 371–372
authority for expenditure (AFE), 157–158
average inflation, 30
average rate of return (AROI), 310–311
average recovery factor, 146

B

back-in option, 246, 265
balloon payments, 74
Bank of Japan, 37
barrel, 149
base case economic analysis, 173
base estimate, 173
base lines, 179
base year and ROI approach, 339–340
base year and ROR approach, 341–342
basis of design (BOD), 169
before-tax cash flow models
 about, 111–113
 capital expenditure (CAPEX), 154–162
 cash flow analysis and inflation, 192–193
 cash flow analysis of foreign investments, 191–192
 cash flow estimation bias, 190–191
 cash flow model, 113–115
 cash flow spreadsheet, 195–198

- cost estimate types, 168–170
- cost of capital, 193–195
- data required for project evaluation, 116–117
- leasing aspects, 178–185
- operating expenditures (OPEX), 162–168
- product pricing, 149–154
- product stream forecasting, 117–149
- project costs, actual variations, 170–174
- severance and ad valorem taxes, 185–190
- transfer pricing, 174–178
- before-tax weighted average cost of capital (WACCBT), 371
- biases in forecasts, 112–113, 190
- Bindemann, Kirsten, 246
- block, 245
- bonus payments, 266
- boundary conditions, 293
- breakeven point, 306
- Brent crude (North Sea), 151–152
- Brigham, E. F., 324
- budget estimate, 169–170
- budgeting techniques, 304
- Bureau of Ocean Energy Management, Regulation and Enforcement. *See* U.S. Minerals and Management Service (MMS)

C

- Canada, 179, 181
- capital, 301
- capital budget, 301
- capital budgeting, 301, 303
- capital budgeting techniques
 - about, 301–305
 - accounting approaches, 306–311
 - discounted cash flow approaches, 311–355
 - discounting frequency effect, 355–360
 - financing mix treatment, 360–375
- capital budget plans, 5
- capital expenditure (CAPEX)
 - about, 154
 - categories of, 114
 - drilling cost estimating, 155–158
 - facility costs, 158–162
 - geological and geophysical (G&G) costs, 155
- capital expenditure (CAPEX) planning, 4–6, 302
- capital expenditure budget, 5
- capital investment analysis, 111
- capital investments, 388
- capital repatriation, 265
- capital structure, 194
- Cartesian coordinate graph paper, 137
- cash flow, 113
- cash flow analysis and inflation, 192–193
- cash flow analysis of foreign investments, 191–192
- cash flow diagram, 45
- cash flow estimation bias, 190–191
- cash flow forecasts, 111–112
- cash flow model, 113–115
- cash flow spreadsheet, 195–198, 258
- cash flow timing, 358
- casing point, 183
- causes of inflation, 23
- central banks, 37
- China, 22
- circularity, 267
- Collarini, C. R., 324
- commercial software packages, 195
- common denomination, 47
- comparable risk, 46
- compounding interest calculations, 47
- compound interest, 39
- conceptual development plans, 169
- concession, 245
- concessionary system, 249–250
- concessionary system sample cash flow spreadsheet, 251–254
- conditional sales contract, 398
- conflicting selection situation, 405, 413
- constant costs, 193
- constant debt ratio schedule, 362, 373
- constant dollars, 29
- constant percentage decline, 123
- constant periodic debt payments, 373
- constant principal payments, 373
- consumer price index (CPI), 26–30
- contingencies and allowances, 172–174
- continuous compounding, 43
- continuous flow of funds, 81
- CONTINUUM (commercial software package), 195
- contract area, 245
- contract arrangement types
 - about, 247–248
 - concessionary system, 249–250
 - concessionary system sample cash flow spreadsheet, 251–254
 - contract initiation and award, 270–271
 - government and contractor take, 271–273
 - production sharing agreements (PSAs), 254–258
 - production sharing and royalty tax system elements, 263–270
 - rate of return (ROR) contracts, 274
 - risk service contracts, 273

