

Yinshun Wang

Fundamental Elements of
Applied
Superconductivity
in Electrical Engineering



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FUNDAMENTAL ELEMENTS OF APPLIED SUPERCONDUCTIVITY IN ELECTRICAL ENGINEERING

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Yinshun Wang

North China Electric Power University, China



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Preface

Since its discovery, superconductivity and its applications have become one of the most active frontiers in modern science and technology. With the progress in exploration and research of superconductivity over nearly half a century, the practical NbTi and Nb₃Sn superconducting wires were successfully fabricated in the 1960s. Superconducting technology, especially superconducting magnet technology, was then put into applications. However, it is difficult for superconductors to be extensively used since they must work at a liquid helium temperature of 4.2 K.

Although the alternating current (AC) losses of superconducting windings is much lower than those of conventional copper windings, the effect of 1 W power consumption generated at a liquid helium temperature is at least equivalent to 500 W consumption of cooling power at room temperature. Therefore, the AC loss is not fully compensated for by reduction in AC losses, and the operating cost of superconducting electrical equipment is expensive, except in direct current (DC) applications. Until the 1980s, the AC application of the Low Temperature Superconductors (LTS) in a power system had not made substantial progress.

Since the High Temperature Superconductor (HTS) was discovered in 1986, the application of HTS electrical equipment operating at the liquid nitrogen temperature of 77 K came into being, and superconducting power technology was hoped to be applied in electrical power systems.

Great progress was made in development of HTS materials in the late 1990s, and practical HTS tapes were commercially realized. Research on superconducting power technology has made important and significant progress in many countries. At present, many superconducting equipment prototypes, such as superconducting cable, superconducting transformer, superconducting Fault Current Limiter (FCL), superconducting motor/generator, superconducting magnetic energy storage and other superconducting equipment, have been developed and demonstrated. At present, several groups of HTS cable prototypes operate in live grids. Superconducting technology has also found important applications in information technology, traffic transportation, scientific instrument, medical technology, national defence, large scientific projects and other fields besides the energy field.

Superconducting power technology is highly comprehensive and interdisciplinary, and related to superconducting technology, electric power technology, cryogenic insulation, cryogenic refrigeration, materials science and technology, etc. At the present, it is a promising research field of new science and high technology, with important scientific significance and application prospects in power systems. At the same time, superconducting power technology will be one of the key technologies in the future Smart Grid. It is predicted that this technology will become a practical technology of extensive scale and play an important role in energy saving, emission reduction, low carbon economy, renewable energy resources, and in other fields.

This book briefly introduces the basic theory of superconductivity. According to the knowledge structure and the order required in application of superconducting technology, electromagnetic properties of practical superconducting materials, stability, AC losses, processing technology, measurement of critical current and n values by contact-free methods, cryogenic insulation, cryostat and refrigeration, current

leads and flux pump, are presented respectively. The principles and structures of various superconducting equipment are also described. Finally, high- T_c superconducting (HTS) cables, and superconductivity applications in power systems, are systematically described to show how the basic technologies described elsewhere in the book fit together. The content of the book focuses on the fundamental elements of applied superconductivity in electrical engineering. A feature of this book is that experimental technology is added to related chapters together with the introduction of fundamental theoretical and technological principles.

There are 12 chapters in the book. The first chapter briefly introduces applications of superconducting power technology with several superconducting apparatus used in power systems. Chapter 2 presents the basic theories and critical parameters of superconductors. Chapter 3 describes mechanical and electromagnetic properties of superconducting materials. Chapter 4 introduces the stability and quench characteristics of superconducting materials and magnets, and protection technology of superconducting magnets. Chapter 5 systematically describes various AC losses of superconducting in commercial frequency power, which includes hysteresis loss, magnetic flux flow loss, coupling and eddy current losses, and methods of measuring AC loss. Chapter 6 briefly lists the preparation techniques of practical superconducting materials. Chapter 7 presents theory and measurements of critical current and n values in practical HTS tapes by contact-free methods, and their evaluation and calculation of non-uniformity are also included. Chapter 8 concerns the insulation characteristics of some cryogenic gas, cryogenic liquid, organic insulation film materials, inorganic insulating materials and cryogenic adhesive. Chapter 9 mainly shows the heat-conduction theory, cryogenic device design and cryogenic refrigeration technology. Chapter 10 systematically introduces the design principles and methods of various current leads, including conductor-cooled current leads, gas-cooled lead, Peltier current lead (PCL) and the hybrid current lead, the applications of superconducting persistent current switch (PCS) and superconducting flux pump technology. Chapter 11 presents basic structures of several superconducting apparatus in power systems. As in the case of the application of superconductivity in a power system, Chapter 12 systematically describes the design of HTS cable.

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Because of my limited knowledge, it is very hard to avoid some omissions and even mistakes, so it is my pleasure to receive your criticisms and corrections.

Yinshun Wang
State Key Laboratory for Alternate Electrical Power System with Renewable Energy Sources
Key Laboratory of HV and EMC Beijing
North China Electric Power University

*Beijing, China
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Abbreviations and Symbols

Abbreviations

Abbreviations	Meanings
1D	One-dimensional
3D	Three-dimensional
2D	Two-dimensional
1G	BSCCO-2223 conductors made by PIT process
2G	YBCO-123 coated conductors
AC	Alternating current
ACSR	Aluminium Cable Steel Reinforced
AMSC	American Superconductor Corporation, MA, USA
ASTM	American Society for Testing and Materials
AVR	automatic voltage regulator
BCS	Barden Cooper Schrieffer
BSCCO-2212	$\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$
BSCCO-2223	$(\text{Bi,Pb})_2\text{Sr}_2\text{Ca}_2\text{Cu}_3\text{O}_{10}$
CAES	Compressed air energy storage
CB	Circuit breaker
CC	Coated conductor
CCl_4	Carbon tetrachloride
CD	Cold dielectric
CH_4	Liquid Methane
CICC	cable-in-conduit conductor
CO	Carbon monoxide
COV	Coefficient of variance
CSC	Current source converter
CSM	Critical state model
CTC	Continuously transposed cable
CVD	Chemical vapor deposition
DC	Direct current
DP	Double pancake
DSC	Dynamic synchronous condenser
EC	Equivalent circuit

Abbreviations	Meanings
EM	Electromagnetic
EMF	Electromagnetic field
EMS	Maglev Electromagnetic System
FACTS	Flexible alternating current transmission system
FC	Field cooling
FCL	Fault current limiter
FDM	Finite difference method
FEM	Finite element method
FES	Flywheel energy storage
FRP	Fiber reinforced plastics
GFRP	Glass fiber reinforced plastics
G-M	Gilford–McMahon
GHe	Gas – helium
GN ₂	Gas – nitrogen
GNe	Gas – neon
H ₂	Hydrogen
He	Helium
H _c	Critical magnetic field
HM	Hysteresis machine
HTS	High temperature superconductor
HV	High voltage
HVDC	High voltage direct current
IBAD	Ion beam assisted deposition
IEC	International Electrotechnical Commission
InnoST	Innova Superconductor Technology, Beijing, China
INSTEC	International Superconductivity Technology Center, Tokyo, Japan
ISD	Inclined substrate deposition
ITER	International Thermonuclear Experimental Reactor
JJ	Josephson junction
kA · m	Kilo-ampere current carried in 1 m length of a wire
LHe	Liquid helium
LN ₂	Liquid nitrogen
LNe	Liquid neon
LO ₂	Liquid oxygen
LTS	Low temperature superconductor
L-type	Transformer type
LV	Low voltage
mol	Mole
maglev	Magnetic levitation
MAJ model	Majoros model
MB	Mono-block
MCG	Magneto-cardiogram
MCP	Melt cast process
MEG	Magneto-encephalogram
MFC	Multi-filamentary composite

Abbreviations	Meanings
MgB ₂	Magnesium Diboride
MHD	Magneto-hydrodynamic
MJR	Modified jelly roll
MLI	Multi-layer super-thermal insulation
MOCVD	Metal-organic chemical vapour
MOD	Metal organic deposition
MPMG	Melt-powder-melt-growth
MPZ	Minimum propagation zone
MQE	Minimum quench energy
MRI	Magnetic Resonance Imaging
MTG	Melt-textured growth
N ₂	Nitrogen
Nb ₃ Al	Niobium – aluminum
Nb ₃ Sn	Niobium – tin
NbTi	Niobium – titanium
Ne	Neon
NMR	Nuclear magnetic resonance
NS model	Norris model
Nu	Nusselt number
NZP	Normal zone propagation
OF	Oxygen-free Oil-filled
PCL	Peltier current lead
PCS	Persistent current switch
PD	Partial discharge
PIT	Powder-in-tube
PLD	Pulse laser deposition
PSF	Polysulfone
PM	Permanent magnet
PMP	Powder melt process
PPLP	Laminated Polypropylene Paper
PSS	Power system stabilizer
Pr	Prandtl number
PVC	Polyvinylchloride
PVD	Physical vapour deposition
PWM	Pulse width modulation
QMG	Quench-melt growth
RABiTS	Rolling-assisted biaxially textured substrate
Re	Reynolds number
RF	Radio frequency
RM	Reluctance machine
RMS	Root mean square
R-type	Resistive type
RT	Room temperature
RRR	Residual resistivity ratio

Abbreviations	Meanings
SC	Superconducting cable
SF ₆	Sulphur hexafluoride
SFCL	Superconducting fault current limiter
SFES	Superconducting flywheel energy storage
SI	Super-insulation
SIS	Superconducting insulator superconductor
SMES	Superconducting magnetic energy storage
SNS	Short section of non-superconducting
SQUID	Superconducting quantum interference device
TE	Thermoelectric
Teflon/PTFE	Polytetrafluoroethylene
UPS	Uninterrupted power supply
VSC	Voltage source converter
VSM	Vibrating sample magnetometer
WD	Warm dielectric
XLPE	Cross linked polyethylene
YBCO-123	YBa ₂ Cu ₃ O ₇
YSZ	Ytria – stabilized zirconia
ZFC	Zero-field cooling

Symbols

Symbols	Nomenclature	Units
A	Magnetic vector potential	Wb/m
A	Area	m ²
	Current unit	
<i>a</i>	Half thickness	m
<i>a_f</i>	Size of filament	m
B, <i>B</i>	Magnetic field strength	T
<i>B_c</i>	Critical magnetic field strength	T
<i>B_{c1}</i>	Lower magnetic field strength	T
<i>B_{c2}</i>	Upper critical magnetic field strength	T
<i>B_{dc}</i>	DC magnetic field strength	T
<i>B_f</i>	Characteristic magnetic field strength	T
<i>B_{irr}</i> (T)	Irreversible magnetic field	T
<i>B_m</i>	Amplitude and peak-to-peak value of AC magnetic field strength	T
<i>B_p</i>	Full penetration magnetic field strength	T
<i>b</i>	Normalized magnetic field strength	
	Thickness of sheath	m
<i>b_{ac}</i>	Normalized AC magnetic field strength	
<i>b_{c1}</i>	Normalized lower critical magnetic field strength	
<i>b_{c2}</i>	Normalized upper critical magnetic field strength	
<i>b_{dc}</i>	Normalized DC magnetic field strength	
<i>C</i>	Effective cross-sectional factor	

Symbols	Nomenclature	Units
	Specific heat capacity	J/(kg · K)
	Capacity	F
	Electron charger unit	
	Fitted constant	
C_L	Latent heat	J/kg
C_p	Specific heat capacity with constant pressure	J/(kg · K)
C_v	Specific heat capacity with constant volume	J/(kg · K)
c	Wall thickness	m
	Velocity of light in vacuum (3.0×10^8)	m/s
D	Diameter	m
	Outermost radius of shielding layer of CD cable conductor	m
	diffusivity	
D_e	Hydraulic diameter	m
D_m	Magnetic diffusivity	m ² /s
D_T	Thermal diffusivity	m ² /s
d	Thickness	m
E	Electric field	V/m
	Energy	J
	Young's modulus	N/m ²
$E_{\max}$	The maximum breakdown field	V/m
e	Electron charge (1.6×10^{-19})	C
	Ratio of ellipse axes	
F	force	N
	Capacity unit	
F_L	Lorenz force	N
F_p	Pinning force	N
f	Frictional factor	
	Frequency	Hz
	Volumetric ratio	
	The efficiency of convective heat transfer	
	Cooling efficiency	
f_c	Critical frequency	Hz
f_L	Lorenz force density	N/m ³
f_p	Pinning force density	N/m ³
G	Geometrical factor	
	Giga (10^9)	
	Ohmic heat generation per unit volume of conductor	W/m ³
G_c	Heat generation at critical temperature T_c	W/m ³
g	Gap between tapes	m
	Acceleration of gravity	kg · m/s ²
H	Magnetic field	A/m
H	Inductance unit	
H_c	Critical magnetic field	A/m
H_{c1}	Lower critical magnetic field	A/m
H_{c2}	Upper critical magnetic field	A/m
H_f	Characteristic magnetic field	A/m
H_m	Amplitude of AC magnetic field	A/m
H_p	Full penetration magnetic field	A/m

Symbols	Nomenclature	Units
Hz	Frequency unit	
h	Normalized magnetic field	
	Heat transfer coefficient	$W/(m^2 \cdot K)$
	Parameter	
	Planck's constant (6.63×10^{-34})	$J \cdot s$
	Height	m
h_{ac}	Normalized AC magnetic field	
h_{c1}	Normalized lower critical magnetic field	
h_{c2}	Normalized upper critical magnetic field	
I	Current	A
	Bessel function	
I_c	Critical current	A
I_m	Amplitude of AC current	A
I_n	n-order Bessel function	
i	Normalized AC current	
	Subscript index	
	Imaginary number unit	$i^2 = \sqrt{-1}$
i_{ac}	Normalized AC current	
i_c	Normalized critical current	
i_{dc}	Normal DC critical current	
J	Current density	A/m^2
	Energy unit	$N \cdot m$
J_c	Critical current density	A/m^2
J_e	Engineering current density	A/m^2
J_t	Transport current density	A/m^2
j	Subscript index	
	Imaginary number unit	
K	Effective thermal conductivity	$W/(m \cdot K)$
	Kelvin-temperature unit	
	Constant	
k	Thermal conductivity	$W/(m \cdot K)$
	Wave vector	m^{-1}
	Coupling coefficient	
k_B	Boltzmann constant (1.38×10^{-23})	J/K
L	Length	m
	Self inductance	H
L_0	Lorentz number	$W \cdot \Omega / K^2$
L_c	Critical length	m
L_p	Twist pitch	m
l_p	Perimeter length	m
M	Magnetization	A/m
	Mutual inductance	H
	Mega (10^6)	
	Mass	kg
	Molar mass	kg/mol.
	Mass flow rate	kg/s

Symbols	Nomenclature	Units
m	Magnetic moment	$A \cdot m^2$
	Mass	
	Length unit	
mol	Unit of measurement used in chemistry to express amounts of a chemical substance	
N	Numbers	
	Force unit	
N_F	Density of states	
n	Number of shielding layers	
	Numbers	
n	n value	
	Density of Cooper pair	
P	Power	W
	Pressure	Pa
	Cooling circumference	m
Pa	Stress and pressure unit	
P_c	Coupling loss per unit length	W/m
P_d	Dielectric loss per unit length	W/m
P_e	Eddy current loss per unit length	W/m
P_h	Hysteresis loss per unit length	W/m
P_{pump}	Pumping fluid loss	W
p	Momentum	$kg \cdot m/s$
Q	Heat per unit length	W/m
	Strain energy	J/m^3
	Heat conduction	W
q	Heat conduction per unit length	W/m
	Heat leakage per unit length	
	Heat per unit area	W/m^2
R	Resistance	Ω
	Universal gas constant (8.3145)	$J/(mol \cdot K)$
	Radius	m
R_g	Radius of generation normal zone	m
r	Radius	m
radian	Angle unit	rad.
S	Cross-sectional area	m^2
s	Time unit	
T	Temperature	K
	Tesla-magnetic field unit	
	Period of time	s
T_0	Base temperature – typically the cryogen bath	K
T_c	Critical temperature	K
T_g	Temperature at which power generation starts in composite conductor	K
T_m	Time of field ramp	s
T_{sh}	Sharing temperature	K

Symbols	Nomenclature	Units
T_p	Peak of temperature	K
t	Time	s
	Normalized critical temperature	
t_c	Thickness of normal metal	m
$\text{tg}\delta$	Dielectric loss angle tangent	
t_Q	Quench time for unbalanced normal zone	s
U	Voltage	V
	Potential	V
	Protection function	
u	Velocity of gas molecules	m/s
V	Voltage	V
	Volume	m^3
	Potential difference	V
v	Velocity	m/s
W	Power	W
	Energy unit	
Wb	Magnetic flux unit	$\text{T} \cdot \text{m}^2$
w	Half width	m
x	Distance	m
	Coordinate	
	Fitted parameter	
Y	Yield strength	MPa
y	Coordinate	
Z	Impedance	Ω
	Figure of merit $Z = \eta^2/(k\rho)$	1/K
z	Coordinate	
Greek		
α	Thermal accommodation coefficient	
	Defined parameter	
	Stekly parameter	
β	Specific heat ratio (C_p/C_v)	
	Stability parameter	
χ	Susceptibility	
χ'	Real part of complex susceptibility	
χ''	Imaginary part of complex susceptibility	
δ	Skin depth	m
	Magnetic penetration depth	m
	Dielectric loss angle	rad
ε	Relative permittivity	
	Strain	
	Emissivity/blackness	
ε_b	Bending strain	
ε_0	Vacuum permittivity (8.85×10^{-12})	F/m
ε'	Real part of complex permittivity	F/m
ε''	Imaginary part of complex permittivity	F/m
ϕ	Angle	radian
γ	Solid density	kg/m^3

Symbols	Nomenclature	Units
η	Efficiency	
	Viscosity	Pa · s
	Seebeck coefficient	V/K
	Characteristic rate for tunneling	1/s
	Current margin of HTS cable	
τ	Characteristic time constant	s
	Coupling time constant	
φ	Phase angle	radian
κ	Ginsburg-Landau parameter ($= \lambda/\xi$)	
λ	Penetration depth	m
	Wave length	m
	Filling factor	
μ	Relative permeability	
μ_0	Vacuum permeability ($4\pi \times 10^{-7}$)	H/m
μ'	Real part of complex permeability	H/m
μ''	Imaginary part of complex permeability	H/m
ν	Poission ratio	
	Heat transfer stability factor	
θ	Angle	radian
ρ	Fluid (liquid and gas) density	kg/m ³
	Resistivity	$\Omega \cdot \text{m}$
σ	Stress	Pa
	Stefan-Boltzmann constant (5.67×10^{-8})	W/(m ² · K ⁴)
τ	Characteristic time constant	s
	Decay time constant	s
ω	Angular frequency ($2\pi f$)	Radian/s
	order parameter	
ξ	Coherent length	m
Ψ	Wave function	
Φ	Magnetic flux	Wb
Φ_0	Magnetic flux quantum (2.07×10^{-15})	Wb
Γ	Γ function	
Θ	Temperature of cooling gas	K
Ω	Resistance unit	
$\hbar$	$\hbar/(2\pi)$	J · s
//	Parallel orientation	
$\perp$	Perpendicular orientation	

1

Introduction

In 1911, the physicist H.K. Onnes, of Leiden Laboratory in the Netherlands, was measuring the resistivity of metals at low temperatures. He discovered that the resistance of mercury completely disappeared when the temperature dropped to that of liquid helium (4.2 K). This phenomenon became known as superconductivity. In 1933, German scientists W. Meissner and R. Ochsenfeld found that the magnetic flux completely disappeared from the interior of materials with zero resistance when cooled to 4.2 K in the magnetic field. This zero magnetic field inside a material became known as perfect diamagnetism and is now called the Meissner effect.

In 1962, B.D. Josephson theoretically predicted the superconducting quantum tunneling effect, known as the Josephson effect. This is where a current flows for an indefinitely long time, without any voltage applied, across a device known as a Josephson junction (JJ) consisting of two superconductors coupled by a weak link. The weak link can consist of a thin insulating barrier (known as a superconductor-insulator-superconductor, or S-I-S) junction, and a short section of non-superconducting (S-N-S) metal. Subsequently, P.W. Anderson and J.M. Rowell experimentally confirmed Josephson's prediction.

Since its discovery, the superconductor and its applications have been one of the most active research fields in modern science and technology, due to its unique physical properties of zero resistance, perfect diamagnetism and the quantum tunneling effect. Superconducting technology is mainly applied in electrical engineering and electronics, and these applications and characteristics are listed in Table 1.1.

Table 1.1 shows that superconducting technology is of great value in the fields of energy resources, transportation, scientific instruments, medical care, national defence and large scientific project. Since its discovery, widespread application of the superconductor has become the pursuit of scientists and engineers. Before the 1960s, practical NbTi and Nb₃Sn superconducting wires were not manufactured until nearly half of a century after the discovery of superconductivity. Since then, superconducting technology and application of superconducting magnets are used for laboratory and practical applications. However, the conventional superconductors have not been widely used in power systems, particularly in alternating current (AC) applications, because of their need to operate at 4.2 K.

With the development of NbTi wires, the Magnetic Resonance Imaging (MRI) system has been increasingly used in hospitals for clinical diagnosis since the 1980s. In 1986, a true breakthrough was made in the field of superconductivity by A. Müller and G. Bednorz, researchers at the IBM Research Laboratory in Rüschlikon, Switzerland. They created a brittle copper oxide ceramic compound, the so-called high temperature superconductor (HTS), which presents superconductivity at temperatures above 40 K. Since then, several kinds of HTS have been discovered and the transition temperature from the normal to superconducting state has reached more than 90 K, which is higher than the liquid nitrogen temperature of 77 K.

Table 1.1 Main applications and characteristics of superconducting technology in electrical engineering

	Applications	Characteristics
Superconducting power technology	Transmission power cable	Power transmission with low loss and large capacity
	Fault current limiter (FCL)	Safety and stability of grid
	Superconducting Magnetic Energy Storage (SMES)	Regulation of power quality and stability of the grid
	Transformer	Energy saving and small size
	Motor	Higher efficiency and small size
	Generator	Higher specific power, higher efficiency and increased grid stability
	Current leads	High current density and low heat leakage
Superconducting magnet technology	Magnet with high field strength	Large scientific project in particle and nuclear physics, Nuclear magnetic resonance (NMR), scientific instruments, magnetic separation, material preparation, biological and medical science
	Magnetic levitation (maglev)	Maglev train, maglev propulsion, flywheel bearing and high precision gyroscope

Therefore, superconducting apparatus working at temperatures of 77 K made the widespread use of superconducting technology possible. With the great progress in development of HTS materials in the late 1990s, practical HTS tapes were manufactured and commercialized. The application of superconducting power technology was developed on a large scale, with the support of governments and multinational companies. Many prototypes of superconducting power apparatus, such as cable, transformer, FCL, motor/generator and SMES, were developed and demonstrated. At present, commercial superconducting apparatus, particularly the HTSs, are continuing to be developed with increasing investment from governments and companies. It is believed that a major breakthrough in superconducting technology will continue well into the future.

Applications of superconductors in electrical engineering primarily involve superconducting power technology and superconducting magnet technology. With their transition from normal state to superconducting state and then the largely increased current carrying capacity at high current density and zero resistance, superconducting power technology has developed further [1–3]. Table 1.2 lists the main advantages of superconducting apparatus and their influence on the electrical power industry. Other applications include the dynamic synchronous condensers (DSC), magneto-hydrodynamic (MHD) generation of power, cryogenic capacitors, gyrotrons, and superconducting induction heaters [4–14].

Table 1.2 shows that utilization of superconducting power technology cannot only improve power quality, enhance safety, stability and reliability of the grid system, reduce voltage level, and make a super large-scale power grid possible, but also greatly increases apparatus capacity and transmission capacity, and simultaneously reduces loss of power to the grid. Furthermore, the quality of power from renewable energy resources can be improved by using SMES to which a large grid can be effectively connected.

In comparison with a conventional magnet, a superconducting magnet has many unique advantages, such as no energy consumption, small volume, light weight, greater efficiency, greater thermal stability, longer magnetic field life and easier cooling, and also the ability to generate a high magnetic field in a relatively large space. Superconducting magnet technology has been extensively applied in large science projects, scientific instruments, inductive heating, magnetic separation, traffic, biomedicine and the defence industry [15–17].

Because superconductivity appears only at low temperatures, cryogenic technology is an essential prerequisite for any superconducting apparatus. Maintaining the cryogenic temperature consumes more energy and, in particular, maintaining the helium temperature will consume even more energy, which is a major impediment to the commercialization of low temperature superconductor (LTS) technology in AC operation. However, the cooling technology greatly influences customer perception of the superconductor

Table 1.2 Main characteristics, advantages and influences of several superconducting power apparatus

Apparatus	Characteristics	Advantages and influences on power system
Cable	(1) High transmission power density (2) Low loss, compact size and light weight (3) Small reactance per unit length	(1) Low voltage, large current and high transmission power density (2) Small occupied urban space
Fault current limiter (FCL)	(1) Zero resistance in normal state and large impedance in fault state. (2) Combination of detection, trigger and limiting current (3) Fast response (4) No harmful effect on power grid	(1) Enhanced stability of the grid (2) Improved reliability of power supply (3) Protected electrical apparatus (4) Reduced cost of construction and retrofit (5) Increased transmission capacity of the grid
Transformer	(1) High power density (2) Low loss, small volume and light weight (3) Liquid nitrogen cooling and insulation	(1) Reduced area of installation (2) Fulfilling requirements for environmental protection and energy saving (3) Noninflammable
Generator	(1) High capacity density (2) Low loss, small size and light weight (3) Low synchronous reactance (4) High overload ability	(1) Reduced energy loss and occupied area (2) Improved stability of the power system (3) Compensated reactive power, improved power quality and stability of grid
Motor	(1) High capacity density (2) Low loss, small size and light weight	(1) Reduced energy loss and occupied area (2) Improved efficiency
Magnetic Energy Storage (SMES)	(1) Fast response (2) High conversion efficiency (3) Able to provide high power to grid in short time	(1) Fast power compensation (2) Enhanced dynamic stability of the grid (3) Improved power quality (4) Improved reliability of power supply
Flywheel energy systems with superconducting bearings	(1) High power density and large capacity (2) High conversion efficiency (3) Low loss	
Current lead	(1) Low thermal conductivity (2) High current density	(1) Low heat leakage (2) High efficiency

product and its operational costs, especially its reliability. Except for certain high-value-added applications, such as MRI or NMR, the potential user is inconvenienced by the requirement to transfer liquid cryogen periodically. HTSs require low maintenance and low cooling costs for commercial acceptability, which is achieved since HTS application requires liquid nitrogen temperatures. The eventual widespread introduction of HTSs to power applications will depend on reliable closed-cycle refrigeration systems.

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2

Superconductivity

A superconductor has several main macroscopic characteristics, such as zero resistance, the Meissner effect, the Josephson effect, the isotope effect, anomalous specific heat capacity and abnormal infrared electromagnetic absorption. Relating to its main application in an electrical system, this chapter focuses on three basic properties, namely zero resistance, the Meissner effect and the Josephson effect.

2.1 The Basic Properties of Superconductors

2.1.1 Zero-Resistance Characteristic

The zero resistance characteristic of the superconductor refers to the phenomenon that resistance abruptly disappears at a certain temperature. It is able to transport direct current (DC) without resistance in the superconducting state. If a closed loop is formed by a superconductor in which a current is induced, the induced “persistent current” will show no obvious signs of decay for several years. The upper limit of resistivity measured by the “persistent current” experiment is less than $10^{-27} \Omega \cdot \text{m}$, while a good conventional conductor, such as copper, has a resistivity of $10^{-10} \Omega \cdot \text{m}$ at 4.2 K, which is more than 17 orders of magnitude than that of the superconductor. The typically experimental dependence of resistance on temperature in a superconductor is shown in Figure 2.1, in which the resistivity of the superconductor suddenly falls to zero when the temperature reduces to a certain value below the critical temperature T_c .

The zero resistance property of a superconductor is not the minimal resistance in the usual sense but is equal to zero. This is because carriers are not scattered by the crystal lattice, thus there is no energy dissipation in a superconductor carrying a DC current, which suggests that superconductivity is a kind of macroscopic quantum effect.

Since the discovery of superconductivity in 1911, there have been many attempts to establish a theory to explain this phenomenon, and a number of models describing physical characteristics of superconductors have been established. Some simple and easily understandable models belong to phenomenological theories of which the two-fluid model is a relatively intuitive theory. This model can successfully describe motion of carriers and magnetic field distribution within the superconductor. Combined with the constitutive Maxwell’s electromagnetism equations, the two-fluid model explains some superconducting phenomena such as zero resistance and the Meissner effect. Based on a series of interaction hypotheses between electrons and lattice in quantum mechanics, in 1957, J. Bardeen, L.N. Cooper and J.R. Schrieffer proposed the concept of Cooper pairs and established the well-known Barden–Cooper–Schrieffer (BCS) theory, that is, the superconducting quantum theory that describes superconductivity from the

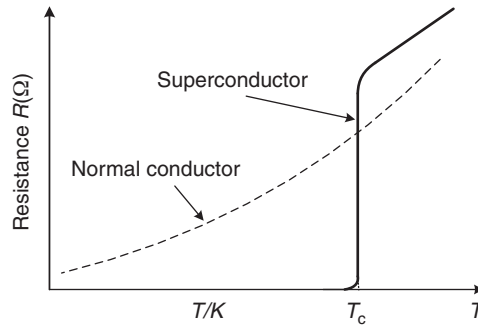


Figure 2.1 Resistance versus temperature curve of superconductors and normal conductors.

microscopic point of view and successfully explains most of superconducting phenomena. In order to easily understand superconductivity, this chapter introduces both the phenomenological two-fluid model and the BCS theory.

2.1.1.1 Two-Fluid Model

The two-fluid model is a phenomenological model based on the following three basic assumptions [1]:

- (i) Carriers consist of superconducting electrons and normal electrons in the superconducting state, the former transporting current without resistance and the latter transporting current with resistance, respectively. Then the carriers' density is composed of superconducting electron density and normal electron density.
- (ii) Superconducting electrons and normal electrons are defined as follows. In the superconducting state, the carrier density of the superconductor n is combined by normal electrons and superconducting electrons:

$$n = n_S + n_N \quad (2.1)$$

where n denotes total carrier density in the superconductor, n_S and n_N refer to superconducting carrier density and normal carrier density, respectively, while e_S and e_N represent superconducting electron charge and normal electron charge, respectively.

- (iii) In the superconductor, normal current density J_N and superconducting current density J_S mutually penetrate and independently transmit. Both can be interchangeable according to different temperatures and magnetic fields, and finally constitute total current density J of the superconductor:

$$J = J_S + J_N \quad (2.2)$$

- (iv) If the velocity of a normal electron is v_N , the normal current density J_N in the superconductor is:

$$J_N = n_N e_N v_N \quad (2.3)$$

Since the normal electrons are scattered by lattice vibrations, impurities or defects, the resistance of the conductor is not zero.

If the velocity of the superconducting electron is v_s , the resistance of the superconductor is zero, because the electrons are not scattered by lattice vibrations, impurities or defects, then superconducting current density J_s in the superconductor is:

$$J_s = n_s e_s v_s \quad (2.4)$$

Based on classical mechanics, if the superconducting electrons with mass m_s and charge e_s are not scattered by lattice vibrations, impurities or defects, they will be accelerated in the electric field E and obey Newton's second law:

$$m_s \frac{dv_s}{dt} = e_s E \quad (2.5)$$

Combining Equations (2.4) and (2.5), we obtain:

$$\frac{\partial J_s}{\partial t} = \frac{n_s e_s^2}{m_s} E \quad (2.6)$$

If superconducting current density J_s is steady (DC), the left-hand side of Equation (2.6) is zero, thus $E = 0$. According to Ohm's law, $J_s = \sigma E$, where σ is the conductivity and should be infinite, namely resistivity $\rho = 0$. However, if superconductor current density J_s varies with time, that is, the left-hand side of Equation (2.6) is not zero, thus $E \neq 0$, then there is an electric field E in the superconductor and it will drive normal electrons, which will cause Joule loss (losses of superconductors on AC conditions will be introduced in Chapter 5). Therefore, this again illustrates that zero resistance of the superconductor, or $E = 0$, only occurs at steady operation (DC).

In order to describe variation of superconducting electron density with temperature T , an order parameter relating to temperature $\omega(T)$ is introduced:

$$\omega(T) = \frac{n_s}{n} \quad (2.7)$$

When temperature T is higher than critical temperature T_c , the superconductor is in a normal state, and the superconducting electron density $n_s(T)$ is 0, then $\omega(T) = 0$. While temperature $T = 0$ K, all electrons convert into superconducting electrons, $\omega(T) = 1$, and the superconducting electron density $n_s(0)$ is equal to n . If temperature T is in the range of $0 < T < T_c$, and the superconductor is in a superconducting state, then the range of the order parameter $\omega(T)$ and carriers density is $0 < \omega(T) < 1$ and $0 < n < n_s$, respectively, and superconducting electrons appear in the superconductor. Thus, the critical temperature T_c , at which the superconductor transfers from the normal state to the superconducting state, may be considered as the corresponding temperature when Gibbs free energy density takes the minimum value within the range of $0 < \omega(T) < 1$ with $T < T_c$.

According to the relationship between Gibbs free energy density and the order parameter $\omega(T)$, if $T < T_c$, dependence of the order parameter of a stable system on temperature is:

$$\omega(T) = 1 - t^4 \quad (2.8)$$

Substituting Equation (2.8) into Equation (2.7), we obtain:

$$n_s(T) = n_s(0)(1 - t^4) \quad (2.9)$$

where $t = T/T_c$ is the normalized temperature.

When the temperature reduces to absolute zero, namely $T = 0$ K, all electrons convert into superconducting electrons. Superconducting electron density decreases as temperature increases. When the temperature rises to the critical temperature, $T = T_c$, and the superconducting electron density is equal to zero, then the superconductor transits to the normal state from the superconducting state, namely, the superconductor quenches.

2.1.1.2 Microscopic Theory – BCS Theory

Although the two-fluid model simply and qualitatively explains the macroscopic superconducting phenomena, as a phenomenological model it cannot fundamentally explain the mechanism of superconductivity. In order to deeply understand superconductivity with the zero-resistance effect, BCS theory that describes superconductivity from a microscopic point of view is introduced according to the Bose–Einstein condensation and interaction theory between electrons and lattice in quantum mechanics. This theory can explain most of superconducting phenomena.

The physical world is composed of two types of elementary particle. One type is the fermion with spins of half an odd integer $s = \pm 1/2, \pm 3/2, \pm 5/2, \dots$; and the other is the boson with spins of integer $s = 0, \pm 1, \pm 2, \pm 3 \dots$. Electrons are fermions with spin $s = \pm 1/2$, while photons are bosons with spin $s = 0$. The spin parameter s is an important basic parameter for describing these microscopic particles. Based on the theory of quantum statistical mechanics, a fermion can only occupy one state and obeys the Fermi–Dirac distribution:

$$f_F(E) = \frac{1}{e^{(E-\mu)/k_B T} + 1} \quad (2.10)$$

where μ denotes chemical potential, E is the electron energy and k_B refers to the Boltzmann constant.

In the case of bosons, more than one boson can occupy one and the same energy state, and obey the Bose–Einstein distribution:

$$f_B(E) = \frac{1}{e^{(E-\mu)/k_B T} - 1} \quad (2.11)$$

If all bosons occupy the same energy state, this phenomenon is known as the Bose–Einstein condensation.

Macro-mechanical laws comply with Newton's laws of mechanics, and macroscopic electromagnetic phenomena obey Maxwell's equations. However, the micro-particle complies with the Schrodinger equation. In classical physics, quantities such as mass, energy, momentum, force, angular momentum and displacement, etc., describe the phenomenon of macroscopic physics. Nevertheless, in microscopic physics, quantities such as mass and spin describe the microscopic particles, while the physical quantities of a single particle such as energy, momentum, force, angular momentum and displacement are meaningless. The laws of microscopic particles are described by wave functions, and the square of absolute value of wave functions represents the probability density of microscopic particles (if the wave function is normalized).

To find the possible energy state of a two-electron system, the wave function of the combined system with an opposed wave vector ($\pm \mathbf{k}$) is:

$$\Psi = \sum_{\mathbf{k}} a_{\mathbf{k}} e^{i\mathbf{k} \cdot \mathbf{r}_1} e^{-i\mathbf{k} \cdot \mathbf{r}_2} = \sum_{\mathbf{k}} a_{\mathbf{k}} e^{i\mathbf{k} \cdot (\mathbf{r}_1 - \mathbf{r}_2)} \quad (2.12)$$

where $\mathbf{r}_1$ and $\mathbf{r}_2$ denote space coordinate vectors of two electrons, respectively, while a_k is the expansion coefficient of the intrinsic plane wave function of free particles. Wave function (2.12) must satisfy the two-electron Schrödinger equation:

$$-\frac{\hbar^2}{2m} (\nabla_1^2 + \nabla_2^2) \psi + V(\mathbf{r}_1, \mathbf{r}_2) \psi = E \psi \quad (2.13)$$

where $V(\mathbf{r}_1, \mathbf{r}_2)$ represents the effective interaction potential between two electrons, $\hbar = h/(2\pi)$, h is Planck's constant and E denotes the Eigenvalue. Substituting Equation (2.12) into Equation (2.13), multiplying both sides by $e^{-i\mathbf{k}' \cdot (\mathbf{r}_1 - \mathbf{r}_2)}$ and integrating over all space by considering orthonormality of the Eigenstates, gives:

$$(E - 2\varepsilon_k) a_k = \sum_{k'} V_{kk'} a_{k'} \quad (2.14)$$

where ε_k is the Eigenstate energy of the single particle and $V_{kk'}$ denotes the expectation (average) value of the interaction potential between a pair of opposite momentum states.

If k is below the Feimi level, k_F , a_k disappears due to the Pauli exclusion principle in electron systems. Furthermore, $V_{kk'}$ is small when k is more than some cut-off term k_C , because the ion-core remains approximately stationary when the electron exits it rapidly. Assuming that the potential $V_{kk'}$ is zero below the Fermi energy E_F and above $E_F + E_C(k_C)$ and is constant $-V$ between E_F and $E_F + (k_C)$, this indicates that two electrons attract each other and the interaction potential energy is negative ($V > 0$). These two electrons are called the Cooper pair, in which two electrons or holes have opposing wave vectors and are bound by an attractive interaction to form an integer spin quasi-particle in a superconductor, in spite of their electric charges having the same sign [2, 3]. For simplicity, the average of interaction potential is assumed to be:

$$V_{k,k'} = \begin{cases} -V & k \geq k_F \quad k' \leq k_C \\ 0 & \text{others} \end{cases} \quad (2.15)$$

Substituting Equation (2.15) into Equation (2.14) and taking the potential energy term out of the summation sign, we find:

$$a_k = -V \frac{\sum_{k=k_F}^{k_C} a_{k'}}{E - 2\varepsilon_k} \quad (2.16)$$

Taking the sum on both sides of Equation (2.16) over k and considering the normalization condition, we obtain:

$$\frac{1}{V} = \sum_{k=k_F}^{k_C} \frac{1}{2\varepsilon_k - E} \quad (2.17)$$

According to quantum static mechanics theory, if the number of quantum states is large, summation can be replaced by integration with the weighted term N_F , where N_F is the state density at the Fermi energy E_F level. Then we have:

$$\frac{1}{V} = N_F \int_{E_F}^{E_F + E_C} \frac{1}{2\varepsilon - E} d\varepsilon \quad (2.18)$$

By integral calculation and series expansion and taking approximation at first level, Equation (2.18) becomes:

$$E \approx 2E_F - 2E_C e^{-2/N_F V} \quad (2.19)$$

or

$$\Delta E \approx -2E_C e^{-2/(N_F V)} \quad (2.20)$$

From Equation (2.19) we conclude that the energy of the two-electron system is smaller than that of two free electrons if there is attractive interaction between them, no matter how weak that attraction is. Universally, the system is always in a state with minimum energy. Thus, despite the charges of the two electrons having the same sign, as long as there is a net attractive interaction, it will always facilitate the formation of electron bound pairs, the Cooper pairs. It should be noted that the occurrence of Cooper pairs is a collective effect rather than a direct combination of two electrons in the superconductor, and the bonding strength depends on the states of all electrons. In classical electromagnetic theory, there is a Coulomb repulsion force between charges with the same sign, which contradicts the concept of Cooper pairs.

According to BCS theory, a pair of electrons is coupled by the interaction between an electron and a phonon, which is equivalent to direct interactions between two electrons, and this coupling makes each electron move toward surroundings of the ion-core with a positive charge due to the Coulomb attractive force. The electron alters the positive charge distribution adjacent to the ion-core when it moves through the lattice, so that a local region with high positive charge distribution forms, which results in attractive interaction with other adjacent electrons. Therefore, there is a possible attractive interaction between those electrons with opposite wave vectors to form an integer spin quasi-particle in spite of their electric charge with the same sign, and Cooper pairs finally form by using the lattice as a media.

There is a remarkable symmetry in Cooper pairs. According to quantum mechanics, the wave function of electrons must be anti-symmetric since they are fermions. However, there are two electrons in each Cooper pair, thus interchange of two electrons does not alter the symmetry, because the sign is changed twice. Because each electron has a spin $s = \pm 1/2$, the spin of the Cooper pair is $s = -1, 0$ or 1 , which indicates that the Cooper pair is a boson. Each electron of the Cooper pair must have two actions. First, it must act as a fermion in order to provide exclusion to make pairing possible. At the same time, it must be a member of the Cooper pair by acting as a boson. Moreover, the Cooper pair is a boson with integer spin $s = -1, 0$ or 1 , which means that an arbitrary number of Cooper pairs can be in the same state, especially in the ground state. If this happens to bosons, it is known as Bose–Einstein condensation. If it happens to a Cooper pair, superconductivity will occur.

To explain superconductivity that results from carriers forming Cooper pairs, we briefly introduce the mechanism of resistance generation in a normal conductor. In conventional conductors, the directional movement of a single electron is affected by inelastic scattering of the lattice, so part of the energy in the electron will be delivered to the lattice, which results in increasing vibration amplitude of the lattice, namely, the temperature rises and produces Joule heat. This is also the origin of resistance in a conventional conductor. Nevertheless, in a superconductor, Cooper pairs act as carriers. When they move directionally, if an electron inelastically collides with the lattice, it will lose part of its energy to the lattice; but the other electron with the opposite wave vector in the Cooper pair will simultaneously obtain the same energy from the lattice by its inelastic collision with the lattice. Consequently, the total net energy of the Cooper pair does not change in the whole scattering process, that is, there is no energy loss. Thus there is no resistance in directional movement of the carriers, and superconductivity occurs.

With temperature increase, thermally excited normal electrons appear near the Fermi surface, and two electrons are generated when each Cooper pair is destroyed. Thus, the number of normal electrons increases with the decreasing number of Cooper pairs. When temperature T reaches its critical temperature

T_c , Cooper pairs disappear so that a superconductor transits to normal state from superconducting state, that is, it quenches.

Although BCS theory is able to successfully explain most of low temperature superconductivity with temperatures below 25 K, it has difficulty in explaining the microscopic mechanism of high-temperature superconductivity. Until now, the microscopic mechanism of high-temperature superconductivity has not been clear, but it is certain that its origin still comes from the bound states of electrons in pairs, that is, the idea of carrier pairing still works. Furthermore, the relevance amongst electrons in HTS is strong and thereby is beyond the range of the weak interactions required by the BCS theory.

2.1.2 Complete Diamagnetism – Meissner Effect

When the superconductor is subjected to a magnetic field, in a non-superconducting state, the magnetic field can penetrate the superconductor and so the inner magnetic field is not zero in its normal state (Figure 2.2(a)) [4]. However, when the superconductor is in a superconducting state, the magnetic flux within is completely excluded from the superconductor, and the inner magnetic field is zero, that is, the superconductor is completely diamagnetic (Figure 2.2(b)). This phenomenon is called the Meissner effect. The superconductor can be suspended in a magnetic field due to its diamagnetism or the Meissner effect (Figure 2.2(b)), in which a YBCO bulk is in a magnetic field produced by conventional magnetic materials (NdFeB permanent). When the YBCO bulk is in a superconducting state, with liquid nitrogen temperature of 77 K, it is suspended in air because the magnetic flux is completely excluded from the YBCO, which results in a magnetic levitation force. We will explain the Meissner effect of superconductors in this section according to phenomenological theory.

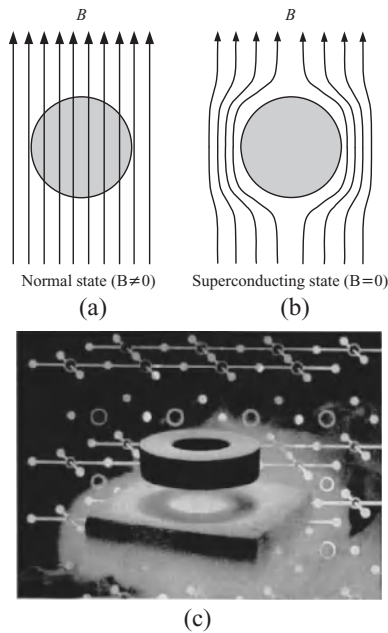


Figure 2.2 Meissner effect and levitation of superconductor: (a) normal state; (b) Meissner state; (c) YBCO bulk levitation.

Since the density and the phase of a Cooper pair varies slowly compared with its size, V.L. Ginzburg and L.D. Landau first proposed that the collective wave function can be defined as $\Psi(r, t) = \sqrt{n}e^{i\varphi}\psi(r, t) = \sqrt{n}e^{i\varphi}$ in superconductors, because all Cooper pairs are in the same local state [5], where $|\Psi|^2 = n$ is the density of the Cooper pairs, and φ denotes the phase of the wave function. In terms of wave function Ψ , current density associated with Cooper pairs may be found by multiplying velocity v with charge $2e$ and then taking the mathematical expectation value by integrating over all space:

$$J = \int \psi^* 2ev \psi dr \quad (2.21)$$

The momentum of particles in an electromagnetic fields is:

$$p = mv + 2eA \quad (2.22)$$

where p , e and m are the momentum of the Cooper pairs, electron charge and mass, respectively, and A denotes the magnetic vector potential.

As mentioned above, momentum and velocity in quantum mechanics should be expressed in the form of operators (quantized):

$$v = -i\frac{\hbar}{m}\nabla - \frac{2e}{m}A \quad (2.23)$$

Then the current density:

$$\begin{aligned} J &= \int \psi^* 2ev \psi dx \\ J &= \int \sqrt{n} \exp(-i\varphi) 2e \left(-i\frac{\hbar}{m}\nabla - \frac{2e}{m}A \right) \sqrt{n} \exp(i\varphi) dx \\ &= \frac{2ne}{m} (\hbar\nabla\phi - 2eA) \end{aligned} \quad (2.24)$$

By taking curl calculation on both sides of Equation (2.24), we obtain:

$$\nabla \times J = -\frac{4ne^2}{m} \nabla \times A = -\frac{4ne^2}{m} B \quad (2.25)$$

Equation (2.25) is called the London equation. Because $\nabla \times B = \mu_0 J$, μ_0 refers to the vacuum permeability, and we obtain:

$$\nabla \times \nabla \times B = \mu_0 \nabla \times J \quad (2.26)$$

substituting Equations (2.25) into Equation (2.26) and then rearranging, we obtain:

$$\nabla^2 B = \frac{4\mu_0 ne^2}{m} B \quad (2.27)$$

Defining:

$$\lambda = \sqrt{\frac{m}{\mu_0 n (2e)^2}} \quad (2.28)$$

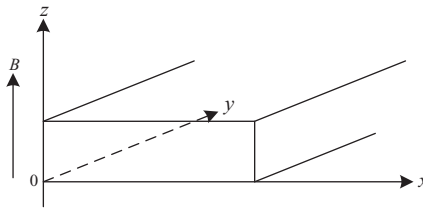


Figure 2.3 Semi-infinite superconductor in uniform magnetic field.

then Equation (2.28) becomes:

$$\nabla^2 B = \frac{1}{\lambda^2} B \quad (2.29)$$

A semi-infinite superconductor is used to simply describe the Meissner effect of superconductors (Figure 2.3). By choosing the Cartesian coordinate system, the superconductor infinitely extends along the y - and z - and positive x - z -axes, respectively. When the superconductor is exposed to the external magnetic field, which is uniform and along the z -axis direction, then Equation (2.29) is simplified to a one-dimensional (1D) differential equation with an x component only:

$$\frac{d^2 B}{dx^2} = \frac{1}{\lambda^2} B \quad (2.30)$$

Boundary conditions are:

$$B(x) = \begin{cases} B_0 & x = 0 \\ 0 & x = \infty \end{cases} \quad (2.31)$$

The solution of Equation (2.30) is:

$$B(x) = B_0 e^{-\frac{x}{\lambda}} \quad (2.32)$$

where λ is called penetration depth. If the Cooper pair density n is about 10^{28} m^{-3} , λ is approximately 10^{-6} cm , and then the current density in the superconductor is:

$$j_y = \frac{B_0}{\mu_0 \lambda} e^{-\frac{x}{\lambda}} = j_0 e^{-\frac{x}{\lambda}} \quad (2.33)$$

Figure 2.4 shows dependence of distributions of magnetic field and current density on penetrated depth in the semi-infinite superconductor. It indicates that the magnetic field and current density will exponentially decay with distance x from the surface and exist only in the thin outer layer of the superconductor with penetrated depth $x \sim \lambda$. When $x > 5\lambda$, either the magnetic field or current density decays almost to zero. The external magnetic field induces a current on the surface of the superconductor, which flows without resistance, and in turn the magnetic field produced by the induced current exactly offsets the external magnetic field B_0 . Then the inner magnetic field B is zero, from the macroscopic point of view, because the penetration depth is so small that it can be ignored when compared with conventional finite size. Therefore, the superconductor presents complete diamagnetism, namely the Meissner effect. Although the magnetic field and induced current in the outer thin layer of the superconductor with

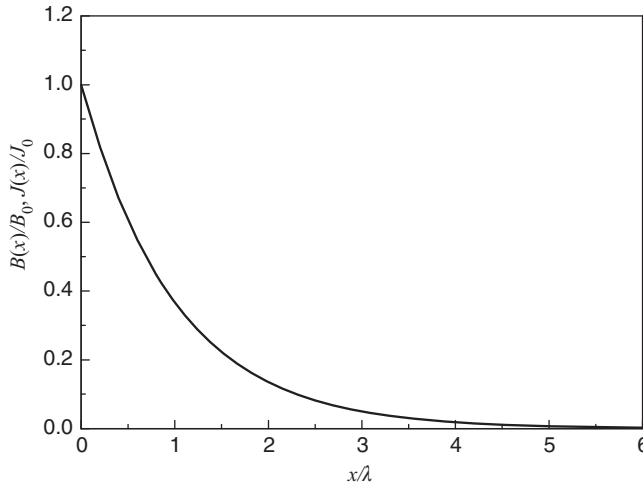


Figure 2.4 Dependence of magnetic field and induced current density on penetration depth.

thickness of penetration depth is not zero, the penetration depth is usually so small that the magnetic field and induced current can be considered as approximately zero, that is, the superconductor fully expels magnetic fields.

In classical electromagnetic theory, the level of the discrete magnetic flux lines density is commonly used to describe the magnitude of the magnetic field to explain the concept of magnetic flux density. But there arises the question of whether the flux exists in discrete or continuous forms within the superconductor?

By considering that the magnetic flux Φ goes through a closed superconducting loop and there is no current density inside it, that is, $J = 0$, according to Equation (2.24), we have:

$$\hbar \nabla \phi = 2eA \quad (2.34)$$

By integrating along the superconductor loop, we obtain:

$$\begin{aligned} \hbar \oint \nabla \phi \cdot dl &= 2e \oint A \cdot dl \\ \hbar \Delta \phi &= 2e \int \nabla \times A \cdot dA = 2e \int B \cdot dA = 2e\Phi \end{aligned}$$

so

$$\Delta \phi = \frac{2e}{\hbar} \Phi \quad (2.35)$$

Since the phase varying along the closed superconducting loop can only be an integer times of 2π , then the following equation must be satisfied:

$$\Delta \phi = 2\pi m \quad (2.36)$$

where m is an integer. Substituting Equation (2.36) into Equation (2.35), we find:

$$\Phi = \frac{2\pi\hbar}{2e}m = \frac{h}{2e}m = \Phi_0 m \quad (2.37)$$

where $\Phi_0 = 2.07 \times 10^{-15}$ Wb. This means that the magnetic flux does not vary continuously and can only increase or decrease by integer times of Φ_0 . This is thus called the flux quantization and Φ_0 is known as a single quantum.

2.1.3 Josephson Effects

As with semiconductor and thermocouple devices, there is the concept of a junction in a superconductor when it joins with an insulator. Two superconductors are separated by a thin insulating layer whose thickness is so small that Cooper pairs can pass through by the tunneling effect [6, 7]. This geometry is called the Josephson junction (Figure 2.5). S_1 and S_2 denote two kinds of superconductors, respectively, and I refers to the thin insulating layer. ψ_1 and ψ_2 are the wave functions of superconductors S_1 and S_2 .

If η is the characteristic rate for tunnelling through the central thin insulating layer, according to the time-dependent Schrödinger equation, the rate of change of wave functions can be found as:

$$i\hbar \frac{\partial \psi_1}{\partial t} = \hbar\eta\psi_2 \quad (2.38a)$$

$$i\hbar \frac{\partial \psi_2}{\partial t} = \hbar\eta\psi_1 \quad (2.38b)$$

Let $\Psi_1(r, t) = \sqrt{n_1}e^{i\varphi_1}$ and $\Psi_2(r, t) = \sqrt{n_2}e^{i\varphi_2}$, n_1 and n_2 correspond to the density of Cooper pairs of superconductors on both sides of the junction, φ_1 and φ_2 are the corresponding phases of two wave functions, and $\hbar = h/(2\pi)$ is Planck's constant. Substituting ψ_1 and ψ_2 into Equation (2.38a), we obtain:

$$i\hbar \frac{1}{2\sqrt{n_1}} \frac{\partial n_1}{\partial t} e^{i\varphi_1} + i\hbar \sqrt{n_1} e^{i\varphi_1} i \frac{\partial \varphi_1}{\partial t} = \hbar\eta \sqrt{n_2} e^{i\varphi_2}$$

After rearranging, we have:

$$\frac{\partial n_1}{\partial t} + 2in_1 \frac{\partial \varphi_1}{\partial t} = -2i\eta \sqrt{n_1 n_2} e^{i(\varphi_2 - \varphi_1)} \quad (2.39)$$

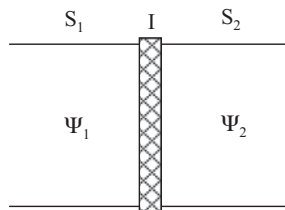


Figure 2.5 Geometry of Josephson junction.

Its real part is:

$$\frac{\partial n_1}{\partial t} = 2\eta\sqrt{n_1 n_2} \sin(\varphi_2 - \varphi_1) \quad (2.40)$$

Since current density J flowing through the junction is proportional to the rate of change of Cooper pair density n_1 , J is proportional to $\partial n_1 / \partial t$. If superconductors S_1 and S_2 on both sides of the junction are exactly the same, n_1 is approximately equal to n_2 , so by combining all constants into a coefficient J_0 , we obtain:

$$J = J_0 \sin(\varphi_2 - \varphi_1) \quad (2.41)$$

This is completely different from the current density of a normal conductor and junction. A current density with patterns of a sine wave relate to the quantum phase difference going through the junction to which there is no applied voltage. This phenomenon is called the DC Josephson effect.

Now applying a DC voltage V to both sides of the junction and adding the energy term to the Hamiltonian of the Cooper pair with a charge of $2e$, we have the time-dependent Schrodinger equations:

$$i\hbar \frac{\partial \psi_1}{\partial t} = \hbar\eta\psi_2 - eV\psi_1 \quad (2.42a)$$

$$i\hbar \frac{\partial \psi_2}{\partial t} = \hbar\eta\psi_1 + eV\psi_2 \quad (2.42b)$$

Substituting the wave functions ψ_1 and ψ_2 into Equation (2.42a), we obtain:

$$\frac{\partial n_1}{\partial t} + 2in_1 \frac{\partial \phi_1}{\partial t} = -2i\eta\sqrt{n_1 n_2} e^{i(\phi_2 - \phi_1)} + i \frac{2eV}{\hbar} n_1 \quad (2.43)$$

Its imaginary part is:

$$\frac{\partial \phi_1}{\partial t} = -\eta\sqrt{\frac{n_2}{n_1}} \cos(\varphi_2 - \varphi_1) + \frac{eV}{\hbar} \quad (2.44)$$

Similarly, substituting wave functions ψ_1 and ψ_2 into Equation (2.42b), we obtain:

$$\frac{\partial \phi_2}{\partial t} = \eta\sqrt{\frac{n_1}{n_2}} \cos(\varphi_1 - \varphi_2) - \frac{eV}{\hbar} \quad (2.45)$$

Let superconductors on both sides of the junction be the same, then n_1 is approximately equal to n_2 and by subtracting Equations (2.44) from Equation (2.45), we get:

$$\frac{\partial(\varphi_2 - \varphi_1)}{\partial t} = -\frac{2eV}{\hbar} \quad (2.46)$$

Integrating Equation (2.46) over time and substituting it into Equation (2.43), after reorganizing and taking the real part, we obtain:

$$J = J_0 \sin\left(-\frac{2eV}{\hbar}t\right) \quad (2.47)$$

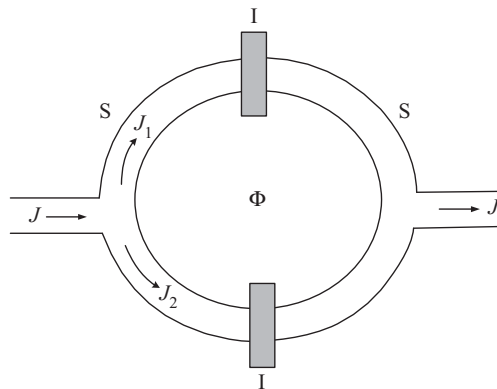


Figure 2.6 Geometry of two identical Josephson junctions connecting in parallel to form SQUID.

Equation (2.47) shows that there is an AC current whose frequency is proportional to the product of the fundamental constant e/h , even when the DC voltage is applied. This phenomenon is called the AC Josephson effect.

If two Josephson junctions are connected in parallel, a loop is formed, known as the superconducting quantum interference device (SQUID) (Figure 2.6). According to Equation (2.35), the phase difference around the loop must be $2e\Phi/\hbar$. In addition, it is known that the total current is the sum of the two branches; each one is a sine function of the phase difference of the Josephson junctions, and the combined current of the two branches is:

$$\begin{aligned}
 J &= J_0 \left[\sin \left(\varphi_0 + \frac{e}{\hbar} \Phi \right) + \sin \left(\varphi_0 - \frac{e}{\hbar} \Phi \right) \right] \\
 &= 2J_0 \sin(\varphi_0) \cos \left(\frac{\pi \Phi}{\Phi_0} \right)
 \end{aligned} \tag{2.48}$$

where φ_0 denotes the overall phase. Due to the interference between the two currents, the total current is a periodic function of the magnetic flux, which is why a device with this structure is called an interferometer. The minimum flux of the SQUID loop is a single flux quantum, $\Phi_0 = h/2e = 2.07 \times 10^{-15}$ Wb.

The Josephson effect is an important basis of superconducting electronics applications and has widespread applications in many instruments such as voltage reference, superconducting cavities, superconducting filters and SQUIDs. For example, it is the most sensitive method to detect a magnetic field, by using SQUID to measure current oscillations. SQUID can convert many other small signals in the magnetic field, so it can also be employed to measure these small current signals in clinical diagnosis, such as the magnetocardiogram (MCG) and the magnetoencephalogram (MEG). Superconducting tunnelling junctions can also be used as basic devices for high-speed, low-dissipation logic families.

2.2 Critical Parameters

Generally, there are three basic critical parameters, that is, critical temperature T_c , critical field H_c and critical current density J_c , which are most important parameters in applications of superconductors.

2.2.1 Critical Temperature T_c

The superconductor shows superconductivity when its temperature is below a certain value, that is, the temperature at which the superconductor transfers to a superconducting state from a normal state. This temperature is called the critical temperature and is denoted by T_c . In general, the superconducting transition usually occurs in a temperature range near T_c , which is called the temperature transition width represented by ΔT_c . In metal or alloy superconductors with high purity, a single crystal and stress free, ΔT_c is smaller than 10^{-3} K; however, transition width ΔT_c of practical HTS materials is usually in the range of 0.5 to 1 K, owing to their intrinsic characteristics, such as internal inhomogeneity, weak link, granularity and defects.

2.2.2 Critical Field H_c

Superconductors lose their superconductivity when the magnetic field strength exceeds a certain value in the external magnetic field. The magnetic field strength that causes a superconductor to lose its superconductivity is called the critical field strength and is denoted by H_c . When the temperature is less than critical temperature T_c , H_c is a function of temperature and continuously increases with temperature decrease. Like the critical temperature T_c , there is also a field transition width ΔH_c in the vicinity of H_c when the superconductor transfers from normal state to superconducting state. For a practical superconductor, there are usually two critical fields, namely the upper critical field H_{c2} and the lower critical field H_{c1} . When the field H is less than H_{c1} , the superconductor is in the Meissner state; however, when the field H is larger than H_{c2} , the superconductor is in the normal state; while the field H is between H_{c1} and H_{c2} , the superconductor is in the mixed state. These properties will be discussed in more detail in the next section.

2.2.3 Critical Current Density J_c

Although a superconductor can transport current without resistance, its ability is limited. It also loses its superconductivity if the transport current increases above a certain value, called the critical current I_c . In practical applications, it is more convenient to use current density than transport current, so that the corresponding current density is defined as critical current density J_c . As the transport current increases, transition of the superconductor does not jump to the normal state from the superconducting state. Usually critical current I_c refers to the maximum direct current that can be regarded as flowing without resistance in the superconductor, and the criteria for this condition are that the electric field strength E is $1 \mu\text{V/cm}$ or the resistivity ρ is $10^{-13} \Omega\cdot\text{m}$. The critical current I_c continuously decreases with increase of temperature T and magnetic field B .

The three basic critical parameters T_c , H_c , and J_c of superconductors are not independent of each other, there being a strong correlation between them. Figure 2.7 shows the relationship between the three critical parameters. Any point within the volume enclosed by a curved surface (J_c , T_c , H_c) and three planes (T_c , J_c), (J_c , H_c), (H_c , T_c) is in superconducting state S ; any point outside the volume is in normal state N , and any point on the curved surface with (J_c , T_c , H_c) is at critical state C . At present, the discovered critical temperature T_c of T_1 -based HTS can reach up to 135 K and its critical magnetic field B_c is more than 25 T, while its theoretical maximum value is more than 100 T. Table 2.1 lists some main critical parameters of several types of superconductors [8], which include crystal structure, critical temperature T_c , upper magnetic field B_{c2} ($B_{c2} = \mu_0 H_{c2}$) at temperature $T = 0$ K predicted by the Ginzburg–Landau (G-L) theory, penetration depth λ and the coherent length ξ .

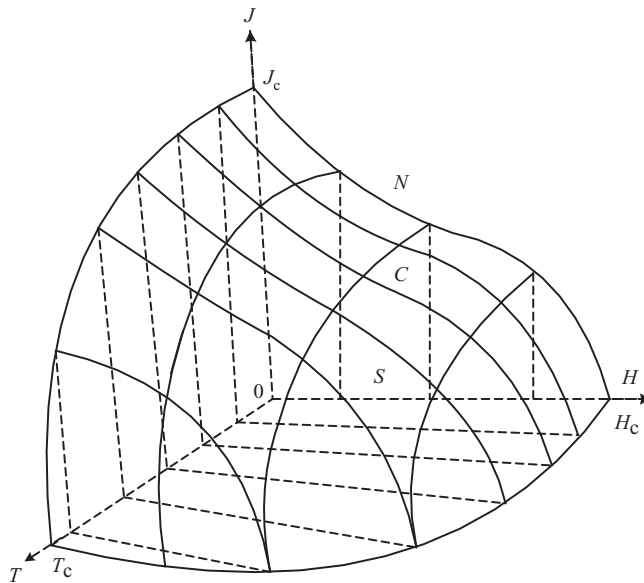


Figure 2.7 Critical parameters and their relationships.

2.3 Classification and Magnetization

2.3.1 Coherence Length

In Section 2.1.2, we introduced the Meissner effect of superconductors and the concept of magnetic field penetration depth λ according to the London equations. In order to clearly explain the purpose of classification of superconductors, another important microscopic parameter, coherence length ξ , is described in this section.

Based on the BCS theory, superconductivity results from formation of Cooper pairs, which act as carriers without resistance. However, the binding energy between two electrons of Cooper pairs is weak, but the correlation distance of two electrons ξ is long. ξ is called the coherent length and can reach up to 10^{-4} cm, which is more than 10^4 times that of the lattice size based on the calculation of second-order phase transition theory in a superconductor. Therefore, the superconducting correlation is a long-range interaction and can occur in space spanning many lattices. Furthermore, there are possibly many Cooper pairs in the same space.

By introducing non-local electrodynamics into superconductivity and developing the London theory, Pippard proposed the concept of the superconducting coherence length ξ . According to the London equation, the penetration depth λ of a superconductor is constant and depends on material properties, as well as on temperature. Based on the Ginzburg–Landau theory and experimental results, corrections on the London penetration depth can be made:

$$\lambda(T) = \lambda(0) \left[1 - \left(\frac{T}{T_c} \right)^4 \right]^{-1/2} \quad (2.49)$$

where $\lambda(0)$ refers to the penetrating depth of superconductors when the temperature is 0 K. The second column from the right in Table 2.1 lists penetration depths of several superconducting materials at 0 K.

Table 2.1 Macro- and microscopic characteristic parameters of several superconductors

Superconductors	Crystal structure	Lattice constant nm	T_c K	$\mu_0 H_{c2}(0\text{ K})$ T	$\lambda(0\text{ K})$ nm	$\xi(0\text{ K})$ nm
NbTi	A2		9.3	13	300	4
V ₃ Ga	A15	0.4816	15	23	90	2~3
V ₃ Si	A15	0.4722	16	20	60	3
Nb ₃ Sn	A15	0.5289	18	23	65	3
Nb ₃ Al	A15	0.5187	18.9	32		
Nb ₃ Ga	A15	0.5171	20.3	34		
Nb ₃ (Al ₇₅ Ge ₂₅)	A15		20.5	41		
Nb ₃ Ge	A15	0.5166	23	38	90	3
NbN	B1		16	15	200	5
V ₂ (Hf,Zr)	C15		10.1	24		
PbMo ₆ S ₈	Chevrel		15	60	200	2
MgB ₂	Hexagon	0.3086	39	~16(ab) ~2.5(c)	140	5.2
La _{1.85} Sr _{0.15} CuO _{4-δ}	I4/mmm	0.3779	40	50	80(ab) 400(c)	~4(ab) 0.7(c)
YBa ₂ Cu ₃ O _{7-δ} (YBCO, Y123)	Pmmm	0.3884	90	670(ab) 120(c)	150(ab) 900(c)	~2(ab) 0.4(c)
Bi ₂ Sr ₂ CaCu ₂ O _{8-δ} (Bi-2212)	A2aa	0.541	90	280(ab) 32(c)	300(ab)	~3(ab) 0.4(c)
(Bi,Pb) ₂ Sr ₂ Ca ₂ Cu ₃ O _{10+δ} (Bi-2223)	Perovskite (Orthogonal)	0.539	110			
Tl ₂ Ba ₂ CaCu ₂ O _{8+δ} (Tl-2212)	I4/mmm	0.3856	110	2.926	215(ab)	~2.2(ab)
Tl ₂ Ba ₂ Ca ₂ Cu ₃ O _{10-δ} (Tl-2223)	I4/mmm	0.385	125	3.588	205(ab) 480(c)	0.5(c) 1.3(ab)
HgBa ₂ Ca ₂ Cu ₃ O _{8+δ}	Pmmm	0.385	133	1.585	160	1.42(ab)

Note: (ab) denotes the *ab* plane of crystal structure; (c) refers to the *c*-axis direction perpendicular to the *ab* plane, the symbol “~” means “close to”.

A Cooper pair reflects the collective effect of the entire electron system with which lattice ions couple, and the coupling strength is determined by the states of all electrons. Theories and experiments show that the superconducting coherence length relates to temperature, and by considering the influence of temperature, the coherence length of a superconductor is approximately:

$$\xi(T) = \xi(0) \left(1 - \frac{T}{T_c}\right)^{-1/2} \quad (2.50)$$

where $\xi(0)$ denotes the coherence length of a superconductor with temperature 0 K. The magnitudes of coherence length of several superconductors are also presented in last column of Table 2.1.

2.3.2 Classifications

It was experimentally found that some superconductors in magnetic fields do not allow penetration of magnetic flux with magnetic field increase before they lose superconductivity, even if the magnetic field is more than its critical magnetic field. Conversely, other superconductors permit penetration of the magnetic field into their partial regions, which results in their interiors showing local interlacement with the normal state and the superconducting state simultaneously, even though their resistance remains at zero. Thus, the superconductors are classified into two types [9].

According to the Ginzburg–Landau theory, superconductors can be classified into two categories based on the ratio of penetration depth λ to coherence length ξ . By defining the Ginzburg–Landau parameter κ as:

$$\kappa = \frac{\lambda(T)}{\xi(T)} \quad (2.51)$$

if $\kappa < 1/\sqrt{2}$, superconductors have positive interface energy and are called Type I superconductors; conversely, if $\kappa > 1/\sqrt{2}$, superconductors have negative interface energy and are defined as Type II superconductors. In order to visually understand the coherence length and penetration depth of superconductors, Figure 2.8 shows the coherence length ξ and penetration depth λ of Type I and Type II superconductors. $n_s(r)$ (the square of the absolute value of the wave function) and $B(r)$ separately denote the spatial distribution of superconducting electron density $n_s(r)$ (the square of the absolute value of the wave function) and the magnetic flux density (magnetic induction) at the interface between the normal and superconducting regions. The regions with $B(0)$, $n_s(0)$ correspond to the normal region (N) and r superconducting region (S), and the mixed area with combination of $B(r)$ and $n_s(r)$ refers to the interface of normal and superconducting regions.

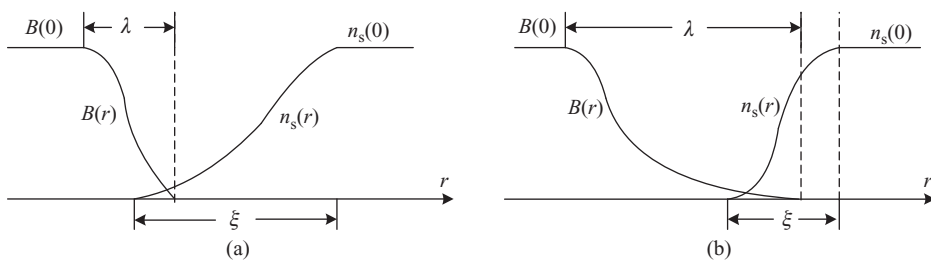


Figure 2.8 Penetration depth and coherence length of superconductors and their interfaces: (a) Type I superconductor; and (b) Type II superconductor.

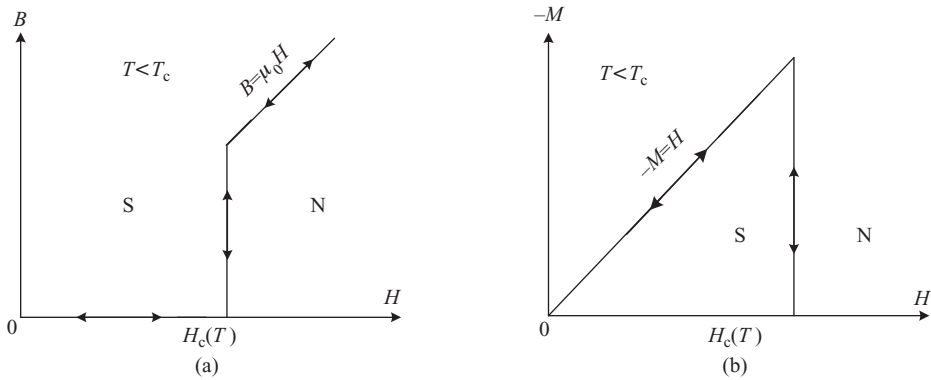


Figure 2.9 Magnetic field (a) and magnetization (b) of Type I superconductor.

2.3.3 Type I Superconductor and Magnetization

Type I superconductors, also known as Pippard superconductors or soft superconductors, have only one critical magnetic field B_c . If the temperature is below the critical temperature ($T < T_c$) and external magnetic field $B < B_c$, the superconductor is in the Meissner state, namely it shows perfect diamagnetism and the magnetic field inside is zero, and magnetization M is equal to magnetic field strength H and both N and H are opposite in direction, that is, $M = -H$. As the external magnetic field increases and reaches the critical magnetic field $B_c (= \mu_0 H_c)$, the superconductor immediately turns into the normal state from the superconducting state. It should be noted that the critical magnetic field B_c is a function of temperature. All of those already discovered metal superconductors belong to the Type I superconductors, except for vanadium, niobium and technetium.

Figures 2.9(a) and (b) show the magnetic field and magnetization of Type I superconductors at normal (N) and superconducting (S) states, respectively. As illustrated in Figure 2.9(b), magnetization of a Type I superconductor is reversible and there are only two states, that is, superconducting (Meissner) and the normal states. Its superconducting phase diagram is shown in Figure 2.9(a). Generally, a Type I superconductor is not useful in a high magnetic field because of its too low critical magnetic field $B_c (= \mu_0 H_c)$. In addition, because the transport current can only flow on the thin layer of its surface with thickness of penetration depth $\lambda \sim 10^{-6}$ cm, there is no current flowing through its entire body, so the Type I superconductor has little practical value in superconducting magnets and power applications.

2.3.4 Type II Superconductor and Magnetization

Type II superconductors are classified into two categories: ideal Type II superconductor and non-ideal Type II superconductors [9, 10].

2.3.4.1 Ideal Type II Superconductors

An ideal Type II superconductor is also colloquially called a “clean” superconductor. Unlike the Type I superconductor, the Type II superconductor has two critical magnetic fields, which are separately defined as the lower critical magnetic field $B_{c1} (= \mu_0 H_{c1})$ and the upper critical magnetic field $B_{c2} (= \mu_0 H_{c2})$, when subjected to temperature T below the critical temperature T_c . Similarly, both of them are also dependent on temperature. When the external magnetic field B satisfies $B < B_{c1}$, the superconductor

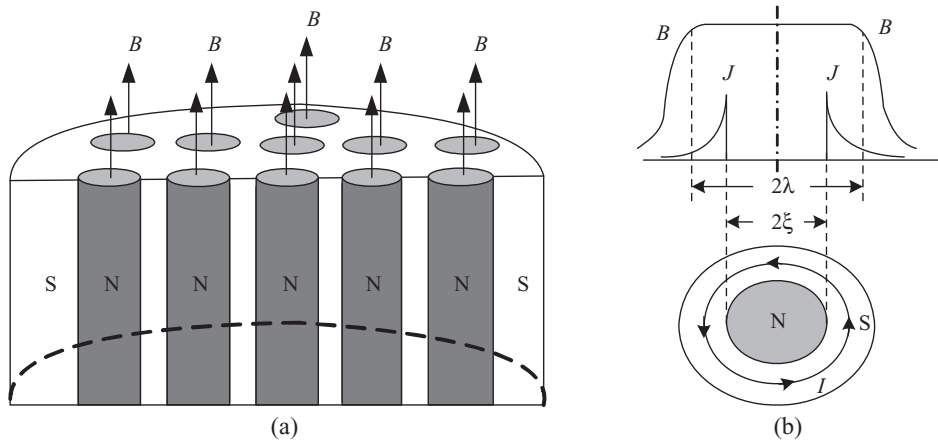


Figure 2.10 Type II superconductors in the mixed state: (a) mixed state; (b) distribution of magnetic field and current density in a unit consisting of normal and superconducting regions.

stays in the Meissner state (S_1) and then has full diamagnetism with a zero magnetic field throughout. When the external magnetic field B is in the range of $B_{c1} < B < B_{c2}$, the superconducting state and normal state coexist, which is known as a mixed state (S_2), and the flux lines can go through the normal region inside the superconductor, which is known as the flux vortex area.

Figure 2.10(a) shows a Type II superconductor in the mixed state, in which the shaded areas denote normal region (N) through which the magnetic field can pass, while other parts are superconducting regions (S). Figure 2.10(b) presents a unit consisting of normal and superconducting regions in the mixed state, in which the normal region is located in the central section with size 2ξ , and λ refers to the penetrating depth. A magnetic field can pass through the normal area, and the vortex screening current forms outside of the central region. This is why the mixed state is sometimes called the vortex state. The magnetic field gradually decreases until it reduces to zero outwards from the central region, that is, the magnetic field is completely screened.

When the external magnetic field B further increases, the superconducting region (S) gradually reduces and the normal area (N) expands; if $B \geq B_{c2}$, the superconducting state completely returns into the normal state (N). Figure 2.11 shows the magnetic field and magnetization of an ideal Type II superconductor, in which both of them are reversibly similar with the Type I superconductor. The phase diagram of an ideal Type II superconductor is illustrated in Figure 2.12(b). There are three states: two superconducting state (S_1 -Meissner state) and mixed state (S_2), and one normal state (N).

Figure 2.13 shows the dependence of critical current density J_c of an ideal Type II superconductor on the magnetic field. Critical current density J_c decreases with increase of external magnetic field B ($= \mu_0 H$). As the external magnetic field becomes more than its lower critical magnetic field B_{c1} , critical current density J_c disappears. Since the crystal structure of ideal Type II superconductors is relatively complete, and there are no defects, the flux lines are uniformly arranged, so the vortex currents around flux lines cancel each other out, thus there is no net current passing through the inside of the superconductor, that is, its body current I_v is zero, $I_v = 0$. To some extent, the ideal Type II superconductor is similar to the Type I superconductor regarding its ability to carry current (Figure 2.14). The current only exists on the surface of a superconductor with thickness of penetration depth so that the body of an ideal Type II superconductor has no ability to transport current.

Figure 2.14 (a) shows that the ideal Type II superconductor is the same as the Type I superconductor with respect to transporting current, since there is only a small area at its surface with a current-carrying

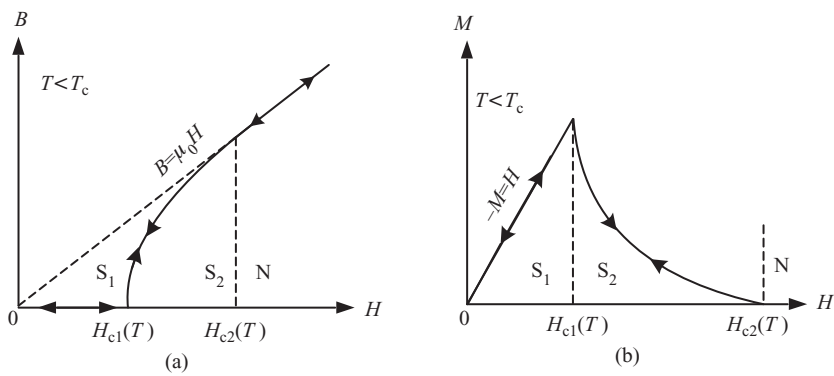


Figure 2.11 (a) Magnetic field and (b) magnetization of ideal Type II superconductors at temperature T .

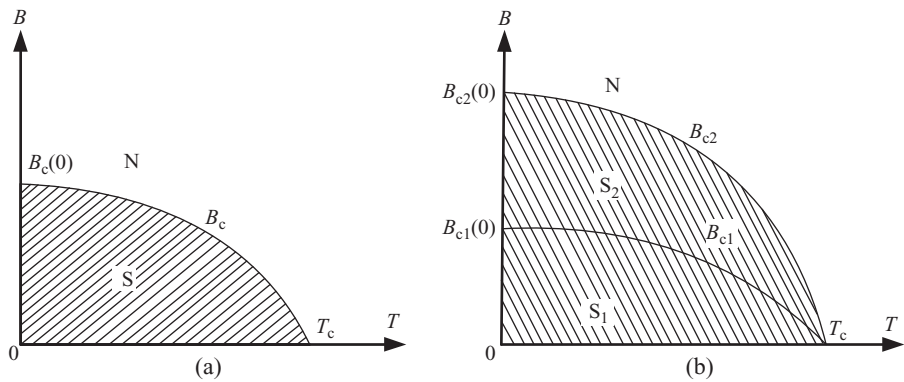


Figure 2.12 Phase diagrams of superconductors: (a) Type I superconductor; (b) Type II superconductor.

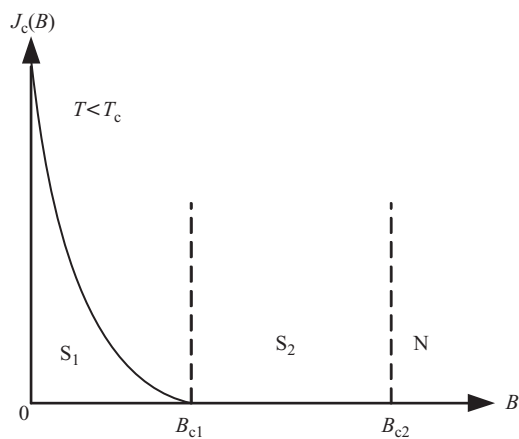


Figure 2.13 Dependence of critical current density J_c on magnetic field in ideal Type II superconductors.

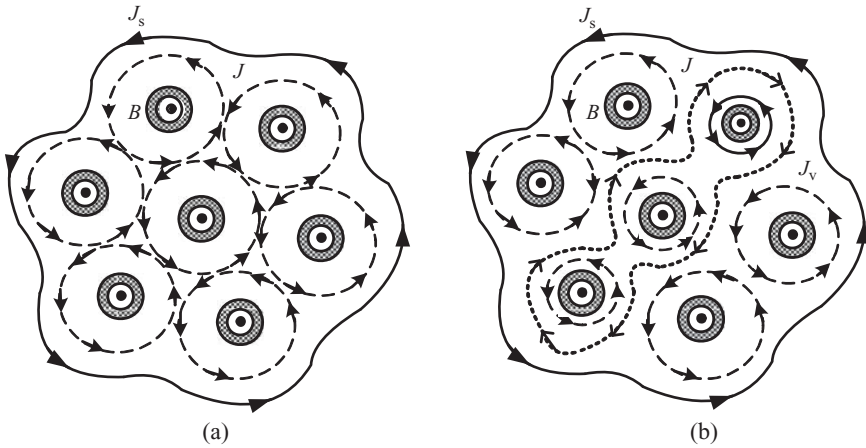


Figure 2.14 Distribution of carrying current in superconductors: (a) Type I superconductor; (b) Type II superconductor. Solid line on surface refers to surface current density J_s , dashed line denotes vortex current density J and the dot line represents the body current density J_v . Shaded and white regions are normal and superconducting, respectively. Symbol $\odot$ indicates magnetic field direction.

capacity, while within the superconductor there is no ability to transport current with a useful capacity. However, applications of an ideal Type II superconductor in superconducting magnets and electrical power apparatus are greatly limited because its lower critical field B_{c1} is low. Practical superconductors are non-ideal Type II superconductors, and the majority of compound superconductors, such as NbTi, Nb₃Sn, MgB₂ and HTS oxide ceramics, belong to the non-ideal Type II superconductors.

2.3.4.2 Non-Ideal Type II Superconductors

The non-ideal Type II superconductors, also known as hard superconductors or “dirty” superconductor, are similar to Type I superconductors in some respects, such as both have a mixed state and two critical magnetic fields with lower H_{c1} and upper H_{c2} , but their magnetization is irreversible and it can trap flux with remanent magnetism characteristics (Figure 2.15). When the non-ideal Type II superconductor is in a superconducting state, it remains in the Meissner state (S_1) with complete diamagnetism when increasing the external magnetic field H and keeping $H < H_{c1}$ simultaneously. Thus, the magnetic field inside the superconductor is zero, and the magnetization and the magnetic field strength H are equal but with opposite directions, that is, $-M = H$. If the external magnetic field continues to increase to the range of $H_{c1} < H < H_{c2}$, the superconductor will be in the mixed state (S_2) and the magnetic flux penetrates into the superconductor.

The relationship of magnetization with magnetic field is non-linear and also irreversible. When the external magnetic field H continuously increases until $H > H_{c2}$, and the superconductor loses superconductivity and recovers to the normal state (N), then the magnetic field uniformly and completely penetrates into the entire body, that is, $-M = 0$. When reducing the external magnetic field B , the varying path of magnetization with the magnetic field is not in accordance with the path when the magnetic field is increased. Even if the external magnetic field H is reduced to zero, the magnetic field and magnetization inside the superconductor are not zero, indicating that there is still magnetization and a magnetic field, which are called remanent magnetization M_r and magnetic field B_r , respectively. If the external magnetic field changes in a cycle, then the magnetization curve is a nonlinear closed curve, that is, a hysteresis loop (Figure 2.15(b)).

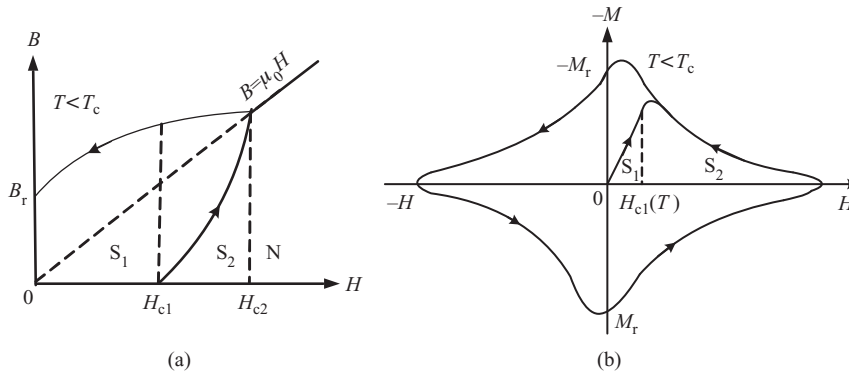


Figure 2.15 (a) Magnetic field and (b) magnetization of Type II non-ideal superconductor at the temperature $T < T_c$.

The irreversible magnetization curve is due to crystal defects, impurity phases, inhomogeneity and radiation damage within the non-ideal Type II superconductors, which have a close relationship with the processing of superconductors. These defects cause a pinning effect on the flux and hinder flux lines from entering and leaving the superconductor. The region where flux lines exist is in the normal state and is called the pinning centre, but the region through which the current flows stays in the superconducting state. As presented in Figure 2.14(b), unlike Type I superconductors, vortex current densities J around the flux lines cannot completely cancel each other out because the arrangement of flux lines is not uniform, thus there is a “net” body current density inside the superconductor besides the current density J_s on its surface. Therefore, the whole superconductor has the ability to transport current.

Although the lower critical magnetic field B_{c1} is usually low, the upper critical magnetic field B_{c2} is much higher, so that the superconductors have strong carrying current capacity in a wide range of magnetic fields. Compared with the ideal Type II superconductor with almost zero carrying current ability, the non-ideal Type II superconductors in the mixed state (S_2) still has a strong current carrying capacity. Figure 2.16 shows the relationship of critical current density J_c with magnetic field B at some temperature $T < T_c$.

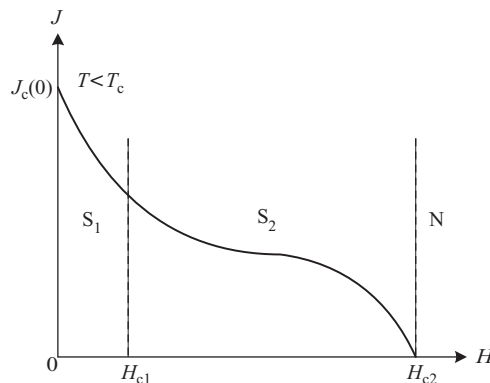


Figure 2.16 Dependence of critical current density J_c on magnetic field in non-ideal Type II superconductors at temperature $T < T_c$.

All superconducting materials suitable for applications are non-ideal Type II superconductors. In superconducting power applications, we focus on current carrying capacity without resistance at certain temperatures and magnetic fields. In this book, if not specified, thereafter all the superconductors referred to non-ideal superconductors.

The more pinning centers there are in superconductors, the stronger the pinning effect and the carrying capacity. Therefore, some special processes such as cold work, doping and radiation are usually chosen to improve the abilities of the pinning flux and current carrying in the production of practical superconducting materials.

2.4 Measurement Technologies of Critical Parameters

Critical temperature T_c , critical current density J_c (critical current I_c) and the critical magnetic field B_c (the critical magnetic field strength H_c), including upper B_{c2} and lower B_{c1} critical magnetic fields, are three basic parameters for describing superconducting properties. Since the three parameters are not independent and there are reciprocal influences amongst them, they are important for applications. This section will only introduce the regular measurement methods of critical temperature in a zero magnetic field (self field) and critical current density in an applied magnetic field. As for the measurement method of a critical magnetic field, it is difficult to measure the upper critical magnetic field B_{c2} , which is too high (e.g., the upper critical magnetic field B_{c2} of HTS may be >100 T at 4.2 K), thus we focus on simply introducing the measurement method of the lower magnetic field B_{c1} without transport current.

2.4.1 Cryogenic Thermometers

To meet the requirements of various applications, practical thermometers of different levels have been developed. Since functions of these devices are related to their electrical properties, they are often called electric thermometers. Table 2.2 gives important parameters of some thermometers commonly used in cryogenic experiments [11], such as applicable temperature range, repeatability and reproducibility, and time lag. In practical applications, appropriate temperature sensors should be chosen to meet the growing requirements of a variety of cryogenic measurement according to their absolute accuracy, sensitivity, reproducibility, lag time, thermal and magnetic effects, costs and sizes, etc.

2.4.2 Measurement of Critical Temperature

As the temperature decreases to or below its critical temperature T_c , a phase transition in the superconductor from normal to superconducting states occurs. In the transition process, some physical characteristic parameters will change significantly. Thus, if we can measure the temperatures corresponding to a sudden variation of these characteristic parameters, the critical temperature T_c can be directly determined [12, 13]. These characteristic parameters mainly include resistivity, DC and AC susceptibility, specific heat, etc. Only the common electric and magnetic methods are introduced in this section for measuring critical temperatures of superconductors with sudden variation of electromagnetic parameters, while other methods, such as the specific heat method, are not included, and readers can consult related references.

Since the critical temperature of superconductors is related to the magnetic field (Figure 2.7), it should refer to the temperature in some fixed magnetic field. In order to introduce the measurement methods, electric and magnetic measurement methods of the critical temperature are described under conditions of zero external magnetic field (self field) and zero transport current, respectively.

Table 2.2 Characteristic parameters of cryogenic thermometer

Types	Name	Range/K	Reproducibility/K	Accuracy/K	Lag time/s	Others
Practical thermometers	Platinum resistor	10~900	$10^{-3}\sim 10^{-4}$	$10^{-2}\sim 10^{-3}$	0.1~10	Performance Stability, Magnetoresistive effect
	Germanium resistor	1~100	$10^{-3}\sim 10^{-4}$	$10^{-2}\sim 10^{-3}$	0.1~10	Good performance used under 20K Magnetoresistive effect
	Rhodium iron resistor	0.5~300	$10^{-3}\sim 10^{-4}$	$10^{-2}\sim 10^{-3}$	0.01~10	Magnetoresistive effect
	Carbon resistor	0.5~300	0.1~0.5	$10^{-1}\sim 10^{-2}$	0.01~10	Cheap, high cryogenic sensitivity, poor stability
	p-n joint diode	2~400	$10^{-1}\sim 10^{-2}$	$\sim 10^{-2}$	0.01~10	Cheap, convenient, high sensitivity, poor reproducibility, high magnetoresistive effect
Thermodynamics thermometer	Constantan-copper thermocouple	70~300	$\sim 10^{-2}$	0.1	<1	Cheap, convenient, good reproducibility
	Nickel chromium- Constantan thermocouple	20~1000	$10^{-1}\sim 10^{-2}$	0.1	<1	Cheap, convenient, low magnetoresistive effect
	Gold-iron thermocouple	1~300	$\sim 10^{-2}$	$10^{-1}\sim 10^{-2}$	<1	Simple, high response speed, magnetoresistive effect
	Copper-iron-copper thermocouple	1~300	$\sim 10^{-2}$	$10^{-1}\sim 10^{-2}$	<1	Similar with gold-iron thermocouple
	Constant volume gas thermometer	4~1000	10^{-3}	10^{-3}	0.1~100	High precision, complicated in use and calculation
	He vapour pressure thermometer	1.8~5	$10^{-3}\sim 10^{-4}$	2×10^{-3}	0.1~100	Small temperature range, good accuracy, simpler and more convenient than gas thermometers
	H ₂ vapour pressure thermometer	14~33	$\sim 10^{-3}$	1×10^{-2}	0.1~100	
	N ₂ vapour pressure thermometer	63~126	$10^{-2}\sim 10^{-3}$	1×10^{-2}	0.1~100	
	O ₂ vapour pressure thermometer	54~155	$10^{-2}\sim 10^{-3}$	1×10^{-2}	0.1~100	

2.4.2.1 Electrical Method

Carrying current without resistance is one of the basic characteristics of superconductivity, so the simplest and most intuitive measurement methods of critical temperature can be realized by measuring the relationship of resistance and temperature. The critical temperature of a superconductor is thus obtained by measuring temperature at which its resistivity suddenly reduces to approximate zero based on some criteria.

2.4.2.2 Four-Probe Technique

The four-probe technique is a simple and intuitive approach for accurately measuring the resistance of a conductor, and its circuit and connection are shown in Figure 2.17. There are two pairs of connections in the specimen. One pair are current leads, 1 and 1', both of them directly connected with a power supply providing current for the specimen; the others are voltage taps, 2 and 2', connected with a voltmeter, recording voltage of the two taps. The current leads are located in the medial region of the voltage leads.

Since there is contact resistance between the superconducting specimen and the current leads, Joule heat is generated at the joint between current leads and specimen, when the specimen transports current. The voltage taps are thus separated by a certain distance from the joint, in order to eliminate the effect of contact resistance on the voltage between 2 and 2' of the specimen. Consequently, contact resistances between current leads and the specimen have no influence on the measured voltage of voltage taps 2 and 2'. Although there are also contact resistances between the voltage taps and the specimen, the branch current in the closed circuit consisting of specimen segments between 2 and 2' and voltmeter can be neglected compared with current flowing from the specimen due to the high input impedance of voltmeter. Therefore, contact resistances have no effect on measurement of the voltage. According to Ohm's law, once the voltage between 2 and 2' is measured, the resistance between the voltage taps 2 and 2' in the specimen can be obtained by dividing the voltage by the current flowing through the specimen.

In the experimental process, the applied transport current should be as small as possible to reduce the cryogen dissipation, especially if liquid helium is employed. By cooling the specimen from room temperature and recording the temperature and voltage when a small current is applied, the dependence of resistance on temperature can be obtained (Figure 2.18), by which the critical temperature T_c of the superconducting specimen in a zero external magnetic field can be found.

2.4.2.3 Critical Temperature

When a superconductor transfers to the superconducting state from the normal state by decreasing its temperature, first its resistance is gradually reduced and finally returns to zero suddenly in a small

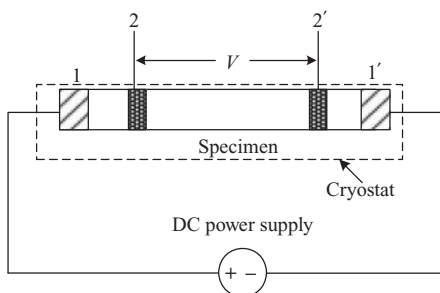


Figure 2.17 Four-probe technique.

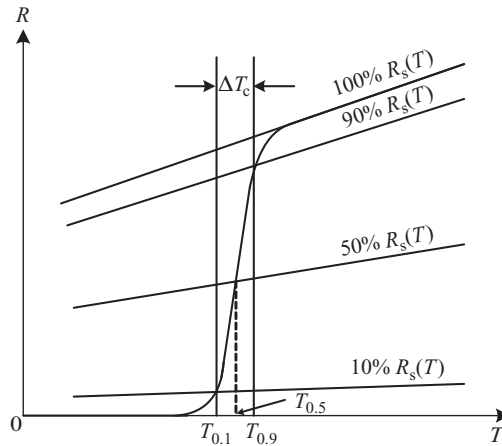


Figure 2.18 Definition of critical temperature and its transition width.

temperature range rather than at a fixed point of temperature (Figure 2.18) [12]. The curve in the figure shows the relationship between resistance and temperature. In order to define the critical temperature of superconductors, first draw a tangent line $R_s(T)$ in the normal state part of the R-T curve before transition and define the values of all points in the tangent line as 100%, then draw three straight lines at 90, 50 and 10% of its values in the tangent line $R_s(T)$, so that there are three intersecting points A, B and C by three straight lines intersecting with the tested R-T curve.

Consequently, there are three temperatures corresponding to three intersecting points, called $T_{0.9}$, $T_{0.5}$ and $T_{0.1}$, respectively, where $T_{0.5}$ is defined as the critical temperature T_c of the superconductor, that is, $T_c = T_{0.5}$. The temperature ΔT_c difference between $T_{0.9}$ and $T_{0.1}$ is defined as the temperature transition width ΔT_c , namely $\Delta T_c = T_{0.9} - T_{0.1}$. Usually, ΔT_c is less than 3% of T_c .

2.4.2.4 Magnetic Method

Superconductors are in the normal state and do not present any magnetic characteristics when the temperature is above critical temperature. As with the magnetization curve of the conventional magnetic materials, the magnetization curve of superconductors is also irreversible when they are in a superconducting state. However, when the temperature is below the critical temperature, superconductors present magnetic characteristics and their susceptibility varies suddenly. Therefore, the critical temperature of a superconductor can also be measured by employing magnetization methods of conventional magnetic materials.

(1) DC Susceptibility Method

There are many methods for measuring the susceptibility, such as DC susceptibility method, AC susceptibility method, vibrating sample magnetometer (VSM), SQUID, specific heat method, etc. VSM and SQUID are generally used to measure macroscopic electromagnetic properties, especially in superconducting material. Since both VSM and SQUID are commercially mature devices, this section briefly introduces the principles of these two devices.

(i) VSM

VSM is mainly composed of a magnet, a magnetic field control component, a detecting component, a pick-up coil and a magnetic moment detecting component, variable temperature and temperature control devices and a vibration system. A magnetic field is induced in a specimen by installing it in an external magnetic field. If magnetic material is placed in the pick-up coil of the VSM and vibrates in a sinusoidal manner, there is an induction voltage signal in the pick-up coil due to variation of flux in the sample and the voltage signal is proportional to the sample's magnetic moment, so that VSM can be employed to measure the magnetization and the susceptibility of magnetic materials and superconductors.

(ii) SQUID

Properties of SQUID have been described in Section 2.1.3 as an introduction to Josephson junctions, and its working principles as a whole system will be introduced in this section. As with VSM, SQUID mainly consists of a magnet, a detecting system, and temperature variation and temperature control systems. The detecting system includes a magnetic flux converter, a SQUID component, an output system and a flux lock-in amplifier. A magnetic flux converter consists of a pick-up coil and a signal coil. The sample is installed near the pick-up coil and the signal coil close to the SQUID component, and the pick-up coil is connected with the signal coil to form a closed loop. When the sample moves in the vicinity of the pick-up coil, the flux variation in the pick-up coil is proportional to its magnetization. Since the SQUID component is sensitive to flux variation, magnetic characteristics of the sample can be accurately measured.

SQUID has high sensitivity, which is two orders of magnitude higher than that of the VSM, and its reliability and reproducibility are also high. But the measurement speed of SQUID is lower than that of VSM, because it cannot be used to measure in the same manner as VSM in the process of sweeping the field.

(2) AC Magnetic Susceptibility Method

Although the four-probe technique is the most direct and simple way to measure the critical temperature of a superconductor, it requires that the superconductor must carry current. As for some superconductors in the form of powder or bulk, or superconductors with granularity, for example the HTS, the transport current is an intergranular current, and such a current cannot serve as a transport current [14]. However, in a small sample or large transport current, it is difficult to connect current leads to the sample, since the four-probe method requires four leads to be soldered onto the sample, but then the superconducting sample is possibly damaged.

The AC susceptibility measurement method is often used to determine AC susceptibility by measuring the magnetic flux in a pick-up coil when the sample is placed in an AC magnetic field with low frequency (several Hz to several kHz) and low flux density (of the order of 1 mT, or even below). However, the AC susceptibility measurement is a contact-free method in which leads and injection current are not required, and it can also measure the magnetic field generated by the intragranular current of the superconducting sample. Thus, the AC susceptibility method is widely used to measure intragranular and intergranular critical currents of a superconductor besides the critical temperature.

In order to simply and conveniently explain the principles of measuring the critical temperature of superconductors by the AC susceptibility method, the infinite superconducting slab model is taken as an example to introduce the relationship between resistivity and susceptibility. As shown in Figure 2.19, the thickness of an infinite superconducting slab is a , its x -axis is along the thickness, the slab in y - z -plane is infinite, and the applied AC magnetic field is in z -axis. Assume that the applied magnetic field is a time-harmonic field with angular frequency ω , namely $B(0,t) = B_0 \exp(-j\omega t)$. According to

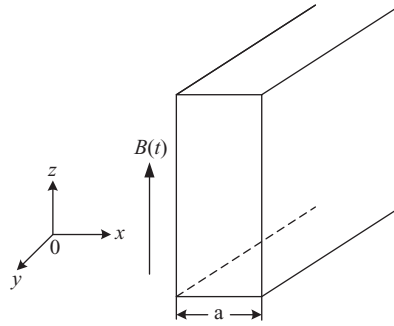


Figure 2.19 Infinite superconducting slab with thickness a in time-harmonic magnetic field.

Maxwell equations, we obtain the magnetic field within the superconducting slab by using the separate variable method:

$$B(x, t) = B_0 \frac{(e^{ikx} + e^{ik(a-x)})}{(1 + e^{ikd})} \quad (2.52)$$

where $k = (1+i)/\delta$, δ is skin depth:

$$\delta = \sqrt{\frac{2\rho}{\mu_0\omega}} \quad (2.53)$$

Susceptibility is then:

$$\tilde{\chi} = \chi' - i\chi'' \quad (2.54)$$

Its imaginary and real parts are, respectively:

$$\chi' = \frac{1}{u} \frac{shu + \sin u}{chu + \cos u} - 1 \quad (2.55)$$

$$\chi'' = \frac{1}{u} \frac{shu - \sin u}{chu + \cos u} \quad (2.56)$$

where

$$u = \frac{a}{\delta} = \sqrt{\frac{\omega\mu_0 a^2}{2\rho}} \quad (2.57)$$

Equations (2.55) and (2.56) indicate that there is an exact corresponding relationship between susceptibility resistivity. If the resistivity ρ tends to 0, u closes to infinity, thus χ' and χ'' tends to -1 and 0, respectively. Conversely, if the resistivity ρ tends to infinity, u closes to 0, thus both χ' and χ'' tend to 0. Based on measurements of the real part of susceptibility at different temperatures, the temperature corresponding to $\chi' = -1$ is the critical temperature of the superconductor. It should be noted that the superconducting transition refers to zero resistivity rather than indicating whether the superconductor stays in Meissner state.

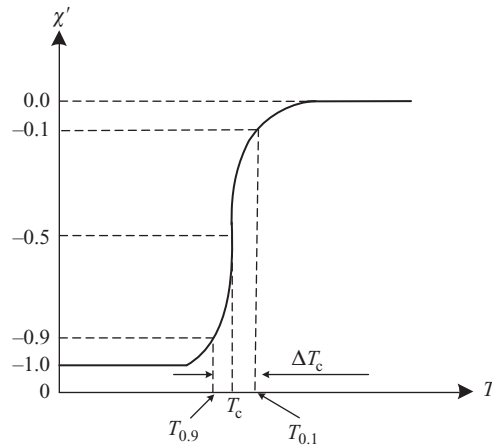


Figure 2.20 Relationships between real part of AC susceptibility and temperature.

According to measurements of the real part of the superconducting magnetic susceptibility at different temperatures, the temperature at $\chi' = -1$ is the critical temperature of the superconductor. It should be noted that the change here refers to zero resistivity change, and not to indicate whether the superconductor stays in the Meissner state. The Meissner state is a DC electromagnetic characteristic determined by measuring magnetization of the superconductor in the DC zero-field cooling (ZFC).

The curve in Figure 2.20 shows the relationship between the real part of AC susceptibility and temperature T [15]. As with the four-probe electric measurement method, the transition of susceptibility corresponds to a temperature range, so the critical temperature T_c and the transition width ΔT_c can be determined by a method similar to electric measurement methods. First, draw a straight line along the line with normal state segment at $\chi' = 0$, and then draw the other three straight lines at $\chi' = -0.1$, -0.5 and -0.9 , respectively. Three intersection points appear at the latter three lines intersecting with the $\chi'-T$ curve, and they correspond to three temperatures $T_{0.1}$, $T_{0.5}$ and $T_{0.9}$, where $T_{0.5}$ is defined as the critical temperature and the transition width is the difference ΔT_c between $T_{0.1}$ and $T_{0.9}$, that is, $\Delta T_c = T_{0.1} - T_{0.9}$.

2.4.3 Measurement of Critical Current I_c

If the temperature is below its critical temperature, the superconductor is in a superconducting state and can carry current with zero resistance, but its ability to transport current without resistance is limited, that is, its ability to transport current is limited. When the transport current is larger than a certain value, the superconductor will also transfer from the superconducting state to the normal state, which is usually referred to as quench of the superconductor. The maximum DC current that a superconductor can transfer with zero resistance is called the critical current I_c . There are many methods of measuring the critical current of a superconductor, amongst which the electrical and the magnetic methods of measurement of critical currents are now introduced.

2.4.3.1 Electrical Method

The most intuitive method for measuring critical current of a superconductor is by directly measuring resistance or voltage. If temperature and magnetic field are below their critical values, the critical current

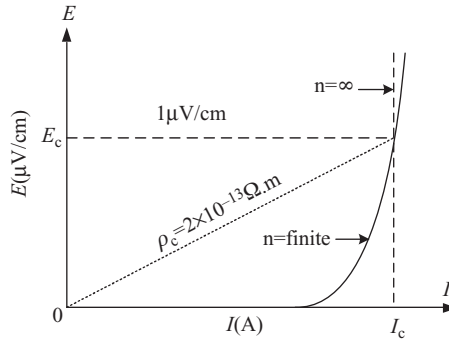


Figure 2.21 Electric field strength versus transport current, $E = U/L$, L refers to length of superconductor.

of the superconductor corresponds to the value of the transport current at which the resistance or voltage of the superconductor abruptly increases when the transport current is increased from zero. Not only does the critical current depend on temperature but also on the magnetic field (Figure 2.7). To illustrate these measurement methods, only the measurement of the critical current at a certain temperature T ($T < T_c$) in a zero magnetic field (self field) is introduced.

(1) Definition of Critical Current

Like the measuring principle of critical temperature, there is a transition process from the superconducting state to the normal state, that is, from zero resistance or zero voltage to a certain resistance or voltage. It is impossible for the transition to occur abruptly in a practical superconductor; the ideal model described in Equation (2.58) only denotes the “ideal conductor”, of which the resistance or voltage suddenly appear with $n = \infty$ (Figure 2.21):

$$U = \begin{cases} 0 & I \leq I_c \\ \infty & I > I_c \end{cases} \quad (2.58)$$

where U and I_c denote voltage and critical current of the “ideal superconductor”, respectively.

Generally, the voltage–current relationship of a practical superconductor approximately satisfies the power-law model, as indicated by:

$$U = U_c \left(\frac{I}{I_c} \right)^n \quad (2.59)$$

where U_c denotes the voltage across a longitudinal superconductor with $I = I_c$, and n is the exponential index, the so-called n value and relates to the temperature and magnetic field (Equation (2.60)). It also characterizes the acute degree of transition of a superconductor from superconducting state to normal state. Obviously, $n = 1$ and $n = \infty$ correspond respectively to conventional conductors and the “ideal superconductor” described by Equation (2.58), that is, there are only two states at which the resistances are either zero or infinite:

$$n = \frac{U_{\text{eff}}(B, T)}{k_B T} \quad (2.60)$$

where $U_{\text{eff}}(B, T)$ is the effective pinning potential of superconductors and is a function of magnetic field and temperature. It also relates to the processing technology of a superconductor and with its intrinsic properties, and k_B is the Boltzmann constant.

Generally, Equation (2.59) is converted into the form of voltage per unit length of the superconductor, so:

$$E = E_c \left(\frac{I}{I_c} \right)^n \quad (2.61)$$

where units of E and E_c are $\mu\text{V/cm}$. In Figure 2.21, the curve corresponding to “ $n = \text{finite}$ ” is the exponential curve described by Equation (2.61).

Criteria are needed to determine the critical current and n value of a superconductor. According to international standards (IEC standards), there are two equivalent criteria that are employed in the measurement of a critical current by the electrical method. These two criteria are respectively the electric field strength criterion with $E_c = 1 \mu\text{V/cm}$ (or $E_c = 0.1 \mu\text{V/cm}$) and the resistivity criterion with $\rho_c = 2 \times 10^{-13} \Omega\cdot\text{m}$ (or $\rho_c = 2 \times 10^{-14} \Omega\cdot\text{m}$). In addition, the n value can also be obtained by fitting the E – I curve with power-law, with E in the range of $0.1 \mu\text{V/cm} < E < 1 \mu\text{V/cm}$.

In Figure 2.21, the horizontal dashed line $E_c = 1 \mu\text{V/cm}$ represents the electric field criterion and the dotted line through the origin represents the resistivity criterion, respectively.

(2) Measurements of Critical Current

As with the circuit in measuring critical temperature of a superconductor, the four-probe method is also used to measure critical current. Current flowing through the sample is supplied by the DC power (Figure 2.18). After the sample is cooled to a temperature T below its critical temperature, it is energized with a current, and curves representing the voltage and the current are recorded simultaneously. The transport current corresponding to the voltage per unit length of sample of $E_c = 1 \mu\text{V/cm}$ or its resistivity of $\rho = 2 \times 10^{-13} \Omega\cdot\text{m}$ is exactly the critical current at temperature T and its self field. In general, the n value of low temperature superconductor (LTS) is large, usually more than 25, which means that the voltage increases sharply when the current increases near its critical value, so that the critical current can be used to fully describe the current-carrying capacity of LTS.

However, the n value of HTS is usually no more than 18 at liquid nitrogen temperatures, due to its characteristics of granularity, weak-link, etc., that is, the voltage increases relatively slowly when the current increases near its critical value. Consequently, the critical current itself is not enough to describe HTS electromagnetic characteristics, so the effect of the n value must also be taken into account [16–19]. Especially, the n value of HTS cannot be ignored in applications of superconducting magnets and superconducting electric power. The definition and effect of the n value will be discussed in more detail in Chapters 3 and 7.

2.4.3.2 Magnetic Method

Because the magnetic measuring method of a critical current is based on the critical state model (CSM) of the superconductor, it is necessary to briefly introduce the CSM to better understand the principle of measuring critical current by the magnetic measurement method.

(1) Critical State Model

Since practical superconductors belong to non-ideal Type II superconductors, there is a pinning force exerted on the fluxon of the pinning centre in the superconductor. When the pinning force balances out the electromagnetic force (Lorentz force), it is then close to its maximum value and this is called the

critical state. There are many kinds of CSM, amongst which the Bean model is the simplest and most intuitive [9]. This model indicates that the critical current density of a superconductor is independent of the magnetic field, and current density J in a superconductor can take only three values, either zero ($J = 0$) or critical current density ($J = \pm J_c$). Specifically speaking, the current density J is zero in the region of a superconductor without magnetic field penetration, but it is $\pm J_c$ in the region into which a magnetic field penetrates, that is:

$$J = \begin{cases} 0 & B = 0 \\ \pm J_c & B \neq 0 \end{cases} \quad (2.62)$$

According to Maxwell equations:

$$\nabla \times B = \pm \mu_0 J_c, \quad (2.63)$$

then the distributions of the magnetic field can be obtained according to Equation (2.63).

To illustrate distributions of magnetic field and current density in a superconductor calculated by the Bean model, a finite superconducting slab with thickness $2a$ is considered for the sake of simplicity. The slab is subjected to a uniform external magnetic field (Figure 2.22). The Cartesian coordinate is adopted, in which the x -axis is along its thickness and the y - z plane extends infinitely, and the external magnetic field is along the z -axis. Based on the Bean model, Figures 2.22(a) and (b) show distributions of the magnetic field and the current density in a superconducting slab subjected to an external magnetic field ($B \neq 0$) without transport current ($I = 0$), carrying transport current ($I \neq 0$) and without external magnetic field ($B = 0$), respectively, where I refers to surface current density with unit of A/m, that is, transport current per unit width along the z -axis.

First, consider the case of a slab subjected to a uniform external magnetic field B without surface transport current density, $I = 0$, where I refers to the transport current (Figure 2.22(a)), and Equation (2.63) converts to the 1D differential equation, by considering the boundary conditions:

$$B(-a) = -B(a) = B \quad (2.64)$$

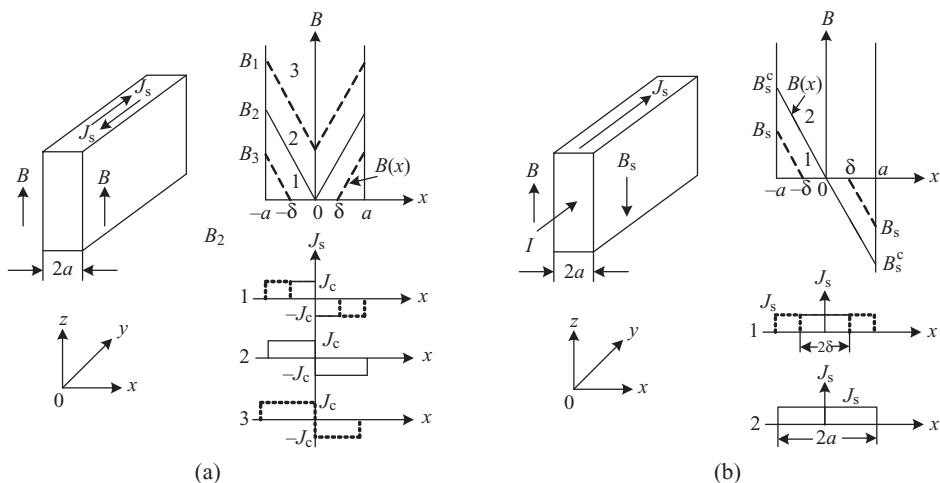


Figure 2.22 Distributions of magnetic field and current density based on Bean model: (a) $B \neq 0$, $I = 0$; (b) $B = 0$, $I \neq 0$.

The distribution of the magnetic field within the superconductor can be found as:

$$B(x) = \mu_0(H - J_c(|x| - a)), \quad |x| \leq a \quad (2.65)$$

where H is the magnetic field strength and $B = \mu_0 H$. Since the external magnetic field B is along the z -axis, the magnetic field $B(x)$ within the superconductor is also along the z -axis. Magnetic fields generated by induced current densities J_s at both surfaces of slab must be opposite to an external magnetic field, due to the shielding effect of induced current density on external magnetic field, so that the induced current densities J_s are parallel to the x - y -plane of the superconductor and are opposite in the left- and right-hand parts. Compared with the external magnetic field, the magnetic field within the slab is symmetrical (upper part of Figure 2.22(a)). The intersection point of dashed line 1 with the x -axis is $(\pm\delta, 0)$, which denotes the position where the magnetic field penetrates and is distant from the centre O of the slab. There are two symmetrical regions, $(-a, -\delta)$ and (δ, a) , into which the external magnetic field penetrates, and their current densities are equal to the critical current with opposite directions, namely $J_s = \pm J_c$. Here the symbol “ $\pm$ ” denotes opposite directions of current densities, and the magnetic fields at the left- and right-hand parts of the slab are symmetrically distributed with the same magnitude and direction. However, the magnetic field and the current density in the region of $(-\delta, \delta)$ are zero, and the current distribution is shown in lower right-hand part of Figure 2.22(a).

As the external magnetic field increases to a certain value at which the magnetic field arrives at the centre O (see solid line 2), then the magnetic field fully penetrates through the slab. At this moment, the magnetic field is called the fully penetrated magnetic field B_p , usually abbreviated to penetrated magnetic field, here $B_p = \mu_0 J_c a$, according to Equation (2.65) by taking $x = \pm a$ and $B(x) = 0$. The magnetic field occupies the whole of the slab and the current density in the entire slab is equal to the critical current density $J_s = \pm J_c$. The corresponding current density distribution is shown as the solid line in the right-hand lower part of Figure 2.22(a).

If the external magnetic field, B further increases, and the magnetic field in the slab and at its centre also linearly increase (see dashed line 3 in right-hand lower part of Figure 2.22(a)). As long as the magnetic field does not exceed the upper critical magnetic field B_{c2} , the current density in the slab theoretically keeps at the critical current density, $J_s = \pm J_c$.

Where the slab carries transport current I without an external magnetic field, $B = 0$, and where transport current I refers to the surface current density in units of A/m, that is, the current is along the z -axis (Figure 2.22(b)), and Equation (2.63) becomes a 1D differential equation, since the transport current is along the y -axis direction, the right-hand side of Equation (2.63) only takes a positive sign. By considering the boundary conditions:

$$B(-a) = -B(a) = B_s \quad (2.66)$$

where B_s refers to the magnetic field produced by the transport current on the slab surface, $B_s = \mu_0 I/2$. When the surface transport current increases to the critical current of $I = I_c$, the magnetic field on the slab surface reaches its maximum, $B_s^c = \mu_0 I_c/2$, thus the magnetic field distribution inside slab can be obtained as:

$$B(x) = \pm \mu_0(H - J_c(|x| - a)) \quad |x| \leq a \quad (2.67)$$

where positive and negative signals “ $\pm$ ” correspond to the magnetic field distributions at both the left- and the right-hand sides of the slab (Figure 2.22(b)). If the transport current is along the $+y$ -axis, the magnetic field at both halves are equal in amplitude and opposite in direction. The transport current always flows first along slab surface and then gradually penetrates into the slab with increasing transport current. As shown in Figure 2.22(b), current density in the slab distributes symmetrically through both the left- and right-hand parts, where dotted line 1 denotes the distribution of the magnetic field in the slab when the surface transport current I is smaller than its critical value I_c and δ represents the distance

between the position where the magnetic field produced by the transport current penetrates and the centre O. The surface current density I is equal to its critical I_c value in the regions of $(-a, -\delta)$ and (δ, a) , through which the magnetic field penetrates, respectively.

Both of surface current densities are the same in magnitude and direction; conversely, the magnetic fields in the two regions are equal in magnitude but opposite in direction, that is, $B(x) = -B(-x)$. The magnetic field and surface current density are zero in the region $(-\delta, \delta)$, and the distribution of current density is shown in the lower right-hand part 1 of Figure 2.22(b). With transport current increasing, the magnetic field generating by the transport current correspondingly increases. If the magnetic field increases to a certain value at which the magnetic field arrives at the centre O from both of the left- and right-hand sides simultaneously, as illustrated by the solid line 2 in the right-hand upper part of Figure 2.22 (b), at this moment the slab is fully penetrated and the surface transport current density I equals to its surface critical current density I_c , that is, $I = I_c$, as indicated by solid line 2 in the lower right-hand part of Figure 2.22(b). Then the magnetic field produced by transport current equals the penetration field of the slab, namely $B_p = \mu_0 I_c / 2$, and the magnetic field in the slab distributes symmetrically at both sides of the central axis with opposite direction.

Distributions of current density and magnetic field within the superconductor can be simply and approximately described by the Bean model, whose prerequisite is the assumption that the critical current density J_c is a constant and independent of magnetic field B . However, the critical current density depends on the magnetic field in practical cases. Also, the Bean model can simplify the complicated mathematical calculations and is often used to calculate electromagnetic properties of superconductors, such as flux jump, quench and propagation, AC losses, etc., which will be introduced in more detail in Chapters 4 and 5. Currently, there are a variety of superconducting CSMs; one is the typical Kim model, which illustrates that the critical current density is inversely proportional to the magnetic field B . Even though the Kim model reflects the effect of the magnetic field on the critical current, it cannot fully explain the experimental facts. Although there are some discrepancies between experimental results and predictions according to the Bean model, it is still the most widely used at present because of its simplicity and intuition and it is easy to understand.

(2) Magnetic Method on Measurement of Critical Current

According to 1D approximation and the Bean model [20, 21], the relationship of the critical current density with magnetization in a superconductor is:

$$J_c(B) = \frac{M_- - M_+}{2a} = \frac{\Delta M}{2a} \quad (2.68)$$

where M_- and M_+ denote the magnetization of a superconductor in the magnetization curve or hysteresis loop with descending and ascending magnetic field B , respectively, and a refers to the average half thickness of a superconducting sample in the direction perpendicular to the magnetic field. If the hysteresis loop is in perfect symmetry, namely $M_- = -M_+$, Equation (2.68) can be simplified to become:

$$J_c(B) = \frac{M(B)}{a} \quad (2.69)$$

If a superconductor is magnetized in a magnetic field, there is a shielding current (induced current) in the superconductor and the shielding current generates a magnetic moment. Magnetization M (magnetic moment per unit volume) or hysteresis loop of superconductor can be easily measured by using SQUID or VSM, thus the critical current density of a superconductor is found according to Equation (2.68). Figure 2.23 shows a typical hysteresis loop of a superconductor. According to Equation (2.69), the critical current density of a magnetic field at H_1 , H_2 , H_3 , and H_4 can be calculated, respectively.

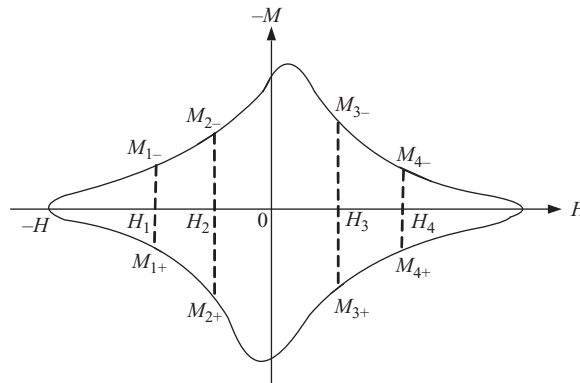


Figure 2.23 Typical hysteresis loop of superconductor.

The critical current density can also be ascertained by measuring the peak values of the imaginary part χ'' of AC susceptibility χ . Assuming that an AC magnetic field is superimposed onto a DC magnetic field in an AC magnetization measurement, the applied DC magnetic field and amplitude of the AC magnetic field are respectively B_{dc} and B_{ac} , and satisfy the condition of $B_{dc} \gg B_{ac}$. Under this condition, the dependence of the imaginary part of AC susceptibility on amplitude B_{ac} of an AC magnetic field can be measured at certain frequencies. If the amplitude of an AC magnetic field B_{ac} is equal to a certain value B_0 , the imaginary part χ'' of AC susceptibility χ takes its maximum, thus the critical current density of the superconductor in the DC magnetic field B_{dc} , can be obtained as:

$$J_c(B_{dc}) = \frac{2B_0}{\mu_0 d} \quad (2.70)$$

where d is the thickness of the superconductor sample.

As with the magnetic measurements of critical temperature, the magnetic measurement of critical current density is also a contact-free method. There are no current leads, voltage taps and Joule heat resulting from contact resistance between specimen and current leads is the electric measuring method of critical current. However, there is no AC loss in measurement of DC magnetization; the hysteresis and eddy current losses are small, even in measurement of AC susceptibility, because the amplitude of an AC magnetic field component is low (with 10^{-4} order) and induced shielding current density is also small. In addition, because variation of temperature is easily controllable, measurements of hysteresis loop $M(T)$ and AC susceptibility $\chi(T)$ are also easily realized by scanning the temperature. Therefore, the critical current density $J_c(B, T)$ of a superconductor as a function of temperature and magnetic field can be easily and conveniently obtained.

Although measurement of critical current density by the magnetic method is convenient and fast, it should be pointed out that this method is based on the Bean model. The critical current density measured by a magnetic field is different from the transport critical current density in non-uniform or anisotropic superconducting materials,

It should be noted that although the magnetic method to measure the critical current density is convenient and fast, this method is based on the Bean CSM. For non-uniform or anisotropic superconducting materials, the measured critical current density is different from the transport critical current density and relates to the magnetic field direction. Moreover, the critical current density measured by the magnetic measuring method is often higher than that measured by the electrical measuring method. However, critical current density measured by the magnetic method is the sum of intragranular and intergranular

critical current densities for a superconducting material with granularity, while the intragranular critical current density is usually at least one order higher in magnitude than that of the intergranular partner, especially for HTS materials [14].

Critical current density measured by the magnetic method can reflect the trapped flux, flux pinning, flux creep and other properties of superconducting materials. Because only the intergranular critical current density has the ability to transport macroscopic current, the critical current density measured by the electrical method is the same as the intergranular critical current density measured by the magnetic method. In practical applications, the magnetic method is not enough to determine the critical current density, so appropriate methods should be adopted to measure the critical current density according to the specific applications.

2.4.4 Measurement of Critical Magnetic Field

As mentioned above, three basic critical parameters, critical temperature T_c , critical current density J_c and the critical magnetic field B_c (B_{c1} , B_{c2}) of superconductors are not independent of each other, so the corresponding temperature must be specified in measurements of the critical magnetic field. This section focuses on the measuring methods of a critical magnetic field at a certain fixed temperature T smaller than critical temperature T_c . There are also two methods for measuring a critical magnetic field of a superconductor according to its property of carrying current without resistance and the Meissner effect when it is in a superconducting state. These two methods are classified as the electrical method and the magnetic method.

2.4.4.1 Electrical Method

The electrical measuring method of a critical magnetic field in superconductors is also the most intuitive and simplest one, in which a four-probe technique is used (Figure 2.18). The difference is that the sample is placed in the DC external magnetic field and supplied with a small current far below its critical current in a fixed temperature lower than its critical value. Applying a small current to the sample at fixed temperature and then gradually increasing the external magnetic field, the resistance and magnetic field are simultaneously recorded. When the resistance of the sample suddenly appears (Figure 2.24), the magnetic field $B_{0.5}$ is its critical magnetic field B_c to which resistance $R_{0.5}$ corresponds, and the

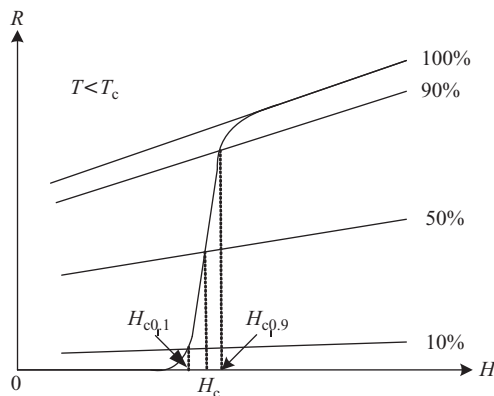


Figure 2.24 Definition of critical magnetic and its transition width.

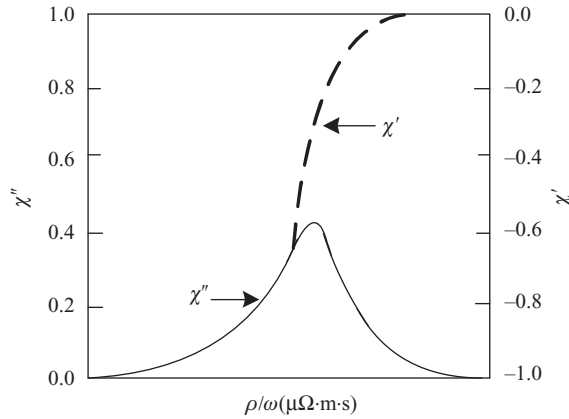


Figure 2.25 Relationship between AC susceptibility and resistivity.

determinations of the critical magnetic field and the magnetic transition width are similar to that of the critical temperature already discussed in Section 2.3.2.

2.4.4.2 Magnetic Method

The critical magnetic field is obtained by measuring the AC susceptibility according to the AC susceptibility method employed in the measurement of critical temperature. Figure 2.25 illustrates the relationship between conductor resistivity ρ , angular frequency ω of an AC magnetic field and AC susceptibility when the resistivity ρ is large, the AC susceptibility $\chi (\chi' - i\chi'')$ tends to be zero, and where i is an imaginary unit. If resistivity ρ is relatively small, AC susceptibility is finite. If angular frequency is fixed, when the resistivity ρ tends to zero, and the skin depth (Equation (2.53)) δ also tends to zero, then χ' , the real part of AC susceptibility χ tends to -1 , which indicates that the sample closes to a perfect diamagnetic state – the Meissner state – and reflects the ability of magnetic shielding of superconductors. The magnetic field is the lower critical magnetic field B_{c1} with $\chi' = -1$. However, when the resistivity ρ tends to zero, the imaginary part of AC susceptibility χ'' also tends to zero, which shows that the loss does not occur and then an AC magnetic field is completely repelled from the superconductor (because AC loss is proportional to the imaginary part χ'' of magnetic susceptibility χ , and related content of AC loss will be described in Chapter 5).

However, if the resistivity ρ is large enough, the skin depth δ is much larger than the sample's size d , that is $\delta \gg d$, and the real part χ' of magnetic susceptibility χ tends to zero, which illustrates that there is no diamagnetism in the sample. Meanwhile, the imaginary part of χ'' is also approximate to zero and then the loss is zero because the induced current in the sample is perfectly inhibited and the magnetic field completely penetrates the sample with the large resistivity ρ . From the right-hand side of Figure 2.25 [22], we note that AC susceptibility χ is small and changes very little with resistivity ρ , when ρ is so large that the penetration depth δ is greater than the sample size d . Thus, the AC susceptibility method is not suitable for measuring the critical current and the critical magnetic field of superconductors with large resistivity ρ .

At a certain temperature T ($T < T_c$), the superconducting sample is put into a magnet and is in a superconducting state, and AC susceptibility $\chi (\chi' - i\chi'')$ is measured with increasing magnetic field. When the magnetic field reaches the critical magnetic field of the superconducting sample, Type I superconductors will turn from a superconducting state to a normal state with resistance. The absolute

value of the real part χ' of AC susceptibility χ will increase sharply and the imaginary part χ'' will also rapidly increase from zero, then the corresponding magnetic field is the critical magnetic field $B_c(T)$ of a Type I superconductor at temperature T .

Type II superconductors have two critical magnetic fields, that is, the lower magnetic field B_{c1} and the upper magnetic field B_{c2} . If the temperature T is below critical value T_c , AC susceptibility $\chi(\chi' - i\chi'')$ is separately measured when the magnetic field increases gradually. When the real part χ' starts to increase from -1 in the Meissner state, the imaginary part χ'' becomes larger than zero, the superconducting sample is in the Meissner state and the corresponding magnetic field is the lower magnetic field B_{c1} . Then the magnetic field is increased continuously. When the real part χ' increases from a minus value and close to zero, the imaginary part χ'' begins to slowly decrease from its peak value. At this moment, the magnetic field is the upper magnetic field B_{c2} at temperature T . It should be noted that the surface of the superconducting sample should be smooth and clean, to ensure that there is no surface barrier and flux pinning on the surface of the sample, so that the measurement error is minimized.

Irreversible magnetic field $B_{irr}(T)$, a function of temperature T , is an important parameter of practical superconducting materials. The magnetic field of the superconductor carrying current must be smaller than the irreversible magnetic field $B_{irr}(T)$, which is generally lower than the upper critical magnetic field B_{c2} in applications. Also the irreversible magnetic field $B_{irr}(T)$ can be measured by the AC susceptibility method. The irreversible magnetic field refers to the magnetic field applied to the superconductor in which the pinning force is zero and the fluxon begins to flow freely, then the critical current density of the superconductor is zero and superconductors cannot transport current with zero resistance if the applied magnetic field is more than the irreversible magnetic field. As for the conventional LTS, the irreversible magnetic field $B_{irr}(T)$ is close to its upper critical magnetic field $B_{c2}(T)$ and is generally recognized as approximately equal to $B_{c2}(T)$. However, the irreversible magnetic field $B_{irr}(T)$ of HTS is much smaller than its upper critical magnetic field $B_{c2}(T)$, so that the irreversible magnetic field $B_{irr}(T)$ is an important parameter and needs to be considered in HTS electrical applications. It is necessary to briefly introduce this method to determine the irreversible magnetic field of HTS materials.

Usually there are two modes for measuring the irreversible magnetic field, that is, the ZFC and field cooling (FC). First, the sample is placed in a cryostat with a preset temperature T ($T < T_c$) and is subjected to a DC magnetic field B_{dc} , then susceptibility χ and temperature T with FC mode are measured when the temperature increases to the temperature at which sample is in a normal state. Next, the sample is cooled to a preset temperature T again and the applied DC magnetic field is kept at B_{dc} . Susceptibility is recorded with increasing temperature in the FC mode, and so the relationship between susceptibility and temperature can be obtained. Since the superconductor at low temperature has a strong ability of flux pinning, the applied magnetic field penetrating the sample is so little that the sample shows remarkable diamagnetism. Thus, the real part χ' of susceptibility measured by the ZFC mode is smaller than that measured by the FC mode.

In the FC mode, the flux penetrating the sample at relative higher temperature remains in the sample due to the pinning effect, and diamagnetic ability is small. Magnetization curves are measured with modes of ZFC and FC, respectively. If the two curves almost coincide, namely the magnetization curve becomes irreversible, the corresponding temperature is defined as the irreversible temperature $T_{irr}(<T_c)$. Then the AC susceptibility curves with an applied magnetic field in the modes of FC and ZFC are measured again, and the magnetic field at which these two AC susceptibility curves (χ vs. B) intersect is the irreversible magnetic field B_{irr} at that temperature T .

In addition, there are other methods for measuring the irreversible magnetic field. For example, irreversible magnetic field B_{irr} can be determined by measuring the peak value of an imaginary part χ'' of AC susceptibility, that is, a relational curve χ versus T of AC susceptibility and temperature is measured and then the temperature T_p corresponding to the peak value of the imaginary part χ'' of AC susceptibility χ can be obtained. When the real part χ' of AC susceptibility χ is -0.05 ($\chi' = -0.05$), the upper critical magnetic field $B_{c2}(T)$ is approximately equal to the irreversible magnetic field, $B_{irr}(T) = B_{c2}(T) = B(T_p)$.

It is noted that the upper critical magnetic field B_{c2} of HTS at 4.2 K is extremely high and can exceed 100 T predicted by calculation, so the upper critical magnetic field B_{c2} of HTS cannot be measured, because so high a constant magnetic field is not available at present. A high upper critical field cannot be measured directly but can be obtained by combining indirect measurement with theoretical calculations. For example, the upper critical magnetic field B_{c2} at a low temperature such as $T = 4.2$ K, can be theoretically calculated based on its results measured at relative high temperature T ($T < T_c$), at which it can easily be measured.

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3

Mechanical Properties and Anisotropy of Superconducting Materials

Superconductors are subjected to mechanical stress in practical applications, for example, they are pre-stressed in processing of the device. In the cooling process they are subjected to thermal contraction stress, and during operation they will be also be subjected to electromagnetic force (Lorentz force). In addition, superconducting devices often take the form of a coil in which the magnitudes and directions of the magnetic fields from point to point vary with different applications. In particularly, due to intrinsic structural characteristics, critical current and n values of high temperature superconductor (HTS) materials show a strong anisotropy. Mechanical properties and anisotropy of the practical superconducting materials are thus the important parameters in superconducting electrical applications.

3.1 Mechanical Properties

Unlike HTS materials, the conventional low temperature superconducting (LTS) materials, such as NbTi, Nb₃Sn and Nb₃Al, are alloys and have properties of isotropy of critical current and with a higher mechanical strength. However, in HTS materials, the oxide ceramics are more brittle and so have poor mechanical properties. The current-carrying capacity will degrade when the mechanical stress (tensile stress, bending stress) reaches a certain value. This section focuses on dependence of critical current on the tensile and bending stresses in superconducting materials.

3.1.1 General Description of Mechanical Properties

Mechanical characteristics of metals or alloys are usually described by those parameters which refer to mechanical strength, including yield strength, tensile stress (tensile strength), strain (tensile strain, bending strain), specific elongation, section shrinkage, hardness, impact toughness, etc. Based on the content of this book and the practical application of superconducting materials, only some common parameters, including tensile stress and tensile strain as well as bending strain and their effects on critical currents, are introduced here.

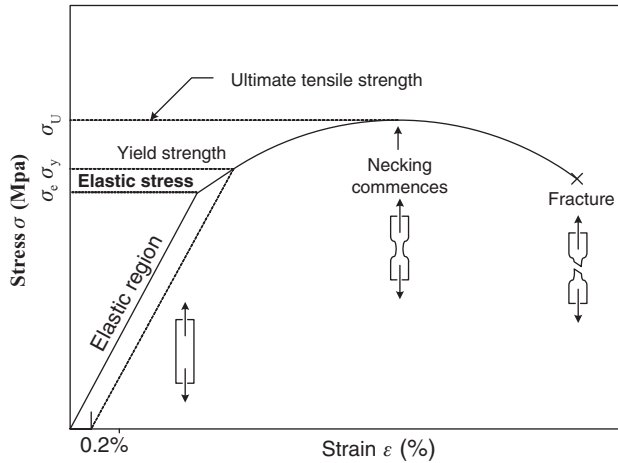


Figure 3.1 Stress-strain curve of ductile materials while stressed axially. (Reproduced with permission from Richard P. Reed and Alan F. Clark, “Mechanical Properties.” Materials at low temperatures. Materials Park, OH: ASM International, 247 © 1983.)

When subjected to tensile stress, a typical metallic material usually experiences four stages, which are elastic, plastic, necking and fracture stages, respectively. The relationship between axial stress σ and strain ε is shown in Figure 3.1 [1]. When an axial force is applied in the elastic stage, the material first deforms elastically, and the corresponding maximum elastic stress is defined as σ_e ; it returns to its original size and shape when the force is removed. This results in the straight-line response of the material shown at the beginning of the figure. The slope of this section in the curve determines the relative stiffness of the material and is defined as the Young’s modulus E . The plastic stage starts at the end point of the elastic (linear) region and this important point indicates the onset of yielding where the material starts to plastically deform. The stress corresponding to this onset point is designated as the yield strength σ_y .

In the necking stage, a local part of the specimen significantly contracts and its stress reduces after stress increases to the maximum σ_u . As a result, the specimen finally fractures at the necking position.

The combination of yield with necking stages is the plastic stage at which the specimen cannot recover to its original length, so permanent deformation takes place. For superconducting materials, the maximum stress and strain should be limited in the range of the elastic stage, to make sure that its normal current carrying capacity does not deteriorate. Therefore, the tensile and bending properties of superconducting materials are usually described within the range of elasticity. For most structural designs, this is the stress level to be avoided. Parts made of superconducting materials are often so sized that the maximum stress σ_u they will need to withstand is no more than half the material’s yield strength, giving a safety margin of two. Values of yield strength are often determined by use of the 0.2% offset method (i.e., 0.2% more strain than that at the end of the elastic region).

3.1.2 Tensile Properties

In superconducting power applications, current-carrying superconductors are inevitably exerted by a combination of pre-stress, thermal shrinkage stress and electromagnetic force. The current carrying capacity of superconductors is not affected if the stress is smaller than a certain critical value σ_c ($\sigma_c < \sigma_e$) above which it will irreversibly degrade [2–6]. Therefore, the mechanical property of superconducting

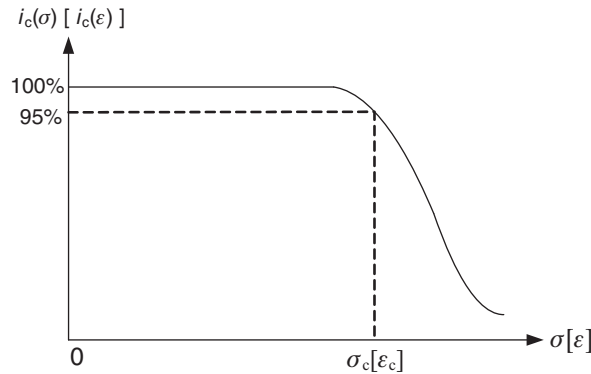


Figure 3.2 Dependence of normalized critical current on the tensile stress/strain.

materials is one of the key factors that needs to be considered in the structural design of superconducting power apparatus.

Figure 3.2 shows the dependence of normalized critical current on tensile stress (strain). The normalized critical current $i(\sigma)$ [or $i(\varepsilon)$] is the ratio of critical current $I_c(0)$ without stress to one with tensile stress (strain) $I_c(\sigma)$ [or $I_c(\varepsilon)$], that is, $i_c(\sigma) = I_c(\sigma)/I_c(0)$ [or $i_c(\varepsilon) = I_c(\varepsilon)/I_c(0)$]. Generally, $\sigma_c(\varepsilon_c)$, the critical stress (strain) is defined as the critical value at which the normalized critical current $i_c(\sigma)$ [or $i_c(\varepsilon)$] decreases to 95%. When the stress (strain) is smaller than the critical values, the normalized critical current $i(\sigma)$ [or $i(\varepsilon)$] is considered to be independent of stress (or strain).

3.1.3 Bending Properties

In superconducting electrical applications, superconductors often appear in the form of coil in which the bend radius of the materials in the process of winding must be limited within a certain range. The bending strain of superconducting material is defined as:

$$\varepsilon_b = \frac{t}{2r} \quad (3.1)$$

where ε_b , t and r are bending strain, thickness and bending radius of the superconductors, respectively.

The definition of a normalized critical current $i_c(r)$ is similar to the tensile case, that is, it is the ratio of critical current $I_c(r)$ with radius r to that of I_c without bending. The relationship of a normalized critical current $i_c(r)$ and $i_c(\varepsilon_b)$ of superconductors to bending radius r (ε_b strain) is shown in Figure 3.3, which indicates that the normalized critical current $i_c(r)$ and $i_c(\varepsilon_b)$ keeps at constant 1, while the bending radius r (strain ε_b) is larger than a certain value and then gradually reduces until the bending radius is smaller than that certain value. Also, as with the tensile case, the critical bending radius r_c (strain ε_{bc}) is defined with a certain value at which the normalized critical current $i_c(r)$ and $i_c(\varepsilon_b)$ reduces to 95%.

Therefore, the mechanical parameters, such as tensile stress (strain) and bending radius (strain), are also important to qualify practical superconductors, besides the critical currents in the self field and different magnetic fields [7–9]. Table 3.1 presents the typical geometrical and mechanical characteristic parameters for the second-generation coated conductor (known as 2G) YBCO(344S) fabricated by the American Superconductor Corporation (AMSC) [10]. These parameters are essential for the design of the superconducting power apparatus. When calculating the stress and strain under rated and fault

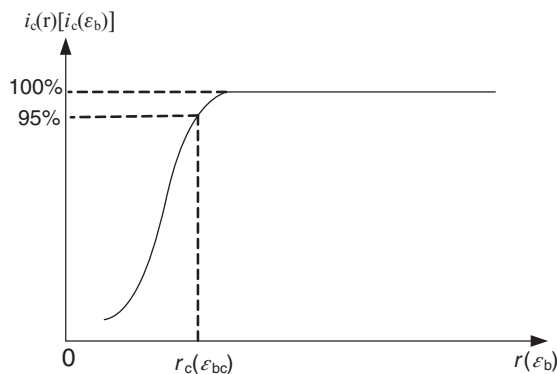


Figure 3.3 Dependence of normalized critical current on bending radius.

operation conditions, whether the apparatus need additional reinforcement and protection in order to avoid mechanically destruction needs to be determined.

3.2 Electromagnetic Anisotropy

As mentioned in the previous chapter, the critical current of superconductors continuously decreases with increases in the magnetic field and with temperature. Since the conventional LTS is an alloy, the decrease of the critical current only relates to the magnitude of the magnetic field and is independent of the orientation of the magnetic field if the temperature is constant. The so-called anisotropy of superconducting material means that its critical current depends not only on the magnitude of the magnetic field but also on its orientation if the temperature is constant. Therefore, it can be said that the LTS is isotropic. However, HTS has strong anisotropy because it is an oxide ceramic material with granularity.

Thereafter, the anisotropy of superconductors mainly refers to HTS if there is no specific explanation in the book. The anisotropic parameters mainly refer to the anisotropy of the critical current under the magnetic field with varying magnitudes and orientations. In addition, transition from a superconducting state to a normal state in a superconductor is generally described by fitting the voltage-current relationship with the power-law model, in which the power index n is called the n value, which is used to indicate the transition degree. The larger the n value, the more abrupt the transition. Since the transition from a superconducting state to a normal state in LTS is much higher than HTS, the n value in LTS is much higher than in HTS (Figure 3.4). The horizontal axis refers to the normalized transport current $I(I/I_c)$, the ratio of transport current to the critical current, and the vertical axis represents the voltage per centimetre in the superconductor. If the critical currents of LTS are equal to those in HTS, the n_L and n_H are the n

Table 3.1 Typical geometrical and mechanical characteristic parameters of 2G YBCO CC (344S) fabricated by AMSC

Geometrical and mechanical parameters	Min/Max thickness (mm)	Min/Max. width (mm)	Min bending radius/mm	Max tensile		
				stress σ /MPa	Max tensile force F /kg	Max tensile strain ϵ
values	0.36/0.44	4.24/4.55	17.5	200	20	0.3%

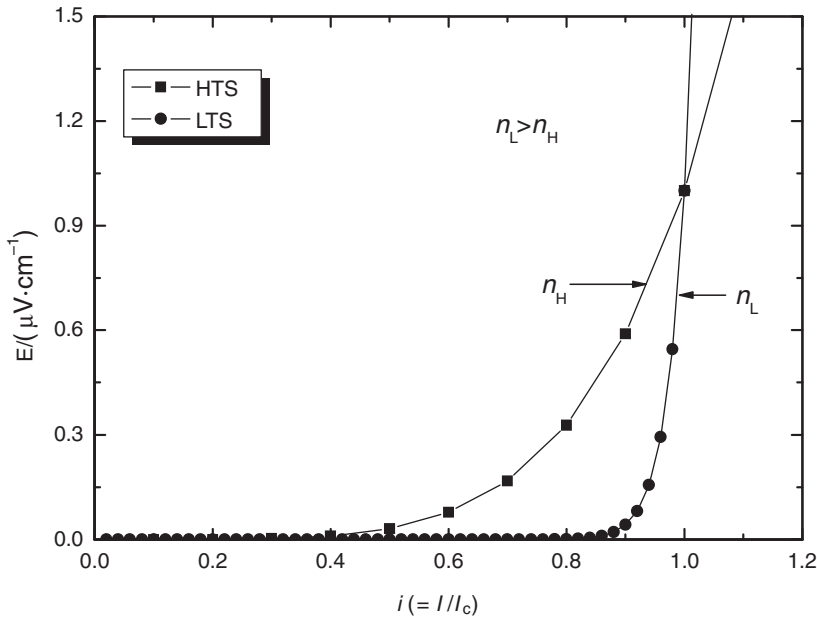


Figure 3.4 Typical current-voltage curves in LTS and HTS.

values in LTS and HTS, respectively. The n value is generally calculated by fitting current-voltage curves of superconductors with the power law model within the range of $0.1 \mu\text{V/cm} \leq E \leq 1 \mu\text{V/cm}$:

$$E = E_c \left(\frac{I}{I_c(B, T)} \right)^{n(B, T)} \quad (3.2)$$

where $E_c = 1 \mu\text{V/cm}$ is the criterion defining the critical current I_c . Critical current $I_c(B, T)$ and n value $n(B, T)$ suggests that both of them are functions of magnetic field B and temperature T .

As with properties of critical current I_c , the n value also continuously reduces with magnetic field B and temperature. The n value is independent of orientations of the magnetic field in LTS; but in HTS it relates not only to the magnitude of the magnetic field but also to its orientation if the temperature is constant. The n value is isotropic in LTS and anisotropic in HTS. However, the n value in LTS is not considered because it is large (in general < 25), but the n value in HTS is taken into account due to its relatively small value (generally < 18) [11]. Therefore, anisotropy of n value in HTS should be included simultaneously with that of the critical current I_c .

3.2.1 Anisotropy of Critical Current in HTS Materials

HTS has strong anisotropy due to its granular structure [12], and a typical anisotropy of critical current is shown in Figure 3.5, in which the insert illustrates the orientations of magnetic field B relating to the HTS tape surface. $B_{\parallel}$ and $B_{\perp}$ refer to the components parallel and perpendicular to the tape width surface, respectively. Obviously, the perpendicular component on the critical current has more effect than the parallel one.

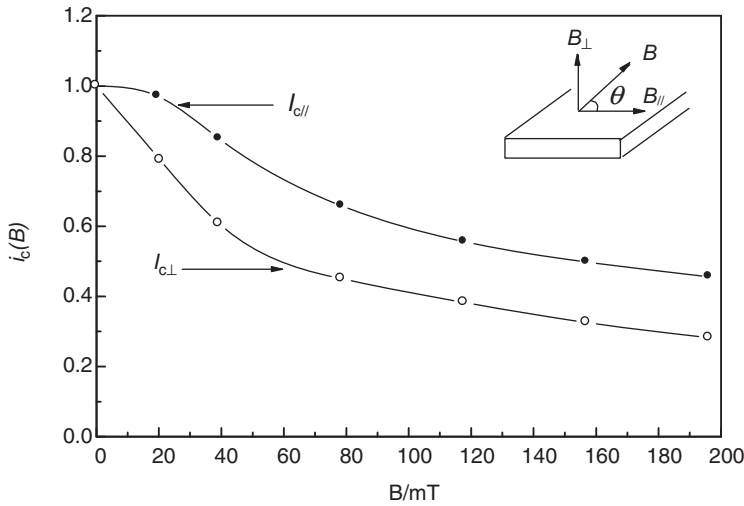


Figure 3.5 Anisotropy of critical current in HTS tapes.

3.2.2 Anisotropy of Critical Current in 1G HTS Tape

Currently, commercial HTS tapes mainly refer to Bi-based and Re-based materials, the former defined as the first-generation HTS tape (1G tape) and the latter defined as the second-generation HTS tape (2G tape). There are no exact and unified expressions for the anisotropic characteristics of 1G superconductor materials. However, there are several kinds of empirical models by which the anisotropy of critical currents can be described in magnetic fields with varying magnitude and orientations at a temperature of 77 K [13–16]. In the DC magnetic field B , the relationship between critical current and magnetic field with some magnitude and orientation is given by:

$$I_c(B, \theta) = \frac{I_{c0}}{48 - 6.8e^{-B_{||}/B_0} - 40.2e^{-B_{\perp}/B_0}} \quad (3.3)$$

where $I_c(0)$ is the critical current in the self field, $B_{||}$ and $B_{\perp}$ refer to the parallel and perpendicular components in absolute values to the width of the tape, respectively, θ indicates the angle between the magnetic field and the widest surface (insert in Figure 3.5), and $B_0 = 1$ T is a constant. At a temperature of 77 K and low magnetic field, Equation (3.3) is in agreement with experimental results, but it deviates from the experimental results in a higher magnetic field ($B > 1$ T).

According to G-L theory, with the introduction of the effective mass tensor model [17], the relationship of critical current with magnetic field is described as:

$$I_c(B, \theta) = I_c(\varepsilon_{\theta}, B) \quad (3.4)$$

where

$$\varepsilon_{\theta} = \sqrt{\varepsilon^2 \cos^2 \theta + \sin^2 \theta} \quad (3.5)$$

where ε_{θ} is a parameter relating to angle θ , and:

$$\varepsilon^2 = m_{ab}/m_c = (B_{c2}^c/B_{c2}^{ab})^2 \quad (3.6)$$

where ε is anisotropic parameter, and m_{ab} and m_c are effective masses for a superconductor crystal along the plane a - b and c -axes of grains in HTS, respectively.

In the temperature range of 20 K through 77 K, theoretical results are in agreement with experimental results, but there is a large deviation between them if the temperature is lower than 20 K. Thus, Equation (3.6) is ineffective at cryogenic temperatures under 20 K.

When taking flux lines consisting of the Josephson vortex and the pancake vortex into account [18], the two-dimensional (2D) vortex model indicates that the relationship between critical current and magnetic field can be described by:

$$I_c(\theta, B) = I_c(B \sin \theta) \quad (3.7)$$

Although there is no adjustable parameter in Equation (3.7), which is simple and explicit, the calculated results deviate seriously from the experimental results when the angle is relatively large. The dependence of critical current on the magnetic field are related to microscopic model parameters, except for Equation (3.3), thus these models cannot be used in applications. When ignoring concrete microscopic mechanism of HTS structure, Equation (3.8) is able to describe the empirical dependence of critical current on magnetic field in simple and explicit analytical expressions [19], in which there is no adjustable parameter and all parameters are obtained by experiment:

$$I_c(B, \theta) = \frac{I_{c//}(B_{//})I_{c\perp}(B_{\perp})}{I_c(0)} \quad (3.8)$$

where $B_{//} = B \cos \theta$, $B_{\perp} = B \sin \theta$, $I_{c//}(B_{//})$ and $I_{c\perp}(B_{\perp})$ correspond to dependences of critical currents on parallel and perpendicular components of the magnetic field, respectively. $I_c(0)$ is the critical current in the self field. If $I_{c//}(B_{//})$ and $I_{c\perp}(B_{\perp})$ are measured experimentally, the critical current $I_c(B, \theta)$ in any magnitude and orientations of the magnetic field can be calculated by substituting them into Equation (3.8). The relations of critical current with angle θ are shown in Figure 3.6 with the condition

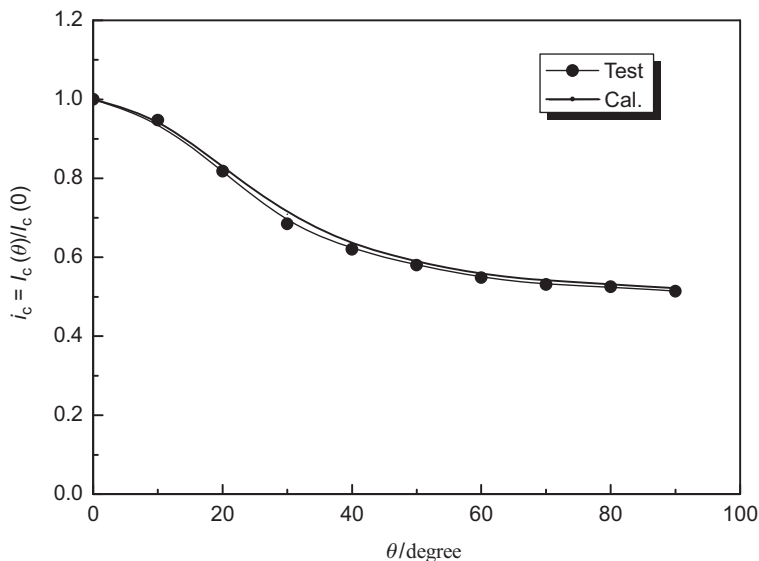


Figure 3.6 Critical current versus orientation with fixed magnetic field.

of a fixed magnetic field B , in which the two curves correspond to the results from measurements and calculations based on Equation (3.8), respectively. It is indicated that both of them are in agreement in the lower magnetic field. It should be noted that both the experimental and calculated results are in agreement only at a temperature of 77 K and a low magnetic field of less than 0.15 T. When the magnetic field is higher than 0.15 T, there is no guarantee of its effectiveness due to shortage of experimental data. However, the magnetic field is generally about 0.15 T in superconducting power applications, therefore Equation (3.8) is still useful and important for the design and operation of superconducting power apparatus.

The relationship of critical current to any temperature T and magnetic field B with any orientation θ can be found empirically by [20, 21]:

$$I_c(T, B, \theta) = I_{c0,77} F(T) G(T, B, \theta) \quad (3.9)$$

where $I_{c0,77}$ refers to critical current in the self field, at a temperature of 77 K. In the temperature range of 20 K through 110 K, the function $F(T)$ relating to temperature is:

$$F(T) = \begin{cases} 5.92 - 0.065T & (T < 75K) \\ 3.69 - 0.035T & (T > 75K) \end{cases} \quad (3.10)$$

In the magnetic field range of 0 T through 5 T, function $G(T, B, \theta)$ is obtained by fitting the experimental data:

$$G(T, B, \theta) = \begin{cases} [1 + |(B \sin \theta)/B_0(T)|^{\alpha(T)}]^{-1} & (\theta < \theta_c) \\ [1 + |(B \sin \theta_c)/B_0(T)|^{\alpha(T)}]^{-1} & (\theta > \theta_c) \end{cases} \quad (3.11)$$

where θ_c is a critical angle with a value of about 6° and then closes to the grain dislocating angle of Bi-based superconducting tapes. The power function $\alpha(T)$ is determined by:

$$\alpha(T) = 0.2116 + 8.3 \times 10^{-3}T + (12 + 0.3T) \times 10^{-4}I_{c0} \quad (3.12)$$

where B_0 is the characteristic magnetic field and is given by:

$$B_0(T) = 3 \times 10^{-2} + (32 - 0.393T) \times 10^{-4}I_{c0} \quad (3.13)$$

where I_{c0} is the critical current of superconducting tape in the self field and at temperature T .

In Equations (3.9) to (3.13), the fitting polynomial formulae are too complicated and only suitable in the calculation of a critical current in a magnetic field within the range of 0 T through 5 T with any orientation, thus they are not applicable for calculation of the critical current if the magnetic field is more than 5 T.

In cases of temperature T and magnetic field B , another empirical equation for calculating critical current is given by [22]:

$$I_c(T, B) = I_c(77, 0) \left(\frac{I_c(T, B_\perp)}{I_c(77, 0)} \right) \left(\frac{I_c(T, B_{//})}{I_c(T, 0)} \right) \quad (3.14)$$

where $I_c(77, 0)$ indicates the critical current in the self field and a temperature of 77 K. Equation (3.14) also has no adjustable parameter but needs to measure critical currents $I_c(T, B)$ of perpendicular and parallel magnetic field components across all ranges of temperature T . To some extent, Equation (3.14) is a generalization of Equation (3.8).

If the temperature is 77 K, the magnetic field is in the range of 0.5 T through 2 T and the angle θ between magnetic B and the wide surface of the 1G HTS tape is in the range of 0° through 70° , the empirical formula for calculation of critical current is [23]:

$$I_c(B, \theta) = 21.2 \exp[-f(\theta)B] \quad (3.15)$$

where function $f(\theta)$ relating to angle θ is determined by:

$$f(\theta) = 2.02 \times 10^{-8}\theta^5 - 2.95 \times 10^{-6}\theta^4 + 1.04 \times 10^{-4}\theta^3 + 1.09 \times 10^{-3}\theta^2 + 3.0 \times 10^{-2}\theta + 1.7 \quad (3.16)$$

3.2.3 Anisotropy of Critical Current in 2G HTS Tape

The 2G HTS tape usually refers to a tape with the superconducting film ReBCO (Re = Y, Sm, Nd, etc.). Similar to the electromagnetic anisotropy of the 1G HTS tape, a 2G HTS tape also presents anisotropy of a critical current in a magnetic field with magnitudes and orientations, that is, its critical current continuously decreases with increasing magnitude and changing orientation of the magnetic field, and depends not only on magnitude but also on orientation of the magnetic field. The degradation of critical current in the perpendicular component of the magnetic field is more serious than that in the parallel component. Compared with the 1G HTS tape, the anisotropic degree of critical current in a 2G HTS tape is superior to the 1G HTS tape. That is, the degrading degree of the critical current in the perpendicular magnetic field is less than in the 1G HTS tape, but it is a little stronger in the parallel magnetic field than the 1G HTS tape.

Different from several empirical and analytical expressions of critical current anisotropy in 1G HTS tape, there is less analytical formula for the critical current anisotropy of 2G HTS. In the range of a low magnetic field, the empirical formula frequently used is [24]:

$$I_c(B) = I_{c0} \left(1 + \frac{\sqrt{\gamma^{-2}B_{//}^2 + B_{\perp}^2}}{B_0} \right)^{-\alpha} \quad (3.17)$$

where implications of $B_{//}$ and $B_{\perp}$ are the same as for the 1G HTS tape, I_{c0} is the critical current in the self field, $B_0 = 20$ mT, $\alpha = 0.65$ and $\gamma = 5$.

As for the YBCO tape, when angle θ is 45° or so, Kramer's law can be adopted to obtain an approximate expression to describe the critical current anisotropy of the YBCO tape by fitting the experimental data with Kramer's law [25]. Then the relationship of critical current density with temperature, magnitude and orientation of the magnetic field is:

$$J_c(B, T, \theta) = \frac{C}{B \sin \theta} \left[\frac{B \sin \theta}{B_c(T)} \right]^{0.9} \left[1 - \frac{B \sin \theta}{B_c(T)} \right] (1 - t)^{2.65} \exp(-3.8\sqrt{t}) \quad (3.18)$$

$$B_c(T) = 150 (1 - t)^{1.5} \exp(-4\sqrt{t}) \quad (3.19)$$

where t is the normalized temperature, $t = T/T_c$, $B_c(T)$ the upper critical magnetic field and C the fitting constant.

It should be noted that Equation (3.18) is not suitable for conditions of $\theta = 0^\circ$ and $\theta = 90^\circ$, since it is obtained by fitting the experimental data, which are acquired in neither the magnetic fields parallel nor perpendicular to the YBCO tape.

In addition, if the range of the magnetic field is low (<0.15 T), the anisotropy of the 2G HTS tape is weak at a temperature of 77 K, and the Kim-like model [26] can be used to describe the relationship of critical current to magnetic field [14]:

$$I_c(B) = I_{c0} \times \begin{cases} 1 - 3.13B - 433.8B^2 + 7007.8B^3 & B \leq 0.03T \\ \left(1 + \frac{B}{0.069}\right)^{-1} & B > 0.03T \end{cases} \quad (3.20)$$

where I_{c0} refers to the critical current in the self field. Although there are no adjustable parameters in Equation (3.20), it is still in a polynomial form and so cannot ensure calculating precision.

Based on Lorentz statistical distribution of the pinning encountered by a vortex and by taking the effects of anisotropic pinning on critical currents [27, 28], the anisotropy of critical currents can be described by:

$$J_c(B, \theta) = \frac{J\Gamma}{\sin^2 \theta + \Gamma^2 \cos^2 \theta} \quad (3.21)$$

where critical current density $J_c(B, \theta)$ refers to that under magnetic field B , whose orientation to the wide surface of the HTS tape is θ (inset in Figure 3.5), parameter J depends on the pinning force density, and parameter Γ may be found by statistical parameters of Lorentz distributions. Two of them can be easily found by experiments in perpendicular and parallel external magnetic fields. For example, in cases of parallel $\theta = 0^\circ$ and perpendicular magnetic fields $\theta = 90^\circ$, Equation (3.21) now becomes:

$$\begin{cases} J_{c//}(B) = \frac{J}{\Gamma} & (\text{parallel magnetic field}) \\ J_{c\perp}(B) = J\Gamma & (\text{perpendicular magnetic field}) \end{cases} \quad (3.22)$$

where $J_{c//}(B)$ and $J_{c\perp}(B)$ represent the critical current densities of the HTS tape in parallel and perpendicular magnetic fields under external magnetic field B . By solving Equation (3.22), the parameters J and Γ can be found as:

$$\begin{cases} \Gamma = \sqrt{\frac{J_{c\perp}(B)}{J_{c//}(B)}} \\ J = \sqrt{J_{c//}(B)J_{c\perp}(B)} \end{cases} \quad (3.23)$$

After substituting Equation (3.22) into Equation (3.21), the critical current density $J_c(B, \theta)$ can be given by:

$$J_c(B, \theta) = \frac{J_{c\perp}(B)}{\sin^2 \theta + [J_{c\perp}(B)/J_{c//}(B)] \cos^2 \theta} \quad (3.24)$$

It is shown that the critical current density $J_c(B, \theta)$ can be found under external magnetic field B with any orientation θ to the HTS width surface, as long as its two critical current densities of $J_{c//}(B)$ and $J_{c\perp}(B)$ are measured in the special cases of parallel and perpendicular magnetic fields B . Consequently, Equation (3.24) is also simple because there are no adjustable parameters. It is also useful for applications, since the manufacturers generally provide the dependences of critical current densities on magnetic field in parallel and perpendicular orientations, respectively.

The critical current of the HTS with a circular cross-section and that of the conventional LTS are isotropic, and their critical current often presents in the form of the Kim-like model [21]:

$$I_c(B) = \frac{I_c(0)}{a + be^{-B/B_0}} \quad (3.25)$$

where $I_c(0)$ is the critical current of the superconductor in the self field. For HTS, $B = (B_{ab}^2 + B_c^2)^{1/2}$, B_{ab} and B_c are magnetic field components parallel to the a - b -plane and c -axis of superconducting grains, respectively; a , b and B_0 are fitted constants and need to be determined by experiment.

3.2.4 Anisotropy of Critical Current in Bi-2212 Wire

The critical temperature of superconducting material Bi-2212 is lower than 1G and 2G HTS. The practical Bi-2212 conductor is always fabricated with a circular cross-section and multifilamentary wire or cable, thus it has no anisotropy and is mainly used in superconducting magnets. The relationship of critical current density with magnetic field and temperature approximately is:

$$J_c(B, T) = J_0 (1 - t)^\gamma \left[(1 - \chi) \frac{B_0}{B + B_0} + \chi \exp(-\beta b) \right] \quad (3.26)$$

where J_0 and B_0 are fitted constants; t and b are normalized temperature and normalized magnetic field, respectively, $t = T/T_{c0}$ and $b = B/B_c(T)$, critical magnetic field $B_c(T)$ is a function of temperature T where T is not fixed, and:

$$B_c(T) = B_{c0} \exp(-\alpha t) \quad (3.27)$$

where T_{c0} is the critical temperature of Bi-2212 in a zero magnetic field, and B_{c0} is the critical magnetic field of Bi-2212 at temperature of 0 K.

Also, the critical temperature T_c is a function of magnetic field B :

$$T_c(B) = \begin{cases} -45.62B + 87.1 & (B < 1 \text{ T}) \\ 42.8 \exp(-0.015B) & (B \geq 1 \text{ T}) \end{cases} \quad (3.28)$$

In Equations (3.26) and (3.27), all of the constants are respectively: $B_{c0} = 465.5 \text{ T}$, $T_{c0} = 87.1 \text{ K}$, $J_0 = 5.9 \times 10^8 \text{ A/m}^2$, $B_0 = 1 \text{ T}$, $\alpha = 10.33$, $\beta = 6.76$, $\gamma = 1.73$ and $\chi = 0.27$.

3.2.5 Anisotropy of n Value for HTS Tape

As already mentioned, the n value of the HTS is much lower than that of the conventional LTS. In general, superconductivity of superconductors with higher n values are superior to those with lower n values. However, the former quench more easily than the latter. Unlike the LTS, the critical current of the HTS cannot exclusively reflect its current-carrying characteristics and thus the n value must be considered in order to evaluate completely the current-carrying properties with zero resistivity [29]. Therefore, the critical parameters describing HTS should include the n value as well as the critical current or critical current density.

The n value is a power index obtained by fitting the voltage-current relationship into the measurement by the conventional four-probe method. Similar to the relationship of critical current to magnetic field, the n value relates not only to the magnitude but also to the orientation of the magnetic field. Compared with the anisotropy of the critical current in HTS tapes, there are few reports concerning anisotropy of n values.

Similar to the anisotropy of critical current in HTS, the anisotropy of the n value can be also described by [13–16]:

$$n(B, \theta) = \frac{n_0}{30.8 - 7.5e^{-B_{//}/B_0} - 22.3e^{-B_{\perp}/B_0}} \quad (2.29a)$$

$$n(T, B, \theta) = n_0 \frac{0.4552 (1.586 - 8 \times 10^{-3}T)}{0.4522 - \log G(T, B, \theta)} \quad (2.29b)$$

where n_0 is the n value of HTS in the self field and at temperature 77 K, other parameters are the same as in Equation (3.3) and function $G(T, B, \theta)$ is the same as in Equation (3.11).

In any temperature and magnetic field, another empirical formula for describing the n value is:

$$n(T, B) = n_0 \left(\frac{I_c(T, B)}{I_c(T, 0)} + x \left(1 - \frac{I_c(T, B)}{I_c(T, 0)} \right) \right) \quad (3.30)$$

where n_0 is a temperature-independent constant; and x is also a constant and approximately equal to 1.

To the author's knowledge, there are few reports describing the n value anisotropy of 2G tapes and thus there is almost no empirical and analytical expression for it. In the range of a magnetic field lower than 0.5 T, the n value of a 2G tape is almost independent of magnitude and orientation of the magnetic field. However, it significantly depends on magnitude and orientation of the magnetic field if it is higher than 0.5 T, and there is also no simple analytical and empirical formula in this respect.

Similar to the critical current, the n value of conventional LTS and HTS with circular cross-sections are also isotropic, and their n values are often described by the Kim-like model [21]:

$$n(B) = \frac{n(0)}{c + de^{-B/B_0}} \quad (3.31)$$

where $n(0)$ is the n value of superconductor in the self field. For HTS, $B = (B_{ab}^2 + B_c^2)^{1/2}$, in which B_{ab} and B_c are magnetic field components parallel to the a - b -plane and c -axis of superconducting grains, respectively; c , d and B_0 are fitted constants and need to be determined by experiment.

Another empirical analytical formula, describing the anisotropy of the n value in the 1G HTS tape, is expressed as [25]:

$$n(B, \theta) = 16.8 \exp(-g(\theta)B) \quad (3.32)$$

where function $g(\theta)$ relating with angle θ is:

$$g(\theta) = 1.81 \times 10^{-9}\theta^5 + 5.1 \times 10^{-7}\theta^4 - 4.87 \times 10^{-5}\theta^3 + 1.29 \times 10^{-3}\theta^2 + 6.0 \times 10^{-2}\theta + 0.97 \quad (3.33)$$

3.2.6 Anisotropy of Critical Current Density in HTS Bulk

The bulk HTS mainly refers to ReBCO HTS, in which Re is rare-earth elements such as Y, Sm and Nd. The bulk HTS generally is made up of single grains with cylindrical pucks or disks with a diameter as large as 10 centimetres. Its special microstructure, consisting of the alternating stack of superconductive CuO_2 layers and almost insulating block layers, results in remarkable anisotropic behaviour in the present bulk HTS. Due to this anisotropic behaviour arising from the intrinsic pinning and other defects in the bulk HTS, the flux-line curvature will always occur when the bulk HTS is placed in a magnetic field. Consequently, the critical current density J_c of the bulk HTS is anisotropic and strongly dependent on

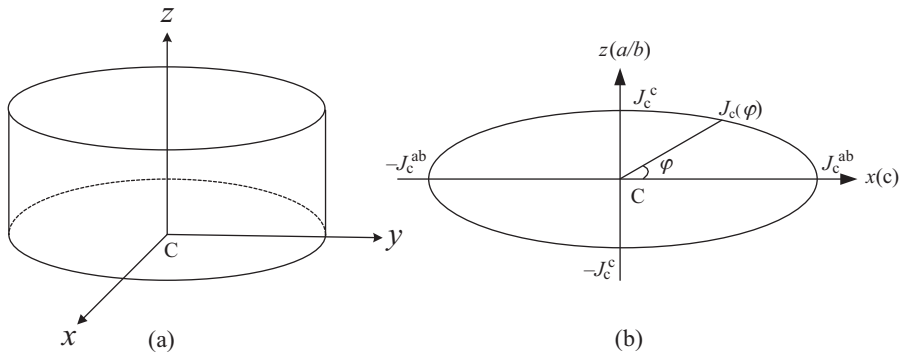


Figure 3.7 (a) Structure of bulk HTS with c -axis parallel to the z -axis; (b) anisotropy of critical current with elliptical model.

the orientation of the applied field [30–34]. That is, the value of the critical current density flowing in the a - b -plane J_c^{ab} is larger than along the c -axis J_c^c . There are a few models to formulate the dependence of the critical current density J_c on the angle φ between the orientation of the local applied magnetic field and the c -axis.

Figure 3.7(a) shows the geometrical structure. A simple model of anisotropic critical current is proposed as:

$$J_c(\varphi) = J_c^{ab} \left(\cos^2 \varphi + \frac{\sin^2 \varphi}{\alpha} \right) \quad (3.34)$$

where α is the out-of-plane anisotropic ratio α of the critical current density J_c^{ab} to J_c^c , namely $\alpha = J_c^{ab}/J_c^c$.

An elliptical model always used to describe the anisotropy of ferromagnetic materials is adopted in angular dependence of critical current J_c in bulk HTS (Figure 3.7(b)). The elliptical model can be expressed as:

$$\left(\frac{J_{cx}}{J_c^{ab}} \right)^2 + \left(\frac{J_{cz}}{J_c^c} \right)^2 = 1 \quad (3.35)$$

where J_{cx} and J_{cz} are the induced current densities in the a - b -plane and parallel to the c -axis, respectively. When taking anisotropic ratio α into account, the critical current density $J_c(\varphi)$ is obtained:

$$J_c(\varphi) = \sqrt{J_{cx}^2 + J_{cz}^2} = J_c^{ab} \sqrt{\cos^2 \varphi + \left(\frac{\sin^2 \varphi}{\alpha} \right)} \quad (3.36)$$

3.3 Critical Current Characteristics of LTS Materials

LTS is a kind of metal-alloy and is isotropic. The conventional practical LTS materials are NbTi and Nb₃Sn composite superconducting wires or tapes. Thus the dependence of critical current density on magnetic field is simpler in HTS than in LTS, namely the critical current density depends on the magnitude of the magnetic field only and is independent of its orientations.

3.3.1 Dependence of Critical Current Density of NbTi on Magnetic Field

When the applied magnetic field is about 5 T, the dependence of the critical current of NbTi on the magnetic field and temperature approximately obeys the Morgan formula:

$$J_c(B, T) = J_{\text{ref}} \left[1 - \frac{0.315319 (T - 4.2) + 0.01528 (T - 4.2)^2 - 0.00161 (T - 4.2)^3}{1 - 0.163089 (B - 5)} \right] \cdot \left[\frac{1 - 0.231741 (B - 5)}{1 - 0.021249 (B - 5) - 0.020418 (B - 5)^2} \right] \quad (3.37)$$

where J_{ref} is the critical current density of NbTi at a temperature of 4.2 K and a magnetic field of 5 T, that is, $J_{\text{ref}} = J_c(5 \text{ T}, 4.2 \text{ K}) = 9.15 \times 10^8 \text{ A/m}^2$.

The relationship of critical temperature with magnetic field in NbTi wire can be described by the Lubell formula [35]:

$$T_c(B) = \begin{cases} 9.2 \left(1 - \frac{B}{14.5} \right)^{0.59} & B < 10 \text{ T} \\ 9.2 \left(1 - \frac{B}{14.8} \right)^{0.59} & B \geq 10 \text{ T} \end{cases} \quad (3.38)$$

The relationship between magnetic field and critical temperature is:

$$B_{c2}(T) = B_{c2}(0) \left[1 - \left(\frac{T}{T_c} \right)^\gamma \right] \quad (3.39)$$

where $B_{c2}(0)$, T_c and γ are all constants. With NbTi, $B_{c2}(0) = 14.5 \text{ T}$, $T_c = 9.2 \text{ K}$, and $\gamma = 1.7$.

3.3.2 Dependence of Critical Current Density of NbTi on Magnetic Field and Temperature

For NbTi wire, the normalized magnetic field b and the normalized temperature t are defined respectively as $b = B/B_{c2}$ and $t = T/T_{c0}$, where B_{c2} is the upper critical magnetic field and T_{c0} is the critical temperature of superconductors NbTi, then the relation of the critical current density with magnetic field and temperature can be approximated as:

$$J_c(B, T) = J_{c,\text{ref}} \frac{C}{B} b^\alpha (1 - b)^\beta (1 - t^\gamma)^\chi \quad (3.40)$$

where C is a fitted constant, parameters α and β describe the reduction of critical current density with increase in the magnetic field, and α is 0.57 and β is 0.9, respectively; parameters γ and χ characterize the decrease of the critical current density with the increase of temperature, $\gamma = 1.7$ and $\chi = 2.32$, respectively; and J_{ref} is the same as that in Equation (3.37).

The relationship between the upper critical magnetic field and temperature is:

$$B_{c2}(T) = B_{c2}(0) (1 - t^\gamma) \quad (3.41)$$

For the superconductor NbTi, Equations (3.39) and (3.41) are the same. The dependence of critical temperature on the magnetic field is calculated by:

$$T_c(B) = T_{c0}(1 - b)^{1/\gamma} \quad (3.42)$$

For the superconductor NbTi, the related constants T_{c0} , $B_{c2}(0)$ γ are the same as those of Equation (3.39).

3.3.3 Dependence of Critical Current Density of Nb₃Sn on Magnetic Field

For superconductor Nb₃Sn, the critical current density is sensitive to mechanical strain ε , as well as to the effect of the magnetic field and temperature. The relationship of its critical temperature with the magnetic field is approximately linear:

$$T_c(B) = T_{c0}(1 - b) \quad (3.43)$$

where T_{c0} is the critical temperature of superconductor Nb₃Sn when the magnetic field is zero, $T_{c0} = 18.3$ K, $b = B/B_{c2}$ is the normalized magnetic field and $B_{c2}(0)$ is the upper critical magnetic field of the Nb₃Sn superconductor at temperature $T = 0$ K. When the temperature is T , the relationship of the upper critical magnetic field of superconductor Nb₃Sn with the temperature is the same as Equation (3.37), but the parameters are different: that is, $B_{c2}(0) = 27.9$ T, $T_c = 18.3$ K and $\gamma = 0.62-0.72$. In the temperature range $3.4 \text{ K} \leq T \leq 7 \text{ K}$, γ is 0.62, while in the temperature range $7 \text{ K} < T \leq 11.5 \text{ K}$, γ is 0.72.

Since superconductor Nb₃Sn is sensitive to the strain ε in the temperature and magnetic fields, the effects of strain ε on critical current density, critical temperature and upper critical magnetic field must be considered in practical applications of Nb₃Sn wire or tape, then its upper critical current and upper critical magnetic field are modified as:

$$T_{c0} = T_{c00} (1 - a\varepsilon^\alpha)^{1/3} \quad (3.44)$$

$$B_{c2}(0) = B_{c20} (1 - a\varepsilon^\alpha) \quad (3.45)$$

where ε is the absolute value of mechanical stress. T_{c00} is the critical temperature of Nb₃Sn in the free state or without mechanical stress and magnetic field, and B_{c20} is the upper critical magnetic field at temperature 0 K. In two- and three-components of Nb₃Sn wire, T_{c00} equals 24 K and 28 K, B_{c20} is 16 T and 18 T, respectively; and a is a fitted constant:

$$a = \begin{cases} 1250, & \varepsilon > 0 \\ 900, & \varepsilon < 0 \end{cases} \quad (3.46)$$

With a combination of the above parameters, the dependence of critical current density on magnetic field and temperature in Nb₃Sn is:

$$J'_c(B, T, \varepsilon) = C \frac{1}{\sqrt{b}B_{c2}} (1 - b)^2 (1 - t^2)^2 \quad (3.47)$$

where C is a fitted constant. It is respectively $2.22 \times 10^{10} \text{ A}\cdot\text{T}/\text{m}^2$ and $1.34 \times 10^{10} \text{ A}\cdot\text{T}/\text{m}^2$ for two- and three-components of Nb₃Sn wire, and the upper critical magnetic field is:

$$B_{c2}(T) = B_{c2}(0) (1 - t^2) \left(1 - \frac{t}{3}\right) \quad (3.48)$$

If the magnetic field is low, Equation (3.47) will be divergent. In order to prevent this confusion, the total critical current density $J_c(B, T, \varepsilon)$ of superconductor Nb_3Sn is often defined as:

$$\frac{1}{J_c(B, T, \varepsilon)} = \frac{1}{J'_c(B, T, \varepsilon)} + \frac{1}{J_{c0}} \quad (3.49)$$

where parameter J_{c0} is given by the Equation (3.50):

$$J_{c0}(T) = J_{c0} (1 - t^2)^2 \quad (3.50)$$

where J_{c0} is the critical current density of Nb_3Sn at temperature 0 K. In two- and three-components of Nb_3Sn wire, J_{c0} is $4.62 \times 10^{10} \text{ A/m}^2$ and $3.35 \times 10^{10} \text{ A/m}^2$, respectively.

Compared with the HTS, either NbTi or Nb_3Sn transmit more sharply from the superconducting state to the normal state, that is, the voltage–current relationship is very steep. If it is fitted by the power law, the power n value would be generally greater than 25. Therefore, the difference between the critical currents defined by the criteria of $E_c = 1 \text{ } \mu\text{V/cm}$ and $E_c = 0.1 \text{ } \mu\text{V/cm}$ is very small. Thus, an n value is not a necessary critical parameter for conventional LTS. As long as the critical currents are known, they generally meet the requirement of applications.

3.4 Irreversible Fields of Superconducting Materials

Irreversibility of the magnetic field is also an important parameter for practical superconducting materials. Especially, the magnetic field of a current-carrying superconductors must be lower than the irreversible magnetic field in practical applications. In general, the irreversible magnetic field is lower than the upper critical magnetic field B_{c2} and much higher than the lower magnetic field B_{c1} . Figure 3.8 shows the

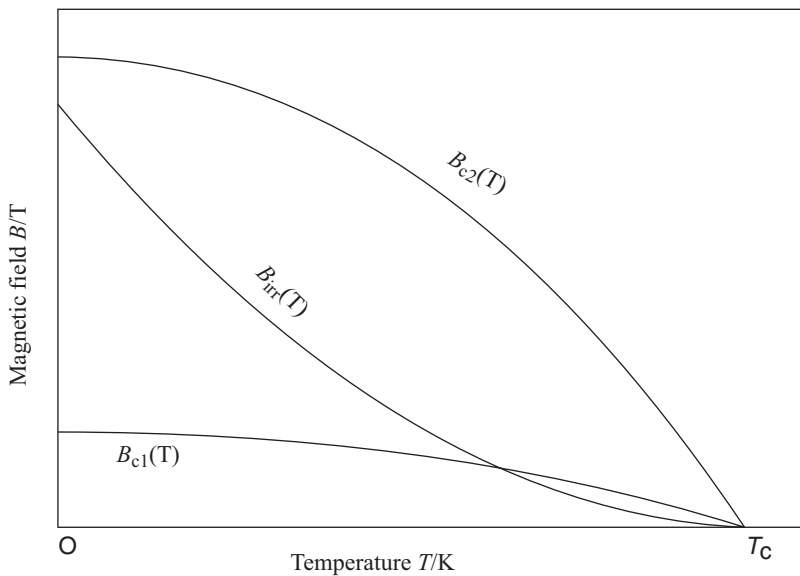


Figure 3.8 Relationships of the lower critical magnetic field, irreversible field and the upper critical magnetic field with temperature.

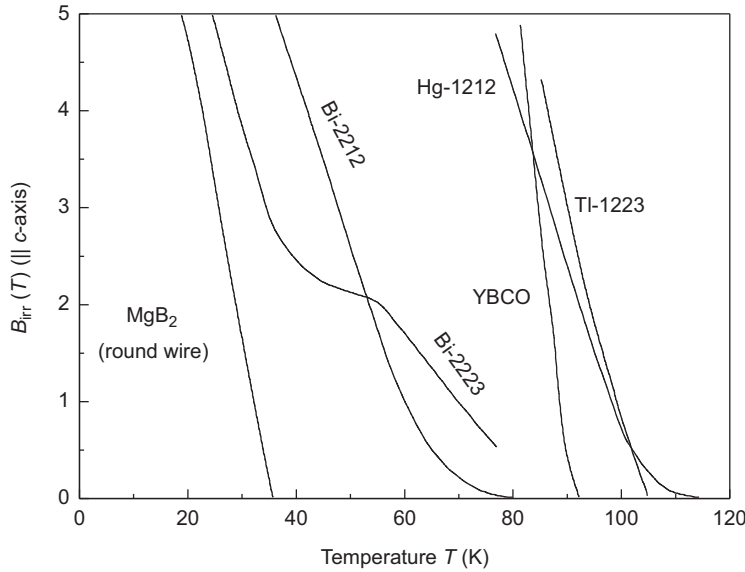


Figure 3.9 Plot of irreversible fields against temperature. (Reproduced with permission from Yinshun Wang, “Bases of Applied Superconductivity in Electrical Engineering,” Science Press, 63, © 2011.)

dependence of the lower magnetic field B_{c1} , irreversible magnetic field B_{irr} and upper magnetic field B_{c2} on temperature. Therefore, the target magnetic field strength generated by superconductors must be less than the irreversible magnetic field strength B_{irr} in the design of superconducting applications.

Fortunately, the upper critical magnetic field and irreversible field of the HTS material are higher than the magnetic field of the superconducting power apparatus, indicating that both of them can meet the requirements of HTS electrical applications. The relations of irreversible magnetic field B_{irr} with the temperature for several HTS films and Mg_2B wire are illustrated in Figure 3.9. It is shown that Bi-2212 has applications only at temperatures below the liquid nitrogen range. However, Bi-2223 reaches the liquid-nitrogen range but is limited to low-field applications. B_{irr} of YBCO, Hg-1212 and Tl -1223 have higher values and so can be used in both high and low magnetic fields at liquid-nitrogen temperature.

Table 3.2 lists the irreversible magnetic field and upper critical magnetic field for Bi- and Y-based HTS materials in temperature of liquid helium and liquid nitrogen [36], respectively.

3.5 Critical Temperature of Several Kinds of HTS Materials

Unlike LTS, the HTS are oxide ceramics with granular characteristics and strong anisotropy. In addition, the transition range of the critical temperature is wide. Relation of critical current density with temperature is:

$$\frac{J_c(T)}{J_c(0)} = \left(1 - \frac{T}{T_c}\right)^x \quad (T < T_c) \quad (3.51)$$

where $J_c(0)$ is a fitted constant and equivalent to the critical current density of HTS at a temperature of 0 K. The critical temperature and index x of several HTS materials are presented in Table 3.3.

Table 3.2 Magnetic field irreversibility of three kinds of HTS materials

Condition	Characteristics	YBCO	Bi-2212	Bi-2223
Temperature	Critical temperature T_c /K	92	85	110
Film structure in 4.2 K	Upper critical magnetic field B_{c2}/T	~300	85	100
	Irreversibility magnetic field B_{irr}/T	>30	>30	>30
Film structure in 77 K	Upper critical magnetic field B_{c2}/T	~56	~35	20
	Irreversibility magnetic field B_{irr}/T	>10	~0.005	~0.2

Table 3.3 Critical temperature and index x of several HTS materials

Materials	Critical temperature T_c /K	Index x
Bi-2223/Ag	105 ~ 110	1.4
Bi-2212/Ag	85 ~ 92	1.8
Tl-1223film/YSZ	107	1.0
Y-123 RABiTS	93	1.2

3.6 Thermodynamic Properties of Practical Superconducting Materials

In this section, the thermodynamic characteristics of practical superconducting materials and some commonly-used structural materials are introduced with cryogenic temperatures, respectively, to describe practical applications in the design of superconducting apparatus.

3.6.1 Thermal and Mechanical Characteristics of Practical Superconducting Materials

Conventional LTS for practical applications are metal-alloys, the most common ones being NbTi and Nb₃Sn. At liquid-helium temperature, their Young's modulus' are 162 MP and 82 MPa, respectively, and their ultimate tensile strengths are 2200 MP and 420 MPa, respectively. Since HTS materials are oxide ceramics, their thermodynamic properties are poor, and they are mechanically brittle. Table 3.4

Table 3.4 Density and some thermodynamic parameters of several HTS materials

Materials	T/K	YBCO	Bi-2212	Bi-2223	BPSCCO	Ag
Density $\gamma/(\text{kg}\cdot\text{m}^{-3})$	300	6380	5350	6500	4350	10500
Young's modulus E/Mpa	300	97	39	96	54	76
	77	97	—	—	54	—
Ultimate tensile strength σ_U/Mpa	300	50	20	—	70	370
Yield strength σ_y/Mpa	300	—	20	—	—	55
Specific heat $C/(\text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1})$	120	245	220	240	192	200
Thermal conductivity $k/(\text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1})$	200 K	2.9	6.0	1.1	1.4	400
	T_c	3.0	5.5	1.0	1.9	—
	77	3.0	5.6	1.0	1.9	420
Resistivity $\rho/(\mu\Omega\cdot\text{m})$	300	10	60	—	8	0.017
	T_c^*	10	60	—	8	0.004

Note: T_c^* refers to the temperature just above its critical temperature.

Table 3.5 Thermal conductivity of YBCO coated conductors

Temperature/K	Thermal conductivity [W/(K·m)]				
	20	50	90	110	295
YBCO123 (<i>ab</i>) 123 phase (melt-textured)	14	27	22	21	~18
YBCO123(<i>c</i>) 123 phase(melt-textured)	3.5	4.4	3.2	3.0	~2.8
YBCO(<i>ab</i>)123+40%211phase(melt-textured)	10	19	16	15	~14
YBCO sintered	5	8	5	5	5

lists density and some thermodynamic parameters of several HTS materials at temperatures 300 K, 200 K and 120 K, and a temperature just above their critical temperature T_c^* [1].

The specific heat capacity of Bi-2223 in a wide range of temperatures can be approximated by using Equation (3.52) in units of J/(kg·K):

$$C = \begin{cases} 4.5683 \times 10^{-3} T^3 & (T \leq 10K) \\ -3.088 + 0.64996T + 8.23239 \times 10^{-3} T^2 + 3.2406 \times 10^{-4} T^3 & (10K \leq T \leq 40K) \\ -58.32 + 3.18672T - 7.8786 \times 10^{-3} T^2 + 6.5556 \times 10^{-6} T^3 & (40K \leq T \leq 300K) \end{cases} \quad (3.52)$$

The density of Bi-2233 (BSCCO) is $\gamma = 6500 \text{ kg/m}^3$, and the density of BPSCCO is $\gamma = 4350 \text{ kg/m}^3$. By multiplying γ with Equation (3.52), the volumetric heat capacity $C_p = \gamma C$ can be calculated in units of J/(m³·K). Its thermal conductivity k is given by:

$$k_{\text{Bi2223}}(T) = \begin{cases} 0.02T & (T \leq 55K) \\ \left(0.474 + 8.43 \times 10^{-3} T + 3.25 \times 10^{-4} T^2 - 6.595 \times 10^{-6} T^3 \right) & (55K < T \leq 77K) \\ \left(0.195 + 9.424 \times 10^{-3} T + 3.4 \times 10^{-4} T^2 - 6.237 \times 10^{-6} T^3 \right) & (77K < T < 113K) \\ \left(4.749 - 0.102T + 9.901 \times 10^{-4} T^2 - 4.167 \times 10^{-6} T^3 \right) & (113K \leq T < 200K) \end{cases} \quad (3.53)$$

There is no analytical expression for thermal conductivity of YBCO-coated conductor (YBCO CC). Thermal conductivity of YBCO-coated conductors at different temperature are listed in Table 3.5 in units of W/(K·m).

For practical YBCO CC, heat capacity and thermal conductivity fabricated by different manufacturers are different. In pure YBCO material, its density γ is 6380 kg/m^3 . As an example, two kinds of YBCO CC of 344C (copper-reinforced) and 344SS (stainless steel-reinforced) are commercially manufactured by the AMSC. In the range of temperatures of 2 K through 300 K, the specific heat capacity of the former is approximately described in units of J/(kg·K) by:

$$C(T) = \begin{cases} 1.1 - 0.4T + 5 \times 10^{-2} T^2 - 4 \times 10^{-5} T^3 & (2K \leq T < 50K) \\ -125.2 + 5.6T - 1.9 \times 10^{-2} T^2 & (50 \leq T < 100K) \\ 80 + 2T - 3 \times 10^{-3} T^2 & (100K \leq T \leq 300) \end{cases} \quad (3.54)$$

and its thermal conductivity k is approximately given in units of W/(m·K) by:

$$k(T) = \begin{cases} -3.5332 + 9.6273T - 0.1282T^2 + 5 \times 10^{-4} T^3 & (2K < T \leq 50K) \\ 208.45 + 0.2165T - 5 \times 10^{-6} T^2 & (50 < T < 300K) \end{cases} \quad (3.55)$$

In the range of temperatures 2 K through 300 K, the specific heat capacity and thermal conductivity of the latter YBCO CC 344SS in units of J/(kg·K) and W/(kg·K) are respectively approximately found:

$$C(T) = \begin{cases} 0.265 - 2.5 \times 10^{-2}T + 3.35 \times 10^{-2}T^2 - 3 \times 10^{-6}T^3 & (2K < T \leq 50K) \\ -120 + 4.6T - 1.0 \times 10^{-2}T^2 & (50 < T \leq 100K) \\ 45 + 2.1T - 3 \times 10^{-3}T^2 & (100K < T \leq 300) \end{cases} \quad (3.56)$$

and

$$k(T) = \begin{cases} -2.7641 + 5.2919T - 0.1143T^2 & (2K < T \leq 20K) \\ 58.9552 - 0.6221T + 4.5 \times 10^{-3}T^2 & (20K < T \leq 85K) \\ 32.767 + 5.74 \times 10^{-2}T - 6 \times 10^{-7}T^2 & (85K < T \leq 300K) \end{cases} \quad (3.57)$$

The relationship of specific heat capacity of NbTi with temperature is given in units of J/(kg·K) [37]:

$$C(T) = 0.152 + 2.10 \times 10^{-3}T^3 \quad (3.58)$$

The density γ of NbTi is 6550 kg/m³, and the volumetric specific capacity in units of J/(m³·K) can be found by multiplying γ with Equation (3.58).

In accurate design and high magnetic field applications, the magnetic field has certain effects on specific capacity. Let T_{sh} be the sharing temperature of superconductors:

$$T_{sh} = T_b + (T_c - T_b) \left(1 - \frac{I}{I_c}\right) \quad (3.59)$$

where T_b and I are the temperature of the cryogenic bath and the transport current, respectively, then the volumetric heat capacity of NbTi in units of J/(m³·K) can be simply expressed as:

$$\gamma C = \begin{cases} 812T + 12.9T^3 & (T > T_{sh}) \\ 812T \frac{B}{B_{c0}} + 42.73T^3 & (T \leq T_{sh}) \end{cases} \quad (3.60)$$

where B_{c0} is upper critical magnetic field strength at a temperature of 0 K, that is, $B_{c0} = B_{c0}(0 \text{ K})$, and B is the magnetic field strength.

For the commercial two-component composite NbTi/Cu, dependence of its volumetric specific capacity on temperature T and magnetic field B in unit of J/(m³·K) can be described by:

$$\gamma C(T) = \frac{1}{f+1} [(6.75f + 50.55) T^3 + (97.43f + 69.81B) T] \quad (3.61)$$

where f is the volume ratio of Cu to NbTi in the composite conductor.

Thermal conductivity k of NbTi at different temperatures T in units of W/(kg·K) is:

$$k(T) = \begin{cases} 0.38887 \times T^{0.1532} - 0.371 & (4K < T \leq 6K) \\ 0.13957 \times T^{0.782} - 0.4262 & (6K < T \leq 100K) \end{cases} \quad (3.62)$$

For superconductor Nb₃Sn, its volumetric specific capacity γC in the vicinity of its critical temperature $T_c (= 18.3 \text{ K})$ in units of J/(m³·K) is approximately [38]:

$$\gamma C = 10.324T + 10.24T^3 \quad (3.63)$$

Table 3.6 Specific heat capacity of superconductors Nb₃Sn at different temperatures

Temperature T/K	Specific capacity C [J/(kg·K)]	Temperature T/K	Specific capacity C [J/(kg·K)]
4	0.12	50	97
6	0.42	70	140
8	0.98	100	200
10	4.23	120	220
15	8.96	150	232
20	17.1	200	250
25	28	250	256
30	45	300	262
40	70	400	274

Note: Specific heat capacity with unit of J/(kg·K) can be converted into volumetric specific capacity in unit of J/(m³·K) by multiplying density γ with specific heat capacity.

In the vicinity of its critical temperature $T_c (= 18.3 \text{ K})$, its thermal conductivity k is:

$$k(T) = \begin{cases} 6.8 \times 10^{-4}T^3 + 8 \times 10^{-4}T^2 - 1.217 \times 10^{-2}T + 0.0308 & (2.5K < T \leq 9.5K) \\ -2.1 \times 10^{-3}T^3 + 9.18 \times 10^{-2}T^2 - 1.0891T + 4.525 & (9.5K < T \leq 19.5) \\ -3.1 \times 10^{-3}T^2 + 0.1702T + 0.3985 & (19.5K < T \leq 30K) \end{cases} \quad (3.64)$$

Since the thermal conductivity of Nb₃Sn is much smaller than in conventional metals, the thermal conductivity of the composite Nb₃Sn conductor above its critical temperature may be neglected in calculations of actual heat conduction. The thermal conductivity of the composite Nb₃Sn conductor is mainly determined by normal metal matrices. In a wide range of temperatures, specific heat capacity of superconductors Nb₃Sn is typically listed in Table 3.6.

For multi-components composite superconducting conductors, calculations of the heat capacity and thermal conductivity are described in Appendix A.1.

3.6.2 Thermal Contraction of Superconducting Materials

Since superconducting materials operate at cryogenic temperatures, they must endure thermal contraction from room temperature to cryogenic temperature, or from cryogenic temperature to room temperature. Therefore, superconducting materials should often be enforced with combinations of other structural materials in actual applications. Taking the thermal contraction of superconducting material into account, it can be mechanically matched with other structural materials so that thermal stress can be eliminated. The coefficient of linear expansion is used to describe the thermal contraction characteristics of the superconducting material from room temperature to indicated temperature T :

$$\alpha = \frac{1}{L} \frac{dL}{dT} \quad (3.65)$$

where L is the length of the superconductor at the temperature T . Another important concept is the linear contraction, which is defined as:

$$\frac{\Delta L}{L_0} = \frac{L_0 - L}{L_0} \quad (3.66)$$

where L_0 is the length of the superconductor at room temperature and L is the length of a superconductor when cooled to the temperature T . Table 3.7 lists linear contractions at a range of temperatures 4 K through 200 K and coefficient of linear expansion at room temperature (293 K) for the conventional LTS and HTS materials, respectively [39].

Table 3.7 Contraction properties of superconducting materials

Materials	$\Delta L/L(\%)$							$\alpha(\times 10^{-6} \text{ K}^{-1})$
Temperature T/K	4	40	77	100	150	200	250	293
Nb ₃ Sn	0.16	0.16	0.14	0.13	0.095	0.065	0.030	7.6
Nb ₃ Sn(10vol%)/Cu	0.30	/	0.28	/	/	/	/	/
Nb-45Ti	0.188	0.184	0.169	0.156	0.117	0.078	0.038	8.2
Nb-Ti/Cu	0.265	0.262	0.247	0.231	0.179	0.117	0.054	12.5
Bi-2212 <i>ab</i> -plane	0.152	0.150	0.139	0.132	0.106	0.074	0.036	8.3
Bi-2212 <i>c</i> -axis	0.295	0.289	0.266	0.250	0.199	0.136	0.064	15.1
Bi-2223/Ag tape	/	0.31	0.30	0.28	0.22	0.15	0.07	13
Bi-2223 <i>ab</i> -plane	0.15	0.15	0.14	0.13	0.11	0.07	0.04	8.3
Bi-2223 <i>c</i> -axis	0.30	0.29	0.27	0.25	0.20	0.14	0.060	15
Bi-2223 tape*	0.22	/	0.23	/	/	/	/	16
YBCO <i>a</i> -axis	/	/	0.12	0.12	0.10	0.070	0.04	7.4
YBCO <i>b</i> -axis	/	/	0.16	0.15	0.13	0.10	0.05	9.6
YBCO <i>c</i> -axis	/	/	0.34	0.33	0.25	0.17	0.09	17.1

*75 vol.%Ag alloy.

Table 3.8 Contraction properties of matrix and substrate materials

Materials		$\Delta L/L_{293\text{K}-77\text{K}} [\%]$	$\Delta L/L_{293\text{K}-4.2\text{K}} [\%]$	$\alpha_{293\text{K}} [10^{-6} \text{ K}^{-1}]$
Oxygen-free copper		0.302	0.324	16.7
Silver		0.37	0.41	18.5
Aluminum		0.415	0.393	22.5
Bronze (Cu-10wt%Sn)		0.35	0.38	18.2
Brass		0.34	0.37	17.5
Nickel		0.212	0.224	12.5
Iron		0.190	0.198	11.5
Nb		0.13	0.143	7.1
Silicon		0.022	0.023	2.32
Soft solder		0.480	0.514	25.5
Strainless steel SS 304		0.28	0.29	15.1
Strainless steel SS 304L		0.28	0.31	15.5
Strainless steel SS 316		0.28	0.30	15.2
Monel copper nickel alloy		0.24	0.25	14.5
Iron nickel alloy	Fe-36Ni	~0.037	0.038	3
	Fe-9Ni	0.188	0.195	11.5
Hastelloy Calloy		0.216	0.218	10.9
Inconel 718alloy		0.22	0.24	13.0
Titanium Aluminum Alloy	Ti-6%Al-4%V	0.163	0.173	8.0
	Ti-5%Al-2.5%V	0.17	0.20	8.3
YSZ(Y-stabilized Zirconia)		—	—	10.3
Quarz (// optical axis)		0.104	—	7.5
Diamond		0.024	0.024	1.0
Al ₂ O ₃		0.078	0.079	5.4
MgO		0.137	0.139	10.2
α -SiC		0.03	—	3.7
SrTiO ₃		—	—	11.1
LaAlO ₃		—	—	12.6
NdGaO ₃		—	—	7.8
AlN	// <i>a</i> -axis	0.032	—	3.7
	// <i>c</i> -axis	0.025	—	3.0

Table 3.9 Mechanical properties of some metal substrates

	Temperature	Ultimate tensile strength	Yield strength	Young's modulus
Materials	T/K	σ_U/Mpa	σ_Y/Mpa	E/Mpa
Copper(Annealing)	295	315	280	70
	77	415	380	77
Aluminum	295	160	70	70
	77	310	90	100
Silver (Annealing)	295	—	—	71
	77	310	90	100
Stainless steel 304L	295	550	200	190
	77	1450	260	200
Stainless steel 304H	295	1030	620	190
	77	1860	1050	200
Stainless steel 316	295	1290	1100	185
	77	1790	1400	195
Inconel 625	295	900	850	220
	77	1160	1040	230
Inconel 718	295	1500	1280	179
	4	1900	1490	182

In actual applications, both practical LTS and HTS materials are composite superconducting conductors made of pure superconductors and normal metal or alloy, the so-called matrix. The matrix materials in LTS are often metals with high thermal conductivity and low resistivity, such as copper, aluminium, silver, gold or alloy. The matrix of 1G HTS, such as in Bi-2223 tapes, are silver, silver alloy and stainless steel reinforced materials. However, the matrix or substrate of 2G HTS tape is complicated compared with conventional LTS wire and 1G HTS tape.

Table 3.8 lists the linear contraction and coefficient of linear expansion for several conventional matrix or substrate materials of practical LTS and HTS materials when cooling from room temperature 293 K to liquid nitrogen temperature (77 K) and liquid helium temperature (4.2 K), respectively.

Table 3.9 lists the mechanical properties of some common metal substrates at room temperature 295 K, liquid nitrogen temperature 77 K and cryogenic temperature 4 K, in order to understand the mechanical properties of practical composite superconducting materials.

Mechanical properties of other metals or alloys, non-metallic materials, insulation materials and structural materials can be found in other professional handbooks.

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4

Stability of Superconductors

As introduced in Chapter 2, the Type-I superconductor has no practical application due to its low critical magnetic field B_c and poor current-carrying ability. Although the upper critical magnetic field B_{c2} of the ideal Type-II superconductor is several times higher than the critical magnetic field B_c of the Type-I superconductor, the magnetic field range of its mixed state is too narrow and there is flux-flow resistance in that state. Consequently, its current-carrying ability at zero resistance is very low, so an ideal Type-II superconductor has no practical applications. However, a non-ideal Type-II superconductor has a high upper critical magnetic field B_{c2} , though its magnetization curve is irreversible and so has a hysteresis effect [1–3]. The upper critical magnetic field B_{c2} of the conventional low temperature superconductor (LTS) and high temperature superconductor (HTS) are up to 25 T and more than 100 T, respectively. In addition, the magnetic field range of mixed states in HTS is wide, therefore all of the practical superconductors are restricted to the non-ideal Type-II superconductors.

A superconductor always takes the form of coils or their analogues in practical applications. A current-carrying superconductor is subjected to an electromagnetic force in magnetic fields generated by other turns of the windings. If the superconductor is not firmly fixed, it will undergo mechanical movement. Consequently, heat will be produced due to frictional sliding, the so-called mechanical disturbance. Other disturbance results from the flux-jump generated by the superconductor itself. In this chapter, the stable theory of the superconductor is introduced, mainly based on the flux-jump theory and mechanical disturbances, and methods to improve the stability of the superconductor are subsequently described. The stabilities of the superconductor include adiabatic stability (also known as enthalpy stability), dynamic stability and cryostability.

In general, the three-dimensional (3D) constitutional equation for heat conduction is described by:

$$\gamma C \frac{\partial T}{\partial t} = \nabla(k \nabla T) + G_j + G_d - G_q \quad (4.1)$$

where T is temperature, k presents thermal conductivity, γC refers to volumetric thermal capacity, γ indicates the density, C is specific heat and $G_j = \rho j^2$ is the Joule heat per unit volume. Usually, C and k are not constant and dependent on temperature T . ρ is the conductor's normal-state electrical resistivity and j its current density; G_d is the heat term per unit volume resulting from magnetic heat (flux jump) and mechanical losses; G_q is the cooling term per unit volume and does not exist in adiabatic approximation. A simplified or modified heat conduction Equation (4.1) is often used in several sections of this chapter.

4.1 Critical States

In Section 2.3.3 of Chapter 2, distributions of magnetic field and current density in superconductors are illustrated by adopting the simplest model, that is, Bean's critical state model (CSM). The critical state of a superconductor is briefly introduced in this section to illustrate the relationships between magnetic flux motion, the pinning effect and flux jumping phenomenon in the superconductor [4, 5]. The so-called critical state indicates that the distributions of magnetic field and current density (including transport and induced current densities) allow the Lorentz force (or flux driving force) and flux pinning force to balance.

In magnetic field B , the superconductor per unit volume with current carrying density J is subjected to the Lorentz force:

$$F_L = J \times B = \frac{1}{\mu_0} [\nabla \times B] \times B \quad (4.2)$$

where μ_0 is permeability of vacuum and F_L represents the Lorentz force acting on the flux line; its direction is pointing from high flux density to low density. Essentially the magnetic pressure of non-uniform flux distribution is always trying to drive the non-uniform flux lattice to a uniform distribution, so that the flux gradient becomes zero and the superconductor has no current carrying ability. Therefore, there should be pinning centres, which bind flux motion in the superconductor to maintain stable non-uniform distribution of the flux. To improve the current carrying ability of the superconductor, flux pinning centres are often generated by introducing defects and impurities with techniques such as doping, heat treatment and irradiation, and the gradient of flux vortices or non-uniform flux line lattices are thus formed in the superconductor. It is assumed that F_p is the pinning force exerted on the flux by pinning centres in unit volume of the superconductor without taking thermally activated motion of the flux into account. If pinning force F_p is smaller than the Lorentz force F_L , that is, $|F_L| > |F_p|$, there is no flux motion. However, the flux will move if pinning force F_p is larger than the Lorentz force F_L , that is, $|F_L| < |F_p|$. When the superconductor is in the state in which the Lorentz force F_L is equal to the pinning force F_p , that is, $F_L = -F_p$, then the flux does not move; this state is known as the critical state, and current density J is now called the critical current density J_c . Different forms of J_c result in different CSMs. In Section 2.3, J_c is described as a constant and is independent of magnetic field B , which is the simplest critical state model known as Bean's CSM. The critical current density J_c continuously decreases with magnetic field B in all real superconductors, and Bean's CSM assumes J_c to be independent of the magnetic field B to simplify calculations. Consequently, other CSMs, such as the Kim model, Kim-like model or power-law model should be adopted according to the situation.

4.2 Adiabatic Stabilization

Many experiments illustrate that the flux jump phenomenon frequently occurs in a superconductor. When this occurs in the magnetic field, the magnetic field in the superconductor is zero due to the action of the screening current. With an increase of magnetic field, the screening current area in the superconductor expands and the area with zero magnetic field decreases until the magnetic field fully penetrates into the superconductor. When the magnetic field reaches a certain value, the screening current is no longer stable, which means a large amount of flux suddenly enters or leaves the superconductor, which results in a temperature rise. If the temperature exceeds its critical temperature, the superconductor will transit from a superconducting state to a normal state, namely the so-called superconductor quench. The process of the flux line abruptly entering or exiting the superconductor is called the flux jump phenomenon (Figure 4.1). According to its magnetization curve, the superconductor exhibits instability when the flux perpendicularly increases or decreases abruptly.

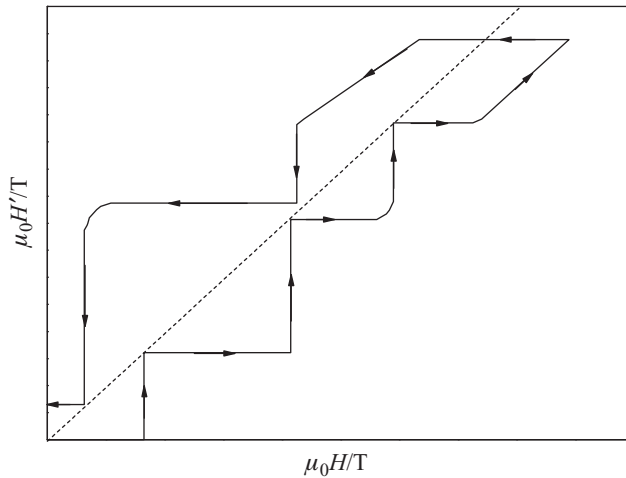


Figure 4.1 Flux jump phenomenon with flux abruptly entering or exiting the superconductor.

There are three causes of flux jump. The first results from the difference between the magnetic diffusivity D_M and thermal diffusivity D_T inside the superconductor. The thermal diffusion coefficient D_T of the superconductor is about 3 to 4 orders of magnitude lower than conventional metal copper or aluminium in a composite superconductor. Nevertheless, its magnetic diffusion coefficient D_M is about 3 to 4 orders of magnitude higher than that of copper or aluminium. For example, magnetic diffusivity D_M of NbTi and Nb₃Sn are 10^3 to 10^4 times higher than that of copper or aluminium, but their thermal diffusivity D_T are 10^{-4} to 10^{-3} times lower than that of copper or aluminium. Thus, the magnetic diffusion process is slow and the thermal diffusion process is fast in normal metals. However, the magnetic diffusion process is fast and the thermal diffusion process is low in superconductors.

Second, there are different dependences of temperature between the flux pinning force and the Lorentz force inside the superconductor. The pinning force of a superconductor decreases with increase of temperature but the Lorentz force is independent of temperature. When the flux pinning force is equal to the Lorentz force, the superconductor is in the critical state at a certain temperature. Sometimes occasional external disturbances can cause a temperature rise ΔT_1 in a superconductor, which then leads to a decrease in the pinning force. If the pinning force is less than the Lorentz force, the critical current density will reduce and then the flux will move. Based on Lorentz's law, the pinning force acts on the motion of the flux line and hinders its movement, which generates viscous resistance and thereby dissipation of energy to form heat energy. This increase in heat energy sequentially brings about a further rise of temperature in the superconductor, so that the positive feedback cycle comes into being. These cycling processes are illustrated in Figure 4.2, in which Q and Φ refer to heat and flux inside the superconductor, respectively.

If the temperature rise of the superconductor is within $\Delta T_2 < \Delta T_1$, the amount of moving flux becomes less and the temperature rise also becomes smaller. Negative feedback then forms, the flux stops moving sequentially, and the critical current density recovers and returns to its original level. This kind of flux jump is called the incomplete (or partial) flux jump. However, if $\Delta T_2 > \Delta T_1$, the amount of moving flux increases and the temperature rises continuously. The phenomenon of the flux jump uninterruptedly takes place, heat generates after each flux jump, and accumulation of heat thus causes the rise in temperature. Positive feedback forms and finally results in quench of the superconductor, called the complete flux jump.

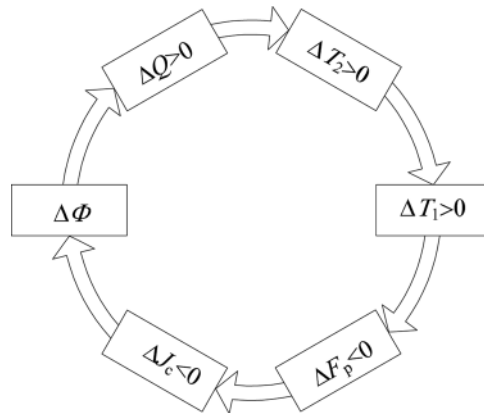


Figure 4.2 Positive feedback process of flux jump.

The third reason for a flux jump results from flux annihilation in an area of low magnetic field, which appears on the magnetization curve. Magnetization suddenly reduces when the magnetic field reverses near the lower critical field H_{c1} . As seen in Figure 4.3, FA exhibits a strange concave shape near the lower critical field $\pm H_{c1}$ in the magnetization curve. The occurrence of flux annihilation (FA) causes flux lines to change abruptly, which results in positive feedback and forms the cycle of flux jump until a complete quench of the superconductor has occurred.

The following sections introduce several methods for preventing thermo- and magnetic-instability. The method to limit the amount of flux motion is called adiabatic stabilization (or intrinsic stabilization). A way of limiting temperature rise is by taking measures to dissipate heat, called cryostability. The matrix material with high thermal conductivity is used in the process of the composite superconductor, from which the generated heat can be easily removed. This measure is known as dynamic stability. Material with higher capacity is used as a matrix to limit temperature rise, called enthalpy stabilization. Of course,

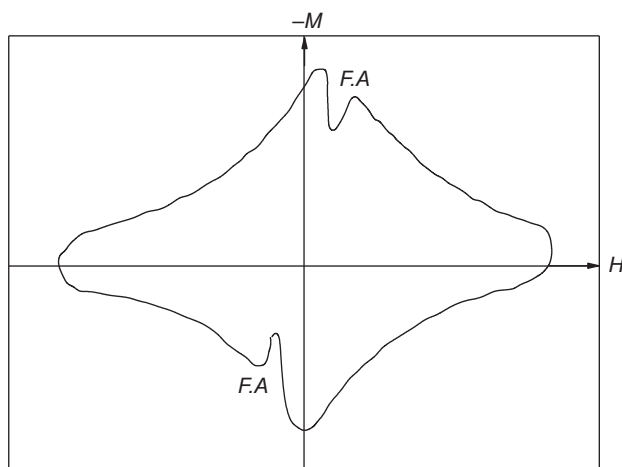


Figure 4.3 Typical flux-annihilation in low field area.

if there are materials whose flux pinning force becomes stronger with increasing temperature, these materials themselves are stable. Unfortunately, to date, these materials have not yet been discovered.

4.3 Adiabatic Stability with Flux Jump

As introduced in Section 4.2, thermal diffusivity of a superconductor is about four orders of magnitude less than magnetic diffusivity, and its heat diffusion velocity is much lower than magnetic diffusion velocity. Thus, the thermo- and magnetic-instability resulting from a flux jump can be approximately treated as adiabatic conditions in the heat conduction process. For convenience, a half infinite superconducting slab without carrying current and in a magnetic field is first considered (Figure 4.4). It extends infinitely along the y - z plane and its width is $2a$. Figure 4.4(a) illustrates the geometrical configuration and magnetic field profile of the superconducting slab, and Figure 4.4(b) exhibits magnetic field distributions due to temperature rise after the flux penetrates the superconducting slab. The solid and dashed lines represent the magnetic field distribution before and after temperature disturbance, respectively in Figure 4.4(b). Origin of the coordinates is at the centre of the superconducting slab.

Assume that the heat term ΔQ is added to the superconducting slab, which is thermally isolated from the environment and its y - z plane is parallel to the uniform magnetic field B . ΔQ will cause a temperature rise ΔT , and critical current density of the superconducting slab simultaneously reduces to [2]:

$$\Delta J_c = J_c \frac{\Delta T}{T_c - T_b} \quad (4.3)$$

where T_c and T_b are the critical temperature and bath temperature of the superconducting slab, respectively. J_c is the critical current density of the superconducting slab at certain temperature T and magnetic field B .

The screening current enters the area into which flux flows with the decrease of critical current density, then critical current density begins to decay and consequently produces heat. The flux penetrates into the superconductor slab and is redistributed at the same time. In Figure 4.4(b), the solid line shows that the initial flux distribution inside the superconductor is obtained by Bean's CSM (complete penetration), $B(x) = \mu_0 J_c(a-x)$, and the dashed line is the flux distribution inside the superconducting slab after the

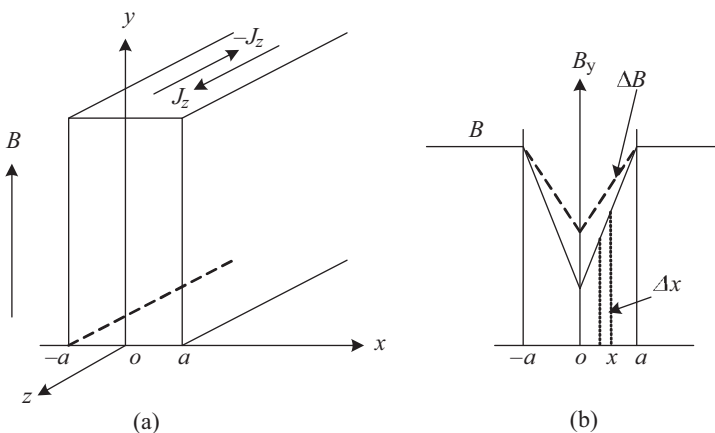


Figure 4.4 Magnetic field distribution of half-infinite slab due to temperature rise: (a) geometrical configuration and magnetic field; (b) magnetic field distribution before and after temperature rise.

temperature rise. According to the symmetry of the superconducting slab, only the right-hand part is necessarily calculated. Voltage $E(x)$ is induced inside the superconductor due to a change in flux. In the right-hand half of the superconducting slab range of $(0, a)$, a slice with width Δx is selected at an arbitrary point x . In this area, the flux change is $\Delta\varphi(x)$ and current is $J_c\Delta x$, and the generated heat $\Delta q(x)$ is:

$$\Delta q(x)\Delta x = \int I(x)E(x)dt = J_c\Delta x\Delta\varphi(x) \quad (4.4)$$

Flux change can be calculated from the magnetic field change in the space range of $(0, x)$, and the electric field E is zero at the centre $x = 0$:

$$\Delta\varphi(x) = \int_0^x \Delta B(x)dx = \int_0^x \mu_0\Delta J_c(a-x)dx = \mu_0\Delta J_c\left(ax - \frac{x^2}{2}\right) \quad (4.5)$$

Then, the heat per unit volume averaged over the superconducting slab is:

$$\begin{aligned} \Delta Q &= \frac{1}{a} \int_0^a \Delta q(x)dx = \frac{1}{a} \int_0^a \mu_0 J_c \Delta J_c \left(ax - \frac{x^2}{2}\right) dx \\ &= \mu_0 J_c \Delta J_c \frac{a^3}{3} \end{aligned} \quad (4.6)$$

Substituting Equation (4.3) into Equation (4.6) and rearranging, we obtain:

$$\Delta Q = \frac{\mu_0 J_c^2 a^2 \Delta T}{3(T_c - T_b)} \quad (4.7)$$

When taking the initial applied heat ΔQ_s into account, the heat balance equation of the whole slab becomes:

$$\Delta Q_s + \frac{\mu_0 J_c^2 a^2}{3(T_c - T_b)} \Delta T = \gamma C \Delta T \quad (4.8)$$

where C and γ are specific heat and density, respectively. Therefore, the effective volumetric capacity per unit volume γC_e is:

$$\gamma C_e = \frac{\Delta Q_s}{\Delta T} = \gamma C - \frac{\mu_0 J_c^2 a^2}{3(T_c - T_b)} \quad (4.9)$$

where the first term is volumetric capacity of the superconductor slab; the second term is energy stored in the screening currents. When the effective capacity of the superconducting slab is reduced, a small heat disturbance will cause the temperature to rise rapidly. If the two terms are equal, then the ultimate catastrophe occurs, and the effective volumetric capacity of the superconducting slab approaches to zero. Even if the disturbance ΔQ_s is small, the temperature rise ΔT may increase without limit, which leads to a flux jump and complete quench. If the second term is much less than the first term, there is little difference between the effective volumetric capacity γC_e and the volumetric capacity γC of the superconducting slab, and even if there is disturbance of heat source ΔQ_s , the temperature rise ΔT is finitely small. Disturbance energy can be absorbed by the superconducting slab, and the occurrence of a

flux jump is effectively restrained. For the convenience of description, parameter β is defined as:

$$\beta = \frac{\mu_0 J_c^2 a^2}{\gamma C (T_c - T_b)} \quad (4.10)$$

If $\beta < 3$, or half thickness of the superconductor slab satisfies Equation (4.11), which ensures that the effective volumetric γC_e is larger than zero, then a flux jump can be avoided:

$$a < \sqrt{\frac{3\gamma C (T_c - T_b)}{\mu_0 J_c^2}} \quad (4.11)$$

β is called the adiabatic stability parameter, and Equation (4.11) is defined as the adiabatic stability criterion due to the assumption that there is no heat exchange with the surrounding environment in the calculation. In addition, Equation (4.11) shows that the half-thickness a should be less than the value determined by the right-hand side of the equation. In order to gain a perceptual knowledge on the size of a , two examples are given here. If the critical current density J_c of an NbTi superconductor is 1.5×10^9 A/m² in magnetic field $B = 6$ T and surrounding temperature is $T_b = 4.2$ K, its density is $\gamma = 6.2 \times 10^3$ kg/m³ and specific heat is $C = 0.89$ J/(kg·K), and critical temperature is $T_c = 6.5$ K, then the half-thickness of the superconducting slab can be obtained as $a < 115$ μ m by substituting these parameters into Equation (4.11). This means that the flux jump can be avoided in principle so long as this condition $a < 115$ μ m is satisfied.

If the half thickness a is just below 115 μ m, spontaneous flux jump does not occur, but the effective volumetric capacity γC_e is still remarkably small, so the superconducting slab is still sensitive to other heat disturbances. To be on the safe side, one half of the value determined by criterion Equation (4.11) is usually chosen in practical applications of superconducting wire when the effective volumetric capacity will be 75% of its original value. Consequently, the actual size of practical superconducting wire a should be 57.5 μ m in this example. As for superconductor Nb₃Sn, the critical current density is $J_c = 2 \times 10^{10}$ A/m² in the magnetic field of 2 T and surrounding temperature of $T_b = 4.2$ K, its volumetric heat capacity is $\gamma C = 1.2 \times 10^3$ J/(m³·K), and critical temperature is $T_c = 16$ K. According to Equation (4.11), one half of the size of the superconducting slab is $a < 9$ μ m, which assures that the flux jump can be avoided.

Since specific heat C increases rapidly with temperature rise, the rate of temperature rise will be effectively reduced. Thus, a certain rate of temperature can improve the ability to prevent flux jumping. When the flux jump starts at T_b and then stops before temperature T reaches a critical temperature T_c , this phenomenon is called partial flux jump.

If the external magnetic field rises from zero, flux will enter the superconductor. Assuming that penetration depth is δ (Section 2.3.3), the magnetic field strength at which flux jump first occurs is defined as the characteristic magnetic field B_{FJ} . Flux jump will first occur when external magnetic field $B = B_{FJ}$. By replacing half thickness a with penetration δ , the characteristic magnetic field is:

$$B_{FJ} = \mu_0 J_c \delta = \sqrt{3\mu_0 \gamma C (T_c - T_b)} \quad (4.12)$$

Flux jump occurs in the range that penetrates magnetic field $B_p > B_{FJ}$. However, if the penetrated magnetic field $B_p < B_{FJ}$, and the stored energy by screening current is small enough, the flux jump will not occur.

Figure 4.5 shows the experimental results of a characteristic magnetic field for a NbTi cylinder with a diameter of 10 mm [2]. The solid line represents the calculated results based on Equation (4.12) in a magnetic field with critical temperature $T_c = 8.6$ K; the dashed line indicates the change of the

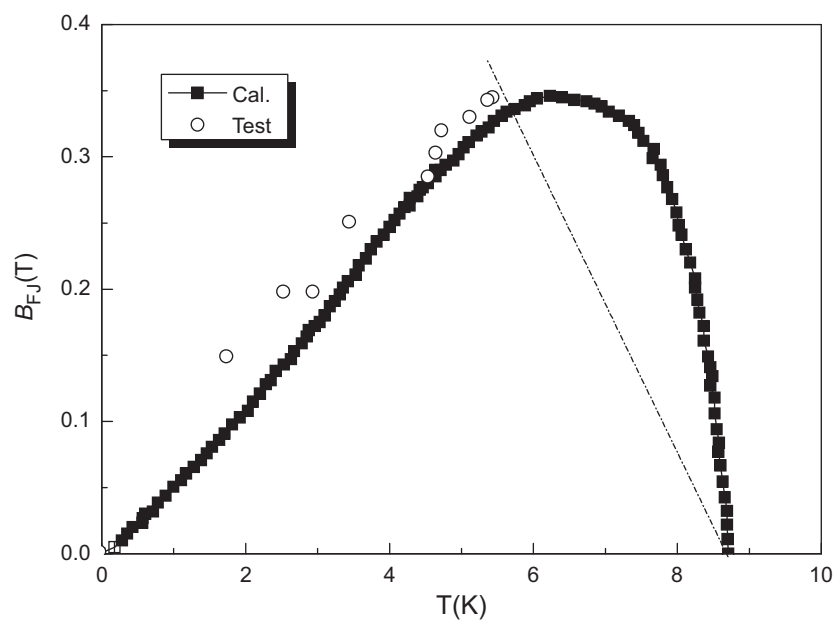


Figure 4.5 Experimental and calculated values of characteristic magnetic field B_{FJ} and their dependence on temperature.

penetration field at the centre of the test sample with temperature. Both theoretical calculations and experimental results illustrate that the characteristic magnetic field also depends on temperature.

Only the half thicknesses of conventional LTS conductors such as NbTi and Nb₃Sn are described above, and their sizes are in the range of several micrometers through several tens of micrometers. Therefore, superconductors are fabricated in forms of fine filaments by which flux jump can be avoided, and is one of the measures for improving the thermo-magnetic stability of a superconductor. To better understand how to avoid flux jump for 1G Bi-2223 and 2G ReBCO materials, the largest sizes for these two classes of HTS superconductors are listed in Tables 4.1 and 4.2, respectively [6], calculated by using the thermo-magnetic stabilization criterion Equation (4.11).

Table 4.1 Sizes of Bi-2223 in temperature range of 20 K through 80 K

	Critical current I_c	Critical current density J_c	Heat capacity C	Critical size a_c
Temperature T/K	A	MA·m ⁻²	kJ·kg ⁻¹ ·K ⁻¹	mm
4	228	1932	0.8	0.2
10	217	1839	7.7	0.7
20	194	1644	68.5	2.4
30	163	1381	240	5.5
40	135	1144	534	9.8
50	108	915	881	16
60	80	678	1219	25
70	53	449	1540	43
80	24	203	1825	103

Table 4.2 Sizes of YBCO at temperatures of 20 K and 77 K

	20 K	77 K
Critical current density $J_c/(\text{A}\cdot\text{m}^{-2})$	1.0×10^{10}	1.0×10^9
Resistivity $\rho/(\Omega\cdot\text{m})$	1.5×10^{-6}	5.0×10^{-6}
Thermal conductivity $k/(\text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1})$	1.8	6.7
Volumetric heat capacity $\gamma C/(\text{J}\cdot(\text{m}^{-3}\cdot\text{K}^{-1}))$	0.86×10^6	0.96×10^6
Critical size a_c/mm	0.4	4.95

From Table 4.1, the critical size a_c of Bi-2223 is 43 mm at temperature 77 K and 103 mm at 80 K, respectively, several orders more than LTS, such as NbTi and Nb₃Sn. Table 4.2 shows that the critical size of ReBCO materials at temperature 77 K is 4.95 mm and at 20 K is 4 mm; the former is one order higher in magnitude than the latter. Consequently, the higher the temperature, the more adiabatic stability can be improved.

At present, the width of commercial 1G HTS and 2G HTS tapes is less than or equal to 10 mm, and their thickness is less than 0.5 mm. According to the adiabatic stability criterion, they are stable if operating at 77 K, and a superconductor with a single core can still satisfy the requirement of adiabatic stability. Of course, when comprehensively considering other factors, such as alternating current (AC) losses and mechanical properties, 1G HTS Bi-2223 wire with a single core cannot meet the requirements of practical applications. Practical multifilamentary Bi-2223 wire is usually processed by the powder-in-tube (PIT) technique.

The volumetric heat capacity of HTS wire is about two orders higher in magnitude than the conventional LTS at the temperature of 77 K and increases rapidly with temperature rise. Temperature rise may hinder occurrence of flux jumping. Therefore, HTS has a better adiabatic stabilization than LTS, due to its wide range of operating temperatures.

4.4 Self-Field Stability

In Section 4.2, thermo-magnetic stability is considered for the superconductor without carrying transport current in an external magnetic field. Nevertheless, even without an external magnetic field, practical superconductors always carry transport current that can produce a magnetic field, the so-called self field. In this section, the thermo-magnetic stability in the self field is presented. Assume that the cross-section of a superconductor is circular (Figure 4.6) [2], and its radius is a .

When the superconductor transports current I , magnetic fields produced at its surface and inside are:

$$B(r) = \begin{cases} \frac{a}{r}B_0 - \frac{\mu_0\lambda J_c}{2r}(a^2 - r^2) & (0 < r \leq a) \\ \frac{\mu_0 I}{2\pi r} & (r \geq a) \end{cases} \quad (4.13)$$

where B_0 is the magnetic field at the surface, $B_0 = \mu_0 I/(2\pi a)$, and λ is the filling factor, that is, the ratio of the superconductor's volume to the total volume. The superconductor in this section is a pure superconductor, so its filling factor $\lambda = 1$. In order to conveniently discuss the stability of a composite conductor in the self field, thereafter the filling factor λ is still formally inserted before the critical current density J_c . The superconductor has no heat exchange with its surroundings, and thermo-magnetic stability of the self field is described by adding a heat source ΔQ_s to the superconductor. It is assumed that the added heat and generated heat inside the superconductor are uniformly distributed across the whole cross-section. The superconductor has a temperature rise ΔT because of adding heat ΔQ_s , which results in the critical current density to decrease ΔJ_c . Flux then enters the inner part of the superconductor,

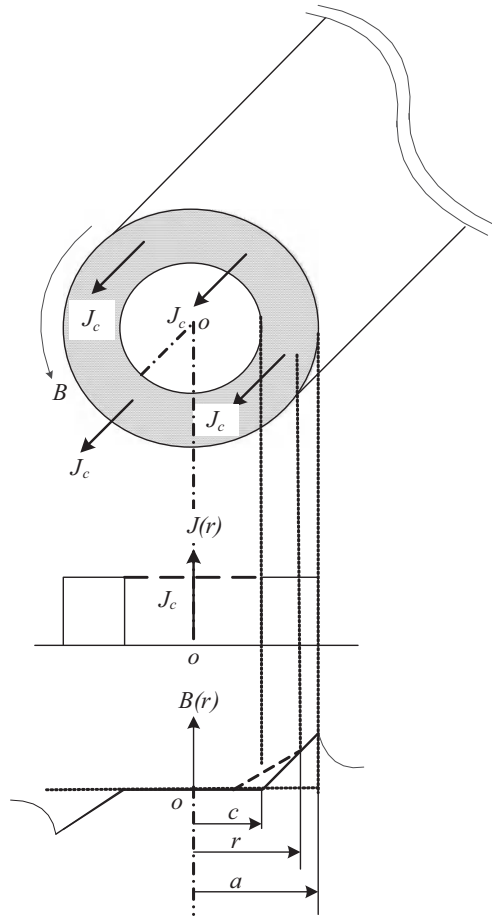


Figure 4.6 Distribution of magnetic field and current in superconductor with circular cross-section and transport current I .

When critical current density J_c reduces by ΔJ_c , the change in flux in the region of radius r is:

$$\Delta\varphi(r) = \int_c^r \Delta B(r) dr = \frac{\mu_0 \lambda \Delta J_c}{2} \left(a^2 \ln \frac{r}{c} - \frac{r^2}{2} + \frac{c^2}{2} \right) \quad (4.14)$$

where c is the difference between radius of the superconductor and depth of current penetrating the surface of the superconductor, which is determined by $I = \lambda J_c \pi (a^2 - c^2)$.

The energy per unit volume generated by changing flux is described by:

$$\begin{aligned} \Delta Q &= \frac{1}{\pi a^2} \int_0^a \Delta\varphi(r) \lambda J_c 2\pi r dr \\ &= \mu_0 \lambda^2 J_c \Delta J_c a^2 \left(-\frac{1}{2} \ln \varepsilon - \frac{3}{8} + \frac{\varepsilon^2}{2} - \frac{\varepsilon^4}{8} \right) \end{aligned} \quad (4.15)$$

where $\varepsilon = c/a$. As with the change of the critical current density expressed in Section 4.3, substituting Equation (4.3) into Equation (4.15) gives the heat balance equation:

$$\Delta Q_s + \frac{\mu_0 \lambda^2 J_c^2 a^2}{T_c - T_b} \Delta T \left(-\frac{1}{2} \ln \varepsilon - \frac{3}{8} + \frac{\varepsilon^2}{2} - \frac{\varepsilon^4}{8} \right) = \gamma C \Delta T \quad (4.16)$$

The effective volumetric heat capacity γC_e is:

$$\gamma C_e = \frac{\Delta Q_s}{\Delta T} = \gamma C - \frac{\mu_0 \lambda^2 J_c^2 a^2}{T_c - T_b} \left(-\frac{1}{2} \ln \varepsilon - \frac{3}{8} + \frac{\varepsilon^2}{2} - \frac{\varepsilon^4}{8} \right) \quad (4.17)$$

When the effective volumetric heat capacity goes to zero, that is, $\gamma C_e = 0$, flux jump occurs. A stabilization parameter is defined as β_t :

$$\beta_t = \frac{\mu_0 \lambda^2 J_c^2 a^2}{\gamma C (T_c - T_b)} = \left(-\frac{1}{2} \ln \varepsilon - \frac{3}{8} + \frac{\varepsilon^2}{2} - \frac{\varepsilon^4}{8} \right)^{-1} \quad (4.18)$$

If i , also called the normalized the transport current, is defined as the ratio of transport current to the critical current, then:

$$i = \frac{I}{I_c} = \frac{\lambda J_c \pi (a^2 - c^2)}{\lambda J_c \pi a^2} = 1 - \frac{c^2}{a^2} = 1 - \varepsilon^2 \quad (4.19)$$

Substituting Equation (4.19) into Equation (4.18):

$$\beta_t = \frac{\mu_0 \lambda^2 J_c^2 a^2}{\gamma C (T_c - T_b)} = \left(-\frac{1}{4} \ln(1 - i) - \frac{3}{8} + \frac{(1 - i)}{2} - \frac{(1 - i)^2}{8} \right)^{-1} \quad (4.20)$$

When the transport current I approaches a value less than the critical current I_c , $i \rightarrow 1-$, Equation (4.20) tends to 0, that is, $\beta_t \rightarrow 0+$, which are completely different from the situation of the screen current induced by the external magnetic field (Section 4.3), because the transport current must remain constant in the instable conditions of the self field. When the transport current is close to the critical current I_c , a slight rise in temperature will be sufficient to cause $I_c(T_b + \Delta T) < I_t$, then the superconductor quenches and cannot return to its superconducting state. Figure 4.7 shows the dependence of stabilization parameter β_t in the self field on the normalized transport current. Clearly, when normalized transport current i is close to 1, stabilization parameter β_t tends to zero, $\beta_t \rightarrow 0$.

To qualitatively understand the self-field stability of a superconductor, the NbTi composite conductor is taken as an example. The critical current density of the NbTi composite conductor is $J_c = 1.5 \times 10^9$ A/m² in magnetic field $B = 6$ T and temperature $T = 4.2$ K, the radius of superconducting filament is $a = 0.25$ mm, its filling factor is $\lambda = 0.4$, volumetric heat capacity is $\gamma C = 2.7 \times 10^3$ J/(m³·K), critical temperature is $T_c = 6.5$ K, and the environment or bath temperature is $T_b = 4.2$ K. Substituting all these parameters into Equation (4.18), $\beta_t = 4.5$ is obtained. However, under adiabatic conditions, the corresponding normalized transport current is $i = 0.89$ (Figure 4.7), at which point flux jump of the superconducting filament will take place. If further improvements on ability of transport current and prevention of thermo-magnetic instability are required, other stabilization methods, such

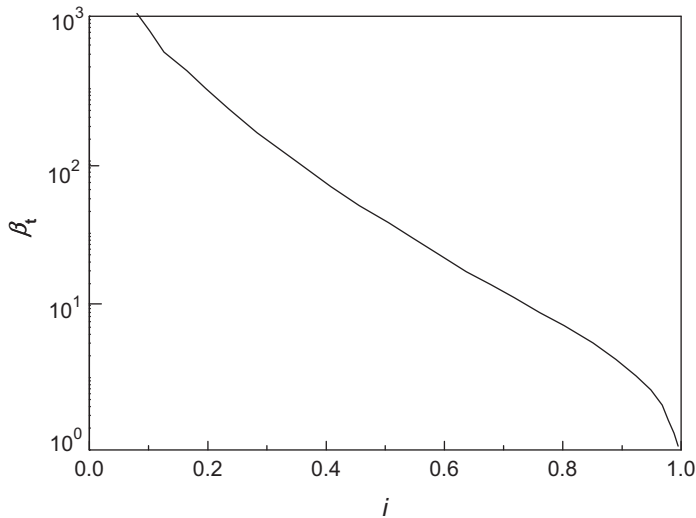


Figure 4.7 Dependence of self-field stable parameter β_t on normalized transport current i .

as dynamic stabilization, enthalpy stabilization and cryogenic stabilization methods, should be adopted simultaneously.

4.5 Dynamic Stability

Only the thermo-magnetic characteristics of the superconductor itself are considered in theories of adiabatic stabilization with flux jump, in which there is no heat exchange with the surroundings and heat transfer is also neglected in the process of flux motion. These assumptions are a good approximation for most superconducting materials. However, the cost is high for processing superconducting filaments of tens of micrometers in diameter in practical engineering applications, and also the technique is complicated and difficult. No matter whether they are HTS or LTS, composite superconducting filamentary conductors are fabricated by embedding superconducting multi-filaments into stabilization materials with low resistivity and high thermal conductivity, which can increase the heat exchange between the superconductor and the surrounding stabilizing matrix materials. Thus, the heat is transferred from the superconductor of which the thermal stability can be improved.

The processing techniques of practical superconducting wire will be mentioned in Chapter 6, and they have been developed to improve thermo-magnetic and dynamic stability. These stabilization matrix materials may hinder flux motion. They can greatly reduce the released heat on one hand, and increase the ability of dissipating heat on the other. In general, the stabilizing matrix materials with low resistivity and high thermal conductivity refer to copper, aluminium, silver or alloys. Their thermal diffusion velocity is fast but magnetic diffusion velocity is slow. However, the superconductor has higher magnetic diffusion velocity but lower thermal diffusion velocity. If both of them are combined, velocity of heat transferring into the surrounding environment will increase and meanwhile velocity of magnetic diffusion will decrease. If the rate of generated heat in the superconductor is smaller than that of transferring heat from the superconductor due to flux motion or heat disturbance, the flux jump will be hindered, which will ensure thermo-magnetic stability of superconductor. This is the principle of the dynamic stabilization method in a superconductor [7–9].

Table 4.3 Thermal diffusivity, magnetic diffusivity and thermoelectricity of some HTSs and LTSs at 4.2 K

Materials	Thermal conductivity k W/(m·K)	Volumetric heat capacity γC J/(m ³ ·K)	Resistivity ρ $\Omega \cdot m$	Thermal diffusivity D_T m ² /s	Magnetic diffusivity D_m m ² /s
NbTi	1.1×10^{-1}	5.4×10^3	6.0×10^{-7}	2.0×10^{-5}	4.8×10^{-1}
Nb ₃ Sn	4×10^{-2}	1.2×10^3	7.0×10^{-7}	3.3×10^{-5}	5.6×10^{-1}
Bi-2212	0.5	2.2×10^3	1.0×10^{-6}	2.3×10^{-4}	8.0×10^{-1}
Bi-2223	0.1	1.65×10^3	1.0×10^{-6}	6.0×10^{-5}	8.0×10^{-1}
YBCO(77 K)	10	1.0×10^6	1.0×10^{-6}	1.0×10^{-5}	8.0×10^{-1}
Cu(B = 2 T)	1.0×10^3	6.2×10^3	2.0×10^{-10}	3.0×10^{-1}	1.6×10^{-4}
Cu(B = 6 T)	2.6×10^2	6.2×10^3	4.2×10^{-10}	2.3×10^{-1}	3.3×10^{-4}
Ag	1.15×10^3	1.58×10^3	1.48×10^{-10}	7.0×10^{-1}	1.2×10^{-4}

If the temperature, current density and magnetic field are time variable, the constitution equation of a superconductor for describing diffusion of heat and magnetic field and current density are, respectively:

$$\begin{cases} \nabla^2 T = \frac{1}{D_T} \frac{\partial T}{\partial t} \\ \nabla^2 B = \frac{1}{D_m} \frac{\partial B}{\partial t} \\ \nabla^2 J = \frac{1}{D_m} \frac{\partial J}{\partial t} \end{cases} \quad (4.21)$$

where $D_T = k/(\gamma C)$ and $D_m = \rho/\mu_0$ are heat diffusivity and (electro-) magnetic diffusivity, respectively; k is heat conductivity and γC is volumetric heat capacity. The half-infinite superconductor slab with a width of $2a$ was introduced in Section 4.3. Equation (4.21) can be solved by the method of separation of variables; their solutions have the same form:

$$\begin{Bmatrix} T(x) \\ B(x) \\ J(x) \end{Bmatrix} = \sum_{n=0}^{\infty} A_n \sin\left(\frac{n\pi x}{2a}\right) \exp\left(-\frac{n^2 \pi^2 D_i}{4a^2} t\right) \quad (4.22)$$

The dominant terms on the right-hand side of Equation (4.22) will decay with time, and its characteristic time constant τ_i is defined as:

$$\tau_i = \frac{4a^2}{\pi^2 D_i} \quad (4.23)$$

In Equations (4.22) and (4.23), D_i represents thermal diffusivity or magnetic diffusivity and τ_i is the characteristic time constant of thermal diffusion or magnetic diffusion. Table 4.3 gives the thermal diffusivity, magnetic diffusivity and thermoelectricity for commonly used HTS and LTS at 4.2 K [8].

4.5.1 Stability of Composite Superconducting Slab with Cooled Side

Consider a composite superconductor with a geometrical structure, as shown in Figure 4.8, of which the central part with width $2a$ is a superconductor, and both sides with width d are the normal conductor used as copper stabilization material. The widest surface of copper stabilization directly contacts with the cryogen, and it is assumed that the transport current is close to the critical current I_c . In order to study the stability of the composite superconducting slab, a heat pulse with energy ΔQ_s per unit volume is

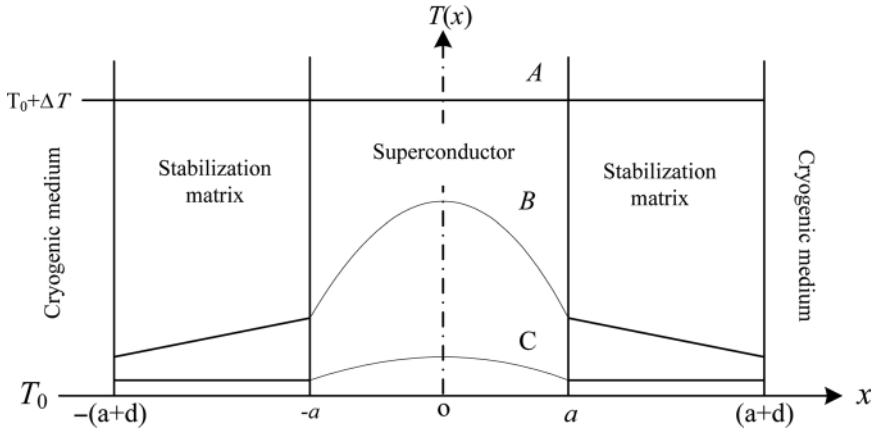


Figure 4.8 Geometrical structure and temperature distribution in wide edge-cooled composite superconductor.

applied to the composite superconducting slab, which causes a uniform temperature rise ΔT , which then leads to a reduction of the critical current density ΔJ_c . Part of the transport current will transfer from the superconductor to the stabilization material. If the generated heat is less than that transferred to the cryogen in the process of transferring current, temperature rise ΔT begins to decrease, finally reducing to zero. This process is shown by curves A, B and C in Figure 4.8 [7]. After the heat pulse disturbance, the temperature of copper rapidly recovers because thermal conductivity of copper is much greater than the superconductor. Recovery of the superconductor is mainly determined by thermal diffusivity of the superconductor. Assuming that the temperature rise ΔT causes the reduction of critical current density J_c by ΔJ_c , the reduced current enters the bypass of copper stabilization materials, so that an electrical field E is generated as:

$$E = \frac{\lambda}{1 - \lambda} \rho_m \Delta J_c \quad (4.24)$$

where λ is the filling factor, that is, the percentage of the superconductor in the entire composite conductor, $\lambda = a/(a + d)$; the expression of ΔJ_c is the same as Equation (4.3), and ρ_m is the resistivity of the stabilization matrix. Then the heat generation power of composite conductor per unit volume is:

$$G = \lambda J_c E \quad (4.25)$$

Substituting Equations (4.3) and (4.24) into Equation (4.25), we obtain:

$$G = \frac{\lambda^2}{1 - \lambda} \rho_m J_c^2 \frac{\Delta T}{T_c - T_b} \quad (4.26)$$

Because stabilization matrix copper directly contacts with the cryogen, the heat power per unit volume transferring to the cryogen is:

$$P = \frac{ph\Delta T}{A} \quad (4.27)$$

where p , h and A are the cooling perimeters of a composite superconductor, the heat transfer coefficient between composite conductor and cryogen, and the cooling cross-section area, respectively.

Temperature distribution inside the copper stabilization matrix is uniform due to its high thermal conductivity. Based on the heat balance Equation (4.1), we obtain:

$$\gamma C \frac{\partial \Delta T}{\partial t} = \frac{\lambda^2}{1-\lambda} \rho J_c^2 \frac{\Delta T}{T_c - T_b} - \frac{ph \Delta T}{A} \quad (4.28)$$

where γ and C are density and specific heat capacity of the copper stabilization material, respectively.

By considering the initial condition at the moment $t = 0$, the temperature rise ΔT is ΔT_0 , and the solution of Equation (4.28) is:

$$\Delta T = \Delta T_0 \exp(-\beta_s t) \quad (4.29)$$

where parameter β_s is defined as the stable parameter:

$$\beta_s = \frac{1}{\gamma C} \left[\frac{p}{A} h - \frac{\lambda^2 \rho_m J_c^2}{(1-\lambda)(T_c - T_b)} \right] \quad (4.30)$$

If the stable parameter β_s is larger than zero, $\beta_s > 0$, temperature rise ΔT will exponentially decay and become zero. The temperature returns to its original balanced state and the superconductor is stable. According to the condition of $\beta_s > 0$, the stable criterion of the composite conductor is then:

$$\alpha = \frac{\rho_m A \lambda^2 J_c^2}{ph(1-\lambda)(T_c - T_b)} \leq 1 \quad (4.31)$$

Equation (4.31) is also called the Stekly criterion of cryogenic stabilization, in which α is defined as the Stekly parameter, which is determined by temperature margin, heat transfer coefficient and cooling perimeter.

Since thermal conductivity of the superconductor is much lower than that of the copper stabilization matrix, the temperature distribution is not uniform inside the superconductor, that is, the temperature gradient is not zero. The thermal power per unit volume generated in the superconductor is approximately:

$$G_s = J_c E = \frac{\lambda}{1-\lambda} \rho_m J_c^2 \frac{\Delta T}{T_c - T_b} \quad (4.32)$$

assuming that the transport current density J of the superconductor is close to the critical current J_c .

When considering the non-uniform distribution of temperature in the superconductor, the heat transfer equation with variable temperature then is:

$$\gamma_s C_s \frac{\partial \Delta T}{\partial t} = \frac{\partial}{\partial x} \left(k_s \frac{\partial \Delta T}{\partial x} \right) + \frac{\lambda}{1-\lambda} \rho_m J_c^2 \frac{\Delta T}{T_c - T_b} \quad (4.33)$$

where γ_s , C_s and k_s are the density, thermal conductivity and the specific heat capacity, respectively.

Supposing that the solution of Equation (4.33) has the following form:

$$\Delta T(x) = \sum_{n=0}^{\infty} b_n \exp(-a_n t) \cos \left(\frac{(2n+1)\pi x}{a} \right) \quad (4.34)$$

Substituting Equation (4.34) into Equation (4.33), the coefficient a_n in the exponential decay term is:

$$a_n = \frac{1}{\gamma C_s} \left[\frac{(2n+1)^2 \pi^2}{a^2} k_s - \frac{\lambda \rho_m J_c^2}{(1-\lambda)(T_c - T_b)} \right] \quad (4.35)$$

If $a_n > 0$, the temperature inside the superconductor decays with time and is able to return to the stable state. Since the first term ($n = 0$) in the series solution is dominant, only this term is taken into account:

$$a_0 = \frac{1}{\gamma_s C_s} \left(\frac{\pi^2}{a^2} k_s - \frac{\lambda \rho_m J_c^2}{(1-\lambda)(T_c - T_b)} \right) \quad (4.36)$$

If $a_0 > 0$, temperature rise ΔT of the superconductor will decay with time and the superconductor becomes stable. According to Equation (4.36), the half thickness of the superconductor must satisfy:

$$a < \pi \sqrt{\frac{k_s (T_c - T_b) (1 - \lambda)}{\lambda J_c^2 \rho_m}} \quad (4.37)$$

Equation (4.37) represents the dynamic stabilization criterion of a composite superconductor, and is determined by a combination of thermal conductivity k_s of the superconductor, resistivity of matrix material resistivity ρ and temperature margin $(T_c - T_b)$. The less the resistivity ρ_m , the larger the superconductor size. It should be noted that thermal conductivity is not independent of resistivity for metallic materials, and both of them follow the Wiedemann–Franz Law:

$$k \rho_m = L_0 T \quad (4.38)$$

where L_0 is the Lorentz constant, and $L_0 = \pi^2 k_B^2 / (3e^2) = 2.45 \times 10^{-8} (\text{W} \cdot \Omega) / \text{K}^2$, in which k_B and e are the Boltzman constant and electron charge, respectively.

Taking the composite superconducting wire NbTi/Cu as an example, its critical current density J_c at temperature $T_b = 4.2$ K and magnetic field $B = 2$ T is 5×10^9 A/m² [10], the thermal conductivity is $k_s = 0.11$ W/(m·K), critical temperature is $T_c = 8.2$ K, filling factor is $\lambda = 0.4$, and resistivity of copper is $\rho_m = 2 \times 10^{-10}$ Ω·m. By substituting all of these parameters into Equation (4.37), we obtain a result of $a < 59$ μm, which is similar to the adiabatic stabilization size discussed in Section 4.3. We notice that the adiabatic stabilization criterion is determined by the volumetric specific capacity of the superconductor in order to avoid flux jump, whereas the dynamic stabilization criterion, discussed in this section, depends not only on the thermal conductivity of the superconductor but also on the resistivity of stabilization material. Equation (4.36) is obtained by choosing the superconducting slab as the calculation model. If the cross-section of the superconductor is circular, Equation (4.37) is also suitable with certain modification and correction. For example, the radius of superconducting filament is a_f , whose relationship with half-thickness a of the superconducting slab is:

$$a_f < \sqrt{8}a \text{ (filament)} \quad (4.39a)$$

$$a_f < \sqrt{3}a \text{ (slab)} \quad (4.39b)$$

Two stable criteria may be used to calculate the superconductor size, that is, the flux jump stabilization criterion Equation (4.11) and the dynamic stabilization criterion Equation (4.37). If the two results are different according to these two criteria, the smaller one should be taken as the optimal size. Of course, that will result in more expensive fabrication costs and a stricter demand on processing techniques.

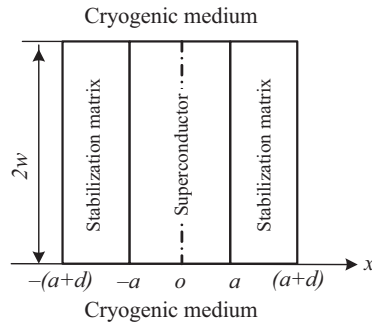


Figure 4.9 Cross-section of composite superconductor with edge-cooled slab.

4.5.2 Stability of Composite Superconducting Slab with Cooled Edge

The geometrical cross-section of the composite superconducting slab with a cooled edge is shown in Figure 4.9, in which the central part with thickness $2a$ is the superconductor, both side parts with thickness d are the stabilization matrix, that is, a normal conductor copper, and these three parts have the same width of $2w$. The edges of the copper matrices and the superconductor are directly in contact with the cryogen, when assuming that the transport current I is close to its critical current I_c . A pulse heat per unit volume ΔQ_s is applied to the composite superconducting slab to study its stability, which causes a uniform temperature rise ΔT across its cross-section and consequently results in reduction of critical current density ΔJ_c . The heat generated in a composite superconducting slab is given by Equation (4.7). By assuming that resistivity of the stabilization matrix is ρ_m and resistivity of the superconductor in normal state is ρ_n , then the effective resistivity of the composite superconducting slab is:

$$\frac{1}{\rho_{\text{eff}}} = \frac{1 - \lambda}{\rho_m} + \frac{\lambda}{\rho_n} \quad (4.40)$$

where λ is the filling factor. Since matrix materials are usually metals or alloys with high thermal conductivity and low resistivity, such as copper, aluminium, silver, etc., their resistivity is much less than that of a superconductor in the normal state, that is, $\rho_n \geq \rho_m$, thus:

$$\rho_{\text{eff}} \approx \frac{\rho_m}{1 - \lambda} \quad (4.41)$$

The characteristic time constant of magnetic diffusion is:

$$\tau = \frac{4w^2}{\pi^2 D} = \frac{4w^2}{\pi^2 (\rho_{\text{eff}} / \mu_0)} = (1 - \lambda) \frac{4\mu_0 w^2}{\pi^2 \rho_m} \quad (4.42)$$

The heat balance equation of the composite superconductor is:

$$\Delta Q_s + \lambda \frac{\mu_0 J_c^2 w^2}{3(T_c - T_b)} \Delta T - \frac{h\tau}{w} \Delta T = \gamma C \Delta T \quad (4.43a)$$

Substituting Equation (4.42) into Equation (4.43a), we obtain:

$$\Delta Q_s + \lambda \frac{\mu_0 J_c^2 w^2}{3(T_c - T_b)} \Delta T - (1 - \lambda) \frac{4\mu_0 wh}{\pi^2 \rho_m} \Delta T = \gamma C \Delta T \quad (4.43b)$$

where the third term on the left-hand side of the equation represents the heat transfer to the cryogen, which directly contacts with both edges of a composite conductor, so that the heat value is taken as negative; h is the heat transfer coefficient between the composite superconductor and the cryogen. The effective volumetric heat capacity γC_e of the composite superconductor is given by:

$$\gamma C_e = \frac{\Delta Q_s}{\Delta T} = \gamma C - \lambda \frac{\mu_0 w^2 J_c^2}{3(T_c - T_b)} + (1 - \lambda) \frac{4\mu_0 wh}{\pi^2 \rho_m} \quad (4.44)$$

If the composite superconductor is stable, γC_e must be larger than zero [11]. The critical half-width of the composite superconductor can be obtained by solving Equation (4.44):

$$w_c = \sqrt{\frac{3\gamma C(T_c - T_b)}{\lambda \mu_0 J_c^2} + \left[\left(\frac{1 - \lambda}{\lambda} \right) \frac{6(T_c - T_b)h}{\pi^2 \rho_m J_c^2} \right]^2} + \left(\frac{1 - \lambda}{\lambda} \right) \frac{6(T_c - T_b)h}{\pi^2 \rho_m J_c^2} \quad (4.45)$$

Comparing Equations(4.45) to Equation (4.11), the first term in the square root of Equation (4.45) with $\lambda = 1$ is the same as the right-hand side of Equation (4.11), which represents the adiabatic term. The second term represents the cooled term. According to Equation (4.45), we avoid flux jump and gain stable parameter β_s :

$$\beta_s = \frac{\mu_0 \lambda^2 J_c^2 w^2}{\gamma C(T_c - T_b)} = 3 \left(1 + \frac{4\nu}{\pi^2} \right) \quad (4.46)$$

where the dimensionless parameter ν , the so-called heat transfer factor, is given by:

$$\nu = \frac{hw}{\gamma C D_m} = \frac{hw\mu_0(1 - \lambda)}{\rho_m \gamma C} \quad (4.47)$$

The processing method of dynamic stabilization in this section is especially suitable for windings or coils with the structure of double pancakes (DP) wound by Nb₃Sn tapes and HTS tapes.

Taking composite Nb₃Sn superconducting tapes as an example, width $2w$ and thickness $2a$ of Nb₃Sn are 5 mm and 15 μ m, respectively, thickness of copper matrix $2d$ is 135 μ m, critical current density J_c is 2×10^9 A/m² in the magnetic field $B = 2$ T, and temperature $T = 4.2$ K. The critical temperature T_c is 16 K, the averaged volumetric heat capacity γC and heat transfer coefficient h are 10^3 J/(m³·K) and 5×10^4 W/(m²·K), respectively. Parameter $\nu = 707$ can be found by substituting these quantities in Equation (4.47), and the parameter $\beta_s = 288$ is obtained by substituting $\nu = 707$ into Equation (4.46). By comparing the results in Section 4.3, the adiabatic stabilization parameters β_s increase up to $\beta_s = 288$ from $\beta_s = 3$ by transferring heat to the cryogen. According to the above parameters, the critical current density J_c , determined by the adiabatic stabilization condition of the composite superconductor is increased to $J_c = 3.3 \times 10^9$ A/m² from $J_c = 3.4 \times 10^8$ A/m² by recalculating Equations (4.10) and (4.46). Its practical current carrying ability is significantly improved.

4.5.3 Dynamic Stability of Current-Carrying Composite Superconductor Slab

In Section 4.3, we showed that the adiabatic stability criterion can be obtained by adopting the adiabatic stabilization method when a single superconducting slab is in a magnetic field parallel to its widest face. However, the magnetic field is not always perpendicular to the superconducting wide face in practical applications, and superconducting tapes are usually closely wound by the tight winding method. Therefore, the flux jump in a set of superconducting slabs or tapes and perpendicular magnetic field is completely unstable under adiabatic conditions. There are often cooling channels along the edge of superconducting slabs or tapes in a real situation, which allows the cryogen to pass through conveniently; or metal slices with high heat conductivity and low resistivity are inserted between the slabs or tapes and in direct contact with the cryogen to improve the cooling environment.

To discuss the dynamic stability of a set of current-carrying superconducting slabs, we assume that there is a set of current-carrying superconducting slabs, each with width $2w$ and thickness $2a$, and the magnetic field is perpendicular to their widest face (Figure 4.10). Figures 4.10(a) and (b) show a geometrical cross-section of a slab in a perpendicular magnetic field and magnetic field distribution across a cross-section without transport current $I = 0$, respectively. Similarly, Figures 4.10(c) and (d) are the geometrical cross-section and magnetic field distribution with transport current $I \neq 0$.

It is also assumed that a heat pulse ΔQ_s per unit volume is applied to the superconducting composite slabs, and the temperature rises by ΔT and critical current density decreases by ΔJ_c , which gives a current of $\lambda \Delta J_c / (1 - \lambda)$ flowing through a metal stabilization material bypass, then Joule heat produced in the metal stabilization material transfers into the cryogen. When there is no transport current, that is, $I = 0$, the distribution of penetrating flux is symmetrical about the centre line (Figure 4.10(b)). Nevertheless, when there is transport current I flowing through the composite slabs, the flux distribution is no longer symmetrical, and flux lines move towards the left from point O to point O' (Figure 4.10(d)).

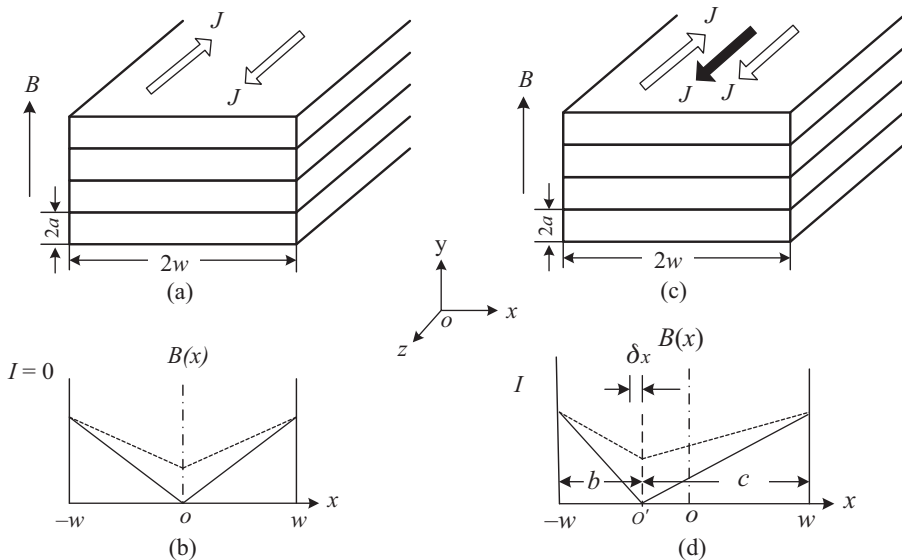


Figure 4.10 Geometrical cross-section and magnetic field distribution in slabs with and without transport current in perpendicular magnetic field: (a) geometrical cross-section without transport current; (b) magnetic field distribution without transport current; (c) geometrical cross-section with transport current; (d) magnetic field distribution with transport current.

If the critical current of the superconductor is I_c , and the normalized transport current is defined as $i = I/I_c$, the two widths of the left- and right-hand sides become b and c , respectively. It is easy to prove that both of them are:

$$\left. \begin{aligned} b &= (1 - i)w \\ c &= (1 + i)w \end{aligned} \right\} \quad (4.48)$$

Referring to the same method mentioned in Section 4.3, the energy consumption per unit volume on both sides is:

$$\Delta Q = \frac{1}{4wa} \int_0^b \Delta \varphi(r) \lambda J_c 2a dx + \frac{1}{4wa} \int_0^c \Delta \varphi(r) \lambda J_c 2a dx \quad (4.49)$$

Substituting Equations (4.5) into (4.49), we obtain:

$$\Delta Q = \frac{\mu_0 \lambda^2 J_c^2 w^2 \Delta T}{6 (T_c - T_b)} [(1 - i)^3 + (1 + i)^3] = \frac{\mu_0 \lambda^2 J_c^2 w^2 \Delta T}{3 (T_c - T_b)} (1 + 3i^2) \quad (4.50)$$

Considering the size of two widths, as shown in Equation (4.48), there should be two characteristic time constants according to Equation (4.23). As a reasonable and conservative approximation, the characteristic time constant for current decay is dominated by the larger one-dimensional (1D) c of Equation (4.48), and then the characteristic time constant is:

$$\tau_s = \frac{4w^2}{\pi^2 D_m} (1 + i)^2 = \frac{4\mu_0 w^2}{\pi^2 \rho_n} (1 - \lambda) (1 + i)^2 \quad (4.51)$$

where ρ_n is resistivity of the superconducting slab after quench.

As with Equation (4.42), the heat balance equation of the composite superconductor is:

$$\Delta Q_s + \frac{\mu_0 \lambda^2 J_c^2 w^2 \Delta T}{3 (T_c - T_b)} (1 + 3i^2) - (1 - \lambda) (1 + i)^2 \frac{4\mu_0 wh}{\pi^2 \rho_n} \Delta T = \gamma C \Delta T \quad (4.52)$$

Then the effective volumetric heat capacity is:

$$\gamma C_e = \frac{\Delta Q_s}{\Delta T} = \gamma C - \frac{\mu_0 \lambda^2 J_c^2 w^2 \Delta T}{3 (T_c - T_b)} (1 + 3i^2) - (1 - \lambda) (1 + i)^2 \frac{4\mu_0 wh}{\pi^2 \rho_n} \Delta T \quad (4.53)$$

To assure stability of the superconductor, volumetric heat capacity γC_e must be larger than zero, and the stabilization parameter β_s is:

$$\beta_s = \frac{\mu_0 \lambda^2 J_c^2 w^2}{\gamma C (T_c - T_b)} = \frac{3}{1 + 3i^2} \left[1 + \frac{4}{\pi^2} (1 + i)^2 \nu \right] \quad (4.54)$$

where ν is the same as in Equation (4.46). Figure 4.11 shows the relationship of stable parameter β_s and the normalized transport current i . For different values of parameter ν , the increase of transport current will lead to the decrease of stable parameter β_s . If the composite superconductor has better ability of heat exchange with its surroundings, when $\nu > 10$, stable parameter β_s is almost independent of the normalized current i . At this point, the ratio of β_s to ν is approximately a constant:

$$\frac{\beta_s}{\nu} = \frac{\lambda^2 J_c^2 \rho_m}{h (T_c - T_b) (1 - \lambda)} \approx \frac{12}{\pi^2} \approx 1.216 \quad (4.55)$$

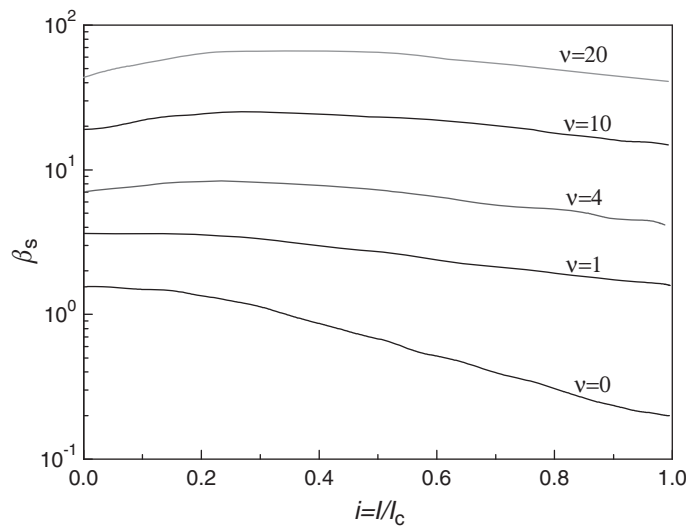


Figure 4.11 Dependence of stabilization parameter β_s on normalized transport current i with different heat transfer factor ν .

When ν is relative small, the stable parameter β_s with transport current can be reduced to one-quarter of its original value; while $\nu = 0$ and I approaches to 1, the stable parameter β_s tends to zero (Figure 4.7). However, when $\nu = 0$, $i \rightarrow 1$, and the stable parameter β_s tends to 0.35 (Figure 4.11). This is because we did not consider the central line reverting and moving to the left with distance δx caused by temperature rise ΔT in calculating Equation (4.46) (Figure 4.10). When i is close to 1, point O moves across the boundary. Thus, the current-carrying capacity of the superconductor is lower than the transport current I , which leads to Joule heat being generated continuously. If there is no cryogenic cooling, that is, $\nu = 0$, superconductor quench will occur.

4.5.4 Dynamic Stability of Current-Carrying Composite Superconductor with Circular Cross-Section

In Section 4.4, the adiabatic stability of a current-carrying superconductor with circular cross-section in the self field was discussed. Now we will generalize the results of dynamic stability. The schematic view is shown as Figure 4.6. When a heat pulse ΔQ_s per unit volume is supplied to the superconductor, its temperature rises by ΔT and the critical current density decreases by ΔJ_c , which makes the transport current $\lambda \Delta J_c (1 - \lambda)$ flow through the metal stabilization matrix bypass, in which λ is the filling factor, and then the heat produced in the metal stabilization matrix material transfers into the cryogen. The radius of the circular region across which the transport current flows is r , $c < r < a$; here a and c are the same as in Section 4.4. Taking the symmetry of the superconductor into account, cylindrical coordinates are used to analyse its dynamic stability. Assuming that the transport current flows along the z -axis, components of the current density in radius and circumference are zero, that is, $J_r = J_\varphi = 0$, according to thermal-magnetic diffusive Equation (4.21), we have:

$$\frac{\partial^2 J_z}{\partial r^2} + \frac{1}{r} \frac{\partial J_z}{\partial r} = \frac{1}{D_m} \frac{\partial J_z}{\partial t} \quad (4.56)$$

Letting the solution of Equation (4.56) be:

$$J_z(r, t) = J(r) \exp\left(-\frac{D_m a^2}{\alpha^2} t\right) \quad (4.57)$$

where α is a constant.

Substituting Equations (4.57) into (4.56), we obtain:

$$\frac{d^2 J}{dr^2} + \frac{1}{r} \frac{dJ}{dr} + \frac{a^2}{\alpha^2} J = 0 \quad (4.58)$$

Clearly, Equation (4.58) is a zero-order Bessel equation; its general solution is:

$$J(r) = AJ_0\left(\frac{\alpha r}{a}\right) + DY_0\left(\frac{\alpha r}{a}\right) \quad (4.59)$$

where J_0 and Y_0 are the Bessel functions of the first and second kind, respectively, and A and D are constants.

By using boundary conditions:

$$\begin{aligned} E = \rho J = 0 & \quad r = c \\ \frac{\partial B}{\partial t} = -\nabla \times E = \rho \frac{dJ}{dr} = 0 & \quad r = a \end{aligned} \quad (4.60)$$

constants A and D can be determined by substituting these boundary conditions into Equation (4.59), the solution of which can be simulated. The characteristic time-constant τ_j is acquired from Equation (4.57):

$$\tau_j = \frac{a^2}{\alpha^2 D_m} \quad (4.61)$$

Taking Equation (4.60) into consideration and adding the heat transfer term to Equation (4.16), we obtain:

$$\begin{aligned} \Delta Q_s + \frac{\mu_0 \lambda^2 J_c^2 a^2}{T_c - T_b} \Delta T \left(-\frac{1}{2} \ln \varepsilon - \frac{3}{8} + \frac{\varepsilon^2}{2} - \frac{\varepsilon^4}{8} \right) - \frac{2\pi a}{\pi a^2} h \tau_j \Delta T &= \gamma C \Delta T \\ \gamma C_e = \frac{\Delta Q_s}{\Delta T} = \gamma C - \frac{\mu_0 \lambda^2 J_c^2 a^2}{T_c - T_b} \Delta T \left(-\frac{1}{2} \ln \varepsilon - \frac{3}{8} + \frac{\varepsilon^2}{2} - \frac{\varepsilon^4}{8} \right) + \frac{2\pi a}{\pi a^2} h \tau_j \Delta T & \end{aligned} \quad (4.62)$$

To maintain the stability of superconductor, the right-hand side of Equation (4.62) must be greater than zero, then stability parameter β_s is defined as:

$$\beta_s = \frac{\mu_0 \lambda^2 J_c^2 a^2}{\gamma C (T_c - T_b)} = \left(1 + \frac{2\nu}{\alpha^2} \right) \left(-\frac{1}{2} \ln \varepsilon - \frac{3}{8} + \frac{\varepsilon^2}{2} - \frac{\varepsilon^4}{8} \right)^{-1} \quad (4.63)$$

where $\varepsilon = c/a$ and ν are given by Equation (4.47). Comparing the stability parameter β_s in Equation (4.18), there is an extra factor $(1 + 2\nu/\alpha^2)$ in the stability parameter indicated in Equation (4.63).

As for the composite superconductor NbTi mentioned in Section 4.3, its critical current density is $J_c = 1.5 \times 10^9$ A/m², the radius of superconducting wire is $a = 0.25$ mm, filling factor is $\lambda = 0.4$, volumetric heat capacity is $\gamma C = 2.7 \times 10^3$ J/(m³·K), its critical temperature is $T_c = 6.5$ K, the bath temperature is $T_b = 4.2$ K, resistivity of stabilization matrix of copper is $\rho_m = 3.5 \times 10^{-10}$ Ω·m, and heat transfer

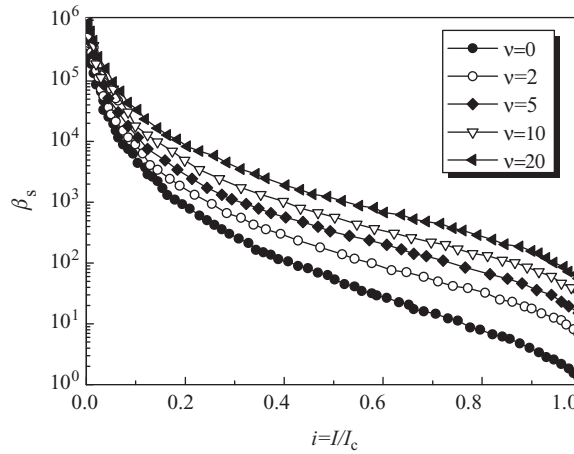


Figure 4.12 The stabilization parameter β_s versus normalized transport current i .

coefficient is $h = 3.5 \times 10^5 \text{ W/(m}^2\cdot\text{K)}$. Substituting these parameters into Equation (4.47), we confirm the heat transfer stability factor ν is 12. If there is an insulating layer coated onto the surface of the composite superconductor, with a thickness of $10 \text{ }\mu\text{m}$ and a thermal conductivity $k = 5 \times 10^{-2} \text{ W/(m}\cdot\text{K)}$, the heat transfer coefficient between the composite superconductor and its surroundings k is $h = 5 \times 10^5 \text{ W/(m}^2\cdot\text{K)}$, and we can also find $\nu = 2$ by substituting these parameters into Equation (4.47). The dependence of stabilization parameter β_s on normalized density I is shown in Figure 4.12 [12]. When stabilization parameter β_s is 6.6 with heat transfer stability factor under adiabatic stabilization condition $\nu = 0$, flux jump occurs at normalized $i = 0.83$.

However, if dynamic stabilization measures are taken, limitation on operation current can be alleviated significantly. Even if the composite superconductor is indirectly cooled and then $\nu = 1.2$ is chosen, a flux jump will take place at $i = 0.96$. If the composite superconductor is directly cooled by the cryogen without an insulation layer on its surface and thus $\nu = 1.2$ is used, the stable transport current may be up to critical current I_c , that is, $i = 1$. Therefore, as long as stabilization conditions can be fully satisfied in the design of a multifilamentary stabilization composite superconductor, instability in the self field can also be restrained.

Considering the size effect of superconducting filaments, their radii or half thickness must satisfy the following conditions in order to ensure stability of flux jump:

$$a < \sqrt{8}d \text{ (filament)} \quad (4.64)$$

$$a < \sqrt{3}d \text{ (slab)} \quad (4.65)$$

where d is determined by Equation (4.66):

$$d = \left(\frac{k(T_c - T_b)(1 - \lambda)}{\lambda J_c^2 \rho_m} \right)^{1/2} \quad (4.66)$$

where k , T_c and J_c are thermal conductivity, critical temperature and critical current density of the superconductor filaments, respectively; λ , ρ_m and T_b are filling factor, resistivity of stabilization matrix and surrounding bath temperature, respectively. Equation (4.66) is the so-called dynamic stability condition.

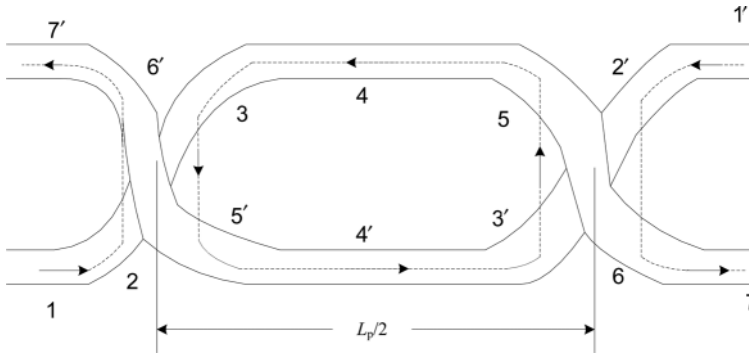


Figure 4.13 Twisting view of two superconducting wires.

Not only is the stability of the superconductor closely related to its thermal conductivity k but also the resistivity ρ_m of the stabilizer material. Although conditions of Equations (4.64) through (4.66) cannot guarantee a composite superconductor with absolute stability, they must be satisfied for stabilization in all possible cases, and they are indispensable prerequisite conditions to maintain stability of composite superconductors.

Superconducting wire with multifilaments can satisfy conditions for adiabatic stability of flux jump in uniform and constant magnetic fields. If the superconductor is in a time-varying magnetic field, full or partial electromagnetic coupling between filaments may occur due to electromagnetic induction, and then the filaments are equivalent to a bulk superconductor of large size. Even if sizes of the superconducting filaments are fabricated with several or several tens of micrometers, which can satisfy the stability criterion of flux jump, electromagnetic “size” is larger than that determined by Equation (4.66) for retaining flux jump, consequently the flux jump is still unavoidable. Therefore, on the premise with multi-filaments and in application of a time-varying magnetic field, twisting superconducting filaments are the effective measure and method for decoupling superconducting filaments, eliminating flux jump and maintaining adiabatic stability.

Figure 4.13 illustrates the twisting view of two superconducting wires, in which L_p refers to the twist pitch and will be introduced in detail in Section 5.9 of Chapter 5. The dashed lines with arrows represent the eddy current induced by a time-varying magnetic field, and arrows show the direction of eddy current. Taking a look at the path of 1-2-3-4-5-6-7, directions of eddy currents are inconsistent and reversed for each L_p . Similarly, reversal directions of eddy currents are also inconsistent by looking along the path of 1'-2'-3'-4'-5'-6'-7'. Thus, although the superconducting wire is in the time-varying magnetic field for a long time, it is effectively “cut” into many sections, each of which is of length L_p from the point of view of eddy currents.

The twist pitch of a superconductor is related to the resistivity ρ_m of stabilization materials, critical current density J_c , filament diameter d_f and the rate of the time-varying magnetic field dB/dt , and the critical length of the superconductor is:

$$L_c = 2 \left(\frac{\rho_m J_c d_f}{dB/dt} \right)^{1/2} \quad (4.67)$$

The calculation of Equation (4.67) will be discussed in Section 5.9 of Chapter 5, where the coupling loss and eddy current loss will be described in detail. Here, we only give the results. As long as the actual twist pitch L_p is less than L_c , instability and coupling loss of the superconductor will be eliminated to a large extent. Generally, by selecting $L_p = L_c/2$, $L_c/5$, $L_c/10$, the rules of adopting twist pitch are that twist

pitch L_p equals a fraction of critical length L_c , such as $L_c/2$, $L_c/5$ or $L_c/10$. Taking stable superconducting wire NbTi with copper stabilizer as an example, its critical current density J_c is 5×10^9 A/m², resistivity is $\rho_m = 3 \times 10^{-10}$ $\Omega \cdot m$, filament diameter d_f is 20 μm , and rate of time varying magnetic field dB/dt is 10 T/s. The critical length L_c of the superconductor is found to be 3.46 mm by substituting these parameters into Equation (4.67).

Because the magnetic field produced by the eddy current is the reverse to the applied magnetic field, the stability of the superconductor in the time-varying magnetic field can be ensured by twisting filaments. However, although twisting can effectively decrease the eddy currents, coupled currents and increase the stability of the superconductor, the superconductor always appears in the form of coils in actual engineering applications, and too small a twist pitch will damage the thin filaments in the superconducting strands and so cannot guarantee their mechanical stability. Though a smaller twist pitch in a transverse magnetic field can reduce coupling between filaments in strands, electromagnetic coupling of the longitudinal magnetic field will effectively strengthen them.

If an isolating layer with high resistivity is coated onto the surface of the composite superconducting strands, such as chromium or nickel, electromagnetic coupling between filaments can be effectively limited. Furthermore, coupling between superconducting filaments can be fully eliminated by twisting if they are completely isolated. Unfortunately, to date, this complete isolating technique amongst superconducting filaments is still not realized in their processing. If quench occurs in a strand of a composite superconductor, the current between strands cannot transfer effectively due to redistribution of this current, thus stability of the composite superconductor is greatly reduced and minimum quench energy (MQE) is also decreased. To design a perfect composite superconductor with high stability, we need to analyse accurately the related electrodynamics process and current distribution amongst filaments. Indeed, a complicated 3D structure of a composite superconductor needs to be taken into account, since the coupling current may transfer amongst strands along its whole length. The contact resistance between the superconducting strands is determined by void factors. All of these need to be numerically calculated in detail.

4.6 Cryostability

Cryostability is the study of the balance of heat power generated due to the superconductor quench and heat transfer power of the cryogen. If the heat power produced after quench is smaller than the heat transfer power of the cryogen, the temperature of the quench zone in the superconductor gradually decreases and then returns to the initial operating temperature, so that the quench zone recovers from the normal state to the superconducting state also. A method of cryostabilization is to research how to effectively transfer the heat produced due to superconductor quench to cryogen in time and take effective measures to make the heat power of the superconductor less than the cooled power of the cryogen.

In cryostability of superconductors, the composite superconductor is processed by adopting metallic materials with high thermal conductivity and low resistivity as the stabilization matrix, such as copper, aluminium, silver and alloys. Meanwhile, the composite superconductor is immersed in the cryogen with which the composite superconductor is directly in contact and is cooled by the boiling heat transfer of the cryogen [9]. If a local part of the composite superconductor transits from the superconducting state to the normal state due to some heat disturbance, part of the transport current will flow into the stabilization matrix by shunt, since the resistivity of the superconductor in the normal state is more than the stabilization matrix, so that heat power generated in the composite superconductor is effectively reduced. Heat generated in the composite superconductor results in a rise in its temperature and finally is transferred to the cryogen. If heat power is less than the heat transfer power, the composite superconductor recovers from the normal state to the superconducting state. This stabilization method, called cryostability, was first proposed by Stekly in 1965.

The following assumptions are usually made in the research of cryostability of a composite superconductor:

- (1) The superconductor is in a stable magnetic field B and with a cryogen with constant temperature T_b .
- (2) Temperature in cross-section of the superconductor equals the temperature on its surface and is uniformly distributed.
- (3) Thermal and electric contacts between the superconductor and stabilization matrix are ideal, and there are no thermal and electric resistances on their boundaries.
- (4) Normal resistivity of the superconductor is far greater than the stabilization matrix to which the transport current completely transfers when quench occurs in the superconductor.
- (5) Resistivity of the stabilization matrix is independent of temperature T and magnetic field B .
- (6) Heat transfer coefficient h is dependent on temperature T , while heat in the superconductor transfers to the cryogen.

4.6.1 Stekly Parameter

The superconductor is in direct contact with the cryogen during cryogenic stabilization, and the cooling effect is thus closely related to the heat transfer ability of the cryogen. Generally, the cryogenic media commonly used are liquid helium and liquid nitrogen, and their boiling temperatures are 4.2 K and 77 K in one atmosphere and the corresponding latent heats are 2.55 kJ/L and 160.62 kJ/L, respectively. There are two heat transfer states between the superconductor and cryogen: nucleate boiling and film boiling. Compared with film boiling, nucleate boiling has a higher heat transfer ability because the superconductor is in direct contact with the cryogen. However, due to evaporation of the cryogen, a gas film forms between the superconductor and the cryogen in the film boiling state. The heat transfer ability of cryogenic gas is much lower than cryogenic liquid, so that the cooling ability of film boiling is less than that of nucleate boiling. Generally, a small heat disturbance leads to nucleate boiling and large heat disturbance will bring about film boiling.

Assuming that the transport current density of a composite superconductor is J_t , its critical current density is J_c , and $J_t < J_c$. When the external heat disturbance is large enough, it will result in a temperature rise and then reduction of critical current density, so that its temperature is higher than current sharing temperature T_{sh} in the superconductor and stabilization matrix. T_{sh} is the temperature at which the transport current begins to transfer to the stabilization matrix from the superconductor, the transport current density J_t is then larger than its critical current density J_c , and part of the transport current will transfer to the stabilization matrix, resulting in an electric field E along the direction of transport current in the composite superconductor:

$$E = \frac{\lambda}{1 - \lambda} \rho_m (J_t - J_c(t)) \quad (4.68)$$

where ρ_m is the resistivity of the stabilization matrix. The heat power of the composite superconductor per unit volume is:

$$g = \lambda J_t E = \frac{\lambda^2}{1 - \lambda^2} \rho_m J_t (J_t - J_c(t)) \quad (4.69)$$

Below the critical temperature T_c , the dependence of critical current density on temperature is approximately linear:

$$J_c(T) = J_{c0} \frac{T_c - T}{T_c - T_b} \quad (4.70)$$

where J_{c0} is the critical current density when the temperature is T_b . Substituting Equations (4.70) and (4.68) into Equation (4.69), we obtain:

$$g = \frac{\lambda^2 \rho_m J_t^2}{1 - \lambda} \left(1 - \frac{J_{c0}}{J_t} \frac{T_c - T}{T_c - T_b} \right) \quad (4.71)$$

The process of heat power generation in the composite superconductor is as follows: when the operating temperature of superconductor T is less than the current sharing temperature T_{sh} , all of the transport current flows into the superconductor without resistance, and there is no generation of heat power. Nevertheless, the current transfers to the stabilization matrix from the superconductor and then Joule heat is generated when operating temperature T is greater than the current sharing temperature T_{sh} but lower than its critical current T_c , and heat power linearly varies with temperature T . When temperature T is more than its critical temperature T_c , the transport current will completely transfer to the stabilization matrix, and the heat term is expressed as [2]:

$$g = \begin{cases} 0 & T \leq T_{sh} \\ \frac{\lambda^2 \rho_m J_t^2}{1 - \lambda} \left[1 - \frac{J_c}{J_t} \left(1 - \frac{T - T_b}{T_c - T_b} \right) \right] & T_{sh} < T \leq T_c \\ \frac{\lambda^2 \rho_m J_t^2}{1 - \lambda} & T > T_c \end{cases} \quad (4.72)$$

where λ is the filling factor of the composite superconductor.

The composite superconductor directly contacts with the cryogen, from which heat transfer power per unit volume to the composite superconductor is:

$$Q = \frac{p}{A} h (T - T_b) \quad (4.73)$$

where p , A and h are cooling perimeter, cooling cross-section and heat transfer coefficient, respectively. The superconductor fully quenches at $T = T_c$, and the transport current fully transfers to the stabilization matrix. With transport current density equal to critical current density, that is, $J_t = J_c$, the generated Joule heat power and heat transfer power are respectively:

$$g_c = \frac{\lambda^2 \rho_m J_c^2}{1 - \lambda} \quad (4.74)$$

$$Q_c = \frac{p}{A} h (T_c - T_b) \quad (4.75)$$

The Stekly parameter α is defined as the ratio of Q_c to g_c :

$$\alpha = \frac{g_c}{Q_c} = \frac{\rho_m \lambda^2 J_c^2 A}{(1 - \lambda) p h (T_c - T_b)} \quad (4.76)$$

In order to guarantee cryostability, the Stekly parameter must satisfy the inequality $\alpha \leq 1$.

Equation (4.76) is similar to dynamic stabilization criterion (Equation 4.31), in which the assumption of the transport current I_t is close to the critical current I_c is made in analysing dynamic stability. However, there is no such strict condition in analysing cryostability (Equation 4.76). If heat generation power is equal to heat transfer power at this time, that is, $\alpha = 1$, the composite superconductor reaches a thermal equilibrium state. In order to guarantee cryostability, the Joule heat power must be smaller than

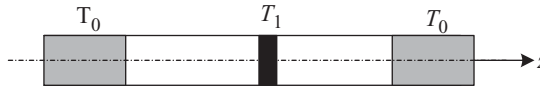


Figure 4.14 1D model of a composite superconducting rod.

the heat transfer power, that is, the Stekly parameter should satisfy inequality $\alpha < 1$, then temperature of the composite superconductor is able to decrease and become lower than the critical temperature T_c , so the transport current can return to the superconductor and recover to the superconducting state without resistance.

In the analysis process of cryostability, heat transferred by the conductor is neglected. Therefore, the Stekly cryostability criterion of $\alpha < 1$ is relatively conservative.

In order to consider the effect of heat conduction on cryostability analysis, we take a simple 1D composite superconductor as an. Figure 4.14 illustrates a composite superconducting rod. The 1D model is reasonable when assuming that its radius is small compared with its length and thus all of the characteristics on the whole cross-section are the same. The z -axis is in the axial direction, and the temperature at the two ends is constant T_0 , while the temperature in the heated central region is T_1 .

The 1D heat conduction equation along the superconducting rod is:

$$\frac{d}{dz} \left(k(T) \frac{dT}{dz} \right) = \frac{p}{A} h(T) - g(T) \quad (4.77)$$

where the term on the left-hand side refers to heat conduction, the first term and the second term on the right-hand side are cooling from coolant term and heating term, respectively; $k(T)$, depending on temperature T , is the thermal conductivity of the composite superconducting rod, while A and p are respectively the cross-section and cooled perimeter of the rod. The $h(T)$ in cooling term is described by:

$$h(T) = \frac{A}{p} \rho_c C_c \frac{dT}{dt} \quad (4.78)$$

where ρ_c , C_c and T are separately the density, specific heat and temperature of the coolant at constant pressure.

By substituting quantity of heat flux S :

$$S = k(T) \frac{dT}{dz} \quad (4.79)$$

and making the following transformation:

$$\frac{dS}{dz} = \frac{dS}{dT} \frac{dT}{dz} = \frac{dS}{dT} \frac{S}{k(T)} \quad (4.80)$$

Substituting Equation (4.80) into Equation (4.77), we find:

$$S \frac{dS}{dT} = k(T) \left(\frac{p}{A} h(T) - g(T) \right) \quad (4.81)$$

Integrating Equation (4.81) with temperature, we obtain:

$$(S_1^2 - S_0^2) = 2 \int_{T_0}^{T_1} k(T) \left(\frac{p}{A} h(T) - g(T) \right) dT \quad (4.82)$$

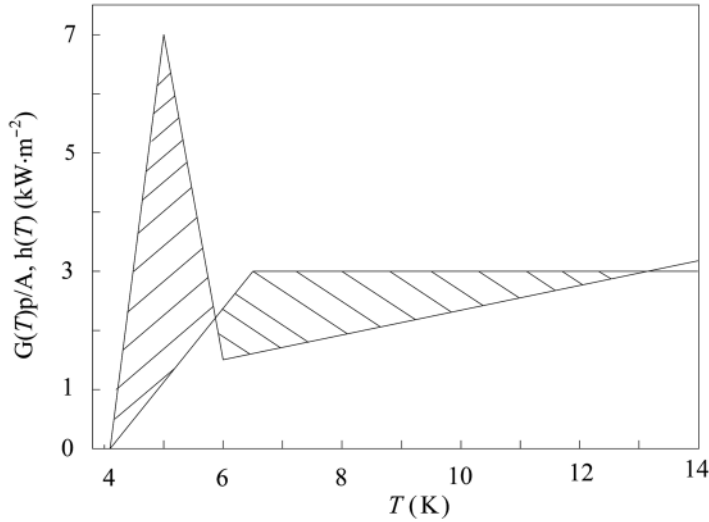


Figure 4.15 Thermal equilibrium in hot normal zone and cold edges of superconducting zone.

where S_0 and S_1 are heat fluxes corresponding to temperature T_0 and T_1 , respectively. If the superconducting rod is long enough, the temperature at its two ends far from the heat source is constant and equal to the surrounding cryogen where there is no temperature gradient, that is, $dT/dz = 0$. Therefore, the temperature at the heat exchange interface reaches equilibrium when the point is far from the heat source, then $S_0 = S_1 = 0$. When heat power and heat transfer to the cryogen reach equilibrium at the heat exchange interface, then:

$$\int_{T_0}^{T_1} \left(h(T) - \frac{A}{p} g(T) \right) k(T) dT = 0 \quad (4.83)$$

If thermal conductivity k is independent of temperature T , Equation (4.83) becomes:

$$\int_{T_0}^{T_1} \left(h(T) - \frac{A}{p} g(T) \right) dT = 0 \quad (4.84)$$

Equation (4.84) is the so-called equal-area theorem, which can be illustrated by simple geometric graphics, that is, the two areas enclosed by heating and cooling curves are equal. Figure 4.15 shows curves of heat transfer and heat generation, which satisfy the equal-area theorem, while the NbTi composite superconducting conductor is in the magnetic field $B = 6$ T and at an operating temperature of boiling helium $T = 4.2$ K with critical temperature of $T_c = 6.5$ K. It also shows that thermal equilibrium reaches between the hot normal zone in the superconductor and the cold superconducting zones at both sides. If the integration area of heat generation is larger than the corresponding one in Figure 4.15, the equilibrium state is disturbed, temperature in the superconductor will rise and the normal zone further expands. However, if the integration area of heat generation is smaller than the corresponding one in Figure 4.15, the temperature in the superconductor will decrease and the normal zone further reduces until it disappears, so that the superconductor completely recovers to its superconducting state. Thus, the normal zone is surrounded by a cold superconducting zone. Figure 4.15 gives the stabilization condition of the normal zone, which is defined as the recovering condition of cold edges.

The aim of superconductor stabilization is for the superconductor to recover from its normal state to the superconducting state after heat disturbance. Cryogenic stabilization criterion interprets how to realize superconductor recovery from the normal state to superconducting state by adopting matrix or different operating current density. However, it is excessively conservative with respect to local disturbance because heat exchange between superconductor and cryogen is only considered in cryogenic stabilization criterion, while the thermal conductivity is not taken into account.

However, the composite superconductor is densely wound in actual applications and does not directly contact with the cryogen due to its surface insulation, so heat conduction plays an important part. In order to study the effect of heat conduction on cryogenic stabilization, stability analysis of the superconducting magnet or coil should be performed using 3D heat conduction models. Before analysing the heat conduction effect, three concepts related to stability of the superconductor should first be introduced:

- (1) Minimum propagation zone (MPZ) criterion – a minimum area in a composite superconductor, which reaches the normal state first due to an initiating disturbance in the presence of a transport current. Then the quench action propagates and causes thermal runaway of the entire superconductor [13].
- (2) Minimum quench energy (MQE) criterion – minimum disturbance energy, which is able to cause full quench of the superconductor and equals to the sum of energy stored in the normal zone and external disturbance energy.
- (3) Normal zone propagation (NZP) velocity criterion – speed with which the interface between the normal zone and the superconducting zone propagates along the superconductor or through an assembly after a local zone has been driven into the normal state.

These three quantities are important parameters for measuring the dynamic stability of a composite superconductor and superconducting magnet or coil.

4.6.2 One-Dimensional Normal Zone Propagation

To understand the concept of MPZ, we treat a current-carrying superconducting rod as a simple 1D model to describe local quench and propagation characteristics under conditions of heat point disturbance. A section of the superconductor with transport current density J_c is shown in Figure 4.16, in which temperature T at the local hot point in the central part resulting from local heat disturbance exceeds its critical temperature T_c . To simplify the calculation, it is assumed that the temperature of the hot point is

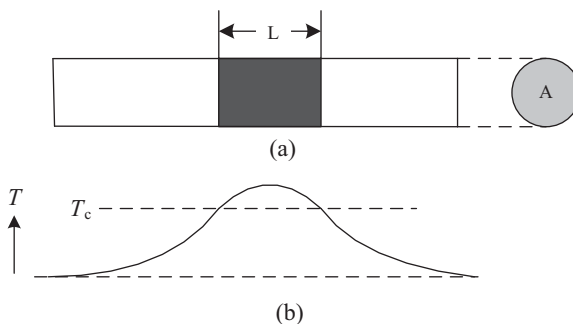


Figure 4.16 Normal zone and temperature distribution resulting from point disturbance in a superconducting rod: (a) normal zone produced by heat disturbance in current carrying superconductor; (b) temperature distribution along axis.

$T = T_c$ and then this zone transits to the normal state, and consequently heat $J_c A \rho L$ is generated, where A , ρ and L are the cross-section, resistivity and normal zone length of the superconductor, respectively. The hot point zone propagates along the rod to both ends. Suppose that the temperature gradient is $(T_c - T_b)/L$, and T_b is the temperature at some point without heat disturbance. In adiabatic conditions, all heat disturbances are exerted on the superconductor and then its temperature rises. According to heat balance equation:

$$2kA \left(\frac{T_c - T_b}{L} \right) = J_c^2 \rho A L \quad (4.85)$$

k is the thermal conductivity of the superconductor, so:

$$L = \left[\frac{2k (T_c - T_b)}{J_c^2 \rho} \right]^{1/2} \quad (4.86)$$

If the heat disturbance exceeds the heat removed by cooling, the normal zone continuously extends. Conversely, the normal zone gradually decreases until it fully disappears, so that superconductor returns to its superconducting state. L in Equation (4.86) is the length of the 1D normal propagation zone.

4.6.3 Three-Dimensional Normal Propagation Zone and Minimum Quench Energy

In practical applications, the superconductor always appears in the form of a coil or winding, and the normal zone should propagate in 3D directions. Figure 4.17(a) shows the cross-sectional view of a superconducting coil and Figure 4.17(b) represents a superconducting wire used in coil or winding and treated approximately as a rod. Thermal conductivity k is anisotropic in the z -axis and radial direction, which are separately represented by k_z and k_r . This assumption is reasonable because the insulation amongst turns of coils is essential.

Taking approximation of steady state with adiabatic conditions and neglecting other heat sources, in cylindrical coordinates, the heat energy balance Equation (4.1) becomes:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r k_r \frac{\partial T}{\partial r} \right) + \frac{\partial}{\partial z} \left(k_z \frac{\partial T}{\partial z} \right) + \lambda_w G(T) = 0 \quad (4.87)$$

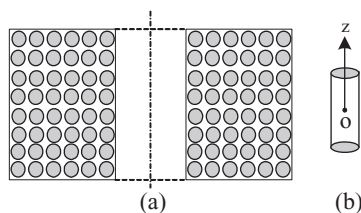


Figure 4.17 3D heat conduction model of magnet coil: (a) schematic cross-section of coil; (b) section of superconducting wire in coil with cylindrical coordinate.

where λ_w is the volume ratio of the composite superconductor to coil and $G(T)$ is the Joule heat term of the current carrying superconductor:

$$G(T) = \frac{\lambda^2 \rho_m J_t^2}{1 - \lambda} \left(\frac{T_c - T}{T_c - T_b} \right) = G_c \left(\frac{T_c - T}{T_c - T_b} \right) \quad (4.88)$$

where G_c is independent of temperature:

$$G_c = \frac{\lambda^2 \rho_m J_t^2}{1 - \lambda} \quad (4.89)$$

For simplicity, we assume that thermal conductivity k is independent of temperature, and Equation (4.87) changes into Equation (4.90) by making the transformation of $\alpha^2 = k_r/k_z$, $R = r/\alpha$:

$$\frac{1}{R} \frac{\partial}{\partial R} \left(R \frac{\partial T}{\partial R} \right) + \frac{\partial^2 T}{\partial z^2} + \frac{\lambda_w G(T)}{k_z} = 0 \quad (4.90)$$

The first two terms of Equation (4.90) are similar with operator ∇^2 in cylindrical coordinates. It is assumed that temperature T has spherical symmetry in coordinate transformation, so Equation (4.90) becomes:

$$\frac{d^2 T}{dR^2} + \frac{2}{R} \frac{dT}{dR} + \frac{\lambda_w G(T)}{k_z} = 0 \quad (4.91)$$

making transformation of $\phi = T - T_g$ and $x = R/R_g$ in Equation (4.91) where R_g is determined by:

$$R_g = \pi \left[\frac{k_z (T_c - T_g)}{\lambda_w G_c} \right]^{1/2} \quad (4.92)$$

where T_g , closing to current sharing temperature T_{sh} , shows that current carrying superconducting conductor with current density J begins to generate Joule heat when the temperature of the superconductor rises at T_g , which is determined by:

$$T_g = T_c - (T_c - T_b) J/J_c \quad (4.93)$$

Substituting Equation (4.93) into Equation (4.91), we have:

$$\frac{d^2 \phi}{dx^2} + \frac{2}{x} \frac{d\phi}{dx} + \pi^2 \phi = 0 \quad (4.94)$$

Considering adiabatic boundary condition, when $x = 0$, $d\phi/dx = 0$, the solution of the Equation (4.94) is:

$$\phi = \frac{A}{x} \sin \pi x \quad (4.95)$$

where A is a constant that needs to be determined, and when $x = 1$, Equation (4.95) is zero for any A , that is, $T = T_b$. In spherical coordinates, heat generates in the entire region of a sphere with radius R_g , and this region is the so-called MPZ. When a normal zone produced by heat disturbance or heat source is smaller than the MPZ and the heat removed exceeds the heat generated, the disturbed zone

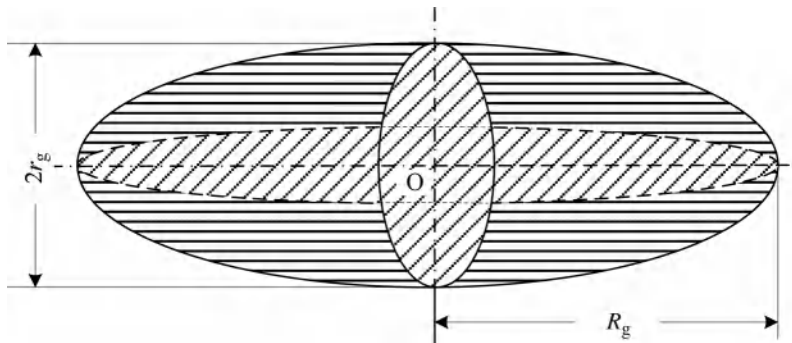


Figure 4.18 3D MPZ of ellipsoid.

will shrink and eventually disappear. However, when a normal zone produced by heat disturbance or heat source is larger than the MPZ and the heat removed is lower than the heat generated, the disturbed zone will continuously extend until the whole superconductor quenches. In spherical coordinates, the MPZ is an ellipsoid in which the major semi-axis R_g is along the axis of the superconductor, while its side cross-section is a circle with radius of $r_g = \alpha R_g$ (Figure 4.18), which is a schematic of the 3D normal zone.

In order to determine the energy needed to form a MPZ, constant A in Equation (4.95) needs to be found, which can be determined by boundary condition $R = R_g$. When $R > R_g$, heat energy balance Equation (4.90) becomes:

$$\frac{d^2 T}{dx^2} + \frac{2}{x} \frac{dT}{dx} = 0 \quad (4.96)$$

Its general solution is:

$$T = \frac{B}{x} + C \quad (4.97)$$

where B and C are constants. Solutions of steady state should satisfy the boundary condition $T = T_b$ on the surface of the superconducting coil. The expansion or disappearance of the normal zone is uniquely determined by local conditions. A rigorous solution is required to solve the time-variant equation, which is rather complicated. Here, a simple but not so strict method by artificially adding a cooled boundary at interfaces with radius R_m ($R_m > R_g$) is first adopted, then we find its steady state solutions. After taking the cooled boundary condition into account, the radius of the new boundary region R_m is assumed to be mR_g ($m > 1$). Let $y = T/T_b$, then by using conditions that dT/dx and T must be continuous at $x = 1$ ($R = R_g$), substituting these quantities into Equations (4.94) and (4.95), we obtain:

$$y = \frac{m\beta}{(m-1)\pi x} \sin \pi x + \beta + 1 \quad (x < 1) \quad (4.98)$$

$$y = 1 + \beta - \frac{m\beta}{m-1} \left(1 - \frac{1}{x}\right) \quad (1 < x < m) \quad (4.99)$$

where

$$\beta = (T_g - T_b)/T_b \quad (4.100)$$

To determine the MQE, we calculate the energy required to form a normal zone and then take its minimum value. In the inner and outer regions of the boundary with R_g , energy per unit volume change is described by Equation (4.101) with the assumption that dependence of volumetric heat capacity on temperature is cubic, that is, $\gamma C \sim T^3$:

$$\Delta E = \frac{\gamma C_b (T^4 - T_b^4)}{4T_b^4} = \gamma H_b (y^4 - 1) \quad (4.101)$$

where C_b and H_b are the specific heat capacity and enthalpy at temperature T_b , respectively. In the heat zone of $R < R_g$, the normalized energy needed to heat is the integration of Equation (4.101) over the ellipsoidal normal zone:

$$e_g = \frac{E_g}{E_b} = 3 \int_0^1 x^2 (y^4 - 1) dx \quad (4.102)$$

where

$$E_b = \frac{4\pi}{3} \alpha^2 R_g^3 \gamma H_b \quad (4.103)$$

Equation (4.103) is the enthalpy in the hot zone of an ellipsoid at temperature T_b . By integrating Equation (4.102), the normalized energy per unit volume is:

$$e_g = \left[\frac{m\beta}{\pi(m-1)} \right] (v^4 + 3.8v^3 + 9v^2 + 11.6v + 6.3) - 1 \quad (4.104)$$

where, v is determined by Equation (4.105):

$$v = \frac{\pi(1+\beta)(m-1)}{\beta m} \quad (4.105)$$

In the annular region of $R_g < R < mR_g$, the normalized energy per unit volume is given by:

$$\begin{aligned} e_h &= \frac{E_h}{E_b} 3 \int_1^m x^2 (y^4 - 1) dx \\ &= \left(\frac{m\beta}{m-1} \right)^4 \left[3 - \frac{3}{m} + 12\eta \ln m + 18\eta^2 (m-1) + 6\eta^3 (m^2-1) + \eta^4 (m^3-1) \right] - (m^3-1) \end{aligned} \quad (4.106)$$

where

$$\eta = \frac{m-1-\beta}{m} \quad (4.107)$$

Therefore, the normalized energy for establishing a normal propagation zone is the sum of the energy described by Equations (4.104) and (4.106), that is, $e_t = e_g + e_h$. For small m , increase of peak temperature results in a temperature rise in the hot zone; for larger m , energy in the annular region increases. The minimum e_t is chosen as the optimized estimation of establishing the 3D normal propagation zone, that is, it is the minimum disturbance energy causing quench.

For example, it is assumed that the superconducting coil made of composite superconductor NbTi operates in the magnetic field $B = 6$ T and temperature $T = 4.2$ K; its other parameters are as follows:

critical current density $J_c = 1.5 \times 10^9$ A/m², operating current density $J = 0.8J_c = 1.2 \times 10^9$ A/m², filling factor of composite superconductor $\lambda = 0.3$, resistivity of copper stabilization matrix material $\rho_m = 3.5 \times 10^{-10}$ $\Omega \cdot m$, heat power density $G_c = 6.6 \times 10^7$ W/m³, critical temperature $T_c = 6.5$ K, current sharing temperature $T_{sh}(T_g) = 4.66$ K, and filling factor of coil $\lambda_w = 0.5$. Substituting all of these parameters into Equation (4.92), we obtain $r_g = 1.67$ cm. The volume of MPZ is then $V = 4\pi/(3\alpha^2 R_g^2) = 1.95 \times 10^{-3}$ m³ with $\alpha = 1$. If some materials are inserted between turns of the superconducting coil and volumetric ratios of filling materials, the NbTi superconductor and copper stabilization matrix are separately 0.5, 0.15 and 0. and average volumetric heat capacity is $\gamma C = 2.7 \times 10^3$ J/(m³·K). Only if the temperature of the superconductor rises from the operating temperature 4.2 K to the current sharing temperature 4.66 K, can MPZ be established and the energy needed then is 2.8×10^{-3} J, which is stored in NPZ, is smaller than MQE.

Although the problem discussed above is the theoretical analysis of the 3D steady state stability, it is based on the approximate prerequisite assumption of a continuous conductor with anisotropy and is only suitable for the condition that the radius of a single filament is smaller than the radius (αR_g) of a minimum NPZ. For a more general analysis, every kind of heat transfer comprehensively needs to be considered in solving heat conduction Equation (4.1). However, there are almost no analytic solutions in practical applications, so a numerical simulation is needed. For calculation of MQE in analysis of stability, the energy needed for setting up the MPZ after the heat disturbance is:

$$E_q = A \int \int G_d(r, t) dr dt \quad (4.108)$$

where G_d is the heat term in Equation (4.1) and A is the cross-section of superconductor.

4.7 NPZ Velocity in Adiabatic Composite Superconductors

NPZ velocity is one of important parameters in superconductor stability and has significance in quench detection and protection of superconducting magnets. The velocities are classified into longitudinal and transversal velocities.