- sample PSC cash flow spreadsheet, 259–263
- contract initiation and award, 270–271
- contractor entitlement, 256
- contractor risk, 273
- contractor take, 271
- control estimate, 170
- conventional cash flow, 319
- conventional debt schedule, 373
- conversion factors, 331
- corporate overheads, 163
- cost categories
 - administration and general overhead, 163
 - evacuation costs, 163–164
 - feedstocks, 162–163
 - insurance costs, 164
 - production costs, 163
 - utilities, 163
- cost depletion, 225–226
- cost estimate types
 - budget estimate, 169–170
 - control estimate, 170
 - optimization study estimate, 169
 - order of magnitude estimate, 168–169
- cost factors, 115
- cost of capital. *See also* discount rate, 193–195
- cost overruns, 170–171
- cost-push inflation, 23–24
- cost recovery (CR), 255–256
- cost recovery ceiling effect, 277–280
- cost recovery oil (cost oil), 255, 278
- CPI for All Urban Consumers (CPI-U), 26
- CPI for Urban Wage Earners and Clerical Workers (CPI-W), 26
- crude oil price volatility, 15–20
- cumulative gas (Gp)-cumulative oil (Np) plot, 138–139
- cumulative net cash flows, 45
- cumulative oil and/or gas production, 118
- cumulative reserves replacement ratio, 18
- cumulative time, 306
- currency risk, 6
- current dollars, 29
- curried interest, 183
- curve shift on log-log paper, 128–131
- cutoff point, 415
- cut-off/threshold requirement, 304

D

- date required for project evaluation, 116–117
- DB function, 235
- DDB function, 239
- debtors and creditors, 25
- debt schedule, 363–366
- decision criteria, 303
- decision making
 - about, 387–389
 - income-producing investments, 403–419
 - lease vs. buy decision analysis, 398–402
 - service-producing projects, 389–398
- decision rules, 403
- decision yardsticks, 326, 390
- decline characteristics, 126
- decline curve analysis, 121–141
- declining balance depreciation, 220–221
- declining balance methods, 218
- deflated costs, 193
- delay rental, 182
- demand, 7–8
- demand, supply, and equilibrium price, 7–14
- demand factors, 9
- demand for capital, 25
- demand for loanable funds, 38
- demand-pull inflation, 23
- demand-shift inflation, 24
- depletion
 - about, 224–225
 - cost depletion, 225–226
 - percentage depletion, 226–228
- depreciable value, 219
- depreciation
 - accelerated cost recovery system, 218–219
 - declining balance depreciation, 220–221
 - in financial statements, 25–26
 - intangible property, 213–214
 - modified accelerated cost recovery system, 214–218
 - straight-line depreciation, 219–220
 - sum-of-the-years'-digits depreciation, 221–223
 - tangible property, 213
 - units of production depreciation, 223
- depreciation, depletion and amortization (DD&A) of assets, 207
- depreciation conventions, 217–218
- depreciation tax shields, 399
- design base, 169
- direct operating expense elements, 165–166
- disbursements (costs), 36
- disbursements flow diagram, 257
- discounted cash flow (DCF), 324
- discounted cash flow (DCF) approaches
 - adjusted net present value (ANPV), 353–355
 - discounted payback period, 311–313
 - growth rate of return (GRR), 350–353
 - internal rate of return (IRR), 316–328
 - long-run marginal cost (LRMC), 331–337

- netback value (NBV) and indexed pricing, 337–350
- net present value (NPV), 313–316
- probability index, 328–330
- unit technical cost (UT), 331–337
- discounted cash flow (DCF) techniques, 193
- discounted cash flow rate of return (DCFROR), 316
- discounted methods, 341
- discounted net cash flows (NCFs), 317
- discounted payback period, 311–313
- discounted profit-to investment ratio (DPIR), 329
- discounted unit technical cost (UT), 332, 333
- discounting frequency effect, 355–360
- discounting future cash flow streams, 47
- discounting production, 333
- discount rate, 37, 47, 193, 313
- discrete period cash flows, 81
- discrete period interest compounding/ discounting, 81
- domestic intangible drilling costs (IDCs), 228
- domestic obligation, 265
- double declining balance depreciation, 220
- Dougherty, E. L., 324
- drainage area, 119
- drilling cost estimating, 155–158
- drilling costs, 114
- drilling rate, 157
- dry hole costs, 228
- dual or multiple rates of return, 319–320