4.7.1 Longitudinal Propagation Velocity

When considering a simple 1D model of a superconductor, the normal zone in the superconductor moves along the longitudinal axis with a constant velocity of U_1 . Figure 4.19 illustrates that the 1D boundary of the normal zone moves along the z -axis with velocity U_1 in adiabatic conditions, where $x < 0$ represents the normal zone, and $x > 0$ and $x = 0$ refer to the superconducting zone and boundary between the normal zone and the superconducting zone, respectively. The heat conduction equation in the normal zone $x < 0$ is:

$$\gamma_n C_n \frac{\partial T_n}{\partial t} = \frac{\partial}{\partial x} \left(k_n \frac{\partial T_n}{\partial x} \right) + \rho_n J^2 \quad (4.109)$$

where T_n , $\gamma_n C_n$, k_n and ρ_n are temperature, volumetric heat capacity, thermal conductivity and resistivity of the normal zone in the superconductor, respectively. In the superconducting zone $x > 0$, the heat conduction equation is:

$$\gamma_s C_s \frac{\partial T_s}{\partial t} = \frac{\partial}{\partial x} \left(k_s \frac{\partial T_s}{\partial x} \right) \quad (4.110)$$

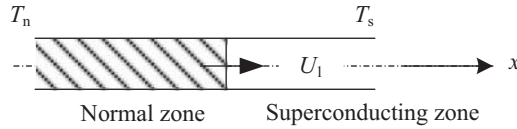


Figure 4.19 1D normal zone boundary moving with constant velocity U_1 .

where T_s , $\gamma_s C_s$ and k_s are temperature, volumetric heat capacity and thermal conductivity of the superconducting zone, respectively. By taking transformation of $z = x - U_1 t$ and then substituting into Equations (4.109) and (4.110), we have:

$$\frac{d}{dz} \left(k_n \frac{\partial T_n}{\partial z} \right) + \gamma_n C_n U_1 \frac{dT_n}{dz} + \rho_n J^2 = 0 \quad (4.111)$$

$$\frac{d}{dz} \left(k_s \frac{dT_s}{dz} \right) + \gamma_s C_s U_1 \frac{dT_s}{dz} = 0 \quad (4.112)$$

Supposing that thermal conductivities of the normal zone and superconducting zone are constants, in the normal zone at $z = 0$, we have $d^2 T_n / dz^2 = 0$, and Equations (4.111) and (4.112) become:

$$\gamma_n C_n U_1 \frac{dT_n}{dz} + \rho_n J^2 = 0 \quad x < 0 \quad (4.113)$$

$$k_s \frac{d^2 T_s}{dz^2} + \gamma_s C_s U_1 \frac{dT_s}{dz} = 0 \quad x > 0 \quad (4.114)$$

The solution of Equation (4.114) is:

$$T_s(z) = A e^{-\beta z} + T_b \quad (4.115)$$

T_b is the operating temperature, which is far from the boundary of the superconductor, $\beta = \gamma_s C_s U_1 / k_s$, $T_s(0) = T_c$, and T_c is the critical temperature of the superconductor. By substituting them into Equation (4.115), we obtain:

$$T_s(z) = (T_t - T_b) \exp(-\beta z) + T_b \quad (4.116)$$

Taking other boundary conditions into account, heat flux must be continuous at boundary $z = 0$ and thus heat flux at both sides is equal, therefore:

$$k_n \frac{dT_n}{dz} \Big|_{z=0^-} = k_s \frac{dT_s}{dz} \Big|_{0^+} \quad (4.117)$$

Substituting Equations (4.113) and (4.116) into Equation (4.117), we obtain:

$$-\frac{k_n \rho_n J^2}{\gamma_n C_n U_1} = -\gamma_s C_s U_1 (T_t - T_b) \quad (4.118)$$

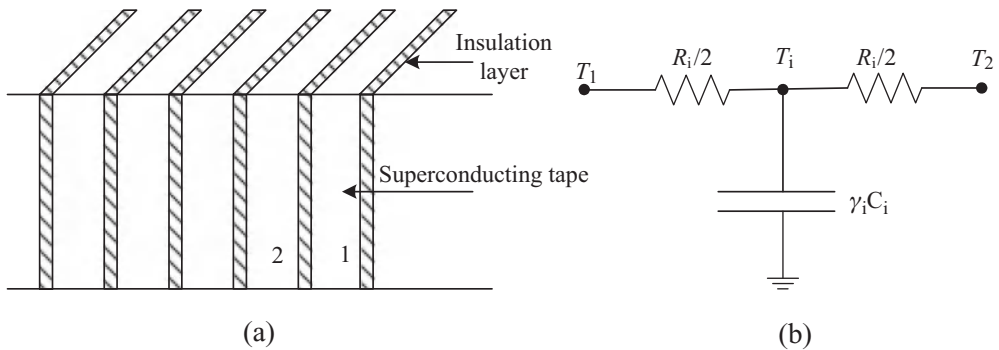


Figure 4.20 Arrangements of insulation and superconducting tapes in superconducting coil and corresponding thermal couple circuit: (a) geometrical configuration; (b) equivalent thermocouple circuit between turns.

Thus, we can obtain the propagation velocity of boundary U_1 between the normal zone and the superconducting zone:

$$U_1 = J \sqrt{\frac{\rho_n k_n}{\gamma_n C_n \gamma_s C_s (T_i - T_b)}} \quad (4.119)$$

4.7.2 Transverse Propagation Velocity

In practical applications, the superconductor always appears in the form of coils or windings in which there is an insulation layer on the surface of the superconducting wire, so that thermal conductivity in longitudinal and radial directions are different. In the superconducting coil, the propagation of the normal zone amongst turns is transverse propagation, and its propagation velocity is defined as U_t . Figure 4.20 shows thermal coupling between a superconductor and insulation layers [13], Figure 4.20(a) illustrates a geometrical arrangement of a superconductor and insulation layer, and Figure 4.20(b) indicates an effective thermal couple circuit with heat transfer from the first turn to the second turn, respectively. T_1 and T_2 are separate temperatures of superconducting tapes 1 and 2; T_i is the temperature of the insulation tape between superconducting tapes 1 and 2; $R_i/2$, which is equal to $A/(kL)$ with units of $\text{m}^2 \text{ K/W}$, represents thermal resistance of the insulation layer between superconducting tapes 1 and 2; and $\gamma_i C_i$ with units of $\text{J}/(\text{m}^3 \cdot \text{K})$ refers to volumetric heat capacity of the insulation layer.

The transverse propagation time τ_t of the normal zone is the time that is required for superconducting tape 2 to raise its temperature from operating temperature T_b to superconducting transition temperature T_t . As for superconducting tape 2, the energy density equation is:

$$\delta \int_{T_b}^{T_t'} \gamma C dT = \int_0^{\tau_t} q_2(t) dt \quad (4.120)$$

where δ , γC stand for the thickness and volumetric heat capacity of the composite superconducting layer, respectively. According to the analogue circuit shown in Figure 4.20(b) and heat flux exchange between superconducting tapes 1 and 2, we obtain:

$$\frac{T_1 - T_i}{(R_i/2)} = \delta_i \gamma_i C_i \frac{dT_i}{dt} + q_2(t) \quad (4.121)$$

where δ_i stands for the thickness of the insulation layer.

Equation (4.121) shows that the heat flux coming from superconducting tape 1 is divided into two; one part is absorbed by insulation tape, and the other transfers to superconducting tape 2 by heat conduction. Assuming that temperature T_2 varies little compared with the temperature T_i in the insulating layer, then, $q_2(t) \approx T_i/(R_i/2)$; T_1 linearly varies with temperature; $T_1 = \theta_1 t$, $R_i = \delta_i/k_i$; k_i is thermal conductivity of insulation layer; and $q_2(t)$ can be found by substituting those quantities into Equation (4.121):

$$q_2(t) = \left(\frac{\theta_1 k_i}{4\delta_i} \right) \left\{ 4t - \tau_i \left[1 - \exp \left(-\frac{4t}{\tau_i} \right) \right] \right\} \quad (4.122)$$

where $\delta_i \gamma_i C_i / k_i$ is called the heat time constant of the insulation layer.

Substituting Equation (4.122) into (4.120), we have

$$\left(\frac{4\delta\delta_i}{\theta_1 k_i} \right) \int_{T_b}^{T'_i} \gamma C dt = 2\tau_i^2 - \tau_i \tau_i + \frac{\tau_i^2}{4} \left[1 - \exp \left(-\frac{4\tau_i}{\tau_i} \right) \right] \quad (4.123)$$

where T'_i depends on a magnetic field B and transport current I_t , and also influences τ_i .

The temperature of the superconductor rises due to Joule heat. The average volumetric heat capacity $(\gamma C)_{\text{avg}}$ and thermal conductivity k_{avg} of a composite superconductor are defined as:

$$(\gamma C)_{\text{avg}} = \sum_{i=1}^n f_i (\gamma_i C_i) \quad (4.124)$$

$$k_{\text{avg}} = \sum_{i=1}^n f_i k_i \quad (4.125)$$

where n represents the number of components consisting of the composite superconductor. f_i , γ_i and C_i are the volumetric ratio, density and specific heat capacity, respectively, of each component in the composite superconductor. The related calculation method of the effective volumetric heat capacity and thermal conductivity of the composite superconductor with multi-components is included in Appendix A.1. Based on the Fourier theory, $(\gamma C)_{\text{avg}}(dT_1/dt) = \rho_m J_m^2$, and ρ_m and J_m are resistivity and transport current density of stabilization matrix in composite superconductor. By substituting $\theta_1 = \rho_m J_m^2 / (\gamma C)_{\text{avg}}$ into Equation (4.123), we obtain:

$$\left(\frac{4\delta\delta_i (\gamma C)_{\text{avg}}}{k_i \rho_m J_m^2} \right) \int_{T_b}^{T'_i} \gamma C dT = 2\tau_i^2 - \tau_i \tau_i + \frac{\tau_i^2}{4} \left[1 - \exp \left(-\frac{4\tau_i}{\tau_i} \right) \right] \quad (4.126)$$

In practical applications, especially for the HTS with low thermal conductivity, we have $\tau_t \gg \tau_i$. Then the thermal time constant in transverse propagation is:

$$\tau_t = \frac{1}{J_m} \sqrt{\frac{2\delta\delta_i (\gamma C)_{\text{avg}}}{\rho_m k_i} \int_{T_b}^{T'_i} \gamma C dT} \quad (4.127)$$

When the thickness of the insulation layer is much less than the thickness of the superconductor, that is, $\delta_i \ll \delta$, the transverse propagation velocity of normal zone is:

$$U_t = \frac{\delta_i + \delta}{\tau_t} \approx \frac{\delta}{\tau_t} = J_m \sqrt{\rho_m k_i \delta} \left(2\delta_i (\gamma C)_{\text{avg}} \int_{T_b}^{T'_i} (\gamma C) dT \right)^{-1/2} \quad (4.128)$$

Taking further approximation $\int_{T_b}^{T_t'} (\gamma C) dT = (\gamma C)_{\text{avg}} (T_t' - T_b)$, Equation (4.128) is simplified as:

$$U_t = \frac{J_m}{(\gamma C)_{\text{avg}}} \sqrt{\frac{\rho_m k_i \delta}{2\delta_i (T_t' - T_b)}} \quad (4.129)$$

Comparing longitudinal propagation velocity Equation (4.119) with transverse propagation velocity Equation (4.129), the ratio of transverse propagation velocity U_t to the longitudinal velocity U_l is proportional to:

$$\frac{U_t}{U_l} \propto \sqrt{\left(\frac{\delta}{\delta_i}\right) \left(\frac{k_i}{2k_n}\right)} \quad (4.130)$$

The transverse and longitudinal propagation velocities of a composite superconductor are driven by the heat power term $\rho_m J_m^2$, while the process of NZP is dominated by the process of thermal diffusion.

4.8 Stability of HTS Bulks

HTS bulks have many applications such as magnetic levitation, motor magnets and permanent magnets due to their strong ability of trapping magnetic flux due to their persistent current with zero resistance. However, the super-current may decay because of relaxation of the non-equilibrium magnetic structures. This phenomenon is known as magnetic flux creep or magnetic relaxation, which is a thermally activated flux motion in which fluxons move from one pinning centre to another. If HTS bulks are used in superconducting devices whose characteristics will greatly degrade, magnetic flux creep originates in a motion vortex out of pinning centers due to thermal activation. The relaxation rate decreases if strong or dense pinning centers are introduced into the HTS sample during processing. The critical current density J_c is determined by its pinning ability. Although there is also magnetic relaxation in LTS, it is so small that its effect on characteristics of LTS equipment can be neglected, thus the flux creep is seldom discussed in LTS. In this section, we will discuss the stability of HTS bulks based on decay of current density and magnetic relaxation.

4.8.1 Evolution of Super-Current Density

A superconducting cylindrical disk with radius R and height h is shown in Figure 4.21, in which the cylindrical coordinate is chosen due to its azimuthal symmetry (Figure 4.21(a)). Assuming that it is magnetized in a non-uniform magnetic field parallel to the z -axis and the critical state extends into the cylindrical disk from its rim, only a radial vortex density gradient will induce the super-current [14]:

$$J_\theta(r) = -\frac{1}{\mu_0} \frac{dB_z(r)}{dr} \quad (4.131)$$

where $B_z(r)$ refers to external magnetic field component along the z -axis. Then magnetization in the z -direction is:

$$M = \frac{1}{R^2} \int_{r_1}^{r_2} J_\theta(r) r^2 dr \quad (4.132)$$

r_1 and r_2 are the radii of the flux gradient boundary at the beginning of magnetic relaxation, respectively. $J_\theta(r)$ denotes induction current density in the azimuthal direction, henceforth subscript θ will be omitted

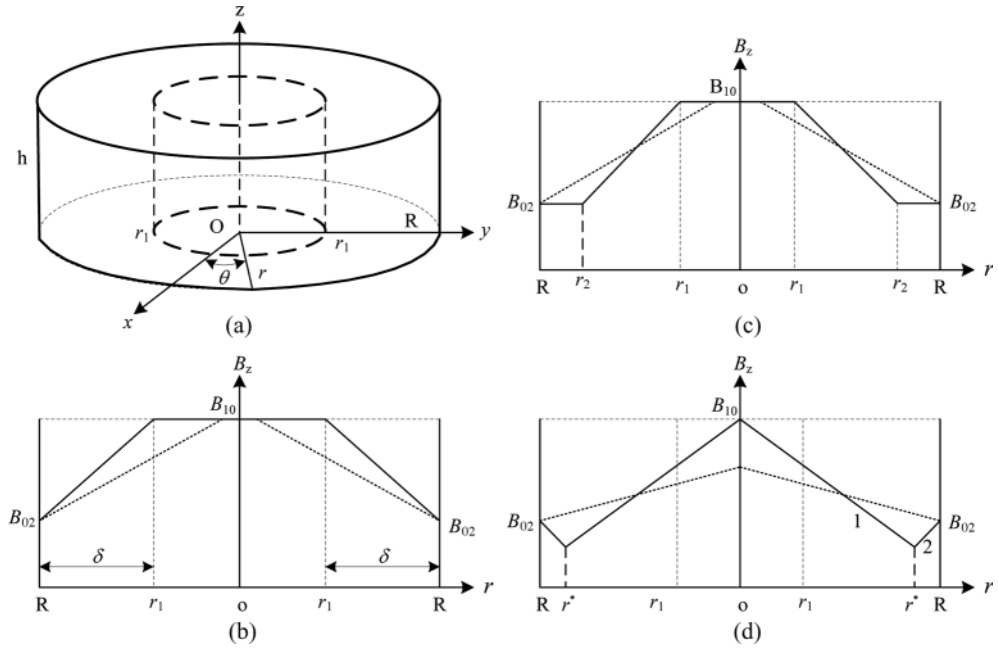


Figure 4.21 Magnetic flux distributions in a cylindrical disk with diameter $2R$ and height h : (a) geometrical configuration; (b) and (c) one gradient induction flux distribution; (d) two gradient induction flux distribution. Solid line (—) and dotted line (---) separately indicate induction flux distributions corresponding to the moment of time t' and line t'' with $t'' > t'$.

in this section for simplicity. Due to strong magnetic relaxation, magnetization of the cylindrical disk decreases immediately after magnetizing. Thus, the critical state with critical current density J_c is merely at initial state which relaxes rapidly, so that the real current density is much less than the critical current density than J_c . The azimuthal current density $J(t)$ at any moment of time t can be expressed as:

$$J(t) = J_0 \left(1 - \frac{k_B T}{U_0} \ln \left(\frac{t}{t_0} \right) \right) \quad (4.133)$$

where t_0 is the relaxation start time at which current density and effective activation energy are J_0 and U_0 , respectively, that is, $J_0 = J(t_0)$, $U_0 = U(t_0)$, k_B denotes the Boltzman constant and $J(t)$ is time-dependent.

4.8.2 Magnetic Relaxation

We suppose that the cylindrical disk is cooled in the magnetic field and changes from B_{01} to B_{02} . The vortices flow through its surface on which the external boundary of the critical state is located [15–17]. Figure 4.21(b) illustrates radial magnetic field distribution in a cylindrical disk. According to Bean's CSM, the critical current flows unidirectionally in the region $r_1 \leq r \leq r_2$; here $r_1 \geq 0$ and $r_2 \leq R$, and by substituting them into Equation (4.132), magnetization of the cylindrical disk is:

$$M = M_{m0} \left(\frac{r_2^3 - r_1^3}{R^3} \right) \quad (4.134)$$

where M_{m0} is the maximum magnetization with unidirectional current flowing through the whole volume, that is, $M_{m0} = J_c(t)R/3$ with $r_1 = 0$ and $r_2 = R$; here $J_c(t)$ denotes that critical current density varies with time t .

Assuming that $\delta(t)$ is the penetration depth of the critical state, $r_2 = R$, $r_1 = R - \delta(t)$ and that $J(t)/J_0$ closes to 1, the magnetic relaxation is approximately:

$$M(t) \approx M_0 \left(1 - C_\delta \frac{k_B T}{U_0} \ln \left(\frac{t}{t_0} \right) \right) \quad (4.135)$$

where $M_0 = M(t_0)$:

$$C_\delta = \frac{3\hat{\delta}_0 - 2\hat{\delta}_0^2}{3 - 3\hat{\delta}_0 + \hat{\delta}_0^2} \quad (4.136)$$

where relation $\delta(t_0) = \delta_0$ refers to initial penetration depth of critical state, and $\hat{\delta}_0 = \delta_0/R$ represents the normalized penetration depth.

If an induction gradient only exists in the annulus region $r_1 < r < r_2$ (Figure 4.21(c)), the current density in the zone away from the surface has the same relaxation as Equation (4.133). Nevertheless, the total magnetic flux remains unchanged in the cylindrical disk, $M(t) = M(t_0)$, which takes a time in the range of $t_0 \leq t \leq t_i$, t_i , determined by Equation (4.133), which represents time essential for emergence of boundary $r_2(t)$ on surface R. But if relaxation takes place in the critical current density, it is then:

$$J_c(t_i) = J_0 \frac{2\hat{\delta}_0}{3 - \sqrt{4\frac{w_0}{\hat{\delta}_0} - 3}} \quad (4.137)$$

where $w_0 = (r_{02}/R)^3$, $\hat{\delta}_0 = (r_{02} - r_{01})/R$ denotes the normalized penetration depth of the critical state, and $(r_{02} - r_{01})$ refers to the interval of the induction gradient zone at the beginning of relaxation.

If the external magnetic field reverses, the opposite vortex density gradient can be set up. The induction magnetic flux distribution in partial penetration of the critical state is presented in Figure 4.21(d). The magnetic flux configuration is indicated by a solid line consisting of two segments 1 and 2, where induction magnetic gradients are the same but opposite in sign. The currents generates opposite magnetic moments circulating around the z -axis of the cylindrical disk whose magnetization becomes bipolar:

$$M_0^* = M_m \left[2 \left(\frac{r^*}{R} \right)^3 - \left(\frac{r_1}{R} \right)^3 - 1 \right] \quad (4.138)$$

where r^* is the boundary between segments 1 and 2 through which the current is passing with reversed direction, that is, the critical state occupies the annulus region $r_1 < r^* < R$.

Supposing that t_b represents the moment at which only bipolar magnetization is conserved, once this time elapses, unipolar magnetization recovers, and t_b is determined by Equation (4.133). During the time range $t_0 \leq t \leq t_b$, bipolar magnetization can be given as:

$$M^*(t_b) = M^* \frac{1}{w_0^*} \left[4\sqrt{\frac{1 + w_0^*}{2}} - 3 \right]^{2/3} \quad (4.139)$$

where $w_0^* = M_0^*/M_{m0}$.

4.9 Mechanical Stability of Superconducting Magnets

Transient disturbances such as flux jump have serious effects on the stability of superconducting magnets, and mechanical disturbance is another kind of disturbance. Flux jump can be eliminated by processing with thin longitudinally extended superconductor or superconducting materials within a composite superconducting wire. Mechanical disturbance mainly results from release of electromagnetic energy due to cracks in filling materials amongst the turns and their movement, as subjected to the electromagnetic force when the superconducting magnet is energized. In the winding process of the magnet, some factors, such as pre-stress, are not large enough or are non-uniform and the arrangement of wire is not dense enough, there is gap between turns, and solidified materials are not properly used, etc., all resulting in movement of the wire in the superconducting magnet. When the superconducting magnet is energized, the wire in the magnet is exposed to a great electromagnetic force and so movement occurs in weak points of the magnet. Wire motion generates heat due to friction and then the temperature of the wire rises, which thereby causes instability and even quench of the superconducting magnet.

As long as wire in the energizing magnet moves only with an order of μm , heat generated due to friction will possibly cause quench of the superconducting magnet, so that mechanical disturbance has the greatest effect on winding processes. For example, current density is $J = 1 \times 10^9 \text{ A/m}^2$ in a magnetic field of $B = 5 \text{ T}$. If the superconducting wire moves $1 \mu\text{m}$ due to the electromagnetic force subjected, the work done by the electromagnetic force per unit volume is 5 kJ/m^3 . If a copper matrix to superconductor volume ratio is 2 in NbTi/Cu composite superconducting wire with volumetric heat capacity of $\gamma C = 1.85 \text{ kJ/(m}^3 \cdot \text{K)}$ at temperature 4.2 K , work done by the motion of $1 \mu\text{m}$ transforms into heat, which will cause a temperature rise of $\Delta T \approx 5/1.85 = 2.7 \text{ K}$ and possibly causes the superconducting magnet to fully quench.

Epoxy or paraffin impregnation solidification methods are commonly used to restrain the motion of the wire in the magnet and thus greatly improve the mechanical stability of the superconducting magnet. These methods are effective, especially for small-scale magnets with low magnetic fields, but they are not suitable for large-scale or high magnetic fields, because the organic impregnation and solidification materials become fragile at cryogenic temperatures and cracks occur due to strong electromagnetic forces and consequently cause mechanical disturbance. The fracture in organic solidification materials is the consequence of the stored elastic deformation energy propagating abruptly in the fragile material. When a fracture appears in organic solidified material, part of the deformation energy transfers to heat. Because there is different thermal contractions between organic material and metal or alloy materials, in the case of uniaxial stress, the deformation energy of solidification material per unit volume is:

$$Q_{\sigma_1} = \frac{\sigma^2}{2Y} = \frac{Y\varepsilon^2}{2} \quad (4.140)$$

where σ , ε and E are the stress, strain and Young's modulus of solidification material, respectively.

In the case of triaxial stress, its deformation energy is:

$$Q_{\sigma_3} = \frac{3\sigma^2(1-2\nu)}{2Y} = \frac{3Y\varepsilon^2}{2(1-2\nu)} \quad (4.141)$$

where ν is Poisson ratio. Taking resin as an example of organic solidified materials, heat contraction of the resin is approximately 0.01141 in the temperature range of 4.2 K to 293 K , Young's modulus is $6.9 \times 10^9 \text{ Pa}$ at 4.2 K and the Poisson ratio is 0.32. By substituting these data into Equations (4.140) and (4.141), Q_{σ_1} and Q_{σ_2} are separately $1.7 \times 10^5 \text{ J/m}^3$ and $1.5 \times 10^6 \text{ J/m}^3$, which are equivalent to a potential energy storage with a large capacity. Extreme destruction will be produced when potential energy releases due to cracking. Thus, the deformation energy of heat contraction should also be simultaneously considered when considering electromagnetic deformation in the design of a superconducting magnet. In addition,

a variety of effective solidification methods must be chosen, so that the production of cracks may be reduced as much as possible.

The mechanical properties are related to structure material and solidified materials in a superconducting magnet, such as Poisson ratio ν , stress σ , strain ε and Young's modulus E . Generally, there are two measures to eliminate the mechanical instability of a superconducting magnet: one is that pre-stress is properly increased in the winding process, so that pre-stress tension is larger than electromagnetic stress and so retains a densely uniform arrangement so as to decrease or eliminate gaps amongst turns during winding; the other is for the magnet to be immersed in epoxy resin or paraffin for solidification, and sometimes glass fibre is mixed with the epoxy resin or paraffin to decrease cracking at cryogenic temperatures. If cooling channels are needed amongst the windings, support must be inserted, and a thin but uniform agent is coated on to each layer to keep turns fastened. These measures can effectively improve the mechanical stability of the superconducting magnet.

4.10 Degradation and Training Effect of Superconducting Magnets

There are two reasons for the critical current degradation and training effect, which significantly affect the characteristics of superconducting magnets [18]. Especially in development at the early stages, these effects perplexed researchers and developers. This section aims to describe these phenomena.

4.10.1 Degradation of Superconducting Magnets

After a magnet is wound by a superconducting material, its performance is always inferior to a superconducting short sample. This phenomenon is called the degeneration of the superconducting magnet.

Figure 4.22 shows dependence of the critical current on the magnetic field in a superconducting short sample material and load line of a superconducting magnet. Critical current $I_c(B)$ continuously decreases with an increase of the magnetic field, as shown in curve 1. As shown in straight line 2, the magnetic field of the superconducting magnet is proportional to the operating current, and straight line 2 is called the load line. The ordinate value at the intersection point P of the two lines corresponds to the critical current of the superconducting wire I_{cP} . When the current of the superconducting magnet increases to the point Q, the corresponding critical current is I_{cQ} , which is often smaller than I_{cP} , and quench of superconducting magnet occurs.

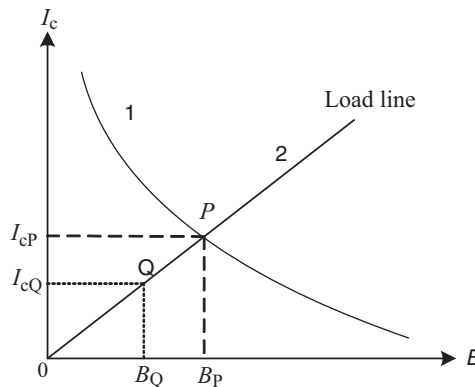


Figure 4.22 Critical current versus magnetic field in the superconducting short sample and load line of superconducting magnet.

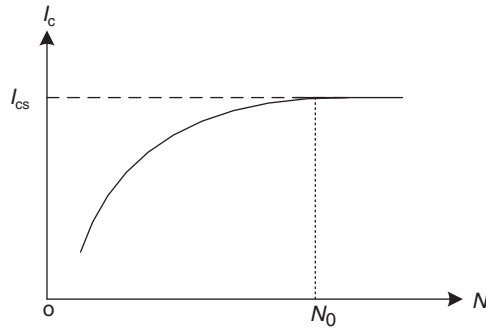


Figure 4.23 Training effect of superconducting magnet.

4.10.2 Training Effects of Superconducting Magnets

When the superconducting magnet is energized again after first quenching, it is found that its critical current is higher. By repeating the process of energizing and quench, every quench will be unlike the former, with the critical current increasing after every quench. Finally its critical current reaches a stable level, and this phenomenon is defined as the training effect of a superconducting magnet – improvement of superconducting characteristics of a superconducting magnet after it has been cycled in temperature or magnetic fields and quenched several times. Figure 4.22 shows the training effect in a superconducting magnet after many quenching cycles, where the horizontal and vertical ordinates represent the times of quench and critical current of the superconducting magnet, respectively, and N_0 denotes quench times over which the critical current of the magnet reaches steady value I_{cs} . Its essence is caused by electromagnetic and mechanical disturbances in the superconducting magnet, such as the flux jump, mechanical effect, etc., which were introduced in Sections 4.2 to 4.8.

Figure 4.23 is a diagram of the training effect of a superconducting magnet quenched many times. In the figure, the abscissa represents the times of quench (training), and the ordinate represents the critical current of the superconducting magnet. When the times of quench for the superconducting magnet exceed N_0 , the critical current of magnet attains a steady value. It is essentially caused by the electromagnetic disturbance and mechanical disturbance of the superconducting magnet, such as the flux jump effect, mechanical effect, etc.

In practical situations, degradation and training effect of a superconducting magnet occur simultaneously. After several times (N_0) of repeated quenching, the performance of the magnet can improve further. The degree of improvement is determined by performance of superconducting materials, structure design, winding and enforcement techniques of the superconducting magnet.

4.11 Quench and Protection of Superconducting Magnets

Quench of superconducting magnets is a serious problem, which can result in burning of the magnets and dangerous explosions, since the energy stored in a magnet suddenly discharges and then liquid coolant (especially for LHe with little latent heat) abruptly evaporates, and the pressure of the cryostat sharply increases. Therefore, it is necessary to describe mechanisms of quench and provide effective protection measures for their safe operation.

4.11.1 Resistance Increase and Current Decay in Quench Processes

Resistance increase and current variation during quench processes are difficult problems because there are complicated nonlinear differential equations to be solved, which generally have no analytical solutions, so we have to resort to numerical solutions. To obtain satisfied approximate solutions, the following hypotheses are made:

- (1) The current density J_0 keeps constant. When the storage energy of superconducting magnet ($LI^2/2$) is completely dissipated, J_0 rapidly reduces to zero.
- (2) Temperature rise is determined by $\int J^2 dt = J_0^2 t_d = \int_{T_0}^T \frac{\gamma C(T)}{\rho(T)} dT = U(T)$, and $U(T) = U(t_0)(T/t_0)^{1/2}$, where t_0 is the reference temperature, t_d and J_0 are the time decay constant of current and the operating current density, respectively.
- (3) Resistance varies linearly with temperature.

First, we should consider the induced voltage at the two ends of a superconducting magnet (almost all the voltage falls into the quench part of the coil) after its quench. In practical applications, when the quench voltage is detected after each quench, the switch between the superconducting magnet and the power supply automatically turns off. Even if the switch does not turn off, induced voltage inside the superconducting coil is opposite to the voltage in the resistance and voltage in the power supply is low, at no more than 20 V, but the induced voltage is generally several hundreds of V or even more than 1 kV, so that the voltage of power supply is usually neglected in calculations.

Figure 4.24 shows voltage distribution and equivalent circuit in the inner coil after quench of the superconducting magnet has occurred, where V_{CS} is voltage of the superconducting coil, R_Q refers to the resistance of the normal zone in the coil and $I(t)$ denotes the current. When neglecting voltage of the power supply, the circuit equation can be written as:

$$V_Q(t) = I(t)R_Q(t) - L \frac{dI(t)}{dt} \quad (4.142)$$

$$L \frac{dI(t)}{dt} = I(t)R_Q(t) \quad (4.143)$$

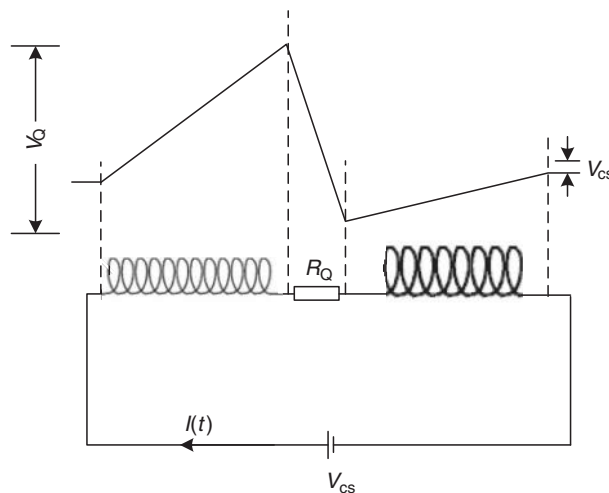


Figure 4.24 Voltage distribution within coil after quenching.

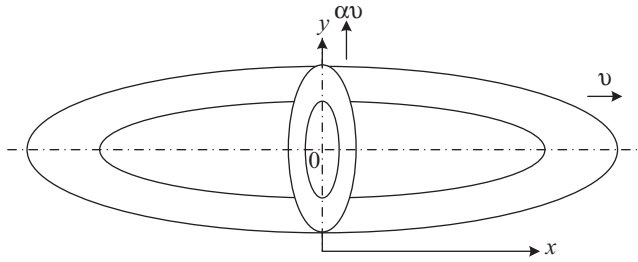


Figure 4.25 Formation and propagating of 3D normal zone.

where L and M are self-inductance of the whole superconducting coil and the mutual inductance between the normal zone and other parts of the coil, respectively. V_Q is terminal voltage of the normal zone in the superconducting coil. Combining Equations (4.142) and (4.133), we obtain:

$$V_Q(t) = I(t)R_Q(t) \left(1 - \frac{M}{L} \right) \quad (4.144)$$

This indicates that the mutual inductance M between the normal zone and other parts of the coil increases with extension of the normal zone; meanwhile, the resistance of the normal zone R_Q increases too, but the current $I(t)$ decreases, so that the inner voltage will rise to its maximum and then decrease.

The current of the superconducting coil exponentially decays after quench occurs, and the normal zone formed is usually an ellipse. Assuming that the boundary temperature of the normal zone is T_b , the highest temperature is at the centre of the normal zone, and the boundary of the normal zone is an isothermal ellipsoid (Figure 4.25). Assuming that the longitudinal propagation velocity of a normal zone in quench time t is v , the length of major axis in the ellipse is $2x = 2vt$, the transverse (short axis) diameter is $2y = 2\alpha vt$, α is the ratio of propagation velocity in the transverse normal zone to that of the longitudinal normal zone, and the resistance of the normal zone increases with the increase of the volume of ellipse.

Assuming that the cross-sectional area of the superconductor is A , the resistivity of the normal state is $\rho(T)$, then resistance of the normal zone is:

$$R = \int_0^x \frac{4\pi x^2 \alpha^2 \rho(T)}{A^2} dx \quad (4.145)$$

where

$$\rho(T) = \rho_0 \left(\frac{T}{T_0} \right) = \rho_0 \left(\frac{U}{U_0} \right)^{1/2} = \rho_0 \frac{J_0^4 \tau^2}{U_0^2} \quad (4.146)$$

where τ is time passed in formation of the normal zone, $\tau = 0$ at boundary of ellipse and $\tau = t$ at the centre of ellipse. In any point within the ellipse, $\tau = t - x/v$. The resistance becomes:

$$R(t) = \int_0^{vt} \frac{4\pi \rho_0 x^2 \alpha^2 J_0^4 (t - x/v)^2}{A^2 U_0^2} dx = \frac{4\pi \rho_0 \alpha^2 J_0^4 U^3 t^5}{30 A^2 U_0^2} \quad (4.147)$$

The time required to consume the total magnetic energy storage ($LI^2/2$) by resistance is defined by t_Q , then:

$$\int_0^{t_Q} I^2 R(t) dt = \frac{1}{2} LI^2 \quad (4.148)$$

Substituting Equation (4.145) into Equation (4.148), we obtain:

$$t_Q = \left[\frac{90 L U_0^2 A^2}{4 \pi J_0^4 \rho_0 \alpha^2 v^2} \right]^{1/6} \quad (4.149)$$

As the boundary is not considered, the actual decay time constant is longer than the characteristic time t_Q , but t_Q can still be regarded as the basic quench parameter. The highest temperature rise can be calculated according to $U(T) = U(t_0)(T/t_0)^{1/2}$:

$$T_{\max} = \frac{J_0^4 t_Q^2 T_0}{U_0^2} = T_0 \left(\frac{90 L A^2 J_0^8}{4 \pi \rho_0 \alpha^2 v^3 U_0^4} \right)^{1/3} \quad (4.150)$$

After superconducting coil quenches, the current decays in the following form:

$$I(t) = I_0 \exp \left(- \int R(t) dt / L \right) = I_0 \exp \left(- \frac{1}{2} \left(\frac{t}{t_Q} \right)^6 \right) \quad (4.151)$$

When neglecting the mutual inductance M between the normal zone and other parts within the superconducting coil and substituting Equation (4.151) into Equation (4.144), we obtain the voltage within the superconducting coil:

$$V = L \frac{dI(t)}{dt} = \frac{L I_0}{t_Q} 3 \left(\frac{t}{t_Q} \right)^5 \exp \left[- \frac{1}{2} \left(\frac{t}{t_Q} \right)^6 \right] \quad (4.152)$$

Because mutual inductance in Equation (4.144) is neglected in calculations of Equation (4.152), the quench voltage within the superconducting coil is overestimated. When $(t/t_Q) = (5/3)^{1/5}$, the voltage takes the maximum value, that is:

$$V_{\max} = \frac{3 L I_0}{t_Q} \left(\frac{5}{3} \right)^{5/6} \exp \left(- \frac{5}{6} \right) \approx \frac{2 L I_0}{t_Q} \quad (4.153)$$

4.11.1.1 Normal Zone Bounded in One Dimension

Considering that the normal zone propagates in 1D (Figure 4.26), it arrives at the boundary $x = \pm a$ from the original point $x = 0$. The normal zone then propagates only in two-dimensional (2D) space, and the rate of increase of resistance decreases with increase of quench time. After time t_a is passed, the propagation area of the normal zone can be approximately regarded as a hollow cylindrical shell, and its disposal route is similar to Equation (4.135), where $a = t_a v$, $x = vt$; when $t < t_a$, and the resistance is directly given by Equation (4.145):

$$R(t) = \int_0^a \frac{4 \pi \alpha^2 x^2 \rho(T)}{A^2} dx + \int_a^x \frac{4 \pi \alpha^2 a x \rho(T)}{A^2} dx \quad (4.154)$$

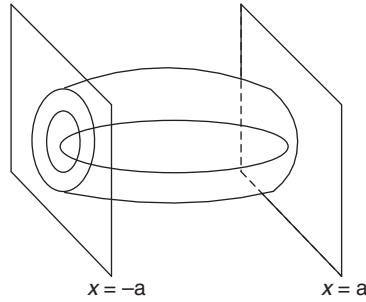


Figure 4.26 NPZ of 1D model.

Taking time integration with Equation (4.154), we obtain:

$$\begin{aligned} \int R(t)dt &= \frac{4\pi\alpha^2\rho_0 J_0^4 v^3}{180A^2 U_0^2} \left\{ \int_0^{t_a} 6t^5 dt + \int_{t_a}^t (15t_a t^4 - 30t_a^3 t^2 + 30t_a^4 t - 9t_a^5) dt \right\} \\ &= \frac{1}{2}L \left[3 \left(\frac{t_a}{t_Q} \right) \left(\frac{t}{t_Q} \right)^5 - 10 \left(\frac{t_a}{t_Q} \right)^3 \left(\frac{t}{t_Q} \right)^3 + 15 \left(\frac{t_a}{t_Q} \right)^4 \left(\frac{t}{t_Q} \right)^2 - 9 \left(\frac{t_a}{t_Q} \right)^5 \left(\frac{t}{t_Q} \right) + 2 \left(\frac{t_a}{t_Q} \right)^6 \right] \end{aligned} \quad (4.155)$$

It is assumed that storage energy of the superconducting coil is totally dissipated by resistance of the normal zone during time t_d , and the resistance R is determined by $\int_0^{t_d} I^2 R dt = \frac{1}{2}LI^2$, where t_d is the effective current decay time. By substituting Equation (4.155) into the storage energy equation, we have:

$$3 \left(\frac{t_a}{t_Q} \right) \left(\frac{t_d}{t_Q} \right)^5 - 10 \left(\frac{t_a}{t_Q} \right)^3 \left(\frac{t_d}{t_Q} \right)^3 + 15 \left(\frac{t_a}{t_Q} \right)^4 \left(\frac{t_d}{t_Q} \right)^2 - 9 \left(\frac{t_a}{t_Q} \right)^5 \left(\frac{t_d}{t_Q} \right) + 2 \left(\frac{t_d}{t_Q} \right)^6 = 1 \quad (4.156)$$

When $t_a \ll t_Q$, we find the resistance:

$$R(t) = \frac{15L}{2t_Q} \left(\frac{t_a}{t_Q} \right) \left(\frac{t}{t_Q} \right)^4 \quad (4.157)$$

Let $t = t_d$, thus we have:

$$\left(\frac{t_d}{t_Q} \right) = \left(\frac{1}{3} \frac{t_Q}{t_a} \right)^{1/5} \quad (4.158)$$

The maximum temperature rise is:

$$T_{\max} = \frac{J_0^4 t_d^2 T_0}{U_0^2} \quad (4.159)$$

After quench, the current decays with:

$$I(t) = I_0 \exp \left(- \int R dt / L \right) \\ = I_0 \exp \left\{ - \frac{1}{2} \left[3 \left(\frac{t_a}{t_Q} \right) \left(\frac{t}{t_Q} \right)^5 - 10 \left(\frac{t_a}{t_Q} \right)^3 \left(\frac{t}{t_Q} \right)^3 + 15 \left(\frac{t_a}{t_Q} \right)^4 \left(\frac{t}{t_Q} \right)^2 \right] \right. \\ \left. - 9 \left(\frac{t_a}{t_Q} \right)^5 \left(\frac{t}{t_Q} \right) + 2 \left(\frac{t_a}{t_Q} \right)^6 \right\} \quad (4.160)$$

If $t_a \ll t_Q$, the expression of current decay is:

$$I(t) = I_0 \exp \left[- \frac{3}{2} \left(\frac{t_a}{t_Q} \right) \left(\frac{t}{t_Q} \right)^5 \right] \quad (4.161)$$

In this case, the maximum voltage in the normal zone is:

$$V_m = \frac{15}{2} \frac{L I_0}{2 t_Q} \left(\frac{t_a}{t_Q} \right)^{1/5} \left(\frac{8}{15} \right)^{4/5} \exp \left(- \frac{4}{5} \right) \approx \frac{2 L I_0}{t_Q} \left(\frac{t_a}{t_Q} \right)^{1/5} \quad (4.162)$$

In Figure 4.26, although the normal zone encounters the boundary at the direction perpendicular to the major axis, it continuously propagates in other 2D space, which is also perpendicular to the major axis direction. Therefore, the expression for t_a is equally effective for calculation of NZP in other directions.

4.11.1.2 Normal Zone Bounded in Two-Dimension

If propagation of the normal zone in 2D space encounters the boundary (Figure 4.27), extension of the normal zone is limited in two directions. Volume of the normal zone can be expressed by a disk with cross-section πa^2 and thickness $v \Delta t$ after time t_a is passed, and its total resistance is:

$$R(t) = \frac{4\pi a^2 \rho_0 J_0^4 v^3}{180 A^2 U_0^2} (30 t_a^2 t^3 - 30 t_a^3 t^2 + 6 t_a^5) \quad (4.163)$$

By using Equation (4.155), and integrating with $t < t_a$, we obtain:

$$\int R(t) dt = \frac{1}{4} L \left[15 \left(\frac{t_a}{t_Q} \right)^2 \left(\frac{t}{t_Q} \right)^4 - 20 \left(\frac{t_a}{t_Q} \right)^3 \left(\frac{t}{t_Q} \right)^3 + 12 \left(\frac{t_a}{t_Q} \right)^5 \left(\frac{t}{t_Q} \right) - 5 \left(\frac{t_a}{t_Q} \right)^6 \right] \quad (4.164)$$

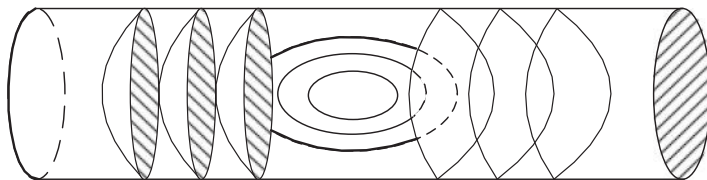


Figure 4.27 NZP of 2D model.

The time that the total storage energy is consumed by the normal zone of the superconducting coil is determined by:

$$\frac{15}{2} \left(\frac{t_a}{t_Q} \right)^2 \left(\frac{t}{t_Q} \right)^4 - 10 \left(\frac{t_a}{t_Q} \right)^3 \left(\frac{t}{t_Q} \right)^3 + 6 \left(\frac{t_a}{t_Q} \right)^5 \left(\frac{t}{t_Q} \right) - \frac{5}{2} \left(\frac{t_a}{t_Q} \right)^6 = 1 \quad (4.165)$$

As for propagation of the 2D normal zone, when time $t_a < t_Q$, Equation (4.165) is approximately equal to:

$$\frac{t_d}{t_Q} \approx \left[\frac{2}{15} \left(\frac{t_Q}{t_a} \right)^2 \right]^{1/4} \quad (4.166)$$

If the normal zone in two directions reaches the boundary with two different times t_a and t_b , and both of the times are less than t_Q , then the decay time of the current is:

$$\frac{t_d}{t_Q} \approx \left(\frac{2}{15} \frac{t_Q}{t_a} \frac{t_Q}{t_b} \right)^{1/4} \quad (4.167)$$

The maximum temperature rise is given by Equation (4.159).

Then the current decay equation is:

$$\begin{aligned} I(t) &= I_0 \exp \left[- \int R(t) dt / L \right] \\ &= I_0 \exp \left\{ - \frac{1}{4} \left[\left(\frac{t_a}{t_Q} \right)^2 \left(\frac{t}{t_Q} \right)^4 - 20 \left(\frac{t_a}{t_Q} \right)^3 \left(\frac{t}{t_Q} \right)^3 + 12 \left(\frac{t_a}{t_Q} \right)^5 \left(\frac{t}{t_Q} \right) - 5 \left(\frac{t_a}{t_Q} \right)^6 \right] \right\} \end{aligned} \quad (4.168)$$

When $t_a < t_Q$, the current decay is approximately equal to:

$$I(t) = I_0 \exp \left[- \frac{15}{4} \left(\frac{t_a}{t_Q} \right)^2 \left(\frac{t}{t_Q} \right)^4 \right] \quad (4.169)$$

If two boundaries encounter different boundaries within different times, then the current decay is:

$$I(t) = I_0 \exp \left[- \frac{15}{4} \left(\frac{t_a}{t_Q} \right) \left(\frac{t_b}{t_Q} \right) \left(\frac{t}{t_Q} \right)^4 \right] \quad (4.170)$$

Then, the maximum quench voltage is:

$$V_m = \frac{3LL_0}{t_Q} \left[5 \left(\frac{t_a}{t_Q} \right) \left(\frac{t_b}{t_Q} \right) \right]^{1/4} \exp \left(- \frac{3}{4} \right) \approx \frac{2.1LL_0}{t_Q} \left[\left(\frac{t_a}{t_Q} \right) \left(\frac{t_b}{t_Q} \right) \right]^{1/4} \quad (4.171)$$

When time t_a and t_b are relatively small, variation of the resistance with time in the normal zone is:

$$R(t) = \frac{15Lt_a t_b t^3}{t_Q^6} \quad (4.172)$$

If t_a is properly selected, orientation between the normal zone and the boundary is insignificant. As with circular cross-sections, we need to multiply the aspect ratio of the ellipse α with the longitudinal cross-section, and then the resistance can be directly obtained.

4.11.1.3 Normal Zone Bounded in Three –Dimension

If the normal zone encounters a boundary in 3D space, the volume of the normal zone can be expressed by a disk with cross-section πa^2 and thickness $v\Delta t$ after time t_a is passed, and its total resistance is:

$$R(t) = \frac{4\pi a^2 \rho_0 J_0^4 v^3}{180A^2 U_0^2} (60t_a^3 t^2 - 90t_a^4 t + 36t_a^5) \quad (4.173)$$

When $t < t_d$, the integration of resistance with time is:

$$\int R(t)dt = \frac{1}{2}L \left[20 \left(\frac{t_a}{t_Q} \right)^3 \left(\frac{t}{t_Q} \right)^3 - 45 \left(\frac{t_a}{t_Q} \right)^4 \left(\frac{t}{t_Q} \right)^2 + 36 \left(\frac{t_a}{t_Q} \right)^5 \left(\frac{t}{t_Q} \right) - 10 \left(\frac{t_a}{t_Q} \right)^6 \right] \quad (4.174)$$

The time required for dissipating the stored energy in the superconducting coil is determined by:

$$20 \left(\frac{t_a}{t_Q} \right)^3 \left(\frac{t_d}{t_Q} \right)^3 - 45 \left(\frac{t_a}{t_Q} \right)^4 \left(\frac{t_d}{t_Q} \right)^2 + 36 \left(\frac{t_a}{t_Q} \right)^5 \left(\frac{t_d}{t_Q} \right) - 10 \left(\frac{t_a}{t_Q} \right)^6 = 1 \quad (4.175)$$

When time $t_a \ll t_Q$:

$$\frac{t_a}{t_Q} \approx \left[20 \left(\frac{t_Q}{t_a} \right)^3 \right]^{1/3} \quad (4.176)$$

If the normal zone encounters the boundary in three directions within three different times t_a , t_b , t_c , and all three are much less than t_Q , then the current decay constant is:

$$\frac{t_a}{t_Q} \approx \left[20 \left(\frac{t_Q}{t_a} \right) \left(\frac{t_Q}{t_a} \right) \left(\frac{t_Q}{t_a} \right) \right]^{1/3} \quad (4.177)$$

The expression of the current decay is:

$$I(t) = I_0 \exp \left\{ -\frac{1}{2} \left[20 \left(\frac{t_a}{t_Q} \right)^3 \left(\frac{t}{t_Q} \right)^3 - 45 \left(\frac{t_a}{t_Q} \right)^4 \left(\frac{t}{t_Q} \right)^2 + 36 \left(\frac{t_a}{t_Q} \right)^5 \left(\frac{t}{t_Q} \right) - 10 \left(\frac{t_a}{t_Q} \right)^6 \right] \right\} \quad (4.178)$$

If time $t_a \ll t_Q$, Equation (4.178) can be simplified as:

$$I(t) = I_0 \exp \left[-10 \left(\frac{t_a}{t_Q} \right)^3 \left(\frac{t}{t_Q} \right)^3 \right] \quad (4.179)$$

When the normal zone reaches the boundary in three directions at three different times, which are much smaller than t_Q , the current decay is:

$$I(t) = I_0 \exp \left[-10 \left(\frac{t_a}{t_Q} \right) \left(\frac{t_b}{t_Q} \right) \left(\frac{t_c}{t_Q} \right) \left(\frac{t}{t_Q} \right)^3 \right] \quad (4.180)$$

As for the normal zone encountering the boundary in three different directions within different times t_a , t_b , t_c , and with all three of them being less than t_Q , the current decay is:

$$I(t) = I_0 \exp \left[-10 \left(\frac{t_a}{t_Q} \right) \left(\frac{t_b}{t_Q} \right) \left(\frac{t_c}{t_Q} \right) \left(\frac{t}{t_Q} \right)^3 \right] \quad (4.181)$$

The inner maximum voltage is:

$$V_m = \frac{30LI_0}{t_Q} \left[\frac{1}{225} \left(\frac{t_a}{t_Q} \right) \left(\frac{t_b}{t_Q} \right) \left(\frac{t_c}{t_Q} \right) \right]^{1/3} \exp \left(-\frac{2}{3} \right) \approx \frac{2.5LI_0}{t_Q} \left[\left(\frac{t_a}{t_Q} \right) \left(\frac{t_b}{t_Q} \right) \left(\frac{t_c}{t_Q} \right) \right]^{1/3} \quad (4.182)$$

It should be noted that here we neglect the mutual inductance M between the normal zone and other parts within the coil and self inductance L in the normal zone in calculation of the quench voltage within the above analyses of the three situations. Consequently, the exact calculation should refer to Equation (4.144) by numerical simulation.

4.11.2 Factors Causing Quench

In the case of enough heat disturbance, superconductivity will disappear, that is, quench occurs, and the superconductor returns to the normal state if the operating parameters of the superconducting magnet exceed their critical parameters such as critical magnetic field B_c , critical current density J_c and critical temperature T_c .

There are many factors that cause a superconducting coil to quench, such as mechanical disturbance, heat disturbance, heat leakage, nuclear radiation, flux jump, and AC loss, etc. Mechanical disturbance (wire motion caused by the electromagnetic force) mainly includes winding deformation or a solidification agent cracking in cryogenic temperature. Heat disturbance includes flux jump within the superconducting wire, AC losses and dissipation caused by current redistribution amongst superconducting strands due to electromagnetic disturbance, heat leakage from current leads and heat conduction from measurement taps. Others mainly include nuclear radiation and beam radiation. Time and energy spectra of all disturbances are illustrated in Figure 4.28, amongst which energy density of AC loss is the highest, while the energy density of heat leakage is the lowest [12]. The period of heat leakage, nuclear radiation and AC loss are longer than one second, and belong to long-term disturbance and can be restrained by related processing technologies. The action time of radiation is in the range of 0.001 s to 100 s, and can be limited by radiation shielding. Whereas energy densities of wire motion and flux jump are approximated to the radiation energy density with a short action time, both of them belong to the transient disturbances.

A superconducting magnet is made of superconducting wire, composite filling material, and former and structural material. Quench phenomenon always occurs first at the locality of the superconducting coil and then propagates throughout the whole magnet. When quench of the superconducting magnet occurs due to disturbance, the transported current in the superconductor transfers to the stabilization matrix of the superconducting wire, resulting in a local temperature rise. In order to restrain this temperature rise, the transported current in the superconducting magnet should rapidly decay and energy stored in the superconducting magnet is absorbed by the superconducting magnet itself or transferred into a protection system. When external disturbance takes place in the superconducting magnet, current sharing occurs in the superconducting wire and the temperature in the superconducting magnet increases and subsequently reaches a maximum value at the end of the disturbance.

After the disturbance disappears, the temperature of the heated point in the superconducting magnet begins to decrease. The quench and recovery process depends on heat generation and heat transfer powers. If energy of the superconducting magnet is absorbed by external protection resistance, the protection

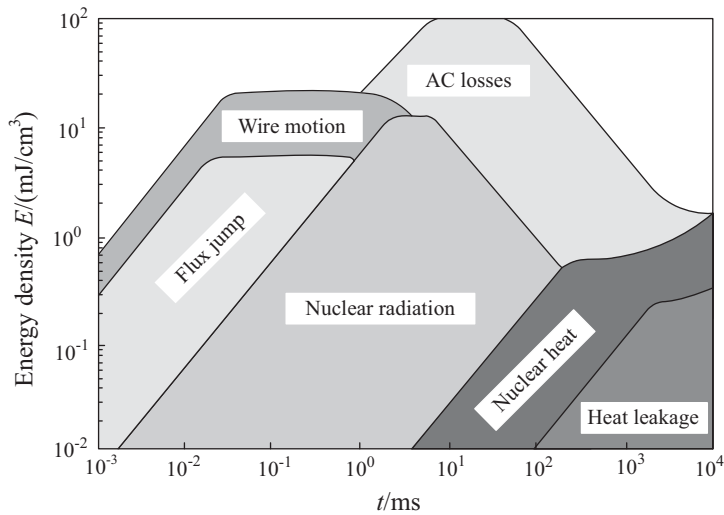


Figure 4.28 Time and energy spectra of several disturbance sources for quench of superconducting magnet. (Reproduced with permission from Yinshun Wang, “Bases of Applied Superconductivity in Electrical Engineering,” Science Press, 108, © 2011.)

circuit may be placed at room temperature or in a cryogenic environment. Some superconducting magnets themselves have large enough enthalpy, so that they can absorb energy stored in the magnet itself, which leads to the temperature being lower than the safety temperature margin of its design value. In practical applications, heat and electromagnetic disturbance with high intensity usually lead to an uncontrollable extension of the normal zone in the superconducting magnet, and that makes the recovery from quench impossible. Studies on the quench of the superconducting magnet include the quench process and related variation of physical parameters and hazard assessment. Meanwhile, effective detection and protection methods have been developed to prevent superconducting magnets from damage during and after quenching. Therefore, the quench protection technique for superconducting magnet is an important part of the superconducting magnet technique.

When quench of a superconducting magnet occurs, energy stored within it will transfer into heat. If the cryostat has enough cooling capacity to absorb the stored energy and the normal zone can extend to most parts of the magnet in a short time, this kind of superconducting magnet does not need external protection; it is enough just to turn off the energizing power supply. But if energy stored in the superconducting magnet is relatively large, it is necessary to protect the superconducting magnet from damage by using an effective external detecting circuit and a protection circuit. The balance of bridge-circuit voltage is commonly used to detect quench of superconducting magnet. However, the balance of bridge voltage is problematic on occasions of a pulse superconducting magnet and large electromagnetic disturbance and other advanced detecting technology can be adopted, such as optical fibre sensing technology, and ultrasonic emission and detection methods.

At present, the developed protection methods are active protection and passive protection technologies, which mainly include external resistance, secondary coupling circuit and multi-sectional external resistance. The most important part of protection circuit is external protection resistance in cryogenic temperatures or at room temperature. Generally, protection resistance should be placed in a room temperature environment. Conversely, it is commonly placed in a cryogenic temperature environment for a superconducting magnet with storage energy smaller than 1 MJ.

The quench protection measures for a superconducting magnet should satisfy the following requirements:

- (1) to reduce releasing energy from the normal zone in windings of the superconducting magnet to prevent superconducting wire from overheating and even damage,
- (2) to decrease terminal voltage of the normal zone of superconducting windings in order to avoid insulating breakdown amongst turns,
- (3) to reduce energy released from inner parts of the superconducting magnet into the cryostat and then prevent the cryogen from tremendous evaporation and high pressure.

In general, protection of the superconducting magnet is mainly classified into two categories: active and passive.

4.11.3 Active Protection

Active protection is also called external protection, in which the energy of the magnet can be rapidly transferred and released when magnet quench occurs.

4.11.3.1 External Resistance Protection

Figure 4.29 shows the equivalent circuit of external resistance protection. The superconducting magnet is placed in a cryostat, and the two terminals of the magnet L_m are connected in parallel with releasing energy resistance R_D outside the cryostat. Both of them are connected in series with switch S, which are connected with power supply SS. Under normal operating conditions, S is turned on, the resistance of the superconducting magnet is zero, and the total current I passes through the superconducting magnet L_m due to its zero resistance. Stored energy in the magnet is then $E_m = L_m I^2 / 2$. Resistance of the normal zone appears when quench occurs and S is turned off. The closed circuit consists of the superconducting

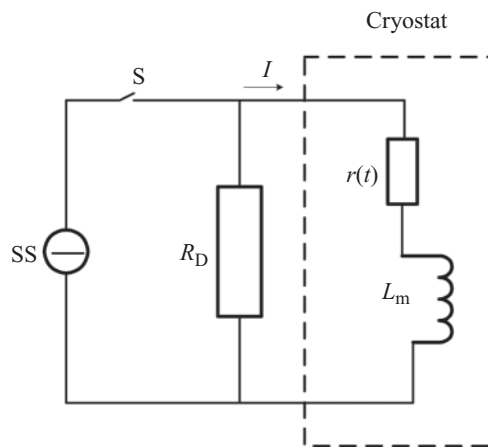


Figure 4.29 Equivalent circuit of external resistance protection method.

magnet, the normal zone resistance $r(t)$ and the external resistance R_D , and in this case the circuit equation is:

$$L_m \frac{dI(t)}{dt} + [r(t) + R_D]I(t) = 0 \quad (4.183)$$

When inner resistance $r(t) \ll R_D$, the current in the closed circuit decays exponentially in accordance with:

$$I(t) = I(0) \exp\left(-\frac{R_D t}{L_m}\right) \quad (4.184)$$

where $I(0)$ is operating current of superconducting magnet at $t = 0$ moment at which it begins quenching.

The voltage generated at terminals of the magnet is:

$$V_D = \left(\frac{A_m}{A}\right) \frac{J^2 E}{U(T_i, T_f)} \quad (4.185)$$

where A_m , A , J , I and E are cross-section of the stabilization matrix of the superconducting wire in the magnet, the total cross-section of the superconducting wire, operating current density, operating current and stored energy, respectively. T_i and T_f are the initial temperature of the hot point in the normal zone and permitted upper limit of the temperature of the hot point in the superconducting magnet; function U is described by:

$$U(T_i, T_f) = \int_{T_i}^{T_f} \frac{\gamma C(T)}{\rho_n(T)} dT \quad (4.186)$$

where γC and ρ_n are volumetric heat capacity and normal resistivity of the superconducting wire in the magnet, respectively. The condition at which the temperature of the superconducting magnet is not more than T_f is:

$$U(T_i, T_f) \geq \left(\frac{A_m}{A}\right) \frac{J_m^2 E_m}{V_D I} \quad (4.187)$$

where V_D is voltage of external resistance R_D .

Almost all of the stored energy E_m is dissipated by the external energy releasing resistance R_D , which provides protection to the superconducting magnet. Here design of the releasing energy resistance is important. Its value should be determined by the maximum temperature rise $T_{\max}$ of the superconducting magnet, and $T_{\max}$ is no more than the temperature that the superconducting magnet can withstand. If R_D is too large, a high voltage will be generated at the terminals of the magnet and possibly lead to destruction of the insulation amongst the turns of magnet. However, if it is too small, the temperature rise of the magnet is possibly higher than what it is able to withstand and even result in burning of the magnet.

4.11.3.2 Bridge Circuit Protection

The bridge circuit protection of a superconducting magnet is based on it being divided into two equal sections. The method of its circuit connection is similar to external resistance protection. Figure 4.30 illustrates the equivalent circuit of bridge protection. There are two branches in the circuit: two arms of the bridge are made of external energy releasing resistances R_{D1} , R_{D2} and self inductances L_{m1} , L_{m2} .

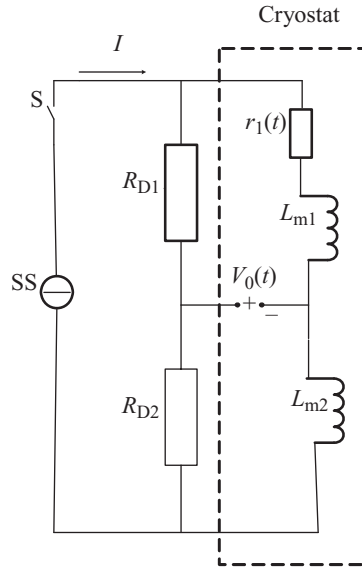


Figure 4.30 Voltage detection circuit with bridge circuit.

Under normal operating conditions and neglecting the mutual inductance between the two sections, the balance of the bridge requires:

$$\frac{R_{D2}}{L_{m2}} = \frac{R_{D1}}{L_{m1}} \quad (4.188)$$

When the bridge circuit balances, the output voltage is zero, $V_0 = 0$. Nevertheless, supposing that quench occurs in the first branch, the normal resistance generated is $r_1(t)$; switch S turns off, a closed circuit consisting of resistances R_{D1} , R_{D2} , $r_1(t)$ and two self inductances L_1 , L_2 of magnet is formed. If $(R_{D1} + R_{D2})$ is much greater than $r_1(t)$, the terminal voltage of magnet is:

$$V_L(t) = L_1 \frac{dI(t)}{dt} + rI(t) + L_2 \frac{dI(t)}{dt} \quad (4.189)$$

The current flows through R_{D1} , R_{D2} is:

$$i_R(t) = \frac{V_L(t)}{R_{D1} + R_{D2}} \quad (4.190)$$

The output voltage of bridge circuit is:

$$V_o(t) = L_1 \frac{dI(t)}{dt} + rI(t) - R_{D1} i_R(t) \quad (4.191)$$

Substituting Equations (4.189) and (4.190) into Equation (4.191), we obtain the output voltage:

$$V_o(t) = \left(\frac{R_{D2}}{R_{D1} + R_{D2}} \right) L_1 \frac{dI(t)}{dt} - \left(\frac{R_{D1}}{R_{D1} + R_{D2}} \right) L_2 \frac{dI(t)}{dt} + \left(\frac{R_{D2}}{R_{D1} + R_{D2}} \right) rI(t) \quad (4.192)$$

In order to make the output voltage $V_o(t)$ proportional to $rI(t)$ only, the first two terms in Equation (4.192) should be zero:

$$\left(\frac{R_{D2}}{R_{D1} + R_{D2}} \right) L_1 \frac{dI(t)}{dt} - \left(\frac{R_{D1}}{R_{D1} + R_{D2}} \right) L_2 \frac{dI(t)}{dt} = 0 \quad (4.193)$$

Rearranging Equation (4.183), we can obtain the balance equation of the bridge circuit Equation (4.188). Then, the output voltage of bridge circuit is:

$$V_o(t) = \left(\frac{R_{D2}}{R_{D1} + R_{D2}} \right) rI(t) \quad (4.194)$$

where $I(t)$ is the decay current of the bridge circuit. Similar to the external energy releasing resistance protection, the choice of releasing resistance R_{D1} , R_{D2} is also essential, and the withstanding temperature rise during superconducting magnet quench is taken as the upper limit of the temperature. The temperature rise generated by a superconducting magnet quench is under the endurance domain, which is the upper limit when two energy releasing resistances are selected.

4.11.3.3 External Resistance Protection with Subdivisions

As shown in Figure 4.31, when dividing the windings of a superconducting magnet into n sections, the energy releasing resistance is R_D , and the self inductances are L_{m1} , L_{m2} , $\dots$, L_{mn} , respectively. The voltage taps of each section winding are externally connected with a voltage amplifier. Under normal operation conditions, the sum $V_o(t)$ of voltage difference between each odd and even section takes the minimum by adjusting the output gain of the amplifier in each section as:

$$V_o(t) = \sum_{i=1}^n [\alpha_{2i-1} V_{2i-1}(t) - \alpha_{2i} V_{2i}(t)] \quad (4.195)$$

where α_{2i-1} and α_{2i} are the amplifier gains of the $(2i-1)$ -th section and the $2i$ -th section, respectively. When quench occurs, the sum $V_o(t)$ of output voltage difference deviates from the smallest output

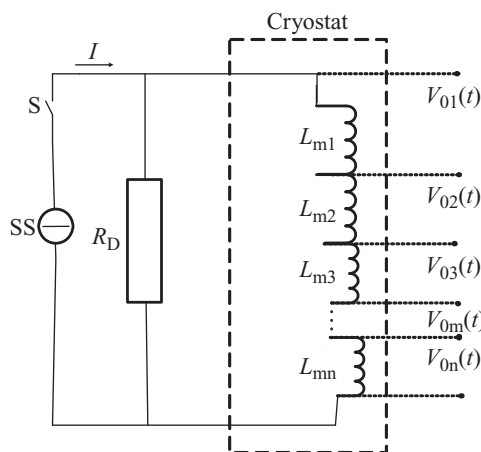


Figure 4.31 Equivalent circuit with subdivisions.

voltage, which is set in advance. The power supply is thereby switched off and then the energy stored in the magnet can be released into the closed circuit consisting of each section of magnet and external energy releasing resistance R_D .

4.11.4 Passive Protection

In the superconducting magnet with closed-loop operation such as a Magnetic Resonance Imaging (MRI) magnet, a Nuclear magnetic resonance (NMR) magnet, a densely winding magnet and a conduction cooled magnet, terminal voltage of the superconducting magnet is strictly limited, and both external releasing resistances of the superconducting magnet are usually placed in a cryostat for protection. If magnet quench occurs, most of the stored energy will transfer to the releasing resistance in the cryostat. The stored energy in the magnet should be released as uniformly as possible and the normal zone should propagate as fast as possible, to ensure that the magnet is not damaged. Thus, the superconducting magnet with multi-coils commonly uses a heating method to speed up the quench propagation velocity.

In the energy releasing circuit, the switch is usually placed in a cryostat. This kind of switch includes a superconducting switch, a cryogenic diode and a transistor. Releasing resistance mainly includes a surface heater and isolated metal tape, which is co-wound with the superconducting wire. It has two actions: it can limit the temperature rise of the hot spot in the magnet and can significantly decrease the inner voltage within the superconducting coil and induction terminal voltage of the superconducting magnet.

4.11.4.1 Coupled Secondary Circuit

In coupled secondary circuit protection, the superconducting magnet acts as a primary winding, and another coupled winding with a short circuit acts as secondary winding. To increase the electromagnetic coupling, generally stainless steel or aluminium tapes are wound amongst the layers of windings of the magnet, since there is good thermal contacts and electromagnetic coupling between them. The energy stored in the magnet is released into the external circuit of the magnet, which causes the superconducting magnet quench with faster velocity and uniformity, so that the quench region increases and the temperature rise of the hot spot is limited. Figure 4.32 shows the equivalent circuit and principle of coupled secondary circuit protection. The inductance and resistance of secondary circuit are L_s and R_s , respectively, the

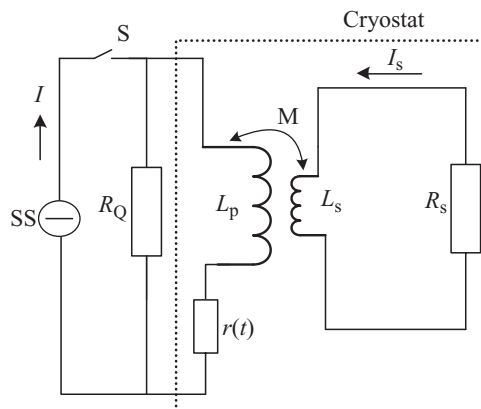


Figure 4.32 Equivalent circuit of coupled secondary protection.

mutual inductance is M , coupling coefficient is k , and R_Q is the resistance of primary superconducting windings in which quench occurs.

The circuit equation is:

$$L_p \frac{dI_p}{dt} + I_p R_Q + M \frac{dI_s}{dt} = 0 \quad (4.196)$$

$$L_p \frac{dI_p}{dt} + I_p R_Q + M \frac{dI_s}{dt} = 0 \quad (4.197)$$

Combining Equations (4.196) and (4.197), we obtain:

$$L_p (1 - k^2) \frac{dI_p}{dt} + I_p R_Q(t) - I_s \frac{MR_s}{L_s} = 0 \quad (4.198)$$

where k is coupling coefficient, $k^2 = M^2/(L_p L_s)$, and when quench begins, $I_s = 0$. As long as the term containing I_s is less than the term containing I_p , Equation (4.198) means that inductance L_p of the superconducting coil reduces to $L_p(1 - k^2)$, due to the coupling of secondary winding after magnet quenches. Thus, the characteristic time t_{Qm} is improved by:

$$t_{Qm} = t_Q (1 - k^2)^{1/6} \quad (4.199)$$

where t_Q represents the characteristic time constant with which energy stored in coil is totally dissipated. In a short time period in which the superconducting coil begins to quench, the coupled current I_s is considered to be so small that it can be assumed to be 0, that is, $I_s \rightarrow 0$, and resistance of the normal zone is:

$$R_Q(t) = \frac{k^2}{[(1 - k^2) \tau_s]} \int R_Q(t) dt \quad (4.200)$$

where τ_s is characteristic constant of secondary winding, $\tau_s = L_s/R_s$. If the quench time is much less than the current decay time constant, it can be assumed that I_s is approximately zero in the entire quench process. If the normal zone propagates only in 1D space and has a relatively small boundary time t_a , we obtain:

$$t_s > \frac{k^2 t_{dm}}{5(1 - k^2)} = \frac{k^2 t_Q}{5(1 - k^2)^{5/6} \left(3 \left(\frac{t_a}{t_Q} \right) \right)^{1/5}} \quad (4.201)$$

Because secondary winding generates heat during the quench process, another function of secondary winding is used to increase the propagation velocity of normal zone. If it has a good thermal contact with the magnet winding (primary winding), it will cause further quench, then the propagation velocity can effectively increase and characteristic time t_Q decreases. This process is called “induced” quench and thus is especially suitable when quench propagates slowly in other zones after the normal zone prematurely encounters the boundary. In order to calculate the “induced” quench, it is assumed that I_p does not change in a short time. If the “induced” quench can occur before the primary current naturally decays, with assumption of $I_p = I_s$ (but $dI_p/dt \neq 0$) and combining Equations (4.197) and (4.198), we

obtain:

$$L_s (1 - k^2) \frac{dI_s}{dt} + I_s R_s - \frac{MI_0 R_Q(t)}{L_p} = 0 \quad (4.202)$$

If the propagation of the normal zone is confined to 2D space, $R_Q(t)$ can be obtained by using Equation (4.172). Substituting into Equation (4.202), we have:

$$\frac{dI_s}{dt} + \frac{I_s}{t_{sk}} = \frac{15MI_0 T_0 t_a t_b t^3}{R_s t_{sk} t_Q^6} \quad (4.203)$$

where $t_{sk} = L_s(1 - k^2)/R_s$ is the corrected time constant of the secondary winding, and the solution of Equation (4.203) becomes:

$$I_s(t) = \frac{15MI_0 t_Q}{R_s} \left(\frac{t_a}{t_Q} \right) \left(\frac{t_b}{t_Q} \right) \left\{ \left(\frac{t}{t_Q} \right)^3 - 3 \left(\frac{t}{t_Q} \right)^2 \left(\frac{t_{sk}}{t_Q} \right) + 6 \left(\frac{t}{t_Q} \right) \left(\frac{t_{sk}}{t_Q} \right)^2 \right\} - 6 \left(\frac{t_{sk}}{t_Q} \right)^3 \left[1 - \exp \left(-\frac{t}{t_{sk}} \right) \right] \right\} \quad (4.204)$$

To find the total energy dissipated in the secondary winding, we need to calculate $E_s = \int I_s^2 R_s dt$. After a series of transformation, we obtain:

$$\frac{E_s (1 - k^2) t_Q^4}{E_0 k^2 t_a^2 t_b^2} = 450 \left\{ \begin{aligned} & \frac{1}{7} \left(\frac{t}{t_Q} \right)^7 \left(\frac{t_{sk}}{t_Q} \right) - \left(\frac{t}{t_Q} \right)^6 \left(\frac{t_{sk}}{t_Q} \right)^2 + \frac{2}{5} \left(\frac{t}{t_Q} \right)^5 \left(\frac{t_{sk}}{t_Q} \right)^3 - 12 \left(\frac{t}{t_Q} \right)^4 \left(\frac{t_{sk}}{t_Q} \right)^4 \\ & + 24 \left(\frac{t}{t_Q} \right)^3 \left(\frac{t_{sk}}{t_Q} \right)^5 - 36 \left(\frac{t}{t_Q} \right)^2 \left(\frac{t_{sk}}{t_Q} \right)^6 + 36 \left(\frac{t}{t_Q} \right) \left(\frac{t_{sk}}{t_Q} \right)^7 + 18 \left(\frac{t_{sk}}{t_Q} \right)^8 \\ & - \left[12 \left(\frac{t}{t_Q} \right)^3 \left(\frac{t_{sk}}{t_Q} \right)^5 + 72 \left(\frac{t}{t_Q} \right) \left(\frac{t_{sk}}{t_Q} \right)^7 \right] \exp \left(-\frac{t}{t_{sk}} \right) - 18 \left(\frac{t_{sk}}{t_Q} \right)^8 \exp \left(-\frac{2t}{t_{sk}} \right) \end{aligned} \right\} \quad (4.205)$$

where $E_0 = L_p I_0^2 / 2$ is the energy stored in the whole superconducting magnet. For simplicity, we consider simple 1D space. Substituting resistance of the normal zone into Equations (4.200) into (4.201), we obtain the current decay with time in the secondary winding as:

$$I_s(t) = \frac{15MI_0 t_Q}{2R_s t_Q} \left(\frac{t_a}{t_Q} \right) \left\{ \left(\frac{t}{t_Q} \right)^4 - 4 \left(\frac{t}{t_Q} \right)^3 \left(\frac{t_{sk}}{t_Q} \right) + 12 \left(\frac{t}{t_Q} \right)^2 \left(\frac{t_{sk}}{t_Q} \right)^2 - 24 \left(\frac{t}{t_Q} \right) \left(\frac{t_{sk}}{t_Q} \right)^3 \right\} - 24 \left(\frac{t_{sk}}{t_Q} \right)^4 \left[1 - \exp \left(-\frac{t}{t_{sk}} \right) \right] \right\} \quad (4.206)$$

Thus, substituting Equation (4.205) and R_s into Equation (4.206), we obtain the total energy E_s dissipated in the secondary winding as:

$$E_s = \int I_s^2 R_s dt \quad (4.207)$$

When the superconducting magnet is energized, secondary windings are slowly turned off to avoid induced quench due to heat generation. In normal operation, the operation current does not change, so there is no induced current in the secondary winding. When quench occurs somewhere in the magnet (primary), current I_s is induced in the secondary winding. Because secondary winding is wound with

normal metal wire and has a good thermal contact with the primary winding (magnet), which is uniformly heated, then quench in most parts of the primary winding is induced, quench propagation velocity increases, energy stored in the magnet is uniformly released into the whole magnet system, so that the concentration of hot spots and possibility of a burnt magnet are effectively avoided.

4.11.4.2 Protection by Subdivisions

As for large-scale magnets such as MRI or NMR high magnetic field magnets, protection by subdivisions and connection with resistance of inner releasing energy can also be used. Figure 4.33 shows the equivalent circuit of protection by subdivisions, where $R_1, R_2, R_3, \dots, R_n$ are resistances in parallel with each section coil, and the corresponding self inductances are $L_1, L_2, L_3, \dots, L_n$, respectively. Each section alone can be connected with resistance in parallel and can use the same method as the parallel resistance in coupled secondary circuit protection, that is, resistances of support structure and former or bifilar resistance co-wound by normal metal tapes with superconducting wire in each section. R refers to resistance of the heater and r denotes the resistance of the small coil by bifilar winding superconducting wire, the so-called resistance of the persistent current switch (PCS), of which the superconducting switch consists. A pair of current leads connected with the power supply is pluggable.

When the superconducting magnet is energized, resistance R is switched on for heating, which leads to quench of the small superconducting coil and then the resistance r appears, and current flows through the circuit consisting of the superconducting magnet and current leads and power supply. When energizing finishes, heating resistance R turns off, and the superconducting switch coil is cooled and transits into the superconducting state, while the temperature is below its critical temperature, that is, $r = 0$. The superconducting switch and magnet form a closed circuit, the so-called closed operation. Then the power supply is switched off and the pluggable current leads are removed from the ends of the magnet in order to reduce heat leakage into the cryostat. When quench occurs in some sections of the magnet, winding of this section and the resistance connected in parallel together form a closed circuit. Since there is coupling

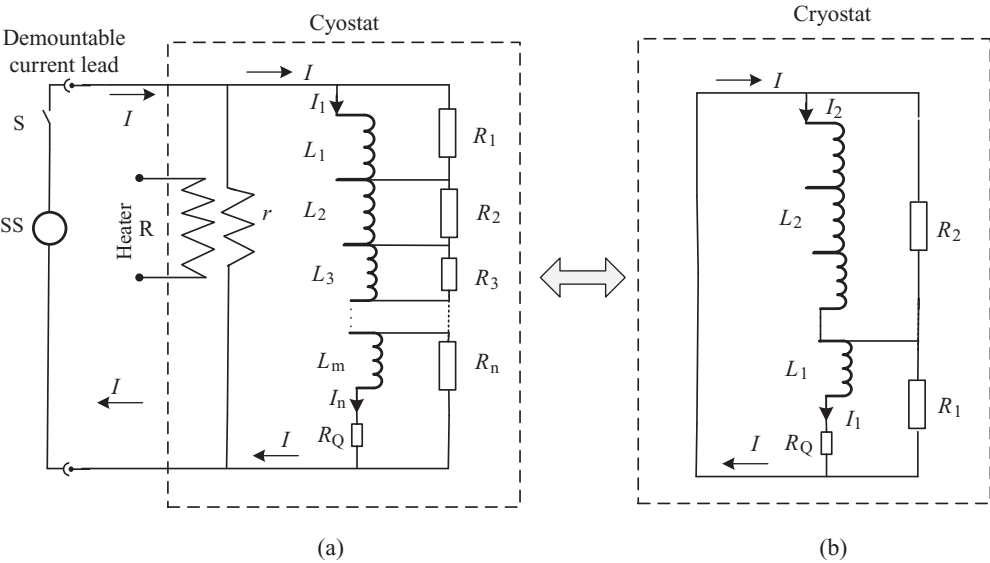


Figure 4.33 Circuits of inner releasing energy resistance protection in multi-sections: (a) protection in subdivisions; (b) equivalent circuit.

amongst sections, coupling current is induced amongst adjacent sections, which induces continuous quench in other sections, so that the superconducting magnet can be heated uniformly, which causes the whole magnet to quench. Consequently, overheating in hot spots is restrained, so the magnet is safely protected.

For simplicity, we assume temporally that the magnet is divided into two sections, that is, $n = 2$. When quench occurs in the second section, current in the first section will decay, but current in the second section can flow continuously through L_2 and R_1 . The decay of current in the quench section is controlled by half of the magnet inductance. According to the equivalent circuit diagram in Figure 4.33, by taking $n = 2$, $I = I_1 + I_2$, the circuit equations are:

$$(I - I_1) R_1 + (I_2 - I) R_2 = 0 \quad (4.208)$$

$$L^2 \frac{dI_2}{dt} + M_{12} \frac{dI_1}{dt} + (I_2 - I) R_2 = 0 \quad (4.209)$$

$$L_1 \frac{dI_1}{dt} + M_{12} \frac{dI_2}{dt} + (I_1 - I) R_1 + I_1 R_Q(t) = 0 \quad (4.210)$$

where M_{12} denotes the mutual inductance between sections 1 and 2, $M_{12} = k(L_1 L_2)^{1/2}$, k is the coupling coefficient, and $R_Q(t)$ is the quench resistance of section 2. If $R_1 = R_2$, $L_1 = L_2$, then $M_{12} = kL_1$. Combining Equations (4.208) to (4.210), we obtain:

$$L_1 (1 - k^2) \frac{d^2 I_1}{dt^2} + [R_1 (1 + k) + R_Q(t)] \frac{dI_1}{dt} + \left[R_Q(t) \frac{R_1}{2L_1} + \frac{dR_Q(t)}{dt} \right] I_1 = 0 \quad (4.211)$$

It is difficult to directly solve this equation. By expanding current $I_1(t)$ in series, supposing that the normal zone is confined to a 2D boundary, that times reaching the boundaries are t_a , t_b respectively, which are much less than t_Q , and the quench resistance $R_Q(t)$ is calculated using Equation (4.172), then the series solution of Equation (4.211) is:

$$I_1(t) = I_0 \left\{ 1 - \frac{15}{2(1-k)} (1+k) \left(\frac{t_a}{t_Q} \right) \left(\frac{t_b}{t_Q} \right) \left[\frac{(t/t_Q)^4}{1+k} - \frac{(t/t_Q)^5}{10(t_k/t_Q)} + \frac{(t/t_Q)^6}{60(t_k/t_Q)^2} - \frac{(t/t_Q)^7}{420(t_k/t_Q)^3} + \dots \right] \right\} \quad (4.212)$$

where $t_k = L_1(1-k)/R_1$. The consumption time of characteristic energy t_{dk} is the time needed for the current to reduce to half of its initial current. When $t_k > t_{dk}$, the consumption time of characteristic energy is:

$$\frac{t_{dk}}{t_Q} \approx \left[\frac{(1-k)}{15} \left(\frac{t_a}{t_Q} \right) \left(\frac{t_b}{t_Q} \right) \right]^{1/2} \quad (4.213)$$

Compared with Equation (4.167), when $k = 0$, decay of the current comes from the coil whose inductance is $L_1 = L/2$, and attenuation in each section is independent. When $k \neq 0$, there is coupling between the two sections and the decay time further decreases.

By adopting the similar method of combining Equations (4.208) to (4.210), we can obtain the current decay in the second section. Also, adopting the similar method of combining Equations (4.198) to

(4.200), we can obtain the current decay in the coil of the second part:

$$I_2 = I_0 \left\{ 1 + \frac{15(1+k)}{2(1-k)} \left(\frac{t_a}{t_Q} \right) \left(\frac{t_b}{t_Q} \right) \left[\frac{k(t/t_Q)^4}{1+k} - \frac{(t/t_Q)^5}{10(t_k/t_Q)} + \frac{(t/t_Q)^6}{60(t_k/t_Q)^2} - \frac{(t/t_Q)^7}{420(t_k/t_Q)^3} \right] + \dots \right\} \quad (4.214)$$

As known from Equations (4.211) and (4.213), when current I_1 decreases, current I_2 increases. While $k \rightarrow 1$, the two currents vary above and below $I = I_0$, and the increase of I_2 is suitable for causing further quench.

Another method for increasing effective propagation velocity of the normal zone is that resistances R_1 and R_2 are also used as heaters, which closely contact with windings of the magnet. For example, the former can be used to replace R_1 or R_2 , and their functions are similar to the “induced” quench of secondary winding. According to Equation (4.208), the current in any resistance is:

$$I_R = \frac{(I_1 - I_2)}{2} \quad (4.215)$$

The heat dissipated in any parallel resistance is:

$$E_R = \int I_R^2 R dt \quad (4.216)$$

According to Equations (4.212) and (4.214), E_R can be found by:

$$\frac{E_R(1-k)}{E_0(1+k)} \left(\frac{t_Q}{t_a} \right)^2 \left(\frac{t_Q}{t_b} \right)^2 = \frac{255}{16} \left(\frac{t}{t_Q} \right)^8 \left[\frac{t}{9t_k} - \frac{t^2}{25t_k^2} + \frac{8t^3}{825t_k^3} - \frac{t^4}{525t_k^4} + \frac{19t^5}{81900t_k^5} - \dots \right] \quad (4.217)$$

where E_0 is energy stored in the whole magnet.

If the magnet is divided into n sections, the mutual inductance amongst sections M_{ij} is:

$$M_{ij} = k\sqrt{L_i L_j} \quad (4.218)$$

where $i, j = 1, 2, 3, \dots, n$; k is the coupling coefficient; and L_i and L_j are the self-inductances of the i -th section and j -th section, respectively.

We assume again that self-inductances of each section in the magnet are distributed uniformly, then $L_1 = L/n$, L is the total self-inductance, and mutual inductance becomes $M_{ij} = kL/n$, so that all self-inductances and mutual inductances are equal, each resistance is R_1 , and all currents are equal. Thus, the circuit can be replaced by two circuit with two asymmetric sections (Figure 4.33(b)), where $R_2 = nR_1$, $L_2 = nL_1[1 + k(n-1)]$, $M_{12} = nkL_1$, and self inductance of the entire magnet is $L = L_1(1+n)(1+nk)$, then the characteristic time of current $I_1(t)$ is:

$$t_{\text{dkn}} = \left[\frac{2(1-k)}{15(1+nk-k)(n+1)} \left(\frac{t_Q}{t_a} \right) \left(\frac{t_Q}{t_b} \right) \right]^{1/2} \quad (4.219)$$

If $n \gg 1$, $k \sim 1$, the decay characteristic time decreases to $n^{1/2}/n^{1/4}$ times of that of the magnet when it is divided into the two sections given by Equation (4.213), so it is beneficial to decrease the highest temperature and the highest inducing voltage by dividing magnet into multi-sections.

The main shortcoming of the passive quench protection method of the superconducting magnet is that almost all the low temperature medium is consumed, and it takes a relatively long time for the magnet to come back to its superconducting state. Taking the protection in sections as an example, if the time of

excitation is t_p , R_s is the total resistance in series, that is, $R_s = R_1 + R_2 + R_3 + \dots + R_n$, then the energy loss that is dissipated in R_s during energizing is:

$$E_{Rs} = E_0 \frac{2L}{R_s t_p} \quad (4.220)$$

In a superconducting magnet system of accelerators, protection of the magnets should adopt the cold-switch method of external energy release, because many superconducting magnets are connected together in series. It must guarantee that no quench happens in large-scale superconducting magnets with low inductance made from cable-in-conduit conductor (CICC) and full stable superconducting magnets cooled by immersion in a cryogen.

4.11.5 Numerical Simulation on Quench

A numerical analysing program has been developed as one of the important methods for quench process simulation of superconducting magnets. At present, there are some commercial and relatively mature numerical analysing software packages, such as Quench, Quench M and QUABER programs, which can be used to solve quench and provide protection for a variety of superconducting coils and can be applied to superconducting coils with or without irons. Not only can they be used in superconducting coils with closed operation mode and connected with a power supply, but can also be used to solve the coupling of single or multi coils. In addition, these software programs can simulate quench within any position and any part of the superconducting coil and quench propagation in 3D space. The magnetic field distribution and inductances of coil can be directly obtained by the finite element method (FEM), and the relationship between the properties of some commonly used materials with temperature and magnetic field can be directly used to simulate their distributions in 3D space of coils.

Numerical simulation on quench of LTS is relatively advanced, and a variety of analytical and numerical analysing methods have been developed and applied in the protection of superconducting magnets. The understanding of quench characteristics for HTS magnets is based on the normal zone diffusion in the winding. Though the quench mechanisms of HTS magnets are similar to their LTS counterparts in many aspects, there are several significant differences between them. The main reasons are that the typical operating temperature of HTS magnet is in the range of 20 K to 70 K, which is much higher than the operating temperature of 4.2 K of conventional LTS; there is a temperature domain of current sharing within 10 K to 100 K in HTS tapes; and current sharing temperature in HTS tapes spans from 10 K to 100 K.

However, the temperature span for LTS is only several K. Current sharing phenomenon in composite superconducting wire exists in both LTS and HTS. The operating temperature of Bi-2223 tapes and ReBCO-coated conductors are usually within the ranges of 4.2 K to 110 K and 4.2 K to 92 K, respectively. Such large temperature spans illustrate that quench of HTS magnets is closely related to Joule heat produced by their early current sharing. Nevertheless, there is a small temperature span of current sharing for LTS. Once the normal zone generates, its quench zone can be transferred to a normal state only by several K. If the typical operating temperature of HTS magnet is limited within 20 K to 70 K, the physical properties of materials in this temperature range are mainly described by critical current density, specific heat capacity (volumetric heat capacity), thermal conductivity and resistance, which are non-linear to temperature and the magnetic field. It is difficult to use analysing methods of quench characteristics in LTS magnets to analyse the process of quench propagation in HTS magnets. At present, some analytical analyses are mostly limited to 1D distribution, and the 1D heat diffusion equation can be transferred to ordinary differential equation for solutions by coordinate transformation. But this method cannot deal with the 3D structure of the HTS magnet and large temperature span of current sharing.

Transient heat diffusion in HTS can be directly obtained by using the finite difference method (FDM) and FEM. FDM can be used to numerically simulate the voltage and temperature distributions of Bi-2223/Ag tapes. We can fully understand quench phenomenon of HTS tapes by means of NZP velocity, temperature and voltage distributions.

4.12 Tests of Stability

There are many factors causing instability of a superconductor, amongst which two factors, namely the intrinsic flux jump and external heat disturbance, are the most important. In this section, we will introduce experiments for measurements of flux jump and some quench parameters of the superconductor, respectively.

4.12.1 Flux Jump Experiments

Flux jump is an intrinsic characteristic of the non-ideal materials of Type-II superconductor. According to properties of flux jump, we will introduce two experiments: one is the variation of magnetization curve caused by a great amount of flux suddenly entering or leaving the superconductor, and the other is the temperature rise caused by the flux jump.

4.12.1.1 Sudden Inrush of Flux

Figure 4.34 shows a superconducting tube made of Nb_3Sn with radius R and wall thickness ΔR . At temperature 4.2 K, an external magnetic field B , parallel to the axis of the tube, is applied. A Hall probe is placed at the centre position O of a hollow cylinder for measuring the magnetic field. When the external magnetic field varies, the magnetic field inside the tube can be measured. The experimental results are illustrated in Figure 4.35. At first, the magnetic field in the tube is zero. When the applied magnetic field increases, the magnetic field in the tube remains at zero due to the shielding effect of Nb_3Sn . With the applied magnetic field increasing to B_1 , as shown in section a , the measured value suddenly increases

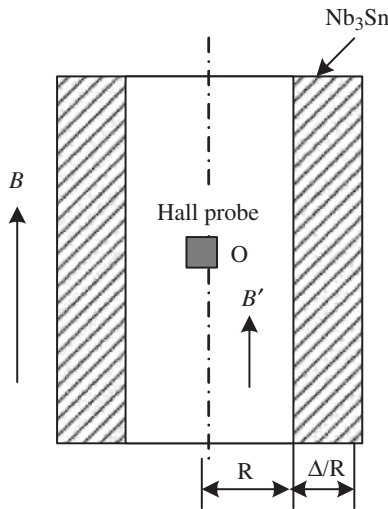


Figure 4.34 Experimental arrangement of superconducting tube and Hall probe.

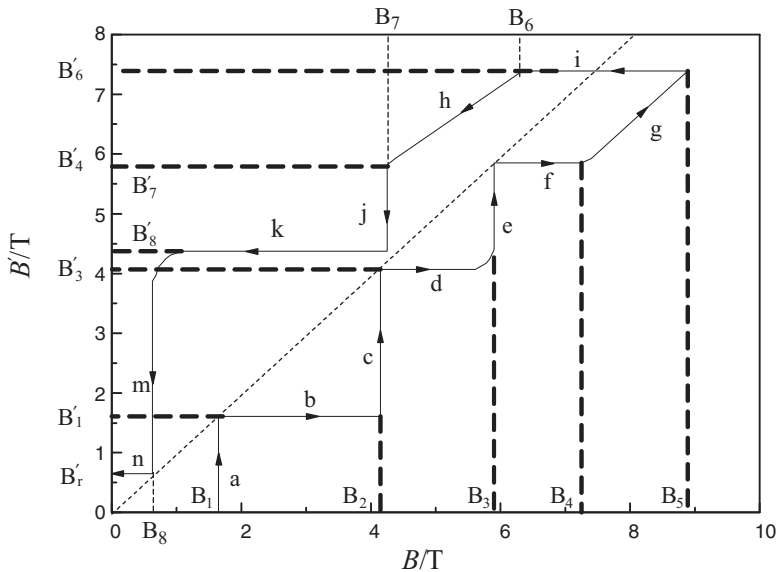


Figure 4.35 Magnetic field in Nb₃Sn tube versus applied external magnetic field.

to B'_1 . The flux abruptly passes through the tube wall and a large amount of flux enters the tube, so that flux jump occurs, and the magnetic field of point O in the tube becomes B'_1 .

If the external magnetic field continuously increases, the magnetic field in the tube is still B'_1 , as shown section *b*. However, when the external applied magnetic field increases to B_2 , as shown section *c*, the flux abruptly passes through the superconducting tube again, and the magnetic field of point O in the tube is B'_1 . With the applied magnetic field increasing, this process is repeated. While the applied field increases to B_5 , a large amount of flux enters the tube. Although the applied magnetic field increases again, no more flux enters the tube due to the repelling effect amongst flux lines in the tube wall, as shown in section *g*, so that the magnetic field increases slowly in this section.

However, when the magnetic field decreases, the observed results are the reverse to those in the increasing process of the magnetic field. When the applied magnetic field decreases along section *i*, no flux leaves the tube in which the magnetic field remains constant; if the applied field keeps reducing, part of flux is repelled slowly by the tube. As the applied magnetic field decreases to B_7 , more flux is repelled, the magnetic field in the tube decreases significantly until it drops to B_8 , and a large amount of flux is repelled by the tube in which the magnetic field decreases suddenly. When the applied magnetic field decreases to zero, the magnetic field in the tube is not zero but B_r , called the remanent magnetic flux resulting from the trapping flux of the tube wall.

4.12.1.2 Temperature Rise Experiment

A pick-up coil with n turns is wound onto the surface of the superconductor NbZr cylinder sample (Figure 4.36). Two terminals of the pick-up coil are connected with a voltmeter. A hole with small diameter is drilled along the central axis of the sample and a thermometer is imbedded into the central point O of the axis to detect temperature variation in the sample. When the sample is cooled to 4.2 K, an external magnetic field parallel to the sample axis is applied at a certain rate. Meanwhile, the terminal voltage of the pick-up coil and temperature at the centre of sample are measured by a voltmeter and a thermometer, respectively.

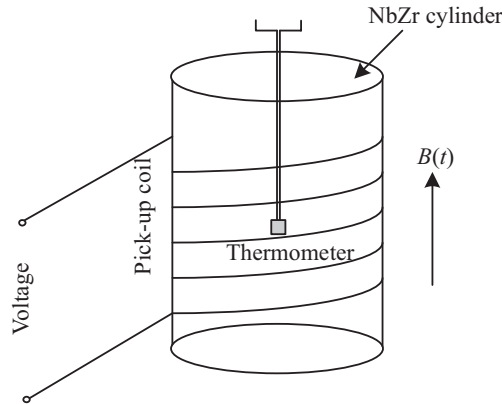


Figure 4.36 Experimental arrangement for measuring voltage variation and temperature rise.

When the magnetic field is applied at a rate of 1.7 T/min, signals of voltage and temperature are recorded separately. The results present that variation profiles of voltage and temperature are similar (Figure 4.37) [19]. Voltage variation equals to $\Delta V = Nd\Phi/dt$, here $d\Phi/dt$ corresponds to the flux variation generated by the applied magnetic field $B(t)$ entering the sample; ΔV appears when the magnetic field $B(t)$ is applied, which means a large amount of flux abruptly enters the inner part of the sample; the acute variation of ΔV shows that the rate of magnetic diffusion is rapid. Simultaneously, a temperature rise with $\Delta T \approx 1$ K appears in the sample, which indicates a large amount of flux inrush into the sample with heat generation and temperature rising, that is, flux jump takes place.

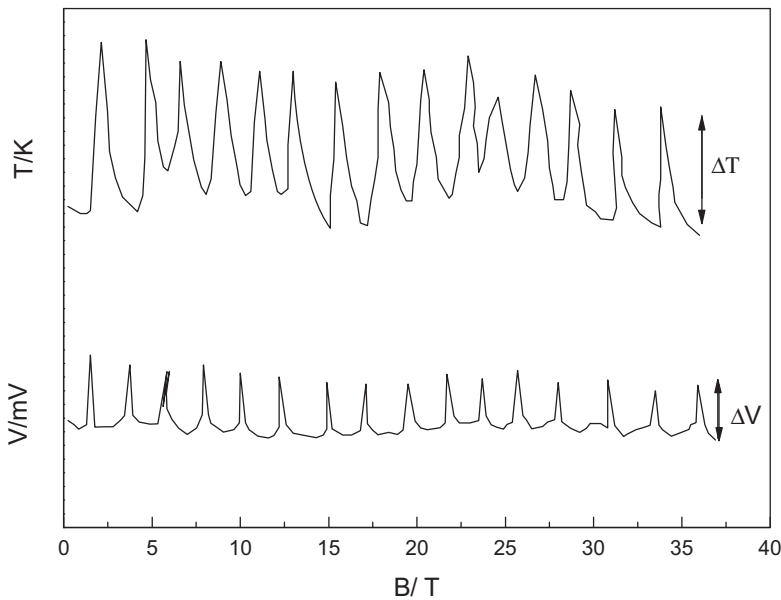


Figure 4.37 Voltage variation and temperature rise. (Reproduced with permission from Yinshun Wang, “Bases of Applied Superconductivity in Electrical Engineering,” Science Press, 121, © 2011.)

However, the variation of ΔT is less acute compared with ΔV , which shows that the thermal diffusion rate is slower than the magnetic diffusion rate. It is also illustrated that the interval of $B(t)$ for each flux jump taking place is approximately the same. Therefore, we conclude that the critical magnetic field is an integral multiple of the magnetic field at which the initial flux jump takes place.

4.12.2 Measurement of Quench Parameters

The main parameters for describing stability of a superconductor are MPZ, NZP velocity and MQE. In this section, we will simply introduce the general method for measuring these three parameters.

Figure 4.38 shows a 1D view for measuring quench parameters of a superconductor rod whose length is much greater than its diameter. Thus, it is reasonable to assume that the temperature distribution is uniform across its cross-section. A heater, wound by bifilar process (generally manganin wire is used) is placed at the central part of the sample. The width of the heater along the sample axis is L and it connects with a pulse power supply. Thermometer T_0 is installed in close contact with the heater.

Five pairs of voltage taps V_1, V_2, V_3, V_4, V_5 and $V'_1, V'_2, V'_3, V'_4, V'_5$ are soldered symmetrically on both sides of the centre, respectively. At each side, the interval between adjacent taps is the same as L_0 . Similarly, four pairs of thermometers T_1, T_2, T_3, T_4 and T'_1, T'_2, T'_3, T'_4 are symmetrically pasted onto the middle section between two voltage adjacent taps on both sides. Taps of each thermometer are installed between two adjacent voltage taps. Both ends of the sample, far away from central heater, are connected with the power supply.

After installing the prepared superconducting sample in a cryogenic environment with which the sample should be thermally isolated if it is directly cooled by immersed into cryogen, measurements can be taken. Measurements include three steps:

- (1) Voltage taps on the sample are connected with voltmeters.
- (2) Nine thermometers are connected to the power supply with constant current and measuring meters; the current should be smaller than 1 mA to reduce the effect on temperature from Joule heat at this time; the thermometer displays the initial temperature of the cryogenic environment.
- (3) The heater is connected to a pulsed power supply with adjustable magnitude of current and duration of time.

Suppose that the resistance of heater is R at cryogenic temperatures. If the output waveform of the pulsed power is a square, and pulse width and current are Δt and current I , respectively, then heat generated in heat transfer to the sample is $E = I^2 R \Delta t$.

When different levels of pulsed energy are supplied to the heater, the temperature between voltage taps and potential of each voltage tap are measured. The variations of voltage and temperature are measured with increased pulsed energy. After a certain number of pulses are supplied, voltage will only appear between V_1 and V'_1 , which indicates that quench occurs at this section, while other sections are still in the superconducting state and quench does not propagate. At this time, L_0 can be estimated as the minimum

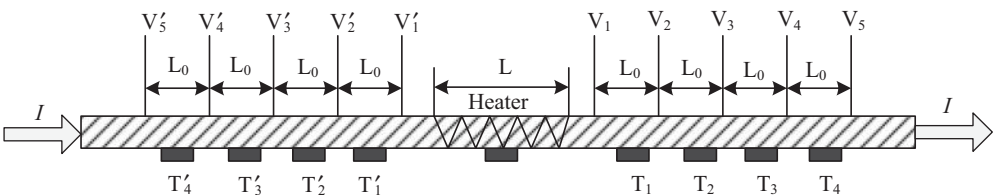


Figure 4.38 1D view of measuring quench parameters of superconductor rod.

propagation size. When that pulse is ended, the minimum pulse energy $E_{\min}$ appears between V_1 and V_2 or V'_1 and V'_1 , and this $E_{\min}$ is approximately the MQE of the superconducting sample at that temperature. According to the time interval between quenching Δt_i (i refers to the time interval during which voltage appears on voltage taps), the quantity L_0 divided by Δt_i is the NZP velocity of the superconducting sample.

Simultaneously, each temperature is recorded to find whether it is more than the critical temperature of the sample, so that both voltage and temperature can be mutually verified for the occurrence of quench. It should be noted that the temperature displayed by the thermometer always lags behind the voltage, because ideal thermal contact between thermometer and sample is impossible and thermal diffusion needs a certain period of time. Of course, the sample can also be connected to a power supply, so the quench and propagation of the sample carrying the transport current can also be measured by the same method. If both the volume and current-carrying ability of the sample are relatively large, the background magnetic field should be applied to the sample in order to reduce the critical current, and also to reduce heat leakage and Joule heat of current leads and their effect on measurement. Afterwards the measurements of quench parameters are performed following the same steps.

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5

AC Losses

In the case of direct current (DC) transmission, there will be no losses in the superconductor due to its zero resistance. However, when carrying an alternating current (AC) or in an AC electromagnetic field, superconductors present some electromagnetic energy losses, namely AC losses, in which the power dissipated in a composite superconductor is due to the presence of a time-varying magnetic field or an electric current. AC losses include time average hysteresis loss of the superconductor, coupling current loss and eddy current loss of the conductor, and eddy current loss of the structural material. AC losses customarily also include the power dissipated in a composite superconductor due to the presence of transient changes in the magnetic field or the current.

Hysteresis loss per cycle is independent of frequency arising in a superconductor under a varying magnetic field. This loss is caused by the irreversible magnetic properties of the superconducting material due to hindering flux line from free motion by flux pinning. For single superconducting filament or bulk, AC losses are mainly hysteresis losses whose mechanism is that the pinning force continuously prevents flux lines from entering or leaving the superconductor in an AC magnetic field. When the operating current is less than critical current I_c , the vortex is completely pinned in superconductors so that only hysteresis loss is generated. When the operating current exceeds critical current I_c , the superconducting vortex can move freely and flux flow loss dominates the AC losses. This assumption makes calculations of AC losses and flux flow losses easier. The total losses can be found by simply adding them together. In fact, there is an overlap between two kinds of losses due to the pinning strength distribution amongst independent pinning centres, which leads to a smooth transition from hysteretic loss to flux flow loss.

Practical superconductors are generally composite conductors consisting of a stabilization matrix made of normal metal or alloy with high thermal conductivity and low resistivity. Thus, there are eddy current losses and coupling losses in the metal matrix of the multifilamentary composite superconductor, besides the hysteretic loss due to induced current in the metal matrix and coupling amongst filaments. Eddy current loss arises in the normal matrix of a composite superconductor or the structural material when exposed to a varying magnetic field, either from an applied field or from a self field. Electric current flows along and between superconducting filaments, which are separated by normal stabilization materials due to the induction in a changing magnetic field. In the normal stabilization matrix of the composite superconductor, the current flows almost perpendicular to the filaments.

AC losses of practical superconducting materials are the sum of several losses such as hysteresis loss, coupling loss and eddy current loss. Because the geometrical structures of most common practical superconducting materials take the form of a cylinder or slab with round or thin rectangular cross-sections, AC losses in an AC magnetic field with various patterns are respectively introduced in this chapter according to the geometrical configurations of the superconductors.

Generally, analytical expressions of AC losses in superconductors are analysed based on the Bean CSM. Only the superconductors with sheet-like or round geometrical structures have analytic expressions for their solutions, while AC loss in superconductors with other geometrical shapes needs to be solved by numerical solutions. Fortunately, most of the practical superconducting materials have slab or round cross-sections, so AC losses are discussed based on these two cross-sections and can meet the requirements of most practical applications.

5.1 AC Losses of Slab

If the superconductor is in a superconducting critical state, the relationship between the critical current and the magnetic field has a variety of models such as the Bean CSM, the Kim model and the index model. The Bean CSM is the simplest, so that analytic solution of AC losses can be obtained. For simplicity and easy understanding, AC losses are calculated based on the Bean CSM throughout this book.

5.1.1 Slab in Parallel AC Magnetic Field

Let us assume that thickness and width of an infinite superconducting slab are $2a$ and $2w$, respectively, and the slab is in an alternating magnetic field $B_{ac}(t)$, parallel to the wide surface of the superconductor without a transport current (Figure 5.1). Current I is induced in the slab along the $\pm z$ -axis direction due to the AC magnetic field along the y -axis direction, and the origin of the coordinates locates at the centre of the superconducting slab.

According to the Bean CSM, current density J in the slab is the critical current density J_c in the region penetrated by flux and is zero in the region not penetrated by flux. That is, critical current density J_c is either $\pm J_c$ or 0 if it is in an alternating magnetic field. In order to understand the Bean CSM and the flux distribution, a simple situation in which the external magnetic field is increasing continuously with time is considered. According to this model, critical current density J_c is either constant or zero and independent of the magnetic field. Magnetic field distribution in the slab is described by the Maxwell equation:

$$\nabla \times B = \pm J_c \text{ or } 0 \quad (5.1)$$

As shown in Figure 5.1, $B = (0, B, 0)$ according to the Cartesian coordinate system. The magnetic field distribution inside the slab can be obtained by solving Equation (5.1).

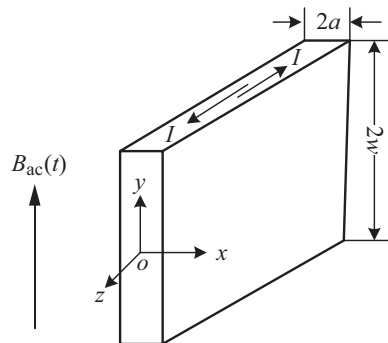


Figure 5.1 Geometry and shielding current of slab in parallel magnetic field.

Figure 5.1 shows that only the external magnetic field parallel to the y-axis is applied, $B = (0, B, 0)$. The magnetic field distribution inside the superconductors can be obtained by solving Equation (5.1) as:

$$B(x) = B_0 - \mu_0 J_c |x| \quad (5.2)$$

where B_0 is the amplitude of the external magnetic field outside the slab, and μ_0 is vacuum permeability. The magnetic field inside the slab decreases with penetration depth. When the external magnetic field is relatively small, the magnetic field is zero at point x_p , that is, $x_p = B_0/(\mu_0 J_c)$, and is called the penetration depth. When the magnetic field B_0 gradually increases until it just reaches the centre of the slab, that is, $B_0 = \mu_0 J_c a$, that indicates that the magnetic field fully penetrates the slab. The external magnetic field at this time is known as the complete penetration magnetic field B_p ($B_p = \mu_0 J_c a$). When the external magnetic field continuously increases, the magnetic field at the centre of the slab also increases.

The distributions of magnetic field and current density in the slab are presented in Figure 5.2. Figure 5.2(a) shows the penetrating process of magnetic field with increasing external magnetic field. Figure 5.2(b) shows the corresponding current distribution in slab. The penetration process of the magnetic field is divided into three steps:

- (1) *Partially penetrated*: When the external magnetic field B_1 is less than the penetration field B_p , that is $B_1 < B_p$, the penetration depth is $(a - x_p)$, and the current density within the penetrated region with width $(a - x_p)$ on both sides of the central axis is equal to the critical current density J_c .
- (2) *Fully penetrated*: When the external magnetic field B_2 equals the penetration field B_p , that is, $B_2 = B_p$, the current density is equal to the critical current density J_c inside the whole slab, $J = J_c$.
- (3) *Over penetrated*: When the external magnetic field B_3 is larger than the penetration field B_p , that is $B_1 > B_p$, the current density J is equal to the critical current density J_c , that is, $J = J_c$, which is same as step (2) due to the Bean CSM.

As with a continuously increasing external magnetic field, the external magnetic field distribution and critical current distribution in the slab can also be obtained when the external magnetic field decreases continuously. Figure 5.3 shows the varying processes of the magnetic field and the corresponding current density with the magnetic field reducing gradually from larger than the penetration magnetic field to smaller than the penetration magnetic field. Since variation of the external magnetic field is negative, the induced current is reversed.

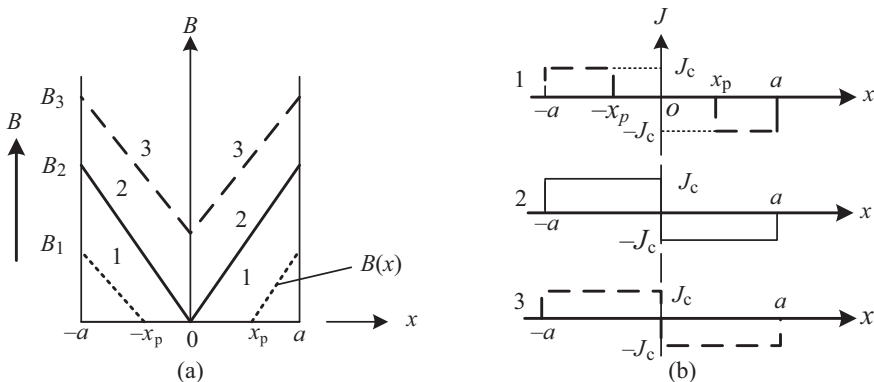


Figure 5.2 Magnetic field and current distributions with external magnetic field increasing in slab: (a) magnetic field distribution; (b) current density distribution.

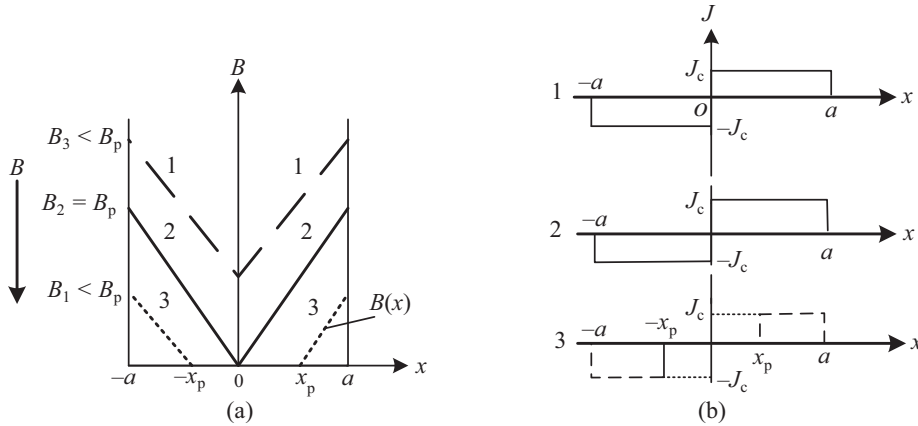


Figure 5.3 Magnetic field and current distributions with external magnetic field decreasing in slab: (a) magnetic field distribution; (b) current density distribution.

If the AC external magnetic field is sinusoidal, that is, $B_{ac}(t) = B_m \sin(\omega t)$, where B_m is amplitude of the AC external magnetic field, ω is angular frequency and f is the frequency, $\omega = 2\pi f$. If the AC magnetic field is parallel to the wide surface of the slab [1], the AC loss per unit length (W/m) is:

$$P_{//} = fCA \frac{2B_m^2}{\mu_0} \left(\frac{b_{ac}}{3} \right) \quad b_{ac} \leq 1 \quad (5.3)$$

$$P_{//} = fCA \frac{2B_m^2}{\mu_0} \left(\frac{1}{b_{ac}} - \frac{2}{3b_{ac}^2} \right) \quad b_{ac} \geq 1$$

where CA is the effective cross-sectional area related to the geometric structure of the superconducting slab, A denotes total cross-section of the slab and equals to $4wa$, $B_p = \mu_0 J_c a$ is the complete penetration magnetic field, and b_{ac} represents normalized magnetic field, $b_{ac} = B_m/B_p$.

5.1.2 Slab in Perpendicular AC Magnetic Field

When the external AC magnetic field is perpendicular to the wide surface of an infinite slab, the slab does not carry a transport current (Figure 5.4), and its width and thickness are $2w$ and $2a$, respectively. If the external AC magnetic field is also sinusoidal [2], the hysteresis loss of slab per unit length (W/m) is:

$$P_{\perp} = 4Kf \frac{w^2 \pi}{\mu_0} B_m^2 \frac{1}{b_{ac}} \left(\frac{2}{b_{ac}} \ln(\cosh b_{ac}) - \tanh(b_{ac}) \right) \quad (5.4)$$

where K is a constant relating with the structure of the superconducting slab, $b_{ac} = B_m/B_f$ is also known as the normalized magnetic field, and B_f refers to characteristic magnetic field $B_f = 2a\mu_0 J_c/\pi$.

5.1.3 Self-Field Losses

While the aspect ratio of an infinite slab is very small or its width is much larger than its thickness (i.e., HTS tape), the slab can be approximated to a thin sheet structure (Figure 5.5). When it carries a current,

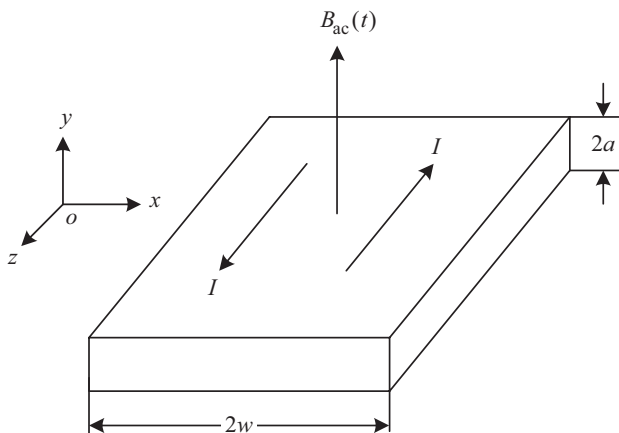


Figure 5.4 Geometry and shielding current of a slab in perpendicular magnetic field.

the slab will generate a magnetic field, the well-known self field, even if there is no external magnetic field. If the current is AC, the self field is naturally an AC magnetic field, so that AC loss, or the so-called self-field loss, is also generated inside the superconductor.

If the slab transports sinusoidal AC $I(t) = I_m \sin(\omega t)$, here I_m is the amplitude of the AC, the slab is in a self-magnetic field only, and self loss per unit length (W/m) is [3]:

$$P_s = f \frac{\mu_0}{\pi} I_c^2 \begin{cases} [(1 - i_{ac}) \ln(1 - i_{ac}) + (1 + i_{ac}) \ln(1 + i_{ac}) - i_{ac}^2] & i_{ac} < 1 \\ 2 \ln 2 - 1 & i_{ac} \geq 1 \end{cases} \quad (5.5)$$

where I_c is the critical current of the slab in the self field, i_{ac} is normalized current $i_{ac} = I_m/I_c$, and I_m is the amplitude of AC.

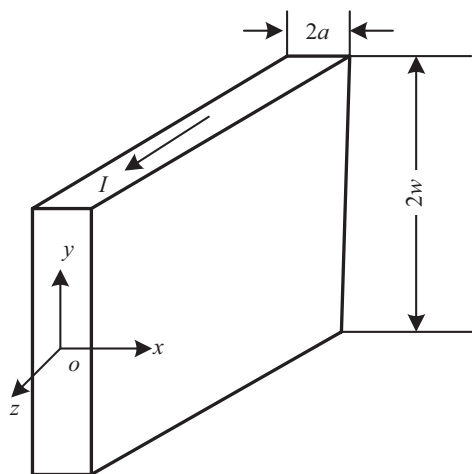


Figure 5.5 Geometrical configuration of slab carrying transport AC current.

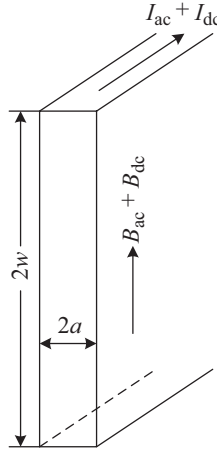


Figure 5.6 AC/DC transport current and magnetic fields parallel to wide slab surface.

5.1.4 Slab-Carrying DC and AC Currents Located in Parallel DC/AC Magnetic Fields

An infinite slab with thickness $2a$ and width $2w$ carries a DC current I_{dc} and AC current $I_{ac}(t) = I_m \sin(\omega t)$ and is in a DC magnetic field B_{dc} and AC magnetic field $B_{ac}(t) = B_m \sin(\omega t)$, which are parallel to the wide surface of the slab, and the AC current $I_{ac}(t)$ has the same phase as the AC magnetic field. Figure 5.6 shows the geometric structure of the slab, the transport current and orientation of the magnetic fields. The total transport current and magnetic field are:

$$I(t) = I_{ac}(t) + I_{dc}, \quad B(t) = B_{ac}(t) + B_{dc} \quad (5.6)$$

If the magnetic field fully penetrates the slab, the penetration magnetic field is $B_p = \mu_0 J_c a$. For simplicity, the normalized currents and normalized magnetic fields are defined as:

$$i_{ac} = \frac{I_m}{I_c}, \quad i_{dc} = \frac{I_{dc}}{I_c}, \quad b_{ac} = \frac{B_m}{B_p}, \quad b_{dc} = \frac{B_{dc}}{B_p} \quad (5.7)$$

According to the varying ranges of the critical current, penetration field, current amplitude and magnetic field amplitude, AC losses (W/m) per unit length of the slab can be expressed as [4–8]:

(i) if $i_{ac} \leq b_{ac} \leq 1 - i_{dc}$, the loss (W/m) per unit length is:

$$P_{//} = CAf \frac{2B_p^2}{3\mu_0} (b_{ac}^3 + 3b_{ac}i_{ac}^2) \quad (5.8)$$

(ii) if $i_{ac} \leq 1 - i_{dc} \leq b_{ac}$, the loss (W/m) per unit length becomes:

$$P_{//} = CAf \frac{2B_p^2}{3\mu_0} \left[b_{ac} (3 + i_{ac}^2 + 3i_{dc}^2) - 2(1 - (i_{ac} + i_{dc})^3 + 3i_{ac}i_{dc}) + \frac{6i_{ac}}{(b_{ac} - i_{ac})} (i_{ac} + i_{dc})(1 - i_{ac} - i_{dc})^2 - \frac{4i_{ac}^2}{(b_{ac} - i_{ac})^2} (1 - i_{ac} - i_{dc})^3 \right] \quad (5.9)$$

(iii) if $1 - i_{dc} \leq i_{ac} \leq b_{ac}$; $i_{dc} - 1 \leq i_{ac} \leq i_{dc} + 1$, the loss (W/m) per unit length is:

$$P_{//} = CAf \frac{2B_p^2}{3\mu_0} \left(\frac{1 + i_{ac} - i_{dc}}{2i_{ac}} \right) [b_{ac} (1 + i_{ac} - i_{dc})^2 + 3 (b_{ac} - i_{ac}) (1 - i_{ac} + i_{dc})] \quad (5.10)$$

(iv) if $1 - i_{dc} \leq i_{ac} \leq b_{ac}$; $i_{dc} + 1 \leq i_{ac}$, the loss (W/m) per unit length is:

$$P_{//} = CAf \frac{2B_p^2}{3\mu_0} \left(4 \frac{b_{ac}}{i_{ac}} \right) \quad (5.11)$$

(v) if $1 - i_{dc} \leq i_{ac} \leq b_{ac}$; $i_{dc} + 1 \leq i_{ac}$, the loss (W/m) per unit length is:

$$P_{//} = CAf \frac{2B_p^2}{3\mu_0} (i_{ac}^3 + 3i_{ac}b_{ac}^2) \quad (5.12)$$

(vi) if $1 - i_{dc} \leq i_{ac}$; $b_{ac} \leq i_{ac}$; $i_{dc} - 1 \leq i_{ac} \leq i_{dc} + 1$, the loss (W/m) per unit length is:

$$P_{//} = CAf \frac{2B_p^2}{3\mu_0} \left(\frac{1 + i_{ac} - i_{dc}}{2i_{ac}} \right)^3 (i_{ac}^3 + 3i_{ac}b_{ac}^2) \quad (5.13)$$

(vii) if $1 - i_{dc} \leq i_{ac}$; $b_{ac} \leq i_{ac}$; $i_{dc} + 1 \leq i_{ac}$, the loss (W/m) per unit length is:

$$P_{//} = CAf \frac{2B_p^2}{3\mu_0} \left(\frac{1}{i_{ac}} \right)^3 (i_{ac}^3 + 3i_{ac}b_{ac}^2) \quad (5.14)$$

(viii) if $i_{ac} \leq i_{dc} - 1$, the loss (W/m) per unit length is:

$$P_{//} = 0 \quad (5.15)$$

Equation (5.15) shows that only the hysteresis loss of a superconductor per unit length is zero under the condition of DC transport current $I_{dc} \geq I_{ac} + I_c$, and the superconducting slab is thus completely quenched and transfers to its normal state, being equivalent to a resistor. Consequently, the superconducting slab presents resistive Joule loss as with a normal conductor. The interesting case is that DC magnetic field has no influence on the AC losses in the thin slab.

5.1.5 Slab-Carrying AC and DC Currents

Consider an infinite superconducting slab with width $2w$ and thickness $2a$ transferring AC current $I_{ac}(t) = I_m \sin(\omega t)$ and DC current I_{dc} simultaneously. The width is much larger than the thickness, that is, $w \gg a$. Figure 5.7 shows the geometric structure of the slab, transport current and self-magnetic fields induced by DC and AC transport currents. The total transport current is:

$$I(t) = I_{dc} + I_{ac}(t) \quad (5.16)$$

For simplicity, normalized AC and DC currents are defined respectively as:

$$i_{ac} = \frac{I_m}{I_c}; \quad i_{dc} = \frac{I_{dc}}{I_c} \quad (5.17)$$

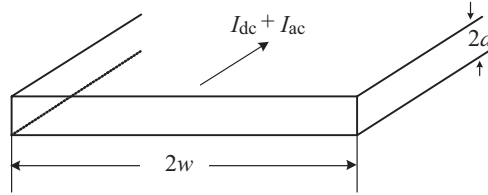


Figure 5.7 Infinite superconducting slab with transport AC and DC currents.

According to magnitudes of normalized AC current i_{ac} and normalized DC current i_{dc} , AC losses can be expressed as the four following ways [9]:

- (i) If $i_{ac} \leq 1 - i_{dc}$, the self-field AC loss (W/m) per unit length is:

$$P_s = f \frac{\mu_0 I_c^2}{2\pi} \left\{ \begin{aligned} &[(1 - i_{ac} - i_{dc}) \ln(1 - i_{ac} - i_{dc}) + (1 + i_{ac} + i_{dc}) \ln(1 + i_{ac} + i_{dc}) - (i_{ac} + i_{dc})^2] \\ &\pm [(1 - |i_{ac} - i_{dc}|) \ln(1 - |i_{ac} - i_{dc}|) + (1 + |i_{ac} - i_{dc}|) \ln(1 + |i_{ac} - i_{dc}|) \\ &\quad - (i_{ac} - i_{dc})^2] \end{aligned} \right\} \quad (5.18)$$

where the sign $\pm$ corresponds to the two cases of $i_{ac} > i_{dc}$ and $i_{ac} < i_{dc}$, respectively.

- (ii) If $1 - i_{dc} \leq i_{ac}$; $i_{dc} - 1 \leq i_{ac} \leq i_{dc} + 1$, the self-field loss (W/m) per unit length is:

$$P_s = f \frac{\mu_0 I_c^2}{2\pi} \{ 2 \ln 2 - 1 \pm [(1 - |i_{ac} - i_{dc}|) \ln(1 - |i_{ac} - i_{dc}|) + (1 + |i_{ac} - i_{dc}|) \ln(1 + |i_{ac} - i_{dc}|) - (i_{ac} - i_{dc})^2] \} \quad (5.19)$$

where the sign $\pm$ also corresponds to the two cases of $i_{ac} > i_{dc}$ and $i_{ac} < i_{dc}$.

- (iii) If $i_{dc} + 1 \leq i_{ac}$, the self-field loss (W/m) per unit length is:

$$P_s = f \frac{\mu_0 I_c^2}{\pi} (2 \ln 2 - 1) \quad (5.20)$$

- (iv) If $i_{ac} \leq i_{dc} - 1$, the self-field loss (W/m) per unit length is:

$$P_s = 0 \quad (5.21)$$

Equation (5.21) indicates that the self-field loss is zero, which means that the superconductor completely quenches. When $i_{ac} \leq i_{dc} - 1$, that is, $I_{dc} \geq I_c + I_{ac}$, the superconducting slab always operates in the normal state, with transport current much higher than its critical current. Equation (5.21) only represents the hysteresis loss per unit length if it is zero and does not mean that the superconducting slab has no loss; the loss of the superconducting slab is only Joule loss. In addition, even in the case of $i_{ac} + i_{dc} < 1$, the self-field loss is also affected by the DC transport current.

5.1.6 Slab with AC Transport Current in Perpendicular AC Magnetic Field

Figure 5.8 shows the geometric structure and magnetic field orientations of a superconducting infinite thin slab, which carries an AC current and is placed in a perpendicular AC magnetic field simultaneously.

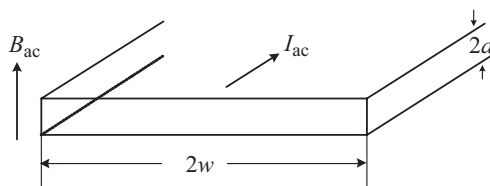


Figure 5.8 Infinite thin superconducting slab with transport AC current in perpendicular AC magnetic field.

Both of the AC transport current and the AC magnetic field have the same phase. Its width and thickness are $2w$ and $2a$, respectively, and the width is much larger than the thickness $w \gg a$, and $I_{ac}(t) = I_m \sin(\omega t)$, $B_{ac}(t) = B_m \sin(\omega t)$. The characteristic magnetic field B_f in the perpendicular magnetic field is defined as:

$$B_f = \mu_0 J_c (2a) / \pi \quad (5.22)$$

where J_c is critical current density of the thin slab:

$$I_c = 4J_c a w \quad (5.23)$$

Similarly, normalized current i_{ac} and normalized magnetic field b_{ac} are defined as:

$$i_{ac} = \frac{I_m}{I_c}, b_{ac} = \frac{B_m}{B_f} \quad (5.24)$$

Depending on the magnitudes of the AC magnetic field and AC transport current, AC losses of a thin slab with AC current and perpendicular to the AC magnetic field can be expressed in the following four ways [10]:

(i) If $i_{ac} < \tanh(b_{ac})$, the AC loss (W/m) per unit length is:

$$P_{\perp} = f \frac{\mu_0 I_c^2}{\pi} \left[\begin{aligned} & 2 \coth^{-1} \left(\frac{(1+p_0)(1-p_0) + a_0^2}{\sqrt{(1+p_0)^2 - a_0^2} \sqrt{(1-p_0)^2 - a_0^2}} \right) \\ & - \frac{1}{4} \left((1+p_0) \sqrt{(1+p_0)^2 - a_0^2} + (1-p_0) \sqrt{(1-p_0)^2 - a_0^2} \right) \\ & \quad \times \left(\cosh^{-1} \left(\frac{1+p_0}{a_0} \right) + \cosh^{-1} \left(\frac{1-p_0}{a_0} \right) \right) \\ & + \frac{1}{2} \left(\sqrt{(1+p_0)^2 - a_0^2} - \sqrt{(1-p_0)^2 - a_0^2} \right) \\ & \quad \times \left((1+p_0) \cosh^{-1} \left(\frac{1+p_0}{a_0} \right) + (1-p_0) \cosh^{-1} \left(\frac{1-p_0}{a_0} \right) \right) \\ & + \frac{1}{4} \left(\sqrt{(1+p_0)^2 - a_0^2} - \sqrt{(1-p_0)^2 - a_0^2} \right)^2 \\ & - \frac{1}{2} \left(\sqrt{(1+p_0)^2 - a_0^2} - \sqrt{(1-p_0)^2 - a_0^2} \right) \\ & \quad \times \left(\sqrt{(1+p_0)^2 - a_0^2} + \sqrt{(1-p_0)^2 - a_0^2} \right) \end{aligned} \right] \quad (5.25)$$

where parameters a_0 , p_0 , c and p are defined as:

$$a_0 = \frac{c}{w}, \quad p_0 = \frac{p}{w} \quad (5.26a,b)$$

$$c = w \frac{\sqrt{1 - i_{ac}^2}}{\cosh(b_{ac})}, \quad p = w i_{ac} \tanh(b_{ac}) \quad (5.27a,b)$$

If the transport AC current is zero, $I_{ac}(t) = 0$, that is, the AC loss of a thin slab is only the loss in the perpendicular AC magnetic field, Equation (5.25) can be simplified to become Equation (5.4).

(ii) If $i_{ac} > \tanh(b_{ac})$, the AC loss (W/m) per unit length is:

$$P_{\perp} = f \frac{\mu_0 I_c^2}{\pi} \left[\begin{aligned} & -2 \coth^{-1} \left(\frac{(1 + p_0)(1 - p_0) + a_0^2}{\sqrt{(1 + p_0)^2 - a_0^2} \sqrt{(1 - p_0)^2 - a_0^2}} \right) \\ & - \frac{1}{4} \left((1 + p_0) \sqrt{(1 + p_0)^2 - a_0^2} - (1 - p_0) \sqrt{(1 - p_0)^2 - a_0^2} \right) \\ & \quad \times \left(\cosh^{-1} \left(\frac{1 + p_0}{a_0} \right) - \cosh^{-1} \left(\frac{1 - p_0}{a_0} \right) \right) \\ & + \frac{1}{2} \left(\sqrt{(1 + p_0)^2 - a_0^2} + \sqrt{(1 - p_0)^2 - a_0^2} \right) \\ & \quad \times \left((1 + p_0) \cosh^{-1} \left(\frac{1 + p_0}{a_0} \right) + (1 - p_0) \cosh^{-1} \left(\frac{1 - p_0}{a_0} \right) \right) \\ & - \frac{1}{4} \left(\sqrt{(1 + p_0)^2 - a_0^2} + \sqrt{(1 - p_0)^2 - a_0^2} \right) \end{aligned} \right] \quad (5.28)$$

When the perpendicular AC field does not exist, that is, $b_{ac} = 0$, the AC loss Equation (5.28) is simplified to self-field loss, as shown by Equation (5.5).

(iii) When the amplitude of AC current is equal to critical current, that is, $I_m = I_c$, the AC loss (W/m) per unit length is:

$$P_{\perp} = f \frac{\mu_0 I_c^2}{\pi} [2 \ln 2 - 1 + 2 \ln(\cosh(b_{ac}))] \quad (5.29)$$

(iv) If $i_{ac} = \tanh(b_{ac})$, the AC loss (W/m) per unit length is:

$$P_{\perp} = f \frac{\mu_0 I_c^2}{\pi} i_{ac} [(1 + i_{ac}^2) \tanh^{-1}(i_{ac}) - i_{ac}] \quad (5.30)$$

5.1.7 Slab in AC and DC Magnetic Fields

Figure 5.9 shows the geometric structure and magnetic field orientation of an infinite slab perpendicular to the AC and DC magnetic fields. The width and thickness are $2w$ and $2a$, respectively, $w \gg a$, and the AC magnetic field $B(t) = B_m \sin(\omega t)$. The total magnetic field is:

$$B(t) = B_{ac}(t) + B_{dc} \quad (5.31)$$

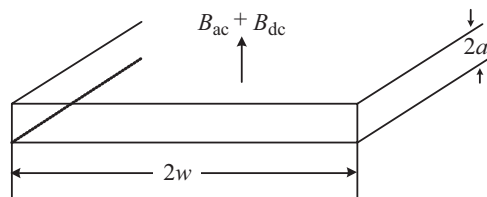


Figure 5.9 Infinite superconducting slab with perpendicular AC and DC magnetic fields.

Similarly, for convenience, normalized AC and DC magnetic fields are defined as:

$$b_{ac} = \frac{B_m}{B_f}, b_{dc} = \frac{B_{dc}}{B_f} \quad (5.32)$$

where B_f is the characteristic magnetic field and is similar to Equation (5.22).

In the AC and DC perpendicular magnetic fields, the AC loss (W/m) per unit length is [9]:

$$P_{\perp} = f \frac{\mu_0 I_c^2}{2\pi} \left\{ \begin{aligned} &[2 \ln (\cosh (b_{ac} + b_{dc})) - (b_{ac} + b_{dc}) \tanh (b_{ac} + b_{dc})] \\ &\pm [2 \ln (\cosh |b_{ac} - b_{dc}|) - |b_{ac} - b_{dc}| \tanh |b_{ac} - b_{dc}|] \end{aligned} \right\} \quad (5.33)$$

where the sign $\pm$ corresponds to the two cases of $b_{ac} > b_{dc}$ and $b_{ac} < b_{dc}$, respectively.

5.1.8 Flux-Flow Loss of Slab with Combinations of AC and DC Transport Currents in Perpendicular and Parallel AC and DC Magnetic Fields

When the operating current in a superconducting slab is close to and just below the critical current, besides the hysteresis losses, flux flow losses P_{ff} can be generated due to the flux flow. In this section, AC loss due to flux flow resistance is introduced in detail. An infinite slab with width $2w$ and thickness $2a$ carries AC and DC transport currents and is placed in parallel and perpendicular the AC and DC magnetic fields (Figure 5.10). The AC transport current and both of the AC magnetic fields have the same phase. The total currents and magnetic fields are:

$$I(t) = I_{ac}(t) + I_{dc}, B(t) = B_{ac}(t) + B_{dc} \quad (5.34)$$

The AC current and AC magnetic field are sinusoidal, that is, $I_{ac}(t) = I_m \sin(\omega t)$ and $B_{ac}(t) = B_m \sin(\omega t)$.

If there exists an AC transport current I and an AC parallel or perpendicular magnetic field B , the flux flow electric field E_{ff} is induced on the surface of the slab and then flux flow loss P_{ff} is generated.

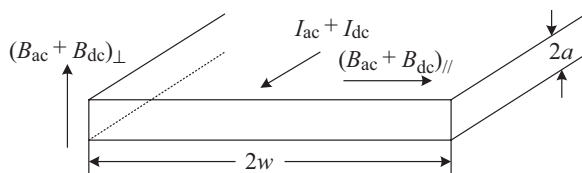


Figure 5.10 Infinite slab carrying AC/DC transport current placed in AC/DC perpendicular and parallel magnetic fields.

Because magnetic field B is proportional to current I , $k = B/I$ is a constant, then the flux flow electric field is [11]:

$$E_{ff} = \rho_{ff} \frac{I}{4aw} \cosh\left(\frac{I}{I_v}\right) \exp\left(\frac{N_v(k)}{N_{v0}}\right) \quad (5.35)$$

where ρ_{ff} is the resistivity of flux vortex motion, and I_v and N_{v0} are parameters relating to the intensity and density of pinning centres, respectively. If the magnetic field is parallel to the wide surface of the slab and we let $k_{//} = B_{//}/I$, N_v is:

$$\begin{cases} N_v(k = k_{//}) = \frac{\mu_0}{\Phi_0} \left(\frac{\ln 2}{\pi} + \frac{2a}{8w} \right); & k_{//} < \frac{\mu_0}{8w} \\ N_v(k = k_{//}) = \frac{\mu_0}{\Phi_0} \left(\frac{\ln 2}{\pi} + \frac{k_{//} 2a}{\mu_0} \right); & k_{//} > \frac{\mu_0}{8w} \end{cases} \quad (5.36)$$

where Φ_0 is the flux quantum, and $\Phi_0 = 2.07 \times 10^{-15} \text{ T}\cdot\text{m}^2$.

If the magnetic field is perpendicular, $k = k_{\perp}$, and the corresponding N_v is:

$$\begin{cases} N_v(k = k_{\perp}) = \frac{\mu_0}{\Phi_0} \left(\frac{\ln 2}{\pi} + \frac{2a}{8w} \right); & k_{\perp} < \frac{\mu_0 \ln 2}{2\pi w} \\ N_v(k = k_{\perp}) = \frac{\mu_0}{\Phi_0} \left(\frac{\ln 2}{\pi} + \frac{k_{\perp} 2w}{\mu_0} \right); & k_{\perp} > \frac{\mu_0 \ln 2}{8\pi w} \end{cases} \quad (5.37)$$

Defining reference parameter N_{vref} equals to μ_0/Φ_0 . Figure 5.11 illustrates the relationship between $N_v(k)/N_{vref}$ and both of the parallel and perpendicular magnetic fields. When the current is increasing, the parameter with the perpendicular magnetic field is much larger than that with the parallel magnetic field, and here the width and thickness of the slab are 3 mm and 0.3 mm, respectively.

Flux flow loss is related to the waveform of the AC current. If the AC current waveform is sinusoidal:

$$I(t) = I_{dc} + I_{ac}(t) = I_{dc} + I_m \sin(\omega t) \quad (5.38)$$

where I_{dc} , I_{ac} and ω denote the magnitude of the DC current, the amplitude of the AC current and AC angular frequency, respectively. The flux flow loss per unit length (W/m) is:

$$P_{ff} = \frac{1}{T} \int_0^T E_{ff}(t) I(t) dt \quad (5.39)$$

where T is the period of the sinusoidal AC current. Substituting Equations (5.34) to (5.37) into Equation (5.39), we find:

$$P_{ff} = \rho_{ff} \frac{I_v^2}{4aw} \exp\left(\frac{N_v(k)}{N_{v0}}\right) \left[\left(\frac{I_{dc}}{I_v} \right)^2 \cosh\left(\frac{I_{dc}}{I_v}\right) I_0\left(\frac{I_m}{I_v}\right) + 2 \frac{I_{dc} I_m}{I_v^2} \sinh\left(\frac{I_{dc}}{I_v}\right) I_1\left(\frac{I_m}{I_v}\right) + \left(\frac{I_m}{I_v} \right)^2 \cosh\left(\frac{I_{dc}}{I_v}\right) I'_1\left(\frac{I_m}{I_v}\right) \right] \quad (5.40)$$

where $I_n(x)$ is the modified Bessel function of the first kind and order n , $I'_n(x)$ is the first derivative of $I_n(x)$. According to Equation (5.40), similar to Joule loss, flux flow loss P_{ff} is independent of frequency.

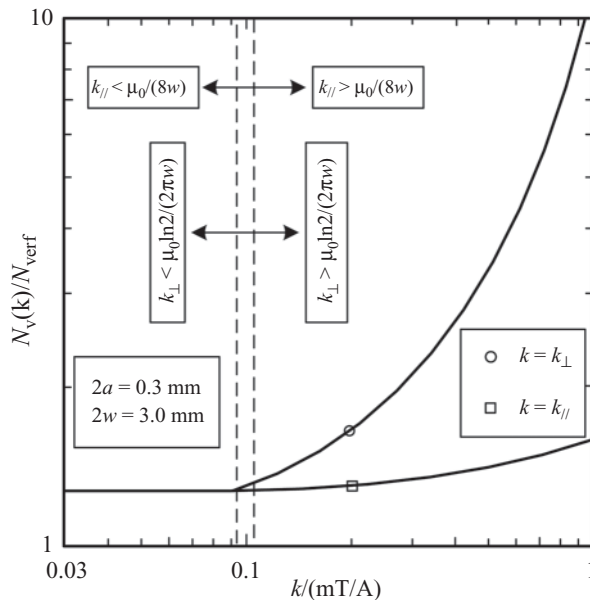


Figure 5.11 $N_v(k)/N_{vref}$ versus $k_{//}$ and $k_{\perp}$.

When the external magnetic field is zero and only an AC current exists, that is, $I_{dc} = 0$, Equation (5.40) can be simplified to:

$$P_{ff} = \rho_{ff} \frac{I_v^2}{4aw} \exp\left(\frac{N_v(k)}{N_{v0}}\right) \left(\frac{I_m}{I_v}\right)^2 I_1'\left(\frac{I_m}{I_v}\right) \quad (5.41)$$

When only a DC current exists, that is, $I_{ac} = 0$, Equation (5.40) becomes:

$$P_{ff} = \rho_{ff} \frac{I_v^2}{4aw} \exp\left(\frac{N_v(k)}{N_{v0}}\right) \left(\frac{I_{dc}}{I_v}\right)^2 \cosh\left(\frac{I_{dc}}{I_v}\right) \quad (5.42)$$

Defining reference flux flow loss P_{ref} as:

$$P_{ref} = \rho_{ff} \frac{I_v^2}{4aw} \quad (5.43)$$

Again, we define the normalized flux flow loss $p_{ffis} = P_{ff}/P_{ref}$, normalized AC current $i_{ac} = I_m/I_v$, and normalized DC current $i_{dc} = I_{dc}/I_v$. Figure 5.12 shows the relationship between normalized flux flow loss p_{ff} and normalized AC current i_{ac} with different DC normalized currents $i_{dc} = 0, 0.4, 0.8, 1.2, 1.6$ and 2.0 . The flux flow loss increases with the increase of normalized DC current. If $i_{ac} < 0.2$ and $i_{dc} > 0.4$, flux flow loss is independent of the normalized AC current i_{ac} . When $i_{ac} > 0.2$, the flux flow loss increases with the increase of the normalized AC current i_{ac} . Figure 5.13 shows the relationship between normalized flux flow loss p_{ff} and normalized DC current i_{dc} at $i_{ac} = 0, 0.4, 0.8, 1.6$, and 2.0 , respectively. According to Figure 5.13, the trend of normalized flux flow loss is similar with those indicated in Figure 5.12.

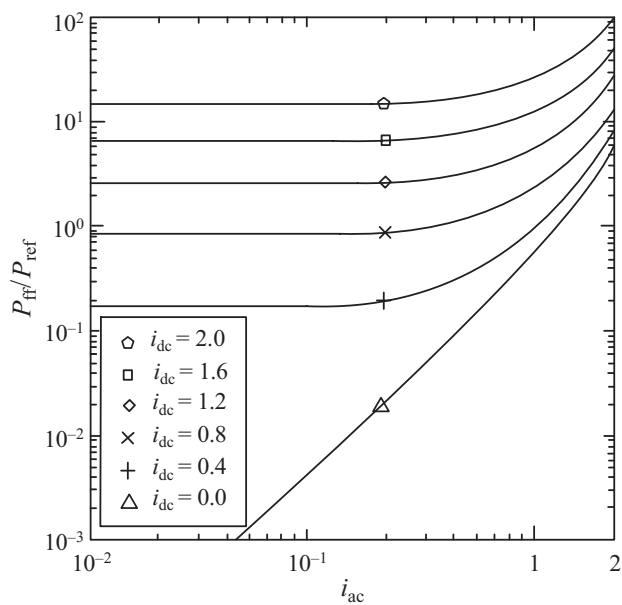


Figure 5.12 Normalized flux flow loss versus normalized AC current for different normalized DC currents.

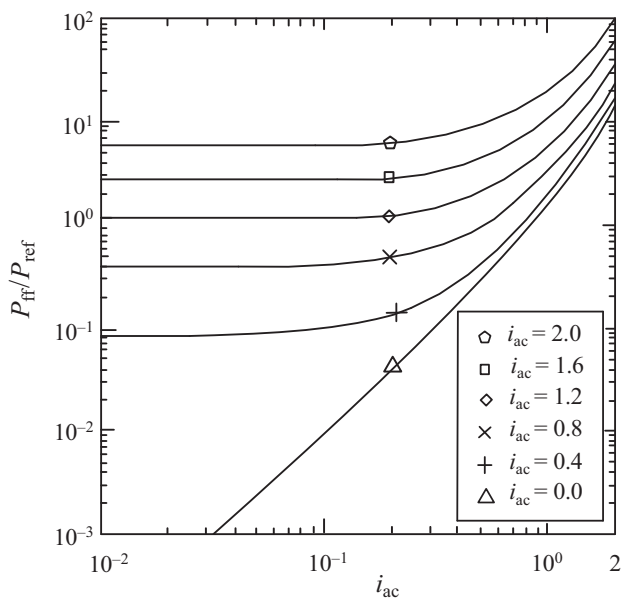


Figure 5.13 Normalized flux flow loss versus normalized DC current for different normalized AC currents.

If the temperature varies but is still below critical temperature T_c , the above calculation methods are also valid by just correcting the parameter I_v to:

$$I_v(T) = I_{v0} \left(1 - \frac{T}{T_0} \right) \quad (5.44)$$

where parameters I_{v0} and T_0 are constants and are related to the superconducting materials.

5.1.9 Total AC Losses of Slab with any AC/DC Current and AC/DC Magnetic Field

More generally, a superconducting slab with width $2w$ and thickness $2a$ carries an AC and DC current and is simultaneously in the AC and DC magnetic fields whose orientations are arbitrary, as shown in the dashed block diagram of Figure 5.14, where $w \gg a$. In order to calculate the total AC loss due to a combination of AC/DC current and AC/DC magnetic fields, the magnetic field B with any orientation can be decomposed into two components $B_{//}$ and $B_{\perp}$, which are parallel and perpendicular to the slab wide surface, respectively:

$$\begin{aligned} B_{ac//} &= B_m \cos \theta_{ac}, & B_{ac\perp} &= B_m \sin \theta_{ac} \\ B_{dc//} &= B_{dc} \cos \theta_{dc}, & B_{dc\perp} &= B_{dc} \sin \theta_{dc} \end{aligned} \quad (5.45)$$

where B_{ac} and θ_{ac} are the AC magnetic field and its angle between the AC magnetic field and the wide surface of the slab, respectively. Similarly, B_{dc} and θ_{dc} are the DC magnetic field and the angle between the DC magnetic field and the wide surface of the slab.

By using the same calculation method described in the above sections, total hysteresis losses P_{tot} (W/m) per unit length can be found by taking the sum of various losses:

$$P_{tot} = P_{//} + P_{\perp} + P_{ff} \quad (5.46)$$

Note that flux floss loss P_{ff} should be zero if the operating current is far away from its critical current I_c .

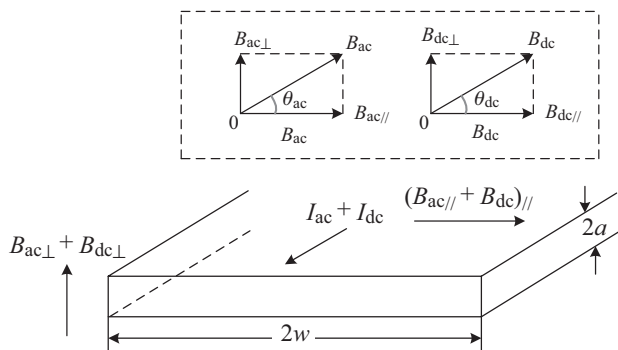


Figure 5.14 Slab with carrying AC/DC current in AC/DC magnetic field of various orientations.

5.2 AC Losses of Concentric Cylinder

As in Section 5.1, for the sake of simplicity and to obtain analytic solutions, the Bean CSM is also used to calculate AC losses of superconductors with circular cross-sections. Although the geometric structure of a superconductor with a circular cross-section is simple, its critical current is isotropic and calculation of the critical current is simple, but the analytic solutions of AC losses is more difficult compared with the superconducting slab described in Section 5.1. Even the analytic expressions cannot be obtained so it is essential to find the AC losses of the superconducting rod by numerical methods.

5.2.1 Rod in Longitudinal AC Magnetic Field

A superconducting rod with radius a and without a transport current is placed in an AC magnetic field $B_{ac}(t)$, which is parallel to the axis of the rod and is customarily called the longitudinal magnetic field (Figure 5.15). The amplitude of the AC magnetic field is smaller than the full penetration field (introduced later in this section). The current density J is the density of the shielding current induced by the AC magnetic field in the rod. By using cylindrical coordinates, the magnetic field is along the z -axis, and the induced current density J is along the circumferential (angular) direction.

According to the Bean CSM, the current density J is the critical current density J_c in the flux penetration region of rod, but the current density J is zero in the region that is not penetrated by flux. The penetration magnetic field B_p is defined as $B_p = \mu_0 J_c a$, where a denotes the radius of the rod. If the AC magnetic field is sinusoidal, $B(t) = B_m \sin(\omega t)$, B_m and ω are amplitude and angular frequency of the

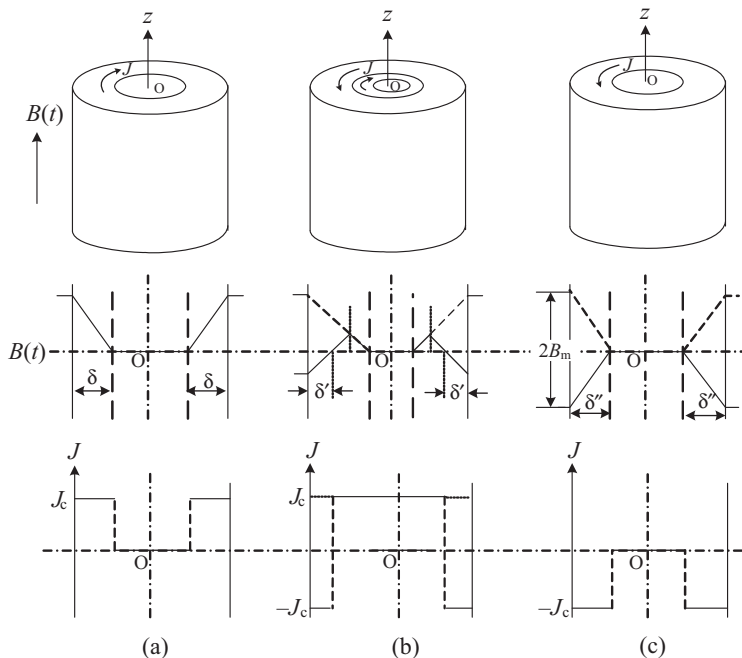


Figure 5.15 Distributions of magnetic field and induced current density in rod in longitudinal AC magnetic field: (a) partial penetrated; (b) magnetic field reduced and reversed with $|B| < B_p$; (c) magnetic field reaches its maximum with $|B_m| < B_p$, just before rising again.

AC magnetic field, respectively. The distributions of magnetic field and current density can be described in accordance with the following three situations:

- (1) When the amplitude of the AC magnetic field is less than full penetration field, $B_m < B_p$, the magnetic field cannot completely penetrate the rod, and the distributions of the magnetic field and the current density are shown in Figure 5.15(a). There are two sub-regions with boundary $r = (a - \delta)$, the inner and outer regions. The outer one is the annular region with width δ , which is the penetration depth of the magnetic field from the rod surface. The magnetic field B decreases linearly within the rod along its radius, and current density J is equal to critical current J_c . The inner one has a circular cross-section with radius $(a - \delta)$, in which both the magnetic field and current density are zero. If the amplitude of the AC magnetic field is greater than full penetration field, $B_m > B_p$, and the magnetic field fully penetrates the rod, that is, the front of the magnetic field passes through the centre of the rod, the magnetic field linearly decreases with penetration depth, and current density J is critical current J_c in the whole cross-section.
- (2) When amplitude B_m of the AC magnetic field decreases from its maximum value, the cross-section is also divided into two sub-regions with boundary $r = (a - \delta')$. The induced current density J in the outer annular region with width of penetration depth δ' is reversed and is equal to critical current $-J_c$, δ' is determined by $B_m = \mu_0 J_c \delta'$ and the current density in the inner region does not change with J_c . The distributions of the magnetic field and current density are shown in Figure 5.15(b).
- (3) After fully penetrated $|B_m| > B_p$, the current density J is the critical current $-J_c$ in the entire cross-section. When magnetic field is less than that of the penetration field, $|B_m| < B_p$, and increases again from the smallest value $-B_m$, the cross-section consists of two regions with boundary $r = (a - \delta'')$. The induced current density J in the outer annular region with width of penetration depth δ'' is reversed and is equal to critical current $-J_c$, δ'' is determined by $B_m = \mu_0 J_c \delta''$ and the current density J in the inner region is zero since there is no magnetic field in cross-section with radius $(a - \delta'')$. The distributions of magnetic field and current density are shown in Figure 5.15(c) [12].

If the longitudinal AC magnetic field is sinusoidal, AC loss (W/m) per unit length is:

$$\begin{aligned}
 P_{lm} &= fCA \frac{2B_m^2}{\mu_0} \left(\frac{2b_{ac}}{3} - \frac{b_{ac}^2}{3} \right) \quad b_{ac} \leq 1 \\
 P_{lm} &= fCA \frac{2B_m^2}{\mu_0} \left(\frac{2}{3b_{ac}} - \frac{1}{3b_{ac}^2} \right) \quad b_{ac} \geq 1
 \end{aligned} \tag{5.47}$$

where CA is the effective cross-sectional area related to the geometric structure of the superconducting rod, A is the total cross-sectional area of the rod (πa^2), $B_p = \mu_0 J_c a$ is the fully penetration field, and $b_{ac} = B_m/B_p$ is the normalized magnetic field.