E

- east-to-west ranges, 179
- economic life, 302
- economic limit (EL), 121–122, 140
- economic principles
 - about, 6
 - consumer price index, 26–30
 - crude oil price volatility, 15–20
 - demand, supply, and equilibrium price, 7–14
 - energy independence, 20–22
 - inflation, 22–26
- economic rent tax (ERT), 282–283
- economic yardsticks, 303
- EFFECT function, 84
- effective interest rate, 42
- effective IRR, 356
- efficient fiscal regime, 275–277
- Ekofisk crude, 151
- elasticity of supply and demand, 13
- elastic supply curve, 13
- embedded functions, 92
- employee benefits, 163
- end of period cash flows, 81
- energy constants conversion, 331
- energy independence, 20–22
- enhanced oil recovery (EOR), 246
- equal payment compound amount factor, 59
- equal payment periods, 73
- equal payment present worth factor (P/A), 58
- equal payment series capital recovery factor (A/P), 65, 73
- equal payment series sinking fund factor, 67
- equal periodic payments, 362, 364–366
- equal principal payments, 74, 362
- equal service life, 395
- equilibrium market price, 7
- equilibrium price, defined, 12
- equity financing, 36
- equity participation, 265
- equity rate of return, 350
- equity residual method (NWER), 361, 366–369, 373–374
- equivalent basis, 36
- equivalent cost analysis, 390–392
- escalation factor, 192
- estimate accuracy, 171–172
- EUREKA (commercial software package), 195
- European Central Bank (ECB), 37
- evacuation costs, 163–164
- Excel (spreadsheet program), 48, 83, 195
- Excel spreadsheet functions
 - DB function, 235
 - DDB function, 239
 - EFFECT function, 84
 - embedded functions, 92
 - FMT function, 90
 - FV function, 84
 - goal seek function, 336
 - INTRATE function, 85
 - IPMT function, 90
 - IRR function, 318
 - MID function, 335
 - NPER function, 85
 - NPV function, 85
 - PPMT function, 91
 - PV function, 91
 - RATE function, 85
 - SLN function, 235
 - SYD function, 242
 - VDB function, 239
 - XNPV function, 91
- excess cost recovery (ECR), 255
- excess profits tax (EPT), 274
- expected value and decision trees, 31
- exploratory fields (wildcats), 146
- exponential decline, 123–126

exponential declines in tight sands, 133
 export tax (ET), 282–283
 eXpress (commercial software package), 195

F

facility costs, 114, 158–162
 federal funds rate, 37
 Federal Reserve (Fed), 37
 feedstocks, 162–163
 fees and bonuses, 264
 f-factor, 348
 financial models, 36
 financial resources, 36
 financing and technology, 265
 financing mix treatment
 about, 360–362
 adjusted net present value (NPVA), 372–373
 after-tax weighted average cost of capital method, 366–369
 Arditti-Levy method (NPVAL), 371–372
 comparison of methods, 373–375
 equity residual method (NWER), 366–369
 project debt repayment schedule, 362–366
 fiscal marksmanship, 271
 fiscal system designs, 280–293
 fiscal terms, effects of various
 about, 274–275
 cost recovery ceiling effect, 277–280
 efficient fiscal regime, 275–277
 fiscal system designs, 280–293
 Fisherian intersection, 409–410
 fixed income groups, 24
 flat fiscal regime, 275
 FMT function, 90
 formation damage, 132
 formation thickness, 119
 fractured tight (low permeability)
 formations, 127
 fracture production, 132
 French curve method, 127–128
 front-end loading index, 272
 full (absorption) price, 176
 funds flow and compounding/discounting, 81–83
 funds flow processes, 81
 future sums, 47
 future value of an annuity, 59–62
 future value of annuity due, 59, 62
 future value of present sum, 54–57
 futuristic decisions, 3
 FV function, 84

G

Gambler's Ruin concept, 31
 gas formation volume factor, 120
 gas reserve estimates, 118
 gas reservoir material balance, 142–146
 gearing ratio, 194
 general depreciation system (GDS), 214
 geological and geophysical (G&G) costs, 114
 global crude oil imports and OPEC's net supply, 18
 goal seek function, 336
 gold plating, 292
 government and contractor take, 271–273
 government license, 245
 government take, 115, 271, 280
 grants of rights, 263–264
 gross revenues, 114, 269
 growth rate of return (GRR), 350–353
Gulf Times, 16
 Gulf War, 15

H

half-year convention, 217
 harmonic decline, 131
 health, safety, and environment (HSE)
 projects, 2, 5
 heavy oils, 150
 held by production, 182
 highly gearing, 194
 hole problems, 157
 host government (HG), 245, 280–285, 293
 hyperbolic decline, 126–127
 hyperbolic decline curves, 132

I

IIAPCO, 246
 improved oil recovery, 246
 income-producing investments
 mutually exclusive investment analysis, 406–415
 non-mutually exclusive investment analysis, 415–419
 project screening economics, 401–406
 incremental investments, 301
 incremental IRR, 392, 393
 incremental IRR analysis, 413
 incremental NPV analysis, 413
 independent producer IDC treatment, 228
 indexed netback pricing, 347–348
 indexed pricing, 347
 Indonesian National Oil Company, 246
 inefficient fiscal regime, 275
 inelastic supply curve, 13

inflation, 22–26, 35
inflation costs, 193
initial gas in place (IGIP), 118
initial oil in place (IOIP), 118
insurance costs, 164
intangible drilling costs (IDCs), 228–235
intangible property, 213–214
integrated producer IDC treatment, 228
interest (concept)
 about, 37–38
 nominal and effective interest rate, 42–44
 simple and compound interest, 39–42
interest only loan payments, 74
interest rate, calculation of unknown, 72
intermediate market, 178
intermediate price, 177
internal rate of return (IRR). *See also* rate of return (ROR)
 about, 316–319
 characteristics of, 326–328
 Collarini on, 324
 dual or multiple rates of return, 319–320
 limitations of, 392
 reinvestment calculation in, 322–326
 and reinvestment of incomes, 325, 327
 risk probability effects, 327
 vs. accounting rate of return (ARR), 341
internal yield, 316
International Energy Agency (IEA), 21
international national oil company (INOC), 246, 280
international oil company (IOC), 246
international petroleum economics
 about, 245–247
 contract arrangement types, 247–274
 fiscal terms, effects of various, 274–293
INTRATE function, 85
investment amount, 302
investment efficiency, 329
investment proposal classification, 388
investment ROI, 339
investor's method, 316
IPMT function, 90
IRR function, 318

J

Johnston, Daniel, 246
joint working interest, 182

K

known unknowns, 172

L

landman, 178
land owner (lessor), 182
land ownership identification system, 179–181
law of demand, 9
law of supply, 10
lease, 245
lease bonus, 182
leasehold income, 182
lease operating expenditure model. *See* operating expenditures (OPEX)
lease vs. buy decision analysis, 398–402
leasing, advantages of, 398–399
leasing aspects
 about, 178–179
 land ownership identification system, 179–181
 ownership interest computation, 181–185
least squares regression, 143
lessee, 398
lessor, 398
level debt schedule, 362, 363–364, 373
light oils, 150
linear flow characteristics, 133
linear interpolation, 50, 395
loanable funds, 38
loan amortization schedule, 73–81
local currency, 191
local tax regimes, 192
location factor, 115
logging surveys, 157
long-run marginal cost (LRMC), 123, 331–337, 342–347
long-term plans, 4
low gearing, 194
low-permeability (tight) gas reserves, 131–136

M

macroeconomics, 6
maintenance costs, 164
major projects, 5
management control, 263
marginal efficiency of capital, 316
marginally profitable fields, 275–276
market price, 175, 177
market speculation, 18
market volatility, 18
material balance techniques, 138
matrix production, 132
measures of investment wealth, 303, 304
measures of profitability, 303

median estimate, 173
 Merak Peep (commercial software package), 195
 metes and bounds system, 180
 microeconomics, 6
 MID function, 335
 mid-month convention, 217
 mid-period cash flows, 82
 mid-quarter convention, 217–218
 milestone payments, 170
 mineral deed, 178
 mineral rights ownership, 245
 minor projects, 5
 mixed cash flow assumptions in first year, 358
 modified accelerated cost recovery system, 214–218
 modified internal rate of return (MIRR), 350
 money income, 26
 money-of-the day costs, 193
 multinational projects evaluation, 353
 mutually exclusive investment analysis, 406–415
 Myers, Dill and Bautista (MNB) model, 400