5.2.2 Rod in Transverse AC Magnetic Field

A superconducting rod with diameter $2a$ is placed in a perpendicular (customarily called transverse) magnetic field and does not carry a transport current. Figure 5.16 shows the magnetic field and induced shielding current density in the rod when amplitude of the AC magnetic field is less than the penetration field. If the variation of the magnetic field is small, the shielding current is first induced on the surface of the rod and screens its interior from the influence of magnetic field variation. That is, the current on the rod surface generates a uniform magnetic field whose magnitude is equal but opposite to the transverse magnetic field. The current distribution is the cosine or overlapping elliptic. Figure 5.16(a) shows distributions of the magnetic field and induced shielding current density in the rod when the

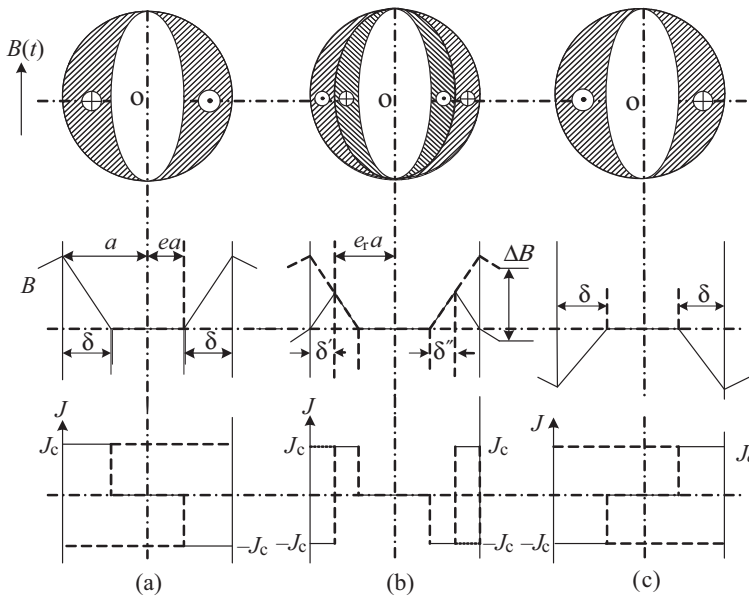


Figure 5.16 Distributions of magnetic field and induced current density across cross-section of a rod in a small AC transverse magnetic field: (a) patterns of magnetic field and current density when magnetic field increases, (b) patterns of magnetic field and current density when magnetic field decreases; (c) patterns of magnetic field and current density when magnetic field decreases its minimum just before increasing again.

amplitude of the transverse magnetic field is small. The region of rod cross-section is then partially penetrated out of the ellipse with a major axis length of $2a$ and a minor axis length of $2(a - \delta)$, where δ is the maximum penetration depth. If we define e as the ratio of the minor axis to the major axis of the ellipse, then $(a - \delta) = ea$. In the ellipse, the magnetic field and current density are zero and induced current density is the constant critical current J_c according to the Bean CSM.

Figure 5.16(b) indicates the distributions of the magnetic field and the induced shielding current density across the rod when the transverse magnetic field decreases from its original amplitude and the rod is not fully penetrated. The cross-section of the rod is divided into two sub-regions with boundary ellipse whose major axis length is still $2a$ and minor axis length is $2(a - \delta')$. The inner ellipse with major axis length $2a$ and minor axis length $2(a - \delta'')$ is in the virgin state without magnetic field and current density. Here, δ'' denotes the penetration depth of the original magnetic field. The inner and outer regions have opposite current densities and magnetic fields. Figure 5.16(c) illustrates the distributions of the magnetic field and induced current density through the cross-section of the rod when the transverse magnetic field continues to decrease from its minimum value before rising again. Here the amplitude of transverse magnetic field is still small, that is, the rod is still not fully penetrated. In this case, the transverse magnetic field is reversed and the distributions of the magnetic field and induced current are opposite to that shown in Figure 5.16(a) [1].

If the transverse AC magnetic field is sinusoidal $B_{ac}(t) = B_m \sin(\omega t)$, the magnetic field across the rod is:

$$B = \frac{4\mu_0 J_c}{2\pi} \int_0^{\pi/2} \cos \theta d\theta \int_{r_e}^a dr \quad (5.48)$$

where r_e is the minor axis half length of the inner elliptical boundary:

$$r_e = \frac{ea}{(\cos^2 \theta + e^2 \sin^2 \theta)^{1/2}} \quad (5.49)$$

By integrating Equation (5.48), we obtain:

$$B = \frac{2\mu_0 J_c a}{\pi} \left[1 - \frac{e \sin^{-1} (1 - e^2)^{1/2}}{(1 - e^2)^{1/2}} \right] \quad (5.50)$$

The magnetic moment per unit volume, that is, the magnetization, is:

$$M = \frac{4}{3\pi} \mu_0 a (1 - e^2) \quad (5.51)$$

Hysteresis loss (W/m) of the rod per unit length is:

$$P_{\text{tfm}} = CAf \frac{2B_m^2}{\mu_0} \left\{ \frac{8}{3b_{\text{ac}}^2} \int_{e_m}^1 \left[e_r - \frac{\sin^{-1} (1 - e_r^2)^{1/2}}{(1 - e_r^2)^{1/2}} \right] de_r - \frac{4}{3b_{\text{ac}}} (1 - e_m^2) \right\} \quad (5.52)$$

where CA is the effective cross-sectional area of the rod ($C\pi a^2$) and C is an equivalent constant. For multifilamentary composite superconductor, C is a constant and smaller than 1 and is related to the volume of the superconductor and the matrix material. In a single core pure superconductor, C is equal to 1, and e_m is the value of e corresponding to Equation (5.51) when the rod is fully penetrated and magnetization takes its maximum M_m . $b_{\text{ac}} = B_m/B_f$ is the normalized penetration transverse magnetic field, if B_f is defined as:

$$B_f = \frac{2\mu_0 J_c a}{\pi} \quad (5.53)$$

Also, B_f is called the characteristic magnetic field which is similar to that described in Section 5.1, where the superconducting slab is placed in a perpendicular magnetic field.

When $B_m < B_f$:

$$b_{\text{ac}} = \frac{B_m}{B_f} = 1 - \frac{e_m \sin^{-1} (1 - e_m^2)^{1/2}}{(1 - e_m^2)^{1/2}} \quad (5.54)$$

According to Equation (5.54), e_m can be obtained by numerical solution, and then by substituting e_m into Equation (5.52), we can find the AC loss when the transverse magnetic field partially penetrates the rod.

If $b_{\text{ac}} = 1$, $e_m = 0$, and by directly substituting them into Equation (5.52), the AC loss (W/m) per unit length can be obtained:

$$P_{\text{tfm}} = CAf \frac{1.246 B_m^2}{\mu_0} \quad (5.55)$$

If $b_{ac} > 1$, the transverse magnetic field fully penetrates the rod, and AC loss (W/m) per unit length is:

$$P_{\text{tfm}} = CAf \frac{2B_m^2}{\mu_0} \left(\frac{4}{3b_{ac}} - \frac{0.71}{b_{ac}^2} \right) \quad (5.56)$$

5.2.3 Rod in Transverse AC Magnetic Field and Carrying DC Transport Current

In Section 5.2.2, AC losses of superconductors with circular cross-section and carrying no transport current in the transverse AC magnetic field were discussed. Consider the case where DC power supplies DC current I_0 to the superconducting rod and the transverse AC magnetic field is sinusoidal, $B_{ac}(t) = B_m \sin(\omega t)$, and its action is to alter the magnetization current of the rod only (Figure 5.17). Here the calculation of AC loss is complicated and so far there is no analytical expression available [13].

However, when amplitude of the transverse AC magnetic field B_m is much larger than the penetration magnetic field B_f , that is, $b_{ac} \gg 1$, the AC loss (W/m) per unit length of the rod is:

$$P_{\text{tfc}} = CAf \frac{4}{3} \frac{B_m^2}{2\mu_0} \frac{1}{b_{ac}} g \left(\frac{I_0}{I_c} \right) b_{ac} \gg 1 \quad (5.57)$$

where the definition of the normalized magnetic field b_{ac} is similar to that described in Section 5.1, due to it carrying DC transport current I_0 simultaneously; modification of the penetration magnetic field B_f becomes:

$$B_f = B_f(0) \left(1 - \frac{I_0}{I_c} \right) \quad (5.58)$$

where $B_f(0)$ is determined by Equation (5.53), that is, the penetration magnetic field without a DC transport current.

Function $g(I_0/I_c)$ is given by:

$$g \left(\frac{I_0}{I_c} \right) = \left(1 - \frac{y^2}{a^2} \right)^{3/2} + \frac{3}{2} \frac{y^2}{a^2} \left(1 - \frac{y^2}{a^2} \right)^{1/2} + \frac{3}{2} \frac{y}{a} \sin^{-1} \frac{y}{a} \quad (5.59)$$

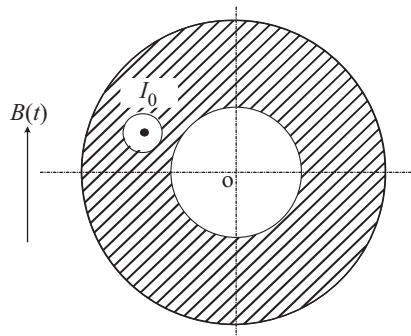


Figure 5.17 Cross-section of superconducting rod carrying transport DC current in AC transverse magnetic field.

Parameter y is related to transport current I_0 , critical current I_c and the radius a of the rod, and is determined by:

$$\frac{I_0}{I_c} = \frac{2}{\pi} \left(\frac{y}{a} \sqrt{1 - \frac{y^2}{a^2}} + \sin^{-1} \frac{y}{a} \right) \quad (5.60)$$

Equation (5.60) has no analytic solution, and y can be calculated only by a numerical approach. By substituting Equation (5.60) into Equations (5.59) and (5.57), we obtain the AC loss if the amplitude B_m of the AC transverse magnetic field is much larger than the penetration field B_f .

If the AC transverse magnetic field partially penetrates the rod, that is, $b_{ac} < 1$, the average AC loss (W/m) per unit length is:

$$P_{\text{fic}}(I_0, B_m) = P_{\text{tfm}}(0, B_m) \quad (5.61)$$

where $P_{\text{tfm}}(0, B_m)$ is determined by Equation (5.52).

When the AC transverse magnetic field fully penetrates the rod, that is, $b_{ac} > 1$, the average AC loss (W/m) per unit length is:

$$P_{\text{tfm}}(I_0, B_m) = P_{\text{tfm}}(0, B_m) + \left(1 - \frac{1}{b_{ac}} \right) F(I_0, B_m) \quad (5.62)$$

where function $F(I_0, B_m)$ is an unknown function. But if $I_0 = I_c$, Equation (5.62) is equivalent to Equation (5.57). $F(I_0, B_m)$ can be calculated only by means of experimental or numerical simulation in other cases.

5.2.4 Rod in Self-Magnetic Field

When a superconducting rod carries an AC transport current, the AC loss can also be generated due to the AC self-magnetic field induced by the AC transport current, which is customarily called the self-field loss without an AC magnetic field. Figure 5.18 shows distributions of current and magnetic field in the superconducting rod with diameter $2a$ and amplitude of transport current smaller than its critical current, that is, $I_m < I_c$, and patterns of the self-magnetic field forms a series of concentric circles around its axis. When carrying an AC transport current, these concentric circles will move toward or away from the superconducting rod along the radial direction with a varying self-magnetic field.

There are two sub-regions with boundary $r = c$ in the cross-section of the rod. Generally, the electric field is first induced on the superconductor surface, so that the variations of the current or magnetic field distribution begin at the surface and then both diffuse into the outer region with increase in transport current. In the magnetic field diffusion region, that is, the outer region, transport current flows with critical current I_c and there is no current in the inner region with no magnetic field diffusion. Since the viscous force resulting from the pinning effect acts on the flux lines of the self-magnetic field, AC loss will be generated when the flux lines of the self-magnetic field enter or leave the rod. Generally, the electric field is first induced on the superconductor surface, so that the variations of current or magnetic field distribution begin from the surface and then both diffuse gradually into the interior regions with the increase in transport current. In the magnetic field diffusion annular region with width of $(a - c)$, transport current flows as critical current I_c and there is no current in the inner region $r = c$ without magnetic field diffusion.

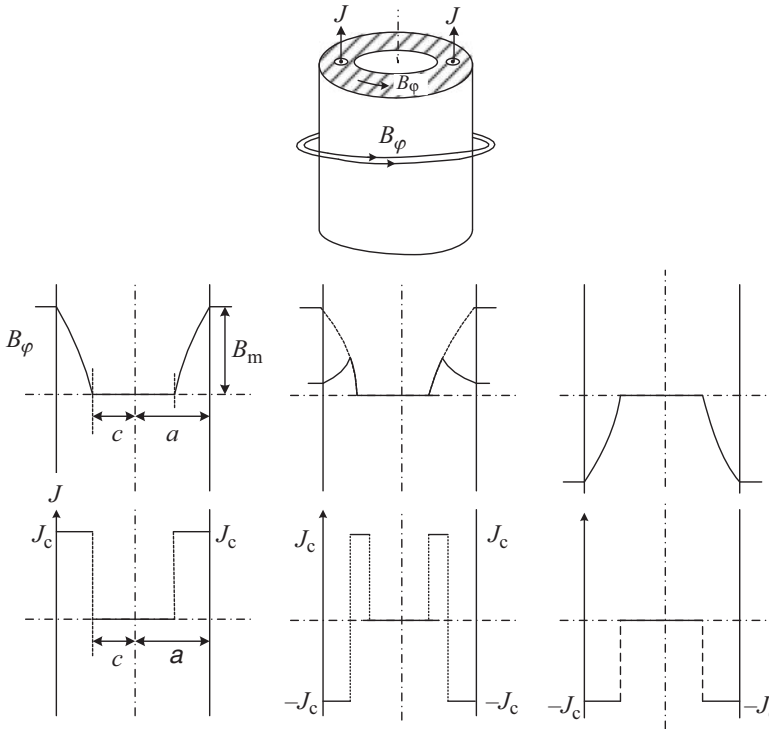


Figure 5.18 Distribution of self magnetic field and current density in rod carrying small AC current.

If the AC transport current is sinusoidal, that is, $I(t) = I_m \sin(\omega t)$, by defining normalized current $i_{ac} = I_m/I_c$, AC loss per unit length (W/m) of a rod with a circular cross-section in the self field is [12]:

$$P_{sf} = f \frac{\mu_0}{\pi} I_c^2 \begin{cases} \left[(2 - i_{ac}) \frac{i_{ac}}{2} + (1 - i_{ac}) \ln(1 - i_{ac}) \right] & i_{ac} < 1 \\ 1/2 & i_{ac} \geq 1 \end{cases} \quad (5.63)$$

If the cross-section of superconducting cylinder is elliptical, the calculation method of AC loss is exactly the same as in Equation (5.63) with a circular cross-section. If the AC transport current does not alter its direction, just cycles in accordance with process of $0 \rightarrow I_m \rightarrow 0 \rightarrow I_m \rightarrow 0 \dots$, self-field AC loss (W/m) per unit length becomes:

$$P'_{sf} = f \frac{\mu_0}{\pi} I_c^2 \begin{cases} \left[i_{ac} \left(2 - \frac{i_{ac}}{2} \right) + 4 \left(1 - \frac{i_{ac}}{2} \right) \ln \left(1 - \frac{i_{ac}}{2} \right) \right] & i_{ac} < 1 \\ \frac{3}{2} - 2 \ln 2 & i_{ac} \geq 1 \end{cases} \quad (5.64)$$

Obviously, the result from Equation (5.64) is smaller than that from Equation (5.63).

5.2.5 Rod-Carrying AC Transport Current in AC Transverse Magnetic Field with Same Phase

In practical applications, superconductors often carry an AC transport current and are placed in an AC transverse magnetic field with the same phase. Consider the case where the superconducting rod carries sinusoidal AC current $I(t) = I_m \sin(\omega t)$ and is simultaneously subjected to the sinusoidal AC transverse magnetic field with the same phase, $B(t) = B_m \sin(\omega t)$. The calculation of AC loss is difficult, except for some extreme cases, but it is important in practice. For example, the AC winding or coil in the superconducting apparatus in which the superconducting wire carries AC transport current, is subjected to an AC magnetic field with the same phase produced by other turns. As shown in Figure 5.19, the superconducting rod carries an AC current in the AC transverse magnetic field. Penetration magnetic field B_p is given by Equation (5.53). The magnetic field generated by an AC transport current in the circumferential direction on the surface of the rod is:

$$B_\varphi(t) = \frac{\mu_0 I(t)}{2\pi a} = \frac{\mu_0 I_m}{2\pi a} \sin(\omega t) = B_{sf} \sin(\omega t) \quad (5.65)$$

where $B_{sf} = \mu_0 I_m / (2\pi a)$ is the amplitude of the AC magnetic self field generated by the AC transport current on the surface of the rod.

AC loss per unit length (W/m) in combination with AC transport current and transverse AC magnetic field with the same phase can be described by following six cases [13]:

(1) If $B_m \ll B_{sf}$ (low field):

$$P_{tcm} = P_{sf} \left\{ 1 + \frac{8}{3} \frac{i_{ac}^3}{b_{sfm}^2} [i_{ac}(2 - i_{ac}) + 2(1 - i_{ac}) \ln(1 - i_{ac})]^{-1} \right\} \quad (5.66)$$

where P_{sf} is self-field AC loss of the rod without an AC magnetic field and only carrying an AC current, determined by Equation (5.63). The normalized current is $i_{ac} = I_m / I_c$, and here I_c denotes the critical current of the superconducting rod. The normalized self-magnetic field is $b_{sfm} = B_{sf} / B_m$, that is, the ratio of the self-field amplitude to the amplitude of the transverse AC magnetic field.

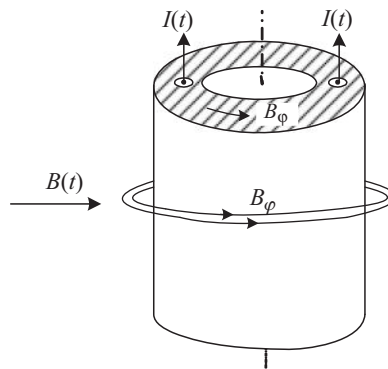


Figure 5.19 Rod carrying AC transport current and subject to AC transverse magnetic field with small current and magnetic field.

(2) If $B_m < B_{sf} < B_p$:

$$P_{tcm} = P_{sf} \left(1 + \frac{3}{b_{sfm}^2} \right) \quad (5.67)$$

where P_{sf} is the same as in (1).

(3) If $B_{sf} < B_m < B_p$:

$$P_{tcm} = P_{tfm} (1 + 3b_{sfm}^2) \quad (5.68)$$

where P_{tfm} is determined by Equation (5.52), that is, the AC loss in the transverse magnetic field without an AC transport current.

(4) If $B_{sf} < B_p < B_m < 2B_p$:

$$P_{tcm} = P_{tfm} \left(1 - \frac{2}{3b_{ac}} \right)^{-1} g(i_{ac}, b_{ac}, b_{sfm}) \quad (5.69)$$

where P_{tcm} is also determined by Equation (5.52). The normalized magnetic field is $b_{ac} = B_m/B_p$, here B_p is the penetration magnetic field given by Equation (5.53). Function $g(i_{ac}, b_{ac}, b_{sfm})$ is defined as:

$$g(i_{ac}, b_{ac}, b_{sfm}) = \left[1 + \frac{1}{3} i_{ac}^2 - \frac{2}{3b_{ac}} (1 - i_{ac}) (1 + i_{ac} + i_{ac}^2) + 2b_{sfm}^2 \frac{(1 - i_{ac})^2}{(1 - b_{sfm})} - \frac{4}{3} \frac{b_{sfm}^2}{b_{ac}} \frac{(1 - i_{ac})^3}{(1 - b_{sfm})^2} \right] \quad (5.70)$$

(5) If $2B_p < B_m$:

$$P_{tcm} = P_{tfm} \frac{2}{\pi} \frac{1}{i_{ac}} g(x) \quad (5.71)$$

where function $g(x)$ is:

$$g(x) = \left[3x - \frac{2}{3} x^3 - \frac{1}{5} x^5 - (1 - x^2)^{3/2} \sin^{-1} x \right] \quad (5.72)$$

Parameter x is determined by:

$$\frac{\pi}{2} i_{ac} = x \sqrt{1 - x^2} + \sin^{-1} x \quad (5.73)$$

where $0 \leq x \leq 1$.

(6) If $B_p \ll B_m$ (AC magnetic field is far larger compared with B_p):

$$P_{tcm} = P_{tfm} \left(1 + \frac{i_{ac}^2}{3} \right) \quad (5.74)$$

Table 5.1 Correction factors of different geometric shapes and different magnetic field orientations

Shape and magnetic field orientations	Aspect ratio	Shape factor
slab/parallel field ($B_{\parallel}$)	$w/d \ll 1$	1
Superconductor with square cross-section	$w/d=1$	~ 2
superconductor with circular cross-section	$w/d=1$	2
slab/perpendicular field ($B_{\perp}$)	$w/d \gg 1$	w/d

Note: w and d denote the width and thickness of superconducting slab, respectively.

5.2.6 Flux-Flow Losses of Rod-Carrying AC/DC Transport Currents Subjected to AC/DC Magnetic Field

Flux flow loss, which is independent of frequency, will also be generated due to the flux flowing when the transport current of a superconducting rod is close to its critical current with a combination of AC/DC transport currents and simultaneously subjected to AC/DC magnetic fields with the same phase besides the hysteresis loss. Flux-flow loss of a superconducting slab was introduced in Section 5.1.8, in which the analytic expressions were obtained. Nevertheless, like the analysis of AC loss in a rod, the calculation of the flux-flow loss in a rod is more difficult, and there have been no analytic solutions available to date. Therefore, to obtain precise solutions, numerical simulations are needed.

Fortunately, although we cannot obtain solutions for the flux-flow loss of the rod, the flux-flow loss of a superconducting slab has accurate analytic solutions. It has been shown that the losses of a superconductor with circular or rectangular cross-sections can be obtained after correcting the calculation of a superconducting slab by shape factors, which are good approximations for applications.

Shape correction factors of superconductors with different geometric cross-sections and different magnetic field orientations are listed in Table 5.1 [14], which introduces how to obtain flux-flow loss P_{ff} of superconductors with circular or rectangular cross-sections by correction shape factors.

The total loss P_{tot} (W/m) per unit length of a superconducting rod can be calculated by the method of calculating AC loss introduced in Section 5.2.

$$P_{tot} = P_{sf} + P_{tcm} + P_{tfc} + P_{ff} \quad (5.75)$$

5.3 AC Losses of Hybrid Concentric Cylinder

Figure 5.20 is the cross-section of a cylindrical rod consisting of two kinds of superconductors in a large transverse AC magnetic field (fully penetrated). As shown in the figure, their critical current densities are J_{c1} and J_{c2} , respectively, and the inner and outer radii of the outer hollow cylindrical superconductor are separately R_i and R_o . The applied transverse magnetic field is sinusoidal, $B(t) = B_m \sin(\omega t)$, and the hybrid superconductors do not carry current. When the applied transverse magnetic field is so small that the hollow cylindrical superconductor is partially penetrated, the solution is the same as that introduced in Section 5.2.2. If both of them are fully penetrated, the penetration field B_p is:

$$B_p = \frac{2\mu_0}{\pi} [J_{c1} (R_o - R_i) + J_{c2} R_i] \quad (5.76)$$

If $B_m \gg B_p$, the AC loss (W/m) per unit length is approximately:

$$P_{im} \approx CAf \frac{4}{3} B_m R_o \left[J_{c1} \left(1 - \left(\frac{R_i}{R_o} \right)^3 \right) + J_{c2} \left(\frac{R_i}{R_o} \right)^3 \right] \quad (5.77)$$

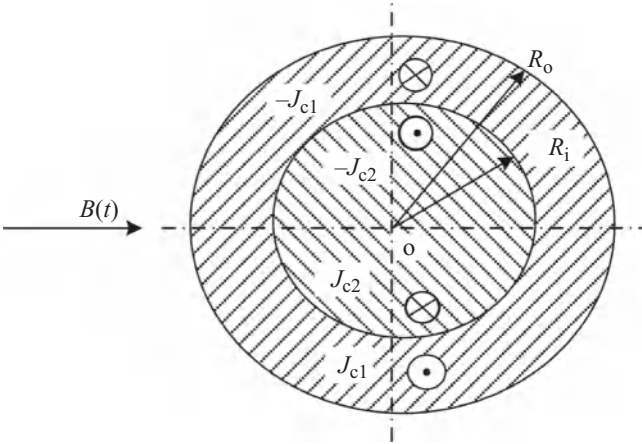


Figure 5.20 Cross-section of current distribution of hybrid cylindrical superconductor with hollow cylinder and rod in large AC transverse magnetic field.

If $J_{c2} = 0$, Figure 5.20 represents a hollow cylinder with thickness $(R_o - R_i)$, and AC loss (W/m) per unit length in the transverse magnetic field is approximately:

$$P_{tm} \approx CAf \frac{4}{3} B_m R_o J_{c1} \left(1 - \left(\frac{R_i}{R_o} \right)^3 \right) \tag{5.78}$$

There is another simple calculation method for AC losses of a hollow cylinder in a large transverse magnetic field. When the amplitude of a transverse AC magnetic field is far larger than the penetration field B_p , $B_m \gg B_p$, AC loss of a hollow superconducting cylinder is the difference of AC losses between the two cylinders with radii of R_o and R_i , respectively (Figure 5.21). The calculation method of a cylindrical superconductor in a transverse magnetic field is given by Equation (5.56) in Section 5.2.2.

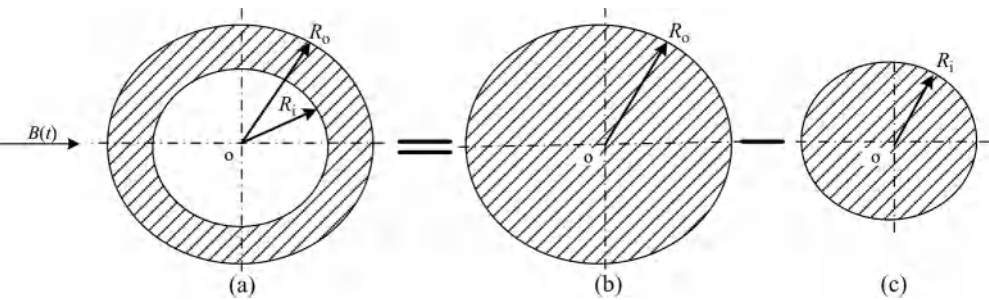


Figure 5.21 Cross-sections of hollow cylinder for AC loss calculation in AC transverse magnetic field: (a) hollow cylinder with thickness $(R_o - R_i)$; (b) cylinder with diameter R_o , (c) cylinder with diameter R_i .

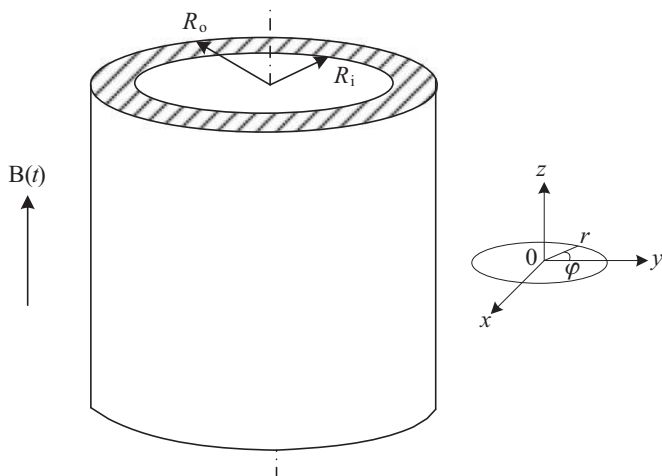


Figure 5.22 Superconducting tube subjected to AC longitudinal magnetic field.

5.4 AC Losses of Concentric Hollow Cylinder in Longitudinal Field

Figure 5.22 shows an infinite superconducting tube with inner and outer radii of R_i and R_o , respectively. The AC magnetic field is sinusoidal $B(t) = B_m \sin(\omega t)$ and is parallel to the tube axis, customarily called the longitudinal magnetic field. A cylindrical coordinate system is adopted [15]. According to the Bean CSM, AC loss (W/m) per unit length is:

$$P_{ml} = CAf \begin{cases} \frac{4B_m^2}{3\mu_0(R_o + R_i)} b_{ac} \left(R_o - \frac{b_{ac}}{2} (R_o - R_i) \right) & B_m \leq B_p \\ \frac{4B_p B_m}{3\mu_0(R_o - R_i)} \left(\frac{R_o^2 + R_i^2 + R_o R_i}{R_o + R_i} \right) - \\ \frac{2B_p^2}{3\mu_0(R_o - R_i)^2} \left(\frac{R_o^3 + R_o^2 R_i + R_o R_i^2 - 3R_i^3}{R_o + R_i} \right) & B_m > B_p \end{cases} \quad (5.79)$$

where $B_p = \mu_0 J_c (R_o - R_i)$ is the penetration magnetic field, and J_c is the critical current density. The normalized magnetic field is $b_{ac} = B_m/B_p$.

If $R_i = 0$, Equation (5.79) returns to Equation (5.47), that is, the equation for the AC loss of the superconducting rod in the AC longitudinal magnetic field.

5.5 AC Losses for Large Transverse Rotating Field

In a rotating magnetic field, the AC loss calculation of a superconductor with a circular cross-section is interesting. In a motor, the rotor rotates in the DC magnetic field with the angular frequency $\omega = 2\pi f$ and often carries a DC transport current. As shown in Figure 5.23, the superconducting cylinder carrying a DC current I_o rotates counter-clockwise at angular velocity ω and is exposed to a transverse constant

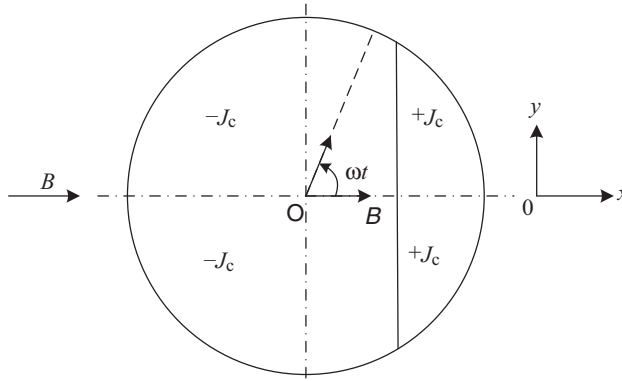


Figure 5.23 Current distribution of cylinder in large transverse rotating magnetic field.

magnetic field. In the superconducting cylinder [13], the rotating magnetic field can be decomposed into two AC transverse magnetic field components with the phase difference of 90° , that is:

$$B(t) = B_m (i \cos \omega t + j \sin \omega t) \quad (5.80)$$

where i and j denote the unit vectors along the x -axis and y -axis in the cross-section of the superconducting cylinder with Cartesian coordinates.

Assuming that the transverse DC magnetic field $B = B_m$ is much larger than the penetration magnetic field B_p , namely, the normalized magnetic field $b_{ac} = B_m/B_p \gg 1$, AC loss (W/m) per unit length is:

$$P_{tfc} = CAf \frac{8}{3\pi} \frac{B_m^2}{2\mu_0} \frac{1}{b_{ac}} g\left(\frac{I_0}{I_c}\right) \quad (5.81)$$

where CA is the effective cross-section of the cylinder and A is its real cross-section, function $g(I/I_c)$ is determined by Equation (5.59) in Section 5.2.3, and I_c is critical current of the superconducting cylinder.

5.6 AC Losses with Different Phases between AC Field and AC Current

In Sections 5.1 to 5.4, AC losses of superconductors under conditions of carrying AC current, exposed to an AC magnetic field, and a combination of a carrying AC current and exposed to an AC magnetic field with the same phase was introduced [5, 16–18]. Where there is a phase difference (phase shift) between the AC transport current and the AC magnetic field, the calculation of AC losses is not relevant. Generally, we will not encounter this question in common superconducting apparatus. However, the phase difference between the AC transport current and the AC magnetic field generated by other phases appears in warm-dielectric (WD) superconducting cables and generators or motors. However, this kind of AC loss calculation is too complicated.

There are some approximate analytical expressions for the AC loss calculation of a superconducting slab carrying an AC current and subjected to an AC magnetic field parallel to its wide surface with a different phase. As for AC loss calculations of a superconducting slab carrying current and exposed to an AC magnetic field perpendicular to its wide surface with a different phase and a superconducting rod carrying an AC current and exposed to its transverse AC magnetic field with a different phase, there are no analytical expressions at present, so numerical are required in accordance with the current-voltage characteristics of superconductors. This section focuses on AC loss of a superconducting slab for carrying

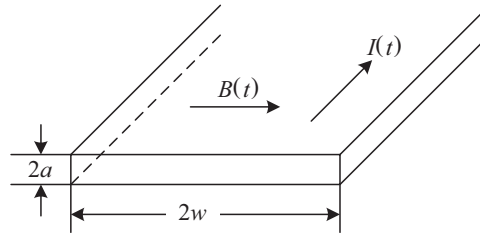


Figure 5.24 Geometrical structure of slab carrying AC current and exposed to parallel AC magnetic field.

current and exposed to an AC magnetic field parallel to its wide surface with a different phase, both of them with the same frequency.

5.6.1 Slab-Carrying Current Exposed to AC Magnetic Field Parallel to its Wide Surface with Different Phases

Figure 5.24 illustrates the geometric structure and magnetic field orientation of a finite superconducting slab carrying an AC transport current $I(t)$ and simultaneously exposed to an AC magnetic field $B(t)$ parallel to its wide surface with a phase difference of δ . If the width and thickness of the slab are $2w$ and $2a$, respectively, and its critical current density is J_c , then the penetration magnetic field is $B_p = \mu_0 J_c a$. Magnetic fields near the upper and lower surfaces of the slab are separately [13]:

$$\begin{aligned} B_1(t) &= B(t) + B_I(t) = B_m \sin(\omega t) + B_{Im} \sin(\omega t + \delta) \\ B_2(t) &= B(t) - B_I(t) = B_m \sin(\omega t) - B_{Im} \sin(\omega t + \delta) \end{aligned} \quad (5.82)$$

where $B_I(t)$ is the magnetic field generated by the AC transport current, that is, the self field. ω is the angular frequency, and B_m and B_{Im} are the amplitudes of the applied AC magnetic field and the self-magnetic field, respectively.

Analyses on AC losses are performed in the following three cases:

- (1) If the amplitude of the AC magnetic field is larger than the penetration field $B_m > B_p$, AC loss (W/m) per unit length is:

$$P_\delta = CAf \frac{B_p}{4\mu_0} \int_0^T \left[\left| \frac{dB(t)}{dt} \right| \left(1 + \frac{I^2(t)}{I_c^2} \right) + 2 \frac{dB_I(t)}{dt} \frac{I(t)}{I_c} \right] dt \quad (5.83)$$

where CA is the effective cross-section of the slab, A is the geometrical cross-section, and C is the effective coefficient. T is period of the AC current or AC magnetic field.

- (2) If the amplitude of the AC magnetic field is smaller than the penetration field, $B_m < B_p$, and both the current and the magnetic field are zero at the initial state and zero moments, AC loss (W/m) per unit length is:

$$P_\delta = CAf \frac{1}{48\pi^2 \mu_0^2 B_p} \int_0^T \left[\left| \frac{\partial}{\partial t} (B(t) + B_I(t))^3 \right| + \left| \frac{\partial}{\partial t} (B(t) - B_I(t))^3 \right| \right] dt \quad (5.84)$$

- (3) If a slab is partially penetrated, $B_m < B_p$, the magnetic field enters the slab from the upper and lower surfaces at the initial moments of t_1 and t_2 . AC loss is also related to the initial current and magnetic

field. Assuming that the initial magnetic field entering the slab from the upper and lower surfaces, and the self field are $B(t_2)$, $B_I(t_2)$, $B(t_1)$ and $B_I(t_1)$, respectively, then AC loss (W/m) per unit length is:

$$P_\delta = CAf \frac{1}{192\pi^2 \mu_0^2 B_p} \int_0^T \left\{ \left[\frac{\partial}{\partial t} (B(t) + B_I(t) - B(t_2) - B_I(t_2))^3 \right] + \left[\frac{\partial}{\partial t} (B(t) - B_I(t) - B(t_1) + B_I(t_1))^3 \right] \right\} dt \quad (5.85)$$

Generally, this kind of AC loss calculation is complicated and no analytic solutions can be found, so numerical integration is required.

5.6.2 Slab-Carrying Current Exposed to Parallel AC Magnetic Field at One Side with Different Phases

In a superconducting solenoid with dense winding, it is assumed that the cross-sectional size of the superconducting slab is much less than the radius of the superconducting solenoid and the superconducting solenoid is infinite in length. The inner side of the slab on the innermost layer within the coil is affected by a combination of the self-magnetic field and the applied parallel magnetic field, while its outer side is affected by the applied parallel magnetic field only. Figure 5.25 shows the main cross-section of the solenoid and AC magnetic fields, including the applied parallel magnetic field and the self-magnetic field. In the coil, width and thickness of the superconducting slab are $2w$ and $2a$, respectively. Supposing that patterns of AC current and applied magnetic field are cosines, and the phase difference is δ , namely:

$$\begin{aligned} I(t) &= I_m \cos(\omega t + \delta) \\ B(t) &= B_m \cos(\omega t) \end{aligned} \quad (5.86)$$

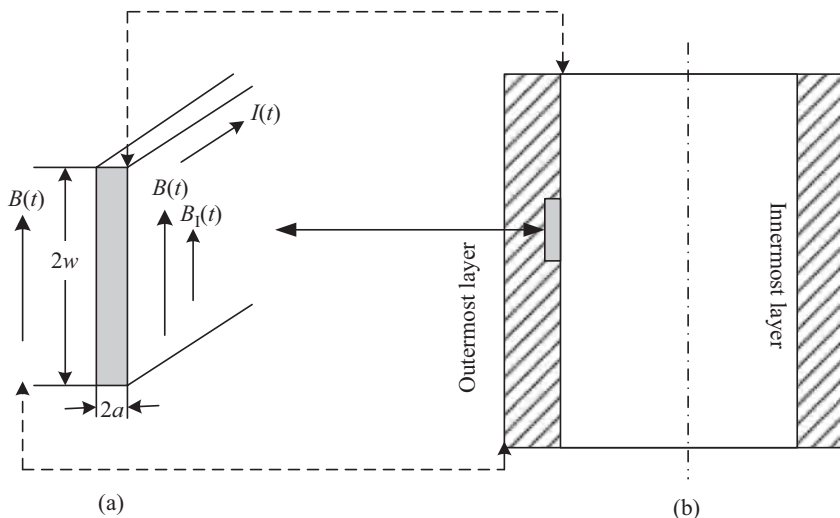


Figure 5.25 Slab on innermost layer of infinite solenoid exposed to AC magnetic field; (a) geometrical structure and patterns of AC magnetic fields on both of sides; (b) main cross view of infinite solenoid.

The magnetic self field on the right-hand surface of the slab generated by an AC transport current in the solenoid is:

$$B_1(t) = \frac{\mu_0 I(t)}{4w} = \frac{\mu_0 I_m}{4w} \cos(\omega t + \delta) \quad (5.87)$$

The amplitude of the self field on its wide surface is $B_{1m} = \mu_0 I_m / (4w)$, and by neglecting the magnetic field generated by the solenoid, the magnetic fields on right-hand and left-hand surfaces of the slab are:

$$\begin{aligned} B_1(t) &= B_m \cos \omega t + B_{1m} \cos(\omega t + \delta) \\ B_2(t) &= B_m \cos \omega t \end{aligned} \quad (5.88)$$

Defining the normalized magnetic field $b_{ac} = B_m / B_p$ and the normalized transport current $i_{ac} = I_m / I_c$, after rearranging, Equation (5.88) becomes:

$$\begin{aligned} B_1(t) &= B_{1m} \cos(\omega t + \alpha) \\ B_2(t) &= B(t) = B_{2m} \cos(\omega t) \end{aligned} \quad (5.89)$$

where

$$\begin{aligned} B_{1m} &= \sqrt{B_m^2 + B_{1m}^2 + 2B_m B_{1m} \cos \delta} \\ \alpha_1 &= \tan^{-1} \left(\frac{B_{1m} \sin \delta}{B_m + B_{1m} \cos \delta} \right) \end{aligned} \quad (5.90)$$

$$\begin{aligned} B_{2m} &= \sqrt{B_m^2 + B_{1m}^2 - 2B_m B_{1m} \cos \delta} \\ \alpha &= \tan^{-1} \left(\frac{B_{1m} \sin \delta}{B_m - B_{1m} \cos \delta} \right) \end{aligned} \quad (5.91)$$

The effective penetration magnetic field is:

$$B_p^* = \frac{B_{1m}^2 - 2B_{1m}B_{2m} \cos \alpha + B_{2m}^2 - 4B_p^2}{-2B_{1m} \cos \alpha + 2B_{2m} - 4B_p} \quad (5.92)$$

where $B_p = \mu_0 J_c a$ is the penetration magnetic field of the slab without current and exposed to a parallel AC magnetic field.

The AC losses are analysed in accordance with the following three situations:

- (1) If $B_m < B_p^*$, namely the slab is partially penetrated, AC loss (W/m) per unit length is:

$$P_\delta = CAf \frac{1}{3\mu_0 B_p} (B_{1m}^3 + B_{2m}^3) \quad (5.93)$$

where CA is the effective cross-section of the slab, A is the geometrical cross-section and C is the effective coefficient. T is the period of the AC current or the AC magnetic field. According to Equation (5.93), AC loss has nothing to do with the phase difference between the AC current and the AC magnetic field. The AC loss calculation is equivalent to the current and AC magnetic field having the same phase.

(2) If $B_m > B_p^*$, that is, the slab is fully penetrated with $\alpha < 0$, AC loss (W/m) per unit length is:

$$P_\delta = CAf \frac{1}{24\mu_0 B_p} \left\{ \begin{aligned} & B_{1m}^3 [5 + 3 \cos (2\alpha + 2\omega t_0)] + B_{2m}^3 [5 + 3 \cos (2\omega t_0)] \\ & - B_{1m}^2 B_{2m} \left[\begin{aligned} & 3 + 3 \cos (2\alpha + \omega t_0) \\ & + \cos (2\alpha) + \cos (2\alpha + 3\omega t_0) \end{aligned} \right] \\ & - B_{1m} B_{2m}^2 [4 \cos \alpha + 3 \cos (\alpha - \omega t_0) + \cos (\alpha + 3\omega t_0)] \\ & + 12 B_p B_{1m}^2 \sin^2 (\alpha + \omega t_0) + 12 B_p B_{2m}^2 \sin^2 (\omega t_0) \end{aligned} \right\} \quad (5.94)$$

If $B_m > B_p^*$, and the slab is penetrated fully with $\alpha > 0$, AC loss (W/m) per unit length becomes:

$$P_\delta = CAf \frac{1}{24\mu_0 B_p} \left\{ \begin{aligned} & B_{1m}^3 [5 + 3 \cos (2\alpha + 2\omega t_0)] + B_{2m}^3 [5 + 3 \cos (2\omega t_0)] \\ & - B_{1m}^2 B_{2m} \left[\begin{aligned} & 4 + 3 \cos (2\alpha + \omega t_0) \\ & + \cos (2\alpha + 3\omega t_0) \end{aligned} \right] \\ & - B_{1m} B_{2m}^2 [3 + 3 \cos (\alpha - \omega t_0) + \cos (2\alpha) + \cos (\alpha + 3\omega t_0)] \\ & + 12 B_p B_{1m}^2 \sin^2 (\alpha + \omega t_0) + 12 B_p B_{2m}^2 \sin^2 (\omega t_0) \end{aligned} \right\} \quad (5.95)$$

where ωt_0 is:

$$\omega t_0 = \begin{cases} \cos^{-1} \left(\frac{B_{1m} - 2B_p}{B_{2m}} \right) & \alpha \leq \alpha_0 \\ \cos^{-1} \left(\frac{B_{2m} - 2B_p}{B_{1m}} \right) & \alpha > \alpha_0 \end{cases} \quad (5.96)$$

α_0 is determined by:

$$\alpha_0 = \cos^{-1} \left(\frac{B_{2m} - 2B_p}{B_{1m}} \right) - \cos^{-1} \left(\frac{B_{1m} - 2B_p}{B_{2m}} \right) \quad (5.97)$$

5.6.3 AC Losses of Slab-Carrying AC Current and Exposed to Symmetrical Parallel AC Magnetic Field with Different Phases

Referring to the description in Section 5.6.1, patterns of the AC transport current and applied magnetic field are cosines. The magnetic fields on both sides are different from Equation (5.82). As shown in Figure 5.24, magnetic fields near the upper and lower surfaces of the superconducting slab are:

$$\begin{aligned} B_1(t) &= B_m \cos \omega t + B_{1m} \cos (\omega t + \delta) \\ B_2(t) &= B_m \cos \omega t - B_{1m} \cos (\omega t + \delta) \end{aligned} \quad (5.98)$$

The moment at which the maximum electric field generating on the surface of the slab is:

$$t_{\max, \pm} = \frac{2\pi}{\omega} \tan^{-1} \left(\left| \frac{\pm 1 + h \cos (\alpha)}{h \sin (\alpha)} \right| \right) \quad (5.99)$$

where h is the effective normalized magnetic field, $h = B_m/B_{lm}$, namely the ratio of amplitudes of applied AC magnetic field and self-magnetic field generated by AC transport current. The phase angle corresponding to Equation (5.99) is:

$$\theta_{\max, \pm} = \omega t_{\max, \pm} \quad (5.100)$$

If the slab is partially penetrated, that is, $b_{ac} = B_m/B_p < 1$, the AC loss (W/m) per unit length is:

$$P_\delta \propto [\sin(\theta_{\max, +}) + h \sin(\theta_{\max, +} + \delta)]^3 + [-\sin(\theta_{\max, -}) + h \sin(\theta_{\max, -} + \delta)]^3 \quad (5.101)$$

Figure 5.26 shows the relationship of normalized AC losses with different normalized AC magnetic field and different phase differences [16–19]. It is indicated that AC loss is independent of phase difference when the amplitude of the AC magnetic field is smaller than or far greater than the penetration field, that is, $B_m < B_p$ or $B_m \gg B_p$. However, if the differences between the amplitude of the AC magnetic field and penetration field are fairly small, namely B_m approaches to B_p , the phase difference between the AC current and the AC magnetic field will have significant effects on AC losses. Especially, if the phase difference closes to 90 degrees, AC loss takes the minimum value.

Figure 5.27 presents the dependence of normalized AC loss on phase difference. In the figure, the dots denote experimental results of AC losses of an HTS tape in an AC magnetic field with amplitude $B_m = 6.3$ mT, and the curved line refers to the calculated results. We can see from the figure that the theoretical calculations are in agreement with experimental results. We can also find the approximate relationship

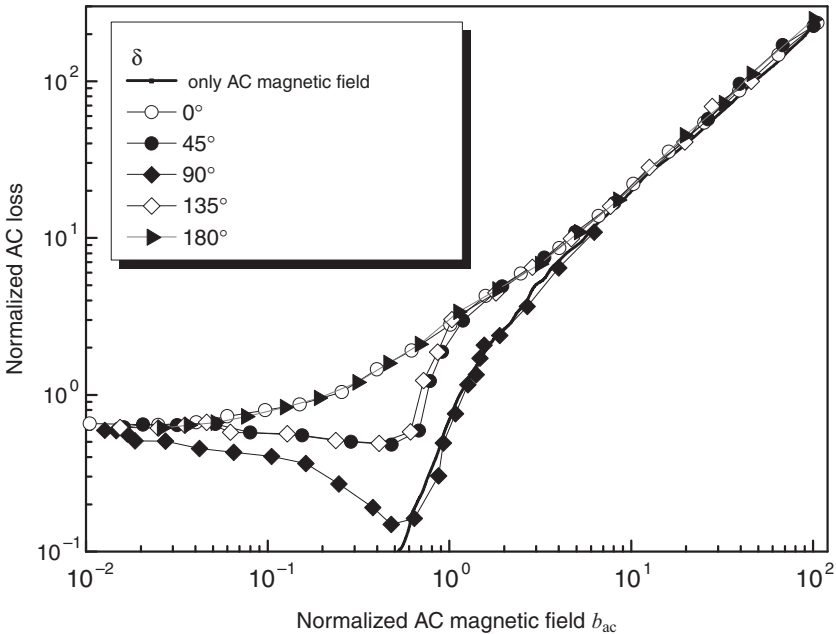


Figure 5.26 Normalized AC losses with normalized magnetic field and phase differences.

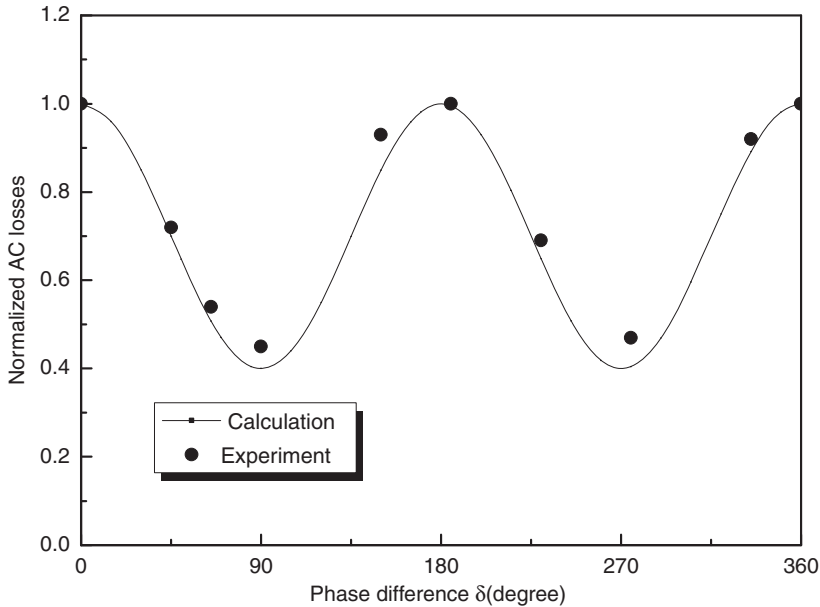


Figure 5.27 Normalized AC loss versus phase differences.

between the normalized AC loss and phase difference when the amplitudes of the AC magnetic field and AC current are invariable [20, 21]:

$$P_{\delta} = a + b \cos(2\delta) \quad (5.102)$$

where a and b are constants relating with the amplitudes of the AC magnetic field and the AC transport current.

If the slab carries an AC transport current and is simultaneously exposed to an AC magnetic field perpendicular to its wide surface with different phases, there are no analytic expressions of AC losses, so we need to calculate the results by a numerical method according to the electromagnetic properties of the superconducting slab. For example, an infinite slab carries an AC current and is exposed to an AC magnetic field perpendicular to its wide surface with different phases, and then the following exponential law of current-voltage characteristic should be used:

$$E = E_c \left(\frac{J}{J_c} \right)^n \quad (5.103)$$

According to the Maxwell equation:

$$\frac{\partial A}{\partial t} = E - \nabla V \quad (5.104)$$

where A and E are magnetic and electric vectors, respectively, and ∇V denotes the gradient of potential along the slab. Then the distributions of the electric field and the current density in the slab can be calculated. Substituting them into Equation (5.105) and integrating over the whole cross-section

of the slab, AC loss within the AC magnetic field with any orientations and phase differences can be obtained:

$$P_{\delta} = \int E \cdot J dS \quad (5.105)$$

The integral path is along the whole cross-section of the superconducting slab.

5.7 AC Losses for other Waves of AC Excitation Fields

In previous sections of this chapter, AC loss of superconductors with sinusoidal AC transport current and AC magnetic field are introduced only at power frequency, but other waveforms of AC current and AC magnetic field are not of concern, such as exponential decay, triangular and trapezoidal patterns, etc. [22] Often there are cases in which the pulsed superconducting magnet is always energized by different pulsed currents. This section will discuss AC loss of other commonly used AC magnetic field waveforms, besides the sinusoidal waveform. For simplicity, we only introduce the AC loss calculation method in the simplest case of an infinite slab with width $2w$ and thickness $2a$ and exposed to an AC magnetic field parallel to its wide surface (Figure 5.1). Other commonly used magnetic field waveforms include the exponential decay waveform, triangular waveform and trapezoidal waveform. Figure 5.28 illustrates the schematics of these three magnetic field waveforms.

The magnetic field expression of the exponential decay waveform is:

$$B(t) = B_m \exp\left(-\frac{t}{\tau_m}\right) \quad (5.106)$$

where τ_m denotes the time decay constant, and B_m is the amplitude of the AC magnetic field. The other two waveforms are shown in Figures 5.28(b) and (c). For these three different waveforms, the AC losses exposed to three kinds of AC magnetic fields parallel to the slab wide surface are discussed in the following two cases:

- (1) If a slab is partially penetrated, that is, $B_m \leq B_p$, the normalized AC magnetic field $b_{ac} = B_m/B_p \leq 1$, AC loss (W/m) per unit length with exponential decay magnetic field is:

$$P_{//} = CA \frac{B_m^2}{2\mu_0} \frac{b_{ac}}{5\tau_m} \quad (5.107)$$

where CA is the effective cross-section of the slab, A is the geometrical cross-section and C is the effective coefficient. T is the period of the AC current or the AC magnetic field. B_p is the penetration magnetic field, here $B_p = \mu_0 J_c a$, and J_c refers to the critical current density of the superconducting slab.

AC loss (W/m) per unit length with triangular waveform or trapezoidal waveforms is:

$$P_{//} = CA \frac{B_m^2}{2\mu_0} \frac{b_{ac}}{5\tau_m} \quad (5.108)$$

- (2) If a slab is fully penetrated, that is, $B_m > B_p$, the normalized AC magnetic field $b_{ac} = B_m/B_p > 1$, AC loss (W/m) per unit length with exponential decay magnetic field is:

$$P_{//} = CA \frac{B_m^2}{2\mu_0} \frac{1}{5\tau_m b_{ac}} \quad (5.109)$$

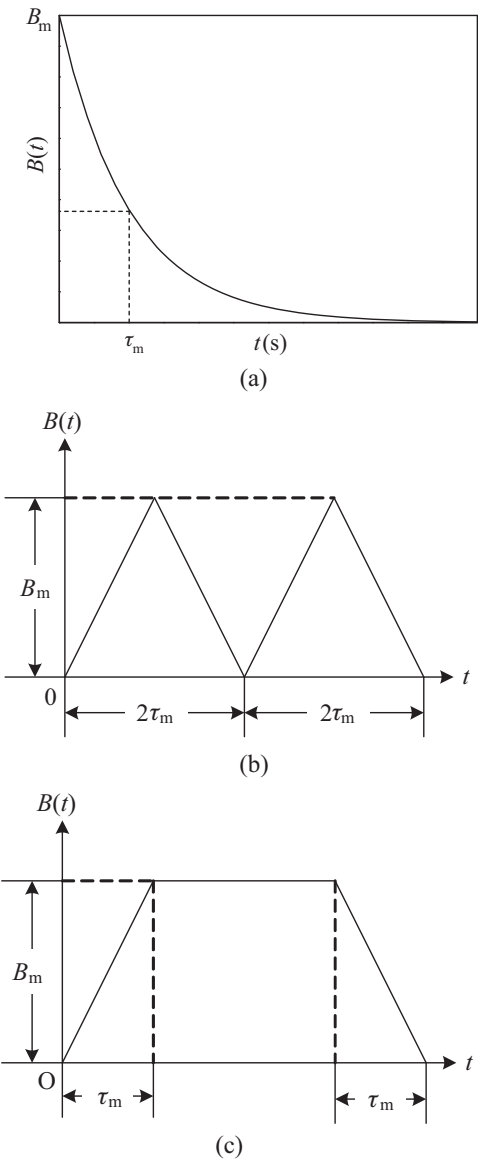


Figure 5.28 Waveforms of AC magnetic fields: (a) exponential decay; (b) triangular; and (c) trapezoidal.

AC loss (W/m) per unit length with triangular waveform or isosceles trapezoidal waveform is:

$$P_{//} = CA \frac{B_m^2}{\mu_0} \frac{1}{2\tau_m b_{ac}} \tag{5.110}$$

In addition, it is indicated that AC losses caused by AC magnetic fields from both triangular and trapezoid waveforms are the same.

5.8 AC Losses for other Critical State Models

In previous sections of this chapter, AC losses were all calculated based on the Bean CSM, that is, the critical current density J in superconductors is either zero or a constant J_c that is independent to the magnetic field. However, the critical current density J_c of superconductors closely depends on the magnetic field. The commonly used critical models mainly include the Kim model and the Kim-like model. We will take an AC magnetic field with sinusoidal waveform $B(t) = B_m \sin(\omega t)$ as an example. AC losses of an infinite superconducting slab and rod will be discussed according to different relationships of critical current density J_c and magnetic field B .

5.8.1 Kim Model

In the Kim model, the dependence of critical current density J_c on the magnetic field is:

$$J_c(B) = \frac{\beta}{B^p} \quad (5.111)$$

where β and p are constants.

- (1) A superconducting slab with width $2w$ and thickness $2a$, is subjected to a magnetic field parallel to its wide surface (Figure 5.1) [23]. AC loss (W/m) per unit length is:

$$P_{l/m} = CAf \frac{4(p+1)}{(p+2)} \frac{B_s^2}{\mu_0} \left(\frac{B_m}{B_s} \right)^{p+3} F_1 \quad (5.112)$$

B_s is determined by:

$$\left(\frac{B_s}{\mu_0} \right)^{p+1} = (p+1) \beta a \quad (5.113)$$

where function F_1 is defined by:

$$F_1 = \left(\frac{1}{2} \right)^s \int_0^1 [(1+x^q)^s - (1-x^q)^s] dx - \frac{1}{p+3} \quad (5.114)$$

and

$$q = p+1, s = \frac{(p+2)}{p+1} \quad (5.115)$$

If $0 \leq p \leq 1.5$, then F_1 becomes:

$$F_1 \approx \frac{(p+2)}{6} \quad (5.116)$$

Then, Equation (5.112) becomes:

$$P_{l/m} = CAf (p+1) \frac{2B_s^2}{3\mu_0} \left(\frac{B_m}{B_s} \right)^{p+3} \quad (5.117)$$

If $p = 0$, then $s = 2$ and $J_c = \beta$, the Kim model becomes the Bean CSM, $B_s = \mu_0 J_c a$ is the same as the penetration magnetic field B_p , and Equation (5.117) is in agreement with Equation (5.3) with the partial penetration case.

- (2) If a superconducting rod with radius a (Figure 5.15), is exposed to a longitudinal magnetic field, the AC loss (W/m) per unit length is:

$$P_{//m} = CAf \frac{8\pi a^2 (p+1) B_s^2}{\mu_0 (p+2)} \left[\left(\frac{B_m}{B_s} \right)^{p+3} F_1 - \left(\frac{B_m}{B_s} \right)^{2p+4} F_2 \right] \quad (5.118)$$

where

$$\left(\frac{B_s}{\mu_0} \right)^{p+1} = (p+1) \beta a \quad (5.119)$$

Function F_1 is given by Equation (5.14), and function F_2 is expressed by:

$$F_2 = \left(\frac{1}{2} \right)^s \int_0^1 [(1+x^q)^s (1-x^q) + (1+x^q) (1-x^q)^s] dx \quad (5.120)$$

If $0 \leq p \leq 1.5$, then $F_2/F_1 \approx 0.5$, $F_1/(p+2) \approx 1/6$, AC loss Equation (5.118) becomes:

$$P_{//m} = f \frac{4\pi a^2 (p+1) B_s^2}{3\mu_0} \left[\left(\frac{B_m}{B_s} \right)^{p+3} - \frac{1}{2} \left(\frac{B_m}{B_s} \right)^{2p+4} \right] \quad (5.121)$$

- (3) If an AC magnetic field is perpendicular to a superconducting rod with radius a (Figure 5.15), AC loss (W/m) per unit length is:

$$P_{\perp m} = \frac{8\pi a^2 B_s^2}{3\mu_0} (p+1) \left[\left(\frac{B_m}{B_s} \right)^{p+3} - \frac{1}{2} \left(\frac{B_m}{B_s} \right)^{2p+4} \right] \quad (5.122)$$

where

$$\left(\frac{B_s}{\mu_0} \right)^{p+1} = 2(p+1) \frac{\beta a}{\pi} \quad (5.123)$$

Similarly, if $p = 0$ and $J_c = \beta$, the Kim model returns to the Bean CSM, and according to Equation (5.123), $B_s = 2\mu_0 J_c a / \pi = B_p$, it is also in complete agreement with Equation (5.53).

5.8.2 Kim–Anderson Model

Similar to Section 5.8.1, considering a superconducting slab with width $2w$ and thickness $2a$, a sinusoidal AC magnetic field is applied parallel to the wide surface. The dependence of the critical current on the magnetic field follows the Kim–Anderson CMS:

$$J_c(B) = J_{c0} \frac{B_0}{B + B_0} \quad (5.124)$$

where J_{c0} is the critical current density of the superconductor in the self-field, B_0 is a fitting constant, and B denotes the absolute value of the applied magnetic field. Considering the current-voltage power

law of the slab, when the amplitude of the AC magnetic field is large enough [24], AC loss (W/m) per unit length is:

$$P_{//m} = P_{//0} \frac{\ln(1+b)}{b} \quad (5.125)$$

where $b = B_m/B_0$, is the effective normalized magnetic field, and $P_{//0}$ represents the calculated AC loss of the slab according to the Bean CSM, seen in Equation (5.3).

5.8.3 Voltage-Current Power-Law Model – Nonlinear Conductor Model

The relationship of voltage and current approximately follows the power law for a practical superconductor, so that the AC loss calculation may be more exact by using this model.

5.8.3.1 AC Losses in AC Magnetic Field

Considering an infinite superconducting slab with width $2w$ and thickness $2a$, a sinusoidal AC magnetic field parallel to the wide surface is applied (Figure 5.29) [25]. By directly adopting the current-voltage power law model, the superconductor is thought of as two extreme special cases of a normal conductor with $n = 1$ and an ideal superconductor with $n = \infty$ (Equation (5.103)).

When the amplitude of an AC magnetic field is low and smaller than the penetration magnetic field, that is, $B_m < B_p$, $B(t) = B_m \sin(\omega t)$. In Cartesian coordinates, when the magnetic field is low, the magnetic field and current in the superconducting slab are:

$$B(r, t) = (0, B(z), 0) \quad (5.126)$$

$$j(r, t) = (j(z), 0, 0) \quad (5.127)$$

Jakob Rhyner studied AC loss of a superconducting slab based on the Maxwell equations and the voltage-current relation Equation (5.103) of superconductors. The generalized penetration depth is defined as:

$$z^* = \frac{B_m}{(\mu_0 J_c)} \left(\frac{\mu_0 J_c E_c}{B_m^2 \omega} \right)^{\frac{1}{n+1}} \quad (5.128)$$

When $n \rightarrow \infty$, $z^* \rightarrow B_m/(\mu_0 J_c)$, which is precisely the magnetic field penetration depth x_p of a superconducting slab according to the Bean CSM, seen in Section 5.1. If $n = 1$, Equation (1.128) becomes:

$$z^* = \left(\frac{E_c}{\mu_0 J_c \omega} \right)^{1/2} = \left(\frac{\rho}{\mu_0 \omega} \right)^{1/2} = \sqrt{2} \delta \quad (5.129)$$

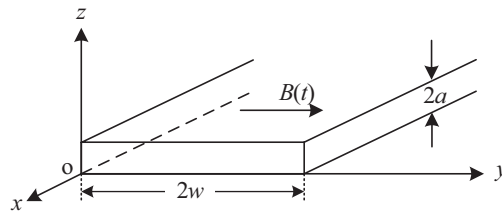


Figure 5.29 Slab exposed to AC parallel magnetic field.

where δ is the skin depth of a normal conductor, Equation (5.128) can be rewritten as:

$$z^* = \frac{B_m}{(\mu_0 J_c)} \left(\frac{[E_c (\omega B_m)]}{[B_m / (\mu_0 J_c)]} \right)^{\frac{1}{n+1}} = x_p f(n) \quad (5.130)$$

$$\text{where } f(n) = \left(\frac{[E_c (\omega B_m)]}{[B_m / (\mu_0 J_c)]} \right)^{\frac{1}{n+1}}$$

The first term in Equation (5.130) is the penetration depth of the Bean CSM. The second term is the correction item by value n . After a series of approximations and derivations, AC loss (W/m) per unit length of slab is:

$$P_{//} = CAf \frac{B_0^3}{2\mu_0^2 J_c} \left(\frac{E_0 J_c \mu_0}{\omega B_0^2} \right)^{1/(1+n)} (1.33 + 3.11n^{-0.55}) \quad (5.131)$$

where CA is the effective cross-section of the slab, A is geometrical cross-section and C is effective coefficient.

Experimental results show that Equation (5.131) is in good agreement with the experiment. All the parameters in theoretical derivation are independent of the external AC magnetic field under the prerequisite condition that the amplitude of the AC magnetic field must be less than the penetration magnetic field B_p , where $B_p = \mu_0 J_c a$. When $n = 1$ and $n = \infty$, Equation (5.131) separately becomes:

$$P_{//} = \begin{cases} CAB_m^2 \omega^{1/2} & (n = 1) \\ CAB_m^3 \omega & (n = \infty) \end{cases} \quad (5.132)$$

The former represents the normal skin effect (very high frequency), while the latter denotes the AC loss of the slab in accordance with the Bean CSM, when not fully penetrated by the magnetic field.

5.8.3.2 Self-Field Losses

Consider the case of AC loss of a superconductor carrying an AC sinusoidal current in the self field, $I(t) = I_m \sin(\omega t)$. By using the voltage and current power-law model of Equation (5.103), AC loss (W/m) per unit length is:

$$\begin{aligned} P_{sf} &= CAf \int_0^{\frac{2\pi}{\omega}} E(I(t)) I(t) dt \\ &= CAf E_c I_m^n i_{ac}^n \int_0^{\frac{2\pi}{\omega}} \sin^{n+1}(\omega t) dt = CAf E_c I_m^n i_{ac}^n g(n) \end{aligned} \quad (5.133)$$

where i_{ac} is normalized current, $i_{ac} = I_m / I_c$, here I_c denotes the critical current of the superconductor and:

$$g(n) = \int_0^{\frac{2\pi}{\omega}} \sin^{n+1}(\omega t) dt \quad (5.134)$$

If n is odd, then the self-field AC loss (W/m) per unit length is:

$$P_{sf} = [E_c I_c l_{ac}^{n+1}] \frac{n}{n+1} \cdot \frac{n-2}{n-1} \cdot \frac{n-4}{n-3} \cdots \frac{5}{6} \cdot \frac{3}{4} \cdot \frac{1}{2} \quad (5.135)$$

If I_m is invariant and n increases, the self-field loss of P_{sf} will decrease, which could explain why the AC loss of superconductors with high n -values tends to decrease but the AC loss of superconductors with low n -values tends to increase with n value increasing. If $n = 1$, Equation (5.135) is the Joule loss power of a DC current I_m . If $n = 19$, the serial item is equal to 0.176. For $n > 19$, there is little change in Equation (5.135). Therefore, for HTS with a low n -value, the influence of the n -value on AC losses must be also taken into account in addition to the critical current.

5.8.4 Combination of Kim-Anderson Model and Voltage-Current Power-Law Model

The width and thickness of a superconducting slab are $2w$ and $2a$, respectively (Figure 5.1). The sinusoidal AC magnetic field is parallel to the wide surface, the relationship of critical current and magnetic field follows the Kim-Anderson CMS Equation (5.124), and the current-voltage relation obeys Equation (5.103). When the amplitude of the magnetic field is large [24], AC loss (W/m) per unit length is:

$$P_{//m} = P_{//0} \left(\frac{w\omega B_m}{E_c} \right)^{\frac{1}{n}} K(b, n) \quad (5.136)$$

where $P_{//0}$ is the same as $P_{//}$ of Equation (5.3), and function $K(b, n)$ is:

$$K(b, n) = \frac{2n}{(1+2n)b} \left\{ \begin{aligned} & \frac{\sqrt{\pi}\Gamma\left(1+\frac{1}{2n}\right)}{b\Gamma\left(\frac{1+n}{2n}\right)} - n - \frac{\pi^{3/2}\csc\left(\frac{\pi}{2n}\right)}{b\Gamma\left(1-\frac{1}{2n}\right)\Gamma\left(\frac{1-n}{2n}\right)} \\ & + \frac{(b^2-1)F_1\left[1, 1, 2-\frac{1}{2n}, 1-b^2\right]}{(2n-1)} \\ & - \frac{(b^2-1)\pi^{3/2}\csc\left(\frac{\pi}{2n}\right)F_1\left[\frac{1}{2}, 1, 2-\frac{1}{2n}, 1-b^2\right]}{2b\Gamma\left(\frac{1-n}{n}\right)\Gamma\left(2-\frac{1}{2n}\right)} \end{aligned} \right\} \quad (5.137)$$

where function $F_1(a, b, c, z)$ is a hypergeometric function, $\Gamma(x)$ is a gamma function, and b is the effective normalized magnetic field $b = B_m/B_0$.

Now considering two extreme cases:

- (1) $n \rightarrow \infty$, AC loss can be obtained according to the Kim-Anderson model (Equation (5.124)).
- (2) $B_0 \rightarrow \infty$, critical current density is independent of the magnetic field, which is in agreement with the Bean CSM, but the n value is finite, so that the AC loss (W/m) per unit length is:

$$P_{//m} = P_{//0} \left(\frac{w\omega B_m}{E_c} \right)^{\frac{1}{n}} K_1(n) \quad (5.138)$$

where

$$K_1(n) = \frac{\sqrt{\pi} \Gamma\left(\frac{1}{2n}\right)}{2(1+2n) \Gamma\left(\frac{3n+1}{2n}\right)} \quad (5.139)$$

If both $n \rightarrow \infty$ and $B_0 \rightarrow \infty$ are taken into account simultaneously, Equations (5.137) and (5.139) separately tend to be:

$$\begin{aligned} K(b, n) \Big|_{n \rightarrow \infty} &= \frac{\ln(1+b)}{b} && \text{Kim - Anderson model} \\ K_1(n) \Big|_{n \rightarrow \infty} &= 1 && \text{Bean CMS} \end{aligned} \quad (5.140)$$

The voltage-current ($E - J$) curve shows the characteristic of the superconductor transferring from superconducting state to normal state, and the n value reflects the transferring degree. The higher of the n value, the sharper the transferring degree is. Generally, the n value of HTS is lower than LTS due to granular characteristics and the weak link of HTS. Strictly speaking, there are errors in the accurate analysis of AC loss in HTS by using the Bean CSM. Numerical methods are commonly used for numerical simulation and quantitative research on AC losses of HTS apparatus based on some analysing methods such as the Brandt analysis method, finite element method (FEM), boundary element (BEM) method and nonlinear diffusion method. The simplest model, first proposed by Brand in 1966, is the penetration analysis method for distributions of current density and magnetic field in HTS by taking the $E - J$ relation of superconductors into account.

5.9 Other AC Losses

In Chapter 6, we will introduce the processing technology of practical superconducting materials with the structure of composite superconductors consisting of normal stabilization matrix and superconducting thin filaments or films. Therefore, eddy current will be induced in normal metal of the composite superconductors when exposed to AC magnetic field so that eddy current losses and coupling losses can be induced in normal metal and amongst filaments through metals. When composite superconductors are exposed to AC magnetic fields with sinusoidal waveforms, the generated eddy currents and coupling losses will be introduced in this section.

5.9.1 Eddy Current Losses

As introduced in Chapter 4, superconducting filaments are usually embedded into normal metal with high thermal conductivity and low resistivity, in order to improve the stability of superconductors. In an AC magnetic field, an eddy current will be generated in the metal matrix, which results in generation of an eddy current loss. If ρ is the resistivity of the metal matrix material, the skin depth of the eddy current is:

$$\delta = \sqrt{\frac{2\rho}{\mu_0\omega}} \quad (5.141)$$

where ω is the angular frequency of the sinusoidal AC magnetic field. If the size of the superconductor is greater than the skin depth, the internal region of the superconductor will be shielded, and the amplitude of the internal magnetic field is lower than the external magnetic field, so that the eddy current loss will be smaller than the calculating results.

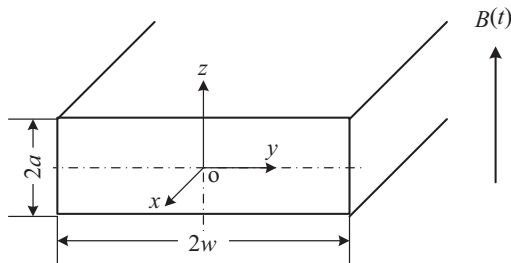


Figure 5.30 Infinite metal slab exposed to perpendicular AC magnetic field.

As shown in Figure 5.30, an infinitely long normal metal slab with width $2w$ and thickness $2a$ is exposed to an AC magnetic field $B(t)$, and its resistivity is ρ . Within Cartesian coordinates, the x -axis extends along its length, and y - and z -axes are along the width and thickness of the slab, respectively. The AC magnetic field $B(t)$ is perpendicular to its wide surface, namely parallel to the z -axis. Consequently, the electric field and current (eddy current) are induced in the x -axis, which results in eddy current loss in the slab. According to the geometrical structure of the metal slab and the Maxwell equation:

$$-\frac{dE_x}{dy} = \frac{dB}{dt} \quad (5.142)$$

Based on its symmetry, the electric field component in the x -axis is zero, that is, $E_x(y=0) = 0$, then:

$$E_x = y \frac{dB}{dt} \quad (5.143)$$

Eddy current loss (W/m) per unit length is:

$$p_e = 2a \int_{-w}^w E_x J_x dy = 2a \int_{-w}^w \frac{E_x^2 dy}{\rho} = \frac{2a}{\rho} \int_0^w y^2 \left(\frac{dB}{dt} \right)^2 dy = \frac{2aw^3}{3\rho} \left(\frac{dB}{dt} \right)^2 \quad (5.144)$$

If an AC magnetic field is a sinusoidal, namely $B(t) = B_m \sin(\omega t)$, the eddy current AC loss (W/m) per unit length is:

$$P_e = \frac{1}{T} \int_0^T p_e dt = \frac{2a}{T} \int_0^T \frac{2aw^3 \omega^2 B_m^2}{3\rho} \cos^2 \omega t dt = (4aw) \frac{(w\omega B_m)^2}{6\rho} \quad (5.145)$$

or

$$P_e = \frac{2\omega^2 B_m^2 w^3 a}{3\rho}$$

In the following, eddy current losses are discussed in AC magnetic fields with exponential decay waveform, triangular waveform and trapezoidal waveform. As shown in Figures 5.28(a)–(c), the eddy current losses (W/m) per unit length in the three kinds of magnetic field waveforms are:

$$P_e = CA \begin{cases} \frac{\beta^2}{24\rho_m \tau_m^2} & (\text{exponent pattern}) \\ \frac{\beta^2}{12\rho_m \tau_m^2} & (\text{triangular or trapezoidal pattern}) \end{cases} \quad (5.146)$$

where

$$\beta^2 = ((2a) B_{m\perp 2a})^2 + ((2w) B_{m\perp 2w})^2 \quad (5.147)$$

where $B_{m\perp 2a}$ and $B_{m\perp 2w}$ are amplitudes of the magnetic field components perpendicular to the thickness and the width of the composite superconducting slab, respectively. CA is the effective cross-section of the slab, A is the geometrical cross-section and C is the effective coefficient.

If the composite superconducting tape only transports an AC current, the eddy current loss is created in the metal sheath due to the self field. Dependence of the eddy current loss on frequency has two distinct frequency regions, it is proportional to ω^2 at low frequency and to $\omega^{1/2}$ at high frequency (> 1 kHz). The transition border between the two frequency regions occurs at a critical frequency f_c , at which half-thickness of the normal sheath b ($b < a$, depending the filling factor λ and equal to $(1 - \lambda)a$, if the sheath and superconductor in the composite tape have the same width w) is equal to skin depth δ [26, 27]. This critical frequency f_c is given by:

$$f_c = \frac{\rho}{\pi \delta^2 \mu_0} \quad (5.148)$$

The eddy current loss (W/m) in a normal sheath of the composite superconducting tapes can be approximately expressed as:

$$P_e = 4 \frac{(\mu_0 \omega I_m)^2 a^3}{\rho L} \quad (5.149)$$

where L denotes the outer perimeter of the superconducting region within the gross cross-sectional area of the composite superconducting tapes, and I_m is the amplitude of the AC transport current.

Clearly, the eddy current loss is proportional to the square of frequency and is different from the hysteresis loss, which is proportional to the frequency. In a perpendicular field, a denotes the thickness and w represents the width. However, in the case of a parallel field, a denotes the width and w refers to the thickness. At frequency $f = 50$ Hz, eddy current loss in a perpendicular field is much higher than in a parallel field, and it is large enough only in a high parallel magnetic field. In a composite superconductor, mutual screening between the superconductor and matrix materials will reduce AC loss and eddy current loss. Since eddy current loss is inversely proportional to the effective resistivity of the composite superconductor, the resistivity of the matrix material should be increased in order to effectively reduce the eddy current losses.

As for present practical 2G HTS (ReBCO CC), its geometric structure is relatively simple and consists of three layers: the substrate layer (one of them is the buffer layer), the ReBCO layer and the metal protection layer, which is generally composed of Ni or Ni alloy with some ferromagnetic characteristics. Consequently, the AC loss calculation of 2G HTS is complicated because ferromagnetic loss should not be ignored, besides the hysteresis loss in the superconducting film and the eddy current loss in substrate. The calculations of AC loss and ferromagnetic loss of ReBCO CC are introduced in detail in Appendix A.2.

5.9.2 Penetration Loss in Transverse AC Magnetic Field

In a transverse AC magnetic field, the induced shielding current flows in the saturated region across the cross-section in multifilamentary composite superconductors. Figure 5.31 shows the saturation zone and induced distribution on a cross-section of a composite superconductor. The loss caused by the magnetic flux penetration from the saturated region to its boundary is called the penetration loss. Therefore, the loss in the saturation region should be included in the process of loss calculation. So far, there is no theory for

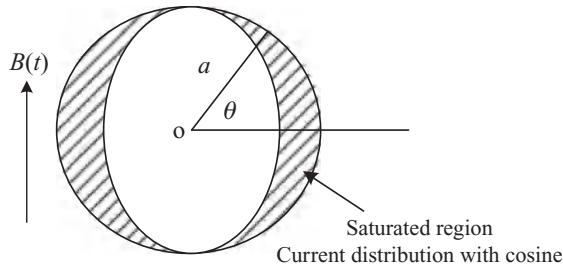


Figure 5.31 Cross-section and current distribution in saturated and virgin regions.

calculating the full penetration loss. But, if a composite superconductor is one with a single core and is subjected to a low AC magnetic field, penetration loss is similar to the hysteresis loss. An induced current with a magnitude of its critical current density and with an approximate cosine waveform will flow in the saturated surface layer of the superconductor. Penetration loss in twisted composite superconductors can be approximately analysed by hysteresis loss of solid superconducting wires with the same diameter and having a difference $(B(t) - B_i(t))$ between internal and external magnetic fields.

Therefore, the loss of the saturation zone should be considered in calculations. So far, the theory of complete penetration loss has not yet been discovered. As for the single superconducting filament in a low AC magnetic field, penetration loss is somewhat similar to hysteresis loss: a current with a roughly cosine waveform and with a magnitude the same as its critical current density flows in the skin layer of the superconductor. Penetration loss in twisted hybrid superconductors can be approximated as the hysteresis loss of solid superconducting wires with the same diameter and with a difference between internal and external magnetic fields.

In order to illustrate this calculation method, consider that the AC magnetic field has a triangular waveform with rising and trailing edges of $dB/dt = B_m/T_m$ (Figure 5.28(b)), and the difference between the internal and external magnetic fields is $B - B_i = B_{mr}/T_m$. In the trailing edge, the differences of magnetic fields are equal in magnitude but opposite in direction. The result is the same with a single superconducting filament in an AC magnetic field with amplitude $B'_m = 2B_{mr}/T_m$, and the effective normalized magnetic field is:

$$b'_{ac} = \frac{\pi B_m}{2\mu_0 \lambda J_c a} \frac{\tau}{T_m} \quad (5.150)$$

The penetration loss (W/m) per unit length is:

$$P_{tfm} = CAf \frac{B_m^2}{2\mu_0} \left\{ \frac{8}{3b'^2_{ac}} \int_{e_m}^1 \left[e_r - \frac{\sin^{-1}(1 - e_r^2)^{1/2}}{(1 - e_r^2)^{1/2}} \right] de_r - \frac{4}{3b'_{ac}} (1 - e_m^2) \right\} \quad (5.151)$$

For a cosine AC magnetic field, the difference between internal and external magnetic fields is:

$$B(t) - B_i = \frac{B_m \omega \tau}{[(\omega \tau)^2 + 1]^{1/2}} \cos(\omega t + \delta) \quad (5.152)$$

where the phase angle δ is determined by $\tan(\delta) = 1/(\omega\tau)$, so that the effective magnetic field amplitude is:

$$B'_m = \frac{B_m \omega \tau}{[(\omega\tau)^2 + 1]^{1/2}} \quad (5.153)$$

and the effective normalized magnetic field is:

$$b'_{ac} = \frac{\pi B_m \omega \tau}{4\mu_0 \lambda J_c a [(\omega\tau)^2 + 1]^{1/2}} \quad (5.154)$$

Penetration loss in a transverse sinusoidal AC magnetic field can be obtained by substituting Equation (5.154) into Equation (5.151).

5.9.3 Twist Pitch

An infinite composite superconducting slab consists of three layers and is exposed to an AC magnetic field $B(t)$. Both the upper and lower layers are superconducting slabs having the same size of length L , width b and thickness $2a$, and the middle layer is a normal conductor with thickness w but having the same length and width as the superconductor slabs (Figure 5.32). In Cartesian coordinates, the magnetic field is along the $-z$ -axis. An AC magnetic field is independent of position and only a function of time, and the x - and y -axis are along the wide and the thickness directions, respectively.

A shielding current is induced in the AC magnetic field, and a voltage is generated in the normal conductor layer. The voltage generated in the area S surrounded by eddy current at the location coordinate x is:

$$E = -\frac{d\phi}{dt} = -\frac{dB}{dt}S = -\frac{dB}{dt}(wx) \quad (5.155)$$

In the range of width δx at x , the current is:

$$I(x + \delta x) - I(x) = \delta I(x)$$

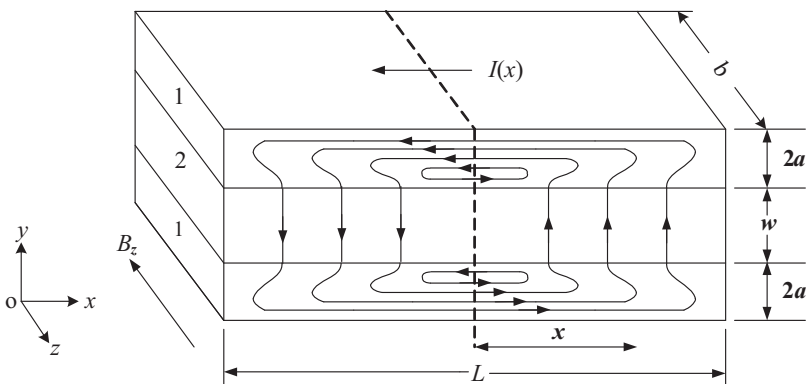


Figure 5.32 Induced shielding current distribution of composite, 1- and 3-superconductors, and 2-normal conductors.

According to Ohm's law, $\delta I(x) = -dB/dt(wxb\delta x)/(\rho w)$, ρ denotes the resistivity of the normal conductor layer, and the differential equation can be approximated as:

$$\frac{dI(x)}{dx} = -\frac{dB}{dt} \frac{bx}{\rho} \quad (5.156)$$

Assuming that the current on the boundary of the composite superconductor is zero, that is, $I(x = L/2) = 0$, its solution is:

$$I(x) = \frac{dB}{dt} \frac{b}{\rho} \left(\frac{L^2}{4} - x^2 \right) \quad (5.157)$$

The maximum current takes place at $x = 0$ and is equal to the critical current $I_c(0) = 2abJ_c$, where J_c is critical current density of superconducting layer. The induced current is distributed across the whole cross-section. Substituting $I_c(0)$ into Equation (5.155) and defining the length of the composite superconductor as:

$$L_c = 2 \left(\frac{2a\rho J_c}{dB/dt} \right)^{1/2} \quad (5.158)$$

If the composite superconductor is twisted with a twist pitch L_p , and $L_p < L_c$, most of induced current can be mutually counteracted or reaches its minimum or is even completely eliminated (Figure 5.33), in which there are two wires that are twisted with pitch L_p . If the transverse AC magnetic field is sinusoidal, that is, $B(t) = B_m \sin(\omega t)$, the minimum twist pitch L_p can be selected as:

$$L_p = 2 \sqrt{\frac{\rho_m J_c (2a)}{\omega B_m}} \quad (5.159)$$

5.9.4 AC Losses in Longitudinal AC Magnetic Field

Taking a twisted multifilamentary composite wire with a circular cross-section as an example, its radius and twist pitch are L_p and a , respectively (Figure 5.34). The current reverses at $r = r_m$. A sinusoidal AC magnetic field is applied longitudinally, that is, parallel to the axial, $B(t) = B_m \sin(\omega t)$.

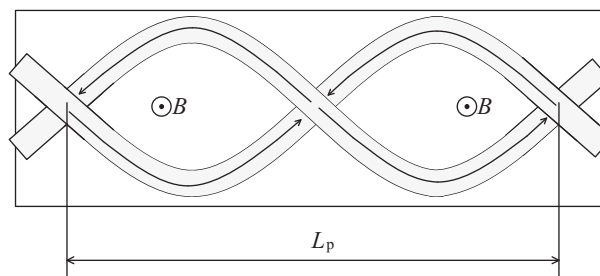


Figure 5.33 Twist pitch of composite superconductor in transverse AC magnetic field.

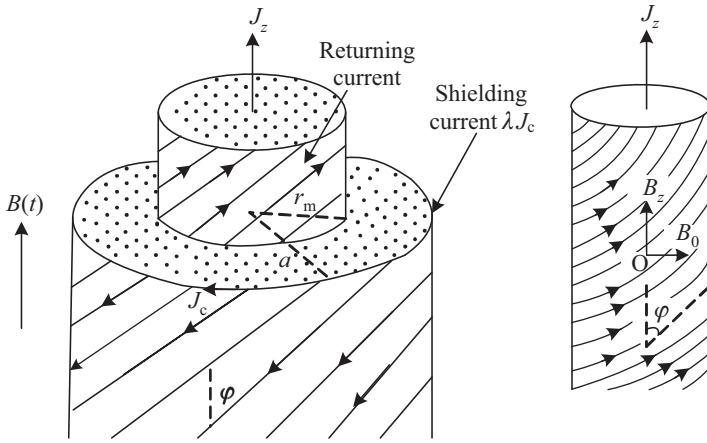


Figure 5.34 Induced current and magnetic field with twisted multifilamentary composite exposed to longitudinal AC magnetic field.

Using cylindrical coordinates, the axis is along the z -axis, the ratio of axial magnetic field to angular magnetic field is constant, that is:

$$\frac{B_\theta}{B_z} = \frac{\mu_0 J_z r}{2B(t)} = \tan \varphi = \frac{2\pi r}{L_p} \quad (5.160)$$

The twist pitch L_p is determined by:

$$L_p = 2 \left(\frac{2a\rho_{\text{eff}}J_c}{dB/dt} \right)^{1/2} \quad (5.161)$$

So the current density in the axis direction is:

$$J_z = \frac{4\pi B(t)}{\mu_0 L_p} \quad (5.162)$$

According to the Bean CSM, when it is fully penetrated, the current density inside and outside the region of the composite is $J_z = \lambda J_c$, where λ is the filling factor of the composite. Defining the longitudinal penetration magnetic field as:

$$B_{pl} = \frac{\mu_0 \lambda J_c L_p}{2\pi} \quad (5.163)$$

the current reverses at radius $r = r_m$, which is determined by the condition that the net circumferential current is zero, that is:

$$\lambda J_z \pi r_m^2 = \lambda J_c \pi a^2 - \lambda J_c \pi r_m^2 \quad (5.164)$$

so that:

$$r_m = a(1 + b_1)^{-1/2} \quad (5.165)$$

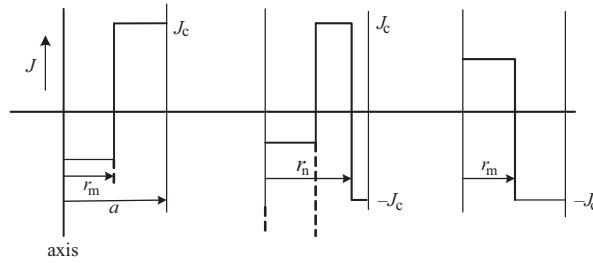


Figure 5.35 Current distribution of twisted multifilamentary composite exposed to AC longitudinal magnetic field after it is raised to maximum and then reduced to minimum.

Where b_1 is the normalized AC magnetic field:

$$b_1 = \frac{B_m}{B_{pl}} = \frac{2\pi B_m}{\mu_0 \lambda J_c L_p} \quad (5.166)$$

Figure 5.35 shows the distribution of induced current in the composite with the axial AC magnetic field increasing to the maximum and then reducing to the minimum. The time-varying magnetic field induces a reverse current layer near the outermost boundary of the composite wire, which reduces the internal reverse current. In the radial range of $r_m < r < r_n$, the current density maintains $J = J_c$. If the external magnetic field varies with ΔB , the current reverses at radius r_n of the outer layer; here r_n can be determined by:

$$r_n = \left[1 + \frac{\Delta B}{B_{pl}} \frac{1}{(1 + b_1)} \right]^{1/2} \quad (5.167)$$

AC loss (W/m) per unit length is:

$$P_l = CAf \begin{cases} \frac{B_m^2}{2\mu_0} \left(\frac{2\pi a}{L_p} \right) \left[\frac{b_1}{3(1 + b_1)^2} \right] & B_m \leq B_{pl} \\ \frac{B_m^2}{2\mu_0} \left(\frac{2\pi a}{L_p} \right) \left[\frac{1}{2b_1} - \frac{5}{12b_1^2} \right] & B_m > B_{pl} \end{cases} \quad (5.168)$$

where CA is the effective cross-section of the slab, A is the geometrical cross-section and C is the effective coefficient. Equation (5.168) gives AC loss of the composite superconductor in a longitudinal AC magnetic field.

5.9.5 Coupling Losses

Figure 5.36 shows that the current (eddy current) is induced amongst the filaments of the twisted multifilamentary composite superconductor with circular cross-section in a transverse AC magnetic field. Radius of the composite superconductor is a , AC magnetic field is sinusoidal $B(t) = B_m \sin(\omega t)$ and twist pitch is L_p .

At point Q, the angle $\theta = (2\pi/L_p)x$, $z = a \sin[(2\pi/L_p)x]$, and the induced voltage along path SQ is:

$$V = 2a \frac{dB_i}{dt} \sin \left(\frac{2\pi}{L_p} x \right) \frac{L_p}{2\pi} = 2 \frac{dB_i}{dt} \left(\frac{L_p}{2\pi} \right) z \quad (5.169)$$

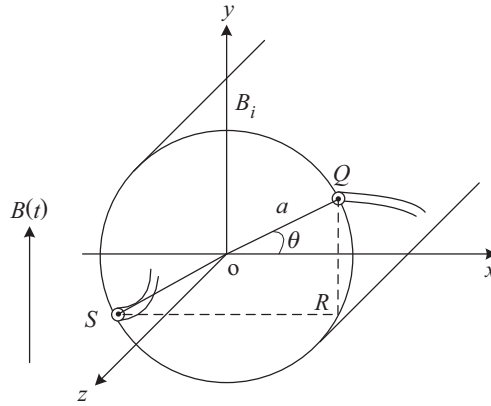


Figure 5.36 Twisted multifilamentary composite with circular cross-section in AC transverse magnetic field. S and Q denote different positions on surface, and B_i is internal magnetic field.

The electric field along the length direction (z -axis) is:

$$E_z = 2 \frac{dB_i}{dt} \left(\frac{L_p}{2\pi} \right) \quad (5.170)$$

The current density along the z -axial direction is:

$$J_z = \frac{dB_i}{dt} \left(\frac{L_p}{2\pi} \right) \frac{1}{\rho_{\text{eff}}} \quad (5.171)$$

where ρ_{eff} is the transverse resistivity of the composite superconductor, and the coupling loss (W/m) per unit length is:

$$P_c = CAfJ_z^2 \frac{1}{\rho_{\text{eff}}} = CAf \left(\frac{dB_i}{dt} \right)^2 \left(\frac{L_p}{2\pi} \right)^2 \frac{1}{\rho_{\text{eff}}} \quad (5.172)$$

The effective transverse resistivity is determined by:

$$\frac{1}{\rho_{\text{eff}}} = \frac{1}{\rho_t} + \frac{w}{a\rho_m} + \frac{aw}{\rho_m} \left(\frac{2\pi}{L_p} \right)^2 \quad (5.173)$$

where w is the thickness of the current layer. As shown in Figure 5.37, J_F and J_θ are current densities amongst superconducting filaments and in the matrix, respectively; in general, $w \ll a$. The range of transverse resistivity ρ_t is given by:

$$\rho_m \frac{1 - \lambda}{1 + \lambda} \leq \rho_t \leq \rho_m \frac{1 + \lambda}{1 - \lambda} \quad (5.174)$$

where λ is the filling factor, and ρ_m refers to the resistivity of matrix. The upper and lower limits of Equation (5.170) represent two extreme situations, one where there are no contact resistances between the superconducting filaments and the matrix materials, and another where both of them are fully insulated or there are high resistances between them.

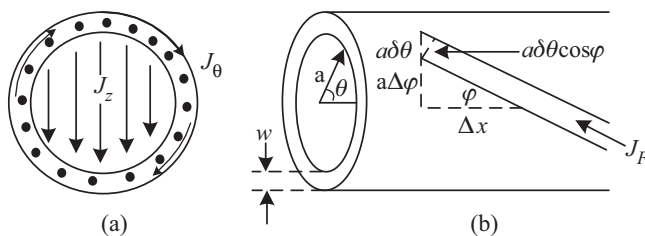


Figure 5.37 Coupling current density in multifilamentary composite with circular cross-section: (a) circumferential and vertical current density on cross-section; (b) side view for calculation of coupling current amongst filaments in outer layer with radius a .

The magnetic field inside the composite superconductor is:

$$B_i = B(t) - \frac{dB_i}{dt} \tau \quad (5.175)$$

where τ is the characteristic time constant that implies the decay time of the shielding current after the external AC magnetic field is removed:

$$\tau = \frac{\mu_0}{2\rho_{\text{eff}}} \left(\frac{L_p}{2\pi} \right)^2 \quad (5.176)$$

Coupling loss (W/m) of the composite superconductor per unit length is:

$$P_c = CAf \left(\frac{dB_i}{dt} \right)^2 \frac{1}{\rho_{\text{eff}}} \left(\frac{L_p}{2\pi} \right)^2 = CAf 2 \frac{dB_i}{dt} \tau \quad (5.177)$$

where CA is the effective cross-section of the slab, A is the geometrical cross-section and C is the effective coefficient.

- (1) As for the pattern of AC magnetic field is triangular waveform, if the rising time $T_m > \tau$, $dB_i/dt = dB/dt$. Coupling loss (W/m) per unit length is:

$$P_c = CAf \frac{B_m^2}{2\mu_0} \frac{8\tau}{T_m} \quad (5.178)$$

When the difference between T_m and τ is very small, the coupling loss will be slightly reduced. If the magnetic field declines for a long time and then begins to rise, the coupling loss (J/m) per unit length becomes:

$$P_c = CAf \frac{B_m^2}{2\mu_0} \frac{8\tau}{T_m} \left\{ 1 - \frac{\tau}{T_m} \left(1 - \exp \left(-\frac{T_m}{\tau} \right) \right) \right\} \quad (5.179)$$

- (2) If the AC magnetic field rises exponentially, then:

$$B(t) = B_m \left(1 - \exp \left(-\frac{t}{T_m} \right) \right) \quad (5.180)$$

Hence, the inner magnetic field is:

$$B_i = B_m \left\{ 1 - \left[T_m \exp\left(-\frac{t}{T_m}\right) - \tau \exp\left(-\frac{t}{\tau}\right) \right] (T_m - \tau)^{-1} \right\} \quad (5.181)$$

Substituting Equation (5.179) into Equation (5.175) and integrating and taking the average in a cycle, coupling loss can be obtained. Because it is too cumbersome, the calculation results are omitted here.

- (3) If the pattern of an AC magnetic field is a trapezoid, and both of the rising and trailing times are T_m , coupling AC loss (J/m) per unit length is:

$$P_c = CAf \frac{B_m^2}{2\mu_0} \frac{4\tau}{T_m + \tau} \quad (5.182)$$

- (4) If the pattern of the AC magnetic field is sinusoidal, namely, $B(t) = B_m \sin(\omega t)$, by substituting $B(t)$ into Equation (5.175), the inner magnetic field can be obtained as:

$$B_i = \frac{B_m}{2[(\omega\tau)^2 + 1]^{1/2}} \cos(\omega\tau - \delta) \quad (5.183)$$

where the phase angle δ is determined by $\tan\delta = \omega\tau$.

Then the coupling loss (W/m) per unit length is:

$$P_c = CA \frac{B_m^2}{2\mu_0} \frac{1}{2\tau} \frac{(\omega\tau)^2}{(\omega\tau)^2 + 1} \quad (5.184)$$

Equations (5.180) and (5.145) show that coupling loss and eddy current losses are proportional to the square of the AC magnetic field frequency in a sinusoidal AC magnetic field, and it is difficult to distinguish them in actual measurements. In addition, Equation (5.179) indicates that the amplitude of the inner magnetic field is less than that of the external AC magnetic field, which reduces the hysteresis loss of superconducting filaments in a composite superconductor, as discussed in Sections 5.1 and 5.2. However, if the AC magnetic field oscillates near the constant magnetic field B_0 , such as $B(t) = B_0 + B_m \sin(\omega t)$, so the magnetic field varies in the range of $B_0 \pm B_m$, the inner magnetic field B_i and coupling loss can still be calculated by the methods introduced above. Different from hysteresis loss, coupling loss depends on the waveform of the applied magnetic field and is proportional to the square of frequency.

When a multifilamentary composite superconductor without twisting is exposed to a sinusoidal magnetic field, a coupling current may result from coupling amongst filaments. Since the coupling current transversely flows through the normal conductor located amongst the filaments, coupling loss per unit length is then generated as:

$$P_c = \lambda \frac{AB_m^2}{2\mu_0} \left[\frac{n_s \omega^2 \tau}{1 + (\omega\tau)^2} \right] \quad (5.185)$$

where λ is the filling factor, n_s denotes the shape factor of the superconducting filaments, A is the cross-sectional area of the superconductor, and τ is the characteristic time constant of the coupling current and determined by P_c taking maximum at the frequency $f = f_m$, that is, $\tau = (2\pi f_m)^{-1}$.

In a composite superconducting slab with rectangular cross-section (Figure 5.1), the AC magnetic field is parallel to the wide surface, and the characteristic time constant of the coupled current is:

$$\tau_{//} = \frac{\mu_0 L_p^2}{16 \rho_{\text{eff}}} \frac{a}{w} \quad (5.186)$$

However, if the AC magnetic field is perpendicular to the wide surface, the characteristic time constant of the coupled current becomes:

$$\tau_{\perp} = \frac{7 \mu_0 L_p^2}{480 \rho_{\text{eff}}} \frac{w}{a} = \frac{7}{30} \left(\frac{w}{a} \right)^3 \tau_{//} \quad (5.187)$$

where ρ_{eff} is the effective resistivity of the composite superconductor:

$$\rho_{\text{eff}} = \frac{1 - \lambda}{1 + \lambda} \rho_m \quad (5.188)$$

or

$$\rho_{\text{eff}} = \frac{1 + \lambda}{1 - \lambda} \rho_m \quad (5.189)$$

where ρ_m is the resistivity of the matrix material of the composite superconductor. Equation (5.186) corresponds with the ideal contact between the superconducting filament and the matrix material, namely the contact resistance is zero. Equation (5.189) corresponds with complete insulation between the superconducting filament and matrix material. The general situation is that the effective resistivity is in between Equations (5.188) and (5.189).

5.9.6 Measures for Reducing AC Losses

Based on frequency characteristics analyses, there are three kinds of AC losses in composite superconductors: hysteresis loss proportional to the frequency, flow resistance loss independent of frequency, and eddy current and coupling losses proportional to the square of frequency. In actual measurements, eddy current loss and coupling loss are indistinguishable. The method of reducing hysteresis loss is to reduce the size of the superconductor by the filamentary process, which is similar with adiabatic stability of the superconductor and eliminating flux jumping. Eddy current loss and coupling loss can be effectively reduced by increasing the transverse resistivity of the composite superconductor and inserting an obstacle layer with high resistivity between superconducting filaments and matrix material, and twisting and transposing are also effective process measures for reduce coupling loss and eddy current loss. For conventional LTS superconductors, twisting and transposing processes are mature, since they are alloys with strong mechanical characteristics. Nevertheless, because the HTS is oxide-ceramic and brittle, the twisting and transposing techniques are premature at present. Figure 5.38 shows the principle of transposing and the actual view of 2G HTS tapes, but this process has not been used in commercial manufacturing [28].

Table 5.2 lists some usual technical measures for reducing AC loss of superconductors when the critical current is unaffected.

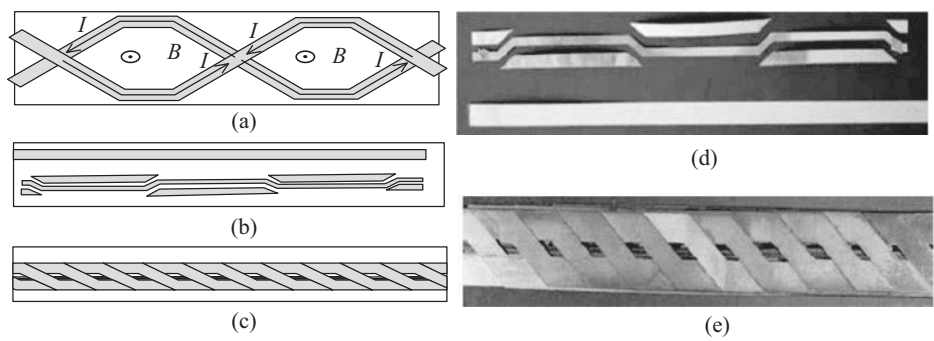


Figure 5.38 Transposition of HTS tapes: (a) principle of transposition; (b) intermediate step; (c) transposing process; (d) intermediate step process of superconducting tape; (e) finished transposing strand. (Reproduced with permission from Yinshun Wang, “Bases of Applied Superconductivity in Electrical Engineering,” Science Press, 169, © 2011.)

5.10 Measurements of AC Loss

Generally, there are three kinds of methods for measuring AC loss of superconductors: magnetic method, electrical method and calorimetric method. The magnetic method is realized by measuring the magnetization of the superconductor and is suitable for measuring AC loss of smaller superconducting samples, such as SQUID and the vibrating sample magnetometer (VSM), in which the hysteresis loop and imaginary part of susceptibility are measured. The electrical method is performed by the approaches of electronic circuit and is especially suitable for measuring AC loss of smaller and shorter samples or sample coils.

There are two main calorimetric methods for measuring AC loss of superconductors: one measures the temperature of the superconductor with a cryogenic thermometer or thermocouple, and the other directly measures evaporation volume of the cryogen. The former is suitable for AC loss measurement of short superconducting samples in a simple electromagnetic environment. The latter is applicable for AC loss measurement of large-scale sample coils and superconducting power apparatus, but is not suitable for small superconducting samples due to its lower precision. Compared with the electrical method, the calorimetric method has the disadvantages of low accuracy and low speed, but its merit is suitable for any complicated electromagnetic environment.

5.10.1 Magnetic Method

There are two methods for hysteresis loss measurement of a superconductor: the hysteresis loop method and the imaginary part of AC susceptibility method.

Table 5.2 Technologies for reducing AC loss

Filament coupling	Hysteresis loss in parallel field	Hysteresis loss in perpendicular field	Hysteresis loss in self field
fully coupled	filament twisted multifilament, short twist pitch	narrow filament	thin narrow filament filament transposition
partly coupled	high aspect ratio high resistivity matrix material, filling insulation barrier amongst filaments		low aspect ratio
completely decoupled	high effective resistivity, low aspect ratio thin filament	high effective resistivity narrow filament	thin narrow filament

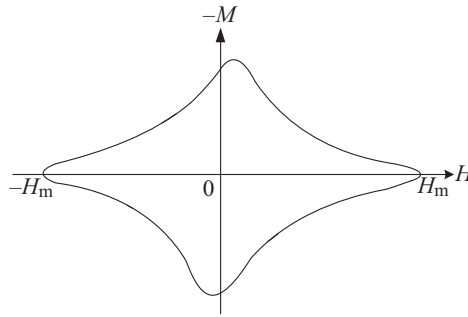


Figure 5.39 Typical hysteresis loop of superconductor.

5.10.1.1 Hysteresis Loop

After the hysteresis loop with the maximum magnetic field H_m is measured at a certain temperature T ($T < T_c$) (Figure 5.39), the hysteresis loss per unit length (W/m) can be calculated by integrating over the hysteresis loop:

$$P_h = CAf\mu_0 \oint H_e dM = -CAf\mu_0 \oint M dH_e \quad (5.190)$$

where H_e is the AC magnetic field applied to the superconductor, M refers to the magnetization, CA is the effective cross-section of the superconductor, A is the geometrical cross-section, and C is the effective coefficient, at low frequency, $C = 1$.

The hysteresis loop method usually includes the standard SQUID method and VSM method. Temperature, magnitude, orientation and pattern of the magnetic field can be easily selected.

5.10.1.2 AC Susceptibility Method

A superconducting rod is subjected to a combination of DC magnetic field B_0 and superimposed AC magnetic field $B(t)$ (Figure 5.40). If an AC magnetic field is in the form of sinusoidal $B(t) = B_m \sin(\omega t)$, and both magnetic fields are parallel to the axis of the rod, then the total external magnetic field applied to the superconductor is:

$$B(t) = B_0 + B_m \sin \omega t \quad (5.191)$$

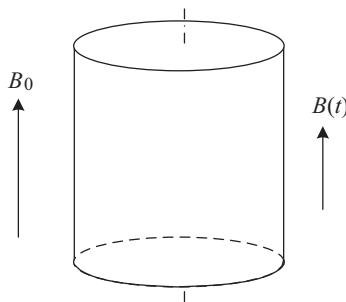


Figure 5.40 Superconducting rod subjected to constant DC B_0 and AC magnetic field $B(t)$.

The time average of an AC field component in the superconductor is defined as B_{avg} , whose Fourier expansion is:

$$B_{\text{avg}} = \sum_{n=1}^{\infty} (\mu'_n \cos(n\omega t) + \mu''_n \sin(n\omega t)) B_m \quad (5.192)$$

where

$$\mu = \mu' - i\mu'' \quad (5.193)$$

$$\mu'_n = \frac{\omega}{\pi B_m} \int_0^T B_{\text{avg}} \cos(n\omega t) dt, \mu''_n = \frac{\omega}{\pi B_m} \int_0^T B_{\text{avg}} \sin(n\omega t) dt \quad (5.194)$$

where T is the period of the AC magnetic field. According to Equation (5.188), hysteresis loss (W/m) per unit length of the rod is:

$$\begin{aligned} P_h &= CAf \oint B_T(t) dM = \frac{CAf}{\mu_0} \oint B_T(t) d\bar{B} = \frac{CAf}{\mu_0} \oint B_T(t) d\bar{B} \\ &= \frac{CAf}{\mu_0} \int_0^T (B_0 + B_m \sin \omega t) \frac{d\bar{B}}{dt} dt \\ &= \frac{CAf}{\mu_0} \int_0^T \left\{ n\omega B_m \sum_{n=1}^{\infty} [\mu''_n \cos(n\omega t) - \mu'_n \sin(n\omega t)] \right. \\ &\quad \left. + n\omega B_m^2 \cos(\omega t) \sum_{n=1}^{\infty} [\mu''_n \cos(n\omega t) - \mu'_n \sin(n\omega t)] \right\} dt \end{aligned} \quad (5.195)$$

In Equation (5.195), the first term with summation of integrations is zero, and all integration terms are also zeroes except for the integration term with $n = 1$ in the second summation term. By integrating Equation (5.195) and replacing μ'_1 with μ'' , the relationship between hysteresis loss (W/m) per unit length and the imaginary part of AC susceptibility is:

$$P_h = CAf \frac{\pi B_m^2}{\mu_0} \mu'' \quad (5.196)$$

Therefore, hysteresis loss of the superconductor can be obtained by measuring its imaginary part of AC susceptibility. The AC susceptibility method is also a mature method, so that it is often used to measure characteristics of conventional magnetic materials.

5.10.2 Electrical Method

The electrical method uses electronic circuits and lock-in-amplifier techniques to measure AC losses, which include hysteresis loss exposed to an AC magnetic field and a self-field loss, respectively.

5.10.2.1 Pick-up Coils

The pick-up coil method is generally applied to measure the hysteresis loss of short superconducting samples or sample coils in an AC magnetic field. Figure 5.41 shows an equivalent circuit for measurement

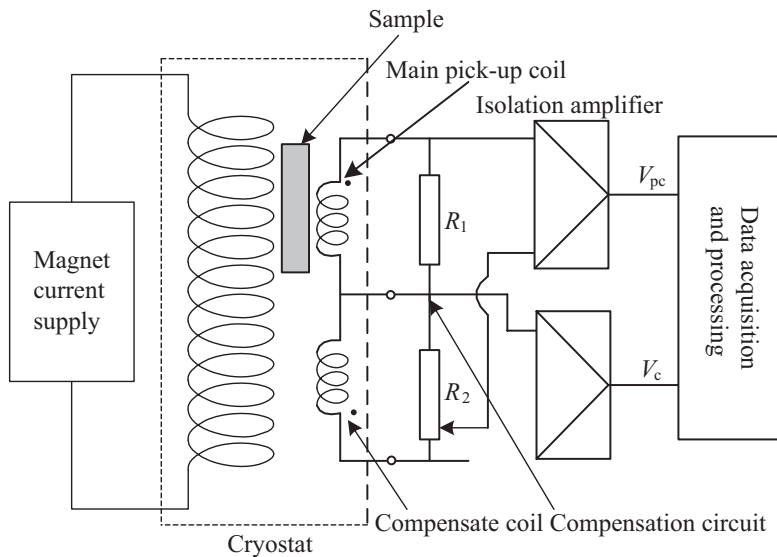


Figure 5.41 Test circuit arrangement for measuring hysteresis loss in AC magnetic field.

of AC losses. The measurement circuit consists of an AC power supply, cryostat, AC superconducting magnet, pick-up coil, compensated coil (cancelled coil), compensated circuit (non-inductive resistance), isolation amplifiers and data acquisition and processing parts [29].

The short sample or coil is concentrically placed between the pick-up coil and the compensated coil. The compensated coil is in the innermost part and plays two roles: the first measures the AC magnetic field and the second compensates for induced voltage of the pick-up coil. The non-inductive resistance is used for adjusting the voltage signal to zero when there are no samples being tested. In general, the heights of the pick-up coil and compensated coil are equal, while the short sample or tested coil is at least three times that of the pick-up coil or compensated coil in height. Figure 5.42 shows the set-up of a pick-up coil, compensated coil and sample coil. The heights of the pick-up coil and compensated coil are h_p and h_c , respectively. The height of the sample coil or the sample is h_s , and $h_s \geq 3h_p = 3h_c$. The radius of the sample coil is R , and distance between the sample coil and the pick-up coil or the compensated coil is a . At certain frequencies, the root mean square (RMS) B_{rms} of the AC magnetic field generated by AC superconducting magnets can be measured by the compensated coil. The RMS value of the induced voltage signal difference between the pick-up coil and the compensated coil is $V_{rms} = (V_p - V_c)_{rms} = (V_{p-c})_{rms}$. By substituting it into Equation (5.197), AC losses (W/m) per unit length in an AC magnetic field can be obtained as:

$$P_h = \frac{hV_{rms}B_{rms}}{\mu_0NL} \quad (5.197)$$

where $h = h_p$ is the height of pick-up coil, N denotes number of turns of the pick-up coil and L refers to length of the sample in the pick-up coil.

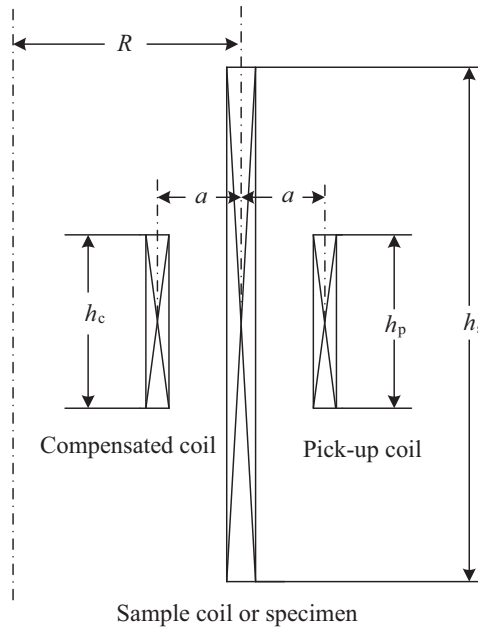


Figure 5.42 Arrangement of specimen and pick-up coils.

In measurement and processing of the voltage signals, it must be guaranteed that the phase of an AC magnetic field must be the same as the induced voltage signal in the pick-up coil. However, there is a possible geometrical error in this measurement method because of the different arrangement of pick-up and compensated coils. Consequently, geometrical correction is needed for correction by multiplying the geometrical factor G with Equation (5.195). The calculation of geometrical factor G is described in Appendix A.3.

5.10.2.2 Electrical Method – Lock-in-Amplifier Technique

When the superconductor transports an AC current, the AC loss will be generated, which is called the self-field loss. It is usually measured by the lock-in amplifier technique. Figure 5.43 shows the principle and set-up used to measure the self-field loss by applying the lock-in amplifier with a four-probe method. Devices include AC power supply, cryostat, non-inductive voltage divider resistance, compensated coil, lock-in amplifier and acquisition system.

In the cryostat, the superconducting sample is short and straight, and is so connected as to use the standard four-probe method. The length of the superconducting sample between the voltage taps is L . In order to eliminate the induced voltage component, the voltage taps are arranged in an “8” shape. The non-inductive resistor plays two roles: one is used to measure the RMS value of the transport current I_{rms} flowing through the superconductor and the other is used to provide a resistive reference voltage signal for the lock-in amplifier. The compensated coil should be adjustable. In order to eliminate the induced voltage signal component (the phase difference with the resistive voltage signal is 90 degrees), the voltage taps of the superconducting sample and the compensated coil are connected in anti-series and then connected with input terminals of a lock-in amplifier. By using the lock-in amplifier to measure the RMS value of the loss voltage component, which has the same phase with resistive

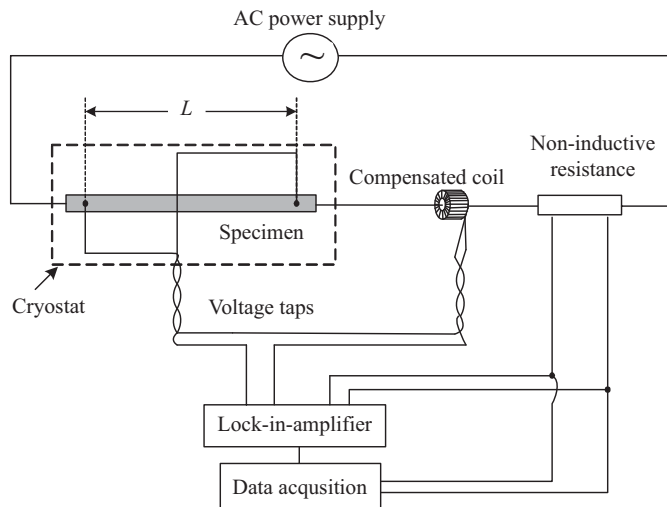


Figure 5.43 Principle and arrangement set-up for measuring self-field losses.

reference voltage signal, the self-field AC loss (W/m) of the superconducting sample per unit length can be found as:

$$P = \frac{V_{\text{rms}} I_{\text{rms}}}{L} \quad (5.198)$$

where V_{rms} and I_{rms} are RMS values of loss voltage and the AC transport current of the superconductor between two voltage taps, respectively.

5.10.2.3 Losses of Superconductor Carrying AC Current in AC Magnetic Field

Figure 5.44 shows the set-up for AC loss measurement of a superconductor carrying an AC current and located in a magnetic field simultaneously. The magnetic field can be either AC or DC. In the case of an AC magnetic field, the AC magnetic field and AC transport current are in-phase. The measuring system consists of an AC power supply for the AC magnet, an AC power supply for the sample, an AC magnet, pick-up coil, two compensated coils and a voltage signal measuring device for loss voltage signals, such as a lock-in amplifier. For the calculation of magnetization loss, both the pick-up voltage $(V_{\text{p-c}})_{\text{rms}}/(V_{\text{c}})_{\text{rms}}$ and the magnetic field B_{rms} must be measured. Similarly, the transport-current loss is obtained using the sample voltage V_{rms} and the transport current I_{rms} . The AC external magnetic field and AC self field directly induce voltages in the pick-up coil/compensated coil and in the loop formed by the voltage taps. Since the induced voltage generated in the sample and the pick-up coil is much higher than the loss voltage component, three methods are generally adopted for eliminating or minimizing the induced voltage signals:

- (1) geometric arrangement method (such as “8”-shaped voltage tap arrangement);
- (2) compensated circuit method (compensated coil); and
- (3) lock-in amplifier technique.

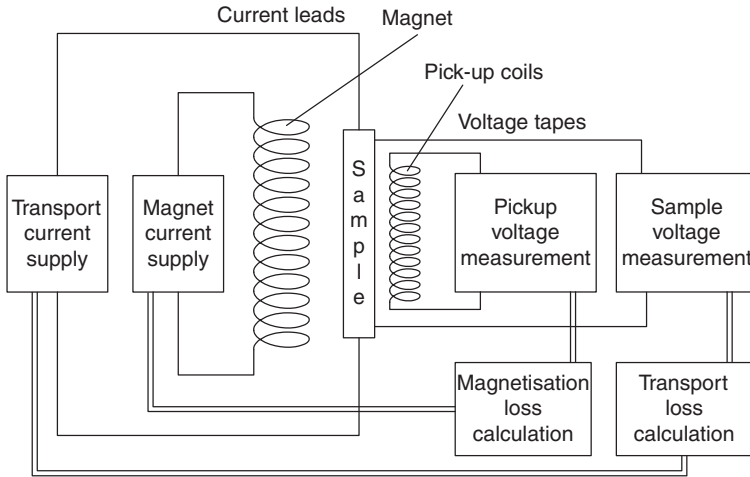


Figure 5.44 AC loss measurement set-up for combination of AC transport current and AC magnetic field.

The data processing of AC loss measurement in a superconducting sample carrying an AC current and being subjected to an AC magnetic field are similar to Equations (5.196) and (5.195), respectively. The sum of two losses is the total loss of the superconducting sample carrying an AC current simultaneously exposed to an AC magnetic field in-phase.

5.10.3 Thermal Method

If a superconductor carries an AC current or is subjected to an AC magnetic field, the AC losses will be generated and then cause a temperature rise in the superconductor. Therefore, AC loss can be found by measuring the temperature rise of the superconductor within the adiabatic environment or by measuring the evaporation amount of the cryogen. Generally, there are two calorimetric methods for measuring AC losses of superconductors. One measures temperature rise and the other measures evaporation volume of the cryogen (also called calorimetry). AC loss measured by both methods is the total AC loss.

5.10.3.1 Measuring Temperature

A superconducting sample with length L is placed in the cryostat, the temperature of the cryogen in the cryostat is T_b , and the middle segment of the sample is adiabatically treated and thus there is no heat exchange with the environment. The two ends are immersed in the cryogen with constant temperature (environment temperature T_b) and the cryogenic thermometer or the temperature difference thermocouple is symmetrically attached to the middle segment of the sample (Figure 5.45) [30]. Two voltage taps are soldered on both sides of the adiabatic section for loss calibration.

As shown in Figure 5.46, the superconducting sample adiabatically locates in the cryogenic environment. The two ends directly contact with the cryogen for keeping the temperature T_b constant. If the transverse size of the sample is much smaller than its length, it can be analysed by the one-dimensional static heat conduction equation:

$$\frac{d^2T}{dx^2} = -\frac{P}{kS(1-\lambda)} \quad (5.199)$$

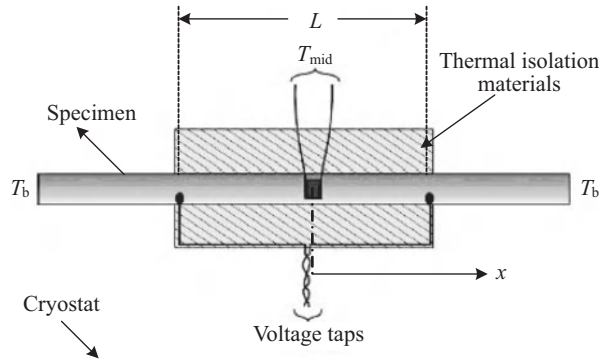


Figure 5.45 Principle of measuring AC loss of superconducting sample by measuring temperature rise.

where P is the heat loss of the sample per unit length, λ is the filling factor of the superconductor, and k and S denote its thermal conductivity and cross-sectional area of superconductor, respectively. Considering the boundary condition, $T(L/2) = T(-L/2) = T_b$, and supposing that the thermal conductivity is approximately invariant with temperature closing to temperature T_b , the solution of Equation (5.197) is:

$$T(x) = T_b - \frac{P}{2kS(1-\lambda)} \left[\left(\frac{L}{2} \right)^2 - x^2 \right] \quad (5.200)$$

The temperature distribution is a hyperbolic function of the position, and the temperature T_{mid} is the highest at the centre of the middle segment. Therefore, once the measurement of temperature distribution is completed, AC loss P can be obtained. When the temperature is measured by a temperature difference thermocouple, one terminal of the thermocouple should contact with the centre of the sample and the other should be installed at the end of the sample at constant temperature T_b . According to Equation (5.200), the temperature difference between the centre and the end can be found as:

$$\Delta T_{\text{mid}} = |T(0) - T_b| = \frac{P}{8kS(1-\lambda)} L^2 \quad (5.201)$$

Therefore, if the temperature difference is measured exactly, the heat loss can be obtained.

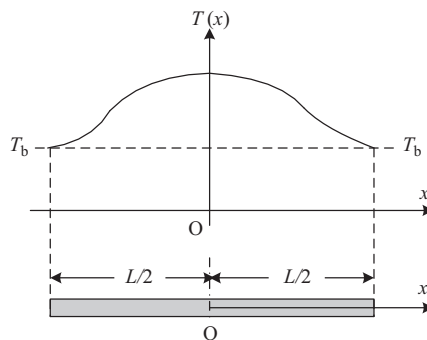


Figure 5.46 Temperature distribution of specimen with constant temperature at both ends.

Equation (5.200) is obtained under conditions that the temperature changes little and the thermal conductivity k is independent of temperature. However, if the temperature difference range is large and then the thermal conductivity k strongly depends on the temperature, Equation (5.199) cannot be directly used for calculating AC loss. In this case, the measurement method is still effective, but the measurement system must be calibrated first and then AC loss can be finally determined by this calibration after measuring the temperature rise.

The thermometer is calibrated with the following steps: a DC magnetic field with a high enough strength is applied to the superconducting sample so that the critical current density of the sample is greatly reduced. Simultaneously, DC power supply provides transport current I (less than the critical current in the self field but higher than the critical current with the applied DC magnetic field, that is, the specimen is in a normal state) to the specimen, and the relationship $P(T_{\text{mid}})$ between Joule loss $P(=IV)$ and temperature T recorded by a thermometer is obtained by measuring the temperature of the central segment and voltage V at both ends of the specimen. Thus, the calibration of the relationship between the losses with the temperature is complete. Then, after removing the DC magnetic field and DC transport current, the temperature is measured again with the same steps as transporting DC current when the superconducting sample is applied with an AC current by an AC power supply. According to the calibrated relation, AC loss of the superconducting sample can be obtained.

The other calibration method is performed by a heating resistance R bifilar wound at the centre of the superconducting sample and supplying heating power P_c to the resistance R , then the effective thermal conductivity k is:

$$(kS)_{\text{eff}} = -\frac{1}{2(1-\lambda)} \frac{P_c}{(dT/dx)_c} \quad (5.202)$$

where $(dT/dx)_c$ denotes average temperature difference at the centre of the sample after applying heating power P_c to the central segment. The relationship of the centre temperature T_{mid} of the superconducting sample and the heating power P_c of heating resistance can be obtained by combining Equation (5.199) with Equation (5.201) as Equation (5.202), thus the calibration of AC loss by the temperature measuring method is completed:

$$P = -4 \frac{P_c}{(dT/dx)_c} \frac{T_{\text{mid}}}{(1-\lambda)L^2} \quad (5.203)$$

5.10.3.2 Calorimetric Method

The calorimetric method is an AC loss measurement method that directly measures the gas flow of the evaporating cryogen due to AC loss. The principle of this method is simple. If the superconductor carries an AC transport current, AC loss is generated in the superconductor. If the volumetric heat capacity of the superconductor is γC , AC loss can be obtained by the enthalpy difference of the cryogen:

$$P = CAf \int_{T_b}^{T_m} \gamma C(T) dt = (CAf) [H(T_m) - H(T_b)] \quad (5.204)$$

where CA denotes the effective cross-sectional area of the superconducting sample, f is the frequency of the AC current or the AC magnetic field, and T_m refers to the average temperature rise of the superconductor. The AC loss is:

$$P = CAf \frac{(T_m - T_b)}{R_{th}} \quad (5.205)$$

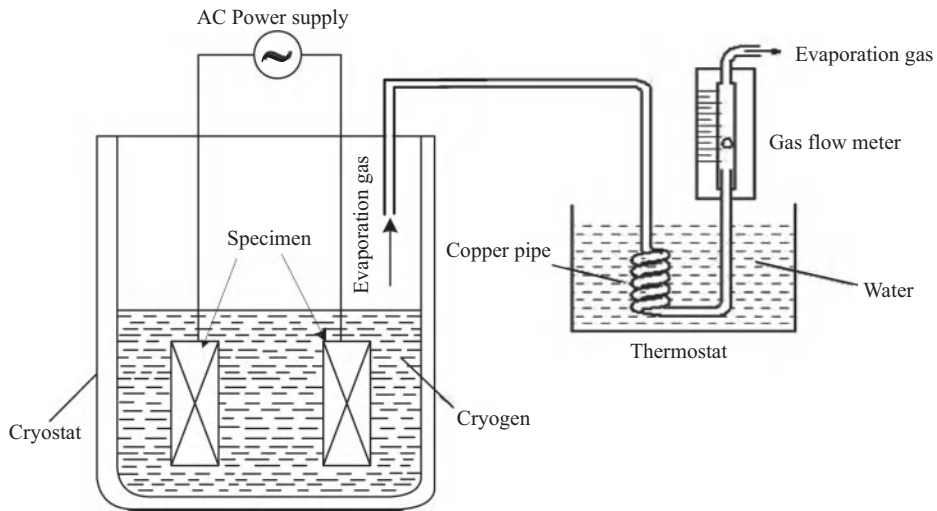


Figure 5.47 Set-up arrangements for AC loss measured with calorimetry. (Reproduced with permission from Yinshun Wang, “Bases of Applied Superconductivity in Electrical Engineering,” Science Press, 177, © 2011.)

where R_{th} stands for the thermal resistance of the superconducting sample. In general, the thermal resistance of the superconductor is difficult to measure precisely, so that the accurate AC loss can be found by directly calibrating the relationship between heat power and flow rate of evaporation of cryogen gas.

Figure 5.47 shows the set-up for measuring AC loss by the calorimetric method (evaporation gas method) [31]. The superconducting specimen is either a coil or a long sample. When the specimen is energized by an AC current, AC loss is generated and causes gas evaporation of the cryogen, and then the evaporation gas passes through the coil made of copper pipe which is immersed into a water tank (heat exchanger) to keep the temperature of evaporation gas constant. After passing through the heat exchanger, flow rate and amount of evaporation are measured by a gas flow meter. Based on flow rate of evaporation gas and latent heat of the cryogen, AC loss of the superconducting sample can be obtained.

The alternative method is realized by directly calibrating the flow rate of evaporation gas. A heater with known power is installed into the cryostat and different power is applied to the heater, then the amount of evaporation gas is measured by a flow rate meter. Thus we can obtain the relationship between heat power and flow rate of evaporation and the calibration is completed. When the superconducting sample carries a different AC current, flow rate of evaporation gas due to the generated AC loss can be recorded by the calibrated flow meter. Based on the relationship between the flow rate of the flow meter and heating power, the AC loss generated in the superconducting sample can be finally determined.

The calorimetric method can be used for AC loss measurement of superconductors in a variety of AC electromagnetic fields. The measured loss is the total loss, which includes resistive loss, flux-flow loss, hysteresis loss, coupling loss and eddy current loss. This method can also be used for AC loss measurement under an AC magnetic field and simultaneous AC current with different phases. Compared with other methods, measuring accuracy of the calorimetric method for small superconducting samples is low; however, and it is more suitable for AC loss measurement of large superconducting samples. In addition, compared with electrical method, the calorimetric method takes a much longer time.

Table 5.3 Sensitivities of AC loss measuring method

Measuring method	The minimum measurable loss $P_{\min}/(\text{W/m})$	Magnetic field orientation	Transport current I
Magnetic method	10^{-6}	AC parallel	—
	10^{-5}	AC perpendicular	—
Electrical method	10^{-5}	AC parallel	DC
	10^{-5}	AC perpendicular	AC
Calorimetric method	10^{-4}	—	AC
	10^{-4}	—	AC
	10^{-2}	AC perpendicular	AC
	10^{-4}	AC parallel	—

5.10.4 Comparison of Electrical with Calorimetric Measuring Method

Electrical and magnetic methods of AC loss measurements in superconductors have many advantages, such as fast rate and high accuracy, but they are easily affected by complicated electromagnetic disturbance, so they are more suitable for AC loss measurement of the superconducting short sample. Compared with the electrical method, the calorimetric method has at least two advantages. First, it has a wide range of applications, experimental set-up is simple, and there is no need for complicated electric circuits and instruments with high precision. It is especially suitable for complicated electromagnetic environments such as any patterns and orientations of AC and DC magnetic fields or AC and DC magnetic field simultaneously carrying an AC current, and even applicable for AC current and magnetic fields either in-phase or out-of-phase. Second, the calibration of the calorimetric method is much easier to understand and visualize than the electrical method.

However, the calorimetric method itself also has some disadvantages. The measurement process takes a long time because heat propagation and temperature variation are much slower than with the electromagnetic propagation. For the conventional LTS, the calorimetric method is easily realized because cryogen liquid helium is used as the cryogen with very low latent heat and then evaporates more easily. As for HTS operating at liquid nitrogen temperature of 77 K, the latent heat of liquid nitrogen is great, more than 60 times of that of liquid helium, so that it is not so easy to evaporate. Therefore, the calorimetric method is only suitable for the AC loss measurement of larger HTS coils or long HTS samples compared with LTS.

Table 5.3 lists sensitivities of measurement of electrical, magnetic and calorimetric methods. It shows that the sensitivities of the electrical method and the magnetic method are high, with almost the same resolution level. However, the sensitivity of the calorimetric method is two to three orders of magnitude lower than the other two methods. For small superconducting samples with low AC loss, the electrical method and magnetic method are superior to the calorimetric method, because of the simple electromagnetic environment. For a large superconducting specimen or coil with higher AC loss, the calorimetric method is superior because of the complicated electromagnetic environment. For the sake of clarity, Table 5.4 lists the applicable scope of AC loss measurement techniques in superconducting samples.

5.11 AC Losses Introduction of Superconducting Electrical Apparatus

In superconducting electrical apparatus, such as superconducting cables, superconducting transformers and superconducting current limiters, the magnetic field amplitudes are generally no more than 0.2 T, while magnetic field amplitude in superconducting motors is usually greater than 1 T. The AC loss requirement in superconducting cable is in the range of 0.15 to 0.65 W/(kA·m), and in other superconducting apparatus it is less than 0.43 W/(kA·m). Table 5.5 lists the magnetic field range of a variety of superconducting electrical installations, where the magnetic fields of superconducting cables and

Table 5.4 Scope of applied AC loss measurement techniques

Tested sample	Electrical method	Magnetic method	Thermal measuring method	
			Temperature method	Calorimetric method
Measuring speed	fast	fast	relatively fast	slow
Short sample	suitable	suitable	relatively suitable	unsuitable
Local measurement (such as films)	unsuitable	unsuitable	suitable	unsuitable
Large superconducting specimen in complicated electromagnetic environment	unsuitable	unsuitable	unsuitable	suitable

Table 5.5 Magnetic field amplitudes of superconducting electrical apparatus

Amplitude range of magnetic field	Superconducting electrical apparatus
>1 T	motors, generators, SMES, inductive FCL
0.1 ~ 0.5 T	transformers
<0.1 T	cables, FCL

superconducting current limiters are the minimum, less than 0.1 T. The magnetic field amplitude of superconducting transformers is in the range of 0.1 T to 0.5 T. At present, the operating temperature of superconducting electrical apparatus such as superconducting motors, inductive superconducting FCL and superconducting magnetic energy storages (SMES), should be lower than the liquid N₂ temperature for current HTS materials, since their magnetic field amplitudes are more than 1 T.

Table 5.6 lists magnetic field parameters and AC losses of Bi-based superconducting materials with cross-section (3 × 0.2) mm² and Y-based superconducting materials with cross-section (5 × 0.002) mm², commonly used in superconducting electrical apparatus. According to these data, two kinds of HTS materials are suitable for superconducting cables and transformers. It is a challenge that either Bi-based or Y-based HTS materials are applied to AC superconducting motors operating at the liquid nitrogen temperature of 77 K, because their AC losses are nearly two orders higher than that of the superconducting cables and transformers [32].

Table 5.6 AC losses of superconducting electrical apparatus

Superconducting electrical apparatus		Cables	Transformers	Motors
frequency of AC magnetic field f /Hz		50	50	20
magnetic field component parallel to wide tape $B_{//}$ /T		0.01	0.1	1
magnetic field component perpendicular to wide tape $B_{\perp}$ /T		—	0.01	1
Bi-2223 tape (3 × 0.2) mm ²	AC loss caused by $B_{//}$ /T $P/I_c/[W/(kA) \cdot m]$	0.1	1.0	4
	AC loss caused by $B_{\perp}$ /T $P/I_c/[W/(kA) \cdot m]$	—	1.5	60
YBCO film (5 × 0.002) mm ²	AC loss caused by $B_{//}$ /T $P/I_c/[W/(kA) \cdot m]$	0.001	0.01	0.04
	AC loss caused by $B_{\perp}$ /T $P/I_c/[W/(kA) \cdot m]$	—	2.5	100

Table 5.7 Applications of superconducting electrical apparatus at power frequency 50 Hz

Temperature T/K	4.2 (LHe)	20 (LH ₂)	77 (LN ₂)
cooling efficiency η	1 : 700	1 : 70	1 : 15
superconducting filament diameter $2a$ with μm order and in perpendicular magnetic field			
AC cables ($B = 0.02 T_{\text{rms}}/\mu\text{m}$)	5	50	220
transformers ($B = 0.15 T_{\text{rms}}/\mu\text{m}$)	0.3	3	15
generators/motors ($B = 1.5 T_{\text{rms}}/\mu\text{m}$)	0.03	0.3	1.5
required twist length L_p in mm; wire with circular cross-section, alloy matrix material, effective resistivity of the matrix material $\rho = 10^{-10} \Omega\cdot\text{m}$			
cables/mm	7	25	45
transformers/mm	1	3	6
generators/motors/mm	0.1	0.3	0.6
Required twist length L_p in mm; tape with Ag matrix material, effective resistivity of matrix material $\rho = 10^{-10} \Omega\cdot\text{m}$, magnetic field parallel to wide tape			
cables/mm	15	60	120
transformers/mm	2	7	15
AC generators/motors/mm	Either in parallel or perpendicular to magnetic field, twisting cannot meet requirements		

In the applications of superconducting electrical apparatus, the requirements on processing technologies, range of magnetic fields and AC losses, cooling efficiency, etc. are listed in Table 5.7. It is indicated that superconducting electrical apparatus operating at 4.2 K do not have economic feasibility in respect of the processing technologies or the requirements on AC loss. Whether the superconductor is twisted or not, superconducting motors cannot meet the requirements of AC operation at 4.2 K, 30 K and 77 K, which is the main reason why only rotor winding (DC) is fabricated by superconducting materials at present.

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6

Brief Introduction to Fabricating Technologies of Practical Superconducting Materials

All practical superconducting materials are non-ideal Type II superconductors. They not only have higher critical current density J_c but also much higher upper critical magnetic field B_{c2} , due to their high irreversible magnetic properties relating to crystal lattice defects. Thus, the non-ideal Type II superconductors are suitable for a wide range of applications in superconducting magnets and superconducting power apparatus, so they have attracted considerable interest. Since the 1950s through 1960s, metallic alloy superconductors, such as V_3Si , V_3Ga , Nb_3Sn , $NbTi$ and, Nb_3Al , were discovered, and thousands of superconducting compounds have also been discovered. Many of them are intermetallic compounds, metallic and non-metallic inorganic compounds and a few organic molecule compounds. The critical temperature of these superconducting compounds is about 20 K.

At the end of 1986, oxide superconductors with a critical temperature higher than liquid nitrogen temperature (77 K) were discovered, and MgB_2 compound superconductors were discovered in 2001. Compound superconductor materials are classified into two types of low temperature superconducting (LTS) and high temperature superconducting (HTS) materials; the former has a critical temperature usually lower than 25 K and the latter is with a critical temperature generally higher than 25 K. At present, the practical HTS materials include Bi-based superconductor materials such as Bi-2223, Bi-2212 and Y-based superconductors such as ReBCO, in which Re refers to rare-earth metals Y, Sm and Nd, amongst which Bi-2223 and Bi-2212 composite tapes are called the 1G HTS materials and the ReBCO coated conductors are called 2G HTS materials.

Superconductor materials are bases of applied superconducting technology. Practical superconducting materials not only have high critical temperatures and high current-carrying capacity in high magnetic fields, but also have stronger mechanical strength, lower AC losses, higher stability, lower cost, etc.

Tables 6.1 and 6.2 list electromagnetic characteristic parameters of some typical and practical LTS and HTS materials, respectively [1, 2].

Currently, there are two main applications in electrical engineering with commercial superconducting materials: one application is to produce high or strong magnetic fields at low temperatures and the other is to produce electric apparatus used in electrical power systems with HTS. For the application of high magnetic fields, $NbTi$ and Nb_3Sn have good electromagnetic properties, and can be processed to form wire with multi-core and thin filaments by mature techniques. They can also be fabricated into compound

Table 6.1 Main critical parameters of practical LTS materials

Materials	Crystal structure	T_c/K @ 0 T	B_{c2}/T @ 4.2 K	$J_c/(A \cdot cm^{-2})$ @ 4.2 K
Nb ₃ Sn	A15 cubic	18.1~18.5	22~25	$3 \sim 5 \times 10^5$ @ 10 T
Nb ₃ Al		18.4~18.9	29.5	5×10^5 @ 10 T
V ₃ Ga		14.6~14.8	20~22	$2 \sim 10 \times 10^5$ @ 10 T
Nb ₃ Ge		23.2	37	4×10^5 @ 10 T
Nb ₃ Ga		20.3	33	1×10^4 @ 15 T
Nb ₃ (Ge·Al)	Body-centered cubic	21	41	7×10^5 @ 10 T
NbTi		9.25	11.8	$1 \sim 5 \times 10^6$ @ 3 T
NbN		16~17	29	4×10^5 @ 12 T
MgB ₂	P6/mm Hexagonal	39	15	$\sim 10^5$

superconducting cable by twisting and transposing techniques so that their mechanical, electromagnetic and thermal stability can be greatly improved. Nowadays, high-quality commercial superconducting wire is available by large-scale production. Commercial superconducting material NbTi occupies more than 95% of the total market.

The technologies of Nb₃Sn and Nb₃Al superconducting wires are much more complex than NbTi wire, and both of them are mainly applied in high magnetic fields (>10 T) for superconducting magnet applications. There are currently several international manufacturers who can produce high-quality practical LTS materials, including IGC corporation in America, Oxford Instrument Corporation in the UK, Furukawa Electric Cooperation, Hitachi and Mitsubishi Cooperation in Japan, Alstom Corporation in France, Outokumpu Corporation in The Netherlands, Luvata Corporation in Italy, Vacuumschmelze GmbH (European Advanced Superconductors (EAS)) in Germany and Western Superconducting Technologies Co., Ltd. in China.

Although MgB₂ superconducting material was discovered as late as 2001, it is now commercially produced and has applications in superconducting magnets for middle- and low-magnetic fields. To date, there are two companies who can provide commercial MgB₂ wires, one is Columbus Corporation in Italy and other is Hyper Tech Corporation in America. As for HTS, they are mainly used for electrical power apparatus with low magnetic fields, such as superconducting cables, transformers fault current limiters (FCL), motors/generators and other industrial devices. They are also used for hybrid magnets with a high magnetic field of 25 T at a temperature of 4.2 K. In recent years, several corresponding prototypes are successfully demonstrated in live grids in many countries, amongst which the 138 kV class superconducting cable hundred of metres long is still operating in New York.

Table 6.2 Main critical parameters of practical HTS materials

Conditions	Characteristics	YBCO	Bi-2212	Bi-2223
Film @ 4.2 K	Critical temperature T_c/K	92	85	110
	Upper critical magnetic field B_{c2}/T	~ 300	85	>100
	Irreversible magnetic field B_{irr}/T	>30	>30	>30
	Critical current density in self field $J_{c0}/(A \cdot mm^{-2})$	5×10^5	2×10^4	1×10^5
	Critical current density $J_{c0}/(A \cdot mm^{-2})$ @ 0.1 T parallel field	5×10^5	2×10^4	2×10^4
Film @ 77 K	Upper magnetic field B_{c2}/T	~ 56	~ 35	>20
	Irreversibility B_{irr}/T	>10	~ 0.005	~ 0.2
	Critical current density in self field $J_{c0}/(A \cdot mm^{-2})$	2×10^5	1×10^3	1×10^4
	Critical current density $J_{c0}/(A \cdot mm^{-2})$ @ 0.1 T parallel field	4×10^4	0	1×10^4

At present, many companies have the ability to manufacture commercial Bi-based HTS, the so-called 1G HTS wires, and the representative manufacturers include American Superconductor Corporation (AMSC) and Superpower Inc. in American, Advance Superconducting Technique Corporation in Germany, Sumitomo Electric in Japan, Innova Superconductor Technology (InnoST) and Western Superconducting Technologies Corporation in China. However, several international companies, including AMSC and Superpower in American, International Superconductivity Technology Center (INSTEC), Sumitomo Electric and SWCC Showa Co., Ltd in Japan and Theva Corporation in Germany, can commercially provide YBCO CC, the so-called 2G HTS wires.

Based on the progress of applications of practical superconducting materials, this chapter briefly introduces the processing technology of commercial LTS wires, HTS wires and HTS bulk. The LTS materials refer to NbTi, Nb₃Sn and Nb₃Al composite wires, and the HTS materials include the 1G HTS tapes, 2G HTS tapes, HTS bulk and MgB₂ wire, and the readers can deepen their understanding of geometrical structures of the practical superconducting materials. There are many processing technologies in preparation of superconducting materials, such as the composite, bronze, internal tin, chemical vapour deposition (CVD), physical vapour deposition (PVD), in-situ, ex-situ, powder metallurgy, including powder-in-tube (PIT) surface diffusion, external diffusion, tube, infiltration, jelly roll and modified jelly roll (MJR) processes, etc. According to practical superconducting materials, this chapter focuses on a brief introduction of preparation technologies and processes for several kinds of common commercial LTS, MgB₂ and HTS materials.

6.1 NbTi Wire

The preparation of an NbTi alloy wire is often accomplished by composite fabrication process technology. Several kinds of basic materials are assembled together, their cross-sections are continuously reduced by drawing, rolling and other mechanical processing, then they are heat treated, and the composite wires with superconducting filaments in matrix are finally achieved [3–8]. At first, the optimal ratio of Ti in a mixture of Nb and Ti alloy is generally in the range of 46 to 50%, in order to obtain the high-quality alloy NbTi ingot. The Nb billet and high-purity Ti billet, prepared by the vacuum electron beam melting technique, are chosen as raw materials. The macroscopic non-uniformity of components, such as Ti spots and Nb-rich regions or non-melting Nb bulk, etc., can be eliminated by repeated vacuum arc melting.

After heat treatment at homogenizing high temperatures, microscopic non-uniformity, resulting from dendritic segregation, can also be eradicated, so that the prepared NbTi alloy has high uniformity and low impurity content, which is favourable to make the entire alloy composition close to compositions of Nb47%Ti and Ti-rich microscopic fluctuations of chemical composition in the range of 1.0 to 1.5%. As a result, the uniform deposition α -Ti is achieved and dispersion phase precipitates, which can improve the flux-pinning force and critical current density. Mechanical and plastic characteristics of the superconductors are also improved, which ensures a superconductor with nanometer class and ultimate thin NbTi filaments, and also ensures it meets the requirements for processing ultra-thin (diameter < 1 μ m) multifilamentary wires.

Due to their good ductility of NbTi alloys and Cu, they are easily machined and shaped. The manufacturing processes of NbTi/Cu multifilamentary wire generally include preparation of NbTi alloy, processing alloy rod, combining and processing of multifilamentary wire, and heat treatment. The manufacturing process is shown in Figure 6.1, where the first row of figures shows the cross-sectional view of the second row. The detailed process is as follows:

- (a) NbTi alloy, which has been repeatedly casted, is put into:
- (b) Cu tube;
- (c) copper sheath is welded and sealed in a vacuum;

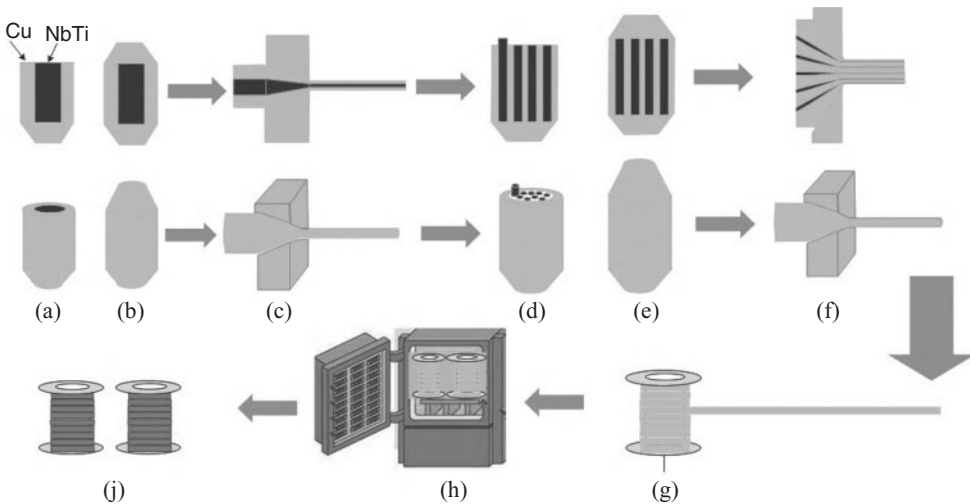


Figure 6.1 Processing of multifilamentary NbTi composite wire: (a) NbTi ingot placed into Cu tube; (b) welded and sealed; (c) drawing; (d) bundle placed in Cu tube; (e) welded and sealed; (f) drawing; (g) winding onto spool; (h) heat treatment; (j) natural cooling.

- (d) single core NbTi/Cu billet is made to have a smaller diameter by drawing and extrusion for the first time;
- (e) single-core billet is cut into multi-sections with single core put into the Cu tube again;
- (f) the process is repeated (c);
- (g) the process is repeated (b) and (c) many times, and the multi-filamentary NbTi/Cu composite wire is obtained;
- (h) the wire is wound onto the ceramic former;
- (i) it is placed into the furnace for heat treatment, at a temperature in the range of 380 °C to 400 °C for about 100 hours;
- (j) a practical multi-filamentary NbTi/Cu composite wire is finally obtained.

Figure 6.2 shows a typical cross-section of a commercial multifilamentary NbTi/Cu composite wire. Currently, these wires with a filament diameter smaller than 10 μm are successfully fabricated, which not only improves the stability of the wire but also reduces its AC losses to a greater extent.

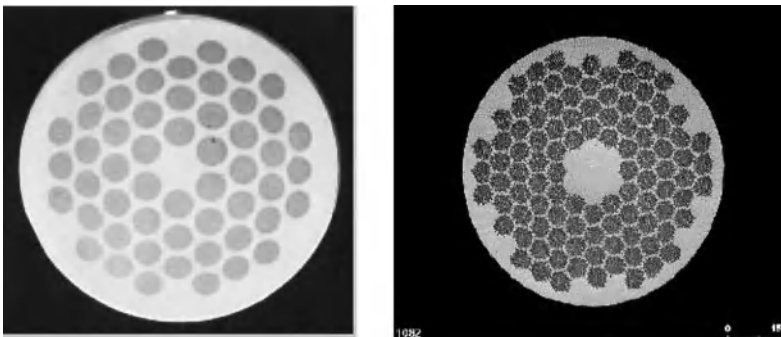


Figure 6.2 Cross-section of multifilamentary NbTi/Cu composite wire (left: multifilamentary standard conductor with 54 fine filaments; right: multifilamentary conductor with 8670 ultra fine filaments).

In principle, the requirements for metal matrices of composite superconducting materials NbTi are low resistivity and high thermal conductivity. However, not all of these materials are practically suitable for processing multifilamentary composite NbTi wire. For example, pure Al is softer than Nb and NbTi, so that there is no feasible way to process an Al matrix. But the mechanical characteristics of metals Cu, Nb and NbTi alloys are similar and can match each other, so they can be easily processed together. The multi-filamentary composite NbTi/Cu wire is so far the most successful representative of practical superconducting materials, and its market share in superconducting materials of strong magnetic fields is more than 95%.

6.2 Nb₃Sn Wire

Nb₃Sn is niobium tin intermetallic compound has a crystal structure of Al₃B, and its mechanical strength is low; it is brittle so that it cannot be manufactured by using mature technology similar to the processing of NbTi composite wire. Based on the excellent ductility of Nb and Sn, several preparation technologies of multi-filamentary Nb₃Sn composite wire have been developed, such as bronze, internal tin, CVD, in-situ and ex-situ processes [9–16]. The bronze process is the most widely used and is the most advanced manufacturing technology in processing multifilamentary Nb₃Sn composite wire. This section only describes the preparation process of Nb₃Sn composite wire by the bronze process. With this process, Nb₃Sn is generated by direct reaction of Nb filaments with bronze. The bronze process includes the two categories of internal diffusion process and external diffusion process.

6.2.1 Internal Diffusion Process

A composite raw conductor is formed by embedding an Nb rod in bronze Cu-Sn materials. The composite wire consisting of Nb multifilaments and bronze is fabricated by the process of vacuum welding and sealing of the sheath, extrusion and drawing process, which is even used in processing multifilamentary NbTi/Cu composite wire; then it is heat treated in a furnace at a temperature range of 650 °C to 700 °C for 50 hours to 100 hours. The process of preparing multifilamentary Nb₃Sn composite wire by using the internal diffusion process is shown in Figure 6.3. Because there is no stabilization copper in the composite, the outer surface should be covered by a copper layer before heat treatment to improve the stability of multifilamentary Nb₃Sn composite wire. But copper is easily contaminated in heat treatment, so a barrier layer of Ta or Nb should be added between the copper and bronze to prevent the Sn from diffusing into the covered Cu layer. There is another internal diffusion process in which the Nb rod is replaced by a Nb tube and the bronze- or Sn-rich alloy is inserted into the centre of the composite. The Nb tube itself plays the role of barrier layer (Figure 6.4). Figure 6.5 is an enlarged typical cross-section of a commercial multifilamentary Nb₃Sn/Cu composite wire.

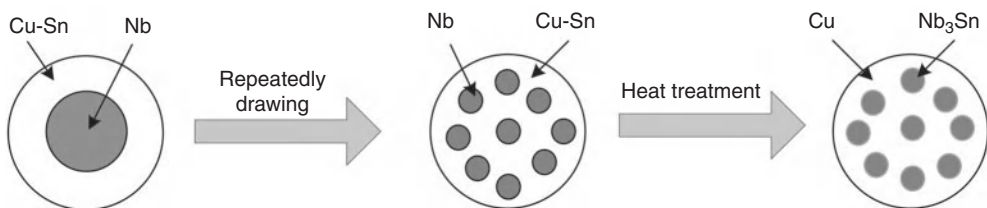


Figure 6.3 Preparation of multifilamentary Nb₃Sn composite wire by internal diffusion process.

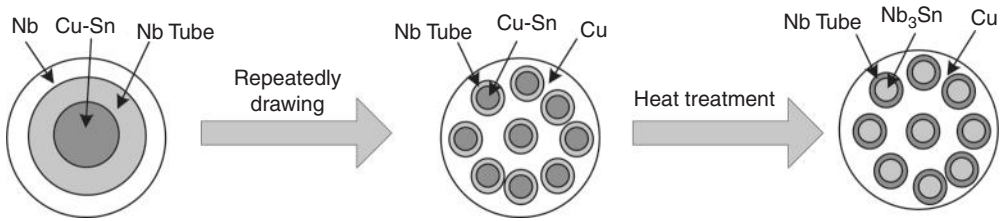


Figure 6.4 Preparation of multifilamentary Nb_3Sn composite wire by internal diffusion process with adding copper stabilizer.

6.2.2 External Diffusion Process

After the Nb rod is embedded in a pure copper tube, a composite consisting of multifilamentary Nb and Cu can be fabricated by the process of vacuum welding and sealing of the sheath, extrusion and drawing, which is the same as in the manufacture of multifilamentary composite NbTi/Cu wire. A layer of Sn is plated onto the outside of the composite and then heat treated in a furnace at 650°C to 700°C for 50 hours to 100 hours. Finally, the multifilamentary Nb_3Sn composite wire is generated by Sn diffusing through the Cu layer and reacted with Nb filaments. The external diffusion process is shown in Figure 6.6.

Because of its weak mechanical strength, multifilamentary Nb_3Sn composite wire, directly fabricated by the bronze process, can be used in applications of superconducting coils in large-scale scientific projects. In order to avoid mechanical damage to Nb_3Sn filaments in the winding process in mid- and small coils with high magnetic fields, the “wind and react technique” is usually adopted as the fabrication method for a superconducting coil, where a conductor containing component elements of required Nb and Sn is wound into a coil and subsequently heat treated to form the Nb_3Sn superconductor. Special attention should ensure that the surface of the composite conductor is covered by insulation materials, which should not be destroyed when exposed to high temperature heat treatment.