N

national oil company (NOC), 246, 280
 negative cash flow (NCF), 306
 negotiated market price, 178
 net advantage to leasing (NAL), 399
 net after-tax cash flow, 207, 208
 netback pricing, 338
 netback value (NBV) and indexed pricing advantages and disadvantages of these approaches, 349–350
 base year and ROI approach, 339–340
 base year and ROR approach, 341–342
 indexed netback pricing, 347–348
 long-run marginal cost (LRMC) approach, 342–347
 net cash flow (NCF), 45, 280
 net income, 208
 net present value (NPV), 193, 280, 313–316
 net present value (NPV) profile, 315
 net revenue interest (NRI), 181
 New York Mercantile Exchange, 152
 nominal and effective interest rate, 42–44
 nominal costs, 193
 nominal dollars, 29
 nominal IRR, 356
 nominal net cash flow, 193
 nonconventional cash flow, 319–320
 non-linear interpolation, 317–318
 non-mutually exclusive investment analysis, 415–419

non-operating interests, 181
 nonresidential real property, 216
 north-to-south townships, 179
 NPER function, 85
 NPV function, 85

O

offshore production facilities, 159
 oil and/or gas reserves, 119
 oil company objectives, 3
 oil formation volume factor, 119
 oil reserve estimates, 118
 OPEC reference basket (ORB), 152
 operating benefits, 302
 operating cash flow and after-tax weighted cost of capital, 361
 operating expenditures (OPEX)
 abandonment costs and sunk costs, 166–167
 about, 162–166
 cash expended category, 114
 opportunity costs, 35, 167–168
 optimization study estimate, 169
 order of magnitude estimate, 168–169
 ordinary annuity, 57, 59
 ordinary annuity interest, 58
 Organization of Petroleum Exporting Countries (OPEC), 17
 original gas in place (OGIP), 118
 original oil in place (OOIP), 118, 119
 overall growth rate vs. project rate of return, 324
 overriding royalty interest (ORRI), 182
 overrun allowance, 173
 overrun sensitivity, 173
 ownership in fee, 178
 ownership interest computation, 181–185

P

patents or copyrights, 213–214
 payback period, 306–310
 payout benchmarks, 403
 percentage depletion, 226–228
 percent depletion, 208, 225–227
 periodic compounding, 43
 period interest compounding/discounting, 82
 periods (time), 84
 Petrocalc3 (commercial software package), 195
 Petroconsultants Australasia Pty, Ltd., 275
 petroleum costs, 255
 physical constants conversion, 331

- planning
 - capital expenditure, 4–6
 - defined, 2–3
 - management role in, 4
 - planning difficulties, 25
 - plot cash flow, 113
 - pooled working interest, 182–183
 - porosity and water saturation (Sw fraction), 119
 - positive net revenues, 307
 - power law model, 160
 - PPMT function, 91
 - present sums, 47
 - present value
 - of an annuity, 57–59
 - of cash surplus, 313
 - of a deferred annuity, 62–64
 - of future sum, 49–54
 - of shifted gradient series, 70–71
 - of uniform gradient, 68–70
 - present value index (PVI), 329
 - present value ratio (PVR), 329–330
 - present value ratio (PVR) and profitability index (PI), 394
 - present worth, 313
 - present worth factor (P/F), 49
 - present worth of future cash flows, 49
 - price of natural gas, 154
 - price shifts in demand and supply, 13
 - primary terms, 182
 - prime lending rate, 37
 - principal, 37
 - principal meridians, 179
 - probability index, 328–330
 - production behavior, 146
 - production bonus, 266
 - production costs, 163, 164
 - production forecast by analogy, 145–149
 - production payment interest (PPI), 183
 - production quotas, 20
 - production sharing agreements (PSAs)
 - cash flow spreadsheet, 258
 - cost recovery (CR), 255–256
 - disbursements flow diagram, 257
 - profit oil (PO) sharing and taxation, 256
 - royalties, 254
 - production sharing and royalty tax system elements, 263–270
 - production sharing contract (PSC) cash flow spreadsheet sample, 259–263
 - production taxes. *See also* severance taxes, 185
 - productive horizon (reservoir), 117
 - product pricing, 149–154
 - product stream forecasting
 - about, 117–119
 - cumulative gas (Gp)-cumulative oil (Np) plot, 138–139
 - curve shift on log-log paper, 128–131
 - decline curve analysis, 121–141
 - exponential decline, 123–126
 - French curve method, 127–128
 - gas reservoir material balance, 142–146
 - harmonic decline, 131
 - hyperbolic decline, 126–127
 - low-permeability (tight) gas reserves, 131–136
 - production forecast by analogy, 145–149
 - rate-accumulation production plot, 136–138
 - volumetric calculations, 119–120
 - watercut vs. cumulative plot, 140–141
 - profitability analysis, 302
 - profitability index (PI), 328–330, 394
 - profitability indicators, 303, 355–356
 - profitable process or investment, 302
 - profit indicators, 303
 - profit oil (PO), 255
 - profit oil (PO) sharing and taxation, 256
 - profit oil (PO) split and taxation, 256
 - profit planning
 - capital expenditure planning, 4–6
 - management role in planning, 4
 - oil company objectives, 3
 - role of, 2–3
 - progressive fiscal regime, 275
 - progressive royalty, 282
 - project costs, actual variations
 - contingencies and allowances, 172–174
 - cost overruns, 170–171
 - estimate accuracy, 171–172
 - project debt repayment schedule, 362–366
 - project rate of return vs. overall growth rate, 324
 - project screening economics, 401–406
 - property classes, 215
 - purposive decisions, 3
 - PV function, 91
- ## Q
- quality deduction, 151
- ## R
- ranges, 179
 - ranking criteria, 388
 - ranking of proposals, 415
 - rate-cumulative production plots, 136–138
 - RATE function, 85
 - rate of inflation, 22, 25
 - rate of return (ROR), 37, 268–270, 280, 316

rate of return (ROR) analysis, 392–393
 rate of return (ROR) contracts, 274
 rate-time curves, 121
 rate time plots vs. rate-cumulative
 production plots, 136
 real costs, 193
 real dollars, 29
 real income, 23, 26
 real net cash flows, 193
 receipts (revenues), 36
 recovery factor, 146, 147
 recovery property, 218
 rectangular survey system, 179
 reference crudes, 152
 refinery product groups, 150–151
 refinery yields, 150
 regressive fiscal regime, 275
 reinvestment calculation, 322–326
 relinquishment requirement, 264
 renewable sources, 21
 replacement decisions, 389
 reserves life index, 20
 residential property, 215
 resource rent tax (RRT), 274
 return on investment (ROI), 310
 revenue conversion, 331
 revenue-producing projects, 6
 R-factor, 267
 ringfencing, 265, 278
 risk
 of contractor, 273
 incorporation of attitudes toward, 31
 and uncertainty, 30–31
 risk-free rate of return, 36
 riskiness of foreign investments, 192
 risk preference, 47
 risk service contracts, 273
 royalties, 182, 254
 royalty owners (small producers), 225
 rule of 70, 23
 running casing, 157