Multifilamentary Nb_3Sn composite wires are generally applied in ultra-high magnetic fields ($> 20\text{ T}$) of superconducting magnets and commercial nuclear magnetic resonance (NMR) magnets. Compared with NbTi/Cu composite wire, Nb_3Sn composite wire Nb_3Sn has a narrow range of applications, and less consumer and market share.

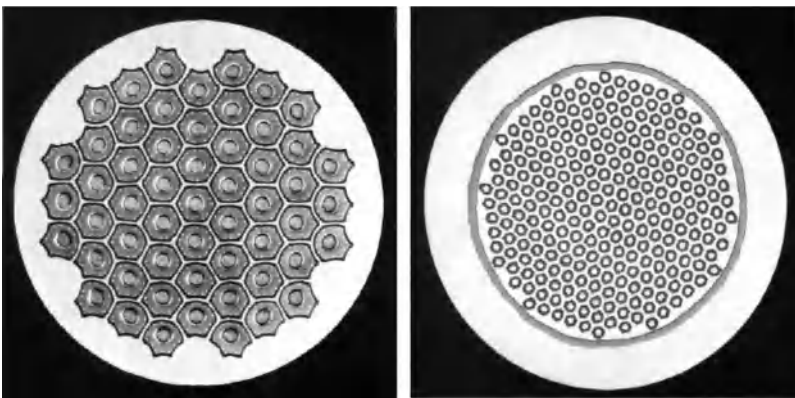


Figure 6.5 Typical enlarged cross-section of multifilamentary $\text{Nb}_3\text{Sn}/\text{Cu}$ composite wire.

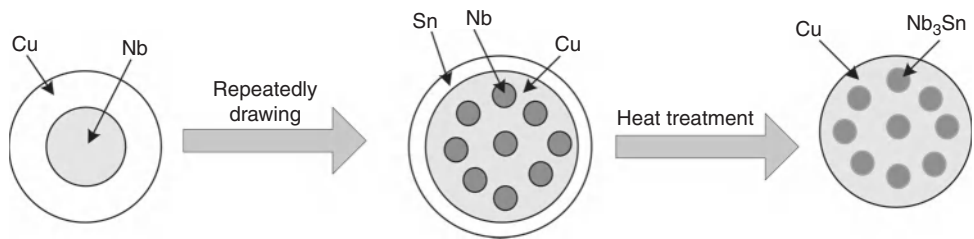


Figure 6.6 Preparation of multifilamentary Nb₃Sn composite wire by external diffusion process.

6.3 Nb₃Al Wire

Both Nb₃Sn and Nb₃Al belong to the A15 compound superconductors. Similar to Nb₃Sn, Nb₃Al has some disadvantages, such as weak mechanical strength, poor dynamic properties and is difficult to fabricate. However, its critical magnetic field is much greater than Nb₃Sn (Table 6.1). Nb₃Al has a strain of about 0.8% and is higher than that of Nb₃Sn (~0.5%). Consequently, Nb₃Al has an advantage over Nb₃Sn in manufacture of superconducting magnets with high magnetic fields. The preparation techniques of multifilamentary Nb₃Al composite wires mainly include the jelly roll process, liquid quenching method, continuous electron gun irradiation method and internal diffusion method, etc. In this section, only the jelly roll process, used to fabricate filamentary Nb₃Al composite wire, is introduced.

After a compound conductor is fabricated by concentrically wrapping Nb and Al foil onto a Nb rod, a compound conductor with a small cross-section is obtained by extruding, drawing and then assembling it into the Nb tube again [11–17]. Finally, the multifilamentary composite wire is completed by repeatedly extruding and drawing. An electric current is applied to the composite wire in a vacuum chamber, so that it is abruptly heated to a temperature of about 1960 °C, then it is put into liquid metal Ga for quick quenching. After heat treatment of the prepared composite wire again, the multifilamentary Nb₃Al composite wire is finally fabricated. The process of the jelly roll technique is shown in Figure 6.7.

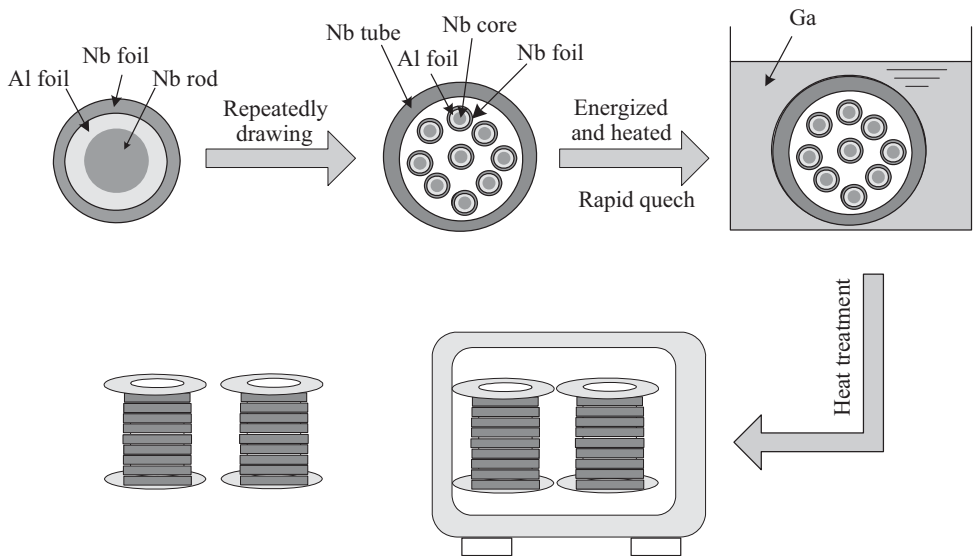


Figure 6.7 Preparation of multifilamentary Nb₃Al composite wire by jelly roll process.

Similar to multifilamentary Nb_3Sn composite wire, multifilamentary Nb_3Al composite wire can be directly applied in large-scale superconducting coils with high magnetic fields. However, it is generally used in mid- and small-scale superconducting coils by the “wind-and-react technique”. After the coils are heat treated, the Nb_3Al phase with multifilaments finally forms. Multifilamentary Nb_3Al composite wire is mainly applied in superconducting magnets with ultra-high magnetic fields (>20 T), but its application range and market share is limited, even less than Nb_3Al .

6.4 MgB_2 Wire

The MgB_2 superconductor with a critical temperature of 39 K was discovered in 2001, and its chemical compositions and structure are relatively simple. Different from oxide HTS, the cost of compound MgB_2 is low and easily fabricated commercially. Compared with conventional LTS, its critical temperature is higher, so it can be applied in the temperature range of 20 to 30 K, which can be cooled directly by a G-M refrigerator with conduction cooling. Although MgB_2 was discovered later than LTS and HTS [18], it can be processed by the mature preparation methods used in HTS, so that the practical MgB_2 wire and tape are successfully manufactured in a short period of time.

There are more techniques for preparing MgB_2 wire or tapes [19–28], amongst which the PIT process is the most successful and can be applied in large-scale industrial production. The basic process of the PIT technique includes preparation of precursor, vacuum welding and sealing of iron sheath, forming composite tube, swaging, drawing, ball milling and rolling. Based on different types of precursors, the processing techniques are classified into two categories in preparation. One of them is the ex-situ process, in which the commercial MgB_2 powder can be directly placed into an iron tube and then drawn and rolled. Figure 6.8(a) presents the process of multifilamentary MgB_2 wire or tape by the ex-situ process. The other is the in-situ process, in which the powders of Mg and B are put into an iron tube with the stoichiometric ratio of Mg powder and B powder, and the MgB_2 wire is finally produced by extruding, drawing, ball milling, rolling and heat treatment. Figure 6.8(b) shows the process of preparing multifilamentary MgB_2 wire and tape by the in-situ process. Since the MgB_2 wire with high critical current density is easily achieved by the in-situ process, and this process is commonly used to prepare multifilamentary MgB_2 composite wire or tape. The sheath material is not limited to metal Fe, and other metallic materials or alloys can also be used as sheath materials as long as they do not react with the MgB_2 phase, such as Nb, NbZr and stainless steel.

Practical multifilamentary MgB_2 composite wire with a round cross-section can be fabricated without the intermediate rolling process. Hyper Tech Company in America can commercially manufacture multifilamentary MgB_2 wire with a round cross-section, but Columbus in Italy fabricates multifilamentary MgB_2 tape with a large aspect ratio. Figures 6.9(a) and (b) show the two multifilamentary MgB_2 composite conductors with different geometrical cross-sections, respectively.

6.5 BSCCO Tape/Wire

The first generation (1G) HTS tapes refer to Bi-based superconducting tapes with chemical structure $\text{Bi}_2\text{Sr}_2\text{Ca}_2\text{Cu}_3\text{O}_{10}$ (Bi-2223) and critical temperature of 115 K. Its upper critical magnetic field B_{c2} is greater than 100 T at a temperature of 4.2 K. The Bi-2223 superconductor has strong anisotropy. Because it is an oxide ceramic with a granular structure, the transport current must go through the grain boundary, so the transport critical current density J_c , that is, intergranular critical current density, is much lower than intragranular J_{cm} . The intergranular critical current density J_c depends on a weak link amongst the grains. However, the degrading pinning energy at relative high temperatures also results in serious flux creep, which further reduces transport critical current density J_c .

Since the Bi-2223 superconductor has a lamellar structure, a high-quality texture can be obtained with the help of mechanical deformation and heat treatment [29–33]. Moreover, the existence of a

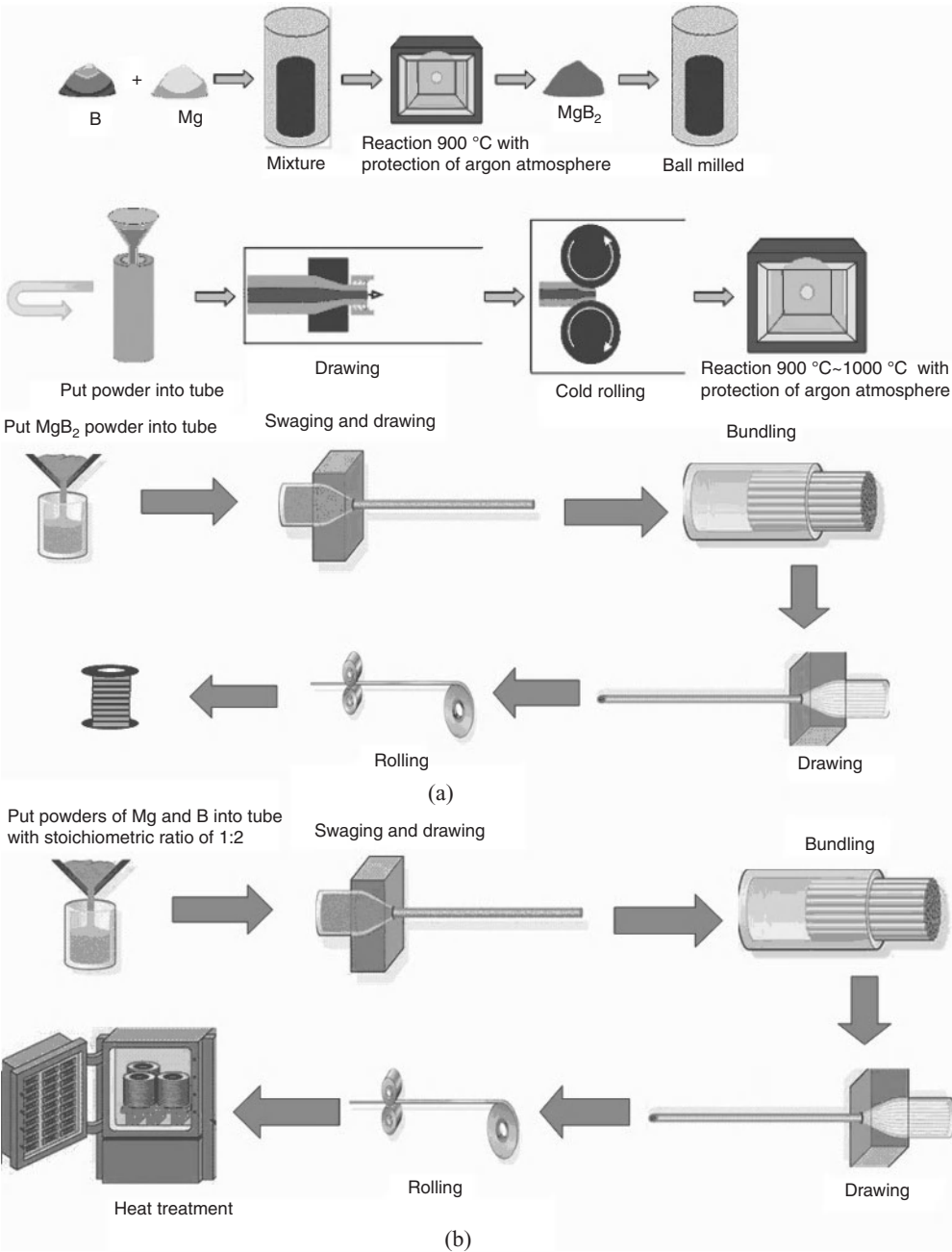


Figure 6.8 Fabrication of MgB_2 wire or tape by PIT technique: (a) by ex-situ process; (b) by in-situ process.

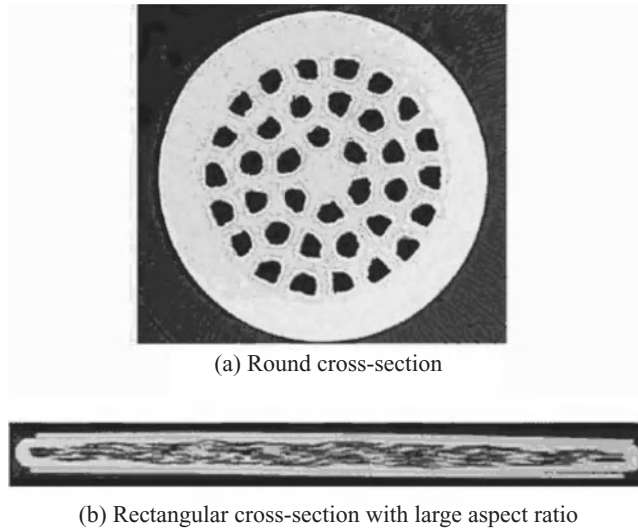


Figure 6.9 Multifilamentary MgB_2 composite conductors with: (a) round; and (b) flat rectangular cross-sections. (Reproduced with permission from Yinshun Wang, “Bases of Applied Superconductivity in Electrical Engineering,” Science Press, 193, © 2011.)

liquid phase during heat treatment can promote its densification and bridge the flaws generated in the deforming processing, so that the connection amongst grains will be improved. Similar to MgB_2 , Bi-2223 tape is also generally fabricated by the PIT technique to produce long Bi-2223 tapes. The process is shown in Figure 6.10. By placing Bi(Pb)-Sr-Ca-Cu-O powders into a metal tube made of Ag or Ag alloy with a certain stoichiometric ratio, it is fabricated as a thin rod with a single core by swaging and drawing. Many of the prepared thin rods with single cores are bundled and then put into a metallic tube with a larger diameter. The multifilamentary conductor is completed by repeatedly drawing and cold rolling. Finally, the prepared tape with multifilaments is put into a furnace for heat treatment, and the atmosphere in the furnace is controllable. Heat treatments are performed one or more times within

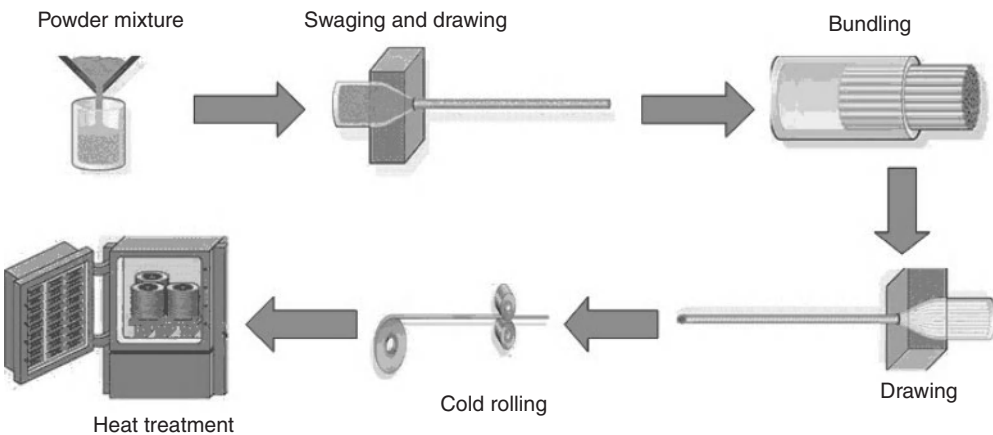


Figure 6.10 Processing of BSCCO tape by PIT technique.

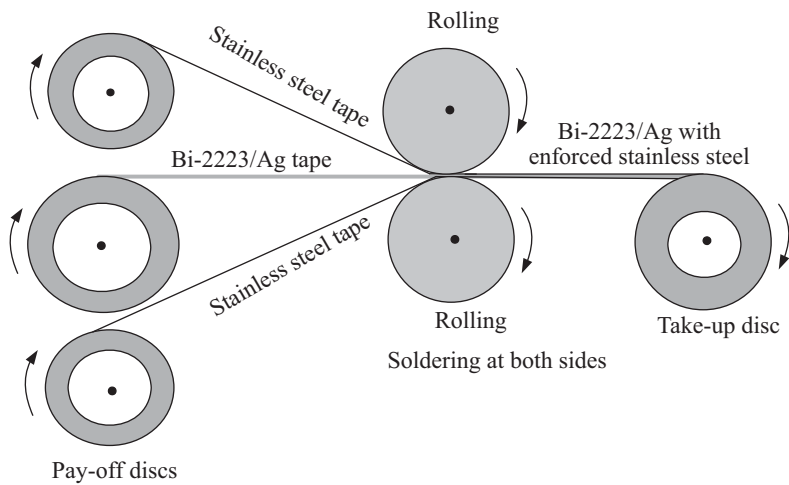


Figure 6.11 Process of multifilamentary Bi-2223/Ag with enforced stainless steel at both sides.

protective atmospheres, which makes the precursor powders convert into the high temperature phase Bi-2223, thus the multifilamentary Bi-2223 tapes are finally fabricated.

In the process of production, the alloys with high mechanical strength, such as AgMg, AgAu and AgPd, are commonly adopted as sheath materials in order to increase the mechanical strength of HTS tape. Another method for improving mechanical strength is that the two thin stainless steel tapes are soldered at both sides of the HTS tapes after the tapes are finished. Thus their mechanical strength can reach more than 250 MPa. Figure 6.11 presents the multifilamentary Bi-2223/Ag with enforced stainless steel fabricated by AMSC. The cross-sections of multifilamentary Bi-2223/Ag tapes with different sheath materials are shown in Figure 6.12. Figure 6.12(a) shows a cross-section of multifilamentary Bi-2223/Ag tape with an Ag sheath with a stress of about 100 MPa; Figure 6.12(b) indicates a cross-section of multifilamentary Bi-2223/Ag tape with an Ag alloy sheath with a stress of about 150 PMa; Figure 6.12(c) presents a cross-section of a multifilamentary Bi-2223/Ag tape with enforced stainless steel sheath with a stress of more than 250 MPa.

Although Bi-2223 tape has been commercialized and made great progress in applications for electric power systems and magnets, its price is still high and it will be difficult to reduce costs further because

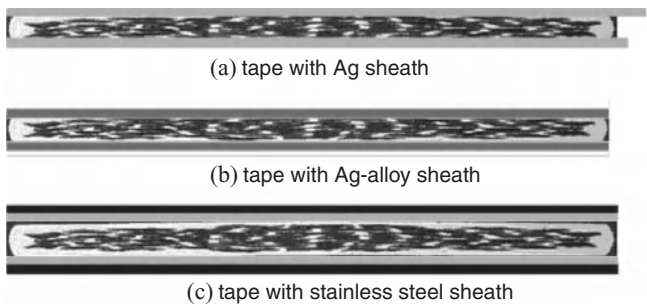


Figure 6.12 Cross-sections of multifilamentary Bi-2223/Ag tape with different sheaths.

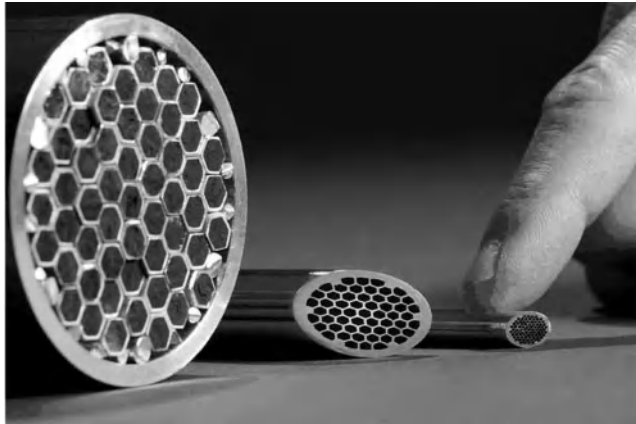


Figure 6.13 Sizes of multifilamentary Bi-2212/Ag with round cross-section.

sheath material Ag is expensive. Moreover, its critical current is strongly anisotropic at a temperature of 77 K and degrades seriously in a perpendicular magnetic field. Therefore, most of the international HTS tape suppliers stopped fabricating 1G HTS tapes and now focus on developing 2G HTS tapes, such as YBCO CC, SmBCO CC and NdBCO CC.

Bi-2212 superconducting material belongs to Bi-based HTS [34, 35]. The stoichiometric ratio of the Bi-based system is $\text{Bi}_2\text{Sr}_2\text{Ca}_{n-1}\text{Cu}_n\text{O}_{2n+4}$ ($n = 2$), whose superconducting transition temperature is about 85 K. Its critical current density J_c has superior performance at low temperatures (< 20 K) and high fields. Compared with conventional LTS, dependence of critical current density J_c of Bi-2212 on the magnetic field is weak at low temperatures, thus it can be used to fabricate a central magnet to be inserted into a conventional LTS magnet to create a strong magnetic field over 23.5 T, the so-called hybrid magnet. The multifilamentary Bi-2212/Ag wire with a round cross-section and hexagonal multi-strands are presented in Figures 6.13 and 6.14, respectively.

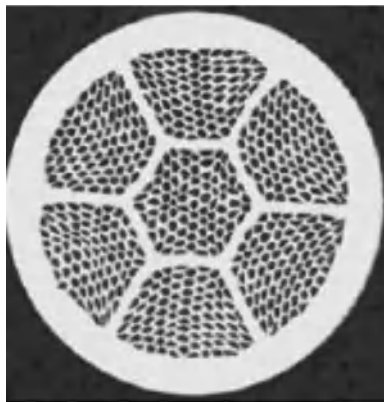


Figure 6.14 Cross-section of multifilamentary Bi-2212/Ag with hexagonal multi-strands.

6.6 YBCO Tape

Although mass production of Bi-based HTS tapes has been realized, the high cost and performance deficiencies, amongst which there exists the serious degradation of critical current density J_c in the perpendicular magnetic field at a temperature of 77 K, make it difficult to achieve large-scale applications. Compared with B-based HTS tape, YBCO has superior electromagnetic characteristics with high irreversibility of the magnetic field. It also displays a higher critical current density J_c and weak anisotropy in high magnetic fields and at a liquid nitrogen temperature of 77 K. Because of the weak link amongst YBCO grains, it is difficult to fabricate Y-based superconducting tapes by PIT techniques. Preparation methods of Y-based HTS tapes [36–45], such as depositing and coating, are the main trend of fabricating practical HTS materials applied in electrical engineering. Since the properties of Y-based HTS tapes strongly depends on the microstructure of biaxial texture, high-quality YBCO films can be prepared only on the biaxial textured substrate or isolated layer by epitaxial growth technology.

The main obstacle for achieving high performance YBCO tapes is the weak link amongst grains, and misorientation angle in adjacent intercrystalline YBCO is a key factor as to whether it can carry large transport current. Therefore, the large misorientation angle should be avoided and the weak link between superconducting phases should be eliminated as much as possible. At present, the coated technique for preparing a YBCO conductor with a high critical current density j_c has been achieved, and the schematic geometrical structure of YBCO CC and sizes of components are displayed in Figures 6.15(a) and (b), respectively. Many samples of YBCO CC were fabricated by ion beam assisted deposition (IBAD)/pulsed

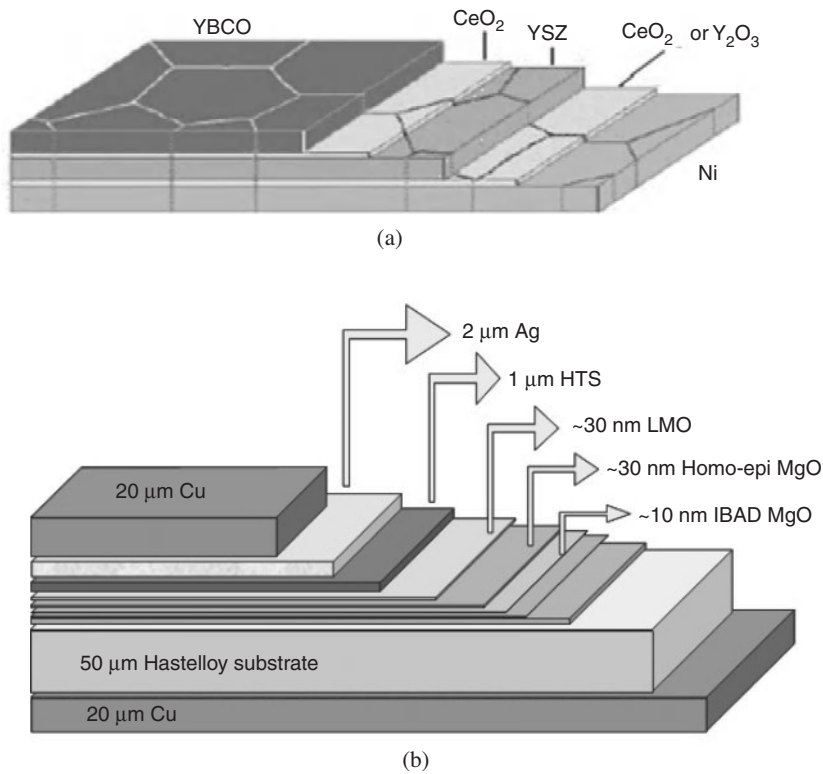


Figure 6.15 Geometrical structure of YBCO CC: (a) schematic configuration; and (b) geometrical sizes.

laser deposition (PLD), rolling-assisted biaxially textured substrate (RABiTS)/PLD, and their critical current density j_c of 22 MA/cm² in the self field and at a temperature of 77 K is easily achieved. Currently, development of Y-based tapes mainly focuses on preparations of substrate and textured isolated layers and depositions of YBCO superconducting films with high critical current density J_c .

6.6.1 Substrate and Textured Insulated Layer

There are many techniques for preparation of substrate and textured isolated layers, which are the most important basic materials for preparation of YBCO CC. Four common processes are briefly introduced in this section:

- (1) *Ion beam assisted deposition (IBAD)*. The IBAD method adopts non-magnetic Ni alloy substrate with high mechanical strength, and a strong structured Zirconia (YSZ) layer is achieved by ion beam sputtering, which is currently a popular process for preparation of 2G HTS tapes.
- (2) *Rolling-assisted biaxially textured substrate (RABiTS)*. After high-quality biaxially textured Ni substrate is achieved using the RABiTS technique, CeO₂ films are deposited on the Ni substrate by PLD and then a buffer layer of YSZ and CeO₂ films are epitaxially grown on the Ni substrate. With the requirement of multilayer isolated layers, not only the manufacturing cost increases but also the manufacturing process becomes more complex. In addition, the rolling process demands more strict environmental conditions, but it can facilitate the deposition process of high performance YBCO films. Thus, the technique is still used extensively.
- (3) *Inclined substrate deposition (ISD)*. Without ion beam assistance, deposition materials can only deposit on the metal substrate at a certain angle, and a textured isolated layer still can be obtained. This process is characterized by high deposition rate, but its degree of textured orientation is not high.
- (4) *Textured silver or silver alloy substrate process*. Silver is currently the unique substrate material that does not require an isolated layer in fabrication of HTS tapes. Furthermore, research indicates that silver doping is advantageous to improve YBCO superconductivity performance and to reduce the surface resistance. But complicated and expensive processing are its disadvantages.

6.6.2 Deposition of Superconducting Layer with High Critical Current Density

At present, there are many processes by which high-quality YBCO films can be deposited onto flexible metal substrates. The processes mainly include PLD, metal-organic chemical vapour deposition (MOCVD), metal organic deposition (MOD) and the sputtering method, as well as the spray decomposition and sol-gel methods, amongst which PLD is the most widely used. Currently, the critical current of the short YBCO samples, prepared by PLD, is up to 400 to 500 A/cm. However, the PLD technique is not suitable for large-scale industrial production, because it demands not only highly expensive apparatus with high vacuum and large volume, but also an industrial laser source. The sputtering method also encounters the difficulty of a high vacuum system.

The MOCVD technique only demands a lower vacuum system by which the YBCO film can be generated at a fast rate and has the potential for mass production of YBCO films. But precursors and the vapour deposition source are expensive. Trifluoroacetic acid-metal organic deposition (TFA-MOD) does not need a vacuum system and thus has attracted increasing attention. The principle of this method is that the metal substrate is first coated with a fluorine-containing solution and then a high-quality YBCO film is finally formed by two heat treatment processes, and the method is shown as Figure 6.16. There are four characteristics in TFA-MOD:

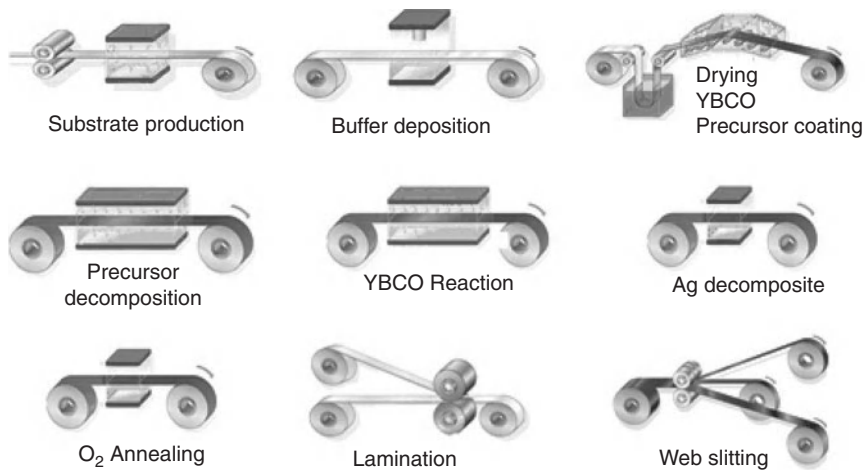


Figure 6.16 Preparation of YBCO CC by TFA-MOD process.

- (1) relatively simple processes, including coating process and two heat treatment processes only;
- (2) all the precursor can be used to 100% and the composition is adjustable;
- (3) vacuum system is not required; and
- (4) deposition with large area can be realized.

Obviously, TFA-MOD is suitable for the industrialization manufacture of YBCO CC with high performance and low cost.

Figure 6.17 shows a typical cross-section of 344 YBCO CC manufactured by AMSC, with a length of more than 200 m. The I_c is more than 100 A at 77 K in the self field.

At present, besides AMSC, the international suppliers of 2G HTS tape mainly include Superpower in American, Sumitomo Electric and SWCC Showa in Japan and Theva in Germany. All of them can produce 2G HTS tapes with a length of more than 200 m and a critical current of more than 300 A/cm. Therefore, 2G HTS (YBCO CC) becomes the main superconducting material for research and development of electrical power applications in the future.

6.7 HTS Bulk

The HTS bulk includes ReBCO (Re refers to rare-earth elements, such as Y, Sm, Nd) and Bi-2212 bulk. Similar to YBCO CC, the ReBCO bulk exhibits a high irreversible field at a temperature of 77 K. At present, although it is impractical to make long lengths of bulk HTS, it is possible to make single grains with cylindrical disk or pucks as large as several tens of centimetres in diameter and several tens of millimetres in height, which is sufficient to create powerful trapped magnetic fields inside the



Figure 6.17 Geometrical cross-section of typical YBCO CC.

bulk components. There are several processes for fabricating the bulk HTS [46–48], which mainly include melt-texture-growth (MTG), powder melt process (PMP), quench-melt growth (QMG)/melt-powder-melting-growth (MPMG) for ReBCO bulk and melt cast process (MCP) processes for Bi-2212 bulk.

6.7.1 Melt-Texture-Growth (MTG) Process

Melt-texture-growth (MTG) is a process by which well-orientated oxide superconductors can be obtained by unidirectional solidification after partial melting at high temperatures. At present, MTG is the most significant technique used in the preparation of bulk ReBCO. First, the common solid state sintering method is used to prepare bulk $Y_1Ba_2Cu_3O_x$ (Y123 phase) with stoichiometry of the YBCO superconducting phase, which is then heated to 1020 °C so that the 123 phase melts and decomposes into 211 + Cu-rich liquid phases. Y211 is non-superconducting second-phase Y_2BaCuO_5 . When the temperature rises to about 1100 °C, the Y123 phase decomposes into the Y211 phase + Cu-rich liquid phase and then slowly cools directionally at a rate of 2 to 10 °C/h. Peritectic reaction takes place between the solid state Y211 phase and the Cu-rich liquid phase at a temperature of 900 °C. When the temperature decreases to 400 °C, the Y123 superconducting phase regenerates after oxidation treatment is performed.

6.7.2 Quench-Melt-Growth (MTG) Process/Melt-Powder-Melt-Growth (MPMG) Process

MTG and MPMG are developed from MTG. Compared with MTG, heat treatment by quench in the temperature range of 1200 to 1500 °C, in which the Y_2O_3 + Cu-rich liquid phases exists, is added in QMG. After quench, the temperature rises to 1100 °C again. Thereafter, heat treatment is the same as for MTG. MPMG is a modification of QMG. Compared with QMG, there is an added pulverization process by which the sample is crushed into a fine powder and then pressed into a block after high temperature quench of the liquid phase. In the process of pulverization, thick Y_2O_3 is refined and homogenized. Uniform distribution of fine Y_2O_3 makes the subsequent Y211 grains become more fine and uniform.

6.7.3 Powder-Melting-Process (PMP)

Three kinds of powders Y_2BaCO_5 (Y11), $BaCuO_2$ and CuO are used as precursors. They enter the region of the Y211 + liquid by abruptly heating the mixed powder billets in the temperature range of 1000 to 200 °C. Then the Y123 phase of the textured *c*-axis is prepared with directional condensation at a temperature gradient of 10 to 20 °C/cm and a cooling rate of 0.5 to 0.8 °C/h. The textured Y123 phase sample is finally prepared by pressure moulding. PMP has two distinctive features, one is size and distribution of Y211 grains that can be conveniently controlled, and the other is that the heat treatment at more than 1000 °C is omitted.

At present, melt texturing is the most significant technique used for preparation of HTS bulk with size of several tens and several hundreds of millimetres in diameter and length, respectively. ReBCO bulks can be fabricated into several different shapes, such as rod, bar, tube, hollow cylinder and tubular with cylindrical shape. Some typical shapes are shown in Figures 6.18(a–d). Sometimes the enforced materials, such as fibre glass plastic with epoxy resin, and stainless steel, will be added to the outer surface of the cylindrical disk or inner and outer surfaces of a hollow cylinder, in order to increase their mechanical strength, because they are brittle and do not have enough strength when energized.

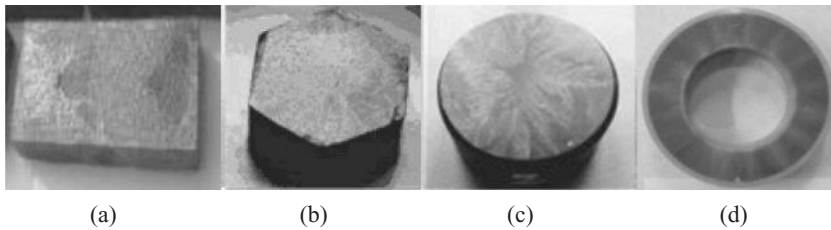


Figure 6.18 Typical shapes of ReBCO bulks: (a) single YBCO grains; (b) hexagonal shape; (c) cylindrical disk (Reproduced with permission from Zhiqiang Hua, “Process of HTS materials”, General Research Institute for nonferrous Metals. <http://www.grinm.com.cn/p466.aspx>, <http://www.grinm.com.cn/584-1498-11384.aspx> (accessed October 28, 2012); (d) hollow cylinder with thin outer tube of fibre reinforced plastic (the outer surface layer) (Reproduced from M. Tomita, Y. Fukumoto, K. Suzuki, A. Ishihara, M. Muralidhar. “Development of a Compact Magnet System Based on High T_c Superconductors,” QR of RTRI, 53, 3, 155–159, Aug. 2012 © 2012 RTRI).

6.7.4 Melt Cast Process (MCP)

The melt cast process (MCP) starts with the heating of a mixture of binary oxides in a metal ratio of 2 : 2 : 1 : 2, together with SrSO_4 in a temperature range of 980 °C through 1200 °C until homogeneous melting is achieved. In order to minimize contamination resulting from crucible materials, the heating process should be as short as possible. The obtained melt is then poured into moulds in which it solidifies and slowly cools to room temperature.

The melt cast materials consist of a more or less coarse-grained multi-phase mixture, which contains an eutectic Cu_2O and an earth alkaline mutation. Only one subsequent annealing step transforms the cast bodies into a composite material consisting of pure two-layer phase and SrSO_4 . The formation of the superconducting phase is favoured by the involvement of a partial melt during annealing. The demands on heating conditions depend on individual size and thickness of the form parts. Generally, the heating is performed between temperatures of 750 °C and 850 °C.

The shape of melting cast bulk depends on the mould used during shaping. The solidified melt after casting is a hard and brittle ceramic material. However, bulk parts are comparatively soft and can be machined by techniques applicable to metals, such as sawing, filing, drilling, mill-cutting and plating



Figure 6.19 Two typical shapes of Bi-2212 bulks used as current leads with different sizes commercially fabricated by Nexans: (a) rods; and (b) tubes (reproduced from “Superconducting materials”, Nexans, <http://www.nexans.de/eservice/navigation/NavigationPublication.nx?publicationId=-31165&obsOrigin=BOOKMARK&language=en&CZ=Germany>). (accessed October 27, 2012).

after heat/oxygen treatment. Thus, shaping of the conductors can also be accomplished by mechanical methods. To date, there are several shapes of Bi-2212 bulk with several tens and hundreds of millimetres in diameter and length, such as rod, bar, tube, ring, hollow cylinder and even conical tubes. Figure 6.19 indicates some typical batch production samples with different shapes. Similar to the ReBCO bulks, some enforced materials should be necessarily added onto the inner and outer surfaces of the bulk so that their mechanical characteristics can be strengthened.

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7

Principles and Methods for Contact-Free Measurements of HTS Critical Current and n Values

The development of high temperature superconducting (HTS) tapes has recently made great progress, and Bi-based tapes (1G) and YBCO-coated conductor (2G) are commercially fabricated with practical lengths. Application of HTS in electric power apparatus has been an important achievement, and various superconducting devices are demonstrated in the grid, laying solid foundations for their commercialization. However, since their intrinsic microscopic defects such as weak link, granularity and small second phase likely exists, critical current and n value of HTS tapes in practical lengths are impossibly homogeneous, which has significant influence on safety, stability and efficiency of the HTS apparatus. Therefore, critical current and n value are two important parameters describing inhomogeneity of HTS tapes, and are also two important indices for evaluating quality of practical long HTS tapes. This chapter will focus on the main progress in inhomogeneity of critical current and n value measurements of HTS tapes using contact-free methods. The statistical analytical methods evaluating the inhomogeneity of critical current and n value are suggested. They can provide essential references for design and operation of HTS apparatus.

7.1 Measurement Introduction of Critical Current and n Values

With advances in production technology of Bi-based (1G) and YBCO-coated conductor (2G), their electromagnetic and mechanical characteristics are significantly improved. HTS tapes of practical length are commercially fabricated, which greatly promotes their application in electrical power apparatus. Compared with conventional devices, HTS electrical devices have many advantages such as small volume, light weight, no fire hazard, environmentally-friendly, and so important progress in development of HTS electrical apparatus has been made in recent years. Also, HTS devices are important components in future smart grids, and increasing attention is being paid to them worldwide. At present, many HTS apparatus, such as cable, transformer, fault current limiter (FCL) and superconducting magnetic energy storage (SMES) are demonstrated in the live grid [1–15], amongst which the HTS cable has operated for a long time in the power grid [16].

The critical current I_c and n value are not only the essential parameters of practical superconducting materials but also important characteristic indices for evaluating current carrying ability and superconductivity. With its high n value ($n > 25$), the critical current value of the low temperature superconductor (LTS) is precise enough to describe its current-carrying ability and superconductivity, while the n value seems unnecessary. However, since intrinsic microscopic defects exist in HTS, the transient phenomenon from the superconducting state to the normal conducting state is slower than with conventional LTS, which also means that the n value ($n < 18$) of HTS is smaller than the LTS value [17, 18]. In general, superconductors with a high n value have superconductivity superior to those with a low n value. But the former superconductor is predicted to reach the current limit of quenching sooner than the latter [19]. Consequently, the single parameter of critical current I_c cannot fully reflect the current-carrying property of HTS tapes, and the n value must be simultaneously taken into account for comprehensive evaluation of their superconducting characteristics [20].

Normally, the critical current I_c is experimentally measured by the conventional four-probe technique and determined by a definition of criterion with electrical field $E_c = 1 \mu\text{V/cm}$ or resistivity $\rho_c = 2 \times 10^{-13} \Omega\cdot\text{m}$. When transport direct current (DC) is supplied to superconductors; the n value is defined by fitting the voltage current (or $E-I$) curve with the power law model in the range of $0.1 \mu\text{V/cm}$ through $1 \mu\text{V/cm}$. Although the critical current I_c and n value are easily obtained by the four-probe technique for short HTS samples, it is difficult or almost impossible to measure both parameters in each section of practical long HTS tapes by this method. This chapter presents the progress in measurements of critical current I_c and n value on HTS tapes by contact-free methods and the statistical analysis methods describing inhomogeneity of critical current I_c and n values in practical long HTS tapes are proposed.

7.2 Critical Current Measurements of HTS Tape by Contact-Free Methods

The contact-free methods for critical current I_c of HTS tapes mainly include the remanent field, alternating current (AC) inductive, mechanical, magneto-optical, magnetic relaxation, SQUID and vibrating sample magnetometer (VSM) methods.

7.2.1 Remanent Field Method

When a superconductor is in an environment at a temperature lower than its critical temperature and a DC constant magnetic field is applied, the magnetic field enters and circulation current is induced inside it. Because the superconductor has zero resistance and flux-pinning effect, the magnetic field is pinned and the magnetic field exists even if the DC magnetic field is removed. The so-called remanent field is proportional to critical current I_c . Therefore, the critical current distribution on practical long HTS tapes can be realized by measuring the remanent field distribution above the HTS with magnetic sensors (generally the Hall probe) after calibrating them [21–23]. Since no transport current is supplied and there are no voltage taps soldered onto the HTS tapes, measurement of critical current I_c with the remanent field method is usually called the contact-free or non-destructive method.

Generally, if the distance between the HTS tape and the Hall probe is much larger than its thickness or the aspect ratio (ratio of width to thickness) or the practical HTS tape is more than 10, it is reasonable for the tape to be considered as an infinite strip. After the external magnetic field is removed, the relative geometrical location and size of the HTS tape, the Hall probe and the critical current I_c are shown in Figure 7.1, where the direction of the x -axis is along the width of the tape, the z -axis refers to the vertical

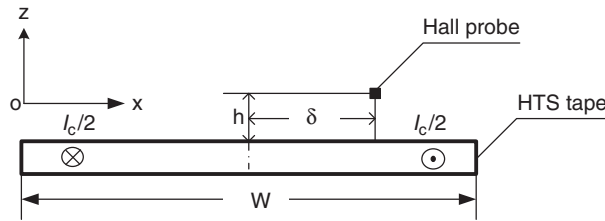


Figure 7.1 Relative locations in HTS tape and Hall probe.

tape surface, and the coordinate origin is at the tape centre [24]. Then the magnetic field along the z -axis at location $(\pm\delta, h)$ above the tape is described by:

$$B_z(h, \delta) = \frac{\mu_0 I_c}{4\pi W} \left\{ \log \frac{[h^2 + (W/2 + \delta)^2][h^2 + (W/2 - \delta)^2]}{[h^2 + \delta^2]^2} \right\} \quad (7.1)$$

where I_c , W , h and δ are critical current, width, horizontal distance between Hall probe and HTS tape centre, respectively. According to Equation (7.1), the magnetic field along the z -axis is proportional to critical current I_c at a fixed position (δ, h) . Consequently, the magnetic field continuously measured by the Hall probe can reflect the critical current distribution along the practical HTS length after calibrating the Hall probe.

Figure 7.2 shows measurement of the critical current of HTS tapes by the remanent field method [25], in which the HTS tape passing through reel 2 is immersed in a cryostat with liquid nitrogen (LN_2) and so is in a superconducting state. When it is passed through a DC magnetic field, the remanent field is produced in the HTS tape and detected by the Hall probe above it. Finally, the tape returns to reel 1, and measurement on inhomogeneity of critical current along the length is complete.

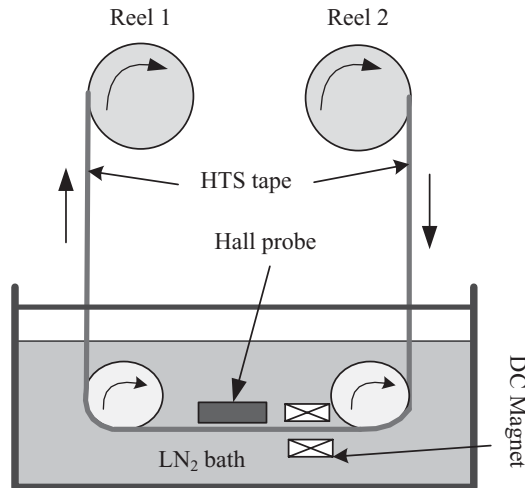


Figure 7.2 Critical current measurement with remanent field.

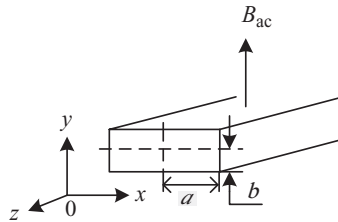


Figure 7.3 Geometrical cross-section and AC field orientation.

7.2.2 AC Magnetic Field-Induced Method

7.2.2.1 Fundamental Component Method

The HTS tape is placed in the background of an AC magnetic field perpendicular to its wide surface on which the current is induced and then an AC magnetic field is produced. The AC magnetic field is measured by the Hall probe and reflects the critical current of the HTS tape [26–29]. The geometrical cross-section and AC field orientations are indicated in Figure 7.3, where the width and thickness of the HTS tape are $2a$ and $2b$, respectively.

Figure 7.4 shows the measuring of critical current in an HTS tape. The magnetic field is measured by a Hall probe, and the AC magnetic field is supplied by a magnetization coil. In the perpendicular field, the HTS tape is segmented into n elements along its width, then a magnetic field in the y -axis direction is given by:

$$B_y(r) = \frac{\mu_0}{2\pi} \sum_i \frac{J_i dx dy \sin \phi(r, r_i)}{|r - r_i|} \quad (7.2)$$

where $\phi(r, r_i)$ is the angle between the AC field and the x -axis, and J_i is the current density in the i -th element.

Figure 7.5 presents continuous measurement of a critical current by an AC magnetic field induced method. The set-up mainly consists of an AC magnetization coil, power amplifier, function generator, lock-in amplifier and a Hall probe. The lock-in amplifier measures the in- and out-of-phase first Fourier components of the field, which correspond to the shielding and remanent fields above the HTS tapes, respectively. The shielding field component is proportional to the induced current and the remanent field component is proportional to the critical current. For sinusoidal alternating field measurements, when the amplitude of the out-of-phase first Fourier component of the field perpendicular to the wide surface

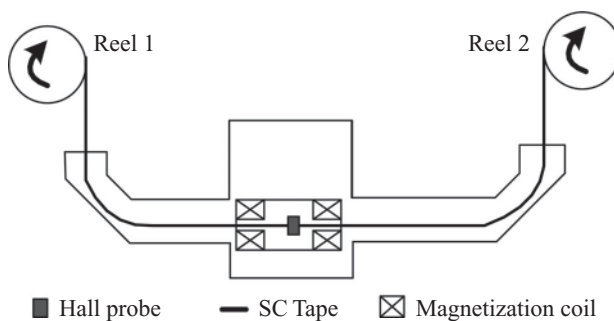


Figure 7.4 Continuous measurement of critical current in HTS tape.

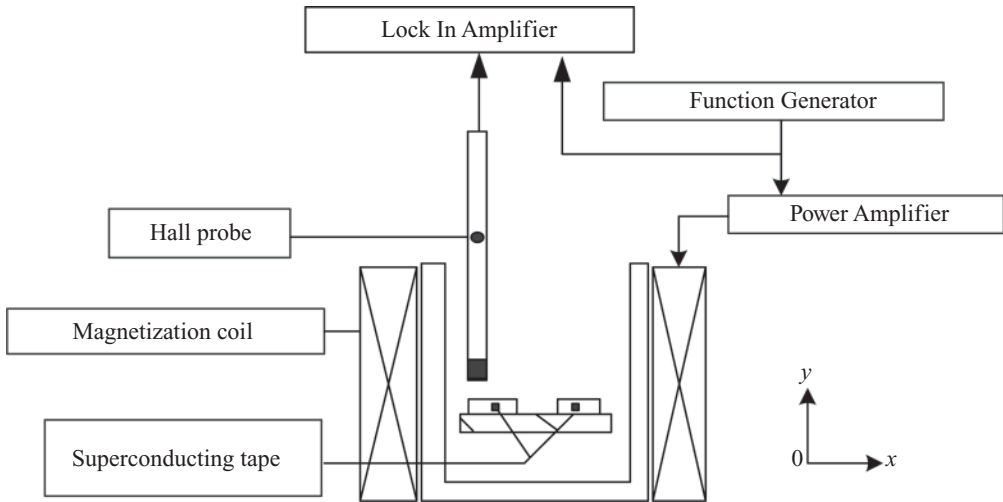


Figure 7.5 Continuous measurement of critical current by AC magnetic field induced method.

of the HTS tape is larger than the fully penetrated field, its profile deviates from the sinusoidal applied field. Based on the Bean's CSM, the current density J_i in the superconductor is equal to its critical current J_c in the penetrated area. According to Equation (7.2), the critical current density J_c can be numerically calculated; the measurements of critical current distribution of the HTS tape are thereby realized. However, the critical current distribution on the HTS tape can also be obtained by directly calibrating the first Fourier out-of-phase component of the field.

7.2.2.2 Third Harmonic Component Voltage Method

The measurement arrangement, mainly consisting of an AC driven coil and a pick-up coil, is shown in Figure 7.6. When the amplitude of the AC field supplied by the drive coil is larger than that of the full penetrated field, the third-harmonic voltage component is induced above the HTS tape [30–32]. It is proportional to the critical current of the HTS tape, and the relationship is described by:

$$E_{3\text{avg}} = (3\sqrt{3}\pi/4)\mu_0 f d^2 J_c \quad (7.3)$$

where f and d are the frequency of the AC magnetic field and thickness of the HTS tape, respectively; $E_{3\text{avg}}$ refers to the root mean square (RMS) of the third-harmonic electrical field induced above the HTS tape; J_c is the critical current and μ_0 is the vacuum permeability. Therefore, the critical current density of the HTS tape can be detected by measuring the induced third-harmonic voltage method. But the method is relatively complicated, in which the lock-in amplifier technique is used to measure exactly the amplitude and phase of the third-harmonic voltage, in order to avoid the effect of the first-harmonic voltage on results.

7.2.3 Mechanical Force Method

The principle of this method comes from the basic properties of superconductors. When the HTS tape, particularly the YBCO coated conductors (YBCO CC) is placed in a DC magnetic field supplied by

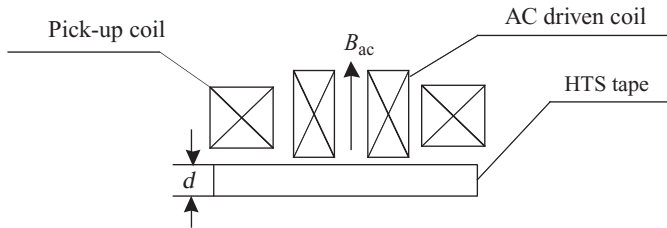


Figure 7.6 Measurement of third harmonic voltage.

a DC magnet or a permanent magnet, there is a shielding current induced. Figure 7.7 shows the J_c measurement apparatus, in which the magnetic field is supplied by a permanent magnet. Because of the induced shielding current and pinning force, repulsive force (F_r) and attractive force (F_a) are induced between the permanent magnet and HTS tape by shielding current and pinning force when the permanent magnet is nearing to or leaving the tape, respectively. The shape of the permanent magnet is cylindrical and d refers to its diameter, and L represents the distance between the down-end of the HTS tape.

When the permanent magnet is nearing the HTS tape, that is, decreasing distance L , the shielding current is induced, thus there is a repulsive force F_r between the magnetic field induced by the shielding current and the permanent magnet. While the permanent magnet leaves the HTS tape, that is, the distance increases, an attractive force F_a between tape and permanent magnet results from the remanent pinning field. Consequently, the repulsive force F_r and attractive force F_a can be measured by using a high-resolution load sensor when changing the distance L . An effective maximum repulsive force F_{mr} could be determined from an extrapolated F_r versus L curve at $L = 0$ mm [33–35]. The critical current density J_c is almost proportional to the F_{mr} normalized by tape thickness $2b$:

$$J_c \propto \frac{F_{mr}}{2b} \quad (7.4)$$

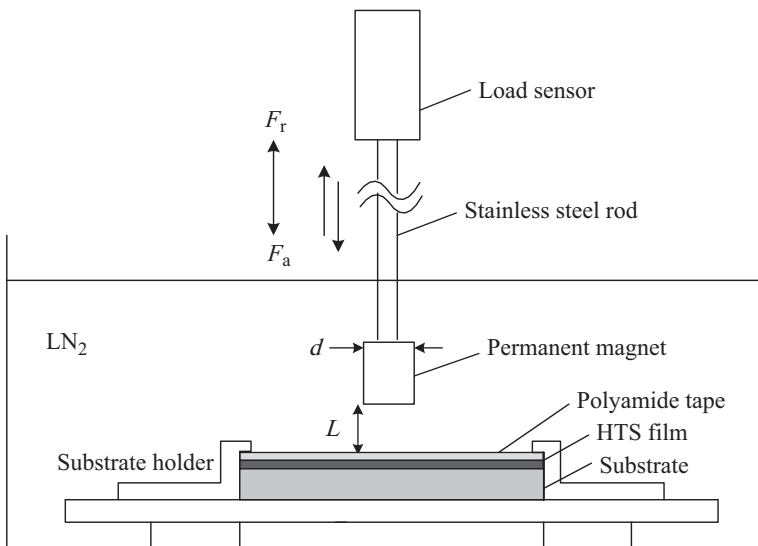


Figure 7.7 J_c measurement arrangement.

Thus, the critical current can be easily evaluated by measuring the maximum repulsive force exerted on the permanent magnet. The set-up is mainly composed of tensile apparatus, high-resolution load sensor and cryostat. The load sensor and tensile apparatus are installed at the upper end of the permanent magnet, and the HTS tape is immersed in LN_2 in the cryostat. This contact-free method is simple, and complicated electronic devices are not necessary; but it has disadvantages such as low resolution, slow measurement speed and is not suitable for fast measurement and large amounts of HTS tape.

There are also other contact-free measurement methods for critical current of HTS tape, such as the magneto-optical method [36], and the magnetic relaxation method by using SQUID and VSM [37], but they are only available for short samples and not suitable for engineering application, and thus are not introduced in this book.

7.3 n Value Measurements of HTS Tape by Contact-Free Methods

Compared with LTS, the transient phenomenon of HTS from the superconducting state to the normal conducting state is much slower, which means that the n value of HTS is smaller than the LTS value. Therefore, the n value in HTS is an important parameter in addition to those parameters such as critical current I_c , T_c , B_c . Especially, it is essential for the HTS tape with a relatively small n value, such as smaller than 12. Under these circumstances, the critical current I_c cannot fully describe the ability of current carrying. Therefore, it is necessary to measure the n value in addition to the critical current I_c . Although the n value is important for design and operation of HTS electrical apparatus [20], the HTS manufacturers usually do not provide any information on this parameter.

Since the n value is a fitted parameter that is obtained by empirically fitting the E - I curve with the power law model, the measurement of the n value must be performed with more data points in the E - I curve. At present, the AC-induced method is generally used in the contact-free method of n value.

7.3.1 Hysteretic Loss Component – Varying Amplitude Method

The contact-free measurement of the n value is performed by the pick-up coil method in conventional hysteretic losses. Figure 7.8 illustrates the transverse AC magnetic field that is parallel to the wide surface of the HTS tape. If the amplitude of the AC magnetic field is smaller than or equal to the full penetration field, the AC loss component of voltage difference between the pick-up coil and the compensated coil is given by [38]:

$$U''_{\text{rms}} \propto B_m^{2-\frac{2}{1+n}} = B_m^\alpha \quad (7.5)$$

where B_m is the amplitude of the AC magnetic field and U''_{rms} refers to the hysteretic loss component of the voltage. The relation between the n value and α is:

$$n = \frac{\alpha}{2 - \alpha} \quad (7.6)$$

If $B_m \leq B_p$ is satisfied, as long as U''_{rms} is acquired with variable B_m , then α is obtained by fitting the curve of U''_{rms} versus B_m with the power law. According to Equation (7.6), the n value can be calculated.

The arrangement and measurement circuit for the loss component of the voltage and n value are illustrated in Figure 7.9. The AC magnetic field, parallel to the wide surface of HTS tape C, is supplied by a pair of Helmholtz coils A and B. D and E are pick-up coil and compensated coil, respectively. The loss voltage component U''_{rms} is acquired with digital voltage multi-meter DVM1, and the magnitude of the AC magnetic field is calibrated by measuring the voltage of the standard resistor F with digital

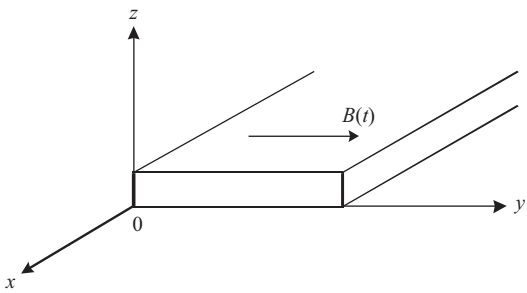


Figure 7.8 Configuration of transverse AC field and HTS tape.

voltage multi-meter DVM2. DVM1 can also be replaced by a lock-in amplifier for measuring the loss component of voltage. The amplitude of the AC magnetic field can be changed by adjusting the current of the AC power, and contact-free measurement on the n value can be realized. With this method, it is not necessary to change the frequency and only the current needs to change in the process of measurements.

It should be noted that the AC magnetic field must be parallel to the wide surface and the amplitude of the AC field must be equal to or smaller than the full penetrated field of the HTS tape in the whole process of measurement.

7.3.2 Fundamental Component Method – Varying Frequency

Similar to the set-up in measuring critical current, the shielding current is induced in an AC magnetic field, and then the shielding and the remanent fields are produced due to the flux-pinning effect in the HTS tape. The in- and out-of-phase first Fourier components of the field correspond to the shielding

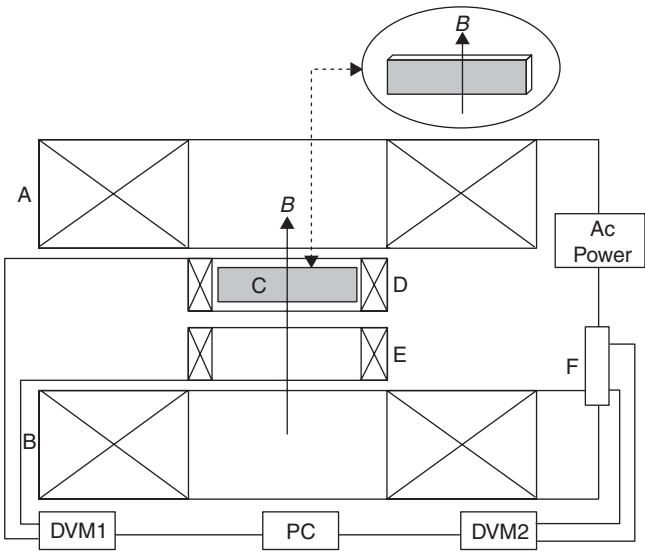


Figure 7.9 Arrangement for measurement of n value.

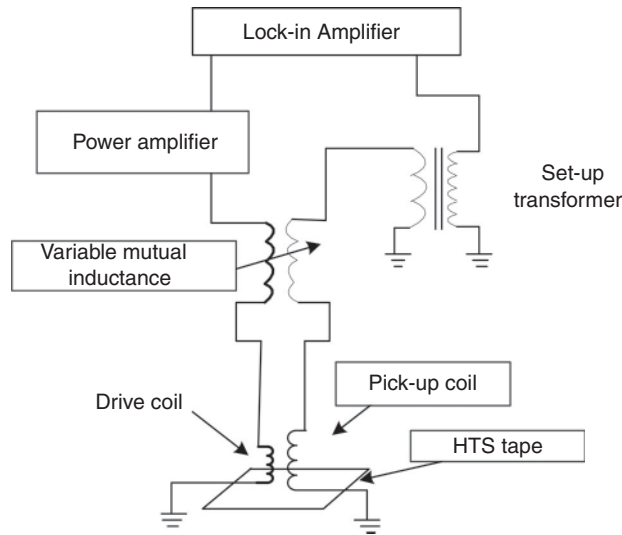


Figure 7.10 Block diagram of measuring system of n values.

and remanent fields above the HTS tapes, respectively. The set-up is shown in Figure 7.5. The shielding and remanent fields of HTS tape are measured by the Hall probe, pick-up coil and lock-in amplifier [30–32, 38–40]. The relationship between the in-phase first Fourier component of shielding field B_s and frequency is described by:

$$B_s(f) = cf^{1/n} \quad (7.7)$$

where c is the constant coefficient, and n refers to the value of the HTS tape. The contact-free method on the n value is realized by measuring the in-phase shielding field component with varying frequency before fitting the relation of B_s versus f .

7.3.3 Third Harmonic Component Voltage Method

When the amplitude of the AC magnetic field is larger than the fully penetrated field of the HTS tape, the third-harmonic component of voltage, being in the same phase as the AC magnetic field, will be induced in the pick-up coil above the surface of the HTS tape. The measurement system is shown in Figure 7.10 [31]. The system consists of a lock-in amplifier, power amplifier, set-up transformer, variable mutual inductance, drive coil and pick-up coil. The pick-up coil is wound outside the drive coil. Similar to the apparatus used in the measurement of the critical current, the third-harmonic voltage can be easily measured by the lock-in amplifier. Since the third-harmonic voltage is proportional to the current in the drive coil and the electrical field in the HTS tape, the current in the drive coil is proportional to the critical current density of the HTS tape [30, 31], therefore, we have:

$$V_3/(fI_m) \propto I_m^n/\sqrt{2} \quad (7.8a)$$

where V_3 , f , I_m are the third-harmonic voltage, frequency and current amplitude in the drive coil, respectively.

The relationship between induced electrical field above the HTS tape and the driven current obeys the power law:

$$E \propto I_0^n \quad (7.8b)$$

Thus, the $E \sim I_0^n$ relation can be obtained by measuring the current in the drive coil, and then the n value of the HTS tapes can be finally calculated by fitting the V_3 and I_0 with varying frequency f .

7.4 Analysis on Uniformity of Critical Current and n Values in Practical Long HTS Tape

Although the longitudinal distributions of critical currents and n values along the entire HTS tape length are realized by contact-free methods, it is impossible for them to be exactly the same due to intrinsic microscopic defects. The parameters, describing inhomogeneity of critical currents and n values and evaluating the quality of the HTS tape, need to be determined [41]. The inhomogeneity of critical currents has an important influence on the design and operating efficiency of the HTS electrical apparatus. Figure 7.11 presents the typical critical current distributions of HTS tapes 200 m in length after calibrating by the contact-free method [24]. At present there are no internationally unified standards for evaluating the inhomogeneity of critical currents and n values in HTS tapes, but it is reasonable for design and development of HTS electrical apparatus to adopt statistical methods to analyse them.

Gauss and Weinbull statistical analytical methods are the two main methods for describing critical current inhomogeneity of practical long HTS tapes.

7.4.1 Gauss Statistical Method

The simple Gauss statistical method is adopted to analyse the local critical currents that are taken as random variables based on the central limit theorem. The Gauss probability of local critical currents is

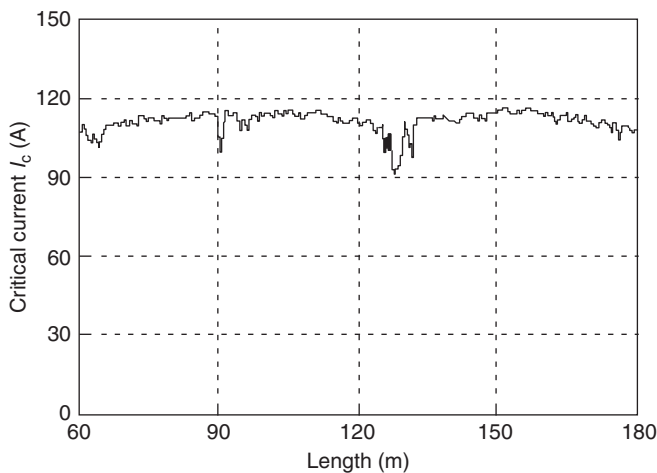


Figure 7.11 Typical inhomogeneity of critical currents in 200 m-order HTS tapes measured by contact-free method. (Reproduced with permission from Yinshun Wang, “Bases of Applied Superconductivity in Electrical Engineering,” Science Press, 211, © 2011.)

described by [24]:

$$p(I_c) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(I_c - \bar{I}_c)^2}{2\sigma^2}\right) \quad (7.9)$$

where I_c , $\bar{I}_c$ and σ are local critical current, the mean (average) critical current and standard deviation of long HTS tapes, respectively. The coefficient of variance (COV) of critical currents is defined as:

$$COV_I = \frac{\sigma}{\bar{I}_c} \times 100\% \quad (7.10)$$

COV_I is the parameter for evaluating the inhomogeneity of critical currents in HTS tapes. Though the Gauss statistical analytical method is simple and concise, special attention must still be paid. If the method is adopted in a rigid and strict manner, an incorrect judgement can possibly be made. For example, if the local critical current is too small or even zero in some local position on a long HTS tape, the entire HTS tape may be considered as unqualified, since superconducting characteristics are determined by the weakest local section. In this case, the variation of mean $\bar{I}_c$ and standard deviation σ are not obviously affected. Therefore, if this case is met, it is unnecessary to perform the Gauss statistical analysis and the HTS tape may be directly judged as unqualified.

Similarly, the Gauss statistical method, just as in the analysis on critical currents, is also adopted in analysing inhomogeneity of n values, that is, the mean $\bar{n}$ and coefficient of variation (COV_n) of n values are regarded as the parameters describing inhomogeneity of n values in practical long HTS tapes.

7.4.2 Weibull Statistical Method

In order to avoid the shortcomings that the Gauss statistical method cannot reflect the minimum critical current $I_c^{\min}$, it is necessary to choose the Weibull statistical method to deal with this inhomogeneity [42]. The Weibull statistical distribution of critical currents is expressed by:

$$p(I) = \frac{m}{\alpha^m} (I - I_c^{\min})^{m-1} \exp\left(-\left(\frac{I - I_c^{\min}}{\alpha}\right)^m\right) \quad (7.11)$$

where α and m are fitting parameters, and $I_c^{\min}$ is the minimum critical current on HTS tape. Integrating Equation (7.11) with the range of I_c through $I_c^{\min}$, the distribution function of critical currents is described by:

$$\varphi(I_c) = 1 - \exp\left(-\left(\frac{I_c - I_c^{\min}}{\alpha}\right)^m\right) \quad (7.12)$$

where the α , m , $I_c^{\min}$ are three parameters describing the inhomogeneity of critical currents along the length of practical long HTS tapes. The same method can be used in analysis of n values if the random variables of critical currents are simply replaced by n values.

Although the contact-free methods to measure the critical current and n value of practical long HTS tapes are relatively mature and feasible, to date there are few reports on real devices of critical currents measured by contact-free methods, except in references [23, 34, 43], yet there are many test results using contact-free methods based on different principles. However, up to the present, there is no reports concerning real devices for measurement of n values by a contact-free method. Nor are there reports on results of n value distribution along practical long HTS tapes. There may be two possible reasons: (i) the speed of measurement of n values by contact-free methods is low; and (ii) the manufacturers of

HTS tapes are reluctant to provide these parameters. They only provide the critical currents measured from end-to-end and never provide the distribution and inhomogeneity of critical currents along the longitudinal HTS tapes.

Since the n value is a fitted parameter, it can be obtained exactly only by acquiring a large quantity of data. Consequently, measurement of n values by contact-free methods takes a long time compared with critical current measurements. If the period of measurement is not of concern, the distribution of n values along any long HTS tapes can be realized. The inhomogeneous parameters, such as COV_n , α , m and $n^{\min}$, that are the same as those used in the analysis of critical currents, can be adopted by using the Gauss and Weibull statistical methods, respectively.

It has been shown numerically by research that based on HTS AC application and consideration of AC losses, the inhomogeneity of critical currents along the length must be limited to less than 10% and 5% in single and multi-parallel HTS tapes, respectively. Furthermore, the effect of inhomogeneity of n values on AC losses and stability needs to be studied in the future.

7.5 Next Measurements of Critical Currents and n Values by Contact-Free Methods

According to evaluation of the market of HTS application, HTS apparatus will be commercialized in the coming 10 years and will also be the important component of future smart grids. HTS application on electrical engineering will become a large-scale high technology industry. Therefore, the inhomogeneity of critical currents and n values, indices for evaluating the quality of practical long HTS tapes, has important practical significance.

With contact-free methods, the inhomogeneous distributions of critical currents and n values in practical long HTS tapes can be obtained. The parameters, describing their inhomogeneity, can be obtained by adopting Gauss and Weibull statistical methods and used to evaluate the quality, current carrying characteristics and AC losses of practical long HTS tapes.

Although the measuring principles of the inhomogeneity of critical currents and n values in HTS tapes by contact-free methods are relatively mature, the critical current and n value depend not only on the magnitude of the magnetic field but also on its orientation due to the strong anisotropy of HTS tapes. However, there are no measurement principles and results of contact-free methods for inhomogeneity of critical currents and n values under magnetic field with different magnitudes and orientations, are the research works to be carried out in the future.

There are no readily available international statistical analysis methods and standards for characterizing the inhomogeneity and distribution of critical currents and n values in practical long HTS tapes. Critical current I_c and n values of the HTS tapes, and the problem as whether the parameters of the related standard deviation, COV and the minimal critical current I_c can fully characterize the inhomogeneity properties of the critical current I_c and n value of the HTS tape, are both the problems that need to be further studied.

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8

Cryogenic Insulating Materials and Performances

Superconducting electrical apparatus mainly refer to high temperature superconductor (HTS) electrical appliances operating at a temperature above the liquid helium temperature (LHe @ 4.2 K), which include the superconducting fault current limiter (SFCL), cable, transformer, motor/generator, magnet, etc. The superconducting magnet and the superconducting magnetic energy storage (SMES) generally work in the DC mode at a temperature of LHe or liquid Ne. This range of temperatures can also be realized by conduction cooling with a cryo-cooler to replace the cryogenes. Superconducting electrical apparatus are closely related to cryogenic insulation. The requirements of superconducting electrical apparatus on cryogenic insulation materials can be summarized as:

- (1) *Mechanical properties*: sufficiently high tensile strength, elasticity modulus and appropriate elongation to reduce the space occupied.
- (2) *Electrical properties*: low dielectric loss and high dielectric strength, such as withstand, breakdown and partial discharge (PD) voltages.
- (3) *Thermal performance*: thermal performance of insulating materials in a cryogenic environment is important. Heat contraction is a significant parameter in the design of the insulating structure, and the structure should match other components of the superconducting electrical apparatus in this respect. In superconducting windings, insulating materials should have high dynamic stability, mechanical properties, thermal stability and thermal conductivity, so as to cool the windings easily. Good thermal insulation is required in outer layers of insulating material.
- (4) *Radiation*: withstanding property is required in some special applications, such as the insulating material used in superconducting coils in a fusion device.

8.1 Insulating Properties of Cryogenic Gas

Cryogenic gas refers to the gas whose temperature at triple point is lower than 120 K, which mainly includes He, H₂, O₂, N₂, Ne, Ar, CO and some organic compound gases. In cryogenic electrical apparatus, especially in superconducting electrical apparatus, LHe and LN₂ are frequently used as the cooling cryogen in practical application since they are safe, non-inflammable and environmentally friend. LH₂ is also sometimes used as a cryogen in other applications, such as cooling of hydraulic generators. Therefore, insulating properties of cryogenic gas are essential to cryogenic electrical design.

Table 8.1 Some thermodynamic parameters of gases He, H₂ and N₂

Thermodynamic parameters	Gas He (4.2 K)	Gas H ₂ (20 K)	Gas N ₂ (77 K)
Thermal conductivity k /[W/(m·K)]	1.0×10^{-2}	0.322	0.765
Specific heat C /[J/(kg·K)]	6.0×10^3	2.43	0.249
Density ρ /[kg/m ³]	8.0	1.27	4.2
Thermal diffusion speed v /[cm ² /s]	2.0×10^{-3}	/	/

8.1.1 Insulating Properties of Common Cryogenic Gas

In a gas-insulated medium, the breakdown strength is an important index which reflects the insulating ability of the gas insulation. This section focuses on the breakdown characteristics of He, H₂ and N₂ at cryogenic temperature. Insulating properties of other conventional gases are also briefly discussed.

8.1.1.1 Breakdown Characteristics in Uniform Electric Field

In a uniform electric field, breakdown voltage U_b of gases grows with the increase of distance d between electrodes under conditions of a certain temperature T and pressure P . If the electrode distance d is fixed, the breakdown voltage U_b rises when gas pressure P is low. Subsequently, with an increase in pressure P , breakdown voltage U_b drops; when pressure P increases to a certain value, U_b decreases to a minimum value and subsequently increases gradually with a further increase of pressure P . Breakdown voltage U_b is a function of the product of gas pressure and electrode distance ($U_b = f(Pd)$), which is the so-called Paschen's Law [1]. At room temperature, the breakdown characteristics of He, H₂ and N₂ follow this rule. Table 8.1 lists some thermodynamic characteristic parameters of the three gases at one atmospheric pressure. Figure 8.1 shows their Paschen's curves at room temperature.

At low temperatures, the breakdown strength of air, He, H₂ and N₂ obeys Paschen's Law before they are liquefied within the ranges of temperature T and ρd (Table 8.2), where ρ is the density of gas.

In the suitable scope of Paschen's Law, breakdown strength rises with the increase of gas density ρ (pressure) and electrode distance. When the electrical field $E < 15$ kV/mm, breakdown voltage U_b of

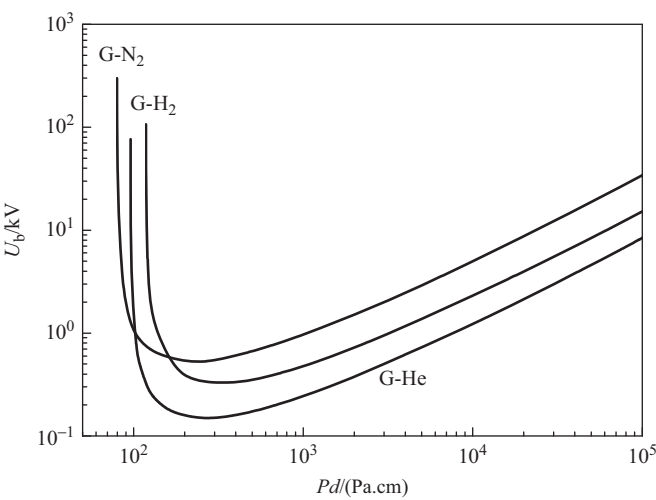


Figure 8.1 Paschen's curve of He, H₂ and N₂.

Table 8.2 Suitable scope of Paschen’s Law

Gas name	Range of ρd (kg·m ⁻²)	Range of temperature T
air	<12	293 ~ 143 K
	<15	293 ~ 113 K
He	<65	<20 K
	$E < 150$ kV/cm	<10 K
	$\rho < 20$ kg/m ³	
H ₂	P = 1 ~ 5 bar, d = 0.4 ~ 12.0 mm	
N ₂	<15	293 ~ 193 K

Note: E denotes breakdown strength and 1 bar = 10⁵ Pa.

gas He increases exponentially with the density. When density $\rho < 20$ kg/m³, breakdown voltage U_b is proportional to $\rho^{0.6}$, and inversely proportional to $d^{0.1}$. As for gases H₂ and N₂, when pressure increases, their breakdown voltage U_b has a similar relationship with gas density ρ and electrode distance d , but the influence of density ρ and electrode distance d on breakdown voltage U_b of gas H₂ is more obvious. For example, when pressure grows from 1 bar to 5 bar, breakdown voltage U_b of gas H₂ increases by two-fold, but U_b of gas N₂ only rises by 30%. However, breakdown voltage U_b of gas under high pressure deviates from Paschen’s Law no matter what the cryogen or temperatures; particularly, the degree of the deviation for gas He is large.

8.1.1.2 Breakdown Characteristics in Non-Uniform Electric Field

Unlike the breakdown characteristics in a uniform electric field, breakdown strength E of gas He shows an apparent polarity effect in a non-uniform electric field. The greater the electrode distance and the lower

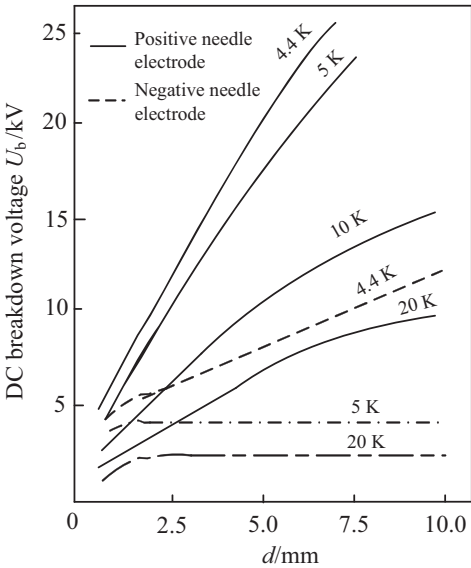


Figure 8.2 Breakdown voltage of He and electrode distance in non-uniform electric field (Electrodes: Needle-plate type, temperature: 4.4 K ~ 20 K).

the temperature, the greater the breakdown voltage ratio of the positive to negative needle electrode is. Figure 8.2 shows the relationship of gas He breakdown voltage with electrode distance and polarity in a non-uniform DC electric field. The electrode structure is a needle-plate type and the temperature ranges between 4.4 K and 20 K. For example, the ratio is as high as 5 at temperature 5 K when the electrode distance is 7 mm. This polarity effect is opposite to the case of electronegative gas and gas N_2 containing impurities. When using a positive needle electrode, the breakdown voltage U_b of gas He rises regularly with the increase of electrode distance, which is related to the space charge formed around the needle electrode.

When breakdown occurs, the breakdown field strength at the needle is a function of electrode distance, that is, the longer the electrode distance, the more the space charge contributes to the rise in breakdown voltage. Nevertheless, when using a negative needle electrode, breakdown voltage U_b of gas He is low and almost independent of changes in electrode distance, which results from positive and negative ions bombarding the negative needle electrode so that emission of secondary electrons increases. This kind of secondary emission not only reduces breakdown voltage U_b of gas He, but also is the reason that the shape of the needle electrode has a great influence on breakdown voltage. Therefore, in insulation design of superconducting electrical apparatus, partial concentration of the electric field caused by structure and shape of conductor should be avoided.

8.1.1.3 Influence Factors of Breakdown Voltage

After quench of a superconductor, the cryogen vaporizes with a rise in temperature, so that the dependence of gas breakdown voltage on temperature is important in the design of superconducting electrical apparatus. Figure 8.3 illustrates the relationship of peak AC breakdown voltage of gases He, N_2 with temperature in a uniform electric field, indicating that breakdown voltage increases with temperature decrease. If the temperature of gas He reduces to 4.4 K from room temperature, the breakdown voltage rises at least 60 times as its density grows with decreased temperature. In different temperature ranges, breakdown voltages are also vary. In the conditions of fixed ρ (that is, volume) and electrode distance d ,

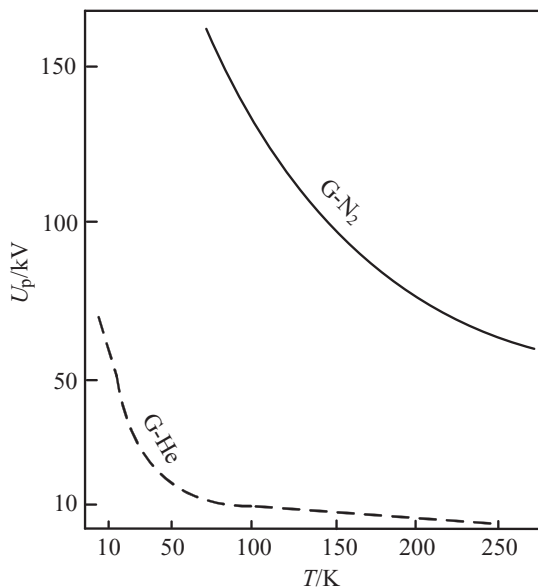


Figure 8.3 Curve between temperature and breakdown voltage of He, N_2 .

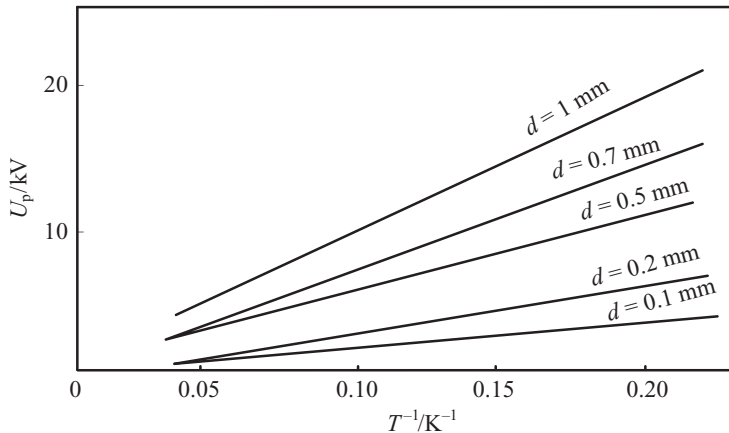


Figure 8.4 Relationship between AC breakdown voltage of gas He and reciprocal of temperature in non-uniform electric field (distance d : needle-plate electrode, frequency f : 50 Hz).

breakdown voltage of gas He is minimum in the temperature range of 10 K to 20 K. Also, the impulse voltage of gas He is much lower than gas N_2 over a wide temperature range.

In a non-uniform electric field, the influence of temperature on AC breakdown voltage resembles that in a uniform AC electric field. Figure 8.4 presents the relationship between the peak value of breakdown voltage and reciprocal of temperature at power frequency 50 Hz. When the temperature decreases from 20 K to 4.4 K, the DC breakdown voltage of gas He is inversely proportional to temperature, which illustrates that Paschen's Law holds in this temperature range. However, in a small temperature range of 4.4 K to 4.2 K, it is shown that the breakdown voltage rises, showing an obvious polarity effect.

Besides temperature, many other factors, such as voltage type (impulse, AC, DC), radiation and surface smoothness of the electrode also has an influence on breakdown voltage of cryogenic gas. All these factors should therefore be taken into account in the design of the insulation structure of superconducting apparatus.

8.1.1.4 Breakdown Characteristics in Vacuum

Cryogenic insulation in a vacuum generally means that the insulation is placed in an environment with a vacuum of less than 10^{-3} Pa [2]. Theoretically, the highest electrical insulation strength occurs in the vacuum state, where breakdown occurs at an electric field of 10^4 kV/cm. In practice, breakdown occurs in an electric field of several hundreds kV/cm. The breakdown voltage is related to factors such as pressure, electrode material, electrode surface area, electrode distance, frequency, temperature, spark discharge treatment and surface smoothness of the electrode. At cryogenic temperatures, a breakdown strength in vacuum is approximately proportional to the square root of electrode distance. In a small range of electrode gaps, breakdown voltage is the highest, up to 800 kV to 1000 kV/cm. The breakdown strength is 200 kV to 500 kV/cm at practical electrode distance. In addition, breakdown strength is obviously affected by electrode materials. Table 8.3 lists the DC breakdown voltage of different electrode materials on conditions better than 10^{-4} Pa vacuum and different temperatures with electrode distance of $d = 0.5$ mm. It is known that the breakdown voltage of copper and aluminium is relatively low, while the breakdown voltage of stainless steel is the highest.

Although a high vacuum presents good performance in respect of electrical and thermal insulation, the breakdown voltage drops sharply with a decrease of vacuum degree. However, it is also difficult to

Table 8.3 Breakdown voltage for vacuum of different electrode materials and temperatures

Electrode materials	Room	LN ₂ /77.3 K	LHe/4.2 K
	temperature/300 K		
Aluminum/kV	30	35	37
Copper/kV	20	24	26
Niobium/kV	32	37	39
Stainless steel/kV	43	60	62

keep a container with a high vacuum degree for long time, and if the stability to retain the vacuum is low, it is hard for the vacuum itself to guarantee the long-term reliability of insulation.

8.1.2 *Insulating Properties of Other Gases*

Some gaseous compounds containing halogen elements, such as sulphur hexafluoride (SF₆) and carbon tetrachloride (CCl₄), have higher electrical strengths. Electrical strength of SF₆ is 2.3 to 2.5 times that of air and electrical strength of CCl₄ is 2.4 to 2.6 times that of air under the same conditions of pressure and electrodes. Therefore, using high electrical strength gases in terminal insulation can efficiently increase gap breakdown voltage and reduce terminal size. If high electrical strength gases are used in terminal insulation of electrical apparatus, breakdown voltage of the gap can be effectively improved and terminal size can be simultaneously reduced, and then the terminal structure becomes more compact. Stability of SF₆ is high and has a strong arc extinguishing performance, besides a high dielectric strength. A liquefaction temperature of SF₆ is as low as −63.8 °C. Applications of CCl₄ are limited because it easily liquefies, even at room temperature.

8.2 **Insulating Characteristics of Cryogenic Liquid**

The insulating properties of cryogenes vary and depend on pressure, not temperature. This section aims to introduce those characteristics of commonly used cryogenes.

8.2.1 *Comparison of Cryogenes*

A cryogen provides a low temperature operating environment for the superconductor and also acts as electrical insulating medium simultaneously. There are only tens of liquid-state materials below 90 K, all others being in a solid state, so the choice of cryogen is limited. Table 8.4 gives the main properties of several common cryogenes as coolants. It is shown that LH and LNe are expensive, while LH₂ and LO₂ are inflammable and potentially explosive. Although carbon monoxide (CO) and liquid methane (CH₄) are cheap, they are toxic and explosive. Only LN₂ is low-cost, safe, non-inflammable and easy to use for thermal and electrical insulations or cooling; it is also environmentally friendly, so is the ideal cryogen.

8.2.2 *Electrical Properties of Cryogenes*

Currently, practical and commonly used cryogenes only include LHe, LN₂ and LH₂ in cryogenic electrical applications. Table 8.5 lists the main thermodynamic parameters of these three cryogenes. Practically, only LHe and LN₂ are commonly used as cryogenes in superconducting electrical application, because LH₂ is explosive.

Table 8.4 Properties of several cryogenes

Cryogenes	LHe	LH2	LNe	LN ₂	CO	LO ₂	CH ₄
Boiling point /K	4.2	20	27	77	82	90	~ 110
Dangerous	no	yes	no	no	yes	yes	mid
Cost	high	lower	Very high	low	low	lower	low
Penalty coefficient ^a	500 ~ 100	50 ~ 100	40 ~ 80	6 ~ 11	/	7	5

^aDissipation power per unit refrigerating capacity.

Table 8.5 Some thermodynamic parameters of LHe, LH₂ and LN₂

Performances		LHe	LH ₂	LN ₂
Boiling point @ one atm/K		4.2	20.4	77.4
Temperature range with liquid state/K		<5.2	14 ~ 33	63 ~ 126
Pressure/MPa		0.226	0.007 ~ 1.3	0.0114 ~ 3.35
@ boiling point	Latent heat/10 ⁶ J/m ³	2.5	32	160
	Viscosity/(μPa.s)	36	140	150
	Density/(kg/m ³)	125	710	810
	Volumetric specific	0.56	0.57	1.65
	heat capacity/[10 ⁶ J/(m ³ ·K)]			

8.2.2.1 Permittivity

At one standard atmosphere and different temperatures, permittivity of several cryogenes are larger than air (Table 8.6), except for LHe whose permittivity is close to air in the temperature range of 1.83 K to 4.21 K. When solid-state insulation materials are immersed into cryogenes, most of the electric field will be concentrated on the cryogenes because permittivity of the most solid-state insulation materials are much higher than cryogenes, so that breakdown easily occurs in the cryogen.

Permittivity of cryogen ϵ relates not only to temperature but also pressure. Taking LHe as an example, there is a characteristic temperature T_λ at which the slope of the ϵ - T curve changes (Figure 8.5). If $T > T_\lambda$, permittivity ϵ decreases with a rise in temperature T , and there is a linear relationship between ϵ and T . When $T < T_\lambda$, ϵ is almost independent of temperature T . Permittivity of LH₂ increases linearly as pressure rises, and permittivity of LN₂ is independent of pressure.

8.2.2.2 Dielectric Loss

Dielectric loss of insulating materials is an important parameter in AC superconducting electrical apparatus. Only electric polarization exists in most cryogenes, but their dielectric loss tangents $\tan \delta$ are too low to be accurately measured at present. With conditions of an electric field lower than 20 kV/cm, its approximate value is smaller than 10^{-6} , even by using the most accurate measuring method.

Table 8.6 Permittivity of several cryogenes

Cryogen	LHe					LN ₂		LH ₂		LO ₂	LNe	LAr
Temperature/K	4.21	2.24	2.19	2.15	1.83	77.33	63.1	20.4	14.0	90.2	27.2	87.3
Permittivity ϵ	1.0469	1.0563	1.0563	1.0565	1.0562	1.431	1.467	1.231	1.259	1.48	1.19	1.54
	1.048	1.055	1.055	1.055	1.055	1.433	/	1.227	1.248	/	/	/

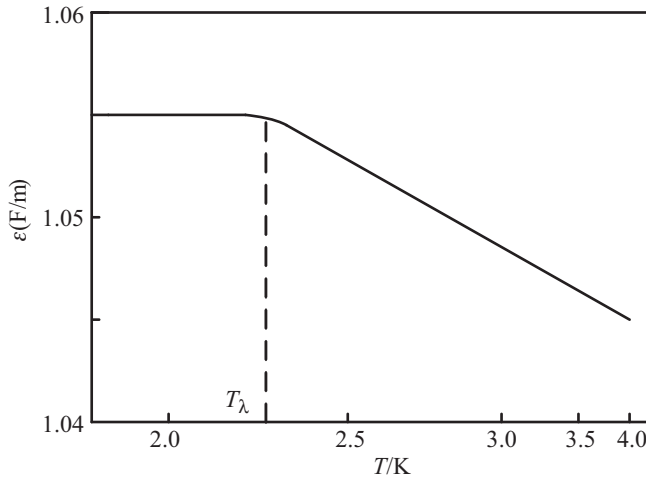


Figure 8.5 Curve of permittivity versus temperature in LHe.

In a high electric field, the dielectric loss tangents $\tan \delta$ increases rapidly. The relationship between dielectric loss tangent $\tan \delta$ and electric field E of LHe and LN_2 and can be expressed as:

$$\tan \delta = A - e^{-bE} \quad (8.1)$$

where A and b are constants.

The relationship of the dielectric loss tangent with the electric field is complicated for LH_2 , because its dielectric loss tangent is also influenced by the duration of action voltage. The dielectric loss tangent slightly increases with increasing voltage duration. Moreover, the dielectric loss tangent of LHe, LH_2 and LN_2 increases with a rise of applied measurement frequency. Figure 8.6 shows the curve of the dielectric loss tangent $\tan \delta$ varying with measurement frequency at different temperatures. The lower the temperature, the higher $\tan \delta$; the higher the measurement frequency, the larger $\tan \delta$ also. It is worth noting that dielectric loss tangent $\tan \delta$ of LHe is discontinuous around T_λ .

8.2.2.3 Breakdown Voltage

Breakdown voltages of several commonly used cryogens are listed in Table 8.7, in which the breakdown voltage of LHe is the lowest, LN_2 is the highest and LH_2 is between LHe and LN_2 .

In a uniform electric field, the breakdown voltage of cryogens relates to electrode distance. Figure 8.7 shows the dependence of breakdown field strength of several cryogens on electrode distance at different pressures and a power frequency of 50 Hz. The figure shows that breakdown voltage increases with increasing electrode distance. Relationships of breakdown voltage and electrode distance generally obey the exponential equation:

$$U_b = Kd^n \quad (8.2)$$

where K is constant, d denotes electrode distance, n is an exponent depending on experimental conditions, and their typical values are listed in Table 8.8.

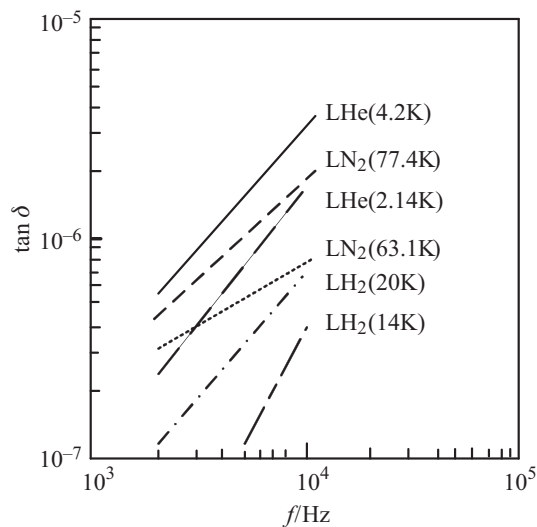


Figure 8.6 Curve of $\tan \delta$ changing with measurement frequency for LHe, LH₂ and LN₂.

In a non-uniform electric field, the relationship between breakdown voltage and electrode distance for LHe and LN₂ is shown in Figure 8.8. LN₂ breakdown voltage by using ASTM (American Society for Testing and Materials) standard electrodes is about one-third of that using a sphere-plate electrode. Under testing conditions of ASTM electrodes, the relationship between breakdown voltage and electrode distance approximately satisfies:

$$U_b = 15.93d^{0.676} \tag{8.3}$$

Table 8.7 Breakdown voltage of several cryogens

Cryogens	Shape of electrodes	Gap/mm	Temperature/K	Breakdown voltage U_b [peak]/(kV/mm)
LHe	sphere	0.05 ~ 0.15	4.2	72 [DC]
	Steel sphere with diameter 3.17 mm	0.15	4.2	105[DC]
		0.5		55[DC]
		1.0	4.2	40[DC]
	Steel sphere with diameter 12.7 mm	0.19		32.7[AC @ 60 Hz]
		0.5		31.9[AC @ 60 Hz]
LAr	Brass sphere with diameter 12.7 mm	1.0		29.4[AC @ 60 Hz]
		0.02 ~ 0.10	87.3	101[DC]
				140[DC]
				150[DC]
LN ₂	Gold – platinum sphere with diameter 5 mm	0.02 ~ 0.10	77.3	224[DC]
				160[DC]
	Tungsten sphere with diameter 5 mm-plate	0.05 ~ 0.15	65	
	Steel sphere with diameter 12.7 mm	0.19	77.3	80.5[AC @ 60 Hz]
		0.19	65	101[AC @ 60 Hz]
LH ₂	Steel sphere with diameter 12.7 mm	0.19	20	74.5[AC @ 60 Hz]
		0.19	14	134[AC @ 60 Hz]

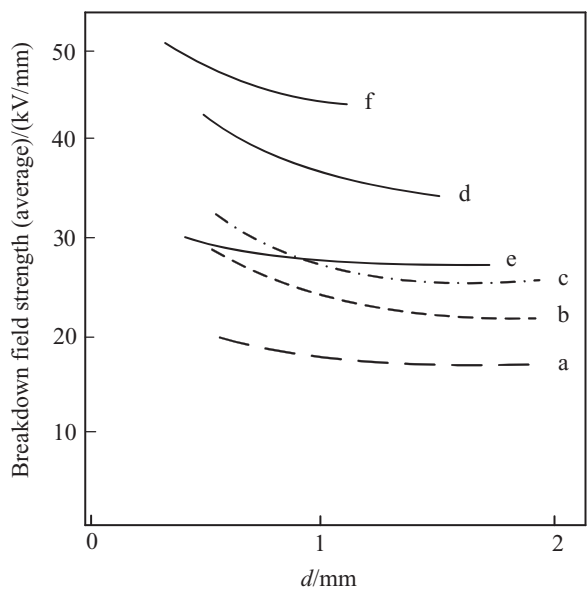


Figure 8.7 AC average breakdown strength and electrode distance (spherical electrode with diameter 62.5 mm: (a) LHe (10^5 Pa); (b) He (3×10^5 Pa); (c) LH₂ (10^5 Pa); (d) LH₂ (3×10^5 Pa); (e) LN₂ (10^5 Pa); and (f) LN₂ (3×10^5 Pa)).

Under testing conditions of a sphere-plate electrode, the relationship between breakdown voltage and electrode distance is:

$$U_b = 43.6d^{0.78}$$

(8.4)

The influence of temperature on the breakdown voltage of cryogenics is different above and below the temperature of boiling point of the cryogen. The temperature effect is not obvious above the boiling point, while the breakdown voltage rises with decrease of temperature below the boiling point. LN₂ is an exception, since its breakdown voltage changes discontinuously around temperature T_λ . When the $T < T_\lambda$, the relationship of breakdown voltage with electrode distance is:

$$U_b = A + Kd^{-n}$$

(8.5)

where A is constant and K and n have the same meaning as in Equation (8.2).

The breakdown voltage of cryogenics also depends on pressure and so rises with increasing pressure. Figure 8.9 shows the varying relationship of breakdown voltage with pressure for LHe, LH₂ and LN₂ with electrode distance $d = 1$ mm. It indicates that the effect of pressure on breakdown voltage in LN₂

Table 8.8 Exponent *n* values under different experimental conditions

Experimental conditions	Steel sphere electrodes with diameter 17 mm			Sphere-plate electrodes		
	$d \leq 1.25$ mm	$d = 25 \sim 100$ μ m	$d \geq 10$ mm	$d > 2$ mm	$d \leq 3.75$ mm	$d \leq 2.5$ mm
<i>n</i>	0.9	0.8 ~ 0.65	0.5	0.78	1	1

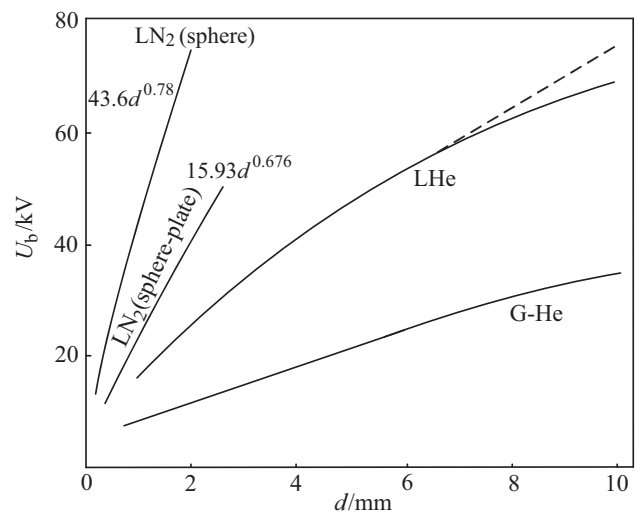


Figure 8.8 Relationship between breakdown voltage and electrode distance in LHe and LN2 in non-uniform electric field.

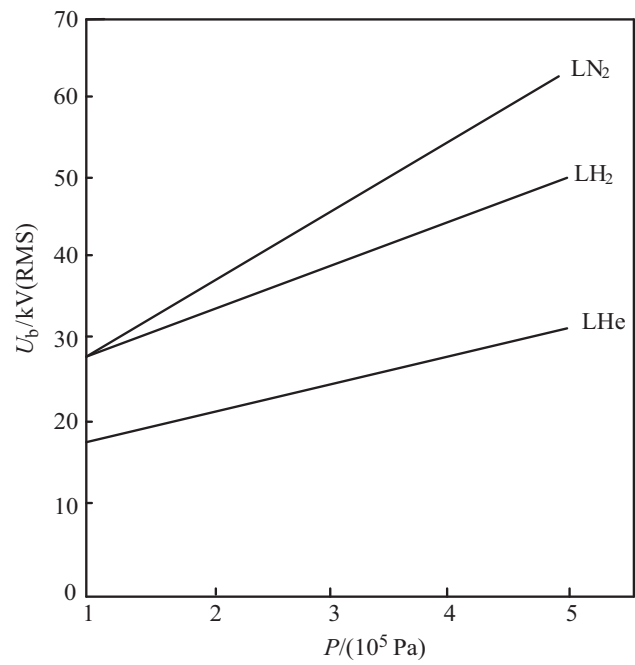


Figure 8.9 Breakdown voltages versus pressure in LHe, LH₂ and LN₂.

Table 8.9 LN₂ breakdown voltage using different electrode materials

Electrode materials		
Anode (positive) materials	Cathode (negative) materials	Breakdown strength/(kV/mm)
Aluminium	Aluminium	114
	Steel	129.8
	Brass	77.1
	Copper	18
Steel	Aluminium	94
	Steel	148
	Brass	70.0
	Copper	35.0
Brass	Aluminium	85.1
	Steel	87.9
	Brass	83.1
Copper	Aluminium	140.3
	Steel	117.3
	Copper	104.7

is greatest, and its breakdown voltage nearly doubles in the pressure range 10^5 to 4×10^5 Pa. The effect of pressure on breakdown voltage in LH₂ is smaller than in LN₂ and the effect in LHe is the least.

Similar to insulation properties for cryogenic gas, some factors, such as electrode material, electrode shape, surface smoothness and area of electrode surface, also have significant influences on the breakdown voltage in liquid cryogenes. Table 8.9 gives the breakdown voltage of LN₂ by using different electrode materials. Breakdown voltage is the highest if hard metals are used as electrodes such as steel and stainless steel; it is the lowest when soft metals such as copper and aluminium are used as the electrodes. If brass is chosen as the electrode, the breakdown voltage of LN₂ is almost not affected by negative the electrode material. Breakdown voltages of other cryogenes, such as LO₂ and LAr, are also related to electrode materials (Table 8.10).

The magnitude of the electrode surface area also has an influence on the breakdown voltage of cryogen, which decreases with increasing electrode area. Taking LN₂ as an example, the relationship of breakdown with electrode area is:

$$U_b = 21.2 - 6.8 \log S \quad (8.6)$$

where unit of breakdown voltage is kV and S is the electrode area. The effect of the electrode area on breakdown voltage reduces with a rise in pressure. Figure 8.10 shows dependence of breakdown voltage of LN₂ on electrode area at electrode distance $d = 0.5$ mm under different pressures.

Table 8.10 LO₂ and LAr breakdown voltage using different electrode materials

Electrode materials	LO ₂ /(kV/mm)	LAr/(kV/mm)
Stainless steel	238	140
Brass	144	101
Copper	181	140
Gold	124	116
Platinum	200	110

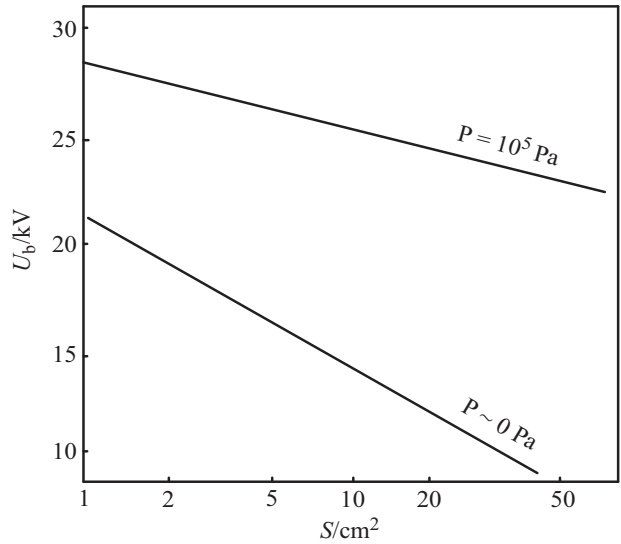


Figure 8.10 Relationship between LN₂ average breakdown voltage and electrode area.

In the measurement of breakdown voltage by using a needle-plate electrode, breakdown voltage can be related to either of the electrode polarities. Breakdown voltage measured by positive needle polarity is obviously higher than that measured by negative needle polarity. As for LHe, the highest value of the breakdown voltage ratio of positive to negative polarity may be up to 4 (Figure 8.11). There are also similar polarity effects in LN₂ as in LHe (Figure 8.12). This phenomenon is contrary to that in the transformer oil dielectric, in which breakdown voltage of negative needle polarity is higher than that of positive polarity.

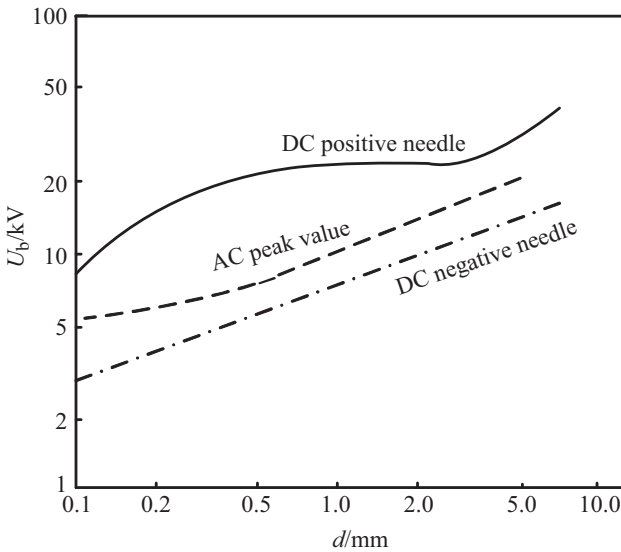


Figure 8.11 Positive and negative polarity breakdown voltages in LN₂.

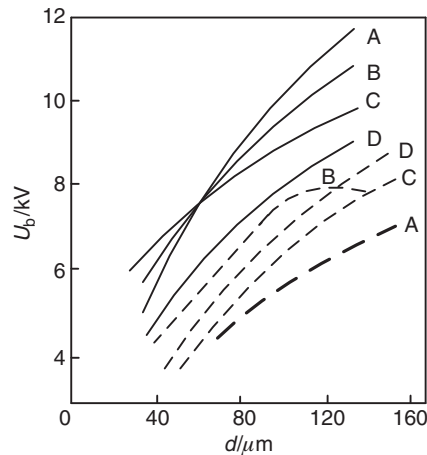


Figure 8.12 LN₂ positive and negative polarity breakdown voltages (solid line: positive polarity; dashed line: negative polarity; A: Pb; B: steel; C: brass; D: Cu).

In addition, voltage types also have great influence on the breakdown voltage of cryogenics. Figure 8.13 indicates the relationships of breakdown voltages with electrode distances for LHe and LN₂ in AC and impulse electric fields, respectively. The impulse breakdown voltage is much higher than AC breakdown voltage, because the action duration of the impulse voltage is too short to form bubbles in cryogenics. Of course, impulse breakdown voltage also relates to the uniformity of the electric field.

In a uniform electric field, impulse breakdown voltage and DC breakdown voltage are approximately the same. However, in a non-uniform electric field, DC breakdown voltage is lower than that in a uniform electric field, and the impulse breakdown voltage is higher than that in a uniform electric field. Figures 8.14 and 8.15 show impulse breakdown voltages of LHe and LN₂ without boiling, respectively. The reason is that the effective area of spherical electrode, especially the needle electrode, is much smaller than that of the plate electrode, so only a few bubbles form in the impulse process. Therefore, the impulse breakdown voltage in a non-uniform field increases obviously.

Previously, the cryogenic insulation properties of cryogenics were discussed when in their stationary situation. In practical applications of superconducting power apparatus, forced-flow circulation cooling is necessary (Section 9.4.4, Chapter 9) and cryogenics flow at a certain speed in circulation, so breakdown voltages can be affected. Taking LN₂ as an example, under the flowing conditions the average breakdown voltage is 22.88 kV, while the lowest and highest breakdown voltage is 16.25 kV and 27 kV, respectively.

However, average breakdown voltage is 17.90 kV in the stationary status, and the lowest and highest breakdown voltages are separately 11.75 kV and 21 kV. Obviously, breakdown voltage of flowing LN₂ in its flowing state is much higher than in its stationary state, which is similar with the phenomenon observed in common transformer oil. Flowing of the cryogen hinders electrically weak areas caused by electrical field concentrations from forming, which results in reducing cumulative probability of low breakdown voltage. The distribution of breakdown voltages obeys the normal distribution.

8.3 Insulating Properties of Organic Insulating Films

Cryogenic insulating materials are mainly classified into three categories, that is, polymer materials, composite materials and ceramics. In applications of superconducting power apparatus, insulating mate-

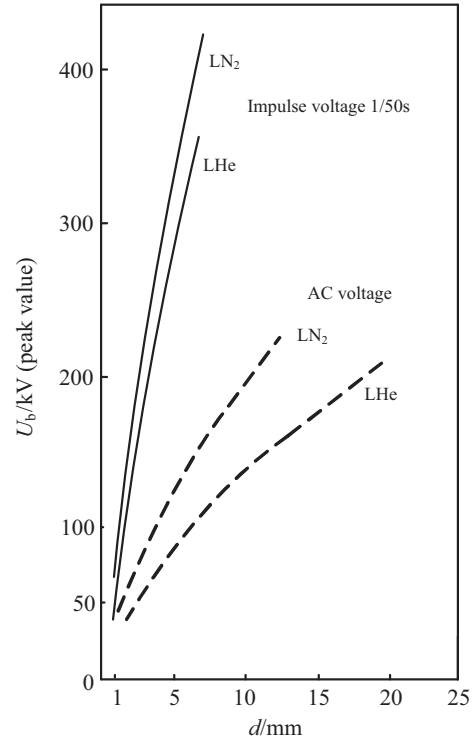


Figure 8.13 Curves of impulse and AC breakdown voltages versus electrode distances in LHe and LN_2 .

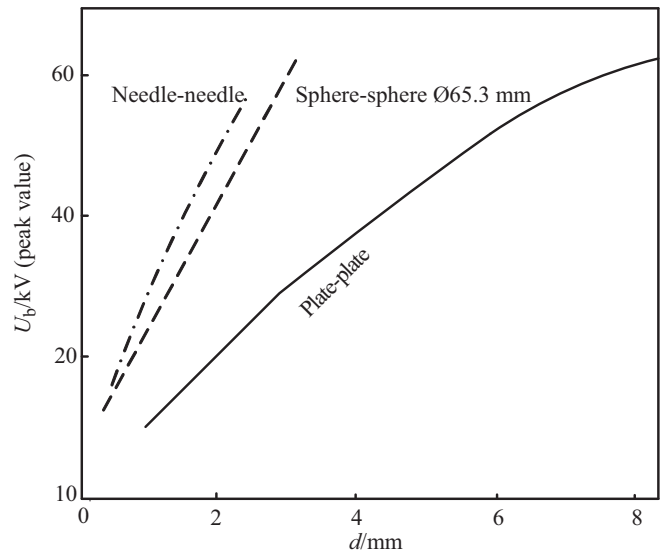


Figure 8.14 Impulse breakdown voltage and electrode distance in LHe without boiling ($500\text{ }\mu\text{s}$ positive half-wave pulse, $P = 0.9 \times 10^5\text{ Pa}$).

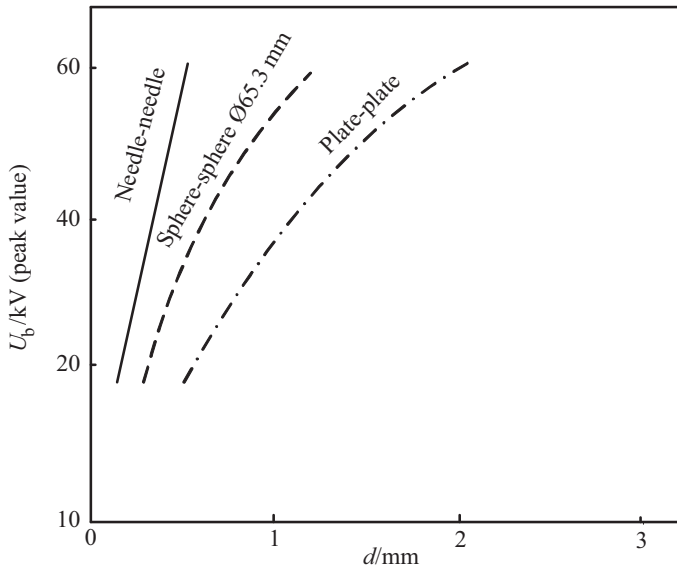


Figure 8.15 Impulse breakdown voltage versus electrode distance of non-boiling LN₂.

rials mainly include respectively polymer films, adhesives and structural materials, on which this section focuses. In order to reduce insulation occupation, we need insulation materials in film form, which have good mechanical and insulating properties [3, 4]. In order to reduce insulation occupation, these film insulating materials with good thermodynamic and insulating properties are needed. The film thickness and number of layers used in superconducting apparatus can be greatly reduced if they have high tensile strength, so the insulation occupation can be further reduced. Thin film is often used in insulations amongst turns or layers and in power cable for applications of superconducting electrical engineering. Currently, there are no special cryogenic insulation materials for superconducting apparatus, and the film insulation materials now used in cryogenics are basically chosen from conventional insulating films. In recent years, Sumitomo of Japan developed a new insulation film material, the so-called Polypropylene Laminated Paper (PPLP), which is fabricated by pressing porous pulp-paper and polypropylene film. Its geometrical structure resembles sandwiches in appearance, polypropylene film being inserted between two layers of kraft paper. PPLP is ideal for dipping and draining, so can effectively prevent gaps from forming and thus reduces the occurrence of PD. Since polypropylene film has high dielectric strength and good mechanical properties at cryogenic temperatures, the composite PPLP not only has good dipping and draining properties but also high dielectric strength. Therefore, PPLP is an appropriate film material for used in cryogenic insulation and is especially suitable for insulation of CD HTS cable.

8.3.1 Thermodynamic Properties of Organic Films

Since thermal contraction is one of the important parameters of cryogenic insulation materials, it should be first considered in the design of the insulation structure of superconducting electrical apparatus. Thermal contraction mostly occurs before the temperature reaches LN₂ temperature during the cooling

Table 8.11 Thermal contractions of several films at LN₂ temperature

Film names	Thickness/ μm	Change of length $\Delta L/\text{mm}$	Thermal contraction $\Delta L/L_0/\%$
Teflon	80	0.97	1.94
Polyimide	30	0.25	0.5
Polyester (Dupond)	50	0.07	0.14
Polyester (Dupond)	80	0.135	0.27
Polyester (Dupond)	190	0.175	0.35
Polyester (Russia)	50	0.195	0.39
Polypropylene	/	/	2.0
Teonex [®] PEN	/	/	0.3
Polyethylene	/	/	2.59
Glass cloth tape	100	0.03	0.06
Polyethylene with low density	/	/	1.6
Cross-linked polyethylene plastic	/	/	1.6
Ethylene propylene rubber	/	/	1.1

process, and thermal contraction is relatively small after the temperature is lower than LN₂ temperature [5]. It is only about 10% of that at higher than LN₂ temperature.

Table 8.11 shows thermal contractions of several film materials at LN₂ temperature. Thermal contractions and elongations of some films at temperature 4 K are listed in Table 8.12 [6]. When thermal contraction is slightly larger than elongation, the film easily cracks. If the elongation is more than 10 times that of thermal contraction, it seldom cracks.

Thermal conductivity of polymer is small, about 0.1% of that of conventional metals, and decreases with decreasing temperature. Its thermal conductivity at temperature 4 K is approximate 1 to 10% of that at room temperature. Similar to thermal conductivity, the specific heat of a polymer decreases with decreasing temperature. Table 8.13 gives the thermal conductivity and other physical parameters of several films at LN₂ temperature [7, 8].

At room temperature, polymer films have good elongation, but become hard and brittle at low temperatures. Especially below 77 K, the molecular chain of the polymer is frozen, so its elongation decreases, but tensile strength and elastic modulus grows, and brittleness also increases. Table 8.14 shows the mechanical properties of several films at different temperatures. Also mechanical properties of films at low temperature are influenced by other factors, such as water absorption, soaking time, numbers of

Table 8.12 Thermal contraction and elongation of films at 4 K

Materials	Thickness/ μm	Thermal contraction /%		Elongation /%	
		longitudinal	Transverse	longitudinal	Transverse
Polyethylene with low density	101.6	2.740	2.690	2.8	1.02
Polyethylene with high density	19.05	2.595	2.519	3.3	1.27
Polyamide (Nylon – 11)	38.1	1.924	1.852	3.1	1.61
Tensile polyethylene I with high density	38.1	1.759	1.173	3.1	1.76
Tensile polypropylene	31.75	1.755	0.255	2.3	1.31
Tensile polyethylene II with high density	101.6	1.706	1.255	3.1	1.80
Polysulfone (PSF)	127	1.160	1.070	3.0	2.58
Polyimide	127	0.484	0.458	5.8	12.0
Polyester	76.2	0.479	0.471	6.1	12.7
Polycarbonate	60.96	0.474	0.471	10.8	23.0

Table 8.13 Thermal conductivity k of several films at LN₂ temperature

Materials	Polyimide	Teflon	Polypropylene	Polyester	Teonex® PEN	Polyethylene
Thermal conductivity k [W/(m·K)]	0.15	0.22	/	0.9	0.27	/
Density γ [g/cm ³]	1.29 ~ 1.42	2.1	0.91 ~ 0.93	0.95	1.38	1.35
glass transition temperature T_g [K] ^a	472 ~ 670	399	255	/	340	/

^aGlass transition temperature T_g refers to the temperature when glass transition occurs, which is the characteristic temperature of polymers. It is the upper limit of temperatures used in amorphous thermoplastics and the lower limit of temperatures used in rubber.

thermal cycling and radiation. The stronger the water absorption is, the worse the mechanical properties are. Mechanical properties of most of films are not affected by soaking time, except polyamide and polypropylene. Thermal cycling has some influence on the mechanical properties of polyamide. When numbers of thermal cycles are more than 200, polyamide tends to become stabilized. Generally, tensile strength of polymer, particularly Teflon and polycarbonate, is easily and obviously affected by radiation, but polyamide and polystyrene have good radio resistance.

8.3.2 Resistivity of Organic Films

Because the motion of impurity ions becomes weak in low temperatures, resistivity of a polymer is higher than that at room temperature. Resistivity of most polymer materials is more than 10¹⁵ Ω·m at temperatures of 4.2 K and 77 K, except for Teflon. Resistivity of several polymers at different temperatures is listed in Table 8.15.

8.3.3 Permittivity of Organic Films

Permittivity of organic polymer materials is determined by the chemical structure, and the permittivity of polar polymer is higher than that of non-polar polymer. Permittivity of both polar polymer and non-polar polymer varies when temperature decreases. When the temperature decreases from room temperature to LHe temperature, relative permittivity of non-polar polymers is almost unchanged and remains in the range of 2 to 2.2. However, variation of permittivity of polar polymers with temperature is comparably complicated. Table 8.16 lists permittivity of several insulation films at low temperatures. Figure 8.16 shows dependences of permittivity of several common insulation films on temperature at a measurement frequency of 75 Hz. It indicates that the permittivity of six insulation films, which include polyethylene, polypropylene, Teflon, polystyrene, polyimide and organic silicone mica foil, slightly reduces with increasing temperature. Permittivity of other insulation film materials remains unchanged at temperatures below 100 K and increases with temperature rising above 100 K. In general, frequency has small influence on the permittivity of polymer, so is ignored.

8.3.4 Dielectric Loss

Dielectric loss of insulation materials is mainly caused by ionic conductivity and dipole orientation polarization. The motion of ions in materials weakens and the relaxation time increases at low temperature, so that dipole orientation cannot keep up with the change of the AC electric field. Therefore, dielectric loss of polymers at low temperatures is lower than that at room temperature. At an LHe temperature 4.2 K, dielectric loss tangents of non-polarity and polarity polymers are of the magnitude order of 10⁻⁶ and 10⁻⁴, respectively. Dielectric loss tangent of polarity polymer is about 10% of that at room temperature. Dielectric loss tangents of several kinds of thin films and papers commonly used in electrical engineering in room, LHe and LN₂ temperatures are listed in Table 8.17 [1].

Table 8.14 Mechanical properties of several films

Films	Thickness/mm	Elongation /%		Yield strength /MPa		Tensile strength /MPa		Elastic Modulus /MPa	
		temperature/K	temperature/K	temperature/K	temperature/K	temperature/K	temperature/K	temperature/K	temperature/K
Polyethylene with low density	0.076	293	77	4.2	293	77	4.2	293	77
Polyethylene with density	0.019	528	4.4	2.98	2	66	110	91	2344
Polyamide	0.040	322	3.60	3.13	6	102	109	724	3620
Polyimide adhesive tape	0.051	355	19.4	2.85	17	139	148	731	5757
Polysulfone (PSF)	0.051	71.2	5.91	3.39	22	112	199	1744	4302
Polyimide	0.051	63.6	19.8	10.9	39	99	128	1848	4447
Polycarbonate	0.076	55.2	20.1	5.78	43	94	152	2779	3654
Polyester	0.076	32.8	12.0	6.10	48	110	194	3378	4854
Teflon	/	114	5.75	10.75	64	168	187	3930	6633
Poly aryl amide	/	480	6.5	3.5	/	/	/	38.6 kPa	/
Polypropylene	/	21	3.5	3.0	/	/	/	29.6 kPa	0.1
Teonex® PEN	/	/	30 ~ 100	/	/	/	/	110 kPa	/
Cross-linked polyethylene plastic	/	/	44 ~ 66	/	85 ~ 120	/	/	102 kPa	/
Ethylene propylene rubber	/	/	7.1	/	110 ~ 180	/	/	100 ~ 140	/
			6.7	/	/	/	/	140 ~ 250	/
			/	/	/	/	/	140 ~ 180	/
			/	/	/	/	/	142	2270
			/	/	/	/	/	140	2430

Table 8.15 Resistivity of several polymers at different temperatures (unit: $\Omega\cdot\text{m}$)

Films	300 K	~ 80 K	~ 5 K
Polyimide	2×10^{13}	$>2 \times 10^{15}$	$>2 \times 10^{15}$
Polycarbonate	3×10^{13}	$>2 \times 10^{15}$	$>2 \times 10^{15}$
Polyester	2×10^{15}	$>2 \times 10^{15}$	2×10^{15}
Teflon	3×10^{13}	4×10^{12}	/
Polypropylene	$>2 \times 10^{13}$	$>2 \times 10^{15}$	/
Teonex [®] PEN	$>2 \times 10^{16}$	$>2 \times 10^{16}$	/
Polyethylene	$>10^{15}$	$10^{15} \sim 10^{16}$	/
Cable paper	1.39×10^{13}	$>3.0 \times 10^{13}$	/
Polyethylene plastic	/	2.97×10^{14}	/
Polypropylene Laminated paper	/	2.91×10^{14}	/

Temperature has a great influence on the dielectric loss tangent of some polymers. With temperature decrease, the dielectric loss tangent of many thermoplastic polymers increases around the glass transition temperature with decreasing temperature; dielectric loss tangent of other polymers tends to increase below 20 K. Figure 8.17 shows the relationship of the dielectric loss tangent of several common insulation films and papers changing with temperature. But the dielectric loss tangent of polyethylene amongst non-polarity polymers is small at low temperature. Peak value of the dielectric loss tangent appears around 100 K and the dielectric loss tangent is the lowest around 77 K. In addition, there is a small peak value of the dielectric loss tangent around helium temperature 4.2 K (Figure 8.18), which shows the relationship between dielectric loss tangent and frequency for polyethylene film with high density at and below 4.2 K.

Dielectric loss tangents of some non-polarity polymers, such as polypropylene and Teflon, are less than 10^{-5} at low temperatures. At low temperature 4.2 K, dielectric loss tangent of some polarity polymers,

Table 8.16 Permittivity of several insulation films at low temperatures

Films	Temperature/K	Thickness/mm	Permittivity
Polyamide (Nylon-11)	4.2	0.040	/
Polycarbonate	4.2	0.076	2.9
Polyester	4.2	0.076	2.5
Polyethylene with High density	4.2	0.019	2.3
Polyethylene with low density	4.2	0.076	2.3
Polyimide	4.2	0.051	3.1
Polyimide coated F-48	4.2	0.051	2.5
Polysulfone (PSF)	4.2	0.051	2.5
Teflon	77	/	2.0 ~ 2.2
Polypropylene	77	/	2.0 ~ 2.2
Teonex [®] PEN	77	/	2.9
Cross-linked polyethylene plastic	77	/	2.3
Ethylene propylene rubber	77	/	2.7
Aramid paper (Nomex)	77	/	3.1
Cellulose fiber paper	77	/	2.21
Cable paper	77	/	2.21
Cross-linked polyethylene	77	/	2.3
Polypropylene laminated paper	77	/	2.21
Biaxially orientated polypropylene laminated paper	/	/	2.6
Silicone rubber	300	/	2.6 ~ 3.1

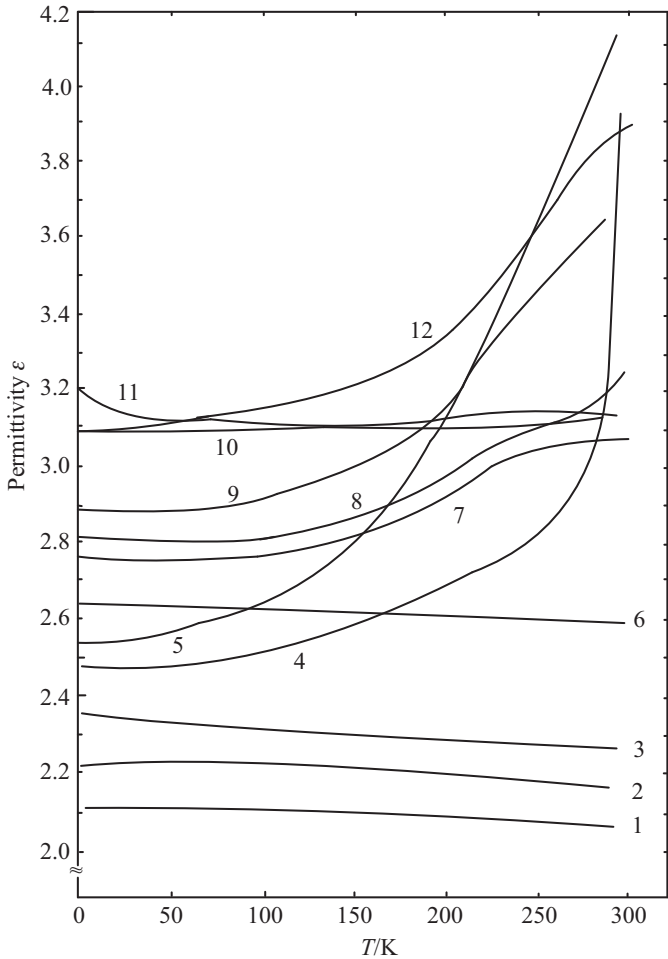


Figure 8.16 Permittivity versus temperature for several insulation films. 1. Teflon; 2. Polyethylene; 3. Polypropylene; 4. Chloroprene rubber; 5. Polyamide; 6. Polystyrene; 7. Polyester film (Mellinex); 8. Polyester film (Mylar); 9. cable paper; 10. Polyimide; 11. organic silicon mica foil; 12. aromatic polyamide paper.

for example polypropylene, is about 2×10^{-4} , which is approximately 100 times larger than that of polyethylene.

8.3.5 Breakdown Voltage

In the low temperature range, there are different breakdown properties amongst polymers. The breakdown voltage of polarity polymers rises with temperature decrease, but influence of temperature on breakdown voltage of non-polarity polymers is small. Table 8.18 shows the breakdown strength of several insulation films in LN_2 ; here the type of electrode used is a $\Phi 2.5$ sphere-plate electrode. Table 8.19 lists the breakdown voltage and withstand voltage of several commercial polymer insulating films fabricated by Chinese companies at LN_2 temperature and power frequency of 50 Hz.

Table 8.17 Dielectric loss tangents of several polymers

Films	4.2 K	77 K	293 K
Polyethylene film I (extrusion)	1.5×10^{-5}	1.5×10^{-5}	2×10^{-4}
Polyethylene film II (extrusion)	2×10^{-5}	4×10^{-5}	5×10^{-4}
Polyethylene paper	3×10^{-5}	2×10^{-4}	5×10^{-4}
Polypropylene paper	/	/	3×10^{-4}
Teflon	5×10^{-6}	7×10^{-6}	2×10^{-4}
Teflon- hexafluoropropylene	3×10^{-5}	8×10^{-5}	2×10^{-4}
Polyamide (Nylon-66)	/	/	1×10^{-2}
Polyamide (Nylon-11)	3×10^{-5}	3×10^{-5}	3×10^{-2}
Polyimide	5×10^{-5}	/	1×10^{-3}
aromatic polyamide (Nomex)	8×10^{-4}	1×10^{-3}	1×10^{-3}
Polycarbonate	1×10^{-4}	/	5×10^{-4}
polyester film	2×10^{-4}	3×10^{-4}	2×10^{-3}
Poly 2,6- two phenyl oxygen	/	/	1×10^{-4}
Polysulfone (PSF)	3×10^{-5}	/	8×10^{-4}
Cable paper @ 1 kHz	6×10^{-4}	2×10^{-3}	/
Cable paper @ 50 Hz	/	/	1.4×10^{-3}
Coated-F46-teflon	9×10^{-5}	/	/
polyether sulphone without tension	4.2×10^{-6} (800 V)	/	/
Teonex® PEN film	/	/	4×10^{-3}
Polyethylene plastics with low density	/	5×10^{-5}	$\sim 10^{-4}$
Cross-linked polyethylene plastic	/	$<7 \times 10^{-5}$	5×10^{-4}
Ethylene propylene rubber	/	3.5×10^{-4}	4×10^{-3}
Cellulose fibre paper	/	$<10^{-3}$	1.4×10^{-3}
Polypropylene laminated paper	/	$<10^{-4}$	8×10^{-4}
Biaxially orientated polypropylene laminated paper	/	7×10^{-4}	/
Silicone rubber	/	/	2×10^{-3}

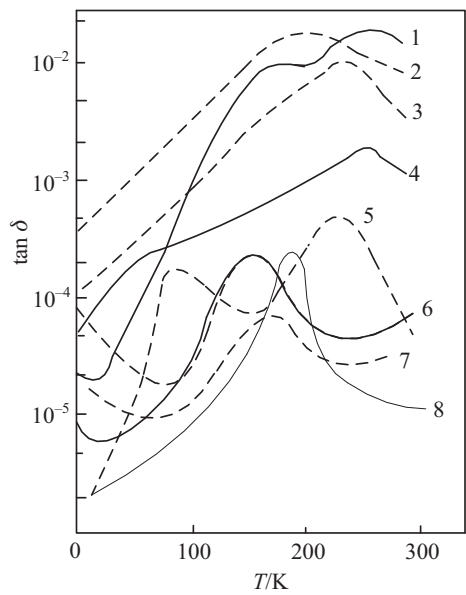


Figure 8.17 Relationship between dielectric loss angle tangent and temperature for several common films. 1. Polyamide; 2. insulation paper; 3. Polyester; 4. Polyimide; 5. Ethylene-propylene rubber; 6. Polyethylene paper; 7. high-density polyethylene; 8. Teflon.

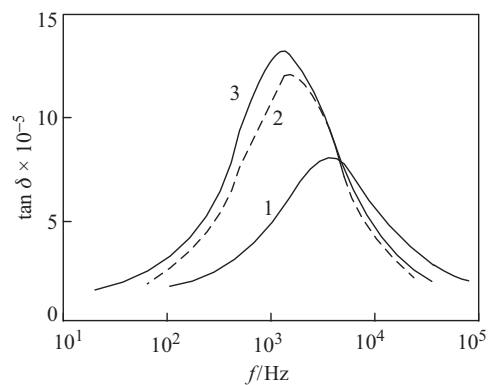


Figure 8.18 Relationship between dielectric loss tangent and frequency for polyethylene film with high density. 1. 4.2 K; 2. 2.06 K; 3. 1.73 K.

Table 8.18 Breakdown strength of several insulation films and papers in LN₂

Films	Thickness (mm)	AC breakdown strength @ 50 Hz		Impulse breakdown voltage (1/40 μs)		Ratio of impulse to AC
		U_b /kV	E_b /(kV/mm)	U_b /(kV)	E_b /(kV/mm)	
Polyethylene plastics with high density	0.05	14	280	17	340	1.2
	0.1	20	200	23	230	1.2
	9.2	25	125	33	165	1.3
Polyethylene plastics with low density	0.1	22	220	/	/	/
	0.15	27	180	/	/	/
	0.2	30	150	/	/	/
Polyethylene paper	0.125	6.3	50	7.5	60	1.2
	0.235	15.3	65	18	76	1.2
Polyamide	0.025	6.2	248	16	640	2.5
Aromatic polyamide paper	0.08	6	75	12	150	2.0
Polycarbonate	0.1	11	110	18	180	1.6
Polyester	0.1	10.3	103	16	160	1.5
Polysulfone (PSF)	0.1	16	160	22	220	1.4
Poly 2,6- two phenyl phenyl oxygen	0.12	20	167	35	290	1.7
Teflon	0.095	8.0	89	/	/	/
Polypropylene	/	/	>150	/	/	/
Teonex® PEN film	/	/	>210	/	/	/
Polypropylene laminated paper	/	/	103.78	/	217.6	2.1
Polyimide	/	/	199.13		329.5	1.65
Cross-linked polyethylene	/	/	35	/	/	/
Ethylene propylene rubber	/	/	29	/	/	/
aromatic polyamide paper (Nomex)	/	/	35	/	/	/
Cellulose paper	/	/	55	/	/	/
Biaxially orientated polypropylene laminated paper	/	/	50 ~ 55	/	/	/
Cable paper	/	/	66 ~ 77	/	113 ~ 138	1.8
Silicone rubber ^a	/	/	18	/	/	/

^a refers to room temperature.

Table 8.19 Breakdown strength and withstand strength in LN₂ at power frequency

Films	Thickness (μm)	Breakdown voltage U_b (kV)	1 min with stand strength with one minute E_b (kV/mm)	Breakdown strength E_b (kV/mm)
Eastern 6012 Polypropylene	15	1.869	97.3	124.60
Eastern 6021(SL-1) Polyester	100	8.87	74	88.7
Eastern 6050 Polyimide	25	5.21	147.2	208.4
Changzhou Polypropylene synthetic paper	160	15.67	83.3	97.94
Changzhou biaxial tensile polypropylene	40	7.02	104.2	175.5
Penglai Polyimide H65	50	7.44	123	148.8
Penglai Polyimide H67	2×25	5.9	98.4	118
Teda Tecpoly IH biaxially tensile PI films	25	/	140	180

Moreover, the cryogen has important influences on breakdown voltage of films. A dielectric discharge has been observed in experiments of AC and DC breakdown characteristics both in cryogens and in air.

The influence of cryogen discharge on breakdown voltage of insulation films is greater than that at low temperatures. Figure 8.19 gives peak values of AC breakdown strength for seven kinds of commonly used insulation thin films, with thickness 100 μm in different cryogens and in air at power frequency 50 Hz. However, the pressure of the cryogens also has an effect on the breakdown voltage of thin films. Especially, the influence of pressure on AC breakdown voltage is remarkable, but its effect on impulse breakdown voltage is relatively small. In order to improve the breakdown strength of the insulation system in engineering applications, the pressure of the cryogen is sometimes raised to prevent PD.

Breakdown voltage values vary when using different types of voltage for the same solid medium. Due to the short duration of the action of impulse voltage, there is not enough time to develop the sustained thermal effect, so breakdown always occurs in the form of voltage breakdown. Therefore, impulse breakdown voltage is higher than AC breakdown voltage, and the impact coefficient, namely the ratio of impulse to AC voltage, is larger than 1. Under conditions of DC voltage, the measured breakdown voltage is higher than that of its AC counterpart, because there is less heat generation and PD is relatively weak. Impact coefficients of non-polarity polymer films and synthetic papers are slightly larger than 1, but they are more than 2 for polarity polymer cable papers and films.

Because the probability of forming cracks and non-uniformity increases in thicker films, it is difficult for heat generated by dielectric loss in thicker films to be transferred to the outside. The radial depth of air gap around butt joints is so large that the cryogen occupies a large space in multilayer insulation structure wound by thick films, so the breakdown strength of thinner films is higher than thicker films in low temperatures. AC or impulse breakdown voltage in polymer films approximately obeys the exponential relation with its thickness:

$$U_b \propto d^n \quad (8.7)$$

where n is a constant relating with PD. The smaller the n value is, the greater influence of surface PD on instantaneous discharge of solid material. If $n = 1$, it is considered that there is no PD. Table 8.20 shows the n values of several polymer insulation films and papers. For different media, the degree of the breakdown strength decreasing with increase of insulation thickness is different. The breakdown strength of films decreases significantly at the temperature of LHe.

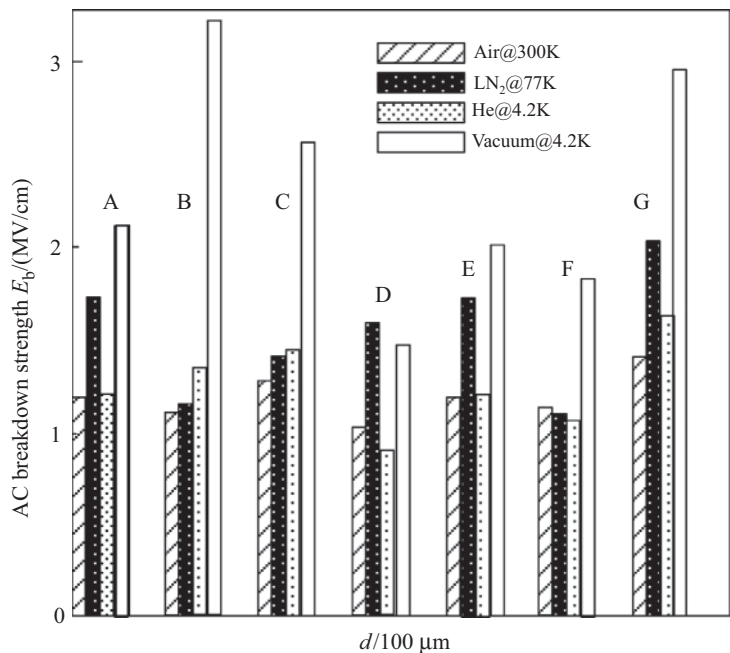


Figure 8.19 AC breakdown strength (peak) of several films in different cryogens. A. Polyester; B. PTFE; C. Teflon-hexafluoropropylene; D. Polyethylene; E. Polyamide; F. Polyvinyl chloride; G. Polyimide. (Reproduced with permission from Yinshun Wang, “Bases of Applied Superconductivity in Electrical Engineering,” Science Press, 237, © 2011.)

In addition, if an insulation layer of a certain thickness is wound by thin films, the relationship between breakdown voltage and thickness satisfies Equation (8.7), which is called the stacked effect. n values of several films are also listed in Table 8.20.

8.3.6 Electrical Ageing Characteristics

The electrical ageing phenomenon is one of the important characteristics of insulation materials, but there is generally no such phenomenon at low temperature because the temperature remains constant. A bubble is easily generated in the cryogen because its latent heat in low-temperature insulation materials is small; moreover, there are possible cracks in polymer materials because they contract and are brittle

Table 8.20 n values of several polymer insulation films and papers

Films	Thickness effects		Stacked effect	
	AC breakdown	Impulse breakdown	AC breakdown	Impulse breakdown
Polyethylene	0.40 ~ 0.43	0.47	0.43	0.56
Polyester	0.49	0.36	0.60	0.45
Polyethylene paper	0.90	1.0	0.72	0.90
Aromatic polyamide paper	0.58	0.53	0.53	0.66

Table 8.21 Inception PD voltages of several insulating films at 77 K

Films	Polypropylene laminated paper	Polyimide	Polyethylene with low density	Cable paper	Ethylene propylene rubber	Aromatic polyamide paper (Nomex)	Biaxially orientated polypropylene laminated paper
Inceptive PD voltages/kV	15 ~ 24	~ 14	~ 15	~ 13	20	15 ~ 24	20

at low temperatures, so that ageing problems of cryogenic insulation caused by PD and tree discharge resulting from bubble and crack must be considered.

PD refers to the phenomenon that local discharge occurs in part of the insulation materials when voltage is applied. Research of PD at room temperature is comparably mature, but it is not the case at low temperature. Inception PD voltage is an important parameter for evaluating the performance to withstand discharging. Inception PD voltage of insulation materials at low temperature depends on the pressure and temperature of cryogen, gap size and location, properties and thickness of thin films, etc. Generally speaking, increasing the pressure can hinder cryogens from forming bubbles whose breakdown strength increases under high pressure. Thus inception PD voltage increases and the maximum amount of discharge decreases at low temperature and high pressure.

Tree discharge is the partial breakdown phenomenon caused by electrode protrusion or cracks, micro-gaps or impurities in solid insulation materials. In general, inception tree discharge voltage at low temperatures is much higher than at room temperature. For example, an inception tree discharge voltage of polyethylene measured using a needle electrode at a temperature 77 K is 6 to 7 times higher than that at room temperature, which is almost equal to the AC inception tree discharge voltage, but the developing speed of the tree discharge is lower than at room temperature. Since polyethylene contracts during cooling, the needle closely contacts with the polyethylene and the injected space charge possibly reduces the concentration of the local field strength near the needle tip. Table 8.21 shows inception PD voltages of several insulating films at a temperature of 77 K and Table 8.22 lists inception tree discharge voltages of polyethylene at room temperature and 77 K when AC voltage, AC impulse and DC voltages are applied.

PD results in an increase of dielectric loss, with insulation ageing even breaking down and reducing the lifetime of insulation materials. Stability of insulation materials under conditions of long-term applied voltage is characterized by insulation lifetime, which is one of the important parameters in insulating design. At low temperature, the relationship of the voltage with the insulation lifetime approximately obeys the negative exponential law:

$$t = KU^{-n} \quad (8.8)$$

Table 8.22 Inception tree discharge voltages of polyethylene at room temperature and at 77 K

Temperature T/K	293	77
AC inception tree discharge voltages (RMS ^a)/kV	7	42
Impulse inception tree discharge voltages/kV	+25 -45	+45 -70
DC inception tree discharge voltages /kV	+35 -40	>+50 <-50

^aRMS denotes Root of Mean Square.

Table 8.23 Lifetime parameters of composite insulation tapes immersed in LN₂

Composition	Thickness/ μm	No. of layers	Artificial gap /mm	Pressure P/kPa	n
Calendering polyethylene paper	110	3	/	/	50
Calendering Polyethylene paper Core model	120	5	1	500	22
Cable paper	120 30	6 ~ 11 6 ~ 10	butt joint gap No or one layer with $\phi 12$ mm gap	/	56 50 ~ 80 without gap 25 with gap
Polyethylene film					
Polyethylene paper	100	4	One layer with $\phi 5$ mm gap	/	70
Poly 2.6- two phenyl phenyl oxygen Core model	157	6	1	/	80
Aromatic polyamide paper, core model	80	6	butt joint gap		100
Polyimide film, core model	50	6	1	/	11
Polyester film, core model	23 75 125	12 6 6	butt joint gap butt joint gap	/	14 16 18
Polyethylene film, core model	75 125	6 6	1 butt joint gap	/	17

where t is insulation lifetime; U denotes the applied voltage; and K refers to parameter relating with experimental conditions. Table 8.23 gives the n values of several kinds of composite insulating tapes, most of them in the range 10 to 90. Ageing time at low temperatures is longer and stability is higher than at room temperature.

In practical applications of cryogenic electrical engineering, various factors, such as operating temperature, pressure, insulation level, mechanical strength, physical properties, chemical stability, dielectric loss and costs, should be comprehensively considered and analysed in the choice of the insulation materials in order to determine the optimized insulation materials.

8.4 Cryogenic Insulating Paints and Cryogenic Adhesive

In superconducting apparatus, some components, such as leads or taps, sensors and thermometers must be bonded to certain locations of the apparatus. Most of normal adhesives crack at cryogenic temperature and so the bonded components may fall off. This section introduces several adhesives that are applicable at LHe temperature. Table 8.24 lists the main properties of several common cryogenic adhesives.

8.4.1 Epoxy Resin

Epoxy resin is the most commonly used cryogenic adhesive and can be used as a structural material. One of these resins is Stycast-1266, which has a high strength and is transparent with small viscosity at liquid

Table 8.24 Main properties of several cryogenic adhesives

Adhesives	Temperature range (K) ^a	Forms	Solidified process	Cost	Advantage	Disadvantage
Epoxy-Nylon	56 ~ 355	Film with Non-woven felt support	Contact pressure $\sim 1.72 \times 10^5$ Pa, temperature 150 ~ 180 °C, 1 h	high	High shear, peeling, strength	Sensitive to moisture
Epoxy-Phenolic aldehyde	56 ~ 533	Glass cloth support	Contact pressure $2.76 \sim 6.89 \times 10^5$ Pa, 160 ~ 180 °C, 40 min ~ 1 h	high	High strength, low creeping	Brittle, low peeling strength
Epoxy-Polyamide	56 ~ 338	Double component solution, casting film	Room temperature 3 ~ 5 days, 200 °C, 3 ~ 5 min	low	Solidification at room temperature	Sensitive to moisture, brittle at mid-temperature
Chloroprene rubber-Phenolic aldehyde	123 ~ 355	single component solution, film, tape	Contact pressure $1.72 \sim 34.47 \times 10^5$ Pa, 150 ~ 232 °C, 40 ~ 60 min	middle	Excellent antiknock and fatigue	Low glass strength
Polyurethane	56 ~ 333	Double component solution/solid	Contact pressure $0.345 \sim 34.47 \times 10^5$ Pa, Room temperature or 150 ~ 230 °C, 45 ~ 60 min	low	high glass strength	Sensitive to moisture before solification
Vinyl acetate-Phenolic aldehyde	197 ~ 473	Film, solution, powder with and without support	Contact pressure $\sim 6.89 \times 10^5$ Pa, 135 ~ 180 °C, 15 ~ 30 min	low	Convenient for using	Low peeling strength
Organo-silicone	56 ~ 368	Single/double component paste, pressure-sensitive tape	Room temperature, 15 minutes ~ 48 h	high	Good flexibility in cryogenic temperature, solidification at room temperature	Low strength

^aRefers to the temperature range when 50% strength at room temperature is retained.

state. It also bonds well with most metals and can solidify at room temperature [9, 10]. Meanwhile, it has disadvantages such as a high thermal expansion coefficient, which can contract up to 1% of volume from room temperature to LN₂ temperature. In addition, another common adhesive is Stycast-1269, which has lower dielectric loss comparing with Stycast-1266.

Stycast2850FT and Stycast2850GT have low thermal expansion coefficients, which are close to copper and brass, respectively. Both of the epoxy resins have high strength, can tightly combine with metals and can withstand 200 °C within a short time. At temperature 77 K, their breakdown strength can reach 100 kV/mm with permittivity of 2.96 and dielectric loss tangent of $1.5\text{--}2.0 \times 10^{-3}$.

8.4.2 GE7031 Varnish

GE7031 varnish is a yellow-brownish vinyl modified phenol resin that can withstand thermal expansion and cooling contraction, so that it is commonly used as a heat transfer adhesive in long-term cryogenic temperatures. For example, it is used in cementation between taps and heat sink made of a cooper block. It can occasionally be used in practical engineering that requires not only good heat conduction but also excellent insulation.

8.4.3 Polyvinyl Acetal Adhesive and other Cryogenic Adhesives

Polyvinyl acetal adhesive is one kind of cryogenic adhesive whose properties are similar to varnish GE7031, but anhydrous alcohol is used as its solvent. Other cryogenic adhesives include DW-4 and DW-3 produced by Shanghai Synthetic Resin Institute (China) and M-bond 43-B, 600, 610 produced by Company Vishay (USA), and their properties in cryogenic temperatures are also adequate.

8.5 Structural Materials for Cryogenic Insulation

Polymer materials (plastics), composite materials and ceramic materials can be used as low-temperature insulation structural materials. They are widely used in cryogenic structural materials of superconducting electrical apparatus, which include mechanical support components, cryostats, insulation paper, inter-turn and inter-layer insulations of superconducting windings, ground insulation, and component bonding and potting. Cryogenic insulation structural materials, with shapes of plates, rods, pipes, tubes, rings and other irregular components, are fabricated by coating adhesives on fibre clothes and then wrapped onto moulds with various shapes and directly laminating. Fibre cloth is made of glass fibre, while adhesives mainly include epoxy resin, polyurethane, phenolic resin, benzene silane, fluorine cluster and polystyrene-imidazole.

8.5.1 Polymer Materials

As one of polymer materials, fluorine cluster (polyimide, PTFE, polyurethane, polyethylene) is commonly used in sealing, piston ring and rotation bearing. Their electrical, thermal and mechanical properties at cryogenic temperatures have been described in Section 8.3. Nylon, belonging to polyamide plastic cluster with many varieties, is also widely used as a cryogenic insulation structural material. Nylon 6 with a melting temperature of 215 °C and Nylon 66 with melting temperature of 264 °C are fabricated in the shape of bars, and are suitable for support components of thermal insulation because their thermal conductivity at LN₂ temperature is 5 to 10 times lower than that of other materials. For example, both can be used as thermal insulation support pieces between the inner and outer walls of a metal Dewar tube and suspension of the hosting specimen.

Table 8.25 Main mechanical properties of epoxy resin produced by the Rutherford Appleton Laboratory (UK)

Trademarks	Thermal contraction 4 ~ 300 K	Fracture strength/GPa 4 ~ 77 K	Yield strength/MPa 4 ~ 77 K	Young modulus /GPa	Glass transition temperature/°C
RAL227	0.01	128	87	3.0	127
RAL229	0.012	116	63	2.9	94
RAL230	0.012	99	59	2.6	63
RAL231	—	72	43	2.0	46
RAL71A	0.012	—	—	2.9	43
RALHY918	0.011	139	114	3.4	125

8.5.2 Epoxy Resin Composites

Epoxy resins, or composites based on epoxy resin, are generally used in cryogenic insulation structural materials. In the application of electrical insulation, their electrical properties are mainly described by volumetric resistivity, breakdown strength, permittivity and dielectric loss tangent. Volumetric resistivity of epoxy resin is about $10^{16} \Omega\cdot\text{cm}$, while its breakdown strength and dielectric loss tangent are higher than 23 kV/mm and in the range of $4.9 \times 10^{-3} \sim 4.6 \times 10^{-2}$, respectively. Its relative permittivity is generally in the range of 3.7 to 5.1. Requirements on their mechanical properties are mainly high elastic modulus, high strength and high fracture toughness. Elastic modulus and strength of epoxy resin composites are mainly determined by mechanical properties of the reinforcement (fibre). The reinforcement can also adjust the thermal expansion coefficient of the composite. For example, thermal expansion coefficients of carbon fibre reinforced epoxy composites can be negative.

Currently, epoxy resin is mainly used in inter-turn and inter-layer insulations of superconducting windings, ground insulation, component bonding and potting, etc. Its parameters, such as solidifying temperature, flowability, mechanical properties, thermal expansion coefficient and thermal conductivity, play a key role in the performance of superconducting magnets. Therefore, appropriate formulations should be chosen according to different operating conditions. Table 8.25 shows the main mechanical properties of epoxy resin produced by the Rutherford Appleton Laboratory (UK).

Nowadays, the insulating structural materials used in large-scale LTS apparatus are manufactured by the process of impregnated epoxy resin with pressure. Epoxy resin composite is widely used in insulation structural material in cryogenic insulations. Epoxy composite reinforced by glass fibre, which is usually called FRP (fibre reinforced plastics), is an excellent cryogenic insulation material. It can keep good dielectric properties at high frequency and is not affected by electromagnetic action. At present, the cryostat used in the AC application of HTS electrical apparatus is generally made from FRP. Table 8.26 gives the main thermal and electrical properties of two typical FRP materials.

Table 8.26 Main thermal and electrical properties of FRP

Trademarks	Temperature T	Resistivity ρ	Breakdown strength E	Thermal conductivity k	Contraction ε	Fracture strength F
	K	$\Omega\cdot\text{m}$	kV/mm	W/(m·K)	%	MPa
G-10CR	298 ~ 77	$\sim 10^{14}$	~ 45	0.3 ~ 0.9	0.2 ~ 0.6	420
	77 ~ 4.2	$\sim 10^{17}$	~ 48	0.05 ~ 0.35	0.2 ~ 0.8	830
G-11CR	298 ~ 77	$\sim 10^{15}$	~ 48	0.3 ~ 0.7	0.2 ~ 0.6	460
	77 ~ 4.2	$\sim 10^{17}$	~ 48	0.06 ~ 0.32	0.2 ~ 0.6	900

Because laminated insulating materials are fabricated by coating adhesives on fibre cloth, which is then wrapped onto moulds and laminated, their thermodynamic and electrical properties present anisotropy. Particularly, thermal conductivity, specific heat capacity, contraction, tensile strength and elastic modulus show obvious anisotropy. Since the fatigue effect of laminated insulation materials is mainly determined by properties of adhesives, accumulated strain of most epoxy resins is lower than glass fibre, even at room temperature. This results in adhesive fracturing, even if the load is far lower than the limiting load of laminated products. It also causes fatigue in glass fibre when stress is lower than its limiting strength. In addition, radiation including ultraviolet radiation can also effect the lifetime of laminated products. There are some differences in properties of FRP materials due to adding different glass fibers in processing. For example, E glass fibre reinforced epoxy composites can be used as electrical insulation because of their low alkali content and high surface resistance. However, although S glass fibre reinforced epoxy composite has high strength and modulus, its cost is high. Kevlar fibre has the lowest density, but is much more expensive than E and S glass fibres.