S

sales taxes, 197
 salvage value (Sv), 219, 390, 392
 Saudi Arabian (Arab light), 151
 Saudi Arabian crude, 152
 savers, 24
 Schallheim, J. J., 398
 scope changes, 171
 screening criteria, 387
 S-curve, 170
 Securities and Exchange Commission
 (SEC), 1, 192
 seismic options, 265
 semi-logarithmic graph paper, 137
 service life breakdown analysis, 394–395
 service life extension, 396
 service-producing projects
 about, 389–398
 capital plans, 5
 equivalent cost analysis, 390–392
 present value ratio (PVR) and
 profitability index (PI), 394
 rate of return (ROR) analysis, 392–393
 service life breakdown analysis,
 394–395
 unequal life service-producing
 alternatives, 395–398
 severance and ad valorem taxes, 185–190
 severance taxes, 197
 shifted gradient, 70
 shifts in equilibrium price, 13
 short-term plans, 4
 signature bonus, 266
 simple and compound interest, 39–42
 simple interest, 39
 simple interest calculations, 75
 simulation exercises, 31
 simulation in decision analysis, 31
 Singapore quotations, 152
 single lump payments, 74
 single payment compound amount factor,
 54
 sizing model, 160
 sliding scale factors, 267–268
 sliding scales, 266–267
 SLN function, 235
 Society of Petroleum Evaluation Engineers
 (SPEE), 185
 sour crude, 151
 spreadsheet applications, 83–94, 235–243
 stable fiscal regime, 275
 stagflation, 24
 state and local taxes, 197
 state take, 115
 statistics and probability concepts, 31
 Stermole, Franklin J., 320, 324
 stock tank barrel (Stb), 149
 stock tank oil initially in place (STOIP), 118
 straight-line depreciation method, 218
 strategic plans/planning, 3, 4
 structural inflation, 24
 sulfur content, 151
 sunk costs, 114, 166
 supply, defined, 7–8
 supply and demand curves, 8–11
 supply factors, 10
 supply of loanable funds, 38
 surface owner, 178
 surrender requirement, 264
 sweet crude, 151
 SYD function, 242

T

tactical plans, 4
 tangible property, 213
 tax credits, 167, 399
 tax deductible items, 207
 tax liability, 208
 Tax Reform Act of 1986, 228
TC-10: State and Local Oil and Gas Severance and Production Taxes (API), 186
 technical service agreement (TSA), 273
 10-K report, 1
 time-adjusted measure of profitability, 304
 time value of money
 about, 35–36, 46–48
 annuity from future value, 67–68
 annuity from present value, 64–67
 calculation of unknown interest rate, 72
 cash flow diagram, 45
 funds flow and compounding/
 discounting, 81–83
 future value of an annuity, 59–62
 future value of present sum, 54–57
 interest, 36, 37–44
 loan amortization schedule, 73–81
 present value of a deferred annuity, 62–64
 present value of an annuity, 57–59
 present value of future sum, 49–54
 present value of shifted gradient series, 70–71
 present value of uniform gradient, 68–70
 ROI approach, 342
 spreadsheet applications, 83–94
 time zero dilemma, 95–96
 time zero, 45, 47, 358
 time zero dilemma, 95–96
 total cash flows and pre-tax weighted cost of capital, 361
 townships, 179
 transfer at market price, 177–178
 transfer pricing
 about, 174–176
 general formula for computing, 177
 transfer at market price, 177–178
 transfer price at cost, 176
 trip time, 157
 true lease, 398
 turnover (gross revenues), 269

U

ultimate oil and/or gas recovery, 119
 ultimate oil and/or gas reserves, 118, 146
 uncertainty, 30, 46
 uncertainty and risk, 30–31
 undiscounted methods, 341
 undiscounted unit technical cost (UT), 333
 unequal life investments, 404
 unequal life service-producing alternatives, 395–398
 unitary elastic supply curve, 13
 United States Geodetic Survey (USGS), 179
 unitized working interest, 183
 unit of production method. *See also* long-run marginal cost (LRMC), 215, 342
 units of production depreciation, 223
 unit technical cost (UT), 331–337
 unit total cost, 332
 unknown unknowns, 172
 Urals-Mediterranean, 152
 U.S. Bureau of Labor Statistics (BLS), 26
 U.S. dollars, 191
 U.S. Information Agency, 152
 U.S. Minerals and Management Service (MMS), 178
 utilities, 163

V

VDB function, 239
 venture participation, 31
 volumetric calculations, 119–120

W

wasting assets, 224
 water cut vs. cumulative oil plot, 140–141
 weighted average cost of capital (WACC), 194, 313, 360, 371
 well completion, 157
 wells, API numbers for, 180
 Weston, J. F., 324
 West Texas Intermediate (WTI), 151–152
 windfall profits tax (WPT), 274
 work commitment, 265
 working (operating) interests (WI), 181

XYZ

XNPV function, 91

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About the Author

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