With the development of superconducting electrical technology, new requirements on solid insulation materials are raised. For example, more attention is paid to boron fibers, whiskers and inorganic nanoparticle reinforced plastics, etc. There are some new commercial FRP insulation materials, such as G10CR and G11CR, which are US improvements based on G10 and G11. Other commercial FRP have also been developed in China, such as 6911, 252–650, 648–650, 618–650, 7101–650, etc.

8.6 Inorganic Insulating Materials

Commonly used inorganic insulation materials at cryogenic temperatures mainly include glass, quartz, ceramic and mica, in which glass is used as a reinforcement material in fibers, textiles, non-woven clothes, felts and microspheres, or is compounded from filament insulations.

8.6.1 Thermodynamic Properties of Glasses

The glasses commonly used in cryogenic temperatures include alkali-free glass, quartz glass and alkali glass. Table 8.27 lists some thermodynamic properties of glass at temperature 4.2 K. Thermal conductivity and specific heat capacity of glass reduces with decrease of temperature, and its thermal conductivity is almost independent of temperature in the temperature range of 5 to 20 K.

Permittivity of glass is approximately equal to 6, which is so high that its applications in electrical engineering are limited. However, permittivity depends on frequency and temperature. The loss peak of fused quartz glass appears at a frequency of 200 Hz. The loss is much higher at a frequency of 400 MHz. If the frequency is fixed, the dielectric loss tangent increases with rise of temperature, and the dielectric loss tangent shows a relative wide range of peak values. The relationship between the dielectric loss tangent of quartz glass and temperature at a frequency of 32 kHz is shown in Figure 8.20. A sharp peak of dielectric loss tangent in pure quartz glass appears at a temperature of about 40 K, and its dielectric loss tangent at other temperatures is approximate 10^{-4} . Table 8.28 gives permittivity and dielectric loss tangent of quartz glass at cryogenic temperature with different frequencies.

Table 8.27 Some thermodynamic properties of glass at 4.2 K

Materials	Thermal conductivity k W/m·K	Specific heat C J/(kg·K)	Thermal linear expansion α ($L-L_0$)/ $L_0 \times 10^{-4}$
Pyrex glass	0.1	23	48.5
Fused glass	50 ~ 400	25.1	-9.6

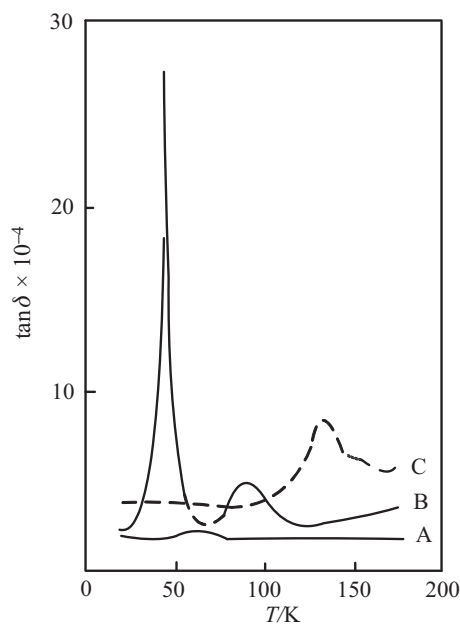


Figure 8.20 Dependences of dielectric loss tangent on temperature. A. pure quartz; B. quartz containing Na^+ ions; C. quartz containing K^+ ions.

8.6.2 Electrical Properties of Ceramics

Ceramics mainly include alumina oxide ceramics, manganese lead tantalate ceramics and high permittivity ceramics. Generally, the heat capacity of ceramic material is large but its thermal conductivity is small, so that ceramic material is an ideal thermal insulating material. In addition, permittivity of ceramic materials is usually high, which is much larger than that of conventional insulation materials. Permittivity of manganese lead tantalate ceramics and high permittivity ceramics is in the

Table 8.28 Permittivity and dielectric loss tangent of quartz at 4.2 K

Frequency f/MHz	34	90	136	232	284
$\tan \delta \times 10^{-4}$	2.0	1.5 ~ 2.0	2.2 ~ 3.1	1.8 ~ 3.0	1.0 ~ 2.0
ε			4.4		

Table 8.29 Thermal and electrical properties of ceramic materials at room temperature

Ceramics	Temperature T/K	Thermal expansion coefficient (10^{-6} K^{-1})	Thermal conductiv- ity/ k $[\text{W}/(\text{m}\cdot\text{K})]$	Resistivity $\rho \text{ } (\Omega\cdot\text{cm})$	Permittivity ε	Dielectric loss tangent $\tan \delta$
TiO_2	293	6.8	6	$\sim 10^9$	90	/
ZrO_2	293	10	1.7	10^6	16	/
Al_2O_3	293	7.2	17	10^{14}	9.4	4×10^{-4} @ (1 MHz)
BN	293	0.2 ~ 2.9	57	10^{14}	1.6	8×10^{-4} @ (1 MHz)
AlN	293	4.4	140 ~ 320	10^{14}	8.9	8×10^{-4} @ (1 MHz)

Table 8.30 Thermodynamic properties of several ceramic properties compared with FRP and SUS in the temperature range 4 ~ 77 K

Materials	Density $\gamma/(\text{g}/\text{cm}^3)$	Fracture strength σ/MPa	Thermal conductivity k [W/(m·K)]	$\sigma/(\gamma \cdot k)$ [(m·K·s)/kg] $\times 10^3$	Fracture toughness $K_{Ic}/(\text{MPa}\cdot\text{m}^{1/2})$
14.5Ce-ZrO ₂	5.77	730	0.6	211	12
16.5Ce-ZrO ₂	5.69	720	0.6	211	13
50CeZTA	4.62	710	0.6	256	6.5
FRP-T60	1.50	1050	0.15	4667	3 ~ 5
FRP-P	2.00	978	0.29	1686	3 ~ 5
304 SUS	8.03	490	5	12	300

range of 100 to 2000. Therefore, ceramic materials are often used in DC applications but are not suitable for AC applications. Table 8.29 lists thermodynamic and electrical parameters of ceramic materials at room temperature. Thermodynamic properties of several ceramic materials and their comparisons with FRP and 344 SUS in cryogenic temperatures are given in Table 8.30. The table indicates that thermal conductivity, density and fracture toughness of ceramics are smaller than SUS but their fracture strength is larger than SUS, while their thermal conductivity, density and fracture toughness are close to FRP and their fracture strength is smaller than FRP.

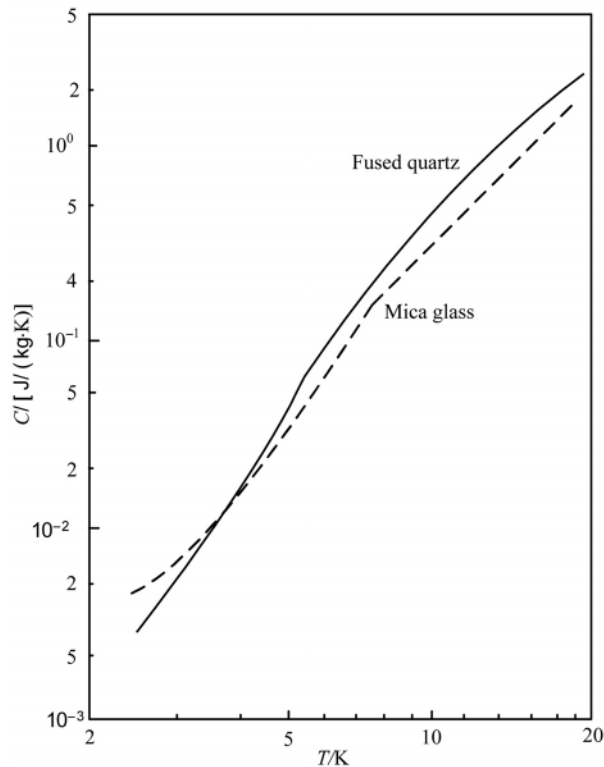


Figure 8.21 Relationship between specific heat capacity and temperature of mica glass and melt-condensed quartz at cryogenic temperature.

8.6.3 Thermodynamic and Electrical Properties of Mica Glass

Mica mainly refers to mica glass made from fluorophlogopite and boro-alumino-silicate glass under high temperatures and high pressures by controlling phlogopite crystallization. This material has small porosity and does not take in air. However, its chemical stability is high and its thermal expansion coefficient is similar to metals. Since mica possesses good thermal shock resistance and excellent DC and AC insulation properties, high-precision components can be made from mica by the common process used in fabricating metal components. Figure 8.21 presents the relationship between the specific heat capacity of mica glass and fused quartz and the cryogenic temperature. Specific heat capacity of mica glass is little different compared with that of fused quartz, which illustrates that specific heat capacity of mica glass is mainly determined by glass.

Thermal conductivity of mica glass increases with a rise of temperature. Figure 8.22 shows the relationship of thermal conductivity of mica glass and fused quartz with temperature. It is illustrated that the thermal conductivity of mica is proportional to the cube of temperature T , if the temperature is below 5 K.

Permittivity and dielectric loss tangent of mica glass relates with temperature, and both their dependences on temperature are shown Figure 8.23. In the temperature range of 2 K to 300 K, permittivity

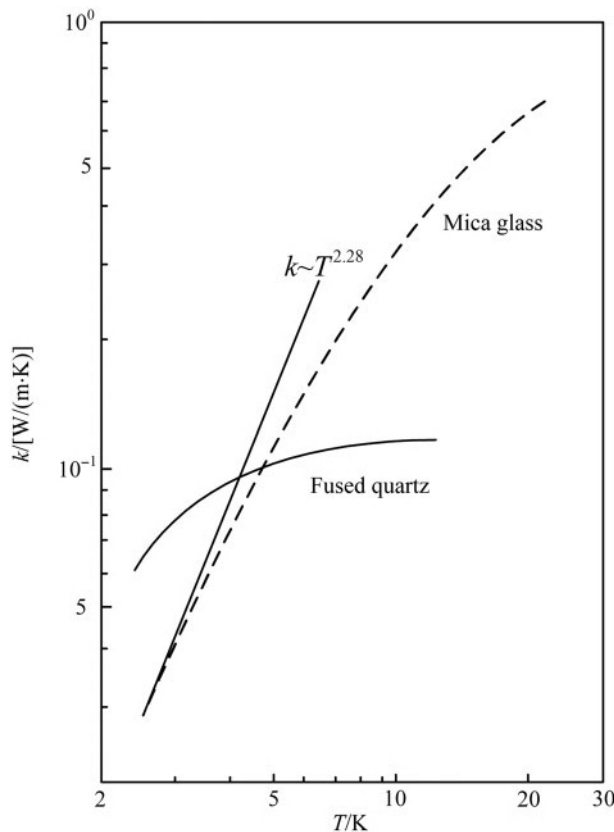


Figure 8.22 Relationship between thermal conductivity and temperature of mica glass and melt-condensed quartz at low temperature.

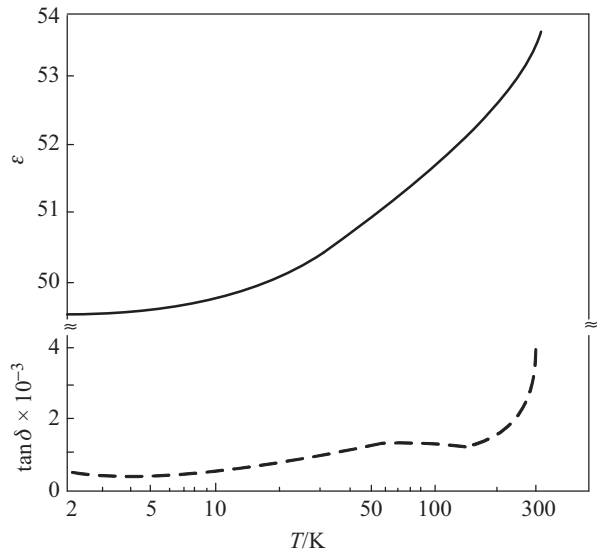


Figure 8.23 Permittivity and dielectric loss tangent of mica glass changing with temperature (sample thickness 0.33 mm).

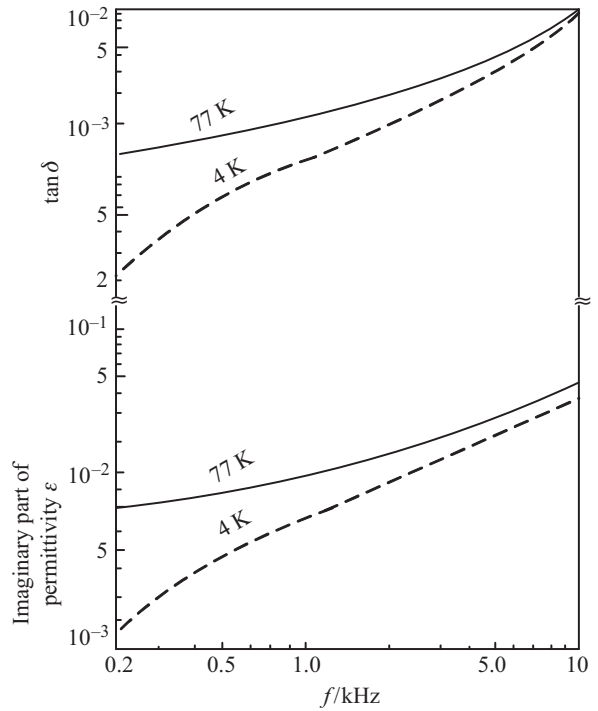


Figure 8.24 Dependences of imaginary part of permittivity and dielectric loss tangent of mica glass on frequency.

increases with a rise of temperature. Conversely, dependence of its dielectric loss tangent on temperature is relatively small in that temperature range.

The frequency also has an obvious influence on permittivity and the dielectric loss tangent of mica. Figure 8.24 shows the dependences of permittivity and dielectric loss tangent on frequency at temperatures of 4 K and 77 K. The dielectric loss tangent increases by about 10 times in the frequency range 0.2 kHz to 10 kHz at temperature 77 K, but increases by about two orders of magnitude in the same frequency range at temperature 4 K.

At room temperature, the DC breakdown strength of mica glass is about 1.45 MV/cm and is no lower than 2.24 MV/cm at temperature 4 K.

Besides the introduced three inorganic solid insulation materials, there are other inorganic solid insulation materials, such as glassy carbon, opal and microcrystalline materials. Because of their limited application at cryogenic temperatures, they are not introduced in this chapter.

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9

Refrigeration and Cryostats

Cryogenic devices provide a low temperature environment by transferring a cryogen or connecting with a cryogenic refrigerator. Cryogenic refrigeration technology means that some objects can be cooled in a certain time and space by artificial methods, to a certain temperature below that of their environment, thus their temperature can be kept at low values at which they will operate. Generally speaking, the cryogenic region refers to the temperature range of 4.2 K to 120 K. Cryostat and cryogenic refrigeration technology are widely applied in many fields, such as agriculture, industry, national defence, biology and medicine, energy, etc. With the rapid development of high temperature superconducting (HTS) electrical power apparatus, there are more strict requirements imposed on cryostat and refrigeration technology. Fabrication of cryostats and cryogenic refrigerators with reliable, efficient and convenient performance and low cost is one of the key technologies for their practical applications.

Besides meeting the requirement of voltage grade, HTS electrical power apparatus should also have large power capacity per volume as well as high reliability in their applications in the power system. Cryogenic technology relating with HTS electric power apparatus should have the following characteristics:

- (1) wide cryogenic temperature range; within the temperature range of 20 K to 77K;
- (2) large cooling capacity; cooling power of 100 through several thousand watts is demanded in a temperature range of 40 K to 80K;
- (3) high reliability of refrigerator and refrigerating system;
- (4) ability to withstand high voltage;
- (5) not influenced by magnetic fields;
- (6) well thermal and magnetic coupling with the superconductor; and
- (7) convenience in operating and maintaining the cryogenic system.

There are usually two refrigerating methods that are applicable in HTS electric power apparatus. One method is closed-circulating by connecting a refrigerator with the HTS apparatus through which the cryogen flows (including conduction-cooled without cryogen) and another is performed by directly immersing the apparatus into cryogens such as LHe or LN₂. In the former method, the cryogen is first cooled by refrigeration and the superconducting apparatus is cooled by the cold cryogen in the closed-circulating system. There is no refrigerating system involved in the latter method, and the superconducting apparatus is cooled by evaporation of the cryogen after it has been immersed in the cryogen. This chapter

will briefly introduce the cryostat as well as the technology of cryogenic refrigeration involved in HTS electrical power apparatus.

9.1 Cryogenics

The cryogen is mainly used as a cooling source for cryogenic devices. The boiling temperature of cryogenics is much lower than room temperature. The commonly used cryogenics include liquid helium (LHe), liquid nitrogen (LN₂), liquid hydrogen (LH₂), liquid neon (LNe) and liquid oxygen (LO₂), etc. Table 9.1 shows the main thermodynamic properties of various cryogenics. Since LH₂ and LO₂ are dangerous, flammable and can easily explode, LNe is rare and expensive, so that LHe and LN₂ are generally chosen as cryogenics in superconducting power applications. Furthermore, LHe is also rare and expensive, but LN₂ is rich in air of which it occupies 78%, and it is cheap and environmentally-friendly without any pollution, thus it is the optimized cryogen in HTS power applications. Currently, large-scale and industrial production of these cryogenics is realized.

He-3 and He-4 are two isotopes of the helium family. The content of He-4 in air is less than 0.005%, and the content of He-3 is even less. If a certain cryogen is used as the cooling source, its boiling temperature must be lower than the critical temperature of the superconductor. For example, the boiling temperature of LN₂ (77.3 K) is lower than the critical temperature of HTS materials, and the boiling temperature of LHe (4.2 K) is lower than the critical temperature of LTS materials, MgB₂ and iron-based superconductors, so that LN₂ and LHe can be used as cryogenics of HTS and LTS materials, respectively.

In superconducting power applications, a temperature lower than the boiling temperature of the cryogenics is sometimes required, which can be realized by direct refrigerator cooling or by decreasing the boiling temperature of the cryogenics by reducing their pressure, because their boiling temperature increases steadily with the increase of pressure. Figure 9.1 shows the variation relationship between the saturated vapour pressure of LHe and temperature. Despite the boiling temperature of He-3 being lower than He-4, He-3 is rare, consequently He-4 is generally adopted as the cryogen [1].

Figure 9.2 illustrates the relationship between saturated vapour pressures of LNe as well as LH₂ and temperature. These two cryogenics are often used in the temperature range of 20 K or so.

The dependences of saturated vapour pressure of LN₂ and LO₂ on temperature are shown in Figure 9.3 [2]. If the practical operating temperature required is slightly lower than the boiling temperature, by decreasing saturated pressure this lower temperature can be obtained. This is useful, especially for a strong magnetic field of an LTS magnet operating below 4.2 K and electrical applications of HTS apparatus operating below 77 K, respectively.

Table 9.1 Thermodynamic properties of several cryogenics

Parameters/units	He-3	He-4	H ₂	Ne	N ₂	O ₂
Molecular weight	3.016	4.0026	2.016	20.18	28.0134	32.0
Boiling temperature/K	3.1971	4.2221	20.3905	27.102	77.344	90.188
Triple point	/	/	13.951	24.551	63.146	54.361
Fluid density/(kg/m ³)	59.3	125	70.96	1207	808.6	1142
Vapor density/(kg/m ³)	8.16	16.89	1.311	9.499	4.614	4.4756
Latent heat/(kJ/kg)	7.52	20.413	445.6	87.03	198.64	212.3
Latent heat/(kJ/L)	0.51	2.552	31.62	105.04	160.62	242.4
Sensible heat (from boiling temperature to 300 K)/(kJ/kg)	/	1543	3510	282	234	194
0 °C @ 1 atm volume ratio of gas to liquid with same mass	443	700.3	789.9	1341	646.5	799.2

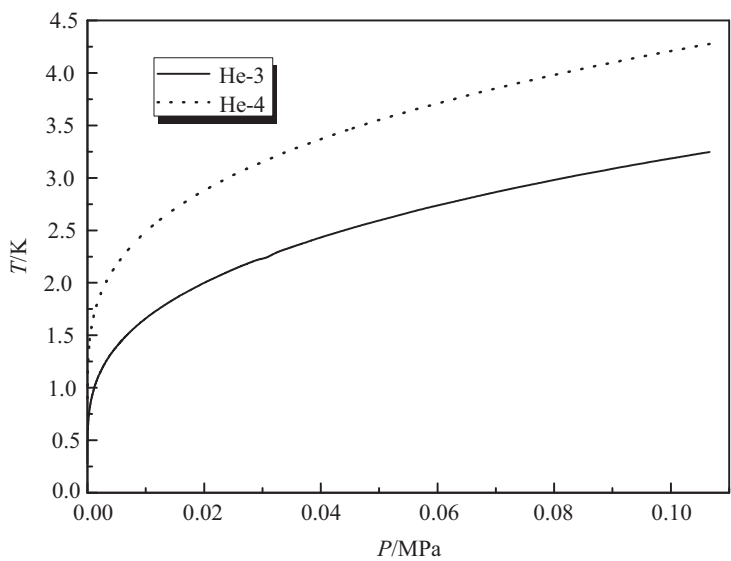


Figure 9.1 Change of saturated vapour pressure of liquid helium with temperature.

9.2 Cryostat

The cryostat, the so-called Dewar, was invented by the British scientist James Dewar. It is used to store cryogenes and is a thermal-insulation device consisting of double walls with a vacuum between them. The simple geometrical structure of the Dewar is illustrated in Figure 9.4. To understand the design principle of the cryostat, the basic heat transfer theory is briefly introduced in this section.

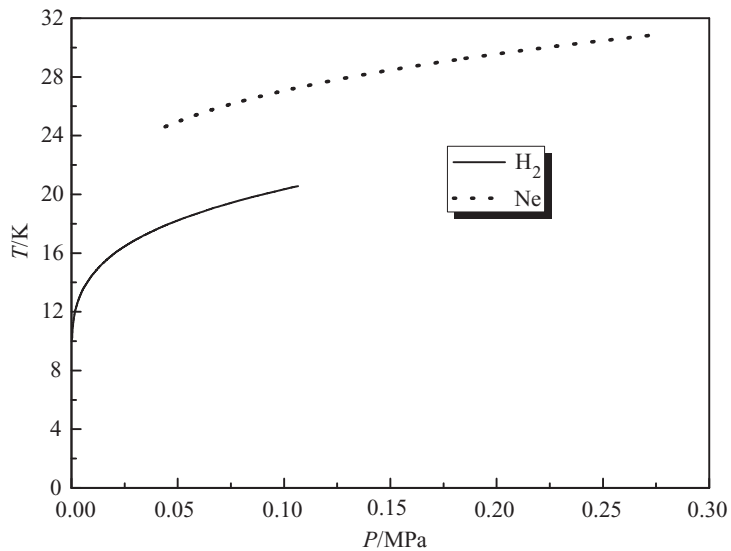


Figure 9.2 Dependences of saturated vapour pressure of LNe and LH_2 on temperature.

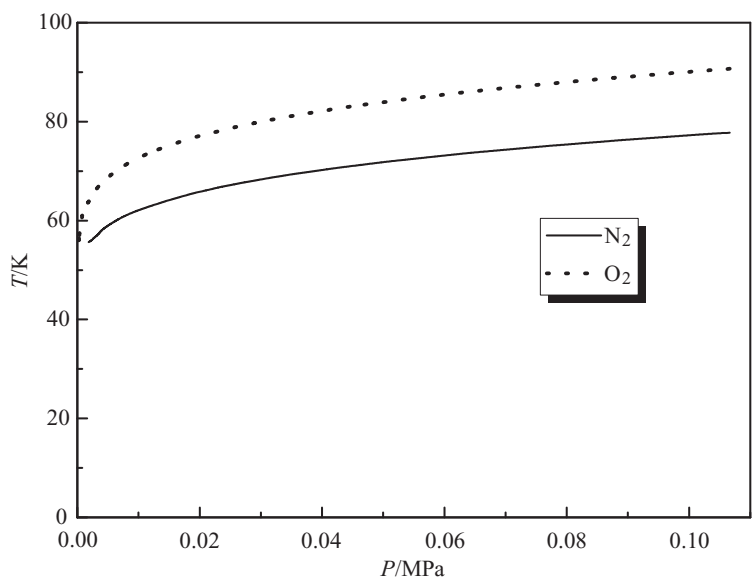


Figure 9.3 Relationship of saturated vapour pressure of LN₂ and LO₂ with temperature.

9.2.1 Cryogenic Thermal Insulation

There are three basic categories of heat transfer: conduction, convection and radiation. Conduction is realized by means of molecular agitation within a material without any motion of the material as a whole. Since heat transferred by conduction in a solid is proportional to its cross-section, it can be greatly decreased by reducing the cross-section of a solid. Convection results from mass motion of a

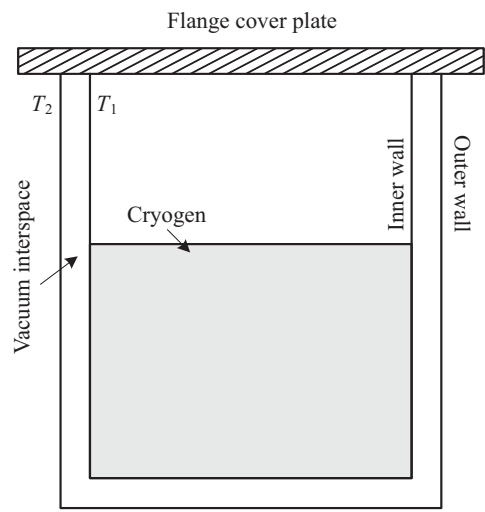


Figure 9.4 Structure of cryostat.

fluid (including gas and liquid, such as air or water) when the heated fluid is forced to move away from the source of heat, thus carrying energy away with it. The heat can be transferred automatically from regions with high temperature to those with low temperature by convection. Heat transfer by convection can be effectively reduced by increasing the degree of vacuum, that is, reducing the number of molecules in the gas. Radiation comes from emission of electromagnetic waves which carry energy away from the emitting object. As long as the temperature of any object is higher than 0 K, it will radiate heat outwards. A smooth surface such as a metal foil can reflect radiation back and then reduce and prevent heat leakage caused by radiation.

A traditional cryostat is made from stainless steel because of its high mechanical strength [3]. Its inner wall can be very thin so that conduction heat can be effectively reduced. Stainless steel has high density and less outgassing properties, so it can maintain a high degree of vacuum for a long time and then can effectively reduce convection heat. In addition, radiation heat can also be effectively prevented by wrapping smooth multilayer foils onto the inner wall as radiation shielding between the inner and outer walls of the cryostat. Finally, if a certain amount of activated carbon is placed between the double layers, it will absorb the residual gas molecules. Therefore, a cryostat made from stainless steel has many advantages such as high strength, maintaining vacuum for a long time, excellent thermal insulation performance, etc. For example, the vacuum degree of a stainless steel cryostat can be kept at 10^{-7} Pa for more than 2 years without any pumping.

9.2.1.1 Solid Conduction

Heat transfer by conduction obeys the Fourier theorem, namely the amount of solid heat conduction is proportional to the temperature gradient and its cross-sectional area perpendicular to the direction of heat conduction:

$$q = -k\nabla T \quad (9.1)$$

where the negative sign “-” means that direction of heat conduction is opposite to the temperature gradient; k is the thermal conductivity, which is usually the function of temperature; and q is the heat flux density in units of W/m^2 , namely heat flow per unit area. Table 9.2 shows the thermal conductivities of some alloys commonly used as cryogenic materials at different temperatures [4].

If the object is a cylinder with a round cross-section and has a radius far smaller than its length, heat conduction at steady-state can be treated as one-dimensional, and heat conduction of the cylinder (W) is

$$Q = \frac{A}{L} \int_{T_1}^{T_2} k(T) dT = \frac{A}{L} \int_{T_1}^{T_2} k(T) dT = \frac{A}{L} \left[\int_4^{T_2} k(T) dT - \int_4^{T_1} k(T) dT \right] \quad (9.2)$$

where A and L represent its cross-section and length, respectively, and T_1 and T_2 are temperatures at the two ends of cylinder, respectively. Sometimes thermal conductivity is given in the form of integrating over temperatures in order to facilitate its use, so that the amount of heat conduction can be directly obtained. $\int_4^T k(T) dT$ of some materials commonly used in cryogenic engineering are listed in Table 9.3.

9.2.1.2 Convection

Generally, convection of heat is realized by flowing of fluids, whether they are liquid or gas. Taking air as an example, convection on a hot surface occurs because hot air expands [5], then becomes less dense

Table 9.2 Thermal conductivities of some alloys and non-metals in units of W/(m·K)

Materials	Composition	2 K	4 K	6 K	10 K	20 K	40 K	80 K	150 K	300 K
Copper	99.999% annealing	/	1900	/	4600	/	1850	590	450	/
Silver	99.999% annealing	/	13000	/	15000	/	970	420	410	410
Aluminium	99.999% annealing	/	1850	/	3550	/	2100	/	/	/
Duralumin	Commodity	/	/	/	/	17	36	60	88	110
Aluminium alloy	94Al, 4.5Mg, 0.7Mn	/	3	/	8.2	/	34	65	80	120
Aluminium alloy	99Al, 0.6Mg, 0.4Si	/	35	/	87	/	270	230	200	200
Copper+Beryllium	90Cu, 10Be annealing	0.9	1.9	2.9	4.9	10.7	21.5	37.1	/	/
Copper+Nickel	90Cu, 10Ni	/	1.1	2.3	5.5	16	32	38	/	/
Copper+Nickel	70Cu, 30Ni	/	0.5	/	2.0	/	12	20	25	/
Gold cobalt	97.9Au, 2.1Co	/	1.0	/	4.0	/	13.5	20	/	/
Constantan	Cu60,Ni40i	/	0.8	/	3.5	/	14	18	20	23
Constantan	Commodity	/	0.84	1.6	3.5	8.8	13	18	20	23
Nickel silver	Commodity	/	0.7	1.3	2.8	7.4	13	17	18	22
Monel alloy	Annealing	/	0.86	1.5	3.0	7.0	12	18	20	24
Monel alloy	Cold drawing	/	0.5	0.81	1.8	4.4	8.4	14	18	22
Nickel	Annealing	/	0.45	0.80	1.7	4.1	8.0	11	13	14
Nickel	Cold drawing	/	0.27	0.47	0.93	2.3	5.0	9.2	12	14
Stainless steel	Commodity	/	0.25	0.4	0.7	2.0	4.6	8.0	11	15
Manganese copper nickel alloy	84Cu,12Mn, 4Ni	/	0.5	/	2.1	/	7.5	13	16	22
Brass	(57Cu,40Zn,3Pb) annealing	1.5	3.4	5.4	9.6	19.3	/	/	/	/
Brass	57Cu,40Zn,3Pb	1.3	2.9	4.6	8.2	17.5	33	53	90	100
Brass	70Cu, 30Zn	/	3.0	/	10	/	37.5	65	85	120
Wood alloy	Commodity	1.0	4.0	7.3	12	17	20	23	/	/
Silicon bronze	Commodity	/	/	/	/	3.4	6.9	14	/	30
Solder	60Sn,40Pb	5	16	26.5	42.5	56	52.5	52.5	50	50
Methyl acrylic acid	Commodity	/	0.056	/	0.062	/	0.10	11.6	/	/
Nylon	Commodity	/	0.0125	/	0.039	/	0.196	0.23	/	/
Teflon	Commodity	/	0.045	/	0.095	/	0.19	0.22	/	/
Varnish GE7031	/	/	0.063	/	0.082	/	/	47	101.5	170
Graphite	/	0.007	/	0.22	/	10.5	/	0.48	0.76	1.1
Heat resistance glass	Pyrex	/	/	/	/	/	/	900	/	/
Sapphire	Single crystal	/	100	/	1000	/	5900	0.48	0.80	1.4
Quartz	/	/	0.095	/	0.12	/	0.25	0.48	0.80	1.4

Table 9.3 $\int_4^T k(T)dT$ of some common cryogenic engineering materials (unit: W/m)

Temperature/K	Glass	Teflon	Nylon	Aluminium	Oxygen-free copper	Stainless steel		Monel alloy cold drawing [70Ni,30Cr]
						Annealing	Cold drawing	
6	0.211	0.113	0.0321	5.3	610	1.33	0.712	1.23
8	0.443	0.262	0.0807	12.9	1450	3.48	1.85	3.29
10	0.681	0.44	0.148	22.9	2520	6.52	3.45	6.29
15	1.31	0.985	0.410	59.4	6140	18.2	9.75	18.1
20	2.00	1.64	0.823	112	11 000	35.8	19.5	36.4
25	2.79	2.39	1.39	181	16 600	59.2	32.5	61.4
30	3.68	3.22	2.08	265	22 800	88.2	48.8	92.9
35	4.71	4.13	2.90	363	28 500	122	68.5	130
40	5.86	5.08	3.85	476	23 800	160	91.8	173
50	8.46	7.16	6.04	736	42 600	247	148	273
60	11.5	9.36	8.59	1040	49 600	345	215	285
70	15.1	11.6	11.3	1390	55 400	452	294	323
76	17.5	13.0	13.1	1620	58 600	519	347	342
80	19.4	13.9	14.2	1770	60 600	566	384	421
90	24	16.8	17.3	2200	65 400	685	484	686
100	29.2	18.7	20.4	2650	70 000	805	593	940
120	40.8	23.7	26.9	3650	78 800	1060	833	1260
140	54.2	28.7	33.6	4780	87 400	1310	1100	1590
160	69.4	33.8	40.5	6030	95 900	1570	1380	1950
180	85.8	39.0	47.5	7380	10 4000	1820	1680	2320
200	103.0	44.2	54.5	8830	112 000	2100	1990	2710
250	150	57.2	72.0	12800	132 000	2800	2810	3730
300	199	70.2	89.5	1710	152 000	3540	3690	4800

and rises. Convection obeys the following:

$$\rho C \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} \right) = \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + q \tag{9.3}$$

where q denotes heat source term; C and ρ are specific heat capacity at constant pressure and mass density of fluid, respectively; and u , v and w are the velocity components along the coordinate direction of x , y and z , respectively. If thermal conductivity k is isotropic and heating item q is absent, Equation (9.3) can be simplified as:

$$\rho C \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} \right) = k \nabla^2 T \tag{9.4}$$

If fluid is a static liquid, and u , v and w are zero, Equation (9.4) can be further simplified as:

$$\rho C \frac{\partial T}{\partial t} = k \nabla^2 T \tag{9.5}$$

Equation (9.5) is similar to Equation (4.21) introduced in Section 4.4. Specific heat capacity C_v at constant volume of a common cryogenic material is shown in Table 9.4 [6].

The purpose of using the cryostat is to keep thermal insulation from the environment and so reduce heat leakage to the minimum. Its geometrical structure is shown in Figure 9.4. There is a high vacuum

Table 9.4 Specific heat capacity C_v of some common cryogenic materials in units of J/(kg K)

Materials	Temperature T/K									
	2	4	6	10	20	40	80	100	200	300
Steel	0.028	0.091	0.23	0.86	7.7	60	205	254	350	386
Aluminium	0.108	0.201	0.50	1.4	8.9	77.5	357	481	797	902
Silver	0.0239	0.124	0.39	1.8	15.5	78	166	187	225	236
Gold	0.025	0.16	0.50	2.2	15.9	57.2	99.2	108.3	123.5	128.5
Platinum	0.074	0.186	0.37	1.12	7.4	38	88	100	127	133
Lead	0.09	0.7	3.0	13.7	53.1	94.4	114	118	125	130
Zinc	0.028	0.11	0.29	2.5	26	125	258	293	367	390
Tin (white)	0.048	0.245	1.27	8.1	40	106	173	189	214	222
Indium	0.141	0.95	3.59	15.5	60.8	141	193	203	225	233
Stainless steel	0.467	0.468	0.546	0.814	6	/	/	251	/	490
Constantan	0.23	0.42	0.73	1.69	6.8	/	190	238	/	/
Manganin	0.146	/	/	/	/	/	/	/	/	/
Solder [50Sn,50Pb]	/	/	/	/	/	101.2	146.8	155.0	173.1	182
Solder [40Sn,60Pb]	0.06	0.60	2.3	11.7	47.5	/	/	/	/	/
Wood Alloy	0.06	0.62	2.9	13.4	46.0	/	/	/	/	/
Pyrex glass	0.025	0.201	0.753	4.19	27.4	/	/	/	/	/
Quartz glass	0.02	0.18	0.71	4.0	24.4	/	218	268	525	738
Teflon	0.3	1.3	4.2	18	76	170	320	386	710	1010
Nylon	0.152	1.22	/	/	/	/	/	/	/	/
Polycrystalline graphite	0.024	0.13	0.35	1.2	6.3	/	/	/	/	/
Epoxy resin I	0.24	2.25	8.2	27.2	81.1	/	/	/	/	/
Epoxy resin 502	0.1999	2.22	/	/	/	/	/	/	/	/
Varnish GE7031	0.31	2.99	7.90	24.5	93	249	510	/	/	/
Surface adhesive 100A	0.237	2.52	/	/	/	/	/	/	/	/
Apiezon grease N	/	/	/	/	/	/	543	654	1177	3590
Apiezon grease T	0.228	2.22	7.24	25.5	94.8	251	521	636	1175	2160

in the space between the inner and outer walls, to reduce the convection of gas molecules. Taking the condition of rarefied gas into account, the relationship between gas convection (W) and vacuum degree (pressure) is:

$$Q = cA_1\alpha P (T_2 - T_1) \tag{9.6}$$

where α and P are the thermal accommodation coefficient and pressure of gas between the inner and outer walls of cryostat, respectively; A_1 denotes the outer wall surface area of cryostat, and T_1 and T_2 refer to temperatures of the inner and outer walls, respectively. Coefficient c (m/(s·K)) can be determined by:

$$c = \frac{\beta + 1}{\beta - 1} \sqrt{\frac{R}{8\pi MT}} \tag{9.7}$$

where $\beta = C_p/C_v$; β is called the specific heat ratio; C_p and C_v are the specific heat capacities at constant pressure and at constant volume of gas, respectively; R is the universal gas constant; and M and T refer to molar mass and temperature, respectively. At room temperature, $T = 300$ K, coefficients (c values) of several common gases with different temperatures are listed in Table 9.5.

At a room temperature of 300 K, the thermal accommodation coefficient α of air is 0.8 to 0.9; it becomes approximate to 1 at temperatures of 77 K and 20 K. Specific heat ratios of several common

Table 9.5 Coefficient c values of several gases

Gases	N ₂	O ₂	H ₂	He	Air	He
Ranges of temperature T_1 and T_2	≤400 K	≤300 K	300 ~ 77 K	300 ~ 4.2 K	<360 K	300 ~ 4.2 K
c /[m/(s·K)]	1.193	1.118	3.961	2.101	1.114	2.101

gases at standard atmospheric conditions are listed in Table 9.6. For example, the specific heat ratio of water vapour at 100 °C is 1.32. Usually, the specific heat ratio of gas depends on temperature. Table 9.7 shows the relationship between specific heat capacity at constant volume and temperature of several gases.

To fully describe thermodynamic properties of gases, thermal conductivities of several common gases at standard atmospheric pressure and 0 °C are listed in Table 9.8 [7].

In order to discuss the calculation of thermal accommodation coefficient α , two infinite slabs parallel to each other are considered (Figure 9.5). Both of their surface temperatures are T_1 and T_2 , respectively, $T_2 > T_1$. If a gas molecule with temperature T_2 collides with the cold surface at temperature T_1 , it transfers a certain amount of energy onto the cold surface. Since it is impossible for a gas molecule to stay on the cold surface for long enough to establish a new heat balance, it will leave the cold surface 1 with a temperature of T'_1 ($T'_1 > T_1$), taking away kinetic energy with the same temperature T'_1 . The gas molecule with temperature T'_1 goes through interlayer space and collides with the warm surface with temperature T_2 again, and then its kinetic energy is equivalent to T'_1 , which is higher than T_1 . Similarly, it is impossible for the gas molecule to stay on the warm surface with higher temperature T_2 for long enough to establish another new heat balance; it will leave the warm surface 2 with a temperature of T'_2 ($T'_2 < T_2$), which is equivalent to its kinetic energy. Trends of the degree of heat balance of the gas molecule colliding with surfaces can be described by thermal accommodation coefficient α :

$$\alpha = \frac{\text{practical transfer energy of gas molecule}}{\text{energy transfer with the maximum possibility}} = \frac{T_i - T_e}{T_i - T_w}$$

(9.8)

where T_i denotes the effective temperature of the incident gas molecule, T_e is effective temperature of the reflection gas molecule, and T_w refers to the temperature of the wall of the cryostat.

According to Equation (9.8) and referring to Figure 9.5, thermal accommodation coefficients of cold and warm surfaces are, respectively:

$$\alpha_1 = \frac{T'_2 - T'_1}{T'_2 - T_1}$$

(9.9)

$$\alpha_2 = \frac{T'_2 - T'_1}{T_2 - T'_1}$$

(9.10)

The temperature difference between cold and warm surfaces is:

$$T_2 - T_1 = \left(\frac{1}{\alpha_1} + \frac{1}{\alpha_2} - 1 \right) (T'_2 - T'_1) = \frac{T'_2 - T'_1}{\alpha}$$

(9.11)

Table 9.6 Specific heat ratios β of several common gases under standard conditions

Gases	Air	N ₂	O ₂	H ₂	Gas with single atom (He, Ne, Ar, etc)	CO ₂	CO
Specific heat ratio β	1.4034	1.405	1.398	1.408	1.67	1.302	1.404

Table 9.7 Relationship between specific heat capacity at constant volume and temperature

Gases	c_v /[J/(kg·K)]
Air	$717.756 \times (1 + 3.45 \times 10^{-5}T + 6.30 \times 10^{-8}T^2)$
N ₂	$735.621 \times (1 + 3.45 \times 10^{-5}T + 6.30 \times 10^{-8}T^2)$
O ₂	$644.349 \times (1 + 3.45 \times 10^{-5}T + 6.30 \times 10^{-8}T^2)$
H ₂	$9656.436 \times (1 + 1.5 \times 10^{-4}T)$
Gases with single atom (Ne, Ne, Ar, etc,)*	12.518/ <i>M</i>
CO ₂	$527.537 \times (1 + 8.11 \times 10^{-4}T - 1.84 \times 10^{-7}T^2)$
CO	$736.039 \times (1 + 3.45 \times 10^{-5}T + 6.30 \times 10^{-8}T^2)$
H ₂ O (100 °C)	$1603.544 \times (1 + 3.39 \times 10^{-4}T - 1.72 \times 10^{-7}T^2)$

**M* refers to molecular weights of gas with single atom.

where α is the comprehensive or effective thermal accommodation coefficient:

$$\alpha = \left(\frac{1}{\alpha_1} + \frac{1}{\alpha_2} - 1 \right)^{-1}$$

(9.12)

If inner and outer surfaces (Figure 9.5) are concentric tubes or a concentric spherical shell, the comprehensive thermal accommodation coefficient becomes:

$$\alpha = \left[\frac{1}{\alpha_1} + \frac{A_1}{A_2} \left(\frac{1}{\alpha_2} - 1 \right) \right]^{-1}$$

(9.13)

where A_1 and A_2 are cross-sectional areas enclosed by cold and warm surfaces, respectively. Although the comprehensive thermal accommodation coefficient is found by thermal accommodation coefficients α_1 and α_2 of cold and warm surfaces, both α_1 and α_2 cannot be obtained by calculation, but only obtained

Table 9.8 Thermal conductivities of common gases at standard atmospheric pressure and 0 °C

Gases	Molecular formula	Molecular weight	Thermal conductivity k /[W/(m·K)]
Hydrogen	H ₂	2.016	0.1742
Helium	He	4	0.1474
Methane	CH ₄	16.031	0.03019
Ammonia	NH ₃	17.031	0.02185
Neon	Ne	20.2	0.04551
Carbon monoxide	CO	28	0.02340
Ethylene	C ₂ H ₂	28.031	0.01704
Nitrogen	N ₂	28.016	0.02428
Air			0.02412
Nitric oxide	NO	30.008	0.02324
Oxygen	O ₂	32	0.02449
Argon	Ar	39.91	0.01662
Carbon dioxide	CO ₂	44	0.01474
Nitrous oxide	N ₂ O	44.016	0.01541
Chlorine	Cl ₂	70.916	0.007658
Krypton	Kr	82.9	0.008876
Xenon	Xe	130.2	0.005192
Freon	CCl ₂ F	137.4	0.00837

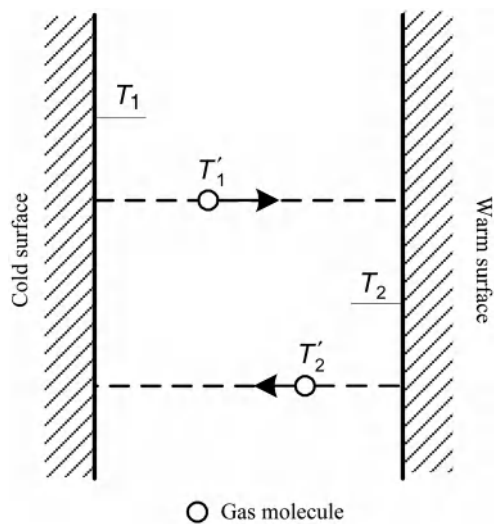


Figure 9.5 “Temperature” of free gas molecule between inner and outer surfaces.

by experimental measurements. They also relate with species of gases, temperature, species of surface materials and roughness of their surface, and are in the range of 0 to 1. If a gas molecule reflects with full diffusion, $\alpha = 1$; if it is a completely specular reflection, $\alpha = 0$. Table 9.9 shows thermal accommodation coefficients (α values) of several gases at different temperatures.

9.2.1.3 Radiation

Any object, if its temperature is higher than 0 K, will radiate heat outward in the form of thermal radiation. Different objects have different abilities of absorption and radiation. For the convenience of discussion, the concept of a black body is defined. If a body can fully absorb all radiant energy falling onto its surface, it is called an ideal black body or absolute black body and abbreviated to black body hereafter. Radiant energy density E_0 (W/m²) of the black body is proportional to absolute temperature T^4 , namely it follows Stefan–Boltzmann law:

$$E_0 = \sigma T^4 \tag{9.14}$$

Table 9.9 Thermal accommodation coefficients (α values) of several gases

Temperature /K	He	Ne	H ₂	Air
300	0.29	0.66	0.29	0.8 ~ 0.9
77	0.42	0.83	0.53	1.0
20	0.59	1.0	0.97	1.0
4	1.0	/	/	/

where $\sigma = 5.67 \times 10^{-8} \text{ W/(m}^2 \cdot \text{K}^4)$ is called as Stefan–Boltzmann constant. In practical engineering application, Equation (9.14) is usually written as:

$$E_0 = C_0 \left(\frac{T}{100} \right)^4 \quad (9.15)$$

where C_0 is defined as the radiation coefficient of an absolute black body, $C_0 = \sigma \times 10^8 \text{ W/(m}^2 \cdot \text{K}^4)$. Since a practical object cannot be a black body, it has three methods for exchanging energy with its surroundings, which include absorption, reflection and transmission. Thus, the practical object is more appropriately considered a grey body whose spectral radiation characteristics do not vary with radiation wavelength. The relationship between radiation energy density of a grey body and temperature is:

$$E = C \left(\frac{T}{100} \right)^4 \quad (9.16)$$

where C is the radiation coefficient of grey body and $C < C_0$.

The ratio of radiation energy of a grey body to a black body at the same temperature is defined as blackness ε :

$$\varepsilon = \frac{E}{E_0} = \frac{C \left(\frac{T}{100} \right)^4}{C_0 \left(\frac{T}{100} \right)^4} = \frac{C}{C_0} \quad (9.17)$$

It is important to determine the radiating capacity of a grey body in practical applications. If the blackness is known, the radiating capacity E of the practical object can be approximately found according to Equation (9.17):

$$E = \varepsilon E_0 = \varepsilon C_0 \left(\frac{T}{100} \right)^4 \quad (9.18)$$

Blackness ε is closely related to the material properties of the object, temperature and surface state (roughness and oxidation extent), which are usually experimentally determined. Table 9.10 lists experimental values of blackness ε of some materials commonly used at low temperature [4].

9.2.2 Basic Classification and Structure of Cryogenic Thermal Insulation

The objective of cryogenic thermal insulation is to reduce the heat transferring to cryogenic devices through conduction, convection and thermal radiation, etc. to as small an amount as possible, so that the cryogenic environment can be maintained for the normal operation of the cryogenic system.

Cryogenic thermal insulation is usually classified into two categories: cryogenic thermal insulations with and without vacuum, in which the latter is also called conventional or cumulated thermal insulations, that is, thermal insulation materials are directly accumulated or bonded outside the surface of devices used at a cryogenic temperature with a certain thickness. However, the former is realized by keeping some vacuum degree in space of the thermal insulation. It includes high vacuum degree thermal insulation, vacuum powder thermal insulation, vacuum multilayer thermal insulation, vacuum multilayer and multi-screen thermal insulation, etc. Figure 9.6 shows the basic structures of several cryogenic thermal insulations and Figure 9.7 illustrates the effective thermal conductivity ranges of various materials in different thermal insulation types and different vacuum degrees.

Table 9.10 Blackness (or emissivity) of some materials used at low temperature

Materials	Surface	ε		
		4 K	77 K	300 K
Copper	foil	/	0.017	0.03
	clean, polished	0.0062 ~ 0.015	0.015 ~ 0.019	0.78
	Severely oxidized	/	/	/
Stainless steel	/	/	0.048	0.08
Brass	clean, polished	0.018 @ 2 K	0.029	0.03
	Severely oxidized	/	/	0.60
Aluminium	clean, polished	0.011	0.018	0.030
	rough	/	/	0.055
	oxidation thickness 1 μm	/	/	0.300
	polyester film with coated aluminium on double sides	/	0.04	/
Silver	Bulk	/	0.0080	0.02
	plating	0.0044	0.0083	0.017
Gold	foil	/	0.01 ~ 0.023	0.02 ~ 0.03
	slab	/	0.026	/
	Copper or stainless steel	/	0.025 ~ 0.027	/
	plated with gold	/	0.02	/
	polyester film coats with gold plated onto double sides	/	/	/
Lead	foil	0.011	0.036	/
	bulk	0.012	0.036	0.05
	oxidized	/	/	0
Zinc	foil	/	0.02	/
	bulk	/	0.026	0.05
	Copper with plated zinc	0.027@20K	0.033	/
Tin	Foil	0.013@2K	0.038 @ 90 K	0.06
	Bulk	0.012	0.013	0.05
	Copper with plated Tin	/	/	0.02
	Glass with plated Tin	/	0.038	/
Solder	/	/	0.032	/
Glass	/	/	0.87 @ 90 K	0.94
Nickel	Foil	/	0.022	0.04
	Polished	/	0.022	0.045
	Polished iron with plated nickel	/	/	/
<i>chromium</i>	slab	/	0.08	0.08
	plated	/	0.08	0.08 ~ 0.26
Cast iron	Polished	/	/	0.21
Monel alloy	/	/	0.11	0.20
Quartz	/	/	/	0.93
Most of non metals	/	/	/	>0.8

Note: Blackness cited in Table 9.10 is measured in the normal orientation of materials. Average of a half-sphere is needed in practical calculations. If the surface of an object is rough, values in the table can be directly used. However, if the surface of a metal is polished, the values in the table should be multiplied by a correction factor of 1.2, in order to correct obvious changes along that orientation.

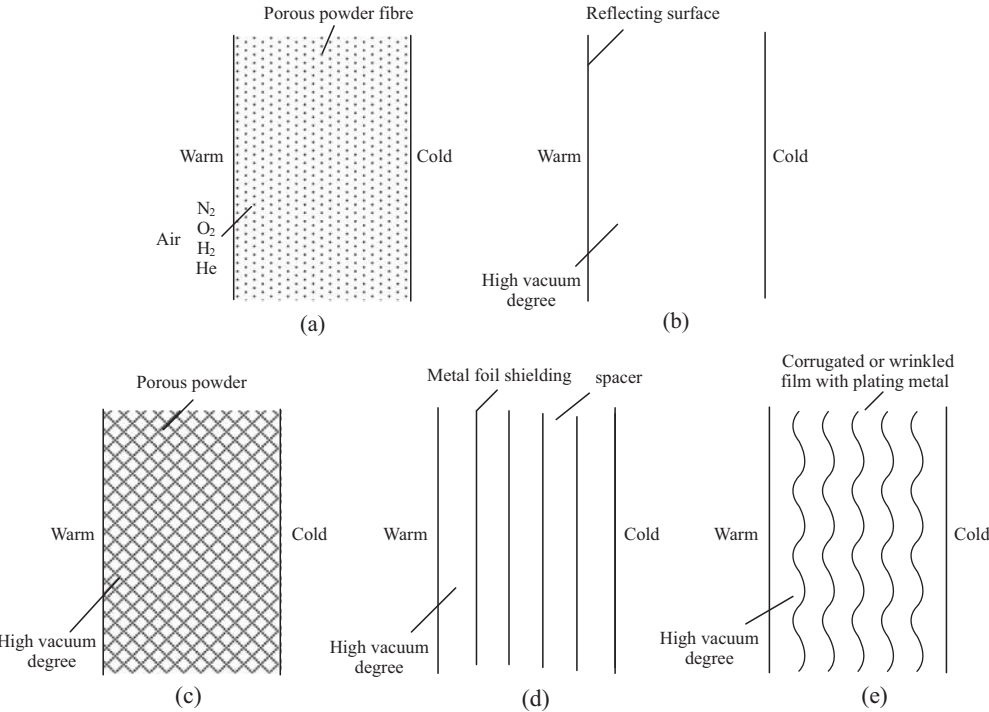


Figure 9.6 Basic forms of cryogenic insulation. (a) conventional thermal insulation, (b) high vacuum thermal insulation, (c) vacuum powder thermal insulation, (d) high vacuum thermal insulation with multi-layers and spacers, (e) high vacuum thermal insulation with multi-screens.

9.2.2.1 Conventional Insulation (Cumulated Insulation)

Conventional thermal insulation, made of materials with light weight and low thermal conductivity, belongs to one of the non-vacuum modes (Figure 9.6(a)). There is a lot of air contained in these materials. The thermal insulation materials are classified into three types of fibre, porous, powder and foam, in which fibrous thermal insulation materials include mineral wool, glass fibre, ceramic fibre,

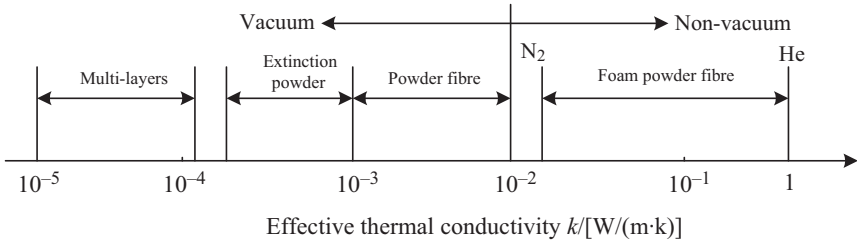


Figure 9.7 Effective thermal conductivities of several materials with different thermal insulation types.

etc.; porous powder materials include magnesium carbonate, diatomite, expanded vermiculite, expanded perlite and gas gel; and foam thermal insulation materials mainly include polyurethane foam, polystyrene foam, glass composition foam, microporous rubber, etc. Conduction in solids and convection of gas are main modes of heat transfer, which account for more than 90% of total heat transfers. These three kinds of thermal insulation materials are used in the interlayers of cryostats or cryogenic devices to reduce the quantity of thermal conduction in the solid.

In order to prevent gas in thermal insulation materials from condensing and then resulting in degradation of thermal insulation performance, some gases, such as nitrogen, hydrogen or helium, whose condensing temperatures are below the temperature of the inner wall of the cryostat, are usually filled into the interlayers of cryostats. Thermal conductivities of several commonly used insulation materials in environments of various gases are shown in Table 9.11. If the cryogenic temperature is higher than the LN₂ temperature of 77 K, nitrogen gas is often chosen as the filling gas. Average thermal conductivities of several thermal insulation materials within the temperature range of room temperature through 77 K surrounded by nitrogen gas are shown in Table 9.12. Table 9.13 lists thermal conductivities of several commonly used porous powder materials in the temperature range of 77 K to 310 K.

This thermal insulation structure has advantages of simple processing technology and low cost, but also has serious disadvantages of high heat leakage and too thick insulation layers. Generally, this thermal insulation mode is rarely chosen if the temperature is lower than the LN₂ temperature of 77 K.

9.2.2.2 High Vacuum Thermal Insulation

High vacuum insulation is realized simply by vacuum insulation, whose structure is shown in Figure 9.6(b). There is not any materials filling the cryostat between the inner and outer walls, in which the degree of vacuum needs to be pumped below 10⁻³ Pa to obtain an ideal thermal insulation effect. Figure 9.8 presents dependences of thermal conductivity of residual gases in an interlayer of the cryostat on the degree of vacuum in high vacuum thermal insulation mode.

Radiation is the main heat leakage source in the high vacuum thermal insulation mode in which heat transfer by radiation is:

$$Q = \frac{\sigma A_1 (T_2^4 - T_1^4)}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2} \left(\frac{1}{\varepsilon_2} - 1 \right)}$$

(9.19)

Table 9.11 Thermal conductivities of several thermal insulation materials in various gas environments

Materials	Density γ (kg/m ³)	Average temperature $T(K)$	Thermal conductivities within environment of filling gases k [W/(m·K)]					
			Kr	CO ₂	Air	N ₂	He	H ₂
Perlite	130	188	/	/	/	0.0325	0.126	0.145
Gas gel	100	188	/	/	/	0.0196	0.062	0.080
Silica gel	93	190	/	/	0.0299	/	0.116	/
Microporous rubber	56	190	0.0102	/	0.0215	/	0.122	/
Mineral wool	150	190	0.0142	/	0.0313	/	0.136	/
Glass wool with diameter 2.58 μm	74	338	/	0.0255	0.0336	/	0.181	/
Glass wool with diameter 0.69 μm	174	338	/	0.0259	0.0356	/	0.126	0.198

Table 9.12 Average thermal conductivities of several thermal insulation materials surrounded by nitrogen (temperature range 77 to 298 K)

Materials	Density $\gamma/(\text{kg/m}^3)$	Particle size (mesh)	Average thermal	
			conductivity $k/[\text{W}/(\text{m}\cdot\text{K})]$	Processes
Magnesium carbonate	160	Powder	0.0456	moisture content 6.7%
Vapour-phase gum	110	/	0.0252	Dry by heat
Silicone gel	446	30 ~ 100	0.0688	Dry by heat
602 Gas chromatography silica gel	290	80 ~ 160	0.0372	/
601 Gas chromatography silica gel	150	Powder	0.0285	/
Cork wood	/	/	0.0690	/
Vermiculite	/	100	0.0621	/
Slag wool	188	flocculence	0.0375	/
Silk floss	145	/	0.0427	/
Glass wool with diameter 15 μm	95	/	0.0421	/
Glass wool with diameter 3 μm	70	/	0.0241	/
Quartz fibre	40 ~ 50	Pellet	0.0329	Dry by heat
Polystyrene	30	bulk	0.0351	/
Casting polyurethane	/	bulk	0.0245	/
Spray polyurethane	/	/	0.0175	/
CO ₂ polyurethane	60	/	0.0265	/
Semi-rigid polyurethane vinyl	214	/	0.0342	/
F-11 polyurethane	40	/	0.0893	/
Urea formaldehyde	30	Snow shaped	0.0266	/

where ε_1 and ε_2 are thermal radiation emissivity (blackness) of inner (cold) and outer (warm) walls in the vacuum interlayer, respectively; T_1 and T_2 are the temperatures of the inner and outer walls, respectively; and A_1 and A_2 correspond to their surface areas.

In order to reduce radiation, the inner and outer walls are usually made from materials with low emissivity, such as copper and aluminium, with polished surfaces to maintain a higher degree of finishing and to increase reflectivity.

High vacuum thermal insulation has advantages of simple structure, compaction, low specific heat capacity and simple manufacturing processes, but its reduction of heat leakage by radiation is limited. In view of long-term engineering applications, this thermal insulation mode has some disadvantages such as higher heat leakage and high frequency of maintenance, which limit its wider applications. An exception

Table 9.13 Thermal conductivities of several porous powder materials (temperature range 77 K to 310 K)

Materials	Density $\gamma/[\text{kg/m}^3]$	Particle size (mesh)	Thermal conductivity $k/[\text{W}/(\text{m}\cdot\text{K})]$	
			Atmospheric pressure	1.33 Pa
High pressure gas gel	104	40 ~ 80	0.0151	0.00159
	124		0.0154	0.00131
Atmospheric gas gel	120	powder	0.0267	0.00143
	170		0.0267	0.00121
Vermiculite	290	40 ~ 80	0.0544	0.00151
	300	80 ~ 100	0.0534	0.00108
Vapor-phase gel	290	80 ~ 120	0.0300	0.00111

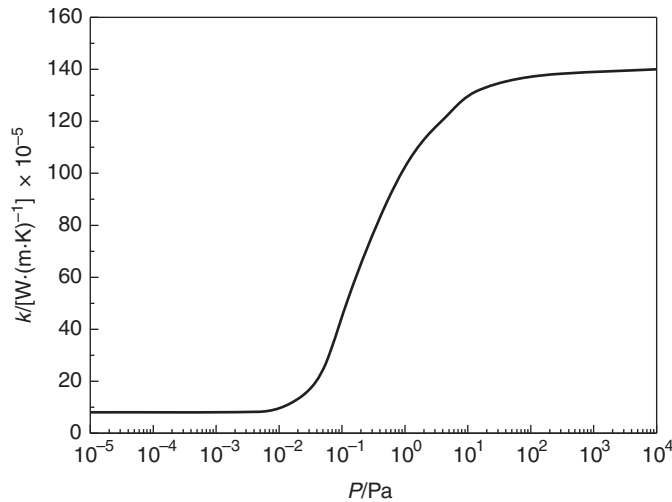


Figure 9.8 Relationship of thermal conductivity of residual gases with degree of vacuum.

is that this method is often used in cryostats made of glass, which is suitable for AC experiments at cryogenic temperatures.

9.2.2.3 Vacuum Porous Thermal Insulation

This thermal insulation method is realized by filling porous insulation powder and fibre material into the interlayer between the inner and outer walls, in which its degree of vacuum is pumped to 1 to 10 Pa. Its structure is shown in Figure 9.6(c). The commonly used porous insulation materials include aerogel, vermiculite, pearlite sand, microspheres, etc. The relationship of thermal conductivity of insulation powders and fibre with residual gas pressure (vacuum degree) in the interlayer is illustrated in Figure 9.9. When pressure of the insulation interlayer is higher than 1.33×10^3 Pa, thermal conductivities of insulation powders and fibre materials change very little. However, if the pressure further decreases, its thermal conductivity linearly reduces with pressure. When the pressure continuously decreases to, for example, lower than 1 Pa, thermal conductivity is almost unchanged. At this stage, heat transfer by convection of gases is smaller than radiation and conduction. Radiation and conduction dominate because pressure has an important influence on the thermal conductivities of insulation powder and fibre. Thus, this insulation method is easily realized due to its low requirement on the degree of vacuum.

Heat conductions of vacuum insulation powder and fibre are determined by:

$$Q = \frac{kA_m (T_2 - T_1)}{\delta} \quad (9.20)$$

where k is the thermal conductivity of thermal insulation materials, A_m denotes the average heat transfer area of the heat insulator, T_2 and T_1 are surface temperatures of warm and cold walls, respectively, and δ refers to the thickness of the thermal insulation layer.

If the shape of the walls is a concentric cylindrical shell, its surface area is:

$$A_m = \frac{A_2 - A_1}{\ln \left(\frac{A_2}{A_1} \right)} \quad (9.21)$$

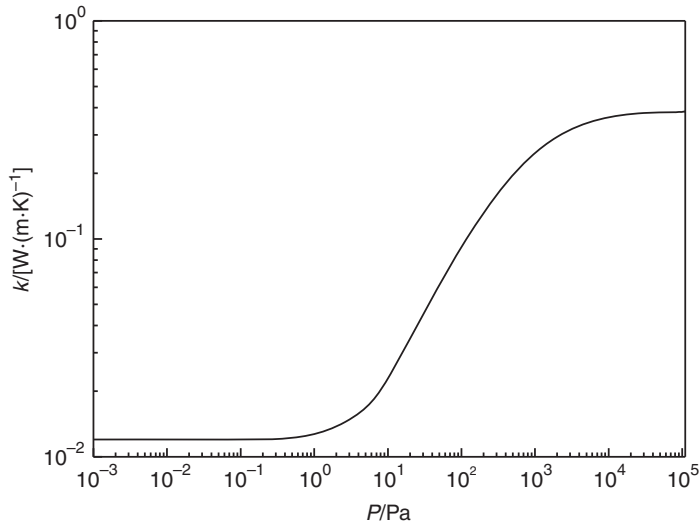


Figure 9.9 Relationship of thermal conductivity of powders with degree of vacuum.

If the shape of walls is a concentric spherical shell, its surface area is:

$$A_m = \sqrt{A_1 A_2} \quad (9.22)$$

Table 9.14 lists thermal conductivities of several commonly used powder and fibre porous insulation materials in different ranges of temperature and degree of vacuum [8].

Although the mode of vacuum porous thermal insulation powder and fibre has low requirements on vacuum, their thermal insulation performance is two orders of magnitude higher than the conventional insulation method and one order of magnitude higher than that of the high vacuum insulation method, respectively. It has many disadvantages, such as a large distance of vacuum interlayer between the inner and outer walls, large insulation space and complicated and bulky structure, which greatly limits its applications in practical engineering.

9.2.2.4 High Vacuum Multilayer Insulation

As shown in Figures 9.6(d) and (e), high vacuum multilayer insulation is realized by wrapping multilayers of foil with spacers or corrugated or wrinkled metal-plated plastic films without spacers in between the insulation interlayer in order to greatly reduce radiation heat leakage, and at the same time maintaining a high vacuum degree so as to reduce the heat transfer by convection. Figure 9.10 shows high vacuum multilayer insulation, in which n radiation shielding screens are installed in the space between cold and warm walls with temperatures T_1 and T_2 ($T_1 < T_2$), respectively, A_1 and A_2 denotes the surface areas of cold and warm walls, respectively, $A_{s1}, A_{s2}, \dots, A_{sn}$ refer to surface areas of n radiation shielding screens, $Q_1, Q_2, \dots, Q_n$ are radiation heat flow fluxes of corresponding layers, ε_1 and ε_2 are emissivities (blackness) of cold and warm walls, and $\varepsilon_{s1}, \varepsilon_{s2}, \dots, \varepsilon_{sn}$ represent emissivities (blackness) of corresponding n radiation shielding screens. When satisfying the condition of heat balance, $Q_1 = Q_2 = \dots = Q_n$, after installing n radiation shielding screens between two walls with temperatures T_1

Table 9.14 Thermal conductivities of common powdery and fibre porous insulation materials

Materials	Density γ /[kg/m ³]	Vacuum degree/Pa	Ranges of temperature/K	Thermal conductivity k /[W(m·K) ⁻¹]
Perlite (>10 mesh)	140	<0.13	300 ~ 76	1.06×10^{-3}
Perlite (30 ~ 80 mesh)	135	<0.13	300 ~ 76	1.26×10^{-3}
Perlite (>30 mesh)	106	<0.13	300 ~ 76	1.83×10^{-3}
Perlite	80 ~ 96	<0.13	300 ~ 20.5	0.7×10^{-3}
Perlite	80 ~ 96	filling gas He	300 ~ 20.5	0.1004
Perlite	80 ~ 96	filling gas N ₂	300 ~ 20.5	0.032
Silicone rubber	80	<0.13	300 ~ 76	2.72×10^{-3}
Silicone rubber (doping Al powder 15 ~ 45%)	96	<0.13	300 ~ 76	0.61×10^{-3}
Glass fibre	118	0.26	422	0.57×10^{-3}
Glass fibre felt	63	1.3	297	1.44×10^{-3}
Glass fiber felt	128	0.13	297	1.00×10^{-3}
Glass fiber felt	128	1.46	297 ~ 77	0.71×10^{-3}
Polystyrene foam	32	Atmospheric pressure	300 ~ 76	0.027
Polystyrene foam	72	Atmospheric pressure	283	0.040
Cellular glass	128 ~ 160	Atmospheric pressure	200	0.057
Diatomite	320	<0.13	278 ~ 20.5	1.11×10^{-3}
Polyurethane foam	26	Atmospheric pressure	297	0.021
Polyurethane foam	96	Atmospheric pressure	297	0.038
Polyurethane foam	80	Atmospheric pressure	297	0.035

and T_2 ($T_1 < T_2$), the heat flow flux is:

$$Q = \frac{\sigma A_1 (T_{n+2}^4 - T_1^4)}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2} \left(\frac{1}{\varepsilon_2} - 1 \right) + \sum_{i=1}^n \left(\frac{A_1}{A_{si}} \right) \left(\frac{1}{\varepsilon_{i1}} + \frac{1}{\varepsilon_{i2}} - 1 \right)}$$

(9.23)

If emissivity ε (blackness) and surface area of each insulation radiation shielding screen are identical, Equation (9.23) becomes:

$$Q = \frac{1}{n + 1} \frac{\varepsilon}{2 - \varepsilon} \sigma A_1 (T_{n+2}^4 - T_1^4)$$

(9.24)

If $n = 0$, and there is no radiation shielding screen in the vacuum interlayer between the two walls, radiant energy becomes thermal radiation in the vacuum thermal insulation mode described by Equation (9.17).

It is always expected that the emissivity of the radiation shielding screen is as low as possible and its surface is clean and smooth. When the metal foils are adopted, their thickness, that is, their cross-sections should be as small as possible to reduce longitudinal thermal conduction. The requirements for spacer

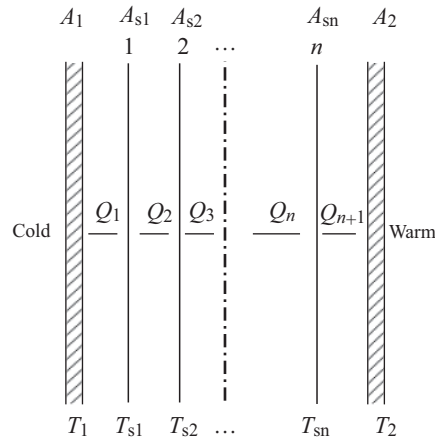


Figure 9.10 High vacuum multilayer insulation.

materials are as follows: their thermal conductivity should be as low as possible, their outgassing ability is low, they should have a certain mechanical strength, and their thickness should be as small as possible. Usually it is finished by covering the spacer materials with one layer of metal foil, which commonly include alkali-free glass fibre cloth, glass fibre paper, nylon mesh fabric cloth, plant fabric paper, etc.

A combination of multilayer thermal insulation can usually be realized by alternatively wrapping and overlapping glass fibre cloth or nylon fabric mesh cloth with aluminium foil and directly wrapping with composite materials such as polyester film with sprayed aluminium on a single surface or aluminium foil paper. In general, composite materials are corrugated in order to reduce the transverse contact area of heat transfer and to increase the contact thermal resistance. Effective thermal conductivity and heat flux (thermal radiation per unit area) of some shielding screen materials commonly used in combination modes with different temperature ranges, different thickness, layer numbers and degree of vacuum, are shown in Table 9.15.

In multiplayer thermal insulation, the influence of the degree of vacuum on thermal insulation performance is two-fold: one is the apparent vacuum degree, namely the vacuum degree measured by the vacuum gauge connected with the thermal insulation space, which has the influence on thermal insulation performance; the other is the influence of the degree of vacuum of the inner layer of the multilayer thermal insulation on thermal insulation performance. The relationship between the degree of vacuum of the thermal insulation space and the effective thermal conductivity of the multiplayer thermal insulation is illustrated in Figure 9.11. When the degree of vacuum is low, namely above 10 Pa, it has little influence on thermal conductivity; but if the degree of vacuum is in the range of 10^{-2} Pa to 10 Pa, thermal conductivity decreases rapidly with increase in the degree of vacuum, and when the degree of vacuum is lower than 10^{-3} Pa, thermal conductivity tends to be constant. Therefore, the apparent degree of vacuum in multiplayer thermal insulation should be at least better than 10^{-2} Pa.

In addition, when the density of the thermal insulation layer, that is, the number of multilayer insulation shielding screens per unit thickness increases, radiative heat transfer will be decreased; but the thickness of thermal insulation is not the thicker the better. If the thickness increases too much, radiation heat transfer will be reduced; however, the heat conduction of a solid will increase because the longitudinal cross-section (parallel to insulation plane) increases and then the contact thermal resistance will reduce. Consequently, there is an optimal thickness in the design of the thermal insulation layers, which needs to be optimized according to the properties of practical insulation materials and structure of the cryostat.

Table 9.15 Effective thermal conductivity and heat flux of some common multilayer insulation materials

Materials and combinations	Layers	Total thickness /mm	Vacuum degree/mPa	Temperature range/K	Effective thermal conductivity k /[W/(m·K)]	Heat flux q / (W/cm ²)
40 μm aluminium foil + 25 μm glass fabric cloth	129	26.5	18.7	77 ~ 300	9.71×10^{-5}	4.16×10^{-5}
20 μm aluminium foil + 15 μm glass fabric cloth	50	30	2.27	77 ~ 300	2.51×10^{-4}	5.38×10^{-4}
2 μm polyester film with spraying aluminium on both sides + 20 mesh Nylon cloth	71	24.4	6.68	77 ~ 300	1.67×10^{-4}	4.29×10^{-4}
2 μm polyester film with spraying aluminium on both sides + 12 μm cellophane	71	25.6	6.68	77 ~ 300	2.03×10^{-4}	4.91×10^{-4}
1 μm polyester film with spraying aluminium on single side + 50 μm plant fibre paper	31	8.9	5.34	77 ~ 300	1.09×10^{-4}	7.58×10^{-4}
GS-80 thermal insulation material	10	2.55	14.7	77 ~ 381	2.48×10^{-5}	2.04×10^{-4}
GS-80 thermal insulation material	30	2.00	12.8	77 ~ 310	6.77×10^{-6}	8.05×10^{-5}
Aluminium foil paper	10	2.70	14.9	77 ~ 303	3.83×10^{-5}	3.10×10^{-4}
20 μm aluminium foil + 120 μm carbon paper (34%)	10	9.5	4.1	77 ~ 293	7.14×10^{-6}	1.163×10^{-4}
1 μm polyester film with spraying aluminium on both sides + 120 μm carbon paper	10	8.5	1.87	77 ~ 293	7.86×10^{-6}	1.42×10^{-4}
8.7 μm aluminium foil + carbon paper	10	3.24	/	77 ~ 300	1.53×10^{-6}	1.09×10^{-4}

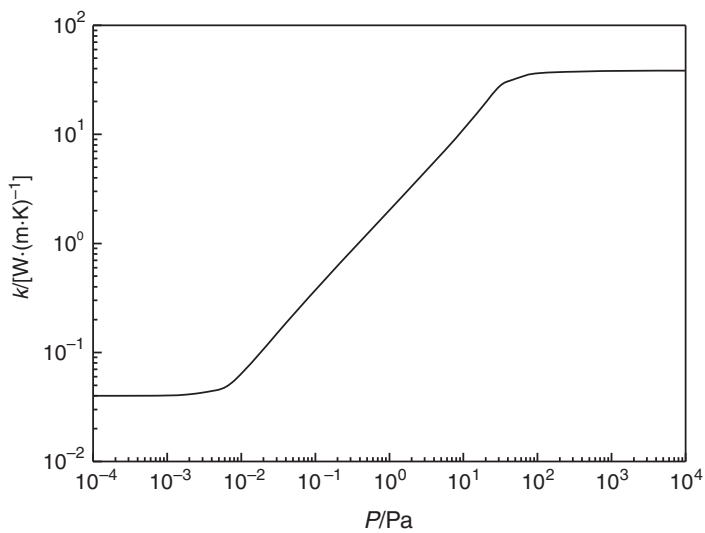


Figure 9.11 Dependence of apparent vacuum degree on effective thermal conductivity.

Longitudinal thermal conductivities of several typical shielding screen materials at different temperatures are listed in Table 9.16.

Longitudinal thermal conductivity of aluminium-plated film is one of several hundredths of aluminium foil. In addition, some factors, such as boundary temperatures, compression load (pre-stress before wrapping, own weight, prop compression) can cause uneven local pressure of thermal insulation layers, so that the contact area amongst layers increases, resulting in the increase of effective thermal conductivity of multilayer thermal insulation and then increasing heat conduction. The related content may refer to professional design handbooks of cryogenic thermal insulation and heat transfer technology.

Despite cryostats being made from dense textured materials, which are usually stainless steel materials and can maintain a vacuum for a long time, there is still the phenomenon of natural gas release between the inner and outer walls of the cryostat, and multiplayer thermal insulation materials between the vacuum interlayer of cryostat can also release residual gas. Thus the cryostat needs to be vacuum pumped at regular intervals. Table 9.17 shows typical natural outgassing characteristics of stainless steel.

In AC applications, the cryostat should be made from non-metallic materials to avoid inducing eddy current. Usually, the non-metallic material is glass fibre reinforced plastics (GRP). Although GRP has a dense texture and its mechanical strength is equivalent to stainless steel, gas can permeate through it, so that the vacuum is hard to maintain for long time periods unless regular pumping action is taken. Gas permeability of several non-metallic materials is showed in Table 9.18.

Table 9.16 Longitudinal thermal conductivity of several typical shielding screen materials

Materials	Thickness/ μm	Longitudinal thermal conductivity $k/[\text{W}\cdot(\text{m}\cdot\text{K})^{-1}]$		
		300 K	77 K	20 K
Aluminium foil	6.3	1.91	2.54	25.4
Polyester film	6.3	0.90×10^{-3}	8.58×10^{-4}	5.33×10^{-4}
Aluminium film	12.7	3.83×10^{-3}	5.1×10^{-3}	5.1×10^{-2}
Aluminium-plated film	/	4.75×10^{-3}	5.98×10^{-3}	5.153×10^{-2}

Table 9.17 Typical natural outgassing rate of stainless steel

Processing method	Outgassing rate/(Torr·L/s·cm ²) ^a
Long-term exposed to atmosphere without any treatment, pumping 1 h	2×10^{-7}
Cleanly removing oil without heating and drying, pumping 4 h	2×10^{-9}
Cleanly removing oil with heating and drying at 250 °C, pumping 15 h	1×10^{-12}
Heating and drying 1000 °C in vacuum furnace for 3 h, vacuum degree is 2×10^{-6} Torr	10^{-14}

^a1 Torr = 133.32 Pa.

Based on outgassing rate and permeability of materials from which the cryostat is made, gas adsorbency needs to be installed between the inner and outer walls of the cryostat when the vacuum multilayer thermal insulation mode is adopted (Figure 9.12(a)). Gas adsorbents can absorb residual gas released from the walls of the cryostat and thermal insulation materials so that a high degree of vacuum is maintained to reduce heat transfer. Usually, activated carbon and molecular sieve materials are adopted as gas adsorbents. Performance parameters of activated carbon are shown in Table 9.19. It should be noted that activated carbon cannot be adopted as a gas adsorbent of the cryostat in the cryogenic thermal insulation mode if the cryostat is used to fill strong oxidants such as LH₂, LO₂, etc.

Table 9.18 Gas permeability of several non-metallic materials

Materials	Types of gas		Thickness d/mm	Pressure difference $\Delta P/\text{Pa}$	Gas permeability $\text{K}/[\text{cmHg}\cdot\text{s}/\text{mm}]^a$
GRP	N ₂		0.525	8.4×10^4	5.1×10^{-11}
	He			8.5×10^4	3.7×10^{-9}
	H ₂			8.5×10^4	4.5×10^{-10}
Films of three layers with combination of polyethylene with resin type	He		0.079	3.5×10^4	6.0×10^{-12}
	CO ₂			3.4×10^4	1.0×10^{-11}
	O ₂			3.5×10^4	8.8×10^{-11}
Composite film with combination of polyester fiber and polyethylene	1#	He	0.055	4.4×10^4	2.4×10^{-9}
		O ₂		6.7×10^4	3.2×10^{-11}
		N ₂		6.7×10^4	1.2×10^{-10}
	2#	He	0.016	6.7×10^4	5.2×10^{-10}
		O ₂		8.6×10^4	1.5×10^{-11}
		N ₂		6.7×10^4	1.2×10^{-11}
	3#	He	0.016	8.6×10^4	1.0×10^{-9}
		O ₂		6.7×10^4	3.2×10^{-10}
		N ₂		3.5×10^4	3.2×10^{-11}
	Nylon taffeta	air	0.183	8.5×10^4	2.4×10^{-10}
He		5.1×10^4		1.2×10^{-9}	
N ₂		8.5×10^4		5.6×10^{-10}	
O ₂		6.4×10^4		2.3×10^{-10}	

^a1 atm = 76 cmHg = 760 Torr.

Table 9.19 Typical performance of activated carbon adsorbents

Gases	Absorption gas capacity /(cm ³ /g)									
	273 K		265 K		195 K		77 K			
	133.3 Pa	1333 Pa	133.3 Pa	1333 Pa	133.3 Pa	1333 Pa	133.3 Pa	1333 Pa		
He	/	/	/	/	/	/	2.78×10^{-3}	2.78×10^{-2}		
H ₂	2.2×10^{-3}	2.78×10^{-2}	/	/	7.7×10^{-3}	7.7×10^{-2}	2.78×10^{-3}	2.78×10^{-2}		
Ar	5.8×10^{-2}	0.581	7.6×10^{-2}	0.764	0.21	2.00	2.63	36.3		
N ₂	3.3×10^{-2}	0.318	/	/	0.396	3.65	8.45	46.5		
CO	3.6×10^{-2}	0.359	/	/	0.794	7.10	/	/		
CO ₂	0.497	4.67	/	/	/	/	/	/		
CH ₄	0.115	1.12	0.249	2.37	/	/	/	/		
C ₂ H ₄	0.985	8.71	/	/	/	/	/	/		
NH ₃	1.086	10.05	/	/	/	/	/	/		
Kr	0.34	3.40	0.497	3.81	2.93	15.03	/	/		
Xe	1.58	9.32	2.46	12.10	16.0	60.0	/	/		

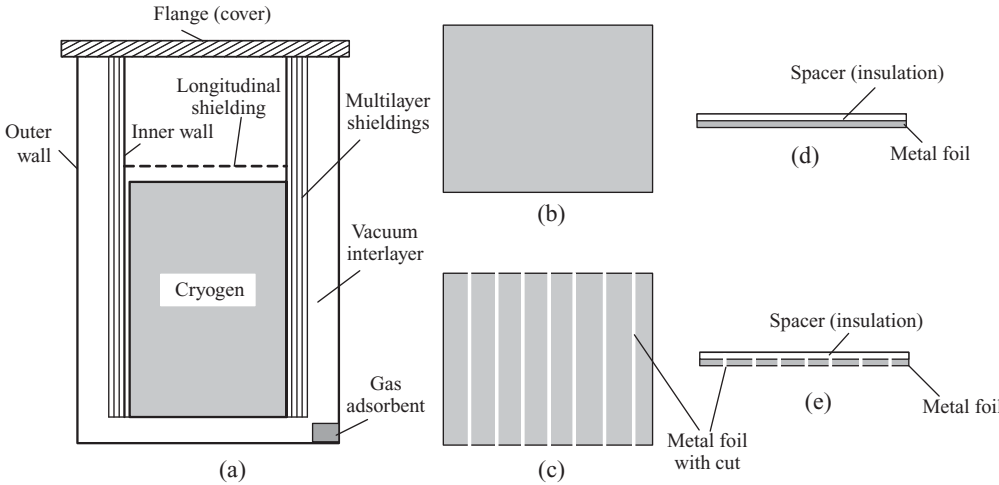


Figure 9.12 Cryostat and shielding screen: (a) structure of cryostat; (b) shielding materials without cut; (c) shielding materials with cut; (d) cross-section of (b); (e) cross-section of (c).

The metallic cryostat described above is applicable for DC and non-inductance AC applications. In AC applications, three factors should be considered in the design of the cryostat, besides those discussed above. First, the walls of the cryostat cannot be made from metallic materials, because these will induce eddy current in the AC magnetic field, and eddy current loss will be generated in the metals. To avoid the generation of eddy currents, the walls of the cryostat materials should use non-metallic materials such as FRP and glass. The latter is fragile so is not usually adopted in a cryostat of practical superconducting power apparatus.

Second, there must be cuts in the metal foil of the shielding screen insulation in the interlayer of the vacuum (Figures 9.12(c) and (e)), so that there will be no large eddy currents induced in the metal foils, to avoid eddy current loss. Finally, activated carbon will affect the insulation characteristics in AC applications because it is a semiconductor. For example, activated carbon is prohibited in the cryostat of superconducting transformer windings. It should be replaced by non-metal or non-semiconductor insulator gas adsorbents, such as molecular sieves. A suction property of the type X13 molecular sieve used as gas absorption capacity of several gases is listed in Table 9.20.

Structure of the cryostat with the mode of a vacuum multilayer thermal insulation has the advantages of compact size, small heat leakage, long duration in maintaining vacuum, etc. Currently, it is the optimal structure in terms of thermal insulation effect, so it is widely used in cryogenic engineering and applied superconductivity.

If there is no other explanation given, structures of cryogenic vessels or devices introduced in this book hereafter refer to those with vacuum multilayer thermal insulation mode.

Table 9.20 Maximum gas absorption rate and capacity of 13X molecular sieves

Gases	N ₂	Air	Ar	H ₂
Maximum absorption rate /[L/(s·g)]	0.8	0.48	0.27	0.01
Absorption capacity/[Pa·L/g]	8198	7598	6665	ignored

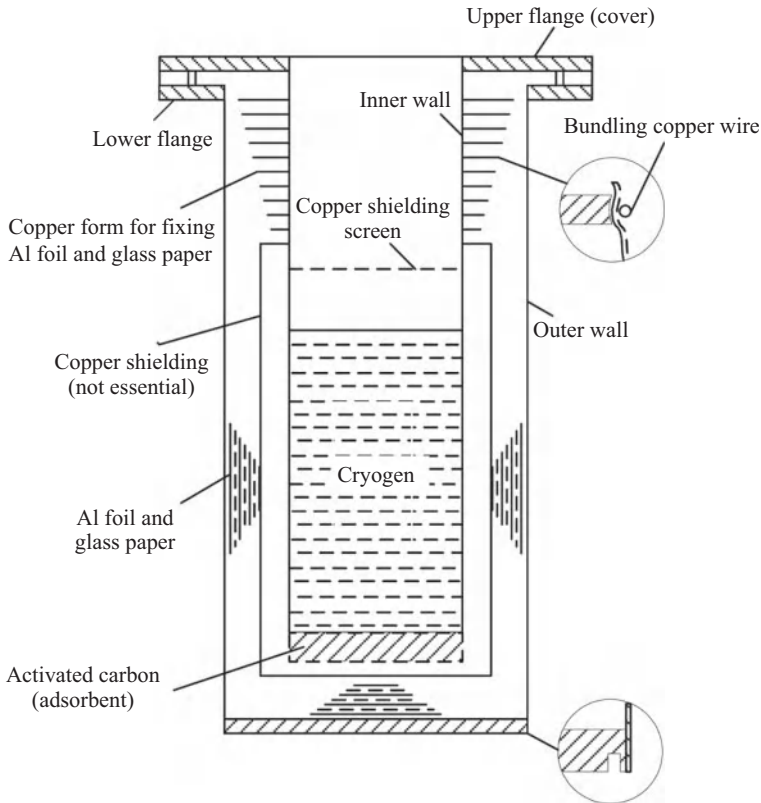


Figure 9.13 Structure of vertical and cylinder-shaped cryostat with large diameter.

9.2.3 Structure Design of Cryostats

Cryostats can be divided into cryostats applied in apparatus and storage cryostats according to different uses. The former provides cryogenic environments for apparatus, such as superconducting devices. They are generally designed in the form of vertical cylinders with a large diameter so that the superconducting devices and other cryogenic devices are conveniently installed. Their geometrical structures are illustrated in Figure 9.13. The latter are mainly used for storage and transportation of cryogenes. Their structures are usually round tanks, with a small neck diameter in order to reduce heat leakage by radiation. Their geometrical structure is indicated in Figure 9.14. Material of the inner and outer wall of the cryostat is usually stainless steel. The upper parts are connected by two flanges, which are sealed together with a washer between them. The bottom part of the inner wall of the cryostat is thicker in order to improve mechanical strength. A high vacuum multilayer thermal insulation mode between the interlayer of the inner and outer walls is realized by wrapping films with sprayed aluminium on both sides and spacers on the inner wall.

Preliminary design steps of cryostats are as follows:

9.2.3.1 Selection of Design Parameters

- (1) *Pressure design.* Since the space between the inner and outer walls of the cryostat is a vacuum interlayer, there is a large pressure difference at both sides of two walls, namely atmospheric

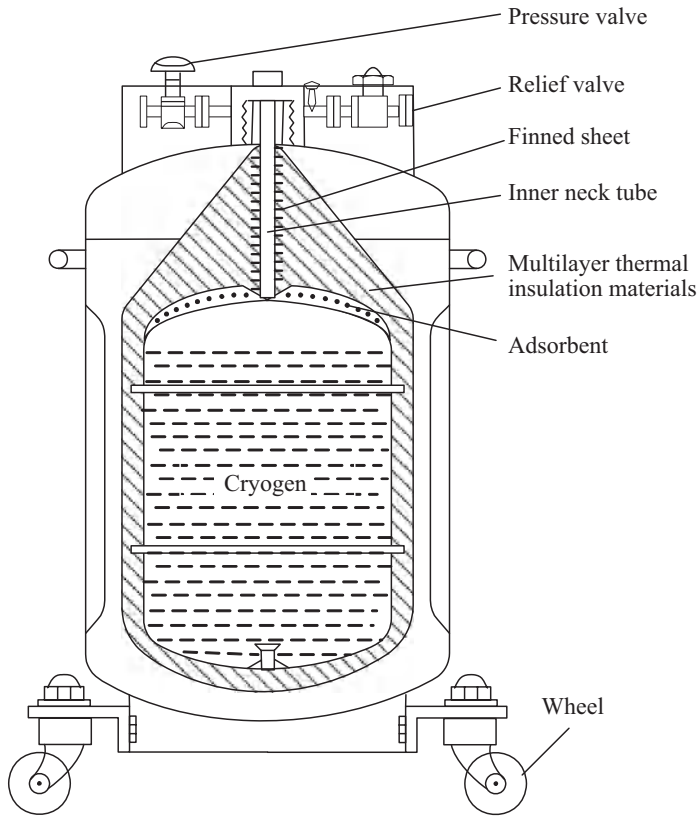


Figure 9.14 Tank for storage and transportation of cryogen.

pressure exerts on one side and the other side is faced with high vacuum. The maximum pressure difference of 10.1 N/cm² is selected in the design of cryostat.

- (2) *Determination of additional wall thickness c .* Additional wall thickness can be determined by Equation (9.25):

$$c = c_1 + c_2 + c_3 \tag{9.25}$$

where c_1 is the negative deviation of thickness, c_2 is the corrosion allowance determined by corrosion of cryogen and lifetime of cryostats, and c_3 is the additional wall thickness needed for the stainless steel plate or pipe due to the tensile elongation experienced when the heads are pressed.

- (3) Allowable stress σ is selected from the minimum one of the following parameters:

$$\sigma = \frac{\sigma_b}{n_b}; \quad \sigma = \frac{\sigma_s}{n_s}; \quad \sigma = \frac{\sigma_D}{n_D}; \quad \text{or} \quad \sigma = \frac{\sigma_n}{n} \tag{9.26}$$

where σ is allowable stress (N/cm²), σ_b denotes the minimum strength limit (N/cm²), σ_s and σ_D refer to yield limit (N/cm²) and endurance limit (N/cm²) at operating temperature, respectively, σ_n is creep limit (N/cm²) at operating temperature; and n_b , n_s , n_D and n_n are safety factors.

- (4) *Determination of welding seam coefficient φ .* A double-sided butt welding seam is selected.

9.2.3.2 Calculation Steps

(1) Thickness (mm) of inner wall of the cylinder can be calculated by:

$$s = \frac{PD_i}{2[\sigma]^t \phi - P} + c \quad (9.27)$$

Stress is corrected by:

$$\sigma' = \frac{P(D_i + (s - c))}{2(s - c)} \quad (9.28)$$

where s is thickness (mm) of the inner wall of the cylinder; P denotes the design pressure (N/cm²), D_i refers to inner diameter (mm) of the cylinder wall, c is the additional wall thickness (mm), ϕ represents the welding seam coefficient, and σ_t and $[\sigma]^t$ are the calculated stress of the wall's allowable stress of material at operating temperature (N/cm²), respectively. If $\sigma_t < [\sigma]^t$, namely the mechanical strength is enough, the maximum allowable operating pressure (N/cm²) of the cylinder wall is:

$$P_{\max} = \frac{2[\sigma]^t \phi (s - c)}{D_i + (s - c)} \quad (9.29)$$

(2) Thickness (mm) of outer wall of cylinder is determined by:

$$S = D_i^{0.6} \left(\frac{mPL}{2.59E} \right)^{0.4} \quad (9.30)$$

where m is the stable coefficient (available by referring to related handbooks), E denotes the elastic modulus (N/cm²) of material, and L is the calculated length of cylinder wall.

(3) *Design of thermal insulation layer.* High vacuum multiplayer thermal insulation mode is adopted.

9.2.3.3 Design of Top Flange (Cover)

If the diameter of the cryostat is not too large, its top flange cover can take the simple plate structure, so that design and fabrication are relatively easy and it is easy to install other components onto the cover, such as current leads, taps, pull-rods, liquid level metre, etc. If stainless steel materials are chosen, the thickness (mm) of typical circular flange cover is:

$$t = D_c \left(\frac{KP}{[\sigma]^t \phi} \right)^{1/2} + c \quad (9.31)$$

where D_c is the diameter of cover, K is the structural characteristic coefficient, which can be obtained by referring to related handbooks, and P is the design pressure (N/cm²).

In AC applications, the inner and outer walls of cryostats must be made from non-metallic material, such as glass and GRP. But the cryostat made from glass is only applicable in a laboratory and is not useful in engineering due to its fragility. GRP is a good choice for fabricating a cryostat used in practical AC applications. Meanwhile, metal foils used as multiplayer thermal insulation should have cuts to avoid the generation of eddy currents. When electrical insulation is considered, the organic thermal insulation should be used as gas adsorbent, such as a molecular sieve, instead of using semiconductor materials, such as activated carbon and aluminium oxide. The structure of this cryostat is shown as Figure 9.13.

In addition, non-metallic cryostats are usually made of a fibre reinforced composite, in which GRP is a good choice. The daily evaporation rate of cryogens in non-metallic cryostats is lower than in metallic cryostats, because the conduction heat leakage of non-metallic materials is much smaller compared with metallic ones. However, some literature experimentally reported that the outgassing rate of GRP tends to be stable with $2.137 \times 10^{-4} \text{ Pa}\cdot\text{m}^3/(\text{s}\cdot\text{m}^2)$ in vacuum and temperature 25°C after 40 h; If the GRP has been sufficiently aged, its outgassing rate can be decreased to $6.3 \times 10^{-6} \text{ Pa}\cdot\text{m}^3/(\text{s}\cdot\text{m}^2)$ at temperature 6°C .

The outgassing rate of GRP is 3 to 6 orders of magnitude higher than that of stainless steel. Meanwhile, the gas permeability of non-metallic materials can reach $6 \times 10^{-12} \sim 10^{-9} (\text{cm Hg}\cdot\text{s}/\text{mm})$, and that of metallic materials can be nearly zero, so is usually neglected. Thus, compared with a metallic cryostat, the good thermal insulation performance of non-metallic cryostats can be maintained in the long term. To improve the vacuum maintenance time, several effective measures can be taken. They are:

- (1) coating some dense texture materials on FRP to reduce its outgassing rate;
- (2) vacuum baking the components;
- (3) attaching barrier films;
- (4) metal-plating film on surface of vacuum;
- (5) controlling glue content, pressing force and uniformity of dipping glue, doping graphite and quartz powders into epoxy resin by a reasonable processing technology.

In addition, since outgassing is an intrinsic characteristics of FRP, the vacuum interlayer of cryostats must be regularly pumped, which is different from a cryostat made from dense texture metallic materials, such as stainless steel.

9.2.4 Cryogenic Transfer Lines and Flexible Pipes

Some cryogens with low latent heat, such as LHe, cannot be transported from the cryogenic storage tank to operating cryostats by conventional metallic pipes. Cryogenic transfer lines with special thermal insulation are always used in order to reduce the dissipation of cryogens during transportation. Especially, LHe transported by transfer lines with poor thermal insulation performance may be totally gassified, with the result of no LHe output. Furthermore, dissipation of cryogens should be as low as possible during the cooling of transfer lines themselves when transporting cryogens. To meet all those requirements, the Dewar pipes with high vacuum thermal insulation are commonly used as transfer lines.

Usually, the transfer line is U-shaped. Figure 9.15 shows the structure of one U-shaped transfer line for transporting cryogens, where the inlet is inserted into the storage container of the cryogen and the outlet is inserted into the operating cryostat. The process of transporting the cryogen into the operating cryostat through a cryogenic transfer line is illustrated in Figure 9.16.

The inner and outer walls of the transfer line are usually made from stainless steel or nickel silver material with low thermal conductivity to reduce the solid heat transfer and number of welding ports. Parts of the outer wall with room temperature and middle parts of the inner wall sometimes also adopt materials with low emissivity such as copper, so that heat transfer by radiation can be effectively reduced. Similar to the vacuum multilayer thermal insulation of the cryostat, multilayer aluminium-plated or polished polyester films are wrapped onto the outer surface of the inner wall of the transfer line to decrease radiation heat leakage. In addition, small amounts of adsorbents, such as activated carbon, should be placed at the inlet of the vacuum interlayer to maintain a high vacuum. Some spacer pieces are installed at the corners or near the corrugated pipe in the vacuum interlayer of the transfer line, which can prevent the inner and outer walls from contacting with each other, which would form a thermal short-circuit and lead to an increase of heat leakage. Generally, spacer pieces are made from those materials with low thermal conductivity, such as epoxy, Teflon, etc.

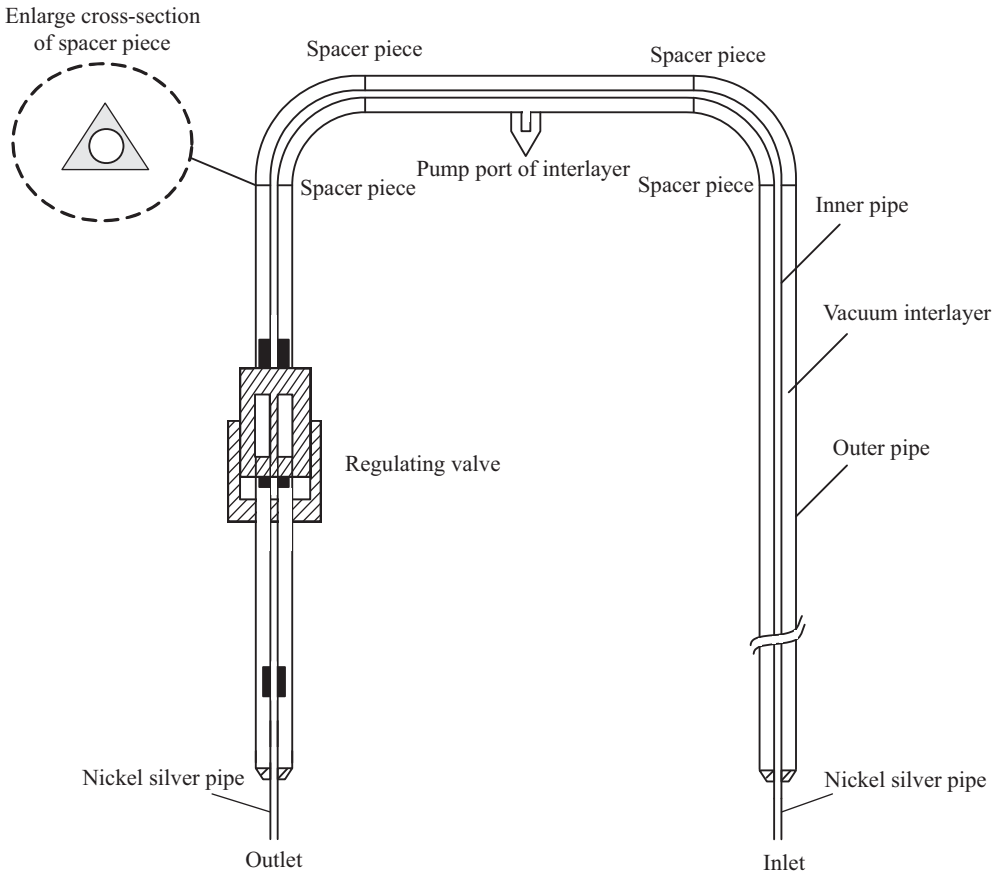


Figure 9.15 U shape of cryogenic transfer line.

The typical structure of a spacer piece is illustrated in the top left-hand corner of the enlarged dotting circle in Figure 9.15. The diameter of the central hole is a little larger than the outer diameter of the inner pipe to make sure that the inner pipe can pass smoothly through. Spacer pieces are located and positioned by point soldering on the surface of the outer wall of the inner pipe. Their cross-sectional size is slightly smaller than the inner diameter of the outer pipes to reduce as much as possible the contact (point contacting) between the spacer pieces and the inner wall of the outer pipe, so that minimum heat leakage will be guaranteed. The thermal contraction of the inner pipes of the transfer line should be considered in practical design and manufacture; though the contraction is a mere 2.8 mm/m, it cannot be ignored. To compensate for this kind of contraction stress, some sections of the inner and outer pipes of the transfer line are usually corrugated by stainless steel pipes instead of straight pipes, so the transfer line becomes flexible. Figure 9.17 shows the main view of the flexible cryogenic Dewar pipe, whose inner and outer pipes consist of corrugated stainless steel pipes, and there is a vacuum interlayer between the inner and outer pipes. Multilayer thermal insulation films are wrapped onto the outer surface of the inner pipe. This kind of cryogenic Dewar pipe has been widely applied in superconducting cables.

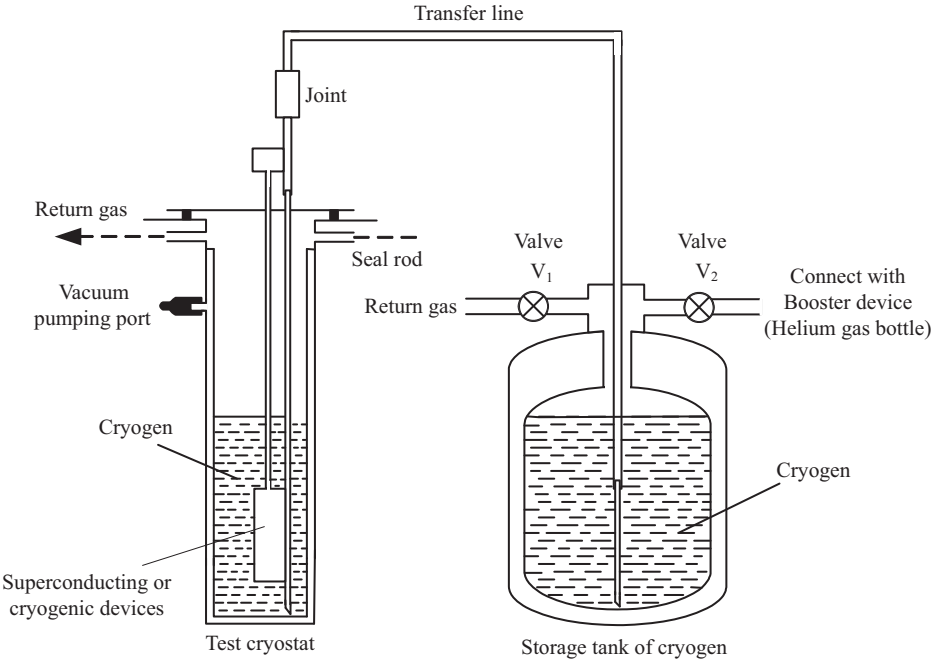


Figure 9.16 Process of transportation of cryogen into operating cryostat through cryogenic transfer line.

9.2.5 Ultra-Cryogenic Cryostat with Dual-Cryostat Structure

Ultra-cryogenics generally refers to the temperature below that of LHe (4.2 K), which is usually achieved by the reducing pressure cooling method. Saturated gas temperature decreases with the reduction of gas pressure (Figure 9.1). When the temperature of LHe is lower than 2.172 K, it will become superfluid helium (viscosity is zero), which results in automatic generation of climbing LHe film and leads to a

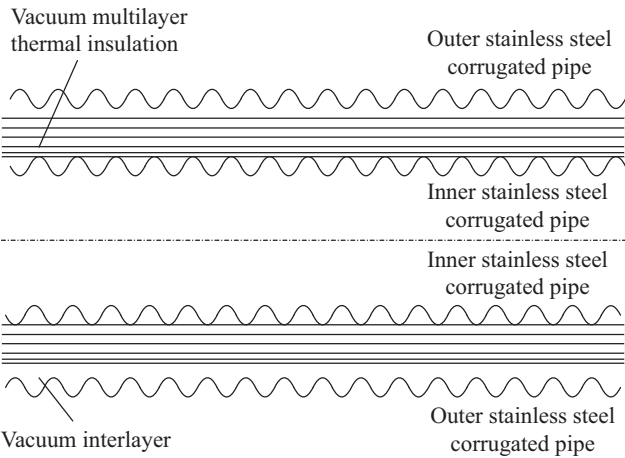


Figure 9.17 Structure of flexible cryogenic Dewar pipe.

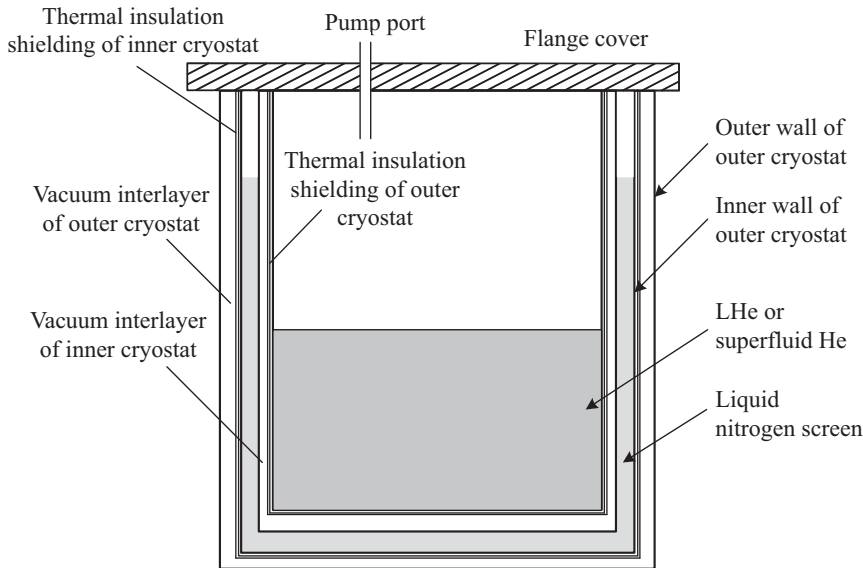


Figure 9.18 Structure of ultra-cryogenic cryostat.

large amount of dissipation of LHe. Based on this, the ultra-cryogenic cryostat is usually designed with the structure of a double cryostat, interlayer space between walls of double cryostats filled with LN_2 , the so-called cooling screen. Figure 9.18 shows the ultra-cryogenic cryostat, which is usually applied in large-scale superconducting magnet engineering, for example, the large-scale LTS magnet made from cable-in-conduit conductor (CICC) conductors.

9.3 Refrigeration

Refrigeration or cryogenics and superconducting technology are twins in superconducting power application, both sharing the relationship of interdependency, inter-coordination and mutual promotion of development. With development of superconducting power technology, higher requirements are suggested for cryogenics and refrigeration techniques. Cryogenic refrigeration technology with reliable, efficient, convenient and low-cost performance become some of the key technologies of superconducting power applications. It is essential to understand cryogenic refrigeration technology for the application of superconducting power apparatus. This section will briefly introduce principles of cryogenic refrigeration and performances of several commonly used commercial refrigerators.

9.3.1 Principle of Refrigeration and Performance of Refrigerators

9.3.1.1 Carnot Cycle

There are two heat sources in the Carnot cycle, and their temperatures are T_1 and T_2 ($T_2 > T_1$), respectively (Figure 9.19). The Carnot cycle consists of four processes:

- (I) working medium (e.g., gas) expands reversibly at constant temperature T_2 , and heat is absorbed from warm heat source Q_2 ;

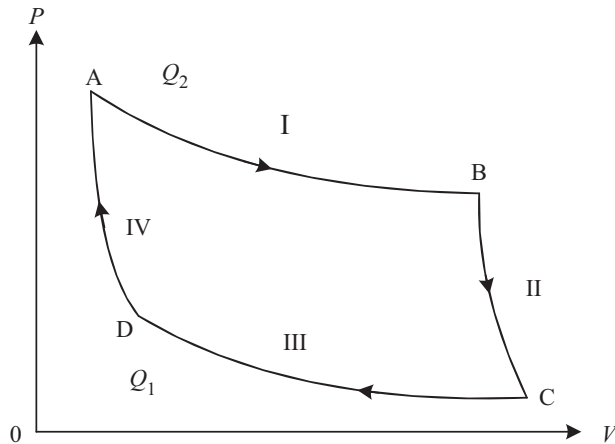


Figure 9.19 Ideal Carnot cycle.

- (II) the temperature T reduces from T_2 to T_1 by adiabatically reversible expansion;
- (III) working medium is compressed at constant temperature T_1 , and heat releases to cold heat source Q_1 ;
- (IV) temperature T rises from T_1 to T_2 by adiabatically reversible compression, system recovers and the cycle is finished.

Refrigerating capacity and refrigeration coefficient of the Carnot cycle are:

$$Q_0 = R(T_2 - T_1) \ln \left(\frac{V_2}{V_1} \right) \quad (9.32)$$

$$\beta = \frac{T_1}{T_2 - T_1} \quad (9.33)$$

The Carnot cycle has the highest efficiency, but it is just an ideal thermodynamics cycling system. The aim of introducing the Carnot cycle is to provide referential comparison for refrigeration efficiency of refrigerators based on principles of other thermodynamic cycles.

9.3.1.2 Stirling Cycle and Stirling Refrigerators

The Stirling cycle consists of two isothermal processes and two isochoric processes, whose ideal cycle is shown in Figure 9.20. I and III are isothermal processes, and II and IV are isochoric processes. The theoretical cooling capacity of Stirling cycle is:

$$Q_0 = RT_3 \ln \left(\frac{V_1}{V_2} \right) \quad (9.34)$$

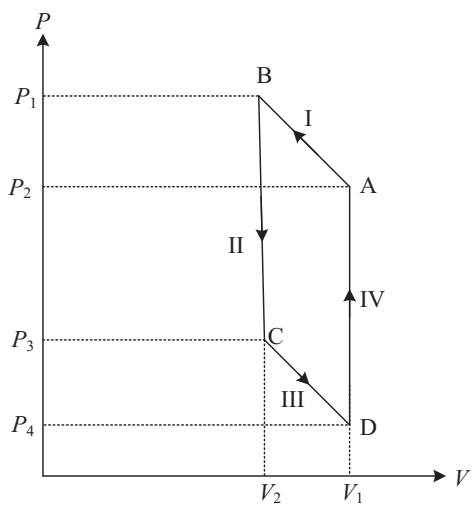


Figure 9.20 Ideal Stirling cycle.

and refrigeration efficiency is:

$$\beta = \frac{T_3}{T_1 - T_3}$$

(9.35)

Stirling refrigerators were successfully developed based on the Stirling cycle, whose refrigeration temperature can be as low as 12 K. Currently, Stirling refrigerators are mainly produced by Netherlands Stirling Cryogenics & Refrigeration BV, with two series and four kinds of products: one-stage single-cylinder, one-stage four-cylinders (Figure 9.21), two-stages single-cylinder (Figure 9.22) and two-stage four-cylinders. Performance parameters of their refrigerators are listed in Table 9.21. Stirling refrigerators are suitable for application of HTS power apparatus with operating temperatures in the range 20 K to 77 K and refrigerating capacity ranges of 50 W to 500 W.

9.3.1.3 Reverse Brayton Cycle and Reverse Brayton Refrigerators

The Reverse Brayton cycle contains two isobaric processes and an isentropic process (Figure 9.23), where I and III are isobaric processes and II and IV are isentropic processes, respectively.

Table 9.21 Main performances of several commercial Stirling refrigerators

Types	Models	Refrigeration capacity/W		Power dissipation/kW	Cooling water rate/(L/h)	Weights/kg	Continuous operating time/h
		20 K class	80 K class				
One stage	SPC-1	/	1030	12	750	600	6000
	SPC-4	/	4250	45	4000	1255	6000
Two stages	SPC-1T	50 W	150 W	12	750	750	6000
	SPC-4T	200 W	600 W	45	3750	1400	6000

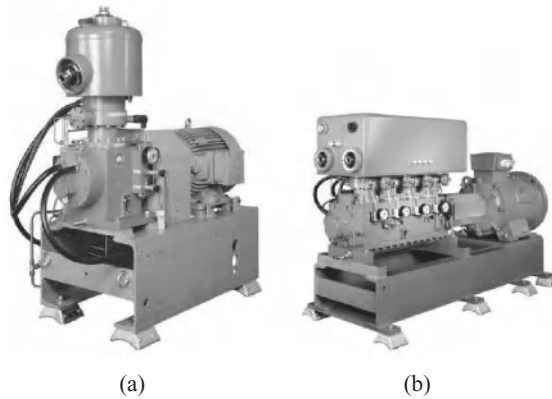


Figure 9.21 One-stage single-cylinder and one-stage four-cylinder of Stirling refrigerators: (a) single-cylinder; (b) four-cylinder (Reproduced with permission from Stirling Cryogenic BV, //www.stirlingcryogenics.com/products/cryogenic-cooling (accessed October 29, 2012)).

Theoretical refrigerating capacity is:

$$Q_0 = \frac{k}{k-1} RT_c \frac{\left(\frac{T_r}{T_c}\right) \left[\left(\frac{P_2}{P_1}\right)^{\frac{\beta-1}{\beta}} - 1 \right]}{\left(\frac{T_r}{T_c}\right) \left(\frac{P_2}{P_1}\right)^{\frac{\beta-1}{\beta}} - 1} \quad (9.36)$$

where β is specific heat ratio (c_p/c_v).



Figure 9.22 Two-stages and single-cylinder of Stirling refrigerators (Reproduced with permission from Stirling Cryogenic BV, //www.stirlingcryogenics.com/products/cryogenic-cooling (accessed October 29, 2012)).

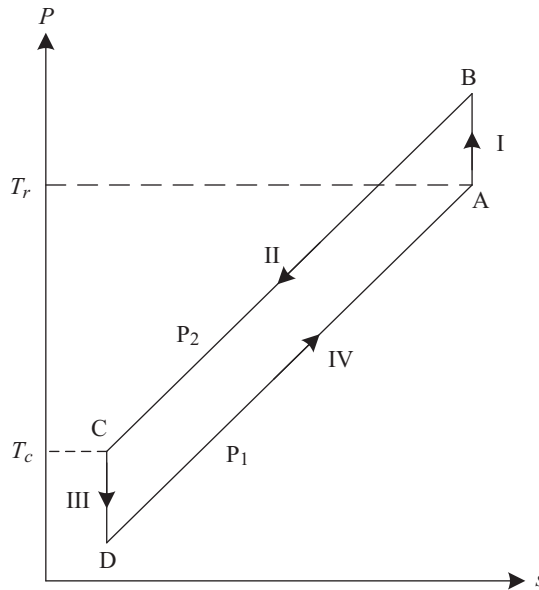


Figure 9.23 Ideal Reverse Brayton cycle.

The ideal refrigerating coefficient of the cycle is:

$$\eta = \frac{1}{\left(\frac{T_r}{T_c}\right) \left(\frac{P_2}{P_1}\right)^{\frac{\beta-1}{\beta}} - 1} \quad (9.37)$$

Refrigerators, manufactured based on the Reverse Brayton cycle, are called Reverse Brayton refrigerators, whose refrigerating temperature range is 4.2 K to 120 K. Their maximum refrigerating capacity of a single refrigerator is 20 kW at temperature 4.5 K, and they are widely used.

Refrigerators manufactured based on the Reverse Brayton cycle are known as Reverse Brayton refrigerators, whose refrigeration temperatures range from 4.2 K to 120 K. At 4.5 K, their single unit maximum refrigeration capacity can be 20 kW, and they are widely used in the LTS magnet of large-scale scientific engineering and hydroxide fuels liquefaction devices for aerospace, etc. At present, these refrigerators are mainly provided by Air Liquide and Linde Kryotechnik AG. Compared with Stirling refrigerators, Reverse Brayton refrigerators have some disadvantages, such as low efficiency, complicated structure, huge system, demanding much more auxiliary apparatus, large occupation of site area and strict operating requirements, even though they have large refrigerating capacity. Therefore, they are mainly suitable for applications of refrigerating capacity higher than 200 W/20 K and large-scale LTS magnets as well as gas liquefaction plants. Generally, there are no ready-made products, so a special design is needed according to each user's requirements.

9.3.1.4 G-M Refrigeration Cycle and G-M Refrigerator

The Gilford–McMahon (G-M) refrigeration cycle is composed of four processes: adiabatically raising pressure, isopiestic filling, adiabatically discharging and isopiestic exhausting (Figure 9.24). High

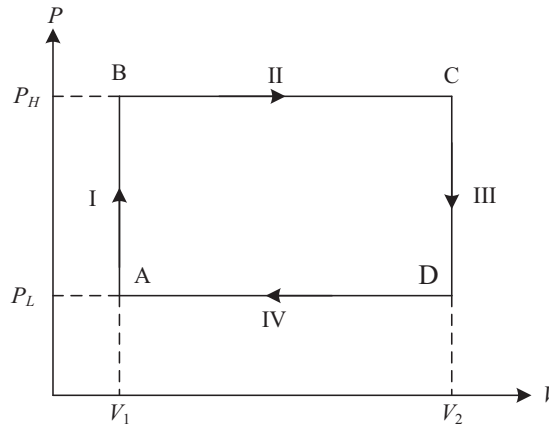


Figure 9.24 Ideal G-M refrigeration cycle.

pressure gas in a container is made to discharge into space at low pressure, that is, the gas expands, so that refrigeration effect by cooling is obtained by consuming the internal energy of gas itself. After one cycle, the theoretical cooling capacity is:

$$Q_0 = (P_H - P_L) nV \quad (9.38)$$

$$\eta = \frac{Q_0}{W} \quad (9.39)$$

where W is power consumption in one cycle:

$$W = (m_1 + m_2) \frac{k_B R T_a}{\nu - 1} \left[1 - \left(\frac{P_H}{P_L} \right)^{\frac{\beta-1}{\beta}} \right] \quad (9.40)$$

where m_1 and m_2 are the mass of the gas compressed in and discharged out of the expander, respectively, T_a is environment temperature of gas before entering the compressor, n is mole number, $n = m/M$, m is gas mass, and M is molar mass of gas.

The thermodynamic cycle of a G-M refrigerator contains four processes: pressure boost, inflation, deflation expansion and exhaustion. A G-M refrigerator is composed of a compressor and cooling components of the refrigerator. Figure 9.25 is a photograph of a G-M refrigerator; the left part shows the compressor of the refrigerator, and the right upper and lower parts are the cooling parts and the cryostat, respectively. The device in the cryostat is directly cooled by the cold head connecting with it. At present, G-M refrigerators have two refrigerating modes: the single-stage G-M refrigerator whose refrigerating temperature can reach LN₂ temperature (77 K), and the double-stage G-M whose refrigerating temperature can be as low as the LH₂ temperature (20 K) or the LHe temperature (4.2 K).

In commercial grades of G-M refrigerators, the single-stage G-M refrigerator can provide a cooling capacity of more than 600 W at LN₂ temperature (77 K); double-stage G-M refrigerator can provide a cooling capacity in the temperature range of LNe (20 K) and LN₂ (77 K) simultaneously, while other double-stage G-M refrigerators can separately provide a cooling capacity in the temperature range of 30 to 50 K and 4.2 K, at which they can continuously operate without faults for more than 10 000 hours. The main refrigerating performances and minimum refrigerating temperature without load of the G-M refrigerator are listed in Tables 9.22 and 9.23, respectively.

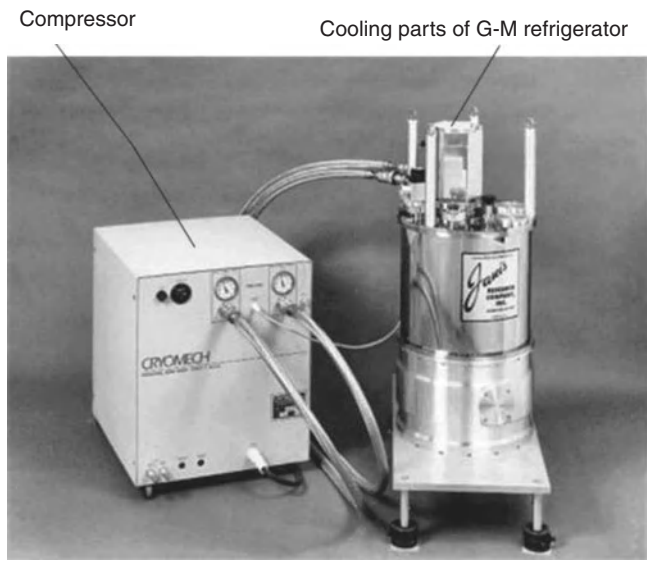


Figure 9.25 G-M refrigerator.

Although the refrigeration coefficient of a G-M refrigerator at LN_2 temperature is only 10% to 14% of that of the Carnot cycle, it is widely applied in many fields, such as cryoelectronics, infrared detection and superconducting apparatus because of its advantages of simple structure, small size, slight vibration and convenience of use. Currently, the main manufacturers of G-M refrigerators are the VTI-Cryogenics company of American, the Cryomech company, the Sumitomo company of Japan and the Leybold company; furthermore, the G-M refrigerator is the unique internationally cryogenic refrigerator with industrializing mass production.

9.3.1.5 Principle of Pulse-Tube Refrigerator

Refrigeration of a pulse-tube refrigerator is by inflation and deflation of high and low pressure gas towards the cavity of a pulse tube. Figure 9.26 illustrates the principle of the pulse-tube refrigerator, which is mainly composed of compressor, regenerator, gas piston, gas reservoir, etc. The refrigerating temperature of a pulse-tube refrigerator with a single-stage orifice and gas storage can be lower than 49 K, and its performance was not really comparable to that of a G-M refrigerator until 1990, and after then it has been modified to the double inlet type.

Since the pulse-tube refrigerator adopts a passive phase modulation mode, its performance has a close relationship with the phase adjuster and adjustable parameter, and the efficiency of the refrigerator is different. Generally, efficiency of pulse-tube refrigerators is lower than that of other refrigerators,

Table 9.22 Refrigerating capacity of commercial G-M refrigerators

Temperature/K	Single-stage /W	Double-stage /W	Double-stage at 4.2 K/W
77	600 W	<100 W	/
50	/	/	<30
20	/	<15 W	/
4.2 K	/	/	<1.5 W

Table 9.23 Minimum refrigerating temperature of commercial G-M refrigerators

Types	Single-stage/W	Double-stage/W	Double-stage at 4.2 K/W
First stage	~ 10 K	~ 45 K	~ 30 K
Second stage	/	7 ~ 10 K	2.6 K

meanwhile, it is rarely applied in the cooling system of superconducting power apparatus, since pulse-tube refrigerators with large cooling capacity have not become advanced enough until now.

9.3.1.6 Principle of Joule-Thomson Refrigerator

The Joule-Thomson (JT) refrigerator mainly consists of four parts: compressor, counterflow heat exchanger, JT valve and a reservoir (Figure 9.27). The gas is at room temperature and there is a pressure of 1 bar at the inlet of the compressor. The pressure is 200 bar after compression. Gas then enters the warm side of the counterflow heat exchanger with high pressure, where it is precooled. After JT expansion, it is liquefied at cryogenic temperature and a pressure of 1 bar. The liquid leaves the system at the bottom of the reservoir and the residual gas flows to the cold side of the counterflow heat exchanger with low pressure. It leaves the heat exchanger at room temperature. Since the JT refrigerator is a relatively simple type of refrigerator, it is widely applied to liquefy natural gas on a large scale.

9.3.2 Choice of Refrigerator Suitable for Superconducting Power Apparatus

Comparison of refrigeration efficiency amongst several refrigerating methods in different temperature ranges is shown in Figure 9.28. The operating temperature in HTS power application is in the range of LN₂ (77 K), and for this the Stirling refrigerator has the highest efficiency and is about 32% of that of the Carnot cycle amongst all refrigerators suitable for HTS power apparatus.

There are several choices of refrigerators for HTS power apparatus, the main factors, such as reliability, operation simplicity and environment adaptability (i.e., cooling water, precooling LN₂, noise, occupational area, etc.) are first considered, then their efficiency, which plays an important role in reducing the operating cost of HTS power apparatus for large-scale refrigerating systems with several hundreds of kW input power.

9.4 Cooling Technologies of Superconducting Electric Apparatus

In general, there are five cooling methods in superconducting power apparatus: open-cycle cooling with immersion, closed-cycle cooling by reducing pressure, closed-type cooled with immersion, forced-flow circulation cooling and direct cooled by refrigerator.

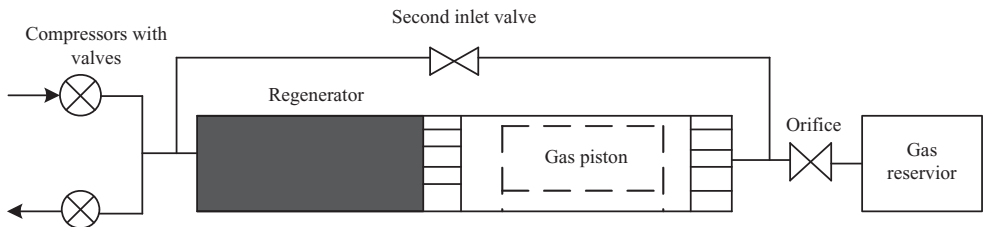


Figure 9.26 Principle of pulse-tube refrigerator.

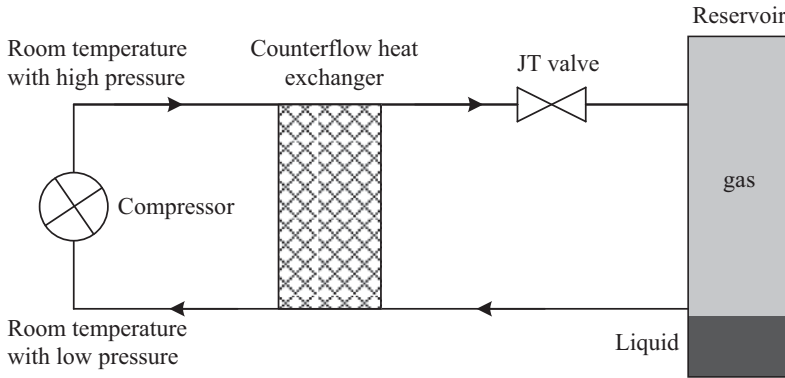


Figure 9.27 Principle of JT refrigerator.

9.4.1 Open-Cycle Cooling

Open-type cooling with immersion is realized by immersing superconductors in the cryogen (i.e., LN_2), so cryogen vapour is completely discharged into the atmosphere without recycling. Immersion cooling refers to superconducting apparatus that are directly immersed in the cryogen and then cooled to cryogenic temperature by the latent heat of phase change of the cryogen when it is vaporizing (Figure 9.29).

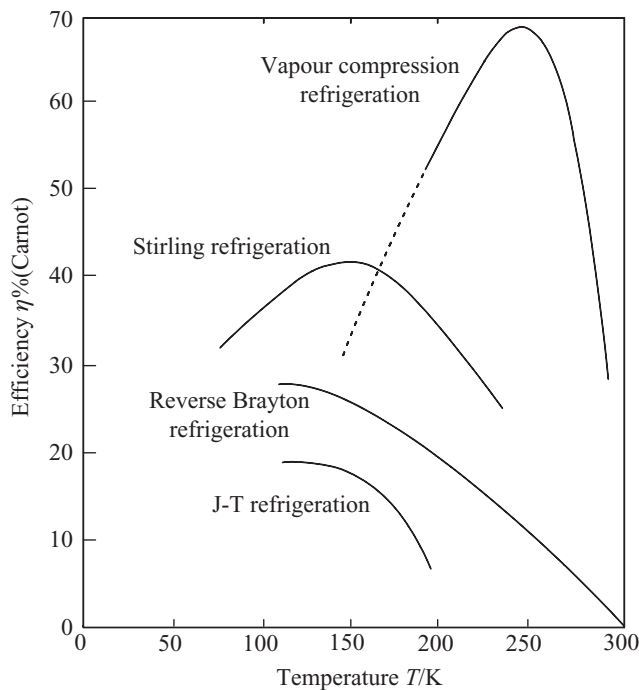


Figure 9.28 Refrigeration efficiency comparisons of several refrigerators.

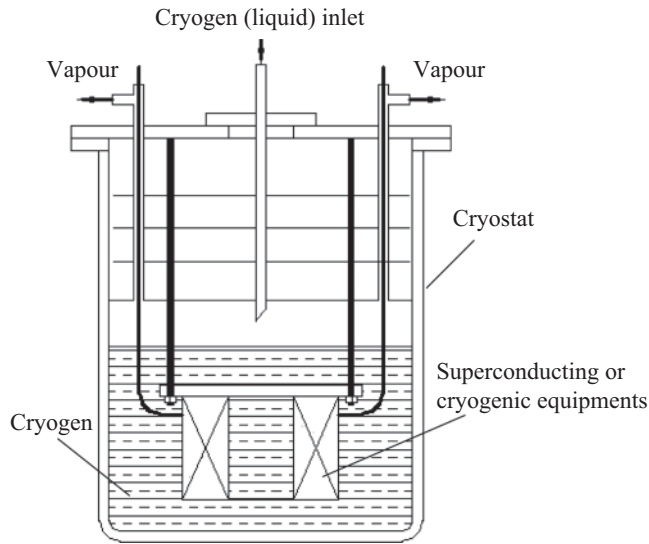


Figure 9.29 Open-cycle cooling with immersion.

Immersion cooling techniques can be classified into LHe, LN_2 and LNe cooling, according to different cryogens. As for HTS magnets, LN_2 and LNe are commonly used as the cryogen for immersion cooling. Most of HTS power apparatus operate in the LN_2 temperature range (i.e., 60 K–90 K); only a few HTS magnets, such as the HTS generator/motor and SMES, operate in the lower temperature range, such as the LNe temperature range (~ 27 K) to improve their efficiency, reliability and engineering current density. Although this cooling method is relatively simple, it is only suitable for laboratory use and not for applications of large-scale superconducting power engineering.

9.4.2 Closed-Cycle Cooling by Reducing Pressure

Similar to the cooling method described in Section 9.2.5, closed-type vacuum cooling with immersion aims to reduce the boiling temperature of the cryogen by lowering its pressure. Its principle procedure is illustrated in Figure 9.30, where cooling of the cryogen in the cycle is achieved by evaporation of the cryogen with negative pressure in sub-cooler 2, which is connected with a pump. Pressure of sub-cooler 2 is then reduced because cryogenic gas is pumped out of the cryostat, so that the cryogen temperature decreases. The degree of sub-cooling is controlled and realized by the pressure of reserved cryogen storage tank 1 and sub-cooler 2.

Though this cooling method is simple, it is not suitable for applications where long-term operation is required; since cooling is realized by vaporization of the cryogen and much more cryogen is consumed, the cryogen needs to be replenished continuously.

9.4.3 Closed-Cycle Cooling by Refrigerator

Closed-cycle cooling with immersion means that superconducting apparatus are directly cooled by immersion in the cryogen, but cryogenic vapour is re-condensed into cryogenic liquid by the refrigerator and returned to the cryostat, so little or no cryogen is consumed in the cryostat system of the

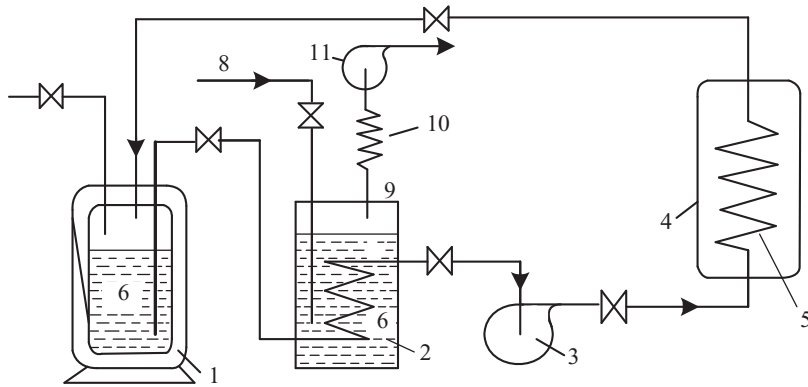


Figure 9.30 Vacuum cooling with immersions: 1. reserved tank of cryogen; 2. sub-cooler; 3. cryogenic pump; 4. cryostat (cooled object); 5. heat exchanger; 6. cryogen; 7. pressurised gas; 8. replenished cryogen; 9. vapour of cryogen; 10. air heat exchanger; 11. gas bump. (Reproduced with permission from Yinshun Wang, “Bases of Applied Superconductivity in Electrical Engineering,” Science Press, 287, © 2011.)

superconducting apparatus. Closed-cycle cooling with immersion for HTS apparatus is shown in Figure 9.31, whose structure is equivalent to inserting a volatile vapour condenser at the top of the typical open-type cooling system with an immersion system.

The refrigerator combines with a cooling system with immersion by thermo-siphon circulation of cryogen. After the cryogen in the cooling system evaporates, vapour moves up into the refrigerator and is then liquefied by the cold head of the refrigerator (also known as a cold exchanger), and then the liquefied cryogen automatically flows back into the cryostat system due to gravity. If the cooling capacity of the refrigerator is equal to the thermal load of the combination of cooled apparatus and condenser, it is not essential for the cryostat system to replenish the cryogen liquid, so that operation costs and maintenance of cooled apparatus can be greatly reduced. Therefore, closed-type cooling with immersion, which combines the advantages of open-type cooling with immersion and direct cooling by the refrigerator as well as simple operation, is the optimal cooling method in applications of superconducting power apparatus and is suitable for long-term operation. Because the cold head of the refrigerator is set in the same cryostat as the superconducting apparatus, whose high voltage insulation is hard to deal with, this cooling method is only suitable for occasions where the apparatus are of the mid- and low voltage classes.

9.4.4 Forced-Flow Circulation Cooling

Sub-cooled forced-flow circulation cooling is mainly applied in large-scale superconducting power apparatus, such as superconducting cable, transformer, FCL, etc. Differing from superconducting power apparatus with mid- and low voltages and small scale, cooling in large-scale superconducting power apparatus needs to overcome viscous resistance of cryogen in the cooling process; meanwhile, it also needs to avoid deterioration of the local cooling effect due to there being no vapour generated after the cryogen absorbs heat in the process of long-distance flowing. Consequently, forced-flow circulation cooling is an optimal choice. A diagram of forced-flow circulation cooling for superconducting power apparatus is presented in Figure 9.32.

The cryogenic system mainly contains a refrigerator, heat exchanger, cryopump, cryogenic storage tank, etc. The cooling system mainly consists of three parts: superconducting power apparatus, cooling part and cryogen storage tank. The former two parts are connected by flexible thermal insulation pipes

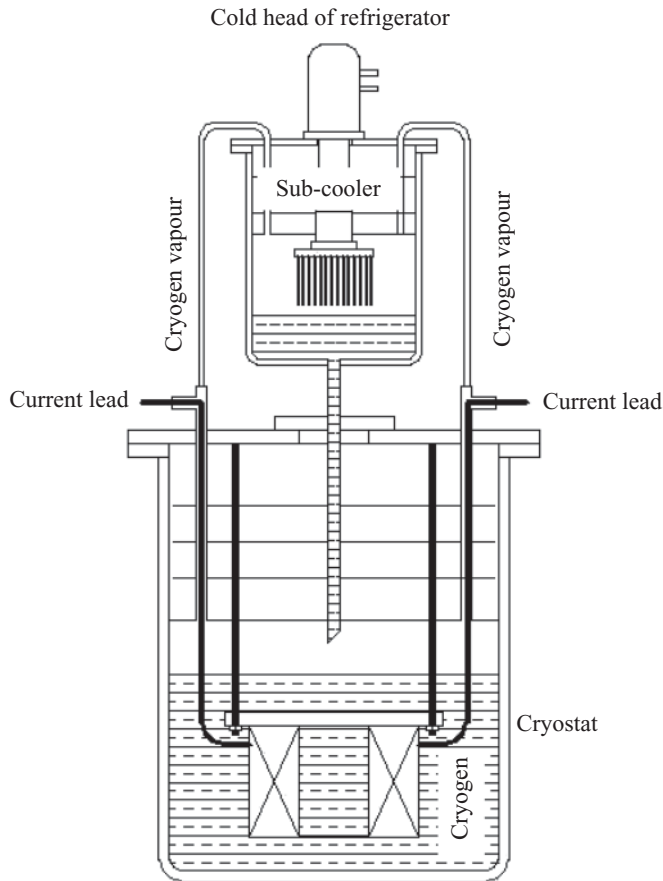


Figure 9.31 Closed-cycle cooling with immersion of liquid nitrogen for superconducting power apparatus.

(transfer line) through which the sub-cooled liquid cryogen circulates. Current leads, pressure reducing valves, volatile vapour pipes and transfer lines are installed on the top cover of the cryostat of the superconducting power apparatus; the inlet of sub-cooled cryogen is placed at the bottom of the cryostat, and the outlet of sub-cooled cryogen is installed above the top cover of the superconducting power apparatus of the cryostat, in order to ensure the cryogen returns to the refrigerating system.

The sub-cooled cryogen with 1 atmospheric pressure is input into the bottom of the cryostat of the superconducting power apparatus. The sub-cooled cryogen with higher temperature circulates back into the refrigerating system and exchanges heat with the cryogen cooled by the refrigerator in the heat exchanger system. Then the cooled cryogen is transferred into the cryostat of the superconducting power apparatus again by the cryopump. Pressure P_2 of the superconducting power apparatus is made higher than P_1 of the refrigerating system by controlling gas injection of the intermediate gas steel bottle with high pressure and pressure reducing valve. The cryogen in the cryostat of the superconducting power apparatus can automatically return to the refrigerating system due to the pressure difference ($P_2 - P_1$).

A refrigerating system is composed of a cryostat, cryopump and refrigerator. The cold head of the refrigerator is connected with a heat exchanger and both of them are placed in the sub-cooled cryogen that can exchange heat with the cryogen circulating from the cryostat of the superconducting power

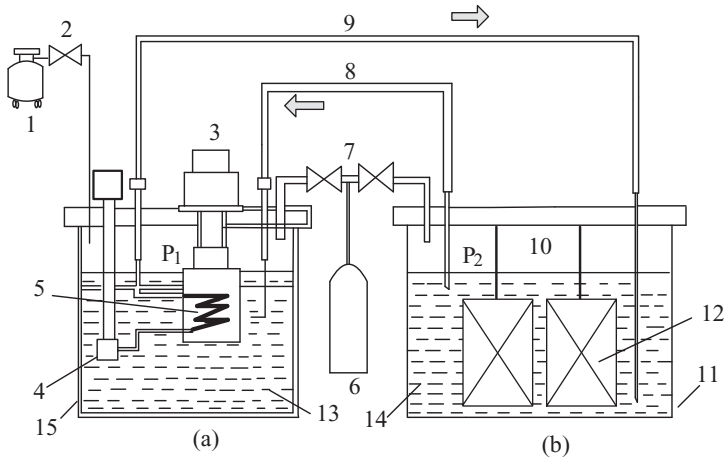


Figure 9.32 Forced-flow cooling process: (a) refrigerating system; (b) cryostat and superconducting power apparatus. 1. cryogen storage tank; 2. cryogenic valve; 3. cold head of refrigerator; 4. cryopump; 6. gas steel bottle; 7. reducing pressure valve; 8. output of cryogen; 9. input of cryogen; 10. current leads; 11. cryostat; 12. superconducting power apparatus; 13. sub-cooled cryogen; 14. cryogen; 15. cryostat.

apparatus. Then, the sub-cooled cryogen is circulated into the cryostat of the superconducting power apparatus by the cryopump.

Control parts are composed of pressure controller, liquid level meter, thermometer and cryopump. The pressure controller can keep the pressure of the cryostat of the superconducting power apparatus slightly higher than 1 atmospheric pressure; the liquid level meter is used to monitor the liquid height of the cryogen in the cryostat of the superconducting power apparatus to ensure that the entire superconducting power apparatus is in the superconducting state. Temperature sensors are separately installed on top, middle and bottom of the superconducting power apparatus to detect the variation of temperature and then refrigerating capacity is controlled so that the superconducting power apparatus can operate normally.

A closed-type circulation system with forced-flow type cooling by using a refrigerator has many advantages, such as little maintenance of the cryogenic system, high efficiency, low noise, low cost, etc. Furthermore, it is suitable for applications of the superconducting power apparatus of high voltage rating, because the superconducting power apparatus is separated from the refrigerating system so the low voltage cryogenic electric insulation can be easily realized.

9.4.5 Direct Cooling by Refrigerator

Based on the development of HTS current leads with low heat leakage and technology progress on miniaturization of the G-M refrigerator, direct cooling by refrigeration has developed and promotes its applications in superconducting power apparatus and superconducting devices. Direct cooling of a superconducting power apparatus by a refrigerator is completed by solid heat conduction, which changes traditional cooling modes of immersion and convective heat transfer. Figure 9.33 shows the structure of direct cooling superconducting apparatus by a G-M refrigerator, which mainly consists of superconducting apparatus, HTS current leads, G-M refrigerator, radiation shielding screen, etc. The superconducting apparatus is directly connected with the cold head of the refrigerator by oxygen-free copper with high thermal conductivity and thus is cooled by conduction. It has many advantages, such as easy operation, cryogen-free, no complicated devices for the cryogen, compact size, maintenance free,

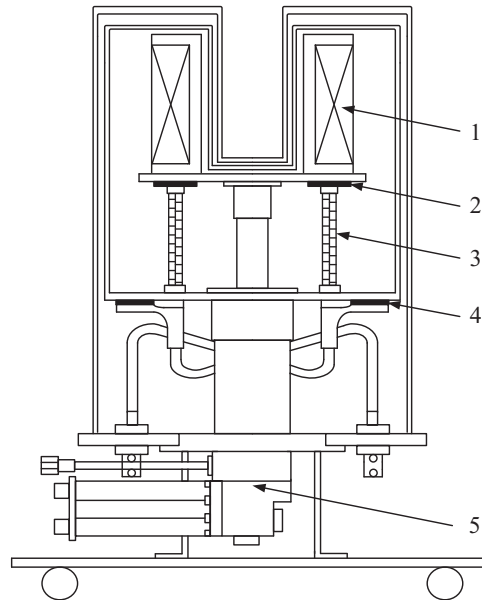


Figure 9.33 Structure of superconducting magnet direct cooled by G-M refrigerator refrigerating: 1, superconducting magnet; 2, electric insulation washer; 3, HTS current leads; 4, electric insulation washer; 5, G-M refrigerator.

low operation costs, etc. In addition, LN_2 below a temperature of 64 K becomes solid, which cannot be used as a cryogen; only LNe is a suitable choice as a cryogen under 64 K.

But, unlike the convenient LN_2 , LNe needs to be recycled due to its high price. However, the refrigerator can simultaneously provide cooling capacity in temperature ranges of LHe, LN_2 and LH_2 , so that it is easily used in HTS power apparatus which operate in temperature ranges below 64 K. HTS power apparatus, such as the HTS generator or motor, can operate in the temperature range 20 K to 30 K, and their performances can be greatly improved compared with the temperature range of LN_2 . At present, the direct cooling technique by refrigeration is mainly applied in superconducting magnets or small devices with DC and low voltage, but is not suitable for applications of superconducting power apparatus with AC and high voltage.

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10

Power Supplying Technology in Superconducting Electrical Apparatus

Superconducting power apparatus operate at low temperatures and need to obtain power from a power supply at room temperature. The external power supply must be connected to the superconducting apparatus by a pair of current leads through which the temperature spans a range from room temperature to 77 K or 4.2 K. Therefore, current leads are usually the maximum heat leakage from room temperature to low temperature, which determines the operating costs of the superconducting power apparatus.

In addition to conduction heat leakage, the Joule heat, generated in current leads operating at high current capacity, is another source of heat loss in cryogenic systems, because the superconducting electrical apparatus often operates at higher currents in the range of several amperes through several tens of thousands of amperes. In order to reduce the heat loss of current leads to a minimum, it is always expected that the sum of conduction heat and Joule heat of current leads should be as small as possible. However, thermal conductivity is the reciprocal of resistivity according to the Wiedemann–Franz law. For the same shape of conventional metal current lead, if it has low thermal conductivity, resistivity increases and then Joule heat also increases though the conduction heat leakage decreases. Instead, if it has low resistivity, although the Joule heat greatly reduces, its thermal conductivity increases and then conduction of the heat leakage also increases.

In addition, for current leads made of the same kinds of materials, when taking its cross-section and length into account, the Joule heat decreases and the conduction heat leakage simultaneously increases if its cross-section increases. When the cross-section is reduced, the situation is exactly reversed. We try to reduce the heat leakage of current leads by all means, so several types of current leads, such as gas-cooled current leads, a hybrid combination of high temperature superconductors (HTS) and conventional current leads, and Peltier current leads (PCL) are developed at the same time, optimizing the structure design of conduction-cooled current leads. The heat leakage markedly reduces and then the cooling load is effectively decreased so that the efficiency improves to a great extent.

When superconducting electrical apparatus (usually referring to as a superconducting magnet) operates in direct current (DC) mode, the persistent current switch (PCS) combined with current leads of demountable structure are adopted. After the superconducting magnet is excited, PCS is closed (superconducting state) and the superconducting magnet can operate in the closed loop, and then the current

leads are removed to minimize the heat input. Current flows continuously in the magnet, so that an external supply power is not necessary.

Another method for reducing heat leakage of current leads is by using flux-pump technology with no current leads. The flux pump is realized by the full-wave chopping technology of an external transformer, in which the primary winding connects with an external power supply and a secondary one connects with the PCS and superconducting magnet.

10.1 Current Leads

Based on structure and cooling methods, current leads can be classified into several types. They can be divided into conduction-cooled current leads and gas-cooled leads according to cooling modes. There are two kinds of demountable current leads and fixed leads according to methods connected with superconducting magnet. They can be classified into conventional conduction-cooled current leads, HTS current leads, PCLs, etc., based on the materials used. The design of various types of current leads, such as conventional conduction-cooled current leads (including demountable current leads), gas-cooled current leads, HTS current leads, hybrid current leads, combination of conventional metal and HTS, and PCLs are described in this section. The general heat equilibrium equation for describing current leads in units of W/m is:

$$\nabla[kA\nabla T] + G_j + G_d - G_q = 0 \quad (10.1)$$

where k is the thermal conductivity, A refers to the cross-section, G_j stands for the Joule heating term; G_d represents the heat term resulting from magnetic thermal effect (flux jump), mechanical loss and alternating current (AC) losses, etc. and G_q is the cooling term.

10.1.1 Conduction-Cooled Current Leads

Figure 10.1 shows the operation of conduction-cooled current leads connected to superconducting electrical apparatus, which are cooled by direct immersion in the cryogen; the current leads are made from oxygen-free copper. Current leads are connected with both external supply power SS and superconducting electrical apparatus located in the cryogenic environment, and temperatures on the current leads span a range from room temperature to cryogenic temperature. The heat flow of a current lead is shown in Figure 10.2, where temperatures on the left and right ends are T_L and T_H , respectively, and L refers to its length. In the conduction-cooled (adiabatic) mode, $G_d = G_q = 0$. If a current lead transports current I , there is a Joule heat term $G_j = \rho I^2/A$. The origin of the coordinate is at the low temperature end [1].

The heat conduction Equation (10.1) of current leads becomes:

$$\nabla[k(T)A\nabla T] + \frac{\rho(T)I^2}{A} = 0 \quad (10.2)$$

where, $k(T)$, $\rho(T)$ and A are thermal conductivity, resistivity and cross-section of current lead, respectively; and I denotes the transport current.

If length L is much larger than the size of the cross-section, Equation (10.2) can be described as a one-dimensional (1D) thermal conduction model, and Equation (10.2) becomes:

$$\frac{d}{dx} \left[k(T)A \frac{dT}{dx} \right] + \frac{\rho(T)I^2}{A} = 0 \quad (10.3)$$

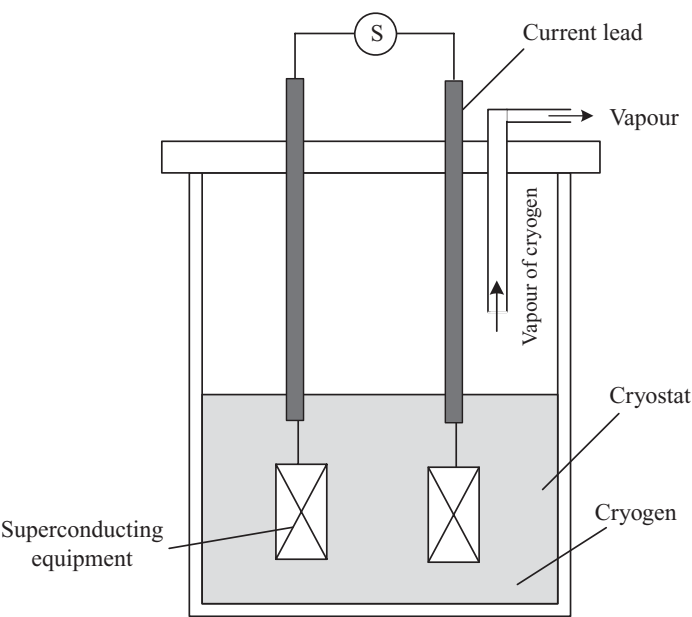


Figure 10.1 Conduction-cooled current leads.

According to Fourier’s law of thermal conduction, the heat leakage of the current leads is:

$$Qdx = kAdT$$

(10.4)

that is:

$$\frac{dx}{kA} = \frac{dT}{Q}$$

(10.5)

Combining Equations (10.3) through (10.5), we obtain:

$$Q(T) = \sqrt{Q_L^2 - 2I^2 \int_{T_L}^T k\rho dT}$$

(10.6)

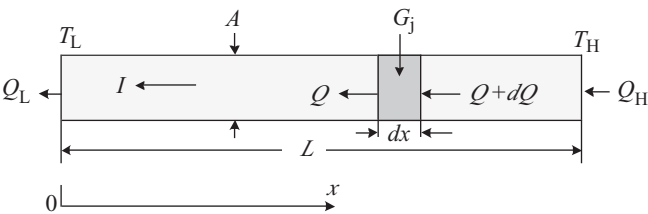


Figure 10.2 1D model of current lead.

The ratio of length to cross-section is:

$$\frac{L}{A} = \int_{T_L}^{T_H} \frac{k}{\sqrt{Q_L^2 - 2I^2 \int_{T_L}^T k\rho \cdot dT}} dT \quad (10.7)$$

There are three operation modes in the current lead: optimized operation, under-current operation and over-current operation.

10.1.1.1 Optimized Operation

When the current lead is the transporting current I , the minimum heat leakage Q_L to cryostat can be achieved by taking the minimum of the Equation (10.6):

$$(Q_L)_{\min} = I \sqrt{2 \int_{T_L}^{T_H} k\rho \cdot dT} \quad (10.8)$$

By substituting Equation (10.8) into Equation (10.7), we can obtain the optimized operation current I_{opt} :

$$\left(\frac{IL}{A}\right)_{\text{opt}} = \int_{T_L}^{T_H} \frac{k}{\sqrt{2 \int_{T_L}^{T_H} k\rho \cdot dT}} dT \quad (10.9)$$

That is:

$$I_{\text{opt}} = \frac{A}{L} \int_{T_L}^{T_H} \frac{k}{\sqrt{2 \int_{T_L}^{T_H} k\rho \cdot dT}} dT \quad (10.10)$$

When the transport current I operates in accordance with Equation (10.10), the heat leakage Q_L from the current leads to the cryostat side can be minimal.

10.1.1.2 Under-Current Operation

If the transport current I is smaller than the current given by Equation (10.10), which is called the undercurrent operation:

$$I < I_{\text{opt}} = \frac{A}{L} \int_{T_L}^{T_H} \frac{k}{\sqrt{2 \int_{T_L}^{T_H} k\rho \cdot dT}} dT \quad (10.11)$$

when the temperature of current leads is T , the heat flow is:

$$Q(T) = \sqrt{Q_L^2 - 2I^2 \int_{T_L}^T k\rho \cdot dT} \quad (10.12)$$

and the ratio of length to its cross-section is:

$$\frac{L}{A} = \int_{T_L}^{T_H} \frac{k}{\sqrt{Q_L^2 - 2I^2 \int_{T_L}^{T_H} k\rho \cdot dT}} dT \quad (10.13)$$

10.1.1.3 Over-Current Operation

If the transport current I is larger than the current determined by Equation (10.10), which is called the over-current operation:

$$I > I_{\text{opt}} = \frac{A}{L} \int_{T_L}^{T_H} \frac{k}{\sqrt{2 \int_T^{T_H} k\rho \cdot dT}} dT \quad (10.14)$$

the heat flow of current leads at the location x with temperature $T(x)$ is:

$$Q[T(x)] = \begin{cases} \sqrt{Q_L^2 - 2I^2 \int_{T_L}^T k\rho \cdot dT} & 0 \leq x \leq L_P \\ -\sqrt{2I^2 \int_T^{T_P} k\rho \cdot dT} & L_P \leq x \leq L \end{cases} \quad (10.15)$$

where L_P is the length of the current leads, to which the highest temperature T_P corresponds and is determined by:

$$Q_L^2 = 2I^2 \int_{T_L}^{T_P} k\rho \cdot dT \quad (10.16)$$

With conditions of over-current operation, the ratio of length to cross-section is:

$$\frac{L}{A} = \int_{T_L}^{T_H} \frac{k}{\sqrt{Q_L^2 - 2I^2 \int_{T_L}^T k\rho \cdot dT}} dT + \int_{T_H}^{T_P} \frac{k}{\sqrt{2I^2 \int_T^{T_P} k\rho \cdot dT}} dT \quad (10.17)$$

Under the conditions of optimized operation, under-current operation and over-current operation, temperature distribution in the current lead are shown separately in Figure 10.3. In situations of under- and optimized operation, the temperature of the current lead is always lower than the room temperature T_H . However, the highest temperature in the current lead is not room temperature T_H but the temperature T_P at position L_P of the current lead, and $T_H > T_P$.

Figure 10.4 presents the relationship between the heat leakage from current lead to low-temperature side and the length as well as the cross-section based on calculation of oxygen-free copper current leads with RRR = 20, where RRR refers to Residual Resistivity Ratio, and the temperature span is from 77 K to 300 K [2]. Under the condition of over-current operation, the current lead will burn out if the current of over-current operation and L/A are more than 0.7 kA and 100 cm^{-1} , respectively.

10.1.2 Approximate Design of Conduction-Cooled Current Lead

Since thermal conductivity k , resistivity ρ and specific heat capacity C of the current lead are functions of temperature T , there are no exact analytical solutions in optimization design of the current leads so that

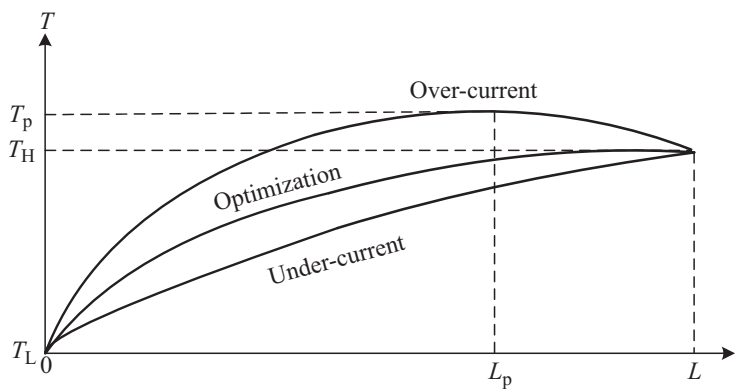


Figure 10.3 Temperature distributions with three conditions: under-current, optimized, and over-current operations.

the numerical calculation must be used. Nevertheless, the accurate analytical solutions can be obtained if some simple approximations are made. Several common approximate solutions are described in this section.

10.1.2.1 Constant Thermal Conductivity and Resistivity

Assume that the average thermal conductivity k and the resistivity ρ over the range of operation temperature are taken in the design of current lead, that is, both of them are constant, by substituting

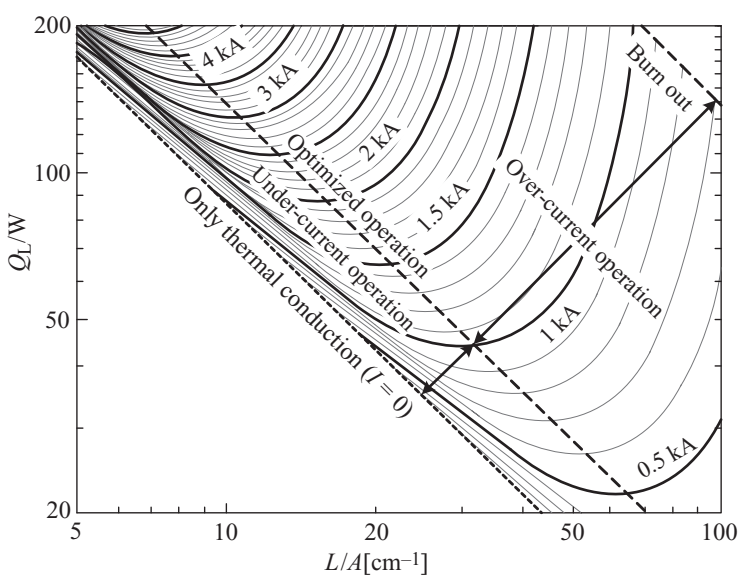


Figure 10.4 Relationships of heat leakage with ratio of length to transport current in lead made from oxygen-free copper, with RRR = 20 in different operations.

them into Equations (10.8) and (10.10), the minimum heat leakage to the low-temperature end and the optimized operation current are separately:

$$(Q_L)_{\min} = I\sqrt{2k\rho(T_H - T_L)} \quad (10.18)$$

$$I_{\text{opt}} = \frac{A}{L}\sqrt{\frac{2k}{\rho}(T_H - T_L)} \quad (10.19)$$

If the current lead is transporting AC current, the design of the optimized operation is described by:

$$\left(\frac{L}{A}\right)_{\text{opt}} = \frac{1}{I_{\text{rms}}}\sqrt{\frac{2k}{\rho}(T_H - T_L)} \quad (10.20)$$

where I_{rms} is the root mean square (RMS) value of the AC current, that is, the optimized design of the AC current lead can be found if the RMS of the AC current replaces the DC current I .

By substituting the constant thermal conductivity k and resistivity ρ into Equations (10.2) and (10.13) or Equations (10.16) and (10.17), then:

$$\frac{L}{A} = \sqrt{\frac{2k}{\rho I^2}} \left[\sqrt{\frac{Q_L^2}{2I^2 k \rho}} - \sqrt{\frac{Q_L^2}{2I^2 k \rho} - (T_H - T_L)} \right] \quad (10.21)$$

$$Q_L = \frac{\rho I^2 L}{2A} + \frac{kA}{L}(T_H - T_L) \quad (10.22)$$

If the current lead is transporting the AC current, the heat leakage can be found by directly integrating Equation (10.22) over a cycle:

$$Q_L = \frac{\rho L}{2A} \int_0^\tau I^2 dt + \frac{kA}{L}(T_H - T_L)\tau \quad (10.23)$$

where τ is the period of one cycle of the AC current.

10.1.2.2 Wiedemann–Franz Approximation

Thermal conductivity k and resistivity ρ of a metal conductor obeys the Wiedemann–Franz law: $k\rho = L_0 T$, where L_0 denotes the Lorentz number, and $L_0 = 2.45 \times 10^{-8} \text{ W}\cdot\Omega\cdot\text{K}^{-2}$. At the room temperature, resistivity ρ and thermal conductivity k approximately meet:

$$\frac{\rho}{\rho_H} = \left(\frac{T}{T_H}\right)^n, \quad \frac{k}{k_H} = \left(\frac{T}{T_H}\right)^{1-n} \quad (10.24)$$

In the temperature range of liquid nitrogen through room temperature, $n \approx 1$, so that k is approximately a constant and resistivity ρ is proportional to temperature. According to the Wiedemann–Franz law:

$$\rho = \frac{L_0}{k} T = \alpha T, \quad \alpha = \frac{L_0}{k} \quad (10.25)$$

Then the solution of Equation (10.3) is:

$$T(x) = \frac{T_L \sin(\beta x) + T_H \sin[\beta(L-x)]}{\sin(\beta L)}, \quad \beta = \frac{I}{A} \sqrt{\frac{\alpha}{k}} \quad (10.26)$$

The heat leakage to the low-temperature end is:

$$Q_L = -kA \frac{dT}{dx} \Big|_{x=0} = I\sqrt{k\alpha} \frac{T_H - T_L \cos \delta}{\sin \delta}, \quad \delta = \sqrt{\frac{\alpha}{k}} \frac{LI}{A} \quad (10.27)$$

- (1) *Optimized operation*: By taking the limit value of Q_L , that is, $dQ_L/d(L/A) = 0$, we find the optimal operation current I_{opt} and the minimum heat leakage $Q_{L\text{min}}$:

$$\left(\frac{LI}{A}\right)_{\text{opt}} = \frac{k}{\sqrt{L_0}} \cos^{-1} \left(\frac{T_L}{T_H}\right) \quad (10.28)$$

$$Q_{L\text{min}} = I\sqrt{L_0(T_H^2 - T_L^2)} \quad (10.29)$$

- (2) *Under-current operation*: As for under-current operation, that is, $I < I_{\text{opt}}$, based on Equations (10.12) and (10.13), heat leakage of the current lead and the ratio of length to its cross-section L/A are respectively:

$$Q_L = I\sqrt{L_0} \left[T_H \csc \left(\frac{IL\sqrt{L_0}}{kA} \right) - T_L \cot \left(\frac{IL\sqrt{L_0}}{kA} \right) \right] \quad (10.30)$$

$$\frac{L}{A} = \frac{k}{I\sqrt{L_0}} \left(\sin^{-1} \frac{T_H}{T_P} - \sin^{-1} \frac{T_L}{T_P} \right) \quad (10.31)$$

where T_P stands for a “virtual” peak temperature and is defined by:

$$T_P = \sqrt{\frac{Q_L^2}{I^2 L_0} + T_L^2} \quad (10.32)$$

where the “virtual” peak temperature $T_P < T_H$.

10.1.2.3 Over-Current Operation

In the case of over-current operation, $I > I_{\text{opt}}$, the heat leakage calculation of Equation (10.15) is similar to the under-current operation of Equation (10.30), but $T_P > T_H$. According to Equation (10.17), the ratio of length to cross-section L/A is:

$$\frac{L}{A} = \frac{k}{I\sqrt{L_0}} \left[\cos^{-1} \left(\frac{T_H}{T_L} \right) + \cos^{-1} \left(\frac{T_L}{T_H} \right) \right] \quad (10.33)$$

If the operation current is the AC current, $I = I(t)$ and its period of cycle is τ , then the heat leakage load of the current lead is:

$$Q_L = \sqrt{L_0} \left[T_H \int_0^\tau I \csc \left(\frac{IL\sqrt{L_0}}{kA} \right) dt - T_L \int_0^\tau I \cot \left(\frac{IL\sqrt{L_0}}{kA} \right) dt \right] \quad (10.34)$$

Thus, Q_L can be optimized by taking $dQ_L/d(L/A) = 0$, and the optimized result is:

$$\frac{T_L}{T_H} = \frac{\int_0^\tau I^2 \csc \left[\frac{I\sqrt{L_0}}{k} \left(\frac{L}{A} \right)_{\text{opt}} \right] \cot \left[\frac{I\sqrt{L_0}}{k} \left(\frac{L}{A} \right)_{\text{opt}} \right] dt}{\int_0^\tau I^2 \csc^2 \left[\frac{I\sqrt{L_0}}{k} \left(\frac{L}{A} \right)_{\text{opt}} \right] dt} \quad (10.35)$$

10.1.2.4 Calculation of Peak Temperature T_P

In the situation of over-current operation, according to Figure 10.2, the energy balance equation is:

$$Q \cdot dQ = -L_0 I^2 T \cdot dT \quad (10.36)$$

When considering the Wiedemann–Franz law $k\rho = L_0 T$ and heat conduction Equation (10.4) of the Fourier law, we can obtain Equation (10.37) by integrating Equation (10.36) over a temperature range of T_L through T_P :

$$(Q_P^2 - Q_L^2) = -L_0 I^2 (T_P^2 - T_L^2) \quad (10.37)$$

At peak temperature T_P , the temperature gradient satisfies $dT/dx = 0$, thus:

$$Q_P = -kA \frac{dT}{dx} \Big|_{T=T_P} = 0 \quad (10.38)$$

Combining Equations (10.30) with (10.27), we can find the peak temperature T_P :

$$T_P = \csc \left(\frac{IL\sqrt{L_0}}{kA} \right) \sqrt{T_H^2 + T_L^2 - 2T_H T_L \cos \left(\frac{IL\sqrt{L_0}}{kA} \right)} \quad (10.39)$$

The calculation of peak temperature is only suitable for over-current operation, that is $T_P > T_H$. Contrary to the “virtual” peak temperature in under-current operation, that “virtual” peak temperature is just a calculated parameter.

In a practical situation, the Lorentz number L_0 usually is not a constant but increases with a rise in temperature. Figure 10.5 shows the dependence of the Lorentz number on temperature in copper, aluminium and silver. Therefore, the Wiedemann–Franz law deviates from the actual cases [3]. In addition, there is no analytical expression between the Lorentz number and temperature, so the exact calculation needs numerical solution.

10.1.2.5 Mcfee Approximation

Another approximate method for optimal design of a current lead was proposed by Mcfee in 1959. In non-optimal operation condition of a current lead, it is assumed that the heat leakage load to the

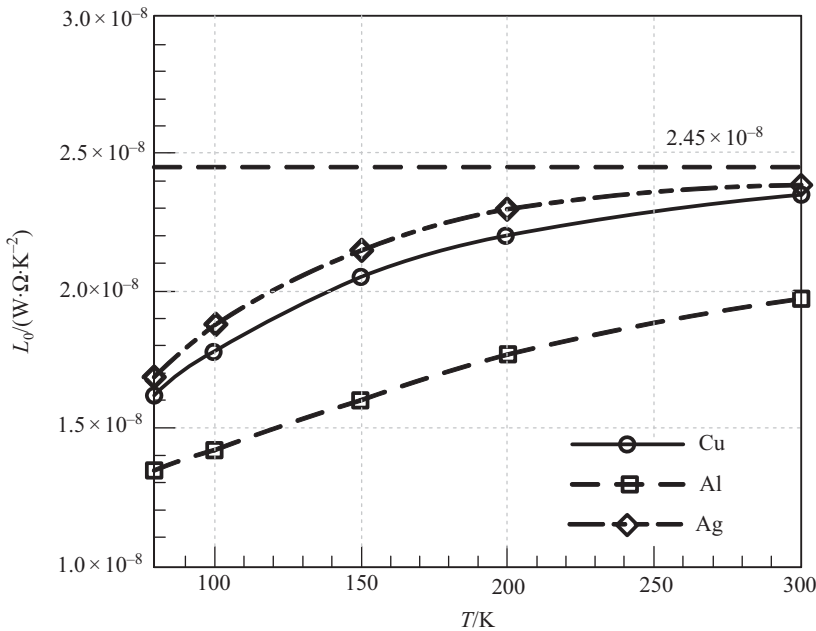


Figure 10.5 Dependences of Lorentz number on temperature.

low-temperature side is introduced by:

$$Q_L = \frac{(Q_L)_{\min}}{2} \left(\frac{I}{I_{\text{opt}}} + \frac{I_{\text{opt}}}{I} \right) \quad (10.40)$$

where $(Q_L)_{\min}$ and I_{opt} are determined by Equations (8.10) and (10.10), both of them are approximated by the Wiedemann–Franz law to simplify calculation (the optimized design results are already shown in Equations (10.28) and (10.29)). By substituting them into Equation (10.40), we have:

$$Q_L = \frac{\sqrt{L_0(T_H^2 - T_L^2)}}{2} \left[\frac{L\sqrt{L_0}}{kA \cos^{-1}(T_L/T_H)} I^2 + \frac{kA}{L\sqrt{L_0}} \cos^{-1}\left(\frac{T_L}{T_H}\right) \right] \quad (10.41)$$

$$\left(\frac{L}{A} \right)_{\text{opt}} = \frac{k}{I\sqrt{L_0}} \cos^{-1}\left(\frac{T_L}{T_H}\right) \quad (10.42)$$

If the current lead transports the AC current with a period of cycle τ , the heat leakage load to the low-temperature side is found by integrating Equation (10.41) in one cycle:

$$Q_L = \frac{\sqrt{L_0(T_H^2 - T_L^2)}}{2} \left[\frac{L\sqrt{L_0}}{kA \cos^{-1}(T_L/T_H)} \int_0^\tau I^2 dt + \frac{kA}{L\sqrt{L_0}} \cos^{-1}\left(\frac{T_L}{T_H}\right) \tau \right] \quad (10.43)$$

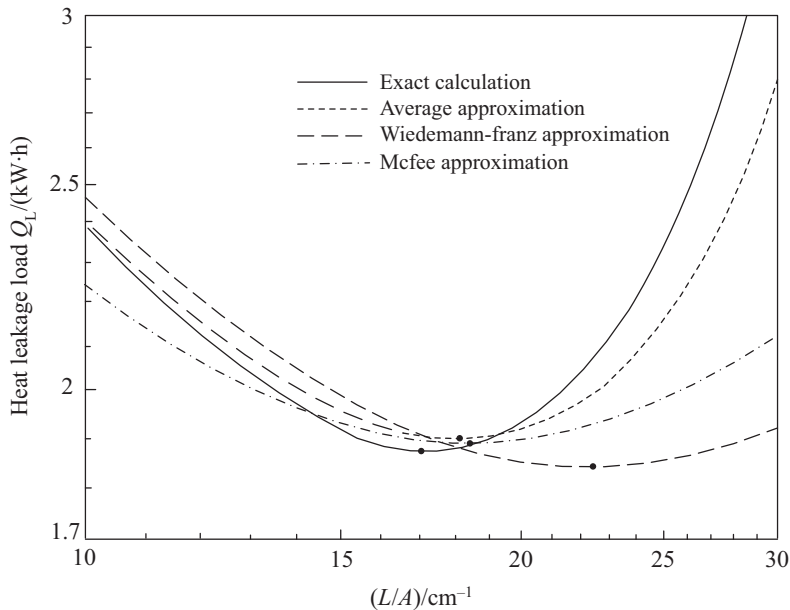


Figure 10.6 Comparisons of heat leakage load by different approximations.

Differentiating L/A in Equation (10.42), $dQ_L/d(L/A) = 0$, the optimized result of the current lead in AC operation can be directly obtained:

$$\left(\frac{L}{A}\right)_{\text{opt}} = \frac{k}{I_{\text{rms}}\sqrt{L_0}} \cos^{-1}\left(\frac{T_L}{T_H}\right) \quad (10.44)$$

where I_{rms} is the RMS value of the AC current.

In order to understand the three approximate and accurate calculations for optimal design of the current lead, Figure 10.6 illustrates optimal results and their comparison between three approximate and accurate calculations of total cooling load for a day. It is indicated that the Wiedemann–Franz approximation is close to the accurate calculated results, while the results calculated by averaging thermal conductivity and resistivity over temperature deviate from calculated results to a large extent, and the results by the Mcfee approximation is between that of the two methods.

10.1.3 Demountable Current Leads

Demountable current leads usually have conduction-cooled structures with the same optimal design as their conduction-cooled partners, but they must be used in conjunction with the superconducting PCS, which will be introduced in Section 10.2. There is a difference between common conduction-cooled current leads and demountable ones, which are detachable from the superconducting electrical apparatus (generally DC superconducting magnets). Superconducting electrical apparatus are connected with PCS in parallel, and PCS opens before exciting the superconducting magnet by the power supply. After exciting is finished, the PCS closes, and the demountable current leads are disconnected from the superconducting electrical apparatus and pulled away, then the superconducting electrical apparatus operates in a closed-loop manner. Therefore, the heat leakage of current leads can be completely eliminated.

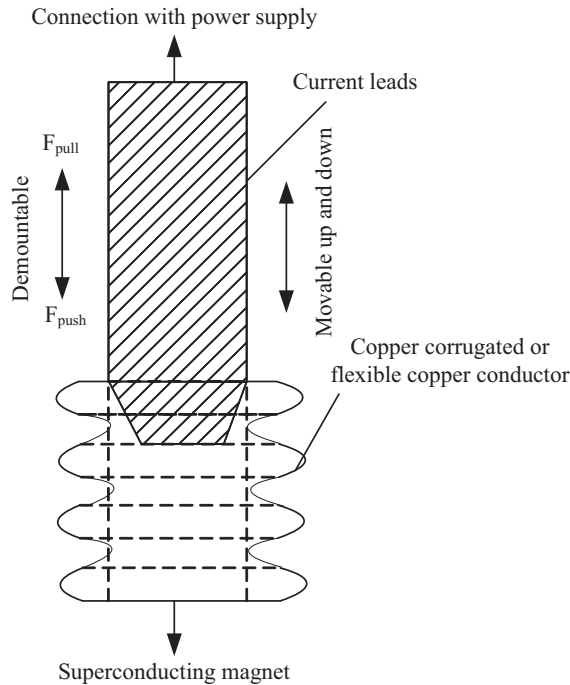


Figure 10.7 Structure of single demountable current lead.

The structure of demountable current leads is shown in Figure 10.7. The top of a demountable current lead directly connects with the power supply, and its bottom connects with the superconducting electrical apparatus through a corrugated copper pipe or flexible copper conductor, which is fixed to the superconducting electrical apparatus. The current lead is pressed into the corrugated copper pipe or flexible copper conductor to form a current path. After exciting of the superconducting electrical apparatus is finished, the PCS closes, and current leads are pulled away from the corrugated copper pipe or flexible copper conductor so that the heat leakage can be completely eliminated. The demountable current leads are widely used in most MRI (NMR) and other commercial superconducting magnets.

10.1.4 Gas-Cooled Current Leads

If the superconducting electrical apparatus are cooled by immersion in the cryogen and are operated in a large current condition, the structure of gas-cooled current leads is usually adopted in order to reduce heat leakage to the cryostat. In this case the cooling capacity of boiling-off gas from the cryogen is made full use of to cool the current leads [4], which can effectively reduce the temperature gradient along the current leads and then decrease the heat leakage to the cryogen. Figures 10.8 and 10.9 show two simple structures of gas-cooled current leads. In Figure 10.8, a sleeve is installed outside the current lead and connects with a flange of the cryostat, which is vacuum sealed, and the boiling-off gas can only flow through the sleeve, so that the outer surface of the current lead is first cooled by the cooling gas. Generally, some radiators made from metals with high thermal conductivity are welded onto the surface of the current leads by increasing the cooling contact areas and decreasing the longitudinal flow velocity of the cooling gas, so that current leads can fully exchange heat with the cooling gas. Another type of

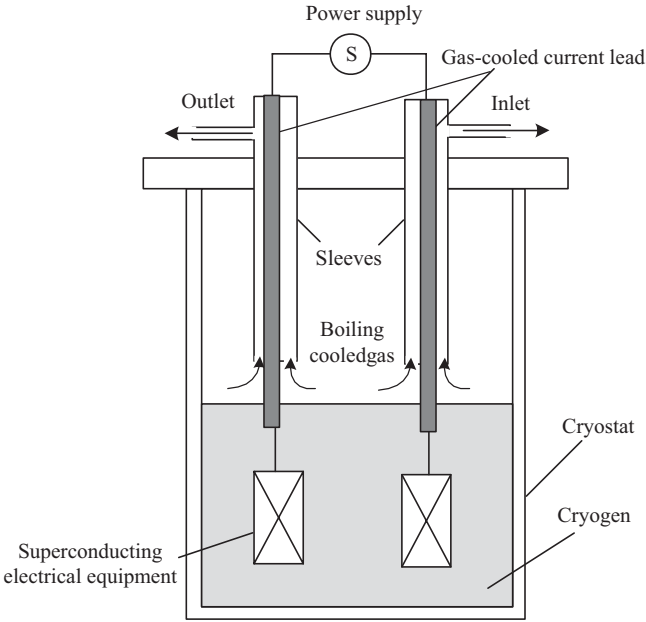


Figure 10.8 Structures of gas-cooled current leads with sleeves.

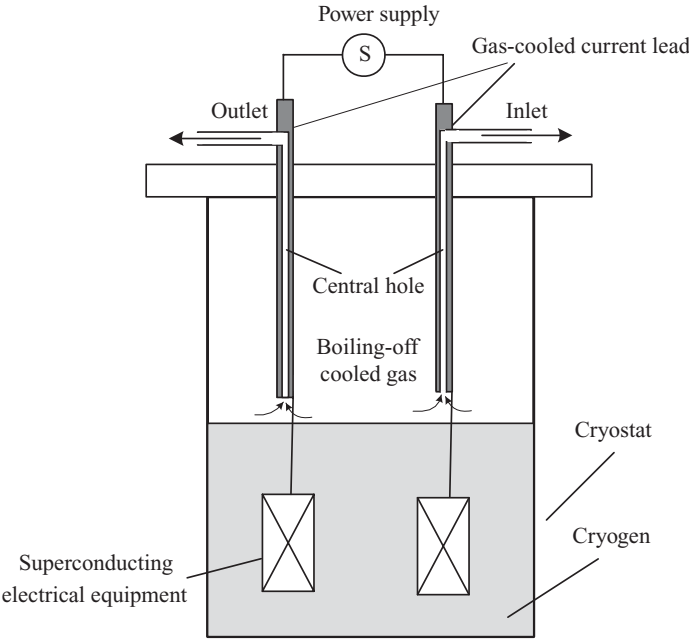


Figure 10.9 Structures of gas-cooled current leads with central holes.

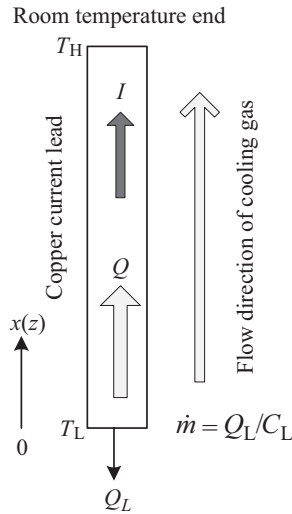


Figure 10.10 Single gas-cooled current lead section.

gas-cooled current lead is illustrated in Figure 10.9. A hole is drilled at the centre of the current lead and directly opens onto the outside of the cryostat, so boiling gas can only flow through the hole because the cryostat is vacuum sealed. Usually it is difficult to manufacture this type of structure of long gas-cooled current leads, but it can be realized by the welding process of multi-segments with central holes.

The structure of a gas-cooled current lead made from copper is shown in Figure 10.10. The lower end is at a low temperature and contacts with the cryogen, the upper end is at room temperature and contacts with the power supply. The coordinate origin locates at the low temperature end, and L denotes the length of current lead with A standing for its cross-section. If the transversal dimension of the current lead is much smaller than its longitudinal length, the current lead can be treated as a 1D model of heat conduction. In the heat conduction Equation (10.1), heating term $G_d = 0$, and G_J is the Joule heat term. In the condition of a gas-cooled current lead, the cooling term G_q is:

$$G_q = Ph(T - \Theta) = f \dot{m} C_g(\Theta) \frac{d\Theta}{dt} \quad (10.45)$$

where P denotes the cooling perimeter, Θ is temperature of cooling gas; $\dot{m} = dm/dt = Q_L/C_L$ represents the evaporation mass rate of cooling gas, C_L and Q_L stand for the latent heat of the cryogen and the heat leakage to the low temperature end, respectively, f is convective heat transfer efficiency, here $0 \leq f \leq 1$, $f = 0$ corresponds to conduction current lead discussed in Section 10.1.1, and h refers to the heat transfer coefficient between current lead and cooling gas and is determined by:

$$h = Nu \frac{\lambda_g}{De} \quad (10.46)$$

where Nu is Newsselt number; λ_g is the thermal conductivity of cooling gas, $De = 4A/P$ is the hydraulic diameter of current lead, and A and P stand for cross-section and perimeter of current lead, respectively. By substituting Equations (10.45) and (10.46) into Equation (10.1), with the 1D model, we have:

$$\frac{d}{dx} \left(k(T)A \frac{dT}{dx} \right) + \frac{\rho I^2}{A} - Ph(T - \Theta) = 0 \quad (10.47)$$

or

$$\frac{d}{dx} \left(k(T) A \frac{dT}{dx} \right) + \frac{\rho I^2}{A} - f m C_g(\Theta) \frac{d\Theta}{dt} = 0 \quad (10.48)$$

Take the following transformations:

$$dz = \frac{I dx}{kA} \quad (10.49)$$

$$q = \frac{Q_L}{I} \quad (10.50)$$

$$u = \frac{C_p}{C_L} \quad (10.51)$$

$$\alpha = \frac{fqu}{2} \quad (10.52)$$

Combining with the Wiedemann–Franz law $k\rho = L_0 T$, Equation (10.48) becomes:

$$\frac{d^2 T}{dz^2} - 2\alpha \frac{dT}{dz} + L_0 T = 0 \quad (10.53)$$

Considering the boundary conditions of $T(0) = T_L$, $T(z_1) = T_H$, the solution of Equation (10.53) is:

$$T(z) = T_H \frac{e^{\alpha z} \sin(\beta z)}{e^{\alpha z_1} \sin(\beta z_1)} + T_L e^{\alpha z} [\cos(\beta z) - \sin(\beta z) \cot(\beta z_1)] \quad (10.54)$$

where

$$\beta = \sqrt{L_0 - \alpha^2} \quad (10.55)$$

$$z_1 = \frac{I}{A} \int_0^L \frac{dx}{k} \quad (10.56)$$

The minimum heat leakage from the gas-cooled current leads to the low-temperature end is:

$$Q_L = Iq = I \left. \frac{dT}{dz} \right|_{z=0} = \frac{T_H \beta}{e^{\alpha z_1} \sin(\beta z_1)} + T_L [\alpha - \beta \cot(\beta z_1)] \quad (10.57)$$

If $f = \alpha = 0$, Equations (10.54) and (10.57) correspond to the conduction-cooled with help of transforming Equation (10.49), that is, Equations (10.26) and (10.27). Taking the zero temperature gradient at the room temperature end with $T = T_H$ into account, namely $dT/dz = 0$, the temperature distribution of the current lead is:

$$T_H [\alpha + \beta \cot(\beta z_1)] = \frac{T_L \beta e^{\alpha z_1}}{\sin(\beta z_1)} \quad (10.58)$$

$\alpha = f = 0$ corresponds to the optimized design of the conduction-cooled current leads, so Equations (10.57) and (10.58) become:

$$\cos(\beta_0 z_{1\text{opt}}) = \frac{T_L}{T_H}, \Rightarrow \cos\left(\frac{\sqrt{L_0} LI}{kA}\right)_{\text{opt}} = \frac{T_L}{T_H}, \Rightarrow \left(\frac{LI}{A}\right)_{\text{opt}} = \frac{k}{\sqrt{L_0}} \cos^{-1}\left(\frac{T_L}{T_H}\right) \quad (10.59)$$

$$(Q_L)_{\min} = Iq \Big|_{f=0} = I\beta_0 \sqrt{T_H^2 - T_L^2} = I\sqrt{L_0 (T_H^2 - T_L^2)} \quad (10.60)$$

where $\beta_0 = \sqrt{L_0}$, Equations (10.59) and (10.60) are exactly in agreement with the optimized design Equations (10.28) and (10.29) of the conduction-cooled current lead by taking the Wiedemann–Franz law approximation.

10.1.5 HTS Current Leads

Because thermal conductivity of HTS materials is several orders of magnitude lower than copper and there is no Joule heat generation when they operate in DC mode, HTS materials have low conduction heat leakage to the cryostat, so that HTS current leads are widely used in superconducting magnets operating in liquid helium temperature, especially mostly applicable for conduction-cooled superconducting magnets [5, 6]. As for superconducting apparatus with high transport current and operating at liquid helium temperature, such as a large-scale superconducting magnet used in International Thermonuclear Experimental Reactor (ITER), the HTS current leads are applicable for operating in the temperature range of 4.2 K through 77 K.

The structure of HTS current leads is shown in Figure 10.11(a). The current lead consists of two sections: the upper section is made from normal metal and its upper end is at room temperature; the lower section is made from HTS materials and connects with the superconducting apparatus.

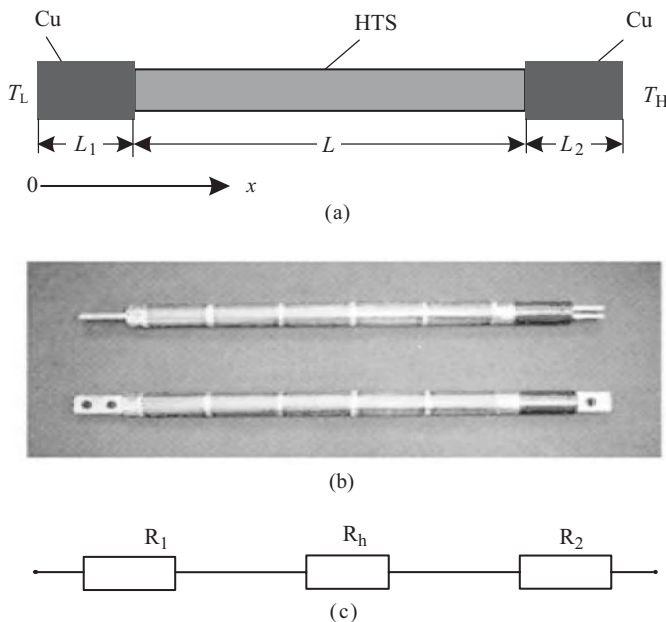


Figure 10.11 Structure of HTS current lead with two components: (a) schematic geometry; (b) practical picture of HTS current leads; and (c) equivalent circuit.

Figure 10.11(b) illustrates a practical HTS current lead [6]. The connected part between the upper and lower sections is cooled by the first-stage cold head of a cryo-cooler whose temperature is below the critical temperature of HTS materials. There is no Joule heat generation in HTS, since the section of HTS operates in a superconducting state. Because oxide HTS materials have a low thermal conductivity (e.g., the average thermal conductivity of Bi-2223 material is only one-third of stainless steel in the temperature range of 4 K through 44 K), the current lead with a structure of two components can remarkably reduce the heat leakage from the room temperature end to the low temperature end. Currently, the heat leakage can be reduced to below one-tenth of the gas-cooled copper current lead (total cooling power can be reduced to one-fifth). Theoretically, the longer the HTS current leads are, the smaller the heat leakage to the low temperature end is.

If the transverse dimension of the HTS current lead is much smaller than its length, the 1D heat conduction equations can be used to describe the HTS current lead. Four cases are discussed.

10.1.5.1 Conduction-Cooled with DC Operation

Under the condition of DC mode, the heat conduction equation of the conduction-cooled HTS current leads is:

$$\begin{cases} \frac{d}{dx} \left[k_i(T) A_i \frac{dT}{dx} \right] + G_{ji} = 0 \\ \frac{d}{dx} \left[k_h(T) A_h \frac{dT}{dx} \right] + G_{jff} = 0 \end{cases} \quad (10.61)$$

The boundary condition is that the temperature in junction $x = x_0$ between normal and HTS sections is the same, that is, $T_i(x_0) = T_h(x_0)$; here the subscript $i = 1, 2$ refers to sections of normal metal at upper and lower ends of the HTS section in the current leads, respectively. A_i and k_i correspond to cross-section and thermal conductivity of both normal metal sections, subscript h stands for HTS section, A_h and k_h are separately cross-section and thermal conductivity of the HTS section, and G_{ji} and G_{jff} are Joule heat term and flux flow loss term of normal metal and HTS sections:

$$\begin{cases} G_{ji} = \frac{\rho_i I^2}{A_i} \\ G_{jff} = E_c \left(\frac{I}{I_c} \right)^n I \end{cases} \quad (10.62)$$

where ρ_i is the resistivity of normal metal at the end of current lead, I_c , n and E_c stand for the critical current, n value and critical criterion of the HTS section respectively, and E_c generally stands for $1 \mu\text{V/cm}$.

10.1.5.2 Conduction-Cooled under AC Operation

There is no Joule loss when the HTS section operates in DC conditions; however, the hysteresis loss will be generated if the HTS section operates in AC mode. Therefore, the heat conduction equation in AC mode becomes:

$$\begin{cases} \frac{d}{dx} \left[k_i(T) A_i \frac{dT}{dx} \right] + G_{ji} = 0 \\ \frac{d}{dx} \left[k_h(T) A_h \frac{dT}{dx} \right] + G_{jff} + G_d = 0 \end{cases} \quad (10.63)$$

with following boundary conditions: the temperature at the junction $x = x_0$ between normal metal and HTS sections are equal, that is, $T_i(x_0) = T_H(x_0)$, where G_d is the AC losses (hysteresis loss) of the HTS section. For the analysis on AC losses of superconductors, please refer to Chapter 5.

10.1.5.3 Gas-Cooled in DC Mode

The 1D heat conduction equation of gas-cooled HTS current leads under DC operating conditions is described by:

$$\begin{cases} \frac{d}{dx} \left[k_i(T) A_i \frac{dT}{dx} \right] + G_{ji} - G_q = 0 \\ \frac{d}{dx} \left[k_h(T) A_h \frac{dT}{dx} \right] + G_{jff} - G_q = 0 \end{cases} \quad (10.64)$$

with the boundary conditions: the temperature in the junction $x = x_0$ between normal metal and HTS sections is continuous, that is, $T_i(x_0) = T_H(x_0)$, where G_d is the gas-cooled cooling term (Equation (10.45)).

10.1.5.4 Gas-Cooled in AC Mode

Similarly, the 1D heat conduction equation of the gas-cooled HTS current leads under AC operating condition is:

$$\begin{cases} \frac{d}{dx} \left[k_i(T) A_i \frac{dT}{dx} \right] + G_{ji} - G_q = 0 \\ \frac{d}{dx} \left[k_h(T) A_h \frac{dT}{dx} \right] + G_{jff} + G_d - G_q = 0 \end{cases} \quad (10.65)$$

with the boundary condition: the temperature in the junction $x = x_0$ between normal metal and HTS sections is continuous, namely, $T_i(x_0) = T_H(x_0)$, where G_d is the AC loss. Meanwhile, resistivity and thermal conductivity of normal metal and HTS materials depend on temperature, and the critical current I_c and n value of the HTS materials depend not only on the temperature but also on the magnetic field and its orientation, which were introduced in Chapter 3. It is difficult to analytically solve Equations (10.61), (10.63) through (10.65), therefore, to solve them numerical calculations are needed.

Usually, HTS current leads are made from HTS materials, such as Bi-2223 rods, Bi-2212 rods, texture-orientated Y-123 bulk, and Bi-2223 tapes with silver or silver-alloy sheaths. The critical temperature T_c of a Bi-based superconductor is high but its thermal conductivity is low. However, the thermal conductivity and critical current I_c of Y-based superconductors are high, and their I_c decreases with increasing magnetic field, but this reduction of critical current I_c is lower than Bi-based superconductors. Thus, the Y-based superconductor has an advantage over the Bi-based superconductor in high magnetic field applications. The critical current I_c of superconducting tapes is higher than their bulk colleagues, but their thermal conductivity is also much higher, so that the heat leakage of the HTS current lead made from HTS tapes is much larger than those made from HTS bulk partners for the same design operation current.

However, because HTS bulks have low thermal conductivity and are brittle, magnetic-thermal instability may result from their large dimensions and cracks are easily formed after heating-cooling cycles, so that HTS tapes are still preferred for use in situations of large current operation. Due to the high thermal conductivity of HTS tapes, their length is longer than their HTS bar partners, so the HTS tapes are helically wound into the form of a cylinder to reduce the length of the current leads. The thermal

conductivity of HTS tapes can be effectively reduced by using silver-based alloy and decreasing the cross-section of the matrix or sheath. In addition, because critical current density J_c of HTS strongly depends on temperature and the temperature of the lower part is lower than the upper part in the HTS current lead, the current lead structure with variable cross-sections can further reduce the heat leakage of the HTS current leads.

10.1.6 Peltier Thermoelectric (TE) Effect

As shown Figure 10.12, there are two kinds of semiconductor materials, A and B, and the junction between A and B is at the environmental temperature T_0 . There is a gap in the middle of B with which the power supply connects, so that a closed circuit forms and the current flows in the closed circuit. Let J_N be the current density. If J_N exists, heat flux J_Q must flow into and out of two junctions in order to maintain isothermal conditions T_0 , which means that the function of Figure 10.12 is similar to the heat flux pump. The heat in one side can be removed from the other side and this process is reversible [7–9]. This effect is called the Peltier thermoelectric effect, which can be applied to create refrigeration.

If the electrical potential gradient ∇V and temperature gradient ∇T simultaneously exist in a semiconductor materials, current I and heat flux Q comply with:

$$I = -(\nabla V + \eta \nabla T) \frac{A}{\rho} \tag{10.66}$$

$$Q = -kA \nabla T + I \eta T \tag{10.67}$$

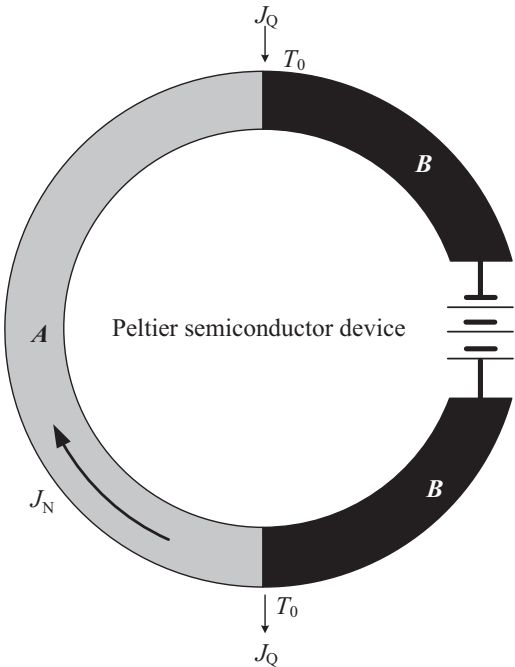


Figure 10.12 Principle of Peltier thermoelectric effect.

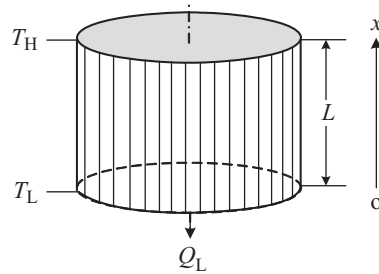


Figure 10.13 1D model of Peltier element.

where η , ρ and k are the Seebeck coefficient, resistivity and thermal conductivity, respectively, and A refers to the cross-section. There are two types of semiconductors: the n -type with $\eta < 0$ and the p -type with $\eta > 0$. The energy balance equation meets the following:

$$\nabla \cdot Q = I \cdot (-\nabla V) \quad (10.68)$$

The cross-sectional temperature of the Peltier semiconductor element is assumed to be evenly distributed (Figure 10.13), and A is the cross-sectional area of the Peltier element and L is its length. If the maximum temperature difference on both sides of the TE element is maintained, the optimization current I , as obtained from the theoretical calculation, should be:

$$I_{\text{opt}} = \eta \frac{T_L}{r_e} = \frac{K_e}{\eta} \left\{ \left[1 + 2Z \left(T_H + \frac{Q_L}{K_e} \right) \right]^{1/2} - 1 \right\} \quad (10.69)$$

where

$$K_e = k \frac{A}{L}, \quad r_e = \rho \frac{L}{A}, \quad Z = \frac{\eta^2}{k\rho} \quad (10.70)$$

For the typical thermal-electronic semiconductor materials such as BiTe, BiSb and Cu, their heat transfer coefficients near liquid nitrogen temperature 77 K are shown in Table 10.1 [10].

According to the data in Table 10.1, it is calculated that the optimized transport currents are 43 A and 140 A, at which BiTe and BiSb can generate the maximum temperature differences, respectively. The temperature at the cold side is:

$$T_L = \frac{1}{\eta I + K_e} \left(\frac{1}{2} r_e I^2 + K_e T_H + Q_L \right) \quad (10.71)$$

Table 10.1 Heat transfer coefficients of BiTe, BiSb and Cu

Materials	Types	Thermal conductivity k [W/(K·m)]	Resistivity ρ [$\mu\Omega\cdot\text{m}$]	Seebeck coefficient η [$\mu\text{V/K}$]
BiTe	p-type	1.78	2.20	75
BiSb	n-type	3.80	1.10	-160
Cu	/	600	2×10^{-3}	0.1

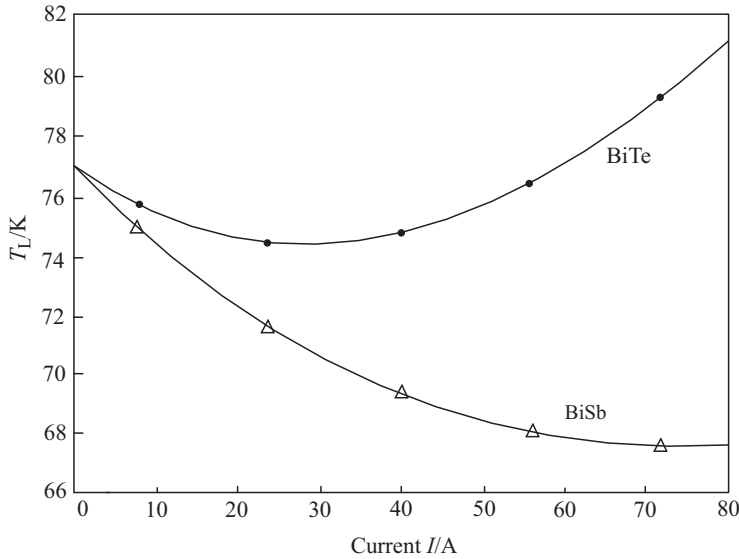


Figure 10.14 Calculated curves of cold side temperature versus transport current in BiTe and BiSb.

where T_H is the temperature at the warm side. In order to calculate the temperature of the cold side, take $T_H = 77\text{K}$, and $Q_L = R_{\text{int}} I^2$, where R_{int} is the resistance of semiconductor elements.

Figures 10.14 and 10.15 show the relationships between temperatures of the cold side and transport currents in semiconductor material BiTe and BiSb by theoretical calculations and experimental results [10]. Both of the profiles are similar, but temperature at the cold side in the calculated curve is higher than the experimental ones, which results from the assumption that the heat transfer coefficient, thermal conductivity, electrical conductivity and Seebeck coefficient are independent of temperature in the calculation process. Therefore, accurate calculation and analysis should be performed by taking the non-linear relationship between the heat transfer coefficients and temperature, to make the calculated results consistent with the experimental ones. When the transport current is 30 A, BiTe has the maximum temperature difference, but the temperature at the cold side in BiSb continues to drop with the increase of transport current.

10.1.7 Gas-Cooled Peltier Current Leads (PCL)

Considering that the current lead is made by connecting the Peltier element with conventional metals, and its length is much larger than its transverse dimensions, then its calculation can be approximated as a 1D model [4, 11–16]. Figure 10.16 shows the structure of the Peltier gas-cooled current leads. The low temperature side of the current lead is a metal segment in contact with the cryogen at a temperature of T_L , and the Peltier element is installed at the warm side. The lower side of the Peltier element also connects with the metal, and the junction temperature is T_J . The temperature of the top side of the current lead, namely the upper side of the Peltier element, is T_H . The heat leakage to the low temperature side is Q_L , and A_1 and A_2 are the cross-sections of the metal segment and the Peltier semiconductor element, respectively. Then the heat term of the Peltier semiconductor element is:

$$G_d = -I \left(T \frac{d\eta}{dx} \right) = -I \left(T \frac{d\eta}{dT} \right) \left(\frac{dT}{dx} \right) \quad (10.72)$$

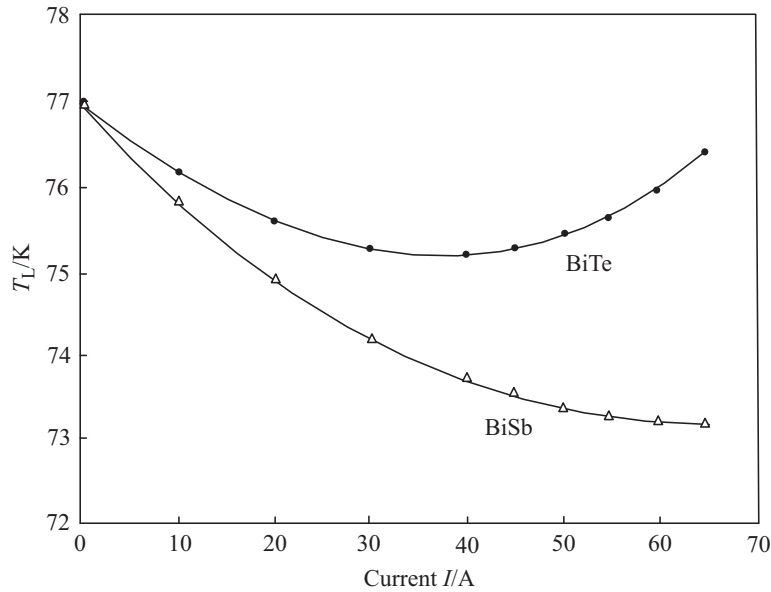


Figure 10.15 Experimental curves of cold side temperature versus transport current in BiTe and BiSb.

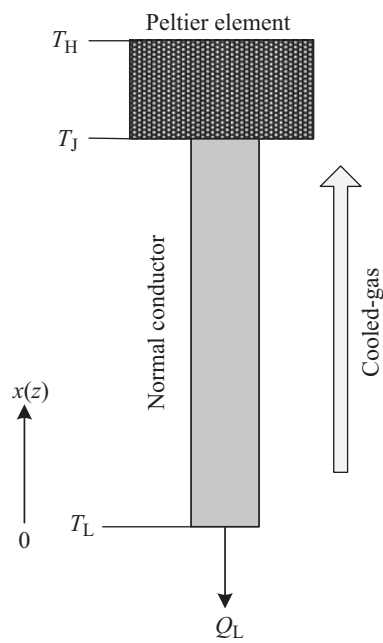


Figure 10.16 Gas-cooled PCL.

By substituting heating term G_d and cooling term G_q into the 1D heat conduction Equation (10.1) with 1D, it becomes:

$$\frac{d}{dx} \left(kA \frac{dT}{dx} \right) - I \left(T \frac{d\eta}{dT} \right) \frac{dT}{dx} + \frac{I^2 \rho}{A} - Ph(T - \Theta) = 0 \quad (10.73a)$$

$$\frac{d}{dx} \left(kA \frac{dT}{dx} \right) - I \left(T \frac{d\eta}{dT} \right) \frac{dT}{dx} + \frac{I^2 \rho}{A} - f \dot{m} C_p \frac{dT}{dx} = 0 \quad (10.73b)$$

where η is the Seebeck coefficient of the Peltier element, and definitions of other parameters are the same as those in Section 10.1.3. By making transformations similar to Equations (10.49) and (10.50), Equation (10.73b) changes into:

$$\frac{d^2 T}{dz^2} - \left(T \frac{d\eta}{dT} + 2\alpha \right) \frac{dT}{dz} + k\rho = 0 \quad (10.74)$$

In the next steps, each segment of the current lead will be analysed in detail.

10.1.7.1 Metal Segment

The normal metal segment is below the Peltier element. The effect of the Seebeck coefficient η is neglected since it is small (Table 10.1). According to the Wiedemann–Franz law $k\rho = L_0 T$, Equation (10.74) becomes:

$$\frac{d^2 T}{dz^2} - 2\alpha \frac{dT}{dz} + L_0 T = 0 \quad (10.75)$$

Equations (10.75) and (10.53) are formally identical, so that solutions of the two equations are also the same. Taking boundary conditions of $T(0) = T_L$, $T(z_1) = T_J$, and T_J replaces T_H in Equations (10.57) and (10.59), then we have:

$$q_L = \frac{dT}{dz} \Big|_{z=0} = \frac{T_J \beta}{e^{\alpha z_1} \sin(\beta z_1)} + T_L [\alpha - \beta \cot(\beta z_1)] \quad (10.76)$$

In the case of the conduction-cooled current lead ($f = \alpha = 0$), heat leakage to the low temperature side is:

$$q_L = (T_J - T_L \cos(\beta_0 z_1)) \frac{\beta_0}{\sin(\beta_0 z_1)} \quad (10.77)$$

10.1.7.2 Peltier Element Segment

In the TE element segment, thermoelectric parameters are usually considered to be independent of temperature and the average of temperatures is taken as the operation temperature. Numerical calculation by Mahan shows that numerical solutions are in good agreement with those of linear differential equation if the dimensionless figure of merit Z_T [$Z_T = \eta^2 T / (\rho k)$] does not exceed 9. Therefore, the terms related with η of Equation (10.74) can be approximated to zero, thus Equation (10.74) becomes:

$$\frac{d^2 T}{dz^2} - 2\alpha \frac{dT}{dz} + M_0 = 0 \quad (10.78)$$

where $M_0 = k\rho$, and the boundary conditions are:

$$T(0) = T_J, \quad T(z_2) = T_H, \quad z_2 = IL_2 / (kA_2) \quad (10.79)$$

Existing theoretical and experimental results indicate that heat leakage is minimal if the temperature gradient of the Peltier element at room temperature is zero; therefore, the first boundary condition of the linear differential Equation (10.75) changes to:

$$\left. \frac{dT}{dz} \right|_{z=z_2} = 0 \quad (10.80)$$

The solution of Equation (10.75) is:

$$T(z) = T_H + \left(\frac{M_0}{2\alpha} \right) (z - z_2) + \left(\frac{M_0}{4\alpha^2} \right) \{1 - \exp[2\alpha(z - z_2)]\} \quad (10.81)$$

Then junction temperature T_J between the normal metal element and the Peltier element is:

$$T_J = T_L - \left(\frac{M_0}{2\alpha} \right) z_2 + \left(\frac{M_0}{4\alpha^2} \right) [1 - \exp(-2\alpha z_2)] \quad (10.82)$$

Another boundary condition is that the heat flow at the junction is continuous, as shown in Equation (10.68), namely:

$$\left(-\frac{dT}{dz} \right)_{z=z_1} \Big|_{CD} = \left(-\frac{dT}{dz} \right)_{z=0} + \eta T_J \Big|_{TE} \quad (10.83)$$

where subscripts CD and TE refer to the upper segment of the normal metal segment and bottom of the Peltier element, respectively, and z_1 and β are separately determined by Equations (10.55) and (10.56). The temperature T_J at the junction can be calculated by:

$$(\alpha + \eta + \beta \cot(\beta z_1)) T_J = \left(\frac{M_0}{2\alpha} \right) [1 - \exp(-2\alpha z_2)] + \frac{T_L \beta \exp(\alpha z_1)}{\sin(\beta z_1)} \quad (10.84)$$

For conduction-cooled PCLs ($f = \alpha = 0$), the temperature distribution is:

$$T(z) = T_H - M_0 \frac{(z - z_2)^2}{2} \quad (10.85)$$

The junction temperature between the normal metal section and the Peltier element is:

$$T_J = T_H - M_0 \frac{z_2^2}{2} \quad (10.86)$$

The continuous condition of heat flow Equation (10.83) becomes:

$$(\eta + \beta_0 \cot(\beta_0 z_1)) T_J = M_0 z_2 + \frac{T_L \beta_0}{\sin(\beta_0 z_1)} \quad (10.87)$$

The heat leakage q to the low temperature side can be calculated by finding T_J and substituting it into Equations (10.76) or (10.77), thus q is the function of z_1 and z_2 . Taking constraints of Equation (10.84), q is only a function of z_1 or z_2 , and then the optimization design for the current lead can be made.

10.1.7.3 Optimization Design

The relationship of heat leakage per unit transport current of the gas-cooled current lead with z_1 and z_2 can be obtained by substituting Equations (10.82) into Equations (10.76) and (10.84):

$$q = T_L (\alpha - \beta \cot(\beta z_1)) + \frac{\left\{ T_H - \left(\frac{M_0}{2\alpha} \right) z_2 + \left(\frac{M_0}{4\alpha^2} \right) [1 - \exp(-2\alpha z_2)] \right\} \beta}{\exp(\alpha z_1) \sin(\beta z_1)} \quad (10.88)$$

$$T_H - \left(\frac{M_0}{2\alpha} \right) z_2 + \left(\frac{M_0}{4\alpha^2} \right) [1 - \exp(-2\alpha z_2)] = \frac{\left(\frac{M_0}{2\alpha} \right) [1 - \exp(-2\alpha z_2)] + \frac{T_L \beta \exp(\alpha z_1)}{\sin(\beta z_1)}}{\alpha + \eta + \beta \cot(\beta z_1)} \quad (10.89)$$

According to definitions of Equations (10.52) and (10.55), α and β are related to heat leakage q per unit transport current, therefore, Equations (10.85) and (10.86) are functions of q . If z_1 (or z_2) is known, q or z_2 (or z_1) can be obtained through Equations (10.85) and (10.86). The minimum exact solution of q and corresponding z_1 (or z_2) can be achieved by numerical solution.

As for the conduction-cooled current lead ($f = \alpha = 0$), by substituting them into heat flow constraint Equations (10.83), (10.88) and (10.89), we obtain:

$$q = \left[T_H - \frac{M_0 z_2^2}{2} - T_L \cos(\beta_0 z_1) \right] \frac{\beta_0}{\sin(\beta_0 z_1)} \quad (10.90)$$

$$T_H - \frac{M_0 z_2^2}{2} = \frac{M_0 z_2 + \frac{T_L \beta_0}{\sin(\beta_0 z_1)}}{\eta + \beta_0 \cot(\beta_0 z_1)} \quad (10.91)$$

For the full metal gas-cooled current lead ($f = 0, \alpha \neq 0$), constraints for heat leakage and temperature are:

$$q = \frac{T_H \beta}{\exp(\alpha z_1) \sin(\beta z_1)} + T_L [\alpha - \beta \cot(\beta z_1)] \quad (10.92)$$

$$T_H [\alpha + \beta \cot(\beta z_1)] = \frac{T_L \beta \exp(\alpha z_1)}{\sin(\beta z_1)} \quad (10.93)$$

The last constraint is that the gradient at the room temperature side in the metal section is zero, namely, $(dT/dz)|_{z=z_1}$. As for the full-metal conduction-cooled current leads ($f = 0, \alpha = 0$), optimization of the conduction-cooled current lead can be realized by substituting them into Equations (10.92) and (10.93):

$$q = \beta_0 \sqrt{T_H^2 - T_L^2} = \sqrt{L_0} \sqrt{T_H^2 - T_L^2} \Leftrightarrow Q_L = Iq = I \sqrt{L_0} \sqrt{T_H^2 - T_L^2} \quad (10.94)$$

$$\cos(\beta_0 z_1) = \frac{T_L}{T_H} \Leftrightarrow \left(\frac{L_1 I}{A} \right)_{\text{opt}} = \frac{k}{\sqrt{L_0}} \cos^{-1} \left(\frac{T_L}{T_H} \right) \quad (10.95)$$

which are identical to Equations (10.8) and (10.9) discussed in Section 10.1.1.

Figure 10.17 illustrates the geometrical relations of z_2 and q with z_1 for the PCL and the full-copper current lead under conduction-cooled ($f = 0$) and ideal gas-cooled ($f = 1$) conditions, and the corresponding optimization properties are given in Table 10.2. The two columns on the right-hand side of Table 10.2 list the optimized design parameters of full-copper current leads with cooling efficiency f

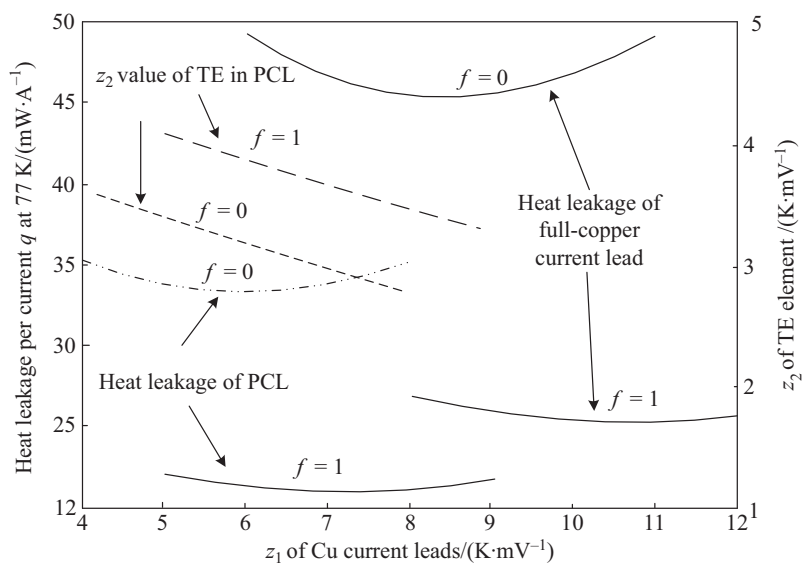


Figure 10.17 Relationships of z_2 and q versus z_1 for PCL and full-Cu current leads at $f = 0$ and 1 .

varying between 0 and 1. Compared with the conduction-cooled current lead ($f = 0$), the heat leakage of the gas-cooled current lead is reduced by about 43.96%. Similarly, heat leakage of the PCL is reduced by more than 16% compared with the full-copper current lead. Even if the cooling efficiency f is low, the PCL can effectively reduce the heat leakage.

Figure 10.18 illustrates the temperature variation profile of the PCL and the full-Cu current lead when conduction-cooled ($f = 0$) and ideal gas-cooled ($f = 1$), when they operate at the optimized design. The horizontal axis represents the normalized length, 0 indicates the junction position of the Peltier element and copper section, and the positive values represent the normalized length of the PCL. The horizontal axis is shifted to the left by unity, namely the negative values denote the normalized length of the full-Cu current lead, and the temperature at the junctions is continuous and equal to T_j . If gas-cooled technology is used in the current leads, the temperature is reduced along the whole current lead and then the heat

Table 10.2 Optimized parameters for PCL and full-Cu current leads with and without gas-cooling

f	PCL				Full-Cu current leads		
	z_1 [K/V]	z_2 [K/V]	T_j [K]	q [mW/A]	z_1 [K/V]	q [mW/A]	χ [%]*
0	5900	3207	216.97	33.461	8380	45.384	26.27
0.2	6200	3321	213.93	29.698	8960	38.743	23.54
0.4	6460	3420	211.31	26.829	9460	34.125	21.38
0.6	6700	3506	209.08	24.557	9920	30.543	19.60
0.8	6900	3588	206.92	22.704	10320	27.721	18.10
1	7100	3659	205.12	21.160	10700	25.533	16.8
36.76%**					4 α 3.96%**		

Note: * χ stands for the reduced percentage of heat leakage of PCL compared with full-Cu current lead.

**These two numerals show the decreased percentage of heat leakage at $f = 1$ as comparing with $f = 0$.

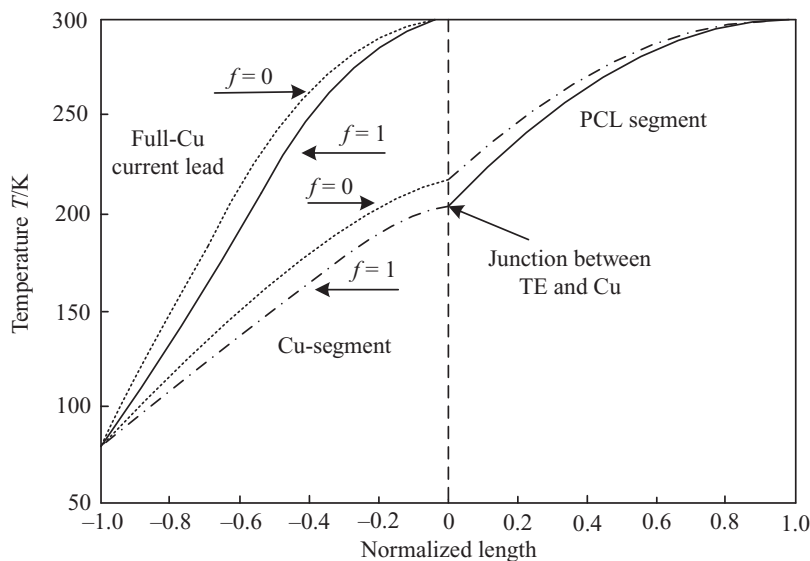


Figure 10.18 Temperature distribution of PCL and full-Cu current leads at $f = 0$ and $f = 1$.

leakage to the low temperature side decreases, which can greatly improve the operating efficiency of the superconducting electrical apparatus.

Figure 10.19(a) is a photograph of a PCL, which is intended to apply in the 1.5 kA HTS DC cable [17]. At present, PCL technology is premature and mainly used in DC superconducting magnet technology

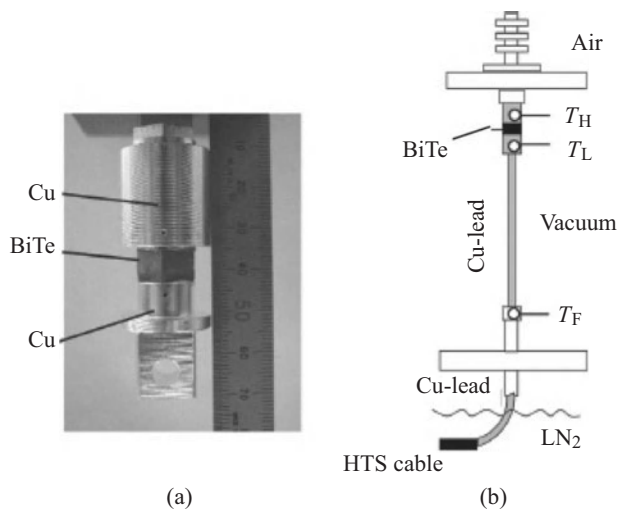


Figure 10.19 View of PCL intended for use in 1.5 kA DC HTS cable: (a) photograph of practical PCL used in DC HTS cable; and (b) installation of PCL in practical DC HTS cable. (Reproduced with permission from Yinshun Wang, “Bases of Applied Superconductivity in Electrical Engineering,” Science Press, 312, © 2011.)

[18]. The development of PCL applications in DC superconducting electrical apparatus has only been conducted for a short time, so further investigations is required.

10.2 Superconducting Switch

Although most of the superconducting DC apparatus (such as superconducting magnets) are usually connected to the external power supply in operation, the power supply is only needed when the superconducting magnet is in the stage of exciting and discharging. In DC operation mode, the power supply is not necessary for the superconducting magnet, and its role is to compensate for the Joule loss of current leads only. The superconducting switches (SCs) are often used for stable and long-term DC operation of the superconducting magnet, and are also known as persistent current switch (PCS).

PCS has many advantages, such as no power supply, no heat leakage and Joule loss when it is used in the superconducting magnet with closed-loop operation. Figure 10.20 shows the operation of a superconducting magnet with the help of PCS. At the beginning of magnet excitation, the heater is powered and the superconducting switch is in a normal state, and the superconducting magnet is excited by the power supply. When the target current is achieved, the power to the heater is switched off, the superconducting switch recovers to superconducting state, and the power supply is no longer essential. At this time, the power supply is turned off, and the magnet current forms a closed superconducting loop in combination with the magnet and the PCS. Then, the current leads are unplugged from the terminals of the superconducting magnet and removed from the cryostat, so that the heat leakage from the current leads to the cryostat is eliminated.

Resistance of the superconducting switch is an important parameter in its design, and special attention should be paid to the resistance in protection and loss of the superconducting magnet in its exciting and discharging stage. If the superconducting magnet is excited with a constant rate within the period of τ , the energy loss is:

$$E_R = E_0 \frac{2L}{RT} \quad (10.96)$$

where E_0 and L are the energy stored in the superconducting magnets and inductors, and R stands for the resistance of the superconducting switch, which is the sum of the jointing resistance between the

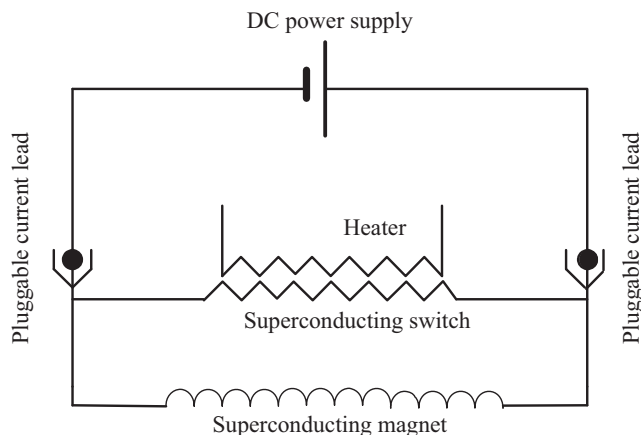


Figure 10.20 Principle operation circuit with combination of PCS and superconducting magnet.

superconducting switch and the magnet and the resistance of the superconducting wire itself used to wind the switch when quench takes place. After the resistance R of the switch is determined, the cross-section of superconducting wire, its length, and other geometric parameters can be chosen. After quench of the superconducting switch, the safety of the superconducting switch should be ensured, that is, it is in the permitted temperature range within the period of discharge so as to avoid burning the superconducting switch due to high temperatures.

10.2.1 Design of LTS Switch

The LTS switch is usually wound by two components of NbTi/CuNi composite wire, since resistivity of NiCu is 2 or 3 orders of magnitudes higher than that of copper. CuNi can provide a large enough resistance after quench of NbTi/CuNi wire to reduce the volume of the superconducting switch and to improve the transfer efficiency as well as to increase the switching-off speed [19]. However, the effect of the n value (flux flow resistance) does not need to be considered because HTS has a high enough n value.

Assuming that the jointing resistance of the superconducting switch is R_j , when the superconducting switch operates in a closed loop, an equivalent circuit is a simple R - L serial circuit and its equation is:

$$L \frac{dI}{dt} + R_j I = 0 \quad (10.97)$$

where L is the inductance of the superconducting magnet. Assuming that the initial conditions are $t = t_0$ and $I(t_0) = I_0$, the solution of Equation (10.97) is:

$$I(t) = I_0 \exp\left(-\frac{R_j}{L}t\right) = I_0 \exp\left(-\frac{t}{\tau}\right) \quad (10.98)$$

where the current decay time constant $\tau = L/R_j$, and the jointing resistance of R_j can be determined according to the period required by stability of the superconducting magnet. Based on requirements of MRI imaging, homogeneity or stability of the magnetic field required is below 10 (ppm [parts per million]) in the range of a sphere of radius 1 cm, then R_j must be small enough. For example, if inductance of a magnet is $L = 10$ H, the jointing resistance R_j must be in the range of $10^{-11} \Omega$, then the decay time constant is $\tau = L/R_j = 10^{12} \text{ s} \approx 3.16 \times 10^3$ years, which will meet the requirement of the magnetic field stability. Therefore, the equation can be expanded by omitting the terms higher than quadratic terms:

$$I(t) = I_0 \left(1 - \frac{R_j}{L}t\right) \quad (10.99)$$

Then, the jointing resistance is:

$$R_j = \frac{\Delta I}{I_0} \frac{L}{\Delta t} \quad (10.100)$$

Thus, the jointing resistance can be found from the magnetic field (or current) decay rate. Currently, LTS switch technology is relatively advanced and the switches can be commercially manufactured. Superconducting switches are widely used in commercial superconducting magnets with high stability and high uniformity, as well as long-term operation.

10.2.2 Design of HTS Switch

The n value of HTS materials (generally < 18) is usually much smaller than that of LTS materials (often > 25) due to their granularity, weak link, second phases, etc. The effects of the n value, namely the magnetic flux flow resistance, cannot be ignored. Compared with the LTS switch, the HTS switch is more complicated [20]. Assuming that the jointing resistance of the HTS switch is R_j , then its circuit equation is:

$$L \frac{dI}{dt} + R_{joint} I + E_c L_0 \left(\frac{I}{I_c} \right)^n = 0 \quad (10.101)$$

where L_0 denotes the length of HTS wire wound in the HTS switch, E_c is critical current criteria, and I_c refers to critical current. The current decays in closed-loop mode, and the solution of the Equation (10.101) is:

$$I(t) = (I_0^{n-1} + \alpha) \exp \left(\frac{(n-1)t}{\tau} - \alpha \right)^{1/(1-n)} \quad (10.102)$$

where

$$\alpha = \frac{E_c L}{R_j I_c^n} \quad (10.103)$$

According to Equation (10.102), current decay not only depends on the time constant τ but also closely relates to the n value of the HTS switch. Therefore, the effect of the n value (the flux flow resistance) on current decay must be taken into account besides that of the jointing resistance.

Although research and development of HTS switch technology have been performed for more than 20 years, it is still immature and cannot yet meet the requirements of commercial applications. Figure 10.21 shows the dependence of current decay on time for respective LTS and HTS switches with closed-loop operations [21]. The inserted figure illustrates the central magnetic field of the superconducting switch

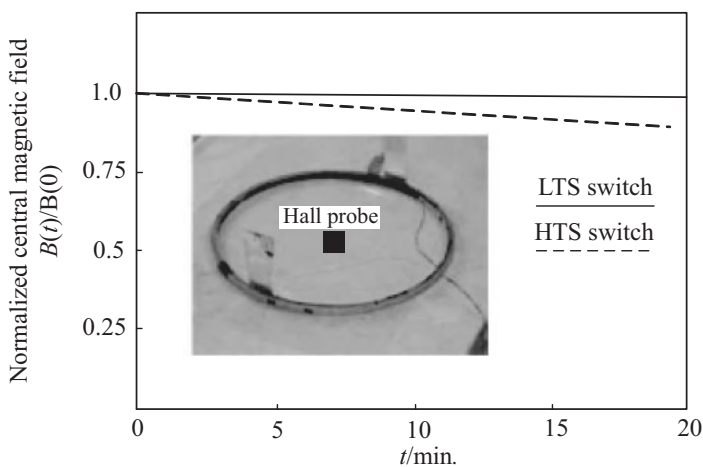


Figure 10.21 Current decays for respective LTS and HTS switches.

measured by the Hall probe, so that the degree of decay in the magnetic field directly reflects the current decay. It is indicated that the current decay of the LTS switch is much slower than the HTS switch, which means that its magnetic field stability is also much higher.

10.2.3 Fabrication of Superconducting Switches

Fabrication of a superconducting switch mainly includes winding and soldering techniques. LTS materials have good mechanical properties, but mechanical properties of HTS materials are relatively poor, so that winding and soldering procedures of LTS and HTS switches are different. A superconducting switch should have low jointing resistance and zero inductance, and there will be no magnetic field generated when the winding is wound in a bifilar way.

10.2.3.1 Winding Process

Since a superconducting switch is often applied in closed-loop operation in a DC superconducting magnet system with high uniformity and stability, the switch is usually wound in the form of a small coil, which thus will not produce a magnetic field. Even if there is some small magnetic field, it will not disturb the magnetic field generated by the superconducting magnet itself. Therefore, the superconducting switch should be bifilarly wound without inductance (Figure 10.22) [19–22]. Figures 10.22(a) and (b) are the diagram of the non-inductive or bifilar winding process and a photograph of a practical superconducting switch wound by NbTi/Cu superconducting wire, respectively. Double NbTi/Cu superconducting wires, folded at the middle position of a single wire, are wound onto the former to form the non-inductive coil. The current flows in the opposite direction when it is excited, so that the total magnetic field is zero, because the two magnetic fields are mutually cancelled.

10.2.3.2 Joint of Superconducting Wire

As mentioned above, compared with HTS tapes, LTS wires have better mechanical properties, and joints of LTS wire with low resistance is relatively easy to realize. It is difficult to joint the HTS tapes with low

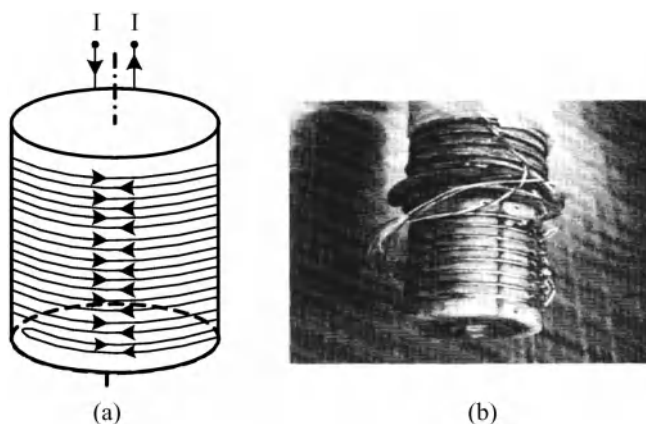


Figure 10.22 Superconducting switch: (a) diagram of process of bifilar winding; and (b) photograph of superconducting switch.

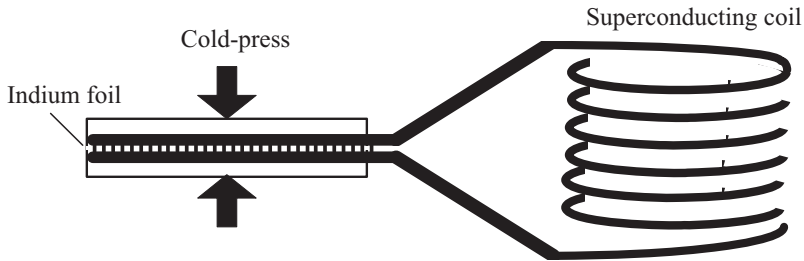


Figure 10.23 Process of cold-press technology.

resistance to meet the application requirement. There are several processes for joining LTS wires with low contact resistances, such as cold-press forming, jointing by solder, laser welding, etc. Figure 10.23 illustrates the commonly used welding process of LTS wires by cold-press forming. A layer of softer metal foil such as Indium foil is inserted between two joints in order to fully contact with each other and to avoid strain concentration. It is easily realized that jointing resistance is less than $10^{-11} \Omega$ by this process.

The jointing process of HTS tapes is complicated, and Figure 10.24 shows a flow diagram of this process. Taking multifilamentary stainless-steel enforced Bi-2223/Ag tapes as an example, the HTS tape is first heated, so the thin stainless-steel layers are melted away when the temperature reaches 200°C . Bi-2223 superconducting multi-cores are exposed where the solder PbSn layer is laid. Then another segment of the HTS tape is prepared by the same process. Both sides of tapes with solder PbSn are made to contact with each other face-to-face, which are heated and pressed simultaneously. The welding process is finished after the tape returns to room temperature. This method is only performed in small-scale specimens and to achieve a jointing resistance smaller than $10^{-9} \Omega$ is difficult. So this process is not able to meet engineering requirements. Therefore, the jointing technology of HTS tapes is still immature and needs further study.

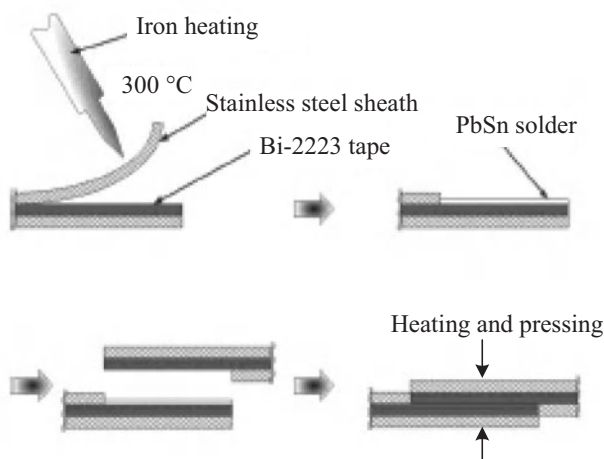


Figure 10.24 Soldering process of HTS tapes.

10.3 Flux Pump

The superconducting flux pump, also known as the superconducting power supply, is an inductive power supply device. The basic idea is that the entire magnet system is in the superconducting state and the current leads are not necessary, and the power supply is obtained by pumping the induced current into the superconducting magnet with the help of the electromagnetic induction method [23, 24].

10.3.1 Principle of Superconducting Flux Pump

Figure 10.25 sketches the operation of a superconducting flux pump consisting of a C-shaped iron magnet, which can rotate around its own axis, a superconducting magnet and a superconducting sheet, all of them in the same cryostat [1]. The superconducting magnet connects with a superconducting sheet whose critical magnetic field is lower than the rotating iron magnet. When the iron magnet rotates above the superconducting sheet, a quench spot swept by the iron core onto the superconducting sheet appears because its critical magnetic field is lower than the magnetic field generated by the iron core. The magnetic flux $\Delta\phi$ generated by the iron magnet penetrates through the normal spot on the superconducting sheet, and the superconducting magnetic circuit is still in superconducting state, except for the normal spot through, which the magnetic flux $\Delta\phi$ passes. When the rotating iron magnet arrives at point P on the superconducting materials composed of superconducting current leads and coils through which the magnetic flux $\Delta\phi$ cannot pass, the magnetic flux $\Delta\phi$ cannot be driven normally and is then frozen because the critical magnetic field of the superconducting material at point P is higher than that of the iron magnet. Consequently, the induced current is generated in the superconducting magnetic circuit and

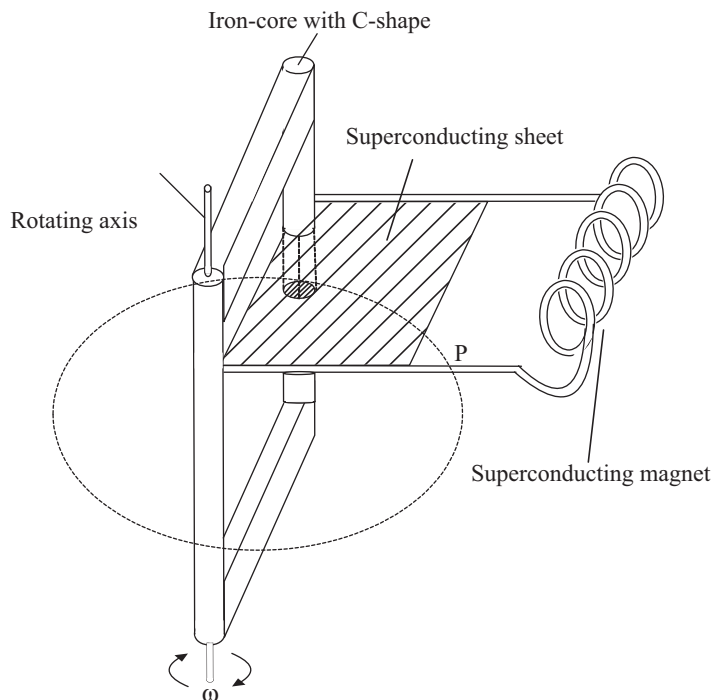


Figure 10.25 Principle scheme of magnetic flux pump.

its magnitude is related to the quantity of the magnetic flux $\Delta\phi$. The magnetic flux in the superconducting magnetic circuit increases by $\Delta\phi$ when the iron magnet rotates by one cycle. If the rotating frequency is f , the average output voltage of the power supply is $f\Delta\phi$, so that the function of the power supply without current leads is realized in the entire superconducting circuit.

Although the structure and principle of the superconducting magnetic flux pump are simple and intuitive, it is difficult to improve on its efficiency, since there is serious eddy current loss in the normal region through which the magnetic flux passes. Because of this, other types of magnetic flux pumps without moving parts are suggested, amongst which a full-wave transformer-rectifier type of superconducting magnetic flux pump is typical.

10.3.2 Transformer-Type Superconducting Magnetic Flux Pump

Figure 10.26 illustrates the principal circuit of a full-wave rectifier-type superconducting magnetic flux pump connected with a superconducting magnet [25]. The equivalent circuit of the superconducting magnetic flux pump composed of a transformer with air core and superconducting switches 3 and 2 is shown in the dotted frame. The two diodes connected with each other in anti-parallel mode play a protective role in the superconducting magnet. The external circuit consists of a power supply for the magnet, superconducting switch 1 and two anti-parallel protective diodes (not shown in this figure).

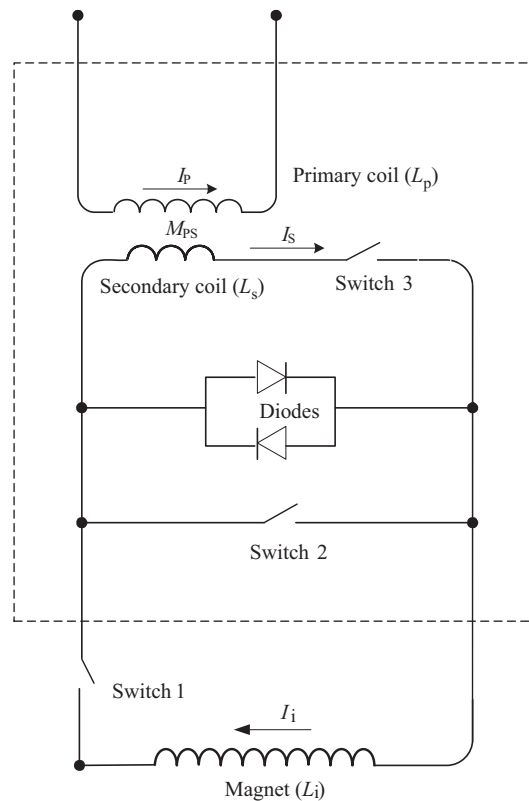


Figure 10.26 Principal circuit of full-wave rectifier-type flux pump for charging superconducting magnet.

The operating steps are carried out in the following order: first, superconducting switch 2 is opened and switches 1 and 3 are closed, so primary coil L_p is excited and current I_s is induced in the secondary coil and flows through the superconducting magnet L_i ; then, the secondary coil is short-circuited by closing superconducting switch 2 and current I_s passes through superconducting switch 2; and finally, superconducting switch 3 is opened and the secondary coil L_s discharges, so the current I_p in the primary coil decreases to zero. In this way, charging in one cycle is finished. The next cycle is carried out by the following steps:

- (1) Closing the superconducting switches 2 and 3, the induced current I_{s1} , which is the same as the first cycle, flows through superconducting switch 2, since the secondary coil L_s of the transformer is short-circuited.
- (2) Increasing current I_p of the primary coil, the current I_{s2} induced in the secondary coil is larger than the first pumped current I_{s1} , but the current through superconducting switch 2 is small and equal to the difference between I_{s2} and I_{s1} .
- (3) Switching off the superconducting switch 2, the current I_L passing through the superconducting magnet increases, because the current difference between I_{s2} and I_{s1} is forced to zero.
- (4) Closing superconducting switch 2 and secondary coil of transformer is short circuited.
- (5) Opening superconducting switch 3, the secondary coil of the transformer discharges, and current I_p in the primary coil decreases to zero. Then, closing superconducting switch 3 again, the next cycle is performed until the current of the superconducting magnet reaches its rated current.

The efficiency of the full-wave rectifier-type flux pump is also not high because all superconducting switches are thermally excited. However, there are not any moving components in this type of magnetic flux pump, and it has significant application value to meet the requirements of the high stability of superconducting magnets, particularly as it has potential application for compensating current decay of HTS magnet caused by flux creep.

10.3.3 HTS Permanent Magnetic Flux Pump

Power supply technology by superconducting magnetic flux pump for superconducting coils was discussed in former sections, but the magnetic flux pump for the superconducting permanent magnets has not been involved. In recent years, some research groups put forward thermally activated superconducting magnetic flux pump technology for the HTS bulk ReBCO permanent magnet, which provides a new application for exciting a bulk HTS permanent magnet. Soft magnetic materials are used in excitation of this magnetic flux pump at or near their Curie temperatures. The applicable soft materials used at low temperature are $\text{Ni}_{1.5}\text{[Cr}^{\text{III}}(\text{CN})_6]$ or Gadolinium. The hard magnetic material NdFeB is also used, as it can be rapidly demagnetized and is reversible at low temperature. Gadolinium experiences transition from ferromagnetic to paramagnetic at a temperature of about 294 K. Figure 10.27 presents the measured dependence of magnetization of an NdFeB permanent magnet on temperature [26]. It indicates that magnetization of an NdFeB permanent magnet rapidly decreases to below 150 K. Thus, the variations of the magnetic field changes can be obtained by controlling temperature.

Figure 10.28 depicts the set-up for thermal excitation of a superconducting magnetic flux pump combined with a permanent magnet [26]. When it is heated to room temperature, it excites the hard magnetic material NdFeB or soft magnetic material Gadolinium, which become a permanent magnets at room temperature. Then the heater is turned off and the refrigerator is turned on, so the temperature decreases. When the temperature is below 150 K, magnetization of hard magnetic material NdFeB or soft magnetic material Gadolinium are sharply reduced, which is equivalent to scanning the magnetically superconducting permanent magnet. Superconducting bulk traps magnetic flux and becomes a permanent magnet, and one cycle is completed. In the next step, the heater is switched on again and the NdFeB

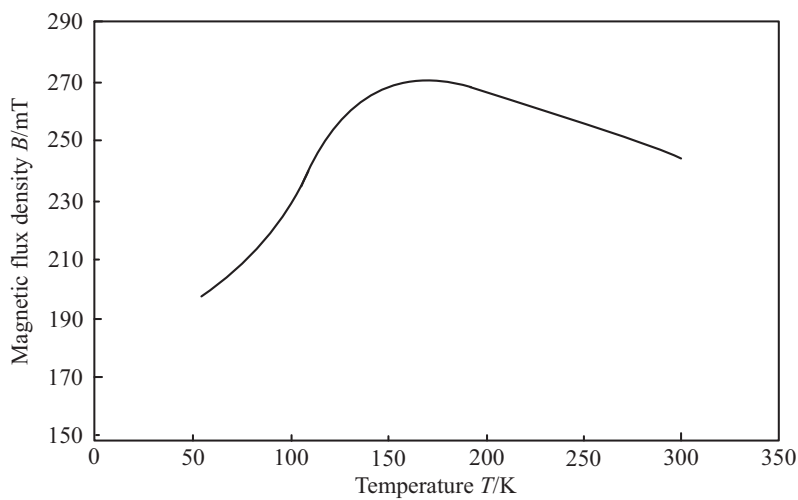


Figure 10.27 Magnetic flux density plots against temperature for permanent NdFeB.

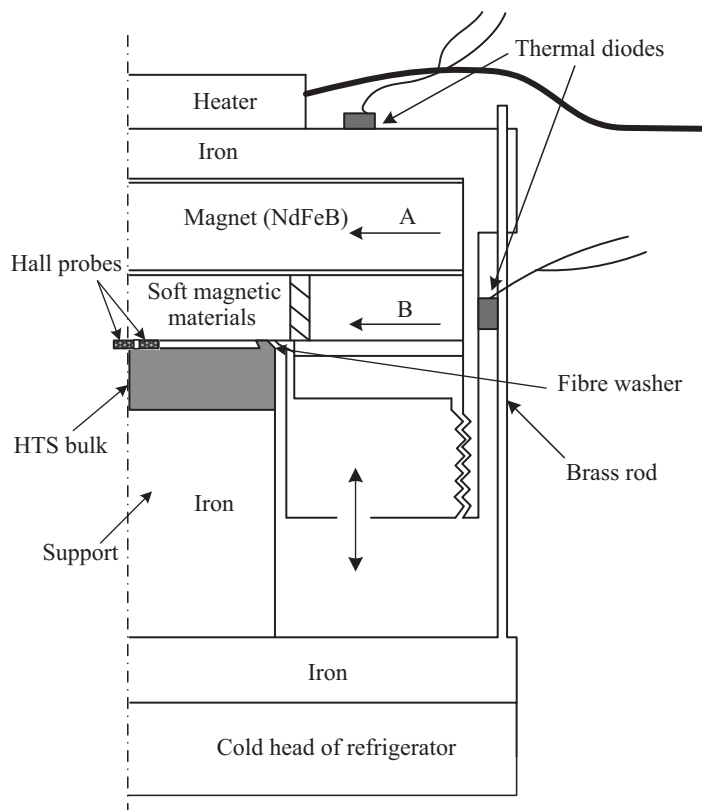


Figure 10.28 Set-up of thermal excitation of magnetic flux pump combined with HTS bulk magnet.

or Gadolinium is heated, so their magnetization increases and they become a normal magnet. After turning off the heater, the temperature of NdFeB or Gadolinium decreases again by switching on the refrigerator, and their magnetization decreases due to the magnetic flux change, which is equivalent to exciting the HTS bulk magnet again. After several cycles, the HTS permanent magnet can finally trap the maximum flux, and a maximum magnetic field at a certain temperature below critical temperature can be achieved.

According to reports in recent years, HTS YBCO bulk can capture 21 T magnetic field at a temperature of 20 K, so that the thermally activated flux pump technology provides an effective and promising way for applications of HTS bulk (HTS permanent magnet), such as a HTS permanent magnet with a high magnetic field and MHD magnets. Currently, research and development on this technology is still at the laboratory stage, and its practical applications need further study.

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11

Basic Structure and Principle of Superconducting Apparatus in Power System

High temperature superconducting (HTS) materials have found a wide range of sensitive and reliable technological applications, amongst which the power system is one of the important applicable fields. Generally, the superconducting apparatus of the power system are operating at low magnetic field and liquid nitrogen (LN_2) temperatures. In recent years, many prototypes of HTS electrical power apparatus, such as transmission cable, fault current limiter (FCL), transformer, motor/generator, magnets for superconducting magnetic energy storage (SMES), direct current (DC) reactor, etc., have been successfully demonstrated worldwide, and rich experience has been accumulated for future contemporary and practical applications. This chapter focuses on the introduction of the basic structure and principles of several kinds of HTS electrical power apparatus, to provide a useful reference for designing HTS electrical power apparatus.

11.1 Cable

At present, electrical power is mainly delivered by ACSR (Aluminium Cable Steel Reinforced) in overhead lines and XLPE (cross-linked polyethylene) underground cables. With the rapid development of the economy in modern society, a much higher capacity in transmission and distribution lines with the increase of electricity consumption is required. It is almost impossible to build new power delivery systems in cities and towns with a high population density. Due to its zero resistance and high current density with low inductance, the transfer capacity of the HTS superconducting cable is about 3 to 5 times that of traditional XLPE cable with the same dimensions, and both the power transmission loss and construction cost of sites can be reduced. Therefore, the HTS cable has potential application over short- and mid-distances. This chapter focuses on the basic structures of HTS cables, and their detailed principle and design will be introduced in Chapter 12, where HTS applications in electrical power systems will be described.

The categories of HTS cables are divided into alternating current (AC) and DC by electrical sources. Based on their architecture, HTS cables are traditionally classified into two types in terms of efficiency, stray electromagnetic field (EMF) generation, and reactive power characteristics. The first type refers

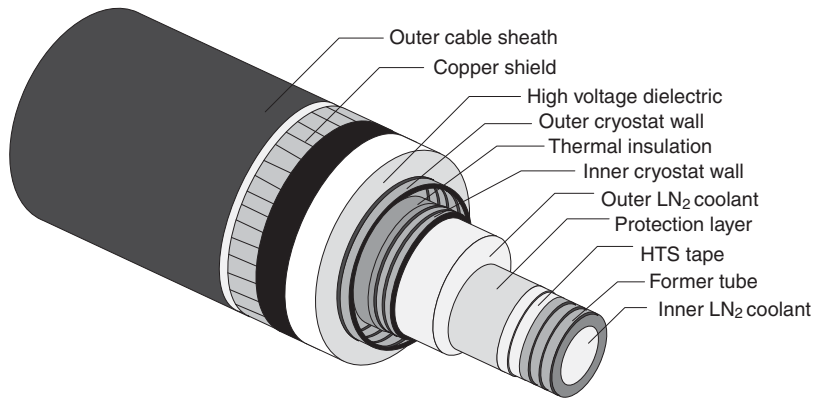


Figure 11.1 Structure of single-phase WD cable.

to warm dielectric (WD) cable [1], and the second type is the cold dielectric (CD) cable, including the single phase and Triax three-phase cable [2].

Figure 11.1 shows the configuration of the WD HTS cable mainly consisting of HTS conductor, cryostat, dielectric insulation and protection layers. The single conductor is made from HTS tapes wound around a former, which may be a flexible core or stranded copper/aluminium strands with or without a hollow centre in order to protect the HTS layers during a fault. The cryostat composed of inner and outer corrugated pipes between which multi-layers of thermal insulation are wound around the inner pipe, and the space between maintains a vacuum by pumping. Similar to the conventional XLPE cable, the dielectric insulation is located outside the cryostat. LN_2 flows in the tube type former and outer HTS layers, depending on the type of former. The outermost layer is composed of copper shielding and protection such as polyvinylchloride (PVC). Consequently, the dielectric loss is not cooled by LN_2 . Not only can WD type HTS cables preserve conventional cable dimensions and processing insulation techniques as well as using traditional mature dielectric materials, but also limited HTS tapes are used because there is no HTS shielding layers, so are cost-effective and efficient in the design of cooling and operation systems.

Since there are no HTS shield layers in the WD HTS cable, it will produce magnetic interaction with phases in the vicinity of the cable and thus will require phase separation to reduce the eddy current and eddy current loss in AC operation mode. However, an eddy current is also induced and eddy current loss generated in the inner and outer walls of the cryostat in AC operation mode. In addition, higher inductance is another drawback of the WD HTS cable. However, it is difficult for the cable conductor and its cryostat to connect with a refrigeration system, since both of them are equi-potential. Thus, the voltage level and power transfer capacity of WD HTS cable is limited and only applicable for mid- and low voltages. Currently, the WD HTS configuration is rarely adopted, so thereafter the HTS cable refers to CD cable unless otherwise specified.

Figure 11.2 presents a single HTS cable with CD design. Similar to the conductor of WD cable, HTS tapes are wound around a flexible copper strand with or without a hollow core. Electrical insulation is wrapped over the conductor of the HTS layers, and a shielding layer of HTS tapes is wound over the electrical insulation layers. Both of them act as shielding layers in a single-phase cable conductor. The geometrical structure of the cryostat is the same as for WD cables, except that the copper shielding over the outer wall of the cryostat is omitted; and the cryostat is zero potential, which makes it easier to connect to a refrigeration system. Since LN_2 flows between the outer and inner ducts of the cable, it cools not only the HTS conductor but also dielectric insulations. According to its structure, there is no magnetic field generation outside the cable conductor, because the induced current in the shielding layer is equal to that in the conductor layer with an opposite phase of 180 degrees (opposite direction).

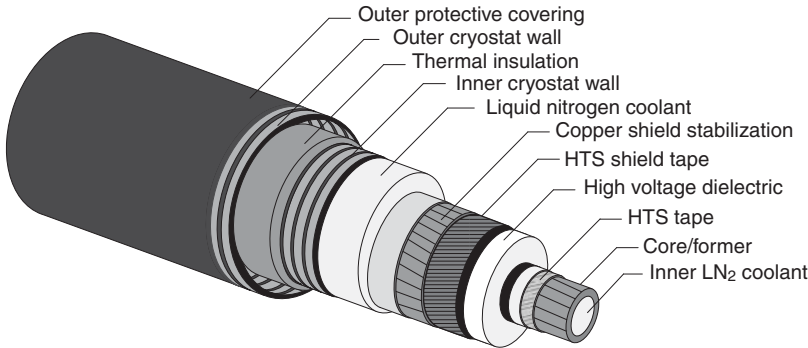


Figure 11.2 Structure of single-phase CD cable.

Compared with WD HTS cable in AC operation mode, CD HTS cable has several advantages, such as no eddy current induced in the cryostat, no magnetic interaction amongst phases, low inductances, high voltage levels, higher current and higher carrying capacity, reduced AC losses, and the complete suppression of any stray EMF outside the cable assembly. The inductance of a CD HTS cable is also much lower than that of a conventional cable. Since the power flow in a circuit is inversely proportional to its impedance, the CD HTS cable could carry a much higher load. Of course, CD HTS cable also has some disadvantages over WD HTS cable, such as employing almost double the amount of HTS tapes due to extra shielding HTS layers and higher cost. In addition, dielectric loss occurs during LN₂ cooling of the electrical insulation.

The other type of CD HTS cable is the Triax cable in which three phases of CD HTS conductors are contained in a single cable with the same former made of flexible hollow copper strands and located in a single cryostat, and its structure is shown in Figure 11.3. The three phases in a single cable are constructed in a “sandwich” configuration, in which three electrically insulated layers of HTS tapes are wound around a flexible former with a hollow core. In order to provide protection against the occurrence of an over-current fault, copper tapes are co-wound with HTS tapes in each phase. This kind of cable also has the lowest impedance and employs the least amount of HTS tape as compared with a single CD cable. The Triax HTS cable has reduced AC losses and no stray magnetic field. Similar to the WD HTS cable, the Triax HTS cable requires about half the HTS tape of a single CD HTS cable, because no HTS shielding layers are required. Furthermore, it occupies less space and needs only one cryostat, since a

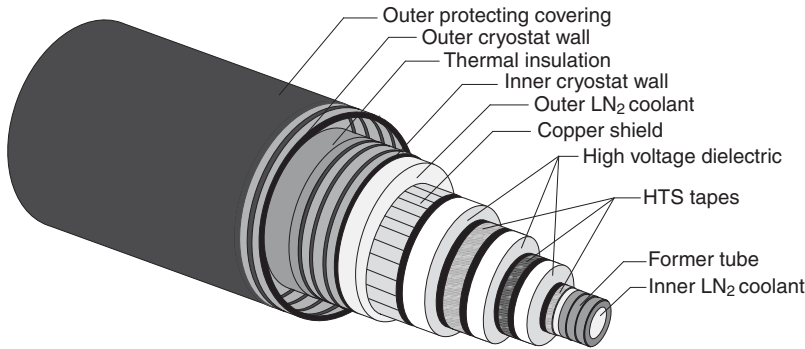


Figure 11.3 Structure of three-phase Triax CD cable.

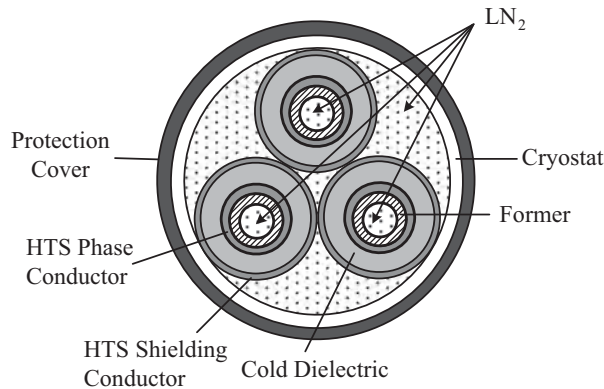


Figure 11.4 Three-phase CD HTS cable in same cryostat.

single cable has the same function as three separate cables. The cold LN_2 flows through the hollowed central former and the outer space between the cable and inner wall of the cryostat, so the entire cable is cooled. There are some drawbacks, such as dielectric loss during cooling by LN_2 , which is similar to the single CD HTS cable. According to its configuration, the Triax CD HTS cable is applicable for mid- and low voltage levels like the WD HTS cable.

Each phase of the CD HTS cable can be housed in an individual cryostat (Figure 11.2), which is available for any voltage level, and a set of three-phase CD HTS cables could share a common cryostat (Figure 11.4), which is applicable for mid- and low voltage levels.

11.2 Fault Current Limiter

With the development of industrialization and growth of electric power generation, as well as an increased interconnection of the network at super high voltage levels, the installation capacity of the electric power system rapidly grows, which results in a sharp increase of short-circuit current and capacity during a fault. A power grid is an extremely complicated system consisting of multiple generators, motors, transformers, and switchgears and transmission lines. It is inevitable for an electric power grid to experience extreme natural events including various faults caused by lightning, short circuits, grounding, etc. Consequently, serious unexpected events such as fire, apparatus and facility damage and even blackout may occur if these large currents are not properly limited and controlled by power system security. In order to protect the power grid system, there are FCL devices such as circuit breakers, inductors and fuses. Circuit breakers are installed and used to isolate the fault and to cut off the fault current; however, it takes a minimum breaking time to cut and sometimes even fails to break.

Although inductors connected in series in a line can limit the fault current by their impedance, their impedance also limits power transfer capacity during normal operations and delays recovery following a fault and simultaneously increases the reactance. Finally, the fuse is a simple and cheap FCL device, but manual intervention is required to replace it after a fault event, which leads to a prolonged interruption of power. However, the fuse is only applicable at low voltage levels and so not suitable for high voltage levels. Therefore, a large short-circuit fault current already becomes one of the bottlenecks restricting the development of the modern power grid system. Effectively limiting a short-circuit fault current not only suppresses over-large capacity of the power grid system during a fault, but also greatly alleviates the design standards of electrical apparatus, such as cable, transformer, generator and circuit breaker, so that their cost can be reduced.

According to their special characteristics, such as zero-resistance and the Meissner effect, a superconducting fault current limiter (SFCL) can provide the most attractive function of limiting the fault current in the power grid. If the fault current exceeds its critical current, the superconductor will quench and its resistance dramatically increases, so the short circuit current is limited. This chapter will focus on principles and configurations of several SFCLs.

11.2.1 Classifications

Based on the sharp phase transition of HTS from zero resistance at the superconducting state with normal current to a finite resistance at normal state with higher currents than its critical current, there are various types of SFCLs, such as resistive (R-type) [3], saturable core [4], bridge [5], hybrid [6], transformer (L-type) [7], magnetic shielding [8] and three-phase reactor types [9]. If a power system is in normal operation, the SFCL operates with or without a low impedance, which has almost no effect on the power system. However, while the over current appears due to a short circuit fault, the SFCL acts as a large impedance in line so that the fault current is effectively limited to a desired level lower than the trip current of the circuit breakers, so the power system is effectively protected from damage.

11.2.2 Resistive Type

The resistive type (R-type) of the SFCL is the simplest of several types of SFCLs, according to HTS inherent characteristics with and without resistance, respectively in superconducting and normal states. Figure 11.5 shows the conceptual circuit of the R-type SFCL, where the HTS components are connected in series with the line being protected. In normal operation, the HTS component presents a small voltage drop because of its zero resistance, so has a negligible effect on the power grid since there is only a small flux-flow resistance in AC operation. During the occurrence of a short circuit, the current surpasses its critical current, which results in quenching of the HTS component, that is, it transfers to normal state from superconducting state, SFCLs resistance sharply increases, and thus the fault current will be limited by quench resistance R_{sc} . The impedance of the SFCL is described by:

$$Z_s = \begin{cases} 0 & (normal) \\ R_{sc} & (fault) \end{cases} \quad (11.1)$$

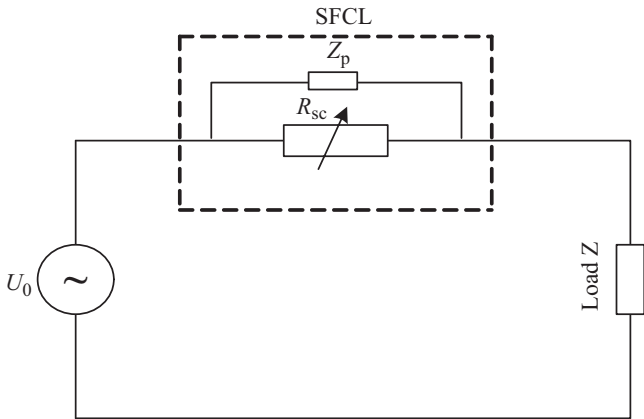


Figure 11.5 Equivalent circuit of R-type SFCL.

R-type SFCL has the simplest structure and fastest response rate, as well as a low voltage drop in normal operation. In order to protect the SFCL from damage when there is a serious fault, sometimes it is shunted by a parallel impedance Z_p . Nevertheless, the R-type SFCL has some drawbacks, amongst which is that it takes a longer recovery time of more than several seconds from normal state to superconducting state due to the large amount of Joule loss after quench, which possibly cannot satisfy the requirements of a conventional power grid system. Another disadvantage is that it has finite resistance and should quench simultaneously before the first peak of short circuit current is reached, and also the refrigeration cost is relatively high due to AC losses from HTS elements and heat leakage from current leads in operation. These disadvantages become a challenge for its applications in a power grid system. The elements of the R-type SFCL can be bifilarly wound by HTS wires or fabricated with HTS bulk by bifilar laser-cut, such as a Bi-2212 rod. At present, there are several prototypes of R-type SFCLs in a real power grid, but they are only demonstrated at mid- and low voltage levels.

11.2.3 Saturated Iron Core Type

The concept of the saturated iron-core type SFCL is mainly composed of two perfect AC primary windings and one DC secondary winding with an iron core for each phase (Figure 11.6). Each AC primary winding is wound by conventional copper wire, while the DC secondary winding is fabricated by HTS tapes. The auxiliary other elements are fast switch, DC supply and dump modulus consisting of dump resistance and zinc oxide lightning arrester. Two AC windings, connected in series with a power line, are separately wound onto iron cores with reverse directions, and then both of them generate opposite AC magnetic flux, so that a magnetic flux generated by DC winding is in the same and opposite directions to the two windings, respectively. The magnetization of SFCLs is a superposition of two iron cores. Figure 11.7 presents the plot of magnetic flux linkage against AC current. When there is a permanent fault in the power system, the fast switch turns off, and the dump modulus absorbs the magnetic energy and reduces terminal voltage of DC winding.

In normal operation, large ampere turns created by the DC in the secondary HTS winding drive the iron core into over-saturation, so the AC magnetic flux generated in the AC windings cannot enable the iron core to be separated from the over-saturated region, which reduces the impedance of the primary AC sides to almost that of air-core windings. At this time, SFCL operates in region 1 of Figure 11.7. During

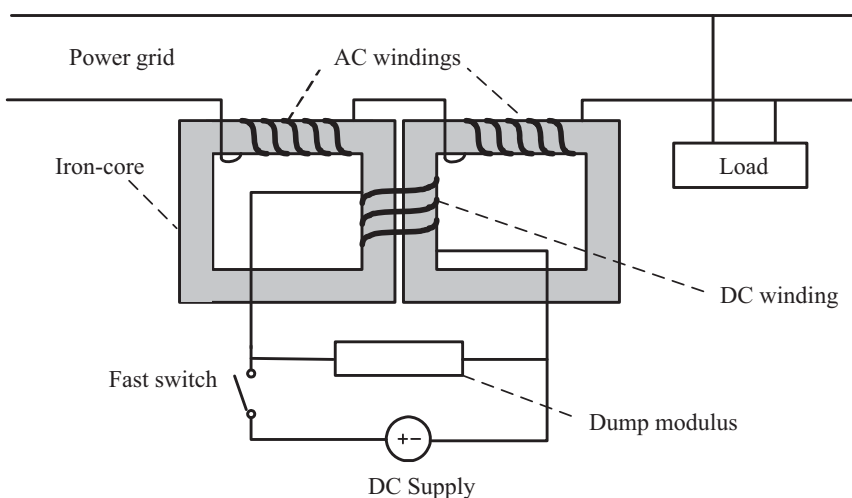


Figure 11.6 Conceptual circuit of saturated iron core type SFCL.

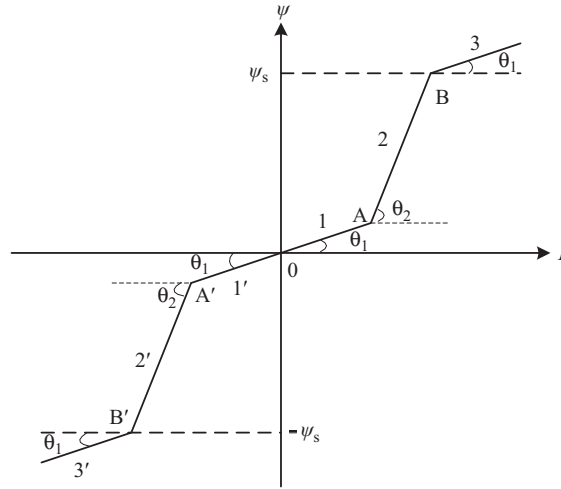


Figure 11.7 Plot of magnetic flux linkage in SFCL iron core against AC current.

a fault, a large fault current deeply demagnetizes the iron core and drives it from the over-saturated to unsaturated state in the linear region 2, which will increase the impedance of the primary AC windings. The increased impedance then automatically limits the fault current to a desired level. Since the SFCL consists of two identical AC windings with an iron core and both of them connect with opposite polarity, the fault current during both the positive and negative half cycle can be limited. If the fault current continuously increases and the AC magnetic flux linkage is larger than that created by the DC winding, the iron core will enter a reversed saturated state, and the SFCL will operate in region 3 of Figure 11.7, in which it will lose its limiting ability. However, this can be avoided by pre-setting the capacity of the SFCL according to the maximum fault current of the practical power grid.

The limiting impedance of the saturated iron core type SFCL can be expressed as:

$$Z_S = \begin{cases} j\omega L_1 \propto j\omega \tan \theta_1 & (\text{normal}) \\ j\omega L_2 \propto j\omega \tan \theta_2 & (\text{fault}) \\ j\omega L_3 \propto j\omega \tan \theta_1 & (\text{no limiting ability}) \end{cases} \quad (11.2)$$

where L_1 , L_2 and L_3 stand for the inductances of SFCL in normal, fault and opposite saturated state, respectively; ω refers to the angular frequency of the AC current and j is an imaginary unit. The saturated iron-core type of the SFCL is passive and immediately triggered, and the DC current flows in the DC winding in which there is no AC losses. In addition, the DC winding does not experience quench under normal operation and fault conditions, thus the SFCL operates with a fast response and there is no recovery problem. The other advantages are that the cryostat associated iron core remains near the ground, HTS windings operate at low DC voltage, so that the problem of electric insulation is easy to deal with, each AC winding nevertheless experiences significant system voltage during a fault and fail-safe operation, and high voltage design is similar to that of mature transformers, which makes the SFCL available in high and super-high voltage levels.

However, it has some drawbacks, which include bulky assembly with a heavy weight and large volume of iron core, and extra voltage drop in the inserted impedance in normal operation due to two AC windings. There is an AC current induced by two AC windings so that the AC voltage and current will be created in the DC HTS windings and AC losses will be generated, and although both of them are small during normal operation, they cannot be ignored during a fault. Finally, high maintenance cost is

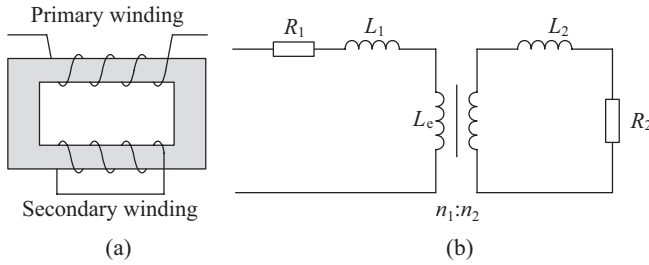


Figure 11.8 Transformer type SFCL: (a) schematic principle view; (b) equivalent circuit. R_1 , L_1 and R_2 , L_2 are leakage resistances and inductances in primary and secondary, respectively; L_e denotes the excited magnetic inductance in iron core.

incurred during operation since the DC power supply always provides current to the DC winding. There is also an induced AC ripple current in the cryostat of the HTS winding during a fault if the cryostat is made of metal, such as stainless steel, which appears as one turn of the secondary winding. If the cryostat is made from non-metals, such as fibre reinforced plastics (FRP), the vacuum between the inner and outer walls of the cryostat is difficult to maintain for long, as described by Section 9.2 of Chapter 9.

11.2.4 Transformer Type

The transformer type of SFCL consists of primary winding wound by copper wire and short circuited secondary winding wound by HTS wire (Figure 11.8(a)). Figure 11.8(b) indicates its equivalent circuit. The terminals of the primary winding are directly connected with the grid line. In normal operation, it is set so that the critical current of the secondary winding is smaller than its rated current, so that it is in a superconducting state without resistance. Impedance of this type of SFCL is merely determined by the leakage impedance of the primary winding and thus is small due to the short circuit of the secondary winding. During a fault, the secondary winding quenches since the current flowing through it exceeds its critical current, so that impedance in the secondary winding rapidly increases. Therefore, effective impedance of the SFCL rapidly increases, which will limit the fault current. Based on Figure 11.8(b), the impedance of the SFCL can be expressed as:

$$Z_s = \begin{cases} Z_1 & (normal) \\ Z_1 + n^2 \frac{Z_e Z_2}{Z_e + n^2 Z_2} & (fault) \end{cases} \quad (11.3)$$

where $n = n_1/n_2$ is the transformation ratio of the transformer, n_1 and n_2 denote the turn number of primary and secondary windings, respectively. $Z_1 = R_1 + j\omega L_1$ is the leakage impedance of the primary winding, $Z_e = j\omega L_e$ is the excited magnetic leakage impedance of the transformer, and $Z_2 = R_2 + j\omega L_2$ is the leakage impedance of the secondary winding.

There is no heat conduction in the secondary winding since the current leads are not necessary, so that the heat leakage is reduced. However, the secondary winding quenches and then creates Joule loss during a fault, and the recovery from normal state to superconducting state takes a long time. This type of SFCL is seldom adopted in practical applications.

11.2.5 Shielded Iron Core Type

The concepts of a shielded iron core SFCL is presented in Figure 11.9. It consists of an iron core, a primary winding wound by conventional wire and a secondary winding made from an HTS tube. Its

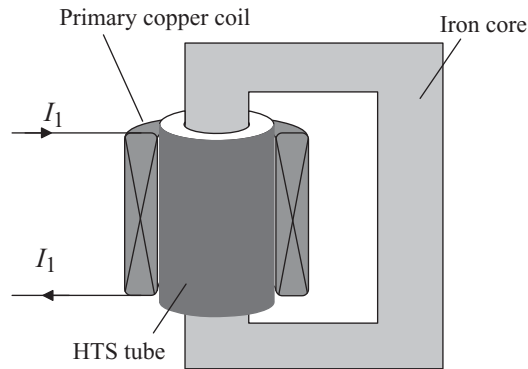


Figure 11.9 Principle of shielded iron core SFCL. (Reproduced with permission from J. R. Hull “Applications of high-temperature superconductors in power technology,” *Report Progress in Physics*, 66, 1865–1886, 2003. © 2003 IOPscience.)

equivalent is similar to Figure 11.8(b). Three of them are concentrically installed. This configuration is similar to a single transformer with shorted secondary windings. In normal operation, secondary winding of the HTS tube is in a superconducting state, and the flux created by the primary winding will induce shielding current (which is lower than its critical current) in the shorted HTS tube. The flux generated by the shielding current is opposite to that created in the primary winding in the iron core and offsets this flux, so there is no flux penetrating across the HTS tube. Consequently, the effective impedance of the primary winding, and the net impedance of a transformer with a secondary winding short circuit, is low. However, during a fault, the HTS tube quenches and its flux shielding effect on the iron core from primary winding is lost, since the induced shielding current is more than its critical current; thus, the flux created by the primary winding enters the iron core. Therefore, a great amount of flux linking the primary winding increases its impedance to a pre-set high value, and then the fault current is effectively limited to a desired value. The expressions of the impedances of the shielded iron-core type SFCL are the same as in Equation (11.3).

Since this device only needs a short HTS tube, which is equivalent to a secondary winding with one turn, the current leads are unnecessary, as the conduction heat leakage is absent. Besides the insertion of finite impedance in the line during normal operation, it has other drawbacks, such as a bulky iron core similar to the transformer type, and difficulty in uniformly quenching and rapid recovery from normal state to superconducting state after a fault. Consequently, this type of SFCL is rarely chosen.

11.2.6 Bridge Type

The bridge type of SFCL mainly consists of four diodes (D_1 – D_4), a bias HTS winding L_{sc} and a bias DC power supply U_b (Figure 11.10). The bridge is made of four diodes, and the DC power supply U_b provides the bias DC current I_{sc} for the HTS winding L_{sc} . The initial value of the bias current I_0 is equal to I_{sc} by adjusting the voltage U_b and is larger than the amplitude I_{ac} of the normal line current, so that the four diodes (D_1 – D_4) are always turned on. In normal operation, the device does not present any impedance for line current I_{ac} except for a small positive voltage drop in the bridge. During a fault, amplitude I_{ac} of the line current is greater than I_0 , diodes D_3 – D_4 turn off and turn on in positive and negative half waves of the line current I_{ac} , respectively; however, diodes D_1 – D_2 separately turn on and off in positive and negative half waves of the line current I_{ac} . Therefore, HTS winding L_{sc} is automatically

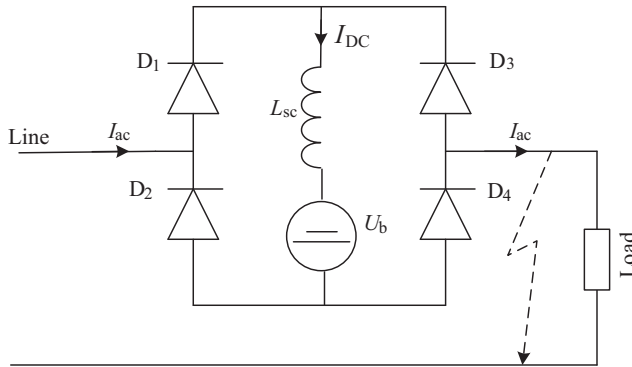


Figure 11.10 Equivalent circuit of bridge-type SFCL.

inserted in series to the line and thus limits the fault current due to impedance $Z_s = j\omega L_{sc}$ of the HTS winding. The limited current impedance of the bridge type SFCL is described by:

$$Z_s = \begin{cases} 0 & (\text{normal}) \\ j\omega L_{sc} & (\text{fault}) \end{cases} \quad (11.4)$$

Since the HTS winding operates in DC mode, there are no AC losses and its operation current I_{sc} can be easily achieved by adjusting the power supply U_b . In addition, the HTS winding does not quench so there is no recovery problem from normal state to superconducting state. However, the bridge type SFCL is applicable at mid- and low voltage level, but it is difficult to use at high voltage levels.

11.2.7 Hybrid Type

The hybrid type SFCL mainly consists of a conventional transformer with a variable coupling magnetic circuit and a triggering HTS winding fabricated by bifilar technique. There are two kinds of hybrid type SFCLs, the first is the serial type in which the triggering HTS winding is connected in series with the secondary winding of the transformer (Figure 11.11(a)); and the second is the parallel type, which is connected in parallel with the secondary winding of the transformer (Figure 11.11(b)). In the serial type, the primary and secondary windings are oppositely wound in order to increase their coupling degree, thus the device shows small impedance in normal operation. During a fault, the current in the secondary winding rapidly increases, and the HTS winding will lose its superconductivity and present resistance R_{sc} .

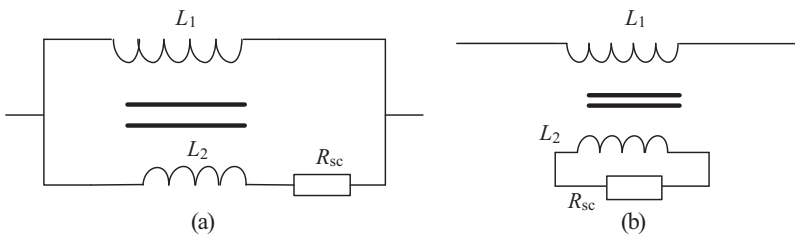


Figure 11.11 Circuits of hybrid type SFCLs: (a) serial type; (b) parallel type.

because it connects with the secondary winding. The resistance automatically inserts to the secondary side, therefore the fault current is limited to a desired level. The expressions of impedance in the serial type of SFCL are:

$$Z_s = \begin{cases} 0 & (normal) \\ \frac{j\omega L_1 R_{sc}^2 + \omega^2 L_1 R_{sc} (L_1 + L_2 - 2M)}{R_{sc}^2 + \omega^2 L_1 (L_1 + L_2 - 2M)} & (fault) \end{cases} \quad (11.5)$$

where L_1 and L_2 denote the inductances of the primary and secondary windings, respectively, M stands for mutual inductance between the primary and secondary windings and $M = k(L_1 L_2)^{1/2}$, k is the coupling coefficient, and R_{sc} refers to the resistance of HTS winding after quench. If the primary winding perfectly couples with the secondary winding, $k = 1$, then the impedance Z_s simplifies to $Z_s = j\omega L_1$ during a fault.

As for the parallel type of hybrid type SFCL, in normal operation, the secondary side of the transformer is short circuited, and the impedance is only determined by the leakage impedance of the primary winding and thus is small. During a fault, the HTS winding will quench since the induced current is more than its critical current, which leads to an increase in the impedance of the transformer, so that the fault current is limited. The expression of the parallel hybrid type SFCL obeys:

$$Z_s = \begin{cases} Z_1 & (normal) \\ Z_1 + \frac{Z_e n^2 (Z_2 + R_{sc})}{Z_e + n^2 (Z_2 + R_{sc})} & (fault) \end{cases} \quad (11.6)$$

where $Z_1 = R_1 + j\omega L_1$ is the leakage impedance of primary winding, R_1 and L_1 are its resistance and inductance, $Z_e = j\omega L_e$ is the impedance of the iron core, $Z_2 = R_2 + j\omega L_2$ is the leakage impedance of the secondary winding, R_2 and L_2 are its resistance and inductance, R_{sc} denotes the resistance of HTS winding after quench, $n = n_1/n_2$ represents the transformation ratio of the transformer, and n_1 and n_2 are number of turns in the primary and secondary windings.

Although the current in the secondary winding can be set to a value small enough by adjusting the transformation ratio of the transformer, there is still the recovery problem of HTS winding after fault. Therefore, researchers seldom develop this kind of SFCL.

11.2.8 Three-Phase Reactance Type

The three-phase reactance type SFCL is composed of three similar HTS windings, which are wound onto a toroidal iron core (Figure 11.12). In normal operation, the sum of the currents in the three-phase windings is zero and the flux does not change in the iron core because they are in a balanced condition. Two types of fault, symmetrical and asymmetrical, are introduced here. In the former case, the inductive reactance component in limiting current impedance has a small function, so quench resistance R_{sc} plays the main role in limiting current. The limiting impedance may be described by:

$$Z_s = \begin{cases} 0 & (normal) \\ R_{sc} & (fault) \end{cases} \quad (11.7)$$

In the latter case, the impedance of the limiting current is expressed as:

$$Z_s = \begin{cases} 0 & (normal) \\ j\omega \mu \frac{N^2 S}{L} + R_{sc} & (fault) \end{cases} \quad (11.8)$$

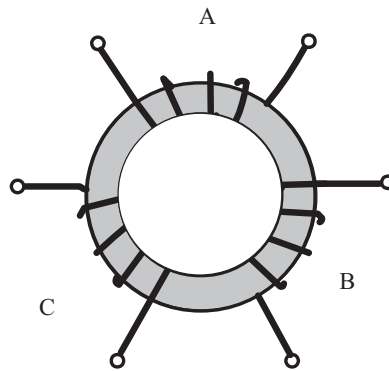


Figure 11.12 Principle of three-phase reactance type SFCL.

where N denotes the turn number of each winding, μ and S refer to permeability and cross-section of the iron core, respectively, L stands for the perimeter of the central line of the iron core, and R_{sc} represents the resistance of each winding after quench. According to Equations (11.7) and (11.8), the current-limiting ability of the three-phase reactance type SFCL in an asymmetrical fault is stronger than that in a symmetrical fault. This kind of SFCL is bulky and complicated in fabricating of the iron core and cryostat. There is an AC loss in the HTS winding, since it operates in AC mode. It is thus rarely used in practical applications.

Currently, only three types of SFCL prototypes, namely resistive type, saturated iron core type and bridge type, are widely developed and demonstrated at the medium and high voltage scales in the power grid, while other conceptual types of SFCLs are researched in principle. However, the saturated iron core type is the most prospective in applications of high voltage scale, since the quench is not necessary and there is no electrical insulation in the HTS winding. The high voltage exists only in AC windings, which can be processed by the mature technique of a conventional transformer, though it is bulky with a large size and heavy weight.

11.3 Transformer

The transformer is the indispensable electric apparatus in a power grid system. Transformers can convert electric power from one voltage level to another, such as from mid-voltage electrical power generation to high voltage for transmission, and from high voltage to low voltage for consumption. A conventional transformer is mainly composed of normal conductor windings, iron core, bushings, etc. Its losses include Joule loss in windings, hysteresis loss in iron core, and eddy current losses in clamps and enclosure shell resulting from leakage of the magnetic field. Although the efficiency may reach about 99%, there is still a huge wastage of power. Currently, the manufacture of the conventional transformer is mature, but it is difficult to further reduce the losses and then improve efficiency. However, it is possible to reduce losses in the transformer by replacing normal conductor windings with HTS windings having no resistance so as to reduce the Joule loss.

11.3.1 Configuration

Compared with conventional transformer, the HTS transformer has a similar basic structure to conventional transformers. Figure 11.13 illustrates the conceptual configuration of the HTS transformer.

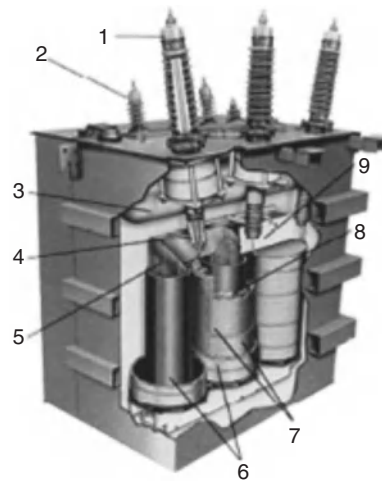


Figure 11.13 Conceptual configuration of three-phase HTS transformer: 1. 4. high voltage (HV) bushing; 2. low voltage (LV) bushing; 3. LN₂ transfer line; 5. iron core; 6. non-metallic cryostat; 7. primary/secondary windings; 8. LN₂; 9. iron core clamps. (Courtesy of Oak Ridge National Laboratory, U.S. Dept. of Energy.)

HTS windings are contained in non-metallic (i.e., FRP) cryostats and operate at cryogenic temperature [10, 11]. The iron core operates at room temperature since it creates losses at the cryogenic temperature, even at no-load, and it is wasteful to remove them by refrigeration. As with the typical oil-filled conventional transformer, a single metallic warm shell should enclose cryostats and the iron core to provide protection against the environment.

11.3.2 Advantages

Since its windings are wound by HTS wire with high current density, which is about two orders of magnitude more than that of the copper, both the size and weight of the HTS transformer will be greatly reduced. Thus, the problems of occupation requirement, transportation and installation can be alleviated. The zero resistance of HTS windings results in the improvement of efficiency. Due to the HTS winding operating at constant cryogenic temperature, degradation of electrical insulation is absent and so has a longer life. In addition, the HTS transformer is non-flammable, safe and environmentally friendly because inexpensive LN₂ is employed as the cryogen and dielectric insulation media, and also nitrogen is rich in air and so causes no damage. The system security can be also improved.

11.3.3 Further Key Technology

Although there is no Joule loss in HTS windings, the hysteresis loss cannot be neglected because they operate in AC mode. At present, commercial HTS wire mainly includes B-based tapes and ReBCO coated conductors, and AC losses are still unacceptable because the dissipated power at LN₂ temperature is at least 11 times that of room temperature. Therefore, the continuously transposed cable (CTC), employed for windings with low loss is essential. However, it is difficult to fabricate long enough CTC by transposing HTS tapes. Due to tape shape with a higher aspect ratio of more than 20 in present commercial HTS wire, the point discharge is high amongst turns, which is a challenge for a high voltage class transformer. If the insulation thickness of each turn increases and its breakdown strength then increases, the capability

of heat dissipation reduces so that the thermal stability may be deteriorated. Finally, the cryostat housing of the HTS windings must be made from non-metallic materials without magnetism. Currently, the FRP is the most optimal candidate. Compared with a cryostat made from stainless steel, the FRP cryostat has some drawbacks, such as higher heat leakage, difficulty in maintaining low vacuum degree for any length of time due to its inherent out-gassing characteristic, sealing up, etc.

Some prototypes of HTS transformers with a voltage lower than 110 kV level were successfully demonstrated in power grid systems in recent years, but there are no reports on the HTS transformers with higher voltage levels and large capacity. Therefore, it will take a longer time for HTS transformers to be applied to practical power grid systems.

11.4 Rotating Machine-Generator/Motor

Traditionally, a rotating machine refers to a generator and motors. The conventional rotating machines employ copper or aluminium windings on rotor and stator. Due to Joule losses created in both windings, their efficiency is difficult to improve further. If the windings are replaced by the HTS windings without resistance and operating at high current density, that is, it becomes a HTS rotating machine, then its electrical losses and also its size and weight will be significantly reduced. In principle, there is no difference between the conventional and HTS rotating machines [12, 13].

11.4.1 Configuration

There are two types of electric rotating machines, that is, induction and synchronous types. Since both of the two windings on rotor and stator of the induction motor carry AC currents, which result in large AC losses and high operating cost as well as low efficiency, they are not optimal candidates to be made from HTS windings, even though the induction motor is popular in industry. However, the common configuration of the synchronous type machine has two windings. One of them lies on the rotor and carries a DC current, and simultaneously is exposed to the DC magnetic field because the AC magnetic field generated by stator winding is synchronous with the rotor. The other is located on the stator and carries the AC current. Therefore, HTS is applicable for winding on a rotor in a synchronous AC machine, and the conventional copper winding is suitable for the windings on the stator. Thereafter, the rotating machine will refer to the synchronous AC machine, unless otherwise specified.

Figure 11.14 indicates a typical configuration of an AC synchronous machine (generator), which mainly includes HTS field winding (rotor winding), stator winding, shaft, torque tube, damping tube, thermal radiation shielding, cryostat, magnetic field shielding, etc. The HTS field winding (rotor winding) generates a magnetic field in the copper armature winding (stator winding), whose magnitude is typically twice of that of a conventional one. The HTS motor (and generator) has no iron core on the rotor and non-metallic teeth in the stator, so there is no iron core loss and saturation problem, even if the magnetic field in the air gap is more than the saturated field of the iron core. The copper armature winding (stator winding) on the stator lies just outside the air gap or is embedded in non-metallic teeth in some applications to provide mechanical support. The HTS field winding and copper armature winding operates in cryogenic and room temperatures, respectively. The HTS field windings experience the full rated torque of a machine.

Torque in a cryogenic environment is transferred to the shaft with room temperature by a warm magnetic shield. Space outside the field winding is a vacuum in which the multi-layer thermal radiation shielding layers are installed, so that the heat leakage to cryogenic environment by radiation can be effectively reduced and then the efficiency is greatly improved. The damping tube reduces the effect of the AC magnetic field created in the stator winding on rotor winding in the transient process besides the function like that in a conventional rotating machine, so that the stability and security of the rotor winding can be improved. At the same time, the damping tube is also the outer wall of the cryostat. Because the

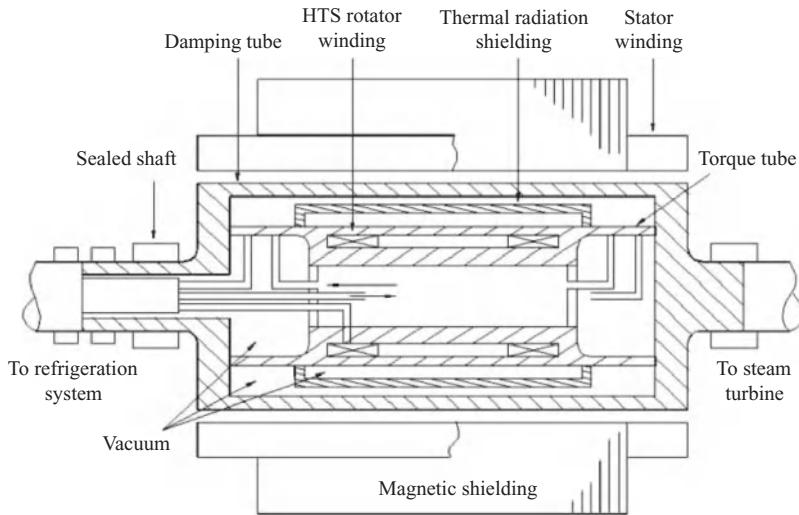


Figure 11.14 Conceptual configuration of synchronous HTS machine.

magnetic field is high and the iron core is neglected in the winding regions, the ferromagnetic materials are commonly employed to shield the magnetic field outside the stator winding in order to form the magnetic circuit of armature winding and prevent the magnetic field from leaking out.

11.4.2 Advantages

The HTS rotating AC machine has a number of advantages over its conventional type partner. First, it is compact and lighter, that is, its volume and weight are about 30 to 50% of the conventional machines because there is no iron core in the windings as well as both operating magnetic field and current density are high, so its transportation is more rapid. Second, the synchronous reactance is small, at about $\frac{1}{2}$ to $\frac{1}{5}$ of a conventional machine; so that its output voltage is stable, the varying range of excited current is small, and the requirements on automatic voltage regulator (AVR) and power system stabilizer (PSS) can be alleviated. In addition, the phase difference in the power system caused by the reactance is also small, thus synchronous torque increases.

Third, it has higher efficiency because the Joule loss is absent in the rotor windings, which leads to significant operating costs, and it has a longer life since the rotor winding operates at a constant cryogenic temperature without thermal cycling. There are almost no harmonics in the terminal voltage and lower vibrations as well as noise. Finally, the range of output power can be increased as compared to the conventional machine with the same capacity. The range of the reactive power of a generator depends on its internal reactance and excited current. Because there is no saturated problem of the iron core, the machine can be operated in the over-excited state with zero power factor as long as the design capacity of excited winding is enough, thus it can also be used as a dynamic synchronous condenser (DSC).

Of course, the HTS synchronous rotating machine also has drawbacks, one of which is that the HTS winding made of present commercial HTS wires must operate below temperatures of about 30 K, due to their significant critical current degradation at LN_2 temperature and high magnetic field. Consequently, operating cost is high, and it is also difficult to further improve on its efficiency.

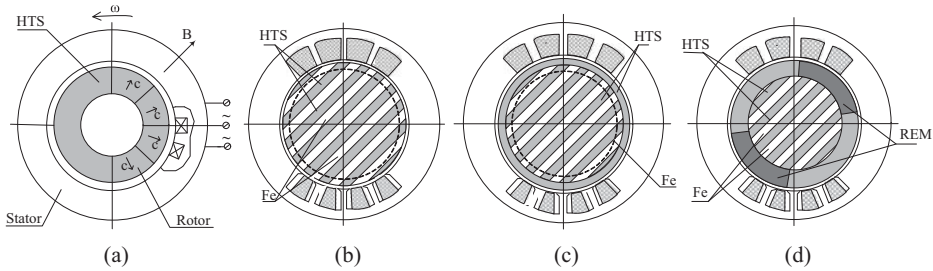


Figure 11.15 HTS permanent motors: (a) hysteresis HTS bulk motor; (b) reluctance HTS bulk motor with compound rotor; (c) trapped field HTS bulk motor; (d) composed HTS bulk motor with PMs (REM denotes rare earth magnets). (Reproduced with permission from L. K. Kovalev, K.V. Hushin, V. T. Penkin, et al. "HTS electrical machines with YBCO bulk and Ag-BSCCO plate-shape HTS elements: recent results and future development," *Physica C* 354, 34–39, 2001. © 2001 Elsevier.)

11.4.3 Electric Machine with HTS Bulk

With development and performance improvement of HTS bulk, several novel types of electric motors have made great progress. Some prototypes are demonstrated. These electric motors with HTS bulk elements are mainly classified into four types: hysteresis machine (HM), reluctance machine (RM), trapped field machine and composed machine. Their structures are shown in Figure 11.15 [12]. The specific output power per unit weight of HTS bulk machines is qualitatively about three to six times more than their conventional partners if operating at LN_2 temperature. HM is developed according to the hysteresis characteristics of HTS bulk, and its schematic configuration is presented in Figure 11.15(a). Its rotor is composed of HTS cylindrical components which can be prepared with a cylinder or glued from circular sectors in such a way that the c -axis of the HTS components is in a radial orientation. Contrary to the conventional electric motor, the torque of the HM electric motor comes from repulsion of magnetic poles induced and exerted to the rotor by the rotating magnetic field of the stator windings and is determined by the interaction of intergranular and intragranular currents in the HTS bulk rotor with rotating magnetic field. In addition, the torque is proportional to total hysteresis loss of the HTS rotor and is independent of its angular velocity. Figure 11.15(b) illustrates the structure of a two-pole HTS RM motor with a layered HTS compound rotor. In the HTS RM machine, HTS bulk elements, possessing strong diamagnetic properties in superconducting state, are used to replace the non-magnetic materials commonly used in their conventional partner.

Due to their strong flux pinning effects, bulk HTS materials can trap high magnetic fields in the superconducting state; for example, the newly developed YBCO bulk is able to trap a magnetic field with a magnitude of 2 T and 21 T at LN_2 and 20 K temperatures, respectively, which are much more than that of the conventional permanent magnet (PM) NdFeB with about 1 T on the surface. Figure 11.15(c) shows the construction of the trapped field HTS bulk motor, where the central core consisting of anisotropic HTS bulk and the ferromagnetic core provide a certain orientation of magnetization and most of the trapped field is confined in the region surrounding the HTS ring. Figure 11.15(d) shows the configuration of the composed HTS motor, which is mainly made from HTS bulks and PM, which is in the rotor core. The PM is located at the sides of the central anisotropic core. This kind of motor, a combination of HTS bulk, iron and PM elements, has a higher power factor and output power compared with its conventional partners.

11.4.4 Applications

Although HTS machines could be smaller, lighter and more efficient than the conventional copper and PM machines, their economic feasibility is contingent on the availability of low-cost HTS materials and

reliable and affordable cooling systems. Their construction and operation cost must be low enough to make HTS machines competitive with conventional copper and PM machines. Currently, the refrigeration system and HTS materials are more expensive, so that the HTS machines have no obvious economic advantages over their conventional partners in practical applications. However, they can have more possible applications on some special occasions, such as ship propulsion, motors and generators for central power stations, wind turbine generators, and synchronous condensers. In civil applications, the wind turbine generator is the most possible application, because it is difficult to increase the capacity of conventional wind turbine generators, which are limited by size and weight as well as gear boxes. Direct drive wind turbine generators with a capacity of more than 10 MW will be the most feasible for offshore applications in the near future.

In addition, the HTS bulk machines are promising in civil applications, such as superconducting application in power systems, cryogenic engineering, aerospace and superconducting gyroscopes. They are used as driving devices for cryogenic pumps of liquid fuel and cryogenics [14, 15]. For example, they can be employed to transport liquid fuels such as LH_2 and LO_2 in aerospace fields, and to transport cryogenics of open or closed cycling in the operation of superconducting devices such as LHe and LN_2 .

There are still some challenges for the spread of their applications. At present, the HTS rotating machines must operate at a temperature much lower than the LN_2 temperature, that is, about 30 K, since the critical currents of the current commercial HTS materials are seriously degraded in high magnetic fields and they show strong anisotropic at LN_2 temperature. However, the rotor rotates at high speed at cryogenic temperatures, so the cryostat with low heat leakage is another problem. The support materials and the torque tube, as well as the shaft in the HTS machine, must have strong enough mechanical properties. HTS windings and bulks should have high stability because they must withstand AC magnetic fields in electromagnetic dynamic and transient processes.

11.5 Superconducting Magnetic Energy Storage (SMES)

Sometimes, electrical power quality of the commercial power system cannot meet the requirements of special users. In the current power grid system there exist voltage fluctuations, undesirable harmonics resulting from other system loads and momentary interruption or voltage sags due to faults in the power system. In order to improve the power quality of local plants or the electric grid, where a short interruption of power may possibly result in a long shutdown and even serious economic loss, as well as substantial impact on social life, SMES magnets are considered to be qualified for this purpose.

11.5.1 Principle and Basic Topology

SMES, an inductance element, can directly store the magnetic energy, which can be returned to the electric grid or other load [16]. As well known, if the inductance of the SMES magnet is L and carries a DC current I , the stored energy is described by:

$$E = \frac{1}{2}LI^2 \quad (11.9)$$

Since the magnitude of the transport current density of the superconducting wire with no resistance is about two orders higher than that of the conventional copper conductor, its stored energy density is high and reaches 100 MJ/m^3 . Compared with other modes of energy storage, such as chemical battery, compressed air energy storage (CAES), pumped-storage hydroelectricity and flywheel energy storage (FES), the SMES magnet has a number of advantages, such as high transformation efficiency of about 95%, fast response time of millisecond order, high power density, long service time and simple maintenance and is pollution free.

Generally, the SMES system mainly consists of a superconducting magnet (coil), cryostat, current converter (power regulating system) and monitoring and controlling system (controller). The superconducting magnet energy storing system transforms electric energy to magnetic energy. The cryostat provides a cryogenic environment for the superconducting magnet. The current converter changes AC current into DC current and vice versa, and controls energy exchange between the two types of energy. The controller system plays an important role in controlling actions of the SMES system.

11.5.1.1 Superconducting Magnet

As a key element of the SMES system, the superconducting magnet is generally in solenoidal or annular form. If the superconducting tape is used, the superconducting magnet is conveniently composed of double pancakes, which are connected in series or parallel, for example, the HTS wire always presents with a tape shape. Although the former is simple and easily fabricated, it has a higher stray magnetic field (leakage magnetic flux). However, the latter has a complicated structure, even though it has a lower stray magnetic field. The design work of the superconducting magnet mainly includes the design of its structure, magnetic field distribution, transport current margin, monitoring and protection technique, AC losses, stability, current leads, etc.

The superconducting must receive special attention to protect it from destruction during quench when faults may occur. In addition, the AC losses and stability must be considered in detail in the design of the superconducting magnet because it operates in AC mode during charging and discharging when the SMES system is applied in the grid power system, so that it must also withstand dynamic operation conditions.

11.5.1.2 Cryostat System

Since the superconducting magnet operates in DC mode, there is no resistance and Joule heat, so that the cryostat can be manufactured by metal materials, and stainless steel is the optimal choice because of its high mechanical strength, low conductivity and high density. The design and structure of the cryostat, as well as cooling modes of the superconducting magnet, were introduced in Chapter 9.

11.5.1.3 Power Regulating System

Because the power grid operates in AC mode and the superconducting magnet operates in DC mode, the intermediate transformation device is essential between both of them. The power regulating system controls energy transformation between the superconducting magnet and power grid, and thus is called the bridge of power transformation between stored energy elements and the grid system. Currently, a converter with pulse width modulation (PWM), based on a full control type switch device, is generally used in a power regulating system. It is able to independently and quickly control the active and reactive power in four quadrants and has the advantages of low harmonic components and fast dynamic response. According to its topology, the converter is classified into two types: current source converter (CSC) and voltage source converter (VSC), as shown in Figures 11.16(a) and (b), respectively, where the superconducting magnet can directly connect with the DC side of the CSC. However, it cannot directly connect with the VSC, and this indirect connection is realized by inserting a chopper between the superconducting magnet and the VSC.

11.5.1.4 Monitoring and Controlling System

Monitoring and controlling the system consists of signal acquisition parts and controlling elements. The system extracts information from the grid system and controls power output of the superconducting

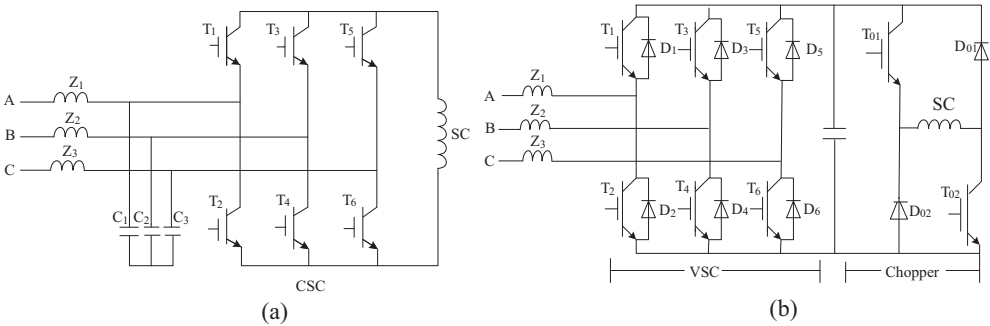


Figure 11.16 Topology of current and voltage source converters: (a) current source converter; (b) voltage source converter.

magnet according to the needs of the grid system. The signal acquisition partly monitors various parameters of the superconducting magnet and grid system and provides basic data for the controller in order to analyse the system state. Based on the state of the grid system, the controller calculates the system's power demand, and then the superconducting magnet can charge or discharge by regulating its terminal voltage with the help of the converter. The controller must match the dynamic processes of the power grid in order to realize effective control. The controller of the SMES system is divided into external and internal loops. The external-loop controller, or the main controller, provides reference values of active and reactive power, which the internal-loop controller requires, while the internal-loop controller provides a triggering signal for the converter switch according to the reference data provided by the external controller.

11.5.2 Application in Grid System

The SMES can increase stability of the power grid system. As a flexibly controlled active power source, the SMES can regulate damping and synchronous torques of the system, so it can resolve the problems of non-periodic out-of-step and oscillation out-of-step and shorten the transient transition process after disappearance of fluctuation, so as to make the grid system rapidly recover to a stable state. In addition, the SMES can flexibly regulate reactive power and provide voltage support for the transmission lines to prevent voltage collapse during a fault and to inhibit further damage from the incident.

Since the SMES can release and absorb power, it can reduce the load fluctuation and decrease impact on the grid system due to the varying output power of generator, so it serves as an uninterrupted power supply (UPS) for some sensitive loads and significant apparatus. Simultaneously, it also can improve power quality.

Due to its rapid charging and discharging performance, the SMES is also applicable for occasions of dispersed power, such as wind and solar energy power generation, to smooth the fluctuation of power output in a distributed power generation system and to effectively inhibit the fluctuation and flicker of voltage due to distributed power generation system connection with the power grid. Therefore, SMES can improve controllability of the distributed power generation system connecting with the grid system.

In recent years, with rapid development of power electric devices, such as the flexible alternating current transmission systems (FACTS) have made great progress and started to enter the commercial market. If the SMES combines with FACTS, it is predicted that a new technology will soon appear.

There are also some drawbacks of SMES applied to the power grid. During exchanging energy with the grid, the SMES charges and discharges, thus it operates in quasi-AC mode, and the AC losses will be a problem. However, the magnetic field of the HTS magnet is low at LN₂ temperature, which limits the

magnitude of stored energy. If the operation temperature reduces to the LHe temperature, the cooling cost and price of HTS materials are expensive, so that it has significant economic disadvantage. The significant challenge is that the protection is difficult for the superconducting magnet because it must connect with CSC or VSC, and the protection techniques for the superconducting magnet of SMES cannot be fully copied from conventional superconducting magnets.

11.6 Superconducting Flywheel Energy Storage (SFES)

A typical flywheel energy storage (FES) system consists of a rotor, which is suspended by bearings inside a vacuum chamber to reduce friction and is connected to a combination electric motor and electric generator [17, 18]. FES operates by accelerating the rotor (flywheel) to rotate at high speed and maintain (equivalent to storing) the energy in the system in the form of mechanical energy. If energy is extracted from the system, the flywheel's rotating speed will decrease due to the conservation of energy. Conversely, if energy is added to the system, the energy will accelerate the flywheel and its speed increases. Like the action of SMES, SFES is also a bridge of power transformation between stored energy elements and the grid system, if it is used in the grid system.

11.6.1 Principle and Structure

If the magnetic levitating bearing is replaced by a superconductor in FES, FES becomes a superconducting flywheel energy storage (SFES), so SFES is a combination of superconducting magnetic levitation bearing and FES. Its conceptual structure is shown in Figure 11.17. The magnetic levitation unit is composed of HTS bulks and PM, with PM bearing the flywheel. During storage of energy, the electric machine drives the flywheel rotating with high speed, which continuously increases until energy storage stops, so that the electrical energy is transformed into mechanical energy, which is stored in the rotating system. However, if the external system needs energy, the electric machine acts as a generator, and the flywheel drives the electrical machine to rotate, thus mechanical energy is converted into electrical energy output

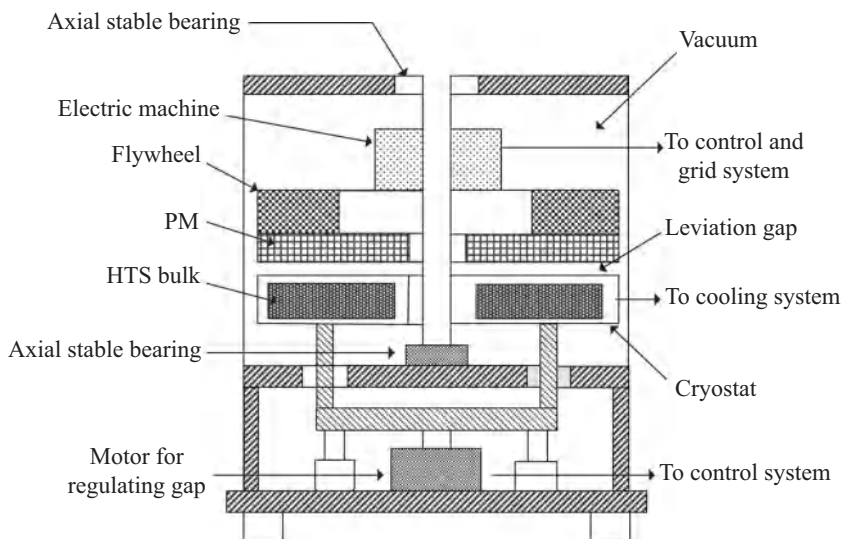


Figure 11.17 Structure of SFES.

to the external system. Consequently, the rotational speed of the flywheel gradually reduces until the mechanical energy runs out.

SFES mainly consists of four systems: flywheel, magnetic levitation system, electric machine system and cryostat and vacuum system.

11.6.1.1 Flywheel

The flywheel is a rotating object with a fixed axis. If its moment of inertia and rotating angular speed are I and ω , respectively, its stored energy is expressed as:

$$E = \frac{1}{2}I\omega^2 \quad (11.10)$$

According to Equation (11.10), there are two methods to increase the stored energy E , one to increase the moment of inertia I and the other to increase the rotating angular speed ω . It is clear that increasing angular speed ω is an effective measure, since the stored energy is proportional to the square of ω .

11.6.1.2 Magnetic Levitation System

When a superconductor is in the superconducting state, the flux will be fully expelled out of the superconductor due to the Meissner effect. Therefore, an electromagnetic repulsive force is created between the superconductor and PM, resulting in the superconductor stably levitating in the magnetic field generated by PM. In FES, the levitating element is a magnetic levitating bearing. Magnitude of the levitating force strongly depends on the pinning force of the superconductor, that is, its critical current density. The larger the pinning force, the stronger the force needed by the flux to penetrate into the superconductor, and then the levitating force becomes larger. At present, the superconductor used in the SFES system is generally the HTS bulk, such as the YBCO bulk, which has strong pinning centres. The commercial YBCO bulk has a disk with a diameter of several centimetres and thickness of several millimetres. Up to now, the superconductors in almost all prototypes of SFESs are YBCO bulk.

11.6.1.3 Electric Machine System

The electric machine is a key element in the SFES system. It must be able to act as a motor or a generator under conditions of fixed rotating direction. During storage of energy, rotating speed of the flywheel continuously increases; however, it gradually decreases during release of energy, thus the frequency conversion technology should be adopted in order to keep constant frequency at output terminals, which can be realized by power electronic devices.

11.6.1.4 Cryostat and Vacuum System

Different from the conventional FES, the superconductor in the SFES works in a cryogenic environment, so the cryostat is essential. If the magnetic field is perfectly uniform, the superconductor rotates in the DC magnetic field, therefore, the cryostat can be made from metal, generally stainless steel being used to fabricate the cryostat. Similar to the conventional FES, a vacuum environment is also required to reduce the friction between air molecules and rotating parts.

11.6.2 Application in Grid System

The SFES system is one of the most promising electro-mechanical energy storage systems with the advantages of long life, high energy density and low rotational loss due to non-contact superconductor bearings. If it is used in the power grid, SFES is similar to SMES. In the aerospace field, the SFES

system has potential applications, for it can replace batteries and effectively control the performance of spacecraft with the help of the momentum generated by the flywheel. In addition, the SFES system may be applied in train locomotives and power grid systems of subways, in which SFES can exchange energy with the grid system during locomotive accelerating and braking. In this way, the locomotive can be operated with constant frequency, so the working environment is improved and the lifetime of the motor is prolonged. The cost of electricity generated from renewable sources can be greatly reduced by managing the unbalanced electricity generation with SFES.

In addition, HTS bulks are oxide ceramics and so are brittle, and their stiffness does not reach the levels required by turbine bearing. Stability of HTS bulk is another problem due to its flux creep, which reduces the levitation force. These factors are challenges for widespread applications of SFES.

11.7 Other Industrial Applications

In previous sections of this chapter, applications of superconducting materials in electric power systems were emphasized. This section will briefly introduce their applications in other industrial areas involving the applications in high as well as low magnetic fields. The former mainly includes the superconducting magnet used in NMR, propulsion and magneto-hydrodynamics (MHD), while the latter mainly involves magnetic separation, induction heating and maglev in transport, etc.

Magnets have spread their applications in various fields, including industrial areas, scientific research and medical use. Traditionally, the electric magnet is wound by copper or low temperature superconducting (LTS) wires. The superconducting magnets, fabricated by the superconducting wire, have a number of advantages over conventional magnets, such as small volume and light weight, high efficiency, and their magnetic field is higher, more uniform and more stable, etc. The high current density without resistance leads to magnets of smaller size and lower weight with about 40 to 80% of that of conventional magnets, as well as high efficiency and energy saving. However, superconducting magnets can operate in a closed loop by using PCL, so the stability of the magnetic field is high with a resolution of more than 10^{-7} T/h. According to the requirements of different applications, shapes of superconducting magnets may be different, and most basic shapes are solenoid, double-pancake, toroidal, track, saddle, horseshoe and wiggler.

11.7.1 High Magnetic Field

A high magnetic field, generally more than 5 T, is an important necessity in certain scientific instruments and magnetic propulsion, such as ITER, NMR, ship propulsion and MHD power generation. Currently, a magnet field under 25 T is realized by a superconducting hybrid magnet consisting of an LTS magnet and an HTS inserted magnet [19]. The HTS application in a high magnetic field operates in the range of 20 K to 30 K temperatures and can be cooled by cryogen-free cryo-cooler conduction, which is convenient for the user. However, the HTS magnet in applications of high magnetic field is infeasible at LN_2 temperature, because HTS materials are strongly anisotropic and their critical currents are seriously degraded at LN_2 temperature.

11.7.1.1 NMR

In a constant magnetic field B , a nucleus will spin around the magnetic field with Larmor frequency $\omega = \gamma B$, where γ is the gyromagnetic ratio. If the radio frequency (RF) field is applied and is perpendicular to the constant magnetic field, the NMR phenomenon occurs when the RF frequency from a coil is equal to the Larmor frequency, so that the energy of the RF field is completely absorbed by the nucleus. Therefore, NMR can probe the environment of nuclear spins to provide remarkable chemical and spatial

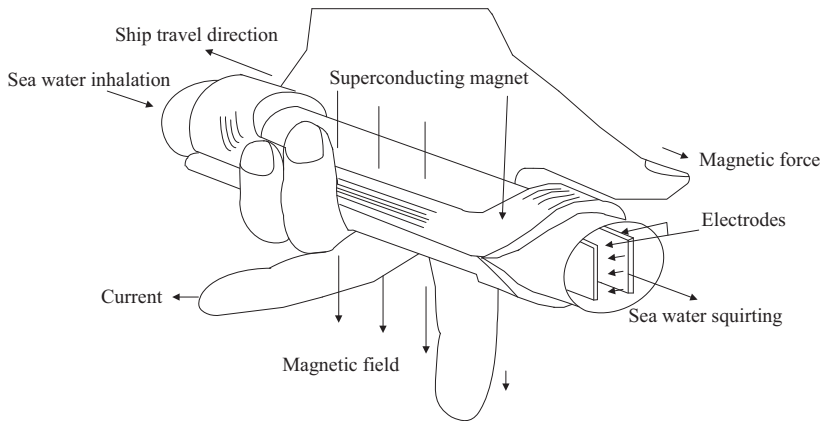


Figure 11.18 Principle of magnetic propulsion.

information. The resolution and sensitivity of the instrument depends on the magnitude of magnetic field. The largest NMR spectrometers have a constant magnetic field in the order of 20 T.

Since NMR can probe the inner parts by contact-free and non-destructive methods with resolutions of 10^{-12} to 10^{-13} , it is widely used in medicine, chemistry, biology, physics, genetics, petroleum, agriculture, etc. It is also the most successful application of superconducting magnets in civil applications.

11.7.1.2 Magnetic Propulsion

Figure 11.18 shows the principle of magnetic propulsion in the sea. The system consists of a superconducting magnet made of a pair of saddles, electrodes and a power supply. When the superconducting magnet carries current I_0 , the magnetic field B_0 is generated, and then the sea water will experience a magnetic force, that is, Lorentz force $F = I \times B$ if a current I perpendicular to the magnetic field B is applied. Consequently, the magnet system will experience a force opposite to the Lorentz force and be driven forward. It has an advantage over the conventional drive in that it has no moving parts. Although several prototypes were demonstrated, the velocity is too low, thus the magnetic propulsion has not been commercially realized yet.

11.7.1.3 Magneto-Hydrodynamic Power Generation

As in the relationship between motor and generator, the reversed application of magnetic propulsion is MHD power generation. Also, as illustrated in Figure 11.18, if the plasma with no electric current supplied replaces the sea water and flows through the magnet, the positive and negative ions in the plasma will experience the Lorentz force with opposite directions and are collected on the electrodes on which a potential will be created, so that electric power is generated. The MHD power generation can be realized with the help of the tides, called tidal power generation. Currently, the superconducting magnet made from permanent HTS bulk is also developed in principle.

11.7.2 Low Magnetic Field

Compared with the high magnetic field, the low magnetic field application of the superconducting magnet is more extensive in industrial areas, such as magnetic separation, induction heating, etc. In view

of saturation of the iron core, there is no iron core in the high magnetic field of a superconducting magnet, which always has an air core. However, if the requirement of the magnetic field is not high (<1.5 T) in low magnetic field applications, the superconducting magnets with an iron core are good candidates.

11.7.2.1 Magnetic Separation

A magnetic separation apparatus, known as a magnetic separator, is developed in such a way that some materials with magnetic properties will experience magnetic forces in a magnetic field. Generally, a low magnetic field between 1 T and 2 T is available. However, it is difficult in industrial cases for conventional magnets to generate such magnetic fields, because they will produce high Joule heat, which must be removed by water-cooled recycling. The HTS magnet can easily create such a magnitude of magnetic fields with higher magnetic field gradients, so that the performance of the magnetic separators with the HTS magnet can be improved. Much magnetic material can be effectively and rapidly separated from non-magnetic material. The magnetic field separators have spread their applications to civil industries, such as ores, kaolin clay, water treatment, waste treatment and isotope separations.

11.7.2.2 Inductive Heater

The conventional AC induction heater with its primary coil wound by copper wire is widely used in industry, such as reprocessing of non-ferrous metals, and copper and aluminium billet, etc. Its principle is indicated in Figure 11.19(a). When the AC high frequency current is flowing in the coil, an eddy current will be produced on the surface of the billet, resulting in a large amount of Joule loss (eddy current loss). Therefore, the whole billet is heated due to its high thermal conductivity. The heat is transformed from the electricity. If the coil is wound by an HTS wire carrying a DC current, while the billet is driven by a motor and is rotating with an angular frequency ω , this is equivalent to the billet being exposed to an AC magnetic field, and the same heating function is realized. The heat is transformed from the mechanical energy of the motor. Figure 11.19(b) illustrates the principle of the inductive heater made from HTS wire [20, 21].

Under conditions of adiabatic and constant temperatures in the whole billet, the temperature rise ΔT during time step Δt is found by:

$$\Delta T = \frac{P}{C(T)V} \Delta t \quad (11.11)$$

where $C(T)$ denotes volumetric specific heat and volume of the billet, respectively, and P stands for the heating power.

The inductive heater has substantial potential in efficiency improvements. The efficiency of conventional heater is generally in the range of 50% to 60%. However, efficiency of the inductive heater is

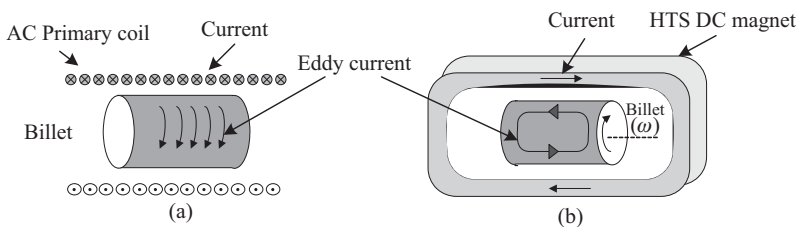


Figure 11.19 Principle of inductive heater. (a) conventional inductive heater; (b) HTS inductive heater.

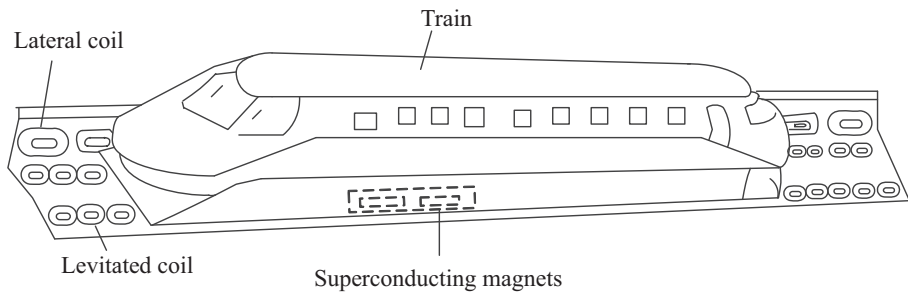


Figure 11.20 Schematic view of maglev train.

more than 90%, because efficiency of the motor is higher, possibly more than 95%. However, there are also other special HTS magnet applications, such as synchrotron, Gyrotron, bending beam and cryogenic capacitor, which are beyond the scope of this book.

11.7.3 Maglev Transportation

Compared with the conventional cold wheeled mass train, magnetically levitated (maglev) trains are predicted to have the potential to be faster, quieter and smoother. Figure 11.20 illustrates the concept of the maglev train, which mainly includes levitated coil, lateral coil, superconducting magnets and train body [22]. The superconducting magnets and propulsion coils are installed at the bottom of the train, and the levitated coils are installed on the track and are always made from metal coils. During locomotion, the magnetic field created by the superconducting magnet induces current in the levitated coils on the track, and the magnetic field generated by the induced current is opposite in direction to that of the train, so that the levitated force between the superconducting magnet and levitated coil is produced. The lateral coils automatically keep the train located at the centre of the guide way. With further propulsion force from the propulsion coil, maglev is realized.

The superconducting magnet can be replaced by HTS bulk (HTS PM) without current leads. Several models were demonstrated. However, there are challenges in commercial application due to the flux-creep effect of the HTS bulk leading to the levitated force being degraded.

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12

Case Study of Superconductivity Applications in Power System-HTS Cable

As introduced in Section 11.1 of Chapter 11, superconducting cable (SC) is classified into two basic configurations, that is, warm dielectric (WD) cable and cold dielectric (CD) cable, including single and Triax three-phase, based on their structure. The electrical source is classified as two kinds: alternating current (AC) and direct current (DC) SC. DC SC is in its development stage to eliminate AC loss in superconductors, and will be applied to high voltage direct current (HVDC) transmission systems. The high temperature superconducting (HTS) cable system mainly consists of HTS cable conductor, cryostat, terminations, cooling system and monitoring system. Due to its drawbacks, WD cable is seldom used. In this chapter, the design of those components in AC and DC HTS cables will systematically be introduced as a case application of HTS materials in a grid power system.

12.1 Design of AC/CD HTS Cable Conductor

The total cross-section of an HTS cable is shown as Figure 12.1, where the components are cable conductor, cryostat and outer protection. The cable conductor consists of a former with or without a hollow core and SC conductor, electrical insulation, SC shielding and protective shielding layers. The former and the protective layer of the shielding copper tapes are respectively soldered with the conductor layer and the shielding HTS layer at both ends of the cable conductor. The cryostat is composed of inner and outer walls, and a thermal insulation layer is in the vacuum space between these walls. The protection layer is made from a PVC (polyvinylchloride) cover on the surface of the outer wall. This section will focus on the design of the cable conductor (Figure 12.2).

12.1.1 Former Size

In a transmission line, a fault is inevitable such as a short circuit, so an SC has to pass a large current securely. Usually, the fault current may reach more than several tens of times the normal current, and this excessive current may exceed the critical current of the SC. In this case, quench may happen and rapid temperature rise may take place, so the HTS cable could be damaged. Therefore, besides conventional

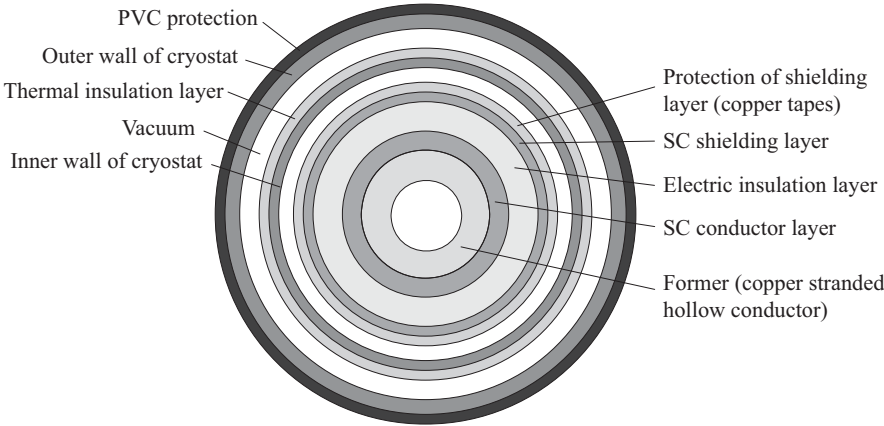


Figure 12.1 Entire cross-section of a single-phase HTS CD cable.

protective measures, a fast circuit breaker (CB) is applied to protect the HTS conductor from quench. The former is expected to protect the cable conductor against fault current so that the security of the cable conductor is ensured before the fast CB switches off in short duration.

During a fault, the internal heat dynamics can be approximately described by heat balance differential Equation (4.1). Because electrical dynamics ends within a short time ($\sim 0.1\text{--}0.2$ s) compared with heat dynamics, the process can be treated as adiabatic, that is, the conductor transfer term and the cooling effect can be disregarded. Therefore, Equation (4.1) simplifies as:

$$\gamma C \frac{\partial T}{\partial t} = G_j$$

(12.1)

where γ and C are separately the density and specific heat of copper, and G_j stands for Joule loss of the cable conductor during a fault:

$$G_j = \rho J^2$$

(12.2)

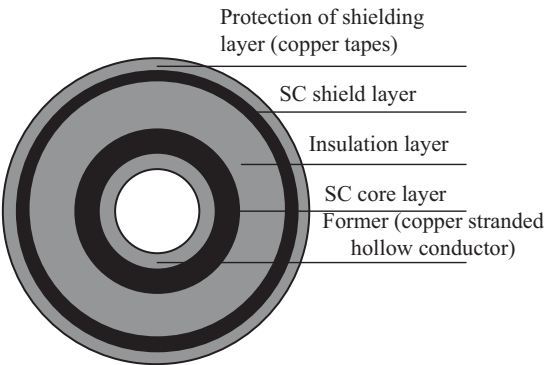


Figure 12.2 Cross-section of a single-phase CD cable conductor.

where ρ is the resistivity of the Cu former in the cable conductor and J denotes fault current density. In order to avoid liquid nitrogen (LN_2) from evaporating due to boiling during a fault, according to Equation (12.1), the cross-sectional area of the former must satisfy:

$$\int_0^{\Delta t_F} \frac{\rho(T)}{A^2} I_F^2 dt = \int_{T_{op}}^{T_P} \gamma C(T) dT \quad (12.3)$$

where Δt_F represents the duration of the fault happening at moment $t = 0$ s before CB switches off; T_{op} stands for the operation temperature of the cable in normal conditions, A is cross-sectional area of the cable conductor and T_P denotes the peak temperature of LN_2 under operation pressure P , which is higher than a standard atmosphere and depends on cycling of LN_2 and length of cable (Figure 9.3). It should be ensured that the peak temperature is less than or equal to the boiling temperature of LN_2 , so that the LN_2 does not evaporate during the fault, to prevent damage to the electrical insulation of the cable, since the breakdown strength of LN_2 gas is much lower than the LN_2 . The worst-case failure of the HTS cable is the breakdown of electrical insulation caused by bubbling of LN_2 inside the electrical insulation layers. Protection by connecting the former in parallel with the conductor HTS layer can avoid the vaporization of LN_2 under pressurized conditions when the short circuit current flows in the HTS cable. After finding the cross-sectional area A of the former by solving Equation (12.3), the temperature rise of the cable conductor and former can be limited below boiling point of the pressurized LN_2 . The formers types, with or without a hollow core, can be adopted according to the rated current and cooling modes after its area A is determined. Thus, the size of the former is obtained.

Strictly speaking, $\rho(T)$ resistivity, cross-sectional area A and volumetric $\gamma C(T)$ are effective parameters of the total cable, and should be calculated by Equations (A1.4) and (A1.6) of Appendix A.1. Three parameters can be approximately replaced by those of the copper former to set up enough margins, respectively, which can assure the security of the cable.

12.1.2 Number of Tapes

Currently, the commercial HTS wires are B- and Y-based tapes with much larger aspect ratios, so that the HTS cable conductor is designed based on configurations of those tapes. If the average critical current of each tape is I_c , the rated current of the cable is I_0 , and the margin is η , then the number N of tapes in conductor should satisfy:

$$N \geq \begin{cases} \frac{\sqrt{2}I_0}{I_c} (1 + \eta) & (\text{AC cable}) \\ \frac{I_0}{I_c} (1 + \eta) & (\text{DC cable}) \end{cases} \quad (12.4)$$

where I_0 denotes the root mean square value of rated current. If the rated current is the DC current, the number should be divided by $\sqrt{2}$. In determining N , some factors should be considered. One of them is the AC losses of the HTS tape; the tolerance of the AC loss of HTS tapes is in the range of 0.15 W/kA·m to 0.6 W/kA·m [1]. The normalized current of each tape, the ratio of current amplitude to its critical current, should be in the range of 0.5 to 0.66, thus it is recommended that the margin η is chosen as about 50%. Other factors include the degradation of critical current in manufacturing and the safety margin.

12.1.3 Number of Layers

Assuming that the inner and outer radii of a former are separately r_0 and r_i , which are determined by its cross-sectional area $A(=\pi(r_0^2 - r_i^2))$, calculated from Equation (12.4), the outer radius r_t of the former

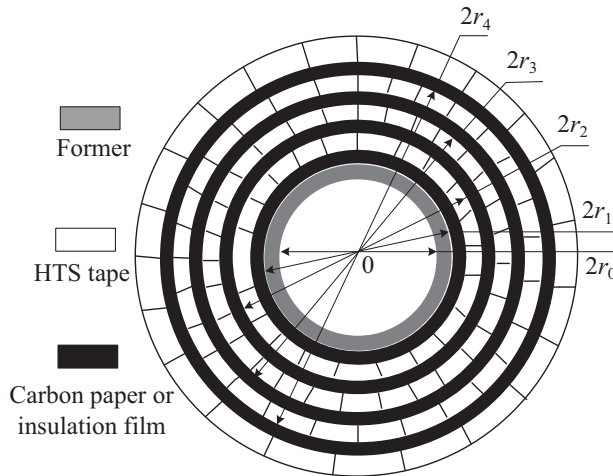


Figure 12.3 Cross-section of cable conductor layers.

after winding carbon paper with thickness t_c onto it is $r_{0f} = r_0 + t_c$. Figure 12.3 shows the cross-section of former and cable conductor layers. There is a layer of semiconductor (carbon) paper between each pair of layers, where carbon paper acts by smoothing the electrical field on the surface of each layer.

Assuming that the thickness of the HTS tape and the carbon paper, including the gap between the layers are t_c and t , respectively, the outer radius of the first conductor layer is:

$$r_1 = r_0 + t_c + (t + t_c) \quad (12.5)$$

The outer radius of the second conductor layer is:

$$r_2 = (r_0 + t_c) + 2(t + t_c) \quad (12.6)$$

By analogy, the outer radius of the i -th conductor layer is:

$$r_i = (r_0 + t_c) + i(t + t_c) \quad (12.7)$$

where, $i = 1, 2, 3 \dots N$.

12.1.4 Number of Tapes on Layer

Figure 12.4 illustrates the configuration of the i -th layer conductor in a multi-layer cable [2–4]. The cross-section and side view of the i -th layer are presented in Figures 12.4(a) and (b). If the width of HTS tape is w , the winding angle and pitch of the i -th layer are θ_i and L_{pi} , respectively, and then the pitch is found by:

$$L_{pi} = \frac{2\pi r_i}{\tan \theta_i} \quad (12.8)$$

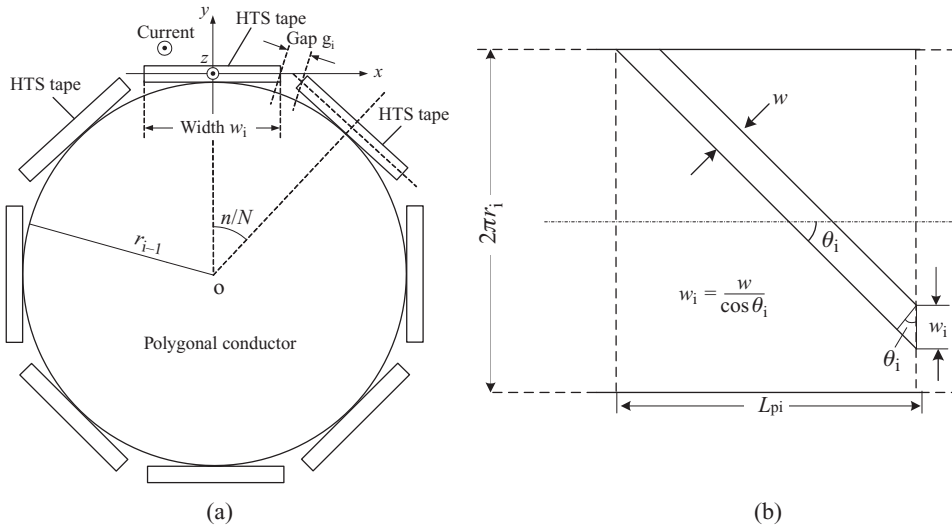


Figure 12.4 i -th layer in multi-layer cable conductor: (a) cross-sectional view of i -th layer; (b) side view of i -th layer with extending perimeter.

If the gap between the tapes in the i -th layer is g_i , the geometrical relationship amongst tape number, tape width, gap, outer radius of $(i-1)$ -th layer and thickness of tape obeys the identity:

$$\left(r_{i-1} + t + \frac{t_c}{2}\right) \tan\left(\frac{\pi}{N_i}\right) = \frac{g_i}{2 \cos\left(\frac{\pi}{N_i}\right)} + \frac{w}{2 \cos \theta_i} \quad (12.9)$$

where $i = 1, 2, 3 \dots N$, N_i is the number of HTS tapes in the i -th layer, w is tape width, g_i denotes the gap between tapes in the i -th layer, r_{i-1} is the outer radius of the $(i-1)$ -th layer, θ_i stands for the winding angle in the process of the cable conductor and will be discussed in the next section, and t and t_c are the thickness of the carbon paper between layers and thickness of HTS tape, respectively. Based on Equation (12.9), the number of HTS tapes in each layer can be determined.

If the ends of the HTS cable conductor are fixed, the gap between tapes should satisfy the following inequality:

$$g_i \geq \frac{\varepsilon_i}{N_i} 2\pi r_i \left(1 + \frac{L_{pi}^2}{(2\pi r_i)^2}\right) \quad (12.10)$$

where ε_i is the free thermal contraction of the HTS tape; r_i , N_i and L_{pi} are respectively the radius, number of tapes and pitch of the i -th layer in the cable conductor.

12.1.5 Insulation

As shown in Figure 12.5, the radial thickness of electrical insulation is the difference between outer radii of the cable conductor and insulation layer. In insulation design of the cable, the relationship of radial

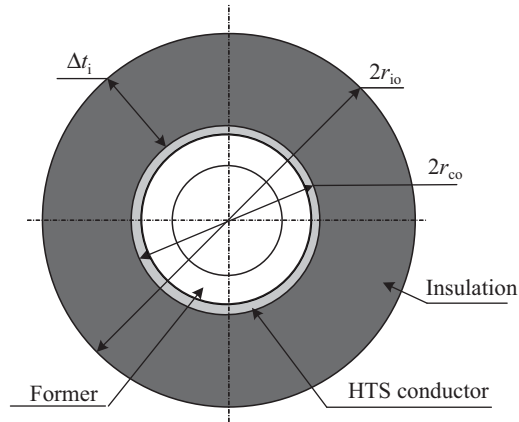


Figure 12.5 Genetic cross-section of cable conductor and insulation layer.

thickness to the outer radius of the cable conductor is theoretically expressed as:

$$\Delta t_i = r_{co} \left[\exp \left(\frac{V_m / \sqrt{3}}{r_{co} E_m} \right) - 1 \right] \quad (12.11)$$

where Δt_i is radial thickness of the insulation layer, r_{co} is outer radius of the cable conductor, V_m stands for the maximum system voltage, and E_m is the minimum breakdown strength of insulation materials.

In practical application of the cable, four kinds of testing voltages should be satisfied, which are long-term withstand voltage, lightning impulse voltage, power frequency withstand voltage and partial discharge (PD) voltage. Therefore, the voltage of insulation design must satisfy three types of withstand voltage [5].

12.1.5.1 PD Insulation

According to PD voltage requirements, the radial thickness of the insulation layer is provided by:

$$\Delta t_{ip} = r_{co} \left[\exp \left(\frac{V_m / \sqrt{3} K_1 K_2 K_3}{r_{co} E_{mp}} \right) - 1 \right] \quad (12.12)$$

where Δt_{ip} is radial insulation thickness for PD, coefficients K_1 , K_2 and K_3 are the AC degradation coefficient, the AC temperature coefficient and the AC design margin, respectively, amongst which coefficient K_2 equals to 1 because the HTS conductor operates at a constant temperature of LN₂, E_{mp} denotes the minimum PD inception strength, and V_m stands for the system maximum operation voltage at which PD occurs.

12.1.5.2 AC Withstand Insulation

Based on AC withstand requirements, the radial thickness of the insulation layer obeys:

$$\Delta t_{ia} = r_{co} \left[\exp \left(\frac{V_{ac} / \sqrt{3} K_1 K_2 K_3}{r_{co} E_{ma}} \right) - 1 \right] \quad (12.13)$$

where Δt_{ia} is the radial insulation thickness for AC withstand voltage, coefficients K_1 , K_2 and K_3 are ageing coefficient, AC temperature coefficient and AC design margin, respectively, E_{ma} represents the minimum AC breakdown strength, and V_{ac} refers to the maximum AC withstand voltage.

12.1.5.3 Impulse Insulation

In impulse design, the radial thickness of the insulation layer is found by:

$$\Delta t_{ii} = r_{co} \left[\exp \left(\frac{V_{imp} K_1 K_2 K_3}{r_{co} E_{mim}} \right) - 1 \right] \quad (12.14)$$

where Δt_{ii} is the radial insulation thickness for impulse voltage, V_{imp} denotes the impulse withstand voltage, coefficients K_1 , K_2 and K_3 are impulse degradation coefficient, impulse temperature coefficient and impulse design margin, respectively, and E_{mim} is the minimum impulse breakdown strength of insulation materials.

As in Equation (12.12), the temperature coefficient K_2 in Equations (12.13) and (12.14) equals to 1 in the design of the HTS cable conductor. According to the three radial thicknesses found from Equations (12.12) to (12.14), it is reasonable that the final radial thickness Δt_i is the maximum one amongst three values, that is, $\Delta t_i = \text{Max.}\{\Delta t_{ip}, \Delta t_{ia}, \Delta t_{ii}\}$. Currently, the electrical insulation materials are Laminated Polypropylene Paper (PPLP) and Nomex paper in most of the demonstrated prototypes of the HTS cable.

12.1.6 Shielding and Protection Layers

In the configuration of AC CD cable, the shielding layers are wound by the HTS tapes, which have the same number of layers as the conductor layers, because the shielding layers transport the same current as conductor layers in normal operation. The protection layer, wound by copper tape, has the same cross-sectional area of the former in order to protect the shielding HTS layers from damage during a fault. In shielding layers, the number of layers and number of tapes on each layer are determined by the same method as the conductor layers. As illustrated in Figure 12.6, the size of the protection layer is found by taking the same area of the former:

$$\pi (r_{so}^2 - r_{si}^2) = A \quad (12.15)$$

where r_{so} and r_{si} are the outer and radii of the protection layer, respectively, and r_{si} is equal to the sum of the outer radius of the HTS shielding and thickness of carbon paper t , A refers to the cross-sectional area of the former and is also the area of the protection layer.

12.2 Electromagnetic Design of AC/CD Cable Conductor

After determining the numbers of layers and tapes in each layer, the winding parameters, such as winding pitch or angle, need to be found for cable fabrication. In this section, the uniformity of current distribution amongst layers, AC losses, etc. will be introduced according to the equivalent circuit (EC) model.

12.2.1 Range of Winding Angle (Pitch)

Since the HTS cable operates at LN₂ temperature, there is a large temperature difference between room temperature and LN₂ temperature. Since the thermal contraction of components in the HTS cable is

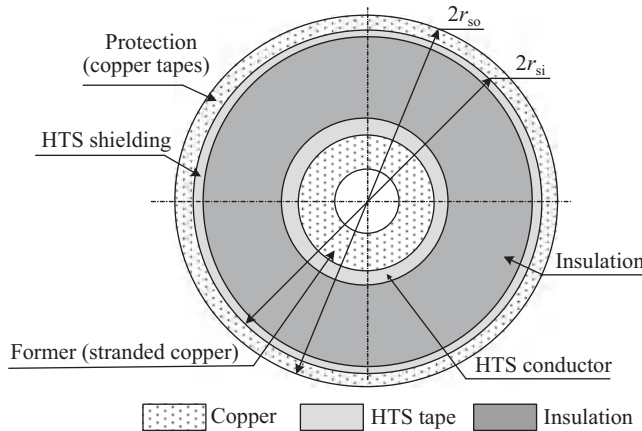


Figure 12.6 Generic cross-section of HTS cable conductor.

different from each other during cooling, the range of winding pitch must be determined in order to avoid damaging these elements before processing the cable.

Taking the mechanical characteristics of the HTS tapes and the size into account, the winding spiral angle should satisfy:

$$\sin^{-1} \left(\sqrt{\frac{\varepsilon_t - \varepsilon_s - \varepsilon_p}{\varepsilon_r - \varepsilon_p}} \right) \leq \theta_i \leq \sin^{-1} \left(\sqrt{\frac{r_i}{R}} \right) \quad (12.16)$$

where ε_t denotes the free contraction of the HTS tape, ε_s is the tension strain, ε_p stands for the change in rate of winding spiral pitch and ε_r is the thermal radial contraction during cooling in the range of room temperature to LN_2 temperature, R and r are the critical bending radius of the HTS tapes and radius of the HTS i -th layer of the cable conductor on which the HTS tapes are wound.

However, the HTS tapes of the i -th layer experience strain when winding around the former with winding pitch L_{pi} , and the strain must satisfy the inequality [6]:

$$\frac{\pi t}{L_{pi}} \cos \theta_i < \varepsilon_{\max} \quad (12.17)$$

where L_{pi} and θ_i are respectively the winding pitch and angle in the i -th layer of the cable conductor, t denotes the thickness of the HTS tape, and the relationship between both of them obeys Equation (12.8), and $\varepsilon_{\max}$ stands for the maximum permitted relative tensile strain of the HTS tapes. It is indicated that the strain reduces with decrease of the twist angle (or twist pitch rise) and with the increase of the twisting diameter.

12.2.2 Design of CD Cable Conductor

The layers from the inner are numbered from layer 1 to layer n and from layer 1 to layer m in the conductor and shielding layers, respectively. Therefore, if the winding angle is determined, the winding pitch L_{pi} can be found according to Equation (12.8).

It is important to realize the uniformity of current distribution of each layer to reduce the AC loss of the HTS cable. For optimization of the HTS cable, the cable twist pitches have to be adjusted for each

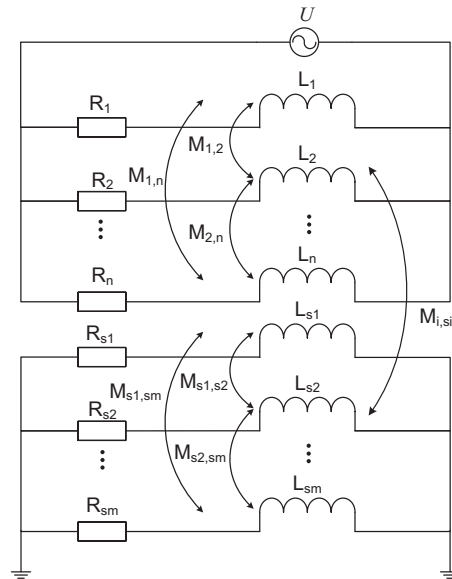


Figure 12.7 EC current of a single HTS CD cable with n layers of conductor and m layers of shielding layer. (Reproduced with permission from T. Masuda, Y. Ashibe, M. Watanabe, et al. "Development of a 100m, 3-core 114MVA HTSC cable system," *Physica C*, 372–376: 1580–1584, 2002 © 2002 Elsevier.)

layer. The equivalent EC model of HTS CD cable is shown in Figure 12.7. According to this model, the matrix equation is described per unit by:

$$\begin{bmatrix} R_1 I_1 \\ R_2 I_2 \\ \vdots \\ R_n I_n \\ R_{s1} I_{s1} \\ R_{s2} I_{s2} \\ \vdots \\ R_{sm} I_{sm} \end{bmatrix} + j\omega \begin{bmatrix} L_1 & M_{1,2} & \cdots & M_{1,n} & M_{1,s1} & M_{1,s2} & \cdots & M_{1,sm} \\ M_{2,1} & L_2 & \cdots & M_{2,n} & M_{2,s1} & M_{2,s2} & \cdots & M_{2,sm} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ M_{n,1} & M_{n,2} & \cdots & L_n & M_{n,s1} & M_{n,s2} & \cdots & M_{n,sm} \\ M_{s1,1} & M_{s1,2} & \cdots & M_{s1,n} & L_{s1} & M_{s1,s2} & \cdots & M_{s1,sm} \\ M_{s2,1} & M_{s2,2} & \cdots & M_{s2,n} & M_{s2,s1} & L_{s2} & \cdots & M_{s2,sm} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ M_{sm,1} & M_{sm,2} & \cdots & M_{sm,n} & M_{sm,s1} & M_{sm,s2} & \cdots & L_{sm} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \\ I_{s1} \\ I_{s2} \\ \vdots \\ I_{sm} \end{bmatrix} = \begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_n \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (12.18)$$

$$I_1 + I_2 + \cdots + I_n = -(I_{s1} + I_{s2} + \cdots + I_{sm}) \quad (12.19)$$

where n and m are the layer numbers of conductor and shielding layers, respectively, and U_i ($i = 1, 2, \dots, n$) denotes the voltage drop of each layer of the conductor. The voltage drops of all shielding layers are zero, because they are grounded at both of ends, I_i is the transport currents flow in the i -th layer of the conductor and shielding layer, L_i stands for the self-inductance of the i -th layer in per unit length of conductor and the i -th layer in shielding layer, M_{ij} denotes the mutual inductances between the i -th layer and the j -th layer in per unit length of conductor and shielding layers, and R_i represents the resistance

between the i -th layer and jointing ends by taking the average over the entire length of the cable conductor, which consists of AC losses of the conductor and jointing resistances. These parameters, resistances, self and mutual inductances, refer to those in per unit length. ω is the angular frequency and j on the left-hand side is an imaginary unit. In Equation (12.17), the effects of the former and protection layer made of copper are ignored, because they have negligible transport current during normal operation [7].

The self inductance of i -th layer and mutual inductance (H/m) between i -th and j -th layers per unit length are separately expressed as:

$$L_i = \frac{\mu_0 \pi r_i^2}{L_{pi}^2} + \frac{\mu_0 \ln(D/r_i)}{2\pi} \quad (12.20)$$

$$M_{i,j} = M_{j,i} = \alpha_i \alpha_j \frac{\mu_0 \pi r_i^2}{L_{pi} L_{pj}} + \frac{\mu_0 \ln(D/r_j)}{2\pi} \quad (r_i < r_j) \quad (12.21)$$

where $i, j = 1, 2, \dots, n + m$, D is the outermost radius of the shielding layer, α_i and α_j are constants $+1$ or -1 , depending on the winding directions, as indicated in Figure 12.8, where the conductor and shielding layers consist of three layers, and the winding direction is opposite in the conductor and shielding layers, in which the corresponding constants α_1 and α_2 (or α_{s1} and α_{s2}) have opposite signs. To reduce AC losses, the insulation layer is inserted between the HTS layers in both conductor and shielding layers. That is, if the two layers are wound in the same direction around the former, the sign of the constants are the same, that is, $\alpha_1 \alpha_2 = +1$. In the opposite winding direction, the product of the two constants is negative, that is, $\alpha_1 \alpha_2 = -1$. L_{pi} denotes the winding pitch of the i -th layer around the former.

Equation (12.21) holds when $r_i < r_j$; if $r_i > r_j$, r_i is replaced by r_j in the first term and r_j is replaced by r_i in the second term. The self and mutual inductances are derived in detail in Appendix A.4.

If the winding pitches of every layer are the same, the self inductances and mutual inductances of the outer layers become smaller than those of the inner layers. In this case, the current that flows in the outer layer must become larger than the inner layer, so that the current distributions are non-uniform, which will lead to large AC losses. In order to make the current distribute with uniformity, a method of equalizing the impedance of each layer by varying winding pitches is effective, because the inductance of the outer layer is enhanced by adjusting the winding pitch of the each layer without any extra loss.

Since all of the layers are connected in parallel at both ends, the voltage drop on each layer of the conductor is the same, that is, $U_1 = U_2 = \dots = U_n = U_0$. Generally, the impedance resulting from the joint

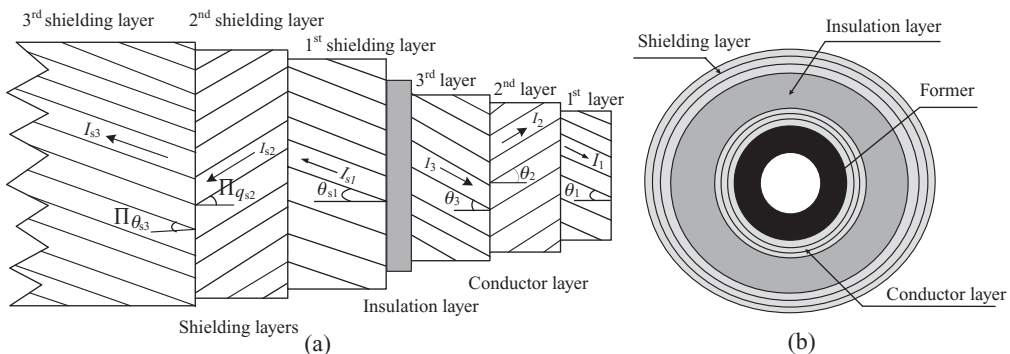


Figure 12.8 HTS cable consisting of two conductor layers and two shielding layers with different winding directions: (a) side view, (b) cross-section.

resistance and AC losses per unit length is much smaller than the inductance, so that uniform current distribution can be realized when ignoring the resistances R_i ($i = 1, 2, \dots, n$) and R_{si} ($i = 1, 2, \dots, m$) in Equation (2.18). Based on the condition of equalizing currents in each layer of the conductor and shielding layers, that is:

$$\begin{cases} I_1 = I_2 = \dots = I_n & (\text{conductor layers}) & (12.22a) \\ I_{s1} = I_{s2} = \dots = I_{sm} & (\text{shielding layers}) & (12.22b) \\ (I_1 + I_2 + \dots + I_n) = -(I_{s1} + I_{s2} + \dots + I_{sm}) & (\text{perfect shielding}) & (12.22c) \end{cases} \quad (12.22)$$

Equation (12.22) is the optimized rule for uniformity of current distribution design of HTS CD cable.

By taking the optimized rule into account and by varying the winding pitch of each layer [8–10], compositions of self and mutual inductances in Equation (2.18) should satisfy:

$$L_i + \sum_{j=1}^{n+m} M_{ij} = L_{sk} + \sum_{j=1}^{n+m} M_{skj} \quad (12.23)$$

By solving the simultaneous Equation (12.22) with many iterations, we can obtain the pitches of each layer with convergence of uniform current distributions amongst conductor layers and shielding layers, respectively. After obtaining the pitches, the winding angle of each layer can also be found according to Equation (12.8) and must be in the range determined by Equations (12.16) and (12.17) for mechanical safety of the HTS cable. However, the corresponding adjustment on determination of number of tapes on each layer should be made according to Equation (12.9).

12.3 Analysis on AC Losses of DC HTS Cable

12.3.1 Magnetic Field Analysis

Based on the spiral pitch, current flow and winding directions of each layer, magnetic fields induced by each layer have two components, that is, an azimuthal magnetic field component at the outer region of each layer and an axial component at the inner region of the cable axis [11, 12]. The two components are shown in Figure 12.9. The magnetic field of the i -th layer is decided

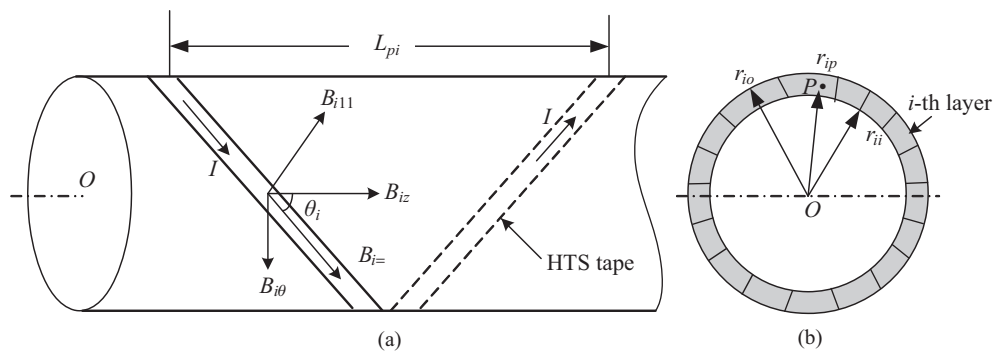


Figure 12.9 Parallel and longitudinal magnetic field components in i -th layer of cable: (a) main view, (b) cross-section.

by currents of the other layers and itself. The axial component of the i -th layer is generated by current flowing in the outer layers of the i -th layer, and the azimuthal component is generated by currents in the inner layers. Therefore, azimuthal and axial components of magnetic fields are separately expressed as:

$$B_{i\theta} = \mu_0 \left(\frac{1}{2\pi r_{ip}} \sum_{k=1}^{i-1} I_k + \left(\frac{r_{ip}^2 - r_{ii}^2}{r_{io}^2 - r_{ii}^2} \right) \frac{I_i}{2\pi r_{ip}} \right) \quad (12.24)$$

$$B_{iz} = \mu_0 \left(\sum_{k=i+1}^n \alpha_k \frac{I_k}{L_{pk}} + \left(\frac{r_{io} - r_{ip}}{r_{io} - r_{ii}} \right) \frac{I_i}{L_{pi}} \beta_i \right) \quad (12.25)$$

where μ_0 is vacuum permeability, r_{ii} and r_{io} are inner and outer radii of the i -th layer, r_{ip} denotes the distance from the axis at a point in the region of the i -th layer ($r_{ii} \leq r_{ip} \leq r_{io}$). α_k refers to direction constant reflecting current flowing and the winding direction in the k -th layer, both of them have constant values of $+1$ or -1 . $\alpha_k = +1$ and $\alpha_k = -1$ respectively correspond to the direction of the winding around the former as clockwise or anti-clockwise, and n is the number of HTS tapes in the i -th layer.

Due to the spiral structure, azimuthal field $B_{i\theta}$ and axial B_{iz} field components are divided into parallel and longitudinal field components, which are described by Equations (12.26) and (12.27). $B_{i//}$ is known as transversal and $B_{i=}$ is called the longitudinal magnetic field, and both of them are perpendicular and parallel to the transport current but parallel to the wide surface of the tape, respectively:

$$B_{i//} = B_{iz} \sin \theta_i - B_{i\theta} \cos \theta_i \quad (12.26)$$

$$B_{i=} = B_{iz} \cos \theta_i + B_{i\theta} \sin \theta_i \quad (12.27)$$

where $i = 1, 2, \dots, n+1, \dots, n+m$, θ_i is the winding angle between the i -th layer tape and axis of the cable conductor, and L_{pi} is the winding pitch, which is determined by radius r_{ip} and θ_i , in the relation of $\tan \theta_i = 2\pi r_{ip} / L_{pi}$.

12.3.2 AC Losses of HTS CD Cable Conductor

According to the geometrical structure and elements of the HTS CD cable, there are four kinds of AC losses, namely hysteresis loss, eddy current and coupling losses created in the HTS conductor and shielding layers, eddy current loss in former, and dielectric loss in insulation layers.

12.3.2.1 Hysteresis Loss

As introduced in Chapter 5, the hysteresis loss exists in the superconductor when it is exposed to an AC magnetic field or carries an AC current. Generally, when the HTS cable conductor transports an AC current, each tape on the cable conductor transports an AC current and meanwhile is exposed to the AC magnetic field generated by other tapes. Based on Equations (12.26) and (12.27), there are two AC magnetic field components, that is, the transversal $B_{i//}$ and longitudinal $B_{i=}$. Generally, the hysteresis loss created by the AC transversal magnetic field $B_{i//}$ is dominant, while the hysteresis loss created by the AC longitudinal field $B_{i=}$ can be neglected. According to Equations (5.8) to (5.14) with the Bean model, if one superconducting tape carries an AC current and is simultaneously exposed to an AC parallel in-phase

magnetic field with conditions of $I_{DC} = B_{DC} = 0$, the hysteresis loss in each layer of the HTS cable conductor with units of W/m is found by:

$$P_{hi} = N_i \frac{2fCAB_p^2}{3\mu_0} \begin{cases} (i_{ac}^3 + 3b_{ac}^2 i_{ac}) & (b_{ac} < i) \\ (b_{ac}^3 + 3b_{ac} i_{ac}^2) & (i_{ac} < b_{ac} < 1) \\ \left(b_{ac} (3 + i_{ac}^2) - 2(1 - i_{ac}^3) + 6i_{ac}^2 \frac{(1 - i_{ac})^2}{(b_{ac} - i_{ac})} - 4i_{ac}^2 \frac{(1 - i_{ac})^3}{(b_{ac} - i_{ac})^2} \right) & (b_{ac} > 1) \\ (b_{ac} (3 + i_{ac}^2) - 2(1 - i_{ac}^3)) & (b_{ac} >> 1) \end{cases} \quad (12.28)$$

where N_i denotes the number of the HTS tapes in the i -th layer of the cable conductor around the former, $i = 1, 2, \dots, n + 1, \dots, n + m$, n and m are numbers of HTS layers on the conductor layer and shielding layer, respectively, f refers to power frequency, CA is the effective cross-sectional area relating to the geometric structure of the HTS tape, $b_{ac} = B_{i//}/B_p$ and $i_{ac} = I_{ac}/I_c$ are the normalized AC magnetic field and the normalized AC transport current of each tape, B_p is the penetrated magnetic field, $B_p = \mu_0 J_c t_c / 2$, J_c and t_c are separately the critical current density and the thickness of the HTS tape, and I_c presents the average critical current of each tape of i -th layer, that is:

$$I_c = \sum_{i=1}^{N_i} I_{ci} / N_i \quad (12.29)$$

Defining the normalized hysteresis loss P_{hi} of each tape on the i -th layer as:

$$P_{hi} = \frac{P_{hi}}{N_i \frac{2fCAB_p^2}{3\mu_0}} \quad (12.30)$$

the normalized hysteresis loss p_i is plotted against the normalized magnetic field with different normalized transport AC current in the i -th layer of the HTS cable conductor (Figure 12.10). If the normalized transport current is $i_{ac} > 0.5$, the normalized hysteresis loss p_i is almost independent of the normalized AC magnetic field b_{ac} , when $b_{ac} < 0.2$.

Therefore, the total hysteresis loss P_h of the HTS CD cable conductor in unit length with units of W/m is expressed as:

$$P_h = \sum_{i=1}^{n+m} P_{hi} \quad (12.31)$$

Based on the other models, the mono-block (MB) and Majoros (MAJ) models, the hysteresis losses of the HTS cable are introduced in Appendix A.5.

12.3.2.2 Eddy Current Loss in HTS Tape

In the present configuration of the HTS cable conductor, since the transversal magnetic field $B_{i//}$ is parallel to the wide surface of the HTS tape, that is, perpendicular to the side surface with width t_c , according to

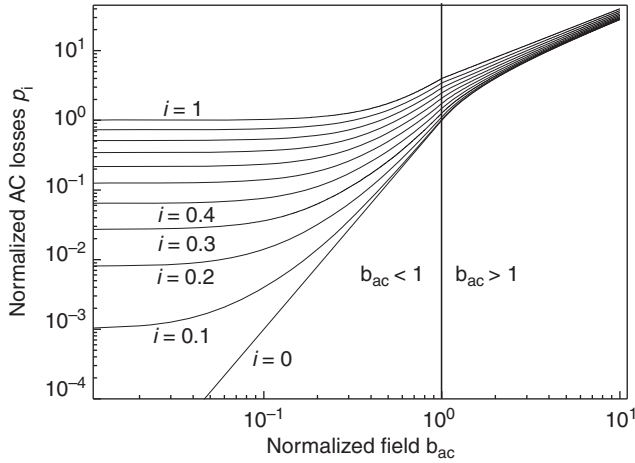


Figure 12.10 Plot of normalized hysteresis loss p_i of each tape in i -th layer against normalized AC magnetic field b_{ac} with different normalised AC transport current i_{ac} .

Equation (5.145), the eddy current loss (W/m) of each tape with dimensions of width w and thickness t_c is given by:

$$P_{ei} = \begin{cases} (1 - \eta_c) \frac{\omega^2 B_{mi}^2 w t_c^3}{12\rho} & (12.32a) \\ 4 \frac{(\mu_0 \omega I_{im})^2 t_c^3}{\rho L} & (12.32b) \end{cases} \quad (12.32)$$

where Equations (12.32a) and (12.32b) denote the eddy current losses in the i -th HTS tape of the HTS cable conductor due to the AC magnetic field and the AC current, respectively; η_c stands for the volumetric ratio of the superconducting component to the entire composite superconducting tape, ρ is the resistivity of the matrix (sheath) of the superconducting tape, B_{mi} refers to the amplitude of the AC magnetic field $B_{i//}$ (determined by Equation (12.24)) perpendicular to the side surface with a width t_c of the i -th tape, I_{im} is the amplitude of AC currents flowing in the i -th HTS tape; and $\omega = 2\pi f$ is the angular power frequency; here f means power frequency. Then the total eddy current loss (W/m) per unit length of all the HTS tapes are the sum of each tape:

$$P_e = \sum_{i=1}^{N_T} P_{ei} \quad (12.33)$$

where N_T is the total number of HTS tapes of the HTS CD conductor, that is:

$$N_T = \sum_{i=1}^{n+m} N_i \quad (12.34)$$

$(n+m)$ refers to total layers of the HTS CD cable, amongst which the numbers of conductor layers and shielding layers are n and m , respectively.

Where the HTS tape is ReBCO with a magnetic substrate, for a detailed calculation of eddy current loss, please refer to Appendix A.2.

12.3.2.3 Coupling Loss in HTS Tape

If the HTS tapes are multi-filamentary, such as Bi-2223/Ag, and are exposed to the AC magnetic field, the current (eddy current), the so-called coupling current, is induced amongst the filaments in a transverse AC magnetic field. Due to the normal metal amongst filaments in a composite HTS tape, the coupling loss (W/m) of each tape is generated and given by:

$$P_{ci} = \eta_{ci} \frac{AB_{mi}^2}{2\mu_0} \left[\frac{n_s \omega^2 \tau}{1 + (\omega\tau)^2} \right] \quad (12.35)$$

where η_{ci} is same as in Equation (10.30), n_s denotes the shape factor of the superconducting filaments, A is the cross-sectional area of the superconductor, B_{mi} is the amplitude of the AC magnetic field in which the i -th tape exists, and τ is the characteristic time constant of the coupling current and determined by P_{ci} taking its maximum at the frequency $f = f_m$, that is, $\tau = (2\pi f_m)^{-1}$ [13]. Therefore, the total coupling loss in the HTS CD cable conductor is the sum of each tape:

$$P_c = \sum_{i=1}^{N_T} P_{ci} \quad (12.36)$$

If the HTS tape is a ReBCO-coated conductor, it has no HTS filaments, and the superconductor is a thin film with thickness of about 1 μm , so there is no coupling and so coupling loss is absent. Fortunately, the eddy current loss and coupling loss of HTS tapes at LN_2 temperature and power frequency are negligible compared with hysteresis loss. Consequently, the AC loss in the HTS cable conductor is approximately equal to the hysteresis loss, so the eddy current and coupling loss are neglected in the design of the HTS cable conductor.

12.3.2.4 Eddy Current Loss in Former and Shielding Protection

According to the geometrical structure of the HTS CD conductor, the HTS tapes are wound around the Cu stranded former in order to protect the HTS conductor layers from burning out or being destroyed during a fault. The Cu former is exposed to AC magnetic fields created in the conductor and shielding layers (Equations (12.14) and (12.25)), thus an eddy current is induced in the former, which then leads to eddy current loss. The eddy current loss (W/m) is expressed as [14]:

$$P_{ef} = \frac{\pi f^2}{4\rho_f} \left(\frac{B_{fm}}{\sqrt{2}} \right)^2 \frac{(r_{fo}^2 - r_{fi}^2)^2 (r_{fo}^2 + r_{fi}^2)}{r_{fo}^2} \quad (12.37)$$

where r_{fo} and r_{fi} are the outer and inner radii of the Cu former, respectively, B_{fm} , determined by Equation (12.24), is the amplitude of the AC magnetic field in the former, and ρ_f denotes its resistivity.

However, there is also a protective Cu layer on the HTS shielding layer to prevent it from being destroyed during a fault. However, the eddy current loss is absent in the protective Cu layer where there is no magnetic field because the current induced in the shielding layer has the same magnitude and is in the opposite direction to the transport current flowing in the cable conductor layer. Also, the eddy current loss in the Cu former is small in normal operation compared with the hysteresis one, so it is seldom taken into account in the design of prototype HTS CD cables.

12.3.2.5 Dielectric Loss

There is electric insulation between the conductor layer and shielding layer, and the insulation is at the same temperature for both layers in normal operation. Thus, the dielectric loss (W/m) is created in the insulation layer and given by:

$$P_d = U_0^2 \omega C \tan \delta = U_0^2 \omega \frac{2\pi \varepsilon_0 \varepsilon}{\ln(r_{i0}/r_{ii})} \tan \delta \quad (12.38)$$

where U_0 is voltage to ground and $U_0 = U_i/\sqrt{3}$, U_i is the phase voltage of the three-phase cable, ω denotes power frequency, C is dielectric capacity of cable conductor per unit length, r_{ii} and r_{i0} are inner and outer radii of the electrical insulation layers, respectively, $\tan \delta$ stands for the dielectric loss tangent at the operating temperature of the cable, and ε_0 and ε are separately the vacuum and relative dielectric constants.

The total electric losses (W/m) of a single phase HTS CD cable conductor in unit length can be calculated by taking the sum of Equations (12.31), (12.33), (12.36), (12.37) and (12.38), and these losses need to be taken away by the cryogenics, such as LN_2 nitrogen, so as to maintain normal operation of the HTS cable.

12.4 Design of AC WD HTS Cable Conductor

According to the structure of the WD HTS cable conductor, its cable conductor without shielding layer is similar to the CD partner. The design of the equalizing current is also similar to the CD partner. In its design, the shielding layer is absent, and the design is completed by taking $I_{si} = R_{si} = L_{si} = M_{si,j} = M_{i,sj} = 0$ in Equation (12.17) with $V_1 = V_2 = \dots = V_n$, because all of the layers are connected in parallel. The corresponding EC model and matrix equation are shown in Figure 12.11 and Equation (12.39):

$$\begin{pmatrix} R_1 I_1 \\ R_2 I_2 \\ \vdots \\ R_n I_n \end{pmatrix} + j\omega \begin{pmatrix} L_1 & M_{12} & \cdots & M_{1n} \\ M_{21} & L_2 & \cdots & M_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ M_{n1} & M_{n2} & \cdots & L_n \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{pmatrix} = \begin{pmatrix} U_1 \\ U_2 \\ \vdots \\ U_n \end{pmatrix} \quad (12.39)$$

where R_i , L_i and M_{ij} are joint resistance, self and mutual inductances per unit length within the i -th layer ($i, j = 1, 2, \dots, n$), respectively. The difference is the choice of parameter D in self and mutual inductance. In the CD cable, D refers to the outermost radius of the shielding layer, while in a WD cable, D is replaced by D' , which is described as the space range of the magnetic field. It is determined mainly by the size of the thermal insulation layer and material properties of the protection layer. For the designing steps, which are the same as the CD ones, please refer to the CD HTS cable conductor in Sections 12.1 to 12.3. Due to absence of a shielding layer and protective Cu stranded layer, here are no hysteresis loss, eddy current loss and coupling loss in HTS tapes and no eddy current loss terms in protective Cu stranded layers. Consequently, the total AC losses of a WD HTS cable conductor are the sum of the hysteresis loss, eddy current loss and coupling loss in HTS tapes of the cable conductor, and the eddy current loss in the former. In this section, only the extra loss terms are introduced.

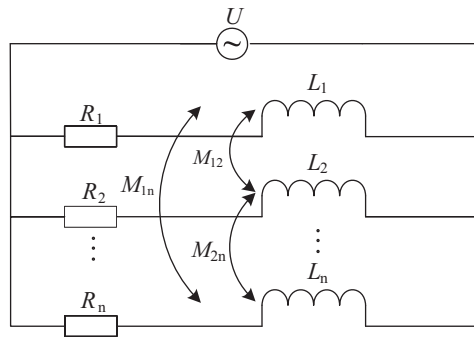


Figure 12.11 EC of WD HTS cable.

12.4.1 Eddy Current Loss in Cryostat

Since HTS shielding layers are absent, there is a magnetic field distributed around the WD cable conductor. Then the cryostat, made from two stainless steel corrugated tubes between which there exists a vacuum space and thermal insulation, is exposed to the AC magnetic field. The inner wall of the cryostat dominates the eddy current loss (W/m) in unit length, which is given by:

$$P_{\text{wall}} = \frac{\mu_0^2 \omega^2 d^3 I_m^2}{8\pi \rho r} \quad (12.40)$$

where d denotes the thickness of the inner wall of the cryostat, I_m is the amplitude of transport current of the WD HTS cable conductor, ρ refers to the resistivity of the inner wall, and r is the average radius of the inner wall of the cryostat. This eddy current loss is also dissipated at a cryogenic temperature of LN_2 .

12.4.2 Dielectric Loss

Based on the geometrical structure of the WD HTS cable conductor, the electric insulation is the same as in the conventional cross-link polyethylene (XLPE) cable, and the electric insulation operates at room temperature. The calculation also follows Equation (10.36), but the difference is that the dielectric loss tangent $\tan\delta$ is taken at room temperature.

12.5 Design of DC HTS Cable Conductor

The DC HTS cable operates in DC mode, and there are theoretically no AC losses due to its zero resistance and also no dielectric loss in the HTS cable conductor, so that there is very little difference in loss terms between WD and CD types of DC HTS cable conductors. The EC model of the DC HTS cable conductor is the same as that of the AC WD cable. However, the design is seemingly simpler than in the AC HTS cable. The DC cable conductor must be designed according to the AC mode because there are the ripple current components (combination of small AC current superimposed on a large DC current component). Strictly speaking, the DC HTS cable is a “quasi-AC cable”.

The EC model of the DC HTS cable conductor is similar to the WD AC HTS cable conductor, because the HTS shielding layers are unnecessary. In principle, the design rules employed in the WD HTS cable

conductor may be used. However, according to our research, the uniformity rule with $I_1 = I_2 = \dots = I_n$ (n refers to the number of layers) employed in CD and WD HTS cable conductors should be replaced by the equal ratio or normalized current rule, that is:

$$\frac{I_{ac1}}{I_{c1}(B_{dc1})} = \frac{I_{ac2}}{I_{c2}(B_{dc2})} = \dots = \frac{I_{acn}}{I_{cn}(B_{dcn})} \quad (12.41)$$

where n denotes the number of layers, $I_{ac1}, I_{ac2}, \dots, I_{acn}$ stand for the amplitudes of the ripple current components for respective layers, $B_{dc1}, B_{dc2}, \dots, B_{dcn}$ are the DC magnetic fields of corresponding HTS layers created by DC current components in the DC cable and can be determined by Equation (12.24), and $I_{c1}(B_{dc1}), I_{c2}(B_{dc2}), \dots, I_{cn}(B_{dcn})$ are the corresponding critical current of HTS layers in those layers in the DC magnetic field $B_{dc1}, B_{dc2}, \dots, B_{dcn}$. In this book, the Equation (12.41) is temporarily called the "Equal ratio" rule to distinguish it from the uniform current rule of Equation (12.22).

The DC current components cannot be adjusted by changing the winding pitch or angles of self and mutual inductances, by which the AC current components can be regulated. According to the introduction in Section 5.1.4 of Chapter 5, the hysteresis loss (W/m) per unit length of the i -th layer of the DC HTS CD cable conductor is expressed as:

$$P_{hi} = N_i \frac{2B_p^2}{3\mu_0} C A f \begin{cases} b_{ac}^3 + 3b_{ac}i_{ac}^2 & (i_{ac} \leq b_{ac} \leq 1 - i_{dc}) \\ \left[b_{ac} (3 + i_{ac}^2 + 3i_{dc}^2) - 2(1 - (i_{ac} + i_{dc})^3 + 3i_{ac}i_{dc}) \right. \\ \quad \left. + \frac{6i_{ac}}{(b_{ac} - i_{ac})} (i_{ac} + i_{dc})(1 - i_{ac} - i_{dc})^2 \right. \\ \quad \left. - \frac{4i_{ac}^2}{(b_{ac} - i_{ac})^2} (1 - i_{ac} - i_{dc})^3 \right] & (1 - i_{dc} \leq i_{ac} \leq b_{ac}; i_{dc} - 1 \leq i_{ac} \leq i_{dc} + 1) \\ \left(\frac{1 + i_{ac} - i_{dc}}{2i_{ac}} \right) [b_{ac}(1 + i_{ac} - i_{dc})^2 + 3(b_{ac} - i_{ac})(1 - i_{ac} - i_{dc})] & (i_{ac} \leq 1 - i_{dc} \leq b_{ac}) \\ 4 \frac{b_{ac}}{i_{ac}} & (1 - i_{dc} \leq i_{ac} \leq b_{ac}; i_{dc} + 1 \leq i_{ac}) \\ i_{ac}^3 + 3i_{ac}b_{ac}^2 & (b_{ac} \leq i_{ac} \leq 1 - i_{dc}) \\ \left(\frac{1 + i_{ac} - i_{dc}}{2i_{ac}} \right)^3 (i_{ac}^3 + 3i_{ac}b_{ac}^2) & (1 - i_{dc} \leq i_{ac}; b_{ac} \leq i_{ac}; i_{dc} - 1 \leq i_{ac} \leq i_{dc} + 1) \\ \left(\frac{1}{i_{ac}} \right)^3 (i_{ac}^3 + 3i_{ac}b_{ac}^2) & (1 - i_{dc} \leq i_{ac}; b_{ac} \leq i_{ac}; i_{dc} + 1 \leq i_{ac}) \\ 0 & (i_{ac} \leq i_{dc} - 1) \end{cases} \quad (12.42)$$

where C is geometry factor and A is the cross-section of HTS tape, and N_i is the number of HTS tapes in the i -th layer. Equation (12.42) means that the hysteretic loss is zero, since the superconductor is completely resistive; the normalized current and normalized magnetic field are respectively represented by:

$$i_{ac} = \frac{I_{ac}}{I_c(B_{dc})}, \quad i_{dc} = \frac{I_{dc}}{I_c(B_{dc})}, \quad b_{ac} = \frac{B_{ac}}{B_p}, \quad b_{dc} = \frac{B_{dc}}{B_p} \quad (12.43)$$

$B_p = \mu_0 J_c(B_{dc})t/2$ is the fully completed magnetic field, t is the thickness of the HTS tape, $I_c(B_{dc})$ denotes the critical current at the DC magnetic field B_{dc} , I_{dc} is the DC transport current of each tape, and B_{dc} is the DC magnetic field produced by the DC current flowing in other tapes.

The total hysteresis loss (W/m) of the DC HTS cable conductor can be found by taking the sum of hysteresis loss in every layer:

$$P_h = \sum_{i=1}^n P_{hi} \quad (12.44)$$

where n denotes the layer number of the DC HTS cable conductor.

Based on the Block model and the Bean model, if the current does not reverse its direction but oscillates with varying magnitudes, the AC losses (W/m), the so-called self-field loss, in whole the conductor with elliptical or circular cross-section is equivalently given by:

$$P'_{sf} = f \frac{\mu_0}{\pi} I_c^2 \begin{cases} \left[i_{ac} \left(2 - \frac{i_{ac}}{2} \right) + 4 \left(1 - \frac{i_{ac}}{2} \right) \ln \left(1 - \frac{i_{ac}}{2} \right) \right] & i_{ac} < 1 \\ \frac{3}{2} - 2 \ln 2 & i_{ac} \geq 1 \end{cases} \quad (12.45)$$

where I_c is the total critical current of the whole DC HTS conductor, I_{ac} is the amplitude of the AC current component superimposed onto the total transport DC I_{dc} , and i_{ac} is the normalized transport current with $i_{ac} = I_{ac}/I_c$.

As an example, a single 10 KA DC HTS cable made of four layers is designed, based on the design rule Equation (12.41). Figure 12.12 exhibits its conceptual configurations. The HTS tape with cross-section size of 4 mm $\times$ 0.28 mm is adopted, and its critical current is 150 A at self field and 77 K. The thickness of insulation between layers is 0.05 mm, and the gap between tapes is 0.1 mm in each layer. The diameter of the form is 35 mm.

Then the “equal ratio” design is calculated by substituting Equations (12.41) into Equation (12.39) with $n = 4$. After many iterations, by taking $R_i = 0$ (since the inductances are three order of magnitude more than the pure resistances in the HTS conductor), here R_i represents the resistance between the i -th layer and jointing ends by taking an average over the entire length of the cable conductor. The main parameters of this conductor are listed in Table 12.1.

Figure 12.13 shows the AC loss of the DC HTS cable at different ripple current ratio $i = I_{ac}/I_{0DC}$, here I_{ac} is the amplitude of ripple current component and $I_{0DC} = 10$ kA [15]. According to the Block model, the conventional uniform current design and the new design rule, it shows that hysteresis loss of a DC HTS cable designed by the “equal ratio” rule of Equation (12.41) is the lowest compared with the uniform current design and the Block model. Therefore, this design rule, that is, the “equal ratio” rule by taking the ratios of ripple AC currents to critical currents in each layer as equal, instead of the uniform

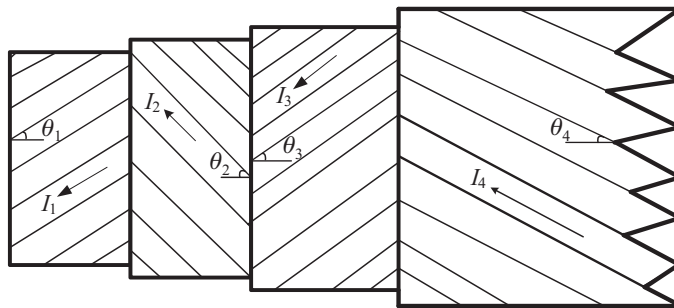


Figure 12.12 Configuration of 10 KA DC HTS cable with four layers.

Table 12.1 Main parameters of 10 kA DC cable conductor

Design principle	Equation (12.22a)	Equation (12.41)
Rated current/kA	10	10
No. of layers	4	4
No. of tapes	96	96
No. of each layer	23, 23, 25, 25	23, 23, 25, 25
Winding angle (°)	13.50 −15.50 13.75 −17.95	12.50 −14.10 12.95 −15.90
Form dia. /mm	35	35
Inn/Outer dia./mm	35.15/37.75	35.15/37.75

current design adopted in pure AC transport current transmission, is more suitable for the DC HTS cable conductor.

As an example, the main design parameters of one typical HTS cable prototype, already successfully demonstrated in a live grid, are listed in Table 12.2 [16].

12.6 Design of Cryostat

The cryostat of an HTS cable, providing the cryogenic environment for the HTS cable conductor, consists of an inner cold wall and an outer warm wall. The walls are usually made from a corrugated stainless steel tube, so that the cryostat is flexible and can withstand thermal contraction. As described in Section 9.2, there are three types of heat source, that is, conduction, convection and radiation. Figure 9.17 in Chapter 9 shows its side cross-sectional view. The conduction heat can be reduced by decreasing the cross-section and thermal conductivity. Stainless steel has high strength and low thermal conductivity, so is an optimal material for the cryostat of the HTS cable and its wall thickness can be small enough. Space between the two walls is in a vacuum for reducing convection heat transfer and is filled with multi-layer

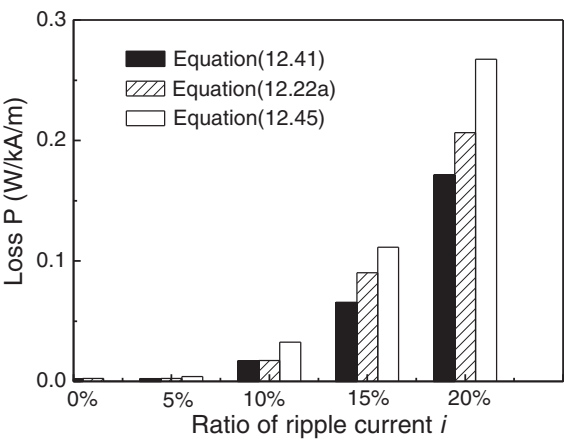


Figure 12.13 AC losses of three models with different levels of ripple current at power frequency of 50 Hz with DC capacity of 10 kA.

Table 12.2 Main specifications of 77kV/1kA single-phase CD HTS cable (Reproduced with permission from M. Ichkawa, S. Torri, T. Takahashi, et al. “Quench properties of 500-m HTS power cable”. *IEEE Transaction on Applied Superconductivity*, 17, 2: 1668–1671, 2007. © 2007 IEEE).

Items	Configuration		Outer diameters/mm
Conductor layer	Former	SUS spiral tube	28
		Hollow Cu stranded	
Electrical insulating layer	superconductors	Ag sheathed Bi-2223 tape	30
	Electrical insulation	Polypropylene laminated paper (PPLP)	48
Magnetic field shielding layer	Shielding superconductor	Ag sheathed Bi-2223 tape	58
Thermal insulating layer	Protective layer	Cu braided wire	/
	Vacuum thermal insulating layer	Evacuated multi-layer thermal insulation with SI	
Protective layer	Outer tube	SUS corrugated tube	124
	Over sheath	PVC	133
Length/m			500

super-thermal insulation (MLI) to reduce radiation heat transfer. The basic structure of the cryostat is depicted in Figure 12.14.

It should be noted that the inner wall of the cryostat is designed to withstand a high positive pressure, depending on practical condition and length of the HTS cable. The thermal loads include thermal conduction through cryostat walls from room temperature to LN₂ temperature, convection thermal load by gas molecules in the space between the inner cold and outer warm walls, and the radiation heat. The latter two loads depend on the quantity of the MLI and the type of mechanical support between the walls. The conduction heat of spacer support between the inner and outer walls should also be considered (as presented in Figure 9.15 of Chapter 9), even though its cross-section is much smaller and heat conduction is also much smaller compared with other conduction heats. The design of the cryostat can be made by taking Equations (9.2), (9.6) and (9.24) into account according to practical sizes of the cryostat.

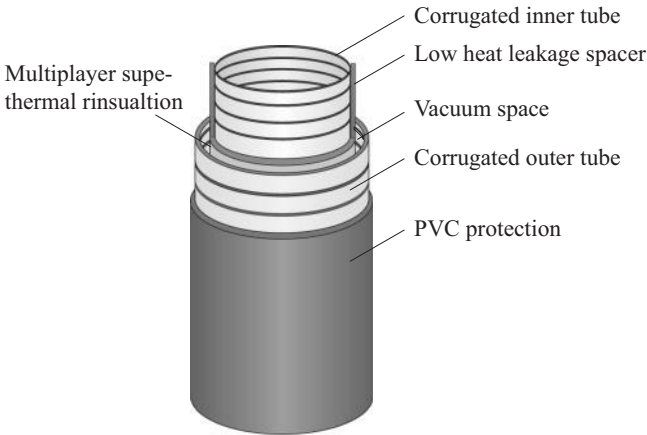


Figure 12.14 Configuration of cryostat for HTS cable.

The less the thermal heat leakage, the better the optimized design. For simplicity, the local heat transfer (W/m) per unit length of HTS cable cryostat with flexible corrugated double-wall construction can be approximated by [17]:

$$Q_{\text{cstat}} = \frac{2\pi k_{\text{eff}} (T_{\text{RT}} - T_i(x))}{\ln(r_{\text{co}}/r_{\text{ci}})} \quad (12.46)$$

where k_{eff} , for a commercially available typical vacuum-insulated flexible cryostat with corrugated walls of 8×10^{-4} W/(m·K), is the field installation thermal conductivity, T_{RT} and $T_i(x)$ refer to the local room temperature and local LN₂ temperature at the location of x along the length of the HTS cable respectively, and r_{co} and r_{ci} denote respectively the inner and outer radii of the cryostat corrugated walls. Generally, the temperature difference driving this heat transfer term is more than 220 K for the outer cryostat corrugated wall.

12.7 Manufacture of CD HTS Cable Conductor

The components of the CD HTS cable conductor consists of the former, HTS conductor layer, dielectric insulation layer, HTS shielding layer and protective Cu stranded layer, amongst which carbon paper is wound. The dielectric insulation layer is wound by insulation tapes with a certain width and gap between them. Currently, the PPLP is commonly used in HTS cable conductors. In order to retain the insulation characteristics, the width and gap must be determined before manufacturing and the parameters are discussed in Sections 12.1 to 12.5. Taking the PPLP as an example, the suitable bending-radius ratio R/r of the cable can be satisfied by [18]:

$$\frac{R}{r} = \frac{W}{G} \quad (12.47)$$

where W denotes the PPLP cutting width, G is the butt gap size between PPLP tapes, and R and r are the bending radius and radius of the cable conductor, as shown in Figure 12.15. The ratio of R/r is typically about 25.

The schematic outline of the manufacturing process of the HTS cable is shown in Figure 12.16 [19], which is a modified manufacturing process of a conventional oil-filled (OF) cable. First, the HTS conductor layer and the HTS shield layer are wound respectively, as indicated in Figures 12.16(a) and (b), where the carbon paper is wound around the former. HTS tape spools are rotated in sync with the core movement to wind tapes at a constant pitch. The tapes are kept at an allowable tension by tension

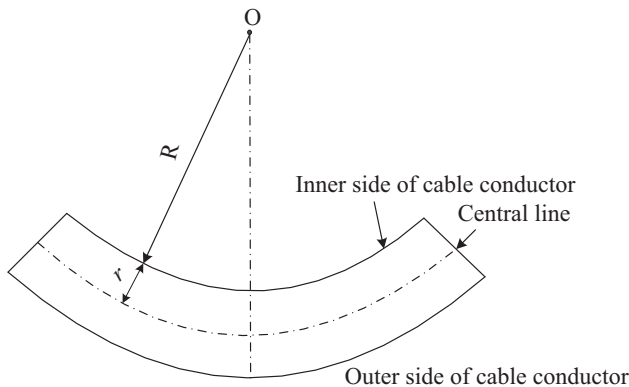


Figure 12.15 Bending view of HTS cable conductor.

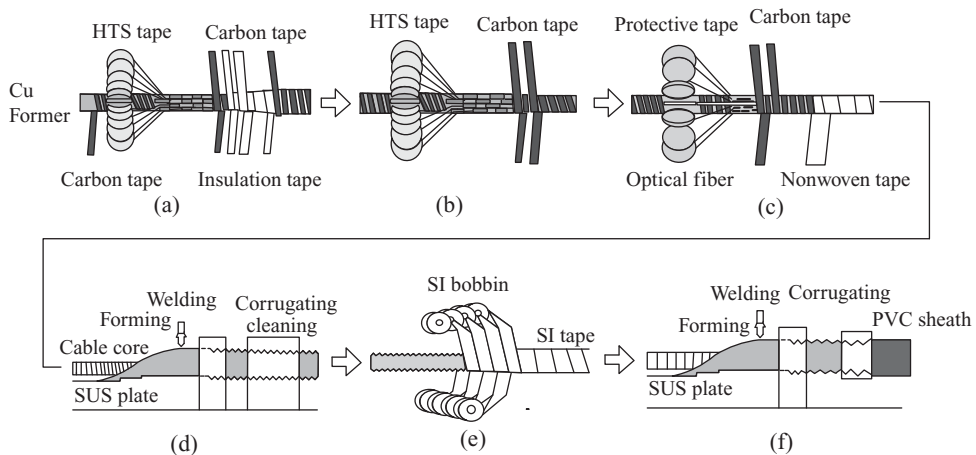


Figure 12.16 Manufacturing processes of HTS cable: (a) HTS tapes and PPL papers wound to form a conductor layer and an electric insulation layer, respectively; (b) HTS tape wound around the conductor to produce HTS shielding layer; (c) optical fibres wound around the shield layer with protective tape; (d) manufacturing process of the inner corrugated pipe; inner pipe is continually cleaned to remove surface oil; (e) SI tapes wound around the inner pipe under optimum conditions; (f) manufacturing process of the outer corrugated pipe; PVC sheath is extruded after the pipe manufacturing. (Reproduced with permission from S. Mukoyama, S. Maruyama, M. Yagi, et al. "Development of 500 m HTS power cable in super-ACE project," *Cryogenics*, 45, 11–15, 2005 © 2005 Elsevier.)

controls and at a predetermined bend-diameter to avoid excessive strain on the manufacturing apparatus of the conductor layer and the shield layer.

Second, many PPL paper tapes are wound onto the conductor layer to form the insulation layer by using the paper wrapping line of the OF cable. The insulation layer was simultaneously formed by a series line after the HTS conductor layer is made (Figure 12.16(a)). Carbon tapes are wound above and under the conductor layer and the shield layer to alleviate electrical field concentration. Optical fibres that are covered by a stainless steel pipe are wound together with braided tapes onto the cable core to measure longitudinal temperature distribution (Figure 12.16(c)). Winding of the fibres also requires a tension control because the fibre is weak as in the HTS tape. The braided tapes layer protects the fibre and the inside of the core against lateral pressure. An inner pipe and an outer pipe of the heat insulation pipe are made of stainless steel plate by processes of plate curving, seam welding, pipe corrugating and surface cleaning (Figure 2.16(d)). The heat-insulation pipe needs to be air tight to retain an ultra-high vacuum in the heat-insulating layer. Therefore, a pressure test, an air-tight test, a liquid penetrating test, an X-ray transmission test and an eddy-current test are carried out rigorously to ensure high reliability of the welded seam.

In addition, it needs to be confirmed that the pipes have no leaks with a detection sensitivity of 10^{-9} Pam³/s by a helium leak test. The new washing equipment that cleans the surface of the pipe is introduced in the manufacturing line of the heat-insulation pipe, because dirt such as oil on a surface of the inner pipe will make the vacuum poor. Super-insulation (SI) tapes are wound around the inner pipe to form the heat insulation layer. Moreover, the winding machine that is able to wind the SI tapes is prepared for cable manufacture (Figure 12.16(e)). In the final step (Figure 12.16(f)), the outer pipe of the heat-insulation pipe is formed by a similar method to the inner pipe and the PVC sheath is formed by extrusion on the outer pipe.

In principle, the HTS cable with any length can be commercially manufactured by the typical process shown in Figure 12.16.

12.8 Bending of HTS Cable

In the process of HTS cable manufacture, and also moving the cable from the manufacturer and installing it to a real grid, the HTS cable needs to be wound onto the drum many times. After bending on a drum of a certain diameter, the HTS tapes of the cable undergo extra strain. The relationship amongst maximum strain, drum diameter and winding pitch and angle obeys the following [6]:

$$\varepsilon_{i\max} = \frac{\pi t}{L_{pi}} \cos \theta_i + \frac{L_{pi}}{2\pi R_d} \sin \theta_i \quad (12.48)$$

where $\varepsilon_{i\max}$ and t are the maximum permitted strain and thickness of the HTS tape, L_{pi} and θ_i denote the winding pitch or angle of the i -th HTS layer, and R_d refers to the radius of the drum on which the HTS cable is wound. From Equation (12.47), the strain increases with the rise of winding pitch (angle) and drum radius R_d and is opposite to that described by Equation (12.17). Reduction of the winding pitches increases the initial strain during the cabling process, but reduces the strain when a cable is bent or wound onto a drum. Furthermore, the drum diameter of the HTS cable also needs to meet the requirements of Equations (12.46) and (12.47). Therefore, a reasonable compromise of the drum diameter, the minimum drum diameter or the extent of bending allowed for the cable, should be found by taking the minimum one satisfying Equations (12.46) and (12.47) to select the proper winding pitch.

12.9 Termination and Joint

Terminations are located at both ends of the HTS cable, to connect the HTS cable and the normal temperature power line. Because of the large differences in temperature between the HTS cable and the outer environment, terminations have to sustain the temperature difference and pump out heat from joint resistance and thermal load. Also, joints between two HTS cables is inevitable due to their limited length and transport problem. This section focuses on the introduction of termination and joint.

12.9.1 Termination

Each HTS cable has two terminations to connect the cold end of the cable with the bushing interface at the room temperature components. The termination includes a current lead connecting the cable conductor with a normal conducting bushing and the cryostat. The current lead acts as the transition from LN₂ temperature to room temperature, by taking into account the dielectric requirements of the cable and the terminations and meanwhile minimizing the thermal conduction from the room temperature end to the cold end. The terminations have to handle the flow of LN₂ passing through the cable.

Figure 12.17 illustrates the basic structures of terminations of a single-phase HTS cable. There are some significant differences in terminations between WD and CD HTS cables. First, there is a high voltage (HV) insulator in the termination of the WD HTS cable, because the cryostat of the WD HTS cable has the same potential as the cable conductor. Thus it must be insulated from termination and refrigeration systems, which are at ground potential (Figure 12.26(a)) [20]. However, the HV insulator is not essential in the termination of the CD HTS cable, since its cryostat is at ground potential and can directly connect with the refrigeration system. These terminations have been successfully built and installed for several prototypes of AC HTS cable demonstrated in the real grid.

The bushing is the one used in a conventional XLPE cable. There are several types of cable bushings with various voltage levels at which the HTS cables are applicable. Another important element is the current lead, which spans from room temperature to LN₂ temperature. Although designs of several types of current leads were already introduced in detail in Chapter 10, a conduction cooled Cu current lead in termination of the HTS cable is applicable for practical applications. The optimal design of the current leads is found by Equation (10.44) in AC operation mode in order to minimize heat leakage (combination

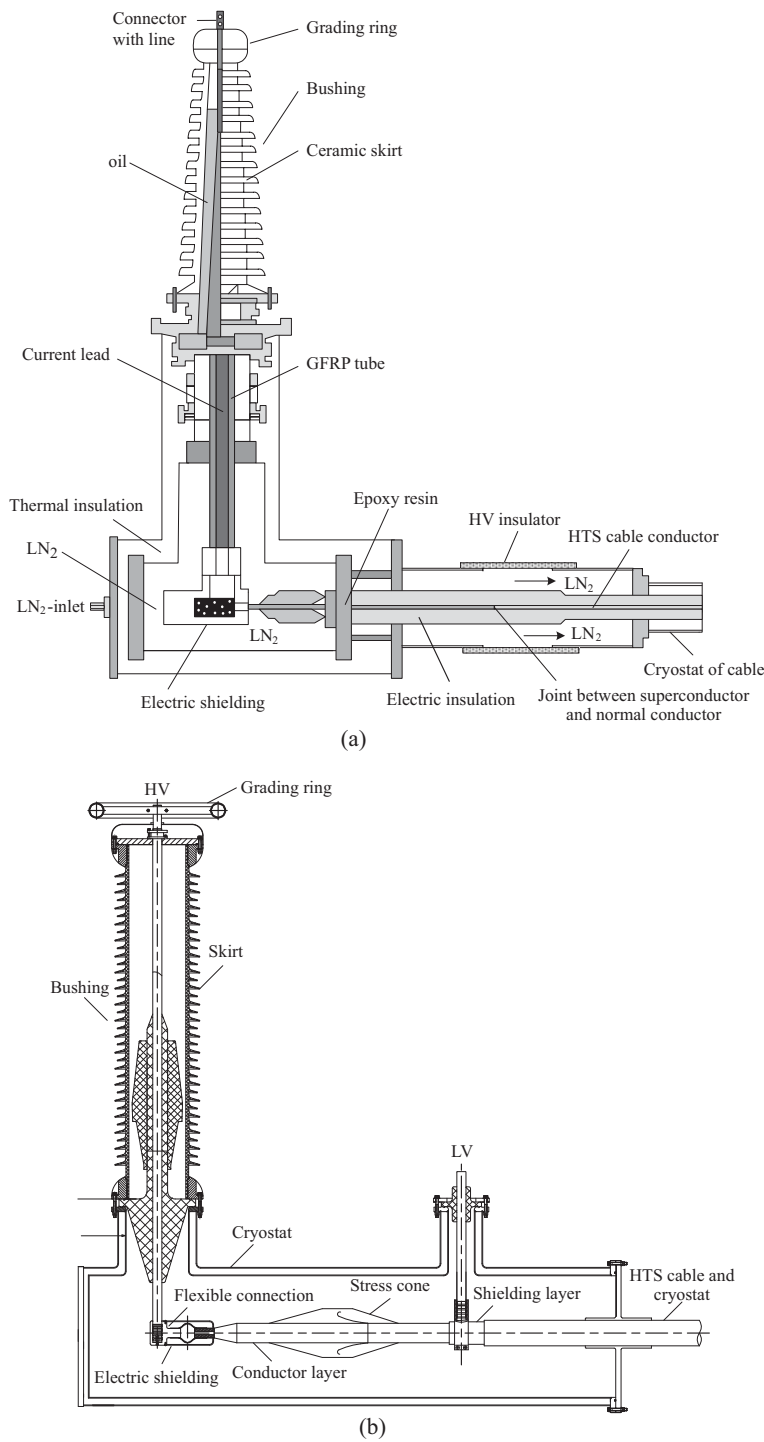


Figure 12.17 Basic structures of terminations of single phase HTS cable: (a) WD cable; (b) CD cable.

of the conduction heat and Joule heat) to the LN_2 environment. Currently, the optimal heat leakage of Cu current lead is about $43 \text{ W/kA}\cdot\text{m}$. Attention should be paid to the cross-sectional area of the Cu current leads by taking the skin effect into account, because one end of the current lead is at LN_2 temperature, where its resistivity is much lower than at room temperature.

The skin depth of normal conductor is given by:

$$\delta = \sqrt{\frac{2\rho}{\omega\mu_0}} \quad (12.49)$$

where ρ and μ_0 are the resistivity of the normal conductor and vacuum permeability, respectively, and ω ($2\pi f$) refers to the angular frequency of the AC current. Therefore, the transversal size of the current lead must be smaller than two times that of the skin depth. For example, the resistivity of a typical oxygen-free copper is respectively $0.21 \times 10^{-8} \Omega\cdot\text{m}$ and $1.70 \times 10^{-8} \Omega\cdot\text{m}$ at 295 K and 77 K, and the skin depths are 3.26 mm and 9.28 mm at power frequency $f = 50 \text{ Hz}$, so that the diameter of the Cu rod or thickness of the Cu bar should be respectively lower than 6.52 mm and 18.72 mm at LN_2 temperature if they are used as current leads. If a larger size of current lead is required in view of the current capacity of the HTS cable, the current lead should be made from the split conductor in order to meet the requirements of having enough cross-sectional area.

The design of the cryostat of the termination is the same as the cryostat of the HTS cable conductor, by taking Equations (9.2), (9.6) and (9.24) into account based on the practical size of the cryostat used in termination.

In addition, although the optimized current lead can also be used in DC HTS cables, the PCL (described in Chapter 9), is the most suitable for a DC HTS cable. If it is used in a HTS DC cable, the heat leakage to the LN_2 environment is about one-third of the Cu current lead [21]. Unfortunately, the current capacity of PCL is not large enough for application in a real grid.

12.9.2 Joint

The HTS cable joint consists of the former joint, the HTS conductor joint, the joint insulation, the HTS shield joint and the Cu stabilization joint. Before the hollow former is welded, a Cu tube with diameter a little smaller than the hollow is inserted into the holes of the formers at jointing locations, then the joint is embodied by welding (Figure 12.18).

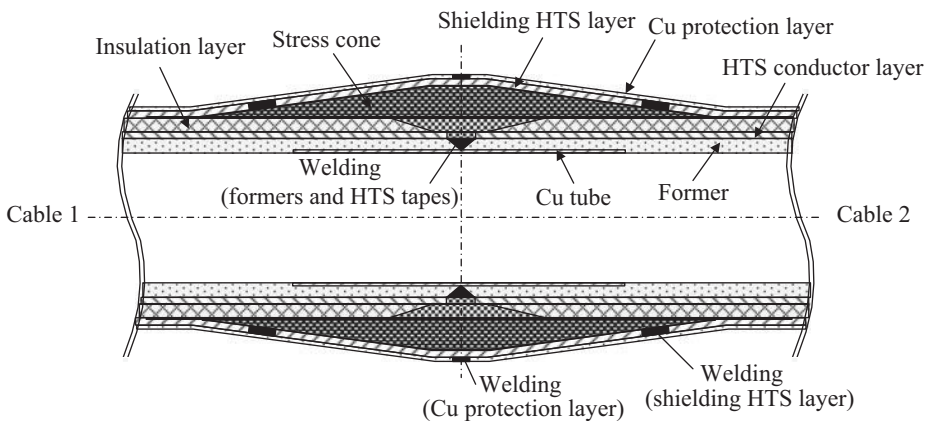


Figure 12.18 Conceptual joint of formers.

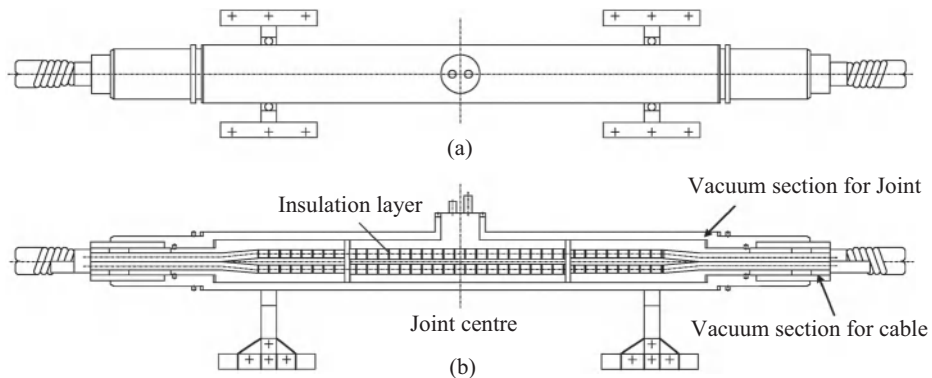


Figure 12.19 HTS joint: (a) top view; (b) main view. (Reproduced with permission from M. Yagi, S. Mukoyama, N. Amemiya, et al. "Development of a 10 m long 1 KA 66/77 KV YBCO HTS cable with low AC loss an a joint electrical resistance," *Superconductor Science and Technology*, 22, 085003 (9pp), 2009 © 2009 2003 IOPscience.)

The HTS conductor joint and HTS shield joint are constructed using the layer-to-layer splicing method with several centimetres of overlap. Each layer of the HTS conductor includes one layer-to-layer joint. However, the HTS shield includes two layer-to-layer joints in one layer because the HTS shield around the joint centre is removed during the construction of the former joint, the HTS conductor joint and the joint insulation. During construction of the joint, the insulation layers are wound by hand, which is similar to jointing conventional XPLE cable on site.

Similar to the compact jointing used in a conventional vault, the joint of the cryostat of the HTS cable is presented in Figure 12.19 [22]. Figures 12.19(a) and (b) are the top and main view of the jointing cryostat section, respectively. In this configuration, the vacuum of the joint and cable parts is maintained to reduce heat leakage, and the vacuum section of the joint and the cable is separated to maintain a high vacuum state in the cables during construction of the joint. The electric insulation in the jointing part is naturally thicker than the original insulation layer, in order to retain enough electrical strength, which is similar to the jointing conventional XLPE cable completed on site.

12.10 Circulating Cooling System and Monitoring System

The cooling facility is one of the important components of the HTS cable system to provide sufficient low temperature at various operating conditions for the HTS cable to maintain its superconductivity. Electrical and thermal status of the HTS cable system needs to be checked in real time during operation, thus the monitoring system is essential for its reliable operation.

12.10.1 Cooling System

12.10.1.1 Flowing Arrangement

Due to thermal heat and AC losses in an HTS DC/AC cable in normal operation, the cable must be cooled by fluid LN_2 circulation, and heat and losses are absorbed by the coolant. There are two types of cooling arrangement (Figure 12.20). To save space, it is desirable for the coolant to return to its source within the cable itself rather than having a separate return pipe (Figure 12.20(a)). This arrangement may give rise to thermal short-circuiting due to finite thermal conductivity between coolant streams flowing in opposite directions. In general, inner and outer coolants in each cable are at different temperatures and there is heat flowing through the dielectric. If thermal conductivity of the dielectric is sufficiently

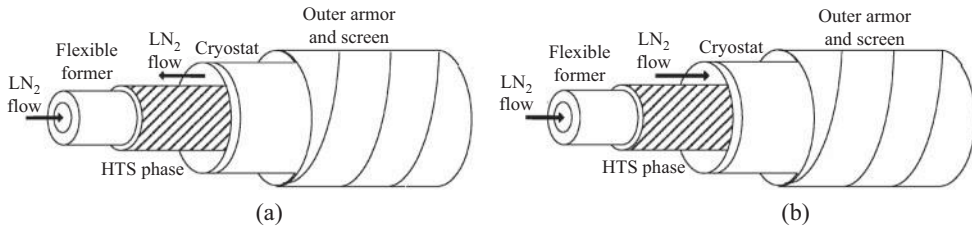


Figure 12.20 Cooling arrangement of HTS cable conductor: (a) Counter-flow; and (b) single direction flow.

large, the temperature of the turn-point of the inner coolant will exceed that at the outlet of the outer coolant and, in the worst case, may exceed the critical temperature of the superconductors. The obstacle is that the thermal conductance of the dielectric materials at LN₂ temperature is about 10 times larger than that at LHe temperature. Therefore, the cooling arrangement with counter-flow cooling illustrated in Figure 12.20(a) is questionable if it is used in a long HTS cable. It is suitable only for the relatively short HTS cable, such that the length is less than 1000 m [23]. For the detailed descriptions, please refer to Appendix A.6.

Therefore, the cooling arrangement with the single flowing method is a good candidate for a long HTS cable. Currently, the cooling arrangements of several HTS cable prototypes, successfully demonstrated in a real grid, are the ones with a single direction of flow cooling (Figure 12.20(b)), where the coolant LN₂ can flow longitudinally only along the cable axis in one direction.

12.10.1.2 Cooling System

The cooling system is composed of the HTS cable system (HTS conductor, cryostat), terminations (current leads, bushing, cryostat), refrigerators, heat exchanger, pumps (operating at room temperature and LN₂ temperature), LN₂ tank, transfer line, valves, etc. The flow chart is shown in Figure 12.21, where the LN₂ flows through the LN₂ line, HTS cable, pump and heat exchanger, in which there is a back-up cooling system by open loop cooling with reducing pressure to avoid the fault condition of the refrigerator.

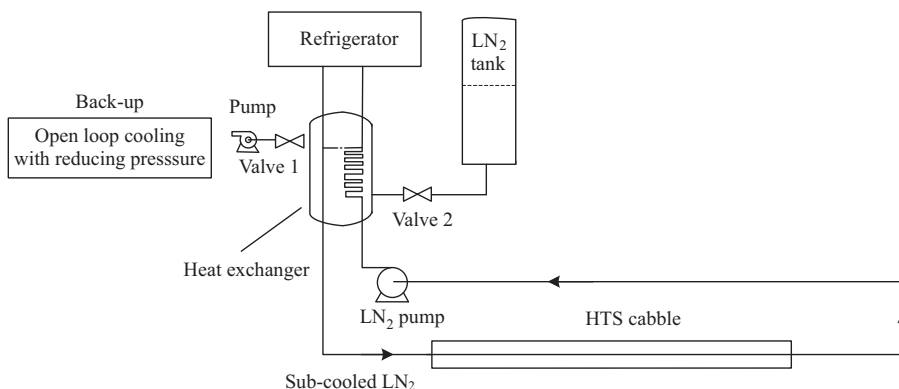


Figure 12.21 Flow chart of cooling system for HTS cable system.

12.10.1.3 Thermal Load

There are different thermal loads in CD and WD HTS cables according to their structures. In the former cable, the major components of thermal loads include AC losses of cable conductor, dielectric loss in dielectric insulation, thermal conduction through cryostat walls from room temperature to LN₂ temperature, depending on quantity of the MLI and the type of mechanical support between the cold and warm walls, LN₂ pumping loss along cable and transfer line, and other heat loads including pump heat and thermal conduction of valves, etc. In the latter cable, there is an extra eddy current loss resulting from the inner wall of the cryostat without HTS shielding layer due to the WD structure with dielectric insulation outside the cryostat, but the dielectric loss and AC loss in the shielding layer are absent in a cooling environment compared with the CD partner. Besides, in a CD HTS cable, the thermal load only comes from the cryostats of cable conductor, terminator, pump and valves, because there are no AC losses if the ripple current is not taken into account.

In the previous paragraph, the thermal loads, such as AC losses from the HTS cable conductor, heat leakages from the cryostats of the cable conductor and terminations, central joints and current leads, were introduced in detail. The other two thermal loads, LN₂ pumping loss and pump thermal load, are not of concern. In this section, these two thermal loads are introduced in order to determine the cooling power of refrigerators required in the cooling system of the HTS cable.

12.10.1.4 LN₂ Pumping Loss

Because there is a velocity gradient across the cross-sectional area of the LN₂ transfer line and the cryostat of the HTS cable, the LN₂ pumping loss results from the viscous force during flowing. In the turbulent regime due to corrugated inner wall of cryostat, the Reynolds number is described by:

$$\text{Re} = \frac{\rho v D_e}{\eta} \quad (12.50)$$

where η denotes viscosity of fluid, D_e stands for the hydraulic diameter of flow channel, which is equal to a ratio of four times the cross-sectional area to perimeter, and ρ and v are the density and velocity of fluid LN₂, respectively.

The pressure drop and temperature difference in the cooling path is expressed as:

$$\Delta P = \frac{f \rho v^2}{2 D_e} L = f \frac{M^2 L}{2 \rho S^2 D_e} \quad (12.51)$$

$$\Delta T = \frac{Q}{C \rho v S} \quad (12.52)$$

where v , ρ , f and C are velocity, density, frictional factor and specific heat of LN₂, respectively, S refers to equivalent cross-sectional area, L stands for the cooling length, Q represents heat loss in the cable, and M is the mass flow. Generally, the frictional factor f is obtained by checking the table of dependence of f on Re. If Re is in the range of 10^4 to 10^5 , the frictional factor f is determined by:

$$f = 0.096 \text{ Re}^{-0.2} \quad (12.53)$$

The friction factor f for cryogenic liquid flow in corrugated bellows has been suggested to be four times that of a smooth pipe [24]. On other configurations of smooth conduit and corrugated conduit as well as shell conduit, the dependences of friction factor f 's on the Re can be empirically predicted by some classical formulas [25], but even they are not exactly predicted by the experimental results.

Due to the viscous flow, the LN₂ pumping loss Q_{pump} (W) is given by:

$$Q_{\text{LN}} = vS\Delta P \quad (12.54)$$

However, by combining Equation (12.51) with (12.52) and taking the permissible temperature rise ΔT_m into account, the cooling length L can be obtained by [19]:

$$L = \left(\frac{2\Delta P (\Delta T_m - T_d) C^2 \rho S^2 D_e}{f P^2} \right)^{1/3} \quad (12.55)$$

where ΔT_m and T_d are separately determined by:

$$\Delta T_m = \Delta T + T_d \quad (12.56)$$

$$T_d = T_1 - T_2 = \frac{P_{\text{ac}} \ln(r_2/r_1) + P_d \ln(2r_2/(r_1 + r_2))}{2\pi k} \quad (12.57)$$

where T_2 and T_1 are the temperatures of cable conductor and coolant LN₂ whose temperature is a little higher than T_2 due to the dielectric loss of the insulation, AC losses of HTS tapes and low thermal conductivity of electric insulation, k refers to the thermal conductivity of insulation materials, r_1 and r_2 stand for the outer radii of the cable conductor and insulation layers, respectively, and P_{ac} and P_d are respectively the AC loss of the cable conductor and the dielectric loss.

12.10.1.5 Thermal Load of LN₂ Pump

The LN₂ pump, providing driving power for circulating LN₂ in a cooling system, is a significant device to take away the heat load created in the HTS cable and to ensure a low temperature environment of HTS cable with superconductivity. The input power of the LN₂ pump, that is, the shaft-power, consists of two components. One component, known as the fluid power, provides driving power for fluid LN₂ to overcome the frictional force of LN₂ flowing in cryostats. Another component is transformed into heat, which transfers partly to the environment by the bearing and shaft of the motor and turbine and partly to the LN₂ coolant. The fluid power P_{pump} (W) is determined by total hydraulic head and flow rate:

$$P_{\text{pump}} = G\rho gh \quad (12.58)$$

where G refers to flow rate (m³/s), ρ (kg/m³) stands for density of fluid LN₂, g (m/s²) is acceleration of gravity, and h (m) represents the pump lift.

Other thermal loads come from LN₂ valves and the LN₂ tank. These devices can be purchased from the manufacturers. Because they are commercially manufactured in accordance with products standards, their thermal loads are readily available and so not discussed in this book.

After determination of the total AC losses from the HTS cable conductor and thermal loads from the cryostats, current leads, transfer line, LN₂ pumping loss and LN₂ pump, the gross power is calculated and the suitable types of refrigerators with high efficiency in operating range of temperatures can be correctly chosen for the cooling system (Figure 9.27).

12.10.2 Monitoring System

Various electric and thermal variables need to be measured and displayed in real time, in order to ensure the normal and reliable operation of a cooling system. The electric variables mainly include voltages and

currents, while the thermal variables are temperatures of the various components, such as HTS cable inlet and outlet, and refrigerator's inlet and outlet, etc. The other variables are flow rates, pressures at inlet and outlet of the HTS cable, LN_2 levels, concentration of GO_2 and GN_2 , etc. If the open loop cooling with a reduced pressure system is used, the concentration of GN_2 must be monitored, even though nitrogen is a colourless, odourless, stable and nonflammable gas that occurs naturally and constitutes approximately 80% of the Earth's atmosphere and, under normal conditions, poses no threat to health or safety. But there are two principal risks associated with nitrogen: (i) oxygen deficiency, and (ii) extreme low temperature.

The release of high concentrations of nitrogen into the environment in enclosed spaces can result in asphyxiation by displacement of oxygen. Raised concentrations of nitrogen may cause a variety of respiratory symptoms, and at high concentrations, unconsciousness or death may occur. Direct contact with LN_2 can cause frost-bite type injuries, and must be avoided. In leaks into a confined room, the oxygen content of the atmosphere has to be maintained over 19% for security reasons. An oxygen sensor should be implemented with alarm levels and linked to ventilation fans. However, the pressure must be measured in real time to monitor slow leakage as well as drops in vacuum pressure. Besides, possible danger of explosion can take places due to the large amount of LN_2 evaporation during short-circuit faults of power lines.

Each cryostat has a LN_2 level regulation system and a supplementary nitrogen level sensor that ascertains the security level measurement and controls directly the circuit breakers. It is essential to detect those electrical, thermal, concentration, pressure and flow rate by using a series of sensors along the length of a LN_2 -filled HTS cable system. In the event of a system failure, a signal can be sent to issue an alarm and/or shut off the flow of LN_2 into the cable if this is deemed appropriate.

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Appendix

A.1 Calculations of Volumetric Heat Capacity, Thermal Conductivity and Resistivity of Composite Conductor

Assuming that a composite conductor is composed of n kinds of components, and the volumetric heat capacity of the i -th component is $(\gamma_i C_i)$, here γ_i and C_i are density and heat capacity of the i -th component, respectively; and k_i and ρ_i are separately the thermal conductivity and resistivity of the i -th component. The volumetric ratio of the i -th component to composite conductor is f_i , $i = 1, 2, \dots, n$. If the heat Q and current I flow transversely, as shown in Figure A.1, which shows the serial model for calculating effective volumetric heat capacity, thermal conductivity and resistivity of the entire composite conductor, the minimum effective volumetric heat capacity and thermal conductivity as well as resistivity of the composite conductor are respectively:

$$\frac{1}{(\gamma C)_{eff}} = \sum_{i=1}^n \frac{f_i}{\gamma_i C_i} \quad (A.1)$$

$$\frac{1}{k_{eff}} = \sum_{i=1}^n \frac{f_i}{k_i} \quad (A.2)$$

$$\rho_{eff} = \sum_{i=1}^n \frac{\rho_i}{f_i} \quad (A.3)$$

If heat Q and current I flow longitudinally through the composite conductor, as indicated in Figure A.2, which shows the parallel model for calculation of its volumetric heat capacity, thermal conductivity and resistivity, the maximum effective volumetric heat capacity and thermal conductivity as well as resistivity of the composite conductor are respectively:

$$(\gamma C)_{eff} = \sum_{i=1}^n f_i (\gamma_i C_i) \quad (A.4)$$

$$k_{eff} = \sum_{i=1}^n f_i k_i \quad (A.5)$$

$$\frac{1}{\rho_{eff}} = \sum_{i=1}^n \frac{f_i}{\rho_i} \quad (A.6)$$

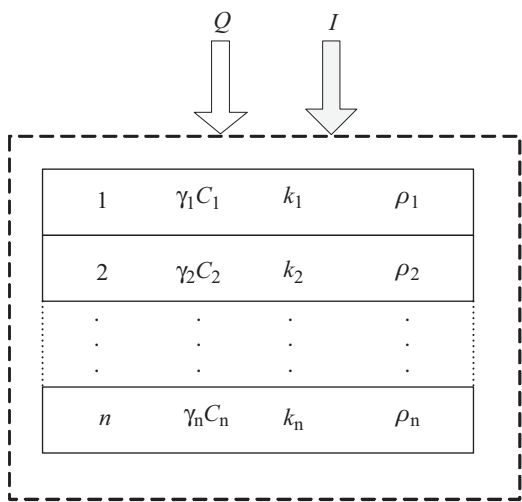


Figure A.1 Parallel model of heat Q and current I flowing transversely through composite conductor.

A.2 Eddy Current Loss of Practical HTS Coated Conductor (YBCO CC)

Figure A.3 shows the geometrical structure of a typical high temperature superconducting (HTS) YBCO CC, the so-called 2G HTS tapes. The origin of the coordinate is located at the centre of the superconducting layer, and the orientation of the coordinate axis is marked in the figure. YBCO CC consists of three layers: substrate layer, YBCO superconducting layer and metal stabilizer. Their widths are all $2a$, and their thicknesses are d_f , d_s , and d_c , respectively [1, 2]. The substrate material is usually made from Ni or Ni-alloy, which is ferromagnetic.

The alternating current (AC) losses of YBCO CC consist of three components: hysteresis loss P_h in the superconductor layer, eddy current loss P_e in the metal stable layer and iron loss (also hysteresis loss) P_f of the substrate due to its ferromagnetism. The hysteresis losses of superconductors in various types

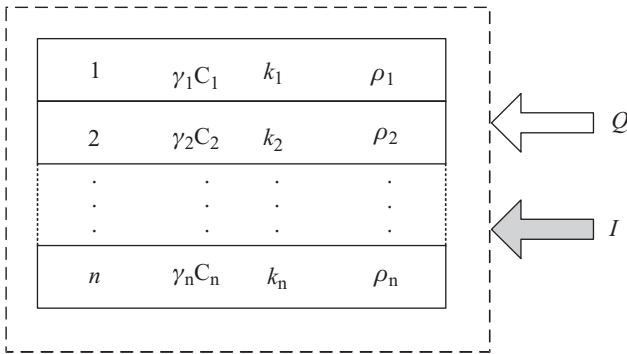


Figure A.2 Heat and current flowing through composite conductor in longitudinal direction.

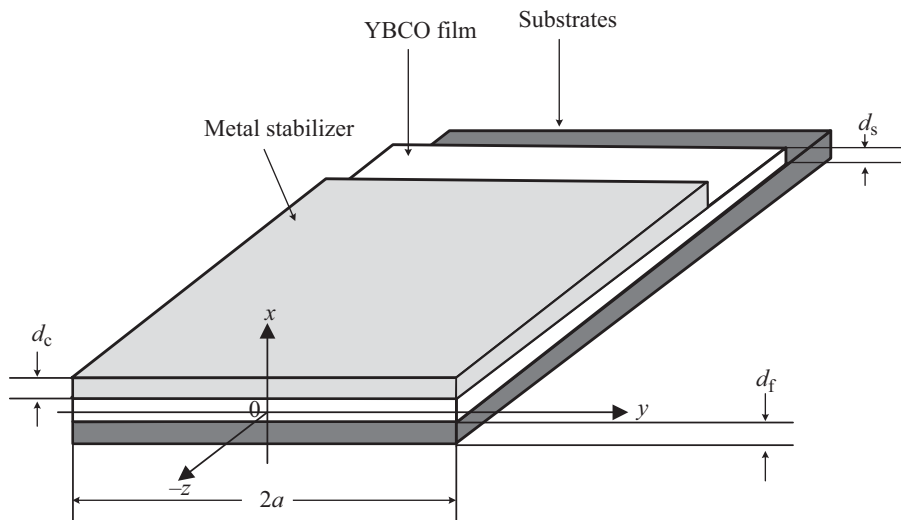


Figure A.3 Geometrical structure of a typical YBCO CC.

of AC magnetic fields and with transporting AC current were already discussed in detail in Chapter 5. Therefore, this section mainly focuses on the eddy current loss and the iron loss of the YBCO CC.

A.2.1 Eddy Current Loss with Transporting Alternating Current

Assuming that YBCO CC transports a sinusoidal current:

$$I(t) = I_m \sin(\omega t) \quad (\text{A.7})$$

where I_m is amplitude, $\omega = 2\pi f$ is angular frequency, f denotes frequency and eddy current loss (W/m) of YBCO CC in the metal stable layer is:

$$P_e = \frac{4\mu_0^2 d_c a \omega^2}{\pi^3} I_c^2 h\left(\frac{I_m}{I_c}\right) \quad (\text{A.8})$$

where I_c is critical current of YBCO CC and ρ stands for resistivity of the stable metal layer. Let $x = I_m/I_c$, then:

$$h(x) = \int_0^x du \sqrt{xu - u^2} \left\{ 1 - \sqrt{1 - u^2} - \frac{u}{2} \ln\left(\frac{1-u}{1+u}\right) + \frac{1}{8} \left[\ln\left(\frac{1+u}{1-u}\right) \right] \right\} \quad (\text{A.9})$$

A.2.2 Eddy Current Loss of YBCO CC Exposed to Perpendicular AC Magnetic Field

Supposing that YBCO CC is exposed to an AC magnetic field $B(t)$, which is perpendicular to its wide surface; the magnetic field is along the x -direction (Figure A.3) and the AC magnetic field is still

sinusoidal:

$$B(t) = B_m \sin(\omega t) \quad (\text{A.10})$$

where B_m is amplitude of AC magnetic field, $\omega = 2\pi f$ is angular frequency, so eddy current loss (W/m) of YBCO CC in the metal stable layer becomes:

$$P_{el} = \frac{8\mu_0}{3\pi} \frac{d_c a^3 \omega^2}{\rho} H_f^2 h \left(\frac{H_m}{H_f} \right) \quad (\text{A.11})$$

where $B_m = \mu_0 H_m$, $H_f = J_c d_s / \pi$ is the characteristic magnetic field of YBCO CC, and $J_c = I_c / (2ad_s)$ is its critical current density. Let $x = H_m / H_f$, then:

$$h(x) = \int_0^x du \sqrt{xu - u^2} \left(1 - \frac{3}{\cosh^2 u} + \frac{2}{\cosh^3 u} \right) \quad (\text{A.12})$$

A.2.3 Eddy Current Loss Exposed to Parallel AC Magnetic Field

Assuming that YBCO CC is exposed to an AC magnetic field parallel to its wide surface, namely along the y-direction (Figure A.3). Eddy current loss of YBCO CC has a similar form to Equation (A.11) if some exchanging between $2a$ and d_c , and substitutions between H_f and H_p in Equation (A.11) are made by:

$$2a \leftrightarrow d_c \quad \text{and} \quad H_f \leftrightarrow H_p = J_c \frac{d_s}{2} \quad (\text{A.13})$$

where $H_p = J_c d_s / 2$ is the full penetrated field of YBCO CC. So that the eddy current loss per unit length is:

$$P_{e//} = \frac{2\mu_0^2}{3\pi^3} \frac{ad_c^3 \omega^2}{\rho} H_p^2 h \left(\frac{H_m}{H_p} \right) \quad (\text{A.14})$$

A.2.4 Iron Losses of Substrate

Because the substrate layer of YBCO CC is often ferromagnetic, there is also iron loss induced in the substrate under conditions of the AC magnetic field, besides the hysteresis loss in the YBCO layer and eddy current loss in the stabilizer. However, there is no analytical expression for the iron loss in YBCO CC, but the iron loss can be described by empirical expressions. The iron loss per unit length is:

$$P_f = Cf \left[\coth(kB)^m - \frac{1}{(kB)^m} \right] \quad (\text{A.15})$$

where C , k and m are constants related to geometric parameters and electrical properties of the substrate layers of YBCO CC. For example, as for the substrate of commercial YBCO CC, if we choose $Ni_5\%W$ as its substrate, $a = 1$ cm, $d_f = 75$ μm , then $m = 1.2$. If unit of B is mT, iron loss of YBCO CC is:

$$P_f = 2f \left[\coth(0.28B)^{1.2} - \frac{1}{(0.28B)^{1.2}} \right] \quad (\text{A.16})$$

A.3 Calculation of Geometrical Factor G

The schematic electric circuit to measure the AC losses of superconducting wires exposed to an AC magnetic field at cryogenic temperature by a pick-up coil method was shown in Figure 5.41. The measuring system consists of a power supply, AC magnet, cryostat, pick-up coils (main coil, compensated coil), compensated circuit and data acquisition system. Theoretically, the AC loss in an AC magnetic field can be obtained according to Equation (5.190) by measuring the magnetization. However, there is a geometrical error in measured magnetization M due to imperfect detection using the pick-up coils. Consequently, a geometrical correction factor G should be multiplied to Equation (5.190), then the theoretical AC loss expression (W/m) Equation (5.190) should be replaced by:

$$P_h = -\mu_0 C A f G \oint M_0 dH_e \quad (\text{A.17})$$

where μ_0 is vacuum permeability and M_0 is actual magnetization induced in the specimen. CA is the cross-sectional area of the specimen and f denotes frequency of the AC external magnetic field. Factor G denotes the geometrical error and is dependent only upon geometrical configurations and sizes of main coil, compensation coil and sample (coil), as well as their relative positions. Consequently, it is possible to measure AC losses fairly accurately if the factor G approaches unity.

The configurations of sample and pick-up coils in the pick-up-coil method are illustrated in Figure A.4 [3]. Figure A.4a shows the configuration in the coaxial arrangement, Figures A.4(b and c) are those arranged in the symmetrical arrangements. It is assumed that the specimen (coil) with radius R and height $2h_s$ can be approximated to a very thin cylinder magnetized uniformly. An apparent AC loss P_h per unit length (W/m) in an AC external magnetic field H_e measured by this method may be obtained by estimating the interlinkage flux into the pick-up coils due to magnetic dipole of the specimen (coil). The theoretical expression of geometrical factor G is given by:

$$G = \frac{1}{8\pi R h} \begin{cases} \int_0^{2\pi} \{g(R+a, \varphi) - [(R+a)/(R-a)]^2 g(R-a, \varphi)\} d\varphi & (\text{A.18a}) \\ \int_0^{2\pi} \{g(R+a, \varphi) - g'(R+a, \varphi)\} d\varphi & (\text{A.18b}) \\ \int_0^{2\pi} g(R+a, \varphi) d\varphi & (\text{A.18c}) \end{cases} \quad (\text{A.18})$$

where the parameters and functions are respectively described by:

$$h = \begin{cases} h_{pc} & \text{if } h_s \geq h_{pc} \\ h_s & \text{if } h_s < h_{pc} \end{cases} \quad (\text{A.19})$$

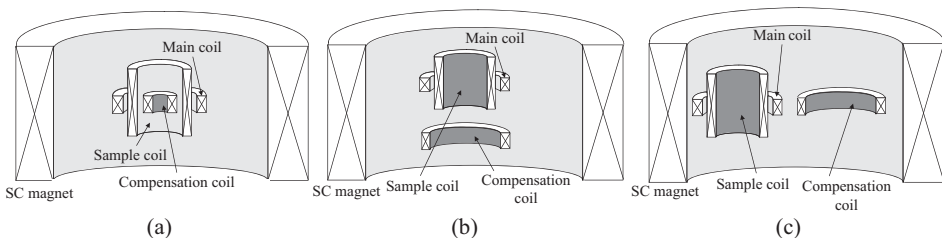


Figure A.4 Geometrical configurations of sample coil and pick-up coils: (a) coaxial; (b) plane-symmetrical; and (c) axis-symmetrical arrangements. (Reproduced with permission from K. Kajikawa, M. Iwakuma and K. Funaki. "Influence of geometrical configuration on AC loss measurement with pick-coil method," IEEE Transaction on Applied Superconductivity, 9, 2, 746–749, 1999. © 1999 IEEE.)

$$g(r, \varphi) = \left[f(r, \varphi, z, z_0) \right]_{z=-h_{pc}}^{z=h_{pc}} \Big|_{z_0=-h_s}^{z_0=h_s} \quad (\text{A.20})$$

$$g'(r, \varphi) = \left[f(r, \varphi, z, z_0) \right]_{z=-d-h_{pc}}^{z=-d+h_{pc}} \Big|_{z_0=-h_s}^{z_0=h_s} \quad (\text{A.21})$$

$$f = \left\{ \left[\frac{(2R^2 + r^2 - 3Rr \cos \varphi)}{(R^2 + r^2 - 2Rr \cos \varphi)} \right] \ell \right\} r \cos \varphi \quad (\text{A.22})$$

$$\ell = \sqrt{R^2 + r^2 + (z - z_0)^2 - 2Rr \cos \varphi} \quad (\text{A.23})$$

where a is the difference distance between the mean radii of the specimen (coil) and each concentric pick-up coil (i.e., main coil and compensation coil), $2h_p$, $2h_c$ and $2h_s$ are respectively the heights of the main coil, compensation coil and sample (coil), d stands for the distance between the centres of the main coil and compensation coil; in Figure A.4a, the d is zero. It is clearly shown that the geometrical factor G is only the function of geometrical parameters R , a , h_c , h_p , h_s or d .

Based on the IEC 61788-8 ed2 [4], the standard arrangement of the coiled specimen and pick-up coils (main coil and compensation coil) should be determined by satisfying the condition:

$$|G - 1.00| < 1\% \quad (\text{A.24})$$

If V_p and V_c represent the terminal voltages of the main coil and compensational coil, k is the ratio of output signal to the input signal in the resistive voltage divider and is adjustable (Figure 5.41). It can be ensured that $(V_p - kV_c)$ equals zero when there is no sample (coil) by regulating the adjustable resistance R_2 (Figure 5.41), so that the background loss is cancelled. Simultaneously, the external magnetic field H_e is measured by numerically integrating V_c , since the magnetic field created by the magnetization of the sample (coil) is much smaller than the external magnetic field H_e .

By using the pick-up coil method, the AC loss per unit length (W/m) in the external AC magnetic field is obtained by measuring the voltages from the pick-up coil and compensation coil:

$$P_h = -\frac{A_s}{V_s n_p} \frac{G}{T} \int_0^T (V_p - kV_c) H_e dt \quad (\text{A.25})$$

where A_s and V_s are separately the sample volume and cross-sectional area surrounded by the main coil, G is geometrical factor, n_p denotes the turn number per unit length of main pick-up coil, and T represents the period of AC external field H_e .

A.4 Derivation of Self and Mutual Inductances of CD Cable

A.4.1 Self Inductance of Layer

As shown in Figure A.5, it is assumed that an infinitely long CD cable consists of a single conductor layer and a single shielding layer, and the thickness of both layers is infinitely thin. Their radii are r_i and D , respectively. The magnetic fields created in the inner and outer regions of the conductor by the cable are described by [5]:

$$B_{in} = \frac{\mu_0 I_{con}}{L_p} \quad (r < r_i) \quad (\text{A.26})$$

$$B_{out} = \begin{cases} \frac{\mu_0 I_{con}}{2\pi r} & (r_i < r < D) \\ 0 & (r > D) \end{cases} \quad (\text{A.27})$$

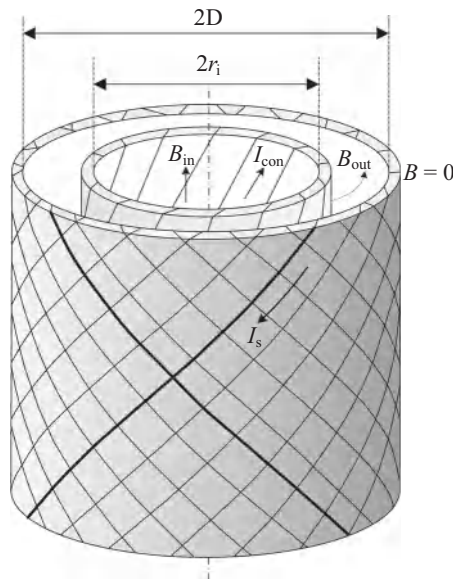


Figure A.5 Magnetic field of CD cable.

where μ_0 is vacuum permeability, I_{cond} is current flowing through the inner conductor layer, D_e and D are outer diameters of the conductor layer and shielding layer, respectively, r refers to the radial coordinate and distance from the axis, and L_p is the winding pitch of the layer. Magnetic field B_{in} and B_{out} are along axial and tangential (circumferential) directions.

The enclosed magnetic field energy density w_m in a vacuum with unit of J/m^3 is given by:

$$w_m = \frac{B^2}{2\mu_0} \quad (\text{A.28})$$

Consequently, the magnetic field energy per unit length (J/m) is found by:

$$W_m = \frac{1}{2\mu_0} \left[\int_0^{r_i} \left(\frac{\mu_0 I_{\text{con}}^2}{L_p} \right)^2 2\pi r dr + \int_{r_i}^D \left(\frac{\mu_0 I_{\text{con}}^2}{2\pi r} \right)^2 2\pi r dr \right] \quad (\text{A.29})$$

where the first and the second terms on the right-hand side of Equation (A.29) are the magnetic field energy stored in the inner and outer regions of the conductor layer in unit length, respectively. However, the magnetic field energy (J/m) stored in an inductance L per unit length (H/m) with current I_{con} is described by:

$$W_m = \frac{1}{2} L I_{\text{con}}^2 \quad (\text{A.30})$$

Combining Equation (A.30) with the calculated results after integrating Equation (A.29), we obtain the self inductance of the i -th layer as:

$$L_i = \mu_0 \frac{\pi r_i^2}{L_{pi}^2} + \mu_0 \frac{\ln(D/r_i)}{2\pi} \quad (\text{A.31})$$

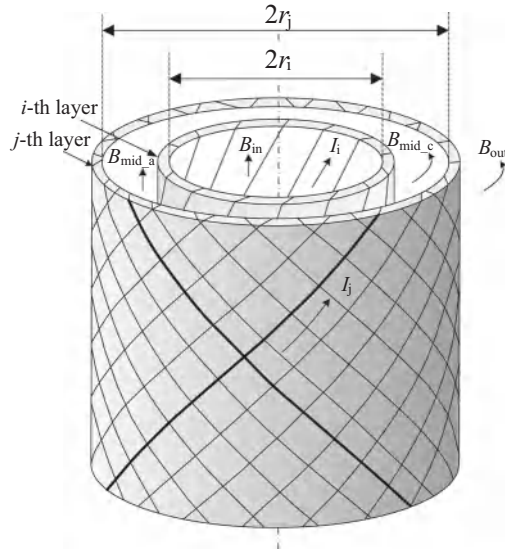


Figure A.6 Generic view of cable conductor consisting of i -th and j -th layers.

A.4.2 Mutual Inductances amongst Layers

A typical schematic view of an infinitely long cable, consisting of the i -th and j -th layers with infinite thin thickness, is depicted in Figure A.6, where $i, j = 1, 2, \dots, N$, N refers to the total numbers of layers, including conductor layers and shielding layers.

According to Ampere's law, the magnetic fields of the inner, intermediate and outer regions are separately expressed as:

$$B_{in} = \alpha_i \frac{\mu_0 I_{con}}{L_{pi}} + \alpha_j \frac{\mu_0 I_j}{L_{pj}} \quad (r < r_i) \quad (\text{A.32})$$

where orientation of B_{in} is along the axis, L_{pi} and L_{pj} are winding pitches of the i -th and j -th layers, respectively; and α_i and α_j are constants (+1 or -1), depending on the relative winding directions. If the two layers are wound with the same orientation around the former, the sign of the constants are the same, that is, $\alpha_i \alpha_j = +1$. In the case of opposing twist, the constants are each other's negation, that is, $\alpha_i \alpha_j = -1$:

$$B_{mid} = \begin{cases} B_{mid\_c} = \frac{\mu_0 I_i}{2\pi r} \\ B_{mid\_a} = \frac{\mu_0 I_j}{L_{pj}} \end{cases} \quad (r_i < r < r_j) \quad (\text{A.33})$$

where B_{mid} has two components, B_{mid_c} and B_{mid_a} , which are circumferential and axial magnetic field components:

$$B_{out} = \frac{\mu_0 (I_i + I_j)}{2\pi r} \quad (r > r_j) \quad (\text{A.34})$$

Similar to B_{in} , the outer magnetic field B_{out} is also circumferential.

The stored energy per unit length (J/m) by each component are separately given by:

$$W_{mi} = \frac{1}{2\mu_0} \int_0^{r_i} \left(\alpha_i \frac{\mu_0 I_i}{L_{pi}} + \alpha_j \frac{\mu_0 I_j}{L_{pj}} \right)^2 2\pi r dr \quad (A.35)$$

$$W_{mid} = \frac{1}{2\mu_0} \int_{r_i}^{r_j} \left[\left(\frac{\mu_0 I_i}{2\pi r} \right)^2 + \left(\frac{\mu_0 I_j}{L_{pj}} \right)^2 \right] 2\pi r dr \quad (A.36)$$

$$W_{mout} = \frac{1}{2\mu_0} \int_{r_j}^D \left(\frac{\mu_0 (I_i + I_j)}{2\pi r} \right)^2 2\pi r dr \quad (A.37)$$

where W_{mi} , denotes energy stored in the volume inside the two layers, W_{mid} refers to energy stored in the volume between the two layers and W_{mout} is the energy stored in the volume from the outer layer to the return path (not shown in Figure A.6). Due to the assumption of infinite thin thickness of each layer, the energy stored within each layer is ignored.

The total enclosed magnetic field energy is the sum of the three components:

$$W_m = W_{mi} + W_{mid} + W_{mout} \quad (A.38)$$

However, the total energy per unit length (J/m) stored in self and mutual inductances can be expressed as:

$$W_m = \frac{1}{2} L I_i^2 + M_{ij} + \frac{1}{2} L I_j^2 \quad (A.39)$$

where L_i , L_j and M_{ij} are self-inductances of the i -th layer, j -th layer and mutual inductance between the two layers, respectively; and I_i and I_j denote separately the currents flowing through the two layers.

Combining Equations (A.38) with (A.39) and considering the self inductance of each layer, the mutual inductance per unit length of cable conductor between the i -th and j -th layers can be given by:

$$M_{ij} = M_{ji} = \alpha_i \alpha_j \frac{\mu_0 \pi r_i^2}{L_{pi} L_{pj}} + \frac{\mu_0}{2\pi} \ln \left(\frac{D}{r_j} \right) \quad (r_j > r_i) \quad (A.40)$$

if $r_j < r_i$, then r_i is replaced by r_j in the first term and the r_j is replaced by r_i in the second term.

A.5 Other Models for Hysteresis Loss Calculations of HTS Cable

There are three types of simple models for hysteresis loss calculations of AC/DC HTS cable according to the Bean model. Those models include mono-block (MB) model, Norris (NS) model and Majoros (MAJ) model (Figure A.7).

Based on the MB model, the hysteresis loss of the HTS cable conductor in units of W/m is given by [6]:

$$Q_{MB} = f \frac{\mu_0 I_c^2}{2\pi h^2} [(2 - i_{ac} h) i_{ac} h + 2 (1 - i_{ac} h) \ln (1 - i_{ac} h)] \quad (A.41)$$

where f refers to the power frequency; I_c is the critical current of a superconducting cylinder whose thickness is the same as that of HTS layers wound by HTS taps; i_{ac} is the normalized transport current;

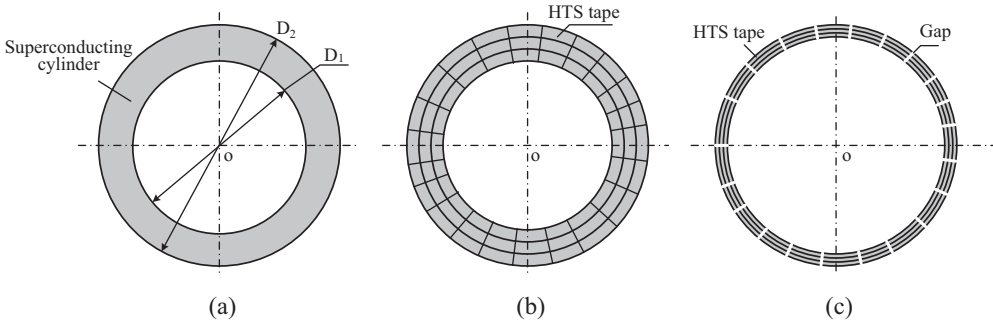


Figure A.7 Cross-sectional geometry of three model cables: (a) MB model; (b) NS model; (c) MAJ model.

$i_{ac} = I_m/I_c$, I_m denotes the amplitude of the AC transport current; and h is a parameter defined by:

$$h = \frac{(D_2^2 - D_1^2)}{D_2^2} \quad (\text{A.42})$$

where D_2 and D_1 stand for the outer and inner diameters of the superconducting cylinder, respectively.

According to the NB model, the hysteresis loss of the HTS cable conductor in units of W/m is found by:

$$Q_{NB} = fN \frac{\mu_0 I_{cc}^2}{\pi} \left[(1 - i_{ac}) \ln(1 - i_{ac}) + (1 + i_{ac}) \ln(1 + i_{ac}) - i_{ac}^2 \right] \quad (\text{A.43})$$

where $I_{cc} = I_c/N$, N represents the total number of thin HTS tapes in all layers of the cable conductor; I_{cc} is the average critical current of each tape; $i_{ac} = I_t/I_c$; and I_t and I_c refer to the amplitude of the transport current and critical current in this layer, respectively.

Based on the MAJ model, in which the monolayer cable is composed of circularly arranged superconducting thin sheets, the hysteresis loss is described by:

$$Q_{MAJ} = fN \frac{\mu_0 I_{cc}^2 g^2}{\pi w^2} \left[2 \ln \left| \cos \left(\frac{1}{2} \pi i_{ac} \right) \right| + \frac{1}{2} \pi i_{ac} \tan \left(\frac{1}{2} \pi i_{ac} \right) \right] \quad (\text{A.44})$$

where g is the gap between the superconducting tapes, w denotes the width of the tapes, and I_{cc} stands for the critical current of the layer on the cable. Other quantities are the same as those in Equation (A.43).

A.6 Cooling Arrangements

Three layouts of flow charts are considered in this section for estimating the feasibility of a cooling system for long HTS cable operation according to the analytical models proposed by [7].

A.6.1 Counter-Flow Cooling

The counter-flow cooling arrangement is conceptually exhibited as Figure A.8. Compared with the parallel flowing arrangement shown in Figure 12.20(a), it has an advantage over the parallel flowing mode shown in Figure 12.20(b), since it saves space and has less thermal load by allowing the coolant to

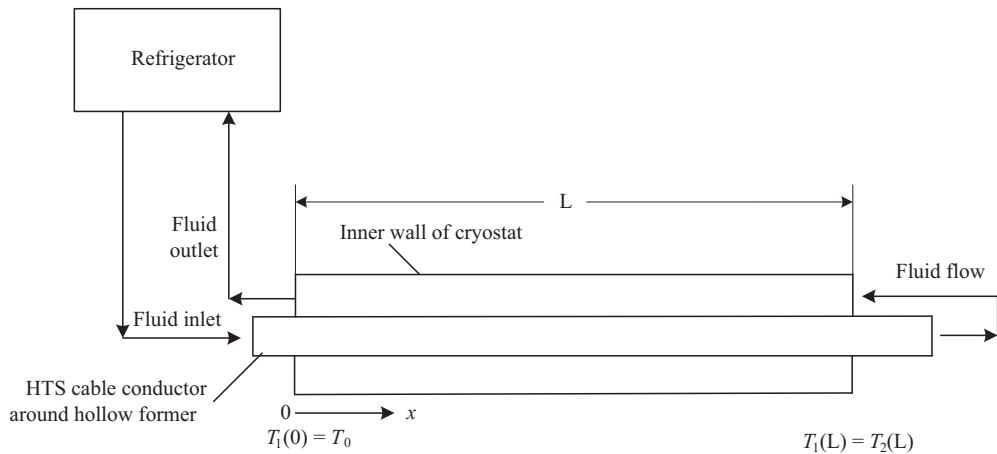


Figure A.8 Flow chart of analytical model with counter-flow.

return to its source within the cable itself rather than by a separate return pipe. However, this arrangement could possibly give rise to thermal short-circuiting if the thermal conductivity between liquid nitrogen (LN_2) in-flow and counter-flow is not small enough.

Figure A.8 shows the flow chart of a one-dimensional (1D) analytical model with counter-flow mode. It is assumed that the mass flow of LN_2 is the same in parallel-flow and counter-flow directions, and its specific heat, thermal conductivity, viscosity and frictional factor, as well as heat transfer coefficient are independent of temperature.

Heat balance of this analytical model is illustrated in Figure A.9, where heat Q per unit length of cable conductor (W/m) is created by viscous friction and electrical dissipation, and equally enters parallel-flow and counter-flow with $Q/2$, respectively. Heat Q_0 per unit length of cable conductor (W/m) denotes heat leakage from the exterior and only finds its way into the counter-flow. $q(x)$ stands for heat leakage per unit length flows from counter-flow to parallel-flow. It is also supposed that thermal conductivity k (W/(m·K)) between parallel-flow and counter-flow is linear, and k , Q and Q_0 are independent of location x .

In conditions of steady-state and turbulent flow, and by ignoring conduction and pressure drops (i.e., the LN_2 pumping loss) along the cable conductor, the heat balance equations can be expressed as:

$$MC_p \frac{dT_1}{dx} = q(x) + \frac{Q}{2} \quad (\text{A.45})$$

$$-MC_p \frac{dT_2}{dx} = -q(x) + \frac{Q}{2} + Q_0 \quad (\text{A.46})$$

where mass flow rate M is defined by:

$$M = \frac{(Q + Q_0)L}{C_p \Delta T} \quad (\text{A.47})$$

where ΔT is designed temperature margin, and $q(x)$ is described by:

$$q(x) = 2\pi K [T_2(x) - T_1(x)] \quad (\text{A.48})$$

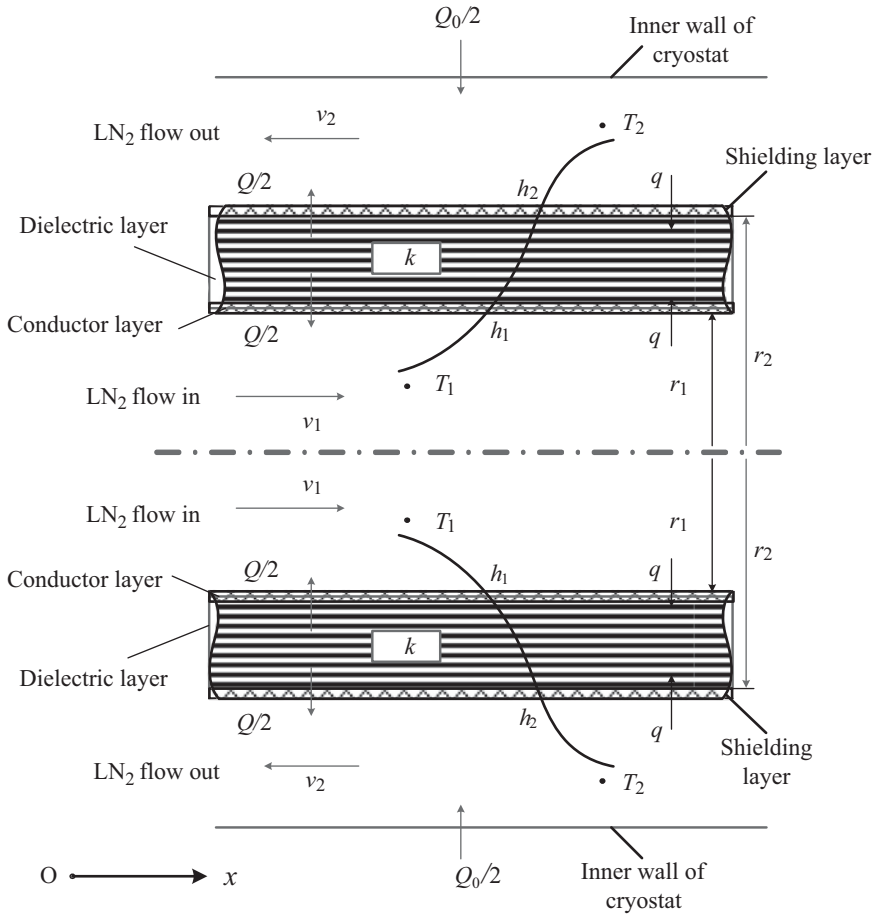


Figure A.9 Heat balance model.

and K is defined by:

$$\frac{1}{K} = \frac{1}{h_1 r_1} + \frac{\ln(r_2/r_1)}{k} + \frac{1}{h_2 r_2} \quad (\text{A.49})$$

where k is the thermal conductivity of dielectric insulation, r_1 and r_2 are separately radii of LN₂ parallel-flow and counter-flow channels, h_1 and h_2 are their corresponding heat transfer coefficients; here h_1 and h_2 , refers to those of the LN₂ fluid in the inner central (flow-in) and outer annular (counter-flow) regions.

The boundary condition is then depicted by:

$$\begin{cases} T_1(0) = T_0 \\ T_1(L) = T_2(L) \end{cases} \quad (\text{A.50})$$

where L refers to the length of cable conductor and T_0 is the temperature at the inlet. In the turbulent region, the Dittus–Boelter expression is the general heat transfer correlation and given by:

$$Nu = \frac{hD_e}{k_{LN}} = 0.023Re^{0.8}Pr^{0.4} \quad (A.51)$$

where Nu is the Nusselt number, D_e stands for the hydraulic diameter of the flow channel, k_{LN} is the thermal conductivity of LN_2 ; Re is the Reynolds number and Pr is the Prandtl number. The D_e , Re and Pr are found by:

$$D_e = \frac{4S}{l_p} \quad (A.52)$$

$$Re = \frac{\rho_{LN}vD_e}{\eta_{LN}} \quad (A.53)$$

$$Pr = \frac{\eta_{LN}C_p}{k_{LN}} \quad (A.54)$$

where S and l_p are cross-sectional area and perimeter of flowing channel; η_{LN} , ρ_{LN} , C_p and v are viscosity, density, specific heat and velocity of LN_2 , respectively.

By considering the boundary condition of Equation (A.50), the solutions of Equations (A.45) and (A.46) are:

$$T_1(x) = -\frac{\pi K(Q + Q_0)}{C_p^2 M^2} x^2 + \frac{1}{C_p M} \left[\frac{2\pi K(Q + Q_0)}{C_p M} L + \frac{Q}{2} \right] x + T_0 \quad (A.55)$$

$$T_2(x) = -\frac{\pi K(Q + Q_0)}{C_p^2 M^2} x^2 + \frac{1}{C_p M} \left[\frac{2\pi K(Q + Q_0)}{C_p M} L - \frac{Q + 2Q_0}{2} \right] x + T_0 + \frac{(Q + Q_0)}{C_p M} L \quad (A.56)$$

Taking the design of the conceptual cable as an example, the design with parameters listed in Tables A.1 and A.2, the longitudinal temperature distributions are shown in Figure A.10 under conditions of different thermal conductivity $k = 0.05, 0.1, 0.25$ and 0.5 in units of $W/(m \cdot K)$, as well as mass flow $M = 0.25$ kg/s and $L = 2500$ m by a combination of Equations (A.48) to (A.56). It is shown that the maximum temperature in the cable conductor exceeds its critical temperature with $k > 0.1$, that is, it is impossible to cool down an HTS cable with counter-flow if the thermal conductivity of the dielectric

Table A.1 Main parameters for thermal design of an example cable

Items	Data
Inner diameter of former r_1 /mm	45
Outer diameter of former r_2 /mm	65
Inner diameter of cryostat	80
Heat leakage from exterior Q_0 /(W/m)	0.85
Heat from AC loss Q /(W/m)	1.39
Inlet temperature T_0 /K	70
Outlet temperature T /K	77
Outlet pressure P (MPa)	0.6
Mass flow M /(kg/s)	0.25
Cooling distance L /km	2.5

Table A.2 Thermo-physical characteristics of LN₂ at 77 K and 0.1 MP

Items	Data
Specific heat C_p /(kJ/(kg·K))	2.038
Thermal conductivity k_{LN} /(W/(m·K))	0.146
Viscosity η_{LN} /(μPa·s)	159.3
Prandlt number Pr	2
Density ρ /(kg/m ³)	806

insulation is higher than 0.1. Although the maximum temperature may decrease if there is an increase in the mass flow M , M is also limited by the pressure drop condition in the cable (Equation (12.51)).

A.6.2 Counter-Flow Cooling with Sub-Cooled Station

In order to effectively cool down the long HTS cable by the counter-flow mode and endure normal operation, another counter-flow cooling with a sub-cooled station is installed at the other end of the cable (Figure A.11). The heat balance Equations (A.45) and (A.46) are still effective, but boundary conditions are changed to:

$$\begin{cases} T_1(0) = T_0 \\ T_2(L) = T_0 \end{cases}$$

(A.57)

As similar steps, the solutions are given by:

$$T_1(x) = -\frac{\pi K(Q + Q_0)}{C_p^2 M^2}x^2 + \frac{2\pi K}{C_p M}\frac{(Q + 2Q_0)L}{2} + \frac{\pi K}{C_p M}(Q + Q_0)L^2 \frac{x}{2\pi KL + C_p M} + \frac{Q}{2C_p M}x + T_0$$

(A.58)

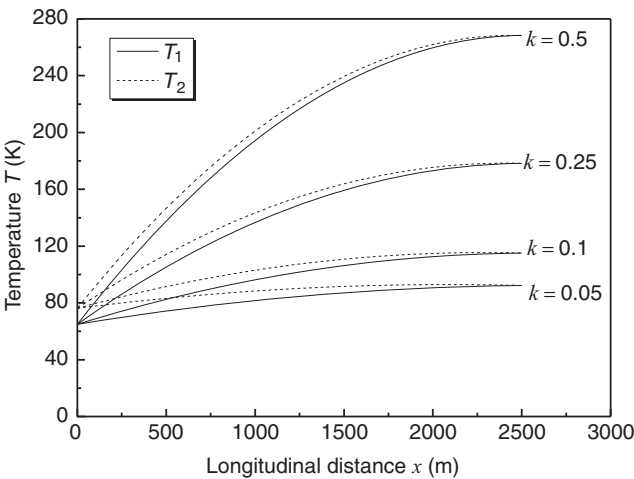


Figure A.10 Longitudinal temperature distribution with counter-flow ($k = 0.05, 0.1$ and 0.5 W/(m·K), $M = 0.25$ kg/s, $L = 2500$ m).

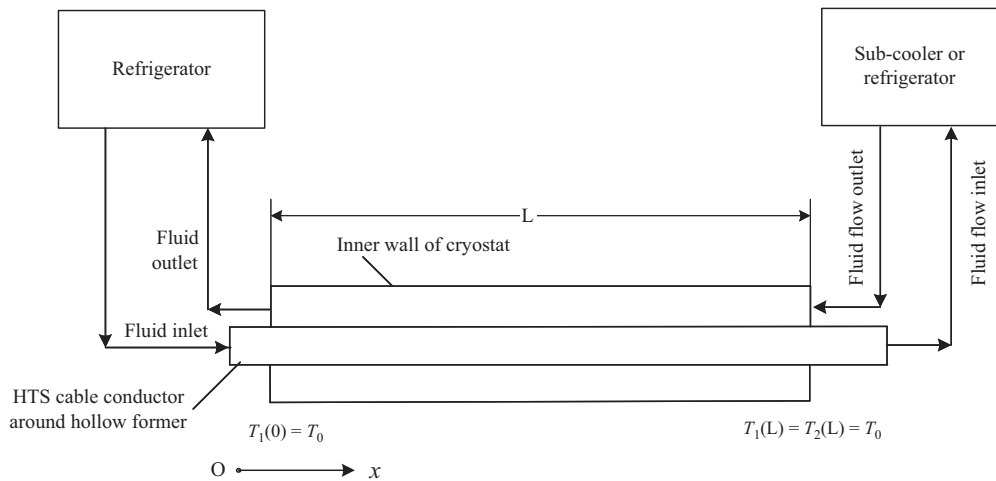


Figure A.11 Flow chart of cooling with sub-cooled station.

$$T_2(x) = -\frac{\pi K(Q + Q_0)}{C_p^2 M^2} x^2 + \left(\frac{2\pi K}{C_p M} \frac{(Q + 2Q_0)L}{2} + \frac{\pi K}{C_p M} (Q + Q_0)L^2 - \frac{Q + 2Q_0}{2C_p M} \right) x + \frac{(Q + 2Q_0)L}{2} + \frac{\pi K}{C_p M} (Q + Q_0)L^2 + T_0 \quad (\text{A.59})$$

Figure A.12 shows the longitudinal temperature distribution along the cable length with the same thermal conductivity values k 's of dielectric insulation and mass flow rate M . The figure illustrates that the temperature rise is reduced so that the situation is effectively improved. Moreover, the thermal conductivity of dielectric insulation k should also satisfy the condition of smaller than $0.1 \text{ W/(m}\cdot\text{K)}$. However, some space must be set aside for the sub-cooler station in the middle locations between the cryogenic stations; clearly, this arrangement is feasible for a longer HTS cable but not an economical candidate for the shorter length of HTS cable. Consequently, it is concluded that the counter-flow cooling mode is only feasible for a shorter HTS cable.

A.6.3 Parallel-Flow Cooling

The parallel cooling arrangement is shown in Figure A.13 (also referred to in Figure 12.20(b)). The heat balance Equations (A.45) and (A.46) become:

$$M_1 C_p \frac{dT_1}{dx} = q(x) + \frac{Q}{2} \quad (\text{A.60})$$

$$M_2 C_p \frac{dT_2}{dx} = -q(x) + \frac{Q}{2} + Q_0 \quad (\text{A.61})$$

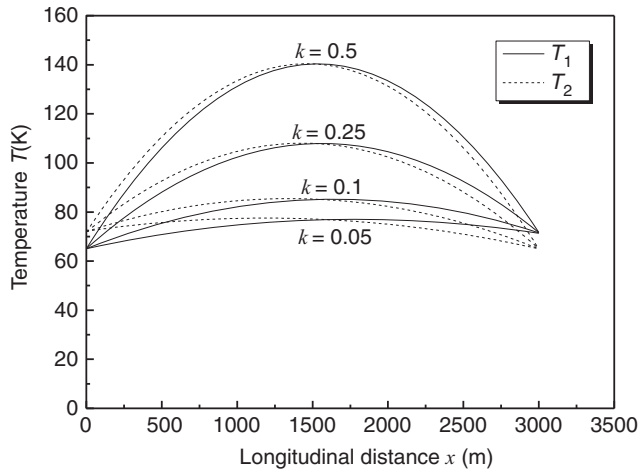


Figure A.12 Longitudinal temperature distribution with counter-flow sub-cooled station at other end of cable ($k = 0.05, 0.1$ and 0.5 W/(m·K) , $M = 0.25 \text{ kg/s}$, $L = 2500 \text{ m}$).

where M_1 and M_2 are the mass flow rates of LN_2 flowing through the inner region and the outer annular region, respectively, and satisfy $M = M_1 + M_2$, M is the mass flow rate of refrigerator defined by Equation (A.47). The boundary conditions obey:

$$T_1(0) = T_2(0) = T_0 \quad (\text{A.62})$$

By combining Equations (A.60) with (A.61) and taking the boundary conditions of (Equation A.62) into account, the solutions are:

$$T_1(x) = \frac{Q + Q_0}{C_p M_1} x + T_0 - \frac{(Q_0 + Q/2)\beta}{2\pi K(1 + \beta)} - \beta \left(\frac{(Q + Q_0)\beta}{(1 + \beta)^2 2\pi K} - \frac{Q_0 + Q/2}{(1 + \beta) 2\pi K} \right) e^{-\frac{2\pi K(1 + \beta)}{C_p M_1 \beta} x} - \frac{(Q + Q_0)\beta}{C_p M_1(1 + \beta)} x + \frac{(Q + Q_0)\beta^2}{(1 + \beta)^2 2\pi K} \quad (\text{A.63})$$

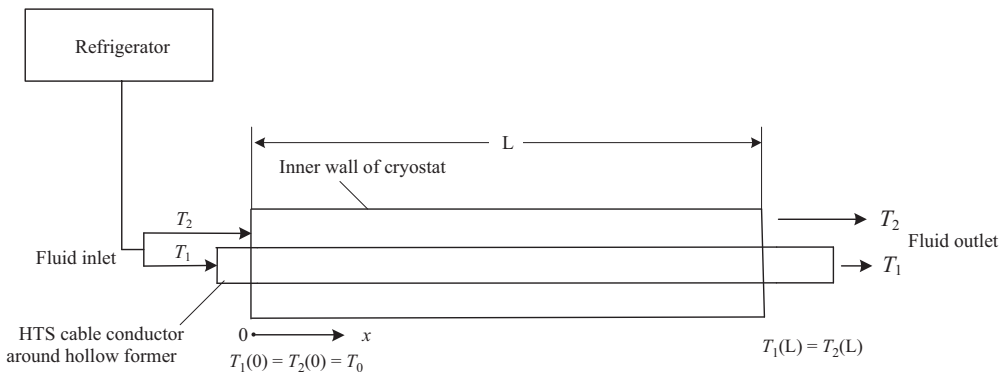


Figure A.13 Flow chart of parallel-flow cooling.

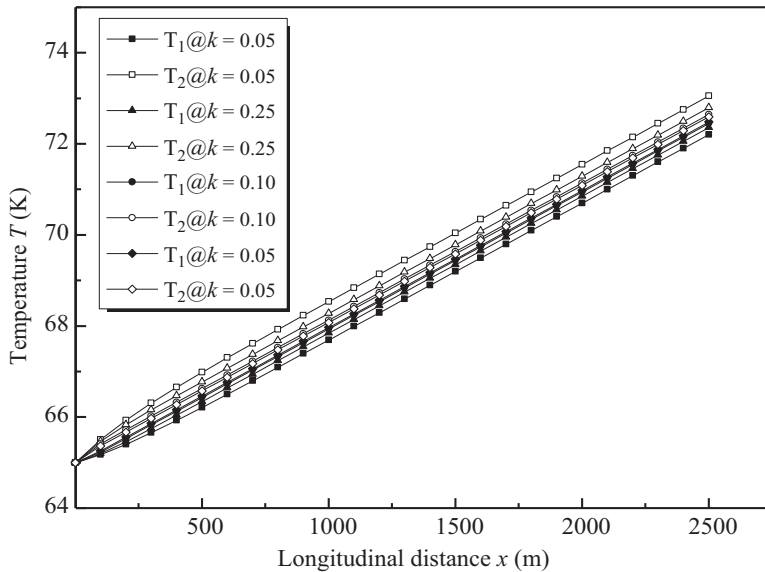


Figure A.14 Longitudinal temperature distribution with parallel-flow ($k = 0.05, 0.10$ and 0.5 W/(m·K), $M = 0.25$ kg/s, $L = 2500$ m).

$$T_2(x) = T_0 + \frac{(Q_0 + Q/2)}{2\pi K(1 + \beta)} + \left(\frac{(Q + Q_0)\beta}{(1 + \beta)^2 2\pi K} - \frac{Q_0 + Q/2}{(1 + \beta) 2\pi K} \right) e^{-\frac{2\pi K(1 + \beta)}{c_p M_1 \beta} x} + \frac{(Q + Q_0)}{c_p M_1 (1 + \beta)} x - \frac{(Q + Q_0)\beta}{(1 + \beta)^2 2\pi K} \quad (\text{A.64})$$

where

$$\beta = \frac{M_2}{M_1} = \frac{S_2}{S_1} \sqrt{\frac{f_1 D_{e2}}{f_2 D_{e1}}} \quad (\text{A.65})$$

where D_{e1} and D_{e2} are hydraulic diameters of the inner cross-sectional area S_1 and the outer annular cross-sectional area S_2 , respectively; f_1 and f_2 stand for the frictional factor of LN₂ flowing through cross-sections S_1 and S_2 .

During derivation of Equation (A.65), Equation (12.51) and equal pressure drops ($\Delta P_1 = \Delta P_2$) at ends of the cryostat are used; therefore, dependences of mass flow rates M_1 and M_2 on M are described by:

$$M = M_1 + M_2, \quad M_1 = \frac{M}{(1 + \beta)}, \quad M_2 = \frac{\beta}{(1 + \beta)} M \quad (\text{A.66})$$

Figure A.14 presents the longitudinal current distributions along the cable length with the same thermal conductivity values k 's of dielectric insulation and mass flow rate M . The figure illustrates that the temperature rise is very small compared to Figures A.10 and A.12, and the temperature difference is less than 1 K. In addition, the temperature rise has little dependence on different thermal conductivity

$k = 0.05, 0.1 \text{ W/(K}\cdot\text{m)}, 0.25 \text{ W/(K}\cdot\text{m)}$ and $0.5 \text{ W/(K}\cdot\text{m)}$. Also, the smaller the thermal conductivity k of the dielectric insulation, the greater the difference of T_1 and T_2 ; therefore, parallel cooling is the optimal amongst three cooling arrangements for a longer HTS cable, since it has no special requirement on thermal conductivity k of dielectric insulation materials.

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